Transferring Einstein-Podolsky-Rosen State Cross Frequency Bands via Cavity Electro-Opto-Mechanical Converters

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ABSTRACT In order to transfer two-mode quantum state from optical input to microwave output, we propose a scheme based on double cavity electro-opto-mechanical converters. Principles and operation modes of cavity electro-opto-mechanical converter are introduced in detail, and theoretical model of transferring Einstein-Podolsky-Rosen state via our scheme is built. The dynamics and corresponding scattering matrices of our scheme are analyzed carefully. Meanwhile, Ulhmann fidelity is utilized to quantify the similarity of initial state and final state, which also stands for the performance of our scheme. Influences of coupling rates and optical input bandwidth on fidelity are well discussed, and some significant factors are also taken into consideration, such as transmission spectrum, interaction rates and decoherence rates. Furthermore, Wigner distributions of initial state and final state as well as the von Neumann entropy are obtained to verify the performance of our scheme. The results show that our scheme could transfer the Einstein-Podolsky-Rosen state from optical input to microwave output with high fidelity (0.74), and the entropy ratio between initial and final states is about 0.78, which is quite close with the fidelity. Besides, Wigner distributions also show that initial state and final state are slightly different in phase structures. In case of strong coupling rates, ultralow temperature and narrow input bandwidth, the performance of our scheme could be further enhanced.

INDEX TERMS Cavity electro-opto-mechanical converter, Einstein-Podolsky-Rosen state, optical-microwave cross frequency bands interaction, quantum entanglement, two-mode quantum state transfer.

I. INTRODUCTION
Cavity electro-opto-mechanical (EOM) converter is a novel achievement in cavity optomechanics. As it connects quantum behaviors of both optical regime and microwave regime via interaction with mechanical resonance, it shows potentials in preparing and observing quantum phenomena conveniently. In recent years, this excellent quantum platform has been widely studied and applied in various fields [1]–[4].

Intra-cavity or itinerant state transfer is a significant ability of cavity EOM converter. Various theoretical and experimental work have been done to explore this feature [5]–[8]. These work show transferring single mode quantum state with high fidelity could be achieved via cavity EOM converter from optical regime to microwave regime. However, rare researches are done to explore two mode or multimode quantum state transfer via cavity EOM converters. In fact, such researches are necessary for broadening the application potentials of cavity EOM converter.

Einstein-Podolsky-Rosen (EPR) state is a typical two-mode state as well as a widely studied entanglement state [9], [10]. If this state could be well transferred from optical signals to microwave signals, it will be beneficial for relevant fields like quantum computing [11] and quantum communication network [12]. This would also help to introduce quantum feature into fields like radar, imaging, and navigation, which are designed to operate in microwave regime. This paper aims at transferring EPR state from optical input to microwave output via double cavity EOM converters, and proposes the theoretical model of the two-mode quantum state transferring process. The performance is quantified with Ulhmann fidelity and discussed in case of weak coupling and strong coupling respectively. Besides, the Wigner distributions and von Neumann entropy of initial state and final state are also obtained and compared to verify the performance intuitively. It shows
TABLE 1. Denotations of main parameters in cavity.

| Cavity/vibration mode | Microwave cavity | Mechanical oscillator | Optical cavity |
|-----------------------|------------------|-----------------------|----------------|
| uncertainty           | \( \hat{C}_M \)   | \( \hat{m} \)         | \( \hat{C}_O \) |
| Cavity/vibration mode frequency | \( \omega_m \) | \( \omega_o \) | \( \omega_O \) |
| Damping rate          | \( \kappa_M \)   | \( \gamma \)         | \( \kappa_O \) |
| Leakage to external channel | \( \kappa'_M \) | -                     | \( \kappa'_O \) |
| Driving mode frequency | \( \omega_{s,M} \) | -                     | \( \omega_{s,O} \) |
| Detuning of frequency | \( \Delta_M \)   | -                     | \( \Delta_O \) |

that a high fidelity (0.74) EPR state transfer is achievable via our scheme.

