Identification of perturbation modes and controversies in ekpyrotic perturbations

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(October 31, 2018)

If the linear perturbation theory is valid through the bounce, the surviving fluctuations from the ekpyrotic scenario (cyclic one as well) should have very blue spectra with suppressed amplitude for the scalar-type structure. We derive the same (and consistent) result using the curvature perturbation in the uniform-field (comoving) gauge and in the zero-shear gauge. Previously, Khoury \textit{et al.} interpreted results from the latter gauge condition incorrectly and claimed the scale-invariant spectrum, thus generating controversy in the literature. We also correct similar errors in the literature based on wrong mode identification and joining condition. No joining condition is needed for the derivation.

PACS numbers: 04.20.Dw, 98.80-k, 98.80.Cq, 98.80.Hw

\section*{I. INTRODUCTION}

The issue of scalar-type structure generated in the recently proposed ekpyrotic scenario is shrouded with controversies by two opposing camps \cite{1–3} and \cite{4–10}; for an introduction to the scenario, see \cite{11}. The main point of \cite{1} is that the dominating solution viewed in the zero-shear hypersurface (gauge) in the collapsing phase happens to show a scale-invariant spectrum. However, this mode was identified in \cite{5,6} as a transient mode in the subsequent expanding phase, thus uninteresting. In this work we wish to add some additional points to \cite{6}. We will show that the same blue spectrum is generated even in the zero-shear gauge by identifying the mode relevant in the later expanding phase. Apparently, the same final observable spectrum should be derived independently of the gauge conditions used, and our result confirms it. We also point out that the possible scale-invariant spectra and others argued in \cite{2,3,12} are errored by identifying wrong modes (often based on \textit{ad hoc} joining conditions) which are transient in the expanding phase, thus irrelevant.

Before we embark on studying the evolution of structures through bounce using the linear perturbation theory, we would like to state clearly the provisions we need. In \cite{7} Lyth has clearly shown that the linear perturbation theory \textit{breaks down inevitably} as the model approaches the singularity in a singular bounce (if such a bounce is possible at all, \cite{13}), see also §VI of \cite{8}. Somehow, this strong conclusion is unluckily ignored by many authors \cite{1,2}. If the bounce is singular we cannot rely on the linear perturbation theory. Thus, in the ekpyrotic scenario and other bouncing models considered in this paper we will explicitly \textit{assume} that the bounce occurs before the linear theory breaks down, and this \textit{requires} the bounce to be smooth and nonsingular. Although the authors of \cite{1} claimed that the bounce in the ekpyrotic scenario to be singular, assuming a nonsingular bounce in such a scenario is legitimate, particularly if we consider the currently unknown physics near the bounce. Since the consequence of the singular bounce is clearly resolved in \cite{7} \textit{i.e.}, the linear theory fails!\textit{) investigating the remaining window with nonsingular bounce would be important to clearly resolve the remaining issue.}

In a single component fluid or field, the scalar-type perturbation is described by a second-order differential equation with two solutions (modes). In the large-scale limit (to be defined later) we can often derive a general asymptotic solution with two modes, see eq. \eqref{eq:7}. In an expanding phase we can identify clearly which ones are relatively growing (C-mode) and decaying (d-mode). If the initial condition is imposed at some early expanding epoch the decaying mode is transient in time, and naturally we are only interested in the relatively growing mode. If we introduce a collapsing phase before the early big-bang phase, however, the conventional growing and decaying classification can be often reversed. Still, if the large-scale conditions are met \textit{and of course, if the linear theory as well as the classical gravity are intact}, the general solutions in eq. \eqref{eq:7} remain valid throughout the transition. Thus, in our observational perspective situated in expanding phase we are interested in the initial condition imposed on the C-mode, even if it \textit{was} subdominating \textit{(relatively decayin}g) compared with the other mode when the initial condition was imposed. Although we made this point clear in \cite{6}, in this work we will reinforce it by deriving concretely the C-mode initial conditions coming from the quantum vacuum fluctuations in the two gauge conditions used previously. In this way, we hope we could clear some of the controversies concerning ekpyrotic scenario and others in the literature. §II and III are reviews. §IV contains our main results with consequences analysed in §V. We set $c \equiv 1 \equiv h$. 

