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Quark-Hadron Phase Transition in DGP Brane Gravity with Bulk Scalar Field

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Abstract. A DGP brane-world framework is picked out to study quark-hadron phase transition problem. The model also includes a bulk scalar field in agreement with string theory prediction. The work is performed using two formalisms as: smooth crossover approach and first order approach, and the results are plotted for both branches of DGP model. General behavior of temperature is the same in these two approaches and it decrease by passing time and expanding Universe. Phase transition occurs at about micro-second after the big bang. The results show that transition time depends on brane tension value in which larger brane tension comes to earlier transition time.

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1 Introduction

Recently, brane-world cosmology has attracted a huge attention of scientists. The model, which is inspired from string/M theory, was introduced by Randall and Sundrum in 1999 [1]. Based on this picture, the Universe is a four-dimensional hypersurface (the brane) which is embedded in five-dimensional space-time (the bulk). All standard matter and their interaction are confined on the brane and only gravity could propagate along fifth dimension. This new picture of the Universe brings cosmologists some attractive opportunities to overcome their problems. Field equations and the evolution equations related to this model are derived in [2]. It is shown that the equations contain some corrected terms with respect to their corresponding equation in standard four-dimension cosmology. The main corrected term is quadratic of energy density which describes a completely different behavior for the Universe in high energy regime. On the other hand, in low energy regime, the quadratic term could be ignored against to the linear term of energy density and comes back to standard form. In 2000, another model of brane-world was proposed by Dvali, Gabadadze and Porrati which is known as DGP brane-world [3]. In this model, the brane is embedded in five-dimension Minkowski space-time with infinite extra dimension. The main difference of DGP model with RS model is the presence of Einstein-Hilbert term in brane action which is known as a quantum correction due to coupling of confined brane matter to bulk gravity [4, 5]. Recovery of standard cosmology is somewhat different with respect to RS model. The standard cosmology is recovered in small length scale and for large length scale we encounter modified equation with new behavior for the Universe. This length scale are compare with a crossover scale which is predicted by the model. Cosmological consequence of the model has been studied.
in [6] which display a self-accelerating solution for the Universe (refer to [5] for other studies in DGP model and [7] for a review). Besides that, it is well-known that the DGP model is divided to two branches related to the value of \( \epsilon \) parameter which appears in the Friedmann equation (\( \epsilon = -1 \) or \( \epsilon = +1 \)). In brief and for original model of DGP, these two branches can be explained as bellow:

- \( \epsilon = -1 \): In late time, under some condition, the Friedmann equation could describe a fully five-dimension regime and have a transition from four-dimension regime to five-dimension regime. This branch of the model is known as normal branch [4].

- \( \epsilon = +1 \): The late time behavior of this case is different. Friedmann equation never goes to five-dimension regime, and if we assume that the energy density decreases by passing time, the model describes an inflationary solution. Then, \( \epsilon = +1 \) branch is able to give a quintessence-like scenario. This branch of the DGP model is known as self-acceleration branch [4].

Between these two branches, the self-acceleration branch undergoes a ghost instability problem. It seems that this problem appears in quantum level.

There is another interesting models in standard cosmology, known as scalar field models, which leads to some attractive consequences. The problem of scalar field in brane-world cosmology received huge interest as well. According to string theory suggestion, it is possible to have a scalar field in the bulk which is free to propagate in extra dimension. Therefore, it seems more consistence to have a bulk scalar field instead of brane scalar field.

On the other hand, according to the standard model of cosmology, as the Universe expanded and cooled it passed through a series of symmetry-breaking phase transitions which might have generated topological defects [8]. This early Universe phase transition could have been of first, second, or higher order, and it has been studied in detail for over three decades [9]. We note that the possibility of no phase transition was considered in Ref. [10]. The phase transition using both first order formalism and crossover formalism has been considered in standard cosmology [11] with viscosity effect [12] that causes a kind of non-conservation relation. Furthermore, the first order and the crossover approaches to phase transitions were studied in the brane-world [8, 13], DGP brane-world [14], and in Brans-Dicke models of brane-world gravity [15] without any bulk-brane energy transfer.

In the present work, we are going to study the problem of quark-hadron phase transition in early time of the Universe evolution in the DGP brane-world model including bulk scalar field. Difference of Friedmann equation certainly leads to new results about the time of transition, with attention to this fact that presence of bulk scalar field in this model is more consistence with string theory prediction. The problem of phase transition will be considered in two formalisms. In the first formalism, we take smooth crossover approach and study the temperature evolution in high and low temperature regimes. In second case, first order phase transition formalism is taken, and temperature evolution is investigated before, during and after phase transition. Above topics will be studied for both branches of the DGP model. It might seem that this is unnecessary for self-acceleration branch because of its ghost instability problem. However, it should be mentioned that this problem appears at the quantum level, where the theory of gravity seems unclear in general as well. Therefore, it is thought that this is so soon to abandon the self-acceleration branch. Also, there are so many other works which
they have studied different feature of self-acceleration branch, which it means that this branch still has scientists interests. So we believe it is. First, phase transition will be considered for self-acceleration branch. It is trying to explain every step of work and driving equation. The main equations and processes of the work, are expressed generally which could be used for both branches and only different is in the value of $\epsilon$. In second stage, phase transition will be studied for normal branch. Since the general equations have already derived, it is preferred to ignored the detail and we just talk about the results.

This paper is organized as follows: In Sec. 2, we introduce the model and derive the main equations of model. We study quark-hadron phase transition in high and low temperature regimes in our model using smooth crossover approach; and we review the first-order phase transition and consider it in our model toward investigate of Temperature evolution before, during and after phase transition for $\epsilon = 1$ in Sec. 3, and for $\epsilon = -1$ in Sec. 4 and Sec. 5 summarizes our results and compares with the previous results of other works.

