ABSTRACT

Decision making algorithms, in practice, are often trained on data that exhibits a variety of biases. Decision-makers often aim to take decisions based on some ground-truth target that is assumed or expected to be unbiased, i.e., equally distributed across socially salient groups. In many practical settings, the ground-truth cannot be directly observed, and instead, we have to rely on a biased proxy measure of the ground-truth, i.e., biased labels, in the data. In addition, data is often selectively labeled, i.e., even the biased labels are only observed for a small fraction of the data that received a positive decision. To overcome label and selection biases, recent work proposes to learn stochastic, exploring decision policies via i) online training of new policies at each time-step and ii) enforcing fairness as a constraint on performance. However, the existing approach uses only labeled data, disregarding a large amount of unlabeled data, and thereby suffers from high instability and variance in the learned decision policies at different times. In this paper, we propose a novel method based on a variational autoencoder for training a policy in an online process. Using synthetic data, we empirically validate that our method converges to the representations to learn a policy in an online process. Using synthetic data, we empirically validate that our method converges to the optimal (fair) policy.

1 INTRODUCTION

The extensive literature on fair machine learning has focused primarily on studying the fairness of the predictions by classification models [1, 17, 23, 56, 57] deployed in critical decision-making scenarios. Consider a university admissions process where the goal is to admit students based on their true potential, which may not be directly observable. We assume that a student’s ground-truth potential is independent of and unbiased by, their assignment to various socially salient groups - defined by sensitive characteristics (race, gender) protected by anti-discrimination laws [7]. That is, we hold it as self-evident that students of different socially salient groups are endowed with a similar (equal) distribution of potential. In practice, ground-truth potential cannot be measured directly and remains unobserved. Instead, we rely on proxy labels, which we assume to contain information about the ground truth. However, due to structural discrimination, these proxy labels are often biased measures of ground truth. For example, prevailing societal discrimination may result in students with similar ground-truth potential but different assigned genders having a very different distribution of university grades (proxy labels). This phenomenon is termed label bias [52] and has been studied extensively in [1, 23, 57, 58]. These fairness studies assume that independent and identically distributed (i.i.d.) labeled data is available for training. However, in decision-making scenarios, data may also suffer from selection bias.
bias [36]. That is, we only observe the labels of a small fraction of the data which received positive decisions. For example, we observe (biased) university grades of only the admitted students, resulting in biased, non-i.i.d. labeled data.

In decision-making scenarios affected by both label bias and selection bias, Kilbertus et al. [31] show that to learn the optimal policy, it is necessary to move from learning fair predictions (e.g., predicting grades) to learning fair decisions (e.g., deciding to admit students). To tackle label bias, the authors introduce fairness constraints in the optimization problem. To address selection bias, they propose to learn stochastic, exploring decision policies in an online learning process, where a new decision policy is learned at each time-step. To get unbiased loss estimates from non-i.i.d. labels, the authors further rely on inverse propensity scoring (IPS) [27].

However, the approach ignores unlabeled data. A large fraction of data in the learning process may remain unlabeled due to receiving a negative decision (e.g., students denied admission). Using only labeled data, the approach suffers from high instability and variance in the learning process. In particular, i) the method may give very different outcomes to the same individual, depending on the random initializations of the learning process, and ii) the method may give the same individual very different outcomes at different points in time.

In this paper, we propose a novel online learning process for fair decision-making that leverages both labeled and unlabeled data. Our method learns fair representations of all data using latent variable models in an attempt to capture the unobserved and unbiased ground truth information. In turn, these representations are used to learn a policy that approximates the optimal fair policy (according to the unobserved ground truth). Importantly, as shown in our experiments, our approach leads to a stable, fair learning process, achieving decision policies with similar utility and fairness measures across time and training initializations.

Our primary contributions in this paper are listed below:

1. We propose a novel two-phase decision-making framework that utilizes both labeled and unlabeled data to learn a policy that converges to the optimal (fair) policy with respect to the unobserved and unbiased ground truth.

2. We present a novel policy learning framework, FairAll that relies on a VAE (a latent variable model) architecture to significantly reduce the need for bias correction of selective labeling.

3. Through theoretical analyses and empirical evaluation on synthetic data, we show that the VAE from our FairAll framework is able to learn an unbiased data representation that captures information from the ground truth.

4. Through extensive evaluations based on real-world data, we show that FairAll, compared to prior work, offers a significantly more effective and stable learning process, achieving higher utility and fairness.

1.1 Related Work

Fair Classification. There exists a variety of approaches for fair classification to tackle biased labels. In-processing methods optimize for correct predictions under additional fairness constraints [1, 17, 56, 57]. This requires formulating differentiable fairness constraints and often lead to unstable training [13]. Pre-processing methods instead utilize representation learning first to learn a fair data representation. This representation is then used for downstream predictive tasks [58]. Different methods for fair representation learning have been brought forward, including variational autoencoders (VAEs) [38, 40], normalizing flows [5], and generative adversarial networks [53]. To enforce independence between the learned representation and the sensitive attribute, some methods condition deep generative models on the sensitive features [14, 21, 39, 40], revert to disentanglement [14], perform adversarial training [21, 39, 50] or add regularization, like Maximum-Mean-Discrepancy [21, 38]. While most work on fair representation learning focuses on satisfying group fairness notions [14, 38], some have also considered individual fairness [46] and counterfactual fairness [21]. Recently, contrastive learning for fair representations has attracted much attention [41]. However, it requires the definition of a similarity measure and meaningful data augmentations. This is non-trivial, especially for tabular data. While some recent work [4] exists, further research is needed.

Although all of the works above use fully labeled training data, some have studied fair classification in the presence of partially labeled data [38, 59, 60]. Further, all of these works assume access to a biased proxy label and not the ground truth. Zafar et al. [55] considered analyzing fairness notions separately, assuming access to a ground-truth label. Further, the notions of biased observed proxies and unbiased, unobserved ground-truth (denoted as construct spaces) were discussed in [16, 19]. Note that all of these studies also assume i.i.d. data. But, in most real-world scenarios, the semi-labeled data is not i.i.d. (selection bias [36]). Wick et al. [52] perform an initial fairness-accuracy analysis of classifiers with respect to label and selection bias. Our work, similar to [31] aims to tackle both label and selection bias while transitioning from a static classification setting to online decision-making.

Fair Online Decision Making. Recent works [8, 31] have started exploring fairness in online decision-learning processes in the presence of partially labeled non-i.i.d. data. In such settings, convergence to the optimal policy requires exploration and stochastic policies [31]. Kilbertus et al. [31] use an extra fairness constraint in the loss to trade-off between utility and fairness. Additionally, they correct for selection bias in training using inverse propensity scoring (IPS) [27] on the entire loss function, which, unfortunately, can introduce additional variance. Bechavod et al. [8] derive an oracle-efficient bandit algorithm to learn an accurate policy while explicitly controlling the exploration-exploitation trade-off, and thus the variance. However, both approaches disregard a major portion of the data that receives the negative decision and remain unobserved. We show how this data contain useful information about the underlying data distribution. In our approach, we posit utilizing this unlabeled data to reduce the need for IPS. We empirically validate how this helps in faster convergence to an optimal decision policy while providing high utility and fairness during training.1

1In Section 5 we compare our method to [31]. Note that [8] is a theoretical work that provides neither experimental results nor an implementation and thus prevents us from comparing to them at the baseline.
2 BACKGROUND AND PROBLEM SETTING

Let us consider a university admission decision-making process inspired by [35], which we will use as a running example. We use uppercase letters for random variables and lowercase letters for their assignments. With \( p \), we optionally refer to a probability distribution or a probability mass function. Let \( S \) be a random variable indicating a sensitive attribute of an individual describing their membership in a socially salient group (e.g., gender). For simplicity we assume binary \( S \in \{-1, 1\} \). Let \( X \in \mathbb{R}^n \) be a set of \( n \) non-sensitive features that are observed (e.g., high school grades), and may be influenced by \( S \). The university aims to take an admission decision \( D \in \{0, 1\} \) based on a ground truth target \( Y \) (e.g., intellectual potential) [16, 19]. For simplicity we assume \( Y \in \{0, 1\} \). Importantly, throughout this paper, we assume that \( Y \perp S \) (e.g., potential is equally distributed across social groups), such that an optimal policy decides \( D \perp S \).\(^2\)

2.1 Label Bias and Selection Bias

Label Bias. In practice, the ground truth \( Y \) often remains unobserved (as it cannot be directly measured). Instead, as shown in Figure 1a, we observe a different label \( \tilde{Y} \) (e.g., semester grades) that is assumed to contain information on \( Y \) along with measurement noise. We refer to this label as the proxy label. For simplicity, we assume \( \tilde{Y} \in \{0, 1\} \). A data-generative process exhibits label bias, if \( \tilde{Y} \perp S \), i.e., the proxy target is biased by the sensitive attribute [52]. For example, the same potential may result in higher grades for one demographic group over another due to existing structural discrimination. Figure 1a highlights our assumed data generative process with biased labels. Recall that we aim to take decisions according to \( Y \perp S \). However, in the biased label scenario, both \( X \) and \( \tilde{Y} \) are biased by \( S \). A policy that maps \( X \) (and potentially \( S \)) to \( \tilde{Y} \) will thus – in the absence of fairness constraints – take biased decisions.

Selection Bias. In practice, algorithms often also need to learn from partially labeled data, where labels \( \tilde{Y} \) are observed only for a particular (usually positive) decision. This is called the selective labels problem [36]. For example, a university only knows whether a student gets good semester grades if it accepts the student in the first place. Let these decisions be taken according to policy \( \pi \), which may be biased and not optimal. For example, for an individual with features \((x, s)\), a decision \( d \) may be taken according to \( d \sim \pi(x, s) \).\(^3\)

Then a labeled data point \((x, s, \tilde{y})\) observed under a policy \( \pi \) is not an i.i.d. sample from the true distribution \( p(X, S, \tilde{Y}) \). Instead, the data is sampled from the distribution induced by the probability of a positive decision \( \pi(d = 1 | x, s) \) such that: \( p_{\pi}(X, S, \tilde{Y}) \propto p(\tilde{Y} | X, S)\pi(D = 1 | X, S)p(X, S) \) [31].