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\section*{II. INITIAL CONDITIONS}

The initial conditions for the ekpyrotic scenario are derived in the following way. To be consistent with the investigation of the ekpyrotic scenario, we specify the initial conditions at the moment the curvature perturbation \textit{equals} the zero shear in the 

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\section*{III. \textit{AD HOC} JOINING CONDITIONS}

In the linear perturbation theory, the \textit{ad hoc} joining conditions are the most common tools for investigating the perturbations in the ekpyrotic scenario. The joining conditions are needed to connect the solution in the expanding phase with the solution in the collapsing phase.

\subsection*{A. Zero-shear Hypersurface (gauge) in the Collapsing Phase}

In the zero-shear hypersurface (gauge) in the collapsing phase, the initial condition is imposed at some early expanding epoch. The initial condition is given by \textit{ad hoc} joining conditions, which are typically specified as the curvature perturbation \textit{equals} the zero shear in the zero-shear hypersurface (gauge).

\subsection*{B. Uniform-field (Comoving) Gauge in the Expanding Phase}

In the uniform-field (comoving) gauge, the initial condition is imposed on the initial hypersurface (gauge) in the expanding phase. The initial condition is given by \textit{ad hoc} joining conditions, which are typically specified as the curvature perturbation \textit{equals} the zero shear in the zero-shear hypersurface (gauge).

\subsection*{C. \textit{AD HOC} JOINING CONDITIONS}

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II. BASIC EQUATIONS AND GENERAL LARGE-SCALE SOLUTIONS

We consider the scalar-type perturbation in a flat Friedmann world model supported by a minimally coupled scalar field. Our metric convention follows Bardeen’s in [14]:

\[
ds^2 = -a^2(1 + 2\alpha)d\tau^2 - 2a^2\beta_\alpha d\eta dx^\alpha + a^2 \left[ g_{00}^{(3)}(1 + 2\varphi) + 2\gamma_{0i}\beta \right] dx^i dx^j,
\]

and \( \chi \equiv a(\beta + \alpha^2) \); an overdot and a prime indicate time derivatives based on \( t \) and \( \eta \), respectively, with \( dt \equiv a d\eta \). The background is described by

\[
H^2 = \frac{8\pi G}{3} \left( \frac{1}{2} \dot{\phi}^2 + V \right), \quad \ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0, \tag{2}
\]

where \( H \equiv \frac{\dot{a}}{a} \). The basic perturbation equations are defined in [15,16]:

\[
u = \frac{4\pi G}{k^2} \left( \frac{v'}{z} \right), \quad \ddot{v} + \left( k^2 - \frac{z''}{z} \right) v = 0, \quad u'' + \left( k^2 - \frac{(1/z)''}{1/z} \right) u = 0, \tag{4}
\]

where \( z \equiv a\dot{\phi}/H \), and

\[
u = a\delta \phi, \quad \varphi_{,\phi} \equiv \varphi - (\dot{H}/\dot{\phi})\delta \phi \equiv -(H/\dot{\phi})\delta \phi, \quad \nu = -\varphi_{,\phi}, \quad \varphi_{,\chi} = \varphi - H\chi. \tag{5}
\]

\( \varphi_{,\phi} \) and \( \varphi_{,\chi} \) are gauge-invariant combinations which are equivalent to the curvature perturbation \( \varphi \) in the uniform-field gauge \( (\delta \phi \equiv 0, \text{equivalently the comoving gauge}) \) and in the zero-shear gauge \( (\chi \equiv 0) \), respectively [14]. The perturbed action was derived in [16]

\[
\delta^2 S = \frac{1}{2} \int \left( (v'^2 - v|^{a}v_{,\alpha} + \frac{z''}{z}v^2) \right) d^3x d\eta. \tag{6}
\]

In the large-scale limit, meaning for negligible \( k^2 \) terms, eq. (4) has general solutions [15,16,8]

\[
\varphi_{,\phi}(k, \eta) = C(k) - d(k) \frac{k^2}{4\pi G} \int\eta d\eta, \quad \varphi_{,\chi}(k, \eta) = 4\pi Gc(k) \frac{H}{a} \int\eta z^2 d\eta + \frac{H}{a} d(k). \tag{7}
\]

To the higher-order in the large-scale expansion each of the four solutions have \( [1+\sum_{n=1,2,3...}c_n(k|\eta)|^{2n}] \) factor with \( c_n \) differing for the four cases. We emphasize the general nature of these solutions in the large-scale limit. These are exact solutions of the spatial curvature perturbation \( \varphi \) in the respective hypersurfaces (gauges) valid as long as the \( k^2 \) terms in eq. (4) are negligible; thus valid for general (time-varying) potential \( V(\phi) \). Similar general solutions exist for the fluid situation for general (time-varying) equation of state \( P(\mu) \), and even for the generalized gravity theories [17].

