Time Modulation of Orbital Electron Capture Decays of H–like Heavy Ions

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According to experimental data at GSI, the rates of the number of daughter ions, produced by the nuclear K–shell electron capture (EC) decays of the H-like $^{140}$Pr$^{68+}$, $^{142}$Pm$^{60+}$ and $^{122}$Sb$^{52+}$ ions, are modulated in time with periods $T_{EC}$ of the order of a few seconds, obeying an $A$-scaling $T_{EC} = A/200$, where $A$ is the mass number of the mother nuclei, and with amplitudes $a_{EC} \simeq 0.21$. In turn, the positron decay mode of the H-like $^{142}$Pm$^{60+}$ ions showed no time modulation of the decay rates. As has been shown in Phys. Rev. Lett. 103, 062502 (2009) and Phys. Rev. Lett. 101, 182501 (2008), these data can be explained by the interference of two massive neutrino mass–eigenstates, called the “GSI Oscillations”, and in turn, the positron decay mode of the H–like heavy ions, carried out in the Experimental Storage Ring (ESR) at GSI in Darmstadt [1]–[4], should explain the absence of the time modulation of the $\beta^+$ decay rates of H–like heavy ions, are modulated in time with periods $T_{EC}$ of the order of a few seconds, obeying an $A$-scaling $T_{EC} = A/200$, where $A$ is the mass number of the mother ion, 2) the modulation amplitude for all observed decays $a_{EC} \simeq 0.21$ and 3) the $\beta^+$ decay rates have no time modulated terms. In addition all theoretical approaches, describing the experimental data on the K–shell electron capture (EC) and positron ($\beta^+$) decay rates of the H–like heavy ions [1]–[4], should explain the absence of the time modulation of the EC–decay rates of bound atoms $^{140}$Pr$^{68+}$, $^{142}$Pm$^{60+}$, and $^{180}$Re, measured in [5].

As has been shown in [7], [11], the experimental data of the EC and $\beta^+$ decay rates of the H–like heavy ions [1]–[4], called the “GSI Oscillations”, and the observations reported in [3], [6] can be explained only following the hypothesis of the interference of the massive neutrino mass–eigenstates $|\nu_\alpha\rangle$ [7], [8], defining the neutrino $|\nu_\alpha\rangle$ with a lepton charge $\alpha = e, \mu$ or $\tau$ as a coherent superposition of massive neutrino mass–eigenstates $|\nu_\alpha\rangle = \sum_j U_{\alpha j}^* |\nu_j\rangle$ [12]. Neither the magnetic field of the ESR [10] (see also [13]) nor the mass splitting of the H–like mother ions [3] [11] (see also [1]) enable to explain the experimental data [1]–[4].

The amplitude of the EC–decay mode of the H–like heavy ions, carried out in the Experimental Storage Ring (ESR) at GSI in Darmstadt [1]–[4], should explain the absence of the time modulation of the $\beta^+$ decay rates of H–like heavy ions, are modulated in time with periods $T_{EC}$ of the order of a few seconds, obeying an $A$-scaling $T_{EC} = A/200$, where $A$ is the mass number of the mother ion, 2) the modulation amplitude for all observed decays $a_{EC} \simeq 0.21$ and 3) the $\beta^+$ decay rates have no time modulated terms. In addition all theoretical approaches, describing the experimental data on the K–shell electron capture (EC) and positron ($\beta^+$) decay rates of the H–like heavy ions [1]–[4], should explain the absence of the time modulation of the EC–decay rates of bound atoms $^{140}$Pr$^{68+}$, $^{142}$Pm$^{60+}$, and $^{180}$Re, measured in [5].

As has been shown in [7], [11], the experimental data [1]–[4], called the “GSI Oscillations”, and the observations reported in [3], [6] can be explained only following the hypothesis of the interference of the massive neutrino mass–eigenstates $|\nu_\alpha\rangle$ [7], [8], defining the neutrino $|\nu_\alpha\rangle$ with a lepton charge $\alpha = e, \mu$ or $\tau$ as a coherent superposition of massive neutrino mass–eigenstates $|\nu_\alpha\rangle = \sum_j U_{\alpha j}^* |\nu_j\rangle$ [12]. Neither the magnetic field of the ESR [10] (see also [13]) nor the mass splitting of the H–like mother ions [3] [11] (see also [1]) enable to explain the experimental data [1]–[4]. The discussions of the “GSI oscillations” as the interference of massive neutrino mass–eigenstates were proposed in [14].

