GENERAL RELATIVISTIC MACHIAN UNIVERSE

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Abstract

The Machian Universe, is usually described with Newtonian Physics. We give an alternative General Relativistic picture for Mach’s Universe. As such, we show that, in the correct Machian limit, Schwarzschild’s metric is coherent with Robertson-Walker’s, on condition that there be a cosmological constant, or the Universe’s rotation — or both. It is now confirmed that the Universe is accelerating, so the former condition applies. The latter was also confirmed one more time with the recently discovered NASA space probes anomalies. From Kerr-Lense-Thirring solution, we find an inverse scale-factor dependent angular speed; we then, show that the cosmological ”constant” may have Classically originated from a centrifugal acceleration field.
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1. INTRODUCTION

The purpose of this paper is (a) to review the Machian picture of the Universe, according to a gravitational theory made with Special Relativity plus Newtonian gravity; (b) to check a possible relation between Schwarzschild’s metric, and Robertson-Walker’s, from the viewpoint of the Machian picture in (a); (c) to analyze Lense-Thirring metric, and to interpret the angular speed derived from it, in terms of the Machian picture above (frame-dragging); (d) to state the equivalence between a cosmological (lambda) ”constant” field of accelerations, and the centrifugal one, originated by the rotation of the Universe; (e) to conclude about the Machian limit of General Relativity Theory, with a $R^{-1}$ dependent angular speed, as also calculated by this author in other prior papers.

The Machian intersection of General Relativity with Newtonian Mechanics, is not necessarily in the low or weak gravitational limit.

We consider an extension for a paper by Barrow (Barrow, 1988): while he showed that Newtonian Cosmology is equivalent with its General Relativistic version, we shall show that Schwarzschild’s metric and Robertson-Walker’s, yield the same results in the Machian limit. This is novel in the literature, and extends our recently published papers (Berman, 2007b; 2008; 2008a,b), where we have hinted on the rotation of the Universe. This last hypothesis could explain the Pioneer anomaly, or, otherwise, the Pioneer anomaly could be the experimental justification for the rotation of the Universe. Our explanation of the Pioneer anomaly as a Machian effect of rotation, was intended to explain, that an ubiquitous field of centripetal accelerations of cosmological and Machian origin, existed for any observer in the Universe. A confirmation of my theory was made by NASA, because not only the two Pioneer’s 10 and 11 were subject to such anomaly, but also other space probes, except those in closed orbits around the Sun. In fact, this shows that my explanation applies well, and there is no contradiction with the inexistence of the anomaly in closed orbits, because such motion is local, not cosmological. Remember that hyperbolic orbits extend to infinity.
Berman (2008; 2008a,b) showed that Robertson-Walker’s metric has hidden a rotational state in addition to its expanding nature. As the cosmological constant has now a relevant place in Modern Cosmology, both conditions (non-null rotation and non-null cosmological constant) are possible. It will be shown that, as well as Barrow’s proof for the equivalence between Newtonian and General Relativistic cosmologies, there is a kind of equivalence between Schwarzschild’s metric and Robertson-Walker’s: the intersection represents the Machian limit.

It will be studied, that the rotation of the Universe may respond for the existence of a cosmological “constant”, and that Kerr-Lense-Thirring metric points out to an angular speed varying with the inverse of the scale-factor.

2. THE NEWTONIAN-MACHIAN PICTURE

Barrow (1988) has shown that, with the adoption of conservation of energy, according to Newtonian Mechanics, plus conservation of energy, according to Thermodynamics, we obtain the usual Robertson-Walker’s field equations for energy density and cosmic pressure. The reason given is that: a) according to General Relativity, local Physics means Newtonian gravitation; b) Robertson-Walker’s metric is homogeneous, so that, each point is equivalent to any other, and thus, Newtonian gravitation applies everywhere.

