Lepton flavor non-universality in $b \to s\ell^+\ell^-$ processes

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We explore a scenario of New Physics entering the description of $B \to K^{(*)}\mu\mu$ decay through couplings to the operators $O_{9,10}^c$, satisfying $C'_9 = -C'_{10}$. From the current data on $\mathcal{B}(B_s \to \mu\mu)$ and $\mathcal{B}(B \to K\mu\mu)_{[15,22] GeV^2}$, we obtain constraints on Re$C'_{10}$ and Im$C'_{10}$ which we then assume to be lepton specific, and find $R_K = \mathcal{B}(B \to K\mu\mu) / \mathcal{B}(B \to K\mu\mu)_{[1,6] GeV^2} = 0.88(8)$, consistent with recent value measured at LHCb. A specific realization of this scenario is the one with a scalar leptoquark state $\Delta$, in which $C'_{10}$ is related to the mass of $\Delta$ and its Yukawa couplings. We then show that this scenario does not make any significant impact on $B_s - \bar{B}_s$ mixing amplitude nor to $\mathcal{B}(B \to K\nu\bar{\nu})$. Instead, it can modify $R_{K^*} = \mathcal{B}(B \to K^*\mu\mu) / \mathcal{B}(B \to K^*\mu\mu)_{[1,6] GeV^2}$, which will soon be experimentally measured and we find it to be $R_{K^*} = 1.11(8)$, while $R_{K^*}/R_K = 0.27(19)$. A similar ratio of forward-backward asymmetries also becomes lower than in the Standard Model.

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I. INTRODUCTION

The $b \to s$ transitions were in the focus of many theoretical and experimental studies during the last two decades due to the possibility to constrain potential New Physics (NP) contributions at low energies. With LHC7 and LHC8 runs direct searches for NP became available. This gives us an excellent opportunity to question the appearance of physics beyond Standard Model (SM). At low energies $B$-factories and the LHCb experiment provided flavor physics community with a lot of rather precise results on $b \to s$ transitions. The LHCb experiment has observed slight discrepancies between the SM predictions and the experimental results for the angular observables in $B \to K^*\mu^+\mu^-$ decay. This effect has been attributed to NP, although the tension might be a result of the SM QCD effects. Recently, another anomaly in $b \to s\ell^+\ell^-$ transition has been found in the ratio of the branching fractions,

$$R_K = \frac{\mathcal{B}(B \to K\mu^+\mu^-)_{q^2 \in [1,6] GeV^2}}{\mathcal{B}(B \to K\ell^+\ell^-)_{q^2 \in [1,6] GeV^2}}. \quad (1)$$

LHCb Collaboration measured this ratio for the square of dilepton invariant mass in the bin $1 GeV^2 \leq q^2 \leq 6 GeV^2$, and found $[1]$,

$$R_{K^{LHCb}}^{LHCb} = 0.745 \pm 0.036, \quad (2)$$

lower than the SM prediction, $R_{K}^{SM} = 1.0003 \pm 0.0001$, in which next-to-next-to-leading QCD corrections, as well as $1/m_b$ corrections have been included $[2]$. In other words, the LHCb result points towards a $2.6 \sigma$ effect of the lepton flavor universality violation.

Furthermore, the combined data analysis of the $B_s \to \mu^+\mu^-$ events gathered at LHCb and CMS resulted in $\mathcal{B}(B_s \to \mu^+\mu^-) = (2.8^{+0.7}_{-0.6}) \times 10^{-9} [3]$, in good agreement with the SM prediction $\mathcal{B}(B_s \to \mu^+\mu^-) = (3.65 \pm 0.23) \times 10^{-9} [4]$. This offers an excellent probe of $b \to s\mu^+\mu^-$ transition in the light of SM and gives rather tight constraints on parameter space of many models of NP. The $R_K$ anomaly has been approached in the literature in different ways: either by using the effective Lagrangian approach or in a specific model of NP. For example the effective Lagrangian approach used in references $[5-7]$ indicated that in order to understand the measured value of $R_K$ one must include

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the effects of NP, and that the effects of non-perturbative QCD alone could not explain such a large deviation of $R_K$ from unity [5–19]. In particular, it was found that the NP contribution most likely affects $C_9$, $C_{10}$ or $C'_9$, $C'_{10}$ effective Wilson coefficients, and that some kind of lepton flavor universality violation is needed, e.g. $C_9^\mu \neq C_9^e$ [9, 20]. In order to determine whether $R_K$ anomaly is due to NP in electron or/and muon couplings through a combined analysis of several decay modes, it is very important to have a high precision knowledge of hadronic form factors [15–17], which can be computed in the region of large $q^2$s by means of numerical simulations of QCD on the lattice [21–23].

In this study we first use a model independent approach, assuming that NP contributes at low energies to an operator that is a product of a right-handed quark and a left-handed muon current. In the language of $b \to s\mu\mu$ effective Hamiltonian such a situation corresponds to a combination of Wilson coefficients $C'_9$ and $C'_{10}$, and that they obey $C'_9 = -C'_{10}$. Decays to the final states with electron-positron pair are instead governed by the SM only. We consider simultaneously the constraints posed by $B(B \to K\mu^+\mu^-)$ and $B(B_s \to \mu^+\mu^-)$ on such a scenario, and then predict the $R_K$ as well as $R_{K^*}$. We discuss other observables which might serve as additional probes of the observed lepton-flavor universality violation.

A specific realization of the scenario we discuss in this paper is a model with a light scalar leptoquark $\Delta$ with quantum numbers (3)\,SU(2)\,L × U(1)\,Y (3, 2, 1/6). It indeed verifies the relation, $C'_9 = -C'_{10}$, and leads to a consistency with the measured value of $R_K$. The features of this leptoquark state have been already described in the literature [24]. While there is no theoretical motivation to forbid leptoquark contributing to $b \to s\mu\mu$ decays, simultaneous presence of both muonic and electronic couplings could be problematic because they would, together, induce lepton flavor violation in $B_s \to e\mu$ and $\mu \to e\gamma$ decays. It is interesting that the flavor physics constraints at low energies agree and are complementary with the constraints obtained from the direct experimental searches at LHC. Furthermore, the atomic parity violation experiments provided a strong constraint on the interaction of the down-quark–electron interaction with the leptoquark state [24, 25], while the couplings to muons appear to be less constrained via $B(K_L \to \mu^+\mu^-e^+\bar{e}^-) < 4.7 \times 10^{-12}$ [24, 26]. We therefore assume in our analysis that in the $b \to s\ell^+\ell^-$ processes only the muons can interact with the leptoquark state. A few other leptoquark states have been discussed in the literature [5, 8, 13, 15] as possible candidates to contribute to the $R_K$ anomaly. However, the leptoquark with quantum numbers (3, 2, 1/6) has a desired feature that it can be light without destabilizing the charm (27). Notice also that another light leptoquark scalar state, not mediating the proton decay, is (3)\,SU(2)\,L × U(1)\,Y (3, 2, 1/6) as well as (5)\,SU(2)\,L × U(1)\,Y (5, 3, 2). Decays to the final states with electron-positron pair are instead governed by the SM only. We further summarize our findings in Sec. V.

