SEISMOLOGY OF TRANSVERSELY OSCILLATING CORONAL LOOPS WITH SIPHON FLOWS

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Received 2010 November 30; accepted 2011 January 28; published 2011 February 15

ABSTRACT

There are ubiquitous flows observed in the solar atmosphere of sub-Alfvénic speeds; however, after flaring and coronal mass ejection events flows can become Alfvénic. In this Letter, we derive an expression for the standing kink mode frequency due to siphon flow in coronal loops, valid for both low and high speed regimes. It is found that siphon flow introduces a linear, spatially dependent phase shift along coronal loops and asymmetric eigenfunctions. We demonstrate how this theory can be used to determine the kink and flow speed of oscillating coronal loops with reference to an observational case study. It is shown that the presence of siphon flow can cause the underestimation of magnetic field strength in coronal loops using the traditional seismological methods.

Key words: magnetic fields – magnetohydrodynamics (MHD) – Sun: corona

1. INTRODUCTION

The measurement of plasma flow speed in the solar atmosphere has traditionally been estimated with Doppler shift using spectrometers on board, e.g., Solar and Heliospheric Observatory (SOHO) and Hinode (e.g., Brekke et al. 1997; Winebarger et al. 2002). Some attempts have also been made by tracking features thought to be associated with flow using imagers (Winebarger et al. 2001; Chae et al. 2008). It has been found that sub-Alfvénic flows of around 100 km s⁻¹ are ubiquitous in the corona. However, less frequent but faster flows have been detected in the vicinity of flaring events and coronal mass ejections (CMEs; e.g., Innes et al. 2001; Harra et al. 2005), even into the Alfvénic regime of 10³ km s⁻¹ (e.g., Innes et al. 2003). Some of the reported flows are of siphon type, i.e., the flow is unidirectional from one end of the loop to the other. Spectroscopic identifications of siphon flows can be found, for example, in Teriaca et al. (2004) with the Solar Ultraviolet Measurement of Emitted Radiation (SUMER)/SOHO and in Tian et al. (2008) with SUMER/SOHO and the Extreme Ultraviolet Imaging Spectrometer (EIS)/Hinode. Very clear identifications of siphon flows through imaging observations were reported by Doyle et al. (2006) with the Transition Region and Coronal Explorer (TRACE) and Tian et al. (2009) with STEREO.

As well as generating fast flows, flares and CMEs can also cause standing kink oscillations in coronal loops (see, e.g., Aschwanden et al. 1999; Nakariakov et al. 1999), so it is natural to develop magnetohydrodynamic (MHD) theory that models the interaction of these waves with the flow. Since the dynamics of coronal plasma are dominated by magnetic fields, in general, the flow direction will be magnetic field aligned. Using Doppler shift, there may be large uncertainty in estimating flow speeds along coronal structures such as loops due to the line-of-sight effects. In the most extreme case, if the direction of flow is perpendicular to the line of sight, there will be no flow detected at all. Furthermore, if a loop with field aligned flow is oscillating with a kink mode, this adds further difficulty in measuring the flow speed using Doppler shift. In this Letter, we want to demonstrate how the problem of estimating the flow speed in oscillating coronal loops can be addressed by implementing the technique of magnetoseismology.

Coronal seismology is an indirect way to obtain the magnitudes of fundamental plasma parameters exploiting the observed oscillatory properties of the solar atmosphere. This idea was proposed by Uchida (1970) and Roberts et al. (1984) and up to now has successfully provided estimates of the magnetic field strength (Nakariakov & Ofman 2001; Van Doorsselaere et al. 2008) and density scale height (Andries et al. 2005; Verth et al. 2008) in the corona. Also, Arregui et al. (2007) and Goossens et al. (2008) established lower limits to the sub-resolution transverse inhomogeneity in coronal loops and upper limits to internal Alfvén speeds.

In this Letter, for the first time, we demonstrate how the flow speed along oscillating coronal loops can be determined from observations of standing kink modes. We theoretically investigate the effect of a constant siphon flow on stationary transverse waves and link this model to an observational case study.

