A Novel Analytical Method Suitable for Coupled Electromagnetic Field of Circuit

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Abstract—A novel analytical method suitable for coupled electromagnetic field of a circuit is proposed in this paper. In a high frequency circuit and high-frequency converter, skin effects are obvious, and the variations in resistance and inductance values depend on frequency. In addition, the voltage and current distribution changes of a high frequency circuit generated with a high-frequency converter during dynamic switching process are complicated and depend on time. A novel analytical method suitable for coupled electromagnetic field of circuit in parameter optimization design of high-frequency circuit and high-frequency converter is proposed in this paper. The proposed method considers the influence of skin effect and coupled electromagnetic field on parameter variation simultaneously. According to the law between parameter variation and line length, the calculation process of parameter optimization will be simpler and more effective.

1. INTRODUCTION

Multiphysics analysis has become an analytical method widely used in various research fields. The electromagnetic coupling relationship of space multiphysics can be represented by a series of Maxwell’s equations.

In high frequency circuit, eddy current phenomena are common [1], and skin effect and proximity effect are obvious [2, 3]. When a conductor size is larger than the skin depth δ, the change of resistance and inductance values depends on frequency. The accurate prediction of eddy current loss and current and voltage distribution with skin effect are important [4, 5]. In addition, the multi-physics analysis of high frequency converter system is becoming more important in improving efficiency and reliability. Among them, the relationship between multi-physics and skin effect caused by transient voltage and current changes is an important problem to be solved [6, 7]. In the process of analysis, PEEC model and finite-element method are proposed for skin effect [8, 9].

In a high-frequency circuit and high-frequency converter, dynamic switching process can cause complex electromagnetic coupling relationship and circuit parameter changes, and traditional skin effect formulas are not applicable.

Thus, some novel circuit model analytical methods are proposed [10–13]. A dual-loop model based on boost and buck converter is proposed, and the steady-state field of typical frequency point is simulated in frequency domain [14]. However, the distribution law of high frequency electromagnetic field cannot be used to accurately optimize loop parameters.

So a simplified calculation method of resistance and inductance parameter optimization considering the influence of high frequency electromagnetic field needs to be proposed in order to balance complex voltage and current changes in circuits.
For a high-frequency circuit, complex voltage and current changes caused by the variations in line impedance value depend on frequency and skin effect. For a high-frequency converter, complex voltage and current changes are caused by complex coupling electromagnetic field in dynamic switching process.

2. COUPLED ELECTROMAGNETIC FIELD OF CIRCUIT

Maxwell’s equations represent a complex electromagnetic coupling relationship in conductors and circuits. It can be seen from Equation (1) that the coupling relationship is related to both circuit frequency and field parameters. On the contrary, conductor and circuit parameter optimization can be solved based on the influence of frequency and space electromagnetic field on the conductor and circuit.

\[
\begin{align*}
\text{curl}\mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \\
\nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \\
\text{div}\mathbf{D} &= \rho \\
\nabla \cdot \mathbf{D} &= \rho \\
\text{curl}\mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\text{div}\mathbf{B} &= 0 \\
\nabla \cdot \mathbf{B} &= 0
\end{align*}
\]

(1)

where \( \mathbf{E} \) indicates the electric field, \( \mathbf{H} \) the magnetic field, \( \mathbf{D} \) the electric flux density, \( \mathbf{B} \) the magnetic flux density, \( \mathbf{J} \) the current density, \( \varepsilon \) the dielectric constant, \( \sigma \) the conductivity, and \( \mu \) the permeability.

Equation (1) uses two symbolic expressions to illustrate the complex electromagnetic field relationship.

Available from Equation (1),

\[
\begin{align*}
\nabla \times \mathbf{H}(\mathbf{r}, t) &= \sigma \mathbf{E}(\mathbf{r}, t) + \frac{\partial \varepsilon \mathbf{E}(\mathbf{r}, t)}{\partial t} \\
\nabla \times \mathbf{E}(\mathbf{r}, t) &= -\frac{\partial \mu \mathbf{H}(\mathbf{r}, t)}{\partial t} \\
\nabla \times \nabla \times \mathbf{E}(\mathbf{r}, t) &= -\sigma \mu \frac{\partial \mathbf{E}(\mathbf{r}, t)}{\partial t} - \varepsilon \mu \frac{\partial^2 \mathbf{E}(\mathbf{r}, t)}{\partial t^2} \\
\nabla \times \nabla \times \mathbf{H}(\mathbf{r}, t) &= -\sigma \mu \frac{\partial \mathbf{H}(\mathbf{r}, t)}{\partial t} - \varepsilon \mu \frac{\partial^2 \mathbf{H}(\mathbf{r}, t)}{\partial t^2}
\end{align*}
\]

(2)

\( \mathbf{E}(\mathbf{r}, t) \) and \( \mathbf{H}(\mathbf{r}, t) \) are related to vector radius and time.