Schwarz has pointed out that as a smooth bounce has to violate the weak energy condition if space-time is flat, \( z^2 \propto \mu + P \) will have (at least two) zeros; this means that \( 1/z \) will be ill defined and the higher order corrections will have singular coefficients, thus the long wavelength expansion becomes inconsistent, [19]. To achieve a bounce, in §V.C of [8] we have used an additional presence of an exotic matter \( X \) with negative energy density. Thus, such an \( X \)-matter cannot dominate even during the bouncing phase. In [8] we have shown that if we concentrate on the evolution of curvature perturbation, assuming near adiabatic initial condition in the collapsing phase, eq. (4) for \( u \), thus our solution for \( \varphi_{,\phi} \) in eq. (7) as well, remains valid. In this context, \( z^2 \) goes through vanishing points at least twice in the bouncing phase, and indeed, in that case the next order large-scale expansion includes \( (1/z^2)d\eta \)-order terms which are ill defined; one such term already appears in the \( d \)-mode of \( \varphi_{,\phi} \) in eq. (7) which is exactly the next order contribution.

As parts of the series solutions in eq. (7) are ill defined for \( z = 0 \), in such a case we should go back to our original equations (forms before we combine to make a second-order equation). One such original equation is conveniently available in the second equation of eq. (4) which shows that, for \( z = 0 \) we have \( (\varphi_{,\phi}/H) = 0 \). Thus, for \( z = 0 \) we have an exact solution: \( \varphi_{,\phi} \propto H/a \). Notice that our eq. (7) includes the above solution as a case! Therefore, we conclude that throughout the bounce (including \( z = 0 \) points), our leading order asymptotic solution for \( \varphi_{,\phi} \) in eq. (7) remains valid. Thus, the ill defined higher order corrections in the series expansion do not cause any practical problem in the perturbations.

III. POWER-LAW EXPANSION

A field with an exponential potential supports power-law expansion/contraction of the scale factor [18]

\[ a \propto |t|^p \times |\eta|^{p/(1-p)}, \quad V = -\frac{p(1-3p)}{8\pi G} e^{-\sqrt{4\pi G/P} \phi}, \quad H/\dot{\phi} = \sqrt{4\pi G/p}. \]

In the power-law case eq. (4) leads to Bessel equations for \( \nu \) and \( v \) with different orders. Using the quantization based on the action formulation in eq. (6), we have the exact mode function solutions (\( p \neq 1 \) [20,6]

\[
\varphi_{,\phi}(k, \eta) = \frac{H}{\dot{\phi}} \left[ \frac{\sqrt{4\pi G/|\eta|}}{2a} \left[ c_1(k)H_{v_1}^{(1)}(x) + c_2(k)H_{v_2}^{(2)}(x) \right] \right], \quad \varphi_{,\chi}(k, \eta) = \frac{H}{k} \left[ \frac{\sqrt{4\pi G/|\eta|}}{\sqrt{p}} c_1(k)H_{v_1}^{(1)}(x) + c_2(k)H_{v_2}^{(2)}(x) \right], \tag{8}
\]

\[
\nu_1 \equiv \frac{3p-1}{2(p-1)}, \quad \nu_2 \equiv \frac{p+1}{2(p-1)}, \tag{9}
\]
where $x \equiv k|\eta|$. The quantization condition implies $|c_2|^2 - |c_1|^2 = \pm 1$ depending on the sign of $\eta$, [6].

