Construction of bipartite and unipartite weighted networks from collections of journal papers

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This work presents a model that allows the study of research specialties through the manifestations of the specialty’s social and epistemological processes in a collection of journal papers. Collections of papers are modeled as coupled bipartite networks interlinking 7 types of entities. Matrix-based link weight functions are introduced to calculate weighted bipartite networks and weighted unipartite co-occurrence networks in the collection of papers. These weight calculation methods, when used in conjunction with unweighted bipartite growth models, produce simple growth models for weighted networks in collections of papers.

I. INTRODUCTION

A collection of journal papers is a database of papers that comprehensively samples the journal literature of a scientific specialty. As such, the social and epistemological processes of the specialty are manifested in the complex network of linkages among entities within the collection of papers. These manifestations are studied by bibliometricians and subject matter experts to assess the state of research in a specialty, and such studies are used to advise managers and policy makers in both government and industry to facilitate research management.

It is important to develop both complex network models and network analysis tools that can be applied to collections of papers. Such tools must be used for the problem of predicting how the underlying processes of a research specialty are manifested in a collection of papers, and more importantly, to perform the inverse problem of modeling research specialty processes from their manifestations in collections of papers. Examples of useful information about research specialties to be extracted from collections of papers include: 1) identifying social structures such as research teams, groups of experts, and leaders of ‘schools of thought’, 2) identifying knowledge structure, such as research subtopics, base knowledge, and exemplars, and 3) identifying temporal trends and events such as discoveries, emergence of new specialties and research teams, knowledge accretion, and creation and obsolescence of concepts and exemplars.

This paper introduces a structural model of coupled networks in collections of journal papers and proposes a construction method for bipartite and unipartite weighted networks from such collections. The methods presented here constitute an important step in the effort to apply the developing science of complex networks theory to collections of papers and eventually to the study of scientific specialties as complex social networks and knowledge networks.

As complex networks, collections of papers have three distinguishing characteristics: 1) they are formed from coupled networks of many different types of entities, e.g., papers, references, authors, 2) both unipartite and bipartite networks in collections of papers are best expressed as weighted networks, where strength of linkage between pairs of entities is expressed as a positive real link weight, and 3) collections of papers are best represented as collections of bipartite networks.

To date, the phenomenon of coupled networks has received little attention in the physics literature. Zheng and Ergun [50] model the simultaneous growth of two loosely coupled sections of a unipartite network and show conditions for power-law link distributions in the crosslinks between network sections. Borner, et al, model the simultaneous growth of citation networks and author collaboration networks by modeling behavior of authors [12].

In contrast to the paucity of research on coupled networks, recently a great deal of study has been focused on weighted networks. Yook, et al [49], originally investigated growing weighted networks using preferential attachment rules and random attachment rules. Newman [38] showed that weighted networks could be expressed as multigraphs, and explained how this treatment allows generalization of many analysis techniques of unweighted networks to weighted networks. Barrat, et al [7], studied a large weighted author collaboration network, and the weighted world airline network, and showed that these networks have differences in correlations of node degrees to strength and clustering. Other studies focus on the statistical properties of weighted networks [3, 9, 11, 26], transport models of weighted networks [5, 21, 22], or growth models of weighted networks [4, 8, 10, 16, 20]. Fan, et al [19], and Li, et al [30], gathered a collection of papers on the specialty of econophysics, and studied a weighted unipartite collaboration network of authors from that collection.

On the topic of bipartite networks, recently several papers have reported on structural models and growth models. Ergun [18] models the human sexual contact network

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as a bipartite graph, with growth having preferential attachment rules similar to a Yule process. Ramasco, et al., present a bipartite Yule model for paper to author networks [39]. Guillaume and Latapy [24] also present a bipartite Yule model and propose a method of deriving a bipartite expression of any unipartite network. Morris [32] proposes the use of general bipartite Yule processes for entity-type pairs in collections of journal papers, and gives examples for paper to reference networks and paper to author networks. Morris [32] also gives a detailed analysis of a bipartite Yule model for paper to reference networks that models heavily cited exemplar references in emerging specialties. Goldstein, et al., [23] and Morris, et al., [35] propose bipartite Yule models for paper to author networks that model the success-breeds-success phenomenon for teams of authors.

As shown in Figure 1, a collection of journal papers constitutes a series of coupled bipartite networks. As diagrammed in the figure, a collection of papers contains 6 direct bipartite networks: 1) papers to paper authors, 2) papers to references, 3) papers to paper journals, 4) papers to terms, 5) references to reference authors, and 6) references to reference journals. Additionally, there are 15 indirect bipartite networks in collections of papers as defined by the diagram. Examples of interesting indirect networks are paper authors to reference authors, and paper journals to reference journals networks, which can be used for author co-citation analysis [46] and journal co-citation analysis [31] respectively.

This paper introduces a formal matrix-based treatment of coupled bipartite structures in collections of papers. This treatment is used to calculate the weights of indirect bipartite networks and is extended to calculation of weights of unipartite co-occurrence networks in the collection. For example, the proposed method can be used to calculate the weights of a bipartite paper author to reference network, or, it can be used to find weights of the unipartite co-occurrence network of authors that link to common papers (a co-authorship network).

The proposed matrix-based technique is similar to multi-port analysis using ABCD parameters in electrical networks [13]. The method is also very similar to methods used in multi-layer neural networks [25].

In conjunction with simple bipartite Yule growth models [33], the proposed weight calculation method produces simple models of weighted network growth, growing as it does from unweighted direct links that occur as papers are added to the collection.

II. COLLECTIONS OF JOURNAL PAPERS

A. Research specialties

A research specialty is a self-organized social organization whose members tend to study a common research topic, attend the same conferences, publish in the same journals, cite each other’s work, and belong to the same social networks that are known as invisible colleges [15]. Thomas Kuhn, the pioneer of the study of research processes, considered specialties to be quite small, ”100 members, sometimes considerably less” [29].

The processes that drive research specialties are twofold: 1) social processes of research teams, communication networks, and collaboration, and 2) epistemological processes of the discovery, emergence, accretion, and obsolescence of knowledge. As described by Kuhn, the distinguishing feature of a specialty is its paradigm, which is the researchers’ ”way of thinking” about their problem: models, analytical techniques, validation standards and so forth. Progress in a specialty is characterized by long and stable periods of puzzle-solving within the specialty’s paradigm, punctuated by discoveries that accompany the overthrow and/or creation of new paradigms [29]. This characteristic of specialties is similar to punctuated equilibrium phenomena [17] that characterize self-organizing systems [6].

