\( \Delta M_K \) and \( \epsilon_K \) in the left-right supersymmetric model

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We perform a complete analysis of \( \Delta S = 2 \) processes in the kaon system and evaluate \( \Delta M_K \) and \( \epsilon_K \) in the left-right supersymmetric model. We include analytic expressions for the contributions of gluinos, neutralinos and charginos. We obtain general constraints on off-diagonal mass terms between the first two generations of both down-type and up-type squarks. In the down-squark sector, we compare the results with gluino-only estimates. In the up-squark sector, we find a complete set of bounds on all combinations of chirality conserving or chirality flipping parameters. Finally, we comment on the size of the bounds obtained by imposing left-right symmetry in the squark sector.

PACS number(s): 12.60.Jv, 13.25.Es, 14.40.Aq

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1 Introduction

Flavour changing neutral currents (FCNC) and charge-parity (CP) violation are still outstanding problems of electroweak theories. In the standard model (SM), both are understood to originate from the Cabibbo-Kobayashi-Maskawa (CKM) matrix, which mixes quark flavours. The smallness of such phenomena is understood in terms of the smallness of the off-diagonal elements of the CKM matrix. Extensions of the SM, such as supersymmetry, introduce new FCNC and CP parameters, which are a priori large. Restricting such parameters to fit within the experimental results puts important constraints on the supersymmetric parameters and drastically reduces the parameter space.

Numerous analyses of such phenomena have been carried out in the context of the minimal supersymmetric extension of the standard model (MSSM) [1]. Some of these analyses have some model-independent features, but the particle content is MSSM. In a series of recent papers, we investigated the constraints imposed on the parameter space of the left-right supersymmetric model (LRSUSY) from FCNC and CP violation in the B system [2]. The LRSUSY has gained considerable interest in the literature for several interesting phenomenological features, not least of which is its ability to provide a solution to the SUSY-CP problem [3]. The Hermitean structure of Yukawa matrices and soft-supersymmetry breaking parameters above the right-handed breaking scale insures the smallness of the electric dipole moments, a known problem in the MSSM. However, off-diagonal elements of these parameters are allowed to develop CP phases, which play a role in B and K decay parameters.

In this work we propose to extend the analysis of the flavor changing parameters in the B system to K system, in particular to the study of $\Delta M_K$ and $\epsilon_K$. There are several important differences between the B system and K system. The K system is subject to many more hadronic uncertainties than the B system, in particular the mass
difference $\Delta M_K$ is affected by long distance contributions, which are difficult to estimate reliably. However, improved QCD corrections in the calculation of $K^0 - \bar{K}^0$ mixing [4], NLO corrections to the $\Delta S = 2$ Hamiltonian [5], and lattice calculations of the full set of parameters entering the kaon mixing matrix elements [6], have raised the theoretical accuracy of predicting kaon parameters. Additionally, the kaon system would place restrictions on different off-diagonal mass parameters and as such, provide complimentary information about flavor violation in the LRSUSY. The determination of $\epsilon_K$ would provide a more complete picture on the constraints on CP violation in this model.

Our paper is organized as follows: in section II we give a brief description of the model and sources of CP and flavor violation. Complete analytical results for gluino, neutralino, gluino-neutralino and chargino contributions to $\Delta M_K$ and $\epsilon_K$ are presented in section III, followed by a numerical analysis in section IV. We reach our conclusions in section V, and in the appendix we give a summary of chargino and neutralino mixing in the LRSUSY, as well as the loop and vertex functions used, for self-sufficiency of the paper.

2 Description of the LRSUSY Model

The minimal left-right extension to the supersymmetric standard model is based on the gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The matter fields of the model consist of three families of quark and lepton chiral superfields with the following transformations under the gauge group

$$
Q = \begin{pmatrix} u \\ d \end{pmatrix} \sim (3, 2, 1, \frac{1}{3}), \quad Q^c = \begin{pmatrix} d^c \\ u^c \end{pmatrix} \sim (3^*, 1, 2, -\frac{1}{3}),
$$

$$
L = \begin{pmatrix} \nu \\ e \end{pmatrix} \sim (1, 2, 1, -1), \quad L^c = \begin{pmatrix} e^c \\ \nu^c \end{pmatrix} \sim (1, 1, 2, 1),
$$

where the numbers in the brackets denote the quantum numbers under $SU(3)_C$, $SU(2)_L$, $SU(2)_R$ and $U(1)_{B-L}$ respectively. The Higgs sector consists of the bidoublet and triplet
Higgs superfields

\[ \Phi_1 = \begin{pmatrix} \Phi^0_{11} & \Phi^+_1 \\ \Phi^-_1 & \Phi^{0*}_2 \end{pmatrix} \sim (1, 2, 2, 0), \quad \Phi_2 = \begin{pmatrix} \Phi^0_{21} & \Phi^+_2 \\ \Phi^-_2 & \Phi^{0*}_2 \end{pmatrix} \sim (1, 2, 2, 0), \]

\[ \Delta_L = \begin{pmatrix} \frac{1}{\sqrt{2}} \Delta_L^- & \frac{1}{\sqrt{2}} \Delta_L^- \\ \frac{1}{\sqrt{2}} \Delta_L^- & \frac{1}{\sqrt{2}} \Delta_L^- \end{pmatrix} \sim (1, 3, 1, -2), \quad \delta_L = \begin{pmatrix} \frac{1}{\sqrt{2}} \delta^L_\tau & \frac{1}{\sqrt{2}} \delta^L_\tau \\ \frac{1}{\sqrt{2}} \delta^L_\tau & \frac{1}{\sqrt{2}} \delta^L_\tau \end{pmatrix} \sim (1, 3, 1, 2), \]

\[ \Delta_R = \begin{pmatrix} \frac{1}{\sqrt{2}} \Delta_R^- & \frac{1}{\sqrt{2}} \Delta_R^- \\ \frac{1}{\sqrt{2}} \Delta_R^- & \frac{1}{\sqrt{2}} \Delta_R^- \end{pmatrix} \sim (1, 1, 3, -2), \quad \delta_R = \begin{pmatrix} \frac{1}{\sqrt{2}} \delta_R^+ & \frac{1}{\sqrt{2}} \delta_R^+ \\ \frac{1}{\sqrt{2}} \delta_R^+ & \frac{1}{\sqrt{2}} \delta_R^+ \end{pmatrix} \sim (1, 1, 3, 2). \quad (2) \]

The bidoublet Higgs superfields appear in all LRSUSY and serve to break the symmetry down to the \( SU(2)_L \times U(1)_Y \) group. Supplementary Higgs representations are needed to break \( SU(2)_R \) symmetry spontaneously at high \( \Lambda_R \): the choice of the adjoint representation is necessary to support the seesaw mechanism. Note that the number of fields in the Higgs sector is doubled with respect to the non-supersymmetric version to ensure anomaly cancellations. The most general superpotential involving these superfields is

\[ W = Y^Q Q^T \Phi_i \tau_2 Q^c + Y^L L^T \Phi_i \tau_2 L^c + i(\bar{Y}_{LR} L^T \tau_2 \Delta_L L + Y_{LR} L^T \tau_2 \Delta_R L^c) \]

\[ + \mu_{LR} [Tr(\Delta_L \delta_L + \Delta_R \delta_R)] + \mu_{ij} Tr(i \tau_2 \Phi_i^T i \tau_2 \Phi_j) + W_{NR}, \quad (3) \]

where \( W_{NR} \) denotes (possible) non-renormalizable terms arising from higher scale physics or Planck scale effects [7]. These terms are necessary for \( R \)-parity conservation of the ground state, for the case in which SUSY breaking scales are above \( M_{W_R} \) [8]. In addition, the potential also includes soft supersymmetry breaking terms

\[ \mathcal{L}_{soft} = A_q^i Q^i Q^T \Phi_i \tau_2 \tilde{Q}^c + A_L^i Y^L L^T \Phi_i \tau_2 L^c + i A_{LR} Y_{LR}(L^T \tau_2 \Delta_L L + L^T \tau_2 \Delta_R L^c) \]

\[ + m_{\Phi_i}^2 \Phi_i^\dagger \Phi_j + [m_{L_i}^2 \bar{L}_i L_j + (m_{R_i}^2 \bar{R}_i R_j)] + [m_{Q_i}^2 \bar{Q}_i Q_j + (m_{Q_R}^2 \bar{Q}_R Q_R,j)] \]

\[ - M_{LR}^2 [Tr(\Delta_R \delta_R) + Tr(\Delta_L \delta_L) + h.c.] - [B_{ij} \Phi_i \Phi_j + h.c.]. \quad (4) \]