Corbett-Davies et al. [12] have shown that deterministic decisions (e.g. taken by thresholding) are optimal with respect to \( \tilde{Y} \) for i.i.d. data. However, Kilbertus et al. [31] demonstrated that, if labels are not i.i.d., we require exploration, i.e., stochastic decision policies. Such policies map features to a strictly positive distribution over \( D \). This implies that the probability of making a positive decision for any individual is never zero. Exploring policies are trained in an online fashion, where the policy is updated at each time step \( t \) as \( \pi_t \). Moreover, we typically learn a policy from labeled data by minimizing a loss that is a function of the revealed labels. However, if labels are non-i.i.d., it is necessary to perform bias correction to get an unbiased loss estimate. A common technique for such bias correction is inverse propensity score (IPS) weighting [27]. It divides the loss for each labeled datum by the probability with which it was labeled, i.e., received a positive decision under policy \( \pi \). However, this bias correction may lead to high variance in the learning process, when this probability is small.

2.2 Measures of Interest: Utility, Fairness, and Their Temporal Stability

Assuming an incurred cost \( c \) for every positive decision (e.g., university personnel and facility costs) [12], the decision-maker aims to maximize its profit (revenue – costs), which we call utility. We define utility \( U = D(Y - c) \) as a random variable that can take on three values \( U \in \{ -c, 1 - c, 0 \} \) depending on decision \( D \). A correct positive decision results in a positive profit of \( 1 - c \) (admitting students with high potential leads to more success and funding), an incorrect positive decision results in a negative profit \( -c \) (sunk facility costs), and a negative decision (rejecting students) in zero profit. The utility of a policy is then defined as the expected utility \( U \) with respect to population \( p(X, S, Y) \) and policy \( \pi \).

\(^2\)Note, depending on which label \( Y \) refers to, \( Y \perp S \) may not always hold in practice. See Section 6 for a discussion of this assumption.

\(^3\)A policy always takes as input features of an individual. Here, we assume the features to be \((X, S)\). However, they could also be only \( X \) or some feature representation.
**Definition 2.1 (Utility of a Policy [31]).** Given utility as a random variable \( U = D(Y - c) \), we define the utility \( UT(\pi) \) as the expected overall utility \( UT(\pi) := \mathbb{E}_{x \in \mathcal{X}, y \sim p(X, S, Y)} [\pi(D = 1 | x, s) \ (Y - c)] \), where decision and label are \( D, Y \in \{0, 1\} \), and \( c \in (0, 1) \) is a problem specific cost of taking a positive decision.

Note, we defined UT with respect to ground truth target \( Y \). However, as mentioned above, in most practical settings, we only observe proxy \( \tilde{Y} \) and can thus only report \( UT \), i.e., the expected utility measured with respect to proxy \( Y \).

As detailed above, we are interested in taking decisions according to \( Y \), where \( Y \perp S \) (e.g., potential is equally distributed across sensitive groups). A policy that takes decisions based on \( Y \) satisfies counterfactual fairness [35] and demographic parity [17]. This follows directly from the fact that \( Y \) is a non-descendant of \( S \). Any policy \( \pi \) that is a function of the non-descendants of \( S \) (namely \( Y \)) is demographic parity and counterfactually fair [35]. The notion of DP fairness for a policy \( \pi \) requires the proportion of decision \( d \) to be the same across all social groups:

**Definition 2.2 (Demographic Parity Unfairness of a Policy [17]).** We define the demographic parity unfairness (DPU) of a policy \( \pi \) with respect to sensitive attribute \( S \in \{-1, 1\} \) and decision \( D \in \{0, 1\} \):

\[
\text{DPU}(\pi) = \mathbb{E}_{x \sim p(X \mid S = 1)} [\pi(D = 1 | x, S = 1)] - \mathbb{E}_{x \sim p(X \mid S = -1)} [\pi(D = 1 | x, S = -1)]
\]

Correspondingly, a policy is counterfactually fair if it assigns the same decision to an individual in the observed (or, factual) world as well as in a counterfactual world, in which the individual belongs to a different sensitive group.

**Definition 2.3 (Counterfactual Unfairness of a Policy [35]).** The counterfactual unfairness (CFU) of policy \( \pi \) with respect to a factual individual belonging to \( S = s \) with features \( x^f \) and the decision \( D \in \{0, 1\} \) can be defined as:

\[
\text{CFU}(\pi) = \mathbb{E}_{x^f, s \sim p(X, S) \mid x^f \neq x^{CF}} [\pi(D = 1 | x^f, s)] - \mathbb{E}_{x^f, s \sim p(X, S) \mid x^f = x^{CF}} [\pi(D = 1 | x^f, s)]
\]

Here, \( x^f \) refers to the non-sensitive features of an individual in the factual world with sensitive attribute \( s \), and \( x^{CF} \) refers to the non-sensitive features of the same individual in a counterfactual world, where its sensitive attribute is \( s' \neq s \).

Note, from [35] that satisfying counterfactual fairness implies satisfying demographic parity but not vice-versa. Further, counterfactual analysis requires hypothetical interventions on \( S \) and exact knowledge of the causal generation process. While estimation techniques for real-world data exist [30, 47], in this paper, we only analyze for synthetic data (with access to the true exogenous variables and the structural equations). See Appendix C.4 for more details.

The above metrics allow assessing the performance of one particular policy. However, the online policy learning process outputs, over \( T \) training steps, the set of policies \( \Pi_T = \{\pi_1, \ldots, \pi_T\} \). Assume we wish to stop the learning process from time \( t_1 \). Can we reliably deploy any policy \( \pi_{t_2} \)? Inspired by prior work on temporal fairness [10, 22], we propose a new notion of temporal variance (TV) for a policy learning process. TV indicates how much a metric \( M \) (e.g., utility, fairness) varies for the set of policies across some time interval \([t_1, t_2]\).

**Definition 2.4 (Temporal Variance of a Policy Learning Process).** We define the temporal variance (TV) of the outcome of a policy learning process \( \Pi_T = \{\pi_1, \ldots, \pi_T\} \) in time interval \([t_1, t_2]\) with respect to metric \( M \) as:

\[
\text{TV}_M(\Pi_T) = \sqrt{\frac{1}{t_2 - t_1} \sum_{t=t_1}^{t_2} [(M(\pi_t) - \mu_M)^2]}
\]

where \( \mu_M = \frac{1}{t_2 - t_1} \sum_{t=t_1}^{t_2} M(\pi_t) \) denotes the temporal average for the metric \( M \) over the time interval \([t_1, t_2]\).

High TV denotes an unstable learning process, where policies of different time steps achieve different utility and fairness levels for a fixed group of people. For example, policy \( \pi_{t_1} \) may treat the same group of individuals very different compared to \( \pi_{t_2} \). A low TV on the other hand indicates a stable learning process, where policies of different time steps achieve similar utility and fairness levels. Hence, it is safe to stop the learning process any time after \( t_1 \).

Note, \( \mu_M \) measures the average metric value (e.g., utility, fairness) over the time interval \([t_1, t_2]\).

### 2.3 Variational Autoencoder

Deep generative models (DGMs), like Variational Autoencoders (VAEs) [33, 45], Normalizing Flows, [44] and Generative Adversarial Networks [20] are latent variable models (LVMs) that *estimate complex data distributions* by capturing hidden structures in the latent space \( Z \).

VAE is one of the most prominent DGMs. It jointly learns a probabilistic generative model \( p_{\theta}(X \mid Z) \) (encoder) and an approximate posterior estimator \( q_{\phi}(Z \mid X) \) (decoder). Encoder and decoder are parameterized by neural networks. As the marginal likelihood \( p(x) = \int p(x, z)dz \) is intractable, a VAE is trained by maximizing the evidence lower bound (ELBO) of the observations \( x \sim p(X) \), consisting of the expected log-likelihood and the posterior-to-prior KL divergence:

\[
\log p(x) \geq \mathbb{E}_{z \sim q_{\phi}(Z \mid x)} \log p_{\theta}(x \mid z) - \text{KL}(q_{\phi}(Z \mid x) || p(Z))
\]

\[
\text{ELBO}(\theta, \phi; x) = \text{Exp. log-likelihood} - \text{KL divergence}
\]

### 3 LEARNING TO DECIDE FAIR

Let us assume that the data \( p(X, S, Y) \) has a generative process as shown in Figure 1a\(^6\). Recall that a decision-maker ideally aims to take decisions \( d \) according to the unbiased ground truth \( Y \), i.e., \( d \sim p(Y) \) \[^{31}\]. However, in practice, \( Y \) remains unobserved. Consider

\[^{6}\text{Note, Figure 1a is the same as the causal model presented as the Scenario 3: University success in [35].}
\[^{7}\text{Note, this is formulated abusing notation for simplicity. Here, decision \( D \) is a deterministic function of \( Y \), i.e., for a data point \( d = y \), where \( y = \pi(Y) \).}
\]
for now a setting with label bias but no selection bias. We have access to i.i.d. samples from the underlying distribution and the observable label is a biased proxy Ỹ. As per Figure 1a, observed features X and proxy labels Ỹ both contain information about Y, but are biased by S (label bias), i.e., XLS, ỸLS. Hence, a policy that takes decisions \( d_{\text{prox}} \sim p(Ỹ | X, S) \) from such biased observed data is unfair.

Assuming access to only biased data, we posit using a conditional latent variable model for fair decision making. As we theoretically show, with the help of a conditional latent variable model (LVM), it is possible to learn a latent representation Z that: i) is independent of the sensitive S, i.e., ZLS and ii) captures the information contained in Y, up to the noise of the observed features and the approximation error of the LVM.

We assume observed features X and proxy labels Ỹ are generated by Y, S and an independent noise variable \( \tilde{E} \in \mathbb{R} \).

**Lemma 1.** Assume the observed \( \{X, Ỹ\} \) is a bijective function of the ground-truth Y, sensitive S and noise E, with S, Y, E being pairwise independent. Then, the conditional data entropy is \( H(X, Ỹ | S) = H(Y) + H(Ỹ | Y, S) = H(Y) + H(E) \).

So, the conditional data distribution \( p(X, Ỹ | S) \) captures the information of the unobserved ground-truth Y, up to the extent of noise E. Next, we consider approximating the underlying data distributions via LVMs.

**Lemma 2.** Given a latent variable model conditional on \( S \) and input data \( \{X, Ỹ\} \), having encoder \( q_\theta(Z | X, Ỹ, S) \) and decoder \( p_\theta(X, Ỹ | Z, S) \), the mutual information between latent variable Z and the conditional data distribution \( p(X, Ỹ | S) \) is \( I(Z; X, Ỹ | S) = H(X, Ỹ | S) - \Delta \) with approximation error \( \Delta \).

