In this work, we investigate the impact of recent anomalous $(g - 2)_\mu$ measurement with $\sim 4\sigma$ deviation from the SM, probing its effect on a light GeV scale fermionic dark matter (DM). The $(g - 2)_\mu$ anomaly can be readily explained in the a beyond the SM (BSM) $U(1)_{L_\mu-L_\tau}$ scenario, where only a portion of hitherto allowed parameter space can explain the anomaly. This constraint impacts the enhancement of the neutrino floor, the neutrino background in the DM direct detection experiments. The GeV scale DM is severely constrained by the $(g - 2)_\mu$ result as it restricts the $Z'$ mass in the range of 20 – 200 MeV. That restriction results in absence of s-channel resonant annihilation of the GeV scale DM, therefore resulting in over abundance of the DM relic. Even if t-channel annihilation aided by large couplings can bring the relic density in the observed range, direct detection cross section shoots up. Super-GeV DM gets almost ruled out where as sub-GeV DM gets severely constrained.
I. INTRODUCTION

The SM predicts the muon magnetic moment which helps in its spin-interaction with external magnetic field is measured in the tree level as 2 in the unit of Bohr magnetons. This is called gyromagnetic ratio $g$ which is 2 ($g = 2$) for spin angular momentum. Quantized picture shows these as a measure of interaction $\mu^+\mu^-\gamma$, which can take radiative corrections inside the SM. All higher order computations are done within the SM. Experimental measurement of $(g - 2)$ for leptons can be a window to probe beyond the SM (BSM) physics. Recently $g - 2$ for the muon has been measured very precisely in the Fermilab experiment and the BSM virtual corrections can well contribute to this observable. The preferred observable that we work on is defined as $a_\mu = \frac{g - 2}{2}$. The Fermilab based muon $(g-2)$ measurement experiment has reported an deviation from the SM prediction as $\Delta a_\mu$ as [1, 2]

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (251 \pm 59) \times 10^{-11}$$

, which is equal to 4.2$\sigma$ deviation away from the SM. This is already a confirmation of previous similar observation for $(g - 2)_\mu$ in Brookhaven experimental observation [3].

Another remarkable aspect that motivates us to look for BSM signature is the absence of a dark matter candidate inside the SM. The majority ($\sim 85\%$) of the matter element of the Universe is inert non-luminous matter called dark matter (DM), interacting only through gravitational interaction. Astrophysical observations like galactic rotational curves and gravitational lensing etc are proof of its presence. Various DM candidates were incorporated in different beyond the standard model (BSM) theories like Inert Higgs Doublet, Right handed neutrino and Super-symmetry, to name a few. But till date, no conclusive observational evidence of of such a particle is found either in the LHC, specifically designed to probe the TeV scale physics, or in the DM direct and indirect detection experiments.

Particle candidates for DM are well motivated by the WIMP (weakly interacting massive particle) miracle, where a DM is expected at the TeV scale with interaction strength typical to have correct DM relic density. The search for the DM particles are on through different DM direct detection experiments, albeit with a renewed vigor directed to find DM particles at a lower mass scale. The GeV scale DM is particularly interesting from the prospect of their experimental probe. In this scale DM is too light to be significantly sensitive to the LHC searches, which in general has played a crucial role in constraining the WIMPs. Moreover, this GeV mass DM is heavy enough to affect any cosmological considerations. Therefore, in light DM sector at the GeV scale, constraints were relaxed, compared both to the ultralight DM and the WIMPs. It is imperative to probe the the GeV regime with innovative probes. In this work we look to probe the GeV scale DM with the help of an anomalous result that itself is a motivation to go beyond the SM.

We seek to look into any plausible connection between two completely different approach towards BSM physics. We take $U(1)_{L_\mu-L_\tau}$ model as it can explain $(g - 2)_\mu$ anomaly contribution through its $Z'\mu^+\mu^-\gamma$ vertex. Simultaneously, this model can accommodate different DM candidates. We have taken a Dirac fermion DM candidate ($\chi$), which is explored here in detail in the context of its viability in the lighter side of the spectrum, i.e. in the GeV scale. We focus particularly in the DM mass range of $0.1 - 10$ GeV. One reason behind working with GeV scale DM is that, in the DM detection process, this set up modifies the neutrino floor background significantly in the GeV range DM mass. Even as at the face of it, these two appear as non-correlated aspects of BSM physics, GeV scale Dirac fermion DM phenomenology is intricately connected to the recent anomalous measurement of $(g - 2)_\mu$ in the $U(1)_{L_\mu-L_\tau}$ model. Different dark matter phenomenological implications of the $(g - 2)_\mu$ observation in the $U(1)_{L_\mu-L_\tau}$ model were discussed already in the literature in Refs. [4-12]. Another important DM related phenomenon that we had explored earlier in another work [13], is regarding the enhancement of the neutrino floor, the standard background to the DM signal events in a dark matter direct detection experiment. The neutrino floor is also affected by the $(g - 2)_\mu$ anomaly, as it rules out a part of the parameter space where we can get maximal enhancement of the neutrino floor, therefore suppressing the neutrino floor enhancement. The GeV scale Dirac fermion dark matter phenomenology gets severely constrained from the constraint coming from anomalous $(g - 2)_\mu$ measurement. The $(g - 2)_\mu$ constraint restricts the $Z'$ mass below 200 MeV, while putting an upper bound on the BSM $U(1)$ gauge coupling. These constraints reduces the effectiveness of DM annihilation, putting stringent constraints to avoid over production of the DM relic density. Restriction on $Z'$ mass bars $Z'$ mediated s-channel resonant DM annihilation to the SM particles. Another annihilation channel $\text{DDDM} \rightarrow Z'Z'$ can have significant annihilation to satisfy DM relic density constraints only for large couplings. Large couplings are viable only if relic density constraints are taken into account, where as that lead to large DM-nucleus scattering cross section values. A number of DM direct detection experiments like Xenon1T, CDMS, DarkSide, CRESST etc are looking for possible dark matter signals. Any null result in these puts upper bounds on the DM-nucleus scattering cross section. Here larger direct detection cross section for the GeV scale DM will either rule them out or will make them very sensitive to any future update of the experiments. Otherwise feasible GeV scale Dirac fermion DM case becomes very constrained with the involvement of deviations of $(g - 2)$ of the muons.

In section II the $U(1)_{L_\mu-L_\tau}$ model is discussed introducing a fermionic DM candidate, while already existing constraints are listed. We explain and exhibit the parameter space where the $(g - 2)_\mu$ can be explained in this
scenario. The section III is dedicated to the neutrino floor computation, while pointing out the modification of the neutrino floor while we take the \((g-2)\) constraint is taken into account. In section IV, we discuss relic density constraints on the GeV scale Dirac fermion DM, with and without presence of the constraints coming from \((g-2)\). In the next section V, we impose further constraints coming from the DM direct detection experiments along with a lower bound from the neutrino floor to study the effects of \((g-2)\) constraints on the existing parameter space. In final section VI, we summarize our results and chart possible road ahead.

