A Non-Self-Referential Paradox in Epistemic Game Theory

AHMAD KARIMI

Department of Mathematics
Behbahan Khatam Alanbia University of Technology
P.O.Box 61635–151, Behbahan, Iran
karimi@bkatu.ac.ir

January 26, 2016

Abstract

In game theory, the notion of a player’s beliefs about the game players’ beliefs about other players’ beliefs arises naturally. In this paper, we present a non-self-referential paradox in epistemic game theory which shows that completely modeling players’ epistemic beliefs and assumptions is impossible. Furthermore, we introduce an interactive temporal assumption logic to give an appropriate formalization of the new paradox. Formalizing the new paradox in this logic shows that there is no complete interactive temporal assumption model.

Keywords: Non-Self-Referential Paradox · Brandenburger-Keisler Paradox · Yablo’s Paradox · Temporal Assumption Logic · Interactive Models.

1 Introduction

In 2006 Adam Brandenburger and H. Jerome Keisler presented a two-person and self-referential paradox in epistemic game theory [4]. They show that the following configuration of beliefs can not be represented: Ann believes that Bob assumes that Ann believes that Bob’s assumption is wrong. The Brandenburger-Keisler paradox (BK paradox) shows impossibility of completely modeling players’ epistemic beliefs and assumptions. Brandenburger and Keisler present a modal logic interpretation of the paradox [4]. They introduce two modal operators intended to represent the agents’ beliefs and assumptions. In [15] Pacuit approaches the BK paradox from a modal logical perspective and presents a detailed investigation of the paradox in neighborhood models and hybrid systems. In particular, he shows that the paradox can be seen as a theorem of an appropriate hybrid logic.
The BK paradox is essentially a self-referential paradox and similarly to any other paradox of the same kind can be analyzed from a category theoretical or algebraic point of view. In [1] Abramsky and Svesper analyze the BK paradox in categorical context. They present the paradox as a fixed-point theorem, which can be carried out in any regular category, and show how it can be reduced to a relational form of the one-person diagonal argument due to Lawvere [13] where he gave a simple form of the (one-person) diagonal argument as a fixed-point lemma in a very general setting.

Başkent in [2] approaches the BK paradox from two different perspectives: non-well-founded set theory and paraconsistent logic. He shows that the paradox persists in both frameworks for category-theoretical reasons, but with different properties. Başkent makes the connection between self-referentiality and paraconsistency.

On the other hand, Stephen Yablo [19] introduced a logical paradox in 1993 that is similar to the liar paradox where he used an infinite sequence of statements. Every statement in the sequence refers to the truth values of the later statements. Therefore, it seems this paradox avoids self-reference. In this paper, we present a non-self-referential multi-agent paradox in epistemic game theory which we call “Yablo-like Brandenburger-Keisler paradox”.

In an interactive temporal assumption logic we study interactive reasoning: how agents reason about the beliefs of other agents over time. In this paper, we introduce an interactive temporal assumption logic to give an appropriate formalization of the Yablo-like Brandenburger-Keisler paradox.

2 Brandenburger-Keisler Paradox

Brandenburger and Keisler introduced the following two person Russell-style paradox [2, 4, 15]. Beliefs and assumptions are two main concepts involved in the statement of the paradox. An assumption is the strongest belief. Suppose there are two players, Ann and Bob, and consider the following description of beliefs:

Ann believes that Bob assumes that Ann believes that Bob’s assumption is wrong.

A paradox arises when one asks the question “Does Ann believe that Bob’s assumption is wrong?” Suppose that answer to the above question is “yes”. Then according to Ann, Bob’s assumption is wrong. But, according to Ann, Bob’s assumption is Ann believes that Bob’s assumption is wrong. However, since the answer to the above question is “yes”, Ann believes that this assumption is correct. So Ann does not believe that Bob’s assumption is wrong. Therefore, the answer to the above question must be “no”. Thus, it is not the case that Ann believes that Bob’s assumption is wrong. Hence
Ann believes Bob’s assumption is correct. That is, it is correct that Ann believes that Bob’s assumption is wrong. So, the answer must have been “yes”. This is a contradiction!

Brandenburger and Keisler [4] introduce belief models for two players to present their impossibility results. In a belief model, each player has a set of states, and each state for one player has beliefs about the states of the other player. We assume the reader is familiar with the belief models and formulation of BK paradox in modal logic, but for the sake of accessibility, we list the main notations, definitions and theorems which will be referred to later on. For more details, we refer the reader to [4, 15].

