Research on Liquid Characteristic Sensing Technology of Quartz Tuning Fork Based on COMSOL Multiphysics

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Abstract. In this paper, the relationship between the liquid characteristic parameters and the resonance characteristics of the quartz tuning fork is studied through impedance analysis and computer simulation. Firstly, the three-dimensional model of the quartz tuning fork driven by AC voltage is established using finite element analysis software, and the vibration mode and frequency response of the quartz tuning fork were analyzed. Then, the impedance spectrum of the quartz tuning fork in the air is measured by experiment, and its equivalent circuit parameters are estimated. This result is consistent with the equivalent circuit parameters estimated by simulation, which proves the effectiveness of the simulation. Finally, the values of the geometric parameters $A$ and $B$ are calculated by the frequency response of the quartz tuning fork in the liquid and used for the calculation of the density and viscosity of the liquid with unknown characteristics. The research provides a theoretical basis and experimental support for the design of quartz tuning fork liquid characteristic sensors.

1. Introduction

Real-time monitoring of liquid characteristic parameters plays an essential role in the production industries such as oil and electricity, and transportation industries such as automobiles and aviation [1]. There is indisputable evidence: 80% of equipment failures are caused by abnormal wear due to lubrication failures, and the analysis of the state of lubricating oil can provide early warning of the machine failure [2]. Therefore, real-time monitoring of the lubricating oil status and realizing oil replacement according to quality can reduce breakdowns and costs, which is beneficial to ecological protection.

In the past several decades, the quartz tuning fork (QTF) based on flexural mode has received a great deal of interest due to its excellent performance in the detection of density and viscosity [3, 4]. Some literature has already investigated the real-time monitoring of the liquid characteristics of the quartz tuning fork. It has been reported in [5] that the physical and chemical properties of liquids were measured by the low-frequency commercial quartz tuning forks, but the density and viscosity cannot be measured separately. A novel algorithm for calculating liquid density and viscosity through resonance frequency and quality factor has been proposed in [6]. In [7], a sensor system that uses a tuning fork to measure the viscosity and density of liquids simultaneously has been developed for measuring downhole fluids. However, the existing research is on the use of quartz tuning forks, and few studies have analyzed the liquid characteristic sensor of quartz tuning fork itself. Finite element analysis is widely used in the analysis and design of the sensor, and it can achieve the visualization of...
complex results [8]. Some studies use finite element analysis to analyze the characteristics of the sensor. The quantitative analysis of the relationship between the input and output in a piezoelectric angular velocity sensor are analyzed by finite element analysis [9]. The pressure distribution inside the pressure sensor of the DTET quartz resonator is explained in [10] by the finite element simulation.

In this paper, the relationship between the liquid characteristic parameters and the resonance characteristics of the quartz tuning fork is studied through impedance analysis and simulation. The vibration mode and frequency response of the quartz tuning fork are analyzed by the COMSOL Multiphysics software. The resonance characteristics of quartz tuning forks in the air are studied through simulation and experiment.

The resonance characteristics of the quartz tuning fork in liquid are analyzed through simulation.

2. Simulation modeling

2.1. Finite element model of the quartz tuning fork
The vibration mode and frequency response of quartz tuning fork and the frequency response in liquids with various characteristics are simulated by the finite element analysis software COMSOL Multiphysics. A solid model of quartz tuning forks is established according to the geometric dimensions of commercial quartz tuning forks, as shown in Fig. 1(a). The dimensions of the quartz tuning fork are listed in Table 1, where \( L \) is the length of tuning fork, \( L_1 \) is the length of fork arms, \( w_1 \) is the width of fork arms, \( w_2 \) is the gap between arms and \( d \) is the thickness of fork. The quartz tuning fork resonator is made of x-cut quartz crystal rotating through 5° about the x-axis. The quartz tuning fork selects the piezoelectric material quartz built in the COMSOL Multiphysics with a density of 2651 kg/m³. The bottom of quartz tuning fork loading fixed constraint, and the electrodes configuration is shown in Fig. 1(b). The electric potential diagram quartz tuning fork is shown as Fig. 1(b), '+' and '-' represent the positive and negative electrodes, respectively.