Recently, [12], Gal has criticised the results obtained in [1]. The main aim of this letter is to clarify all problems, which have prevented Gal from accepting our results.

Frequencies of “GSI oscillations”, caused by interferences of massive neutrino mass–eigenstates, should be inversely proportional to neutrino energies, i.e. to $Q$-values of EC–decays of H–like heavy ions, but not to the mass number $A$ of mother ions [13].

The amplitude of the EC–decay $m \to d + \nu_e$, caused by a Gamow–Teller $1^+ \to 0^+$ transition of the mother ion $m$ from the ground hyperfine state $|1s\rangle$ into the daughter ion in the stable ground state and the electron neutrino $\nu_e$, a coherent superposition $|\nu_e\rangle = \sum_j U_{ej}^* |\nu_j\rangle$ of massive neutrino mass–eigenstates $|\nu_j\rangle$ with masses $m_j$, is a function of time $t$ defined by [13]

$$A(m \to d \nu_e)(t) = \sum_j U_{ej} A(m \to d \nu_j)(t),$$

where the amplitude $A(m \to d \nu_j)(t)$ of the $m \to d + \nu_j$ transition is calculated with the Hamilton operator of weak interactions given by [7] [8]

$$H_W^{(j)}(t) = \frac{G_F}{\sqrt{2}} v_{ud} \int d^3 x [\bar{\psi}_e(x) \gamma^\mu (1 - g_A \gamma^5) \psi_a(x)] \times [\bar{\psi}_a(x) \gamma_\mu (1 - \gamma^5) \psi_e(x)],$$

with standard notation [7] [8] [10]. It is equal to

$$A(m \to d \nu_j)(t) = - \delta_{M_F} \frac{3}{4} \sqrt{2} M_m \times M_{GT} \langle \psi_1^Z \rangle \sqrt{2 E_d(q_j) E_j(k_j)} e^{i \frac{\Delta E_j - i \varepsilon}{\Delta E_j - i \varepsilon}} \times \Phi_d(k_j + q_j),$$

where $\Delta E_j = E_d(q_j) + E_j(k_j) - M_m$ is the energy difference of the final and initial state, $M_m$ is the...
mother ion mass, \( E_d(q_i^j) \) and \( E_j(k_j) \) are the energies of the daughter ion and massive neutrino \( \nu_j \) with 3-momenta \( q_i^j \) and \( k_j \), respectively, \( M_{\text{GT}} \) is the nuclear matrix element of the Gamow–Teller transition \( m \to d \) and \( \langle \psi_{i,s}^{(Z)} \rangle \) is the wave function of the bound electron in the H–like heavy ion \( m \), averaged over the nuclear density \( \rho \).

Suppose that in the EC–decay \( m \to d + \nu_e \) the momenta of the daughter ions \( q_i^j \) produced in the decay channels \( m \to d + \nu_j \), are precisely measured. In case that the differences of momenta \( |q_i^j - q_j^j| \) are larger then the momentum resolutions \( |\delta q_i^j| \), i.e. \( |q_i^j - q_j^j| \gg |\delta q_i^j| \), where \( q_i^j \) is a 3–momentum of a daughter ion in the decay channel \( m \to d + \nu_i \) for \( i \neq j \), all decay channels \( m \to d + \nu_j \) are distinguished experimentally and the EC–decay rate should never show time modulation. This agrees with the assertion pointed out in Ref. [17].

