Density of condensates of two-component Bose-Einstein condensates restricted between two planar walls

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Abstract. The density of condensates of a binary mixture of Bose gases restricted by two parallel planar walls is investigated within the framework of Cornwal-Jackiw-Tomboulis effective action approach in improved Hartree-Fock approximation. It results that the density of condensates strongly depend not only on the distance between two walls but also on the interspecies interaction strength and they are equal their expectation values of the field operators after adding a term associated with the quantum fluctuations.

1. Introduction
One of the most important quantities in studies on Bose-Einstein condensate (BEC) in both theory and experiment is the density of condensate. Based on it, all of static and dynamic properties of the BEC can be easily investigated.

The well-known theory of the BEC proposed by Gorss and Pitaevskii (GP) [1, 2], where the temperature is assumed to be zero so that all of atoms are condensed. The ground state is described by the wave function, which is the solution of a nonlinear differential equation called the GP equation. The density of condensate is defined as square of the wave function and called the profile [2, 4]. However, it is reported that we have never achieve the absolute zero temperature and even so, a number of particles will be in the excited state instead of the ground state [5]. Based on the GP theory, the density of condensate in the BEC have been investigated by many authors, for example, see Refs. [6, 7, 8] and so on. In a simple approximation called double-parabola approximation [9] and triple-parabola approximation [10], this aspect was also studied.

Taking into account the quantum fluctuations, quantum field theory is proposed in several levels of the accuracy. In the one-loop approximation, the density of condensate in the ideal and weakly interacting Bose gas was investigated [11]. The Cornwal-Jackiw-Tomboulis (CJT) effective action approach [12] can be applied to consider the contribution of two-loop diagrams. The authors of Ref. [13] employed this method to consider the scenarios of phase transition in a binary mixture of Bose gases (BECs).

The finite-size effect in the BEC(s) produces many amazing changes in properties of the BEC(s). Within the framework of the GP theory, the density of condensate of the BEC has
been researched [14, 15] and [16, 17] for BECs. One of the most interesting consequence of the finite-size effect is the Casimir effect, which has been studied widely in many scopes of physics. In the BEC field, this effect was investigated in the one-loop approximation [14, 15, 18], in which the expectation value of the field operator is assumed independence of the coordinate. The results showed that the expectation value of the field operator is not only independent of the coordinate but also of the size of system, which is usually the distance between two planar walls filled by the Bose gas. By the same way, this effect was also studied in BECs [19]. In higher approximation, called improved Hartree-Fock (IHF) approximation, the expectation value of the field operator was considered in BECs with lower-order terms in the momentum integrals [20]. The contribution of the higher-order terms is taken into account to investigated the density of condensate of the BEC [21]. The resulting the expectation value of the field operator depends on both the coordinate and the distance between two planar walls. In this paper, the density of condensate of the dilute Bose gas confined between two parallel plates is considered in the IHF approximation with the present of the high-order terms in the momentum integrals, which is named the higher-order improved Hartree-Fock (HIHF) approximation.

This paper is organized as follows. In Section 2, the density of condensate is derived in improved Hartree-Fock approximation. Section 3 devote the density of condensate of a weakly interacting Bose gas in the HIHF approximation. Conclusions are given in Section 4.

2. Density of condensate in the improved Hartree-Fock approximation

We start by considering a two-component Bose-Einstein condensates, which is described by the Lagrangian [2, 4],

\[ \mathcal{L} = \sum_{j=1,2} \psi_j^* \left( -i\hbar \frac{\partial}{\partial t} - \frac{\hbar^2}{2m_j} \nabla^2 \right) \psi_j - V, \]

with GP potential

\[ V = \sum_{j=1,2} \left( -\mu_j |\psi_j|^2 + \frac{g_{jj}}{2} |\psi_j|^4 \right) + g_{12} |\psi_1|^2 |\psi_2|^2, \]

