Cosmological Constraints on $\Omega_m$ and $\sigma_8$ from Cluster Abundances Using the GalWCat19 Optical-spectroscopic SDSS Catalog

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Abstract

We derive cosmological constraints on the matter density, $\Omega_m$, and the amplitude of fluctuations, $\sigma_8$, using GalWCat19, a catalog of 1800 galaxy clusters we identified in the Sloan Digital Sky Survey–DR13 spectroscopic data set using our GalWeight technique to determine cluster membership. By analyzing a subsample of 756 clusters in a redshift range of 0.045 $\leq z \leq$ 0.125 and virial masses of $M \geq 0.8 \times 10^{14}\, h^{-1}\, M_{\odot}$, with mean redshift of $z = 0.085$, we obtain $\Omega_m = 0.310^{+0.027}_{-0.023} \pm 0.041$ (systematic) and $\sigma_8 = 0.810^{+0.036}_{-0.036} \pm 0.035$ (systematic), with a cluster normalization relation of $\sigma_8 = 0.430\,\Omega_m^{-0.55}$. There are several unique aspects to our approach: we use the largest spectroscopic data set currently available, and we assign membership using the GalWeight technique, which we have shown to be very effective at simultaneously maximizing the number of bona fide cluster members while minimizing the number of contaminating interlopers. Moreover, rather than employing scaling relations, we calculate cluster masses individually using the virial mass estimator. Since GalWCat19 is a low-redshift cluster catalog we do not need to make any assumptions about evolution either in cosmological parameters or in the properties of the clusters themselves. Our constraints on $\Omega_m$ and $\sigma_8$ are consistent and very competitive with those obtained from non-cluster abundance cosmological probes such as cosmic microwave background, baryonic acoustic oscillation (BAO), and supernovae. The joint analysis of our cluster data with Planck18+BAO+Pantheon gives $\Omega_m = 0.315^{+0.013}_{-0.011}$ and $\sigma_8 = 0.810^{+0.011}_{-0.001}$.

Unified Astronomy Thesaurus concepts: Cosmological parameters (339); Galaxy clusters (584); Cosmological models (337); Galaxy cluster counts (583); Large-scale structure of the universe (902)

1. Introduction

In the current picture of structure formation, galaxy clusters arise from rare high peaks of the initial density fluctuation field. These peaks grow in a hierarchical fashion through the dissipationless mechanism of gravitational instability with more massive halos growing via continued accretion and merging of low-mass halos (White & Frenk 1991; Kauffmann et al. 1999, 2003). Galaxy clusters are the most massive virialized systems in the universe and are uniquely powerful cosmological probes. The cluster mass function (CMF), or the abundance of galaxy clusters, is particularly sensitive to the matter density of the universe $\Omega_m$ and $\sigma_8$, the rms mass fluctuation on the scale of $8\, h^{-1}\,\text{Mpc}$ at $z = 0$ (e.g., Wang & Steinhardt 1998; Battye & Weller 2003; Dahle 2006; Wen et al. 2010).

Cosmological analyses have been performed using samples of galaxy clusters constructed from galaxy surveys (e.g., Rozo et al. 2010; Kirby et al. 2019; DES Collaboration et al. 2020), X-ray emission (e.g., Vikhlinin et al. 2009; Mantz et al. 2015), and thermal Sunyaev–Zel’dovich (SZ) signals (e.g., Bocquet et al. 2019; Zubeldia & Challinor 2019). These cluster abundance studies showed that $\Omega_m$ varies from $\sim0.2$ to 0.4 and $\sigma_8$ varies from $\sim0.6$ to 1.0. The discrepancies or tensions among these various studies is basically dependent on the accuracy of cluster mass estimation. Cluster mass can be calculated from cluster dynamics using, for example, the virial mass estimator (e.g., Binney & Tremaine 1987), the weak gravitational lensing (Wilson et al. 1996; Holljen et al. 2009), and the application of the Jeans equation for the gas density calculated from the X-ray analysis of the galaxy cluster (Sarazin 1988). It can also be estimated from other observables, the so-called mass proxies, which scale tightly with cluster mass, such as X-ray luminosity (e.g., Pratt et al. 2009), optical luminosity, or richness (e.g., Yee & Ellingson 2003; Simet et al. 2017), and the velocity dispersion of member galaxies (e.g., Biviano et al. 2006; Bocquet et al. 2015). Generally, most of these methods introduce large systematic uncertainties, which limits the accuracy of estimating cluster masses (e.g., Wojtak & Łokas 2007; Mantz et al. 2016).

Cosmological analyses of galaxy cluster abundance introduce a degeneracy between $\Omega_m$ and $\sigma_8$. Large ongoing and upcoming wide and deep-field imaging and spectroscopic surveys at different redshifts, such as DES (Abbott et al. 2018a), eROSITA (Merloni et al. 2012), Vera C. Rubin Observatory (LSST Science Collaboration et al. 2009), and Nancy Grace Roman Telescope (Ak ese et al. 2019), will simultaneously increase the precision of measuring the cosmological parameters and break the degeneracy between them. This is because $\Omega_m$ evolves slowly while $\sigma_8$ evolves strongly with redshift. Also, these galaxy surveys at different redshifts are significant to study the evolution of the CMF, which is critical to measuring structure growth, and therefore can be used to constrain properties of dark energy (e.g., Haiman et al. 2001; Mantz et al. 2008). Introducing advanced methods is essential to analyzing these surveys. One of these methods is the GalWeight technique (Abdullah et al. 2018, hereafter Abdullah+18), which can be applied to the available and upcoming spectroscopic databases of eBOSS (Raichoor et al. 2017), DESI (Levi et al. 2019), and Euclid (Euclid Collaboration et al. 2019) to construct cluster catalogs. These catalogs provide an unlimited data source for a wide range of astrophysical and cosmological applications.
In addition, there are independent cosmological probes to constraining the cosmological parameters that can be applied alongside or in combination with galaxy cluster abundance. The anisotropies in the cosmic microwave background (CMB) are an independent probe of cosmological parameters (e.g., Hinshaw et al. 2013; Planck Collaboration et al. 2016). The likelihoods of the $\Omega_m - \sigma_8$ confidence levels (CLs) introduced by the CMB and CMB are almost orthogonal to each other, which means combining these measurements will eliminate the degeneracy between $\Omega_m$ and $\sigma_8$ and shrink the uncertainties. Other independent cosmological probes that are used to constrain $\Omega_m$ and $\sigma_8$ include cosmic shear, galaxy-galaxy lensing, and angular clustering (e.g., Abbott et al. 2018b; van Uitert et al. 2018). The likelihoods of the $\Omega_m - \sigma_8$ CLs introduced by these probes are almost parallel to those introduced by the CMB. Moreover, the two cosmological probes of baryon acoustic oscillations (BAO, e.g., Eisenstein et al. 2005) and supernovae (SNe, e.g., Perlmutter et al. 1999) can be used to constrain $\Omega_m$ only (independent of $\sigma_8$).