Hence, the information captured by the latent Z reduces the uncertainty about the conditional distribution \( p(X, Ỹ | S) \) up to the error. We refer to Appendix A.2 for the detailed proof following [2]. Combining the two lemmas, we get:

\[
I(Z; X, Ỹ | S) = \frac{H(Y)}{\text{information of Y}} + \frac{H(E)}{\text{noise}} - \frac{\Delta}{\text{approx. error}} (1)
\]

The two lemmas together show that using a conditional LVM to model the observed data \( p(X, Ỹ | S) \) allows us to learn a latent variable Z that captures the information of the unobserved ground-truth Y, up to the extent of noise (note that \( \Delta \) is also dependent on \( E \)). Consequently, a policy that learns to make decisions using the latent Z with respect to the proxy Ỹ would, in fact, make decisions based on the information contained in Y (up to the effect of noise \( H(E) \)).

As pointed out in Section 2, following [35], a policy deciding based on Y satisfies counterfactual fairness and demographic parity. Hence, a policy \( \pi \) mapping from Z to Ỹ tackles label bias and satisfies both fairness notions without the need for additional constraints (up to the distortion due to \( H(E) \) and \( \Delta \)). Following, in Section 4, we propose a pipeline to learn a fair policy using unbiased representations Z from non-i.i.d. data that suffer from both label and selection bias.

4 OUR APPROACH

In this section, we propose a novel online fair policy learning framework FairAll for tackling both biased and selective labels. Our FairAll framework consists of: i) a fair representation learning step that relies on a VAE-based model (illustrated in Figure 1b) trained on both labeled and unlabeled data and; ii) a policy learning approach that leverages the learned fair representations to approximate the optimal fair policy according to the ground truth Y. Both steps of our framework, i.e., the VAE and the policy, are continually optimized as more data becomes available through the development of previous policies (i.e., in an online manner). Note, in taking decisions based on a fair representation, our policy mitigates label bias; in learning a stochastic policy in an online manner, we allow for exploration during training, which mitigates selection bias [31]. Specifically, we correct label bias by conditioning the VAE on sensitive S, while we correct selection bias by weighting our online learning loss with IPS.

In the following, we first detail how to use both labeled and unlabeled data to learn a fair representation and then describe decision learning with policy \( \pi \). Lastly, we present an overview of our entire fair online policy learning pipeline and propose a method to further exploit unlabeled population information.

4.1 Learning a Fair Representation

Following the result in Eq. 1, we aim to learn latent Z that is both informative of Y and independent of the sensitive attribute S, i.e., ZLS. Consider an online setting in which we have access to a dataset \( \mathcal{A}_t \) (applicants) at each time step \( t \). The partitioning of \( \mathcal{A}_t \) into labeled data \( \mathcal{A}_t^L \) (accepted applicants) and unlabeled \( \mathcal{A}_t^U \) (rejected applicants) is invoked by policy \( \pi_t \). To ease notation, we will, in the following, consider a particular time step \( t \) and omit the subscript.

Let us recall that for each data observation, we only observe the proxy label Ỹ if the previous policy made the positive decision \( D = 1 \) (labeled data), and the actual value of the ground truth remains unobserved. However, we can leverage the fact that the utility with respect to the proxy label, \( U = D(Ỹ - c) \), is always observed.\(^8\) This allows us to learn an unbiased latent representation Z from both labeled and unlabeled data.

**VAE-Based Fair Representation Learning.** Specifically, we build on previous work on semi-supervised and conditional VAEs [32, 44, 49] to approximate the conditional distribution \( p_\theta(X, Ỹ | S, D) = \int p_\theta(X, Ỹ, Z | S, D) dZ \) (generative model), and the posterior over the fair latent representation \( q_{\phi, \omega}(Z | X, S, D = 1) \approx \int q_{\phi}(Z | X, S, \hat{u}, D = 1) q_{\omega}(\hat{u} | X, S, D = 1) d\hat{u} \).

The inference model contains an encoder \( q_\phi \) and a separate classifier model \( q_\omega \). Note, we condition the inference model on \( D = 1 \) (see Appendix C.1 for an overview) and thus introduce the classifier to predict the label for any unlabeled data point. We optimize the model parameters (\( \theta, \phi \), and \( \omega \)) by minimizing the following

\(^8\) As per Section 2.2, utility U can take three values dependent on the decision, hence providing a value for accepted and rejected applicants.
where the latter terms corresponds to the ELBO capturing the goodness of fit of the VAE, and the first term measures the accuracy of the classifier model that estimates the utility of labeled data. The goodness of fit of the VAE; and the first term measures the accuracy on decision costs, false negatives are weighed by the lost profit cross-entropy loss to train the classifier $q$. Based on the above, we can learn a VAE optimized as:

$$
\mathcal{K}(\theta', \phi'; x, s) = \mathbb{E}_{z \sim q_{\phi'}(z \mid x, s)} \left[ \log p_{\theta'}(x \mid z, s) \right] - KL \left( q_{\phi'}(Z \mid x, s, d = 1) || p(Z) \right)
$$

(6)

4.4 FairAll Overview

Our FairAll learning framework is illustrated in Figure 2 and consists of two phases. In the first step (Phase I), we learn a fair representation using an unsupervised VAE trained in an offline manner using only unlabeled data. We then use the resulting model to initialize the parameters of the semisupervised VAE via transfer learning. In a second step (Phase II), we enter the online decision-making process, where at each time step, we first update our semisupervised VAE using both labeled and unlabeled data, and then update the decision policy $\pi$.

5 EXPERIMENTAL RESULTS

In this section, we evaluate our fair policy learning framework with regard to i) its convergence to the optimal policy; ii) its training effectiveness until convergence; and iii) its deployment performance after convergence. We can evaluate (i) on synthetic data only, and evaluate (ii) and (iii) on real-world datasets.
Baseline and Reference Models. We perform rigorous empirical comparisons among the following learning frameworks:

- **FairAll (I+II):** Our complete proposed learning framework including Phase I (i.e., offline unsupervised representation learning) and Phase II (online semisupervised representation and policy learning).
- **FairAll (II):** Baseline approach that make use of only Phase II of the proposed FairAll. This approach allows us to evaluate the impact of Phase I, i.e., fully unlabeled data.
- **FairLab (I+II):** Baseline approach that consist of unsupervised Phase I and a fully supervised Phase II using only the IPS-weighted ELBO on labeled data. It allows us to evaluate the importance of unlabeled data in Phase II.
- **FairLog [31]:** Competing approach that minimizes the IPS weighted cross entropy (Eq. 5), denoted by \( \mathcal{L}_{\text{UnfairLog}} \), with a Lagrange fairness constraint, i.e., \( \mathcal{L}_{\text{UnfairLog}} + \lambda \cdot \text{DPU} \) with DPU as defined in Def. 2.2.
- **UnfairLog [31]:** Unfair reference model, corresponding to FairLog without fairness constraint, i.e., \( \lambda = 0 \). It allows us to measure the cost of fairness.

We refer to the Appendix for a detailed description of baselines and competing methods (Appendix D.5), details on the hyperparameter selection (Appendix D.2) and other practical considerations (Appendix C).

**Metrics.** We measure observed proxy utility \( \bar{U}\bar{T} \) (Def. 2.1 w.r.t. \( \bar{Y} \)) and demographic parity unfairness DPU (Def. 2.2) on i.i.d. test data on both synthetic and real-world datasets. Further, on synthetic data with access to the ground truth generative process, we report counterfactual unfairness CFU (Def. 2.3)\(^{10}\) as well as the unobserved ground truth utility UT (Def. 2.1 w.r.t. \( Y \)). For the real-world settings, we report effective \( \bar{U}\bar{T} \) [31], which is the average \( \bar{U}\bar{T} \) accumulated by the decision-maker on the training data up to time \( t \). Similarly, we report effective DPU. We also report the temporal variance (Def. 2.4) of \( \bar{U}\bar{T} \) and DPU over time interval \( t = [125, 200] \).

**Datasets.** We report results on one synthetic and three real-world datasets (more details in Appendix B):

- Synthetic, where \( X \) contains 2 features, with Gender \( S \) and grades after university admission \( \bar{Y} \) (note that the ground truth intellectual potential \( Y \) is considered unobserved and only used for evaluation).
- COMPAS [3, 37], where \( X \) contains 3 features, with Race \( S \) and no recidivism \( \bar{Y} \).
- CREDIT [15], where \( X \) contains 19 non-sensitive features, with Gender \( S \) and credit score \( \bar{Y} \).
- MEPS [24], where \( X \) contains 39 non-sensitive features, with Race \( S \) and high healthcare utilization \( \bar{Y} \).

**Optimal Policies.** In our Synthetic setting, where observed \( X_{LS} \) and unobserved \( K_{LS} \), the optimal unfair policy (OPT-UNFAIR) takes decisions \( d \sim p(\bar{Y} \mid X, S) \) and the optimal fair policy (OPT-FAIR) decides \( d \sim p(\bar{Y} \mid K) \). OPT-UNFAIR can be approximated with access to i.i.d. samples from the posterior distribution, while OPT-FAIR additionally requires access to unobserved \( K \). See Appendix C.3 for details.

**Setup.** We assume access to the proxy labels. Since labeled data is often scarce in practice, we assume an initial HARSH policy which labels around 10-18% of the data that we see in Phase II at \( t = 0 \). We report results for a lenient policy in Appendix E. For details on initial policies, see Appendix D.1. The decision cost is \( c = 0.1 \) for MEPS and \( c = 0.5 \) for all other datasets. See Appendix E.4 for a case study on the impact of the cost value. Wherever applicable, Phase I was trained over a large number of epochs. Phase II was trained for 200 time steps with the same number of candidates in each step. All policy training were done over 10 independent random initializations. For a full description of the experimental setup, see Appendix D. Our code is publicly available\(^{11}\).

\(^{10}\)See Appendix C.4 for further details on generating counterfactuals on the synthetic dataset.

\(^{11}\)https://github.com/mrateike/fairall
Figure 3: Utility ($\tilde{U}T$) with respect to the proxy, demographic parity unfairness (DPU), and counterfactual unfairness (CFU) on the Synthetic dataset. FairAll (I+II) converges to the optimal fair policy in utility and fairness while being more fair than FairLog.

5.1 Can We Reach the Optimal Fair Policy?

In this section, we use synthetic data to evaluate if, given enough time-steps, the different learning methods yield policies that converge to the optimal (fair) policy (OPT-FAIR) both in terms of proxy utility and fairness.

