Instabilities with polyacrylamide solution in small and large aspect ratios Taylor-Couette systems

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Abstract. We have investigated the stability of viscoelastic polyacrylamide solution in Taylor-Couette system with different aspect ratios. The first instability modes observed in a Taylor-Couette system with Γ = 10 were TVF and WVF, as for Newtonian fluid. At higher Taylor numbers moving vortices occur, a wavy mode with non-stationary vortex size. In the Taylor-Couette system with Γ = 45.9 we note a coexistence of various instability modes. In addition to TVF, counterpropagating waves developed at the transition from the base state flow. At higher Taylor number values Taylor vortices of different sizes occurred. Reduced amplitude Wavy vortex flow has also been observed.

1. Introduction

The Taylor-Couette system consists of two independently rotating coaxial cylinders with a working fluid in the annular gap between them. In the case of Newtonian fluids various routes from purely azimuthal base flow to turbulence are well characterized and reviewed [1, 2]. The basic flow becomes unstable against Taylor vortex flow, as second instability mode a Wavy vortex flow (WVF) occurs. Depending of the aspect ratio the WVF bifurcates to modulated WVF or directly to turbulent Taylor vortex flow.

Experimental studies of viscoelastic Taylor-Couette flows show different results depending on the rheological characteristics of the fluids used. A theoretical and experimental review of instabilities in viscoelastic Taylor-Couette flows is given in [3]. One of the main properties of viscoelastic solutions is the elasticity characterized by the elastic number. It defines the ratio between solution relaxation time $\lambda$ and viscous diffusion time $\tau_\nu$: $\epsilon = \frac{\lambda}{\tau_\nu} = \frac{\lambda}{\nu d^2}$. Very dilute solutions have low elasticity where inertia forces prevail, whereas the dynamic of semidilute and concentrated solutions is dominated by the elasticity.

Groisman and Steinberg [4, 5, 6, 7, 8] used polyacrylamide solutions with a constant polymer concentration and different solvent viscosities. They observed the existence of a Rotating Standing Wave. Baumert and Muller [9] reported comparable counterpropagating spirals for a Boger fluid and Crumeyrolle et al. [10] for a shear-thinning PEO solution. Satchwell and Mullin [11] investigated the four/six-cell primary flow exchange mechanism in a viscoelastic shear-thinning Taylor-Couette flow. They reported a very long time-scale phenomenon connected with a gradual shift of a codimension-2 point.
To the extent of our knowledge there is no explicit work about the influence of the aspect ratio on the instability modes of dilute polymer solutions in the Taylor-Couette system. We have performed an experimental study of the polyacrylamide solution with a weak shear-thinning behaviour in the Taylor-Couette system with two different aspect ratios. We found that the aspect ratio has significant influence on the instability modes.

The publication is organized as follows: the experimental setups are described in section 2. In section 3 are presented experimental results of the investigations with polyacrylamide solution in section 3 which are discussed in section 4. Section 5 contains concluding remarks.

2. Experimental setup

We performed our experimental studies in two Taylor-Couette systems with different geometric parameters all with the outer cylinder fixed. In the first Taylor-Couette system, the inner cylinder has a radius of $a = 40\text{mm}$, the outer one $b = 50\text{mm}$. The gap size between both cylinders is $d = b - a = 10\text{mm}$ and extends over a length of $L = 459\text{mm}$. The outcomes of this are the radius ratio $a/b = 0.8$ and the aspect ratio $\Gamma = L/d = 45.9$. The second system has the following parameters: $a = 25\text{mm}$, $b = 50\text{mm}$ and $L = 250\text{mm}$. This results in the radius ratio $\eta = 0.5$ and the aspect ratio $\Gamma = 10$. Control parameter in both experimental setups is the angular frequency $\Omega$ of the inner cylinder.

