One-loop four-graviton amplitude in eleven-dimensional supergravity

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Abstract

We find explicit expression for the one-loop four-graviton amplitude in eleven-dimensional supergravity compactified on a circle. Represented in terms of the string coupling (proportional to the compactification radius) it takes the form of an infinite sum of perturbative string loop corrections. We also compute the amplitude in the case of compactification on a 2-torus, which is given by an $SL(2, \mathbb{Z})$ invariant expansion in powers of the torus area. We discuss the structure of quantum corrections in eleven-dimensional theory and their relation to string theory.

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1. Introduction

Recent suggestions indicate that $D = 11$ supergravity is a low-energy effective field theory of a more fundamental M-theory [1,2] (for reviews see [3,4,5]). One expects that various properties of ten-dimensional string theories may be understood from eleven-dimensional perspective.

Most of known relations between type IIA string theory and M-theory, viewed as its strong-coupling limit, are restricted to BPS states. A surprising recent observation [6] is that the \textit{tree-level} type II string correction $\zeta(3)\alpha'^3 R^4$ [7,8] can be interpreted as originating from a \textit{one-loop} $D = 11$ supergravity contribution. Our aim below will be to compute the one-loop four-graviton amplitude in $D = 11$ supergravity compactified on a circle and to demonstrate that it has the structure of an infinite sum of perturbative higher-loop string corrections. This suggests that the one-loop quantum $D = 11$ theory (with a proper UV cutoff implied by string theory) may contain information about certain string corrections to all orders in string coupling.

The reason why the $D = 11$ amplitude has this form may be understood as follows. The one-loop contribution to the effective action of $D = 11$ supergravity compactified on a circle of radius $R_{11}$ can be represented as the one-loop correction in type IIA $D = 10$ supergravity plus an infinite sum of one-loop contributions of massive Kaluza-Klein modes (0-brane supermultiplets). That sum may be represented as a local series using inverse mass expansion, $\sum M^{-2n}C_n$. Since the masses of Kaluza-Klein modes are proportional to inverse string coupling [2], $M \sim R_{11}^{-1} \sim g_s^{-1}$, the contribution of Kaluza-Klein modes has the structure of a sum of perturbative higher-loop closed string corrections, $\sum_n g_s^{2n}C'_n$. This suggests that some perturbative string theory results may be reproduced in the ‘dual’ formulation of the theory, in which certain solitons (0-branes) play a central role.

The scattering amplitude computed below corresponds to external gravitons with vanishing values of the 11-th component of momentum $p_{11}$. Using $D = 11$ Lorentz invariance it is, in principle, straightforward to generalise the final expression for the amplitude to the case when external momenta are arbitrary, subject only to the zero-mass on-shell condition in $D = 11$. The resulting amplitude with $p_{11} = \text{fixed}$ may then be interpreted as a one-loop correction to the scattering of 0-branes in $D = 10$ and may be of interest from the point of view of testing Matrix theory [9]. In particular, one should be able to analyse the one-loop $D = 11$ supergravity contribution to the phase shift, which was previously obtained only in a semiclassical (eikonal) approximation (see [10] and refs. there).

In Section 2 we shall make some general remarks on cutoff dependence of the $D = 11$ supergravity effective action, suggesting that certain curvature invariants should play a special role in both $D = 11$ and $D = 10$ theories. The one-loop four-graviton amplitude in $D = 11$ supergravity on a circle will be computed explicitly in Section 3.1. The amplitude in the supergravity compactified on a 2-torus will be found in Section 3.2. In Section 4 we shall discuss possible relation of these amplitudes to perturbative and non-perturbative contributions in string theory.
2. Higher order corrections in $D = 11$ theory and their relation to string theory

Let us start with some comments on the structure of higher-loop terms in low-energy $D = 11$ supergravity effective action and their relation to string theory. We shall consider the $D = 11$ theory compactified on a circle of radius $R_{11}$ with the action

$$S = -\frac{1}{2\kappa^2_{11}} \int d^{11}x \sqrt{-g} \, \mathcal{R} + \ldots , \quad \kappa^2_{11} = 16\pi^5 l_{11}^9 ,$$

(2.1)

where $l_{11}$ is the $D = 11$ Planck scale. The two parameters of the compactified $D = 11$ theory $R_{11}$ and $\kappa_{11}$ are related to the string scale $l_{10} = \sqrt{\alpha'}$ and the string coupling $g_s$ (defined as a ratio of the fundamental string and D-string tensions) by

$$l_{11} = (2\pi g_s)^{1/3} \sqrt{\alpha'} , \quad R_{11} = g_s \sqrt{\alpha'} , \quad \kappa_{10}^2 = \frac{\kappa^2_{11}}{2\pi R_{11}} = 64\pi^7 g_s^2 \alpha'^4 ,$$

(2.2)

$$\alpha' = \frac{l_{11}^3}{2\pi R_{11}} , \quad g_s^2 = \frac{2\pi R_{11}^3}{l_{11}^3} .$$

The $D = 11$ supergravity is UV divergent, so one needs to introduce a cutoff $\Lambda_{11}$. Since the $D = 11$ and $D = 10$ supergravities are related by dimensional reduction, $\Lambda_{11}$ should be proportional to a cutoff $\Lambda_{10}$ in type IIA $D = 10$ supergravity. The two cutoffs may be related, e.g., by comparing the divergent terms in the one-loop effective actions in $D = 11$ and $D = 10$ supergravities. The $D = 10$ supergravity is a low-energy limit of type IIA string theory, so its effective cutoff is $\Lambda_{10} \sim \frac{1}{\sqrt{2\Lambda_{11}}}$. Expressed in terms of the (proper-time) cutoff $\Lambda_{10}$, the cutoff $\Lambda_{11}$ is given by

$$R_{11} \Lambda_{11}^3 = a \Lambda_{10}^2 , \quad \Lambda_{10} = \frac{1}{2\pi \alpha'} ,$$

