Multi-spin errors in the optical control of a spin quantum memory

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We study a quantum memory composed of an array of charged quantum dots embedded in a planar cavity. Optically excited polaritons, i.e. exciton-cavity mixed states, interact with the electron spins in the dots. Linearly polarized excitation induces two-spin and multi-spin interactions. We discuss how the multi-spin interaction terms, which represent a source of errors for two-qubit quantum gates, can be suppressed using local control of the exciton energy. We show that using detuning conditional phase shift gates with high fidelity can be obtained. The cavity provides long-range spin coupling and the resulting gate operation time is shorter than the spin decoherence time.

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a. Introduction

In the last few years there have been great advances towards quantum information processing in the solid state. Yet, there are many theoretical and practical problems that remain to be addressed. In particular, there is not yet a solid state systems for which all the feasibility criteria for quantum computing (i.e. decoherence, reliable one- and two-qubit operations, scalable qubit, initialization and read-out) have been simultaneously demonstrated. Lately, electron spins in semiconductors, localized either in low-dimensional nanostructures, i.e. QDs or in impurities, are increasingly receiving attention as qubits due to their very long decoherence time, which is typically of the order of $T_2 \approx 3 \mu s$. The long coherence time of the electron spin is due to its weak interaction with the environment, which on the other hand makes its control more demanding. In this framework, optical techniques are very promising since in this case the control is realized using an optically active ancillary excited state, e.g. a trion state in quantum dots, leading to a control that can be obtained in picoseconds. Optical initialization\cite{10,11} single qubit measurement\cite{12,13} and selective one-qubit control of QD’s spin\cite{10,11} have been already demonstrated. Similar experiments on impurity states have also been carried out\cite{12,13}

The two-qubit control represents a more challenging task. Optically mediated long range spin interaction in a cavity system has been explored theoretically only for two QDs\cite{14,15}.

In this paper, we show that an array of charged QDs embedded in a planar cavity (see Fig. 1) is a good candidate for a controllable quantum memory. We extend the previous works on polariton mediated spin coupling\cite{16,17} to the case of many dots, which leads to the appearance of multi-spin Ising-like coupling terms. We consider a system in which the energy of the ancillary states on each dot can be controlled independently, for instance using gates on each dot. We calculate the fidelity of the phase gate of two spins being in resonance with the cavity mode and show that by controlling the detuning of the remaining dots, gates with very small error can be obtained. Errors due to multi-spin terms in the case of quantum dot directly coupled by wavefunction overlap have also been studied recently\cite{18}. The model of multi-spin coupling is also applicable to similar systems like e.g. superconducting qubits embedded in a cavity, for which the two-qubit control has been demonstrated in a recent experiment\cite{19}.

b. Polariton-Spin Hamiltonian

Our assumptions for the system studied are the following: (i) the trion energy $\Delta_{tr}$ of each dot can be independently controlled e.g. by applying a local voltage\cite{20}, (ii) the quantum dots are well separated so there is not direct overlap of the trion wavefunction, (iii) each dot can be occupied only by one additional exciton, (iv) the heavy-hole light-hole splitting is large enough that only the heavy-hole exciton is taken into account, and (v) the cavity is ideal. The role of the cavity is to enhance the range of the interaction between dots\cite{21} and their spins\cite{22}. The Hamiltonian describing the memory can therefore be written as ($\hbar = 1$ throughout}
the off-diagonal terms operator of exciton on the jth dot at position $R_j$ with polarization $\alpha$, $a_{\alpha q}^{\dagger} (a_{\alpha q})$ is the creation (annihilation) operator of the photon with two-dimensional momentum $q$, $g_{qj} = g e^{-q^2 \beta^2} e^{iqR_j}$ is the dot-photon coupling constant with $\beta$ being the effective dot size, $J_S$ is the energy difference between trion states with parallel and antiparallel spins as schematically shown in Fig. 1. $S_{iz}$ is the z-component of the electron spin in the jth QD, and $P_{zj} = C_{i\alpha}^{\dagger}C_{j\alpha} - C_{j\alpha}^{\dagger}C_{i\alpha}$ is the operator corresponding to the z component of the exciton polarization. A $\sigma^+ (\sigma^-)$ polarized photon creates a bright exciton with $\downarrow (\uparrow)$ electron spin in the growth (z) direction. For excitons in III-V confined systems the possible values of the electron spin are $\sigma^s = \pm 1/2$ and the heavy hole spin are $\sigma^h = \pm 3/2$.

