Modeling the competing effects of the immune system and EMT on tumor development

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Abstract

Preceding and during cancer progression, the immune system is engaged in complex sets of interactions with epithelia that affect tumor incidence and prognosis. Inflammation also has significant effects on gastrointestinal and pancreatic tumors. Moreover, epithelial cell plasticity can significantly alter disease trajectories (for better or worse) via epithelial-to-mesenchymal transition (EMT). Several of the same pathways that regulate EMT are involved in tumor-immune interactions, yet little is known about the mechanisms and consequences of these regulatory processes. Here we introduce a multiscale evolutionary model to describe the interplay between tumor-immune-EMT interactions and their impact on epithelial tissues. Through in silico analysis of large patient cohorts, we found controllable regimes that maximize cancer-free survival. Delaying tumorigenesis depended crucially on properties of the mesenchymal phenotype: its growth rate and its immune-evasiveness. We performed analysis of pancreatic cancer data from The Cancer Genome Atlas to compare clinical measurements with model predictions. We found, in broad agreement with the model, that association with EMT worsened the survival probabilities of inflammation-associated cancer. These results offer novel means to delay disease onset by regulating properties of EMT, and demonstrate the importance of studying cancer-immune interactions in light of EMT.

1 Introduction

Cancer and the immune system interact in myriad ways. Beyond direct tumor-immune cell interactions, the immune system modulates the tumor microenvironment (TME): responding to
local inflammatory signals, targeting mutated cells, and eradicating tumor cells to potentially restore homeostasis in a local context [1]. Recent breakthroughs in the field of immunotherapy are beginning to produce therapies that have had a significant impact on patient health and survival [2, 3].

Tumor cells interact with the immune system by initiating a sequence of events that results in the activation of the adaptive immune response. This is accomplished by the presentation of antigens recognized by innate immune cells that are transported to lymph nodes where T cells (and other components) can be activated [4]. The tumor also engages in cellular processes that indirectly shape the TME, for example by releasing transforming growth factor beta (TGF-β), thus shifting the TME towards a tumor-supportive environment by enhancing immunosuppression via activation of T regulatory cells (Tregs) [4].

The effects of the immune system on a tumor can be broadly summarized into two modules. The cytotoxic branch of the immune system, such as natural killer cells (NKs) and cytotoxic T cells (CTLs), seek out and lyse tumor cells. Upon carrying out their effector functions, these cytotoxic cells lose efficacy or deactivate entirely [5]. The regulatory branch of the immune system (Tregs and other factors), inhibits the effective functioning of the cytotoxic branch [6]. In addition, the immune system plays important regulatory roles in pre-cancerous tissue, where inflammation can increase the probability of tumorigenesis (and/or decrease the time to cancer), particularly in tumors originating in gastrointestinal and pancreatic tissues [7, 8]. Recent work raises some questions about the relationship between inflammation and cancer, suggesting that under certain conditions inflammation may not be oncogenic but rather onco-protective [9].

Epithelial-to-mesenchymal transition (EMT) describes a reversible process by which cells with an epithelial phenotype transition into cells with a mesenchymal phenotype. While epithelial cells are in part defined by their adhesion to neighbors, mesenchymal cells show much greater ranges of motility, and may display stem-like properties [10], although controversy regarding 'stemness' and EMT remains [11, 12]. Recent work has shown that – rather than being a binary process – at least two stable intermediate states exist on the EMT axis [13, 14]. Ongoing investigations into the plasticity and stability of EMT overlap with discussions elsewhere, e.g. of discrete vs. continuous processes during cell differentiation [15]. Intermediate states have emerged as a central mechanism by which cell fates (and the noise inherent within them) can be controlled [16–18].

While EMT has for some time been assumed to impact metastasis, more recently EMT-related phenotypes have been linked to other aspects of both tumor initiation and progression [10]. TGF-β is a master regulator of EMT [19], linking it to the signaling networks involved in tumor-mediated immune responses, since Tregs release TGF-β upon arriving at the tumor site [20]. Thus, even by considering only a single signaling factor, we find these three components (the tumor, the immune system, and EMT) to be linked. It therefore strikes us as a priority to develop models to understand how interactions between each of these components affect cancer incidence and progression.

Two features of the mesenchymal phenotype are of particular relevance in the context of cancer and the immune system: i) mesenchymal cells are less susceptible to immune clearance [20]; and ii) mesenchymal cells proliferate on average at a slower rate than epithelial cells. The immune evasiveness of mesenchymal cells is well-established: as a cell is targeted by cytotoxic immune cells for clearance, a physical connection between the two cells must be established before the lysing event. This immunological synapse is dependent upon the surface proteins,
Modeling the competing effects of the immune system and EMT on tumor development

such as the T-cell receptors of the major histocompatibility complex, of the target cell, and in
the case of mesenchymal cells there is preliminary evidence that these surface markers are
down-regulated in such a way that the immune synapse is more difficult to form [20]. Below
we refer to this phenotype as mesenchymal immune evasion (MIE).

