4-D Quantum Dilaton Gravity
During Inflation
and Renormalization at One loop

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Abstract

We consider 4D quantum-dilaton gravity with the most general coupling in a homogeneous and isotropic universe, especially an inflationary one, which is essentially characterized by an exponentially expanding scale factor with time. We show that on the inflationary background this theory can be miraculously renormalized, at least at the one-loop level, which must be an effective theory during the inflation of the un-constructed complete quantum theory of gravity.

1 Introduction and Summary

At present, no complete unified theory which includes quantum gravity has yet been constructed. However if we believe in the existence of such a theory, and wish to know the history of our universe controlled by this theory, we must find it as soon as possible. Our point of view is that such a

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fundamental theory of quantum gravity is either string theory or another theory like it. We do not place any restrictions on which is true. Furthermore, we do not remove the possibility of quantum gravity to be a local field theory with renormalizability. In any case there is an effective theory of fundamental quantum gravity for low energy or special background.

A candidate for a consistent theory of quantized gravity is string theory. A low-energy effective theory of a string below the Plank scale represented by a metric and a dilaton is well known [1]. Such an effective action arises in the form of a power-series type of a slope parameter ($\alpha'$); the standard point of view is that the higher orders in such an expansion correspond to higher energies. From this point of view, at a lower energy scale the action for gravity has the form of a lower derivative dilaton action. The Einstein gravity coupled to scalars is nonrenormalizable as naive power counting, and higher derivative gravity is a renormalizable [2]; however, it is not unitary within a perturbation scheme [3]. Of course, it is not strange that a useful local field theory of gravity covering all energy regions does not exist, though its possibility cannot be rejected. It is important to know whether a renormalizable local field theory of gravity constructed by metric exists or not, and what type of environment would allow its existence. It is well known [4] that the standard cosmological model works well for the red shift, cosmic microwave background, and nucleosynthesis. However, its naive version cannot explain some problems: flatness and horizon. An inflationary model was proposed by Guth [5] in order to settle these problems, which was implemented by means of a first-order phase transition. Unfortunately, it cannot be a realistic model, as Guth himself has pointed out. After a time, other models were proposed with inflation as a second order phase transition [6] or “chaotic” inflation [7], which have fine-tuned parameters that depend on fundamental theory before the inflation begins. The inflation model can be well explained within the framework of the grand unified theory (GUT) of particles at least qualitatively [8]. In the GUT model all interactions, except for gravity,
may be unified on a scale (GUT scale) below the Plank scale, where the gauge symmetry of GUT spontaneously breaks down to one of the standard models of elementary particles with some field which has a non-zero vacuum expectation value. Naively, it seems that this field can be identified with the field driving inflation. However, quantitatively such a model cannot be constructed well while being consistent with observational data concerning the inhomogeneity of the microwave background radiation [8]. Thus, in some model the inflationary phase transition is supposed to begin above the GUT scale, say, the Plank scale [3, 4]. At this point, a reliable discussion of inflation can be realized only after a quantum theory of gravity is constructed. For instance, within the framework of the string theory the cosmology near to the Plank scale has been discussed [10, 11, 12, 13, 14, 15]; furthermore, the relation between the inflationary universe phase to usual A Friedman-universe phase is considered [10, 12, 14] using the fact that scale factor duality [11, 16] is a subset of the T duality [17, 18], where the dilaton is identified as being the field driving inflation.

Our standpoint is that this dilaton drives inflation, however, since our action is more general than a low-energy effective action of a string we do not restrict our scalar field to the usual dilaton in string theory. We consider an arbitrary action with a metric and a scalar field called “dilaton” having two derivatives, and consider the inflation driven by the dilaton near to the Plank scale where quantum effect of gravity is very important.

Until quite recently, from long time ago, several studies were performed to calculate the divergence of effective action of four-dimensional gravity [19, 20, 21, 22]. In the pure Einstein action case without a cosmological term, it was originally calculated at the one-loop level by t’Hooft and Veltman [11]. They found that the action is not renormalizable off mass shell, but is finite on mass shell at the one-loop level. Furthermore, although the pure Einstein action with a cosmological constant is renormalizable [20], if one introduces matter fields the one loop renormalizability is lost, even on mass shell. Recently
we considered the divergence of the effective action, which is the most general class with less than two derivatives for a scalar and a metric, while explicitly leaving functions \( A, B, \Lambda \) arbitrary. Classically, theory (1) can be classified into two cases according to whether they have conformal symmetry or not. In the later case, on an arbitrary back-ground space-time we found models which are finite in the case without a cosmological term, and are renormalizable in the case with it by fine-tuning of functional form of \( A(\phi), B(\phi), \Lambda(\phi) \) at the one loop level on mass shell. This property is the same as pure Einstein gravity with or without a cosmological term, although there is a matter coupling to the metric in our model. On the other hand, an analysis of quantum theory was considered [15, 24, 25] on maximal symmetric space-time where a number of Killing vectors exist. It includes De Sitter space-time, which is a limiting case of the usual inflationary universe which has an exponentially expanding scale factor.

