Measurements of the CKM angle $\gamma$ in tree-dominated $B$ decays at LHCb

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Abstract. The LHCb experiment is a general purpose forward spectrometer operating at the Large Hadron Collider, optimized for the study of $B$ and $D$ hadrons. LHCb collected $\sim 1$ fb$^{-1}$ of integrated luminosity during 2011 data taking, which provides an unprecedentedly large sample of $B$ hadron decays to final states involving charmed hadrons. These decays offer many complementary ways to measure the angle $\gamma$ of the CKM triangle. We present a selection of new world-leading measurements of $CP$-violating observables in these decays, using both time-independent and, for the first time, time-dependent analyses. A first LHCb combination of the time-independent measurements to provide a value of $\gamma$ is also presented.

1. Introduction
The CKM parameter $\gamma$ is currently the least well-constrained angle of the CKM triangle. Combinations by the CKMFitter [1] and UTFit [2] collaborations, using all existing measurements, give $\gamma = (66 \pm 12)^\circ$ and $\gamma = (76 \pm 10)^\circ$ respectively. Measurements of $\gamma$ can be made using $B$ decays that are mediated only by tree-level transitions, for example $B$ decays to open charm mesons. Since these transitions are unlikely to be affected by New Physics, they provide a “standard candle” to measure the Standard Model (SM) value of $\gamma$. The value of $\gamma$ measured in this way can then be compared to values measured using $B$ decays mediated by loop-level transitions, for example $B$ decays to light charmless hadrons. A significant difference between these values would indicate a New Physics contribution to the loop processes.

This document presents the latest results from the LHCb experiment [3] measuring $CP$-violating observables in hadronic $B$ decays to open charm. Results from time-independent analyses of $B^+ \to \bar{D}^0 K^+$ and $B^+ \to \bar{D}^0 \pi^+$ decays, with various final states of the $\bar{D}^0$ meson, are reported. These results are combined to give a first LHCb value for $\gamma$. A further measurement, not included in the $\gamma$ combination, is also reported: the first measurement of the $CP$-violating observables in $B^0_s \to D^{\mp} s K^\pm$, using a time-dependent analysis. All results are based on data corresponding to $\sim 1$ fb$^{-1}$ of integrated luminosity, collected in 2011 at a centre-of-mass energy of 7 TeV.

2. Time-independent analyses of $B^+ \to \bar{D}^0 K^+$ and $B^+ \to \bar{D}^0 \pi^+$
In general, time-independent methods of measuring $\gamma$ from hadronic $B$ decays to open charm take advantage of the interference between $b \to c$ and $b \to u$ transitions at tree-level, where the $D$ final state is accessible to both $D^0$ and $\bar{D}^0$. Aside from the weak phase $\gamma$, these analyses also
measure the hadronic parameters $r_{B(D)}$ and $\delta_{B(D)}$, where the ratio of the favoured to suppressed $B(D)$ decay amplitudes is $r_{B(D)}e^{i\delta_{B(D)}}$. The method to extract these hadronic parameters and $\gamma$ depends on the $D$ final state. The three methods discussed in this document are usually referred to by the initials of the authors of the paper(s) that first proposed the method: the “GLW” method [4], the “ADS” method [5] and the “GGSZ” method [6].

2.1. GLW Analysis

The GLW method uses $D$ decays to $CP$-eigenstates. The most experimentally accessible such decays are $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$. Since these states have a positive $CP$-eigenvalue, the decays are denoted by $D_{CP^+}$. The two main observables in the GLW method are the average partial rate $R_{CP^+}$, and the asymmetry $A_{CP^+}$, defined as

$$R_{CP^+} = \frac{2\Gamma(B^- \to D_{CP^+}K^-) + \Gamma(B^+ \to D_{CP^+}K^+)}{\Gamma(B^- \to D^0 K^-) + \Gamma(B^+ \to D^0 K^+)}$$

