Topical Review

Relaxation of nonequilibrium quasiparticles in mesoscopic size superconductors

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Abstract

Rapid development of micro- and nanofabrication methods have provoked interest and enabled experimental studies of electronic properties of a vast class of (sub)micrometer-size solid state systems. Mesoscopic-size hybrid structures, containing superconducting elements, have become interesting objects for basic research studies and various applications, ranging from medical and astrophysical sensors to quantum computing. One of the most important aspects of physics, governing the behavior of such systems, is the finite concentration of nonequilibrium quasiparticles, present in a superconductor even well below the temperature of superconducting transition. Those nonequilibrium excitations might limit the performance of a variety of superconducting devices, like superconducting qubits, single-electron turnstiles and microrefrigerators. On the contrary, in some applications, like detectors of electromagnetic radiation, the nonequilibrium state is essential for their operation. It is therefore of vital importance to study the mechanisms of nonequilibrium quasiparticle relaxation in superconductors of mesoscopic dimensions, where the whole structure can be considered as an ‘interface’. At early stages of research the problem was mostly studied in relatively massive systems and at high temperatures close to the critical temperature of a superconductor. We review the recent progress in studies of nonequilibrium quasiparticle relaxation in superconductors including the low temperature limit. We also discuss the open physical questions and perspectives of development in the field.

Keywords: mesoscopic superconductivity, nonequilibrium superconductivity, relaxation of nonequilibrium quasiparticles, energy imbalance, charge imbalance

(Some figures may appear in colour only in the online journal)

1. Introduction

Mesoscopic-size superconducting hybrid structures have become increasingly important devices in applications ranging from medical and astrophysical sensors to quantum computing due to their minimal energy dissipation at low temperatures. The conversion of electric current at an interface between different materials is a common process in any hybrid structure. Of particular interest are boundaries with a superconductor where electric current converts from single
electrons to Cooper pairs [1]. At nanoscales the whole system might behave as an ‘interface’ if the dimension(s) are comparable to the characteristic relaxation length.

In a superconductor at a finite temperature there are always nonpaired electrons called equilibrium quasiparticles. In the presence of additional disturbance their concentration can be increased by nonequilibrium quasiparticles. There can be deviations from equilibrium (nonequilibrium modes) of charge, energy and/or spin degrees of freedom. Rapid development of micro- and nano-fabrication methods facilitated the fabrication of devices and circuits with dimensions of the order of relaxation lengths of these nonequilibrium modes. The nonequilibrium quasiparticles might limit the performance of a variety of nanoscale superconducting devices with dimensions comparable to corresponding relaxation scales, such as refrigerators based on normal metal (N)—insulator (I)—superconductor (S) junctions [2–7], NIS refrigerators with a ferromagnetic (F) interlayer [8–10], superconducting resonators [11–14], superconducting qubits [15–24], single-electron hybrid turnstiles [25, 26], SFS $\pi$-junctions [27–30], nanorings [31–33] and many other devices. The effect has been notoriously called quasiparticle poisoning. On the contrary, in some applications, like various types of photon detectors and bolometers [34–38], the nonequilibrium state is essential for their operation.

The problem of quasiparticle removal in superconducting devices can be partly solved by introducing the so called quasiparticle traps away from the junction region, either by using normal-metal layers covering the superconducting electrode [39–42] or the local energy gap suppression by an external magnetic field [43, 44]. Quasiparticles are then trapped by the region with no energy gap (or suppressed gap). Other possibility is an alternative device design immune to quasiparticle overheating [45].

The phenomena of relaxation of nonequilibrium quasiparticles attracted attention in the mid-1970s resulting in an impressive number of papers (for references see section 3). Those early experiments were mainly performed on sandwich-type flat structures not adequate for spatially resolved studies. Agreement between the experiment and theory was established reliably, mainly in the high temperature limit $T \to T_c$. The understanding of the opposite limit $T \ll T_c$ is still far from being satisfactory. In this topical review we focus on our works, where for the first time we have measured relaxation lengths for the charge and energy nonequilibrium modes on the same hybrid microstructure at ultra-low temperatures. We compare these results with new recent results of nonlocal supercurrent measurements this review in section 7 and discuss the nonequilibrium quasiparticle injection in section 8. We present our principle results concerning the energy (section 9) and charge (section 10) imbalance relaxation lengths. In section 11 we present new experimental results on nonlocal supercurrent measurements for charge imbalance length determination. In section 12 we review recent results on nonequilibrium electron cooling in NIS refrigerators and discuss the problems of quasiparticle poisoning and evacuation. Finally, in section 13 we give a summary and an outlook on possible future developments in the field.

2. Nonequilibrium quasiparticles in superconductors

In figure 1 we schematically represent an energy spectrum of a superconductor: excited states, separated from the ground state by the energy gap $\Delta$ and equilibrium charge carriers (Cooper pairs) at a Fermi level $E_F$. A certain number of unpaired electrons (equilibrium quasiparticles) are always present in a superconductor at any finite temperature $T < T_c$ due to a thermal activation (figure 1(a)). These equilibrium excitations does not contribute to the electronic transport and, consequently the resistance of the superconductor (dc biased) equals to zero. The occupancy of excited states can be expanded at the same arbitrary temperature $T < T_c$ by creating nonequilibrium quasiparticles. If such quasiparticles are created due to processes connected with the Cooper pair breaking (for example, irradiation by photon energies larger than the energy gap, $\hbar \omega > \Delta$), they symmetrically occupy excited states with respect to a Fermi momentum $p_F$ (figure 1(b)). If transport measurements are conducted fast enough than the finite voltage drop (i.e. the resistance) in the superconductor can be registered at a time scale smaller than corresponding relaxation times. However, there appears to be an even more nontrivial deviation from the equilibrium, the so called charge imbalance, which is an asymmetrical occupation of the excitation spectrum (figure 1(c)). It can be created, for example, by electron injection from a normal metal into a superconductor. Depending on a polarity of an applied voltage, electron-like quasiparticles with the momentums $p > p_F$ (as shown in figure 1(c)) as well as hole-like quasiparticles with the momentums $p < p_F$ can be created. Historically, several notations were introduced for both excitation types
3. Quasiparticle relaxation times

The first systematic research in the field of nonequilibrium superconductivity began in the early seventies. A highly detailed state of this matter can be found in profound reviews [51, 52] and [53]. Despite the significant progress in understanding of the physics of processes, that was achieved in the following decades [54–58], a number of questions are still open. The main subject of this topical review is the studies of current states of a superconductor. We will concentrate on questions connected with charge imbalance, which is typical for a current injection of nonequilibrium excitations (so called transverse nonequilibrium mode, see section 4). Nevertheless, charge imbalance is always accompanied by the excitation of energy imbalance (the longitudinal mode, see section 4), which, for example, is related to the electron cooling phenomenon. Both excitations will be discussed in this review.

