JET PHYSICS AT LEP AND SLC

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Abstract

Experimental results on jet physics at LEP and SLC are reviewed and compared with perturbative QCD predictions. The discussion includes determinations of the strong coupling $\alpha_S$, measurements of event shape distributions and jet cross sections, studies of subjet multiplicities and tests of QCD coherence.
1. Introduction

The large electron-positron colliders LEP and SLC have provided many detailed studies of hadronic final states. These include jet cross sections, direct-photon production, hadron spectra and, more recently, studies of fragmentation functions and their scaling violations. Most of these topics are discussed in several review papers (see, for instance, Refs. [1-4]). In this contribution, rather than presenting a general review on hadronic physics at LEP and SLC, I concentrate only on jet physics. The reason is the following.

On the experimental side, LEP and SLC are remarkable machines for studying hadronic interactions. They operate at a very high centre-of-mass energy, $\sqrt{s} \simeq 91$ GeV, and provide a very large event statistics (due to the large cross section at the $Z^0$ resonance) and a very clean environment (as in any $e^+e^-$ collider, the final state is not contaminated by the debris of initial-state hadrons).

On the theoretical side, if we consider jet physics (i.e. high-energy hadronic observables which are infrared and collinear safe) we are dealing with a very predictive theory, namely perturbative Quantum Chromodynamics (QCD). We have at our disposal a well defined theoretical framework (with essentially only one free parameter, the strong coupling $\alpha_s$) and quite accurate calculations.

Combining these experimental and theoretical facilities, we are thus in a position to perform ‘precision tests’ of QCD. The aim of the present contribution is to show that at LEP/SLC we can investigate strong-interaction physics to an accuracy better than 10%, which is quite an achievement in this field.

The outline of this review is as follows. In Sect. 2, I recall the general theoretical features of jet observables and their treatment within perturbative QCD. Section 3 is devoted to the measurements of $\alpha_s$ from fully inclusive quantities, namely the hadronic widths of the $Z^0$ boson and the $\tau$ lepton. More detailed tests of QCD from event shape distributions are considered in Sect. 4. Here, I present results based on analyses in two-loop order and including the all-order resummation of logarithmically enhanced contributions. A summary of $\alpha_s$ determinations at LEP and SLC is also included in this Section. Section 5 deals with the definition of jets and studies of jet cross sections. A discussion on subjet multiplicities, QCD coherence and properties of quark and gluon jets is presented in Sect. 6. Finally, in Sect. 7, I briefly summarize the requirements for further progress in the field.

2. Jet observables in perturbative QCD

Jet quantities are, by definition, hadronic observables which turn out to be infrared and collinear safe. In other words, their actual value does not vary if the final state changes by the addition of one more particle which is soft or collinear to another particle.

The main theoretical feature of jet observables is that they can be computed in QCD perturbation theory. This is not a trivial statement. In general, evaluating QCD Feynman diagrams in terms of partons (quarks and gluons), one finds integrals which are divergent in the low-momentum and/or small-angle regions. In the case of jet quantities, the coherent sum over different soft and collinear partonic states leads to the cancellation of these
divergences. As a result, jets observables are finite (calculable) at the partonic level order by order in perturbation theory.

Denoting by \( R \) a generic (dimensionless) jet variable, we can compute its corresponding perturbative expansion in the form

\[
R = 1 + \alpha_S(Q) R_1 + \alpha_S^2(Q) R_2 + \ldots .
\]  

(1)

The first non-trivial term \((\alpha_S R_1)\) in Eq. (1), the second term \((\alpha_S^2 R_2)\), etc. represent respectively the leading-order (LO) contribution to \( R \), the next-to-leading order (NLO) contribution and so on. The typical energy scale, at which the observable \( R \) is considered, is denoted by \( Q \).

The expansion parameter in Eq. (1) is \( \alpha_S(Q) \), the QCD running coupling. It is the sole (apart from quark masses) free parameter in the theory. QCD does not predicts its actual value but only its energy behaviour. Indeed, due to the property of asymptotic freedom, the running coupling \( \alpha_S(Q) \) decreases as the energy \( Q \) increases, according to the approximate logarithmic behaviour \( \alpha_S(Q) \sim (\beta_0 \ln Q^2/\Lambda^2)^{-1} \), \( \Lambda \) being the fundamental scale of QCD.

Asymptotic freedom is due to the non-abelian gauge structure of the theory. Thus, the experimental observation of the running of \( \alpha_S(Q) \) according to QCD is a fundamental test of the underlying gauge theory. For this reason, in the past few years much effort has been devoted to measurements of \( \alpha_S(Q) \) (see Refs. [3,11] and Secs. 3 and 4).

The QCD running of \( \alpha_S \) implies that the effective coupling \( \alpha_S(Q) \) is small at high energy \( Q \). This property justifies the use of perturbation theory for predicting jet observables, at least at asymptotic energies. However, just because of its perturbative nature, the QCD running can be hidden in higher-order corrections by the replacement \( \alpha_S(Q) = \alpha_S^{(0)}[1 + K(Q) \alpha_S(Q) + \ldots] \), \( \alpha_S^{(0)} \) being the value of \( \alpha_S \) at a fixed (and arbitrary) energy scale. It follows that a LO calculation gives only the order of magnitude of a certain observable. The accuracy of the perturbative QCD expansion is instead controlled by the size of the higher-order corrections. Any definite QCD prediction thus requires (at least) a NLO definition of \( \alpha_S \) and a NLO perturbative calculation.

In the following I shall consider the NLO definition of \( \alpha_S \) as given in the \( \overline{\text{MS}} \) renormalization scheme. In this scheme the relation between \( \alpha_S(Q) \) and the QCD scale \( \Lambda_{\overline{\text{MS}}} \) is

\[
\alpha_S(Q) = \frac{1}{\beta_0 \ln Q^2/\Lambda_{\overline{\text{MS}}}^2} \left[ 1 - \frac{\beta_1}{\beta_0^2} \frac{\ln(Q^2/\Lambda_{\overline{\text{MS}}}^2)}{\ln Q^2/\Lambda_{\overline{\text{MS}}}^2} + \mathcal{O} \left( \frac{\ln^2(Q^2/\Lambda_{\overline{\text{MS}}}^2)}{\ln^2 Q^2/\Lambda_{\overline{\text{MS}}}^2} \right) \right] ,
\]  

(2)

where \( 12\pi\beta_0 = 33 - 2N_f \), \( 24\pi^2\beta_1 = 153 - 19N_f \) and \( N_f \) is the number of quarks whose mass is smaller than \( Q \).

As for the power series expansion in \( \alpha_S \) beyond the LO, one has to be more precise than in Eq. (1). In general, the perturbative series has the following structure

\[
R(\alpha_S(\mu), Q/\mu; y) = 1 + \alpha_S(\mu) R_1(y) + \alpha_S^2(\mu) \left[ R_2(y) - R_1(y) \beta_0 \ln Q^2/\mu^2 \right] + \mathcal{O}(\alpha_S^3(\mu) \ln^2 Q^2/\mu^2, \alpha_S^3(\mu)) ,
\]  

(3)

where \( Q \) is the typical energy scale of the process and \( y \) denotes any other kinematic scale involved in the definition of the observable \( R \).
In Eq. (3) we have made explicit the dependence on the renormalization scale $\mu$ which has to be introduced in any higher-order calculation in order to control (regularize) ultraviolet divergences. Obviously, the observable $R$ is a renormalization group invariant quantity if computed to all orders in $\alpha_S$. The $\mu$-dependence is an artifact of the truncation of the perturbative expansion at a fixed order in $\alpha_S$. This implies that

1. the renormalization scale dependence is formally an higher-order effect (i.e. $dR/d\ln\mu = \mathcal{O}(\alpha^3_S)$ in Eq. (3)) and, correspondingly, 
2. higher-order terms have an explicit $\ln(Q/\mu)$-dependence (see the $\mathcal{O}(\alpha^3_S \ln^2 Q^2/\mu^2)$ term in Eq. (3)).

