Effect of thermal wall condition on the dissimilarity of momentum and heat transfer in pulsating channel flow

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Abstract
Pulsating turbulent channel flows under constant temperature difference (CTD) condition and uniform heat flux heating (UHF) condition are studied by direct numerical simulation (DNS). The main objective of the present study is to clarify how the dissimilarity between momentum transfer and heat transfer appeared in the pulsating flows, in which the dissimilarity may originate from the CTD condition dissimilar to the no-slip condition of the velocity field rather than from the thermo-fluid physics under pulsation. Simulations have been performed for three pulsation frequencies under the friction Reynolds number at steady-state, $Re_{\tau} = 300$. Comparing the phase-averaged quantities under CTD and UHF conditions, it is found that the frequency dependence of the temperature oscillations in the near-wall region is almost the same regardless of the thermal boundary condition although the time-averaged temperature profiles are different. As a result, the ratio of Stanton number to friction factor, which works as a barometer of the analogy, changes during the pulsation period at both CTD and UHF conditions. Besides, the oscillation amplitude becomes larger as the pulsation frequency increases. Therefore, it was confirmed that the dissimilarity appears regardless of the thermal boundary condition. In addition, turbulent Prandtl number shows similar cyclic behavior to the ratio of Stanton number to friction factor. Time variations of each component constituting turbulent Prandtl number reveal that increasing dissimilarity at the high frequency is mainly attributed to the amplified oscillation of velocity gradient near the wall, where Reynolds shear stress and turbulent heat flux are kept at around the time-averaged values because the near-wall vortex structures cannot follow the rapid change of flow rate.

Keywords : Turbulent heat transfer, Channel flow, Boundary condition, Pulsating flow, Dissimilarity, Direct numerical simulation

1. Introduction

Turbulent transport phenomena under the flow subjected to imposed oscillations play a key role in a lot of natural and engineering situations, such as the bloodstream in large arteries and reciprocating IC engines. Since many of them are characterized by momentum and heat transfer in unsteady turbulent flows, which show significantly complex behaviors due to the cyclic acceleration and deceleration, it is essential to understand the effect of flow oscillation on the momentum/heat transfer. The oscillating flow is classified into two kinds of flow, one is pulsating flow (non-zero mean flow) and the other is a reciprocating flow (zero mean flow) depending on the presence or absence of mean flow. In this study, we discuss momentum transfer and heat transfer in pulsating flows. The pulsating flows with heat transfer are governed by the bulk Reynold number (mean flow), Prandtl number, pulsation frequency, and pulsation amplitude. From an engineering perspective, it is required to estimate accurately the wall friction and the wall heat transfer in a wide range of these control parameters.

Pulsating turbulent flows have been widely investigated by experiments and numerical simulations. About the flow field, many studies have been conducted to understand the fluid dynamics of unsteady turbulent flows. When the ratio
of the oscillation amplitude to the bulk mean velocity $a_{\text{ab}}$ is smaller than unity, which is so-called current-dominated flow, Tardu et al. (1993) shows that the flow is controlled by the pulsation frequency. In current-dominated flows, the frequency dependence of the wall shear stress is described well (Mao and Hanratty, 1986; Tardu and Binder, 1993; Tardu, et al., 1994). However, their studies remain qualitative and have not been evaluated quantitatively. In addition to the experiments, numerical simulations using DNS (Kawamura, et al., 1999; Weng, et al., 2016) and LES (Scotti and Piomelli, 2001) have also provided important knowledge of pulsating flows. With consideration for the variety of the control parameters and the interaction between the oscillation and mean flow, many problems remain unsolved, so that, the modeling of pulsating turbulent flows is still difficult and the dynamics of the modulated boundary layer due to flow pulsation are still unclear.

On the other hand, pulsating flows with heat transfer have been investigated in terms of heat transfer enhancement. Until now, heat transfer enhancement or suppression has been reported under the various control parameter conditions (Ishino, et al., 1996; Habib, et al., 1999, 2002; Wang and Zhang, 2005; Elshafei, et al., 2008; Li, et al., 2012; Fukuchi, et al., 2015). However, these results could not provide general understandings of the effects of pulsating flow on heat transfer phenomena. Shiibara, et al. (2017) pointed out that these numerous studies contain conflicting findings of the pulsation effect on the enhancement or suppression of heat transfer. They also insisted that the measurement of heat transfer distribution and its fluctuation is needed to clarify the heat transfer phenomena of pulsating flows. They measured heat transfer fluctuation subjected to a sudden acceleration and deceleration of flow using high-speed infrared thermography with a heated thin foil and showed that the heat transfer is greatly affected by the turbulence structure in pulsating flow.

