Reverse Topological Indices of Some Molecules in Drugs Used in the Treatment of H1N1

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Abstract: Topological descriptors/indices are very significant in the study of intrinsic properties of chemical graphs and nanostructures. These indices are used to extract information from the symmetry of molecular graphs and are based on some of the graph invariants such as the vertex degree, distance, and spectrum. This article computes reverse degree-based topological indices of two commonly used antiviral drugs, namely Oseltamivir and Zanamivir. We also discuss the graphical behavior of the indices of these compounds.

Keywords: molecular graph; topological indices; topological polynomials; H1N1; Oseltamivir; Zanamivir.

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1. Introduction

The study of causes and treatments of various infectious diseases has been one of the primary areas of study of modern scientific research in medicine. During one such study, conducted in 2009, scientists discovered a new influenza A virus subtype, H1N1 (A/H1N1). This novel H1N1 virus, first isolated in the United States (U. S.), had a rare combination of influenza genes that had never been seen before in an animal or a human. The first human infection of the virus was detected in California and soon later in other parts of U. S. The virus was found to cause severe respiratory infection in humans. It was found to spread rapidly through the air. The Centers for Disease Control and Prevention (C.D.C.), U.S., initially recommended using influenza antiviral medicines to treat and prevent the virus. However, in a matter of days, the situation got worse, and the virus started spreading quickly to other parts of the owing to the World Health Organisation (W. H. O.) declaring it a pandemic. Research studies were carried out to find a vaccine that would control the spread of the virus. Subsequently, about five different 2009 H1N1 influenza vaccines were approved by the end of 2009.
To date, various vaccines have been introduced and adopted by different countries worldwide to keep the virus spread under control. Currently, two classes of antiviral drugs are globally approved to treat influenza in humans, namely the adamantanes (amantadine and rimantadine) and the neuraminidase inhibitors (Oseltamivir, Zanamivir, and Peramivir). However, the widespread use of these antiviral drugs as monotherapies has resulted in the rise of influenza viruses resistant to both drug classes [1-4]. Due to this, only the neuraminidase inhibitors (Oseltamivir and Zanamivir) are presently prescribed for influenza. Combination therapy with Oseltamivir and Zanamivir is reasonable because these agents bind within the neuraminidase active site differently[5]. The likelihood of selecting viruses dually resistant to Oseltamivir and Zanamivir is low. As a result, combining these two drugs in therapy could be a viable option for combating resistance. The chemical structure of Oseltamivir and Zanamivir is as given in Figure 1.

![Chemical structures of Zanamivir (left) and Oseltamivir (right).](image)

Figure 1. Chemical structures of Zanamivir (left) and Oseltamivir (right).

We consider hydrogen suppressed molecular graphs of compounds since the vertices representing hydrogen atoms do not contribute graph isomorphism.

![Molecular graphs of Zanamivir (left) and Oseltamivir (right).](image)

Figure 2. Molecular graphs of Zanamivir (left) and Oseltamivir (right).

Topological descriptors, also known as topological indices, associated with chemical graphs are commonly used mathematical tools in modern literature that are significant in predicting the intrinsic chemical and physical properties of the corresponding chemical compounds and nanostructures. Various aspects, such as boiling point, strain energy, rigidity, and fracture toughness, are found to be correlating with these indices. These indices are based on some of the invariants in a graph, such as the degree of vertices/edge and distance. Wiener indices, Randić indices, Hosoya indices, and Zagreb indices are some of the commonly known topological indices in literature.
The reverse vertex degree is defined by $\mathcal{R}\Psi(G) = \Delta(G) - \Psi(m) + 1$, where $\Psi(m)$ and $\Delta(G)$ are the degree of the vertex $m$ and maximum vertex degree respectively. Zhao et al. in [6] computed the reverse degree-based topological indices like the reverse general Randić index, the reverse Balaban index, the reverse atom bond connectivity index, the reverse geometric index, the reverse Zagreb type indices, and the reverse augmented Zagreb index for this metal-organic networks TM-TCNB. The expression for these indices is as given in Table 1.

