The hyperfine transition in light muonic atoms of odd Z

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The hyperfine (hf) transition rates for muonic atoms have been re-measured for select light nuclei, using neutron detectors to evaluate the time dependence of muon capture. For $^{19}$F $\Lambda_h = 5.6 (2)$ $\mu$s$^{-1}$ for the hf transition rate, a value which is considerably more accurate than previous measurements. Results are also reported for Na, Al, P, Cl, and K; that result for P is the first positive identification.

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I. INTRODUCTION

There is still much uncertainty about the hyperfine transition rate for muonic atoms in the 1s ground state. Different techniques often give different rates, and there are many inconsistencies. This topic was boosted into prominence 30 years ago by the Chicago group of Winston, Telegdi and co-workers [1,2] who studied $^{19}$F and established the major properties of this effect. Even today, $^{19}$F remains the best understood example, because of the convenient time constant of about 180 ns.

When a $\mu^-$ stops in a target, it forms a muonic atom, and quickly cascades down to the 1s level on a time scale of $10^{-12}$ s which is effectively instantaneous for any detector. If the nucleus has no spin, there is a single ground state and the muon awaits its fate via the weak interaction decay ($\mu^- \rightarrow e^- \nu_e \nu_\mu$) or via nuclear capture. For the light elements this occurs within a few $\mu$s and can be studied with a variety of detectors. If the nucleus has a non-zero spin however, the situation is more complicated because there are two hyperfine levels of the 1s state, separated by an energy varying between a few eV and a keV or so. If the transition between these levels occurred via an M1 photon, the rate is too slow to be observed. However, the Chicago group showed that Auger emission can speed up the transition rate a thousand fold and bring it to the time scale of nanoseconds, which is within the range of standard detectors. The hyperfine transition can be detected in any nucleus by observing the depolarization of the $\mu^-$ via the detection of its decay electron. However, the $\mu^-$ has a very small residual polarization in the atomic state (< 10% for nuclei with spin), and in addition the $\mu^-$ can be depolarized by magnetic interactions or by nearby radicals. Nevertheless, several experiments have been carried out successfully.

If the nucleus has an odd Z, the $\mu^-$ is magnetically coupled to the odd proton. Now the capture probability in the $\mu^-p$ system, is $660$ s$^{-1}$ for the singlet state, but only $12$ s$^{-1}$ for the triplet state. Thus in one hyperfine state the total muon capture probability is approximately proportional to $(1 + Z/2)$ and in the other proportional to $(Z/2)$. Thus nuclear capture is highly sensitive to the hyperfine state, and the hyperfine transition can be followed by detecting neutrons from the capture events. The effect can also be observed via the time-dependence of the decay electron, but

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in light nuclei the normal decay rate dominates and thus dilutes the signal. Winston \[2\] as well as Suzuki et al. \[3\] were able to observe this effect in \(^{19}\)F, but for other nuclei the signal is too small.

The hyperfine transition can also be observed by detecting specific \(\gamma\)-rays resulting from nuclear capture. Some transitions are highly sensitive to the initial hyperfine state and, for certain spin combinations, this is also true for even-Z nuclei such as \(^{13}\)C. Several nuclei have been studied by Gorringe and coworkers \[4,5\]. In addition Wiaux \[6\] has found a very large sensitivity for the 320 keV \(\gamma\)-ray from muon capture in \(^{11}\)B. An unexpected observation by Gorringe et al., was that the hyperfine rate in metallic sodium was \(\Lambda_h = 15.5 \pm 1.1 \text{ } \mu\text{s}^{-1}\) \[7\], but when in NaF, the sodium hyperfine rate was only \(\Lambda_h = 8.4 \pm 1.9 \text{ } \mu\text{s}^{-1}\) \[8\], the fluorine hyperfine rate was \(\Lambda_h = 4.9 \pm 1.2 \text{ } \mu\text{s}^{-1}\), consistent with Winston’s measurement of \(\Lambda_h = 6.1 \pm 0.7 \text{ } \mu\text{s}^{-1}\) \[2\], and our own measurement of \(\Lambda_h = 5.6 \pm 0.2 \text{ } \mu\text{s}^{-1}\) in LiF. This raised the possibility that there might be a difference between metals and insulators, or between different molecular species. Such a difference could be due to different recombination times, or an unusual sensitivity to the binding energy of the valence electrons. However both possibilities seem unlikely.