However, the $\mu$-dependence can be numerically large if i) the perturbative functions $R_1(y), R_2(y), \ldots$ are large, and/or ii) $\mu$ is very different from the typical energy scale $Q$ of the process (in this case the $\ln Q/\mu$-terms in Eq. (3) are large and may spoil the convergence of the perturbative expansion). For these reasons, assuming a well-behaved perturbative expansion, $\mu$ should be set equal to $Q$ and $\mu$ variations (typically by a factor of four) around this value can be used for estimating the theoretical uncertainty due to uncalculated higher-order contributions.

Before discussing in detail jet measurements at LEP/SLC, I have to add a last general comment about non-perturbative effects. Any hadronic observable has necessarily a non-trivial non-perturbative component because of the hadronization of quarks and gluons. Therefore, in general we should write

$$ R = R_{\text{pert}}(\alpha_S(\mu), Q/\mu; y) + \mathcal{O}((1/Q)^p), $$

where the last term represents the non-perturbative contribution. One of the main properties of the jet observables, ensuing from their infrared and collinear safety, is that the non-perturbative component is suppressed by some inverse power of the energy as the energy increases (i.e. $p \geq 1$ in Eq. (4)). Therefore, in principle, the hadronization effects are negligible at asymptotic energies. In practice, since we are dealing with experiments at finite energy, we should try to estimate the size of the non-perturbative corrections.

At present there are essentially two methods for estimating non-perturbative contributions. The first method is based on the Operator Product Expansion (OPE) \[12]\ and consists in relating the non-perturbative term in Eq. (3) to the matrix elements of some local operators. These matrix elements are not computable in perturbation theory but are universal, in the sense that their value does not depend on the particular jet quantity considered. Therefore one can estimate them from low-energy measurements (or from lattice calculations) and evaluate the corresponding correction to the jet observables.

The second method is based on Monte Carlo event generators \[13-16]\. The Monte Carlo generators used at LEP/SLC consist of two different components. In the perturbative component, starting from the hard energy scale of the process, one generates a parton shower describing quark and gluon production according to (approximate) QCD matrix elements. In the non-perturbative component, at a certain scale of the order of 1 GeV, partons from the shower are converted into hadrons according to some phenomenological hadronization model. The hadronization parameters are tuned in order to reproduce the experimental data. Therefore, comparing jet quantities as obtained by Monte Carlo simulations both from partons at the end of the QCD shower and from particles after hadronization, one is able to estimate the size of the non-perturbative corrections.
3. Fully inclusive quantities

Fully inclusive quantities are jet observables depending on a single momentum scale. Their perturbative expansion in Eq. (3) is thus a simple power series in $\alpha_S$ with constant ($y$-independent) coefficients $R_n$. Moreover, due to the highly inclusive nature of these quantities, they are expected to be marginally affected by non-perturbative corrections ($p \geq 2$ in Eq. (4)). For these reasons, these jet observables are particularly suitable for $\alpha_S$ determinations. Two of them have been investigated at LEP so far: the hadronic width of the $Z^0$ boson and the hadronic branching ratio of the $\tau$ lepton.

3.1 Hadronic branching ratio of the $Z^0$ boson

The hadronic branching fraction $R_Z$ of the $Z^0$ boson is defined by the ratio of the hadronic and leptonic widths:

$$R_Z = \frac{\Gamma_{\text{had}}(M_Z)}{\Gamma_{\text{lep}}(M_Z)},$$

and is computable in terms of electroweak and QCD corrections as follows

$$\Gamma_{\text{lep}}(M_Z) = \Gamma_{\text{ew}}^V + \Gamma_{\text{ew}}^A,$$

$$\Gamma_{\text{had}}(M_Z) = \Gamma_{\text{ew}}^V R_{\text{QCD}}^{V} + \Gamma_{\text{ew}}^A R_{\text{QCD}}^{A}.$$  

Here $\Gamma_{\text{ew}}^{V,A}$ include all the known electroweak corrections [17] within the standard model.

The QCD contributions $R_{\text{QCD}}^{V,A}$ are evaluated starting from the imaginary part of $\Pi^{(i)}(Q^2)$, the correlation functions of the vector ($i = V$) and axial ($i = A$) currents. These quantities are completely known up to next-to-next-to-leading order (NNLO) in perturbation theory (i.e. to relative accuracy $O(\alpha_S^3)$ with respect to the lowest-order approximation) [13,20], including all the relevant corrections due to finite quark masses [21,22].

In spite of these very accurate theoretical results, last year at the EPS Conference I pointed out [9] that there were some discrepancies in the actual numerical implementations of these calculations. The effect of these discrepancies turned out to be of the same size as the expected theoretical uncertainties.

This problem has been considered in a recent analysis by the authors of Ref. [17]. They have shown that the numerical discrepancies are eliminated after full updating of all the programs [23]. Moreover, they have provided a simple effective (factorized) formula for $R_Z$ and have carefully estimated its theoretical accuracy.

The QCD corrections $R_{\text{QCD}}^{V,A}$ to the vector and axial part of the electroweak current are different. Therefore, in general, starting from Eqs. (3) and (7) one cannot write down a simple power series in $\alpha_S$ for the ratio $R_Z$ in Eq. (3). Nonetheless, in the proper range of electroweak parameters, one can derive an approximate factorized expression for $R_Z$. The

$\dagger$The $O(\alpha_S^3)$-singlet part of the axial current, which was previously missing, has been computed recently [18].
effective formula obtained in Ref. [17] is:

\[ R_Z \simeq R_0 \left[ 1 + 1.060 \frac{\alpha_S}{\pi} + 0.90 \left( \frac{\alpha_S}{\pi} \right)^2 - 15 \left( \frac{\alpha_S}{\pi} \right)^3 \right], \]

\[ R_0 = 19.943, \tag{8} \]

where \( \alpha_S \) stands for \( \alpha_S(M_Z) \). In the range \( 0.10 < \alpha_S < 0.15 \), Eq. (8) agrees with the full perturbative formula for \( R_Z \) to an accuracy better than \( \Delta \alpha_S < 0.0001 \).

Using the expression (8), one can translate theoretical uncertainties on the value of \( R_Z \) into corresponding uncertainties on \( \alpha_S(M_Z) \). The estimate in Ref. [17] gives

\[ \Delta \alpha_S(M_Z) = \pm 0.002 \text{(ew)} \pm 0.002 \text{(QCD)} \text{ }^{+0.004}_{-0.003} (M_t, M_H), \tag{9} \]

The electroweak error (ew) is mainly due to electroweak corrections to the \( Z^0 \to b\bar{b} \) vertex. The QCD error is dominated by unknown higher orders in perturbation theory and is estimated by varying the renormalization scale \( \mu \) within the range \( M_Z/4 < \mu < M_Z \). Note that non-perturbative contributions to \( R_Z \) are negligible because of the large value \( M_Z \simeq 91 \text{ GeV} \) of the \( Z^0 \) mass: from the OPE analysis the non-perturbative contributions turn out to be proportional to \( (1/M_Z)^4 \) (even including some additional corrections of the type \( \Lambda^2/M_Z^2 \), one obtains \( \Delta \alpha_S < 0.001 \)).

The dominant source of uncertainty in Eq. (9) is due to the values of the masses of the top quark \( (M_t) \) and Higgs boson \( (M_H) \). The coefficients in Eq. (8) refer to \( M_t = 150 \text{ GeV} \) and \( M_H = 300 \text{ GeV} \), whilst the corresponding errors in Eq. (9) are obtained by considering variations in the range \( 200 \text{ GeV} < M_t < 100 \text{ GeV}, 1 \text{ TeV} < M_H < 60 \text{ GeV} \).

Including the data collected in the 1993 run, the updated LEP average for \( R_Z \) is \( R_Z = 20.795 \pm 0.040 \) [24]. From this value and using Eq. (8) one gets:

\[ \alpha_S(M_Z) = 0.124 \pm 0.006 \text{(exp.)}^{+0.005}_{-0.004} \text{(th.)}, \tag{10} \]

where the experimental error is dominated by the event statistics.

This result is perfectly consistent with the value \( \alpha_S(M_Z) = 0.126 \pm 0.005 \text{(exp.)} \pm 0.002 (M_H) \) (and \( M_t(\text{GeV}) = 173 \pm 13 \text{(exp.)} \pm 19 (M_H) \)) obtained from a standard-model fit to the \( Z^0 \) lineshape and asymmetries at LEP [24]. However the latter is more dependent on the parameters of the electroweak theory (in particular, \( M_t \)).