Postulating the existence of a definite energy spectrum of the superconductor in figure 1, it can be assumed from general considerations that some typical time hierarchy, that describes the nonequilibrium modes relaxation into a ground state, should exist. Apparently, the smallest scale can be the time of an electron–electron interaction $\tau_{ee}$. Electrons are not thermalized at shorter times and, consequently cannot be described by the definite distribution function. A nonequilibrium population of the energy spectrum which corresponds both to a longitudinal (figure 1(b)) and a transverse (figure 1(c)) mode may take place after the thermalization. Obviously, equilibrium Cooper pairs are formed from electrons which moments are equal in absolute values but are opposite in the sign (with respect to the Fermi momentum $p_F$). They cannot be formed directly from a type of spectrum illustrated in figure 1(c): first of all the numbers of electron-like and hole-like excitations should be equal at typical relaxation times of a charge imbalance $\tau_Q$. A parallel process is an inelastic quasiparticle scattering with a phonon emission and an absorption which is defined by the time $\tau_s$. And finally, an inelastic recombination of two quasiparticles which leads to the forming of an equilibrium Cooper pair is described by the typical time $\tau_r$, see figure 2 [59]. In the most general case all these three processes are inelastic and do require a presence of a subsystem which allows the energy exchange, for example, phonons. It is remarkable, that in case of a gap anisotropy and/or a presence of a finite current and/or magnetic impurities the relaxation of a charge imbalance may occur due to the elastic processes [60]. Obviously, with a temperature drop this relaxation channel should become the dominant one.

4. Basic equations

Nonequilibrium distribution functions $f_L$ and $f_T$ for longitudinal and transverse modes, respectively, can be defined within the framework of a Keldysh formalism [61–66]. The matrix Keldysh–Green’s function (which is $2 \times 2$ matrix in Nambu space) is given by the following expression,

$$\hat{G}^K = \hat{g}^R \hat{f} - \hat{f} \hat{g}^A,$$

where $\hat{g}^R$ and $\hat{g}^A$ are, correspondingly, the retarded and advanced Green’s functions in Nambu space, and $\hat{f}$ is the matrix distribution function having only diagonal elements

$$\hat{f} = f_L + \sigma_z f_T,$$

where $f_L(T)$ are odd (even) components (scalars) with respect to the Fermi surface, and $\sigma_z$ is the Pauli matrix. Collecting $\hat{g}^R$, $\hat{g}^A$, $f_L$, and $f_T$ into the matrix $\hat{G}$, we obtain

$$\hat{G}^K = \hat{g}^R \hat{f} - \hat{f} \hat{g}^A,$$

where $\hat{g}^R$ and $\hat{g}^A$ are, correspondingly, the retarded and advanced Green’s functions in Nambu space, and $\hat{f}$ is the matrix distribution function having only diagonal elements

$$\hat{f} = f_L + \sigma_z f_T,$$
$g^4$ and $G^K$ into the symbolic $4 \times 4$ matrix in Keldysh–Nambu space, one can write the diffusive equations of nonequilibrium superconductivity (see [61] for the comprehensive review). Obtaining $f_{LT}$ from these equations is the central question of the nonequilibrium superconductivity. We note that the normal distribution function in such representation is expressed as

$$f = \frac{1}{2} (1 - f_T - f_L).$$  \hspace{1cm} (3)

Matrix distribution function can be expressed through the electron and hole population numbers, $n_e$ and $n_h$, respectively,

$$\hat{f} = 1 - 2 \begin{pmatrix} n_e & 0 \\ 0 & n_h \end{pmatrix}.$$  \hspace{1cm} (4)

In thermal equilibrium, for instance in the reservoirs at voltage $V$, it can be expressed by the Fermi functions

$$f_{eq} = \begin{pmatrix} 1 - 2f_e(-E + eV) & 0 \\ 0 & 2f_e(-E + eV) - 1 \end{pmatrix},$$

$$f_{eq} = \begin{pmatrix} \tanh \frac{E + eV}{2k_BT} & 0 \\ 0 & \tanh \frac{E + eV}{2k_BT} \end{pmatrix},$$  \hspace{1cm} (6)

where $e$ is the electron charge, $E$ is the quasiparticle energy, counted from the Fermi energy $E_F$, $T$ is the temperature, and $k_B$ is the Boltzmann constant. Equilibrium values of $f_{LT}$ can be expressed as,

$$f_{LT}^{eq} = \begin{pmatrix} \tanh \frac{E + eV}{2k_BT} + \tanh \frac{L - eV}{2k_BT} \\ \tanh \frac{E + eV}{2k_BT} - \tanh \frac{E - eV}{2k_BT} \end{pmatrix},$$  \hspace{1cm} (7)

$$f_{LT}^{eq} = \begin{pmatrix} \tanh \frac{E + eV}{2k_BT} + \tanh \frac{E - eV}{2k_BT} \\ \tanh \frac{E + eV}{2k_BT} - \tanh \frac{E - eV}{2k_BT} \end{pmatrix}. $$  \hspace{1cm} (8)

From equations (4) and (5) it can be seen a clear physical explanation for $f_L$ and $f_T$ distribution functions. The deviations of $f_L$ from equilibrium produces more (or fewer) quasiparticles equally on both holelike and electronlike branches of the quasiparticle spectrum of the superconductor, while the deviation of $f_T$ from equilibrium produces more electrons than holes or vice versa (therefore this class of disequilibrium is often called ‘charge’ or ‘branch’ imbalance). From the last equation, equation (8), one can see that the equilibrium value of the transverse mode is zero at zero voltage bias.

In this review we will mostly consider the problem of nonequilibrium quasiparticle injection from a normal metal lead into a superconductor in normal metal—insulator—superconductor (NIS) tunnel junctions. The flow of electric current in NIS tunnel junctions is accompanied by the heat transfer from the normal metal into the superconductor. The electric current within the Keldysh formalism is given by the following equation [61],

$$I = \frac{g_N}{e} \int_0^\infty D_T \nabla f_T \, dE,$$  \hspace{1cm} (9)

where $g_N$ is the normal conductance of the normal metal lead per unit length, and $D_T = \text{Tr}(1 - \sigma_z g^R \sigma_z g^L)$ is a dimensionless diffusion coefficient. The energy current within the Keldysh formalism is defined as [61],

$$Q = \frac{g_N}{e^2} \int_0^\infty E D_L \nabla f_L \, dE,$$  \hspace{1cm} (10)

where $D_L = \text{Tr}(1 - \sigma_z g^R \sigma_z g^L)$. The spectral electric current is proportional to the transverse mode gradient (which is created, for example, by the applied voltage), while the spectral energy current is proportional to the gradient of the longitudinal mode. The heat current out of the normal metal lead in an NIS junction (the cooling power) is given by [7],

$$P = -IV - Q.$$  \hspace{1cm} (11)

It is due to selective tunneling of high-energy quasiparticles out of the normal metal which is induced by the superconducting energy gap. The heat $P$ taken from the N electrode is then released in the superconductor electrode, thus the full heat production in both electrodes is equal to the Joule heating, $IV$. 

