The $B \rightarrow \pi K$ puzzle and the Bulk Randall-Sundrum model

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Abstract

The recent measurements of the direct CP asymmetries ($A_{CP}$) in the penguin-dominated $B \rightarrow K \pi$ decays show some discrepancy from the standard model (SM) prediction. While $A_{CP}$ of $B^+ \rightarrow \pi^0 K^+$ and that of $B^0 \rightarrow \pi^- K^+$ in the naive estimate of the SM are expected to have very similar values, their experimental data are of the opposite sign and different magnitudes. We study the effects of the custodial bulk Randall-Sundrum model on this $A_{CP}$. In this model, the misalignment of the five-dimensional (5D) Yukawa interactions to the 5D bulk gauge interactions in flavor space leads to tree-level flavor-changing neutral current by the Kaluza-Klein gauge bosons. In a large portion of the parameter space of this model, the observed nonzero $A_{CP}(B^+ \rightarrow \pi^0 K^+)-A_{CP}(B^0 \rightarrow \pi^- K^+)$ can be explained only with low Kaluza-Klein mass scale $M_{KK}$ around 1 TeV. Rather extreme parameters is required to explain it with $M_{KK} \approx 3$ TeV. The new contributions to well-measured branching ratios of $B \rightarrow K \pi$ decays are also shown to be suppressed.

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1. INTRODUCTION

The study of $B$ meson decays at Belle and BaBar [1] have been a crucial probe of the standard model (SM), especially its CP violation part. Recently the $B \to K\pi$ decay system has drawn a lot of interest due to the discrepancy between the SM predictions and the measurements [2–6]. There are nine measurements of the four decays of $B^+ \to \pi^+ K^0$, $B^+ \to \pi^0 K^+$, $B^0 \to \pi^- K^+$, and $B^0 \to \pi^0 K^0$: four branching ratios (BR), four direct CP asymmetries $A_{\text{CP}}$, and one mixing-induced CP asymmetry $S_{\text{CP}}$. The 2008 data of these nine measurements are in Table I.

$$
\begin{array}{|c|c|c|c|}
\hline
\text{Mode} & \text{BR} [10^{-6}] & A_{\text{CP}} & S_{\text{CP}} \\
\hline
B^+ \to \pi^+ K^0 & 23.1 \pm 1.0 & 0.009 \pm 0.025 & \\
B^+ \to \pi^0 K^+ & 12.9 \pm 0.6 & 0.050 \pm 0.025 & \\
B^0 \to \pi^- K^+ & 19.4 \pm 0.6 & -0.098^{+0.012}_{-0.011} & \\
B^0 \to \pi^0 K^0 & 9.8 \pm 0.6 & -0.01 \pm 0.10 & 0.57 \pm 0.17 \\
\hline
\end{array}
$$

TABLE I. Experimental data for $B \to \pi K$; BR’s, direct CP asymmetries $A_{\text{CP}}$, and mixing-induced CP asymmetry $S_{\text{CP}}$ [7, 8].

We focus on the direct CP asymmetries $A_{\text{CP}}$ of $B^+ \to \pi^0 K^+$ and $B^0 \to \pi^- K^+$. In the SM, the dominant contribution to each decay amplitude comes from the strong penguin contribution $P$. The color-suppressed tree contribution $C$ may be smaller than the color-favored tree contribution $T$ by a factor of the small parameter $\lambda = |V_{us}| \simeq 0.22$. Therefore, both $B^+ \to \pi^0 K^+$ and $B^0 \to \pi^- K^+$ could have $A_{\text{CP}}$ given by the interference between $T$ and $P$ in the leading order, as shown in Eq. (1). The direct CP asymmetries of two decay modes are expected to be the same size with the same sign within the naive estimate of the SM. As can be seen in Table I, however, the observation is quite different from this naive SM prediction: $A_{\text{CP}}(B^+ \to \pi^0 K^+)$ is non-zero positive, while $A_{\text{CP}}(B^0 \to \pi^- K^+)$ is negative. Known as “$B \to \pi K$ puzzle”, this discrepancy has brought extensive attentions, leading to model-independent studies as well as new physics (NP) effect studies in the literature.

In this paper, we study this $A_{\text{CP}}$ puzzle in the framework of the custodial bulk Randall-Sundrum (RS) model [9]. In the simplest implementation of the RS model, featuring an

\footnote{Albeit statistically less significant with larger error compared with BELLE and BaBar data, the CLEO collaboration observed negative mean value for this asymmetry \cite{7}, $A_{\text{CP}}(B^+ \to \pi^0 K^+) = -0.29 \pm 0.23 \pm 0.02$.}
$SU(2)_L \times U(1)_Y$ bulk gauge symmetry and a minimal brane-localized Higgs sector, this $B \rightarrow K\pi$ puzzle was studied, showing the difficulty to solve the puzzle under the experimental constraints [10]. As a five-dimensional (5D) warped model with all the SM fields in the bulk (except for the Higgs boson field), the bulk RS model provides very attractive explanations for both gauge hierarchy and fermion mass hierarchy [11–21]. To ensure the $SU(2)$ custodial symmetry, we adopt the model with the bulk gauge symmetry of $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X$, induced from AdS$_5$/CFT feature [9].

Since the 5D Yukawa interaction is not generally flavor-diagonal in this model, the flavor-changing-neutral-current (FCNC) is generated at tree level, mediated by Kaluza-Klein (KK) gauge bosons [22, 23]. The 5D Yukawa couplings $\lambda_{5ij}$ and the bulk Dirac mass parameters $c_f$’s determine all the FCNC processes in principle. Without a priori information about the model parameters, however, this model lacks the prediction power.