In our earlier study, we conducted the direct numerical simulation (DNS) of pulsating turbulent channel flow at constant wall temperature difference (CTD) condition to clarify the effect of flow pulsation on momentum transfer and heat transfer (Yamazaki, et al., 2018, 2020). Various instantaneous, time-averaged and phase-averaged quantities were obtained to discuss the correlation among the velocity field, thermal field, and spatiotemporal vortex structures, which are generally difficult to be captured in experimental studies. As a result, we reported that the dissimilarity between momentum and heat transfer, which was evaluated based on the deviation from the Reynolds analogy, was found to appear in the pulsating turbulent flow. That is, in the pulsating flow, the phase-averaged ratio of Stanton number to friction factor changes periodically and the resultant time-averaged ratio over the pulsation period becomes larger than that of steady (non-pulsating) turbulent flow. As for the dissimilarity, many researchers have investigated it to achieve energy-efficient heat transfer by controlling the flow field, based on various control strategies (e.g. Hasegawa and Kasagi, 2011; Uchino, et al., 2017). In this context, our result suggests that the flow pulsation is one of the ways to achieve a dissimilar control of momentum transfer and heat transfer. In the CTD condition, however, the boundary conditions of velocity and temperature fields are not similar. Therefore, the dissimilar boundary condition may become a source of dissimilarity between momentum and heat transfer in our previous study. As for the effect of thermal boundary conditions on the dissimilarity, Suzuki, et al. (1988) investigated a turbulent boundary layer being disturbed by a cylinder and reported that dissimilar transport appears in both similar and dissimilar boundary conditions. Like this, a comparison of the pulsating flows in the similar and dissimilar boundary conditions is important to study the difference of thermal field caused by the boundary condition and to clarify the dynamics leading to the dissimilarity. Therefore, in the present study, we conducted DNS in pulsating turbulent channel flows under uniform heat flux heating (UHF) condition in addition to CTD condition, to investigate the effect of thermal boundary conditions on the dissimilar transport of momentum and heat in the pulsating flows.

2. Numerical method
2.1 Governing equation and numerical condition

In this study, direct numerical simulation (DNS) was performed in the fully developed channel flow driven by a spatially uniform pressure gradient. The dimension of the computational domain shown in Fig. 1 is defined as $6.4 \delta$, $2 \delta$, $3.2 \delta$, along x-, y-, z-axis, using channel half-width $\delta$. The governing equations for an incompressible fluid are following continuity and Navier-Stokes equations for the flow field.

$$\frac{\partial u_i}{\partial x_i} = 0$$  \hspace{1cm} (1)
Here, the variables shown in superscript (*) are non-dimensionalized by channel half-width $\delta$ and friction velocity at steady-state $u_\tau$, and those in superscript (+) are non-dimensionalized by density $\rho$, kinematic viscosity $\nu$, and $u_\tau$. The first term of the right-hand side in Eq. (2) is a spatially uniform pressure gradient as the driving force of flow pulsation defined as follows.

$$-\frac{\partial p^+}{\partial x} = 1.0 + A \cdot \cos(\omega^+ \tau)$$  

where $A$ is an amplitude of pressure gradient and $\omega^+$ is the non-dimensional pulsation frequency defined as $\omega^+ = \omega \sqrt{\nu / u_\tau^2}$. The boundary conditions for the flow field are periodic conditions for uniform directions ($x, z$) and no-slip condition on the solid walls. To study the effect of pulsating frequency, we tested $\omega^+ = 0.0044, 0.017, 0.035$ to cover a wide range of pulsating flow regimes that is called as “low frequency”, “intermediate frequency” and “high frequency” regimes by Scotti & Piomelli (2001) and Weng, et al. (2016). These pulsation conditions are shown in Table 1, where $A$ in Eq. (3) is chosen to keep the ratio of oscillation amplitude to bulk mean velocity $a_{ob} \approx 0.19$ among different frequencies.

For thermal fields in CTD and UHF conditions, we introduce the following two types of energy equations (Seki, et al., 2004). Note that the temperature field is non-dimensionalized using friction temperature at steady-state $\theta_\tau (= q_w / \rho C_p u_\tau)$, where $q_w$ and $C_p$ are the wall heat flux and specific heat at constant pressure, respectively.

$$\frac{\partial \theta^+}{\partial \tau} + u_j^+ \frac{\partial \theta^+}{\partial x_j} = \frac{1}{Re_\tau \cdot Pr} \frac{\partial^2 \theta^+}{\partial x_j^2} - \frac{\Delta T^+}{2} u_j^+$$  
in CTD condition

$$\frac{\partial \theta^+}{\partial \tau} + u_j^+ \frac{\partial \theta^+}{\partial x_j} = \frac{1}{Re_\tau \cdot Pr} \frac{\partial^2 \theta^+}{\partial x_j^2} + \frac{u_j^+}{u_b^+}$$  
in UHF condition

For energy equation, Eqs. (4) and (5) are used for CTD and UHF conditions, respectively. Where, $\Delta T^+$ is the non-dimensional temperature difference between upper wall and lower wall in CTD condition, and $u_b$ is the bulk mean velocity. Here, the non-dimensional temperature is defined as follows in each condition.

$$T^+(x^+, y^+, z^+) = \frac{\Delta T^+}{2} y^* + \theta^+(x^+, y^*, z^*)$$  
in CTD condition