We assume throughout the paper that the light is linearly polarized. This choice simplifies considerably the multi-spin problems since it makes all multi-spin terms of odd order identically zero. The coupling of the cavity to the external electromagnetic field is described using the quasi-mode Hamiltonian

$$H_{\text{L}} = \sum_{q \alpha} (V_{q\alpha} e^{iqz L} a_{\alpha q} + \text{h.c.})$$

where $V_{q\alpha}$ is the laser-cavity coupling constant proportional to the cavity area $\sqrt{\lambda}$.

c. Multi-Spin Hamiltonian

The effective spin Hamiltonian can be calculated introducing the level shift operator $R(\omega_{L})$ as

$$\hat{H}_s = \hat{P} R(\omega_{L}) \hat{P} = \hat{P} \hat{H}_{\text{L}} - \frac{Q}{\omega_{L} - \hat{H}_Q} \hat{H}_L \hat{P},$$

where

$$\hat{P} = \sum_{\lambda} |\lambda\rangle \langle \lambda| \otimes |0\rangle \langle 0|$$

is the projection operator on the subspace of all spin states $\lambda$ and zero [one] excitation. Assuming the rotating wave approximation and linearly polarized laser light propagating perpendicularly to the cavity plane ($q = 0$) then $H_{\text{L}} = V_{10} a_{10} + V_{10} a_{01} + \text{h.c.}$. By solving first the polariton problem for $q = 0$ and both polarizations, we can write $\hat{P} |\alpha \uparrow (\downarrow)\rangle = \omega_{\alpha} |\alpha \uparrow (\downarrow)\rangle$ and $|\alpha \downarrow (\uparrow)\rangle = (\sum_{\gamma} v_{\alpha \gamma} C_{0\gamma(1)}^{\dagger}) |\alpha \gamma\rangle + \sum_{\gamma} v_{\alpha \gamma} a_{0\gamma(1)}^{\dagger}) |0\rangle$ where $Q$ is a reciprocal lattice vector of the dot lattice. Then the matrix element between the spin states reads

$$R_{\alpha \hat{\alpha}} = \sum_{\alpha \beta} v_{\alpha 0} v_{\beta 0}^{\ast} \sum_{\gamma \beta} \frac{V_{\alpha 0}^{2}}{2} \langle \alpha \gamma | (\omega_{L} - \hat{H})^{-1} |\beta \gamma\rangle.$$ 

The off-diagonal terms $\langle \lambda| (\omega_{L} - \hat{H})^{-1} |\lambda'\rangle$ are zero since all spin dependent terms are proportional to $S_z$. This allows us to calculate the energies in Eq. (3) exactly. Perturbation theory can also be applied and, assuming linearly polarized light, only even contributions ($\sim J_S^{(2n)}$) are nonzero giving

$$\hat{H}_T = \sum_{j \neq j} j^{(2)}_{ij} S_{iz} S_{jz} + \sum_{i > j > k > l} j^{(4)}_{ijkl} S_{iz} S_{jz} S_{kz} S_{lz} + \cdots$$

where the coupling constants are renormalized to take into account multiple scattering, e.g.

$$j^{(2)}_{ij} = J_{12}^{(2)} + J_{21}^{(2)} \pm \sum_{i \neq j \neq k \neq l} j^{(4)}_{ijkl} S_{iz} S_{jz} S_{kz} S_{lz} \pm \cdots$$

where $P$ indicates a permutation of all the indices. The z-coupling constants can be explicitly expressed as

$$j^{(n)}_{i_1 \cdots i_n} = J_{12}^{n} V_{L}^{2} (C_{i_n}^{\dagger})^{*} T_{i_1 i_2} \cdots T_{i_{n-1} i_n} C_{i_n}^{+}$$

in terms of the photon-exciton coupling function and exciton inter-dot transfer probability (see scheme in Fig. 2).

