Analysis of student errors in integral concepts based on the indicator of mathematical competency using orthon classification

Yani Ramdani¹, Nety Kurniati¹, Erwin Harahap¹, Euis Setiawati², Nia Kurniati¹, Henriady de Keizer³

¹ Universitas Islam Bandung, Indonesia
² Widyaiswara of Education and Learning Center of the Ministry of Religion, Indonesia.
³ Politeknik Pajajaran Bandung Indonesia, Indonesia

Email: yani_ramdani@ymail.com

Abstract. This paper reveals findings from a research that investigates the errors of students' understanding of integral concepts. Research design is quasi-experimental. Data collection is done through test instruments that are used for two purposes. The first objective is to find out the level of student error in solving problems. The second objective is to analyze and explain the results of student performance. Respondents of this study were two hundred students who took courses in Integral Calculus. Error level data were calculated using descriptive statistics and analyzed using Orthon theory. The findings show that the highest average level of student error in integral concepts based on learning strategies, early knowledge of mathematics, gender, and the educational background in the Scientific Debate class is arbitrary errors with mathematical competencies which are adaptive reasoning include: (1) estimating answers and process of solutions, and use patterns and relationships to analyze mathematical situations; (2) compile and test the conjecture; (3) follow the rules of inference and compile valid arguments, check the validity of arguments. In the conventional class, the level of highest error is Executive Errors with mathematical competencies are productive dispositions, namely the ability to always see mathematics positively, beneficially, and meaningfully.

1. Introduction

Errors in learning are a very important part of learning outcomes. For more than a decade, there has been an increasing interest in research on student errors and misunderstandings in mathematics [1-3]. Errors of students understanding in mathematical concepts have an impact on understanding other mathematical concepts.

There is a call for teachers to accept mistakes rather than avoid them [1]. Such thinking is based on the justification that errors in mathematics are equitable and systematic [4]. Errors are the result of students' mathematical thinking and are therefore make sense and reasonable for students [2]. Mistakes are seen as negative as an aberration, a result of some confusion in students, and as an unfavorable event that must be eliminated and avoided at any time [5]. Most teachers have linked errors with factors related to students such as lack of knowledge, lack of adequate preparation, student attitudes and students' psychological situation.[5,6].
The results of conception or misunderstanding previously used by students to interpret phenomena, events, and situations in constructing their knowledge in the classroom [7]. So mistakes are seen as make sense for students, as a result of reasoning that has been formed in students' thinking. The context of mathematical knowledge that already exists in students usually raises the belief that work is correct [2, 8]. The misperception is equally invisible with inconsistent explanations about the nature of errors in mathematics. Errors show students' reasoning abilities and become an important part of mathematics instructional [9, 2]. The results of the study are able to describe or explain the next step to interpret errors by finding out the thoughts of students who contribute to mistakes [2]. Some perceptions show an initial understanding of the steps in making mistakes, for example seeing mistakes as part of mathematics and student learning [2]. Therefore, the mathematics teacher must be able to express and explain some common mistakes from the learning outcomes. Booth's research in this field has highlighted some of these errors and misconceptions [10-13].

This study aims to analyze student errors in problem-solving of integral concept based on indicators of mathematical competence. Integral is an important concept in mathematics. Student mastery in integral concepts contributes to the development of engineering, science, and industry such as the use of oil droplets from the tank to determine the amount of leakage, the use of space shuttle, Endeavor to determine the height achieved and the use of knowledge about energy consumption to determine the energy used [14]. The understanding of mathematics is very important as a strong basis for further study in various disciplines, including engineering, business, and finance [15]. In addition, the skills and knowledge of mathematics have been considered important for studying at universities, especially in health sciences [16, 17]. The importance of integral concepts is not yet in line with learning outcomes. The students' understanding of integral concepts has not reached the complete limit in groups as indicated by an average value of 59.20 [18]. One effort to enhancement capabilities in the integral concept is to encourage students to build knowledge independently. The teachers must encourage students to undergo cognitive processes such as those of scientists, including asking questions, forming hypotheses, designing exploration, obtaining data, drawing conclusions, redesigning exploration, and finally, form and revise the theory [19]. But this condition can lead to errors in understanding students' mathematical concepts.

