Gravitational wave signature of proto-neutron star convection

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GW signal in CCSNe

Bounce signal:
- only fast rotating models
- $\Delta t \sim 5$ ms
- $f \sim 600$-900 Hz
- $h \sim 10^{-21}$ @ 10 kpc

Post-bounce “SN” signal:
- g-modes, SASI, convection
- $\Delta t \sim 0.1$-1- s
- $f \sim 50$-2000 Hz
- $h \sim 10^{-23}$-$10^{-22}$ @ 10 kpc

PNS convection signal:
- Computationally expensive
- $\Delta t \sim 10$-50- s
- $f \sim ?$
- $h \sim ?$

Richers et al 2017
Torres-Forné et al 2019

Onset of collapse
Bounce and shock formation
Onset of explosion
End of the convective phase
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Modelling the PNS convective zone

**Input:**
- Temperature profile
- Density profile

**Transport coefficients:**
- Kinematic viscosity $\nu$
- Thermal diffusivity $\kappa$
- Magnetic diffusivity $\eta$

**Hypothesis:**
- Spherical geometry
- Adiabatic stratification
- Low Mach convection
- 2nd order diffusion approximation for the neutrino transport
- Electrical conductivity of degenerate, relativistic electrons

**Boundary conditions:**
- Mechanical: stress-free / no slip
- Thermal: fixed entropy flux
- Magnetic: perfect conductor ($B_{//}$) / pseudo-vacuum ($B_{\perp}$)

**Orders of magnitude**
\[
\begin{align*}
\Phi_o & \sim 10^{52} \text{ erg/s} \\
\rho_o & \sim 10^{13} \text{ g/cm}^3 \\
\nu_o & \sim 10^{10} \text{ cm}^2/\text{s} \\
\kappa_o & \sim 10^{12} \text{ cm}^2/\text{s} \\
\eta_o & \sim 10^{-3} \text{ cm}^2/\text{s}
\end{align*}
\]
Early and late time background models

Source
Lorenz Hüdepohl’s PhD thesis
Prometheus-Vertex code
1D model + MLT
LS220 EoS
27 M\textsubscript{\odot} progenitor
PNS baryonic mass 1.78 M\textsubscript{\odot}

Method
1. stability determined according to the Schwarzschild criterion
2. deduce the shell geometry
3. fit the background profile \((\bar{\rho}, \bar{T})\)
The MHD anelastic equations

\begin{align*}
[d] &= r_o - r_i, \\
[t] &= d^2/\nu_o, \\
[S] &= d \partial S/\partial r|_{r_o}, \\
[p] &= \Omega \nu_o, \\
[B] &= \sqrt{\Omega \mu_0 \eta_o}
\end{align*}

\[ \nabla \cdot B = 0 \]

\[ \frac{\partial B}{\partial t} = \nabla \times (u \times B) - \frac{1}{Pm} \nabla \times (\eta \nabla \times B) \]

\[ 0 = \nabla \cdot (\ddot{\theta} u) \]

\[ \frac{D u}{D t} = -\nabla \left( \frac{p}{E \ddot{\theta}} \right) - \frac{2}{E} e_z \times u - \frac{Ra d \ddot{T}}{Pr} S e_r + F_\nu + \frac{1}{EPm \ddot{\theta}} \left( \nabla \times B \right) \times B \]

\[ \frac{D S}{D t} = \frac{1}{Pr \ddot{\theta} \ddot{T}} \nabla \cdot \left( \kappa \ddot{\theta} \ddot{T} \nabla S \right) \]

Heat flux

\[ + \frac{Pr}{Ra \ddot{\theta} \ddot{T}} \left( \frac{\eta}{Pm^2 E} (\nabla \times B)^2 + Q_\nu \right) \]

Ohmic heating

Viscous heating

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GW signal of PNS convection
## 3D MHD direct numerical simulations

### Control parameters

| Parameter               | Formula |
|-------------------------|---------|
| Prandtl number          | $Pr = \nu_o/\kappa_o$ |
| Magnetic Prandtl number | $Pm = \nu_o/\eta_o$ |
| Ekman number            | $E = \frac{\nu_o}{\Omega d^2}$ |
| Rayleigh number         | $Ra = \frac{\bar{T}_o d^3}{\nu_o \kappa_o} \frac{\partial S}{\partial r} \bigg|_{r_o}$ |