Define a closed circuit loop \( l_1 \) and coordinates \( x, y, z \) in Fig. 1 [14].

Figure 1. Space electromagnetic field distribution of a circuit loop.

The dynamic voltage added in \( l_1 \) is \( V \), and the dynamic current flowing through \( l_1 \) is \( I \). The components of electric and magnetic field vector on coordinate axis are: \( \mathbf{E}_x, \mathbf{E}_y, \mathbf{E}_z, \mathbf{E}_{-x}, \mathbf{E}_{-y}, \mathbf{E}_{-z} \) and \( \mathbf{H}_x, \mathbf{H}_y, \mathbf{H}_z, \mathbf{H}_{-x}, \mathbf{H}_{-y}, \mathbf{H}_{-z} \). In Fig. 1, the center coordinate of \( l_1 \) is 0, and the induced magnetic field in center has a symmetrical and radial effect on \( l_1 \), such as \( \Delta \mathbf{H}_{z1}, \Delta \mathbf{I}_1 \). The change of
induced magnetic field causes the change of induced electric field, such as $\Delta E_x \sim x_5$, $\Delta E_y \sim y_5$. In $l_1$, $-l_{en1}/2 \leq x \leq l_{en1}/2$, $-l_{en2}/2 \leq y \leq l_{en2}/2$.

Equation (2) can be converted to numerical calculation expression,

$$
\begin{align*}
\frac{\partial^2 E(r, t)}{\partial r^2} &= \sigma \mu \varepsilon \frac{\partial E(r, t)}{\partial t} + \varepsilon \mu \frac{\partial^2 E(r, t)}{\partial t^2} \\
\frac{\partial^2 H(r, t)}{\partial r^2} &= \sigma \mu \varepsilon \frac{\partial H(r, t)}{\partial t} + \varepsilon \mu \frac{\partial^2 H(r, t)}{\partial t^2}
\end{align*}
$$

(3)

Assuming $E(r, 0) = H(r, 0) = 0$, the Laplace transform of Equation (3) is,

$$
\begin{align*}
\frac{\partial^2 E(r, s)}{\partial r^2} &= (\sigma \mu s + \varepsilon \mu s^2) E(r, s) \\
\frac{\partial^2 H(r, s)}{\partial r^2} &= (\sigma \mu s + \varepsilon \mu s^2) H(r, s)
\end{align*}
$$

(4)

In Fig. 2, when $H_z(r, s) \neq 0$, it is incident perpendicularly from above the center of the plane loop $l_1$, which is $\theta = 90^\circ$ with respect to the plane loop $l_1$. Due to the symmetry of $E(r, s) = E(-r, s)$ and $H(r, s) = H(-r, s)$, the boundary conditions in $l_1$: $E_x(r, s) = 0$, $|x| - |y| \neq 0$, $\theta = 90^\circ$, $-l_{en1}/2 \leq x \leq l_{en1}/2$, $-l_{en2}/2 \leq y \leq l_{en2}/2$, solve $E(r, s)$ and $H(r, s)$, where $a_1$ and $b_1$ are related to the incident angle $\theta$ in Equation (5),

$$
\begin{align*}
E(r, s) &= 2a_1 \cosh \left( \sqrt{\frac{(\mu \sigma s + \varepsilon \mu s^2) \cdot \sqrt{x^2 + y^2 + z^2}}{\sin \theta}} \right) \hat{r}, a_1 = A_1 \sin \theta, \theta = 90^\circ \\
H(r, s) &= 2b_1 \cosh \left( \sqrt{\frac{(\mu \sigma s + \varepsilon \mu s^2) \cdot \sqrt{x^2 + y^2 + z^2}}{\sin \theta}} \right) \hat{r}, b_1 = B_1 \sin \theta, \theta = 90^\circ \\
E(r, s) &= E_x(r, s) \hat{x} + E_y(r, s) \hat{y} + E_z(r, s) \hat{z} \\
H(r, s) &= H_x(r, s) \hat{x} + H_y(r, s) \hat{y} + H_z(r, s) \hat{z} \\
E_z(r, s) &= 0
\end{align*}
$$