The most puzzling result to date has been the observation of a hyperfine transition in \(^{14}\)N by Ishida et al. \[9\], using the \(\mu^-\)SR technique to observe the muon depolarization. They detected a relaxation rate of 0.092(33) \(\mu\text{s}^{-1}\) of which about 0.016 \(\mu\text{s}^{-1}\) is due to the different loss rates from the hyperfine states. This leaves 0.076(33) \(\mu\text{s}^{-1}\) which may be from a hyperfine transition. The problem is that the energy difference between the hyperfine states is only 7.4 eV, whereas, in the carbon atom, the least bound electron is bound by 11.3 eV, i.e. this energy is the ionization potential (the \(\mu^-\) is close to the nucleus, so the \(^{14}\)N muonic system appears to the electrons to be more like a \(^{14}\)C nucleus). If the pseudo \(^{14}\)C atom has formed a “C”N bond, the ionization potential is 14.3 eV, which makes the matter worse. We note that Wiaux found a slower hyperfine rate in \(^{11}\)B in comparison to a \(\mu\)SR experiment, so we suggest that the observation in \(^{14}\)N was caused by an additional depolarization mechanism, not the hyperfine transition.

Because of the various inconsistencies in this field, it was decided to re-investigate several light nuclei, using the more reliable technique of neutron detection. A comparison was also desirable between Na and NaH, Al and LiAlH\(_4\), and K and KH, to see if any difference could be detected. Unfortunately, no satisfactory result was obtained for KH. It should be noted that any muon captured in hydrogen is immediately transferred to another atom because the \(\mu p\) system is neutral. Thus NaH, for example, is effectively pure sodium in an insulating environment.

II. EXPERIMENTAL TECHNIQUE

The experiment was carried out in the M9B channel at TRIUMF, which includes a superconducting solenoid. For muons of 60 MeV/c the stop rate was \(\approx 10^4 \text{ } \text{s}^{-1}\) with an electron contamination of 2 % and a pion contamination of \(\leq 0.2 \%\). The muons passed through 2 plastic scintillators, and stopped in various targets; a third large plastic scintillator was used as a veto to define a muon stopped in the target. A mu-metal shield around the targets reduced the ambient magnetic field from 1.5 to 0.1 gauss. Table II lists the properties of the targets.

The neutron detectors were four cylindrical liquid scintillators, 2 of NE213, 1 of NE224 and 1 of BC501A (equivalent to NE213). They were arranged in a symmetrical array at 45°, 135°, 225°, and 315° to the beam, in order to minimize muon rotation effects. Plastic scintillators were placed in front of each detector to veto charged particles (such as decay electrons). The counters had a timing resolution of better than 5 ns full width half maximum (FWHM) for \(\gamma\)-rays. The counters were about 30 cm from the target so a \(\gamma\)-ray time of flight is 1 ns.

Pulse shape discrimination was used to distinguish neutrons from \(\gamma\)-rays. Two different modules were used at different times, but the discrimination was always set conservatively \[10\]. Sources of \(^{60}\)Co and AmBe were used to set up the system. If \(\gamma\)-rays are detected during the experiment, a prompt peak is clearly distinguishable, so it is straightforward to monitor the electronics on-line.

The time of arrival of an event with respect to the stopping muon was measured by both a 5 \(\mu\text{s}\) and a 10 \(\mu\text{s}\) full scale time to digital converter (TDC). These TDCs have been tested and do not contribute significantly to the errors in the experiment.

Data acquisition was carried out by a VAX station 3200 and a PDP-11 frontend processor (starburst). Data collected from the CAMAC modules were written to 8 mm tapes for later analysis. Further technical details of the
To test the overall system several targets were chosen for which there is no hyperfine transition, or for which the transition is too fast to be observed. The time of arrival of the neutrons could then be fitted by the formula:

$$N(t) = \left(\frac{1}{2} \text{erf} \frac{t - t_0}{\sqrt{2S}} + \frac{1}{2}\right)Ce^{-Dt} + B \quad (1)$$

where D is the total disappearance rate for the element under study, B is a flat background, $t_0$ is the mid-point of the rise-time curve, and S is a folding of the time of flight spread and instrumental time resolution.

