Normalized Rotational Multiple Yield Surface Framework (NRMYSF) stress-strain curve prediction method based on small strain triaxial test data on undisturbed Auckland residual clay soils

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Abstract. Small strain triaxial test measurement is considered to be significantly accurate compared to the external strain measurement using conventional method due to systematic errors normally associated with the test. Three submersible miniature linear variable differential transducer (LVDT) mounted on yokes which clamped directly onto the soil sample at equally 120° from the others. The device setup using 0.4 N resolution load cell and 16 bit AD converter was capable of consistently resolving displacement of less than 1µm and measuring axial strains ranging from less than 0.001% to 2.5%. Further analysis of small strain local measurement data was performed using new Normalized Multiple Yield Surface Framework (NRMYSF) method and compared with existing Rotational Multiple Yield Surface Framework (RMYSF) prediction method. The prediction of shear strength based on combined intrinsic curvilinear shear strength envelope using small strain triaxial test data confirmed the significant improvement and reliability of the measurement and analysis methods. Moreover, the NRMYSF method shows an excellent data prediction and significant improvement toward more reliable prediction of soil strength that can reduce the cost and time of experimental laboratory test.

1. Introduction
The prediction of stress-strain behaviour of soil always is helpful when laboratory experimental works are complicated and expensive. [1] has proposed a unique relationship between minimum mobilized friction angle $\phi_{\text{min mob}}$ and axial strain $\varepsilon_A$ irrespective of the effective stress applied to a consolidated drained triaxial test. Therefore, if $\phi_{\text{min mob}}$ is plotted against $\varepsilon_A$ for experiments conducted at various effective stresses, the curve of all experiments will overlap to indicate a unique relationship. An intrinsic property of soil obtained from shear strength against effective stress plotting allows good prediction of shear strength at various effective stresses. [2] applied the prediction of stress-strain response based on curvilinear shear strength envelope using remoulded saturated specimens of granite residual grade VI from Rawang. The predicted stress-strain responses were satisfactory compared to the laboratory data for 50 kPa, 100 kPa and 200 kPa of effective stresses.

Small strain triaxial test allows more reliable measurement and investigation of soil stiffness under working conditions where pre-failure behaviour is generated [3]. Local measurement of soil sample deformation in the small strain triaxial system has eliminated most of systematic errors, such as bedding and other effects at the top and bottom platens of the triaxial test setup [4]. However, the experimental
setup of small strain triaxial test is complicated and cumbersome compared to the conventional triaxial test. Therefore the stress-strain response prediction using curvilinear shear strength envelope will allows reliable estimation of shear strength parameters without actually conducting the experiment. This paper discusses the new approach of prediction method using improvised curvilinear shear strength envelope, called normalized curvilinear shear strength envelope. The new proposed method will be applied on small strain triaxial test data using undisturbed Auckland residual clay soils.

2. Small strain triaxial test
The interest in pre-failure behaviour of soil has been generated by the observations that most of the soil under working condition is subjected to strains smaller than 0.1% [3;5]. Significant improvement has been achieved in the small strain testing since it was developed in early 1970s. Scholey et al. [6] have done a review on small strains instrumentations and characterized five components for the ideal system; the testing system must be at low cost, simplicity of installation and operation, does not interfere with soil behavior, highly accurate and capable of resolving very small strains ( < ± 10⁻³%) and for cyclic systems, low hysteresis and rapid response.

2.1 Small strain triaxial test instrumentation and modifications
Three submersible miniature linear variable differential transducers (LVDTs) were used in the study. The linear range of displacement that can be measured by LVDTs is about ±0.64 mm. From the preliminary test, it was observed that the displacement can be measured by this particular LVDT was 3.5 % strain at maximum, and therefore external LVDT is still required to enable measurement of larger strain level. The transducer is 16.8 mm length and only 5 g in weight. The used of 16 bit A/D converter in the system produced the resolution better than 1µm over a 100 mm gauge length. It enables the data acquisition system to record 1520 numbers of data for every one second interval for this experimental setting.

Internal load cell with a capacity of 3 kN was used in this particular test. It was able to resolve forces less than 0.5 N resulting in the resolution of axial stress less than 0.1 kPa for 75 mm diameter specimen. The load cell was connected to 19 mm diameter loading piston, which was ridden through the low friction sleeve on top of Lucite chamber. A stainless steel restrain cap attached to the bottom of internal load cell to restrict a deviation of sample during the saturation and consolidation process thus significantly minimize the error induced by the rotation of the top cap.