2. EQUILIBRIUM MODEL

Consider an equilibrium model of a cylindrical axisymmetric flux tube of radius $R$ with constant axial magnetic field $B_0$, and a density contrast of $\rho_i/\rho_e$. The subscripts “i” and “e” refer to the internal and external parts of the tube, respectively. The length of the tube is $L$ and it is assumed that inside there is a unidirectional constant flow $U$ and no flow outside.

The stationary solution is constructed by superposing two propagating waves traveling in opposite directions. The effect of flow on propagating waves causes a frequency Doppler shift, a phenomena studied in the past by many authors, e.g., Goossens et al. (1992). In particular, it was found that, in the zero-$\beta$ limit, a valid approximation for the coronal plasma frequency is given by the following expression:

$$\omega = \frac{k \rho_i - \rho_e}{\rho_i + \rho_e} U \pm k c_s \sqrt{1 - \frac{\rho_i \rho_e}{(\rho_i + \rho_e)^2} \frac{U^2}{c_s^2}},$$

where $c_s$ the kink speed for a static equilibrium within the thin tube regime ($R \ll L$). The corresponding eigenfunctions (see, for example, Edwin & Roberts 1983) are expressed in terms of Bessel functions in the radial direction and have a sinusoidal dependence in the longitudinal direction.

To find a stationary solution, we impose that the forward (+ sign) and backward (− sign) waves must have the same frequency. This condition can only be satisfied if the corresponding
longitudinal wavenumbers are different, and according to Equation (1) they are given by

$$k_{\pm} = \frac{\rho_i}{\rho_i + \rho_e} U \pm c_k \sqrt{1 - \frac{\rho_o \rho_i}{(\rho_i + \rho_e)^2} U^2}.$$  

(2)

Note that in general $k_+ > 0$ and $k_- < 0$ (we do not consider KH-unstable modes here, i.e., we assume that the square root is always positive).

We concentrate on the $z$-dependence of the stationary solution, neglecting the radial dependence because we are assuming the thin tube limit. The study of the full MHD eigenvalue problem would require a more involved analysis. The transversal displacement of the tube axis is thus

$$\xi(t, z) = A \sin(\omega t - k_+ z) + B \sin(\omega t - k_- z),$$  

(3)

with $A$ and $B$ constants determined by boundary conditions. Imposing line-tying conditions at the footpoints, i.e.,

$$\xi(t, z = 0) = 0,$$  

(4)

$$\xi(t, z = L) = 0,$$  

(5)

from the first condition we find that $A = -B$. Using this relation and the second condition, we obtain the following dispersion relation:

$$\omega = k_0 c_k \left(1 - \frac{\rho_i}{\rho_i + \rho_e} \frac{U^2}{c_k^2}\right),$$  

(6)

where

$$k_0 = n \frac{\pi}{L}, \quad n = 1, 2, \ldots.$$  

(7)

The natural eigenfrequencies of the flux tube are given by Equation (6) (see Taroyan 2009 for the equivalent result for a pure Alfvén wave, and the recently published work of Ruderman 2010 for a more general study). This expression gives the explicit dependence of frequency on $U$. If we compare with the eigenfrequencies of the static case, $\omega = k_0 c_k$, it turns out that flow always leads to a frequency reduction, i.e., an increase in the period of oscillation.

