Research on Tracking Control of A Quadrotor UAV Based on Model Predictive Control and PD Control

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Abstract. In order to solve the problem of path tracking of a quadrotor UAV, this paper proposes a track tracking control method which combines Model Predictive Control algorithm and PD control method. Model Predictive Control algorithm can generate control input for formation flight and track the specified trajectory. PD control can achieve rapid response to attitude and adjust error quickly. The simulation results verify the effectiveness of the proposed control method.

1. Introduction
The quadrotor UAV is a kind of vertical take-off and landing aircraft, which is lifted and controlled by four rotors. It is an underactuated system which has six degrees of freedom and four control inputs. Due to its wide application in the fields of military surveillance, intelligence acquisition and rescue missions, the automatic control of UAV has attracted the attention of many scholars in recent years [1].

In recent years, scholars have developed many control algorithms. The authors of [3] proposed a control method combining model predictive control and PID control, which can have good effects on the height and attitude stability of the quadrotor UAV under different disturbed and undisturbed conditions. The authors of [4] proposed a path tracker for the quadrotor UAV based on model predictive control and adaptive backstepping, which can track the reference trajectory better. The authors of [5] proposed a method to stabilize the quadrotor UAV using sliding mode control, which can robustly stabilize the quadrotor UAV. The authors of [6] proposed two flight control strategies, PID algorithm and LQR algorithm, and compared them. The authors of [7] proposed a tracker design method that combines nonlinear $H_{\infty}$ control and MPC algorithm. This method can effectively deal with the influence of continuous disturbance and track the reference track better. The authors of [8] proposed a control rate of the attitude subsystem based on an exponential observer, which can effectively solve the effect of input saturation. The authors of [9] proposed a fractional order improved super twisting proportional-integral-derivative sliding mode control, which has high control performance, efficiency and anti-interference ability.

To solve the path tracking problem of the quadrotor UAV, model predictive control algorithm and PD control are adopted in this paper. Model predictive control (MPC) is a feedback control scheme in
which the optimal sequence is calculated online, only the first control input in the sequence is applied, and the optimization is repeated in each subsequent step. PD can realize quick response to posture.

The chapters of this paper are arranged as follows: Section II introduces the UAV dynamic model. Section III describes the design of the controller. Section IV presents the simulation results to verify the effectiveness of the proposed method. Section V gives the conclusion.

2. Mathematical model

The motion of the quadrotor UAV is an extremely complex process. To obtain the dynamic equation of the UAV, it is necessary to analyze and integrate the different motion states of the UAV. In this case, two different coordinate systems need to be established: the geographic coordinate system $E(O_x, O_y, O_z)$ and the body coordinate system $B(O_x, O_y, O_z)$. Finally, transforming the corresponding relation between the two coordinate systems by Newton-Euler method, so as to obtain the dynamics equation of the quadrotor UAV. Figure 1 shows that the Euler angle of the UAV is defined as the roll angle $\gamma$ round the X axis, the pitch angle $\theta$ round the Y axis and the yaw angle $\psi$ round the Z axis. Therefore, the transformation matrix of the body coordinates and geographic coordinates can be obtained as follows:

$$R_{eb} = \begin{bmatrix}
\cos \theta \cos \psi & \sin \theta \sin \gamma \cos \psi - \sin \psi \sin \gamma \cos \theta & \sin \theta \cos \gamma \cos \psi + \sin \gamma \sin \psi \\
\cos \theta \sin \psi & \sin \theta \sin \gamma \sin \psi + \cos \gamma \cos \psi & \sin \theta \cos \gamma \sin \psi - \cos \gamma \sin \psi \\
-\sin \theta & \sin \gamma \cos \theta & \cos \gamma \cos \theta
\end{bmatrix}$$

(1)

The motion equation of the quadrotor UAV can be obtained by Newton's second law and Euler equation, as:

$$F = m\ddot{P}$$

(2)

$$M = \frac{dH}{dt}$$

(3)

where F represents the combined force of the quadrotor UAV; M represents the mass of the quadrotor UAV; H represents the angular momentum of the quadrotor UAV; $P = (x, y, z)^T$ represents the spatial position of the quadrotor UAV in geographical coordinates.