**Definition 1.** A belief model is a two-sorted structure 

\[ \mathcal{M} = (U^a, U^b, P^a, P^b, ...) \]

where \( U^a \) and \( U^b \) are the nonempty universe sets (for the two sorts), \( P^a \) is a proper subset of \( U^a \times U^b \), \( P^b \) is a proper subset of \( U^b \times U^a \), and \( P^a, P^b \) are serial, that is, the sentences

\[ \forall x \exists y P^a(x,y) \quad \text{and} \quad \forall y \exists x P^b(y,x) \]

hold. To simplify notation, \( x \) is always a variable of sort \( U^a \) and \( y \) is a variable of sort \( U^b \). The members of \( U^a \) and \( U^b \) are called states for Ann and Bob, respectively. \( P^a \) and \( P^b \) are called possibility relations. We say \( x \) believes a set \( Y \subseteq U^b \) if \( \{ y : P^a(x,y) \} \subseteq Y \), and \( x \) assumes \( Y \) if \( \{ y : P^a(x,y) \} = Y \). ⊗

For an arbitrary structure \( \mathcal{N} = (U^a, U^b, P^a, P^b, ...) \), by the first order language for \( \mathcal{N} \) we mean the two-sorted first order logic with sorts for \( U^a \) and \( U^b \) and symbols for the relations in the vocabulary of \( \mathcal{N} \). Given a first order formula \( \varphi(u) \) whose only free variable is \( u \), the set defined by \( \varphi \) in \( \mathcal{N} \) is the set \( \{ u : \varphi(u) \text{ is true in } \mathcal{N} \} \). In general, by a language for \( \mathcal{N} \) we will mean a subset of the set of all formulas of the first order language for \( \mathcal{N} \). Given a language \( \mathcal{L} \) for \( \mathcal{N} \), we let \( \mathcal{L}_a, \mathcal{L}_b \) be the families of all subsets of \( U^a, U^b \) respectively which are defined by formulas in \( \mathcal{L} \). Also, The diagonal formula \( D(x) \) is the first order formula

\[ \forall y \ (P^a(x,y) \rightarrow \neg P^b(y,x)). \]

This is the formal counterpart to the intuitive statement: “Ann believes that Bob’s assumption is wrong.”

**Definition 2.** Let \( \mathcal{M} \) be a belief model, and let \( \mathcal{L} \) be a language for it. The model \( \mathcal{M} \) is **complete** for \( \mathcal{L} \) if each nonempty set \( Y \in \mathcal{L}_b \) is assumed by some \( x \in U^a \), and each nonempty \( X \in \mathcal{L}_a \) is assumed by some \( y \in U^b \). ⊗
In their paper [4], Brandenburger and Keisler prove that there is no belief model which is complete for a language $L$ which contains the tautologically true formulas, $D(x)$, $\forall y P^b(y, x)$ and formula built from the formal counter-part of the BK paradox.

They also present modal formulations of the BK paradox by presenting a specific modal logic, called interactive assumption logic, with two modal operators for “belief” and “assumption”. For semantical interpretations Brandenburger and Keisler use Kripke models [4]. Pacuit [15] presents a detailed investigation of the BK paradox in neighborhood models and in hybrid systems. He shows that the paradox can be seen as a theorem of an appropriate hybrid logic.

3 Yablo’s Paradox

To counter a general belief that all the paradoxes stem from a kind of circularity (or involve some self-reference, or use a diagonal argument) Stephen Yablo designed a paradox in 1993 that seemingly avoided self-reference [18][19]. Since then much debate has been sparked in the philosophical community as to whether Yablo’s Paradox is really circular-free or involves some circularity (at least hidden or implicitly); see e.g. 3 5 7 11 12 16 17 20. Unlike the liar paradox, which uses a single sentence, this paradox applies an infinite sequence of statements. There is no consistent way to assign truth values to all the statements, although no statement directly refers to itself. Yablo considers the following sequence of sentences $\{S_i\}$:

