Exotic States in the Dynamical Casimir Effect

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We consider the interaction of a qubit with a single mode of the quantized electromagnetic field and show that, in the ultrastrong coupling regime and when the qubit-field interaction is switched on abruptly, the dynamical Casimir effect leads to the generation of a variety of exotic states of the field, which cannot be simply described as squeezed states. Such effect also appears when initially both the qubit and the field are in their ground state. The non-classicality of the obtained exotic states is characterized by means of a parameter based on the volume of the negative part of the Wigner function. A transition to non-classical states is observed by changing either the interaction strength or the interaction time. The observed phenomena appear as a general feature of nonadiabatic quantum gates, so that the dynamical Casimir effect can be the origin of a fundamental upper limit to the maximum speed of quantum computation and communication protocols.

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In 1948 Casimir [1] predicted that two uncharged conducting parallel plates, experience an attractive force. This phenomenon, known as (static) Casimir effect, is explained in terms of quantum mechanical vacuum fluctuations of the electromagnetic field. Indeed the two plates impose boundary conditions to the field such that the density of electromagnetic modes between the plates depends on their distance $d$. The plates affect the virtual photons which constitute the field, thus generating a net attractive force proportional to $d^{-4}$. The static Casimir effect, experimentally demonstrated for the first time by Sparnay in 1958 [2], plays a very important role both in fundamental physics investigations and in understanding the basic limits of nanomechanical technologies, often with surprising results [3–5]. A similar evolution can be foreseen for the dynamical Casimir effect (DCE) [6, 7], for quantum computation and communication protocols.

The DCE concerns the generation of real photons from the vacuum due to time-dependent boundary conditions, for instance when the distance $d(t)$ between the two plates changes in time. The DCE is closely related [8] to other quantum vacuum amplification mechanisms such as the Unruh effect [9] and the Hawking radiation [10]. The DCE has been recently demonstrated experimentally in superconducting circuits [11, 12], in the framework of circuit quantum electrodynamics (circuit-QED) [13, 14].

In the paradigmatic model of a qubit-oscillator system, within the rotating wave approximation (RWA) the ground state is the product of the qubit’s ground state and the oscillator’s vacuum state. Starting from an initial state with both the qubit and the oscillator in their ground states, within the RWA the state remains unchanged even when the interaction is turned on. On the other hand, the inclusion of the interaction terms beyond RWA leads to squeezed or cat states containing virtual photons [15–17], which can be released as real photons under abrupt switching of the matter-field coupling [15, 18]. The RWA is a good approximations for a two-level atom (qubit) in a resonant cavity (oscillator), where the ratio between the frequency $\Omega$ of the Rabi oscillations between the two relevant states of the atom and the cavity frequency $\omega$ is typically $10^{-6}$ [19]. On the other hand, terms beyond the RWA cannot be neglected in circuit-QED experiments, where one can enter the so-called ultrastrong coupling regime, in which the two frequencies $\Omega$ and $\omega$ become comparable [20, 22]. Such regime is of great interest for quantum computation, as high-speed operations are needed to perform a large number of quantum gates within the decoherence time scale, as requested to operate fault tolerantly [23, 24].

In this paper we study the dynamics of a qubit-oscillator system, in the nonadiabatic regime in which the coupling is switched on and off suddenly. We find that, by varying the interaction strength and the interaction time, a great variety of exotic states of the field are generated, both with and without measurement of the final state of the qubit, even when in the initial state both the qubit and the oscillator are in their ground state. The non-classicality of such states is characterized by means of a negativity parameter based on the volume of the negative part of the Wigner function [25, 26]. We show that such parameter indicates a transition to non-classical states by varying either the interaction strength or the interaction time.

We stress the important differences between the problem considered here and the standard QED: (i) we consider a single mode rather than an infinite number of modes; (ii) the quantization volume (i.e., the volume of the cavity) is fixed and the limit of infinite volume is not taken at the end of the computations; (iii) while in
standard QED the interaction is usually switched on adiabatically to avoid transient phenomena, here we switch on and off the interaction abruptly and focus exactly on transient phenomena. Moreover, differently from cavity QED, where the quantization volume is usually larger than $\lambda^3$, with $\lambda$ wave length of the single mode, in circuit QED by means of microstrips one can achieve quantization volumes much smaller than $\lambda^3$. As a consequence, the ultrastochastic regime is accessible and the terms beyond the RWA cannot be neglected.

