THERMAL CONDUCTION IN MAGNETIZED TURBULENT GAS

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ABSTRACT

Using numerical methods, we systematically study in the framework of ideal MHD the effect of magnetic fields on heat transfer within a turbulent gas. We measure the rates of passive scalar diffusion within magnetized fluids and make the comparisons (1) between MHD and hydrodynamic simulations, (2) between different MHD runs with different values of the external magnetic field (up to the energy equipartition value), and (3) between thermal conductivities parallel and perpendicular to the magnetic field. We do not find apparent suppression of diffusion rates by the presence of magnetic fields, which implies that magnetic fields do not suppress heat diffusion by turbulent motions.

Subject headings: galaxies: clusters: general — ISM: general — MHD — turbulence

1. ASTROPHYSICAL MOTIVATION

It is well known that astrophysical fluids are turbulent and that magnetic fields are dynamically important. One characteristic of the medium that magnetic fields and turbulence may substantially change is the heat transfer.

There are many instances when heat transfer through thermal conductivity is important. For instance, thermal conductivity is essential in rarefied gases in which radiative heat transfer is suppressed. This is exactly the situation that is present in clusters of galaxies. It is widely accepted that ubiquitous X-ray emission due to hot gas in clusters of galaxies should cool significant amounts of the intracluster medium (ICM) and this process, however, is important for many regions within the interstellar medium (ISM) and for supernova remnants.

Gas in clusters of galaxies is magnetized, and the conventional wisdom suggests that the magnetic fields strongly suppress thermal conduction perpendicular to their direction. Realistic magnetic fields are turbulent, and the issue of the thermal conduction in such a situation has been long debated. A recent paper by Narayan & Medvedev (2001) obtained estimates for the thermal conductivity of turbulent magnetic fields, but those estimates happen to be too low to explain the absence of cooling flows for many of the clusters of galaxies (Zakamska & Narayan 2003). Narayan & Medvedev (2001) treat the turbulent magnetic fields as static. In hydrodynamical turbulence it is possible to neglect plasma turbulent motions only when the diffusion of electrons that is the product of the electron thermal velocity

$$v_{\text{elect}}l_{\text{mfp}}$$

and the electron mean free path in plasma

$$l_{\text{mfp}}$$

i.e.,

$$v_{\text{elect}}l_{\text{mfp}}$$

is greater than the turbulent velocity

$$v_{\text{turb}}$$

times the turbulent injection scale

$$l_{\text{inj}}$$

i.e.,

$$v_{\text{turb}}l_{\text{inj}}$$

If such scaling estimates are applicable to heat transport in magnetized plasma, the turbulent heat transport should be accounted for heat transfer within clusters of galaxies. Indeed, data for

$$v_{\text{elect}}l_{\text{mfp}}$$

given in Zakamska & Narayan (2003; Narayan & Medvedev 2001) provide the classical Spitzer (1962) diffusion coefficient

$$\kappa_{\text{Sp}} = \frac{1}{5} \times 10^{30} \text{ cm}^2 \text{ s}^{-1}$$

for the inner region of

$$R \sim 100 \text{ kpc}$$

and

$$\kappa_{\text{Sp}} = \frac{1}{5} \times 10^{25} \text{ cm}^2 \text{ s}^{-1}$$

for the very inner region of

$$R \sim 10 \text{ kpc}$$

for Hydra A. If turbulence in the cluster of galaxies is of the order of the velocity dispersion of galaxies, while the injection scale is of the order of 20 kpc, the diffusion coefficient is

$$v_{\text{inj}}l_{\text{mfp}} \sim 3.1 \times 10^{30} \text{ cm}^2 \text{ s}^{-1}$$

where we take

$$v_{\text{turb}} \sim 500 \text{ km s}^{-1}$$

Earlier numerical studies by Cho, Lazarian, & Vishniac (2002) revealed a good correspondence between hydrodynamic motions and motions of fluid perpendicular to the local direction of magnetic field. To what extent heat transfer in a turbulent medium is affected by a magnetic field is the subject of the present study. To solve this problem, we systematically study the passive scalar diffusion in a magnetized turbulent medium, compare results of MHD and hydrodynamic calculations, and investigate the heat transfer perpendicular and parallel to the mean magnetic field for magnetic fields of different intensities.

