POSSIBLE EFFECTS OF A COSMOLOGICAL CONSTANT ON BLACK HOLE EVOLUTION

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Abstract

We explore possible effects of vacuum energy on the evolution of black holes. If the universe contains a cosmological constant, and if black holes can absorb energy from the vacuum, then black hole evaporation could be greatly suppressed. For the magnitude of the cosmological constant suggested by current observations, black holes larger than \( \sim 4 \times 10^{24} \) g would accrete energy rather than evaporate. In this scenario, all stellar and supermassive black holes would grow with time until they reach a maximum mass scale of \( \sim 6 \times 10^{55} \) g, comparable to the mass contained within the present day cosmological horizon.

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The past several years have presented us with two important and intriguing developments concerning the nature of our universe:

[A] Observations of distant supernovae [1] strongly suggest that the Hubble expansion departs from that expected for a purely matter dominated cosmology. The leading explanation for this departure is a cosmological constant contribution to the energy density of roughly half the critical density, i.e., \( \rho_V \approx \frac{\rho_{\text{cr}}}{2} \approx 10^{-29} \text{ g/cm}^3 \, h^2 \approx (0.003 \text{ eV})^4 \, h^2 \), where \( h \) is the present day Hubble constant in units of \( 100 \text{ km s}^{-1} \text{ Mpc}^{-1} \) so that \( 0.4 < h < 1 \). The corresponding energy scale of the vacuum is thus \( T_{\text{vac}} = 0.003 \text{ eV} \, h^{1/2} \).

[B] The observational evidence for black holes has passed a threshold of firmness so that black holes can now be considered as “discovered”. This observational evidence can be found in three different settings: the three million solar mass black hole in the center of our galaxy [2], supermassive black holes in the centers of external galaxies [3], and stellar mass black holes within our galaxy [4]. Thus far, however, no evidence has been found for smaller black holes [5], which presumably have a primordial origin.

Given the existence of both black holes and a cosmological constant, an interesting physical process can potentially occur: The black holes can accrete energy from the vacuum and grow larger with time [6]. The usual conceptual description of a cosmological constant is that seemingly empty space is not really empty, but rather is continually seething with virtual particles, which must contribute a net positive energy density. Within this picture, the virtual particles can be accreted by black holes. Given the (almost) one-way nature of a black hole’s event horizon, more energy will enter the black hole than will be released and the black hole can gain energy and thereby grow larger. In this letter, using the explicit assumption that this accretion process is viable, we explore the possible effects of a non-vanishing
cosmological constant on the future evolution of black holes.

Building on earlier work [7, 8], Mallett [6] performed the relativistic calculation of an evaporating black hole embedded within a background space-time endowed with a cosmological constant. The original motivation was to determine the effects of the vacuum energy on the evaporation of black holes during the inflationary epoch, but the results apply to the present case as well. For a black hole radiating into a background universe with a vacuum energy contribution, the line element can be written in advanced time coordinates in the form

$$ds^2 = -[1 - \frac{2GM(v)}{r} - \chi^2 r^2]d\tau^2 + 2dvdr + r^2d\Omega^2. \quad (1)$$

Unless explicitly stated otherwise, we work in units with $\hbar = 1 = c$ and hence $G = 1/M_{\text{pl}}^2$. The parameter $\chi$ sets the magnitude of the cosmological constant and is defined by the relation

$$\chi = \left(\frac{2\pi^3}{45}\right)^{1/2} \frac{T_{\text{vac}}^2}{M_{\text{pl}}} \quad (2)$$

and where $T_{\text{vac}}$ is the effective temperature scale of the cosmological vacuum energy ($T_{\text{vac}} \approx 0.003 \text{ eV} \approx 34 \text{ K}$ for the presently suspected cosmological constant).

Following Mallett [6], we can write the effective luminosity of a black hole living within this space-time in the phenomenological form

$$L = 4\pi(r_{\Lambda H}^-)^2\{T_H^4 - T_{\text{vac}}^4\}. \quad (3)$$

where $r_{\Lambda H}^-$ is the inner apparent horizon and $T_H$ is the usual Hawking temperature (this equation represents a conjecture, rather than a rigorous derivation, so we ignore dimensionless constants of order unity). The first term represents the Hawking radiation flux [9] flowing outward from the black
hole, whereas the second term represents an inward accretion of energy from the vacuum [6]. The apparent horizons are determined by the condition
\[ 1 - \frac{2GM(v)}{r} - \chi^2r^2 = 0. \] (4)

For black holes of astrophysical interest, the mass lies in the range \(1M_\odot < M_{bh} < 10^{10}M_\odot\). For these hole masses and the suggested value of \(T_{\text{vac}}\), the inner apparent horizon \(r_{\text{AH}}\) is close to the Schwarzschild radius \(r_S = 2GM\) and the Hawking temperature is close to the usual result \(T_H = 1/8\pi GM\) [9, 10, 11].