### 3.2 Hadronic branching ratio of the \( \tau \) lepton

An independent determination of \( \alpha_S \) in NNLO can be obtained from the hadronic width of the \( \tau \) lepton. This quantity is indeed theoretically related to the current correlation functions \( \Pi^{(i)}(Q^2) \) via a simple momentum sum rule [25]. The corresponding hadronic branching ratio is given by

\[ R_\tau = \frac{B(\tau \to \nu_\tau + \text{had.})}{B(\tau \to \nu_\tau \bar{\nu}_\tau)} = R^{(6)}(1 + \delta_{\text{pert.}} + \delta_{\text{non-pert.}}), \tag{11} \]

\[ ^2\text{This formula is valid for massless leptons. Corrections due to finite lepton masses can easily be taken into account [17].} \]
where \( R^{(0)} \) is the parton model value and the perturbative QCD correction (as evaluated from the three-loop calculation of \( \Pi^{(i)}(Q^2) \), described in the previous subsection) is

\[
\delta_{\text{pert.}} = \frac{\alpha_S(M_\tau)}{\pi} + 5.2 \left( \frac{\alpha_S(M_\tau)}{\pi} \right)^2 + 26.4 \left( \frac{\alpha_S(M_\tau)}{\pi} \right)^3. \tag{12}
\]

Unlike the case of \( R_Z \), the non-perturbative contributions are potentially sizeable at a scale as low as the \( \tau \) mass \( M_\tau = 1.78 \) GeV. Using the OPE and estimating the relevant matrix elements by means of QCD sum rules, the non-perturbative term in Eq. (11) is found to be

\[
\delta_{\text{non-pert.}} = -0.02 \pm 0.01, \tag{13}
\]

which is quite a small contribution (although of the same order as the \( O(\alpha_s^3) \) term in Eq. (12), since \( \alpha_s(M_\tau) \sim 0.3 \)).

Although this estimate is not completely unambiguous, at present we are more confident on the value of \( \delta_{\text{non-pert.}} \). As a matter of fact, the estimated value in Eq. (13) has been found consistent with the hadronic mass spectrum of the \( \tau \) lepton as measured by the ALEPH Collaboration [25].

The average value of the results published by the four LEP experiments [27] is \( R_\tau = 3.617 \pm 0.034 \). Using this value and the theoretical predictions in Eqs. (11)-(13) one obtains

\[
\alpha_S(M_\tau) = 0.36 \pm 0.02(\text{exp.}) \pm 0.04(\text{th.}) \tag{14}
\]

The way of estimating the theoretical uncertainties on the value of \( \alpha_S \) from \( \tau \) decay is still a matter of discussion [7,28]. The main points regard the validity of the OPE close to the resonance region [29] and possible corrections to the (NNLO) perturbative running of \( \alpha_S \) for scales as low as \( M_\tau \) [30]. The theoretical error quoted in Eq. (14) takes into account the uncertainty on \( \delta_{\text{non-pert.}} \) in Eq. (13), the variation of the renormalization scale between 1 GeV and 3 GeV and the effect of adding to Eq. (12) a contribution of order \( \pm 100(\alpha_S/\pi)^4 \).

As expected from QCD, the value of \( \alpha_S(M_\tau) \) in Eq. (14) is significantly larger than the value of \( \alpha_S(M_Z) \) obtained from \( R_Z \). Extrapolating the result (14) from the \( \tau \) mass to the \( Z^0 \) mass using the perturbative QCD running (see Eq. (2)), one finds

\[
\alpha_S(M_Z) = 0.122 \pm 0.002(\text{exp.}) \pm 0.004(\text{th.}) \tag{15}
\]

where the relative size of the errors is decreased because of the logarithmic dependence of \( \alpha_S \) on the energy.

4. Event shape distributions

The most detailed QCD tests performed so far at \( e^+e^- \) colliders are based on studies of shape variables and jet cross sections.

\[\text{§This average value does not include two new measurements of } R_\tau \text{ by the ALEPH and CLEO Collaborations [4]. These results are still preliminary and differ each other by almost three standard deviations.}\]
The shape variables are global jet observables characterizing the structure of the hadronic final states. One of these variables is the thrust $T$, which is defined as

$$T = \max_n \frac{\sum_i |\mathbf{p}_i \cdot \mathbf{n}|}{\sum_i |\mathbf{p}_i|},$$

and thus maximizes the total longitudinal momentum (along the unit vector $\mathbf{n}$) of the final-state particles $p_i$ in a given event. For a two-jet event we have $T = 1$, whilst a spherical event has $T = 1/2$. Many other shape variables can be defined \[31\], with the only constraint of being infrared and collinear safe observables.

In the following I denote by $y$ a generic shape variable whose two-jet limit corresponds to the region $y \to 0$ (for instance, $y = 1 - T$). The corresponding shape-variable event fraction is defined by

$$R(\alpha_S(\mu), Q/\mu; y) = \int_0^y dy' \frac{1}{\sigma} \frac{d\sigma}{dy'},$$

where $Q$ is the $e^+e^-$ centre-of-mass energy and $\mu$ is the renormalization scale.

The event fraction in Eq. (17) is computable in QCD perturbation theory in the form of Eq. (3). Note that now the coefficients $R_1(y), R_2(y), \ldots$ are not $c$-numbers but functions of the actual value $y$ of the shape variable. Therefore, unlike the case of the fully inclusive quantities, by studying shape variable distributions one can not only measure $\alpha_S$ but also perform more effective tests of the QCD matrix elements. The price which has to be paid is that, at comparable energy, the non-perturbative effects are stronger than for the completely inclusive observables. Any time that the degree of inclusiveness is degraded, the effective power $p$, which controls the energy behaviour of the non-perturbative component (see Eq. (4)), decreases. According to the naive expectation, the hadronization corrections to the event shapes are nominally of the order $\Lambda/Q$. This expectation is in agreement with the non-perturbative effects estimated by Monte Carlo generators and has been confirmed by calculations of renormalon contributions \[32\].

4.1 QCD studies to complete $\mathcal{O}(\alpha_S^2)$

The conventional approach for studying shape variables is based on the comparison between data and NLO QCD calculations. For all the relevant shape variables the $\mathcal{O}(\alpha_S)$ and $\mathcal{O}(\alpha_S^2)$ coefficients $R_1(y)$ and $R_2(y)$ in Eq. (3) have been evaluated numerically\footnote{In the case of the energy-energy correlation function, the numerical disagreement among different calculations \[31,33\] has been solved recently \[34\] in favour of Ref. \[31\].} by Kunszt and Nason \[31\], using the two-loop matrix elements computed in Ref. \[35\]. In these analyses the renormalization scale $\mu$ is usually set equal to $Q$. Note that, due to the explicit $y$-dependence of the coefficients $R_n(y)$, by varying $\mu$ one can modify the shape in $y$ of the QCD predictions. Therefore, a more empirical approach is often considered: $\mu$ is left as a free parameter and fitted to the data together with $\alpha_S$.

A summary \[11\] of $\alpha_S$ determinations from NLO calculations at LEP and SLC \[36-38\] is presented\footnote{The most recent (and complete) results from the SLD Collaboration are not reported in Fig. 1. However, their inclusion does not change the average value in Eq. (18).} in Fig. 1. All these measurements are pretty consistent and the average value
(taking into account experimental and theoretical correlations) for $\alpha_S$ is

$$\alpha_S(M_Z) = 0.119 \pm 0.001\text{(exp.)} \pm 0.006\text{(th.)}. \quad (18)$$

The error is dominated by theoretical uncertainties (common to all the measurements) due to hadronization corrections and higher-order contributions. The hadronization effects are estimated via Monte Carlo event generators by comparing the corresponding results at parton and hadron level. The overall size of these corrections is typically between 5 and 15% (see, for instance Fig. 2). The relative effect among different Monte Carlo generators is instead of the order of few percent and is usually assigned as the corresponding hadronization uncertainty.