II. EFFECTIVE HAMILTONIAN AND BASIC FORMULAS

The processes with flavor structure $(\bar{s}b)\,(\bar{\mu}\mu)$ at scale $\mu = \mu_b = 4.8$ GeV are governed by dimension-6 effective Hamiltonian [28]:

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \left[ \sum_{i=1}^{6} C_i(\mu) \mathcal{O}_i(\mu) + \sum_{i=7, \ldots, 10} (C'_i(\mu)\mathcal{O}_i(\mu) + C''_i(\mu)\mathcal{O}'_i(\mu)) \right]. \tag{3}$$

The contributions of the charged-current operators $\mathcal{O}_{1,2}$, QCD penguins $\mathcal{O}_{3,\ldots,6}$, and the electromagnetic (chromomagnetic) dipole operators $\mathcal{O}_7$ ($\mathcal{O}_8$) will be assumed to be saturated by the SM. On the other hand, operators involving a quark and a lepton current will contain the SM and potential NP contributions. The basis of operators may be further extended to account for possible (pseudo)scalar or tensor operators [21], whereas for the purposes of this work the following operators will suffice:

$$\mathcal{O}_7 = \frac{e}{g^2} m_b (\bar{s}\sigma_{\mu\nu}P_Rb) F^{\mu\nu}, \quad \mathcal{O}_8 = \frac{1}{g} m_b (\bar{s}\sigma_{\mu\nu}G^{\mu\nu}P_R b),$$
$$\mathcal{O}_9 = \frac{e^2}{g^2} (\bar{s}\gamma_\mu P_Lb)(\bar{\ell}\gamma^\nu\ell), \quad \mathcal{O}_{10} = \frac{e^2}{g^2} (\bar{s}\gamma_\mu P_Lb)(\bar{\ell}\gamma^\mu\gamma_5\ell). \tag{4}$$

Here $P_{L/R} = (1 \mp \gamma_5)/2$, while $e$ is the electromagnetic and $g$ the color gauge coupling. $F^{\mu\nu}$ and $G^{\mu\nu}$ are the electromagnetic and color field strength tensors, respectively. The basis is further extended by the wrong-chirality operators, $\mathcal{O}_{9,10}$, which are related to $\mathcal{O}_{9,10}$ by replacing $P_L \leftrightarrow P_R$ in the quark current.
A. $B \to K\mu^+\mu^-$

In calculating the amplitude for the $B \to K\mu^+\mu^-$ decay it is convenient to group the combinations of Wilson coefficients multiplying the same hadronic matrix element. Namely, the operators $O_{1-6}$ mix at leading order into $O_{7,8,9}$ and it is customary to define effective Wilson coefficients as $[31]$:

\begin{align}
C_{T,9}^{\text{eff}}(\mu_b) &= \frac{4\pi}{\alpha_s} C_7 - \frac{1}{3} C_3 - \frac{4}{9} C_4 - \frac{20}{3} C_5 - \frac{80}{9} C_6, \\
C_{9}^{\text{eff}}(\mu_b) &= \frac{4\pi}{\alpha_s} C_9 + Y(q^2), \\
C_{10}^{\text{eff}}(\mu_b) &= \frac{4\pi}{\alpha_s} C_{10}, \quad C_{7,8,9,10}^{\text{eff}}(\mu_b) = \frac{4\pi}{\alpha_s} C_{7,8,9,10}^b,
\end{align}

where the function $Y(q^2)$ at NLL can be found in Ref. [32]. We also incorporate the NNLL mixing of $O_1$ and $O_2$ into $O_7$ and $O_9$ as calculated in Ref. [33]. The Wilson coefficients on the right-hand sides are evaluated at $\mu = \mu_b$. For the sake of readability we will from here on discuss only the effective Wilson coefficients that will be addressed simply as “Wilson coefficients” and denoted without the “eff” label. The values of the SM Wilson coefficients at scale $\mu_b$ are $C_7 = -0.304$, $C_9 = 4.211$, and $C_{10} = -4.103$ [31, 32, 34].

The decay spectrum as a function of the invariant mass of the muon pair is given by

$$
\frac{d\Gamma}{dq^2}(B \to K\mu^+\mu^-) = 2a_{\mu}(q^2) + \frac{2}{3}c_{\mu}(q^2),
$$

where $q^2 = (p_{\mu^-} + p_{\mu^+})^2$, while functions $a_{\mu}(q^2)$, $c_{\mu}(q^2)$ are combinations of Wilson coefficients and hadronic form factors and their explicit expressions can be found in Ref. [21] and in the Appendix of the present paper in the limit of $m_\ell \to 0$. The rate depends on the sums of the Wilson coefficients of opposite chiralities, $C_7 + C_7^b$, $C_9 + C_9^b$, $C_{10} + C_{10}^b$, from what follows that even in principle we cannot determine the chirality of the quark-current in $B \to K\mu^+\mu^-$. Definitions of the hadronic form factors are relegated to the Appendix. We employ the form factors calculated in the unquenched lattice simulation using non-relativistic formulation of the $b$ quark and staggered fermion formulation for the light quarks [22]. We use the $z$-expansion to parameterize the form factors and take into account the statistical errors given by the covariance matrix of the parameters, both given in [22]. However, we neglect additional systematic errors that should come on top of the ones contained in the covariance matrix. The correlations between form factor parameters are propagated onto observables of interest, namely we can construct $\chi^2$ statistic for $B(B \to K\mu^+\mu^-)$ and $R_K$, that are functions of the form factor parameters, as well as the Wilson coefficients.

