Quantum steering—a strong correlation to be verified even when one party or its measuring device is fully untrusted—not only provides a profound insight into quantum physics but also offers a crucial basis for practical applications. For continuous-variable (CV) systems, Gaussian states among others have been extensively studied, however, mostly confined to Gaussian measurements. While the fulfillment of Gaussian criterion is sufficient to detect CV steering, whether it is also necessary for Gaussian states is a question of fundamental importance in many contexts. This critically questions the validity of characterizations established only under Gaussian measurements like the quantification of steering and the monogamy relations. Here, we introduce a formalism based on local uncertainty relations of non-Gaussian measurements, which is shown to manifest quantum steering of some Gaussian states that Gaussian criterion fails to detect. To this aim, we look into Gaussian states of practical relevance, i.e. two-mode squeezed states under a lossy and an amplifying Gaussian channel. Our finding significantly modifies the characteristics of Gaussian-state steering so far established such as monogamy relations and one-way steering under Gaussian measurements, thus opening a new direction for critical studies beyond Gaussian regime.
making an excellent candidate for quantum information processor. A wide range of quantum systems can process quantum information on CVs including light fields, collective spins of atomic ensembles, and motional degrees of trapped ions, Bose-Einstein condensates, mechanical oscillators. An important regime to address CV systems both theoretically and experimentally is the one dealing with Gaussian states, Gaussian measurements and Gaussian operations. Although the Gaussian regime has many practical advantages due to its experimental feasibility, it has also been known that non-Gaussian operations and measurements are essential for some tasks such as CV nonlocality test and universal quantum computation. On the other hand, there are also cases that Gaussian states and operations provide an optimal solution, e.g. classical capacity under Gaussian channels, quantification of entanglement and quantum discord. These examples indicate that Gaussian regime can be sufficient for the characterization of certain quantum correlations of Gaussian states. In particular, Gaussian criterion is both sufficient and necessary to detect quantum entanglement for Gaussian states.

So far, most of the studies on the steering of Gaussian states have been confined to Gaussian measurements, which established numerous characteristics of Gaussian-state steering. A remarkable example is a two-mode Gaussian state, where steering criterion based on local uncertainty relations involving non-Gaussian measurements cannot manifest steering, but non-Gaussian measurements can. For this purpose, we employ a formulation of steering criterion based on a set of local orthogonal observables, which can generally be true beyond the Gaussian regime. For instance, a special class of non-Gaussian measurements, i.e., higher-order quadrature amplitudes, was employed to study steerability of Gaussian states, which did not show any better performance than Gaussian measurements. Kogias and Adesso further conjectured that for all two-mode Gaussian states, Gaussian measurements are optimal. It is a conjecture of both fundamental and practical importance to prove or disprove.

In this Article we demonstrate that there exist two-mode Gaussian states for which Gaussian measurements cannot manifest steering, but non-Gaussian measurements can. For this purpose, we employ a formulation of steering criterion based on local uncertainty relations involving non-Gaussian measurements. We study steerability of mixed Gaussian states, specifically, Gaussian states under a lossy and an amplifying channel that represent typical noisy environments. Our counter-examples to the aforementioned conjecture imply the breakdown of monogamy relations and one-way steerability emerging under the restriction to Gaussian measurements and point out a critical need for more studies beyond Gaussian regime.

**Results**

**Gaussian vs. non-Gaussian steering criterion.** Let us define two orthogonal quadrature amplitudes $X_i = \frac{1}{\sqrt{2}} (\hat{a}_i + \hat{a}^\dagger_i)$, $\hat{P}_i = \frac{1}{\sqrt{2}} (\hat{a}_i - \hat{a}^\dagger_i)$, where $\hat{a}_i$ and $\hat{a}^\dagger_i$ are the annihilation and the creation operator for $i$-th mode. The covariance matrix $\gamma_{AB}$ of a bipartite $(M+N)$-mode Gaussian state $\rho_{AB}$ is then given by

$$\gamma_{AB} = \begin{pmatrix} \gamma_A & C \\ C^T & \gamma_B \end{pmatrix}$$

whose elements are

$$\gamma_{AB} = \begin{pmatrix} a & c_1 & 0 \\ 0 & a & -c_2 \\ c_1 & 0 & b \\ 0 & -c_2 & 0 \\ 0 & 0 & b \end{pmatrix}.$$  

Without loss of generality, we may set $c_1 \geq |c_2| \geq 0$.

