Neutrino-nucleus reactions and effective field theory

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Abstract

Effective field theory is believed to provide a useful framework for describing low-energy nuclear phenomena in a model-independent fashion. I give here a brief account of the basic features of this approach, some of its latest developments, and examples of actual calculations carried out in this framework.

1 Introduction

Nuclear weak-interaction processes play important roles in many astrophysical phenomena and also in terrestrial experiments designed to detect astrophysical neutrinos. It is obviously desirable to have reliable estimates of the cross sections for these processes. I wish to describe here some of the recent developments in our endeavor to obtain such estimates. I limit myself here to nuclear weak processes involving relatively low energy-momentum, and, as far as calculational methods are concerned, I’ll talk about SNPA, EFT and EFT*. These terms may seem to have popped out of alphabet soup, but their meaning will become clear as we go along.

SNPA stands for the standard nuclear physics approach, which has been used extensively in describing a large class of nuclear properties; a brief recapitulation of SNPA will be given later. Recently there have been many important applications of effective field theory (EFT) to low-energy nuclear phenomena. I would like to survey some prominent features of nuclear EFT. A viewpoint that I believe worth advocating is that SNPA and EFT can play complementary roles. We have recently developed a version of EFT which allows us to take advantage of the merits of these two approaches. This new method, to be referred to as EFT*, will also be explained in some detail below.

As concrete examples of physical observables calculated in these methods, I consider the following three processes: (i) neutrino-deuteron reactions for solar neutrino energies; (ii) solar pp fusion; (iii) solar Hep fusion. I should explain why these processes are of particular current interest.

At SNO a 1-kiloton heavy water Cerenkov counter is used to detect the solar neutrinos. SNO can monitor the following neutrino-deuteron reactions

\[ \nu_e + d \rightarrow e^- + p + p, \quad \nu_x + d \rightarrow \nu_x + p + n, \quad \bar{\nu}_e + d \rightarrow e^+ + n + n, \quad \bar{\nu}_x + d \rightarrow \bar{\nu}_x + p + n, \]

and the pure leptonic reaction \( \nu_x + e^- \rightarrow \nu_x + e^- \). Here \( x \) stands for a neutrino of any flavor (e, \( \mu \) or \( \tau \)). The recent SNO experiments have established that the total solar neutrino flux (summed over all flavors) agrees with the prediction of the standard solar model, whereas the electron neutrino flux from the sun is significantly smaller than the total solar neutrino flux. The amount of deficit in the electron neutrino flux is consistent with what used to be known as the solar neutrino problem. These results of the SNO experiments have given “smoking-gun” evidence for the transmutation of...
solar electron neutrinos into neutrinos of other flavors. It is obvious that a precise knowledge of the \( \nu d \) reaction cross sections is of primary importance in interpreting the existing and future SNO data.

The pp fusion reaction

\[
p + p \rightarrow d + e^+ + \nu_e
\]  

is the most basic solar nuclear reaction that essentially controls the burning rate of the sun, and hence the exact value of its cross section is a crucial input for any further developments of solar models. Meanwhile, the Hep fusion reaction

\[
p + ^3\text{He} \rightarrow ^4\text{He} + e^+ + \nu_e
\]  

is important in a different context. Although the sun very rarely uses Hep to produce \(^4\text{He}\), Hep generates highest-energy solar neutrinos whose spectrum extends beyond the maximum energy of the \(^8\text{B}\) neutrinos. So, even though the flux of the Hep neutrinos is small, it can distort the higher end of the \(^8\text{B}\) neutrino spectrum [3], and this distortion can affect the interpretation of the results of a recent Super-Kamiokande experiment [4].

2 Calculational frameworks

2.1 Standard nuclear physics approach (SNPA)

As is well known, the phenomenological potential picture has been highly successful in describing many kinds of nuclear phenomena. In this picture an A-nucleon system is described by a Hamiltonian of the form

\[
H = \sum_i t_i + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \cdots,
\]

where \(t_i\) is the kinetic energy of the \(i\)-th nucleon, \(V_{ij}\) is a phenomenological two-body potential between the \(i\)-th and \(j\)-th nucleons, \(V_{ijk}\) is a phenomenological three-body potential, and so on. Since the interactions involving three or more nucleons are known to play much less important roles than the two-body interactions, we shall be mainly concerned with \(V_{ij}\). Once the Hamiltonian \(H\) is specified, the nuclear wave function \(|\Psi\rangle\) is obtained by solving the Shrödinger equation

\[
H|\Psi\rangle = E|\Psi\rangle.
\]

It is to be noted that the progress of numerical techniques for solving eq.5 has reached such a level [5] that the wave functions of low-lying levels for light nuclei can now be obtained practically with no approximation (once the validity of eq.[5] is accepted). This frees us from the “usual” nuclear physics complications that arise as a result of truncation of nuclear Hilbert space down to certain model space (such as shell-model configurations, cluster-model trial functions, etc.)

