Collective Choice Theory in Collaborative Computing

Walter D Eaves
February 1, 2008

Abstract

This paper presents some fundamental collective choice theory for information system designers, particularly those working in the field of computer–supported cooperative work. This paper is focused on a presentation of Arrow’s Possibility and Impossibility theorems which form the fundamental boundary on the efficacy of collective choice: voting and selection procedures. It restates the conditions that Arrow placed on collective choice functions in more rigorous second–order logic, which could be used as a set of test conditions for implementations, and a useful probabilistic result for analyzing votes on issue pairs. It also describes some simple collective choice functions. There is also some discussion of how enterprises should approach putting their resources under collective control: giving an outline of a superstructure of performative agents to carry out this function and what distributing processing technology would be needed.

1 Collective Choice in Information Systems

1.1 Naming Services

1. Windows NT

If one uses a system then, from time to time, one might receive an event message saying that “The Browser Has Forced an Election...” [MSK99a, MSK99b].

2. The Internet and the Domain Naming Service DNS

Internet connected systems could not function without the Domain Naming Service and this too relies upon elections: individual system administrators choose when their name–server is to authoritative or not [Wel99, Ricy] and which name–servers it will rely upon.

The difference between the two systems is that Windows NT is designed to manage the naming of relatively small domains and can use a direct election amongst all its naming components, the browsers. This is represented in figure 1, where the master browser is elected by the itself and the other browsers.

The browsers apply the same fitness criteria in choosing their master. There can be no conflict in policy.
Figure 1: Browser programs choosing a master browser

The Internet’s DNS has to rely upon a loosely co-ordinated database of name servers. This is represented in figure 1. Here the Administrator of each DNS chooses which other DNS it will use to resolve names. In this example, for all names other than their own, the Superior Administrator is chosen as authoritative.

The Administrators need not apply the same fitness criteria in choosing their superior DNS. There may be conflicts in policy.

They can therefore attempt malevolent actions collectively if they so wish, for example:

Consider an electronic commerce web site. The user’s web browser makes a secure connection to the site, providing a protected channel. If the DNS entry for the server’s address was replaced by one indicating an attacker’s address, the browser will connect to the malicious site, possibly without the user’s knowledge. In this scenario, the DNS spoofer could monitor the traffic over the “secure” connection, since the secure connection would actually be to the spoofer, and forward the transaction data to the real website or process the traffic itself [Wel99].

Without human intervention, computer programs have wholly predictable behaviour and cannot possess ulterior motives: people can. (The problems that can arise from badly managed networks are described to a much greater extent in [EFL+97].)
Figure 2: DNS Administrators choosing a superior DNS
But even if computer programs have predictable behaviour they may not apply the same fitness criteria in making choices. This may lead to conflicts in policy.

1.2 Groups choosing policies

Most people will have come across moderated newsgroups and mailing lists. Potentially these services could be made self-managing and would be simple examples of a computer-supported working environments.

1. Joining the Group

A policy decision is needed to determine whether or not an individual should be allowed to join a particular group and take the rights and privileges enjoyed by its members. A collective choice, probably by a membership committee, is made based upon the applicant's credentials.

The procedure is usually carried out using a set of recommendations. The individual requesting the rights fills in a form stating his credentials. People are assigned to check the applicant’s trustworthiness, qualifications and so forth. If the membership committee is satisfied, someone is instructed to assign the applicant to the group.

2. Expulsion from the Group

Should the membership committee decide to expel an individual from the group, that, too, would be a collective policy decision.

A situation that could clearly arise is for a number of individuals to infiltrate a group, subvert it by having themselves elected to the membership committee and then expelling all the members of the group who are not sympathetic to the infiltrators. Of course, those individuals might also do this legitimately, if the selection of the membership committee reflected the views of the current membership. What legitimates actions is a wider consensus.

1.3 Lattices and Access Control

Organizing the membership of groups is a sub-process needed for the formation of lattices of membership classes for an access control system, first put forward by Denning in [Den76]. The only provably safe access control systems are those that are based on Mandatory Access Control, MAC, schemes, also as described by Denning [Den82].

1.3.1 Operating Systems

Discretionary Access Control, DAC, schemes are used in most multi-user operating systems such as VAX-VMS [Cor84] and Unix [Cur90] and resource-sharing operating systems such as Windows NT [Jum98]. These are not as discretionary as one might think:

- Individual users are allocated to groups
- Privileged User(s): the “super-user” or “Administrator” set group memberships.
It could be described as a dictatorial Discretionary Access Control scheme. The only latitude that individuals have is to be able to grant or deny access rights to members of their own group or to everyone. The privileged user can undo any access control operations performed by any individual.

To use some better terminology: subjects are entities who may possess access rights and objects are those entities to which subjects have rights to use. An operating system that uses a MAC scheme is represented in figure 3 and one that uses a DAC scheme is represented in figure 4.

1. MAC Scheme

This is a simple scheme. The administrator classifies all the subjects and the objects and it classifies some subjects above other subjects so that higher subjects can access everything that lower subjects can.

2. DAC Scheme
This is more sophisticated. The administrator classifies all the subjects and can denote that they belong to certain a Group. Every subject belongs to the group of Everyone.

Individual subjects own some objects and can choose to grant access to

- Either: their groups (or groups)
- Or: to Everyone

Groups do not own anything and neither can the group Everyone. In figure 4, the object $A$ is owned by one of the subjects and has allowed access to the group its owner belongs to and to Everyone. The object $B$ can be accessed by the group, but not by Everyone.

There is degree of autonomy granted to the subjects in that they may classify objects to be accessible to the members of the groups they belong to, but they may not choose which group, or groups, they belong to. Neither may they change ownership of an object they own.

1.3.2 Database Management Systems DBMS

Some DBMSs have more flexible DACs which allow some individuals to be more privileged than others by granting them the right to grant rights. This feature is available in some DBMSs that support the Structured Query Language, SQL, which was based upon System–R, which is described by Denning in \[\text{Den89}\].

In effect, this is the same as the DAC scheme for operating systems, see figure 4, but the owner can grant access to subjects other than those in its group and it can grant to others the right to grant access, but not to those to whom access has been explicitly denied by the owner.

1.4 General Resource Management Systems

Information processing systems can be thought of as general resource management systems and the Open Distributed Processing ODP standards, in particular the prescriptive model, \[\text{ISO97}\], describe how an enterprise modelling language could be used to state the relationships between resource owners and users as behavioural contracts given in terms of a set of permissions, prohibitions and obligations. This can be seen as a generalization of the MAC and DAC schemes, but the ODP standards are only reference models and each information system should contain some component that embodies its enterprise model. This is a wholly new superstructure to an information processing system: there are some simple class relationships describing the ODP model set out in appendix A, these rather vaguely state the information model for the superstructure. Some mechanisms that could be used for applying policy, implementing the administrators, has already been proposed \[\text{Eav99b}\].

Recently, the Java programming environment has provided a powerful language for expressing permissions \[\text{SUN98}\]. However, it assumes the existence of some agent which would negotiate the contracts between resource owner and user. There have been only a few efforts made to develop arbitrator agents which could generate such contracts.

\[1\text{A POSIX} \text{[Lew91]} \text{requirement, Unix typically does allow ownerships to be changed.}\]
Figure 4: Discretionary Access Control Scheme
There are some system proposals which attempt to apply abstract behavioural rules in terms of concrete permissions: in papers by Minsky [Min89, Min95]. Minsky’s treatment is for information processing systems in general, but a paper by Rabitti et al. [RBKW91] describes authorization generation mechanisms which support a lattice model of authorization policy for an object–oriented database. The innovation of the system is that it generates authorization policy as it operates. Authorization is viewed as having three dimensions:

**Expression** Authorizations specified by users, which are known as *explicit* and those that are derived by the system as known as *implicit*.

**Direction** An authorization can be *positive*, stating what may be done, or *negative* stating what may not be done.

**Strength** An authorization may be *strong*, in which case it may not be over-ridden, or *weak*, in which it can.

This model has been extended [BW94] and a recent contribution by Castano [Cas97] introduces metrics that can be used to generate concrete permissions from more abstract specifications, including:

- Operation compatibility
- Individuality similarity co-efficient
- Authorization compatibility
- Semantic correspondence
- Clustering of Individuals

Although Bertino *et al.* attempt to produce mechanical means of generating authorization policy no-one would seriously expect a system to be driven wholly by mechanical recommendation, it would require choices to be made by people and, if that is the case, then a suitable collective choice procedure must be found.

### 1.5 Summary

Information processing systems are not mechanical systems. They represent the interests of people and the agents that comprise an information processing system will require policies that represent people’s interests which will be used in formulating behavioural contracts between agents that own resources and the agents that use them.

### 2 Issues in Collective Choice Theory

The principal difficulty in collective choice theory is that if a group of people have to choose between more than two issues, there is no choice procedure they can adopt that is *not* open to abuse. By abuse, it is meant:

- Denial of service: some agent can exercise a veto on any policy proposed.
- Enforced service: some agent can force a policy upon others.

An abuse takes the form of insincere behaviour: an agent acts not to fulfil his own interests, but to prevent others from fulfilling theirs.

This simple example, known as the Voting Paradox might help:

**Scenario 2.1 (Denial of Service by Policy Cycle).** Three agents, \(x\), \(y\) and \(z\), have to choose between three services: \(A\), \(B\) and \(C\). \(x\) and \(y\) rank sincerely, but \(z\) ranks the policies to prevent \(x\) and \(y\) from reaching a compromise, i.e. policy \(A\), see table 1.

| Agent | Ranking |
|-------|---------|
| \(x\) | \(A > B > C\) |
| \(y\) | \(C > A > B\) |
| \(z\) | \(B > C > A\) |

Table 1: Policy cycle used to deny service

Just to clarify terminology: an election is an expression of collective choice and the policy chosen by an election is the outcome of what statisticians might call a trial. In a trial, there are a number of choices available to each voter, or individual, taking part. A policy is usually chosen with regard to an issue; the proposal that a policy should be followed regarding an issue is called a motion.