in which the field operator \( \psi_j(\vec{r}, t) \) has the expectation value \( \psi_{j0} \) plays the role of the order parameter; \( \hbar \) is the reduced Plack’s constant; the chemical potential and atomic mass of \( j \)-component are denoted by \( \mu_j \) and \( m_j \), respectively. The particles of species \( j \) and \( j' \) interact weakly via s-wave scattering, quantified by a positive scattering length \( a_{jj'} \) and a coupling constant

\[ g_{jj'} = 2\pi \hbar^2 a_{jj'} \left( \frac{1}{m_j} + \frac{1}{m_{j'}} \right) > 0, \]

and the interspecies interaction are determined by

\[ g_{jj} = \frac{4\pi \hbar^2 a_{jj}}{m_j} > 0. \]

Let \( \psi_{j0} \) be the expectation value of the field operator, in tree-approximation, minimizing the potential (2) with respect to the order parameter, in broken phase one has

\[ \psi_{10}^2 = \frac{g_{12}^2 \mu_1 - g_{12} \mu_2}{g_{11} g_{22} - g_{12}^2}, \quad \psi_{20}^2 = \frac{g_{11} \mu_2 - g_{12} \mu_1}{g_{11} g_{22} - g_{12}^2}. \]
In order to consider in the Hartree-Fock (HF) approximation, the field operator is decomposed in terms of the order parameter and two real fields $\psi_1, \psi_2$ associated with the quantum fluctuation of the field \cite{11},

$$\psi_j \rightarrow \psi_{j0} + \frac{1}{\sqrt{2}}(\psi_{j1} + i\psi_{j2}).$$

(6)

Inserting (6) into Lagrangian (1) yields the free Lagrangian

$$\mathcal{L}_0 = \sum_{j=1,2} \left( -\mu_j \psi_{j0} + \frac{g_{jj}}{2} \psi_{j0}^4 \right) + \frac{g_{12}}{2} \psi_{10}^2 \psi_{20}^2,$$

(7)

which gives us the inversion propagator in tree-approximation in momentum space

$$D^{-1}_{j0}(k) = \begin{pmatrix} \frac{\hbar^2 k^2}{2m_j} + 2g_{jj} \psi_{j0}^2 - \omega_n & -\omega_n \\ -\omega_n & \frac{\hbar^2 k^2}{2m_j} \end{pmatrix},$$

(8)

in combining with (5). Here we denote $\vec{k}$ the wave vector, $\omega_n = 2\pi n/\beta$, $n = 0, \pm 1, \pm 2, \ldots$ stands for the Matsubara frequency for boson. At temperature $T$ one has $\beta = 1/k_B T$ with $k_B$ being the Boltzmann constant. Let the determinants of the inversion propagator be zero \cite{22}

$$\det D_{j0}^{-1}(k) = 0,$$

one gets the dispersion relation

$$E_j(k) = \sqrt{\frac{\hbar^2 k^2}{2m_j} \left( \frac{\hbar^2 k^2}{2m_j} + 2g_{jj} \psi_{j0}^2 \right)}.$$

(9)

It is obvious that there are Goldstone bosons associated with $U(1) \times U(1)$ breaking.

We now look for the CJT effective potential in double-bubble approximation. To do this, substituting the decomposition (6) into Lagrangian (1) one arrives at the interaction Lagrangian \cite{20},

$$\mathcal{L}_{\text{int}} = \frac{1}{\sqrt{2}} \sum_{j=1,2} \left[ g_{jj} \psi_{j0} \psi_{j1} + g_{12} \psi_{j0} \psi_{j2} \right] (\psi_{j1}^2 + \psi_{j2}^2) + \frac{1}{8} \sum_{j=1,2} g_{jj} (\psi_{j1}^2 + \psi_{j2}^2)^2$$

$$+ \frac{g_{12}}{4} (\psi_{11}^2 + \psi_{12}^2)(\psi_{21}^2 + \psi_{22}^2).$$

(10)

and the CJT effective potential can be easily read off from (10)

