Comparative Analysis of Strain Fields in Step-graded Buffers of Different Design, based on $\text{In}_x\text{Al}_{1-x}\text{As}$ Ternary Solutions

Abstract

Two heterostructures with the step-graded buffers of different design grown on (001) GaAs substrates by molecular beam epitaxy were employed to reveal applicability of an extension of phenomenological approach developed for the description of strain relief in single layer heterostructures to multilayer thin film systems. Difference in the design of buffers provided to the formation of dislocation free layers of different thickness. The determination of the residual strains in the epitaxial layers was done using reciprocal space mapping performed with a triple-axes X-ray diffractometer Smart Lab 9kW and the following processing of data obtained within the linear theory of elasticity. It was established that, despite the different design of buffers the character of strain spatial distributions in them was similar. It gives possibility to attract a phenomenological rule to describe the strain relief in the final constructive elements of both heterostructures. A correction for a work hardening in the phenomenological rule governing the strain relief in single layer heterostructures was performed.

Keywords: Triple-axes X-ray diffractometry; Reciprocal space mapping; Elastic strain; Step-graded buffer, Spatial strain distribution; Work hardening

Introduction

Hetero structures applied to ultrahigh frequency (UHF) electronics devices (for example, high electron mobility transistors–HEMTs) are created, as a rule, on a single crystal substrate GaAs of (001) orientation and consist of a metamorphic (MM) buffer aiming to remove a mismatch between the substrate and device active layers including a quantum well (QW). The MM-buffer may have a different design, for example, it may be step-graded [1-3] or linear graded with an increasing value of the lattice mismatch [2,3]. Very often, the MM-buffer has such additional elements as a healing layer or an inverse step [4]. During the successive growth of MM-buffer layers, the strain relief occurs, which is accompanied by the generation of misfit dislocations and the propagation of threading dislocations into heterostructure top layers [5]. The MM-buffer should prevent the penetration of threading dislocations into the device active layers. The creation of MM-buffer is based on possibility of the system to form a dislocation free layer, which is, in its turn, a platform for following the healing layer or the inverse step. The theoretical prediction of the formation of such a dislocation free layer was done in [6].

This model considered MM-buffer with a continuous increasing lattice misfit due to the increase of the concentration of an alloying element. When moving this system to the equilibrium, the formation of dislocation free layer at the top region of buffer gives an energetic gain for the system [3-6]. Details concerning the mechanism of strain relief are currently revealed for single heteroepitaxial layers. It was established that there are three stages of strain relief [7]. In the first one, the process of relaxation is slow because only the bending of the dislocations penetrating from the substrate to the epitaxial layer takes place. The second stage of relaxation is the fast stage due to the multiplication of misfit dislocations; this stage occurs if the thickness of layer is more than 100 nm. The third stage manifests an inhibition of dislocation multiplication and occurs for much thicker layers due to the work hardening. For the fast stage of strain relief, there are some numerical relations between a residual strain and a thickness of single heteroepitaxial layer. Thus, Dunstan with
collaborators in the series of articles [8-10] gave some evidence concerning the existence of an inverse proportion between the residual compressive strain and the layer thickness of 100-
1000 nm. Such a relation between strains and thicknesses was explained in the framework of a “geometrical approach” taking into account the character of the spatial distribution of misfit
dislocations in the region between the substrate and the growing
epitaxial layer. By contrary, in [11] there was established that the residual compressive strain varies in an inverse proportion to the
root square of layer thickness. It means that there is an energetic
limit for an elastic strained thin film, which governs the strain
relief in single layer hetero structures. Both these approaches
may be described numerically and each of them is characterised
by its own phenomenological constant. Such a description of
strain relief in single layer hetero structures may be considered
as a phenomenological approach. This article is aimed to reveal
possibility of the application of the phenomenological approach
to describe the strain relief in multilayer systems, particularly,
in MM step-graded buffers, and to find some numerical criterions
for the formation of dislocation free layer. We believe that the
comparative analysis of the structural parameters of step-graded
MM-buffers of different design is a key to solve this problem.
Two step-graded MM-buffers of different design are the subjects
of this investigation. The principle difference between two MM-
buffers concluded in their final constructive elements: first of
them was terminated by the healing layer and second one had
in its structure the inverse step. Moreover, the buffers had different
thicknesses of their steps. The epitaxial layers of both
step-graded MM-buffers were based on In,Al,Ga. As ternary solid
solutions. X-ray reciprocal space mapping and transmission
electron microscopy (TEM) were involved in this study. Below, the
hetero structures with MM-buffers we will mark as metamorphic
HEMTs – MHEMTs.