## 2 Field Equation of Brane

Introducing action of the model is a good and common point for beginning. In this model, action contains two main part as brane action and bulk action

$$S = S_{\text{bulk}} + S_{\text{brane}}.$$  \hfill (2.1)

The bulk action is defined as

$$S_{\text{bulk}} = \int_{\mathcal{M}} d^5x \sqrt{-g} \left[ \frac{1}{2\kappa_5^2} R + (5) L_B \right],$$ \hfill (2.2)

where $(5)L_m$ indicates the matter of bulk which in this work defined by $(5)L_B = -\nabla_A \phi \nabla^A \phi - V(\phi) - \Lambda$ and $(5)R$ is five-dimensional Ricci scalar related to five-dimensional metric $(5)g_{AB}$. $\kappa_5^2$ is related to five-dimensional Planck mass by $\kappa_5^2 = 8\pi G_5 = M_5^{-3}$, where $G_5$ is the five-dimensional Newtonian gravitational constant. The bulk is filled by two component as cosmological constant $\Lambda$ and scalar field $\phi$. On the other hand, we have brane action which generally is expressed as following

$$S_{\text{brane}} = \int_{\mathcal{M}} d^4x \sqrt{-g} \left[ \tilde{K} + L_{\text{brane}}(g_{\alpha\beta}, \psi) \right],$$ \hfill (2.3)

where $\tilde{K} = [K]/\sqrt{-g}\kappa_5^2$ and $[K]$ is exterior curvature on each side of brane and is known as Gibbons-Hawking term. $L_{\text{brane}}$ is general form of brane matter Lagrangian. $g_{\mu\nu}$ is induced metric on brane which is defined as $g_{\mu\nu} = \delta^A_{\mu} \delta^B_{\nu} (5)g_{AB}$. Taking variation of action with respect to five-dimensional metric leads one to the five-dimension Einstein field equation which is derived as

$$(5) G_{AB} = \left[ (5)T_{AB} + \tau_{AB} \delta(y) \right],$$ \hfill (2.4)

$(5)T_{AB}$ indicates the total bulk energy momentum tensor and are defined as

$$(5)T_{AB} \equiv -2\frac{\delta(5)L_B}{\delta(5)g^{AB}} + (5)g_{AB}(5) L_B,$$ \hfill (2.5)
which is sum of bulk cosmological constant and scalar field energy-momentum tensor,

\[(5)T_{AB} = (5)T_{AB}^{(\phi)} + (5)T_{AB}^{(\Lambda)}.\]  

(2.6)

Note that in above relations we set \(\kappa_5 = 1\), and we keep this contract for the rest of this work. Each one of bulk energy-momentum tensor is obtained as

\[T_{AB}^{(\phi)} = \nabla_A \phi \nabla_B \phi - g_{AB} \left( \frac{1}{2} g^{CD} \nabla_C \phi \nabla_D \phi + V(\phi) \right),\]  

(2.7)

\[T_{AB}^{(\Lambda)} = -\Lambda g_{AB}.\]  

(2.8)

In Eq. (2.4), \(\tau_{\mu\nu}\) on the right hand side of field equation defines the effective energy-momentum tensor of brane and is expressed as

\[\tau_{\mu\nu} \equiv -2 \frac{\delta L_{\text{brane}}}{\delta g_{\mu\nu}} + g_{\mu\nu} L_{\text{brane}}.\]  

(2.9)

Following the same process as [16], one can obtained the effective field equation of brane as

\[G_{\mu\nu} = \frac{2}{3} \left[ (5)T_{RS} g^R_{\mu} g^S_{\nu} + g_{\mu\nu} \left( (5)T_{RS} n^R n^S - \frac{1}{4} (5)T \right) \right] + \kappa_5^4 \pi_{\mu\nu} - E_{\mu\nu},\]  

(2.10)

where \(n^A\) is unit normal vector and \(\pi_{\mu\nu}\) is expressed as

\[\pi_{\mu\nu} = -\frac{1}{4} \tau_{\mu\alpha} \tau_{\nu}^\alpha + \frac{1}{12} \tau_{\mu\nu} + \frac{1}{8} g_{\mu\nu} \tau_{\alpha\beta} \tau^{\alpha\beta} - \frac{1}{24} g_{\mu\nu} \tau^2\]  

(2.11)

above relation contains quadratic term of energy-momentum tensor therefore it produced quadratic term of energy density which is the main difference of brane world scenario with standard four dimension equation. The other modified term of field equation Eq. (2.10), is \(E_{\mu\nu}\) which is picture of bulk Weyl tensor on brane; this term includes some information about bulk gravitation and is obtained as

\[E_{\mu\nu} = (5)C_{MRNS} n^M n^N g^R_{\mu} g^S_{\nu}.\]  

(2.12)

On the other hand, scalar field equation of motion is obtained by taking variation of action with respect to scalar field, which is read as following

\[(5)\Box \phi = V'(\phi),\]  

(2.13)

where \((5)\Box\) indicates five-dimensional D’Alambert, and prime denotes derivative with respect to scalar field.

2.1 DGP brane world scenario

The interaction between matter on brane and bulk gravity produce some quantum correction on brane and there is induced gravity on brane. Based on Dvali, Gabadadze and Porrati this quantum correction is appear as Ricci scalar in brane action[3]. Therefore, we have

\[L_{\text{brane}} = \frac{1}{2\mu^2} R - \lambda + L_{\text{m}}.\]  

(2.14)
where $\mu^2 = M_4^{-2}$ and $M_4$ is four-dimensional Planck mass. Note that it is a generalized version of original DGP model which is obtained by taking $\lambda = \Lambda = 0$. In this work the effective brane energy-momentum tensor is derived

$$\tau^\mu_\nu = -\lambda \delta^\mu_\nu + T^\mu_\nu - \mu^{-2} G^\mu_\nu. \quad (2.15)$$

