INTERSTELLAR SCINTILLATIONS OF POLARIZATION OF COMPACT SOURCES

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ABSTRACT

We demonstrate that the measurement of fluctuations of polarization due to the Galactic interstellar scintillations may be used to study the structure of the radiation field at compact radio sources. We develop a mathematical formalism and demonstrate it on a simple analytical model in which the scale of the polarization variation through the source is comparable to the source size. The predicted amplitude of modulation of the polarized radiation flux is \((\sim 20\%) \pi m_{scatt}\), where \(\pi\) is the characteristic degree of polarization of radiation at the source and \(m_{scatt}\) is the typical modulation index due to scattering, i.e., \(m_{scatt} \approx 1\) for diffractive scintillations and \(m_{scatt} < 1\) for refractive scintillations.

Subject headings: ISM: general — scattering — techniques: interferometric — techniques: polarimetric

1. INTRODUCTION

Interstellar scintillations (ISSs) have been used in studies of Galactic and extragalactic compact radio sources (see, e.g., Cordes et al. 1985, 1986), mostly pulsars and also active galactic nuclei (see, e.g., Ghosh & Gopal-Krishna 1990 and references therein), some compact extragalactic radio sources (e.g., Rickett et al. 1995), \(\gamma\)-ray burst remnants (Goodman 1997; Waxman, Kulkarni, & Frail 1998), and the accretion disk around the Galactic center (Lo et al. 1998). However, this technique is (mostly) limited to the measurements of the intensity variations, while the information contained in the polarization properties is usually lost. In this Letter, we develop a formalism of the ISS of polarization, which will allow us to study the structure of the radiation field at the source which is difficult to obtain by other techniques because of resolution limits (see, e.g., Medvedev & Loeb 1999 for an application to \(\gamma\)-ray bursts).

Using a simple model of the source radiation, we analytically calculate modulation indices. We demonstrate that the amplitude of fluctuations of polarization is maximum when the angular correlation scale of radiation at the source (which is often the source size) is comparable to the typical scale of “speckles” produced at the observing plane by the interstellar scattering and amounts up to \((\sim 20\%) \pi m_{scatt}\), where \(\pi\) is the typical degree of polarization at the source and \(m_{scatt}\) is the typical modulation amplitude, which is \(\approx 1\) for diffractive ISSs (DISSs) and usually \(\sim 0.1\) for refractive ISSs (RISSs).

2. POLARIZATION SCINTILLATION FORMALISM

The ISSs arise when fluctuations in the electron density randomly modulate the refractive index of a medium. As a result of diffraction (for DISSs) and random focusing (for RISSs) of electromagnetic waves, a point source produces a pattern in an observing plane which consists of randomly located bright and dim spots—speckles—and is observed as fluctuations of the source brightness when an observer moves through the pattern. The characteristic correlation length of the pattern \(r_s(\lambda)\) is determined by the regime of scattering, statistical properties of the interstellar turbulence, and the observing wavelength \(\lambda\) (for more details, see a nice review by Narayan 1992). It is convenient to introduce the “angular” size of a speckle, \(\theta_s = r_s/D\), where \(D\) is the distance to the scattering screen, and we define \(\theta_s\) to be the angular size of a source.

Let us assume that the emission is polarized and the sense of polarization varies through the source, so that the net (average) polarization vanishes. If \(\theta_s \ll \theta_g\), the source is effectively pointlike. In this regime the intensity variations are large, while the polarization signal is weak due to averaging. In contrast, when \(\theta_s \gg \theta_g\), different elements in the source produce identical patterns, which are offset by \(D\theta\), according to the element angular separation \(\delta\theta\). The overall pattern is, thus, obtained from the superposition (convolution) of individual, incoherent patterns. An observer will measure fluctuations of the direction of polarization, while the intensity exhibits less modulation. When \(\theta_s \gg \theta_g\), the offsets become much larger than the speckle size, so that the speckles are multiply overlapped and both intensity and polarization fluctuations smear out.