(2.3)

where $a$ is a numerical constant. Eq. (2.2) implies that

$$\Lambda_{11} = a^{1/3} l_{11}^{-1} \sim \kappa_{11}^{-2/9} ,$$

(2.4)

i.e. that $\Lambda_{11}$ depends only on $\kappa_{11}$ and not on $R_{11}$. This has a natural ‘membrane-theory’ interpretation: just as the $D = 10$ cutoff $\Lambda_{10}$ is proportional to the square root of the string tension $T_1 = \frac{1}{2\pi \alpha'} = \frac{1}{2\pi l_{10}^2}$, the $D = 11$ cutoff $\Lambda_{11}$ is proportional to the cubic root of the membrane tension

$$\Lambda_{11} = (2\pi a T_2)^{1/3} , \quad T_2 = \frac{1}{2\pi l_{11}^3} = \frac{1}{4\pi^2 g_s \alpha'^{3/2}} .$$

(2.5)

The general structure of the cutoff-dependent part of the effective action of $D = 11$ supergravity at the $L$-loop level is

$$S_{L,\infty} = \kappa^2_{11}(L-1) \sum \Lambda_{11}^n (\ln \Lambda_{11})^l \int d^{11}x \sqrt{-g} \, \mathcal{R}^m ,$$

(2.6)
where $\mathcal{R}^m$ stands for all possible scalars built out of curvature and its covariant derivatives which have length dimension $-2m$. On dimensional grounds,

$$n + 2m = 9(L - 1) + 11.$$  \hspace{1cm} (2.7)

Note that purely logarithmic divergences ($n = 0$) may appear only at even loop orders and have $m = 10, 19, ...$

At the one-loop order, the leading $\mathcal{R}^m$ ($m = 0, 1, 2, 3$) divergences cancel out \[14\], so that

$$S_{1\infty} \propto \int d^{11}x \sqrt{-g} \Lambda_{11}^3 \mathcal{R}^4.$$  \hspace{1cm} (2.8)

The presence of the cubic $\mathcal{R}^4$ divergence in $D = 11$ supergravity is implied \[14\] by the presence of quadratic $\mathcal{R}^4$ divergence in the $D = 10$ supergravity, which, in turn, can be found as the $\alpha' \to 0$ limit \[15,16\] of the one-loop string-theory contribution $\frac{1}{\alpha'} \int d^{10}x \sqrt{-g} \mathcal{R}^4$ \[17,18\].

Eq. (2.8) may, in principle, contain also a linear divergence $\Lambda_{11} \mathcal{R}^5$ which would correspond to the logarithmic divergence in $D = 10$ supergravity or to a finite one-loop term $\ln \alpha' \mathcal{R}^5$ in the string theory effective action. Such $\mathcal{R}^5$ terms should be built out of five powers of the curvature: terms like $\nabla^2 \mathcal{R}^4$ are absent since the string theory four-graviton amplitude does not contain the corresponding (momentum)\(^{10}\) term \[16\].

An uncompactified $D = 11$ M-theory (having $D = 11$ supergravity as its low-energy approximation) is suggested to be a strong-coupling limit of type IIA string theory \[3\]. Let us suppose that there are special terms $f(g_s)\mathcal{R}^m$ in the string theory effective action which do not receive corrections beyond certain order $L$ in string loop expansion. Then their coefficients will have simple power-like (or ‘perturbative’) dependence on $g_s$ in the limit of $g_s \gg 1$, i.e. $f(g_s) \sim g_s^{2(L-1)}$. Such terms must then have a natural $D = 11$ theory interpretation. Using this logic, one may be able to obtain certain constraints on possible terms in the effective action of M-theory. As we will argue below, such special terms in the string-theory action may correspond to covariant $\mathcal{R}^m$ terms in the uncompactified $D = 11$ theory only if $m = 3k + 1$, $k = 0, 1, 2, ...$

Using (2.2), (2.4)

$$\int d^{11}x \to 2\pi R_{11} \int d^{10}x , \quad \kappa_{11}^2 \sim g_s^3 , \quad R_{11} \sim g_s \quad \Lambda_{11} \sim g_s^{-1/3} ,$$

and $ds_{11}^2 = dx_{11}^2 + g_{\mu\nu}dx^\mu dx^\nu$ one finds

$$\kappa_{11}^{-2(L-1)} \Lambda_{11}^n \int d^{11}x \sqrt{-g} \mathcal{R}^m \to g_s^{2(m-4)} \int d^{10}x \sqrt{-g} \mathcal{R}^m .$$

\[1\] We shall ignore terms depending on 3-form field $C_3$ and gravitino. The structure of terms depending on $C_3$ is restricted by the invariance of the supergravity action \[12\] under $C_3 \to -C_3$, $t \to -t$ \[13\].
In the last relation we have used (2.4). The condition \( \frac{1}{3}(m - 4) = k - 1 \) where \( k \) is an integer (effective loop order in string theory) implies

\[
m = 3k + 1, \quad n = 9L - 6k, \quad k = 0, 1, 2, \ldots.
\]

Thus the terms in the \( D = 11 \) action related to the special string-theory terms with coefficients which have ‘perturbative’ dependence on \( g_s \gg 1 \) are

\[
k_{11}^{2(L-1)} \Lambda_{11}^{9L-6k} \int d^{11} x \sqrt{-g} R^{3k+1} \sim l_{11}^{6k-9} \int d^{11} x \sqrt{-g} R^{3k+1},
\]

where we have used (2.4). \(^{2}\)

One may arrive at the same restriction on powers of curvature invariants in the \( D = 11 \) theory (i.e. \( m = 1, 4, 7, 10, \ldots \)) by an independent argument. In general, local perturbative contributions to the string-theory effective action are given by series of terms in expansion to the eleven-dimensional Lagrangian only if it scales like \( g_s^{2k-2} \) at strong coupling, the non-renormalization of the \( \sqrt{-g} R^m \) term after the reduction to \( D = 10 \) should be \( e^{2(k-1)\phi} \). Indeed, relating the \( D = 11 \) metric to the \( D = 10 \) string-frame metric by \( ds_{11}^2 = e^{4\phi/3} dx_{11}^2 + e^{-2\phi/3} ds_{10}^2 \) we find that \( (\sqrt{-g} R^m)_{11} \) reduces to \( e^{2(m-4)\phi/3} (\sqrt{-g} R^m)_{10} \) so that the required condition is \( m - 4 = 3(k - 1) \) or \( m = 3k + 1 \).

