Inflation of fireballs, the gluon wind and the homogeneity of the HBT radii at RHIC

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Abstract. We solve analytically the ellipsoidally expanding fireball hydrodynamics with source terms in the momentum and energy equations, using the non-relativistic approximation. We find that energy transport from high $p_t$ jets of gluons to the medium leads to a transient, exponential inflation of the fireballs created in high energy heavy ion collisions. In this transient, inflatory period, the slopes of the single particle spectra are exponentially increasing, while the HBT radius parameters are exponentially decreasing with time. This effect is shown to be similar to the development of the homogeneity of our Universe due to an inflatory period. Independently of the initial conditions, and the exact value of freeze-out time and temperature, the measurables (single particle spectra, the correlation functions, slope parameters, elliptic flow, HBT radii and cross terms) become time-independent during the late, non-inflatory stages of the expansion, and they satisfy new kind of scaling laws. If the expansion starts with a transient inflation caused by the gluon wind, it leads naturally to large transverse flows as well as to the simultaneous equality, and scaling behaviour of the HBT radius parameters, $R_{side} \approx R_{out} \approx R_{long} \approx t_f \sqrt{T_f/m}$. With certain relativistic corrections, the scaling limit is $\tau_f \sqrt{T_f/m}$, where $m_t$ is the mean transverse mass of the pair.

Keywords: inflation, fireball hydrodynamics, relativistic heavy ion collisions, single-particle spectra, Bose-Einstein correlations

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1. Introduction

Recently, there were a number of QCD based perturbative calculations that indicated the energy loss of gluon jets with high transverse momentum due to multiple scattering during the course of their penetration through hot and dense hadronic matter. This phenomena, the jet quenching goes back to papers of Gyulassy, Plümer and Wang [1,2], recently studied in the papers of Gyulassy, Lévai and Vitev (GLV) [3]. It is clear from these works, that the jet quenching mechanism results in the
energy loss or the depletion of the number of hadrons from the high transverse momentum part of the spectrum. Due to the conservation of the total energy, this implies that the number of particles with small total momentum has to increase. See ref. [4] for the summary of recent developments in this field from the point of view of perturbative QCD.

In this paper, we focus on the phenomenological consequences of energy and momentum transfer to the soft, hydrodynamically behaving, small transverse momentum part of the single particle spectrum. We present a new family of solutions of non-relativistic hydrodynamics with source terms. This family of solutions is a generalization of the results of refs. [6, 5, 7] to ellipsoidally symmetric, expanding fireballs that are subject to energy and momentum transfer from the non-hydrodynamically behaving modes. A constant rate of energy pumping is shown to lead to an exponential inflation of the principal axis of the fireballs, while momentum transfer leads only to linear increase. These effects are correlated in time to the transition of high energy gluons through the nuclear medium, hence, by default, they are also transient phenomena, existing only in the initial phase of the expansion. We point out the similarity of this mechanism to the existence of a transient inflatory period in the expansion of the early Universe.

Inflation of the Universe after the Big Bang leads to a homogeneous and flat solution of Einstein’s equation. We show here that an inflation of hadronic fireballs, caused by the gluon wind in the initial stage of the Little Bangs of heavy ion collisions also leads to the simultaneous equality, spherical symmetry and scaling of the effective, measurable source sizes (frequently referred to as the HBT radii). This effect connects the physics of the smallest observable scales to the physics of the largest observable ones, underlining the universal nature of the laws of physics and the scale independence of the local conservation laws, governing the hydrodynamics of these expansions.

