Radiative decays of pseudoscalar ($P$) and vector ($V$) mesons and the process $e^+e^- \rightarrow \eta'\rho$

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Radiative decays of pseudoscalar and vector mesons are calculated in the framework of the chiral Nambu-Jona-Lasinio (NJL) model. We use the amplitude for triangle quark loops of anomalous type. In evaluating these loop integrals we use two methods. In the first one, we neglect the dependence of external momenta by reproducing the Wess-Zumino-Witten terms of effective chiral meson Lagrangian. In the second method, we take into account the momentum dependence of loop integrals omitting their imaginary part. This makes it possible to allow for quark confinement. As applied both the methods is in qualitative agreement with each other and with experimental data. The second method allows us to describe the electron-positron annihilation with production of $\eta'$ and $\rho$ mesons in the center of mass energy range from 1.6 to 3.5 GeV. The comparison with the recent experimental data is presented.

I. INTRODUCTION

In the local NJL model [1], [2], [3] meson interactions are described in terms of quark loops. If one neglects the dependence of external momenta in the corresponding integrals, the result will not violate chiral symmetry. In this way we can reproduce effective chiral Lagrangian corresponding to $U(3) \times U(3)$ symmetry [4], [2], [5]. In this lagrangian strong interaction vertices are expressed in terms of logarithmically divergent parts of corresponding loop amplitudes. Radiative interactions of mesons are described in terms of quark Feynman diagrams of anomalous type which do not contain ultraviolet divergencies. We use these anomalous quark loops for description of radiative mesons decays.

However for production of pseudoscalar and vector mesons in electron-positron collisions we should keep the external momentum dependence of the amplitudes. In this case, we encounter a serious problem of providing quark confinement condition. To solve this problem, one usually uses a nonlocal version of the NJL model, which involves the relevant formfactors for description of the interaction between mesons and quarks. The choice of formfactors leads to the functional unambiguity. Among the different ways to introduce these formfactors we should mention the QCD approach [6] and the formfactors which arises from the instanton model [7]. It is necessary to mention the models suggested by G. Efimov [8], Yu. Simonov [9], Roberts and so on.

It should be noted that a different form of formfactors leads to a different behavior of amplitudes in the physical region. This becomes essential at large values of external momenta. It is the reason why we apply rather a simple and rough method which consists in exact calculation of amplitudes and neglection of their imaginary parts to avoid the production of free quarks. As a result, we obtain a rather satisfactory description of radiative decay widths of vector and pseudoscalar mesons. This fact allows us to hope that this approach can be applied to describe the processes $e\bar{e} \rightarrow \eta'\rho$, $e\bar{e} \rightarrow \eta'\pi^+\pi^-$ which can be measured in a series of existing and planned experiments with colliding electron-positron beams [10], [11], [12], [13].

For the problems of the last type we hope to obtain only qualitative results in the center of mass energy range $1 – 3$ GeV. In our approach we do not introduce any quark-meson formfactors.

Describing the decays of light mesons we ignore the dependence of corresponding loop integrals of external momenta – Approximation I. The problem of confinement in that approximation is automatically solved. In the case of heavy mesons we keep exact external momentum dependence of relevant loop amplitudes and neglect a possible imaginary part – Approximation II. Approximation II is used further for description of processes at electron-positron colliders.
is described by the amplitude of one loop with quark (see Fig. 1): 

\[ C \]

where \((abcd)\) quarks are involved; \(C\) the angle \(\theta\) where 

\[ \rho = \frac{5}{3} \text{ sin} \theta \]

\[ \lambda = \frac{5}{3} \text{ cos} \theta \]

\[ \bar{g} = \frac{5}{3} \text{ diag} (1, 1, 1, 1) \]

\[ \text{II. RADIATIVE DECAYS OF VECTOR AND PSEUDOSCALAR MESONS} \]