In this paper, we aim to derive the CMB and the cosmological parameters $\Omega_m$ and $\sigma_8$ using a subsample of 756 clusters (SelFMC) obtained from the GalWCat19 cluster catalog as we discuss below in detail. The GalWCat19 (Abdullah et al. 2020, hereafter Abdullah+20) catalog was derived from the Sloan Digital Sky Survey-Data Release 13 spectroscopic data set (hereafter SDSS-DR13,5 Albareti et al. 2017). The clusters were first identified by looking for the Finger-of-God effect (see Jackson 1972; Kaiser 1987; Abdullah et al. 2013). The cluster membership was constructed by applying our own GalWeight technique, which was specifically designed to simultaneously maximize the number of bona fide cluster members while minimizing the number of contaminating interlopers (Abdullah +18). In Abdullah+18, we applied our GalWeight technique to MDPL2 and Bolshoi N-body simulations and showed that it was >98% accurate in correctly assigning cluster membership. The GalWCat19 catalog is at low-redshift for which the effects of cluster evolution and cosmology are minimal. Finally, the cluster masses were calculated individually from the dynamics of the member galaxies via the virial theorem (e.g., Limber & Mathews 1960; Abdullah et al. 2011), and corrected for the surface pressure term (e.g., The & White 1986; Carlberg et al. 1997). A huge advantage of our approach relative to mass proxy methods is that it returns an estimate of the total cluster mass (dark matter and baryons) without making any assumptions about the internal complicated physical processes associated with the baryons (gas and galaxies). The publicly available GalWCat19,6 contains 1800 clusters at redshift $z \leq 0.2$, which is one of the largest available samples that used a high-quality spectroscopic data set.

The paper is organized as follows. In Section 2, we describe in more detail how we created the GalWCat19 cluster catalog. In Section 3, we investigate the volume and mass incompleteness of GalWCat19 to obtain a mass-complete local subsample of 756 clusters (SelFMC) used to constrain $\Omega_m$ and $\sigma_8$. In Section 4, we compare our complete sample with theoretical models to constrain the cosmological parameters $\Omega_m$ and $\sigma_8$. We investigate how systematics affect the recovered cosmological constraints and compare our results with recent results constrained from some cosmological probes and summarize our conclusions in Section 5. Throughout the paper we adopt $\Lambda$CDM with $\Omega_m = 1 - \Omega_\Lambda$, and $H_0 = 100~h$ km s$^{-1}$ Mpc$^{-1}$.

2. The GalWCat19 Cluster Catalog

In this section, we summarize how we created the GalWCat19 cluster catalog. Full details may be found in Abdullah+20. Using photometric and spectroscopic databases from SDSS-DR13, we extracted data for 704,200 galaxies. These galaxies satisfied the following set of criteria: spectroscopic detection, photometric and spectroscopic classification as a galaxy (by the automatic pipeline), spectroscopic redshift between 0.001 and 0.2 (with a redshift completeness >0.7, Yang et al. 2007; Tempel et al. 2014), r-band magnitude (reddening-corrected) <18, and the flag SpecObj.zWarning is zero indicating a well-measured redshift.

Galaxy clusters were identified by the well-known Finger-of-God effect (Jackson 1972; Kaiser 1987; Abdullah et al. 2013). The Finger-of-God effect causes a distortion of line-of-sight velocities of galaxies in the redshift-phase space due to the cluster potential well. As described in Abdullah+20, we calculated the membership of each cluster as follows. We first calculated the galaxy number density within a cylinder of radius 0.5 $h^{-1}$ Mpc and height 3000 km s$^{-1}$ centered on a galaxy. Second, we sorted all galaxies descending from highest to lowest number densities with the condition that the cylinder has at least eight galaxies. Third, starting with the galaxy with the highest number density, we applied the binary tree algorithm (e.g., Serra et al. 2011) to accurately determine a cluster center ($\alpha_0$, $\delta_0$, $z_0$) and a phase-space diagram. Fourth, we applied the GalWeight technique (Abdullah+18) to galaxies in the phase-space diagram out to a maximum projected radius of 10 $h^{-1}$ Mpc and a maximum line-of-sight velocity range of ±3000 km s$^{-1}$ to identify cluster membership. In Abdullah+18, we showed that the cumulative completeness of the FOG algorithm, which we tested using the Bolshoi simulation (Klypin et al. 2016) was approximately 100% for clusters with masses $M_{200} > 2 \times 10^{14} h^{-1}M_\odot$, and ∼85% for clusters with masses $M_{200} > 0.4 \times 10^{14} h^{-1}M_\odot$.

The virial mass of each cluster was estimated by applying the virial theorem to the cluster members, under the assumption that the mass distribution follows the galaxy distribution (e.g., Giuricin et al. 1982; Merritt 1988). The estimated mass was corrected for the surface pressure term which, otherwise, would overestimate the fiducial cluster mass (e.g., The & White 1986; Binney & Tremaine 1987; Carlberg et al. 1997). The cluster virial mass was calculated at the virial radius within which the cluster is in hydrostatic equilibrium. The virial radius is approximately equal to the radius at which the density $\rho = \Delta_{200}/\rho_s$, where $\rho_s$ is the critical density of the universe and $\Delta_{200} = 200$ (e.g., Carlberg et al. 1997; Klypin et al. 2016). Abdullah+20 showed that the cluster mass estimates returned by the virial theorem after utilizing the GalWeight technique (Abdullah+18) performed very well in comparison to most other mass estimation techniques described in Old et al. (2015). In particular, our procedure was applied to two mock catalogs (HOD2 and SAM2) recalled from Old et al. (2015). We found that the rms differences of the recovered mass by GalWeight relative to the fiducial cluster mass were 0.24 and 0.32 for HOD2 and SAM2, respectively. Also, the intrinsic scatter in the recovered mass was ∼0.23 dex for both catalogs. Moreover, the uncertainty of the virial mass estimator is calculated using the limiting fractional uncertainty $\pi^{-1}\sqrt{2}\ln N/N$ (Bahcall & Tremaine 1981).
The scatter and bias in the recovered mass using the virial mass estimator are caused by some factors, including (i) the assumption of hydrostatic equilibrium, projection effect, and possible velocity anisotropies in galaxy orbits, and the assumption that halo mass follows light (or stellar mass); (ii) the presence of substructure and/or nearby structure such as a cluster, or supercluster, to which the cluster belongs, or filament (e.g., Merritt 1988; Fadda et al. 1996); (iii) the presence of interlopers in the cluster frame due to the triple-value problem, for which there are some foreground and background interlopers that appear to be part of the cluster body because of the distortion of phase space (Tonry & Davis 1981; Abdullah et al. 2013); and (iv) the identification of cluster center (e.g., Girardi et al. 1998; Zhang et al. 2019).

The 1800 GalWCat19 clusters range in redshift between 0.01–0.2 and in mass between (4.4–14) × 10^13 h^−1 M_☉. The GalWCat19 catalog contains a large number of cluster parameters including sky position, redshift, membership, velocity dispersion, and mass at overdensities Δ = 500, 200, 100, 5.5. The 34,471 member galaxies were identified within the radius at which the density is 200 times the critical density of the universe. The galaxy catalog provided the coordinates of each galaxy and the ID of the cluster that the galaxy belongs to. The catalog was made publicly available at the following website: https://mohamed-elhashash-94.webself.net/galwcat/.