**Experimental**

**Sample preparation**

Molecular beam epitaxy (MBE) was employed to create two
MHEMTs with the step-graded buffers of different design. A
Riber 32 MBE system was used for the fabrication of MHEMTs.
MHEMTs consisted of InAlAs/InGaAs/InAlAs active layers and
six-layered MM-buffers. The layers covering up the upper steps of
MM-buffers, the healing layer (MHEMT 1) or the inverse step
(MHEMT 2), had close values of the molar fraction of In. In
these constructive elements of MHEMTs the molar fraction of
In, $x$, was equal to 0.39 and 0.394, correspondingly. MBE
was performed at a constant temperature of substrates. MHEMT 1
was grown on a standard semi-isolated (001) GaAs substrate,
while MHEMT 2 was grown on the vicinal surface of GaAs
substrate with a deviation angle of 2° from (001) plane. The step-
to-step change in the molar fraction of In in In, Al, As ternary
solutions for the first five steps of MM-buffers was achieved in
the process of non-interrupted epitaxial growth at a constant
Al-source temperature. At this stage of heterostructure growth,
the thickness of each layer was equal to 0.1 μm for MHEMT 1 and 0.2
μm for MHEMT 2. During growth, the temperature of substrates
was equal to 380°C for MHEMT 1 and 400°C for MHEMT 2.
The barriers layers of both MHEMTs were grown at the higher
temperatures of the substrates: 480 and 500°C, correspondingly.
The growth rate of epitaxial layers was equal to 0.5 μm/h. The
layer by layer growth was interrupted for two minutes prior to
the growth of the healing layer (MHEMT 1) or the inverse step
(MHEMT 2). The growth was also interrupted for five minutes
when transiting to the regime of high temperature growth.
The composition of the epitaxial layers was controlled by regulating
the temperature of In, Al, Ga, as and Si molecular sources
based on the calibrated temperature dependence of molecular
fluxes. The common feature of two MHEMTs was the equality
of thicknesses of the final constructive elements: the healing layer
(MHEMT 1) and the inverse step (MHEMT 2). The thicknesses
of the barriers layers in both MHEMTs were also equal. The details
concerning the process of epitaxial growth and the technologival
characteristics of the constructive elements of MHEMTs are
presented in Table 1. X-ray diffraction measurements were
performed with a Smart Lab 9 kW X-ray diffractometer in
the three-axial configuration. The diffractometer operated in
the step-by-step mode of X-ray recording using the Cu K$_{α1}$

