What can we learn from global spin alignment of $\phi$ meson in heavy-ion collisions?

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We propose that a significant positive deviation from 1/3 for the spin density matrix element $\rho_{00}$ of the $\phi$ meson may indicate the existence of a mean field of the $\phi$ meson generated in heavy-ion collisions. This explains why STAR preliminary data for the $\phi$ meson’s $\rho_{00}$ are much larger than 1/3 while the data of $\Lambda$ and $\bar{\Lambda}$ polarization seem not to allow such a significant and positive deviation. The contribution may be from the polarization of the strange quark and antiquark through the $\phi$ field, an effective mode of the gluon field in strong interaction. We show that $\rho_{00}$ for the $\phi$ meson is a good analyzer for fields even if they may strongly fluctuate in space-time.

I. INTRODUCTION

The rotation and spin polarization are inherently correlated and can be converted from one to another in materials as manifested in the Barnett effect [1] and the Einstein–de Haas effect [2]. One of the most recent examples is that an electric voltage from the spin-current is observed to be generated from the vortical motion in a liquid metal [3]. In ultra-relativistic heavy-ion collisions (HIC), a huge orbital angular momentum (OAM) can also be generated mainly along the direction perpendicular to the reaction plane [4, 9] (see, e.g. [10], for a recent review). Such a huge OAM is distributed into the hot and dense quark matter and converted to global polarization of hadrons through the spin-orbit coupling [4, 9, 11] in a microscopic approach or spin-vorticity coupling in a macroscopic approach [12–17]. The STAR collaboration has recently measured a non-vanishing global polarization of $\Lambda$ hyperons in Au+Au collisions at $\sqrt{s_{NN}} = 7.7 – 200$ GeV [18, 19]. Accompanying a huge OAM in HIC, a strong magnetic field is also formed, pointing to the same direction [20–27]. The OAM and magnetic field lead to chiral effects of massless fermions: the chiral magnetic effect (CME) which probes the topological fluctuation of quantum chromodynamics vacuum [28–30] (see, e.g. [31], for a recent review) and the chiral vortical effect (CVE) [32–39] which probes the vorticity field of the fluid. One of the most active research in HIC experiments is to search for the CME [40–48]. However, the CME has not been observed due to dominant backgrounds. Furthermore, no direct and definite effects from electric and magnetic fields have been found so far. The challenge comes from the fact that the lifetime of the electric and magnetic field is so short ($\lesssim 1$ fm/c) that they can be regarded as a pulse.

While the polarization of $\Lambda$ can be measured by its weak decay, the polarization of vector mesons cannot be measured since they mainly decay through strong interaction. However, the spin alignment of a vector meson can only be measured through $\rho_{00}$, the 00-element of its spin density matrix, encoded in the angular distribution of its decay daughters [5, 49]. If $\rho_{00} \neq 1/3$, the distribution is anisotropic and the spin of the vector meson is aligned to the spin quantization direction. In 2008, the STAR collaboration measured $\rho_{00}$ for the vector meson $\phi(1020)$ in Au+Au collisions at 200 GeV, which is consistent to 1/3 indicating no spin alignment within errors [50]. Recent STAR’s preliminary data for the $\phi$ meson’s $\rho_{00}$ or $\rho_{00}^\phi$ at lower energies show a significant POSITIVE deviation from 1/3, which is far beyond our current understanding of the polarization [51]. In this note, we will show that such a large POSITIVE deviation of $\rho_{00}^\phi$ from 1/3 may imply the existence of a mean field for the $\phi$ meson in heavy ion collisions.

II. CONVENTIONAL UNDERSTANDING FOR SPIN ALIGNMENT OF $\phi$ MESON

The 00-element of the spin density matrix $\rho_{00}$ for the vector meson enters the angular distribution of its decay daughter as

$$\frac{dN}{d\cos\theta} = \frac{3}{4} \left[ (1 - \rho_{00}) + (3\rho_{00} - 1) \cos^2\theta \right],$$

where $\theta$ is the angle between the daughter’s momentum and the spin quantization direction [5, 49]. The STAR preliminary data imply that $\rho_{00}^\phi > 1/3$ and significantly deviate from 1/3. In the coalescence or combination model the $s$ and $\bar{s}$ quark form a $\phi$ meson, and $\rho_{00}^\phi$ is related to the polarization $P_s$ and $P_{\bar{s}}$ for $s$ and $\bar{s}$ respectively,