3. Cluster Mass Function

The GalWCat19 catalog is not complete in either volume or mass. In Section 3.1, we analyze GalWCat19 to develop an appropriate selection function of our sample, which is used to correct for the volume incompleteness. Also, in Section 3.2, we compute the CMF derived from GalWCat19 and compare it with the CMF calculated from the MDPL2 simulation (described in the next paragraph) to obtain a mass-complete subsample (SelGMC) used to constrain the cosmological parameters Ω_m and σ8.

The MDPL2 is an N-body simulation of 3840^3 particles in a box of comoving length 1 h^−1 Gpc, mass resolution 1.51 × 10^8 h^−1 M_☉, and gravitational softening length 5 h^−1 kpc (physical) at redshifts from the suite of MultiDark simulations (see Table 1 in Klypin et al. 2016). It was run using the L-GADGET-2 code, a version of the publicly available cosmological code GADGET-2 (Springel 2005). It assumes a flat ΛCDM cosmology, with cosmological parameters Ω_Λ = 0.693, Ω_m = 0.307, Ω_b = 0.048, n = 0.967, σ8 = 0.823, and h = 0.678 (Planck Collaboration et al. 2014). Halos and subhalos have been identified with ROCKSTAR (Behroozi et al. 2013a) and merger trees constructed with CONSISTENT TREES (Behroozi et al. 2013b). The catalogs are split into 126 snapshots between redshifts z = 17 and z = 0. We downloaded the snapshot (hlist_0.91520.list) with z ~ 0.09, which is consistent with the mean redshift of the GalWCat19 sample.

3.1. GalWCat19 Completeness

The GalWCat19 catalog is incomplete in the distribution of clusters with respect to comoving distance (redshift), and in the distribution of clusters with respect to mass. In this section, we discuss such incompleteness and how to make corrections.

The completeness in comoving volume (redshift) of the GalWCat19 catalog can be investigated by calculating the abundance of clusters predicted by a theoretical model and comparing it with the abundance of GalWCat19 clusters. We adopt the functional form of Tinker et al. (2008; hereafter Tinker08) to calculate the halo mass function (HMF); see Section 4.1 for more details and consequently the predicted abundance of clusters.

The integrated abundance of clusters as a function of redshift for the GalWCat19 sample, N(<z), is presented in the upper left panel of Figure 1. Note that N(<z) is calculated for the clusters with redshift z ≥ 0.04 to remove the effect of nearby regions where the cosmic variance has a large effect due to the small volume. The plot shows that the catalog is matched with the prediction of Tinker08 for z ≤ 0.09. Also, the fractional error of N(<z) relative to the expectation of Tinker08, (N(<z)obs − N(<z)model)/N(<z)model), for each model and the expected Poisson noise (gray shaded area) are presented in the lower left panel. The plot shows that the scatter relative to each model is nearly constant (around zero) for z ≤ 0.09 before it blows up after this redshift limit. This indicates that GalWCat19 is approximately complete in volume for z ≤ 0.09 (or equivalently comoving distance of D ≤ 265 h^−1 Mpc for the ΛCDM universe with Ω_m = 0.3). We call this volume-complete subsample NoSelPVC.

Similarly, the integrated abundance of clusters as a function of cluster mass, N (> M), is presented in the upper right panel of Figure 1 in comparison to five Tinker08 models and the scatter is presented in the lower right panel. The plot shows that the data is matched with the models of Ω_m = [0.20, 0.305, 0.40] with σ8 = 0.825 better than the models of Ω_m = 0.305 and σ8 = [0.725, 0.925]. Even though it is not an easy task to specifically determine the mass threshold at which the catalog is complete, the three matched models indicate that GalWCat19 is approximately complete for log(M) ≥ 13.9 h^−1 M_☉. We discuss the systematics of adapting this mass threshold on our analysis in Section 5.1. The large scatter at the high-mass end is due to the small number of massive clusters, while the large scatter at the low mass end comes from the incompleteness of GalWCat19.

In order to correct for the incompleteness in volume of GalWCat19 each cluster should be weighed by S(D), where S is the selection function at a distance D. Figure 2 introduces the normalized number density N_n(D), defined as the cluster number density normalized by the average number density calculated for clusters within comoving distance D < 265 h^−1 Mpc, for all clusters and for five mass bins as described in Table 1. The distribution of points in Figure 2 can be described by an exponential function that represents the selection function S(D). It has the form

\[ S(D) = a \exp \left[ -\left( \frac{D}{b} \right)^\gamma \right]. \]  

The parameters a, b, and γ are determined by applying the chi-squared algorithm using the Curve Fitting MatLab Toolbox. The best-fit values of these parameters are a = 1.07 ± 0.12, b = 293.4 ± 20.7 h^−1 Mpc, and γ = 2.97 ± 0.90 with an rms error of 0.15. Note that the normalization a is greater than unity because of the scatter and the effect of the cosmic variance.

9 We use CMF for mass functions derived from observations and HMF for mass functions computed by theoretical models.
However, we apply the selection function with the condition that

$$\sum D_1.$$ We should be cautious when using $$\sum D$$ at large distances. This is because $$S(<z_{\text{obs}}) / S(<z_{\text{model}})$$ drops to $$0.01$$ as demonstrated in Figure 2, which means that a distant cluster would be weighted at least 100 times as a nearby cluster. This will overestimate or overcorrect the number of clusters at large distances, and consequently the estimated CMF will be noisy.

Thus, in order to avoid the overcorrection and the noisiness of CMF we restrict our sample to a maximum comoving distance of $$D < 365$$ (or $$z < 0.125$$) for which $$\sum$$.

It is well-known that the cluster number density of a given mass decreases with redshift for a 100% complete sample because of the HMF evolution effect. Thus, the CMF should be

Figure 1. GalWCat19 completeness. Left: the black line shows the integrated abundance of clusters as a function of redshift for the GalWCat19 catalog. The dashed lines present the expectation of complete samples estimated by Tinker08 for five different cosmologies as shown in the legend. Right: the black line shows the integrated abundance of clusters as a function of mass. The dashed lines present the expectation of complete samples estimated by Tinker08 for five different cosmologies as shown in the legend. The fractional error $$(N(<z_{\text{obs}}) - N(<z_{\text{model}})) / N(<z_{\text{model}})$$ is shown in the lower panels. The gray shaded areas represent the expected Poisson noise.

Figure 2. Selection function of GalWCat19 cluster sample. Colored points show the normalized number density of the five mass bins described in Figure 1. The black line shows an exponential form describing the selection function $$S(D)$$, which is fitted with the data. The scatter of data relative to the exponential form is presented in the lower panel.

### Table 1

| Mass Bin ($h^{-1} M_\odot$) | Number of Clusters | Average Number Density ($10^{-5} h^3 \text{Mpc}^{-3}$) | Color |
|-----------------------------|--------------------|------------------------------------------------------|-------|
| 13.6–15.2                  | 1800               | 5.6                                                  | Black |
| 13.6–13.8                  | 527                | 2.2                                                  | Blue  |
| 13.8–14.0                  | 461                | 1.5                                                  | Green |
| 14.0–14.2                  | 411                | 1.0                                                  | Red   |
| 14.2–14.5                  | 326                | 0.7                                                  | Cyan  |
| 14.5–15.2                  | 75                 | 0.2                                                  | Magenta |

**Note.** Columns: (1) the mass bin in units of log M ($h^{-1} M_\odot$); (2) the number of clusters in each mass bin; (3) the average number density calculated for clusters within comoving distance $$D < 265 h^{-1}$$ Mpc in each mass bin; (4) the color of the number density profile as shown in the right panel of Figure 1.