There is large freedom in choosing possible forms of \(V_{ij}\) apart from a well-established requirement that, as the inter-nucleon distance \(r_{ij}\) becomes sufficiently large, \(V_{ij}\) should approach the one-pion exchange Yukawa potential. For the model-dependent short-range part of \(V_{ij}\), one needs to assume certain functional forms and fix the parameters appearing therein by demanding that the solutions of eq.[5] for the \(A=2\) case reproduce the nucleon-nucleon scattering data (typically up to the pion-production threshold energy) as well as some of the deuteron properties. There are by now several so-called modern high-precision phenomenological N-N potential that can reproduce all the existing two-nucleon data with normalized \(\chi^2\) values close to 1. These potentials differ significantly in the ways they parametrize short-range physics, and, as a consequence, they exhibit substantial difference in their off-shell behavior. To what extent this arbitrariness may affect the observables of our concern will be discussed below.
In normal circumstances, nuclear responses to external electroweak probes are given, to good approximation, by one-body terms; these are also called the impulse approximation (IA) terms. To obtain higher accuracy, however, one must also consider exchange current (EXC) terms, which represent the contributions of nuclear responses involving two or more nucleons. In particular, if for some reason the IA contributions are suppressed, then it becomes essential to take account of the EXC contributions. These exchange currents (usually taken to be two-body operators) are derived from one-boson exchange diagrams, and the vertices featuring in the relevant diagrams are determined to satisfy the low-energy theorems and current algebra [6]. We refer to a formalism based on this picture as the **standard nuclear physics approach** (SNPA). (This is also called a potential model in the literature.) Schematically, the nuclear matrix element in SNPA is given by

\[
\mathcal{M}^{SNPA}_{fi} = \langle \Psi_f | \sum_{\ell} O_{\ell} + \sum_{\ell<m} O_{\ell m} | \Psi_i \rangle ,
\]

where the initial (final) nuclear wave function, \( \Psi_i \) (\( \Psi_f \)), is a solution of eq.(5), and \( O_{\ell} \) and \( O_{\ell m} \) are, respectively, the one-body and two-body transition operators for a given electroweak process.

SNPA has been used extensively for describing nuclear electroweak processes in light nuclei, and general good agreement found between theory and experiment [5] gives a strong indication that SNPA practically captures much of the physics involved.

### 2.2 Effective field theory (EFT)

Although SNPA has been scoring undeniable successes in correlating and explaining a vast variety of data, it is still important from a formal point of view to raise the following issues. First, since hadrons and hadronic systems (such as nuclei) are governed by quantum chromodynamics (QCD), one should ultimately be able to relate SNPA with QCD, but this relation has not been established. In particular, while chiral symmetry is known to be a fundamental symmetry of QCD, the formulation of SNPA is largely disjoint from this symmetry. Secondly, in SNPA, even for describing low-energy phenomena, we start with a “realistic” phenomenological potential which is tailored to encode short-range (high-momentum) and long-range (low-momentum) physics simultaneously. This mixing of the two different scales seems theoretically dissatisfactory and can be pragmatically inconvenient. Thirdly, in writing down a phenomenological Lagrangian for describing the nuclear interaction and nuclear responses to the electroweak currents, SNPA is not equipped with a clear guiding principle; it is not clear whether there is any identifiable expansion parameter that helps us to control the possible forms of terms in the Lagrangian and that provides a general measure of errors in our calculation. To address these and other related issues, a new approach based on EFT was proposed [7] and it has been studied with great intensity; for reviews, see, [8]-[12].

The general idea of EFT is in fact very simple. In describing phenomena characterized by a typical energy-momentum scale \( Q \), we expect that we need not include in our Lagrangian those degrees of freedom that pertain to energy-momentum scales much higher than \( Q \). This expectation motivates us to introduce a cut-off scale \( \Lambda \) that is sufficiently larger than \( Q \) and we classify our fields (to be generically represented by \( \phi \)) into two groups: high-frequency fields \( \phi_H \) and low-frequency fields \( \phi_L \). By eliminating (or integrating out) \( \phi_H \), we arrive at an effective Lagrangian that only involves \( \phi_L \) as explicit dynamical variables. Using the notion of path integrals, the effective Lagrangian \( \mathcal{L}_{\text{eff}} \) is related to the original Lagrangian \( \mathcal{L} \) as

\[
\int [d\phi] e^{i \int d^4x \mathcal{L}(\phi)} = \int [d\phi_L] e^{i \int d^4x \mathcal{L}_{\text{eff}}(\phi_L)} .
\]
all possible monomials of \( \phi_L \) and their derivatives that are consistent with the symmetry requirements dictated by \( L \). Since a term involving \( n \) derivatives scales like \((Q/\Lambda)^n\), the terms in \( \mathcal{L}_{\text{eff}} \) can be organized into a perturbative series in which \( Q/\Lambda \) serves as an expansion parameter. The coefficients of terms in this expansion scheme are called the low-energy constants (LECs). Insofar as all the LEC’s up to a specified order \( n \) can be fixed either from theory or from fitting to the experimental values of the relevant observables, \( \mathcal{L}_{\text{eff}} \) serves as a complete (and hence model-independent) Lagrangian to the given order of expansion.

