Observable isocurvature fluctuations from the Affleck-Dine condensate

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March 27, 2022

Abstract

In D-term inflation models, Affleck-Dine baryogenesis produces isocurvature density fluctuations. These can be perturbations in the baryon number, or, in the case where the present neutralino density comes directly from B-ball decay, perturbations in the number of dark matter neutralinos. The latter case results in a large enhancement of the isocurvature perturbation. The requirement that the deviation of the adiabatic perturbations from scale invariance due to the Affleck-Dine field is not too large then imposes a lower bound on the magnitude of the isocurvature fluctuation of about \(10^{-2}\) times the adiabatic perturbation. This should be observable by MAP and PLANCK.

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The quantum fluctuations of the inflaton field give rise to fluctuations of the energy density which are adiabatic [1]. However, in the minimal supersymmetric standard model (MSSM), or its extensions, the inflaton is not the only fluctuating field. It is well known that the MSSM scalar field potential has many flat directions [2], along which a non-zero expectation value can form during inflation, leading to a condensate after inflation, the so-called Affleck-Dine (AD) condensate [3]. The AD field is a complex field and, in the currently favoured D-term inflation models [4] on which we focus in this letter, is effectively massless during inflation. Therefore both its modulus and phase are subject to fluctuations. In D-term inflation models the phase of the AD field receives no order $H$ corrections after inflation and so its fluctuations are unsuppressed [5]. Because the subsequent evolution of the phase of the AD condensate generates the baryon asymmetry [3], the fluctuations of the phase correspond to fluctuations in the local baryon number density, or isocurvature fluctuations, while the fluctuations of the modulus give rise to adiabatic density fluctuations. We will show that the adiabatic fluctuations may in fact dominate over the inflaton fluctuations, with potentially adverse consequences for the scale invariance of the perturbation spectrum, thus imposing an upper bound on the amplitude of the AD field. As a consequence, there is a lower bound on the isocurvature fluctuation amplitude. The magnitude of this lower bound will depend on the nature of the AD field. D-term inflation models require that $d > 4$, where $d$ is the dimension of the non-renormalizable superpotential terms stabilizing the potential, in order to avoid thermalizing the AD field too early [5], whilst R-parity conservation, required to eliminate dangerous renormalizable B and L violating terms from the MSSM, rules out odd values of $d$. Therefore we will focus on the $d = 6$ direction, in particular the $u^c d^c d^c$ direction, in the following. We will show that the isocurvature fluctuations will most likely be observable in forthcoming cosmic microwave background (CMB) satellite experiments.

An important point is that the AD condensate is not stable but typically breaks up into non-topological solitons [6, 7] which carry baryon (and/or lepton) number [8, 9] and are therefore called B-balls (L-balls). This is a generic feature which is not realized only if the fluctuations take the AD field into certain leptonic $d = 4$ ("H_u L") directions. The formation of the B-balls takes place with an efficiency $f_B$, likely to be in the range 0.1 to 1 [10]. Hence the AD isocurvature fluctuations are inherited by the B-balls. The properties of the B-balls depend on SUSY breaking and on the flat direction along which the AD condensate forms. We will consider SUSY breaking mediated to the observable sector by gravity. In this case the B-balls are unstable but long-lived, decaying well after the electroweak phase transition has taken place [11], with a natural order of magnitude for decay temperature $T_d \sim \mathcal{O}(1) \text{ GeV}$. This assumes a reheating temperature after inflation, $T_R$, of the order of 1 GeV. Such a low value of $T_R$ is necessary in D-term inflation models because the natural magnitude of the phase
of the AD field, $\delta_{\text{CP}}$, is of the order of 1 in D-term inflation and along the d=6 direction AD baryogenesis implies that the baryon to entropy ratio is $\eta_B \sim \delta_{\text{CP}}(T_R/10^9 \text{ GeV})$ \[1\]. It is significant that a low reheating temperature can naturally be achieved in D-term inflation models, as these have discrete symmetries in order to ensure the flatness of the inflaton potential which can simultaneously lead to a suppression of the reheating temperature \[1\].