For the description of interaction of mesons with quarks we use the NJL model lagrangian \([2, 3]\):

\[
L_{\text{int}} = \bar{q} \left[ eQ\dot{A} + (i\gamma_5) (g_u\lambda_u\eta_u + g_s\lambda_s\eta_s) + \frac{g_P}{2} \left( \lambda_3 \dot{\phi}_0 + \lambda_u \dot{\omega} + \lambda_s \dot{\phi} \right) \right] q,
\]

where \(\bar{q} = (\bar{u}, \bar{d}, \bar{s})\) where \(u, d, s\) are the quark fields, \(Q = \text{diag}(2/3, -1/3, -1/3)\) is the quark charge matrix, \(\lambda_u = (\sqrt{2}\lambda_0 + \lambda_3) / \sqrt{3}, \lambda_s = (-\lambda_0 + \sqrt{2}\lambda_3) / \sqrt{3}\) where \(\lambda_i\) are the Gell-Mann matrices and \(\lambda_0 = \sqrt{2/3} \text{ diag}(1, 1, 1)\). \(g_u = m_u/f_\pi, g_s = m_s/f_\pi\) are the meson-quark coupling constants which are evaluated by Goldberger-Treiman relation \((m_u = 263 \text{ MeV}, m_s = 407 \text{ MeV} \text{ are quark masses} [14]), \text{ and } f_\pi = 92.4 \text{ MeV} \text{ is the pion decay constant and } f_\pi = 1.3 f_\pi\). \(g_P = 5.94\) is the \(\rho \rightarrow 2\pi\) coupling constant.

Physical states of \(\eta\) and \(\eta'\) mesons are obtained after taking into account of singlet-octet mixing of \(\eta_u\) and \(\eta_s\) with the angle \(\theta = 51.3^\circ [2, 15]\):

\[
\eta = -\eta_u \sin \theta + \eta_s \cos \theta,
\]

\[
\eta' = \eta_u \cos \theta + \eta_s \sin \theta.
\]

We will consider the following processes: \(\rho(\omega) \rightarrow \eta\gamma, \eta' \rightarrow \rho(\omega)\gamma, \phi \rightarrow \eta(\eta')\gamma\).

The vector meson decay

\[ V(p_1) \rightarrow \gamma(p_2) + P(p_3), \]

is described by the amplitude of one loop with quark (see Fig. 1):

\[
M_{V \rightarrow P\gamma} = \frac{i}{(2\pi)^2} e g_P g_V C_{PV} M_q J(p_1^2, 0, p_3^2) (e_1 e_2 p_1 p_2),
\]

where \((abcd) \equiv \varepsilon_{\alpha\beta\gamma\delta} a^\alpha b^\beta c^\gamma d^\delta, g_V = g_\rho/2, g_P = g_u\) if light quarks go through the loop and \(g_P = g_s\) if strange quarks are involved; \(C_{PV}\) is the flavour-color multiplier corresponding to quark-meson interaction, \(C_{\eta\omega} = 2 \sin \theta, C_{\eta\rho} = 6 \sin \theta, C_{\eta'\omega} = 2 \cos \theta, C_{\eta'\rho} = 6 \cos \theta, C_{\eta\phi} = 4 \cos \theta, C_{\eta'\phi} = 4 \sin \theta\); \(M_q\) is the loop quark mass and

\[
J(p_1^2, p_2^2, p_3^2) = \text{Re} \left( \int \frac{dk}{i\pi^2} \frac{1}{(M_q^2 - k^2 - i0) (M_q^2 - (k + p_2)^2 - i0) (M_q^2 - (k - p_3)^2 - i0)} \right) = \text{Re} \left( \int_0^1 dx \int_0^{1-x} dy \frac{1}{M_q^2 - xy p_1^2 - yz p_2^2 - xz p_3^2 - i0} \right),
\]

where \(z = 1 - x - y\). In the heavy quark approximation (Approximation I) we obtain

\[
J(p_1^2, p_2^2, p_3^2) = \frac{1}{2M_q^2},
\]
In this approximation a wide set of decays of light mesons was described in \cite{2} and the results were found to be in good agreement with the experimental data. The matrix element square can be written in the form:

\[ |M_{\text{V} \rightarrow \gamma\gamma}|^2 = \frac{e^2 g_p^2 g_V^2 C_{PV}^2}{(2\pi)^3} \left( M_q, J(M^2_V, 0, M^2_P) \right)^2 \left[ \frac{1}{2} \left( M^2_V - M^2_P \right) \right]^2. \]  

