Numerical modeling of the stress state of constructions from fibrous composites

A M Polatov¹, A M Ikramov², S I Pulatov² and S M Gaynazarov²

¹Professor, NUUz, Uzbekistan, Tashkent, University street, 4, 100174
²Associate professor, NUUz, Uzbekistan, Tashkent, University street, 4, 100174

E-mail: asad3@yandex.ru

Abstract. The paper is devoted to numerical modeling of the deformed state of physically nonlinear transversely isotropic bodies with an aperture. To solve the problem of the theory of plasticity, a simplified theory of small elastoplastic deformations for a transversely isotropic body is used. In the work, to describe the anisotropy of the mechanical properties of transversely isotropic materials, a structural-phenomenological model is used. It allows you to represent the source material in the form of a complex of two jointly working isotropic materials. The main material is considered from the standpoint of continuum mechanics. The fiber material is oriented along the anisotropy direction of the starting material. It is assumed that the fibers perceive only the axial tensile-compression forces and are deformed together with the main material. The presentation of fibrous composites in the form of homogeneous anisotropic materials with effective mechanical parameters allows a fairly accurate calculation of structures with stress concentrators. Based on a simplified theory and finite element method, a computer model of nonlinear deformation of fibrous composites is constructed. The influence of the configuration of holes and a rectangular crack on the distribution of deformation and stress fields in the vicinity of these concentrators is investigated.

1. Introduction

Currently, there are numerous models describing the behavior of isotropic materials under ductility conditions. These models also allow the analysis of structural deformations of fibrous materials.

Composite materials mechanics is one of very rapidly developing research areas, which obtained significant theoretical and experimental results. However, non-linear strain processes of composite materials with concentrators are not well investigated yet. Modern developments in mathematical modeling of transversely-isotropic materials’ elastic-plastic strain process cannot be considered as complete. Wide implementation of composite materials has led to the emergence of new fields in science related to the study of elastic-plastic materials strain [1]. In [2] study an elastic-plastic stress analysis is carried out for long silicon carbide fiber reinforced magnesium metal matrix composite with a square hole by using finite element technique. The expansion of plastic zone and residual stress are determined in the symmetric cross-ply and angle-ply laminated plates for small deformations. The results show that the plastic regions at the plate edges expand in the direction of the fiber, but at the border of the hole expand toward the diagonal of the hole. In monograph [3] was given, an analysis of the concentration and distribution of stresses in isotropic, orthotropic and layered composite plates
with a central round hole subjected to transverse static load. In [4] is study an elastic-plastic strain analysis is carried out for fibrous composites by using numerical modeling. Application of homogeneous transversely-isotropic model was chosen based on problem solution of a square plate with a circular hole under uniaxial tension. The influence of holes on stress state of complicated configuration transversely-isotropic bodies has been studied. The influences of the fibrous structure on stress concentration in vicinity of holes on boron/aluminum D16, used as an example. In [5] presents experimental and theoretical study of the mechanical and strength properties of orthotropic composite materials. Fiberglass composites based on epoxy laminates are investigated. The paper compares experimental results and numerical simulation for flat specimens of an orthotropic composite material in the case of uniaxial deformation. Adequate results on the modeling of deformation of orthotropic composite materials are obtained.

In this paper, to describe the anisotropy of the mechanical properties of transversely isotropic materials, a structural-phenomenological model is used. It allows you to represent the source material in the form of a complex of two jointly working isotropic materials. The main material is considered from the standpoint of continuum mechanics.

The fiber material is oriented along the anisotropy direction of the starting material. It is assumed that the fibers perceive only the axial tensile-compression forces and are deformed together with the main material. The presentation of fibrous composites in the form of homogeneous anisotropic materials with effective mechanical parameters allows a fairly accurate calculation of structures with stress concentrators. It is assumed that the fiber thickness and the gap between the fibers are several orders of magnitude smaller than the radius of the hole or the length of the crack.

An elastoplastic medium an inhomogeneous continuous material is considered. The material consists of two components: reinforcing elements and a matrix (or binder). The matrix provides the joint work of the reinforcing elements. In fibrous materials, the deformation of an elastoplastic matrix provides loading of fibers of high strength.