Higher-order contributions are estimated from renormalization scale variations. As a matter of fact, the peculiar feature of these QCD tests and $\alpha_S$ determinations to $\mathcal{O}(\alpha_S^2)$ is the following [36,38]. Far away from the two-jet region, the NLO expression (3) with a renormalization scale $\mu \simeq Q$ gives good fits to the data. On the contrary, reasonable fits in the two-jet region can be achieved only by using renormalization scales $\mu$ much smaller than $Q$ (as small as few GeV!) and values of $\alpha_S$ smaller than those obtained in the multi-jet region (Fig. 3).

Fig. 1: Compilation of measurements of $\alpha_S(M_Z)$ from event shapes, jet rates, energy correlations and scaling violations, in $\mathcal{O}(\alpha_S^2)$, at LEP and SLC [11].
Fig. 2: DELPHI data on the heavy jet mass $M_H$ (the mass of the heavier hemisphere with respect to the thrust axis) compared with the $O(\alpha_S^2)$ QCD predictions (with a renormalization scale factor $f \equiv \mu^2/Q^2 = 0.25$). The hadronization corrections applied to the data are shown below the distribution.

Fig. 3: Dependence of $\alpha_S(M_Z)$ (solid curves) and $\chi^2$/d.o.f. (dashed curves) on $x_\mu \equiv \mu/Q$ for $O(\alpha_S^2)$ fits to the OPAL data on thrust ($T$), heavy jet mass ($M_H$), total ($B_T$) and wide ($B_W$) jet broadening.

The reason for this strong scale dependence of the shape-variable event fractions is that
the corresponding perturbative functions \( R_n(y) \) in Eq. (3) are large in the two-jet region \((y \to 0)\). For instance, in the case of thrust the actual calculation gives \[39\]

\[ R_T(y = 1 - T) \to 1 - C_F \frac{\alpha_S}{\pi} \ln^2(1 - T) + \frac{1}{2} \left( C_F \frac{\alpha_S}{\pi} \right)^2 \ln^4(1 - T) + \mathcal{O}(\alpha_S^3 \ln^6(1 - T)) \, . \] (19)

The double logarithmic contributions \( \alpha_S \ln^2(1 - T) \), \( \alpha_S^2 \ln^4(1 - T) \) . . . are due to the bremsstrahlung spectrum of soft and collinear gluons. Although infrared and collinear singularities cancel in jet observables upon adding real and virtual contributions, in the two-jet limit real emission is strongly inhibited. The ensuing mismatch of real and virtual corrections generates logarithmically-enhanced terms which spoil the convergence of the perturbative expansion in \( \alpha_S \).

Observables dominated by two-jet configurations are thus affected by a large and systematic theoretical uncertainty due to higher-order logarithmic corrections. Reliable predictions can be obtained only computing these corrections and, if possible, resumming them to all orders in \( \alpha_S \).

4.2 QCD studies using resummed calculations

A detailed understanding of logarithmically enhanced terms now exists for many shape variables \[40-44\], namely those for which the small-\( y \) logarithms \( L = \ln 1/y \) exponentiate \[45\]. The shape variable event fraction \( R(\alpha_S, y) \) can thus be written as follows (in order to simplify the formulae I set \( \mu = Q \))

\[ R(\alpha_S; y) = C(\alpha_S) \Sigma(\alpha_S, L) + D(\alpha_S; y) \, , \] (20)

where

\[ C(\alpha_S) = 1 + \sum_{n=1}^{\infty} C_n \alpha_S^n \]

\[ \ln \Sigma(\alpha_S, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{n+1} G_{nm} \alpha_S^n L^m \]

\[ = L \, g_1(\alpha_S L) + g_2(\alpha_S L) + \alpha_S \, g_3(\alpha_S L) + \cdots \, , \] (21)

and \( D(\alpha_S; y) \) vanishes as \( y \to 0 \) order by order in perturbation theory. The word exponentiation refers to the fact that the terms \( \alpha_S^n L^m \) with \( m > n + 1 \) are absent from \( \ln R(\alpha_S; y) \), whereas they do appear in \( R(\alpha_S; y) \) itself. In the expression (20) the singular \( \ln y \) dependence is entirely included in the effective form factor \( \Sigma \). The function \( g_1 \) resums all the leading contributions \( \alpha_S^n L^{n+1} \), while \( g_2 \) contains the next-to-leading logarithmic terms \( \alpha_S^n L^n \), and \( g_3 \) etc. give the remaining subdominant logarithmic corrections \( \alpha_S^n L^m \) with \( 0 < m < n \).

Equation (21) represents an improved perturbative expansion in the two-jet region. Once the functions \( g_i \) have been computed, one has a systematic perturbative treatment of the shape distribution throughout the region of \( y \) in which \( \alpha_S L \ll 1 \), which is much larger than the domain \( \alpha_S L^2 \ll 1 \) in which the \( \alpha_S \) perturbative expansion (3) is applicable. Furthermore, the resummed expression (21) can be consistently matched with fixed-order calculations. In particular, one can consider the next-to-leading logarithmic approximation
(NLLA) as given by the functions $g_1$ and $g_2$ and combine them with the $\mathcal{O}(\alpha_S^2)$ results in eq. (3) (after subtracting the resummed logarithmic terms in order to avoid double counting), to obtain a prediction (NLLA+$\mathcal{O}(\alpha_S^2)$) which is everywhere at least as good as the fixed-order result, and much better as $y$ becomes small.

Fig. 4: (a) Measured distributions of thrust ($\tau = 1 - T$), heavy jet mass ($\rho = M_H^2/Q^2$), total ($B_T$) and wide ($B_W$) jet broadening, compared with fits to the $\mathcal{O}(\alpha_S^2)$ and to the resummed NLLA+$\mathcal{O}(\alpha_S^2)$ calculations, with a renormalization scale factor $\mu/Q = 1$ in
both cases. The (b) hadronization and (c) detector corrections applied to the data are also shown.

Extensive experimental studies based on NLLA+$\mathcal{O}(\alpha_s^2)$ calculations have been carried out during the last two years \[38,46,47\]. As expected \[40\] from the improved theoretical accuracy of these predictions, it has been shown that the resummed calculations have a reduced dependence on the renormalization scale and, in particular, remove the need to choose ‘unphysical’ (very small) renormalization scales in the two-jet region (see, for instance, Fig. 4).

\[\text{Fig. 5: Compilation of measurements of } \alpha_s(M_Z) \text{ from LEP and SLC, using resummed NLLA+$\mathcal{O}(\alpha_s^2)$ calculations [11].}\]

A partial summary of $\alpha_s(M_Z)$ from resummed calculations is given in Fig. 5 \[11\]. This summary does not include a very recent and detailed analysis performed by the SLD Collaboration \[38\]. Combining the new SLD result ($\alpha_s(M_Z) = 0.118 \pm 0.006$) with those of the LEP experiments, I obtain the (correlated) average value

$$\alpha_s(M_Z) = 0.122 \pm 0.002\text{(exp.)} \pm 0.005\text{(th.)} .$$ (22)

This result is in good agreement with that obtained from analyses in $\mathcal{O}(\alpha_s^2)$ alone and has a comparable uncertainty. Note however that the theoretical errors are treated differently. All the central values in Fig. 5 refer to the same renormalization scale value $\mu = q$ and scale uncertainties are evaluated by varying $\mu$ by (approximately) a factor of four.
around $Q$. This range includes the best-fit values for $\mu$: renormalization scales very different from $Q$ are not only theoretically disfavoured but they also fail in describing the data. Non-perturbative effects are again estimated from Monte Carlo event generators. However, since modern Monte Carlo simulations operate by generating parton configurations of arbitrarily large multiplicity (typically much larger than the maximum of four partons involved at order $O(\alpha_s^3)$), they are better suited to estimating the hadronization corrections for resummed calculations. In summary, using resummed calculations there is much less freedom to define the central value of $\alpha_S$ and its theoretical uncertainty and a more consistent QCD picture emerges.