$$\rho_{00}^\phi \approx \frac{1}{3} - \frac{4}{9} P_s P_{\bar{s}},$$

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if $P_s$ and $P_\bar{s}$ are both small. In a simple model, the spin polarization of $\Lambda$ and $\bar{\Lambda}$ is carried by $s$ and $\bar{s}$ respectively, so we have $P_\Lambda = P_s$ and $P_{\bar{\Lambda}} = P_{\bar{s}}$. Therefore $\rho_{00}^\phi$ in (2) is approximately

$$\rho_{00}^\phi \approx \frac{1}{3} - \frac{4}{3} P_\Lambda P_{\bar{\Lambda}} \leq \frac{1}{3},$$

(3)

where $P_\Lambda$ and $P_{\bar{\Lambda}}$ can be estimated by using the STAR data $P_\Lambda \approx (1.08 \pm 0.15 \pm 0.11)\%$ and $P_{\bar{\Lambda}} \approx (1.38 \pm 0.30 \pm 0.13)\%$. (4/9) $P_s P_{\bar{s}} \approx 6.6 \times 10^{-5}$. So the STAR data for $P_\Lambda$ and $P_{\bar{\Lambda}}$ seem to imply that $\rho_{00}^\phi$ cannot be significantly larger than 1/3, which contradicts the STAR preliminary data on $\rho_{00}^\phi$. We will show that the key to reconcile such a conflict is that $P_s$ and $P_{\bar{s}}$ will have additional contributions which have never been considered before.

III. SPIN POLARIZATION IN VORTICITY AND ELECTROMAGNETIC FIELD

We take $xz$ plane as the reaction plane with one nucleus moving along $+z$ direction at $x = b/2$ while the other nucleus moving along $-z$ direction at $x = -b/2$. The OAM is along $+y$ direction.

From Eq. (64) in Ref. [49] the spin polarization vector (normalized to 1) for massive fermions (upper sign) and anti-fermions (lower sign) in the vorticity and electromagnetic field is

$$P_s^\mu(x,p) = \frac{1}{2m} \left( \omega_{\mu}^{\nu} + \frac{1}{E_p T} Q F^{\nu\mu} \right) p_\nu \left[ 1 - f_F D(E_p \mp \mu) \right],$$

(4)

where $Q$ is the electric charge of the fermion, $p_\mu = (E_p, \mathbf{p})$ denotes the four-momentum for fermion or anti-fermion with $E_p \equiv \sqrt{\mathbf{p}^2 + m^2}$ being the energy of the fermion or anti-fermion, $\omega_{\mu}^{\nu} = \frac{1}{2} \varepsilon^{\mu} \sigma_{\rho\sigma} \partial_\sigma$ is the dual thermal vorticity tensor with the thermal vorticity tensor given by $\omega_{\mu}^{\nu} = \frac{1}{2} \varepsilon^{\mu} \sigma_{\rho\sigma} \partial_\sigma$ with $\beta = 1/T$, $F^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$ is the dual electromagnetic field strength tensor, and $f_F D$ is the Fermi-Dirac distribution. The electric and magnetic field as three-vectors are defined as $E^i = E_i - F^0$ and $B^i = B_i = -\frac{1}{2} \varepsilon_{ijk} F^{jk}$ with $i, j, k = x, y, z$. In a similar way, one can define the thermal vorticity three-vector $\omega^i = \omega_i = \omega_i^{00}$, the 'magnetic' part of the thermal vorticity tensor, and the 'electric' part of the thermal vorticity tensor $\varepsilon^i = \varepsilon_i = \omega_i^{00}$, which are $\omega = \frac{1}{2} \varepsilon \times (\mathbf{u})$ and $\varepsilon = -(1/2) [\partial_i (\mathbf{u}) + \nabla (\beta u^0)]$ in three-vector forms.

Applying Eq. (4) to the strange and anti-strange quark $s$ and $\bar{s}$, we obtain the polarization along the $y$ direction

$$P_{s/s,\bar{s}/\bar{s}}^y(t,x,\mathbf{p}_{s/s}) = \frac{1}{2} \omega_y + \frac{1}{2m_s} \mathbf{y} \cdot (\varepsilon \times \mathbf{p}_{s/s})$$

$$\pm \frac{Q_s}{2m_s T} B_y \pm \frac{Q_{\bar{s}}}{2m_{\bar{s}} T} \mathbf{y} \cdot (\mathbf{E} \times \mathbf{p}_{s/s}),$$

(5)

where $Q_s = -e/3$ is the electric charge of the $s$ quark ($e > 0$), and we have taken the non-relativistic limit $E_p \approx m_s$ and the Boltzmann limit $1 - f_F D(E_p \mp \mu) \approx 1$. The last term of Eq. (5) is the spin-orbit term for quarks in electromagnetic fields, the similar term is the key to the nuclear shell structure if applying to nucleons in meson fields [52, 53].