Having outlined the basic idea of EFT, we now discuss specific aspects of EFT as applied to nuclear physics. The underlying Lagrangian \( L \) in this case is the QCD Lagrangian \( \mathcal{L}_{\text{QCD}} \), whereas, for the typical nuclear physics energy-momentum scale \( Q \ll \Lambda_\chi \sim 1 \text{ GeV} \), the effective degrees of freedom that would feature in \( \mathcal{L}_{\text{eff}} \) are hadrons rather than the quarks and gluons. It is a non-trivial task to apply the formal definition in eq.(7) to derive \( \mathcal{L}_{\text{eff}} \) written in terms of hadrons starting from \( \mathcal{L}_{\text{QCD}} \); the hadrons cannot be straightforwardly identified with the low-frequency field, \( \phi_L \) in eq.(7), in the original Lagrangian. At present, the best one could do is to resort to symmetry considerations and the above-mentioned expansion scheme. Here chiral symmetry plays an important role. We know that chiral symmetry is spontaneously broken, generating the pions as Nambu-Goldstone bosons;\(^2\) or chiral symmetry is realized in the Goldstone mode. This feature can be incorporated by assigning suitable chiral transformation properties to the Goldstone bosons and writing down all possible chiral-invariant terms up to a specified chiral order \( \nu \). The above consideration presupposes exact chiral symmetry in \( \mathcal{L}_{\text{QCD}} \). In reality, \( \mathcal{L}_{\text{QCD}} \) contains small but finite quark mass terms, which explicitly violate chiral symmetry and lead to a non-vanishing value of the pion mass \( m_\pi \). Again, there is a well-defined method to determine what terms are needed in the Goldstone boson sector to represent the effect of explicit chiral symmetry breaking \( \nu \). These considerations lead to an EFT called chiral perturbation theory (\( \chi \text{PT} \)) \(^{14, 15} \). The successes of \( \chi \text{PT} \) in the meson sector are well known; see, \( \text{e.g.}, \) \(^8 \).

A problem we encounter in extending \( \chi \text{PT} \) to the nucleon sector is that, as the nucleon mass \( m_N \) is comparable to the cut-off scale \( \Lambda_\chi \), a simple application of expansion in \( Q/\Lambda \) does not work. This problem can be circumvented by employing heavy-baryon chiral perturbation theory (HB\( \chi \text{PT} \)), which essentially consists in shifting the reference point of the nucleon energy from 0 to \( m_N \) and in integrating out the small component of the nucleon field as well as the anti-nucleonic degrees of freedom. An effective Lagrangian in HB\( \chi \text{PT} \) therefore involves as explicit degrees of freedom the pions and the large components of the redefined nucleon field. HB\( \chi \text{PT} \) has as expansion parameters \( Q/\Lambda_\chi \), \( m_\pi/\Lambda_\chi \) and \( Q/m_N \). Since \( m_N \approx \Lambda_\chi \), it is convenient to combine chiral and heavy-baryon expansions and introduce the chiral index \( \bar{\nu} \) defined by \( \bar{\nu} = d + (n/2) - 2 \). Here \( n \) is the number of fermion lines that participate in a given vertex, and \( d \) is the number of derivatives (with \( m_\pi \) counted as one derivative). A similar power counting scheme can also be introduced for Feynman diagrams as well. According to Weinberg \(^7 \), the contribution of a Feynman diagram that contains \( N_A \) nucleons, \( N_E \) external fields, \( L \) loops and \( N_C \) disjoint parts can be shown to scale like \((Q/\Lambda)^\nu\), where the chiral index \( \nu \) is defined as

\[
\nu = 2L + 2(N_C - 1) + 2 - (N_A + N_E) + \sum_i \bar{\nu}_i, \tag{8}
\]

with the summation running over all the vertices contained in the Feynman diagram. HB\( \chi \text{PT} \) has been used with great success to the one-nucleon sector \(^8 \).

