Gauge Amplitude Identities by On-shell Recursion Relation in S-matrix Program

Bo Feng \textsuperscript{a}, Rijun Huang \textsuperscript{b}, Yin Jia \textsuperscript{b}

\textsuperscript{a} Center of Mathematical Science, Zhejiang University, Hangzhou, China
\textsuperscript{b}Zhejiang Institute of Modern Physics, Physics Department, Zhejiang University, Hangzhou, China

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Using only the Britto-Cachazo-Feng-Witten (BCFW) on-shell recursion relation we prove color-order reversed relation, \( U(1) \)-decoupling relation, Kleiss-Kuijf (KK) relation and Bern-Carrasco-Johansson (BCJ) relation for color-ordered gauge amplitude in the framework of S-matrix program without relying on Lagrangian description. Our derivation is the first pure field theory proof of the new discovered BCJ identity, which substantially reduces the color ordered basis from \((n-2)!\) to \((n-3)!\). Our proof gives also its physical interpretation as the mysterious bonus relation with \( \frac{1}{z} \) behavior under suitable on-shell deformation for no adjacent pair.

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INTRODUCTION

S-matrix program\textsuperscript{[1]} is a program to understand the scattering amplitude of quantum field theory based only on some general principles, like the Lorentz invariance, Locality, Causality, Gauge symmetry as well as Analytic property. The significance of this approach is its generality: results so obtained do not rely on any detail information of theories, such as the Lagrangian description of theories.

However, exactly because its generality with so little assumptions, there are not much tools available and its study is very challenging. One big step along S-matrix program is the idea of S-matrix program and its generality has inspired many works, one of them is the work of Benincasa and Cachazo\textsuperscript{[5]}. In the paper, by assuming the applicability of BCFW recursion relation they have easily re-derived many well known (but difficult to prove) fundamental facts in S-matrix, such as the Non-Abelian structure for gauge theory and all matters couple to gravity with same coupling constant.

In this paper we will focus on one fundamental object in gauge theory, i.e., the color-cyclic-ordered tree-level gluon amplitudes, which are gauge invariant, dynamical building blocks with Lie-algebra structure having been stripped away. More explicitly we will use S-matrix framework to discuss following four identities among these primary amplitudes: (1) Color-order reversed relation \( A(1,2,...,n) = (-)^{n-1}A(n,n-1,...,1) \); (2) The \( U(1) \)-decoupling relation \( \sum_{\sigma \in \text{cyclic}} A_n(1,\sigma(2,3,...,n)) = 0 \); (3) The Kleiss-Kuijf relation \( A_n(1,\alpha,n,\beta) = (-1)^{n\beta} \sum_{\sigma \in \text{OP}(\alpha,\beta)} A_n(1,\sigma,n) \). (2)

where Order-Preserved(OP) sum is over all permutations of the set \( \alpha \cup \beta^T \) where the relative ordering in each set \( \alpha \) and \( \beta^T \) (which is the reversed ordering of set \( \beta \)) is preserved. The \( n_\beta \) is the number of \( \beta \) elements. (4) The BCJ-relation\textsuperscript{[7]}

\[ A_n(1,2,\alpha,3,\beta) = \sum_{\sigma \in \text{POP}(\alpha,\beta)} A_n(1,2,3,\sigma).F_\beta \quad (3) \]

where Partial-Order-Preserved(POP) sum is over all permutations of set \( \alpha \cup \beta \) with preserving the relative ordering inside the set \( \beta \). The \( F_\beta \) are some dynamical factors and explicit definition can be found in \textsuperscript{[7]}. These four identities have been understood from different perspectives. The properties (1) and (2) can be shown from Lie-algebra structure. Property (3) is inspired from string theory and then shown in field theory\textsuperscript{[8]} using different color decomposition. Property (4) is conjectured through the Jacobi-identity but has only been proved from string theory\textsuperscript{[9]} (see further study\textsuperscript{[10]}). These four identities, especially the KK and BCJ relations, contain unexpected important properties of gauge theory. Our proof in S-matrix frame unifies the treatment of them all and makes them hold in general ground. Especially our proof is the first pure field theory proof of BCJ relation. Furthermore
our method can be applied to the field theory understanding of another very important Kawai-Lewellen-Tye(KLT) relation [11], which has only been shown from string theory. The importance of BCJ and KLT relations lies in the mysterious observation: on-shell gravity likes the square of gauge theory while their off-shell Lagrangian descriptions are completely different (one is normalizable and another one, unnormalizable). Understanding these observations in field theory will help us with the searching of consistent quantum gravity theory, which is still one of most fundamental open problems in physics.