2. There blows the gluon wind ...

Consider the non-relativistic hydrodynamical problem, as given by the continuity, Euler and energy equations, appended with source terms in the momentum and the energy conservation laws:

\[ \partial_t n + \nabla (n \vec{v}) = 0, \]  
\[ \partial_t \vec{v} + (\vec{v} \nabla) \vec{v} = -\nabla (p + p_{G})/(mn), \]  
\[ \partial_t \epsilon + \nabla (\epsilon \vec{v}) + p \nabla \vec{v} = j_G, \]

where \( n \) denotes the particle number density, \( \vec{v} \) stands for the non-relativistic (NR) flow velocity field, \( \epsilon \) for the energy density, \( p \) for the pressure and in the following the temperature field is denoted by \( T \). The above set of equations are supplemented by the equations of state (EoS). The parameters of the EoS vary in various domains of the \((T, \mu)\) plane, corresponding to various phases of matter. A general, and
analytically solvable family of equations of state is given by

\[ p = \lambda_p nT - B, \quad \epsilon = \lambda_e nT + B. \] (4)

The parameters \( \kappa = \lambda_e/\lambda_p \) and \( B \) characterize the EoS for a broad variety of materials: e.g. a non-relativistic ideal gas yields \( \kappa = 3/2 \) and \( B = 0 \). Softening of the EoS can be modeled by increasing the values of \( \kappa \) in certain temperature domains. A parameter \( B > 0 \) can be used to describe a kind of generalized bag model equation of state (vacuum pressure) and to describe phase transitions, similarly to the class of models considered in refs. \cite{5, 6}. Hence we present generalizations of the results in refs. \cite{5, 6} and references therein, for a possible description of phase transitions during the time evolution of the exploding fireballs.

The source terms \( p_G \) and \( j_G \) in eqs. (2,3) stand for the contribution of the gluon wind that blows from the center of the collision zone through the material for a finite period of time. This wind does not carry conserved charges, and we assume that the energy and the momentum transfer is proportional to the number density \( n \) of the fluid, as well as to the average local velocity of the fluid elements. We assume that the gluons are proceeding essentially with the speed of light, so the relative velocity distribution depends only on the breadth of the local momentum distribution, which in turn is characterized with the magnitude of the temperature \( T \). We define a model that is analytically solvable by assuming that

\[ p_G = j_v(t) nT, \] (5)
\[ j_G = j_E(t) \kappa nT, \] (6)

where the coefficients \( j_v \) and \( j_E \) depend only on the time variable \( t \). The time dependence of the gluon wind can be expressed with the help of the dimensionless functions \( j_v(t) \) and \( j_E(t) \) in such a way that they are non-vanishing only within a finite period of time \( t_1 \leq t \leq t_2 \). The equations of the usual non-relativistic hydrodynamics are recovered in the case of vanishing source terms, \( j_E(t) = j_v(t) = 0 \).

A simple analytic solution of the non relativistic hydrodynamic equations without source terms (i.e. \( j_E = j_v = 0 \)) was given for the spherical and cylindrical symmetric cases in refs. \cite{3} and \cite{10} more than 20 years ago, and a recent series of papers \cite{12, 13, 14, 15} generalized these solutions to various (Gaussian, donut shaped or oscillatory) density and corresponding temperature profiles, for various symmetry classes ranging from spherical to ellipsoidally symmetric cases. Ref. \cite{12} also considered the case of Gaussian density profiles, spherically symmetric expansions and source terms in the energy equation, and pointed out the existence of an inflatory solution with an exponentially increasing radius parameter for the case corresponding to \( j_E(t) = j_0 > 0 \). Here we generalize this solution for the case of ellipsoidal symmetry and study in detail the consequences for the observables.

We choose \( n, v, T \) as the independent variables. We find that the NR hydro equations with source terms are solved by the following self-similar, ellipsoidally
symmetric density, temperature and flow profiles:

\[
n(t,r') = n_0 \frac{V_0}{V} \exp \left(-\frac{r_x'^2}{2X^2} - \frac{r_y'^2}{2Y^2} - \frac{r_z'^2}{2Z^2}\right),
\]

(7)

\[
v'(t,r') = \left(\frac{\dot{X}}{X} r_x', \frac{\dot{Y}}{Y} r_y', \frac{\dot{Z}}{Z} r_z'\right),
\]

(8)

\[
T(t) = T_i(t) \left(\frac{V_0}{V}\right)^{1/\kappa},
\]

(9)

\[
T_i(t) = \kappa T_0 [1 + j_v(t)] \exp \left(\int_{t_1}^{t} j_E(u) du\right)
\]

(10)

where the variables are defined in a center of mass frame \(K'\), but with the axes pointing to the principal directions of the expansion. This is the frame corresponding to the System of Ellipsoidal Expansion (SEE), introduced in ref. [6], where the relationship between the observables in SEE and in the CMS of the fireball were analyzed in great details.