(5)

$$
\begin{align*}
\nabla \times H(r, s) &= \left( \frac{\partial H_z(r, s)}{\partial y} - \frac{\partial H_y(r, s)}{\partial z} \right) \hat{x} + \left( \frac{\partial H_z(r, s)}{\partial x} - \frac{\partial H_x(r, s)}{\partial z} \right) \hat{y} + \left( \frac{\partial H_y(r, s)}{\partial x} - \frac{\partial H_x(r, s)}{\partial y} \right) \hat{z} \\
&= (\sigma + s) (E_x(r, s) \hat{x} + E_y(r, s) \hat{y} + E_z(r, s) \hat{z})
\end{align*}
$$

(6)

In Equation (7), $E_x$, $E_y$, $E_z$ and $H_x$, $H_y$, $H_z$ can be represented using Eqs. (5), (6). The change increments affected by induced magnetic field and electric field are also obtained.
\[ \begin{align*}
\frac{\partial H_x(r, s)}{\partial y} - \frac{\partial H_y(r, s)}{\partial z} &= -2B_1 \sqrt{\sigma \mu s + \varepsilon \mu s^2} (|y| - |z|) \sinh \left( \sqrt{\sigma \mu s + \varepsilon \mu s^2} r \right) = (\sigma + \varepsilon s) E_x(r, s) \\
\frac{\partial H_x(r, s)}{\partial z} - \frac{\partial H_z(r, s)}{\partial x} &= -2B_1 \sqrt{\sigma \mu s + \varepsilon \mu s^2} (|z| - |x|) \sinh \left( \sqrt{\sigma \mu s + \varepsilon \mu s^2} r \right) = (\sigma + \varepsilon s) E_y(r, s) \\
\frac{\partial H_y(r, s)}{\partial x} - \frac{\partial H_z(r, s)}{\partial y} &= (\sigma + \varepsilon s) E_z(r, s) = 0 \\
\frac{\partial E_x(r, s)}{\partial y} - \frac{\partial E_y(r, s)}{\partial z} &= 2A_1 \sqrt{\sigma \mu s + \varepsilon \mu s^2} |z| \sinh \left( \sqrt{\sigma \mu s + \varepsilon \mu s^2} r \right) = -\mu s H_x(r, s) \\
\frac{\partial E_x(r, s)}{\partial z} - \frac{\partial E_z(r, s)}{\partial x} &= -2A_1 \sqrt{\sigma \mu s + \varepsilon \mu s^2} |z| \sinh \left( \sqrt{\sigma \mu s + \varepsilon \mu s^2} r \right) = -\mu s H_y(r, s) \\
\frac{\partial E_y(r, s)}{\partial x} - \frac{\partial E_z(r, s)}{\partial y} &= -2A_1 \sqrt{\sigma \mu s + \varepsilon \mu s^2} (|x| - |y|) \sinh \left( \sqrt{\sigma \mu s + \varepsilon \mu s^2} r \right) = -\mu s H_z(r, s)
\end{align*} \]

When \( H_x(r, s) \neq 0 \) and \( 0 < \theta < 90^\circ \) with respect to the plane loop \( l_1 \), due to \( \theta \neq 90^\circ \), \( H_x, H_y \) and \( E_z \) will change so that the symmetry of \( E(r, s) \) and \( H(r, s) \) will also change. The boundary conditions in \( l_1: E_z(r, s) \neq 0, |x| - |y| \neq 0, 0 < \theta < 90^\circ, -len1/2 \leq x \leq len1/2, -len2/2 \leq y \leq len2/2 \), solve \( E(r, s) \) and \( H(r, s) \), where \( a_1 \) and \( b_1 \) are related to the incident angle \( \theta \) in Equation (9),