If, then, Newtonian Physics means also cosmological Physics, we must take a look, as we shall do below, on the possibility that Schwarzschild’s metric includes a limit for the validity of Robertson-Walker’s metric. We shall indeed show that for a Machian Universe, in the sense of Berman’s zero-total-energy approach (Berman, 2007; 2007a; 2007b; 2008; 2008b), this requirement is fulfilled, so that, in fact, ”local” means ”global” phenomena.

The importance of the present paper resides in the demonstration that the Machian-Newtonian view of the Universe, coincides in a certain sense, with the Machian-General Relativistic picture, but of necessity, we have to consider either the existence of a cosmological constant or, of a spin of the Universe — or both.

Consider a Newtonian description of a rotating sphere (the ”causally connected Universe”), endowed with a cosmological constant additional energy density. The total energy is given by:
\[ E = E_i + E_g + E_L + E_\Lambda = 0 \]  \hspace{1cm} (1)

where, the right hand side energy terms represent inertia \( Mc^2 \), gravitation \( -\frac{GM^2}{R^2} \), rotation \( \frac{L^2}{MR^2} \) and cosmological constant’s \( \frac{\Lambda R^3}{6G} \), where \( L, \Lambda \) and \( R \) stand respectively for the Universe’s total spin, cosmological ”constant”, and its ”radius”. If we consider that relation (1) remains time-invariant \( \dot{E} = 0 \), the only solution involving a time-varying scale factor \( \dot{R} \neq 0 \), implies the following generalised Brans-Dicke equalities:

\[ \frac{GM}{c^2R} = \gamma_G \]  \hspace{1cm} (2)

\[ \frac{GL}{c^2R^2} = \gamma_L \]  \hspace{1cm} (3)

\[ \Lambda R^2 = \gamma_\Lambda \]  \hspace{1cm} (4)

where all the \( \gamma \)'s are constants. This resembles Brans-Dicke approximate Machian relation (Brans and Dicke, 1961). Of course, another possibility would be \( \dot{R} = 0 \), with another set of equalities.

In consequence,

\[ R \propto M \propto L^{1/2} \propto \Lambda^{-1/2} \]  \hspace{1cm} (5)

The Machian picture, generally means that (2), (3), (4) and, thus, (5) apply to the Universe, on condition that we remember the following:

**First**: (sphericity postulate) The Universe, for each and any observer, resembles a "ball", of approximate spherical shape, of radius \( R \) and mass \( M \).

**Second**: (egocentric observers’ postulate). Each observer finds himself in the center of the "ball".

**Third**: (observers’ democratic principle). As a consequence, the Universe is homogenous so that, any location is equivalent to any other one.
Fourth: (intersection of Newtonian gravity with Special Relativity). We have associated Einstein’s relation from Special Relativity with Newtonian gravity. General Relativity theory, would be similarly treated by Special Relativity, but substituting Newtonian gravity, by other field equations.

3. GENERAL RELATIVISTIC-MACHIAN PICTURE

Local Physics extends globally with the same ”laws” – if Newtonian Physics is valid locally, it is also valid globally; but, we shall show that, if General Relativity is applied locally through Schwarzschild’s metric, it must be equivalent to Robertson-Walker’s in the large. This should be the General Relativistic Machian picture.

Let us show that the above criterion is possible. Consider Schwarzschild’s metric with a cosmological term:

\[ ds^2 = g_{00} dt^2 - g_{00}^{-1} dr^2 - d\sigma^2 \]

where,

\[ g_{00} = 1 - \frac{2GM}{c^2 R} + \frac{GMAR}{3c^2} \]

If we associate the Schwarzschild’s \( g_{00} \) with the Machian Brans-Dicke equalities above, so that, any point of space is equivalent to any other, at each one, we have a kind of Schwarzschild’s \( g_{00} \), given by:

\[ g_{00} = 1 - 2\gamma G + 4\gamma A = \Gamma > 0 \]

where \( \Gamma \) is a constant. A constant \( g_{00} \), can be described as Robertson-Walker’s temporal coefficient (it can be always made equal to one).

