Scaling Dark Energy in a Five-Dimensional Bouncing Cosmological Model

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We consider a 5-dimensional Ricci flat bouncing cosmological model in which the 4-dimensional induced matter contains two components at late times - the cold dark matter (CDM)+baryons and dark energy. We find that the arbitrary function \( f(z) \) contained in the solution plays a similar role as the potential \( V(\phi) \) in quintessence and phantom dark energy models. To resolve the coincidence problem, it is generally believed that there is a scaling stage in the evolution of the universe. We analyze the condition for this stage and show that a hyperbolic form of the function \( f(z) \) can work well in this property. We find that during the scaling stage (before \( z \approx 2 \)), the dark energy behaves like (but not identical to) a cold dark matter with an adiabatic sound speed \( c_s^2 \approx 0 \) and \( p_x \approx 0 \). After \( z \approx 2 \), the pressure of dark energy becomes negative. The transition from deceleration to acceleration happens at \( z_T \approx 0.8 \) which, as well as other predictions of the 5D model, agree with current observations.

Keywords: scaling solution, dark energy, big bounce

1. Introduction

In recent decades, the observations of high redshift Type Ia supernovae have revealed that our universe is undergoing an accelerated expansion rather than decelerated expansion \(^1\), \(^2\), \(^3\). Meanwhile, the discovery of Cosmic Microwave Background (CMB) anisotropy on degree scales together with the galaxy redshift surveys indicate \( \Omega_{\text{total}} \approx 1 \) \(^4\) and \( \Omega_m \approx 1/3 \). All these results strongly suggest that the universe is permeated smoothly by 'dark energy' which has a negative pressure and violates the strong energy condition. The dark energy and accelerating universe have been discussed extensively from different points of view \(^5\), \(^6\), \(^7\). In principle, a natural explanation to the cosmic acceleration is the cosmological constant. However, there exist serious theoretical problems such as the fine tuning problem and coincidence problem. To overcome the fine tuning problem, a self-interacting scalar field, dubbed quintessence, with an equation of state (EOS) \( w_\phi = p_\phi / \rho_\phi \) was introduced, where \( w_\phi \) is time varying and negative. Thus, by properly choosing the

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forms of the potential \( V(\phi) \), desired behaviors of the quintessence can be obtained: (i) with a negative pressure which drives the universe accelerated expansion; (ii) with an energy density that was much smaller than that of the ordinary matter (and radiation) at early times (due to the constrains from primordial nucleosynthesis and structure formation) and is comparable to the latter at recent times. To resolve the coincidence problem, tracker solutions or scaling solutions were designed in which there exists a stage where the ratio of potential and kinetic energies of the scalar field remains constant approximately. In this way, all the initial ‘information’ could be eliminated. So, in the past, if the ratio of the densities of dark energy and matter is approximately constant, the coincidence problem could be avoided.

It has been drawn great attention to the idea that our universe is a 4-dimensional hypersurface embedded in a higher dimensional world as is in Kaluza-Klein theories and in brane world scenarios. Here we consider the 5-dimensional Space-Time-Matter (STM) theory\(^{11}\). This theory is distinguished from the classical Kaluza-Klein theory by that it has an non-compact fifth dimension and that it is empty viewed from 5\(D\) and sourceful viewed from 4\(D\). That is, in STM theory, the 5\(D\) manifold is Ricci-flat which implies that the five-dimensional space-time is empty, the matter of our universe is induced from the fifth dimension, and the 4\(D\) hypersurface is curved by this induced matter. Mathematically, this approach is supported by Campbell’s theorem\(^{12}\) which says that any analytical solution of Einstein field equation of \(N\) dimensions can be locally embedded in a Ricci-flat manifold of \((N+1)\) dimensions, though there is an argument recently concerning Campbell’s theorem\(^{13}\).

