Lambda hypernuclei and neutron star matter in a chiral SU(3) relativistic mean field model with a logarithmic potential

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(Dated: October 1, 2009)

We develop a chiral SU(3) symmetric relativistic mean field (RMF) model with a logarithmic potential of scalar condensates. Experimental and empirical data of symmetric nuclear matter saturation properties, bulk properties of normal nuclei, and separation energies of single- and double-Λ hypernuclei are well explained. The nuclear matter equation of state (EOS) is found to be softened by σζ mixing which comes from determinant interaction. The neutron star matter EOS is further softened by Λ hyperons.

I. INTRODUCTION

The equation of state (EOS) of dense baryonic matter is one of the keys in nuclear physics as well as in physics of compact stars [1–14]. In dense matter, various forms of matter are expected to appear such as the hyperon admixture [4–10], meson condensation [11, 12], baryon rich quark gluon plasma [13], and color superconductor [14]. Among these exotic forms of matter, hyperons are expected to appear at relatively small densities, ~ (2 – 3)ρ0. In addition to the variety of particle species, partial restoration of chiral symmetry is also expected in nuclear medium, and it would modify the properties of dense matter significantly. Therefore, it is desired to respect both hypernuclear physics information [15–19] and chiral symmetry [20–22] in constructing the dense matter EOS.

It is widely believed that hyperons should emerge such as in the neutron star core [4–6, 9, 10] and/or during the black hole formation [23], as the baryon density increases in the hadronic matter. In neutron stars, the hyperon admixture causes the softening of EOS, and reduces the maximum mass of neutron stars. It also increases the proton fraction, which may promote faster cooling processes [24]. In black hole formation processes, hyperons are abundantly produced due to temperature or density effects, and shorten the duration time of neutrino emission [23]. Hyperon admixture is governed by the hyperon potentials in nuclear matter, which may be determined from the hyperon separation energies from hypernuclei and hyperon production spectra. From this point of view, we need to adopt a theoretical framework which can explain both nuclear matter and finite nuclear properties.

The chiral symmetry is another important ingredient in dense matter. It is a fundamental symmetry of QCD with massless quarks, and its spontaneous symmetry breaking generates constituent quark and thus hadron masses [20]. Hadron properties and EOS would be modified in nuclear matter due to the partial restoration of the chiral symmetry [21]. Thus it is preferable for hadronic many-body theories to possess the chiral symmetry, including its spontaneous breaking and partial restoration at finite densities.

We have recently developed a chiral SU(2) relativistic mean field (RMF) model with a logarithmic σ potential in the form of − log σ [25], which is derived from the strong coupling limit of lattice QCD (SCL-LQCD) [26, 27]. If we naively include the vector meson in the linear σ model (φ4 theory), the chiral symmetry is found to be restored below the normal nuclear density (Lee-Wick vacuum, or chiral collapse) [28, 29]. There are several attempts to avoid this problem, some of which result in having instability at large σ values [30] or too stiff EOS [31]. In the SCL model, we do not have any instability in the σ potential, and the obtained nuclear matter EOS is found to be reasonably soft. In addition, the bulk properties of finite nuclei (binding energies and charge rms radii) are well described. Then it is desired to extend the chiral SU(2) RMF model to the SUf(3) version in order to describe hypernuclear systems. We expect that this extension enables us to get detailed information of various hypernuclei.

In this paper, we introduce a chiral SU(3) RMF model, abbreviated as SCL3 RMF in the later discussion, extended from the previous chiral SU(2) RMF (SCL2 RMF) [25]. Several model parameters are constrained by the chiral symmetry through the hadron masses and vacuum condensates. We determine meson-Λ coupling constants by fitting existing Λ hypernuclear data and the SUf(3) symmetric relation for vector couplings. We show that we can reproduce the separation energies of single Λ hypernuclei (SΛ) [18] and the ΛΛ bond energy (∆EΛΛ) in 6ΛHe [19] by choosing the coupling constants appropriately. The EOS of symmetric matter is found to be softened by the scalar meson with hidden strangeness, ζ = ¨ss, which couples with σ through the determinant interaction representing the effects of U1(1) anomaly [32, 33]. We also discuss the neutron star matter and its maximum mass in this RMF model.

It is generally preferable to derive the dense matter EOS from bare baryon-baryon interactions.
Non-relativistic calculations based on the variational method [34, 35] and the g-matrix [36, 37] have been carried out based on realistic bare baryon-baryon interactions. Unfortunately, EOS only with two-body nucleon-nucleon interaction does not reproduce the nuclear matter saturation point, and we need to include three-body forces, which plays significant roles at high densities. In the relativistic Brückner Hartree-Fock (RBHF) theory [38], empirical nuclear matter saturation is explained quantitatively, and the EOS in RBHF is approximately reproduced in RMF with non-linear ω interaction, which is introduced to simulate the high density behavior of the vector potential [39]. In this work, we follow the latter standpoint: We start from the Lagrangian with several model parameters and determine these parameters by fitting existing data. As a result, the scalar and vector potentials in nuclear matter are found to be consistent with the RBHF results, then we expect that the results with hyperons would be also meaningful.

This paper is organized as follows. In Sec. II, we introduce a chiral SU(3) potential derived from SCL-LQCD in RMF as an extension from the chiral SU(2) potential. In Sec. III, we investigate the properties of symmetric nuclear matter, normal nuclei, and Λ hypernuclei, and extend to a chiral SU(3) potential derived from SCL-LQCD [25, 27, 40]. Extension to a chiral SU(3) potential would be also meaningful. In Sec. III, we investigate the properties of symmetric nuclear matter, normal nuclei, and Λ hypernuclei, and extend to a chiral SU(3) potential derived from SCL-LQCD [25, 27, 40]. Extension to a chiral SU(3) potential would be also meaningful. In Sec. IV, we summarize how to derive the chiral potential in SCL-LQCD.

II. CHIRAL SU(3) RMF MODEL

The energy density as a function of the chiral condensate, abbreviated as the chiral potential here, describes the spontaneous chiral symmetry breaking and its partial restoration through the chiral condensates, and these chiral condensates determine hadron masses. In Ref. [25], Tsubakihara and Ohnishi proposed to apply the logarithmic chiral SU(2) potential derived from the strong coupling limit of lattice QCD (SCL-LQCD) and developed the SCL2 RMF model. In Subsec. II A, we briefly summarize how to derive the chiral potential in SCL-LQCD [25, 27, 40]. Extension to a chiral SU(3) potential is described in Subsec. II B.

