Research on Mathematical Model of Environmental Assessment

Yunna Han
Modern College of Northwest University, Xian 710130, China
ynnahan@163.com

Abstract. The assessment methods are important contents in environmental quality assessment; each assessment method is based on one mathematical model. Through the analysis of various mathematical models, it is found that the selection of assessment methods in environmental assessment has very important guiding significance. By introducing four types of common mathematical models in current environmental assessment, this paper discusses the principles, features and applicable scope of various models, and briefly describes the application conditions of assessment methods in various models in environmental assessment.

Keywords: Environmental assessment, mathematical model.

1. Introduction
With the development of environmental science, mathematics has played an increasingly important role in the field of environmental science. The object of mathematics research in environmental assessment is to explore the mathematical laws existing in environmental problems. The mathematical model is used to study environmental quality problems; we first need to abstract environmental problems into mathematical models in order to study this problem with mathematical tools. This paper introduces and analyzes the mathematical model in environmental assessment.

2. Overview of Environmental Assessment
Environmental assessment is a qualitative and quantitative description of the pros and cons of each element of the environment. Environmental assessment is to explore the value of environmental quality, not to evaluate the quality of the environment itself. It is to evaluate the satisfaction of environmental quality and the needs of human society for survival and development. The object of environmental assessment is the relationship between environmental quality and human survival and development needs, it can also be said that environmental assessment explores the social significance of environmental quality. Environmental assessment provides the scientific basis for comprehensive management of environmental damage, environmental planning and management.

How to choose a suitable environmental assessment method is the key to correctly and objectively assess environmental quality and distinguish environmental functions. Environmental assessment targets and objectives are different, and the selected assessment parameters or indicators are different, therefore, the assessment method must be reasonably selected according to the assessment objects, aims, and the nature and characteristics of the assessment parameters. Conversely, there are good or bad for
various assessment methods, their starting points and focuses are different, and their characteristics and application scope are also different, therefore, different types of environmental assessment are applicable to different assessment models. In the environmental assessment work, we must deeply understand the different characteristics and applicable scopes of various assessment models, and choose and use them scientifically and reasonably.

3. **Common Mathematical Models in Environmental Assessment**

According to the assessment modes, common mathematical models can be divided into three categories: assessment of chemical indicators, assessment of biological indicators, and assessment of mathematical models, among them, mathematical model assessment is widely used in practical work because it can objectively, quantitatively and comprehensively reflect the real situation of environmental quality. The theories of mathematical models for environmental assessment at home and abroad are mostly based on random probability theory and mathematical statistics. Common methods include quality index assessment models and mathematical statistics assessment models, etc. In addition to the randomness, environmental information also has features such as ambiguity and grayness, so fuzzy mathematical assessment models and gray system assessments are also the most common mathematical models in environmental assessment. These methods are different from each other, but there is no clear boundary and they can be used comprehensively. In addition to the above common models, many new methods have appeared in recent years, such as artificial neural networks, matter-element analysis and assessment methods, projection pursuit technology, genetic algorithms and ant colony algorithms, set pair analysis and rough set theory. Several main mathematical models in environmental assessment are as follows:

Green proposed a mathematical model for air pollution assessment in 1966:

\[
I_1 = a_1 S_{SO_2}^{b_1} = 84.0 \ SO_{0.431} \\
I_2 = a_2 C_{COH}^{b_2} = 26.6 \ C^{0.576} \\
I = \frac{1}{2} (I_1 + I_2) = 0.5(84.0 \ SO_{0.431}) + 26.6 \ C^{0.576}
\]

In the formula: \(I_1\) is SO\(_2\) pollution index; \(I_2\) is COH pollution index; \(I\) is comprehensive pollution index of air; \(S\) is SO\(_2\) measured average concentration; \(C\) is measured average smoke coefficient (C Oh unit/100ft); \(a_1, a_2, b_1, b_2\) is coefficient and index.

Mathematical model of air quality index:

\[
ORAQI = [5.7 \sum_{i=1}^{5} \left( \frac{C_i}{S_i} \right)^{1.37}]
\]

In the formula, 5 is the number of evaluation parameters; 5.7, 1.37 is the coefficient and index; \(C_i\) is the measured concentration of the ith pollutant; \(S_i\) is the air quality standard value of the ith pollutant formulated by the U.S. Environmental Protection Agency.