II. TRANSFERRING SCHEME UTILIZING DOUBLE CAVITY EOM CONVERTERS

A cavity EOM converter is a hybrid quantum platform where a mechanical oscillator is connected to an optical F-P cavity and a microwave LC circuit cavity simultaneously in superconducting environment. The common form of mechanical oscillator is a silicon plate with part of the circuit on it. The silicon plate is the movable mirror of the optical cavity, and the part of the circuit is the movable plate of a capacitor in microwave LC circuit. The outer surface of the silicon plate dose not have coating material on it, while its inner surface is coated with highly reflective material, which only allows the transmission of desired cavity mode. The mass and size of the oscillator are in the level of ng and nm respectively. This unique structure enables the interaction pattern as optical photon-mechanical resonance phonon-microwave photon. According to the cavity detuning, the cavity EOM converter can be operated in two different modes: state transfer mode (\( \Delta_M = \Delta_O = \omega_m \)) [13] and microwave-optical entanglement preparation mode (\( \Delta_M = -\Delta_O = \omega_m \)) [14]. A typical schematic of cavity EOM converter operated in the state transfer mode can be depicted as Fig. 1.

We would like to explain the denotations of this paper first. The detuning of each cavity can be expressed as \( \Delta_j = \omega_j - \omega_{j,M} \). The subscript \( j = M, O \) stands for the parameter of microwave or optical wave in the whole paper. For simplicity, we won’t repeat the quotation of the \( j = M, O \). Other parameters of the cavity EOM converter and their denotations are also listed in Tab. 1.

Here we treat the cavity mode as the sum of steady state amplitude and quantum fluctuation around it: \( \hat{C}_j = C_j + \delta \hat{C}_j \). The coupling rate between cavities and mechanical resonator can be expressed as \( G_j = g_j \sqrt{\hat{C}_j} \), where \( g_j \) is single photon-phonon coupling rate and \( C_j \) is the steady state amplitude of corresponding cavity mode. The optical input and microwave output are denoted as \( \hat{O} \) and \( \hat{M} \), respectively. In fact, the working bandwidth is very narrow, which can be seen later in the analysis of scattering. For simplicity, we only consider the fundamental mode in each cavity and mechanical oscillator.

When the frequency detuning setting is \( \Delta_M = -\Delta_O = \omega_m \), the cavity EOM converter is operated in the M-O entanglement preparation mode, which is realized by a parametric down-conversion process and a beam-splitter process. When the frequency detuning setting is \( \Delta_M = \Delta_O = \omega_m \), the cavity EOM converter is operated in the intra-frequency energy transfer and state transfer mode as shown by Fig.1, which is facilitated by two beam-splitter processes. The detailed explanations on the intra-frequency energy transfer and state transfer mode and its physical processes can be found in Appendix A.

Here in this paper, we apply two cavity EOM converters which are both operated in the state transfer mode (\( \Delta_M = \Delta_O = \omega_m \)) with the same parameter setting. Our scheme can be depicted as Fig.2. Each beam of EPR state optical input passes through a cavity EOM converter, experiencing a high fidelity state transfer process and converting into EPR state microwave output. Here we aim to make both converters operated in a symmetric manner and obtain the same efficiency. Of course, if two converters have different efficiency, it will degrade the performance as the quantum state is two-mode state and asymmetry will lead to part of the correlation information missing in the transfer process. In fact, we can hardly fabricate two completely identical converters in practical view, and thus we can never achieve 100% state transfer. However, we can restrict difference between two converters to a degree which is tiny enough, and we could still achieve a rather high fidelity. There are several ways to restrict asymmetry, such as nano-scale finishing, balanced pumps, feedback control and so on.

