Logarithmic Operators in AdS$_3$/CFT$_2$

Alex Lewis Department of Mathematical Physics, National University of Ireland,
Maynooth, Republic of Ireland. e-mail: alex@thphys.may.ie

Abstract

We discuss the relation between singletons in AdS$_3$ and logarithmic operators in the CFT on the boundary. In 2 dimensions there can be more logarithmic operators apart from those which correspond to singletons in AdS, because logarithmic operators can occur when the dimensions of primary fields differ by an integer instead of being equal. These operators may be needed to account for the greybody factor for gauge bosons in the bulk.
One particularly interesting example of the AdS/CFT correspondence \[1, 2, 3\] is the AdS\(_3\)/CFT\(_2\) correspondence, which relates supergravity on AdS\(_3\) \(\times\) S\(_3\) to a 2-dimensional CFT. One advantage of this is that 2-dimensional conformal field theories are very well understood, and that makes AdS\(_3\) especially suitable for studying the relation between singletons on AdS and logarithmic conformal field theories (LCFT), since almost all previous work on LCFT has concentrated on the 2-dimensional case.

According to \[2, 3\], at the boundary of AdS\(_{D+1}\) we have a coupling between bulk fields \(\Phi_i(\vec{x}, z)\) and boundary fields \(O_i(\vec{x})\), \(\int d^{D+1}x \Phi_i O_i\), where the boundary fields are subject to the boundary condition \(\Phi_i(\vec{x}, z) = \lambda_i(\vec{x}, R)\), with \(z = R\) the boundary of AdS\(_{D+1}\). The relation between correlation functions in CFT\(_D\) and the bulk supergravity action is
\[
\langle e^{\sum_i \int d^D x \lambda_i O_i} \rangle = e^{-S[\{\Phi_i\}]} \tag{1}
\]
This relation was used in \[4, 5\] to show that, if there are singletons in AdS\(_{D+1}\), the theory on the boundary is in fact an LCFT. A theory of free singletons is formulated in terms of a dipole-ghost pair of fields \(A\) and \(B\) which satisfy \[6\]
\[(\partial^\mu \partial_\mu + m^2)A + B = 0, \quad (\partial^\mu \partial_\mu + m^2)B = 0 \tag{2}\]
these fields have the bulk AdS action
\[
S = \int d^{D+1}x \sqrt{g} \left( g^{\mu\nu} \partial_\mu A \partial_\nu B - m^2 AB - \frac{1}{2} B^2 \right) \tag{3}
\]
The fields \(A\) and \(B\) couple to boundary fields \(C\) and \(D\) and using eq. \[4\] the two-point functions of \(C\) and \(D\) are found to be (see \[3\] for details of the calculation)
\[
\langle C(x)C(y) \rangle = 0, \quad \langle C(x)D(y) \rangle = \frac{c}{|x - y|^{2\Delta}}, \quad \langle D(x)D(y) \rangle = \frac{1}{|x - y|^{2\Delta}}(d - 2c \ln |x - y|) \tag{4}
\]
with the dimension \(\Delta\) given by \(\Delta(\Delta - D) = m^2\), \(c = \Delta(2\Delta - D)\) and \(d = 2\Delta - D\). These are the usual two-point functions for logarithmic operators in CFT \[4, 8\]. These correlation functions occur if the Hamiltonian (in two dimensions, the Virasoro generator \(L_0\)) is non-diagonalizable, and has the Jordan form
\[
L_0|C\rangle = h|C\rangle, \quad L_0|D\rangle = h|D\rangle + |C\rangle \tag{5}
\]
and similarly for \(\bar{L}_0\), where for singletons we have \(h = \bar{h}\) and so \(\Delta = 2h\). The theories with this type of operators are called Logarithmic CFTs and their properties have been studied extensively \[8\] since they were introduced in \[7\]. Applications of LCFT to strings and D-brane scattering were developed in \[3, 10\]. A recent paper relevant to AdS\(_3\) is \[11\]
\[ \langle O_i(x_1)O_j(x_2)O_j(x_3)O_i(x_4) \rangle = \sum_{kl} \frac{f_{ij}^k f_{ij}^l}{|x_{12}|^{\Delta_i + \Delta_j} |x_{34}|^{\Delta_i + \Delta_j}} F(x) \]  

