Complex linear superfield as a model for Goldstino

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Abstract

We propose a Goldstino model formulated in terms of a constrained complex linear superfield. Its comparison to other Goldstino models is given. Couplings to supersymmetric matter and supergravity are briefly described.

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1 Introduction

Since the pioneering work of Volkov and Akulov [1, 2] in which they proposed the Goldstino action, there have appeared alternative formulations in which the Goldstino is described in terms of constrained $\mathcal{N} = 1$ superfields. The most famous constructions are: (i) Roček’s model [3] realized in terms of a constrained chiral superfield; (ii) the Lindstrøm-Roček model [4] realized in terms of a constrained real scalar superfield; (iii) the Samuel-Wess model [5] which is formulated using a constrained spinor superfield. What is missing in this list of Goldstino models is a realization involving a complex linear superfield. The present note is aimed at filling this gap. Our notation and conventions correspond to [8].

2 Constrained complex linear superfield

A complex linear superfield $\Gamma$ obeys the only constraint $\bar{D}^2 \Gamma = 0$, and can be used to provide an off-shell description for the scalar multiplet (non-minimal scalar multiplet) [8, 9]. A modified complex linear superfield, $\Sigma$, is defined to satisfy the constraint

$$-\frac{1}{4} \bar{D}^2 \Sigma = f, \quad f = \text{const}.$$  \hspace{1cm} (1)

Here $f$ is a parameter of mass dimension 2 which, without loss of generality, can be chosen to be real. The above constraint naturally occurs if one introduces a dual formulation for the chiral scalar model

$$S[\Phi, \bar{\Phi}] = \int d^4x d^2\theta d^2\bar{\theta} \Phi \bar{\Phi} + \left( f \int d^4x d^2\theta \Phi + \text{c.c.} \right),$$  \hspace{1cm} (2)

with $\Phi$ being chiral. The general solution to the constraint (1) is

$$\Sigma(\theta, \bar{\theta}) = e^{i\theta_{\sigma a} \bar{\theta}_{\dot{a} \sigma}} \left( \phi + \theta \psi + \sqrt{2} \bar{\theta} \bar{\rho} + \theta^2 F + \bar{\theta}^2 f + \theta^a \bar{\theta}^\dot{a} U_{a\dot{a}} + \theta^2 \bar{\theta} \bar{\chi} \right).$$  \hspace{1cm} (3)

The free action for the complex linear superfield is

$$S[\Sigma, \bar{\Sigma}] = -\int d^4x d^2\theta d^2\bar{\theta} \Sigma \bar{\Sigma} = -\int d^4x \left( f^2 + FF - \phi \Box \bar{\Phi} + i \rho \partial \bar{\rho} - \frac{1}{2} \xi \chi - \frac{1}{2} \bar{\xi} \bar{\chi} - \frac{1}{2} \bar{U}^a U_a \right).$$  \hspace{1cm} (4)

\footnote{According to the general theory of the nonlinear realisation of $\mathcal{N} = 1$ supersymmetry [2, 6, 7], the Akulov-Volkov action is universal in the sense that any Goldstino model should be related to the Akulov-Volkov action by a nonlinear field redefinition. However, the superfield models [3, 4, 5] are interesting in their own right, in particular in the context of supergravity [4, 5].}
where we have introduced
\[ U^a = U^a + 2i \partial^a \phi, \quad \chi = \chi - \frac{1}{2} \partial \bar{\psi}. \]  
(5)

It is seen from the component expression for \( S[\Sigma, \bar{\Sigma}] \) that \( \phi \) and \( \rho \) are physical fields while the rest of the fields are auxiliary.

It turns out that the above action is suitable to describe the Goldstino dynamics provided \( \Sigma \) is subject to the following nonlinear constraints:
\[ \Sigma^2 = 0, \]
\[ -\frac{1}{4} \Sigma \bar{D}^2 D_a \Sigma = f D_a \Sigma. \]  
(6)
(7)

The constraints can be seen to be compatible. Using (1), the second constraint can be rewritten in the form:
\[ i \Sigma \partial_{\alpha} \bar{D}^\alpha \Sigma = -f D_a \Sigma. \]  
(8)

Any low-energy action of the form
\[ S_{\text{eff}} = \int d^4x d^2\theta d^2\bar{\theta} K(\Sigma, \bar{\Sigma}) \]  
reduces to (4) if \( \Sigma \) is subject to the nilpotent condition (6).

