High-Precision Direction-of-Arrival Estimations Using Digital Programmable Metasurface

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Commonly, phased array antennas are used to provide spatial resolution for direction of arrival (DOA) estimations. However, the phase-array-based DOA estimations usually require complicated system architecture and signal processing. Digital programmable metasurfaces (DPMs) can realize flexible spatial modulations of electromagnetic (EM) waves without using massive transceivers, and hence can be used for DOA estimations with much simplified architectures. Here, a mechanism of DPM-based DOA estimations at Ka band is proposed. A reflective DPM is designed to produce a series of dual beams in random, so as to establish a sensing matrix and sample the incident wave. Orthogonal matched pursuit algorithm is used to estimate DOA from the sparsely sampled datasets. The performances of the DPM-based DOA estimations are demonstrated by simulations and experiments for the cases of single source and double sources. This work promotes the development of DPMs and is expected to find important applications in radar and wireless communication systems, as well as the joint radar and communication systems.

1. Introduction

As one of the key technologies of radar and wireless communications, direction of arrival (DOA) estimation has been studied for many years. Several algorithms have been proposed to improve the resolution of DOA estimation, such as multiple signal classification (MUSIC) algorithm[1] and estimating signal parameters via rotational invariance techniques (ESPRIT) algorithm,[2] as well as a lot of optimized algorithms such as maximum likelihood (ML),[3] weighted subspace fitting (WSF) algorithm,[4] and orthogonal propagator (OP) algorithm.[5] Basically, all these algorithms are based on the hypothetic premise that phase-array antennas are used in the DOA estimations.[6,7] However, the array antennas are usually accompanied with complex hardwares and high costs.

Digital programmable metasurfaces (DPMs), being consisted of massive digital meta-unit structures, can modulate electromagnetic (EM) waves in space, time, and frequency domains, and hence have been explored in many areas such as dynamic beam forming,[8-11] active polarization regulation,[14-16] active holographic imaging,[17] intelligent sensing,[18,19] and space-time modulations.[20-28] These researches have shown great potentials of DPMs in radar and wireless communication systems. Due to the capability of dynamic controlling the EM waves, DPM can be a perfect hardware platform to provide the necessary spatial resolution for the DOA estimations. In term of hardware design, the DPM-based DOA estimations possess the simplified system architectures and reduced system costs; in terms of signal processing, they are well suited for the compressed sensing (CS) theory[29-34] and the orthogonal matched pursuit (OMP)[35,36] algorithm, which has been widely used to recover signals from sparsely sampled datasets. Hence, the DPM-based DOA estimation can innovate the traditional DOA estimations in both hardware design and signal processing. Several metasurface-based DOA estimations have been proposed recently.[37,38] Wang and Caloz[37] proposed to obtain the full amplitude and phase distributions on the metasurface at the same time by applying space-time modulations. With the amplitude and phase distributions, DOA estimations can be performed by using any conventional digital algorithm, such as MUSIC and ESPRIT. Lin et al.[18] used completely random radiation patterns to build the sensing matrix; however, the completely random radiation patterns are more vulnerable to discrete errors of the programmable metasurface.

In this work, a mechanism is proposed to realize high-precision DOA estimation based on a DPM. The OMP algorithm is used to recover the information of the sources. Random dual beams instead of completely random radiation patterns are used in this mechanism to produce the sensing matrix and sample the incident wave. The orthogonality of the sensing matrix is guaranteed by specifying different directions of the dual beams. A reflective DPM working at 28 GHz is fabricated and is used to perform the DOA estimations for single source and double...
sources. The measured results are in very good agreements with the calculations, validating the effectiveness of the presented mechanism of DOA estimation. Compared with existed works, the contributions of our work can be summarized as following: 1) we propose to use random dual beams rather than completely random radiation patterns to build the sensing matrix. Because array theory is very efficient for single beam or dual beams, but is not that accurate for completely random patterns due to discrete errors of the programmable metasurface and 2) the programmable metasurface in our work is work at Ka band. Hence, the presented prototype in our work can be directly used in millimeter wave communications.

