Constraining the Randall-Sundrum modulus in the light of recent PVLAS data

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Abstract

Recent PVLAS data put stringent constraints on the measurement of birefringence and dichroism of electromagnetic waves travelling in a constant and homogeneous magnetic field. There have been theoretical predictions in favour of such phenomena when appropriate axion-electromagnetic coupling is assumed. Origin of such a coupling can be traced in a low energy string action from the requirement of quantum consistency. The resulting couplings in such models are an artifact of the compactification of the extra dimensions present inevitably in a string scenario. The moduli parameters which encode the compact manifold therefore play a crucial role in determining the axion-photon coupling. In this work we examine the possible bounds on the value of compact modulus that emerge from the experimental limits on the coupling obtained from the PVLAS data. In particular we focus into the Randall-Sundrum (RS) type of warped geometry model whose modulus parameter is already restricted from the requirement of the resolution of gauge hierarchy problem in connection with the mass of the Higgs. We explore the bound on the modulus for a wide range of the axion mass for both the birefringence and the dichroism data in PVLAS. We show that the proposed value of the modulus in the RS scenario can only be accommodated for axion mass \( \gtrsim 0.3 \) eV.

1 Introduction

Theories with extra spatial dimensions have drawn considerable attention in recent times. Various implications of the presence of such dimensions and their observable consequences

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are being studied both in the collider and cosmological/astrophysical experiments. One of
the primary motivations to consider the presence of such extra dimension emerge from string
theory, which gives us a perturbatively finite quantum theory of gravity at the expense of
bringing in several extra spatial dimensions. Despite many theoretical successes, string theory
so far has failed to make any contact with the observable universe. To establish contact with
the present low energy world one considers the low energy field theory limit of a string theory.
The resulting field theory corresponds to the massless sector of a ten dimensional supergravity
(SUGRA) multiplet coupled to super-Yang Mills. A suitable compactification of the extra
dimensions then results into an effective low energy supergravity action in four dimensions. It
is therefore worthwhile to explore some generic observable features of such an action which may
provide some indirect evidence whether or not a string inspired supergravity model is a correct
description of our low energy world. Apart from areas like cosmological/astrophysical and high
energy collider experiments to look for the signature of stringy effects, it is also important to
study various purely laboratory based experiments which can provide complementary results.
In this work we focus into those experiments which make use of the conversion of axions or
any other low-mass (pseudo)scalar particles into photons in the presence of an electromagnetic
field. These include the Brookhaven-Fermilab-Rutherford-Trieste (BFRT) experiment [1], the
Italian PVLAS experiment [2] and several other experiments such as Q & A [3], BMV [4] etc.
which are either already in progress or in the process of being built up. All these experiments
are expected to produce axions from polarized laser beams, which are allowed to propagate in a
transverse, constant and homogeneous magnetic field. There has been theoretical prediction of
the possibility of modifying the polarization of light propagating through transverse magnetic
field due to its coupling with the pseudo-scalar axions [5]. The resulting birefringence and the
dichoism of the vacuum can be tested in these experiments, which in turn will constrain the
parameter space of a given model involving the mass of this light (pseudo)scalar particle and
its coupling to the electromagnetic field.

Other ongoing experiments looking for the (pseudo)scalar-photon mixing include LIPSS [6], ALPS [7], GammeV [8] and OSQAR [9]. Axion induced rotation of polarization plane of polarized prompt gamma ray radiation from gamma ray bursts (GRBs) have been analyzed recently in Ref.[10]. Several other implications of this axion-two photon coupling have been explored in various works [11]. Such coupling, which is related to the moduli parameters of the extra dimension in a string inspired action, therefore can be constrained by experiments.

The low energy four dimensional effective field theory action of string theory [12, 13, 14], namely the supergravity action, contains two massless fields, viz., a second rank antisymmetric tensor field (Kalb-Ramond (KR) field [15]) as well as a scalar field called dilaton. The interpretation of the KR field strength [16] as a torsion in the background spacetime [17], inevitably implies the study of electromagnetism in a spacetime with torsion. The three form field strength of the two form KR field is identified with the spacetime torsion and in this context, the gauge $U(1)$ Chern-Simons term that appears naturally on account of gauge anomaly-cancellation in the supergravity theory plays a crucial role in establishing a gauge-invariant coupling of the KR field (or, torsion) with the electromagnetic field [16]. Furthermore the dual of the Kalb-Ramond field strength in four dimensions in a string inspired model is the derivative of a scalar field called axion. Thus an axion-electromagnetic coupling arises naturally in string theory from the requirement of quantum consistency of the underlying supergravity theory. Certain physically observable phenomena, especially in the cosmological scenario, may result from such KR-electromagnetic coupling [18, 19, 20, 21, 22, 23, 24]. One such phenomenon of particular interest, argued to be induced by the axion-electromagnetic coupling [25]-[27], is a frequency-independent cosmic optical rotation of the plane of polarization of a linearly polarized synchrotron radiation from high redshift galaxies as well the birefringence effects in electromagnetic waves [28, 29, 30, 31, 32].
In this paper we aim to address these issues in a string inspired model and test it against the PVLAS experiment to probe into the role of extra dimensions in respect to these phenomena. Some interesting features of KR background in extra dimensional theories have already been explored in several works [33, 34, 35]. As will be shown later that an effective axion-photon coupling in four dimensions, is determined by Planck mass($M_p$) and the appropriate parameters encoding the nature of compactification of extra dimensions. We shall here focus into a specific axion-electromagnetic coupling induced by compactification proposed in Randall-Sundrum warped brane world model. Randall-Sundrum model has successfully resolved the problem of fine tuning of the Higgs boson mass against large radiative correction (originated from the large hierarchy between the electroweak and the Planck scale) without bringing in any new scale in the theory. Apart from supersymmetry, Randall-sundrum model perhaps is the most successful theory to resolve this longstanding problem which has always been an embarrassment for the standard model of elementary particles. We therefore examine the role of RS model in a different context namely the laboratory experiment like PVLAS. It should be noted here that there are other particles motivated by string theory which can produce observable effects similar to that of axion, in experiments such as PVLAS involving the propagation of light in external magnetic field [36]. Millicharged particles in the context of RS models could also provide similar effects [37]. Recently, PVLAS results have been analyzed in the context of a chameleon field whose properties depend on the environment [38].