A typical fit for Mg is illustrated in Figure 1. The values for S and $t_0$ are presented in Table II and show the tendency for heavier elements to have a slower time of flight for the neutrons. As $t_0$ refers to the time of arrival of the $\gamma$-rays, one can add the $\gamma$-ray time of flight (0.97 ns) to obtain the average neutron time of flight ($\Delta t$) and average neutron energy ($E$). From S one can remove the counter timing resolution to obtain the spread in energy of the neutrons ($\Delta E$). These values were used as guidance in analyzing the elements with a hyperfine effect. Note that these values for $E$ and $\Delta E$ are for the neutrons detected in this experiment, and do not constitute measurements of these parameters for muon capture in general.

By taking the time derivative of the data such as that shown in Figure 1, one can get an idea of the neutron time of flight and detector resolution effects. Figure 2 shows this time derivative spectra for Au, brass, Si, and S. One can clearly see in brass and Au that a single error function (or gaussian) is not enough. We found that the function:

$$N(t) = \left[\frac{1}{2} \text{erf} \frac{t - t_1}{\sqrt{2S}} + \frac{1}{2}\right] + R \left(\frac{1}{2} \text{erf} \frac{t - t_1}{\sqrt{2S_1}} + \frac{1}{2}\right) Ce^{-Dt} + B \quad (2)$$

seems to give a better fit for those elements. The effect is not seen in Si and S, due to limited statistics.

For most elements there was a small correction (a few percent) for background from muons stopping in carbon in the scintillators. In addition, a few targets contained two elements viz LiF, (CF$_2$)$_n$, LiCl, and CCl$_4$. We used teflon, i.e. (CF$_2$)$_n$, and CCl$_4$ as test cases to ensure that the corrections could be adequately applied. For such molecular targets one needs to know the atomic capture ratio. The Fermi-Teller Z law is far from adequate and a Z$^{3/2}$ variation gives a better approximation [11]. However, far more satisfactory is to use an actual measurement as the empirical variation is significant.

For LiF the atomic capture ratio (Li/F) has been measured to be 0.28(3) by Zinov et al. [12] and 0.10(8) by Wilhelm et al. [13] for muons, and 0.10(1) [14] and 0.22(2) [15] for pions, which should be the same. As Zinov et al. disagree with later measurements for several other molecules, we take the pion values and use 0.16(12) with a liberal error. This value agrees with the empirical capture probability for muons determined by von Egidy and Hartmann which is 0.18(5) for LiF [11]. For (CF$_2$)$_n$, Martoff et al. [15] found that (89.4 ± 0.7) % of pions capture on fluorine (i.e. an atomic capture ratio C/F of 0.24(2) in the normal definition, which takes into account the relative number of atoms. (The Z$^{3/2}$ law gives 0.87; von Egidy and Hartmann [11] do not give a value for carbon, but using the Martoff value, and a measurement of CO$_2$ [17], we estimate the C capture probability is 0.33(9) in the von Egidy-Hartmann scheme). For CCl$_4$ there is no measurement but using our value of the C capture probability we obtain C/Cl = 0.25(7) i.e. (5.8 ± 1.6) % of muons stop in the carbon of CCl$_4$. The Z$^{3/2}$ rule gives C/Cl = 0.71, i.e. 15 % of muons stop in the carbon of CCl$_4$. For LiCl the atomic capture ratio, as measured by Daniel et al. [18] is 0.19(8).

Although these values are very uncertain, the situation is not as bad as it might appear. Since the lighter elements produce many fewer neutrons, the corrections turn out to be fairly minor. Hence, the number of stops in an element for which capture occurs (most of which produce neutrons), are 0.53% for Li, 7.8% for C, 33 % for F and 75 % for Cl [3]. The effective background, therefore, from lithium in LiF is 0.25 %, which is negligible in comparison to other problems. We assume the neutron multiplicity to be the same for these elements.

III. EXPERIMENTAL RESULTS

The results for elements with a hyperfine effect are fitted to two different functions; one with a single error function viz
and NaF results.