Figure 1. Quadrotor dynamics diagram.
Assuming that the UAV is a completely symmetrical rigid body, and without considering the air resistance and friction, by the derivation and simplification of (1), (2) and (3), it can be obtained:

\[
\begin{align*}
\dot{x} &= \frac{U_1 (\cos \gamma \sin \theta \cos \psi + \sin \gamma \sin \psi)}{m} \\
\dot{y} &= \frac{U_1 (\cos \gamma \sin \theta \sin \psi - \sin \gamma \cos \psi)}{m} \\
\dot{z} &= \frac{U_1 (\cos \theta \cos \psi)}{m} - g \\
\dot{\phi} &= \frac{(I_y - I_z) \dot{\psi} + U_2}{I_x} \\
\dot{\theta} &= \frac{(I_z - I_x) \dot{\psi} + U_3}{I_y} \\
\dot{\psi} &= \frac{(I_x - I_y) \dot{\psi} + U_4}{I_z}
\end{align*}
\]  

(4)

where \( U_1 \) is the upward pulling force, and \( U_2, U_3 \) and \( U_4 \) are rolling moment, pitching moment and yaw moment respectively.

3. Controller design

![Figure 2. The control structure.](image)

Figure 2 shows the control structure for realizing the flight path tracking of the quadrotor UAV. According to the reference trajectory provided by the trajectory generator, in the outer loop, a predictive controller is used to control the translational motion of the quadrotor UAV. In the inner loop, designing the PD controller for realizing the attitude control.

3.1. MPC controller design

Model predictive control not only fully considers the current and future states, but also can handle input and output constraints. Therefore, this paper designs a model predictive controller to realize tracking control of the UAV. The position control of a quadrotor UAV can be divided into \( x, y \) direction control and \( z \) direction control. Through the control quantity \( U_1 \), the height control quantity can be derived, and then the control input in the \( x \) and \( y \) directions can be obtained.

Linearized the equation (4) and establish the continuous system control model under the framework of the state space equation, as:

\[
\rho = f (\rho, u)
\]

(5)
where \( \rho = [x(t) \ v_1(t) \ y(t) \ v_2(t) \ z(t) \ v_3(t)]^T \) represents the state space vector, and \( v_1(t), v_2(t) \) and \( v_3(t) \) are the components of the UAV’s centroid linear velocity on the \( x, y \) and \( z \) axes, respectively.

From (4) and (5), we can get:

\[
\dot{\rho}(t) = \begin{bmatrix}
\dot{x}(t) \\
\dot{v}_1(t) \\
\dot{y}(t) \\
\dot{v}_2(t) \\
\dot{z}(t) \\
\dot{v}_3(t)
\end{bmatrix} =
\begin{bmatrix}
v_1(t) \\
\frac{u_x(t)}{m} \\
v_2(t) \\
\frac{u_y(t)}{m} \\
v_3(t) \\
\frac{(\cos \theta(t) \cos \gamma(t))}{m} \frac{U_{x_r}(t)}{m} - g
\end{bmatrix} =: \rho_r(t) \tag{6}
\]

with:

\[
u_x(t) = \cos \gamma \sin \theta \cos \psi + \sin \gamma \sin \psi \]
\[
u_y(t) = \cos \gamma \sin \theta \sin \psi - \sin \gamma \cos \psi \tag{7}
\]

For the given reference trajectory, it can be described by the reference trajectory of the UAV. The general expression of the reference trajectory point \((\rho_r, u_r)\) is as follows:

\[\dot{\rho}_r = f(\rho_r, u_r) \tag{8}\]

where \( \rho_r = [x_r(t) \ v_{1r}(t) \ y_r(t) \ v_{2r}(t) \ z_r(t) \ v_{3r}(t)]^T \) represents the reference state, is the system reference control input. Considering that there is no external interference in the reference state space, it can be obtained by (4):

\[
U_{x_r} = m \cdot (\ddot{x}_r + g), \quad u_{x_r} = \frac{\ddot{x}_r \cdot m}{U_{x_r}}, \quad u_{y_r} = \frac{\ddot{y}_r \cdot m}{U_{x_r}} \tag{9}
\]

In order to track the reference trajectory more accurately, an integral link is introduced in equation (6).

\[
\dot{\chi}(t) = \begin{bmatrix}
\dot{x}(t) \\
\dot{u}_x(t) \\
\int \dot{x}(t) dt \\
\dot{y}(t) \\
\dot{u}_y(t) \\
\int \dot{y}(t) dt \\
\dot{z}(t) \\
\dot{u}_z(t) \\
\int \dot{z}(t) dt
\end{bmatrix} =
\begin{bmatrix}
x(t) - x_r(t) \\
u_x(t) - u_{x,r}(t) \\
\int (x(t) - x_r(t)) dt \\
y(t) - y_r(t) \\
u_y(t) - u_{y,r}(t) \\
\int (y(t) - y_r(t)) dt \\
z(t) - z_r(t) \\
w_{x}(t) - w_{x,r}(t) \\
\int (z(t) - z_r(t)) dt
\end{bmatrix} =: \chi(t) \tag{10}
\]

