SINR and Capacity Analysis for Multiuser MIMO Interference Channels

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Abstract. Multiple-Input-Multiple-Output (MIMO) has a significant effect on wireless communications due to providing high capacity and low BER as compared to the conventional wireless communications with SISO. Due to transmitting a large number of independent signal from different users in different space-time approach, the BER performance suffers inferiority leading to decreasing in the system reliability. So, our work is based on maximizing the SINR of a multiuser (MU)-MIMO system by sending separate data of different users from one base station (BS) via a number of antennas with different time slots to a number of mobile stations (MS) each with more than one antenna technique. The proposed work considers one user as a desired user and others as interferer users when these data are propagating in multipath fading channel. BER performance as well as channel capacity for various diversity systems such as 2×2 and 4×4 are simulated. The numerical results obtained by computer simulation show that implementing 4×4 MIMO system gives gain of 1 dB as compared to the system implemented with 2×2 MIMO. The results show that when the 10 users communicate with the BS required SINR of 18 dB at 10^{-2} BER while for 20 users it requires a SINR of 20 dB at BER of 10^{-2}. We can conclude that the BER is decreased and the capacity is increased with increasing the number of antennas when the system is under multiuser and multipath interferences.

Keywords. MIMO, Multiuser, Interference, Capacity.

1. Introduction

When wireless communication technologies grow, (QoS) as well as capacity are of prime importance. To ensure effective contact communication over a mobile radio channel, a system needs to resolve multipath fading, polarization mismatch, and interference. The move to low-power handheld transceivers exacerbates all of these problems. But increasing the low power run into the regulations of International Telecommunication Union (ITU) where forbid the increasing of power to the specified limits due to its risk. Even with more bandwidth is allocated, competition for higher data rate services and an ever-increasing number of users should encourage service providers to find ways to increase their system ability [1]. In [2-4], the Hybrid MIMO channel with the millimetre wave bandwidth system is checked using a versatile phased antenna array. They proposed a zero Intermediate Frequency (IF) 16 antennas millimetre wave transceiver that performs analogue baseband beamforming to ensure high robustness and low parasite element sensitivity, as operations are conducted at low frequencies. In [5], The SINR distributions of the scenario uplink with zero forcing beamforming and the selection of the
antennas at the Aggregate Gain of Interference (AGG) receiver studied. Multivariate Beta type II distributions model the downlink SINR with perfect and imperfect CSI and multivariate Wishart distributions model the uplink SINR with perfect and imperfect Channel State Information (CSI). The empirical results of the proposed SINR distributions analytical results are compared to the Monte-Carlo simulations. In [6], the downlink transmission performance of linearly pre-coded multi-user MIMO systems is investigated. ZF, RZF, and MMSE linear precoding schemes are considered in this research, the BER performance of a KM multi-user MIMO system is investigated. Simulation findings showed that, in terms of the BER, a RZF precoded multi-user MIMO system is a superior performance than ZF and MMSE of corresponding precoded systems. In [7], A MIMO Detection of Multi-user Uplink is proposed through Parallel Access. They developed simultaneous signal detection techniques from multiple users for the uplink multi-user MIMO channel. The proposed UL MU-MIMO ZF / MMSE decoupling method gives a new perspective for implementing multi-user detection algorithms. The signals from different users can be isolated as if the other users didn’t exist and were identified in parallel. In recent years, the demands for high data rate leads to necessity of using a diversity technique to increase the system’s capacity, so multiple antennas technique at the base station and mobile stations is required to improve overall system performance. The main challenge for MIMO channel when taking into consideration the complexity and performance is the system's detector. The maximum likelihood (ML) decoder including sphere decoder (SD) which is widely used in MIMO. In spite of their optimality, these types of detectors are not convenient due to their complexity. However, using of MMSE and ZF is suitable for MIMO channel due to their acceptable optimality and less complexity [8,9,10]. In multi-user scenario, high data rates can be provided when multiple antennas at transmitter and receiver are used [11]. These users can share the same radio frequencies. MIMO is a major area of interest within the field of wireless communications, which is heightened the reliable of the system. However, observations have indicated a serious decline in the BER performance when the more than one user send their information simultaneously. One major issue in MU-MIMO system is the multipath and multiuser interference as well as capacity issue [12-16]. So, the use of an efficient detector can enhance system performance, even by increasing the number of cell users. The main contribution of this paper is to analyse and evaluate the SINR for the multi-user MIMO system considering one user as a desired user and others as interferer users as well as evaluate the system capacity.