FIG. 2: Diagram illustrating multiple scattering events that lead to a multi-spin coupling $J_{8,15,13,10}$, as derived in Eq. (4).

Let us now consider two dots labeled by $\{1,2\}$ with a small detuning with respect to the lowest cavity mode, i.e. $\Delta x_{1,2} = \Delta p$. The remaining quantum dots are detuned by a larger amount: $\Delta x_{j \neq 1,2} > \Delta p$, as schematically plotted in Fig. 1(a). Dots shifted off-resonance by a DC Stark shift will also have a weaker light-dot coupling $g$ due to the decrease of the electron-hole overlap. However, in order to have a conservative estimate of the error we neglect this effect.

We have used the following parameters: $\beta = 35$ nm, $g = 70$ meV, $\eta = 50$ meV, $V_{10} = V_{01} = 0.9$ meV, $\omega_{L} = \omega_{q=0}$, $J_{S} = 0.39$ meV, and exciton detuning up...
to ΔX = 20 meV, which is about the upper limit for a Stark shift that can be obtained in current experiments. In the numerical calculation we consider a finite system with 9 dots and we used periodic boundary conditions in order to match the excitonic states in the dots with the continuous two-dimensional photon modes.

The dependence of different multi-spin terms on the detuning is shown in Fig. 3 where we separate the terms that involve the two dots nearly resonant with the cavity from the others. We plot J12 + J21 (solid black) and ∑_{ij≠\{1,2\}} [J_{ij}] (black dashed) for n = 2 spin terms. The contributions, that renormalize the effective coupling between 1 and 2 (J_{R}^{(n)}) from contributions that involve only the dots strongly detuned from the cavity (J_{O}^{(n)}) are separated for multi-spin terms (n = 4, n = 6). For instance, for n = 4 the resonant (off-resonant) terms are defined as J_{O}^{(4)} = ∑_{ijkl} |J_{ijkl}| (J_{O}^{(4)} = ∑_{ijkl} |J_{ijkl}| - |J_{R}^{(4)}|). This definition enables us to better estimate the contribution of the off-resonant terms. In fact, even if the magnitude of the individual terms J_{ijkl}^{(4)} is very small (e.g. 10^{-15} for J_{ijkl}^{(6)}) we get a sizeable effect due to the large number of n-dot combinations (∼ \left(\binom{9}{N_{D}}\right)). Note that there is almost no dependence on the detuning for the resonant terms and a strong decrease for the off-resonant terms (J_{R}^{(n)} ∼ ΔX^{-(n-1)}) as expected from the form of the coupling in Eq. 4.

