Private Secure Coded Computation

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Abstract—We introduce a variation of coded computation that protects the data security and the master’s privacy against the workers, which is referred to as private secure coded computation. In private secure coded computation, the master needs to compute a function of its own dataset and one of the datasets in a library exclusively shared by the external workers. After recovering the desired function, the workers should not know which dataset in the library was desired by the master or obtain any information about the master’s own data. We propose a private secure coded computation scheme for matrix multiplication, namely private secure polynomial codes, based on private polynomial codes for private coded computation. By simulation, we show that the private secure polynomial codes achieve better computation time than private polynomial codes modified for private secure coded computation.

I. INTRODUCTION

In a distributed computing system where a master partitions a massive computation into smaller sub-computations and distributes these sub-computations to several workers in order to reduce the runtime to complete the whole computation, some slow workers can be bottleneck of the process. These slow workers are called stragglers and mitigating the effect of these stragglers is one of the major issues in distributed computing. Recently, a coding technique was introduced for straggler mitigation [1]. In [1], for a matrix-vector multiplication, the matrix is \((n, k)\)-MDS coded and distributed to \(n\) workers so that each encoded matrix is assigned to one worker. Each worker multiplies the coded submatrix by a vector and returns the multiplication to the master. After \(k\) out of \(n\) workers return their multiplications, the master can decode the whole computation. Since the computation of the slowest \(n - k\) workers is ignored, at most \(n - k\) stragglers can be mitigated. This kind of approach to distributed computing is referred to as coded computation. Several follow-up studies of coded computation were proposed [2] - [4].

In this paper, we introduce a variation of coded computation that considers both of the master’s privacy and data security against the workers, which is referred to as private secure coded computation. In the private secure coded computation, the master requires distributed computing on a function \(f\) of its own data \(A\) and specific data \(B_D\) included in a library \(B\), which is exclusively shared by external workers. For each worker, the master encodes \(A\) with an encoding function \(g^D_A\), sends encoded data to the worker, and requests the worker to encode \(B\) with an encoding function \(g^D_B\) and compute a function \(f_W(g^D_A(A), g^D_B(B))\). After the master recovers the result of desired function \(f(A, B_D)\) from the computation results of \(f_W\) returned by the workers, the workers should not be able to identify that \(B_D\) is desired by the master, which would imply that the master’s privacy is protected. The workers also should not obtain any information about the master’s data \(A\), which would imply that the data security is protected. Private secure coded computation will be explained in further detail in Section II.

As a motivating example of the private secure coded computation, we may consider a user who employs an artificial intelligence (AI) assistant, e.g. Google Assistant or Siri, with its mobile. We assume that the user can request a recommendation from an AI assistant of an item which is included in one of \(M\) categories that the AI assistant can recommend, e.g. movies, games, restaurants, and so on. We refer to the \(M\) categories as a library \(B\) and denote them by \(\{B_k\}_{k=1}^M\) such that \(B = \{B_k\}_{k=1}^M\). We also assume that the user stores its preference parameter \(A\). When the user requests a recommendation from the AI assistant of an item in a category \(B_D\), the assistant encodes \(A\) and sends encoded data to several distributed workers, e.g. data centers, for recovering \(f(A, B_D)\) in a distributed way. After recovery, the AI assistant can decide the recommended item based on \(f(A, B_D)\). We assume that the AI assistant does not share the user preference parameter \(A\) with workers and that the user can delete the recommendation service usage record right after the item is recommended so that the AI assistant does not identify the user’s recommendation service usage pattern.

In this example, the data security of \(A\) against the workers is protected by encrypting \(A\) while encoding it. However, encrypting \(A\) cannot protect the user’s privacy. Generally, the user uses this recommendation service according to its life cycle. That is, if the workers track the recommendation service usage records, the user’s life cycle is revealed to them, which implies that the user’s privacy has been invaded. We remark that this privacy invasion on the user’s life cycle is related to \(B\), not \(A\). Therefore, encrypting \(A\) cannot protect the user’s privacy on the life cycle. In order to protect the user’s privacy, the workers should not know that a particular \(B_D\) is desired by a user, which motivates the private secure coded computation.