Using Eq. (2.15), one can find out the effective field equation of DGP brane

$$\left(1 + \frac{\lambda}{6\mu^2}\right) G^\mu_\nu = -\Lambda_4 g^\mu_\nu + \mu^2 \tilde{T}^\mu_\nu + \frac{\lambda}{6} T^\mu_\nu + \frac{1}{\mu^2} G^\mu_\nu - E^\mu_\nu, \quad (2.16)$$

where

$$\Lambda_4 = \frac{1}{2} \left[ \Lambda + \frac{1}{6} \lambda^2 \right] \quad (2.17)$$

$$\pi^{(T)}_\mu^\nu = \frac{1}{4} T^\alpha_\mu T^\nu_\alpha + \frac{1}{12} g_\mu^\nu T^\alpha_\beta T^\alpha_\beta - \frac{1}{24} g_\mu^\nu T^2, \quad (2.18)$$

$$\pi^{(G)}_\mu^\nu = \frac{1}{4} G_\mu^\rho G^\rho_\nu + \frac{1}{12} G^\mu_\alpha G^\nu_\alpha + \frac{5}{8} g_\mu^\nu g_\alpha^\beta g_\alpha^\gamma g_\beta^\gamma - \frac{1}{24} g_\mu^\nu G^2, \quad (2.19)$$

$$G_\mu^\nu = \frac{1}{4} \left( G_\mu^\rho \tau^\rho_\nu + \tau^\rho_\mu G^\rho_\nu \right) - \frac{1}{12} \left( \tau G_\mu^\nu + G \tau_\mu^\nu \right) - \frac{q_\mu^\nu}{2} \left( G_\alpha^\beta \tau^\alpha_\beta - \frac{G}{3} \right). \quad (2.20)$$

$\Lambda_4$ is the effective cosmological constant of brane which same as RS-II model can be taken as zero. $\tilde{T}^\mu_\nu$ is the energy-momentum tensor related to the bulk scalar field which appears in brane field equation and display the effect of bulk on brane evolution. This tensor is derived as

$$\tilde{T}^\mu_\nu = \frac{l_{DGP}}{3} \left( 4 \phi_\mu \phi_\nu + \left[ \frac{3}{2} (\phi^2) - \frac{5}{2} g_\mu^\rho g_\nu^\rho \phi_\alpha \phi_\beta - 3V(\phi) \right] g_\mu^\nu \right), \quad (2.21)$$

where $l_{DGP} = \kappa_5^2/2\mu^2$ is the crossover length scale between the 4D and 5D regimes in the DGP brane model. The conservation equation is given by following

$$D^\nu T^\mu_\nu = 0 \implies \dot{\rho} + 3H(\rho + p) = 0, \quad (2.22)$$

where $D$ indicates covariant derivative with respect to $g_\mu^\nu$ metric.

### 2.2 Evolution Equation

Up to now, the field equation of the model was derived and we are on a place to obtained evolution equation describing behavior of the Universe. The (0-0)-component of field equation is

$$\left(1 + \frac{\lambda}{6\mu^2}\right) G_{00} = -\Lambda_4 g_{00} + \mu^2 \tilde{T}_{00} + \frac{\lambda}{6} T_{00} + \frac{1}{\mu^2} G_{00} - E_{00}, \quad (2.23)$$

Following [1] and for simplicity we take four-dimensional cosmological constant to zero. To get evolution equation, we should emphasize the metric. In this work we adopt a spatially flat FLRW metric which is described by

$$ds^2 = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j + dy^2. \quad (2.24)$$
Then, the (0-0)-component of Eqs. (2.18), (2.19), (2.20) and (2.21) lead one to

\[
\tilde{T}_{00} = \rho_B = l_{DGP}(\dot{\phi}^2/2 + V(\phi)) \quad (2.25)
\]

\[
\pi^{(T)}_{00} = \frac{\rho^2}{12} \quad (2.26)
\]

\[
\pi^{(G)}_{00} = \frac{1}{12}G_{00}G^{00} \quad (2.27)
\]

\[
G_{00} = -\frac{1}{6}\rho G_{00} \quad (2.28)
\]

where \( G_{00} = 3H^2 \). To derive Eq. (2.25), we impose a boundary condition as \( \partial_y \phi \big|_{y=0} = 0 \) [17]. Inserting above results in Eq. (2.23), leads to

\[
-\frac{3}{4\mu^4}H^4 + 3 \left(1 + \frac{1}{6\mu^2(\rho + \lambda)}\right)H^2 = \mu^2\rho_B + \frac{\lambda}{6}\rho + \frac{1}{12}\rho^2 - \frac{\zeta}{a^4}. \quad (2.29)
\]

Finally one can easily derive the modified Friedmann equation as

\[
H^2 = 2\mu^4 \left[\chi + \epsilon \sqrt{\chi^2 - \frac{1}{3\mu^4}} \left[\mu^2\rho_B + \frac{\lambda}{6}\rho \left(1 + \frac{\rho}{2\lambda}\right)\right]\right] \quad (2.30)
\]

where \( \chi = (1 + (\rho + \lambda)/6\mu^2) \) and \( \epsilon = \pm 1 \). It should be noted that the contribution of dark radiation has been ignored finding above relation.

To consider phase transition in this model we need to know the function of scalar field. The exact form of scalar field is derived by solving Eq. (2.13); although finding the solution encounter with difficulty. However, one can apply some appropriate assumption to derive scalar field. Based on inflationary scenario, scalar field potential dominates the universe and a quasi-de Sitter expansion occurs while scalar field move very slowly to the minimum of its potential. After inflation, scalar field begin to oscillate in minimum of potential and other standard particle could be produced. Therefore, one can conclude that, the potential of scalar field could be ignored after inflation and reheating of the Universe evolution. According to this argument, the five-dimensional scalar field equation of motion on brane could be read as

\[
\ddot{\phi} + 3H\dot{\phi} - \partial_{yy}\phi + V'(\phi) = 0. \quad (2.31)
\]

To solve above differential equation, one need to assume an ansatz for \( \partial_{yy}\phi \). According to [18], we take it as \( \partial_{yy}\phi = -\dot{\phi} \). Therefore, the scalar field could be derived as \( \dot{\phi}^2/2 = \phi_0 a^{-3} \), where \( \phi_0 \) is constant of integration. Then, the bulk energy density on brane can be expressed by

\[
\rho_B = l_{DGP}\phi_0 a^{-3}. \quad (2.32)
\]

3 Phase Transition for \( \epsilon = +1 \) Branch

As beginning, we are going to study the \( \epsilon = +1 \) branch of the model for quark-hadron phase transition.
3.1 Lattice QCD Phase Transition

One of definite prediction of QCD is the existence of a phase transition from a quark-gluon plasma phase to hadron gas phase. The transition could be first, second or higher order; or it only can be a crossover with a rapidly change in some of observable which strongly depend on the values of quark masses. Todays, there is two kind of phase transition which is popular among scientist and seen in papers: 1. First order phase transition, 2. Crossover transition. In this section we are going to study phase transition utilizing smooth crossover approach.

