Charged configurations in (A)dS spaces

James T. Liu\textsuperscript{1}* and W. A. Sabra\textsuperscript{2}†

\textsuperscript{1} Randall Laboratory, Department of Physics, University of Michigan, Ann Arbor, MI 48109–1120
\textsuperscript{2} Center for Advanced Mathematical Sciences (CAMS) and Physics Department, American University of Beirut, Lebanon.

Abstract

We construct new backgrounds of \(d\)-dimensional gravity with a negative cosmological constant coupled to a \(m\)-form field strength. We find a class of magnetically charged anti-de Sitter black holes with \(m\)-dimensional Einstein horizon of positive, zero or negative curvature. We also construct a new magnetic co-dimension four brane for the case of \(m = 3\). This brane obeys a charge quantization condition of the form \(q \sim L^2\) where \(q\) is the magnetic 3-form charge and \(L\) is the AdS radius. In addition, we work out some time-dependent solutions.

\textsuperscript{*}email: jimliu@umich.edu
\textsuperscript{†}email: ws00@aub.edu.lb
1 Introduction

Many recent developments in M-theory have resulted from the AdS/CFT conjecture and its generalizations. In such models, the (at least asymptotically) anti-de Sitter geometry appears to play an important role in the proper formulation of a gauge/gravity dual (see, e.g., [1] and references therein). Hence there has been much interest in obtaining a better understanding of supergravity backgrounds that are asymptotic to AdS. An important class of such solutions include Schwarzschild-AdS as well as \( R \)-charged black holes. In general, such black holes, especially the non-extremal solutions, have dual descriptions corresponding to gauge theories at finite temperature. This has led to new insights on the thermodynamics of gauge theories, as well as signatures of phase transitions.

In addition to AdS black holes [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13], backgrounds which interpolate between one or more extrema of gauged supergravity have also received attention [14, 15, 16, 17, 18, 19, 20, 21, 22, 23]. For such solutions, one may isolate a ‘radial’ direction and write the bulk metric in a warped product form

\[
\text{ds}^2 = e^{2A(r)} g_{\mu\nu} dx^\mu dx^\nu + dr^2.
\]

(1.1)

On the gauge theory side, such solutions may be given a physical interpretation in terms of a renormalization group flow, where \( r \) may be interpreted as an energy scale [24]. Of course, this class of solutions is not entirely distinct from that of AdS black holes; both cases may be written in terms of a radial \( r \)-coordinate, and in general the near-horizon behavior of a black hole (or brane) reduces to the product of a metric of the form (1.1) times an ‘internal’ space of positive, zero or negative curvature.

In general, we are interested in developing a systematic treatment of brane solutions in AdS. However, with the exception of black holes and domain walls, little is known about such objects. One of the reasons for this is that, unlike for solutions which are asymptotically Minkowski, the AdS background introduces a second length scale (namely the AdS radius) in addition to the scale of the object itself (e.g. the Schwarzschild radius). This results in more complicated profiles for the explicit solutions. In addition, extended objects in AdS would appear to be sensitive to the cosmological force originating from the curvature of spacetime. This creates some difficulties with supersymmetry, although ones that may be overcome, since special classes of strings and branes in AdS have been constructed [23, 25, 26, 27] (see also [28, 29, 30, 31]).

Motivated by this desire to more fully develop backgrounds of relevance to AdS/CFT, in this paper, we examine a large class of magnetic solutions in anti-de Sitter space. One important note, however, is that while gauged supergravity is perhaps most relevant for AdS/CFT, for the most part we forgo supersymmetry altogether. It is of course known that magnetically charged black holes in AdS are generically non-supersymmetric. Hence it would be reasonable to expect that we must similarly give up supersymmetry when considering more general magnetic branes. To motivate our setup, we recall that in the asymptotically Minkowski case, the relevant fields supporting a \( p \)-brane solution are generally a dilatonic scalar \( \phi \) and \( m \)-form field strength \( F_{(m)} \) (carrying either electric or magnetic charge) as well as the metric \( g_{\mu\nu} \). The system may be described by an effective
Lagrangian of the form

\[ e^{-1} \mathcal{L} = R - \frac{1}{2} (\partial \phi)^2 - \frac{1}{2 \cdot m!} e^{a \phi} F_{(m)}^2, \tag{1.2} \]

where \( a \) is a constant parametrizing the dilaton coupling. While this is not necessarily a complete model in itself, these fields may be suitably embedded in an appropriate supergravity framework, provided \( a \) is chosen accordingly. In this manner, the resulting \( p \)-branes may be seen as supersymmetric objects preserving some fraction of supersymmetry.

In the present case, to obtain solutions which are asymptotically AdS, we simply add a negative cosmological constant to the system (1.2). Although in general the dilaton cannot be completely ignored, we nevertheless choose as a simplification to turn off the dilaton. Even in this case, we find a rich set of solutions, which presumably capture the main features of the magnetic brane solutions.