In the coalescence model, the polarization of $\Lambda$ or $\bar{\Lambda}$ in its rest frame is given by [49]

$$P_{\Lambda/\bar{\Lambda}}^y(t,x) = \frac{1}{3} \int \frac{d^3y}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} \left| \psi_{\Lambda/\bar{\Lambda}}(\mathbf{q}, \mathbf{r}) \right|^2 \times \left[ P_{s/s}^y(t,x,\mathbf{p}_1) + P_{s/s}^y(t,x,\mathbf{p}_2) + P_{s/s}^y(t,x,\mathbf{p}_3) \right]$$

$$= \frac{1}{2} \omega_y \pm \frac{Q_s}{2m_s T} B_y,$$

(6)

where $\psi_{\Lambda/\bar{\Lambda}}(\mathbf{q}, \mathbf{r})$ are wave-functions of $\Lambda/\bar{\Lambda}$ in momentum space with the normalization condition $\int d^3q d^3r \left| \psi_{\Lambda/\bar{\Lambda}}(\mathbf{q}, \mathbf{r}) \right|^2 = (2\pi)^6$, and internal momenta of three quarks are denoted as $\mathbf{p}_1 = \mathbf{p}/2 + \mathbf{q}$, $\mathbf{p}_2 = \mathbf{p}/2 - \mathbf{q}$ and $\mathbf{p}_3 = -\mathbf{r}$ which satisfy $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = 0$ in the rest frame of $\Lambda/\bar{\Lambda}$. In the square bracket of Eq. (6), $P_{s/s}^y(t,x,\mathbf{p}_1)$ means that $\mathbf{p}_1$ is the momentum of the s/p quark in $\Lambda/\bar{\Lambda}$ (the momenta of two light quarks/antiquarks are then $\mathbf{p}_2$ and $\mathbf{p}_3$), and $P_{s/s}^y(t,x,\mathbf{p}_2)$ and $P_{s/s}^y(t,x,\mathbf{p}_3)$ have similar meanings. Comparing Eq. (6) with Eq. (5), we see that there are no contributions from $\varepsilon$ and $\mathbf{E}$ in $P_{\Lambda/\bar{\Lambda}}^y$. The reason is that both $\varepsilon$ and $\mathbf{E}$ terms in $P_{s/s}^y$ are linearly proportional to $\mathbf{p}$, so these terms in the square bracket of Eq. (6) are vanishing due to $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = 0$ in the rest frame of $\Lambda$ and $\bar{\Lambda}$.
The 00-element of the spin density matrix for the $\phi$ meson is calculated by [49]

$$
\rho_{00}(t, x) \approx \frac{1}{3} - \frac{4}{9} \int \frac{d^3p}{(2\pi)^3} P^\mu_\phi(p) P^\nu_\phi(-p) |\psi_\phi(p)|^2,
$$

(7)

where $\psi_\phi(p)$ is the wavefunction in momentum space for the $\phi$ meson with the normalization $\int d^3p |\psi_\phi(p)|^2 = (2\pi)^3$, and we have put $p_\mu = p$ and $p_\nu = -p$ in the center of mass frame of $\phi$. Note that it is the correlation between $P^\mu_\phi(p)$ and $P^\nu_\phi(-p)$ [54] that is essential to resolve the puzzle in $\rho_{00}$. Inserting [5] into (7) and taking an average of $\rho_{00}(t, x)$ over the fireball volume $V$ and the polarization time $t$ with an effective temperature $T_{\text{eff}}$, we obtain

$$
\rho_{00}^\phi \approx \frac{1}{3} - \frac{4}{9} \langle P^\mu_\phi P^\nu_\phi \rangle + \frac{1}{27m^2_\phi} \langle p^2 \rangle \langle \epsilon^2 \rangle \epsilon^2 \epsilon^2 \\
\quad - \frac{e^2}{243m^4_\phi T^2_{\text{eff}}} \langle p^2 \rangle \langle E^2_z + E^2_y \rangle,
$$

