Full-heavy tetraquark states and their evidences in the LHCb di-\(J/\psi\) spectrum

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In the framework of a nonrelativistic potential quark model (NRPQM) for heavy quark system, we investigate the mass spectrum of the \(P\)-wave tetraquark states of \(cc\bar{c}\bar{c}\) and \(bb\bar{b}\bar{b}\). The Hamiltonian contains a linear confinement potential and parameterized one-gluon-exchange potential which includes a Coulomb type potential and spin-dependent potentials. The full-heavy tetraquark system is solved by a harmonic oscillator expansion method. With the same parameters fixed by the charmonium and bottomonium spectra, we obtained the full spectra for the \(S\) and \(P\)-wave heavy tetraquark states. We find that the narrow structure around 6.9 GeV recently observed at LHCb in the di-\(J/\psi\) invariant mass spectrum can be naturally explained by the \(P\)-wave \(cc\bar{c}\bar{c}\) states. Meanwhile, the observed broad structure around 6.2 \(\sim\) 6.8 GeV can be consistently explained by the \(S\)-wave states around 6.5 GeV predicted in our previous work. Some contributions from those suppressed low-lying \(P\)-wave states around 6.7 GeV are also possible. Other decay channels are implied in such a scenario and they can be investigated by future experimental analysis. Considering the large discovery potential at LHCb, we give our predictions of the \(P\)-wave \(bb\bar{b}\bar{b}\) states which can be searched for in the future.

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Introduction—Searching for genuine exotic hadrons beyond the conventional quark model has been one of the most important initiatives since the establishment of the nonrelativistic constituent quark model in 1964 [1, 2]. Benefited from great progresses in experiment, many candidates of exotic hadrons have been found since the discovery of \(X(3872)\) by Belle in 2003 [3]. Recent reviews of the status of experimental and theoretical studies can be found in Refs. [4–10]. While many observed candidates have been found located in the vicinity of \(S\)-wave open thresholds, no signals for overall-color-singlet multiquark states have been indisputably established due to difficulties of distinguishing them from hadronic molecules [10]. Recently, the tetraquarks of all-heavy systems, such as \(cc\bar{c}\bar{c}\) and \(bb\bar{b}\bar{b}\), have received considerable attention. Since the light quark degrees of freedom cannot be exchanged between two heavy mesons at leading order, the color interactions between the heavy quarks (antiquarks) should be dominant at short distance and they may favor to form genuine color-singlet tetraquark configurations rather than loosely bound hadronic molecules. Furthermore, such exotic states may have masses and decay modes significantly different from other conventional states, thus, can be established in experiment.

Early theoretical studies of the full-heavy tetraquark states can be found in the literature [11–16]. A revival of this topic driven by the experimental progresses can be found by the intensive publications recently [17–37]. Physicists are very concerned with the stability of the tetraquark \(cc\bar{c}\bar{c}\) (\(T_{cc\bar{c}\bar{c}}\)) and \(bb\bar{b}\bar{b}\) (\(T_{bb\bar{b}\bar{b}}\)) states. If the \(T_{cc\bar{c}\bar{c}}\) or \(T_{bb\bar{b}\bar{b}}\) states have relatively smaller masses below the thresholds of heavy charmonium or bottomonium pairs [18–25], they may become “stable” because no direct decays into heavy quarkonium pairs through quark rearrangements would be allowed. However, some studies showed that stable bound tetraquark states made of \(cc\bar{c}\bar{c}\) or \(bb\bar{b}\bar{b}\) may not exist [11, 15, 27–35] because the predicted masses are large enough for them to decay into heavy quarkonium pairs. Due to these very controversial issues, experimental evidence for such exotic objects would be crucial for our understanding of the underlying dynamics.

Very recently, the LHCb Collaboration reported their preliminary results on the observations of \(T_{cc\bar{c}\bar{c}}\) states [38]. Using the the full Run1 and Run2 LHCb data of 9 fb\(^{-1}\) the di-\(J/\psi\) invariant mass spectrum is studied at \(p_T > 5.2\) GeV/c. A broad structure above threshold ranging from 6.2 to 6.8 GeV and a narrower structure at about 6.9 GeV are observed with more than 5 \(\sigma\) of significance level. There are also some vague structures around 7.2 GeV to be confirmed. While these clear structures may be evidences for genuine tetraquark \(cc\bar{c}\bar{c}\) states, they can also set up experimental constraints on theoretical models of which the successful interpretations and most importantly the early predictions should bring a lot of insights to the underlying dynamics.