Table 1  Pulsation frequency and pressure gradient amplitude used in DNS ($Re_\tau = 300$).

| Pulsation frequency $\omega^+$ | Pressure gradient amplitude $A$ |
|-------------------------------|-------------------------------|
| 0.0044                        | 4.13                          |
| 0.017                         | 16.5                          |
| 0.035                         | 33.0                          |

Fig. 1 Configuration of DNS and coordinate system.
fluctuation components from time-averaged quantities and phase-averaged quantities are defined as follows.

\[ T^*(x^*, y^*, z^*) = \frac{dT^*_b}{dx} x^* - \Theta^*(x^*, y^*, z^*) \]  

in UHF condition

Where \( T^*_b \) is the bulk mean temperature. The thermal wall boundary conditions in both CTD and UHF conditions are given as follows.

\[ \Theta^*(x^*, 0, z^*) = \Theta^*(x^*, 2, z^*) = 0 \]  

Note that in this study, the temperature fluctuation is assumed to be zero on the both walls at UHF condition.

The above governing equations are solved with a finite difference method on a collocated grid system. Spatial derivatives were basically approximated by a 2nd-order central difference. For the convective term of momentum equations, a skew-symmetric form of 2nd-order central difference was applied to assure conservation of kinetic energy. For the time integration and pressure-velocity coupling, a fractional step method was used with the 3rd-order Adams-Bashforth method, except for the thermal diffusion term to which the Crank-Nicolson scheme was applied.

The Reynolds number effect in the pulsating flows appears in two terms of Eq. (2). One is the viscous diffusion term that is proportional to the inverse of \( Re_\omega \). The other is a driving force term, \( -dP^*/dx^* \), where \( Re_\omega \) is implicitly contained in the form of nondimensional frequency parameter, \( \omega^* = \omega \tau_s / u_{\tau_s}^2 = (\omega \delta^2/\nu)/Re_\omega^2 = Wo^2 / Re_\omega^2 \), where \( Wo \) is Womersley number. Since \( \omega^* \) depends on \( Re_\omega \), it is difficult to discuss the Reynolds number effect alone in pulsating flows. Despite the complexity, as mentioned in introduction, \( \omega^* \) is known to be a dominant parameter in current-dominated pulsating flows, which is defined as the flow at \( a_{ub} = A_{ub} / \tau_s < 1 \) where \( A_{ub} \) is the amplitude of bulk mean velocity. Since all flow conditions of this study are current-dominated (\( a_{ub} \approx 0.19 \)), Reynolds number alone is expected to have minor effects on pulsating flow behavior. This indicates that important flow characteristics of current-dominated pulsating flows under different pulsation frequencies can be examined by DNS at a moderate Reynolds number. Thus, considering the availability of public DNS databases for validation, we have set \( Re_\omega = 300 \) that can keep the pulsating flows in a fully turbulent regime under the pulsation amplitude given by \( a_{ub} \approx 0.19 \).

The Prandtl number \( Pr \) is set to 0.713 for comparison to our previous study at CTD condition (Yamazaki, et al., 2018, 2020). The numbers of grids are \((N_x, N_y, N_z) = (215, 169, 215)\). Spatial resolution is evaluated at \( \Delta x^+ = 9.0, 0.4 < \Delta y^+ < 5.9, \Delta z^+ = 4.5 \), using the friction velocity at steady-state. Since the friction velocity varies during flow pulsation, we have checked the cyclic variation of it and then confirmed that the peak value at highest frequency remains less than twice the steady state. Since the current grid resolution gives \( y^+ = 0.4 \) at a first grid point from the wall at \( Re_\omega = 300 \), our grid gives enough resolution even at the maximum wall shear stress at highest frequency. The time step is set to \( \Delta t^+ = 1.46 \times 10^4 \) in order to keep the Courant number less than 0.35. It is worth noting that current DNS code was validated in Yamazaki, et al. (2020), where time-averaged profiles of streamwise velocity and Reynolds shear stress show good agreements with the DNS databases at \( Re_\omega = 150 \) and 300 by Iwamoto, et al. (2002).

### 2.2 Data processing

To evaluate quantities in fully developed pulsating channel flows, we introduce two types of averaging techniques, one is the time-average and the other is the phase-average. For flow and thermal fields that are statistically homogeneous in the streamwise and the spanwise directions, the time-average and phase-average can be defined as follows.

\[ \bar{f}(y) = \frac{1}{t_{total} L_z L_y} \int_{t_{total}} \int_{L_y} \int_{L_z} f(x, y, z, t) dx dz dt \]  

\[ \langle f \rangle(y,t) = \frac{1}{N L_y L_z} \sum_{n=1}^{N} \int_{L_z} \int_{L_y} f(x, y, z, t + \frac{2\pi n}{\omega^*}) dx dz \]  

Where \( t_{total} \) is the total time of integration, and \( N \) is the total number of periods over which the quantity has integrated. In this study, the flow and thermal fields are integrated from \( t^* = 20 \) to remove the effect of the initial transient period, and we confirmed that the phase averaged quantities did not vary sufficiently by setting \( N \) to 100 cycles or more. The fluctuation components from time-averaged quantities and phase-averaged quantities are defined as follows.