A solution of 39.4% saccharose and 1% NaCl (Merck Eurolabs, Germany) in deionised water is used as a solvent for the polymer as described in [12]. In addition, 0.1% formalin is added to the solution for better durability. Polyacrylamide (PAAm) (Polysciences, $M_w = 18 \cdot 10^6$g/mol) is added at a concentration of 80 ppm. Viscosity measurements with an AR2000 rheometer (TA Instruments) exhibit a weak shear-thinning behaviour of the polymer solution (figure 1). This can be described with the Power-Law $\eta(\dot{\gamma}) = K \cdot \dot{\gamma}^{n-1}$ with $K = 7.11 \cdot 10^{-3}\text{Pas}$ and $n = 0.976$. The Newtonian solvent viscosity is determined as $\eta_S = 5.66 \cdot 10^{-3}\text{Pas}$.

### Figure 1. Shear-rate dependence of dynamic viscosity $\eta$ for the PAAm solution.

Therefore we used the effective Reynolds number $Re = (\Omega ad\rho)/\eta(\dot{\gamma})$ and the effective Taylor number $Ta = Re \sqrt{d/a}$ as dimensionless control parameters.

For flow visualization Iriodin is used in the small Taylor-Couette system with $\Gamma = 10$. The fluid flow behaviour is recorded with a video camera (720x576 pixel, 26.8 pixel/cm, 25 frames/s). To visualize the structures with white light in the large Taylor-Couette system ($\Gamma = 45.9$), 1% of Kalliroscope AQ 1000 (Kalliroscope Corp., USA) has been added into polymer solution. An area CCD camera (Basler A641f, Basler AG, Germany) records the reflected light intensity $I(z)$ along the axial direction (40 pixel/cm, 12 frames/s). From the recorded videosequences, space-time diagrams $I(z, t)$ are provided at $x = d/2$. 

The analysis of the space-time diagrams is based on a 2D Fourier analysis and complex demodulation, which gives the amplitude and phase of the modulations of a particular mode. As a generic example, if \( e^{i(q_0 x - f_0 t)} \) is modulated by a function \( M(x, t) \), the resulting space-time signal is \( I(x, t) = \text{Re}\{M(x, t) \cdot e^{i(q_0 x - f_0 t)}\} \). Filtering the spectrum of \( I(x, t) \) around \((q_0, f_0)\), followed by Fourier inverse transform gives \( |M(x, t)| e^{i(q_0 x - f_0 t + \text{Arg}(M(x, t)))} \), where \(|M(x, t)|\) is the amplitude and \( \text{Arg}(M(x, t)) \) the phase perturbation. A description of the procedure is given in [13, 14].

3. Results
The observed flow states in the Taylor-Couette system with \( \Gamma = 45.9 \) and \( \eta = 0.8 \) are represented in a stability diagram (figure 2). The Couette flow becomes simultaneously unstable to Taylor vortex flow (TVF) and Standing waves (SW) (figure 4). This suggests that the investigated states are close to a codimension-2 point. In the lower part of the Taylor-Couette system the TVF starts to oscillate in the form of a weak wavy outflow. This periodic oscillation can be observed until the onset of Wavy vortex flow (WVF). The SW are superseded at \( \text{Ta}=61.5 \) by Taylor vortices with different wavenumber (TVF 2, figure 5). In the lower part of the Taylor-Couette system the vortices have a unique size with the wavenumber \( q = 3.08 \). In contrast, the vortices are compressed in the middle part. In the upper part of Taylor-Couette system asymmetric vortex pairs occur as shown schematically in figure 6. For those vortices a mean wavenumber \( q = 2.94 \) was determined from the spatial spectrum. At \( \text{Ta}=68.3 \) a reduced amplitude Wavy vortex flow (WVF*) occurs in the middle of the Taylor-Couette system. The transition to Wavy vortex flow (WVF) appears at \( \text{Ta}=80.9 \) and starts in the area of the asymmetric Taylor vortices. The WVF then propagates downward. The total number of vortices is reduced as compared to the pattern below \( \text{Ta}=80.9 \).

