The paper presents a novel algorithm for designing technological parameters by which one optimize power losses and induction in SMC. The advantage of the presented algorithm consists in the bicriteria optimization: minimization of losses and maximization of induction. The crucial role in the presented algorithm plays scaling.

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I. INTRODUCTION

Recently the algorithm for designing values of the hardening temperature and the compaction pressure in production process of Soft Magnetic Composites (SMC) has been derived by using concept of Pseudo State Equation [1]. In equilibrium thermodynamics the equation of state relates between thermodynamic parameters. For instance in the case of gas-liquid system they are: temperature, pressure and volume of considered material. By an analogy to equation of state we consider a phenomenological relation between technological parameters and physical properties of the material. Such approach for SMC is possible due to topology of the completed set of scaled power losses characteristics. The most important features of this topology are the following [2],[3]:

- set of characteristics consists of one variable smooth functions:

\[ \frac{P_{\text{tot}}}{(B_m)^\beta} = F\left( \frac{f}{B_m^\alpha} \right) , \]

where \( P_{\text{tot}} \) is density of power loss, \( B_m \) is pick of induction, \( f \) is frequency of electromagnetic field wave, \( F(\cdot) \) is a function determined experimentally, \( \alpha \) and \( \beta \) are scaling exponents,

- considered set is continuous which means that if for each characteristic one defines a cover having arbitrary small radius then this cover will contain an infinite number of characteristics. Each characteristic is determined by \( \alpha \) and \( \beta \) exponents. These are functions of the technological parameters \( T \) and \( p \)

\[ \alpha = \alpha(T, p), \quad \beta = \beta(T, p), \]

where \( T \) and \( p \) are hardening temperature and compacsion pressure, respectively.

- Characteristics do not cross each other except at origin point \( \frac{f}{B_m} = 0 \), which is the common point for all of them.

- All the characteristics are monotonic increasing functions of \( \frac{f}{B_m} \)
These properties have enabled us to introduce measure of power loss $V(T, p)$ which was average of characteristics with respect to $B_m$ [1]:

$$V(T, p) = \frac{1}{\phi_{max} - \phi_{min}} \int_{\phi_{min}}^{\phi_{max}} \frac{P_{tot}(f)}{B_m^3} d\left(\frac{f}{B_m}\right),$$

where the integration domain was common for all characteristics. Basing on the topological properties of the characteristics’ set and on (2) as well as on (3) the Pseudo-State Equation for Soft Magnet Composites has been derived. This equation has enabled us to determine optimal values of the technological parameters [1]. However, the described optimization concerns only the power losses. Whereas, in designing processes optimization of induction is also important. Goal of this paper is to extend the described in [1] algorithm for optimization both power losses and induction.

II. EXPERIMENTAL DATA

Specimens were produced by cold pressing under pressure of 500...900 MPa. The specimens made of Somaloy 500 powder were cured at a temperature of 400...600°C for 30 minutes in air atmosphere. The specimens used in experiments were ring-shaped with a square cross-section. The specimens had the following dimensions: external diameter 55 mm, internal diameter 45 mm and thickness 5 mm. Total power loss density $P_{tot}$, expressed in watts per kilogram (W/kg), were obtained from measurements of the AC hysteresis cycle according to IEC Standards 60404-6 using the system AMH-20K-HS produced by Laboratorio Elettrofisico Walker LDJ Scientific. Total power losses $P_{tot}$ were measured at maximum flux density $B_m = 0.1...1.3$ T over a frequency range from 10 to 5000 Hz. During measurements of the total power losses $P_{tot}$, the shape factor of the secondary voltage was equal to 1.111 ± 1.5%. Maximum measurement error of the total energy losses was equal to 3%. In order to optimize the magnetic properties, the magnetic inductions $B$ at fixed magnetic field $H$ equal to 1000 A/m have been determined. These values were obtained from measurements of the DC magnetization curve according to IEC Standards 60404-4 using the same measuring system.