Results. Figure 3 reports $\tilde{U}T$, DPU and CFU on test data across time. Recall that FairLab (I+II) uses unlabeled data only in Phase I, FairAll (II) only in Phase II, and FairAll (I+II) in both Phase I and Phase II. FairAll (II) starts convergence after approximately 20 steps. FairLab (I+II) starts at higher utility but then exhibits slower convergence behavior and has not fully converged at $t = 200$. FairAll (I+II) instead starts at a higher utility and at $t = 200$ yields utility and fairness (both in demographic parity and counterfactual fairness) values close to OPT-FAIR. Regarding fairness, FairAll (II) does not converge to OPT-FAIR and exhibits high variance in DPU and CFU both across seeds and over time. Instead, FairAll (I+II) and FairLab (I+II) both rely on unsupervised fair representation learning (Phase I) to converge to a value very close to the optimal one. This can be explained by the fact that FairLog enforces DPU constraints, and, as shown empirically, DPU does not imply CFU. In contrast, FairAll (I+II) learns a fair representation $Z$ that achieves CFU and, as a result, also DPU (see Section 3). This evaluation concludes that i) all fair models approximately converge to the optimal utility; ii) unlabeled data helps in faster convergence; iii) utilizing Phase I leads to significantly lower unfairness; iv) our approach FairAll, compared to FairLog, achieves convergence while satisfying both counterfactual and demographic parity fairness notions.

5.2 Do We Actually Trade-off Utility for Fairness?

In Figure 4, we observe that FairAll yields higher fairness but lower observed proxy utility $\tilde{U}T$ compared to the unfair reference model UnfairLog. This observation is often referred to as a fairness-accuracy trade-off [6], which assumes that fairness comes at the cost of utility (or accuracy in predictive settings). As pointed out in [16, 52], if utility is a function of biased labels, then the utility measurement is also biased. Recall the assumption that a decision-maker aims to take decisions based on $Y$ and aims to maximize UT. For example, potential ($Y = 1$) drives a successful career, not high university grades ($\tilde{Y} = 1$). In the synthetic setting, we can measure unbiased ground truth utility $UT$.

Results. In Figure 4, we observe that both FairAll and UnfairLog achieve a similar level of ground truth utility UT, while FairAll reports significantly less discrimination (DPU). This suggests that a decision-maker may not actually trade-off fairness and (true) utility, although we observe lower proxy utility. Note that despite being fair, FairAll (and FairLog) do not reach the UT of the optimal (fair) policy OPT-FAIR. This could be due to the noise $\varepsilon$ in the dataset, which is known to the optimal policy, but is naturally not captured by the models. An in-depth discussion of the phenomenon is outside the scope of this paper, and we refer the reader to the literature [11, 16, 52].

5.3 How Effective Is the Learning Process?

We have shown that FairAll asymptotically outputs a policy that approximates the optimal. Now we investigate how much it costs the decision-maker in terms of utility and the society in terms of fairness to learn this policy. We evaluate the effective proxy $\tilde{U}T$, and DPU that the online learning process accumulates across time on real-world datasets.

Results. Table 1 summarizes the results for several real-world datasets. FairAll (I+II) consistently accumulates more utility and less unfairness during the online learning process compared to the other approaches. Note that FairLab (I+II), which uses only labeled data in Phase II, accumulates less utility and more unfairness than FairAll (II), which skips Phase I but uses both labeled and unlabeled data in Phase II. This suggests that joint training on labeled and unlabeled data in Phase II significantly improves learning of both a fair representation and policy. Furthermore, FairAll (I+II) outperforms FairAll (II), suggesting that using unlabeled data in
5.4 How Do the Learned Policies Perform During Deployment?

In this section, we analyze how a given strategy, when applied to the population of interest, is expected to perform in the long run. To this end, we compare the performance of the resulting strategy at each time step using an i.i.d. test set.

Result. Figure 5 shows how utility $\bar{UT}$ and unfairness $DPU$ evolve over time for COMPAS. Results on the other real-world datasets are in Appendix E. FairAll (II) achieves both higher utility and higher unfairness compared to FairLab (I-II) with significantly higher variance. This suggests that unlabeled data in Phase I benefit fairness, while unlabeled data in Phase II benefit utility. FairAll (I-II) provides significantly higher utility and lower unfairness than the other learning models after approximately 50 time steps. Moreover, FairAll (I-II) even provides higher utility than even the unfair reference model UnfairLog while being as fair as FairLog. This empirically confirms the importance of unlabeled data in both Phase I and Phase II to achieve high test utility and fairness in real-world scenarios.

5.4.1 Can We Reliably Stop Learning at Any Time? Assume that we want to stop the policy learning process from time $t_1$. Can we stop the learning process and deploy the policy at any time $t \geq t_1$? We study this by measuring how much utility (fairness) of the output policies vary over time interval $[t_1, t]$ for a fixed test dataset. A low temporal variance ($TV$) indicates stable behavior, such that it is safe to terminate the learning process at any time. However, when the variance is high, the decision-maker must carefully select the best stopping point. High TV leads to unstable policies that may lead to low utility and/or high fairness. In addition, we measure the temporal average $\mu$ of utility (unfairness). It is desirable for a learning process to have both low variance and high (low) $\mu$ for utility (unfairness).

Results. In Table 2, we report $TV_{DPU,\bar{UT}}$ for unfairness and $TV_{\bar{UT},\bar{DPU}}$ for utility (see Def. 2.4). Compared to FairLab (I-II) and FairAll (II), FairAll (I-II) provides similar TV values. However, FairAll (I-II) exhibits a better temporal average utility and
fairness. For CREDIT, for example, note that FairLab (I+II) has a lower TV$_{DPU}$ and a higher average unfairness level $\mu_\text{DPU}$ than FairAll (I+II). Compared to FairLog, our method FairAll (I+II) is much more stable. It has lower TV$_{DPU}$ and TV$_{UT}$ as well as a higher average utility $\mu_\text{UT}$ and lower unfairness $\mu_\text{DPU}$.

6 DISCUSSION OF ASSUMPTIONS AND OUTLOOK

In this section, we discuss the main assumptions, limitations, and potential consequences of our proposed framework.

Assumptions on the Data Generation Process. In this work, we assume that the true data generation process follows Figure 1a and that $S$ is a social construct like gender or race. This follows the understanding that discrimination is based on social constructs and not biological differences [28]. We assume that each individual can be assigned to a social group within a social construct (e.g., gender) and that $S$ is a root node in the graphical model. While being common [35], this is a debated modeling assumption [6, 28]. Furthermore, we assume that we observe a biased proxy variable $\tilde{Y}$, and that an unobserved unbiased ground truth $Y$ exists that is independent of $S$. This means that there are no innate differences between social groups with respect to $Y$. However, see Appendix F.1 for examples where $Y\perp S$. We advise any practitioner using our pipeline to carefully assess whether these assumptions hold in their case.

Assumptions of Our Policy Learning Pipeline. First, we assume access to a large unlabeled dataset of i.i.d. samples from the population of interest for our pipeline (Phase I). For example, a university may have access to the grades of all students who graduated from high school in a given time period. We also assume access to sensitive information, which, in the real world, may conflict with privacy regulations (e.g., the principle of data minimization). In this paper, we show that access to a large unlabeled dataset of sensitive information not only increases the utility of the decision-maker but also fairness. We hope that this contributes to the debate between fair algorithmic decision-making and privacy.

Second, we assume a decision-maker has an unlimited budget at each time step, i.e., it can take as many positive decisions as desired.

A related line of work [29, 34], deals with fairness in selection problems, where candidates compete for a limited number of positions, or with pipeline settings [18], where candidates enter the decision process one at a time. This is an interesting direction for future research.

Third, we assume that the underlying data distribution does not change over time and thus is not affected by the decisions. This assumption does not necessarily hold in the real world. While it exceeds the scope of this paper, it would be interesting to extend our pipeline to address distribution shift as a consequence of the decision-making process.

Lastly, in Phase II we learn a stochastic policy at each time step and use it to collect new data. We follow Kilbertus et al. [31] in their call for a general discussion about the ethics of non-deterministic decision-making.

Assumptions on Fairness Metrics. While the ethical evaluation of the applicability of a particular fairness notion in a specific context lies outside of our scope, we give an overview of when the use of our pipeline may be helpful in practice. We evaluate fairness based on the demographic parity (DP) notion [17]. Heidari et al. [25] map DP to Rawl’s moral understanding, according to which unequal treatment may only be justified on the basis of innate potential or ambition, not gender or race. Within this framework, the underlying assumption of DP is that individuals should receive utility solely based on the factors for which they are accountable. In this paper, such factors are assumed to be captured by the unobserved ground truth label $Y$. Hertweck et al. [26] show that one should enforce DP not only if socio-demographic groups have similar innate potential at birth, but in some cases even if unjust life biases lead to differences in realized abilities. Wachter et al. [51] similarly argue that DP and counterfactual fairness are bias transforming metrics that acknowledge historical inequalities and assume that certain groups have a worse starting point than others. However, they also warn that, e.g., giving an individual a loan that they cannot repay can exacerbate inequalities.