![Figure 2. Stability diagram for \( \Gamma = 45.9 \). CF: Couette flow, SW: Standing waves, TVF: Taylor vortex flow, OS: TVF with oscillations, TVF 2: TVF with different wavelength, WVF* reduced amplitude WVF](image)

![Figure 3. Stability diagram for \( \Gamma = 10 \). CF: Couette flow, TVF: Taylor vortex flow, WVF: Wavy vortex flow, mov. vortices: moving vortices](image)

The occurring flow states in the small Taylor-Couette system with \( \Gamma = 10 \) are shown in the stability diagram (figure 3). A Newtonian-like transition from basic flow to TVF and WVF is observed. Further increase of the shear rate leads to moving vortices as shown in figure 8. The fundamental structure of those flow pattern is a WVF with a non-stationary vortex size. Fast small-scale structures are embedded inside the main structure. The frequencies of the main flow pattern and the small-scale structures were identified in the temporal spectrum (figure 8 c) with \( f_0 = 0.234 H z \) and \( f_A = 1.08 H z \). Further analysis of the flow pattern showed that amplitudes of left and right wave are not balanced (figure 9). The axial wavenumber remained constant with \( q = 3.15 \) in the observed flow pattern for \( \Gamma = 10 \).
Figure 4. Space-time-plot and time averaged amplitude of left and right wave of the flow pattern for $\Gamma = 45.9$, $Ta = 41.7$.

Figure 5. Cross-section of TVF with different wavenumber $q$ (TVF 2) for $\Gamma = 45.9$, $Ta = 64.7$. i: inner cylinder, o: outer cylinder, left: bottom, right: top.

Figure 6. Schematic representation of asymmetric Taylor vortex pairs ($\Gamma = 45.9$). Arrows indicate the in- and outflow between the vortices.

Figure 7. Space-time diagram of WVF for $\Gamma = 45.9$, $Ta = 82.7$.

4. Discussion

We have investigated the behaviour of Newtonian silicone oil (Baysilone M3, GE Bayer Silicones, Germany) in $\Gamma = 10$ as a reference. We have observed TVF and WVF ($Ta_{c,TVF} = 68.1$, $Ta_{c,WVF} = 443.9$) as expected for a Newtonian fluid. For the PAAm solution, the base Couette flow exhibited instability to TVF at $Ta_{c,TVF} = 43$. Hence a destabilising effect on the Couette flow has been observed for $\Gamma = 10$. The same holds true for $\Gamma = 45.9$, in which Couette flow disappeared at $Ta_{c,TVF} = 38.2$ for the PAAm solution whereas the Newtonian critical value for TVF is 47.3. The flow patterns however, are different, namely for $\Gamma = 10$ the TVF is similar to the Newtonian case, whereas the $\Gamma = 45.9$ experiment exhibited different coexisting patterns, including Standing waves. For the second instability, we note that for $\Gamma = 10$ the TVF is indeed stabilized as WVF onset is delayed to $Ta_{c,WVF} = 480.2$. But for $\Gamma = 45.9$ the WVF appears at lower values of $Ta$ than the bifurcation takes place from TVF to WVF in the
Newtonian case. The moving vortices occurred as higher instability mode for $\Gamma = 10$, were not reported before. The embedded small-scale structures inside might be appeared as a result of the wide gap ($\eta = 0.5$). In the Taylor-Couette system with $\eta = 0.8$ this flow patterns could not be observed.

Figure 8. a) Space-time diagram of moving vortices for $\Gamma = 10$, $Ta=673$; b) zoom of the space-time diagram; c) temporal spectrum of the diagram with fundamental frequency $f_0$ and frequency $f_A$ of embedded small-scale structures

Figure 9. Amplitude profile of left and right wave of moving vortices for $\Gamma = 10$, $Ta=673$