$$V_{CJT}^\beta = \sum_{j=1,2} \left( -\mu_j |\psi_{j0}|^2 + \frac{g_{jj}}{2} |\psi_{j0}|^4 \right) + g_{12} |\psi_{10}|^2 |\psi_{20}|^2$$

$$+ \frac{1}{2} \int_\beta \text{tr} \left\{ \sum_{j=1,2} \left[ \ln D_j^{-1}(k) + D_{j0}^{-1}(k)D(k) \right] - 2 \mathbb{I} \right\} + \frac{3g_{11}}{8} (P_{11}^2 + P_{22}^2) + \frac{g_{11}}{4} P_{11} P_{22}$$

$$+ \frac{3g_{22}}{8} (Q_{11}^2 + Q_{22}^2) + \frac{g_{22}}{4} Q_{11} Q_{22} + \frac{g_{12}}{4} (P_{11} Q_{11} + P_{12} Q_{22} + P_{22} Q_{22} + P_{22} Q_{22}),$$

(11)

where $D(k)$ is the propagator in the double-bubble approximation and notations

$$\int f(k) = \frac{1}{\beta} \sum_{n=-\infty}^{+\infty} \int \frac{d^3 \vec{k}}{(2\pi)^3} f(\omega_n, \vec{k}),$$

$$P_{aa} = \int_\beta D_{1aa}(k), \quad Q_{bb} = \int_\beta D_{2bb}(k),$$

$$P_{ab} = \int_\beta D_{1ab}(k), \quad Q_{ab} = \int_\beta D_{2ab}(k),$$

$$\int D(k) = \int_\beta D_{11}(k), \quad \int_\beta D_{22}(k).$$
are employed. It is confirmed in our previous papers [20] that the CJT effective potential (11) breaks the Goldstone theorem. In order to restore it, the method proposed by Ivanov et. al. [23] is invoked therefore the CJT effective potential (11) is replaced by a new one [21],
\[ \tilde{\mathcal{V}}_{\beta}^{\text{CJT}} = \sum_{j=1,2} \left( -\mu_j |\psi_{j0}|^2 + \frac{g_{jj}}{2} |\psi_{j0}|^4 \right) + g_{12} |\psi_{10}|^2 |\psi_{20}|^2 \\
+ \frac{1}{2} \int \text{tr} \left\{ \sum_{j=1,2} \left[ \ln D_j^{-1}(k) + D_j^{-1}(k)D(k) \right] - 2 \right\} \int \frac{d^3 k}{(2\pi)^3} \left[ \ln D_0^{-1}(k) + D_0^{-1}(k)D(k) \right] - 2 \right\} + \frac{g_{11}}{8} (P_{11}^2 + P_{22}^2) + \frac{3g_{11}}{4} P_{11}P_{22} \\
+ \frac{g_{22}}{8} (Q_{11}^2 + Q_{22}^2) + \frac{3g_{22}}{4} Q_{11}Q_{22} + \frac{g_{12}}{4} (P_{11}Q_{11} + P_{12}Q_{22} + P_{22}Q_{11} + P_{22}Q_{22}). \] (12)

As it was pointed out in Ref. [20], this CJT effective potential (12) ensures that the Goldstone theorem is valid and the Goldstone bosons have the new dispersion relation [21],
\[ E_j(k) = \sqrt{\frac{h^2 k^2}{2 m_j} \left( \frac{h^2 k^2}{2 m_j} + M_j^2 \right)}, \] (13)

with \( M_j \) being the effective mass and the approximation associated with the effective potential (12) is called the IHF approximation. The inversion propagator is now
\[ D_j^{-1}(k) = \left( \frac{h^2 k^2}{2 m_j} + M_j^2 - \omega_n \frac{h^2 k^2}{2 m_j} \right). \] (14)

