ON CONSTRAINING A TRANSITING EXOPLANET’S ROTATION RATE WITH ITS TRANSIT SPECTRUM

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Abstract

We investigate the effect of planetary rotation on the transit spectrum of an extrasolar giant planet. During ingress and egress, absorption features arising from the planet’s atmosphere are Doppler shifted by order the planet’s rotational velocity (≈1–2 km s−1) relative to where they would be if the planet were not rotating. We focus in particular on the case of HD 209458b, which ought to be at least as good a target as any other known transiting planet. For HD 209458b, this shift should give rise to a small net centroid shift of ≈60 cm s−1 on the stellar absorption lines. Using a detailed model of the transmission spectrum due to a rotating star transited by a rotating planet with an isothermal atmosphere, we simulate the effect of the planet’s rotation on the shape of the spectral lines, and in particular on the magnitude of their width and centroid shift. We then use this simulation to determine the expected signal-to-noise ratio (S/N) for distinguishing a rotating from a nonrotating planet, and assess how this S/N scales with various parameters of HD 209458b. We find that with a 6 m telescope, an equatorial rotational velocity of ≈2 km s−1 could be detected with a S/N ≈ 5 by accumulating the signal over many transits over the course of several years. With a 30 m telescope, the time required to make such a detection is reduced to less than 2 months.

Subject headings: astrophysics — astrochemistry — planetary systems — radiative transfer — stars: atmospheres — stars: individual (HD 209458)

Online material: color figures

1. INTRODUCTION

Since the early 1990s, more than 200 planets have been discovered orbiting other stars. Roughly four-fifths of the planets that have been discovered are at least half as massive as Jupiter, and about a quarter of the known planets orbit extremely close to their parent star (0.1 AU). The Jupiter-mass planets that are in close orbits are highly irradiated by their stars and are therefore called “hot Jupiters.”

Twenty hot Jupiters transit along the line of sight between Earth and their star (Charbonneau et al. 2000; Henry et al. 2000; Konacki et al. 2003, 2004, 2005; Bouchy et al. 2004; Pont et al. 2004; Udalski et al. 2002a, 2002b, 2002c; 2003, 2004; Alonso et al. 2004; Bouchy et al. 2005 b; McCullough et al. 2006; O’Donovan et al. 2006; Bakos et al. 2006, 2007a, 2007b; Collier Cameron et al. 2006; Burke et al. 2007; Gillon et al. 2007; Johns-Krull et al. 2007). Observations of the transiting planets have confirmed their similarity to Jupiter by revealing that these are gas giant planets, with radii comparable to, or somewhat larger than, Jupiter’s (Charbonneau et al. 2000; Henry et al. 2000; Gaudi 2005).

The effects of the tidal torques experienced by an orbiting body have been studied for a long time—for an early seminal analysis, see Goldreich & Peale (1966). Such torques tend to synchronize a satellite’s rotation rate to its orbital rate, and if the torque is sufficient this synchronization is achieved and the orbiter is said to be “tidally locked,” as the Earth’s Moon is. The hot Jupiter class of extrasolar planets are thought to orbit sufficiently close to their stars that their tidal-locking timescales are much shorter than the ages of the planets. The planets, then, are expected to be tidally locked to the stars, with one hemisphere in permanent day and the other in permanent night (Harrington et al. 2006).

A tidally locked hot Jupiter will have a permanent sharp contrast in temperature between the substellar point and the night side, which must have a profound influence on the atmospheric dynamics. Showman & Guillot (2002) make simple predictions of the day/night temperature difference (≈500 K) and the speed of winds (up to ≈2 km s−1), and their detailed, three-dimensional simulations agree with their estimates. Shallow-water simulations by Cho et al. (2003) predict longitudinally averaged zonal wind speeds of up to 400 m s−1, with local winds approaching 2.7 km s−1 (under some assumptions). Simulations by Cooper & Showman (2005) predict a super-rotational jet (i.e., blowing eastward, where north is defined by the right-hand rule) that blows the hottest part of the planet downstream by about 60° from the substellar point. Their simulations predict supersonic winds exceeding 9 km s−1 at high latitudes, high in the atmosphere (where the optical depth is low), and winds exceeding 4 km s−1 at pressures near the photosphere. A Spitzer Space Telescope phase curve for v Andromedae b rules out a phase shift as large as 60° between the substellar point and the hottest spot (Harrington et al. 2006), but a Spitzer phase curve for HD 189733b favors a ~30° shift for that planet (Knutson et al. 2007 a), so it remains unclear to what extent available data indicate very strong photospheric winds.

Transmission spectroscopy is a way to probe the atmospheres of these planets. Charbonneau et al. (2002) were the first to detect an absorption feature in what is probably the atmosphere of HD 209458b, when they found that the effective radius of the planet increases slightly at the wavelength of a strong sodium absorption doublet (the Na D lines) at ~590 nm. In addition, Vidal-Madjar et al. (2003, 2004) have reported a number of absorption features in HD 209458′s transit spectra that are due to various species (including hydrogen Lyα, neutral carbon, oxygen, and sulfur, and some ionization states of carbon, nitrogen, and silicon) in a hot exosphere that is probably only loosely bound to the planet. Intriguingly, through analyzing the red and near-IR portion of HD 209458b’s transit spectrum Barman (2007) found a 10 σ detection of atmospheric water vapor. Several measurements of the planet’s emission spectrum, however, have found results that seem to be inconsistent.

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with high water abundance high in the atmosphere (Grillmair et al. 2007; Richardson et al. 2007; Swain et al. 2007). Initial work by Seager & Sasselov (2000) and a comprehensive study by Brown (2001, hereafter B01) have described various other considerations that should affect the details of transit spectra, including the orbital motion of a planet (a few tens of kilometers per second in the radial direction), the rotation of the planet (a few kilometers per second at the equator, according to the hypothesis that the planet is tidally locked), and winds on the planet’s surface (in B01’s analysis, up to ~1 km s\(^{-1}\)). These physical effects should tend to broaden or impose Doppler shifts on absorption features due to the planet’s atmosphere. B01 constructed an impressively detailed model of radiative transfer through a hot Jupiter’s atmosphere, assuming various models of zonal wind flow superimposed on an equatorial bulk rotation speed of \(v_{eq} = 2\) km s\(^{-1}\), which is approximately the value for HD 209458b under the assumption that it is tidally locked in its 3.5 day orbit. He finds the height of the cloud deck to be the most important parameter that affects the transmission of light through the planet’s atmosphere.

The original discovery of the roughly Jupiter-mass planet in a close, ~4 day orbit around 51 Pegasus (Mayor & Queloz 1995) prompted interest in the dynamics and structure that must govern a highly isolated gas giant planet (Guillot et al. 1996). Observations of the transiting hot Jupiters heightened this interest when they revealed a puzzling feature of these planets: at least several of them are a bit puffier than Jupiter, with diameters ranging from slightly larger than Jupiter’s to as much as ~80% larger. It is not clear what allows some planets to maintain such large radii. It has been suggested that if a Jovian planet migrates very quickly, from its presumed formation location at least several AU from its star to its eventual several day orbit, then it might reach its final home before it has cooled enough to shrink to Jupiter’s radius. Accordingly, some authors have investigated the migration processes that lead gas giant planets to such close orbits as have been found (e.g., Trilling et al. 2002). Others have investigated various ways in which a gas giant could either be heated once it ends up near its star or otherwise maintain sufficient internal energy to sustain its inflated size (Guillot & Showman 2002; Burrows et al. 2003, 2007; Laughlin et al. 2005; Bodenheimer et al. 2003; Guillot 2005; Chabrier & Baraffe 2007). Although various physical mechanisms have been suggested as the apparently missing energy source that allows the unexpectedly large radii sometimes seen, the lesson of these investigations in toto is that it is not easy to explain the inflated sizes, either in terms of the greater stellar flux that these planets experience by virtue of being so close to their stars, or in terms of their evolutionary migratory histories. A recent paper by Winn & Holman (2005) proposes that, contrary to the commonly accepted paradigm, hot Jupiters might be trapped in a Cassini state with large obliquity, in which the spin axis precesses in resonance with the orbit, but lies nearly in the orbital plane. Such a state might be stable against perturbation, and yet be able to generate sufficient internal energy to increase a gas giant planet’s radius to the observed values. In light of an even more recent analysis by Levraud et al. (2007), however, it appears that the probability of capture into a Cassini state 2 resonance is quite small for a planet with semimajor axis \(a < 0.1\) AU. Furthermore, Fabrycky et al. (2007) argue that even if a planet is captured into Cassini state 2, it is likely to remain there for a time that is short relative to the age of the system.