7 CONCLUSION

In this paper, we considered the problem of learning optimal fair decision policies in the presence of label bias and selection bias.
Table 2: Temporal variance and means of utility ($TV_{UT}, \mu_{UT}$) and demographic parity unfairness ($TV_{DPU}, \mu_{DPU}$). Metrics are measured on the time interval $t = [125, 200]$ on real-world datasets. We report the mean over ten runs with the standard deviation in brackets. For TV, lower values are better. For $\mu$ higher (lower) is better for UT (DPU). Values multiplied by 100 for readability.

| Model      | COMPAS       | CREDIT       | MEPS       |
|------------|--------------|--------------|------------|
|            | $TV_{DPU} (\mu)$ | $\mu_{DPU} (\mu)$ | $TV_{UT} (\mu)$ | $\mu_{UT} (\mu)$ | $TV_{DPU} (\mu)$ | $\mu_{DPU} (\mu)$ | $TV_{UT} (\mu)$ | $\mu_{UT} (\mu)$ |
| FairAll (I-II) | 1.0 (0.7) | 4.3 (3.1) | 0.4 (0.3) | 8.6 (0.7) | 3.0 (2.0) | 5.0 (5.0) | 1.0 (0.7) | 18.8 (1.3) | 1.8 (1.6) | 5.1 (3.1) | 0.2 (0.1) | 7.9 (0.5) |
| FairAll (II)  | 2.8 (1.7) | 7.8 (4.0) | 0.8 (0.5) | 6.6 (1.0) | 2.7 (2.4) | 4.6 (3.0) | 0.7 (0.6) | 18.2 (1.5) | 3.8 (2.9) | 6.4 (4.8) | 0.3 (0.2) | 7.4 (0.5) |
| FairLab (I-II)| 0.7 (0.6) | 4.3 (3.1) | 0.3 (0.3) | 4.2 (0.9) | 1.7 (1.3) | 9.1 (5.7) | 0.5 (0.4) | 14.9 (1.4) | 0.7 (0.6) | 4.0 (2.3) | 0.3 (0.2) | 5.9 (0.7) |
| FairLog     | 1.8 (1.4) | 4.5 (3.8) | 0.5 (0.4) | 4.3 (1.1) | 3.6 (2.0) | 7.6 (4.4) | 1.3 (1.1) | 17.1 (2.3) | 2.6 (2.8) | 5.4 (3.3) | 0.5 (0.4) | 6.4 (0.7) |

Prior work that attempted to solve the problem by learning stochastic exploratory decision policies in an online process neglects a large amount of unlabeled data and suffers from high variance in the learned policies over time. In this work, we proposed a novel two-phase framework that leverages both labeled and unlabeled data to learn stable, fair decision policies. We introduced a practical sequential decision-making pipeline FairAll that uses the latent representations of a variational autoencoder to learn, over time, the optimal fair policy according to the unobserved ground truth. In line with our assumptions, the decision policies learned by FairAll satisfy both the notion of counterfactual fairness and demographic parity without requiring additional fairness constraints. Through theoretical analysis and experiments with synthetic data, we validate that FairAll converges to the optimal (fair) policy with respect to the unobserved ground truth both in terms of utility and fairness. Compared to the prior work, we show how our modeling approach helps us to be counterfactual and demographic parity fair. On real-world data, we show how FairAll provides not only a significantly more effective learning method, but also higher utility, higher fairness, and a more stable learning process than the existing approach.

In comparison to baseline models, we also show the importance of using unlabeled data in both phases to achieve a more accurate, fair, and stable decision learning process.

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A PROOFS

A.1 Proof Lemma 1

Proof. Let $V = \{X, \tilde{Y}\}$. Considering $V = f(S, Y, E)$, where $S, Y$ and noise $E$ are pairwise independent. We want to determine $H(V \mid S)$:

$$I(V, Y \mid S) = H(V \mid S) - H(V \mid Y, S)$$

Equating (7) and (8) gives:

$$H(V \mid S) - H(V \mid Y, S) = H(Y \mid S) - H(Y \mid X, S)$$

$$H(V \mid S) = H(Y \mid S) - H(Y \mid V) + H(V \mid Y, S)$$

Assuming $f$ is a bijective function with a joint probability mass function $p(y, s, e) = \rho(y)\rho(s)\rho(e)$, we get:

$$H(Y \mid V) = H(Y \mid f(S, Y, E))$$

$$= - \sum_{y,s,e} p(y, s, e) \log \frac{\rho(y, s, e)}{\rho(y, s)}$$

$$= - \sum_{y,s,e} p(y, s, e) \log 1 = 0$$

$$H(V \mid Y, S) = H(f(S, Y, E) \mid Y, S)$$

$$= - \sum_{y,s,e} p(y, s, e) \log \frac{\rho(y, s, e)}{\rho(y, s)}$$

$$= - \sum_{y} p(y) \sum_{s} p(s) \sum_{e} p(e) \log p(e) = - \sum_{e} p(e) \log p(e)$$

$$= H(E)$$

This leads to:

$$H(Y \mid V) = H(Y) - H(Y \mid V) + H(V \mid Y, S)$$

$$= H(Y) + H(E)$$

\[ \square \]

A.2 Proof Lemma 2

In this proof we follow [2]. Let $V = \{X, \tilde{Y}\}$. Here $\rho$ denotes the original data distribution, $\rho_\theta$ refers to the distribution induced by the decoder, and $q_\phi$ refers to the distribution induced by the encoder of the latent variable model.

Proof.

$$I(V, Z \mid S)$$

$$= \int \int \int_{\rho(z, s)} p(v \mid z, s) \log \frac{p(v \mid z, s)}{\rho(v \mid z, s)} dz, ds$$

$$= \int \int \int_{\rho(z, s)} p(v \mid z, s) \rho_\theta(v \mid z, s) \log \frac{\rho_\theta(v \mid z, s)}{\rho(v \mid z, s)} dz, ds$$

$$= \int \int \int_{\rho(z, s)} q_\phi(z \mid v, s) \rho_\theta(v \mid z, s) \log \frac{\rho_\theta(v \mid z, s)}{\rho(v \mid z, s)} dz, ds$$

$$\geq \int \int \int_{\rho(z, s)} q_\phi(z \mid v, s) \rho(v \mid z, s) \log \rho(v \mid z, s) - \int \int \int_{\rho(v \mid z, s)} p(v \mid z, s) \log p(v \mid z, s) dz, ds$$

$$= \int \int \int_{\rho(z, s)} q_\phi(z \mid v, s) \rho(v \mid z, s) \log \rho(v \mid z, s) dz, ds$$

Thus $I(V \mid S; Z) = H(V \mid S) - \Delta$ with distortion $\Delta$ that measures the approximation error or reconstruction ability of the latent variable model.

Note that we obtain the inequality using:

$$KL(p(v \mid z, s) \mid \rho_\theta(v \mid z, s)) \geq 0$$

$$\int p(v \mid z, s) \log \rho_\theta(v \mid z, s) \geq \int p(v \mid z, s) \log p(v \mid z, s)$$

\[ \square \]

B DATASETS

B.1 Synthetic Dataset

Our synthetic dataset Synthetic follows [35] and models the Law school admission scenario. It includes two continuous observed variables (test-scores LSAT, GPA), one sensitive variable (Gender S), and one unobserved confounding variable (hidden potential $Y$ expressed as talent (knowledge) K). The label $\tilde{Y}$ indicates whether a student passes or fails the first year of Law school and is determined
We select three real-world datasets for our analysis of fair decision-making. This willingness is unobserved and assumed to be independent of the social construct (i.e., ground truth label $Y$). We may then aim to learn a decision-making algorithm, which may decide whether or not to offer an inmate a career management that have a high genuine medical needs. This helps ensure appropriate attention and care to reduce overall costs and improve the quality of services. A decision-maker may wish to make decisions based on the unobserved actual medical needs of individuals (i.e., ground truth label $Y$ independent of race $S$). Instead, we may observe a person’s number of hospital visits during a particular time period (i.e., proxy label $\tilde{Y}$). We may assume that the more visits, the higher the likelihood that the person has a high need for medical care. We assume the number of visits to be dependent on the sensitive attribute, for example, due to unequal access to health care [48]. Our goal may then decide to provide individuals with care management that have a high genuine medical needs ($Y$) not a high number of hospital visits ($\tilde{Y}$).

C.2 Real-world Datasets

We select three real-world datasets for our analysis of fair decision-making. These datasets have different complexity in terms of heterogeneity of features and the total number of features. We show our results on the COMPAS recidivism dataset [3, 37], CREDIT (German) Dataset [15] and MEPS health-expenditure Dataset [24] (the 2015 MEPS Panel 20 dataset [9]). The characteristics of these datasets are listed in table 3. We obtain all datasets using the AIF360 tool [9]. We describe a possible and simplified decision scenario for each dataset in the following.

COMPAS. For the COMPAS recidivism dataset, we may aim to decide whether or not to offer an inmate a reintegration program. A decision-maker may wish to offer integration support to inmates with high innate potential for reintegration. This potential is unobserved and assumed to be independent of the social construct (i.e., ground truth label $Y$ independent of race $S$). We observe if a person has committed a crime within two years after trial (recidivism). This is the observed proxy label $\tilde{Y}$. We assume arrest to be dependent on the sensitive attribute. For example, members of a particular racial group may be profiled and arrested at a significantly higher rate. We may then aim to learn a decision-making algorithm, which decides according to potential ($Y$) not arrest ($\tilde{Y}$).

As explained in the paper, we pre-processed the label $\tilde{Y}$ of the COMPAS dataset such that $\tilde{Y} = 1$ indicates no recidivism. This is done in order to match our assumption that a person with a positive ground truth label should receive a positive decision.

CREDIT. For the CREDIT dataset, we may aim to decide whether or not to give a credit to an individual. A decision-maker may like to provide credit to people who have a high willingness to repay. This willingness is unobserved and assumed to be independent of the social construct (i.e., ground truth label $Y$ independent of gender $S$). Instead, we observe a credit risk score (i.e., proxy label $\tilde{Y}$) and reject individuals with poor credit risk scores. We assume that societal discrimination (e.g., gender wage gaps) causes risk scores to disadvantage certain genders $S$, even if an individual is honest and willing to repay. Our goal may then be to create a fair decision-making system that lends to individuals depending on their willingness to pay back ($Y$) not risk score ($\tilde{Y}$).

MEPS. Care management often uses datasets such as MEPS Health [24] to predict individuals who have increased health care needs. This helps ensure appropriate attention and care to reduce overall costs and improve the quality of services. A decision-maker may wish to make decisions based on the unobserved actual medical needs of individuals (i.e., ground truth label $Y$ independent of race $S$). Instead, we may observe a person’s number of hospital visits during a particular time period (i.e., proxy label $\tilde{Y}$). We may assume that the more visits, the higher the likelihood that the person has a high need for medical care. We assume the number of visits to be dependent on the sensitive attribute, for example, due to unequal access to health care [48]. Our goal may then decide to provide individuals with care management that have a high genuine medical needs ($Y$) not a high number of hospital visits ($\tilde{Y}$).