The other facet of mesenchymal cells is their slower proliferation rate, and this is tied
to the observation that mesenchymal cells have been speculate to exhibit a more “stemness”
phenotype. While many cell behaviors are associated with stemness, here we target the specific
reduction in proliferation rate [21] (which, under certain conditions can be called quiescence,
though we avoid this term as it can be controversial, and is not a necessary assumption to
the model developed below). Cancer cells are of course characterized by some degree of
unregulated proliferation, thus it is important to explore the effects of differential proliferation
rates within the cancer cell population. In our model, we refer to the reduced proliferative
capacity associated with a mesenchymal cell phenotype as mesenchymal growth arrest (MGA).

Mathematical oncology, that is, mathematical models of cancer incidence, progression,
and treatment, has become a well-developed field; many models have offered insight into the
cellular interactions underlying cancer and its interplay with the immune system, including
classic [22–24] and more recent works [25–38]. These studies have increased our understanding
of how tumors grow in the presence of various immune components, and how treatment regimes
can be designed to maximize the efficacy of cytotoxicity while minimizing risks to the patient.
However, to our knowledge no models have addressed how the effects of EMT alter interactions
between the immune system and cancer, and the subsequent implications for treatment.

Here we develop a model with the goal of studying interactions between the tumor, the
immune system and EMT. We seek to describe a set of crucial molecular and cellular interactions
in epithelial tissue cells, including effects due to DNA damage and mutation, to investigate
the probability that cancer will occur and, if so, when. A recent model of cancer-immune
interactions [9] described the effects of the TME on the risk of cancer, and we build on the
core cell cycle component of this model, adding significant new interactions to the immune
component of the model (which was previously modeled by a single interaction), as well
as adding the effects of EMT. We seek to understand whether this more complex immune
module will change our understanding of inflammatory effects on the tumor, and how the
epithelial-mesenchymal axis influences these.

In the next section we develop the model and give explanation of the intuition behind
each component. We then go on to analyze its behavior. Global sensitivity analysis identifies
parameters that are crucial for tumorigenesis. We then study these in more depth, focusing
on the competing effects of EMT and of the immune system on tumorigenesis: we find that
EMT intricately regulates tumorigenesis and that under certain regimes a careful balance of
EMT- and immune-driven processes can significantly prolong cancer-free survival. To test these
predictions, we analyze data from the Cancer Genome Atlas (TCGA), and find evidence for the
synergistic effects predicted by the model for patients with pancreatic cancer.

2 Methods

We develop an agent-based model to describe the relationships between cancer, the immune
system, and EMT, building on cell-cycle and tissue-cell components of [9]. The agents in the
model are the tissue cells with tumorigenic potential. These can either be mutation-free or, resulting from DNA damage during the cell cycle, can have any combination of three possible pathway mutations (shown in Fig. 1A). If the pathway is mutated, the component is colored red and is marked with ‘X’. A single mutation is sufficient for the cell to be considered as a mutant cell.

We model immune cells as continuous variables, appropriate since here it is not necessary to keep track of single cell effects. The cytokine TGF-β is continuous within the TME, i.e. the system is well-mixed. Tissue cells can take on either epithelial or mesenchymal phenotypes in a plastic manner: these phenotypes depend on both the TME and cell-intrinsic factors. While the score is continuous, a threshold determines if a given cell is labeled as epithelial or mesenchymal. In Fig. 1A, the EMT state is given by the color of the cell. Even though each cell does have an EMT score on a spectrum of values, only the discrete state influences its interactions in the model.

2.1 Tissue Cells

The tissue cells at all times have associated with them two important quantities: which combination of mutations they carry and an EMT score. The model has three pathways that have the potential to mutate over the course of a simulation. There is a proliferation pathway that when mutated increases the probability of the cell proliferating. There is an apoptosis pathway that when mutated decreases the probability of the cell undergoing apoptosis. There is an immune evasion pathway that when mutated decreases the probability that an immune cell can effectively clear the mutated cell. The EMT score will be exposited in Section 2.3.

2.2 The Immune System

The immune system is modeled as having three active immune cell types: NKs, CTLs, and Tregs. The NKs and CTLs act on the system by recognizing mutated cells and clearing them. Upon successful clearance, they themselves are then deactivated and removed from the immune population. Tregs act on the system by suppressing the effector functions of NKs and CTLs, making it less likely they can clear mutated cells. In addition to this, Tregs release TGF-β which further shapes the TME by pushing tissue cells towards a mesenchymal phenotype.

2.3 EMT

Each tissue cell has an EMT score between 0 and 1. When the score is above a fixed threshold value, the cell is labeled as mesenchymal. Otherwise, it is below the threshold and labeled as epithelial. For the purposes of the model, a cell in an epithelial state is considered in the base state, and one in a mesenchymal state will have some of its parameters updated. In particular, mesenchymal cells benefit from increased immune evasion but suffer from decreased proliferation. Note, that because the immune system can only clear mutated cells, the increased immune evasion only benefits mutated mesenchymal cells.
2.4 Model Simulation

2.4.1 Initial conditions

Every model run starts with a fixed amount of cells, \( N_0 \). However, different parameter values will lead to different steady states for the population of tissue cells. In light of this, a warmup period of 1000 cell cycles is used in which time no mutations are allowed to happen. Thus, the only immune cells present are NKs. After the warmup cycles are complete, a mutagenic event is simulated in which all the tissue cells experience a random increase in their probability to mutate. From then on, a cell which proliferates also either mutates or increases its probability of mutating later on.