(8)

where we have used $\langle p \rangle = 0$, $\langle p_{z,x} \rangle = (1/3) \langle p^2 \rangle \phi$, $\langle p_{z,p_x} \phi \rangle = 0$, with $\langle a(p) \phi \rangle \equiv (2\pi)^{-3} \int d^3p |\psi_\phi(p)|^2 a(p)$ being the mean value of a momentum function $a(p)$ in the $\phi$ meson wave function in momentum space, and replaced $T_{\text{eff}}$ by the effective temperature $T_{\text{eff}}$ of the fireball. From the $\phi$ meson wave function in the quark potential model [55, 56], we have $\langle p^2 \rangle_\phi \approx 0.18 \text{ GeV}^2 \approx 9.45 m^2_\pi$ with $m_\pi \approx (2/3)m_{\pi^+} / (1/3)m_{\pi^0} \approx 138.05 \text{ MeV}$. Using Eq. (6), the second term in the right-hand side of Eq. (8) is denoted as $c_\Lambda \equiv -(4/9) \langle P^\mu_\phi P^\nu_\phi \rangle$:

$$
c_\Lambda = \frac{e^2}{9m^2_\phi T_{\text{eff}}} \langle E^2_z + E^2_y \rangle.
$$

(9)

We see that the contribution to $\rho_{00}^\phi$ from the vorticity is always negative while that from the magnetic field is always positive. We also see that the magnitudes of $\langle \omega^2 \rangle$ and $\langle B^2 \rangle$ are constrained by the data of $P_\Lambda$ and $P_\Sigma$, but this is not the case for $\langle \epsilon^2 \rangle$ and $\langle E^2_z + E^2_y \rangle$ in Eq. (8).

We denote the third and fourth term in the right-hand side of Eq. (8) as $c_\epsilon$ and $c_E$ respectively. Note that all these terms are either positive or negative definite, which is a good feature of $\rho_{00}$. The $c_\Lambda$ term has two contributions: the vorticity contribution is negative and the magnetic field contribution is positive, they are both in the order $10^{-5}$ to $10^{-4}$ according to the simulations using hydrodynamic models [57, 58] and transport models [21, 22], respectively. The $c_\epsilon$ term provides a positive contribution to $\rho_{00}$ but not constrained by the data of $\Lambda$ polarization. This term comes from the fluid vorticity and can be estimated by the hydrodynamic simulation. We use CLVisc [57, 58], a (3+1)D viscous hydrodynamic model, to calculate $\langle \epsilon^2 \rangle$ at the freezeout. The numerical results show $\langle \epsilon^2 \rangle \sim 10^{-4}$. Using the constituent quark mass for $m_\pi$ of about 450 MeV, $c_\epsilon$ is even more suppressed. The $c_E$ term is from the electric field which is also absent in the $\Lambda$ polarization [6] and therefore not constrained by the data of $\Lambda$ polarization. The peak value for $c_E \equiv e\sqrt{\langle E^2_z + E^2_y \rangle}$ is about $m^2_\pi$ according to the simulation based on the Parton-Hadron-String Dynamics (PHSD) transport model [59] which includes a dynamical generation of retarded electromagnetic fields [21, 22], where we set $m_\pi \approx 450 \text{ MeV}$ and $T_{\text{eff}} \approx 100-300 \text{ MeV}$ for Au+Au collisions in the collision energy range 20-200 GeV. Then we obtain $c_E \sim 10^{-5}$, which cannot give a large deviation of $\rho_{00}$ from 1/3.

**IV. SPIN POLARIZATION IN A MESON FIELD OF $\phi$**

Like the electromagnetic field, a mean field of the $\phi$ meson, if exists, can also polarize $s$ and $\bar{s}$ and contribute to $\rho_{00}^\phi$. The role of the mean field of vector mesons in the polarization of the Lambda hyperon was proposed in Ref. [60]. The electric and magnetic part of the $\phi$ meson field $E_\phi$ and $B_\phi$ can be obtained by the field potential $\phi^\mu$ in the same way as for the electromagnetic field: $F^\mu_\phi = \partial^\mu \phi^\nu - \partial^\nu \phi^\mu$. This is in analogy with the vector dominance model similar to the baryon current in Ref. [60]. $\phi^\mu$ can be approximately proportional to the current density of the strangeness quantum number, $\phi^\mu \approx -(g_\phi / m^2_\phi) J_\mu^s$, known as the current-field identity in the vector dominance model [61, 62]. Here $m_\phi$ is the $\phi$ meson mass, and $g_\phi$ is the coupling constant of the $s$ quark to the $\phi$ meson in the quark-meson model [63, 66].