In this paper we consider a class of five-dimensional cosmological models\(^9\) of the STM theory. This class of exact solutions satisfies the 5\(D\) Ricci-flat equations \( R_{AB} = 0 \) and is algebraically rich because it contains two arbitrary functions \(\mu(t)\) and \(\nu(t)\). It was shown\(^9\) that several properties characterize these 5\(D\) models: (i) The 4\(D\) induced matter could be described by a perfect fluid plus a variable cosmological ‘constant’. (ii) By properly choosing the two arbitrary functions, both the radiation-dominated and matter-dominated standard FRW models could be recovered. (iii) The big bang singularity of the 4\(D\) standard cosmology is replaced by a big bounce at which the universe reaches to a finite and minimal size. Before the bounce, the universe contracts; after the bounce, it expands. Also, the evolution of the universe containing three components was discussed in\(^{10}\). In this paper we assume that the universe is permeated smoothly by CDM+baryons \(\rho_m\) as well as dark energy \(\rho_x\) with pressure \(p_x = w_x \rho_x\) at late time. We will show that a scaling solution can be obtained which is different from the conventional scaling solutions and late-time attractors: (i) There exists a scaling stage before the transition from decelerated to accelerated expansion, and so the coincidence problem could be resolved; (ii) During this stage, the dark energy has the equation of state \(w_x \approx 0\) and behaves like a cold dark matter - this would provide a unified model for dark matter and dark energy as the Chaplygin gas model does\(^{15}\). Given an appropriate form of
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f(z), we will find that before z ≈ 2, the dark energy behaves like a ‘tracker’ solution and mimics CDM+baryons. The transition from deceleration to acceleration is at zT ≈ 0.8, which agrees with recent cosmological observations.

This paper is organized as follows. In section 2, we assume that the induced matter of the universe contains two components: CDM+baryons and dark energy. We write down Einstein field equations and the equation of states (EOS) of dark energy. In section 3, we rewrite these equations via redshift z, then the arbitrary functions µ(t) contained in the 5D solution corresponds to a function f(z), which is found to play a similar role as the potential V(ϕ) in quintessence and phantom models. With use of a kind of hyperbolic function of f(z), a scaling stage is obtained. Also, we discuss the evolution of the dimensionless density parameters, the EOS of dark energy, the deceleration parameter and the clustering of dark energy. Section 4 is a conclusion.

2. Dark energy in the 5D model

Within the framework of STM theory, a class of exact 5D cosmological solution was presented by Liu and Mashhoon in 1995. Then, in 2001, Liu and Wesson restudied the solution and showed that it describes a cosmological model with a big bounce as opposed to a big bang. The 5D metric of this solution reads

\[ dS^2 = B^2 dt^2 - A^2 \left( \frac{dy^2}{1 - kr^2} + r^2 d\Omega^2 \right) - dy^2 \quad (1) \]

where \( d\Omega^2 \equiv (d\theta^2 + \sin^2 \theta d\phi^2 ) \) and

\[ A^2 = (\mu^2 + k) y^2 + 2\nu y + \frac{\nu^2 + K}{\mu^2 + k}, \]
\[ B = \frac{1}{\mu} \frac{\partial A}{\partial t} \equiv \frac{\dot{A}}{\mu}. \quad (2) \]

Here \( \mu = \mu(t) \) and \( \nu = \nu(t) \) are two arbitrary functions of t, k is the 3D curvature index (k = ±1, 0), and K is a constant. This solution satisfies the 5D vacuum equation \( R_{AB} = 0 \). So, the three invariants are

\[ I_1 \equiv R = 0, I_2 \equiv R^{AB}R_{AB} = 0, \]
\[ I_3 = R_{ABCD}R^{ABCD} = \frac{72K^2}{A^8}. \quad (3) \]

The invariant \( I_3 \) in Eq. (3) shows that K determines the curvature of the 5D manifold. Here we should mention that because the 5D field equations of the STM theory are always sourceless, i.e., \( G_{AB} = 0 \) or \( R_{AB} = 0 \), there is no need to introduce a higher-dimensional Newtonian constant or a higher-dimensional Planck mass in the theory. This is one of the main differences between STM theory and other Kaluza-Klein theories.
Using the 4D part of the 5D metric (1) to calculate the 4D Einstein tensor, one obtains

\[ (4) G^0_0 = \frac{3 (\mu^2 + k)}{A^2}, \]
\[ (4) G^1_1 = (4) G^2_2 = (4) G^3_3 = \frac{2\dot{\mu} \mu}{AA} + \frac{\mu^2 + k}{A^2}. \] (4)

In the previous work \(^9\), the induced matter was set to be a conventional matter plus a time variable cosmological ‘constant’. In this paper, we assume that the induced matter contains two parts: one is CDM+baryons \(\rho_m\) with pressure \(p_m = 0\), the other is the dark energy \(\rho_x\) with pressure \(p_x\). So, we have

\[ 3 \left(\mu^2 + k\right) A^2 = \rho_m + \rho_x, \]
\[ \frac{2\dot{\mu} \mu}{AA} + \frac{\mu^2 + k}{A^2} = -p_x, \] (5)

where

\[ p_x = w_x \rho_x. \] (6)