One of fundamental concepts in particle physics is QCD phase transition and incredibly becomes relevant to any study which concern early universe. Recently lattice QCD calculation for two quark flavors suggest that QCD makes a smooth crossover.

Lattice QCD is a new approach which allows one to systematically study the non-perturbative regime of the QCD equation of state. Using supercomputers, the QCD equation of state was computed on the lattice in [19] with two light quarks and a heavier strange quark. In [19], the data for energy density \( \rho(T) \), pressure \( p(T) \), trace anomaly \( \rho - 3p \) and entropy \( s \) have been formed and in this work we used this data in Sec.3 of the main work. Recent information on lattice QCD in high temperature could be found in [21]. In high temperature, as it was expected, we there is radiation like behavior (equations (38) and (39) in the main paper).

The trace anomaly can be accurately calculated in the high temperature region, while in the low temperature region it is affected by possibly large discretization effects. Therefore to construct realistic equation of state we could use the lattice data for the trace anomaly in the high temperature region, \( T \geq 250 \text{MeV} \), and use HRG model in the low temperature region, \( T \leq 180 \text{MeV} \). (for more detail refer to [22]). The HRG result for trace anomaly can be performed by the simple form

\[
\frac{I(T)}{T^4} \equiv \frac{\rho - 3p}{T^4} = a_1 T + a_2 T^3 + a_3 T^4 + a_4 T^{10},
\]  

(3.1)

In lattice QCD the calculation of the energy density, pressure and entropy density usually proceed through the calculation of the trace anomaly. Finally the energy density and pressure could be obtained as

\[
\rho(T) = 3a_6 T^4 + 4a_1 T^5 + 2a_2 T^7 + \frac{7a_3}{4} T^8 + \frac{13a_4}{10} T^{14},
\]  

(3.2)

\[
p(T) = a_0 T^4 + a_1 T^5 + \frac{a_2}{3} T^7 + \frac{a_3}{4} T^8 + \frac{a_4}{10} T^{14},
\]  

(3.3)

the calculation of equation of state by recently obtained results of lattice QCD simulation has been briefly explained above.

In this section we are going to study phase transition using crossover approach for two main regime as high temperature regime and low temperature regime.

3.1.1 High Temperature Regime

As it was mentioned above, in high temperature regime one can use lattice QCD data for trace anomaly in order finding realistic equation of state. It is found out that there is a radiation like behavior and we have a simple form of equation of state as

\[
\rho(T) \simeq \alpha T^4,
\]  

(3.4)

\[
p(T) \simeq \sigma T^4,
\]  

(3.5)
where $\alpha = 14.9702 \pm 009997$ and $\sigma = 4.99115 \pm 004474$. Inserting Eqs. (3.4) and (3.5) in conservation equation (2.22), the Hubble parameter and scale factor could be obtained as

$$H = -\frac{4\alpha}{3(\alpha + \sigma)} \frac{T'}{T}, \quad a(T) = e^{\frac{4\alpha}{3(\alpha + \sigma)}}. \quad (3.6)$$

Substituting above relation for Hubble parameter in Friedmann equation (2.30), leads to the differential equation of temperature which is expressed by

$$\dot{T} = -\frac{3(\alpha + \sigma)T}{4\alpha} \times \left\{ 2\mu^4 \left[ \chi + \epsilon \sqrt{\chi^2 - \frac{1}{3\mu^4} \left[ \mu^2 \rho_B + \frac{\lambda}{6} \rho \left( 1 + \frac{\rho}{2\lambda} \right) \right]} \right] \right\}^{\frac{1}{2}}, \quad (3.7)$$

where displays the behavior of temperature with respect to the cosmic time. The differential equation (3.7) is solved numerically and the results have been depicted in Fig.(1). It could be found out that the phase transition occur at about $2 - 2.5$ micro-second after big bang. Transition time depends on the brane tension value in which larger transition time is obtained for larger values of brane tension.

![Figure 1](attachment:image.png)

**Figure 1.** $T$ versus $t$ according to high temperature region (for $250 < T < 750$ MeV) of the smooth crossover procedure in the DGP brane gravity including bulk scalar field. The constant parameters are taken as: $\lambda = 10^{10}$MeV$^4$, $l_{DGP} = 10^{11}$MeV$^{-1}$, $\kappa_5 = \sqrt{2\mu^2 l_{DGP}} = 1$MeV$^{-3/2}$, $\phi_0 = 2 \times 10^6$MeV$^2$ and $\epsilon = 1$.

### 3.1.2 Low Temperature Regime

In the regime the situation is somewhat different with high temperature regime. The trace anomaly is affected by large discretization effects and is not a suitable approach obtaining equation of state. However the HRG is still a good model achieving a realistic equation of state as mentioned. In the HRG scenario, the confinement phase of QCD is treated as a non-interacting gas of fermions and bosons [22]. The idea of the HRG model is to implicitly account for the strong interaction in the confinement phase by looking at the hadronic resonances only, since these are basically the only relevant degrees of freedom in that phase. The results of HRG is parametrized for trace anomaly so that

$$\frac{I(T)}{T^4} = \frac{\rho - 3p}{T^4} = a_1 T + a_2 T^3 + a_3 T^4 + a_4 T^{10}, \quad (3.8)$$

where $a_1 = 4.654$ GeV$^{-1}$, $a_2 = -879$ GeV$^{-3}$, $a_3 = 8081$ GeV$^{-4}$, $a_4 = -7039000$ GeV$^{-10}$. Through the calculation of the trace anomaly $I(T) = \rho(T) - 3p(T)$ in lattice QCD, the
parameters energy density, pressure, entropy density is estimated using the usual thermodynamics identities. The difference of pressure at two temperature is expressed as the integral of trace anomaly which is described by following relation

$$\frac{p(T)}{T^4} - \frac{p(T_{\text{low}})}{T_{\text{low}}^4} = \int_{T_{\text{low}}}^{T} \frac{dT'}{T'^{\alpha}} I(T'),$$  \hspace{1cm} (3.9)

where \(p(T_{\text{low}})\) could be ignored because of exponential suppression for sufficiently small value of lower integration limit. On the other hand, the energy density could be obtained as \(\rho(T) = I(T) + 3p(T)\). Therefore, the energy density and pressure in this regime is expressed by