\(^{2}\) The same condition is found by demanding that the dilaton dependence of the \( \sqrt{-g} R^m \) term after the reduction to \( D = 10 \) should be \( e^{2(k-1)\phi} \).

\(^{3}\) Let us note that supersymmetry may also impose certain constraints on possible \( R^m \) curvature invariants. The \( R^m \) invariants that originate from the full (on-shell) superspace integral \( \int d^{11} x d^2 \theta D^{2p} W^m, \quad W \sim \tilde{\theta}^2 \mathcal{R} + \ldots \), have \( m = 16 + p \) (combined with (2.7) with \( n = 0 \), this gives further restriction on possible purely-logarithmic counterterms: \( m + p = 9L - 14 \)) \(^{13}\). This condition includes \( m = 3k + 1 \geq 16 \) for \( p = 3k - 15 \). The terms with \( m = 3k + 1 < 16 \) (i.e. \( \mathcal{R}^4, \mathcal{R}^7 \), etc.) should correspond to super-invariants constructed as integrals over parts of superspace.
\( \mathcal{R}^4 \) term made in \cite{Kaluza} to the case of \( k > 1 \). Thus we conjecture that all \( \mathcal{R}^{3k+1} \) terms should not receive contributions beyond the \( k \)-th loop order in type IIA string perturbation theory.

At the same time, contributions to \( \mathcal{R}^{3k+1} \) terms at lower loop orders in string perturbation theory are not excluded (as they will be subleading in the \( g_s \to \infty \) limit). Their \( D = 11 \) origin should be in the finite ‘Casimir-type’ \( R_{11}^{-n} \) terms, which accompany \( \Lambda_{11}^n \)-terms when the \( D = 11 \) effective action is computed in the space with one circular dimension. For example, the one-loop \( \Lambda_{11}^3 \mathcal{R}^4 \) term in the case of finite radius \( R_{11} \) is replaced by \((\Lambda_{11}^3 + c_1 R_{11}^{-3})\mathcal{R}^4 \) \cite{Kaluza}. In general,

\[
\kappa_{11}^{2(L-1)} R_{11}^{-n} \int d^{11} x \sqrt{-g} \mathcal{R}^m \to g_s^{2q} \int d^{10} x \sqrt{\mathcal{R}^m}, \quad q = m - 3L - 2,
\]

where we have used (2.7). Remarkably, if \( m = 3L + 1 \) as in (2.9), (2.10), then \( q = -1 \), i.e. we conclude that the term \( \kappa_{11}^{2(L-1)} (\Lambda_{11}^n + c R_{11}^{-n}) \int d^{11} x \sqrt{-g} R^m \) in the \( D = 11 \) effective action corresponds to a sum of \( L \)-loop and tree-level \( \mathcal{R}^{3L+1} \) terms in the \( D = 10 \) string theory effective action. For example, like the one-loop \( D = 11 \) terms \((\Lambda_{11}^3 + c_1 R_{11}^{-3})\mathcal{R}^4 \), which correspond to a sum of one-loop and tree-level terms in \( D = 10 \) \cite{Kaluza}, the two-loop terms \( \kappa_{11}^2 (\Lambda_{11}^6 + c_2 R_{11}^{-6}) \mathcal{R}^7 \) should correspond to a sum of two-loop \((\kappa_{11}^2 \alpha' \mathcal{R}^7)\) and tree-level \((\frac{\alpha'^5}{\kappa_{11}^2} \mathcal{R}^7)\) terms in string theory.

3. One-loop four-graviton amplitude in \( D = 11 \) supergravity

Deriving the one-loop four-graviton amplitude directly from the component formulation of \( D = 11 \) supergravity \cite{Kaluza} would be quite complicated. Fortunately, there is a short-cut way using the known expression \cite{Polchinski} for the one-loop \( D = 10 \) closed superstring 4-point amplitude. It was shown in \cite{Green} that the one-loop graviton scattering amplitude in \( D \leq 8 \) maximal supergravities can be obtained as a limit \((\alpha' \to 0, R \to 0, \kappa_D \text{fixed})\) of the amplitude of \( D = 10 \) string theory compactified on a torus. To find the amplitude in \( D = 10 \) type II supergravity theory one should take \( \alpha' \to 0 \lim \) treating \( 1/\alpha' \) as a proper-time UV cutoff \cite{Green}. The resulting expression is formally the same as for \( D < 8 \) \cite{Green} (where the \( \alpha' \to 0 \) limit is regular), but it still depends on \( \alpha' \) via the cutoff (and it is quadratically divergent for \( \alpha' \to 0 \)).

\footnote{The existence of terms in uncompactified type II string theory action which receive corrections only at one specific loop order was conjectured in \cite{Green}. Examples of such terms are known in the case of \( N = 2, D = 4 \) supersymmetric compactifications of type II string theory \cite{Green}.}

\footnote{It may seem that in the compactified case one should have \( \Lambda_{11}^n \to \Lambda_{11}^n + a_1 \frac{\Lambda_{11}^{n-1}}{R_{11}} + \ldots + a_n \frac{1}{R_{11}} \). However, the presence of \( \frac{\Lambda_{11}^{n-k}}{R_{11}} \) terms is ruled out on the grounds of locality of UV divergences.}
According to \[15\]

\[ A_4^{(D)} = \left( \frac{2\pi R}{\sqrt{\alpha'}} \right)^{D-10} \frac{\kappa_{10}^2}{\alpha'} \int_1^{\infty} d\tau_2 \frac{\tau_2}{\tau_2^{D-1}} F(s,t;\tau_2) \]

\[ = \kappa_D^2 \int_0^{\infty} d\tau \frac{\tau}{\tau^{D-1}} F(s,t;\tau) . \] (3.1)