In the present paper we consider the action (1) on a homogeneous and isotropic four-dimensional background space-time (spatially maximal symmetric space-time), and calculate the divergence of the effective action. We especially consider the divergence on inflationary background space-time (maximal symmetric one), and found that one-loop divergence can be renormalized. With a special choice of \( A(\phi), B(\phi), \Lambda(\phi) \), (1) is equivalent to \( R^2 \) gravity, by which we show that this case is renormalizable, and consider a renormalization group analysis on the inflationary back-ground [26].

This paper is organized as follows. In section 2, considering model (1) in a homogeneous and isotropic four-dimensional background space-time, we found that the classical equations of motion of the metric are the “Hubble” equation and the energy momentum-conservation law, that the equation

\[
S = \int d^4x \sqrt{-g} \left\{ A(\phi)g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + B(\phi)R - 2B(\phi)\Lambda(\phi) \right\}
\]  

(1)

\(^2\)In Ref [22, 23], we used a convention \( C(\phi) = -2B(\phi)\Lambda(\phi) \)
of motion of the dilaton ($\phi$) means the state of universe being defined by a “state” equation between the energy density and the pressure driven by dilaton, and that there is a maximal symmetric solution as the simplest one. In section 3 we discuss our calculation of the divergence of effective action of (1) on homogeneous and isotropic four dimensional background space-time using the back-ground field method and the Schwinger-DeWitt technique. Next, we show that the structure of the divergence on maximal symmetric space-time is miraculously simple, and renormalizable, at least at the one-loop level.

2 Analysis in Classical level

We consider gravity with the general coupling to a scalar in which the action is. For a special classical back-ground it may be realized in the very early universe. First, in this section we analyze this theory at the classical level.

2.1 Classical Equation of Motion

The classical equations of motion for $g_{\mu\nu}$ and $\phi$ are

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu} \quad \text{(for } g_{\mu\nu}) \quad (2)$$

and

$$B_1 R - 2(\Lambda A_1) - A_1 (\nabla \phi)^2 - 2A (\Box \phi) = 0 \quad \text{(for } \phi), \quad (3)$$

where

$$T_{\mu\nu} :=$$

$$- \left( \left( \frac{B_2}{B} - \frac{A}{2B} \right) (\nabla \phi)^2 + \frac{B_1}{B} (\Box \phi) \right) g_{\mu\nu} + \left( \frac{B_2}{B} - \frac{A}{B} \right) (\nabla_\mu \phi)(\nabla_\nu \phi) + \frac{B_1}{B} (\nabla_\mu \nabla_\nu \phi). \quad (4)$$

In this paper we restrict $B \neq 0$ and $A \neq \frac{B_2^2}{2B}$, where we write $X_n := \frac{d^n \phi}{d\phi^n}$ for any function $X(\phi)$.
In the next subsection we consider a homogeneous and isotropic universe (maximally symmetric three-
dimensional space) with the Robertson-Walker metric, which is a realistic model of a hot universe. Also in the following subsection we consider a maximally symmetric space-time, which is the simplest model of the universe during inflation.

## 2.2 Maximally Symmetric Space

It is well known that in D-dimensional space-time there are a maximum of \( \frac{D(D+1)}{2} \) Killing vectors. That space-time is called a maximally symmetric space-time. It is specified by

\[
g_{\mu\nu} = \eta_{\mu\nu} + \frac{K x_\mu x_\nu}{1 - K x_\lambda x^\lambda}, \quad \eta = \text{diag}(\pm 1, \pm 1, \cdots), \quad K \text{ is constant and } x_\mu := \eta_{\mu\lambda} x^\lambda. \tag{5}
\]

Thus,

\[
\Gamma^{\rho}_{\nu\lambda} = K x^{\rho} g_{\nu\lambda}, \tag{6}
\]

\[
R_{\mu\nu\alpha\beta} = K (g_{\mu\alpha} g_{\nu\beta} - g_{\nu\alpha} g_{\mu\beta}), \tag{7}
\]

\[
R_{\mu\nu} = (D - 1)K g_{\mu\nu}, \tag{8}
\]

and

\[
R = D(D - 1)K. \tag{9}
\]

Furthermore, if \( K \) and the signature of \( g_{\mu\nu} \) are fixed, any metric with a maximal symmetry can be transformed to above the formula by some general coordinate transformation. In the next subsection we consider \( D = 3 \) and \( \eta = \text{diag}(-1, -1, -1) \) case; in the subsequent subsection we consider the \( D = 4 \) and \( \eta = \text{diag}(+1, -1, -1, -1) \) case.