**Notation:** We use uppercase italic letters for matrices and lowercase bold-italic letters for vectors. ( )^T stands for the transpose, ( )^H stands for conjugate transpose and ( )^{-1} stands for inverse matrix operations.

2. Research Method

2.1. System Description and Mathematical Model

As set out in the previous section, most communication systems deal with multiuser systems, where each user may share the same radio frequencies. Figure 1 illustrates a configuration of multiple users’ communication system where a one base station communicates with multi mobile stations, where each station in both sides provided with multiple antennas to enhance the system performance. As shown in Figure 1, the same resources such as frequency and time are explored by K users one of them is the desired user and the others are interferer user. Suppose that the BS has multiple antennas (N) and each mobile station has multiple antennas represented by (M).
Figure 1. Multiple users MIMO environment.

Let us consider $K$ users ($0, 1, 2, \ldots, K$) each equipped with $M$ antennas, and the Base Station (BS) is equipped with $N$ antennas. If user 0 is the desired user and the other users 1, 2, $\ldots$, $K$ are interferer users. Let the information symbols transmitted to the $K$ users are $s_0$, $s_1$, $s_2$, $\ldots$, $s_K$, where $s_0$, $s_k$ represent the symbols transmitted to user 0 and $s_k$, respectively, represent the symbols transmitted to user $K$. In downlink channel (DL), $s \in \mathbb{C}^{N \times 1}$ is the transmitted signal from the BS and the received signal $y \in \mathbb{C}^{M \times 1}$ at the $u^{th}$ user $u = 0, 1, 2, \ldots, K$. Now, let us consider $H_{DL} \in \mathbb{C}^{M \times N}$ is the channel gain between the BS and the MS for the $u^{th}$ user. However, the transmitted signal on the downlink is,

$$s(m) = \sum_{l=0}^{K} s_l(m)$$  \hspace{1cm} (1)

where $s(m)$ is the $m^{th}$ transmitted symbol.

The signal received for the $u^{th}$ user on DL which transmitted through channel with channel gain $H$ is

$$y_u = H_{DL} s + n_u, \quad u = 0, 1, 2, \ldots, K$$  \hspace{1cm} (2)

where $n_u \in \mathbb{C}^{M \times 1}$ is the additive white Gaussian noise for the user $u$. Hence, the overall system can be described as,

$$\begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_K \end{bmatrix} = \begin{bmatrix} H_{DL0} \\ H_{DL1} \\ \vdots \\ H_{DLK} \end{bmatrix} s + \begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_K \end{bmatrix}$$  \hspace{1cm} (3)

Equation (3) can be written as,

$$Y_{DL} = H_{DL} s + N_{DL}$$  \hspace{1cm} (4)

Downlink (DL) channel for multiuser system is shown in Figure 2.
However, User 0 got the signal
\[ y_0(m) = H_{DL0}(0)s_0(m) + H_{DL0}(1)s_1(m) + H_{DL0}(2)s_2(m) + \ldots + H_{DL0}(L-1)s_k(m) + n_0(m) \quad (5) \]
where \( L \) is the number of multipath components.

Similarly, for users 1, 2, ..., and \( k \)
\[ y_1(m) = H_{DL1}(0)s_0(m) + H_{DL1}(1)s_1(m) + H_{DL1}(2)s_2(m) + \ldots + H_{DL1}(L-1)s_k(m) + n_1(m) \quad (6) \]
\[ y_k(m) = H_{DLk}(0)s_0(m) + H_{DLk}(1)s_1(m) + H_{DLk}(2)s_2(m) + \ldots + H_{DLk}(L-1)s_k(m) + n_k(m) \quad (7) \]

Hence, this is an intersymbol interference (ISI) channel,
\[ y(m) = \sum_{d=0}^{L-1} \sum_{k=0}^{K} H_{DLk}(d)s_k(m) + n(m) \quad (8) \]
where \( H_{DLk}(d) \) is the \( dth \) multipath component.