This work has a broad astrophysical impact. Clusters of galaxies is just one of the examples in which nonradiative heat transfer is essential. This process, however, is important for many regions within galactic interstellar medium, e.g., for supernova remnants.

2. NUMERICAL METHODS

We use a third-order hybrid essentially nonoscillatory (ENO) upwind shock-capturing scheme to solve the ideal MHD equations. To reduce spurious oscillations near shocks, we combine two ENO schemes. When variables are sufficiently smooth, we use the third-order Weighted ENO scheme (Jiang & Wu 1999) without characteristic mode decomposition. When the opposite is true, we use the third-order Convex ENO scheme (Liu & Osher 1998). We use a three-stage Runge-Kutta method for
The energy injection scale \( l_{inj} \) is \( \sim 1/2.5 \) of a side of the numerical box. The scalar field follows the continuity equation

\[
\frac{\partial \psi}{\partial t} + \nabla \cdot (\psi \mathbf{v}) = 0.
\]

We are concerned mainly with time evolution of \( \sigma_i \) (i = x, y, and z):

\[
\sigma_i^2 = \left( \int (x_i - \bar{x}_i)^2 \psi(x, t) dx \right) \left( \int \psi(x, t) dx \right)^{-1},
\]

where \( \bar{x}_i = \left[ \int \int (x_i \psi(x, t) dx dx) \right] \left[ \int \psi(x, t) dx \right]^{-1} \). Common wisdom was that the mean magnetic field suppresses diffusion in the direction perpendicular to it. If this is the case, we expect to see \( \sigma_z < \sigma_i \). Otherwise, we will get \( \sigma_i \sim \sigma_z \).

We inject passive scalars after turbulence is fully developed. Figure 1a shows when we inject the passive scalars. For the hydrodynamic run with \( M_s \) (sonic Mach number) \( = 0.3 \) and \( 192^3 \) grid points (thick solid line), we inject passive scalars 5 times. The injection times are marked by arrows. We also mark the injection times by arrows for the MHD run with \( V_s = 1 \), \( M_s \sim 0.3 \), and \( 192^3 \) grid points [thin solid line for \( \psi(x, t) dx dx \) and dashed line for \( \psi(x, t) dx \)].

3. THEORETICAL CONSIDERATIONS

Consider two massless particles in the inertial range. Let the separation be \( l \). The separation follows

\[
\frac{dl^2}{dt} \sim \frac{(l + v_i dt)^2 - (l - v_i dt)^2}{dt} \sim l v_i,
\]

where \( v_i = 1/16 \) of a side of the numerical box and \( x_i \) lies at the center of the computational box. The value of \( v_i \) ensures that the scalar is injected in the inertial range of turbulence.
where we ignore constants of order unity. Using \( \epsilon \sim \nu^3/l_l \), we get
\[
\frac{d l^2}{d t} \sim \nu(\epsilon l)^{1/3},
\]
where \( \epsilon \) is the energy injection rate. This leads to
\[
l^{2/3} - l_0^{2/3} = (C_R)^{2/3} \nu^{1/3}(t - t_0),
\]
where \( C_R \) is the Richardson constant. When \( l \gg l_0 \), we can write
\[
l^2 = C_\nu(t - t_0)^3,
\]
which was first discovered by Richardson (1926).

Recent direct numerical simulations suggest that \( C_R \approx 1 \). Boffetta & Sokolov (2002) obtained \( C_R \approx 0.55 \), Ishihara & Kaneda (2001) obtained \( C_R \approx 0.7 \).

When we inject a passive scalar field as in equation (4), we can write
\[
\sigma^{2/3} - \sigma_0^{2/3} = (C_1)^{2/3} \nu^{1/3}(t - t_0),
\]
where \( \sigma = (\sigma_x^2 + \sigma_y^2 + \sigma_z^2)^{1/2} \) and the dimensionless constant \( C_1 \) is not necessarily the same as \( C_R \). The constant \( C_1 \approx (C_R l_0)^{1/3} \) has dimension. In this Letter, we do not attempt to obtain \( C_1 \) or \( C_R \). Instead, we investigate how \( C_1 \) behaves when we vary \( B_0 \).