2. Design of Reflective DPM

The reflective DPM consists of 400 (20 × 20) elements and is illuminated obliquely by an x-polarized horn antenna with the incident angle of 45°, as shown in Figure 1a. The distance between the center of DPM and the phase center of the horn antenna is 150 mm. Each unit element of DPM consists of three metallic layers and three dielectric layers, as shown in Figure 1b. A positive-intrinsic-negative (PIN) diode (MA4AGP907) is integrated on the top metallic layer. The effective circuits of the PIN diode at different states are given in Figure 1c.[39,40] Figure 1d,e illustrate the reflection coefficients of the unit element when the PIN diode is switched “on” and “off.” It is observed that reflection coefficients have similar amplitudes but opposite phases at 28 GHz, hence forming a binary element at 28 GHz in term of the reflection phases. Multi-bit element can also realized by using varactor diode or multiple PIN diodes, but these schemes need much more complexity DC feeding network, which decrease the flexibility and convenience of the metasurface-based applications.

In Figure 2a,b, the scattering patterns of the element at state “on” and “off” are nearly identical due to their subwavelength size. Although the element patterns are not perfect omnidirectional, but they are approximately omnidirectional in a certain range of elevation angle (around 45°). Figure 2c–f, gives the reflection coefficients for different incident angles. E-Plane means the incident polarization is parallel to conducting direction of PIN diode. H-Plane means the incident polarization is perpendicular to conducting direction of PIN diode. It is observed that, in each case of polarization, the incident angles have little influence on the reflection coefficients; hence, it is safe to perform the design based on the vertical-incident coefficients.

By configuring all elements, the DPM can produce different radiation patterns dynamically. Details of synthesizing aperture codes of the DPM from specified radiation patterns can be found in a previous work.[12] Figure 3 gives an example to produce dual beams by the DPM. Figure 3a is the synthesized aperture code, in which the yellow and blue patches represent the elements that are switched “on” and “off,” respectively. Single cells rather than super cells are used here to design the programmable metasurface. Because a super cell consists of identical cells, this scheme will increase cell period and result in bigger discrete errors for a programmable metasurface with limited aperture size. Figure 3b,c are radiation patterns calculated from the synthesized aperture code, and Figure 3d is the measured result. It is founded that the main lobes are consistent with the calculations, but the side lobes show deviations due to the errors of theoretical models and fabrications. To alleviate the impact...
Figure 2. Scattering patterns of the element at a) “on” state and at b) “off” state. c–f) Reflection coefficients of different incident angles. c) Amplitude of “on” state. d) Amplitude of “off” state. e) Phase of “on” state. f) Phase of “off” state.

Figure 3. An example of designing dual beams by the DPM. a) Digital coding schemes of the dual beams. The dual beams point at \( \theta_1 = -30^\circ \) and \( \theta_2 = 10^\circ \), respectively. b) Calculated three-dimensional far-field pattern. c) Calculated 2D far-field pattern on E-plane. d) Tested result in a microwave anechoic chamber.
resulted from errors of side lobes, we have used random dual beams instead of completely random radiation patterns for the DPM-based DOA estimations.

3. Mathematical Model of DOA Estimation

Assuming that an antenna array composed of N elements is illuminated by M incident sources from the far-field region, the signal of the i-th element received from the m-th source can be expressed as (omitting the time harmonics and noise term)

\[ X_{mi} = s_m e^{j2\pi \left( \frac{x_i}{\lambda} \sin \theta_m \cos \phi_m + \frac{y_i}{\lambda} \sin \theta_m \sin \phi_m \right)} \]  

(1)

where \( s_m \) is the intensity of the m-th source. For the reflective DPM, the signals received by all elements are focused to the receiving horn antenna, as illustrated in Figure 4. Therefore, the signal received from the m-th source at the horn antenna can be expressed as

\[ H_m = \sum_{i=1}^{N} s_m \left( -1 \right)^{q_i} e^{j2\pi \left( \frac{x_i}{\lambda} \sin \theta_m \cos \phi_m + \frac{y_i}{\lambda} \sin \theta_m \sin \phi_m \right) \sqrt{\frac{\lambda^2}{\sin^2 \theta_m} (x_i-x)^2 + (y_i-y)^2} \]  

(2)

where q is set to be 1 or 0, corresponding to the binary code of each element; (\( x, y, z \)) is the relative coordinates of phase center of the horn antenna.