Table 1. Notations and expressions for various reverse topological indices

| Index                              | Notation     | Expression                                                                 |
|------------------------------------|--------------|=============================================================================|
| Reverse Randić index               | $\mathcal{R}R_a(G)$ | $\sum_{mn \in E(G)} [\mathcal{R}\Psi(m) \times \mathcal{R}\Psi(n)]^\alpha$, \(\alpha = 1, -1,\) \(\frac{1}{2}, \frac{1}{2}\) |
| Reverse atomic bond connectivity index | $\mathcal{R}ABC(G)$ | $\sum_{mn \in E(G)} \sqrt{\mathcal{R}\Psi(m) + \mathcal{R}\Psi(n) - 2}$ \(\mathcal{R}\Psi(m) \times \mathcal{R}\Psi(n)\) |
| Reverse geometric arithmetic index  | $\mathcal{R}GA(G)$ | $\sum_{mn \in E(G)} \frac{2\sqrt{\mathcal{R}\Psi(m) \times \mathcal{R}\Psi(n)}}{\mathcal{R}\Psi(m) + \mathcal{R}\Psi(n)}$ |
| Reverse first Zagreb index         | $\mathcal{R}M_1(G)$ | $\sum_{mn \in E(G)} \mathcal{R}\Psi(m) + \mathcal{R}\Psi(n)$ |
| Reverse second Zagreb index        | $\mathcal{R}M_2(G)$ | $\sum_{mn \in E(G)} \mathcal{R}\Psi(m) \times \mathcal{R}\Psi(n)$ |
| Reverse first Zagreb co-index      | $\mathcal{R}\bar{M}_1(G)$ | $2|E(G)|(|V(G)| - 1) - \mathcal{R}M_1(G)$ |
| Reverse second Zagreb co-index     | $\mathcal{R}\bar{M}_2(G)$ | $2|E(G)|^2 - 1 - 2 \mathcal{R}M_1(G) - \mathcal{R}M_2(G)$ |
| Reverse hyper Zagreb index         | $\mathcal{R}HM(G)$ | $\sum_{mn \in E(G)} [\mathcal{R}\Psi(m) \times \mathcal{R}\Psi(n)]^2$ |
| Reverse forgotten index            | $\mathcal{R}F(G)$ | $\sum_{mn \in E(G)} (\mathcal{R}\Psi(m))^2 + (\mathcal{R}\Psi(n))^2$ |
| Reverse Balaban index              | $\mathcal{R}J(G)$ | $\sum_{mn \in E(G)} \frac{m}{m - n + 2} \sum_{mn \in E(G)} \frac{1}{\mathcal{R}\Psi(m) \times \mathcal{R}\Psi(n)}$ |
| Reverse first multiple Zagreb index | $\mathcal{R}PM_1(G)$ | $\prod_{mn \in E(G)} \mathcal{R}\Psi(m) + \mathcal{R}\Psi(n)$ |
| Reverse second multiple Zagreb index | $\mathcal{R}PM_2(G)$ | $\prod_{mn \in E(G)} \mathcal{R}\Psi(m) \times \mathcal{R}\Psi(n)$ |
| Reverse first redefined Zagreb index | $\mathcal{R}ReZG_1(G)$ | $\sum_{mn \in E(G)} \mathcal{R}\Psi(m) + \mathcal{R}\Psi(n)$ \(\mathcal{R}\Psi(m) \times \mathcal{R}\Psi(n)\)$ |
| Reverse second redefined Zagreb index | $\mathcal{R}ReZG_2(G)$ | $\sum_{mn \in E(G)} \mathcal{R}\Psi(m) \times \mathcal{R}\Psi(n)$ \(\mathcal{R}\Psi(m) + \mathcal{R}\Psi(n)\)$ |
| Reverse third redefined Zagreb index | $\mathcal{R}ReZG_3(G)$ | $\sum_{mn \in E(G)} (\mathcal{R}\Psi(m) + \mathcal{R}\Psi(n))(\mathcal{R}\Psi(m) \times \mathcal{R}\Psi(n))$ |

Jung et al. in [7] computed first and second reverse Zagreb indices, first and second reverse hyper Zagreb indices, reverse atomic-bond connectivity index and reverse geometric-arithmetic index for TUC4[m, n]. Wei et al. in [8] computed some reverse topological indices, namely, the reverse general Randić index, the reverse atom bond connectivity index, the reverse geometric arithmetic index, the reverse forgotten index, the reverse Balaban index, and the reverse Zagreb type indices for Remdesivir Compound Used in Treatment of Corona Virus (COVID 19). Rosary in [9] presented some reverse degree-based topological indices for the line graph of Remdesivir. Ravi et al. in [10] analyzed the Quantitative Structure-Property Relationship (QSPR) of the reverse degree-based topological indices. Haoer and Virk in [11] computed reverse Zagreb indices, reverse hyper Zagreb indices, and their polynomials for metal-organic networks. Hashmi et al. in [12] computed the reverse ABC index for different generations of dendrimers.
For a more in-depth look at molecular structure topological indices in chemical graph theory and biology, we refer to [13-24].