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**Figure 2.** Calculated energy dependencies of characteristic relaxation times $\tau_\rho$, $\tau_Q$ and $\tau_r$ expressed in units of the electron–phonon scattering time $\tau_0$. Reprinted figure with permission from [59], copyright 1976 by the American Physical Society.
We mention here that the current transport in NIS junctions is governed not only by single-particle tunneling but also by two-particle (Andreev) tunneling [67, 68]. In this topical review we do not discuss the questions of nonequilibrium current transport in junctions and weak links through the Andreev levels. The reader can see, for example, the following articles [69–77] and references therein.

5. Early experiments and phenomenological models

First experimental works on charge imbalance in superconductors began to appear in the early seventies [60, 78]. The experimental design is illustrated in figure 3: nonequilibrium quasiparticles are injected from a normal metal (N) through a tunnel barrier (I) into a superconductor (S) where a potential difference is measured either by a superconducting electrode (S_p) through a Josephson contact or by a normal electrode (N_p) through a thin tunnel barrier.

A finite magnitude of the excess voltage (see figure 4) detected in these experiments was interpreted as an ohmic contribution of a quasi-normal part enriched with nonequilibrium quasiparticles. It have been shown that: (1) the polarity of the excess voltage depends on the polarity of the injection current; (2) the signal magnitude decreases with the thickness of a superconducting film and (3) the magnitude of the excess voltage tends to infinity when approaching the critical temperature. It is worth saying that due to technical constraints thin-film structures of the sandwich type studied in the seventies had a nonequilibrium superconductor (tin) thickness w significantly smaller than the typical relaxation length of a charge imbalance Λ_Q, w ≪ Λ_Q. Consequently, it was postulated that the nonequilibrium carrier concentration is constant along the whole thickness of the superconductor and a spatial relaxation dependence could be derived indirectly from a weak dependence on the thickness \( \sim \exp(-w/\Lambda_Q) \approx 1 \). The majority of experiments were conducted at temperatures close to the critical one and the agreement with the phenomenological model [60] has been established exactly in this high-temperature limit.

In numerous following works [49] qualitative conclusions of pioneer research [78] were confirmed using various materials and a simple phenomenological model [60] was developed on more serious grounds [53]. In particular it was shown that the current–voltage dependence of a normal metal-insulator-superconductor (NIS) detector \( I(V) \) can be written as:

\[
I = \frac{G_{NN}}{e} \int_{0}^{\infty} \left\{ N_S(E)[f_F(E) - f_S(E - eV)] + [f_{<} - f_{>}^e] \right\} dE,
\]

where \( G_{NN} \) is a tunneling conductance of a junction (detector) in a normal state, \( f_F(E) \) is the Fermi distribution function,
$N(E)$ is the density of states in superconductor, and $f_{k>}$, $f_{k<}$ are the populations of electron-like and hole-like branches of an excitation spectrum, correspondingly.

The magnitude of a charge imbalance $Q^*$ is defined exactly by the population difference of electron-like and hole-like branches,

$$Q^* = 2 \int_0^\infty N_N(E) [f_{k<} - f_{k>}] dE = -2N(0) \delta \mu_S, \quad (13)$$

where $N(0)$ is the single electron density of states at the Fermi level and $\delta \mu_S$ is the deviation of the chemical potential of Cooper pairs from its equilibrium value. It is worth mentioning that according to longitudinal and transverse excitation modes the expression in equation (12) can be rewritten using nonequilibrium distribution functions $f_T^N$ and $f_T^S$, where symbols $N$ and $S$ are related to a normal metal and a superconductor electrode of a tunnel NIS detector,

$$I_d = \frac{G_{NN}}{e} \int_0^\infty N_N(E) [f_T^N - f_T^S] dE, \quad (14)$$

where

$$f_T(E, V_d, T_e) \equiv f_T^N (E, V_d, T_e)$$

$$= \frac{1}{2} \tanh[(E + eV_d)/2k_BT_e] - \frac{1}{2} \tanh[(E - eV_d)/2k_BT_e] \quad (15)$$

is the equilibrium value of $f_T$ in the normal detector, $V_d$ is the detector voltage, $T_e$ is the electron temperature in the detector, and $f_T^S$ is the local nonequilibrium value of $f_T$ in the superconductor.

A simple comparison with the standard (equilibrium) equation for a tunnel current of a NIS junction at a temperature $T$ [79],

$$I(V, T) = \frac{G_{NN}}{e} \int_0^\infty \{N_N(E) [f_{k<} - f_{k>}] \} dE, \quad (16)$$

allows us to conclude that all deviation from the equilibrium is defined by the last term in the integrand of equations (12) and (14). The first term in the integrands of equations (12) and (14) corresponds to a standard (equilibrium) tunnel current equation (16) which depends on the properties of the superconductor via the density of states. The energy dependence of the population differences of electron-like and hole-like branches of an excitation spectrum $[f_{k<} - f_{k>}]$ a priori is unknown and, therefore, calculation of the dependence $I(V)$ for arbitrary displacement values of $V$ is not at all evident. However, the first term in the equation (12) falls out at zero voltage and the so called excess current $I_{es}$ is defined as with the help of equation (13),

$$I_{es} \equiv I(V = 0) = \frac{G_{NN}}{e} \int_0^\infty [f_{k<} - f_{k>}] dE = -\frac{G_{NN}}{e} \delta \mu_S. \quad (17)$$

The equation equation (17) allows us to define the particularly microscopic parameter—the shift of a chemical potential of Cooper pairs $\delta \mu_S$ using easily measured in the experiment values $I_{es}$ and $G_{NN}$.

There have recently appeared series of experimental papers where an attempt to develop the aforementioned observations

Figure 5. An experimental design and current–voltage characteristics of the NIS detectors placed at various distances from the NIS injector. Reprinted figure with permission from [82], copyright 2006 by the American Physical Society.

6. Spatially resolved measurement setup

In this section we discuss our experimental research of spatial dependencies of relaxations of the charge and energy imbalances in a superconductor at temperatures significantly lower than the critical temperature, $T \ll T_c$ [84].

Multiterminal nanostructures were fabricated using electron beam lithography and ultra high vacuum evaporation of aluminum (superconductor, S) and copper (normal metal, N) separated by naturally-grown aluminum oxide (I) [85] (figure 6). The sample layout was similar to the layout, described in [80–83] and [86, 87]. Electrons were injected from a normal
Figure 6. Top panel: the design of a sample and measurements. The electron temperature of the injector made from a normal metal is measured by the NIS junction ‘thermometer’. The relaxation of nonequilibrium quasiparticles is measured either by a single NIS detector or by a pair of NIS junctions equidistant from the injector at the opposite sides of the superconductor (the NISIN configuration). Low panel: a scanning probe microscope image of the sample fragment. One can see the injector (the left massive configuration). Low panel: a scanning probe microscope image of the sample fragment. One can see the injector (the left massive electrode) and a pair of NIS detectors on the Si/SiO$_2$ substrate. Reprinted figure with permission from [84], copyright 2011 by the American Physical Society.

metal through a tunnel junction into a superconductor. The typical thickness of the aluminum, detectors and the injector was 25, 40 and 80nm, and the line width was 400, 180 and 1600nm respectively. The critical temperature of the aluminum microstrip was $T_c \approx 1.35$ K and electron mean free path was $l \approx 20$ nm. The tunnel resistance of a large-area NIS injector was selected to be sufficiently low $\approx 3 \, \text{k}\Omega$ in order to provide a high ‘pumping’ quasiparticle current while a tunnel resistance of narrow NIS detectors was about $50 \, \text{k}\Omega$. In both cases the tunnel conductance of our junctions was low enough in order to neglect the proximity effect.

