From new states of matter to a unification of light and electrons

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Abstract

For a long time, people believe that all possible states of matter are described by Landau symmetry-breaking theory. Recently we find that string-net condensation provide a mechanism to produce states of matter beyond the symmetry-breaking description. The collective excitations of the string-net condensed states turn out to be our old friends, photons and electrons (and other gauge bosons and fermions). This suggests that our vacuum is a string-net condensed state. Light and electrons in our vacuum have a unified origin – string-net condensation.

1 Introduction

Quantum theory of condensed matter was dominated by two main themes. The first one is Fermi liquid theory.[1] The second theme is Landau symmetry-breaking theory.[2, 3] Fermi liquid theory is a perturbation theory around a particular type of ground states – the states obtained by filling single-particle energy levels. Fermi liquid theory has very wide applications. It describes metals, semiconductors, magnets, superconductors, etc. Landau symmetry-breaking theory provides a deep insight into phase and phase transition. It points out that the reason that different phases are different is because they have different symmetries. A phase transition is simply a transition that changes the symmetry. Not so long ago Landau symmetry-breaking theory is believed to describe all possible phases, such as crystal phases, ferromagnetic and anti-ferromagnetic phases, superfluid phases, etc., and all of the phase transitions between them.

Condensed matter theory is a very successful theory. It allows us to understand properties of almost all forms of matter. As a result, one starts

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to get a feeling of completeness, and a feeling of seeing the beginning of the end of the condensed matter theory. However, through the researches in last 20 years, a different picture starts to emerge. It appears that what we have seen is just the end of beginning. There is a whole new world ahead of us waiting to be explored.

A peek into the new world is offered by the discovery of fractional quantum Hall (FQH) effect.[4] Another peek is offered by the discovery of high $T_c$ superconductors.[5] Both phenomena are completely beyond the two themes mentioned above. Rapid and exciting developments in FQH effect and in high $T_c$ superconductivity resulted in many new ideas and new concepts. Looking back at those new developments, it becomes more and more clear that, in last 20 years, we were actually witnessing an emergence of a new theme in condensed matter physics. The new theme is associated with new states of matter and new class of materials. This is an exciting time for condensed matter physics. The new paradigm may even have an impact in our understanding of fundamental questions of nature.

Why FQH effect starts a new theme condensed matter physics? We know that there are many different FQH states, but they all have the same symmetry. So we cannot use Landau symmetry-breaking theory to describe different orders in FQH states. It was proposed that FQH states contain a new kind of order — topological order [6, 7]. Topological order is new since it has nothing to do with symmetry breaking, long range correlation, or local order parameters. None of the usual tools that we used to describe a phase applies to topological order. Despite this, topological order is not an empty concept since it can be described by a new set of tools, such as the number of degenerate ground states [8, 9], quasiparticle statistics [10, 11, 12], and edge states [13, 14].

Amazingly, FQH effect is not the first evidence that indicates the presence of the third new theme. The first sign showed itself 150 years ago even before the first two themes were introduced.

According to Landau symmetry-breaking theory, the gapless collective excitations in a state are determined by the symmetry-breaking order in the state. Those excitations correspond to various waves (the fluctuations of order parameter) in the state. If particles organize themselves into a lattice, the resulting crystal order breaks translation symmetry and leads to sound waves in crystals. The ferromagnetic order that breaks the spin rotation symmetry give rise to spin waves. In fact, all known waves are originated this way from symmetry breaking, except for the electromagnetic wave described by the Maxwell equation!

When Maxwell equation was first introduced, people firmly believed that
Figure 1: Maxwell’s ether: a mechanical model that might give rise to Maxwell equation and electromagnetic waves.

any wave must be correspond to motion of something. So people want to find out what is the origin of the Maxwell equation? The motion of what gives rise electromagnetic wave? Maxwell himself have tried to invent a mechanical model so that the electromagnetic wave correspond to the motion of the mechanical parts in the model (see Fig. 1).[15] However, Maxwell’s mechanical model does not really work. Furthermore, according to Landau symmetry-breaking theory, fluctuating order parameter can never produce waves that are described by Maxwell equation. Thus if we believe that Landau symmetry-breaking theory describe all possible states of matter (ie all possible organizations of particles), we will conclude that there is no way to produce electromagnetic wave by properly organizing particles.

However, after the discovery of the FQH states and the associated topological order, we find that Landau symmetry-breaking theory does not describe all possible organizations of particles. So there is still hope. The Maxwell equation may arise from a new kind of organizations of particles that are beyond Landau symmetry-breaking theory. Those new organizations of particles will correspond to new states of matter or a new class of materials. This motivates us to revisit the old question: what organization of particles can give rise to Maxwell equation? How to find those new states of matter? How to make those new materials?