$$A_{CP^+} = \frac{\Gamma(B^- \to D_{CP^+}K^-) - \Gamma(B^+ \to D_{CP^+}K^+)}{\Gamma(B^- \to D_{CP^+}K^-) + \Gamma(B^+ \to D_{CP^+}K^+)}$$

where the $D^0$ or $\overline{D}^0$ refers to a quasi-flavour-specific $D$ final state, which in practice is usually $D^0 \to K^-\pi^+$ or $\overline{D}^0 \to K^+\pi^-$. These observables are related to the physics parameters via

$$R_{CP^+} = 1 + r_B^2 + 2C_f r_B \cos \delta_B \cos \gamma, \quad A_{CP^+} = \frac{2C_f r_B \sin \delta_B \sin \gamma}{R_{CP^+}},$$

where $C_f$ is a coherence factor that is equal to 1 for two-body $D$ final states (such as $K^+K^-$ or $\pi^+\pi^-$), and is between 0 and 1 for multi-body $D$ final states (such as $K^+K^-\pi^+\pi^-$). For a multi-body final state, $C_f = 1$ corresponds to the case where the final state is always reached via intermediate resonances (for example $\phi\phi^0$), such that it can be considered as a two-body decay. The value $C_f = 0$ corresponds to the case where the final state has no contributions from intermediate resonances. For multi-body $D$ decays, the $C_f$ value typically lies somewhere between these two extremes. In this case the various paths to the final state can interfere, which is why the decay is not “coherent”. Equivalent expressions for $R_{CP^+}$ and $A_{CP^+}$ exist for the $B^+ \to D_{CP^+}\pi^+$ decay, but the asymmetry there is expected to be negligible, due to the very small value of $r_B$. LHCb has measured the GLW observables using the $K^+K^-$ and $\pi^+\pi^-$ final states of the $D$ [7]. The mass spectra of selected events in the GLW $D$ final states (split by bachelor particle type and charge) are shown in Fig. 1. Stringent particle identification (PID) requirements are used to suppress background from the Cabibbo-favoured $D\pi$ final state to the $DK$ final state. In Figs. 1-3, the green line represents the $D\pi$ events (including the $D\pi$ background to $DK$ remaining after the PID requirements), the red line represents the $DK$ events, and the shaded contribution represents the partially reconstructed background. In Figs. 2 and 3, the pink dashed line represents the background from $B_s^0 \to DK^+\pi^-$ events where the pion is not reconstructed. A clear asymmetry between $B^+$ and $B^-$ can be seen in the $DK$ final state, for both $D$ final states. After correcting for the (small) detector and production asymmetries, and averaging over the $D$ final states, the average partial rate and asymmetry for $DK$ are measured to be

$$R_{CP^+} = 1.007 \pm 0.038 \pm 0.012, \quad A_{CP^+} = 0.145 \pm 0.032 \pm 0.010,$$

where the first error is statistical, and the second systematic. The $A_{CP^+}$ value is nonzero with a significance of 4.5 standard deviations.
2.2. ADS Analysis

The ADS method uses $D$ decays to quasi-flavour-specific final states, where the decay to one charge combination is Cabibbo-favoured, and the decay to the opposite charge combination is doubly-Cabibbo-suppressed. The most experimentally accessible such pair are the two-body decays $D^0 \to K^-\pi^+$ (Cabibbo-favoured) and $D^0 \to K^+\pi^-$ (doubly-Cabibbo-suppressed). The two main observables in the ADS method are defined similarly to those in the GLW method:

$$R_{ADS} = \frac{\Gamma(B^- \to (K^+\pi^-)_D K^-) + \Gamma(B^+ \to (K^-\pi^+)_D K^+)}{\Gamma(B^- \to (K^-\pi^+)_D K^-) + \Gamma(B^+ \to (K^+\pi^-)_D K^+)},$$