However, HB\( \chi \text{PT} \) cannot be applied in a straightforward manner to nuclei that contain more than one nucleon. The reason is that nuclei involve very low-lying excited states, and the existence of this small energy scale upsets the original counting rule \(^7 \). This is analogous to a problem one encounters in ordinary quantum mechanics when a system allows for low-lying intermediate states that spoil

\(^2\)We limit ourselves here to SU(2)×SU(2) chiral symmetry.
perturbation expansion. Weinberg proposed to avoid this difficulty as follows. Classify Feynman diagrams into two groups, irreducible and reducible diagrams. Irreducible diagrams are those in which every intermediate state has at least one meson in flight; all others are classified as reducible diagrams. We then apply the above-mentioned chiral counting rules only to irreducible diagrams. The contribution of all the two-body irreducible diagrams (up to a specified chiral order) is treated as an effective potential (to be denoted by $V_{ij}^{\text{EFT}}$) acting on nuclear wave functions. Meanwhile, the contributions of reducible diagrams can be incorporated by solving the Schrödinger equation

$$H_{\text{EFT}}^\Psi >_{\text{EFT}} = E >_{\text{EFT}},$$

where

$$H_{\text{EFT}} = \sum_i A t_i + \sum_{i<j} A V_{ij}^{\text{EFT}},$$

We refer to this two-step procedure as nuclear $\chi$PT, or, to be more specific, nuclear $\chi$PT in the Weinberg scheme.\(^3\)

To apply nuclear $\chi$PT to a process that involves (an) external current(s), we derive a nuclear transition operator $T$ by evaluating the complete set of all the irreducible diagrams (up to a given chiral order $\nu$) involving the relevant external current(s). To preserve consistency in chiral counting, the nuclear matrix element of $T$ must be calculated with the use of nuclear wave functions which are governed by nuclear interactions that represent all the irreducible A-nucleon diagrams up to $\nu$-th order. Thus, a transition matrix in nuclear EFT is given by

$$M_{f_i}^{\text{EFT}} = <\Psi_f^{\text{EFT}}|\sum_i^A O_i^{\text{EFT}} + \sum_{\ell<m}^A O_{\ell m}^{\text{EFT}}|\Psi_i^{\text{EFT}}>,$$

where the superscript, “EFT”, means that the relevant quantities are obtained according to EFT as described above. If this program is carried out exactly, it would constitute an ab initio calculation. We note that in EFT we know exactly at what chiral order three-body operators start to contribute to $T$, and that, to chiral orders relevant to the applications described below, there is no need for three-body operators. With this understanding, we have retained only one- and two-body operators in eq.(11). This unambiguous classification of transition operators according to their chiral orders is a great advantage of EFT, which is missing in eq.(6).

I should point out that there exists an alternative form of nuclear EFT based the power divergence subtraction (PDS) scheme. The PDS scheme proposed by Kaplan, Savage and Wise in their seminal papers\(^9\) uses a counting scheme (often called Q-counting) that is different from the Weinberg scheme. An advantage of the PDS scheme is that it maintains formal chiral invariance, whereas the Weinberg scheme loses manifest chiral invariance. In many practical applications, however, this formal problem is not worrisome up to the chiral order of our concern, viz., the chiral order up to which our irreducible diagrams are to be evaluated. Although many important results have been obtained in the PDS scheme (for a review, see e.g.\(^11\)), I concentrate here on the Weinberg scheme, as this is a framework in which our own work has been done.

I also remark that, if we are interested in low-energy nuclear phenomena the typical energy-momentum scale of which is $Q \ll m_\pi$, even the pions may be regarded as “heavy” particles and can be eliminated from $L_{\text{eff}}$.

### 2.3 Hybrid EFT

In the preceding subsection we emphasized the formal merits of nuclear EFT. In actual calculations, however, the following two aspects need to be considered. First, it is still a big challenge to generate,\(^3\) This is often called the $\Lambda$-counting scheme\(^10\).
strictly within the EFT framework, nuclear wave functions the accuracy of which is comparable to that of SNPA wave functions. Secondly, as mentioned earlier, the chiral Lagrangian, $L_{\text{eff}}$, is definite only when the values of all the relevant LECs are fixed, but there may be cases where this condition cannot be readily met. A pragmatic solution to the first problem is to use in eq. (11) wave functions obtained in SNPA; we refer to this eclectic approach as hybrid EFT. Thus a nuclear transition matrix element in hybrid EFT is given by

$$M_{fi}^{\text{hyb-EFT}} = \langle \Psi_{SNPA}^{f} | \sum_{\ell} O_{\ell}^{\text{EFT}} + \sum_{\ell<m} O_{\ell m}^{\text{EFT}} | \Psi_{SNPA}^{i} \rangle ,$$

Since, as mentioned, the NN interactions that generate SNPA wave functions reproduce accurately the entirety of the two-nucleon data, the adoption of eq. (12) is almost equivalent to using the empirical data themselves to control the initial and final nuclear wave functions. In the purely theoretical context of deriving the nuclear interactions based on EFT, hybrid EFT may be deemed as a “regression”. But, if our goal is to obtain a transition matrix element as accurately as possible with the maximum help of available empirical input, then hybrid EFT does have a legitimate status, so long as the afore-mentioned off-shell problem and the contributions of three-body (and higher-body) interactions are properly addressed. These two points will be discussed later in this talk.