THE COLOR-ORDER REVERSED RELATION

One basic observation of [5] is that color-ordered three particle amplitude is completely fixed by Lorentz symmetry and satisfy \( A(1, 2, 3) = (-)A(3, 2, 1) \) without using any Lie-algebra property. Using the BCFW recursion relation with pair \((n, 1)\), we get

\[
A(n, \beta_1, \ldots, \beta_{n-2}, 1) = \sum_{i=1}^{n-3} A(\tilde{n}, \beta_1, \ldots, \beta_i, -\tilde{P}_i) \frac{1}{P_i^2} A(\tilde{P}_i, \beta_{i+1}, \ldots, \beta_{n-2}, 1) = \sum_{i=1}^{n-3} (-)^{n-1} A(\tilde{n}, \beta_{n-2}, \ldots, \beta_{i+1}, \tilde{P}_i) \frac{1}{P_i^2} (-)^{1+2} A(-\tilde{P}_i, \beta_{i}, \ldots, \beta_{1}, \tilde{n}) = (-)^n A(\beta_{n-2}, \beta_{n-1}, \ldots, \beta_{1}, n).
\]

where \( A(1, 2, 3, 4, 5) = A(1, P_{23}, 4, 5) + A(1, P_{24}, 5) \) with \( n = 4 \) case is easy to check after using the color-reversed relation in the BCFW expansion. To get more idea of proof, let us present example of \( n = 5 \) given in (4). At each line we use (1) to expand left hand side into the right hand side. To make formula compact we have used, for example \( P_{523} \) to represent amplitude \( A(\tilde{5}, 2, 3, -\tilde{P}_{523})/s_{523} \). Then we use the KK-relation for the four gluon part, i.e., \( A(\tilde{5}, 2, 3, -\tilde{P}_{523}) = A(\tilde{5}, 2, 3, -\tilde{P}_{523}) A(P_{523}, 1, 4)/s_{523} \). By our purposely arrangement, it is easy to see that the sum of each column at the right hand side is zero after we use the \( (U(1)) \)-decoupling equation for \( n = 3 \) and \( n = 4 \) by induction.

THE KK-RELATION

Having the experience of \( n = 5 \), the proof for general \( n \) by induction is again by BCFW expanding each amplitude first, then regrouping every piece into \( U(1) \)-identity for the lower \( m \). For example, with \((1, 2)\)-shift the expansion of a general amplitude

\[
A_n(\hat{k}, \ldots, \hat{i}, k, \ldots, i, 1, k + 1, \ldots, n) = A_n(\hat{k}, \ldots, \hat{i}, 3, \ldots, k, \tilde{i}, \tilde{k} + 1, \ldots, n) + \sum_{\{i, \ldots, j\} \subseteq \{1, \ldots, n\}} A_n(\hat{k}, \ldots, \hat{i}, [3, \ldots, k], \tilde{i}, \tilde{k} + 1, \ldots, n - 1) + \ldots + A_n(k, \ldots, k, \hat{i}, \hat{k}, \ldots, \hat{i}, \tilde{k}, \tilde{i}, \tilde{k} + 1, \ldots, n),
\]

where \([i, \ldots, j]\) means sum of all divisions between legs \( i \) to \( j \) and \((k, t)\) means there is \( t \) particles in front of \( \hat{k} \). It can be checked that with fixed \( t \), the sum of \( k \) is indeed the \( U(1) \)-decoupling identity with lower \( m \) and is zero. Having all possible \( t \) we get the identity for \( n \), thus finished the proof.

THE \( (U(1)) \)-DECOUPLING RELATION

The demonstration of our proof. With \((1, 5)\)-shifting we can expand \( A(1, 2, 3, 4, 5) \) as

\[
A(1, 2, 3, 4, 5) = A(1, P_{23}, 4, 5) + A(1, P_{24}, 5) + 0 + 0 + 0 = A(1, P_{52}, 3, 4) + A(1, P_{53}, 4) + 0 + 0 + 0 = A(1, 4, P_{52}, 3) + A(1, 4, P_{53}) + A(1, P_{452}, 3)
\]

\[
A(4, P_{12}, 3) + A(4, 1, -P_{41}, P_{41}, 2, 5, 3)
\]