\[ \begin{align*}
E_x(r, s) &= -2B_1 \sin \theta \sqrt{\sigma \mu s + \varepsilon \mu s^2} (|y| - |z|) \sinh \left( \sqrt{\sigma \mu s + \varepsilon \mu s^2} r \right), \ 0^\circ < \theta < 90^\circ \\
E_y(r, s) &= -2B_1 \sin \theta \sqrt{\sigma \mu s + \varepsilon \mu s^2} (|z| - |x|) \sinh \left( \sqrt{\sigma \mu s + \varepsilon \mu s^2} r \right), \ 0^\circ < \theta < 90^\circ \\
E_z(r, s) &= -2B_1 \cos \theta \sqrt{\sigma \mu s + \varepsilon \mu s^2} |z| \sinh \left( \sqrt{\sigma \mu s + \varepsilon \mu s^2} r \right), \ 0^\circ < \theta < 90^\circ \\
H_x(r, s) &= 2A_1 \sqrt{\sigma \mu s + \varepsilon \mu s^2} |z| \sinh \left( \sqrt{\sigma \mu s + \varepsilon \mu s^2} r \right) \\
&\quad + 2A_1 \cos \theta \sqrt{\sigma \mu s + \varepsilon \mu s^2} |y| \sinh \left( \sqrt{\sigma \mu s + \varepsilon \mu s^2} r \right), \ 0^\circ < \theta < 90^\circ \\
H_y(r, s) &= 2A_1 \sqrt{\sigma \mu s + \varepsilon \mu s^2} |z| \sinh \left( \sqrt{\sigma \mu s + \varepsilon \mu s^2} r \right) \\
&\quad - 2A_1 \cos \theta \sqrt{\sigma \mu s + \varepsilon \mu s^2} |x| \sinh \left( \sqrt{\sigma \mu s + \varepsilon \mu s^2} r \right), \ 0^\circ < \theta < 90^\circ \\
H_z(r, s) &= 2A_1 \sin \theta \sqrt{\sigma \mu s + \varepsilon \mu s^2} (|x| - |y|) \sinh \left( \sqrt{\sigma \mu s + \varepsilon \mu s^2} r \right), \ 0^\circ < \theta < 90^\circ
\end{align*} \]
\begin{equation}
\left\{
\begin{aligned}
0 < E_x (r, s) &= -2B_1 \sin \theta \frac{\sqrt{\sigma \mu + \varepsilon \mu^2}}{\mu s r} \sinh \left( \sqrt{\sigma \mu + \varepsilon \mu^2} r \right) \\
&< -2B_1 \frac{\sqrt{\sigma \mu + \varepsilon \mu^2}}{(\sigma + \varepsilon) r} \sinh \left( \sqrt{\sigma \mu + \varepsilon \mu^2} r \right) \\
0 < E_y (r, s) &= -2B_1 \sin \theta \frac{\sqrt{\sigma \mu + \varepsilon \mu^2}}{(\sigma + \varepsilon) r} \sinh \left( \sqrt{\sigma \mu + \varepsilon \mu^2} r \right) \\
&< -2B_1 \frac{\sqrt{\sigma \mu + \varepsilon \mu^2}}{(\sigma + \varepsilon) r} \sinh \left( \sqrt{\sigma \mu + \varepsilon \mu^2} r \right) \\
0 < E_z (r, s) &= -2B_1 \cos \theta \frac{\sqrt{\sigma \mu + \varepsilon \mu^2}}{(\sigma + \varepsilon) r} \sinh \left( \sqrt{\sigma \mu + \varepsilon \mu^2} r \right) \\
&< -2B_1 \frac{\sqrt{\sigma \mu + \varepsilon \mu^2}}{(\sigma + \varepsilon) r} \sinh \left( \sqrt{\sigma \mu + \varepsilon \mu^2} r \right) \\
\end{aligned}
\right.
\end{equation}

Assuming in $l_1$, the initial value of applied dynamic voltage is $V_0$, and the initial value of applied dynamic current is $I_0$,

\begin{equation}
\left\{
\begin{aligned}
\oint_{l_1} (E_x (r, s) + E_y (r, s)) dl &= -\Delta V \\
\oint_{l_1} H_z (r, s) dl &= -\Delta I \\
V^* &= V_0 - \Delta V \\
I^* &= I_0 - \Delta I \\
Z^* &= V^*/I^*
\end{aligned}
\right.
\end{equation}

According to Equations (8)-(11), the variation of voltage and current, and the variation of line impedance with frequency caused by space electromagnetic field distribution of $l_1$ can also be obtained.