Errors in solving mathematical problems such as integral concepts are still mostly done by students. Errors of students in an understanding of integral concepts can be influenced by many factors such as teacher competence, instructional strategies, mathematical early abilities and the ability of students to communicate their knowledge both orally and writing. The teacher considers errors not solely because of students, but also because of other factors that arise from teaching and the nature of the subject [20]. The teacher considers the error useful for further investigation in mathematics instructional. Some teachers respond by ignoring mistakes while others make efforts to get involved with mistakes [2]. Errors are a normal part of learning as a result of prior knowledge that is not understood by students. Levels of error must be measured to avoid fatal errors and improve student concepts. The measurement of error level is classified using Orton’s theory, namely: (1) Structural errors; (2) Arbitrary errors; or (3) Executive errors [21].

2. Method

2.1 Participant

The population in this research is students of Department of Mathematics and Statistics in Indonesia, the same level as the Department of Mathematics and Statistics Universitas Islam Bandung (Unisba). The subjects of the research were students taking Integral Calculus courses (N = 200).
2.2 Data Analysis

Data on error rates were analyzed using proportion techniques then grouped using Orton's theory. The mistakes and errors of students have grouped into three categories, namely: (1) Structural errors: arise from several errors in seeing the relationships involved in the problem or on the graph some of the principles that are important for solving problems. (2) Arbitrary errors: errors do not match the rules or appear accidentally and errors in making calculations from the delimiter. (3) Executive errors: involves manipulating errors even though the principle involved is understood.

3. Result and Discussion

3.1. Mathematical Competency Indicators in Integral Concept based on Instructional Strategies

The measured mathematical competency indicators include: (1) Understanding the concept \(X_1\), namely concepts understanding, operations, and relations. (2) Procedure fluency \(X_2\), namely the ability to implement procedures in a manner of flexible, accurate, efficient and appropriate. (3) Strategic competencies \(X_3\), namely the ability to formulate, present, and solve mathematical problems. (4) Adaptive reasoning \(X_4\), namely the capacity to think logically, reflect, explain, proposing justification. (5) Productive disposition \(X_5\) namely the ability to always see mathematics positively, beneficially, and meaningfully.

To identify and describe comprehensively errors, mistakes, or shortcomings and difficulties experienced by students in terms of the process of problem-solving, an analysis of the students' answers is wrong, wrong means that it is not in accordance with the demands of the questions, and incomplete. For students who do not answer at all callings are made to explore the difficulties faced by these students. The classification of mistakes made by students is as follows: (1) Structural errors with indicators of mathematical competence are understanding concepts, procedures fluency, and strategic competence. (2) Arbitrary errors with indicators of mathematical competence are adaptive reasoning. (3) Executive errors with indicators of mathematical competence are productive dispositions. Table 1 presents the value of the average mathematical competencies for each indicator based on the instructional strategy.

| Indicator | Class          | N   | Mean  | Std. Deviation | Std. Error Mean | Level of Errors |
|-----------|----------------|-----|-------|----------------|-----------------|-----------------|
| \(X_1\)   | Scientific Debate | 103 | 75.96 | 18.20          | 1.79            | 24.04           |
|           | Conventional    | 97  | 70.72 | 16.49          | 1.67            | 29.28           |
| \(X_2\)   | Scientific Debate | 103 | 89.18 | 15.82          | 1.56            | 10.82           |
|           | Conventional    | 97  | 75.92 | 14.99          | 1.52            | 24.08           |
| \(X_3\)   | Scientific Debate | 103 | 71.41 | 15.39          | 1.52            | 28.59           |
|           | Conventional    | 97  | 61.59 | 12.17          | 1.24            | 38.41           |
| \(X_4\)   | Scientific Debate | 103 | 72.06 | 16.62          | 1.64            | 27.94           |
|           | Conventional    | 97  | 69.68 | 14.62          | 1.48            | 30.32           |
| \(X_5\)   | Scientific Debate | 103 | 76.55 | 12.18          | 1.20            | 23.45           |
|           | Conventional    | 97  | 68.94 | 11.83          | 1.20            | 31.06           |

3.2. Level of Student Errors in Integral Concept based on Instructional Strategies

Table 2 presents the value of the average level of error in mathematical competencies for each indicator based on the instructional strategy.

| Orthon Classification | Scientific Debate Strategy | Instructional of Conventional |
|----------------------|----------------------------|------------------------------|
| Structural Errors    | 21.15                      | 30.59                        |
| Arbitrary Errors     | 27.94                      | 30.32                        |
| Executive Errors     | 23.45                      | 31.06                        |
Based on Table 2, it appears that the highest error in the Scientific Debate strategy is the Arbitrary Errors classification with indicators of mathematical competence is adaptive reasoning of 27.9417%. In conventional learning, the highest error is in the classification of Structural errors of 30.58877% with indicators of mathematical competence are (1) Understanding of concepts namely concepts understanding, operations, and relations. (2) Procedure fluency, namely the ability to implement procedures in a manner of flexible, accurate, efficient and appropriate. (3) Strategic competencies, namely the ability to formulate, present, and solve mathematical problems.