III. POWER LOSSES AND INDUCTION PSEUDO-STATE EQUATIONS

Optimization of the power losses was basing on the topological properties of the characteristics. Now situation is much simpler because for optimization of magnetic properties we have selected induction $B_{1000}$ for the fixed magnetic field $H=1000$ (A/m). We have chose this value because the magnetic permeability of the soft magnetic composites reaches a maximum values around this magnetic field. We expect that the Pseudo-State Equation will properly describe induction at this point as a function of $T$ and $p$. For the further considerations we assume that also induction obey the scaling. In order to justify this assumption we refer to the two phenomenons: invariance of power losses (area of the hysteresis loop) with respect to scaling and invariance of the hysteresis loop with respect to the scaling. Therefore, for the two criteria optimization problem: minimization of the power losses and maximization of the induction for the fixed magnetic field we will use the two Pseudo-State Equations of the general forms:
\[ V \left( \frac{T}{T_c}, \frac{p}{p_c} \right) = \left( \frac{p}{p_c} \right)^\gamma \cdot \Phi(X), \quad (4) \]

\[ X = \frac{T}{(p_c)^{\delta}}, \quad (5) \]

\[ B_{1000} \left( \frac{T}{T'_c}, \frac{p}{p'_c} \right) = \left( \frac{p}{p'_c} \right)^{\gamma'} \cdot \Lambda(X'), \quad (6) \]

\[ X' = \frac{T}{(p_c')^{\delta'}}, \quad (7) \]

where \( \Phi(\cdot) \) and \( \Lambda(\cdot) \) are arbitrary functions to be determined. \( \gamma, \delta, \gamma', \delta' \) and \( T_c, p_c, T'_c, p'_c \) are scaling exponents and scaling parameters respectively, to be determined.

In the case of the power losses pseudo-state equation all calculations concerning modeling of \( \Phi(\cdot) \) and fitting of scaling exponents as well as model parameters have been done in [1]. The most important result for this consideration was the derivation of an infinite set of solutions for the technological parameters which minimize the power losses:

\[ \frac{T}{(p_c)^{\delta}} = 19, 75. \quad (8) \]

IV. INDUCTION PSEUDO-STATE EQUATION

Task of this Section is to derive an analogous condition to (8) for \( T \) and \( p \) which now lead to maximum of \( B_{1000} \). Deriving in [1] the form for \( \Phi(\cdot) \) we have revealed the two phases of Somaloy500: low losses and high losses ones. Therefore in considerations of the Induction Pseudo-State Equation we have to take into account this phase separation. Measurement data of \( B_{1000} \) v.s. \( T \) and \( p \) are separated into these two phases in TABLE I. Horizontal line between \( B_{1000} = 0, 414(T) \) and \( B_{1000} = 0, 425(T) \) indicates the crossover between the Low Losses phase and the High Losses one. This transition is very well visible by jump of \( V(T, p) \) function around the separation line [1]. For each phase we assume an independent brunch of the Pseudo-State Equation in the forms of the Padé approximants. In order to simplify notations we introduce the following abbreviations:

\[ \tau' = \frac{T}{T'_c}, \quad \pi' = \frac{p}{p'_c}, \quad X' = \frac{T}{(p_c')^{\delta'}} = \frac{\tau'}{(\pi')^{\delta'}}. \quad (9) \]

Expressing \( \Lambda(\cdot) \) in (6) by the Padé approximant we get the following form for the Induction Pseudo-Equation of State:

\[ B_{1000}(\tau', \pi') = \pi'^\gamma \left( \frac{G_0}{1 + D_1 X' + D_2 X'^2 + D_3 X'^3 + D_4 X'^4} \right) \]

\[ + \left( \frac{G_1}{1 + D_1 X + D_2 X^2 + D_3 X^3 + D_4 X^4} \right) X', \quad (10) \]

where \( G_0, \ldots, G_4, D_1, \ldots, D_4 \) are parameters of the Padé approximante. All parameters have to be determined from the data presented in TABLE I. Corresponding pseudo-state equation for the power losses has been derived in [1]:

\[ V(\tau, \pi) = \pi^\gamma \left( \frac{G_0}{1 + D_1 X + D_2 X^2 + D_3 X^3 + D_4 X^4} \right) \]

\[ + \left( \frac{G_1}{1 + D_1 X + D_2 X^2 + D_3 X^3 + D_4 X^4} \right) X. \quad (11) \]
TABLE I. Somaloy 500. Measure of Induction $B_{1000}$ v.s. hardening temperature $T$ and compaction pressure $p$ for magnetic field $H=1000$ (A/m).

| Temperature ($K$) | Pressure (MPa) | Induction (T) |
|------------------|----------------|---------------|
| 723,15           | 800            | 0.378         |
| 773,15           | 900            | 0.496         |
| 773,15           | 700            | 0.483         |
| 673,15           | 800            | 0.335         |
| 773,15           | 600            | 0.467         |
| 823,15           | 800            | 0.546         |
| 773,15           | 500            | 0.414         |
| 741,15           | 764            | 0.425         |
| 773,15           | 750            | 0.489         |
| 773,15           | 800            | 0.504         |
| 773,15           | 650            | 0.469         |
| 773,15           | 725            | 0.467         |
| 873,15           | 800            | 0.568         |

