The perturbative odderon intercept\textsuperscript{a}

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We present our recent results on the odderon intercept in perturbative QCD, obtained through the solution of the Baxter equation and investigation of the spectrum of the relevant constant of motion.

1 Introduction

An interesting problem of QCD is to understand the behaviour of the theory in the Regge limit of large energy, fixed momentum transfer. In the Leading Logarithmic Approximation the leading pole in the \( C = +1 \) channel is the famous BFKL pomeron\textsuperscript{1}. Later this was generalized to the channel odd under charge conjugation (\( C = -1 \)) — the odderon\textsuperscript{2}. In contrast to the BFKL case, however, the value of the intercept remained unknown despite the discovery of conformal symmetry and integrals of motion\textsuperscript{3}.

The hard odderon may be observed in such processes as \( \gamma^* \gamma^* \rightarrow \eta_c + X \), \( \gamma^* p \rightarrow \eta_c + X \), \( \gamma^* \gamma^* \rightarrow \eta_c \eta_c \). A number of recent papers\textsuperscript{7} calculated the cross sections in the approximation where the odderon was modelled by an exchange of three (non-reggeized) gluons. The cross sections were small, and hence it is interesting to see if reggeization could lead to a significant enhancement at high energies, which could perhaps be investigated experimentally.

Recently substantial progress\textsuperscript{4,5} has been made with the reduction of the problem to solving the Baxter equation

\[
(\lambda + i)^3 Q(\lambda + i) + (\lambda - i)^3 Q(\lambda - i) = (2\lambda^3 + q_2 \lambda + q_3)Q(\lambda)
\]

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for physical values of the constants of motion $q_2$ and $q_3$. The intercept then follows through a logarithmic derivative. A number of approximation techniques for solving this equation have been tried. In our work an exact method of solving the Baxter equation for arbitrary values of the parameters $q_2$ and $q_3$ was developed. This reduced the problem to finding the physical spectrum of $q_3$ ($q_2$ is fixed by group theory to be $q_2 = h(1-h)$ where $h$ is the conformal weight).

Ideally one could do this directly, by requiring that the solution of the Baxter equation $Q(\lambda)$, gives rise to a normalizable, single-valued wavefunction (see below). However, the explicit relation between wavefunctions and the functional forms of the solutions $Q(\lambda)$ is very indirect. In particular it is still unknown how does the normalizability requirement explicitly look like in terms of $Q(\lambda)$.

In our second paper we pursued a more direct approach, using standard physical requirements which are imposed on the wave function to find the allowed values of $q_3$ and the explicit form of the wave function. This form can be useful when considering the coupling of the odderon to external probes. In this talk we would like to present the main results of these papers and give our conclusions on the intercept of the odderon.

2 Solution of the Baxter equation

The Baxter equation poses a number of difficulties. It is a nonlocal functional equation which possesses polynomial solutions only for integer values of the conformal weight $h = n > 3$. In contrast to the BFKL case there is no easy analytical continuation to the physically most interesting case of $h = 1/2$. Quasiclassical methods involve an expansion in $1/h$ and use powerful methods to continue to $h = 1/2$, the precision is, however, difficult to control. Our aim was to find an expression for the solution directly for $h = 1/2$ (or indeed also for more general $h$) in a form which could be numerically calculated to an arbitrary precision.

The starting point was the contour integral representation used in:

$$Q(\lambda) = \int_C \frac{dz}{2\pi i} z^{-i\lambda-1}(z-1)^{i\lambda-1} \tilde{Q}(z)$$

where $\tilde{Q}(z)$ satisfies a $3^{rd}$ order differential equation $D\tilde{Q}(z) = 0$. The expression satisfies then the Baxter equation only if the contour $C$ is chosen as such that the boundary terms arising from integration by parts cancel out. It turns out that for $h = 1/2$ it is impossible to choose such a contour because the curve always ends up on a different sheet of the Riemann surface of the integrand. To remedy this we extended the ansatz to a sum of two integrals:

$$\int_{C_1} \frac{dz}{2\pi i} K(z, \lambda) \tilde{Q}_1(z) + \int_{C_{II}} \frac{dz}{2\pi i} K(z, \lambda) \tilde{Q}_2(z)$$

where the contours are independent (see e.g. figures in). The functions $\tilde{Q}_1(z)$ and $\tilde{Q}_2(z)$ are both solutions of $D\tilde{Q}(z) = 0$, and thus depend initially on 6 free parameters. We now impose the condition of cancellation of boundary terms. This gives 3 equations, leaving us with $6 - 3 = 3$ parameters. Now one notes that since $D\tilde{Q}(z) = 0$ has a solution holomorphic at infinity, it gets integrated out to zero in each of the integrals in (3). This leaves us with only $3 - 2 = 1$ parameter which is just an irrelevant normalization. The above procedure leads therefore to a unique solution within our ansatz (3). This solution can be calculated to an arbitrary precision by including a sufficient number of terms in power series expansions of the $\tilde{Q}_i(z)$’s. This being done we will move on, in the next section, to consider the problem of finding the physical values of $q_3$.
3 Quantization of $q_3$

The wavefunction $\Psi(z, \bar{z})$ can be decomposed into the following sum:

$$\Psi(z, \bar{z}) = \sum_{i,j} \overline{u}_i(\bar{z}) A^{(0)}_{ij} u_j(z),$$

(4)

where $u_i(z)$ and $\overline{u}_j(\bar{z})$ are eigenfunctions (analytic in the whole complex plane apart from some cuts) of the $3^{rd}$ order ordinary differential operators $\hat{q}_3$ and $\overline{\hat{q}}_3$ with eigenvalues $q_3$ and $\overline{q}_3$. We will later also consider the $(-q_3, -\overline{q}_3)$ sector which is necessary for Bose symmetry of the wavefunction.

