Cosmic Ray Momentum Diffusion in Magnetosonic versus Alfvénic Turbulent Fields

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Abstract. The acceleration of energetic particle transport in high amplitude magnetosonic and Alfvénic turbulence is considered using the Monte Carlo particle simulations which involve integration of particle equations of motion. We derive the momentum diffusion coefficient \( D_p \) in the presence of anisotropic turbulent wave fields in the low-\( \beta \) plasma, for a flat and a Kolmogorov-type turbulence spectrum. We confirm the quasilinear result (cf. Schlickeiser & Miller (1997)) of enhanced values of \( D_p \) due to transit-time damping resonance interaction in the presence of isotropic fast-mode waves as compared to the case of slab Alfvén waves of the same amplitude. The relation of \( D_p \) to the turbulence amplitude and anisotropy is investigated.

Key words: cosmic rays – magnetohydrodynamic turbulence – interstellar medium – Fermi acceleration

1. Introduction

Many astronomical objects (extragalactic radio sources, supernova remnants, solar flares) emit radiation with non-thermal spectra. These emissions are often connected with the existence of a hot turbulent magnetized plasma providing conditions for particle acceleration by MHD turbulence. It was first shown by Hall and Sturrock (1967) and by Kulsrud and Ferrari (1971) that charged particles can be accelerated by MHD turbulence having wavelengths long compared to the particle gyration radius. For example, the importance of a Fermi-like acceleration mechanism was proved for extragalactic radio sources (Burn 1975; De Young 1976; Blandford and Rees 1978; Achterberg 1979 and Eilek 1979) and for the second phase acceleration in solar flares (Melrose 1974; Ramaty 1979). The interaction between the waves and particles is determined primarily by resonances \( \omega - k_{\parallel} v_{\parallel} = n \Omega \) between particles characterized by their parallel velocity \( v_{\parallel} \) and gyrofrequency \( \Omega \) and undamped waves characterized by their frequency \( \omega \) and wavenumber \( k_{\parallel} \). At such interactions energies vary in a diffusive way, and over a long timescale a net energy gain results in the process of stochastic acceleration. For \( n \neq 0 \), a particle in its parallel rest frame \(( v_{\parallel} = 0) \) resonates with those waves that it sees at an integral multiple of a gyrofrequency and for \( n = 0 \) at zero frequency. Then, a particle feels a net force which is not averaged out by the phase mixing (Stix 1962). It is a magnetic analogy of the Landau damping and is associated with the first-order change in \( \delta |B| \). This type of coupling is not observed in the case of Alfvén waves but becomes important for the magnetosonic waves containing compressive components of \( \delta B \). The interaction between the particle magnetic momentum and the parallel gradient of the magnetic field is called transit-time damping (cf. discussions by Lee and Völk 1975, Achterberg 1981, Miller et al. 1996). Recently, Schlickeiser & Miller (1997) presented a quasilinear derivation of cosmic ray transport coefficients in the presence of MHD waves, including the isotropic fast-mode turbulence. For the isotropic Kolmogorov turbulence they demonstrated that the Fokker-Planck coefficients depend both on the transit-time damping and the gyro-resonance interactions. For cosmic ray particles with \( v \gg V_A \) and a vanishing turbulence cross helicity the momentum diffusion coefficient in the fast-mode turbulence is mainly determined by the transit-time damping contribution, leading to a more effective stochastic acceleration in comparison to the process in the presence of the pure Alfvénic turbulence.

The aim of the present paper is to study the momentum diffusion coefficient \( D_p \) in the presence of non-linear \(( \delta B \geq B_0) \) magnetosonic and Alfvén waves where the quasilinear approximations do not apply. We also study the influence of the degree of wave propagation anisotropy and of the wave spectral slope on the value of \( D_p \). To consider these problems the Monte Carlo simulations in-

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\textsuperscript{1} The notation is explained in Appendix A
volving derivations of particles trajectories in the space filled with finite amplitude fast-mode and Alfvén waves are applied. Anisotropic wave distributions are modeled by choosing their wave vectors from finite opening cones directed along the mean magnetic field $B_\parallel$. For our simulations we adopt fast-mode or Alfvén mode turbulence with flat ($q = 1$) and Kolmogorov-type ($q = 5/3$) power law spectrum $\propto k^{-q}$ above the minimum wavenumber $k_{\text{min}}$. We confirm a substantial increase of $D_p$ for the fast-mode waves in comparison to the Alfvén waves of the same amplitude if the nearly perpendicularly propagating waves ($k \perp B_\parallel$) are included.