d. Fidelity of a conditional phase shift gate

We will now explicitly estimate the error in the implementation of a conditional phase shift gate due to multi-spin interaction terms. The conditional phase gate (PG), is a universal two-qubit gate, i.e. can realize universal quantum computation when combined with single qubit operations. In the computational basis \{\{↓↓\},\{↓↑\},\{↑↓\},\{↑↑\}\} the PG can be written as diagonal matrix with elements U_{PG} = {1,1,1,-1}. Assuming the Ising-like interaction between two spins ∼ S_{z1}S_{z2}, the following sequence gives the PG U_{PG} = e^{π/4}[S_{z1} + S_{z2}][−2S_{z1}S_{z2}] with [P] = e^{π/2P}. A quantitative measure of the gate quality can be been given using the gate fidelity defined as \(F = |⟨ψ|U_{P}^†U_{R}|ψ⟩|^2\), where U_{I} is the ideal gate matrix, and U_{R} is the real gate matrix, i.e. one that includes the effects of multi-spin terms. \(Ψ\) is an arbitrary initial pure state, and \(|⟨Ψ|Ψ⟩|^2\) indicates averaging over all pure states. Working in the basis of full spin-Hamiltonian eigenstates \{\phi_{i}\} (with \(2^{N_{D}}\) states), we can define an eigenvector fidelity as \(F_{i} = |⟨ψ_{i}|U_{P}^†U_{R}|ψ_{i}⟩|^2\). Since the total Hamiltonian does not allow for spin flip processes, the fidelity can be expressed as \(F = |N_{D}^{-1} \sum_{i} F_{i}|^2\). In order to calculate the fidelity, we calculate the dynamics exp(-iH_{R}t_{C}), where the time t_{C} is optimized so to obtain maximal fidelity. The gate can be described as follows: (i) two selected dots \{1,2\} are brought adiabatically into resonance with the cavity by controlling the exciton energy with local electric field, (ii) the laser is switched on for a time t_{C}, and (iii) dots are brought back into the off-resonant state.

The calculated error \(E = 1 - F\) as a function of the detuning and lattice constant of the dot array is shown in Fig. 4. The fidelity F increases at larger detuning ΔX since only the two selected dots \{1,2\} remain in resonance with the cavity and the multi-spin coupling with the other dots is suppressed. On the other hand, increasing the lattice constant decreases the fidelity since the exciton transfer, even if considerably enhanced by the cavity, decreases with distance. The strong dependence of the fidelity on the detuning reflects the competition between the resonant and off-resonant terms as shown in Fig. 3. Furthermore, note that the maximal value of the individual dot detuning is limited by the inter dot separation a. Then the fidelity function \(F(Δ, a)\) can be used to select an optimal lattice constant.

Another important characteristic of the PG is its operation time, i.e. the time during which the spin-interaction is switched on. The operation time increases with increasing detuning since the spin-spin coupling increases.

FIG. 3: (Color online) The logarithmic plot of \(J_{R}^{(n)}\) (solid) and \(J_{O}^{(n)}\) (dashed) [see text for details] as a function of the detuning ΔX in a 3×3 array of charged QDs with ΔP = 1 meV and the lattice constants a = 100 nm for n = 2 (black), n = 4 (red), and n = 6 (blue) are shown.

FIG. 4: (Color online) Logarithmic plot of the error E as a function of the detuning ΔX and lattice constant a in a Phase Gate between two most distant dots in a 3×3 array of charged QDs, ΔP = 1 meV.
~ $J_{12}$ decreases. Note that the time $t_C$ grows like $\Delta^2$, following the dependence of the resonant terms in Eqs. 6 and 7. Typical values of the operation times are $t_C = 100 \text{ps}$ ($t_C = 450 \text{ps}$) for $a = 100 \text{nm}$ ($a = 1300 \text{nm}$) [$\Delta_X = 20 \text{meV}$]. These characteristic gate times are shorter than the spin decoherence time $T_2$, which is of orders of at least $\mu$s.

e. Conclusions We have studied an array of charged quantum dots embedded in a planar cavity as a candidate for the realization of a spin quantum memory. We have shown that optical excitation can be used to control the spins and implement quantum gates. The optical excitation couples many dots in the quantum memory, and multi-spin interaction terms beyond the ideal two-spin interaction are generated. We have shown that the multi-spin terms can induce errors in the gate operation even if their value is small, due to their multiplicity. These error can be corrected by a local control of the excitonic resonance on each dot. In the control scheme we also include a planar cavity that modifies the photon density of states by providing a spectral region where dots do not couple to radiation. The present control scheme can be applied to other similar solid state systems like e.g., superconducting qubits embedded in a cavity.

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