Specialties create their own literature, i.e., a body of journal papers and books that broadly focus on the specialty’s research topic. We define a collection of papers as a list of journal papers that constitutes a comprehensive sample of a specialty’s journal literature. As a working definition, define a collection of papers as a database of records, one record per paper, that contains information about the individual papers in such a list.

Although the range of size of such collections is large, the size of such collections is much smaller than the immense databases of papers that are often studied in the physics literature. Morris [32], using back-of-envelope style approximations, suggests that collections of papers should range from as few as 100 papers to as many as 5000 papers. Huge heterogeneous datasets, such as the SPIRES database [40], 20 years of PNAS papers [12], 100 years of Physical Review journals [41], or all the chemistry publications of the Netherlands [45], are not collections of papers as defined here, because they all sample
more than one specialty’s literature. Despite this conceptual constraint, the weight calculation method proposed here can still be applied to such huge collections.

B. Definition of collections of journal papers

For discussion in this paper, a collection of journal papers is a database where each record corresponds to a journal paper. For each paper, its associated authors, cited references, journal, index terms and publication year are listed. Furthermore, for each reference, a reference author, reference journal, and reference year are listed. As defined here, collections of papers are constructed to comprehensively sample the literature of a scientific specialty. For our purposes, collections of papers are typically collected and analyzed as weighted networks and those weights can be calculated from the paths of direct links that connect entities of the two partitions of interest.

A collection of papers can be considered as a network of bibliographic entities of various entity-types [36]. Bibliographic entities may correspond to physical entities in the real world, and more than one bibliographic entity may correspond to the same physical entity. For example, a paper and a reference in a collection of papers may both correspond to the same physical paper in the real world.

It is common in studies of networks in journal literature to match references to papers to build a model of “papers citing papers”, usually referred to as a citation network [2]. There are both methodological and theoretical reasons to avoid this type of treatment: 1) on one hand, a collection of papers typically has 20 times more references than papers, making such citation network models grossly incomplete because unmatched papers and references (including references corresponding to books), have unknown incoming and outgoing links, 2) the second problem is that references, especially highly cited references, can be considered as concept symbols [32, 44], and therefore should be considered as separate entity-types from papers, which merely represent undifferentiated research reports. Figuratively, it is inappropriate to use an ”apples-citing-oranges” model when the actual network is ”apples-citing-apples.” Further discussion of citation networks is outside the scope of this paper.

For our proposed structural model of collections of journal papers presented in this paper, we will limit our discussion to a model comprised of 7 entity-types: 1) papers, 2) paper authors, 3) paper journals, 4) index terms, 5) references, 6) reference authors and 7) reference journals. Index terms are terms supplied by authors or abstract services to associate with papers for search and classification purposes. Paper authors are the authors of papers, while reference authors are the authors associated with references. Paper journals are the journals that papers are published in, while reference journals are the journals associated with references. References corresponding to books, films, web pages, and eprint archive articles have no associated reference journal.

Using the 7 entity-types given in our structural model, Figure 1 illustrates that a collection of journal papers constitutes a series of coupled bipartite networks. As noted in Section I, there are 6 direct bipartite networks and 15 indirect bipartite networks in this structural model. These indirect bipartite networks are best analyzed as weighted networks and those weights can be calculated from the paths of direct links that connect entities in the two partitions of interest.

Note the fictitious collection of papers in the Appendix. The source file for this collection, which consists of 4 papers, is listed in ISI tagged file format. See footnote [52]. The extracted entities for this collection consists of 4 papers, 3 paper authors, 4 paper journals, 7 index terms, 10 references, 6 reference authors, and 7 reference journals. These entities and their corresponding index numbers are listed in the Appendix.

III. BIPARTITE NETWORKS IN COLLECTIONS OF JOURNAL PAPERS

A. Dyad definitions

In a dyad, the two entities can be: 1) like entities, that is, entities of the same entity-type, or 2) unlike entities, that is, entities of different entity-types. Direct links are defined as direct associations. A paper has direct links to its authors (paper authors), its associated index terms, the references the paper cites, and the journal the paper was published in. A reference is directly linked to the papers that cite it, the author associated with the reference (reference author), and the journal that is associated with the reference (reference journal).

Indirect links are links between two unlike entities that occur over a path of two or more direct links. For example, a paper author is indirectly linked to a reference author if he or she authors a paper that cites a reference that is associated with that reference author.

The first entity of interest in a dyad is the primary entity while the other entity is the secondary entity. Designation of primary entity-type and secondary entity-type in direct and indirect bipartite networks is arbitrary and is assumed to be based on the interest of the investigator.
For co-occurrence networks, the primary and secondary entity-types are explicitly defined, as will be explained in Section IV F. Co-occurrence links are between like primary entities and occur when both entities link to the same secondary entity. For example, two papers have a co-occurrence link when they both cite a common reference, or, in another example, two paper authors have a co-occurrence link if they coauthor a paper. In co-occurrence links the like entities of the dyad are primary entities, while the unlike entities to which they co-link are the secondary entities.

### B. Dyad identifier notation

Table I lists the conventions used here to denote entity-type variables within a collection of papers. The variables $x_1$, $x_2$, and so forth will be used to denote unspecified entity-types. Dyad notation is used to specify dyad types in the collection of papers. The symbols of primary and secondary entity-types associated with dyads are separated by a comma and placed between square brackets, e.g., $[x_1,x_2]$, where $x_1$ denotes the primary entity-type, and $x_2$ denotes the secondary entity-type. This notation will be referred to as the dyad identifier, and will be used as a suffix to variables to specify the entity-types of interest. However, the dyad identifier will be dropped to reduce clutter in the notation when the primary and secondary entity-types are obvious from context. Some examples of the use of dyad identifiers:

- $O[p,r]$ denotes an occurrence matrix listing the links of papers, the primary entity-type, to references, the secondary entity-type.
- $C[ap,p]$ denotes the co-occurrence matrix listing the co-authorship counts of pairs of paper authors, the primary entity-type, in papers, the secondary entity-type.