The inward accretion flow implied by equation (3) can be motivated by a simple conceptual argument analogous to that often used for the Hawking effect. Consider the volume of space located just outside the black hole horizon. The volume is filled with virtual particles with characteristic energy \(T_{\text{vac}}\) and wavelength \(\lambda = 1/T_{\text{vac}}\). If a given particle has a large uncertainty \(\Delta r\) in its radial position, then it could happen to lie within the black hole horizon and can be accreted directly. So let’s suppose that the particle does not have a large uncertainty in its radial position. In particular, it must be localized so that \(\Delta r < \lambda\). Then the uncertainty principle implies that the radial momentum \(p_r\) obeys the ordering \(p_r \approx \Delta p_r > 1/\Delta r > 1/\lambda = T_{\text{vac}}\). On average, the momenta for half of such particles will be directed radially inward. This inward momentum implies a net accretion flux of \(\sim np_r/2\), where \(n \approx T_{\text{vac}}^3\) is the number density of the particles. With this flux, and with the effective area \(4\pi(r_{\text{AH}})^2\) of the black hole horizon, the net accretion rate becomes
\[ \dot{M} = 4\pi(r_{\text{AH}})^2T_{\text{vac}}^4 \approx 3 \, \text{g yr}^{-1} \, h^2 \left( \frac{M_{bh}}{1M_\odot} \right)^2, \] (5)
in accordance with equation (3).
For comparison, recall that a de Sitter space with no black holes produces a thermal radiation bath with an effective temperature of $T_{\text{dS}} \sim \chi \sim T_{\text{vac}}^2/M_{\text{pl}}$ [10, 12]. This radiation results from a Hawking-like effect in which the cosmological horizon at $r \approx 1/\chi$ emits nearly thermal radiation into the universe. Black holes will also accrete this energy [6], and hence equation (3) should contain an additional term $\propto \chi^4$. This radiation is less energetic than the vacuum energy scale $T_{\text{vac}}$ by nearly 31 orders of magnitude, however, and its contribution to black hole accretion is negligible in this present context.

For completeness, we also note that black holes absorb energy from the cosmic background radiation field. At the present epoch, these microwave background photons have an effective temperature of $T_{\text{cmb}} = 2.74 \text{ K} \approx 0.00024 \text{ eV}$, about ten times smaller than $T_{\text{vac}}$. As a result, this additional contribution to the accretion flux is approximately $10^4$ times smaller than that due to the $T_{\text{vac}}^4$ term. The accretion of energy from the cosmic background will become increasingly less important as the universe expands and the photons redshift. In contrast, the energy density of the vacuum remains constant.

This problem contains an important critical mass scale $M_C$. Sufficiently small black holes will experience Hawking evaporation and are relatively unaffected by the presence of the cosmological constant. For large black holes, however, the Hawking temperature $T_H$ is less than $T_{\text{vac}}$ and such black holes can accrete energy from the vacuum rather than evaporate. The critical mass scale $M_C$, obtained by setting $T_H = T_{\text{vac}}$, has a value of

$$M_C = \frac{1}{8\pi GT_{\text{vac}}} \approx 4 \times 10^{24} \text{ g} \approx 2 \times 10^{-9} M_\odot.$$  

(6)

This mass scale is about the same as that of Titania, the largest moon in
the Uranian system [13]. Thus, all black holes more massive than Titania will accrete energy rather than evaporate. The Schwarzschild radius of such a critical mass black hole would be rather small, only about 6 microns.

A second characteristic mass scale exists. For sufficiently large black hole masses, the cubic equation (4) has no real positive solutions and the apparent horizons disappear. This condition defines a second critical mass $M_*$ given by

$$M_* = \frac{1}{3\sqrt{3\chi G}} \approx 6 \times 10^{55} \text{g} \approx 3 \times 10^{22} M_\odot,$$

where we have used the presumed magnitude of the cosmological constant. This critical mass scale, with the mass equivalent of $3 \times 10^{79}$ protons, is thus roughly comparable to the mass contained within the present-day cosmological horizon.