Eqs. (12) and (14) give us the momentum integrals
\[ P_{11} = \frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} \sqrt{\frac{h^2 k^2/2m_1 + M_1^2}{h^2 k^2/2m_1 + M_1^2}}, \]
\[ P_{22} = \frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} \sqrt{\frac{h^2 k^2/2m_1 + M_1^2}{h^2 k^2/2m_1 + M_1^2}}, \]
\[ Q_{11} = \frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} \sqrt{\frac{h^2 k^2/2m_2 + M_2^2}{h^2 k^2/2m_2 + M_2^2}}, \]
\[ Q_{22} = \frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} \sqrt{\frac{h^2 k^2/2m_2 + M_2^2}{h^2 k^2/2m_2 + M_2^2}}. \] (15)

From (12) it is easy to derive the gap equations
\[ -\mu_1 + g_{11} \psi_{10}^2 + g_{12} \psi_{20}^2 + \Sigma_2^{(1)} = 0, \]
\[ -\mu_2 + g_{22} \psi_{20}^2 + g_{12} \psi_{10}^2 + \Sigma_2^{(2)} = 0. \] (16)

The effective masses are solution of the and Schwinger-Dyson (SD) equations
\[ M_1^2 = -\mu_1 + 3g_{11} \psi_{10}^2 + g_{12} \psi_{20}^2 + \Sigma_1^{(1)}, \]
\[ M_2^2 = -\mu_2 + 3g_{22} \psi_{20}^2 + g_{12} \psi_{10}^2 + \Sigma_1^{(2)}, \] (17)

in which
\[ \Sigma_1^{(1)} = \frac{1}{2} (g_{11} P_{11} + 3g_{11} P_{22} + g_{12} Q_{11} + g_{12} Q_{22}), \]
\[ \Sigma_1^{(2)} = \frac{1}{2} (g_{22} Q_{11} + 3g_{22} Q_{22} + g_{12} P_{11} + g_{12} P_{22}), \]
\[ \Sigma_2^{(1)} = \frac{1}{2} (3g_{11} P_{11} + g_{11} P_{22} + g_{12} Q_{11} + g_{12} Q_{22}), \]
\[ \Sigma_2^{(2)} = \frac{1}{2} (3g_{22} Q_{11} + g_{22} Q_{22} + g_{12} P_{11} + g_{12} P_{22}). \] (18)
The pressure is defined as

$$P = -\hat{V}_{\beta}^{CJT} \bigg|_{\text{at minimum}}.$$  \hfill (19)

Using Eqs. (16)-(18), the pressure (19) can be easily obtained

$$P = -\sum_{j=1,2} \left( -\mu_j |\psi_{j0}|^2 + \frac{g_{jj}}{2} |\psi_{j0}|^4 \right) - g_{12}|\psi_{10}|^2|\psi_{20}|^2 - \frac{1}{2} \int_{\beta} \text{tr} \left[ \ln D_1^{-1}(k) + \ln D_2^{-1}(k) \right]$$

$$- \frac{1}{2} \left( -M_1^2 - \mu_1 + 3g_{11}\psi_{10}^2 + g_{12}\psi_{20}^2 \right) P_{11} - \frac{1}{2} \left( -\mu_1 + g_{11}\psi_{10}^2 + g_{12}\psi_{20}^2 \right) P_{22}$$

$$- \frac{1}{2} \left( -M_2^2 - \mu_2 + 3g_{22}\psi_{20}^2 + g_{12}\psi_{10}^2 \right) Q_{11} - \frac{1}{2} \left( -\mu_2 + g_{22}\psi_{20}^2 + g_{12}\psi_{10}^2 \right) Q_{22}$$

$$- \frac{g_{11}}{8} (P_{11}^2 + P_{22}^2) - \frac{3g_{11}}{4} P_{11} P_{22} - \frac{g_{22}}{8} (Q_{11}^2 + Q_{22}^2) - \frac{3g_{22}}{4} Q_{11} Q_{22}$$

$$- \frac{g_{12}}{4} (P_{11}Q_{11} + P_{11}Q_{22} + P_{22}Q_{11} + P_{22}Q_{22}).$$  \hfill (20)