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conditions with a threshold on the neutron energy and an energy dependent efficiency for the neutron detector. Thus off no neutrons, and to levels which give off 2 neutrons). The experiment is for specific experimental previous calculations, which were made for the total capture rates (including transitions to bound levels which give neutron-detection experiments. A word of caution should be given concerning the comparison of the asymmetry with γ-rays all travel at the same speed). Thus γ-ray experiments observe large asymmetries (and γ-rays all travel at the same speed). Thus γ-ray experiments with lower statistics can effectively compete with neutron-detection experiments. A word of caution should be given concerning the comparison of the asymmetry with previous calculations, which were made for the total capture rates (including transitions to bound levels which give off no neutrons, and to levels which give off 2 neutrons). The experimental asymmetries are for specific experimental conditions with a threshold on the neutron energy and an energy dependent efficiency for the neutron detector. Thus one would not expect an exact equivalence, but the comparison is interesting.

We see that, on the whole, the errors on the hyperfine rate are somewhat large. The two results which are most convincing are F and Na, but the latter is not sufficient to resolve the disagreement between the earlier sodium metal, and NaF results.

\[
N(t) = \left[ \frac{erf(t-t_0)}{\sqrt{2S}} + 1 \right] \left[ Ce^{-Dt} (1 - Ae^{-Ht}) + Fe^{-Gt} + Pe^{-Qt} \right] + B
\]

and another more complex function with two error functions:

\[
N(t) = \left[ \frac{1}{2} \frac{erf(t-t_0)}{\sqrt{2S}} + \frac{1}{2} + R \left( \frac{1}{2} \frac{erf(t-t_1)}{\sqrt{2S_1}} + \frac{1}{2} \right) \right] \left[ Ce^{-Dt} (1 - Ae^{-Ht}) \right]
+ Fe^{-Gt} + Pe^{-Qt} + B
\]

where H is the hyperfine rate, A is the hyperfine asymmetry, F and G are contributions from carbon in the counters, P and Q are contributions from other elements (if appropriate), and B is a flat background. The parameters F, G, P, and Q were calculated and fixed. These components make no substantial contribution to the data analysis. By using these two equations we can get a handle on the systematic errors. In Equation 4 we tie the S_1 and t_1 values to the S and t_0 for two cases, using fits to the brass and the Au data. For LiF the data were of sufficient quality that all the other parameters (t_0, S, C, D, A, H) could be fitted freely. In this case the hyperfine effect is well separated from the rise-time effect caused by the neutron time of flight effects. The best fit is illustrated in Figure 2 and the values of S and t_0 are given in Table 24. They are consistent with the non-hyperfine elements but slightly different.

This is a salutary warning because we shall need to use both Equations (3) and (4) to get a handle on the systematic errors due to the time of flight and timing resolution effects. Our recommended values are the averages of these two methods. One would think that one could get the values of S and t_0 from the time spectra of the neighbouring 0^+ nuclei. This turns out to be a bad assumption. First let us understand the physics behind this hypothesis. Nuclei without a hyperfine effect are 0^+ nuclei, often even-even nuclei such as ^16O, ^24Mg, ^28Si, and ^40Ca. These are tightly bound. However odd Z nuclei like ^19F, ^23Na, ^27Al, and ^31P tend to have N=Z+1 and so, transforming a proton to a neutron proceeds to a nucleus even more neutron rich and more likely to fall apart. The typical neutron energy spectrum after muon capture is composed of two components, an evaporation spectrum peaked at about 1.5 MeV followed by a high energy tail starting around 5 or 6 MeV [19]. Setting a threshold at 10 MeV Kozlowski et al. [20] have shown that high energy neutrons constitute the following fraction: 26(5)% in ^16O, 19(3)% in ^28Si, 11(2)% in ^40Ca and 11(2)% in Pb. Thus for light elements the high energy component is very important, and it is not surprising if it is sensitive to details of the nuclear structure of the product nucleus. The results of our experiment confirm that there are significant variations from nucleus to nucleus.