The first class of solutions we obtain are \( d \)-dimensional magnetically charged AdS black holes that are simply the magnetic duals of the well known electrically charged Reissner-Nordstrom-AdS black holes. In some cases, these black holes may be viewed as magnetic solutions in a gauged supergravity context [32]. However, except for the charged quantized black holes which we indicate, the magnetic solutions generally break all supersymmetries.

We also obtain a more interesting class of magnetic co-dimension four branes supported by a 3-form field strength. These solutions satisfy a charge quantization relation of the form \( q \sim L^2 \) where \( q \) is the magnetic charge and \( L \) is the AdS radius. This is similar to the case of magnetic co-dimension three branes found in [33, 34].

In addition to the static brane solutions, it is also straightforward to construct new cosmological backgrounds by taking an appropriate time-dependent ansatz. We are interested in such spaces where the \( m \)-form field strength has a magnetic flux turned on. In general, these backgrounds may be obtained by analytic continuation of their static counterparts. We construct new cosmological solutions in de Sitter space as well as in anti-de Sitter space.

In the following section, we outline the basic setup for obtaining magnetic \( m \)-form solutions, and explicitly construct \( d \)-dimensional magnetically charged AdS black holes. In section three, we generalize this by considering branes with extended longitudinal dimensions. In particular, we construct the above mentioned class of co-dimension four branes. In section four, we examine possible near horizon brane geometries. Then in section five we turn to the case of time-dependent backgrounds. Finally, in the last section, we conclude with some speculation on the embedding of these solutions in gauged supergravities, and the possibility of thereby obtaining new supersymmetric backgrounds with flux.
2 Black holes with magnetic charge

Our starting point is $d$-dimensional gravity with a $m$-form field strength, $F_{(m)} = dA_{(m−1)}$, and negative cosmological constant. The Lagrangian has the form

$$e^{-1}L = R - \frac{1}{2 \cdot m!} F_{(m)}^2 + (d - 1)(d - 2)g^2,$$  \hspace{1cm} (2.1)

with resulting equations of motion

$$R_{MN} = S_{MN} \equiv \frac{1}{2(m - 1)!} \left( F_{(m) MN} - \frac{m - 1}{m(d - 2)} g_{MN} F_{(m)}^2 \right) - (d - 1)g^2 g_{MN},$$ \hspace{1cm} (2.2)

$$d \ast F_{(m)} = 0.$$ \hspace{1cm} (2.3)

For $m = 2$, this reduces to the familiar Einstein-Maxwell system with a cosmological constant. In this case, (2.1) may often be interpreted in the context of $d$-dimensional gauged supergravity. However, we are mainly interested in higher form field strengths, where there is no obvious supergravity generalization. Because of the absence of supersymmetry, we are forced to work with the second order equations of motion when looking for solutions to this model.

We begin with a construction of magnetic black holes in this system. These may be viewed as charged generalizations of the models found in [35]. Since we seek a magnetic configuration with $m$-form field strength, we take a natural metric ansatz

$$ds^2 = -e^{2A(r)} dt^2 + e^{2B(r)} dr^2 + r^2 h_{ij}(y) dy^i dy^j,$$ \hspace{1cm} (2.4)

where $i, j = 1, \ldots, m$ are $m$ directions on an Einstein manifold $\mathcal{M}^m$, with metric $h_{ij}$ depending on the coordinates $y^i$. In particular, we take

$$R_{ij}(h) = k(m - 1) h_{ij},$$ \hspace{1cm} (2.5)

where $k = 1, 0, -1$ corresponds to elliptic, flat and hyperbolic horizon metrics. Clearly the dimension of our spacetime is given by $d = m + 2$.

For the metric ansatz (2.3), the non-vanishing Ricci components are

$$R_{tt} = e^{2A-2B} \left( A'' + A'^2 - A'B' + \frac{m}{r} A' \right),$$

$$R_{rr} = -A'' - A'^2 + A'B' + \frac{m}{r} B',$$

$$R_{ij} = R_{ij}(h) + e^{-2B} h_{ij} \left( r B' - r A' - m + 1 \right),$$ \hspace{1cm} (2.6)

where $R_{ij}(h)$ is given in (2.4). For a magnetic solution, we take

$$F_{(m)} = q \, d^m y,$$ \hspace{1cm} (2.7)

where $d^m y$ is the volume form on $\mathcal{M}^m$. This yields

$$F_{(m) ij} = (m - 1)! \frac{q^2}{r^{2(m-1)}} h_{ij}, \quad F_{(m)}^2 = m! \frac{q^2}{r^{2m}}.$$
For this black hole ansatz, we therefore have

\[ S_{tt} = e^{2A} \left( \frac{(d-3)q^2}{2(d-2)r^{2(d-2)}} + (d-1)g^2 \right), \]

\[ S_{rr} = -e^{2B} \left( \frac{(d-3)q^2}{2(d-2)r^{2(d-2)}} + (d-1)g^2 \right), \]

\[ S_{ij} = -r^2 h_{ij} \left( -\frac{q^2}{2(d-2)r^{2(d-2)}} + (d-1)g^2 \right), \]