(7)

where \( x = x_{12}x_{34}/x_{13}x_{24} \) and \( F(x) \) has an expansion in powers of \( x \), without any logarithmic singularity. If there are logarithmic operators however, the OPE has to be modified and we have instead

\[ O_i(x_1)O_j(x_2) \sim \frac{1}{|x_{12}|^{\Delta_i + \Delta_j - \Delta}} (D + C \ln |x_{12}|^2) + \cdots \]  

(8)

which together with the two-point functions for \( C \) and \( D \) leads to four point functions of the form \([7]\), but with \( F(x) \xrightarrow{x \to 0} x^{2\Delta} \ln x \). Indeed, logarithmic singularities have been found in four point functions calculated in supergravity on AdS\(_5\) \([12, 13]\), and it is possible that these could be an indication that there is an LCFT on the boundary of AdS\(_5\). However, these logarithms could also be accounted for as the perturbative expansion of anomalous dimensions in CFT\(_4\), with no need for logarithmic operators \([14]\). The clearest evidence for the existence of logarithmic operators in AdS/CFT comes from calculations of grey-body factors in AdS\(_3\). Since grey-body factors are related to two-point functions in CFT, logarithms here are a clear indication that we have logarithmic operators on the boundary.

The grey-body factor (or absorption cross section) for a field in AdS\(_3\) which couples to a field \( O(x) \) in the CFT on the boundary is related to the two point function in the CFT by \([15, 16]\)

\[ \sigma_{abs} = \frac{\pi}{\omega} \int d^2x [\mathcal{G}(t - i\epsilon, x) - \mathcal{G}(t + i\epsilon, x)] \]  

(9)

where \( \mathcal{G}(t, x) = \langle O(x,t)O(0) \rangle \) is the thermal Green’s function in imaginary time. This can be determined from the periodicity in imaginary time and the singularities of the Green’s function \([15]\), which if \( O \) is a primary field with weights \( h, \bar{h} \), are given by

\[ \langle O(t, x)O(0) \rangle \sim \frac{C_O}{x_+^{2h}x_-^{2\bar{h}}} \]  

(10)

with \( x_\pm = t \pm x \). \( \mathcal{G}(t, x) \) has the form

\[ \mathcal{G}(t, x) = C \left( \frac{\pi T_+ R}{\sinh(\pi T_+ x_+)} \right)^{2h} \left( \frac{\pi T_- R}{\sinh(\pi T_- x_-)} \right)^{2\bar{h}} \]  

(11)

for a BTZ black hole with mass \( M = r_+^2 - r_-^2 \), angular momentum \( J = 2r_+ r_- \), left and right temperatures \( T_\pm = (r_+ \pm r_-)/2\pi \), and Hawking temperature given by \( 2/T_H = 1/T_+ + 1/T_- \) \([17]\). The absorption cross section is then \([15, 16]\)

\[ \sigma_{abs}(h, \bar{h}) = \frac{\pi C (2\pi T_+ R)^{2h-1} (2\pi T_- R)^{2\bar{h}-1}}{\omega \Gamma(2h) \Gamma(2\bar{h})} \sinh \left( \frac{\omega}{2T_H} \right) \left| \Gamma \left( h + i \frac{\omega}{4\pi T_+} \right) \Gamma \left( \bar{h} + i \frac{\omega}{4\pi T_-} \right) \right|^2 \]  

(12)

This expression can be obtained either using the effective string method for supergravity \([15]\) or using the Ads/CFT correspondence \([16, 18]\).
A large number of classical calculations of absorption cross sections have given results which are consistent with (12) (or a similar expression for fermions) [15], including calculations for several fields for the BTZ black hole [13, 14]. However, in [19] the cross section for gauge bosons with spin 2, which couple to fields with \( h, \bar{h} = (2,0) \) or \((0,2)\) on the boundary was found to have logarithmic corrections which cannot be accounted for by eq. (12). In [20], the grey-body factor for singletons was calculated, and while this does have a logarithmic correction to the cross section (12), it was found that it still does not give the correct cross section for the gauge bosons. The question we would like to address in this letter is, are there other kinds of logarithmic operators in AdS\( _3 /\text{CFT}_2 \), and can they correctly account for the greybody factor for the gauge bosons?