Additional flavor and CP violation is generated in the non-supersymmetric version of the model through a right-handed version of the Cabibbo-Kobayashi-Maskawa matrix, \( K_R \), which is measurable in left-right models, but not in the SM. Additional constraints can
be imposed if this matrix is different from the one in the left-handed sector. However, here we are interested mainly in the sources of flavor and CP breaking coming from the supersymmetric sector, so we demand manifest left-right symmetry $K_R = K_L$. Then the effects of the right-handed currents are suppressed by the mass of the heavy $W_R$ boson. Much less is known about the right-handed supersymmetric partners of the gauge bosons and their contributions to flavor and CP violation could be significant. In the quark-squark sector, flavor violation arises mainly from the Yukawa couplings $Y_Q$ and the trilinear scalar mixing parameters $A_Q$, through the squark mass matrices.

In what follows, we parameterize all the unknown soft breaking parameters coming mostly from the scalar mass matrices using the mass insertion approximation [9]. We choose a basis for fermion and sfermion states in which all the couplings of these particles to neutral gauginos are flavor diagonal and parametrize flavor changes in the non-diagonal squark propagators through mixing parameters $(\delta_{ij}^q)_{AB}$, where $i, j = 1, 2, 3$ and $A, B = L, R$. The dimensionless flavor mixing parameters used are

$$
(\delta_{ij}^q)_{LL} = \frac{(m_{\tilde{q},ij}^2)_{LL}}{m_{\tilde{q}}^2}, \quad (\delta_{ij}^q)_{RR} = \frac{(m_{\tilde{q},ij}^2)_{RR}}{m_{\tilde{q}}^2},
$$

$$
(\delta_{ij}^q)_{LR} = \frac{(m_{\tilde{q},ij}^2)_{LR}}{m_{\tilde{q}}^2}, \quad (\delta_{ij}^q)_{RL} = \frac{(m_{\tilde{q},ij}^2)_{RL}}{m_{\tilde{q}}^2}, \quad (5)
$$

where $m_{\tilde{q}}^2$ is the average squark mass and $(m_{\tilde{q},ij}^2)_{AB}$ are the off-diagonal elements which mix squark flavors for both left- and right- handed squarks with $q = u, d$. 
3 The analytic formulas

3.1 Effective Hamiltonian for the $\Delta S = 2$ process in the LR-SUSY

The contributions of the left-right supersymmetric model to the $\Delta S = 2$ process are given by the effective Hamiltonian

$$\mathcal{H}_{\Delta S=2}^{\text{eff}} = \sum_i [C_i(\mu)Q_i(\mu) + \tilde{C}_i(\mu)\tilde{Q}_i(\mu)].$$

where the relevant operators entering the sum are

\begin{align*}
Q_1 &= \bar{d}_L^\alpha \gamma_\mu s^\alpha_L \bar{d}_L^\beta \gamma_\mu s^\beta_L, \\
\tilde{Q}_1 &= \bar{d}_R^\alpha \gamma_\mu s^\alpha_R \bar{d}_R^\beta \gamma_\mu s^\beta_R, \\
Q_2 &= \bar{d}_L^\alpha \gamma_\mu s^\alpha_L \bar{d}_R^\beta \gamma_\mu s^\beta_R, \\
\tilde{Q}_2 &= \bar{d}_R^\alpha \gamma_\mu s^\alpha_R \bar{d}_L^\beta \gamma_\mu s^\beta_L, \\
Q_3 &= \bar{d}_L^\alpha \gamma_\mu s^\alpha_R \bar{d}_R^\beta \gamma_\mu s^\beta_R, \\
\tilde{Q}_3 &= \bar{d}_R^\alpha \gamma_\mu s^\alpha_R \bar{d}_L^\beta \gamma_\mu s^\beta_L, \\
Q_4 &= \bar{d}_L^\alpha \gamma_\mu s^\alpha_L \bar{d}_R^\beta \gamma_\mu s^\beta_L, \\
\tilde{Q}_4 &= \bar{d}_R^\alpha \gamma_\mu s^\alpha_R \bar{d}_L^\beta \gamma_\mu s^\beta_R.
\end{align*}

where $q_{R,L} = P_{R,L}q$ with $P_{R,L} = (1 \pm \gamma_5)/2$, and $\alpha, \beta$ are color indices. The Wilson coefficients $C_i$ and $\tilde{C}_i$ are initially evaluated at the electroweak or soft supersymmetry breaking scale, then evolved down to the scale $\mu$. Because of the left-right symmetry, we must consider all contributions from both chirality operators.

The $K^0 - \bar{K}^0$ mixing is mediated through the box diagrams shown in Fig. 1. Below we give a comprehensive list of all box diagram contributions for $C_i$ and $\tilde{C}_i$. The notations of various vertices, mixing matrices and functions are defined in the appendix.

$$C_i = C_1^g + C_1^{\tilde{g}s} + C_1^{s\tilde{s}} + C_1^{s\tilde{s} },$$

6
Figure 1: Leading supersymmetric box diagrams contributing to $K^0 - \bar{K}^0$ mixing, $A, B, C, D = (L, R)$. Crossed diagrams (not shown) are also included in the calculation.

with

\[
C_{1}^{\tilde{g}} = -\frac{\alpha_s^2}{m_{\tilde{q}}^2} x_{\tilde{q}\tilde{q}} \left( \frac{1}{9} F(x_{\tilde{q}\tilde{q}}, 1) - \frac{11}{36} G(x_{\tilde{q}\tilde{q}}, 1) \right) (\delta_{12}^d)_{LL}^2,
\]

\[
C_{2}^{\tilde{g}} = -\frac{\alpha_s^2}{m_{\tilde{q}}^2} x_{\tilde{q}\tilde{q}} \frac{17}{18} F(x_{\tilde{q}\tilde{q}}, 1)(\delta_{12}^d)_{RL}^2,
\]

\[
C_{3}^{\tilde{g}} = \frac{\alpha_s^2}{6m_{\tilde{q}}^2} x_{\tilde{q}\tilde{q}} F(x_{\tilde{q}\tilde{q}}, 1)(\delta_{12}^d)_{RL}^2,
\]

\[
C_{4}^{\tilde{g}} = -\frac{\alpha_s^2}{m_{\tilde{q}}^2} x_{\tilde{q}\tilde{q}} \left[ \left( \frac{7}{3} F(x_{\tilde{q}\tilde{q}}, 1) + \frac{1}{3} G(x_{\tilde{q}\tilde{q}}, 1) \right) (\delta_{12}^d)_{LL}(\delta_{12}^d)_{RR} + \frac{11}{18} G(x_{\tilde{q}\tilde{q}}, 1)(\delta_{12}^d)_{LR}(\delta_{12}^d)_{RL} \right],
\]