In Equation (2), the aperture code of the DPM can be defined arbitrarily. If the DPM is configured by L kinds of aperture codes, then the horn antenna will sample the receiving signals L times. This procedure can be expressed by the following matrix

\[ W_L = H_{LM} S_M \]  

(3)

where \( W_L = [w_1, w_2, \ldots, w_L]^T \) is the sampling vector; \( H_{LM} \in \mathbb{R}^{L \times M} \) is the sensing matrix; and \( S_M \in \mathbb{R}^{M \times 1} \) is the unknown sparse source vector whose nonzero elements represent the sources. OMP algorithm can be used to evaluate \( S_M \). Essentially, the OMP algorithm is a series of orthogonal projections on the sensing matrix, and the projection coefficients constitute the estimated source vector. Hence, the sensing matrix plays an important role in the process of an efficient DOA estimation. In the case of the DPM-based DOA estimations, it means that the choice of coding schemes of the DPM is important for efficient DOA estimations. Basically, random coding schemes should be used to improve orthogonality of the sensing matrix; however, completely random radiation patterns produced by the DPM are more vulnerable to interferences from errors of theoretical models and fabrications. Therefore, by comprehensive considerations, we have built the sensing matrix in this work by designing coding schemes, which produce random dual beams. Each dual-beam pattern contains two main lobes pointing at different directions randomly, and all the dual-beam patterns cover the whole testing space to guarantee the efficiency of DOA estimations.

4. Simulations and Experiments

The sensing matrix is established by 40 sets of aperture codes of DPM, and the correlation coefficients of the sensing matrix \( H_{LM} \) are given in Figure 5a. It shows that different columns of the sensing matrix possess low correlation coefficients. Figure 5b displays the spatial distributions of the radiation fields corresponding to the 40 sets of aperture codes. For simplicity, the random dual beams radiated from DPM are defined on the horizontal plane, and so are the sources. Figure 5c shows the averaged radiation energy of all 40 radiation patterns. It can be observed that random dual beams have covered the testing scope from -60° to 60°.

To illustrate the procedures of DPM-based DOA estimations, Figure 5d–i give the simulated results when single and double sources illuminate the DPM. For the case of single source, the incident angle is 30°; for the case of double sources, the incident angles are 30° and -20°. In both cases, the DPM is configured by 40 coding schemes to sample the incident wave for 40 times. All the sampling dates form an observation vector. Obviously, when the incident source are located in the angular regions of main lobes, more energy will be received by the DPM, as indicated by the peaks in Figure 5d,g. Based on the sensing matrix and observation vectors, projection method and OMP algorithm can be used to perform the estimations. Figure 5e,f give the estimated DOA of single source, and Figure 5h,i give the estimated DOA of double sources. It is observed that, although direct projections can give DOA estimations, but OMP algorithm can give much more distinct estimations due to the orthogonal characteristic. In the case of double sources, the OMP algorithm has used the assumption that the number of sources have already been known.

We established a testing system in a microwave anechoic chamber to verify the DPM-based DOA estimations, as shown in Figure 6. Here, the DPM is used as the receiver and is configured by 40 sets of coding schemes. For the case of single
A horn antenna is used as the transmitter and the incidence angle is set to be $25^\circ$. The receiver and the transmitter are connected to a vector network analyzer. For the case of two sources, two horn antennas are used as the transmitters. The two transmitters are connected to the same port of the vector network analyzer through a power combiner so that the vector network analyzer can simultaneously receive the energy from the two transmitters. The incident angles of the two transmitters are randomly set to be $34^\circ$ and $13^\circ$. In both cases, the distance between the transmitters and the DPM are 90 cm.

During each test, the DPM is configured by one of the 40 sets of coding schemes, and the vector network analyzer records the corresponding scattering parameter ($S_{21}$). It is noted that, due to multiple reflections between the sources and the DPM, time gate has to be set in the vector network analyzer in each test so that the recorded scattering parameter contains only single incident value. By 40 times of tests, the observation vector $\mathbf{W}_L$ can be obtained. Figure 6c,d are the observation vectors of single source obtained by calculations and tests, respectively. Very good consistency can be observed from these two curves.