The calculations reported in Refs. [20, 21] seem to render support for hybrid EFT. There, the nuclear matrix elements in the $A=2$ systems for one-body operators (or IA terms) calculated with the use of EFT-generated wave functions were found to be very close to those calculated with the SNPA wave functions. Thus EFT and hybrid EFT should give practically the same IA matrix elements. Meanwhile, it is generally expected that the ratio of the two-body EXC contributions to those of the IA operators should be much less sensitive to the details of the nuclear wave functions than the absolute values are. It therefore seems reasonable to rely on $\chi$PT for deriving transition operators and evaluate their matrix elements using the realistic wave functions obtained in SNPA, and in this sense hybrid EFT is more than a mere expedient.

The issue of possible unknown LECs will be discussed in connection with EFT* in the next subsection.

### 2.4 EFT*

Hybrid EFT can be used for complex nuclei ($A = 3, 4, ...$) with essentially the same accuracy and ease as for the $A=2$ system.\(^4\) We should reemphasize in this connection that, in $A$-nucleon systems ($A \geq 3$), the contributions of transition operators involving three or more nucleons are intrinsically suppressed according to chiral counting, and hence, up to a certain chiral order, a transition operator in an $A$-nucleon system consists of the same EFT-based 1-body and 2-body terms as used for the two-nucleon system. Then, since SNPA provides high-quality wave functions for the $A$-nucleon system, one can calculate $M_{fi}^{\text{hyb-EFT}}$ with precision comparable to that for the corresponding two-nucleon case.

Now, in most practical cases, the one-body operator, $O_{\ell}^{\text{EFT}}$, is free from unknown LECs. So let us concentrate on the two-body operator, $O_{\ell m}^{\text{EFT}}$, and suppose that $O_{\ell m}^{\text{EFT}}$ under consideration contains an LEC (call it $\kappa$) that cannot be determined with the use of $A=2$ data alone. It is possible that an observable (call it $\Omega$) in a $A$-body system ($A \geq 3$) is sensitive to $\kappa$ and that the experimental value of $\Omega$ is known with sufficient accuracy. Then we can determine $\kappa$ by calculating $M_{fi}^{\text{hyb-EFT}}$ responsible for $\Omega$ and adjusting $\kappa$ to reproduce the empirical value of $\Omega$. Once $\kappa$ is fixed this way, we can make

\(^4\)Here I am ignoring “purely technical” complications that can grow in actual numerical calculations for higher-$A$ systems.
predictions for any other observables for any other nuclear systems that are controlled by the same transition operators. When hybrid EFT is used in this manner, we refer to it as EFT*.

EFT* is the most efficient existing formalism for correlating various observables in different nuclei, using the transition operators controlled by EFT. A further notable advantage of EFT* is that, since correlating the observables in neighboring nuclei is likely to serve as an additional renormalization, the possible effects of higher chiral order terms and/or off-shell ambiguities can be significantly suppressed by the use of EFT*.

I will come back to this point later, when we discuss concrete examples.

3 Numerical results

We now discuss the applications of the above-described calculational methods to the three processes of our concern: pp fusion, Hep fusion, and the ν-d reaction. A common feature of these reactions is that a precise knowledge of the Gamow-Teller (GT) transition matrix elements is crucial in estimating their cross sections. We therefore concentrate on the GT transitions. I will show here, following Refs. [24, 25, 26], that the idea of EFT* can be used very nicely for this group of reactions.

We can argue (see, e.g., [26]) that 1-body IA operators for the GT transition can be fixed unambiguously from the available 1-body data. As for the 2-body operators, to next-to-next-to-next-to-leading order (N^3LO) in chiral counting, there appears one unknown LEC that cannot be at present determined from data for the A=2 systems. This unknown LEC, denoted by \( \hat{d}_R \) in [19], parametrizes the strength of contact-type four-nucleon coupling to the axial current. Park et al. [24, 25, 26] noticed that the same LEC, \( \hat{d}_R \), also features as the only unknown parameter in the calculation of the tritium β-decay rate \( \Gamma_{\beta} \), and they proposed to use EFT* to place a constraint on \( \hat{d}_R \) from the experimental information on \( \Gamma_{\beta} \). Since the empirical value of \( \Gamma_{\beta} \) is known with high precision, and since the accurate wave functions of \(^3\)H and \(^3\)He are available from a well-developed variational calculation in SNPA [27], we can determine \( \hat{d}_R \) with sufficient accuracy for our purposes. Once the value of \( \hat{d}_R \) is determined this way, we can carry out parameter-free EFT* calculations for pp-fusion [24, 26], Hep fusion [25, 26], and the ν-d reactions [29]. I present here a brief summary of the results of these calculations.