Taylor series expansion is adopted at the reference trajectory points \((\chi_r, u_r)\) and higher-order terms are ignored, and the following is obtained:
\[
\dot{\chi} = f(\chi, u) + \frac{\partial f(\chi, u)}{\partial \chi} (\chi - \chi_r) + \frac{\partial f(\chi, u)}{\partial u} (u - u_r)
\] (11)

Subtracting the reference state from the above formula to get the displacement error model:

\[
\dot{\chi} = \dot{\chi} - \dot{\chi}_r = \frac{\partial f(\chi, u)}{\partial \chi} (\chi - \chi_r) + \frac{\partial f(\chi, u)}{\partial u} (u - u_r)
\] (12)

In order to be able to apply the error model to the design of the model predictive controller, discretizing the above formula:

\[
\chi(k+1) = A(k)\chi(k) + B(k)\bar{u}(k)
\] (13)

where \(\bar{u}(k) = u(k) - u_r(k)\), \(A(k)\) is the Jacobian matrix related to \(\chi(k)\) and \(B(k)\) is the Jacobian matrix related to \(\bar{u}(k)\).

The error model can be divided into two sub-models: the height error model and the error model in the x and y directions. The height error model and the Jacobian matrix expressions in the x and y directions are as follows:

\[
A_z = \begin{bmatrix}
1 & \Delta t & 0 \\
0 & 1 & 0 \\
\Delta t & 0 & 1
\end{bmatrix}
\] (14)

\[
B_z = \frac{\Delta t}{m} \cos \theta(k) \cos \gamma(k)
\] (15)

\[
A_{xy} = \begin{bmatrix}
1 & \Delta t & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
\Delta t & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & \Delta t & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & \Delta t & 0 & 1
\end{bmatrix}
\] (16)

\[
B_{xy} = \begin{bmatrix}
0 & 0 & 0 & \Delta t & \frac{U_1(k)}{m} \\
0 & 0 & 0 & 0 & \frac{U_1(k)}{m} \\
0 & 0 & 0 & \Delta t & \frac{U_1(k)}{m} \\
0 & 0 & 0 & 0 & \frac{U_1(k)}{m}
\end{bmatrix}
\] (17)

where \(\Delta t\) represents the sampling time.

According to the previous analysis, the problem of path tracking is to find a suitable control input so that the tracking error approaches zero. So, the problem of path tracking can be transformed into a
secondary planning problem. For height position control, the control input $U_1$ will be obtained by the following expression:

$$
\text{min } J_c = [\hat{\chi}_c - \hat{\chi}_c]^T Q_x [\hat{\chi}_c - \hat{\chi}_c] + [\hat{u}_c - \hat{u}_c]^T R_x [\hat{u}_c - \hat{u}_c]
$$

(18)

where $Q_x$ and $R_x$ are the weight coefficients of the state variable and the control variable respectively, and both are positive definite weighted diagonal matrices. $N_p$ is the prediction horizon.

$\hat{\chi}_c$, the predictions of the model output, is calculated by (13), (14) and (15)

$$
\hat{\chi}_c = \hat{A}_c (k|k) \chi_c(k|k) + \hat{B}_c (k|k) \hat{u}_c(k|k)
$$

(19)

where $\hat{u}_c(k|k) = U_1(k) - U_{1r}(k)$ and $\chi_c(k)$ is the error vector of height state. The height reference vector is as follows:

$$
\hat{\chi}_c = \begin{bmatrix}
\chi_{1c}(k+1|k) - \chi_{1c}(k|k) \\
\vdots \\
\chi_{1c}(k+N_p|k) - \chi_{1c}(k|k)
\end{bmatrix}, \quad \hat{u}_c = \begin{bmatrix}
U_{1r}(k+1|k) - U_{1r}(k-1|k) \\
\vdots \\
U_{1r}(k+N_p-1|k) - U_{1r}(k-1|k)
\end{bmatrix}
$$

(20)

where $N_{up}$ is the control horizon.