Experiments were performed in a $^3$He/$^4$He dilution refrigerator located inside the electromagnetically shielded room using analog preamplifiers connected with the external measuring instruments through a system of radio-frequency filters. A typical measuring network consists of two circuits: an injector and a detector (figure 6, top panel). Two complementary configurations were used for the quasiparticle detection: a single NIS junction, or a pair of tunnel junctions (NISIN) which were equidistant from the injector. The second configuration was found to be more stable with respect to parasitic potentials. However, it was obvious that the NISIN configuration cannot be used for the charge imbalance detection, since the signals from two NIS junctions compensate and extinguish each other due to the junctions reverse polarity with respect to the superconductor.

A special consideration was given to the thermalization of the system. In order to prevent the normal injector from overheating it was made sufficiently massive. An additional NIS electrode (marked as a ‘thermometer’ in figure 6 and placed at a distance of $\approx 1 \, \mu\text{m}$ from the NIS injector) was used for measuring the electron temperature of the injector $T_e^i(I)$. The current–voltage characteristic of this ‘thermometer’ $I_T(V_T, I = \text{const})$ was measured for fixed injector current values $I$ and the electron temperature was calculated from the fitting of experimental current–voltage characteristics by theoretical curves for the NIS junction when the electrode disequilibrium could be neglected, equation (16). After calibrating the injector the dependence of the electron temperature on the injection current has been established $T_e^i(I, T = \text{const})$, and the thermometer circuit was disconnected for the rest of experiments. Here and below the $I_T(V_T)$, $I_e(V_e)$ and $I_d(V_d)$ dependencies denote current–voltage characteristics of the thermometer, the injector and the detector, respectively.

7. Temperature parameters

It is instructive to summarize the definitions of various parameters of the dimensionality ‘temperature’ hereafter used in this review. The parameter $T$ corresponds to the bath temperature measured by the two resistors made of RuO$_2$, which are thermally and mechanically reliably connected to the walls of the mixing chamber and to the massive copper sample holder. Both sensors were calibrated by a nuclear orientation thermometer and during the measurements their readings deviated less than by a couple of mK.

$T_e^i$ and $T_d^i$ denote electron temperatures of the injector $(i)$ and the detector $(d)$ made from a normal metal (copper). Both values were defined from the fitting of the experimental $I_e(V_e)$ and $I_d(V_d)$ dependencies by an expression for the tunnel current of the NIS junction when nonequilibrium effects could be neglected, given by equation (16). The electron temperature of the injector depends on the injection current $T_e^i(I, T = \text{const})$ due to the Joule heating. Despite sufficiently low currents used in the experiments this trivial phenomenon can lead to tangible overheating at ultralow temperatures. At the lowest bath temperatures $T \approx 20$ mK and maximal injection currents $I \approx 1 \, \mu\text{A}$ the overheating of the injector subsystem was $\delta T^i_e \equiv T_e^i - T \approx 100$ mK. It should be noted that in the discussed experiments $T_e^i$ is the highest ‘real’ temperature of the whole sample. Several parameters with the ‘temperature’ dimension will sufficiently exceed $T_e^i$ which is the evidence of their ‘effective’ nature, as it will be shown below.

The electron–phonon interaction is very weak at ultralow temperatures [3], and a heat conductivity of a long superconducting sample part is exceptionally low. Therefore, the heat spreading effect from the ‘hot’ injector along
the superconducting sample should be small enough. Subsequently, it is reasonable to consider that the phonon temperature of the remote detectors should not significantly deviate from the bath temperature \( T \). On the contrary, the electron temperature of the normal metal lead of an NIS detector \( T^f \), determined by fitting the experimental \( I_s(V_{sd}, I_i = 0) \) dependencies at the zero injection current by the equation (16), is always lower than the bath temperature \( T \). This phenomenon is typical for galvano-magnetic measurements at ultralow temperatures and is related to the inevitable heating of the electronic subsystem by the electromagnetic radiation which enters through junction leads. This heating channel can be reduced with the help of different kinds of the high frequency filtration but can never be completely eliminated. At the lowest bath temperatures \( T \approx 20 \, \text{mK} \) and zero injection currents \( I_i \approx 0 \), an increase of electron temperature \( \delta T^f \equiv T^f - T \) in NIS detectors varied from 10 to 40 mK for different junctions. This is a quite worthy result which is the evidence of the quality of the used high-frequency filters. Since each detector is isolated from the ‘hot’ injector by two tunnel NIS junctions divided by the extended superconducting sample ‘body’ which has a low heat conductivity and a good thermal connection with the substrate (Si/SiO\(_2\)), it is reasonable to assume that the electron temperature of the detector does not depend on the injection current \( I_i \) and is defined only by the level of the electromagnetic noise, which results in the expression for the superconductor electron temperature: 

\[
T^e_{d}(I_i > 0, T) \approx T^e_{d}(I_i = 0, T).
\]

The \( T^e_{d} \) denote the superconductor electron temperature. Strictly speaking, \( T^e_{d} \) should be determined from a complicated energy balance equations [3], which parameters, for example a metal-substrate boundary heat conductivity, are usually not accurately defined. From the very general considerations it can be expected that the thermodynamic electron temperature in a superconductor, which enters into the nonequilibrium distribution function \( f^d_{S}(E, T^e_{d}) \), should be higher than the phonon temperature \( T_{\text{phonon}} \approx T \) at finite injection currents \( I_i > 0 \). However, it should be immediately noted that the \( T^e_{d} \) parameter cannot be determined in the context of the phenomenological formalism employed in this review. A deeper microscopic approach (which is as far as we know currently absent) should be used to determine the nonequilibrium and essentially asymmetric with respect to the chemical potential distribution function \( f^d_{S}(E, T^e_{d}) \). Otherwise, any arbitrary value \( T^e_{d} \) substituted in a symmetric function \( f^d_{S}(E, T^e_{d}) \), for example the equilibrium function, gives exactly the same tunnel current of an NIS junction.