The greybody factor for the gauge bosons with spin \( s = 2 \) in AdS\( _3 \), in the low temperature limit \( \omega \gg T_\pm \), was found to be [19]

\[
\sigma_{gb}^{ab} = \pi^2 \omega R^2 [1 + \omega R s \ln(2\omega R s)]
\]

In the low temperature limit, eq. (12) becomes, up to a normalization which is proportional to \( C(\Delta = h + \bar{h}) \),

\[
\sigma_{abs}(h, \bar{h}) \sim \omega^{2\Delta - 3}
\]

So that the second term in eq. (13) is an indication that the gauge bosons cannot just couple to ordinary primary fields on the boundary. The greybody factor for a singleton can also be calculated from eq. (9), using the relation

\[
\langle D(t,x)D(0) \rangle = \frac{\partial}{\partial \Delta} \langle C(t,x)D(0) \rangle,
\]

since \( \langle C(t,x)D(0) \rangle \) is the same as the two point function for an ordinary primary field.

The greybody factor for a singleton is therefore given by \( \sigma_{abs}^s = \partial \sigma_{abs}(h, \bar{h}) / \partial \Delta \) [20], and so

\[
\sigma_{abs}^s = \frac{\pi C}{\omega} \frac{(2\pi T_+ R)^{2h-1}(2\pi T_- R)^{2\bar{h}-1}}{\Gamma(2h)\Gamma(2\bar{h})} \left| \sinh \left( \frac{\omega}{2T_H} \right) \right| \left( \frac{\omega}{4\pi T_+} \right) \left( \frac{\omega}{4\pi T_-} \right)^2 \left[ \frac{1}{C} \left( \frac{\partial C}{\partial \Delta} + \ln(2\pi T_+ R) + \ln(2\pi T_- R) - \psi(h) - \psi(2\bar{h}) \right) + \frac{1}{2} \left( \psi \left( h + i \frac{\omega}{4\pi T_+} \right) + \psi \left( h - i \frac{\omega}{4\pi T_+} \right) + \psi \left( \bar{h} + i \frac{\omega}{4\pi T_-} \right) + \psi \left( \bar{h} - i \frac{\omega}{4\pi T_-} \right) \right) \right]
\]

Which in the low temperature limit reduces to

\[
\sigma_{abs}^s \sim \omega^{2\Delta - 3} (2 \ln(\omega R) + c')
\]

In an LCFT we always have the freedom to shift \( D \) by \( D \rightarrow D + \lambda C \), which leaves eq. (3) invariant, and this can be used to choose any value for the constant \( c' \). However, comparing eqs. (16) and (13), we can see that the logarithmic term in (13) is multiplied by an extra factor of \( \omega R \) and is thus of a sub-leading order compared to (16). The gauge boson cannot therefore be represented by a singleton in AdS\( _3 \) [20]. However, we cannot immediately conclude, as was said in [20], that the gauge boson has nothing to do with the AdS/LCFT correspondence, because there is potentially a much richer spectrum of logarithmic operators in a two dimensional LCFT than has been considered so far. The logarithmic operators we have looked at so far arise when the dimensions of two of the
primary fields $O_\lambda$ in the OPE (3) become degenerate, which leads to logarithms in the four-point functions and the OPE has to be modified to include the logarithmic pair $C$ and $D$, as in eq. (3). In fact, logarithms will also arise in the four point function if two of the primary fields have dimensions which are not equal, but differ by an integer, so that it is a descendant of one primary field which becomes degenerate with the other primary field. This is because the function $F(x)$ in the four-point function (7) usually satisfies a Fuchsian differential equation, such as a hypergeometric equation, and when there are no degenerate dimensions the solutions have the form