| MHEMT | Layer number | $X_{m}$ | Layer thickness, μm | Layer destination | Substrate temperature, °C |
|-------|--------------|---------|---------------------|-------------------|--------------------------|
| 1     | 1            | 0.15    | 0.1                 | First step of buffer | 380 |
|       | 2            | 0.23    | 0.1                 | Second step of buffer | 380 |
|       | 3            | 0.29    | 0.1                 | Third step of buffer | 380 |
|       | 4            | 0.35    | 0.1                 | Fourth step of buffer | 380 |
|       | 5            | 0.39    | 0.1                 | Fifth step of buffer | 380 |
|       | 6            | 0.39    | 0.2                 | Healing layer       | 380 |
|       | 7            | 0.39    | 0.2                 | Barrier layer       | 480 |
| 2     | 1            | 0.10    | 0.2                 | First step of buffer | 400 |
|       | 2            | 0.20    | 0.2                 | Second step of buffer | 400 |
|       | 3            | 0.30    | 0.2                 | Third step of buffer | 400 |
|       | 4            | 0.39    | 0.2                 | Fourth step of buffer | 400 |
|       | 5            | 0.48    | 0.2                 | Fifth step of buffer | 400 |
|       | 6            | 0.394   | 0.2                 | Inverse step        | 400 |
|       | 7            | 0.394   | 0.2                 | Barrier layer       | 500 |
irradiation. A Ge single crystal with (002) orientation was employed as an analyzer crystal. Two reflections, 004 and 224 (at glancing exit geometry), were recorded in the regime of the so-called ω-2θ scanning, which implies the use of Bragg–Brentano technique with the variation of sample position relatively the Bragg maximum of substrate. During recording, X-ray reflected radiation was detected along the scattering vector \( H_{2θ} \): \( H_{001} \) (for symmetric recording) and \( H_{224} \) (for asymmetric recording). The initial position for 004 symmetric recording corresponded to the Bragg maximum of the substrate \((2θ=6)\), where \( θ \) is a deviation of the substrate from the Bragg position. Such a mode of recording allowed us to reveal minor X-ray maxima for all layers of MM-buffers. For asymmetric recording, the scanning along the scattering vector \( H_{224} \) is achieved by the variation of hkl values that gives the decomposition of the scattering vector \( H_{224} \) along the directions [001] and [110] of the reciprocal space. On the basis of the ω-2θ scanning, it is possible to plot the so-called reciprocal space maps, which represent the positions of minor X-ray maxima in the reciprocal space. The use of the technique of reciprocal space mapping have some advantages in comparison with the conventional technique of rocking curves because the latter is invalid in the case of the spatial disorientation of epitaxial layers relative the substrate. The effect of spatial disorientation between an epitaxial layer grown on the vicinal surface of substrate and the corresponding substrate plane is a well-known fact [12]. The angular parameters of spatial disorientation of epitaxial layers in heterostructures grown on the vicinal surface of (001) GaAs substrate were presented in [12,13,14]. The actual axes of reciprocal space for epitaxial films grown on (001) GaAs substrates are the axis [110] (axis Y) and the axis [001] (axis Z). The antinodes of iso-concentration contours revealed in the reciprocal space maps correspond to the interfluence maxima of X-ray radiation. The coordinates of the antinodes (X-maxima) are expressed by the vectors \( q_{110} \) and \( q_{001} \), which are deviations (along the corresponding reciprocal space axes) of minor X-ray maxima from the major X-ray maximum \( H^0_{001} \) corresponding to the substrate. The knowledge of \( q_{110} \) and \( q_{001} \) allows us to determine the vertical and lateral lattice parameters for all epitaxial layers.

**Characterization of MHEMTs microstructure**

The structural investigation of MHEMTs was performed by TEM with a Jeol JEM-2100 operating at an accelerating voltage of 200 kV. Figure 1 shows cross-sectional bright-field electron microscopy images obtained for two investigated MHEMTs. It is seen that, despite the interruption of growth, layers 5, 6 and 7 in MHEMT 1 do not have interphase boundaries and we can consider them as a single phase. This combined layer of MHEMT 1 does not have threading dislocations and, consequently, the elastic strain developed in it should be significantly higher than that in the internal layers of MHEMT 1. Similar situation, the formation of dislocation free layer, is realized for layer 5 in MHEMT 2. The inverse step and the barrier layer of this MHEMT (layers 6 and 7) are the special elements of heterostructure design; similar layer 5 these layers do not have threading dislocations. The role of these constructive elements in the formation of strain fields in MHEMT 2 will be discussed below. It is important that, the electron microscopy image taken from MHEMT 2 demonstrates the inverse step and the barrier layer as a single structural layer.

**Reciprocal space maps and their processing**

Reciprocal space maps for MHEMTs under study plotted on the basis of 004 and 224 reflections are presented in Figure 2 (MHEMT 1) and Figure 3 (MHEMT 2). The maps for MHEMT 1 manifest five minor X-ray maxima whose values of \( [9] \) (in ascending order) correspond to layers 1, 2, 3, 4 and 5 (6, 7). Attention is drawn to the fact that layers 5, 6 and 7 are characterized by a single reflex that indicates the affinity of the structural parameters of these layers. The similar situation is realized for layers 6 (the inverse step) and 7 (the barrier layer) of MHEMT 2; these layers are also characterized by a single X-ray maximum. So, the reciprocal space maps are in agreement with the MHEMTs microstructure revealed by TEM. The arrangement of minor X-ray maxima in the reciprocal space maps for MHEMT 2 is more complicated than that for MHEMT 1 due to their partial overlap and absence of sharp X-ray maximum corresponding to layer 4. To find all positions of interference maxima in the map obtained for 004 reflection we use the procedure of the modelling of X-ray scans by a set of Gaussians [14]. Taking into account that for MHEMT 2 there is no difference in microstructure between the inverse step and the barrier layer Figure 1b we considered the strong central X-ray peak at the reciprocal space maps for MHEMT 2.
as the peak corresponding simultaneously to both constructive elements. This circumstance facilitates the processing of the map plotted on the basis of 004 reflection for MHEMT 2 (the expansion of X-ray scan along the axis Z on six Gaussians) and allows us to determine the values of $q_{004}^{004}$ and $q_{004}^{224}$ for layers 3, 4, and 6(7). For all maps the coordinates of distinct defined X-ray maxima, the values of $q_0$ and $q_x$ were determined by searching for the maximal value of X-ray reflected radiation counts using a special option of the Origin 15 software. The $q_{004}^{004}$, $q_{004}^{224}$ and $q_{224}^{224}$ values (excepting the value of $q_{224}^{224}$ for layer 4 in MHEMT 2) are listed in Table 2. The complete characterization of the structural state of layer 4 in MHEMT 2 was done using the specific features of the MHEMT 2 arrangement and will be described below.