Because the B-ball is essentially a squark condensate, in R-parity conserving models its decay produces both baryons and neutralinos (\(\chi\)), which we assume to be the lightest supersymmetric particles (LSPs), with $n_\chi \simeq 3n_B$ \[10, 12\]. This case is particularly interesting, as the simultaneous production of baryons and neutralinos may help to explain the remarkable similarity of the baryon and dark matter neutralino number densities \[10, 12\]. With B-ball decay temperatures $T_d \sim \mathcal{O}(1) \text{ GeV}$, the decay products no longer thermalize completely and, so long as $T_d$ is low enough that they do not annihilate after B-ball decay \[12\], retain the form of the original AD isocurvature fluctuation. Therefore in this scenario the cold dark matter particles can have both isocurvature and adiabatic density fluctuations, resulting in an enhancement of the isocurvature contribution relative to the baryonic case. On the other hand, if the neutralinos from B-ball decays annihilate, the neutralino contribution to the isocurvature fluctuation will be erased, leaving only the baryonic contribution. Although we will be primarily interested in the neutralino isocurvature fluctuation case, we will also comment on the purely baryonic case in the following.

Isocurvature perturbations have been studied previously \[13\], in particular in the context of axion models \[14, 15\]. The isocurvature perturbations give rise to extra power at large angular scales but are damped at small angular scales. The amplitude of the rms mass fluctuations in an $8h^{-1} \text{ Mpc}^{-1}$ sphere, denoted as $\sigma_8$, is about an order of magnitude lower than in the adiabatic case. Hence COBE normalization alone is sufficient to set a tight limit on the relative strength of the isocurvature amplitude. Small isocurvature fluctuations are, however, beneficial, in that they improve the fit to the power spectrum in $\Omega_0 = 1 \text{ CDM}$ models with a cosmological constant \[14\] (or $\Omega_0 = 1$, $\Lambda = 0 \text{ CDM}$ models with some hot dark matter \[13\]). For instance, in the context of axion models it has been found \[14\] that an $\Omega_0 = 1$ mixed fluctuation model with a relative isocurvature perturbation amplitude of 5%, $\Omega_a = 0.4$ and $\Omega_\Lambda = 0.6$ would give a very good fit to the data. However, the isocurvature fluctuations seem to require a large axion decay constant, which is already excluded unless there is considerable late entropy production \[14\]. The Affleck-Dine case we consider here is more economic, in the sense that it requires only the particles of the MSSM.

In D-term inflation models, the AD field $\Phi = \phi e^{i\theta}/\sqrt{2} \equiv (\phi_1 + i\phi_2)/\sqrt{2}$ remains effectively massless during inflation. Therefore the real fields $\phi_i$ are subject to quantum
fluctuations with
\[ \delta \phi_i(x) = \sqrt{V} \int \frac{d^3k}{(2\pi)^3} e^{-i k \cdot x} \delta_k , \] (1)

where \( V \) is a normalizing volume and where the power spectrum is the same as for the inflaton field,
\[ \frac{k^3 |\delta_k|^2}{2\pi^2} = \left( \frac{H_I}{2\pi} \right)^2 , \] (2)

where \( H_I \) is the value of the Hubble parameter during inflation. Thus, for given background values \( \bar{\theta} \) and \( \bar{\phi} \), (with \( \bar{\theta} \) naturally of the order of 1) one finds
\[ \left( \frac{\delta \theta}{Tan(\theta)} \right)_k = \frac{H_I}{Tan(\theta) \bar{\phi}} = \frac{H_I k^{-3/2}}{\sqrt{2} Tan(\theta) \bar{\phi}_I} , \] (3)