High-resolution transit spectra that have high signal-to-noise ratios (S/Ns) will allow us to distinguish between various models of orbit, rotation, and weather, as discussed by B01. Because the orbit is known to high accuracy, and the predictions of the effects of weather (or climate) are highly uncertain, as described above, we focus in this paper on the much more easily predicted effect of a planet’s rotation on a transit spectrum. If we neglect winds, then the large-obliquity Cassini state described by Winn & Holman (2005) should have a spectral signature that is very similar to that of a nonrotating model. In contrast, the rotation of a tidally locked planet should impose a Doppler distortion on spectral lines arising from the planet’s atmosphere that is roughly an overall redshift during ingress, as the planet is just entering the stellar disk, and a similar distortion that is roughly an overall blueshift during egress, as the planet is just exiting the disk. During mid-transit, the spectral distortion is more similar to rotational broadening. In the present investigation, we address whether there is any hope that these spectral distortions from tidally locked rotation can be observed. In our study, we focus only on the sodium doublet detected by Charbonneau et al. (2002). As we show below, the sensitivity of a measurement of rotation scales with the square root of the number of lines under consideration. Model spectra from, e.g., Sudarsky et al. (2003) and Barman (2007), predict a strong potassium doublet at ~770 nm, strong water absorption features in the near-infrared, and a handful of near-UV lines. If some of these are confirmed in the atmosphere of a transiting planet, they will provide a modest increase in S/N. Since the sodium lines are expected to be the strongest, however, it seems unlikely that observing multiple lines will yield a boost in S/N by more than a factor of a few.

We emphasize that it may not be at all justified to neglect winds. It is quite likely that there are super-rotational winds on hot Jupiters, which are probably necessary to heat the “night” side. As indicated above, some models predict, and the observed phase curve for HD 189733b suggests, that at the photosphere these winds might be significantly (100% or more) greater than the equatorial rotation rate, and therefore might contribute importantly to the Doppler distortion induced by the motion of the planet’s atmosphere. Nevertheless, in order to isolate the contribution of rotation, we do neglect winds in this study. The Doppler distortions that we predict can therefore probably be taken as a lower bound on the distortions that would be observed for a tidally locked transiting hot Jupiter.

We find that the spectral shifts induced by rotation will be difficult to detect with current technology, but perhaps not insurmountably so, at least with technology that might be available in the not-too-distant future. The measurements we describe are limited by a paucity of photons. As such, their signal-to-noise ratio will be enhanced by a bright star and a puffy planet (i.e., a planet with a large scale height). HD 209458 is at least a magnitude brighter than any other star with a known transiting planet except HD 189733, and its planet is larger than HD 189733b; so HD 209458b should be a better target than any other known transiting planet except possibly HD 189733b. In this paper, we model the HD 209458b system because it is the best-studied system, and it is unlikely that any currently known planets would be significantly better targets. In a single transit, observations of HD 209458 with a 6 m telescope that has a high-resolution (>50,000) optical spectrograph with good throughput (~18%) could only show the influence of tidally locked rotation at the ~0.2 \(\sigma\) level. With ultrahigh resolution (700,000) and good throughput (~4%) this effect would still only show up at the ~0.6 \(\sigma\) level. In less than a year, the signal of rotation could be present at 5 times the noise (S/N = 5). Of course, a telescope with larger collecting area, higher spectral resolution, or better throughput would cause the signal to be apparent at that significance level in less time.

Other studies have approached the problem of determining the rotation rate from a different angle. Seager & Hui (2002) and Barnes & Fortney (2003) suggest that an oblate spheroid will have
a transit light curve different from that of a perfect sphere, and so measuring the oblateness from transit photometry will provide a handle on the rotation rate. The oblateness is somewhat degenerate with several other parameters that are not perfectly known, however, so they conclude that it would be difficult to actually determine that rotation rate in this manner. The method we describe here could eventually prove to be an important complement to other observations to constrain the rotation rate.

In the remainder of this paper, we address this idea in detail. One complication that we discuss below is that the technique of this paper is not immune from several near-degeneracies among the many attributes of transiting extrasolar planets that influence light curves or spectra. Although it is likely that current or near-future instruments will be sensitive enough that the spectral distortion imposed by HD 209458b’s rotation (if it is tidally locked) is visible, it might still be very challenging to discern the fingerprint of rotation from other attributes that affect the spectra at a similar level. In this paper, we tackle the forward problem of calculating the amount of distortion that is caused by rotation. The inverse problem—determining from observations whether a planet is tidally locked—is more difficult and should be the topic of a future study.

The structure of the rest of this paper is as follows: In § 2 we describe qualitatively what happens to the starlight received on Earth when a planet transits its star; we give a rough order of magnitude estimate of the magnitude and detectability of the spectral distortions caused by tidally locked rotation; and we briefly describe some technological progress and remaining challenges relevant to our task of deducing bulk motions in a planet’s atmosphere from transit spectra. In § 3 we describe our computational model of a transit spectrum. In § 4 we describe the results of our model according to various assumed input parameters. In § 5 we discuss the scaling of S/N on various model parameters, and we address the prospects of actually observationally determining whether a transiting planet is tidally locked. In § 6 we conclude by describing various ways to boost our predicted S/N to a more optimistic value.

2. OVERVIEW OF THE PROBLEM

The practical feasibility of the investigation we undertake depends on a few factors: understanding the various detailed processes that affect the starlight that reaches Earth when a planet transits its star; the magnitude of the distortion that tidally locked rotation induces; and the technology available to measure such distortions. In this section, we give an overview of these three factors—in particular, in § 2.2 we give a simple estimate of the results that we later (in § 4) calculate in detail.

2.1. Relevant Processes

A planet transiting in front of its star affects the starlight that ultimately reaches Earth in many ways. The motion of the planet’s atmosphere (rotation and winds) is a small perturbation on top of several more dominant effects. We therefore summarize below the physical processes that are at least as significant as the effect of tidally locked rotation. Figure 1 schematically represents this situation, and captures nearly all of the processes described below: a rotating planet (of exaggerated relative size) transits in front of a rotating star. The figure depicts a snapshot partway through ingress, when half of the planet is in front of the stellar disk. The white circle indicates a hypothetical sharp demarcation between the opaque part of the planet (black) and the optically thin part, labeled “Atmosphere” (described further below).

1. Geometric occultation.—The largest effect is an overall dimming by a factor of roughly the ratio of the area of the planet to that of the star: $(R_p/R_\ast)^2$. Since stars are not perfectly uniform disks, but instead tend to darken toward the limb at most visible wavelengths, the fractional dimming due to being in the planet’s shadow tends to be slightly less than the ratio of the areas when the planet is near the edge of the stellar disk and slightly more than this ratio when the planet is near the center.

