A General Approach for the Neutrino-Antineutrino Oscillation in an NCG Curved Space-time

H Aissaoui¹ N Mebarki¹ and O Mebarki¹,²
¹Laboratoire de Physique Mathématique et Subatomique, Physics Department, Mentouri University, Constantine, Algeria. ²Physics Department, Skikda University, Skikda Algeria
E-mail: aissaoui_h@yahoo.com

Abstract. A general prescription to study the neutrino-antineutrino oscillation is presented in an expanded as well as static non-commutative space-time background. It is shown that besides the curvature effect of the space-time non-commutativity may play a crucial role in explaining this oscillation.

1. Introduction
During the last few years many efforts have been devoted to the study the non-commutative space-time and its possible physical applications namely in quantum and elementary particle physics and cosmology [1]-[17]. The most important one are those related to the non-commutative curved space-time (NCG). The motivation was that the space-time non-commutativity could be significant at the Planck scale (the same scale where quantum gravity effects become important) and very sensible to search for signatures in the cosmological observations [18]. Moreover, the observed anisotropies of the cosmic microwave background (CMB) may be caused by the non-commutativity of space-time geometry (NCG) [19],[20]. Thus, the mechanism of gravity and space-time non-commutativity will play a major role in explaining some of the outstanding phenomena like the neutrino-antineutrino oscillation. This neutrino asymmetry can be considered as a consequence of the leptogenesis namely in the early universe affecting the present energy density, cosmic microwave background etc. [1]. The goal of this paper is to give a general prescription to study the neutrino-antineutrino oscillation in an arbitrary NCG (expanded or static). In section 2, we present the general formalism and theoretical approach. In section 3, we derive the analytical expression of the difference between the neutrino and antineutrino number density. For an illustration, we will consider an example of the NCG FRW universe. Finally in section 4, we draw our conclusions.

2. General prescription and theoretical approach
Following the approach of ref.[21], in a non-commutative gauge theory of gravitation the structure of the space-time is determined by the relation:

\[ [X^{\mu}, X^{\nu}] = i\theta^{\mu\nu} \]  \hspace{1cm} (1)

To whom any correspondence should be addressed.
where $\Theta^{\mu v} = -\Theta^{v \mu}$ ($\mu, \nu = 1, 4$) are constant (canonical) parameters. The gauge fields for the non-commutative case are denoted by $\omega_{\mu}^{AB}(x, \Theta)$ and can be expanded in power series of the non-commutative parameter $\Theta$ as:

$$\omega_{\mu}^{AB} = \omega_{\mu}^{AB} - i \Theta^{\nu \rho} \omega_{\mu \rho}^{AB} + \Theta^{\nu \rho} \Theta^{\tau \xi} \omega_{\mu \rho \xi}^{AB} + \ldots$$

(2)

where

$$\omega_{\mu \rho}^{AB} = \frac{1}{4} \left\{(\omega_{\mu}, \partial_{\rho} \omega_{\mu} + R_{\rho \mu})\right\}^{AB}$$

(3)

and

$$\omega_{\mu \rho \xi}^{AB} = \frac{1}{32} \left\{\left(\omega_{\mu}, \partial_{\rho} \omega_{\mu} + R_{\rho \mu}\right)\right\}^{AB}$$

(4)

with the notations

$$\{\alpha, \beta\}^{AB} = \alpha^{AC} \beta_{C}^{B} + \beta^{AC} \alpha_{C}^{B}$$

(5)

and the covariant derivative $D_{\mu} R_{\rho \sigma}^{AB}$ is such that:

$$D_{\mu} R_{\rho \sigma}^{AB} = \partial_{\mu} R_{\rho \sigma}^{AB} + (\omega_{\rho}^{AC} R_{\sigma \mu}^{DB} + \omega_{\rho}^{BC} R_{\sigma \mu}^{DA} ) \eta_{CD}$$

(6)

If we have a vanishing commutative torsion, one can show that the components of the tetrad fields $\hat{e}_{\mu}^{a}$ take the form:

$$\hat{e}_{\mu}^{a} = e_{\mu}^{a} - i \Theta^{\nu \rho} e_{\mu \rho}^{a} + \Theta^{\nu \rho} \Theta^{\tau \xi} e_{\mu \rho \xi}^{a} + \ldots$$

(7)

where

$$e_{\mu \rho}^{a} = \frac{1}{4} \left\{(\omega_{\mu} e_{\rho}^{ab} + (\partial_{\rho} \omega_{\mu}^{ac} + R_{\rho \mu}^{ac}) e_{\nu}^{cd} ) n_{\nu \mu} \right\}^{a b}$$