It is known that fibrous material and a transversely isotropic medium are equivalent concepts. In this regard, when solving the problem of physically nonlinear deformation of fiber composites, the theory of small elastoplastic deformations for a transversely isotropic medium is used [6].

The paper noted that when considering a reinforced composite, the stiffness of the reinforcing elements of significantly exceeds the stiffness of the binder material. In this case, it becomes possible to use the simplified deformation theory of plasticity. The simplified theory allows the theory of small elastoplastic deformations to be applied to solve specific applied problems. The essence of simplification lies in the following assumption: with simple stretching of the composite in the direction of the axis of transversal isotropy and the direction perpendicular to it, plastic deformations do not occur. The application of the simplified theory is based on the fact that for the reinforced composite under consideration, the stiffness of the reinforcing elements significantly exceeds the stiffness of the binder.

2. Materials and methods
The boundary value problem of the theory of elasticity for anisotropic bodies consists of the equilibrium equation:

$$\sigma_{ij,j} + X_i = 0, \quad x_i \in V$$

Hooke's generalized law

$$\sigma_{ij} = C_{ijkl} : \varepsilon_{kl},$$

Cauchy relations

$$\varepsilon_{kl} = \frac{1}{2} \left( \frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k} \right)$$
and boundary conditions

\[ u_i \bigg|_{\Sigma_i} = u'^i, \quad x_i \in \Sigma_i, \quad i = 1, 2 \]

\[ \sum_{j=1}^3 \sigma_{ij} n_j \bigg|_{\Sigma_2} = S'_j, \quad x_j \in \Sigma_2, \quad j = 1, 2 \]

where

\( u_i \) – displacement vector components;

\( X_i, S_i \) – volumetric and surface forces;

\( \Sigma_i, \Sigma_2 \) – constituent surfaces \( \Sigma \) of volume \( V \);

\( n_j \) – external normal to the surface \( \Sigma_i \) of volume \( V \);

\( C_{ijkl} \) – tensor of elastic constants.

In the case of the deformation theory of plasticity of transversely isotropic media, the defining relation (2) is replaced by relation (6). Relation (6) is a decomposition of the stress tensor into the spherical and deviator parts [6]:

\[ \sigma_{ij} = \bar{\sigma}(\delta_{ij} - \delta_{ij}^3) + \sigma_{33}\delta_{ij}^3 + \frac{P_u}{p_u} p_{ij} + \frac{Q_u}{q_u} q_{ij}, \]

where

\[ \bar{\sigma} = (\lambda_2 + \lambda_4) \tilde{\theta} + \lambda_3 \epsilon_{33}, \quad \sigma_{33} = \lambda_3 \tilde{\theta} + \lambda_1 \epsilon_{33}, \]

\[ P_u = 2\lambda_4(1 - \pi) p_u, \quad Q_u = 2\lambda_5(1 - \chi) q_u. \]

In the ratios (6-8), the following notation is used:

\( \lambda_i \) – elastic constants of transversely isotropic materials,

\( \pi, \chi \) – experimentally determined functions,

\[ p_u = \frac{1}{\sqrt{2}} p_{ij} p_{ij}, \quad q_u = \frac{1}{\sqrt{2}} q_{ij} q_{ij}, \]

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where

\[ p_{ij} = \epsilon_{ij} + \frac{\tilde{\theta}}{2}(\delta_{ij}^3 - \delta_{ij}), \quad q_{ij} = \epsilon_{ij}^3 \delta_{ij}^3 + 2\epsilon_{33}^3 \delta_{ij}^3 - (\epsilon_{ij}^3 \delta_{ij}^3 + \epsilon_{33}^3 \delta_{ij}^3), \]

\[ P_u = \sigma_{ij} + \bar{\sigma}(\delta_{ij}^3 - \delta_{ij}), \quad Q_u = \sigma_{ij} + \bar{\sigma}(\delta_{ij}^3 - \delta_{ij}). \]