We now turn our attention to the time dependence of the amplitude of oscillations along the loop. Using the fact that $A = -B$ and Equation (6), we have that Equation (3) can be written, after some algebra, in the following elementary form:

$$\xi(t, z) = C \sin k_U z \cos(\omega t + k_U z),$$  

(8)

where $C$ is an arbitrary constant, and we have introduced the wavenumber due to flow,

$$k_U = k_0 \frac{\rho_i}{\rho_i + \rho_e} c_k \sqrt{1 - \frac{\rho_o \rho_i}{(\rho_i + \rho_e)^2} U^2}.$$  

(9)

When there is no flow, $U = 0$, we recover the standing wave pattern of the static case, being a solution separable in time and space. When there is flow, the solution is no longer separable but the interpretation is straightforward. The term with the cosine in Equation (8) represents a propagating wave traveling in the opposite direction of the flow with an effective wavenumber, which is simply linearly proportional to flow speed in the slow flow regime ($U \ll c_k$). The term with the sine is just the envelope of the traveling wave, satisfying the line-tying conditions at the footpoints. Key to the present investigation, Equations (8) and (9) show that there are two main observational signatures of siphon flow in a coronal loop kink standing wave:

Signature 1. There is a linear phase dependence of the standing kink mode along the loop.

Signature 2. In one full period, the eigenfunctions of the standing kink mode will mostly exhibit an asymmetry about the center of the loop.

If either Signature 1 or 2 is present in data, then this may be an indication that there is a siphon flow present, but if both are present this is a stronger indication of siphon flow. Figure 1 illustrates the dependence of amplitude along a coronal loop at different times due to constant siphon flow, illustrating Signature 2 of siphon flow. It is worth noting that in the absence of siphon flow Signature 1 may simply indicate the presence of a propagating kink wave. The correct interpretation of data must be done on a case-by-case basis, taking into account the loop geometry and all estimated wave parameters.

3. EXPRESSIONS FOR SIPHON FLOW AND KINK SPEED

Usually, the data analysis of standing kink oscillations is based on a fit of the loop displacement at a given point along the loop of the following form:

$$\xi = \xi_0 \cos(\omega t + \phi) e^{-t/\tau_D}.$$  

(10)

The parameters that are determined from the fit are $\xi_0$, $\omega$, $\phi$, and $\tau_D$. The damping time, $\tau_D$, is not of interest in this work since a damping mechanism is most likely required to produce the attenuation of the signal with time. Regarding the other parameters, it is important to point out that it is possible, depending on the data set, to derive their values along some portion of the loop and not just at a single point (see, e.g., Verwichte et al. 2010).

Usually, the fundamental mode is reported in the observations, meaning that $n = 1$ in Equation (7) but there is also possible evidence of higher overtones (see, e.g., Verwichte et al. 2004). The loop length, $L$, can be estimated using a circular shape or a three-dimensional reconstruction, meaning that $k_0$ is well determined (up to the uncertainties in $L$).

When there is no flow, the determination of $\omega$ (calculated using the fit) allows us, together with the value of $k_0$, to calculate
the kink speed of the tube, i.e., \( c_k = \omega / k_0 \). However, with only estimates for \( \omega \) and \( k_0 \), we cannot determine the kink speed if flows are present. We also need additional information about the phase \( \phi \) and plasma densities, \( \rho_i \) and \( \rho_e \). From Signature 1 of siphon flow in Section 2, the phase should show a linear dependence with position along the loop with gradient \( k_U \) and this can be estimated with a linear fit to data. Also, the values of \( \rho_i \) and \( \rho_e \) can be estimated from observed intensity, since emission measure is approximately proportional to density squared if one considers the intensity of a clean emission line formed in the transition region or corona. In imaging observations, usually a passband is wide enough to include several emission lines with different formation temperatures and the integrated intensity of this passband is no longer simply proportional to the density squared. The density diagnostics is possible if you have observations of density-sensitive line pairs. But this usually needs spectroscopic observations (e.g., SUMER, the Coronal Diagnostic Spectrometer, and EIS).

Once \( \omega \), \( k_0 \), \( k_U \), \( \rho_i \), and \( \rho_e \) are determined from observation, it is straightforward to calculate the two magnitudes of interest, namely, \( U \) and \( c_k \). We only need to combine Equations (6) and (9) to find

\[
U = \frac{\rho_i + \rho_e}{\rho_i} \frac{k_U}{k_0^2 - k_U^2} \omega
\]

and

\[
c_k^2 = \frac{1}{2} \frac{\omega^2}{k_0^2} + \frac{\rho_i}{\rho_i + \rho_e} U^2 + \frac{1}{2} \frac{\omega^2}{k_0^2} \frac{\rho_i^2}{(\rho_i + \rho_e)^2} U^2.
\]

Note that Equation (11) contains the information about the flow direction as well, if \( k_U > 0 \) (\( k_U < 0 \)) then we have that \( U > 0 \) (\( U < 0 \)) assuming that \( k_0 > k_U \).