II. MUON \(g-2\) IN \(U(1)_{L_\mu - L_\tau}\)

We first discuss the \(U(1)_{L_\mu - L_\tau}\) model briefly outlining the constraints on this prior to the muon \((g-2)\) anomalous measurement. Then it is explored how the inclusion of this new constraint can restrict the model parameter space.

\(U(1)_{L_\mu - L_\tau}\) Model: Constraints and DM

In this article we consider the \(U(1)_{L_\mu - L_\tau}\) model to study the feasibility of simultaneously explaining \((g-2)\), dark matter relic density and modification in neutrino floor. The \(U(1)_{L_\mu - L_\tau}\) is one of the three simplest \(U(1)\) extension in SM that does not induce any extra gauge anomalies [14–17]. The other two models being \(U(1)_{L_\mu - L_\tau}\) and \(U(1)_{\mu - L_\tau}\).

In \(U(1)_{L_\mu - L_\tau}\) model, the new \(U(1)\) symmetry can be broken spontaneously by a new complex scalar \(S\). The symmetry breaking causes the \(U(1)\) associated vector field \(Z'\) to acquire mass through its interaction with \(S\). The new Lagrangian terms are given by,

\[
L_{\text{new}} = -\frac{1}{4} Z'^{\mu \nu} Z'^{\mu \nu} + \sum_i \bar{\ell}_i \gamma^\mu \left( -g_{\mu - \tau} Y'_i Z'_{\mu} \right) \ell_i + \left( D_\mu S \right) (D^\mu S) + \mu_S^2 S^\dagger S + \lambda_S \left( S^\dagger S \right)^2 + \lambda_{SH} \left( S^\dagger S \right) H^\dagger H
\]

(1)

here \(\mu_S^2\) and \(\lambda_S\) are bilinear and quartic self interactions for \(S\) respectively. \(S\) couples with the SM Higgs \(H\) via quartic coupling \(\lambda_{SH}\). The \(Z'\) boson interacts with fermions by total coupling \(g_{\mu - \tau} Y'_i = g_{\mu - \tau} \left( L_\mu - L_\tau \right)\).

In this model \(Z'\) does not have tree-level interactions with quarks and electrons. The interactions are mediated by \(Z/\gamma - Z'\) loop given by the diagram in Fig. 1.

![FIG. 1: Lepton loop through which \(Z' - Z/\gamma\) mixing is induced in \(U(1)_{L_\mu - L_\tau}\) model.](image)

\[
\delta_{ij}^{\mu \nu} = \frac{1}{(2\pi^2)} \left[ -l^\mu l^\nu + g^{\mu \nu} l^2 \right] \int_0^1 dx \left( \log \frac{x(x-1)l^2 + m_{l(i)}^2}{x(x-1)l^2 + m_{l(j)}^2} \right) x(1-x)
\]

(2)

where, \(l\) is momentum transfer, \(m_{l(i)}\) is mass of \(i(j)^{th}\) flavour lepton in the loop.

The model can incorporate a vector like fermion (VLF), \(\chi\), without introducing an extra gauge anomaly. The DM Lagrangian can take the form:

\[
L_{DM} = \bar{\chi} (i \gamma^\mu D_\mu - m) \chi.
\]

The \(Z'\) also acquires interaction with the DM candidate \(\chi\) through the covariant derivative \(D_\mu \chi = \left( \partial_\mu - iq_\chi g_{\mu - \tau} \right) \chi\) present in the DM kinetic term. The interactions between \(Z'\) and fermions, dark or otherwise are highlighted by the interaction terms.

\[
L_i - L_j = -g_{\mu - \tau} (g_\chi \bar{\chi} \gamma^\mu \chi + \bar{l}_i \gamma^\mu l_i - \bar{l}_j \gamma^\mu l_j + \bar{\nu}_i \gamma^\mu L \nu_i - \bar{\nu}_j \gamma^\mu L \nu_j) Z'_\mu \ .
\]

Here \(g_\chi \times g_{\mu - \tau}\) is the total gauge charge of VLF under \(U(1)_{L_\mu - L_\tau}\). As \(Z'\) interacts with both DM candidate \(\chi\) and leptons: \(\mu\) and \(\tau\), it effectively acts as a portal between dark matter and SM particles. Therefore DM candidate, \(\chi\)
contributes to dark matter relic density through s-channel annihilation process $\bar{\chi}\chi \to Z' \to \ell\ell (\bar{\nu}_l\nu)$ and t-channel process $\bar{\chi}\chi \to Z'Z'$. For $m_{\chi} \leq m_{Z'}$, s-channel is the dominant contribution to DM relic density whereas for $m_{\chi} \geq m_{Z'}$, t-channel remains the dominant contribution. $\chi$ annihilation to leptons in high gravitational regions in our galaxy. The annihilation can be detected through final state radiation (FSR) in form of photon flux in Fermi-LAT experiment [18]. Also $\mu$ and $\tau$ leptons along with their anti particles in final states can further decay to electron and positron before reaching Earth. Electron and positron excess has been measured at DAMPE [19] and AMS02 [20] experiments respectively. We shall refrain from discussing indirect detection of dark matter in detail. We shall discuss the direct detection of dark matter through a recoil of target nucleus in detail in the following sections.

We refer to Ref. [21] for detailed understanding of $U(1)_{L_\mu-L_\tau}$ model constraints. In the electron beam dump experiments [22–24] and the proton beam dump experiments [25–27], an accelerated beam of electrons and protons respectively falls on a target material. Secluded photons can be produced with electron and proton as final state radiations and through the decay of mesons which are produced when energetic proton hit the target material. As the $Z'$ interactions are loop suppressed in this model, electron and proton beam dump constraints are weaker. TEXONO [28] experiment measures elastic scattering of $\bar{\nu}_e$ produced at Kuo-sheng Nuclear power reactor with CsI(Tl) crystal array target. As the incident neutrino is $\bar{\nu}_e$, TEXONO experimental constraints are diminished for the model. Experiments like CCFR [29], Charm-II [30] provide constraints for the model derived from the neutrino trident process $\nu_\mu Z \to \nu_\mu \mu^+\mu^-$. Borexino [31] measures the scattering of solar neutrinos with a liquid scintillator. Borexino and Charm-II provide us the most stringent constraints in the region of interest, i.e. for $m_{Z'}$ range 1-200 MeV. Currently COHERENT experiment CEρNS measurements are preliminary and constraints remain comparatively weaker. In future, with increased exposure COHERENT experiment will be able to probe currently available parameter space for $(g - 2)_{\mu}$. In addition, constraints derived from astrophysical observations and meson decays for an ultra light $Z'$ ($m_{Z'} \leq 1\, \text{eV}$), have been studied in Ref. [32].