Recall that, for \( k \) users, the MIMO channel becomes,
\[
\begin{bmatrix}
Y_0(m) \\
Y_1(m) \\
\vdots \\
Y_k(m)
\end{bmatrix} = \begin{bmatrix}
\sum_{d=0}^{L-1} \sum_{k=0}^{K} H_{DL0} \\
\sum_{d=0}^{L-1} \sum_{k=0}^{K} H_{DL1} \\
\vdots \\
\sum_{d=0}^{L-1} \sum_{k=0}^{K} H_{DLk}
\end{bmatrix} \begin{bmatrix}
S_0(m) \\
S_1(m) \\
\vdots \\
S_k(m)
\end{bmatrix} + N_{DL}(m) \quad (9)
\]

Where \( N_{DL}(m) = \sum_{k=0}^{K} n_k(m) \)
The mean of MPI is

\[
\text{error} = \|Y_{DL} - H_{DL}s\|^2
\]

Consider the case when the number of antennas at MS is equal or more than at BS i.e. \(N \geq M\), so, we have more equations than unknowns. Almost, all possible transmit vectors \(\hat{s}\) choose the minimum vectors.

\[
\hat{s} = (H_{DL}^T H_{DL})^{-1} H_{DL} Y_{DL}
\]

The SINR of the system under study can be computed as follows:

- Signal components: \(signal = \frac{H_{D0}^T H_0(0) s_0 + H_{D1}^T H_0(1) s_0 + \ldots + H_{DL}^T H_0(L-1) s_0}{E(s_0)}\)

\[
= \frac{|H_0(0)|^2}{E(s_0)} s_0 + \frac{|H_0(1)|^2}{E(s_0)} s_0 + \ldots + \frac{|H_0(L-1)|^2}{E(s_0)} s_0 = \sum_{d=0}^{L-1} H_0(d) s_0
\]

So, by taking the expected value of the equation (14), the signal power becomes

\[
\text{signal power} = E(\sum_{d=0}^{L-1} H_0(d) s_0) = \sum_{d=0}^{L-1} H_0(d) s_0 E(s_0)
\]

\[
= \sum_{i=0}^{L-1} H_0(i) p_0
\]

Multipath Interference (MPI):

\[
I(m) = s_d(0) \sum_{d=1}^{L-1} \sum_{i=1}^{R-1} \sum_{k=0}^{K} H_{DLd}(d) s_k(r-d) s_k(r)
\]

Where \(R\) is the delayed sequences and \(s_k(r-d)\) is independent of \(s_k(r)\).

However,

\[
I(m) = s_d(0) \sum_{d=1}^{L-1} \sum_{k=0}^{K} H_{DLd}(d) s_k(00)(d)
\]

The mean of MPI is
\[
E(I) = s_0(0)E\left( \sum_{d=1}^{L-1} \sum_{k=0}^{K} H_{DL}(d) s_k(0)(d) \right)
\]
\[
E(I) = s_0(0)\sum_{d=1}^{L-1} H_{DL}(d) \cdot 0 = 0.
\] (18)

The variance of the MPI is represented as
\[
E(I^2) = E(s_0(0))^2\sum_{d=1}^{L-1} \sum_{k=0}^{K} H_{DL}(d)E(s_k(0)(d))^2 = P_0 \sum_{d=1}^{L-1} \sum_{k=0}^{K} H_{DL}(d) \cdot 1
\]
\[
E(I^2) = P_0\sum_{d=1}^{L-1} \sum_{k=0}^{K} H_{DL}(d)
\]
\[
E(I^2) = P_0\|H_{DL}\|^2 - P_0|H_{DL}|^2
\] (20)

Multiuser Interference (MUI)
\[
M = \sum_{r=1}^{R-1} \sum_{k=1}^{K-1} H_{DL}(k)s(r-k)s(k)
\]
\[
E(M) = \left( \sum_{r=1}^{R-1} \sum_{k=1}^{K-1} H_{DL}(k)s(r-k)s(k) \right) = \sum_{r=1}^{R-1} \sum_{k=1}^{K-1} H_{DL}(k)E(s_0(k)) = \sum_{r=1}^{R-1} \sum_{k=1}^{K-1} H_{DL}(k) \cdot 0 = 0
\] (21)
\[
E(M^2) = \sum_{r=1}^{R-1} \sum_{k=1}^{K-1} |H_{DL}(k)|^2 s(r-k)^2 = |H_{DL}(k)|^2 \sum_{k=1}^{K-1} p(k)
\] (23)

From the above equations of MPI and MUI, the total interference becomes
\[
W = I + M
\]
\[
W = P_0\|H_{DL}\|^2 - P_0|H_{DL}|^2 + |H_{DL}(k)|^2 \sum_{k=1}^{K-1} p(k)
\] (25)