The interaction between a two-level system and a single mode of the quantized electromagnetic field is described by the time-dependent Hamiltonian ($\hbar = 1$) \[ H(t) = H_0 + H_I(t), \]

\[ H_0 = -\frac{1}{2} \omega_0 \sigma_z + \omega \left( a^\dagger a + \frac{1}{2} \right), \]

\[ H_I(t) = f(t) \left[ g \sigma_+ (a^\dagger + a) + g^* \sigma_- (a^\dagger + a) \right], \tag{1} \]

where $\sigma_i$ ($i = x, y, z$) are the Pauli matrices, $\sigma_\pm = \frac{1}{2} (\sigma_x + i \sigma_y)$ are the rising and lowering operators for the two-level system: $\sigma_+ |g\rangle = |e\rangle$, $\sigma_+ |e\rangle = 0$, $\sigma_- |g\rangle = 0$, $\sigma_- |e\rangle = |g\rangle$; the operators $a^\dagger$ and $a$ create and annihilate a photon: $a^\dagger |n\rangle = \sqrt{n+1} |n+1\rangle$, $a |n\rangle = \sqrt{n-1} |n-1\rangle$, $|n\rangle$ being the Fock state with $n$ photons. We assume sudden switch on/off of the coupling: $f(t) = 1$ for $0 \leq t \leq \tau$, $f(t) = 0$ otherwise. For simplicity’s sake, we consider the resonant case ($\omega = \omega_0$) and the coupling strength $g \in \mathbb{R}$. The RWA is obtained when we neglect the term $\sigma_+ a^\dagger$, which simultaneously excites the two-level system and creates a photon, and $\sigma_- a$, which de-excites the two-level system and annihilates a photon. In this limit, Hamiltonian (1) reduces to the Jaynes-Cummings Hamiltonian [27], with a switchable coupling. We set $\omega = 1$, so that in the RWA the interaction time needed to transfer an excitation from the qubit to the field or vice versa ($|e\rangle |0\rangle + |g\rangle |1\rangle$) is $\tau = \pi/2g$.

We consider as initial condition the state $|\Psi_0\rangle = |g\rangle |0\rangle$, i.e. the tensor product of the ground state of the two-level system and of the oscillator vacuum, so that within RWA such state is the ground state of the overall system and therefore its dynamical evolution must be ascribed to the terms beyond the RWA. We compute numerically the qubit-field state after the interaction time: $|\Psi(\tau)\rangle = c_g(\tau) |g\rangle |\phi_g(\tau)\rangle + c_e(\tau) |e\rangle |\phi_e(\tau)\rangle$, where $|\phi_g\rangle$ and $|\phi_e\rangle$ are normalized states of the field.

By changing the interaction strength $g$ and the interaction time $\tau$ we can generate a great variety of states of the field, both in the unconditional case and in the conditional case in which the final qubit state is measured, for instance in the $\{ |g\rangle, |e\rangle \}$ basis. In the first case, the field state reads $\rho = \text{Tr}_q |\Psi\rangle \langle \Psi | = c_g^2 |\phi_g\rangle \langle \phi_g | + c_e^2 |\phi_e\rangle \langle \phi_e |$, where $\text{Tr}_q$ denotes the trace over the qubit subsystem; in the latter case, we obtain the (pure) states $\rho_g = |\phi_g\rangle \langle \phi_g |$ or $\rho_e = |\phi_e\rangle \langle \phi_e |$. In Fig. 11 we show the Wigner function $W(x,p)$, with $x$ and $p$ position and momentum operators for the harmonic oscillator, at $g = 0.5$, $g = 1$, and $g = 1.5$ and for different values of $\tau$, both for the unconditional state $\rho$ and for the conditional states $\rho_g$ and $\rho_e$. Note that in all these instances, even at $g = 0.5$ (first row of Fig. 11), the field is not simply in a squeezed state, since there are negative components of the Wigner function. At largest interaction strengths ($g = 1.5$ in the second and $g = 1$ in the third and fourth rows of Fig. 11) we obtain exotic states of the field, very different from the squeezed states usually associated with the dynamical Casimir effect.

In Fig. 2 we show the populations $p_i (\rho = \sum_{i,j} \rho_{ij} |i\rangle \langle j |)$, and $p_i = \rho_{ii}$ of the states $\rho$, $\rho_g$ and $\rho_e$ for $g = 0.5$ and $g = 1.5$, with $\tau = \pi/2g$. An interesting feature of these plots is that for the conditional states $\rho_g$ and $\rho_e$ only the states with respectively an even and an odd number of photons are populated. This follows from the fact that Hamiltonian (11) conserves the parity $\Pi = \sigma_z (-1)^{a^\dagger a}$ of the excitations. Fig. 2 shows a significant population of states with a rather large number of photons, demonstrating the relevance of the DCE for the parameter values here considered.