![Figure 1. Quartz tuning fork (a) COMSOL model and (b) electrodes configuration.](image)

| Table 1. The dimensions of the quartz tuning fork. |
|---|---|---|---|---|
| \( L(\text{mm}) \) | \( L_1(\text{mm}) \) | \( w_1(\text{mm}) \) | \( w_2(\text{mm}) \) | \( d(\text{mm}) \) |
| 6 | 3.75 | 0.6 | 0.3 | 0.365 |

2.2. Analysis of vibration mode and frequency response of quartz tuning fork
The eigenfrequency and frequency domain research of quartz tuning fork are studied by the piezoelectric module of COMSOL Multiphysics. The eigenfrequency and vibration mode of the quartz tuning fork is analyzed by the analysis of eigenfrequency, as shown in Fig. 2. Fig. 2(d) shows that the vibration mode of the quartz tuning fork is out-phase when the eigenfrequency is 32.339 kHz. The
The relative error between the value of the eigenfrequency and the resonant frequency of the quartz tuning fork resonator is 1.31%, which proves the effectiveness of the model of the quartz tuning fork.

![Figure 2](image1.png)

Figure 2. The six different vibration modes of the quartz tuning fork.

According to the range of eigenfrequency, the range of sweeping frequency is from 0 kHz to 100 kHz. The susceptance-frequency curve of the quartz tuning fork is obtained by the frequency response analysis, as shown in Fig. 3. The vibration mode and eigenfrequency corresponding to the peak value are shown as illustrations in Fig. 3. It can be seen from the figure that the vibration mode with a frequency of 32.339 kHz without interference from adjacent frequency points is a single out-phase vibration mode.

![Figure 3](image2.png)

Figure 3. The susceptance-frequency curve of the quartz tuning fork.
3. Parameter extraction

3.1. Equivalent circuit parameter extraction by experiment

To analyze the resonance characteristics of the quartz tuning fork, a commercial quartz tuning fork with a resonant frequency of 32.768 kHz is used in the experiment. The laboratory temperature is controlled at 25 ± 0.1°C to avoid the influence of temperature on the resonant frequency of the quartz tuning fork through the air conditioner. The impedance spectrum of the quartz tuning fork is measured by the TH2829C impedance analyzer, and the parameter setting of the impedance analyzer is shown in Table 2.

| NO | Property          | Parameter   |
|----|-------------------|-------------|
| 1  | Sweeping point    | 801         |
| 2  | Driving AC voltage| 10 mV       |
| 3  | Sweeping frequency range | 30-35 kHz , 20-100 kHz |

The impedance spectrum of the quartz tuning fork is shown in Fig. 4. As shown in Fig. 4(a), series resonant frequency $f_s$ and parallel resonant frequency $f_p$ are 32.7625 kHz and 32.8 kHz, respectively. The reactance-frequency curve is shown in Fig. 4(b).

![Figure 4. The impedance spectrum of the quartz tuning fork.](image)

The quartz tuning fork is analyzed by the equivalent circuit [6], as shown in Fig. 5.

![Figure 5. Equivalent circuit of the quartz tuning fork.](image)

The motional branch impedance $Z_0(\omega)$ of quartz tuning fork can be expressed as

$$Z_0(\omega) = R_0 + j(\omega L_0 - \frac{1}{\omega C_0})$$

(1)
The total impedance $Z(\omega)$ is

$$Z(\omega) = Z_0(\omega) / \frac{1}{j\omega C_p}$$  \hspace{1cm} (2)$$

where $\omega$ is the excitation angular frequency.

The value of circuit parameters including $C_p$, $R_0$, $C_0$ and $L_0$ in the equivalent circuit of the quartz tuning fork is obtained by processing the impedance characteristics measured by impedance analyzer. As shown in Fig. 4(b), the reactance can be expressed as $X = \frac{1}{j\omega C_p}$ far away from resonance, and susceptance is the reciprocal of reactance. Therefore, the capacitance $C_p$ can be estimated by linear fitting the curve of reactance or susceptance far away from resonance. The equivalent resistance corresponding to the series resonant frequency $f_s$ in the impedance spectrum is the value of $R_0$. The series resonant angular frequency and parallel resonant angular frequency of the quartz tuning fork can be expressed as

$$\omega_s = \frac{1}{\sqrt{L_0 C_0}}$$  \hspace{1cm} (3)$$

$$\omega_p = \frac{C_p + C_0}{L_0 C_0 C_p} = \omega_s \sqrt{1 + \frac{C_0}{C_p}}$$  \hspace{1cm} (4)$$

Then the value of $C_0$ and $L_0$ can be estimated by the series resonant and parallel resonant angular frequency with the equation (3) and equation (4). The equivalent circuit parameters of the quartz tuning fork in the air are extracted by calculation, as shown in Table 3.