Without considering the constraints, the formula (18) is minimized, and the optimal control rate is obtained as follows:

$$
U_1(k) = U_{1r}(k) + \hat{u}_c(k|k)
$$

(21)

According to the receding horizon principle of MPC, only select the first element. So the following control signal is applied to the UAV: $U_1(k) = U_{1r}(k) + \hat{u}_c(k|k)$.

For the x and y direction tracker design is the same as the z direction design method, and its control rate expression is similar to (21), that is:

$$
u_{xy}(k) = \hat{u}_{xy}(k|k) + u_{xy}(k)
$$

(22)

where $\hat{u}_{xy}(k|k) = [\hat{u}_x(k|k) \hat{u}_y(k|k)]^T$.

3.2. PD controller design

The attitude control has a great influence on the flight of the quadrotor UAV. Due to the advantages of PD control, such as fast response speed and error adjustment speed, this paper adopts PD control method to control the attitude of the quadrotor UAV. $\theta_\psi$ and $\gamma_\psi$ can be obtained by (7):

$$
\gamma_\psi = \arcsin(\sin \psi, u_\psi - \cos \psi, u_\psi)
$$

$$
\theta_\psi = \arcsin(\cos \psi, u_\psi + \sin \psi, u_\psi)
$$

(23)

The PD control system as shown in Figure 3.

PD is defined:

$$
\begin{align*}
U_2 &= K_{\rho_\psi} (\gamma_\psi - \gamma) + K_{\dot{\gamma}_\psi} (\dot{\gamma}_\psi - \dot{\gamma}) \\
U_3 &= K_{\rho_0} (\theta_\psi - \theta) + K_{\dot{\theta}} (\dot{\theta}_\psi - \dot{\theta}) \\
U_4 &= K_{\rho_\psi} (\psi_\psi - \psi) + K_{\dot{\psi}} (\dot{\psi}_\psi - \dot{\psi})
\end{align*}
$$

(24)
In this section, there is simulation that used to verify the performance of the tracker. The reference path is:

\[
\begin{align*}
x_r &= \frac{1}{2} \cos \left( \frac{\pi t}{10} \right), \\
y_r &= \frac{1}{2} \sin \left( \frac{\pi t}{10} \right), \\
z_r &= 3 - 2 \cos \left( \frac{\pi t}{10} \right), \\
y_r &= 0 \text{rad}
\end{align*}
\]  

(25)

The parameters of model predictive controller as follows:

\[
\begin{align*}
N_p &= 10, \\
N_{ap} &= 10, \\
R_{xy} &= \text{diag}(0.1,0.1), \\
R_z &= 0.01, \\
Q_{xy} &= \text{diag}(2.4,0.1,0.8,2.4,0.1,0.8), \\
Q_z &= \text{diag}(100,4,7)
\end{align*}
\]  

(26)

The parameters of PD controller as follows:

\[
\begin{align*}
k_{p1} &= 0.5, \\
k_{d1} &= 3, \\
k_{p2} &= 1, \\
k_{d2} &= 0.5, \\
k_{p3} &= 0.05, \\
k_{d3} &= 0.5
\end{align*}
\]  

(27)

The three-dimensional trajectory tracking is shown in Figure 4. Figure 5, Figure 6 and Figure 7 show the position of x, y and z directions respectively, and the virtual path is approach the reference path in 3 seconds.

Figure 8 shows the simulation of UAV attitude tracking. Figure 9, Figure 10 and Figure 11 shows the position error of x, y and z direction respectively, and the error can approach 0 in 3 seconds.

A good path tracking for the quadrotor UAV flight is showed in Figure 4, Figure 5, Figure 6 and Figure 7.
Figure 6. Position of y direction

Figure 7. Position of z direction

Figure 8. Orientation

Figure 9. Position error of x direction

Figure 10. Position error of x direction

Figure 11. Position error of x direction

Figure 12 shows the comparison of MPC method and LQR method in tracking straight-line trajectory. It can be seen from the figure that MPC method’s effect of tracking is better than LQR’s effect of tracking. Besides, MPC method can track the reference trajectory in a shorter distance.
5. Conclusion
This paper proposes a trajectory tracking control method for a quadrotor UAV that combining model predictive control and PD control strategies. The model predictive controller calculates a future control sequence, which makes the tracking error of the quadrotor UAV close to 0. And we design PD controller to realize the quick response to track the reference attitude. At last, the effectiveness of the scheme is proved by the MATLAB simulation results.

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