However, this is not the case with the “GSI oscillations” [1]. [2]. The wave function of the detected daughter ion should be taken in the form of a wave packet, since the time differential detection of the daughter ions from the EC–decays with a time resolution \( \tau_d \approx 0.32 \text{s} \) introduces energy \( \delta E_d \sim 2\pi/\tau_d \) and 3–momentum \( |\delta q_d| \sim 2\pi/\tau_d \nu_d \) uncertainties, where \( \nu_d \) is the velocity of the daughter ion in the ESR. [2]. Such a smearing is described by the wave function \( \Phi_d(k_j + q_j^i) \). In the rest frame of the H–like mother ion the energy and momentum uncertainties are equal to \( \delta E_d \sim 2\pi/\tau_d \approx 1.85 \times 10^{-14} \text{eV} \) and \( |\delta q_d| \sim 2\pi \nu_d Q_{\text{EC}}/M_d \) uncertainties, where \( \gamma = 1.432 \) is the Lorentz factor of the H–like mother ions [1], \( \nu_d = Q_{\text{EC}}/M_d \) is a velocity of the daughter ion and \( Q_{\text{EC}} \) is the \( Q \)-value of the EC–decay, equal to the momentum of the daughter, and \( M_d \) is the mass of the daughter ion. For the EC–decay \( ^{140}\text{Pr}^{58+} \to ^{140}\text{Ce}^{58+} + \nu_e \) the \( Q \)-value is equal to \( Q_{\text{EC}} = 3348(6) \text{keV} \) [16]. This gives \( |\delta q_d| \sim 7.21 \times 10^{-10} \text{eV} \).

Due to energy and momentum conservation in every EC–decay channel \( m \to d + \nu_j \) the energy and momentum of massive neutrino \( \nu_j \) are equal to

\[
E_j(k_j) \approx Q_{\text{EC}} + \frac{m_j^2}{2M_m}, \quad |k_j| \approx Q_{\text{EC}} - \frac{m_j^2}{2Q_{\text{EC}}},
\]

where \( Q_{\text{EC}} = M_m - M_d \). The differences of energies and momenta of neutrino mass–eigenstates are

\[
\omega_{ij} = E_i(k_i) - E_j(k_j) = \frac{\Delta m_{ij}^2}{2M_m},
\]

\[
k_{ij} = |k_i| - |k_j| = -\frac{\Delta m_{ij}^2}{2Q_{\text{EC}}}.\]

In turn, \( \omega_{ij} \) and \( k_{ij} \) determine also the recoil energy and 3–momentum differences of the daughter ions.

For two massive neutrino mass–eigenstates and the EC–decay of \( ^{140}\text{Pr}^{58+} \) we get

\[
\omega_{21} = \frac{\Delta m_{21}^2}{2M_m} = 8.40 \times 10^{-16} \text{eV},
\]

\[
|k_{21}| = \frac{\Delta m_{21}^2}{2Q_{\text{EC}}} = 3.27 \times 10^{-11} \text{eV},
\]

where we have set \( \Delta m_{21}^2 = 2.19 \times 10^{-4} \text{eV}^2 \). Since \( \delta E_d \gg \omega_{21} \) and \( |\delta q_d| \gg |k_{21}| \), the daughter ions, produced in the two decay channels \( m \to d + \nu_1 \) and \( m \to d + \nu_2 \), are indistinguishable [2]. This is the origin of the coherence in the EC–decays \( m \to d + \nu_e \) of the H–like heavy ions, measured in GSI [2].

Thus, in GSI experiments the observed daughter ion \( d \) is a nucleus with energy \( E_d(q_i^j) \) and 3–momentum \( \vec{q}_i^j \) for all decay channels \( m \to d + \nu_j \). The amplitude of the \( m \to d + \nu_e \) decay reads

\[
A(m \to d \nu_e)(t) = -\delta_{M_F, -\frac{1}{2}} \sqrt{3\sqrt{2}M_mM_{\text{GT}}} \times \langle \psi_{i,s}^{(Z)} \rangle \sum_j U_{e,j} \sqrt{2E_d(q_i^j)E_j(k_j)} e^{i(\Delta E_j - i\epsilon)t} \times \frac{\Phi_d(k_j + q_i^j)}{\Delta E_j - i\epsilon},
\]

where \( \Delta E_j = E_d(q_i^j) + E_j(k_j) - M_m \). The EC–decay rate is related to the expression

\[
\lim_{\epsilon \to 0} \frac{d}{dt} \frac{1}{2 \sum_{M_F} |A(m \to d \nu_e)(t)|^2 = 3M_m|M_{\text{GT}}|^2
\]