The general solution to the constraint (6) fixes \( \phi \) and two of the auxiliary fields
\[ f \phi = \frac{1}{2} \rho^2, \quad f \psi_\alpha = \frac{1}{\sqrt{2}} U_{a\alpha} \bar{\rho}^a, \quad f F = \frac{1}{\sqrt{2}} \bar{\chi} \rho + \frac{1}{4} U^a U_a. \]  
(9)

Taking into account the second constraint, eq. (7), fixes all of the components as functions of the Goldstino \( \bar{\rho} \)
\[ f \phi = \frac{1}{2} \rho^2, \quad \sqrt{2} f^2 \psi_\alpha = -i \rho^2 (\partial \bar{\rho})_\alpha, \quad f^3 F = \rho^2 (\partial_a \bar{\rho} \bar{\sigma}^a \partial_b \bar{\rho}), \]
\[ f U_{a\bar{a}} = 2i (\sigma^a \bar{\rho})_a \partial_a \bar{\rho} \bar{\beta}, \quad f^2 \bar{\chi}_\bar{a} = \sqrt{2} (\rho \bar{\sigma}^a \rho^b \partial_a \bar{\rho} \bar{\beta} - \frac{1}{2} (\square \bar{\rho}^2) \bar{\rho} \bar{\beta}). \]  
(10)

Note that the simplicity of these solutions follows from the fact that the two constraints depend only on \( \Sigma \) and not \( \bar{\Sigma} \).

The Goldstino action that follows from (3) and (11) is
\[ S[\rho, \bar{\rho}] = -\frac{1}{2} \int d^4x \left( \kappa^{-2} + \langle \omega + \bar{\omega} \rangle + \kappa^2 (\partial^a \rho^2 \partial_a \bar{\rho}^2 + 4 \langle \omega \rangle \langle \bar{\omega} \rangle) + \kappa^4 (2 \langle \omega \rangle \langle \partial^a \rho^2 \partial_a \bar{\rho}^2 + 4 \langle \omega \rangle \langle \bar{\omega} \rangle + 4 \langle \bar{\omega} \rangle^2 - 2 \langle \omega \rangle^2 - \rho^2 \square \rho^2 \rangle + c.e.) + \kappa^6 (\rho^2 \bar{\rho}^2 \square \rho^2 \bar{\rho}^2 - 8 \langle \omega \rangle^2 \langle \omega \rangle^2 - 8 \langle \omega \rangle^2 \langle \bar{\omega} \rangle^2 \right), \]  
(11)
where, to ease the comparison with the standard literature on nonlinearly realized supersymmetry, we have introduced the coupling constant $\kappa$ defined by $2\kappa^2 = f^{-2}$. We have also introduced the notation
\[
\omega^b_a = i\rho \sigma^b \partial_a \bar{\rho}, \quad \bar{\omega}^b_{\bar{a}} = i\bar{\rho} \bar{\sigma}^b \partial_{\bar{a}} \rho,
\]
and let $\langle M \rangle$ denote the matrix trace of any matrix, $M = (M^b_a)$, with Lorentz indices.

The above action proves to be the same as the component action described by Samuel and Wess [5] (see, e.g., eq. (41) of [10]). The reason for this will be explained shortly.

It is instructive to compare the constraints (6) and (7) with those corresponding to Roček’s Goldstino action [3]. The latter model is described by a chiral scalar $\Phi$, 
\[
\bar{D}_{\dot{\alpha}} \Phi = 0 ,
\]
constrained as follows:
\[
\Phi^2 = 0 ,
\]
\[
-\frac{1}{4} \Phi \bar{D}^2 \Phi = f \Phi , \quad f = \text{const} ,
\]
where the parameter can also be chosen real. The constraint (16) mixes $\Phi$ and its conjugate $\bar{\Phi}$, while (7) involves $\Sigma$ only. The complete solution to the constraints (15) and (16) can be found in [3, 10].