The current through a cylindrical conductor with radius $r_c$ can be expressed as [1],

\begin{equation}
I (s) = \frac{\pi r_c \sigma}{2} \int_{-r_c}^{r_c} E (r, s) dr = \frac{2\pi r_c \sigma I}{\sqrt{\sigma \mu + \varepsilon \mu^2}} \sinh \left( \sqrt{\sigma \mu + \varepsilon \mu^2} \cdot r_c \right)
\end{equation}
The voltage \( V(s) \) across conductor is \( V(s) = E(r_c, s)l \), and the impedance value of conductor is \( Z(s) \),

\[
Z(s) = \frac{V(s)}{I(s)} = \frac{2a_1 \cosh\left(\sqrt{\sigma \mu s + \varepsilon \mu s^2} r_c\right)}{\frac{2\pi r_c a_1 \sigma}{\sqrt{\sigma \mu s + \varepsilon \mu s^2}} \sinh\left(\sqrt{\sigma \mu s + \varepsilon \mu s^2} r_c\right)}
\]

\[
= \frac{l}{\pi r_c \sigma} \frac{\sqrt{\sigma \mu s + \varepsilon \mu s^2} r_c}{\lambda} \coth\left(\frac{\sqrt{\sigma \mu s + \varepsilon \mu s^2} r_c}{\lambda}\right)
\]

(13)

\[
\lambda \coth \lambda = 1 + \lambda^2/3 - \lambda^4/45 + 2\lambda^6/945 - \lambda^8/4725 + \cdots = \sum_{\theta=0}^{\infty} \frac{2^{2\theta} B_{2\theta}}{(2\theta)!} \lambda^{2\theta}
\]

(14)

When variable \( \lambda \) is introduced, \( Z(s) \) can be converted into the form of Equation (16). \( B_{2\theta} \) is Bernoulli number as Equation (15).

\[
\frac{z}{e^z - 1} = \sum_{\theta=0}^{\infty} \frac{B_{2\theta} z^{\theta}}{\theta!}
\]

(15)

where \( B_0 = 1, B_1 = -1/2, B_2 = 1/6, B_3 = 0, B_4 = -1/30, B_5 = 0, B_6 = 1/42, B_7 = 0, B_8 = -1/30, B_9 = 0, B_{10} = 5/66, B_{11} = 0, B_{12} = -691/2730, B_{13} = 0, B_{14} = 6/7, B_{15} = 0, B_{16} = -3617/510 \).

When \( s = 0 \), DC resistance \( R_0 = 1/\pi r_c^2 \sigma \), characteristic frequency \( \omega_c = 1/\mu \sigma \) make \( \beta = \omega/\omega_c \). If \( \beta = \omega/\omega_c = 1 \) and \( \varepsilon = 0 \) at high frequency, \( \lambda \) is,

\[
\lambda = \sqrt{\sigma \mu s + \varepsilon \mu s^2 r_c} = \sqrt{\left(j \frac{\omega}{\sigma \mu} - \varepsilon \left(\frac{\omega}{\sigma \mu}\right)^2\right)} r_c = \sqrt{\left(j \frac{\omega}{\omega_c} - \varepsilon \left(\frac{\omega}{\omega_c}\right)^2\right)} r_c
\]

(16)

\[
Z(j\omega_c)/R_0 = 1 + r_c^2 j/3 + r_c^4/45 = 1.0222r_c^4 + j0.3333r_c^2\ (\varepsilon = 0)
\]

Table 1 shows the changes in impedance value at different frequency ratios \( \beta \).

**Table 1.** Impedance value for cylindrical conductor with radius \( r_c \).

| Frequency ratio \( \beta \) | Various impedance value |
|-----------------------------|------------------------|
| \( \beta = 1 \)            | \( 1.0222r_c^4 + 0.3333r_c^2 \) |
| \( \beta = 5 \)            | \( 1.5555r_c^4 + j1.6665r_c^2 \) |
| \( \beta = 10 \)           | \( 3.2222r_c^4 + j3.3333r_c^2 \) |
| \( \beta = 20 \)           | \( 9.8888r_c^4 + j6.6666r_c^2 \) |
| \( \beta = 50 \)           | \( 56.5555r_c^4 + j16.6665r_c^2 \) |

As shown in Fig. 3 and Table 1, for a cylindrical conductor with radius \( r_c \), when \( \beta \leq 10 \), \( R/R \), and \( X/R \) tend to be equal; the gap between \( R/R \) and \( X/R \) increases when \( \beta \geq 10 \), and the trend of resistance ratio \( R/R \) gradually increases when \( \beta \geq 20 \). It can be seen that the impedance value has different changing trends at different frequency ratios \( \beta \) and different radii \( r_c \).
Figure 3. Various impedance value with different frequency ratio $\beta$ and radius $r_c$.