$$A_{ADS} = \frac{\Gamma(B^- \to (K^+\pi^-)_D K^-) - \Gamma(B^+ \to (K^-\pi^+)_D K^+)}{\Gamma(B^- \to (K^-\pi^+)_D K^-) + \Gamma(B^+ \to (K^+\pi^-)_D K^+)},$$

are are related to the physics parameters via

$$R_{ADS} = r^2_B + r^2_D + 2C_f r_BR_D \cos(\delta_B + \delta_D) \cos\gamma,$$

$$A_{ADS} = \frac{2C_f r_BR_D \sin(\delta_B + \delta_D) \sin\gamma}{R_{ADS}},$$

where $C_f$ is again a coherence factor analogous to the GLW case. Equivalent expressions exist for the decay with the bachelor pion. LHCb has measured the ADS observables using the $K^-\pi^+$ and $K^+\pi^-$ final states of the $D$ [7]. The mass spectra of selected events in the ADS $D$ final states (split by bachelor particle type and charge) are shown in Fig. 2. A clear asymmetry between $B^+$ and $B^-$ can be seen for the doubly-Cabibbo-suppressed $D$ final state, in both the

![Figure 1](image1.png)

**Figure 1.** Left: Invariant mass distributions for $B^\pm \to [K^\pm K^-]_D h^\pm$ candidates, where $h$ represents a charged kaon or pion. The top (bottom) plots show the $h = K(h = \pi)$ final state, split by charge. Right: the equivalent plots for $B^\pm \to [\pi^\mp\pi^-]_D h^\pm$ candidates.

![Figure 2](image2.png)

**Figure 2.** Left: Invariant mass distributions for $B^\pm \to [K^\pm\pi^-]_D h^\pm$ candidates, where $h$ represents a charged kaon or pion. The top (bottom) plots show the $h = K(h = \pi)$ final state, split by charge. Right: the equivalent plots for $B^\pm \to [K^\mp\pi^\pm]_D h^\pm$ candidates.
DK and Dπ final states. After correcting for the (small) detector and production asymmetries, the average partial rates and asymmetries for DK and Dπ are measured to be

\[
R_{ADS(\pi)} = 0.00410 \pm 0.00025 \pm 0.00005, \quad A_{ADS(\pi)} = 0.143 \pm 0.062 \pm 0.011,
\]

\[
R_{ADS(K)} = 0.0152 \pm 0.0020 \pm 0.0004, \quad A_{ADS(K)} = -0.52 \pm 0.15 \pm 0.02,
\]

where the first error is statistical, and the second systematic. The DK decay with the doubly-Cabibbo-suppressed D final state is observed for the first time, with a significance of 10 standard deviations. The \(A_{ADS(\pi)}(A_{ADS(K)})\) value is nonzero with a significance of 2.4(4.0) standard deviations. Considering the \(A_{ADS(K)}\) measurement together with the \(A_{CP^+}\) measurement from the GLW analysis, \(CP\)-violation is observed in \(B^\pm \to DK^\pm\) decays for the first time, with a significance of 5.8 standard deviations.

The ADS method can also be applied to the multi-body decays \(D^0 \to K^-\pi^+\pi^-\pi^+\) (Cabibbo-favoured) and \(D^0 \to K^+\pi^-\pi^-\pi^+\) (doubly-Cabibbo-suppressed). Since the D decay parameters are different to the two-body case, complementary information is gained by performing this analysis in addition to the two-body analysis. LHCb has measured the ADS observables using the \(K^-\pi^+\pi^-\pi^+\) and \(K^+\pi^-\pi^-\pi^+\) final states of the D [8]. The mass spectra of selected events in the doubly-Cabibbo-suppressed D final state (split by bachelor particle type and charge) are shown in Fig. 3. There is some indication of an asymmetry between \(B^+\) and \(B^-\), in both the DK and Dπ final states. After correcting for the (small) detector and production asymmetries, the average partial rates and asymmetries for DK and Dπ are measured to be