$S_1 : \forall k > 1; S_k$ is untrue,
$S_2 : \forall k > 2; S_k$ is untrue,
$S_3 : \forall k > 3; S_k$ is untrue,
$
$
The paradox follows from the following deductions. Suppose $S_1$ is true. Then for any $k > 1$, $S_k$ is not true. Specially, $S_2$ is not true. Also, $S_k$ is not true for any $k > 2$. But this is exactly what $S_2$ says, hence $S_2$ is true after all. Contradiction! Suppose then that $S_1$ is false. This means that there is a $k > 1$ such that $S_k$ is true. But we can repeat the reasoning, this time with respect to $S_k$ and reach a contradiction again. No matter whether we assume $S_1$ to be true or false, we reach a contradiction. Hence the paradox. Yablo’s paradox can be viewed as a non-self-referential liar’s paradox; it has been used to give alternative proof for Gödel’s first incompleteness theorem [8][11]. Recently in [9][10], formalization of Yablo’s paradox and its different versions in Linear Temporal Logic (LTL) yield genuine theorems in this logic.
4 A Non-Self-Referential Paradox in Epistemic Game Theory

In this section, we present a non-self-referential paradox in epistemic game theory. Unlike the BK paradox which uses one single statement on belief and assumption of agents, this paradox consists of two sequences of players (or agents) and a sequence of statements of agent’s belief and assumptions. Indeed, this paradox shows that not every configuration of agents’ beliefs and assumptions can be represented. Let us consider two infinite sequence of players \{A_i\} and \{B_i\}, with the following description of beliefs:

\[
\begin{array}{cc}
A_1 & B_1 \\
A_2 & B_2 \\
A_3 & B_3 \\
\vdots & \vdots \\
\end{array}
\]

For all \(i\), \(A_i\) believes that \(B_i\) assumes that for all \(j > i\), \(A_j\) believes that \(B_j\)’s assumption is wrong.

A paradox arises when one asks the question “Does \(A_1\) believe that \(B_1\)’s assumption is wrong?”

Suppose that the answer to the above question is “no”. Thus, it is not the case that \(A_1\) believes that \(B_1\)’s assumption is wrong. Hence \(A_1\) believes \(B_1\)’s assumption is correct. That is, it is correct that for all \(j > 1\), \(A_j\) believes that \(B_j\)’s assumption is wrong. Specially, \(A_2\) believes that \(B_2\)’s assumption is wrong. On the other hand, since for all \(j > 2\), \(A_j\) believes that \(B_j\)’s assumption is wrong, one can conclude that \(A_2\) believes \(B_2\)’s assumption is correct. Therefore, at the same time \(A_2\) believes that \(B_2\)’s assumption is both correct and wrong. This is a contradiction!

If the answer to the above question is “yes”. Then according to \(A_1\), \(B_1\)’s assumption is wrong. But, according to \(A_1\), \(B_1\)’s assumption is that “for all \(j > 1\), \(A_j\) believes that \(B_j\)’s assumption is wrong”. Thus, there is \(k > 1\) for which \(A_k\) believes that \(B_k\)’s assumption is correct. Now we can apply the same reasoning we used before about \(A_k\) and \(B_k\) to reach the contradiction! Hence the paradox. This paradox is a non-self-referential multi-agent version of the BK paradox.
5 Interactive Temporal Assumption Logic

In this section, we introduce an Interactive Temporal Assumption Logic (iTAL) to present an appropriate formulation of the non-self-referential Yablo-like BK paradox. The interactive temporal assumption language $L_{iTAL}$ contains individual propositional symbols, the propositional connectives, the linear-time operators $\circ, \square, \Diamond$ and the epistemic operators “Believe” and “Assumption”: for each pair of players $ij$ among Ann and Bob, the operator $B^{ij}$ will be beliefs for player $i$ about $j$, and $A^{ij}$ is the assumption for $i$ about $j$.

In words, $B^{ij}\phi$ means that the agent $i$ believes $\phi$ about agent $j$, and $A^{ij}\phi$ is that the agent $i$ assumes $\phi$ about agent $j$. The temporal operators $\circ, \square$, and $\Diamond$ are called next time, always (or henceforth), and sometime (or eventuality) operators, respectively. Formulas $\bigcirc \phi, \square \phi,$ and $\Diamond \phi$ are typically read “next $\phi$”, “always $\phi$”, and “sometime $\phi$”. We note that $\Diamond \phi \equiv \neg \Box \neg \phi$.