For Na, Al, P, Cl, and K we do not have sufficient data to allow a free parameter search, so we must average the single error function results with the two error function results, otherwise we do not know whether the results are from both time of flight and hyperfine effects. Notice for LiF and (CF_2)_n this does not have to be done, because in F the capture rate asymmetry is large enough and the hyperfine rate is slow enough that the hyperfine rate can be distinguished from the time of flight effects. One way out of the dilemma for the other elements is to raise the energy threshold on the neutron detector. However, the hyperfine asymmetries are very small, and statistics becomes a problem. The other way is to do a full study of the neutron spectrum but this is a major project. We have therefore been forced to simply compare results from different fitting functions, using shape information from the spectra in Figure 2. The difference between the fits gives us an estimate of the systematic error. In Table 11 we present some of these fits to illustrate the problem; Figure 3 shows two fits for Na which are both quite satisfactory. In Table 24 our final recommended values are compared with previous results and the hyperfine asymmetries are compared in Table 25 to various theories, and previous experiments. Note that in Table 24 the Cl and K results did not use these error functions; since their asymmetry values were < 0, the hyperfine effect could be easily distinguished from the neutron time of flight effects. The most important feature is that apart from ^19F and to some extent ^23Na, the hyperfine asymmetries are small, and that creates part of the problem. The γ-ray experiments observe large asymmetries (and γ-rays all travel at the same speed). Thus γ-ray experiments with lower statistics can effectively compete with neutron-detection experiments. A word of caution should be given concerning the comparison of the asymmetry with previous calculations, which were made for the total capture rates (including transitions to bound levels which give off no neutrons, and to levels which give off 2 neutrons). The experimental asymmetries are for specific experimental conditions with a threshold on the neutron energy and an energy dependent efficiency for the neutron detector. Thus one would not expect an exact equivalence, but the comparison is interesting.
The phosphorus result is worth a short discussion. An initial polarization measurement by Egorov et al. \cite{21} claimed that an asymmetry had been observed. The measurement was from 0 to 2.4 $\mu$s implying no fast depolarization. Note also that the upper level is $F=1$ and the lower level $F=0$, so that if a fast hyperfine transition does occur, there can be no residual polarization. Later asymmetry measurements by Hutchinson et al. \cite{22} and Babaev et al. \cite{23} observed no asymmetry and set limits at 10% of the value observed by Egorov et al., thus indicating that the hyperfine transition was fast, as suggested by Winston \cite{2} who calculated $58 \mu$s$^{-1}$, i.e. $\tau = 17$ ns. This scenario was consistent with the data of Gorringe et al. \cite{5} using $\gamma$-ray detection, but no positive observation was obtained. Again, our own result has systematic difficulties but is the first positive identification of the hyperfine transition in phosphorus. All the other hyperfine rates are consistent with earlier results.

The comparison of Na and NaH in our data is interesting, but not of sufficient quality to resolve the questions about the sodium hyperfine rate, similarly for Al and LiAlH$_4$, though the trend is for the hyperfine rates to be similar.

In conclusion, we have measured the hyperfine rate in P for the first time, and obtained a more accurate value for F. Even though neutron detection appears to be the most sensitive technique for measuring the hyperfine transition rate, the problems with the neutron spectrum and the spread of the time of flight to the detectors have made it very difficult to observe fast rates. Further experiments should use a higher energy threshold (and take considerably greater statistics). An alternative technique would be to use a $\gamma$-ray detector with a high resolution, but a faster time response than a HPGe detector.

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FIG. 1. The time spectrum of neutrons from muon capture in Mg, fitted to equation 1. The data are presented with the muon disappearance rate divided out.
FIG. 2. The time derivative spectra of neutrons from muon capture in Au, brass, Si and S fitted to Equation 1 and Equation 2.
FIG. 3. The time spectrum of LiF, with the decay divided out. The hyperfine transition in fluorine is clearly visible with its time constant of about 180 ns.
FIG. 4. The time spectrum of Na, with the decay divided out for the one error function case (top) and the two error function case (bottom).
| Z  | Target         | Thickness (g/cm²) | Number of Events in Time Spectrum |
|----|----------------|-------------------|----------------------------------|
| 3.9| LiF            | 2.70              | $1.2 \times 10^6$                |
| 6.9| (CF₂)ₙ         | 2.25              | $3.5 \times 10^5$                |
| 11 | NaH            | 1.14              | $1.5 \times 10^6$                |
| 11 | Na             | 1.65              | $1.5 \times 10^6$                |
| 12 | Mg             | 1.61              | $2.2 \times 10^5$                |
| 13 | Al             | 4.69              | $1.2 \times 10^6$                |
| 13 | LiAlH₄         | 1.51              | $1.9 \times 10^5$                |
| 14 | Si             | 3.55              | $3.0 \times 10^5$                |
| 15 | P              | 2.79              | $2.8 \times 10^6$                |
| 17 | LiCl           | 2.60              | $7.6 \times 10^5$                |
| 17 | CCl₄           | 6.18              | $4.9 \times 10^5$                |
| 19 | K              | 1.78              | $8.4 \times 10^5$                |
| 29,30| Brass (Cu+Zn)| 2.79              | $1.8 \times 10^5$                |