### C. Bipartite networks

Bipartite networks are comprised of two distinct partitions of nodes, where all links in the network are from entities in the first partition to entities in the second partition. For our purposes, the first partition exclusively holds entities of some entity-type, while the other partition exclusively holds entities of some other entity-type. As an example, Figure 2 shows a diagram of a bipartite network of a partition of papers linked to a partition of references. Note that links only occur between papers and references and that there are no links between pairs of papers or pairs of references.

Assume the diagrammatic convention as shown in Figure 3, that entities of $x_1$, the primary entity-type, are the entities in the group on the left and the entities of $x_2$, the secondary entity-type, are the entities in the group to the right. There are $nx_1$ primary entities and $nx_2$ secondary entities. The strength of the link between $x_1$ entity $i$ and $x_2$ entity $j$ is the link weight, $o_{ij}[x_1,x_2]$.

### D. Occurrence matrices

Mathematically, the links in a bipartite network are described by a rectangular adjacency matrix, which we’ll define as an occurrence matrix. This is an $nx_1$ by $nx_2$ matrix that lists all the link weights between the entities of the two partitions:

$$O[x_1,x_2] = \begin{bmatrix} o_{11} & o_{12} & \cdots & o_{1nx_2} \\ o_{21} & \ddots & \vdots \\ \vdots & \ddots & \vdots \\ o_{nx_1,1} & \cdots & o_{nx_1nx_2} \end{bmatrix} \quad (1)$$

Figure 3 shows how the links in a bipartite network correspond to elements in its occurrence matrix. There is a bipartite network for every possible pair of entity-types in the collection of papers. Occurrence matrices for entity-type pairs with direct relations are derived directly from the tables in the collection’s database. For the example

| TABLE I: Variable conventions used for entities in collections of papers. |
|---------------------------|-------------------|
| $p$: paper                | $r$: reference    |
| $ap$: paper author        | $ar$: reference author |
| $jp$: paper journal       | $jr$: reference journal |
| $yp$: paper year          | $yr$: reference year |
| $t$: term                 | $x$: unspecified entity |

Prefix ‘$n$’ to any entity variable to denote the number of entities in the collection of that entity-type, e.g., $np$ denotes the number of papers in the collection.
partite network. We first find a cascade of networks between two different types of entities as a weighted bi-
ables is equivalent to transposing the occurrence matrix.

partitions and all other partitions as the define the extreme left and right partitions as the
work is coupled to a reference to paper network that is
pled bipartite networks. Figure 5 shows an example of
are comprised of a series of two or more cou-
networks collection of papers in the Appendix.
ence network through common papers using the example
an author to paper network coupled to a paper to refer-
cence network through common papers. This example is taken from the
evexample collection in the Appendix.

collection of papers discussed in this paper, the occur-
cence matrices for the 6 direct bipartite networks in the
collection are given in the Appendix. Occurrence matri-
ces for entity-type pairs with indirect links are calculated
cascading bipartite networks of direct links, as will be
shown later.

Note the following property of occurrence matrices:

\[
O[x_1, x_2] = O[x_2, x_1]^T
\]

(2)

Using dyad identifier notation, exchanging the vari-
ables is equivalent to transposing the occurrence matrix.

E. Coupled and cascaded bipartite networks

Coupled bipartite networks are pairs of bipartite net-
works that share a common partition. Figure 4 shows
an author to paper network coupled to a paper to refer-
cence network through common papers using the example
collection of papers in the Appendix. Cascaded bipartite
networks are comprised of a series of two or more cou-
ned bipartite networks. Figure 5 shows an example of
such a cascade, where a reference author to reference
network is coupled to a reference to paper network that is
in turn coupled to a paper to paper author network. We
define the extreme left and right partitions as the outer
partitions and all other partitions as the inner partitions.

Assume that we are interested in describing the links
between two different types of entities as a weighted bi-
partite network. We first find a cascade of networks
where the two entity-types of interest are the outer parti-
tions. Then it is necessary to apply some algorithm that
meaningfully reduces the indirect links between pairs of
opposite outer entities as weights in a bipartite network
joining those outer entities. Intuitively, we want pairs of
outer entities that have many indirect links through the
inner partitions to have more weight than those pairs of
outer entities with few or no connecting links.

For example, suppose that we wish to find a weighted
bipartite network between reference authors and paper
authors for the purpose of conducting author co-citation
analysis [46]. We can find a cascade of bipartite networks
as shown in Figure 5, where reference authors are linked
to their references, the references are linked to the pa-
pers that cite them, and those papers are linked to the
paper authors that authored them. The weights of a bi-
partite network of reference authors to paper authors are
found by finding the indirect links between each reference
author and paper author through references and papers,
and applying an algorithm that produces a weight from
those identified indirect links. The more indirect links be-
tween a reference author and a paper author, the more
weight should be assigned to the link between them in
the resulting bipartite network.

IV. ALGORITHM FOR CONSTRUCTION OF WEIGHTED BIPARTITE NETWORKS

A. Reducing a cascade of bipartite networks to a single weighted bipartite network

Given a cascade of bipartite networks with occurrence
matrices \(O[x_1, x_2], O[x_2, x_3], \ldots, O[x_{n-1}, x_n]\), this cas-
cade can be reduced to a single bipartite network with
occurrence matrix \(O[x_1, x_n]\) listing the link weights be-
tween the \(x_1\) entities and the \(x_n\) entities in the network.
The proposed weight algorithm is iterative and works by
sequentially reducing two adjacent networks to a single
network, then reducing that weighted network and its ad-
B. Reducing adjacent coupled bipartite networks to a single weighted bipartite network

Consider a pair of coupled bipartite networks, with entity-types $x_1$, $x_2$, and $x_3$, as shown in Figure 6. Occurrence matrices $O[x_1, x_2]$ and $O[x_2, x_3]$ enumerate the links in the two bipartite networks in this figure. Each link in the figure is labeled with its corresponding occurrence matrix element. There are $nx_1$, $nx_2$, and $nx_3$ entities of the entity-types $x_1$, $x_2$, and $x_3$ respectively. A pair of links that connects an $x_1$ entity to an $x_3$ entity is defined as a path. Figure 7, part (a) shows a path from $x_1$ entity $i$ to $x_3$ entity $j$, connected through $x_2$ entity $k$ by links $o_{ik}[x_1, x_2]$ and $o_{kj}[x_2, x_3]$. There are $nx_2$ possible paths from $x_1$ entity $i$ to $x_3$ entity $j$ as shown in Figure 7 part (b).