Definition 3. Formulas in $L_{iTAL}$ are defined as follows:

$$\phi := p \mid \neg \phi \mid \phi \land \psi \mid \bigcirc \phi \mid \square \phi \mid B^{ij}\phi \mid A^{ij}\phi$$

For semantical interpretations we introduce an appropriate class of Kripke models.

Definition 4. An iTAL-Model is a Kripke structure

$$\mathcal{W} = (W, \mathbb{N}, \{P_n : n \in \mathbb{N}\}, U^n, U^b, V),$$

where $W$ is nonempty set, $\mathbb{N}$ is the set of natural numbers, for each $n \in \mathbb{N}$, $P_n$ is a binary relation $P_n \subseteq W \times W$ and $U^n, U^b$ are disjoint sets such that $(U^n, U^b, P^n, P^b)$ is a belief model, where $U^n \cup U^b = W$, $P_n = P_n \cap U^n \times U^b$, and $P^b_n = P_n \cap U^b \times U^n$. $V : \text{Prop} \rightarrow 2^{\mathbb{N} \times W}$ is a function mapping to each propositional letter $p$ the subset $V(p)$ of Cartesian product $\mathbb{N} \times W$. Indeed, $V(p)$ is the set of pairs $(n, w)$ such that $p$ is true in the world $w$ at the moment $n$.

The satisfiability of a formula $\varphi \in L_{iTAL}$ in a model $\mathcal{W}$, at a moment of time $n \in \mathbb{N}$ in a world $w \in W$, denoted by $\mathcal{W}_n w \models \varphi$ (in short; $(n, w) \models \varphi$), is defined inductively as follows:

- $(n, w) \models p \iff (n, w) \in V(p)$ for $p \in \text{Prop},$
- $(n, w) \models \neg \varphi \iff (n, w) \not\models \varphi,$
- $(n, w) \models \varphi \land \psi \iff (n, w) \models \varphi$ and $(n, w) \models \psi,$
- $(n, w) \models \bigcirc \varphi \iff (n + 1, w) \models \varphi,$
- $(n, w) \models \square \varphi \iff \forall m \geq n \ (m, w) \models \varphi,$
- $(n, w) \models B^{ij}\varphi \iff (n, w) \models U^n \land \forall z [(P_n(w, z) \land (n, z) \models U^d) \rightarrow (n, z) \models \varphi)],$
\[
(n, w) \models A^{ij} \varphi \iff (n, w) \models \text{U}^c \land \forall z[(P_n(w, z) \land (n, z) \models \text{U}^d) \leftrightarrow (n, z) \models \varphi)].
\]

Let \(x\) has sort \(U^a\), \(y\) has sort \(U^b\) and \(\varphi\) is a statement about \(y\). Intuitively, \((n, x) \models B^{ab} \varphi\) says that “in time \(n\), \(x\) believes \(\varphi(y)\)”, and \((n, x) \models A^{ab} \varphi\) says that “in time \(n\), \(x\) assumes \(\varphi(y)\)”. A formula is valid for \(V\) in \(W\) if it is true at all \(w \in W\), and satisfiable for \(V\) in \(W\) if it is true at some \(w \in W\).

In an interactive assumption model \(W\), we will always suppose that \(D\) is a propositional symbol, and \(V\) is a valuation in \(W\) such that \(V(D)\) is the set
\[
D = \{(n, x) \in W \times \mathbb{N} : (\forall y \in W)[P_n(x, y) \rightarrow \neg P_n(y, x)]\}.
\]

In the rest of paper, we present our formulation of the non-self-referential Yablo-like BK paradox in the interactive temporal assumption setting. The thought is that we can make progress by thinking of the sequences of agents in Yablo-like BK paradox not as infinite families of agents but as a two individual agents that their belief and assumption can be evaluated in lots of times (temporal states) in an epistemic temporal model. Thus, the emergence of the Yablo-like BK paradox should be the same as the derivability of a particular formula in the epistemic temporal logic.

Let us have a closer look at the Yablo-like BK paradox:

For all \(i\), \(A_i\) believes that \(B_i\) assumes that

for all \(j > i\), \(A_j\) believes that \(B_j\)’s assumption is wrong.