### 2.1 Two Policy Issues

This could also be described as a two outcome trials. These arise when there are only two policies which can result: the choice is to accept a policy or not.

1. **Number of Choices is always three**
   Although there are two policies, there are three choices: one can vote *For* or *Against*. One may also be given the explicit right to *Abstain*, and, by doing so, state that one cannot vote for or against. One may also choose *Not to Vote*.

2. **Abstentions and Not Voting**
   Usually in referenda, there is no option to abstain and those who do not vote are considered to have abstained. This assumption is legitimate if one is sure that all individuals who are entitled to vote have been informed that they may do so and have made a decision not to. In most business processes, this would not be the case.

   In what follows, it is assumed that all abstentions are explicitly made and that individuals who do not vote have excluded themselves.

3. **Choice Function**
   Simple majority rule is the usual method for resolving two policy elections, but it by no means the only one. Two policy collective choice is dealt with in some detail in appendix B.
What is required from a choice function is that it is not open to abuse and it is decisive. Two policy issues can not be abused because it can be shown to be the case that the majority have chosen the policy. The only difficulty is resolving ties.

2.2 Three policy issues

A three–outcome trial, for example, one of $A, B$ or $C$ must be chosen. The voting procedures described here are explained at greater length in [Saa94] and there is some documentation on voting methods at [VOT98]. What follows illustrates some of the problems that arise from using them.

1. Number of Choices can vary

   (a) Four Choices
      One can organize the election so that are four choices: any of $A, B$ or $C$ and to abstain.
   (b) Seven (or Eleven) Choices
      One could also allow voters to express a choice between their first two preferences and to abstain. So a vote might be: $(A, B)$ which implies that $A > B > C$.
      One can also allow voters to state they are indifferent between their first two choices, but prefer them to the third; an example of this kind of vote is: $(A = B) > C$.
   (c) Thirteen (or Seven) Choices
      Also one can allow voters to express their choices as a ranking over all three policies, in two ways:
      - Strong ordering: no statements of indifference allowed.
      - Weak ordering: statements of indifference are allowed.
      The former allows seven choices of rankings, the latter thirteen.

2. Choice Functions

   (a) Simple Majority Rule
      This could only be used when the voters are presented with four choices and clearly would not work:
      - Three voters: a tie can result from a policy cycle and can be used by one voter to deny service, see table 1.
      - Seven voters: if three vote $A$, two $B$ and two $C$, then, even though a majority did not want $A$, $A$ is chosen.
   (b) Single Transferable Vote
      This could be used when the voters are presented with seven or eleven choices. It suffers from the same problem as the next procedure.
   (c) Hare Voting System and Borda Preferendum
      These two can be used if one presents to the voters thirteen or seven choices; they, and the Single Transferable Vote procedure, all suffer from the same fault, [Dor73, Fis73], which is that voting is affected
by irrelevant alternatives. This is best illustrated by using the results of a Borda Preferendum which has each voter rank their alternatives in order, see table 2.

If the voters $i$, $j$ and $k$ are asked to rank the four policies $w$, $x$, $y$ and $z$, then the order is $w > x > y > z$, but if asked to choose between $w$, $y$ and $z$ then $(w = y) > z$, but it was clear that $w$ was preferred over $y$. The anomaly being that an irrelevant policy, $x$, serves to differentiate between relevant ones.

(d) Condorcet Procedure

This procedure is often employed in committees but also suffers from irrelevant alternatives affecting the selection of a final choice. (There are some good examples of how a Condorcet procedure can be abused in [Saa94].) It is simply a series of two policy elections: $(A, B)$, $(A, C)$ and $(B, C)$. If any policy beats the other two, then it is chosen. If there is a policy cycle then there is no Condorcet winner. It may also allow an irrelevant alternative to beat a potential Condorcet winner.

It will be shown that there is no satisfactory choice function for more than two policy issues. The next note shows that the number of different orderings for a given number of issues increases dramatically.

### 2.3 $n$ policy issues

The total number of different weak orderings for $n$ policies can be calculated as follows:

1. Generate all the partitions [Ski90, p. 56] of $n$.

2. Calculate the number of permutations for each partition, call this $N$(partitions).

3. For each partition find the number of number of ways in which the policies could be allocated to the elements of the partition, $N$(policies).

4. Multiply $N$(partitions) by $N$(policies) for each partition and sum them together.

$$\sum_{\text{partitions}} N(\text{policies}) \times N(\text{partitions}) \quad (1)$$

A Mathematica [Ins99] package is available [Eav99a] that performs the calculation. Table 3 lists the total number of different preference orders for up to 6 policies and clearly shows how large the search space becomes.
2.4 Sincere and Sophisticated Voting

Sophisticated voting utilizes some strategy whereby a voter does not vote for their first choice to ensure that their least–preferred policy is not chosen. For example, an electorate of seven votes sincerely for three policies $x, y$ and $z$ thus: $4x, 3y$ and $2z$, then $x$ would be chosen. However, the $z$ voters may prefer $y$ to $x$ so their sophisticated vote is for $y$.

An interesting example of a sophisticated vote is global abstention. If a motion is formulated which requires a choice between $A$ or $B$, but all voters prefer $C$ which is not proposed, a sophisticated response is a global abstention.

Unfortunately, sophisticated voters enjoy an advantage over sincere voters, but, to do so, they must formulate their own voting policy, which usually requires that they have some information as to the relative strengths of the different coalitions within an electorate and their choice of voting policy would, presumably, be decided by a sincere vote amongst them. One of the attractions of presenting an electorate with a complex agenda—of more than two issues—is that they are less able to formulate strategies amongst themselves, so that complex agendas should elicit more sincere voting, but the likelihood of a policy cycle arising is greater, as is made clear in a later section, §5 and by equation (2). Sincere voting would allow voters’ underlying values to more precisely determined, which one would hope, would in the long–run be a more stable basis for decision–making.

3 Collective Choice Mathematical Model

This is more rigorous presentation of collective choice theory. This following section introduces the notation that will be used to formalize the conditions that are placed on collective choice functions.

3.1 Some Notation

Notation 3.1 (Relations, Preferences and Their Ordering: $\succ$). is a preference ordering over a set of objects in a finite set $X = \{x_1, \ldots, x_n\}$ constructed thus:

$R$ is an instance of a class of binary relations between any two objects. To state that $x_1$ is related to $x_2$ in some way, one would write: $x_1 R x_2$. The particular relation might be any of the following $\succ, \succeq, \preceq, \equiv$. At this stage, $R$ is taken to be transitive and connected.
is a statement of an individual’s preference order or preferences over the elements of $X$. It is a tuple, i.e. a vector, with exactly $\#X$ elements, with each element being of the form $x_i R x_j$, where $R$ is instantiated to one of the values that the class might take. The ordering is assumed to be consistent for whatever qualities $R$ possesses. There must be at least one statement of preference for each element of $X$, even if that statement is one of indifference. It is assumed that such a preference ordering is consistent with the qualities of $R$. The power set of $\succ$ is $X \times X$.

**Notation 3.2 (Policies and Voters).** Some simple set definitions are needed.

$I$ is the set of voters or individuals $I = \{1, \ldots, i, \ldots, n\}$.

$X$ a non-empty set is the universal set of social alternatives, or policies, at least one of which must be chosen by the voters.

$\mathcal{X}$ is a subset of the power set $\mathcal{P}(X)$ of $X$; it is a non-empty set of non-empty subsets of $X$ and describes the potential feasible policy sets of $X$.

$Y$ is an element of $\mathcal{X}$. It is the set of policies that are presented to an electorate for them to vote on: the proposal set.

$\vec{D}$ is a preference profile of all voters, it will be called a vote, but will contain more than just the voters’ preferences on the elements of the proposal set. It contains the preference orders of all the individuals in the society for all alternatives in $X$. For the $n$ individuals, if individual $i$ is presumed to have a preference order $\succ_i$, $\vec{D}$ can be written as the $n$-tuple $(\succ_1, \succ_2, \ldots, \succ_n)$ of preference orders on $X$.

$\mathcal{P}(\vec{D})$ is the power set of all votes, feasible and infeasible. For a given set of policies and a given set of individuals only a subset of these votes will occur.

$(Y, \vec{D})$ is an ordered pair called the situation. It is the feasible set of policies, $Y$, presented to the electorate, and a vote $\vec{D}$.

The important word is feasible. Only some votes will be feasible given the preferences held by voters; therefore only some policy sets will be feasible.

**Definition 3.1 (Sincere and Sophisticated).** The following function definitions clarify how voters make up their minds and form their preference orders. They are, therefore, purely notional and one or the other is performed by each individual, $i$. How a preference order is formed is dependent on whether the individual votes sincerely or is sophisticated.

If an individual, or population, is voting sincerely, then:

- $\text{sincere}: I \times X \mapsto X \times X$ e.g. $\succ_i = \text{sincere}(i, X)$
- $\text{sincere}: X \mapsto \mathcal{P}(\vec{D})$ e.g. $\vec{D} = \text{sincere}(X)$
If an individual is a sophisticated voter, then a new set of histories is needed: \( \mathcal{D} \)—and its power set \( \mathcal{P}(\mathcal{D}) \)—which is the set of all votes that have taken place.

\[
\text{sophisticated: } I \times X \times \mathcal{P}(\mathcal{D}) \mapsto X \times X \text{ e.g. } \succ_i = \text{sophisticated}(i, X, \mathcal{D})
\]

\[
\text{sophisticated: } X \mapsto \mathcal{P}(\mathcal{D}) \quad \mathcal{D} = \text{sophisticated}(X, \mathcal{D})
\]

**Definition 3.2 (Promotion and Demotion).** When stating conditions it is useful to construct votes from other votes. These may be elements of \( \mathcal{P}(\mathcal{D}) \) that are infeasible.