Under Gaussian measurements, a Gaussian state $\rho_{AB}$ is non-steerable from $B$ to $A$ iff

$$\gamma_{AB} + i\Omega_{AB} \otimes \theta_0 \geq 0,$$

where $\theta_0$ is a $2N \times 2N$ null matrix.

We now introduce our non-Gaussian steering criterion based on a set of local orthogonal observables, which we use to detect the steerability of two-mode Gaussian states.

**Lemma:** Let us choose a collection of $n^2$ observables $\{A^{(l)}\} = \{\lambda_k, \lambda_{kl}^\dagger\}$ $(k, l = 0, 1, \ldots, n - 1)$ where

$$\lambda_k = |k\rangle \langle k|$$

and

$$\lambda_{kl}^\dagger = |k\rangle \langle l + |l\rangle \langle k|/\sqrt{2},$$

where $k < l$. 

**References:**

1. [Reference 1]
2. [Reference 2]
3. [Reference 3]
4. [Reference 4]
5. [Reference 5]
6. [Reference 6]
7. [Reference 7]
8. [Reference 8]
9. [Reference 9]
10. [Reference 10]
11. [Reference 11]
12. [Reference 12]
13. [Reference 13]
14. [Reference 14]
15. [Reference 15]
16. [Reference 16]
17. [Reference 17]
18. [Reference 18]
19. [Reference 19]
20. [Reference 20]
21. [Reference 21]
22. [Reference 22]
23. [Reference 23]
24. [Reference 24]
25. [Reference 25]
26. [Reference 26]
27. [Reference 27]
28. [Reference 28]
29. [Reference 29]
30. [Reference 30]
31. [Reference 31]
32. [Reference 32]
33. [Reference 33]
34. [Reference 34]
35. [Reference 35]
36. [Reference 36]
37. [Reference 37]
38. [Reference 38]
39. [Reference 39]
40. [Reference 40]
41. [Reference 41]
42. [Reference 42]
43. [Reference 43]
44. [Reference 44]
45. [Reference 45]
46. [Reference 46]
47. [Reference 47]
48. [Reference 48]
49. [Reference 49]
50. [Reference 50]
\[
\lambda_{kl} = \frac{|k - l|}{\sqrt{2l}} (k < l).
\]  

Here \(|k\) refers to a Fock state of number \(k\). Note that each of \(\lambda_{kl}\) and \(\lambda_{lk}\) represents a complete projective measurement individually. That is, \(\lambda_{kl}\) represents a two-outcome projective measurement, assigning \(+1\) if the tested state is detected in \(|k\) and 0 otherwise. On the other hand, \(\lambda_{lk}\) represents a three-outcome projective measurement, assigning \(\pm 1\) if detected in the two eigenstates of \(\lambda_{kl}\) and 0 otherwise. These observables satisfy the orthogonal relations \(\text{Tr}(A_{(a)}^{(n)} A_{(b)}^{(n)}) = \delta_{ab}\) and the sum of variances must satisfy the uncertainty relation,  

\[
\sum_j \delta^2(A_j^{(n)}) \geq (n - 1)\langle I_n \rangle,
\]

where \(\delta^2(A_j^{(n)}) = \langle(A_j^{(n)})^2\rangle - \langle A_j^{(n)} \rangle^2\) represents each variance and \(\langle I_n \rangle = \sum_{|k\rangle} \langle k| k \rangle\) the projection to the Hilbert space of truncated \(n\)-levels. The proof of Lemma is given in Supplementary Information.