### Input

- $Ra/Ra_c \sim 10$
- $Pr = 0.1$
- $Pm \sim 5 \quad (\ll 10^{14})$
- $E \equiv P_{rot} \in [1 \text{ ms}, 10^2 \text{ ms}]$

### Output

Gravitational signal computed with the quadrupole approximation
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Dipole field strength

$B_{\text{dip}} \ [G]$

Rotation period [ms]

Strong field scaling

$E_B/E_k$

$R_o^{-1}$

Magnetostrophic balance : Lorentz-Coriolis

$\frac{E_B}{E_k} \propto R_o^{-1}$

$R_o = \frac{u}{d\Omega}$

Raynaud et al. 2020
Typical cases: slow versus fast rotation

- P=175 ms
  - Alpha-omega dynamo
  - Not realistic spectrograms!
  - NB: fixed background!

- P=2.1 ms
  - Strong field dynamo
  - Magnetar formation
  - (Raynaud et al., 2020)
Amplitude scaling

 Scaling relation for slow/fast regimes

$$ h_{\text{slow}} \propto \frac{2G}{De^4} r_{\text{mid}}^2 M_{\text{conv}} \left( \frac{\gamma}{2\sigma_{\text{mlt}}} + 1 \right) \frac{d^{-2/3}}{c^2} \left( \left| \partial_r \ln \tilde{T} \right| \frac{\Phi_0}{4\pi r_{\text{mid}}^2} \right)^{4/3} $$

$$ h_{\text{fast}} \propto \frac{2G}{De^4} r_{\text{mid}}^2 M_{\text{conv}} \left( \frac{\gamma}{2\sigma_{\text{mlt}}} + 1 \right) \frac{d^{2/5}}{c^2} \left( \left| \partial_r \ln \tilde{T} \right| \frac{\Phi_0}{4\pi r_{\text{mid}}^2} \right)^{4/5} \Omega^{8/5} $$

Decay of the convective flux

Background time evolution

Rossby number

$$ \text{Ro} = \frac{u}{d\Omega} $$

$$ h_{\text{rms}} / h_{\text{slow}} $$

$$ \Phi \left[ \text{erg cm}^{-1} \right] $$

$$ \text{Time [s]} $$

$$ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 $$

$$ 10^{-2} \quad 10^{-1} \quad 10^0 \quad 10^1 \quad 10^2 $$

$$ \frac{\text{Myr}}{\text{cm}} $$

$$ D = 10 \text{ [kpc]} $$

$$ t = 0.2 \text{ [s]} $$

$$ t = 5 \text{ [s]} $$

hydro

MHD

GW signal of PNS convection

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Frequency scaling: slow rotation

Broad spectrum due to convection

Peak frequency scales with the turnover frequency $U/d$

Broad spectrum due to convection
Fast rotation: spectra

Hydro

$P = 1.6 \text{ [ms]}$

LHP1.6

$D = 10 \text{ [kpc]}$

MHD

$P = 1.0 \text{ [ms]}$

LMP1.0

$D = 10 \text{ [kpc]}$

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GW signal of PNS convection
Fast rotation: frequency scaling

$t = 5 \text{ s post bounce}$  $1 \text{ ms} < \text{Period} < 6 \text{ ms}$

Signature of inertial modes
Increasing rotation rate
Peaks scale with the PNS rotation frequency

$Ra=0$ models
(Decaying turbulence)

Signature of inertial modes

$Ra=0$ models
(Decaying turbulence)
Strong field dynamo signature?

Hypothesis: $m=1$ Rossby mode modified by toroidal magnetic field?

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Conclusions

Raynaud et al. (arXiv:2103.12445)

Slow rotation \((Ro \gg 1)\)
- broad spectrum
- peak scales with \(f_{\text{turn}}\)
- weak impact of B field

Fast rotation \((Ro \ll 1)\)
- \(h_{\text{rms}}\) strongly increases
- complex spectra
- peaks scale with \(f_{\text{rot}}\)
- inertial modes
- low frequency signature of strong field dynamo

Limitations
- consider only one background model
- no continuous evolution of the PNS cooling (no realistic GW template)
- convective zone only (no \(g\)-modes)

Perspectives: detectability
- use amplitude/frequency scalings to rescale the signal as a function of the background evolution