Finally, the last parameter with the ‘temperature’ dimension, \( T^\ast \), characterizes the superconductor energy gap \( \Delta \). As it will be shown further, the injection of nonequilibrium quasiparticles into a superconductor among other effects leads to a gap suppression. Therefore two alternative descriptions are possible: either to obtain the dependence \( \Delta(I_i) \) directly from the experimental data, or the gap suppression can be ‘converted’ into the effective temperature \( T^\ast \) using the standard (equilibrium) BSC theory. In the latter case the \( T^\ast \) parameter indicates which equilibrium temperature \( T = T^\ast \) corresponds to the nonequilibrium gap \( \Delta(I_i > 0) = \Delta_{\text{BSC}}(I_i = 0, T^\ast) \). Obviously, the \( T^\ast \) value is only a convenient parameter for the description of a gap suppression and has no direct link with the ‘true’ thermodynamic temperature \( T^e_{d} \). As it will be shown further the injection of nonequilibrium charge carriers into a superconductor can lead to various phenomena. In particular, the longitudinal mode (the energy imbalance) cannot be fully described only by the gap suppression or, alternatively, by the rise of the effective temperature \( T^\ast \).

8. Nonequilibrium quasiparticle injection

The nonequilibrium quasi-particle injection into a superconductor can lead to a deviation of the density of states (DOS) \( N_S(E) \) and the distribution functions \( f^d_{S}(E, T^e_{d}) \) from their equilibrium values as it was discussed earlier. The distribution function can be found by the deconvolution of the experimental tunnel current–voltage characteristics under certain conditions [88, 89].

In a general case, in order to find the distribution function of a superconductor the self-consistent solution of a Keldysh–Usadel equation is needed [61], which is an exceptionally complicated task. We are going to use a simplified approach for an interpretation of our results by postulating the identity of functional forms of the distribution function and the density of states with their equilibrium values,

\[
f_S(E, i^e_{\Delta}, \mu_S) = \frac{1}{\exp[(E - \mu_S)/k_B T^e_{d}] + 1}, \tag{18}
\]

\[
N_S(E, \Delta, \Gamma) = \frac{\text{Re}(E + i\Gamma/2)}{\sqrt{(E + i\Gamma/2)^2 - \Delta^2}}. \tag{19}
\]

The deviation of \( f^d_{S} \) from its equilibrium value formally corresponds to an increase of the temperature \( T^e_{d} \) above the bath temperature \( T \), while the deviation of \( f^d_{S} \) corresponds to a finite value of the chemical potential \( \mu_S \) measured with respect to the Fermi level. Formally, if the expression equation (18) correspond nominally to the reality, the three parameters \( T^e_{d} \), \( \mu_S \) and \( \Gamma \) should be sufficient for describing the nonequilibrium state of the superconductor. However, as mentioned previously, no symmetric function, including the Fermi function equation (18), cannot be even qualitatively used for determining the tunnel current of the NIS junction where a superconducting electrode is in a nonequilibrium state. The only role using the inapplicable expression equation (18) is to obtain a finite value of a chemical potential \( \mu_S \), which is physically connected with the charge imbalance. For the analysis of the experimental data we are going to use three following fitting parameters in our phenomenological approach. The \( \Gamma \) parameter, which defines the broadening of the density of states [90, 91], the superconductor energy gap \( \Delta \), and the effective chemical potential \( \mu_S \) [92, 93]. All these parameters can be defined from the experimental \( I_s(V_{sd}) \) and \( dI_s/dV_{sd} \) dependencies. It is worth mentioning once again that the thermodynamic temperature of a superconductor \( T^e_{d} \) cannot be defined in the framework of this phenomenological approach.

Since the injected nonequilibrium quasiparticles have to relax at certain times (or, equivalently, at certain distances),
it should be expected that the fitting parameters have to depend both on a pumping level (of the energy $eV_i$ or the current $i=VG_{NN}$) and on the distance between the detector and the injector $L_{sd}$. The $I_d(V_d)$ dependence of a NIS detector in the presence of nonequilibrium injection is given by equation (14).

The shape of the current–voltage characteristic $I_d(V_d)$ depends on the quasiparticle injection level (figure 7). By fitting the experimental $I_d(V_d)$ and $dI_d/dV_d$ data the relevant energy, temperature and spatial dependencies $\Gamma(eV_i,T,L_{sd})$ and $\Delta(eV_i,T,L_{sd})$ can be determined (figures 8 and 9). The smearing of the current–voltage characteristic at values of bias close to the gap (the smothering of ‘corners’ at $eV_i \approx \Delta(I_i)$) is determined by the $\Gamma(I)$ parameter included in the expression for the superconductor density of states equation (19) and related to the finite lifetimes of quasiparticle excitations [90, 91]. For a given injection current $I_i$ the temperature dependence $\Gamma(T,I_i=\text{const},L_{sd}=\text{const})$ (right inset in figure 8) most probably originates from the more intensive quasiparticle relaxation due to the inelastic scattering on phonons.

The total number of nonequilibrium quasiparticles injected into the superconductor lead per unit time is proportional to the injection current $I_i$ (or alternatively to the injection energy $eV_i$) but only part of them $\sim \exp(-L_{sd}/\sqrt{D\tau_f})$ reaches the detector located at a distance of $L_{sd}$ from the injector. Here $D=\frac{1}{3}v_F\ell$ is the diffusion coefficient, and $v_F$ is the Fermi velocity. At low temperatures the quasiparticle recombination time $\tau_f$ can be estimated as

$$\tau_0/\tau_f \sim (T/T_c)^{1/2} \exp(-\Delta/k_bT),$$  \hspace{1cm} (20)$$

where $\tau_0$ is a characteristic time of the electron–phonon scattering [59, 94]. These rather simple considerations qualitatively explain the observed spatial, energy and temperature dependencies of the splitting parameter $\Gamma$ (figure 8). However, microscopic models are required for the quantitative analysis which have to include the contribution of the electromagnetic environment [95].

9. Energy imbalance (longitudinal mode)

Neglecting the phase of the superconductor order parameter, the gap equation for the pairing potential $\Delta$ to be solved self-consistently with the Keldysh–Usadel equation, is

$$\Delta = \lambda \int_0^\infty f_+^E(E,L_{sd})\text{Re}(F)dE,$$ \hspace{1cm} (21)$$

where $\lambda$ is the electron–phonon interaction constant, $F$ is the anomalous Green’s function (pair amplitude), and $f_+^E(E,L_{sd})$ is the local nonequilibrium value of the longitudinal component $f_+$ of the nonequilibrium distribution function [61]. For any arbitrary energy value $E$ the function $f_+^E(E,L_{sd})$ is always smaller than its equilibrium value $\text{tauh}(E/k_bT)$ [96], thereby reducing $\Delta$ (figure 9(a)). This effect increases with the bath temperature rise (figure 9(a), inset). Using the well-known temperature dependence of the BCS model we can describe the gap suppression due to the quasiparticle injection as the rise of some effective temperature $T^*$.