4.3 Summary of measurements of $\alpha_S$ at LEP and SLC

The measurements of $\alpha_S$ at LEP and SLC can be summarized as follows (Fig. 6). We have two largely independent determinations of $\alpha_S$ at the scale $M_Z$ from $R_Z$ and event shape distributions in NLLA+$O(\alpha_s^3)$. They respectively give (see Eqs. (10) and (22)):

$$\alpha_S(M_Z) = 0.124 \pm 0.008 ,$$  \hspace{1cm} (23)

$$\alpha_S(M_Z) = 0.122 \pm 0.006 .$$  \hspace{1cm} (24)

A third determination of $\alpha_S$ at the scale $M_\tau$ is provided by $R_\tau$:

$$\alpha_S(M_\tau) = 0.36 \pm 0.05 .$$  \hspace{1cm} (25)

Fig. 6: Summary of measurements of $\alpha_S$ from $R_Z$ and $R_\tau$ at LEP, and from event shape distributions at LEP and SLC.

The two values of $\alpha_S(M_Z)$ in Eqs. (23) and (24) are very consistent with each other, thus proving that perturbative QCD is extremely successful in describing hadronic interactions at the energy $\sqrt{s} \approx 91$ GeV of the $Z^0$ boson mass. Moreover, as shown in Fig. 6, combining
these two determinations of \( \alpha_S(M_Z) \) with that of \( \alpha_S(M_\tau) \) from \( \tau \) decay, we have a strong evidence for the QCD running of \( \alpha_S \) from LEP (and SLC) data alone! Assuming the QCD running of Eq. (2), the value of \( \alpha_S(M_\tau) \) in Eq. (25) corresponds to \( \alpha_S(M_Z) = 0.122 \pm 0.005 \). Therefore, considering that the corresponding energy scales differ by almost two orders of magnitude \( (M_Z \approx 50 M_\tau) \), the agreement among these determinations of \( \alpha_S \) is a very significant test of QCD.

Let me also recall that the measurements of \( \alpha_S \) at LEP/SLC are in good agreement with those obtained from other processes. The world summary of \( \alpha_S \) presented by S. Bethke at the Montpellier Conference [11] is reported in Fig. 7. The data points denoted by circles at the extremes of the plot correspond to determinations performed at LEP/SLC. Further evidence for the QCD running is due to the measurements at intermediate values of the energy \( Q \), which are obtained mainly from deep inelastic lepton-hadron scattering (DIS). The values of \( \alpha_S(M_Z) \) from DIS are slightly smaller than those from LEP/SLC and lead to the consistent (and dominated by theoretical uncertainties) world average \( \alpha_S(M_Z) = 0.117 \pm 0.006 \) [4,6-11].

Fig. 7: Summary of measurements of \( \alpha_S \) from different processes and comparison with perturbative QCD expectations [11].

5. Jet cross sections

A jet is qualitatively defined as a collimated spray of high-energy hadrons. It is likely to be produced by hard scattering of partons and thus it can be regarded as a universal signal of parton dynamics at short distances. However, for the purpose of performing accurate quantitative studies, one needs a precise definition of jet. Essentially, one has to specify how low-energy particles are assigned to jets, in order to have infrared finite cross sections.
5.1 Jet algorithms

The standard jet definition in $e^+e^-$ annihilation [48,31] amounts to introducing a dimensionless resolution variable $y_{ij} = d_{ij}/Q^2$ for every pair $(i,j)$ of particles (jets). Then the particles (jets) with the minimum $y_{ij}$ are merged until a fixed resolution $y_{\text{cut}}$ is reached ($y_{ij} > y_{\text{cut}}$). The final-state particles in each event are therefore classified in a well defined number of jets, depending on the resolution $y_{\text{cut}}$. The main feature of this definition is that the corresponding iterative procedure of clustering type provides an unambiguous and exhaustive assignment of particles to jets.

Several jet clustering algorithms for $e^+e^-$ annihilation are available [31,49]. Different jet algorithms are specified by the definition of the dimensionful resolution variable $d_{ij}$. As a result of many theoretical and phenomenological investigations carried out in the last few years [43,49-51], the theoretically favoured resolution variable turns out to be:

$$d_{ij}^{(k_\perp)} = \min 2(E_i^2, E_j^2)(1 - \cos \theta_{ij}) . \quad (26)$$

Here $E_i$ and $E_j$ are the particle (jet) energies in the $e^+e^-$ centre-of-mass frame and $\theta_{ij}$ is their relative angle. This resolution variable reduces to the minimal relative transverse momentum $k_{\perp}\sqrt{s}$ in the limiting case of small relative angles ($\theta_{ij} \to 0$). For this reason the algorithm is known as $k_{\perp}$-algorithm.

In order to motivate the preference for the $k_{\perp}$-algorithm, let me compare its features with those of an older jet definition as given by the JADE algorithm [48]. In the latter the jet resolution variable is essentially the invariant mass $d_{ij}^{(J)} = 2E_iE_j(1 - \cos \theta_{ij}) \simeq (p_i + p_j)^2$ of the pair of particles (jets).

The resolution variables $d_{ij}^{(k_{\perp})}$ and $d_{ij}^{(J)}$ both vanish in the soft limit $E_i, E_j \to 0$ (the two algorithms are thus infrared safe) but they behave differently for small (and finite) particle energies. It follows that they treat soft radiation in a different way. In particular, since $d_{ij}^{(J)}$ depends on the product $E_iE_j$, the JADE algorithm prefers to merge soft particles first, even if they are far apart in angle. In the case of the $k_{\perp}$-algorithm, the resolution variable $d_{ij}^{(k_{\perp})}$ is instead diagonal with respect to particle energies (i.e. the product $E_iE_j$ is replaced by $E_i^2$ or $E_j^2$). Hence, soft particles are merged with the energetic particle closest in angle. The different jet classification is clearly shown by the L3 event reported in Fig. 8 [1]. One can say that the JADE algorithm induces strong attractive kinematic (due to the jet definition and independent of the underlying dynamics) correlations among soft particles. On the contrary, the $k_{\perp}$-algorithm, where these correlations are absent, leads to the several advantages: i) it avoids a non-intuitive classification of events and an unnatural assignment of particles to jets (soft and wide-angle jets); ii) it reduces the size of the non-perturbative corrections due to the hadronization process (since soft particles are merged with the energetic particle closest in angle, soft fragmentation products are likely to be assigned to the ‘parent’ jet); iii) theoretical calculations in perturbative QCD are more reliable, because multi-parton kinematics does not dominate multi-parton dynamics in higher perturbative orders.

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**This algorithm was first discussed at the Durham Workshop on Jet studies at LEP and HERA, December 1990 [52], and is sometimes referred to as the Durham algorithm.**

††A more detailed discussion can be found in Ref. [50].
The last point is particularly evident in the small-$y_{\text{cut}}$ region, where the QCD perturbative expansion of jet cross sections is dominated by large double logarithmic corrections of the type $\alpha_S^n \ln^m y_{\text{cut}} (m \leq 2n)$. These contributions, whose origin is due to the emission of soft and collinear gluons (as in the case of event shapes in the two-jet region), can be resummed to all orders in $\alpha_S$ if jets are defined using the $k_\perp$-algorithm. On the contrary, in the case of the JADE algorithm, the attractive kinematic correlations among soft particles prevent the implementation of the resummation procedure\cite{53}.

\textbf{Fig. 8}: A 3-jet event as ‘seen’ by (left) the JADE algorithm and (right) the $k_\perp$-algorithm.

One more attractive feature of the $k_\perp$-algorithm is its applicability to processes involving initial-state hadrons (i.e. deep inelastic lepton-hadron scattering, photoproduction processes and hadron-hadron collisions). The generalization to hadron collisions of $e^+e^-$-clustering algorithms is by no means trivial, because one has to face the problem of dealing with the soft remnants of the incoming hadrons and factorize them from high-$p_\perp$ jets produced by hard scattering of partons. As proposed in Ref.\cite{54}, resolution variables of transverse-momentum type are particularly suitable for this purpose and, actually, a $k_\perp$-clustering algorithm for hadron collisions has been set up in Refs.\cite{54,55}. Its use for studying jet physics at HERA and at the Tevatron collider may offer some advantages.
with respect to standard cone algorithms.\cite{54,57}

Fig. 9: Comparison between ALEPH data on the differential two-jet rate $D_2(y_{\text{cut}} = y_3)$ and resummed $+\mathcal{O}(\alpha_s^2)$ calculations corrected (solid curve) for hadronization effects.

5.2 Jet rates and multiplicities

In the last few years, the $k_\perp$-algorithm has replaced the original JADE algorithm in most of the jet analyses carried out in $e^+e^-$ annihilation, including determinations of $\alpha_s$ and studies of jet topology and coherence effects.