Naturally associated with $\Sigma$ and $\bar{\Sigma}$ are the spinor superfields $\Xi_\alpha$ and its conjugate $\bar{\Xi}_{\dot{\alpha}}$ defined by
\[
\Xi_\alpha = \frac{1}{\sqrt{2}} D_\alpha \bar{\Sigma} , \quad \bar{\Xi}_{\dot{\alpha}} = \frac{1}{\sqrt{2}} \bar{D}_{\dot{\alpha}} \Sigma .
\]
Making use of the constraints (11), (6) and (7), we can readily uncover those constraints which are obeyed by the above spinor superfields. They are
\[
\bar{D}_{\dot{\alpha}} \Xi_\alpha = \kappa^{-1} \varepsilon_{\dot{\alpha} \dot{\beta}} ,
\]
\[
D_\alpha \Xi_{\dot{\alpha}} = 2i\kappa \Xi^{\dot{\beta}} \partial_{\dot{\alpha}} \Xi_{\dot{\dot{\beta}}} ,
\]
where, as above, $2\kappa^2 = f^{-2}$. These are exactly the constraints given in [5], so we recognise $\Xi_\alpha$ as the Samuel-Wess superfield. This connection is discussed in more detail in the next sections. It appears that the Goldstino realization in terms of $\Sigma$ and $\bar{\Sigma}$ is somewhat more fundamental than the one described by eqs. (18) and (19).
3 Comparison to other Goldstino models

The two most basic Goldstino models start with the nonlinear Akulov-Volkov (AV) supersymmetry \[\delta \eta \lambda = \frac{1}{\kappa} \eta \lambda - i\kappa (\lambda \sigma^b \bar{\eta} - \eta \sigma^b \bar{\lambda}) \partial_b \lambda,\] \hspace{1cm} (20)

and the chiral nonlinear AV supersymmetry \[\delta \eta \xi = \frac{1}{\kappa} \eta \xi - 2i\kappa (\xi \sigma^a \bar{\eta}) \partial_a \xi.\] \hspace{1cm} (21)

The latter supersymmetry first appeared in [11] before being discussed in [6] and [3]. It was then central to the approach of Samuel and Wess [5] that we discuss below.

As discussed in [12], the AV supersymmetry is naturally associated with a real scalar superfield (also known as “vector superfield” in the early supersymmetry literature), while the chiral AV supersymmetry is associated with a chiral scalar. Constraints that eliminate all fields but the Goldstino have previously been given for both of these types of superfields. The first was for the chiral scalar, \(\Phi\), where Roček [3] introduced the constraints (15) and (16). The relevant constraints for the real scalar,

\[V^2 = 0, \hspace{1cm} V D^a \bar{D}^2 D_\alpha V = 16 f V,\] \hspace{1cm} (22)

were given by Lindström and Roček [4]. The first constraint in both of these sets is a nilpotency constraint, while the second is such that the free action is equivalent to a pure \(F\)- or \(D\)-term respectively. This latter property is not one possessed by the second constraint (7) for the complex linear superfield.

The constraints for both the chiral and real scalar superfields were solved in [5] in terms of the spinor Goldstino superfield

\[\Xi_\alpha(x, \theta, \bar{\theta}) = e^{\delta \theta \xi_\alpha(x)}.\] \hspace{1cm} (23)

The actions of the supercovariant derivatives \(D_\alpha\) and \(\bar{D}_\dot{\alpha}\) on \(\Xi_\alpha\) follow from the supersymmetry transformation (21) and are exactly the constraints presented in (18) and (19). The solutions for the constrained superfields that were given in [5] are

\[2 f \Phi = -\frac{\kappa^2}{4} \bar{D}^2 (\Xi^2 \Xi^2), \hspace{1cm} 2 f V = \kappa^2 \Xi^2 \Xi^2.\] \hspace{1cm} (24)

From these solutions, it is straightforward to check that \(f V = \Phi \bar{\Phi}\).
It is interesting to note that exactly the same solutions work,

\[ 2f\Phi = -\frac{\kappa^2}{4} \bar{D}^2 (\Lambda^2 \bar{\Lambda}^2), \quad 2fV = \kappa^2 \Lambda^2 \bar{\Lambda}^2, \]  

if we replace \( \Xi \) with the spinor Goldstino superfield that follows from the normal AV supersymmetry (see, e.g., [13])

\[ \Lambda_{\alpha}(x, \theta, \bar{\theta}) = e^{\delta_{\bar{\theta}}_{\alpha}} \lambda_{\alpha}. \]  