$$\Delta(T^*,I_i,L_{sd}) = \Delta_{BCS}(T^*,I_i=0,L_{sd})$$

(figure 9(b)). It is reasonable to assume that together with the spatial relaxation of the injected quasiparticles the gap suppression (or alternatively the rise of the $T^*$) should also decay at a certain characteristic distance $\Lambda_{T^*}$,

$$T^* = T^*(0)\exp(-L_{sd}/\Lambda_{T^*}).$$ \hspace{1cm} (22)$$

The experimental data analysis results in a huge (on the microscopic scale) value of the characteristic length $\Lambda_{T^*} = 40 \mu m \pm 20 \mu m$ (figure 9(b), inset). A relatively big error $\sim 50\%$
is related to the weak temperature dependence $\Delta_{\text{BCS}}(T)$ at low temperatures $T \ll T_c$ and to a limited number of experimental points given by the quantity of NIS detectors in tested samples (figure 6). It is worth mentioning once again that $T^\ast$ is nothing more than a convenient parameter which has no direct relation to the thermodynamic temperature of the superconductor, $T_\text{th}$. It should be noted that in the limit of strong quasiparticle pumping and weak electron–electron interaction a highly nonequilibrium state can be realized. In this case the thermodynamic temperature $T_\text{th}$ introduction is quite problematic. However, the energy gap value $\Delta(I)$ or, alternatively $T^\ast(I)$ can be obtained from experimental current–voltage characteristics $I_d(V_d, I = \text{const})$.

We assume the energy gap supression $\Delta(I)$ (figure 9) and the density of states smearing $\Gamma(I)$ (figure 8) to be the manifestations of the same phenomenon—the energy imbalance. It is likely that in a more general (microscopic) model both excitation modes of nonequilibrium charge carries (the longitudinal and the transverse) are going to be entangled and consistently describe the dependencies $\Delta(I)$ and $\Gamma(I)$. Within the framework of the standard BCS theory the current–voltage characteristic of the NIS detector junction (expression equation (14)) is defined exclusively by the distribution function $f_\text{n}(E, V, T^\circ)$ in the normal electrode and depends on the parameters of the superconductor only via the value of the energy gap $\Delta$. The temperature of the superconductor $T^\circ$ contributes to the current–voltage characteristic only through the temperature dependence $\Delta(T^\circ)$. In an equilibrium state $T^\circ = T^\circ = T_\text{phonon} = T$ the experimentally observed smearing of the current–voltage characteristic of the NIS junction is only due to the finite temperature of the normal electrode. Our approach for the interpretation of the experimental data differs from the standard BCS theory due to the introduction of the splitting parameter $\Gamma(I)$ included in the expression for the density of states equation (19), and contributed to the additional (not thermal) smearing of the current–voltage $I_d(V_d)$ dependence, equation (14). The use of the ‘equilibrium’ tunnel current expression (first term in equation (14)) with the modified density of states (equations (18) and (19)) allows us to separate the influence of two contributions, $\Delta(I)$ and $\Gamma(I)$. The gap supression $\Delta(I)$ by the finite injection current (figure 9) leads to a ‘sharp’ current–voltage characteristics of the detectors $I_d(V_d, I_d > 0)$, but with a smaller value of the gap $\Delta(I) > 0 < \Delta(I = 0)$. The contribution of the finite
smearing of the density of states $\Gamma(I)$ manifests itself as the ‘smoothing’ of the current–voltage characteristics at voltages $eV_d \simeq \pm \Delta(I)$, which cannot be explained by increase of normal electrode temperature, $T^N$, within reasonable limits.

The current–voltage characteristics of the detectors measured at zero injection currents can be sufficiently accurately described by the equilibrium expression for the NIS junction current equation (16) under the assumption of the finite but rather small Dynes smearing of the density of states $\Gamma(I_0) = 0/\Delta(I_0) \simeq 0.02$ (figure 7(b)) in agreement with the existing literature data [95]. However, at finite injection currents the experimental current–voltage characteristics $I_d(V_d, I_0 > 0, T)$ cannot be described within the standard BCS theory taking into account that corresponding values have increased above the bath temperature, $T^N \ll T_c$. At sufficiently low temperatures $T' \ll T_c$ the shape of the experimental current–voltage characteristics $I_d(V_d, I_0 > 0, T')$ is qualitatively different from the equilibrium dependencies obtained at higher temperatures $I_d(V_d, I_0 = 0, T'' > T')$ (figure 7(b)). If we assume at a bath temperature $T$ that the current–voltage characteristic broadening is explained solely by the rise of the effective temperature $T^f \equiv T'' > T$ than the same temperature value should be also substituted into the BCS dependence $\Delta(T, T_c) = \Delta_{BCS}(I_0, T^f)$. However, it turns out that following this description, it should be assumed that the temperature of the superconductor is considerably higher than the ‘hottest’ system point—the normal injector: $T^f \equiv T'' > T$. This effect has been observed both in early works on flat structures of the ‘sandwich’ type [60, 78] and in the recent research on multiterminal NIS structures [80–83]. Within the phenomenological description the nonzero value of the excess current is related to the deviation of the Cooper pairs chemical potential $\mu_S$ from its equilibrium value (expression equation (17)), see figure 10. The typical example of the experimental current–voltage characteristic of the NIS detector at the voltage shifts smaller than the gap, $eV_d < \Delta$, at various quasi-particle injection levels is shown in figure 11(a). As it was expected, the excess current $I_{dex}$ increases with the pumping energy rise and changes the sign with the change in the polarity of the injection current. The effect weakens with increasing distance between the detector and the injector $L_d$ (figure 11(b)) and with the bath temperature rise (figure 11(c)). The weak asymmetry of the excess current with respect to the zero injection current (figure 11(a)) is probably related to the presence of parasitic residual thermo-electric potentials in the measuring circuit due to significant temperature gradients between the preamplifiers (at a room temperature) and the sample.

Once again we would like to note that in NISIN detector geometry (figure 6) the excess current $I_{dex}$ is absent due to the signal compensation from the two NIS junctions connected with the opposite polarity with respect to the superconductor. The NISIN geometry was used only as a complementary configuration in the experiments on the energy mode relaxation (figure 7(a)).

10. Charge imbalance (transverse mode)

Let’s now move on to the analysis of another interesting phenomenon manifested at the same experiments on the injection of nonequilibrium quasiparticles—the charge imbalance. As was discussed above, see equation (12), the population difference between the electron-like and the hole-like spectrum branches of the excitation spectrum $f_{S\uparrow} \neq f_{S\downarrow}$ leads to the appearance of the finite NIS detector current at a zero $V_d$ referred to as the excess current, $I_{dex} \equiv I_d(V_d = 0)$. This effect has been observed both in early works on flat structures of the ‘sandwich’ type [60, 78] and in the recent research on multiterminal NIS structures [80–83]. Within the phenomenological description the nonzero value of the excess current is related to the deviation of the Cooper pairs chemical potential $\mu_S$ from its equilibrium value (expression equation (17)), see figure 10. The typical example of the experimental current–voltage characteristic of the NIS detector at the voltage shifts smaller than the gap, $eV_d < \Delta$, at various quasi-particle injection levels is shown in figure 11(a). As it was expected, the excess current $I_{dex}$ increases with the pumping energy rise and changes the sign with the change in the polarity of the injection current. The effect weakens with increasing distance between the detector and the injector $L_d$ (figure 11(b)) and with the bath temperature rise (figure 11(c)). The weak asymmetry of the excess current with respect to the zero injection current (figure 11(a)) is probably related to the presence of parasitic residual thermo-electric potentials in the measuring circuit due to significant temperature gradients between the preamplifiers (at a room temperature) and the sample.
The solution of the diffusion equation for the charge imbalance $Q^*$ leads to the expression for the excess current:

$$I_{d}^{ex} = \frac{I_e^e \Lambda Q^* G_{NN}}{2e^2N(0)\bar{\sigma}} \exp(-L_{id}/\Lambda Q^*).$$  