In this article, we compute the reverse topological indices for the molecular structures of Oseltamivir and Zanamivir.

2. Reverse Topological Indices of Zanamivir used in the Treatment of H1N1

The edge set of the Zanamivir compound molecular graph is partitioned into four sets based on the reverse degree of end vertices. The first edge set of the partition contains one edge, where $\mathcal{R}_\psi(m)=3$ and $\mathcal{R}_\psi(n)=2$. The second edge set of the partition contains nine edges $mn$, where $\mathcal{R}_\psi(m)=2$ and $\mathcal{R}_\psi(n)=1$. The third edge set of the partition contains eight edges $mn$, where $\mathcal{R}_\psi(m)=1$ and $\mathcal{R}_\psi(n)=3$. The fourth edge set of the partition contains five edges $mn$, where $\mathcal{R}_\psi(m)=1$ and $\mathcal{R}_\psi(n)=1$. The tabular representation of the edge set of the partitioning technique based on reverse degree is depicted in Table 2.

| (R$\psi$(m), R$\psi$(n)) | Frequency |
|-------------------------|-----------|
| (3,2)                   | 1         |
| (2,1)                   | 9         |
| (1,3)                   | 8         |
| (1,1)                   | 5         |

**Theorem 2.1.** The reverse Randić index, for different values of $\alpha$, of the molecular graph of Zanamivir compound is given by:

$$\mathcal{R}_{\alpha}R_1(G) = \sum_{mn \in E(G)} (\mathcal{R}_\psi(m) \times \mathcal{R}_\psi(n))^\alpha,$$

Proof. From Table 2, we compute the reverse Randić index as below,

For $\alpha = 1$, $\mathcal{R}_{1}R_1(G) = (1)(3 \times 2) + (9)(2 \times 1) + (8)(3 \times 1) + (5)(1 \times 1) = 53$.

For $\alpha = -1$, $\mathcal{R}_{-1}R_1(G) = 5 + \frac{9}{2} + \frac{8}{3} + \frac{5}{1} = 12.33$.

For $\alpha = \frac{1}{2}$, $\mathcal{R}_{\frac{1}{2}}R_1(G) = (1)(\sqrt{3} \times 2) + (9)(\sqrt{2} \times 1) + (8)(\sqrt{3} \times 1) + (5)(\sqrt{1} \times 1) = 34.03$.

For $\alpha = -\frac{1}{2}$, $\mathcal{R}_{-\frac{1}{2}}R_1(G) = \frac{1}{\sqrt{3} \times 2} + \frac{9}{\sqrt{2} \times 1} + \frac{8}{\sqrt{3} \times 1} + \frac{5}{\sqrt{1} \times 1} = 16.39$.

**Theorem 2.2.** The reverse atomic bond connectivity index, reverse geometric arithmetic index of the molecular graph of Zanamivir compound are given by:

$$\mathcal{R}_{ABC}(G)=13.60 \quad | \quad \mathcal{R}_{GAC}(G)=21.39$$

Proof. From Table 2, we compute the reverse atomic bond connectivity index as below,

$$\mathcal{R}_{ABC}(G) = \sum_{mn \in E(G)} \sqrt{\frac{\mathcal{R}_\psi(m) + \mathcal{R}_\psi(n) - 2}{\mathcal{R}_\psi(m) \times \mathcal{R}_\psi(n)}}$$

$$\mathcal{R}_{ABC}(G) = (1) \sqrt{\frac{3 + 2 - 2}{3 \times 2}} + (9) \sqrt{\frac{2 + 1 - 2}{2 \times 1}} + (8) \sqrt{\frac{1 + 3 - 2}{1 \times 3}} + (5) \sqrt{\frac{1 + 1 - 2}{1 \times 1}} = 13.60.$$
\[
\mathcal{RGA}(G) = \sum_{mn \in E(G)} \frac{2\sqrt{\psi(m) \times \psi(n)}}{\psi(m) + \psi(n)},
\]
\[
\mathcal{RGA}(G) = (1)\frac{2\sqrt{3 \times 2}}{3 + 2} + (9)\frac{2\sqrt{2 \times 1}}{2 + 1} + (8)\frac{2\sqrt{1 \times 3}}{1 + 3} + (5)\frac{2\sqrt{1 \times 1}}{1 + 1} = 21.39.
\]