The properties of radiation are then fully described by four scalar quantities, the Stokes parameters, which are additive for incoherent (incoherent) radiation fluxes (Rybicki & Lightman 1979). They are the intensity \(I\), \(Q\), \(U\), and \(V\). The quantities \(Q\) and \(U\) describe linear polarization, and \(V\) describes circular polarization. The degree of polarization is defined as

\[
\pi = I_{pol}/I = (Q^2 + U^2 + V^2)^{1/2}/I,
\]

where \(I_{pol}\) is the polarized intensity. Note that the polarized and total intensities are proportional to the integrated fluxes from the source which are directly measurable. We assume that scintillations do not affect polarization properties of radiation. It has been demonstrated (Simonetti, Cordes, & Spangler 1984)

2 Let us introduce the Fresnel scale \(r_s = (\lambda D/2\pi)^{1/2}\) (where \(D\) is the distance to the scattering screen) and the diffractive length \(r_{diff}\), which represents the transverse separation for which the root mean square phase difference (produced by scattering) is equal to 1 radian. In the weak scattering regime \(r_{diff} \gg r_s\), the speckle size is of order of the Fresnel scale, \(r_s \sim r_{diff}\). In the strong scattering regime \(r_{diff} \ll r_s\), the diffractive and refractive effects are distinct (Goodman & Narayan 1985). For the short-term DISS, the speckle size is \(r_s \sim r_{diff}\), while for the longer term RISS, this is \(r_s \sim r_{diff} = r_s^D I_{diff} \gg r_{diff}\).

Note that the apparent angular size of the scattered point source image is \(\theta_{app} \sim r_{diff}/D\) for both DISS and RISS.

1 If the net degree of polarization is nonzero, one can always subtract the constant contribution. The variation of this polarized contribution is identical to the total flux variation.
that birefringence in the interstellar medium is negligible in discussion of scattering. Also, the effect of angular Faraday depolarization across a source is weak for the observing frequencies above ~10 MHz (Linfield 1996).

Let us assume that a point source is located at infinity. It illuminates a thin phase-changing screen located at a distance $D$ from an observer plane $(x,y)$ and produces a speckle pattern on this plane. We introduce the normalized intensity distribution in the pattern and its Fourier transform as follows:

$$W(x,y) = \frac{I_p}{\langle I_p \rangle} \equiv \frac{I_p - \langle I_p \rangle}{\langle I_p \rangle} \equiv W(k_x,k_y),$$

where angle brackets denote a spatial average, $\equiv$ denotes a Fourier conjugated pair, and the “calligraphic” letters are used to denote the Fourier transformed quantities. Since the Stokes parameters are additive, the point source speckle patterns of $Q$, $U$, and $V$ are also described by $W$. The statistics of speckles are characterized by the autocorrelation function of the normalized intensity fluctuations,

$$M(x,y) = \langle W(x_0,y_0)W(x_0 + x,y_0 + y) \rangle = \langle |W(k_x,k_y)|^2 \rangle = M(k_x,k_y),$$

the Fourier transform of which is called the power spectrum of the intensity fluctuations.

The intensity distribution produced by an extended source is obtained from the convolution of the intensity distribution of a point source with the source surface brightness distribution $P_0(\theta_0,\theta_0)$,

$$I(x,y) = W(x,y) * P_0(\theta_0,\theta_0)$$

$$\equiv \int \int W(x-D\theta_x,y-D\theta_y)P_0(\theta_x,\theta_y)d\theta_x d\theta_y$$

$$\equiv W(k_x,k_y)P(k_x,k_y) = \tilde{I}(k_x,k_y),$$

where $(\theta_x,\theta_y)$ are angular coordinates in the sky, and we normalize the total flux to unity, i.e., $\langle I \rangle = \int \int P_0(\theta_x,\theta_y)d\theta_x d\theta_y = 1$. In equation (4), the convolution theorem has been used. The amplitude of intensity fluctuations due to scintillations is determined by the modulation index:

$$m_l = (\hat{P})^{1/2} = \sqrt{\int \int |\tilde{I}|^2 d^2k} = \sqrt{\int \int M |\theta|^2 d^2k},$$

where the last expression follows from Parseval’s theorem and we denote $d^2k = dk_x dk_y$. Analogously, we define the distributions and modulation indices of the other Stokes parameters.

The amplitude of fluctuations of the polarized intensity is described by

$$m_{QUV} = (m_Q^2 + m_U^2 + m_V^2)^{1/2}.$$ 

By analogy with equation (1), we formally introduce a measure of the “degree” of polarization, $m_* = m_{QUV}/m_l$.