The LHCb collaboration measured partial branching fractions below and above the region of charmonium resonances. For the $q^2 > 15$ GeV$^2$ region we can predict the partial branching ratio using form factors determined on the lattice that are largely free from extrapolation errors and parameterization dependence. Thus we will use [35],

$$
\mathcal{B}(B^+ \to K^+\mu^+\mu^-)|_{q^2[15,22]}\text{GeV}^2 = (8.5 \pm 0.3 \pm 0.4) \times 10^{-8},
$$

as an experimental constraint, where the errors quoted are statistical and systematic, respectively. In our analysis we will sum the two and treat the observable with a Gaussian $\chi^2$.

B. $B_s \to \mu^+\mu^-$

This decay receives contributions from operators with axial, scalar, and pseudoscalar lepton currents, and, owing to the pseudoscalar nature of the $B_s$ meson, the wrong-chirality Wilson coefficients will affect the decay with opposite sign. In the absence of (pseudo)scalar operators, the amplitude is proportional to the difference $C_{10} - C_{10}^b$.

$$
P = \frac{2m_\mu}{m_{B_s}}(C_{10} - C_{10}^b),
$$

and the “theoretical” branching ratio is expressed as

$$
\mathcal{B}(B_s \to \mu^+\mu^-)_{\text{th}} = B_0 |P|^2, \quad B_0 = \frac{f_{B_s}^2 m_{B_s}^3 G_F^2 |V_{tb}V_{ts}|^2}{16\pi^3} \sqrt{1 - \frac{4m_\mu^2}{m_{B_s}^2}}.
$$

For the decay constant of the $B_s$ meson we take $f_{B_s} = (228 \pm 8)$ MeV, consistent with the average made by FLAG [36]. Due to $B_s - \bar{B}_s$ oscillations and relatively large $y_s = \Delta\Gamma_s/(2\Gamma_s)$ in the $B_s$ sector, the measured branching fraction
actually corresponds to a time-integrated rate of the oscillating $B_s$ system to $\mu^+\mu^-$ [37]. In effect, the value reported by the experimentalists is different from $\mathcal{B}(B_s \rightarrow \mu^+\mu^-)^{\text{th}}$:

$$\mathcal{B}(B_s \rightarrow \mu^+\mu^-)^{\text{exp}} = \frac{E_0}{1 - y_s} \left[ |P|^2 + y_s \text{Re}(P^2) \right].$$ \hfill (10)$$

Latest average of the LHCb and CMS measurements of $B_s \rightarrow \mu^+\mu^-$ branching fraction is [3]

$$\mathcal{B}(B_s \rightarrow \mu^+\mu^-)^{\text{exp}} = (2.8^{+0.7}_{-0.6}) \times 10^{-9}.$$ \hfill (11)$$

The relative decay width difference $\Delta \Gamma_s$ has been determined from LHCb simultaneous measurement of total width $\Gamma_s$ and width difference $\Delta \Gamma_s$ in decay channels $B_s \rightarrow J/\psi P^+P^-$ [38]. The above determined value agrees very well with the HFAG and PDG averages [26, 39]. In the fits we use the values for $\Gamma_s$ and $\Delta \Gamma_s$ reported by LHCb with summed statistical and systematic errors

$$\Delta \Gamma_s = (0.0805 \pm 0.0123) \text{ps}^{-1}, \quad \Gamma_s = (0.6603 \pm 0.0042) \text{ps}^{-1},$$ \hfill (12)$$

with correlation coefficient $-0.3$ [38].

III. NEW PHYSICS IN $C'_9 = -C'_{10}$ AND PREDICTION FOR $R_K$

We focus now on the SM extensions that affect the effective Hamiltonian solely by a single operator that is a product of right-handed quark current with a left-handed lepton current. In our operator basis it corresponds to a linear combination $O'_9 - O'_{10}$ implying

$$C'_9(\Lambda) = -C'_{10}(\Lambda),$$ \hfill (13)$$

where $\Lambda$ is a scale where NP degrees of freedom are integrated out. An explicit example of such a scenario can be made in a leptoquark model that will be discussed in Section IV. If Eq. (13) holds at scale $\Lambda$ it is neccessary to run the Wilson coefficients down to the low scale $\mu_s$ using the renormalization group equations. Under QCD renormalization group the two operators do not run, keeping the constraint (13) intact [40].

Thus we have, at low energies, a SM modification that satisfies

$$C'_9 = -C'_{10},$$ \hfill (14)$$

where $C'_{9,10}$ are scale invariant, modulo small QED corrections.

In Fig. 1 we show in gray the $1\sigma$ region in the $C'_{10}$ plane as obtained from the fit to the partial branching fraction of $B^+ \rightarrow K^+\mu^+\mu^-$, cf. Eq. (7). The $1\sigma$ region is defined here as $\chi^2 < 2.30$. The width of the “donut” reflects both experimental and form factor uncertainties. The SM point in the parameter space is marked with a dot and exhibits a tension with the measurement with $\chi^2 = 3.9$. In Fig. 1 the $1\sigma$ region (defined as before) of fit to the $\mathcal{B}(B_s \rightarrow \mu^+\mu^-)$ according to Eq. (11) is depicted in blue. In this case the SM point is in comfortable agreement with the observable ($\chi^2 = 0.7$). Then we perform combined fit to all of the above quantities and find the best value to be $\chi^2_{\text{min}} = 2.26$, which is substantially better than the SM point with $\chi^2_{\text{SM}} = 4.6$. The green patch is defined by $\chi^2 < \chi^2_{\text{min}} + 1 = 3.26$ and will be used below to predict the $R_K$.

Assuming that the effective Hamiltonian (3), tailored for $b \rightarrow s e^+e^-$, receives only SM contributions, unlike $b \rightarrow s\mu^+\mu^-$ that also receives NP contributions from $C'_{9,10}$, we can now predict the value of $R_K$. In $R_K$ the uncertainties of the hadronic form factors cancel out to a large extent in the ratio and the formula boils down to:

$$R_K(C'_{10}) = 1.001(1) - 0.046 \text{Re}[C'_{10}] - 0.094(3) \text{Im}[C'_{10}] + 0.057(1)[C'_{10}]^2.$$ \hfill (15)$$

Remaining uncertainties are indicated by the numbers in parentheses. In Fig. 2 we show contours of constant $R_K$ in the $C'_{10}$ plane using the formula (15) with central values for the coefficients. By dark gray we indicate the region corresponding to the measured value of $R_K$. In the same figure we plot again the $1\sigma$ prediction of $C'_{10}$, also shown in Fig. 1. We see an appreciable overlap with the measured $R_K$. Mapping the fitted region (green) to $R_K$ we obtain the prediction

$$R_K^{\text{pred}} = 0.88 \pm 0.08,$$ \hfill (16)$$

which is indeed in good agreement with $R_K^{\text{LHCb}} = 0.745 \pm 0.009 \pm 0.036$ [1].