A. Chiral SU(2) Potential from SCL-LQCD (SCL2)

The lattice QCD action consists of the pure Yang-Mills part and the fermionic part. The pure Yang-Mills part is proportional to $1/g^2$, where $g$ is the bare QCD coupling. In SCL-LQCD ($g \rightarrow \infty$), we can ignore the pure Yang-Mills action terms, and only those terms including fermions $S_F$ should be kept in the action. The fermionic action with staggered fermions in the chiral limit is written in the lattice unit as,

\[
S_F[\chi, \bar{\chi}, U] = \frac{1}{2} \sum_{x, \mu} \eta_\mu(x) \left[ \bar{\chi}(x) U_\mu(x) \chi(x + \hat{\mu}) - \bar{\chi}(x + \hat{\mu}) U_\mu^\dagger(x) \chi(x) \right],
\]

where $\eta_\mu(x) = (-1)^{x_0 + x_1 + \cdots + x_{n-1}}$ represents the staggered factor. After integrating out link variables $U_\mu$ in the leading order of $1/d$ expansion, we obtain the following partition function $Z$,

\[
Z = \int \mathcal{D}[\chi, \bar{\chi}, U] \exp \left( -S_F[\chi, \bar{\chi}, U] \right)
\]

\[
\simeq \int \mathcal{D}[\chi, \bar{\chi}] \exp \left[ \frac{1}{2} \sum_{x, y, \alpha, \beta} M_{\alpha\beta}(x)V_M(x,y)M(y)_{\beta\alpha} \right]
\]

\[
= \int \mathcal{D}[\chi, \bar{\chi}, \sigma] \exp \left( -S_\sigma[\chi, \bar{\chi}, \sigma] \right),
\]

\[
S_\sigma = \frac{1}{2} \sum_{x, y, \alpha, \beta} \sigma(x)_{\alpha\beta} V_M(x,y) \sigma(y)_{\beta\alpha} + \sum_{x, y, \alpha, \beta} \sigma(y)_{\alpha\beta} V_M(y,x) M(\beta\alpha),
\]

The mesonic composites are denoted as $M_{\alpha\beta}(x) = \chi^\dagger_\beta(x) \chi^\alpha_\alpha(x)$, and the auxiliary fields $\sigma_{\alpha\beta}$ are related to the expectation values of the mesonic composites and written as $\langle \sigma_{\alpha\beta}(x) \rangle = -\langle M_{\alpha\beta}(y) \rangle$. In these equations, the superscript $a$ denotes color and the subscripts $\alpha$ and $\beta$ show the flavor. The lattice mesonic inverse propagator $V_M(x,y)$ is given as $V_M(x,y) = \sum_\mu (\delta_{y,x+\hat{\mu}} + \delta_{y,x-\hat{\mu}})/4N_c$. From the first to the second line in Eq. (2), the one-link integral formula, $\int dU_{ab} U_{ab}^\dagger = \delta_{aa} \delta_{bb}/N_c$ has been used.

Here, we substitute the auxiliary fields with the static and uniform scalar $\Sigma_{\alpha\beta}$ and pseudoscalar $\Pi_{\alpha\beta}$ matrices as the mean field ansatz,

\[
\sigma_{\alpha\beta}(x) = \Sigma_{\alpha\beta} + i\varepsilon(x) \Pi_{\alpha\beta},
\]

where $\varepsilon(x) = (-1)^{x_0 + x_1 + \cdots + x_3}$. Since fermions are decoupled in each space-time point, the grassmann integral can be easily evaluated and the effective free energy is obtained up to a constant as,

\[
V_\chi(\sigma, \pi) = \frac{1}{2} \left\{ (\text{tr} |\sigma V_M \sigma|) - N_c \langle \log \det(V_M \sigma) \rangle \right\}
\]

\[
= b_\sigma \text{tr} \left[ M^\dagger M \right] - a_\sigma \log \det \left[ M^\dagger M \right],
\]

\[
b_\sigma = \frac{d}{2N_c}, \quad a_\sigma = N_c,
\]

where $\langle \cdots \rangle$ denotes the space-time average, $d = 4$ is the space-time dimension, and $M$ represents the meson matrix, $M = \Sigma_{\alpha\beta} + i\Pi_{\alpha\beta}$, in which the $\varepsilon(x)$ factor is replaced with unity in $\sigma_{\alpha\beta}$.

While the coefficients $b_\sigma$ and $a_\sigma$ are fixed in the lattice unit in Eq. (5), they depend on the lattice spacing and the
scaling factor connecting the meson field and the quark condensate, which should be chosen for $\sigma$ and $\pi$ to be in the canonical form. Furthermore, $n_f$ species of staggered fermions corresponds to $N_f = 4n_f$, and the coefficient modification may not be trivial when we take $N_f = 2$ for SU(2) or $N_f = 2 + 1$ for SU(3). Thus we stipulate them as parameters to obtain physical meson masses.

In SU(2), meson matrix is given as $M = (\sigma + i\tau \cdot \pi)/\sqrt{2}$. Requiring that the chiral potential has a minimum at $\sigma = f_\pi$ and fitting the pion mass $m_\pi$, one parameter $m_\pi$ is left as a free parameter. Then, the chiral potential is given as,

$$V_\chi = -\frac{a_\sigma}{2} \log (\det MM^\dagger) + \frac{b_\sigma}{2} \text{tr}(MM^\dagger) - c_\sigma \sigma$$

$$= -a_\sigma \log(\sigma^2 + \pi^2) + \frac{b_\sigma}{2} (\sigma^2 + \pi^2) - c_\sigma \sigma$$

$$\simeq -2a_\sigma f_{SCL}(\frac{\sigma}{f_\pi}) + \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\pi^2 \pi^2,$$

$$f_{SCL}(x) = \log(1 - x) + x + \frac{x^2}{2},$$

where $\varphi_\sigma = f_\pi - \sigma$, and the explicit chiral symmetry breaking term $-c_\sigma \sigma$ is introduced. We have omitted pion self-energy terms and constants in the third line in Eq. (7). Parameters $a_\sigma, b_\sigma, c_\sigma$ are given as,

$$a_\sigma = \frac{f_\pi^2}{4} (m_\sigma^2 - m_\pi^2), \quad b_\sigma = \frac{1}{2} (m_\sigma^2 + m_\pi^2), \quad c_\sigma = f_\pi m_\pi^2,$$

With the above logarithmic $\sigma$ potential, full chiral symmetry restoration is suppressed because of the repulsive contribution of $V_\chi$ at small $\sigma$. The present treatment of SCL-LQCD is referred to as the zero temperature treatment, where $V_\chi$ diverges at $\sigma \to 0$. In the finite temperature treatment of SCL-LQCD [41], the divergent behavior of $V_\chi$ disappears, while $V_\chi$ has a finite negative derivative at $\sigma \to 0$. This finite negative derivative is enough to suppress full chiral restoration at finite density, since the nucleon Fermi integral contribution behaves as $\rho_\sigma \sigma^2$ and we always have a minimum at a finite $\sigma$ value. Therefore, we suppose that the present chiral potential $V_\chi$ would be a good starting point to investigate cold nuclear matter and nuclei.