**Theorem 2.3.** The reverse first Zagreb index, reverse second Zagreb index, reverse first Zagreb co-index and reverse second Zagreb co-index of the molecular graph of Zanamivir compound are given by:

\[
\mathcal{RM}_1(G) = 74, \quad \mathcal{RM}_2(G) = 53, \quad \mathcal{RM}_4(G) = 938, \quad \mathcal{RM}_6(G) = 968.
\]

**Proof.** From Table 2, we compute the reverse first Zagreb index as below,

\[
\mathcal{RM}_1(G) = \sum_{mn \in E(G)} \psi(m) + \psi(n),
\]
\[
\mathcal{RM}_1(G) = (1)(3 + 2) + (9)(2 + 1) + (8)(3 + 1) + 5(1 + 1) = 74.
\]
From Table 2, we compute the reverse second Zagreb index as below,

\[
\mathcal{RM}_2(G) = \sum_{mn \in E(G)} \psi(m) \times \psi(n),
\]
\[
\mathcal{RM}_2(G) = (1)(3 \times 2) + (9)(2 \times 1) + (8)(3 \times 1) + 5(1 \times 1) = 53.
\]
From Table 2, we compute the reverse first Zagreb co-index as below,

\[
\mathcal{RM}_4(G) = 2|E(G)|(|V(G)| - 1) - \mathcal{RM}_1(G),
\]
\[
\mathcal{RM}_4(G) = 2|23|(|23 - 1|) - 74 = 938.
\]
From Table 2, we compute the reverse second Zagreb co-index as below,

\[
\mathcal{RM}_6(G) = 2|E(G)|^2 - \frac{1}{2} \mathcal{RM}_1(G) - \mathcal{RM}_2(G),
\]
\[
\mathcal{RM}_6(G) = 2|23|^2 - \frac{1}{2}(74) - 53 = 968.
\]

**Theorem 2.4.** The reverse hyper Zagreb index, reverse forgotten index and reverse Balaban index of the molecular graph of Zanamivir compound are given by:

\[
\mathcal{RM}_M(G) = 254, \quad \mathcal{RF}(G) = 148, \quad \mathcal{RJ}(G) = 188.49.
\]

**Proof.** From Table 2, we compute the reverse hyper Zagreb index as below,

\[
\mathcal{RM}_M(G) = \sum_{mn \in E(G)} [\psi(m) \times \psi(n)]^2,
\]
\[
\mathcal{RM}_M(G) = (1)(3 + 2)^2 + (9)(2 + 1)^2 + (8)(3 + 1)^2 + (5)(1 + 1)^2 = 254.
\]
From Table 2, we compute the reverse forgotten index as below,

\[
\mathcal{RF}(G) = \sum_{mn \in E(G)} (\psi(m))^2 + (\psi(n))^2,
\]
\[
\mathcal{RF}(G) = (1)(3^2 + 2^2) + (9)(2^2 + 1^2) + (8)(3^2 + 1^2) + (5)(1^2 + 1^2) = 148.
\]
From Table 2, we compute the reverse Balaban index as below,

\[
\mathcal{RJ}(G) = \frac{m}{m - n + 2} \sum_{mn \in E(G)} \frac{1}{\sqrt{\psi(m) \times \psi(n)}},
\]
\[
\mathcal{RJ}(G) = \left(\frac{23}{23 - 23 + 2}\right)\left(\frac{1}{\sqrt{3 \times 2}} + \frac{9}{\sqrt{2 \times 1}} + \frac{8}{\sqrt{3 \times 1}} + \frac{5}{\sqrt{1 \times 1}}\right) = 188.49.
\]
Theorem 2.5. The reverse first multiple Zagreb index, reverse second multiple Zagreb index of the molecular graph of Zanamivir compound is given by:

\[
\mathcal{RPM}_1(G) = 43200 \quad \mathcal{RPM}_2(G) = 12960
\]

**Proof.** From Table 2, we compute the reverse first multiple Zagreb index as below,

\[
\mathcal{RPM}_1(G) = \sum_{mn \in E(G)} R\psi(m) + R\psi(n),
\]

\[
\mathcal{RPM}_1(G) = (1)(3 + 2) \times (9)(2 + 1) \times (8)(3 + 1) \times (5)(1 + 1) = 43200.
\]