### Table 3. The circuit parameters of the quartz tuning fork in air extracted by experiment.

| Parameter | $f_s$ | $f_p$ | $R_0$ | $L_0$ | $C_0$ | $C_p$ |
|-----------|------|------|------|------|------|------|
| Value     | 32.7625 kHz | 32.8000 kHz | 115.178 kΩ | 6050 H | 0.0039 pF | 1.7 pF |

3.2. **Equivalent circuit parameter extraction by simulation**

The frequency response of the quartz tuning fork in the air is simulated because air is also a special liquid [11]. The frequency response of the quartz tuning fork in the air at 25 degrees is recorded. The ranges of sweep frequency in frequency domain simulation are from 30 to 35 kHz and from 40 to 100 kHz, respectively. The impedance and susceptance are shown in Fig. 6 as a function of frequency. As shown in Fig. 6(a), the series resonant frequency of quartz tuning fork working in the air is 32.650 kHz. The relative error between the experiment and simulation is 0.34%, which proves the effectiveness of the simulation method. The susceptance curve far away from the resonance is shown in Fig. 6(b), the lumped parameters of the quartz tuning fork liquid characteristic sensor can be extracted based on the simulation results.
Figure 6. The simulation of the quartz tuning fork in air (a) impedance and (b) susceptance as a function of frequency.

Table 4. The comparison between the circuit parameter of the quartz tuning fork in air extracted by experiment and simulation.

| Parameter | \(f_s\) (kHz) | \(f_p\) (kHz) | \(R_0\) (k\(\Omega\)) | \(L_0\) (H) | \(C_0\) (pF) | \(C_F\) (pF) | Relative Error |
|-----------|--------------|--------------|-----------------|---------|-----------|--------|---------------|
| Experiment | 32.7625      | 32.8000      | 115.178         | 6050    | 0.0039    | 1.70   | 0.34%         |
| Simulation | 32.6500      | 32.6860      | 114.279         | 6253    | 0.0038    | 1.73   | 0.34%         |

The equivalent circuit parameter extracted by the Fig. 6 are compared with the experiment results, as shown in Table 4. The Relative Error in Table 4 can be expressed as \(\frac{|x_0 - x_0|}{x_0} \times 100\%\), where \(x_0\) and \(x_0\) are the value extracted by simulation and experiment respectively. As shown in Table 4, the results extracted by simulation have a great agreement with the experiment. It shows that the working properties of the quartz tuning fork in liquid can be simulated by the COMSOL Multiphysics software.

4. Frequency response of qtf in liquid

The circuit parameter of the quartz tuning fork will change when the quartz tuning fork oscillates in the liquid, as shown in Fig. 7. The impedance \(Z_i(\omega)\) introduced by surrounding fluids is defined as [12]

\[
Z_i(\omega) = i\omega A \rho + B \sqrt{\omega \eta (1+i)}
\]  

where \(\rho\) is the density of the liquid, \(\eta\) is the viscosity of the liquid, \(A\) and \(B\) are the geometric factors that depend only on the resonator geometry and mode of oscillation [12].

Figure 7. Equivalent circuit of the quartz tuning fork in liquid.
Therefore, to realize the measurement of liquid characteristic parameters, the impedance characteristic changes of quartz tuning fork oscillating in liquids with different characteristics should be analyzed.

The values of parameters $A$ and $B$ are estimated by the impedance characteristic of a quartz tuning fork oscillating in a liquid with known characteristics. When the quartz tuning fork immersed in a liquid, the real and imaginary parts of the motional branch impedance can be expressed as

$$
\text{Re}[Z] = R_0 + B\sqrt{\rho \eta \omega}
$$

$$
\text{Im}[Z] = \omega L_0 - \frac{1}{\omega C_0} + \omega A \rho + B\sqrt{\rho \eta \omega}
$$

The equation of calculation of the parameters $A$ and $B$ can be written as follows:

$$
A = \frac{1/(\omega C_0) - \omega L_0 - \text{Re}[Z_i] + R_0}{\omega \rho}
$$

$$
B = \frac{\text{Re}[Z_i] - R_0}{\sqrt{\rho \eta \omega}}
$$

where $\omega_i$ is the series resonant angular frequency of a quartz tuning fork in a liquid with known characteristics, $\text{Re}[Z_i]$ is the equivalent resistance when resonating in a liquid with known characteristics.