### B. Chiral SU(3) potential from SCL-LQCD (SCL3)

In order to apply the logarithmic chiral potential to hypernuclear systems, it is necessary to include mesons with hidden strangeness ($\bar{s}s$) such as $\zeta$ and $\phi$ in addition to mesons made of $u$ and $d$ quarks, $\sigma, \omega$ and $\rho$. Here, we replace meson matrix $M$ with that of SU(3),

$$M = \begin{pmatrix}
M_{11} & a_0^+ + i\pi^+ & \kappa^0 + iK^0 \\
\kappa^- + iK^- & a_0^- + i\pi^- & M_{22} \\
M_{12} & M_{21} & \kappa^0 + iK^0
\end{pmatrix},$$

where

$$M_{11} \equiv \frac{1}{\sqrt{2}} [(\sigma + i\eta) + (a_0^0 + i\varphi_0)],$$

$$M_{22} \equiv \frac{1}{\sqrt{2}} [(\sigma + i\eta) - (a_0^0 + i\varphi_0)].$$

In a similar way to the previous subsection, the chiral SU(3) potential in SCL-LQCD may be given as,

$$V_\chi = -\frac{a'_\sigma}{2} \log (\det M'M'^\dagger) + \frac{b'_\sigma}{2} \text{tr}(MM^\dagger)$$

$$- c_\sigma \sigma - c_\zeta \zeta + V_{KMT},$$

where the explicit chiral symmetry breaking effects are included in $c_\sigma \sigma$ and $c_\zeta \zeta$ terms. Since the strange quark mass is not small compared with $f_{\pi}$ and $f_{\zeta}$, we have taken account of its effects also in the shift of the meson matrix, $M' = M + \text{diag}(0,0,\delta_s)$. This shifted meson matrix plays the role of the constituent quark mass in Eq. (3) and appears in the logarithmic term of the chiral potential.

The Kobayashi-Maskawa-'t Hooft interaction term [32, 33] is represented in a form of the determinant of meson matrix,

$$V_{KMT} = -d' \left( \det M + \det M' \right).$$

This KMT interaction represents the $U_A(1)$ anomaly effects. Without $V_{KMT}$, the above chiral potential is invariant under $U_L(3) \times U_R(3)$ transformation in the chiral limit, $\delta_s = c_\sigma = c_\zeta = 0$. In the real world, $U_A(1)$ symmetry is broken by the anomaly. Kobayashi and Maskawa [32] proposed the above determinant interaction term, and this term is derived as the instanton induced quark interaction vertex by 't Hooft [33].

Now we shall decompose the chiral potential $V_\chi$ in Eq. (13) into meson mass terms and interaction terms.

$$V_\chi = -a' \log(\sigma^2 + \zeta^2) + b' \left( \sigma^2 + \zeta^2 \right) - d' \sigma^2 \zeta - c_\sigma \sigma - c_\zeta \zeta$$

$$+ \frac{1}{2} \sum_\alpha m_\alpha^2 \phi_\alpha^2 + \delta V$$

$$= \frac{1}{2} m_\sigma^2 \varphi_\sigma^2 + \frac{1}{2} m_\zeta^2 \varphi_\zeta^2 + V_{\zeta}(\varphi_\sigma, \varphi_\zeta)$$

$$+ \frac{1}{2} \sum_\alpha m_\alpha^2 \phi_\alpha^2 + \delta V(\varphi_\sigma, \varphi_\zeta, \{\phi_\alpha\}) + \text{const.},$$

$$V_{\sigma\zeta} = -a'' \left( 2f_{SCL}(\frac{\varphi_\sigma}{f_\sigma}) + f_{SCL}(\frac{\varphi_\zeta}{f_\zeta}) \right) + \xi_{\sigma\zeta} \varphi_\sigma \varphi_\zeta,$$

where $\varphi_\sigma = f_\pi - \sigma$ and $\varphi_\zeta = f_\zeta - \zeta$ show the deviation of $\sigma$ and $\zeta$ from their vacuum expectation values, respectively, and $V_{\sigma\zeta}$ denotes the interaction energy density.
In the logarithmic potential, shifted vacuum expectation value of $\zeta$ appears, $f_\zeta' = f_\zeta + \delta_\zeta$. Other meson fields than $\sigma$ and $\zeta$ are shown by $\phi_\alpha$, and $m_\alpha$ and $\delta V$ represent their masses and interaction terms. We have ignored the third order term $(\varphi_\sigma)^3 \varphi_\zeta$ coming from the determinant interaction. This term does not change the chiral potential significantly around the vacuum, but it makes the system unstable at large values of $\sigma$ and $\zeta$. This is because we do not have polynomial terms such as $\sigma^4$ and $\zeta^4$, which stabilizes the chiral potential in the $\phi^4$ theory. Compared with the case of SU(2), where all of $\text{tr}(MM'^1)$, $\text{det} M$ and $\text{det} M'^1$ are proportional to the same combination, $\sigma^2 + \pi^2$, we have several different terms from $\log(\text{det} M'^{2''})$, $\text{tr}(MM'^1)$ and $\text{det} M + \text{det} M'^1$ in SU(3).

In Eq. (15), the $\sigma\zeta$ mixing appears in the quadratic form of the meson fields, thus we have to diagonalize the mass matrix to obtain observed $\sigma$ and $\zeta$ meson masses as,

$$
\frac{1}{2} \begin{pmatrix} \varphi_\sigma & \varphi_\zeta \\ \varphi_\sigma^* & \varphi_\zeta^* \end{pmatrix} \begin{pmatrix} m_\sigma^2 & \xi_{\sigma\zeta} \\ \xi_{\sigma\zeta}^* & m_\zeta^2 \end{pmatrix} \begin{pmatrix} \varphi_\sigma & \varphi_\zeta \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \varphi_\sigma' & \varphi_\zeta' \end{pmatrix} \begin{pmatrix} M_{\sigma}^2 & 0 \\ 0 & M_{\zeta}^2 \end{pmatrix} \begin{pmatrix} \varphi_\sigma' & \varphi_\zeta' \end{pmatrix}. \quad (17)
$$

Five out of six ($a'$, $b'$, $c_\sigma$, $c_\zeta$, $d'$ and $\delta_\zeta$) parameters in this chiral potential are fixed by fitting observed meson masses of $m_\sigma$, $m_K$ and $M_\zeta$, and vacuum expectation values of $\sigma$ and $\zeta$ ($f_\sigma$ and $f_\zeta$). Relevant meson masses are related to the parameters ($a'$, $b'$, $d'$, $\delta_\zeta$) as,

$$
m_\sigma^2 = b' - \frac{2a'}{f_\pi} - 2d' f_\zeta, \quad (18)$$

$$
m_\sigma^2 = b' + \frac{2a'}{f_\pi} - 2d' f_\zeta, \quad (19)$$

$$
m_K^2 = b' - \frac{\sqrt{2a'}}{f_\pi f_\zeta} - \sqrt{2d'} f_\pi, \quad (20)$$

$$
m_\zeta^2 = b' + \frac{a'}{f_\pi^2}; \quad (21)$$

$$
\xi_{\sigma\zeta} = -2d' f_\pi. \quad (22)
$$

We regard $m_\sigma$ as a model parameter, and give $a'$ and $b' - 2d' f_\zeta$ as in the case of the SCL2 model,

$$
a' = \frac{f_\pi^2}{4} \left( m_\sigma^2 - m_\zeta^2 \right) = a_\sigma, \quad (23)$$

$$
b' - 2d' f_\zeta = \frac{1}{2} \left( m_\sigma^2 + m_\zeta^2 \right) = b_\sigma. \quad (24)
$$

We assign the observed $\zeta$ as $f_\zeta(980)$, and the parameters $b'$, $d'$ and $\delta_\zeta$ are determined to reproduce $m_\zeta = 138$ MeV, $m_K = 496$ MeV and $M_\zeta = 980$ MeV.