The plan of the paper is as follows: we begin with a brief description of the axion-photon coupling which emerges naturally in a string inspired model. We shall then consider the Randall-Sundrum warped braneworld model and explain the nature of the resulting axion-photon interaction. This will be followed by an analysis of the optical rotation and the birefringence that result from such interaction. We shall then compare our result against the experimental findings of the PVLAS experiments. Finally we shall conclude with the possible future directions
2 Axion-photon coupling in a string inspired model

The string theoretic low energy effective action of Einstein-Kalb-Ramond-electromagnetic system is,

$$ S_d = \int d^d x \sqrt{-G} \left[ M^{d-2} R - \frac{1}{12} H_{ABC} H^{ABC} - \frac{1}{4} F_{AB} F^{AB} \right] $$

(1)

where,

$$ \bar{H}_{MNP} = \partial_{[M} B_{NP]} + \frac{1}{M^{d/2-1}} A_{[M} F_{NP]}.$$  

(2)

The $U(1)$ gauge field strength corresponding to $A_C$ is given as, $F_{CD} = \partial_{[C} A_{D]}$. The three form field strength corresponding to the second rank antisymmetric Kalb-Ramond field $B_{MN}$ in general is given as $H_{MNP} = \partial_{[M} B_{NP]}$. However the requirement of $U(1)$ gauge anomaly cancellation leads us to redefine $H_{MNP}$ by $\bar{H}_{MNP}$ with the appropriate electromagnetic Chern-Simon term as described in Eq.(2) above. $\mathcal{R}$ is the d-dimensional scalar curvature, $M$ is the d-dimensional Planck constant and $\sqrt{-G}$ is the determinant of the $d$-dimensional spacetime metric.

In order to get an effective four dimensional action from the $d$-dimensional action, we need to compactify the extra $d - 4$ dimensions. Keeping the Randall-Sundrum model in mind, let us consider the cases of one extra dimension only. After compactification the effective action in four dimensions becomes

$$ S_4 = \int d^4 x \sqrt{-g} \left[ M_p^2 R - \frac{1}{12} \bar{H}_{\mu
u\rho} \bar{H}^{\mu
u\rho} - \frac{1}{4} F_{\mu
u} F^{\mu\nu} \right] $$

(3)

where,

$$ \bar{H}_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} + \frac{\beta}{M_p} A_{[\mu} F_{\nu\rho]}.$$  

(4)
In this case the parameter $\beta$ is determined by the geometry of extra dimension and its moduli and the compactification scale. $M_p$ is the four dimensional Planck mass and $\sqrt{-g}$ is the determinant of the 4-dimensional spacetime metric. The equations of motion of the Kalb-Ramond (KR) and electromagnetic fields, obtained from the above action are,

$$D_\mu \tilde{H}^{\mu\nu\rho} = 0 ; \quad D_\mu F^{\mu\nu} = \frac{\beta}{M_p} \tilde{H}^{\nu\rho\gamma} F_{\rho\gamma}.$$  

(5)

The corresponding Bianchi identities are

$$D_\mu \tilde{F}^{\mu\nu} = 0 ; \quad \epsilon^{\mu\nu\gamma\delta} \partial_\mu H_{\nu\gamma\delta} = 0.$$  

(6)

In the above set of equations, $D_\mu$ is the covariant derivative and the dual $\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\gamma\delta} F_{\gamma\delta}$. Now, in four dimensions the third rank KR field strength tensor can be written in terms of a massless scalar field called axion through the duality relation,

$$\tilde{H}^{\mu\nu\rho} = \epsilon^{\mu\nu\rho\delta} \partial_\delta a.$$  

(7)

In terms of massless axion field $a$, Eqs.(5) and (6) yield

$$\Box a = \frac{1}{2} F^{\mu\nu} \tilde{F}_{\mu\nu},$$  

(8)

$$D_\mu F^{\mu\nu} = -\frac{2\beta}{M_p} \partial_\mu a \tilde{F}^{\mu\nu}.$$  

(9)

So, from the string inspired low energy effective action, we have an axion-photon coupling which depends on the moduli parameters of the compact space and the four dimensional Planck mass. We reiterate that our main objective in this paper is to explore the observable consequences of this coupling in recent PVLAS experiments.