\[ f'(x, y, z, t) = f(x, y, z, t) - \bar{f}(y), \quad f^*(x, y, z, t) = f(x, y, z, t) - \langle f \rangle(y,t) \]  

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The phase-averaged quantities can be divided into a time-average part, an oscillation part at forcing frequency, and a part including the contribution of other frequency components (Scotti and Piomelli, 2001).

\[
\langle f \rangle (y, t) = \overline{f}(y) + A_f(y)\cos(\omega_f^* t^* + \phi_f(y)) + \varepsilon_f(y, t)
\]  

(12)

Where \(A_f\) is the amplitude of the oscillation part, \(\phi_f\) is the phase lag, and \(\varepsilon_f\) is the contribution of other frequency components. In this study, \(A_f\) and \(\phi_f\) are determined by the least-squares method to minimize the \(L^2\) norm of \(\varepsilon_f\).

It should be noted that streamwise velocity and other quantities in the pulsating flow field are not a pure sine function, however, it is known that they mostly show a peak value in their frequency spectra at the frequency of the imposed oscillation in previous experiment (Mao and Hanratty, 1986) and simulation (Weng, et al., 2016) of pulsating flows.

3. Results and Discussion

At first, the cyclic variations of the phase-averaged bulk Reynolds number \(<Re_b>\) (= \(<u_b(2\delta)/\nu)>\) are shown in Fig. 2 as a reference. Note that the time variation of \(<Re_b>\) shows a phase delay relative to the pressure gradient and it takes the maximum value and the minimum value at 90 degrees and 180 degrees, respectively. Figure 3 shows the time-averaged temperature profile at CTD condition (\(T/\Delta T\)) and UHF condition (\(T/T_c\)), where \(T_c\) represents the temperature at the channel center. Major difference in the two temperature profiles consists in the zero or non-zero temperature gradient at the channel center. Also, similarity of the UHF boundary condition to the no-slip condition is confirmed from the temperature profile similar to the streamwise velocity profile. At both conditions, the time-averaged temperature is unchanged from steady-state regardless of the pulsation frequency. This indicates that the pulsation does not affect the time-averaged temperature field.

About the oscillation field, on the other hand, Fig. 4 shows the oscillation amplitude of temperature \(A\omega^*\) obtained by the least-squares method (Eq. (12)) under both conditions. From these figures, the oscillation amplitude of temperature can be divided into two regions, the near-wall region (\(y^+ \leq 20\)), and the outer layer (\(y^+ \geq 20\)). The profiles in both regions at both conditions show the same frequency dependence, in which the oscillation amplitude decreases as the frequency increases. Especially, the oscillation amplitudes in the near-wall region for both conditions are significantly similar. However, the difference between CTD and UHF conditions is observed from the outer layer to the channel center at \(\omega^* = 0.0044\). The difference in the outer layer is due to the temperature gradient at the channel center in CTD condition, and the reason will be described later.
In addition, Fig. 5 shows the phase lag of temperature oscillation $\phi$ relative to the pressure gradient. From this figure, it is found that the temperature from the conductive sublayer to buffer layer ($y^+ \leq 10$) has a constant phase lag although it changes with different amplitudes as shown in Fig. 4. It can be seen that the phase lag starts to change dramatically from $y^+ \approx 20$, of which location agrees with the valley of the amplitude profiles. These facts indicate that the temperature oscillation due to the flow pulsation forms a node at the valley point, like a node in standing waves, owing to the propagation of the temperature oscillation from the wall toward the channel center. From the wall to the node point, the temperature oscillation is generated like a standing wave, which varies in the same phase with different amplitudes. This behavior is consistent in all cases, but the oscillation amplitude becomes smaller as the pulsation frequency increases. In the region from the node to the channel center, on the other hand, the temperature oscillation starts to propagate with phase lag. As seen in Fig. 5, the phase lag starts to increase from the node to channel center. This suggests the propagation speed is decelerated in this region. As the deceleration of propagation speed increases with the frequency, the amplitude of temperature oscillation decays from the node to channel center.

Since the oscillation part of temperature variations is not necessarily a pure sine function with the forcing frequency, in Fig. 6, we evaluate the contribution of other frequencies using the ratio of $||\omega||$ to the oscillation.
amplitude $A_\theta$, where the $L^2$ norm of error $||\mathcal{E}_f||$ is defined as follows:

$$\|\mathcal{E}_f\| = \left(\frac{1}{t_p} \int_0^{t_p} e(t)^2 dt\right)^{1/2} \tag{13}$$

Here, $t_p$ is the pulsation period. This figure shows that the contribution of other frequency components is much smaller than that of the forcing frequency except at $y^+ \approx 20$ for $\omega^+ = 0.0044$ and $y^+ \leq 100$ for $\omega^+ = 0.035$, where the amplitude of oscillation part is very small and the approximation to a pure sine function becomes difficult.