where \( \phi_I \) is the value of \( \phi \) when the perturbation leaves the horizon. After inflation, during the inflaton oscillation dominated period, the mass squared of the magnitude of the AD field will receive an order \( H^2 \) correction, which must be negative in order to have a non-zero \( \phi \) and so AD baryogenesis [2], whilst its phase receives no order \( H \) corrections. Therefore, the magnitude of the AD field \( \Phi \) remains at the non-zero minimum of its potential until \( H \simeq m_S \), where \( m_S \sim 100 \text{ GeV} \) is the SUSY breaking scalar mass, whence it begins to oscillate and the baryon asymmetry \( n_B \propto Sin(\theta) \) forms. Since \( \bar{\theta} \) and \( \delta \theta \) remain constant until \( H \simeq m_S \), we have
\[ \left( \frac{\delta n_B}{n_B} \right)_k = \left( \frac{\delta \theta}{Tan(\theta)} \right)_k \] (4)

with \( \delta \theta/Tan(\bar{\theta}) \) given by Eq. (3).

We first consider the case where the adiabatic perturbation is mostly due to the inflaton. The adiabatic perturbation is determined by the invariant \( \zeta = \delta \rho / (\rho + p) \) with \( \delta \rho = V' \delta \phi \). During inflation, when all the fields are slow rolling, one finds [15]
\[ \zeta_{\text{adiab}} = \frac{3}{4} \delta^{(a)} = \frac{9}{\sqrt{2}} \frac{H_I^3}{V'} k^{-3/2} , \] (5)

where \( \delta \gamma \equiv \delta \rho_\gamma / \rho_\gamma \). For super-horizon size isocurvature fluctuations \( \delta \rho / \rho = 0 \), so that
\[ m_x \delta n_x + m_B \delta n_B + 4(\rho_\gamma + \rho_\nu) \delta T / T = 0 \] (here \( \rho_\gamma \) and \( \rho_\nu \simeq 0.68 \rho_\gamma \) are respectively the photon and the neutrino densities, and we assume for simplicity that there are no massive neutrinos). We then find that in the presence of both adiabatic (\( \delta (n_x / s) = 0 \)) and isocurvature (\( \delta (n_x / s) \neq 0 \)) fluctuations
\[ \frac{\delta T}{T} = -\frac{\rho_x \delta^{(i)}_x + \rho_B \delta^{(i)}_B}{3(\rho_\chi + \rho_B) + 4(\rho_\gamma + \rho_\nu)} , \] (6)

where \( \delta x = \delta n_x / n_x \) for non-relativistic particles \( x \). The isocurvature fluctuations of the LSPs are related to the baryonic isocurvature fluctuations by \( \delta n^{(i)}_x = 3f_B \delta n^{(i)}_B \), with
\( \delta n_B \) given by Eq. (3). In the linear perturbation theory adiabatic and isocurvature fluctuations evolve independently so that the total perturbation is just the sum of the two.

The total LSP number density is the sum of the thermal relic density \( n^{(th)}_\chi \) and the density \( n_B = 3 f_B n_B \) originating from the B-ball decay. Using Eq. (3), the isocurvature fluctuation imposed on the CMB photons is then found to be

\[
\delta(i) = 4 \frac{\delta T}{T} = -4 \left( 1 + \frac{m_B}{3 f_B m_\chi} \right) \frac{\rho^{(B)}_\chi \delta(i)_B}{3(\rho_\chi + \rho_B) + 4(\rho_\gamma + \rho_\nu)}
\]

\[
\approx -4 \frac{1}{3} \left( 1 + \frac{m_B}{3 f_B m_\chi} \right) \left( \frac{\Omega_\chi - \Omega^{(th)}_\chi}{\Omega_m} \right) \delta(i)_B \equiv -4 \frac{3}{3} \omega \delta_B(i)_B \ , \tag{7}
\]

where \( \rho^{(B)}_\chi \) is the LSP mass density from the B-ball, \( \Omega_m \) (\( \Omega_\chi \)) is total matter (LSP) density (in units of the critical density), and \( \delta_B(i)_B \) is given by Eq. (4). To obtain the last line in Eq. (7), we have used the fact that \( \rho_\gamma \) is negligible. In the notation of reference [15] and using Eq. (5) we can write

\[
\beta \equiv \left( \frac{\delta(i)_{\gamma}}{\delta(i)_{\gamma}} \right)^2 = \frac{1}{9} \omega^2 \left( \frac{M^2 V'(S)}{V(S)\Tan(\theta)\phi} \right)^2 , \tag{8}
\]

where \( S \) is the inflaton field with a potential \( V(S) \) and \( M \equiv M_{Pl}/\sqrt{8\pi} \).