Note that the contribution from $s$ and $\bar{s}$ to $J_\mu^s$ is negative and positive, respectively. The strangeness current density in the central rapidity region is assumed to be a function of time and space

$$
J_\mu^s(t, x) = (\rho_s, J_s) = (\rho_s, j_s^x, j_s^y, j_s^z),
$$

(10)
It must satisfy strangeness conservation \( \partial_i J_{\mu}^i = 0 \) with the condition \( \int d^3 x \rho_4 (t, x) = 0 \). The electric and magnetic part of the \( \phi \) field that contribute to the spin alignment along \( +y \) direction are given by

\[
\begin{align*}
E_{\phi} &= \hat{\mathbf{z}} \frac{g_\phi}{m_\phi} \bar{E}^z_{\phi} + \hat{\mathbf{x}} \frac{g_\phi}{m_\phi} \bar{E}^x_{\phi}, \\
B_{\phi} &= \hat{\mathbf{y}} \frac{g_\phi}{m_\phi} \left( \frac{\partial j_s^z}{\partial x} - \frac{\partial j_s^x}{\partial z} \right),
\end{align*}
\]

where \( \bar{E}^i_{\phi} = \bar{E}_{\phi,i} \equiv \nabla_i \rho_4 + \partial_j j_s^i / \partial t \) with \( i = x, y, z \). The \( z \) component of \( J_s \) in \( \text{(10)} \) is the result of the difference in the parton distribution function for \( s \) and \( \bar{s} \) in nucleons: \( s(x_B) \neq \bar{s}(x_B) \) in different regions of \( x_B \), where \( x_B \) is the momentum fraction (Bjorken variable) carried by \( s \) and \( \bar{s} \) in the proton. Although the uncertainty in extracting \( s(x_B) \) and \( \bar{s}(x_B) \) in the nucleon sea from experimental data \[67\] is large, there are strong evidences \[69, 70\] for \( s(x_B) \neq \bar{s}(x_B) \). Extensive theoretical studies have been done on the asymmetry of \( s(x_B) \) and \( \bar{s}(x_B) \) in the past 30 years \[71\].

In nucleus-nucleus collisions, this feature is responsible for the non-zero strangeness current \( j_s^z \) which may depend on time. We have also generalized this feature by introducing \( \rho_4, j_s^z \) and \( j_s^y \) in Eq. (\text{10}).

Then the contribution from the \( \phi \) meson field can be obtained from Eqs. \[6\text{–}9\] by replacements: \( \mathbf{B} \to \mathbf{B}_\phi \), \( \mathbf{E} \to \mathbf{E}_\phi \) and \( Q_s = -\frac{1}{2} e \to g_\phi \). Now \( P_{s/\bar{s}}^y \) in Eq. \[5\] have two additional terms: \( \pm g_\phi B_x^y (2m_s t) \) and \( \pm g_\phi \mathbf{y} \cdot (\mathbf{E}_\phi \times \mathbf{p}_{s/\bar{s}}) / (2m^2 T) \).

Correspondingly, \( P_{\Lambda/\bar{\Lambda}}^y (t, x) \) in \[6\] has an additional term \( \pm g_\phi B_x^y (2m_s t) \) which is constrained by the data. We see that it is \( \mathbf{B}_\phi \) instead of \( \mathbf{E}_\phi \) that contributes to \( P_{\Lambda/\bar{\Lambda}}^y (t, x) \). Equation (\text{8}) becomes

\[
\rho_{00}^\phi \approx \frac{1}{3} + c_\Lambda + c_z + c_E + c_\phi,
\]

where \( c_\phi \) is from the \( \phi \) field

\[
c_\phi = \frac{g^2_\phi}{27 m^2_s T_{\text{eff}}} \left[ 3 \langle B_{\phi,y}^2 \rangle - \frac{\langle P^2 \rangle_\phi}{m^2_s} \left( \langle E_{\phi,z}^2 \rangle + \langle E_{\phi,x}^2 \rangle \right) \right].
\]

