Technicolor in hot and cold phases

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Abstract. We review recent developments on models of dynamical electroweak symmetry breaking. We first consider determination of the conformal window, i.e. the phase diagram, of cold gauge theory and how this can be applied to classify phenomenologically viable models of walking technicolor type. In the second part of the talk we consider finite temperature phase diagram of a generic walking type theory.

1. Introduction

Recently models of dynamical electroweak symmetry breaking [1, 2, 3, 4], i.e. technicolor, have received much attention. A general property of phenomenologically viable technicolor theories is that they be of walking type. This assumes that there are two widely different energy scales, $\Lambda_T$ and $\Lambda_{ET} \sim 10^3 \Lambda_T$, between which the coupling constant of the theory evolves very slowly; the theory is almost conformal towards the infrared. Below $\Lambda_T$ and above $\Lambda_{ET}$ the coupling runs similarly to asymptotically free theories (like QCD). In (Extended) Technicolor theories $\Lambda_T = \Lambda_{TC} \approx 246$ GeV and $\Lambda_{ET} = \Lambda_{ETC}$. The need for walking behavior comes from the requirement that the successes of the Standard Model should not be spoiled: the contributions from new physics to the electroweak precision parameters should be small and the contributions to flavor changing neutral current interactions should be suppressed.

The breakthrough with respect to earlier models has been the realization that one can achieve near conformal dynamics with a very small number of flavors using matter in higher dimensional representations [5, 6, 7, 8]. These results were obtained by considering the phase diagram of cold gauge theory as a function of number of colors and flavors and fermion representations [5], which will be the first topic of this talk; after that we will discuss finite temperature phase diagram of a generic walking technicolor theory using gauge/gravity duality.

2. Phase diagrams at $T = 0$

Let us first review how zero temperature phase diagrams of gauge theories can be constructed and how they help to identify plausible candidates for walking technicolor; we consider only non-supersymmetric theories. Generally, the evolution of a non-abelian gauge coupling is governed by two complementary effects: By the anti-screening of gauge bosons and by the screening of matter fields. If the former dominates the theory is in an asymptotically free QCD-like phase with confinement and chiral symmetry breaking in the vacuum, while if the latter dominates
Consider the two-loop beta function, \( \beta(g) = -\frac{\beta_0}{16\pi^2}g^3 - \frac{\beta_1}{(16\pi^2)^2}g^5 \). Asymptotic freedom is lost when \( \beta_0 \) becomes negative and this signals the phase transition into non-abelian Coulomb phase. The loss of asymptotic freedom defines the upper boundary of the conformal window. On the other hand, if \( g^* = -\frac{16\pi^2}{\beta_1} \) is positive, i.e. \( \beta_1 < 0 \) while \( \beta_0 > 0 \), the theory has a fixed point, \( \beta(g^*) = 0 \) and the theory is in the infrared conformal phase, i.e. inside the conformal window. Finally, if \( \beta_1 > 0 \), then the theory is asymptotically free and in a QCD-like phase with confinement and chiral symmetry breaking. However, in addition to the existence of a fixed point in the two-loop beta function one needs to take into account that at sufficiently strong coupling, chiral symmetry breaking is triggered and the theory cannot flow into the fixed point. Approximate solutions to Schwinger-Dyson equation for the fermion two-point function [9] suggest that the critical coupling for chiral symmetry breaking is \( g_c^2 = \frac{4\pi^2}{3C_2(R)} \), where \( R \) denotes the fermion representation under investigation. With \( g_c^2 \) and \( g^* \) defined as above, the condition \( g_c^2 = g^* \) determines an estimate for the lower boundary of the conformal window. For walking one wants to be slightly below the lower boundary of the conformal window so that \( g_c^2 \lesssim g^* \).