In the simplest D-term inflation model, the inflaton is coupled to the matter fields \( \psi^- \) and \( \psi^+ \) carrying opposite Fayet-Iliopoulos charges through a superpotential term \( W = \kappa S \psi^- \psi^+ \). At one loop level the inflaton potential reads

\[
V(S) = V_0 + g^4 \xi^4 \frac{\kappa^2 S^2}{32\pi^2} \ln \left( \frac{Q^2}{Q^2} \right) \ ; \quad V_0 = \frac{g^2 \xi^4}{2} , \tag{9}
\]

where \( \xi \) is the Fayet-Iliopoulos term and \( g \) the gauge coupling associated with it. COBE normalization fixes \( \xi = 6.6 \times 10^{15} \text{ GeV} \). In addition, we must consider the contribution of the AD field to the adiabatic perturbation. During inflation, the potential of the \( d = 6 \) flat AD field is simply given by

\[
V(\phi) = \frac{\lambda^2}{32M^6} \phi^{10} . \tag{10}
\]

With \( \rho = V(S) + V(\phi) \) and \( \rho + p = \dot{S}^2 + \dot{\phi}^2 \) one finds, taking both \( S \) and \( \phi \) to be slow rolling fields with \( \dot{S} = -V'(S)/(3H_I) \) and \( \dot{\phi} = -V'(\phi)/(3H_I) \), that the invariant \( \zeta \) is now

\[
\zeta_{\text{adiab}} \propto \frac{V'(\phi) + V'(S)}{V'(\phi)^2 + V'(S)^2} \delta \phi \ , \tag{11}
\]

where we have used the fact that both fields are massless, so that \( \delta S = \delta \phi \). Thus the field which dominates the spectral index of the perturbation will be that with the
largest value of $V'$ and $V''$. The index of the power spectrum is given by $n = 1 + 2\eta - 6\epsilon$, where $\epsilon$ and $\eta$ are defined as

$$\epsilon = \frac{1}{2} M^2 \left(\frac{V'}{V}\right)^2, \quad \eta = M^2 \frac{V''}{V}. \quad (12)$$

The present lower bounds imply $|\Delta n| \lesssim 0.2$. (This bound will be much improved by future satellite missions). In the case where the derivatives with respect to the inflaton dominate (for which the potential is dominated by $V_0$ for all $\xi < M$), $|\Delta n| = 1/N \approx 0.02$ for $N \sim 50$. Once the derivatives with respect to the AD field dominate, the spectral index increases rapidly with $\phi$; from $\eta (\epsilon)$, $|\Delta n|$ is proportional to $\phi^8 (\phi^{18})$. The condition for the AD field to dominate the spectral index is that $\phi > \text{Max}(\phi_{c1}, \phi_{c2})$, where

$$\phi_{c1} \simeq 0.64 (g^3 \lambda^{-2} \xi^4 M^5)^{1/9} \quad (13)$$

and

$$\phi_{c2} \simeq 0.48 \left(\frac{g}{\lambda}\right)^{1/4} (M \xi)^{1/2}, \quad (14)$$

and where we have used the fact that during the slow roll-over $S_N^2 \simeq g^2 M^2 N/(4\pi^2)$. The inflaton derivatives will dominate once $\phi < \text{Min}(\phi_{c1}, \phi_{c2})$. (In practice $\phi_{c1}$ and $\phi_{c2}$ only differ by a factor of less than 2). As a result of the rapid increase of the spectral index once the AD derivatives dominate, the condition that the spectral index is acceptably close to scale invariance essentially reduces to the condition that it is dominated by the inflaton. The lower bounds on $\beta$ corresponding to $\phi_{c1}$ and $\phi_{c2}$ are then

$$\beta > \beta_{c1} \simeq 6.5 \times 10^{-3} g^{4/3} \lambda^{1/3} \omega^2 T \text{an}(\bar{\theta})^{-2} \quad (15)$$

and

$$\beta > \beta_{c2} = 2.5 \times 10^{-2} g^{3/2} \lambda^{1/2} \omega^2 T \text{an}(\bar{\theta})^{-2}. \quad (16)$$

(For most values of the couplings the latter leads to a slightly more stringent lower bound). Thus significant isocurvature fluctuations are a definite prediction of the AD mechanism.

