Anticrossing of optical modes in coupled microcavities

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Abstract. In this work we investigate the reflectance spectra of the GaAs/AlGaAs/AlAs-based coupled microcavities consisting of two cavity layers separated and surrounded by distributed Bragg reflectors (DBRs). The observed optical modes are described by the coupled harmonic oscillators model with parameters derived from the modelling of the layered structure by the transfer matrix method. The theory explains the observed anticrossing behaviour of optical modes with position on the sample, being able to reveal the cavities thickness gradients.

1. Introduction
The model of coupled harmonic oscillators is one of the most important in physics. This model can be used to describe exciton-polaritons – quasiparticles resulting from the strong coupling between the exciton resonance in the quantum well and the photon mode of the resonator, such as a Bragg microcavity [1] or a planar waveguide [2].

A related phenomenon is the strong coupling between two Bragg microcavities. In a layered structure consisting of the bottom Distributed Bragg Reflector (DBR), the first cavity, the intermediate DBR, the second cavity, and the upper DBR, a strong coupling between two photon modes of these cavities can be observed. In the reflection spectra the strong coupling is manifested in the appearance of two dips demonstrating anticrossing behaviour with the change of the spectral positions of photon modes. Such microcavity design provides a great tunability of these modes through changing optical thicknesses of cavities. Furthermore, similar to exciton-polariton optical properties of these structures are observable at room temperature. In our case a different gradient of cavities thicknesses made it possible to observe anticrossing of optical modes when moving along the sample. Coupled microcavities structures have found a few applications involving use as refractive index sensors [3], parametric oscillators [4] for possible use for infrared to terahertz generation using exciton-polaritons [5, 6], potential use in polariton lasers or optical memory elements [7] and can be theoretically exploited in quantum processing for generation of hyper-entangled photon pairs [8].

Possibilities to use coupled microcavities for its common and potential applications requires practicing of structures’ characterization methods. For instance, an ability of fabrication of such structures based on ZnTe and an analysis of acquired reflectance spectra are presented in the
paper [9]. An analogous structure containing quantum wells (QWs) inside cavities is studied in [10] and the coupling of three oscillators (two optical and one electronic) was shown.

In this paper we studied the coupled Bragg microcavities (CMC) consisting of III-V layers (GaAs/Al(Ga)As-based) made by molecular beam epitaxy. In order to characterize the structure’s properties reflectance spectroscopy was used allowing us to observe the avoided crossing of optical modes. The observed phenomena were explained in the model of coupled oscillators. A precise calculation of structure’s spectra carried out using transfer matrix method made it possible to link the experimental anticrossing data with the gradient of cavities’ thicknesses.

2. Coupled harmonic oscillators
Consider two coupled mechanical oscillators (figure 1), which are bodies with masses $m_1$ and $m_2$ attached to springs with constants $k_1$ and $k_2$ and connected by a spring with constant $k$. In absence of damping the equations of motion for these two oscillators could be written based on the Newton’s second law:

\[
\begin{align*}
  m_1 \ddot{x}_1 &= -k_1 x_1 - k(x_1 - x_2), \\
  m_2 \ddot{x}_2 &= -k_2 x_2 - k(x_2 - x_1).
\end{align*}
\] (1)

In the absence of coupling ($k = 0$) eigenfrequencies of the oscillators are $\omega_i^0 = \sqrt{k_i/m_i}$ and $\omega_2^0 = \sqrt{k_2/m_2}$ respectively. In the coupling regime ($k \neq 0$) the trial solutions of ordinary differential equations (1) are: $x_{\pm}(t) = A_{\pm} \exp(-i\omega_{\pm} t)$. The new eigenfrequencies $\omega_{\pm}$ leading to nontrivial solutions could be found by substituting the trial solutions in (1):

\[
\omega_{\pm}^2 = \frac{1}{2} \left( \omega_1^2 + \omega_2^2 \pm \sqrt{(\omega_1^2 - \omega_2^2)^2 + 4\Gamma^2 \omega_1 \omega_2} \right),
\] (2)

where $\omega_i = \sqrt{k_i/m_i}$, $i = 1, 2$, and coupling strength $\Gamma = \sqrt{k_1/m_1 \sqrt{k_2/m_2}}$. In the absence of losses, the strong coupling regime leads to a splitting between modes $\Delta \omega = \omega_+ - \omega_-$. If the frequencies of unperturbed modes coincide $\omega_1 = \omega_2$, the splitting reaches a minimum $\Delta \omega = \Gamma$.

In studied coupled microcavities structure $\omega_1$ and $\omega_2$ are defined by cavities’ optical thicknesses and the coupling strength is dependent on the number of layers’ pairs of the intermediate reflector [12]. In order to determine the relations between these parameters and parameters of the structure it is essential to solve Maxwell’s equations for exact structure.

Alternatively, an analogous to number 2 relation can be also derived using transfer matrix formalism [7, 13] in more detail.

![Figure 1. Two mechanical oscillators with masses $m_1$ and $m_2$ connected by springs with constants $k_1$ and $k_2$ to the wall, and by a spring with constant $k$ to each other.](image-url)
Figure 2. Scanning electron microscopy image of the CMC sample cross section. The light grey areas are Al$_{0.15}$Ga$_{0.85}$As and GaAs. The dark grey areas correspond to AlAs. *Unintentional cavity.