2. Quasi-linear momentum diffusion coefficient

The quasi-linear theory treats the effect of the weakly perturbed magnetic field as perturbations of orbits of particles moving in the average background field. Schlickeiser (1989) has considered the quasilinear transport and acceleration parameters for cosmic ray particles interacting resonantly with Alfvén waves propagating parallel to the average magnetic field. The transport equation can be derived from the Fokker-Planck equation by a well-known approximate scheme (Jokipii 1966, Hasselmann & Wibberenz 1968) which is commonly referred to as the diffusion-convection equation for the pitch-angle averaged position of the particles. Schlickeiser (1989) obtained the momentum diffusion coefficient $D_p$ is predominantly determined by the transit-time damping and the spatial diffusion coefficient is determined only by the gyroresonance interactions. Finally for the flat ($1 < q \leq 2$) and the steep ($q \geq 2$) fast-mode turbulence spectra (eq. 2.1) they obtained

$$D_p(1 < q \leq 2) \approx \frac{\pi q q \Gamma(q) \Gamma(2 - \frac{q}{2})}{2q^{1+4}q^27^{3/3+5/2}} \left(\frac{\delta B}{B}\right)^2 |\Omega|(k_{\text{min}}) q^{1-2} \frac{V_A^2 p^2}{v^3-q} . (2.2)$$

Recently, Schlickeiser & Miller (1997) considered cosmic ray particles interacting with oblique fast-mode waves propagating in a low-$\beta$ plasm. In the cold plasma limit the fast and slow magnetosonic waves merge to the fast-mode waves with the same dispersion relation $\omega^2 = V_A^2 k^2$. For such waves they demonstrated that the rate of adiabatic deceleration vanishes, and the momentum diffusion coefficient and spatial diffusion coefficient $k_{\parallel}$ can be calculated as pitch angle averages of the functions determined by the non-vanishing Fokker-Planck coefficient $D_{\mu \nu}$ and by $D_{\mu \nu} = c^2 p^2 D_{\mu \nu}(\mu)$, where $\epsilon = V_A^2 / v$. Adopting isotropic fast-mode turbulence with a power-law turbulence spectrum they obtained

$$D_{\mu \nu} = \frac{\pi |\Omega| (q-1)(1-\mu^2)}{4} \left(\frac{\delta B}{B}\right)^2 (k_{\text{min}} R_L)^{q-1} \left[ f_T(\mu) + f_G(\mu) \right] , (2.3)$$

which is the sum of transit-time damping ($f_T$) and gyroresonance interaction ($f_G$) contributions. The form of $f_T$ admits the pitch angle scattering by transit-time damping of super-Alfvénic particles with pitch-angles contained in the interval $\epsilon \leq |\mu| \leq 1$. In the interval $|\mu| \leq \epsilon$, where no transit-time damping occurs, the gyroresonance ($n \neq 0$) interactions provide a small but finite contribution to the particle scattering rate. As a result the momentum diffusion coefficient $D_p$ is determined only by the transit-time damping contribution and the spatial diffusion coefficient is determined only by the gyroresonance interactions. Finally for the flat ($1 < q \leq 2$) and the steep ($q \geq 2$) fast-mode turbulence spectra (eq. 2.1) they obtained

$$D_p(1 < q \leq 2) \approx \frac{\pi q q \Gamma(q) \Gamma(2 - \frac{q}{2})}{2q^{1+4}q^27^{3/3+5/2}} \left(\frac{\delta B}{B}\right)^2 |\Omega|(k_{\text{min}}) q^{1-2} \frac{V_A^2 p^2}{v^3-q} . (2.2)$$

and

$$D_p(2 \leq q \leq 6) \approx \frac{\pi q q \Gamma(q) \Gamma(2 - \frac{q}{2})}{2q^{1+4}q^27^{3/3+5/2}} \left(\frac{\delta B}{B}\right)^2 |\Omega|(k_{\text{min}}) q^{1-2} \frac{V_A^2 p^2}{v^3-q} . (2.2)$$

respectively.

3. Description of simulations

The approach applied in the present paper is based on numerical Monte Carlo particle simulations. The general procedure is simple: test particles are injected at random positions into a turbulent magnetized plasma and their trajectories are followed by integrating the particle’s equations of motion. Due to the presence of waves, particles move diffusively in space and momentum. By averaging over a large number of trajectories one derives the diffusion coefficients for turbulent wave fields. In the simulations we consider relativistic particles with $v \gg V_A$ and use dimensionless units (cf Appendix A): $\delta B \equiv \delta B / B_0$ for magnetic field perturbations, $1/\Omega_o$ for time, $k/k_{\text{res}}$ for wave vector and $p_0^2 \Omega_o$ for the momentum diffusion coefficient.