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\(^4\)In this paper we use the convention \( \Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} (g_{\mu\sigma,\nu} + g_{\nu\sigma,\mu} - g_{\mu\nu,\sigma}) \), \( R^{\rho}_{\sigma\mu\nu} = \Gamma^{\rho}_{\sigma\nu,\mu} + \Gamma^{\rho}_{\mu\lambda} \Gamma^{\lambda}_{\sigma\nu} - (\mu \leftrightarrow \nu) \), \( R_{\mu\nu} = R^{\rho}_{\mu\rho\nu} \).
2.3 Homogeneous and Isotropic Universe

A homogeneous and isotropic universe is a spatially maximally symmetric subspace of the whole four-dimensional space-time. In this space-time the Robertson-Walker metric is convenient, which is

$$ds^2 = dt^2 - a(t)^2 \left( \frac{1}{1-kr^2}dr^2 + r^2d\Omega^2 \right),$$

(10)

where $a(t)$ is a scale factor and $k = \pm 1, 0$. Then,

$$R_{ijkl} = \kappa_i (g_{ik}g_{jl} - g_{il}g_{jk}), \quad R_{0j0l} = -\kappa_s g_{jl},$$

(11)

$$R_{\mu\nu} = \begin{pmatrix} 3\kappa_s & 0 \\ 0 & (\kappa_s + 2\kappa_t)g_{ij} \end{pmatrix},$$

(12)

and

$$R = 6 (\kappa_s + \kappa_t),$$

(13)

where $\kappa_s(t) := \frac{\dot{a}}{a}$, $\kappa_t(t) := (\frac{\dot{a}}{a})^2 + \frac{k}{a^2}$, and a dot means a time derivative. In a maximally symmetric three-dimensional space, a rank-2 tensor is proportional to a metric ($g_{ij}$) with a coefficient which depends only on time, and a scalar depends only on time. Therefore, the energy momentum tensor ($T_{\mu\nu}$) is very simple:

$$T_{\mu\nu} = \begin{pmatrix} -\rho(t) & 0 \\ 0 & p(t)g_{ij} \end{pmatrix}.$$  

(14)

The equations of motion of $g_{\mu\nu}$ and $\phi$ are equivalent to the following equations:

(i) “Hubble” equation:

$$\left( \frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} = \frac{\rho + \Lambda}{3}$$

(15)

(ii) Energy conservation law:

$$\dot{\rho} + \dot{\Lambda} = -3\frac{\dot{a}}{a}(\rho + p)$$

(16)

(iii) “State” equation:

$$\left( B_1^2 + \frac{2A_1BB_2}{A} + 2B(A-2B_2) \right) \rho - (3B_1^2 + 2AB) p + 2B_1(B_1A - BA_1) = 0$$

(17)

7
(iv) Relation between $\rho$, $p$ and $\phi$:

$$\rho = -\frac{A}{2B} \dot{\phi}^2,$$

$$p = \left( \frac{B_2}{B} - \frac{A}{2B} \right) \dot{\phi}^2 + \frac{B_1}{B} \ddot{\phi}.$$ 

(18)

2.4 Inflationary Universe

The essence of the inflationary model is that during inflation some scalar field is in a “wrong” vacuum where the scale factor grows (quasi)exponentially with time. The rapid expansion of the universe is almost exponential, where sufficient time forces the universe to be flat, and the correlated horizon to be large, which is represented by an almost constant $H$ during inflation.

In this paper, $\rho + p$ is a non-negative constant of space-time. If we write

$$H := \sqrt{\frac{\rho + \Lambda}{3}},$$

(19)

and solve (i) and (ii), we obtain

$$a(t)^2 = \left( \frac{\cosh Ht}{H} \right)^2 \quad \text{for } k = +1,$$

$$a(t)^2 = (\text{constant} \cdot \exp Ht)^2 \quad \text{for } k = 0,$$

and

$$a(t)^2 = \left( \frac{\sinh Ht}{H} \right)^2 \quad \text{for } k = -1.$$  

(20)

One can find that this space-time is four-dimensional maximally symmetric,

$$R_{\mu\nu\alpha\beta} = H^2 (g_{\mu\alpha}g_{\nu\beta} - g_{\nu\alpha}g_{\mu\beta}).$$  

(21)

To solve (iii) and (iv) we can find a solution for $\rho$ and $p$ (or $\phi$).