There are two limiting cases: $f_B \gg m_B/3 f_B m_\chi$, for which $\omega = 1$, and $f_B \ll m_B/3 f_B m_\chi$, for which $\omega = \Omega_B/\Omega_m$. The latter corresponds to the case where the B-balls form very inefficiently or where the neutralino contribution to the isocurvature perturbation is erased by annihilations [10, 12]. In this case only the baryonic isocurvature perturbation remains.

The actual lower limit on $\beta$ depends on the unknown couplings $g$ and $\lambda$, as well as on $\bar{\theta}$. To obtain an estimate for $\beta_{c2}$ in the case where the dark matter neutralinos come directly from B-ball decay, let us adopt the following values: $g \simeq g_{\text{GUT}} \simeq$
Figure 1: The relative difference $\Delta C_l/C_l$ between the purely adiabatic and a mixture of adiabatic and isocurvature angular power spectra with $\beta = 0.001$ (dotted line) and $\beta = 0.0001$ (solid line) for a purely CDM $\Omega = 1$ model (with $\Omega_B = 0.05$, $h = 0.5$ and the spectral index $n = 1$). Shown is also the projected PLANCK error level, averaged over ten multipoles (dashed line).
0.7, \( \lambda \simeq 1/5! = 0.008 \) (corresponding to non-renormalizable interaction with physical strength set by \( M \[3, 11\] \)), \( \tan(\bar{\theta})^2 \simeq 1 \) (corresponding to \( \bar{\theta} \simeq \pi/4 \)) and, assuming \( f_B \) is not too small, \( \omega \simeq 1 \). We then find that \( \beta > \beta_{c2} \simeq 1.3 \times 10^{-3} \), with the conservative lower bound perhaps an order of magnitude smaller, \( \beta \sim 10^{-4} \). For the purely baryonic case, the value of \( \omega \) depends on \( \Omega_B \) and \( \Omega_m \). Nucleosynthesis combined with the current best estimate of the expansion rate (\( 0.6 \lesssim h \lesssim 0.87 \[17\] \)) implies that \( 0.006 \lesssim \Omega_B \lesssim 0.036 \). Thus with \( \Omega_m = 1 \) (0.4) we obtain that \( \omega \) for neutralino case is \( 30-150 \) (10-60) times larger than in the purely baryonic case. Therefore in the baryonic case the corresponding value of \( \beta \) is two to four orders of magnitude smaller. Thus there is a significant enhancement of the isocurvature perturbation in the case where the dark matter neutralinos come from B-ball decay without subsequent annihilations. It should be emphasized that there is no physical reason to expect \( \phi \) to be close to its upper bound, so \( \beta \) may, in general, be expected to be much larger than these lower bounds. In this case, even without the neutralino enhancement, the purely baryonic isocurvature fluctuation may well be important.