In order to give ”power” to the above, we must understand that:

(a). the radial coordinate may be associated with the scale-factor in the Machian limit.
(b). we can always reparametrize the metric coordinates by making:

\[ dx^i \equiv R^2(t)dx^i \quad \text{,} \quad (i = 1, 2, 3) \]
and,

\[ dt' \equiv dt \]

We now see why Robertson-Walker’s metric (at least, the flat case) is a kind of isomorphic to Schwarzschild’s and, as we shall see below, its rotating analogues, in the Machian limit.

The reader can check the above procedure from Poisson’s book (Poisson, 2004).

We also see why Berman (2007c), has hinted that the Universe is to be considered a white (or, generally speaking, black) hole, when General Relativity with R.W.’s metric with cosmological constant, is taken into account. The fact that there is a limit where both metrics yield the same picture, has a similarity with Barrow’s equivalence between Newtonian and General Relativistic Cosmologies.

4. LENSE-THIRRING METRIC AND MACH’S PRINCIPLE

In order to match two spacetime metrics, each one valid in separate but adjacent regions, there is a "thin-shell" formalism (Poisson, 2004). So, we first "freeze" the expansion of the Universe, keeping track only of the rotation; the final results, in what refers to the rotation of the Universe, are independent of the expansion, sufficing to remember that in Cosmology, what we shall call as \( R \) bellow, is in fact an increasing function of time.

The "thin-shell" formalism (Poisson, 2004), begins with a comparison of exterior and interior metrics for a rotating shell. In the low rotation limit, the Kerr-metric gives origin Lense-Thirring description of the exterior of a rotating shell:

\[
\begin{align*}
    ds_{\text{out}}^2 &= -f \, dt^2 + f^{-1} \, dr^2 + r^2 \, d\Omega^2 - \frac{4GMa}{r} \sin^2 \theta \, d\phi \, dt \\
    \text{where,} \\
    f &= 1 - \frac{2GM}{r} \\
    \text{With decreasing values for } r, \text{ we shall reach a } "\text{cut off}" \text{ radial value,} \\
    r &= R
\end{align*}
\]
under which the shell is located, and where we find the "induced" metric, as viewed from the exterior,

\[ ds_\Sigma^2 = -F dt^2 + R^2 d\Omega^2 - \frac{4GMa}{R} \sin^2 \theta \ d\phi \ dt \]

(12)

where,

\[ F = 1 - \frac{2GM}{R} \]

(13)

The rotation parameter \( a \) is given in terms of the angular momentum \( J \) and the mass \( M \), by,

\[ a^2 = \frac{J^2}{M^2} \]

(14)

Consider now that we make the above metric, to include an opposite rotating speed, by means of a diagonalization, and described by,

\[ \psi = \phi - \omega_0 t \]

(15)

so that, the diagonalization becomes,

\[ ds_\Sigma^2 \approx -F dt^2 + R^2 d\bar{\Omega}^2 \]

(16)

where,

\[ d\bar{\Omega}^2 \equiv R^2 \left( d\theta^2 + \sin^2 \theta d\psi^2 \right) \]

(17)

The angular speed \( \omega_0 \), is found to be,

\[ \omega_0 = \frac{2GM}{R^3} a \]

(18)

We now write the interior metric, which should be “cut-off” at the same radial distance, which we now write as \( \rho = R \),

\[ ds_{int}^2 \approx -F dt^2 + d\rho^2 + \rho^2 d\bar{\Omega}^2 \]

(19)

By working with a perfect fluid model, Poisson shows that the shell must move with angular speed \( \omega \), given by,
\[ \omega = \omega_0 \left[ \frac{1 - F}{(1 - \sqrt{F})(1 + 3\sqrt{F})} \right] \] \hspace{1cm} (20)

We now consider an observer at constant \( \psi \), moving then, with angular speed,
\[ \omega_0 = \frac{d\phi}{dt} \] \hspace{1cm} (21)