\[
R_{ADS(\pi)}^{K^3\pi} = 0.00639 \pm 0.00036, \quad A_{ADS(\pi)}^{K^3\pi} = 0.13 \pm 0.10,
\]

\[
R_{ADS(K)}^{K3\pi} = 0.0124 \pm 0.0027, \quad A_{ADS(K)}^{K3\pi} = -0.42 \pm 0.22,
\]

where the error includes both statistical and systematic uncertainties. The Dπ and DK decays with the four-body doubly-Cabibbo-suppressed D final state are both observed for the first time, with a significance of >10 standard deviations for Dπ, and 5.1 standard deviations for DK.

2.3. GGSZ Analysis
The GGSZ method uses D decays to self-conjugate three-body final states, with the simplest cases being \(D \to KS\pi^+\pi^-\) and \(D \to KS^+K^-\). The analysis compares the Dalitz plots of the D decay for \(B^+ \to DK^+\) and \(B^- \to DK^-\). This can be thought of as combining the GLW and ADS analyses into one simultaneous analysis, since the final state includes contributions

![Figure 3](image_url)

**Figure 3.** Invariant mass distributions for \(B^\pm \to [K^+\pi^+\pi^+\pi^-]_Dh^\pm\) candidates, where \(h\) represents a charged kaon or pion. The top (bottom) plots show the \(h = K(h = \pi)\) final state, split by charge.
from decays with different levels of Cabibbo-suppression (for example the Cabibbo-favoured $D^0 \to K^+ - \pi^+$ and the doubly-Cabibbo-suppressed $D^0 \to K^+ + \pi^-$). This gives the analysis access to enough observables to allow an unambiguous determination of $\gamma$ and $r_B$ (which is not the case for the GLW or ADS analyses alone).

To measure physics parameters (such as $\gamma$) using this method, information is needed on how the $D$ decay amplitude varies over the Dalitz plot. The LHCb analysis described here uses CLEO-c measurements [9] of the strong phase variation as input. The Dalitz plot is binned in regions of similar strong phase. The bins are numbered from $-N$ to $N$, where $N = 8$ for $D \to K_S \pi^+\pi^-$ and $N = 2$ for $D \to K_S K^+K^-$. The number of events in the $i^{th}$ bin of the Dalitz plot is given by

$$N_{\pm i}^{+} = h_{B^+}[K_{x_i} + (x_i^2 + y_i^2)K_{\pm i} + 2\sqrt{K_{i}K_{-i}}(x_i c_{\pm i} \mp y_i s_{\pm i})]$$

for $B^+$,

$$N_{\pm i}^{-} = h_{B^-}[K_{x_i} + (x_i^2 + y_i^2)K_{\pm i} + 2\sqrt{K_{i}K_{-i}}(x_i c_{\pm i} \pm y_i s_{\pm i})]$$

for $B^-$,

where $h_{B^\pm}$ is a normalisation factor, the $K_i$ are the number of events in bin $i$ of the Dalitz plot of a flavour-tagged $D^0 \to K_S \pi^+\pi^-$ decay, and the $c_i(s_i)$ are the cos(sin) of the strong phase difference in bin $i$, taken from the CLEO-c measurement. The analysis thus measures the observables $x_\pm$ and $y_\pm$, which are related to the physics parameters via

$$x_\pm = r_B \cos(\delta_B \pm \gamma), \quad y_\pm = r_B \sin(\delta_B \pm \gamma).$$

LHCb has performed the GGSZ Dalitz analysis using the $K_S \pi^+\pi^-$ and $K_S K^+K^-$ final states of the $D$ [10]. The mass spectra of selected events are shown in Fig. 4.

