Physical conditions in thin laminar-convective accretion flows

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Abstract. The physical conditions of convection appearance in laminar accretion flows with microscopic transport coefficients are examined. Hot sparse ionised flow with periods below an hour found to be optically thin and have convective layer. Cold sparse molecular flow with period about a year found to be optically thin too and are fully convective. Ranges of temperature, density and period of optically thick laminar accretion flow are shown.

1. Introduction

Disk accretion is a very common phenomenon in the Universe: planets are born in accretion disks near young stars, accretion disks near black holes are sources of bright X-ray radiation. Observation behaviour of accretion disks in different sources are described in the terms of standard accretion disk model [1]. The standard model of disk accretion assumes turbulent viscosity, but the nature of the turbulence is still under discussion (see, e.g. a review [2]). From purely hydrodynamic point of view, Keplerian flows are stable against small perturbations according to the classical Rayleigh criterion, and various mechanisms giving rise to turbulence in Keplerian accretion discs have been discussed. For example, recently in an attempt to search for purely hydrodynamic mechanisms of turbulence in shear flows, we have revisited the problem of turbulence appearance in thin Keplerian disks from small perturbations in non-ideal fluids with microscopic transport coefficients (viscosity and heat conductivity [3, 4, 5]).

In paper [6] (hereafter MPS17) we found a solution of vertical structure of stationary shear accretion flows with microscopic transport coefficients — dynamic viscosity $\eta$ and heat conductivity. Three cases were considered: optically thin molecular flow, optically thin ionised flow and optically thick ionised flow. In optical thin cases we assumed that radiation cooling is much smaller than viscous heating. It was found that optically thin molecular flow is fully convective unstable, while optically thin ionised flow has convective layer. Optically thick ionised flow is found to be convectively stable. Convection instability can be a trigger of turbulence, so the structure of such disks was found.

In this paper we examine physical conditions corresponded to found disk structures. Optically thin and thick cases are different in vertical optical depth along vertical axis. Moreover, MPS17...
neglected radiation in optical thin case, it corresponds to the case when radiation cooling rate is much smaller than viscous heating rate. Below we show the parameter space corresponding to these conditions: temperature, density, and orbital period ranges.

2. Energy dissipation

We can neglect the effects of radiation cooling only if radiation energy loss rate is much smaller than viscous energy generation rate.

Viscosity energy heating $\epsilon_{\text{visc}}$ of thin accretion disk reads [1, 7]:

$$\epsilon_{\text{visc}} = \eta_{\text{visc}} \left( \frac{d\Omega}{dr} \right)^2,$$

(1)

where $r$ is the distance to central object, $\Omega$ is the angular velocity of the accretion flow. For the Keplerian flow $\Omega \sim r^{-3/2}$ this equation can be rewritten using orbital period $P = 2\pi/\Omega$:

$$\epsilon_{\text{visc}} = \frac{9\pi^2}{P^2} \eta_{\text{visc}}.$$

(2)

2.1. Fully ionised gas

In this section we assume that the considered gas is electron-proton plasma.

Viscosity $\eta_{\text{ion}}$ of fully ionised hydrogen reads [8]:

$$\eta_{\text{ion}} = 2.21 \cdot 10^{-15} \frac{T^{5/2}}{\ln \Lambda} \text{g cm}^{-1} \text{s}^{-1} \text{K}^{-5/2},$$

(3)

where $\ln \Lambda$ is the Coulomb logarithm,

$$\Lambda = \frac{2}{3e^3} \left( \frac{(k_B T)^3}{\pi n} \right)^{1/2},$$

(4)

e is the elementary charge, $k_B$ is the Boltzmann constant.