\[
C_{5}^{\tilde{g}} = \frac{\alpha_s^2}{m_{\tilde{q}}^2} x_{\tilde{q}\tilde{q}} \left[ \left( \frac{5}{9} G(x_{\tilde{q}\tilde{q}}, 1) - \frac{1}{9} F(x_{\tilde{q}\tilde{q}}, 1) \right) (\delta_{12}^d)_{LL}(\delta_{12}^d)_{RR} - \frac{5}{6} G(x_{\tilde{q}\tilde{q}}, 1)(\delta_{12}^d)_{LR}(\delta_{12}^d)_{RL} \right],
\]

\[
C_{1}^{\tilde{\chi}_i^{0}} = -\frac{\alpha_s G_{12}^{*i} G_{12} G(x_{\tilde{q}\tilde{q}}, x_{\tilde{\chi}_i^{0}})}{3m_{\tilde{q}}^2} \sum_{i=1}^{9} \left[ G_{12}^{*i} G_{12}^{*i} \sqrt{x_{\tilde{q}\tilde{q}} F(x_{\tilde{q}\tilde{q}}, x_{\tilde{\chi}_i^{0}}) (\delta_{12}^d)_{LL}^2} \right],
\]

\[
C_{2}^{\tilde{\chi}_i^{0}} = -\frac{\alpha_s G_{12}^{*i} G_{12} G(x_{\tilde{q}\tilde{q}}, x_{\tilde{\chi}_i^{0}})}{3m_{\tilde{q}}^2} \sum_{i=1}^{9} G_{12}^{*i} G_{12}^{*i} \sqrt{x_{\tilde{q}\tilde{q}} F(x_{\tilde{q}\tilde{q}}, x_{\tilde{\chi}_i^{0}}) (\delta_{12}^d)_{RL}^2},
\]

\[
C_{3}^{\tilde{\chi}_i^{0}} = -\frac{\alpha_s G_{12}^{*i} G_{12} G(x_{\tilde{q}\tilde{q}}, x_{\tilde{\chi}_i^{0}})}{3m_{\tilde{q}}^2} \sum_{i=1}^{9} G_{12}^{*i} G_{12}^{*i} \sqrt{x_{\tilde{q}\tilde{q}} F(x_{\tilde{q}\tilde{q}}, x_{\tilde{\chi}_i^{0}}) (\delta_{12}^d)_{RL}^2},
\]
\[ C_4^\tilde{\chi}^0 = \frac{\alpha_s \alpha_W}{m^2_\tilde{q}} \sum_{i=1}^{9} \left\{ (G_{DR} G_{DL}^j + G_{DR}^* G_{DL}^* j) G(x_{\tilde{q}0}, x_{\tilde{\chi}_i^0}) 
 - 2(G_{DL}^i G_{DR}^* j + G_{DL}^* j G_{DR}^i) \sqrt{x_{\tilde{q}0}} F(x_{\tilde{q}0}, x_{\tilde{\chi}_i^0}) (\delta_{12}^d)_{LL} (\delta_{12}^d)_{RR} 
 - (G_{DL}^i G_{DL}^* j + G_{DR}^* G_{DR}^i) G(x_{\tilde{q}0}, x_{\tilde{\chi}_i^0}) (\delta_{12}^d)_{RL} (\delta_{12}^d)_{LR} \right\}, \]
\[ C_5^\tilde{\chi}^0 = \frac{\alpha_s \alpha_W}{3 m^2_\tilde{q}} \sum_{i=1}^{9} \left\{ \left[ 2 \left( G_{DL}^i G_{DR} G_{DL}^j + G_{DL}^* G_{DR}^j \right) \sqrt{x_{\tilde{q}0}^0} F(x_{\tilde{q}0}, x_{\tilde{\chi}_i^0}) \right] (\delta_{12}^d)_{LL} (\delta_{12}^d)_{RR} \right\}, \]
\[ C_1^\tilde{\chi}^0 = \frac{\alpha_W}{2 m^2_\tilde{q}} \sum_{i,j=1}^{9} x_{\tilde{q}0} \left( G_{DL}^j G_{DL} G_{DL}^j G_{DL} G(x_{\tilde{q}0}, x_{\tilde{\chi}_i^0}) \right) (\delta_{12}^d)_{LL}^2, \]
\[ C_2^\tilde{\chi}^0 = -\frac{\alpha_W}{m^2_\tilde{q}} \sum_{i,j=1}^{9} x_{\tilde{q}0}^i G_{DL}^j G_{DR} G_{DL}^j G_{DR} G_{DL}^j G_{DL} G(x_{\tilde{q}0}, x_{\tilde{\chi}_i^0}) (\delta_{12}^d)_{RL}, \]
\[ C_3^\tilde{\chi}^0 = -\frac{\alpha_W}{2 m^2_\tilde{q}} \sum_{i,j=1}^{9} x_{\tilde{q}0}^i G_{DL}^j G_{DR} G_{DL}^j G_{DR} G_{DL}^j G_{DL} G(x_{\tilde{q}0}, x_{\tilde{\chi}_i^0}) (\delta_{12}^d)_{LR}, \]
\[ C_4^\tilde{\chi}^0 = -\frac{\alpha_W}{2 m^2_\tilde{q}} \sum_{i,j=1}^{9} x_{\tilde{q}0}^i \left\{ \left[ G_{DL}^j G_{DL}^* G_{DR} G_{DL}^j G_{DL} G(x_{\tilde{q}0}, x_{\tilde{\chi}_i^0}) \right] (\delta_{12}^d)_{RR} \right\}, \]
\[ C_5^\tilde{\chi}^0 = \frac{\alpha_W}{6 m^2_\tilde{q}} \left\{ \sum_{h,k=1}^{3} K_{h2}^* (\delta_{hk})_{LL} K_{k1} \right\}^2 \sum_{i,j=1}^{4} x_{\tilde{q}0}^i G_{UL}^i G_{UL}^* G_{UL}^i G_{UL}^* G_{UL}^i G_{UL}^* G_{UL}^* G(x_{\tilde{q}0}, x_{\tilde{\chi}_i^0}), \]
\[ C_1^\tilde{\chi}^- = -\frac{\alpha_W}{m^2_\tilde{q}} \sum_{h,k=1}^{3} K_{h2} (\delta_{hk})_{RL} K_{k1} \sum_{i,j=1}^{4} x_{\tilde{q}0}^i G_{UR}^i G_{UL}^i G_{UR}^i G_{UL}^* G_{UL}^* G_{UL}^* G_{UL}^* G(x_{\tilde{q}0}, x_{\tilde{\chi}_i^0}), \]
\[ C_2^\tilde{\chi}^- = 0, \]
\[ C_3^\tilde{\chi}^- = \frac{\alpha_W}{m^2_\tilde{q}} \sum_{h,k=1}^{3} K_{h2} (\delta_{hk})_{RL} K_{k1} \sum_{i,j=1}^{4} x_{\tilde{q}0}^i G_{UR}^i G_{UL}^i G_{UR}^i G_{UL}^* G_{UL}^* G_{UL}^* G_{UL}^* G(x_{\tilde{q}0}, x_{\tilde{\chi}_i^0}), \]
\[ C^\tilde{\chi}_- = \frac{\alpha_V^2}{12m_q^2} \sum_{h,k,m,n=1}^{3} K_{h2}^* (\delta_{hk})_{RL} K_{k1} K_{m2}^* (\delta_{mn})_{LR} K_{n1} \]
\[ \times \sum_{i,j=1}^{4} x_{\tilde{q} \tilde{\chi}_i} G_{UR}^i G_{UL}^{*i} G_{UR} G(x_{\tilde{q} \tilde{\chi}_i}, x_{\tilde{\chi}_j}) \]
\[ C^\tilde{\chi}_+ = -\frac{\alpha_V^2}{m_q^2} \sum_{h,k,m,n=1}^{3} K_{h2}^* (\delta_{hk})_{LL} K_{k1} K_{m2}^* (\delta_{mn})_{RR} K_{n1} \]
\[ \times \sum_{i,j=1}^{4} x_{\tilde{q} \tilde{\chi}_i} G_{UR}^i G_{UL}^{*i} G_{UR} G(x_{\tilde{q} \tilde{\chi}_i}, x_{\tilde{\chi}_j}) \]  