4. OBSERVATIONAL CASE STUDY

There have been a number of observational studies that have shown a gradient in phase along coronal loops oscillating with the standing kink mode, e.g., Verwichte et al. (2004), Van Doorsselaere et al. (2007) and De Moortel & Brady (2007). However, it is unclear from these studies whether the observed phase difference is due to siphon flow or wave propagation since there is only evidence of Signature 1 in these cases. For this reason, we instead focus on the particular case study of Verwichte et al. (2010), since both Signatures 1 and 2 are present in the data, providing a stronger argument for the siphon flow interpretation. Using combined TRACE and EIT observations, Verwichte et al. (2010) analyzed the standing kink mode generated in a coronal loop that was part of a large arcade. The coronal loop, positioned off-limb, starts oscillating after an M1.5 GOES level flare and associated CME occurs at about 11:04 UT. Regarding Signature 1, Verwichte et al. (2010) fitted a linear function to the observed phase along a 120 Mm portion of the loop (about 18% of the total loop length \( L = 680 \) Mm) and this is shown in Figure 6 of their paper. Evidence of Signature 2, an asymmetry in the amplitude about the loop half-length, is shown in Figure 8.

Verwichte et al. (2010) perform a fit to the loop displacement as in Equation (10). The values of the parameters of interest (see Table 1 of their paper) are \( \omega = 2.60 \times 10^{-3} \) s\(^{-1} \) (\( P = 2418 \) s) and \( k_0 = 4.62 \times 10^{-3} \) Mm (\( L = 680 \) Mm, and \( n = 1 \)). The authors also provide a linear fit to the phase as a function of position along the loop (denoted by \( s \)), which is

\[
\phi(s) = 0.93 - 3.67 \times 10^{-3} \left( \frac{s}{1 \text{ Mm}} - 80 \right) \text{ (rad)},
\]

meaning that \( k_U = -3.67 \times 10^{-3} \) Mm\(^{-1} \). Using Equation (11) we find that \( U = -1445 \) km s\(^{-1} \), while from Equation (12) we obtain that \( c_k = 1610 \) km s\(^{-1} \) (assuming a density contrast of \( \rho_i/\rho_e = 5 \)). Using the static model we find that \( c_k = 563 \) km s\(^{-1} \), almost three times smaller than the inferred value assuming a siphon flow. The value of the flow speed is negative, meaning that the flow would be toward the footpoint shown in the lower image of Figure 1 of Verwichte et al. (2010).

It is relatively simple to estimate the errors in the calculations of the flow and kink speeds. These errors are due to the uncertainties in the quantities that appear in Equations (11) and (12). Typically, the errors are \( \delta L = 0.05 L \), \( \delta P = 0.05 P \), and \( \delta (\rho_i/\rho_e) = 0.4 \), and also assume that \( \delta k_U = 0.1 k_U \).

Applying the error propagation formula in Equations (11) and (12), we find that the flow is \( 1445 \pm 755 \) km s\(^{-1} \) and the kink speed is \( 1610 \pm 903 \) km s\(^{-1} \). In this example, the errors in the speeds are not small and are specially sensitive to the uncertainty in \( k_U \). It is worth noting that the calculated speeds and errors are based on the assumption of constant flow speed along the loop. It is clear that, for this particular example, the assumption of a constant siphon flow significantly increases the inferred kink speed (almost by a factor of three). Therefore, the presence of siphon flow has an important implication for the method of estimating magnetic field strength along coronal loops by Nakariakov & Ofman (2001), which assumes a static equilibrium. In the present example, the magnetic field strength would therefore be underestimated by a factor of three.