$$F(x) \sim x^{\Delta_i} \sum_{n=0}^{\infty} a_n x^n$$

but when two of the dimensions differ by an integer, say $\Delta_2 = \Delta_1 + N$, the second solution instead has the form

$$F(x) \sim x^{\Delta_1} \sum_{n=0}^{\infty} (a_n x^n + b_n x^n \log x)$$

in this case we again have a logarithmic pair with the higher of the two dimensions which as before make the Hamiltonian non-diagonalizable, as in eq. (3). In addition the $C$ field satisfies in both cases the usual condition for a primary field

$$L_n |C\rangle = 0, \quad n \geq 1$$

However, in the earlier situation where two primary fields became degenerate, $D$ also satisfied this condition, while in the case where we have two fields whose dimension differs by an integer $N$ we have instead

$$(L_1)^N |D\rangle = \beta |C'\rangle, \quad L_n |D\rangle = 0, \quad n \geq 2$$

where $C'$ is another primary field, with conformal weights $(h - N, \bar{h})$, and $\beta$ is some constant. $C$ is now not really a primary field, but rather a descendant of $C'$: $|C\rangle = \sigma_{-N} |C'\rangle$, where $\sigma_{-N}$ is some combination of Virasoro generators and, in general, the other generators of the chiral algebra of the CFT, of dimension $N$. Eq. (19) then implies that $C$ must be a null vector of the CFT, that is $[L_n, \sigma_{-N}] = 0$ for $n \geq 1$ (21) (which is why the two-point function $\langle CC\rangle = 0$). This type of logarithmic operator therefore cannot exist with any dimension, but only with those dimensions for which there are null vectors of the algebra. Because of this, we can only have these generalized logarithmic operators in 2-dimensional CFT, and we do not expect them in AdS$_{D+1}$ for $D > 2$.

The logarithmic pair $C$ and $D$ still have the same correlation functions (4), and $C'$ is just an ordinary primary field with the usual two point function

$$\langle C'(x_+, x_-)C'(-0) \rangle \sim \frac{1}{x_+^{2(h-N)}x_-^{2\bar{h}}}$$

so it seems that these new fields cannot give us anything new when we compute greybody factors. However, it is easy to see that we can reproduce the greybody factor for the gauge bosons (13) if we assume that they correspond not to one of the fields $C$, $D$ or $C'$ in the LCFT, but to a linear combination of all three. This might happen, for example,
if the bosons can be thought of as arising from the fusion of two primary fields, since \(C\), \(D\) and \(C'\) must always appear together in any OPE. Then if \(C'\) has dimension \(\Delta = 2\), as is expected for the gauge bosons \([20]\), and \(C', D\) form a representation of the type discussed above with \(N = 1\), the greybody factor will have exactly the right form, with the logarithmic term being of sub-leading order. Of course, this would imply that the representation which includes the primary field \(C'\) must have a null vector at level 1. This would be true if \(C'\) has \((h, \bar{h}) = (0, 2)\) (or \((2, 0)\)), as then \(L_{-1}|C'\rangle\) (or \(\bar{L}_{-1}|C'\rangle\)) is a null vector. If \(C'\) has \((h, \bar{h}) = (1, 1)\), there could still be a null; vector if, for example, the CFT on the boundary has a conserved current for which \(J_{-1}|C'\rangle = 0\).

Now that we know there could be fields in an LCFT on the boundary that give the correct greybody factor for the gauge bosons, the next question we address is, what sort of fields in the bulk can couple to these fields on the boundary? To answer this question, we start by reviewing how the conformal weights of fields on the boundary determine the mass and spin of fields in the bulk when there are no logarithmic operators. We write the metric for AdS\(_3\) in the form

\[
\begin{align*}
\mathrm{ds}^2 &= l^2 \left( -\cosh^2 \rho \mathrm{d}t^2 + \sinh^2 \rho \mathrm{d}\phi^2 + \mathrm{d}\rho^2 \right) \\
&= \frac{l^2}{2} \left( -\cosh^2 \rho \partial_t^2 + \sinh^2 \rho \partial\phi^2 + \partial\rho^2 \right)
\end{align*}
\] (22)