2. Stellar wobble.—The primary spectral effect of the planet orbiting the star is the radial velocity wobble induced by the planet’s gravity. This periodic spectral shift is of course in effect during the transit, when, for a close-in planet like HD 209458b, it has an influence on the order of $\sim\pm10$ m s$^{-1}$. This effect is a redshift as the planet begins to transit across the disk (during ingress) and a blueshift during egress.

3. Rossiter-McLaughlin effect.—A more subtle effect arises because, during the transit, the planet moves across—and therefore blocks—parts of the star that have different recessional velocities. If (as is expected) the planet’s orbit is aligned with the star’s spin, then during ingress the planet is blocking a part of the star that is slightly blueshifted, and during egress it is blocking a part of the star that is slightly redshifted. Figure 1 illustrates the planet blocking some of the bluest parts of the star during ingress.

The parts of the star that are occulted during ingress/egress have spectra that are blue/redshifted by a velocity that is approximately the equatorial rotational speed of the star, or about $\sim1-2$ km s$^{-1}$ for a Sun-like star. As the figure indicates, during ingress/egress the integrated spectrum of the remaining (unblocked) parts of the star is on average slightly redder/bluer than it would be if the planet

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4 The planet is above the star’s equator to represent a slight inclination in its orbit.
were entirely transparent. Therefore, during ingress the centroids of stellar lines are shifted slightly to the red, and during egress the centroids are correspondingly shifted to the blue.

This so-called Rossiter-McLaughlin effect (RME), described originally by Rossiter (1924) and McLaughlin (1924) in the case of eclipsing binary stars, adds to the shifts already caused by the radial velocity induced by the planet’s gravity, described in process 2 above. The RME has been described in depth more recently in the context of extrasolar planets by Ohta et al. (2005), Giménez (2006), and Gaudi & Winn (2007). These centroid shifts are expected to be comparable in magnitude to the radial velocity wobble from the planet’s gravity, and can be roughly estimated as

$$|\delta v_{\text{R-M}}| \approx 1 \text{ km s}^{-1} \times (R_p/R_*)^2 \approx 10 \text{ m s}^{-1}.$$ 

In fact, the amount of the shift can be predicted precisely for a given orientation of the planet’s orbit, and so measuring the shift is tantamount to measuring the alignment between the star’s spin and the planet. Three years ago, Winn et al. (2005) first found that the spin of HD 209458 and the orbital plane of its planet are nearly aligned. The degree of alignment has been measured for two other systems—Winn et al. (2006) found that the spin of HD 189733 and its planet are also nearly aligned, and Narita et al. (2007) measured a misalignment between these two vectors by ~30° ± 20° in the TrES-1 system.

4. Planet’s atmospheric opacity.—Furthermore, a gas giant planet’s opacity surely does not have a perfectly sharp discontinuity at an outer boundary, as a billiard ball does. Instead, it has an extended atmosphere in which the opacity must vary more or less smoothly. There may be a cloud layer, below which the planet is entirely opaque to tangential rays and above which the opacity varies smoothly. Most critical to our investigation, at a given radius, the planet’s opacity to tangential lines of sight must vary with wavelength, depending on the contents of its atmosphere. At wavelengths of strong atomic or molecular transitions, the planet’s atmosphere will be more opaque than at other wavelengths. As a result, the effective radius of the planet, or the radius at which the optical depth along a tangential ray is of order unity, is greater at some wavelengths than at others. These effects have been described in detail by B01.

5. Planet’s orbital motion.—The motion of the planet’s atmosphere must influence the transit spectrum in several delicate ways. As B01 points out, there are three main mechanisms by which the motion of a planet’s atmosphere relative to its star can affect the spectrum: the planet’s orbital velocity along the line of sight, the planet’s (possibly tidally locked) rotation, and winds in its atmosphere. The largest effect of these three is the orbital velocity, which imposes a bulk blueshift/redshift during ingress/egress of ~15 km s⁻¹ to spectral lines arising from the planet’s atmosphere. These shifts are of opposite sign to the radial velocity wobble and to the shifts from the RME, and therefore tend to lessen the apparent RME slightly.

6. Planet’s atmospheric motion.—The most dynamically interesting (and subtlest) effects are those caused by the planetary rotational velocity and atmospheric winds. Since a tidally locked planet rotates in the same sense as it orbits, the rotational velocity of its outside edge has the same sign as its orbital velocity, and the rotational velocity of its inside edge has the opposite sign. As a result, during the beginning of ingress and the end of egress, when only the inside edge of the planet is over the star, tidally locked rotation will impose a spectral distortion that is in the opposite sense of that caused by the bulk orbital velocity described in process 5 above, and that is in the same sense as the RME: the distortions are roughly equivalent to a relative redshift during ingress (graphically represented in Fig. 1) and a relative blueshift during egress. During mid-transit, with some parts of the rotating planet’s atmosphere moving toward and other parts away from the star relative to an otherwise identical but nonrotating planet, the overall influence of the planet’s rotation is approximately equivalent to rotational broadening.

Winds complicate the picture even further. It is likely that winds tend to rush from the substellar hot spot to the colder night side of the planet. With the substellar point on the opposite side of the planet from Earth during a transit, this corresponds to winds rushing toward us at several hundred to several thousand meters per second. This would tend to blueshift the spectrum throughout the transit. Zonal wind bands, somewhat similar to those on Jupiter but with much higher speeds, or other more detailed winds can have an even more intricate effect.

7. Additional effects.—If a transiting planet were to have non-zero orbital eccentricity, or rings, these could complicate a measurement of rotation rate. Nonzero eccentricity would break the symmetry between ingress and egress. Still, if the orbit were well known, this could be modeled and taken into account. It seems unlikely that a hot Jupiter could maintain rings: icy rings would sublime, and, if not continuously replenished, dusty/rocky rings would quickly succumb to the Poynting-Robertson effect (Poynting 1903; Robertson 1937). But if, somehow, a ring were to find a way to persevere around a transiting hot Jupiter, it could confound—perhaps hopelessly—a measurement of rotation. The consequences of rings for the RME are addressed in Ohta et al. (2006). Saturn’s rings are nearly 4 times the area of the planet, so for a planet (with equatorial rings that are as relatively large as Saturn’s) whose orbit is tilted an angle of 0.1 (in radians) from edge-on, the rings would be ~40% the area of the planet, which would increase the RME by ~40%. Uncertainty about the presence and size of a ring introduces an uncertainty in the size of the RME effect that is probably larger than the size of the rotation effect. Furthermore, a ring would occlude a (small) part of the planet’s atmosphere, which would (slightly) reduce the strength of the rotation signal.

Other interesting phenomena that primarily affect a transit light curve, rather than the spectrum, include star spots (Silva 2003), atmospheric lensing (Hui & Seager 2002), and finite-speed-of-light effects (Loeb 2005). Although Winn & Holman (2005) describe a possible configuration (Cassini state 2) that would produce a spectral signature that is nearly identical to what would be expected from a nonrotating planet, the likelihood that any hot Jupiters are in such a configuration might be low, and it seems quite likely that some transiting planets are not in this state. Nonetheless, the motion of a transiting planet’s atmosphere—rotational, wind, or other—is clearly interesting, and the basic technique that we describe below is applicable to any model of atmospheric motion.