However, we apply the selection function with the condition that $$S(D) \leq 1$$.

We should be cautious when using $$S(D)$$ at large distances. This is because $$S(D \gtrsim 500) h^{-1}$$ Mpc drops to $$\gtrsim 0.01$$ as demonstrated in Figure 2, which means that a distant cluster would be weighted at least 100 times as a nearby cluster. This will overestimate or overcorrect the number of clusters at large distances, and consequently the estimated CMF will be noisy. Thus, in order to avoid the overcorrection and the noisiness of CMF we restrict our sample to a maximum comoving distance of $$D \leq 365$$ (or $$z \leq 0.125$$) for which $$S(D) \lesssim 0.2$$.

It is well-known that the cluster number density of a given mass decreases with redshift for a 100% complete sample because of the HMF evolution effect. Thus, the CMF should be
scaled or corrected by an evolution function, $S_{\text{evol}}(D)$. For a sample with a broad range of redshifts, the only way to take the evolution into account is to calculate this function. However, the disadvantage of this approach is that the correction is model dependent: the measured HMF (i.e., CMF) is a convolution of the true HMF and theoretical estimate of $S_{\text{evol}}(D)$. However, for a sample with a narrow range of redshifts (as in our case) we show in Appendix A that the evolution effect is less than 3% for clusters in the redshift range of $0.045 \leq z \leq 0.125$. In Appendix B, we discuss the effect of adopting this redshift interval in our results.

3.2. Estimating the Mass Function

In this section, we compute the CMF, $dn(M)/d\log(M)$, and its corresponding cumulative mass function, $n(>M)$, which are estimated for a $\Lambda$CDM cosmology with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$. The CMF is defined as the number density of clusters per logarithmic cluster mass interval. Also, the cumulative CMF is defined as the number density of clusters more massive than a given mass $M$.

Mathematically, the CMF, weighted by the selection function $S$, is given by

$$\frac{dn(M)}{d \log M} = \frac{1}{d \log M} \sum_i \frac{1}{V} \frac{1}{S(D_i)} ,$$

where $D_i$ is the comoving distance of a cluster $i$, and $V$ is the comoving volume, which is given by

$$V = \frac{4\pi}{3} \Omega_{\text{sky}} (D_2^3 - D_1^3) ,$$

where $\Omega_{\text{sky}} = 41.253$ deg$^2$ is the area of the sky, $\Omega_{\text{sur}} \approx 10,000$ deg$^2$ is the area covered by GalWCat19, and $D_1$ and $D_2$ are the minimum and maximum comoving distances of the cluster sample.

Figure 3 introduces the cumulative CMF computed from GalWCat19. The black line is the CMF computed from the MDPL2 simulation (for the snapshot hlist_0.91520.list at $z \sim 0.9$ or $D \sim 265$; Klypin et al. 2016). The blue points introduce the CMF for NoSelFVC without the correction of $S(D)$, since this sample is already complete in volume (see Section 3.1 and Figure 1). The red points represent our CMF corrected by $S(D)$ for $D \leq 365$ h$^{-1}$ Mpc ($z \sim 0.125$). Comparing the CMF estimated by the NoSelFVC subsample with that derived from the MDPL2 simulation indicates that the sample is approximately complete in mass for $\log(M) \gtrsim 13.9$ h$^{-1}$ $M_\odot$, while it drops lower than the CMF of MDPL2 at the low-mass end. Also, our CMF, corrected by $S(D \leq 365)$, is in good agreement with the CMF derived from NoSelFVC with a scatter of 0.026 dex. The mass completeness of GalWCat19 is discussed in Section 3.1 and Figure 1. In Appendix B, we show that the results of deriving the cosmological parameters from NoSelFVC are consistent with that derived from SelFMC. This indicates that weighting each cluster in our sample by $S(D \leq 365)$ introduced in Section 3.1 and Equation (1) is sufficient to correct for the volume incompleteness of GalWCat19.

Therefore, our final subsample, corrected by $S(D)$ is restricted by $\log(M) \gtrsim 13.9$ h$^{-1}$ $M_\odot$ and $0.045 \leq z \leq 0.125$. The number of clusters of this subsample is 756, which represents $\sim 42\%$ of the GalWCat19 sample. We use this subsample to constrain $\Omega_m$ and $\sigma_8$ and call it a fiducial SelFMC sample.

4. Implications for Cosmological Models

In Section 4.1, we discuss the prediction of HMF from the theoretical framework. In Section 4 we derive the constraints on the cosmological parameters $\Omega_m$ and $\sigma_8$, and discuss the degeneracy between these two parameters.

4.1. Prediction of HMF

The number of dark matter halos per unit mass per unit comoving volume of the universe, HMF, is given by

$$\frac{dn}{d \ln M} = f(\sigma) \frac{\rho_0}{M} \left| \frac{d \ln \sigma}{d \ln M} \right| ,$$

here $\rho_0$ is the mean density of the universe, $\sigma$ is the rms mass variance on a scale of radius $R$ that contains mass $M = 4\pi \rho_0 R^3/3$, and $f(\sigma)$ represents the functional form that defines a particular HMF fit.

Assuming a Gaussian distribution of mass fluctuation, Press & Schechter (1974) used a linear theory to derive the first theoretical model (hereafter PS) of HMF. While fairly successful in matching the results of N-body simulations, the PS formalism tends to predict too many low-mass clusters and
too few high-mass clusters. More recently proposed theoretical models provide better approximations to the output from N-body simulations (e.g., Jenkins et al. 2001; Sheth et al. 2001; Warren et al. 2006; Tinker & Wetzel 2010; Bhattacharya et al. 2011; Behroozi et al. 2013a).

In this paper, we adopt the functional form proposed by Tinker et al. (2008) (hereafter Tinker08) as our form of the HMF. This approach assumes universality of the HMF across the cosmological parameter space considered in this work, and uses a fitting function that was calibrated against N-body simulations. The Tinker08 model is formally accurate to better than 5% for the cosmologies close to the ΛCDM cosmology and for the mass and redshift range of interest in our study (e.g., Vikhlinin et al. 2009). Although the formula has been calibrated using dissipationless N-body simulations (i.e., without the effect of baryons), hydrodynamic simulations suggest that these have negligible impact for clusters with masses as high as those considered here (e.g., Rudd et al. 2008; Velliscig et al. 2014; Bocquet et al. 2016). Finally, note that the Tinker08 model is defined in spherical apertures enclosing overdensities similar to the mass we derive for the GalWCat19 observed clusters.