3.1. The Wave Field Models

In the modelling we consider a superposition of 384 MHD waves propagating oblique to the average magnetic field $B_\parallel = B_\parallel \hat{e}_z$. The wave propagation angle with respect to $B_\parallel$ is randomly chosen from a uniform distribution within a cone (‘wave-cone’) along the mean field. For a given simulation two symmetric cones are considered centered along $B_\parallel$, with the opening angle $2\alpha$, directed parallel and anti-parallel to the field direction. The same number of waves is selected from each cone in order to model the
symmetric wave field. Related to the wave 'i' the magnetic field fluctuation vector $\delta B^{(i)}$ is given in the form:

$$\delta B^{(i)} = \delta B_0^{(i)} \sin(k^{(i)} \cdot r - \omega^{(i)}t) .$$  \hspace{1cm} (3.1)

The electric field fluctuation related to a particular wave is given as $\delta E^{(i)} = -V^{(i)} \wedge \delta B^{(i)}$ where $V^{(i)}$ is a wave velocity.

For Alfvén waves (A) we consider the respective dispersion relation

$$\omega^2_A = k^2 V^2_A \hspace{1cm} (3.2)$$

where $V_A = B_o/\sqrt{4\pi \rho}$ is the Alfvén velocity in the field $B_o$. The wave magnetic field polarization is defined by the formula

$$\delta B_A = \delta B_A(k, \omega_A) (k \times \hat{e}_z) k^{-1}_1 .$$  \hspace{1cm} (3.3)

In low-$\beta$ plasma the fast-mode magnetosonic waves (M) propagate with the Alfvén velocity and the respective relations are:

$$\omega^2_M = k^2 V^2_A \hspace{1cm} (3.4)$$

$$\delta B_M = \delta B_M(k, \omega_M) (k \times (k \times \hat{e}_z)) k^{-1}_1 k^{-1}_2 .$$  \hspace{1cm} (3.5)

For all our simulations we adopt $V_A = 10^{-3}c$. One should be aware of the fact that the considered turbulence model is unrealistic at large $\delta B$ and the present results cannot be considered as the exact ones. In particular, in the presence of a finite amplitude turbulence the magnetic field pressure is larger than the mean field pressure and the wave phase velocities can be greater than the assumed here $V_A(B_o)$.

3.2. Spectrum of the turbulence

In our simulations we consider a power turbulence spectrum where the irregular magnetic field, obtained from the energy density which is defined in equation (2.1), in the wave vector range $(k_{\text{min}}, k_{\text{max}})$ can be written

$$\delta B(k) = \delta B(k_{\text{min}}) \left( \frac{k}{k_{\text{min}}} \right)^{-q/2}$$  \hspace{1cm} (3.6)

where $k_{\text{min}} = 0.08 (k_{\text{max}} = 8.0)$ corresponds to the considered longest (shortest) wavelength and $q$ is the wave spectral index. In the present simulations we consider the flat spectrum with $q = 1$ and the Kolmogorov spectrum with $q = 5/3$. We included the flat spectrum because of our earlier simulations (Michalek & Ostrowski 1996) when we considered the Alfvén waves with $q = 1$. On the other hand such kind of turbulence spectrum is very convenient for numerical simulations due to presence of a big number of short waves. For the flat spectrum wave vectors are drawn in a random way from the respective ranges: $2.0 \leq k \leq 8.0$ for 'short' waves, $0.4 \leq k \leq 2.0$ for 'medium' waves and $0.08 \leq k \leq 0.4$ for 'long' waves. The respective wave amplitude are drawn in a random manner so as to keep constant

$$\sum_{i=1}^{384} (\delta B_0^{(i)})^2 \leq \delta B,$$  \hspace{1cm} (3.7)

where $\delta B$ is a model parameter, and, separately in all mentioned wave-vector ranges

$$\sum_{i=128}^{256} (\delta B_0^{(i)})^2 \leq \sum_{i=129}^{256} (\delta B_0^{(i)})^2 \equiv \delta B, \hspace{1cm} \sum_{i=257}^{384} \equiv \frac{\delta B}{\sqrt{3}}.$$  \hspace{1cm} (3.8)

Thus the wave energy is uniformly divided into all wave-vector ranges. As a second "realistic one" turbulence model we consider one with the Kolmogorov spectrum. The observed spectra in the interplanetary space often take a such form (Jokipii 1971). In that case all wave vectors are drawn in a random manner from the whole considered range ($0.08 \leq k \leq 8.0$) but the amplitudes $\delta B^{(i)}$ are fitted according to Kolmogorov law (2.1) and to keep the formula (3.7). In such a turbulence spectrum most of the energy is carried by 'long' waves. In the discussion below we will consider four different turbulence fields: i. Alfvén waves with a flat spectrum - AF, ii. Alfvén waves with a Kolmogorov spectrum - AK, iii. Fast-mode waves with the flat spectrum - MF, iv. Fast-mode waves with the Kolmogorov spectrum - MK.