So far in this model the axions are taken as massless. However, various stringy perturbative/non-perturbative corrections may produce mass for the axion. Without entering into the details of
its origin, we consider here the mass of the axion as a free parameter and choose its value in the range which can be probed by the experiments under consideration. In that case the string inspired effective 4-dimensional low energy action becomes,

$$S = \int \sqrt{-g} d^4x [M_p^2 R - \frac{1}{2} (\partial_{\mu} a \partial^\mu a - m_a^2 a^2) - \frac{\beta}{2M_p} a F_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}]$$  

where, $M_p$ is the 4-dimensional Planck scale, $m_a$ is the mass of pseudoscalar axion field $a$ and $F_{\mu\nu}$ is the total electromagnetic field strength. We now calculate the exact form of $\beta$ for Randall-Sundrum warped geometric model.

3 Randall-Sundrum brane world model and its effect on the axion-photon coupling

Here we consider a specific five dimensional warped geometry model proposed by Randall and Sundrum (RS). The minimal version of such a model is described in a five dimensional bulk AdS spacetime where the extra coordinate $y$ is compactified on a $S^1/Z_2$ orbifold. We define $y = r_c \phi$, where $r_c$ is the radius of $S^1$ and $\phi$ is the corresponding angular coordinate. $Z_2$ orbifolding restricts the value of $\phi$ between 0 and $\pi$. Two branes, viz., the hidden and visible branes, are located at two orbifold fixed points $\phi = 0$ and $\phi = \pi$ respectively. The line element of the corresponding background

$$ds^2 = e^{-2\sigma(y)} \eta_{\alpha\beta} dx^\alpha dx^\beta + r_c^2 d\phi^2$$

(11)

describes a non-factorizable geometry with an exponential warping over a flat ($\eta_{\alpha\beta}$) four dimensional submanifold. The warp factor is given in terms of the parameter $\sigma = kr_c \phi$, where $r_c$ is the compactification radius and $k$ is of the order of the higher dimensional Planck scale $M$. The four dimensional Planck scale $M_p$ is related to the five dimensional Planck scale $M$.
as: \( M_p^2 = \frac{M_3^3}{R} \left( 1 - e^{-2kr_c \pi} \right) \). The exponential warp factor causes a suppression of a scalar mass from the Planck scale to TeV scale on the visible brane, located at \( \phi = \pi \), as

\[
m = m_0 e^{-kr_c \pi}.
\]

Thus for \( k \sim M_p \) and \( r_c \sim 1/M_p \) such that \( kr_c \sim 11.7 \), the scalar mass on the brane \( m \sim \text{TeV} \) for \( m_0 \sim M_p \). Therefore the fine tuning problem in connection with the scalar Higgs mass is resolved geometrically without introducing any intermediate scale in the theory.

As an extension to this model, we include the second rank antisymmetric KR field in the bulk along with gravity. Just as graviton mode, the KR field, being a closed string excitation, can enter into the five dimensional bulk spacetime whereas all the standard model fields being open string modes are confined on the visible 3-brane. The Randall-Sundrum compactification of the free Einstein-Kalb-Ramond Lagrangian has already been studied extensively in [34, 35]. Similar kind of compactification for the bulk \( U(1) \) gauge field has been studied in [39] and also elucidated in [35] in the context of cosmic optical activity. Starting from the Einstein-KR action we write down the Eq.(1) in \( d = 5 \) as follows,

\[
S_5 = \int d^5 x \sqrt{-G} \left[ M^3 R - \frac{1}{12} \hat{H}_{ABC} \hat{H}^{ABC} - \frac{1}{4} F_{AB} F^{AB} \right]. \tag{13}
\]

Let us consider the KR field action in 5-dimension as follows

\[
S_H = \frac{1}{12} \int d^5x \sqrt{-G} H_{MNL} H^{MNL} \tag{14}
\]

where \( \sqrt{-G} = e^{-4\sigma} r_c \). This action has KR gauge invariance \( \delta B_{MN} = \partial_M \Lambda_N \). We use the KR gauge fixing condition to set \( B_{4\mu} = 0 \). Therefore the only non vanishing KR field components are \( B_{\mu\nu} \) where \( \mu, \nu \) runs from 0 to 3. These components in general are functions of both compact and non-compact coordinates. One thus gets

\[
S_H = \frac{1}{12} \int d^4x \int d\phi r_c e^{2\sigma(\phi)} \left[ \eta^{\mu\alpha} \eta^{\nu\beta} \eta^{\lambda\gamma} H_{\mu\nu\lambda} H_{\alpha\beta\gamma} - \frac{3}{r_c^2} e^{-2\sigma(\phi)} \eta^{\mu\alpha} \eta^{\nu\beta} B_{\mu\nu} \partial_\phi^2 B_{\alpha\beta} \right]. \tag{15}
\]
Applying the Kaluza-Klein decomposition for the Kalb-Ramond field:

\[ B_{\mu\nu}(x, \phi) = \sum_{n=0}^{\infty} B_{\mu\nu}^n(x) \chi^n(\phi) \frac{1}{\sqrt{r_c}} \]  