Coefficients of the linear terms in $\sigma$ and $\zeta$ are determined to reproduce the vacuum expectation values,

$$
c_\sigma = f_\pi \left( b' - \frac{2d'}{f_\pi} - 2d' f_\zeta \right) = f_\pi m_{\sigma}^2, \quad (25)$$

$$
c_\zeta = b' f_\zeta - \frac{a'}{f_\zeta^2} - \frac{f_\pi^2}{f_\zeta} d'. \quad (26)
$$

Once we fix the parameters in the chiral SU(3) potential, masses of other scalar and pseudoscalar mesons are determined as shown in Appendix A. Calculated masses of these mesons are tabulated in Table I. They are in reasonable agreement with experimental values except for $\kappa$.

In Fig. 1, we show the energy density as a function of $\sigma$ in the SCL3 model (solid curve) is compared with those in the linear $\sigma$ $(\phi^4$, open-triangles), SCL2 (dashed curve), and TM1 (dotted curve) models.

![Energy density as a function of $\sigma$](image)

**FIG. 1:** (Color online) Energy density in vacuum as a function of $\sigma$ in the SCL3 model (solid curve) is compared with those in the linear $\sigma$ $(\phi^4$, open-triangles), SCL2 (dashed curve), and TM1 (dotted curve) models.

In Fig. 1, we show the energy density as a function of $\sigma$. We compare the SCL3 results with those in SCL2 [25], TM1 [39] and the linear $\sigma$ $(\phi^4$) models. We adopt the parameter $m_\sigma = 690$ MeV, which reproduces the bulk properties of normal nuclei as explained later, and optimal $\varphi_\zeta$ value is chosen for each $\sigma$. When we only consider the quadratic term in $\varphi_\sigma$, the energy density behaves as $m_\sigma^2 \varphi_\sigma^2/2$. Thus the energy density in SCL3 can be twice larger than the results in SCL2 and TM1, in which $m_\sigma$ is around 500 MeV, while the calculated results shows similar values around $\sigma = f_\pi$. This is because optimal $\kappa$ value is chosen and reduces the energy density in SCL3. In Fig. 2, we show the energy surface as a function of $\varphi_\sigma$ and $\varphi_\zeta$. The optimal value of $\varphi_\zeta$ is modified from zero to a finite value by the $\sigma\zeta$ coupling from the KMT interaction, and the energy density is reduced by this mixing.

### C. SCL3 RMF model

We incorporate the chiral SU(3) potential $V_{\sigma\zeta}$ discussed in the previous subsections into the SU(3) RMF. We consider the following SU(3) RMF Lagrangian, which describes baryons which couple with $\sigma$ and $\zeta(= s\bar{s})$ scalar
TABLE I: Parameters and masses of vector mesons as functions of $m_\sigma$. Parameters are determined by fitting $\pi, K$ and $\zeta$ masses ($m_\pi = 138$ MeV, $m_K = 496$ MeV, $M_\zeta = 980$ MeV), and vacuum condensate of $\sigma$ and $\zeta$ ($f_\pi = 92.4$ MeV, $f_\zeta = 94.5$ MeV).

| $m_\sigma$ (MeV) | $M_\sigma$ (MeV) | $M_\eta$ (MeV) | $M_{\eta'}$ (MeV) | $M_\sigma^*$ (MeV) |
|-----------------|----------------|----------------|-------------------|------------------|
| 630             | 1108.4         | 536.0          | 1069.7            | 321.7            |
| 640             | 1108.3         | 535.7          | 1062.4            | 344.6            |
| 650             | 1108.1         | 535.4          | 1054.9            | 366.4            |
| 660             | 1108.0         | 535.0          | 1047.2            | 387.2            |
| 670             | 1107.8         | 534.6          | 1039.4            | 407.3            |
| 680             | 1107.5         | 534.1          | 1031.3            | 426.7            |
| 690             | 1107.3         | 533.6          | 1023.1            | 445.5            |
| 700             | 1107.0         | 533.1          | 1014.6            | 463.8            |
| 710             | 1106.8         | 532.5          | 1006.0            | 481.6            |
| 720             | 1106.4         | 531.8          | 997.1             | 499.0            |
| 730             | 1106.1         | 531.1          | 988.0             | 516.0            |

Exp. $980 \pm 20 672 \pm 40 547.85 957.78 400-1200$

\[ M_\sigma^* = M_\sigma - g_{\sigma \omega} \phi - g_{\omega \omega} \phi \zeta, \]

\[ U_{\mu}^\mu = g_{\omega \omega} \omega + g_{\rho \rho} R, \]

where $V_{\mu \nu}(V = \omega, R, \phi)$ shows the field tensor of the vector meson $V$, and $\phi_S = f_S - S$ ($S = \sigma, \zeta$) represents the deviation of the scalar meson field $S$ from its vacuum expectation value. The $\omega^2$ term in Eq. (27) is introduced to simulate the behavior of the vector potential at high densities in the RBHF theory [38, 39]. The term $V_{\alpha \beta}(\phi_S, \phi_\zeta)$ is the scalar self-interaction, and we adopt the chiral SU(3) potential $V_{\alpha \beta}$ in Eq. (15) in SCL3.

In this Lagrangian, we have one parameter $m_\sigma$ in $V_{\alpha \beta}$, one parameter $c_{\zeta}$ for the $\omega$ self-interaction, and meson-baryon coupling constants $g_{\alpha \beta}$ in the meson-baryon interaction. We assume that (1) nucleon mass is fully generated by the chiral condensate, $M_N = g_{\sigma N} f_\pi$, (2) the vector couplings obey the SU(3) relation [15, 42], and (3) nucleon does not couple with hidden strangeness mesons ($\zeta$ and $\phi$) [43, 44]. Suppressed $N \phi$ coupling is understood in the Okubo-Zweig-Iizuka (OZI) rule [43], while the scalar meson-nucleon coupling $g_{\sigma N}$ may violate the OZI rule [44]. Thus the assumption of $g_{\zeta N} = 0$ may be regarded as an working hypothesis.

Under these assumptions, we have four meson-baryon coupling constants, $g_{\omega N}, g_{\rho N}, g_{\eta N}, g_{\zeta N}$, as model parameters. We have totally six parameters $(m_\sigma, c_\zeta, g_{\omega N}, g_{\rho N}, g_{\eta N}, g_{\zeta N})$. For parameters relevant to normal nuclear properties $(m_\sigma, c_\zeta, g_{\omega N}, g_{\rho N})$, first we give $m_\sigma$ and fix $g_{\omega N}$ and $c_\zeta$ by fitting the saturation point. Next from the binding energies of Sn and Pb isotopes, $m_\sigma$ and $g_{\rho N}$ are obtained. The separation energies of A hypernuclei mainly reflect the core-A potential depth, $U_\Lambda \sim -30$ MeV, thus the combination $g_{\sigma N} \phi (\rho_0) + g_{\zeta N} \phi_\zeta (\rho_0)$ is obtained from this fitting procedure. Finally, the ratio of $g_{\sigma N}$ and $g_{\zeta N}$ is determined.
TABLE II: Parameter set determined from saturation point of symmetric nuclear matter, binding energies and charge rms radii of normal nuclei, $\Delta A$ of single $\Lambda$ hypernuclei and $\Delta B_{\Lambda\Lambda}$ in $\Lambda\Lambda$He. Input constants adopted in this paper are also shown. We adopt the saturation point, $(\rho_0, E_0/A) = (0.150 \text{ fm}^{-3}, -16.3 \text{ MeV}).$

| Parameter | Value |
|-----------|-------|
| $m_\sigma$ (MeV) | 690 |
| $\delta_\pi$ (MeV) | 32.81 |
| $g_{aNN}$ | 0.1195 |
| $g_{pNN}$ | 2.94 |
| $g_{2NN}$ | 4.54 |
| $g_{3NN}$ | 3.40 |
| $g_{3\Lambda}$ | 5.17 |

from the $\Lambda\Lambda$ bond energy in the double $\Lambda$ hypernucleus. All parameters are tabulated in Table II.