(2.8)

where we have made use of the fact that \( d = m + 2 \). Since the magnetic ansatz automatically solves the \( F_{(m)} \) equation of motion, we only need to worry about the Einstein equations. These equations are

\[ A'' + A'^2 - A'B' + \frac{d-2}{r} A' = e^{2B} \left( \frac{(d-3)q^2}{2(d-2)r^{2(d-2)}} + (d-1)g^2 \right), \]

\[ A'' + A'^2 - A'B' - \frac{d-2}{r} B' = e^{2B} \left( \frac{(d-3)q^2}{2(d-2)r^{2(d-2)}} + (d-1)g^2 \right), \]

\[ \frac{(A' - B')}{r} + \frac{d-3}{r^2} = e^{2B} \left( -\frac{q^2}{2(d-2)r^{2(d-2)}} + (d-1)g^2 + \frac{k(d-3)}{r^2} \right). \]

(2.9)

In order to solve the Einstein equations, we note that the first two equations in (2.9) imply the simple condition

\[ A' + B' = 0. \]

(2.10)

Combining this with the last equation of (2.9), we obtain the following expression

\[ \partial_r (r^{d-3}e^{-2B}) = -\frac{q^2}{2(d-2)} \frac{1}{r^{(d-2)}} + (d-1)g^2 r^{d-2} + k(d-3)r^{d-4}, \]

(2.11)

which can be easily integrated to give the solution

\[ e^{-2B} = k - \frac{\mu_0}{r^{d-3}} + \frac{q^2}{2(d-2)(d-3)} \frac{1}{r^{2(d-3)}} + g^2 r^2, \]

\[ e^{2A} = e^{2A_0} e^{-2B}. \]

(2.12)

While the general solution has two integration constants, \( A_0 \) and \( \mu_0 \), the former may be set to zero by a simple rescaling of the time coordinate. Thus we are left with a single non-extremality parameter \( \mu_0 \), as well as the magnetic charge \( q \). The magnetic black hole solution then has the standard form

\[ ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 h_{ij}(y)dy^i dy^j, \]

\[ F_{(d-2)} = q d^{d-2} y, \]

(2.13)

where

\[ f(r) = k - \frac{\omega dM}{r^{d-3}} + \frac{q^2}{2(d-2)(d-3)} \frac{1}{r^{2(d-3)}} + g^2 r^2. \]

(2.14)
Following [35], we have defined

\[ \omega_d = 16\pi G \frac{\omega_d}{(d - 2)\text{Vol}(\mathcal{M}^{d-2})}, \]  

(2.15)

where Newton’s constant has been restored. For vanishing charge, \( q = 0 \), this reduces to the solution of [35].

It ought to be apparent that these black holes are the magnetic duals of the well-known electrically charged solutions of the Einstein-Maxwell system with cosmological constant, given by the Lagrangian

\[ e^{-1} \mathcal{L} = R - \frac{1}{4} F^2 + (d - 1)(d - 2)g^2. \]  

(2.16)

The electric charged black holes are supported by a vector potential

\[ A(1) = \frac{q}{(d - 3)r^{d-3}}dt, \]  

(2.17)

with corresponding field strength \( F(2) = \frac{q}{r^2}dt \wedge dr \). An important difference between the electric and magnetic viewpoints, however, is that of supersymmetry. While the extremal electrical solutions usually preserve some fraction of supersymmetry in a corresponding gauged supergravity theory, this is not generally the case for the magnetic solutions. Even assuming that the Lagrangian (2.1) admits a supersymmetric generalization, it can be shown that generically the magnetic black holes break all of the supersymmetries.

This issue with supersymmetry may be illustrated for the special case of \( m = 2 \), which corresponds to black holes in four dimensions. Here, the metric function \( f(r) \) takes the form

\[ f(r) = k - \frac{\omega_4 M}{r} + \frac{q^2}{4r^2} + g^2 r^2. \]  

(2.18)

The 2-form field strength can either be electric, \( F(2) = q \frac{r^{-(d-2)}}{r^2} dt \wedge dr \), or magnetic, \( F(2) = q dy^1 \wedge dy^2 \). For \( k = 1 \), the supersymmetric electric \( R \)-charged black holes may be obtained by taking the BPS condition \( q = \omega_4 M \), so that \( f(r) \) has the extreme Reissner-Nordstrom-anti de Sitter form

\[ f(r) = \left( 1 - \frac{q}{2r} \right)^2 + g^2 r^2, \quad F(2) = \frac{q}{r^2} dt \wedge dr. \]  

(2.19)

This solution, however, is non-supersymmetric when viewed as a magnetic black hole. On the other hand, for the particular choice

\[ \omega_4 M = 0, \quad q^2 = \frac{k^2}{g^2}, \]  

(2.20)

the magnetic solution has the form

\[ f(r) = (gr)^2 \left( 1 + \frac{k}{2g^2r^2} \right)^2, \quad F(2) = q dy^1 \wedge dy^2. \]  

(2.21)
This is the solution which was found sometime ago by Romans \[32\]. In the context of four-dimensional gauged $N = 2$ supergravity, this solution is BPS and preserves a quarter of the supersymmetry. It was furthermore shown in \[32\] that this ‘cosmological black hole’ solution with charge quantization given by (2.20) is the unique magnetic solution that preserves some fraction of the supersymmetries.