(16)

and demanding that in four dimensions an effective action for \( B_{\mu\nu} \) should be of the form

\[ S_H = \int d^4x \sum_{n=0}^{\infty} \left[ \eta^{\mu\alpha} \eta^{\nu\beta} \eta^{\lambda\gamma} H_{\mu\nu\lambda}^n H_{\alpha\beta\gamma}^n + 3m^2_n \eta^{\mu\alpha} \eta^{\nu\beta} B_{\mu\nu}^n B_{\alpha\beta}^n \right] \]  

(17)

where \( H_{\mu\nu\lambda}^n = \partial_{[\mu} B_{\nu\lambda]}^n \) and \( \sqrt{3}m_n \) gives the mass of the nth KK mode of the KR field, one obtains

\[-\frac{1}{r_c^2} \frac{\partial^2 \chi^n}{\partial \phi^2} = m^2_n \chi^n e^{2\sigma}. \]  

(18)

The \( \chi^n(\phi) \) field satisfies the orthogonality condition

\[ \int e^{2\sigma(\phi)} \chi^m(\phi) \chi^n(\phi) d\phi = \delta_{mn}. \]  

(19)

Defining \( z_n = e^{\sigma(\phi)}m_n/k \), the above equation reduces to

\[ \left[ z_n^2 \frac{d^2}{dz_n^2} + z_n \frac{d}{dz_n} + z_n^2 \right] \chi^n = 0. \]  

(20)

This has the solution

\[ \chi^n = \frac{1}{N_n} [J_0(z_n) + \alpha_n Y_0(z_n)]. \]  

(21)

The zero mode solution [34, 35] of \( \chi \) therefore turns out to be

\[ \chi^0(\phi) = C_1|\phi| + C_2. \]  

(22)

However, the condition of self-adjointness leads to \( C_1 = 0 \) and leaves the scope of only a constant solution for \( \chi^0(\phi) \). Using the normalization condition, one finally obtains

\[ \chi^0 = \sqrt{kr_c} e^{-kr_c \pi}. \]  

(23)
This result clearly indicates that the massless mode of the KR field is suppressed by a large warp factor on the visible 3-brane. In a similar way one can express the electromagnetic field in bulk first by decomposing it into Kaluza-Klein modes

\[ A_\mu(x, \phi) = \frac{1}{\sqrt{r_c}} \sum_{n=0}^{\infty} A^n_\mu(x) \xi^n(\phi). \]  

The solution for the massless mode of the \( U(1) \) gauge field \([39]\) reads as

\[ \xi^0 = \frac{1}{\sqrt{2\pi}}. \]  

Using the zero mode solutions for both fields, the interaction term in Eq.(13) turns out to be

\[ S_{\text{int}} = \frac{1}{M_p^2} \int d^4x \int d\phi \left[ e^{2\sigma} r_c \eta^{\mu\alpha} \eta^{\nu\beta} H_{\mu\nu\lambda} A_{[\alpha} F_{\beta\gamma]} + \frac{6}{r_c} \eta^{\mu\alpha} \eta^{\nu\beta} (\partial_\phi B_{\mu\nu}) A_{[\alpha} (\partial_\phi A_{\beta]} \right]. \]  

Now using the Kaluza-Klein decomposition for both the fields described earlier, one obtains

\[ S_{\text{int}} = \frac{1}{M_p^2} \int d^4x \int d\phi \left[ e^{2\sigma} \frac{1}{\sqrt{r_c}} \sum_{n,m,l=0}^{\infty} \chi^n \xi^m \xi^l \eta^{\mu\alpha} \eta^{\nu\beta} \eta^{\lambda\gamma} H^n_{\mu\nu\lambda} A^m_{\alpha} F^l_{\beta\gamma]} + \frac{6}{r_c} \sum_{n,m,l=0}^{\infty} (\partial_\phi \chi^n) \chi^m (\partial_\phi \xi^l) \eta^{\mu\alpha} \eta^{\nu\beta} B^n_{\mu\nu} A^m_{\alpha} A^l_{\beta]} \right]. \]  

The part of the above action containing the massless modes only is given as \([35]\)

\[ S_{\text{int}} = \frac{1}{M_p^2} \int d^4x \int d\phi \frac{e^{2\sigma}}{\sqrt{r_c}} \chi^0 (\xi^0)^2 H_{\mu\nu\lambda} A^{[\mu} F_{\nu\lambda]}, \]  

where \( \chi^0 = \sqrt{kr_c e^{-kr_c x}}, \xi^0 = \frac{1}{\sqrt{2\pi}} \) and \( A^\mu = \eta^{\mu\nu} A_\nu. \)