Note that the average is taken over the space-time volume. In deriving \[\text{12}\] we have assumed that there are no correlations among different fields (fluid field, electromagnetic field, \( \phi \) field), e.g., between fluid and electromagnetic field, between \( \mathbf{B} \) and \( \mathbf{B}_\phi \), and between \( \mathbf{E} \) and \( \mathbf{E}_\phi \), etc. We have also assumed that there is no correlation between the electric and magnetic part of the same field. The important feature of Eq. (\text{13}) is that \( c_\phi \) has positive contribution from \( B_{\phi,s} \) and negative contribution from \( E_{\phi,s} \) in the form of field squares which are not constrained by \( P_{\Lambda/\bar{\Lambda}}^y (t, x) \). We note that Eq. \[\text{13}\] is for \( \rho_{00}^\phi \) in the \( y \) direction, one can obtain \( \rho_{00}^\phi \) in the \( x \) or \( z \) direction as well. For \( \rho_{00}^\phi \) in the \( x \) direction, one can just replace \( \omega_x \), \( B_y \) and \( B_x^y \) in \( c_\Lambda \) by \( \omega_x \), \( B_x \) and \( B_x^y \) respectively, and replace \( \varepsilon_x \), \( E_x \) and \( E_x^2 \) in \( c_z \), \( c_E \) and \( c_\phi \) by \( \varepsilon_y \), \( E_y \) and \( E_y^2 \) respectively.

As we have shown in Sec. \[\text{11}\] that \( c_\Lambda, c_z \) and \( c_E \) in Eq. \[\text{12}\] are negligibly small compared with \( 1/3 \) for Au+Au collisions in the collision energy range 20-200 GeV. If the data show that \( \rho_{00}^\phi \) is larger than \( 1/3 \) by at least a few percent, according to our model, the deviation may possibly be from \( c_\phi \) involving the magnetic part of the \( \phi \) field. A good feature of \( \rho_{00}^\phi \) is that each contribution is in square up to a sign, so it is either positive or negative definite. This property does not depend on the procedure of taking an average or on choices of parameters. It exists even for fluctuating fields (the vorticity, electromagnetic and \( \phi \) field). Therefore \( \rho_{00}^\phi \) is a good analyzer for fields even if they may fluctuate strongly in space-time.

We can estimate in a simple model the dominant contribution to \( \rho_{00}^\phi \) from the last term of Eq. \[\text{12}\]. We choose the effective temperature as \( T_{\text{eff}} \propto n_{\tau}^{-1/3} (dn_{ch}/d\eta)_{\eta=0}^{1/3} \), where \( n_{\tau} \sim s_{NN}^{1/3} \) and \( (dn_{ch}/d\eta)_{\eta=0} \sim -0.4 + 0.39 \ln s_{NN} \) is the pseudorapidity density of charged particles at the central pseudorapidity \( \eta = 0 \) and the collision energy \( s_{NN}^{1/2} \) should take the dimensionless number when expressed in the unit GeV \[30\]. We set \( T_{\text{eff}} = 300 \text{ MeV} \) at \( s_{NN}^{1/2} = 200 \text{ GeV} \) for calibration. In this way the collision energy behavior of \( \rho_{00}^\phi \) is solely from \( T_{\text{eff}} \) which is a strong assumption in this order of magnitude estimate. As an approximation, we assume that \( \partial J_{\phi}/\partial t \) and \( \nabla \times \mathbf{J}_s \) do not depend on the collision energy. We set the values of the following parameters: \( m_s = 450 \text{ MeV} \) and \( G_s^{(y)} = (2.05, 3.08, 5.13) m^4_s \) where \( G_s^{(y)} \equiv g_\phi \left[ 3 \langle B_{\phi,y}^2 \rangle - \frac{\langle P^2 \rangle_\phi}{m^2_s} \left( \langle E_{\phi,z}^2 \rangle + \langle E_{\phi,x}^2 \rangle \right) \right] \).