4. **Assessment Model of Environmental Quality Index**

4.1. **Single factor assessment index**

This assessment mode is to compare the actual measurement result of an assessment parameter with the assessment standard, it reflects the over-standard situation of the pollutant or the score value of the assessment index, namely the following formula is used:

\[
Pi = \frac{Ci}{Si}
\]

In the formula: \(Pi\) is the single factor assessment index of a certain pollutant or assessment index; \(Ci\) is the actual measured value of a certain pollutant or assessment index, \(Si\) is the assessment standard value of a certain pollutant or assessment index.

The advantage of this assessment method is that it can reflect the excess of each pollutant and is the basis of other assessment models; its disadvantage is that it cannot give a general conclusion, and it is not suitable for comprehensive assessment of environmental quality with multiple indicators and factors.
4.2. Comprehensive index assessment method

This assessment mode is to mathematically process the single pollution index mode to establish a mathematical formula to make it generally reflect environmental quality. The comprehensive index method is the most common in environmental quality assessment, and it is widely used in various environmental quality assessments such as the atmosphere, soil, water body, and ecology. The common methods and their respective characteristics are as follows: as shown in Table 1:

| type                      | formula                                      | characteristics                                                                 |
|---------------------------|----------------------------------------------|-------------------------------------------------------------------------------|
| simple superposition method | $PI = \sum_{i=1}^{n} P_i$                    | The method believes that the geological environment is the result of the combined action of various factors, so effect and influence of multiple factors must be greater than the effect and influence of any one of these elements. With all assessment parameters. The sum of relative values of all assessment parameters can reflect the comprehensive quality of environmental factors. |
| arithmetic mean method     | $PI = \frac{1}{n} \sum_{i=1}^{n} P_i$        | The basic starting point is that the influence of various factors on environmental quality is equal weight. Based on PI value and size, grading standards are compared with to determine the environmental quality condition. |
| weighted average method    | $PI = \frac{1}{n} \sum_{i=1}^{n} W_i P_i$    | The introduction of weight $W_i$ can reflect the different effects of different environmental elements on the environment. |
| square root of quadratic sum method | $PI = \sqrt{\sum_{i=1}^{n} P_i^2}$        | It not only highlights the highest sub-index, but also takes into account the influence of remaining sub-indexes that are greater than 1. |
| root-mean-square method    | $PI = \frac{1}{n} \sum_{i=1}^{n} P_i$       | It is basically the same as the above formula, and its calculated value is less than the sum of the sub-indexes. |
| integral value method      | $M = \sum_{i=1}^{n} a_i$                    | $M$ is the total score of an assessment unit; $a_i$ is the score of $i$ assessment factor; $n$ is the assessment factor number. |

5. Mathematical Statistical Model

With the improvement of environmental geology research, the accumulated background information of geological environment is more and more abundant, and the environmental monitoring data is more comprehensive, systematic, especially the development of calculation technology, which makes it possible to establish various mathematical statistical models and carry out statistical calculation to analyze and solve environmental problems in addition to qualitative analysis of a large number of data and monitoring data. So far, all the mathematical statistics methods can be applied to environmental science to a certain extent. There are three main types of mathematical statistical models applied to environmental quality assessment: (1) cluster analysis model, it uses mathematical method to quantitatively determine the relationship between the clustering objects and classify them. This model is applicable to the environmental quality zoning and grading assessment not determined by the grading standard. Fuzzy clustering and grey clustering in cluster analysis are often used to evaluate the
environmental quality of water and soil, (2) principal component analysis is a statistical method that transforms many elements of environmental quality into a few comprehensive indicators, it can be used to quantitatively describe the environmental quality, find out its dominant factors, build independent comprehensive variables from multiple variables under the condition of no loss or little loss of information, and select reasonable and effective environmental factors as assessment indicators, factor analysis model, which uses the measured multiple indicators (multi-dimensional vector) and the lines of a few potential indicators The method of factor analysis is often used in the comprehensive assessment of water pollution, which can reduce the assessment index and grasp the main contradiction for analysis and assessment under the condition of less information loss. The mathematical statistical model is used for environmental quality assessment, and its operation is generally completed on the computer by using the existing program.