For the simplified transversely isotropic theory of plasticity, the relations between the invariants of the stress and strain tensors are written in the form [6]:

\[ \bar{\sigma} = (\sigma_{11} + \sigma_{22})/2. \]
\begin{align}
\bar{\boldsymbol{\sigma}} &= \left( \lambda_2 + \lambda_4 \right) \bar{\boldsymbol{\theta}} + \lambda_3 \varepsilon_{33}, \\
\sigma_{33} &= \lambda_3 \bar{\boldsymbol{\theta}} + \lambda_4 \varepsilon_{33}, \\
P &= P(p), \quad Q = Q(q),
\end{align}

Here the functions $P$ and $Q$ have the form:

\begin{align}
P &= 2\lambda_4 (1 - \pi(p)) p, \quad Q = 2\lambda_3 (1 - \chi(q)) q, \tag{14}\end{align}

where $\pi(p)$ and $\chi(q)$ – plasticity functions type Ilyushin [7], whose values in the elastic zone are equal to zero. In the elastic region, the parameters $\sigma_{ij}$ are determined by Hooke's law. In the field of plastic deformations, they are determined on the basis of the deformation theory of plasticity. Plastic strain zones are determined based on the Mises criterion.

For the simplified transverse isotropic theory of plasticity, the relationship between stresses and strains is described by the relations:

\begin{align}
\sigma_{11} &= (\lambda_2 + \lambda_4) \bar{\boldsymbol{\theta}} + \lambda_3 \varepsilon_{33} + \frac{P_u}{p_u} p_{11}, \quad p_{11} = \frac{\varepsilon_{11} - \varepsilon_{22}}{2}, \\
\sigma_{22} &= (\lambda_2 + \lambda_4) \bar{\boldsymbol{\theta}} + \lambda_3 \varepsilon_{33} + \frac{P_u}{p_u} p_{22}, \quad p_{22} = \frac{\varepsilon_{22} - \varepsilon_{11}}{2}, \tag{15}
\end{align}

\begin{align}
\sigma_{33} &= \lambda_3 \bar{\boldsymbol{\theta}} + \lambda_4 \varepsilon_{33}, \quad \bar{\boldsymbol{\theta}} = \varepsilon_{11} + \varepsilon_{22}, \quad \sigma_{12} = \frac{P_u}{p_u} p_{12}, \quad p_{12} = \varepsilon_{12}, \\
\sigma_{13} &= \frac{Q_u}{q_u} q_{13}, \quad q_{13} = \varepsilon_{13}, \quad \sigma_{23} = \frac{Q_u}{q_u} q_{23}, \quad q_{23} = \varepsilon_{23}.
\end{align}

The mechanical parameters of a transversely isotropic material are related to the $\lambda_i$ modules by the following relationships:

\begin{align}
\lambda_1 &= E'_f \left( 1 - \mu_f \right)/l; \quad \lambda_2 = E'_f \left( \mu_f + k \mu_f^2 \right)/\left[ \left( 1 + \mu_f \right)/l \right]; \\
\lambda_3 &= E_f \mu_f/l; \quad \lambda_4 = G_f = E_f'; \\
\lambda_5 &= G'_f; \quad l = 1 - 2\mu_f^2k; \quad k = E_f'/E'_f. \tag{16}
\end{align}

Here $\mu_f$ and $\mu'_f$ – are the effective Poisson's ratios, $E_f$ and $E'_f$ – are the effective Young module, also $G_f$ and $G'_f$ – are the effective longitudinal shear module, respectively, along the plane and axis of the transversal isotropy.

To calculate the effective mechanical parameters of fibrous materials in the work, the ratios given in [8] are used. They make it possible to take into account the internal structure of the material for calculating periodically inhomogeneous materials based on the asymptotic averaging method, and suitable for any property values and volume fractions of components. Their use also allows one to take into account the radial interaction of the components caused by the difference in the Poisson's ratios of the matrix and fiber. It should be noted that in the case of high stiffness and a large volume fraction of fiber, the expression for determining the shear modulus $G$ during deformation in the transverse direction does not take into account the geometric structure of the composite and gives an underestimated value for the effective modulus in the case of high stiffness and a large volume fraction of fiber.
3. Results and discussion

3.1. Determination of the effective composite mechanical parameters

Table 1 shows the values of the effective modules of a unidirectional composite – boron/aluminum alloy. The results were obtained on the basis of the expressions given in [8] for various values of the volumetric content of fiber in the composite.