In addition to the Maxwell equation, there is an even stranger equation, Dirac equation, that describes wave of electrons (and other fermions). Electrons have Fermi statistics. They are fundamentally different from the quanta of other familiar waves, such as photons and phonons, since those
quanta all have Bose statistics. To describe the many-electron wave, the amplitude of the wave must be anticommuting Grassman numbers, so that the wave quanta will have Fermi statistics. Since electrons are so strange, few people regard electrons and the electron waves as collective motions of something. People accept without questioning that electrons are fundamental particles, one of the building blocks of all that exist. However, from a condensed matter physics point of view, all low energy excitations are collective motion of something. If we try to regard photons as collective modes, why can’t we regard electrons as collective modes? So we can ask a similar question for electrons: what organization of particles can give rise to Dirac equation and Fermi statistics? Are Fermi statistics and gauge structure fundamental laws of nature, or are they emergent phenomena and a natural consequence of a particular organization of simple bosonic particles?

2 New states of matter from string-net condensations

A recent study provides an answer to the above questions. [16, 17, 18] We find that if particles form large strings and if those strings form a liquid state, then the collective motion of the such organized particles will correspond to waves described by Maxwell equation and Dirac equation. The strings in the string liquid are free to join and cross each other. As a result, the strings look more like a network (see Fig. 2). For this reason, the string liquid is actually a liquid of string-nets, which is called string-net condensed state.  

The particles that form the string-nets are bosons. So in the string-net picture, the Maxwell equation and Dirac equation emerge from local bosonic models. The electric field and the magnetic field in the Maxwell equation are called gauge fields. The string-
Figure 3: A hand-waving way to understand why fluctuations of condensed string-net have only two transverse modes. (a) A transverse motion of a string results in a new state and leads to a collective excitation. (b) A motion along the string does not result in any new states. Such a motion does not lead to any collective excitations.

But why the waving of string-nets produces waves described by the Maxwell equation? We know that the particles in a solid organized into a regular lattice pattern. The waving of such organized particles produces a compression wave and two transverse waves. The particles in a liquid have a more random organization. As a result, the waves in liquids lost two transverse modes and contain only a single compression mode. The particles in a string-net liquid also have a random organization, but in a different way. The particles first form string-nets and string-nets then form a random liquid state. Due to this different kind of randomness, the waves in string-net condensed state lost the compression mode and contain two transverse modes.

net liquids demonstrate how gauge fields emerge from local bosonic models. Many closely related earlier works led to such a picture of the Maxwell equation. String structures appear in the Wilson-loop characterization[19] of gauge theory. The Hamiltonian and the duality description of lattice gauge theory also reveal string structures.[20, 21, 22, 23]. Lattice gauge theories are not local bosonic models and the strings are unbreakable in lattice gauge theories. String-net theory points out that even breakable strings can give rise to gauge fields.[24] So we do not really need strings. Bosonic particles themselves are capable of generating gauge fields and the associated Maxwell equation. This phenomenon was discovered in several bosonic models[25, 26, 27, 28] before realizing their connection to the string-net liquids. The connection between ends of strings and Fermi statistics is a more recent realization. [29]
verse modes. Such a wave (having only two transverse modes) is exactly the electromagnetic wave described by the Maxwell equation.

To see how electrons appear from string-nets, we would like to point out that if we only want photons and no other particles, the strings must be closed strings with no ends. The fluctuations of closed strings produce only photons. If strings have open ends, those open ends can move around and just behave like independent particles. Those particles are not photons. In fact, the ends of strings are nothing but electrons.

How do we know that ends of strings behave like electrons? First, since the waving of string-nets is an electromagnetic wave, a deformation of string-nets correspond to an electromagnetic field. So we can study how an end of a string interacts with a deformation of string-nets. We find that such an interaction is just like the interaction between a charged electron and an electromagnetic field. Also electrons have a subtle but very important property — Fermi statistics, which is a property that exists only quantum theory. Without their Fermi statistics, the electrons in atoms would have a very different organization. All atoms would have very similar chemical properties and behave like a noble gas. Amazingly, the ends of strings reproduce this subtle quantum property of Fermi statistics. [29, 16] Actually, string-net liquids explain why Fermi statistics should exist.

We see that string-nets naturally explain both light and electrons. In other words, string-net theory provides a way to unify light and electrons. [17, 18] So, the fact that our vacuum contains both light and electrons may not be a mere accident. It may actually suggest that the vacuum is indeed a string-net liquid.