The KR-EM part of the 4d effective action (without the curvature term) therefore becomes

\[ S_{\text{eff}} = \int d^4x \left[ M_p^2 R - \frac{1}{12} H^{\mu\nu\lambda} H_{\mu\nu\lambda} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right], \]  

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where
\[ H_{\mu\nu\gamma} = H_{\mu\nu\gamma} + \sqrt{\frac{k}{M_3^3}} e^{k r c \pi} A_{[\mu} F_{\nu\gamma]} \]. \quad (30)

We have thus explicitly determined the KR-Maxwell coupling in the proposed string inspired RS scenario. If we now compare the above equation Eq.(30) with Eq.(4), we observe that the parameter \( \beta \) in the effective four dimensional axion-photon coupling is determined by the RS compactification as,
\[ \beta = \sqrt{\frac{k}{M_p}} e^{k r c \pi}. \] \quad (31)
Thus determining the axion-photon coupling in a RS compactified string inspired model, we now explore the theoretical predictions of such coupling on rotation of plane of polarization as well as birefringence for an electromagnetic wave propagating in a transverse magnetic field.

4 Optical rotation and birefringence in string inspired model

In this section we explicitly estimate the rotation angle of the plane of polarization and the birefringence due to the axion-photon coupling that has been shown to arise naturally in a string inspired model.
Recall the string low energy action,
\[ S = \int \sqrt{-g} d^4 x [M_p^2 R - \frac{1}{2}(\partial_\mu a \partial^\mu a - m_0^2 a^2) - \frac{\beta}{2M_p} a F_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}] \] \quad (32)

\[ F_{\mu\nu} = F_{\mu\nu}^{ext} + \partial_\mu A_\nu - \partial_\nu A_\mu \] \quad (33)
where \( F_{\mu\nu}^{ext} \) corresponds to the external magnetic field and \( A_\mu \) is the vector potential associated with the light wave.
Now, the classical equations of motion for the axion and electromagnetic fields obtained from the above action are

\[
\Box + m_a^2) a = -\frac{2\beta}{M_p} \mathbf{E} \cdot \mathbf{B} \tag{34}
\]
\[
\nabla \cdot \mathbf{E} = \frac{2\beta}{M_p} \nabla a \cdot \mathbf{B}
\]
\[
\nabla \times \mathbf{B} - \dot{\mathbf{E}} = \frac{2\beta}{M_p} [\mathbf{E} \times \nabla a - \mathbf{B} \dot{a}]
\]
\[
\nabla \cdot \mathbf{B} = 0
\]
\[
\dot{\mathbf{B}} + \nabla \times \mathbf{E} = 0. \tag{35}
\]

In the presence of an external magnetic field \(\mathbf{B}_0\), the magnetic field \(\mathbf{B}\) in the above set of equations can be written as \(\mathbf{B} = \mathbf{B}_{\text{wave}} + \mathbf{B}_0\), where \(\mathbf{B}_{\text{wave}}\) is the magnetic field associated with the electromagnetic wave. In our analysis we will use the gauge condition \(\nabla \cdot \mathbf{A} = 0\). Taking the propagation direction of the electromagnetic wave to be orthogonal to the external magnetic field \(\mathbf{B}_0\) and specifying [26] the condition \(\mathbf{A}_0 = 0\) the coupled axion-photon equations in the linear order in \(\phi\) and \(\mathbf{A}\) turn out to be,

\[
\Box \mathbf{A} + \frac{2\beta}{M_p} \frac{\partial a}{\partial t} \mathbf{B}_0 = 0, \tag{36}
\]
\[
(\Box + m_a^2) a - \frac{2\beta}{M_p} \frac{\partial \mathbf{A}}{\partial t} \cdot \mathbf{B}_0 = 0. \tag{37}
\]

However, from the above equation it is clear that only the component of \(\mathbf{A}\) parallel to \(\mathbf{B}_0\) is affected. Thus for the linearly polarized wave, the orthogonal component of the vector potential with respect to \(\mathbf{B}_0\) can be written as

\[
\mathbf{A}_\perp(t, x) = \exp[-i(\omega t - \kappa \cdot x)], \tag{38}
\]
\[
\kappa \cdot \mathbf{B}_0 = 0. \tag{39}
\]
where $\omega$ and $\kappa$ respectively are the energy and wave number of the initial beam ($|\kappa| = \omega$). Our solution ansatz for the parallel component of the $A$ with respect to the external magnetic field $B_0$ and the axion field is

$$A_\parallel = A_0 \exp[-i(\omega' t - \kappa \cdot x)],$$

$$a = a_0 \exp[-i(\omega' t - \kappa \cdot x)].$$

So, in order to have the consistent solutions we solve the corresponding secular equation

$$(\kappa^2 - \omega'^2)(\kappa^2 + m_a^2 - \omega'^2) - \frac{4\beta^2 B_0^2}{M_p^2} \omega'^2 = 0.$$  

(42)