Now suppose that there are two players, namely Ann and Bob. Assume that \(A_i\) and \(B_i\) are the counterparts of Ann and Bob in the \(i^{th}\) temporal state. Then infinitely many statements in the Yablo-like BK paradox can be represented in just one single formula using temporal tools:

\[
\Box[\text{Ann believes that Bob assumes that} \quad (\Diamond \Box (\text{Ann believes that Bob’s assumption is wrong}))]
\]

The interpretation of above formula in English can be seen as: “Always it is the case that Ann believes that Bob assumes from the next time henceforth that Ann believes that Bob’s assumption is wrong”. We note that Ann and Bob in each state refer to their belief and assumptions in the next temporal states; not the state that they are in.

**Theorem 1.** In an interactive temporal assumption model \(W\), if \(\Box(A^{ab}U^b)\) is satisfiable, then

\[
\Box[B^{ab}A^{ba}(\Box \Diamond D)] \rightarrow \Box D
\]

is valid in \(W\).
Proof. Since $\Box (A^{ab} U^b)$ is satisfiable, there exist $k \in \mathbb{N}$ and $\bar{x} \in W$ for which

$$(k, \bar{x}) \models \Box (A^{ab} U^b).$$

This means that $\forall l \geq k \ (l, \bar{x}) \models A^{ab} U^b$. So, for all

$l \geq k \ (l, \bar{x}) \models U^a \land \forall y \ [P^a_l(\bar{x}, y) \land (l, y) \models U^b \leftrightarrow (l, y) \models U^b].$ Thus

$$\forall l \geq k \ (l, \bar{x}) \models U^a \land \forall y \ [P^a_l(\bar{x}, y)]. \quad (1)$$

Assume $(k, \bar{x}) \models \Box B^{ab} A^{ba}(\Box \Box D)$; we show that $(k, \bar{x}) \models \Box D$. To this end, for a moment, suppose that $(k, \bar{x}) \not\models \Box D$. So, there is $m \geq k$ for which $(m, \bar{x}) \models \neg D$. We can assume that $m = k$. Thus, $(k, \bar{x}) \models \neg D$ which means that $\exists \bar{y} \ [P^a_k(\bar{x}, \bar{y}) \land P^b_k(\bar{y}, \bar{x})]$. By (1) for $l = k$, $(k, \bar{x}) \models U^a \land \forall y \ [P^a_k(\bar{x}, y)]$. Since $(k, \bar{x}) \models \Box B^{ab} A^{ba}(\Box \Box D)$, then

$$\forall l \geq k \ (l, \bar{x}) \models B^{ab} A^{ba}(\Box \Box D)$$

$$\implies \forall l \geq k \ (l, \bar{x}) \models U^a \land \forall y \ [P^a_l(\bar{x}, y) \land (l, y) \models U^b \rightarrow (l, y) \models A^{ba}(\Box \Box D)]$$

$$\implies \forall l \geq k \ \forall y \ (l, y) \models A^{ba}(\Box \Box D)$$

$$\implies \forall l \geq k \ \forall y \ ((l, y) \models U^a \land \forall w \ [P^a_l(y, w) \land (l, w) \models U^a \leftrightarrow (l, w) \models \Box \Box D]).$$

Therefore, since $P^a_k(\bar{y}, \bar{x})$, one can conclude that $(k, \bar{x}) \models \Box \Box D$. So, we have $\forall l \geq k + 1 \ (k, \bar{x}) \models \Box \Box D$, thus $\forall l \geq k + 1 \ [P^a_l(\bar{x}, \bar{y}) \rightarrow \neg P^b_k(\bar{y}, \bar{x})]$. In particular, for $k + 1, P^a_{k+1}(\bar{x}, \bar{y}) \rightarrow \neg P^b_{k+1}(\bar{y}, \bar{x})$. By (1), since $P^a_{k+1}(\bar{x}, \bar{y})$ then

$\neg P^b_{k+1}(\bar{y}, \bar{x})$. On the other hand, since $(k + 1, \bar{x}) \models \Box \Box D$, then $P^b_{k+1}(\bar{y}, \bar{x})$ which is a contradiction! Therefore, $(k, \bar{x}) \models \Box D$ which shows that

$$\Box [B^{ab} A^{ba}(\Box \Box D)] \rightarrow \Box D$$

is valid in $W$. \hfill $\Box$

**Theorem 2.** In any interactive temporal assumption model $W$, the formula

$$\neg \Box [B^{ab} A^{ba}(U^a \land \Box \Box D)]$$

is valid.