Correspondingly, the calculated and the tested DOA are also consistent with each other, as shown in Figure 6g,h that are recovered by projection method, and Figure 6k,l that are recovered by OMP algorithm. For double sources, Figure 6e,f give the calculated and tested observation values, respectively. Again, the calculations and measurements are in very good agreements with each other. Figure 6i,j are estimated DOA of double sources by using projection method, and Figure 6m,n are estimated results by using OMP algorithm. According to the experiments, the OMP algorithm used only 40 samples to recover the incident angles of single and double sources that are located in angular region from $-60^\circ$ to $60^\circ$, and the evaluation error is less than $1^\circ$, thus validating the correctness and effectiveness of the presented mechanism of the DPM-based DOA estimation.

5. Conclusion
In this work, we presented a CS mechanism to realize the high-precision DOA estimations based on DPM at the Ka
Digital coding schemes that producing random dual beams are used to design the sensing matrix, and the OMP algorithm is used to estimate the incident angles of the sources. The measured results are in very good agreement with the calculated results. Both calculations and measurements have shown that the proposed DPM-based mechanism can effectively achieve the DOA estimations of multiple sources, hence showing great potentials in the fields of radar, wireless communications, as well as joint radar and communications.

Figure 6. Experiments of DOA estimations. a,b) Experimental setup for the DOA estimation of single source and double sources, respectively. c,d) The observation values of single source obtained by calculations and tests. e,f) The observation values of double sources obtained by calculations and tests. g,k) The calculated DOA estimations of single source. h,j) The tested of DOA estimations of single source. i,m) The calculated DOA estimations of double sources. j,n) The tested of DOA estimations of double sources.

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Conflict of Interest
The authors declare no conflict of interest.

Data Availability Statement
The data that support the findings of this study are available from the corresponding author upon reasonable request.