The roots corresponding to $\omega'$ are

$$\omega'^2 = \kappa^2 + \delta_{\pm}$$

(43)

where

$$\delta_{\pm} = \frac{1}{2} \left\{ m_a^2 + \frac{4\beta^2 B_0^2}{M_p^2} \pm \left[ \left( m_a^2 + \frac{4\beta^2 B_0^2}{M_p^2} \right)^2 + \frac{16\kappa^2 \beta^2 B_0^2}{M_p^2} \right]^{1/2} \right\}.$$ 

(44)

Using the initial boundary conditions,

$$A_\parallel(t = 0, x = 0) = 1 ; \quad a(t = 0, x = 0) = 0$$

(45)

the solution becomes,

$$A_\parallel = A_1 e^{-i(\omega t - \kappa \cdot x)} + A_2 e^{-i(\omega' t - \kappa \cdot x)}$$

(46)

where, the integration constants are

$$A_1 = \frac{-\delta_- \omega_+}{\delta_+ \omega_- - \delta_- \omega_+},$$

$$A_2 = \frac{\delta_+ \omega_-}{\delta_+ \omega_- - \delta_- \omega_+}.$$ 

(47)

(48)
To establish a connection with the experimental set up, we consider the initial \((t = 0)\) electromagnetic field to be linearly polarized and making an angle \(\alpha\) with the external magnetic field \(B_0\), so that

\[
A(t = 0) = \cos \alpha \ i + \sin \alpha \ j. \tag{49}
\]

While travelling through the region of external magnetic field, the resulting interaction causes the wave solution to have the form after \(t = \ell\) as

\[
A(t) = \cos \alpha \ A_{\parallel}(t) \ i + \sin \alpha \ \exp(-i\omega t) \ j. \tag{50}
\]

Therefore, the amplitude part of \(A_{\parallel}\) becomes (up to a common phase factor with respect to the orthogonal component \(A_{\perp}\))

\[
A_{\parallel}(\ell) = A_1 \exp[-i\theta_+] + A_2 \exp[-i\theta_-], \tag{51}
\]

where

\[
\theta_+ = \frac{\delta_+ \ell}{2\kappa}; \quad \theta_- = \frac{\delta_- \ell}{2\kappa}. \tag{52}
\]

So, from the above set of expressions Eq.(49) and Eq.(51), we see that the vector potential describes an ellipse with the major axis at an angle

\[
\alpha(\ell) = \alpha + A_1 A_2 \sin^2 \left(\frac{\Delta \theta}{2}\right) \sin 2\alpha, \tag{53}
\]

with the external magnetic field \(B_0\). Here \(\Delta \theta \equiv \theta_+ - \theta_-\). Similarly the extra phase difference developed due to interaction with the axion field is

\[
\Phi = \tan^{-1} \left[ \frac{A_1 \sin \theta_+ + A_2 \sin \theta_-}{A_1 \cos \theta_+ + A_2 \cos \theta_-} \right]. \tag{54}
\]

Now, Eq.(53) yields the expression for the optical rotation of the plane of polarization of the electromagnetic wave as \(\epsilon = \alpha(\ell) - \alpha\) and similarly the Eq.(54) gives the expression for the
ellipticity $\mathcal{E}$ as the ratio of the minor to major axis.

$$\epsilon(\ell) = A_1 A_2 \sin^2 \left( \frac{\Delta \theta}{2} \right) \sin 2\alpha,$$

(55)

$$\mathcal{E}(\ell) = \frac{1}{2} \tan^{-1} \left[ \frac{A_1 \sin \theta_+ + A_2 \sin \theta_-}{A_1 \cos \theta_+ + A_2 \cos \theta_-} \right].$$

(56)

These are the two quantities which establish the direct link with the experimental data. We now proceed to estimate them in the context of PVLAS experiments.

5 Probing the moduli parameters using Laser experiments

As discussed in the previous sections, the purely laboratory based experimental search for ultralight (pseudo)scalar particles are devised on the basis of the prediction that the polarization properties of light, propagating in a constant and transverse magnetic field, can be changed because of the couplings of these particles with two photons [25]. In these class of experiments, it is possible to make accurate measurements on the modification of the polarization state of a light beam. In a practical experiment a laser beam is reflected back and forth N times between two mirrors, in a constant magnetic field of strength $B_0$ which is orthogonal to the beam direction. If the distance between the subsequent reflections is $\ell$ then the total length travelled by the laser beam in the magnetic field is $L = N\ell$. The laser beam is linearly polarized to start with and after traversing a distance $L$, which is usually of the order of a few kilometers, it is possible to measure very small ellipticity and change in the rotation of the polarization plane.

The vacuum magnetic birefringence (or in other words the acquired very small ellipticity of the linearly polarized light) predicted by the QED, is due to the dispersive effect produced
by the virtual electron-positron pair as discussed by Heisenberg and Euler [40]. The ellipticity produced this way serves as the background event for the experiment looking for birefringence or dichroism produced by (pseudo)scalar particle. The QED contribution to the ellipticity can be written as

\[
E = N \frac{B_0^2 \ell \alpha^2 \omega}{15m_e^4},
\]

where \( \alpha = 1/137 \) is the fine-structure constant, \( \omega \) is the photon energy and \( m_e \) the electron mass. Here we have assumed that the polarization vector of the initially linearly polarized beam makes an angle 45° with the direction of the external magnetic field. If we take a laser beam with a wavelength \( \lambda = 1550 \text{ nm} \), \( B_0 = 9.5 \text{ T} \) and \( N \ell = 25 \text{ km} \) then the resulting ellipticity from Eq.(57) is \( 2 \times 10^{-11} \text{ rad} \) [41].