Now consider an orthogonal \(n^2 \times n^2\) matrix \(O_n\) in the truncated space, satisfying \(O_n^T O_n = O_n O_n^T = I_n\). Then the set of observables \(\{\lambda_{kl}\}\) under the transformation \(A_j^{(n)} = \sum_{l} O_{jl} A_l^{(n)}\) also satisfies the uncertainty relation in equation (8) since  

\[
\sum_j \delta^2(A_j^{(n)}) = \sum_{j,l} O_{jl}^T O_{jl} A_j^{(n)} A_j^{(n)} = \sum_l \langle A_l^{(n)} \rangle^2 = n\langle I_n \rangle, \quad \sum_j \langle A_j^{(n)} \rangle^2 = \sum_{j,l} O_{jl}^T O_{jl} \langle A_j^{(n)} \rangle = \langle A_j^{(n)} \rangle^2.
\]

This invariance property will be used later.

We now obtain a non-steering condition based on these orthogonal observables.

**Theorem:** If a two-mode quantum state \(\rho_{AB}\) is non-steerable from B to A, it must satisfy the inequality  

\[
\sum_j \delta^2(A_j^{(n)} \otimes 1 + g I \otimes B_j^{(n)}) \geq (n - 1)\langle I_n \rangle.
\]

for any real \(g\), where the observables \(\{A_j^{(n)}\}\) satisfy the uncertainty relation in Eq. (8). Its proof is given in Supplementary Information.

We next introduce a useful criterion originating from the inequality (9) that can be readily computable.

**Proposition:** If a two-mode state \(\rho_{AB}\) is non-steerable from B to A, the correlation matrix \(C^{\text{TLOOs}}_{\rho_{AB}}\) with elements  

\[
\{C^{\text{TLOOs}}_{\rho_{AB}}\}_{j} = \langle A_{(a)}^{(n)} \otimes B_{(b)}^{(n)} \rangle - \langle A_{(a)}^{(n)} \rangle \langle B_{(b)}^{(n)} \rangle
\]

constructed with \(n\)- and \(n'\)-level truncated local orthogonal observables (TLOOs) must satisfy  

\[
\|C^{\text{TLOOs}}_{\rho_{AB}}\|_{tr} \leq \sqrt{\left(\langle I_n \rangle - \sum_{j=1}^{n} \langle A_{(a)}^{(n)} \rangle^2\right) \left(n'\langle I_{n'} \rangle - \sum_{j=1}^{n'} \langle B_{(b)}^{(n')} \rangle^2\right)}.
\]

The proof of Proposition in Supplementary Information clearly shows that if a given state violates the inequality (10), it is steerable from B to A, and also that there exist a set of TLOOs that violates the inequality (9) as well. We give some details on how to calculate the expectation values of TLOOs for a given state in Supplementary Information.

**Application to Gaussian states.** Let us consider a two-mode squeezed-vacuum (TMSV) state as an initial state with its covariance matrix \(\gamma_{AB}^{\text{TMSV}}\), given by  

\[
\begin{bmatrix}
\cosh 2r & \sqrt{\eta} \sinh 2r & 0 & 0 \\
0 & \cosh 2r & 0 & -\sqrt{\eta} \sinh 2r \\
\sqrt{\eta} \sinh 2r & 0 & \eta \cosh 2r + 1 - \eta & 0 \\
0 & -\sqrt{\eta} \sinh 2r & 0 & \eta \cosh 2r + 1 - \eta
\end{bmatrix}
\]

If \(\eta > \frac{1}{2}\), one can readily check that the Gaussian criterion detects steering from B to A for all states regardless of squeezing. However, if the transmission rate is below \(\eta = \frac{1}{2}\), steering is impossible from B to A under Gaussian measurements. On the other hand, the steering from A to B is always possible regardless of \(\eta > 0\) for a nonzero squeezing. In contrast, as we show in Fig. 1, one can detect steerability of noisy Gaussian states from B to A even below \(\eta = 1/2\) using non-Gaussian measurements based on 2-level [Fig. 1(a)] and 3-level [Fig. 1(b)] TLOOs. For this calculation, we need the density matrix elements in Fock basis, \(\rho_{nm,n'm'}\equiv \text{Tr}\{\rho \}_{n_1} \otimes | n_2 \rangle \langle n_2 |\}_{n_2}\), which are given in Methods for completeness.