There are two points to be noted in view of the chiral symmetry in the above Lagrangian. We omit pseudoscalar mesons, and we do not require chiral symmetry in baryon-scalar meson couplings. In order to satisfy the chiral symmetry in the meson sector with scalar mesons, we need pseudoscalar mesons as chiral partners. However, the expectation values of pseudoscalar mesons disappear in the mean field treatment with parity fixed single particle baryon states. In principle, it is necessary to require the chiral symmetry to baryon-scalar meson couplings [45, 46]. It is possible to construct an SU(3) chiral symmetric coupling term of scalar and pseudoscalar mesons with baryons [45], but we have to assume baryons transform as nonet and only D-type coupling appears. For octet baryons having both D- and F-type couplings, it is necessary to introduce two-types of baryons [46], which is out of the scope in this paper.

III. FINITE NUCLEI AND NUCLEAR MATTER

A. Nuclear matter and normal nuclei

First we discuss the EOS of symmetric nuclear matter. There are three relevant parameters, $m_\sigma$, $g_{aNN}$ and $c_\omega$. For a given $m_\sigma$ value, latter two are determined by fitting the saturation point $(\rho_0, E_0/A) = (0.15 \text{ fm}^{-3}, -16.3 \text{ MeV}/A)$. In Fig. 3, we show calculated energy per nucleon ($E/A$) in TM1, SCL2, and the present model (SCL3) with $m_\sigma = 690$ MeV. When we adopt $m_\sigma \sim 700$ MeV, the EOS in SCL3 is considerably softer than those in TM1 and SCL2. We also find the SCL3 EOS is in good agreement with the variational calculation results by Friedman and Pandharipande (FP) [34], especially at around $\rho_0$. SCL3 EOS has rather soft incompressibility $K \sim 211.0$ MeV and this result is comparable with the empirical incompressibility $K = 210 \pm 30$ MeV [47].

Nuclear matter EOS at several $\rho_0$ has been probed in heavy-ion collisions [48–50]. In Fig. 4, we show the region of pressures consistent with the experimental flow data analyzed by using the Boltzmann equation model [48]. Danielewicz, Lacey and Lynch suggested the range of the incompressibility $167 \text{ MeV} \leq K \leq 380 \text{ MeV}$ in the density range $2\rho_0 \leq \rho_B \leq 5\rho_0$ [48]. Other theoretical model calculations [49, 50] also explain flow data at AGS and SPS energies with $K \simeq 300$ MeV. The calculated pressure in SCL3 is consistent with the pressure range suggested in Ref. [48].

The EOS softening is caused by the $\zeta$ meson, which couples with $\sigma$ through the determinant interaction. In Fig. 2, we show the density dependence of the equilibrium point in $(\phi_\sigma, \phi_\zeta)$ plane. Equilibrium values at $\rho_B/\rho_0 = 0 \sim 5$ are shown with points. We find that the system evolves along with the valley where $\zeta$ values are finite. This $\zeta$ variation reduces the quadratic part of the mesonic energy density, while both $\phi_\sigma$ and $\phi_\zeta$ contribute...
repulsively in the higher order term, \( V_{\sigma \zeta} \). As a result, the energy gain from the scalar mesons is suppressed a little, and it leads to a smaller vector coupling to reproduce the saturation point. Cancellation of smaller scalar and vector potentials leads to a softer EOS in SCL3.

The density dependence of the scalar and vector potentials in SCL3 are found to be qualitatively consistent with the RBHF results [38] at low densities, \( \rho_0 < 0.3 \text{ fm}^{-3} \). In Fig. 5, we show the scalar and vector potentials, \( U_N^{s} = M_N^{s} - M_N = -g_{\sigma N} \varphi_\sigma \) and \( U_N^{v} = g_{\omega N} \omega \) in symmetric nuclear matter, as functions of density in SCL3 in comparison with those in SCL2, TM1, and RBHF. Scalar and vector potentials grows almost linearly with \( \rho_0 \) at very low densities, and they are suppressed at higher densities in RBHF via the exchange and correlation, and the relativistic normalization [38]. In RMF models, suppression is caused by the non-linear terms in \( \sigma \) and \( \omega \). The RBHF results are between SCL2 and SCL3, and the density dependence at \( \rho_0 < 0.3 \text{ fm}^{-3} \) is well reproduced with SCL3. Once these potentials are given, similar EOSs are obtained at low densities; the difference appears from the density dependence of the vector potential. As shown in Fig. 5, the vector potential in SCL3 is suppressed more strongly than in RBHF. Since the \( \omega^4 \) term is introduced to mimic the density dependence in RBHF, it may be necessary to include other types of non-linear interaction terms for vector field provided that the density dependence in RBHF gives the convergent result in the hole-line expansion.

For normal finite nuclei, binding energies per nucleon and charge rms radii are controlled by two parameters, \( m_\sigma \) and \( g_{\rho N} \). We determine them by fitting experimental data of binding energies and charge rms radii of some stable nuclei (\( ^{12}\text{C}, ^{16}\text{O}, ^{40}\text{Ca}, ^{48}\text{Ca}, ^{58}\text{Ni} \) and \( ^{90}\text{Zr} \)) and Sn and Pb isotopes. In Fig. 6, we show the calculated binding energies per nucleon (\( B/A \)) of C, O, Si, Ca, Ni, Zr, Sn and Pb isotopes, and \( B/A \) and charge rms radii for some stable nuclei are shown in Tables III and IV, where we also tabulate NL1 [51], NL3 [52], and non-chiral RMF (TM [22, 39]) results.

From these results, we find that SCL3 RMF model well describes the bulk properties of normal nuclei with the parameters shown in Table II, especially those of Sn and Pb isotopes which we can treat as spherical. Since \( B/A \) in these nuclei apparently reflects the character of EOS around \( \rho_0 \), our choice of the saturation point and adopted values of parameters (\( m_\sigma = 690\text{MeV}, g_{\rho N} = 4.54 \)) is appropriate in explaining these data.