In our previous works [23, 24], we show that if we adopt two simple and natural assumptions, we can fix the model parameters and extract the necessary information for the calculations. The first assumption is that the 5D Yukawa couplings are universal, $\lambda_{5ij} \simeq \lambda_5 \sim O(1)$. The second assumption is that fine tuning is not allowed when explaining the observed SM mixing matrices (CKM and PMNS). Here our restriction is at the level of no order-changing by cancellation. Mild fine tuning is permitted in this setup. With the given $\lambda_{5ij}$’s and $c_f$’s based on the two assumptions, we study the bulk RS model effects on the $B \rightarrow K\pi$ decay. We will show that this model can explain the discrepancy between the observed $A_{CP}$ in the $B \rightarrow \pi K$ decay and the SM prediction with the KK mass scale around 1 TeV. These NP effects give suppressed contributions to the well observed BR’s.

The organization of the paper is as follows. In Sec. II, we briefly review the current status of the measurements of four $B \rightarrow K\pi$ decays. In Sec. III, we summarize the custodial bulk RS model and formulate the bulk fermion sector. After presenting two naturalness assumptions, we determine all the bulk Dirac mass parameters. Section IV deals with the effects of this model on $A_{CP}$’s of $B \rightarrow K\pi$ decays. We conclude in Sec. V.

II. SHORT REVIEW OF $B \rightarrow \pi K$ DECAYS

In the SM, the $B \rightarrow \pi K$ decays are dominated by the $\bar{b} \rightarrow \bar{s}$ QCD penguin diagrams. The electroweak penguin and the tree contributions are next dominant. The current experimental
data in Table I show that the branching ratios are very precisely measured. The observations are more precise than the SM theoretical estimates such as QCD factorization and the perturbative QCD [25].

The $B \to \pi K$ decay amplitudes can be written in terms of topological amplitudes up to $\lambda^2$ scale:

$$A(B^+ \to \pi^+ K^0) = P' - \frac{1}{3} P'^C_{EW}, \quad (1)$$

$$\sqrt{2}A(B^+ \to \pi^0 K^+) = -P' - T' - P'^{C}_{EW} - \frac{2}{3} P'^C_{EW} - C',$$

$$A(B^0 \to \pi^- K^+) = -P' - T' - \frac{2}{3} P'^C_{EW},$$

$$\sqrt{2}A(B^0 \to \pi^0 K^0) = P' - P'^T_{EW} - \frac{1}{3} P'^C_{EW} - C'. \quad (2)$$

The primes denote the $\bar{b} \to \bar{s}$ transition. The color-favored (color-suppressed) tree diagrams are represented by $T'$ ($C'$), and the $P'^{(C)}_{EW}$ is the electroweak (color-suppressed electroweak) penguin.

The penguin diagram $P'$ is the sum of three up-type ($u,c,t$) quark contributions:

$$P' = \lambda_u \tilde{P}_u + \lambda_c \tilde{P}_c + \lambda_t \tilde{P}_t$$

$$= \lambda_t (\tilde{P}_t - \tilde{P}_c) + \lambda_u (\tilde{P}_u - \tilde{P}_c)$$

$$\equiv P'_{tc} + P'_{uc}. \quad (2)$$

where $\lambda_i \equiv V^*_{ib}V_{is}(i = u,c,t)$, and the unitarity of the CKM matrix is used for the second equality. Here, the phase of $\lambda_t \equiv V^*_{tb}V_{ts}$ is $\sim \pi$ within the SM. We also expect the following hierarchies from theoretical calculations in the SM [26–28]:

$$O(1) \quad |P'_{tc}|,$$

$$O(\lambda) \quad |T'|, |P'_{EW}|,$$

$$O(\lambda^2) \quad |C'|, |P'_{uc}|, |P'^C_{EW}|. \quad (3)$$

If we define

$$\Delta A_{CP} \equiv A_{CP}(B^+ \to \pi^0 K^+) - A_{CP}(B^0 \to \pi^- K^+), \quad (4)$$

the SM predicts very small $\Delta A_{CP}$, which is contradictory to the experimental data in Table I. This discrepancy possibly suggests the existence of the NP contribution, especially in the CP violating phases. If the NP contribution is the source of the discrepancy, it should be of the order of $\lambda$ or more.
The effective Hamiltonian for $B \to \pi K$ can be written in operator expansion \[29\]:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left( \sum_{p=u,c} \lambda_p (C_1 Q^p_1 + C_2 Q^p_2) - \lambda_t \sum_{i=3}^{10} C_i Q_i \right) .$$

(5)

The operators are defined by

$$Q^p_1 = (\bar{b}_i p_i)_{V-A} (\bar{q}_j s_j)_{V-A}, \quad Q^p_2 = (\bar{b}_i p_j)_{V-A} (\bar{q}_j s_i)_{V-A},$$

$$Q_3 = (\bar{b}_i s_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V-A}, \quad Q_4 = (\bar{b}_i s_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A},$$

$$Q_5 = (\bar{b}_i s_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V+A}, \quad Q_6 = (\bar{b}_i s_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A},$$

$$Q_7 = (\bar{b}_i s_i)_{V-A} \sum_q \bar{q}_j q_j (\bar{q}_j q_j)_{V+A}, \quad Q_8 = (\bar{b}_i s_j)_{V-A} \sum_q \bar{q}_j q_i (\bar{q}_j q_i)_{V+A},$$

$$Q_9 = (\bar{b}_i s_i)_{V-A} \sum_q \bar{q}_j q_j (\bar{q}_j q_j)_{V-A}, \quad Q_{10} = (\bar{b}_i s_j)_{V-A} \sum_q \bar{q}_j q_i (\bar{q}_j q_i)_{V-A},$$

(6)

where $i, j$ are color indices, $e_q$ is the electric charge of the quark, $(\bar{q}_i q_2)_{V=\pm} = \bar{q}_1 \gamma_\mu (1 \pm \gamma_5) q_2$ and $q = u, d$.