Our motivation for the choice of TLOOs in low-photon numbers is rather natural. For a low transmission rate \(\eta\), the mode B resides in the Fock space of low photon numbers and this is particularly true for a small initial
are detected where, whereas it is so from, and where in equation (3). In this case, steering is possible from B to A (from A to B). The dashed gray straight line $\eta = 1/2$ is the bound under which Gaussian steering criterion cannot detect steering from B to A. Regions above dotted green curves represent the two-mode Gaussian states with which a positive key extraction is possible using CV OSDIQKD of ref. 45.

Figure 1. Detection of steerability for a lossy Gaussian state based on the criterion in equation (10) using (a) 2-level TLOOs and (b) 3-level TLOOs, respectively. Initially, a two-mode squeezed state with squeezing parameter $r$ is prepared and only the mode B undergoes a lossy channel with transmittance rate $\eta$. The shaded red (blue) regions in the figures manifest a successful detection of steering from B to A (from A to B). The regions above dotted green curves represent the two-mode Gaussian states with which a positive key extraction is possible using CV OSDIQKD of ref. 45.

squeezing $r$. It is then expected that the information on correlation exists largely in the low-photon Fock space. For the case of 2-level TLOOs in Fig. 1(a), the detection range under our steering test, for which Gaussian criterion fails, turns out to be $0 < r < 0.869$ (red shaded region below dashed line). On the other hand, for the case of 3-level TLOOs in Fig. 1(b), it turns out to be $0.364 < r < 0.987$ (red shaded region below dashed line). One might then be interested to see if a truncated TMSS with a proper normalization in a genuinely low-dimensional Hilbert space can also show steering in a similar way. We find that it does not detect steerability beyond Gaussian criterion. In a sense, our TLOOs constructed with low-dimensional Fock states obtain a coarse-grained information space can also show steering in a similar way. We find that it does not detect steerability beyond Gaussian criterion. In a sense, our TLOOs constructed with low-dimensional Fock states obtain a coarse-grained information on higher-order terms, not completely ignoring them, in an on-off fasion. (See the paragraph below equation (7).)

Other Gaussian states of experimental relevance are TMSVs under an amplifying channel. Let mode $B$ undergo the amplification channel with a gain factor $G \geq 1$, i.e. $b_{\text{out}} = \sqrt{G}b_{\text{in}} + \sqrt{G} - 1 \hat{v}$, where $\hat{v}$ represents a vacuum noise. Then the output covariance matrix is given by $a = \cosh 2r, c_1 = c_2 = \sqrt{G}\sin 2r$, and $b = G\cosh 2r + G - 1$ in equation (3). In this case, steering is possible from A to B using Gaussian criterion in the range of $1 \leq G < \frac{2\cosh 2r}{\cosh 2r + 1}$, whereas it is so from B to A in all ranges of $G$ for a nonzero $r$. On the other hand, if we choose 2-level TLOOs in each party and test the violation of our steering criterion in Eq. (10), we find that the states with $G = \frac{2\cosh 2r}{\cosh 2r + 1} + \epsilon$ are detected where $0 \leq \epsilon \leq 0.05$ for $0 < r < 0.65$, where a nonzero $\epsilon$ indicates that there are some amplified Gaussian states the steering of which is detected not via Gaussian criterion but via our non-Gaussian measurements.

Discussion

Aside from its fundamental interest, our result can have some practical implications as well. First, it has been known that there exists a strict monogamous property of steering under Gaussian measurements\textsuperscript{41,44}. That is, if Bob can steer Alice’s system, Eve cannot steer it simultaneously. In contrast, our result provides a clear counter-example to this monogamy relation. For example, a loss channel can be modeled by a beamsplitter as shown in Fig. 2. Then, if Bob possesses a field of fraction $\eta$, Eve takes the remaining fraction $1 - \eta$. In the case of e.g. $\eta = 0.55$, Bob can steer Alice’s system via Gaussian measurements. At the same time, however, Eve can also steer Alice’s system via non-Gaussian measurements because it corresponds to the case of $1 - \eta = 0.45$ at which steering is possible as shown in Fig. 1. Therefore, there occurs a possibility of simultaneous steering if not restricted to the same Gaussian measurements. A similar argument can be given to the case of amplified states.

