Gauge fields, massless modes and topology of gauge fields in multi-band superconductors

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(Received July 14, 2013)

Multi-phase physics is a new physics of multi-gap superconductors. Multi-band superconductors exhibit many interesting and novel properties. We investigate the dynamics of the phase-difference mode and show that this mode yields a new excitation mode. The phase-diference mode is represented as an abelian vector field. There are massless modes when the number of gaps is greater than three and the Josephson term is frustrated. The fluctuation of phase-difference modes with non-trivial topology leads to the existence of a fractional-quantum flux vortex in a magnetic field. A superconductor with a fractional-quantum flux vortex is regarded as a topological superconductor with the integer Chern number.

KEYWORDS: multi-band superconductor, phase-difference mode, fractional vortex, massless mode

1. Introduction

There are interesting and profound analogies between particle physics and superconductivity. This was first pointed out by Y. Nambu, and he invented a concept of spontaneous symmetry breaking in particles physics [1, 2]. The global $U(1)$ phase invariance is spontaneously broken in superconductors. It is well known that the gapless Goldstone mode exists when the continuous symmetry is spontaneously broken. Superconductivity is most familiar phenomenon that occurs as a result of spontaneous symmetry breaking. The Ginzburg-Landau free energy describes a spontaneous breaking of $U(1)$ symmetry. The order parameter is written as $\psi = |\psi|e^{i\theta}$ for any real angle $\theta$ in the range $0 \leq \theta \leq 2\pi$. Any choice of $\theta$ would have exactly the same energy that implies the existence of a massless Nambu-Goldstone boson. This changes qualitatively when the Coulomb interaction between the electrons is included. The Coulomb repulsive interaction turns the massless mode into a gapped plasma mode [3]. Therefore the mode that originates from the phase variable $\theta$ does not play an important role in single-band superconductors. This would change qualitatively again in multi-gap superconductors because the multi-phase mode variables will produce new excitation states.

Multi-phase physics is a new physics of multi-gap superconductors. The study of multi-gap superconductors stemmed from works by Kondo [4] and Suhl et al. [5]. An additional phase invariance will bring about novel phenomena. The phase-difference mode would yield new phenomena [6–10] and new excitation modes in multi-gapped superconductors. The negative isotope effect in Fe pnictides is an example of the multi-band effect [11, 12]. The existence of fractionally quantized flux vortices is very significant and attractive. The fluctuation of phase-difference mode leads to half-quantum flux vortices in two-gap superconductors [13–15]. A generalization to a three-gap superconductor results in very attractive features, that is,
chiral states with time-reversal symmetry breaking and the existence of fractionally quantized vortices [16–22].

2. Gauge fields and the free energy

Superconductivity is phenomenologically described by the Ginzburg-Landau free energy. We first consider the Ginzburg-Landau free energy density of a two-band superconductor without the Josephson term in a magnetic field:

\[
f = (\alpha_1 |\psi_1|^2 + \alpha_2 |\psi_2|^2) + \frac{1}{2}(\beta_1 |\psi_1|^4 + \beta_2 |\psi_2|^4) + \frac{h^2}{2m_1} \left| \nabla - ie^{*}c A \right| \psi_1|^2 + \frac{h^2}{2m_2} \left| \nabla - ie^{*}c A \right| \psi_2|^2 + \frac{1}{8\pi} (\nabla \times A)^2, \tag{1}\]

where \(\psi_j \ (j = 1, 2)\) are the order parameters and \(e^{*} = 2e\). This functional is not invariant under the transformation:

\[
\psi_j \rightarrow \exp \left( i\frac{e^{*}}{hc} \theta_j \right) \psi_j, \quad A \rightarrow A + \nabla \chi. \tag{2}\]