Considering the influence of space coupled electromagnetic field around circuit loop on circuit parameters, the change of line impedance is shown as Eq. (17),

$$\begin{align*}
\Delta Z_y(r, s) &= \frac{B_1}{2\pi r_c A_1} \frac{1}{\left(\frac{\text{len}1/2 - |y|}{\mu s x^2}\right)} \left(-\text{len}2/2 \leq y \leq \text{len}2/2\right) \\
\Delta Z_x(r, s) &= \frac{B_1}{2\pi r_c A_1} \frac{1}{\left(\frac{\text{len}2/2 - |x|}{\mu s x^2}\right)} \left(-\text{len}1/2 \leq x \leq \text{len}1/2\right) \\
\Delta I_y &= 4\pi r_c A_1 \sin \theta \sqrt{\frac{\sigma + \varepsilon}{\mu s^2}} \sinh \left(\sqrt{\frac{\sigma + \varepsilon \mu s^2}{\mu r}}\right) \\
\Delta I_x &= 4\pi r_c A_1 \sin \theta \sqrt{\frac{\sigma + \varepsilon}{\mu s^2}} \sinh \left(\sqrt{\frac{\sigma + \varepsilon \mu s^2}{\mu r}}\right) \\
\Delta V_y &= \Delta Z_y \cdot \Delta I_y \\
\Delta V_x &= \Delta Z_x \cdot \Delta I_x \\
V^* &= V_0 - \Delta V \\
I^* &= I_0 - \Delta I
\end{align*}$$

In Eq. (17), the change in impedance is independent of $\theta$, so the following analysis considers $\sin \theta = 1$. When characteristic frequency $\omega_c = 1/\mu \sigma$, make $\beta = \omega/\omega_c = 1$ and $\varepsilon = 0$ at high frequency, and the changes of line impedance, voltage, and current are as follows,

$$\begin{align*}
\Delta Z_y(y, \beta) &= j \frac{B_1}{2\pi r_c A_1 \sigma^2} \frac{1}{\left(\frac{\text{len}1/2 - |y|}{\beta y^2}\right)} \left(-\text{len}2/2 \leq y \leq \text{len}2/2\right) \\
\Delta Z_x(x, \beta) &= j \frac{B_1}{2\pi r_c A_1 \sigma^2} \frac{1}{\left(\frac{\text{len}2/2 - |x|}{\beta x^2}\right)} \left(-\text{len}1/2 \leq x \leq \text{len}1/2\right) \\
\Delta I_y &= 4\pi r_c A_1 \sigma \sqrt{\frac{1}{\beta r}} \sinh \left(\sqrt{\frac{1}{\beta r}}\right) \\
\Delta I_x &= 4\pi r_c A_1 \sigma \sqrt{\frac{|x| - \text{len}2/2}{\beta r}} \sinh \left(\sqrt{\frac{|x| - \text{len}2/2}{\beta r}}\right) \\
\Delta V_y(x, y, \beta) &= \frac{2B_1}{\sigma} \frac{y^2}{\left(\frac{\text{len}1/2 - |y|}{\beta y^2}\right)} \cdot \frac{j\beta (\text{len}1/2 - |y|)}{\sqrt{\beta r}} \sinh \left(\sqrt{\frac{1}{\beta r}}\right) \\
\Delta V_x(x, y, \beta) &= \frac{2B_1}{\sigma} \frac{x^2}{\left(\frac{\text{len}2/2 - |x|}{\beta x^2}\right)} \frac{j\beta (|x| - \text{len}2/2)}{\sqrt{\beta r}} \sinh \left(\sqrt{\frac{1}{\beta r}}\right)
\end{align*}$$

(18)
\[
\frac{\sinh \frac{\lambda}{\lambda}}{\lambda} = 1 + \frac{\lambda^2}{3!} + \frac{\lambda^4}{5!} + \frac{\lambda^6}{7!} + \cdots 
\] (19)

In Fig. 4, the voltage change of line impedance is proportional to \( |x|^2 + |y|^2 \) and frequency; the current change of line impedance is proportional to \( (|y| - |x|) + (|x|^2 + |y|^2) \) and frequency; the change of line impedance is proportional to \( (|x|^2 + |y|^2)/(|y| - |x|) \) and frequency. It can be seen that changing line impedance at different locations can optimize the influence of space coupled electromagnetic field around circuit loop on circuit parameters, which can reduce spurious parameter loss and improve efficiency.