### 3.3. Level of Student Errors in Integral Concept based on Instructional Strategies and PAM

Table 3 presents the average value of mathematical competencies for each indicator based on learning strategies and early mathematical knowledge (PAM).

| PAM    | Scientific Debate Strategy | Conventional Class |
|--------|-----------------------------|--------------------|
| High   | X_1: 90.95, X_2: 100.00, X_3: 83.00, X_4: 89.24, X_5: 89.83 | X_1: 82.4, X_2: 68.15, X_3: 77.17, X_4: 74.27 |
| Middle | X_1: 73.50, X_2: 88.45, X_3: 70.48, X_4: 75.76, X_5: 68.68 | X_1: 74.91, X_2: 60.96, X_3: 67.57, X_4: 67.62 |
| Low    | X_1: 68.36, X_2: 80.864, X_3: 62.86, X_4: 59.36, X_5: 66.01 | X_1: 71.2, X_2: 55.07, X_3: 68.5, X_4: 67.26 |

Table 4 presents the value of the average level of error of mathematical competencies for each indicator based on instructional strategies and early knowledge of mathematics (PAM).

| PAM    | Scientific Debate Strategy | Conventional Class |
|--------|-----------------------------|--------------------|
| High   | X_1: 9.05, X_2: 0.00, X_3: 17.00, X_4: 10.76, X_5: 10.17 | X_1: 17.60, X_2: 31.85, X_3: 22.84, X_4: 25.73 |
| Middle | X_1: 26.50, X_2: 11.55, X_3: 29.52, X_4: 29.3, X_5: 24.24 | X_1: 25.09, X_2: 39.04, X_3: 32.43, X_4: 32.38 |
| Low    | X_1: 31.64, X_2: 19.136, X_3: 37.14, X_4: 40.64, X_5: 33.99 | X_1: 28.80, X_2: 44.93, X_3: 31.5, X_4: 32.74 |

Table 5 presents the value of the average level of error of mathematical competencies for each indicator based on instructional strategies and early knowledge of mathematics of student using the Orthon classification.

| Orthon Classification | Level of Errors on the Scientific Debate Strategy | Level of Errors in Conventional Classes |
|-----------------------|-----------------------------------------------|----------------------------------------|
| High                  | 8.68, Structural Errors                       | 22.52, High                             |
| Low 1                 | 22.52, Arbitrary Errors                       | 29.3, Medium                            |
| Low 2                 | 40.64, Arbitrary Errors                       | 22.48, Low                              |
| Low 3                 | 29.3, Executive Errors                        | 31.82, High                             |
| Low 4                 | 40.64, Executive Errors                       | 22.48, Medium                           |
| Low 5                 | 29.3, Executive Errors                        | 31.82, Low                              |
| Low 6                 | 40.64, Executive Errors                       | 22.48, Low                              |
| Low 7                 | 29.3, Executive Errors                        | 31.82, Low                              |
| Low 8                 | 40.64, Executive Errors                       | 22.48, Low                              |
| Low 9                 | 29.3, Executive Errors                        | 31.82, Low                              |
| Low 10                | 40.64, Executive Errors                       | 22.48, Low                              |
| Low 11                | 29.3, Executive Errors                        | 31.82, Low                              |
| Low 12                | 40.64, Executive Errors                       | 22.48, Low                              |
| Low 13                | 29.3, Executive Errors                        | 31.82, Low                              |
| Low 14                | 40.64, Executive Errors                       | 22.48, Low                              |
| Low 15                | 29.3, Executive Errors                        | 31.82, Low                              |
| Low 16                | 40.64, Executive Errors                       | 22.48, Low                              |

Based on Table 5, it appears that the highest error in the Scientific Debate strategy is an adaptive reasoning indicator that is an error not according to the rules or arises by chance and error in the low group is Arbitrary Errors of 40.64% in adaptive reasoning indicators that take calculations from the boundary. In a conventional class, the highest error was in the low PAM group, namely structural errors of 36.65% including (1) Understanding of concepts namely concepts understanding, operations, and relationships. (2) Procedure fluency, namely the ability to implement procedures in a manner of flexible, accurate, efficient and appropriate. (3) Strategic competencies, namely the ability to formulate, present, and solve mathematical problems.
3.4. Level of Student Errors in Integral Concept based on Instructional Strategies and Gender

Table 6 presents the value of the average of mathematical competencies for each indicator based on instructional strategy and gender.