2.2. Preview of Results

A rough estimate of the velocity shift that is imposed during ingress to the centroids of the stellar Na D lines by the planet’s tidally locked rotation (on top of the RME and the shift from the planet’s orbital velocity, both of which would be present even if the planet were not rotating) is the following:

$$\delta v \approx \left(\cos \phi / \pi/2\right) \left(1 R_p^2 / 2 R^2\right) \left(2 \pi R_\text{eq} \Pi \text{atm} / \pi R_p^2 \right) v_\text{eq}$$

$$\approx 0.64 \times 1\% \times 15\% \times 2000 \text{ m s}^{-1}$$

$$= 1.9 \text{ m s}^{-1}.$$ (1)
In this equation, \( \phi \) is a planet-centered azimuthal angle, \( R_p \) and \( R \) are the planet’s and star’s radius, respectively, \( \Pi_{\text{atm}} \) is the height of the planet’s atmosphere, and \( v_\text{eq} \) is the equatorial rotation speed. The rotation speed at angle \( \phi \) is \( v_\text{eq} \cos \phi \). We take the average of \( \cos \phi \) from \(-\pi/2\) to \(\pi/2\) to get the average planetary rotation speed. We have used \( \Pi_{\text{atm}} = 7500 \) km, or 15 times the presumed scale height of 500 km, because the sodium lines are so heavily saturated that at the assumed abundance and cloud deck height in our model the line cores do not become optically thin until that height. Burrows et al. (2004) and Fortney (2005) describe how the optical depth along tangential rays is greater than the optical depth on normal rays. The product

\[
\delta_{\text{atm}} \approx \left( \frac{1}{2} \frac{R_p^2}{R^2} \right) \left( \frac{2\pi R_p \Pi_{\text{atm}}}{\pi R_p^2} \right) = \left( \frac{R_p}{R} \right)^2 \left( \frac{\Pi_{\text{atm}}}{R_p} \right)
\]

is the ratio of the area of the portion of the planet’s atmosphere that is in front of the star halfway through ingress to the total area of the disk of the star. Based on this estimate, we expect a maximum velocity shift of \( \delta v \approx 190 \) cm s\(^{-1}\). If we take into account that HD 209458b’s orbit is actually slightly inclined relative to the line of sight, the cosine average decreases to \(\sim 0.45\), and the total estimate decreases to \(\sim 140 \) cm s\(^{-1}\). This estimate is in reasonably good agreement with the centroid shifts predicted by the full model calculation below (\(\sim 60\) cm s\(^{-1}\)); the difference between the estimates is most likely due to the difference between the shapes of the stellar and planetary lines.

We now estimate the S/N for the detectability of this effect in an observation of duration \( \Delta t \), with a telescope that has diameter \( D \) and throughput efficiency \( \eta \). The signal is the distortion of the spectrum relative to a nonrotating planet, and for now we assume that the noise is dominated by photon noise. If a spectrum \( F' [\lambda] \) with a symmetric absorption feature of depth \( \Delta F \) centered at \( \lambda_0 \) is redshifted by an amount \( \Delta \lambda \) to \( \tilde{F}' [\lambda] \equiv F'[\lambda - \Delta \lambda] \), what is the integrated absolute difference \( \int F' - \tilde{F}' \) over some wavelength range \( 2 \lambda_0 \) centered on \( \lambda_0 \)? If the absorption feature is wide compared with \( \Delta \lambda \), then by symmetry,

\[
S = \int_{\lambda_0 - \Delta \lambda}^{\lambda_0 + \Delta \lambda} F[\lambda] - F[\lambda - \Delta \lambda] \, d\lambda \\
= \int_{\lambda_0 - \Delta \lambda}^{\lambda_0 + \Delta \lambda} 2 \left( F[\lambda] - F[\lambda - \Delta \lambda] \right) \, d\lambda; \quad (2)
\]

and if \( \Delta \lambda \) is small then

\[
S \approx 2 \Delta \lambda \int_{\lambda_0}^{\lambda_0 + \Delta \lambda} F' [\lambda] \, d\lambda \\
\approx 2 (\Delta \lambda) \Delta F. \quad (3)
\]

We may now estimate the S/N of our effect (for a single absorption line) using the lesson of equation (3), provided we know the absolute normalization of the stellar spectrum (the number of photons per unit wavelength). A spherical blackbody of radius \( R \), and temperature \( T_* \), at distance \( d \) from the telescope, has a photon number flux at wavelength \( \lambda \) of

\[
\frac{dN}{d\lambda} \sim B [\lambda, T_*] \left( \frac{1}{hc} \right) \left( \frac{\pi R^2}{d^2} \right) \eta \pi (D/2)^2 \]

\[
= \frac{\pi^2 c}{2 \lambda^4 (\exp \{ \hbar c/(\lambda k T_*) \} - 1) \eta} \left( \frac{R_*}{d} \right)^2, \quad (4)
\]

where \( B_\lambda \) is the Planck function. Since the fractional decrease in the spectrum at the line center is approximately \( \delta_{\text{atm}} \), we may express the parameter \( \Delta F \) from equation (3) as \( \Delta F \approx \delta_{\text{atm}} (dN/d\lambda) \). Similarly, since the rms velocity shift during ingress is \( (\lambda^2)^{1/2} \sim (2000 \text{ m s}^{-1}) = 1000 \text{ m s}^{-1} \), we may express the parameter \( \lambda \) as \( \lambda \sim (\lambda^2/c) \times \lambda_0 \). The distortion (the signal) from a single line can therefore be estimated as

\[
S = \left\{ 2 (\Delta \lambda) \left( \frac{\delta_{\text{atm}}}{\lambda_0} \frac{dN}{d\lambda} \right) \right\} \Delta t \\
= \frac{\pi^2 c}{2 \lambda^4 (\exp \{ \hbar c/(\lambda k T_*) \} - 1) \eta} \left( \frac{R_*}{d} \right)^2 \left( \Delta \lambda \right) (\Delta t). \quad (5)
\]

The shot noise is the square root of the number of photons in a wavelength range \( 2 \lambda \), roughly equal to the FWHM of the line, or about \( 7 \) km s\(^{-1}\) for a heavily saturated line such as the Na D lines under consideration:

\[
N \sim \sqrt{\frac{dN}{d\lambda} (2 \lambda) (\Delta t)} \\
\sim \sqrt{\frac{\pi^2 L c (\Delta t)}{2 \lambda^4 (\exp \{ \hbar c/(\lambda k T_*) \} - 1) \eta} \left( \frac{R_*}{d} \right)^2} \quad (6)
\]

We estimate the total S/N arising from a single absorption line, during an ingress integration of duration \( \Delta t \), to be roughly

\[
\frac{\delta_{\text{atm}}}{\lambda_0} \frac{dN}{d\lambda} \left( \frac{\Delta \lambda}{\lambda} \right) \left( \frac{\sqrt{c \Delta t}}{L} \right) \left( \frac{R_*}{d} \right)^2 \sqrt{\eta} \\
\sim (6.6 \times 10^{-4}) (3.3 \times 10^{-6}) (2.1 \times 10^{10}) (5.0 \times 10^{-3}) \sqrt{\eta} \\
\sim 2.3 \sqrt{\eta}. \quad (7)
\]

The above calculation uses parameters for HD 209458 and its planet, a Na D line, and a 6 m telescope: \( \lambda = 600 \) nm, \( \Delta t = 1000 \) s, \( R_* = 7.3 \times 10^{10} \) cm, \( T_* = 6100 \) K, \( d = 47 \) pc, and \( D = 600 \) cm. For two identical absorption lines, we gain a factor of \( 2^{1/2} \) in S/N, and for egress we gain another factor of \( 2^{1/2} \), giving a total one-transit S/N of roughly \( 4.6 \eta^{1/2} \), not counting the additional signal available during mid-transit (see further discussion below). This S/N ratio is in principle independent of the spectral resolution of the spectograph, for sufficiently high spectral resolution. For low spectral resolution, however, the S/N could be lower than this estimate (below, we conclude that the S/N loses its dependence on resolving power for spectral resolution \( \gtrsim 500,000 \)).