The extreme $k = 1$ Reissner-Nordstrom-anti de Sitter black hole, (2.19), generalizes to arbitrary dimensions. However, the supersymmetric magnetic solution, (2.21), is restricted to four dimensions. For five dimensions ($m = 3$), there is instead a charged quantized magnetic black hole solution of the form

$$f(r) = (gr)^2 \left( 1 + \frac{k}{3g^2 r^2} \right)^3,$$

$$F(3) = q \, dy^1 \wedge dy^2 \wedge dy^3,$$

where

$$\omega_5 M = -\frac{k^2}{3g^2}, \quad q^2 = \frac{4k^3}{9g^4}. \quad (2.23)$$

It would be tempting to view this as a BPS solution to gauged supergravity in five dimensions. However this interpretation is not at all clear, since in this case the 3-form field strength ought to satisfy odd-dimensional self-duality equations \[36, 37, 38, 39\], as opposed to the standard equations of motion considered here.

It is interesting to note that, for $m \leq 3$, the charge quantized magnetic solutions (2.21) and (2.22) follow a harmonic function-like form

$$f(r) = (gr)^2 \left( 1 + \frac{k}{m \, g^2 r^2} \right)^m, \quad m \leq 3. \quad (2.24)$$

This no longer holds for $m \geq 4$, as then (2.14) can no longer be factored in this manner.

### 3 Magnetically charged Branes

In this section we are interested in constructing magnetic $p$-brane solutions for the system described by the Lagrangian (2.1). As in the previous section, it is natural to take a magnetic $m$-form ansatz $F(m) = q \, d^m y$ where $d^m y$ is the volume form on an $m$-dimensional Einstein manifold $\mathcal{M}^m$. This motivates us to take a metric ansatz

$$ds^2 = e^{2A(r)} g_{\mu\nu}(x)dx^\mu dx^\nu + e^{2B(r)}dr^2 + r^2 h_{ij}(y)dy^i dy^j. \quad (3.1)$$

Here $\mu, \nu = 1, \ldots, n$ are $n$ longitudinal directions of a manifold with metric $g_{\mu\nu}$. The spacetime dimension is thus given by $d = n + m + 1$. For this ansatz, the non-vanishing Ricci components are

$$R_{\mu\nu} = R_{\mu\nu}(r) - e^{2A-2B} g_{\mu\nu} \left( A'' + n A' - B' + \frac{m}{r} A' \right),$$

$$R_{rr} = -n \left( A'' + A'^2 - A'B' \right) + \frac{m}{r} B',$$

$$R_{ij} = R_{ij}(r) + e^{-2B} h_{ij} \left( r B' - nr A' - m + 1 \right). \quad (3.2)$$
This reduces to the previous case, (2.5), when \( n = 1 \). We now assume the Einstein conditions

\[
R_{\mu\nu}(g) = \lambda g_{\mu\nu}, \quad R_{ij}(h) = k(m - 1)h_{ij}.
\]

As a result, for the magnetic \( m \)-form ansatz, the Einstein equations become

\[ A'' + A'(nA' - B' + \frac{m}{r}) = e^{2B} \left( \frac{(m - 1) q^2}{2(d - 2)r^{2m}} + (d - 1)g^2 + \lambda e^{-2A} \right), \]

\[ n(A'' + A'^2 - A'B') - \frac{m}{r} B' = e^{2B} \left( \frac{(m - 1) q^2}{2(d - 2)r^{2m}} + (d - 1)g^2 \right), \]  

(3.4)

\[ \frac{1}{r}(nA' - B') + \frac{m - 1}{r^2} = e^{2B} \left( -\frac{mq^2}{2(d - 2)r^{2m}} + (d - 1)g^2 + \frac{k(m - 1)}{r^2} \right). \]

Before proceeding, we note that these expressions may be slightly simplified. Subtracting the first two equations gives

\[ A'' - A'B' - \frac{m}{(n - 1)r}(A' + B') = -\frac{\lambda}{n - 1} e^{2B-2A}. \]  

(3.5)

Alternatively, we may eliminate \( A'' \) from the first two equations to obtain

\[ nA'^2 + \frac{m}{(n - 1)r}(nA' + B') = \frac{n\lambda}{n - 1} e^{2B-2A} + e^{2B} \left( \frac{(m - 1) q^2}{2(d - 2)r^{2m}} + (d - 1)g^2 \right). \]  