2.2. Capacity of Multiuser MIMO System

In SVD, the channel matrix is decomposed as \( H = U\Sigma V^H \), where the upper script (H) is the Hermitian matrix which is equal to conjugate transpose of the matrix. Remember that the received signal equal to \( \tilde{H}_{DL}s + \tilde{N}_{DL} \). However,
\[
Y_{DL} = \sum_{k=0}^{K-1} (U\Sigma V^H)_{DL}s + N_{DL}
\] (31)

where \( \Sigma = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_L \end{bmatrix} \)

At the receiver, the received signal is processed by multiply by \( U^H \) then,
\[
U^HY_{DL} = \sum_{k=0}^{K-1} U^H(U\Sigma V^H)_{DL}s + U^HN_{DL}
\] (32)

Let \( U^HY_{DL} = \tilde{Y} \) and \( U^HN_{DL} = \tilde{N} \), then
\[
\tilde{Y} = \sum_{k=0}^{K-1} \Sigma V^H s + \tilde{N}
\] (33)

Also, the precoding process for information signal is generated as \( s = VS \), therefore
\[
\tilde{Y} = \sum_{k=0}^{K-1} \Sigma s + \tilde{N}
\] (34)

However, transmitting \( t \) information symbols in parallel (spatial multiplexing), then the above system can be parallelized as
\[ \begin{bmatrix} \tilde{Y}_1 \\ \vdots \\ \tilde{Y}_t \end{bmatrix} = \sum_{k=0}^{K-1} \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_t \end{bmatrix} \begin{bmatrix} \tilde{s}_1 \\ \vdots \\ \tilde{s}_t \end{bmatrix} + \begin{bmatrix} \tilde{N}_1 \\ \vdots \\ \tilde{N}_t \end{bmatrix} \]  

(35)

This system can be written as

\[ \tilde{Y}_1 = \sum_{k=0}^{K-1} \sigma_1 \tilde{s}_1 + \tilde{N}_1 \]
\[ \tilde{Y}_2 = \sum_{k=0}^{K-1} \sigma_2 \tilde{s}_2 + \tilde{N}_2 \]
\[ \vdots \]
\[ \tilde{Y}_t = \sum_{k=0}^{K-1} \sigma_t \tilde{s}_t + \tilde{N}_t \]  

(36)

3. Simulation Results and Discussion

A QPSK is used as a modulation scheme that provides two bits per second for each user to compare the difference between different diversity techniques over a Rayleigh fading channel. It is assumed the BS transmits to different users but one of them is the desired user and others are interferer users, also the BS and mobile stations equipped with multiantenna system, in this paper we assumed 2×2 and 4×4 MIMO channel. A Matlab was utilized to carry out the simulation results of MU-MIMO communication system. In Figure 3 when the number of antennas on both sides is increased, there is a clear trend of decreasing the BER. 4×4 MIMO is performed better than 2×2 MIMO and SISO schemes, where at \(10^{-3}\) BER required SINR of 14 dB in 4×4 MIMO while it requires more power for both 2×2 MIMO and SISO. The ZF and or MMSE equalizers added to wireless MIMO systems to optimize the post detection signal to interference plus noise ratio (SINR).

A simulation results shown in Figure 4, for multiuser MIMO scheduling and precoding is presented here. The simulator supports scheduling methods, the precoding methods including ZF and MMSE. The results in Figure 4 refer to ADMM, Alternating Direction Multipliers Method. ADMIN is an iterative algorithm that has a superior linear detectors with a limited number of iterations when there is a limited ratio between the number of antennas in BS and users. In the first iteration, ADMIN calculates the linear MMSE solution, which is necessary when the ratio between BS antennas numbers and users is very high. With successive increases the number of users in Figure 5 there is a clear trend of decreasing the BER performance for 4TX,4RX MIMO-MU with MMSE equalization using QPSK modulation which requiring more SINR. The mean score of increasing the number of transmitting and receiving antennas is to maximize the data rate. There is a significant increasing in capacity in Figure 6 for high SINR when a 4×4 MIMO-MU system is used as compared to systems with less number of antennas at transmitter and receiver. The mean score of increasing the number of transmitting and receiving antennas is to maximize the data rate. There is a significant increasing in capacity in Figure 6 for high SINR when a 4×4 MIMO-MU system is used as compared to systems with less number of antennas at transmitter and receiver.
Figure 3. Simulation results for $4 \times 4$ MIMO, $2 \times 2$ MIMO and SISO systems.