The time dependent scale parameters are denoted by \((X,Y,Z)\) the typical volume of the expanding system is \(V = X Y Z\), and the initial temperature and volume are \(T_0 = T(t_0)\) and \(V_0 = V(t_0)\), and \(n_0\) is a constant. The time evolution of the radius parameters \(X, Y\) and \(Z\) is equivalent to the classical motion of a particle in a non-central, time dependent potential:

\[
\ddot{X} X = \ddot{Y} Y = \ddot{Z} Z = \frac{T_i(t)}{m} \left(\frac{V_0}{V}\right)^{1/\kappa},
\]

(11)

which correspond to a Hamiltonian motion of a mass point at \((X,Y,Z)\) driven by the following non-central, time dependent potential:

\[
H = \frac{P_x^2 + P_y^2 + P_z^2}{2m} + T_i(t) \left(\frac{V_0}{V}\right)^{1/\kappa},
\]

(12)

where \((P_x,P_y,P_z) = m(\dot{X},\dot{Y},\dot{Z})\). See refs. [5, 7] for a similar problem, the solution of non-relativistic fireball hydrodynamics for self-similarly expanding, ellipsoidally symmetric case without source terms in the energy and the Euler equations.

In all the cases, the normalization constant of the temperature increases in time corresponding to reheating during inflation, given by eq. (10). This is due to the external sources in the energy and momentum balance equations. This effect competes with a decreasing factor, governed by the expansion of the system, the rate of which is controlled by the parameter \(\kappa\). Note that for relativistic, massless particles, the speed of sound is simply \(c_s^2 = 1/\kappa\).

From this Hamiltonian formulation, the effects of the equation of state and the effects from the source terms in the energy and momentum balance equations can be qualitatively analyzed, and numerical solutions are also very easily obtained.
Softening the equation of state decreases slope of the repulsive potential in the corresponding mechanical problem. As the coordinates \((X, Y, Z)\) correspond to the characteristic scales of the expanding ellipsoids in the hydrodynamical system, softening the equation of state means slowing down the expansion. It is very interesting to observe the phenomena of a “stall” during a first order hadronization transition of a quark-gluon plasma \([20, 21, 22, 23]\). In our case, the acceleration of the fluid does not stop even for the softest equations of state, \(1/\kappa \to 0\), although the rate of expansion slows down substantially, according to eq. (11).

3. Inflation generates strong transversal flows

From the analysis of the solution of fireball hydrodynamics with inflation, it follows that three different effects may lead to the speeding up of the transverse expansion:

\(i\) Hardening the equation of state (corresponding to a decrease in the value of \(\kappa\)). This does not change the type of the expansion, the coordinates \((X, Y, Z)\) increase as power-law type of functions of the time, as the slope of the potential term in eq. (12) is increased. However, decreasing \(\kappa\) makes the algebraic coefficients of the expansion larger.

\(ii\) Momentum transfer. A positive value of the factor \(j_{\nu}(t)\) is also effective in increasing the height of the potential, however, the effect is only linear in \(j_{\nu}\).

\(iii\) Energy transfer. This is the most effective method to inflate the fireball, as the increase in the volume changes its form from a characteristic power-law type of expansion to an exponential shape. Similar effect was discussed for the case of spherically symmetric fireballs in ref. [12].

4. Inflation, the gluon wind and scaling of HBT radii

The observables: the shape, the slope parameters of the single particle spectra, the various flow coefficients: first, second and third flows, as well as the various HBT radius parameters were calculated analytically in ref. [13] for the case when the source terms were all set to zero. We found that the introduction of the source terms to the energy and the Euler equation does not change the form of the solution, only the time evolution of the source parameters is modified. Hence, all the results of ref. [13] are generalized to the case of inflating expansions. We present some numerical examples of the time evolution of the observables, using equations (8-37) of ref. [13], that we do not recapitulate here. Although the steps are trivial, the results are literally surprising.