TABLE II. Values of S and $t_0$ from non-hyperfine targets. Also given are the average energy $\overline{E}$, and the energy spread $\Delta E$ of the neutron events used for the timing measurements.

| Z  | Target      | S (ns)       | $t_0$ (ns)       | $\Delta t$ (ns) | $\overline{E}$ (MeV) | $\Delta E$ (MeV) |
|----|-------------|--------------|-----------------|-----------------|-----------------------|------------------|
| 12 | Mg          | 4.63 ± 0.24  | 4.22 ± 0.23     | 5.19 ± 0.23     | 16.5 ± 1.6            | 27 ± 4           |
| 14 | Si          | 5.26 ± 0.17  | 5.14 ± 0.17     | 6.11 ± 0.17     | 11.9 ± 0.7            | 20 ± 2           |
| 16 | S           | 4.84 ± 0.12  | 4.35 ± 0.14     | 5.32 ± 0.14     | 15.7 ± 0.9            | 25 ± 2           |
| 29,30| Brass (Cu+Zn)| 5.92 ± 0.11  | 9.72 ± 0.11     | 10.69 ± 0.11    | 3.9 ± 0.1             | 14.9 ± 0.7       |
| 79 | Au          | 6.93 ± 0.08  | 13.36 ± 0.07    | 14.33 ± 0.07    | 2.2 ± 0.1             | 10.4 ± 0.3       |
TABLE III. Values of S and \( t_0 \) for hyperfine targets, and their effect on the systematic uncertainties of H and A. The different results for H and A for each target should be taken as a measure of the systematic uncertainties.

| Z | Target | S (ns) | \( t_0 \) (ns) | Type of Error Functions | H (\( \mu s^{-1} \)) | A (unitless) |
|---|--------|-------|------------|-----------------------|-----------------|------------|
| 9 | LiF    | 5.38 ± 0.18 (varied) | 5.30 ± 0.17 (varied) | 1 | 5.6 ± 0.2 | 0.288 ± 0.006 |
|   |        | 4.92 ± 0.15 (varied) | 4.81 ± 0.15 (varied) | 2 | 5.7 ± 0.3 | 0.290 ± 0.006 |
|   |        | 5.31 ± 0.20 (varied) | 5.87 ± 0.18 (varied) | 2 | 5.6 ± 0.2 | 0.287 ± 0.006 |
| 9 | (CF\(_2\))\(_n\) | 5.03 ± 0.31 (varied) | 5.18 ± 0.29 (varied) | 1 | 5.2 ± 0.4 | 0.33 ± 0.01 |
|   |        | 4.68 ± 0.26 (varied) | 4.59 ± 0.27 (varied) | 2 | 5.2 ± 0.4 | 0.33 ± 0.01 |
|   |        | 4.89 ± 0.36 (varied) | 5.71 ± 0.31 (varied) | 2 | 5.2 ± 0.4 | 0.33 ± 0.01 |
| 11 | NaH   | 5.0 ± 0.2 (varied) | 5.0 ± 0.1 (varied) | 1 | 26 ± 4 | 0.15 ± 0.02 |
|   |        | 4.31 ± 0.22 (varied) | 4.51 ± 0.13 (varied) | 2 | 32 ± 4 | 0.19 ± 0.03 |
|   |        | 4.94 ± 0.16 (varied) | 5.54 ± 0.22 (varied) | 2 | 26 ± 5 | 0.15 ± 0.02 |
| 11 | Na    | 4.7 ± 0.1 (varied) | 4.5 ± 0.2 (varied) | 1 | 39 ± 7 | 0.18 ± 0.04 |
|   |        | 4.23 ± 0.13 (varied) | 3.77 ± 0.32 (varied) | 2 | 49 ± 10 | 0.24 ± 0.06 |
|   |        | 5.21 ± 0.23 (varied) | 4.75 ± 0.14 (varied) | 2 | 34 ± 6 | 0.15 ± 0.03 |
| 13 | Al    | 5.13 ± 0.13 (varied) | 3.52 ± 0.38 (varied) | 1 | 68 ± 9 | 0.30 ± 0.06 |
|   |        | 4.63 ± 0.13 (varied) | 2.73 ± 0.30 (varied) | 2 | 75 ± 7 | 0.37 ± 0.04 |
|   |        | 5.21 ± 0.23 (varied) | 4.75 ± 0.14 (varied) | 2 | 71 ± 9 | 0.31 ± 0.06 |
| 13 | LiAlH\(_4\) | 5.04 ± 0.31 (varied) | 2.37 ± 0.45 (varied) | 1 | 90 ± 6 | 0.50 ± 0.04 |
|   |        | 4.96 ± 0.23 (varied) | 4.66 ± 0.85 (varied) | 2 | 53 ± 25 | 0.22 ± 0.13 |
| 15 | P     | 6.51 ± 0.07 (varied) | 5.7 ± 0.23 (varied) | 1 | 44 ± 8 | 0.12 ± 0.03 |
|   |        | 5.90 ± 0.07 (varied) | 4.57 ± 0.19 (varied) | 2 | 62 ± 6 | 0.23 ± 0.03 |
|   |        | 6.73 ± 0.11 (varied) | 5.45 ± 0.20 (varied) | 2 | 76 ± 5 | 0.27 ± 0.02 |