Here and in what follows we omit the standard kinematic factor \( K \sim (\text{momentum})^8 \) in the expressions for the four-graviton amplitude and ignore the overall normalization coefficient. In eq. (3.1) \( \kappa_D^2 = (2\pi R)^{D-10} \kappa_{10}^2, \quad \tau \equiv \alpha' \tau_2, \) and

\[ F(s,t;\tau) = \int [d\rho] e^{-\tau M(s,t;\rho)} , \] (3.2)

\[ \int [d\rho] e^{-\tau M(s,t;\rho)} \equiv \int_0^1 d\rho_1 \int_0^{\rho_1} d\rho_2 \int_0^{\rho_2} d\rho_3 e^{-\tau M(s,t;\rho)} + 5 \text{ terms that symmetrise } s, t, u \]

\[ M(s,t;\rho) \equiv s\rho_1 \rho_2 + t\rho_3 \rho_2 + u\rho_1 \rho_3 + t(\rho_1 - \rho_2) , \quad s + t + u = 0 . \] (3.3)

The dependence on the cutoff \( \alpha' \to 0 \) disappears in \( D < 8 \) (where maximal supergravities are one-loop finite), but remains in \( D = 10 \)

\[ A_4^{(10)} = \frac{\kappa_{10}^2}{\sqrt{\alpha'}} \int_0^{\infty} \frac{d\tau}{\tau^{D-1}} F(s,t;\tau) \sim A_{10}^2 + \text{finite part} . \] (3.4)

Here \( \tau \equiv \alpha' \tau_2 = \frac{1}{2\pi} \) is related to the standard proper-time parameter \( t \) so that the effective proper-time cutoff is \( \Lambda_{10} = \frac{1}{\sqrt{2\pi \alpha'}} = \frac{1}{\sqrt{2\pi \alpha'}} \).

3.1. \( D = 11 \) supergravity compactified on a circle

As follows from the string-theory derivation in \[15\] (and is obvious from the proper-time integral representation of (3.4)) the amplitude in the case of \( D = 10 \) supergravity compactified on a circle is given essentially by the \( D = 9 \) supergravity expression (3.4) with only one modification: the factor of the sum over the Kaluza-Klein modes \( \sum_m e^{-\pi \tau m^2\rho^2_{10}} \) should be introduced under the integral over \( \tau \).

Being a consequence of the underlying supersymmetry, the same correspondence pattern applies to the 4-point amplitudes of any pair of maximal supergravities obtained by dimensional reduction, irrespective of their dimension and relation to string theory. The four-graviton amplitude in \( D = 11 \) supergravity compactified on a circle (with all external particles having ten-dimensional polarisations and \( p_{11} = 0 \)) is thus given by eq. (3.4) with an extra Kaluza-Klein factor, i.e.

\[ A_4^{(11)} = \kappa_{11}^2 (2\pi R_{11})^{-1} A_4(s,t) , \] (3.5)
\[ A_4(s, t) = \sum_{m=-\infty}^{\infty} \int_{\epsilon_{11}}^{\infty} \frac{d\tau}{\tau^2} e^{-\frac{\pi m^2}{R_{11}^2}} F(s, t; \tau), \quad \epsilon_{11} = \Lambda_{11}^{-2}, \]  

where \( F \) is defined in (3.2). Because of the sum over the Kaluza-Klein modes, the \( \tau \)-integral here has a stronger (cubic instead of quadratic, cf. (3.4)) divergence, as appropriate to the \( D = 11 \) theory.

The resulting amplitude (3.6) is in agreement with the general expression for the \( D = 11 \) supergravity four-graviton amplitude suggested (on the basis of a somewhat different reasoning) in \([6]\). Our aim below will be to study the structure of this amplitude, going beyond the leading (momentum) terms considered in \([6]\).

The integrand in the amplitude eq. (3.6) can be expanded in powers of \( M 

\[ A_4(s, t) = \sum_{m=-\infty}^{\infty} \int_{\epsilon_{11}}^{\infty} \frac{d\tau}{\tau^2} e^{-\frac{\pi m^2}{R_{11}^2}} \int [d\rho] \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \tau^k M^k(s, t). \]  

Let us separate the first \((k = 0)\) term \( A_4^{(a)} \) in the expansion,

\[ A_4(s, t) = A_4^{(a)} + A_4^{(b)}(s, t), \quad A_4^{(a)} = \sum_{m=-\infty}^{\infty} \int_{\epsilon_{11}}^{\infty} \frac{d\tau}{\tau^2} e^{-\frac{\pi m^2}{R_{11}^2}}. \]  

The term \( A_4^{(a)} \) was considered in \([3]\). Performing first the Poisson resummation, one obtains

\[ A_4^{(a)} = R_{11} \sum_{w=-\infty}^{\epsilon_{11}} \int_0^{1/\epsilon_{11}} d\hat{\tau} \hat{\tau}^{1/2} e^{-\pi w^2 R_{11}^2 \hat{\tau}}, \quad \hat{\tau} \equiv \tau^{-1}. \]  

Splitting the sum in (3.9) into the \( w = 0 \) and \( w \neq 0 \) parts

\[ A_4^{(a)} = A_4^{(a0)} + \tilde{A}_4^{(a)}, \]  

we find that \( \tilde{A}_4^{(a)} \) is finite, while \( A_4^{(a0)} \) is the UV divergent contribution

\[ A_4^{(a0)} = R_{11} \int_0^{1/\epsilon_{11}} d\hat{\tau} \hat{\tau}^{1/2} = \frac{2}{3} R_{11} \epsilon_{11}^{-3/2} = \frac{2}{3} R_{11} \Lambda_{11}^3. \]  

Thus finally

\[ A_4^{(a)} = \frac{2}{3} R_{11} \Lambda_{11}^3 + \frac{\zeta(3)}{\pi R_{11}^2}. \]  

The cutoff-independent part \( A_4^{(b)} \) in (3.8) can be written as \( A_4^{(b)}(s, t) = A_4^{(b0)}(s, t) + \tilde{A}_4^{(b)}(s, t), \) with \( A_4^{(b0)} \) representing the \( m = 0 \) contribution to the sum in (3.7), i.e.