(3.6)

We are thus left with two coupled non-linear first order equations, namely (3.6) and the last equation of (3.4), as well as a second order equation, (3.5). However, the second order equation is in fact redundant, as it may be obtained by differentiation of the first order equations. To see this explicitly, we may first eliminate \( B' \) from the first order equations to obtain

\[ e^{-2B} \left( n(n - 1)r^2 A'^2 + 2mn rA' + m(m - 1) \right) = \]

\[ n\lambda r^2 e^{-2A} - \frac{1}{2} \frac{q^2}{r^{2(m-1)}} + (d - 1)(d - 2)g^2 r^2 + km(m - 1). \]  

(3.7)

Taking a derivative and using (3.6) to eliminate the term proportional to \( q^2 \), we find

\[ 2(n + (n - 1)rA') \left( A'' - A'B' - \frac{m}{n - 1} \frac{1}{r}(A' + B') + \frac{\lambda}{n - 1} e^{2B-2A} \right) = 0, \]  

(3.8)

which proves the claim, at least provided \( m + (n - 1)rA' \neq 0 \), or equivalently \( e^{2A} \neq c_0r^{-(2m)/(n-1)} \) (for some constant \( c_0 \)).

Our goal, thus, is to solve the coupled first order equations for \( A(r) \) and \( B(r) \) with appropriate boundary conditions. However, before we do so, let us note that for vanishing magnetic charge, the Einstein equation is simply \( R_{MN} = -(d - 1)g^2 g_{MN} \). However, the symmetry of the ansatz, (3.1), precludes the maximally symmetric AdS\(_d\) vacuum. Instead,
we note that the Einstein condition may be solved by choosing $\lambda = (m - 1)g^2k$, so that the metric (for $q = 0$) takes on the form

$$ds^2 = \frac{dt^2}{k + g^2r^2} + r^2 \left( g^2g_{\mu\nu}(x)dx^\mu dx^\nu + h_{ij}(y)dy^idx^j \right), \quad (3.9)$$

where $\hat{k} = (m - 1)k/(d - 2)$.

Although this vacuum would not be supersymmetric (in a gauged supergravity context), it nevertheless leads to an asymptotically anti-de Sitter geometry of the form

$$ds^2 \approx g^2r^2 (g_{\mu\nu}dx^\mu dx^\nu + g^{-2}h_{ij}dy^i dy^j) + g^{-2}\frac{dt^2}{r^2}. \quad (3.10)$$

For non-zero magnetic charge, we note that, asymptotically the magnetic field strength, ($2.7$), falls off as $F_{(m)}^2 \sim 1/r^{2m}$. Thus even when $q \neq 0$, it is natural to expect the magnetic brane solution to be asymptotic to (3.10), at least up to corrections of the order $O(1/r^2)$. Hence we require that

$$e^{2A} \sim e^{-2B} \sim (gr)^2 \quad \text{as} \quad r \to \infty. \quad (3.11)$$

Formally, the system of first order equations, (3.6) and the last equation of (3.4), admits a two-parameter family of solutions; in addition, there are of course the inputs $\lambda$ (the cosmological constant on the longitudinal space) and $q$ (the magnetic charge) as well as the discrete parameter $k = 1, 0, -1$. While we have been unable to find an explicit solution for arbitrary values of $\lambda$ and $q$, we note that the equations simplify if we take the longitudinal space to the Ricci-flat, $\lambda = 0$. In this case, we may combine (3.6) with the last equation of (3.4) to eliminate $A'$. This results in a first order equation

$$f'' - \frac{4m}{n - 1}f f' - \frac{4m(d - 1)}{n - 1} \frac{1}{r^2}f^2 - \frac{4}{r}f' \left( (d - 1) + \frac{k(m - 1)}{(gr)^2} \right) = 0,$$

where

$$f = \frac{1}{(gr)^2}e^{-2B}, \quad (3.13)$$

and $\tilde{q}$ is a rescaled (dimensionless) magnetic charge

$$\tilde{q}^2 = \frac{n}{2(d - 2)}g^{2(m - 1)}q^2. \quad (3.14)$$

In general one may seek numerical solutions to (3.12) subject to the boundary condition $f(r) \to 1$ as $r \to \infty$. However closed form solutions may be obtained for the special cases of $m = 2$ and $m = 3$, provided a form of charge quantization is imposed.
The $m = 2$ ($d = n + 3$) case, corresponding to Einstein-Maxwell theory with a cosmological constant, has recently been investigated in [33, 34]. Given the charge quantization condition

$$\tilde{q}^2 = \frac{k^2}{(n + 1)^2},$$

we find a solution to (3.12) of the form

$$e^{-2B} = (gr)^2 \left(1 + \frac{k}{(n + 1)g^2r^2}\right)^2.$$  \hspace{1cm} (3.16)

This may then be inserted into the last equation of (3.4) to obtain an equation for $A$, which may be solved to give

$$e^{2A} = (gr)^2 \left(1 + \frac{k}{(n + 1)g^2r^2}\right)^{\frac{n+1}{n}}.$$  \hspace{1cm} (3.17)

This agrees with the magnetic co-dimension three solutions obtained in [33, 34].