The density of condensates in the IHF approximation is determined by

$$\rho_j = \frac{\partial P}{\partial \mu_j}.$$  \hfill (21)

Plugging Eq. (20) into (21) one obtains the density of condensates in the IHF approximation

$$\rho_1 = \psi_{10}^2 + \frac{1}{2} (P_{11} + P_{22}),$$

$$\rho_2 = \psi_{20}^2 + \frac{1}{2} (Q_{11} + Q_{22}).$$  \hfill (22)

3. Influence of the compactification on the density of condensates

We now consider the two-component Bose-Einstein condensates filling space between two larger parallel planar walls. These planar walls are perpendicular to 0z-axis at a distance $\ell$ and this distance is very small than the size of each wall. The compactification in $z$-direction leads to the quantization of the wave vector

$$k^2 \rightarrow k_{\perp}^2 + k_n^2,$$  \hfill (23)

in which the wave vector component $k_{\perp}$ is perpendicular to 0z-axis and $k_n$ is parallel with 0z-axis. Along 0z direction, the periodic boundary condition is applied [21], the $k_n$ component of the wave vector has the form

$$k_n = \frac{2\pi n}{\ell}, \quad n \in \mathbb{Z}.$$  \hfill (24)

In this paper, we adopt the dimensionless quantities for the purpose of seeking for simplicity: the distance between two plates is scaled by the healing length $\xi_j = \hbar / \sqrt{2mg_{jj}n_{j0}}$ with $n_{j0}$ being the bulk density of $j$-component thus the dimensionless distance is $L_j = \ell / \xi_j$. The dimensionless wave vector is $\kappa_j = k \xi_j$ and Eq. (23) becomes

$$\kappa_j^2 \rightarrow \kappa_{j\perp}^2 + \kappa_{jn}^2,$$  \hfill (25)
with the dimensionless form of (24) being \( \kappa_j = n/L_j \), \( L_j = L_j/(2\pi) \). In terms of these dimensionless quantities, the momentum integrals (15) are rewritten in form

\[
P_{11} = \frac{1}{2\xi_1^2} \int \frac{d^3\kappa_1}{(2\pi)^3} \frac{\kappa_1}{\sqrt{\kappa_1^2 + M_1^2}}, \quad P_{22} = \frac{1}{2\xi_1^2} \int \frac{d^3\kappa_1}{(2\pi)^3} \frac{\sqrt{\kappa_1^2 + M_1^2}}{\kappa_1},
\]

\[
Q_{11} = \frac{1}{2\xi_2^2} \int \frac{d^3\kappa_2}{(2\pi)^3} \frac{\kappa_2}{\sqrt{\kappa_2^2 + M_2^2}}, \quad Q_{22} = \frac{1}{2\xi_2^2} \int \frac{d^3\kappa_2}{(2\pi)^3} \frac{\sqrt{\kappa_2^2 + M_2^2}}{\kappa_2}.
\]

(26)

Because of the quantization of the wave vector (25), the momentum integration perpendicular to the plates is replaced by a discrete sum

\[
\int \frac{d^3\kappa_j}{(2\pi)^3} f(\kappa_j) \rightarrow \sum_{n=-\infty}^{+\infty} \int \frac{d^2\kappa_{j\perp}}{(2\pi)^2} f(\kappa_{j\perp}, \kappa_{jn}),
\]

therefore Eqs. (26) become

\[
P_{11} = \frac{1}{2\xi_1^2} \sum_n \int_0^L \frac{d^2\kappa_{1\perp}}{(2\pi)^2} \sqrt{\frac{\kappa_{1\perp}^2 + (2\pi n/L_1)^2}{\kappa_{1\perp}^2 + (2\pi n/L_1)^2 + M_1^2}},
\]