These functions promote and demote a policy within a vote.

\[
\text{promote: } \mathcal{P}(\mathcal{D}) \times X \mapsto \mathcal{P}(\mathcal{D}) \text{ e.g. } \mathcal{D}' = \text{promote}(x, \mathcal{D})
\]

And similarly,

\[
\text{demote: } \mathcal{P}(\mathcal{D}) \times X \mapsto \mathcal{P}(\mathcal{D}) \text{ e.g. } \mathcal{D}' = \text{demote}(x, \mathcal{D})
\]

demote() would be implemented as follows:

1. Every preference not involving \( x \) is unchanged:

   \[
   \forall x', y' \in X[(x' \neq x, y' \neq x, x' \succ_i y' \in \succ_i, x' \succ_i' y' \in \succ_i') \\
   \rightarrow (x' \succ_i y' \leftrightarrow x' \succ_i' y')]
   \]

2. Everything that involves \( x \) is unchanged if \( x \) if preferred over something else; or is changed so that \( x \) is now preferred over the other policy

   \[
   \forall y' \in X[x > y' \in \succ_i \rightarrow x > y' \in \succ_i']
   \]

   or

   \[
   \forall y' \in X[x = y' \in \succ_i \rightarrow x > y' \in \succ_i']
   \]

Along the same lines, two other variants of promote and demote can be defined, which promote or demote for a particular voter on a particular policy and resolve any conflicts.

\[
\mathcal{D}' = \text{promote}(i, x, \mathcal{D}) \text{ and } \mathcal{D}' = \text{demote}(i, x, \mathcal{D})
\]

**Definition 3.3 (Collective Choice Function).** Is a function that maps each situation to a subset of the feasible subsets for that situation. The collective choice function \( F \) yields the choice set of the proposal set.

\[
F: \mathcal{P}(\mathcal{D}) \mapsto \mathcal{X} \quad F(Y, \mathcal{D}) \subseteq Y
\]

Typically the choice set will contain only one policy, the one that is preferred over all others. The chosen policy can then be removed from \( Y \) and the next policy found. In the event of a tie—if ties are tolerated—the choice set will contain the tied policies.
Remark 3.1 (Quorums). A quorum is usually taken to be the minimum number of voters that can demand that a policy be a legitimate choice. However, it may be that case that there is a quorum, but all votes, bar one, are abstentions and that the choice of that single individual becomes mandatory.

For the time being, this anomaly should be noted, and there will be references in the text to the validity of a policy decision.

3.2 Conditions on a Collective Choice Function

These conditions prescribe the behaviour of a collective choice function\(^2\). These are derived from Arrow’s work\(^{[Arr63]}\) and have been the subject of considerable debate. This rendering is original and, it is hoped, is more explicit, self-contained and rigorous than that given in Arrow’s work. Each condition is expressed as a deduction rule in second-order logic: if the premises are fulfilled then the conclusion is required i.e. it is expected behaviour. The conditions therefore constitute tests for an implementation of a collective choice function.

A more succinct rendering can be found in a paper by Batteau et al.\(^{[PB81]}\). It requires some familiarity with Arrow’s conditions and the theory of games\(^3\), but has the advantage of relating Arrow’s conditions to work in games theory and also to requirements on the behaviour of collective choice functions. This latter task has been carried out very successfully by Fishburn\(^{[Fis74]}\), but only for two issue collective choice functions. The paper by Batteau et al. only addresses collective choice functions that use strong orderings.

There are five conditions in all. There is a brief description of the meaning of each.

**Condition 3.1 (Admissible Orderings).** This is a specification that the function need only operate on what are called admissible orderings, an individual’s ordering is admissible if it alone satisfies the collective choice function, \(\text{viz.}\)

\[
\forall Y \in \mathcal{X} \\
\forall i \exists \vec{D} [\vec{D} = (\succ_i), \emptyset \neq F(Y, \vec{D}) \subset Y] \\
\exists \vec{D}' [\vec{D}' = (\succ_1, \ldots, \succ_i, \ldots, \succ_n), \emptyset \neq F(Y, \vec{D}) \subset Y]
\]

By specifying that individual voters must present orderings that are proper subsets of \(Y\), this eliminates orderings that are completely indifferent or are cyclical, e.g. \(X = \{x, y, z\}, \succ_i = (x > y, y > z, z > x)\)—so it is a condition on the vote set as well as on the collective choice function. In practice, it would be best to ensure that the orderings in \(\vec{D}\) are well-formed.

The choice set \(F()\) cannot be empty and it must be a proper subset of \(Y\). If it is neither of these then there is either global indifference or a policy cycle.

**Condition 3.2 (Monotonicity).** or “positive association of social and individual values”\(^{[Arr63]}\) p. 25: put simply if the individuals want something and choose it for their society; if, in a later vote, more individuals choose it, then,

\(^2\)The more common term is social choice function, but this due to its origin in social welfare economics.

\(^3\)See for example, \(^{[BOS82]}\).
**ceterus paribus**, it will be chosen for society again, or, more formally:

\[
\forall Y \in X \\
\forall \vec{D} \exists S[F(Y, \vec{D}) = S] \\
\forall x \in S \exists \vec{D}'[\vec{D}' = \text{promote}(x, \vec{D})] \\
\exists S'[S' = F(Y, \vec{D}'), x \in S']
\]

**Condition 3.3 (Independence).** or “independence of irrelevant alternatives” [Arr63, p. 27] requires that the collective choice function return a choice set regardless of any individual’s preferences for policies that are not explicitly part of the proposal set. This means that individuals may take on or discard values, or they may change their values regarding other matters, but these changes should not effect those values that have not changed. Formally, this can be expressed thus:

\[
\forall Y \in X \\
\exists X, X'[Y \subseteq X, Y \subseteq X', X \neq X'] \\
\forall \vec{D}, \vec{D}'[\vec{D} = \text{sincere}(X), \vec{D}' = \text{sincere}(X'), F(Y, \vec{D}) = F(Y, \vec{D}')] \\
\exists \vec{D}'[\vec{D}' = \text{demote}(x, \vec{D})] \\
\exists S'[S' = F(Y, \vec{D}'), x \in S']
\]

This condition on the implementation of the collective choice function is probably unimportant in practice; normally, the input to the collective choice function is \(\vec{D}\) which only contains the preferences on the contents of \(Y\), but as can be deduced from the discussion of the effect of an irrelevant alternative in the Borda preferendum, see table 2, these can affect a preference order.

The following two conditions are more contentious. They are different from the other conditions in that they need not be applicable to all issues and there are two types of tests one can apply.

**Condition 3.4 (Non–imposition).** There is no bias in the collective choice function that causes it, on some issues, to yield a choice set that is insensitive to voters’ preferences. The first test is a unilateral test, viz.

\[
\exists Y \in X \\
\forall \vec{D} \exists S[F(Y, \vec{D})] \\
\forall x \in S \exists \vec{D}'[\vec{D}' = \text{promote}(x, \vec{D})] \\
\exists S'[S' = F(Y, \vec{D}'), x \in S']
\]

That is, it should be possible on a particular set of issues to construct a vote that does not return a particular policy for all votes that select that policy. The second test is used in the event of a tie between some policies to ensure that the collective choice function does not prefer one policy over the other.

\[
\exists Y \in X[\#Y \geq 3] \\
\exists S[\#S \geq 1, F(Y, \vec{D}) = S] \\
\forall y, z \in Y \setminus S[y \neq z] \\
\forall w \exists Y'[-(w \in Y), Y' = Y \cup \{w\}] \\
\exists \vec{D}'[\vec{D}' = \text{sincere}(Y')] \\
\exists S'[S' = F(Y', \vec{D}'), (y \in S' \land \neg(z \in S')) \lor (z \in S' \land \neg(y \in S'))]
\]
It might not be immediately clear from this formulation but this is an exact statement of the irrelevant alternative anomaly observed in the Borda preferendum, see table 2.

**Condition 3.5 (Non–dictatorial).** There is no one individual whose choice on some issues is always returned by the collective choice function, a dictator, nor is there any one individual who can reject some policies, a vetoer. (Unfortunately, there is some contention about the use of the term “one individual”, see the discussion following.)

1. **Unilateral Tests**

These test whether it is possible to overcome a dictator’s choice or a vetoer’s rejection. The dictator or vetoer is placed in position 1 for convenience.

(a) Dictator

$$\exists Y \in \mathcal{X}$$

$$\exists S \exists \vec{D} [\vec{D} = (\succ_1), \ S = F(Y, \vec{D})]$$

$$\forall x \in S \exists \vec{D}' [\forall \succ_i [i \neq 1, \succ_i = \text{demote}(x, \succ_i)], \ \vec{D}' = (\succ_1, \ldots, \succ_n)]$$

$$\not\exists S'[S' = F(Y, \vec{D}'), S' \cap S \neq \emptyset]$$

(b) Vetoer

$$\exists Y \in \mathcal{X}$$

$$\exists \vec{D} \exists S' [\vec{D} = (\succ_1), \ S' = Y \setminus F(Y, \vec{D})]$$

$$\forall x \in S' \exists \vec{D}' [\forall \succ_i [i \neq 1, \succ_i = \text{promote}(x, \succ_i)], \ \vec{D}' = (\succ_1, \ldots, \succ_n)]$$

$$\not\exists S''[S'' = Y \setminus F(Y, \vec{D}'), S'' \cap S' \neq \emptyset]$$

2. **Tie–Breaking Tests**

Subtler tests are those that are applied in the event of a tie.