The basic jet measurements one can consider are the $n$-jet rates $R_{n-\text{jet}} = \sigma_{n-\text{jet}}/\sigma_{\text{TOT}}$, defined by the ratio between the cross section $\sigma_{n-\text{jet}}$ for producing $n$ jets (at the resolution scale $y_{\text{cut}}$) and the total cross section $\sigma_{\text{TOT}}$. In Fig. 9, the ALEPH data \cite{46} on the differential two-jet rate $D_2(y_{\text{cut}}) = dR_{2-\text{jet}}(y_{\text{cut}})/dy_{\text{cut}}$ are compared with the corresponding QCD predictions in $\mathcal{O}(\alpha_s^2)$ and including the all-order resummation of the logarithmic contributions $\alpha_s^n \ln^{2n} y_{\text{cut}}$ and $\alpha_s^n \ln^{2n-1} y_{\text{cut}}$ \cite{43}. The value of $\alpha_s$ extracted from this analysis contributes an entry in the list reported in Fig. 5.

As a further example of jet studies using the $k_\perp$-algorithm, let me consider the average jet multiplicity $\langle n_{\text{jet}} \rangle = \sum_{n \geq 2} n R_{n-\text{jet}}$. This measurement can be considered as an alternative QCD test with respect to the mean multiplicity of hadrons produced in $e^+e^-$ annihilation. In the case of the hadron multiplicity one is interested in its dependence on the $e^+e^-$ centre-of-mass energy (see, for instance, Sect. 6.2 in Ref. \cite{1}). Here, at a fixed centre-of-mass energy, one can instead study the jet multiplicity as a function of the jet
An appealing feature of $\langle n_{\text{jet}} \rangle$ is that, unlike the hadron multiplicity, it is an infrared safe observable and thus its absolute normalization (and not only its dependence on the energy) is computable in perturbation theory. Moreover, the increase of the jet multiplicity is highly sensitive to non-abelian effects, i.e. to multiple jet production by gluon cascades. This effect is particularly evident in the small-$y_{\text{cut}}$ region where the QCD calculation, to leading accuracy in the double logarithmic expansion parameter $a \equiv (\alpha_S/2\pi) \ln^2 y_{\text{cut}}$, predicts:

$$\langle n_{\text{jet}} \rangle - 2 \approx C_F a \left[ 1 + \frac{1}{12} C_A a + \frac{1}{360} C_A^2 a^2 + \cdots \right].$$

(27)

Note that in this approximation the deviation of Eq. (27) from the double logarithmic behaviour $\alpha_S \ln^2 y_{\text{cut}}$ is entirely due to non-abelian (i.e. proportional to powers of the gluon charge $C_A = N_c$) contributions. The QCD predictions for $\langle n_{\text{jet}} \rangle$ in NLLA+$O(\alpha_S^3)$ [44] have been successfully tested by the L3 and OPAL Collaborations [46] (see Fig. 10).

5.3 Other jet studies

The gauge structure of QCD is completely determined by the fact that the gauge group is $SU(N_c)$ with $N_c = 3$ colours, and that the quarks belong to the fundamental representation. This fixes completely the relative magnitude of all the couplings of the theory. Roughly speaking, to the splitting processes $q \to qg$, $g \to gg$ and $g \to q\bar{q}$, one can associate splitting probabilities respectively proportional to $C_F = (N_c^2 - 1)/2N_c$, $C_A = N_c$, and $T_R = 1/2$. Events with more than four final-state jets in $e^+e^-$ annihilation give access to all the relevant splitting processes and thus multijet events offer the possibility of performing very detailed tests of the gauge structure of the theory.

Angular correlations within four-jet events have been investigated by the LEP Col-
In these investigations, the values of the colour factors $C_F, C_A, T_R$ in the two-loop matrix elements are left as free parameters and fitted to the LEP data. The results of the fits are consistent with the colour gauge group $SU(3)$ whilst the only other groups with three quark colours (namely, $SO(3)$ and the abelian toy model introduced in Ref. [60]) are excluded. Unfortunately, since only $O(\alpha_s^2)$ matrix elements are at present available for these four-jet studies, the ensuing calculations are just LO predictions. Therefore it is very difficult to assess the theoretical accuracy of these analyses.

Many more studies of jet properties have been performed at LEP and SLC. Among them, the results, which have been recently reported on the evidence for differences between quark and gluon jets, are particularly relevant. This and other jet topics are reviewed elsewhere. In the next section, I discuss in some detail a study of the internal structure of jets based on the measurement of the subjet multiplicity inside jets. I think this measurement is quite interesting for addressing issues like QCD coherence and properties of quark and gluon jets.

6. Sub-jet multiplicity and QCD coherence

6.1 Quark and gluon jets

One of the main features of perturbative QCD is the different colour charge carried by quarks and gluons. The bremsstrahlung spectra for the emission of a soft gluon from a hard quark and from a hard gluon differ only in their relative normalization. The emission probability from a quark (or an antiquark) is proportional to the square $|T_q|^2 = C_F = 4/3$ of the (non-abelian) quark charge $T_q$, whilst the emission probability from a gluon is proportional to the square $|T_g|^2 = C_A = 3$ of the gluon charge $T_g$. Among the quantities which are easily measurable in the experiments, the average particle (parton) multiplicity is the most sensitive to the radiation of soft gluons. If we compare the particle multiplicities $N_g$ and $N_q$ in a gluon and a quark jet, we may thus expect a ratio $N_g/N_q$ which is approximately unity at low energies and increases with the energy. At asymptotic energies (in practice, as soon as $N_g, N_q \gg 1$) this ratio should be equal to the ratio of the (squares of the) colour charges, $N_g/N_q \simeq C_F/C_A = 9/4$, i.e. larger than a factor of two.

This naïve expectation has two main problems, besides possibly large subasymptotic corrections. One problem is related to the hadronization effects. The hadron multiplicity is not an infrared safe observable and thus it cannot be computed in QCD perturbation theory. In other words, we have poor theoretical control over the non-perturbative contribution to the ratio of the hadron multiplicities: some systematic difference in the hadronization of quark and gluon jets could significantly contribute to a deviation from the asymptotic multiplicity ratio.

The other problem is related to the definition and identification of quark and gluon jets. As discussed at the beginning of Sect. 5, the quantitative definition of jet is a highly non-trivial issue, which can be solved only by introducing jet-finding algorithms. The problem is amplified in the case of a distinct definition of quark and gluon jets because, although we

\*The quark and gluon colour charges $T_q$ and $T_g$ are $SU(3)$ colour matrices respectively in the fundamental and adjoint representations.
observe jet-like objects, quarks and gluons are not experimentally accessible. Thus, quark jets and gluon jets can be singled out by comparing different processes in which only quarks or gluons contribute at the level of the naïve parton model. Alternatively, in a given process one can anti-tag a gluon jet by identifying heavy-quark decays \[61,62\]. No matter how quark and gluon jets are defined, their definition is theoretically biased by our underlying partonic picture. More quantitatively, we can say that quark and gluon jets cannot be defined in a universal way to an accuracy better than \(\mathcal{O}(\alpha_S)\), whether due to the dependence on the process or the dependence on the observable. Moreover, although nominally of \(\mathcal{O}(\alpha_S)\), this dependence can be numerically large because typically enhanced by logarithmic coefficients (as discussed in Sect. 5.1, even the double logarithmic contributions \(\alpha_S \ln^2 y_{\text{cut}}\) to the jet rates do depend on the jet definition).

Some years ago \[64\], we propose a method aimed at investigating differences between quark and gluon jets by overcoming the problems discussed so far. The idea is to select two-jet and three-jet event samples with the \(k_T\)-algorithm at a fixed value of the jet resolution parameter \(y_{\text{cut}} \equiv y_1\). Then the \(k_T\)-algorithm is again applied by clustering subjets with a smaller resolution \(y_{\text{cut}} \equiv y_0 \leq y_1\) and counting the number of subjets in each jet sample.