From Table 2, we compute the reverse second multiple Zagreb index as below,

\[
\mathcal{RPM}_2(G) = \sum_{mn \in E(G)} R\psi(m) \times R\psi(n),
\]

\[
\mathcal{RPM}_2(G) = (1)(3 \times 2) \times (9)(2 \times 1) \times (8)(3 \times 1) \times (5)(1 \times 1) = 12960.
\]

Theorem 2.6. The reverse first redefined Zagreb index, reverse second redefined Zagreb index, and reverse third redefined Zagreb index of the molecular graph of Zanamivir compound are given by:

\[
\mathcal{RReZG}_1(G) = 35 \quad \mathcal{RReZG}_2(G) = 15.7 \quad \mathcal{RReZG}_3(G) = 190
\]

**Proof.** From Table 2, we compute the reverse first redefined Zagreb index as below,

\[
\mathcal{RReZG}_1(G) = \sum_{mn \in E(G)} \frac{R\psi(m) + R\psi(n)}{R\psi(m) \times R\psi(n)}.
\]

\[
\mathcal{RReZG}_1(G) = \frac{3 + 2}{3 \times 2} + \left(\frac{3 + 2}{3 \times 2}\right)(9)(2 + 1) \times (8)(3 + 1) \times (5)(1 + 1) = 35.
\]

From Table 2, we compute the reverse second redefined Zagreb index as below,

\[
\mathcal{RReZG}_2(G) = \sum_{mn \in E(G)} \frac{R\psi(m) \times R\psi(n)}{R\psi(m) + R\psi(n)}.
\]

\[
\mathcal{RReZG}_2(G) = \frac{3 \times 2}{3 + 2} + \left(\frac{3}{1 + 2}\right)(9)(2 + 1) \times (8)(3 + 1) \times (5)(1 \times 1) = 15.7.
\]

From Table 2, we compute the reverse third redefined Zagreb index as below,

\[
\mathcal{RReZG}_3(G) = \sum_{mn \in E(G)} (R\psi(m) + R\psi(n))(R\psi(m) \times R\psi(n)).
\]

\[
\mathcal{RReZG}_3(G) = (1)(3 \times 2)(3 + 2) + (9)(1 \times 2)(1 + 2) + (8)(1 \times 3)(1 + 3) + (5)(1 \times 1)(1 + 1) = 190.
\]

3. Reverse Topological Indices of Oseltamivir Drug Used in Treatment of H1N1

The edge set of the Oseltamivir compound molecular graph is partitioned into five sets based on the reverse degree of end vertices. The first edge set of the partition contains three edges \(mn\), where \(R\psi(m) = 3\) and \(R\psi(n) = 2\). The second edge set of the partition contains one edge \(mn\), where \(R\psi(m) = 2\) and \(R\psi(n) = 2\). The third edge set of the partition contains eleven edges \(mn\), where \(R\psi(m) = 2\) and \(R\psi(n) = 1\). The fourth edge set of the partition contains four edges \(mn\), where \(R\psi(m) = 1\) and \(R\psi(n) = 3\). The fifth edge set of the partition contains three edges \(mn\), where \(R\psi(m) = 1\) and \(R\psi(n) = 1\). The tabular representation of the edge set of the partitioning technique based on reverse degree is depicted in Table 3.

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**Table 3.** Reverse edge set the partition of the Oseltamivir compound molecular graph.

| (ℛψ(m),ℛψ(n)) | (3,2) | (2,2) | (2,1) | (1,3) | (1,1) |
|-----------------|-------|-------|-------|-------|-------|
| Frequency       | 3     | 1     | 11    | 4     | 3     |

**Theorem 3.1.** The reverse Randić index, for different values of α, of the molecular graph of Oseltamivir compound is given by:

\[
RR_α(G) = \sum_{mn \in E(G)} [ℛψ(m) \times ℛψ(n)]^α,
\]

*Proof.* From Table 3, we compute the reverse Randić index as below,

For \( α = 1\),
\[
RR_1(G) = (3)(3 \times 2) + (1)(2 \times 2) + (11)(2 \times 1) + (4)(1 \times 3) + (3)(1 \times 1), = 59.
\]

For \( α = -1\),
\[
RR_{-1}(G) = \frac{3}{2} + \frac{1}{3} + \frac{11}{3} + \frac{4}{3} + \frac{3}{3} = 10.58.
\]