6. Fuzzy Mathematical Model
Environment is a multi-dimensional system composed of multi elements; each element has many different forms of qualitative and quantitative changes, which is a very complex system. Although people have established some classical mathematical models and methods about the classification, recognition, assessment and prediction of environmental quality, and can also give some quantitative description; all of these descriptions are to use the exact mathematical concepts to describe the objects that are not exact in nature. Therefore, in environmental science research, it is impossible and unnecessary to be absolutely accurate. In more cases, it is inevitable to use a certain degree of fuzziness.

Fuzzy mathematics is a science to study and deal with the phenomenon of fuzziness. "Fuzziness" mainly refers to the "one is the other" nature of the difference of objective things in the intermediate transition. It uses the degree of membership to describe the intermediary transition of differences, and it is a kind of description of fuzziness with accurate mathematical language. The fuzzy mathematical models in environmental quality assessment include fuzzy comprehensive assessment method, fuzzy close degree assessment method and fuzzy probability method. Among them, the fuzzy comprehensive assessment method is the most common one of the environmental quality assessment methods; it is not only used in the quality assessment of single environmental factors such as water quality and soil, but also widely used in the multi-level, multi factor and multi index environmental quality assessment, especially in the geological environmental quality assessment. The quality of this kind of assessment object is not easy to be divided, and the assessment grade limit is fuzzy, which involves a lot of complex phenomena and fuzzy concepts, so the fuzzy mathematical model can evaluate this kind of environmental object very well. However, the simple fuzzy comprehensive assessment method is relatively rough, which is often controlled by some indexes with larger weight, so there is a phenomenon of misjudgment, especially when there are many assessment factors.

7. Grey System Assessment Model
A system with clear and partially clear information is called a gray system. In the study of environmental systems at various levels, we often encounter systems with non-deterministic quantities, as well as systems with small samples due to human, material, and technical constraints, coupled with the special nature and extreme complexity of environmental systems. Often with atypical distribution or incomplete and severely disturbed data, especially when assessing the quality of geological environment, the extracted data of various traits is even more so, which makes many fruitful mathematical methods in geological environment research It is extremely difficult to perform mathematical analysis on such problems. Grey system theory provides a means to solve this problem. The gray system theory is based on a white mathematical method from a system perspective. It allows gray factors, gray coefficients, and gray relationships in the system. It is proposed to use gray numbers, gray intervals, gray equations, and gray groups to describe non-deterministic quantities system. Grey relational analysis and grey clustering are the two most common models in grey systematic review.

Grey correlation analysis is a quantitative method to analyze the degree of correlation between various factors in the system. The basic idea of gray correlation analysis is to judge the relative degree
of correlation based on the similarity between the ordered series of curves, takes into account the integrity of the curve in the interval under study and the particularity of the points. This method has no special requirements for the number of samples, nor does it restrict the data to have a strict distribution, and the calculation amount is small, and the quantitative results of the gray correlation degree do not appear to be inconsistent with the qualitative analysis results, it is often used in water quality assessment.

Gray clustering, like fuzzy clustering, is an extension of ordinary clustering methods. Grey clustering is a new clustering method formed by introducing grey theory in cluster analysis. Fuzzy clustering assessment can obtain dynamic clustering map from course to fine and a series of clustering results, but it cannot give out which results are consistent with the actual situation, it needs to artificially determine a confidence level to determine the results. The gray cluster does not need to give a critical judgment in advance, and can directly obtain the cluster assessment result. Grey clustering assessment model is a typical quantitative analysis method in environmental quality assessment, which is widely used in environmental quality assessment of soil, water and atmosphere.

8. Conclusion
This paper introduces several types of mathematical models commonly used in current environmental quality assessment, these models are mature in application, and other assessment models are mostly based on these models. The appropriate assessment method should be selected in accordance with the specific environmental assessment, so that the scientific, reasonable and objective environmental assessment result can be obtained.

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