\[
\times \left| \langle \psi_{i,s}^{(Z)} \rangle \sum_j |U_{e,j}|^2 2E_d(q_i^j)E_j(k_j) 2\pi \delta(\Delta E_j) \times |\Phi_d(k_j + q_i^j)|^2 + \sum_{i,j} U_{e,i}^* U_{e,j} \sqrt{2E_d(q_i^j)E_j(k_j)} \times \sqrt{2E_d(q_i^j)E_j(k_j)} \Phi_d^*(k_j + q_i^j) \Phi_d(k_j + q_i^j) \times [2\pi \delta(\Delta E_j) + 2\pi \delta(\Delta E_j) + \cos(\omega_{ij}t)\right].
\]

This expression reproduces Eq. (8) in our paper [2] with the same frequencies \( \omega_{ij} = \Delta m_{ij}^2/2M_m \).

As has been mentioned in [2], the first term in Eq. (8) is the sum of the two diagonal terms of the transition probability into the states \( d + \nu_1 \) and \( d + \nu_2 \), describing the incoherent contribution of massive neutrino mass–eigenstates, while the second term defines the interference of states \( \nu_i \) and \( \nu_j \) with \( i \neq j \) causing the periodic time dependence with the frequency \( \omega_{ij} \) equal to

\[
\omega_{ij} = \Delta E_i - \Delta E_j = E_d(q_i^j) + E_i(k_i) - M_m
\]

\[
- E_d(q_i^j) - E_j(k_j) + M_m = E_i(k_i) - E_j(k_j) = \frac{\Delta m_{ij}^2}{2M_m}.
\]
Thus, we argue that the interference term, produced by a coherent contribution of massive neutrino mass–eigenstates with a frequency inversely proportional to the mass number of the mother ion, can be observed only due to energy and momentum uncertainties, introduced by the time differential detection of the daughter ions [2].

Due to the smallness of neutrino masses for the calculation of the EC–decay rate we can take the massless limit everywhere except the modulated term $U^*_{\alpha j} U_{\alpha j}(\omega_{ij}t)$ [3]. Since the 3–momenta of the massive neutrino mass–eigenstates and the 3–momenta of the daughter ions differ only slightly from the $Q$–value of the EC–decay, we can set $\mathbf{\tilde{k}}_i \simeq \mathbf{k}_i$ and $|\Psi_d(\mathbf{k} + \mathbf{q})|^2 = V(2\pi)^3 \delta^3(\mathbf{k} + \mathbf{q})$, where $V$ is a normalisation volume [3]. Using the definition of the EC–decay rate

$$\lambda_{EC}(t) = \frac{1}{2\Lambda \nu} \int \frac{d^3q}{(2\pi)^3 2E_d} \frac{d^3k}{(2\pi)^3 2E_v} \times \lim_{\varepsilon \to 0} \int \frac{d\varepsilon}{t_2} \sum_{M_F} |A(m \to d \nu_e)(t)|^2,$$

we obtain the following expression for the time modulated EC–decay rate [5]

$$\lambda_{EC}(t) = \lambda_{EC}(1 + a_{EC} \cos(\omega_{12} t)), \quad (11)$$

where $\omega_{21} = \Delta m_{21}^2/2\Lambda \nu, a_{EC} = \sin 2\theta_{12}$ and $\lambda_{EC}$ has been calculated in [13]. The EC–decay rate Eq. (11) is calculated for the matrix elements $U_{\alpha j}$ of the mixing matrix $U$, taken at $\theta_{13} = 0$ [3] (see also [12]). In the laboratory frame the EC–decay rate is time modulated with a frequency $\omega_{EC} = \omega_{21}/\gamma$. Thus, the period $T_{EC}$ of the time modulation is

$$T_{EC} = \frac{2\pi}{\omega_{EC}} = \frac{2\pi \Lambda \nu}{\Delta m_{21}^2}. \quad (12)$$

For the experimental data on the periods of the time modulation [1]–[4] we get $\Delta m_{21}^2 = 2.19 \times 10^{-4} \text{eV}^2$ [7].