For \( α = \frac{1}{2}\),
\[
RR_{\frac{1}{2}}(G) = \frac{3}{2} + \frac{1}{\sqrt{2}} + \frac{11}{\sqrt{2}} + \frac{4}{\sqrt{2}} + \frac{11}{\sqrt{2}} = 14.81.
\]

**Theorem 3.2.** The reverse atomic bond connectivity index reverse geometric arithmetic index of the molecular graph of Oseltamivir compound is given by:

\[
RABC(G) = 13.87 \quad | \quad RGA(G) = 23.11
\]

*Proof.* From Table 3, we compute the reverse atomic bond connectivity index as below,

\[
RABC(G) = \sum_{mn \in E(G)} \sqrt{ℛψ(m) + ℛψ(n) - 2 / ℛψ(m) \times ℛψ(n)}
\]

\[
RABC(G) = (3)\frac{3 + 2 - 2}{3 \times 2} + (1)\frac{2 + 2 - 2}{2 \times 2} + (11)\frac{2 + 1 - 2}{2 \times 1} + (4)\frac{1 + 3 - 2}{1 \times 3} + (3)\frac{1 + 1 - 2}{1 \times 1}
\]

\[
= 13.87.
\]

From Table 3, we compute the reverse geometric arithmetic index as below,

\[
RGA(G) = \sum_{mn \in E(G)} 2 \sqrt{ℛψ(m) \times ℛψ(n) / ℛψ(m) + ℛψ(n)}
\]

\[
RGA(G) = (3)\frac{2\sqrt{3 + 2}}{3 + 2} + (1)\frac{2\sqrt{2 + 2}}{2 + 2} + (11)\frac{2\sqrt{2 + 1}}{2 + 1} + (4)\frac{2\sqrt{1 + 3}}{1 + 3} + (4)\frac{2\sqrt{1 + 1}}{1 + 1} = 23.11.
\]

**Theorem 3.3.** The reverse first Zagreb index, reverse second Zagreb index, reverse first Zagreb co-index and reverse second Zagreb co-index of the molecular graph of Oseltamivir compound are given by:

\[
RM_1(G) = 74 \quad | \quad RM_2(G) = 59 \quad | \quad RM_1(G) = 850 \quad | \quad RM_2(G) = 872
\]

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\textbf{Proof.} From Table 3, we compute the reverse first Zagreb index as below,
\[RM_1(G) = \sum_{mn \in E(G)} R\psi(m) + R\psi(n),\]
\[RM_1(G) = (3)(3 + 2) + (1)(2 + 2) + (11)(2 + 1) + (4)(1 + 3) + (3)(1 + 1) = 74.\]
From Table 3, we compute the reverse second Zagreb index as below,
\[RM_2(G) = \sum_{mn \in E(G)} R\psi(m) \times R\psi(n),\]
\[RM_2(G) = (3)(3 \times 2) + (1)(2 \times 2) + (11)(2 \times 1) + (4)(1 \times 3) + (3)(1 \times 1) = 59.\]
From Table 3, we compute the reverse first Zagreb co-index as below,
\[RM_1(G) = 2|E(G)|(|V(G)| - 1) - RM_1(G),\]
\[RM_1(G) = 2|22|(|22| - 1) - 74 = 850.\]
From Table 3, we compute the reverse second Zagreb co-index as below,
\[RM_2(G) = 2|E(G)|^2 - \frac{1}{2} RM_1(G) - RM_2(G),\]
\[RM_2(G) = 2|22|^2 - \frac{1}{2}(74) - 59 = 872.\]

\textbf{Theorem 3.4.} The reverse hyper Zagreb index, reverse forgotten index and reverse Balaban index of the molecular graph of Oseltamivir compound are given by:
\[RHM(G)=266 \quad | \quad RF(G)=269 \quad | \quad RJ(G)=162.93\]
\textbf{Proof.} From Table 3, we compute the reverse hyper Zagreb index as below,
\[RHM(G) = \sum_{mn \in E(G)} [R\psi(m) \times R\psi(n)]^2,\]
\[RHM(G) = (3)(3 + 2)^2 + (1)(2 + 2)^2 + (11)(2 + 1)^2 + (4)(1 + 3)^2 + (3)(1 + 1)^2 = 266.\]
From Table 3, we compute the reverse forgotten index as below,
\[RF(G) = \sum_{mn \in E(G)} (R\psi(m))^2 + (R\psi(n))^2,\]
\[RF(G) = (3)(3^2 + 2^2) + (1)(2^2 + 2^2) + (11)(2^2 + 1^2) + (4)(1^2 + 3^2) + (3)(1^2 + 1^2) = 269.\]
From Table 3, we compute the reverse Balaban index as below,
\[RJ(G) = \frac{m}{m - n + 2} \sum_{mn \in E(G)} \frac{1}{\sqrt{R\psi(m) \times R\psi(n)}},\]
\[RJ(G) = \left(\frac{22}{22 - 22 + 2}\right) \left(\frac{3}{\sqrt{(3 \times 2)}} + \frac{1}{\sqrt{(2 \times 2)}} + \frac{11}{\sqrt{(2 \times 1)}} + \frac{4}{\sqrt{(1 \times 3)}} + \frac{3}{\sqrt{(1 \times 1)}}\right) = 162.93.\]