TABLE II. Somaloy 500, low-losses phase. Values of the induction pseudo-state equation’s parameters and the Padé approximant’s coefficients of (10).

| $\gamma'$ | $\delta'$ | $T'$ | $p'$ | $\tilde{G}_0$ | $\tilde{G}_1$ | $\tilde{G}_2$ |
|-----------|-----------|------|------|---------------|---------------|---------------|
| 1.114     | 0.499     | 32.19| 32.84| 784.41        | 764.05        | -276.06       |
| $\tilde{G}_3$ | $\tilde{G}_4$ | $\tilde{G}_5$ | $\tilde{D}_1$ | $\tilde{D}_2$ | $\tilde{D}_3$ | $\tilde{D}_4$ |
| 0.6486    | 2.9005    | 2.8373 | 3.975 | -43.412      | -2.4315      | 3.6486       |

V. ESTIMATION OF THE INDUCTION PSUDO-STATE EQUATION’S PARAMETERS

In [1] it has been revealed the sudden change of $V$ between two points: [773, 15; 500, 0] and [742, 15; 764, 0] which has been interpreted as crossover between two phases: the low-losses phase and the high-losses phase. This effect is not seen in the induction magnitude. However in order to have compact description of the power losses and the induction we take that into account and we divide the data of Table I into two subsets corresponding to the revealed two phases, respectively. Minimizations of $\chi^2$ for both phases have been performed by using MICROSOFT EXCEL 2010, where

$$\chi^2 = \sum_{i=1}^{N} \left( B(t_i', \pi_i') - \pi_i' \gamma' \tilde{G}_0 + \tilde{G}_1 X_{1i}' + \tilde{G}_2 X_{2i}'^2 + \tilde{G}_3 X_{3i}'^3 + \tilde{G}_4 X_{4i}'^4 \right)^2,$$

where $N = 7$ and $N = 6$ for the low-losses and high-losses phases, respectively. Table II and Table III present estimated values of the model parameters for the low-losses and high-losses phases, respectively.

TABLE III. Somaloy 500, high-losses phase. Values of the induction pseudo-state equation’s parameters and the Padé approximant’s coefficients of (10).

| $\gamma'$ | $\delta'$ | $T'$ | $p'$ | $\tilde{G}_0$ | $\tilde{G}_1$ | $\tilde{G}_2$ |
|-----------|-----------|------|------|---------------|---------------|---------------|
| 1.1146    | 0.4992    | 32.19| 25.83| 808.91        | 747.44        | -266.95       |
| $\tilde{G}_3$ | $\tilde{G}_4$ | $\tilde{G}_5$ | $\tilde{D}_1$ | $\tilde{D}_2$ | $\tilde{D}_3$ | $\tilde{D}_4$ |
| 0.5846    | -3.001    | -9.968 | 7.1432 | -42.852      | -2.5142      | 0.5846       |
VI. OPTIMIZATION OF INDUCTION AND POWER LOSSES

In the optimization of the power losses problem \[1\] we have accepted the Low Losses Phase solutions whereas the High Losses Phase solutions where rejected. However, it is not clear whether for pressure characteristics such a choice would be correct solution. As we have proved in the recent paper the binary relations are invariant with respect to scaling \[1\]. This enables us to present all scaled characteristics in the one picture Fig.1 and make the following conclusion. All considered pressure characteristics of the High Losses Phase are covered by the set of the Low Losses Phase characteristics. Therefore for further considerations we limit our searchin to the Low Losses Phase. For this purpose we draw part of the phase diagram of Somaloy500 corresponding to the Low losses Phase Fig. 2. The experimental dimensionless constant 19,75 has been derived from Fig.2 as a value of \(\tau \pi^{-\delta}\) which was coordinate of minimum of scaled power losses’ measure \(V \pi^{-\gamma}\). According to \[5\] \(X = 19.75\). Unfortunately, due to plato around 19.75 the uncertainty of \(X\) is large which would influence the further calculations. Therefore, in order to solve this problem we assume that \(X\) is a variable running around 19.75. This variable will be used in the optimization procedure under consideration. As it has been mentioned above \[3\] the revealed condition for the power losses optimum was the following:

\[
T = X T_c \left( \frac{p}{p_c} \right)^\delta,
\]

where the values of \(T_c, p_c, \delta\) are displayed in Table [IV]

Substituting \[13\] to \[7\] we derive the following formula for \(X'\):

\[
X' = X' p^{(\delta - \delta') - \delta'} \frac{T_c}{T' c} \left( \frac{p'}{p_c} \right)^{\delta'}
\]

Expressing \(T\) and \(X'\) by \(p\) and \(X\) according to \[13\] and \[15\] we derive final forms for
FIG. 2. Scaled $V$ v.s. scaled temperature in the low losses phase.