2.4.2 Tissue cell fate

During each cell cycle, every cell is stochastically assigned a fate from the following options:

- proliferation
- apoptosis
- immune clearance (by NKs or CTLs)
- rest in \( G_0 \)

For each cell, a weight is chosen for each option and these are normalized to determine what each cell does during the cell cycle.

**Proliferation** There are four factors that contribute to the weight of a cell to proliferate. The first is a base proliferation rate that all cells have, \( p \). Second, if the cell has a mutation in the proliferation pathway \( (\delta_P = 1) \), then the weight for proliferation is proportionally increased by \( \Delta_P \). Third, if the cell is mesenchymal \( (\zeta = 1) \), then the weight for proliferation is proportionally decreased by \( \Delta_{MGA} \), which stands for mesenchymal growth arrest. Fourth, there is a negative feedback of the cells on their own proliferation which is quantified by a Hill factor as a function of the tissue cell population, \( N_C \), with EC50 term \( K_0 \). In total, the weight for proliferation is given by

\[
\rho_P = p(1 + \delta_P \Delta_P)(1 - \zeta \Delta_{MGA}) \frac{K_0}{K_0 + N_C}.
\]

(2.1)

**Apoptosis** There are two factors that contribute to a cell’s weight for undergoing apoptosis. There is a basal apoptosis rate that all cells experience, \( d_C \) for death. Second, if the cell has a mutation in the apoptosis pathway \( (\delta_A = 1) \), then the weight for undergoing apoptosis is proportionally decreased by \( \Delta_A \). In total, the weight for apoptosis is given by

\[
\rho_A = d_C(1 - \delta_A \Delta_A).
\]

(2.2)

**Immune Clearance** For both NK clearance and CTL clearance, the weights are built with the same factors but have different parameter values for NK and CTLs. First of all, the cell needs to be mutated \( (\delta_{MUT} = 1) \). Second, there is a Hill factor that captures the probability of
Modeling the competing effects of the immune system and EMT on tumor development

an immune cell finding and interacting with the given tissue cell with EC50 term \( K_1 \). Third, NKs and CTLs have their own efficacy parameters, \( E_{NK} \) and \( E_{CTL} \), which can be understood as the rate of immune clearance given an immune cell has found the mutated cell. Fourth, there is a decreasing Hill factor based on the number of Tregs present with EC50 term \( K_2 \). Finally, there are two factors that proportionally decrease the weight of immune clearance depending on if the cell has an immune evasion mutation (\( \delta_{IE} = 1 \)) or if it is mesenchymal (\( \zeta = 1 \)) with respective decreases \( \Delta_{IE} \) and \( \Delta_{MIE} \). In total, the weight of NK clearance is given by

\[
\rho_{NK} = \delta_{MUT} \frac{N_{NK}}{N_C/K_1 + N_{NK}/1 + N_{Treg}/K_2} (1 - \delta_{IE}\Delta_{IE})(1 - \zeta\Delta_{MIE})
\] (2.3)

A similar formula holds for CTLs with only the number of CTLs and their efficacy being different from the above equation.

**Rest in G_0** The weight associated with quiescence is taken as 1 except in the case of mesenchymal cells. For mesenchymal cells, the decrease in the probability of proliferation leads to an increase in the probability of quiescence. Hence, the overall weight for quiescence is given by

\[
\rho_R = 1 + \zeta p(1 + \delta_P\Delta_P)\Delta_{MGA} \frac{K_0}{K_0 + N_C}
\] (2.4)

The reason for adding that term is due to the understanding that overall mesenchymal cells proliferate less as individual cells rest longer in the \( G_0 \) phase.

### 2.4.3 Completing the Cell Cycle

After the cell fates are determined and the results reflected in the system, there are a few things that happen before the system moves on to a new cell cycle. First, the NK and CTL populations are reduced by the number of mutated cells they cleared. This represents the fact that individual immune cells lose efficacy as they carry out their effector functions. Second, all proliferating cells have a cell-specific probability of undergoing a pathway mutation. If they do, one is randomly chosen among the three pathways and the pathway in that cell mutates. If the cell does not undergo a mutation, then its probability of mutation during subsequent cell cycles increases. Finally, the EMT values for each cell are updated. This depends on the cells current EMT score and how much TGF-\( \beta \) is currently in the system. Each cell receives an amount of TGF-\( \beta \) given by

\[
\tau_i = \frac{\tau_{max}}{N_C + 1 + K_3/\tau} + X_i, \quad X_i \sim N(0, \sigma^2).
\] (2.5)

The first term divides up the total TGF-\( \beta \) among all \( N \) cells evenly and a Hill function on this quantity determines how much TGF-\( \beta \) each cell thus receives. Then, randomness is introduced to the dynamics through the i.i.d. variables \( X_i \), where \( \sigma \) is a parameter in the model. If this quantity is large enough relative to the EMT score of the cell, the cell will undergo EMT. If not, then the cell undergoes the reverse process, MET, and its EMT score will decrease accordingly. Each cell then is relabeled as either epithelial or mesenchymal depending on its new EMT score and whether it is below or above the mesenchymal threshold. Thus, there are two main factors...
that determine if a cell will end a cell cycle as mesenchymal: concentration of TGF-β in the system and the current EMT score of the cell.