Turning now to the $m = 3$ case, we find that a closed form solution exists provided we impose a charge quantization condition

$$\tilde{q}^2 = \frac{16n^2k^3}{(n + 1)^3(n + 2)^3}.$$  \hspace{1cm} (3.18)

This time, the solution for $B$ is somewhat more complicated, and is given by

$$e^{-2B} = (gr)^2 \left(1 + \frac{2k}{(n + 1)(n + 2)g^2r^2}\right) \left(1 + \frac{2nk}{(n + 1)(n + 2)g^2r^2}\right)^2,$$  \hspace{1cm} (3.19)

whereas that for $A$ has the simple form

$$e^{2A} = (gr)^2 \left(1 + \frac{2nk}{(n + 1)(n + 2)g^2r^2}\right)^{\frac{n+2}{n}}.$$  \hspace{1cm} (3.20)

Note that for $n = 1$, both the $m = 2$ and $m = 3$ solutions reduce to magnetic black holes with quantized magnetic charge, given by (2.24). Thus these solutions may be viewed as a larger family of ‘cosmological magnetic branes’ generalizing the cosmological black holes of [32].

Although we have searched for a generalization to arbitrary $m$, we have as yet been unsuccessful. In particular, it can be shown that no terminating series solution for $f$ given in (3.13) exists for $m \geq 4$, regardless of the charge $\tilde{q}$, while assuming the asymptotics of (3.11). Of course, this result is not entirely unexpected, as even for $n = 1$, factorization of the metric function, (2.24), is only possible for $m \leq 3$.  

9
4 The near-horizon solution

In the previous sections, we have considered a transverse space metric of the form

$$ds^2 = e^{2B} dr^2 + r^2 d\Omega_k^2,$$  \hspace{1cm} (4.1)

corresponding to a foliation of space in terms of hypersurfaces with curvature $k = 1, 0$ or $-1$ (corresponding to the horizon geometry of the black object). On the other hand, assuming a regular horizon with finite area (i.e. horizon radius $r_0 > 0$), it is instructive to examine the near horizon solution. In this case, it is natural to take a direct product metric

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu + h_{ij}(y)dy^i dy^j,$$  \hspace{1cm} (4.2)

where $\mu, \nu = 1, \ldots, d-m$ and $i, j = 1, \ldots, m$. This essentially corresponds to a Freund-Rubin compactification with magnetic field strength given by

$$F_{(m)} = q \, d^m y.$$  \hspace{1cm} (4.3)

Similar to (2.7), this yields

$$F_{(m)}^2_{ij} = (m-1)! \, q^2 h_{ij}, \quad F_{(m)}^2 = m! \, q^2.$$  \hspace{1cm} (4.4)

The resulting Einstein equations are then

$$R_{\mu\nu}(x) = - \left[ \frac{m-1}{2(d-2)} q^2 + (d-1) g^2 \right] g_{\mu\nu},$$

$$R_{ij}(y) = \left[ \frac{d-m-1}{2(d-2)} q^2 - (d-1) g^2 \right] h_{ij}.$$  \hspace{1cm} (4.5)

As a consequence, the scalar curvature satisfies

$$R = \frac{d-2m}{2(d-2)} q^2 - d(d-1) g^2.$$  \hspace{1cm} (4.6)

The equations, (4.5), are simply Einstein conditions for the ‘longitudinal’ and ‘transverse’ dimensions, with cosmological constants

$$\Lambda_{d-m} = - \left[ \frac{m-1}{2(d-2)} q^2 + (d-1) g^2 \right],$$

$$\Lambda_m = \left[ \frac{d-m-1}{2(d-2)} q^2 - (d-1) g^2 \right].$$  \hspace{1cm} (4.7)

Note that the longitudinal space is always AdS (since $\Lambda_{d-m} < 0$). However, the transverse space may have either sign for $\Lambda_m$, depending on the relative strength of $q^2$ versus $g^2$. It is also possible to set $\Lambda_m$ to zero, although without supersymmetry this may only be viewed as a fine tuning. The freedom to adjust $\Lambda_m$ may be understood by recalling that, starting from a $d$-dimensional theory without cosmological constant, the Freund-Rubin
compactification yields a geometry of the form \( \text{AdS} \times \text{Sphere} \). Turning on an overall negative cosmological constant in the initial higher dimensional theory then contributes an additional negative factor for both the AdS and sphere curvatures. For a sufficiently large negative cosmological constant, the sphere is then replaced by hyperbolic space.