\textbf{Theorem 3.5.} The reverse first multiple Zagreb index, reverse second multiple Zagreb index of the molecular graph of Oseltamivir compound is given by:
\[RPM_1(G)=190080 \quad | \quad RPM_2(G)=57024.\]
\textbf{Proof.} From Table 3, we compute the reverse first multiple Zagreb index as below,
\[RPM_1(G) = \prod_{mn \in E(G)} R\psi(m) + R\psi(n),\]
\[RPM_1(G) = (3)(3 + 2) \times (1)(2 + 2) \times (11)(2 + 1) \times (4)(1 + 3) \times (3)(1 + 1) = 190080.\]
From Table 3, we compute the reverse second multiple Zagreb index as below,
\[ \mathcal{RPM}_2(G) = \prod_{mn \in E(G)} \mathcal{R}\psi(m) \times \mathcal{R}\psi(n), \]
\[ \mathcal{RPM}_2(G) = (3)(3 \times 2) \times (1)(2 \times 2) \times (11)(2 \times 1) \times (4)(1 \times 3) \times (3)(1 \times 1) = 57024. \]

**Theorem 3.6.** The reverse first redefined Zagreb index, reverse second redefined Zagreb index, and reverse third redefined Zagreb index of the molecular graph of Oseltamivir compound is given by.

\[ \mathcal{RZG}_1(G) = \sum_{mn \in E(G)} \frac{\mathcal{R}\psi(m) + \mathcal{R}\psi(n)}{\mathcal{R}\psi(m) \times \mathcal{R}\psi(n)} \]
\[ \mathcal{RZG}_1(G) = (3) \frac{3 + 2}{3 \times 2} + (1) \frac{2 + 2}{2 \times 2} + (11) \frac{2 + 1}{2 \times 1} + (4) \frac{1 + 3}{1 \times 3} + (3) \frac{1 + 1}{1 \times 1} = 31.33. \]

From Table 3, we compute the reverse second redefined Zagreb index as below,

\[ \mathcal{RZG}_2(G) = \sum_{mn \in E(G)} \frac{\mathcal{R}\psi(m) \times \mathcal{R}\psi(n)}{\mathcal{R}\psi(m) + \mathcal{R}\psi(n)} \]
\[ \mathcal{RZG}_2(G) = (3) \frac{3 \times 2}{3 + 2} + (1) \frac{2 \times 2}{2 + 2} + (11) \frac{2 \times 1}{2 + 1} + (4) \frac{1 \times 3}{1 + 3} + (3) \frac{1 \times 1}{1 + 1} = 16.43. \]

From Table 3, we compute the reverse third redefined Zagreb index as below,

\[ \mathcal{ZG}_3(G) = \sum_{mn \in E(G)} (\mathcal{R}\psi(m) + \mathcal{R}\psi(n))(\mathcal{R}\psi(m) \times \mathcal{R}\psi(n)) \]
\[ \mathcal{ZG}_3(G) = (3)(3 + 2)(3 \times 2) + (1)(2 + 2)(2 \times 2) + (11)(2 + 1)(2 \times 1) + (4)(1 + 3)(1 \times 3) + (3)(1 + 1)(1 \times 1) = 226. \]

**4. Graphical Comparison of Zanamivir and Oseltamivir**

To understand the similarities between the biological and statistical behavior of the two chemical compounds, we have provided results for topological indices for the structure of Zanamivir and Oseltamivir. Moreover, we have drawn Figures 3-8 for the structure of Zanamivir and Oseltamivir to review the graphical behavior of topological indices computed above to predict physicochemical and biological properties.