4. Results

The results of the simulations are presented at the figures 1 and 2. In Fig. 1 on the successive panels, the simulated momentum diffusion coefficients $D_p$ for the Alfvén and the magnetosonic turbulence are presented. The derived diffusion coefficients $D_p$ are given for different wave-cone opening angles and for different turbulence amplitudes. In all cases a systematic increase of $D_p$ with the amplitude occurs, but the rate of this increase diminishes at larger non-linear $\delta B$. We suspect the considered effect reflects partial particle trapping in high-amplitude turbulence, decreasing randomness of the particle momentum variation. There are no significant differences between results obtained for both types turbulence spectrum, $q = 1$ or $5/3$. For the Alfvénic turbulence an increase of the wave-cone opening angle $\alpha$ provides waves with smaller phase velocities and leads to decreasing values of $D_p$. Additionally, the values of the wave vector amplitude $k$ decrease leading to less efficient coupling mediated by the cyclotron resonance. The trend is independent of the considered turbulence amplitude. In the case of fast-mode waves a more complicated relation is observed. For the flat turbulence spectrum a small increase of the angle $\alpha$ leads to a slight
decrease of $D_p$ due to mentioned weakening of cyclotron resonance. However, the appearance of more waves propagating nearly perpendicular to the ordered magnetic field inverts this trend by allowing the transit time-damping resonance to become more effective. For the Kolmogorov turbulence an increase in opening angles is always followed by an increase of $D_p$, but, again reaches the maximum in the presence of perpendicular waves. The described non-monotonic variations are clearly seen at low wave amplitudes in Fig. 1, while for large amplitudes the phenomenon is less pronounced. Generally, for large amplitudes of the magnetosonic waves the value of $D_p$ does not strongly depend on the opening angle of the waves. The simulation errors can be evaluated from comparison of the results for $\alpha = 0^\circ$ of Alfvén and fast mode waves, which should coincide. In Fig. 1 only the clarity the results for $\alpha = 0^\circ$, $40^\circ$...
and 90° are presented, but the ones calculated by us for α = 30° and 60° are consistent with the above description.

In order to explain this non-monotonic behavior one can relate to the quasi-linear derivations of Schlickeiser & Miller (1997). For n = 0 the resonance condition for the transit-time damping may be written as \( v_\parallel = \omega/k_\parallel \) and, with the dispersion relation (4), \( V_A/v_\parallel = k_\parallel/k \). It is clear that for \( V_A \ll v_\parallel \) particles can effectively interact with waves at wide range of \( v_\parallel \) only if the waves with \( k_\parallel \gg k_\parallel \) are present. This fact explains the effective transit-time damping interactions for magnetosonic waves with isotropic space distribution of waves. Then, the \( D_p \) increase results mainly from the presence of waves propagating quasi-perpendicular to the mean magnetic field.

In our simulations we adopted highly relativistic particles with \( v = 0.99c \gg V_A \), so that this effect is the most pronounced.

In order to verify this behaviour in more detail we considered the interaction of relativistic particles with waves propagating in narrow cones with respect to the ordered magnetic field. We performed simulations for cones \( 0° \pm 5°, 45° \pm 5°, 85° \pm 5° \) and results are shown at Fig. 2. Only the presence of waves propagating nearly perpendicular to the mean magnetic field leads only to significant increase of \( D_p \). For these waves the transit-time damping resonance is responsible for the wave particle coupling, while the waves propagating at small angles or parallel to the average magnetic field only contribute to the gyroresonance interaction.