(8)

and

$$e_{\mu \rho \xi}^{a} = \frac{1}{32} \left\{ -\omega_{\mu}^{ab} \left(D_{\rho} R_{\tau \mu}^{cd} + \partial_{\rho} R_{\tau \mu}^{cd}\right) e_{\nu \rho}^{ed} \eta_{dm} - \left(\omega_{\nu \rho}^{ac} \partial_{\mu} e_{\rho \xi}^{a} + (\partial_{\rho} \omega_{\mu}^{ac} + R_{\rho \mu}^{ac}) e_{\rho \xi}^{a} \right) \right\}^{\eta_{bc}}$$

(9)

A real NCG metric $\hat{g}_{\mu \nu}$ but not symmetric is defined as:

$$\hat{g}_{\mu \nu} = \frac{1}{2} \eta_{ab} \left(\hat{e}_{\mu}^{a} \hat{e}_{\nu}^{b} + \hat{e}_{\nu}^{b} \hat{e}_{\mu}^{a} \right)$$

(10)

Here the superscript ‘+’ denotes the complex conjugate and “*” the star product.
### 3. Neutrino-Antineutrino oscillation in NCG

In a curved non-commutative Seiberg-Witten space-time, the generalized Dirac Lagrangian density \( L \) is shown to have the following expression

\[
L = \sqrt{-g} \bar{\psi} \gamma^\mu \partial_\mu \psi + \gamma^\mu \gamma^5 \hat{B}_a \psi - m \bar{\psi}
\]

(\( \gamma^\mu \)'s are the Dirac matrices) and \( \bar{\psi} \) an NCG four components Dirac spinor. The explicit form of the four vector \( \hat{B}_a \) is given by:

\[
\hat{B}_a = \epsilon_{aef} \gamma^a \gamma^f
\]

The symbol \( \epsilon_{aef} \) is the totally antisymmetric tensor. Then, one can show (as in ref.[22]), that the dispersion relations for left and right chiral fields are:

\[
(p_a \pm \hat{B}_a)^2 = m^2, \quad a = 0,3
\]

The upper (resp. lower) sign corresponds to a particle (resp. anti-particle) with four momentum \( p_a = (E, \vec{P}) \) and mass \( m \). Thus the energies \( E_A \) and \( E_\bar{A} \) for the particle and antiparticle are:

\[
E_A = \sqrt{\vec{P}^2 + m^2 + \hat{B}_0}, \quad E_\bar{A} = \sqrt{\vec{P}^2 + m^2 + \hat{B}_0}
\]

Notice that the particle and anti-particle propagating in the presence of a non-commutative curved space-time have different energies. This is quite expected since as it is argued from many authors like the one of ref.[22] that when we have an universe which is spherically symmetric, the energy gap between the particle and antiparticle disappears. In the NCG case, the non-commutativity has generated an inhomogeneity and the spherical symmetry will be lost. Now, one can show that the difference between the particles and anti-particles NCG number density \( \Delta n_{NCG} \) in the early universe is:

\[
\Delta n_{NCG}(\hat{r}, \hat{\theta}, \theta, \beta) \approx \frac{8g}{2\pi^2} T^3 \sum_{k=0}^\infty \frac{(-1)^k}{k^3} \sin(k\hat{B}_0 / T)
\]

For an illustration and as an example and to simplify the calculations, we choose the only non-vanishing constant components of the non-commutativity parameter \( \Theta^{23} = \Theta \). We consider a flat space spherically symmetric, isotropic and homogeneous FRW universe. We set \( \hat{r} = t/t_0, \hat{r} = \hat{r}/r_0 \) (\( r_0 \) and \( t_0 \) are the present radius and time respectively) and take a power law formula of the form \( R(t) = \hat{r}^\beta \) for the scale factor \( R(t) \). Using a Maple package, the induced \( \hat{B}_0 \) field is shown to have the following expression:

\[
\hat{B}_0 = \Theta \left[\begin{array}{cc}
-2a^{\beta} \hat{r}^{\beta - 6} + \frac{1}{4} \hat{r}^{\beta - 2} a^{\beta} + \frac{\cos^2 \theta}{\hat{r} \sin \theta} (1 + \hat{r}) + \frac{1}{4} a^{\beta} \hat{r}^{\beta - 2} \sin \theta \\
- \frac{\hat{r}^{-\beta}}{2 \hat{r} \sin \theta} + \frac{\hat{r}^{-\beta}}{4 \hat{r}} \cot g^2 \theta
\end{array}\right]
\]
Notice that $\hat{B}_0$ is proportional to $\Theta$ as it was expected (a pure NCG quantity).

4. Conclusions
Throughout this paper, we have find out a new mechanism to explain the oscillation is proposed (more details are under study). We have presented a general prescription to generate the neutrino-antineutrino oscillation in a curved non-commutative space-time quantified in the number density difference. The prescription goes as follows:

i) we choose the commutative curved space-time;

ii) we use the Maple package code (which we have developed) to determine the Non-commutative Vierbeins, spin connections, etc.;

iii) we calculate the quantity $\hat{B}_0$;

iv) we determine the difference number density.

As an example, we have considered a non-static universe (FRW).

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