Then we use the KK-relation for the four gluon part, i.e., \( A(\hat{4}, \hat{1}, 2, \hat{P}_{35}) = -A(\hat{1}, 2, 4, \hat{P}_{35}) = A(\hat{1}, 4, 2, \hat{P}_{35}) \) as well as the one for \( A(\tilde{P}_{41}, 2, 5, 3) \) to get (notice we have used the color-order reverse relation)

\[
A((1, 2, 4, P_{15}) + (1, 4, 2, P_{35}) | 3, 5) + A(1, 4, 3, P_{25} | 2, 5)
\]

\[
+ A(1, 2, P_{12}, 4, 3, 5) + A(1, 4 | P_{41}, 2, 3, 5) + (P_{41}, 3, 2, 5)
\]

It is easy to see that among these six terms, \( T_1 + T_4 = A(1, 2, 4, 3, 5) \), \( T_2 + T_5 = A(1, 4, 2, 3, 5) \) and \( T_3 + T_6 = A(1, 4, 3, 2, 5) \), thus by the recombination we have produced the KK-relation for \( A(1, 2, 5, 3, 4) \).

Having above example, the proof of the general case \( A(1, \{\alpha_1, \ldots, \alpha_k\}, n, \{\beta_1, \ldots, \beta_m\}) \) with \((1, n)\)-shifting is done first by expanding as
where two cases \((i = 0, j = m)\) and \((i = k, j = 0)\) should be excluded from the summation. Now we use the induction for each component, i.e., \(A(\beta_{j+1}, \ldots, \beta_m, 1, \alpha_1, \ldots, \alpha_n, P_{ij}) = (-)^{m-j} \sum \sigma_i A(1, \sigma_i, P_{ij})\) and similarly for the second factor. With some calculations like previous example of \(n = 5\), it is easy to see that for each given set \(\{i, j, \sigma_i, \overline{\sigma}_i\}\), \((6)\) gives a term at the right hand side of \((2)\) with legitimate ordering and BCFW splitting. Thus if we can show that number of terms for both expansions are same, the proof is done.

To count terms, it is easy to see that there are \(C_{i+m-j}^i\) and \(C_j^{i-k-m}\) terms for each factor respectively at the right hand side of \((6)\). Thus the total number of terms at the right hand side of \((6)\) is

\[
-\frac{2(m+k)!}{m!k!} + \sum_{i=0}^k \sum_{j=0}^m \frac{(i+k-i)! (j+k-i)!}{i!(j!m-k)!}.
\]

where \(-\frac{2(m+k)!}{m!k!}\) counts the two excluded cases. The right hand side of KK-relation \((2)\) will be \(\frac{1}{(k+m)!}(k+m-1)\) after we have used the BCFW to expand each amplitude into \((k+m-1)\) terms. These two numbers match up as it should be.

### THE BCJ RELATION

The BCJ relation \((4)\) is more complicated since the appearance of various dynamical factors \(s_{ij}\). In its most general form, the set \(\alpha, \beta\) can be arbitrary. However, we want to show that all other equations are redundant except the one where the set \(\alpha\) has only one element, which we call the "fundamental BCJ-relation". More accurately we want to show that if these fundamental BCJ-relation are true, combining with \(U(1)\)-decoupling relation and KK-relation we can express any amplitude by \((n-3)!\) amplitudes of the form \(A(1, 2, 3, \sigma(4, n))\). This is exact the same statement given by general BCJ-relation.

To show that, let us start from the configuration \(A(1, 2, \{t_1\}, t_2, \{t_3, \ldots, t_{n-3}, 3\})\), i.e., the particle 3 is at the location \(n\) at the left hand side of the fundamental BCJ-relation. By the expansion at the right hand side of BCJ-relation, particle 3 will have two locations at each equation: one is at the location \(n\) and one is at the location \(n-1\). There are \((n-3)!\) equations, thus we can use them to solve all configurations of 3 at the location \(n\) by the one at the location \(n-1\). At the next step we consider the configuration at the left hand side of fundamental BCJ-relation with 3 at the location \(n-1\). By the expansion of the BCJ-relation at the right hand side we see now that 3 can be located at \((n-1)\) and \((n-2)\), thus we can solve 3 at the location \((n-1)\) by the one at location \((n-2)\). Iterating this procedure we can solve 3 at the location 5 by the one at the location 4 and finally we solve the one at the location 4 by the one at the location 3.