Data security in coded computation was studied in previous works [5] - [7]. In these works, the master has both of two matrices \(A\) and \(B_D\) whereas the workers do not have any library. The master wants to compute a matrix multiplication \(AB_D\) using the workers. Since the workers do not have their
own library, the master’s privacy against the workers is not considered in this system model.

The master’s privacy considered in private secure coded computation was previously considered in private information retrieval (PIR) problem [10]. There have been various works on PIR and its variations [11] - [14]. However, in PIR literature, the master does not have its own data A and the desired matrix B_D is merely downloaded from the databases. The master’s privacy in coded computation was firstly considered in [8] and the coded computation model that considers the master’s privacy was referred to as private coded computation.

In this paper, we propose a private secure coded computation scheme for matrix multiplication, based on polynomial codes. We refer to this scheme as private secure polynomial codes. We refer to this scheme as private coded computation based on polynomial codes [9] was proposed, which was referred to as private polynomial codes.

In this paper, we propose a private secure coded computation scheme for matrix multiplication, based on polynomial codes. We refer to this scheme as private secure polynomial codes. For the data security, the master jointly encode a and a random matrix R into polynomial codes where the random matrix R is exclusively owned by the master and concealed to the workers, which was previously proposed in [5]. The idea for protecting the master’s privacy is based on private polynomial codes in [8]. In simulation results, we show that the private secure polynomial codes achieves better computation time than private polynomial codes modified for private secure coded computation.

Notation : We use $[N]$ to denote a set comprised of N elements, 1 to N. A set comprised of M elements, N + 1 to N + M is denoted by $[N + 1 : N + M]$.

II. SYSTEM MODEL

In this section, we describe a system model of private secure coded computation. There is a master who has its own dataset A, where A is an element (matrix) in a vector space $\mathbb{V}_1$ over a field $\mathbb{F}$. There are also N external workers $\{W_i\}_{i=1}^N$, and these workers share a library B which consists of M different datasets $\{B_k\}_{k=1}^M$. Each dataset B_k is an element (matrix) in a vector space $\mathbb{V}_2$ over the same field $\mathbb{F}$. The master needs distributed computing on a function $f$ of A and one of M datasets $\{B_k\}_{k=1}^M$ in library B, where $f : (\mathbb{V}_1, \mathbb{V}_2) \rightarrow \mathbb{V}_3$ for a vector space $\mathbb{V}_3$ over the same field $\mathbb{F}$. We denote the desired dataset by B_D. Therefore, the whole computation desired by the master is denoted by $f(A, B_D)$. Since we consider private coded computation for matrix multiplication, $f(A, B_D) = AB_D$ in this paper.

The whole computation is converted into several sub-computations and assigned to the workers. Each worker returns its sub-computation result to the master. When sufficient number of sub-computation results are returned to the master, the master can recover the whole computation $f(A, B_D)$ based on the received sub-computation results. We denote the minimum number of sub-computation results to recover $f(A, B_D)$ by K which was referred to as recovery threshold in [9]. The slowest N - K workers become stragglers, since they do not return their sub-computations results. After the master recovers the whole computation, each worker should not be able to obtain any information about A or identify that B_D is desired by the master, thus protecting the data security and the master’s privacy. In this paper, we assume that the workers do not collude with each other so that each worker does not know which sub-computations are assigned to, computed by, and returned by the other workers.