\[
f(\sigma, z) = A \left[ \frac{\sigma}{b} \right]^{-a} + 1 \exp(-c/\sigma^2) \tag{5}\]

where \(A = 0.186(1 + z)^{-0.14}, a = 1.47(1 + z)^{-0.06}, b = 2.57(1 + z)^{-0.1}, c = 1.19, \) and \(\ln \alpha(\Delta_{\text{in}}) = [75/(\ln(\Delta_{\text{in}}/75))]^{1/2},\) and \(\sigma^2\) is the mass variance defined as

\[
\sigma^2(M, z) = \frac{g(z)}{2\pi} \int P(k) W^2(kR) k^2 dk \tag{6}
\]

\(P(k)\) is the current linear matter power spectrum (at \(z = 0\)) as a function of wavenumber \(k, W(kR) = 3[\sin(kR) - kR \cos(kR)]/(kR)^3\) is the Fourier transform of the real-space top-hat window function of radius \(R,\) and \(g(z) = \sigma_8(z)/\sigma_8(0)\) is the growth factor of linear perturbations at scales of 8 \(h^{-1}\) Mpc, normalized to unity at \(z = 0.\)

The current linear power spectrum \(P(k)\) is defined as \(P(k) = B k^{n_8} T^2(k),\) where \(T(k)\) is the transfer function, \(B\) is the normalization constant and \(n\) is the spectral index. Usually the normalization \(B\) is calculated from the cosmological parameter \(\sigma_8\) (e.g., Reiprich & Böhringer 2002; Murray et al. 2013). The function \(k^8\) imprints the primordial power spectrum during the epoch of inflation. The transfer function \(T(k)\) quantifies how this primordial form is evolved with time to the current linear power spectrum on different scales. The transfer function \(T(k)\) is calculated using the public Code for Anisotropies in the Microwave Background (CAMB; Lewis et al. 2000). The quantities \(\Omega_m\) and \(\sigma_8\) are the main cosmological parameters that define the HMF. The other parameters do not strongly affect the HMF and thus we fix them during the calculation of the HMF as described below (e.g., Reiprich & Böhringer 2002; Bahcall et al. 2003; Wen et al. 2010).

4.2. Constraining \(\Omega_m\) and \(\sigma_8\)

The HMF is calculated using the publicly available HMFcalc\(^{11}\) code (Murray et al. 2013). The code provides about 20 fitting functions that can be used to calculate the HMF. In this paper, in order to constrain \(\Omega_m\) and \(\sigma_8,\) we use Tinker08 (Equation (5)) as discussed above. We calculate the HMF by allowing \(\Omega_m\) to range between [0.1, 0.6] and \(\sigma_8\) between [0.6, 1.2], both in steps of 0.005. We keep the following cosmological parameters fixed: the CMB temperature \(T_{\text{CMB}} = 2.725\) K, baryonic density \(\Omega_b = 0.0486,\) and spectral index \(n = 0.967\) (Planck Collaboration et al. 2014), at redshift \(z = 0.089\) (the mean redshift of GalWCat19).

In order to determine the best-fit mass function and constrain \(\Omega_m\) and \(\sigma_8\) we use a standard \(\chi^2\) procedure

\[
\chi^2 = \sum_{i=1}^{N} \frac{(y_{o,i} - y_{m,i})^2}{\sigma_i^2} \tag{7}
\]

where the likelihood, \(L(y|\sigma_8, \Omega_m)\), of data (CMF) given a model (HMF) is

\[
L(y|\sigma_8, \Omega_m) \propto \exp \left( -\chi^2(y|\sigma_8, \Omega_m) / 2 \right) \tag{8}
\]

\(y_o\) and \(y_m\) are the data and model cumulative mass functions at a given mass and \(\sigma\) is the statistical uncertainty of the data.

Using the fiducial SelFMC sample of 756 clusters with \(\log(M) \geq 13.9\) and 0.045 \(\leq z \leq 0.125,\) the best-fit parameters for the minimum value of \(\chi^2\) are \(\Omega_m = 0.310^{+0.025}_{-0.029}\) and \(\sigma_8 = 0.810^{+0.039}_{-0.034}\) for Tinker08 at redshift \(z = 0.085.\) In Section 5.1 we discuss the systematics of cluster mass uncertainty, mass threshold, and selection function.

The banana shape in Figure 4 shows the well-known degeneracy between \(\sigma_8\) and \(\Omega_m.\) The relationship between \(\sigma_8\) and \(\Omega_m\) is often expressed as

\[
\sigma_8 = \alpha \Omega_m^\beta. \tag{9}
\]

The parameters \(\alpha, \beta,\) and \(\delta\) are determined by applying the \(\chi^2\) algorithm using the Curve Fitting MatLab. The best-fit values of

![Figure 4. Likelihood contour map of \(\chi^2\) in the \(\sigma_8-\Omega_m\) plane derived from the SelFMC cluster catalog. The black star represents the best-fit point for \(\Omega_m\) and \(\sigma_8\) which minimizes the \(\chi^2\) value. Ellipses show 1σ, 2σ, and 3σ confidence levels, respectively. The dashed yellow line represents the best-fit \(\sigma_8-\Omega_m\) relation as shown in the legend.](image-url)
these parameters are $\alpha = 0.425 \pm 0.006$ and $\beta = -0.550 \pm 0.007$ with an rms error of 0.005 for the Tinker08 model.

We now ask the question—how do $\Omega_m$ and $\sigma_8$ contribute individually to the HMF? In other words, why do cluster abundance studies introduce a degeneracy between $\Omega_m$ and $\sigma_8$? The degeneracy occurs because a low abundance of massive clusters could be caused either by a small amount of matter in the universe (a low value of $\Omega_m$) or small fluctuations in the density field (a low value of $\sigma_8$). Similarly, a high abundance of massive clusters could be caused either by a large amount of matter in the universe (a high value of $\Omega_m$) or large fluctuations in the density field (a high value of $\sigma_8$). Therefore, it is possible to obtain the same abundance of massive clusters by fixing one parameter and varying the other one. Figure 5 introduces two sets of HMFs calculated by Tinker08. The first set is shown on the left panel for five different values of $\Omega_m = [0.1 \ 0.2 \ 0.3 \ 0.4 \ 0.5]$, while fixing $\sigma_8 = 0.8$. The second set is shown on the right panel for five different values of $\sigma_8 = [0.6 \ 0.7 \ 0.8 \ 0.9 \ 1.0]$ while fixing $\Omega_m = 0.3$. As expected, increasing the matter density of the universe increases the number of clusters of all masses. But increasing the rms mass fluctuation increases the number of high-mass clusters more dramatically than the number of low-mass clusters. In other words, $\sigma_8$ is very sensitive to the high-mass end of the HMF.

5. Discussion and Conclusion

In this section, we investigate how systematics affect the recovered cosmological constraints from our analysis (Section 5.1). We compare our constraints on the cosmological parameters $\Omega_m$ and $\sigma_8$ with those obtained from cluster abundance studies (Section 5.2). We also compare our constraints with those obtained from other cosmological probes, which we refer to as non-cluster cosmological probes (Section 5.3).

5.1. Systematics

In constraining $\Omega_m$ and $\sigma_8$ in Section 4.2, we only account for the statistical uncertainty of the estimated cumulative CMF using the fiducial SelFMC sample. In this section, we discuss the systematics due to mass uncertainty, mass threshold, and parameterization of the selection function.