\[
P_{22} = \frac{1}{2\xi_1^2} \sum_n \int_0^L \frac{d^2\kappa_{1\perp}}{(2\pi)^2} \sqrt{\frac{\kappa_{1\perp}^2 + (2\pi n/L_1)^2}{\kappa_{1\perp}^2 + (2\pi n/L_1)^2 + M_1^2}},
\]

\[
Q_{11} = \frac{1}{2\xi_2^2} \sum_n \int_0^L \frac{d^2\kappa_{2\perp}}{(2\pi)^2} \sqrt{\frac{\kappa_{2\perp}^2 + (2\pi n/L_2)^2}{\kappa_{2\perp}^2 + (2\pi n/L_2)^2 + M_2^2}},
\]

\[
Q_{22} = \frac{1}{2\xi_2^2} \sum_n \int_0^L \frac{d^2\kappa_{2\perp}}{(2\pi)^2} \sqrt{\frac{\kappa_{2\perp}^2 + (2\pi n/L_2)^2}{\kappa_{2\perp}^2 + (2\pi n/L_2)^2 + M_2^2}}.
\]

(27)

Note that in the above equations, the dimensionless effective mass is defined as \( M_j = M_j/\sqrt{g_{jj}^2} \). In addition, the integrations over the perpendicular components of the wave vectors are ultraviolet divergent so that a momentum cut-off \( \Lambda \) is introduced. The sum over the parallel component \( \kappa_j \) of the wave vector can be dealt by using the Euler-Maclaurin formula [24],

\[
\sum_{n=0}^{\infty} \theta_n F(n) - \int_0^{\infty} F(n) \, dn = -\frac{1}{12} F'(0) + \frac{1}{720} F''(0) - \frac{1}{30240} F^{(5)}(0) + \cdots,
\]

(28)

with

\[
\theta_n = \begin{cases} 
1/2, & \text{if } n = 0; \\
1, & \text{if } n > 0.
\end{cases}
\]

After performing the integration and then taking the sum, let the cut-off \( \Lambda \) tends to infinity one has [20],

\[
P_{11} = Q_{11} = 0, \quad P_{22} = \frac{g_{11} m_1^2 M_1}{12\hbar^2 \ell}, \quad Q_{22} = \frac{g_{22} m_2^2 M_2}{12\hbar^2 \ell}.
\]

(29)
According to Eq. (22), in order to calculate the density of condensates one needs to know the order parameters. To do that, the momentum integrals (29) are plugged into (16) and (17). The gap and SD equations in dimensionless form are

\[
\begin{align*}
-1 + \phi_1^2 + K \phi_2^2 + \frac{m_1 g_{11} M_1}{24 h^2 \ell} + K \frac{m_2 g_{22} M_2}{24 h^2 \ell} &= 0, \\
-1 + \phi_2^2 + K \phi_1^2 + \frac{m_2 g_{22} M_2}{24 h^2 \ell} + K \frac{m_1 g_{11} M_1}{24 h^2 \ell} &= 0, \\
-1 + 3\phi_1^2 + K \phi_2^2 + \frac{3m_1 g_{11} M_1}{24 h^2 \ell} + K \frac{m_2 g_{22} M_2}{24 h^2 \ell} &= \mathcal{M}_1^2, \\
-1 + 3\phi_2^2 + K \phi_1^2 + \frac{3m_2 g_{22} M_2}{24 h^2 \ell} + K \frac{m_1 g_{11} M_1}{24 h^2 \ell} &= \mathcal{M}_2^2,
\end{align*}
\]  

(30)

where the reduced order parameter \( \phi_j = \frac{\psi_j}{\sqrt{n_j}} \) is introduced and the dimensionless gas parameter \( K = \frac{g_{12}}{\sqrt{g_{11} g_{22}}} \) is defined. Solving Eqs. (30) one easily finds the solution in dimensionless form

\[
\begin{align*}
\phi_1 &= \frac{1}{K + 1} + \frac{m_1 g_{11}}{12 \sqrt{2(K + 1) h^2 \ell}}, \\
\phi_2 &= \frac{1}{K + 1} + \frac{m_2 g_{22}}{12 \sqrt{2(K + 1) h^2 \ell}}, \\
\mathcal{M}_1^2 &= \mathcal{M}_2^2 = \frac{2}{K + 1}.
\end{align*}
\]  