\[ A_4^{(b0)}(s, t) = \int [d\rho] \int_0^{\infty} \frac{d\tau}{\tau^2} \left[ e^{-\tau M(s, t)} - 1 + \tau M(s, t) \right], \] 

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\[ \tilde{A}_4^{(b)}(s, t) = 2 \int [d\rho] \sum_{m=1}^{\infty} \sum_{k=2}^{\infty} \frac{(-1)^k m^2}{k!} \int_0^\infty \frac{d\tau}{\tau} e^{-\frac{\pi m^2}{\tau} R_{11}^2} \sum_{k=2}^\infty \frac{(-1)^k}{k!} \tau^k M^k(s, t). \] (3.14)

In the last expression (3.14) we have omitted the term linear in \( M \), which drops out after integrating over \( \rho \) and symmetrising in \( s, t, u \) since \( s + t + u = 0 \). This reflects the absence of logarithmic divergences in the 4-point amplitude in \( D = 10 \) supergravity [16]. To compute \( A_4^{(b)}(s, t) \) in (3.13) we first regularise it by integrating \( \tau \) from 0 to \( \tau_0 \) and then take the limit \( \tau_0 \to \infty \). As a result,

\[ A_4^{(b)}(s, t) = \lim_{\tau_0 \to \infty} \int [d\rho] \left[ \tau_0^{-1} (1 - e^{-\tau_0 M}) + (\gamma - 1 + \ln \tau_0) M(s, t) \right. \]
\[ + \left. M(s, t) \int_{\tau_0}^\infty \frac{d\tau}{\tau} e^{-\tau M(s, t)} + M(s, t) \ln M(s, t) \right] = \mathcal{H}(s, t), \] (3.15)

where
\[ \mathcal{H}(s, t) \equiv \int [d\rho] M(s, t; \rho) \ln M(s, t; \rho) = s \tilde{\mathcal{H}} \left( \frac{s}{t} \right), \] (3.16)

and we used again that terms linear in \( M \) disappear after symmetrisation in \( s, t, u \). The integration over \( \tau \) in (3.14) then gives

\[ \tilde{A}_4^{(b)}(s, t) = \sum_{k=2}^{\infty} \frac{2(-1)^k}{\pi^{k-1} k(k-1)} \sum_{m=1}^{\infty} m^{2-2k} R_{11}^{2k-2} H_k(s, t) = \sum_{k=2}^{\infty} c_k R_{11}^{2k-2} H_k(s, t). \] (3.17)

Here the coefficients \( c_k \) are proportional to values of the Riemann \( \zeta \)-function

\[ c_k = \frac{2(-1)^k}{\pi^{k-1} k(k-1)} \zeta(2k - 2), \] (3.18)

and

\[ H_k(s, t) \equiv \int [d\rho] M^k(s, t; \rho) = s^k \tilde{H}_k \left( \frac{s}{t} \right), \] (3.19)

where \( \tilde{H}_k \) is a polynomial of order \( k \). The integral similar to (3.19) appeared in [15], where it was put into the form

\[ H_k(s, t) = b_k \left[ I_k(s, t) + I_k(t, s) + I_k(s, u) + I_k(u, s) + I_k(t, u) + I_k(u, t) \right], \] (3.20)

\[ b_k = \frac{\sqrt{\pi} \Gamma(k + 1)}{2^{2k+2} \Gamma(k+5/2)}, \]

\[ \text{6 The integral over } \rho \text{ in } \mathcal{H}(s, t) \text{ can be performed explicitly, giving a combination of logarithmic and polylogarithmic functions.} \]
\[ I_k(t, s) = t^{k+1} \int_0^1 dx \frac{(1-x)^{k+1}}{sx-t(1-x)} = -\frac{t^k}{k+2} 2F_1[1, 1, 3+k; 1+\frac{s}{t}] . \]  

(3.21)

For integer \( k > 0 \) the function \( I_k \) reduces to a polynomial plus some combination of logarithmic functions. The latter cancel out in the symmetric combination \( H_k(s, t) \) in eq. (3.20). One is left with a homogeneous polynomial of degree \( k \) in the variables \( s, t, u \) (this follows also from direct computation of the integrals in eq. (3.19) after expanding the binomial).

Combining the above expressions (3.12), (3.15), (3.17), we find

\[ A_4(s, t) = \frac{2}{3} R_{11} \zeta(3) + \frac{s \mathcal{H} \left( \frac{s}{t} \right)}{\pi R^2_{11}} + \sum_{k=2}^\infty c_k R_{11}^{2k-2} s^k \tilde{H}_k \left( \frac{s}{t} \right) . \]  

(3.22)

In the case when all 11 dimensions are non-compact, the amplitude is given by the same universal expression (3.1) (with \( D = 11 \) and 11-dimensional cutoff)

\[ A_4^{(11)} = \kappa^2_{11} \int_{\epsilon_{11}}^{\infty} d\tau \frac{F(s, t; \tau)}{\tau^{3/2}} = \kappa^2_{11} \left[ \frac{2}{3} \Lambda^3_{11} + \frac{4}{3} \sqrt{\pi s^3/2} \tilde{H}_{3/2} \left( \frac{s}{t} \right) \right] , \]  

(3.23)

where \( \tilde{H}_{3/2} \) is defined as in (3.19). For comparison, the corresponding amplitudes (3.1) in uncompactified \( D = 10 \) (3.4) and \( D = 9 \) supergravities have the following explicit form (\( \Lambda_9 = \Lambda_{10} \))

\[ A_4^{(10)} = \kappa^2_{10} \left[ 2\pi \Lambda^2_{10} + s \mathcal{H} \left( \frac{s}{t} \right) \right] , \]  

(3.24)

\[ A_4^{(9)} = \kappa^2_{9} \left[ 2\sqrt{2} \Lambda_9 - 2\sqrt{\pi} \sqrt{s} \tilde{H}_{1/2} \left( \frac{s}{t} \right) \right] . \]  

(3.25)

Thus the third term in (3.22) is the finite part of the contribution of the massless \( D = 10 \) supergravity fields.