5.1.1. Mass Uncertainty

The first uncertainty comes from the difficulty of calculating cluster masses accurately. Generally, masses that are estimated using scaling relations, such as luminosity, richness, temperature, and dispersion velocity–mass relations, introduce large scatter and consequently large systematic uncertainties (e.g., Mantz et al. 2016; Mulroy et al. 2019). Masses that are computed by dynamical estimators are subject to systematic uncertainties (e.g., Wojtak & Łokas 2007; Rozo et al. 2010; Old et al. 2018). However, using the virial theorem, corrected for the surface pressure term, provides a relatively unbiased estimation of cluster masses (e.g., Rines et al. 2010; Ruel et al. 2014), particularly when using a sophisticated interloper rejection technique such as GalWeight (Abdullah+18). Also, the virial mass estimator calculates the total cluster mass including baryonic (gas and galaxies) and dark matter regardless of the internal complex physical processes associated with the baryonic component in clusters. However, the virial mass estimator still introduces scatter in estimating cluster masses (see Section 2). Abdullah+20 showed that the application of the virial mass estimator on two mock catalogs (HOD2 and SAM2) recalled from Old et al. (2015) returned intrinsic scatter of $\sim 0.23$ dex in the recovered mass relative to the fiducial cluster mass. Also, the GalWCat19 catalog introduced the fractional uncertainty (see Section 2) of each cluster mass.

Assuming a normal distribution, we investigate the systematics of the mass uncertainty by generating $\sim 8000$ estimate for each cluster mass using both the fractional uncertainty for each cluster and the intrinsic scatter for the entire sample. In other words, we reanalyze SelFMC $\sim 8000$ times and refit for $\Omega_m$ and $\sigma_8$ for each time. The left panel of Figure 6 introduces the effect of cluster mass uncertainty on the constraints on $\Omega_m$ and $\sigma_8$. Using the fractional uncertainty, we obtain $\Omega_m = 0.305 \pm 0.014$ and $\sigma_8 = 0.816 \pm 0.021$, where the red ellipse represents 68%...
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Abdullah, Klypin, & Wilson

The question now is why are the cosmological constraints derived from many of the cluster abundance techniques in tension with each other? All cluster samples constructed from photometric surveys or detected by the SZ effect do not return an estimate of each cluster’s mass directly. For such samples the cluster mass has to be inferred indirectly from other observables, which scale tightly with cluster mass. Among these mass proxies are X-ray luminosity, temperature, the product of X-ray temperature and gas mass (e.g., Vikhlinin et al. 2009; Mantz et al. 2016), richness (e.g., Yee & Ellingson 2003; Simet et al. 2017), and SZ signal (e.g., Bocquet et al. 2019). To estimate cluster masses for the clusters in these samples it is necessary to follow-up a subset of clusters and calculate their masses using, e.g., weak lensing or X-ray observations. Then, an observable–mass relation can be calibrated for these subsamples. Finally, the mass of each cluster in the sample can be estimated from this scaling relation. However, this reliance on observable–mass proxies introduces significant systematic uncertainties, which is the dominant source of error (e.g., Henry et al. 2009; Mantz et al. 2015) for the reasons explained in the next paragraph.

First, the masses obtained for the follow-up subsample of clusters are often biased. For example, it is known that X-ray mass estimates are typically biased low and so a mass bias factor, (1−β), needs to be introduced and calibrated. Second, the size of the subsample used for calibration is usually small (tens of clusters), which introduces large uncertainties in both the slope and the normalization of the scaling relation. Third, many cluster catalogs span a large redshift range, so evolution (due to

5.2. Comparison with External Data from Cluster Abundance

The left panel of Figure 7 introduces the 68% CL derived from SelFMC in comparison to the results obtained from other cluster abundance studies. Samples of galaxy clusters constructed from galaxy surveys include optical photometric data (e.g., Kirby et al. 2019), X-ray (e.g., Mantz et al. 2015), and SZ (e.g., Zubeldia & Challinor 2019) catalogs as listed in Table 2. The figure shows that the CLs of all cluster abundance studies introduce a degeneracy between $\Omega_m$ and $\sigma_8$ as we discussed in Section 4.2. Also, the CL derived from SelFMC overlaps the CLs obtained from all other results as shown in the figure. Regardless of this overlapping, the right panel of Figure 7 shows that the constraints on $\Omega_m$ and $\sigma_8$ from cluster abundance studies are in tension with each other, even for the studies that use the same type of cluster sample. Specifically, the X-ray independent studies listed in Table 2 introduce different values of $\Omega_m$ and $\sigma_8$, which vary from ~0.22 to 0.40 and 0.71 to 0.89, respectively. Also, the independent studies that use SZ-cluster samples show that $\Omega_m$ and $\sigma_8$ vary from ~0.25 to 0.31 and 0.77 to 0.98, respectively.

Figure 6. Effects of cluster mass uncertainty (left), mass threshold (middle), and selection function (right) on our constraints on $\Omega_m$ and $\sigma_8$. Left: the 68% CLs of our fiducial sample (black), fractional mass uncertainty (blue), and intrinsic scatter of 0.23 (red). Middle: the 68% CLs (green) for varying mass threshold $\log M$ from 13.8 to 14 $h^{-1} M_{\odot}$. Right: the 68% CLs (magenta) due to systematic of the selection function.

5.1.1. Selection Function Parameterization

The constraints on $\Omega_m$ and $\sigma_8$ are affected by parameterization of the selection function. Our selection function depends on three parameters $a$, $b$, and $\gamma$. The normalization $a$ is already fixed to unity. Assuming a normal distribution, the systematic of the selection function is investigated by generating ~8000 pairs of $b$ and $\gamma$, using the uncertainty in $b$ and $\gamma$ (see Section 3.1). For each pair we estimate the best-fit values of $\Omega_m$ and $\sigma_8$. Figure 6 shows the 68% CL for the systematic of the selection function. This analysis rotates the error ellipses slightly compared to our fiducial analysis, but does not affect our results. We obtain $\Omega_m = 0.313 \pm 0.035$ and $\sigma_8 = 0.809 \pm 0.012$, which is consistent with our result of the fiducial sample.

5.1.2. Mass Threshold

The second systematic uncertainty comes from the difficulty of accurately determining the mass threshold at which the sample is mass complete. As discussed in Section 3.1 and Figure 1 the catalog is approximately complete around $\log M \gtrsim 13.9$ [$h^{-1} M_{\odot}$]. However, the mass threshold at which the sample is mass-complete is not accurately specified. Therefore, we investigate the effect of varying the mass threshold $\log M$ between 13.8 and 14.0 [$h^{-1} M_{\odot}$] in steps of 0.05 dex on the recovered cosmological constraints from our analysis. For each mass threshold we calculate the $\chi^2$ likelihood and then we obtain the joint 68% CL of all $\chi^2$ distributions as shown in the middle panel of Figure 6. The plot shows that the best-fit values of $\Omega_m$ and $\sigma_8$ deviate very slightly from the results of the fiducial sample with $\Omega_m = 0.300^{+0.015}_{-0.015}$ and $\sigma_8 = 0.820^{+0.020}_{-0.023}$.