Now let us write down the form of fundamental BCJ-relation for \(n = 4, 5, 6\) as following:

\[
\begin{align*}
0 &= I_4 = A(2, 4, 3, 1)(s_{43} + s_{41}) + A(2, 3, 4, 1)s_{44} + A(2, 3, 4, 5, 1)(s_{45} + s_{41}) + A(2, 3, 4, 5, 1)(s_{45} + s_{41}) + A(2, 3, 5, 4, 1)s_{44} + A(2, 3, 5, 4, 6, 1)(s_{46} + s_{41}) + A(2, 3, 5, 4, 6, 1)(s_{46} + s_{41}) + A(2, 3, 5, 6, 4, 1)s_{44} \quad (8)
\end{align*}
\]

and obviously generalization for general \(n\). There are two observations useful later. The first one is the special relation for \(n = 3\), i.e., \(A(2, 3, 1)s_{33} = 0\). The second one is that we can use momentum conservation to write above relation into dual form, for example, the case \(n = 5\) can be rewritten as

\[
0 = A(2, 3, 4, 5, 1)s_{43} + A(2, 3, 4, 5, 1)s_{44} + A(2, 3, 4, 5, 1)s_{45} + A(2, 3, 4, 5, 1)s_{46} + A(2, 3, 5, 4, 1)s_{44} + A(2, 3, 5, 4, 6, 1)s_{46} + A(2, 3, 5, 4, 6, 1)s_{46} + A(2, 3, 5, 6, 4, 1)s_{44} \quad (8)
\]

Before we present our general proof by induction, let us consider how we can derive the BCJ-relation for \(n = 4\). Starting from the \(U(1)\)-decoupling with \((1, 2)\)-shifting we consider following contour integration expression which is zero by \(U(1)\)-decoupling relation

\[
\int \frac{dz}{z} s_{23}(z)[A(1, 2, 3, 4) + A(1, 3, 4, 2) + A(1, 4, 2, 3)] = 0. \quad (9)
\]

Among these three terms, since the multiplication of factor \(s_{23}(z)\), \(A(1, 2, 3, 4)\) has only one pole contribution at \(z = 0\), thus we have \(T_1 = s_{23}A(1, 2, 3, 4)\). The third term is zero, since \(1, 2\) are not nearby and the large \(z\) limit of amplitude is in fact \(1\). The second term is given by \(T_2 = (s_{23} - s_{23}(z_{123}))A(1, 3, 4, 2) = -s_{13}A(1, 3, 4, 2)\). Putting all results together and using the color-reserved relation we get immediately \(s_{23}A(2, 3, 4, 1) + (s_{23} + s_{43})A(2, 3, 4, 1) = 0\).

Having done for \(n = 4\), we move to the general proof using the induction. To make the step clear, we consider the case \(n = 6\) and arbitrary \(n\) is easily dealt with same method. Taking the \((2, 1)\)-shifting and using the BCFW recursion relation to expand each amplitude in \(I_6\), we will get three different splitting for each amplitude. Let us consider the splitting

\[
I_6^{[2]} = A(2, 4, 3, P_{24}, 3, 5, 6, \overline{1})(s_{43} + s_{45} + s_{46} + s_{41}) + A(2, 3, 5, 6, \overline{1})(s_{43} + s_{45} + s_{46} + s_{41}) + A(2, 3, 5, 6, \overline{1})(s_{43} + s_{45} + s_{46} + s_{41}) + A(2, 3, 5, 6, \overline{1})(s_{43} + s_{45} + s_{46} + s_{41}) + A(2, 3, 5, 6, \overline{1})(s_{43} + s_{45} + s_{46} + s_{41}) + A(2, 3, 5, 6, \overline{1})(s_{43} + s_{45} + s_{46} + s_{41}) + A(2, 3, 5, 6, \overline{1})(s_{43} + s_{45} + s_{46} + s_{41}) + A(2, 3, 5, 6, \overline{1})(s_{43} + s_{45} + s_{46} + s_{41})
\]

