Two-photon width and gluonic component of $\sigma/f_0(600)$

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We analyse data on $\pi\pi$ and $\gamma\gamma$ scattering below 700 MeV within an improved analytic K-matrix model. This model is based on an effective theory with couplings between resonances, hadrons and photons. The two-photon decay of a resonance can proceed through intermediate transition into charged hadrons (here: $\pi^+\pi^-$) and their subsequent annihilation or through a “direct” transition into photons. Our analysis confirms the rather large total radiative width of the $\sigma$ resonance which we find as $(3.9 \pm 0.6)$ keV but suggests its dominance by the $\pi\pi$ rescattering process. This process is not sensitive to the internal structure of the resonance contrary to the direct component which we find small, $(0.13 \pm 0.05)$ keV, and well consistent with the expectations for an unmixed glueball according to the QCD sum rule calculations.

1. INTRODUCTION

The study of scalar mesons and the interpretation of the experimental results is, despite a long lasting effort, still an active field of research with controversies on experimental results and the theoretical interpretation. The lightest scalar mesons have been interpreted as conventional $q\bar{q}$, but also as tetra-quark or molecular states. In addition, there is the definitive expectation within QCD of the existence of scalar glueballs which can mix with the nearby states with quark constituents. Depending on the mass of the glueballs, this affects strongly the interpretation of the spectrum and leads to different scenarios.

The phenomenological analysis attempts to group the spectrum of observed states into appropriate $qq$ or $4q$ multiplets: left over states are possible candidates for glueballs. Also one expects the production of such states to be enhanced in “gluon rich processes”, while they should be suppressed in $\gamma\gamma$ reactions and corresponding rules hold for decays as well.

The existence of glueballs has been predicted long ago in the early time of QCD as consequence of the self-interaction of gluons and first scenarios have been developed already in 1975 [1]. Today there is agreement on the existence of such states and the lightest state to be a scalar meson. Quantitative results are available from Lattice QCD and from QCD sum rules (QSSR). The theoretical results, recalled below, suggest a light scalar glueball with mass around 1 GeV or below.

Among the light particles the $f_0(600)/\sigma$ meson could be such a gluonic resonance. Recent analyses of the $\gamma\gamma \to \pi\pi$ processes have extracted the width of $f_0(600)/\sigma \to \gamma\gamma$ as: $(4.1 \pm 0.3)$ keV [2], $(3.5 \pm 0.7)$ or $(2.4 \pm 0.5)$ keV [3] to $(1.8 \pm 0.4)$ keV [4], while the one from nucleon electromagnetic polarizabilities has given $(1.2 \pm 0.4)$ keV [5]. Such results have been interpreted in [2] as disfavouring a gluonic nature which is expected to have a small coupling to $\gamma\gamma$ [6,7,8,9,10,11].

In our recent paper [12] a resolution of this conflict has been suggested. From the analysis of the $\gamma\gamma \to \pi\pi$ processes in the low energy region below 700 MeV we conclude that they are dominated by the coupling of the photons to charged pions and their rescattering, which therefore can hide any direct coupling of the photons to the scalar.
mesons. Some first results from this study have been presented elsewhere [13].

2. A LIGHT $0^{++}$ GLUEBALL

There are several results suggesting a light scalar glueball:

- **Lattice QCD.** Calculations in the simplified world without quark pair creation (quenched approximation) find the lightest state at a mass around 1600 MeV (recent review in Ref. [14]). These findings lead to the construction of models where the lightest glueball/gluonium mixes with other mesons in the nearby mass range of around 1300-1800 MeV. However, recent results beyond this quenched approximation [15] indicate the lightest state with a large gluonic component to fall into the region around 1 GeV, and therefore, a scheme based on the mixing of meson states with all masses higher than 1300 MeV could be insufficient to represent the gluonic degrees of freedom in the meson spectrum. Further studies concerning the dependence on lattice spacings and the quark mass appear important.