In the case of arbitrarily $A$, $B$ and $\Lambda = -\frac{1}{2}cB$ with negative constant $c$ there is a solution that

$$\dot{\phi} = 0.$$
Then, $\rho = -p = \text{constant} = 0$, which is static, and $H^2 = \frac{4}{3}$.

3 One Loop Divergence of Effective Action

3.1 Back Ground Field method and One-Loop Effective Action

We start with the back-ground field method [27]. We split the fields into background fields $(g_{\mu\nu}, \phi)$ and quantum fields $(h_{\mu\nu}, \varphi)$:

$$
\phi \rightarrow \phi' = \phi + \varphi, \quad g_{\mu\nu} \rightarrow g'_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu},
$$

where the background fields $(g_{\mu\nu}, \phi)$ are a solution of the classical equations of motion. The one-loop effective action is given by the standard general expression,

$$
\Gamma_{\text{1-loop}} = \frac{i}{2} \text{Tr ln } \hat{H} - i \text{Tr ln } \hat{H}_{\text{gh}},
$$

where $\hat{H}$ is the bilinear form of the action (1) with an added gauge fixing term, and $\hat{H}_{\text{gh}}$ is the bilinear form of the ghosts action ($S_{\text{gh}}$). To perform the calculations in the simplest way one needs to introduce a special form of the gauge fixing term:

$$
S_{\text{gf}} = \int d^4x \sqrt{-g} \chi_{\mu} \frac{\alpha}{2} \chi^\mu,
$$

where $\chi_{\mu} := \nabla_{\alpha} h_{\mu}^\alpha + \beta \nabla_{\mu} h + \gamma \nabla_{\mu} \varphi$, $h := h_{\mu}^\mu$, $h_{\mu\nu} := h_{\mu\nu} - \frac{1}{4} h g_{\mu\nu}$ and $\alpha, \beta, \gamma$ are functions of the background dilaton, which can be tuned for our purposes. For instance, if one choose these functions as

$$
\alpha = -B, \quad \beta = -\frac{1}{4}, \quad \gamma = -\frac{B_1}{B},
$$

the bilinear part of the action $(S + S_{\text{gf}} + S_{\text{gh}})$ and the operator ($\hat{H}$ and $\hat{H}_{\text{gh}}$) have an especially simple (minimal) structure:

$$
\left. \left( S + S_{\text{gf}} + S_{\text{gh}} \right) \right|_{\text{bilinear}} = \int d^4x \sqrt{-g} \left( \Phi \hat{H} \Phi^T + c_{\mu} H_{\text{gh}} \hat{c}^\mu \right),
$$

where $c_{\mu}$ is a constant.
where
\[ \hat{H} = \hat{K} \Box + \hat{L}_\mu \nabla^\mu + \hat{M} \quad \text{and} \quad \hat{H}_{\text{gh}} = g^{\mu\nu} \Box + \gamma (\nabla^\mu \phi) \nabla^\nu + \gamma (\nabla^\mu \nabla^\nu \phi) + R^{\mu\nu} \] (27)

Here, \( \Phi = (\tilde{h}_{\mu\nu}, h, \varphi) \), \( c_\mu \) stand for ghosts and \( T \) stands for transposition. The components of \( \hat{H} \) have the following form:
\[ \hat{K} = \begin{pmatrix} \frac{B_4}{4} \delta^{\mu\nu,\alpha\beta} & 0 & 0 \\ 0 & -\frac{B_3}{4} & -\frac{B_4}{8} \\ 0 & -\frac{B_4}{4} & \frac{B_3^2}{2B_1} - A \end{pmatrix} \] (28)

The next problem is to separate the divergent part of \( \text{Tr} \ln \hat{H} \). To do this we rewrite this trace as
\[ \text{Tr} \ln \hat{H} = \text{Tr} \ln \hat{K} + \text{Tr} \ln \left( \hat{1} \Box + \hat{K}^{-1} \hat{L}_\mu \nabla^\mu + \hat{K}^{-1} \hat{M} \right) . \] (31)

We note that the first term does not contribute to the divergences.

### 3.2 One Loop Divergence by Schwinger-DeWitt technique

We now explore the second term in eq.(31) which has the standard minimal form, and can be easily estimated by using the standard Schwinger-DeWitt method \([28, 3]\). The general off shell structure of
a one-loop divergence of the effective action is as follows [22]:

\[
\Gamma_{\text{1-loop, div}} = \frac{1}{16\pi^2\epsilon} \int d^4x \sqrt{-g} \left[ c_c G + c_c G^{\mu\nu\alpha\beta} + c_r R^2 + c_7 R + c_{12} 
+ c_4 R(\nabla \phi)^2 + c_5 R(\Box \phi) + c_6 R^{\mu\nu}(\nabla_\mu \phi)(\nabla_\nu \phi) + c_{11}(\nabla \phi)^2 
+ c_8 (\nabla \phi)^4 + c_9 (\nabla \phi)^2(\Box \phi) + c_{10}(\Box \phi)^2 + \text{(surface term)} \right],
\]