From Eqs.(5) and (6), one obtains the EOS of dark energy

\[ w_x = \frac{\rho_x}{\rho_m} = -\frac{2 \mu \ddot{\mu}}{3 (\mu^2 + k)} \frac{A^2}{A^{(\mu^2 + k)}}. \] (7)

and the dimensionless density parameters

\[ \Omega_m = \frac{\rho_m}{\rho_m + \rho_x} = \frac{\rho_m}{3 (\mu^2 + k) A}, \] (8)
\[ \Omega_x = 1 - \Omega_m. \] (9)

where \(\rho_{m0} = \bar{\rho}_{m0} A_0^3\). The proper definitions of the Hubble and deceleration parameters should be given as \(^9\),

\[ H = \frac{\dot{A}}{A \dot{A}} = \frac{\mu}{A} \] (10)
\[ q(t, y) = -A \frac{d^2 A}{dy^2} \left( \frac{dA}{dy} \right)^2 = -\frac{A \ddot{\mu}}{\mu \dot{A}} \] (11)

from which we see that \(\ddot{\mu} / \dot{\mu} > 0\) represents an accelerating universe, \(\ddot{\mu} / \dot{\mu} < 0\) represents a decelerating universe. So the function \(\mu(t)\) plays a crucial role in determining the properties of the universe at late times. It was pointed out that another arbitrary function \(\nu(t)\) may relate closely to the early epoch of the universe \(^9\).

3. Late time evolution of the cosmological parameters versus redshift

We consider the spatial flat case \(k = 0\). From the solution (2) we see that on a given \(y = constant\) hypersurface we have \(A = A(t)\). So, without loss of generality,
one can always write $\mu = \mu(z)$ and $\nu = \nu(z)$ if $y$ is fixed, where $z$ is the redshift, $A_0 / A = 1 + z$. Now we define $\mu_E^2 / \mu_z^2 = f(z)$ and then we find that Eqs. (7)-(11) reduce to

\begin{align}
wx &= -\frac{1 + (1 + z) \frac{d \ln f(z)}{dz}}{3 - 3\Omega_{m0}(1 + z) \frac{f(z)}{f(0)}}, \\
\Omega_m &= \Omega_{mz} \frac{f(z)(1 + z)}{1 + zE} = \Omega_{m0} \frac{(1 + z) f(z)}{f(0)}, \\
\Omega_x &= 1 - \Omega_{mz} \frac{f(z)(1 + z)}{1 + zE}, \\
n &= \frac{1 + 3\Omega_x w_x}{2} = -\frac{(1 + z) \frac{d \ln f(z)}{dz}}{2}.
\end{align}

Here the subscript $E$ denotes the point where $\Omega_{mz} = \Omega_{xz}$ (which is $1/2$ when the radiation is neglected). As is known from quintessence and phantom dynamical dark energy models, the potential $V(\phi)$ is unfixed. By choose different forms of $V(\phi)$, desired properties of dark energy could be obtained. Now, there is an arbitrary function $f(z)$ in the present 5D model. Different choice of $f(z)$ corresponds to different choice of the potential $V(\phi)$. This enables us to look for desired properties of the universe via Eqs. (12)-(15).

To obtain a scaling solution, there should be a stage where the ratio of the energy densities of dark energy and dark matter is approximately constant. So, the condition for exiting a scaling stage at late time is $\frac{\Omega_m}{\Omega_x} \sim \text{const}$. From Eqs. (13) and (14), this condition leads to

\begin{equation}
\frac{d\left[ f(z)(1 + z) \right]}{dz} \approx 0.
\end{equation}