(9)

where \( x_{ab} = \frac{m_a^2}{m_b^2} \). In the above expressions we have neglected the contributions coming from higgsino vertices because of proportionality to the masses of the first two generations of quarks. The coefficients \( \tilde{C}_i \) are obtained from the coefficients \( C_i \)

\[ \tilde{C}_i^{\tilde{g} \tilde{\chi}_0, \tilde{\chi}_0, \tilde{\chi}^-} = C_i^{\tilde{g} \tilde{\chi}_0, \tilde{\chi}_0, \tilde{\chi}^-}(L \leftrightarrow R). \]

(10)

### 3.2 Hadronic Matrix Elements

The matrix elements of the operators \( Q_i \) between \( K^0 - \bar{K}^0 \) mesons in the vacuum insertion approximation (VIA) [10] are given by

\[ \langle K^0 | Q_1 | \bar{K}^0 \rangle_{VIA} = \frac{1}{3} m_K f_K^2 \]
\[ \langle K^0 | Q_2 | \bar{K}^0 \rangle_{VIA} = -\frac{5}{24} \left( \frac{m_K}{m_s + m_d} \right)^2 m_K f_K^2 \]
\[ \langle K^0 | Q_3 | \bar{K}^0 \rangle_{VIA} = \frac{1}{24} \left( \frac{m_K}{m_s + m_d} \right)^2 m_K f_K^2 \]
\[ \langle K^0 | Q_4 | \bar{K}^0 \rangle_{VIA} = \left[ \frac{1}{24} + \frac{1}{4} \left( \frac{m_K}{m_s + m_d} \right)^2 \right] m_K f_K^2 \]
\[ \langle K^0 | Q_5 | \bar{K}^0 \rangle_{VIA} = \left[ \frac{1}{8} + \frac{1}{12} \left( \frac{m_K}{m_s + m_d} \right)^2 \right] m_K f_K^2 \]  

(11)

where \( m_K = 497.67 \) MeV, \( m_s \) and \( m_d \) are the masses of the \( K^0 \) meson, s and d quark respectively, \( f_K = 159.8 \) MeV is the decay constant of \( K^0 \) mesons. The expressions for \( \tilde{Q}_{1-3} \) are same as those of \( Q_{1-3} \).
To take into account nonperturbative effects, the B parameters, which are scale dependent, are defined as

\[
\begin{align*}
\langle K^0|Q_1|\overline{K}^0\rangle_{V\Lambda} &= \frac{1}{3}m_K f_K^2 B_1(\mu), \\
\langle K^0|Q_2|\overline{K}^0\rangle_{V\Lambda} &= -\frac{5}{24}\left(\frac{m_K}{m_s + m_d}\right)^2 m_K f_K^2 B_2(\mu), \\
\langle K^0|Q_3|\overline{K}^0\rangle_{V\Lambda} &= \frac{1}{24}\left(\frac{m_K}{m_s + m_d}\right)^2 m_K f_K^2 B_3(\mu), \\
\langle K^0|Q_4|\overline{K}^0\rangle_{V\Lambda} &= \frac{1}{4}\left(\frac{m_K}{m_s + m_d}\right)^2 m_K f_K^2 B_4(\mu), \\
\langle K^0|Q_5|\overline{K}^0\rangle_{V\Lambda} &= \frac{1}{12}\left(\frac{m_K}{m_s + m_d}\right)^2 m_K f_K^2 B_5(\mu),
\end{align*}
\]

(12)

with the numerical values at \(\mu = 2\) GeV

\[
\begin{align*}
m_s(\mu) &= 125\text{ MeV}, & m_d(\mu) &= 7\text{ MeV}, \\
B_1(\mu) &= 0.60(6), & B_2(\mu) &= 0.66(4), \\
B_3(\mu) &= 1.05(12), & B_4(\mu) &= 1.03(6), \\
B_5(\mu) &= 0.73(10).
\end{align*}
\]

(13)

The coefficients at the scale of \(\mu\) are given by

\[
C_r(\mu) = \sum_i \sum_s (b_i^{(r,s)} + \eta c_i^{(r,s)}) \eta a_i C_s(M),
\]

(14)

where \(\eta = \alpha_s(M)/\alpha_s(m_t)\) and we have chosen \(M = (m_\tilde{g} + m_\tilde{q})/2\). The numerical coefficients \(a_i, b_i^{(r,s)}, c_i^{(r,s)}\) can be found in Ref. [11].

Putting all the above together, we can calculate the mass difference \(\Delta M_K\) and CP violating parameter \(\epsilon_K\).

\[
\begin{align*}
\Delta M_K &= 2\text{Re}\left\langle K^0|\mathcal{H}_{\text{eff}}^{\Delta S=2} |\overline{K}^0\rangle\right\rangle, \\
\epsilon_K &= \frac{1}{\sqrt{2}\Delta M_K}\text{Im}\left\langle K^0|\mathcal{H}_{\text{eff}}^{\Delta S=2} |\overline{K}^0\rangle\right\rangle.
\end{align*}
\]

(15)

By setting the calculated expressions to their experimental results, we obtain restrictions on the sources of flavor and CP violation in the LRSUSY.
4 Numerical Analysis

In this section we present the results of our analysis for the individual bounds on both $(\delta_{12}^d)_{AB}$ and $(\delta_{12}^u)_{AB}$, obtained by selecting only one source of flavor violation and neglecting interference between different sources. In retrospect, the hierarchical structure of the flavor-violating parameters in the kaon system disfavors accidental cancellations.

We obtain the constraints from $\Delta M_K$ by requiring that the LRSUSY contribution proportional to a single $\delta$ parameter does not exceed the present experimental value. The SM contribution is proportional to the $cs$ and $cd$ elements of the $K$ matrix and thus independent of SUSY contributions. A common assumption is that the SM and supersymmetric contributions simply add, and several bounds are obtained in the literature with this simplification. We consider only supersymmetric effects here, and short-distance effects only, so that, if all the other effects are additive, the constraints we obtain are more conservative than previous analyses. This assumption also avoids introducing extra parameters, such as the relative phase of various contributions and mass parameters for the Higgs bosons into the calculations.

In the case of $\epsilon_K$, the constraints are also obtained requiring the LRSUSY contribution alone to saturate the experimental value for $\epsilon_K$. The SM contribution depends on the phase in the CKM matrix, which is another free parameter, but the SM result for $\epsilon_K$ is always positive. Thus, setting the CKM phase to zero gives conservative bounds on the supersymmetric flavor violation parameters.