The Ulhmann fidelity [15] is a commonly utilized quantity for the state transfer performance. It shows the similarity between the initial state and the final one. As it is also a quantity related with probability and its value satisfies normalization criterion, the fidelity itself is also the efficiency of the state transfer. The Ulhmann fidelity can be expressed as

\[
F \equiv \left( \text{Tr} \left[ \left( \sqrt{\rho_1} \hat{\rho} \sqrt{\rho_1} \right)^{1/2} \right] \right)^2,
\]

where \( \hat{\rho} \) and \( \hat{\rho}_j \) are density matrixes of initial state and final state. As for the EPR state, its joint distribution is the same as the photon distribution in each mode [16], which means

\[
P^{(a,b)}(n) = P^{(a)}(n) = P^{(b)}(n) = \frac{(n)!}{(n + 1)!^{a+b+1}},
\]

where \( P^{(a,b)}(n) \) is the photon number joint distribution of the EPR state; \( P^{(a)}(n) \) and \( P^{(b)}(n) \) are photon number distributions in each mode; \( n \) stands for the photon number and \( a \) \((b)\) stands for one of the entangled beams; and \( (n) \) is the average photon number of corresponding state. Thus, the density matrixes of
initial state and final state can be expressed as [17]

\[
\hat{\rho}_i = \sum_n p_{n,n}^{(a,b)} |n\rangle \langle n|,
\]

(3)

\[
\hat{\rho}_f = \sum_n p_{n,n}^{(a,b)} |n\rangle \langle n|,
\]

(4)

where \(p_{n,n}^{(a,b)}\) and \(p_{n,n}^{(a,b)}\) are the joint distributions for initial state and final state. They can be obtained by (2) with different average photon number as \(\langle n \rangle_i\) and \(\langle n \rangle_f\) respectively.

The average photon number of initial state \(\langle n \rangle_i\) can be controlled via the squeezing degree in the preparation stage. This means we can easily obtain \(\hat{\rho}_i\) by (2) and (3). As for the \(\hat{\rho}_f\), we need to figure out the dynamics in the cavity EOM converters and then obtain the output average photon number \(\langle n \rangle_f\). Both cavity EOM converters share identical input-output relationship. It is convenient to utilize Heisenberg–Langevin equations [18] to study the dynamics of state transfer in each cavity EOM converter, and these relations can be solved by using standard input-output relations [19] in the interaction picture with respect to the frequency of cavity pump, which yields [20]

\[
D_{\text{out}} [\omega] = s [\omega] D_{\text{in}} [\omega] + s' [\omega] D'_{\text{in}} [\omega],
\]

(5)

where \(D_{\text{out}} [\omega]\) represents the output vector; \(D_{\text{in}} [\omega]\) stands for the input vector; \(D'_{\text{in}} [\omega]\) is the vector containing noise photons and phonons; \(s [\omega]\) is a \(3 \times 3\) matrix describing scattering between optical cavity, the microwave cavity and the thermal bath from mechanical dissipation, while \(s' [\omega]\) is a \(3 \times 2\) matrix which describing how noise from the sources of internal cavity loss can influence cavity modes and mechanical resonance. To be specific, we have expressions as

\[
\begin{align*}
D_{\text{in}} [\omega] & = \left( \hat{O}_{\text{in}} [\omega] \quad \hat{M}_{\text{in}} [\omega] \quad \hat{m}_{\text{in}} [\omega] \right)^T, \\
D_{\text{out}} [\omega] & = \left( \hat{O}_{\text{out}} [\omega] \quad \hat{M}_{\text{out}} [\omega] \quad \hat{m}_{\text{out}} [\omega] \right)^T, \\
D'_{\text{in}} [\omega] & = \left( \delta \hat{C}_O [\omega] \quad \delta \hat{C}_M [\omega] \right)^T.
\end{align*}
\]

As for the scattering matrices \(s [\omega]\), it can be easily found that the element \(s_{21} [\omega]\) is the key parameter for the state transfer, which describes the desired input-output process. As \(s_{21} [\omega]\) shows ideal transmission window at frequency near the mechanical resonance \(\omega_m\), we may arrive at the expressions for \(s [\omega]\) at \(\omega = \omega_m\) as (7), as shown at the bottom of the next page, where \(c_O = \kappa_O/(\kappa_O + \kappa'_O)\) and \(c_M = \kappa_M/(\kappa_M + \kappa'_M)\) are corresponding ratios of cavity intrinsic damping rates over the whole damping rates; \(\Gamma_O\) and \(\Gamma_M\) are cooperativity parameters of corresponding cavity with expressions as \(\Gamma_O = G_O \kappa_O\) and \(\Gamma_M = G_M \kappa_M\gamma\), while the hybrid cooperativity \(\Gamma\) can be expressed as \(\Gamma = 1 + 4\Gamma_M c_M + 4\Gamma_O.\) As for the elements of \(s' [\omega]\), they can be obtained via relation \(s'_{ij} [\omega_m] = s_{ij} [\omega_M] \sqrt{k'_i/k_j}\). Detailed deductions for scattering matrices can be found in Appendix B.