(7)

The phase volume of the final state is:

\[ d\Phi_{\gamma\gamma} = \frac{d^3p_3}{2E_3} = \frac{1}{8\pi} \frac{M^2_V - M^2_P}{M^2_V}. \]

(8)

And then the decay width reads as:

\[ \Gamma_{\text{V} \rightarrow \gamma\gamma} = \frac{1}{3} \frac{\alpha}{2^7\pi^4} \left( \frac{M^2_P - M^2_P}{M^2_V} \right)^3 \left[ g_p \ g_V \ C_{PV} \ M_q \ J(M^2_V, 0, M^2_P) \right]^2. \]

(9)

The relevant expression for radiative pseudoscalar meson decays $P \rightarrow V\gamma$ has the form:

\[ \Gamma_{P \rightarrow V\gamma} = \frac{\alpha}{2^7\pi^4} \left( \frac{M^2_P - M^2_V}{M^2_P} \right)^3 \left[ g_p \ g_V \ C_{PV} \ M_q \ J(M^2_V, 0, M^2_P) \right]^2. \]

(10)

In Table I we present the theoretical results for both the methods – Approximation I (6) and Approximation II (5) – and compare them with the relevant experimental data.

In particular, we would like to note that the ratio $R_{\text{th.}} = \Gamma(\phi \rightarrow \eta'\gamma)/\Gamma(\phi \rightarrow \eta\gamma) = 2.49 \times 10^{-3}$ is in qualitative agreement with the result of the KLOE collaboration $R_{\text{exp.}} = (4.70 \pm 0.47 \text{(stat.)} \pm 0.31 \text{(syst.)}) \times 10^{-3}$ \cite{12}.

### III. ASSOCIATIVE PRODUCTION OF PSEUDOSCALAR AND VECTOR MESONS IN ELECTRON-POSITRON ANNIHILATION

The matrix elements of the processes of associative production of pseudoscalar and vector mesons

\[ e^+(p_+) + e^-(p_-) \rightarrow V(p_1) + P(p_3), \]

(11)

where $s = (p_+ + p_-)^2$, $p_+^2 = m^2$, $p_1^2 = M^2_V$, $p_3^2 = M^2_P$, in the lowest order of the QED coupling constant $\alpha$ have the form (see Fig. 2):

\[ M_{PV} = i \frac{4\pi\alpha}{s} J_{\mu}^\text{em} J^{A\mu}, \]

(12)

where the QED lepton current is $J_{\mu}^\text{em} = \bar{v}(p_+)\gamma_\mu u(p_-)$ and the anomalous current has the form

\[ J^{A\mu} = \frac{g_P \ g_V \ C_{PV}}{(2\pi)^3} (e_1\mu p_1 p_2) \ M_q \ J(p^2_1, s, p^2_3), \]

(13)

where $p_+ + p_- = p_2 = p_1 + p_3$ and $e_1$ is the polarization vector of the vector meson (i.e. $(e_1 p_1) = 0$).
The cross section built by general rules is
\[ d\sigma = \frac{1}{8s} \sum |M^{PV}|^2 d\Phi_{PV}, \]
where the phase volume of the final state has the form:
\[ d\Phi_{PV} = \frac{d^3p_1 d^3p_3}{2E_1 2E_3} \frac{1}{(2\pi)^2} \delta^4(p_+ + p_- - p_1 - p_3). \]

As we are concerned with the total cross section only we can use the property of anomalous current gauge invariance and thus rewrite the final state phase integral as
\[ \sum \int J^A_\mu (J^A_\nu)^* d\Phi_{PV} = \frac{1}{3} \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \int |J^A_\mu|^2 d\Phi_{PV}. \]

