Impacts of non-homogeneous nanofluid approach and orientation angle on convection heat transfer within 3D wavy cavity

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Abstract. Steady laminar natural convection and heat transfer inside a tilted 3D wavy cavity filled by water-Al\textsubscript{2}O\textsubscript{3} nanofluid does examined numerically by applying the finite element technique. An isothermal curved heater is located at the right surface of the chamber while the left surface remains in a steady cold temperature. The top and bottom horizontal surfaces are held adiabatic. The governing equations are formulated in the non-dimensional model using the two-component non-homogeneous equilibrium model through transporting phenomena in nanofluids connecting among the influences of Brownian diffusion and thermophoresis. The results of the dimensionless buoyancy-driven parameter, inclination angle, nanoparticle volume fraction and number of undulations were examined. It is shown that the inclination angle, the wavy shape of the surface plus the features of nanofluid are outstanding control parameters to the rate of the heat transfer and the distribution of the fluid flow.

1. Introduction

The significance of natural convection including heat transfer within enclosed mediums own recognizing in many engineering applications toward its diverse applications in solar thermal collectors, ventilation, cooling about electronics and buildings, containment building cooling, exchange of heat, storage tanks, heat exchangers, and dual-pane windows, and various others [1]. A numerical investigation on the natural convection heat transfer in a differentially heated 3D cubical cavity was explained by Fusegi et al. [2]. Wang et al. [3] investigated the problem of 3D natural convection heat transfer numerically in a cubical cavity with high Rayleigh numbers. They concluded that the flow motion turned to be fully turbulent when the Rayleigh number was approached to $10^9$. Gibanov and Sheremet [4] examined the effect of various heat source configurations on the steady natural convection heat transfer into a 3-D cubical hollow employing the finite difference method. They discovered that the addition of Rayleigh number points to the heat transfer augmentation concerning the trapezoidal chamber. Very recently, [5] demonstrated the unsteady natural convection and heat transfer within a 3-D cubical cavity with the presence of trapezoidal heater. They concluded that the increase of the heater length led to an apparent reduction in the average Nusselt number.
Within the field of thermal science, the heat transfer enhancement is the most critical process that engineers are widely attempting for, due to the enormous interests in industry for controlling such procedures [6]. Several concepts have investigated the area of enhancing the heat transfer, an essential technique and innovative idea toward this domain is combining nanometer-sized particles into the core fluid; as a result, the heat transfer is improved. Adding those nanometer-sized solid particles within the variety of sizes 10-50nm into the base liquid is named nanofluid. Nanofluid regards to active liquid that holds suspended nanoparticles sized smaller than 100nm concerning conventional heat transfer, such as, water, ethylene glycol, and oil [7]. Khanafer et al. [8] described an excellent comprehensive investigation on the natural convection heat transfer in a 2D square cavity that partially occupied by nanofluids. Applying the finite volume method, Al-Rashed et al. [9] described the effect of an external magnetic field on natural convection heat transfer in a cubical cavity filled with nanofluid (carbon nanotube-water).

Another concept that used for improving the thermal transmission is the corrugated borders of the cavity. Such a topic is widely investigated by many researchers in many engineering applications, such as nuclear component, micro-electronic devices, electrical and solar collectors, etc. [7,10]. A recent work on the free convection and heat transfer inside a wavy hollow that loaded with water-Al$_2$O$_3$ nanofluid and having an inner solid block was reported by Hashim et al. [11]. They noticed by using the two-phase nanofluid model that the convection heat transfer in the wavy cavity is improved by adding nanoparticles as well as the number of oscillations.

According to the given literature and the best of our knowledge, there are no studies on the laminar free convection inside an inclined 3D wavy chamber filled by water-Al$_2$O$_3$ nanofluid. Thus, the present numerical work aims to study the impacts of non-homogeneous nanofluid approach and orientation angle toward convective heat transfer within a 3D wavy chamber. Indeed, this sort of complex geometry performs a primary role in numerous engineering applications, such as solar energy (solar thermal collector), cooling of electronic devices, heat exchangers, and many others.