(23)

where $N(0) = 1.08 \times 10^{27} 1/J \times m^3$ is the density of state of the aluminum at the Fermi level (in a normal state), $F^e$ is a slowly varying function which is equal to zero at $V_i = 0$ and equal to unity at $eV_i \gg \Delta_i$ [60]. The substitution of corresponding values: the mean free path $\ell \approx 20$ nm, the Fermi velocity $v_F = 1.36 \times 10^6$ m s$^{-1}$, the diffusion coefficient $D = \frac{1}{4}v_F \ell$, and the cross-section of the superconducting channel $\sigma = 25$ nm $\times$ 400 nm provides us with the satisfactory agreement with the experimental dependencies $I_{d}^{ex}(eV_i)$ (figure 11(b)). The obtained value for the relaxation length of the charge imbalance $\Lambda Q^* = \sqrt{D\tau e}$ varies from 3.5 to 6.5 $\mu$m being in a reasonable agreement with the existing data [80–83, 97].

A weak temperature dependence of the charge imbalance relaxation $Q^* \sim I_{d}^{ex} \sim \delta_{12}$ at ultralow temperatures (figure 11(c)) has a rather simple explanation: at the weak effective electron–phonon interaction the only remaining relaxation channel is the elastic scattering by the inhomogeneity and anisotropy of the energy gap and/or the finite value of the supercurrent [60]. In contrast to this (transverse) excitation mode the energy relaxation (the longitudinal mode) always requires the inelastic scattering. At ultra-low temperatures $k_B T \ll \Delta$, the concentration of equilibrium phonons is not sufficient for the effective energy relaxation. The only source of phonons with energies larger than the temperature might be the spreading of the Joule heat from the ‘hot’ injector. However, as was already discussed earlier, even in case of maximum injection currents the increase of the electron temperature of the injector is $\delta T \approx 100$ mK, which is sufficiently smaller than the minimum energy $2\Delta$ required for the formation of a Cooper pair from an electron-like and a hole-like quasiparticles. Since the energy relaxation is slow, as was found above, but still occurs on ‘astronomically’ large (for a superconductor) scales $\Delta_T \approx 40 \mu$m, a corresponding scattering channel has to exist. For example, the emission and absorption of nonequilibrium phonons or photons, can provide an appropriate relaxation mechanism. Naturally this relaxation channel also has to make a finite contribution to the charge imbalance relaxation. The observed considerable difference in the characteristic relaxation scales $\Delta_T < \Delta_T^* \cong \Lambda_T$ confirms our assumption that the elastic scattering is an utterly important relaxation mechanism of the charge imbalance at ultralow temperatures.

A considerable quantitative difference between $\Delta_T$ and $\Lambda_T$ values at ultralow temperatures $k_B T \ll \Delta$ is in the qualitative agreement with previous theoretical calculations [59, 94]. The explicit condition when the elastic scattering can be neglected has been obtained in [98],

$$\frac{T_e - T}{T_e} \ll (\tau_e T_e)^{-2/3},$$  

(24)

where $\tau_e$ is the characteristic time of the energy relaxation (in units $1/K$). This condition allows us to conclude that the
only requirement for $\Lambda_{Q} > \lambda_{E} \simeq \lambda_{T}$ is the high-temperature limit $T \rightarrow T_{c}$. Formally this observation is equivalent to the statement that the relaxation channel of the charge imbalance does not exist in a normal metal and the corresponding time tends to infinity as the temperature is approaching $T_{c}$. The experiments discussed above have been conducted at ultralow temperatures $T \ll T_{c}$, where, to our best knowledge, there are no theoretical predictions to \textit{a priori} relate $\Lambda_{Q}$ with $\Lambda_{T}$.

11. Nonlocal supercurrent measurements for charge imbalance length determination

In this section we discuss another method of the experimental determination of charge-imbalance relaxation length. It can be estimated using a modified Kadin, Smith and Skocpol (KSS) scheme [99] in case of a planar geometry. We refer to recent works, done in the group of Ryazanov [100]. In this paper the detection of a nonlocal critical current was demonstrated in mesoscopic SNS (Al–Cu–Al) Josephson junctions with several spatially separated normal metal (Cu) injectors connected to one of the superconducting Al wires. Similar structure is shown in figure 12(a). To measure the nonlocal voltage at low temperatures $T \ll T_{c}$, we need to use superconducting leads just near the junction (wite lines in figure 12(a)). In order to describe the interplay of charge imbalance and Josephson effect in the realized mesoscopic system in the low-temperature limit, a two-channel charge-imbalance KSS model was proposed [99]. This approach was simplified for the case of low-frequency processes and extended to study the effect of nonequilibrium quasiparticle flow in Josephson SNS junctions by Kaplunenko, Ryazanov, and Schmidt [101]. The authors assume that nonequilibrium processes in a superconductor including the conversion of a quasiparticle flow into a pair current can be described reasonably by means of an equivalent circuit introduced by Kadin, Smith, and Skocpol for the explanation of phase-slip-center behavior. Recently, in [100] this model was modified for the planar Josephson structures geometry.

To determine the charge imbalance length in the multi-terminal planar structure, Golikova et al derived a simple equation [100],

$$\Lambda_{Q} = \frac{d_{\text{inj}} - d_{\text{inj}}}{\ln(I_{c}^{3}/I_{c}^{1})},$$

(25)

where $\Lambda_{Q}$ is the charge-imbalance relaxation length, $d_{\text{inj}}$ and $d_{\text{inj}}$ are the distances between planar Josephson junction and injector probes, correspondingly (figure 12(a) white lines), $I_{c}^{3}$ and $I_{c}^{1}$ are the critical currents of planar Josephson junction in case of nonlocal measurements scheme (figures 12(c)–(f)).

The results presented in figure 12 were recently obtained in the group of Stolyarov by a measurement technique similar to that of [100]. Figure 12(a) shows a scanning electron microscopy (SEM) image of our sample, together with a measurement scheme. The submicron planar structures were fabricated by means of electron beam lithography and standard \textit{in situ} shadow e-beam evaporation without breaking the vacuum.

In our case the copper layer thickness was $d_{\text{Cu}} = 30 \text{ nm}$ and the aluminum thickness $d_{\text{Al}} = 250 \text{ nm}$, correspondingly. All injectors were realized as long copper SNS junctions without Josephson coupling. Geometrical size of SNS elements can be seen in atomic force microscope (AFM) image, see figure 12(b).