(a) Dictator

The dictator is preferred in some way. In that, if the dictator changes allegiance, policy changes, but if anyone else it does not.

$$\exists Y \in \mathcal{X} [\# Y \geq 3]$$

$$\exists S [S \subset Y, \# S \geq 1]$$

$$\forall y [y \in Y \setminus S]$$

$$\forall i \exists \vec{D}'_i [\vec{D}'_i = \text{promote}(i, y, \vec{D})]$$

$$\forall i \exists S_i [F(Y, \vec{D}_i) = S_i]$$

$$\not\exists j \forall i [i \neq j, S_i \neq S_j]$$

---

4Vetoer is a noun constructed solely for the purposes of this exposition.
This definition states that “no one individual” can dictate a vote, which would seem to suggest that individuals can change their minds, but must do so en masse. Arrow requires that a deciding set of voters must change their preferences. How many need to be in that set can only be determined by analyzing a vote. It may appear that simple majority rule does not appear to meet this criteria, since, in a close result, any voter can invert the result, but under simple majority rule on two issues, this can be dismissed, because every voter has exactly the same capability, therefore the simple majority becomes the decisive set.

So, as it stands, this condition is still not accurately expressed, it should state that no subset of voters that is not a deciding set can change the outcome. The problem is that the deciding set cannot be known until the votes have been cast and counted.

4 Possibility and Impossibility Theorems

Arrow, who originally addressed the problem of the distribution of social welfare, developed these theorems as general statements about a class of functions which seek to combine hierarchies of preference relations. Such functions would be of great use in any field where a joint policy must be formulated. Clearly, that includes distributed computing and the “globalization” of local access rules to databases to form lattices of information flow, §1.4.

There are two results:

1. Possibility theorem for two-policy elections

Such a collective choice function does exist for elections which have only a choice between two policies.

2. Impossibility theorem for elections having more than two policies

There is, in general, no such collective choice function for elections having more than two policies.

The idea behind the proof of the possibility theorem has already been given in the discussion of deciding sets, condition 3.5, but the same idea is used in the proof of the impossibility theorem.

---

5Also called a preventing set in [PB81] or a winning set as defined in [Isb60]. Some useful rules on winning sets are defined in the latter paper.
4.1 Conflict Resolution Mechanisms

Arrow’s proof for the impossibility theorem consists of analyzing how a collective choice function can choose one preference over another. The notation is as used in §3.3.

**Definition 4.1 (Unanimous Choice).** If the voters unanimously agree that one policy is preferred over all others then that policy is chosen.

\[ \forall Y \in \mathcal{X} \\forall x \in Y \exists \vec{D} \left[ \vec{D} = (\succ_i), x \in F(Y, \vec{D}) \right] \]

Note that with this formulation it is possible to be indifferent to \( x \), but one cannot oppose it. For example, if some voter has \( \succ = (x = y, y > z, x > z) \) then \( \{x, y\} = F(\{x, y, z\}, (\succ)) \).

**Definition 4.2 (Biased Choice).** If pairs of voters contradict one another over a policy, one policy is chosen over the other.

\[ \forall Y \in \mathcal{X} \\forall x', y' \in Y \exists i, j \in I \left[ i \neq j, x' > y' \in \succ_i, y' > x' \in \succ_j \right] \]

Note that this rule can only be applied pair-wise, it cannot be applied to the population as a whole.

**Definition 4.3 (Unresolved Choice).** If pairs of voters contradict one another over a policy, neither policy is chosen.

\[ \forall Y \in \mathcal{X} \\forall x', y' \in Y \exists i, j \in I \left[ i \neq j, x' > y' \in \succ_i, y' > x' \in \succ_j \right] \]

This, too, may only be applied pair-wise.

4.2 Inadequacy of Conflict Resolution Mechanisms

Unanimity, with abstentions, does not resolve any conflicts. If one attempts to do so using one of the other two choice methods, one or more of the conditions will be breached.

1. Biased Choice

   With two voters there is no majority decision, so the collective choice function must prefer:

   (a) Either a policy

   \footnote{This is, in essence the Pareto principle of social welfare. It can be stated as: “social welfare is increased by a change that makes at least one individual better off, without making anybody else worse off.” Clearly, if one abstains then one feels one is not going to be worse off if \( x \) is chosen. If one opposes \( x \) then one would be worse off if \( x \) were chosen.}
(b) Or a particular voter's choice of policy
(c) Or randomly choose one policy

The first two lead to an imposed policy or indicate a dictatorship, respectively; the latter has been suggested \cite{Zec68}, but it is rather arbitrary.

2. Unresolved Choice

This method allows anyone to act as a vetoer.

One might think that one can improve the biased choice method so that it decides in favour of whichever policy has a simple majority over the other. Unfortunately, if one admits a third voter to make the biased choice decisive, then one also allows that third voter to present a third policy choice; in which case, one is attempting to choose between three policies, which is the problem one is trying to solve.

If, in an attempt to overcome this, one requires that there always be more voters than issues, then on some issues at least one voter will be a dictator, or vetoer\footnote{This minority power is often said to be the cause of the instability of proportional representation parliaments.}. The dictator, or vetoer, is acting as a “Kingmaker”. It is possible to stand this dilemma on its head (or feet) and use it as the basis for a collective choice function as in \cite{How89}.

4.3 Are the Conditions Reasonable?

Hopefully, it should now be clear that it is not possible to construct a collective choice function that satisfies all the conditions given above simultaneously. That said, one can argue that the requirements on the collective choice function’s behaviour are too demanding.

1. Decisiveness

Implicit in the definition of the collective choice function is that it is decisive.

2. The Non–imposition and Non–dictatorial Conditions

These conditions come in two forms. The first form is unilateral and quite acceptable, although it may even be desirable that one particular voter has an absolute veto over a policy. The other form of the condition is only invoked in the event of a tie, where the requirement is that no policy or voter be preferred over another. This, it has been seen, is the pivotal difference between simple majority rule for a two policy vote and a three (or more) policy vote. In the two policy vote in the event of there only being one vote separating those for and those against, every voter is equally decisive and will be supported by the majority. In a close three (or more) policy vote, some voters can choose the minority position to force a tie.

The issue is, again, the decisiveness of the collective choice function and would so close a result be acceptable.
3. Independence from Irrelevant Alternatives

The preferendum, see table 2, and Condorcet pairings are both examples of collective choice functions that are non–dictatorial but are not free from the effects of irrelevant alternatives. This is very unfortunate, since an irrelevant alternative is any policy that is ranked lower than the collective winner, but is ranked higher than the collective winner by some voters. This would happen when the voters are assessing the policies with different underlying values which are more abstract.

If one looks at the rankings made in table 2, it is clear that voter \( k \) agrees with the others that \( y > z \) but disagrees with them regarding the merits of both \( y \) and \( z \) over both of \( w \) and \( x \).

4. Monotonicity

It can also be argued that monotonicity should be sacrificed to achieve a consensus. There is an attractive suite of collective decision functions called “Kingmaker Trees” [How89]. These do not demonstrate monotonic behaviour, but can be used to obtain a decision in the presence of sophisticated voting.

4.4 Are three (or more) policy collective choice functions usable?

Whether the conditions Arrow imposed on collective choice functions are reasonable only arises when the votes are close and that a sophisticated voter would vote in such a way that a policy cycle arises. Three (or more) policy votes can give decisive results and it would be very useful to have them resolve issues: can one therefore quantify how reliable a collective choice is? How likely is it that an election has been subverted by sophisticated voters, given the distribution of votes. Referring again to table 2, if it were possible to compare the two sets of votes, impartially, an administrator would be able to make a better choice of final policy. In which case, it would be better to introduce more irrelevant alternatives to make a more certain choice.

Technologically, this is feasible. There are cryptographic algorithms which would allow vote sets to be cast secretly [Sch96], and these could then be assessed by an impartial arbitrator to make the most appropriate choice. The basis for that choice would be probabilistic and an example of a probabilistic criterion that could be employed is given next.

5 Max–Min Probabilities in Condorcet Pairings

There is an interesting paper by Usiskin [Usi64], which quantifies the probabilities for Condorcet pairings. The paper addresses the “Voting Paradox”, but this is slightly misleading, it addresses the organization of votes within committees. It covers the same ground as the seminal works of Black and Farquharson [Bla58, Far69], which describe how committee procedures can be abused, if a policy cycle exists.

In a committee procedure, if there is a policy cycle then for all those voting:

- \( A > B > C \) has exactly \( \frac{1}{3} \) of vote
• as does $C > A > B$
• and $B > C > A$

If the policies are voted on in pairs, then the order in which they are introduced will determine which is chosen, viz.

• $(A \text{ vs } B) \text{ vs } C = C$ ∵ $A \text{ vs } B = A$
• $(C \text{ vs } A) \text{ vs } B = C$ ∵ $C \text{ vs } A = C$

The question that Usiskin resolves is how much more popular than one another can they be. In the example above, they all have probability of beating, or of being beaten, of $\frac{2}{3}$.

Denote by $X_i$ a real–valued random variable that represents the proportion of a simple majority vote received for policy $i$. The probability of a simple majority vote having the outcome that $X_i > X_j$ will be: $P(X_i > X_j)$. A policy cycle will be revealed if the probabilities for all pairs, $P(X_1 > X_2)$, $P(X_2 > X_3)$ and so on, for $n$ policies is non–zero and, finally, $P(X_n > X_1)$ is also non–zero.