In this work, we will show that the mass spectrum of the LHCb observations can fit in the pattern obtained in a nonrelativistic potential quark model (NRPQM) which is developed recently for the heavy tetraquark system [33]. The NRPQM is based on the Hamiltonian proposed by the Cornell model [39], which contains a linear confinement and a one-gluon-exchange (OGE) potential for heavy quark-quark and quark-antiquark interactions. The Cornell model has made great successes in the description of the charmonium and bottomonium spectra with high precision, and been broadly applied to multiquark systems in the literature. In Ref. [33] we adopted the Cornell model for the study of all-heavy tetraquark system with a Gaussian expansion method. The
masses of the $S$-wave all-heavy tetraquark states were predicted there and we found that the $S$-wave $T_{(ccar{c}ar{c})}$ masses should be above the two-charmonium thresholds within a commonly accepted parameter space [33]. This turns out to be consistent with the broad structure around 6.2 to 6.8 GeV. With the same parameters determined in Ref. [33] we will show in this work that the narrow state may arise from the $P$-wave $T_{(ccar{c}ar{c})}$ states.

As follows, we first give a brief introduction to our framework. We then present the full numerical results for the $S$ and $P$-wave $T_{(ccar{c}ar{c})}$ states to compare with the experimental observations. Phenomenological consequence and implications for future experimental studies will be discussed.

Model and method—Apart from the linear confinement, Coulomb type potential, and spin-spin interaction potential for calculating the $S$-wave tetraquark states in the Hamiltonian [33], we include the spin-orbit and tensor potentials here to deal with the first orbital excitation,

\[
V_{ij}^L = -\frac{\alpha_{ij}}{16} \frac{1}{r_{ij}^2} \left( \frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4}{m_im_j} \right) \left( L_{ij} \cdot (S_i + S_j) \right)
\]

\[
-\frac{\alpha_{ij}}{16} \frac{1}{r_{ij}^2} \left( \frac{1}{m_i^2} - \frac{1}{m_j^2} \right) \left( L_{ij} \cdot (S_i - S_j) \right),
\]

where $S_i$ stands for the spin of the $i$-th quark, and $L_{ij}$ stands for the relative orbital angular momentum between the $i$-th and $j$-th quark. If the interaction occurs between two quarks or antiquarks, operator $A_i \cdot A_j$ is defined as $A_i \cdot A_j \equiv \sum_{\lambda=1}^{\infty} \lambda^0 \lambda^0$, while if the interaction occurs between a quark and antiquark, we have $A_i \cdot A_j \equiv \sum_{\lambda=1}^{\infty} -\lambda^0 \lambda^0$, where $\lambda^0$ is the complex conjugate of the Gell-Mann matrix $\lambda^\mu$. The parameters $b_{ij}$ and $\alpha_{ij}$ denote the confinement potential strength and the strong coupling for the OGE potential, respectively. The same model parameters, $m_c = 1.483$ GeV, $\alpha_{cc} = 0.5461$, $\sigma_{cc} = 1.1384$ GeV, and $b_{cc} = 0.1425$ GeV^2, are adopted by fitting the charmonium spectrum as in Refs. [33, 40].

For $T_{(ccar{c}ar{c})}$, there are two kinds of color structures, $(66)_c$ and $(33)_c$. In this work, the relative Jacobi coordinate between these two charm quarks (two antiquark quarks) is defined by $\xi_1 = (r_1 - r_2) / \sqrt{2}$, $\xi_2 = (r_3 - r_4) / \sqrt{2}$, while the relative Jacobi coordinate between cc and c$\bar{c}$ is defined by $\xi_3 = (r_1 + r_2 - (r_3 + r_4)) / 2$. Thus, there are three spatial excitation modes which are denoted as $\xi_1$, $\xi_2$, and $\xi_3$. Their wave functions are defined as $\phi(\xi_i)$ ($i = 1, 2, 3$). According to the requirements of symmetry, there will be 20 different configurations in the $L - S$ coupling scheme, which are listed in Table 1. Apart from the conventional quantum numbers, i.e., $J^{PC} = 0^{-+}, 1^{--}, 2^{++}, 3^{--}$, the $P$-wave can access exotic quantum numbers, i.e., $J^{PC} = 0^{--}, 1^{--}, 2^{--}$.