4.1. Inflating hydro solutions with a gluon wind

In this subsection we present the analysis of the expansion for the initial case of a heavy ion reaction, when energy and momentum transfer to the soft modes is supposed present, as a consequence of the penetrations of the highly energetic, perturbative gluons through the fireball. For simplicity, we assume that the momentum
transfer is negligibly small, \( j_v(t) = 0 \). We assume that the QCD jet-quenching processes and the related energy transport to the soft momentum modes can be characterized by an approximately time independent coefficient \( j_E(t) = j_0 \), valid in a finite time interval \( (t_1, t_2) \). In principle, the functions \( j_v(t) \) and \( j_E(t) \) should be determined from QCD based Monte-Carlo simulations, if a realistic simulation is attempted. Here our aim was not yet a detailed fitting of the data, but the qualitative understanding of the behaviour of measurables, hence we analyzed the behaviour of the observables for these reactions. As high pt gluons are emitted from the beginning of the high energy heavy ion collisions, it is convenient to choose \( t_1 = 0 \). The end of the hard collisions can be estimated by the maximum of the penetration time, \( t_2 \approx 1.12A^{(1/3)/\gamma} \). For clarity, we investigate analytically the case when the fireball is initially spherical, \( X = Y = Z = R \).

From eq. (12), we find that the radius of the fireball inflates exponentially, and the coefficient of the temperature also increases exponentially,

\[
R(t) = R_0 \exp\left(\frac{\kappa}{3 + 2\kappa} j_0 t\right),
\]

\[
T_i(t) = T_{i,0} \exp(j_0 t),
\]

\[
T(t) = T_i(t) \left(\frac{R_0}{R}\right)^{3/\kappa} = T_{i,0} \exp\left(\frac{2\kappa}{3 + 2\kappa} j_0 t\right).
\]

Hence the local temperature \( T(t) \) increases exponentially for all \( \kappa > 0 \) values, because the exponentially increasing volume term cannot compensate the even faster, exponential increase of the \( T_i(t) \) factor. This effect is referred to as reheating during the inflation, or reheating caused by the gluon wind.

Let us consider how such a scenario effects the experimentally measurable slope parameters, \( T_{\text{eff}} \) and the HBT radius parameters, \( R_{\text{eff}} \). Following the lines of refs. [6, 12], these observables are given as

\[
T_{\text{eff}} = T + m R^2 = (T_0 + \frac{\kappa}{3 + 2\kappa} m R_0^2) \exp(\frac{2\kappa}{3 + 2\kappa} j_0 t),
\]

\[
R_{\text{eff}} \approx \frac{1}{j_0} \left(\frac{\kappa}{3 + 2\kappa}\right) \sqrt{\frac{T_0}{m}} \exp\left(-\frac{\kappa}{3 + 2\kappa} j_0 t\right),
\]

which implies that the effective slope parameters increase, while the effective homogeneity regions decrease exponentially with increasing time. These features of the inflatory period remain qualitatively similar for non-spherical initial conditions too, as shown in Figs. 1 and 2.

Hence inflation by the gluon wind may lead to the development of strong transverse flows as well as small HBT radius parameters. However, after the penetration of the nuclei through each other, the production mechanism for high momentum gluons comes to an end, and this implies that the inflation by the gluon wind will be over. After this time, the expansion will be driven by the hydrodynamical equations without source terms.
Inflation of fireballs ...

Some numerical examples for asymmetric initial conditions are shown on Fig. 1 and Fig. 2.

4.2. Asymptotic hydro without gluon wind effects

Let us observe, that the analytic results indicate a fast transverse flow at the end of the inflatory period.