Usually it was considered that MHD turbulence is different from its hydrodynamic counterpart. However, Cho et al. (2002) recently showed that motions perpendicular to the local mean fields are hydrodynamic to high order. This means that many turbulent processes are as efficient as hydrodynamic ones. For example, Cho et al. (2002) numerically showed that the cascade timescale in MHD turbulence follows hydrodynamic scaling relations (see also Maron & Goldreich 2001). The similarity between magnetized and unmagnetized turbulent flows motivates us to speculate that turbulent mixing is also efficient in MHD turbulence. This is why we can use equation (11), which is derived from hydrodynamic turbulence. It is worth noting that these facts are consistent with a recent model of fast magnetic reconnection in turbulent medium (Lazarian & Vishniac 1999).

4. RESULTS

In Figures 1b and 1c, we compare the time evolution of \( \sigma \) in the hydrodynamic case and in the MHD case. In the MHD case, the Alfvén velocity of the mean field (\( V_A = 1 \)) is slightly larger than the rms fluid velocity (\( v_{\text{turb}} \sim 0.7 \)). This is the so-called sub-Alfvénic regime. Since \( V_A \sim v_{\text{turb}} \), the turbulence is strong. The results show that turbulent diffusion is faster in the hydrodynamic case. However, Figure 1d implies that this is due to reduction in velocity. Note that \( v_{\text{turb}} \sim 1 \) in the hydrodynamic case and \( v_{\text{turb}} \sim 0.7 \) in the MHD case (see Fig. 1a).

Figure 1d shows that there are good relations between \( \sigma^{2/3} \) and \( t - t_0 \). The slopes correspond to the constant \( C_2 \) in equation (11). The slopes are not very sensitive to \( V_A \) or \( M_\nu \).

Figures 1e and 1f show that the diffusion rate does not strongly depend on the direction of the mean field. When the mean magnetic fields are strong (as in Figs. 1e and 1f), the local magnetic field at any given point in the computation box has a preferred direction (i.e., \( x \)-direction). On average, the angle between the magnetic field and the \( x \)-axis is around \( \tan^{-1}(b/B_0) \sim 30^\circ \), where we use \( b \sim 0.6 \) (see Fig. 1a) and \( B_0 = 1 \). Therefore, the parallel and perpendicular conductivity based on the mean field is statistically the same as that based on the local magnetic field.

The validity of equation (7) enables us to write
\[
\kappa_{\text{dynamic}} = C_{\text{dyn}} l_{\text{inj}} v_{\text{turb}},
\]
where \( C_{\text{dyn}} \) is a constant of order unity. This is the effective diffusion by turbulent motions suitable for scales larger than \( l_{\text{inj}} \). The value of \( C_{\text{dyn}} \) remains almost constant for \( B_0 \) of up to equipartition value, \( B_0/(4\pi \rho)^{1/2} \sim v_{\text{turb}} \sim b/(4\pi \rho)^{1/2} \). The exact value of \( C_{\text{dyn}} \) is uncertain. In hydrodynamic cases, \( C_{\text{dyn}} \) is of the order of \( \sim 0.3 \) (see Lesieur 1990 and references therein).

5. ASTROPHYSICAL IMPLICATIONS

We have shown that turbulence motions provide efficient mixing in MHD turbulence. In this section, we show that this process is as efficient as that proposed by Narayan & Medvedev (2001) for some clusters.

We summarize models of thermal diffusion in Figure 2. In the classical picture, thermal diffusion is highly suppressed in the direction perpendicular to \( B_0 \). Transport of heat along wandering magnetic field lines (Narayan & Medvedev 2001) partially alleviates the problem. But the applicability of Narayan & Medvedev’s model is a bit restricted—their model requires a strong (i.e., \( V_A \sim B_0/(4\pi \rho)^{1/2} \sim v_{\text{turb}} \)) mean magnetic field. In the Galaxy, there are strong mean magnetic fields. But, in the ICM, this is unlikely. When the mean field is weak, the scales smaller than the characteristic magnetic field scale \( \sim b/(4\pi \rho)^{1/2} \) may follow the Goldreich & Sridhar model (1995). However, this requires further studies. Our turbulent mixing model gives the same \( \kappa_{\text{dynamic}} \) regardless of magnetic field geometry.