As mentioned above, the frequency dependence of temperature profiles is almost similar in the near-wall region regardless of the thermal boundary condition although the time-averaged profiles are different. Next, Fig. 7 shows the time variation of Nusselt number $\langle Nu \rangle$ for CTD condition ($\langle Nu \rangle = 2Re_s/\theta^+ \cdot \partial <T^+>/\partial y^+|_{wall}$) and UHF condition ($\langle Nu \rangle = 2Re_s/\theta^+ \cdot \partial <T^+>/\partial y^+|_{wall}$). For both conditions, $\langle Nu \rangle$ changes during one cycle and the behaviors are similar while the absolute value of the Nusselt number varies depending on the thermal wall condition. Nusselt number in both conditions shows an apparent phase delay as the frequency increases, and the phase angles at peak values

Fig. 6 Ratio of $L^2$ norm of error to oscillation amplitude in temperature oscillation obtained by the least-squares method, in (a) CTD and (b) UHF conditions. The contribution of other frequency components is sufficiently small except at $y^+ \approx 20$ for $\omega^+ = 0.0044$ and $y^+ \leq 100$ for $\omega^+ = 0.035$.

Fig. 7 Time variations of phase-averaged Nusselt number $\langle Nu \rangle$ in (a) CTD (reported by Yamazaki, et al. (2018, 2020)) and (b) UHF conditions. Color symbol: phase-averaged value, Color solid line: time-averaged value. The time variations of Nusselt number at both boundary conditions are similar even though the absolute values are different.
mostly agree with each other, which are consistent with the phase lags of the temperature oscillations in the near-wall region. And also, the oscillation amplitude of $<\text{Nu}>$ decreases as the frequency increases. This is because the oscillation amplitude of temperature in the near-wall region decreases in the same manner as shown in Fig. 4. This indicates that the temperature profile and its oscillation amplitude ($A\theta$), which show dissimilar profiles at the channel center between CTD and UHF conditions, have a minor effect on the time variation of wall heat transfer, so that the time variations of $<\text{Nu}>$ have no clear difference between CTD and UHF conditions.

Based on the results of the Nusselt number in Fig. 7, the phase-averaged ratio of Stanton number ($<\text{St}> = <\text{Nu}>/(<\text{Re}_b> \cdot \text{Pr})$) to friction factor ($<\text{C}_f> = 2\tau_w/\rho <u_b>^2$), $<\text{St}>/<\text{C}_f>/2$ is shown in Fig. 8 for (a) CTD and (b) UHF conditions, respectively. Here, $\tau_w$ is the wall shear stress defined by the molecular viscosity and velocity gradient at the wall. Note that $<\text{St}>/<\text{C}_f>/2$ should remain constant regardless of the bulk Reynolds number if analogy between momentum transfer and heat transfer holds at each phase. Therefore, the ratio works as a barometer of the analogy. In CTD condition reported by Yamazaki, et al. (2018, 2020), $<\text{St}>/<\text{C}_f>/2$ changes depending on the phase due to the flow pulsation. And, the oscillation amplitude increases as the frequency increases. This phenomenon was referred to as “time-local dissimilarity” between momentum and heat transfer by the authors. As for the time-averaged value over one cycle of pulsation, it agrees with that of a steady case (flow without pulsation) at $\omega^* = 0.0044$ but surprisingly those at $\omega^* = 0.017, 0.035$ are higher than that of a steady case. This means the dissimilarity appears not only in each phase but also in time-average. As shown in Fig. 8 (b), the result obtained in UHF condition shows the same behavior as that in CTD condition, that is, $<\text{St}>/<\text{C}_f>/2$ changes depending on the phase, and the oscillation amplitude becomes larger as the frequency increases. In addition, the time-averaged value also tends to be higher than that of a steady case as the frequency increases. Table 2 summarizes the time-averaged friction factor ($<\text{C}_f> = 2\tau_w/(\rho <u_b>^2)$), Stanton number ($<\text{St}> = \text{Nu}/(\text{Re}_b \cdot \text{Pr})$), and the ratio ($<\text{St}>/<\text{C}_f>/2$). This table shows that $<\text{St}>/<\text{C}_f>/2$ also becomes higher than that of a

| $\omega^*$ | Friction factor $<\text{C}_f>$ | Stanton number $<\text{St}>$ | $<\text{St}>/<\text{C}_f>/2$ |
|-----------|------------------|-------------------|------------------|
| 0.0       | 0.00698          | 0.00255           | 0.730            |
| 0.0044    | 0.00697          | 0.00256           | 0.736            |
| 0.017     | 0.00679          | 0.00254           | 0.747            |
| 0.035     | 0.00660          | 0.00254           | 0.770            |

Table 2  Time-averaged values of friction factor, Stanton number, and the ratio.
steady case as $\omega^+$ increases in both thermal boundary conditions. Namely, the dissimilarity between momentum transfer and heat transfer does not originate from the dissimilarity of boundary condition but from a phenomenon specific to pulsating flow.