We analyze cases in which the supersymmetric partners have masses around the weak scale, so we will assume relatively light superpartner masses. We choose all trilinear scalar couplings in the soft supersymmetry breaking Lagrangian to be universal: $A_{ij} = A\delta_{ij}$ and $\mu_{ij} = \mu_H\delta_{ij}$, and we fix $A$ to be 100 GeV and the Higgsino mixing parameter $\mu_H = 200$ GeV throughout the analysis.
To constrain \((\delta_{12}^{d})_{AB}\), we include contributions from gluino, neutralino and gluino-neutralino graphs. Because of the large number of neutralinos in the LRSUSY, the last two contributions are what distinguishes this model from the MSSM. We set \(M_L = M_R = 500 \text{ GeV}\), and allow the squark and gluino masses to vary. We obtain bounds on \(\text{Re}(\delta_{12}^{d})_{AB}(\delta_{12}^{d})_{CD}\) with \(AB, CD = LL, RR, LR, RL\) from \(\Delta M_K\). The bounds obtained are presented in Table 1. We include in brackets, for self-check and comparison, the bounds obtained by including only gluino contributions. The results are in agreement with previous evaluations available in the literature. Specifically, compared with Ref. [11], the bounds on \((\delta_{12}^{d})_{LL}\) are slightly weaker, mainly due to that we set the SM contribution zero, while Ref. [11] subtracted the SM contribution (to the coefficient \(C_1\)). The results share the same general features as the bounds obtained in the MSSM [11], that the restrictions on the \(\text{Re}(\delta)\) parameters become stronger with decreasing \(x = m_{\tilde{g}}^2/m_{\tilde{q}}^2\). There is, to a good approximation, left-right symmetry in the flavor-violating parameters, and the chirality flipping parameters are more restricted than the chirality conserving parameters.

As expected from the MSSM evaluations, the gluino diagram dominates. Neutralino contributions are suppressed by \((\alpha_W/\alpha_s)^2\) compared with gluino contributions, thus they are negligible; while gluino-neutralino contributions are suppressed by \(\alpha_W/\alpha_s\) and, from Table 1, they contribute at most 10% for heavy gluino and light neutralino masses. These features persist for bounds on \(\text{Im}(\delta_{12}^{d})_{AB}(\delta_{12}^{d})_{CD}\) coming from \(\epsilon_K\), presented in Tables 2. We include there too, in brackets, the gluino-only contributions.

We present next restrictions on \((\delta_{12}^{u})_{AB}(\delta_{12}^{u})_{CD}\) coming from the chargino sector. It is there that we expect to encounter the most significant deviations from the MSSM, since the LRSUSY includes a right-handed gaugino. This analysis allows restrictions of a full complement of up-type squark flavor violating mass insertions, similar to the ones obtained for the down-type squark sector. In Table 3, we present the bounds obtained on the real parts of the mass insertions, under the assumption that only one insertion
dominates. The results obtained are almost same as the corresponding ones in the MSSM [13], where only bounds on $\delta_{12}^{LL}$ were given. All constraints become weaker with increasing squark mass $m_{\tilde{q}}$. The bounds appear to be approximately left-right symmetric. $\text{Im}(\delta_{12}^{u})_{AB}$ shown in Table 4 share the general features for $\text{Re}(\delta_{12}^{u})_{AB}$, except that there the bounds are stronger by about a factor of 10. The bounds on $\text{Im}(\delta_{12}^{u})_{LL}$ are in agreement with the bounds found in Ref. [13]. We expect these to be further strengthened by an analysis of $\epsilon'/\epsilon$ [14].

In LR models, one cannot generally assume that there is only one type of mass insertion at a time. The bounds obtained in the above considerations, by allowing only one non-zero mass insertion at a time, show approximate left-right symmetry. Thus it is worthwhile to consider possible bounds which respect the left-right symmetry. If we let $\delta_{LR}^{d,u} = \delta_{RL}^{d,u}$ and $\delta_{LL}^{d,u} = \delta_{RR}^{d,u}$, the parameters reduce to $\text{Re}(\text{Im})(\delta_{12}^{d,u})_{LR}$ and $\text{Re}(\text{Im})(\delta_{12}^{d,u})_{LL}$. Effectively, this means that we are allowing two mass insertions at the same time. The bounds are shown in Table 5-8. They are much more restrictive than the ones obtained by allowing only one source of flavor violation because several contributions from different sources are added together. For both $\delta_{LL}^{d,u}$ and $\delta_{LR}^{d,u}$, the bounds follow from the most restrictive bounds from the previous tables, coming from $\sqrt{|\text{Re}(\text{Im})(\delta_{12}^{u,d})_{LL}(\delta_{12}^{u,d})_{RR}|}$ and $\sqrt{|\text{Re}(\text{Im})(\delta_{12}^{u,d})_{LR}(\delta_{12}^{u,d})_{RL}|}$. These bounds are purely a consequence of left-right symmetry and have no counterpart in other models one could compare with.

5 Conclusions

We have performed a complete analysis of the $K^0 - \overline{K}^0$ mixing in a general left-right supersymmetric model, where flavor violation is parametrized in a model-independent way using the mass insertion approximation. We have provided general upper bounds on mass insertions for either the up or down squark sectors, under the assumption that
the supersymmetric contributions alone do not exceed the experimental value of $\Delta M_K$.

This analysis provides bounds on $\text{Re}(\delta_{12}^d)_{AB}$ and $\text{Re}(\delta_{12}^u)_{AB}$, $A, B = L, R$. Similarly, for $\text{Im}(\delta_{12}^d)_{AB}$ and $\text{Im}(\delta_{12}^u)_{AB}$, we obtain upper bounds by requiring that the supersymmetric contributions do not exceed the experimental values for $\epsilon_K$, i.e. setting the CKM phase to zero. The bounds for the down-squark sector are obtained including gluino-neutralino and neutralino diagrams (in addition to gluino only) and are similar to those coming from the MSSM. In the up-squark sector, the bounds are much weaker than in the neutral sector, justifying the assumption of gluino domination of this process. The bounds on $\text{Re}(\delta_{12}^u)_{LL}$ and $\text{Im}(\delta_{12}^u)_{LL}$ are comparable to the ones obtained in the MSSM. In this sector, we also obtain, for the first time, bounds on the real and imaginary parts of all combinations of chirality conserving or flipping parameters. As a general feature, the imaginary parts are bound to be smaller than their real counterparts by a factor of 10.

If we impose left-right symmetry in the squark mass matrix, $\delta_{LR}^{d,u} = \delta_{RL}^{d,u}$ and $\delta_{LL}^{d,u} = \delta_{RR}^{d,u}$, the bounds are dramatically different and better than bounds on single mass insertion by one or more orders of magnitude. A realisation of such a scenario would be a promising sign for the manifestation of left-right supersymmetry.

Acknowledgements

This work was funded in part by NSERC of Canada (SAP0105354).
Appendix

For self-sufficiency, we list the mass-squared matrices for charginos and neutralinos, relevant Feynman rules and functions used for this calculation.

The terms relevant to the masses of charginos in the Lagrangian are

\[ \mathcal{L}_C = -\frac{1}{2} (\psi^+, \psi^-) \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix} \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + \text{H.c.}, \]  

(16)

where \( \psi^+ = (-i\lambda_1^L - i\lambda_2^R, \tilde{\phi}^+_{u1}, \tilde{\phi}^+_{d1})^T \) and \( \psi^- = (-i\lambda_1^L - i\lambda_2^R, \tilde{\phi}^-_{u2}, \tilde{\phi}^-_{d2})^T \), and

\[ X = \begin{pmatrix} M_L & 0 & g_{L\kappa_u} & 0 \\ 0 & M_R & g_{R\kappa_u} & 0 \\ 0 & 0 & 0 & -\mu_H \\ g_{L\kappa_d} & g_{R\kappa_d} & -\mu_H & 0 \end{pmatrix}, \]  

(17)

where we have taken, for simplification, \( \mu_{ij} = \mu_H \delta_{ij} \). The chargino mass eigenstates \( \chi_i \) are obtained by

\[ \tilde{\chi}^+_i = V_{ij} \psi^+_j, \ \tilde{\chi}^-_i = U_{ij} \psi^-_j, \ \text{for } i, j = 1, \ldots, 4, \]  

(18)

with \( V \) and \( U \) unitary matrices satisfying

\[ U^* X V^{-1} = M_D. \]  

(19)

The diagonalizing matrices \( U^* \) and \( V \) are obtained by computing the eigenvectors corresponding to the eigenvalues of \( X^\dagger X \) and \( XX^\dagger \), respectively.