(32)

where \( \epsilon := D - 4 \) is a dimensional parameter; \( c_c, c_c, c_r \), and \( c_4, \cdots c_{12} \) are some functions of \( A, B, \Lambda \).

\[ G = R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \]

is the Gauss-Bonnet topological invariant and \( C_{\mu\nu\alpha\beta} \) is the Weyl tensor \((C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta} = R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 2 R_{\mu\nu} R^{\mu\nu} + \frac{1}{3} R^2)\).

In a homogeneous and isotropic space-time this structure can be written as follows.

\[
\Gamma_{\text{1-loop, div, R-W}} := \frac{1}{16\pi^2\epsilon} \int d^4x dt \sqrt{-g} \left[ a_1 (\kappa_s + \kappa_t)^2 + a_2 \kappa_s \kappa_t + \frac{\Lambda}{3} a_3 (\kappa_s + \kappa_t) + \left( \frac{\Lambda}{3} \right)^2 a_4 
+ \left( b_1 (\kappa_s + \kappa_t) + b_2 \kappa_s + \frac{\Lambda}{3} b_3 \right) \dot{\phi}^2 + \frac{\Lambda}{3} b_4 \ddot{\phi} + c_1 \dot{\phi}^4 + c_2 \dot{\phi}^2 \ddot{\phi} + c_3 \dddot{\phi}^2 \right] + \text{(s.t.)},
\]

(33)

where \( \kappa_s := \frac{\dot{a}}{a} \), \( \kappa_t := \left( \frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} \), and the explicit structure of \( a_1 \cdots c_3 \) is expressed in Appendix.

### 3.3 Divergence on Inflationary Background and Renormalization

Here, we consider an inflationary universe where \( \kappa_s = \kappa_t = H^2 = \frac{\Lambda}{3} = -\frac{\epsilon B}{\phi} \). Thus the structure of the divergence terms is very simple, such that

\[
\Gamma_{\text{1-loop, div, D-S}} := \Gamma_{\text{1-loop, div, R-W}} \bigg|_{R_{\mu\nu\alpha\beta} = \frac{\epsilon}{g_{\mu\nu} g_{\alpha\beta} - g_{\nu\beta} g_{\mu\alpha}}, \dot{\phi} = 0} = \frac{1}{16\pi^2\epsilon} \int d^4x \sqrt{-g} \left[ -\frac{371}{90} \Lambda(\phi)^2 \right].
\]

(34)

Remarkably, all of the dependences of \( A(\phi) \) and the derivative of \( B(\phi) \) canceled out (or all \( \delta, \omega \) dependence was cancelled). The on-shell structure of the effective action is a very simple, such that
we can easily renormalize that divergence by only renormalizing the function $B(\phi)$ (or $\Lambda(\phi)$); $c$ and $\phi$ do not have to be renormalized.

$$B_0(\phi) := \mu^2 \left( 1 - \frac{c}{16\pi^2} \frac{371}{180} \right) B(\phi), \quad (35)$$

where $\mu$ is the renormalization scale and $B_0$ is a bare function.

4 Conclusion and Discussion

In this paper we considered the general action (1) on homogeneous and isotropic four-dimensional background space-time, and used the Robertson-Walker metric, which is a realistic model of a hot universe. It is a spatially maximally symmetric space-time in which all quantities depend only on time. First, we considered this action at the classical level. We found that the classical equations of motion concerning the metric are the “Hubble” equation and the energy momentum conservation law, and that the equation of motion of dilaton is the “state” equation between the energy density and a pressure driven by dilaton. Since we have four parameters and five equations, it seems at first that there are no solutions. However, if we set $\Lambda = -\frac{1}{2}cB$ with a negative constant $(c)$ on the inflationary background, we were able to find a solution with constant dilaton, zero density, zero pressure, and $H^2 = \frac{\Lambda}{3}$. This solution is independent of time, and the background is the maximally symmetric four-dimensional space-time. Next, we considered this action at the quantum level. A one-loop calculation was carried out for the model (1) using the background field method. This calculation is formally the same as [22] but background fields must follow a solution of the equations of motion on homogeneous and isotropic space-time. We pulled a bilinear form out of action (1) with a gauge fixing term added, and out of the ghost action. Such a form is sufficient to calculate the effective action at one-loop levels. We have fixed a gauge for the minimal one to cancel the derivative terms, except for the d’Alembertian terms; we were then able to apply the standard Schwinger-DeWitt method.
to estimate the divergence of effective action. We found that the divergence is constructed by the scale factor of universe, the dilaton field, \( \delta = -\frac{B}{H} \), \( \omega = \frac{1}{H} \left( A - \frac{3}{2} \frac{B^2}{H} \right) \), and their time derivatives. The spatial distinction of the universe between open, flat and closed depends only on \( \kappa \). In this background the structure of the divergence is so complicated that it seems that the divergence of the effective action cannot be removed by renormalizing any quantity in the theory. If we consider an inflationary background (maximal symmetric one) especially, the structure of the divergence, however, turned out to be miraculously simple, such that there is only one term, which depends only on the cosmological function \( \Lambda(\phi) \) under a general function form of \( A(\phi), B(\phi) \); also, the divergence of the effective action can be removed by multiplicatively renormalizing only \( \Lambda(\phi) \). All of the dependences of \( A(\phi) \) and the derivative of \( B(\phi) \) were canceled. Such miraculous cancellations cannot be accidental. This suggests that the cancellations also occur in the higher loop structure of the divergence of the effective action during inflation, and that these cancellations may represent an effective appearance of some symmetry of the un-constructed complete quantum gravity on the inflationary background.