The topological amplitudes are written in terms of the Wilson coefficients in the standard operator basis as \[29, 30\]

$$A(B^+ \to \pi^+ K^0) = -\lambda_t \left[ \left( a_4 - \frac{1}{2} a_{10} \right) + r^K \left( a_6 - \frac{1}{2} a_8 \right) \right] A_{\pi K},$$

$$\sqrt{2} A(B^+ \to \pi^0 K^+) = A(B^0 \to \pi^- K^+) - \left[ \lambda_u a_2 + \frac{3}{2} \lambda_t (a_7 - a_9) \right] A_{K\pi},$$

$$A(B^0 \to \pi^- K^+) = - \left[ \lambda_u a_1 - \lambda_t (a_4 + a_{10}) - \lambda_t r^K \left( a_6 + a_8 \right) \right] A_{\pi K},$$

$$\sqrt{2} A(B^0 \to \pi^0 K^0) = A(B^+ \to \pi^+ K^0) + \sqrt{2} A(B^+ \to \pi^0 K^+) - A(B^0 \to \pi^- K^+),$$

(7)

where $a_i = C_i + C_{i+1}/3$ with $+(-)$ sign for odd (even) $i$. We can specify each penguin contributions as \[30\],

$$P_{tc}' = -\lambda_t \left[ a_4 + r^K a_6 \right] A_{\pi K},$$

$$P_{\text{EW}}' = \frac{3}{2} \lambda_t \left[ a_7 - a_9 \right] A_{K\pi},$$

$$P_{\text{EW}}'^C = -\frac{3}{2} \lambda_t \left[ a_{10} + r^K a_8 \right] A_{\pi K},$$

$$T' = \lambda_u a_1 A_{\pi K},$$

$$C' = \lambda_u a_2 A_{K\pi} ,$$

(8)

where $r^K = 2m_{K}^2/m_b(m_u + m_d)$, $m_q = (m_u + m_d)/2$, $A_{\pi K(K\pi)} = G_F(m_B^2 - m_{\pi(K)}^2)F_0^{\pi(K)} f_{K(\pi)}/\sqrt{2}$, and $F_0^{\pi(K)} \approx 0.3$ are semileptonic form factors for $B$ decays \[31\].
III. REVIEW OF THE BULK RANDAL-SUNDRUM MODEL

The RS model is based on a 5D warped spacetime with the metric \[ ds^2 = \frac{1}{(kz)^2} (dt^2 - dx^2 - dz^2), \] (9)
where the fifth dimension \( z \) is compactified between \( 1/k < z < 1/T \). Here \( k \simeq M_{Pl} \) and \( T \) is the natural cut-off of the gauge theory at TeV scale. Two boundaries \( z_{UV} = 1/k \) and \( z_{IR} = 1/T \) are called the Planck brane and the TeV brane, respectively.

For \( SU(2) \) custodial symmetry, we adopt the model suggested by Agashe et.al. in Ref. [9], based on the gauge structure of \( SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X \). The custodial symmetry is guaranteed by the bulk \( SU(2)_R \) gauge symmetry. The bulk gauge \( SU(2)_R \) symmetry is broken into \( U(1)_R \) by the orbifold boundary conditions on the Planck brane: charged \( SU(2)_R \) gauge fields have \((-+)\) parity. The \( U(1)_R \times U(1)_X \) is spontaneously broken into \( U(1)_Y \) on the Planck brane and the Higgs field localized on the TeV brane is responsible to the breaking of \( SU(2)_L \times U(1)_Y \) to \( U(1)_{EM} \).

The action for a 5D gauge fields is

\[ S_{gauge} = \int d^4x dz \sqrt{G} \left[ -\frac{1}{4} g^{MP} g^{NQ} F_{MN}^a F_{PQ}^a \right], \] (10)

where \( G \) is the determinant of the AdS metric \( g^{MN} \), and \( F_{MN}^a = \partial_M A_N^a - \partial_N A_M^a + g_5 f^{abc} A_M^b A_N^c \). The 5D action for the gauge interactions of a bulk fermion \( \hat{\Psi}(x,z) \equiv \Psi(x,z)/(kz)^2 \) is

\[ S_{int} = \int d^4x dz \sqrt{G} \frac{g_{5i}}{\sqrt{2k}} \bar{\Psi}(x,z) i\Gamma^M A_M^a(x,z) T^a \hat{\Psi}(x,z), \] (11)

where \( g_{5i} \) is the 5D dimensionless gauge coupling (\( g_{5s}, g_{5L}, g_{5R}, g_{5X} \)) for each gauge group \( (SU(3)_c, SU(2)_L, SU(2)_R, U(1)_X) \) and \( \Gamma^M = (\gamma^\mu, i\gamma_5) \).

A bulk gauge field \( A_\nu(x,z) \) and a bulk fermion field \( \hat{\Psi}(x,z) \) are expanded in terms of KK modes by

\[ A_\nu(x,z) = \sqrt{k} \sum_n A^{(n)}_\nu(x) f^{(n)}_A(z), \]
\[ \hat{\Psi}(x,z) = \sqrt{k} \sum_n [\psi_L(x) f_L(z) + \psi_R(x) f_R(z)], \] (12)

where the mode functions of \( f^{(n)}_A(z) \) and \( f^{(0)}_L(z,c) = f^{(0)}_R(z,-c) \) are referred to Ref. [18]. Here \( c \) is defined through the 5D Dirac mass \( m_D = \text{sign}(y)ck \) and \( z = e^{k|y|}. \) Note that a
massless SM fermion corresponds to the zero mode with \((++)\) parity. Since \(\Psi_L\) has always opposite parity of \(\Psi_R\), the left-handed SM fermion is the zero mode of a 5D fermion whose left-handed part has \((++)\) parity (the corresponding right-handed part has automatically \((-\cdash-\cdash)\) parity which cannot have a zero mode).