The frequency response of the quartz tuning fork working in the water with a density of 997.03 kg/m$^3$ and viscosity of 0.8926 Cp through simulation is shown in Fig. 8. Meanwhile, the coordinate of the series resonance point is shown in Fig. 8. The values of parameters $A$ and $B$ are calculated by the resonant frequency $f_i$ and equivalent resistance $R$, as shown in Table 5.

![Figure 8. The frequency response of the quartz tuning fork working in the water.](image)

**Table 5.** The values of parameters $A$ and $B$.

| Fluid | Density $\rho$ (kg/m$^3$) | Viscosity $\eta$ (Cp) | Resonant frequency $f_i$ (kHz) | Equivalent resistance $R$ (k$\Omega$) | $A$ $(\Omega \cdot cm^3 \cdot s \cdot g^{-1})$ | $B$ $(\Omega \cdot cm^{3/2} \cdot s^{1/2} \cdot (g \cdot Cp)^{1/2})$ |
|-------|--------------------------|----------------------|-------------------------------|----------------------------------|---------------------------------|---------------------------------|
| Water | 997.03                   | 0.8926               | 29.138                        | 2144904                          | 1591.85                         | 5030.81                         |
The density and viscosity of other fluids with unknown properties can be detected after the parameters $A$ and $B$ were obtained. The density $\rho$ and viscosity $\eta$ of a liquid can be estimated as shown in the equations below.

\[
\rho = \frac{1/(\omega_s C_0) - \omega_s L_0 - \text{Re}[Z] + R_0}{A\omega_s} \quad (10)
\]

\[
\eta = \frac{A((\text{Re}[Z] - R_0)/B)^2}{1/(\omega_s C_0) - \omega_s L_0 - \text{Re}[Z] + R_0} \quad (11)
\]

where $\omega_s$ is the series resonant angular frequency of a quartz tuning fork in a liquid with unknown characteristics, $\text{Re}[Z]$ is the equivalent resistance when resonating in a liquid with unknown characteristics.

The frequency response curves of the quartz tuning fork in water with different densities and viscosities and its corresponding resonant frequency and equivalent resistance are shown in Fig. 9. The density and viscosity of the fluid with unknown properties can be calculated by the corresponding frequency and equivalent resistance with the parameters of $A$ and $B$, and have a comparison with the standard value, as shown in Table 6. Table 6 depicts that the density results have an agreement with the standard, and the viscosity results have a small error. The error may be due to the thickness of the electrode and the slight difference between the simulation and the actual process.

![Figure 9. The frequency response of quartz tuning fork in water with different properties.](image)

Table 6. Density and viscosity of water with different properties are calculated by FEM.

| Fluids  | Density ($kg / m^3$) | Viscosity ($Cp$) | Density ($kg / m^3$) | Viscosity ($Cp$) |
|---------|----------------------|------------------|----------------------|------------------|
| Water1  | 997.54               | 0.9365           | 997.37               | 0.8922           |
| Water2  | 996.76               | 0.8718           | 996.70               | 0.8935           |
| Water3  | 995.92               | 0.8142           | 995.68               | 0.8934           |

5. Conclusion

In this research, the relationship between the resonance characteristics of the quartz tuning fork and the liquid characteristic parameters (including density and viscosity) is obtained by impedance analysis. A three-dimensional model of the quartz tuning fork driven by AC Voltage is established by COMSOL Multiphysics, combining a commercial quartz tuning fork resonator with a resonant frequency of 32.768 kHz. The eigenfrequency of the quartz tuning fork is 32.339 kHz analyzed by the vibration mode and frequency response, which proves the effectiveness of the model of the quartz tuning fork. By comparing the equivalent circuit parameters extracted from the resonance
characteristics of the quartz tuning fork in the experimental and simulation results, it is known that the relative error does not exceed 3.3%. It shows that the liquid characteristic sensing of the quartz tuning fork can be analyzed by the finite element analysis software. The values of the geometric parameters $A$ and $B$ are calculated by the frequency response of the quartz tuning fork in the liquid and used for the calculation of the density and viscosity of the liquid with unknown characteristics. The results show that the density results have good consistency, while viscosity results have a small error. The error may be due to the thickness of the electrode and the slight difference between the simulation and the actual process. The influence of the material and thickness of the electrode and temperature on the measurement of liquid density and viscosity can be considered in the next research.

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