The master encodes A for each worker W_i. We denote the encoding function of A for the worker W_i and desired matrix B_D by $g_{A,W_i}^D$, where $g_{A,W_i}^D : \mathbb{V}_1 \rightarrow \mathbb{U}_1$ for a vector space $\mathbb{U}_1$ over the same field $\mathbb{F}$. The master sends the encoded data $g_{A,W_i}^D(A)$ to the worker W_i and also sends the queries for requesting W_i to encode the library B. We denote the encoding function of the worker W_i for the library B = $\{B_k\}_{k=1}^M$ and the desired dataset B_D by $g_{B,W_i}^D$, where $g_{B,W_i}^D : \mathbb{V}_2^M \rightarrow \mathbb{U}_2$ for a vector space $\mathbb{U}_2$ over the same field $\mathbb{F}$. The master also sends the queries to the worker W_i to compute a function of $g_{A,W_i}^D(A)$ and $g_{B,W_i}^D(B)$ and return the computation result of the function to the master. That is, the worker W_i computes $f_{W_i}^D(g_{A,W_i}^D(A), g_{B,W_i}^D(B))$. We denote the function of W_i by $f_{W_i}^D : (\mathbb{U}_1, \mathbb{U}_2) \rightarrow \mathbb{V}_3$ for a vector space $\mathbb{U}_3$ over the same field $\mathbb{F}$. Without considering where the sub-computation result comes from, we denote the ith sub-computation result returned to the master by $S_i$, where $S_i$ is an element in the vector space $\mathbb{V}_3$. After K sub-computation results $\{S_i\}_{i=1}^K$ across the N workers are returned to the master, the master can recover the whole computation $f(A, B_D)$ by decoding $\{S_i\}_{i=1}^K$. If we denote the decoding function at the master by $d_D : \mathbb{U}_3^K \rightarrow \mathbb{V}_3$, the decoding function $d_D$ should satisfy the constraint given by $d_D(S_1, S_2, \ldots, S_K) = f(A, B_D)$.

The master’s privacy is protected when none of the workers can identify index D of the desired dataset B_D after the master recovers the whole computation. Since the privacy we consider is information-theoretic privacy, the privacy constraint for each worker W_i can be expressed as

$$I(D; Q^D_i, g_{A,W_i}^D(A), f_{W_i}^D(g_{A,W_i}^D(A), g_{B,W_i}^D(B)), B) = 0,$$

where $Q^D_i$ denotes the queries that the master sends to the worker W_i for encoding $g_{B,W_i}^D(B)$ and computing $f_{W_i}^D(g_{A,W_i}^D(A), g_{B,W_i}^D(B))$.

For a simpler expression, we denote $g_{A,W_i}^D(A)$ and $f_{W_i}^D(g_{A,W_i}^D(A), g_{B,W_i}^D(B))$ by $C_i^D$ and $R_i^D$, respectively, so that the privacy constraint becomes

$$I(D; Q_i^D, C_i^D, R_i^D, B) = 0. \quad (1)$$

Similarly, the data security constraint for each worker W_i can be expressed as

$$I(A; Q_i^D, C_i^D, R_i^D, B) = 0. \quad (2)$$

The overall process of the private secure coded computation is depicted in Fig. 1.

III. PRIVATE SECURE POLYNOMIAL CODES

In this section, we propose private secure polynomial codes for matrix multiplication. We describe the scheme with an illustrative example and generally describe the private secure polynomial codes.
A. Illustrative Example

We assume that the master has a matrix \( \mathbf{A} \in \mathbb{F}_q^{r \times s} \) for sufficiently large finite field \( \mathbb{F}_q \) and that there are 12 non-colluding workers \( \{W_n\}_{n=1}^{12} \) where each worker has a library of two matrices \( \mathbf{B}_1, \mathbf{B}_2 \in \mathbb{F}_q^{r \times t} \). As in Section II, we denote the library by \( \mathbf{B} \). Let us assume that the master wants to compute \( \mathbf{AB}_1 \) using \( \{W_n\}_{n=1}^{12} \) while hiding that the master desires \( \mathbf{B}_1 \) from the workers. The matrix \( \mathbf{A} \) can be partitioned into two submatrices \( \mathbf{A}_0, \mathbf{A}_1 \in \mathbb{F}_q^{r \times s/2} \) so that \( \mathbf{A} = \begin{bmatrix} \mathbf{A}_0 \\ \mathbf{A}_1 \end{bmatrix} \)

and each of \( \mathbf{B}_1, \mathbf{B}_2 \) are partitioned into two submatrices \( \mathbf{B}_{k,1}, \mathbf{B}_{k,2} \in \mathbb{F}_q^{r \times 2} \), \( k \in [2] \), so that \( \mathbf{B} = \begin{bmatrix} \mathbf{B}_{0,1} & \mathbf{B}_{0,2} \\ \mathbf{B}_{1,1} & \mathbf{B}_{1,2} \end{bmatrix} \).