2. Methodology

2.1 Mathematical Analysis

The steady three-dimensional (3D) natural convection inside a wavy-walled chamber with length $L$, width $L/2$ and thickness $L/2$, as described in Fig. 1. The right wavy surface is taking the following form $1 - A(1 - \cos(2N \pi V))$. An isothermal curved heater is located at the right surface of the chamber while the left surface remains in a steady cold condition. Those top and bottom horizontal surfaces are taken adiabatic. The chamber borders remain impermeable; the space inside the wavy region is loaded with water-Al$_2$O$_3$ nanofluid. The Boussinesq approximation holds applicable. Through the attention of those hypotheses, continuity, momentum and energy equations of the convection flow can be addressed in the dimensionless scheme as [7]:

2.1.1 Governing Equations

\[ \nabla \cdot \mathbf{V} = 0, \]
\[ \mathbf{V} \cdot \nabla \mathbf{V} = -\nabla P + \frac{\rho_f}{\rho_{nf}} \frac{\mu_f}{\mu_{nf}} \nabla^2 \mathbf{V} + \frac{(\rho_f \beta_f)_{nf}}{\rho_{nf} \beta_f} \frac{1}{Pr} \text{Ra} \cdot \theta, \]
where $\textbf{V}$ explains the non-dimensional velocity vector $(U, V, W)$.

The governing equations of Eqs. (1)-(4) are moulded into dimensionless forms by employing the following non-dimensional variables:

\begin{align*}
X, Y, Z & = \frac{x, y, z}{L}, \quad \textbf{V} = \frac{\textbf{u} L}{v_f}, \quad P = \frac{\rho L^2}{\rho_0 v_f^2}, \quad \phi^* = \frac{\phi}{\phi}, \\
D_b^* & = \frac{D_b}{D_b^0}, \quad D_r^* = \frac{D_r}{D_r^0}, \quad \delta = \frac{T_c - T_h}{T_h - T_c}, \quad \theta = \frac{T - T_h}{T_h - T_c}.
\end{align*}

Figure 1. Physical design regarding convection inside a tilted wavy chamber (a) 2D-xy-plane including the coordinate system and (b) 3D cavity.

2.1.2 Boundary Conditions

The corresponding conditions concerning the dimensionless boundary for Eqs. (1)-(4) do provided by:

\begin{align*}
U = V = 0, \quad \text{toward all the surfaces of the cavity.} \quad (6)
\end{align*}

On the right hot surface :

\begin{align*}
\textbf{V} = 0, \quad \frac{\partial \phi^*}{\partial \textbf{V}} = -\frac{D_r^*}{D_b^*} \cdot \frac{1}{N_{br}} \cdot \frac{1}{1 + \delta \theta} \frac{\partial \theta}{\partial \textbf{V}}, \quad \theta = 1, \quad (7)
\end{align*}
On the left cold surface:
\( \mathbf{V} = 0, \quad \frac{\partial \phi}{\partial \mathbf{V}} = -\frac{D^*}{D_b} \frac{1}{N_{BT}} \frac{1}{1 + \partial \theta / \partial \mathbf{V}}, \quad \theta = 0, \) \hspace{1cm} (8)

On the the adiabatic top surface:
\( \mathbf{V} = 0, \quad \frac{\partial \phi}{\partial \mathbf{V}} = 0, \quad \frac{\partial \theta}{\partial \mathbf{V}} = 0, \) \hspace{1cm} (9)

On the the adiabatic bottom surface:
\( \mathbf{V} = 0, \quad \frac{\partial \phi}{\partial \mathbf{V}} = 0, \quad \frac{\partial \theta}{\partial \mathbf{V}} = 0, \) \hspace{1cm} (10)

The used thermophysical properties concerning the nanofluid are: heat capacitance \((\rho C_p)_{nf}\), effective thermal diffusivity \((\alpha_{nf})\), effective density \((\rho_{nf})\), the thermal expansion coefficient \((\beta_{nf})\), thermal conductivity ratio \((\frac{k_{nf}}{k_f})\) and dynamic viscosity ratio \((\frac{\mu_{nf}}{\mu_f})\) which can be explained as the following [19, 15]:

\[
(\rho C_p)_{nf} = (1-\phi)(\rho C_p) + \phi(\rho C_p),
\]

\[
\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}},
\]

\[
\rho_{nf} = (1-\phi)\rho + \phi\rho,
\]

\[
(\rho\beta)_{nf} = (1-\phi)(\rho\beta) + \phi(\rho\beta),
\]

\[
\frac{k_{nf}}{k_f} = 1 + 4.4\text{Re}^0.4\text{Pr}^{0.66}\left(\frac{T}{T_f}\right)^{10}\left(\frac{k_p}{k_f}\right)^{0.03}\phi^{0.66},
\]

\[
\frac{\mu_{nf}}{\mu_f} = 1 \left(1 - 34.87 \left(\frac{d_p}{d_f}\right)^{0.3} \phi^{1.03}\right).
\]