### B. A hypernuclei

Next we study single- and double-\( \Lambda \) hypernuclei in SCL3 RMF model. In single- and double-\( \Lambda \) hypernuclei, we have four adjustable parameters, \( g_{\sigma \Lambda}, g_{N \Lambda}, g_{\omega \Lambda}, \) and \( g_{\phi \Lambda} \). Here, we assume that the vector couplings obey the lowest order SU(3) symmetric relation [15, 42],

\[
\mathcal{L}_{NV} = \sqrt{2} \left( g_{\sigma} \text{tr} \left( M_N \right) \text{tr} \left( \bar{B} M_N \right) + g_D \text{tr} \left( \bar{B} \{ M_N, B \} \right) \right) + g_r \text{tr} \left( \bar{B} \{ M_N, B \} \right) + g_2 \text{tr} \left( \bar{B} M_B \right) + g_2 \text{tr} \left( \bar{B} M_B \right) \right),
\]

where \( g_D = (g_1 + g_2)/2 \) and \( g_r = (g_1 - g_2)/2 \). In this form of vector coupling, \( g_\omega \Lambda \) and \( g_{\phi \Lambda} \) are already fixed by the vector coupling constants with nucleons, \( g_{\rho N} \) and
TABLE III: Experimental and theoretical binding energies and of stable nuclei. The results obtained from the SCL3 model are compared with those obtained from TM1, [39] TM2, [39] NL1, [51] NL3, [52] and SCL2 [25] models and with experimental data.

| Nucleus exp. | SCL3 | SCL2 | TM1 | TM2 | NL1 | NL3 |
|--------------|------|------|-----|-----|-----|-----|
| $^{12}$C      | 7.68 | 6.91 | 7.09 | 7.68 | -   | -   |
| $^{16}$O      | 7.98 | 8.11 | 8.06 | 7.92 | 7.95 | 8.05 |
| $^{28}$Si     | 8.45 | 7.85 | 8.02 | 8.47 | 8.25 | -   |
| $^{40}$Ca     | 8.55 | 8.64 | 8.57 | 8.62 | 8.48 | 8.56 | 8.55 |
| $^{48}$Ca     | 8.67 | 8.58 | 8.62 | 8.65 | 8.70 | 8.60 | 8.65 |
| $^{58}$Ni     | 8.73 | 8.44 | 8.54 | 8.64 | 8.70 | 8.68 |
| $^{90}$Zr     | 8.71 | 8.67 | 8.69 | 8.71 | 8.71 | 8.70 |
| $^{114}$Sn    | 8.52 | 8.51 | 8.51 | 8.53 | 8.52 | 8.51 |
| $^{196}$Pb    | 7.87 | 7.84 | 7.87 | 7.87 | 7.89 | -   |
| $^{208}$Pb    | 7.87 | 7.87 | 7.87 | 7.87 | 7.89 | 7.88 |

TABLE IV: Same as Table III but for charge rms radii.

| Nucleus exp. | SCL3 | SCL2 | TM1 | TM2 | NL1 | NL3 |
|--------------|------|------|-----|-----|-----|-----|
| $^{12}$C      | 2.46 | 2.47 | 2.43 | 2.39 | -   | -   |
| $^{16}$O      | 2.74 | 2.63 | 2.62 | 2.67 | 2.74 | 2.73 |
| $^{28}$Si     | 3.09 | 3.06 | 3.04 | 3.07 | 3.03 | -   |
| $^{40}$Ca     | 3.45 | 3.43 | 3.44 | 3.44 | 3.50 | 3.48 | 3.47 |
| $^{48}$Ca     | 3.45 | 3.46 | 3.46 | 3.45 | 3.50 | 3.44 | 3.47 |
| $^{58}$Ni     | 3.77 | 3.78 | 3.77 | 3.76 | -   | 3.73 | 3.74 |
| $^{90}$Zr     | 4.26 | 4.26 | 4.27 | 4.27 | 4.27 | 4.29 |
| $^{114}$Sn    | 4.63 | 4.61 | 4.62 | 4.61 | -   | 4.61 | 4.61 |
| $^{196}$Pb    | 5.48 | 5.48 | 5.47 | 5.47 | -   | 5.47 | -   |
| $^{208}$Pb    | 5.50 | 5.54 | 5.54 | 5.53 | 5.57 | 5.58 |

$g_{\rho N}, g_{\omega A}, g_{\sigma A}$ are given by

$g_{\rho N} = \frac{5}{6} g_{\omega N} - \frac{1}{2} g_{\rho N}, \quad g_{\phi A} = \frac{\sqrt{3}}{6} (g_{\omega N} + 3g_{\rho N}) \cdot (31)$

Scalar coupling constants, $g_{\sigma A}$ and $g_{\sigma A}$, are then reproduced to reproduce existing data of $S_A$ and the AA bond energy, $\Delta E_{\text{BA}},$ observed in the Nagara event, $^{6}$ He [19]. In Fig. 7, we show the calculated results of $S_A$ with the parameter set in Table II. Here, we evaluate the zero-point kinetic energy, $E_{\text{ZPE}},$ with a harmonic-oscillator wave function as $E_{\text{ZPE}} = \frac{1}{4} \cdot 41 A_{\text{core}}^{-1/3}$ MeV. The results of the $S_A$ for $p, d$ and $f$ levels are the weight-averaged ones of the spin-orbit partners.

The scalar potential of $\Lambda$ is given in the form of linear combination of the coupling constants and chiral condensates,

$U_{\Lambda}^\rho (\rho_B) = -[g_{\sigma A} \rho_{\sigma A} (\rho_B) + g_{\phi A} \rho_{\phi A} (\rho_B)] \cdot (32)$

In Fig. 8, we show the single particle potential for $\Lambda$, defined as the sum of the scalar and vector potentials, $U_{\Lambda} = U_{\Lambda}^\rho + U_{\Lambda}^{\mu}$, where the temporal component of the vector potential $U_{\Lambda}^{\mu}$ is defined in Eq. (29). This sum roughly corresponds to the Schrödinger equivalent potential for $\Lambda$, $U_{\Lambda}^{\text{SEP}} = U_{\Lambda}^\rho + (E/M_{\Lambda}) U_{\Lambda}^{\mu}$. As in the previous studies [15–17], $U_{\Lambda}$ amounts to be around $-30$ MeV at $\rho_0$. Calculated $S_A$ values are very similar as far as the scalar potentials at $\rho_0$ are the same.

The is splitting between $p_{1/2}^\Lambda$ and $p_{3/2}^\Lambda$ in $^{12}$C is calculated to be 900 keV in the present treatment. This result is larger than the observed small is splitting [53]. A small is splitting would be obtained by including tensor type coupling between meson and $\Lambda$ hyperon [54], since this coupling directly corresponds to is force when we translate the Dirac equation in RMF into a Schrödinger equivalent form. This coupling, however, reduces only the is splitting and does not change the average loca-
and charge neutrality condition, we calculate the energy density ($\varepsilon$) and pressure ($P$), and we solve the Tolman–Oppenheimer–Volkoff (TOV) equation,

$$\frac{dP}{dr} = -\frac{[P(r) + \varepsilon(r)][M(r) + 4\pi r^3 P(r)]}{r(r - 2M(r))}. \quad (35)$$

where $M(r)$ denotes the mass inside the radius $r$. Here, we neglect nuclear crust for simplicity.

In Fig. 10, we show the NS matter EOS. We compare the results of SCL3 RMF model only with nucleons (SCL3) and with $\Lambda$ hyperons (SCL3Λ). We also show the NS matter EOS in SCL2 [25], TM1 [39], NL1 [51], NL3 [52] and IOTSY [8], where $\Lambda$ hyperons are not taken into account except for IOTSY. The IOTSY RMF model is based on TM1 model and includes all octet hyperons ($\Lambda, \Sigma, \Xi$). We find that the NS matter EOS in SCL3 is softer than those in other RMF EOSs without hyperons. Especially, when we include $\Lambda, \Lambda$ hyperons appear at around $2\rho_0$ and NS matter EOS including $\Lambda$ hyperons becomes further softer.