Also, to obtain an accelerating universe at late times, some mechanics is needed to break the scaling state steeply and make the dark energy dominated. In a hyperbolic function $k(\phi) = k_{min} + \tanh(\phi - \phi_1) + 1$ as a leap kinetic term to achieve the desired features of quintessence is discussed. So, in our case, to have a scaling dark energy, we let $f(z)$ being of a similar form,

\begin{equation}
f(z) = \frac{\tanh[a(z - b)] + c}{1 + z},
\end{equation}

where, $a$, $b$ and $c$ are arbitrary constants and can be estimated by observation data, i.e. by using $\Omega_{mz} = 0.5$, $\Omega_{m0} = 0.273^{+0.042}_{-0.041}$, $q_0 = -0.67 \pm 0.25$. Then we obtain the values of $z_E$, $a$ and $b$ as

\begin{equation}
a = \frac{1}{2z_E} \ln \frac{(1 + \delta)(1 + \Delta)}{(1 - \delta)(1 - \delta)} , \quad b = \frac{z_E \ln \frac{1 + \Delta}{1 + \delta}}{\ln \frac{1 + \delta(1 + zE)}{1 + \delta}},
\end{equation}

where, $\delta = 1 + z_E - c$ and $\Delta = c - (1 + zE)\frac{\Omega_{m0}}{\Omega_{mz}}$. Using the equation $q_0 = -\frac{1}{3} \frac{d\ln f(z)}{dz}{\bigg|}_{z=0}$, we can solve $z_E$ numerically by leaving $c$ as a free parameter which could be determined by the constraints from structure formation (In this paper we
take $c = 1.45$). So, the function $f(z)$ describes the evolution of CDM+baryons and dark energy completely at late time.

Under the choice of the function $f(z)$ in the form of (17), the ratio of the dimensionless density parameters $\Omega_m$ and $\Omega_x$ becomes constant when $z > z_s$, that is to say, the dark energy is scaling with the matter. And the EOS of dark energy is $w_x \approx 0$, though it is negative in nature. Thus we conclude that just not a long time ago, the EOS became negative and the dark energy began to drive the universe to accelerate. For different values of $\Omega_{m0}$ and $q_0$, the parameters $z_T$, $z_E$ and $w_{x0}$ are listed in Table 1. We can see that the transition point from deceleration to acceleration is at $z_T \approx 0.8$, which is compatible with current observations. Also, for a larger $\Omega_{m0}$, a larger absolute value of $w_{x0}$ is needed to drive the universe accelerating. For the case of $\Omega_{m0} = 0.232$, $q_0 = -0.45$ (which corresponds to $a = 4.52$ and $b = 0.29$), we plot the evolution of dimensionless density parameters $\Omega_m$, $\Omega_x$, EOS of dark energy $w_x$, and deceleration parameter $q$ versus redshift $z$ in Fig. 1. The adiabatic sound speed of the dark energy is

$$c_s^2 = \frac{w_x^2 \partial p_x}{\partial z} \left( w_x \frac{\partial p_x}{\partial z} - p_x \frac{\partial w_x}{\partial z} \right). \tag{19}$$

where, in terms of the redshift $z$, the pressure $p_x$ is

$$p_x = -\frac{\mu^2}{\Omega_0 f(z)} \left( \frac{(1+z)d\ln f(z)}{dz} + 1 \right)^2$$
$$= H_0^2 f(0) \left( \frac{(1+z)d\ln f(z)}{dz} + 1 \right). \tag{20}$$

We plot the adiabatic sound speed $c_s^2$ in Fig. 2, where $a = 4.52$, $b = 0.29$ and $c = 1.45$. From Fig. 2 we find that the adiabatic sound speed of the dark energy

| Parameters | For $\Omega_{m0} = 0.232$ |
|------------|--------------------------|
| $q_0$      | -25 -35 -45 -55 -65 -75 -85 -95 |
| $z_E$      | 1.03 .82 .71 .63 .57 .52 .48 .45 |
| $z_T$      | .06 .00 .95 .94 .92 .89 .86 .83 |
| $w_{x0}$   | -.65 -.74 -.82 -.91 -1.00 -1.09 -1.17 -1.26 |

| Parameters | For $\Omega_{m0} = 0.273$ |
|------------|--------------------------|
| $q_0$      | -25 -35 -45 -55 -65 -75 -85 -95 |
| $z_E$      | .71 .61 .54 .48 .44 .40 .37 .35 |
| $z_T$      | .06 .88 .88 .87 .84 .82 .79 .76 |
| $w_{x0}$   | -.69 -.78 -.87 -.96 -1.05 -1.15 -1.24 -1.33 |