**Muon $g - 2$**

![BSM contribution to the muon (g-2) through a Z' mediated loop diagram.](image)

In $U(1)_{L_\mu-L_\tau}$ model, we get a contribution to muon-$(g - 2)$ at one loop level due to the additional $Z'$ boson, which is given by:

$$
\Delta a_\mu = \frac{g_{\mu-\tau}^2}{8\pi^2} \int_0^1 dx \frac{2m_\mu^2x^2(1-x)}{x^2m_\mu^2 + (1-x)m_{Z'}^2}
$$

(4)

The Brookhaven National laboratory(BNL) E821 experiment had earlier shown a $\Delta a_\mu^\text{BNL} = 279(76) \times 10^{-11}$ [3], thus resulting in a $3.7\sigma$ deviation of from the SM value. Recently Fermilab National Laboratory(FNAL) has confirmed the BNL results by reporting a deviation of $\Delta a_\mu^\text{FNAL} = 230(69) \times 10^{-11}$. The two results has thus put a combined discrepancy of $4.2\sigma$ from the SM with $\Delta a_\mu^\text{FNAL+BNL} = 251(59) \times 10^{-11}$ [1, 2]. This result increases the prospects for a light $Z'$ boson in the masses of around 10-100 MeVs. The left plot of Figure 3 shows the allowed $\Delta a_\mu$ band as a function of the $m_{Z'}$ mass and the coupling $g_{\mu-\tau}$ utilizing this combined BNL-FNAL result. In calculating this $\Delta a_\mu$, the Standard Model value of $g - 2$ is taken from the data driven method utilizing perturbative QCD [33]. This particular result, although opens prospects for new physics, however has to be taken with a grain of salt because of another SM muon g-2 calculation by Budapest-Marseille-Wuppertal (BMW) collaboration [34] utilizing lattice QCD.
results, providing us with the value, $\Delta \mu^{\text{LO-HVP (lattice)}} = 107(69) \times 10^{-11}$. If this result is indeed true, the discrepancy of the the combined BNL-FNAL result with the SM comes within $2\sigma$. This reduces the prospects of looking for new physics models.

For the sake of completeness, we have shown allowed value of parameter space for such a scenario in the right plot of Figure 3. The $(g - 2)_\mu$ constraint restricts the $Z'$ mass to a small range of $20 - 200$ MeV while for higher masses the region explaining the anomaly is already ruled out from other experiments. When we include the lattice QCD computation of SM $(g - 2)_\mu$ values, the deviation from the SM are smaller with larger uncertainty which prefers smaller $g_{\mu - \tau}$ values along with larger $m_{Z'}$, broadening and lowering in coupling the allowed parameter range overall.

III. NEUTRINO FLOOR MODIFICATION: $(g - 2)_\mu$ EFFECTS

As we discussed previously, in $U(1)_{\mu - \tau}$ model, the additional $Z'$ boson interacts with $\nu_\mu$ and $\nu_\tau$. Therefore non standard coherent elastic neutrino nucleus scattering(CE$\nu$NS) can be induced in the model through $Z' - \gamma$ mixing.

Differential scattering cross-section

Total neutrino-nucleus differential scattering cross-section in $U(1)_{\mu - \tau}$ is given by,

$$\frac{d\sigma_{\mu - \tau}}{dE_r} = \frac{d\sigma_{\text{SM}}}{dE_r} - \frac{m_N G_f Q_{\nu N\mu - \tau} Q_{\nu N} \left(1 - \frac{E_r m_N}{2E_r^2}\right) F^2(E_r)}{\sqrt{2} \pi (2E_r m_N + m_{Z'}^2)} + \frac{m_N Q_{\nu N\mu - \tau}^2 \left(1 - \frac{E_r m_N}{2E_r^2}\right) F^2(E_r)}{2\pi (2E_r m_N + m_{Z'}^2)^2},$$

where $\frac{d\sigma_{\text{SM}}}{dE_r}$ is the contribution from SM, given as,

$$\frac{d\sigma_{\text{SM}}}{dE_r} = G_f^2 m_N^2 Q_{\nu N}^2 \left(1 - \frac{E_r m_N}{2E_r^2}\right) F^2(E_r).$$

Here $G_f$ is the Fermi constant, $Q_{\nu N} = N - (1 - 4\sin^2 \theta_w)Z$ is effective weak hyper-charge in the SM for the target nucleus with $N$ neutrons and $Z$ protons and $F(E_r)$ is the Helm form factor (Ref. [35]). The effective coupling between neutrino and nucleus for $U(1)_{\mu - \tau}$ model can be written as,

$$Q_{\nu N\mu - \tau} = g_{\mu - \tau}^2 \frac{2\alpha_{EM}}{\pi} \delta_{\mu\tau} Z$$

where $g_{\mu - \tau}$ is coupling given in Eq. 3, $\alpha_{EM}$ is the fine structure constant and $\delta_{\mu\tau}$ is the scalar part of loop factor given in equation 3.
Neutrino and Dark Matter Rate equation

CEνNS rate equation is given by,
\[
\frac{dR_{\nu-N}}{dE_r} = \frac{\epsilon}{m_N} \int_{E_{\nu_{\min}}^{\nu}} A(E_r) \frac{d\phi_{\nu}}{dE_{\nu}} \left| \frac{d\sigma(E_{\nu}, E_r, \nu_{\beta})}{dE_r} \right| P(\nu_\alpha \rightarrow \nu_\beta, E_{\nu}) dE_{\nu}
\]  

(8)

Here \( \epsilon \) is the exposure of the experiment measured in units of mass \( \times \) time, \( A(E_r) \) is the detector efficiency and is set to one in following calculations. \( E_{\nu_{\min}}^{\nu} \) is minimum incident neutrino energy required to produce a detectable recoil for a material nucleus of mass \( m_N \) with energy \( E_r \), which in the limit of \( m_N \gg E_{\nu} \) can be written as,
\[
E_{\nu_{\min}}^{\nu} = \sqrt{\frac{m_N E_r}{2}}.
\]

(9)

Here, \( \frac{d\sigma(E_{\nu}, E_r, \nu_{\beta})}{dE_r} \) is \( \beta \) flavor dependent neutrino-nucleus differential scattering cross-section and \( \frac{d\phi_{\nu}}{dE_{\nu}} \) is the incoming neutrino flux of flavor \( \alpha \). The fluxes used in this analysis involve fluxes from solar, atmospheric, diffuse supernova neutrinos, which can be found in Refs. [36, 37].

Dark matter direct detection rate equation

Spin-independent DM-nucleon scattering rate equation is given by,
\[
\frac{dR_{DM-N}}{dE_r} = \epsilon \frac{\rho_{DM} \sigma_0^n A^2}{2m_{DM} \mu_n^2} F^2(E_r) \int_{\nu_{min}} f(v) \frac{d^3v}{v}
\]

(10)

Here \( \epsilon \) is the exposure of the detector given in units of MT (mass x time), \( \rho_{DM} \) is the DM mass \( \mu_n \) is DM-nucleon reduced mass, \( A \) is the mass number of target nuclei, \( \sigma_0^n \) is the DM-nucleon scattering cross-section at zero momentum transfer. \( F(E_r) \) is the Helmholtz form factor.

\[
\int_{\nu_{min}} f(v) \frac{d^3v}{v} = \frac{1}{2v_0 \eta_E} \left[ er\left( \eta_+ \right) - er\left( \eta_- \right) \right] - \frac{1}{\pi v_0 \eta_E} \left( \eta_+ - \eta_- \right) e^{\eta_{esc}^2}
\]

(11)

Here \( \eta_E = \frac{v_E}{v_0}, \eta_{esc} = \frac{v_{esc}}{v_0} \) and \( \eta_{\pm} = min \left( \frac{v_{min}}{v_0} \pm \eta_E, \frac{v_{esc}}{v_0} \right) \), where \( v_0 \) is local galactic rotational velocity, \( v_E \) velocity of Earth with respect to galactic center, \( v_{esc} \) escape velocity of DM from galaxy. We have used values \( v_0 = 220 \text{ km/s}, v_E = 232 \text{ km/s} \) and \( v_{esc} = 544 \text{ km/s} \) in above calculations.