The amount of TGF-β in the next cell cycle is determined by the number of mutated cells, $N_{\text{MUT}}$, and the number of Tregs, $N_{\text{Treg}}$, each one producing a fixed amount of TGF-β. It is given by

$$\tau = \tau_{\text{MUT}} N_{\text{MUT}} + \tau_{\text{Treg}} N_{\text{Treg}}.$$  \hspace{1cm} (2.6)

Finally, the immune populations are updated. The NKs obey the following differential equation:

$$N'_{\text{NK}} = \sigma_{\text{NK}} - d_{\text{NK}} N_{\text{NK}}$$  \hspace{1cm} (2.7)

which is solved and then discretized over the cell cycle ($k$) to give

$$N_{\text{NK}}(k+1) = \left( N_{\text{NK}}(k) - \frac{\sigma_{\text{NK}}}{d_{\text{NK}}} \right) \exp(-d_{\text{NK}} \Delta t) + \frac{\sigma_{\text{NK}}}{d_{\text{NK}}}.$$  \hspace{1cm} (2.8)

For CTLs and Tregs, they rely on mutant cells being cleared before they can be activated. Let $N_{\text{MUT}}^*(k)$ represent the number of mutant cells cleared by the immune system during cell cycle $k$. In addition, Treg recruitment is upregulated by TGF-β, which will be incorporated via a Hill function with EC50 term $K_4$. The following differential equations govern the CTL and Treg populations:

$$N'_{\text{CTL}} = \sigma_{\text{CTL}} N_{\text{MUT}}^* - d_{\text{CTL}} N_{\text{CTL}}$$
$$T_{\text{reg}}' = \sigma_{\text{Treg}} N_{\text{MUT}}^* \frac{\tau}{1 + \tau/K_4} - d_{\text{Treg}} N_{\text{Treg}}$$  \hspace{1cm} (2.9)

Again, solving and discretizing over the cell cycle, we have:

$$N_{\text{CTL}}(k+1) = \left( N_{\text{CTL}}(k) - \frac{\sigma_{\text{CTL}} N_{\text{MUT}}^*(k)}{d_{\text{CTL}}} \right) \exp(-d_{\text{CTL}} \Delta t) + \frac{\sigma_{\text{CTL}} N_{\text{MUT}}^*(k)}{d_{\text{CTL}}}$$
$$N_{\text{Treg}}(k+1) = \left( N_{\text{Treg}}(k) - \frac{\sigma_{\text{Treg}} N_{\text{MUT}}^*(k) \tau(k)}{d_{\text{Treg}}} \frac{1}{1 + \tau(k)/K_4} \right) \exp(-d_{\text{Treg}} \Delta t) + \frac{\sigma_{\text{Treg}} N_{\text{MUT}}^*(k) \tau(k)}{d_{\text{Treg}}} \frac{1}{1 + \tau(k)/K_4}.$$  \hspace{1cm} (2.10)
Modeling the competing effects of the immune system and EMT on tumor development

| Name     | Description                                      |
|----------|--------------------------------------------------|
| $p$      | proliferation rate of tissue cells               |
| $d_C$    | death rate of tissue cells                       |
| $\Delta_{\text{MIE}}$ | mesenchymal immune evasion                      |
| $\Delta_{\text{MGA}}$ | mesenchymal growth arrest                      |
| $\Delta_A$ | mutant cells decreased apoptosis               |
| $\Delta_{\text{IE}}$ | mutant cells increased immune evasion          |
| $\Delta_P$ | mutant cells increased proliferation          |
| $K_0$    | EC50 term for negative feedback of tissue cells on own proliferation |
| $K_1$    | EC50 term for probability of NK cell finding mutant cell |
| $K_2$    | EC50 term for Treg inhibition of cytotoxic functions |
| $K_3$    | EC50 term for how much TGF-$\beta$ each cell has |
| $E_{\text{NK}}$ | rate of NKs clearing mutants                |
| $E_{\text{CTL}}$ | rate of CTLs clearing mutants           |
| $\sigma_{\text{NK}}$ | NK source rate                                 |
| $\sigma_{\text{CTL}}$ | CTL source rate per cleared mutant cell       |
| $\sigma_{\text{Treg}}$ | Treg source rate per cleared mutant cell  |
| $k_{\text{EMT}}$ | EMT/MET rate                                 |
| $\sigma$ | standard deviation of noise in TGF-$\beta$ each cell receives |
| $\tau_{\text{max}}$ | max amount of TGF-$\beta$ any cell can receive |
| $\tau_{\text{MUT}}$ | rate of TGF-$\beta$ production by mutant cells |
| $\tau_{\text{Treg}}$ | rate of TGF-$\beta$ production by Treg        |

Table 1: The model parameter names and descriptions. Note that many of these values are affected by the inflammation state of the system.