The system was pressurized using Dow-Corning #200 silicone oil since the system requires non-conductive cell fluid for unsealed electronic devices. The silicone oil has extremely low viscosity under a wide range of temperature and optically transparent, thus ideal for observation purposes during the test. The oil also does not degrade the O-ring seals or the latex membranes encapsulated the specimens. In addition, its large molecular structure helps prevent penetration through the membrane and eliminates the osmotic pressure gradient across it.

The small strain triaxial apparatus developed for this particular study uses stepper motor to raise up the pedestal in shearing stage. The stepper motor consists of 200 steps per revolution or 1.8 degree per step connected to a ball screw which has a pitch of 5mm. The rotation of stepper motor causes the working surface of the table to raise or lower by 5 mm for each revolution of the screw. The stepper motor is driven from the micro-step controller which is an intelligent device including an RS323 interface to a computer for complete control of unit. This controller divides each step of the motor into 64 parts, so each revolution consists of 12,800 discrete steps. Is assuming no backlash in the system, then each step moves the working surface by 0.390626 micrometres.

Figure 1 shows the diagram of small strain triaxial system developed for this project. Minor modification was made to the LVDTs yokes by gluing sand paper around the yoke’s perimeter. Sand papers attached around the individual yoke significantly improved the reading stability by providing better grip between soil, rubber membrane and the rubber band.

The technique adopted in mounting the LVDTs is relatively simple and cheaper compared to the system described in the [7]. Three yokes were clamp using normal rubber band onto the 75mm diameter
soil specimen at three points, each 120° from the other. To improve the grip of yokes, two rubber band were used for both top and bottom position of yokes. Axial strains were measured over a gauge length of 100mm at the middle of specimen. The advantages of using individual yoke are it can easily accommodate any specimen size and no restriction for specimen barreling at very large strains. Three stainless steel guide rods were used to ensure the position of LVDTs on yokes are vertical in direction. This can be achieved with the top and bottom yokes for each LVDT position are horizontally mounted and parallel to each other. Each designated yoke has a series of hole to assist the alignment and the guide rods were polished to ease the process.

Figure 1. Experimental set up using three submersible miniature LVDTs attached locally on the peripheral of 150 mm height and 75 mm diameter specimen.

Figure 2. Schematic diagram of three-part split mould and the processes of preparing and plastering the ends of sample.
2.2 Sample preparations

Samples of undisturbed Auckland residual clay were taken from the Orewa site, located approximately 37 km in northern part from Auckland City central. Undisturbed samples were obtained from shallow sampling pit by pushing 200 mm in diameter by 200 mm length steel tube, fitted with a low angle cutting shoe into the ground at desired level using a hydraulic jack. The tubes were then recovered and samples ends were levelled and sealed with thin rubber disk between the soil and caps to preserve natural water content. Soil sample was extruded using hydraulic jack and cut into four quarters using band saw. Each quarter of material was trimmed to the 75 mm in diameter and 150 mm length using hand operated soil lathe. To ensure square and parallel ends of the specimen, a special three part split mould was used to cut the soil ends. Top and bottom restrain rings assist in holding the mould when the trimming of specimen’s ends was done. Thin layer of quick-setting plaster will be applied on both ends of the specimens if the height measurement was not consistent, within ± 0.2mm as shown in figure 2. This method has been successfully applied in eliminating the bedding errors for the small strain triaxial testing [4].

3. Small strain triaxial experimental results and discussion

Consolidated undrained (CU) small strain triaxial test were conducted on fully saturated undisturbed Auckland residual clays at four different effective confining pressures. The analysis of experimental data performed under various effective confining pressures is basically finding the relationship between the axial strain \( \varepsilon_A \) and minimum mobilized friction angle \( \phi_{\text{min,mod}} \) value. Details selection of \( \varepsilon_A \) ranging from 0.05 % to 2.0 % strains level produced excellent combined curvilinear shear strength envelope as shown in figure 3. Figure 4 shows the intrinsic properties of undisturbed Auckland residual clays under various \( \varepsilon_A \) considerations based on the combined curvilinear shear strength envelope.

![Combined curvilinear shear strength envelope](image)

Figure 3. Combined curvilinear shear strength envelope for selection of \( \varepsilon_A \) ranging from 0.05 % to 2.0 % strains level.
**Figure 4.** Intrinsic properties of undisturbed Auckland residual soil based on small strain triaxial test.