To solve the Schrödinger equation, we expand the radial part $R_{n\ell}(\xi)$ of spatial wave function $\phi(\xi)$ with a series of harmonic oscillator functions [41]:

\[
R_{n\ell}(\xi) = \sum_{\ell=1}^{n} C_{\ell\ell} \phi_{n\ell}(d_{\ell\ell}, \xi),
\]

where

\[
\phi_{n\ell}(d_{\ell\ell}, \xi) = \left( \frac{1}{d_{\ell\ell}} \right)^{\frac{1}{2}} \left[ \frac{2^{\ell+\nu-\nu}((2\nu+2\ell+1)!)}{\nu!(\nu+\ell)!(\nu+\ell+1)!} \right]^{\frac{1}{2}} \left( \frac{d_{\ell\ell}}{\xi} \right)^{\nu} \times e^{-\frac{d_{\ell\ell}}{\xi}} F\left( -n_{\ell}, l_{\ell} + \frac{3}{2}, \left( \frac{d_{\ell\ell}}{\xi} \right)^2 \right),
\]

(4)

with

\[
d_{\ell\ell} = d_{l} a_{l}^{-1}, (l = 1, \ldots, n),
\]

where $n$ is the number of harmonic oscillator quantum numbers, and $a$ is the ratio coefficient. There are three parameters [$d_l$, $d_n$, $n$] to be determined through the variation method. It is found that with the parameter set [0.068 fm, 2.711 fm, 15] for the $cc\bar{c}\bar{c}$ system, we can obtain stable solutions.

Results and discussion—We focus on the $P$-wave $T_{(cc\bar{c}\bar{c})}$ in this part to understand the di-$J/\Psi$ spectrum observed at LHCB [38]. In Table 1 the eigenvectors for different color configurations of the $P$-wave $T_{(cc\bar{c}\bar{c})}$ states are listed in the last column. The eigenvalues for each configuration and mixing terms between different configurations are listed in the 3rd column. Then, the eigenvalues for the physical states can be extracted by diagonalizing the mass matrix of the last column, and the masses are listed in the 4th column.

Given the total spin of the $cc\bar{c}\bar{c}$ system, $S = 0$, 1, 2, and the orbital angular momentum $L = 1$, the system can couple to states with total angular momentum $J = 0$, 1, 2, 3. For $J = 3$ it is required that all the quark (antiquark) spins are in parallel with the orbital angular momentum, and the orbital excitation is between the $cc$ and $c\bar{c}$ system. The spatial wavefunction is antisymmetric, while the spin wavefunction is symmetric. As a result, the flavor wavefunction must be antisymmetric in order to respect the Fermi statistics with the color wavefunction considered. This suggests that the $J = 3$ state can only have $C = -1$ and is the only pure state of $5P_3 - (33)_c(\xi_1)$. In contrast, for states with $J = 0, 1, 2$ the total spin wavefunction can have all possible configurations, i.e. symmetric, antisymmetric, or mixing of both. It thus allows access to both $C = \pm 1$ states for those states, i.e., $J^{PC} = 0^{+-}, 1^{+-}, 2^{+-}$, where both conventional and exotic quantum numbers are present.

For the $J = 3$ state our calculation shows that it has the largest $L - S$ coupling and significantly large spin-spin coupling. These contributions have raised its mass to be above the averaged values of other multiplets in the first orbital excitation. Apart from the $3^{--}$ state it shows that all the other states have the excitation modes of both $\xi_1$ and $\xi_2$. For each quantum number these two excitation modes have an equal strength because of the flavor symmetry.