### Strain fields in epitaxial layers of MHEMTs

The vectors $q_{004}^{004}$, $q_{004}^{224}$, and $q_{224}^{224}$ allow us to determine the so-called “total” strains (relative to GaAs substrate) of MHEMTs epitaxial layers: \[ \left[ \left( a_s - a_i \right) / a_s \right]_{[001]} \] (based on 004 reflection) and \[ \left[ \left( a_s - a_i \right) / a_s \right]_{[110]} \] (based on 224 reflection), where and $a_i$ are, respectively, the vertical and lateral lattice parameters of layer and $a_s$ is the substrate lattice parameters. In accordance with [15], for the epitaxial layers of MHEMTs created on GaAs substrates the “total” strains are determined by the following relationships.

\[
\begin{align*}
q_{004}^{004} &= \frac{1}{a_i} \left( a_s - a_i \right)_{[001]} \\
q_{224}^{224} &= \frac{1}{a_i} \left( a_s - a_i \right)_{[110]}
\end{align*}
\]

where

\[
Q_i = q_{004}^{004} - q_{004}^{224}.
\]

Eq. (3) takes into account the effect of spatial misorientation of epitaxial layers relative to the substrate. The quantities \[ \left[ \left( a_s - a_i \right) / a_s \right]_{[001]} \] and \[ \left[ \left( a_s - a_i \right) / a_s \right]_{[110]} \] correspond to the total strains $\varepsilon_{33}$ and $\varepsilon_{11}$, which are directed along the main crystallographic axes. (Note, when rotating a cubic lattice by an angle of 45° about the axis [001], the required operation for transition from the axis [110] to the axis [001], the equality between the quantities \[ \left[ \left( a_s - a_i \right) / a_s \right]_{[001]} \] and \[ \varepsilon_{11} \] will be achieved, if the distortion tensor does not have off-diagonal components and the elastic strains $\varepsilon_{33}$ and $\varepsilon_{11}$ are equal. Generally, this condition is accepted default). The knowledge of the values of $E_{33}$ and $\lambda_{11}$ gives us possibility to calculate the lattice misfit $\varepsilon_{33} = (a_s - a_i)/a_s$, where $a_i$ is the lattice parameter of fully relaxed lattice, and the residual compressive elastic strain $\lambda_{11}$ on the basis of Hook law using the linear theory of elasticity. The concept according to which the residual strain in epitaxial layers is elastic is conventional and generally accepted in calculations of the lattice misfit $\varepsilon_{33}$. According to this concept, the stress $\sigma_{33}$ in the [001] direction is assumed to be zero due to the plastic deformation near the interphase boundaries oriented perpendicular to the [001] axis. On the basis of the equality of lateral stresses $\sigma_{11}$ and $\sigma_{22}$, it follows from the Hook law for the crystals of cubic system, elastic strains $\varepsilon_{11}$ and $\varepsilon_{33}$ are subjected to the next relation.

### Table 2 Reciprocal space vectors for the maps obtained on the basis of 004 and 224 reflections.

| MHEMT | Layer | $q_{004}^{004}$ μm$^{-1}$ | $q_{004}^{224}$ μm$^{-1}$ | $q_{224}^{224}$ μm$^{-1}$ |
|-------|-------|----------------|----------------|----------------|
| 1     | 1     | -75.36        | -9.90         | -44.93        |
| 2     | 2     | -111.45       | -5.00         | -67.39        |
| 3     | 3     | -139.32       | -5.03         | -86.96        |
| 4     | 4     | -165.35       | -5.11         | -110.07       |
| 5     | 5(6,7)| -204.70       | -5.00         | -111.71       |
| 6     | 1     | -62.41        | -5.20         | -35.07        |
| 7     | 2     | -109.87       | -5.20         | -61.79        |
| 8     | 3     | -152.47       | -4.63         | -90.63        |
| 9     | 4     | -188.22       | 4.05          | -              |
| 10    | 5     | -256.49       | 6.27          | -120.66       |
| 11    | 6(7)  | -179.00       | 6.27          | -120.66       |
\[
\frac{\varepsilon_{13}}{\varepsilon_{11}} = -\frac{2C_{13}}{C_{11}},
\]