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1 Eq. (13) is broken only by tiny effects from QED renormalization.
Figure 1. Regions in the complex $C'_{10}$ plane that are in 1σ agreement with $B_s \rightarrow \mu^+\mu^-$ (blue), $B \rightarrow K\mu^+\mu^-$ (gray). Green area is a 1σ region from fit to both observables. Black dot is the SM.

Figure 2. Contours of constant $R_K$ are indicated by dashed lines. Gray region represents the 1σ measured range of $R_K$ projected onto the $C'_{10}$ plane, whereas green contour denotes the region allowed by $B_s \rightarrow \mu^+\mu^-$ and $B \rightarrow K\mu^+\mu^-$. Black dot is the SM.
A. Impact on $B \to K^{*}\ell^{+}\ell^{-}$

$B \to K^{*}\ell^{+}\ell^{-}$ is particularly interesting for the NP searches because of the observables that one can construct from the $q^{2}$-dependent coefficients $I_{1-9}$ which appear in the angular distribution,

$$
\frac{d^{4}\Gamma(\bar{B}^{0} \to \bar{K}^{0}\ell^{+}\ell^{-})}{dq^{2} \, d\cos\theta_{\ell} \, d\cos\theta_{K} \, d\phi} = \frac{9}{32\pi} \left[ I_{1}^{2} \sin^{2}\theta_{K} + I_{1}^{*} \cos^{2}\theta_{K} + (I_{2}^{0} \sin^{2}\theta_{K} + I_{2}^{*} \cos^{2}\theta_{K}) \cos 2\theta_{\ell} \right. \\
+ I_{1}^{2} \sin^{2}\theta_{K} \sin^{2}\theta_{\ell} \cos 2\phi + I_{4} \sin 2\theta_{K} \sin 2\theta_{\ell} \sin \phi + I_{5} \sin 2\theta_{K} \sin \theta_{\ell} \cos \phi \\
+ \left. (I_{6}^{0} \sin^{2}\theta_{K} + I_{6}^{*} \cos^{2}\theta_{K}) \cos \theta_{\ell} + I_{7} \sin 2\theta_{K} \sin \theta_{\ell} \sin \phi \right] + I_{8} \sin 2\theta_{K} \sin 2\theta_{\ell} \sin \phi + I_{9} \sin^{2}\theta_{K} \sin^{2}\theta_{\ell} \sin 2\phi \] .

(17)

Differential decay rate is then simply $d\Gamma/dq^{2} = (3I_{1}^{*} + 6I_{1}^{0} - I_{2}^{0} - 2I_{3}^{0})/4$, and the similar expressions can be written for the transverse/longitudinal part of the decay rate, for the forward-backward asymmetry, $A_{FB}(q^{2}) = 3I_{6}^{0}/(4d\Gamma/dq^{2})$, CP-asymmetry, and several other observables. Each of the coefficient functions, $I_{i} \equiv I_{0}(q^{2})$, can be written in terms of transversity amplitudes, $A_{L,R}^{L,R}(q^{2})$, which are related to the respective spin states of the on-shell $K^{*}$-meson, and the amplitude $A_{L,R}^{L,R}(q^{2})$ which is related to the off-shell virtual gauge boson decaying into the lepton pair. The superscripts $L, R$ indicate the chirality of the lepton. Detailed expressions can be found, for example, in Refs. [32, 41–43].

The strategy of looking for the NP effects through a detailed analysis of the angular distribution of $B \to K^{*}\ell^{+}\ell^{-}$ is somewhat plagued by hadronic uncertainties. The observables built up of $A_{L,R}^{L,R}(q^{2})$ turn out to be less sensitive to hadronic uncertainties because they involve the (combinations of) hadronic form factors which appear to be under a rather good theoretical control, especially in the region of small $q^{2}$’s [44] (see also discussion in Ref. [43]). On the other hand, the observables made of $A_{L,R}^{L,R}(q^{2})$ entail the hadronic form factors that are less well understood. Moreover, the latter observables are subject to another kind of hadronic uncertainty, i.e. the one arising from misidentification of the $K\pi$ pairs coming from $B \to K^{*}(\to K\pi)\ell^{+}\ell^{-}$ with those emerging from $B \to K_{s}^{0}(\to K\pi)\ell^{+}\ell^{-}$, where $K_{s}^{0}$ stands for a broad scalar state [47]. Finally, and to avoid problems of the $c\bar{c}$-resonances in the $q^{2}$-spectrum of the decay, a standard strategy is to either work at low $q^{2} < m_{J/\psi}^{2}$ or large $q^{2} > 15 \text{GeV}^{2}$, in which the impact of the $c\bar{c}$-resonances is expected to be small.

With the information obtained in the previous section of this paper, i.e. with $C_{10}^{\prime} = -C_{9}^{\prime}$ extracted from the comparison of the measured $B(B_{s} \to \mu^{+}\mu^{-})$ and $B(B \to K\mu^{+}\mu^{-})$ with the corresponding theoretical expressions, we already showed that we were able to verify the consistency of our result for $R_{K}$ with the one measured at LHCb. With our approach, in which only the decay to muon-pair is modified, we can also predict $R_{K^{*}}$, defined as

$$
R_{K^{*}} = \frac{\Gamma(B \to K^{*}\mu^{+}\mu^{-})_{q^{2} \in [4,6] \text{ GeV}^{2}}}{\Gamma(B \to K^{*}\ell^{+}\ell^{-})_{q^{2} \in [4,6] \text{ GeV}^{2}}} .
\] (18)

as well as the ratio of the two [5, 20], namely,

$$
X_{K} = \frac{R_{K^{*}}}{R_{K}} - 1 .
\] (19)

In Ref. [18] it was shown that the ratio of forward-backward asymmetries integrated between $q^{2} \in [4,6] \text{ GeV}^{2}$ can also be sensitive to lepton flavor universality violation. After defining,

$$
A_{FB}^{\ell}_{[4-6]} = \frac{3}{4} \int_{4 \text{ GeV}^{2}}^{6 \text{ GeV}^{2}} I_{6}^{0}(q^{2}) \, dq^{2} \frac{\Gamma(B \to K^{*}\ell^{+}\ell^{-})_{q^{2} \in [4,6] \text{ GeV}^{2}}}{\Gamma(B \to K^{*}\ell^{+}\ell^{-})_{q^{2} \in [4,6] \text{ GeV}^{2}}},
\] (20)

the ratio of forward-backward asymmetries is then simply,

$$
R_{FB}^{\ell} = \frac{A_{FB}^{\ell}_{[4-6]}}{A_{FB}^{\ell}_{[4-6]}} .
\] (21)