To be specific, the microwave output and the output average photon number \(\langle n \rangle_f\) can thus be expressed as (8) and (9).

\[
\langle n \rangle_f = \left( \hat{M}_{\text{out}} [\omega_m] \hat{M}_{\text{out}} [\omega_m] \right)
\]

(8)

\[
\langle n \rangle_f = \left( \hat{M}_{\text{out}} [\omega_m] \hat{M}_{\text{out}} [\omega_m] \right)
\]

(9)

where \(\langle n_{0,\text{in}} \rangle = \langle n \rangle_i\) and \(\langle n_{0,\text{in}} \rangle\) are input average photon numbers of optical cavity and microwave cavity respectively; \(\langle n_{M,\text{in}} \rangle\) \(\langle n_{0,\text{noise}} \rangle\) \(\langle n_{0,\text{noise}} \rangle\) correspond to numbers of mechanical thermal excitation phonons, thermal photons in optical cavity and microwave cavity respectively, and their values depend on the Planck’s law [21].

Besides, there is one more criterion for efficient state transfer, which guarantees the correlations between cavity modes can be also transferred into outputs. This criterion requires that the photon-phonon interaction rates \(G_j^2/k_j\) should be larger than decoherence rate of mechanical oscillator \(\gamma (\nu_m)\), which ensures that the correlation would not been destroyed by reheatting effect of mechanical oscillator before it is output.

Hence, the fidelity of transferring EPR state from optical input to microwave output can be calculated via (1), and the performance of our scheme can be quantified.

III. RESULTS AND DISCUSSIONS

In order to check the performance of our scheme quantitatively, we assume experimentally achievable values for basic
parameters. Frequencies of optical cavity mode, microwave cavity mode and mechanical resonance are $\omega_O/2\pi = 10 \text{THz}$, $\omega_M/2\pi = 10 \text{GHz}$ and $\omega_p/2\pi = 1 \text{GHz}$; pump powers of optical cavity and microwave cavity are $P_O = 5 \text{ mW}$ and $P_M = 35 \text{ mW}$; $\Delta_M = \Delta_O = \omega_m$ and $c_O = c_M = 0.2$ are satisfied in both cavity EOM converters; the average photon number of initial state $\langle n_i \rangle = 0.5$, and the input average photon number of microwave cavity is set to be $\langle n_{m,\text{in}} \rangle = 0$. As for the environment settings, we assume that cavity temperature and external temperature as $T_E = 20 \text{ mK}$ and $T_B = 293 \text{ K}$ respectively. As for the relation between cavity dimensions and cavity eigenmodes, we discuss this in Appendix C.

Assuming the cavity intrinsic damping rates and mechanical damping rate in both cavity EOM converters satisfy $\kappa^C_0 = \frac{\kappa}{2\pi} = 0.5 \text{ KHz}$ and $\frac{\kappa'}{2\pi} = 30 \text{ Hz}$, we can obtain the fidelity of our scheme versus coupling rates $G_O$ and $G_M$ as Fig.3 shows.

In Fig.3, the fidelity is quite low when both cavity EOM converters are operated in the weak coupling regime, where $G_O$ ($G_M$) is smaller than $\kappa$. As the coupling rates grow, the fidelity increases swiftly and reaches its maximum $F \approx 0.74$ soon in the strong coupling regime ($G_i \gg \kappa$). The physical explanation for this phenomenon can be stated in two aspects.