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both the evolution of the universe and the physical processes of baryons in clusters) in the the scaling relations used to estimate the masses needs to be carefully handled, introducing another source of uncertainty. All of the aforementioned assumptions can introduce large uncertainties in the estimates of cluster mass and consequently the constraints on cosmological parameters. For instance, $\sigma_8$ is specifically very sensitive to the high-mass end of the CMF and any offset of cluster true masses leads to biased estimation of $\Omega_m$. Other observational systematics that introduce additional uncertainties are photometric redshift errors and cluster miscentering.

By using the GalWiCat19 cluster catalog and deriving cluster masses using the virial theorem, we were able to avoid most of the complexities described above. First, we were able to identify clusters, assign membership, and determine cluster centers and redshifts with high accuracy from the high-quality SDSS spectroscopic data set. Second, cluster membership was determined by the GalWeight technique, which has been shown to be $\sim98\%$ accurate in assigning cluster membership (Abdullah+18). Third, a mass for each cluster was determined directly using the virial theorem. Therefore, we were able to recover a total (dark plus baryonic) mass for each cluster and circumvent having to make any assumptions about the complicated physical processes associated with the baryons. It has been suggested that cluster masses estimated via the virial theorem are overestimated by $20\%$. But we note that we have applied a correction for the surface pressure term, which we believe decreases this bias, especially when applied in combination with our GalWeight membership technique (Abdullah+18). Abdullah+20 showed that the virial mass estimator performed well in comparison to the other mass estimators described in Old et al. (2015), and resulted in a relatively low bias and scatter when applied to two semianalytical simulations (see Figure 3 in Abdullah+20). Fourth, since GalWiCat19 is a low-redshift cluster catalog it eliminates the need to make any assumptions about evolution in clusters themselves and evolution in cosmological parameters. Finally, because of the large size of the GalWiCat19 we are able to determine the CMF well and consequently constrain the cosmological parameters $\Omega_m$ and $\sigma_8$ with high precision.

5.3. Comparison with External Data from Noncluster Cosmological Probes

Cosmological parameters can be estimated from different cosmological probes rather than cluster abundance studies. We use measurements of primary CMB anisotropies from both WMAP (9 yr data; Hinshaw et al. 2013) and Planck satellites focused on the TT+lowTEB data combination from the 2018 analyses (Planck Collaboration et al. 2018). We also use angular diameter distances as probes by BAO including the 6dF Galaxy Survey (Beutler et al. 2011), the SDSS Data Release 7 (Ross et al. 2015), and the BOSS Data Release 12 (Alam et al. 2017). Furthermore, we use measurements of luminosity distances from Type Ia supernovae from the Pantheon sample (Sconico et al. 2018). Finally, we use the measurements from a joint analysis of three cosmological probes: cosmic shear, galaxy–galaxy lensing, and angular clustering, including the results of the Kilo Degree Survey and the Galaxies And Mass Assembly survey (KiDS+GAMA; van Uitert et al. 2018) and the first year of the Dark Energy Survey (DES Y1; Abbott et al. 2018b; see Table 2). The left panel of Figure 8 introduces the 68\% CL derived from SelFMC in comparison to the the scaling relations used to estimate the masses needs to be carefully handled, introducing another source of uncertainty. All of the aforementioned assumptions can introduce large uncertainties in the estimates of cluster mass and consequently the constraints on cosmological parameters. For instance, $\sigma_8$ is specifically very sensitive to the high-mass end of the CMF and any offset of cluster true masses leads to biased estimation of $\Omega_m$. Other observational systematics that introduce additional uncertainties are photometric redshift errors and cluster miscentering.

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We define the scatter

$$
\Delta_{pl} = \left( \frac{\Omega_{m,\text{ref}} - \Omega_{m,pl}}{\Omega_{m,pl}} \right)^2 + \left( \frac{\sigma_{8,\text{ref}} - \sigma_{8,pl}}{\sigma_{8,pl}} \right)^2,
$$

(10)

to compare the constraints on $\Omega_m$ and $\sigma_8$ obtained from all cosmological probes, which are listed in Table 2 with that
In this paper, we derived the CMF and the cosmological parameters \( \Omega_m \) and \( \sigma_8 \) at low redshift. We combine our 68% C.L. constraints on \( \Omega_m \) and \( \sigma_8 \) obtained from Planck18 with those obtained from Planck18, BAO, and Pantheon to eliminate the degeneracy of the our likelihood and to remarkably shrink the uncertainties of the cosmological parameters. The joint analysis gives \( \Omega_m = 0.315_{-0.001}^{+0.004} \) and \( \sigma_8 = 0.810_{-0.01}^{+0.013} \).

### 5.4 Conclusion

In this paper, we derived the CMF and the cosmological parameters \( \Omega_m \) and \( \sigma_8 \) using a mass-complete subsample of 756 clusters (SelFMC) obtained from the GalWCat19 cluster catalog, which was constructed from the SDSS-DR13 spectroscopic data set. The advantages of using these catalogs are (i) we were able to identify clusters, assign

| Sample       | Mass Estimation | \( \Omega_m \) | \( \sigma_8 \) | \( \Delta \sigma_8 \) | Reference                  |
|--------------|-----------------|----------------|---------------|-------------------|----------------------------|
| GalWCat19    | Virial theorem  | 0.305^{+0.037}_{-0.042} | 0.810^{+0.053}_{-0.056} | 0.817 | 0.032 | This work                  |

**Notes.**
- The cluster normalization condition parameter, \( S_8 \), is defined as \( S_8 = \sigma_8 (\Omega_m / 0.3)^{0.5} \) as used in the literature.
- \( \Delta \sigma_8 = \sqrt{[\Delta \Omega_m (\Omega_m / 0.3)]^2 + [\Delta \sigma_8 (\Omega_m / 0.3)]^2} \) is the scatter of \( \Omega_m \) and \( \sigma_8 \) obtained from each method listed the table relative to that obtained from Planck18 (Planck Collaboration et al. 2018).
- Mantz et al. (2015) used the combination of luminosity, temperature, gas mass, and lensing mass to estimate cluster masses, which was referred to as Weighting the Giant (WtG).
- \( \Delta \sigma_8 = \sqrt{[\Delta \Omega_m (\Omega_m / 0.3)]^2 + [\Delta \sigma_8 (\Omega_m / 0.3)]^2} \) is the scatter of \( \Omega_m \) and \( \sigma_8 \) obtained from each method listed the table relative to that obtained from Planck18 (Planck Collaboration et al. 2018).
- Cha & Jee (2015) used the combination of luminosity, temperature, gas mass, and lensing mass to estimate cluster masses, which was referred to as Weighting the Giant (WtG).

As discussed above there is a degeneracy between \( \Omega_m \) and \( \sigma_8 \) derived from the CMF at low redshift. We combine our 68% C.L. with those obtained from Planck18 + BAO + Pantheon, to eliminate the degeneracy of the our likelihood and to remarkably shrink the uncertainties of the cosmological parameters. The joint analysis gives \( \Omega_m = 0.315_{-0.001}^{+0.004} \) and \( \sigma_8 = 0.810_{-0.01}^{+0.013} \).
membership, and determine cluster centers and redshifts with high accuracy from the high-quality SDSS spectroscopic data set; (ii) cluster membership was determined by the Gal-Weight technique, which has been shown to be \( \sim 98\% \) accurate in assigning cluster membership (Abdullah et al. 2018); (iii) the cluster masses were calculated individually using the virial theorem and corrected for the surface pressure term; (iv) GalWCat19 is a low-redshift cluster catalog that eliminates the need to make any assumptions about evolution in clusters themselves and evolution in cosmological parameters; (v) GalWCat19 is one of the largest available spectroscopic samples in order to be a fair representation of the cluster population.