The resulting phase diagram for different fermion representations, fundamental and two-index (anti)symmetric ones, is shown in Fig. 1 as a function of numbers of colors and flavors. The shaded regions show the conformal window. For adjoint representation this would be a horizontal band coinciding with the two-index symmetric one at \( N_c = 2 \), and we chose not to plot it explicitly to maintain clarity of the figure. These four representations exhaust the interesting cases: yet higher representations lead to loss of asymptotic freedom already for very low values of \( N_f \) [10], and in particular do not admit asymptotically free \( N_c \to \infty \) limit. We stress the fact that the lower boundary of the conformal window shown in Fig. 1 is a semiquantitative estimate. Various other methods than the one we have used here can be applied [11, 12, 13, 14], and the results are quantitatively similar. Recently there has been also a lot of developments towards determining the conformal window for \( N_c = 2 \) or \( N_c = 3 \) and various fermion representations using lattice simulations [15, 16, 17, 18, 19, 20, 21, 22, 23].

**Figure 1.** A phase diagram of a nonsupersymmetric SU(\( N_c \)) gauge theory as a function of number of colors, \( N_c \) and flavors \( N_f \). The shaded bands are the conformal windows corresponding to different fermion representations. The symbols F and 2(A)S stand for fundamental and 2-index (anti)symmetric representations.
Since phenomenological constraints, most notably the precision $S$-parameter [24], require walking technicolor with as few flavors as possible, Fig. 1 immediately implies two candidate theories: $N_f = 2$ in the two index symmetric representation of either SU(2) or SU(3). The former was proposed as a minimal walking technicolor model [5], and its phenomenological consequences have been investigated in detail both for colliders [25, 26, 27] and for cosmology [28, 29, 30]. A particularly important feature which was established in [6], was that the composite Higgs particle can be parametrically light in comparison to other hadronic states. This is due to the fact that the Higgs scalar is the order parameter of the quantum phase transition to the conformal phase and hence its mass is expected to vanish as $M^2 \sim (N_{c}^2 - N_f^2) \Lambda^2$, where $\Lambda$ is the intrinsic scale of the strong dynamics, as the scaling field $N_f$ approaches the critical value $N_{c}^2$. This result was recently reconsidered in [31].

3. Hot technimatter

Let us then turn to another aspect: thermodynamics of a generic walking theory and gauge/gravity duality which has been proposed to describe various aspects of SU($N_c$) gauge theories. Most of the studies have concentrated on QCD thermodynamics [32, 33, 34], but here we will apply them to walking technicolor.

The model [32, 33] starts from a metric ansatz

$$ds^2 = b^2(z) \left[ -f(z)dt^2 + dx^2 + \frac{dz^2}{f(z)} \right]$$

(1)

plus a scalar field $\phi(z) = \log \lambda(z)$. The three functions $b(z), f(z)$ in the metric and the scalar field $\phi(z)$ are determined from the Einstein equations following from the gravity action (in standard notation)

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[ R - \frac{4}{3} (\partial_{\mu} \phi)^2 + V(\phi) \right].$$

(2)

For the details of these equations and the numerical method of solution, see [33, 35]. An advantage of this gauge/gravity duality approach is that the information on the boundary theory we will need is a knowledge of the beta function of its gauge coupling. We will consider an ansatz for a walking type beta function dependent on few parameters; in principle the parameter values can be obtained if the dynamics of the underlying theory is known.

The beta function ansatz we shall use is

$$\beta(\lambda) = -c \lambda^2 (1 - \lambda)^2 + e, \quad \lambda = N_c g^2,$$

(3)

which is tuned to asymptotic freedom in the UV ($\lambda \to 0$) and to walking near $\lambda = 1$ if $e > 0$ is small. The case $e = 0$ corresponds to a theory with an infrared fixed point (IRFP). A beta function, of course, is scheme dependent while the consequences we derive, an equation of state and associated phase structure, are entirely physical. One can thus say that our model defines the regularisation scheme leading to the coupling constant and its beta function we start from. For the solution of the Einstein equations the potential $V$ must be known, and to begin with, we do not know $V$ but only the beta function. However, we can construct a potential which reproduces the desired beta function, (3), on the boundary as $T \to 0$, i.e. $f \to 1$, and implies confinement at strong coupling. As discussed in [36], a condition for confinement is that the equation $\beta(\lambda) + \frac{3}{2} \lambda = 0$ have a solution; for details of the construction of the potential, see [35]