where the splitting parameter \([2]\) means there are two particles at the left hand side. All terms of \(I_6^{[2]}\) can be divided into two
categories: the one with 4 at the left hand side and the other
one, right hand side. The last three terms with 4 at the right
hand side can be rewritten as
\[ A(2, 3, \tilde{P}_{23} | \tilde{P}_{23}, 5, 4, 6, 1) (s_{45} + s_{46} + s_{41}) \]
\[ + A(2, 3, \tilde{P}_{23} | \tilde{P}_{23}, 5, 4, 6, 1) (s_{46} + s_{41}) \]
\[ + A(2, 3, \tilde{P}_{23} | \tilde{P}_{23}, 5, 6, 4, 1) s_{41} \]
\[ + \{ A(2, 3, \tilde{P}_{23} | \tilde{P}_{23}, 5, 4, 6, 1) + A(2, 3, \tilde{P}_{23} | \tilde{P}_{23}, 5, 4, 6, 1) \}
\[ (s_{41} - s_{41}(z_{23})) \]
By the induction over the second factor we know the sum of
first three lines are zero. The first term of \( I_{6}^{[2]} \) can be rewritten in
dual form as
\[ -s_{24} A(2, 4, \tilde{P}_{24} | \tilde{P}_{24}, 3, 5, 6, 1) \]
\[ = -s_{24}(z_{24}) A(2, 4, \tilde{P}_{24} | \tilde{P}_{24}, 3, 5, 6, 1) \]
\[ - (s_{24} - s_{24}(z_{24})) A(2, 4, \tilde{P}_{24} | \tilde{P}_{24}, 3, 5, 6, 1) \]
where again the first term is zero by induction. Using the
fact \(-s_{24} - s_{24}(z_{24}) = (s_{41} - s_{41}(z_{24}))\), we can put them
together as
\[ I_{6}^{[2]} = (s_{41} - s_{41}(z_{24})) \left\{ A(2, 3, \tilde{P}_{23} | \tilde{P}_{23}, 5, 4, 6, 1) \right\} \]
\[ + A(2, 3, \tilde{P}_{23} | \tilde{P}_{23}, 5, 6, 4, 1) + A(2, 3, \tilde{P}_{23} | \tilde{P}_{23}, 5, 6, 4, 1) \}
\[ + A(2, 4, \tilde{P}_{24} | \tilde{P}_{24}, 3, 5, 6, 1) (s_{41} - s_{41}(z_{24})) \]
By similar manipulations for \( I_{6}^{[3]} \) and sum all three
we finally have
\[ I_{6} = s_{41} \left\{ A(2, 4, 3, 5, 6, 1) + A(2, 3, 4, 5, 6, 1) \right\} \]
\[ + f \int_{z \neq 1} \frac{dz}{z} A(2, 4, 3, 5, 6, 1) + A(2, 3, 4, 5, 6, 1) \]
\[ + f \int_{z \neq 1} \frac{dz}{z} A(2, 4, 3, 5, 6, 1) + A(2, 3, 4, 5, 6, 1) \]
\[ + A(2, 3, 4, 5, 6, 1) + A(2, 3, 4, 5, 6, 1) \right\} \]
(10)
where the contour integration has excluded the contribution
at the pole \( z = 0 \). Using the KK-relation, we can rewrite it as
\[ I_{6} = s_{41} A(2, 4, 3, 5, 6, 1) + f \int_{z \neq 1} \frac{dz}{z} A(4, 2, 3, 5, 6, 1) \]
Now noticing that \((1, 2)\) are not nearby, thus
\[ f \int_{z \neq 1} \frac{dz}{z} A(4, 2, 3, 5, 6, 1) = 0 \]
by the \( \frac{1}{z^2} \) behavior at in-
finity, and we get
\[ -f \int_{z \neq 0} \frac{dz}{z} A(4, 2, 3, 5, 6, 1) = -f \int_{z = 0} \frac{dz}{z} A(4, 2, 3, 5, 6, 1) \]
\[ = -s_{41} A(4, 2, 3, 5, 6, 1) \]
Putting it back we have finally proved \( I_{6} = 0 \). The
proof for general \( n \) will be exact same as the
one with \( I_{6} \) and given by \( s_{41} A_{n}(4, 2, 3, 5, ..., n, 1) - f \int_{z = 0} \frac{dz}{z} A_{n}(4, 2, 3, 5, ..., n, 1) = 0 \).
Let us have some final remarks. In the proof of BCJ relation,
it is crucial that when shifted pair \((i, j)\) are not nearby,
there is a deformation making the amplitude vanishing as \( \frac{1}{z^2} \).

With this better behavior we should have some bonus relation
as found in gravity in \( [13, 14] \). From this paper, now we know the
mysterious bonus relation in gauge theory is nothing, but
the BCJ-relation.

The BCJ-relation has not been explored extensively, but
its potential importance is manifest. It can be used to speed
up amplitude calculation. Furthermore, its generalization to
higher loop \( [15] \) and its relation to gravity \( [16] \) make it impor-
tant for the discussion of finiteness of \( N' = 8 \) supergravity.

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