The path weight associated with a path is calculated from the weights of the path's two links using a path combining function:

$$p_{ij}(k) = f_2(o_{ik}[x_1, x_2], o_{kj}[x_2, x_3]),$$

where $f_2$ is the path weight function, to be defined later.

The resulting link weight from $x_1$ entity $i$ to $x_3$ entity $j$ is calculated from the path weights of all possible paths between those two entities using a path combining function:

$$o_{ij}[x_1, x_3] = f_1\left(p_{ij}(1), p_{ij}(2), \ldots, p_{ij}(nx_2)\right),$$

where $f_1$ is the path combining function, to be defined later. Substituting Equation (3) into Equation (4) gives the link weight function which defines the rules for calculating the weight of the link between the two entities.
Calculating link weights of cascaded bipartite networks:

\[ o_{ij}[x_1, x_3] = f_1 \left( f_2 (o_{i1}, o_{j1}), f_2 (o_{i2}, o_{j2}), \ldots, f_2 (o_{inx}, o_{nxj}) \right). \tag{5} \]

The link weight function of Equation 5 is a matrix function that is used to compute all the \( nx_1 \) times \( nx_3 \) possible weights of the occurrence matrix \( O[x_1, x_3] \) according to the rules for weight computation given by \( f_1 \) and \( f_2 \). Consider Figure 8 which illustrates how the link weight function uses row \( i \) of \( O[x_1, x_2] \) and column \( j \) of \( O[x_2, x_3] \) to produce element \( o_{ij} \) of matrix \( O[x_1, x_3] \). As shown, the function \( f_2 \) is applied to matching elements of the row vector and column vector to produce \( nx_2 \) scalar results. The function \( f_1 \) operates on all these \( nx_2 \) results to produce the final scalar result \( o_{ij}[x_1, x_3] \).

The concepts of 1) bipartite networks of entities, 2) cascaded bipartite networks, and 3) link weight functions, provide a systematic means of finding multiple indirect links between outer entities in cascades of bipartite networks, and combining those multiple links as a weight in a bipartite network between the outer entities. The choice of path weight function and path combining function is generally driven by the application. In the case of cascades of unweighted bipartite networks, matrix multiplication makes a good link weight function because it yields weights that are equal to occurrence counts. For example, for a paper to reference network coupled to a reference to reference author network, matrix multiplication as a link weight function will produce weights, \( o_{ij}[p, ar] \), that are the the number of times paper \( i \) cites reference author \( j \).

In other situations, however, other link weight functions are more appropriate. For example, when reducing cascades of weighted bipartite networks, it is necessary to consider how to compute path weights from the two links in a path. Suppose we have a weighted bipartite network of linguistic terms to papers in a collection of papers. The weights, \( o_{ij}[t, p] \), in this network are the number of times term \( i \) appears in the body of paper \( j \). Now assume this matrix is coupled to a paper to reference author network, and that there is a path from term \( i \) to reference author \( j \) that corresponds to 10 occurrences of term \( i \) in paper \( k \), which cites reference author \( j \) 2 times. If we use multiplication as the path weight function, then this yields \( 10 \times 2 = 20 \) for the path weight. This has no meaning as an occurrence count between term \( i \) and reference author \( j \). In this case we may want to simply use a link weight equal to the number of times reference author \( j \) is cited by paper \( k \), or use a link weight equal to the minimum of the number of times paper \( k \) cites reference author \( j \) and the number of times term \( i \) occurs in paper \( k \). We can also express the two links in the path as electrical conductances and calculate the path weight as the resulting conductance of those two conductances in series.

The next three subsections will describe three link weight functions: 1) matrix multiplication, appropriate for cascades of unweighted networks, 2) the overlap function, appropriate for cascades of weighted occurrence networks, and 3) the inverse Minkowski function, used to compute paths weights as similar to conductances in series.

### C. Link weight function using matrix multiplication

For applications where at least one of the matrix arguments is binary, matrix multiplication is often used as the link weight function because it directly yields weights that are simple occurrence and co-occurrence counts in the resulting reduced bipartite matrix.

If the path weight function \( f_2 \) is defined as a product:

\[ f_2 \left( o_{ik}[x_1, x_2], o_{kj}[x_2, x_3] \right) = o_{ik}[x_1, x_2] \cdot o_{kj}[x_2, x_3] \tag{6} \]

and the path combining function \( f_1 \) is a summation:

\[
\begin{align*}
  f_1 & \left( f_2 \left( o_{i1}[x_1, x_2], o_{j1}[x_2, x_3] \right), \ldots, f_2 \left( o_{inx}[x_1, x_2], o_{nxj}[x_2, x_3] \right) \right) \\
  &= \sum_{k=1}^{nx_2} f_2 \left( o_{ik}[x_1, x_2], o_{kj}[x_2, x_3] \right).
\end{align*} \tag{7}
\]

Then the link weight function is simply standard matrix multiplication:

\[ o_{ij}[x_1, x_3] = \sum_{k=1}^{nx_2} o_{ik}[x_1, x_2] \cdot o_{kj}[x_2, x_3]. \tag{8} \]

As an example, assume that \( x_1, x_2, \) and \( x_3 \) are paper authors, papers and references respectively, taken from the example collection of papers in the Appendix. The binary matrix \( O[ap, p] \), the transpose of \( O[p, ap] \), Equation (A.2), lists the links of the individual paper authors
to each paper, while the binary matrix $O[p, r]$, Equation (A.1), lists the links of individual papers with each reference. Using matrix multiplication:

$$O[ap, r] = O[ap, p] \cdot O[p, r]. \quad (9)$$

This yields:

$$O[ap, r] = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 3 & 1 & 2 & 2 & 2 & 1 & 2 & 1 & 1 \end{bmatrix}. \quad (10)$$

This is a matrix, $O[ap, r]$, in which weight, $o_{ij}[ap, r]$, is the number of times that paper author $i$ cites reference author $j$.