Before doing that, we need to discuss the important role of momentum cutoff in EFT. As emphasized before, the effective Lagrangian \( L_{\text{eff}} \) is, by construction, valid only below the specified cutoff scale \( \Lambda \). Needless to say, this basic constraint should be respected in our nuclear EFT calculations, and for that we must make sure that nuclear intermediate states involved in the computation of eq. (11) do not get out of this constrained world. It is reasonable to implement this constraint by requiring that the two-nucleon relative momentum should be smaller than \( \Lambda \); Park et al. used a Gaussian cutoff function proportional to \( \exp(-\vec{p}^2/\Lambda^2) \) but its detailed form should not be too important. As a reasonable range of the value of \( \Lambda \) we may choose: 500 MeV ≤ \( \Lambda \) ≤ 800 MeV, where the lower bound is dictated by the requirement that \( \Lambda \) should be sufficiently larger than the pion mass (to fully accommodate pion physics), while the upper bound reflects the fact that our EFT is devoid of the ρ meson.

For a given value of \( \Lambda \) within the above range, \( \hat{d}_R \) is tuned to reproduce \( \Gamma_{\beta} \), and then the cross sections for pp-fusion, Hep fusion and the νd reactions are calculated. Before giving a (brief) account of the individual results, I should point out a notable common feature. Although the optimal value of \( \hat{d}_R \) varies significantly as a function of \( \Lambda \), the observables (in our case the above three reaction cross sections) exhibit remarkable stability against the variation of \( \Lambda \) (within the above-discussed physically reasonable range). This stability may be taken as an indication that the use of EFT* for

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5It is also called more effective effective field theory (MEEFT) [20, 21, 24].

6EFT* should be distinguished from an earlier naive hybrid EFT model wherein the short-range terms were dropped altogether using an intuitive argument based on short-range NN repulsion.
inter-correlating the observables in neighboring nuclei effectively renormalizes various effects, such as
the contributions of higher-chiral order terms, mismatch between the SNPA and EFT wave functions,
etc. This stability is essential in order for EFT$^*$ to maintain its predictive power.

Park et al. [24, 26] used this EFT$^*$ method to calculate the rate of pp fusion, $pp \rightarrow e^+\nu_e d$. The
result expressed in terms of the threshold $S$-factor is

$$S_{pp}(0) = 3.94 \times (1 \pm 0.005) \times 10^{-25} \text{ MeV b}. \quad (13)$$

It has been found that $S_{pp}(0)$ changes only by $\sim 0.1\%$ against changes in $\Lambda$, assuring thereby
the robustness of the prediction provided by EFT$^*$. The EFT$^*$ result, eq. (13), is consistent with that obtained
in SNPA by Schiavilla et al. [28]. Meanwhile, the fact that EFT$^*$ allows us to make an error estimate [as given in eq. (13)] is a notable advantage over SNPA. The details on how we arrive at this error estimate can be found in [24, 26]. Here I just remark that the error indicated in eq. (13) represents an improvement by a factor of $\sim 10$ over the previous results based on a simple hybrid EFT [19].

We now discuss the application of the same EFT$^*$ method to the Hep fusion reaction, $^3\text{He} \rightarrow e^+\nu_e^4\text{He}$ [25, 26]. An accurate estimation of this cross section is a particularly challenging task because: (1) the contribution of the leading-order 1-body GT operator is highly suppressed due to the approximate wave function orthogonality, and (2) there is a strong cancellation between the 1-body and 2-body GT matrix elements [33, 27]. Park et al.’s EFT$^*$ calculation [24, 26] give for the threshold $S$-factor

$$S_{Hep}(0) = (8.6 \pm 1.3) \times 10^{-20} \text{ keV b}, \quad (14)$$

where the error spans the range of the $\Lambda$ dependence for $\Lambda = 500$-800 MeV. Again, the EFT$^*$ result agrees with that obtained in SNPA by Marcucci et al. [27]: $S_{Hep}(0) = 9.64 \times 10^{-20}$ keV b. The above-mentioned large cancellation between the 1-body and 2-body contributions in this case amplifies the cutoff dependence of $S_{Hep}(0)$, but the error quoted in eq. (14) is still small enough for the purpose of analyzing the existing Super-Kamiokande data [4].