The non-classicality of the generated field states is characterized by the negativity parameter \[ \delta = \int \int [W(x,p) - W(x,p)] \, dx \, dp. \tag{2} \]

Note that such parameter vanishes for positive-definite Wigner functions, as it is the case for squeezed states. Therefore, a non-zero negativity distinguishes the states of Fig. 11 from the usual squeezed states generated by parametric amplifiers [8]. As shown in Fig. 3 negativity can be increased by either increasing the coupling strength $g$ or the interaction time $\tau$. An interesting feature of this figure is that negativity becomes significantly different from zero only after a finite interaction time $\tau$. The stronger the coupling strength $g$, the smaller is the interaction time requested to obtain non-classical states.

As shown in Fig. 11 there are signatures of a sharp transition to non-classical states, $\delta > 0$, i.e. with negative components of the Wigner function, for instance at $\tau = \tau_c \approx 0.56 \pi$ for $g = 0.4$. A similar behavior is observed when the parameter $\delta$ is drawn as a function of $g$ for a given $\tau$. Note that, by decreasing $\tau$ in Fig. 4 the transition at $\tau = \tau_c$ leads to a drop in the parameter $\delta$ down to a value smaller than $10^{-15}$. Hence, for practical purposes at $g = 0.4$ the Wigner function can be considered as non-negative for $\tau < \tau_c$, even though we cannot exclude that other transitions to states with negligible negativity occur at smaller values of $\tau$.

The results of Figs. 3 and 4 can be interpreted in terms of the time-dependent perturbation theory, starting from the initial state $|g\rangle |0\rangle$ with both the qubit and the field in the ground state. The zeroth order of the Dyson series is in agreement with the RWA result, that
is no transitions occur to other states. The state $|e\rangle|1\rangle$ is coupled to $|g\rangle|0\rangle$ to first order. The Wigner function of the single-photon state $|1\rangle$ exhibits strong negativity. However, its weight in the Dyson series is not sufficient to overcome the positivity of the vacuum state $|0\rangle$. To second order, the state $|g\rangle|2\rangle$ is excited. Such states generates interference terms with $|g\rangle|0\rangle$, which are responsible of the appearance of negative components in the Wigner function. For $g = 0.4$, the second-order threshold to obtain $\delta > 0$ is $\tau = \tau_c^{(2)} \approx 0.60\pi$, while a fourth-order calculation gives $\tau_c^{(4)} \approx 0.58\pi$, in good agreement with the exact numerically computed result $\tau_c = 0.56\pi$.

To summarize, we have shown that a sudden change of the coupling constant in the ultrastrong coupling regime leads to the emergence of exotic states of the electromagnetic field, which cannot be described as squeezed or cat states. Such states are a manifestation of the dynamical Casimir effect, whose first detectable consequence is the emission of real photons. As analyzed in this work, these phenomena are present even if in the initial state both the qubit and the field are in the ground state.

It might appear surprising that, with such initial condition, one can measure final states with both the qubit and the field in an excited state (see the third row of Fig. 2). The energy for such excitations comes from the setup that allows for a fast switching of the interaction. For instance, when a two-level atom enters a cavity, it experiences a braking force and is slowed down (see Fig. 2). The missing part of the kinetic energy is used to generate excited states of the atom and of the cavity.

Since a nonadiabatic variation of the coupling constant
FIG. 2. Populations of the final states $\rho$ (first row), $\rho_g$ (second row), and $\rho_e$ (third row) of the field, for $\tau = \pi/2g$, $g = 0.5$ (left column) and $g = 1.5$ (right column).

FIG. 3. Non-classicality indicator $\delta$ as a function of the interaction strength $g$ and of the interaction time $\tau$ for the unconditional field state $\rho$.

in the ultrastrong coupling regime is a necessary condition to perform fast quantum gates, the DCE appears as a generic feature for high-speed quantum computation and communication protocols. As a result, photons are emitted and the fidelity of quantum gates \[30, 31\] or the capacity of quantum communication channels \[32\] deteriorate. Therefore, the dynamical Casimir effect can be the origin of a fundamental upper limit to the maximum speed of quantum computation or communication protocols.

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