**Figure 5.** Comparison between predicted deviatoric stress values using Rotational Multiple Yield Surface method and laboratory data based on intrinsic properties of Auckland residual soil.
The unique relationship or intrinsic properties of soil allows good prediction of deviatoric stress properties as shown in figure 5. Good agreement between predicted (dotted line) and laboratory data (continuous line) at 60kPa, 220kPa, 500kPa and 800kPa proved that good prediction is applicable to a wide range of stress from low to high effective stresses. A unique relationship $\phi_{min, mob} - \epsilon_A$ irrespective of the value of effective stress applied proves that there is simultaneous rotation about shear strength axis during the compression of specimen in shearing stage. The existence of this unique relationship is the keystone of the Rotational Multiple Yield Surface Framework (RMYSF) framework introduced by [8]. Therefore, prediction of stress-strain behaviour at any effective stress is possible by utilizing this unique relationship. Moreover, the good prediction obtained from this unique relationship can reduce the cost and time of conducting relatively complex small strain triaxial test.

4. Normalized Rotational Multiple Yield Surface Framework (NRMYSF) stress-strain curve prediction

Further improvement of stress-strain behaviour prediction method is proposed in this study by addressing the fact that each individual sample was failed at different strain levels for various effective stresses applied in the experimental works as shown in figure 6. Normally the same type of sample at lower effective stress will fail at lower strains level compared to the sample at higher effective stress. Data analysis of laboratory data shows that value of strain at failure or 60kPa pressure was 1.4% followed by 2 %, 2.3% and 2.5 % strains for 220kPa, 500kPa and 800kPa respectively. Normalized strain is calculated based on the ratio of maximum strains value over maximum strains of failure for respective laboratory data. Figure 7 shows stress-strain curves (dotted lines) of deviatoric stress against axial strains that were calculated based on equation below.

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\text{Normalized strain ratio} = \frac{\text{Maximum strain at failure}}{\text{Maximum strain at failure of (selected stress level)}} \times \text{strains value}(\epsilon_a)
\]

![Stress-strain curves of laboratory small strain triaxial test at various effective confining pressures.](image-url)
**Figure 7.** Stress-strain curves of small strain triaxial laboratory data (continuous line) and normalized stress-strain (dotted lines) curves.

**Figure 8.** Stress-strain curves of renormalized strains (black dotted line) and laboratory data (continuous line).
Table 1. Comparison between laboratory data and predicted deviatoric stress form Rotational Multiple Yield Surface (RMYSF) and Normalized Rotational Multiple Yield Surface (NRMYSF).

| Inversed normalized strains (%) | 220 kPa Effective confining pressure | 500 kPa Effective confining pressure |
|---------------------------------|--------------------------------------|-------------------------------------|
|                                 | Deviatoric stress (kPa) Lab data     | Deviatoric stress (kPa) RMYSF       | Deviatoric stress (kPa) Lab data | Deviatoric stress (kPa) RMYSF |
| 0.5                             | 166.80                               | 160.00                              | 167                             | 213.60                         | 226.50                         | 214 |
| 0.8                             | 197.10                               | 192.00                              | 197                             | 254.46                         | 270.00                         | 254 |
| 1.0                             | 209.80                               | 200.00                              | 210                             | 273.76                         | 284.00                         | 273 |
| 1.2                             | 218.38                               | 207.00                              | 218                             | 287.09                         | 298.50                         | 288 |
| 1.5                             | 225.55                               | 218.00                              | 225                             | 301.60                         | 314.50                         | 301 |

Figure 7 shows deviatoric stresses (dotted lines) plotted against normalized strains curves are slightly deviated from laboratory data (continuous line) except for curve at 800kPa. This deviation is mainly due to the application of normalized strain ratio. The stress-strain plot in figure 8 shows excellent agreement between predicted and laboratory data where the normalized strains value is multiplied with inverse normalized strains ratio. Based on the prediction of deviatoric stress value between the RMYSF and NRMYSF methods as shown in figure 5 and figure 8 respectively, the latter technique has significantly improved the accuracy of shear strength prediction especially for the stress-strain curve at 220 and 500 kPa as shown in table 1.

5. Conclusion
The NRMYSFF method is proven to produce an excellent prediction of stress-strain curve for undisturbed Auckland residual soils compared to the existing RMYSF. The simulated stress-strain curves were exceptionally matched the laboratory data of small strain triaxial test.

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