In a related context, one may wonder if the above breakdown of monogamy relation can have implication on the security of one-sided device independent quantum key distribution (OSDIQKD). Basically, steering is a necessary condition to establish a positive key rate for OSDIQKD. If the monogamy relation does not hold, i.e. a simultaneous steering is possible, the security can be potentially compromised. However, importantly, it is not an arbitrary form of steering but a specific one that matters for a given protocol. For example, the CV OSDIQKD scheme of ref. 45 extracts keys based on specific observables at Alice’s station, which are two orthogonal quadratures $\hat{X}$ and $\hat{P}$ (Fig. 1). If only two observables are considered for the purpose of steering in the trusted party, a simultaneous steering is impossible, which was first shown by Reid\textsuperscript{45}. This monogamy argument is even valid.
regardless of the type of states, whether Gaussian or non-Gaussian. Therefore, if Alice can establish a positive key rate with Bob, Eve cannot.

Second, the interesting phenomenon of asymmetric steering, i.e. Alice steers Bob but Bob cannot steer Alice, which was so far investigated under the restriction to Gaussian measurements\(^\text{15,16}\), must be rigorously reassessed. For example, the recent experiment\(^\text{15}\) studied the case of Gaussian states under a lossy channel and our result demonstrates that the transmission rate \( \eta = 1/2 \) is not a critical value for the one-way steering.

In summary we showed that there exist Gaussian states the steering of which Gaussian measurements cannot detect but non-Gaussian measurements can. To this aim, we introduced a criterion based on local orthogonal observables and their uncertainty relations in a truncated Hilbert space. We have applied this criterion to the case of TMSV under a lossy and an amplifying channel and found that Gaussian measurements are not always optimal to demonstrate steering of Gaussian states. Our result implies that the steering properties known under the restriction to Gaussian measurements must be rigorously reassessed. For example, the important properties such as the strict monogamy relation and asymmetric steering break down beyond Gaussian regime.

Our investigation clearly indicates that we must pursue more studies to completely understand the characteristics of quantum steering even for the restricted set of Gaussian states. We hope our finding here could provide some useful insights into future studies beyond Gaussian measurements and operations.

**Remarks:** Upon completion of this work\(^\text{49}\), we became aware of a related work in ref. 50 that employs pseudo-spin observables to show non-optimality of Gaussian measurements for steering of Gaussian states. We here briefly compare their method with ours particularly in detecting Gaussian states under a loss channel. As shown in Fig. 3, the method of ref. 50 detects steering at a lower level of transmittance \( \eta \) for the squeezing range \( 0 < r < 0.746 \) (\( 0 < r < 0.743 \)) than our 2 (3)-level TLOO criterion. However, it does not detect steering below \( \eta = 0.5 \), which is the case of interest as Gaussian criterion fails, if the squeezing level is rather high as \( r > 0.81 \). On the other hand, our method detects steering in the range of \( 0 < r < 0.869 \) and \( 0.364 < r < 0.987 \) using 2- and 3-level TLOO criterion, respectively, below \( \eta = 0.5 \). Thus, the red and the purple shaded regions in Fig. 2 indicate the advantage of our criteria over the method of ref. 50. In this respect, two approaches are complementary to each other in detecting steering of Gaussian states for which Gaussian criterion fails.
Methods
To calculate the expectation values of the observables in our steering criteria, we need the density matrix elements in Fock basis, i.e., $\rho_{m_1m_2n_1n_2} \equiv \text{Tr}\{\rho|n_1\rangle \langle n_1| \otimes |m_2\rangle \langle m_2|\}$. In particular, a two mode squeezed state (squeezing: $r$), after the mode $B$ undergoes a vacuum noisy channel with transmittance $\eta$, gives

$$
\rho_{0000} = \frac{2}{\cosh (2r) + 1},
$$

$$
\rho_{0011} = \rho_{1000} = \frac{2\sqrt{\eta} \sinh (2r)}{(\cosh (2r) + 1)^2},
$$

$$
\rho_{0022} = \rho_{2000} = \frac{2\eta (\cosh (2r) - 1)}{(\cosh (2r) + 1)^3},
$$