Within this approach the hadronization corrections are under better control because hadrons are replaced by subjets, which are infrared and collinear safe objects. Moreover, \(e^+e^-\) annihilation is a point-like source of \(q\bar{q}\) events which undergo fragmentation through the splitting process \(q\bar{q} \to q\bar{q}g\). Therefore, comparing the two-jet and three-jet samples, one has access respectively to the properties of a mixture of \(q\bar{q}\) jets and \(q\bar{q}g\) jets without further specification and identification of quark and gluon jets.

In particular, one can consider the ratio of the average subjet multiplicities \(M_3\) and \(M_2\) in three-jet and two-jet events. By fixing \(y_1\) and decreasing \(y_0\), the ratio \(M_3/M_2\) is expected to vary in the range:

\[
\frac{3}{2} \leq \frac{M_3}{M_2} = \frac{\mathcal{N}_q + \mathcal{N}_{\bar{q}} + \mathcal{N}_g}{\mathcal{N}_q + \mathcal{N}_{\bar{q}}} \approx \frac{2C_F + C_A}{2C_F} = \frac{17}{8}.
\]

The value \(3/2\) on the left-hand side of Eq. (28) corresponds to \(y_0 = y_1\) and is simply due to kinematics (at \(y_0 = y_1\) subjets and jets coincide and there are exactly 3 jets in the three-jet sample and 2 jets in the two-jet sample). The right-hand side of Eq. (28) is instead approached at \(y_0 \ll y_1\). Actually, the value \(17/8\) is that naïvely expected as \(y_0/y_1 \to 0\) on the basis of the (asymptotic) charge counting rule \(\mathcal{N}_g/\mathcal{N}_q = \mathcal{N}_g/\mathcal{N}_{\bar{q}} \approx C_A/C_F\).

The theoretical \[64\] and experimental \[65\] analyses show that this naïve expectation is not only numerically violated but completely misleading from a physical viewpoint. In any physical process, jets are not produced independently because of colour conservation. This leads to coherence effects that, in the case of the subjet multiplicities, are responsible for a value of the ratio \(M_3/M_2\) which remains well below \(3/2\) (which is the value at the kinematic boundary and not the asymptotic value!) for most of the \(y_0\) range \[64\].

The perturbative QCD predictions obtained in Ref. \[64\] include the resummation of the leading and next-to-leading logarithmic contributions \(\alpha_S^2 L^{2n}\) and \(\alpha_S^3 L^{2n-1}\) (\(L\) standing for both \(L_1 = -\ln y_1\) and \(L_0 = -\ln y_0\)) to all orders in \(\alpha_S\). The actual calculation is quite involved, so that, in the following, I limit myself to presenting the final results and to providing their qualitative interpretation.
6.2 Two-jet and three-jet events: $\Delta \eta$-rapidity profiles

Considering jets produced at the $e^+e^-$ centre-of-mass energy $Q$, let me introduce the transverse-momentum scales $Q_1 = \sqrt{y_1}Q$ and $Q_0 = \sqrt{y_0}Q$. Because of the definition (24) for the resolution variable, jets selected by the $k_t$-algorithm at $y_{\text{cut}} = y_1$ have a natural angular size $\theta_J \simeq 2Q_1/E_J$, $E_J$ being the jet energy (Fig. 11a). This angular size identifies the jet core, i.e. the fragmentation region of the jet: all the subjets (or particles), which are inside a cone of aperture $\theta_J$ around the jet axis, belong to the jet. However, each jet $J$ contains also all the subjets which are relatively soft (with energy much smaller than $E_J$) and are produced at an angle $\theta$ (with respect to the jet axis) in the region $\theta_J \lesssim 2\theta \lesssim \theta_{J,J'}$, $\theta_{J,J'}$ being the angular distance to the nearest jet $J'$ (Fig. 11). This region can be called ‘interjet’ region since its angular extension depends on the topology of the multijet configuration.

Let me thus consider the (pseudo-)rapidity profile $dN/d\eta$ of a jet, that is, the number of subjets as a function of their pseudorapidity $\eta = -\ln \tan(\theta/2)$ with respect to the jet axis.

A two-jet event with resolution $y_{\text{cut}} = y_1$ consists of the subjets radiated by a leading quark and a leading antiquark (Fig. 11a). In the fragmentation region of the quark jet the rapidity profile increases up to a saturation value and then produces a rapidity plateau in the interjet region (Fig. 12). The situation is perfectly symmetric in the antiquark hemisphere and the result for the subjet multiplicity in the two-jet event is [64]:

$$M_2(Q_0, Q_1; Q) = 2 \left\{ N_q(Q_0, Q_1) + C_F H(Q_0, Q_1) \ln \frac{Q}{Q_1} \right\}.$$  (29)

The first term on the right-hand side of Eq. (29) represents the ‘intrinsic’ multiplicity of the quark and antiquark jets, coming from their natural angular regions. This contribution is indeed obtained by integrating the rapidity profile in Fig. 12 over the fragmentation regions.

The subjet multiplicity $N_q(Q_0, Q_1)$ is equal to $\frac{1}{2} \langle n_{\text{jet}}(y_{\text{cut}} = Q_0^2/Q_1^2) \rangle$, $\langle n_{\text{jet}}(y_{\text{cut}} = Q_0^2/Q_1^2) \rangle$ being the total jet multiplicity discussed in Sect. 5.2. As a matter of fact, if $y_1 = 1$ (i.e. $Q_1 = Q \simeq E_J/2$) the fragmentation region extends up to $\eta = 0$ and all the $e^+e^-$ events are identified as two-jet events.

The second term on the right-hand side of Eq. (29) gives the extra interjet multiplicity between the jet cores. Note that each jet contributes to a rapidity plateau whose length and height are respectively $\eta = \ln 2E_J/Q_1 \simeq \ln Q/Q_1$ and $C_F H(Q_0, Q_1) \simeq \partial N_q(Q_0, Q_1)/\partial \ln Q_1$. The height of the plateau is proportional to the square $|T_q|^2 = C_F$.
of the colour charge of the leading quark.

**Fig. 11:** (a) A two-jet event in $e^+e^-$ annihilation. The subjets $k_1$ and $k_2$ are produced respectively in the fragmentation region (I) and in the interjet region (II). (b) Typical kinematic configuration of a three-jet event: (A), (B), and (C) denote the angular regions described in the text.

**Fig. 12:** Rapidity profile of a quark (antiquark) jet in a two-jet event: (I) fragmentation and (II) interjet regions ($\eta_0 = \ln 2E_J/Q_0 \simeq \ln Q/Q_0$, $\eta_1 = \ln 2E_J/Q_1 \simeq \ln Q/Q_1$).

Let me then consider three-jet events. When $y_1 \ll 1$ (as appropriate for a calculation to logarithmic accuracy in $\ln y_1$) a three-jet event has the typical kinematic configuration in Fig. 11b. An antiquark (quark) jet recoils from a quark (antiquark) and a gluon jet which are produced at a small relative angle $\theta_{qg}$ in the opposite hemisphere. The rapidity profile of the antiquark jet is similar to that in the two-jet sample. The situation is instead different in the hemisphere with two jets. One has to examine subjet production in three different angular regions (Fig. 11b): (A) at small angle $\theta$ with respect to the quark-jet axis ($\theta < \theta_{qg}$); (B) at small angle $\theta$ with respect to the gluon-jet axis ($\theta < \theta_{qg}$); (C) at large
angle $\theta$ with respect to both the quark-jet and the gluon-jet axis ($\theta > \theta_{qg}$).

Fig. 13: Rapidity profiles for the angular regions (A), (B) and (C) (see text) of the quark+gluon jet hemisphere in three-jet events ($\eta_{qg} = -\ln \tan \theta_{qg}/2$).

The angular regions (A) and (B) include the jet cores and lead to rapidity profiles (Fig. 13) which are similar to that observed in two-jet events. The only difference between the quark jet (A) and the gluon jet (B) is that the latter has a larger multiplicity $N_g(Q_0, Q_1)$ coming from its fragmentation region and, correspondingly, a higher rapidity plateau. The height of the rapidity plateau of the gluon jet is $C_A H(Q_0, Q_1) \simeq \partial N_g(Q_0, Q_1)/\partial \ln Q_1$, i.e. proportional to the square $|T_g|^2 = C_A$ of the gluon charge.