3.2. \( D = 11 \) supergravity compactified on 2-torus

In the case of compactification on \( T^2 \), one has (cf. (3.5),(3.6))

\[ A_4^{(11)} = \kappa^2_{11} (4\pi^2 R_{10} R_{11})^{-1} A_4^{(a)}(s, t) , \]  

(3.26)

\[ A_4^{(a)}(s, t) = \sum_{m,n=-\infty}^\infty \int_{\epsilon_{11}}^{\infty} d\tau \frac{e^{-\pi\tau \left( \frac{m^2}{R^2_{11}} + \frac{n^2}{R^2_{10}} \right)}}{\tau^{3/2}} F(s, t; \tau) . \]  

(3.27)

As in the circle case, we expand the exponential \( e^{-\tau M} \) in \( F (3.2) \) in powers of \( M \) and separate the \( k = 0 \) term as in (3.8),

\[ A_4^{(a)}(s, t) = A_4^{(a)} + A_4^{(b)}(s, t) , \quad A_4^{(a)} = \sum_{m,n=-\infty}^\infty \int_{\epsilon_{11}}^{\infty} d\tau \frac{e^{-\pi\tau \left( \frac{m^2}{R^2_{11}} + \frac{n^2}{R^2_{10}} \right)}}{\tau^{3/2}} . \]  

(3.28)
The constant part $A_{4T}^{(a)}$ is the one considered in [6], and it can be computed by Poisson resumming in both $m$ and $n$ and integrating over $\tau$,

$$A_{4T}^{(a)} = \frac{2}{3} \mathcal{V} \Lambda_{11}^3 + \frac{\zeta(3) E_{3/2}(\Omega)}{\pi^{1/2}} , \quad (3.29)$$

where

$$\mathcal{V} \equiv R_{10} R_{11}, \quad \Omega = \Omega_2 \equiv \frac{R_{10}}{R_{11}} . \quad (3.30)$$

$\Lambda_{11}$ is the same cutoff as in (3.11) and $E_r(\Omega)$ is the generalised Eisenstein series,

$$E_r(\Omega) = \sum_{(p,q)'} \frac{\Omega^r}{(p^2 \Omega^2 + q^2)^r} = \sum_{(p,q)'} (p^2 \Omega + q^2 \Omega^{-1})^{-r} , \quad (3.31)$$

where the notation $(p,q)'$ means that $p$ and $q$ are relatively prime. As in [25], one can show that for large $\Omega$

$$E_r(\Omega) = \Omega^r + \gamma_r \Omega^{1-r} + O(e^{-2\pi\Omega}) , \quad (3.32)$$

$$\gamma_r = \frac{\sqrt{\pi} \Gamma(r - 1/2) \zeta(2r - 1)}{\Gamma(r) \zeta(2r)} .$$

To calculate $A_{4T}^{(b)}(s,t)$ in (3.28) we decompose it as $A_{4T}^{(b)}(s,t) = A_{4T}^{(b_0)}(s,t) + \tilde{A}_{4T}^{(b)}(s,t)$, with $A_{4T}^{(b_0)}$ representing the $(m,n) = (0,0)$ contribution,

$$A_{4T}^{(b_0)}(s,t) = \int [d\rho] \int_0^\infty \frac{d\tau}{\tau^{3/2}} \left[ e^{-\tau M(s,t)} - 1 + \tau M(s,t) \right]$$

$$= -2\sqrt{\pi} \int [d\rho] \frac{M^{1/2}(s,t)}{\tau^{3/2}} = -2\sqrt{\pi} H_{1/2}^1(s,t) = -2\sqrt{\pi} s^{1/2} \bar{H}_{1/2}^1(\frac{s}{t}) . \quad (3.33)$$

For the remaining part $\tilde{A}_{4T}^{(b)}$ we have (cf. (3.17))

$$\tilde{A}_{4T}^{(b)}(s,t) = \int [d\rho] \sum_{(m,n) \neq (0,0)} \int_0^\infty \frac{d\tau}{\tau^{3/2}} \ e^{-\pi \tau \left( \frac{m^2}{R_{11}^2} + \frac{n^2}{R_{10}^2} \right)} \sum_{k=2}^\infty \frac{(-1)^k}{k!} \pi^k M^k$$

$$= \sum_{k=2}^\infty \frac{(-1)^k \Gamma(k - 1/2)}{\pi^{k-1/2} k!} \sum_{(m,n) \neq 0} \left( \frac{m^2}{R_{11}^2} + \frac{n^2}{R_{10}^2} \right)^{1/2-k} H_k(s,t) , \quad (3.34)$$

or, equivalently (cf. (3.17))

$$\tilde{A}_{4T}^{(b)}(s,t) = \sum_{k=2}^\infty d_k \sum_{(p,q)'} \left( p^2 \Omega + q^2 \Omega^{-1} \right)^{1/2-k} \mathcal{V}^{k-1/2} H_k(s,t) , \quad (3.35)$$

$$d_k = \frac{2(-1)^k \Gamma(k - 1/2)}{\pi^{k-1/2} k!} \zeta(2k - 1) .$$
The total amplitude $A_4T(s, t) = A_4^{(a)} + A_4^{(b)} + \tilde{A}_4^{(b)}$ in the 2-torus case is thus (cf. (3.22))

$$A_4T(s, t) = \frac{2}{3} \mathcal{V} \Lambda_{11}^3 + \frac{\zeta(3)E_{3/2}(\Omega)}{\pi \mathcal{V}^{1/2}}$$

$$- 2\sqrt{\pi} s^{1/2} \bar{H}_{1/2} \left( \frac{s}{t} \right) + \sum_{k=2}^{\infty} d_k E_{k-1/2}(\Omega) \mathcal{V}^{k-1/2} s^k \bar{H}_k \left( \frac{s}{t} \right). \tag{3.36}$$

Written in this form the amplitude is given by an $SL(2, \mathbb{Z})$ invariant expansion in powers of the torus area $\sim \mathcal{V}$.