Therefore, \( \mathbf{AB}_1 = \begin{bmatrix} \mathbf{A}_0 \mathbf{B}_{1,1} + \mathbf{A}_1 \mathbf{B}_{1,2} \\ \mathbf{A}_0 \mathbf{B}_{2,1} + \mathbf{A}_1 \mathbf{B}_{2,2} \end{bmatrix} \). The private secure polynomial codes for \( \mathbf{A}, \mathbf{B}_1 \) and \( \mathbf{B}_2 \) are as follows.

\[
\tilde{\mathbf{A}}(x) = \mathbf{A}_0 + \mathbf{A}_1 x + x R_z x^2, \quad \tilde{\mathbf{B}}_k(x) = \mathbf{B}_{k,1} x^3 + \mathbf{B}_{k,2} x^6,
\]

where \( k \in [2] \), \( R \in \mathbb{F}_q^{r \times 2} \) denotes a random matrix, and \( x \in \mathbb{F}_q \) denotes the variable of polynomials \( \tilde{\mathbf{A}} \) and \( \tilde{\mathbf{B}}_k \).

We denote the evaluations of \( \tilde{\mathbf{A}} \) and \( \tilde{\mathbf{B}}_k \) at \( x = x_i \) by \( \tilde{\mathbf{A}}(x_i) \) and \( \tilde{\mathbf{B}}_k(x_i) \), respectively. For the desired matrix \( \mathbf{B}_1 \) and each worker \( W_i \), the master evaluates \( \tilde{\mathbf{B}}_1 \) at a randomly chosen point \( x_i \) and sends the evaluation \( \tilde{\mathbf{A}}(x_i) \) to the worker \( W_i \). That is, \( g^1_{\mathbf{A}_W, \mathbf{B}_1}(\mathbf{A}) = \tilde{\mathbf{A}}(x_i) \). We assume that the points \( \{x_i\}_{i=1}^{12} \) are distinct from each other. The master also sends the queries \( Q^1_i \) that \( W_i \) needs \( \mathbf{A} \) and \( \tilde{\mathbf{B}}_1 \) with an encoding function \( g^1_{\mathbf{B}_W}(\mathbf{A}) \) and compute a function \( f^1_{W_i}(g^1_{\mathbf{A}_W, \mathbf{A}}, g^1_{\mathbf{B}_W, \mathbf{B}}) \).

The library \( \mathbf{B} \) is encoded as follows. Firstly, for each worker \( W_i \), \( \tilde{\mathbf{B}}_1 \) is evaluated at \( x = x_i \). Secondly, for all of the workers, the undesired matrix \( \tilde{\mathbf{B}}_2 \) is evaluated at a randomly chosen point \( x_{13} \) which is distinct from the points \( \{x_i\}_{i=1}^{12} \). Since the workers do not collude with each other, they cannot notice that \( \tilde{\mathbf{B}}_2 \) is evaluated at an identical point \( x_{13} \) across workers. Finally, for each worker \( W_i \), the encoded library is given by \( g^1_{\mathbf{B}_W}(\mathbf{B}) = \tilde{\mathbf{B}}_1(x_i) + \tilde{\mathbf{B}}_2(x_{13}) \).