The photon splitting effect can also produce an apparent rotation of the plane of polarization of a linearly polarized light [42]. However, the resulting effect is too small to be observed in the laboratory. On the other hand, if the coupling of scalar/pseudoscalar with two photons is sufficiently large then this effect of photon splitting can be significantly enhanced [43, 44].

It is important to note here that any physical mirror appears to be transparent to axions so that only the photon component of the beam is reflected. This essentially sets the axion component of the beam back to zero after each reflection [5]. The resulting effect of \( N \) reflections is that \( E(L) = N E(\ell) \) where, in general, \( N E(\ell) \) is not equal to \( E(N \ell) \).

Thus, in order to take into account the effect of \( N \) reflections appropriately for a multiple-beam-path experiment, we need to multiply the right-hand side of Eq.(55) and Eq.(56) by \( N \) (keeping everything else the same) and on the left-hand side the length \( \ell \) of a single-path is now replaced by the total length \( L = N \ell \).

If we now consider the case of extremely small axion masses, which means \( \theta_+, \theta_- \) and \( \Delta \theta \ll \)
1, the results in Eq.(55) and Eq.(56) can be expanded and gives us

\[ \epsilon(L) = N \frac{B_0^2}{16M^2} \ell^2 \]  \hspace{1cm} (58)

and

\[ \mathcal{E}(L) = N \frac{(B_0 m_a)^2}{48\kappa M^2} \ell^3. \]  \hspace{1cm} (59)

Here the effective inverse coupling constant \( \tilde{M} \) is defined as

\[ \tilde{M} \equiv M_p/2\beta \]  \hspace{1cm} (60)

as can be seen from Eq.(10). These two results in Eq.(58) and Eq.(59) agree with the corresponding expressions given by Eq.(44) in Ref.[5] \(^4\). As we shall see later, these results in the small axion mass limits describe the behaviours of the rotation and the ellipticity as a function of the axion mass and the inverse coupling strength.

In the year 2006 the PVLAS experiment [2] measured a positive value for the amplitude of the rotation \( \epsilon \) of the polarization plane in vacuum with \( B_0 \approx 5 \) T. The result is (with a 3\( \sigma \) uncertainty) \( \epsilon = (3.9 \pm 0.5) \times 10^{-12} \text{ rad/pass} \). However, the new observations reported very recently [45], do not show the presence of a rotation signal down to the levels of

\[ 1.2 \times 10^{-8} \text{ rad at a magnetic field strength of } 5.5 \text{ T} \]  \hspace{1cm} (61)

\[ 1.0 \times 10^{-9} \text{ rad at a magnetic field strength of } 2.3 \text{ T} \]

(at 95\% c.l.) with 45000 passes. In the same experimental environment no ellipticity signal has been detected down to

\[ 1.4 \times 10^{-8} \text{ at a magnetic field intensity of } 2.3 \text{ T} \]  \hspace{1cm} (62)

\(^4\)Note that the relation between \( \epsilon(L) \) in our paper and \( \varepsilon(L) \) in Eq.(44) of Ref.[5] is given by \( \epsilon(L) = \frac{1}{2} \varepsilon(L) \).
These new results exclude the particle interpretation of the previous PVLAS results and impose bounds on the mass and the inverse coupling constant for scalar/pseudoscalar bosons coupled to two photons. It should be noted that for the same experimental situation, the QED effects induce an ellipticity $\sim 1.6 \times 10^{-10}$.

In Fig. 1 we have plotted the exclusion regions in the two-dimensional plane spanned by the parameters axion mass ($m_a$) and the effective inverse coupling constant ($\tilde{M}$) of axion to two photons. The curves have been drawn using our Eq.(55) and Eq.(56) where we have appropriately considered the number of passes (N) and taking into account the limiting values for the rotation and birefringence mentioned in Eq.(61) and Eq.(62). We have also assumed the value of N to be 45000 in the interaction region. From this figure it is evident that the bound on the effective inverse coupling $\tilde{M}$ coming from the absence of rotation is independent of the mass of the axion (for $m_a \lesssim 10^{-3}$ eV). This can be easily seen from Eq.(58) where in the small axion mass limit the rotation is actually independent of the axion mass. The resulting bound is $\tilde{M} \gtrsim 3 \times 10^6$ GeV. One can translate this bound on $\tilde{M}$ into an upper bound on the moduli parameter $\beta$ using the relation given in Eq.(60). Thus, $\beta$ is bounded from above as $\beta \lesssim 1.6 \times 10^{12}$. Using the relation between $\beta$ and the compactification radius $r_c$ as stated in Eq.(31), one gets an upper bound on $r_c$ given by

$$kr_c \gtrsim \frac{1}{\pi} (13 \ln(10) - \ln(6)) \simeq 8.95.$$  \hspace{1cm} (63)