$$
\rho_{0101} = \frac{2(1 - \eta) (\cosh (2r) - 1)}{(\cosh (2r) + 1)^3},
$$

$$
\rho_{1021} = \rho_{2101} = \frac{2\sqrt{\eta} (1 - \eta) \sinh (2r) (\cosh (2r) - 1)}{(\cosh (2r) + 1)^3},
$$

$$
\rho_{1111} = \frac{2\eta (\cosh (2r) - 1)}{(\cosh (2r) + 1)^3},
$$

$$
\rho_{1122} = \rho_{2211} = \frac{2\eta^2 \sinh^2 (2r)}{(\cosh (2r) + 1)^3},
$$

$$
\rho_{2020} = \frac{2(1 - \eta)^2 (\cosh (2r) - 1)^2}{(\cosh (2r) + 1)^3},
$$

$$
\rho_{2121} = \frac{4\eta (1 - \eta) (\cosh (2r) - 1)^2}{(\cosh (2r) + 1)^3},
$$

$$
\rho_{2222} = \frac{2\eta^2 (\cosh (2r) - 1)^2}{(\cosh (2r) + 1)^3},
$$

(11)

while other terms are zero in the basis of $\{|0\rangle, |1\rangle, |2\rangle\}$.

On the other hand, each single mode state in mode $A$ and $B$ is a thermal state, whose expectation values are also necessary to test our criterion. The single-mode thermal states are all diagonal in Fock basis, and the nonzero expectation values for the mode $B$ are given by

$$
\langle B_0 \rangle_B = \langle 0 \rangle \langle 0 | = \frac{1}{1 + \eta} = \frac{2}{\cosh (2r) - \eta + 2},
$$

$$
\langle B_1 \rangle_B = \langle 1 \rangle \langle 1 | = \frac{\eta}{1 + \eta} = \frac{2\eta (\cosh (2r) - 1)}{(\eta \sinh (2r) - \eta + 2)^2},
$$

$$
\langle B_2 \rangle_B = \langle 2 \rangle \langle 2 | = \frac{\eta^2}{1 + \eta} = \frac{2\eta^2 (\cosh (2r) - 1)^2}{(\eta \sinh (2r) - \eta + 2)^3}.
$$

(12)

where $\bar{n}$ is the mean photon number of the initial thermal state. For mode $A$ which does not undergo a lossy channel, we simply set $\eta = 1$ in the above expressions.