To discuss the reason why the dissimilarity appears regardless of the thermal wall condition, hereafter, temperature variance and the time variation over a period of pulsation will be examined in detail.

Figure 9 shows the time-averaged profiles of the temperature variance $\overline{\theta^2} = \overline{\theta^+ \theta^+}$ in (a) CTD and (b) UHF conditions. For comparison, the turbulent kinetic energy at a steady case (flow without pulsation) is shown together. In both thermal boundary conditions, the time-averaged temperature variance of pulsating flow coincides with that of a steady case at all frequencies. The profile of temperature variance in UHF condition is similar to the profile of the turbulent kinetic energy but it in CTD condition is different. However, the profile of temperature variance in both conditions is similar in the near-wall region that is responsible for wall heat transfer.

![Fig. 9 Profiles of the time-averaged temperature variance in (a) CTD and (b) UHF conditions. Symbol: pulsating flow conditions, Red solid line: steady case (without pulsation), Blue dashed line: turbulent kinetic energy at a steady case for comparison. In both thermal boundary conditions, the time-averaged temperature variance of pulsating flow coincides with that of a steady case at all frequencies. The profile of temperature variance in UHF condition is similar to the profile of the turbulent kinetic energy but it in CTD condition is different. However, the profile of temperature variance in both conditions is similar in the near-wall region that is responsible for wall heat transfer.](image)

![Fig. 10 Oscillation amplitude of the temperature variance in (a) CTD and (b) UHF conditions. As $\omega^+$ increases from 0.017 to 0.035, the oscillation amplitude of the temperature variance gradually decreases in both thermal boundary conditions, and their profiles are almost the same. At $\omega^+ = 0.0044$, on the other hand, the oscillation amplitudes in both conditions are different from the logarithmic layer to the channel center ($y^+ \gtrsim 20$).](image)
fields, taking a maximum value at the channel center (out of range of the vertical axis). However, $\theta_k$ in both conditions shows similar profile in the near-wall region that is responsible for wall heat transfer. Since the discussion here holds at all frequencies, it accounts for a reason why both thermal wall conditions give similar results on wall heat transfer at all frequencies.

Figure 10 shows the amplitude of oscillation part ($A_{k,0}$ of Eq. (12)) of the temperature variance. As $\omega^+$ increases from 0.017 to 0.035, the oscillation amplitude of the temperature variance $A_{k,0}$ gradually decreases in both thermal boundary conditions, and their profiles are almost the same. At the low frequency of $\omega^+=0.0044$, on the other hand, the oscillation amplitudes of CTD and UHF conditions show greatly different behavior from the logarithmic layer to the channel center ($y^+\approx 20$). However, their profiles are similar in the near-wall region, and these two distinct regions can be divided at $y^+=20$, of which location is the same as the valley of temperature amplitude (Fig. 4). To discuss the reason why the difference at $y^+\approx 20$ is observed depending on the thermal boundary condition, we will refer to the effect of the mean field on the fluctuation field in temperature. Here, the phase-averaged production term of the temperature variance $\langle k\theta^+\rangle$ in the fully developed pulsating channel flow is expressed as follows.

$$\langle P_{k,0}^+ \rangle = \langle u^+\theta^+ \rangle \frac{\partial\langle T^+ \rangle}{\partial y^+}$$  \hspace{1cm} (14)

Here, the double prime means the deviations from the phase-averaged value shown in Eq. (11). From Eq. (14), the reason why the oscillation amplitude is different in the outer layer at $\omega^+=0.0044$ is that the temperature gradient at the channel center is zero in UHF condition and therefore the production of $\langle k\theta^+ \rangle$ is also zero, while the production of $\langle k\theta^+ \rangle$ exists in CTD condition because of the non-zero temperature gradient at the channel center. This leads to that $E_{k,0}^+$ of CTD condition becomes larger at $y^+>70$ toward the channel center while $E_{k,0}^+$ of UHF condition is decayed toward the channel center. In addition, the difference in the outer layer is also seen between $\omega^+=0.0044$ and $\omega^+=0.017, 0.035$ in CTD condition. This result indicates the effect of pulsation on thermal fields is limited to the near-wall region at higher frequencies as Scotti and Piomelli (2001) reported for flow fields. This is confirmed from Fig. 9 showing the large amplitude of temperature in the outer layer only at $\omega^+=0.0044$ while showing the small amplitude at $\omega^+=0.017, 0.035$ that becomes even smaller than those in the near-wall region.

To see the contribution of other frequency components in the cyclic time variation of $\langle k\theta^+ \rangle$, FFT (Fast Fourier Transform) is further applied to it, and the amplitude of the first to fifth mode is shown in Fig. 11 at (a) $\omega^+=0.0044$ and (b) $\omega^+=0.035$.
and (b) $\omega^+ = 0.035$, where the $\omega^+$ expresses the fundamental mode (first mode) that is identical to the forcing frequency. In the near-wall region ($y^+ = 5.39$), the frequency of the imposed oscillation is dominant regardless of the thermal boundary condition. The oscillation amplitude becomes larger as the pulsation frequency increases. The peak value of $<Pr>$ is seen in the phase from 180 to 225 degrees, which agrees with the phase that gives the peak value of the ratio of Stanton number to friction factor in all frequency cases.