Of course, since we are considering non-supersymmetric models, we could equally well have started with a positive cosmological constant in the \( d \)-dimensional theory. In this case, turning on fluxes would yield a longitudinal space of arbitrary curvature, while the transverse space is always positively curved. In this case, an appropriate fine tuning would yield a geometry of the form \( \text{Minkowski} \times \text{Sphere} \).

5 Cosmological solutions

Until now, we have been considering only static magnetic AdS brane solutions. However, it is straightforward to analyze time dependent de Sitter solutions as well. Here we will obtain a set of cosmological solutions to the system described by the Lagrangian

\[
e^{-1} \mathcal{L} = R - \frac{1}{2} \cdot \frac{m!}{m!} F_{(m)}^2 - (d-1)(d-2)g^2.
\] (5.1)

This is identical to (2.1), except that here the cosmological constant is positive. As a metric ansatz for the time-dependent solutions, we take our total spacetime dimension to be \( d = (n+1) + m \) and assume a factorized form \( K^{1,n} \times \mathcal{M}^m \) where \( K^{1,n} \) is spatially isotropic and \( \mathcal{M}^m \) is Einstein, with metric \( h_{ij} \) with zero, positive or negative curvature.

To be precise, the ansatz is given by

\[
ds^2 = -e^{2B(t)} dt^2 + e^{2A(t)} \left( dx_1^2 + \cdots + dx_n^2 \right) + t^2 h_{ij} dy^i dy^j,
\] (5.2)

where \( i, j = 1, \ldots, m \). This is a natural cosmological version of the original static ansatz given by (3.1). For a magnetic solution, we again take \( F_{(m)} = q d^m y \). For this ansatz, the non-vanishing Ricci components are

\[
R_{tt} = -n(\ddot{\tilde{A}} + \dot{\tilde{A}}^2 - \dot{\tilde{A}} \dot{B}) + \frac{m}{t} \dot{B},
\]

\[
R_{ab} = e^{2A-2B} \delta_{ab} \left( \ddot{\tilde{A}} + n \dot{\tilde{A}}^2 - \dot{\tilde{A}} \dot{B} + \frac{m}{t} \dot{A} \right),
\]

\[
R_{ij} = h_{ij} \left[ e^{-2B} \left( m - 1 - t \dot{B} + nt \dot{A} \right) + k(m-1) \right],
\] (5.3)

so that the Einstein equations take the form

\[
-n(\ddot{\tilde{A}} + \dot{\tilde{A}}^2 - \dot{\tilde{A}} \dot{B}) + \frac{m}{t} \dot{B} = e^{2B} \left( \frac{(m-1)}{2} \frac{q^2}{2(d-2) t^{2m}} - (d-1)g^2 \right),
\]

\[
-\left( \ddot{\tilde{A}} + n \dot{\tilde{A}}^2 - \dot{\tilde{A}} \dot{B} + \frac{m}{t} \dot{A} \right) = e^{2B} \left( \frac{(m-1)}{2} \frac{q^2}{2(d-2) t^{2m}} - (d-1)g^2 \right),
\]

\[
\frac{1}{t}(n \ddot{\tilde{A}} - \dot{B}) + \frac{m-1}{t^2} = e^{2B} \left( \frac{n}{2} \frac{q^2}{2(d-2) t^{2m}} + (d-1)g^2 - \frac{k(m-1)}{t} \right).\] (5.4)
For the special case $n = 1$, the first two equations imply $\dot{A} = -\dot{B}$ and the solution is given by

$$ds^2 = -\frac{dt^2}{f(t)} + f(t)dr^2 + t^2h_{ij}(y)dy^i dy^j,$$

where

$$f(t) = -k - \frac{\mu}{t^{d-3}} - \frac{q^2}{2(d-2)(d-3)} \left( \frac{1}{t^{2(d-3)}} + g^2t^2. \right)$$

(5.6)

For the choice $k = 1$ and $q^2 = \frac{\mu^2}{2} (d - 2) (d - 3)$, the function $f(t)$ becomes

$$f(t) = g^2t^2 - \left( 1 - \frac{\mu}{t^{d-3}} \right)^2,$$

(5.7)

and in this case one obtains the solution presented in [40].

Of course, it should be noted that the cosmological solution (5.5) and (5.6), is formally identical to that of the static Reissner-Nordstrom-de Sitter black hole

$$ds^2 = -g(r)dt^2 + \frac{dr^2}{g(r)} + r^2h_{ij}(y)dy^i dy^j,$$

$$g(r) = k - \frac{\mu}{r^{d-3}} + \frac{q^2}{2(d-2)(d-3)} \left( \frac{1}{r^{2(d-3)}} - g^2r^2. \right)$$

(5.8)

This may be seen by making the substitution $r \leftrightarrow t$ as well as $f(t) \rightarrow -g(r)$ and $\mu \rightarrow -\mu$. However, here we are interested in the region of spacetime given by $f(t) > 0$. This corresponds to the counterpart of the static black hole solution on the other side of the de Sitter horizon. In other words, for the time dependent solution, we focus on the region of spacetime given by $g(r) < 0$, where the roles of $r$ and $t$ are interchanged.