The maximum minimum value will represent how much more popular one policy can be over another so that a policy cycle might still result.

If one then has at least one policy that beats another by an amount that is greater than this, then there can be no policy cycle.

**Theorem 5.1 (Arbitrary Random Variables).** The maximum minimum value for the joint probability distribution of a set of $n$ arbitrary random variables is given by:

$$\max \{ \min [P(X_1 > X_2), \ldots, P(X_{n-1} > X_n), P(X_n > X_1)] \} = \frac{n-1}{n} \quad (2)$$

This is as one would expect, looking at the example given above, where each of $A$, $B$ and $C$ had a probability of $\frac{2}{3}$ of beating the other, if one of these had a greater probability than that, there could be no policy cycle. In the example given above, were there were only three voters, this would mean the voters unanimously agreed on one policy being preferred over at least one other. This may not be the winning policy, it would allow at least one policy to be eliminated, then a two–policy vote can be taken.

This result is rather depressing but one can appreciate its intuitive correctness, because it tells us that the more policies there are, the more difficult it is to have one policy beating all others.

This case of arbitrary random variables does not help in understanding the behaviour of sophisticated voters. (The probabilistic events would all be conditioned by at least the previous result, viz. $P(X_i > X_{i+1} | X_i > X_{i-1})$) and probably would need to be conditioned by all events.

However, Usiskin does present a result which could be used to interpret sincere voting results. The election results $P(X_i > X_j)$ would be based on $X_i, X_j$ being independent random variables

**Theorem 5.2 (Independent Random Variables).** The maximum minimum value for the joint probability distribution of a set of $n$ independent random variables is given by:

$$\max \{ \min [P(X_1 > X_2), \ldots, P(X_{n-1} > X_n), P(X_n > X_1)] \} = b(n)$$

where $b(n + 1) > b(n)$ and $\lim_{n \to \infty} b(n) = \frac{3}{4}$
This latter result is quite encouraging, because if there is an election where at least one vote has a probability of greater than $\frac{3}{4}$ then no policy cycle can exist.

Usiskin also demonstrates a method for formulating the function $b(n)$ and presents some upper and lower bounds.

## 6 Some Collective Choice Functions

It was mentioned above, in the discussion of quorums, that a simple majority rule collective choice would be valid even if only one voter expressed a choice and all the others abstained. Simple majority rule is just one of a number of collective choice functions that could be employed. It is worthwhile just listing the collective choice functions. These are only for two policy votes and, because of Arrow’s conditions, cannot be extended to three (or more) policy votes, but they are insightful to the acceptability of collective choices. This summary follows Fishburn, [Fis74] and the details are contained in appendix B, but suffice to say that when only two policies, $x$ and $y$, are under consideration ternary logic can be used with $x > y$ being 1, $y < x$ being $-1$ and $x = y$ 0. There are some diagrams that illustrate the different types of voting rule and outcomes, figure [10].

From the discussion above, §4.1, a Pareto–optimal collective choice function can also be specified, which is not in the appendix, but is discussed in [ARS98] where it is called “unanimous with abstentions”.

| Rank | Rule                  | Paretian |
|------|-----------------------|----------|
| 1    | Specified Majority    | Yes      |
| 2    | Simple Majority       | Yes      |
| 3    | Specified Majority    | No       |
| 4    | Simple Majority       | No       |

Table 4: Ranking of Binary Voting Rules

**Simple Majority** if $s(\vec{D}) > 1$ then $x > y$ is the collective choice, see (3).

**Non–minority** if $1(\vec{D}) > n/2$ then $x > y$ is the collective choice, see (4). This is a special case of the next type of rule.

**Specified Majority** if $1(\vec{D}) > \alpha n$ then $x > y$, where $\alpha$ is some pre–defined constant in range $(0, 1)$, see (5).

**Absolute Majority** if $1(\vec{D}) > \alpha n$ then $x > y$ and $y > x$ otherwise, where $\alpha$ is some pre–defined constant in range $(0, 1)$, see (6).

**Absolute Special Majority** if $1(\vec{D}) \leq \alpha n$ then $y > x$, see (7).

**Pareto Majority: For** if $-1(\vec{D}) = 0$ and $1(\vec{D}) > 0$ then $x > y$.

**Pareto Majority: Indifference** if $0(\vec{D}) > 0$ and $-1(\vec{D}) = 1 = 0$ then $x = y$.  

23
Absolute special majority is a variant of absolute majority (it is an absolute majority of votes against) and absolute majority is just a variant of specified majority where the complement of the policy is not installed if the required vote count is not reached and, as noted, non-minority rule is a variant of Specified Majority Rule. So all of these can be replaced by that rule.

It is possible to define a Paretian quality which can be added to any voting rule and requires there is no dissenting vote. (Paretian indifference can be thought of as a vote for \( z \equiv (x = y) \) as opposed to \( w \equiv (x > y) \lor (y > x) \), so it is actually a Paretian vote on a different pair of issues: \( z \) and \( w \).) The rules can be ranked in a qualitative order of difficulty of attaining them, see table 4.

7 Summary

As information systems become more sophisticated they will be used to support human decision-making. The prospect of constructing virtual organizations based on how people interact is attractive: they could potentially be more responsive—Miller describes an evolving information processing organization, [Mil95], which develops its internal structure using a genetic algorithm. There are already some prototypes, [HK96, HH94], which share information based on past usage.

This paper acts as a warning that collective decision-making is not something to be taken lightly. Even the most sophisticated voting systems can give rise to erroneous results, §2.2. Sophisticated voters making policy choices could give rise to systems falling into stasis or being subverted to execute the wrong policies. If a discretionary access control mechanism used to control the release of information from databases were put under the control of collective choice functions and determined access rights based on the criteria proposed by Bertino et al., §1.4, it would almost certainly prove to be a vulnerable system.

There is clearly a need for a more sophisticated architecture to deal with access requests which can only be expressed in an enterprise modelling language which would have components similar to that described by ISO in their ODP, [ISO95]. This type of information processing system would need software agents acting on behalf of individuals to ensure that their information is protected. This would necessarily be a probabilistic analysis, based on how trustworthy potential information users are and it may prove expedient to develop systems that are insured against loss or provide degrees of surety, like those proposed by Neumann et al. [LMN97].

These information systems and their users would constitute an economy very much like the everyday commercial world occupied by institutions, corporations and people—only it would be faster, less resource wasteful and, if information system designers integrate the safeguards before they are used, safer.

A ODP Enterprise Entities

These are Booch [Boo94] class diagrams of the relationships that exist between entities in a distributed processing system, or, indeed, any organization as described in [ISO95, Enterprise Modelling Language].

1. Communities, see figure 5
Communities comprise of collections of resources and policies. The community is itself a resource.

2. Enterprise Agents, see figure 6
Performative agents are also resources. There are three kinds of these:

- Administrators
- Arbitrators
- Policy-Makers

Administrators and arbitrators have policies they follow, but policy-makers create policy.

3. Resource Users
These are also agents, but are not performative. They will have their own policies, but they are not explicitly open to arbitration. Resource users may contact administrators prior to using a resource or they may not, it depends on the nature of the resource.

Resource users usually control the policy-makers within communities and this is the case with most societies, since legislative assemblies are usually elected, but this need not be so. Companies are owned by its shareholders who appoint the board of directors, the administrators, but may not use the resources the company makes available. Suffice to say, that, in practice, in most business processes, the resource users have very little influence on the policy-makers.

4. Administrators, see figure 7
Administrators control sets of resources. Each consumable resource has a behaviour and may use other consumable resources. Since there may be
Figure 6: Agents (Policy-maker, Administrator, Arbitrator) and Policy

Figure 7: Administrators, Resources and Behaviour
different administrators vying for the same consumable resources conflicts may arise.

Resource users must obey the policy for using a resource. If there is no suitable policy, then the prospective user must have policy made. (This might seem different from what is observed in most organizations, where one can ask an administrator to apply policy differently in some way: usually by asking the administrator’s superior to become involved. The administrator’s superior is then acting as a policy–maker.)

5. Arbitrators, see figure 8

![Figure 8: Arbitrators and Administrators](image)

Arbitrators control administrators in that they resolve any conflicts that arise between them. They control neither administrators nor resources directly.

6. Policy, see figure 9

Policy is a set of prescriptive statements about the behaviour of a set of resources: usually one sub–set of those resources vis à vis another sub–set. There are three kinds of statement:

- **Permission** what one sub–set *may* do with (or for) the other sub–set.
- **Requirement** (or Obligation) what one sub–set *must* do with (or for) the other sub–set.
- **Prohibition** what one sub–set *must not* do with (or for) the other sub–set.

B Collective Choice Functions

What follows is drawn mostly from [Fis73] (who refers to proofs from his own text [Fis74], and gives a precise definition of the conditions that a collective
Figure 9: Policy, Resources and Behaviour
choice function must fulfill for a particular voting procedure for two policy alternative systems.

**Definition B.1 (Principle of Choice).** The basic materials for collective choice functions are social alternatives (candidates, policies, etc.) and individuals (voters, members, etc.) who have preferences among the alternatives. The idea of a collective choice function is to map a non–empty subset of the potential feasible subset of alternatives to each ordered pair consisting of a potential feasible subset of alternatives and a schedule of the voters’ preferences. The assigned set is often referred to as the *choice set*.

How that mapping is achieved is based on the properties of the collective choice function, which decides whether the choice is:

- Egalitarian
- Weighted
- Representative
- Unbiased (or neutral)
- Decisive
- Unanimous

**B.1 Two Policies**

If each individual has only two policies to choose from, then the policy chosen by the population as a whole will always be one of them, so two policy systems cannot select a set of policies that an individual has not specified.

