Research on seismic absorption performance of adjacent structures with additional tuned mass dampers

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Abstract. In order to investigate the seismic performance of adjacent structures with additional tuned mass dampers, this article takes two adjacent single degree of freedom systems as an example, and takes the transfer rate of displacement response of adjacent structures as performance index, analyses the relationship between damping parameters and structural parameters of adjacent structures with additional tuned mass dampers. The numerical relationship between the structural parameters of adjacent structures with tuned mass dampers, the parameters of tuned mass dampers, and Specific Performance of the adjacent structures in Earthquakes has been found. The optimum structural parameters are obtained by theoretical deduction. Taking displacement response curve as an index, the reliability of the optimum structural parameters is verified.

1. Introduction
In recent years, the world's total population has gradually increased, the proportion of urban population has gradually expanded, and the per capita possession area has gradually decreased. As a result, urban buildings have become more and more dense, and many adjacent structures have been formed due to the small space of buildings. When an earthquake occurs, these adjacent structures may collide with each other, leading to structural damage and collapse. In the Loma Prieta earthquake that occurred in 1989, more than 200 of the 500 buildings surveyed collided [1]. In the 2011 Christchurch Earthquake, the structural damage caused by the collision was investigated, according to the survey results, more than 6% of the structures were severely damaged by collisions [2]. The collision between adjacent buildings caused by earthquakes has resulted in huge loss of life and property. Therefore, it is very meaningful to study the ways to prevent these buildings from colliding with each other and to improve the seismic performance of these buildings. Klein [3], an American, first proposed the idea of connecting adjacent structures to improve the seismic capacity of the structure. Later, he and his colleagues studied the control reaction of the connected structure connected by the energy dissipating component and the semi-active control device under the action of earthquake. SA Anagnostopoulos [4] studied the collision of two adjacent single-degree-of-freedom structures under earthquake action, and proposed the installation of viscoelastic dampers at the collision. Westermo [5] proposed the position in the same plane of adjacent structures. With articulated connecting rods, a number of subsequent studies have been conducted to reduce structural dynamic response by joining adjacent structures. Sun [6] simplified the adjacent structure to a two-degree-of-freedom system with viscous dampers connected by selecting the frequency ratio and mass ratio of adjacent structures as parameters, and the relationship between the mean square value of the structural displacement response and the connected
damping parameters. Chen and Pei [7] made a more indepth research on the basis of their predecessors, and discussed the energy dissipation and damping of the adjacent structures of two adjacent multi-layer steel frame structures connected with viscous dampers and metal dampers. In recent years, domestic and foreign scholars have analyzed the selected research models by adding numerical simulation and experimental research methods by adding damping devices between adjacent structures, and analyzing the rationality of damping control methods from different levels. However, there are few researches have been done on the addition of tuned mass dampers between adjacent structures. The purpose of this paper is to find out the numerical relationship between the structural parameters of adjacent structures with tuned mass dampers, the parameters of tuned mass dampers, and Specific Performance of the adjacent structures in Earthquakes. And to find out the structural parameters of adjacent structures most suitable for installing tuned mass dampers.

2. Numerical model

The Mechanical model of adjacent structures of additional tuned mass dampers is shown in figure 1.

![Mechanical model](image)

**Figure 1. Mechanical model.**

The vibration differential equation corresponding to figure 1 is:

\[ m_a \ddot{x}_a + k_a x_a + c_d (\dddot{x}_a - \dddot{x}_d) + k_d (x_a - x_d) = -m_a \ddot{y} \quad (1) \]

\[ m_b \ddot{x}_b + k_b x_b + c_d (\dddot{x}_b - \dddot{x}_d) + k_d (x_b - x_d) = -m_b \ddot{y} \quad (2) \]

\[ m_d \ddot{x}_d + c_d (\dddot{x}_d - \dddot{x}_a) + k_d (x_d - x_a) + c_d (\dddot{x}_d - \dddot{x}_b) + k_d (x_d - x_b) = -m_d \ddot{y} \quad (3) \]

In the formula, \( x_a, x_b, x_d \) are the displacements of A, B and D relative to the ground, and Y is the displacement of the ground. Suppose the input ground motion is a simple harmonic, let \( y = Y \cdot e^{i\omega t} \), the displacement response of structure A and B under earthquake action is \( x_a = X_a \cdot e^{i\omega t}, x_b = X_b \cdot e^{i\omega t}, X_d = X_d \cdot e^{i\omega t} \). \( Y, X_a, X_b, X_d \) represent the corresponding amplitudes of each vibration. Substitute \( Y, X_a, X_b, X_d \) into expressions (1), (2) and (3), by combining the same items, it is concluded that:

\[ \frac{X_a}{Y} (-m_a \omega^2 + k_a + ic_d \omega + k_d) + \frac{X_d}{Y} (-ic_d \omega - k_d) = m_a \omega^2 \]

\[ \frac{X_b}{Y} (-m_b \omega^2 + k_b + ic_d \omega + k_d) + \frac{X_d}{Y} (-ic_d \omega - k_d) = m_b \omega^2 \]

\[ \frac{X_a}{Y} (-ic_d \omega - k_d) + \frac{X_b}{Y} (-i\omega c_d - k_d) + \frac{X_d}{Y} (-m_d \omega^2 + 2ic_d \omega + 2k_d) = m_d \omega^2 \]
By substituting $\omega_a = (k_a/m_a)^{1/2}$, $\omega_b = (k_b/m_b)^{1/2}$, $\omega_d = (k_d/m_d)^{1/2}$ into the above formulas, we can get the result that:

$$\left| \frac{X_a}{Y} \right| = \frac{GC^2 + BDE + HBC - C^2E}{ABD - C^2B - AC^2}$$  \hspace{1cm} (4)

$$\left| \frac{X_b}{Y} \right| = \frac{EC^2 + AGD + HAC - C^2G}{ABD - C^2B - C^2A}$$  \hspace{1cm} (5)

$$\left| \frac{X_d}{Y} \right| = \frac{ABH + BCE + ACG}{ABD - BC^2 - AC^2}$$  \hspace{1cm} (6)

In the formula: $A = -m_a\omega_a^2 + \omega_a^2m_a + ic_d\omega + \omega_d^2m_d$, $B=-m_b\omega_b^2 + \omega_b^2m_b + ic_d\omega + \omega_d^2m_d$, $C=ic_d\omega + \omega_d^2m_d$, $D=-m_d\omega_d^2 + 2ic_d\omega + 2\omega_d^2m_d$, $E=m_a\omega_a^2$, $G=m_b\omega_b^2$, $H=m_d\omega_d^2$.

Verify the displacement transfer formula:

It is assumed that the characteristic parameters of the structure are: $k_a = 60$, $m_a = 2$, $k_b = 40$, $m_b = 1$, $m_d = 0.2(m_a + m_b)/2=0.3$ \cite{8}.

The natural vibration period and the natural vibration frequency corresponding to the structures A and B are:

$$T_a = \frac{2\pi}{\omega_a} = 1.14s, \quad \omega_a = \sqrt{k_a/m_a} = \frac{5.48rad}{s},$$

$$1 \quad T_b = \frac{2\pi}{\omega_b} = 0.99s, \quad \omega_b = \sqrt{k_b/m_b} = 6.32rad/s.$$

When the damper parameter $k_d = 0$, the displacement transfer function curves corresponding to the structure A, B are shown in Fig. 2 and Fig. 3.

Set two buildings A and B as the main structure, and the dampers of the mass D and the connecting mass and dampers are substructures. When $c_d = 0$ (the substructure damping is 0), it means that the substructure is not attached to the main structure. At this time, A and B are two independent structures. When A and B are subjected to external excitation, they are equivalent to the traditional structures without damping device. When $c_d \to \infty$ of the substructure, it means that the substructure is completely fixed to the main structure (A, B, D are connected as a whole). If A and B are externally excited, they are equivalent to the traditional structure without damping device. In the data and trend of figures 4 and 5, equations (4) and (5) are reasonable.

![Figure 2. Displacement transfer function curve of structure A at $k_d = 0$.](image)
Figure 3. Displacement transfer function curve of structure B at $k_d = 0$.

Figure 4. Displacement transfer function curve of structure A at $k_d = 0$.

Figure 5. Displacement transfer function curve of structure B at $k_d = 0$. 
3. Calculation and analysis of case model

It is still assumed that the characteristic parameters to the structure are:

\[ k_a = 60, k_b = 40. \]

\[ m_a = 2, m_b = 1, m_d = 0.2(m_a + m_b) = 0.3 \] \[ [8]. \]

The natural vibration period and the natural vibration frequency corresponding to the structures A and B are:

\[ T_a = \frac{2\pi}{\omega_a} = 1.14 \text{s}, \quad \omega_a = \sqrt{\frac{k_a}{m_a}} = \frac{5.48 \text{rad}}{s}, \]

\[ T_b = \frac{2\pi}{\omega_b} = 0.999 \text{s}, \quad \omega_b = \sqrt{\frac{k_b}{m_b}} = 6.32 \text{rad/s}. \]

The displacement transmission curves of the two buildings A and B when \( k_d = 0 \) are shown in Fig. 4 and Fig. 5.