To compute the above-mentioned quantities we use the standard values of the Wilson coefficients [34], and include the effect of quark loops in the coefficients $C_{7,9}$ arising from the operators $O_{1,2}$, as calculated in Ref. [33]. We neglect the soft gluon corrections to the charm quark loop at low $q^{2}$, which according to Ref. [52] is reasonable. For the
form factors we use the values computed by means of QCD sum rules on the light cone [53], and neglect the effect of soft gluon corrections $c\bar{c}$. In Fig. 3 we show our results for $R_{K^*}$, $X_K$ and $R_{fb}$ as functions of $\text{Re}[C'_{10}]$. For an easier comparison, in the same plot we also show $R_K$. The range $0.075 \leq \text{Re}[C'_{10}] \leq 0.41$ has been obtained in the previous section of this paper, where we showed that for a given value of $0.075 \leq \text{Re}[C'_{10}] \leq 0.41$ there is a region of allowed $\text{Im}[C'_{10}]$, and therefore instead of curves in Fig. 3 we actually have the corresponding regions of values determined by $\text{Im}[C'_{10}]$. We should emphasize again that the uncertainties related to form factors cancel to a large extent in the ratios. As for the results, we first see that in the scenario with $C'_{10} = -C'_9 \neq 0$, allowing coupling to muons only, and explicitly realized in the model with a $(3,2,1/6)$ leptoquark state, we get

$$ R_K = 0.88 \pm 0.08, \quad R_{K^*} = 1.11 \pm 0.08, \quad X_K = 0.27 \pm 0.19, \quad R_{fb} = 0.84 \pm 0.12, $$

which are obviously different from the values obtained in the SM, $R_{K}^{\text{SM}} = 1.00$, $R_{K^*}^{\text{SM}} = 0.996(5) \approx 1$, $R_{fb} = 0.995(4) \approx 1$, and $X_K = -0.004(5) \approx 0$. Notice, however, that while our value for $R_K$ is lower than the one in the SM, our prediction for $R_{K^*}$ is larger than that obtained in the SM. The measurement of $R_{K^*}$ at LHCb will therefore help to either confirm or discard our model as a viable description of the lepton flavor universality violation. The errors in Eq. (22) are completely dominated by the range of $\text{Re}[C'_{10}]$ and $\text{Im}[C'_{10}]$, while those arising from form factors are reduced in the ratios and induce an uncertainty negligible in comparison with that coming from the variation of $C_{10}$.

Finally, before closing this part of our paper, we need to comment on $P_5(q^2)$, an observable constructed from coefficients of the angular distribution of the $B \to K^* \ell^+ \ell^-$ decay [54], $P_5(q^2) = I_5 / \sqrt{-4I_1^2 I_2^2}$, which has been measured at LHCb, and turned out to be $4\sigma$ away from the value predicted in the SM when integrated over an interval $q^2 \in [4.3,8.68]$ GeV$^2$ [55]. More specifically, the SM value is $\langle P_5^{\text{SM}}(q^2) \rangle = -0.09(5)$, while the measured one is $\langle P_5^{\text{LHCb}}(q^2) \rangle = -0.19(16)$ [55], which can be compactly written as, $\langle P_5^{\text{LHCb}}(q^2) \rangle / \langle P_5^{\text{SM}}(q^2) \rangle = -0.22(18)$. While the interpretation of this discrepancy is somewhat controversial [17, 18, 56], it is nevertheless interesting to check

![Figure 3](image-url)

Figure 3. $R_K$, $R_{K^*}$, $X_K$ and $R_{fb}$, defined in Eq. (11,18,19,21) respectively, are plotted as functions of $\text{Re}[C'_{10}]$, in the range allowed by the measured values of $B(B_s \to \mu^+ \mu^-)$ and $B(B \to K^+ \mu^+ \mu^-)$, at $q^2 > 15$ GeV$^2$. Instead of a curve for each quantity we actually have a region of values, reflecting the fact that for each $\text{Re}[C'_{10}]$ there is a range of allowed values of $\text{Im}[C'_{10}]$, as shown in Fig. 1.
whether or not the leptoquark model used in this paper (and discussed in more details in the following Section) can describe the manifest disagreement between theory and experiment. With the values of $C'_{16} = -C'_9$ discussed above we indeed see that $(P_{51}^{LQ}/|4.3-8.68|)/(P_{51}^{SM}/|4.3-8.68|) < 1$, but with the leptoquark model discussed here we cannot reach very low values. We instead obtain $0.78 \leq (P_{51}^{LQ}/|4.3-8.68|)/(P_{51}^{SM}/|4.3-8.68|) \leq 0.98$. A similar tendency is observed for other bins, and in particular the one corresponding to $q^2 \in [1,6]$ GeV$^2$.

IV. MODEL WITH A SCALAR LEPTOQUARK

In this Section we discuss a specific model in which the scenario discussed above, i.e. $C'_9 = -C'_{16}$, is explicitly realized and involves the presence of a light scalar leptoquark state $\Delta$. More specifically, we choose the leptoquark $\Delta$ to carry the quantum numbers $(3,2,1/6)$ of the SM gauge group. Its couplings to fermions are described by a renormalizable Lagrangian

$$\mathcal{L} = Y_{ij} \bar{L}_i \tau^2 \Delta^* d_{Rj} + \text{h.c.}$$

$$= Y_{ij} \left( -\bar{\ell}_L d_{Rj} \Delta^{(2/3)*} + \bar{\nu}_{Lk} (V_{PMNS})^*_k \bar{d}_{Rj} \Delta^{(-1/3)*} \right) + \text{h.c.} ,$$

where $Y$ is a $3 \times 3$ complex matrix, $L_i$ and $d_{Rj}$ are the lepton doublet and down-quark singlet. Charge eigenstates of the leptoquark doublet are denoted with $\Delta^{(2/3)}$ and $\Delta^{(-1/3)}$ and we will assume that they are degenerate. The second line in the above Lagrangian is written in the fermion mass basis, and a relative PMNS rotation in lepton doublet components has been assigned to the neutrino sector.