There were several optimistic assumptions that went into this estimate. Still, this rough estimate of the degree to which a planet’s rotation influences its transit spectrum indicates that the more in-depth study that we perform below is warranted.

2.3. Available Technology

Detecting the centroid shifts caused by tidally locked rotation (1 m s\(^{-1}\)) will require very precise measurements of stellar transit spectra. Obtaining such high-precision spectra will be quite challenging for a number of reasons, several of which were described in the ground-breaking paper by Butler et al. (1996) that analyzes the limits of Doppler precision. Of particular concern, stellar pulsations and turbulent motions in stellar photospheres can cause small regions of the stellar disk to move at up to 300 m s\(^{-1}\).

\footnote{We write \( \frac{1}{2} \) (2000 m s\(^{-1}\)) because the mean value of \( \cos^2 \) from \(-\pi/2\) to \(\pi/2\) is \(\frac{1}{2}\).}
(Dravins 1985; Ulrich 1991). These motions tend to average out to produce stellar spectra that are largely stable; but it is likely that at least some giant convection cells are not small relative to the size of a planet, and these could introduce a contaminating source of noise when they are located behind the planet or its atmosphere. Butler et al. (1996) reviewed what was then known about the variability of stellar line profiles; the upshot is that line widths may vary by up to several meters per second over several years, but it is not clear to what extent spurious apparent velocity shifts may be induced by convection, and such stellar jitters may prove to be a significant source of noise that would render it difficult to measure sub-meter-per-second velocity shifts. More recently, Bouchy et al. (2005a) have actually achieved sub-meter-per-second accuracy with the HARPS instrument (spectral resolution of 115,000), and they have found a dispersion in night-averaged radial velocity measurements for a particular star (HD 160691) of ~0.4 cm s⁻¹ for nights when they took many (~200) observations. Since in our situation (e.g., taking spectra during ingress) we have minutes, not hours, available, the rms scatter in ingress-averaged radial velocity measurements is likely to be larger than what they found. In addition to the difficulties posed by several systematic sources of noise, achieving sufficient photon statistics will be difficult for two reasons: for a given throughput efficiency η, higher spectral resolution means fewer photons per bin; and η tends to decrease with increasing spectral resolution R_S.

By the mid-1990s, the timeless quest for high-resolution spectrographs reached a milestone at the Anglo-Australian Telescope with the development of UHRF and its resolving power of up to 1,000,000 (Diego et al. 1995). Despite impressive throughput relative to previous endeavors, however, its efficiency was insufficient to obtain the sub-decimeter-per-second Doppler precision on a V ≥ 7 star that would be required for planet searches. With a R_S = 600,000 spectrograph built at Steward Observatory, Ge et al. (2002) obtained stellar spectra with R_S ~ 250,000 and throughput of 0.8%. Furthermore, they predicted that by optimizing their technology they could increase the throughput to 4%. More recently, Ge et al. (2006) detected a new planet, around HD 102195, with the Exoplanet Tracker instrument at Kitt Peak. This instrument has a resolution of R_S ~ 60,000 and a total throughput of 18%. Plans for a spectrograph that has a resolving power of 120,000 on a 30 m telescope (Tokunaga et al. 2006) give cause for optimism that plans for a spectrograph that has a resolving power of 120,000 on a 30 m telescope (Tokunaga et al. 2006) give cause for optimism that it is possible that the HD 209458b system where possible.

The planet is modeled as an inner component that is entirely opaque and an outer component that is isothermal and drops off exponentially. We compute the wavelength-dependent optical depth due to the Na D doublet at ~590 nm in the planet’s atmosphere; important parameters include the temperature and density of the planet’s atmosphere and its Na content. We use the Voigt profile—as described by, e.g., Press & Rybicki (1993)—to calculate τ/λ, the optical depth to absorption along the line of sight.

As the planet transits the star, there are four points of “contact” between the planet and the star (really between their projections on the sky): when the disk of the planet first touches the disk of the star; when the planet is first entirely over the stellar disk; when the planet is last entirely over the stellar disk; and when the planet last touches the stellar disk. We additionally sometimes refer to “1.5th contact” (halfway between first and second contact), and analogously to 2.5th and 3.5th contact.

As described in § 2, the type of distortion that a planet’s rotation imposes relative to a nonrotating planet changes depending on when during the transit the observation is made. During ingress or egress, the rotation of a tidally locked planet’s atmosphere will impose a distortion similar to an overall shift relative to a non-rotating planet: redshift during ingress, and blueshift during egress. When the planet is in mid-transit, in the middle of the stellar disk, the overall distortion to the spectrum imposed by its rotation is akin to a star’s rotational broadening. Since the line centers of the lines we are considering are heavily saturated and therefore flat at their cores, rotational broadening has the somewhat counterintuitive effect of steepening the cores of the profiles while broadening the wings. We discuss this in greater detail in the next section. Although the type of distortion is different during ingress and egress from during mid-transit, it turns out that the amount of distortion, in terms of S/N ratio, is nearly constant throughout transit. This, too, we discuss in § 4.

We simulate the HD 209458b system, with a 1.32 R_J planet in a 3.5 day orbit, orbiting a G0 star at radius 1.05 R_☉ that is 47 pc away. Our model star has the limb-darkening profile that Knutson et al. (2007b) measured for HD 209458. In order to approximate the fits to the data in Charbonneau et al. (2002) we assign our model planet’s atmosphere a sodium content (1% solar) and cloud deck height (0.01 bars) that are comparable to the parameter combinations that result in the best fits in that paper. Finally, we present results at our simulation’s spectral resolution (R_S = 700,000), and we simulate transit events observed using two different lower resolution spectrographs, one with spectral resolution R_S = 50,000 and one with R_S = 150,000. All spectrographs (and associated optical paths) in our simulations have 100% throughput efficiency. In the remainder of this section, we provide a detailed description of our parameterization of the problem.

3.1. Parameters of the Star

The parameters related to the star are listed in Table 1. They are set to match measured parameters for HD 209458, and we use the limb-darkening profile from Knutson et al. (2007b). We normalize the flux to that of a blackbody of temperature T_☉ of the size and at the distance of HD 209458.

3.2. Parameters of the Planet

The parameters related to the planet are listed in Table 2. We model the planet as an inner component that is essentially a biliard ball (completely opaque at all wavelengths) and an outer initial study of the spectral effect of the motion of a transiting planet’s atmosphere, separating rotation from other processes makes the problem more tractable. We set parameter values to match measured values from the HD 209458b system where possible.

The parameters of the planet are listed in Table 2. We model the planet as an inner component that is essentially a biliard ball (completely opaque at all wavelengths) and an outer...
component that is an isothermal atmosphere with scale height
\( H = R_g T_p \mu g \), where \( R_g \) is the gas constant, \( \mu \) is the molar mass, and \( g \) is the acceleration of gravity. The density of our model planet’s atmosphere varies as \( \rho = \rho_0 \exp [(r - R_p)/H] \), where \( R_p \) is the radius of the optically thick part (some authors have called this radius the “cloud deck” [Charbonneau et al. 2002]). This hypothetical cloud deck could cause the planet to be optically thick at a higher altitude than would otherwise be expected, as discussed in, e.g., Richardson et al. (2003) and Sudarsky et al. (2000). The cloud deck causes the optical depth as a function of radius in our model to have a singular discontinuity at radius \( R_p \).

3.3. Spectral Parameters

The parameters pertaining to the shape of the observed spectrum are listed in Table 3. In addition to the location of the planet within the stellar disk, the shape of the stellar spectrum and the wavelength-dependent opacity of the planet’s atmosphere together influence the transmission spectrum.