where \(C_{11}\) and \(C_{13}\) are the elastic stiffness coefficients. The relationship between the total strains \(\varepsilon_0\) (measured directly in an experiment) and the elastic strain \(\varepsilon_{11}\) is expressed by the following relationship [16]

\[
\varepsilon_0 = \bar{\varepsilon}_{11} - \bar{\varepsilon}_0.
\]

Taking into consideration Eq. (4) and Eq. (5) we arrive to

\[
\bar{\varepsilon}_0 = \frac{2N_{12}}{N_{11} + 2N_{12}}(\bar{\varepsilon}_{11} - \bar{\varepsilon}_{0}).
\]

The results of calculation of \(\bar{\varepsilon}_{0}^{(n)}\) and \(\varepsilon_{11}^{(n)}\) (\(n\) is the layer number) based on the measured \(q_{004}^{004}\), \(q_{004}^{004}\) and \(q_{224}^{224}\) values are presented in Table 3. The GaAs lattice parameter was assumed to be 0.565321 nm [17]. The elastic stiffness coefficients \(C_{11}\) and \(C_{13}\) of InAlAs ternary solutions, which must be known to calculate \(\bar{\varepsilon}_{0}^{(n)}\) and \(\varepsilon_{11}^{(n)}\), were obtained on the basis of the Vegard law proceeding from the corresponding data for AlAs and InAs [18]. For MHEMT 2 the values of \(\bar{\varepsilon}_{0}^{(n)}\) and \(\varepsilon_{11}^{(n)}\) were determined on the basis of a linear dependence of \(\varepsilon_0\) on the layer position in space and using Eq. (6). Results presented in Table 3 allows us to conclude that the design of MM-buffer in MHEMT 2 satisfies the philosophy of the inverse step creation [4] because the strain in this constructive element of MM-buffer is practically equal to zero. The existence of the inverse step in MM-buffer of MHEMT 2 does not influence the strain field in the internal layers of the buffer. Because layers 6 and 7 of MHEMT 2 do not contribute the strains into the total strain field, they were excluded from the following analysis. The lattice misfit spatial profiles and strain spatial profiles in both MHEMTs are shown in Figures 4 and 5 correspondingly. The residual strain profiles in MHEMTs are presented in the form of a function of \(\varepsilon_{11}\) on \(\varepsilon_0\). Both dependences are similar and manifest non-zero strains for the internal layers of MM-buffers that is in contradiction with the models predicting the full strain relief in internal layers [19,20]. Such a situation is realized due to the work hardening, which may appear in multilayer systems at sufficiently less epitaxial layer thicknesses in comparison with a single layer heterostructure [20].

Table 3 The lattice misfit, the residual strain and the thickness of the constructive elements of MM-buffers.

| MHEMT | Layer | \(\varepsilon_0\) | \(\varepsilon_{11}\) | Constructive element thickness, \(\mu m\) |
|-------|-------|-----------------|-----------------|-----------------------------------------|
| 1     | 1     | 9.444 \(\times 10^{-3}\) | -1.38 \(\times 10^{-3}\) | 0.1                                     |
|       | 2     | 14.342 \(\times 10^{-3}\) | -1.71 \(\times 10^{-3}\) | 0.1                                     |
|       | 3     | 18.385 \(\times 10^{-3}\) | -1.74 \(\times 10^{-3}\) | 0.1                                     |
|       | 4     | 22.686 \(\times 10^{-3}\) | -1.26 \(\times 10^{-3}\) | 0.1                                     |
|       | 5 (6,7) | 25.801 \(\times 10^{-3}\) | -4.01 \(\times 10^{-3}\) | 0.5                                     |
| 2     | 1     | 7.483 \(\times 10^{-3}\) | -1.48 \(\times 10^{-3}\) | 0.2                                     |
|       | 2     | 13.648 \(\times 10^{-3}\) | -2.21 \(\times 10^{-3}\) | 0.2                                     |
|       | 3     | 19.779 \(\times 10^{-3}\) | -2.29 \(\times 10^{-3}\) | 0.2                                     |
|       | 4     | 25.829 \(\times 10^{-3}\) | -1.51 \(\times 10^{-3}\) | 0.2                                     |
|       | 5     | 31.815 \(\times 10^{-3}\) | -5.78 \(\times 10^{-3}\) | 0.2                                     |
|       | 6 (7) | 25.993 \(\times 10^{-3}\) | 0.04 \(\times 10^{-3}\) | 0.4                                     |