Suppose we wish to find the paper author to reference author occurrence matrix of the example collection of papers in the Appendix. Consulting Figure 1, the direct links from paper authors to reference authors go from paper author to paper to reference to reference author. Calculation of the occurrence matrix, $O[ap, ar]$, from paper author to reference author is performed by the matrix multiplication:

$$O[ap, ar] = O[ap, p] \cdot O[p, r] \cdot O[r, ar]. \quad (11)$$

Using the example paper collection in the Appendix, first find the paper author to reference matrix by multiplying the paper author to paper matrix and the paper to reference matrix. This was done in Equation (10). Then multiply the paper author to reference matrix with the reference to reference author matrix:

$$O[ap, ar] = O[ap, r] \cdot O[r, ar] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 2 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 3 & 1 & 2 & 2 & 1 \end{bmatrix}. \quad (12)$$

The result in Equation (12) gives the desired occurrence matrix of paper authors to reference authors for the example. In this matrix, the weight $o_{ij}[ap, ar]$ is the number of times that paper author $i$ cites reference author $j$.

D. Link weight function using the overlap function

The overlap function is useful for calculating weights of links when reducing cascades of weighted bipartite networks. This is appropriate for calculating bipartite networks involving linguistic terms, and is also useful for calculating weights in co-occurrence networks of reference authors and reference journals.

Think of the two links in a path as conduits, each with a maximum capacity. The maximum capacity of these two conduits in series is equal to that of the conduit with the smallest capacity. Considering this series capacity as the path weight, the path weight function becomes the minimum of the weights of the two links on the path:

$$f_2 = \min(o_{ik}[x_1, x_2], o_{kj}[x_2, x_3]). \quad (13)$$

Using a path combining function that sums the path weights:

$$f_1 = \sum_{k=1}^{n \times 2} f_2(o_{ik}[x_1, x_2], o_{kj}[x_2, x_3]), \quad (14)$$

yields the overlap function [42] as the link weight function:

$$f_1 = \sum_{k=1}^{n \times 2} \min(o_{ik}[x_1, x_2], o_{kj}[x_2, x_3]). \quad (15)$$

This can be defined as a matrix operation ”OVL”:

$$O[x_1, x_3] = OVL(O[x_1, x_2], O[x_2, x_3]). \quad (16)$$
Discussion of the application and characteristics of this function can be found in [27]. As an example, assume that $x_1$, $x_2$, and $x_3$ are linguistic terms, papers and reference authors respectively, as shown in Figure 9. The matrix $O[t, p]$ lists the occurrence counts of the individual terms with each paper:

$$O[t, p] = \begin{bmatrix} 3 & 5 \\ 2 & 6 \\ 1 & 9 \end{bmatrix},$$ (17)

and the matrix $O[p, ar]$ lists the associations of individual papers with each reference author:

$$O[p, ar] = \begin{bmatrix} 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}.$$ (18)

Using the overlap function to calculate the link weights of $O[t, ar]$:

$$O[t, ar] = OVL(O[t, p], O[p, ar])$$

$$O[t, ar] = OVL\left(\begin{bmatrix} 3 & 5 \\ 2 & 6 \\ 1 & 9 \end{bmatrix}, \begin{bmatrix} 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}\right) = \begin{bmatrix} 2 & 7 & 1 \\ 2 & 6 & 1 \\ 1 & 5 & 1 \end{bmatrix}.$$ (19)

### E. Link weight function using the inverse Minkowski function

The inverse Minkowski function, an adaptation of the well-known Minkowski distance metric [14], can be used when it is desired to model path weights as if the link weights were electrical conductances in series. In this case use the inverse Minkowski metric as the path weight function:

$$f_2 = \left[ (o_{ik}[x_1, x_2])^{-p} + (o_{kj}[x_2, x_3])^{-p} \right]^{-\frac{1}{p}}.$$ (20)

where $p$ ranges from zero to positive infinity. Note that, in contrast to the Minkowski metric as normally expressed, the exponents in the inverse Minkowski metric are negative. This function will always generate a path weight that is less than or equal to the smallest link weight in the path, modeling a situation where indirect links tend to be weaker than direct links. Using a path combining function that sums the path weights:

$$f_1 = \sum_{k=1}^{n_{x_2}} f_2(o_{ik}[x_1, x_2], o_{kj}[x_2, x_3])$$ (21)

yields the final inverse Minkowski link weight function:

$$o_{ij}[x_1, x_3] = \sum_{k=1}^{n_{x_2}} \left[ (o_{ik}[x_1, x_2])^{-p} + (o_{kj}[x_2, x_3])^{-p} \right]^{-\frac{1}{p}}.$$ (22)

This can be defined as a matrix operation "INVMINK":

$$O[x_1, x_3] = INVMINK(O[x_1, x_2], O[x_2, x_3]).$$ (23)

When this function is used with $p = \infty$, Equation (20) produces the minimum of its arguments and so reverts to Equation (13), making the inverse Minkowski link weight function revert to the overlap link weight function. When $p = 1$, then the path weight function, Equation (20), becomes:

$$f_2 = \left[ \frac{1}{o_{ik}[x_1, x_2]} + \frac{1}{o_{kj}[x_2, x_3]} \right]^{-1}.$$ (24)

This makes the path weight function produce a value that is twice the harmonic average of the link weights of the path. This is equivalent to calculating the path weight by modeling the link weights as electrical conductances in series.

The inverse Minkowski path weight function always produces a path weight that is less than the smallest weight on the path. This is appropriate in situations where indirect paths should have less weight than direct paths, and mathematically expresses a sensed diffusion, or weakening, of the strength of linkage when linkage is indirect.

### F. Weights in unipartite co-occurrence networks

Co-occurrence networks are weighted unipartite networks of like entities where the links between pairs of entities is the count of the number of common secondary entities that the two primary entities both link to. For example, in a bibliographic coupling network, the nodes are papers, and the link weights are the number of common references cited by each pair of papers. A co-occurrence matrix is the adjacency matrix of a co-occurrence network. For binary occurrence matrices the co-occurrence
matrix can be found by post multiplying the occurrence matrix by its transpose. Using Equation (2):

\[
C[x_1, x_2] = O[x_1, x_2] \cdot O[x_2, x_1],
\]

(25)

where \(C[x_1, x_2]\) is the co-occurrence matrix listing the number of common associations of pairs of \(x_1\) entities with \(x_2\) entities. For example, to calculate the co-occurrence of papers by their links to references using the paper to reference matrix from the example collection in the Appendix, use Equation (A.1):

\[
C[p, r] = O[p, r] \cdot O[r, p] = \begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 
\end{bmatrix}
\]

(26)

The diagonal of the co-occurrence matrix \(c_{ii}[x_1, x_2]\) lists the number of links that each \(x_1\) has with entities of the \(x_2\) entity-type. For example, in the bibliographic coupling matrix, \(C[p, r]\), calculated in Equation (26), the diagonal lists the number of references each papers cites.