In eqs. (3.22) and (3.36) the functions $\bar{H}_k \left( \frac{s}{t} \right)$ with $k = 2, 3, \ldots$ are polynomials of degree $k$, and thus correspond to local higher derivative terms in the one-loop effective action (these are the contributions of the massive Kaluza-Klein modes). The non-local ($D = 10$ massless mode) contributions originate from the $\bar{H}$ term in the circle amplitude case (3.22) or from $\bar{H}_{1/2}$ term in the torus amplitude case (3.36). The latter $\bar{H}_{1/2}$ term has the meaning of the finite part of the amplitude in $D = 9$ supergravity (3.23).

4. Remarks on relation to string theory

Let us now comment on the structure of the amplitudes (3.22) and (3.36), corresponding to the circular and toroidal compactifications of the $D = 11$ supergravity, and their relation to string theory. Expressing (3.22) in terms of the string coupling $g_s$ and the string scale $\sqrt{\alpha'}$ using (2.2), (2.3) we find

$$A_4(s, t) = \frac{a}{3\pi \alpha'} + \frac{\zeta(3)}{\pi \alpha' g_s^2} + s \bar{H} \left( \frac{s}{t} \right) + \sum_{k=2}^{\infty} c_k g_s^{2k-2} \alpha'^{k-1} s^k \bar{H}_k \left( \frac{s}{t} \right). \tag{4.1}$$

The first two constant terms in this amplitude (multiplied by the kinematic factor) correspond to the one-loop and tree-level $\mathcal{R}^4$ terms in the type II string effective action. That the one-loop amplitude in $D = 11$ supergravity effectively includes $\zeta(3)\mathcal{R}^4$ term of string theory may look miraculous: while in string theory this term is produced by exchanges of massive string modes, in $D = 11$ expression it originates from the loop of the Kaluza-Klein modes which are 0-brane solitons of string theory. This fact is suggesting

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7 In the notation of [1], $\frac{4}{3} \Lambda_{11}^3 \equiv C = \frac{\pi^2}{3}$, corresponding to $a = \pi^2$ in our notation. As was argued in [3] using the first two terms in the amplitude (3.36) on the 2-torus, this value is implied by consistency with string theory (T-duality invariance of one-loop term in type II theories compactified on a circle).
that the uncompactified type IIA string theory (‘dual’ to \(D = 11\) theory) may have a reformulation in terms of solitonic objects.\(^8\)

Let us briefly comment on the explicit structure of the \(\mathcal{R}^4\) terms in the effective actions of type IIA and \(D = 11\) theories. In general, one expects the \(\mathcal{R}^4\) terms in \(D = 10\) theory to be a linear combination of the \(D = 10\) terms \(\mathcal{J}_1 \equiv ts ts R^4\) and \(\mathcal{J}_2 \equiv \frac{1}{2} \epsilon_{10} \epsilon_{10} R^4\).\(^9\) \(\mathcal{J}_2\) is the higher-dimensional extension of the Gauss-Bonnet invariant in 8 dimensions \((\epsilon_8 \epsilon_8 \rightarrow - \frac{1}{2} \epsilon_{10} \epsilon_{10})\). Its expansion near flat space \((g_{mn} = \eta_{mn} + h_{mn})\) starts with \(h^5\) terms and thus its coefficient cannot be determined from consideration of the on-shell 4-graviton amplitude only. The sigma-model approach implies\(\)[4,26] that (up to the usual field redefinition ambiguities) the tree-level type II string term is \(L_0 \sim \zeta(3)J_0, \ J_0 = \mathcal{J}_1 + \mathcal{J}_2\). The structure of the kinematic factor in the one-loop type IIA 4-point amplitude with transverse polarisations and momenta \((t_8 + \frac{1}{2} \epsilon_8)(t_8 + \frac{1}{2} \epsilon_8)\) hints that the one-loop \(\mathcal{R}^4\) terms in \(D = 10\) type IIA theory should be proportional to the opposite-sign combination \(\mathcal{J}_1 - \mathcal{J}_2\).\(^10\) Combining with the \(B \mathcal{R}^4\) term\(\)[28], we get \(L_{1A} = \mathcal{J}_1 - \mathcal{J}_2 + b_1 \epsilon_{10} B[trR^4 - \frac{1}{4}(trR^2)^2]\). This can be re-written (using \(\mathcal{J}_1 = 24[t_8 trR^4 - \frac{1}{4} t_8 (trR^2)^2]\)) as a combination of the bosonic parts of the three \(N = 1\) super-invariants\(\)[29] \(I_3 = t_8 trR^4 - \frac{1}{4} \epsilon_{10} BtrR^4, \ I_4 = t_8 trR^2 trR^2 - \frac{1}{4} \epsilon_{10} B(trR^2)^2\) and \(J_0\) provided \(b_1 = -12\). Then \(L_{1A} = -J_0 + 48(I_3 - \frac{1}{4} I_4)\).\(^11\)

While the coefficients of \(I_3\) and \(I_4\) are expected not to be renormalised, there is no reason for this to be true for the coefficient of \(J_0\)\(\)[22] and thus of the \(\mathcal{J}_2\) term. This may preclude one from identifying the \(D = 11\) counterpart of this term as \(\frac{1}{24} \epsilon_{111} \epsilon_{111} R^4\).

Returning to the discussion of the amplitude\(\)[4,11], we observe that not only the two constant terms but also all \textit{momentum-dependent} terms in the \(D = 11\) amplitude\(\)[4,11] have ‘perturbative’ dependence on the type IIA string coupling. It appears as if the \textit{one-loop} four-graviton amplitude in \(D = 11\) supergravity represents a sum of certain perturbative string corrections, containing contributions of \textit{all orders} in the string loop expansion.