3. Coupled microcavities

The CMC sample was grown by the molecular beam epitaxy. The sample consist of three DBRs grown on the GaAs substrate: bottom DBR with 25 pairs of 57.18 nm Al$_{0.15}$Ga$_{0.85}$As and 65.40 nm AlAs layers, intermediate DBR with 23 pairs and upper DBR with 15 pairs of the same compositions. There are two GaAs cavities of nearly the same width of 117.00 nm between bottom and intermediate DBR (cavity 1), intermediate and upper DBR (cavity 2). There is also an unintentional cavity of smaller thickness in the middle of the intermediate DBR. Electron microscopy image of the sample cross section is shown in figure 2.

Normal reflectance spectrum of the sample at room temperature is shown in figure 3, a. A stop-band centred at 1.525 eV is clearly seen. There are two dips located close to each other in the middle of the stop-band corresponding to the coupled cavities’ modes (1 and 2), and separate dip (1.555 eV) originated from the unintentional cavity (*).

We will use the transfer matrix method in the characteristic matrix approach [11] to simulate experimental reflectance spectrum. Let us denote the growth axis as $z$. We will consider the s-polarized light at normal incidence (in $xz$ plane) with electric field $\vec{E}$ perpendicular to the plane of incidence: $\vec{E} = (0, E_y, 0)$ and $\vec{H} = (H_x, 0, H_z)$. In this case electric field $\vec{E}$ and magnetic field $\vec{H}$ at the layer number $n$ could be found from the incident field at $z_0$ in the following way:

$$
\begin{pmatrix}
E_y(z_n) \\
H_x(z_n)
\end{pmatrix} = 
\left( \prod_{j=1}^{n} M_j(z_j - z_{j-1}) \right)^{-1} \cdot 
\begin{pmatrix}
E^0_y(z_0) \\
H^0_x(z_0)
\end{pmatrix}, 
M_j(z) = 
\begin{pmatrix}
\cos(k_0n_jz) & -\frac{i}{n} \sin(k_0n_jz) \\
-i n \sin(k_0n_jz) & \cos(k_0n_jz)
\end{pmatrix},
$$

(3)

where $z_j$ - z-coordinate of an interface between $j$ and $j+1$ layers; $k_0=2\pi/\lambda$; $n_j$ - refractive index of a layer labeled with $j$ ($j=1,2,\ldots,n$); $M_j=M_j(\lambda,n_j,\{z_{j-1} \leq z \leq z_j\})$ - characteristic matrix for $j$-layer. The reflection coefficient $r$ is given by:

$$
r = \frac{(M_{11}^{total} + M_{12}^{total} n_N)n_0 - (M_{21}^{total} + M_{22}^{total} n_N)}{(M_{11}^{total} + M_{12}^{total} n_N)n_0 + (M_{21}^{total} + M_{22}^{total} n_N)},
$$

(4)

where $M^{total} = \prod_{j=1}^{n} M_j(z_j - z_{j-1})$. 


Described procedure was used for simulating the experimental spectrum with refractive indices and extinction coefficients taken from literature [14, 15, 16]. The best fit parameters for the theoretically modelled spectrum shown in figure 3, b are the following: Bragg mirrors thicknesses are 58.88 nm for \( \text{Al}_{0.15}\text{Ga}_{0.85}\text{As} \) and 66.9 nm for AlAs for the upper reflector and half of the intermediate one; and for the second half and the bottom one: 57.16 nm for \( \text{Al}_{0.15}\text{Ga}_{0.85}\text{As} \) and 67.4 nm for AlAs. The accidental cavity thickness was 94.7 nm. This cavity does not influence the coupling of the other two since its spectral position is heavily detuned. Therefore we can neglect the influence of this cavity.

### 4. Anticrossing of optical modes

The reflectance spectra from different points of the sample along the \( x \)-axis demonstrate the change in the spectral positions of the reflection dips \( \omega_+ \) and \( \omega_- \), shown in figure 4, a. The spectral splitting between modes \( \Delta \omega = \omega_+ - \omega_- \) is depicted in figure 4, b. It shows a clear minimum around \( x_{\text{min}} = 2 \) nm. In order to explain the observed anticrossing behaviour we will use the coupled oscillators model described earlier.

Different growth conditions of cavities 1 and 2 lead to their thicknesses gradient and to the difference in the spectral behavior of the corresponding single cavities frequencies \( \omega_1 \) and \( \omega_2 \). We will assume that this behaviour is parabolic with \( x \) for both of the cavities. The splitting minimum clearly corresponds to the case of \( \omega_1 = \omega_2 \) enabling us to determine the coupling strength \( \Gamma = \Delta \omega(x_{\text{min}}) = 4 \) meV. The best fit of \( \omega_1(x) \) and \( \omega_2(x) \) and coupled modes \( \omega_+(x) \) and \( \omega_-(x) \) are shown in figure 4, a by dashed and solid lines correspondingly. Dashed line in figure 4, b shows the theoretical behaviour of the splitting.

The spectral position of the uncoupled modes resonances is connected to the optical thicknesses of a cavities by a relation \( nd_i = \lambda_i/2 \) which enables us to find the approximate cavities thickness gradient shown in figure 5.
5. Conclusion
In this paper, we have studied the phenomenon of a strong coupling between optical modes in a double Bragg microcavity. The splitting of the order of 4 meV allowed us to observe the typical anticrossing behaviour between the coupled modes in the reflection spectra with a change in position on the sample. To explain the observed dependencies, we used the model of coupled oscillators and the transfer matrix method. With their help, the gradient of the thickness of the cavities leading to the anticrossing was determined.

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