5. Discussion and summary

We considered momentum diffusion coefficient \( D_p \) in the presence of oblique Alfvénic and magnetosonic turbulence with amplitudes ranging from small ones up to highly non-linear ones. The influence of the degree of anisotropy and of the waves spectrum slope on \( D_p \) was also studied. Generally, in all cases a systematic increase of \( D_p \) with the wave amplitude is observed. \( D_p \) does not strongly depend on the type of turbulence spectrum. We confirm a substantial increase of \( D_p \) for the fast-mode waves in comparison to the Alfvén waves of the same amplitude if the nearly perpendicularly propagating waves are included. That increase is caused by the transit-time damping which occurs in the presence magnetosonic of waves containing compressive component of \( \delta B \). In the presence of a small amplitude magnetic turbulence a significant increase of \( D_p \), on a factor of 10, is achieved. This agrees well with the estimate of Schlickeiser and Miller (1997) that the coefficient \( D_p \) for the isotropic Kolmogorov-type spectrum in the presence of magnetosonic waves is about a factor \( \ln \frac{\omega}{k_\parallel} = \alpha(10) \) larger than the one for the parallel-propagating Alfvén waves. In the case of our simulations for the Kolmogorov spectra this value is nearly constant for a wide range of waves’ amplitudes and for isotropic wave spectrum our results are consistent with the quasi-linear estimates. The approximation worsens for the largest non-linear amplitudes \( \delta B/B_0 > 1.5 \) when the considered ratio diminishes on about 50%.

We also found that for large amplitudes of magnetosonic waves the difference between \( D_p \) does not strongly depend on the opening angle of the waves. When the strength of the compression is high enough most particles can be reflected before complete penetrating the region of the compression they could only be accelerated due to a simple second order Fermi mechanism. As the amplitude decreases, the number of particles that are reflected decrease. For very small amplitudes particles can be reflect at the wave when their parallel velocities in the wave frame is about zero. That occurs when \( v_\parallel = \omega/k_\parallel \) what is condition for the transit-time damping. Hence, that process could be called small-amplitude Fermi acceleration (Achterberg 1981). We should remember that the wave phase velocity are expected to be larger in realistic turbulence than the assumed here \( V_A(B_0) \). Therefore our results for large \( \delta B \) should be consider rather as the lower limits.

Considerations of the MHD waves propagation oblique to the mean magnetic field (e.g. Tademaru 1969, Lee and Volk 1975) show that such waves are subject to effective processes dissipating their energy. Because of that the con-

![Fig. 2. Examples of the simulated \( D_p \) for anisotropic waves propagating close to the chosen pitch angle (0°, 45°, 85°) for both types of turbulence spectrum and \( \delta B = 0.6 \).](image-url)
considered here effects of the momentum diffusion enhancement due to fast-mode waves can occur only in a volume with acting the turbulence generation force. For example, in vicinity of the strong shock or in a region of the magnetic field reconnection the required fast-mode perpendicular waves are expected to be effectively created.

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A. Summary of notation

\( a_2 = D_{\mu} \) – momentum diffusion coefficients
\( B = B_0 + \delta B \) – magnetic induction vector
\( B_0 \) – regular component of the background magnetic field
\( (B_0 = 1 \) in the simulations\)
\( \delta B \) – turbulent component of the magnetic field
\( c \) – light velocity \( (c = 1 \) in simulations\)
\( D_{\mu\nu} \) – Fokker-Planck coefficient
\( E \) – electric field vector
\( g^2(k) \) – magnetic energy density for given wave
\( e \) – particle charge
\( k_{res} = 2\pi/r_g \)
\( k_\parallel \) – wave vector along mean magnetic field
\( p \) – particle momentum vector
\( q \) – spectral index of waves
\( r_g \) – particle gyro-radius
\( R_L = \frac{v}{\Omega} \) – particle Larmour radius
\( v \equiv c^2p/\varepsilon \) – particle velocity vector
\( V_A \) – the Alfvén velocity in the field \( B_0 \)
\( v_\parallel \) – velocity along mean magnetic field
\( \alpha \) – open angle of MHD waves
\( \gamma \equiv (1 - v^2/c^2)^{-1/2} \) – the Lorentz factor
\( i \) – number of give waves
\( k \) – wave-vector
\( \kappa_\perp \) – transverse (cross-field) diffusion coefficient
\( \kappa_\parallel \) – parallel diffusion coefficient
\( m \) – particle mass \( (m = 1 \) in simulations\)
\( \omega \) – wave frequency
\( \Omega \equiv eB/\gamma mc \) – particle angular velocity
\( \Omega^{(o)} \equiv eB_0/\gamma mc \)
\( \Theta \) – the momentum pitch-angle with respect to \( B_0 \)
\( (\mu \equiv \cos \Theta) \)
\( \varepsilon \) – particle energy
\( \epsilon = \frac{V_A}{\varepsilon} \)

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