The terms relevant to the masses of neutralinos in the Lagrangian are

\[ \mathcal{L}_N = -\frac{1}{2} \psi^0 Y \psi^0 + \text{H.c.}, \]  

(20)
where \( \psi^0 = (-i\lambda_3^L, -i\lambda_3^R, -i\lambda_V, \tilde{\phi}_u^0, \tilde{\phi}_u^0, \tilde{\phi}_d^0, \tilde{\phi}_d^0, \Delta_R, \Delta_R^0)^T \), and

\[
Y = \begin{pmatrix}
M_L & 0 & 0 & \frac{g_{L^u}}{2} & 0 & 0 & -\frac{g_{L^d}}{2} & 0 & 0 \\
0 & M_R & 0 & \frac{g_{R^u}}{2} & 0 & 0 & -\frac{g_{R^d}}{2} & -g_R v_R \sin \phi & -g_R v_R \cos \phi \\
0 & 0 & M_V & 0 & 0 & 0 & 2g_V v_R \sin \phi & 2g_V v_R \cos \phi \\
\frac{g_{L^u}}{2} & \frac{g_{R^u}}{2} & 0 & 0 & 0 & -\mu_H & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\mu_H & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\mu_H & 0 & 0 & 0 & 0 \\
-\frac{g_{L^d}}{2} & -\frac{g_{R^d}}{2} & 0 & 0 & -\mu_H & 0 & 0 & 0 & 0 \\
0 & -g_R v_R \sin \phi & 2g_V v_R \sin \phi & 0 & 0 & 0 & 0 & -\mu_H & 0 \\
0 & -g_R v_R \cos \phi & 2g_V v_R \cos \phi & 0 & 0 & 0 & 0 & 0 & -\mu_H \\
\end{pmatrix},
\]

where \( \sin \phi = \tan \theta_W \). The mass eigenstates are defined by

\[
\tilde{\chi}_i^0 = N_{ij} \psi^0_j \quad (i, j = 1, 2, \ldots 9),
\]

where \( N \) is a unitary matrix chosen such that

\[
N^* Y N^{-1} = N_D,
\]

and \( N_D \) is a diagonal matrix with non-negative entries.

The vertices of neutralino-quark-squark and chargino-quark-squark given by the mixing matrices \( G_{DL,DR} \) and \( G_{UL,UR} \) respectively, where

\[
G_{DL}^j = \left[ \sin \theta_W Q_d N'_{j1} + \frac{1}{\cos \theta_W} (T_d^3 - Q_d \sin^2 \theta_W) N'_{j2} \right. \\
- \left. \frac{\sqrt{\cos 2\theta_W}}{\cos \theta_W} Q_u + \frac{Q_d}{2} N'_{j3} \right] \\
G_{DR}^j = - \left[ \sin \theta_W Q_d N'_{j1} - \frac{Q_d \sin^2 \theta_W}{\cos \theta_W} N'_{j2} \right. \\
+ \left. \frac{\sqrt{\cos 2\theta_W}}{\cos \theta_W} (T_d^3 - Q_d \sin^2 \theta_W) N'_{j3} \right],
\]

\[
G_{UL}^j = V_{j1}^*, \quad G_{UR}^j = U_{j2}.
\]

The relevant two-variable functions from the box diagrams are

\[
(F(x, y), G(x, y)) = x^2 \partial_a \partial_b (F'(a, b, y), G'(a, b, y)) |_{a=b=x},
\]
with

\[
F'(x, y, z) = -\frac{1}{x-y} \left\{ \frac{1}{x-z} \left[ \frac{x}{x-1} \log x - (x \rightarrow z) \right] - (x \rightarrow y) \right\}, \quad (29)
\]

\[
G'(x, y, z) = \frac{1}{x-y} \left\{ \frac{1}{x-z} \left[ \frac{x^2}{x-1} \log x - \frac{3}{2} x - (x \rightarrow z) \right] - (x \rightarrow y) \right\}. \quad (30)
\]

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\[
\begin{array}{|c|c|c|c|}
\hline
& m_\tilde{q} = 200 \text{ GeV} & m_\tilde{q} = 500 \text{ GeV} & m_\tilde{q} = 1000 \text{ GeV} \\
\hline
x & \sqrt{|\text{Re}(\delta_{12}^d)^{2LL}|} & \sqrt{|\text{Re}(\delta_{12}^d)^{2LR}|} & \sqrt{|\text{Re}(\delta_{12}^d)^{2RL}|} \\
\hline
0.25 & 1.1 [1.1] \times 10^{-2} & 3.1 [3.1] \times 10^{-2} & 6.8 [6.8] \times 10^{-2} \\
1.0 & 2.3 [2.6] \times 10^{-2} & 6.3 [7.5] \times 10^{-2} & 1.4 [1.6] \times 10^{-1} \\
4.0 & 1.8 [0.66] \times 10^{-1} & 2.1 [1.8] \times 10^{-1} & 3.9 [4.0] \times 10^{-1} \\
\hline
x & \sqrt{|\text{Re}(\delta_{12}^d)^{2RR}|} & \sqrt{|\text{Re}(\delta_{12}^d)^{2LL} (\delta_{12}^d)^{2RR}|} & \sqrt{|\text{Re}(\delta_{12}^d)^{2LR} (\delta_{12}^d)^{2RL}|} \\
\hline
0.25 & 1.1 [1.1] \times 10^{-2} & 3.1 [3.1] \times 10^{-2} & 6.8 [6.9] \times 10^{-2} \\
1.0 & 2.5 [2.6] \times 10^{-2} & 6.9 [7.5] \times 10^{-2} & 1.5 [1.6] \times 10^{-1} \\
4.0 & 8.9 [6.6] \times 10^{-2} & 5.6 [1.8] \times 10^{-1} & 7.9 [4.0] \times 10^{-1} \\
\hline
x & \sqrt{|\text{Re}(\delta_{12}^d)^{2LL} (\delta_{12}^d)^{2RR}|} & \sqrt{|\text{Re}(\delta_{12}^d)^{2LR} (\delta_{12}^d)^{2RL}|} \\
\hline
0.25 & 4.6 [4.2] \times 10^{-3} & 1.2 [1.1] \times 10^{-3} & 2.5 [2.3] \times 10^{-3} \\
1.0 & 5.4 [5.2] \times 10^{-4} & 1.4 [1.4] \times 10^{-3} & 2.9 [2.8] \times 10^{-3} \\
4.0 & 7.9 [7.8] \times 10^{-4} & 2.0 [2.0] \times 10^{-3} & 4.1 [4.2] \times 10^{-3} \\
\hline
\end{array}
\]