As discussed earlier, the required value of $kr_c$ in order to generate the TeV scale naturally in the Randall-Sundrum scenario is $kr_c = 11.72$. This value of $kr_c$ is in direct conflict with the bound obtained in Eq.(63). Thus, we see that the limits on the rotation of plane of polarization of a plane polarized light coming from the PVLAS experiments, puts severe restrictions on the modulus of the Randall-Sundrum scenario. This essentially means that it is very difficult to address the hierarchy problem in the context of the RS model, particularly in the region of low
Figure 1: Bounds on mass \( (m_a) \) and effective inverse coupling constant \( (\tilde{M}) \) for axion to two photons using the recent PVLAS results for the rotation and the ellipticity. Area below the solid and the dotted curves are disallowed from the data.

axion-mass.

On the other hand, if we consider higher values of the axion mass then the stronger restrictions on the moduli parameters come from the limits on the ellipticity measurements as can be seen from Fig.(1). The resulting bound on the parameter \( \tilde{M} \) is \( \tilde{M} \gtrsim 3 \times 10^3 \) GeV for an axion mass \( m_a \approx 0.3 \) eV. The corresponding limit on the moduli parameters appears to be

\[
kr_c \gtrsim \frac{1}{\pi} (16 \ln(10) - \ln(6)) \simeq 11.15. \tag{64}
\]

This value of \( kr_c \) is more or less in the right ballpark to solve the hierarchy problem in the Randall-Sundrum scenario. Hence we observe that though the RS model is disfavoured as a potential candidate to solve the hierarchy problem in the low axion mass region, it indeed gives the values of the parameters in the required range in the region of larger axion mass.
If we analyze the data for rotation and the ellipticity separately then we can see from Fig. 1 that the ellipticity bound allows the correct value of $kr_c$ (to address the hierarchy problem) also for axion mass $m_a \lesssim 10^{-5}$ eV. Similarly, the rotation bound gives the allowed value of $kr_c$ only in the high axion mass region ($m_a \gtrsim 0.07$ eV).

At this stage it is very important to discuss the bounds on the parameter $\tilde{M}$ coming from astrophysical considerations. There is a very strong constraint $\tilde{M} \gtrsim 10^{10}$ GeV (for $m_a < O$(keV)) from the calculation of stellar energy loss [46] of horizontal branch stars and from the non-observation of axions in helioscopes such as the CERN Axion Solar Telescope (CAST) [47]. This puts much severe restrictions on the parameters of Randall-Sundrum model which in turn make such models disfavoured in the context of a possible resolution of the hierarchy problem. However, it has been shown recently that such strong astrophysical constraints can be evaded under certain assumptions [48]. For example, the axion-two photon vertex can be suppressed at keV energies due to low scale compositeness of the axion [48]. One can also consider the case where the temperature and the matter density inside the stars control the mass and the coupling of the axion. This way axions can acquire an effective mass larger than a few keV which is the typical photon energy. This can suppress the axion production inside the stellar plasmas and relax astrophysical bounds by several orders of magnitude [49]. Since the stringent astrophysical bounds can be evaded, it is very important to look for laboratory experiments where we have control over all the relevant experimental parameters. This is the reason we have studied the constraints coming from the PVLAS experiment on the moduli parameters of the Randall-Sundrum scenario.
6 Conclusions

In this work we have explored the implications of axion-photon coupling in a string inspired Randall-Sundrum model where such coupling emerges inevitably from the requirement of quantum consistency of the model. Randall-Sundrum model, which is advertised to be a viable alternative to supersymmetric theory for offering a possible resolution to the gauge hierarchy problem in standard model, confronts some rigorous test in laboratory experiments like PVLAS because of such axion-photon coupling. Possibility of finding the signature of warped extra dimensional models in controlled laboratory-based experiments is therefore the main motivation of this work. Our results put severe constraint on the modulus of Randall-Sundrum type of model. For experiments like optical rotation of the plane of polarization of an electromagnetic wave, the RS model is disfavoured for axion mass $\lesssim 0.07$ eV, whereas for experiments measuring the ellipticity the value of the modulus reside in the allowed range only for axion mass $\lesssim 10^{-5}$ eV or $\gtrsim 0.3$ eV. However on combining both the experimental results the RS model is shown to be consistent only for axion mass $\gtrsim 0.3$ eV. In conclusion RS model, tested against PVLAS results (and similar such experiments) puts severe bound on the modulus and the axion mass if it has to resolve the hierarchy problem of the standard model. It should be mentioned at this point that one can also explain the KR field strength as a torsion in space-time. In that case the role of additional bulk fields could be important. However, in this work we just extend the original bulk gravity model of RS to include the asymmetric torsion part and find its effect in the context of PVLAS experiment. We have shown that such a field indeed produces interesting effects and determined that to what extent it may explain the experimental data. Analysis presented in this work may now be extended for models with more than one extra warped dimensions\[50\] and also other type of compactification scenarios in extra dimensional models. This might lead to a much deeper understanding of the role of extra dimension and
its possible signature in different laboratory-based experiments.

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