| Parameters | For $\Omega_{m0} = 0.314$ |
|------------|--------------------------|
| $q_0$      | -25 -35 -45 -55 -65 -75 -85 -95 |
| $z_E$      | .52 .45 .40 .36 .33 .31 .28 .26 |
| $z_T$      | .06 .83 .82 .81 .78 .76 .73 .71 |
| $w_{x0}$   | -.73 -.83 -.92 -1.02 -1.12 -1.21 -1.31 -1.41 |
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Fig. 1. The dimensionless density parameters versus redshift $z$: $\Omega_x$ of dark energy (dotted line, the third line from the top), $\Omega_m$ of CDM+baryons (solid line, the first line from the top), EOS of dark energy $w_x$ (dashed line, the forth line from the top), and the deceleration parameter $q$ (dash-dotted line, the second line from the top). The figure is plotted with $a = 4.52$, $b = 0.29$ and $c = 1.45$. In these values, the current values of the cosmological parameters are $\Omega_m^0 = 0.232$, $\Omega_x^0 = 0.768$, $w_x^0 = -0.82$ and $q_0 = -0.45$, respectively. From $z = 10$ to $z = 400$, we find $\Omega_x/\Omega_m \approx 0.02$ which is indeed a scaling.

is $c_s^2 \approx 0$ during the scaling stage. So, it behaves like the conventional matter with an almost zero $w_x$. However, for $z < 2$, the sound speed $c_s^2$ becomes wild and even negative - this is also encountered in other unified dark matter and dark energy models. Note that before $z = 2$ the dark energy with $c_s^2 \approx 0$ and $w_x \approx 0$ behaves like a dark matter (but not identical to it because $w_x$ is negative in nature). However, one should take into account the clustering of dark energy seriously. As did in Ref. 20, we consider a universe containing only the dark energy with $c_s^2 \approx 0$ and the perturbation $\delta = \delta_{\rho_x}/\rho_x$, where $\rho_x$ is the dark energy density. Since $w_x \neq 0$, the Jean’s instability equation is modified,

$$a^2 \frac{d^2 \delta}{da^2} + \frac{3}{2} a A[c_s^2, w_x] \frac{d \delta}{da} + \left(\frac{\kappa^2 c_s^2}{H^2} - \frac{3}{2} B[c_s^2, w_x]\right) \delta = 0,$$

(21)

where $a$ is the FRW scale factor, $H = \frac{\dot{a}}{a}$ (the prime denotes the derivative w.r.t. the conformal time), $A[c_s^2, w_x] = 1 - 5w_x + 2c_s^2$ and $B[c_s^2, w_x] = 1 - 6c_s^2 + 8w_x - 3w_x^2$. In general, the “growing” solution is $\delta \propto a^\gamma$ where $\gamma = \frac{\kappa^2 c_s^2}{12} (1 - \frac{3}{2} A + [(1 - \frac{3}{2} A)^2 + 6B]^{1/2})$. For $c_s^2 = w_x = 0$, $\gamma = 1$, the solution $\delta \propto a$ is the conventional dust-dominated growing solution. For a small but none-zero $c_s^2$ and $w_x$, the dark energy may cluster during scaling. However, the process is slower than the conventional dust-dominated case. And once $w_x < -0.12$ (for all $c_s^2 \geq 0$),
Fig. 2. The evolution of the adiabatic sound speed versus the redshift $z$. The figure is plotted with $a = 4.52$, $b = 0.29$ and $c = 1.45$. In these values, the current values of the cosmological parameters are $\Omega_{m0} = 0.232$, $\Omega_{x0} = 0.768$, $w_{x0} = -0.82$ and $q_0 = -0.45$, respectively.

$\gamma$ will less than zero and the dark energy stop clustering. As does, the growth exponent $f$ is defined as

$$f(a) \equiv \frac{d \ln \delta_k(a)}{d \ln a},$$

(22)

which is roughly $k$-independent for a wide range of $k$. And, the growth factor $g$ of density perturbations between arbitrary $a_1 < a_2$ is defined as

$$g(a_1, a_2) \equiv \frac{\delta_k(a_2)}{\delta_k(a_1)} = \left(\frac{a_2}{a_1}\right)^{\bar{f}}$$

(23)

with a suitably defined average growth exponent $\bar{f}$. For example, the scaling stage begins at the dark matter dominated point $a_{dec}$ (or $z_{dec} = 1100$, decoupled epoch) and ends at $a_{es}$ (or $z = 2$), we have the growth factor $g(a_{dec}, a_{es}) \approx 1100$. After the scaling stage and if there is no clustering as expected, we have $g(a_{es}, a_0) \approx z_{es}^{\bar{f}}$ and $\bar{f} = -\log_{z_{es}}(z_{dec} - z_{es}) = -\log_2(1097) = -10$ roughly. In our case, the evolution of $\gamma$ versus redshift $z$ is plotted in Fig. 3 numerically. In Fig. 3, we find that the singular points exist which correspond to a negative infinity for $\gamma$. So, after this stage, we cannot find any clustering dark energy in the universe at present. However, we must point out that the wild properties of the dark energy might be out of our knowledge and make the dark energy more mysterious.