Note that the value of \( g_\phi \) can be taken from the constraint by the compact star properties in the quark-meson model \[65, 66\]. With these values of parameters the dominant contribution to \( \rho_{00}^\phi \), \( c_\phi \) in Eq. \[\text{12}\], as a function of collision energy in Au+Au collisions is shown in Fig. \[\text{1}\]. We see in Fig. \[\text{1}\] that \( \rho_{00}^\phi \) decreases with the collision energy.
A natural question arises: are the theory and conclusion in this paper valid for another vector meson $K^{*0}(892)$? The answer would be no. There are a few reasons for it. First, due to unequal masses of $\bar{s}$ and $d$, one cannot derive similar formula to Eq. (8) in which terms of vorticity and those of electric and magnetic field are decoupled. Therefore one cannot build up a simple relationship between $\rho_{00}$ for $K^{*0}$, $\rho_{00}^{K^{*0}}$, and the hyperon polarization. In $\rho_{00}^{K^{*0}}$ each contribution can be either positive or negative, so it is not easy to single out a specific contribution from $\rho_{00}$ which belongs to the vorticity, electromagnetic field or mesonic field without ambiguity. Second, the interaction of $K^{*0}$ with the surrounding matter is much stronger than the $\phi$ meson. In this sense, the $\phi$ meson is a cleaner probe than $K^{*0}$ to the state of the fireball. Actually, preliminary data from the ALICE experiment show that $\rho_{00}$ for the $K^{*0}$ meson is less than 1/3 at LHC energies [81, 82], which is very different from the $\phi$ meson. Another question is: what happens for $\rho_{00}$ at LHC energies? From the energy behavior in Fig. 1 we expect that negative $c_\Lambda$ and $c_{\bar{\Lambda}}$ would be comparable to positive $c_\phi$ at LHC energies. In this case, whether $\rho_{00}$ is larger or smaller than 1/3 depends on a fine-tuning of each terms.

V. SUMMARY

Due to the difference in the parton distribution function of $s$ and $\bar{s}$ in high energy proton-proton collisions, the longitudinal momenta carried by $s$ and $\bar{s}$ are not equal. This leads to a non-vanishing collective strangeness current in the beam direction in high energy heavy-ion collisions. We generalize this feature to transverse directions. Such a strangeness current gives rise to a non-vanishing electric and magnetic part of the mean $\phi$ field, $E_\phi$ and $B_\phi$, respectively. Like the magnetic field, $B_\phi$ can also polarize $s$ and $\bar{s}$ through their magnetic moments which contributes to the polarization of $\Lambda$ and $\bar{\Lambda}$, while the contribution from $E_\phi$ is absent and therefore is not constrained by the polarization of $\Lambda$ and $\bar{\Lambda}$. Note that $E_\phi$ can also polarize $s$ and $\bar{s}$ through the spin-orbit coupling, the same coupling that is responsible for the nuclear shell structure at the nucleon level. The contributions from $B_\phi^2$ and $E_\phi^2$ to $\rho_{00}^\phi$ are positive and negative respectively, if the polarization of $s$ and $\bar{s}$ is assumed to be only along the OAM direction ($y$ direction) and if $\phi$ mesons are static (with vanishing momenta). Both $B_\phi^2$ and $E_\phi^2$ are not constrained by the polarization data of $\Lambda$ and $\bar{\Lambda}$. We then propose that a significant deviation of $\rho_{00}^\phi$ from 1/3 could indicate the presence of the $\phi$ field in heavy-ion collisions which polarizes $s$ and $\bar{s}$ in the same way as the electromagnetic field. The contributions are significant even for fluctuating fields. In this sense $\rho_{00}^\phi$ is a good analyzer for fluctuating fields.

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Appendix A: Note added after publication

The main result in Eq. 13 is derived originally from Eq. 7 based on two assumptions or conditions: (a) φ mesons are static with vanishing momenta; (b) The polarization of s and ̅s quarks is only along the OAM direction, i.e. the y direction. We can relax the condition (b) by allowing the polarization of s and ̅s quarks can be along all directions but the spin quantization direction is still in the y direction. In this case Eq. 7 becomes

$$\rho_{00}(t, \mathbf{x}) \approx \frac{1}{3} - \frac{4}{9} \int \frac{d^3p}{(2\pi)^3} |\psi_\phi(p)|^2$$

$$\times \left\{ P^y_s(p)P^y_\bar{s}(-p) - \frac{1}{2} [P^y_s(p)P^\bar{s}_s(-p) + P^\bar{s}_s(p)P^y_s(-p)] \right\}.$$

(A1)