The Dalitz plots of the selected events in the signal region for the $B^\pm \to [K_S \pi^+\pi^-]_{D}K^\pm$ final state are shown for $B^+$ and $B^-$ in Fig. 5. Combining the $K_S \pi^+\pi^-$ and $K_S K^+K^-$ final states,
the measured values of \(x\pm\) and \(y\pm\) are

\[
x_\pm = (0.0 \pm 4.3 \pm 1.5 \pm 0.6) \times 10^{-2},
\]

\[
y_\pm = (2.7 \pm 5.2 \pm 0.8 \pm 2.3) \times 10^{-2},
\]

where the first error is statistical, the second is systematic and the third is due to the uncertainties on the \(c_i\) and \(s_i\) parameters. The total precision on \(x\pm\) and \(y\pm\) from this analysis is similar to that achieved from the equivalent analyses at the B-Factories. From these \(x\pm\) and \(y\pm\) values, the physics parameters can be extracted:

\[
r_B = 0.07 \pm 0.04, \quad \gamma = (44^{+43}_{-38})^\circ,
\]

which have less precision than the equivalent B-Factory results, due to the lower central value of \(r_B\) measured in this sample compared to the B-Factory results.

### 3. Combined Value of \(\gamma\)

The LHCb measurements from the two-body ADS and GLW analyses, the four-body ADS analysis, and the GGSZ analysis have been combined, using a frequentist approach, to obtain a combined LHCb value for \(\gamma\) [11]. In addition to the LHCb measurements, a CLEO-c measurement of the strong phase in \(D \to K\pi\pi\pi\) is also used [12]. The experimental likelihoods are combined as

\[
\mathcal{L}(\vec{\alpha}) = f_i(A^{\text{obs}}_i|A_i(\vec{\alpha_i})),
\]

where \(\vec{A}\) are the experimental observables (such as, \(R_{CP}\), \(x\), etc), and \(\vec{\alpha}\) are the physics parameters (such as \(\gamma\), \(r_B\), etc). Confidence intervals are then obtained from this likelihood in a frequentist way. The confidence intervals for \(\gamma\) are shown in Fig. 6, both from a combination of only the \(DK\) measurements, and from a combination of the \(DK\) and \(D\pi\) measurements. Using only the \(DK\) measurements, the 68% confidence interval for \(\gamma\) is \((71.1^{+16.6}_{-15.7})^\circ\), which has similar precision to the existing constraints from B-Factory measurements. Using the \(DK\) and \(D\pi\) measurements, the 68% confidence interval for \(\gamma\) is \([61.8, 67.8]^{\circ} \cup [77.9, 92.4]^{\circ}\).

### 4. Time-dependent analysis of \(B_0^s \to D^\pm_s K^\pm\)

Since tree diagrams of similar magnitude exist for both \(B_0^s\) and \(B_0^-\) decays to \(D^- K^+\), large interference can occur between them. This allows the channel \(B_0^0 \to D^\pm_s K^\pm\) to be used to
measure $\gamma$, via a time-dependent and flavour-tagged analysis [13]. For the general case of an initially-produced $B_s^0$ or $\bar{B}_s^0$ meson decaying to a final state $f$, the decay rates are given by

$$\frac{d\Gamma_{B_s^0 \rightarrow f(t)}}{dt} = \frac{1}{2} |A_f|^2 (1 + |\lambda_f|^2) e^{-\Gamma_s t} \left[ \cosh \left( \frac{\Delta\Gamma_s t}{2} \right) - D_f \sinh \left( \frac{\Delta\Gamma_s t}{2} \right) \right],$$

$$\frac{d\Gamma_{\bar{B}_s^0 \rightarrow f(t)}}{dt} = \frac{1}{2} |A_f|^2 \left( \frac{p}{q} \right)^2 (1 + |\lambda_f|^2) e^{-\Gamma_s t} \left[ \cosh \left( \frac{\Delta\Gamma_s t}{2} \right) - D_f \sinh \left( \frac{\Delta\Gamma_s t}{2} \right) \right] - C_f \cos (\Delta m_s t) + S_f \sin (\Delta m_s t),$$