An asymptotic solution of the hydrodynamical problem is given by the large time, large volume approximation, which implies

\[ X\ddot{X} = Y\ddot{Y} = Z\ddot{Z} \to 0, \]

\[ \dot{X} \to \dot{X}_a, \quad X \to \dot{X}_a t, \]

\[ \dot{Y} \to \dot{Y}_a, \quad Y \to \dot{Y}_a t, \]

\[ \dot{Z} \to \dot{Z}_a, \quad Z \to \dot{Z}_a t, \]

\[ T \to T_a(t_a/t)^2. \]

Substituting this solution into the expressions for the observables, we obtain a very interesting result, namely that the observables freeze-out in the sense that their value becomes independent from the time of the observation in the asymptotically late period of the expansion, when the inflation effects are already over.

\[ T_x \to m\dot{X}_a^2, \]

\[ T_y \to m\dot{Y}_a^2, \]

\[ T_x \to m\dot{Z}_a^2, \]

\[ R_{\text{side}} \approx R_{\text{out}} \approx R_{\text{long}} \to t_a \sqrt{T_a/m}, \]

independently of the time, for asymptotically large values of the time, which implies that the region of homogeneity becomes spherically symmetric. Due to this reason, it is possible to show, that the cross-terms all vanish

\[ R_{s,o}^2 \approx R_{v,l}^2 \approx R_{s,l}^2 \to 0 \]

in the late stage of the expansion, not only in the system of ellipsoidal expansion [1], but also in the frame of the observation.

Hence the observable part of the expanding fireball becomes spherically symmetric, after the inflation to large source sizes all the initial ellipsoidal geometrical symmetry is washed out from the local regions of homogeneity. Only the thermal scales [23, 25, 24] dominate the observable HBT radii, and these scales become direction independent due to the very nature of the thermal smearing - direction independent.

4.3. Relativistic corrections

We find that the flow field for asymptotically large times approaches a spherically symmetric, Bjorken type flow field, \( u^\mu = x^\mu/\tau = \gamma(1,v) \) with \( v = r/t \). This is
Fig. 1. The effect of the gluon wind on the hydro solution. We plot the evolution of \((X, Y, Z)\), the principal axis of the expanding ellipsoid, and the corresponding measurable HBT radius parameters, \((R_x, R_y, R_z)\) for expanding fireballs in hydrodynamics, with inflation and source terms in the energy equation, due to a gluon wind. The characteristic geometrical sizes increase exponentially with time, while the measurable HBT radii decrease exponentially with time. The characteristic time scales to reach the equality for all the HBT radii become much shorter than on Fig. 3, which is the case without inflation. The initial conditions are \((X_0, Y_0, Z_0) = (5, 6, 2)\) fm, \((X, Y, Z) = (0, 0, 0.3)\), \(m = 940\) MeV, \(T_{i,0} = 94\) MeV and \(j_E = j_0 = 0.3\) c/fm.
Fig. 2. The effect of the gluon wind on the observables for the inflating fireballs, the same solution as on Fig. 1. During the period of the inflation, neither the slopes nor the HBT radii saturate.
Fig. 3. The time evolution of \((X, Y, Z)\), the principal axis of the expanding ellipsoid, and the corresponding measurable HBT radius parameters, \((R_x, R_y, R_z)\) for expanding fireballs in hydrodynamics, without inflation and source terms. The initial conditions are \((X_0, Y_0, Z_0) = (6, 5, 2)\) fm, \((\dot{X}_0, \dot{Y}_0, \dot{Z}_0) = (0, 0, 0)\), \(T_i = 470\) MeV \(m = 940\) MeV. The acceleration period is over after about 10 fm/c, the solution tends to a coasting, ellipsoidal fireball. The HBT radii approach the same, constant and direction independent value, although the actual source sizes keep on expanding and remain direction dependent. Note the qualitative similarity to the results of ref. [25].
Fig. 4. The time evolution of the Hubble constants ($\dot{X}, \dot{Y}, \dot{Z}$) and the measurable slope parameters ($T_x, T_y, T_z$) for the hydro solution shown in Fig. 3. The saturation of the slopes of the single particle spectra happens at the same time as that of the Hubble constants, which happens earlier than the saturation of the measurable HBT radius parameters.
the same flow profile as the Hubble flow of our Universe. Furthermore, due to the spherical symmetry of the asymptotic flow profile, the local lengths of homogeneity will also become spherically symmetric. Due to the large value of the geometrical scales, the lengths of homogeneity will measure only the thermal scales \[15, 16, 13, 24\], and due to the homogeneity of the temperature and the asymptotic flow profile, the volume of homogeneity becomes spherical, and all the thermal length scales will be given by \(R_{\text{out}} \simeq R_{\text{side}} \simeq R_{\text{long}} \simeq t \sqrt{\frac{T(t)}{m}}\). Interestingly, this thermal length scale will become a constant of motion in the late stages of the expansion, as the temperature decreases in time as \(T(t) = T_f (\frac{t_f}{t})^2\), so all the radius parameters tend to \(t_f \sqrt{\frac{T_f}{m}}\).