We have determined parameters characterizing the solution in small and large aspect ratio Taylor-Couette system (Table 1). The specific viscosity $\eta_p/\eta_S$ is obtained from shear viscosity measurements. Since it was not possible to obtain the fluid relaxation time in rheological measurements, hence we use the molecular relaxation time $\tau_M$ defined as $\tau_M = \frac{\eta - \eta_S}{n_V R T}$, with the gas constant $R$ and the amount of polymer per unit volume $n_V$. The ratio between the elastic driving force and the viscous resistance is proportional to $K_M = [\eta - \eta_S]/(\eta S)$ $[\frac{\eta}{\eta S}]^{0.5} \cdot (\frac{\eta}{\eta S})^{0.5} \cdot We$ $[7]$. Baumert and Muller [9, 15] obtained for a Boger fluid $We_c = 2.04$, $K=0.231$ ($\eta = 0.912$) and $We_c = 1.88$, $K=0.3138$ for $\eta = 0.827$. Groisman and Steinberg [7] reported $K=1.36$ for $\eta = 0.829$ and $K=1.64$ for $\eta = 0.796$ in a similiar polyacrylamide solution. Our experiments
revealed that $K=0.025$ for $\eta = 0.5$ and $K=0.136$ ($\eta = 0.8$). Avgousti et al. [16, 17] showed that for an elasticity number $\epsilon \geq 0.01$ there is a transition from axisymmetric to nonaxisymmetric instability modes. According to these theoretical predictions we are below $\epsilon_{\text{crit}}$ in our investigations [cf. Table 1]. Therefore only axisymmetric instability modes should occur. This could be sighted for $\Gamma = 10$. In the large aspect ratio Taylor-Couette system with $\Gamma = 45.9$ we have observed those in the middle of the system, at bottom and top perturbated rolls appear. This suggests we are so close to $\epsilon_{\text{crit}}$ that end effects disturb significantly.

**Table 1.** Specific viscosity $\eta_p/\eta_S$, Weissenberg number, elastic number, molecular relaxation time and $K_M$ calculated with $\eta = \eta_c$.

| $\eta_p/\eta_S$ | We | $\tau_M$ [s] | $\tau_{\nu}$ | $\epsilon \times 10^{-3}$ | $K_M$ [s] |
|-----------------|-----|-------------|--------------|----------------|----------|
| $\Gamma = 10$   | 0.344 | 0.049      | 0.149        | 97.276         | 1.539    | 0.025 |
| $\Gamma = 45.9$ | 0.244 | 0.613      | 0.106        | 17.6           | 6.023    | 0.136 |

For $\Gamma = 10$ we have determined the fundamental frequencies of Wavy vortex flow and moving vortices (Table 2). The dimensionless period is expressed by the viscous diffusion time ($T/\tau_{\nu}$) and the molecular relaxation time ($T/\tau_M$). It increases from WVF to moving vortices.

**Table 2.** Parameter for $\Gamma = 10$.

| $f$ [Hz] | $T/\tau_{\nu}$ | $T/\tau_M$ |
|----------|----------------|------------|
| WVF (Ta=512) | 0.242 | 0.042 | 28.380 |
| Moving vortices (Ta=673) | 0.234 | 0.044 | 29.414 |

The fundamental frequencies and the dimensionless periods $T/\tau_{\nu}$ and $T/\tau_M$ of the observed flow states in the large aspect ratio Taylor-Couette system are summarized in Table 3. With increasing Taylor number elasticity becomes a growing influence on the flow, $T/\tau_M$ decrease.

**Table 3.** Parameter for $\Gamma = 45.9$.

| $f$ [Hz] | $T/\tau_{\nu}$ | $T/\tau_M$ |
|----------|----------------|------------|
| TVF + SW (Ta=38.2) | 0.134 | 0.419 | 70.865 |
| TVF 2 + OS (Ta=61.1) | 0.159 | 0.354 | 59.741 |
| TVF 2 + WVF* + OS (Ta=68.3) | 0.177 | 0.306 | 53.486 |
| WVF (Ta=80.9) | 0.535 | 0.105 | 17.696 |

5. Conclusion
We have experimentaly investigated the instability modes occurring in a dilute PAAm solution in Taylor-Couette systems with aspect ratio $\Gamma = 10$ and $\Gamma = 45.9$. The transition sequence in
the small Taylor-Couette system is similar to a Newtonian fluid. The same inertial instability modes (TVF, WVF) were observed. At higher $Ta$, moving vortices occurred which differ from WVF. In the large Taylor-Couette system the investigations were close to a codimension-2 point, the Couette flow became simultaneously unstable to TVF and SW. A significant variation of the axial wavenumber $q$ was observed in the system with $\Gamma = 45.9$.

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