**Coherence vs. incoherence in two-body electron capture - No interference terms in EC–decay rates of H–like heavy ions** [13]

According to Gal [13], the two–body K–shell electron capture $m \to d$ decays, when only the daughter ions are observed, are driven by the complete set of orthogonal neutrino states $|\nu_{\alpha}\rangle = \sum_j U^*_{\alpha j} |\nu_j\rangle$ with all lepton flavours $\alpha = e, \mu$ and $\tau$. The amplitude of the $m \to d + \nu_\alpha$ transition is equal to [15]

$$A(m \to d \nu_\alpha)(t) = \sum_j U_{\alpha j} U_{\alpha j} A(m \to d \nu_j)(t), \quad (13)$$

where the amplitude $A(m \to d \nu_j)(t)$ is defined by Eq. (3).

Since in GSI experiments neutrinos in the EC–decays of the H–like heavy ions are not detected, Gal proposes to define the probability of the $m \to d$ transition as the incoherent sum of the squared absolute values of the amplitudes of the $m \to d + \nu_\alpha$ transitions [15]

$$P(m \to d)(t) = \sum_{\alpha} |A(m \to d \nu_\alpha)(t)|^2 =$$

$$= \sum_{\alpha} \sum_j U^*_{\alpha j} U_{\alpha j} A^*(m \to d \nu_j)(t) \times A(m \to d \nu_j)(t), \quad (14)$$

where the index $\alpha$ runs over $\alpha = e, \mu$ and $\tau$. Using the orthogonality relation for the matrix elements of the mixing matrix [12]

$$\sum_{\alpha} U^*_{\alpha i} U_{\alpha j} = \delta_{ij} \quad (15)$$

one can arrive at the expression [13]

$$P(m \to d)(t) = \sum_j |U_{\alpha j}|^2 |A(m \to d \nu_j)(t)|^2, \quad (16)$$

which contains no interference term. A similar argument has been recently given by Yazaki [18].

We want to point out here that the use of the complete set of neutrino wave functions $|\nu_{\alpha}\rangle = \sum_j U^*_{\alpha j} |\nu_j\rangle$ with all lepton flavours leaves room for the restoration of the interference terms in the rates of the $m \to d$ transitions.

The amplitude $A(m \to d)(t)$ of the $m \to d$ transition we propose to define as a coherent superposition of the amplitudes $A(m \to d \nu_\alpha)(t)$

$$A(m \to d)(t) = \sum_{\alpha} e^{-i\phi_{\alpha}} A(m \to d \nu_\alpha)(t), \quad (17)$$

where we have introduced arbitrary phases $\phi_{\alpha}$ for neutrinos $\nu_\alpha$, which are responsible for the restoration of the interference term. The amplitudes $A(m \to d \nu_\alpha)(t)$ are determined by Eq. (13). The possibility to describe the amplitude $A(m \to d)(t)$ by a coherent superposition Eq. (17) is obvious, since neutrinos $\nu_\alpha$ are not detected.

The rate of the $m \to d$ transition is related to the expression [2]

$$\frac{\sum_{\alpha} \sum_{M_F} |A(m \to d \nu_\alpha)(t)|^2}{2 \int \frac{d\varepsilon}{t_2} \sum_{M_F} |A(m \to d \nu_\alpha)(t)|^2} =$$

$$= 3 \Lambda \nu |M_{GT}|^2 |\langle \phi_{1s}^{(Z)} \rangle|^2 \times \left( \sum_{\alpha} \sum_j U^*_{\alpha j} U_{\alpha j} e^{-i\phi_{\alpha}} \right)^2 2E_d(\mathbf{q}) E_d(\mathbf{k}_j) 2\pi$$

3
where the coherent contribution of the decay channels

\[ \Phi_d(k_j + \bar{q})^2 + \sum_{\ell > j} \sqrt{2E_d(\bar{q})E_{\ell}(\bar{k}_\ell)} \]

\[ \times \sqrt{2E_d(\bar{q})E_{\ell}(\bar{k}_\ell)} [2\pi \delta(\Delta E'_\ell) + 2\pi \delta(\Delta E'_\ell)] \]

\[ \times \text{Re} \left\{ \sum_\beta U_{\beta\ell}^*U_{\beta\ell} e^{i\phi_{\beta\ell}} \right\}, \] (18)

where indices \( \ell \) and \( j \) denote the neutrino mass–eigenstates, the indices \( \alpha \) and \( \beta \) run over all lepton flavours. The frequencies \( \omega_{\ell j} \) of the time modulation are defined by Eq. 9.