To illustrate the resulting thermodynamics, we consider $c = 3a/2 = 10, e = 0.1$ One has two first order transitions for large enough $c$. Decreasing $T$ from very large values one encounters a first order transition at $T = T_{ET}$ with the structure shown in Fig. 2. Below $T_{ET}$ there follows
Figure 2. Energy density and $3p$ scaled by $T^4$ plotted vs $T/T_{ET}$ for the beta function (3) with $e = 0.1$, $c = 3a/2 = 10$. The normalisation is such that at $T \gg T_{ET}$ both $\epsilon/T^4$ and $3p/T^4$ approach $\pi^2/15$. There is a first order transition at $T = T_{ET}$, metastable branches are dashed and the unstable branch is dotted. There is a second 1st order phase transition at $T = T_T \approx 0.0011T_{ET}$, the details are shown in the inset. In between, for $T_T < T < T_{ET}$ there is a quasiconformal phase with nearly constant $p/T^4$. The thick line below $T_T$ corresponds to the non-black hole low $T$ phase with $p = 0$.

Figure 3. Left: The input beta function (parametrisation (3), dotdashed line) and the output beta function, computed from the numerical solution at $T = 0.73T_{ET}$ (dashed black line) and at the very low $T$ value $T = T_T$ (continuous line), for parameter values shown in the figure. The inset shows how the output beta function at $T = T_T$ crosses the (dotted) confinement line [36], $-\frac{3}{2}\lambda$, at the position of the quasi conformal $\rightarrow$ confined transition. Right: Evolution of the coupling corresponding to the low $T$ output beta function.

a wide quasiconformal phase in which $p/T^4$ is almost constant. For the IRFP case, $e = 0$ this would extend down to $T = 0$; now at about $T \approx 0.01T_{ET}$ pressure starts decreasing and crosses $p = 0$ at some $T = T_T = 0.0011T_{ET}$. Below this temperature the quasi conformal phase becomes metastable (see inset in Fig. 2) and the vacuum phase with $p = 0$ is the stable one. This is a first order transition from a quasiconformal phase to a confining phase, the energy density $\epsilon$ dropping suddenly to zero.

Retaining $c$ and $a$ fixed to their above values, we next study how the phase diagram depends on the parameter $e$ which controls the departure from the limit of exactly infrared conformal theory. See Fig. 4. At the origin, $T = e = 0$, the line of first order transitions, $T_T(e)$, terminates at a second order quantum transition, in which the vacuum theory becomes infrared conformal.
Figure 4. The \((T, e)\) phase diagram for \(c = 3a/2 = 10\). The upper curve shows \(T_{ET}(e)\), the lower \(T_T(e)\), both normalised by \(T_T(0.1)\). The inset shows for what values of \(c\) for various \(e\) the upper ET transition is of first order, for \(c\) below the line the transition is a cross-over. The line to the right of the triple point is the deconfinement transition line.

At finite values of \(e < \sim 0.5\) there are, as a function of temperature, three phases corresponding to confined, quasi-conformal and deconfined matter. When \(e\) increases, the upper transition temperature \(T_{ET}(e)\) decreases slowly, while the lower transition temperature \(T_T(e)\) increases rapidly. At some critical \(e_c\) these lines merge and the quasi-conformal phase disappears; for \(e > e_c\) the phase structure will consist of a single transition line separating deconfined and confined phases. In the case depicted in the figure all transition lines correspond to a first order transition and hence the intersection is a triple point.

The inset in Fig. 4 shows for what values of \(c\) the transition changes from a crossover to a 1st order one, for various \(e\).

4. Conclusions
We considered phase diagrams related to technicolor both at zero and finite temperature. At zero temperature the role of the phase diagrams of gauge theories is to help identify plausible models for walking technicolor, i.e. theories which are naturally near the conformal window in the \((N_c, N_f)\)-plane with modest number of new matter fields. The results obtained using approximative analytic methods provide important input which can be further tested using lattice simulations.

Once such theories, like minimal walking technicolor, are identified their properties can be studied further. Here we considered in particular finite temperature phase diagrams using a method based on gauge/gravity duality treating the strong dynamics in isolation. A natural next step would be to embed the obtained phase structure into the full electroweak theory and study the implications for early universe and its evolution history.

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