The matrix material used is aluminum alloy D16 (boron/aluminum alloy) with elastic constants: 

\[ E = 7.1 \cdot 10^4 \text{ MPa; } \mu = 0.32; \text{ hardening coefficient } \bar{\lambda} = 0.5 \text{ and elastic limit } \sigma_s = 2.13 \cdot 10^5 \text{ MPa.} \]

For boron fiber: 

\[ E' = 39.7 \cdot 10^4 \text{ MPa; } \mu' = 0.21; \text{ tensile strength } \sigma_s' = 2.5 \cdot 10^3 \text{ MPa [9].} \]

In order to study the effect of the fiber volumetric content in a unidirectional composite material, a computational experiment was performed [4], in which the ratios for various values of the fiber volumetric content in the composite are determined.

Carrying out computational experiments allowed us to study the stress-strain state of fibrous composite materials. Studies have confirmed the patterns associated with the influence of the volumetric content of fibers in the composite [10]. Analysis of the calculation results showed that, with fiber volume content in the range from 30% to 60%, the fibrous composites possess elastoplastic characteristics [4].

Table 1. Effective mechanical parameters.

| \(\nu\) | \(E'[\text{MPa}]\) | \(E[\text{MPa}]\) | \(G'[\text{MPa}]\) | \(G[\text{MPa}]\) | \(\mu'\) | \(\mu\) |
|---|---|---|---|---|---|---|
| 25% | 152710 | 89340 | 34010 | 37200 | 0.2881 | 0.2011 |
| 35% | 185320 | 99640 | 38020 | 43110 | 0.2762 | 0.1558 |
| 45% | 217970 | 112610 | 43120 | 50530 | 0.2646 | 0.1144 |
| 55% | 250560 | 129480 | 49790 | 59910 | 0.2537 | 0.0806 |
| 60% | 266820 | 139920 | 53960 | 65510 | 0.2480 | 0.0682 |

3.2. Elastoplastic design analysis with an elliptical hole

To study the effect of an insulated hole on the stress-strain state of a structure from a unidirectional composite, we consider the three-dimensional problem of deformation of a rectangular unit plate (height – 1.0; width – 0.5; thickness – 0.1). The plate is in uniaxial uniform tension along the axis by a distributed load applied at the lower and upper edges (Figure 1).

![Figure 1. The fourth part of the plate section plane 0xz.](image-url)
$\mu' = 0.21$, $\sigma'_t = 2.5 \times 10^5$ MPa (tensile strength). The fibers of the material are oriented along the axis, the volumetric fiber content in the composite is $\nu = 60\%$.

Effective mechanical parameters for an alloy of boron/aluminum: $E = 1.3992 \times 10^5$ MPa, $E' = 2.6682 \times 10^5$ MPa, $G = 0.6551 \times 10^5$ MPa, $G' = 0.5396 \times 10^5$ MPa, $\mu = 0.0682$, $\mu = 0.2480$.

The effect of an elliptical hole on the distribution of deformation and stress fields in the plane of transversal isotropy $0xy$ is studied. As a stress concentrator is considered isolated through hole in the shape of an ellipse ($r_1 = 0.5$; $r_3 = 0.1$). The characteristic points $A$ and $B$ are located in the zones of stress concentration. For an ellipse (Figure 1), this is the point $A$ at the intersection of the contour of the hole with the axis $0x$ and the point $B$ located at the intersection with the axis $0z$;

The three-dimensional elastoplastic problem of uniformly distributed tension of a single plate along the load axis ($P_{yz} = 950$ MPa) is considered. A load is applied at the lower and upper edges of the plate. The plate has an insulated hole in the shape of an ellipse ($r_1 / r_3 = 1/8$).