We now comment that in the regime of strong scattering, the diffractive contribution to $m_{QUV}$ should be subject to similar bandwidth limitations as its contribution to $m_\nu$. A useful way to explain the physical effect is to imagine reversing the roles of source and receiver (a legitimate application of optical reciprocity). The telescope is effectively pointlike, and if it were made into a narrowband transmitter, then a chaotic pattern of fringes (speckles) would be placed on the sky. If the extragalactic object is larger in angular size than these fringes, then intensity variations are quenched because light and dark fringes average out. But polarization variations may not be quenched if they occur on angular scales within the object that are smaller than the fringe spacing. Changing the narrowband transmission frequency shifts the fringes, causing different polarized regions to be amplified. Hence the use of a broad frequency band will quench the diffractive polarization fluctuations, although refractive fluctuations will remain.

3. ANALYTICAL MODEL

An analytical calculation of the modulation indices is a difficult mathematical task. For this reason, we consider the simplest source model. We assume that the source has a uniform surface brightness over a rectangular angular region on the sky: $-\theta_s \leq \theta_x \leq \theta_s, -\theta_s \leq \theta_y \leq \theta_s$, with $\theta_s$ being the angular size (“radius”) of the source. This approximation simplifies considerably the related integrals and provides a good approximation to a circular source. We normalize the total flux to unity. We choose the surface distribution of the Stokes parameters to be $P_0(\theta_0,\theta_0) = 0$ and

$$P_\nu(\theta_0,\theta_0) = \frac{1}{4\theta_s^2} \Theta(\theta_0 - |\theta_0|)\Theta(\theta_0 - |\theta_0|),$$

$$P_\nu(\theta_0,\theta_0) = \pi, P(\theta_0,\theta_0) = \frac{1}{2} \left( \cos \frac{\pi \theta_0}{\theta_s} - \cos \frac{\pi \theta_0}{\theta_s} \right),$$

$$P_\nu(\theta_0,\theta_0) = \pi, P(\theta_0,\theta_0) = \sin \frac{\pi \theta_0}{2\theta_s} \sin \frac{\pi \theta_0}{2\theta_s},$$

where $\pi$ is the “intrinsic” degree of polarization of radiation and $\Theta(x)$ is the Heaviside step function. The functions $P_0$ and $P_\nu$ are shown in Figure 1. These distributions mimic the radiation field which is linearly polarized in the radial direction from the center of the source. We take the intensity power
spectrum to be Gaussian:

$$\mathcal{M}(k_x, k_y) = \pi m_{sc}^2 \theta^2 \exp \left[-\pi^2 \theta^2 D^2 (k_x^2 + k_y^2)\right],$$  \hspace{1cm} (8)

which is normalized so that $\langle |\mathcal{M}|^2 \rangle = m_{sc}$, where $m_{sc}$ is the “typical” modulation index for a point source. We do not specify $\pi$, $\theta$, and $m_{sc}$ here and keep them as free parameters. The degree of polarization $\pi$, depends on the emission mechanism and may be as high as ~70% for synchrotron radiation. The quantities $\theta$ and $m_{sc}$ depend on the regime of scintillations and the wavelength of radiation. Namely, $\theta \sim r_{diff}/D$ and $m_{sc} \sim 1$ for DISSs and $\theta \sim r_{vis}/D$ and $m_{sc} < 1$ for RISSs (see Narayan 1992 and footnote 2).

Using the formalism from the previous section (§ 2), we calculate the modulation indices $m_i$, $m_{UV}$ and $m_v$. Upon lengthy and cumbersome calculations, some details of which are presented in the Appendix, we arrive at the following expressions:

$$m_i^2 = m_{sc}^2 \pi^2 \theta^2 \left(\mathcal{C}(\theta) - \mathcal{A}(\theta)\right)^2,$$  \hspace{1cm} (9a)

$$m_{p}^2 = \frac{\pi^2 m_{sc}^2}{4 \pi^2 \theta^2} \left[\mathcal{C}(\theta) + \mathcal{A}(\theta)\right] \left[\mathcal{C}(\theta) - 2\mathcal{C}(\theta)\right],$$  \hspace{1cm} (9b)