IV. MODE IDENTIFICATION

The Hankel functions can be expanded as [21]

$$H^{(1,2)}_\nu(x) = \sum_{n=0}^\infty \frac{1}{n!} \left( -\frac{x^2}{4} \right)^n \frac{1}{\sin \nu \pi} \left[ \left( \frac{x}{2} \right) \nu \pm ie^{\nu \pi} \frac{\Gamma(\nu + n + 1)}{\Gamma(\nu - n + 1)} \right] \frac{\Gamma(\nu - n + 1)}{\Gamma(\nu + n + 1)}.$$

Notice that, in the small $x$ limit, the first (second) term in the parenthesis dominates for $\nu < 0$ ($\nu > 0$). In eq. (7) the leading orders of the $C$-modes are time independent whereas the leading orders of the $d$-modes behave as $\varphi_{\delta \phi} \propto |\eta|^{2\nu}$ and $\varphi_\chi \propto |\eta|^{2\nu}$. Since

$$\varphi_{\delta \phi} \propto |\eta|^{\nu} H^{(1,2)}_\nu(k|\eta|), \quad \varphi_\chi \propto k^{-1}|\eta|^{\nu} H^{(1,2)}_\nu(k|\eta|),$$

we can easily identify the first and the second terms in the parenthesis of eq. (10) as the $d$-mode and the $C$-mode, respectively.

The lower-bounds of integrations in eq. (7) give rise to terms which can be absorbed to the other modes. Such an ambiguity is removed, for example, by identifying the $C$-mode of $\varphi_{\delta \phi}$ in expanding phase in the large-scale limit when the time-dependent part of the $d$-mode has asymptotically decayed away. The authors of [22] have shown that the $d$-mode does not necessarily decay away immediately after the horizon crossing; in some inflationary models we have the $d$-mode effect not negligible near the horizon-crossing, and the final result can be interpreted as an amplification of the spectrum. Such an amplification occurs because, in expanding phase, it takes some time to have the time-dependent part of $d$-mode be negligible. The point is that there is no such an effect from the $d$-mode while in the asymptotically super-horizon scale. In our case of the bounce the relevant scale remains in the asymptotically super-horizon scale during the bounce, thus the solutions in eq. (7) are well valid, and we do not anticipate any ambiguity arising while in the asymptotically super-horizon scale. Accordingly, later in §IV, we will identify the $C$- and $d$-modes ignoring the contributions from lower-bounds of integrations of eq. (7).

The power spectrum and the spectral index are defined as $P_{\delta} = \frac{k^3}{2\pi^2} |\varphi_k|^2$ and $n_S - 1 \equiv d \ln P_{\delta} / d \ln k$. The spectral indices for the $C$-modes of $\varphi_{\delta \phi}$ and $\varphi_\chi$ can be read as (in the following, we assume the simplest vacuum state choice)

$$(n_S - 1)_{\varphi_{\delta \phi}} = (n_S - 1)_{\varphi_\chi} = \frac{2}{1 - p},$$

Although not interesting (because it becomes transient in an expanding phase) the spectral indices for the $d$-modes are

$$(n_S - 1)_{\varphi_{\delta \phi}} = \frac{4 - 6p}{1 - p}, \quad (n_S - 1)_{\varphi_\chi} = -\frac{2p}{1 - p}.$$ (13)

Notice that the spectral indices of the $C$-mode coincide in both gauge conditions, whereas the ones for the $d$-mode show strong gauge dependence. This is easily understandable from the general solutions in eq. (7): in the power-law expansion case we have $\varphi_{\delta \phi}/\varphi_\chi = 1 + p$ for the $C$-mode; the $C$-mode of $\varphi_{\delta \phi}$ remains constant even under the changing potential whereas $\varphi_\chi$ changes it value. Similarly, for the $d$-mode we have $\varphi_{\delta \phi}/\varphi_\chi = \frac{(p-1)^2(k|\eta|)^2}{3p-1}$, thus we have $(n_S - 1)_{\varphi_\chi} = (n_S - 1)_{\varphi_{\delta \phi}} - 4$.

We note again that the $d$-modes show strong gauge dependence: the $d$-mode of $\varphi_{\delta \phi}$ shows more blue spectrum compared with $\varphi_\chi$. Near singularity, the $d$-mode of $\varphi_\chi$ diverges more strongly compared with the ones in the other gauge conditions [17]; see §III of [8] for a summary. The strong divergence in the zero-shear gauge is known to be due to the strong curvature of the hypersurface (temporal gauge condition) [23]. According to Bardeen the behavior of $\varphi_\chi$ “overstates the physical strength of the singularity”, [23]. Thus, even in the collapsing phase where $d$-mode is the proper growing solution, we should not attach more meaning to the $d$-mode of $\varphi_\chi$ than to the one of $\varphi_{\delta \phi}$.