- **QCD spectral sum rules (QSSR) and Low-energy theorems (LET).** These approaches have given quantitative estimates for the mass and decay properties of the gluonic bound states. In particular, in a combined analysis of subtracted and unsubtracted sum rules a low mass for the bare unmixed gluonium state is obtained [15]:

$$M_{\sigma B} \simeq (0.95 \sim 1.10) \text{ GeV},$$

besides the heavier state around $M_{G} \simeq (1.5 \sim 1.6) \text{ GeV}$. These masses are similar to the unquenched and quenched lattice results respectively, quoted above. The hadronic width in this approach is obtained rather large:

$$\Gamma_{\sigma B \to \pi^+ \pi^-} \simeq 0.7 \text{ GeV},$$

whereas the radiative decay width is found small:

$$\Gamma_{\sigma B \to \gamma \gamma} \simeq (0.2 \sim 0.6) \text{ keV}.$$  

Indeed, this width is considerably smaller than the value of about 5 keV, which is expected for a state $S_2 \equiv (\bar{u}u + \bar{d}d)$ with mass of 1 GeV [11]. On the other hand, QSSR predicts, for a four-quark state having the same mass of 1 GeV, a $\gamma \gamma$ width of about 0.4 eV [11].

- **Phenomenological studies.** A full understanding of the scalar meson spectrum is required if the glueball is to be found as a left over state after the identification of flavour multiplets. Several schemes exist, motivated by the quenched lattice result, where the extra gluonic state is assumed to mix into the three isoscalars $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ [19]. In an alternative approach [17], the lightest $q \bar{q}$ multiplet is formed from $f_0(980)$, $f_0(1500)$, $K^*_0(1430)$ and $a_0(1450)$, and the glueball is identified as the broad object at smaller mass represented by $f_0(600)$; it dominates $\pi \pi$ scattering near 1 GeV but extends from threshold up to $\sim 1800$ MeV (see also ref. [18]). The appearance of this broad object in most gluon rich processes was considered in support of this hypothesis. In this analysis of the spectrum, results from elastic and inelastic $\pi \pi$ scattering as well as from $D$, $B$ and $J/\psi$ decays have been considered [17][19].

3. THE DIRECT CONTRIBUTION

The amplitudes for the reactions $\gamma \gamma \to \pi \pi$ are largely determined by elastic $\pi \pi$ scattering and likewise in the case of multi-channel processes. First, there are the extended unitarity relations corresponding to the Watson theorem for the single channel; secondly, there are dispersion relations where the necessary subtraction terms generate polynomial ambiguities [20]. This general formalism has been applied by Mennessier [21], to obtain the electromagnetic processes $\gamma \gamma \to \pi \pi, K \bar{K}$, given the strong processes $\pi \pi \to \pi \pi, K \bar{K}$. The latter ones are represented by a $K$ matrix model which represents the amplitudes by a set of resonance poles. In that case, the dispersion relations in the multi-channel case can be solved explicitly, which is not possible otherwise. This model can be reproduced by a set of Feynman diagrams, including resonance (bare) couplings to $\pi \pi$ and $K \bar{K}$, and 4-point $\pi \pi$ and $K \bar{K}$ interaction vertices. A subclass of bubble pion loop diagrams including resonance poles in the $s$-channel are resummed (unitarized Born).

The radiative width of the resonances cannot
be predicted due to the polynomial ambiguity, which can be taken into account by introducing as free parameters the “direct couplings” of the resonances to $\gamma\gamma$ in an effective interaction vertex. In the present analysis we have extended the model by the introduction of a shape function which takes explicitly into account left-handed cut singularities for the strong interaction amplitude. This allows a more flexible parametrisation of the $\pi\pi$ data at low energies and improves the high energy behaviour. The model shows different characteristics at low and high energies:

- **Low energy limit: rescattering.** A striking feature of the low energy $\gamma\gamma \to \pi\pi$ scattering is the dominance of the charged over the neutral $\pi\pi$ cross section by an order of magnitude which can be explained by the contribution of the pion-exchange Born term in $\gamma\gamma \to \pi^+\pi^-$. In the process $\gamma\gamma \to \pi^+\pi^-$, the photons cannot couple “directly” to $\pi^0\pi^0$ but through intermediate charged pions and subsequent rescattering with charge exchange. This feature is realized in the analytic model considered here [21] which has it in common with the Chiral perturbation theory [22].