Radiation energy loss rate due free-free and free-bound processes in ionised hydrogen reads [8]

$$\epsilon_{\text{ff}} = 1.57 \cdot 10^{-27} n^2 T^{1/2} \text{erg s}^{-1} \text{cm}^3 \text{K}^{-1/2}.$$

(5)

Radiation energy loss can be neglected when $\epsilon_{\text{ff}}/\epsilon_{\text{visc}} \ll 1$,

$$\frac{\epsilon_{\text{ff}}}{\epsilon_{\text{visc}}} = 8.0 \cdot 10^{-15} P^2 n^2 T^{-2} \ln \Lambda = 0.16 \left( \frac{P}{1\text{ s}} \right)^2 \left( \frac{n}{10^9 \text{ cm}^{-3}} \right)^2 \left( \frac{T}{10^6 \text{ K}} \right)^{-2} \left( \frac{\ln \Lambda}{20} \right).$$

(6)

2.2. Molecular gas

Cold interstellar molecular gas basically consists of $\text{H}_2$ molecules mixed with He, CO, $\text{H}_2\text{O}$ and other atoms and molecules. We assume that viscosity of such a mixture is dominated by $\text{H}_2$.

Viscosity of $\text{H}_2$ molecular gas reads [9]

$$\eta_{\text{mol}} = 2.67 \cdot 10^{-21} \frac{\sqrt{\mu T}}{d^2 \omega^{1/2} (k_B T/\varepsilon)} \text{g cm}^{-1} \text{K}^{-1/2},$$

(7)

where $\mu$ is the gas molecular weight, $\omega^{1/2}$ is the dimensionless (12-6) Lennard-Jones potential, $\varepsilon$ is the depth of potential well, $d$ is the distance at which potential is zero. For $\text{H}_2$ molecule $\mu = 2$, $d = 2.968 \text{Å}$ and $\varepsilon/k_B = 33.3 \text{K}$.  

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Optically thin cold molecular gas is cooled in two channels: rotational lines of molecules and thermal dust cooling. In laminar accretion flow dust should settle and doesn’t mix with gas, so we neglect the second option. Cooling of optically thin sparse molecular gas for temperatures of $\sim 20 - 50$ K is dominated by CO rotational transitions. Approximation of the CO cooling function [10] in optically thin case is given by [11]:

$$\epsilon_{CO} = 2.16 \cdot 10^{-27} X_{CO} n^2 T^{3/2} \text{erg s}^{-1} \text{cm}^3 \text{K}^{-3/2},$$  \hspace{1cm} (8)

where $X_{CO} = n_{CO}/n$ is the CO abundance.

Using a power-law approximation for $T = 10 - 100$ K of Lennard-Jones potential of H$_2$ molecule gas $\omega_{H_2}^{(2,2)} \simeq 1.64 (k_B T/\varepsilon)^{-2/5}$, a ratio between radiation energy loss and viscous heating can be estimated:

$$\frac{\epsilon_{CO}}{\epsilon_{\text{visc}}} \simeq 1.14 \cdot 10^{-26} P^2 T^{3/5} n^2 = 4.5 \cdot 10^{-5} \left( \frac{P}{1 \text{yr}} \right)^2 \left( \frac{T}{10^3 \text{K}} \right)^{3/5} \left( \frac{n}{10^3 \text{cm}^{-3}} \right)^2.$$  \hspace{1cm} (9)

3. Optical depth

The estimation of the vertical optical depth $\tau_0$ of the accretion flow is derived from equation plane values of accreting matter properties: its density $\rho_c$, temperature $T_c$, and opacity $\kappa_c$.

$$\tau_0 \equiv \rho_c z_0 \kappa_c \simeq \frac{3}{2\pi} \sqrt{\frac{m_p k_B T_c}{\mu}} P n_c \kappa_c = 7.3 \cdot 10^{-21} P T^{1/2} n \kappa,$$  \hspace{1cm} (10)

where $m_p$ is the mass of proton, $z_0$ is the half-height of the flow. Here we used the estimation $z_0/r \simeq 3v_s/v_c$ from MPS17.

For both cases of ionised and molecular gas we used interpolation of tabulated opacity for solar abundance [12, 13, 14] obtained using MESA code [15]. However, opacity doesn’t change dramatically for the larger part of the studied range of density and temperature. For the case of ionised gas and $T \gtrsim 10^6$ K opacity is described by Thomson electron scattering $\kappa_T = \sigma_T/m_p \simeq 0.4 \text{cm}^2 \text{g}^{-1}$ that leads to

$$\tau_0 = 2.9 \cdot 10^{-9} \left( \frac{P}{1 \text{s}} \right) \left( \frac{T}{10^6 \text{K}} \right)^{1/2} \left( \frac{n}{10^9 \text{cm}^{-3}} \right) \left( \frac{\kappa}{0.4 \text{cm}^2 \text{g}^{-1}} \right).$$  \hspace{1cm} (11)