C.2.1 Generative and Inference Model

Generative model. Our generative model approximates features $X$ and utility $U$. Note that for unlabeled data ($d = 0$), their actual observed utility is 0. This enables us to easily train the model without any IPS correction.

$$p_\theta(x,u,z|s,d) = p_\theta(x|z,s)p_\theta(u|z,s,d)p(z),$$

where

$$p_\theta(u|z,s,d) = (1 - d) \cdot I[\tilde{u} = 0] + d \cdot p_\theta(u|z,s,d = 1),$$

Inference model. Note that our inference model is always condition on the positive decision ($d = 1$). This helps us encode useful information regarding which individual has actual positive/negative utility. For unlabeled data, we observe $u = 0$, so we do not know the actual positive/negative utility. For estimating the same while avoiding the need for IPS bias correction, we utilize a separate classifier model. This idea comes from the semi-supervised literature [38]. The classifier is trained only with labeled data (with IPS) and approximates if an unlabeled datum ($z$) has positive/negative utility.

$$q_{\omega,\phi}(z|x,s,d = 1) = q_{\omega,\phi}(\tilde{u}|x,s,d = 1)q_\phi(z|x,\tilde{u},s,d = 1)$$

such that

$$q_{\omega,\phi}(z|x,s,d = 1) = \int q_\phi(z|\tilde{u},s,x,d = 1)q_{\omega}(\tilde{u}|x,s,d = 1)d\tilde{u}$$

$$= \mathbb{E}_{q_{\omega}}(\tilde{u}|x,s,d = 1)\{q_\phi(z|\tilde{u},s,x,d = 1)\}$$

where

$$q_\phi(z|x,\tilde{u},s,d = 1) = \mathcal{N}(z|\mu_\phi(\tilde{u},x,s),\text{diag}(\sigma_\phi(\tilde{u},x,s)))$$

$$q_{\omega,\phi}(\tilde{u}|x,s,d = 1) = \text{Bern}(Y_{GR}(x,s))$$
C.2 Monte-Carlo Estimation of KL Divergence

In the binary case (which we assume), the KL-divergence for the unlabeled scenario in our approach as in Eq. (4) can be computed as:

\[
KL = \int q_\theta(z|x, \tilde{u})q_\theta(\tilde{u}|x)duz - \sum_{z} q_\theta(z|x, \tilde{u})q_\theta(\tilde{u}|x)duz
\]

where we set \( u = 1 | x \) and \( u = 0 | x \) for the two cases, respectively.

Since there is no closed form solution, we approximate the KL-divergence using Monte-Carlo:

\[
q_\theta(\tilde{u} = 1 | x) = \frac{1}{K} \sum_{z=0}^{z=q_\theta(z|x,\tilde{u}=1)} \log \left( \frac{q_\theta(\tilde{u} = 1|x)q_\theta(z|x, \tilde{u} = 1) + q_\theta(\tilde{u} = 0|x)q_\theta(z|x, \tilde{u} = 0)}{p(z)} \right)
\]

In the implementation we evaluate the log probability of the first term in Eq. (17) as:

\[
\log \text{sumexp}(x) = \log \left( \log \left( q_\theta(\tilde{u} = 1|x)q_\theta(z|x, \tilde{u} = 1) \right) \right) + \left( \log (q_\theta(\tilde{u} = 0|x)q_\theta(z|x, \tilde{u} = 0)) \right)
\]

C.3 Optimal Utilities for Synthetic Data

In our Synthetic setting we have \( X\), \( Y \) and \( K \) with observed features \( X = \) (LSAT, GPA) and the true hidden factor \( K \), as in Eq. 9. Then we define the optimal unfair policy (OPT-UNFAIR) to take decision \( d \) according to the posterior distribution \( d \sim p(\hat{Y}|X,S) \) and the optimal fair policy (OPT-FAIR) according to \( d \sim p(\hat{Y}|K) \).

While in our synthetic generation process, we do not have access to the posterior distribution, we can generate i.i.d. samples from it. We approximate the posterior distribution by training logistic regression model on these samples. We approximate the unfair policy (OPT-UNFAIR) by training the logistic model \( \hat{Y} = f(X,S) \) and the optimal fair policy (OPT-FAIR) by training the logistic model \( \hat{Y} = g(K) \).

C.4 Counterfactual Generation on Synthetic Data

For a factual individual (with sensitive attribute \( S = s \)), we compute the counterfactual (had the sensitive attribute been \( S = s' \) with \( s' \neq s \)) following the abduction-action-prediction steps by Pearl in [42]. We compute counterfactuals for the Synthetic dataset with access to the generative process (Eq. 9) with the following steps:

1. We sample \( Y \), correspondingly \( K \) and exogenous noise-factors \( \epsilon \) for each Gaussian distribution. Based on the values of \( Y, K, \epsilon \), we compute the factual LSAT, GPA, and \( \hat{Y} \).

2. We intervene by modifying the value of \( S = s \) to \( S = s' \) (e.g. if for the factual \( S = 1 \), then we set \( S = 0 \)).

3. We use the same values of \( Y, K, \epsilon \) and the modified \( S = s' \) and compute the counterfactual LSAT and GPA.

D EXPERIMENTAL SETUP

This chapter provides a complete description of the experiments presented in section 5. We describe the initial policies (D.1), the training process, hyperparameter selection and coding environment (D.2), metrics (D.3), policy model choices (D.4), and baselines (D.5).

D.1 Initial Policies

The initial policy is used at the beginning of our decision-making phase (Phase II at time \( t = 0 \)) before our policy models are trained.
We consider two different types of initial policies: HARSH, which provides a positive decision only to a small fraction of the data (thus resulting in a small number of labeled data points), and LENI, which provides a larger fraction of the data with a positive decision. See also Figure 2. Table 4 shows the acceptance rates for the different policies. Note that because of existing biases in the data, the initial policies are also biased with respect to the sensitive characteristic.

### D.2 Training and Validation

#### D.2.1 Training Parameters

**Dataset Size.** For our synthetic data, we consider 5000 training samples for Phase 1. In addition, we consider 2500 validation and 5000 test samples. For COMPAS, we split the data into 60–40–40 for training-validation-testing. In the case of CREDIT, we split the data into 70–15–15 for training-validation-testing. Finally, for the MEPS dataset, we consider 75–25–25 for training-validation-testing. For each real-world dataset, we further consider 70% of the training data in an unlabeled fashion for Phase 1 pre-training. At the beginning of the decision-making phase (Phase 2), we perform a warmup with 128 samples which are sampled using some initial policy (HARSH, or LENI). Finally, in Phase 2, we consider we get 64 samples at each time-step.

**Training Epochs.** We train Phase I models for 1200 epochs (for the larger MEPS data, we perform 500 epochs instead) for cross-validation. For final training in Phase I, we train for 2000 epochs (500 epochs for MEPS). Before starting the online decision-learning Phase II, we perform a warmup learning where the data is labeled using some initial policy (explained prior). In the warmup stage, we train each model for 50 steps. Finally, in the online decision phase, we train for one epoch for each time step. We perform each evaluation for 200 online decision time-steps. During the decision stage, we ensure that each model has the same batch size. We determine the batch size by splitting the data such that we have 3 batches per epoch, per time-step during training.

**Other Parameters.** For all training, we consider the Adam optimizer. For each dataset, the real variables are standardized to have zero mean and unit variance. In our semi-labeled training model, for the Monte-Carlo estimate of the KL divergence, we consider 100 samples. We further consider 50 samples for ELBO computation for the unlabeled samples.

All models are deep neural networks with a fully-connected architecture. For all neural network models, we consider the ReLU activation function in the hidden layers. For weight initialization, we consider Xavier uniform weight initialization. For our policy models that use the latent space of the VAE, we use the same neural architecture as the classifier models.

For the basic utility-fairness analyses, we consider a cost of 0.5 for the decisions for all datasets, except for MEPS, where we consider 0.1 for positive decisions. We show a cost-utility-fairness analysis separately.

#### D.2.2 Hyperparameter Selection

For each model in our evaluation section, we perform extensive hyperparameter selection across multiple parameters. We list each hyperparameter, the combinations for each dataset and the model in Table 5. We evaluate the models on the held-out validation data by training each model for 5 seeds. We select the best models based on their performance. For VAE based models we analyze the VAE reconstruction performance and the independence of the latent space with respect to the sensitive feature. For Phase II models, we also test the classification performance. Note that we do not perform separate validation for our distinct policy model trained from the latent space of VAE. For each setup, we simply use the same neural architecture as is selected for the classifier model. For the logistic baseline models (UnfairLog, FairLog) we test for classification performance and fairness. The best parameters for each model are shown in Table 6.

#### D.2.3 Coding Environment

All evaluations are run on a cluster with Intel Xeon E5 family of processors using Linux OS. Evaluation jobs are submitted to the cluster such that each experiment run is executed on one CPU. Deep learning implementations are done on PyTorch v1.7 also utilizing the Ignite v0.3.0 library for training. For FairLog, the demographic parity constrained loss is applied using fastorch v0.1.2. All other machine learning classification models are trained using scikit-learn. Real-world fairness datasets are loaded using the AIF360 library.

### D.3 Additional Metrics

In addition to the metrics defined in Section 2, in our empirical evaluation in 5 we report effective utility and effective demographic parity. Effective utility at time $t$ is the average utility that is accumulated by the decision-maker until time $t$ through the learning process. We define effective utility with respect to ground truth $y$. However, in practical settings we often do not have access to $y$. In this case, we compute effective proxy utility $\hat{U}T$ using proxy label $\hat{y}$.

**Definition D.1 (Effective Utility [31]).** Effective utility: the utility realized during the learning process up to time $t$, i.e.,

$$\text{Effect. UT}(t) = \frac{1}{N \cdot t} \sum_{t' = 1}^{t} \sum_{y_i \in D'} (y_i - \hat{c}),$$

| Policy | LENI | HARSH |
|--------|------|-------|
| Dataset | $p(d=1)$ | $p(d=1|s=1)$ | $p(d=1|s=0)$ | $p(d=1)$ | $p(d=1|s=1)$ | $p(d=1|s=0)$ |
| Synthetic | 0.5468 | 0.7674 | 0.3297 | 0.128 | 0.1581 | 0.0979 |
| COMPAS | 0.1995 | 0.7664 | 0.3274 | 0.0124 | 0.1519 | 0.0705 |
| CREDIT | 0.4857 | 0.5846 | 0.2909 | 0.1776 | 0.2142 | 0.1981 |
| MEPS | 0.4680 | 0.7826 | 0.2724 | 0.1119 | 0.1925 | 0.0617 |
Table 5: Hyperparameters for different types of models that were tuned for the best model selection. VAE models hyperparameters are used for FairAll, FairLab (I-II), FairLabVAE-. All combinations are tested on a separate held-out validation set.