The relativistic corrections to this result can be estimated following the lines of reasoning and the conditions given in ref. \[15, 16\] when analyzing the connection between the observables and a relativistic generalization of the source functions. Under certain conditions, spelled out in great details in ref. \[15\] and investigated numerically also in ref. \[17\], the relativistic form of result is

\[
R_{\text{out}} \simeq R_{\text{side}} \simeq R_{\text{long}} \simeq \tau_f \sqrt{\frac{T_f}{m}},
\]

where \(\tau_f = \sqrt{t_f^2 - r_0^2}\) at the time when the accelerationless expansion sets in, \(m_t = \sqrt{m^2 + p^2_x + p^2_z}\) is the transverse mass and \(T_f\) is the value of the local temperature in the center of the transverse plane at the propertime \(\tau_f\).

4.4. Microscopic dynamics

It is interesting to note, that the time evolution of the observables from the ellipsoidally symmetric hydro solution is very much similar to the time evolution given by T. Humanic’s cascade code, where the pions, kaons, nucleons and other hadronic resonances take part in rescattering, starting from a very hot and dense initial state \[25\]. The realization that the HBT radii may reach a saturation limit, but somewhat later, than the single particle spectra, was recently predicted in ref. \[26\], and confirmed in Figs. 3, 4. Strong transverse push was generated also by massive quark matter produced in the middle of Au+Au collisions from a cascade model in ref. \[27\]. It seems that a various gases of massive quanta that undergoes strong rescattering yields qualitatively similar results to the hydrodynamic solution that we described here.

5. Summary and Conclusions

We find that an inflatory period automatically leads to a spherical symmetry for the observable HBT radius parameters in an analytically solvable model of fireball hydrodynamics with source terms motivated by the flux of high-pt gluon wind through the expanding hadronic matter, created in high energy collisions of heavy
ions. The effect is very similar in nature how a flat and spatially homogeneous Universe is obtained after a period of inflation in astrophysics. Our results underline the similarity between the physics of 'Little Bangs' at the smallest experimentally accessible scales and the physics of the 'Big Bang' of our Universe, at the largest observable scales.

We also find, that the form of the observables is exactly the same for cases with or without inflation, and for various changes of the equation of state in the intermediate steps of the time evolution. This implies that various different equations of state from various different initial conditions may lead to exactly the same hadronic final state. Furthermore, the time evolution of the parameters of the hydrodynamical solution is sensitive not only to the equations of state but also to the magnitude of the source terms in the energy and momentum balance equations, which may provide non-equilibrium mechanisms to connect the initial state to a violently exploding final state.

These results imply that i) the hadronic final state of fireball evolution does not remember the path (hence any earlier transient phase of matter) of its time evolution, but ii) given the equation of state, and given the possible source terms in the energy and the Euler equation, the initial state can still be reconstructed, from the observables in the final hadronic state.

Further studies are in progress in for the publication of the ellipsoidal symmetric solutions of relativistic hydrodynamics and for the calculation of the observables corresponding to these relativistic generalizations of the present results.

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