$$m_{v}^2 = \frac{\pi^2 m_{sc}^2}{4 \pi^2 \theta^2} \left[\mathcal{C}(\theta) - \mathcal{A}(\theta)\right]^2,$$  \hspace{1cm} (9c)

where $\theta = 2\theta/\theta_0$ is the size of the object in units of the characteristic speckle scale, and $\mathcal{A}(\theta)$, $\mathcal{C}(\theta)$ and $\mathcal{C}(\theta)$ are complicated expressions involving the error functions of real and imaginary arguments, given by equations (A12)-(A4). Using equations (5) and (6), it is now straightforward to calculate the quantities that are directly measured, namely the amplitude of fluctuations of total and polarized intensity (flux), $m_i$ and $m_{p,v}$. The dependence of the scintillation indices on $\theta$ is presented in Figure 2. Clearly, for a small source size ($\theta \ll \theta_0$) there are no fluctuations of polarization, while the intensity fluctuations are maximum, ~100% $m_{sc}$. As the speckle size approaches the source size, the observed radiation is partially polarized. At the same time, the intensity contrast decreases due to the overlap between the speckles. The fluctuations of polarization peak when $\theta$ is a few times $\theta_0$ at the value of ~20% $\pi m_{sc}$. For a larger source, the fluctuation amplitude of both polarized and total intensity decreases. However, the degree of polarization ($m_i = m_{UV}/m_v$) continuously increases with the increasing source size and asymptotes at ~70% $\pi$.

It is a good approximation because for a power spectrum of electron density fluctuations in the ISM, $|\delta n_e(\xi)|^2 \propto q^2$ (with $q$ being the spatial wavenumber), the inferred value of the index is close to the Kolmogorov theory prediction $\beta = 11/3$ (Armstrong, Rickett, & Spangler 1995). A little higher value, $\beta = 2$, yields the Gaussian law for the phase structure function (the root mean square phase shift along different rays), $D_{max} \propto (\delta n_e)^{1/2}$.

\section*{APPENDIX}
\section*{CALCULATION OF THE SCINTILLATION INDICES}

First we introduce new dimensionless variables $\xi = x/D$ and $\eta = y/D$. We use the following definition of the Fourier transform: $B(k_x, k_y) = \int B(\xi, \eta) \exp \left[-2\pi i(k_x \xi + k_y \eta)\right] d\xi d\eta$, and similarly for other quantities. The Fourier transformed distributions of

\begin{figure}[ht]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Modulation indices of the intensity ($m_i$), polarized intensity ($m_{p,v}$), and “degree” of polarization ($m_v = m_{p,v}/m_i$), as functions of the normalized source size $\theta$ for completely polarized radiation, $\pi = 1$, in the diffractive scattering regime, $m_{sc} = 1$. The thin vertical line marks where $\theta = \theta_0$.}
\end{figure}

4. CONCLUSION

In this Letter we propose a new observational technique—the detection of polarization scintillations—to study the structure of the radiation field at compact radio sources. We have developed a general formalism and demonstrated it on a simple analytical model. For randomly polarized radiation with the degree of polarization at the source typical of synchrotron radiation (~75%), the amplitude of fluctuations of the polarization flux is calculated to be ~15% in the case of DISSs and an order of magnitude lower in the case of RISSs. We should note that these quantitative results were obtained for an oversimplified model and provide an illustration of the predicted effect. Radiation fields of real sources are more complicated and are determined by the properties of emission processes and radiative transfer. The statistical properties of the interstellar turbulence are determined by pumping and cascade mechanisms, and the assumption that the fluctuations are Gaussian is also rather crude. Therefore, we expect that numerical values of the modulation indices computed in a more detailed study may be different from those presented here.