Our CMF closely matches predictions from MultiDark Planck N-body simulations (snapshot hlist.0_91520_list12, with \( z \sim 0.09 \)) for \( \log(M) \gtrsim 13.9 \, h^{-1} \, M_\odot \). Assuming a flat \( \Lambda \)CDM cosmology, we used the publicly available HMFcalc\(^{13}\) code (Murray et al. 2013) to estimate HMFs for the Tinker08 model (Equation (5)). Then, using a standard \( \chi^2 \) procedure, we compared our cumulative mass function to HMFs to determine the best-fit mass function and constrain \( \Omega_m \) and \( \sigma_8 \). We measured \( \Omega_m \) and \( \sigma_8 \) to be \( \Omega_m = 0.310^{+0.025}_{-0.027} \pm 0.041 \) (systematic) and \( \sigma_8 = 0.810^{+0.031}_{-0.036} \pm 0.035 \) (systematic), with a cluster normalization relation of \( \sigma_8 = 0.43\Omega_m^{0.55} \).

The cosmological constraints we derived are very competitive with those recently derived using both cluster abundance studies and other cosmological probes. In particular, our constraints on \( \Omega_m \) and \( \sigma_8 \) are consistent with Planck18+BAO+Pantheon constraints. This remarkable consistency highlights the potential of using GalWCat19 and its subsample SelFMC, which are derived from the SDSS-DR13 spectroscopic data set utilizing the application of GalWeight to produce precision constraints on cosmological parameters. The joint analysis of our cluster data with Planck18+BAO+Pantheon gives \( \Omega_m = 0.315^{+0.011}_{-0.013} \) and \( \sigma_8 = 0.810^{+0.011}_{-0.010} \).

We would like to thank Steven Murray for making his HMFcalc calculator publicly available and also for his guidance in running it. We would also like to thank Jeremy Tinker, Brian Siana, and Benjamin Forrest for their useful comments and help as well as Shahab Alam for providing us with the chain of BOSS-DR12 BAO data. Finally, we appreciate the comments and suggestions of the reviewer, which improved this paper. This work is supported by the National Science Foundation through grant AST-1517863, by HST program number GO-15294, and by grant number 80NSSC17K0019 issued through the NASA Astrophysics Data Analysis Program (ADAP). Support for program number GO-15294 was provided by NASA through a grant from the Space Telescope Science Institute, which is operated by the Association of Universities for Research in Astronomy, Incorporated, under NASA contract NAS5-26555.

**Appendix A Evolution**

In this section, we discuss the evolution effect for a sample of clusters with a narrow redshift range between \( z_1 \) and \( z_2 \) with an average of \( \langle z \rangle \). The HMF depends on the mass and redshift and is given by \( \int_{z_1}^{z_2} n(M, z) \, dz / (z_2 - z_1) \). We test the effect of evolution assuming an analytical model for the evolution of HMF and cosmological model with reasonable parameters. We then take the integral \( \int_{z_1}^{z_2} n(M, z) \, dz / (z_2 - z_1) \) and compare the results with \( n(M, z) \) at \( z = 0.085 \).

\[^{12}\text{https://www.cosmosim.org/data/catalogs/NewMD_3840_Planck1/ROCKSTAR/trees/hlists/}\]

\[^{13}\text{http://hmf.icrar.org/}\]
Figure A1 shows the evolution of the cluster number density expected by Tinker08 for cosmological parameters $\Omega_m = 0.305$ and $\sigma_8 = 0.825$. In the left panel, we plot the HMF times $M/\rho_c$, where $\rho_c$ is the critical density of the universe, to clarify the differences between the models at different redshifts. The right panel shows the scatter of models relative to the expectation at $z = 0.085$ (the mean redshift of the sample).

Figure A1 shows the evolution of the cluster number density expected by Tinker08 for cosmological parameters $\Omega_m = 0.305$ and $\sigma_8 = 0.825$. In the left panel, we plot the HMF times $M/\rho_c$, where $\rho_c$ is the critical density of the universe, to clarify the differences between the models at different redshifts. The right panel shows the scatter of models relative to the expectation at $z = 0.085$ (the mean redshift of the sample).

The evolution is $>15\%$ for $0.0 \leq z \leq 0.125$ for massive clusters, while it drops to $<3\%$ for $0.045 \leq z \leq 0.125$.

Note that we do not neglect the effects of evolution. In other words, we do not assume that the HMF at $z_1$ is (nearly) the same as at $z_2$ (admittedly, there is a 10\%–20\% difference in the most massive $M$). Because we use ratios of these quantities, most of the cosmological parameters (e.g., $\sigma_8$) are canceled for a sensible range (e.g., $\sigma_8 = 0.75$–0.85). We also test other HMF approximations such as Despali HMF (Despali et al. 2016) and obtain the same conclusion. Therefore, we restrict our data (observed clusters) to $0.045 \leq z \leq 0.125$ for which the evolution effect of the number density of clusters is minimal.
Appendix B
Redshift Threshold

In this section we investigate the choice of the redshift interval and the application on the selection function of our results of the fiducial analysis as shown in Figure B1. In the left panel, we fix the upper redshift threshold to 0.125 and decrease the lower redshift threshold from 0.075 to 0.045. The plots indicate that decreasing the lower redshift threshold does not affect our result for the fiducial sample (black ellipse). It also demonstrates that the evolution effect is unremarkable in this small redshift interval. The left panel also introduces the 68% CL of the NoSelFVC sample (dashed brown ellipse), which gives $\Omega_m = 0.295^{+0.033}_{-0.034}$ (5% less than the fiducial value) and $\sigma_8 = 0.815^{+0.049}_{-0.050}$ (1% greater than the fiducial value). The consistency between the results of SelFMC and NoSelFVC demonstrates that applying the selection function for $z \leq 0.125$ does not affect the results of the fiducial analysis and is sufficient to correct for the volume incompleteness of $\text{GalWCat19}$. In the right panel, we fix the lower redshift threshold to 0.045 and increase the upper redshift threshold from 0.125 to 0.16. The plots indicate that increasing the upper redshift threshold significantly affects our constraints on $\Omega_m$ and $\sigma_8$ because applying the selection function to higher redshift ($>0.125$) affects the shape of the CMF by increasing the scatter and noise and overcorrecting the number of clusters at high redshifts.

Figure B1. Effect of adopting the redshift threshold. Left: 68% CLs for three subsamples, fixing the upper redshift threshold to 0.125 and decreasing the lower redshift threshold from 0.075 to 0.045. The dashed brown ellipse represents the 68% CL of the NoSelFVC sample. Right: 68% CLs for three subsamples, fixing the lower redshift threshold to 0.045 and increasing the upper redshift threshold from 0.125 to 0.16.