Using (20), the actions of the supercovariant derivatives on this superfield are [13]

\[ D_{\alpha} \Lambda_{\beta} = \frac{1}{\kappa} \varepsilon_{\beta\alpha} + i\kappa \bar{\Lambda}_{\alpha} \partial_{\alpha} \Lambda_{\beta}, \quad \bar{D}_{\dot{\alpha}} \Lambda_{\beta} = -i\kappa \Lambda_{\alpha} \partial_{\alpha} \bar{\Lambda}_{\dot{\alpha}}. \]  

The projection to the components of (25) immediately reproduces the results of [3] and gives the relation between the constrained superfield Goldstino models and the (chiral) AV Goldstino.

For the complex linear superfield \( \Sigma \), the solution to the constraints (1), (6) and (7) in terms of \( \bar{\Xi}_{\dot{\alpha}} \) is very simple:

\[ 2f\Sigma = \bar{\Xi}^2. \]  

Projection to components yields \( \rho_{\alpha} = \xi_{\alpha} \) and the component solutions (11). So we see that the model proposed in this paper is the natural constrained superfield to associate with the chiral AV Goldstino and the Samuel-Wess superfield (23) can be considered derivative (17). The Roček and Lindström-Roček superfields can both be constructed from the complex linear scalar as

\[ \Phi = -\frac{1}{2} f\kappa^2 \bar{D}^2 (\Sigma \Sigma) \quad \text{and} \quad V = 2f\kappa^2 \Sigma \Sigma. \]  

Unlike the chiral and real superfield cases, the solution of the complex linear constraints in terms of the superfield \( \Lambda_{\alpha} \) is different from that using \( \Xi_{\alpha} \). Some work gives

\[ 2f\Sigma = 4 (\bar{\Lambda}^2 + \frac{\kappa}{4} D^a (\Lambda_{\alpha} \bar{\Lambda}^2) - \frac{\kappa^2}{16} D^2 (\Lambda^2 \bar{\Lambda}^2)) \]

\[ = \bar{\Lambda}^2 (1 - i\kappa^2 (\Lambda \sigma^a \partial_a \bar{\Lambda}) + \kappa^4 \Lambda^2 (\partial_a \bar{\Lambda} \sigma^{ab} \partial_b \bar{\Lambda})). \]  

\[ \]  

4 Couplings to matter and supergravity

Complex linear superfields are ubiquitous in \( \mathcal{N} = 2 \) supersymmetry in the sense that any off-shell \( \mathcal{N} = 2 \) hypermultiplet without intrinsic central charge contains a complex linear
scalar as one of its $\mathcal{N} = 1$ components, see e.g. [14] for a review. This is one of the reasons

The constraints (1) and (7) admit nontrivial generalizations such as

$$-\frac{1}{4}\bar{D}^2\Sigma = X, \quad \bar{D}_\dot{\alpha}X = 0,$$

for some (composite) chiral scalar $X$ possessing a non-vanishing expectation value. Such constraints are compatible with the nilpotency condition (6). This makes it possible to construct couplings of the Goldstino to matter fields. For example, we can choose $X = f + G_1(\varphi) + G_2(\varphi)\text{tr}(W^aW_a)$, where $G_1$ and $G_2$ are arbitrary holomorphic functions of some matter chiral superfields $\varphi$, $W_a$ is the field strength of a vector multiplet and the trace is over the gauge indices. The resulting Goldstino-matter couplings can be compared with those advocated recently by Komargodski and Seiberg [18]. In the approach of [18], the Goldstino is described by a chiral superfield $\Phi$ subject to the nilpotent constraint (15). Matter couplings for the Goldstino in [18] are generated simply by adding suitable interactions to the Lagrangian. In our case, the Goldstino superfield $\Sigma$ also obeys the nilpotency condition $\Sigma^2 = 0$, along with the differential constraints (1) and (7). Matter couplings can be generated by deforming the latter constraints to the form given by eqs. (31) and (32). Similarly to the analysis in section 2, the constraints (9) and (11) can be solved in terms of the Goldstino $\bar{\rho}_\dot{\alpha}$ and two more independent fields $U_{\alpha\dot{\alpha}}$ and $\bar{\chi}_{\dot{\alpha}}$. The latter fields become functions of the Goldstino and matter fields upon imposing the constraint (32). The supersymmetry remains off-shell.