Given enough time, a black hole can accrete energy from the vacuum until it reaches the second critical mass scale $M_*$. The total accretion time $\tau$ is defined to be the time required for a black hole with initial mass $M_{0\text{bh}}$ (above the minimum mass threshold $M_C$) to accrete enough energy to shed its horizons. This accretion time is given by

$$\tau = \frac{1}{4\pi T_{\text{vac}}} \int_{M_{0\text{bh}}}^{M_*} \frac{dM}{r^2(M)} = \left(\frac{2\pi}{15}\right)^{1/2} \frac{M_{\text{pl}}}{8T_{\text{vac}}^2} \frac{(1 - q)^2}{q},$$

where $M_{0\text{bh}}$ is the starting mass of the black hole, $r(M)$ is the inner apparent horizon as defined by equation (4), and $q$ is the root of the equation $q^3 - 3q + 2M_{0\text{bh}}/M_* = 0$. For all known black holes, $M_{0\text{bh}}/M_* \ll 1$, $q \approx 2M_{0\text{bh}}/3M_*$, and the time scale becomes

$$\tau \approx 0.02(M_{\text{pl}}/T_{\text{vac}})^4 M_{0\text{bh}}^{-1} \approx 5 \times 10^{31} \text{yr} h^{-2} \left(\frac{M_{0\text{bh}}}{1M_\odot}\right)^{-1}.$$
to the maximum mass scale $M_* \sim 10^{56}$ g, whereas smaller black holes (with mass $M_{\text{bh}} < M_C$) shrink to the Planck mass $M_{\text{pl}} \sim 10^{-5}$ g through the Hawking effect. Black holes are thus confined to a mass range that is “only” 61 orders of magnitude in extent.

Even though relatively large black holes (with $M_{\text{bh}} > M_C$) accrete energy rather than evaporate, they continue to emit a flux of “ordinary” radiation through the Hawking process. The curvature of space-time near the event horizon gives rise to a nearly thermal spectrum of photons, neutrinos, and gravitons emerging from the hole [14]. With this luminosity $L_H$, the black hole thus follows its usual evolutionary track in the Hertzsprung-Russell diagram, i.e.,

$$L_H = \frac{\sigma_B}{4\pi} T_H^2.$$  \hspace{1cm} (10)

With no accretion, the Hawking temperature $T_H$ increases with time as the black hole mass decreases; with a net accretion, however, the temperature $T_H$ is a decreasing function of time.

In summary, we have explored the possible consequences of nonzero vacuum energy on black hole evolution. In particular, we have considered a scenario in which the vacuum energy can be accreted in accordance with equation (3). If our universe does indeed contain a substantial fraction of its energy density in the form of a cosmological constant contribution, then the long term fate and evolution of black holes can be greatly altered:

[1] All known black holes will never evaporate through the Hawking effect. Instead they will continue to grow larger by accretion of energy from the vacuum (the cosmological constant energy).

[2] The only black holes that can ever be observed to evaporate in the present day universe must lie within the restricted mass range $4 \times 10^{15}$ g
< M_{bh} < 4 \times 10^{24} \text{ g}. Smaller black holes will have evaporated by the current cosmological epoch, whereas larger black holes will accrete energy rather than evaporate. The larger black holes that accrete energy will continue to emit photons, neutrinos, and gravitons through the Hawking process, but with an ever decreasing temperature.

[3] In the long term, if black holes continue to accrete energy and grow larger, the horizons will vanish as the black hole mass approaches a critical mass scale $M_*$ which is comparable to the present day horizon mass scale (the mass equivalent of about $3 \times 10^{79}$ protons).

This evolutionary scenario for black holes rests on the validity of equation (3), which ultimately depends on the nature of the vacuum energy. A full understanding of this issue thus requires a solution to the cosmological constant problem, which remains an open question [15]. This effect greatly changes the long term evolution of black holes, however, and could have important implications for the long term fate of our universe [16]. This present discussion does not address the back reaction, i.e., the effects of black hole accretion on the background cosmological constant. This issue must be addressed to obtain a full understanding of this effect.

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