Table 1 Limits on \(\text{Re}(\delta_{12}^d)_{AB} (\delta_{12}^d)_{CD}\), with A, B, C, D=(L,R) for different values of \(x = m_\tilde{q}^2 / m_\tilde{q}^2\) with one mass insertion each time, including gluino and neutralino contributions and the SM contribution set to zero. The results in brackets represent gluino-only contribution.
\[
\begin{array}{|c|c|c|c|}
\hline
& m_\tilde{q} = 200 \text{ GeV} & m_\tilde{q} = 500 \text{ GeV} & m_\tilde{q} = 1000 \text{ GeV} \\
\hline
x & \sqrt{\text{Im}(\delta_{12}^d)_{\text{LL}}} & \sqrt{\text{Im}(\delta_{12}^d)_{\text{LR}}} & \sqrt{\text{Im}(\delta_{12}^d)_{\text{RL}}} \\
\hline
0.25 & 8.7 \times 10^{-4} & 2.5 \times 10^{-3} & 5.4 \times 10^{-3} \\
1.0 & 1.9 \times 10^{-3} & 5.1 \times 10^{-3} & 1.1 \times 10^{-2} \\
4.0 & 1.4 \times 10^{-2} & 1.7 \times 10^{-2} & 3.2 \times 10^{-2} \\
\hline
x & \sqrt{\text{Im}(\delta_{12}^d)_{\text{RR}}} & \sqrt{\text{Im}(\delta_{12}^d)_{\text{LR}(\delta_{12}^d)_{\text{RR}}}} & \sqrt{\text{Im}(\delta_{12}^d)_{\text{LR}(\delta_{12}^d)_{\text{RL}}}} \\
\hline
0.25 & 8.8 \times 10^{-4} & 2.5 \times 10^{-3} & 5.4 \times 10^{-3} \\
1.0 & 2.0 \times 10^{-3} & 5.5 \times 10^{-3} & 1.2 \times 10^{-2} \\
4.0 & 7.2 \times 10^{-3} & 4.5 \times 10^{-2} & 6.3 \times 10^{-2} \\
\hline
0.25 & 3.7 \times 10^{-5} & 9.7 \times 10^{-5} & 2.0 \times 10^{-4} \\
1.0 & 4.3 \times 10^{-5} & 1.1 \times 10^{-4} & 2.3 \times 10^{-4} \\
4.0 & 6.3 \times 10^{-5} & 1.6 \times 10^{-4} & 3.3 \times 10^{-4} \\
\hline
0.25 & 5.0 \times 10^{-6} & 1.3 \times 10^{-5} & 2.5 \times 10^{-4} \\
1.0 & 8.9 \times 10^{-5} & 2.2 \times 10^{-4} & 4.3 \times 10^{-4} \\
4.0 & 2.0 \times 10^{-4} & 4.9 \times 10^{-4} & 9.2 \times 10^{-4} \\
\hline
\end{array}
\]

Table 2  Limits on \(\text{Im}(\delta_{12}^d)_{AB}(\delta_{12}^d)_{\text{CD}}\) with A, B, C, D=(L,R) for different values of 
\(x = m_\tilde{g}^2/m_\tilde{q}^2\) with one mass insertion each time, including gluino and neutralino contributions and the SM contribution set to zero. The results in brackets represent gluino-only contribution.
| $M_L / m_\tilde{q}$ (GeV) | 300   | 500   | 700   |
|--------------------------|-------|-------|-------|
| 150                      | $6.2 \times 10^{-2}$ | $8.1 \times 10^{-2}$ | $1.0 \times 10^{-1}$ |
| 250                      | $9.0 \times 10^{-2}$ | $1.0 \times 10^{-1}$ | $1.2 \times 10^{-1}$ |
| 350                      | $1.2 \times 10^{-1}$ | $1.3 \times 10^{-1}$ | $1.5 \times 10^{-1}$ |
| 450                      | $1.7 \times 10^{-1}$ | $1.6 \times 10^{-1}$ | $1.7 \times 10^{-1}$ |

| $\sqrt{|\text{Re}(\delta_{12}^u)^2_{LL}|}$ |
|--------------------------|-------|-------|-------|
| 150                      | $6.9 \times 10^{-2}$ | $8.7 \times 10^{-2}$ | $1.1 \times 10^{-1}$ |
| 250                      | $9.9 \times 10^{-2}$ | $1.1 \times 10^{-1}$ | $1.3 \times 10^{-1}$ |
| 350                      | $1.4 \times 10^{-1}$ | $1.4 \times 10^{-1}$ | $1.5 \times 10^{-1}$ |
| 450                      | $1.8 \times 10^{-1}$ | $1.7 \times 10^{-1}$ | $1.8 \times 10^{-1}$ |

| $\sqrt{|\text{Re}(\delta_{12}^u)^2_{RR}|}$ |
|--------------------------|-------|-------|-------|
| 150                      | $1.6 \times 10^{-1}$ | $3.0 \times 10^{-1}$ | $5.0 \times 10^{-1}$ |
| 250                      | $1.3 \times 10^{-2}$ | $2.0 \times 10^{-2}$ | $2.9 \times 10^{-2}$ |
| 350                      | $1.6 \times 10^{-2}$ | $2.3 \times 10^{-2}$ | $3.0 \times 10^{-2}$ |
| 450                      | $1.7 \times 10^{-2}$ | $2.2 \times 10^{-2}$ | $2.9 \times 10^{-2}$ |

| $\sqrt{|\text{Re}(\delta_{12}^u)^2_{LR}|}$ |
|--------------------------|-------|-------|-------|
| 150                      | $8.1 \times 10^{-2}$ | $1.6 \times 10^{-1}$ | $2.6 \times 10^{-1}$ |
| 250                      | $6.9 \times 10^{-3}$ | $1.0 \times 10^{-2}$ | $1.5 \times 10^{-2}$ |
| 350                      | $8.4 \times 10^{-3}$ | $1.2 \times 10^{-2}$ | $1.5 \times 10^{-2}$ |
| 450                      | $8.8 \times 10^{-3}$ | $1.1 \times 10^{-2}$ | $1.5 \times 10^{-2}$ |

| $\sqrt{|\text{Re}(\delta_{12}^u)^2_{LR}|}$ |
|--------------------------|-------|-------|-------|
| 150                      | $9.7 \times 10^{-3}$ | $1.2 \times 10^{-2}$ | $1.5 \times 10^{-2}$ |
| 250                      | $1.4 \times 10^{-2}$ | $1.5 \times 10^{-2}$ | $1.8 \times 10^{-2}$ |
| 350                      | $1.9 \times 10^{-2}$ | $1.9 \times 10^{-2}$ | $2.1 \times 10^{-2}$ |
| 450                      | $2.5 \times 10^{-2}$ | $2.4 \times 10^{-2}$ | $2.5 \times 10^{-2}$ |

**Table 3** Limits on $\text{Re}(\delta_{12}^u)_{AB}$, $\text{Re}(\delta_{12}^u)_{CD}$, with $A$, $B$, $C$, $D=(L,R)$ for different values of $M_L = M_R$ and $m_\tilde{q}$ with one mass insertion each time, with $\tan \beta = 5$ and $\mu_H = 200$ GeV. The bounds are insensitive to $\tan \beta$ in the range of $2 - 30$ and to $\mu_H$ in the range of $200 - 500$ GeV.
Table 4  Limits on $\text{Im}(\delta_{12}^u_{AB})$, with $A$, $B$, $C$, $D=(L,R)$ for different values of $M_L = M_R$ and $m_{\tilde{q}}$ with one mass insertion each time, with tan $\beta = 5$ and $\mu_H = 200$ GeV. The bounds are insensitive to tan $\beta$ in the range of 2 – 30 and to $\mu_H$ in the range of 200 – 500 GeV.
\begin{table}
\begin{tabular}{|c|c|c|c|}
\hline
 & $m_\tilde{q} = 200$ GeV & $m_\tilde{q} = 500$ GeV & $m_\tilde{q} = 1000$ GeV \\
\hline
$x$ & $\sqrt{|\text{Re}(\delta^d_{12})|^2_{LL}}, \quad (\delta^d_{12})_{RR} = (\delta^d_{12})_{LL}$ & & \\
\hline
0.25 & $4.7 \times 10^{-4}$ & $1.2 \times 10^{-3}$ & $2.5 \times 10^{-3}$ \\
1.0 & $5.4 \times 10^{-4}$ & $1.4 \times 10^{-3}$ & $2.9 \times 10^{-3}$ \\
4.0 & $7.9 \times 10^{-4}$ & $2.0 \times 10^{-3}$ & $4.1 \times 10^{-3}$ \\
\hline
$x$ & $\sqrt{|\text{Re}(\delta^d_{12})|^2_{LR}}, \quad (\delta^d_{12})_{RL} = (\delta^d_{12})_{LR}$ & & \\
\hline
0.25 & $8.2 \times 10^{-4}$ & $2.0 \times 10^{-3}$ & $3.9 \times 10^{-3}$ \\
1.0 & $6.9 \times 10^{-3}$ & $8.8 \times 10^{-3}$ & $1.2 \times 10^{-2}$ \\
4.0 & $2.1 \times 10^{-3}$ & $5.9 \times 10^{-3}$ & $1.4 \times 10^{-2}$ \\
\hline
\end{tabular}
\caption{Limits on $\text{Re}(\delta^d_{12})_{AB}(\delta^d_{12})_{CD}$, with $A, B, C, D=(L,R)$ for different values of $x = m_\tilde{g}^2/m_\tilde{q}^2$ with two mass insertions each time, including gluino and neutralino contributions and the SM contribution set to zero.}
\end{table}