Fig. 12 Time variations of phase-averaged turbulent Prandtl number in (a) CTD and (b) UHF conditions. Symbol: phase-averaged value, Line: time-averaged value. The time-variation of phase-averaged turbulent Prandtl number is mainly seen in the near-wall region and their cyclic behaviors are almost the same regardless of the thermal boundary condition. The oscillation amplitude becomes larger as the pulsation frequency increases. The peak value of $<Pr>$ is seen in the phase from 180 to 225 degrees, which agrees with the phase that gives the peak value of the ratio of Stanton number to friction factor in all frequency cases.
The effect of forcing frequency on the cyclic behavior of $<Pr_t>$ is also similar in both conditions. At $\omega^+ = 0.0044$, $<Pr_t>$ basically agrees with the time-averaged value except for the near-wall region that shows a little deviation from the time-averaged value in some phases (e.g., 0.0 and 315 degrees). As the frequency increases, however, the oscillation amplitude of $<Pr_t>$ becomes larger in the near-wall region, and the deviation from the averaged value is observed at most phases. The peak value is seen in the phase from 180 to 225 degrees in all cases. The phase angle giving the maximum $<Pr_t>$ agrees with the phase that gives the peak value of the ratio of Stanton number to friction factor in Fig. 8. Since the effect of thermal wall condition on the $<Pr_t>$ profile is insignificant, it is confirmed that the appearance of the dissimilarity is independent from the thermal wall conditions in the pulsating turbulent flow and heat transfer.

Furthermore, the mechanism yielding the dissimilarity is considered through an investigation of each component in
Eq. (15). The profiles of the second moments, Reynolds shear stress and turbulent heat flux, are shown in Figs. 13 and 14, respectively, at $\omega^* = 0.0044$ and 0.035. Note that turbulent heat flux is shown only for UHF condition because CTD condition gives similar cyclic behavior. At $\omega^* = 0.0044$, Reynolds shear stress and turbulent heat flux show large amplitude in the cyclic change. This is because, in the low frequency, the quasi-steady state of turbulent field is realized at each phase, as reported by Yamazaki, et al. (2020). Consequently, the second moment profiles at each phase are basically determined by the instant flow rate, while showing a phase lag of up to 45 degrees. At $\omega^* = 0.035$, on the other hand, the amplitude becomes smaller, and the near-wall profiles are scaled well by the steady-state friction velocity and temperature. This means that the turbulent eddies producing velocity and temperature fluctuations do not show large change during one cycle since it cannot follow the rapid change of flow rate; i.e. the turbulent eddies seem to be “frozen” as also confirmed later by the vortex structures visualized by Q-criterion. As a result, the turbulent fluctuations under the high frequency pulsation are maintained at around the time-averaged value during one cycle. Note that, although the second moments show complex cyclic behaviors depending on the pulsation frequency, time-averaged values coincide with those of a steady flow case. Next, Fig. 15 shows the amplitudes of velocity and temperature gradients evaluated by Eq. (12). The amplitude of velocity gradient near the wall becomes larger as $\omega^*$ increases because the driving force becomes larger to keep the amplitude of $Re_b$ at the same level. On the other hand, the amplitude of temperature gradient becomes smaller in both thermal wall conditions as $\omega^*$ increases, because of the reduced amplitude of turbulent heat flux as shown in Fig. 14 (b). That is, dissimilar response to the frequency occurs between the velocity and temperature gradients. In light of Eq. (15) closely related to the time-local dissimilarity, the velocity gradient plays a dominant role to increase the dissimilarity at the high frequency, where the second moments and temperature gradient do not change much since the eddy structure is holding the state around time-averaged $Re_b$.

In summary, at the low frequency, the turbulent Prandtl number does not change in time because the quasi-steady state is realized at each phase of pulsation although each component of the turbulent Prandtl number, Eq. (15), changes in time. At the high frequency, on the other hand, the velocity gradient near the wall greatly changes in time while other components do not show large change. Thus, it is concluded that the time variation of turbulent Prandtl number near the wall at the high frequency is mainly caused by the time variation of velocity gradient under “frozen” turbulent eddies.