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[1] R. Schmidt, IEEE Trans. Antennas Propag. 1986, 34, 276.
[2] R. Roy, T. Kailath, IEEE Trans. Acoust. Speech Signal Process. 1989, 37, 984.
[3] P. Stoica, K. C. Sharman, IEEE Trans. Acoust. Speech Signal Process. 1990, 38, 1132.
[4] R. Hamza, K. Buckley, IEEE Trans. Signal Process. 1994, 42, 2520.
[5] J. Munier, G. Y. Delisle, IEEE Trans. Signal Process. 1991, 39, 746.
[6] F. Zardi, P. Nayeri, P. Rocca, R. Haupt, IEEE Antennas Propag. Mag. 2021, 63, 28.
[7] A. Sharma, S. Mathur, IETE Tech. Rev. 2016, 33, 472.
[8] H. Yang, X. Cao, F. Yang, J. Gao, S. Xu, M. Li, X. Chen, Y. Zhao, Y. Zheng, S. Li, Sci. Rep. 2016, 6, 35692.
[9] X. Wan, T. Y. Chen, X. Q. Chen, L. Zhang, T. J. Cui, IEEE Trans. Antennas Propag. 2018, 66, 4942.
[10] X. Wan, M. Q. Qi, T. Y. Chen, T. J. Cui, Sci. Rep. 2016, 6, 20663.
[11] W. Li, T. Qiu, J. Wang, Y. Zheng, Y. Jing, J. Jia, H. Wang, Y. Han, S. Qu, IEEE Trans. Antennas Propag. 2021, 69, 296.
[12] X. Wan, Q. Xiao, Y. Z. Zhang, Y. Li, J. Eisenbeis, J. W. Wang, Z. A. Huang, H. X. Liu, T. Zwick, T. J. Cui, IEEE Antennas Wirel. Propag. Lett. 2021, 20, 381.
[13] J. Shabanpour, S. Beyraghi, A. Cheldavi, Sci. Rep. 2020, 10, 8950.
[14] L. Chen, Q. Ma, Q. F. Nie, Q. R. Hong, H. Y. Cui, Y. Ruan, T. J. Cui, Photon. Res. 2021, 9, 116.
[15] Q. Ma, Q. R. Hong, G. D. Bai, H. B. Jing, R. Y. Wu, L. Bao, Q. Cheng, T. J. Cui, Phys. Rev. Appl. 2020, 13, 021003.
[16] J. Shabanpour, S. Beyraghi, F. Ghorbani, H. Oraizi, ArXiv210102298 Phys. 2021.
[17] L. Li, T. Jun Cui, W. Ji, S. Liu, J. Ding, X. Wan, Y. Bo Li, M. Jiang, C.-W. Qiu, S. Zhang, Nat. Commun. 2017, 8, 197.
[18] Q. Ma, Q. R. Hong, X. X. Gao, H. B. Jing, C. Liu, G. D. Bai, Q. Cheng, T. J. Cui, Nanophotonics 2020, 9, 3271.
[19] Q. Ma, G. D. Bai, H. B. Jing, C. Yang, L. Li, T. J. Cui, Light Sci. Appl. 2019, 8, 98.
[20] X. Wan, C. K. Xiao, H. Huang, Q. Xiao, W. Xu, J. W. Wang, Z. A. Huang, Q. Cheng, S. Jin, T. J. Cui, Adv. Mater. Technol. 2021, 6, 2001254.
[21] L. Zhang, X. Q. Chen, S. Liu, Q. Zhang, J. Zhao, J. Y. Dai, G. D. Bai, X. Wan, Q. Cheng, G. Castaldi, V. Galdi, T. J. Cui, Nat. Commun. 2018, 9, 4334.
[22] J. Zhao, X. Yang, J. Y. Dai, Q. Cheng, X. Li, N. H. Qi, J. C. Ke, G. D. Bai, S. Liu, S. Jin, A. Alù, T. J. Cui, Nat. Sci. Rev. 2019, 6, 231.
[23] J. Y. Dai, J. Zhao, Q. Cheng, T. J. Cui, Light Sci. Appl. 2018, 7, 90.
[24] X. Wan, Q. Zhang, T. Y. Chen, L. Zhang, W. Xu, H. Huang, C. Kun Xiao, Q. Xiao, T. Jun Cui, Light Sci. Appl. 2019, 8, 60.
[25] X. Wang, C. Caloz, Engineering Archive, 2021, DOI: 10.31224/osf.io/pzijr.
[26] T.-Y. Hwang, Y. Choi, Y. Song, N. S. A. Eom, S. Kim, H.-B. Cho, N. V. Myung, Y.-H. Choa, J. Mater. Chem. C 2018, 6, 972.
[27] X. Wang, C. Caloz, IEEE Trans. Antennas Propag. 2021, 69, 286.
[28] J. Shabanpour, Ann. Phys. 2020, 532, 2000321.
[29] D. L. Donoho, IEEE Trans. Inf. Theory 2006, 52, 1289.
[30] G.-H. Chen, J. Tang, S. Leng, Med. Phys. 2008, 35, 660.
[31] M. Lustig, D. Donoho, J. M. Pauly, Magn. Reson. Med. 2007, 58, 1182.
[32] E. J. Candès, M. B. Wakin, IEEE Signal Process. Mag. 2008, 25, 21.
[33] S. Fortunati, R. Grasso, F. Gini, M. S. Greco, K. LePage, EURASIP J. Adv. Signal Process. 2014, 2014, 120.
[34] A. Xenaki, P. Gerstoft, K. Mosgaard, J. Acoust. Soc. Am. 2014, 136, 260.
[35] J. A. Tropp, A. C. Gilbert, IEEE Trans. Inf. Theory 2007, 53, 4655.
[36] T. Hoshikawa, T. Nishimura, T. Ohgane, Y. Ogawa, J. Hagiwara, in 15th Workshop Position. Navig. Commun., IEEE, Piscataway, NJ 2018.
[37] X. Wang, C. Caloz, in IEEE Int. Symp. Antennas Propag. USNC-URSI Radio Sci. Meet., IEEE, Piscataway, NJ 2019, pp. 1613–1614.
[38] M. Lin, M. Xu, X. Wan, H. Liu, Z. Wu, J. Liu, B. Deng, D. Guan, S. Zha, IEEE Internet Things J. 2021, 8, 10187.
[39] L. Di Palma, A. Clemente, L. Dussopt, R. Sauleau, P. Potier, P. Pouliguen, IEEE Antennas Wirel. Propag. Lett. 2016, 15, 560.
[40] A. Clemente, L. Dussopt, R. Sauleau, P. Potier, P. Pouliguen, IEEE Trans. Antennas Propag. 2012, 60, 2260.