In Fig. 11, we show the results of neutron star mass as a function of the central density with SCL3, SCL3Λ, SCL2, TM1, IOTSY, NL1 and NL3 models. While the EOS in SCL3 is much softer than other EOSs, calculated maximum NS mass in SCL3 (without hyperon) is $1.65M_\odot$, which exceeds the precisely observed NS mass, $1.44M_\odot$ [57]. When $\Lambda$ hyperons are included, the maximum mass is calculated to be $1.34M_\odot$ with SCL3Λ and underestimates the observed mass.

This underestimate is caused by the softer EOS, particularly in the high $\rho_n$ region. It is suggested that extra repulsion coming from three-baryon interactions or string-junction model [58] which are repulsive for all baryon universally are needed to surpass the known NS mass data in non-relativistic calculations [36, 37, 59]. In RMF models, the EOS of nuclear matter is stiff enough, and extra repulsion is not generally required to support $1.44M_\odot$. In SCL3 and SCL3Λ, however, the incompressibility is as small as the empirical value and the pressure at high density region is compatible with the estimate in heavy-ion collisions [48]. Thus, we encounter the same problem as in the non-relativistic calculations and have to consider additional repulsions which have a large effect at high densities. One of the candidates of this extra repulsion may come from the $\sigma\omega$ coupling, such as in the term of $\sigma^2\omega^2$ [28, 60]. The $\sigma\omega$ coupling results in reducing of the vector meson mass at high densities, and is found to give very stiff EOS when combined with the linear $\sigma$ model. Inclusion of such coupling may stiffen EOS in high $\rho_n$ region after re-fitting experimental data and it may solve the underestimation of the maximum mass of NS.

One of the problems in the present SCL3 is that the $c_\omega$ value ($c_\omega = 294.9$) is larger than those in TM1 ($c_\omega = 71.3075$) and SCL2 ($c_\omega = 200$) models. The potential term of $\frac{\omega^2}{2}$ strongly suppresses $\omega$ meson field especially at high $\rho_n$ [39]. The lower $c_\omega$ value an RMF model has, the higher neutron star maximum mass the EOS shows as seen in Figs. 10 and 11. When we reduce this parameter by changing $m_\omega$ value and re-fixing all the parameter in

C. Neutron star matter

In previous subsections, we have fixed all parameters for nucleon and $\Lambda$ by fitting the empirical saturation point of symmetric nuclear matter, and the binding energies of normal nuclei and $\Lambda$ hypernuclei. Now we apply the SCL3 RMF model to neutron star (NS) matter.

In NS matter, we require neutrinoless $\beta$–equilibrium and charge neutrality condition,

$$\mu_i = b_i \mu_B - q_i \mu_e, \quad (33)$$

$$\rho_e = \sum_B q_B \rho_{\rho_B}^{(B)} + \sum_I q_I \rho_{\rho_I}^{(I)} = 0. \quad (34)$$

The first equation relate the chemical potentials as, $\mu_n = \mu_p + \mu_e = \mu_\Lambda$ and $\mu_e = \mu_\mu$. Under this equilibrium condition, we calculate the energy density ($\varepsilon$) and

FIG. 9: (Color online) $g_{sA}$ and $g_{lA}$ suggested from experimental data. Red solid line and gray shaded area are determined by reproducing experimental $S_A$ and $\Delta B_{AA}$ in $^6\Lambda_\Lambda$He, respectively. Red point presents the pair of $g_{sA}$ and $g_{lA}$ adopted in latter calculation.
the way we discussed, calculated maximum mass of neutron star should be 1.8 $M_\odot$ on the condition of $m_\sigma = 725$MeV and $c_\omega = 75.66$. With this choice, however, we cannot reproduce the binding energies of Sn and Pb isotopes. Thus, in addition to chiral potential, the form and strength of vector meson potential is also important and should be investigated precisely.

**IV. SUMMARY AND DISCUSSION**

In this paper, we have proposed a chiral SU(3) symmetric RMF (SCL3 RMF) model, and examined its properties in nuclear matter, normal nuclei, $\Lambda$ hypernuclei and neutron star matter. We adopt a logarithmic chiral SU(3) potential, as the energy density as a function of $\sigma$ at $\rho_N = 0$, derived in the strong coupling limit of lattice QCD [26, 27]. The Kobayashi-Maskawa-’t Hooft (KMT) determinant interaction term [32, 33] is also introduced in order to take account of the $U_A(1)$ anomaly. Since the chiral symmetry relates the condensates and hadron masses, the number of parameters are reduced by introducing this symmetry. After fitting $\pi, K, f_\omega, (980)$ masses together with $f_\sigma$ and $f_\omega$, we have only one free parameter, $m_\sigma$, in the vacuum part. Under the assumptions that the nucleon mass are fully generated by the chiral condensate ($M_N = g_{\sigma N} f_\sigma$) and that the nucleon does not couple to $s$-mesons, we determine four parameters relevant to normal nuclei ($m_\sigma, g_{\sigma N}, g_{\omega N}, c_\omega$), by fitting the empirical and experimental data of symmetric nuclear matter saturation point, binding energies and size of normal nuclei. For $\Lambda$ hypernuclei, we assume that the vector couplings obey the lowest order SU$_f(3)$ symmetric relation [15, 42], and the remaining two parameters ($g_{\sigma \Lambda}, g_{\omega \Lambda}$) are determined by fitting the experimental data of the separation energies of single-$\Lambda$ hypernuclei and the $\Lambda\Lambda$ bond energy in the double-$\Lambda$ hypernucleus, $\Lambda\Lambda$He.

We find that the SCL3 model well describes the symmetric nuclear matter properties and the bulk properties of normal nuclei: The equation of state (EOS) is found to be softened by the $\sigma\zeta$ coupling generated by the KMT interaction, and the incompressibility of symmetric nuclear matter is found to be $K \approx 210$ MeV, which is consistent with the empirical value, $K = 210 \pm 30$ MeV [47]. The EOS around $\rho_0$ is in agreement with the results of variational calculations [34], and the pressure in the density region of $2\rho_0 \leq \rho_N \leq 5\rho_0$ is in agreement with the estimates from heavy-ion collision data [48]. The density dependence of the vector potential is close to that in the relativistic Brückner-Hartree-Fock (RBHF) [38] at low densities, $\rho_\sigma < 3\rho_0$. The binding energies of normal nuclei from C to Pb isotopes are reasonably well explained except for the jj—closed shell nuclei. Single- and double-$\Lambda$ hypernuclei are also well described. Separation energies of $\Lambda$ in single-$\Lambda$ hypernuclei, $S_\Lambda$, are mainly determined by the potential depth, and the $\Lambda\Lambda$ bond energy depends on the $\zeta\Lambda$ coupling $g_{\zeta \Lambda}$ more strongly than on the $\sigma\Lambda$ coupling $g_{\sigma \Lambda}$.