We also note that the constraints (31) and (32) can be further generalized to allow for a coupling to an Abelian vector multiplet; this requires replacing the covariant derivatives in (31) and (32) by gauge-covariant ones and turning $X$ into a covariantly chiral superfield, with $X$ and $\Sigma$ having the same $U(1)$ charge.

The constraints (1) and (7) can naturally be generalised to supergravity as

$$-\frac{1}{4}D^2 - 4R)\Sigma = X, \quad \bar{D}_\dot{\alpha}X = 0,$$

for modified linear constraints of the form (31) were first introduced in [15] and naturally appear, e.g., when one considers “massive” off-shell $\mathcal{N} = 2$ sigma-models [16] in projective superspace [17].

4 The complex auxiliary field $F$ contained in $\Phi$ is to be eliminated using its resulting equation of motion, which renders the supersymmetry on-shell.

5 Our conventions for $\mathcal{N} = 1$ supergravity correspond to [8].
for some covariantly chiral scalar $X$. Here $\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}_\alpha, \bar{\mathcal{D}}^\dot{\alpha})$ denote the superspace covariant derivative corresponding to the old minimal formulation $[19]$ for $\mathcal{N} = 1$ supergravity, and $R$ the covariantly chiral scalar component of the superspace torsion described in terms of $R$, $G_{\alpha\dot{\alpha}}$ and $W_{\alpha\beta\gamma}$ (see $[8, 9, 13]$ for reviews). The constraints (33) and (34) have to be accompanied by the nilpotency condition (6). As an example, consider the simplest case when $X$ is constant. We represent $X = (\sqrt{2}\kappa)^{-1} = \text{const}$, where $\kappa$ can be chosen to be real. As a minimal generalization of (17), we now introduce spinor superfields $\Xi_\alpha = \frac{1}{\sqrt{2}} \mathcal{D}_a \bar{\Sigma}$ and $\bar{\Xi}^\dot{\alpha} = \frac{1}{\sqrt{2}} \bar{\mathcal{D}}^\dot{\alpha} \Sigma$. Using the constraints (6), (33) and (34), we can derive closed-form constraints obeyed, e.g., by $\bar{\Xi}^\dot{\alpha}$. They are

$$
\bar{\mathcal{D}}^\dot{\alpha} \Xi_\beta = \bar{\varepsilon}_{\alpha\beta} \left( \frac{1}{\kappa} - \kappa R \Xi^2 \right), \tag{35}
$$

$$
\mathcal{D}_\alpha \Xi^\dot{\alpha} = \kappa \left( 2i \bar{\Xi}^\dot{\beta} \mathcal{D}_\alpha \Xi^\dot{\beta} - G_{\alpha\dot{\alpha}} \Xi^2 \right), \tag{36}
$$

where $G_{\alpha\dot{\alpha}}$ is the supergravity extension of the traceless Ricci tensor (see $[8, 9, 13]$ for more details). The constraints (35) and (36) were introduced by Samuel and Wess $[5]$ as a result of the nontrivial guess work (these constraints are non-minimal generalizations of (18) and (19)). In our approach, these constraints are trivial consequences of the formulation in terms of the complex linear Goldstino superfield.

We believe that our results will provide a useful contribution to the existing literature on Goldstino couplings to supersymmetric matter and supergravity, see $[5, 6, 18, 20, 21, 22]$ and references therein.

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References

[1] D. V. Volkov and V. P. Akulov, “Possible universal neutrino interaction,” JETP Lett. 16, 438 (1972) [Pisma Zh. Eksp. Teor. Fiz. 16, 621 (1972)]; “Is the neutrino a Goldstone particle?,” Phys. Lett. B 46, 109 (1973).