On one hand, $G_i \leq \kappa$ means the energy change rate is slower than its dissipation, which disables the optical photon-microwave photon conversion. On the other hand, $G_O^2/\kappa_0 < \gamma \langle n_m \rangle$ and $G_M^2/\kappa_M < \gamma \langle n_m \rangle$ also show that interaction rates are slower than decoherence rate, which disables the state transfer from input to output. In strong coupling regime, these requirements are well satisfied, and thus high fidelity state transfer is achievable. Assuming $G_O = G_M = G$, optical cavity and microwave cavity in both converters share identical interaction rate as $G_O^2/\kappa_0 = G_M^2/\kappa_M = G^2/\kappa$.

The interaction rate can be plotted versus reduced coupling rate as Fig. 4, and decoherence rates in different cavity temperatures are also shown in Fig.4 for comparison.

It is clear that the interaction rate increases with the coupling rate, and it climbs faster in the strong coupling regime. The larger the coupling rate is, the faster the photons interacts with phonons. It also shows that cavity temperature is not one of the influence factor of interaction rate. On the contrary, the decoherence rate $\gamma \langle n_m \rangle$ relies heavily on the cavity temperature. When the cavity temperature is low enough, the mechanical excitation phonon number is well suppressed and the decoherence rate is very low. Once the cavity temperature

\[
\begin{aligned}
    s_{\omega_m} = & \left( 1 - c_O \left( 2 - \frac{8\Gamma c_O}{\Gamma} \right) \frac{8\sqrt{\Gamma_o \Gamma_M} c_O c_M}{\Gamma} \frac{4i\sqrt{\Gamma_o}}{\Gamma} c_O \right) \\
    & \left( 1 - c_M \left( 2 - \frac{8\Gamma c_M}{\Gamma} \right) \frac{4i\sqrt{\Gamma_M}}{\Gamma} c_M \right).
\end{aligned}
\]

(7)
risers to the level of several $K$, the mechanical excitation phonon number becomes rather large and the decoherence rate will dominate the intracavity process even in the strong coupling regime. We can also find that the decoherence rate is irrelevant with coupling rate. The lower the cavity temperature is, the lower the decoherence rate is. A cavity temperature of 0.02K only corresponds to a decoherence rate of $3 \times 10^3$, which could be basically ignored in strong coupling regime comparing with the interaction rate. This means that the cavity temperature we choose ($T_E = 20$ mK) arouses even slower decoherence than that of 0.02K, and this is a proper temperature for state transfer.

Another issue is how the bandwidth of the optical input in each cavity EOM converter influences the fidelity. The scattering matrix could well explain the input-output relation of our scheme, and the transmission quality is mainly influences by the element $s_{21}$ [$\omega$] as discussed above. We consider its transmission spectrum in weak (strong) coupling regime by setting $G_O = G_M = 10k=1000\gamma$ ($G_O = G_M = 0.5k=1000\gamma$), and its transmission spectrum can be illustrated as Fig. 5(a). There is a perfect transmission window which appears in both weak and strong coupling regime. This transmission window has a very narrow bandwidth centered at $\omega_{in}$, which means the mechanical resonance of balanced cavity EOM converter has a strictly limited bandwidth and the limitation originates from the interaction pattern as optical photon-mechanical phonon-microwave photon. Hence, the optical input bandwidth influences the transmission feature, and the transmission feature limits the performance of microwave output. This reveals the mechanism of how the optical input bandwidth influences the fidelity, and it can be illustrated as Fig. 5(b).

As the optical input bandwidth grows, it degrades the possibility and ability of cavity EOM converter to transfer input state efficiently and the fidelity decreases in both weak and strong coupling regime. When we compare fidelity variances in both cases, we find that the fidelity in the weak coupling case falls faster versus optical input bandwidth than that in the strong coupling case. It is clearly shown that strong coupling is more helpful to maintain a high fidelity than weak coupling. However, when the optical input bandwidth is too broad to transfer, the coupling strength would not make difference as the curves indicate.