The functional is not invariant for any choice of \(\chi\). Let us assume that the phase of \(\psi_j\) is \(\theta_j\): \(\psi_j = e^{i\theta_j} \rho_j\), and define \(\Phi = \theta_1 + \theta_2\) and \(\varphi = \theta_1 - \theta_2\), where \(\rho_j = |\psi_j|\). The free energy is written as

\[
f = (\alpha_1 |\rho_1|^2 + \alpha_2 |\rho_2|^2) + \frac{1}{2}(\beta_1 |\rho_1|^4 + \beta_2 |\rho_2|^4) + \frac{h^2}{2m_1} \left| \nabla - ie^{*}c (A - \frac{hc}{2e^{*}} B) \right| \rho_1|^2 + \frac{h^2}{2m_2} \left| \nabla - ie^{*}c (A + \frac{hc}{2e^{*}} B) \right| \rho_2|^2 + \frac{1}{8\pi} (\nabla \times A)^2, \tag{3}\]

where

\[
B = -\frac{hc}{2e^{*}} \nabla \varphi, \tag{4}\]

and we write \(A - \frac{hc}{2e^{*}} \nabla \Phi\) as \(A\).

It is straightforward to generalize the free energy to an \(N\)-band superconductor. In this case, we have \(N-1\) phase-difference modes. \(N-1\) equals the rank of \(SU(N)\). The rank is the number of elements of Cartan subalgebra, namely commutative generators. Let \(t_1, \ldots, t_{N-1}\) be elements of the Cartan subalgebra of \(SU(N)\). Then, the covariant derivative is

\[
D_\mu = \partial_\mu - i\frac{e^{*}}{hc} A_\mu - i\frac{e^{*}}{hc} \sum_{j=1}^{N-1} B_{\mu}^{j}, \tag{5}\]

and the free energy density (without the Josephson terms) is given by

\[
f = \sum_j \alpha_j |\rho_j|^2 + \frac{1}{2} \sum_j \beta_j |\rho_j|^4 + \frac{h^2}{2m} |D_\mu \psi|^2 + \frac{1}{8\pi} (\nabla \times A)^2. \tag{6}\]

Here, we adopted that masses are the same and \(\psi = (\rho_1, \ldots, \rho_N)^t\) is a scalar field of order parameters. The phase-difference modes \(B_\mu^{j}\) are represented by the diagonal part of \(SU(N)\) nonabelian gauge fields and correspond to the abelian projection of \(SU(N)\) gauge theory by 'tHooft [23].
3. Josephson term and massless modes

There are $N-1$ gauge fields $B_\mu$ in the $N$-gap superconductors. We add the Josephson term to the free energy functional, representing the pair transfer interactions between different conduction bands [4]. The Josephson term is given as $V = -\sum_{i\neq j} \gamma_{ij} |\psi_i| |\psi_j| \cos(\theta_i - \theta_j)$, where $\gamma_{ij} = \gamma_{ji}$ are chosen real. This term obviously loses the gauge invariance of the free energy or the Lagrangian because $\theta_i - \theta_j$ is not gauge invariant. This indicates that the phase-difference modes acquire masses. Hence, in the presence of the Josephson term, the phase-difference modes are massive and there are excitation gaps.

This would change qualitatively when $N$ is greater than 3 [24, 25]. We show that massless modes exist for an $N$-equivalent frustrated band superconductor. Let us consider the Josephson potential given by

$$V = \Gamma[\cos(\theta_1 - \theta_2) + \cos(\theta_1 - \theta_3) + \cos(\theta_1 - \theta_4) + \cos(\theta_2 - \theta_3) + \cos(\theta_2 - \theta_4) + \cos(\theta_3 - \theta_4)],$$

for $N = 4$. We assume that $\Gamma$ is positive: $\Gamma > 0$ which indicates that there is a frustration effect between Josephson couplings. The ground states of this potential are degenerate. For example, the states with $(\theta_1, \theta_2, \theta_3, \theta_4) = (0, \pi/2, \pi, 3\pi/2)$ and $(0, \pi, 0, \pi)$ have the same energy. The Fig.1 shows $V$ as a function of $\theta_1 - \theta_3$ and $\theta_2 - \theta_4$ in the case of $\theta_1 - \theta_3 = \theta_2 - \theta_4$.