Due to the gauge structure of \(SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X\), the right-handed SM fermions belong to a \(SU(2)_R\) doublet, and couple to \(SU(2)_R\) gauge bosons with \((-\cdash+)\) parity. As a result, the whole quark sector is

\[
Q_i = \begin{pmatrix}
u_{iL}^{(++)} \\ d_{iL}^{(++)}
\end{pmatrix}, \quad U_i = \begin{pmatrix}
u_{iR}^{(++)} \\ D_{iR}^{(--)}
\end{pmatrix}, \quad D_i = \begin{pmatrix}U_{iR}^{(--)} \\ d_{iR}^{(++)}
\end{pmatrix},
\]

where \(i\) is the generation index. Dirac mass parameters \((c_{Q_i}, c_{U_i}, c_{D_i})\) determine their mode functions, KK mass spectra, and coupling strength with KK gauge bosons.

On the TeV brane, the SM fermion mass is generated as the localized Higgs field develops its VEV of \(\langle H \rangle = v \simeq 174\) GeV. The SM mass matrix for a fermion \(f(=u,d,\nu,e)\) is

\[
(M_f)_{ij} = v\lambda_{5ij}^f \frac{k}{T} f_R^{(0)}(z,c_{f_R}) f_L^{(0)}(z,c_{f_L}) \bigg|_{z=1/T} \equiv v\lambda_{5ij}^f F_R(c_{f_R}) F_L(c_{f_L}),
\]

where \(i, j\) are the generation indices, \(\lambda_{5ij}^f\) are the 5D (dimensionless) Yukawa couplings, and \(F_L(c) = F_R(-c)\) is defined by

\[
F_L(c) = \frac{f_L^{(0)}(1/T,c)}{\epsilon^{1/2}},
\]

where \(\epsilon = T/k\). The mass eigenstates of the SM fermions involve two independent mixing matrices, defined by

\[
\chi_{fL} = U_{fL}^\dagger \psi_{fL}^{(0)}, \quad \chi_{fR} = U_{fR}^\dagger \psi_{fR}^{(0)}.
\]

Note that the observed mixing matrix is a multiplication of two independent mixing matrices such that \(V_{CKM} = U_{UL}^\dagger U_{DL}\) and \(U_{PMNS} = U_{eL}^\dagger U_{\nu L}\).

If bulk Dirac mass parameters and 5D Yukawa couplings are given \textit{a priori}, we could predict all the mass spectra and mixing matrices as well as the couplings with KK gauge bosons. Without those crucial knowledge, we have to take the opposite way, \textit{i.e.}, deducing them from the observation. The problem is that the number of observations is not enough to fix all the model parameters. In the previous work, we have developed a theory based on the following two natural assumptions:

1. For all fermions, 5D Yukawa couplings have the same order of magnitude \(\lambda_5^f \sim O(1)\).
2. When explaining the observed mixing matrix $V_{CKM} = U_{uL}^\dagger U_{dL}$ and $U_{PMNS} = U_{eL}^\dagger U_{\nu L}$, no order-changing by cancellation is allowed.

The assumption-1 yields anarchy type fermion mass matrix, which naturally explains the large top quark mass $v \simeq 174$ GeV. Other small SM fermion masses are generated by controlling $c$'s. The assumption-2 is consistent with the spirit of no fine-tuning.

In Ref. [18], we have shown that the above two natural assumptions determine the nine bulk mass parameters within a well-defined regions as

\begin{equation}
\begin{aligned}
c_{Q_1} &\simeq 0.61, & c_{Q_2} &\simeq 0.56, & c_{Q_3} &\simeq 0.3^{+0.02}_{-0.04}, \\
c_{U_1} &\simeq -0.71, & c_{U_2} &\simeq -0.53, & 0 &\lesssim c_{U_3} \lesssim 0.2, \\
c_{D_1} &\simeq -0.66, & c_{D_2} &\simeq -0.61, & c_{D_3} &\simeq -0.56. 
\end{aligned}
\end{equation}

Recently phenomenological constraint on the value of $c_{Q_3}$ has been studied, focused on the anomalous coupling of $Zb\bar{b}$ vertex [32]: $c_{Q_3}$ cannot be smaller than 0.3. Combined with our constraint based on the two naturalness assumptions, we consider the case of $c_{Q_3} = 0.3 \sim 0.32$ in what follows.

The quark mixing matrices are

\begin{equation}
(U_{qL})_{ij(i\leq j)} \approx \frac{F_L(c_{Q_i})}{F_L(c_{Q_j})}, \quad (U_{qR})_{ij(i\leq j)} \approx \frac{F_R(c_{A_i})}{F_R(c_{A_j})}, \quad A = U, D.
\end{equation}

Then our mixing matrices show the following order of magnitude behaviors:

\begin{equation}
\begin{aligned}
U_{uL} &\simeq U_{dL} \simeq \begin{pmatrix}
1 & \lambda & \lambda^3 \\
\lambda & 1 & \lambda^2 \\
\lambda^3 & \lambda^2 & 1
\end{pmatrix} + \mathcal{O}(\lambda^4), \\
U_{uR} &\simeq \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & \lambda^2 \\
0 & \lambda^2 & 1
\end{pmatrix} + \mathcal{O}(\lambda^4), \quad U_{dR} \simeq \begin{pmatrix}
1 & \lambda & \lambda^2 \\
\lambda & 1 & \lambda \\
\lambda^2 & \lambda & 1
\end{pmatrix} + \mathcal{O}(\lambda^3).
\end{aligned}
\end{equation}

As shall be shown below, only $U_{uL}$ and $U_{dL}$ make dominant contributions to $B \to K\pi$ decays. Because of high similarity of $U_{uL}$ and $U_{dL}$ to the CKM matrix, we take the following assumptions of

\begin{equation}
(U_{qL})_{ij} = \kappa_{ij} (V_{CKM})_{ij}.
\end{equation}

In order to satisfy our naturalness assumptions, we require

\begin{equation}
\frac{1}{\sqrt{2}} < |\kappa_{ij}| < \sqrt{2}.
\end{equation}
IV. BULK RS MODEL EFFECTS ON $B \to K\pi$ DECAYS