$$\rho(T) = 3\eta T^4 + 4a_1 T^5 + 2a_2 T^7 + \frac{7a_3}{4} T^8 + \frac{13a_4}{10} T^{14},$$ \hspace{1cm} (3.10)

$$p(T) = \eta T^4 + a_1 T^5 + \frac{a_2}{3} T^7 + \frac{a_3}{4} T^8 + \frac{a_4}{10} T^{14},$$ \hspace{1cm} (3.11)

where \(a_0 = -0.112\). Now, let’s consider the behavior of temperature. In this case we are actually considering the times before phase transition where the Universe is in confinement phase and is treated as a non-interacting gas of fermions and bosons. Using conservation equation, the Hubble parameter and scale factor can be derived respectively as

$$H = \frac{-12\eta T^3 + 20a_1 T^4 + A(T)}{3\left[4\eta T^3 + 5a_1 T^3 + B(T)\right]} \dot{T},$$ \hspace{1cm} (3.12)

$$a(T) = \frac{c}{T[60\eta + 75a_1 T + 35a_2 T^3 + 30a_3 T^4 + 21T^40]^{1/3}},$$ \hspace{1cm} (3.13)

where \(A(T) = 14a_2 T^6 + 14a_3 T^7 + \frac{32}{5} T^{13}\) and \(B(T) = \frac{7}{5} a_2 T^7 + 2a_3 T^8 + \frac{7}{5} T^{14}\). Inserting Eq. (3.12) in the Friedmann equation (2.30), the differential equation of temperature could be derived as

$$\dot{T} = -\frac{3\left[4\eta T^4 + 5a_1 T^3 + B(T)\right]}{12\eta T^3 + 20a_1 T^4 + A(T)}$$

$$\times \left\{2\mu^4 \left[\chi \pm \sqrt{\chi^2 - \frac{1}{3\mu^4} \left[\mu^2 \rho_B + \frac{\lambda}{6} \rho \left(1 + \frac{\rho}{2\lambda}\right)\right]}\right]\right\}^{1/2},$$ \hspace{1cm} (3.14)

which expresses the behavior of temperature as a function of cosmic time. The numerical results of the differential equation (3.14) is depicted in Fig.(2) which express that phase transition occur at about 30 – 35 micro-second after big bang. This transition time occur after transition time related to high temperature regime which display a consistence result. Transition time in low temperature regime strongly depends on brane tension value so that it increases by enhancement of brane tension value.

### 3.2 First Order Phase Transition

The HRG model in the temperature interval \(180 \text{MeV} < T < 250 \text{MeV}\) is no longer valid, then the study of quark-hadron phase transition in this temperature interval should be performed using another formalism. Quark-hadron phase transition in QCD is characterized by the singular behavior of partition function, and might be first or second order phase transition.
Figure 2. $T$ versus $t$ according to low temperature region (for $50 < T < 180$ MeV) of the smooth crossover procedure in the DGP brane gravity including bulk scalar field. The constant parameters are taken as: $\lambda = 10^{10}$MeV$^4$, $l_{DGP} = 10^{11}$MeV$^{-1}$, $\kappa_5 = \sqrt{2} \mu^2 l_{DGP}^{-1} = 1$MeV$^{-3/2}$, $\phi_0 = 2 \times 10^6$MeV$^2$ and $\epsilon = 1$.

In this section, we are going to pick out first order phase transition formalism and study the behavior of temperature in DGP brane gravity framework with bulk scalar field in this temperature interval. Based on [23], the equation of state for matter in quark-gluon phase is given by

$$
\rho_q = 3a_q T^4 + U(T), \quad p_q = a_q T^4 - U(T),
$$

(3.15)

where the subscript $q$ indicates the quark-gluon phase of matter, and the constant $a_q$ is given as $a_q = 61.75(\pi^2/90)$. $U(T)$ denotes potential energy density and is expressed by [23]

$$
U(T) = B + \gamma_T T^2 - \alpha_T T^4,
$$

(3.16)

where $B$ is the bag pressure constant, $\alpha_T = 7\pi^2/20$, and $\gamma_T = m_s^2/4$. $m_s$ indicates the mass of the strange quark, which is in the $60 - 200$ MeV range. This form of $U$ comes from a model in which the quark fields interact with a chiral field formed by the $\pi$ meson field together with a scalar field. Results obtained from low energy hadron spectroscopy, heavy ion collisions, and from phenomenological fits of light hadron properties give a value of $B^{1/4}$ between 100 and 200 MeV [8].

After quark-gluon phase, the matter comes to hadron phase, then the cosmological fluid can be taken as an ideal gas of massless pions and nucleons described by the Maxwell-Boltzmann distribution function with energy density $\rho_h$ and pressure $p_h$. The equation of state for matter in this area of the Universe evolution is expressed by following simple relations

$$
p_h = \frac{1}{3} \rho_h = a_\pi T^4,
$$

(3.17)

where $a_\pi = 17.25(\pi^2/90)$. The critical temperature $T_c$ is defined by the condition $p_q(T_c) = p_h(T_c)$ [24]. Taking $m_s = B^{1/4} = 200$ MeV, the critical temperature is

$$
T_c = \left[ \frac{\gamma_T + \sqrt{\gamma_T^2 + 4B(a_q + \alpha_T - a_\pi)}}{2(a_q + \alpha_T - a_\pi)} \right]^\frac{1}{2} \approx 125 \text{ MeV}.
$$

(3.18)

Since the phase transition is first order, all physical quantities, such as the energy density, pressure, and entropy, exhibit discontinuities across the critical curve.
3.2.1 Behavior of Temperature

At first stage of phase transition, we study the behavior of temperature when the matter is in quark-gluon phase. Doing so, we need the scale factor which could be derived using conservation equation and specific form of equation of state. Using Eqs. (3.15), (3.16) and conservation equation (2.22) one can obtained the Hubble parameter as a function of temperature and its first time derivative

\[
H = -\left[\frac{3\alpha_T - \alpha T}{3a_q} + \frac{\gamma T}{6a_q T^2}\right] \frac{\dot{T}}{T},
\]

where we have taken the general form of potential \( U(T) \). Then, integrating above relation gives scale factor as

\[
a(T) = c T^{\frac{\alpha_T - 3\alpha_q}{\alpha q}} \exp\left(\frac{\gamma T}{12a_q T^2}\right).
\]