Figure 12(c) depict the current–voltage (IV) dependences for local (red curve) and nonlocal scheme of measurements for three cases of injector position (blue, black and green yellow, correspondingly). Figure 12(d,e) presents an evolution of IV curves with temperature for two different injectors (inj2 and inj3). The critical current dependance in nonlocal measurements for $I_{c}^{3}$ and $I_{c}^{1}$ allows as to obtain the charge imbalance relaxation length for Al superconducting wires. In our case it varies in the range from 3 $\mu$m to 4 $\mu$m in temperature range from 0.2 K to 0.9 K. Such transport measurements are simple enough for determination of charge imbalance relaxation length in different type superconductors. The obtained value of $\Lambda_{Q} \sim 4 \mu$m is consistent with results, obtained in the group of Arutyunov (see section 10 [84]).

12. Nonequilibrium electron cooling

The subject of nonequilibrium quasiparticle relaxation is of primary importance for the operation of normal metal—insulator—superconductor (NIS) refrigerators (see the review papers [2–5] and references therein). In such devices the flow of the electric current is accompanied by heat transfer from the normal metal into the superconductor, which enables cooling of electrons in the normal metal (see section 4). This phenomenon arises due to selective tunneling of high-energy quasiparticles out of the normal metal which is induced by the superconducting energy gap. Though the principle of operation is rather straightforward, the high cooling power requires high density of nonequilibrium quasiparticles injected into the superconductor and accumulated near the tunnel interface [42, 96]. The consequences are the backtunneling of hot quasiparticles to the normal metal [96, 102], the emission of phonons that partially penetrate the normal metal [42, 102], and the overheating of the superconducting electrode [42]. All these effects reduce the efficiency of NIS refrigerators. To minimize the undesired back-action, relaxation of those nonequilibrium quasiparticles should happen in the superconductor electrode [42, 96].

A micrometer-sized refrigerator, based on a NIS tunnel junction, has been first fabricated by Nahum et al [103]. The authors used a single NIS junction in order to cool a small normal metal strip. Later, Leivo et al [104] noticed that the cooling power is an even function of the applied voltage and fabricated a refrigerator with two NIS junctions arranged in a symmetric series configuration (SINIS). Another important point is that in SINIS geometry the normal metal lead is effectively isolated by two superconductor electrodes from the electromagnetic environment which improve cooling performance.

The quasiparticle poisoning of SINIS refrigerators can be eliminated by a special trick—utilization of a quasiparticle...
drain (so called quasiparticle trap), and by fine tuning the parameters of the NIS junctions tunnel barrier. The drain should be ‘isolated’ from the superconductor with thin tunnel barrier, which from one side lets quasiparticles get efficiently trapped in the drain, while from another side, stops the inverse proximity effect. In particular, it has been demonstrated that electronic cooling can be optimized in specially-designed large area normal metal-insulator-superconductor junctions [105]. The two key ingredients were found to be of high importance: (i) the tunnel barrier transparency for the cooling junctions [106], (ii) the coupling to a quasiparticle drain, through a (separate) tunnel junction [107]. With such provisions temperature reduction of a factor 5, from 150 mK down to 30 mK, and a cooling power of the order of one nanowatt has been achieved. An additional improvement of cooling performance can be reached by utilization of the second-stage SINIS cooler actively evacuating quasiparticles out of the hot superconductor, especially in the low-temperature limit [108]. The working principle of the device is following: back sides of the main cooler are connected to two other SINIS coolers. These second-stage coolers act as ‘active quasiparticle traps’ and help to thermalize the hot superconductor leads of the main device.

Rather unusual result has been recently obtained in electron refrigerators utilizing semiconductor—insulator—superconductor junctions [109]. Such an interface barrier does not increase the junction resistance but strongly reduces the detrimental subgap leakage current. The result was attributed to the Fermi level de-pinning and dopant segregation effects that strongly affect the junction properties at the nanoscale. Due to high transparency and low leakage such semiconductor—insulator—superconductor junctions showed excellent cooling power performance, comparable to that of high power NIS coolers.

13. Summary and outlook

In conclusion, recently there have been performed spatially resolved measurements of the nonequilibrium quasiparticle relaxation in superconducting aluminum. For the first time on the same hybrid microstructures made of aluminum and copper the spatial, temperature and energy characteristics of the energy and charge imbalance were measured at ultralow temperatures, $T \ll T_c$. This imbalance occurs when non-equilibrium quasiparticle excitations are injected into the superconductor form a normal metal through the tunnel barrier.

It was shown that the experimental results can be described by the phenomenological model which assumes the validity of the equilibrium expression for the NIS junction tunnel current. Further, it has been postulated equilibrium functional dependencies of the density of states (DOS) and distribution function of the superconductor, while assumed that the DOS broadening parameter (Dynes parameter) $\Gamma(I_I, L_{id}, T)$, the effective chemical potential of the Cooper pairs $\mu_S(I_I, L_{id}, T)$, and the superconducting gap $\Delta(I_I, L_{id}, T)$ depend on the rate of quasiparticle injection $I_I$ and the distance to the injector junction $L_{id}$ for a given temperature $T$.

It was also shown that spatial relaxations of the nonequilibrium quasiparticle excitations in the aluminum can be described by the exponential dependence with a typical scale of $\Lambda_T = 40 \pm 20 \mu m$ and $\Lambda_Q = 5 \pm 1.5 \mu m$ for the energy (longitudinal) and charge (transverse) imbalance, respectively. It should be stressed that both quantities were measured simultaneously on the same samples and using the same experimental technique, which eliminates various sample and measurement artifacts.
This is the central result of this review. It should be emphasized that both the energy and the charge disequilibrium modes are universal phenomena which should be taken into consideration in a broad class of systems involving electron, spin and/or coherent non-local transport. Despite the reasonable agreement with the phenomenological model, application of the relaxation approximation approach for the essentially spatially inhomogeneous problem is not fully justified at ultra-low temperatures, $T \ll T_c$. A deeper (microscopic) model is required for such a quantitative analysis. We hope that the findings presented in this review will trigger the corresponding research activity.

This result allows us formulate an interesting hypothesis. We have found that in superconducting Aluminum both transversal and longitudinal modes relax on ‘astronomical’ scale for superconductors, where the coherence length 100 nm gives the characteristic scale. Based on this one can make a hypothesis that at some conditions one can detect the coherent transport of nonequilibrium quasiparticles. It can be studied, for example, in Aaronov–Bohm type experiment.

We have also presented new data on nonlocal measurements of charge imbalance relaxation length. The results are consistent with [84], with $\Delta Q_\text{rms} \sim 4 \mu m$. Such large scales of charge and energy imbalance relaxation impose important limits for high-density integration of logical superconducting circuits and should be taken into account. In this paper we have also reviewed recent articles on nonequilibrium electron cooling in NIS refrigerators. We discussed the problems of quasiparticle poisoning and quasiparticle evacuation by so-called quasiparticle traps.

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