We can also check the performance of state transfer in a more intuitive way via Wigner distributions. As each mode of EPR state has the average photon number as

$$\langle n \rangle = \sinh^2(r), \quad (10)$$

thus we can obtain two-mode squeezed amplitude $r$ according to the average number of EPR state. Then, we can further obtain the Wigner distribution in subspaces $\{(X_a, X_b), (Y_a, Y_b)\}$ via the following equations [22]

$$W_{XX}(X_a, X_b) = \frac{2}{\pi} \exp \left[ -e^{-2r}(X_a+X_b)^2 - e^{2r}(X_a-X_b)^2 \right]. \quad (11)$$

$$W_{YY}(Y_a, Y_b) = \frac{2}{\pi} \exp \left[ -e^{-2r}(Y_a-Y_b)^2 - e^{2r}(Y_a+Y_b)^2 \right]. \quad (12)$$

where $X_{ab}$ and $Y_{ab}$ are amplitude and phase quadrature components of mode $a(b)$. As $|n_i| = 0.5$ is given and $|n_f|$ can be calculated by (9), we can obtain specific values of two-mode squeezed amplitudes as $r_f \approx 0.6585$ and $r_f \approx 0.5468$ in strong coupling regime ($G_O = G_M = 10k=1000\gamma$). The two-mode squeezing amplitudes we assume here may be higher than practical values, but this would not influence much as we focus more on the similarity between features of initial and final states. The Wigner distributions of initial state and final state can be thus illustrated as Fig.6.

Comparing Wigner distributions of initial state and final state, both of them exhibit two-mode squeezed feature in subspaces $\{(X_a, X_b), (Y_a, Y_b)\}$. We can easily find that the final state only shows tiny distortion in the distribution profiles comparing with the initial state, which indicates a smaller two-mode squeezed amplitude. Similar distribution profiles also mean a high fidelity state transfer can be achieved by our scheme in strong coupling regime.

Furthermore, we can utilize the von Neumann entropy [23] $E(r)$ of initial and final state to quantify their respective entanglement degree so that we can easily verify the high fidelity transfer of EPR state. The von Neumann entropy $E(r)$ is a widely adopted entanglement quantification and can be expressed as (13) in terms of two-mode squeezed amplitude.

$$E(r) = -\sum_n P_{n,n} \ln P_{n,n} \approx - \sum_n \frac{(\tanh^2 r)^n}{\cosh^2 r} \ln \frac{(\tanh^2 r)^n}{\cosh^2 r}. \quad (13)$$

By (13), it is convenient to obtain the von Neumann entropy of initial state and final state. The results are $E(r_i) \approx 0.9444$ and $E(r_f) \approx 0.7430$, and their ratio turns to be $E(r_f)/E(r_i) \approx 0.7867$, which is very close to the level of aforementioned fidelity in strong coupling regime. The Wigner distributions and the von Neumann entropy both
prove the high fidelity transfer of EPR state from optical input to microwave output can be achieved via our scheme, and the results are reciprocal with the Ulmann fidelity.

As for the possible implementations, we could utilize silicon chip as optical cavity mirror (stabilized or movable) to form convenient Fabry-Perot cavity. The microwave cavity can be realized by superconducting LC circuits with one plate of capacitor on the movable mirror of optical cavity, while the rest of the circuits are stabilized on another silicon chip. Thus, the movable mirror is exact the mechanical oscillator. All
the sizes are supposed to design at nano scale. The ultralow operation temperature could be achieved by ultrahigh vacuum dilution refrigerator. Of course, the maintenance of ultralow operation temperature as 20mK would lead to a high cost. We may try a higher temperature, but the thermal phonon excitation and thermal cavity noise photons would degrade the state transfer fidelity. Hence, the balance between cost and performance should be well considered in implementation such a scheme.

This scheme has broad potential applications in quantum computing, quantum network, quantum state teleportation, quantum interface and so on.

IV. CONCLUSION
Based on two cavity EOM converters, an EPR state transfer scheme is proposed in this paper. The physical dynamics of such two-mode state transfer is analyzed, and the theoretical model of the whole process is built. Ulhmann fidelity, Wigner distributions and von Neumann entropy are applied to discuss the performance of our scheme. In strong coupling regime, our scheme achieve a fidelity as high as $F \approx 0.74$, and the ratio of the von Neumann entropy of initial state and final state turn to be $E(r_f)/E(r_i) \approx 0.7867$. The two-mode squeezing factors of input entanglement and output entanglement are $r_i \approx 0.6585$ and $r_f \approx 0.5468$, which are quite close. Besides, Wigner distributions of optical input and microwave output also display quite close profiles, which intuitively shows the similarity between initial state and final state. In theory, detailed analysis and intensive discussions prove that our proposal is capable of high fidelity transfer of two-mode quantum state like EPR state cross different frequency bands.