Neutrino Floor

Neutrino floor is defined as the value of DM-nucleon scattering cross-section at which ratio of no. of scattering events generated by DM-nucleon scattering to that coming events generated by neutrino background is 2.3 to 1 (at 90% CL). This establishes a borderline below which contribution of neutrino background increases and certainty that the observed events if any are from DM-nucleon scattering decreases. It is calculated by measuring the exposure required to observe a neutrino scattering event and then for the same exposure calculation the value of DM-nucleon scattering cross-section for which we get 2.3 scattering events (For more details see Ref. [13]).

\[
\int_{E_{th}}^{E_{max}} dR_{DM-N} dE_r = \frac{2.3}{1} \int_{E_{th}}^{E_{max}} dR_{\nu-N} dE_r,
\]

This can be reiterated in form of the master equation,

\[
\sigma_0^n = \frac{2.3}{1} \left( \frac{1}{m_N} \int_{E_{\nu_{min}}^{\nu}} \frac{d\phi_{\nu}}{dE_{\nu}} \left| \frac{d\sigma(E_{\nu}, E_r, \nu_{\beta})}{dE_r} \right| P(\nu_\alpha \rightarrow \nu_\beta, E_{\nu}) \frac{d\sigma(E_{\nu}, E_r, \nu_{\beta})}{dE_r} dE_{\nu} \right)
\]

\times \left( \frac{\rho_{DM} A^2}{2m_{DM} \mu_n^2} \int_{E_{th}}^{E_{max}} F^2(E_r) \int_{\nu_{min}} f(v) \frac{d^3v}{v} \right)^{-1}
\]

(12)
(a) $m_{Z'} = 16$ MeV, $g_{\mu-\tau} = 0.0006$, with and without $(g - 2)_{\mu}$ (b) $m_{Z'} = 25$ MeV, $g_{\mu-\tau} = 0.00067$, with $(g - 2)_{\mu}$, max point

(c) $m_{Z'} = 25$ MeV, $g_{\mu-\tau} = 0.00093$, without $(g - 2)_{\mu}$, max point

(d) $m_{Z'} = 25$ MeV, $g_{\mu-\tau} = 0.00093$, without $(g - 2)_{\mu}$, max point

FIG. 4: Neutrino floor projected in the $\sigma_n^0$ vs $m_{DM}$ plane. Comparison of the neutrino floor for the SM (presented by the blue line) and that for U(1)$_{\mu-\tau}$ (presented by the red line). For different detector materials, Ge$^{68}$ and Xe$^{131}$.

Here $E_{\text{max}}^n$ is the maximum recoil energy of DM with mass $m_{DM}$ can produce in a given nucleus. It is written as,

$$2m_{DM} \left( \frac{m_N m_{DM}}{m_N + m_{DM}} \right) v_{\text{esc}}^2$$

Using the master equation 12, we evaluate the modified neutrino floor for the allowed region in Fig 3. It is noticed that enhancement in neutrino floor increases with increasing value of $m_{Z'}$ till 25 MeV for corresponding maximum value of allowed $g_{\mu-tau}$ and starts to decrease with larger masses and becomes negligible around and after $m_{Z'} = 200$ MeV. This can be understood from the profile of excluded region seen in Fig 3 which shows a sharp kink around $m_{Z'} = 25$ MeV. Also the beyond standard model contribution to CEvNS is proportional to $\frac{\sigma_{\mu-\tau}^0}{2m_N E_{\nu} + m_{Z'}^2}$, therefore neutrino floor enhancement decreases with larger $m_{Z'}$ value and lower $g_{\mu-\tau}$. We choose the benchmark points $m_{Z'} = \{16, 25, 50, 200\}$ MeV to show the enhancement in neutrino floor at different DM masses in tabulated form in the Appendix reftable. As the chosen region is also important from $(g-2)_{\mu}$ perspective, we also show neutrino floor enhancement for points $(g - 2)_{\mu}$ allowed couplings for same benchmarks.

In Fig. 4 we show graphs showing modification of theoretically estimated neutrino floor on the $\sigma_n^0 - m_{DM}$ plane. Solid lines exhibit the boundary of DM-nucleon scattering cross-section above which we can be certain (90% C.L.) that ratio of DM-scattering events to that of neutrino-scattering events is greater than 2.3:1. Blue contours signify neutrino floor estimated from Standard Model interactions whereas red contour show neutrino floor for U(1)$_{\mu-\tau}$ model. It can be noticed that modification in neutrino floor is most significant for DM mass below 7 Gev and diminishes to a barely noticeable change for higher masses. This can be understood as below 7 Gev DM mass $\sigma_n^0$ is sensitive to threshold recoil energies below 1 kev and less. Below 1 keV threshold energy CEvNS rate for U(1)$_{\mu-\tau}$ show modification of the same order as seen in neutrino floor below 7 GeV DM mass and for threshold energy greater than 1 keV CEvNS rate modification in U(1)$_{\mu-\tau}$ rapidly diminishes (For detail see Ref. [13]). Top left panel 4a shows modification in neutrino
IV. GEV SCALE DARK MATTER: ROLE OF \((g - 2)_\mu\) ANOMALY

![Diagram of DM candidates](image)

FIG. 5: The annihilation diagrams of DM candidates: The \(Z'\) mediated s-channel annihilation \(\chi\chi \rightarrow \text{SMSM} \) (left) and \(Z'\) mediated t-channel annihilation \(\chi\chi \rightarrow Z'Z'\) (right)

We have argued before that the 100 MeV to 10 GeV mass scale is a very favorable parameter space to survive where we can explore possibility of a light dark matter (DM) candidate. Here we work with a light Dirac fermion as a DM candidate. At mass scales around 100 MeV, DM is sufficiently heavy to contribute anything to cosmological computations. As with \(m_{\text{DM}} \sim 100\) MeV, it cannot remain in thermal equilibrium at the energy scale of Big bang Nucleosynthesis (BBN), contributing minimally to effective degrees of freedom as was indicated in Ref. [38]. On the other end of the spectrum, 10 GeV dark matter is light enough to not get significantly constrained from the ongoing LHC observations, possibly in the jet/lepton plus missing energy search channels.

In the thermal DM regime, weakly interacting massive particle (WIMP) search sensitivity rules out a large part of TeV scale DM masses. The constraints on the GeV scale DM are weaker, compared to those on an ultralight DM and the WIMP paradigm. Here we try to probe this DM territory with the current \((g - 2)_\mu\) measurement in our arsenal, that shows a anomalous deviation \((\sim 4.2\sigma)\)from the SM predicted values.

