Theoretical substantiation of optimum energy efficiency of ultrasonic wet dispersing

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Abstract. The known preliminary experimental results of ultrasonic dispersing of suspension were presented and analyzed. The need for determination optimum modes providing maximum energy efficiency of ultrasonic wet dispersing is justified. The physical mechanism and the theoretical model of ultrasonic dispersing were proposed. The model allows to calculate fraction composition of dispersed particles and to evaluate, that the maximum of energy efficiency exists at fixed intensity for different materials of solid particles.

1. The relevance of the problem

The destruction and dispersion of solid particles when exposed by ultrasonic vibrations is determined mainly by the cavitation process [1–3].

In the publications of various researchers, the fundamental possibility of realizing the ultrasonic dispersion of particles of various substances has been evaluated [1–5]. It has been revealed that ultrasonic dispersing makes it possible to obtain suspensions, the particle sizes of which can be in a wide range, that is from several micrometers to several tens of nanometers.

In the presented publications, the process was investigated mainly experimentally. In the course of the studies carried out, the effect of the concentration of the dispersed phase and the intensity of ultrasonic action on the particle size in the treated suspension was studied.

In the publication [4], it was experimentally proved that ultrasonic dispersion is more energetically efficient in comparison with a ball mill in a wide range of sizes of initial particles that form agglomerates. In the publication of T. Hielscher [5], who is the founder of the company of the same name for the production of ultrasonic equipment, the results of experiments are given to determine the effect of the amount of introduced ultrasonic energy on the efficiency of ultrasonic dispersing of SiO2 (aerosil) in water (figure 1). The experiments were carried out at oscillation frequency 20 kHz.

As follows from figure 1, with an increase in the input energy from 2·10^4 to 10^5 kJ/m^3 (+8·10^4 kJ/m^3) average particles diameter $d_{32}$ is decreased from 0.125 to 0.1 μm (specific interfacial area $\frac{6}{d_{32}}$ is increased from 4.8·10^7 to 6·10^7 m^2/m^3). With an increase in the input energy from 2·10^4 to 10^6 kJ/m^3 (+9.8·10^5 kJ/m^3) average particle size is decreased from 0.125 to 0.09 μm (specific interfacial area is increased from 4.8·10^7 to 6.7·10^7 m^2/m^3).
Figure 1. Dependences of the average volumetric surface diameter of particles $d_{32}$ on the specific input ultrasonic energy $e$ at various mass concentrations of the dispersed phase. $P_V$ – the value of the excess pressure used in the experiments; $P_{V, \text{MAX}}$ – the value of the maximum overpressure that can be created in the experimental setup.

Thus, as follows from the presented results, $+8 \times 10^4$ kJ/m$^3$ to input energy gives increase specific interfacial area $1,2 \times 10^7$ m$^2$/m$^3$, and $+9,8 \times 10^5$ kJ/m$^3$ increases interfacial area in $1,9 \times 10^7$ m$^2$/m$^3$. Calculation of the increase in the specific interfacial surface by 1 kJ/m$^3$ gives the following results:

- $+8 \times 10^4$ kJ/m$^3$ to input energy increases interfacial area in 150 m$^2$/kJ;
- $+9,8 \times 10^5$ kJ/m$^3$ to input energy increases interfacial area in 19.4 m$^2$/kJ.

Ultimately, the results given by T. Hielscher confirm an increase in the degree of grinding with an increase in the input of ultrasonic energy, which can be achieved by increasing the vibration intensity or by a longer dispersion time. However, the same results show that an increase in the input energy above a certain limit leads to a decrease in the increase in the interphase surface per unit of input energy. As a result, the efficiency of the process is decreased and the unit cost of the final product is increased.

The evaluated fact indicates that the optimal modes providing the maximum efficiency of the cavitation process do not guarantee the maximum efficiency of transformation of the structure of the medium during dispersion. Therefore, the evaluation of optimal modes that ensure the maximum increase in the interface area per unit of the introduced ultrasonic energy is one of the urgent tasks.

More detailed experimental studies of the optimal dispersion modes by ultrasonic vibrations have not yet been carried out.

Therefore, it is obvious that for the industrial implementation of the process of ultrasonic dispersion of solid particles in a liquid, new scientific approaches are needed to create a model that allows optimizing the mode of the dispersing process in order to increase the efficiency of the process (the actual ratio of the energy consumption of increasing the surface particles to the total input energy of ultrasonic action).

To develop a model, it is necessary to evaluate the main physical mechanism of ultrasonic dispersion.
The substantiation for the main physical mechanism is described in the next section.

2. Main physical mechanism of ultrasonic dispersing

The processes of breaking the intermolecular bonds of the dispersed phase occur only at the moments of the formation of a high-amplitude shock wave with the formation of a cumulative jet in the phase of bubble collapse. This is due to the fact that only in the collapse phase the pressure in the liquid surrounding the cavitation bubble exceed the ultimate strength of the material of the solid particle.

The cumulative jet formed under the action of the shock wave provokes elastic deformation of the particle, which can lead to the destruction of the latter. In addition, the cumulative jet imparts some momentum to the particle. As a result of the action of the cumulative jet, the particle acquires a certain velocity of spatial movement. This means the possibility of collision of pairs of particles. Moreover, when the relative velocity of the particles at the moment of collision exceeds a certain threshold value, the destruction of the colliding particles is not excluded.

Thus, there are 2 possible physical mechanisms for dispersing solid particles:

- concentration of mechanical stresses in individual independent particles due to elastic compression of a particle when a cavitation bubble collapses near it;
- concentration of mechanical stresses in colliding particles due to a sharp deceleration of particle motion as a result of collision.