Clearly, the lepton flavor universality is explicitly broken by the terms presented in Eq. (23). This might appear questionable because in a similar situation in which the coupling of leptoquark to $\mu c$ would be allowed, the ratio of the electronic and muonic widths of the decay of $J/\psi$ shows no violation of the lepton flavor coupling universality. In particular, the measured $\Gamma(J/\psi \to \mu^+ \mu^-)/\Gamma(J/\psi \to e^+e^-) = 1.0016 \pm 0.0031$ [26] is in excellent agreement with its SM value, 1.00001. 2 That situation is, however, much different from the examples discussed in this paper, because the amplitude for $J/\psi \to \ell^+ \ell^-$ is dominated by the tree-level electromagnetic interaction diagram which is much larger than the weak interaction one, suppressed by $1/m_Z^2$ with respect to the dominant one, and therefore completely negligible. Our leptoquark state is $m_\Delta \gg m_Z$, and its contribution to $J/\psi \to \ell^+ \ell^-$ is even smaller than the weak interaction diagram and cannot make an impact on the decay of charmonia at the present level of accuracy.

Instead, the weak $b \to s \mu^+ \mu^-$ decays in the SM are loop-induced so that the tree level contribution involving couplings to the leptoquark state may become comparable in size to the SM amplitude, which is why the $b \to s \mu^+ \mu^-$ is likely to be more sensitive to the presence of the term described by the lagrangian (23). The relevant leptoquark coupling for the $b \to s \mu^+ \mu^-$ is the product $Y_{\mu b} Y^{*}_{\mu s}$, which enters the Wilson coefficients divided by $m_\Delta^2$. The scalar particle exchange generates scalar operators in the Fierzed basis and those appear as (pseudo)vector currents in the ordinary operator basis [8]:

$$C'_{10} = -C'_9 = -\frac{Y_{\mu b} Y^{*}_{\mu s}}{2 \sqrt{2} G_F V_{tb} V^{*}_{ts} m^2_\Delta} .$$

We assume other elements of Y ukawa matrix $Y$ to vanish. The same state will also contribute at loop level to electro- and chromo-magnetic operators $C'_7(m_\Delta)$ and $C'_8(m_\Delta)$ where these coefficients will be suppressed by electromagnetic $\alpha(m_\Delta)/(4\pi)$ and strong $\alpha_S(m_\Delta)/(4\pi)$ couplings at high scale $m_\Delta$, respectively. We have explicitly checked that these modifications result in negligibly small value of $C'_9$ when compared to the $C'_7$ of SM, cf. Eq. (5). In the remainder of this Section we will analyze additional observables that constrain this leptoquark scenario.

The considered leptoquark state $\Delta$ couples to the neutrinos with the same couplings as to the charged leptons, only modified by a PMNS rotation matrix. Namely, the charge $-1/3$ state will generate $(s \bar{b})(\bar{\nu} \nu)$ operators while the box diagrams will lead to $B_x - B_x$ mixing.

---

2 By explicitly including the lepton mass in the calculation of phase space we obtain

$$\Gamma(J/\psi \to \ell^+ \ell^-) = \frac{16\pi\alpha^2}{27 m_{J/\psi}} \left(1 + \frac{2m^2}{m_{J/\psi}} \right) \sqrt{1 - \frac{2m^2}{m_{J/\psi}} f^2_{J/\psi}}$$

and the effect on the ratio of the electronic and muonic widths is extremely small.
A. Contribution of (3,2,1/6) leptoquark in $B_s - \bar{B}_s$ oscillation frequency

The state (3, 2, 1/6) will induce $\Delta B = 2$ box diagrams with $\mu$ and $\Delta^{(2/3)}$ or $\nu$ and $\Delta^{(-1/3)}$ running in the box. The two contributions of boxes with $\mu$ and $\nu$ are equal in the $m_\mu = 0$ limit and in sum they amount to

$$C^\text{LQ}_{6}(m_\Delta) = -\frac{Y^\mu_{eb}Y^\mu_{es}}{64\pi^2 m^2_\Delta}. \quad (25)$$

The effective $\Delta B = 2$ Hamiltonian is defined as

$$\mathcal{H}_{\text{eff}} = C^\text{SM}_{1}(\bar{b}\gamma_\mu P_L s) (\bar{b}\gamma_\mu P_L s) + C^\text{LQ}_{6}(\bar{b}\gamma_\mu P_R s) (\bar{b}\gamma_\mu P_R s) + \text{h.c.}, \quad (26)$$

where $P_{L/R} = (1 \pm \gamma_5)/2$. The coefficient in Eq. (25) is subject to QCD renormalization and has to be evaluated at scale $\mu_b$. The anomalous dimensions of $C^\text{LQ}_{6}$ is however equal to the one of $C^\text{SM}_{1}$. Therefore the two Wilson coefficients renormalize with the same multiplicative factor between scales $\mu = m_t$, where SM is matched onto effective Hamiltonian (26), and $\mu_b$, where the hadronic matrix elements are computed. Remaining $C^\text{LQ}_{6}$ running from $m_\Delta$ down to $m_t$ is already in the asymptotic regime of QCD and can be safely neglected. The mass difference of the $B_s - \bar{B}_s$ system is then

$$\Delta m_{B_s} = \frac{2}{2m_{B_s}} \left| \frac{G_F^2 m^2_W}{16\pi^2} (V_{tb}^* V_{ts})^2 \eta_B S_0(x_t) + \frac{\eta_B}{4} C^\text{LQ}_{6}(m_\Delta) \right| \langle \bar{B}_s^0 | \bar{b} \gamma_\mu (1 - \gamma_5) s \bar{b} \gamma_\mu (1 - \gamma_5) s | B_s^0 \rangle . \quad (27)$$

By using Eq. (24) we can write

$$C^\text{LQ}_{6}(m_\Delta) = -\frac{G_F^2}{8\pi^2} (V_{tb}^* V_{ts})^2 \alpha^2 m^2_\Delta (C_{10}^\prime)^2 , \quad (28)$$

which, together with $\langle \bar{B}_s^0 | \bar{b} \gamma_\mu (1 - \gamma_5) s \bar{b} \gamma_\mu (1 - \gamma_5) s | B_s^0 \rangle = (8/3) f^2_{B_s} m^2_{B_s} B_{B_s}$, gives

$$\Delta m_{B_s} = \frac{G_F^2 m^2_W}{6\pi^2} \left| (V_{tb}^* V_{ts})^2 f^2_{B_s} m^2_{B_s} B_{B_s} \eta_B S_0(x_t) \right| 1 - \frac{1}{2} \frac{\alpha^2}{S_0(x_t)} \left( C_{10}^\prime \right)^2 m^2_\Delta \frac{m^2_{B_s}}{m^2_W} . \quad (29)$$