**Table 6 Average Value of Mathematical Competency Indicators based on Instructional Strategies and Gender**

| Gender | Scientific Debate Strategy | Conventional Class |
|--------|---------------------------|-------------------|
|        | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_1$ | $X_2$ | $X_3$ | $X_4$ |
| Male   | 69.00 | 85.90 | 70.10 | 74.55 | 69.44 | 68.65 | 63.81 | 63.77 |
| Female | 77.43 | 89.85 | 71.62 | 76.70 | 71.17 | 78.45 | 62.29 | 71.72 | 70.73 |

Table 7 presents the value of the average level of error in indicators of mathematical competency for each indicator based on instructional strategy and gender.

**Table 7 Average Value of the Student Errors in the Mathematical Competency Indicator based on Instructional Strategies and Gender**

| Gender | Scientific Debate Strategy | Conventional Class |
|--------|---------------------------|-------------------|
|        | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_1$ | $X_2$ | $X_3$ | $X_4$ |
| Male   | 31.00 | 14.10 | 25.90 | 24.81 | 30.56 | 31.35 | 40.42 | 36.23 |
| Female | 22.57 | 10.15 | 28.38 | 28.83 | 21.55 | 37.71 | 28.28 | 29.27 |

Table 8 presents the average level of error in mathematical competencies for each indicator based on instructional strategies and gender using the Orthon classification.

**Table 8 Average Value of Student Errors in the Integral Concept based on Instructional Strategies and Gender Using Orthon Classification**

| Orthon Classification | Level of Errors in the Scientific Debate Strategy | Level of Errors in Conventional Classes |
|-----------------------|-----------------------------------------------|----------------------------------------|
|                       | Male                           | Female                     | Male                           | Female                     |
| Structural Errors     | 25.00                         | 20.37                      | 34.11                         | 29.37                      |
| Arbitrary Errors      | 25.45                         | 28.81                      | 36.19                         | 28.28                      |
| Executive Errors      | 24.81                         | 23.30                      | 36.23                         | 29.27                      |

Based on Table 8 it appears that the highest error in the strategy of Scientific Debate is an adaptive reasoning indicator that is an error not according to the rules or arising by chance and the error in the female group is Arbitrary Errors of 28.81% in the adaptive reasoning indicator that takes calculations from the boundary. In a conventional class, the highest error was in the male group, namely Executive errors of 36.23% for indicators of productive disposition, namely the ability to always see mathematics positively, beneficially, and meaningfully.

3.5. Level of Student Errors based on Instructional Strategies and Educational Background of Student

Table 9 presents the average value of mathematical competencies for each indicator based on instructional strategies and student education background.

**Table 9 Average Value of Mathematical Competency Indicators based on Instructional Strategies and Educational Background of Student**

| Educational Background | Scientific Debate Strategy | Conventional Class |
|------------------------|---------------------------|-------------------|
|                        | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ |
| SMA                   | 77.05 | 90.93 | 72.1  | 73.86 | 77.93 | 70.61 | 75.56 | 62.11 | 68.39 | 68.86 |
| MA                    | 73.38 | 84.31 | 68.15 | 68.23 | 73.58 | 79.5  | 74.8  | 65.7  | 71.16 | 71.86 |
| SMK                   | 69.89 | 80.56 | 69.89 | 61.33 | 68.4  | 66.75 | 77.91 | 57.13 | 73.71 | 67.83 |
Table 10 presents the value of the average level of error of mathematical competencies for each indicator based on the instructional strategy and educational background of the students.

| Educational Background | X₁ | X₂ | X₃ | X₄ | X₅ | X₁ | X₂ | X₃ | X₄ | X₅ |
|------------------------|----|----|----|----|----|----|----|----|----|----|
| SHC                    | 22.95 | 9.07 | 27.90 | 26.14 | 22.07 | 29.39 | 24.44 | 37.89 | 31.61 | 31.14 |
| ISHC                   | 26.62 | 15.69 | 31.85 | 31.77 | 26.42 | 20.50 | 25.20 | 34.30 | 28.84 | 28.14 |
| VHC                    | 30.11 | 19.44 | 30.11 | 38.67 | 31.60 | 33.25 | 22.09 | 42.88 | 26.29 | 32.17 |