After encoding the library, each worker \( W_i \) computes a function \( f^1_{W_i}(g^1_{\mathbf{A}_W, \mathbf{A}}, g^1_{\mathbf{B}_W, \mathbf{B}}) = \tilde{\mathbf{A}}(x_i)(\tilde{\mathbf{B}}_1(x_i) + \tilde{\mathbf{B}}_2(x_{13})) \) which is given by

\[
\tilde{\mathbf{A}}(x_i)(\tilde{\mathbf{B}}_1(x_i) + \tilde{\mathbf{B}}_2(x_{13})) = (\mathbf{A}_0 + \mathbf{A}_1 x_i + R_z x^2) \times (\mathbf{B}_{1,1} x_i^3 + \mathbf{B}_{1,2} x_i^6 + \mathbf{B}_{2,1} x_{13}^3 + \mathbf{B}_{2,2} x_{13}^6)
\]

\[
= \sum_{l=0}^{8} Z_l x_i^l,
\]

where \( Z_l \) are as follows.

\[
Z_l = \mathbf{A}_l (\mathbf{B}_{2,1} x_{13}^3 + \mathbf{B}_{2,2} x_{13}^6), \quad Z_1 = \mathbf{R} (\mathbf{B}_{2,1} x_{13}^3 + \mathbf{B}_{2,2} x_{13}^6), \quad Z_2 = \mathbf{R} (\mathbf{B}_{2,1} x_{13}^3 + \mathbf{B}_{2,2} x_{13}^6), \quad Z_5 = \mathbf{R} \mathbf{B}_{1,1}, \quad Z_8 = \mathbf{R} \mathbf{B}_{1,2}.
\]

Since the degree of polynomial \( \tilde{\mathbf{A}}(x_i)(\tilde{\mathbf{B}}_1(x_i) + \tilde{\mathbf{B}}_2(x_{13})) \) is 8 and the evaluating points \( \{x_i\}_{i=1}^{12} \) are distinct from each other, the master can decode the polynomial from the subcomputation results returned by the 9 fastest workers, by polynomial interpolation. As a result, the master obtains the coefficients \( \{Z_l\}_{l=0}^{8} \). The whole computation \( \mathbf{AB}_1 \) can be recovered from the coefficients of \( x^3, x^4, x^6, x^7 \).

B. General Description

In this section, we generally describe the private secure polynomial codes for matrix multiplication. There are \( N \) non-colluding workers \( \{W_n\}_{n=1}^{N} \) and each worker has a library \( \mathbf{B} \) of matrices \( \{\mathbf{B}_k\}_{k=1}^{M} \) where each \( \mathbf{B}_k \in \mathbb{F}_q^{r \times s} \) for sufficiently large finite field \( \mathbb{F}_q \). The master has a matrix \( \mathbf{A} \in \mathbb{F}_q^{r \times s} \) and desires to multiply \( \mathbf{A} \) by one of \( \{\mathbf{B}_k\}_{k=1}^{M} \) in the library \( \mathbf{B} \) while keeping the index of desired matrix \( \mathbf{B}_D \) and content of \( \mathbf{A} \) from all of the workers. Matrix \( \mathbf{A} \) can be partitioned into \( m \) submatrices \( \{\mathbf{A}_k\}_{k=1}^{m} \) in \( \mathbb{F}_q^{r/m \times s/m} \) and each \( \mathbf{B}_k \) can be partitioned into \( n \) submatrices \( \{\mathbf{B}_{k,l}\}_{l=1}^{n} \) in \( \mathbb{F}_q^{r/n \times s/n} \), where \( m, n \in \mathbb{N}^+ \). The whole computation \( \mathbf{A} \mathbf{B}_D \) that the master wants to recover is given by

\[
\mathbf{AB}_D = \begin{bmatrix}
\mathbf{A}_0 \mathbf{B}_{D,1} & \mathbf{A}_0 \mathbf{B}_{D,2} & \ldots & \mathbf{A}_0 \mathbf{B}_{D,n-1} \\
\mathbf{A}_1 \mathbf{B}_{D,1} & \mathbf{A}_1 \mathbf{B}_{D,2} & \ldots & \mathbf{A}_1 \mathbf{B}_{D,n-1} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{A}_{m-1} \mathbf{B}_{D,1} & \mathbf{A}_{m-1} \mathbf{B}_{D,2} & \ldots & \mathbf{A}_{m-1} \mathbf{B}_{D,n-1}
\end{bmatrix}
\]