With the current values for $f_{B_s} = 228(8)$ MeV and $B_{B_s} = 1.33(6)$, as obtained in numerical simulations of QCD on the lattice [36], and $m^2_{B_s}(m_t) = 160^{+5}_{-3}$ GeV [26], we get \footnote{To evaluate $\Delta m^\text{SM}_{B_s}$ we also used $\eta_B = 0.55(1)$ [57], and $S_0(x_t) = 2.25^{+11}_{-9}$, the Inami-Lim function at $x_t = m^2_t/m^2_W$.}

$$\Delta m^\text{SM}_{B_s} = 17.3 \pm 1.4 \text{ ps}^{-1} , \quad (30)$$

which is in excellent agreement with the measured $\Delta m_{B_s} = 17.7(2) \text{ ps}^{-1}$ [26]. With the values of $C_{10}^\prime$ determined in the previous Section, we see that Eq. (29) leads to a very loose upper bound for $m_\Delta$. For example, for Re$[C_{10}^\prime] = 0.15, 0.25, 0.35$, we get $m_\Delta < 42.5, 31.6, 31.8$ TeV, respectively.

B. Impact of (3,2,1/6) leptoquark on $B \rightarrow K \nu \bar{\nu}$

In the presence of leptoquark $\Delta$ the pair of neutrinos in the final state of $B \rightarrow K \nu \bar{\nu}$ may be in any flavor combination. In order to encompass such a possibility we must extend the effective Hamiltonian of Ref. [58] to account for the disparity in neutrino flavors:

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left( C_{L}^{ij} \mathcal{O}_{L}^{ij} + C_{R}^{ij} \mathcal{O}_{R}^{ij} \right). \quad (31)$$

The operators are defined as $\mathcal{O}_{L,R}^{ij} = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_{L,R} b)(\bar{\nu}_i \gamma_\mu (1 - \gamma_5) \nu_j)$. The authors of [58] found that in the SM the Wilson coefficient at next-to-leading order in QCD is

$$C^\text{SM}_{L} \equiv C^{ij}_{L} = -6.38 \pm 0.06 , \quad \text{no sum over } i \text{ implied}. \quad (32)$$
If the leptoquark state \((3, 2, 1/6)\) is present then it will manifest itself in \(B \to K \nu \bar{\nu}\) through right-handed operators:

\[
C'^{ij}_R = \frac{1}{N} \frac{(VY)_{is}(VY)^*_ja}{4m^2_\Delta}, \quad N = \frac{G_FV_{ub}V^*_{us}\alpha}{\sqrt{2}\pi}.
\]  

(33)

Here \(V\) denotes the PMNS matrix. The experimentally accessible decay width of \(B \to K \nu \bar{\nu}\) is a sum of partial widths of \(B \to K \nu \bar{\nu}\). The amplitudes are proportional to the sum of the SM and leptoquark contribution and the two will interfere in the \(B \to K \nu \bar{\nu}\) decays width as

\[
\Gamma(B \to K \nu \bar{\nu}) \sim \sum_{i,j=1}^3 \left| \delta_{ij} C^\text{SM}_L + C'^{ij}_R \right|^2
\]

\[
= 3|C^\text{SM}_L|^2 + |C'^{10}_L|^2 - 2\text{Re}[C^\text{SM}_L C'^{10}_L].
\]  

(34)

\(C'^{10}_L\) is the Wilson coefficient of \(b \to s \mu^+ \mu^-\) that we obtained from the fits to experimental data in the previous Section. Last line of Eq. (34) was obtained by applying the unitarity of matrix \(V\), and assuming that \(Y_{\mu b}\) and \(Y_{\mu s}\) are the only non-zero elements of the matrix \(Y\). Finally, the \(q^2\)-spectrum of this decay reads,

\[
\frac{d\Gamma}{dq^2}(B \to K \nu \bar{\nu}) = \frac{|N|^2}{384\pi^3 m^3_B} f_+(q^2) \left[ \lambda(m^2_B, m^2_K, q^2) \right]^{3/2} \left( 3|C^\text{SM}_L|^2 + |C'^{10}_L|^2 - 2\text{Re}[C^\text{SM}_L C'^{10}_L] \right),
\]  

(35)

where \(q^2\) in this case stands for the invariant mass of the neutrino pair. Notice that the above expression, for \(C'^{10}_L = 0\), confirms Eq. (2.14) of Ref. [58]. The expression (35) can be recast into a product of the SM \(q^2\)-spectrum and a correction factor,

\[
1.01 < \left[ 1 + \frac{1}{3} |C'^{10}_L/C^\text{SM}_L|^2 - \frac{2}{3} \text{Re}[C'^{10}_L/C^\text{SM}_L] \right] < 1.05,
\]  

(36)

where its lower and upper bounds have been derived from the 1\(\sigma\) region of \(C'^{10}_L\), obtained in the previous Section. We learn that the \(B(B \to K \nu \bar{\nu})\) will increase by at most 5\% if leptoquark \(\Delta\) is present.

V. SUMMARY AND CONCLUSION

In this paper we discussed a possibility of constraining a scenario of New Physics affecting the \(b \to s \mu^+ \mu^-\) decays through coupling with the operators \(C'^{9,10}_L\). Such a scenario is explicitly verified in a model with a light scalar leptoquark state, \(\Delta\), carrying the quantum numbers \((3, 2, 1/6)\) of the Standard Model gauge group. In this scenario, \(C'^9 = -C'^{10}_L\) is specific for the muons in the final state. In the leptoquark model discussed in this paper, \(C'^{10}_L\) is related to \(Y_{\mu b}Y_{\mu s}^*/m^2_\Delta\). From the currently available experimental data on \(B(B_s \to \mu^+ \mu^-)\) and \(B(B \to K\mu^+ \mu^-)\) we were able to constrain \(\text{Re}[C'^{10}_L]\) and \(\text{Im}[C'^{10}_L]\), which are then used to compute \(R_K = B(B \to K\mu^+ \mu^-)/B(B \to K\ell^+ \ell^-)\) and \(R_{K^*}/R_K = 1.11(8)\), similarly, in this scenario the ratio of forward-backward asymmetries becomes different from unity. In particular, we find \(R_{\ell b} = (A^\mu_{tb}/A^\mu_{\bar{b}b})|_{[4-6] \text{GeV}}^2 = 0.8(1)\). Furthermore, we checked that a combination of coefficients of the angular distribution of \(B \to K\mu^+ \mu^-\), known as \(P_5^\mu\), and weighted over a specific bin of \(q^2\)'s, indeed becomes smaller than its value predicted in the Standard Model. However, it cannot explain a very low value of \(\langle P_5^\mu \rangle|_{[4.3-8.6]}^2\) measured at LHCb, for which the Standard Model prediction is still a subject to controversies mainly related to the issue of treatment of the charm quark loops.