It is not clear, however, which regions of the moduli spaces of higher genus Riemann surfaces this expression is accounting for. Moreover, while the first two terms in\(\)[4,11] (or \(\mathcal{R}^4\) terms in the type II string effective action) are expected to be unchanged by both

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\(^8\) The relation between tree-level \(\mathcal{R}^4\) term in type IIA theory and one-loop \(\mathcal{R}^4\) term in \(D = 11\) theory is reminiscent of the relation between tree-level \(F^4\) term in type I theory and one-loop \(F^4\) term in heterotic theory\(\)[22].

\(^9\) We follow the notation of\(\)[22] up to the sign change \(\epsilon_{10} \epsilon_{10} \rightarrow - \epsilon_{10} \epsilon_{10}\) due to Minkowski signature used here. Thus here \(J_0 = ts ts R^4 + \frac{1}{2} \epsilon_{10} \epsilon_{10} R^4\).

\(^10\) This is implied also by the discussion in\(\)[27]. We are grateful to E. Kiritsis for clarifying correspondence on the issue of \(\mathcal{R}^4\) terms.

\(^11\) Similar observation was made in\(\)[21], where, however, the possible presence of \(\mathcal{J}_2\) was ignored and thus to be able to represent \(L_{1A}\) as a combination of \(I_3\) and \(I_4\) a different value \(b_1 = -6\) was assumed.
$D = 11$ supergravity and type IIA string higher-loop corrections \cite{21,3}, this may not be true for other $s$, $t$-dependent terms in (4.1). If this is the case, one may be unable to relate the $D = 10$ and $D = 11$ expressions in a simple way.

To relate the torus amplitude (3.26), (3.36) to type IIA and type IIB string theories compactified on a circle, it is useful to consider the corresponding contribution to the effective action of $D = 11$ supergravity compactified on a 2-torus which may be written in the following symbolic form (cf. (2.8))

$$S_1 \propto \int d^3 x \sqrt{-g} \left[ \frac{2}{3} \nu \Lambda_{11}^3 + \pi^{-1} \zeta(3) E_{3/2}(\Omega) \nu^{-1/2} 
+ h_{1/2}(\nabla^2)^{1/2} + \sum_{k=2}^{\infty} h_k E_{k-1/2}(\Omega) \nu^{k-1/2} \nabla^{2k} \right] \mathcal{R}^4. \tag{4.2}$$

One may now relate the $D = 11$ metric $g_{mn}$ and the torus area $(2\pi)^2 \nu$ and the modulus $\Omega$ to the string-frame metrics, couplings and radii of type II string theories compactified on a circle.\cite{12} In terms of type IIB coupling and compactification radius, $\Omega = R_{10} R_{11}^{-1} = g_B^{-1}, \quad \nu = R_{10} R_{11} = \alpha'/3 g_B^{1/3} R_B^{-4/3}$, so that the limit of uncompactified type IIB theory corresponds to $\nu \to 0$ for fixed $\Omega$ \cite{3}. The momentum-dependent terms in (3.36) and the higher-derivative terms in (1.2) disappear in this limit (the third non-local term is also subleading as it does not scale as $R_B$, see below). The remaining second term proportional to the Eisenstein function $E_{3/2}(\Omega)$ was shown in \cite{25} to contain not only the tree-level and one-loop contributions but also the sum of all type IIB D-instanton contributions to the $\mathcal{R}^4$ term (similar results for type IIB theory compactified to 8 dimensions were obtained in \cite{27}). The limit of non-compact type IIA theory is $R_A \to \infty$ for fixed $g_A$, i.e. $R_{10} = \Omega \nu \to \infty$ for fixed $R_{11} = (\nu/\Omega)^{1/2}$. In that limit one recovers the amplitude (1.1) of the $D = 11$ theory compactified on a circle, containing perturbative contributions to all orders in string coupling.

In general, eqs. (3.36) and (1.2) appear to be describing a mixture of perturbative and non-perturbative contributions in type II string theories compactified on a circle. Expressing (1.2) in terms of type IIB parameters and using the expansion (3.32) of $E_\tau(\Omega)$ for large $\Omega$ ($\Omega$ is large for small $g_B$) we find

$$S_1 \propto \int d^3 x \sqrt{-g} B R_{11} \left( \frac{2}{3} \Lambda_{11}^3 R_{11}^2 \right. + \pi^{-1} \zeta(3) \left[ g_B^{-2} + \gamma_{3/2} + O(e^{-\frac{2\pi}{g_B}}) \right] + h_{1/2} R_B^{-1}(\nabla^2)^{1/2} B R_{11} \right)$$

\footnote{Here we shall follow \cite{21,3} and use the standard relations ($\alpha' = 1$): $ds_{11}^2 = e^{4\phi_A/3} ds_{11}^2 + e^{-2\phi_A/3}(R_A^2 + ds_B^2), \quad ds_{11}^2 = ds_B^2, \quad R_A = R_{10} R_{11}^{1/2}, \quad g_A = e^{\phi_A}, \quad g_B = e^{\phi_B} R_{11}^{-1}, \quad R_{11} = g_B^{-2/3} R_B^{2/3}, \quad \int d^3 x \sqrt{-g} R^4 = R_{11}^{-1/2} \int d^3 x (\sqrt{-g} R^4)_B, \quad \nabla^2 = (\nabla^2)_B, \quad \text{etc.}$}
\[ + \sum_{k=2}^{\infty} h_k \left[ 1 + \gamma_{k-1/2} g_B^{2k-2} + O(e^{-\frac{2\pi}{g_B}}) \right] R_B^{-2k} (\nabla^{2k})_B \left( R^4 \right)_B. \] (4.3)

The terms proportional to \( E_{k-1/2}(\Omega) \) thus appear to contain only one-loop and \( k \)-loop parts among perturbative contributions. This is a generalisation of the observation of [25,6] about \( E_{3/2}(\Omega) \nu_{-1/2} R^4 \) term (which contains tree-level and one-loop contributions).

It seems likely that \( O(e^{-\frac{2\pi}{g_B}}) \) terms in the expansion of the functions \( E_{k-1/2}(\Omega) \) are related to non-perturbative type II string theory contributions since they constitute the simplest \( SL(2, \mathbb{Z}) \) invariant completions of the one-loop and \( k \)-loop terms.

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