For $m = 2$, namely Einstein-Maxwell-de Sitter, we get the following solution [41]

$$ds^2 = -(gt)^{-2} \left( 1 - \frac{k}{(n+1)g^2t^2} \right)^{-2} dt^2 + (gt)^2 \left( 1 - \frac{k}{(n+1)g^2t^2} \right)^{\frac{n+3}{2}} d\vec{x}^2 + t^2 d\Omega^2_k,$$

(5.9)

where $d\Omega^2_k$ is the metric of a two-dimensional manifold with $k = 1, 0, -1$. However, the charge quantization is given by

$$q^2 = -\frac{2k^2}{g^2n(n+1)}.$$

(5.10)

Thus this solution has imaginary magnetic charge, and for $n = 2$ may be viewed as a solution of the five dimensional supergravity theory arising from the reduction of IIB* on $dS_5 \times H^5$. These reductions yield five-dimensional de Sitter supergravities, albeit with wrong sign kinetic terms [42].

For the case $m = 3$, we obtain a time-dependent solution in de Sitter space

$$ds^2 = -(gt)^{-2} \left( 1 - \frac{2k}{(n+1)(n+2)g^2t^2} \right)^{-1} \left( 1 - \frac{2nk}{(n+1)(n+2)g^2t^2} \right)^{\frac{n+4}{2}} dt^2$$

$$+ (gt)^2 \left( 1 - \frac{2nk}{(n+1)(n+2)g^2t^2} \right)^{\frac{n+4}{2}} d\vec{x}^2 + t^2h_{ij}(y)dy^i dy^j,$$

(5.11)
with magnetic flux satisfying
\[ q^2 = \frac{32nk^3}{g^4(n+1)^3(n+2)^2}. \tag{5.12} \]

This charge quantization condition is identical in sign with that for the static anti-de Sitter case, namely \((3.18)\). Thus one must take \(k = 1\) in order to obtain real gauge fields in both the AdS and dS cases.

### 6 Discussion

In this paper, we have focused on constructing magnetic brane solutions asymptotic to anti-de Sitter space. For the 0-brane case, we recover the magnetically charged Reissner-Nordstrom-AdS black hole. On the other hand, we have demonstrated that a new class of magnetic \(p\)-branes \((p > 0)\) may be obtained by solution of the non-linear equation \((3.13)\). For the case of \(m = 2\) (Einstein-Maxwell), the co-dimension three branes reproduce the solutions of \([33, 34]\), while for \(m = 3\), we obtain a new analytic solution for co-dimension four branes.

In general, we are unable to work in a supersymmetric framework. This is due to the fact that gauged supergravities (which yield AdS vacua) generally do not contain \(m\)-form field strengths with \(m > 2\) satisfying standard \(m\)-form equations of motion. Instead, such higher form gauge fields satisfy odd-dimensional self-duality equations, at least in the five and seven-dimensional gauged supergravities \([36, 37, 38, 39]\). On the other hand, for \(m = 2\), the Lagrangian \((2.1)\) may generally be given a supersymmetric interpretation with \(F_{(2)}\) transforming as a graviphoton. In this case, it is known that the extreme electrically charged AdS black holes preserve a fraction of the supersymmetries \([32, 2, 3, 4, 5, 6, 12]\). For the magnetic objects, the charge quantized co-dimension three solution \((3.16)\) and \((3.17)\) was shown to be one quarter supersymmetric, both in four dimensions \([32]\) and in general \([34]\).

Although we are not aware of a gauged supergravity with \(m = 3\) and a standard kinetic term of the form \((2.1)\), it is tempting to suggest that the cosmological magnetic brane solution \((3.19)\) and \((3.20)\) is likewise somehow one quarter supersymmetric. In five dimensions, one may be tempted to identify this magnetic \(F_{(3)}\) black hole as dual to an electric \(F_{(2)}\) black hole (which would be supersymmetric in the extremal limit). However, this cannot be the case, as it is not possible to dualize the graviphotons in gauged supergravity. Nevertheless, we hold open the possibility that some dual five-dimensional theory may be formulated where the gauging is accomplished via an antisymmetric tensor with field strength \(F_{(3)} = dA_{(2)}\).

Finally, we note that a class of supersymmetric branes supported by higher form potentials have been constructed using the Liu-Pope ansatz \([43, 44, 45, 46]\). These solutions make explicit use of the odd-dimensional self-duality equations. Thus they appear rather different in structure from the magnetic branes considered above. Furthermore, the Liu-Pope lifted solutions have both electric and magnetic components of the field strengths.
active by virtue of odd-dimensional self-duality. This suggests that, in order to make a connection between the solutions discussed here and the branes of the Lü-Pope type, we would at least have to generalize our magnetic ansatz to include the dyonic case. It would be interesting to see if this is indeed possible.

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