Finally, in the leptoquark model our constraints on the Wilson coefficient \(C'^{10}_L\) can have impact on other physical processes. We checked, in particular, that the contribution to the frequency of oscillation in the \(B_s - \bar{B}_s\) system is insignificant, and that only up to five percent enhancement of \(B(B \to K \nu \bar{\nu})\) can be obtained.
Appendix A: $B \to K^{(*)}$ form factors

For completeness we remind the reader of the standard parameterization of the $B \to K^{(*)}\ell^+\ell^-$ hadronic matrix elements in terms of the relevant form factors,

\[ \langle K(k)|\bar{s}\gamma_{\mu}|bB(p)\rangle = \left[(p+k)_{\mu} - \frac{m_B^2 - m_K^2}{q^2}q_{\mu}\right] f_{+}(q^2) + \frac{m_B^2 - m_K^2}{q^2}q_{\mu}f_{0}(q^2), \]

\[ \langle K(k)|\bar{s}\sigma_{\mu\nu}|bB(p)\rangle = i(p_{\mu}k_{\nu} - p_{\nu}k_{\mu}) \frac{2f_{T}(q^2)}{m_B + m_K}, \]  

(A1)

\[ \langle K^*(k)|\bar{s}\gamma_{\mu}(1 - \gamma_5)|bB(p)\rangle = \varepsilon_{\mu\nu\rho\sigma}^{B*}p^\rho k^\sigma \left(\frac{2V(q^2)}{m_B + m_{K*}} - i\varepsilon_{\mu}^{B}(m_B + m_{K*})A_1(q^2)\right) + i(p_{\mu}k_{\nu})\left(\varepsilon^* \cdot q\right) \frac{A_2(q^2)}{m_B + m_{K*}} + iq_{\mu}(\varepsilon^* \cdot q) \frac{2m_{K*}}{q^2} \left[A_3(q^2) - A_0(q^2)\right], \]

(A2)

\[ \langle K^*(k)|\bar{s}\sigma_{\mu\nu}q^{\nu}(1 + \gamma_5)|bB(p)\rangle = 2i\varepsilon_{\mu\nu\rho\sigma}^{B*}p^\rho k^\sigma T_{1}(q^2) + \left[\varepsilon^* \cdot (m_B^2 - m_{K*}^2) - (\varepsilon^* \cdot q)(2p - q)\right] T_{2}(q^2) + (\varepsilon^* \cdot q) \left[q_{\mu} - \frac{q^2}{m_B^2 - m_{K*}^2}(p + k)_{\mu}\right] T_{3}(q^2), \]

where $2m_{K*}A_3(q^2) = (m_B + m_{K*})A_1(q^2) - (m_B - m_{K*})A_2(q^2)$, and $T_{1}(0) = T_{2}(0)$. In the limit of massless lepton, the $q^2$-dependent functions entering eqs. (6,17), relevant for the present study, read

\[ a_\mu(q^2) = \frac{3}{2}F, \quad I_2 = \frac{1}{4}\left(|A_{L,R}^L|^2 + |A_{L,R}^R|^2\right), \]

\[ I_6^\epsilon = -|A_{0,L,R}^L|^2, \quad I_6^s = 2\text{Re}\left(A_{1,L}^L A_{1,R}^L - A_{1,R}^L A_{1,L}^R\right), \]

(A4)

where

\[ F(q^2) = N\lambda(q^2)\left[|C_9 + C_9'|f_{+}(q^2) + \frac{2m_B}{m_B + m_{K*}}(C_7 + C_7')f_{T}(q^2)\right]^2 + \left|(C_{10} + C_{10}')f_{+}(q^2)\right|^2, \]

\[ A_{L,R}^{L,R}(q^2) = \sqrt{Nq^2\lambda(q^2)}\left[\frac{2m_B}{q^2}(C_7 + C_7') T_1(q^2) + [(C_9 + C_9') \mp (C_{10} + C_{10}')] \frac{V(q^2)}{m_B + m_{K*}}\right], \]

\[ A_{L,R}^{L,R}(q^2) = -\sqrt{Nq^2(m_B^2 - m_{K*}^2)}\left[\frac{2m_B}{q^2}(C_7 - C_7') T_2(q^2) + [(C_9 - C_9') \mp (C_{10} - C_{10}')] \frac{A_1(q^2)}{m_B + m_{K*}}\right], \]

\[ A_{0,L,R}^{L,R}(q^2) = -\sqrt{N}\left[(C_9 - C_9') \mp (C_{10} - C_{10}')] \times \left[(m_B^2 - m_{K*}^2 - q^2)(m_B + m_{K*}) A_1(q^2) - \frac{A_2(q^2)}{m_B + m_{K*}}\right] \frac{\lambda(q^2)}{m_B + m_{K*}} + 2m_B(C_7 - C_7') \left[(m_B^2 + 3m_{K*}^2 - q^2) T_2(q^2) - \frac{\lambda(q^2)}{m_B^2 - m_{K*}^2} T_3(q^2)\right] \right\}, \]

(A5)

and $N = |V_{ub}V_{cb}^*|^2 \frac{\lambda^{1/2}(q^2)}{G_F^2\alpha^2/(3072\pi^5m_B^2)}, \lambda(q^2) = q^2 - (m_B + m_{K*})^2 \left[q^2 - m_{K*}^2\right]$. In the same massless lepton limit, $a_\mu = -c_\mu, I_2^s = 3I_2^s$ and $I_6^\epsilon = -I_6^s$, so that $d\Gamma(B \to K\mu\mu)/dq^2 = 4a_\mu/3$, and $d\Gamma(B \to K^{*}\mu\mu)/dq^2 = 4I_2^s - I_2^s = |A_{L,R}^{L,R}|^2 + |A_{L,R}^{L,R}|^2 + |A_{L,R}^{L,R}|^2$.

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