Relic Density Constraints: Role of \((g - 2)_\mu\) Anomaly

The Dirac fermion DM \((\chi)\) introduced here has coupling with \(U(1)_{\mu-\tau}\) gauge fermion through an interaction term in the Lagrangian like

\[
g_{\mu-\tau} q_\chi \bar{\chi} \gamma^\mu Z'_\mu \chi
\]

, where we take the effective \(Z' - \chi\) coupling as \(g_\chi = g_{\mu-\tau} q_\chi\). The DM annihilation in this scenario happens through three major channels, namely \(\chi\chi \rightarrow \ell\ell\) and \(\chi\chi \rightarrow \nu\nu\), both through s-channel \(Z'\) mediated annihilation and a t-channel \(Z'\) mediated annihilation process \(\chi\chi \rightarrow Z'Z'\), while \(Z'\) can decay to the SM muons and tau leptons. The annihilation processes are depicted in Fig. 5. The s-channel and t-channel annihilation cross sections are computed...
following the Re. [39]. The s-channel diagram contributions to the DM annihilation:

\[
\langle \sigma v \rangle (\chi \bar{\chi} \rightarrow \ell \bar{\ell}) \approx \frac{g_\chi^2 Y'_\ell^2 g_{\mu-\tau}^4}{16 \pi m_\chi^2} \frac{m_\ell^2}{2} \frac{2 m_\ell^2 + m_\chi^2}{(4 m_\chi^2 - m_{Z'}^2)},
\]

\[
\langle \sigma v \rangle (\chi \bar{\chi} \rightarrow \nu \bar{\nu}) \approx \frac{g_\chi^2 Y'_\ell^2 g_{\mu-\tau}^4}{16 \pi m_\chi^2} \frac{m_\chi^2}{2} \frac{2 m_\ell^2 - m_\chi^2}{(4 m_\chi^2 - m_{Z'}^2)},
\]

where \( Y'_\ell = 1 - 1 \) for the muons and tau leptons respectively. The t-channel contribution is computed as

\[
\langle \sigma v \rangle (\chi \bar{\chi} \rightarrow Z' Z') \approx \frac{g_\ell^4 g_{\mu-\tau}^4}{16 \pi m_\chi^2} \left( 1 - \frac{m_{Z'}^2}{m_\chi^2} \right)^{3/2} \left( 1 - \frac{m_{Z'}^2}{2 m_\chi^2} \right)^{-2},
\]

providing total thermally averaged annihilation cross section as

\[
\langle \sigma v \rangle = \langle \sigma v \rangle (\chi \bar{\chi} \rightarrow \ell \bar{\ell}) + \langle \sigma v \rangle (\chi \bar{\chi} \rightarrow \nu \bar{\nu}) + \langle \sigma v \rangle (\chi \bar{\chi} \rightarrow Z' Z').
\]

Initially when the Universe was hot, DM particles were in thermal equilibrium with other SM particles inside the thermal plasma through \( 2 \rightarrow 2 \) annihilation processes. As the Universe cools down as it expands, the DM annihilation does not remain efficient enough to keep them in thermal equilibrium, where two DM particles cannot come together for an annihilation. This is called the thermal freeze out. After the freeze out of the dark matter out of the thermal equilibrium, the remnant density of the dark matter that remains till today, the DM relic density is given as,

\[
\Omega h^2 \approx 1.04 \times 10^9 x_F \text{GeV}^{-1} \sqrt{g} \text{M}_{\text{pl}} \langle \sigma v \rangle,
\]

where \( x_F \) dictates the time or temperature of the thermal freeze out given as \( x_F = \frac{m_\chi}{T_F} \), with \( T_F \) as the freeze out temperature. For the freeze out point fixation and DM relic density computation we closely follow the approximate method we had outlined in Ref. [40]. For GeV scale Dirac fermionic DM the freeze out temperature is specified by \( x_F \sim (17 - 20) \), and this is obtained solving the Boltzmann equation governing the thermal evolution of the DM.

The s-channel annihilation processes contribute significantly to the overall cross section only in the region of resonance i.e. \( m_{Z'} \approx 2 m_\chi \). Therefore a light DM at the mass range of \( 0.1 - 10 \text{ GeV} \) can annihilate resonantly if there is lighter \( Z' \) to assist, through the resonant process. Even as \( Z' \) is allowed for a very light mass range, the constraints on \( g_{\mu-\tau} \) is very tight from existing neutrino experiments like Borexino, Charm-II, flavor physics experiments like BABAR etc.

The s-channel annihilation, apart from the resonant factor, depends on \( g_{\mu-\tau} \) couplings, where the gauge coupling is constrained to have smaller values. Still s-channels can have a significant cross section only through a resonant enhancement. For small \( q_\chi \) values t-channel contribution is not that significant, and we shall obtain a tiny parameter space that does enough DM annihilation not to overproduce DM relic density. Whenever the DM mass and \( Z' \) masses are close to each other satisfying the resonant condition, the s-channel dominates. Due to s-channel annihilation,
even for a very small $g_\chi$ values DM relic density can be satisfied for GeV scale DM when $M_{Z'}$ is allowed to vary in the range of $0.1 - 20$ GeV. Requirement of resonant condition sets an interdependence of DM and extra gauge boson masses, to satisfy total relic density.

When we increase $g_\chi$ values then the t-channel diagram starts contributing more and more to the total annihilation cross section, and therefore we require smaller contributions from the resonant s-channel annihilation. When the strictness of the resonance condition loosens, the DM mass stops being so intricately connected to the chosen $Z'$ mass. Then there is no interrelation between the DM and BSM gauge boson masses. From the plot in Fig. 3 and the discussion of satisfying the deviation of $(g-2)_\mu$, it is observed that the gauge boson coupling vs mass parameter space is even more constrained. The muon $(g-2)$ constraints restrict the $m_{Z'}$ in the range 10 MeV to 200 MeV, while the gauge coupling also gets restricted further to vary in the range $3 - 8 \times 10^{-4}$. This constraint has direct implication on the allowed DM parameter space; specially for a GeV scale DM.

With the $(g-2)_\mu$ constraint being included, the s-channel resonant annihilation gets very restricted. This is because for a $Z'$ in range of masses from 10-200 MeV, only DM masses $\sim m_{Z'}/2$ will be eligible for a resonant annihilation. Only the MeV scale DM of masses 5-100 MeV can therefore annihilate through an s-channel resonance, having significant cross section not to over-populate the Universe with more DM. Another constraining aspect is that along with ruling out higher GeV scale masses for $Z'$, it also rules out $g_{\mu-\tau} \geq 8 \times 10^{-4}$. For these scale of couplings, the s-channel resonant enhancement of cross section is not sufficient enough for proper relic density. The 5 $- 100$ MeV range DM is plausible through s-channel resonant annihilation even after $(g-2)_\mu$, but still there are not many points for moderate $g_\chi$. If we increase our $g_\chi$ values which will be reflected in increasing $g_\chi$ values, the t-channel contribution to the total annihilation increases continually reducing dependence on s-channel annihilation. The t-channel annihilation can only start contributing when $m_\chi > m_{Z'}$. Whenever in the parameter space $m_\chi \leq m_{Z'}$, only the s-channel annihilation is phenomenologically viable.