In a first approximation, the effective material constants of the plasticity function of an equivalent transversely isotropic medium are equal to the material constants of the duralumin matrix. To compare the results, table 2 shows the values of the components of the elastic and elastoplastic state of the plate in the plane of transverse isotropy $0xy$ for the cross section $0xz$. The results characterize the behavior of the matrix material of fibrous composites. Table 2 shows the values of the parameters of the stress-strain state in the vicinity of the point of the elliptical hole. An analysis of the data indicates a significant decrease in the values of strain intensity $u_p$ and stresses $P_u$ in the plane of isotropy due to plastic strains.

| Calculation      | $P_u$, MPa | $\sigma_{xx}$, MPa | $\sigma_{yy}$, MPa | $P_u$, MPa |
|------------------|------------|--------------------|--------------------|------------|
| Elastic          | 0.00398    | -810.63            | -72.47             | 522.09     |
| Elastoplastic    | 0.00327    | -622.23            | -105.28            | 410.95     |

Figure 2 shows the distribution of stress intensity in the plane of isotropy for a plate with a hole in the form of an ellipse at $r_1 / r_3 = 1/8$. The results were obtained from elastic (Figure 2a) and elastoplastic calculations (Figure 2b). The areas of plastic deformations are concentrated in the vicinity of the upper and lower parts of the hole (Figure 2b).

Figure 2. Distributions of strain intensity $P_u$ in a plate with an elliptical hole in elastic (a) and elastoplastic (b) calculations.
3.3. Elastoplastic design analysis with rectilinear crack

Consider the results calculating the stress-strain state of a plate with an isolated notch in the form of a horizontal rectilinear crack are analyzed.

Stress concentrator – isolated straight crack \( l = (0, 1) \). The characteristic points \( A \) and \( B \) are located in the zones of stress concentration: \( A \) – a point at its peak and \( B \) – a coastal point in the middle of the length. In table 3 shows the values of the parameters of the stress-strain state in the plane of isotropy at the crack tip (point \( A \)) and midpoint (point \( B \)). An analysis of the data indicates a significant decrease in the values of strain intensity \( p_u \) and stresses \( P_u \) in the plane of isotropy due to plastic strains.

Table 3. Comparison of calculation results in plate with crack.

| Parameters | At the top of crack (point \( A \)) | At the midpoint (point \( B \)) |
|------------|-----------------------------------|--------------------------------|
| \( p_u, \text{MPa} \) | 0.00232                          | 0.0155                          | 0.0373                          | 0.0338                          |
| \( \sigma_{xx}, \text{MPa} \) | 711.44                           | 492.22                          | –733.41                         | –634.13                         |
| \( \sigma_{yy}, \text{MPa} \) | 293.19                           | 221.15                          | –42.526                         | –72.61                          |
| \( P_u, \text{MPa} \) | 304.29                           | 203.10                          | 488.65                          | 418.18                          |

In the vicinity of the midpoint on the crack faces (Figure 3b), the strain intensities in the isotropy plane \( p_u > p_s \), this corresponds to plastic deformation. However, in the vicinity of the crack apexes (Figure 3a), values \( p_u < p_s \) indicate elastic deformation. This confirms the stretching of the fibrous composite along the axis of transversal isotropy \( 0z \). The region of plastic deformations along the isotropy plane is concentrated in the vicinity of the middle of the crack faces. The areas in the vicinity of the tops of the crack remain elastic.

![Figure 3. Distributions of strain intensity in a plate with a crack in elastic (a) and elastoplastic (b) calculation.](image)

It is established that the analysis of the results of a computational experiment allows us to design a rational structure of fibrous composites, to identify the location of additional holes and to reduce the stress concentration in the vicinity of concentrators.

4. Conclusion

Analysis of the results of computational experiments allows us to draw the following conclusions:

- based on the simplified theory of small elastoplastic deformations of transversely isotropic media and the finite element method, a numerical model is developed for solving three-dimensional problems of elastoplastic deformation of fibrous composites;
• the effects of volumetric fiber content on the deformation of fibrous composite materials have been investigated;
• in plane of isotropy the effect of an elliptical cavity on the stress-strain state of fibrous composites is studied;
• in plane of isotropy the effect of an internal cut in the form of a horizontal rectilinear crack on the stress-strain state of fibrous composites is studied;
• the regions of plastic deformations in the isotropy plane and the redistribution of parameters of the stress-strain state of fibrous composites in the vicinity of stress concentrators are determined.

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