**Definition B.2 (Sets for Two Policies).** The sets can be enumerated quite easily for two policies $x, y$. If a voter prefers $x$ to $y$, then 1 else if $y$ to $x$ then $-1$ else 0 signifies indifference—ternary logic.

- Policies $X = \{x, y\}$
- $\mathcal{X} = \{X\}$
- so write $F(\vec{D})$ for $F(X, \vec{D})$
- $\vec{D} = (D_1, D_2, \ldots, D_n)$ where $D_i \in \{1, 0, -1\}$
- $\mathcal{P}(\vec{D}) \equiv \{1, 0, -1\}^n$ and $\vec{D} \subseteq \{1, 0, -1\}^n$

For the collective choice function over $n$ individuals:

$$F: \{1, 0, -1\}^n \mapsto \{1, 0, -1\}$$

Note that the power set of the preferences is written $\{1, 0, -1\}^n$ as shorthand and is the set of all permutations of vectors of length $n$ where each component can take one of three values—$\#\{1, 0, -1\}^n = 3^n$. When a condition is applied to a preference profile, it is either applied with reference to the power set or the vote set: the power set, although large, is denumerable *a priori*, the vote set is not.
B.2 Egalitarian

Egalitarian
collective choice functions treat each voter’s vote as identical in
effect to every other’s.

Condition B.1 (Strongly Neutral). A collective choice function $F$ is Strongly Neutral if, for all $\vec{D} \in \{1, 0, -1\}^n$:

$$F(-\vec{D}) = -F(\vec{D})$$

(Strongly Neutral)

Condition B.2 (Strongly Monotonic). A collective choice function $F$ is Strongly Monotonic if, for any $\vec{D}, \vec{D}' \in \{1, 0, -1\}^n$:

$$\vec{D} \geq \vec{D}' \Rightarrow F(\vec{D}) \geq F(\vec{D}')$$

$$\vec{D} > \vec{D}', F(\vec{D}') = 0 \Rightarrow F(\vec{D}) = 1$$

(Strongly Monotonic)

Condition B.3 (Egalitarian). A collective choice function $F$ is Egalitarian if for all $\vec{D} \in \{1, 0, -1\}^n$:

$$F(D_1, \ldots, D_n) = F(D_{\sigma(1)}, \ldots, D_{\sigma(n)})$$

if $\sigma$ is a permutation on $\{1, \ldots, n\}$ (Egalitarian)

Theorem B.1 (Conditions for Simple Majority Rule). A collective choice function $F$ implements simple majority rule over two policies and has the following qualities: Strongly Neutral, Strongly Monotonic and Egalitarian.

Definition B.3 (Simple Majority Rule). If $F$ applies ternary logic, it can be implemented with:

$$F(D) \triangleq s(\vec{D})$$

(3)

Condition B.4 (Monotonic). A collective choice function is just Monotonic rather than Strongly Monotonic if, for any $\vec{D}, \vec{D}' \in \{1, 0, -1\}^n$:

$$\vec{D} \geq \vec{D}' \Rightarrow F(\vec{D}) \geq F(\vec{D}')$$

(Monotonic)

Definition B.4 (Non–minority Rule). If a collective choice function is Strongly Neutral, Monotonic and Egalitarian and is implemented thus:

$$F(\vec{D}) = 1 \iff 1(\vec{D}) > n/2$$

$$F(\vec{D}) = -1 \iff -1(\vec{D}) > n/2$$

(4)

then the voting system is known as non–minority rule.

\footnote{Fishburn in \cite{Fis74} uses the term “anonymous” for egalitarian and “dual” for neutral.}
Non–minority rule is just one of a class of neutral, monotonic and egalitarian collective choice functions; they differ in effect from the strongly monotonic simple majority rule by having a “dead–band”. A geometric insight into the specification of a collective choice function can be given using a unit simplex[San94]. There is only one dimension.

\[ \vec{q} = \left( \frac{1(\vec{D})}{n}, \frac{-1(\vec{D})}{n} \right) \]

The election vector \( \vec{q} \) emanates from the origin and will always be within \([-1, 1]\). Under simple majority rule, whichever point, \(-1\) or \(1\), the vector is closest to wins. Under non–minority rule the vector has to be over half \( \frac{1}{2} \) way towards the point, see figure 10. The indecisive region in the centre is symmetric. For non–minority rule, any boundary can be chosen, so long as it is symmetric about the origin.

![Figure 10: Two policy voting systems](image)

It should be clear from figure 10 that a simple majority voting system can determine policy if only one voter is not indifferent to either of the policies, the others abstaining. In this respect, non–minority rule seems to impose a natural quorum, since it requires that at least half of the voters have chosen one or the other policy. In this respect, non–minority rule is less questionable as a decision–making device than simple majority rule. Most electoral systems do in fact operate a non–minority rule system.

**Condition B.5 (Neutral).** A collective choice function is neutral as opposed to strongly neutral, if, for all \( \vec{D} \in \{1, 0, -1\}^n \):

\[ 1(\vec{D}) \neq -1(\vec{D}) \Rightarrow F(-\vec{D}) = -F(\vec{D}) \] (Neutral)
Condition B.6 (Strongly Decisive). A collective choice function is Strongly Decisive if, for all $\vec{D} \in \{1, 0, -1\}^n$:
\[
F(\vec{D}) \neq 0 \quad \text{(Strongly Decisive)}
\]

Condition B.7 (Unanimity unambiguous). A collective choice function is Unanimity unambiguous if:
\[
F(\vec{1}) = 1 \text{ and } F(\vec{-1}) = -1 \quad \text{(Unanimity unambiguous)}
\]

Condition B.8 (Pro–biased). A collective choice function is Pro–biased if, for all $\vec{D} \in \{1, 0, -1\}^n$:
\[
\text{if } \vec{D} = \vec{D}' \text{ except that } (D_i, D'_i) = (0, 1) \text{ for some } i \text{ then } F(\vec{D}) = F(\vec{D}') \quad \text{(Pro–biased)}
\]

The number of electoral ties can be reduced by downgrading Strongly Neutral to Neutral and adding Strongly Decisive. If Neutral is dropped then an electoral preference is given to one policy over the other. The policy that is preferred is usually already in force and is therefore called the incumbent, the other policy is the challenging policy. See figure 10 for the asymmetry of the indecisive region under an absolute majority rule.

Definition B.5 (Absolute Majority Rule). If the collective choice function is no longer Neutral and is made Strongly Decisive, and the function is implemented thus:
\[
F(\vec{D}) = 1 \iff 1(\vec{D}) > \alpha n \quad \text{(5)}
\]
\[
F(\vec{D}) = -1 \iff 1(\vec{D}) \leq \alpha n
\]
for $\alpha \in (0, 1)$
then these are the absolute majority rule functions. A special case is unanimous rule which requires either all votes are $-1$ or $1$.

Definition B.6 (Absolute Special Majority Rule). As for absolute majority rule, but the function is also (Unanimity unambiguous) and (Pro–biased). The collective choice function can be implemented by:
\[
F(\vec{D}) = -1 \iff 1(\vec{D}) \leq \alpha n \quad \text{(6)}
\]
for $\alpha \in (0, 1)$

B.3 Non–Egalitarian: Weighted

The effect of the Egalitarian property is that one voter’s preference can be exchanged for another within the decision profile and it will have no effect on the evaluation of the collective choice function. An alternative system is to use a weighted system, which is often used by some committees, where the chairman is given both a deliberative and a casting vote. Weighted systems have a number of attractions because they can be designed so that:

- No one person can dictate policy to the group.
- Ties can be readily resolved without a further vote.

The simplest way to define a weighted voting function is to use a weighting vector.
**Weighted Voting**

**Definition B.7 (Weighting Vector and Vote).** A weighting vector can be defined thus:

\[ \vec{\rho}^{(n)} \succ 0^{(n)}, \vec{\rho} = (\rho_1, \ldots, \rho_n) \]

where \( \rho_i \geq 0, \rho_i \in \mathbb{Z}_0^+ \) is the weight assigned to each voter.

Then

- **Weighted Vote** \( \vec{\rho} \cdot \vec{D} \)
- **Weight Function** \( W(\vec{\rho}) \triangleq \sum_{i=1}^{n} a_i \)

Weight of \( c \in \{1, 0, -1\} \)

\[ W_c(\vec{D}) \triangleq W((\vec{\rho} \cdot \vec{D}) : D_i = c) = \sum_{i=1}^{n} \rho_i D_i \text{ for those } D_i = c \]

It does not follow that \( n(\vec{\rho}) \geq n \) because it is possible to set any number of \( \rho_i \) equal to zero, but at least one must be non-zero.

**Theorem B.2 (Weighted Majority Function).** A function \( F, F : \{1, 0, -1\}^n \rightarrow \{1, 0, -1\} \), is a weighted majority function if and only if it satisfies **Monotonic, Unanimity unambiguous** and **Neutral**, the weighted majority function can be defined as \( F(\rho \cdot D) = s(\vec{\rho} \cdot \vec{D}) \).

The conditions can be readily tested by substituting \( F(\vec{\rho} \cdot \vec{D}) \) for \( F(\vec{D}) \).

The other systems can be added thus:

1. **Non–minority rule**

\[ F(\vec{\rho} \cdot \vec{D}) = \begin{cases} 1 & W_1(\vec{\rho} \cdot \vec{D}) > \frac{W(\vec{\rho})}{2} \\ -1 & W_1(\vec{\rho} \cdot \vec{D}) \leq \frac{W(\vec{\rho})}{2} \\ 0 & \text{Otherwise} \end{cases} \]

2. **Absolute majority**

\[ F(\vec{\rho} \cdot \vec{D}) = \begin{cases} 1 & W_1(\vec{\rho} \cdot \vec{D}) > \alpha W(\vec{\rho}) \\ -1 & W_1(\vec{\rho} \cdot \vec{D}) \leq \alpha W(\vec{\rho}) \\ 0 & \text{Otherwise} \end{cases} \]

**Dictators and Vetoers** The final choice may be wholly determined by only one of the voters, in which case that voter is either a **dictator** or a **vetoer**.