In Fig. 1 we display the difference between the purely adiabatic power spectrum and the spectra with \( \beta \neq 0 \). We also plot the expected error for the Planck Surveyor Mission, following the estimates in ref. \[18\]. (A similar error is expected for MAP for \( l \lesssim 500 \)). The standard error reads \( (\Delta C_l)^2 = 2(C_l + \delta)^2 /[2l + 1] f_{\text{sky}} \), where \( f_{\text{sky}} \) is the fraction of the sky sampled (we take \( f_{\text{sky}} = 0.65 \)) and \( \delta \) is from the beam, the angular resolution and the sensitivity, as discussed in \[18\]; \( \delta \) becomes non-negligible only for \( l \gtrsim 1000 \) for PLANCK and \( l \gtrsim 500 \) for MAP. One should bear in mind that, in principle, each multipole provides an independent measurement of the spectrum. As can be seen, detecting isocurvature fluctuations at the level of \( \beta \sim 10^{-4} \) should be quite realistic by averaging over a sufficient number of multipole measurements. However, detecting or setting an actual lower limit on \( \beta \) will require a much more careful analysis, which we do not attempt here. Nevertheless, on the basis of Fig. 1, it seems likely that the forthcoming CMB experiments will definitely be able to see isocurvature perturbations in the case where the baryons and neutralinos come directly from the decay of unstable B-balls in the context of D-term inflation models, hence offering a test not only of the inflationary Universe but also of the B-ball variant of AD baryogenesis.

In conclusion, AD baryogenesis in the context of D-term inflation generally implies the existence of isocurvature density fluctuations. In the case where B-balls, which are generally expected to form in AD baryogenesis, decay late enough to produce the observed neutralinos without annihilations, the isocurvature fluctuations should be observable by MAP and PLANCK. Even in the case where only baryonic isocurvature fluctuations arise, there is still a reasonable possibility of observing them, although in this case it is less certain. Thus isocurvature fluctuations are a clear fingerprint
of D-term inflation and Affleck-Dine baryogenesis. In particular, for the case where the neutralino dark matter comes directly from B-ball decays, which allows for an understanding of the remarkable similarity of the baryon and dark matter number densities \[\text{[10, 12]}\], observation of isocurvature perturbations combined with the non-thermal nature of the dark matter neutralino density (testable by observation of the sparticle spectrum \[\text{[12]}\]) would strongly support the late decaying B-ball scenario and D-term inflation, giving us a deep insight into the nature of particle physics and the very early Universe.

**Acknowledgements**

We would like to acknowledge the use of CMBFAST to calculate the angular power spectrum. This work has been supported by the Academy of Finland under the contract 101-35224 and by a European Union Marie Curie Fellowship under EU contract number ERBFM-BICT960567.
References

[1] E.W. Kolb and M.S. Turner, *The Early Universe* (Addison-Wesley, Reading MA, USA, 1990).

[2] See e.g. M. Dine, L. Randall and S. Thomas, *Nucl. Phys.* B458 (1996) 291.

[3] I. A. Affleck and M. Dine, *Nucl. Phys.* B249 (1985) 361.

[4] E. Halyo, *Phys. Lett.* B387 (1996) 43; P. Binetruy and G. Dvali, *Phys. Lett.* B388 (1996) 241.

[5] C. Kolda and J. March-Russell, hep-ph/9802358.

[6] A. Kusenko and M. Shaposhnikov, *Phys. Lett.* B418 (1998) 104.

[7] K. Enqvist and J. McDonald, *Phys. Lett.* B425 (1998) 309.

[8] A. Cohen, S. Coleman, H. Georgi and A. Manohar, *Nucl. Phys.* B272 (1986) 301.

[9] A. Kusenko, *Phys. Lett.* B404 (1997) 285.

[10] K. Enqvist and J. McDonald, hep-ph/9803380 (To be published).

[11] K. Enqvist and J. McDonald, *Phys. Rev. Lett.* 81 (1998) 3071.

[12] K. Enqvist and J. McDonald, hep-ph/9807269 (To be published).

[13] R. Stomper, A. J. Banday and K. M. Gorski, *Ap. J.* 468 (1996) 8.

[14] M. Kawasaki, N. Sugiyama and T. Yanagida, *Phys. Rev.* D54 (1996) 2442; T. Kanazawa, M. Kawasaki, N. Sugiyama and T. Yanagida, astro-ph/9805102.

[15] S. D. Burns, astro-ph/9711303.

[16] D. Lyth and A. Riotto, *Phys. Lett.* B412 (1997) 28.

[17] W. L. Freedman, astro-ph/9706072.

[18] J. R. Bond, G. Efstathiou and M. Tegmark, astro-ph/9702107.