Figure 4. Simulation results for ZF and MMSE detectors.
Figure 5. Simulation results for MU-MIMO system.

Figure 6. Simulation results for Capacity Vs SINR for different MIMO schemes with 5 subscribers.

4. Conclusions
Downlink MU-MIMO performance is considered in the presence of Rayleigh fading. Data is modulated using the QPSK modulator and modulated data then divided into m streams which are simultaneously transmitted by the N transmission antennas for each user. SINR is analyzed numerically and BER. Performance is simulated by computer and it is shown that performance decreases in the range of 1.5 to 2 dB in order to increase the number of users from ten to twenty user. So, when the 10 users communicate with the BS required SINR of 18 dB at 10^-2 BER while for 20 users it requires a SINR of 20 dB at BER of 10^-2.
Also, it is concluded that when as the number of transmitting and receiving antennas increase, the capacity for wireless communication systems is increased. As well as the BER performance is improved i.e. implementing 4x4 MIMO system gives gain of 1 dB as compared to the system implemented with 2x2 MIMO.
References

[1] Bhargava V K 2006 State of the art and future trends in wireless communications: Advances in the physical layer Proc. - 4th Annu. Commun. Networks Serv. Res. Conf. CNSR 2006 3

[2] Blandino S, Mangraviti G, Dessel C, Bourdoux A, Wambacq P and Pollin S 2019 Multi-User Hybrid MIMO at 60 GHz Using 16-Antenna Transmitters IEEE Trans. Circuits Syst. I Regul. Pap. 66 848–58

[3] Dhakal S, Martin P A and Smith P 2019 Hybrid zero-forcing for correlated and semi-orthogonal multi-user MU-MIMO channels IET Commun. 13 3575–81

[4] Maung E N, Shimbo Y, Suganuma H and Maehara F 2019 Low complexity fair user scheduling employing spatial orthogonality for mu-MIMO systems 2019 26th Int. Conf. Telecommun. ICT 2019 437–41

[5] Saki H, Charbit G and Shikh-Bahaei M R 2019 On the SINR Distribution of SWIPT MU-MIMO with Antenna Selection 2019 IEEE ComSoc Int. Commun. Qual. Reliab. Work. CQR 2019 1–6

[6] Saleeb B, Shehata M, Mostafa H and Fahmy Y 2019 Performance Evaluation of RZF Precoding in Multi-User MIMO Systems Midwest Symp. Circuits Syst. 2019-Augus 1207–10

[7] Zu K, Zhu J and Haardt M 2019 Uplink Multi-user MIMO Detection via Parallel Access ICASSP, IEEE Int. Conf. Acoust. Speech Signal Process. - Proc. 2019-May 4365–9

[8] Eraslan E, Daneshrad B and Lou C Y 2013 Performance indicator for MIMO MMSE receivers in the presence of channel estimation error IEEE Wirel. Commun. Lett. 2 211–4

[9] Ngo H Q, Larsson E G and Marzetta T L 2013 Energy and spectral efficiency of very large multiuser MIMO systems IEEE Trans. Commun. 61 1436–49

[10] Email A 2015 Block-Iterative Frequency-Domain Equalizations for SC-IDMA Systems Journal of Engineering, University of Baghdad 21(7) 85-101

[11] Wong K K, Murch R D and Letaief K Ben 2002 Performance enhancement of multiuser MIMO wireless communication systems IEEE Trans. Commun. 50 1960–70

[12] Chen C J and Wang L C 2006 Enhancing coverage and capacity for multiuser MIMO systems by utilizing scheduling IEEE Trans. Wirel. Commun. 5 1148–57

[13] Lee J H and Choi W 2010 Opportunistic interference aligned user selection in multiuser MIMO interference channels GLOBECOM - IEEE Glob. Telecommun. Conf.

[14] Jia L, Hou Y T, Yi S and Sherali H D 2006 Optimization of multiuser MIMO networks with interference GLOBECOM - IEEE Glob. Telecommun. Conf. 7 488–94

[15] Amani N, Wynneersch H, Johannsen U, Smolders A B, Ivashina M V. and Maaskant R 2019 Towards a Generic Model for MU-MIMO Analysis Including Mutual Coupling and Multipath Effects 13th Eur. Conf. Antennas Propagation, EuCAP 2019 1–4

[16] Abdulameer L F, Jignesh J D, Sripati U and Kulkarni M 2014 BER performance enhancement for secure wireless optical communication systems based on chaotic MIMO techniques Nonlinear Dyn. 75