(31)

Defining the reduced density of condensate

\[
\vartheta_j = \frac{\rho_j}{n_j} \quad (32)
\]

then combining Eqs. (31) and (22) yields the reduced density of condensates

\[
\begin{align*}
\vartheta_1 &= \frac{1}{K + 1} + \frac{m_1 g_{11}}{6 \sqrt{2} \sqrt{K + 1} h^2 \ell}, \\
\vartheta_2 &= \frac{1}{K + 1} + \frac{m_2 g_{22}}{6 \sqrt{2} \sqrt{K + 1} h^2 \ell}.
\end{align*}
\]  

(33)

It is not difficult to see that both the density of condensates and square of the order parameters are equal their values in the one-loop approximation after adding a corrected term associated with the contribution of the two-loop diagrams.

\textbf{Figure 1.} The dimensionless distance dependence of the order parameters at \( K = 3 \). The red and blue lines correspond to first component \( (\phi_1) \) and second one \( (\phi_2) \).
Let us illustrate for above calculations by numerical computations for a mixture of Bose gasses, which consists of the first component (rubidium Rb 87) and second one (cesium Cs 133) [25] with parameters $m_1 = 86.909 \text{ u}$, $a_{11} = 100.4a_0$, $m_2 = 132.905 \text{ u}$, $a_{22} = 280a_0$, $a_0 = 0.529 \text{ nm}$, with $u$ and $a_0$ being atomic mass unit and Bohr radius, respectively. Fig. 1 shows the order parameters as a function of the dimensionless distance $L = \ell/\xi_1$ between two plates at $K = 3$. The first thing one easily sees is the fast decay of the order parameters as the distance increases and which approach their values associated with those of the infinite system $\phi_1 = \phi_2 = 1/\sqrt{K+1}$ (black line).

The evolution of density of condensates as a function of the dimensionless distance are sketched in Fig. 2 for the first component (rubidium Rb 87) and Fig. 3 for the second component (cesium Cs 133). In these figures, the solid and dashed lines correspond to the density of condensate and square of the order parameter, black lines associate with those in the one-loop approximation. Similar to the order parameters, the density of condensates decay fast as the distance increases and approach to their values for the infinite system. Both the density of condensates are divergent when the distance tends to zero. There is no doubt that the contribution of the quantum fluctuations into the density of condensates is not small, especially in region of the small distance. Amount of difference between the solid and dashed lines correspond to contribution of the term $(P_{11}+P_{22})^2$ for the first component and $(Q_{11}+Q_{22})^2$ for the second one and which is $\frac{m_jg_{jj}}{12\sqrt{2(K+1)}}\ell$.

4. Conclusions
In the foregoing Section, the density of condensates of two-component Bose-Einstein condensates restricted between two planar walls is investigated in the improved Hartree-Fock approximation. The ultraviolet divergence in integrating over the perpendicular component of the wave vector is eliminated by introducing the momentum cut-off whereas the sum over all value of quantized component of the wave vector is dealt by using Euler-Maclaurin formula.

Our main results are in order:
- The compactification of space strongly affects on the density of condensate of the BECs, especially in region of the small distance. The density of condensates diverge as the distance between two planar walls approaches zero.

Figure 2. (Color online) The evolution of the reduced density of condensate (solid line) and square of the order parameter (dashed line) of the first component (rubidium Rb 87) versus the dimensionless distance at $K = 3$.

Figure 3. (Color online) The evolution of the reduced density of condensate (solid line) and square of the order parameter (dashed line) of the second component (cesium Cs 133) versus the dimensionless distance at $K = 3$. 
- The contribution of the quantum fluctuations into the density of condensates are remarkable. This contribution is not negligible when the distance is sufficiently small.