\begin{table}
\begin{tabular}{|c|c|c|c|}
\hline
 & $m_\tilde{q} = 200$ GeV & $m_\tilde{q} = 500$ GeV & $m_\tilde{q} = 1000$ GeV \\
\hline
$x$ & $\sqrt{|\text{Im}(\delta^d_{12})|^2_{LL}}, \quad (\delta^d_{12})_{RR} = (\delta^d_{12})_{LL}$ & & \\
\hline
0.25 & $3.7 \times 10^{-5}$ & $9.7 \times 10^{-5}$ & $2.0 \times 10^{-4}$ \\
1.0 & $4.3 \times 10^{-5}$ & $1.1 \times 10^{-4}$ & $2.3 \times 10^{-4}$ \\
4.0 & $6.3 \times 10^{-5}$ & $1.6 \times 10^{-4}$ & $3.3 \times 10^{-4}$ \\
\hline
$x$ & $\sqrt{|\text{Im}(\delta^d_{12})|^2_{LR}}, \quad (\delta^d_{12})_{RL} = (\delta^d_{12})_{LR}$ & & \\
\hline
0.25 & $6.7 \times 10^{-5}$ & $1.6 \times 10^{-4}$ & $3.1 \times 10^{-4}$ \\
1.0 & $5.5 \times 10^{-4}$ & $7.1 \times 10^{-4}$ & $9.4 \times 10^{-4}$ \\
4.0 & $1.7 \times 10^{-4}$ & $4.8 \times 10^{-4}$ & $1.1 \times 10^{-3}$ \\
\hline
\end{tabular}
\caption{Limits on $\text{Im}(\delta^d_{12})_{AB}(\delta^d_{12})_{CD}$, with $A, B, C, D=(L,R)$ for different values of $x = m_\tilde{g}^2/m_\tilde{q}^2$ with two mass insertions each time, including gluino and neutralino contributions and the SM contribution set to zero.}
\end{table}
The bounds are insensitive to $\tan \beta$ of 200 GeV. The bounds are insensitive to $\tan \beta$ of 200 GeV. The bounds are insensitive to $\tan \beta$ of 200 GeV. The bounds are insensitive to $\tan \beta$ of 200 GeV. The bounds are insensitive to $\tan \beta$ of 200 GeV. The bounds are insensitive to $\tan \beta$ of 200 GeV.

| $M_L / m_\tilde{q}$ (GeV) | 300 | 500 | 700 |
|----------------------------|-----|-----|-----|
| $\sqrt{|\text{Re}(\delta_{12}^u)_{LL}|}$, $(\delta_{12}^u)_{RR} = (\delta_{12}^u)_{LL}$ | | | |
| 150 | $4.0 \times 10^{-2}$ | $5.6 \times 10^{-2}$ | $7.2 \times 10^{-2}$ |
| 250 | $6.8 \times 10^{-3}$ | $1.0 \times 10^{-2}$ | $1.5 \times 10^{-2}$ |
| 350 | $8.4 \times 10^{-3}$ | $1.1 \times 10^{-2}$ | $1.5 \times 10^{-2}$ |
| 450 | $8.8 \times 10^{-3}$ | $1.1 \times 10^{-2}$ | $1.4 \times 10^{-2}$ |
| $\sqrt{|\text{Re}(\delta_{12}^u)_{LR}|}$, $(\delta_{12}^u)_{RL} = (\delta_{12}^u)_{LR}$ | | | |
| 150 | $9.6 \times 10^{-3}$ | $1.2 \times 10^{-2}$ | $1.5 \times 10^{-2}$ |
| 250 | $7.7 \times 10^{-3}$ | $1.0 \times 10^{-2}$ | $1.3 \times 10^{-2}$ |
| 350 | $9.8 \times 10^{-3}$ | $1.2 \times 10^{-2}$ | $1.5 \times 10^{-2}$ |
| 450 | $1.1 \times 10^{-2}$ | $1.3 \times 10^{-2}$ | $1.6 \times 10^{-2}$ |

**Table 7** Limits on $\text{Re}(\delta_{12}^u)_{AB}(\delta_{12}^u)_{CD}$, with A, B, C, D=(L,R) for different values of $M_L = M_R$ and $m_\tilde{q}$ with two mass insertions each time, with $\tan \beta = 5$ and $\mu_H = 200$ GeV. The bounds are insensitive to $\tan \beta$ in the range of 2 – 30 and to $\mu_H$ in the range of 200 – 500 GeV.

| $M_L / m_\tilde{q}$ (GeV) | 300 | 500 | 700 |
|----------------------------|-----|-----|-----|
| $\sqrt{|\text{Im}(\delta_{12}^u)_{LL}|}$, $(\delta_{12}^u)_{RR} = (\delta_{12}^u)_{LL}$ | | | |
| 150 | $3.2 \times 10^{-3}$ | $4.5 \times 10^{-3}$ | $5.7 \times 10^{-3}$ |
| 250 | $5.5 \times 10^{-4}$ | $8.3 \times 10^{-4}$ | $1.2 \times 10^{-3}$ |
| 350 | $6.7 \times 10^{-4}$ | $9.2 \times 10^{-4}$ | $1.2 \times 10^{-3}$ |
| 450 | $7.0 \times 10^{-4}$ | $9.1 \times 10^{-4}$ | $1.2 \times 10^{-3}$ |
| $\sqrt{|\text{Im}(\delta_{12}^u)_{LR}|}$, $(\delta_{12}^u)_{RL} = (\delta_{12}^u)_{LR}$ | | | |
| 150 | $7.7 \times 10^{-4}$ | $9.7 \times 10^{-4}$ | $1.2 \times 10^{-3}$ |
| 250 | $6.2 \times 10^{-4}$ | $8.4 \times 10^{-4}$ | $1.1 \times 10^{-3}$ |
| 350 | $7.9 \times 10^{-4}$ | $9.9 \times 10^{-4}$ | $1.2 \times 10^{-3}$ |
| 450 | $8.7 \times 10^{-4}$ | $1.1 \times 10^{-3}$ | $1.3 \times 10^{-3}$ |

**Table 8** Limits on $\text{Im}(\delta_{12}^u)_{AB}(\delta_{12}^u)_{CD}$, with A, B, C, D=(L,R) for different values of $M_L = M_R$ and $m_\tilde{q}$ with two mass insertions each time, with $\tan \beta = 5$ and $\mu_H = 200$ GeV. The bounds are insensitive to $\tan \beta$ in the range of 2 – 30 and to $\mu_H$ in the range of 200 – 500 GeV.