In these coordinates the Virasoro generators \(L_0, L_{\pm 1}\), with commutators \([L_0, L_{\pm 1}] = \mp L_{\pm 1}\) and \([L_1, L_{-1}] = 2L_0\), for spin \(s\) fields are \((u = \tau + \phi, v = \tau - \phi)\) \([22, 23]\):

\[
\begin{align*}
L_0 &= i \partial_u \\
L_{-1} &= ie^{-iu} \left( \coth 2\rho \partial_u - \frac{1}{\sinh 2\rho} \partial_v + \frac{i}{2} \partial_\rho - \frac{i}{2} s \coth \rho \right) \\
L_1 &= ie^{iu} \left( \coth 2\rho \partial_u - \frac{1}{\sinh 2\rho} \partial_v - \frac{i}{2} \partial_\rho + \frac{i}{2} s \coth \rho \right)
\end{align*}
\] (23)

and similarly for \(\bar{L}_0, \bar{L}_{\pm 1}\) with \(u \leftrightarrow v\) and \(s \rightarrow -s\). For a primary fields \(\Phi\), the conditions \(L_0\Phi = h\Phi\) and \(\bar{L}_0\Phi = \bar{h}\Phi\), and \(L_1\Phi = \bar{L}_1\Phi = 0\) can then be solved to give \(s = h - \bar{h}\) and

\[
\Phi \sim \frac{e^{-i(hu + \bar{h}v)}}{\cosh \rho} \tag{24}
\]

The second Casimir of \(sl(2, \mathbb{R})\) is

\[
L^2 = \frac{1}{2} (L_1 L_{-1} + L_{-1} L_1) - L_0^2
\]

and similarly for \(\bar{L}^2\), so that, using eqs. \([22]\) and \([23]\), the sum of the two Casimirs is

\[
L^2 + \bar{L}^2 = -l^2 \partial^2 + s^2 \coth^2 \rho
\] (26)

For a primary field, \((L^2 + \bar{L}^2)\Phi = (2h(h - 1) + 2\bar{h}(\bar{h} - 1))\Phi\), so eq. \([20]\) can be written as

\[
\left( -\partial^2 + \frac{s^2}{l^2 \sinh^2 \rho} \right) \Phi = m^2 \Phi
\]

\] (27)
which is the equation of motion for a field with spin \( s \) and mass \( m \) in AdS\(_3\), with the mass

\[
l^2 m^2 = 2h(h - 1) + 2\tilde{h}(\tilde{h} - 1) - s^2 = \Delta(\Delta - 2) \tag{28}
\]

So we can see that the conformal weights \( h \) and \( \tilde{h} \) on the boundary completely determine the mass and spin of the fields in AdS\(_3\) (and vice versa). We can repeat this analysis for a logarithmic pair \( C \) and \( D \) on the boundary. \( C \) satisfies the same conditions as \( \Phi \) above, so we find \( C \sim e^{-i(\mu u + \nu v)} / (\cosh \rho)^{h + \tilde{h}} \). The conditions \( L_0 D = hD + C \) and \( \tilde{L}_0 D = \tilde{h}D + C \) then imply that

\[
D = [u + v + f(\rho)]C \tag{29}
\]

where the function \( f(\rho) \) will depend on what type of logarithmic operator we have. In the simplest case, which we expect to give us singletons, we have \( L_1 D = \tilde{L}_1 D = 0 \), which has the solution

\[
D = [u + v - 2i \ln(\cosh \rho) + \delta]C \tag{30}
\]

where \( \delta \) is an arbitrary constant, which we can set to any value using the freedom to shift \( D \) by an amount proportional to \( C \). Evaluating the second Casimirs gives the equations of motion for the fields in AdS\(_3\) which will couple to \( C \) and \( D \):

\[
\left(-\partial^\mu \partial_\mu + \frac{s^2}{l^2 \sinh^2 \rho}\right) C = m^2 C
\]