In the bulk RS model, the mass eigenstate of a SM fermion is a mixture of gauge eigenstates as in Eq. (16) and we have FCNC mediated by KK gauge bosons. In terms of gauge eigenstates, the four-dimensional (4D) gauge interactions with KK gauge modes $A^{\alpha(n)}_{\mu}$ are

$$\mathcal{L}_{4D} \supset g_{\lambda} \sum_{n=1}^{\infty} \left( \hat{g}_L^{(n)}(c_i) \bar{\psi}_{iL}^{(0)} T^a \gamma^\mu \psi_{iL}^{(0)} + \hat{g}_R^{(n)}(c_i) \bar{\psi}_{iR}^{(0)} T^a \gamma^\mu \psi_{iR}^{(0)} \right) A^{\alpha(n)}_{\mu}, \quad (23)$$

where $g_{\lambda} = g_{\lambda j}/\sqrt{k_L}$ for $j = s, L, R, X$. Since the bulk RS effects are suppressed by the forth power of the KK mass, we consider only the contribution of the first KK mode of gauge bosons. In what follows, therefore, we omit the KK mode number notation $(n)$. Then the effective gauge couplings with the first KK gauge boson are

$$\hat{g}_L(c_f) = \hat{g}_R(-c_f) = \sqrt{k_L} \int dz \kappa \left[ f_L^{(0)}(z, c_f) \right]^2 f_A^{(1)}(z) \equiv \hat{g}(c_f). \quad (24)$$

Note that the effective coupling $\hat{g}(c)$ vanishes if $c = 1/2$.

The values of the bulk mass parameters $c$'s in Eq. (17) fix the $\hat{g}$ values as

$$\hat{g}(c_{Q_1}) = -0.192, \quad \hat{g}(c_{Q_2}) = -0.179, \quad \hat{g}(c_{Q_3}) = 1.797 \sim 1.974, \quad (25)$$
$$\hat{g}(c_{U_1}) = -0.193, \quad \hat{g}(c_{U_2}) = -0.133, \quad \hat{g}(c_{U_3}) = 2.759 \sim 3.948, \quad \hat{g}(c_{D_1}) = -0.193, \quad \hat{g}(c_{D_2}) = -0.192, \quad \hat{g}(c_{D_3}) = -0.179,$$ 

where, for example, $\hat{g} = 1.974$ for $c_{Q_3} = 0.3$ and $\hat{g} = 1.797$ for $c_{Q_3} = 0.32$. It can be seen that $\hat{g}(c_{Q_3})$ and $\hat{g}(c_{U_3})$ are dominant over all the other $\hat{g}$'s of the order of $\lambda$.

The relevant FCNC processes mediated by the first KK gauge bosons are described by the following Lagrangian:

$$\mathcal{L}_{4D} = g_s \left( K_{lm}^{UL} \bar{u}_L \bar{T}^a \gamma^\mu u_m + K_{lm}^{DL} \bar{d}_L \bar{T}^a \gamma^\mu d_m + K_{lm}^{U} \bar{u}_{LR} \bar{T}^a \gamma^\mu u_m + K_{lm}^{D} \bar{d}_{LR} \bar{T}^a \gamma^\mu d_m \right) C_{\mu}^{a(1)} - \frac{1}{2} \left[ g \left( K_{lm}^{UL} \bar{u}_L \bar{\gamma}^\mu u_m - K_{lm}^{DL} \bar{d}_L \bar{\gamma}^\mu d_m \right) W_{3\mu}^{(1)} \right. \quad (26)$$

$$+ \tilde{g} \left( K_{lm}^{U} \bar{u}_{LR} \bar{\gamma}^\mu u_m + K_{lm}^{D} \bar{d}_{LR} \bar{\gamma}^\mu d_m \right) W_{3\mu}^{(1)}$$
$$+ g_X \left( K_{lm}^{UL} \bar{u}_L \bar{\gamma}^\mu u_m + K_{lm}^{DL} \bar{d}_L \bar{\gamma}^\mu d_m + K_{lm}^{U} \bar{u}_{LR} \bar{\gamma}^\mu u_m + K_{lm}^{D} \bar{d}_{LR} \bar{\gamma}^\mu d_m \right) B_{X\mu}^{(1)},$$

where the subscript $l, m$ are the generation indices ($l, m = 1, 2, 3$), and the 4D gauge cou-
plings are \( g_4 = g_5 \sqrt{\kappa L} \). The effective mixing matrices \( K \)'s are

\[
K_{qL}^{ij} = \sum_{k=1}^{3} (U_{qL}^i)_{lk} \hat{g}(c_{Qk}) (U_{qL}^j)_{km}, \quad \text{for } q = u, d,
\]

\[
K_{U}^{ij} = \sum_{k=1}^{3} (U_{uR}^i)_{lk} \hat{g}(-c_{Uk}) (U_{uR}^j)_{km},
\]

\[
K_{D}^{ij} = \sum_{k=1}^{3} (U_{dR}^i)_{lk} \hat{g}(-c_{Dk}) (U_{dR}^j)_{km}.
\]

(27)

We first estimate the value of the elements of \( K \)'s. Since \( \hat{g}(c_{Q1}) \approx \hat{g}(c_{Q2}) \ll \hat{g}(c_{Q3}) \) as in Eq. (25), the dominant elements of \( K_{qL} \) are

\[
K_{qL}^{32} \simeq (U_{dL})_{33} (U_{dL})_{32} \hat{g}(c_{Q3}) \sim \lambda^2 \hat{g}(c_{Q3}),
\]

\[
K_{qL}^{31} \sim \lambda^3 \hat{g}(c_{Q3}), \quad K_{qL}^{11} \sim \hat{g}(c_{Q1}), \quad K_{qL}^{12} \sim \lambda \hat{g}(c_{Q1}).
\]

(28)

In addition, the condition of \( \hat{g}(c_{D1}) \approx \hat{g}(c_{D2}) \approx \hat{g}(c_{D3}) \), up to \( O(\lambda^2) \), leads to diagonal \( K_D \) up to \( O(\lambda^4) \):

\[
K_{D}^{ij(i \neq j)} = 0.
\]

(29)