\( \text{Spec}_\text{Shape} \) is a parameter that can take on the values “Flat,” “Blackbody,” or “Solar,” and determines the rest-frame spectrum of the model stellar photosphere. (The integrated stellar spectrum is the convolution of the rest-frame spectrum with the stellar rotation profile.) When “Flat” is chosen, the rest-frame model stellar spectrum intensity is set to the mean value of the blackbody intensity in the specified wavelength range \([\lambda_{\min}, \lambda_{\max}]\), which, in our simulation, is set to [580 nm, 600 nm]. When “Solar” is chosen, the model stellar spectrum intensity is set to a high-resolution solar spectrum that is normalized to the flux from HD 209458b,6 but the Na D lines in this high-resolution spectrum have been replaced by Gaussian fits to the solar lines.

The planet’s atmosphere has \( N_{\text{abs}} \) absorption features, each of which is due to an element with a given fraction of the solar abundance. In the models presented in this paper, \( N_{\text{abs}} = 2 \): we consider the Na doublet at 588.95 and 589.59 nm, with sodium at fractional abundance \( f_\odot = X_{\text{Na}}/X_{\text{Na}} \odot = 0.01 \) of the solar abundance. Each line is modeled as a Voigt profile, as described in, e.g., Press & Rybicki (1993).

3.4. Parameters of Observing and Computing

The final set of parameters, listed in Table 4, includes those that specify the observer and those that determine how the observation is discretized for the purpose of numerical computation. The model observational setup is determined by three parameters: the telescope’s diameter \( D \) (6 m in our simulations) and efficiency \( \eta \) (100%), and the spectrograph’s spectral resolution \( R_s \) (we set \( R_s \) to 700,000 for the purpose of computing the model, and we rebinned to lower, more easily achieved resolutions—150,000 and 50,000—after computing a model). These three parameters prescribe the sizes of the spectral bins and the rate at which those bins are capable of collecting light.

---

**Table 1**

| Parameter       | Description                  | Value |
|-----------------|------------------------------|-------|
| \( M_p \)       | Planet mass                  | 1.05 \( M_J = 1.31 \times 10^{33} \) g |
| \( R_p \)       | Optically thick planet radius| 1.32 \( R_J = 9.44 \times 10^{7} \) km |
| \( P_0 \)       | Planet pressure at \( R_p \)  | 0.01 bars |
| \( H_p \)       | Planet atmosphere scale height| 500 km |
| \( T_p \)       | Planet atmosphere temperature| 1300 K  |
| \( f_{\text{TL}} \) | Frac. tidal locked rot. rate | 0 or 1 (\( t_{\text{TL}} = 0 \) or 2 km s\(^{-1}\)) |
| \( a \)         | Semimajor axis               | 0.046 \( \text{AU} \) |
| No.\( g \)      | Number of scale heights in atm.| 15 |

Note:—Parameter values are set to match measured values from the HD 209458b system where possible.

**Table 2**

| Parameter       | Description                  | Value |
|-----------------|------------------------------|-------|
| \( M_p \)       | Planet mass                  | 0.69 \( M_J = 1.31 \times 10^{33} \) g |
| \( R_p \)       | Optically thick planet radius| 1.32 \( R_J = 9.44 \times 10^{7} \) km |
| \( P_0 \)       | Planet pressure at \( R_p \)  | 0.01 bars |
| \( H_p \)       | Planet atmosphere scale height| 500 km |
| \( T_p \)       | Planet atmosphere temperature| 1300 K  |
| \( f_{\text{TL}} \) | Frac. tidal locked rot. rate | 0 or 1 (\( t_{\text{TL}} = 0 \) or 2 km s\(^{-1}\)) |
| \( a \)         | Semimajor axis               | 0.046 \( \text{AU} \) |
| No.\( g \)      | Number of scale heights in atm.| 15 |

**Table 3**

| Parameter       | Description                  | Value |
|-----------------|------------------------------|-------|
| \( \text{Spec}_\text{Shape} \) | Shape of star spectrum     | Flat, Blackbody, or Solar |
| \( \lambda_{\min} \) | Min. wavelength in sim.     | 580 nm |
| \( \lambda_{\max} \) | Max. wavelength in sim.     | 600 nm |
| \( N_{\text{abs}} \) | No. abs. features in atm.   | 2 |
| \( f_1 \)       | Fractions solar abund., first line | 0.01 |
| \( f_2 \)       | Fractions solar abund., second line | 0.01 |
| \( A_{\text{TL}} \) | Transition prob. first line | 6.16 \times 10^{-7} \text{ s}^{-1} |
| \( A_{\text{TL},2} \) | Transition prob. second line | 6.14 \times 10^{-8} \text{ s}^{-1} |

Notes.—In parameters that have \( i \) and \( k \) subscripts, \( i \) indicates the lower level (3S\(_1\)\(_2\) for both lines) and \( k \) indicates the upper level (3P\(_1\)\(_2\) for the bluer line and 3P\(_1\)\(_1\) for the redder line). The fractional solar abundance is set to 0.01 in order to achieve modest agreement with data observed for the Na D doublet in HD 209458b’s atmosphere.

**Table 4**

| Parameter       | Description                  | Value |
|-----------------|------------------------------|-------|
| \( D \)         | Telescope diameter           | 2.4 – 30 m |
| \( \eta \)      | Spectroscopic efficiency     | 1.00 |
| \( R_s \)       | Obs. spec. resolution        | 50,000 – 700,000 |
| \( T_{\text{int}} \) | Integration time             | 932.088 s |
| \( R_s \)       | Comp. spec. resolution       | 700,000 |
| \( \Delta t \)   | Time-step in integration     | 50 s  |
| \( n_{\text{obs}} \) | No. of star annuli           | 10 |
| \( n_{\phi} \)   | No. of star azimuthal sections| 16 |
| \( n_{\text{annu}} \) | No. of annuli in p. annulus  | 10 |
| \( n_{\phi} \)   | No. of annul. sec. in p. ann.| 10 |
| \( n_{\text{annu}} \) | No. of planet atm. annuli   | 20 |
| \( n_{\phi} \)   | No. of planet atm. azimuth. sections | 20 |

Note.—Parameter values are set to match measured values from the HD 209458b system where possible.
In order to compute the flux at Earth as a function of wavelength, we begin by dividing the stellar disk into \( n_{\theta s} \) concentric annuli, and we divide each annulus into \( n_{\theta p} \) azimuthal sections. In each section, the redshifted spectrum and the normalization must both be computed.

Knowing the stellar rotation rate and axis, we may calculate the recessional velocity of any point on the star’s surface as a function of its location on the projected stellar disk, and we redshift the spectrum from each part of the star accordingly. When the planet is in transit, we separate the stellar disk into an annulus that contains the planet and the rest of the disk that we treat as described above. The annulus that contains the planet is treated almost as above—divided into \( n_{\theta p} \) subannuli, each of which has \( n_{\phi p} \) azimuthal sections—but the subannuli are incomplete, interrupted by the planet.