1. **Dictator**

**Definition B.8 (Dictator).** A voter, \( j \), is a **Dictator** with regard to a collective choice function \( F \) and a weighting vector \( \vec{\rho} \) if:

For all \( \vec{D} \in \{1, 0, -1\}^n \) such that

\[ \vec{D} = (D_1, \ldots, D_j, \ldots, D_n) \quad \text{(Dictator)} \]

when \( D_j \neq 0 \), \( F(\vec{\rho} \cdot \vec{D}) = D_j \)
Whether a dictator can effect all decision profiles is a condition on the behaviour of the collective choice function and the weighting vector, not on the voters, which is as follows.

**Definition B.9 (Undominated by dictator).** A collective choice function is **Undominated by dictator** if there is no **Dictator**. This can be stated thus:

For all \( \vec{D} \in \vec{D} \)

There is no \( i \) such that

\[
\vec{D} = (D_1, \ldots, D_i, \ldots, D_n),
\]

\( D_i \neq 0 \) and \( F(\vec{D}) = D_i \)  \hspace{1cm} \text{(Undominated by dictator)}

This is not particularly useful, since there may legitimately be a voter whose vote is always in line with the choice of the group as a whole.

2. Vetoer

**Definition B.10 (Vetoer).** The first voter, 1, is a **Vetoer** with regard to a collective choice function \( F \) and a weighting vector \( \vec{\rho} \) if:

For

\[
\vec{D}_1 = (0, 1, \ldots, 1)
\]

\( \vec{D}_{-1} = (0, -1, \ldots, -1) \) \hspace{1cm} \text{(Vetoer)}

\[
F(\vec{\rho} \cdot \vec{D}_1) = 0
\]

\[
F(\vec{\rho} \cdot \vec{D}_{-1}) = 0
\]

(The vetoer is in first position for convenience).

Whether a vetoer can effect those two very specific decision profiles is a condition on the behaviour of the collective choice function and the weighting vector, not on the voters, which is as follows.

**Definition B.11 (Undominated by vetoer).** A group of voters are **Undominated by vetoer** if there is no voter \( j \), such that:

If

\[
D_j = 1, \quad F(\vec{\rho} \cdot \vec{D}_1) = 1, \text{ and}
\]

\[
D_j = -1, \quad F(\vec{\rho} \cdot \vec{D}_{-1}) = -1
\]

but

\[
D_j \neq 1, \quad F(\vec{\rho} \cdot \vec{D}_1) = 0
\]

\[
D_j \neq -1, \quad F(\vec{\rho} \cdot \vec{D}_{-1}) = 0
\]

For all \( \vec{D} \in \vec{D} \).

This, again, is not particularly useful, since there may legitimately be a voter whose always votes against the group.

34
Sensitivity

Definition B.12 (Essential). A voter \( i \) is said to be\textcolor{red}{\textbf{Essential}} with regard to a collective choice function, and weighting vector, if at least one of the following conditions holds:

Either \( F(\vec{\rho} \cdot \vec{D}_1) \neq F(\vec{\rho} \cdot \vec{D}_0) \)

Or \( F(\vec{\rho} \cdot \vec{D}_{-1}) \neq F(\vec{\rho} \cdot \vec{D}_0) \) \textcolor{red}{(Essential)}

Or \( F(\vec{\rho} \cdot \vec{D}_{-1}) \neq F(\vec{\rho} \cdot \vec{D}_1) \)

For at least one of the vectors \( \vec{D}_1, \vec{D}_0, \vec{D}_{-1} \) constructed as follows:

\( \vec{D}_1 = (D_1, \ldots, D_{i-1}, 1, D_{i+1}, \ldots, D_n) \)

\( \vec{D}_0 = (D_1, \ldots, D_{i-1}, 0, D_{i+1}, \ldots, D_n) \)

\( \vec{D}_{-1} = (D_1, \ldots, D_{i-1}, -1, D_{i+1}, \ldots, D_n) \)

where the contents of those vectors can be taken from any of the vectors constructed thus:

\( \vec{D}^{(n-1)} = (D_1, \ldots, D_{i-1}, D_{i+1}, \ldots, D_n) \in \{1, 0, -1\}^{n-1} \)

This condition requires that a voter \textcolor{red}{can} be decisive in at least one decision profile. It prevents a voter from being given so ineffectual a vote that it is never decisive in any election.

B.3.1 Vetoer, Dictator and Essential

By this it is meant safe from dictators and vetoers and sensitive to voters; it is desirable if a collective choice function and weighting vector could be chosen so that for all decision profiles in \( \{1, 0, -1\}^n \) there is no voter who is either a \textcolor{red}{\textbf{Dictator}} or a \textcolor{red}{\textbf{Vetoer}}. It would also be desirable there is at least one voter who is \textcolor{red}{\textbf{Essential}}.

The collective choice functions simple majority, non–minority and absolute majority are demonstrably safe from dictators and vetoers when used under an egalitarian regime so only the weighting vector needs to be checked.

Dictators, Vetoers and Weighting Vectors

1. Weighted majority and weighted non–minority rule

If the collective choice function is either of the above, for \( \tilde{\rho} = (\rho_1, \ldots, \rho_n) \) and the weighting vector has been scaled so that \( \rho_1 \) has weight 1. In which case, the worst case for the dictator is that all vote against, so for the dictator to succeed \( \rho_{\text{dictator}} > \frac{W(\tilde{\rho})}{2} \).

Because both of these collective choice functions are dual; it should be clear \( \rho_{\text{dictator}} = \rho_{\text{vetoer}} \).

Consequently, \( \rho_{\text{max}} < \frac{W(\vec{\rho})}{2} \), is sufficient for both of these.

2. Absolute majority

To succeed, \( \rho_{\text{dictator}} > \alpha \cdot W(\tilde{\rho}) \), so the converse is required.

A vetoer has it easier \( \rho_{\text{vetoer}} > (1 - \alpha) \cdot W(\tilde{\rho}) \), so the converse.
Weighting Vectors that are Essential to Voters  Under egalitarian rule, all voting functions are sensitive to all voters, because if any voter is sensitive, a permutation can put another voter in his place, so again the collective choice function will not be at fault should a system of rule prove insensitive, it will be the weighting vector.

There are two possibilities:

- The voter has a weighting of zero
- The voter has a weighting which can never be decisive

Whether a voter has a non-zero vote can only be tested for; probably by using the weighted majority rule function with all other voters not voting.

Having a vote that is never decisive is more subtle. The weighting vector is as usual, with the lowest rated voter in first position having value 1. Construct a dictator to each voter in the following manner.

\[
\begin{align*}
\rho_1 &= 1 \\
\rho_2 &= \rho_1 + 1 \\
\rho_3 &= \rho_2 + \rho_1 + 1 \\
&\quad \vdots \\
\rho_{n-1} &= \rho_{n-2} + \cdots + \rho_2 + \rho_1 + 1 \\
\rho_n &= \rho_{n-1} + \rho_{n-2} + \cdots + \rho_2 + \rho_1 + 1 \\
\rho_n &= \rho_{n-1} + \rho_{n-2} \\
\rho_n &= 2\rho_{n-1} \\
\rho_n &= 2 \cdot 2 \cdot \cdots \cdot n - 1 \text{ times } 1 \\
\rho_n &= 2^{n-1}
\end{align*}
\]

A necessary conditions on weighting vectors can be set, each \(\rho_i < 2^{n-1}\) if \(W(\vec{\rho}) \geq 2^n - 1\)—geometric progression. There are effectively two choices:

- Set \(n \geq 3\), \(\max \rho_i = 2^{n-1} - 1\), in which case \(W(\vec{\rho}) \geq 2^n - 2\) and all other voters will tie with the largest voter.
- Set \(n \geq 4\), \(\max \rho_i = 2^{n-1} - 2\), in which case \(W(\vec{\rho}) \geq 2^n - 3\) and all other voters will defeat the largest voter.

For \(\vec{\rho} = (\rho_1, \ldots, \rho_i, \ldots, \rho_n)\)

\[
\sum_{j=1}^{n} \rho_j > \frac{1}{2} \sum_{j=1}^{n} \rho_j
\]

B.4 Representative Systems

A representative system can be thought of as a heirachy of voting councils in which the outcomes of votes in lower councils become votes in higher councils. A voter in one of the higher councils may be a voter or a voting council. Each voting council can use weighted majority rule between its members. Voters (or councils) can vote more than once in different councils.
Definition B.13 (Representation). Let a hierarchy of voting councils be defined as a representation \( R \). Let each level of representation be denoted by a suffix, the lowest level being \( R_0 \).

To make the lowest level similar in mathematical structure to higher levels, we shall introduce a selection function \( S_i \) which selects from a preference profile, \( D \), the preference of voter \( i \).

\[
S_i : \{1, 0, -1\}^n \mapsto \{1, 0, -1\} \\
S_i(D) = D_i
\]

\( R_0 \) can then be written as:

\[
R_0 = \{S_1(D), \ldots , S_n(D)\}
\]

So \( R_0 \) is simply the decision profile as a set.

Thereafter, there is a level \( m \in \mathbb{N} \) which is such that:

\[
R_m = \{s(F_1, \ldots , F_K)(R_{m-1})\}
\]

This is effectively voting using a tree structure and is probably the most used organizational control system. Unfortunately, it is proving to be very difficult to analyze. Hopefully, more results will arise.