As has been remarked in [7] and discussed above, the first term in the r.h.s. of Eq. (18) corresponds to the decoherent contribution of massive neutrino mass–eigenstates, whereas the second one is caused by the coherent contribution of the decay channels \( m \to d + \nu_j \).

The EC–decay rates, measured in GSI experiments, take the form [1]

\[ \lambda_{EC}(t) = \lambda_{EC}(1 + a_{EC} \cos(\omega_{EC}t + \phi_{EC})). \] (19)

In order to reproduce the correct value of the EC–decay constant \( \lambda_{EC} \), calculated in [1] in the theory of weak interactions with massless neutrinos, we impose the following constraint on the phases of the neutrino wave functions

\[ \sum_j \left| \sum_\alpha U_{\alpha j} U_{\ell j} e^{-i\phi_{\alpha j}} \right|^2 = 1. \] (20)

Setting the mixing angles \( \theta_{13} = 0 \) and \( \theta_{23} = \pi/4 \) and using the definition of the mixing matrix \( \tilde{U} \) [12], the condition Eq. (20) can be transcribed into the form

\[ \sum_j \left| \sum_\alpha U_{\alpha j} U_{\ell j} e^{-i\phi_{\alpha j}} \right|^2 = 1 \]

\[ - \frac{1}{2} \sin^2 2\theta_{12} \cos(\varphi_{\mu e} - \varphi_{\tau e}) + \frac{1}{\sqrt{2}} \sin 2\theta_{12} \]

\[ \times (\cos \varphi_{\tau e} - \cos \varphi_{\mu e}) \cos 2\theta_{12} = 1, \] (21)

where \( \varphi_{\alpha e} = \varphi_\alpha - \varphi_e \) for \( \alpha = \mu, \tau \).

The interference term is

\[ \sum_{\ell > j} 2 \text{Re} \left\{ \sum_\beta U_{\beta\ell}^*U_{\beta j} e^{i(\phi_{\beta j} - \phi_{\alpha j})} U_{\alpha j} U_{\ell j} e^{i\omega_{\ell j}t} \right\} = \]

\[ = \frac{1}{\sqrt{2}} \sin 2\theta_{12} (\sin \varphi_{\mu e} - \sin \varphi_{\tau e}) \sin(\omega_{21}t). \] (22)

where we have used Eq. (21).

Identifying the rate of the \( m \to d \) transition with the EC–decay rate Eq. (19) we get

\[ \omega_{EC} = \frac{\omega_{21}}{\gamma} = \frac{\Delta m_{21}^2}{2\gamma M_m}, \]

\[ \phi_{EC} = -\frac{\pi}{2}, \]

\[ a_{EC} = \frac{1}{\sqrt{2}} \sin 2\theta_{12} (\sin \varphi_{\mu e} - \sin \varphi_{\tau e}). \] (23)

Thus, we predict that the phase and amplitude of the time modulated term of the EC–decay rate of the H–like heavy ions should be universal and equal to \( \phi_{EC} = -\pi/2 \) and \( a_{EC} = \frac{1}{\sqrt{2}} \sin 2\theta_{12} (\sin \varphi_{\mu e} - \sin \varphi_{\tau e}) \) (see Eq. (23)), respectively.

For the experimental value of the mixing angle \( \theta_{21} = 34^0 12' \) and the modulation amplitude \( a_{EC} = 0.21 \) we get the following system of equations

\[ \begin{cases} 
\sin \varphi_{\mu e} - \sin \varphi_{\tau e} = 0.32 \\
1.75 \cos(\varphi_{\mu e} - \varphi_{\tau e}) + \cos \varphi_{\mu e} - \cos \varphi_{\tau e} = 0 
\end{cases} \] (24)

defining the phase differences \( \varphi_{\mu e} \) and \( \varphi_{\tau e} \). The second equation we have obtained from the condition Eq. (21). One of the solutions of the system Eq. (21) is \( \varphi_{\mu e} \approx 1.75 \text{ rad} \) and \( \varphi_{\tau e} \approx 0.73 \text{ rad} \).