References
1. Nielsen, M. A. & Chuang, I. L. Quantum computation and Quantum Information. (Cambridge University Press, Cambridge, 2000).
2. Brunner, N., Cavalcanti, D., Pironio, S., Scarani, V. & Wehner, S. Bell nonlocality. Rev. Mod. Phys. 86, 419 (2014).
3. Bell, J. On the Einstein Podolsky Rosen paradox. Physics (Long Island City, N.Y.) 1, 195 (1964).
4. Acín, A. et al. Device-Independent Security of Quantum Cryptography against Collective Attacks. Phys. Rev. Lett. 98, 230501 (2007).
5. Pironio, S. et al. Device-independent quantum key distribution secure against collective attacks. New J. Phys. 11, 045021 (2009).
6. Masanes, L., Pironio, S. & Acín, A. Secure device-independent quantum key distribution with causally independent measurement devices. Proc. R. Soc. London, Ser. A 459, 238 (2011).
7. Jozsa, R. & Linden, N. On the role of entanglement in quantum-computational speed-up. Proc. R. Soc. London, Ser. A 459, 238 (2011).
8. Wiseman, H. M., Jones, S. J. & Doherty, A. C. Steering, Entanglement, Nonlocality, and the Einstein-Podolsky-Rosen Paradox. Phys. Rev. Lett. 98, 140402 (2007).
9. Cavalcanti, E. G., Jones, S. J., Wiseman, H. M. & Reid, M. D. Experimental criteria for steering and the Einstein-Podolsky-Rosen paradox. Phys. Rev. A 80, 032112 (2009).
10. Branciard, C., Cavalcanti, E. G., Walborn, S. P., Scarani, V. & Wiseman, H. M. One-sided device-independent quantum key distribution: Security, feasibility, and the connection with steering. Phys. Rev. A 85, 010301 (2012).
11. Piani, M. & Watrous, J. Necessary and Sufficient Quantum Information Characterization of Einstein-Podolsky-Rosen Steering. Phys. Rev. Lett. 114, 060404 (2015).
12. Simth, D. H. et al. Conclusive quantum steering with superconducting transition-edge sensors. Nat. Commun. 3, 625 (2012).
13. Wittmann, B. et al. Loophole-free Einstein–Podolsky–Rosen experiment via quantum steering. New J. Phys. 14, 053030 (2012).
14. Bennet, A. J. et al. Arbitrarily Loss-Tolerant Einstein-Podolsky-Rosen Steering Allowing a Demonstration over 1 km of Optical Fiber with No Detection Loophole. Phys. Rev. X 2, 031003 (2012).
15. Händchen, V. et al. Observation of one-way Einstein–Podolsky–Rosen steering. Nature Photonics 6, 596 (2012).
16. Olsen, M. K. & Bradley, A. S. Bright bichromatic entanglement and quantum dynamics of sum frequency generation. Phys. Rev. A 77, 023813 (2008).
17. Middelley, S. L. W., Ferris, A. J. & Olsen, M. K. Asymmetric Gaussian steering: When Alice and Bob disagree. Phys. Rev. A 81, 022101 (2010).
18. Olsen, M. K. Asymmetric Gaussian harmonic steering in second-harmonic generation. Phys. Rev. A 88, 051802(R) (2013).
19. He, Q. Y. & Reid, M. D. Einstein-Podolsky-Rosen paradox and quantum steering in pulsed optomechanics. Phys. Rev. A 88, 052121 (2013).
20. He, Q. Y. & Ficek, Z. Einstein-Podolsky-Rosen paradox and quantum steering in a three-mode optomechanical system. Phys. Rev. A 89, 022332 (2014).
21. Tan, H., Zhang, X. & Li, G. Steady-state one-way Einstein-Podolsky-Rosen steering in optomechanical interfaces. Phys. Rev. A 91, 052121 (2015).
22. Quintino, M. T., Vertesi, T. & Brunner, N. Joint measurability, Einstein-Podolsky-Rosen steering, and Bell nonlocality. Phys. Rev. Lett. 113, 160402 (2014).
23. Uola, R., Moroder, T. & Gühne, O. Joint measurability of generalized measurements implies classicality. Phys. Rev. Lett. 113, 160403 (2014).
24. Bowles, J., Vertesi, T., Quintino, M. T. & Brunner, N. One-way Einstein-Podolsky-Rosen steering. Phys. Rev. Lett. 112, 200402 (2014).
25. Braunstein, S. L. & van Loock, P. Quantum information with continuous variables, Rev. Mod. Phys. 77, 513 (2005).
26. Reid, M. D. Demonstration of the Einstein-Podolsky-Rosen paradox using nondegenerate parametric amplification. Phys. Rev. A 40, 913 (1989).
27. Reid, M. D. et al. The Einstein-Podolsky-Rosen paradox: From concepts to applications. Rev. Mod. Phys. 81, 1727 (2009).
28. Quantum Information with Continuous Variables of Atoms and Light. (eds Cerf, N. J., Leuchs, N. G. & Polzik, E.S.) (Imperial College Press, London, 2007).
29. Weedbrook, C. et al. Gaussian quantum information, Rev. Mod. Phys. 84, 621 (2012).