To proceed, let us first look into the other $P$-wave states with conventional quantum numbers, i.e., $J^{PC} = 0^{-+}, 1^{--}, 2^{++}$.
0\textsuperscript{+}, 1\textsuperscript{−}, 2\textsuperscript{−}. Three energy levels are expected for the quantum number 0\textsuperscript{+}. In Table I it shows that the highest and lowest mass states are dominated by the (ξ_1, ξ_2) mode. It is interesting to note that the mass of the highest one is about 6.89 GeV which is located at the same mass position as the narrow peak observed by LHCb. By assigning the narrow peak to be the highest 0\textsuperscript{+} state, we find that the mixing pattern turns out to be consistent with the experimental observations:

- By assigning the \( T_{(ccar{c}ar{c})0^{-}→(6891)} \) as the narrow peak, it means that this state will couple to the final-state di-\( J/\psi \) mostly via the couplings of \( 3P_{0^{-}→(6691),ξ_1,ξ_2} \) and \( 3P_{0^{-}→(33),(ξ_1,ξ_2)} \) → \( J/\psi J/\psi \). Notice that these two configurations are in phase in the mixing eigenvector. It implies that the coupling of the lowest-mass state \( T_{(ccar{c}ar{c})0^{-}→(6681)} \) to di-\( J/\psi \) will be suppressed by the out-of-phase mixing between configurations \( 3P_{0^{-}→(6691),ξ_1,ξ_2} \) and \( 3P_{0^{-}→(33),(ξ_1,ξ_2)} \). In this scenario the \( T_{(ccar{c}ar{c})0^{-}→(6891)} \) state can also decay into \( η_{c}\chi_c \) and \( J/\psi h_c \) via an \( S \)-wave transition. However, notice that the \( S \)-wave channels have higher thresholds. The limited phase space can still keep the total width relatively narrow.

- The middle state \( T_{(ccar{c}ar{c})0^{-}→(6749)} \) is dominated by configuration \( 3P_{0^{-}→(33),(ξ_1)} \) which is a \( ξ_1 \) excitation mode. Namely, the orbital angular momentum is between the \( cc \) diquark and \( c\bar{c} \) anti-diquark. Decay of this mode into two charmonia requires the interchange of a \( c \) and \( \bar{c} \) in the spatial wavefunctions which will be suppressed by the wavefunction convolution between the initial and final state hadrons.

- The 2\textsuperscript{+} states follow a similar mixing pattern as that of 0\textsuperscript{+}. It may also contribute to the narrow peak around 6.9 GeV. For this reason as for the 0\textsuperscript{+} states, contributions from the lower mass states to the \( J/\psi J/\psi \) channel will be suppressed.

- The 1\textsuperscript{−} states are forbidden to decay into the di-\( J/\psi \) channel due to the \( C \)-parity conservation. As shown in Table I there are five energy levels predicted with the configuration mixings. These states can be searched in their \( S \)-wave coupling channels such as \( J/\psi \chi_c \) and \( η_c h_c \), or \( P \)-wave coupling channels such as \( J/\psi \eta_\eta \).

The \( P \)-wave \( T_{(ccar{c}ar{c})} \) can have access to exotic quantum numbers such as \( J^{PC}=0^{-},1^{-},2^{-} \) as shown in Table I. Considering the \( C \)-parity conservation, only the 1\textsuperscript{−} states can couple to the di-\( J/\psi \) channel. The following points can be learned:

- The energy levels cover a range from 6.67 to 6.91 GeV. The highest states are both dominated by the \( (ξ_1, ξ_2) \) mode which is similar to the 0\textsuperscript{+} and 2\textsuperscript{−} cases except that the mixing pattern is different. Here, the \( 3P_{1^{-}→(6691),ξ_1,ξ_2} \) and \( 3P_{1^{-}→(33),(ξ_1,ξ_2)} \) configurations are in phase for the \( T_{(ccar{c}ar{c})1^{-}→(6676)} \) but out of phase for \( T_{(ccar{c}ar{c})1^{-}→(6908)} \). Thus, it is possible to assign the narrow structure to \( T_{(ccar{c}ar{c})1^{-}→(6908)} \) which can strongly couple to the di-\( J/\psi \) channel with an opposite sign between these two configurations. Meanwhile, one can understand that the lowest state \( T_{(ccar{c}ar{c})1^{-}→(6676)} \) will be suppressed in its decays into di-\( J/\psi \).

- The middle state \( T_{(ccar{c}ar{c})1^{-}→(6769)} \), which is dominated by \( 3P_{1^{-}→(33),(ξ_1,ξ_2)} \), will also be suppressed due to the quark-anti-quark interchange effects in the wavefunction convolution under the \( ξ_1 \) mode.