Therefore, one can easily derive the expression of temperature evolution by inserting Eq. (3.19) in Friedmann equation (2.30)

\[
\ddot{T} = -\frac{6a_q T^3}{[2(3\alpha_T - \alpha T)T^2 + \gamma T]} \times \left\{2\mu^4 \left[\chi \pm \sqrt{\chi^2 - \frac{1}{3\mu^4} \left[\mu^2 \rho B + \frac{\lambda}{6} \rho \left(1 + \frac{\rho}{2\lambda}\right)\right]}\right]\right\}^{\frac{1}{2}}
\]

Above expression govern on the evolution of temperature with respect to cosmic time. The differential equation is solved numerically and the results are plotted in Fig(3). It could be realized that the quark-gluon phase is finished at about 2.5 – 4.5 micro-second after the big bang which is in agreement with the results of previous section. earlier transition time occur for higher values of brane tension value which express dependence of transition time on brane tension.

*Figure 3.* \( T \) versus \( t \) in the quark-gluon phase with general form of potential energy density has been plotted for different values brane tension \( \lambda \) as: \( 1 \times 10^9 \) (solid line), \( 5 \times 10^9 \) (dashed line), \( 10 \times 10^9 \) (dotted line), \( 20 \times 10^9 \) (dotted-dashed line). The other constant parameters are taken as: \( \lambda = 10^{10}\text{MeV}^4 \), \( l_{DGP} = 10^{11}\text{MeV}^{-1} \), \( \kappa_5 = \sqrt{2\mu^2 l_{DGP}} = 1\text{MeV}^{-3/2} \), \( \phi_0 = 2 \times 10^5\text{MeV}^2 \), \( \epsilon = 1 \) and \( B^{1/4} = 200 \text{ MeV} \).

**Bag Model**

Elastic bag model, which allows quark to move around freely, is one of popular model dealing with quark confinement. In this case the potential energy density is a constant, namely
\[ U(T) = B, \] and equation of state of quark matter describes by \[ p_q = (\rho_q - 4B)/3. \] Going forward by this assumption that bag model provide the quark equation of state, the Hubble parameter is derived as

\[ H = -\frac{\dot{T}}{T} \implies a(T) = \frac{c}{T}. \quad (3.22) \]

Inserting this results for Hubble parameter in Friedmann equation (2.30) leads to

\[ \dot{T} = -T \left\{ 2\mu^4 \left[ \frac{\chi + \epsilon}{\sqrt{\lambda^2 - \frac{1}{3}\mu^4}} \left[ \frac{\mu^2\rho_B}{6}\rho + \frac{\lambda}{6}\rho \left( 1 + \frac{\rho}{2\lambda} \right) \right] \right] \right\}^{\frac{1}{2}}, \quad (3.23) \]

which describe time evolution of temperature as a function of cosmic time. The numerical results of this differential equation has been plotted in Fig. (4). In a comparison, phase transition in bag model occur later than general case and quark-gluon phase is finished at about \( 3 - 5.5 \) micro-second after the big bang. Dependence on brane tension is clear from Fig. (4) which show earlier transition for higher values of brane tension.

\[ \begin{array}{c}
\text{Figure 4.} \\
T \text{ versus } t \text{ in the quark-gluon phase with constant potential energy density has been plotted for different values brane tension } \lambda \text{ as: } 1 \times 10^9 \text{ (solid line), } 5 \times 10^9 \text{ (dashed line), } 10 \times 10^9 \text{ (dotted line), } 20 \times 10^9 \text{ (dotted-dashed line). The other constant parameters are taken as: } \lambda = 10^{10}\text{MeV}^4, \ l_{DGP} = 10^{11}\text{MeV}^{-1}, \ \kappa_5 = \sqrt{2\mu^2l_{DGP}} = 1\text{MeV}^{-3/2}, \ \phi_0 = 2 \times 10^5\text{MeV}^2, \ \epsilon = 1 \text{ and } U(T) = B = 200\text{ MeV}^4. \\
\end{array} \]

### 3.2.2 Evolution of hadron Volume Fraction

During phase transition, the matter energy density decreases from quark energy density, \( \rho_Q \), to hadron energy density, \( \rho_H \). In this time, pressure, enthalpy and entropy remain conserved and temperature at \( T = T_c = 125\text{MeV} \) is conserved as well, and for other constant parameters we have \( \rho_Q = 5 \times 10^9 \text{ MeV}^4, \ \rho_H \approx 1.38 \times 10^9 \text{ MeV}^4 \), and constant pressure \( p_c \approx 4.6 \times 10^8 \text{ MeV}^4 \). Following [8, 14, 15], the parameter \( \rho(t) \) could be replaced by volume fraction of matter in hadron phase, \( h(t) \), defining by

\[ \rho = \rho_H h(t) + (1 - h(t))\rho_Q \quad (3.24) \]

where \( \rho_H \) and \( \rho_Q \) respectively are energy density of hadron and quark. The Hubble parameter could be derived from conservation equation

\[ H = -\frac{1}{3} \frac{(\rho_H - \rho_Q)\dot{h}}{\rho_H + p_c + (\rho_H - \rho_Q)\dot{h}} = -\frac{1}{3} \frac{r\dot{h}}{1 + rh} \quad (3.25) \]
where \( r = (\rho_H - \rho_Q)/(\rho_Q + p_c) \). Integrating Eq. (3.25) gives the scale factor as a function of the hadron volume fraction, \( h(t) \) above relation one can easily obtain scale factor as

\[
a(t) = a(t_c) \left[ 1 + rh(t) \right]^{-1/3}
\]

(3.26)

where we assumed \( h(t_c) = 0 \). So, plugging Eq. (3.25) into Friedmann equation (2.30), the time evolution equation of the matter fraction in the hadronic phase is

\[
\dot{h} = -3 \frac{(1 + rh)}{r} \times \left\{ 2\mu^2 \left[ \chi + \epsilon \sqrt{\chi^2 - \frac{1}{3\mu^4} \left[ \mu^2 \rho_B + \frac{\lambda}{6} \rho \left( 1 + \frac{\rho}{2\lambda} \right) \right]} \right] \right\}^{1/2}
\]

(3.27)

Numerically evaluated of evolution equation of the matter fraction has been plotted in Fig. (5).