The local Nusselt number on the right wavy surface may now be defined as follows:

\[
Nu_{nf} = \frac{k_{nf}}{k_f} \left(\frac{\partial \theta}{\partial \mathbf{V}}\right)_{\theta = 0}.
\]

Lastly, the average Nusselt number evaluated on the right wavy surface and it is provided by:

\[
\overline{Nu}_{nf} = \int_0^1 Nu_{nf} d\mathbf{V}.
\]

2.2 Simulation Method and Validation

The dimensionless governing equations of Eqs. (1)–(4) controlled by the adopted boundary conditions of Eqs. (6)–(10) are determined by the Galerkin weighted and finite element techniques. The computational region does discretized towards small triangular components, as outlined by Fig. 2. Triangular Lagrange finite elements about several forms are employed to each concerning the flow variables inside the computational area. Residuals regarding to any conservation equation hold achieved through replacing the approximations toward the dimensionless governing equations. For explaining the nonlinear expressions into equations of momentum, the algorithm of Newton-Raphson iteration is
applied. The convergence concerning the solution does appropriate if the relative error of each variable meets the following convergence criteria:

$$\frac{\Gamma^{i+1} - \Gamma^i}{\Gamma^{i+1}} \leq \eta,$$

where $i$ denotes the iteration number and $\eta$ stands for the convergence criterion. In this numerical work, the convergence criterion was set at $\eta = 10^{-6}$.

Toward verifying the developed computational code, Fig. 3 explains a comparison of the used nanofluid models for the most pair influenced properties (thermal conductivity and the dynamic viscosity) between the present results and the experimental data of Chon et al. [12] and Ho et al. [13] and numerical one provided by Corcione et al. [14].

**Figure 2.** Grid-points distribution for (a) two-dimensional $XY$-plane and (b) three-dimensional cavity.

**Figure 3.** Validations concerning (a) thermal conductivity ratio of the present work among Chon et al. [12] and Corcione et al. [14] and (b) dynamic viscosity ratio by Ho et al. [13], Corcione et al. [14] and the present work.
3. Results and Discussion

This section of the work explains computational data of the stream function (streamlines), temperature profile (isotherms) and nanoparticles concentration for the inclination angle (\(0 \leq \Omega \leq 120^\circ\)) and undulations number (\(0 \leq N \leq 4\)). The inclination angle in this study is applied to the considered cavity not to the flow only for indicating the clear effects of the orientation angle. Effects of other parameters, such as the dimensionless buoyancy-driven parameter (Rayleigh number) (\(10^4 \leq Ra \leq 10^7\)) and nanoparticle volume fraction (\(0 \leq \phi \leq 0.04\)) are presented. The thermophysical characteristics of water and solid Al\(_2\)O\(_3\) phases are described in Table 1.

| Physical properties | Fluid phase (water) | Al\(_2\)O\(_3\) |
|---------------------|---------------------|------------------|
| \(C_p\), (J/kg K)   | 4178                | 765              |
| \(\rho\), (kg/m\(^3\)) | 993                | 3970             |
| \(k\), (W m\(^{-1}\) K\(^{-1}\)) | 0.628              | 40               |
| \(\beta \times 10^5\), (1/K) | 36.2              | 0.85             |
| \(\mu \times 10^6\), (kg/ms) | 695               | -                |
| \(d_p\), (nm)      | 0.385              | 33               |

Figure 4 describes the 3D plot of differences regarding the flow, temperature field, and distribution of nanoparticles for several orientation angle at \(Ra = 10^6\), \(\phi = 0.02\) and \(N = 3\). Generally, the warm fluid close to the wavy surface produces two streamlines cells with the absence of the orientation angle (\(\Omega = 0\)). Affected by the buoyancy forces, which is resulting in the imposed temperature gradient, streamlines cells appear in the anticlockwise direction, as clearly shown in Figure 4. Isotherms are highly destroyed within the middle of the cavity and next to the wavy wall, while vertical shape dominants on the isotherms patterns. Nanoparticles concentration show a strong behaviour on the top of the cavity due to the buoyancy forces. Imposing a low value of \(\Omega\) tends to influence the fluid flow and temperature distribution. The second cell of the streamlines vanishes, and the flow circulation reduces. The isotherms patterns less destroyed within the centre of the cavity.