Note that these vanishing off-diagonal elements of the right-handed quarks ensure the validity of the operator expansions in the effective Hamiltonian Eq. (5). The diagonal elements are

\[
K_{D}^{11} \sim \hat{g}(c_{D1}), \quad K_{11}^{U} \sim \hat{g}(c_{U1}).
\]

(30)

Finally we have the effective Hamiltonian for \( B \to K \pi \) decay, given by

\[
\mathcal{H}_{RS} \simeq \frac{g^2 K_{32}^{dl}}{8 M_{KK}^2} \left( \bar{b}_i s_j \right)_{\nu N} \left\{ K_{11}^{UL} (\bar{u}_j u_i)_{\nu N} + K_{11}^{UL} (\bar{d}_j d_i)_{\nu N} + K_{11}^{UL} (\bar{u}_j u_i)_{\nu N} + K_{11}^{UL} (\bar{d}_j d_i)_{\nu N} \right\}
\]

\[
- \frac{1}{3} \left( \bar{b}_i s_j \right)_{\nu N} \left\{ K_{11}^{UL} (\bar{u}_j u_i)_{\nu N} + K_{11}^{UL} (\bar{d}_j d_i)_{\nu N} + K_{11}^{UL} (\bar{u}_j u_i)_{\nu N} + K_{11}^{UL} (\bar{d}_j d_i)_{\nu N} \right\}
\]

\[
- \frac{g^2 K_{32}^{dl}}{16 M_{KK}^2} \left( \bar{b}_i s_j \right)_{\nu N} \left\{ K_{11}^{UL} (\bar{u}_j u_i)_{\nu N} - K_{11}^{UL} (\bar{d}_j d_i)_{\nu N} \right\}
\]

\[
- \frac{g^2 K_{32}^{dl}}{16 M_{KK}^2} \left( \bar{b}_i s_j \right)_{\nu N} \left\{ K_{11}^{UL} (\bar{u}_j u_i)_{\nu N} + K_{11}^{UL} (\bar{d}_j d_i)_{\nu N} + K_{11}^{UL} (\bar{u}_j u_i)_{\nu N} + K_{11}^{UL} (\bar{d}_j d_i)_{\nu N} \right\},
\]

(31)

where \( i, j \) are the color indices and \( M_{KK} \) is the first KK gauge boson mass. Here we have included only the most dominant terms proportional to \( K_{32}^{dl} \) since the values of \( \hat{g}(c_{Q3}) \) and \( \hat{g}(c_{U3}) \) are much larger than those of the other \( \hat{g} \)'s.
Matching the NP contribution to Wilson coefficients from Eq. (5) and Eq. (9), we get even $C_i$'s of

$$C_4 = -\frac{B_{RS}}{\lambda_t} g_s^2 (K_{11}^{uL} + 2K_{11}^{dL}), \quad C_6 = -\frac{B_{RS}}{\lambda_t} g_s^2 (K_{11}^{U} + 2K_{11}^{D}),$$

$$C_8 = -2\frac{B_{RS}}{\lambda_t} g_s^2 (K_{11}^{U} - K_{11}^{D}), \quad C_{10} = -2\frac{B_{RS}}{\lambda_t} g_s^2 (K_{11}^{uL} - K_{11}^{dL}),$$

and odd $C_i$'s of

$$C_3 = \frac{B_{RS}}{\lambda_t} \left\{ \left( \frac{g_s^2}{3} + \frac{g^2 + g_X^2}{2} \right) K_{11}^{uL} + 2 \left( \frac{g_s^2}{3} + \frac{g_X^2 - g^2}{2} \right) K_{11}^{dL} \right\},$$

$$C_5 = \frac{B_{RS}}{\lambda_t} \left( \frac{g_s^2}{3} + \frac{g_X^2}{2} \right) (K_{11}^{U} + 2K_{11}^{D}),$$

$$C_7 = \frac{B_{RS}}{\lambda_t} \left( \frac{2g_s^2 + g_X^2}{3} \right) (K_{11}^{U} - K_{11}^{D}),$$

$$C_9 = \frac{B_{RS}}{\lambda_t} \left\{ \left( \frac{2g_s^2}{3} + g^2 + g_X^2 \right) K_{11}^{uL} - \left( \frac{2g_s^2}{3} - g^2 + g_X^2 \right) K_{11}^{dL} \right\}.$$

Here $B_{RS}$ denotes the common NP factor, given by

$$B_{RS} = \frac{\sqrt{2}K_{32}^{dL}}{24GFM_{KK}^2} = \frac{1}{3g^2} \left( \frac{m_W}{M_{KK}} \right)^2 K_{32}^{dL}.$$

The leading NP contributions in the same form as the penguin diagrams of Eq. (8) become

$$P'_{NP} = B_{RS} \left[ \left( \frac{8}{9}g_s^2 - \frac{g_X^2}{6} \right) \left(K_{11}^{uL} + 2K_{11}^{dL} + r^K_{X} (K_{11}^{U} + 2K_{11}^{D}) \right) \right. - \left. \frac{g^2}{6} (K_{11}^{uL} - 2K_{11}^{dL}) \right] \Lambda_{\pi K},$$

$$P'_{EW,NP} = -\frac{3}{2} B_{RS} \left[ g_s^2 \left(K_{11}^{uL} - K_{11}^{dL} - K_{11}^{U} + K_{11}^{D} \right) + g^2 (K_{11}^{U} + K_{11}^{D}) \right] \Lambda_{K \pi},$$

$$P'_{EW,NP}^{C} = \frac{3}{2} B_{RS} \left[ \left( \frac{16}{9}g_s^2 - \frac{g_X^2}{3} \right) \left(K_{11}^{uL} - K_{11}^{dL} + r^K_{X} (K_{11}^{U} - K_{11}^{D}) \right) \right. - \left. \frac{g^2}{3} (K_{11}^{uL} + K_{11}^{dL}) \right] \Lambda_{\pi K}.$$

The SM coupling of $U(1)_Y$ is $g_Y = g_X \tilde{g}/\sqrt{g_X^2 + \tilde{g}^2}$.