In order to sample the planet’s atmosphere, we divide the region that overlaps the star into \( n_{\theta p} \) concentric annuli around the planet’s center, each of which is divided into \( n_{\phi p} \) azimuthal sections. In each section, we must determine the optical depth and multiply by \( \exp(-\tau) \). In calculating the optical depth, we note that in the case that the planet’s rotation axis is entirely normal to the line of sight, if the planet rotates as a solid body then the radial component of its recessional velocity is constant along a ray:

\[
\tau[b_p, \phi, \lambda] = N[b_p] \times \sigma \left[ \frac{\lambda}{1 + (v_p[b_p, \phi]/c)} \right],
\]

where the column density is calculated in terms of a function \( G \) that is specified below: \( N[b_p] = n(b_p, R_{p0}, H) \). In equation (8), \( v_p[b_p, \phi] \) is the recessional velocity of the planet, as a function of radius and azimuth, which depends on the orbit and the rotation. Note that there is a single \( v_p \) along a given line of sight defined by a \((b_p, \phi_p)\) pair only under the assumption of solid body rotation. The rest-frame cross-section \( \sigma[\lambda] \) is computed according to the Voigt profile. The function \( G \) is defined as the following integral:

\[
G[b_p, R_{p0}, H] = \int_{-\infty}^{\infty} \exp \left[ -\frac{b^2 + I^2 - R_{p0}}{H} \right] dl \quad b_p > R_p,
\]

\[
\int_{-\infty}^{\infty} \exp \left[ -\frac{b^2 + I^2 - R_{p0}}{H} \right] dl \quad b_p \leq R_p.
\]

4. MODEL TRANSIT SPECTRA

As described in § 2.2, we seek the expected S/N for distinguishing between the spectrum that would be observed due to a nonrotating planet (or one that is in a Cassini state with its rotation axis nearly in the plane of orbit) and the spectrum that would be observed due to a tidally locked planet. The computed model spectrum \( N[\lambda] \) is the time integral of the instantaneous spectrum \( \dot{N}[\lambda] \) and consists of the number of photons detected per wavelength bin:

\[
N[\lambda] \approx \dot{N}[\lambda] \Delta t_{obs},
\]

for some small exposure time \( \Delta t_{obs} \).

The model signal (of rotation) per bin that we are looking for is the difference between the rotating model spectrum \( N_{\text{rot}} \) and the nonrotating model spectrum \( N_{\text{no rot}} \):

\[
S_b = \left( \dot{N}_{\text{rot}}[\lambda] - \dot{N}_{\text{no rot}}[\lambda] \right) \Delta t_{obs}.
\]

We make the optimistic approximation that the noise per bin is just the photon noise:

\[
N_b = \sqrt{N_{\text{no rot}}[\lambda] \Delta t_{obs}}.
\]

The total S/N in a single exposure, then, is the sum in quadrature of \( S_b/N_b \) for all wavelength bins \( \lambda_i \):

\[
S/N = \sqrt{\sum_{\lambda_i} \left( \frac{N_{\text{rot}}[\lambda_i] - N_{\text{no rot}}[\lambda_i]}{N_{\text{no rot}}[\lambda_i]} \right)^2 \Delta t_{obs}}.
\]

A similar summation in quadrature applies over all exposures. Note that, in principle, the expression in equation (12) is insensitive to the sizes of bins and hence to the spectral resolution \( R_s \), as long as the bins are small relative to the Gaussian width of the absorption feature under consideration. Our simulations indicate that the spectral resolution must be \( \geq 500,000 \) in order for S/N to be nearly independent of \( R_s \).

The effect of rotation, both during ingress and during mid-transit, is illustrated in Figure 2. For illustrative purposes, in this figure we assume a uniform star (flat spectrum, nonrotating, no limb darkening). In Figure 2 (left panels) we show a snapshot during ingress (at 1.5th contact), and in Figure 2 (right panels) we show a snapshot during the middle of a transit (2.5th contact). The quantity plotted is \( R' = R - 1 \) from B01, where

\[
R[\lambda, t] = \frac{N_{\text{in transit}}[\lambda, t]}{N_{\text{out of transit}}[\lambda, t]}.
\]

Figure 2 (bottom panels) shows the difference spectra between the models with a tidally locked planet and the models with a nonrotating planet (\( R_{\text{rot}} - R_{\text{no rot}} \)).

As described in § 2, a planet’s rotation causes the centroids of stellar absorption features to shift relative to a nonrotating planet. In Figure 3, centroid shifts (in velocity units) are plotted as a function of position in transit for a planet transiting in front of a realistic star model with a Sun-like spectrum. The recessional velocity increases roughly sinusoidally during ingress, reaching a peak of about 60 cm s\(^{-1}\) at 1.5th contact. During mid-transit, between second and third contacts, the net velocity shift is much smaller. Egress is nearly perfectly symmetrical with ingress, although the velocity shifts have the opposite sign.

The cumulative and incremental S/N across the transit are shown in the top and bottom panels, respectively, of Figure 4. As a planet proceeds in its transit from first to fourth contact, the S/N builds up steadily throughout the transit, as can be seen in the top panel of this figure. This cumulative increase reflects the steady incremental S/N per second of observation, which, for a planet crossing in front of a uniform star, is shown in the bottom panel. The three curves in the bottom panel represent, in decreasing order of S/N, the incremental S/N curves that are expected for a spectrograph with ultrahigh resolution (the simulation’s resolution \( R_s = 700,000 \)), for a spectrograph with as high resolution as bHROS on Gemini (\( R_s = 150,000 \)), and for a spectrograph with more standard high resolution (\( R_s = 50,000 \)). In the top panel we show the cumulative S/N (assuming the simulation’s resolution) for a planet in front of a uniform star and for a planet in front of a more realistic Sun-like star that rotates once per month, has a limb-darkening profile, and has a solar spectrum. It is apparent in Figure 4 (top) that a spectrograph with our simulation’s resolution would see the effect of rotation at S/N = 7.1\( \eta^{1/2} \) in a single transit in the case of the simplified uniform star, and at S/N = 3.2\( \eta^{1/2} \) in one transit of the realistic
The effect of including in the simulation the realistic features of stellar rotation, limb darkening, and a solar spectrum is therefore to depress the S/N of the effect of tidally locked rotation by a factor of slightly more than 2. Figure 4 (bottom) indicates that our predicted S/N for one transit of a realistic star might be adjusted downward by 50\%, depending on the spectral resolution, indicating a total one transit S/N for the case of the realistic star of about 1.7\(\tilde{\eta}\)\(^{1/2}\). Finally, we do not show the incremental S/N for the realistic star model, but, as the top panel of the figure indicates, a larger proportion of the total signal comes from ingress and egress in the realistic star model than in the uniform star model.

5. DISCUSSION

The S/N scales with the square root of the number of absorption lines under consideration, with the square root of the number of transits, and with several other parameters, as follows:

\[
S/N \sim \left( \frac{N_{\text{abs}}}{2} \right)^{1/2} \left( \sqrt{\text{No. transits}} \right) \left( \frac{D \sqrt{\tilde{\eta}}}{6 \text{ m}} \right) \\
\times 10^{-0.2(V-V_0)} \left( \frac{R_0}{R_p} \right)^{-2} \left( \frac{R_{\text{H}}}{{R_H}} \right) \\
\times \left( \frac{v}{2000 \text{ m s}^{-1}} \right) \left( \psi \left[ R'_S \right] \right) (S/N)_0. \tag{14}
\]

Fig. 2.—Top panels: Snapshot spectra for one of the Na D lines for two different model planets (tidally locked and nonrotating). Bottom panels: Difference between the two model spectra. The quantities plotted are \(R' = R - 1\) (top panels) and \(\Delta R'\) (bottom panels), where \(R(\lambda, t) = N_{\text{abs}}(\lambda, t)/N_{\text{abs}}(\lambda, t_0)\). In the top panels, the solid curve is the tidally locked planet’s transit spectrum, and the dashed curve is the nonrotating planet’s transit spectrum. In the bottom panels, the difference between the rotating and nonrotating planet’s spectra. Left: Halfway through ingress (at 1.5\(^{\text{th}}\) contact). Right: Halfway through the whole transit (2.5\(^{\text{th}}\) contact). [See the electronic edition of the Journal for a color version of this figure.]