Computation of co-occurrences can be viewed, similar to the discussion of Section III E, as the calculation of link weights in a cascade of two bipartite networks. Given a bipartite network of two unlike entity-types, mirror the network across the secondary entity-type partition to obtain a cascade of two networks. For example, the paper to reference network shown in Figure 2 has been mirrored on the references to produce the paper-reference-paper cascade of two bipartite networks shown in Figure 10 (a). Calculating the weights of this cascade using matrix multiplication will produce the co-occurrence counts of papers’ links to references, bibliographic coupling strength [28], as was done in Equation (26).

The same network of Figure 2 can be mirrored on the papers to produce the reference-paper-reference cascade of bipartite networks shown in Figure 10(b). Calculating the link weights in this network using matrix multiplication yields the co-occurrence counts of references links to papers, co-citation strength [43]. Note that each occurrence matrix has two co-occurrence matrices associated with it. Figure 11 illustrates this for a sample paper to reference occurrence matrix, \(O[p, r]\). To the right of \(O[p, r]\) is the square symmetric bibliographic coupling matrix \(C[p, r]\), whose size is number of papers in \(O[p, r]\). Similarly, below \(O[p, r]\) is the square symmetric co-citation matrix, \(C[r, p]\) whose size is the number of references in \(O[p, r]\).

Linguistic terms to paper networks, reference author to paper networks and reference journal to paper networks are weighted networks. Because of this, it is not desirable to calculate their co-occurrence matrices using matrix multiplication because the resulting link weights cannot be interpreted. Noting that calculation of co-occurrence matrices is analogous to computing link weights for a pair of cascaded bipartite networks, as was demonstrated in Figure 10 and the discussion above, other link weight functions can be used to find their co-occurrence matrices. This can be done, for example, using the overlap function of Section IV D.

As an example, assume the paper to linguistic term matrix:

\[
O[p, t] = \begin{bmatrix}
8 & 9 & 5 & 3 & 1 & 0 \\
5 & 4 & 9 & 2 & 0 & 1 \\
0 & 0 & 2 & 6 & 5 & 4 \\
1 & 1 & 0 & 5 & 2 & 5
\end{bmatrix}
\]

(27)

Using the overlap function, the co-occurrence matrix of
papers linked to terms is:

\[
C[p, t] = OVL(O[p, t], O[t, p])
\]

\[
= OVL\left(\begin{bmatrix}
8 & 9 & 5 & 3 & 1 & 0 \\
5 & 4 & 9 & 2 & 0 & 1 \\
0 & 0 & 2 & 6 & 5 & 4 \\
1 & 1 & 0 & 5 & 2 & 5 \\
\end{bmatrix},
\begin{bmatrix}
8 & 5 & 0 & 1 \\
9 & 4 & 0 & 1 \\
5 & 9 & 2 & 0 \\
3 & 2 & 6 & 5 \\
1 & 0 & 5 & 2 \\
0 & 1 & 4 & 5 \\
\end{bmatrix}\right)
\]

\[
= \begin{bmatrix}
26 & 16 & 6 & 6 \\
16 & 21 & 5 & 5 \\
6 & 5 & 17 & 11 \\
6 & 5 & 11 & 14 \\
\end{bmatrix}
\]

(28)

\[
\]

\[
\]

FIG. 11: Diagram showing that each occurrence matrix is associated with a pair of co-occurrence matrices. Upper left matrix is paper to reference occurrence matrix \(O[p, r]\), below is reference co-occurrence matrix relative to papers (co-citation matrix), \(C[r, p]\). Upper right matrix is paper co-occurrence matrix relative to references (bibliographic coupling matrix), \(C[p, r]\).

V. RECURSIVE MATRIX GROWTH

The recursive growth equations presented in this section are a natural outgrowth of the proposed matrix-based mathematical treatment of collections of journal papers. They are useful for the purpose of providing insight into the character of occurrence distributions in the collections, as will be explained.

The basic record in a collection of journal papers is the paper. The collection grows paper by paper in the temporal order of the publication dates of the papers. When a new paper is added, it is associated with the existing entities in the collection and additionally, new entities, e.g., new paper authors or new references, and new terms that enter into the collection.

This section will present a recursive model of the growth of both occurrence and co-occurrence matrices as papers are added to the collection. The recursive model of matrix growth is found by examination of matrix partitions in occurrence and co-occurrence matrices as papers are added to the collection.

It is easiest to consider the growth of an example occurrence matrix. For convenience, the paper-reference matrix will be studied. The results can be easily extended to other occurrence matrices, for example the paper to paper author matrix [35]. In the matrix the rows correspond to papers and are ordered in the sequence of publication of the papers to which they correspond. The columns correspond to references and are ordered...
in the sequence in which their corresponding references first appear. As shown in Figure 12, the matrix contains a descending stair step sequence of ones from its upper left corner diagonally to its lower right corner. This sequence of ones corresponds to the initial appearance of references as papers are added to the collection. Below this diagonal sequence of ones is a roughly lower triangular region sparsely populated with ones that correspond to citations to existing references as each paper is added. Above the diagonal sequence of ones is a roughly upper triangular area of zeros.

Consider the addition of paper \( i \) by \( \Omega_i \), whose size is \( i \) by \( nr_i \), as the paper-reference matrix after the addition of paper \( i \), then consider the addition of paper \( i + 1 \). A new row vector, \( \delta \), is added to \( \Omega_i \). This vector is partitioned into a 1 by \( i \) vector \( \delta_i \) listing the paper's citations to existing references, and a 1 by \( nr_{i+1} - nr_i \) vector of ones occurring in new columns added for the new references that have appeared in paper \( i + 1 \). Figure 12 shows a pictorial representation of this addition. In the new columns, \( \delta \), an \( i \) by \( nr_{i+1} - nr_i \) zero matrix appears. The recursive matrix equation for growth of the paper-reference equation is:

\[
\Omega_{i+1} = \begin{bmatrix} \Omega_i & 0 \\ \delta_i & 1 \end{bmatrix}.
\tag{29}
\]

Figure 13 shows a map of a typical paper-reference matrix, where each dot shows the location of a one in the matrix.