| Model               | Parameter     | Synthetic | COMPAS | CREDIT | MEPS |
|---------------------|---------------|-----------|--------|--------|------|
| VAE-Phase I         | batch-size    | 64, 128, 256 | 64, 128, 256 | 64, 128, 256 | 64, 128, 256 |
|                     | learning-rate | 1e-3, 5e-3, 1e-2 | 1e-3, 5e-3, 1e-2 | 1e-3, 5e-3, 1e-2 | 1e-3, 5e-3, 1e-2 |
|                     | vae-arch      | (32x32), (32x32x32), (32x32), (32x32x32) | (32x32), (32x32x32), (32x32), (32x32x32) | (32x32), (32x32x32), (32x32), (32x32x32) | (32x32), (32x32x32), (32x32), (32x32x32) |
|                     | latent-size   | 2          | 2      | 10, 12, 14, 16 | 20, 22, 25, 27, 30 |
|                     | beta          | 0.7, 0.8, 0.9, 1.0 | 0.7, 0.8, 0.9, 1.0 | 0.7, 0.8, 0.9, 1.0 | 0.7, 0.8, 0.9, 1.0 |
| VAE-Phase II        | learning-rate | 1e-3, 1e-2 | 1e-3, 1e-2 | 1e-3, 1e-2 | 1e-3, 1e-2 |
|                     | clf-arch      | (64x64), (64x64x64), (64x64), (64x64x64) | (64x64), (64x64x64), (64x64), (64x64x64) | (64x64), (64x64x64), (64x64), (64x64x64) | (64x64), (64x64x64), (64x64), (64x64x64) |
|                     | clf-dropout   | 0.0, 0.1, 0.1 | 0.0, 0.1, 0.1 | 0.0, 0.1, 0.1 | 0.0, 0.1, 0.1 |
|                     | alpha         | 1, 5, 10, 15 | 1, 5, 10, 15 | 1, 5, 10, 15 | 1, 5, 15 |
|                     | beta          | 0.7, 0.85, 1.0 | 0.7, 0.85, 1.0 | 0.7, 0.85, 1.0 | 0.7, 0.85, 1.0 |
| UnfairLog           | learning-rate | 1e-3, 1e-2 | 1e-3, 1e-2 | 1e-3, 1e-2 | 1e-3, 1e-2 |
|                     | clf-arch      | (64x64), (64x64x64), (64x64), (64x64x64) | (64x64), (64x64x64), (64x64), (64x64x64) | (64x64), (64x64x64), (64x64), (64x64x64) | (64x64), (64x64x64), (64x64), (64x64x64) |
|                     | clf-dropout   | 0.0, 0.1, 0.1 | 0.0, 0.1, 0.1 | 0.0, 0.1, 0.1 | 0.0, 0.1, 0.1 |
|                     | lambda        | 1–15 | 1–15 | 1–15 | 1–15 |

where \( D_i^t \) is the data in which the policy \( \pi_i \) took positive decisions \( d_i = 1 \), \( N \) is the number of considered examples at each time step \( t \) and \( c \) the problem specific costs of a positive decision.

Effective demographic parity at time \( t \) is the average unfairness accumulated by the decision-maker until time \( t \) while learning the decision policy.

Definition D.2 (Effective Demographic Parity [31]). The demographic parity level realized during the learning process up to time \( t \), i.e.,

\[
\text{Effect. DPU}(t) = \frac{1}{t} \sum_{t' 

where \( D_i^{t'} \) is the set of decisions received by the group of individuals with \( s = -1 \) at time \( t' \) and \( D_i^{t'+1} \) is the set of decisions received by the group of individuals with \( s = +1 \). This is the unfairness accumulated by the decision-maker while learning better policies.

D.4 Policy Model Choices

Section 4 provides a detailed overview of our approach and modeling. We can propose several policy model options in Phase II (i.e., for policy \( \pi_{i+1} \) in Figure 2). The policy model is trained at the end of the time step \( t \) and then applied to the new set of applicants at time \( t + 1 \). We explain the different policy options in the following, including the model \( \pi_{\text{dec}} \) that we chose for all our evaluations in this paper.

D.4.1 Classifier as Policy. One option is to use the classification model \( \tilde{u} \sim q_{\phi}(\tilde{u}|x, s, d = 1) \) to take decision \( d = \tilde{u} \). One benefit is that this model is trained end-to-end while optimizing Eq. 4.1 and hence requires no separate training. At the same time, it is trained only on labeled data (and so trained via an IPS-weighted loss function potentially introducing high variance). We denote this policy by \( \pi_{\text{clf}}^{(x,s)} \). Note that the model is unfair as it directly utilizes the sensitive feature as input (without any fairness constraints).

D.4.2 Decoder as Policy. Another option is to deploy the decoder model of the trained VAE. We denote this model as \( \pi_{\text{dec}}^{(x,s)} \). The decoder has been trained on labeled and unlabeled data. Receiving \( \tilde{u} \sim p_{\theta}(\tilde{u}|z, s, d) \) and taking decision \( d = \tilde{u} \) requires no separate training of a policy. Note that the model is unfair as it directly utilizes the sensitive feature as input (without any fairness constraints).

D.4.3 Policy Using Latent Z. The third option is to take decisions based on the latent variable \( Z \) of our conditional VAE model. Note that following Section 3, we assume \( Z \) to contain the information of \( Y \) and at the same time be independent of sensitive information \( S \) up to an approximation error. Therefore, we assume taking decisions (only) based on \( Z \) to be (approximately) DP and CF fair. See proofs in Appendix A. We train a separate policy model to take decisions only based on \( Z \), i.e., \( \pi_{\text{Z}}^Z : Z \rightarrow \tilde{U} \). There are different options for training this model, i.e., either on labeled data only or on all data.

- Use only labeled data: We train the policy model only on labeled data and correct for selective labeling with IPS. We term this policy \( \pi_{\text{Z}}^\text{label} \).
Table 6: Best selected hyperparameters for all the models, for each dataset. Note that for our methods, the policy model using latent $Z$ utilizes the same hyperparameters as the classifier model.

| Model           | Parameter | Dataset     | Synthetic | COMPAS | CREDIT | MEPS  |
|-----------------|-----------|-------------|-----------|--------|--------|-------|
|                 |           |             |           |        |        |       |
| VAE (Phase I)   | batch-size| 64          | 256       | 128    | 256    |
|                 | learning-rate| 5e-3       | 5e-3      | 1e-3   | 1e-3   |
|                 | vae-arch   | 64x64       | 32x32     | 64x64  | 64x64  |
|                 | latent-size| 2           | 3         | 12     | 20     |
|                 | beta       | 0.8         | 0.8       | 0.8    | 0.7    |
| FairAll (I+II)  | learning-rate| 1e-2       | 1e-3      | 1e-2   | 1e-3   |
|                 | vae-arch   | 64x64       | 32x32     | 64x64  | 64x64  |
|                 | clf-arch   | 32x32x32    | 32x32x32  | 32x32x32 | 100x100 |
|                 | clf-dropout| 0.0         | 0.1       | 0.1    | 0.1    |
|                 | alpha      | 5           | 1         | 5      | 1      |
|                 | beta       | 0.7         | 0.7       | 0.85   | 0.7    |
| FairAll (II)    | learning-rate| 1e-2       | 1e-2      | 1e-2   | 1e-2   |
|                 | vae-arch   | 64x64       | 64x64     | 64x64  | 64x64  |
|                 | clf-arch   | 64x64       | 64x64x64  | 64x64  | 100x100 |
|                 | clf-dropout| 0.1         | 0.0       | 0.1    | 0.0    |
|                 | latent-size| 2           | 2         | 12     | 25     |
|                 | alpha      | 5           | 10        | 1      | 1      |
|                 | beta       | 0.85        | 1.0       | 0.7    | 0.7    |
| UnfairLog       | learning-rate| 1e-2       | 1e-2      | 1e-3   | 5e-3   |
|                 | vae-arch   | 64x64x64    | 32x32x32  | 64x64  | 64x64  |
|                 | clf-arch   | 64x64       | 32x32x32  | 32x32x32 | 64x64   |
|                 | clf-dropout| 0.1         | 0.0       | 0.0    | 0.0    |
| FairLog         | learning-rate| 1e-2       | 1e-2      | 1e-2   | 1e-2   |
|                 | vae-arch   | 64x64       | 64x64     | 64x64  | 64x64  |
|                 | clf-arch   | 64x64x64    | 32x32x32  | 32x32x32 | 64x64   |
|                 | clf-dropout| 0.1         | 0.0       | 0.0    | 0.0    |
| FairLab (I+II)  | learning-rate| 1e-3       | 1e-3      | 1e-3   | 1e-3   |
|                 | vae-arch   | 64x64       | 32x32     | 64x64  | 64x64  |
|                 | latent-size| 2           | 3         | 12     | 20     |
|                 | alpha      | 1.0         | 1.0       | 1.0    | 1.0    |
|                 | beta       | 0.7         | 0.85      | 0.7    | 0.7    |

- **Use the classifier**: We use the classifier model to label all data (both labeled and unlabeled data). Training thus requires no IPS correction. We term this policy $\pi_{\text{clf}}$.
- **Use the decoder**: We use the decoder model to label all data (both labeled and unlabeled data). Training thus requires no IPS correction. We term this policy $\pi_{\text{dec}}$. We use this policy in our evaluations in the main paper, whenever referring to FairAll.

**Practical Considerations.** Figure 7 shows a comparison of different policy models.

For any policy model $\pi^{Z}$, we train a deep fully-connected neural network with the same architecture that we select for the classifier model (from Table 6). Note that selecting hyperparameters for these policy models separately could be further explored in future work.

D.5 Baselines

In our evaluations in Section 5 we compare our method to the baseline method FairLog, and the reference model UnfairLog. Both are state-of-the-art methods as shown in [31]. In addition, in order to show the importance of unlabeled data in the decision-making process, we modify our model to provide two comparative benchmark models, FairLab (I+II) and FairAll (II).

D.5.1 UnfairLog Policy Model. We train a deep neural model $f_{v}$ based on the logistic model from [31]. We optimize the following cost-sensitive cross-entropy loss Eq. (5), which we also refer to as $L^{\text{UnfairLog}}$. We train on labeled data only and correct for the selective labels bias with IPS.

D.5.2 FairLog Policy Model. Following [31], we train a fair version of UnfairLog by adding a fairness constraint with a Lagrangian hyperparameter. We call this model FairLog. This model is trained on labeled data only and uses IPS correction. We train by minimizing the following loss:

$$L^{\text{FairLog}}(v, x, s, u) = L^{\text{UnfairLog}}(v, x, s, u) + \lambda \cdot DPU(v, x, s) \quad (19)$$

where $L^{\text{UnfairLog}}$ refers to Eq. 5, and DPU to Eq. 2.2. The Lagrangian multiplier $\lambda \geq 0$ is tuned for a utility-fairness trade-off. Note, setting $\lambda = 0$ would reduce FairLog to the UnfairLog model.
Figure 7: Different policies in our method across different datasets. Harsh initial policy. Note in our evaluations we utilize $\hat{\pi}_{\text{dec}}$.  