**Figure 5.** Variation of hadron fraction parameter \( h \) versus \( t \) has been depicted for different values brane tension \( \lambda \) as: \( 1 \times 10^9 \) (solid line), \( 5 \times 10^9 \) (dashed line), \( 10 \times 10^9 \) (dotted line), \( 20 \times 10^9 \) (dotted-dashed line). The other constant parameters are taken as: \( \lambda = 10^{10} \text{MeV}^4 \), \( l_{DGP} = 10^{11} \text{MeV}^{-1} \), \( \kappa_5 = \sqrt{2\mu^2 l_{DGP}} = 1 \text{MeV}^{-3/2} \), \( \phi_0 = 2 \times 10^5 \text{MeV}^2 \) and \( \epsilon = 1 \).

**Figure 6.** The figure displays behavior of scale factor as a function of hadron volume fraction. We have set \( \lambda = 10^{10} \text{MeV}^4 \).

The volume fraction increases by passing time and it reach to its maximum value, \( h = 1 \) at about \( 3-9 \) micro-second after the big bang which is in agreement with the results of previous subsection. Dependence of hadron volume fraction on brane tension is clear from Fig.(F03), which displays a faster evolution for larger value of brane tension. After that, the Universe comes to a pure hadronic phase which is discussed in next case.
Evolution of scale factor of the Universe during the QHPT as a function of the hadron volume fraction is expressed in Fig. (6). It is well known that when the QHPT occurs the density of quark gluon plasma decreases but the hadron volume fraction and the scale factor of the Universe increase.

### 3.2.3 Evolution of temperature in pure Hadronic Era

At final stage of phase transition, the Universe comes to a pure hadronic phase, and cosmological fluid is described by following equation of state

\[ \rho_h = 3p_h = 3a_\pi T^4 \]  \hspace{1cm} (3.28)

Using Eq. (2.22), the Hubble parameter and scale factor are

\[ H = \frac{-\dot{T}}{T} \quad \implies \quad a(T) = c\frac{T}{T} \]  \hspace{1cm} (3.29)

and from Friedmann equation (2.30), differential equation of temperature described by

\[ \dot{T} = -T \left\{ 2\mu^4 \left[ \chi + \epsilon \sqrt{\chi^2 - \frac{1}{3\mu^4} \left[ \mu^2 \rho_B + \frac{\lambda}{6} \rho \left( 1 + \frac{\rho}{2\lambda} \right) \right]} \right] \right\}^{\frac{1}{2}} \]  \hspace{1cm} (3.30)

The differential equation could be solved numerically and the results is depicted in Fig. (7).

#### Figure 7. Variation of temperature $T$ versus cosmic time $t$ has been depicted for different values of brane tension $\lambda$ as: $1 \times 10^9$ (solid line), $5 \times 10^9$ (dashed line), $10 \times 10^9$ (dotted line), $20 \times 10^9$ (dotted-dashed line). The other constant parameters are taken as: $\lambda = 10^{10}\text{MeV}^4$, $l_{DGP} = 10^{11}\text{MeV}^{-1}$, $\kappa_5 = \sqrt{2\mu^2 l_{DGP}} = 1\text{MeV}^{-3/2}$, $\phi_0 = 2 \times 10^5\text{MeV}^2$ and $\epsilon = 1$.

The figure indicates that the hadron phase occur at about $10 - 35$ micro-second after the big bang dependence on the value of brane tension. Larger value of brane tension comes to faster temperature rate which is in agreement with the Universe expansion. The results are in good consistence with the previous results.

### 4 Phase Transition for $\epsilon = -1$ Branch

In previous section the problem of phase transition was investigated by using two formalism, for $\epsilon = +1$ branch of DGP model. The case was studied in detail and the steps was explained...
clearly. In this case, we are going to considered the same problem for $\epsilon = -1$ branch of DGP model. The process is same as before, therefore we ignore the detail of work and go straight to results. Quark-hadron phase transition is studied by utilizing two formalisms Lattice QCD and First order phase transition respectively, and the results are mentioned as following sub-sections.

4.1 Lattice QCD Phase Transition

Eq.(3.7) is solved numerically for both high and low temperature regime, for $\epsilon = -1$ branch of the model. The results are plotted in Fig.8. Figure 8(a) is related to high temperature regime, which shows that the temperature decreases by passing time. The phase transition occurs at about $3 - 3.5$ micro-second after the big bang. The numerical results of Eq.(3.14), related to low temperature regime, are displayed in Fig.8(b), which expresses that phase transition occurs at about $85 - 90$ micro-second after the big bang. In comparison with the $\epsilon = +1$ branch, it is realized that, phase transition in both high and low temperature regime for $\epsilon = -1$ branch occurs at later times.

![Graph](image1.png)

(a) High temperature regime

![Graph](image2.png)

(b) Low temperature regime

**Figure 8.** Variation of temperature $T$ versus cosmic time $t$ for high and low temperature regimes of the smooth crossover procedure in the DGP brane gravity including bulk scalar field. The constant parameters are taken as: $\lambda = 10^{10}$MeV$^4$, $l_{DGP} = 10^{11}$MeV$^{-1}$, $\kappa_5 = \sqrt{2}\mu^2l_{DGP} = 1$MeV$^{-3/2}$, $\phi_0 = 2 \times 10^5$MeV$^2$ and $\epsilon = -1$.

4.2 First Order Phase Transition

Quark-hadron phase transition was consider in previous section by using lattice QCD formalism. we are going to consider the problem of phase transition for $\epsilon = -1$ branch of the model by using first-order formalism. In this formalism, phase transition is studied in three step as: before, during and after phase transition. In following, the results of three steps will be discussed.

As first step, we consider behavior of temperature before phase transition. The numerical results of Eqs.(3.21) and (3.23) for $\epsilon = -1$ have been depicted in Fig.9. It is realized that temperature is decreasing by passing time. For general case of potential, the quark-gluon phase is finished at about 5 micro-second after big bang, Fig.9(a), and it finished at about 6 micro-second after big bang for bag model Fig.9(b). Therefore, one could realized that,
according to bag model, phase of quark-gluon is finished later.