If we redefine

$$\tilde{P} = P' + P'_{NP} - \frac{1}{3}(P'_{EW} + P'_{EW,NP}),$$

Eq. (11), ignoring $\mathcal{O}(\lambda^2)$ terms, becomes

$$A(B^+ \rightarrow \pi^+ K^0) = \tilde{P},$$

$$\sqrt{2}A(B^+ \rightarrow \pi^0 K^+) = -\tilde{P} - T' - P'_{EW} - P'_{EW,NP} - P'_{EW,NP}^{C},$$

$$A(B^0 \rightarrow \pi^- K^+) = -\tilde{P} - T' - P'_{EW,NP}^{C},$$

$$\sqrt{2}A(B^0 \rightarrow \pi^0 K^0) = \tilde{P} - P'_{EW} - P'_{EW,NP}.$$
Considering the fact that the NP contribution should not affect the well-measured BR’s of $B \to K\pi$ decays, it is reasonable to assume that all NP contributions should be smaller than $P'_{tc}$. In the case where the strong CP phases of $P'_{NP}$ and $P'_{EW,NP}$ are the same, the ratio of the NP contribution of $P'$ to the SM $P'_{tc}$ in this model is

$$\frac{P'_{NP} - P'_{EW,NP}/3}{P'_{tc}} \sim -\frac{B_{RS}(8g_s^2/3 - g_X^2/2) \{K_{11}^{dL} + r^K K_{11}^{D}\}}{\lambda_t(a_4 + r_X a_6)}$$

$$\sim -\lambda C \left(\frac{300 \text{ GeV}}{M_{KK}}\right)^2,$$

where $K_{32}^{dL} \sim K_{11}^{D} \sim \lambda$, $K_{32}^{dL} \sim \lambda_t \hat{g}(c_{Q3})$ and $|a_4 + r_X a_6| \sim 0.05$ to 0.1. The constant $C$ is $O(1)$ and less than 10 in conservative estimation. The KK mass over 1 TeV suppresses new contribution to the branching ratios of $B \to K\pi$ decays.

On the other hand, the $\Delta A_{CP}$, defined in Eq. (41), can be explained by new contribution to the $P'_{EW}$.

We adopt the simplifying notation of

$$P' = |P'_{tc}| \eta_{NP},$$

where $\eta_{NP} \approx (1 + P'_{NP}/|P'_{tc}| - P'_{EW,NP}/3|P'_{tc}|)$. The contribution of $\eta_{NP}$ to $A_{CP}$ is suppressed, if $M_{KK} > 1$ TeV, $P'_{NP}/|P'_{tc}| \ll \lambda$, and $P'_{EW,NP}/3|P'_{tc}| \ll \lambda$. The NP effect can be written as

$$A(B^+ \to \pi^+ K^0) = -|P'_{tc}| \eta_{NP},$$

$$\sqrt{2} A(B^+ \to \pi^0 K^+) = |P'_{tc}| \eta_{NP}(1 - r_{EW} e^{i\delta_{EW}} - r_T e^{i\delta_T} - r_1 e^{i\phi_1} e^{i\delta_1} - r_2 e^{i\phi_2} e^{i\delta_2}),$$

$$A(B^0 \to \pi^- K^+) = |P'_{tc}| \eta_{NP}(1 - r_T e^{i\delta_T} - r_2 e^{i\phi_2} e^{i\delta_2}),$$

$$\sqrt{2} A(B^0 \to \pi^0 K^0) = -|P'_{tc}| \eta_{NP}(1 + r_{EW} e^{i\delta_{EW}} + r_1 e^{i\phi_1} e^{i\delta_1}),$$

where $r_{EW} \equiv |P'_{EW}/P'_{tc}|$, $r_T \equiv |T'|/P'_{tc}$, $r_1 \equiv |P'_{EW,NP}/P'_{tc}|$, $r_2 \equiv |P'_{EW,NP}/P'_{tc}|$, $\phi_i$’s are the weak phases and $\delta_i$’s are strong phases. The difference between two $A_{CP}$ is

$$\Delta A_{CP} \approx 2r_1 \sin \delta_1 \sin \phi_1,$$

where $r_1$ is

$$r_1 \approx \frac{1}{2g^2} \left| (K_{11}^{uL} - K_{11}^{dL} - K_{11}^{uL} + K_{11}^{D}) g_X^2 + (K_{11}^{uL} + K_{11}^{dL}) g_X^2 \right| \hat{g}(c_{Q3}) |K_{32}^{uL} K_{32}^{dL}| f_\pi \frac{m_W^2}{|a_4 + r_X a_6|} \frac{f_K}{f_K M_{KK}^2}$$

$$\lesssim \frac{1}{2g^2} \left[ (\lambda g_X)^2 + g^2 \right] \frac{m_W^2}{M_{KK}^2} R_1,$$

where $R_1$ is...
FIG. 1. $\Delta A_{CP}$ for $R_1 \sin \delta_1 \sin \phi_1 = 5$ and $R_1 \sin \delta_1 \sin \phi_1 = 20$ when $g \gg \lambda g_X$. The horizontal lines are the observed $\Delta A_{CP}$ with the 1σ experimental error.

where $R_1 = 2\hat{g}(cQ_1)\hat{g}(cQ_3)|\kappa_{33}^*\kappa_{32}|f_\pi/(f_K|a_4 + r_K^0a_6|)$. Here $\kappa_{ij}$ is defined in Eq. [22] and $\lambda \simeq 0.22$. The value of $(K_{11}^{uL} - K_{11}^{dL} - K_{11}^U + K_{11}^D)$ has a suppressed value of $\kappa^2\lambda^2 \hat{g}(cQ_1)$ because of the semi-diagonal mixing matrices of $U_{qL,qR}$ as in Eq. [19], while $K_{11}^{uL} + K_{11}^{dL} \sim 2\hat{g}(cQ_1)$. We have used the mixing parameters in Eqs. [19, 22]. In our choices of input parameters, the range of $R_1$ is from 4 to 20, depending on the effective mixing matrices $K$’s, $|\kappa_{33}^*\kappa_{32}|$ and the SM values of $a_4$ and $a_6$ which are around $0.05 \sim 0.07$.