30. Nha, H. & Carmichael, H. J. Proposed Test of Quantum Nonlocality for Continuous Variables. Phys. Rev. Lett. 93, 020401 (2004).
31. García-Patrón, R. et al. Proposal for a loophole-free Bell test using homodyne detection. Phys. Rev. Lett. 93, 130409 (2004).
32. Lloyd, S. & Braunstein, S. L. Quantum computation over continuous variables. Phys. Rev. Lett. 82, 1784 (1999).
33. Bartlett, S. D. & Sanders, B. C. Efficient classical simulation of optical quantum information circuits. Phys. Rev. Lett. 89, 207903 (2002).
34. Mari, A., Giovannetti, V. & Holevo, A. S. Quantum state majorization at the output of bosonic Gaussian channels. Nat. Comm. 5, 3826 (2014).
35. Giedke, G., Wolf, M. M., Kröger, O., Werner, R. F. & Cirac, J. I. Entanglement of Formation for Symmetric Gaussian states. Phys. Rev. Lett. 91, 107901 (2003).
36. Adesso, G. & Datta, A. Quantum versus Classical Correlations in Gaussian States. Phys. Rev. Lett. 105, 030501 (2010).
37. Pirandola, S., Spedalieri, G., Braunstein, S. L., Cerf, N. J. & Lloyd, S. Optimality of Gaussian Discord. Phys. Rev. Lett. 113, 140405 (2014).
38. Olivieri, H. & Zukrev, W. H. Quantum Discord: A Measure of the Quantumness of Correlations. Phys. Rev. Lett. 88, 017901 (2001).
39. Duan, L. M., Giedke, G., Cirac, J. I. & Zollner, P. Inseparability criterion for continuous variable systems. Phys. Rev. Lett. 84, 2722 (2000).
40. Simon, R. Peres-Horodecki Separability criterion for continuous variable systems. Phys. Rev. Lett. 84, 2726 (2000).
41. Reid, M. D. Monogamy inequalities for the Einstein-Podolsky-Rosen paradox and quantum steering. Phys. Rev. A 88, 062108 (2013).
42. He, Q. Y. & Reid, M. D. Genuine multipartite Einstein-Podolsky-Rosen steering. Phys. Rev. Lett. 111, 250403 (2014).
43. Kogias, I., Lee, A. R., Ragy, S. & Adesso, G. Quantification of Gaussian Quantum Steering. Phys. Rev. Lett. 114, 060403 (2015).
44. Ji, S.-W., Kim, M. S. & Nha, H. Quantum steering of multimode Gaussian states by Gaussian measurements: monogamy relations and the Peres conjecture. J. Phys. A: Math. Theor. 48, 155301 (2015).
45. Walk, N., Wiseman, H. M. & Ralph, T. C. Continuous variable one-sided device independent quantum key distribution. arXiv: 1405.6593/2
46. Kogias, I. & Adesso, G. J. Opt. Soc. Am. B 32, A27 (2015).
47. Ji, S.-W., Lee, J., Park, J. & Nha, H. Steering criteria via covariance matrices of local observables in arbitrary-dimensional quantum systems. Phys. Rev. A 92, 062130 (2015).
48. Simon, R., Mukunda, N. & Datta, B. Quantum-noise matrix for multimode systems: U(n) invariance, squeezing, and normal forms. Phys. Rev. A 49, 1567 (1994).
49. Ji, S.-W., Lee, J., Park, J. & Nha, H. Quantum steering of Gaussian states via non-Gaussian measurements. arXiv. 1511.02649.
50. Wollmann, S., Walk, N., Bennet, A. J., Wiseman, H. M. & Pryde, G. J. Observation of genuine one-way Einstein-Podolsky-Rosen steering. Phys. Rev. Lett. 116, 160403 (2016).

Acknowledgements
This work was supported by an NPRP grant 7-210-1-032 from Qatar National Research Fund.

Author Contributions
H.N. conceived the problem, which was theoretically developed together with S.-W.J. and others. All authors contributed to the analysis of the results and the review of the manuscript.

Additional Information
Supplementary information accompanies this paper at http://www.nature.com/srep

Competing financial interests: The authors declare no competing financial interests.

How to cite this article: Ji, S.-W. et al. Quantum steering of Gaussian states via non-Gaussian measurements. Sci. Rep. 6, 29729; doi: 10.1038/srep29729 (2016).

This work is licensed under a Creative Commons Attribution 4.0 International License. The images or other third party material in this article are included in the article’s Creative Commons license, unless indicated otherwise in the credit line; if the material is not included under the Creative Commons license, users will need to obtain permission from the license holder to reproduce the material. To view a copy of this license, visit http://creativecommons.org/licenses/by/4.0/