Figure 9. Behavior of temperature $T$ versus cosmic time $t$ for general and bag model of first order procedure in the DGP brane gravity including bulk scalar field. The constant parameters are taken as: $\lambda = 10^{10}\text{MeV}^4$, $l_{DGP} = 10^{11}\text{MeV}^{-1}$, $\kappa_5 = \sqrt{2\mu^2 l_{DGP}} = 1\text{MeV}^{-3/2}$, $\phi_0 = 2 \times 10^5\text{MeV}^2$ and $\epsilon = -1$.

When quark-gluon is finished, the Universe comes to a phase which the energy density is a mixture of quark and hadron. Temperature and some other parameters stay constant in this era, and variation of energy densities could be described by volume fraction parameter $h$. The corresponding differential equation is explained by Eq. (3.27), and the numerical results for $\epsilon = -1$ are plotted in Fig. 10. Volume fraction parameter increases by passing time, and gets to its maximum value at about 12 micro-second after the big bang.

Figure 10. Variation of hadron fraction parameter $h$ versus cosmic time $t$ by utilizing first order formalism in the DGP brane gravity including bulk scalar field. The constant parameters are taken as: $\lambda = 10^{10}\text{MeV}^4$, $l_{DGP} = 10^{11}\text{MeV}^{-1}$, $\kappa_5 = \sqrt{2\mu^2 l_{DGP}} = 1\text{MeV}^{-3/2}$, $\phi_0 = 2 \times 10^5\text{MeV}^2$ and $\epsilon = -1$.

After the volume fraction reach the maximum value, $h = 1$, the Universe enters a pure hadronic phase. Temperature decreases again, and its behavior is described by differential equation (3.30). The differential equation is solved numerically, and the results are plotted in Fig. 11. It shows that, hadron phase occurs at about 60 micro-second after big bang.
All of obtained results about phase transition in three steps are in good agreement with each other.

Figure 11. Pure hadronic temperature behavior versus cosmic time in the DGP brane gravity including bulk scalar field by using first order formalism. The constant parameters are taken as: \( \lambda = 10^{10}\text{MeV}^4 \), \( l_{DGP} = 10^{11}\text{MeV}^{-1} \), \( \kappa_5 = \sqrt{2} \mu^2 l_{DGP} = 1\text{MeV}^{-3/2} \), \( \phi_0 = 2 \times 10^7\text{MeV}^2 \) and \( \epsilon = -1 \).

5 Discussion and Conclusion

The main purpose of this paper was investigation of quark-hadron phase transition problem in DGP brane-world framework including bulk scalar field, for both \( \epsilon = +1 \) and \( \epsilon = -1 \) branches. Evolution of physical quantities relevant to physical description of the early times such as energy density, scale factor, and temperature, were considered during this study. After deriving basic evolution equations, work has been done in two main sections, and the problem of phase transition was investigated by utilizing two popular approaches known as: 1. smooth crossover formalism and 2. first order phase transition formalism. At first, we went to the \( \epsilon = +1 \) branch and smooth crossover approach has been selected for studying phase transition. This case divided to two parts related to high and low temperature regimes. In high temperature regime, lattice QCD data for estimation trace anomaly was used to construct a realistic equation of state. As it was expected, cosmological fluid behaves like radiation in high temperature regime and follow a simple form of equation of state. Quark-gluon phase finished at about \( 2\)–\( 2.5 \) micro-second after the big bang. On the other hand, the trace anomaly is affected by large discretization effect in low temperature regime, therefore it is not a good model to construct an appropriate equation of state. However, Hadronic Resonance Gas (HRG) is a well-known to build a suitable equation of state in low temperature regime. The temperature differential equation has been solved numerically and the results expressed a transition time at about \( 30 \)–\( 35 \) micro-second after the big bang. General behavior of temperature is same for both regimes which decreases by passing time and Universe expansion. Phase transition depends on brane tension value, specially in low temperature regime, so that by enhancement of brane tension phase transition occurs earlier. For \( \epsilon = -1 \) the same processes have been repeated. The obtained numerical results were plotted and it was realized that, phase transition in high and low temperature regime respectively occurs at about \( 3 \)–\( 3.5 \) and \( 85 \)–\( 90 \) micro-second after big bang. Therefore, there is a later transition time than \( \epsilon = +1 \) branch.

The results of this case could be compare with [15, 25]. In [15], the authors investigated the problem in RS brane-world model including a Brans-Dicke scalar field. They found out a
transition time at about few microsecond after the big bang for both regimes. In [25], the authors studied the phase transition in RS brane-world including a bulk chameleon like scalar field. This model provided a non-conservation equation of state and their results expressed a phase transition at about nano-second after the big band. However in our case, phase transition occurs at about micro-second after the big bang and displays later transition time in comparison to [15] for both high and low temperature regimes.

In next section, first order approach was used to consider phase transition. The temperature evolution was investigated in three steps: 1. before phase transition (quark-gluon phase); 2. during phase transition; 3. after phase transition (Hadron phase). Differential equations for temperature were solved numerically and the results depicted for both branches. The Universe effective temperature decreased by passing time. Quark-gluon phase took place at about $2.5 - 4.5$ micro-second ($5$ micro-second) after the big bang for $\epsilon = +1$ ($\epsilon = -1$) branch. Phase transition for $\epsilon = +1$ ($\epsilon = -1$) branch, took place at about $3 - 9$ microsecond (12 micro-second) after the big bang. Finally, the Universe describing by $\epsilon = +1$ ($\epsilon = -1$) branch came to a hadronic phase at about $10 - 35$ micro-second (60 micro-second) after the big bang. The acquired results in this case are in agreement with the results of previous section, and generally a consistence results were obtained during this study. Again, transition time in $\epsilon = -1$ branch occurs later than $\epsilon = +1$ branch.

In this case, our results could be compare with [8]. In [8] the authors consider the problem of phase transition using a RS brane-world framework. They consider the problem for different values of brane tension, and their results show a phase transition at about micro-second after the big bang. In our case, phase transition occurs at about micro-second however later than [8]. The transition time in our case was obtained for different values of brane tension and in contrast with [8] we derived later transition time for larger value of brane tension.

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