As papers are added to the collection, note that individual papers collect no links after their initial appearance, while references cumulate links (citations from newly appearing papers) as papers are added. Entity-types that cumulate links in collections of papers usually have a power-law frequency distribution relative to papers. Three such power-law distributions are well-known: 1) papers per paper author distribution (Lotka's law) [47], 2) papers per paper journal distribution (Bradford's law) [47], and papers per reference distribution (reference power law) [37]. Papers, which don’t cumulate links, tend to have exponential tailed distributions relative to other entity-types. Two examples are authors per paper distribution (1-shifted Poisson) [35], and references per paper distribution (lognormal) [32].

The bibliographic coupling matrix, which will be designated \( \beta \), is a symmetric matrix that lists the bibliographic coupling counts of all pairs of papers within the data collection. The diagonal of \( \beta \) contains the counts of the number of references cited in each paper. The bibliographic coupling matrix can be obtained by multiplying the paper-reference matrix by its transpose:

\[
\beta = \Omega \cdot \Omega^T. \tag{30}
\]

The recursive growth equations for the bibliographic coupling matrix can be derived by substituting (29) into (30):

\[
\beta_{i+1} = \Omega_{i+1} \cdot \Omega_{i+1}^T = \\
= \begin{bmatrix}
\Omega_i \cdot \Omega_i^T & \Omega_i \cdot \delta_i^T \\
\delta_i \cdot \Omega_i^T & \delta_i \cdot \delta_i^T + 1 \cdot 1^T
\end{bmatrix} \\
= \begin{bmatrix}
\beta_i & \Omega_i \cdot \delta_i^T \\
\delta_i \cdot \Omega_i^T & m_{i+1}
\end{bmatrix}, \tag{31}
\]

where \( m_{i+1} \) is the number of references cited by paper \( i + 1 \). Figure 14 shows a pictorial representation of a typical bibliographic coupling matrix with the partitions in Equation (31) identified. It is easy to see from Equation (31) and Figure 14 that bibliographic coupling counts between pairs of papers are static, and do not change as more papers are added to the collection.

The co-citation matrix, designated as \( \Gamma \), is a symmetric \( nr \) by \( nr \) matrix that lists the co-citation counts of references.
all pairs of references within the data collection. The diagonal of $\Gamma$ contains the counts of the number of papers that cite each reference. The co-citation matrix can be obtained by multiplying the transpose of the paper-reference matrix by itself:

$$\Gamma = \Omega^T \cdot \Omega.$$  \hspace{1cm} (32)

The recursive growth equations for the co-citation matrix can be derived by substituting Equation (29) into Equation (32):

$$\Gamma_{i+1} = \Omega_{i+1}^T \cdot \Omega_{i+1} + \delta_i^T \cdot \delta_i \cdot \delta_i^T \cdot 1$$

$$= \left[ \begin{array}{c} \Gamma_i^T \cdot \delta_i + \delta_i^T \cdot \delta_i \cdot \delta_i^T \cdot 1 \\ 1^T \cdot \delta_i \cdot \delta_i^T \cdot 1 \end{array} \right].$$ \hspace{1cm} (33)

Figure 15 shows a pictorial representation of a typical co-citation matrix with the partitions in Equation (33) identified. It is easy to see that the co-citation count between two references is not static, but can be increased with the addition of each new paper to the collection.

**VI. EXAMPLE**

An illustrative example of the techniques outlined here uses a collection of 902 papers on the topic of complex network theory. This collection was gathered in 2003 by finding all papers that cite key references in the specialty. A detailed analysis of the paper to reference network for this collection was presented by Morris [32], while analysis of the paper author to paper network for this collection was presented by Goldstein, et al, [23] and Morris, et al, [35].

Figure 16 shows a weighted occurrence matrix, $O[ap, ar]$, for the paper author to reference author network from this collection. In this diagram, the paper authors are rows, reference authors are columns, and the size of the circle at position $(i, j)$ in the diagram is proportional to the link weight from paper author $i$ to reference author $j$. In this case the link weight is equal to the number of times that paper author $i$ cited reference author $j$.

In order to visualize the structure of links in the network, the rows and columns of the matrix have been arranged using a seriation algorithm [34] and clustering dendrograms have been added on the left and top of the figure [36]. The figure is meant to show collaboration groups of paper authors and their links to reference authors as symbols of ‘schools of thought’ [48]. The visualization technique of Figure 16 is explained in Morris and Yen [36].

Only paper authors that authored 6 or more papers were visualized. For clustering paper authors, the co-occurrence matrix of co-authorship counts, $C[ap, p]$, was calculated using matrix multiplication: $C[ap, p] = O[ap, p] \cdot O[p, ap]$. These co-authorship counts were converted to distances and a hierarchical clustering routine was applied to produce the dendrogram on the left of the figure. Groups of paper authors clustered this way can be regarded as ‘research teams.’

Only reference authors that were cited 50 or more times were visualized. For clustering reference authors, the co-occurrence matrix of co-citation counts, $C[ar, p]$, was calculated using the overlap function: $C[ar, p] = O[ar, p] \cdot O[p, r] \cdot O[r, ar]$. These co-citation counts were converted to distances and a hierarchical clustering routine was applied to produce the dendrogram at the top of the figure. Groups of reference authors clustered this way can be regarded as representing ‘schools of thought.’

The paper author to reference author matrix, $O[ap, ar]$, was calculated using matrix multiplication $O[ap, ar] = O[ap, p] \cdot O[p, r] \cdot O[r, ar]$. The matrix clearly shows that dominant reference authors in the specialty, who are cited by authors to represent key ideas in the specialty, are heavily linked across all paper authors. Note that there is evidence of correlation of groups of paper authors to groups of reference authors. For example, pa-
per authors Choi, Hong, Kim and Holme are all heavily connected to reference authors Newman and Watts, while paper authors Pastor-Satorras, Vespignani, Vazquez, and Moreno are all heavily connected to reference authors Pastor-Satorras and Albert.

This example illustrates the usefulness of the matrix-based mathematical treatment of cascades of bipartite networks in collection of journal papers. In the example, we have shown this treatment can be used for construction of weighted unipartite co-occurrence networks for clustering purposes: 1) paper authors linked by co-authorship, and 2) reference authors linked by common papers. Additionally, the method was used to calculate a weighted bipartite network of paper authors to reference authors.

VII. CONCLUSION

We have introduced several valuable methods that can be used to apply complex networks theory to collections of journal papers:

- **The structural model of coupled bipartite networks for collections of papers.** This is a novel model that allows analysis of any bipartite network in the collection in a general, standardized, manner. Further, it allows building a *multi-entity-type* growth model of this system of networks, a technique not generally studied by complex networks researchers.