Opacity of molecular gas for considered parameter range is around the value $0.6 \text{cm}^2 \text{g}^{-1}$ that leads to

$$\tau_0 = 4.3 \cdot 10^{-12} \left( \frac{P}{1 \text{yr}} \right) \left( \frac{T}{10^3 \text{K}} \right)^{1/2} \left( \frac{n}{10^3 \text{cm}^{-3}} \right) \left( \frac{\kappa}{0.6 \text{cm}^2 \text{g}^{-1}} \right).$$  \hspace{1cm} (12)

4. Results

Figure 1 shows splits of parameter space by two inequalities: ratio of radiation cooling to viscous energy generation $\epsilon_{\text{rad}}/\epsilon_{\text{visc}} \lesssim 1$, and characteristic vertical optical depth $\tau_0 \lesssim 1$. For the considered parameter range a region of $\epsilon_{\text{rad}}/\epsilon_{\text{visc}} < 1$ is always inside $\tau_0 < 1$ (dark color). This region satisfies assumptions of MPS17 for the case of optically thin laminar accretion flow with radiation energy loss rate is smaller than viscosity heating.

The region $\epsilon_{\text{rad}}/\epsilon_{\text{visc}} > 1$ and $\tau_0 > 1$ (light color) does not satisfy any conditions considered by MPS17. An accretion flow in this region should cool rapidly by radiation and should be thermal unstable.
Figure 1. Comparison with unity of $\tau_0$ and $\epsilon_{\text{rad}}/\epsilon_{\text{visc}}$ for various values of orbital period $P$, temperature $T$ and density $\rho$. Left panels are for ionised gas and right panels are for molecular gas. Pink (light) shows the parameter space where $\tau_0 < 1$ and $\epsilon_{\text{rad}}/\epsilon_{\text{visc}} > 1$, violet (dark) shows $\tau_0 < 1$ and $\epsilon_{\text{rad}}/\epsilon_{\text{visc}} < 1$, and white shows $\tau_0 > 1$ and $\epsilon_{\text{rad}}/\epsilon_{\text{visc}} > 1$. 
4.1. Ionised gas
Left panels of figure 1 correspond to the case of high temperature sparse ionised accretion flow. We consider periods between 1 and $10^4$ seconds that correspond to the hot central parts of accretion disks near astrophysical black holes with masses from several to millions of solar masses. Condition $\epsilon_{ff}/\epsilon_{visc} = 1$ (the boundary between dark and light colored regions) can be found from equation (6). For the high temperature region the condition $\tau_0 = 1$ (straight line of the boundary between light and white colored regions) can be derived from equation (11) assuming opacity to be constant and equals Thomson value $0.4 \text{ cm}^2 \text{ g}^{-1}$. For the temperature $T \lesssim 10^6 \text{ K}$ absorption is higher than scattering and boundary between optically thick and optically thin regions is more curved. The optically thick region ($\tau_0 > 1$, white color) satisfies assumptions of MPS17 for the case of optically thick ionised laminar flow.

4.2. Molecular gas
Right panels of figure 1 correspond to the case of low temperature sparse molecular accretion flow. We consider periods between 0.1 to 10 years that correspond to cold parts of protoplanetary disks around young stars. The approximation for the condition $\epsilon_{ff}/\epsilon_{visc} = 1$ (the boundary between dark and light colored regions) can be found from equation (9). Note that optical depth of molecular accretion flow is very small even for relatively high density (see equation 12), so the boundary between optically thin and optically thick region is absent on these panels.

5. Conclusions
In this paper we have found space of physical conditions corresponded to laminar accretion flows with microscopic transport coefficients described in MPS17. We considered two cases: ionised and molecular gas flows. In the both cases we found that radiation can neglected only for low density cases because of small cooling rate and small optical depth (i.e. small radiation conductivity). Also, we found conditions corresponded to optically thick ionised flow, and a "transition" zone with small optical depth but high radiation rate.

Acknowledgements
Authors thank Valery Suleimanov for valuable discussion. The work has been supported by RBFR grant 18-32-00553.

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