Fig. 3.—Centroid shift of Na D lines from tidally locked rotation from the beginning to the end of a transit, relative to an identical but nonrotating planet; there is a Sun-like stellar spectrum. The vertical lines denote 1.5\(^{\text{th}}\) and 3.5\(^{\text{th}}\) contact (dotted lines) and second and third contact (dot-dashed lines). Between first and second contact, the spectrum with the rotating planet is redshifted relative to the nonrotating planet by up to about 60 cm s\(^{-1}\); between third and fourth contact, it is blueshifted by the same amount. This plot samples the transit at 60 regularly spaced points. Parameters were chosen to represent the HD 209458 system. [See the electronic edition of the Journal for a color version of this figure.]
In this equation, \( N_{\text{abs}} \) is the number of absorption lines, \( D \) is the telescope’s diameter, \( \eta \) is the efficiency of the instrumental setup, \( V_i \) is the star’s magnitude, \( V_o \) is HD 209458’s magnitude (about 7.6), \( R_s \) is the stellar radius, \( R_p \) is HD 209458’s radius, \( R_p \) and \( H \) are the planet’s radius and scale height, and \( R_{00} \) and \( H_0 \) are HD 209458’s radius and scale height. Also, \( \psi[R_s] \) is a function that characterizes the (nonlinear) response of S/N to the spectral resolution of the spectrograph; for instance, \( \psi[700,000] \approx 1 \), \( \psi[150,000] \sim 0.6 \), and \( \psi[50,000] \sim 0.2 \). Finally, \( (S/N)_0 = 7.1 \) for a uniform star and \( A_{Na}/A_{Na} \approx 0.01 \); \( (S/N)_0 = 3.2 \) for a more realistic (rotating, limb-darkened) star with a solar spectrum.

Culling the signal from mid-transit, however, is much more difficult than doing so from ingress and egress, because the shape of the distortion depends more sensitively on the structure of the planet. For the realistic star, our model predicts \( S/N \sim 1.3\eta^{1/2} \) (with the 6 m telescope, with Poisson-dominated photon noise) for ingress alone, and therefore a factor of \( 2^{1/2} \) greater, or \( S/N \sim 1.8\eta^{1/2} \) for ingress and egress.

In Table 5 we present the number of orbits that must be observed in order to make a 5 \( \sigma \) detection of rotation, for various combinations of parameters. In all cases we assume a realistically achievable optical setup, with spectral resolution \( R_s = 150,000 \) and throughput \( \eta = 4\% \). Improvement in \( R_s \) will yield a modest improvement in \( S/N \), and improvement in \( \eta \) could be quite significant.

In Table 5 we present the total available \( S/N \) for the whole transit, according to our model, while in column (4) (B) we present the available \( S/N \) from just ingress and egress. Columns (5) and (6) are based on the \( S/N \) in column (4).

Finally, as a sanity check, our model can be tested by comparing it to the analysis of Charbonneau et al. (2002). That analysis suggests that the sodium content of a planet’s atmosphere can be determined by comparing the flux in a narrow band centered on the sodium resonance lines with the flux in a wider band surrounding but excluding the lines. In that paper, the decrement in the narrow band containing the sodium features is named \( n_{Na} \), and was measured to be \( 2.32 \times 10^{-4} \). For HD 209458 during the middle of transit. They presented several models, all of which over-predicted the sodium decrement. The model that predicted the smallest magnitude of the decrement had 1% solar metallicity and cloud deck at 0.06 bar; this model predicts \( n_{Na} \sim 3.4 \times 10^{-4} \) in mid-transit. Our model (1% solar metallicity, cloud deck at 0.01 bar) predicts \( n_{Na} \sim 4.1 \times 10^{-4} \) in mid-transit, as shown in Figure 5. We conclude that our model is in reasonable agreement with both Charbonneau et al.’s model (\( \sim 20\% \)) and with the actual data (\( \sim 40\% \)).

### 6. CONCLUSION

Our investigation indicates that, with currently available instruments, it will be difficult to obtain the sensitivity needed to achieve a minimal \( S/N \geq 5 \) detection of tidally locked rotation of the planet HD 209458b. Nevertheless, it appears that the effect of rotation will have a significant (although small) influence on transit spectra taken with current or near-future instruments. Because this influence is so small, it is worth considering ways that the \( S/N \) could be improved, according to the scalings summarized in equation (14).

#### TABLE 5

| Spec_Shape       | Required No. of Transits For 5 \( \sigma \) Detection |
|------------------|------------------------------------------------------|
| Spec_Shape (1)   | \( D \) (m) | \( (S/N)/Tr. \) (A) | \( (S/N)/Tr. \) (B) | Req. No. Transits (A) | Duration (A) |
| Flat (uniform star) | 6          | 1.1 | 0.52 | \( \sim 25 \) | \( \sim 3 \) months |
| Solar            | 6          | 0.48 | 0.27 | \( \sim 110 \) | \( \sim 1.25 \) yr |
| Solar            | 10         | 0.79 | 0.45 | \( \sim 40 \) | \( \sim 5 \) months |
| Solar            | 30         | 2.4 | 1.3 | \( \sim 5 \) | \( \sim 3 \) weeks |

Notes.—Table showing the number of orbits required to achieve a \( S/N = 5 \) detection of tidally locked rotation of HD 209458b, assuming a planetary Na content that is 1% solar. Assumptions: we are monitoring two spectral lines (the Na doublet) with a spectrograph with resolving power \( R_s = 150,000 \), and an efficiency in the optical setup of 0.04. Col. (3), “(S/N)/Tr. (A),” is the S/N per transit, as shown in Figure 4, and col. (4) is the S/N per transit considering only ingress and egress. The “(A)” in cols. (5) and (6) indicates that the numbers of transits and the required duration are tabulated for \( S/N \) values from col. (3).
HD 209458. If a more bloated planet is found to transit a star as bright as HD 209458, such a planet would probably be a more promising target for the type of study described in this paper.

Although the optical transit spectrum of an extrasolar giant planet contains many precious clues regarding the nature of rotation and climate or weather, we emphasize that we have so far addressed only the forward problem: deep, high spectral resolution transit spectra will be distorted by HD 209458b’s tidally locked rotation, if indeed it is tidally locked. But seeing the effect of rotation in a spectrum and knowing that we are seeing the effect of rotation are two very different things. The inverse problem—identifying the rotation rate—will be much more challenging, and is beyond the scope of this paper’s investigation. In brief, it is impossible to decode the clues that the spectrum holds without a Rosetta stone—namely, without already possessing accurate knowledge of abundances in the planet’s atmosphere. We note that when it comes to solving the inverse problem, it may be possible to make do without a perfect Rosetta stone by comparing the star’s spectrum at two different times of transit instead of comparing it to an accurate input model. Specifically, comparing the ingress spectrum to the egress spectrum should eliminate various systematic uncertainties that may affect both ingress and egress spectra. This strategy involves foregoing spectra between second and third contacts, at a cost of a factor of ~2 in S/N, but has the advantage of being more robust with respect to errors in modeling the planetary atmosphere. Furthermore, if detailed analysis along the lines of Charbonneau et al. (2002) should reveal many more planetary absorption features, a large current-generation telescope could detect the motion of an extrasolar planet’s atmosphere in order a month.

A final point to keep in mind is that the technique presented in this paper is not limited to discerning the difference between a tidally locked planet and a nonrotating planet; this technique can be used to investigate the applicability of any model of the motion of a transiting planet’s atmosphere. Some atmospheric circulation models of hot Jupiters predict extremely fast winds, up to ~9 km s⁻¹, or more than 4 times the predicted equatorial rotation rate. It might therefore turn out to be easier to detect winds than to detect rotation. Moreover, if an analysis similar to the one described here were to find a deviation from the hypothesis of tidally locked rotation, this would mean that something interesting is going on—the planet could be rotating roughly as a solid body, but at a different rate; there could be significant nonrotational motion in the atmosphere, or there could be some combination of these phenomena.

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