where $A_f$ is the decay amplitude of a $B_s^0$ decaying to a final state $f$, and $\lambda_f \equiv (p/q)(\bar{A}_f/A_f)$. The decays rate for decays to the CP-conjugate state $\bar{f}$ can be obtained by replacing $A_f$ by $\bar{A}_f = (\bar{f}|T|B_s^0)$, $\lambda_f$ by $\bar{\lambda}_f \equiv (p/q)(\bar{A}_f/\bar{A}_f)$ and $|p/q|^2$ by $|q/p|^2$. For a $B_s^0$ decay, the $D_f$ and $D_{\bar{f}}$ terms are experimentally accessible, due to the relatively large value of $\Delta\Gamma_s$. In the particular case of $B_s^0 \rightarrow D_s^\mp K^{\pm}$, the observables $C \equiv C_f = C_{\bar{f}}$, $S_f$, $D_f$, $S_{\bar{f}}$ and $D_{\bar{f}}$ are related to the physics parameters via

$$C = \frac{1 - r_{D_s K}^2}{1 + r_{D_s K}^2},$$

$$D_f(\bar{f}) = \frac{2r_{D_s K} \cos(\Delta - (\mp(\gamma - 2\beta_s)))}{1 + r_{D_s K}^2},$$

$$S_f(\bar{f}) = \frac{2r_{D_s K} \sin(\Delta - (\mp(\gamma - 2\beta_s)))}{1 + r_{D_s K}^2},$$

where $r_{D_s K} \equiv |A(B_s^0 \rightarrow D_s^- K^+)/A(B_s^0 \rightarrow D_s^+ K^-)|$, is the ratio of the magnitudes of the decay amplitudes of the interfering tree diagrams, $\Delta$ is the strong phase difference between these amplitudes, and $\beta_s$ is the $B_s^0$ mixing phase. Since there are five observables, the parameters $r_{D_s K}$, $\Delta$ and $\gamma - 2\beta_s$ can be unambiguously extracted from this analysis. The value of $\gamma$ can then be determined using external measurements of $\beta_s$, for example from the channel $B_s^0 \rightarrow J/\psi\phi$ [14].

![Figure 6](image-url)

**Figure 6.** Confidence intervals for $\gamma$, resulting from the combination of measurements with $DK$ only (left), and with both $DK$ and $D\pi$ (right).
LHCb has performed this time-dependent and flavour-tagged analysis of $B_s^0 \rightarrow D_s^\pm \pi^\mp$ decays [15]. The Cabibbo-favoured $B_s^0 \rightarrow D_s^\pm \pi^\mp$ decay is used as a control channel. Three separate decay modes of the $D_s^+$ are considered — $K^+ K^- \pi^+$, $K^+ \pi^- \pi^+$ and $\pi^+ \pi^- \pi^+$. The mass spectra of selected events in the $D_s^\mp \pi^\pm$ and $D_s^\mp K^\pm$ final states, including all three final states of the $D_s$, are shown in Fig. 7. The decay time distribution for selected $B_s^0 \rightarrow D_s^\mp K^\pm$ events is shown in Fig. 8. The distribution for all events is shown, along with the flavour-tagged events. Only opposite-side flavour tagging is used, resulting in an effective tagging power of $(1.9 \pm 0.3)\%$. From the fit to this decay time distribution, the $CP$-asymmetry observables are measured to be

\begin{align*}
C &= 1.01 \pm 0.50 \pm 0.23, \\
D_f &= -1.33 \pm 0.60 \pm 0.26, \\
S_f &= -1.25 \pm 0.56 \pm 0.24,
\end{align*}

where the first error is statistical, and the second systematic. This is the first measurement of these observables in the $B_s^0 \rightarrow D_s^\mp K^\pm$ decay. A value for $\gamma$ has not been extracted from
the current analysis, due to the need to understand the correlation matrix for the systematic uncertainties, which is found to substantially affect the determination of $\gamma$.

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