C Funding and Author Details

Research was funded by the Engineering and Physical Sciences Research Council of the United Kingdom. Thanks to Malcolm Clarke, Russell-Wynn Jones and Robert Thurlby.

Walter Eaves  
Department of Electrical Engineering,  
Brunel University  
Uxbridge,  
Middlesex UB8 3PH,  
United Kingdom

Walter.Eaves@bigfoot.com  
Walter.Eaves@brunel.ac.uk

http://www.bigfoot.com/~Walter.Eaves  
http://www.brunel.ac.uk/~eepgwde

References

[Arr63] Kenneth Arrow. Social Choice and Individual Values. Yale University Press, second edition, 1963.

[ARS98] Hajnal Andréka, Mark Ryan, and Pierre-Yves Schobbens. Operators and laws for combining preference relations. World-Wide Web, 1998. http://www.dmi.ens.fr/~vaudenay/spw97/.
[Bla58] Duncan Black. The theory of committees and elections. Cambridge University Press, 1958.

[BO82] Tamer Baser and Geert Jan Olsder. Dynamic Noncooperative Game Theory, volume 160 of Mathematics in Science and Engineering. Academic Press, London, 1982.

[Boo94] Grady Booch. Object-oriented analysis and design with applications. Benjamin/Cummings, 1994.

[BW94] E. Bertino and H. Weigand. An approach to authorization modeling in object-oriented database systems. Data & Knowledge Engineering, 12:1–29, 1994.

[Cas97] Silvana Castano. An approach to deriving global authorizations in federated database systems. In P Samarati and R S Sandhu, editors, Database Security X: Status and Prospects, International Federation for Information Processing. Chapman and Hall, 1997.

[Cor84] Digital Equipment Corp. VAX/VMS DCL commands and lexical functions. Technical Report AA-Z003A-TE, Digital Equipment Corp. (DEC), Maynard, Mass., September 1984.

[Cur90] David A. Curry. Improving the security of your unix system. Technical Report ITSTD-721-FR-90-21, SRI International, 1990. [http://julmara.ce.chalmers.se/Security/security-doc.tar.gz]

[Den76] Dorothy E. Denning. A lattice model of secure information flow. Communications of the ACM, 19(5):236–243, May 1976. Papers from the Fifth ACM Symposium on Operating Systems Principles (Univ. Texas, Austin, Tex., 1975).

[Den82] Dorothy Elizabeth Robling Denning. Cryptography and data security. Addison–Wesley, 1982.

[Den89] D E Denning. Secure databases and safety: some unexpected conflicts. In T Anderson, editor, Safe and Secure Computing Systems, chapter 6. Blackwell, Oxford, 1989.

[Dor79] Gideon Doron. Is the hare voting scheme representative? The Journal of Politics, 41(3):918–922, August 1979. In Research Notes.

[Eav99a] Walter Eaves. mypack.m. Mathematica Package: Available by FTP, January 1999. [ftp://ftp.brunei.ac.uk/eepgwde/mypack.m]

[Eav99b] Walter D Eaves. ODP channel objects that provide services transparently for distributing processing systems. Technical report, e-print archives, April 1999, [http://xxx.lanl.gov/abs/cs.DC/9904020].

[EFL+97] R J Ellison, D A Fisher, R C Linger, H F Lipson, T Longstaff, and N R Mead. Survivable network systems: An emerging discipline. Technical Report CMU/SEI-97-TR-013, Carnegie Mellon University. Software Engineering Institute, November 1997. [http://www.cert.org/research/97tr013.pdf]
[Far69] Robin Farquharson. *Theory of voting*. Blackwell, Oxford, 1969.

[Fis73] Peter C Fishburn. *The Theory of Social Choice*. Princeton University Press, 1973.

[Fis74] Peter C Fishburn. Social choice functions. *SIAM Review*, 16(1):63–90, January 1974.

[HH94] B A Hubermann and T Hogg. Communities of practice: Performance and evolution. World-Wide Web, 1994. [ftp://parcftp.xerox.com/pub/dynamics/multiagent.html].

[HK96] B A Hubermann and M Kaminsky. Beehive: A system for filtering and sharing information. World-Wide Web, 1996. [ftp://parcftp.xerox.com/pub/dynamics/multiagent.html].

[How89] J V Howard. Implementing alternative voting in kingmaker trees. Technical report, STCRED, London School of Economics, 1989.

[Ins99] Wolfram Research Institute. Mathematica. World Wide Web, February 1999. [http://www.wri.com].

[Isb60] J R Isbell. Homogeneous games - II. *Proceedings of the American Mathematical Society*, 11(2):159–161, April 1960.

[ISO95] ISO. Reference model: Architecture (10). In A Herbert, editor, *Information technology - Open distributed processing*, number 903 in Series X Recommendations X.900 to X.1000. International Telecommunication Union, International Telecommunication Union (ITU), Place des Nations, CH-1211 Geneva 20, Switzerland, 1995. [http://info.itu.ch/itudoc/itu-t/rec/x/x500up.html].

[Jum98] James G. Jumes, editor. *Microsoft Windows NT 4.0 Security, Audit, and Control*. Microsoft Technical Reference. Microsoft Press, December 1998. ISBN 157231818X.

[Lew91] Donald A. Lewine. *POSIX programmer’s guide: writing portable UNIX programs with the POSIX.1 standard*. O’Reilly & Associates, Inc., 981 Chestnut Street, Newton, MA 02164, USA, 1991. March 1994 printing with corrections, updates, and December 1991 Appendix G.

[LMN97] Charlie Lai, Gennady Medvinsky, and B. Clifford Neuman. Endorsements, licensing, and insurance for distributed services. In Lee W Knight and Joseph P Bailey, editors, *Internet Economics*, chapter Internet Commerce. MIT Press, 1997.

[May52] Kenneth May. A set of independent necessary and sufficient conditions for simple majority decisions. *Econometrica*, 20:680–684, 1952.

[Mil95] John H Miller. Evolving information processing organizations. Research note, Carnegie Mellon University, Social and Decision Sciences, Carnegie Mellon University, Pittsburgh, PA 15213, 1995. [http://zia.lss.cmu.edu/miller].
[Min89] N. H. Minsky. Law-governed software processes. In D. E. Perry, editor, *Proceedings of the 5th International Software Process Workshop*, pages 98–100, October 1989.

[Min95] Naftaly Minsky. Coordination and trust in open distributed systems. Technical report, Rutgers University, NJ, March 1995. [http://www.cs.rutgers.edu/~minsky/public-papers/trust-paper.ps](http://www.cs.rutgers.edu/~minsky/public-papers/trust-paper.ps).

[MSK99a] Event msg: The browser has forced an election... Microsoft Personal Support Center, January 1999. [http://support.microsoft.com/support/kb/articles/q103/0/42.asp](http://support.microsoft.com/support/kb/articles/q103/0/42.asp), Article ID: Q103042.

[MSK99b] Information on browser operation. Microsoft Personal Support Center, January 1999. [http://support.microsoft.com/support/kb/articles/q102/8/78.asp](http://support.microsoft.com/support/kb/articles/q102/8/78.asp), Article ID: Q102878.

[PB81] Bernard Monjardet Pierre Batteau, Jean-Marie Blin. Stability of aggregation procedures, ultrafilters and simple games. *Econometrica*, 49(2):527–534, March 1981.

[RBKW91] Fausto Rabitti, Elisa Bertino, Won Kim, and Darrel Woelk. A model of authorization for next-generation database systems. *ACM Transactions on Database Systems*, 16(1):88–131, March 1991. [http://www.acm.org/pubs/toc/Abstracts/tods/103144.html](http://www.acm.org/pubs/toc/Abstracts/tods/103144.html).

[Ricly] Craig Richmond. Setting up a basic DNS server for a domain. World–Wide Web, 1993 July. [http://web.syr.edu/~jmwbosus/comfaqs/faq-dns](http://web.syr.edu/~jmwbosus/comfaqs/faq-dns).

[Saa94] Donald Saari. *Geometry of Voting*. Springer, 1994. BLPES JF1001.

[Sch96] Bruce Schneier. *Applied Cryptography: Protocols, Algorithms, and Source Code in C*. John Wiley & Sons, Inc, 1996.

[Ski90] Steven S. Skiena. *Implementing Discrete Mathematics: Combinatorics and Graph Theory with Mathematica*, with programs by Steven Skiena and Anil Bhansali. Addison-Wesley, Reading, MA, USA, 1990.

[SN89] P A Samuelson and W D Nordhaus. *Economics*. McGraw–Hil, 13 edition, 1989.

[SUN98] Java security. World–Wide Web, 1998. [http://java.sun.com/products/jdk/1.2/docs/guide/security/](http://java.sun.com/products/jdk/1.2/docs/guide/security/).

[Usi64] Zalman Usiskin. Max-min probabilities in the voting paradox. *Annals of Mathematical Statistics*, 35(2):857–862, June 1964.

[VOT98] Alternative systems for calculating winners of multi-option votes. World–Wide Web Document Queens University Belfast, Jan 1998. [http://www.qub.ac.uk/mgt/papers/prefer/prefcalc.html](http://www.qub.ac.uk/mgt/papers/prefer/prefcalc.html).

[Wel99] Brian Wellington. An introduction to Domain Name System security. World–Wide Web Document from TISLabs at Network Associates, January 1999. [http://www.nai.com/products/security/tis_research/netsec/dnsssec-paper.html](http://www.nai.com/products/security/tis_research/netsec/dnsssec-paper.html).
Richard Zeckhauser. Majority rule with lotteries on alternatives. 
*The Quarterly Journal of Economics*, 83(4):696–703, 1969. Nov.