Considering the renormalizable condition in the case of the present paper, in the case of \ref{22} and in the case of pure gravity, we can find that all cases are the same; namely, in all cases the one-loop effective action is finite, or renormalizable, in the case of the absence or presence of a cosmological term, respectively. However, there is a difference: in the case of \ref{22} we must tune the functions \( A, B, \Lambda \), and arbitrary backgrounds are allowed. On the other hand, in the case of the present paper, arbitrary functions \( (A, B, \Lambda) \) are allowed, and backgrounds are restricted to a maximally symmetric one. It seems that the property between these models gives a cosmological vision between before inflation, during inflation and after inflation, namely an energy region where the dilaton has an important role with special coupling to the metric in the case of \ref{22}, the region where the dilaton is constant with general coupling to the metric in the case of the present paper, and where the dilaton is decoupled.
to the metric in the case of pure gravity. However, in order to reliably discuss the transitions between these phases we must find a complete quantum gravity, like a string theory, which can explain these phases in a unified way.

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A Appendix

The coefficients of the structure of divergence in equation (33) are the following (with notation $\delta(\phi) := -\frac{d}{d\phi}$ and $\omega(\phi) := \frac{1}{\phi} \left(A - \frac{3B^2}{4\phi^2}\right)$):

$$
a_1 = \frac{19}{2} + 2 \frac{1}{\omega'} + 6 \frac{1}{\omega''} + 4 \frac{\delta_1}{\delta^2 \omega'} + 2 \frac{\delta_1}{\delta^2 \omega''} + 2 \frac{\delta_1^2}{\delta^3 \omega'} ,
$$

$$
a_2 = \frac{99}{5} + 8 \frac{1}{\omega} ,
$$

$$
a_3 = 26 + 4 \frac{1}{3 \omega^2} + 14 \frac{1}{3 \omega} + 8 \frac{\delta_1}{3 \delta^2 \omega^2} + 2 \frac{\delta_1}{3 \delta^2 \omega''} + 4 \frac{\delta_1^2}{3 \delta^3 \omega^2} ,
$$

$$
a_4 = 5 + 2 \frac{1}{9 \omega^2} + 2 \frac{1}{9 \omega} + 4 \frac{\delta_1}{9 \delta^2 \omega^2} + 2 \frac{\delta_1^2}{9 \delta^3 \omega^2} ,
$$

$$
b_1 = -\frac{155}{4} \delta^2 - \frac{\delta^2}{\omega} - \frac{7 \delta^2}{2 \omega'} + 6 \delta^2 + 18 \delta_1 + 9 \frac{\delta_1}{\omega} + \frac{\delta_1^2}{\delta^2 \omega'} + 3 \frac{\delta_1^2}{\delta^2 \omega''} + 2 \frac{\delta_1^3}{\delta^3 \omega'} + 2 \frac{\delta_1^3}{\delta^3 \omega''} \\
+ 3 \frac{\delta_1 \omega_1}{\omega^2} + 4 \frac{\delta_1 \omega_1}{\omega} + \frac{\delta_1}{\delta \omega} + 2 \frac{\delta_1^2 \omega_1}{\delta^2 \omega^2} + \frac{1}{2} \frac{\omega_1}{\omega^3} + \frac{1}{4} \frac{\omega_1}{\omega^2} + \frac{1}{2} \frac{\delta_1}{\omega^3} \\
+ \frac{1}{2} \frac{\delta_1}{\omega^2} + 6 \delta_1 + \frac{\delta_1}{\delta} + 7 \frac{\delta_1}{\omega} + 4 \frac{\delta_1}{\omega^2} + 6 \frac{\delta_1 \omega_1}{\omega} - \frac{\delta_1 \omega_1}{\delta \omega} ,
$$