- **High energy limit: direct production.** On the other hand, at high energies the photon can recombine the constituents of the hadrons. An example is the production of $f_2(1270)$ in $\gamma\gamma \to \pi\pi$. The analytic model [21] predicts a decreasing cross section for this process with pion exchange reaching $< 10\%$ of the observed cross section at the peak of the $f_2$. These considerations require the need of a “direct coupling” of $f_2 \to \gamma\gamma$. Indeed, it is well known that the radiative decays of the tensor mesons $f_2, f'_2, a_2$ are well described by a model with direct coupling to the $q\bar{q}$ constituents according to the $SU(3)$ structure of the nonet with nearly ideal mixing (see e.g. Ref. [23]). We therefore interpret the direct terms in the $\gamma\gamma$ processes as originating from the coupling of the photon to the partonic constituents of the resonances (such as $q\bar{q}, 4q, gg,...$).

4. **ANALYTIC K-MATRIX MODEL**

In the application of this model [21], we first obtain a suitable K-matrix parametrisation of the $\pi\pi$ scattering data, and then determine the direct coupling by comparing the model with the $\gamma\gamma$ results. In the present analysis, we restrict ourselves to the low mass region below 700 MeV where we neglect vector and axial-vector exchanges [21], inelastic channels and D-waves. Furthermore, we assume a pointlike pion-photon coupling, which is expected to be a good approximation in the region where we are working.

We apply the analytic model [21], but we introduce a shape function $f_0(s)$ which multiplies the $\sigma\pi\pi$ coupling. The real analytic function $f_0(s)$ is regular for $s > 0$ and has a left cut for $s \leq 0$. For our low energy approach, a convenient approximation, which allows for a zero at $s = s_A$ and a pole at $\sigma_D > 0$ simulating the left hand cut, is:

$$f_0(s) = \frac{s - s_A}{s + \sigma_D}.$$  

(4)

For simplicity, we don’t include the 4-point coupling term. The unitary $\pi\pi$ amplitude $T^{(4)}_\gamma$ for the isospin $I = 0$ S-wave is written as:

$$T^{(0)}_\gamma(s) = \frac{Gf_0(s)}{s_R - s - Gf(s)} = \frac{Gf_0(s)}{D(s)},$$  

(5)

where $G = g^2_{\pi\gamma}$ is the bare coupling squared. Unitarity determines the imaginary part, while a dispersion relation subtracted at $s = 0$ determines the form of $f(s)$ [12].

Using the S wave amplitude in Eq. (5) we derive the amplitude $T^{(4)}_\gamma$ for the electromagnetic process for isospin $I = 0$ as:

$$T^{(0)}_\gamma = \sqrt{\frac{2}{3}} \alpha \left( f_B + G \frac{f_B}{D} \right) + \alpha \frac{D}{D}.$$  

(6)

The first contribution comes from the Born term for $\gamma\gamma \to \pi^+\pi^-$, the second one from the $\pi\pi$ rescattering and the third one represents the polynomial contribution $P = sF_{\gamma\gamma} \sqrt{2}$ which reflects the ambiguity from the dispersion relations and represents the direct coupling of the resonance to $\gamma\gamma$. The residues at the pole $s_0$ of the rescattering and direct contributions to $T^{(0)}_\gamma$ in Eq. (6) determine the respective branching ratios. Similarly, we construct the $I = 2$ S-wave amplitude $T^{(2)}_\gamma$ and $T^{(2)}$ as well as the cross sections for the $\pi\pi$ and $\gamma\gamma$ scattering processes [21].
Given the $\pi\pi$ amplitude, we can predict the cross sections for $\gamma\gamma \to \pi\pi$ where the only free parameter $F_\gamma$ is related to the strength of the direct coupling $\sigma \to \gamma\gamma$. The fit of the model to the data from Crystal Ball ($\pi^0\pi^0$) and MARK-II ($\pi^+\pi^-$) collaborations is shown in Figs. 1 and 2 for the cases with direct contribution and without (“unitarised Born” : $F_\gamma = 0$). We obtained excellent fits to the neutral pion data with $F_\gamma = -0.090$. A fit to both channels yields $F_\gamma = -0.070$. Then the fit in the charged channel deviates from the data at high-mass; the large systematic errors and the absence of data points in the region 0.40 to 0.55 GeV do not permit a good understanding of this channel. We take as a final estimate $F_\gamma \approx -(0.080 \pm 0.014)$. From the residues at the pole $s_0$ and $\Gamma_{\sigma\to\gamma\gamma} = 2\text{Im}M_\sigma$, we can deduce the “partial” $\gamma\gamma$ widths at the complex pole:

\[
\Gamma_{\sigma\to\gamma\gamma} \approx (0.13 \pm 0.05) \text{ keV},
\]

\[
\Gamma_{\text{resc}} \approx (2.7 \pm 0.4) \text{ keV},
\]

and the total $\gamma\gamma$ width (direct + rescattering):

\[
\Gamma_{\sigma\to\gamma\gamma} \approx (3.9 \pm 0.6) \text{ keV}.
\]

Our result for the direct coupling is comparable with our previous value [14] obtained without using the shape function $f_0(s)$, but not as small as in a parallel analysis [29]. Our total $\gamma\gamma$ width is compatible with the range of values obtained previously [24,16]. These analyses, however, don’t separate the direct term.

The results from QSSR/LET are obtained in the physical region and can better be compared with experiment by using the corresponding results for the “visible meson” on the real axis instead of the results at the complex pole: this can be, either the Breit-Wigner mass and width (see [13]) or the “on shell” mass and width (see e.g. [30]), which are obtained for the mass $M_\sigma$ where the real part of the propagator vanishes $\text{Re}D((M_\sigma)2) = 0$. For this definition, we obtain:

\[
M_\sigma^{\text{os}} \approx 0.92 \text{ GeV}, \quad \Gamma_{\sigma\to\gamma\gamma}^{\text{os}} \approx 1.02 \text{ GeV}, \quad \Gamma_{\sigma\to\pi\pi}^{\text{os, dir}} \approx (1.0 \pm 0.4) \text{ keV}.
\]

As the model is extrapolated here towards energies beyond its validity ($M \sim 1 \text{ GeV}$), we consider these results as a crude approximation.

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5. RESULTS OF THE ANALYSIS

We first determine the parameters of the model for $\pi\pi$ scattering below 700 MeV from the best approximation of our formula to the phase shifts $\delta_0^{(0)}$ and $\delta_0^{(2)}$ obtained in the Roy equation analysis in [24]. For both channels $I = 0$ and $I = 2$ we fit 4 parameters each. We find the pole mass:

\[
M_\sigma \approx (422 - i 290) \text{ MeV}
\]

which is close to the masses in other recent determinations (441 - i 272) MeV [25] or (489 - i 264) MeV [26].
6. COMPARISON WITH QSSR/LET

One can notice that the QSSR and LET predictions for the mass and hadronic widths of a low mass gluonium $\sigma_B$ in Eqs. (1) and (2) are in remarkable agreement with the results in Eq. (10) for an on-shell resonance. The direct $\gamma\gamma$ coupling, which can reveal the photon coupling to the intrinsic quark ($q\bar{q}$, $4q$, . . .) or gluon ($gg$, . . .) constituent structure of the resonance, can be related to the QSSR/LET evaluations of its width through quark or gluon loops as in case of the quark triangle for the pion or the $f_2(1270)$. Our $\sigma \rightarrow \gamma\gamma$ width in Eq. (10) (also the pole value in Eq. (8)) is consistent with the small value predicted in Eq. (3). This “overall agreement” favours a large gluon component in the $\sigma/f_0(600)$ wave function. It disfavours, in particular, a $\bar{q}q$ interpretation because of the larger radiative width to be expected ($\sim 5$ keV). On the other hand, a four-quark interpretation would require a smaller radiative width and is disfavoured as well by our result in Eq. (8) (at the level of 2.5 $\sigma$).

Improvements of our estimates need more precise data below 700 MeV, which could be provided by the KLOE-2 experiment in the future. Furthermore, an extension of the analysis to higher energies will be important.

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