The influence of early dark energy on CMB anisotropy was discussed in $^{21}$. It was pointed out that large $\Omega_x$ is ruled out by cosmic observations. In our case, the dimensionless energy density $\Omega_x$ in scaling stage depends on the parameter $c$ mainly, we can choose the parameters properly to fit observational constraints.
Fig. 3. The evolution of $\gamma$ versus redshift $z$. In the scaling stage, $f \approx 1$ behaves like dark matter. In the breaking stage, three singular points exist. The figure is plotted with $a = 4.52$, $b = 0.29$ and $c = 1.45$. In these values, the current values of the cosmological parameters are $\Omega_{m0} = 0.232$, $\Omega_{x0} = 0.768$, $w_{x0} = -0.82$ and $q_0 = -0.45$, respectively.

From Fig. 1, we find that $\Omega_x \approx 0.02$ at scaling stage which is in compatible with observations. In $^{19}$, a general formula to calculate $\sigma_8$ was given as

$$\frac{\sigma_8(A)}{\sigma_8(B)} \approx \left(\frac{\alpha_{eq}}{\alpha_{eq}}\right)^3(\Omega_0^f(A) - \Omega_0^f(B))^{3/5}(1 - \Omega_0^f(B))^{2/5} \sqrt{\frac{\tau_0(A)}{\tau_0(B)}}$$

(24)

In our case, during the scaling stage, the dark energy behaves like dark matter with a zero pressure. So, comparing the model with the standard cold dark matter (SCDM) ($\sigma_8(SCDM) \approx 0.5 \pm 0.1$), we have

$$\sigma_8(x) = 1100^{-3(0.02-0)/5}\sigma_8(SCDM) = 0.37\sigma_8(SCDM) \approx 0.46.$$  

(25)

Moreover, observations such as cluster abundance constraints yield $^{22}$

$$\sigma_8 = (0.5 \pm 0.1)\Omega_m^{-\gamma}$$

(26)

where $\gamma$ is slightly model dependent and usually $\gamma \approx 0.5$. So, in our case, we have

$$\sigma_8^{clus}(x) = 0.5 \times 0.98^{-0.5} \approx 0.51.$$  

(27)

So, the ratio is

$$\frac{\sigma_8(x)}{\sigma_8^{clus}(x)} \approx 0.92,$$

(28)

which is close to unity and compatible with observations.
4. Conclusion

A general class of $5D$ cosmological models is characterized by a big bounce as opposed to the big bang in $4D$ standard cosmological model. This exact solution contains two arbitrary functions $\mu(t)$ and $\nu(t)$, which are in analogy with the different forms of the potential $V(\phi)$ in quintessence or phantom dark energy models. Also, once the forms of the arbitrary functions are specified, the universe evolution will be determined. In this bounce model, we assume the universe to contain at late time two components: CDM+baryons and dark energy. Instead of choosing the forms of the arbitrary functions $\mu(t)$ and $\nu(t)$, we transform $\mu(t)$ from coordinate time $t$ to redshift $z$. So, the choice of $\mu(t)$ becomes the choice of $f(z)$. Then, we let $f(z)$ to be of the form of a hyperbolic function (17) and study the evolution of the universe. We find that there is a scaling stage before $z \approx 2$, in which the dark energy has constant ratio with CDM+baryons and behaves like a pressureless cold dark matter with $w_x \approx 0$. After that, the pressure of the dark energy deviates from $w_x \approx 0$ and gradually becomes much more negative. The transition from decelerated expansion to accelerated expansion is at $z_T \approx 0.8$ which, together with other predictions of the present $5D$ model, is in agreement with current observations. Thus we see that the arbitrary function $f(z)$ of the $5D$ model plays a similar role as the potential $V(\phi)$ in quintessence or phantom models in eliminating the initial ‘information’, then the coincidence problem is resolved. Also, this kind of model would provide a unified model for dark matter and dark energy.

However, we also see from Fig 2 and 3 that several singular points for $z < 2$. Perhaps, these singularities correspond to some unknown phase transition.

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