- Apart from the di-\( J/\psi \) channel, the \( T_{(ccar{c}ar{c})1^{-}→(6908)} \) can also be searched in its \( S \)-wave decays into \( J/\psi h_c \) or \( η_c\chi_c \). Notice that the thresholds of the \( S \)-wave decay channels are higher than the \( P \) wave. It is possible to keep the state relatively narrow. Similar argument also applies to the assignment of the narrow structure to \( T_{(ccar{c}ar{c})0^{-}→(6891)} \) as mentioned earlier.

It is interesting to note that the masses of these higher states of 0\textsuperscript{−}, 1\textsuperscript{−}, and 2\textsuperscript{−} are almost degenerate. The mass splittings is within 40 MeV which is much smaller than the observed width in experiment, i.e., 80 ~ 168 MeV. Since there is no interference among these states, it is possible that they all can contribute to the observed peak structure. A partial wave analysis in the future will be able to clarify it.

The broad structure above threshold ranging from 6.2 to 6.8 GeV seems to be complicated. The lineshape does not look like a single Breit-Wigner structure. In Fig. 1 (A) apart from the \( P \)-wave states, we also include the \( S \)-wave states as a comparison. It shows that the 0\textsuperscript{+} and 2\textsuperscript{−} states are both located in a range of 6.45 ~ 6.55 GeV. Due to the \( S \)-wave coupling their contributions to the di-\( J/\psi \) channel may produce broad structures. Also, suppressed contributions from those low-lying \( P \)-wave states are also possible. From the experimental invariant mass spectrum, the broad structure can be interpreted as a summed effects of multi-states.

In Fig. 1 (B) with the well determined parameters from our previous work [33], we also give our predictions of the \( P \)-wave \( T_{(b\bar{b}b\bar{b})} \) states. The calculated \( P \)-wave spectra of \( T_{(b\bar{b}b\bar{b})} \) are listed in comparison with the \( S \)-wave states which were obtained in Ref. [33]. One can see that the spectrum of \( T_{(b\bar{b}b\bar{b})} \) exhibits a similar behavior as that of \( T_{(ccar{c}ar{c})} \). However, the hyperfine splitting effects are largely reduced. Note that the mass splitting between the \( (ξ_1, ξ_2) \) -dominant states are driven by the confinement and Coulomb type potentials. Thus, the mass difference between the highest and lowest states with the same \( J^{PC} \) in \( T_{(b\bar{b}b\bar{b})} \) is comparable with that in \( T_{(ccar{c}ar{c})} \). The middle energy levels are more sensitive to the spin-dependent interactions. Interestingly, in the case of \( T_{(b\bar{b}b\bar{b})} \), all the \( P \)-wave energy levels intend to fall into three bands. This may reduce the number of peaking structures in the invariant mass spectra, but get them enhanced by multi-states. This phenomenon may help experimental search for these exotic signals in the future.

**Summary**—By a detailed study of the full-heavy tetraquark spectra in the NRPQM we show that the recently reported results from LHCb in the di-\( J/\psi \) channel have provided a strong evidence for the \( S \) and \( P \)-wave \( T_{(ccar{c}ar{c})} \) states. The narrow structure around 6.9 GeV may arise from the \( P \)-wave states \( T_{(ccar{c}ar{c})0^{-}→(6891)} \), \( T_{(ccar{c}ar{c})1^{-}→(6908)} \), and/or \( T_{(ccar{c}ar{c})2^{-}→(6928)} \). The broad structure covers the mass region of the \( S \)-wave states.
and may arise from $T_{(cc)\bar{c}\bar{c}} (6455)$, $T_{(cc)\bar{c}\bar{c}} (6550)$, and/or $T_{(cc)\bar{c}\bar{c}} (6550)$. Some suppressed contributions from those low-lying P-wave states with $C = +1$ are also possible.

Base on such a scenario, more signals for the $T_{(bb)\bar{b}\bar{b}}$ states should be observed in other decay channels via either S or P-wave transitions, such as $J/\psi \eta$, $J/\psi h$, and so on. As an analogy, the di-$\bar{t}$ decay channel should have potential for the observations of the $T_{(bb)\bar{b}\bar{b}}$ states. Notice that no signals of the $T_{(bb)\bar{b}\bar{b}}$ states are found based on the present statistics at LHCb [38]. This can be due to the low production rates for such heavy objects.