From figures 4 and 5, it can be seen that with the increase of damping coefficient, the amplitude of displacement transfer function decreases first and then increases. The displacement transfer function corresponding to system A and B passes exceeds a certain point when the damping coefficient \( c_d \) changes and is recorded as P and Q points respectively. According to the fixed-point theory, these fixed-point heights should be equal in seismic design.

When the damping parameter \( c_d = 0 \), \( k_d = 0 \), the displacement transfer function of corresponding structure A and B can be obtained from formula (4) and formula (5) as follows:

\[ \left| \frac{X_a}{Y} \right| = \frac{\omega^2}{|\omega^2 - \omega_a^2|}, \quad \left| \frac{X_b}{Y} \right| = \frac{\omega^2}{|\omega^2 - \omega_b^2|} \] \[ (7) \]

When the damping parameter \( c_d = \infty \), \( k_d \) is constant, the displacement transfer function of corresponding structure A and B can be obtained from formula (4) and formula (5) as follows:

\[ \left| \frac{X_a}{Y} \right| = \frac{\omega^2}{|\omega^2 - \omega_\infty^2|}, \quad \left| \frac{X_b}{Y} \right| = \frac{\omega^2}{|\omega^2 - \omega_\infty^2|} \] \[ (8) \]

In the formula: \( \omega_\infty = \sqrt{(k_a + k_b)/(m_a + m_b + m_d)} \).

When the damping parameter \( c_d \) is constant and \( k_d = \infty \), the displacement transfer function of corresponding structure A and B can be obtained from formula (4) and formula (5) as follows:

\[ \left| \frac{X_a}{Y} \right| = \frac{\omega^2}{|\omega^2 - \omega_\infty^2|}, \quad \left| \frac{X_b}{Y} \right| = \frac{\omega^2}{|\omega^2 - \omega_\infty^2|} \] \[ (9) \]

Assuming that P and Q are two fixed points on the displacement transfer function curve of structure A and B, the corresponding \( \omega \) to P and Q should satisfy the equation (10). The \( \omega \) value corresponding to the fixed point solved by the equation (10) should satisfy the formula (11). By substituting the corresponding \( \omega \) at the fixed point into equation (8), the displacement transfer coefficients at the fixed points of A and B structures can be obtained as formula (12) and formula (13) from equation (12).

\[ \text{Formula (13) shows that:} \quad \frac{k_a}{k_b} = \frac{\sqrt{J^2 - 4I^2}}{2I} \]

\[ \text{In the formula:} \quad I = m_am_b, \quad J = m_a^2 + m_b^2 + 2m_am_d + 2m_bm_d + m_d^2 \]

\[ \left\{ \begin{array}{l}
\left| \frac{X_a}{Y} \right|_{c_d=0} = \left| \frac{X_a}{Y} \right|_{c_d=\infty}, \quad \omega = \omega_p \\
\left| \frac{X_b}{Y} \right|_{c_d=0} = \left| \frac{X_b}{Y} \right|_{c_d=\infty}, \quad \omega = \omega_Q 
\end{array} \right. \] \[ (10) \]
\[
\begin{align*}
\omega_p &= \sqrt{\left(\omega_\infty^2 + \omega_a^2\right)/2} \\
\omega_Q &= \sqrt{\left(\omega_\infty^2 + \omega_b^2\right)/2}
\end{align*}
\]

(11)

\[
\begin{align*}
\frac{X_a}{Y}_{\omega=\omega_p} &= \left[(\omega_\infty^2 + \omega_a^2)/(\omega_\infty^2 - \omega_a^2)\right] \\
\frac{X_b}{Y}_{\omega=\omega_Q} &= \left[(\omega_\infty^2 + \omega_b^2)/(\omega_\infty^2 - \omega_b^2)\right]
\end{align*}
\]

(12)

\[
\frac{(k_a + k_b)m_a + k_a(m_a + m_b + m_d)}{(k_a + k_b)m_a - k_a(m_a + m_b + m_d)} = \frac{(k_a + k_b)m_a + k_a(m_a + m_b + m_d)}{(k_a + k_b)m_a - k_a(m_a + m_b + m_d)}
\]

(13)

Figure 6. Displacement transfer function curves of structures A and B at \(k_d = 0\).

The displacement transfer curves of A and B buildings under the satisfactoriness formula (13) are shown in figure 6. It can be seen from the same height of the fixed point that formula (13) is reasonable.

4. Conclusion

- In this paper, the numerical relationship between the structural parameters of adjacent structures with tuned mass dampers, the parameters of tuned mass dampers, and Specific Performance of the adjacent structures in Earthquakes has been find out by theoretical derivation, and the formula (4), (5) are verified by numerical simulation.
- The structural parameters of the adjacent structure which is most suitable for installing the tuned mass damper are obtained by theoretical derivation, and the result is verified by the displacement transfer curve (ie, formula (13)).

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