The polynomial codes for \( \mathbf{A} \) and \( \{\mathbf{B}_k\}_{k=1}^{M} \) are given as follows.

\[
\tilde{\mathbf{A}}(x) = \sum_{l=0}^{m-1} \mathbf{A}_l x^l + R_z x^m, \quad \tilde{\mathbf{B}}_k(x) = \sum_{l=1}^{n-1} \mathbf{B}_{k,l} x^{l(m+1)}.
\]
\[ \{x_i\}_{i=1}^N \] are distinct from each other. The master also sends the queries \( Q_D \) that request \( W_i \) to encode the library \( B \) with an encoding function \( g_{B_w_i}^D \), and compute a function \( f_{B_w_i}^D (g_{B_w_i}^D (A), g_{B_w_i}^D (B)) \).

The library \( B \) is encoded as follows. Firstly, for each worker \( W_i \), \( B_D \) is evaluated at \( x_i \). Secondly, for all of the workers and the undesired matrices \( \{B_k \}_{k \in [M] \setminus D} \), each undesired matrix \( B_k \) is evaluated at a randomly chosen point \( x_{jk} \) which is distinct from the points \( \{x_i\}_{i=1}^N \). Since the workers do not collude with each other, they cannot notice that each \( B_k \) is evaluated at an identical point \( x_{jk} \) across workers. Finally, for each worker \( W_i \), the encoded library is given by \( g_{B_w_i}^D (B) = B_D (x_i) + \sum_{k \in [M] \setminus D} B_k (x_{jk}) \).

After encoding the library, each worker \( W_i \) computes a function \( f_{B_w_i}^D (g_{B_w_i}^D (A), g_{B_w_i}^D (B)) = \tilde{A} (x_i) (B_D (x_i) + \sum_{k \in [M] \setminus D} \tilde{B}_k (x_{jk})) \) which is given by

\[
\tilde{A} (x_i) (B_D (x_i) + \sum_{k \in [M] \setminus D} \tilde{B}_k (x_{jk})) = \sum_{l=0}^{m-1} A_l B_{D,pl} x^l + p(m+1) + \sum_{l=0}^{n-1} R B_{D,pl} x^{pm+mp} + n(m-1) - 1 \sum_{l=0}^{n} z_l x^l, \]

where \( \{z_l\}_{l=0}^{n(m+1)-1} \) are given by

\[
z_l = \sum_{k \in [M] \setminus D} A_l \tilde{B}_k (x_{jk}) \quad \forall l \in [0 : m-1],
\]

\[
z_l = \sum_{k \in [M] \setminus D} R \tilde{B}_k (x_{jk}) \quad \forall l = m,
\]

\[
z_l = A_{l-p(m+1)} B_{D,p} \quad \forall l \in [p(m+1) : p(m+1) + m-1],
\]

\[
z_l = A_{l-p(m+1)} B_{D,p} \quad \forall l \in [p(m+1) + m-1: n].
\]

Since the degree of polynomial \( \tilde{A} (x_i) (B_D (x_i) + \sum_{k \in [M] \setminus D} \tilde{B}_k (x_{jk})) \) is \( mn + n - 1 \) and the evaluating points \( \{x_i\}_{i=1}^N \) are distinct from each other, the master can decode the polynomial from the sub-computation results returned by the \( mn + n \) fastest workers, by polynomial interpolation. As a result, the master obtains the coefficients \( \{z_l\}_{l=0}^{mn+n-1} \). The whole computation \( AB_p \) can be recovered from the coefficients \( \{z_l\}_{l \in [p(m+1) : p(m+1) + m-1], p \in [1 : n-1]} \). Therefore, the recovery threshold \( K \) equals \( mn + n \) and \( d_D (S_1, S_2, \ldots, S_{mn+n}) = AB_p \).