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References
[1] Gross E P 1961 Structure of a Quantized Vortex in Boron Systems *Nuovo Cimento* 20 454.
[2] Pitaevskii L P 1961 Vortex lines in an imperfect Bose gas *Sov. Phys. JETP* 13 451.
[3] Pitaevskii L, Stringari S 2003 *Bose-Einstein Condensation* Oxford University Press, Oxford.
[4] Pethick C J, Smith H 2008 *Bose-Einstein Condensation in Dilute Gases* Cambridge University Press, Cambridge.
[5] Annett J F 2004 *Superconductivity, superfluids, and condensates* Oxford University Press, Oxford.
[6] Ao P and Chui S T 1998 *Phys. Rev. A* 58 4836.
[7] Brankov R A 2002 *Phys. Rev. A* 66 013612.
[8] Mazets I E 2002 *Phys. Rev. A* 65 033618.
[9] Indekeu J O, Lin C -Y, Thu N V, Schaeybroeck B V, Phat T H 2015 *Phys. Rev. A* 91 033615.
[10] Deng Z, Schaeybroeck B V, Lin C -Y, Thu N V, Indekeu J O 2016 Interfacial tension and wall energy of a Bose-Einstein condensate binary mixture: Triple-parabola approximation *Physica A* 444 1027.
[11] Andersen J O 2004 Theory of the weakly interacting Bose gas *Rev. Mod. Phys.* 76 599.
[12] Cornwall J M, Jackiw R, Tomboulis E 1974 Effective action composite operators *Phys. Rev. D* 10 2428.
[13] Phat T H, Hoa L V, Anh N T, and Long N V 2009 Bose-Einstein condensation in binary mixture of Bose gases *Ann. Phys.* 324 2074.
[14] Thu Nguyen Van 2018 The forces on a single interacting Bose-Einstein condensate *Phys. Lett. A* 382 1078.
[15] Biswas S, Bhattacharjee J K, Majumder D, Saha K, Chakravarty N. 2010 Casimir force on an interacting Bose-Einstein condensate *J. Phys. B* 43 085305.
[16] Thu N V, Phat T H and Song P T 2017 *J. Low Temp. Phys.* 186 127.
[17] Takahashi D A, Kobayashi M, Nitta M 2015 Nambu-Goldstone modes propagating along topological defects: Kelvin and ripple modes from small to large systems *Phys. Rev. B* 91 184501.
[18] Schiefle J. and Henkel C. 2009 Casimir energy of a BEC: from moderate interactions to the ideal gas *J. Phys. A* 42 045401.
[19] Thu Nguyen Van and Theu Luong Thi 2017 *J. Stat. Phys.* 168 1.
[20] Thu Nguyen Van and Theu Luong Thi 2019 Finite-size effect on Bose-Einstein condensate mixtures in improved Hartree Fock approximation *Int. J. Mod. Phys. B* 33 1950114.
[21] Thu Nguyen Van and Song Pham 2020 Casimir effect in a weakly interacting Bose gas confined by a parallel plate geometry in improved Hartree-Fock approximation *Physica A* 540 123018.
[22] Floerchinger S and Wetterich C 2009 Superfluid Bose gas in two dimensions *Phys. Rev. A* 79 013601.
[23] Ivanov Yu B, Rick F, Knoll J 2005 Gapless Hartree-Fock resummation scheme for the O(N) model *Phys. Rev. D* 71 105016.
[24] Arfken G B and Weber H J 2005 *Mathematical Methods for Physicists* 6th edn San Diego: Academic.
[25] McCarron D J, Cho H W, Jenkin D L, Koppinger M P, and Cornish S L 2011 *Phys. Rev. A* 84 011603.
[26] Biswas S 2007 Bose-Einstein condensation and the Casimir effect for an ideal Bose gas confined between two slabs *J. Phys. A* 40 9969.