2.4.4 Mutation and tumorigenesis

At the end of each cell cycle, the proportion of tissue cells that are mutated is calculated, and if it is above a certain threshold, a tumorigenic event is recorded and the patient is determined to have cancer. The Time to Cancer is given as the time from the start of the simulation, not including warmup, until tumorigenesis. Simulations run until either a tumorigenic event is recorded or until the maximum number of cycles has been reached.

2.5 Parameter Sensitivity Analysis

To study parameter sensitivity, we implemented the Morris method, a one-step-at-a-time global sensitivity analysis algorithm. The algorithm runs the model numerous times by picking several base points and then varying one parameter at a time from each base point and tracking the difference [39,40]. Each model run involved 1000 patients and the value returned was the area under the survival curve. We ran the algorithm by sampling at 30 points in parameter space and then changing each parameter one time at each point. In the implementation by Sohier et al., using at least 10 points is recommended [40].
To determine initial points in parameter space to sample at, we set prior distributions over each of the parameters. Where possible, these were informed by literature, elsewhere, we made informed approximations and chose relatively uninformative priors. For example, many previous studies have established the parameters of immune population dynamics such as recruitment and death rates [41]. We scaled the model (to reduce the computational burden) to contain on the order of $10^2$ tissue cells, whereas a tumor would have on the order of $10^9$ cells [41]: we thus scaled the relevant parameters for immune populations accordingly. We assigned the priors to be truncated Normal distributions, centered at these estimates. Some of the parameters were also truncated to the right due to physical constraints. The variance for each was set to be twice the mean.

The Morris algorithm then computes the average of the absolute change in the output ($\mu^*$), which in the case of this model is the area under the survival curve.

### 2.6 Analysis of patient survival data from TCGA database

We obtain primary tumor bulk mRNA sequencing and censored survival data for all individuals monitored in the relevant project from the Cancer Genome Atlas (TCGA) from the Genomic Data Commons portal in R, using the package TCGABiolinks. Given $n > 1$ gene set keywords (e.g. "inflammatory" and "emt"), the symbolic names of all msigdb gene sets are searched for matches to these keywords, and matched gene sets are grouped by keyword. For each element in the product $S$ of all keyword groups, the following analysis is performed.

1. The first principle component of the expression across the gene set is obtained for each element of the n-tuple of gene sets.
2. Two clusters of n-dimensional patient vectors are obtained by k-means clustering.
3. The patients are separated by their cluster identity and Kaplan-Meier curves are fit to their corresponding survival data.
4. Under the null hypothesis that there is no difference between the survival of the two groups, a log-rank test is performed.
5. The log-rank test p-value for each element in $S$ is placed in an ordered list, the lowest of which defines the element of $S$ whose composite gene sets are most predictive of patient survival in the given TCGA project or tumor type.

Below we analyze data gathered for pancreatic cancer, denoted by “PAAD” in TCGA.

### 3 Results

#### 3.1 A multiscale agent-based model of EMT-immune-tissue cell interactions to study tumorigenesis

We begin by investigating general features of the model to establish baseline conditions and to assess the impact of various model components on the key measured outcomes: the probability of cancer, and the Time to Cancer. Within the cell cycle, cell fate is determined via a set of rules that are influenced by EMT and immune interactions (Fig. 1A). For example, if a cell undergoes EMT, the probability that it will proliferate is reduced; if a cell gains a mutation in the apoptosis pathway, the probability that it undergoes apoptosis is greatly reduced. The
Modeling the competing effects of the immune system and EMT on tumor development

Figure 1: A. Schematic depiction of agent-based model components; blue/red denote tissue cells with/without mutations. Black arrows depict regulation of the cell fate in each cell cycle. Inset depicts major interactions between the immune system and tissue cells. B. A representative simulation of one patient. The model parameter values used can be found in Supplementary Table 2. The inflammation cycling scheme is represented above the patient dynamics. The vertical dashed line denotes the end of the warmup period. Mut: mutated cells; Mes: mesenchymal cells. C. Survival curve for one cohort of patients with the parameter values given in Supplementary Table 2.
Modeling the competing effects of the immune system and EMT on tumor development

Different means by which the immune system acts on tissue cells are also shown (1A Inset). NKs and CTLs attempt to clear mutated tissue cells, and deactivate upon successfully carrying out their cytotoxic function. Tregs inhibit cytotoxic activity. Tregs also release TGF-β which promotes the recruitment of Tregs, as well as increasing the rate of EMT.