In this work the focus is on DM mass $0.1 - 10$ GeV. We separate the range of interest in two regions I. sub-GeV dark matter ($m_\chi = 0.1 - 1$ GeV) II. super-GeV dark matter ($m_\chi = 1 - 10$ GeV). The sub-GeV and super-GeV DM cases in the context of relic density constraints are depicted in Fig. 6 and Fig. 7 respectively. For the 0.1 $- 1$ GeV DM we plot $g_\chi$ vs $m_\chi$, where smaller $g_\chi$ values are not very favored as that can overproduce the relic. Still for relatively higher masses $\sim 1$ GeV, relatively lower couplings arrange sufficient annihilation, possibly aided by s-channel resonant effects. When the $(g-2)_\mu$ explanation is included, then lower couplings are ruled out to favor only the higher couplings like $g_\chi \sim 0.01 - 0.07$. This is attributed to the absence of s-channel annihilation, t-channel needs higher coupling to have sufficient annihilation. The $m_{Z'} - m_\chi$ plot shows that the region with $m_{Z'} \leq m_\chi$ is sparsely populated due to the absence of t-channel annihilation. This persists even after $(g-2)_\mu$ constraint is applied, significantly restricting the $m_{Z'}$ to a upper limit of 0.2 GeV. Also, higher values of DM easily satisfy the relic density constraint as t-channel process is kinematically more viable with $m_\chi - m_{Z'}$ being sizable. The similar patterns are evident from the plots for the super-GeV DM, with the effects being more pronounced for this case. When only the t-channel contribution is important i.e. once $(g-2)_\mu$ constraint is being imposed, then annihilation decreases with increasing DM mass, pushing the couplings to larger values. Along with that larger $g_\chi$ values also get ruled out after the $(g-2)_\mu$ constraint as it restricts the $g_{\mu-\tau}$ to relatively smaller values compared to non-constrained case. If we look at the $m_\chi - m_{Z'}$ correlation for the $1 - 10$ GeV DM, the $m_{Z'} \geq m_\chi$ part of the parameter space is only due to s-channel contribution,
while region around $m_{Z'} \sim m_\chi$ observes overproduction of the relic and thus ruled out. After the $(g-2)_\mu$ limit imposition this full range gets ruled out allowing only few points around $m_{Z'} \sim 0.1$ GeV. These features are observed in Fig. 7.

V. DM DIRECT DETECTION VS NEUTRINO FLOOR: $(g-2)_\mu$ EFFECTS

The model in contention being leptophilic, does not have any direct coupling between the DM particle and the quarks. Hence any scattering off the nuclei of the form $\chi N \rightarrow \chi N$ can only be rendered through a loop of charged leptons, where photons emitted by virtual leptons couple to the charge of the nucleons inside a nucleus as shown in Fig 8.

The expression for spin-independent cross section for the D-Nucleon scattering can be written as:

$$\sigma_{SI} = \frac{\mu_N^2}{9\pi A^2} \left( \frac{\alpha_{EM} Z g_{\mu-\tau} q_\chi}{\pi m_{Z'}^2} \log \frac{m_\mu^2}{m_\tau^2} \right)^2 \label{eq:16}$$

where $\mu_N$ is the reduced mass of the nucleon-DM system given by $m_N m_\chi/(m_N + m_\chi)$, with $m_N$ being the nuclear mass, $A$ and $Z$ being mass number and atomic number respectively. We stick to $Z = 54$ and $m_N = 129$ GeV consistent with Xenon (Xe) based experiments. As observed above, the enhancement of the neutrino floor is subdued due to anomalous $(g-2)_\mu$ measurement, even more so when the detector is Xenon material based. Here, we stick to the SM neutrino floor for the DM direct detection analysis, as our conclusion is not critically dependent on the neutrino floor enhancement. The region of primary interest lies in DM with masses ranging from 0.1-10 GeV, called GeV scale DM. As shown by the purple-shaded region of the left plot of Fig 9, the primary constraint for sub-GeV DM is provided by the CRESST [41] experiment. For DM in the range 1-10 GeV, the constraints become severe as we go towards higher mass, with the most constraining limits coming from Darkside-50 [42], XENON1T-2018 [43], XENON1T-2019 [44] along with SuperCDMS [45], as shown by the purple-shaded region of the right plot of Fig 9. The neutrino floor is depicted by the grey shaded region in both the plots, with it becoming less constraining as we move towards high DM masses. For the points that go below the neutrino floor, the parameter space associated to them cannot be probed in these set of experiments, where as they are not necessarily ruled out. Concentrating on the sub-GeV scenario, here we are more likely to get a parameter space which satisfies the relic density constraints and the constraints coming from $(g-2)_\mu$.

As already shown in Fig. 3, a $Z'$ mass in the range 10 – 200 MeV is required to satisfy the $(g-2)_\mu$ constraint. This ensures that resonant s-channel contribution to the DM-annihilation cross-section in conjunction with the $g - 2$ constraint can only happen in this part of the parameter space whenever the resonant condition $m_\chi \approx m_{Z'}/2$ is getting satisfied. This leads to restricting the s-channel resonant contribution only to DM mass regime of $m_\chi \leq 0.1$ GeV. In order to decipher the allowed parameter space, we look for direct detection cross section numbers for the $m_{Z'} \sim 0.01 – 2$ GeV, $g_{\mu-\tau} \sim 5 \times 10^{-3} – 5 \times 10^{-4}$ and the DM charge $q_\chi$ from 0.1 to 50. The green points denote the cross sections before the imposition of the constraints coming from $(g-2)_\mu$ when at least 10% of the DM relic density is satisfied. The dark-red points on the top of the green points are the same relic density points after imposing the $\pm 2\sigma$
In $m_\chi$ range $0.1 - 1$ GeV i.e. for a sub-GeV DM, both the resonant s-channel and t-channel contributions to the DM annihilation were there before $(g-2)_\mu$ had constrained the parameter space. Due to efficient annihilation, DM relic density was well within the allowed for moderate $g_{\mu-\tau}, q_\chi$ values. With moderate couplings along with the plausibility of $m_{Z'}$, being as high up to 20 GeV, the direct detection cross section was mostly in the $10^{-45} - 10^{-38}$ cm$^2$ range, gradually increasing with the DM mass. In this range, large portion of parameter space will have cross section lower than the neutrino floor, therefore evading any probe. A large part of the parameter space will present a DM candidate to show up with a cross section above the neutrino floor and below the most conservative direct detection bound in this DM mass region that comes from the CRESST results. Very few points in this region can reach current experimental search sensitivity and they cannot be ruled out from the experiment. With the $(g-2)_\mu$ bounds being included the $Z'$ mass range gets restricted resulting in absence of s-channel contribution. That requires larger $q_\chi$ values to have a significant contribution from t-channel process, which as a consequence increases the direct detection cross section. The $(g-2)_\mu$ constrains the overall parameter space only to allow only a part of it where almost no allowed points have cross section below the neutrino floor. While this is good from the detection perspective, a significant number of points now reach experimental search sensitivity and therefore get ruled out. Any experimental update in the $0.1 - 0.5$ GeV dark matter range can probe presence of dark matter with signal cross section $10^{-40} - 10^{-38}$ cm$^2$.