Subjets produced in the interjet region (C) also contribute to a rapidity plateau (Fig. 13). However, since their emission angle $\theta$ is much larger than $\theta_{qg}$, they cannot resolve two dis-

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Footnote: The gluon multiplicity $N_g(Q_0, Q)$ is equal to one half of the total jet multiplicity one could measure in quarkonium decay into two gluons.
tinct jets. The height of the plateau is thus proportional to the square $|T_q + T_g|^2$ of the total colour charge of the quark and gluon jets. Because of colour conservation $T_q + T_g = -T_{\bar{q}}$ ($T_q$ being the colour charge of the antiquark jet recoiling in the opposite hemisphere), and hence, the height of the plateau is proportional to $|T_q + T_g|^2 = |-T_{\bar{q}}|^2 = C_F$.

**Fig. 14:** $\Delta \eta$-rapidity profiles for (a) two-jet and (b) three-jet events ($L_i = -\ln y_i$, $i = 0, 1$). The suppression at small $\Delta \eta$ in (b) is due to QCD coherence.

The results of this discussion on three-jet events can be combined in the rapidity profile of Fig. 14b. Here $\Delta \eta = \eta - \ln(2E_J/Q_0)$, where $\eta$ is the subjet pseudorapidity with respect to the axis of the parent jet and $E_J$ is the energy of the parent jet. Therefore the $\Delta \eta$-rapidity profile is experimentally measurable without identifying quark and gluon jets. Integrating the $\Delta \eta$-rapidity profile in Fig. 14b, one obtains [64]:

$$M_3(Q_0, Q_1; Q) = M_2(Q_0, Q_1; Q) + N_g(Q_0, Q_1) + C_A H(Q_0, Q_1) \langle \eta \rangle_g.$$  (30)

The term $M_2$ on the right-hand side of Eq. (30) comes from the contribution of the antiquark jet and from the contributions of the angular regions (A) and (C) (which add themselves to reconstruct an entire quark-jet rapidity profile as in Fig. 12) in the opposite hemisphere. The remaining terms on the right-hand side of Eq. (30) are due to the additional gluon jet. Like the quark and antiquark jets, the gluon jet provides an intrinsic and an interjet contribution. However, the latter is restricted to the angular region $\theta_{qg} > \theta > Q_1/E_J$ around the gluon jet axis. Thus the length of the pseudorapidity plateau is $\ln q/Q_1$, where $q = \min(E_q\theta_{qg}, E_g\theta_{qg})$ is the transverse momentum at which the gluon and quark jets are resolved. In Eq. (30), this length is averaged over $q$ ($\langle \eta \rangle_g \equiv \langle \ln q/Q_1 \rangle$) with the appropriate three-jet cross section.

In Fig. 14a, I have also reported the $\Delta \eta$-rapidity profile of the two-jet sample, in order to make easier the comparison with the corresponding profile of the three-jet sample. We can see that the additional gluon jet contribution in Fig. 14b, although asymptotically higher by the canonical factor $C_A/C_F = 9/4$, is always shorter than those of the quark jets if $y_1 \ll 1$. In this case the angle between the quark and the gluon jet is very small and the
gluon jet is strongly squeezed. This suppression of the $\Delta \eta$-rapidity profile in the central region is due to QCD coherence, that is, to destructive interference of the quark and gluon jets. In this large-angle region, the quark and gluon jets act coherently with a colour-charge factor of $|T_q + T_g|^2 = |T_\bar{q}|^2 = C_F$, which is much smaller than $|T_q|^2 + |T_g|^2 = C_F + C_A$, the colour-charge factor that one would consider in the case of independent fragmentation of the two jets.

A general lesson one can learn from this discussion is that quark and gluon jets do not live independently. Because of QCD coherence (colour conservation) their properties depend on their production mechanism and on the actual selection procedure.

6.3 Multiplicities in two- and three-jet events

Figure 15 shows results on the ratio $M_3/M_2$ from LEP [65]. As a consequence of the squeezing of the gluon jet due to QCD coherence, this ratio is predicted to fall off near the kinematic limit $y_0 \approx y_1$ [64]. This prediction is in good agreement with the data. Also shown is the prediction of a toy model which does not take into account coherent effects. In this toy model, where jets contribute according to their energies and colour charges without interference, the multiplicity ratio rises monotonically towards its asymptotic value, in disagreement with the experiments. As a further check on the origin of the decrease in the ratio $M_3/M_2$, one can measure its slope at $y_0 = y_1$ as a function of $y_1$. According to QCD coherence, by increasing $y_1$ the angle $\theta_{qg}$ increases, thus reducing the region in which the quark-gluon destructive interference takes place. The slope of $M_3/M_2$ is thus predicted to decrease at large $y_1$, in agreement with the data [65].

**Fig. 15**: L3 data on the ratio $M_3/M_2$ compared with the QCD predictions in Eqs. (29),(30) (solid curve) and with the predictions of an incoherent toy model (dotted curve).

The measurement of the subjet multiplicity in two-jet and three-jet events can be definitely considered a detailed test of (perturbative) QCD coherence. Since it is based on infrared and collinear safe observables which do not require any identification of the gluon jet, this test of coherence is certainly more ‘model independent’ than the celebrated string
The perturbative QCD predictions for $M_3/M_2$ are confirmed by the LEP data up to $y_0 \simeq 5 \cdot 10^{-5}$. This value of $y_0$ corresponds to a minimal transverse momentum $Q_0 \simeq 2$ GeV at LEP energies. It is thus evident that perturbative QCD fails at smaller values of $y_0$ because the hadronization effects dominate. On the contrary, Monte Carlo event generators can be used for investigating the development of a jet by varying $y_0$ from the formation scale ($y_0 = y_1$) to the scale at which individual hadrons are resolved as subjets ($y_0 \sim 10^{-5}$ at $Q = M_Z$). The measurements of subjet properties performed at LEP have been compared with different Monte Carlo models [65]. These include generators like JETSET [13], HERWIG [14], ARIADNE [15], which implement coherence effects in the parton shower evolution, and generators like COJETS [16], which are based on independent fragmentation. As

Fig. 16: OPAL data on the subjet multiplicity of two-jet events compared with different Monte Carlo models. The fractional difference between models and data is also shown.
shown by the representative study in Fig. 16, all the models describe well the data. This is not surprising: the various (hadronization) parameters have been tuned to describing many hadronic observables and such a tuning can mask the dynamics differences among the models. However, it is interesting to note that, although the agreement with the data is always very good for \( y_0 \approx 10^{-5} \) (i.e. at scales where hadronization effects dominate), the Monte Carlo generators with no QCD coherence perform worse at the true subjet level \( (y_0 > 10^{-5}) \). Typically, the incoherent models produce too much radiation and too many subjets in this phase space region. This analysis is a further evidence in favour of coherence and shows that the study of subjets can be a useful tool for investigating the transition from the perturbative to the non-perturbative phase of QCD.

7. Outlook

As discussed in the previous sections, with the advent of the high-energy and high-statistics experiments at LEP/SLC and of accurate theoretical calculations, ‘high-precision’ tests of QCD have become possible. These tests include measurements of \( \alpha_s \) and its energy dependence, detailed analyses of two-jet and three-jet events and studies of QCD coherence.

Further improvements in the field require mainly progress in the theory. Future \( \mathcal{O}(\alpha_s^3) \) calculations for event shapes and jet cross sections can be very useful to check the consistency of the present NLO analyses \[69\]. Accurate studies of multijet \((n \geq 4)\) final states can be performed only if the corresponding QCD predictions in NLO become available. The increasing capabilities of tagging \( b \)-quark jets by micro-vertex detectors allow to carry out precise experimental studies in this field, but complete NLO calculations are still missing \[70\].

This list does not exhaust the demands to the theory. One of the main problems indeed remains our poor understanding of the hadronization process and, in general, of the non-perturbative region. In order to deal with this problem we need better theoretical methods and more phenomenological investigations. In this respect, studies of infrared and collinear safe observables at the boundary of the perturbative region can be of some help. As briefly discussed at the end of Sect. 6, the small-\( y_0 \) behaviour of the subjet multiplicity in two-jet and three-jet events is an example of this kind of studies. Further calculations and predictions for observables which are suitable for similar investigations are warranted.

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