**Proof.** If not, then $(k, \bar{x}) \not\models \Box [B^{ab} A^{ba}(\Box \Box D)]$ for some sate $(k, \bar{x})$. By Theorem 1 $(k, \bar{x}) \models \Box D$ so $\forall l \geq k, (l, \bar{x}) \models D$, in particular for $l = k$. Thus, $\forall y \ [P^a_k(\bar{x}, y) \rightarrow \neg P^b_k(\bar{y}, \bar{x})]$. On the other hand, $(k, \bar{x}) \models B^{ab} A^{ba} U^a$. By the definitions of $B^{ab}$ and $A^{ba}$ on $U^a$, there is some $\bar{y}$ such that the relation $[P^a_k(\bar{x}, \bar{y}) \land P^b_k(\bar{y}, \bar{x})]$ holds. So, $(k, \bar{x}) \models \neg D$ which is a contradiction! Therefore, $\neg \Box [B^{ab} A^{ba}(U^a \land \Box \Box D)]$ is valid in $W$. \hfill $\Box$
6 Conclusions

In game theory, the notion of a player’s beliefs about the game player’s beliefs about other players’ beliefs arises naturally. We presented a non-self-referential paradox in epistemic game theory which we called “Yablo-like Brandenburger-Keisler paradox”. Arising the paradox shows that completely modeling the players’ epistemic beliefs and assumptions is impossible. We formalized Yablo-like Brandenburger-Keisler paradox in the interactive temporal assumption logic and showed that there is no complete model for the set of all modal formulas built from $U^a, U^b$ and $D$.

References

[1] Abramsky, S. and J. Zvesper (2015). From Lawvere to Brandenburger-Keisler: interactive forms of diagonalization and self-reference. Journal of Computer and System Sciences 81, 799–812.

[2] Başkent, C. (2015). Some Non-classical Approaches to the Brandenburger-Keisler Paradox. Logic Journal of IGPL 23, 533–552.

[3] Beall, J.C. (2001). Is Yablo’s Paradox Non-Circular? Analysis 61, 176–187.

[4] Brandenburger, A. and H. Jerome Keisler (2006). An Impossibility Theorem on Beliefs in Games. Studia Logica 84, 211–240.

[5] Bringsjord, S. and B. van Heuveln (2003). The ‘Mental Eye’ Defence of an Infinitized Version of Yablo’s Paradox, Analysis 63, 61–70.

[6] Bueno, O. and M. Colyvan (2003). Yablo’s Paradox and Referring to Infinite Objects. Australasian Journal of Philosophy 81, 402–412.

[7] Bueno, O. and M. Colyvan (2003). Paradox Without Satisfaction. Analysis 63, 152–156.

[8] Cieśliński, C. and R. Urbaniak (2013). Gödelizing the Yablo Sequence. Journal of Philosophical Logic 42, 679–695.

[9] Karimi, A. and S. Salehi (2014). Diagonalizing by Fixed-Points. Preprint [arXiv:1303.0730]

[10] Karimi, A. and S. Salehi (2014). Theoremizing Yablo’s Paradox. Preprint [arXiv:1406.0134]

[11] Ketland, J. (2004). Bueno and Colyvan on Yablo’s paradox. Analysis 64, 165–172.
[12] Ketland, J. (2005). Yablo’s Paradox and $\omega$-Inconsistency. *Synthese* 145, 295–302.

[13] Lawvere, F.W. (1969). Diagonal Arguments and Cartesian Closed Categories. *Lecture Notes in Mathematics* 92, 134–145.

[14] Leach-Krouse, G. (2014). Yablifying the Rosser Sentence. *Journal of Philosophical Logic* 43, 827–834.

[15] Pacuit, E. (2007). Understanding the Brandenburger-Keisler Paradox. *Studia Logica* 86, 435–454.

[16] Priest, G. (1997). Yablo’s Paradox. *Analysis* 57, 236–242.

[17] Sorensen, R.A. (1998). Yablo’s Paradox and Kindred Infinite Liars. *Mind* 107, 137–155.

[18] Yablo, S. (1985). Truth and Reflection. *Journal of Philosophical Logic* 14, 297–349.

[19] Yablo, S. (1993). Paradox Without Self-Reference, *Analysis* 53, 251–252.

[20] Yablo, S. (2004). “Circularity and Paradox”. in: T. Bolander & V. F. Hendricks & S. A. Pedersen (eds.), *Self-Reference* CSLI Publications; 139–157.