V. CONSEQUENCES

In an ekpyrotic scenario with $0 < p \ll 1$ we have a very blue $n_S - 1 \sim 2$ spectrum for the $C$-mode. Although $n_S - 1 \simeq 0$ for the $d$-mode of $\varphi_\chi$, we are not interested in the $d$-mode. Incidentally, we have $n_S - 1 \simeq 4$ for the $d$-mode of $\varphi_{\delta \phi}$ which better characterizes the physical strength of the growing perturbation during the collapsing phase than $\varphi_\chi$. Our point is that, although the $d$-mode is the relatively growing solution in the collapsing phase, our classification of the $C$- and $d$-modes is based on the general large-scale solutions in eq. (7). The large-scale conditions used to get these solutions are well met during the transition phase in the ekpyrotic scenario. In [8] we have shown analytically that, as long as the linear perturbation is valid, the solutions in eq. (7) remain valid throughout a (smooth and nonsingular) bounce, thus there occurs no mixing for the eventual growing solution in the later expanding phase. Thus, claiming the scale-invariant spectrum based on the $d$-mode of $\varphi_\chi$ is incorrect; see the next paragraph for some technical details. The $C$-modes of both $\varphi_{\delta \phi}$ and $\varphi_\chi$ show the same blue spectra, and the complete spectrum of the $C$-mode and the one for the tensor-type perturbation can be found in [6].

We would like to comment on several minor complications made in [1]. Firstly, the authors of [1] claimed
that by combining the scale-invariance of the $d$-mode of $\phi_X$ before the bounce and the new joining condition introduced by the authors they can derive a scale-invariant final spectrum. This implies that mixing occurs so that the $d$-mode before the bounce sources and dominates the $C$-mode in the subsequent expanding phase. This contradicts with our result based on the general large-scale solutions in eq. (7). It was shown in [5,6] that the well known joining condition based on equations of motion [24] in fact confirms our result: i.e., in the large-scale limit the $C$-mode is affected only by the $d$-mode of the previous era. The new joining condition used in [1] is ad hoc and is not based on proper physical or mathematical arguments, see [6,9]. As we have shown in this paper, and more properly in [8], in order to trace the large-scale evolution of the eventual $C$-mode in expanding phase, we can use the analytic solutions in eq. (7), thus we do not need the joining condition at all. Secondly, in [1] it was emphasized that before the bounce the potential is restored to zero so that expansion rate changes to $p \simeq \frac{1}{3}$. As the perturbation still remains in superhorizon scale during the bounce such a change in the field potential does not affect the already generated perturbation spectrum. We have emphasized that the large-scale general solution in eq. (7) remains valid even under such a changing potential. We are interested only in the $C$-mode and the solution shows that the $C$-mode is not affected by the changing potential. Thirdly, the authors of [1] also stressed that radiation is present after the bounce. In §V.C of [8] we have shown that the evolution of adiabatic (curvature) perturbation is not affected by the changing background equation of state or the presence of multiple component while in the superhorizon scale. Thus, the presence of radiation component after the bounce adds only a minor complication which does not affect the curvature perturbation in the superhorizon scale.

In a similar context, for $p = \frac{2}{3}$ we have $n_S - 1 = 0$ for the $d$-mode of $\phi_{d\phi}$ (in this case we have $n_S - 1 = -4$ for the $d$-mode of $\phi_X$). Identifying this as another possibility for generating a scale-invariant spectrum attempted in [12] is incorrect for the same reason as in the ekpyrotic case; this was pointed out in [6]. For the $C$-mode we have $n_S - 1 = 6$, thus still blue.

Another similar error was made in [3], now in the case of $p = \frac{1}{2}$. In this case we have $n_S - 1 = 4$ for the $C$-modes, and 2 for $\phi_{d\phi}$ and $-2$ for $\phi_X$ for the $d$-modes. Based on the zero-shear gauge authors of [3] claimed that the generated spectrum has $n_S = -1$ for $p = \frac{1}{2}$ and $n_S = 1$ for $p \simeq 0$ (ekpyrotic!), both of which are the ones for the $d$-mode of $\phi_X$, thus irrelevant for the final surviving (observationally relevant) spectrum. Although [3] used a bounce model which differs slightly from the one used in [8], as we have argued, while in the superhorizon scale the final surviving spectrum is independent of the changing background expansion rate. The authors of [3] also considered a radiation dominated era during the quantum generation stage whereas we considered a scalar field dominated era with $p = \frac{1}{3}$. It is well known that the scalar field with an exponential potential can be effectively identified as an ideal fluid with constant $w(= P/\rho)$, thus the two systems coincide for $p = \frac{1}{3}$.