\[
\left(-\partial^\mu \partial_\mu + \frac{s^2}{l^2 \sinh^2 \rho}\right) D = m^2 D + 4(\Delta - 1)C + \Psi \tag{31}
\]

with \( m^2 \) again given by eq. (28). When \( s = 0 \), these are just the expected equations of motion for singleton dipole-pair \( (2) \) (apart from a different normalization of \( C \)), with the expected relation between the singleton mass \( m \) and the dimension of the logarithmic operator \( \Delta \). Thus we can see that the mass and spin in AdS\(_3\) are still completely determined by the data of the LCFT on the boundary when we have singletons and logarithmic operators. This also give us another way of seeing, as was found in \( [5] \) that there can be no logarithmic operators with \( \Delta = 1 \), corresponding to \( m^2 = -1 \).

Now we can use this map between AdS\(_3\) and CFT\(_2\) to see what kind of operators will couple to the other kinds of logarithmic operators. In this case we have three fields \( C, D \) and \( C' \), but since \( C \) and \( C' \) are both primary fields they will both have the same form as before, but with weights \( (h, \tilde{h}) \) for \( C \) and \( (h - N, \tilde{h}) \) for \( C' \). \( D \) is then given by eq. (29) with \( f(\rho) \) a solution of the \((N + 1)\)'th order differential equation \( (L_1)^{N+1} D = 0 \). The second casimirs then give the equations of motion in AdS\(_3\) as

\[
\left(-\partial^\mu \partial_\mu + \frac{s^2}{l^2 \sinh^2 \rho}\right) C = m^2 C
\]

\[
\left(-\partial^\mu \partial_\mu + \frac{s^2}{l^2 \sinh^2 \rho}\right) D = m^2 D + 4(\Delta - 1)C + \Psi \tag{32}
\]

Which is the same as the equations of motion for the singleton except that we have the new field is \( \Psi = L_{-1} L_1 D \). \( \Psi \) will therefore be a descendant of \( C' \), or in AdS\(_3\) it will correspond to some derivative of the field which couples to the primary field \( C' \), and
the action for the singleton (3) should be modified by adding a term which couples the singleton to the new field. We will therefore have an interacting theory instead of a free singleton, with an action of the form

\[ S = \int d^3x \sqrt{g} \left( g^{\mu\nu} \partial_\mu A \partial_\nu B - m^2 AB - \frac{1}{2} B^2 + \lambda A \Psi \right) \]  

(33)

where \(\Psi\) is a derivative of a field with spin \(N\), for a spinless singleton. In addition, it is important that \(C\) is also a descendant in this case, and so the field \(A\) in the above action is also a derivative of the field which couples to \(C'\) and is not a fundamental field itself. This is especially significant for the case when \(N = 1\), since then \(C'\) has no descendant at level 1 except \(C\) itself, and so the action in AdS\(_3\) in this case is the same as for the ordinary singleton, except that \(B\) is now a derivative of a field \(B'\) with spin 1. Of course, we also need to add to the action the kinetic and mass terms for the field \(B'\) to treat this field properly.

Although we have seen that new kinds of logarithmic operators can exist in AdS\(_3\)/LCFT\(_2\), they cannot exist for just any values of \(m^2\) and \(s\) - we have to have a null vector in the CFT on the boundary. This means that to determine if such fields really exist we need to know more about the structure of the CFT, or to calculate four-point functions, from which the OPE could be deduced. However, it seems to be clear that at least one example of this type of operator is needed to give the correct greybody factor for the spin-2 gauge bosons. It is an interesting question why the same interactions cannot be introduced for singletons which do not have special values for the mass, which would lead to a contradiction in the CFT, but is not obviously forbidden from the three-dimensional point of view. Possibly related is the question of why these type of fields can exist in AdS\(_3\) but not in AdS\(_{D+1}\) for \(D > 2\) – since the full Virasoro algebra applies only to CFT in 2 dimensions, there are no null vectors in \(D > 2\) and so these type of logarithmic do not exist, although there can be singletons and the ordinary logarithmic pair of \(C\) and \(D\) in any dimension.

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