Figure 3 The reciprocal space maps plotted for MHEMT 2 on the basis of (a) 004 reflection and (b) 224 reflection. The numerals indicate the number of layer responsible for the appearance of given X-ray maximum.

Figure 4 The lattice misfit spatial profiles in the epitaxial layers of MM-buffers for: (a) MHEMT 1 and (b) MHEMT 2.
Discussion

On the basis of our experimental data, it is possible to show that for two MHEMTs the following relationship is performed

$$\left[ \left( e_{11}^{(6-7)} - e_{11}^{(4)} \right)^2 \right]_{\text{MHEMT1}} = \left[ \left( e_{11}^{(5)} - e_{11}^{(4)} \right)^2 \right]_{\text{MHEMT2}}$$  \hspace{1cm} (7)

where $h_{\text{eff}}$ is the thickness of the dislocation free layer. For MHEMT 1 $h_{\text{eff}}=0.5 \, \mu\text{m}$; for MHEMT 2 $h_{\text{eff}}=0.2 \, \mu\text{m}$. Eq. (7) characterizes numerically the process of strain relief in the final constructive elements of MM-buffer in MHEMTs. The product $\left( e_{11}^{(6-7)} - e_{11}^{(4)} \right)^2 h_{\text{eff}}$ equals 0.0038 nm while the product $\left( e_{11}^{(5)} - e_{11}^{(4)} \right)^2 h_{\text{eff}}$ is equal to 0.0036 nm. The average value of two products equals 0.0037 nm.

In a general form Eq. (7) can be written as

$$\left( e_{11}^{\text{eff}} - e_{11}^{\text{int}} \right)^2 = \frac{k}{h_{\text{eff}}}$$  \hspace{1cm} (8)

where $e_{11}^{\text{eff}}$ the residual strain in the dislocation free layer, $e_{11}^{\text{int}}$ is the residual strain in the internal adjacent layer and $k$ is a phenomenological constant. Attention is drawn to the fact that Eq. (8) is very close to the equation describing the strain relief in a single layer heterostructure. As was shown in [11], the strain relief of epitaxial layer of In Ga As grown on (001) GaAs substrate is governed by the equation

$$e_{11}^{\text{eff}} = \frac{k}{h}$$  \hspace{1cm} (9)

where $h$ is the thickness of epitaxial layer. The phenomenological constant $k$ in Eq. (9) is equal to 0.0037 nm. Eq. (8) and Eq. (9) operate equal values of $k$ that indicates the common mechanism of strain relief in both cases. We may consider Eq. (8) as an extension of Eq. (9) to two layer thin film system where the dislocation free layer plays the role of a substrate. It should be pointed out that, in such a two layers system the strain relief occurs in the internal layer while the dislocation free layer exhibits strong compression. Because the dislocation free layer is sufficiently thinner than a real massive substrate in a single layer heterostructure, it gives a possibility to perform its pseudomorphic growth on the platform of internal adjacent layer. Both layers, the dislocation free layer and the internal adjacent layer, are, in fact, the single equalized system. The determination of interrelations between residual strains in all the layers of multilayer system requires additional investigation.

Conclusion

The performed study gives some evidence concerning the possibility of numerical description of a value of strain in the dislocation free layer of the step-graded MM-buffer in the framework of the phenomenological approach developed for single layer heterostructures. This description has a quadratic form as an inverse proportion between the residual compressive stain in the dislocation free layer and the root square of the layer thickness. The value of the phenomenological constant governing the process of the strain relief in the layers of metamorphic step-graded buffers based on ternary In$_{x}$Al$_{(1-x)}$As solutions coincides with that which controls the strain relief in single layer hetero structures. The numerical expression takes into account a correction for the work hardening in the internal layer adjacent to the dislocation free layer.

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