**Figure 3.** Comparison of the reverse Randić index for different values of α, of the molecular graph of Zanamivir compound(Left). Red, magenta, blue, and green represents \( \mathcal{RR}_1(G), \mathcal{RR}_1(G), \mathcal{RR}_1(G), \) and \( \mathcal{RR}_1(G) \) respectively. Comparison of the reverse atomic bond connectivity index and reverse geometric arithmetic index of the molecular graph of Zanamivir (Right). Red and magenta represents \( \mathcal{RGA}(G) \) and \( \mathcal{RABC}(G) \) index, respectively.
**Figure 4.** Comparison of the reverse first Zagreb index, reverse second Zagreb index, reverse first Zagreb co-index and reverse second Zagreb co-index of the molecular graph of Zanamivir compound (Left). Red, magenta, blue, and green represents $\mathcal{RM}_2(G)$, $\mathcal{RM}_1(G)$, $\mathcal{RM}_1(G)$ and $\mathcal{RM}_2(G)$, respectively. Comparison of the reverse hyper Zagreb index, reverse forgotten index, and reverse Balaban index of the molecular graph of Zanamivir compound (Right). Red, magenta and blue represents $\mathcal{RHM}(G)$, $\mathcal{RJ}(G)$ and $\mathcal{RF}(G)$, respectively.

**Figure 5.** Comparison of the reverse first multiple Zagreb index, reverse second multiple Zagreb index of the molecular graph of Zanamivir compound (Left). Red and blue represent $\mathcal{RP}_M_1(G)$ and $\mathcal{RP}_M_2(G)$ respectively. Comparison of the reverse first redefined Zagreb index, reverse second redefined Zagreb index, and reverse third redefined Zagreb index of the molecular graph of Zanamivir compound (Right). Red, magenta and blue represents $\mathcal{RR}_1ZG_3(G)$, $\mathcal{RR}_1ZG_1(G)$ and $\mathcal{RR}_1ZG_2(G)$.

**Figure 6.** Comparison of reverse Randić index, for different values of $\alpha$, of the molecular graph of Oseltamivir compound (Left). Red, magenta, blue, and green represents $\mathcal{RR}_1(G)$, $\mathcal{RR}_2(G)$, $\mathcal{RR}_{-1/2}(G)$ and $\mathcal{RR}_1(G)$ respectively. Comparison of the reverse atomic bond connectivity index, reverse geometric arithmetic index of the molecular graph of Oseltamivir compound (Right). Red and magenta represents $\mathcal{RGA}(G)$ and $\mathcal{RABC}(G)$ index, respectively.
Figure 7. Comparison of the reverse first Zagreb index, reverse second Zagreb index, reverse first Zagreb co-index and reverse second Zagreb co-index of the molecular graph of Oseltamivir compound (Left). Red, magenta, blue, and green represent $\mathcal{RM}_2(G)$, $\mathcal{RM}_1(G)$, $\mathcal{RM}_1(G)$ and $\mathcal{RM}_2(G)$, respectively. Comparison of the reverse hyper Zagreb index, reverse forgotten index, and reverse Balaban index of the molecular graph of Oseltamivir compound (Right). Red, magenta, and blue represents $\mathcal{RF}(G)$, $\mathcal{RHM}(G)$ and $\mathcal{RI}(G)$ respectively.

Figure 8. Comparison of the reverse first multiple Zagreb index, reverse second multiple Zagreb index of the molecular graph of Oseltamivir compound (Left). Red and blue represent $\mathcal{RPM}_1(G)$ and $\mathcal{RPM}_2(G)$ respectively. Comparison of the reverse first redefined Zagreb index, reverse second redefined Zagreb index, and reverse third redefined Zagreb index of the molecular graph of Oseltamivir compound (Right). Red, magenta and blue represents $\mathcal{RReZG}_3(G)$, $\mathcal{RReZG}_1(G)$ and $\mathcal{RReZG}_2(G)$.

5. Conclusions

In this paper, we have computed the reverse degree-based topological indices, namely the reverse Randić index, reverse atomic bond connectivity index, reverse geometric arithmetic index, reverse Zagreb and reverse redefined Zagreb indices and co-indices, reverse forgotten index, and the reverse Balaban index of the molecular graphs of Oseltamivir and Zanamivir. In addition, we have compared topological indices graphically. Since topological indices can predict several qualities and activities like entropy, critical pressure, boiling point, acentric factor, enthalpy, and so on, our findings can aid in developing new drugs for H1N1 treatment.

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Conflicts of Interest

The authors declare no conflict of interest.

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