The second term in the braces does not give a contribution due to gauge invariance of the lepton current \( J^\text{em}_\mu \). The first term contribution is proportional to
\[ \sum (e_1 \mu p_1 p_2) (e_1^* \mu p_1 p_2) = -\frac{1}{2} \lambda(s, M^2_P, M^2_V), \]
where \( \lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc \) is the well-known triangle function. Thus, the quantity \( |J^A_\mu|^2 \) in (16) does not depend on the vectors \( p_1 \) and \( p_2 \) themselves but only of their squares \( p_1^2 = M^2_P \), \( p_3^2 = M^2_V \). This allows us to calculate the integral over the final state phase volume which, neglecting the masses of leptons, can be written as
\[ \int \frac{d^3p_1 d^3p_3}{2E_1 2E_3} \delta^4(q - p_1 - p_3) = \frac{\pi}{2} \lambda \left( s, M^2_P, M^2_V \right) \frac{s}{s}. \]

Then the total cross section obtains the form:
\[ \sigma(s) = \frac{\alpha^2}{96\pi^3 s^3} \lambda^{\frac{3}{2}} \left( s, M^2_P, M^2_V \right) \left| g^\gamma g^P C^PV M_q J \left( M^2_P, M^2_V, s \right) \right|^2. \]

The differential cross section can be written as:
\[ \frac{d\sigma}{d\Omega} = \sigma(s) \frac{3 \left( 1 + \cos^2 \theta \right)}{16\pi}, \]
where \( \theta \) is the center of mass angle between the direction of 3-momenta of the initial electron \( p_1 \) and the final vector particle momentum direction \( p_1 \).

Let us consider the concrete process \( e^- e^+ \rightarrow \eta' \rho \). The expression for the total cross section (19) implied only contact Feynman diagram, i.e., Fig 2 a. Recalling the possible conversion of virtual photon into vector mesons beyond resonances one must take into account the diagrams presented on Fig 2 b. This leads to the replacement of the factor \( s^{-3} \) in (19) by the following one:
\[ \frac{1}{s^3} \rightarrow \frac{1}{s^3} \left( 1 - \frac{1}{2 \left( 1 - \frac{M^2_P}{s} \right)} \right)^2. \]
The cross section of the process $e^+e^- \rightarrow \eta'(950)\rho$ is drawn on Fig. 3 where the relevant experimental data are also shown. One can conclude that satisfactory agreement within the experimental errors is observed.

Let us now make a prediction for the process $e\bar{e} \rightarrow \eta'\phi$. The relevant correction factor is

$$
\left( \frac{1}{s'^2} - \frac{1}{s^2} \right) \left( 1 - \frac{1}{3\sqrt{2} \left( 1 - \frac{M_{\phi}^2}{s} \right)} \right)^2.
$$

Besides only $s$-quark loop works. The result is given in Fig. 4

IV. CONCLUSION

In this paper, we investigated the radiative decays of vector and pseudoscalar mesons described by the quark loops of anomalous type.

Let us note that in [10] we considered the process $\phi \rightarrow f_0(980)\gamma$ within the same framework of the NJL model. However, there the quark loop contribution was small enough and the main contribution arose from terms of next order of $1/N_c$ expansion (where $N_c$ is the number of colors) – meson loops. In this paper, we have another situation: meson loops are absent totally and only quark loops of anomalous type give a contribution to the amplitude of the process.

Both the approaches (Approximation I and Approximation II) were considered. We show that the application of the NJL model leads to rather satisfactory agreement with the modern experimental data for the radiative decays. That allows us to use the Approximation II to calculate the cross sections of associative vector and pseudoscalar mesons production in the electron-positron annihilation channel in the lowest order of electromagnetic constant. In the last case, the heavy virtual photon converts to the set of pseudoscalar and vector mesons. Two mechanisms must be taken into account: first one with the intermediate virtual photon and the second one which contains the conversion of intermediate photon into vector meson. We give a comparison of our prediction for the process $e\bar{e} \rightarrow \eta'\rho$ with the experimental data of the BABAR collaboration. For the process $e\bar{e} \rightarrow \eta'\phi$ the prediction is given for future experimental data.
FIG. 4: The prediction for cross section $e^+ e^- \rightarrow \eta' \phi$.

For comparison with the experiment the precision of our results is worth mentioning. We should like to notice that for the NJL model results precision is of an order of 20-30%.