For the DM mass range just above a GeV, the $(g-2)_\mu$ constraints are even more tight. Without the $(g-2)_\mu$ constraint being imposed there was certain possibility of DM observations in the cross section range $10^{-43} - 10^{-41}$ cm$^2$, while a significant chunk of points were ruled out by the direct detection experiments like Xenon1T, CDMSLite, DarkSide experiments. After $(g-2)_\mu$ being included the cross section for the allowed points go further higher due to prevalence of higher $q_\chi$ values. This results in all the points allowed by the relic density constraints, getting ruled out by the direct detection search of various experiments named above. Only very few points for DM mass below 2 GeV are allowed. For $m_{DM} \geq 2$ GeV, the DM relic density and the constraint from $g - 2$ cannot be satisfied simultaneously. These results in almost ruling out $1 - 10$ GeV DM from the $(g-2)_\mu$ anomalous results while no tight bounds were there in this mass range earlier.

**VI. SUMMARY AND CONCLUSION**

In this article we have investigated viability of $U(1)_{L_\mu - L_\tau}$ model to simultaneously explain $(g-2)_\mu$ and DM relic density along with possible modification in the neutrino floor. We found that the parameter range $m_{Z'}$ in 10-200 MeV and gauge coupling $g_{\mu-\tau} \sim (0.5 - 1.0) \times 10^{-3}$ remains viable for explaining $(g-2)_\mu$ which is allowed by latest experimental constraints. We have studied this allowed region to explore possible modification of the neutrino floor. It is found that with increasing $m_{Z'}$, neutrino floor reaches maximum enhancement of $2.25$ times and $1.65$ times for Germanium and Xenon based experiments respectively, for a benchmark point $(m_{Z'} = 25$ MeV, $g_{\mu-\tau} = 9.3 \times 10^{-4})$. Above this mass, floor enhancement gradually decreases with increasing $m_{Z'}$. With the inclusion of $(g-2)_\mu$, the benchmark point $(m_{Z'} = 16$ MeV, $g_{\mu-\tau} = 6.0 \times 10^{-4})$ shows the maximum enhancement of $2.15$ times and $1.62$ times for Germanium and Xenon respectively. Next, considering the constraints from CMB and $(g-2)_\mu$, we investigate the DM mass range from 0.1 to 10 GeV for a VLF, $\chi$ as dark matter candidate. The DM candidate $\chi$ contributes to dark matter relic density though the s-channel annihilation process $\bar{\chi} \chi \rightarrow Z' \rightarrow l \bar{l} (\nu \bar{\nu})$ and t-channel process $\bar{\chi} \chi \rightarrow Z' Z'$. For sub-GeV $m_{\chi}$: $(0.1 - 1)$ GeV, without the $(g-2)_\mu$ constraint the parameter space allowed by DM...
relic density range from $g_\chi \approx 0.001 - 1.0$ and $m_{Z'} = 0.01 - 10$ GeV. With the inclusion of $(g - 2)_\mu$ constraint allowed parameter space is more restricted with $m_{Z'} = 0.01 - 0.2$ GeV and $g_\chi$ picking higher values in the range $\approx 0.001 - 1.0$. For super GeV range, we found a similar trend, but with inclusion of $(g - 2)_\mu$ parameter space is even more restricted, with $g_\chi$ being confined to a narrow band of $g_\chi \approx 0.02 - 0.05$. For sub GeV dark matter CRESST experiment excludes $\sigma_{SI} > 10^{-37} - 10^{-38}$ cm$^2$, but still appreciable parameter space in this range is allowed. Studying the DM parameter space for direct detection experiments, we conclude that most of the super GeV parameter space viable for $(g - 2)_\mu$ is ruled out by DarKSide-50, CDMSLite, XENON-1T, with only small parameter space being allowed for DM mass less than 2 GeV.

In conclusion, the model parameter space viable for $g - 2$ experiments can account for a sub GeV VLF dark matter as well as shows 2 times enhancement in the neutrino floor. The region of interest can be worthwhile to probe in upcoming experiments like COHERENT. With increasing exposure, it may become difficult for direct detection experiments to differentiate between signal form dark matter and neutrino background. The conclusion we draw here are valid for the Dirac fermionic DM candidates. A light Majorana fermion candidate can have a new scalar portal for its annihilation, that can open most of the parameter space we rule out due to the muon $(g - 2)$ constraint here. Also a scalar dark matter can be studied in this context. In general GeV scale DM is an interesting region due to its viability, being non-constrained by other constraints. The recent anomalous $(g - 2)$ measurement can play a crucial role here.

ACKNOWLEDGMENTS

We would like to thank Prof. Debajyoti Choudhury for useful discussions. SS thanks Vivekananda Centre for Research (VCR) for providing me with research facilities. MPS would like to thank S.P. Singh for partial financial support.

APPENDIX

1. Enhancement in CEνNS event rate and neutrino floor

I. Benchmark chosen: $Z'$ mass 16 MeV and coupling $g_{\mu-\tau} = 6.0 \times 10^{-4}$

| $m_{DM}$ (GeV) | SM Neutrino floor (cm$^2$) | $U(1)_{\mu-\tau}$ Neutrino floor (cm$^2$) | Enhancement |
|---------------|-----------------------------|---------------------------------|-------------|
| 0.5           | $2.36 \times 10^{-43}$      | $5.22 \times 10^{-43}$          | 2.211       |
| 1             | $5.23 \times 10^{-44}$      | $1.14 \times 10^{-43}$          | 2.17        |
| 5             | $7.05 \times 10^{-45}$      | $1.51 \times 10^{-44}$          | 2.121       |
| 10            | $1.42 \times 10^{-47}$      | $1.74 \times 10^{-47}$          | 1.225       |

II. Benchmark chosen: $Z'$ mass 200 MeV and coupling $g_{\mu-\tau} = 1.64 \times 10^{-3}$

III. Benchmark chosen: $Z'$ mass 50 MeV and coupling $g_{\mu-\tau} = 1.1 \times 10^{-3}$

IV. Benchmark chosen: $Z'$ mass 25 MeV and coupling $g_{\mu-\tau} = 0.93 \times 10^{-3}$

TABLE I: Neutrino floor versus dark matter mass table highlighting modification of neutrino floor for $U_{\mu-\tau}$ with respect to SM.


| $m_{DAM}$ (GeV) | SM Neutrino floor (cm$^2$) | $U(1)_{\mu-\tau}$ Neutrino floor (cm$^2$) | Enhancement |
|----------------|-----------------------------|-------------------------------------|-------------|
| **Germanium based experiments** | | | |
| 0.5 | $2.36 \times 10^{-43}$ | $2.46 \times 10^{-43}$ | 1.04 |
| 1. | $5.23 \times 10^{-44}$ | $5.48 \times 10^{-43}$ | 1.047 |
| 5 | $7.05 \times 10^{-45}$ | $7.44 \times 10^{-44}$ | 1.055 |
| 10 | $1.42 \times 10^{-47}$ | $1.49 \times 10^{-47}$ | 1.049 |
| **Xenon based experiments** | | | |
| 0.5 | $3.91 \times 10^{-43}$ | $4.01 \times 10^{-43}$ | 1.02 |
| 1. | $8.49 \times 10^{-44}$ | $8.74 \times 10^{-43}$ | 1.03 |
| 5 | $1.08 \times 10^{-44}$ | $1.125 \times 10^{-44}$ | 1.04 |
| 10 | $9.27 \times 10^{-48}$ | $9.56 \times 10^{-47}$ | 1.03 |