![Figure 4](image-url)

**Figure 4.** The voltage and current changes of line impedance at different location and frequency. (a) \( \Delta Z_x \) and \( \Delta Z_y \). (b) \( \Delta V_x \) and \( \Delta V_y \). (c) \( \Delta I \).

According to the above analysis, the circuit loop \( l_3-l_2-l_1 \) in Fig. 5, when \( |\Delta x_{1,2}| = |\Delta y_{1,2}| \neq 0, |\Delta y_{1,2}| > |\Delta x_{1,2}| = 0, \) and \( |\Delta x_{1,2}| > |\Delta y_{1,2}| = 0, \) radius \( r \) and the voltage change of \( x \) and \( y \) axis (\( \Delta E_x, \Delta E_y \)) gradually increase in three ways.
Figure 5. Various line impedance of a circuit loop at different locations.

However, the changes of $\Delta V_x$ and $\Delta V_y$ are different by three ways. Because the current change of line impedance is proportional to $\left(||y| - |x|| \right) \ast (|x|^2 + |y|^2)$ and frequency, when $|\Delta y_{1,2}| > |\Delta x_{1,2}| = 0$, $|\Delta x_{1,2}| > |\Delta y_{1,2}| = 0$ and $|\Delta x_{1,2}| \neq |\Delta y_{1,2}|$ and the changes of $\Delta V_x$ and $\Delta V_y$ are greater than the changes when $|\Delta x_{1,2}| = |\Delta y_{1,2}| \neq 0$.

So changing line impedance at different locations can optimize the voltage and current changes affected by space coupled electromagnetic field around circuit loop.

3. SIMULATION RESULTS VERIFICATION

The following utilizes boost circuit and buck circuit topology in Fig. 6 to verify the correctness and effectiveness of above theoretical analysis.

Figure 6. Boost circuit and buck circuit topology with various line impedance at different location.

The results are summarized in Fig. 7, when $Z_1 = Z_2 = Z_3 = 0$ in boost circuit. $V_L = V_d$ is inversely proportional to frequency when $0 \text{kHz} \leq f_s \leq 50 \text{kHz}$, which is proportional to frequency when $50 \text{kHz} \leq f_s \leq 500 \text{kHz}$. $V_i$ is inversely proportional to frequency. Switching loss is proportional to frequency, and the output current and diode loss are inversely proportional to frequency. When $Z_1 = Z_2 = Z_3 = 50 \text{n}$, $200 \text{n}$, $500 \text{n}$, $V_L$ and $V_d$ are proportional to frequency and impedance value; $V_o$ is inversely proportional to frequency and impedance value. The output current and diode loss are inversely proportional to frequency and impedance value. When $Z_1 = Z_2 = Z_3 = 200 \text{n}$, switching loss is proportional to frequency when $0 \text{kHz} \leq f_s \leq 50 \text{kHz}$; switching loss is inversely proportional to frequency when $50 \text{kHz} \leq f_s \leq 500 \text{kHz}$; when $Z_1 = Z_2 = Z_3 = 500 \text{n}$, switching loss is inversely proportional to frequency.

It can be seen from the above analysis that the output efficiency of boost circuit is related to impedance value and switching frequency. When $Z_1 = Z_2 = Z_3$ increase, the switching loss is inversely proportional to frequency, and efficiency tends to be maximum in high frequency.

When $Z_1 \neq Z_2$, $Z_2 \neq Z_3$, $Z_1 \neq Z_3$, change the values of $Z_1$, $Z_2$, $Z_3$, respectively, and compare the $V_L$, $V_d$, $V_o$ difference with $Z_1 = Z_2 = Z_3 = 50 \text{n}$. 
Figure 7. The changes of various variables with different line impedance and frequency in boost circuit. (a) $V_L$, $V_d$ and $V_o$. (b) Power loss value. (c) Efficiency.
It can be seen from Fig. 8 that when $Z_2$ and $Z_3$ increase, the voltage difference is more obvious, and the change of $Z_1$ has less effect on voltage change. The change of voltage conforms to the law analyzed in Fig. 4 and Fig. 5.