The polarization of s and ̅s quarks can be written as

$$P_{s/\bar{s}}(t, \mathbf{x}, \mathbf{p}_{s/\bar{s}}) \approx \frac{1}{2} \omega + \frac{1}{2m_s} (\mathbf{e} \times \mathbf{p}_{s/\bar{s}}) \pm \frac{Q_s}{2m_sT} \mathbf{B} \pm \frac{Q_s}{2m_s^2T} (\mathbf{E} \times \mathbf{p}_{s/\bar{s}})$$

$$\pm \frac{g_\phi}{2m_sT} \mathbf{B} \pm \frac{g_\phi}{2m_s^2T} (\mathbf{E} \times \mathbf{p}_{s/\bar{s}}),$$

(A2)

which has three contributions: the vorticity, electromagnetic, and φ fields. Inserting (A2) into (A1) we obtain ρ_{00}^\phi for static φ mesons

$$\rho_{00}^\phi \approx \frac{1}{3} - \frac{1}{9} \left[ \langle \omega^2_y \rangle - \frac{1}{2} \langle \omega^2_x + \omega^2_z \rangle \right]$$

$$- \frac{Q^2_s}{9m_s^2T_{eff}^2} \left[ \langle \varepsilon^2_y \rangle - \frac{1}{2} \langle \varepsilon^2_x + \varepsilon^2_z \rangle \right]$$

$$+ \frac{g^2_\phi}{9m_s^2T_{eff}^2} \left[ \langle B^2_y \rangle - \frac{1}{2} \langle B^2_x + B^2_z \rangle \right]$$

$$+ \frac{Q^2_s}{27m_s^2T_{eff}^2} \left[ \langle E^2_y \rangle - \frac{1}{2} \langle E^2_x + E^2_z \rangle \right]$$

$$+ \frac{g^2_s}{9m_s^2T_{eff}^2} \left[ \langle B^2_{\phi,y} \rangle - \frac{1}{2} \langle B^2_{\phi,x} + B^2_{\phi,z} \rangle \right]$$

$$+ \frac{g^2_s}{27m_s^2T_{eff}^2} \left[ \langle E^2_{\phi,y} \rangle - \frac{1}{2} \langle E^2_{\phi,x} + E^2_{\phi,z} \rangle \right].$$

(A3)

If ρ_{00}^\phi is dominated by the φ field, we obtain ρ_{00}^\phi for the spin quantization direction being in the y direction as

$$\rho_{00}^\phi \approx \frac{1}{27m_s^2T_{eff}^2} G_s^{(y)},$$

(A4)

where G_s^{(y)} now becomes

$$G_s^{(y)} = \frac{3}{2} Q_{yy}(\mathbf{B}_\phi) - \frac{\langle \mathbf{P}^2 \rangle_\phi Q_{yy}(\mathbf{E}_\phi)}{2m_s^2}$$

$$= g_\phi^2 \left[ 3 \langle B^2_{\phi,y} \rangle + \frac{\langle \mathbf{P}^2 \rangle_\phi}{m_s^2} \langle E^2_{\phi,y} \rangle \right.$$

$$- \frac{3}{2} \langle B^2_{\phi,x} + B^2_{\phi,z} \rangle - \frac{\langle \mathbf{P}^2 \rangle_\phi}{2m_s^2} \langle E^2_{\phi,x} + E^2_{\phi,z} \rangle \bigg].$$

(A5)
Here we have defined the average quadrupole moments for $B_φ$ and $E_φ$ as

\[ Q_{ij}(B_φ) = g_φ^2 \left( 3B_{φ,i}B_{φ,j} - |B_{φ}|^2 \right), \]
\[ Q_{ij}(E_φ) = g_φ^2 \left( 3E_{φ,i}E_{φ,j} - |E_{φ}|^2 \right), \]

with $i, j = x, y, z$. We see in Eq. (A5) that if the spin quantization direction is chosen to be the $y$ direction, the positive contributions to $ρ_{00}$ come from $B_{φ,y}$ and $E_{φ,y}$ while negative ones to $ρ_{00}$ come from $B_{φ,i}$ and $E_{φ,i}$ with $i = x, z$. We can take the difference between $ρ_{00}$ for the spin quantization in the $y$ (out-plane) direction from that in the $x$ (in-plane) direction as

\[ Δρ_{00} = \frac{1}{2πm_s^2T_{\text{eff}}^2} \left[ G_s^y - G_s^x \right] \]
\[ \approx \frac{g_φ^2}{18m_s^2T_{\text{eff}}^2} \left[ 3(B_{φ,y}^2 - B_{φ,x}^2) + \frac{P_y^2}{m_s^2} E_{φ,y}^2 - E_{φ,x}^2 \right]. \]

This can also be tested in future experiments.

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