Figure 5 shows the 2D plot of differences regarding the flow, temperature field, and distribution of nanoparticles for several orientation angle at \(Ra = 10^6\), \(\phi = 0.02\) and \(N = 3\). The anti-clockwise vortex (positive values of \(\Psi\)) of the streamlines takes place within the middle of the wavy chamber (please see the value of the streamlines colour bar). The strength of the flow circulation decreases with an increment of \(\Omega\). Rising the orientation angle up to \(90^\circ\) leads to a strong influenced on the flow and temperature field. Multiple streamlines cells (anti-clockwise and clockwise vortexes) appear next to the wavy surface, which indicated the changing of the heating system. The heating system, in this case, show a hot temperature from the top wavy surface and a cold one from the bottom surface. Due to that, the isotherms patterns occur with almost horizontal lines except the area next to the wavy surface. The concentration of nanoparticles show a clear propagation within the cavity which evaluated by the orientation angle (\(\Omega\)) for \(Ra = 10^6\), \(\phi = 0.02\) and \(N = 3\). Indicate the influence of the Brownian motion of thermophoresis. At the high orientation angle (\(\Omega = 120^\circ\)), the direction of the fluid heat flow changes to the clockwise. The stream function takes a form of single clockwise vortex at the middle of the wavy chamber. The strong concentration of nanoparticles observed at the lower segment of the wavy.

Figures 6 and 7 explain variations of 3D and 2D plots of the streamlines, isotherms, and nanoparticle distribution for various undulations number (\(N\)) at \(Ra = 10^6\), \(\Omega = 30^\circ\) and \(\phi = 0.02\). For vertical flat surfaces (\(N = 0\)), the flow motions appear with a singular anti-clockwise vortex that drags diagonally within the centre of the chamber. Isotherms patterns observed with low distortion and almost a horizontal
shape with the middle of the cavity. Weak concentration of nanoparticles is noted within the distribution evaluated by the orientation angle ($\Omega$) for $Ra = 10^6$, $\phi = 0.02$ and $N = 3$ in $XY$-plane. With the considered cavity with very low propagation. Applying number of undulations on the right surface enhances the flow motions temperature distributions and nanoparticles distribution.

**Figure 4.** Variations of the 3D plot of (left) streamlines, (middle) isotherms, and (right) nanoparticle distribution evaluated by the number of undulations ($N$) for $Ra = 10^6$, $\Omega = 30^\circ$ and $\phi = 0.02$. 

The anti-clockwise vortex of the streamlines shrinks vertically due to the narrow space inside the cavity. A substantial enhancement is recognized toward the force of the flow circulation by the increasing of $N$; please see the values of the streamlines color bar in Figure 7. The isotherms patterns are highly destroyed, especially nearby to the wavy surface influenced by the intense thermal gradient. Increasing the number of undulations is to enhance the concentration of nanoparticles. Strong concentration of nanoparticles was indicated at the top of the cavity and near to the wavy surface. Besides, a high propagation of nanoparticles concentration is noted within the cavity. The strength of the nanoparticles increases from 0.035 to 0.054, with an increment of the number of undulations, as shown in Figures 7(c) and 7(d). The strength of the flow circulation shows a continuous enhancement with the rising of $N$ which affected by the nanofluid intensity, and as a result, the fluid flow circulation enhances.