The inflammation cycling scheme for a typical in silico patient consists of alternating high and low regimes (Fig. 1B); the inflammation schemes modeled will be discussed in detail in the next section. For this patient, after the warmup period, cell mutations are observed at a rate low enough that they are cleared by cytotoxic cells before being able to establish a tumor. However, after approximately 1000 cell cycles (750 days), one mutated cell exhibits clonal growth, and a tumor is established. At this point, large numbers of cells from the adaptive immune system (CTLs and Tregs) are being heavily recruited, and a peak in TGF-β expression is observed. The number of mesenchymal cells increases initially with increasing TGF-β, but decreases after the clone takes over the tissue. This shift back towards an epithelial tissue phenotype occurs even though the expression of TGF-β remains high. After 841 cell cycles, the proportion of mutated cells reaches 50% of the total tissue cells: this is modeled as the threshold defining tumorigenesis. Thus this patient has a Time to Cancer of 841 cell cycles, or 631 days. After cancer incidence, the number of mutated cells continues to grow rapidly and soon makes up 100% of the cell population. A peak in mesenchymal phenotypes is observed following cancer incidence, before most transition back to an epithelial state.

Considering the immune system dynamics, we see that the NK population is approximately constant, while the adaptive populations (CTLs and Tregs) grow quickly and dwarf NKs in number following the accumulation of oncogenic mutations. The adaptive immune populations also appear to exhibit oscillatory behavior, however note that this is not due to intrinsic dynamics but rather due to the inflammation scheme that the patient is undergoing: alternating between 30 cell cycles of high inflammation and 60 cell cycles of low inflammation. Within each of these periods, adaptive immune populations increase or decrease rapidly in accordance with the inflammation state.

The in silico patient described here is given for the purpose of illustrating features of the model. Given the multiple sources of noise in the model, in order to quantify patient dynamics and cancer-free survival rates, below we will simulate large cohorts of patients. In Fig. 1C we simulate the survival curve for a cohort of 500 patients: we see that all the patients survive for approximately 100 cell cycles (75 days). Subsequently, in the approximate range of \( T = [100, 300] \), the cancer onset rate is roughly constant, and after 600 cell cycles, no patients remain cancer free.

### 3.2 Identifying regulatory parameters via Morris global sensitivity analysis

Exploring the parameter spaces of models in systems biology is – in general – a hard problem. Performing Bayesian parameter inference to inform parameter values is advisable wherever possible [42]. Here, a lack of detailed measurements (data for tumor growth dynamics abound, but simultaneous data on the immune dynamics are lacking) preclude inference of the full model. In addition, while inference schemes for agent-based models are developing [43, 44], simulation times remain a hurdle [45]. Parameters for aspects of this model that were studied previously can be constrained [9], however even for these, the new additions to the model could push it into new behavioral regimes. Thus to adequately sample the parameter space of
Modeling the competing effects of the immune system and EMT on tumor development

To assess the sensitivity of the model parameters and thus identify those that are most important in determining cancer-free survival times, we performed Morris one-step-at-a-time (OAT) sensitivity analysis (see Methods). The results of Morris OAT on the 31 model parameters are shown in Figure 2. We see that a subset of parameters demonstrate much higher levels of sensitivity than others. The two most influential according to this analysis are the death rate and the recruitment rate of Tregs, this is most likely due to the dual roles Tregs play in both suppressing the cytotoxic effects of other immune cells and secreting TGF-β, which drives EMT. This ties Tregs to all three components of the model. Since we seek to separate the effects of different model components, we do not choose the parameters influencing Tregs for detailed analysis below.

Many parameters in the model change depending on the inflammation state. For the sake of a naming convention, any parameter that includes “low” in the name represents the parameter value when inflammation is low (baseline). When the inflammation is high, these “low” parameters get scaled by the parameters that include “up” in their names. The ILD and IHD parameters determine the duration of low and high inflammation, respectively. A cell with mutated pathways will be affected by the ΔA, ΔP, and ΔIE parameters, depending on whether the apoptosis, proliferation, or immune evasion pathways are mutated. The mesenchymal cells are influenced by the MGA and MIE parameters. Finally, σ, k_{EMT}, τ_{max}, and K_3 all control how cells transition between the epithelial and mesenchymal states.

Since one goal of our analysis is to assess the specific effects of EMT on immune-cancer
dynamics, the parameters MIE and MGA are of particular interest. In addition, inflammation parameters dictating the cycling scheme are of interest because they are both highly influential on Time to Cancer and capable of being targeted by therapeutic treatments. In terms of Tregs, their secretion of TGF-β is important and we will also study it further.