The calculated maximum neutron star mass in the present SCL3 RMF model underestimates the observed neutron star mass 1.44$M_\odot$. This underestimate would originate from the soft EOS of nuclear matter at high densities. The vector potential is suppressed more strongly than the RBHF results, and the EOS of symmetric matter is softer than the results in the variational calculation [34] and RBHF [34] at high densities. The suppression of the vector potential is caused by the self-interaction term of $\omega$, $-c_\omega \omega^3/4$, whose coefficient is large in SCL3 compared with previous RMF models with this term. Since this term is introduced phenomenologically to simulate the suppression of the vector potential in RBHF [39], it would be necessary to introduce other types of coupling, such as the scalar meson-vector meson coupling, $\sigma^2 \omega^2$ [28, 60]. The scalar-vector coupling acts to modify the in-medium...
vector meson mass [21, 61]. It may be interesting to invoke the results of strong coupling lattice QCD with finite coupling effects, where the plaquette contribution is found to generate the vector potential [62]. Another possibility is to introduce the repulsive three-baryon force, which is widely adopted in non-relativistic theories [34–37].

In this paper, we treat only Λ hyperon as hyperon degrees of freedom and Σ and Ξ hyperons are not included. In our previous work [63], by fitting atomic shift of Σ, which is widely adopted in non-relativistic theories [34–37], we have suggested that Σ hyperon feels rather strong repulsion in nuclear medium. Besides, if we keep the SU(3) relation in vector coupling, Eq. (30), Ξ hyperon also feels strong repulsion in hyperon-rich environment since $\eta_0\Xi$ has the largest value and $\phi$ meson generate repulsion between hyperons. These results for $\Xi^-$ will be reported elsewhere. Since other hyperons than Λ may further soften EOS at high densities, it is necessary to find the mechanism of re-stiffening at high densities, it is necessary to find the mechanism of re-stiffening in order to construct reliable and chiral SU(3) symmetric EOS including all these hyperons. Explicit role of pions [64] is another important subject to study in terms of relativistic nuclear many-body problems. The isospin potential [65] from pion exchange would improve the binding energies of $jj$-closed shell nuclei. In addition, tensor suppression may be play a critical role in EOS at higher densities.

Acknowledgments

We would like to thank Professor Avraham Gal for useful discussions. This work was supported in part by KAKENHI from MEXT and JSPS under the grant numbers, 17070002, 19540252 and 20-4326, Global COE Program "The Next Generation of Physics, Spun from Universality and Emergence", and the Yukawa International Program for Quark-hadron Sciences (YIPQS).

APPENDIX A: THE MASSES OF SCALAR AND PSEUDOSCALAR MESONS

We show the formulae of the scalar and pseudoscalar meson masses except for $\sigma$, $\zeta$, $\pi$ and $K$, which we have already shown in Sec. II. When we adopt expectation value of each meson and mass values of $\pi$, $K$ and $f_0$ as constraints, mass of $a_0$, $\eta$ and $\eta'$ can be represented as a function of parameter $m_\sigma$. These masses can be read as

$$m_{a_0}^2 = b' + \frac{2a'}{f_\pi} + 2d'f_\zeta$$  \hspace{1cm} (A1)

$$m_\eta^2 = b' + \frac{\sqrt{2a'}}{f_\pi f_\zeta} + \sqrt{2}d'f_\pi,$$  \hspace{1cm} (A2)

$$m_{\eta_0}^2 = b' - \frac{2a'}{f_\zeta^2} + 2d'f_\zeta,$$  \hspace{1cm} (A3)

$$m_{\eta_0}^2 = b' - \frac{a'}{f_\zeta},$$ \hspace{1cm} (A4)

$$\xi_{\eta_0} = 2d'f_\pi,$$ \hspace{1cm} (A5)

$$M_{\eta_0}^2 = \frac{(m_{\eta_0}^2 + m_{\eta_0}^2) - \sqrt{(m_{\eta_0}^2 - m_{\eta_0}^2)^2 + 4\xi_{\eta_0}^2}}{2},$$ \hspace{1cm} (A6)

$$M_{\eta'}^2 = \frac{(m_{\eta'}^2 + m_{\eta'}^2) + \sqrt{(m_{\eta'}^2 - m_{\eta'}^2)^2 + 4\xi_{\eta_0}^2}}{2},$$ \hspace{1cm} (A7)

where we use same parameters, such as $a'$, $b'$ and $d'$, defined in Sec. II. We have the mixing term of $\eta$ and $\eta_0$ mesons, thus one have to diagonalize their mass matrix to obtain vacuum masses. We tabulate calculated masses as functions of $m_\sigma$ in Table I.

[1] J. M. Lattimer and F. D. Swesty, Nucl. Phys. A535, 331 (1991).

[2] H. Shen, H. Toki, K. Oyamatsu and K. Sumiyoshi, Nucl. Phys. A637, 435 (1999) [arXiv:nucl-th/9805035].

[3] M. Lutz, Nucl. Phys. A642, 171 (1998) [arXiv:nucl-th/9807040].

[4] A. R. Bodmer, Phys. Rev. D 4, 1601 (1971).

[5] E. Witten, Phys. Rev. D 30, 272 (1984).

[6] A. Gal and C. B. Dover, Nucl. Phys. A585, 1C (1995).

[7] J. Schaffner and I. N. Mishustin, Phys. Rev. C 53, 1416 (1996) [arXiv:nucl-th/9506011].

[8] C. Ishizuka, A. Ohnishi, K. Tsukubaihara, K. Sumiyoshi and S. Yamada, J. Phys. G 35, 085201 (2008) [arXiv:0802.2318 [nucl-th]].

[9] N. K. Glendenning and J. Schaffner-Bielich, Phys. Rev. C 58, 1298 (1998) [arXiv:astro-ph/9808323].

[10] I. Bednarek and R. Manka, J. Phys. G 31, 1009 (2005) [arXiv:hep-ph/0506059].

[11] N. K. Glendenning and J. Schaffner-Bielich, Phys. Rev. Lett. 81, 4564 (1998) [arXiv:astro-ph/98102k84].

[12] A. Ohnishi, D. Jido, T. Sekihara and K. Tsukubaihara, Phys. Rev. C, to appear[arXiv:0810.3531 [nucl-th]].

[13] A. K. Holme, E. F. Staubo, L. P. Csernai, E. Osnes and D. Strottman, Phys. Rev. D 40, 3735 (1989).

[14] R. D. Pisarski and D. H. Rischke, Phys. Rev. Lett. 83, 37 (1999) [arXiv:nucl-th/9811104].

[15] C. B. Dover and A. Gal, Prog. Part. Nucl. Phys. 12, 171 (1985).

[16] D. J. Millener, C. B. Dover and A. Gal, Phys. Rev. C 38, 2700 (1988).

[17] H. Bando, T. Motoba and J. Zofka, Int. J. Mod. Phys. A 5 (1990) 4021.

[18] R. E. Chrien [BNL (PI+, K+) COLLABORATION COLLABORATION], Nucl. Phys. A478, 705C (1988); P. H. Pile et al., Phys. Rev. Lett. 66, 2585 (1991); T. Hasegawa et al., Phys. Rev. C 53, 1210 (1996); O. Hashimoto and H. Tamura, Prog. Part. Nucl. Phys. 57, 564 (2006).

[19] H. Takahashi et al., Phys. Rev. Lett. 87, 212502 (2001).