**TABLE II:** Neutrino floor versus dark matter mass table highlighting modification of neutrino floor for $U_{\mu-\tau}$ with respect to SM.

| $m_{DAM}$ (GeV) | SM Neutrino floor (cm$^2$) | $U(1)_{\mu-\tau}$ Neutrino floor (cm$^2$) | Enhancement |
|----------------|-----------------------------|-------------------------------------|-------------|
| **Germanium based experiments** | | | |
| 0.5 | $2.36 \times 10^{-43}$ | $5.06 \times 10^{-43}$ | 1.33 |
| 1. | $5.23 \times 10^{-44}$ | $7.23 \times 10^{-43}$ | 1.38 |
| 5 | $7.05 \times 10^{-45}$ | $1.01 \times 10^{-44}$ | 1.43 |
| 10 | $1.42 \times 10^{-47}$ | $1.82 \times 10^{-47}$ | 1.28 |
| **Xenon based experiments** | | | |
| 0.5 | $3.91 \times 10^{-43}$ | $4.67 \times 10^{-43}$ | 1.19 |
| 1. | $8.49 \times 10^{-44}$ | $1.03 \times 10^{-43}$ | 1.21 |
| 5 | $1.08 \times 10^{-44}$ | $1.35 \times 10^{-44}$ | 1.25 |
| 10 | $9.27 \times 10^{-48}$ | $1.08 \times 10^{-47}$ | 1.16 |

**TABLE III:** Neutrino floor versus dark matter mass table highlighting modification of neutrino floor for $U_{\mu-\tau}$ with respect to SM.

| $m_{DAM}$ (GeV) | SM Neutrino floor (cm$^2$) | $U(1)_{\mu-\tau}$ Neutrino floor (cm$^2$) | Enhancement |
|----------------|-----------------------------|-------------------------------------|-------------|
| **Germanium based experiments** | | | |
| 0.5 | $2.36 \times 10^{-43}$ | $5.06 \times 10^{-43}$ | 2.14 |
| 1. | $5.23 \times 10^{-44}$ | $1.16 \times 10^{-43}$ | 2.23 |
| 5 | $7.05 \times 10^{-45}$ | $1.61 \times 10^{-44}$ | 2.28 |
| 10 | $1.42 \times 10^{-47}$ | $2.05 \times 10^{-47}$ | 1.43 |
| **Xenon based experiments** | | | |
| 0.5 | $3.91 \times 10^{-43}$ | $6.27 \times 10^{-43}$ | 1.60 |
| 1. | $8.49 \times 10^{-44}$ | $1.40 \times 10^{-43}$ | 1.65 |
| 5 | $1.08 \times 10^{-44}$ | $1.84 \times 10^{-44}$ | 1.70 |
| 10 | $9.27 \times 10^{-48}$ | $1.16 \times 10^{-47}$ | 1.25 |

**TABLE IV:** Neutrino floor versus dark matter mass table highlighting modification of neutrino floor for $U_{\mu-\tau}$ with respect to SM.

[1] B. Abi et al. Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm. *Phys. Rev. Lett.*, 126(14): 141801, 2021. [doi:10.1103/PhysRevLett.126.141801](https://doi.org/10.1103/PhysRevLett.126.141801)

[2] Paolo Girotti. Status of the Fermilab Muon g – 2 Experiment. 2021.

[3] G. W. Bennett et al. Final Report of the Muon E821 Anomalous Magnetic Moment Measurement at BNL. *Phys. Rev. D*, 73:072003, 2006. [doi:10.1103/PhysRevD.73.072003](https://doi.org/10.1103/PhysRevD.73.072003)
[33] T. Aoyama et al. The anomalous magnetic moment of the muon in the Standard Model. *Phys. Rept.*, 887:1–166, 2020. doi:10.1016/j.physrep.2020.07.006

[34] Sz. Borsanyi et al. Leading hadronic contribution to the muon magnetic moment from lattice QCD. *Nature*, 593(7857):51–55, 2021. doi:10.1038/s41586-021-03418-1

[35] J. D. Lewin and P. F. Smith. Review of mathematics, numerical factors, and corrections for dark matter experiments based on elastic nuclear recoil. *Astropart. Phys.*, 6:87–112, 1996. doi:10.1016/S0927-6505(96)00047-3

[36] Louis E. Strigari. Neutrino Coherent Scattering Rates at Direct Dark Matter Detectors. *New J. Phys.*, 11:105011, 2009. doi:10.1088/1367-2630/11/10/105011

[37] J. Billard, L. Strigari, and E. Figueroa-Feliciano. Implication of neutrino backgrounds on the reach of next generation dark matter direct detection experiments. *Phys. Rev.*, D89(2):023524, 2014. doi:10.1103/PhysRevD.89.023524

[38] Manuel Drees and Wenbin Zhao. U(1)\(\mu\)–L\(\tau\) for light dark matter, \(g_{\mu}\)FIX ME!!!–FIX ME!!!2, the 511 keV excess and the Hubble tension. *Phys. Lett. B*, 827:136948, 2022. doi:10.1016/j.physletb.2022.136948

[39] Wolfgang Altmannshofer, Stefania Gori, Stefano Profumo, and Farinaldo S. Queiroz. Explaining dark matter and B decay anomalies with an \(L_{\mu} – L_{\tau}\) model. *JHEP*, 12:106, 2016. doi:10.1007/JHEP12(2016)106

[40] Debasish Borah, Soumya Sadhukhan, and Shibananda Sahoo. Lepton Portal Limit of Inert Higgs Doublet Dark Matter with Radiative Neutrino Mass. *Phys. Lett. B*, 771:624–632, 2017. doi:10.1016/j.physletb.2017.06.006

[41] A. H. Abdelhameed et al. First results from the CRESST-III low-mass dark matter program. *Phys. Rev. D*, 100(10):102002, 2019. doi:10.1103/PhysRevD.100.102002

[42] P. Agnes et al. Low-Mass Dark Matter Search with the DarkSide-50 Experiment. *Phys. Rev. Lett.*, 121(8):081307, 2018. doi:10.1103/PhysRevLett.121.081307

[43] E. Aprile et al. Dark Matter Search Results from a One Ton-Year Exposure of XENON1T. *Phys. Rev. Lett.*, 121(11):111302, 2018. doi:10.1103/PhysRevLett.121.111302

[44] E. Aprile et al. Constraining the spin-dependent WIMP-nucleon cross sections with XENON1T. *Phys. Rev. Lett.*, 122(14):141301, 2019. doi:10.1103/PhysRevLett.122.141301

[45] R. Agnese et al. Search for Low-Mass Dark Matter with CDMSlite Using a Profile Likelihood Fit. *Phys. Rev. D*, 99(6):062001, 2019. doi:10.1103/PhysRevD.99.062001