3.3 Mesenchymal phenotypic properties dramatically change cancer-free survival times

When a cell transitions from an epithelial to a mesenchymal state, two phenotypic cell characteristics change: mesenchymal immune evasion (MIE) and mesenchymal growth arrest (MGA). Both parameters are defined as proportions, and lie in \([0, 1]\), where higher values indicate more mesenchymal-like properties. The parameter \(\Delta_{\text{MIE}}\) is the proportional reduction in the probability that a mutated mesenchymal cell will be subject to immune clearance in a given cycle. The parameter \(\Delta_{\text{MGA}}\) is the proportional reduction in the probability that a mesenchymal cell will proliferate in a given cell cycle, thus is equivalent (strictly in terms of its impact on the cell cycle) to an increased proportion of time spent resting in the \(G_0\) phase. The cytokine TGF-β is also involved in the EMT process (by increasing the probability of EMT). All three of these parameters were found to have large effects on the model by the Morris OAT analysis presented in Section 3.2.

As MIE is increases, the Time to Cancer decreases (Fig. 3A) under all sets of parameters studied: as this subpopulation of cells becomes more resistant to immune clearance, the potential for tumor resilience increases and the tumor can grow faster as a result.

As MGA increases, the Time to Cancer increases, i.e. lower proliferation rates for mesenchymal cells slow cancer growth (Fig 3B). This is not immediately intuitive, since the decreased proliferation rates affect both mutated and non-mutated tissue cells.

TGF-β can be regulated in two ways: its production by mesenchymal cells and its production by Tregs. In Fig 3C, the results of varying Treg TGF-β production are shown, indicating that an increased Treg TGF-β production leads to a shorter Time to Cancer. The two main ways in which TGF-β influences the system is in recruitment of Tregs and in pushing tissue cells to a mesenchymal phenotype. Tregs are modeled as tumor-protective and thus increasing their number will naturally decrease Time to Cancer. Mesenchymal cells are more likely to evade the immune system, so pushing the system towards an overall more mesenchymal phenotype will better protect the cancer and decrease the Time to Cancer.

3.4 A key EMT regime maximizes cancer-free survival time under chronic inflammation

We explored the effects of varying the inflammation state of the patient on cancer-free survival to investigate competing interactions within the TME and their effect on EMT. Patient cohorts were simulated under different inflammation regimes: permanently low inflammation; permanently high inflammation; or variable inflammation. For patients drawn from cohorts in a permanently high inflammatory state, the relationship between mesenchymal parameters and the Time to Cancer is monotonic, i.e. increasing either MIE (Fig. 4A-B) or MGA (Fig. 4C-D) decreases the Time to Cancer. However, under regimes with either temporary or permanent periods of low inflammation, different relationships emerge: a local maximum for the Time to Cancer is found
Figure 3: Effects of mesenchymal cell properties on the time to cancer. Trajectories of one patient per cohort from warmup period ([-1000,0]) to 2000 cell cycles, for $\Delta_{\text{MIE}}$ (A); $\Delta_{\text{MGA}}$ (B); and $\tau_{\text{Treg}}$ (C). D. Survival curves corresponding to changes in MIE (A) for a patient cohort of size 1000; shaded region represents the 95% confidence interval for the evaluated function. E. Survival curves for patient cohort corresponding to changes in MGA (B). F. Survival curves for patient cohort corresponding to changes in Treg TGF-$\beta$ (C).
Figure 4: Effects of inflammation on the time to cancer under different cycling schemes. A-B. As MIE varies, survival curves (each of 200 patients) and corresponding bar plots to summarize the mean Time to Cancer for each cohort are shown. C-D. As MGA varies, survival curves and corresponding bar plots to summarize the mean time to cancer for each cohort are shown.
Modeling the competing effects of the immune system and EMT on tumor development

Figure 5: Summary of the contrasting effects of MIE and MGA on the Time to Cancer.

These striking differences in the mean Time to Cancer – extended by up to one year by controlling mesenchymal growth arrest – have clear therapeutic implications. Our model predicts that reducing the inflammatory state of the TME in three weeks out of every nine would best prolong cancer-free survival in combination with therapies targeting mesenchymal cells. The control of mesenchymal cell proliferation could become an effective anti-cancer strategy.

In contrast, when MIE is varied for different inflammation cycling schemes, regardless of the cycling scheme, increasing the MIE leads to decreases in the Time to Cancer (worse survival probabilities), although in the case where the inflammatory state is permanently high, MIE has little effect on the Time to Cancer. Thus under any inflammation regime with periods of low inflammation, any reductions in the rate of mesenchymal immune evasion will lead to improved patient outcomes.

3.5 Analysis of TCGA data against model predictions reveals that mesenchymal phenotypes reduce cancer-free survival probability

To further assess the effects on tumorigenesis of the mesenchymal properties of immune evasion and growth arrest, we study the joint density plot for each of these parameters against the Time to Cancer (Fig. 5). We found that over the full range of values of $\Delta_{\text{MGA}}$ considered, increasing $\Delta_{\text{MIE}}$ decreases the Time to Cancer. However, for any given value of mesenchymal immune evasion, there is a value of mesenchymal growth arrest that maximizes the Time to Cancer. Moreover, this optimal value increases with increasing MIE. Together these results show that while reducing the ability of a mutated cell to evade the immune system by any amount will improve the probability of cancer-free survival, an optimal value of MGA will prolong survival.