One of our important theoretical assumptions is the absence of the imaginary part of relevant amplitudes. The mechanism of elimination of the imaginary part is tightly connected with the confinement nature and is not considered here. We carry out the elimination "by hand" ("naive confinement"). It is to be noted, however, that if the imaginary part is taken into account, the considerable disagreement with the experimental data will occur in decay case. For instance, if the imaginary part of the amplitude is taken into account, the decay width of $\phi \rightarrow \eta' \gamma$ is $\Gamma^{th.}_{\phi \rightarrow \eta' \gamma} = 0.824$ KeV while the experiment gives $\Gamma^{exp.}_{\phi \rightarrow \eta' \gamma} = 0.265$ KeV (see Table I).

Concerning the singlet-octet angle mixing we use the additional interaction of the 'tHooft type in lagrangian in the NJL model [15], [22], [23]. This approach was widely used in literature [24], [25].

Let us note however that the alternative solution of mixing angle problem was developed in [18], [19], [20], [21]. In this approach two mixing angles appear. The application of the both approach to describe the decays of the pseudoscalar and vector mesons leads to the similar results.

Our results for decays in Approximation I are in agreement with the ones obtained in [17] (compare Table I and Table 2 in [17]).

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[1] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961).
[2] M. K. Volkov, Fiz. Elem. Chast. Atom. Yadra 17, 433 (1986).
[3] M. K. Volkov and A. E. Radzhabov, Phys. Usp. 49, 551 (2006).
[4] S. Gasiorowicz and D. A. Geffen, Rev. Mod. Phys. 41, 531 (1969).
[5] D. Ebert and H. Reinhardt, Nucl. Phys. B271, 188 (1986).
[6] B. A. Arbuzov, M. K. Volkov, and I. V. Zaitsev, Int. J. Mod. Phys. A21, 5721 (2006), hep-ph/0604051.
[7] R. S. Plant and M. C. Birse, Nucl. Phys. A628, 607 (1998), hep-ph/9705372.
[8] G. V. Efimov and M. A. Ivanov, Bristol, UK: IOP (1993) 177 p.
[9] A. Di Giacomo, H. G. Dosch, V. I. Shevchenko, and Y. A. Simonov, Phys. Rept. 372, 319 (2002), hep-ph/0007223.
[10] R. R. Akhmetshin et al., Phys. Lett. B642, 203 (2006).
[11] BABAR, B. Aubert et al., Phys. Rev. D76, 092005 (2007), arXiv:0708.2461 [hep-ex].
[12] KLOE, A. Aloisio et al., Phys. Lett. B541, 45 (2002), hep-ex/0206010.
[13] BES, M. Ablikim et al., Phys. Rev. Lett. 97, 202002 (2006), hep-ex/0607006.
[14] A. E. Radzhabov, M. K. Volkov, and N. G. Kornakov, (2007), arXiv:0704.3311 [hep-ph].
[15] M. K. Volkov, M. Nagy, and V. L. Yudichev, Nuovo Cim. A112, 225 (1999), hep-ph/9804347.
[16] Y. M. Bystritskiy, M. K. Volkov, E. A. Kuraev, E. Bartos, and M. Secansky, Phys. Rev. D77, 054008 (2008), arXiv:0712.0304 [hep-ph].
[17] J. Prades, Z. Phys. C63, 491 (1994), hep-ph/9302246.
[18] P. Ball, J. M. Frere, and M. Tytgat, Phys. Lett. B365, 367 (1996), hep-ph/9508359.
[19] R. Escribano and J. Nadal, JHEP 05, 006 (2007), hep-ph/0703187.
[20] R. Escribano and J. M. Frere, Phys. Lett. B459, 288 (1999), hep-ph/9901405.
[21] R. Escribano and J.-M. Frere, JHEP 06, 029 (2005), hep-ph/0501072.
[22] M. K. Volkov, Phys. Part. Nucl. 24, 35 (1993).
[23] D. Ebert, H. Reinhardt and M. K. Volkov, Prog. Part. Nucl. Phys. 33, 1 (1994).
[24] S. P. Klevansky, Rev. Mod. Phys. 64, 649 (1992).
[25] U. Vogl and W. Weise, Prog. Part. Nucl. Phys. 27, 195 (1991).