APPENDIX A.
The intra-frequency energy transfer and state transfer mode of cavity EOM converter.

The pumps here are utilized to provide adequate energy for transfer process, and their interactions with phonons help to form cavity modes. To be specific, the pump photons would experience anti-Stokes process under $\Delta M = \Delta O = \omega_m$, where a phonon would absorb a pump photon and scatter into a cavity mode photon. In other words, pumps help phonons turning into cavity mode photons in each cavity, while pumps themselves do not interact with cavity modes directly.

In fact, even optical and microwave cavity mode do not interact with each other directly. Each of them interacts with mechanical oscillation directly, forming a tripartite interaction pattern as microwave cavity mode-mechanical oscillation-optical cavity mode. This interaction (or coupling) could be shown by Hamiltonian as

$$H = \hbar G_M (\delta \hat{C}_M \hat{n} + \hat{n} \delta \hat{C}_M^\dagger) + \hbar G_O (\delta \hat{C}_O \hat{m} + \hat{m} \delta \hat{C}_O^\dagger).$$

(A.1)

where both terms are beam-splitter like terms. Both cavity modes interact with mechanical mode by beam-splitter like transformation. If we look into the operator evolution of cavity mode and mechanical mode, we may have a clearer understanding on the physical process of the state transfer in each cavity EOM converter. The physical process can be described as two steps.

Step 1: After optical input photons entering the cavity and scattering into cavity resonance photons, they will interact with mechanical phonons, and this interaction swaps the state of optical input pulse into the mechanical resonance, which can also be easily understood in aspect of the time evolution of the operators. The beam-splitter operation can be expressed as

$$\hat{O}(t) = \cos(G_Ot)\hat{O}(0) + \sin(G_Ot)\hat{m}(0),$$

(A.2)

$$\dot{\hat{m}}(t) = \sin(G_Ot)\hat{O}(0) - \cos(G_Ot)\dot{\hat{m}}(0),$$

(A.3)

where $\hat{O}(0)$ is optical input we are going to transfer and $\dot{\hat{m}}(0)$ is the original state of mechanical resonator. When the interaction time interval is $t_1 = \pi/2G_O$, the operators arrive at

$$\hat{O}(t_1) = \dot{\hat{m}}(0),$$

(A.4)

$$\hat{m}(t_1) = \hat{O}(0).$$

(A.5)

Both equations clearly show that the state of optical input has been swapped to mechanical resonance after interaction for $t_1 = \pi/2G_O$.

Step 2: After the first step, similar transfer occurs between mechanical resonance and microwave output. When the beam-splitter like interaction between mechanical resonance and microwave output lasts $t_2 = \pi/2G_M$ after Step 1, the operators arrive at

$$\dot{\hat{m}}(t_1 + t_2) = \dot{\hat{M}}(0),$$

(A.6)

$$\hat{M}(t_2) = \hat{m}(t_1) = \hat{O}(0).$$

(A.7)

Both equations clearly show that the state of mechanical resonance has been swapped to microwave output after interaction for $t_2 = \pi/2G_M$. Hence, the state of optical input is transferred to the state of microwave output.

APPENDIX B.
Detailed deductions for scattering matrices.