ICM.—As we mentioned earlier, \( \kappa_{\text{sp}} \sim 3 \times 10^9 (kT/keV)^{5/2} \times (n/10^{-3} \text{ cm}^{-3})^{-1} \text{ cm}^2 \text{ s}^{-1} \) and \( \kappa_{\text{dynamic}} \sim 3.1 \times 10^9 (v_{\text{turb}}/500 \text{ km s}^{-1})(l_{\text{inj}}/20 \text{ kpc}) \text{ cm}^2 \text{ s}^{-1} \). The ratio of the two is of the order of unity for the ICM:

\[
f_{\text{ICM}} \equiv \frac{\kappa_{\text{dynamic}}}{\kappa_{\text{sp}}} \sim O(1).
\]

To be specific, for Hydra A, \( f \sim 0.5 \) for the inner region \( (R \sim 100 \text{ kpc}) \) and \( f \sim 8.6 \) for the very inner region \( (R \sim 10 \text{ kpc}) \). For 3C 295, \( f \sim 0.34 \) for the inner region and \( f \sim 24 \) for the very inner region.

Our model deals with thermal diffusion in fully developed MHD turbulence with \( v_{\text{turb}} \sim 500 \text{ km s}^{-1} \). When turbulence is not fully developed or \( v_{\text{turb}} \) is smaller, we expect a lower thermal conductivity. The observed temperature inhomogeneities in several clusters may indicate that turbulence in the clusters is either underdeveloped or very weak.

Local Bubble and Supernova Remnants.—The Local Bubble is a hot \((T \sim 10^8 \text{ K}; kT \sim 100 \text{ eV})\) tenuous \((n \approx 0.008 \text{ cm}^{-3})\) cavity immersed in the interstellar medium (Berghofer et al. 1998; Smith & Cox 2001). Turbulence parameters are uncertain. We take typical interstellar medium values: \( l_{\text{inj}} \sim 10 \text{ pc} \) and \( v_{\text{turb}} \sim 5 \text{ km s}^{-1} \). For these parameters, the ratio of \( \kappa_{\text{dynamic}} \) to \( \kappa_{\text{sp}} \) is

\[
f_{\text{in}} = \frac{\kappa_{\text{dynamic}}}{\kappa_{\text{sp}}} \sim 0.05
\]
Fig. 2.—Models of thermal diffusion. Left: Classical picture. Middle: Narayan & Medvedev (2001). Wandering of field lines provides efficient diffusion \( \kappa \sim v_{\text{inj}} \). But the model assumes \( B_0 \) is of approximate equipartition value. Right: Turbulent diffusion model. Thermal electrons are mixed by turbulent motions, which leads to turbulent diffusion coefficient of \( \kappa_{\text{turb}} \sim l_{\text{inj}} v_{\text{turb}} \). In many astrophysical situations, this coefficient is comparable with the Spitzer value. The figure is the snapshot of the passive scalar field at \( t \approx 3 \) from the MHD run described in Fig. 1c; 1923 grid points, \( M_\ast \sim 0.3 \), and \( V_\ast = 1 \). In the case shown here, the mean field is strong and parallel to the dashed line. In general, mean magnetic fields, weak or moderately strong, do not strongly suppress turbulent motions/diffusion.

for the inside of the Local Bubble. For the mixing layers, it is

\[
\frac{f_{\text{mix}}}{\kappa_{\text{Sp}}} \sim 100, \tag{15}
\]

where we take \( \bar{T} \sim (T, T_h)^{1/2} \sim 10^5 \) K, \( \bar{n} \sim (n, n_h)^{1/2} \sim 0.1 \) cm\(^{-3}\) (Begelman & Fabian 1990), \( T_h \sim 10^4 \) K, \( n_h \sim 1 \) cm\(^{-3}\), \( T_h \sim 10^4 \) K, and \( n_h \sim 0.008 \) cm\(^{-3}\). We expect similar results for supernova remnants since parameters are similar.

6. CONCLUSION

We have shown that magnetic fields (either a random or a mean magnetic field of up to equipartition value) do not suppress turbulent diffusion processes, which implies that the turbulent diffusion coefficient has the form \( \kappa_{\text{dyn}} \sim l_{\text{inj}} v_{\text{turb}} \) in MHD turbulence, as well as in hydrodynamic cases. This result has two important astrophysical implications. First, in the ICM, this turbulent diffusion coefficient is of the same order of the classical Spitzer value. Second, in the face of hot and cold media in the interstellar matter (e.g., the boundary between the Local Bubble and surrounding warm media), this turbulent diffusion coefficient is much larger than the classical Spitzer value.

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