Here, we would like to point out that there are many different kinds of string-net liquids. The strings in those different liquids may have different numbers of types and may join in different ways. For some string-net liquids, the waving of the strings does not correspond to light and the ends of strings are not electrons. Only one kind of string-net liquids give rise to light and electrons. On the other hand, the fact that there are many different kinds of string-net liquids allows us to explain more than just light and electrons. We can find a particular type of string-net liquids which not only gives rise to electrons and photons, but also gives rise to quarks and gluons. [31, 16] The waving of such type of string-nets corresponds to photons (light) and gluons. The ends of different types of strings correspond to electrons and quarks. It might even be possible to design a string-net liquid that produces all elementary particles! In this case, the ether formed by such string-nets
can provide an origin of all elementary particles.

Because of so many different string-net condensed states, the string-net condensed states are much richer than symmetry-breaking states. So the new theory of string-net condensation may even be a richer theory than the Landau symmetry-breaking theory. The study of string-net condensed states and the associated new states of matter may become a new theme in theoretical condensed matter physics.

3 A rotor model with string-net condensation

To explain in more detail what is string-net condensation, let us consider a XXZ spin model on Kagome or pyrochlore lattice (see Fig. 4). The spins in the model carry an integer angular momentum and have only on-site and nearest-neighbor interactions:

\[ H = J \sum_i (S_i^z)^2 + J \sum_{\langle ij \rangle} S_i^z S_j^z + J_{xy} \sum_{\langle ij \rangle} (S_i^x S_j^x + S_i^y S_j^y) \]

To see why the ground state of the above XXZ model is a string-net condensed state, let us first rewrite \( H \) as

\[ H = \frac{J}{2} \sum_I Q_I^2 + J_{xy} \sum_{\langle ij \rangle} (S_i^x S_j^x + S_i^y S_j^y), \quad Q_I = \sum_{\text{star}} S_i^z \]

\(^3\)So far we can use string-net to produce almost all elementary particles, except for two: the \( SU(2) \) gauge boson that is responsible for the weak interaction and the graviton that is responsible for the gravity.
Here we have viewed the Kagome lattice as the links of the honeycomb lattice which is labeled by $I$ (see Fig. 5a). $\sum_{\text{star}}$ is the sum over the three spins next to a vertex $I$ of the honeycomb lattice and $\sum_I$ is the sum over all such vertices. When $J_{xy} = 0$, the model is exactly soluble. One of the ground state is the state with all $S^z_i = 0$ which has zero energy. We will call such a state as a no-string state. There are many other zero energy states. Those states can be constructed starting from the $S^z_i = 0$ state. We first draw a closed loop in the honeycomb lattice and then alternatively increase and decrease $S^z_i$ by 1 along the loop. Such a state is called a closed-string state. One can check that all closed-string states have zero energy and all other states have an energy at least $J_2$. So when $J_{xy}$ is small, the low energy states of the XXZ model are closed-string states. When $J_{xy} \neq 0$, the degeneracy between the closed-string states is lifted. In the small and negative $J_{xy}$ limit, the ground state is an equal amplitude superposition of all closed-string states. Such a state is called a string-net condensed state. It represents a new state of matter. In Ref. [30], it was shown that that the collective excitations above such a string-net condensed state behave like light and are described by Maxwell equation.

We would like to point out that the equal amplitude superposition of all closed-strings represents the simplest string-net condensed state. For such a string-net condensed state, the ends of strings have a Bose statistics. However, if we modify the $J_{xy}$ term in a certain way, the ground state of the rotor model will correspond to more complicated string-net condensation where the amplitudes of different string-net configuration in the ground state may have different signs. [18] For such a more complicated string-net condensed state, the ends of the strings may have a Fermi statistics.
From the simple XXZ model, we see that photons and electrons, just like phonons, can emerge as collective motions of a proper organized particles. Photon and electron do not have to be elementary particles, if our vacuum itself is a string-net condensed state.

4 Potential applications of string-net condensation

The materials described by Landau symmetry-breaking theory have had enormous impact on technology. Ferromagnetic materials that break spin rotation symmetry can be used as the media of digital information storage. A hard drive made by ferromagnetic materials can store so much information that books from whole library can be put in it. Liquid crystals that break rotation symmetry of molecules find wide application in display. Nowadays one can hardly find a household without liquid crystal display somewhere in it. Crystals that break translation symmetry lead to well defined electronic band which in turn allow us to make semiconducting devices. Semiconducting devices make the high tech revolution possible which changes the way we live. String-net condensed states are a new class of materials which are even richer than symmetry breaking states. After seeing so much impact of symmetry-breaking states, one cannot help to imagine the possible potential applications of the richer string-net condensed states.