Brief review about the phenomenological constraints on $M_{KK}$ is in order here. In the minimal custodial RS model with an TeV-brane localized Higgs field and anarchic 5D Yukawa couplings, the constraints from $\epsilon_K$, $b \rightarrow s\gamma$ and $B^0\bar{B}^0$ mixing put the lower bound of KK mass gauge boson above 10 TeV [23, 34–36]. Later, it has been suggested that $M_{KK}$ can be lowered down to $\sim 5$ TeV if we release the Higgs field in the bulk and include the one-loop matching of gauge couplings [37, 38]. However, if we allow mild tuning in the 5D Yukawa couplings, both models with the Higgs filed on the TeV-brane or in the bulk accommodate sizable region in parameter space for $M_{KK} > 3$ TeV [35]. In our model which allows mild tuning in terms of $\kappa$, we accept the phenomenological bound of $M_{KK} > 3$ TeV, which is in marginal reach of the LHC.
FIG. 2. The maximal $\Delta A_{CP}$ for $R_1 \sin \delta_1 \sin \phi_1 = 20$ and $g_X = 2g/\lambda$ and $g_X = 3g/\lambda$. The horizontal lines are the observed $\Delta A_{CP}$ with the 1$\sigma$ experimental error.

In Fig. 1 we plot $\Delta A_{CP}$ for $R_1 \sin \delta_1 \sin \phi_1 = 5, 20$ with $\lambda g_X$ ignored. The horizontal lines are the observed $\Delta A_{CP}$ with the 1$\sigma$ experimental error. As expected, $\Delta A_{CP}$ decreases with increasing $M_{KK}$. For small $R_1 \sin \delta_1 \sin \phi_1 = 5$, the new contribution to $\Delta A_{CP}$ is too small for $M_{KK} > 1$ TeV. In order to explain $\Delta A_{CP}$, the scale $M_{KK}$ should be around 1 TeV even for the maximal value of $R_1$ and CP phases. This low $M_{KK}$ is excluded by other phenomenological constraints.

In Fig. 2 we explore the possibility to explain the observed $\Delta A_{CP}$ with $M_{KK} \simeq 3$ TeV in rather extreme case of large $g_X$. We found that this is feasible only when $U(1)_X$ coupling is large enough such as $g_X \simeq 3g/\lambda$ and the contribution from $g_X$ sector in Eq. (44) is maximal. This large value of $g_X$ is marginally allowed by the pertubativity.

Large $g_X$ gives additional and possibly significant contributions to other experiments. We notice that four quark vertices such as $B^0 - \bar{B}^0$ mixing and $B \rightarrow K\phi$ do not have severe contributions unless $g_X$ is not too larger than $g_s$, because there are 8 gluon modes. We also consider the most stringent lepton number violating processes such as $\mu \rightarrow 3e$ and $B \rightarrow Kl^+l^-$ decays where the dominant RS contribution to decay amplitudes is proportional to $\{g^2 - (3g_X)^2\}\hat{g}(c_{Li})$. These NP contributions are proportional to $\hat{g}(c_{Li})$ where $c_{Li}$'s
$(i = e, \mu, \tau)$ are the bulk Dirac mass parameters for the lepton doublets. Since $\hat{g}/c$ defined in Eq. (23) vanishes if $c = 1/2$, it is required that $c_{Li}$ are extremely close to $1/2$ to suppress the large RS contribution. Also, considering the perturbative nature of $U(1)_{B-L}$ gauge couplings, it may be unnatural to assume $g_X/g \gg 3$.

V. CONCLUSIONS

More precise data of $B \to K\pi$ decays have recently raised many interests. While the branching ratios are well measured and can be explained within the SM, the direct CP asymmetries in the $B^+ \to \pi^0 K^+$ and $B^0 \to \pi^- K^+$ show significant deviation from the SM predictions. This is called the $\Delta A_{CP}$ puzzle in the $B \to K\pi$ decays. We study this discrepancy in the 5D custodial bulk Randall-Sundrum model where all the SM fields are in the bulk: one exception is the localized Higgs boson field.

In this model, the Yukawa interactions with the localized Higgs fields lead to non-zero masses for the SM fermions which are the zeroth modes of the bulk fermion. While the bulk gauge interactions are flavor-diagonal, the Yukawa interactions generally mix the SM fermions of different generations. At tree level, the KK gauge bosons can mediate FCNC. We study these FCNC effects on four $B \to K\pi$ decays. A custodial bulk RS model with two naturalness assumptions has been adopted, where all the 5D Yukawa couplings and the bulk Dirac mass parameters for the SM fermions can be determined.

We showed that the operator expansions in the effective Hamiltonian for $B \to K\pi$ take the same form to leading order since the mixing matrix of the right-handed fermion fields is almost diagonal. To leading order, this model has new contributions to the gluonic penguin amplitude $P'$, color-favored and color-suppressed electroweak penguin amplitudes $P'_{EW}$ and $P'_{EW, C}$. The well measured branching ratios of four $B \to K\pi$ decays remain almost intact if the KK mass scale is above 1 TeV. On the contrary, the new contribution to the color-favored electroweak penguin amplitude $P'_{EW, NP}$ get NP contributions from the gluon KK mode exchange. However its magnitude in a large portion of the model parameter space is too small: the discrepancy of $\Delta A_{CP}$ between the experimental data and the SM prediction can be explained with $M_{KK} \simeq 1$ TeV, which contradicts with other phenomenological constraints. In rather extreme case of very large $g_X$, however, the observed $\Delta A_{CP}$ can be explained with $M_{KK} \simeq 3$ TeV.
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