Assuming there is a siphon flow present, then the flow is in the fast (Alfvénic) flow regime of \( 10^4 \) km s\(^{-1} \). The oscillation event analyzed here takes place in a coronal loop arcade in the vicinity of an M class flare and CME. In similar events, fast flow signatures have often been measured in Doppler shift (e.g., Innes et al. 2001; Harra et al. 2005). After an X class flare Innes et al. (2003) measured similar Alfvénic flow speeds in the range 800–1000 km s\(^{-1} \) in a coronal arcade using combined SUMER and TRACE observations, and it must be emphasized that these Doppler shifts are likely to be underestimates of true field aligned flows due to line-of-sight effects.

Alfvénic flow would cause significant asymmetry about the loop half-length in the eigenfunction (Signature 2) along the loop. Interestingly, the estimated eigenfunction, derived by Verwichte et al. (2010) using EIT, shows that the fundamental mode has such an asymmetry. This is clearly seen in Figure 8(a) (see the region \( s/L \in [0,0.4] \)) where Verwichte et al. (2010) fit the observed displacement (solid line) with that predicted assuming a planar loop model (dashed line). However, according to our model, this asymmetry can be explained by fast axial flow (see Figure 1).

5. SUMMARY AND CONCLUSIONS

Using a simple model of a coronal loop, we have derived an analytical expression for the frequency of oscillation of a thin tube with a constant siphon flow, valid for both slow and fast flow regimes. It was found that calculation of the kink speed and estimation of magnetic field assuming a static model give values that are smaller than the ones obtained in the presence of flow. This calls into question the validity of traditional seismological
methods assuming a static background to estimate the magnetic field along coronal loops.

Contrary to the static case, different positions along the tube oscillate with a different phase, in agreement with the recent results of Ruderman (2010). However, we have pointed out in our work the importance of the phase shift, since it has a linear behavior with the axial coordinate along coronal loops and in the low flow regime it is simply linearly proportional to the flow speed. Furthermore, the presence of flow breaks the symmetry of the eigenmodes about the loop half-length.

According to our model, assuming that the linear phase shift reported along the post-flare coronal loop analyzed by Verwichte et al. (2010) is caused by a siphon flow, the flow would be in the Alfvénic regime. This is not completely unreasonable since in the dynamic post-flare environment of coronal loops such fast flows have been estimated using Doppler shift and feature tracking techniques (e.g., Innes et al. 2001, 2003; Harra et al. 2005). In any case, other cases need to be investigated in order to prove that this technique can also be used with sub-Alfvénic flows.

The most important observed parameter that determines the value of the flow (and kink speed) in our model is the value of $kU$. The value of this parameter was estimated using only a small portion of the loop. If we determine the phase dependence along the whole loop (the Solar Dynamics Observatory should help in this regard), we would have a much more accurate determination of the speeds and a significant reduction in the errors. This more complete information will also allow us to test whether the flow is unidirectional or not, according to the sign of $kU$ at the footpoints.

In reality, flows along coronal loops are much more complex than the simple model proposed in this Letter. In particular, they are probably non-symmetric with respect to the loop apex, and more importantly, they are usually transient (see, for example, Doyle et al. 2006; Tian et al. 2009). Nevertheless, we must understand the simplest possible configuration before additional complications are added. In this regard, we have made the first step in assessing the effect of constant siphon flows on standing kink oscillations.

J.T. acknowledges the Universitat de les Illes Balears for a postdoctoral position and the funding provided under projects AYA2006-07637 (Spanish Ministerio de Educació n y Ciencia) and FEDER funds. G.V. and M.G. acknowledge G.V. and M.G. acknowledge support from K.U. Leuven via GOA/2009-009 and financial support from UIB during their stay at this university. Authors also acknowledge M. Ruderman and E. Verwichte for their helpful comments and the anonymous referee for his/her constructive suggestions.

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