We impose the following obvious physical requirements for the wavefunction: (1) $\Psi(z, \bar{z})$ must be single-valued, (2) $\Psi(z, \bar{z})$ must be normalizable and (3) $\Psi(z, \bar{z})$ must satisfy Bose symmetry. The crucial assumption is the first one. It turns out that normalizability follows automatically (apart from the unphysical case of $q_3 = 0$), and Bose symmetry is easy to implement.

We must examine the requirement of single valuedness near the singular points $z = 0$ and $z = 1$ ($z = \infty$ follows — see discussion in [10]). Using the asymptotic behaviour of the $u_i$'s near $z = 0$ ($u_1(z) \sim z^{1/3}$, $u_2(z) \sim z^{5/6}$, $u_3(z) \sim z^{5/6} \log z + z^{-1/3}$) it is clear that the coefficient matrix $A^{(0)}$ of (4) must have the form

$$A^{(0)} = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & \gamma \\ 0 & \gamma & 0 \end{pmatrix}.$$  

(5)

We thus have initially 3 parameters $\alpha$, $\beta$ and $\gamma$. Now we must impose the same requirement around $z = 1$. To this end we note that the solutions $v_i(z) \equiv u_i(1 - 1/z)$ have similar asymptotics around $z = 1$ to the asymptotics of $u_i$'s around $z = 0$. One can numerically calculate the analytical continuation matrices $u_i(z) = \Gamma_{ij} v_j(z)$.

We must now reexpress the wavefunction $\Psi(z, \bar{z})$ in terms of the $v_i$'s and require that the transformed coefficient matrix $A^{(1)} = \Gamma A^{(0)} \Gamma$ has the same form as (5). This leads to a number of linear homogeneous equations for $\alpha$, $\beta$ and $\gamma$. Because the number of equations is greater than 3, the existence of a nonzero solution fixes both the parameters of the wavefunction $\alpha$, $\beta$, $\gamma$ and the allowed values of $q_3$. If the coefficient matrices $A^{(0)}$ and $A^{(1)}$ coincide, the requirement of Bose symmetry boils down to adding the corresponding wavefunction in the $(-q_3, -\overline{q}_3)$ sector.

$$\Psi(z, \bar{z}) = \Psi_{q_3,q_3^*}(z, \bar{z}) + \Psi_{-q_3, -q_3^*}(z, \bar{z})$$

(6)

4 Results

In [10] a number of possible solutions were found. The requirement of Bose symmetry picked out just those lying on the imaginary axis. We may now plug in those values of $q_3$ into our solution of the Baxter equation to yield the intercepts corresponding to those states. The results are summarized below:

| No. | $q_3$     | $\epsilon_3$ |
|-----|-----------|--------------|
| 1   | $0.20526i$| $-0.49434$  |
| 2   | $2.34392i$| $-5.16930$  |
| 3   | $8.32635i$| $-7.70234$  |

We are thus led to conclude that the odderon state with the highest intercept has $q_3 = \pm 0.20526i$. The intercept of this state reads

$$\alpha_O = 1 - 0.24717\alpha_s N_c / \pi = 1 - 0.16478 \cdot 3\alpha_s N_c / 2\pi$$

(7)
and the wavefunction parameters are $\alpha = 0.7096$, $\beta = -0.6894$ and $\gamma = 0.1457$. Recently this result has been confirmed by M.A. Braun\cite{Braun98} who redid his earlier work on the variational functional using our wave function, and obtained after a formidable numerical calculation $\alpha_O = 1-0.16606-3\alpha, N_c/2\pi$, in excellent agreement with our result following from the Baxter equation.

A second check of our results follows from the new symmetry of the odderon discovered by Lipatov\cite{Lipatov92} which forces the wavefunction parameters to be related by $\gamma = |q_3|\alpha$ which indeed is verified in our case. A more precise check can be made using the very recent asymptotic formula\cite{Lipatov98} for the energy expressed in terms of $q_3$ derived by Lipatov for large $q_3$. We cannot use it for the ground state but for the higher states identified here one obtains $\epsilon_3 = -5.16956$ and $\epsilon_3 = -7.70234$ (for states with $q_3 = \pm 2.34392i$ and $q_3 = \pm 8.32635i$ respectively) which is in perfect agreement with our earlier results obtained using the Baxter equation.

The states with higher $q_3$ give subleading contributions as expected. Also a change in $h$ from $h = 1/2$ to $h = 1/2 + iv$ also lowers the energy. This observation is consistent with the expectation that the dominant contribution should come from the $h \leftrightarrow 1 - h$ symmetric point $h = 1/2$ investigated in this work.

Some further confirmations and results are presented in\cite{Braun98} and in\cite{Braun98}.

As a final point we note that our results suggest that the predictions\cite{Faddeev94} for odderon mediated processes will not be enhanced at high energies. It remains to see if the specific form of the wave function might lead to some interesting effects through the coupling of the ground state of the odderon found here, to external probes.

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