Yet another similar errors were recently added in the literature, [2]. The authors of [2] argued that one cannot ignore the entropy generation near the bounce of the ekpyrotic scenario; if the bounce is singular, we already have stated that the problem cannot be handled using the linear theory. Based on this argument the authors claimed that the conventional joining conditions should be changed. Unless we use the proper joining conditions derived in [24], we can show that the growing (and dominating) $d$-mode in the collapsing phase can easily dominate and source the $C$-mode in the subsequent expanding phase while in the large-scale. In this way, the authors claimed $n_S = 1$ spectrum for the ekpyrotic scenario which comes from the $d$-mode of $\phi_X$. However, we can see that the entropy generation anticipated near bounce would not affect the superhorizon evolution of (the $C$-mode) perturbation. The joining conditions known in the literature give the same result as our present one based on analytic solutions, [5,6]. Perhaps the entropy generation would be important for the background evolution so that we could achieve a smooth and nonsingular bounce as we have investigated using toy models in [8].

The authors of [2] also have claimed, that even for the pre-big bang scenario the final spectrum should pick up the $d$-mode of $\phi_X$ generated in the collapsing phase. For the pre-big bang scenario based on a conformally transformed Einstein frame we have eq. (8) with $p = \frac{1}{3}$; thus we have a vanishing potential. In such a case we have $n_S - 1 = 3$ for the $C$-mode, and 3 for $\phi_{d\phi}$ and $-1$ for $\phi_X$ for the $d$-modes. Based on the same logic as their ekpyrotic case, the authors claimed that the final spectrum should be $n_S = 0$ which is the one for $d$-mode of $\phi_X$. We already have explained what is wrong with such analysis and result. The correct $n_S = 4$ spectrum in Einstein frame was derived in [25]. In the original frame based on the low-energy effective action of string theory the pre-big bang scenario shows a pole-like inflation with $a \propto |t-t_0|^{-1/\sqrt{3}}$. The perturbation spectrum was derived by us in the original frame with $n_S - 1 = 3$, [26]. We also have shown that $\phi_{d\phi}$ is conformally invariant [27]. Thus, it is natural for the final spectra from the two frames (despite their very different descriptions of the background evolutions) to coincide. We note that in the original frame the pre-big bang scenario does not involve a contracting phase, and is just another inflation. In such a case, as in the case of ordinary inflation, the calculation does not require any joining condition.

If the linear perturbation theory is valid throughout, and the bounce is smooth and non-singular (see [8] for several examples) we could rely on solutions in eq. (7) as long as the large-scale conditions are met. In such a case, as emphasized in [6], we do not need to use joining condition which, if we use the ones derived properly,
also gives the same result [5,6,8,9]. Several possibilities to have (smooth and non-singular) bounce models were studied in [8]. In [8], using a toy bounce model based on an exotic matter with a negative energy density we have shown analytically that the pre- and post-bounce results of the $C$-modes of $\varphi_{\delta\varphi}$ and $\varphi_{\chi}$ show the same behaviors as the ones we studied in this work (which ignores the precise physics of the bounce), independently of the presence of the exotic matter (and the bounce itself) introduced to connect the collapsing and the expanding phases, see §V.C in [8].

Equation (12) shows that the only way to get a $n_S - 1 \simeq 0$ spectrum from the power-law expansion based on an exponential potential is to have $p \gg 1$ which is the ordinary power-law expansion or a damped collapsing phase. As pointed out in [6], in the latter case as the model approaches the bouncing phase the comoving scales shrink faster than the Hubble (dynamical) horizon. Thus, the large-scale condition can be violated near the bounce, and we cannot simply trace the perturbation through the bounce, see [8]. Therefore, the only remaining possibility to get an observationally viable spectrum is the former case which is just a well known version of inflation.

Acknowledgments

We thank Robert Brandenberger, Patrick Peter, Nelson Pinto-Neto, and Dominik Schwarz for useful correspondences. This work was supported by Korea Research Foundation grants (KRF-2001-041-D00269).

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