TABLE IV. Muonic hyperfine transition rates in \( \mu s^{-1} \).

| Compound | Theory | Previous Work | This experiment |
|----------|--------|---------------|----------------|
| LiF      | 5.8    | 5.8 ± 0.8 2\[8\] | 5.6 ± 0.2 |
|          |        | 6.3 ± 1.8 \[2\] |               |
|          |        | 4.9 ± 1.2 \[2\] |               |
| NaF      | 14     | 15.5 ± 1.1 \[2\] | 38 ± 9 |
| (CF\(_2\))\(_n\) |        |               | 28 ± 5 |
| Na       | 14     | 41 ± 9 \[2\] | 72 ± 9 |
| NaH      |        |               | 89 ± 10 |
| NaF      | 41     |               | 60 ± 15 |
| Al       | 58     | \( \lambda_\hbar \gg \lambda_- \) \[8\] | 14 ± 27 |
| Red P    | 22     | \( \lambda_- \gg \lambda_\hbar \) \[2\] | 15 ± 29 |
| LiCl     | 8      | 6.5 ± 0.9 \[2\] | 25 ± 15 |
| CCl\(_4\)| 22     |               |               |
| K        | 22     |               |               |

\( \!*\lambda_- \approx 1.1 \mu s^{-1} \)
### TABLE V. The capture rate asymmetry. The theory values are for all captures. This experiment is for neutron detection only (weighted by multiplicity).

| Compound   | BLYP | Primakoff | Uberall | Previous work | This Experiment |
|------------|------|-----------|---------|---------------|-----------------|
| LiF        | 0.24 | 0.36      | 0.36    | 0.36 ± 0.04 (n) | 0.29 ± 0.01     |
|            |      |           |         | 0.25 ± 0.04 (γ) |                |
|            |      |           |         | 0.29 ± 0.02 (nγ) |                |
| (CF₂)ₙ     | 0.24 | 0.36      | 0.36    | 0.33 ± 0.01    |                 |
| Na         | 0.08 |           |         | 0.17 ± 0.05    |                 |
| NaH        | 0.08 |           |         | 0.16 ± 0.03    |                 |
| Al         | 0.09 | 0.14      | 0.22    | 0.34 ± 0.05    |                 |
| LiAlH₄     | 0.09 | 0.14      | 0.22    | 0.35 ± 0.15    |                 |
| Red P      | 0.16 | 0.22      | 0.25    | 0.2 ± 0.1      |                 |
| LiCl       | -0.06 (²⁰Cl) | -0.10 (²⁰Cl) | -0.03 ± 0.06 |                 |
|            | -0.06 (³⁵Cl) |           |         |                 |
| CCl₄       |      |           |         | -0.03 ± 0.06   |                 |
| K          | -0.05 | -0.07    |         | -0.080 ± 0.055 |                 |