The relationship between turbulent eddy motion and heat transfer can be seen in Fig. 16, showing the vortex structures visualized by the iso-surface of Q-criterion and the distribution of Nusselt number at (a, b, c, d) $\omega^* = 0.0044$ and (e, f, g, h) $\omega^* = 0.035$ in UHF condition. Here, vortex structures are visualized only in the lower half region with the same value of Q-criterion; this also applies to Fig. 17. The snapshots correspond to the phases where $\langle Nu \rangle$ shows approximately (b, f) the maximum value, (d, h) minimum value and (a, c, e, g) time-averaged value in Fig. 7, respectively. Note that $\langle Nu \rangle$ and $\langle Re_b \rangle$ are shown in each caption. At $\omega^* = 0.0044$, vortex structures and distribution of $Nu$ show a large cyclic change during flow pulsation; high $Nu$ region in red color changes with the vortex structures
showing a transition from dense to coarse distributions repeatedly. At $\omega^* = 0.035$, on the other hand, both $Nu$ and vortex structures do not change substantially during one cycle, as if the vortex structures were “frozen” to the state at around time-averaged $Re_b$. Figure 17 shows the vortex structures and the distribution of wall friction $\tau^*_w$ at $\omega^* = 0.0044$ and 0.035 at the same moment as in Fig. 16. Note that $<\tau^*_w>$ and $<Re_b>$ are shown in each caption. Similar to $Nu$, the high $\tau^*_w$ region shown by red color changes during one cycle together with the vortex structure at $\omega^* = \ldots$
Fig. 17 Iso-surface of second invariant of velocity gradient tensor (Q-criterion) colored by white and wall friction $\tau_w^+$ distribution on the bottom wall ($y = 0$), at (a, b, c, d) $\omega^+ = 0.0044$ and (e, f, g, h) $\omega^+ = 0.035$ at the same moment as in Fig. 16. Similar to $Nu$, the high $\tau_w^+$ region shown by red color changes during one cycle together with the vortex structure at $\omega^+ = 0.0044$. At $\omega^+ = 0.035$, on the other hand, $\tau_w^+$ changes greatly due to the cyclic change of velocity gradient with a large amplitude, while keeping the shape of the streaky pattern during one cycle. Since the near-wall vortex structure is responsible for the streaky pattern, the unchanged streaky pattern is consistent with the fact that the vortex structures do not change substantially during one cycle. As a whole, the Nusselt number is mainly determined by the eddy structure at the moment, while the friction factor is much affected by the change of the velocity gradient near the wall under the varying flow rate at the high frequency. Consequently, it can be said that the time-local dissimilarity observed in Fig. 8
is caused by the interaction between the change of flow rate and the vortex structures, which affect the wall friction and wall heat transfer in a different manner depending on the frequency regimes.

As for the dissimilarity observed in time-average, the Fukagata-Iwamoto-Kasagi (FIK) identity (Fukagata, et al., 2002) and its extension to heat transfer (Kasagi, et al., 2012) show the time-averaged wall friction and Nusselt number are determined by the laminar component and the profiles of Reynolds shear stress and turbulent heat flux, the latter of which is weighted or unweighted depending on the thermal boundary condition. Thus, it is worth checking the profiles of the second moment to explore the mechanism of dissimilar transport in pulsating turbulent flows in time-average. However, as shown in Figs. 13 and 14, the time-averaged profiles of Reynolds shear stress and turbulent heat flux are almost identical to those of a steady case (flow without pulsation) for all pulsation frequencies. This means that the second moment profiles cannot explain the dissimilarity in time-average. This supports the conclusion that time-local dissimilar transport at high frequency pulsation is mainly caused by the change of velocity gradient adjacent to the wall. Further investigation is needed to clarify the mechanism yielding the dissimilar transport in time-average in pulsating turbulent flows.

4. Conclusions

Pulsating turbulent channel flows at constant temperature difference (CTD) and uniform heat flux heating (UHF) conditions were studied by direct numerical simulation for three pulsating frequencies, \( \omega' = 0.0044, 0.017, 0.035 \), in order to investigate whether the dissimilarity between momentum and heat transfer appears or not under the similar boundary condition between velocity and temperature field. The phase-averaged quantities under both thermal boundary conditions were compared. It is found that the temperature oscillation in the near-wall region, \( y^+ \lesssim 20 \), is almost the same regardless of the thermal boundary condition. Since the time variation of the Nusselt number depends on the temperature oscillation in the near-wall region, this result means the dissimilarity between momentum transfer and heat transfer appears in UHF condition as well as CTD condition. Namely, the dissimilarity in CTD condition is not a phenomenon caused by the dissimilar boundary condition between the velocity field and temperature field but a phenomenon specific to the pulsating flow. Besides, the oscillation of the temperature variance also shows the same cyclic behavior at both thermal boundary conditions in the near-wall region.

To seek the mechanism of the dissimilar transport, cyclic variations of turbulent Prandtl number were examined. It varies from the mean value in the near-wall region, and the oscillation amplitude becomes larger as the pulsation frequency increases. The phase yielding the peak turbulent Prandtl number is almost the same as the phase showing the peak time-local dissimilarity, where the ratio of Stanton number to friction factor takes a maximum, in both thermal wall conditions. Time variations of each component constituting turbulent Prandtl number clarify that the cyclic variation of velocity gradient is strongly amplified at the high frequency, where Reynolds shear stress, turbulent heat flux, and temperature gradient do not change much because the near-wall eddy structure cannot follow the rapid change of flow rate. Therefore, it is concluded that increasing dissimilarity is mainly attributed to the velocity gradient oscillation under the “frozen” eddy structure at the high frequency.

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