[2] V. P. Akulov and D. V. Volkov, “Goldstone fields with spin 1/2,” Theor. Math. Phys. 18, 28 (1974) [Teor. Mat. Fiz. 18, 39 (1974)].

[3] M. Roček, “Linearizing the Volkov-Akulov model,” Phys. Rev. Lett. 41, 451 (1978).

[4] U. Lindström and M. Roček, “Constrained local superfields,” Phys. Rev. D 19, 2300 (1979).
[5] S. Samuel and J. Wess, “A superfield formulation of the non-linear realization of supersymmetry and its coupling to supergravity.” Nucl. Phys. B 221, 153 (1983).

[6] E. A. Ivanov and A. A. Kapustnikov, “Relation between linear and nonlinear realizations of supersymmetry.” Preprint JINR-E2-10765, June 1977. “General relationship between linear and nonlinear realisations of supersymmetry.” J. Phys. A 11, 2375 (1978). “The nonlinear realisation structure of models with spontaneously broken supersymmetry.” J. Phys. G 8, 167 (1982).

[7] T. Uematsu and C. K. Zachos, “Structure of phenomenological Lagrangians for broken supersymmetry.” Nucl. Phys. B 201, 250 (1982).

[8] I. L. Buchbinder and S. M. Kuzenko, “Ideas and Methods of Supersymmetry and Supergravity, or a Walk Through Superspace.” Bristol, UK: IOP (1998).

[9] S. J. Gates, Jr., M. T. Grisaru, M. Roček and W. Siegel, Superspace, or One Thousand and One Lessons in Supersymmetry, Front. Phys. 58, 1 (1983) [arXiv:hep-th/0108200].

[10] S. M. Kuzenko and S. J. Tyler, “On the Goldstino actions and their symmetries,” arXiv:1102.3043 [hep-th].

[11] B. Zumino, “Fermi-Bose supersymmetry,” in Proceedings of 17th International Conference on High-Energy Physics, London UK: Rutherford, 254 (1974).

[12] H. Luo, M. Luo and S. Zheng, “Constrained Superfields and Standard Realization of Nonlinear Supersymmetry,” JHEP 1001 (2010) 043 [arXiv:0910.2110 [hep-th]].

[13] J. Wess and J. Bagger, “Supersymmetry and Supergravity,” Princeton, USA: Univ. Pr. (1992).

[14] S. M. Kuzenko, “Lectures on nonlinear sigma-models in projective superspace,” J. Phys. A 43, 443001 (2010) [arXiv:1004.0880 [hep-th]].

[15] B. B. Deo and S. J. Gates Jr., “Comments on nonminimal N=1 scalar multiplets,” Nucl. Phys. B 254, 187 (1985).

[16] S. M. Kuzenko, “On superpotentials for nonlinear sigma-models with eight supercharges,” Phys. Lett. B 638, 288 (2006) [arXiv:hep-th/0602050].

[17] U. Lindström and M. Roček, “New hyperkähler metrics and new supermultiplets,” Commun. Math. Phys. 115, 21 (1988); “N = 2 super Yang-Mills theory in projective superspace,” Commun. Math. Phys. 128, 191 (1990).

[18] Z. Komargodski and N. Seiberg, “From linear SUSY to constrained superfields,” JHEP 0909, 066 (2009) [arXiv:0907.2441].

[19] J. Wess and B. Zumino, “Superfield Lagrangian for supergravity,” Phys. Lett. B 74, 51 (1978). K. S. Stelle and P. C. West, “Minimal auxiliary fields for supergravity.” Phys. Lett. B 74, 330 (1978). S. Ferrara and P. van Nieuwenhuizen, “The auxiliary fields of supergravity.” Phys. Lett. B 74, 333 (1978).

[20] S. Samuel, J. Wess, “Secret supersymmetry,” Nucl. Phys. B233, 488 (1984).

[21] T. E. Clark and S. T. Love, “Goldstino couplings to matter,” Phys. Rev. D 54, 5723 (1996).

[22] H. Luo, M. Luo and L. Wang, “Nonlinear realization of spontaneously broken N=1 supersymmetry revisited,” JHEP 1002 (2010) 087 [arXiv:1001.5369 [hep-th]].