**Remark 1.** In the private polynomial codes in [8], the master’s own data \( A \) and the library \( B \) are encoded into separate polynomials. That is, \( A \) is polynomial of \( x \) whereas \( \{\tilde{B}_k\}_{k=1}^M \) are polynomials of \( y \). Since the random matrix \( R \) is not considered in the private polynomial codes, \( A \) and \( B \) should not be encoded with same variable \( x \) for protecting the master’s privacy. If \( A \) and \( B \) are encoded with same variable \( x \) without \( R \), the workers obtain non-zero information about \( \tilde{A} (x_i) \). That is, the workers may identify that \( A \) is encoded with \( x \). Since the desired matrix \( B_D \) is also encoded with \( x \), the workers thereby realize that \( B_D \) is desired by the master, thus implying that the master’s privacy is violated. Therefore, compared to the private polynomial codes in [8], our private secure polynomial codes has its own novelty.

**C. Privacy and Security Proof**

To prove that the data security and the master’s privacy are protected in the private secure polynomial codes, we need to show that the privacy constraint in (1) and the data security constraint in (2) are satisfied for every worker. The basic idea for proof is similar to that in [8]. Due to the space limit, we omit the detail of the proof in this paper.

**IV. SIMULATION RESULTS**

In this section, in terms of the computation time consumed for receiving \( K \) sub-computation results across \( N \) workers, we compare the private secure polynomial codes and private polynomial codes modified for private secure coded computation. In private polynomial codes, the data security can be protected by adding \( R \) when encoding \( A \), which is same as private secure polynomial codes. Nevertheless, as explained in Remark 1, since the library \( B \) is encoded with distinct variable \( y \), the workers are still divided into groups in private polynomial codes. Grouping may increase the computation time since the computation time is determined by the slowest group.

We assume that the computation time distribution of each worker is independent of each other and follows the exponential distribution as in [1]. In [1], the computation time is given by

\[
t_{conv} = \frac{1}{K} (\gamma + 1 + \mu \log \frac{N}{N-K}),
\]

where \( \mu \) and \( \gamma \) are straggling parameter and shift parameter, respectively.

Compared to (3), in private secure polynomial codes, the recovery threshold \( K \) equals to \( n(m+1) \) and \( n/(n-1) \) times more computation than directly computing \( AB_D \) is required. Therefore, the computation time of private secure polynomial code is given by

\[
t_{ps} = \frac{1}{(m+1)(n-1)} (\gamma + 1 + \mu \log \frac{N}{N-n(m+1)}).
\]

The computation time of private polynomial codes is given by (9) in [8].

We compare the computation time between three schemes for \( N = 12, M = 4 \), and \( \gamma = \mu = 0.1 \). We set \( n = 2 \), which implies that the workers are divided into 2 groups in private polynomial codes. For fair comparison, we assume that each worker returns only one sub-computation result to the master.
in private polynomial codes, as in private secure polynomial codes. Since $K = n(m + 1)$ in the private secure polynomial codes and private polynomial codes, we set $K$ as even number and vary $K$ from 4 to 10. That is, $m$ is varying from 1 to 4. The comparison result for computation time is given in Fig. 2. Compared to the private polynomial codes, private secure polynomial codes achieves at most $25\%$ reduction in computation time, thus implying that our proposed scheme outperforms the previous works when considering both of the data security and the master’s privacy.

We also compare the computation time for given $N, M, K, \gamma$ and varying $\mu$. We set $N = 12, M = 4, K = 4, \gamma = 1$ and vary $\mu$ from $10^{-1}$ to $10$. As seen in the figure, the private secure polynomial codes outperforms the private polynomial codes, whereas the gap between the two schemes decrease as $\mu$ becomes larger. This is because the effect of stragglers are diminished when $\mu$ becomes larger.

V. CONCLUSION

In this paper, we introduced private secure coded computation as a variation of coded computation that protects the data security and the master’s privacy at the same time. As an achievable scheme for private coded computation, we proposed private secure polynomial codes based on private polynomial codes in private coded computation. By simulation, we compared the private secure polynomial codes and private polynomial codes in terms of computation time, and shown that the proposed scheme outperforms the previous work. In future work, we will further analyze the performance of private secure polynomial codes and compare the performance with actual distributed machines, e.g., AWS or Google Cloud.

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