In the interaction picture with respect to corresponding cavity pumps, The Heisenberg–Langevin equations of each cavity EOM converter can be expressed as

$$\dot{\hat{O}} = i\Delta_O \hat{O} - \frac{\kappa_O}{2} \hat{O} - iG_O \hat{m} - \sqrt{\kappa_O} \hat{O}_\text{in} - \sqrt{\kappa_O} \delta \hat{C}_O,$$

(B.1)

$$\dot{\hat{M}} = i\Delta_M \hat{M} - \frac{\kappa_M}{2} \hat{M} - iG_M \hat{m} - \sqrt{\kappa_M} \hat{M}_\text{in} - \sqrt{\kappa_M} \delta \hat{C}_M,$$

(B.2)

$$\dot{\hat{m}} = -i\omega_m \hat{m} - \frac{\gamma}{2} \hat{m} - iG_O \hat{O} - iG_M \hat{M} - \sqrt{\gamma} \hat{m}_\text{in},$$

(B.3)

and they can be written in matrix form as (B.5).

$$D_{out} [\omega] = s [\omega] D_{in} [\omega] + s' [\omega] D'_{in} [\omega].$$

(B.5)
Taking the standard input-output relation into consideration, we can obtain the scattering matrix \( s[\omega] \) \((3 \times 3)\) as where \( \chi_O[\omega] \) and \( \chi_M[\omega] \) are the susceptibilities of optical and microwave mode, and they can be expressed as

\[
\chi_O[\omega] = \frac{\kappa_O}{\kappa_O + \kappa_D^{\text{eff}}} \quad \text{and} \quad \chi_M[\omega] = \frac{\kappa_M}{\kappa_M + \kappa_D^{\text{eff}}},
\]

As for the parameter \( \nu[\omega] \), it can be expressed as

\[
\nu[\omega] = \left( \frac{1}{8\chi_m[\omega]} + \frac{1}{2} \Gamma_O \chi_O[\omega] + \frac{1}{2} \Gamma_M \chi_M[\omega] \right)^{-1},
\]

where \( \chi_m[\omega] = \frac{x}{\omega} \) and \( \nu \) is the distance between two plates of the movable capacitor; \( m \) is the mass of mechanical oscillator; \( \mu \) is a dimensionless parameter related with capacitors of LC circuits; \( d \) is the distance between two plates of the movable capacitor; and \( h \) is Planck’s constant.

**APPENDIX C.**

Relation between cavity dimensions and its eigenmodes

There are many previous work on the cavity dimensions criteria \([24],[25]\). As we discussed in the main text, the cavity eigenmodes are determined by the rebalanced cavity conditions. To be specific, there is deterministic mathematical relation between cavity dimensions and its eigenmodes. We illustrate this relation with optical cavity and its eigenmodes as an example.

The relation between cavity length \( l \) and the wavelength of cavity modes \( \lambda \) can be written as

\[
l = \frac{n\lambda}{2}, \quad n = 1, 2, 3 \cdots.
\]

Taking \( \omega = c k \) and \( k = \frac{2\pi}{\lambda} \) into consideration, we can rewrite (C.1) as

\[
\omega(l) = \frac{n \pi c}{l}, \quad n = 1, 2, 3 \cdots.
\]

This is the relation between cavity length and cavity eigenmodes under stable conditions. When the cavity balance is changed by the optical pressure on the movable mirror, (C.2) is changed into

\[
\omega(l + x) = \frac{n \pi c}{l + x} \approx \omega_0(1 - \frac{x}{l}),
\]

where \( x \) is the variance of the cavity length and \( \omega_0 \) is the cavity mode described by (C.2). Hence, the influence of cavity dimensions on the cavity eigenmodes can be described by \( \frac{1}{l} \), and it is exact the photon-phonon coupling rate, which means \( g_O = \frac{1}{l} \). These analysis show the relation between cavity dimensions and cavity eigenmodes in optical cavity, and similar results can be obtained in microwave cavity in aspect of plate distance \( d \) and cavity eigenmodes.

As for cavity EOM converter, relation between cavity dimensions and eigenmodes in both cavities can thus be illustrated via the photon-phonon coupling rates, and they can be expressed as

\[
g_M = \frac{\mu \omega_M}{2d} \sqrt{\frac{\hbar}{2m \omega_M}},
\]

\[
g_O = \frac{\omega_O}{l} \sqrt{\frac{\hbar}{2m \omega_O}},
\]

where \( m \) is the mass of mechanical oscillator; \( \mu \) is a dimensionless parameter related with capacitors of LC circuits; \( d \) is the distance between two plates of the movable capacitor; and \( h \) is Planck’s constant.

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