Table 11 presents the value of the average level of error of mathematical competencies for each indicator based on instructional strategies and educational backgrounds namely: Senior High School (SMA), Islamic Senior High School (MA), and Vocational High School (SMK) of students using the Orthon classification. It appears that the highest error in the Scientific Debate strategy is for students with a vocational background and for a conventional class is for students with a high school background. With the average error rate in the Arbitrary Errors classification of 38.67% and 31.61% respectively, errors are not according to the rules or appear accidentally with adaptive reasoning indicators.

| Orthon Classification | Level of Errors on the Scientific Debate Strategy | Level of Errors on Conventional Classes |
|-----------------------|-----------------------------------------------|----------------------------------------|
|                       | SMA   | SMK   | Average | SMA   | MA   | SMK   | Average |
| Structural Errors     | 19.98 | 24.72 | 26.56    | 23.75 | 30.57 | 26.67 | 32.74    | 29.99 |
| Arbitrary Errors      | 26.14 | 31.77 | 38.67    | 32.19 | 31.61 | 28.84 | 26.29    | 28.92 |
| Executive Errors      | 22.07 | 26.42 | 31.6     | 26.7  | 31.14 | 28.14 | 32.17    | 30.48 |

In general, the average value of highest students' errors in integral concepts based on instructional strategies, early knowledge of mathematics, gender, and the educational background in the Scientific Debate class are Arbitrary errors including: (1) estimating answers and process solutions, and using patterns and relationship to analyze mathematical situations; (2) compile and test the conjecture; (3) follow the rules of inference and compile valid arguments, check the validity of arguments. In the conventional class, the highest error is Executive Errors, namely the ability to always see mathematics positively, beneficiably, and meaningfully.

Competence indicator average with education background of SMA includes the concept understanding good; procedure fluency good; strategic competence enough; adaptive reasoning enough and productive disposition excellent. MA includes an understanding of concepts enough; procedure fluency enough; strategic competence enough; Adaptive reasoning enough and productive disposition good. SMK includes an understanding of concepts enough; fluency of procedure enough; strategic competence enough; Adaptive reasoning less and Productive Disposition good [22].

Other classifications of errors of students are structural errors including (1) expressing daily events in mathematical language or symbols; (2) explain ideas, situations, and mathematical relations in writing in the form of images, tables, diagrams, or graphs; (3) use mathematics in other fields of study or daily life; and (4) using connections between mathematical topics, and between mathematical topics and other topics.

The above conditions indicate that students are still weak in indicators of adaptive reasoning and productive dispositions. This is consistent with Brodie's study that systematic, persistent and widespread errors made by students occur in various contexts [2]. From a constructivist perspective, errors are explained as (1) Results of gaps in understanding that threaten the construction of student knowledge and the coherent structure of mathematics [23]. (2) The results of applying previously
acquired and correct knowledge to mathematical situations where such knowledge cannot be applied [6,24]. (3) The result of a misunderstanding is "a conceptual framework that is consistently based on previously acquired knowledge" [25] and makes sense for students because they make conceptual links with the knowledge they have acquired before [26]. Early mistakes and misunderstandings of students in some aspects can cause learning difficulties in their mathematics learning further if left untreated.

4. Conclusion

From the above error analysis, the weaknesses of most students are (1) expressing daily events in mathematical language or symbols; (2) explain ideas, situations, and mathematical relations in writing in the form of pictures, tables, diagrams, or graphs and (3) change a form of mathematical representation to other mathematical representations, this ability requires fairly good algebraic abilities. Another error is that students misrepresent ideas, situations, and mathematical relations in graphical form.

From the results of the analysis of indicators of mathematical competence, then grouped using classification of Orton, where mistake and errors of students are grouped into three categories, namely: Structural errors include: (1) expressing daily events in a language or mathematical symbol; (2) explain ideas, situations, and mathematical relations in writing in the form of images, tables, diagrams, or graphs; (3) use mathematics in other fields of study or daily life; and (4) use connections between mathematical topics, and between mathematical topics with other topics Arbitrary errors include: (1) estimating answers and process solutions, and using patterns and relationships to analyze mathematical situations, (2) compiling and testing conjectures, (3) following inference rules and constructing valid arguments, checking the validity of arguments Executive errors: include: (1) changing a form of mathematical representation to another form of mathematical representation and (2) looks for the connection of one procedure to another procedure in the equivalent representation.

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