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The amplitude of fluctuations, which is the autocorrelation function at zero separation, may now be calculated using Parseval’s theorem (see eq. [5]): \( \langle P^2 \rangle = \langle \tilde{P}(\xi, \eta) \tilde{P}(\xi + \delta \xi, \eta + \delta \eta) \rangle_{\delta \xi = \delta \eta} = \int M(k_1, k_2) \mathcal{P}(k_1, k_2) \mathcal{P}(-k_1, -k_2) dk_1 dk_2 \), and similarly for \( \langle \tilde{Q}^2 \rangle \) and \( \langle \tilde{U}^2 \rangle \). In calculating these quantities, one encounters the following basic integrals:

\[
I_1 = \int_{-\infty}^{\infty} e^{-\alpha^2 \xi^2} \sin^2 \frac{\pi \xi}{\lambda} d\xi, \quad I_2 = \int_{-\infty}^{\infty} e^{-\alpha^2 \xi^2} \sin^2 \left( \frac{\pi (\xi - 1)}{\lambda - 1} \right)^2 d\xi, \quad I_3 = \int_{-\infty}^{\infty} e^{-\alpha^2 \xi^2} \sin^2 \left( \frac{\pi (\xi - 1)}{\lambda - 1} \right)^2 d\xi.
\]

Other integrals can be reduced to the above integrals by noticing that \( \sin \pi (\xi - 1) = \sin \pi (\xi + 1) = - \sin \pi \xi \). To illustrate the method of taking these integrals, consider \( I_2 \) as an example. We write \( I_2 \) as a function of a new parameter, say \( \alpha \), so that it reduces to the original integral for \( \alpha = 1 \), namely \( I_2(\alpha) = \int_{-\infty}^{\infty} e^{-\alpha^2 \xi^2} \sin^2 \left( \alpha \pi (\xi - 1)/(\xi - 1)^2 \right) d\xi \). Differentiating twice with respect to this new parameter, we obtain an integral which can now be evaluated:

\[
I''_2(\alpha) = 2\pi^2 \int_{-\infty}^{\infty} e^{-\alpha^2 \xi^2} \cos 2\pi \alpha (\xi - 1) d\xi = 2\pi^2 \frac{\pi}{a^2} \exp \left( \frac{-\pi^2 \alpha^2}{a^2} \right) \cos \pi \alpha.
\]

One should now integrate \( I''_2(\alpha) \) twice with respect to \( \alpha \) and then set \( \alpha = 1 \). We have \( I_2 = \int_0^1 I''_2(\alpha) d\alpha \). The lower limits in the integrals are taken so as to automatically include constants of integration by taking into account that \( I_2(0) = I''_2(0) = 0 \) by definition. The integrals involving the error functions, which emerge after the first integration, may be taken by integrating by parts and using the fact that \( \text{erf}^2(\xi) = (2/\pi) e^{-\xi^2} \). Finally, the integrals \( I_1 \)–\( I_3 \) become, respectively,

\[
\mathcal{A}(\vartheta) = \frac{\pi^{3/2}}{\vartheta} \left[ e^{-\vartheta^2} - 1 + \pi \vartheta \text{erf} (\vartheta) \right],
\]

\[
\bar{A}(\vartheta) = \frac{\pi^{3/2}}{\vartheta} \left\{ e^{-\vartheta^2} - 1 - \frac{\pi}{2} e^{-\vartheta^2} \left[ \left( \frac{\pi}{\vartheta} - i \vartheta \right) \text{erfi} \left( \frac{\pi}{\vartheta} - i \vartheta \right) + \left( \frac{\pi}{\vartheta} + i \vartheta \right) \text{erfi} \left( \frac{\pi}{\vartheta} + i \vartheta \right) - 2 \frac{\pi}{\vartheta} \text{erfi} \left( \frac{\pi}{\vartheta} \right) \right] \right\},
\]

and

\[
\bar{C}(\vartheta) = \frac{\pi}{4} e^{-\vartheta^2} \left[ \text{erfi} \left( \frac{\pi}{\vartheta} - i \vartheta \right) + \text{erfi} \left( \frac{\pi}{\vartheta} + i \vartheta \right) - 2 \text{erfi} \left( \frac{\pi}{\vartheta} \right) \right],
\]

where \( \text{erf}(x) = 2/\pi \int_0^x e^{-t^2} dt \) is the error function, \( \text{erfi}(z) = 2/\pi \int_0^z e^{-t^2} dt = -i \text{erf}(iz) \) is the error function of imaginary argument, and \( \vartheta = 2\theta_t/\theta_0 \). The scintillation indices are then readily calculated to yield equations (9a)–(9c).

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