We now move to the $\nu$-$d$ reactions, eq. (11). It is useful to give a short but general survey of all the recent results obtained in SNPA, EFT and EFT$^*$, and that’s what I am going to do here. Within SNPA a detailed calculation of the $\nu$-$d$ cross sections, $\sigma(\nu d)$, was carried out by Nakamura, Sato, Gudkov and myself [30, 7] and this calculation was recently updated by Nakamura et al. (NETAL) [32]. As demonstrated in Ref. [33], the SNPA exchange currents for the GT transition are dominated by the $\Delta$-particle excitation diagram, and the reliability of estimation of this diagram depends on the precision with which the coupling constant $g_{\pi N \Delta}$ is known. NETAL fixed $g_{\pi N \Delta}$ by fitting the experimental value of $\Gamma^\beta_{3z}$, the tritium $\beta$-decay rate, and proceeded to calculate $\sigma(\nu d)$. Meanwhile, Butler, Chen and Kong (BCK) [34] carried out an EFT calculation of the $\nu$-$d$ cross sections, using the PDS scheme [16]. The results obtained by BCK agree with those of NETAL in the following sense. BCK’s calculation involves one unknown LEC (denoted by $L_{1A}$), which like $d_R$ in Ref. [26], represents the strength of a four-nucleon axial-current coupling term. BCK therefore determined $L_{1A}$ by requiring that the $\nu d$ cross sections of NETAL be reproduced by their EFT calculation. With the value of $L_{1A}$ fine-tuned this way, the $\sigma(\nu d)$’s obtained by BCK show a perfect agreement with those of NETAL for all the four reactions in eq. (11) and for the entire solar neutrino energy range, $E_\nu \lesssim 20$ MeV. Moreover, the optimal value, $L_{1A} = 5.6 \text{ fm}^3$, found by BCK [34] is consistent with the order of magnitude of $L_{1A}$ expected from the naturalness argument (based on a dimensional analysis), $|L_{1A}| \lesssim 6 \text{ fm}^3$. The fact that an EFT calculation (with one parameter fine-tuned) reproduces the results of SNPA very well strongly suggests the robustness of the SNPA results for $\sigma(\nu d)$.

Even though it is reassuring that the $\nu$-$d$ cross sections calculated in SNPA and EFT agree with each other (in the sense explained above), it is desirable to carry out an EFT calculation that is free

\footnote{For a review of the earlier SNPA calculations, see [31].}
from any adjustable LEC. Fortunately, EFT* allows us to carry out an EFT-controlled parameter-free calculation of the $\nu$-d cross sections, and such a calculation was carried out by Ando et al. [29]. The $\sigma(\nu d)$'s obtained in [29] are found to agree within 1% with $\sigma(\nu d)$'s obtained by NETAL using SNPA [32]. These results show that the $\nu$-d cross sections used in interpreting the SNO experiments [1] are reliable at the 1% precision level, and hence the evidence for neutrino oscillations reported in those experiments is robust against nuclear physics ambiguities.

We note that, as PDS [16] is built on an expansion scheme for transition amplitudes themselves, it does not employ the concept of wave functions. This feature may be an advantage in some contexts, but its disadvantage in the present context is that one cannot readily relate the transition matrix elements for an A-nucleon system with those for the neighboring nuclei; in PDS, each nuclear system requires a separate parametrization. This feature underlies the fact that, in the work of BCK [34], $L_{1A}$ remained undetermined, as no experimental data is available to fix $L_{1A}$ within the two-nucleon systems.

Although the determination of $\hat{d}_R$ from $\Gamma_\beta$ should be good enough for all practical purposes, it is worthwhile to study a possibility to fix $\hat{d}_R$ with the use of an observable belonging to the A=2 systems. A promising candidate is the $\mu$-capture process, $\mu^- + d \to \nu_\mu + n + n$. Although rather large energy-momentum transfers involved in the disappearance of a $\mu^-$ seem to make the applicability of EFT here a delicate issue, we can show that, as far as the hadron sector is concerned, $\mu$-d capture is in fact a reasonably “gentle” process. This is because: (1) the $\nu_\mu$ carries away most of the energy, and (2) there is a large enhancement of the transition amplitude in a kinematic region where the relative motion of the final two nucleons is low enough to justify the use of EFT. According to Ando et al.’s recent study [35], $\mu$-d capture can be useful for controlling $\hat{d}_R$, if the quality of experimental data improves sufficiently. We note that an experiment to measure the $\mu d$ capture rate with 1% precision is planned at the PSI [36].

4 Discussion

In introducing hybrid EFT, we have replaced $|\Psi >^{EFT}$ for the initial and final nuclear states in eq.(11) with the corresponding $|\Psi >^{SNPA}$; see eq.(12). This replacement may bring in a certain degree of model dependence, called the off-shell effect, because the phenomenological NN interactions are constrained only by the on-shell two-nucleon observables.\footnote{In a consistent theory, physical observables are independent of field transformations that lead to different off-shell behaviors, and therefore the so-called off-shell effect is not really a physical effect. In an approximate theory, observables may exhibit superficial dependence on off-shell behavior, and it is customary to refer to this dependence as an off-shell effect.} This off-shell effect, however, is expected to be small for the reactions under consideration, since they involve low momentum transfers and hence are not extremely sensitive to the short-range behavior of the nuclear wave functions. One way to quantify this expectation is to compare a two-nucleon relative wave function generated by the phenomenological potential with that generated by an EFT-motivated potential. Phillips and Cohen [21] made such a comparison in their analysis of the 1-body operators responsible for electron-deuteron Compton scattering, and showed that a hybrid EFT should work well up to momentum transfer 700 MeV. A similar conclusion is expected to hold for a two-body operator, so long as its radial behavior is duly “smeared-out” reflecting a finite momentum cutoff. Thus, hybrid EFT as applied to low energy phenomena is expected to be practically free from the off-shell ambiguities. The off-shell effect should be even less significant in EFT*, wherein an additional “effective” renormalization is likely to be at work (see subsection 2.4).