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| $J^{PC}$ | Configuration | $(H)$ (MeV) | Mass (MeV) | Eigenvector |
|--------|---------------|-------------|------------|-------------|
| 0--    | $^3P_{0}^{-}(66_0_0_{0,1,2})$ | (6751, -132) | (6651) | (-0.7985, -0.6020) |
|        | $^3P_{0}^{-}(33_0_{0,1,2})$ | -132 6827 | (6926) | (-0.6020, 0.7985) |
|        | $^3P_{1}^{-}(66_0_0_{0,1,2})$ | 6746 88 37 | (6681) | (0.82, -0.47, -0.32) |
|        | $^3P_{0}^{-}(33_0_{0,1,2})$ | 88 6825 18 | (6749) | (-0.14, 0.38, 0.91) |
|        | $^3P_{1}^{-}(33_0_{0,1,2})$ | 37 18 6750 | (6891) | (0.55, 0.80, 0.25) |
| 1--    | $^3P_{1}^{-}(66_0_0_{0,1,2})$ | 6733 132 -29 -16 31 | (6636) | (0.82, -0.55, 0.12, 0.06, -0.03) |
|        | $^3P_{1}^{-}(33_0_{0,1,2})$ | 132 6827 -14 -7 26 | (6750) | (0.02, -0.24, -0.96, -0.16, 0.06) |
|        | $^3P_{2}^{-}(33_0_{0,1,2})$ | -29 -14 6754 -3 10 | (6768) | (-0.01, 0.05, -0.17, 0.98, 0.10) |
|        | $^1P_{1}^{-}(33_0_{0,1,2})$ | -16 -7 -3 6770 -19 | (6904) | (-0.48, -0.69, 0.19, 0.02, 0.50) |
|        | $^1P_{1}^{-}(66_0_0_{0,1,2})$ | 31 26 10 -19 6968 | (6993) | (0.31, 0.39, -0.02, -0.11, 0.86) |
| 1--    | $^3P_{1}^{-}(66_0_0_{0,1,2})$ | 6751 -108 9 | (6676) | (0.82, 0.56, -0.05) |
|        | $^3P_{1}^{-}(33_0_{0,1,2})$ | -108 6834 -4 | (6769) | (-0.01, -0.08, -1.00) |
|        | $^3P_{1}^{-}(33_0_{0,1,2})$ | 9 -4 6769 | (6908) | (-0.57, 0.82, -0.06) |
| 2--    | $^3P_{2}^{-}(66_0_0_{0,1,2})$ | 6746 -155 -18 | (6630) | (0.80, 0.59, 0.06) |
|        | $^3P_{2}^{-}(33_0_{0,1,2})$ | -155 6837 9 | (6780) | (-0.01, 0.12, -1.00) |
|        | $^3P_{2}^{-}(33_0_{0,1,2})$ | -18 9 6781 | (6955) | (-0.60, 0.80, 0.10) |
| 2--    | $^3P_{2}^{-}(66_0_0_{0,1,2})$ | 6754 123 12 | (6667) | (0.82, -0.57, -0.06) |
|        | $^3P_{2}^{-}(33_0_{0,1,2})$ | 123 6841 6 | (6783) | (0.00, 0.10, -1.00) |
|        | $^3P_{2}^{-}(33_0_{0,1,2})$ | 12 6 6783 | (6928) | (0.58, 0.81, 0.08) |
| 3--    | $^3P_{3}^{-}(33_0_{0,1,2})$ | (6801) | (6801) | 1 |

TABLE I: Predicted masses for the $P$-wave $ccar{c}$ states. $\xi_1, \xi_2, \xi_3$ are the Jacobi coordinates. $(\bar{\xi}_1, \bar{\xi}_2)$ stands for a configuration containing both $\xi_1$- and $\xi_2$-mode orbital excitations, while $(\bar{\xi}_3)$ stands for a configuration containing $\xi_3$-mode orbital excitation.
FIG. 1: Mass spectra for the $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ systems.