**Figure 5.** Variations of the 2D plot of (left) streamlines, (middle) isotherms, and (right) nanoparticle distribution evaluated by the orientation angle ($\Omega$) for $Ra = 10^6$, $\phi = 0.02$ and $N = 3$ in $XY$-plane.
Figure 8 demonstrates differences of the local Nusselt number ($N_{\text{uf}}$) through the wavy surface ($S$) for diverse (a) $\Omega$ and (b) $N$ at $Ra = 10^6$ and $\phi = 0.02$. The increasing of the orientation angle is enhanced the local heat transfer, as presented in Fig. 8(a). The strongest enhancement on the heat transfer is noted on the lower and upper corners of the wavy surface due to the large temperature gradient. At the lower corner from the curved wall, lower and higher amounts of $\Omega$ ($0$ and $120^\circ$) display a substantial augmentation in the local heat transfer. While the growth of the orientation angle up to $60^\circ$ reveals the rising on the local heat transfer at the upper corner of the wavy surface. Figure 8(b) shows a robust growth of the local heat transfer with the rise of the number of undulations. This due to the growth of the temperature gradient, which indications in the maximizing local heat transfer.

Figure 6. Variations of the 3D plot of (left) streamlines, (middle) isotherms, and (right) nanoparticle distribution evaluated by undulations number ($N$) for $Ra = 10^6$, $\Omega = 30^\circ$ and $\phi = 0.02$. 
Figure 7. Varieties of the 3D plot of (left) streamlines, (middle) isotherms, and (right) nanoparticle distribution evaluated by undulations number (N) for $Ra = 10^6$, $\Omega = 30^\circ$ and $\phi = 0.02$ in $XY$-plane.

Figure 9(a) clarifies the enhancement of the heat transfer among the increasing of the dimensionless buoyancy-driven parameter (Rayleigh number). The is because of the increasing of the thermal gradient following the rising of Rayleigh number. Increasing the number of undulations tends to increase the drag effect on the wavy segments, which raises the energy plus as a result; the rate of the heat transfer increments. This enhancement is distinctly obtained in Figure 9(b). The average Nusselt number seems with an improvement by sinusoidal plots with the increasing of the number of undulations. Also, an optimal enhancement is noted on the average Nusselt number with the increasing of the nanoparticle volume fraction ($\phi$) up to 0.03.
Figure 8. Differences of the local Nusselt number relates to the wavy surface ($S$) for different (a) $\Omega$ and (b) $N$ at $Ra = 10^6$ and $\phi = 0.02$.

Figure 10 reports the variations of the heat transfer rate interface with the orientation angle for different (a) nanoparticle volume fraction and (b) number of undulations at fixed Rayleigh number ($Ra = 10^6$). The average Nusselt number shows a reducing function with the increasing of the orientation angle up to $90^\circ$. However, rising $\Omega$ more than $90^\circ$ indicates an enhancement on the thermal gradient furthermore as a result; convective heat transfer raises. The convection heat transfer growths by the extension of $\phi$ due to the higher thermal conductivity of the nanoparticles. Additionally, the rise in the number of undulations revels a substantial enhancement on the overall heat transfer which observed with the ascension of the orientation angle more than $90^\circ$, as clearly shown in Figure 10(b).

Figure 9. Differences of $\bar{Nu}_{hf}$ with (a) $Ra$ and for various $N$, and (b) $N$ for various $\phi$ at $\Omega = 30^\circ$. 
4. Conclusions

The present study considered the steady laminar natural convection and heat transfer inside a tilted 3D wavy chamber packed with water-Al₂O₃ nanofluid. The governing equations are written in the dimensionless form by employing the two-component non-homogeneous approach toward conveying phenomena in nanofluids and solved numerically using the finite element method. It is determined that the intensity concerning the streamlines, isotherm patterns and nanoparticles distribution boosted with an increment of the number of undulations. The strongest enhancement on the local heat transfer is noted on the lower and upper corners of the wavy surface due to the large temperature gradient. The average Nusselt number appears with a clear reduction with the increasing of the orientation angle up to 90°. However, rising Ω more than 90° indicates an augmentation on the convection heat transfer. The overall heat transfer is increased by the addition of the nanoparticle volume fraction up to 0.03 under the influence of the number of undulations. Furthermore, it is shown that the inclination angle, the wavy shape of the surface including the features of nanofluid are outstanding control parameters for the rate of the heat transfer plus the distribution of the fluid flow.

Acknowledgments

The work was supported by the Universiti Kebangsaan Malaysia (UKM) research grant DIP-2017-010.

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