$$
b_2 = 3 \delta^2 - 3 \frac{\delta^2}{\omega} + 6 \delta_1 + 7 \frac{\delta_1}{\omega} + 4 \frac{\delta_1}{\omega^2} + 6 \frac{\delta_1 \omega_1}{\omega} - \frac{\delta_1 \omega_1}{\delta \omega} ,
$$

$$
b_3 = \frac{4 \delta_1}{3 \omega^3} - \frac{7 \delta^2}{2 \omega^2} + \frac{1 \delta^2}{3 \omega} + \frac{3 \delta^2}{\omega''} + 3 \delta \omega + 3 \frac{\delta_1}{\omega} + 3 \frac{\delta_1}{\omega^2} + 2 \frac{\delta_1^2}{\delta \omega'} + \frac{5}{6} \frac{\delta_1}{\delta^2 \omega^3} - \frac{4 \delta_1 \delta_2}{3 \delta^3 \omega} \\
- \frac{4 \delta_1 \delta_2}{3 \delta^3 \omega} + \frac{4 \delta_1^2 \omega_1}{\delta^3 \omega^3} + 8 \frac{\delta_1 \omega_1}{\delta^2 \omega^2} + \frac{1}{3} \frac{\delta_1 \omega_1}{\delta^3 \omega} + 3 \frac{\delta_1 \omega_1}{\delta^2 \omega^2} + 8 \frac{\delta_1^2 \omega_1}{\delta^3 \omega^3} + 8 \frac{\delta_1 \omega_1}{\delta^3 \omega} \\
+ \frac{7 \delta_1}{3 \omega^2} + \frac{1 \delta_1}{2 \delta} + \frac{4 \delta_1}{3 \delta^2 \omega} + \frac{2 \delta_1}{3 \delta^2 \omega} + \frac{5 \delta_1}{6 \omega^2} + \frac{2 \delta_1}{3 \omega^2} - \frac{1 \delta_2}{3 \omega^2} .
$$
\[ b_4 = \frac{31}{2} \delta + 2 \frac{\delta}{\omega^2} + 6 \frac{\delta}{\omega} + \frac{\delta_1}{\omega} + 4 \frac{\delta_1}{\omega^2} + 4 \frac{\delta_1}{\omega^3} + 2 \frac{\delta_1^2}{\omega^2} + \frac{2}{\delta^3} + \frac{8}{\omega} + 1 \frac{\omega_1}{\omega^2} + 1 \frac{\omega_1}{\omega^2} + \frac{\delta_1}{\omega^2} + \frac{\delta_1}{\omega^2}, \]

\[ c_1 = \frac{33}{16} \delta^4 \omega + \frac{201}{32} \delta^4 + \frac{45}{4} \delta^4 \omega^2 + \frac{1}{8} \frac{\delta^4}{\omega^2} + \frac{1}{2} \frac{\delta^4}{\omega} - \frac{1}{6} \frac{\delta_1}{\omega} - \frac{1}{6} \frac{\delta_1}{\omega^2} + \frac{1}{2} \frac{\delta_1}{\omega^3} + \frac{1}{\delta_1} \frac{\delta_1}{\omega^2} + \frac{1}{4} \frac{\delta_1}{\omega^2} + \frac{1}{6} \frac{\delta_1}{\omega^2} + \frac{1}{4} \frac{\delta_1}{\omega^2} + \frac{3}{\delta_1} \frac{\delta_1}{\omega^2}, \]

\[ c_2 = -\frac{105}{8} \delta^3 \omega - \frac{47}{2} \frac{\delta^3}{\omega^2} + \frac{3}{2} \delta^3 \omega^2 + \frac{3}{8} \frac{\delta^3}{\omega} + \frac{35}{8} \delta \frac{\delta}{\omega^2} + \frac{2}{\delta} \frac{\delta}{\omega} + \frac{1}{4} \frac{\delta}{\omega^2} + \frac{4}{\delta} \frac{\delta}{\omega^2} - \frac{1}{4} \frac{\delta}{\omega^2} + \frac{4}{\delta} \frac{\delta}{\omega^2} + \frac{7}{8} \frac{\delta}{\omega^2} - \frac{3}{4} \frac{\delta}{\omega^2} + \frac{3}{4} \frac{\delta}{\omega^2} + \frac{3}{4} \frac{\delta}{\omega^2} + \frac{3}{4} \frac{\delta}{\omega^2} + \frac{3}{4} \frac{\delta}{\omega^2}, \]

\[ c_3 = \frac{43}{8} \delta^2 + \frac{1}{2} \frac{\delta^2}{\omega^2} + \frac{3}{2} \frac{\delta^2}{\omega} + \frac{3}{2} \frac{\delta^2}{\omega^2} + \frac{3}{2} \frac{\delta^2}{\omega^2} + \frac{1}{2} \frac{\delta^2}{\omega^2} + \frac{1}{2} \frac{\delta^2}{\omega^2} + \frac{1}{2} \frac{\delta^2}{\omega^2} + \frac{1}{2} \frac{\delta^2}{\omega^2} + \frac{1}{8} \frac{\delta^2}{\omega^2}, \]