For the understanding of the physical origin of the phases \( \varphi_{\alpha} \) one can analyse, for example, the mixing matrix for neutrinos \( \nu_\alpha \) with definite lepton charges \( \alpha = e, \mu, \tau \) as coherent superpositions of massive neutrino mass–eigenstates. The wave functions of neutrinos \( \nu_\alpha \), which we use for the calculation of the amplitudes of the EC–decay rates, can be written as \( |\nu_\alpha\rangle = \sum_j \tilde{U}_{\alpha j}^* |\nu_j\rangle \), where the mixing matrix \( \tilde{U}^* \) is defined by

\[ \tilde{U}^* = \begin{pmatrix} e^{i\varphi_e} & 0 & 0 \\ 0 & e^{i\varphi_\mu} & 0 \\ 0 & 0 & e^{i\varphi_\tau} \end{pmatrix} \]

\[ = \begin{pmatrix} e^{i\varphi_e} & 0 & 0 \\ 0 & e^{i\varphi_\mu} & 0 \\ 0 & 0 & e^{i\varphi_\tau} \end{pmatrix} \begin{pmatrix} 0 & 0 & c_{12} \\ 0 & -\frac{s_{12}}{\sqrt{2}} & \frac{s_{12}}{\sqrt{2}} \\ 0 & \frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} \end{pmatrix}. \] (25)

Here \( \tilde{U}^* \) is the standard mixing matrix, calculated for the mixing angles \( \theta_{13} = 0 \) and \( \theta_{23} = \pi/4 \) [12]. In such a representation of the mixing matrix, the phase differences \( \varphi_{\mu e} = \varphi_{e} - \varphi_{e} \) and \( \varphi_{\tau e} = \varphi_{\tau} - \varphi_{e} \) may have the meaning of Majorana phases, caused by CP–violation [12].

Since due to the condition Eq. (21) the limit of equal phases \( \varphi_e = \varphi_\mu = \varphi_\tau = \varphi \) including \( \varphi = 0 \), of wave functions of neutrinos with lepton flavours \( \alpha = e, \mu, \tau \) corresponds to the massless limit of massive neutrino mass–eigenstates \( m_j \to 0 \), being equivalent to the vanishing of the mixing angle \( \theta_{12} \to 0 \), the interference term vanishes for \( \varphi_e = \varphi_\mu = \varphi_\tau = \varphi \) and, of course, for \( \varphi = 0 \) as a partial case.
Final state interference in EC–decay rates is due to the neutrino magnetic moment [15].

The “GSI oscillations” cannot be induced by the magnetic moments of neutrinos, since in this case the experimental data on the EC and $\beta^+$ decays of the H–like ions should show a time modulation with equal periods. This contradicts [1]–[4].

**Conclusive discussion**

We have shown that the appearance of the interference terms in the EC–decay rates of the H–like heavy ions with periods, proportional to the mass of the H–like mother heavy ion $T_{EC} \sim M_m$ but not the $Q$–value of the EC–decay, is due to overlap of massive neutrino mass–eigenstate energies and of the wave functions of the daughter ions in two–body decay channels $m \rightarrow d + \nu_1$ and $m \rightarrow d + \nu_2$, caused by the energy and momentum uncertainties, introduced by the time differential detection of the daughter ions in GSI experiments.

We have shown that the idea that neutrinos with all lepton flavours contribute to the EC–decays of the H–like heavy ions, which has been used by Gal to show the non–existence of interference terms, can be adopted for the derivation of the EC–decay rates with interference terms.

We have pointed out that magnetic moments of massive neutrino mass–eigenstates cannot be responsible for the time modulation of the EC–decay rates of the H–like heavy ions. The most important objective is that such a time modulation should be universal for EC and $\beta^+$ decay rates of all H–like heavy ions. This contradicts the experimental data [1]–[4].

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