To compare these model predictions with experimental studies, we analyzed data from The Cancer Genome Atlas (TCGA) database to study the effects of immune and EMT interactions on prognosis of cancers for which inflammation is known to play an important role, such as colonic or pancreatic cancers [7,8]. Using gene ontologies as a measure of the effects of inflammatory or EMT processes on cancer-free survival, we investigate pancreatic cancer. We note that the
Modeling the competing effects of the immune system and EMT on tumor development

![Figure 6](image.png)

**Figure 6:** A. K-means clustering of pancreatic cancers using gene ontology terms indicative of an EMT signature ($k = 2$). B. Survival plots corresponding to the clustering on EMT. C. K-means clustering of pancreatic cancers using gene ontology terms indicative of EMT and Inflammation signatures ($k = 2$). D. Survival plots corresponding to the clustering on EMT and inflammation.

4 Discussion

Despite the intense interest in cancer and the immune system, and in the effects of EMT on cancer, there has not previously, to the best of our knowledge, been a model developed that
Modeling the competing effects of the immune system and EMT on tumor development

combines these components: cancer, the immune system, and EMT. We see this as an even more pressing need given the shared factors that influence all of these, such as TGF-β. We used an individual cell-based model framework to describe the multiscale processes that can lead to cancer: DNA damage occurs during the cell cycle and this can lead to mutations in pathways that affect the fitness of the cell, which in turn affects the cell population dynamics, which are also influenced by the intrinsic state of the cell (EMT), and extrinsic immune factors (inflammation, cytotoxic cells, etc).

We found that this model recapitulated cancer-free survival dynamics, and via global parameter sensitivity analysis, we identified parameters exerting key control over model behavior. Focusing on these led us to identify that: increasing mesenchymal immune evasion, decreasing mesenchymal growth arrest, or increasing Treg TGF-β production all lead to shorter Times to Cancer. However varying the level of inflammation leads to striking effects: under regimes with periods of low inflammation, an optimal level of mesenchymal growth arrest can improve outcomes and maximize the Time to Cancer.

To test these predictions, we performed unsupervised analysis of pancreatic cancer data from The Cancer Genome Atlas, and looked at survival across two groups: we found that the presence of an EMT signature increased differences in survival between groups in combination with inflammation. We chose to summarize in silico patient simulations with a single parameter: the Time to Cancer, which we found captured the essential characteristics of the model. There are, of course, many tissue trajectories that all result in cancer, and it is our opinion that analysis of the transient cell dynamics in pre-cancerous tissues is a pressing need to shed insight into cellular biomarkers of cancer.

These results represent promising steps in understanding the competing roles of the immune system and EMT during development of epithelial cancers, yet much remains to be done. Further development of the inflammation module of this model is important given the large and sometimes paradoxical roles that the inflammatory state exerts on the epithelia and cancer-free survival (Figs. 2 and 4B, D). Currently, inflammation is modeled as independently cycling between high and low schemes, however many of the agents considered in the model actively contribute to the inflammatory state, thus it would be interesting to consider connecting these components (for example by letting the level of inflammation be conditional on the number of mutant cells in the tissue). Another layer of complexity is revealed by the natural anti-inflammatory role of Tregs. One consequence of the current model is that decreasing the number of Tregs increases the cancer-free survival. Clearly, there exists a trade-off to be accounted for, and adding to the model the main effector function of Tregs could remedy this and add depth to our understanding of the various roles that Tregs play in and around the tumor.

The roles that TGF-β plays throughout the tumor and its microenvironment also warrant further investigation. We found that, below a certain threshold, reduction of TGF-β increases the Time to Cancer (Fig. 3E), thus reducing expression levels of TGF-β in the tumor microenvironment benefits survival. Intriguingly, recent experimental work demonstrated that TGF-β drives tumor suppression in pancreatic cancer by promoting EMT [46]. However, TGF-β is a master regulator implicated in numerous cellular signaling processes, and changing the concentrations of TGF-β even in a local tumor microenvironment could have large off-target effects. Indeed, it has been showns that TGF-β promotes invasion and heterogeneity (although suppresses cell proliferation) in squamous cell carcinoma [47]. Future work thus ought to consider the
effects of targeting signaling factors downstream of TGF-β that still have the ability to modulate epithelial cell dynamics. Towards this end, we are currently developing a larger TGF-β signaling pathway module with appropriate crosstalks to epithelial/mesenchymal/immune cell functions to be incorporated into the model.

A further goal for future work is to explore (and exploit) the heterogeneity of tumor evolution in greater depth: this heterogeneity aids the evasion of the tumor from immune effects. Studying the consequences of decanalization [48] during cancer progression is too-often sidelined, despite evidence supporting its prominence [49–51]. Yet despite these challenges, for which the complexity of the disease may be often in part responsible, great progress has been and continues to be made. As we approach a new generation of immunotherapies, it is these very complexities that we must better understand in order to control or eradicate the disease.

Software availability

The code used to simulate the model, written in MATLAB (Naticks, MA), and R scripts to analyze TCGA data are available at: https://github.com/drbergman/tumor-immune-emt-code.

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