D.5.3 Fair (I-II) Model. We design a comparative benchmark model to understand the importance of unlabeled data in Phase II. In this, we assume our VAE model would use only labeled data in Phase II. We can consider this model to be a natural extension of the FairLog model using VAEs (and Phase I pre-training). This reduces loss in Eq. 4.1 to the labeled ELBO, but with IPS correction:

$$L^{\text{FairLog (I-II)}}(\theta, \phi; x, s, u) = -\frac{1}{\pi(d = 1|x, s)} \left[ \mathbb{E}_{q_{\phi}(Z|x, u, d = 1)} \left\{ \log p_{\theta}(x|s) + \log p_{\theta}(u|z, s, d = 1) \right\} - KL(q_{\phi}(Z|x, u, s, d = 1)||p(Z)) \right]$$

Note that this model does use unlabeled data in the pre-training phase (Phase I), but it does not use unlabeled data in Phase II. It does so by training a VAE model (using Eq. 4.3) and then applying transfer learning. In terms of policy learning, as this model only utilizes labeled data in Phase II, we use $\pi_{\text{label}}^\pi$.

D.5.4 FairAll (II) Model. We design another benchmark model to understand the role of pre-training with unlabeled data in Phase I. We modify our model such that there is no pre-training in Phase I. That is, we do not train a VAE model in Phase I, instead we directly start with Phase II. As such, this model optimizes the Phase II loss in Eq. 4.1. Note that we do not optimize any VAE model in Phase I.

E ADDITIONAL RESULTS

In this section, we show additional results demonstrating that our FairAll method is able to learn more stable and fairer decision policies than comparable approaches. In the main paper, we reported results for the initial policy HARSH and plotted different utility and fairness measures for the real-world datasets COMPAS. In this section, we report results with a more lenient initial policy LENI for all datasets. In addition, we show plots for the real-world datasets CREDIT and MEPS for both initial policies, HARSH and LENI.

E.1 Additional Synthetic Data Analysis

We extend the results in Section 5.1. Figure 8 depicts results for the Synthetic dataset with LENI as initial policy $\pi_0$. Compared to the baseline FairLog, we achieve similar utility convergence, while being significantly fairer regarding DP and CF. Further, the perceived trade-off between utility and fairness continues to be explained away if we were to measure to the unobserved ground truth $y$ instead of the proxy $\hat{y}$.

E.2 Real-World: Effective Decision Learning

Following Section 5.3, we measure the effectiveness of any decision learning process. We compare the methods based on effective (accumulated) utility and DP unfairness (Def. D.1 and D.2) after 200 time steps. Table 7 summarizes the results. We see that across all datasets, our method FairAll is able to provide significantly higher accumulated utility as well as lower unfairness. Especially when comparing to the baseline and reference models UnfairLog and FairLog respectively, we see that our model performs significantly better, outperforming even the unfair UnfairLog model in terms of utility. We also see the clear benefit of using unlabeled data in both Phase I and Phase II. FairAll consistently manages to accumulate higher utility and lower unfairness compared to the benchmarks FairLab (I-II) and FairAll (II).
Table 7: Effective (accumulated) utility and demographic parity unfairness measured during the policy learning process for different real-world datasets after \( t = 200 \) time steps. We consider LENI initial policy here. We assume the cost of a positive decision of 0.5 for all datasets (in MEPS we assume 0.1). We report mean values over the same 10 independent seeds, with the numbers in the brackets representing the deviation. All reported values are scaled up by 100 for improved readability. Our method outperforms the baseline methods in most cases, achieving high utility while being less unfair.

| Model          | COMPAS Effect. UT | COMPAS Effect. DPU | CREDIT Effect. UT | CREDIT Effect. DPU | MEPS Effect. UT | MEPS Effect. DPU |
|----------------|-------------------|--------------------|-------------------|--------------------|-----------------|-----------------|
| FairAll        | 6.4 (0.8)         | 10.5 (0.6)         | 20.7 (0.5)        | 8.7 (1.8)          | 8.1 (0.3)       | 9.9 (1.3)       |
| FairAll (II)   | 5.1 (0.6)         | 10.6 (0.7)         | 19.8 (1.0)        | 10.8 (1.9)         | 7.7 (0.2)       | 9.4 (1.8)       |
| FairLab (I+II) | 3.5 (0.5)         | 10.9 (0.8)         | 16.9 (0.9)        | 10.4 (2.3)         | 5.9 (0.6)       | 10.3 (0.7)      |
| FairLog        | 3.5 (0.5)         | 10.9 (1.0)         | 19.5 (1.1)        | 9.8 (1.9)          | 6.9 (0.4)       | 11.2 (1.0)      |
| UnfairLog      | 4.7 (0.6)         | 15.1 (1.2)         | 21.2 (0.5)        | 11.5 (2.0)         | 7.7 (0.3)       | 19.8 (2.4)      |

E.3 Real-World: Deployment on Test Data

Following Section 5.4, we further show extensive evaluations for the deployment of the different methods as decision-making systems. We see from the adjacent figures below that our method FairAll manages to converge to a significantly better utility level while being temporally more stable. We see similar behavior with respect to DP unfairness. Our method achieves the lower values of unfairness, which, again, is more stable temporally. We see that when considering deployment, FairAll clearly outperforms the fair baseline and also the unfair reference model. We also see the benefits of unlabeled data – FairLab (I+II) and FairAll (II) both fail to provide high, stable measures of utility. These benchmark models also converge to worse levels of unfairness. Further, Table 8 shows how our method provides the best trade-off between temporal variance TV and convergence level \( \mu \) across utility and unfairness.

E.4 Cost Analysis

Finally, we perform a cost analysis for our decision-making algorithm. The cost of making positive decisions in any decision-making scenario depends on the context. It can change from one context to another, and with it, the expected utility and unfairness of any
Figure 11: Different measures for CREDIT data. Initial policy HARSH.

Figure 12: Different measures for CREDIT data. Initial policy LENI.

Figure 13: Different measures for MEPS health data. Initial policy HARSH.

Figure 14: Different measures for MEPS health data. Initial policy LENI.
Table 8: Temporal variance of utility and demographic parity unfairness, TV$_{UT}$ and TV$_{DPU}$, respectively, and the corresponding temporal mean $\mu_{UT}$ and $\mu_{DPU}$, respectively, for time interval $t = [125, 200]$. Results are shown for three real-world datasets. The assumed initial policy is LENI. We report the mean over 10 runs with the standard deviation in brackets. For TV, lower values are better, for $\mu$ higher (lower) is better for $UT$ ($DPU$). Note all reported values are multiplied by 100 for improved readability.

| Model       | COMPAS   | CREDIT   | MEPS     |
|-------------|----------|----------|----------|
|             | TV$_{DPU}$ ($\uparrow$) | $\mu_{DPU}$ ($\downarrow$) | TV$_{UT}$ ($\uparrow$) | $\mu_{UT}$ ($\uparrow$) | TV$_{DPU}$ ($\downarrow$) | $\mu_{DPU}$ ($\downarrow$) | TV$_{UT}$ ($\downarrow$) | $\mu_{UT}$ ($\uparrow$) |
| FairAll     | 1.0 (0.7) | 5.8 (3.1) | 0.3 (0.2) | 8.8 (0.8) | 2.7 (1.6) | 4.9 (3.7) | 0.9 (0.4) | 19.3 (1.2) | 2.4 (2.7) | 4.4 (3.4) | 0.2 (0.1) | 7.9 (0.4) |
| FairAll (II)| 2.9 (1.6) | 7.6 (4.2) | 0.7 (0.6) | 7.0 (0.8) | 2.8 (2.2) | 5.3 (4.0) | 0.8 (0.5) | 19.1 (1.5) | 3.4 (3.2) | 6.2 (5.1) | 0.3 (0.2) | 7.4 (0.4) |
| FairLab (I-II) | 0.5 (0.4) | 4.9 (4.1) | 0.2 (0.1) | 4.9 (1.1) | 2.1 (1.8) | 9.5 (6.8) | 0.5 (0.4) | 14.6 (1.3) | 0.7 (0.4) | 4.8 (1.7) | 0.2 (0.2) | 6.2 (0.6) |
| FairLog     | 1.6 (1.2) | 4.2 (3.8) | 0.5 (0.3) | 4.5 (0.9) | 4.0 (2.5) | 7.9 (5.3) | 1.2 (0.7) | 17.3 (1.7) | 2.2 (1.9) | 5.6 (2.9) | 0.7 (0.5) | 6.4 (0.8) |

learned decision-making algorithm. For example, in a healthcare setting, we might have to consider lower costs for positive decisions. This would ensure we do not reject anyone who needs access to critical healthcare and the necessary facilities. Likewise, in loan settings, we might have to operate with higher costs. It might happen that a decision-maker in the loan scenario needs to account for significantly higher costs whenever they provide a loan. In this section, we show how varying the cost in a decision-making setting could affect the accumulated effective utility and unfairness, if we were to apply our decision-making algorithm in several real-world settings.

Figure 15 shows the effect of cost on the acquired utility and fairness. We see how at lower costs, we can acquire very high profits by accepting most people. As cost increases, naturally, the acquired utility reduces as we start rejecting more and more people. Correspondingly, the DP unfairness shows a more U-shaped relationship to cost. At a very low (high) cost, unfairness is almost zero, as we start almost always accepting (rejecting) every individual. At intermediate values of cost, we make more balanced accept/reject decisions, and thus see slightly higher unfairness. Nonetheless, we believe performing such an analysis might give a better picture of a decision-making context, and help guide deployment and tuning the cost in related settings.

FURTHER DISCUSSION OF ASSUMPTIONS

F.1 Assumptions of the Generative Process

We live in a world where the target of interest $Y$ is not always independent of a social construct $S$. We give two prominent examples where this may not be the case. For example, we can assume that the distribution of willingness to pay back a loan is independent of gender. However, the ability to pay back – which determines the utility of a bank – may depend on gender (e.g., due to the gender pay gap). In other cases, e.g., in the medical domain, discrimination by sex may be justified. On a population level, biological sex and social gender are in general not independent, such that a decision that is dependent on sex may not be independent of gender.
Figure 15: Analysis of effective utility and effective demographic parity (DP) unfairness (at time $t = 200$) on different real-world datasets for different values of the cost of the decision. We assumed initial policy $\pi_0 = \text{HARSH}$.