This rotation is relative to the "fixed" stars, obtained from the exterior metric, which is Lorentzian at infinity, i.e., when \( r \to \infty \), we retrieve Schwarzschild’s metric. In the non-rotating frame, the shell’s angular speed is given by,
\[ \omega_1 = \frac{d\phi}{dt} = \frac{d\psi}{dt} + \omega_0 = \omega_0 \left[ \frac{1 + 2\sqrt{F}}{(1 - \sqrt{F})(1 + 3\sqrt{F})} \right] \] \hspace{1cm} (22)

In the Machian Universe, it is the fixed stars that are no more fixed, i.e., they rotate relative to the shell, and from relations (2), (3), (4) and (5), we have,
\[ F = 1 - 2\gamma G = \text{constant} \] \hspace{1cm} (23)

and, then,
\[ \omega_1 = \alpha \omega_0 \] \hspace{1cm} (24)

For \( \gamma G \approx \frac{1}{2} \), we have \( F \approx 0 \), and, then,
\[ \omega_1 \approx \omega_0 = \frac{2GM}{R^3} a \] \hspace{1cm} (25)

or,
\[ J = a M = \frac{\omega_1}{2} R^3 \] \hspace{1cm} (26)

The Machian spin of the Universe \( L \), was obtained as being proportional to \( R^2 \), so that, the Machian angular speed, is proportional to \( R^{-1} \), according to (26).

The \( R^{-1} \) dependence of the angular speed, has been found by Berman in several other contexts, (Berman, 2007b; 2008a; 2008b; 2008c; 2008d).

5. CLASSICAL ORIGIN FOR THE DARK ENERGY
In last Section, we must remember to say that the condition for the signature of the metric not to be altered, is that \( F > 0 \), or \( \gamma_G < \frac{1}{2} \).

From Raychaudhuri’s equation for a perfect fluid, we obtain the following Robertson-Walker’s metric result,

\[
\ddot{R} = \left[ -\frac{8}{3} (\rho + 3p) + \frac{1}{3} \Lambda \right] R \quad .
\] (27)

It is clear that the cosmological term represents the repulsive acceleration,

\[
a_\Lambda = \frac{1}{3} \Lambda R \quad .
\] (28)

On the other hand, we have shown that the Universe undergoes a rotational state, so that, there is a Machian centrifugal acceleration,

\[
a_{cf} = \omega^2 R \quad .
\] (29)

From the Machian relations (2), (3), (4) and (5), and, from the result of last Section, that the angular speed depends on \( R^{-1} \), we find that both accelerations above, depend on \( R^{-1} \), and are, from the Machian viewpoint, equivalent. Do not forget that \( \Lambda \) and \( \omega^2 \) should depend on \( R^{-2} \).

It does not matter if we say that the Universe rotates, or that the Universe is endowed with a cosmological "constant" term. We would say that the spin of the Universe stands for the Classical origin of the cosmological constant; and we need not refer to Quantum phenomena in order to generate a lambda-Universe.

6. CONCLUSIONS AND PREDICTIONS

The main result of this paper, depends directly on relation (8); this relation implies that local \( g_{00} \) is equal to a positive constant, when applied for the Universe, and this entails either the rotation of the Universe or a given cosmological constant, or both. The latter, implies a deep new look on cosmological theories, and in Berman (2008; 2008a,b), he showed that Robertson-Walker’s metric has a hidden rotational character, along with the usual evolutionary property. The lambda-accelerating Universe was also confirmed by recent observations.
That being the case, we predict that the left-handed Universe, is caused by rotation; global statistical analysis of rotating clusters of galaxies, and chaotic motions in the Universe, must show the left-handed property, due to rotation. All phenomena, like violation of parity and matter-antimatter asymmetry, are explained likewise. A lambda Universe means perhaps a rotating one. This makes us predict that, not only the Universe is left-handed (Barrow and Silk, 1983), but if it will be paid attention, to chaotic phenomena in the Universe, and, also to rotational states of galaxies and clusters of them, a preference for the left-hand must be found. Not only, parity violations, but also barion-antibarion asymmetries (Feynman et al., 1962) will be explained in terms of the said rotation of the Universe.

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