- **The matrix-based method of calculating weighted bipartite networks.** Using the general concept of link weight functions, we have shown
that this matrix-based technique can be applied to cascades of unweighted bipartite networks using matrix multiplication. Additionally, the technique can be applied to cascades of weighted bipartite networks using the overlap function or the inverse Minkowski function.

- **The calculation of weighted unipartite co-occurrence networks.** Considering co-occurrence networks as coupled bipartite networks made by mirroring around a bipartite partition, calculation of weighted co-occurrence networks uses the same matrix-based calculation method as weighted bipartite networks.

- **The construction of simple models of weighted matrix growth.** This structural model of coupled bipartite networks, when considered with unweighted bipartite growth models, such as the bipartite Yule model, yields a simple model of growth of weighted bipartite networks and weighted unipartite co-occurrence networks. Morris [33] has shown that simple bipartite Yule processes effectively simulate the statistics of bipartite and weighted unipartite networks in collections of papers.

The structural model and matrix-based techniques introduced here provide a unified framework of all entities in networks of papers, e.g., paper to author networks that are manifestations of social collaboration processes, or paper to reference networks that are manifestations of epistemological processes such as knowledge accretion and exemplar knowledge in a specialty. Such networks are often studied as decoupled processes despite their almost certain interdependence. For example, note that the paper author to reference author network example of Figure 16 shows correlations between groups of paper authors and groups of reference authors. A realistic model of processes in a research specialty should be able to predict that such correlations will occur, but the model must also predict the characteristics of the paper author to paper network (such as Lotka’s law), and simultaneously predict the characteristics of the paper to reference network (such as the reference power law.) All of these bipartite networks are interdependent and those interdependencies cannot be modeled using simple unipartite or bipartite growth models. The structural model introduced here is a step toward modeling the complex interdependencies in a research specialty.

Furthermore, and importantly, these techniques can be applied to other report-based structures that can be expressed as collections of entities. For example, a collection of intelligence reports about terrorist events can, after application of an entity extraction program, be expressed as a collection of entities: reports, place names, terrorist group leader names, terrorist group names, government officials’ names, and incident types. These entities are linked in a coupled bipartite structure, similar to Figure 1 and analysis of those linkages could produce useful information about networks of terrorists. So the structural model introduced here may allow the study of other self-organizing social organizations as well, through their manifestations in collections of reports.

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**APPENDIX: EXAMPLE COLLECTION OF JOURNAL PAPERS**

1. **ISI tags**

The table below explains the tags used in the ISI source file given in this appendix.

| Tag | Description |
|-----|-------------|
| PT  | Publication type |
| AU  | Author |
| TI  | Title |
| SO  | Source journal |
| ID  | Index terms |
| CR  | Cited reference |
| PY  | Published year |
| VL  | Volume |
| BP  | Beginning page |
| ER  | End of record |

2. **Source file**

Below, in ISI tagged file format, are listed four records comprising a fictitious collection of papers on the fictional specialty of *improbability generation*:

| FN | ISI Export Format |
|----|-------------------|
| PT | J                 |
| AU | Beeblebrox, Z     |
| TI | Review of finite improbability generators |
| SO | Bambleweeny Review |
| ID | FINITE IMPROBABILITY; LIFE; UNIVERSE |
| CR | FORD P, 1996, J LIFE UNIV EVERY, V46, P111 |
|    | MOUSE B, 1997, REV FUT PHYS, V27, P76 |
|    | MOUSE B, 1998, BISTROMATH, V991, P342 |
| PY | 2003 |
| VL | 13 |
| BP | 844 |
| ER | |

| PT | J |
| AU | Beeblebrox, Z |
|    | Dent, A |
| TI | Dentrassi hot tea: a scale free brownian motion generator |
3. Extracted entities

The table below lists the entities extracted from the collection of papers above.

Papers (identified by title)

\begin{align*}
p_1: & \text{ Review of finite...} \\
p_2: & \text{ Dentrassi hot tea: a scale free...} \\
p_3: & \text{ Application of infinite...} \\
p_4: & \text{ Power laws in infinite...} \\
\end{align*}

Paper authors

\begin{align*}
ap_1: & \text{ Beeblebrox, Z.} \\
ap_2: & \text{ Dent, A.} \\
ap_3: & \text{ Prefect, F} \\
\end{align*}

Paper journals

\begin{align*}
jp_1: & \text{ Bambleweeney Review} \\
\end{align*}

References

\begin{align*}
r_1: & \text{ FORD P, 1996, J LIFE UNIV EVERY, V46, P111} \\
r_2: & \text{ MOUSE B, 1997, REV FUT PHYS, V27, P76} \\
r_3: & \text{ MOUSE B, 1998, BISTROMATH, V991, P342} \\
r_4: & \text{ TRILLIAN A, 2000, SIRIAN CYBERN J, V82, P675} \\
r_5: & \text{ TRILLIAN A, 2002, BISTROMATH, V995, P937} \\
r_6: & \text{ BEEBLEBROX Z, 1994, REV FUT PHYS, V24, P923} \\
r_7: & \text{ BEEBLEBROX Z, 2003, BAMBLEWEENY REV, V13, P844} \\
r_8: & \text{ BEEBLEBROX Z, 1989, PRINCIPLES OF IMPROBAPHYSICS} \\
r_9: & \text{ SLARTIBARTFAST B, 2001, GALACT J PHYS, V887, P2846} \\
r_{10}: & \text{ ZARNIWOOP N, 1978, MEGADODO MAG, V564, P23} \\
\end{align*}

4. Occurrence matrices

Below are the occurrence matrices for the direct bipartite networks in the collection of papers above.

Paper to reference network:

\[
O[p, r] = \begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\
\end{bmatrix} \tag{A.1}
\]

Paper to paper author network:

\[
O[p, ap] = \begin{bmatrix}
1 & 0 & 0 \\
1 & 1 & 1 \\
0 & 0 & 1 \\
0 & 0 & 1 \\
\end{bmatrix} \tag{A.2}
\]

Paper to paper journal network:
Reference to reference journal network:

\[
O_{r,jr} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]  

(A.6)

Note that in some cases the paper to terms matrix may weighted when working with abstract or title terms rather than index terms.

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