One possible applications is to use string-net condensed states as media for quantum computing. String-net condensed state is a state with complicated quantum entanglement. As a many-body system, the quantum entanglement in string-net condensed state is distributed among many different particles/spins. As a result, the pattern of quantum entanglements cannot be destroyed by local perturbations. This significantly reduces the effect of decoherence. So if we use different quantum entanglements in string-net condensed state to encode quantum information, the information can last much longer.[32] The quantum information encoded by the string-net entanglements can also be manipulated by dragging the ends of strings around each others. This process realizes quantum computation.[33] So string-net condensed states are natural media for both quantum memory and quantum computation. Such realizations of quantum memory and quantum computation are fault tolerant.[34]
5 Possible experimental realizations of string-net condensations

Right now, we only know how to construct theoretical models that give rise to string-net condensations. The next step is to design realistic materials or to find realistic materials that have string-net condensations.

We have seen that the XXZ model is a simple model with string-net condensation which can give rise to artificial photons. But which experimental system can realize such a XXZ Hamiltonian? One possibility is to use ultra cold atoms confined in optical lattice (see Fig. 6) to realize the XXZ model. Usually, optical lattice is formed by the interference of several laser beams. But we can also use the interference of laser beams with holographic masks to produce more complicated lattices such as the Kagome lattice. The optic lattice has a very useful property that the confining potential of the optic lattice may depend on the atomic spins. By tuning the laser frequency strength, we can tune such a spin dependent confining potential. This allows us to tune the spin-spin interaction between atoms confined in the optical lattice [35] to make the XXZ Hamiltonian.

Frustrated spin systems on Kagome lattice and pyrochlore lattice appear quite commonly in nature. So another possibility is to find the string-net condensed states in those frustrated spin systems. However, the spin Hamiltonians of those systems may not have a form of the XXZ model discussed above. But this does not mean that the string-net condensed states cannot exist in those frustrated systems. String-net condensed states are stable robust phases which exist for a finite range of parameters in the Hamiltonian. The main difficulty is that those more general Hamiltonians are hard to solve. Even if a Hamiltonian supports a string-net condensed ground state, we may not know it. So it is important to develop some simple mean-field methods for string-net condensed states. The mean-field theory will allow us to estimate whether a Hamiltonian has string-net condensation or not.

Figure 6: An optic lattice formed by the interference of several laser beams.
This will allow us to determine from the measured spin interaction if a frustrated spin system has string-net condensations or not. We hope this line of research will result in a list of materials which are likely to have string-net condensations. Experimentalists can then study those materials in detail to check if string-net condensations really do exist in those materials or not.

The third type of promising systems is the Josephson junction arrays.[36] The charge on a superconducting island or the flux through a superconducting ring may play the role of spin degree freedom. Josephson junction array is quite tunable. Recent progress in quantum computing has find ways to reduce the decoherence.[37, 38, 39] So building a Josephson junction array to realized the quantum XXZ model may be possible.

6 Summary

To summarize, string-net liquid represents a different way to understand the deep structure of matter, where the elementary particles are not regarded as the building blocks of everything but as an emergent phenomenon from a deeper structure of our non-empty vacuum. String-net liquid provides an unified origin for almost all the elementary particles. In other words, if we say let there be string-net liquid, we will get almost everything.4 This is not just a fun thing to say. In principle, we can realize string-net liquids in certain materials which will allow us to make artificial elementary particles. So we can actually create an artificial vacuum, and an artificial world for that matter, by making a string-net liquid. This would be a fun experiment to do!

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4We know that superstring theory is a potential theory of everything. One may want to ask what is the difference between the string-net-liquid approach and the superstring approach? Our understanding of the superstring theory has been evolving. According to an early understanding of the superstring theory, all the elementary particles correspond to small segments of superstrings. Different vibration modes of a small superstring result in different types of elementary particles. This point of view is very different from that of the string-net liquid. According to the string-net picture, everything comes from simple bosonic particle. The bosons stick together which from string-nets that fill the whole space. The strings can be as long as the size of universe. Light (photons) correspond to the collective motion of the large string-nets and an electron corresponds to a single end of string. A modern understanding of the superstring theory is still under development. According to Witten, one of the most important questions in superstring theory is to understand what is superstring. [40] So at this time, it is impossible to compare the modern understanding of the superstring theory with the string-net theory. In particular it not clear if the superstring theory can be viewed as a local bosonic model. The string-net theory is fundamentally a local bosonic model.
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