![Figure 8](image)

**Figure 8.** The $V_L$, $V_d$ and $V_o$ changes of boost circuit with $Z_1 \neq Z_2, Z_2 \neq Z_3, Z_1 \neq Z_3$.

The results are summarized in Fig. 9, when $Z_1 = Z_2 = Z_3 = 0$ in buck circuit. $V_L = V_d$ is inversely proportional to frequency; $V_o$ is inversely proportional to frequency. Switching loss is proportional to frequency, and the output current and diode loss are inversely proportional to frequency. When $Z_1 = Z_2 = Z_3 = 50\,\text{n}, 200\,\text{n}, 500\,\text{n}$, $V_L$ and $V_d$ are inversely proportional to frequency and impedance value; $V_o$ is inversely proportional to frequency and impedance value. The output current and diode loss are inversely proportional to frequency and impedance value. When $Z_1 = Z_2 = Z_3 = 200\,\text{n}$, switching loss is proportional to frequency when $0\,\text{kHz} \leq f_s \leq 50\,\text{kHz}$; switching loss is inversely proportional to frequency when $50\,\text{kHz} \leq f_s \leq 500\,\text{kHz}$; when $Z_1 = Z_2 = Z_3 = 500\,\text{n}$, switching loss is inversely proportional to frequency.
Figure 9. The changes of various variables with different line impedance and frequency in buck circuit. (a) $V_L$, $V_d$ and $V_o$. (b) Power loss value. (c) Efficiency.
It can be seen from the above analysis that the output efficiency of buck circuit is related to impedance value and switching frequency. When $Z_1 = Z_2 = Z_3$ increase, the switching loss is inversely proportional to frequency, and efficiency tends to be minimum in high frequency.

When $Z_1 \neq Z_2$, $Z_3 \neq Z_3$, $Z_1 \neq Z_3$, change the values of $Z_1$, $Z_2$, $Z_3$, respectively, and compare the $V_L$, $V_d$, $V_o$ difference with $Z_1 = Z_2 = Z_3 = 50 \, \text{n}$.

It can be seen from Fig. 10 that when $Z_1$ and $Z_2$ increase, the voltage difference is more obvious, and the change of $Z_3$ has less effect on voltage change. The change of voltage conforms to the law analyzed in Fig. 4 and Fig. 5.

Figure 10. The $V_L$, $V_d$ and $V_o$ changes of buck circuit with $Z_1 \neq Z_2$, $Z_2 \neq Z_3$, $Z_1 \neq Z_3$. 
4. COMPARISON RESULTS VERIFICATION

Compare average voltage difference with the proposed method and simulation results under $Z_1 \neq Z_2$, $Z_2 \neq Z_3$, $Z_1 \neq Z_3$, and $Z_1 = Z_2 = Z_3$ condition, and the comparison results are shown in Fig. 11 and Fig. 12.

In Fig. 11 and Fig. 12, when $Z_1$, $Z_2$, and $Z_3$ increase from 50 n to 500 n, the voltage difference is more obvious; $V_d$ is more obvious than that of $V_L$ and $V_o$. The voltage changes of nodes are different.
and affected by space coupled electromagnetic field at different frequencies. So changing line impedance at different locations can optimize the voltage and current changes. Therefore, the analysis of the relationship between space electromagnetic field and voltage and current changes in high-frequency circuit is beneficial to the calculation of node and line voltage changes.

5. CONCLUSION

In high-frequency circuit and parameters optimization of high-frequency converter, the spurious parameters are affected by skin effect and space coupled electromagnetic field changes during switching process. The proposed method based on the relationship between space electromagnetic field and voltage, and current changes in high-frequency circuit. Changing line impedance at different locations can optimize the influence of space coupled electromagnetic field around circuit loop on the voltage and current changes. In addition, changing line impedance at different locations can optimize the influence of space coupled electromagnetic field around circuit loop on circuit parameters and improve efficiency.

In high-frequency circuit and high-frequency converter, the space coupled electromagnetic field distribution is related to switching frequency and affects converter efficiency and power loss. The optimal switching frequency and control time can further optimize efficiency and simplify control process.

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