By expanding $V$ around a minimum $(0, \pi/2, \pi, 3\pi/2)$, we find that there is one massless mode and two massive modes. In fact, for $\theta_1 - \theta_2 = -\pi + \eta_1$, $\theta_2 - \theta_4 = -\pi + \eta_2$ and $\theta_2 - \theta_3 = -\pi/2 + \eta_3$, the potential $V$ is written as $V = \Gamma[-2 + (1/2)\eta_1^2 + (1/2)\eta_2^2 + \cdots]$, where the dots indicate higher order terms. Missing of $\eta_3^2$ means that there is a massless mode and there remains a global $U(1)$ rotational symmetry, indicating that the ground states are continuously degenerate. The gauge field corresponding to $\theta_2 - \theta_3$ represents a massless mode near $(\theta_1, \theta_2, \theta_3, \theta_4) = (0, \pi/2, \pi, 3\pi/2)$. One gauge symmetry is not broken and two gauge symmetries are broken for $N = 4$. The massive modes are represented by linear combinations of $\theta_1 - \theta_3$ and $\theta_2 - \theta_4$. When we expand the potential $V$ near the minimum $(\theta_1, \theta_2, \theta_3, \theta_4) = (0, \pi, 0, \pi)$, we obtain $V = \Gamma[-2 + \frac{1}{2}\eta_1^2 + \cdots]$. This indicates that there are two massless modes and one massive mode (see Fig.2).

![Fig. 1. Josephson potential for the 4-band band as a function of $\theta_1 - \theta_3$ and $\theta_2 - \theta_3$. We set $\theta_1 - \theta_3 = \theta_2 - \theta_4$ in the potential. The flat minimum indicates an existence of zero mode.](image1)

![Fig. 2. Configurations which have the same energy where angles $\theta_j$ are shown by arrows. In (b) and (c) two spins can be rotated with the phase difference fixed to be $\pi$ keeping the energy constant.](image2)
For $\Gamma > 0$, there are two massive modes and $N - 3$ massless modes, near the minimum $(\theta_1, \theta_2, \theta_3, \theta_4, \cdots) = (0, 2\pi/N, 4\pi/N, 6\pi/N, \cdots)$. Near the minimum $(\theta_1, \theta_2, \theta_3, \cdots) = (0, \pi, 0, \cdots)$, we have $N - 2$ massless modes and one massive mode.

4. Non-trivial configuration of gauge fields

The phase-difference gauge field $B$ in the two-gap case is defined as $B = -\frac{\hbar c}{(2e^*)} \nabla \phi$. The half-quantum vortex can be interpreted as a monopole. Let us assume that there is a cut, namely, kink on the real axis for $x > 0$. The phase $\theta_1$ is represented by $\theta_1 = -\frac{1}{2} \text{Im} \log \zeta + \pi$, where $\zeta = x + iy$. The singularity of $\theta_j$ can be transferred to a singularity of the gauge field by a gauge transformation. We consider the case $\theta_2 = -\theta_1$: $\phi = 2\theta_1$. Then we have

$$B = -\frac{\hbar c}{2e^*} \nabla \phi = -\frac{\hbar c}{e^*} \frac{1}{2} \left( \frac{y}{x^2 + y^2}, -\frac{x}{x^2 + y^2}, 0 \right).$$

Thus, when the gauge field $B$ has a monopole-type singularity, the vortex with half-quantum flux exists in two-gap superconductors. The one-form corresponding to $B$ defines the Chern class and the integral of it over the sphere $S^2$ gives the Chern number $C_1$. In general, the gauge field $B$ has the integer Chern number: $C_1 = n$. For $n$ odd, we have a half-quantum flux vortex.

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