TABLE IV. Somaloy 500, low-losses phase. Values of the V pseudo-state equation’s parameters and the Padé approximant’s coefficients of (15) [1].

| $\gamma$   | $\delta$ | $T_c$  | $G_0$   | $G_1$   | $G_2$   |
|------------|----------|--------|---------|---------|---------|
| 0.1715     | 1.2812   | 21.62  | 37.729  | 37.031  | 47.752  |
| $G_3$      | $G_4$    | $D_1$  | $D_2$   | $D_3$   | $D_4$   |
| -1.3764    | -678.26  | 170.80 | 6243.8  | 386.96  | -28.699 |

The pseudo-state equations:

$$V(p, X) = \left( \frac{p}{p_c} \right)^\gamma \frac{\sum_{i=0}^{4} G_i X_i^i}{1 + \sum_{i=1}^{4} D_i X_i^i}, \quad (15)$$

$$B_{1000}(p, X) = \left( \frac{p}{p'_c} \right)^{\gamma'} \frac{\sum_{i=0}^{4} \tilde{G}_i X_i^{i'}}{1 + \sum_{i=1}^{4} \tilde{D}_i X_i^{i'}}, \quad (16)$$

where the values of $G_i, D_i, p_c, \gamma$ are displayed in TABLE IV.

We are ready to start the bicriteria optimization: using the two independent variables $p$ and $T$ the induction $B_{1000}$ will be maximized while the losses measure $V$ will be minimized.

Optimization has been done by SOLVER routine of EXCEL2010 program. The following procedure was executed.

- Assign $B_{1000}$ as a target function and switch on SOLVER for the $max$ mode.
- Assign $p$ and $T$ as independent variables.
- For the selected set of loss values $\{V_1, V_2, \ldots, V_n\}$ calculate the corresponding values of $\{B_{1000, max}\}$.
- Assign $V$ as a target function and switch on SOLVER for the $min$ mode.
TABLE V. Somaloy 500, low-losses phase. Optimum solutions in technological and in physical spaces.

| $p$ (MPa) | $T$ (°C) | $V$ (W/kgT$^{-2}$) | $B_{1000}$ (T) |
|-----------|----------|------------------|----------------|
| 389       | 370      | 14.1             | 0.300          |
| 492       | 407      | 20.0             | 0.356          |
| 584       | 440      | 27.3             | 0.400          |
| 683       | 478      | 40.0             | 0.449          |
| 733       | 499      | 50.0             | 0.479          |
| 764       | 515      | 58.5             | 0.500          |
| 860       | 532      | 70.0             | 0.525          |
| 838       | 549      | 82.9             | 0.550          |
| 906       | 570      | 101              | 0.580          |
| 979       | 584      | 116              | 0.600          |

FIG. 3. Technological optimum curve presenting dependence of optimum temperature vs optimum pressure.

- Keep $p$ and $T$ as the independent variables.
- For the calculated set of induction values $\{B_{1000,\text{max}}\}$ calculate the corresponding values of $V_{\text{min}}$.
- Each time when the particular result is obtained one must check whether $X$ satisfies the following conditions: $18.4 < X < 22.9$. These result from limitations of the presented calculations to the Low Losses Phase presented in Fig. 2.

For each time this procedure has converged to fixed point after two steps. The obtained results are presented in TABLE V. Fig. 3 and Fig. 4 present these results in technological and in physical spaces, respectively. There is one to one correspondence between points of these spaces. The set of points in the presented Figures can be as dense as needed.
VII. CONCLUSIONS

We have presented a possible method for the bicriteria optimization of the chosen physical properties of Soft Magnetic Composites. By this way we have solved the problem mentioned in [3] concerning optimization of losses and induction. The crucial roles in the presented method play scaling and the notion of pseudo-state equation. The created system is as good as the experimental data which have been used for the estimations of models parameters. Therefore, presented here the first version will be improved by forthcoming new experimental data. The derived algorithm is addressed to designers of SMC.

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