Each physical mechanism makes a certain contribution to the frequency of particle destruction and as a result determines the kinetics and efficiency of the ultrasonic dispersing process.

To preliminary estimate the contribution of each of the proposed physical mechanisms to the dispersion of particles, the initial velocity of a particle acquired as a result of the action of a cumulative jet was calculated. Calculations according to the equations of bubble dynamics (Gilmore’s equation), the maximum shock wave pressure amplitude in the core of the cavitation bubble is more than 100 MPa, while the characteristic bubble collapse time (shock wave pulse duration) is 0.1 μs.

These data make it possible to calculate the momentum transferred to the dispersed particle under the action of a high-amplitude shock wave (cumulative jet).

The maximum possible force acting on the dispersed particle when the cavitation bubble collapses (when the cavitation bubble is in the immediate vicinity of the particle) is calculated according to the following expression:

\[
F_{\text{sh}}(t) = \int_0^d 2\pi r \frac{P(t)R(t)}{\sqrt{r^2 + \left(\frac{d}{2} \sqrt{\frac{d^2}{4} - r^2 + h}\right)^2}} dr \leq \int_0^d 2\sqrt{2\pi} \frac{P(t)R(t)}{d} \sqrt{\frac{1}{\pi} - \frac{4r^2}{d^2}} dr = \frac{d}{2} \int_0^1 \frac{P(t)R(t)}{\sqrt{1 - x^2}} dx \approx 25P(t)R(t)d; 
\]

where \(P(t)\) is pressure of the shock wave in the core of the cavitation bubble determined according to the expression \(P(t) = P_{st} \left(\frac{R(t)}{R_{st}}\right)^{3/4}\) (\(P_{st}\) is pressure in the core of the cavitation bubble at its radius \(R_{st}\)), Pa; \(R(t)\) is cavitation bubble radius, m; \(d\) is nominal diameter of a dispersed particle, μm.

The maximum possible momentum transferred to a particle under the action of a shock wave is defined as:
According to the obtained expression, under ultrasonic action with maximum cavitation efficiency (maximum total absorption coefficient of oscillations in a liquid), the dependence of the velocity imparted to the particle by a separate shock wave is as follows (Figure 2).

As follows from the presented dependence, for the range of particle sizes most often subjected to dispersing in practice, the velocity can reach 65 m/s. Obviously, such speeds will lead to collisions of particles with each other. Nevertheless, the velocities of the particles at the moment of their collision will be much less than the velocity of the cumulative jet (several hundred m/s) formed during the collapse of the cavitation bubble.

This indicates that the contribution of the 2nd physical mechanism (deformation of particles as a result of a pair collision) is much less than the contribution of the 1st mechanism (destruction of a particle as a result of the action of the cumulative jet). Before the surfaces of the particles come into contact as a result of the collision, a certain fraction of the kinetic energy of the particles will be spent on elastic compression and rupture of the liquid film located between the surfaces of the particles. Elastic compression and rupture of the liquid film will additionally reduce the energy transferred to the deformation of the particles as a result of the collision.

The physical mechanism will be used for calculation of particles fraction compositions.

3. Result of calculation of ultrasonic dispersing

Based on the identified physical mechanism, according to the kinetic equation proposed by author (1), the fractional composition of particles was calculated as a result of ultrasonic dispersion at constant total energy consumption ($I=\text{const}$; $I$ is ultrasonic influence intensity, W/cm$^2$; $t$ is exposure time, s).

$$p(t) = \int_0^t \left[ F_{\text{id}}(t) - 6\pi\mu\frac{p(t)}{m} \right] dt < \int_0^t \left[ 25P(t)R(t)d - 6\pi\mu\frac{p(t)}{m} \right] dt.$$  

where $n_k$ is number concentration of particles with fixed size $d_k$ (m), m$^{-3}$; $\varsigma$ is factor taking into account fatigue strength and accumulation of microcracks in the particle material; $f$ is ultrasonic oscillations frequency, Hz; $n_{\text{bub}}$ is concentration of cavitation bubbles, m$^{-3}$; $h$ is the minimum distance from the center of a bubble to the surface of a particle at which the particle collapses from the collapse of a given bubble, m.
The fraction compositions for aluminium and coal particles are presented in figures 3, 4. The presented histograms confirm the existence of an intensity optimum at which the maximum increase in the free surface of the particles is achieved at the same energy consumption for different particle substances. For each material of particle the fixed intensity provides maximum of mass fraction of smallest particles (selected by green on histograms) at same introduced energy.

4. Conclusion
The known preliminary experimental results of ultrasonic dispersing of suspension were presented and analyzed. The need for determination optimum modes providing maximum energy efficiency of ultrasonic wet dispersing is justified. The physical mechanism and the theoretical model of ultrasonic dispersing were proposed. The model allows to calculate fraction composition of dispersed particles and to evaluate, that the maximum of energy efficiency exists at fixed intensity for different materials of solid particles.

![Figure 3](image-url) Fraction composition of aluminium particles in water at different ultrasonic intensities and same total introduced energy ($I_t=\text{const}=3600 \text{ J/cm}^2$).

![Figure 4](image-url) Fraction composition of coal particles in water at different ultrasonic intensities.
Figure 4. Fraction composition of coal particles in water at different ultrasonic intensities and same total introduced energy ($I_t=\text{const}=28800\ J/cm^2$).

Acknowledgements
The reported study was supported by the Grant of the President of Russian Federation No. MK-5387.2021.1.1.

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