I now wish to discuss briefly another very interesting development, due to Tom Kuo and his colleagues [37], which can shed much light on the reliability of a hybrid EFT or EFT* calculation.
As mentioned, a “realistic phenomenological” nuclear interaction, $V_{ij}$ in eq.(11), is determined by solving the Schrödinger equation, eq.(5), for the $A=2$ system and fitting the results to the full set of two-nucleon data up to the pion production threshold energy. So, physically, $V_{ij}$ should reside in a momentum regime below a certain cutoff, $\Lambda_c$. In the conventional treatment, however, the existence of this cutoff scale is ignored, and eq.(5) is solved, allowing the entire momentum range to participate. Kuo et al. proposed to construct an effective low-momentum potential $V_{low-k}$ by eliminating (or integrating out) from $V_{ij}$ the momentum components higher than $\Lambda_c$, and calculated $V_{low-k}$’s corresponding to many well-established examples of $V_{ij}$’s. Remarkably, it was found that all these $V_{low-k}$’s give identical results for the half-off-shell T-matrices, even though the ways short-range physics is encoded in these $V_{ij}$’s are highly diverse. This implies that the $V_{low-k}$’s are free from the off-shell ambiguities, and therefore the use of $V_{low-k}$’s is essentially equivalent to employing $V_{ij}^{EFT}$ (that appeared in eq.(10)), which by construction should be model-independent. Now, as mentioned, our EFT* calculation has a momentum-cutoff regulator, and this essentially ensures that the matrix element, $\mathcal{M}_{ji}^{hyb-EFT}$, in eq.(12) is only sensitive to the half-off-shell T-matrices that are controlled by $V_{low-k}$ instead of $V_{ij}$. Therefore, we can expect that the EFT* results reported here are essentially free from the off-shell ambiguities.

It is also worth noting that the calculation of the cross section for “Hen”, $^3\text{He} + n \rightarrow ^4\text{He} + \gamma$, should provide a further check of the reliability of EFT*. Such a calculation is being done by T.-S. Park and Y.H. Song [39].

My final remark in this section is concerned with the LECs, $L_{1A}$ and $\tilde{d}_R$. Chen, Heeger and Robertson [40] have recently reported that a “self-calibrating” analysis of the SNO data allows one to place a model-independent constraint on $L_{1A}$. The $\sigma(\nu d)$’s corresponding to the range of $L_{1A}$ obtained in this analysis are consistent with $\sigma(\nu d)$’s obtained in SNPA and EFT*. However, although the method used in this self-calibrating analysis is very beautiful, the resulting constraint on $L_{1A}$ is still rather loose. From a comparison of the $\sigma(\nu d)$’s calculated in SNPA and the cross sections measured in a reactor anti-neutrino experiment [11], we have known for some time that the theoretical values cannot be off by more than 10%. The results of the “self-calibrating” analysis at present do not provide much improvement over this well-known empirical upper limit of errors.

I have mentioned that both $L_{1A}$ and $\tilde{d}_R$ represent the strength of axial-current-four-nucleon contact coupling. It is to be noted, however, that $L_{1A}$ belongs to pion-less EFT, while $\tilde{d}_R$ to pion-ful EFT. In the pion-ful EFT, because of the strong tensor force, the exchange current involving the deuteron $d$-state is important, and the $s$-wave exchange current arising from the $\tilde{d}_R$ term is separate from this tensor-force effect. By contrast, in the pion-less EFT, the explicit $d$-wave term is a higher-order correction, and hence the $s$-wave $L_{1A}$ term must subsume the strong tensor-force contributions. It would be illuminating to investigate the relation between $L_{1A}$ and $\tilde{d}_R$ from this perspective. Such a study is currently underway [38].

5 Summary

After giving a very limited survey of the current status of nuclear $\chi$PT, I must repeat my disclaimer that I have left out many important topics belonging to nuclear $\chi$PT. Among others, I did not discuss very important studies by Epelbaum, Glöckle and Meißen [12] to construct a formally consistent framework for applying $\chi$PT to complex nuclei ($A = 3, 4, ...$). It should be highly informative to apply this type of formalism to electroweak processes and compare the results with those of EFT*.

Despite the highly limited scope of topics covered, I hope I have succeeded in conveying the message that EFT* is a reliable framework for computing transition amplitudes for a large class of electroweak processes in light nuclei. I also wish to emphasize that, in each of the cases for which both SNPA and EFT* calculations have been performed, it has been found that the result of EFT* supports and
improves the SNPA result.

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