References

[1] M. B. Green, J. H. Schwarz and E. Witten, Superstring Theory, (Cambridge University Press, Cambridge, 1987).

[2] K. S. Stelle, Renormalization of the Higher Derivative Quantum Gravity, Phys. Rev. 16D, (1977), 953.
[3] I. L. Buchbinder, S. D. Odintsov and I. L. Shapiro, *Effective Action in Quantum Gravity*, (IOP, Bristol, 1992).

[4] S. Weinberg, *Gravitation and Cosmology*, (J. Wiley and Sons, 1972).

[5] A. H. Guth, *A Possible Solution to the Horizon and Flatness Problems*, Phys. Rev. **D23**, (1981), 347.

[6] A. Linde, *A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems*, Phys. Lett. **108B**, (1982), 389.

[7] A. Linde, *Chaotic Inflation*, Phys. Lett. **129B**, (1983), 177.

[8] P. J. Steinhardt and M. Turner, *A Prescription for Successful New Inflation*, Phys. Rev. **D29**, (1984), 2162.

[9] A. Linde, *Particle Physics and Inflationary Cosmology*, (Harwood, Chur, Switzerland, 1990).

[10] G. Veneziano, *Strings, Cosmology, ... And A Particle*, CERN-TH.7502/94

[11] G. Veneziano, *Scale Factor Duality For Classical And Quantum Strings*, Phys. Lett. **B265**, (1991), 287.

[12] G. Veneziano, *The Graceful Exit Problem in String Cosmology*, Phys. Lett. **B329**, (1994), 429.

[13] A. A. Tseytlin and C. Vafa, *Element of String Cosmology*, Nucl. Phys. **B372**, (1992), 443.

[14] A. A. Tseytlin, *String Cosmology and Dilaton*, Eric. Theo. Phys. (1992), 202.

[15] A. A. Tseytlin, *Cosmological solutions with dilaton and maximally symmetric space in string theory*, Int. J. Mod. Phys. **D1**, (1992), 223.
[16] A. A. Tseytlin, *Duality and Dilaton*, Mod. Phys. Lett. **A6**, (1991), 1721.

[17] K. Kikkawa and M. Yamasaki, *Casimir Effect in Superstring Theories*, Phys. Lett. **B149**, (1984), 367.

[18] J. H. Schwarz, *Superstring Compactification and Target Space Duality*, Strings: Stony Brook, (1991), 3, [hep-th/9108022].

[19] G. t’Hooft and M. Veltman, *One Loop Divergences in the Theory of Gravitation*, Ann. Inst. H. Poincare. **A20**, 69, (1974).

[20] S. Christensen and M. Duff, *Quantum Gravity with A Cosmological Constant*, Nucl. Phys. **170B**, (1980), 480.

[21] A. O. Barvinski, A. Kamenschik and B. Karmazin, *The Renormalization Group for Non-Renormalizable Theories: Einstein gravity with A Scalar Field*, Phys. Rev. **D48**, 3677, (1993).

[22] I. L. Shapiro and H. Takata, *One Loop Renormalization of the Four-Dimensional Theory for Quantum Dilaton Gravity*, Phys. Rev. **D52**, 2162, (1995).

[23] I. L. Shapiro and H. Takata, *Conformal Transformation in Gravity*, Phys. Lett. **B361**, (1995), 31.

[24] T. Inagaki, S. Mukaiwaga and T. Muta, *A Soluble Model of Four-Fermion Interactions in de Sitter Space*, Phys. Rev. **D52**, (1995), 4267.

[25] I. Antoniadis, J. Iliopoulos and T. N. Tomaras, *One Loop Effective action around De Sitter Space*, [hep-th/9510112].

[26] I. L. Shapiro and H. Takata, in Preparation.
[27] L. F. Abbott, *The Background Field Method Beyond One Loop*, Nucl. Phys. **185B**, (1981), 189.

[28] B. S. DeWitt, *Dynamical Theory of Groups and Fields*, (Gordon and Breach, NY, 1965).