HARMONIC SUPERSPACES FOR THREE-DIMENSIONAL THEORIES

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Abstract

Three-dimensional field theories with $N = 3$ and $N = 4$ supersymmetries are considered in the framework of the harmonic-superspace approach. Analytic superspaces of these supersymmetries are similar; however, the geometry of gauge theories with the manifest $N = 3$ is richer and admits construction of the topological mass term.

1 Introduction

Three-dimensional supersymmetric gauge theories have been intensively studied in the framework of new nonperturbative methods in the field theory [1, 2]. Superfield description of the simplest $D = 3$, $N = 1, 2$ theories and various applications have earlier been discussed in refs. [3 – 10]. The most interesting features of $D = 3$ theories are connected with the Chern-Simons terms for gauge fields and also with duality between vector and scalar fields.

We shall discuss the harmonic-superfield formalism of three-dimensional theories with the extended supersymmetries, which reflects the intrinsic geometry of these theories relevant for quantum description. The formulation of $D = 3$, $N = 4$ superfield theories is analogous to the harmonic formalism of $D = 4$, $N = 2$ theories [11, 12, 13], although the existence of different $SU(2)$ automorphism groups allows one to choose various versions of harmonic superspaces. General $N = 3$ superfields are not covariant with respect to $N = 4$ supersymmetry, but the analytic $N = 3$ superfields are equivalent to the corresponding $N = 4$ superfields. However, interactions with the manifest $N = 3$ require a specific geometric description [14, 15, 16], which does not guarantee conservation of the additional 4-th supersymmetry. The harmonic $N = 3$ superspace of ref. [14] has been based on the use of isovector harmonics, and now we consider the improved version of the harmonic formalism for these theories. Note that field models with $N = 3$ supersymmetry (in distinction with $N = 4$ models) are dual to the dynamics of non-orthogonal intersections of branes [17, 18].

Our conventions for the (2,1)-dimensional $\gamma$-matrices are

\[(\gamma_m)_{\alpha\beta}(\gamma_n)_{\gamma\rho} + (\gamma_n)_{\alpha\beta}(\gamma_m)_{\gamma\rho} = 2\eta_{mn}\delta_{\alpha\gamma}, \quad (\gamma_m)_{\alpha\beta} \equiv \varepsilon_{\alpha\rho}(\gamma_m)^{\rho}_{\beta}, \quad (1.1)\]

where $\eta_{mn}$ is the metric with signature $(1, -1, -1)$ and $\alpha, \beta \ldots$ are the $SL(2, R)$ spinor indices. We also shall use the basic notation of ref. [11] for the isospinor harmonics $u^i_\pm$.

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2 N=4 harmonic superspace

Superfield models with $D = 3$, $N = 4$ supersymmetry can be studied via the dimensional reduction of $D = 4$, $N = 2$ superfield theories. We shall discuss $D = 3$, $N = 4$ harmonic superspace by analogy with refs. [11, 12]. Let $z^M = (x^{\alpha\beta}, \theta^a_k)$ be the central coordinates of the general $D = 3$, $N = 4$ superspace $SS^4_3$, where $k, l \ldots$ and $a, b \ldots$ are indices of the automorphism groups $SU_L(2)$ and $SU_R(2)$, respectively. In the superspace without central charges, the relations between basic spinor derivatives are

$$\{D^k_\alpha, D^l_\beta\} = 2i\varepsilon^{kl}\varepsilon_{\alpha\beta},$$

where $D^k_\alpha = (D^k_\alpha, \bar{D}^k_\alpha)$ and $\partial_{\alpha\beta} = (\gamma^m)_{\alpha\beta}\partial_m$.

The superfield constraints of $N = 4$ super-Yang-Mills theory $SYM^4_3$ can be written as follows:

$$\{\nabla^k_\alpha, \nabla^l_\beta\} = 2i\varepsilon^{kl}\varepsilon_{\alpha\beta}\nabla_{\alpha\beta} + \varepsilon_{\alpha\beta}\varepsilon^{kl}W_{ab},$$

where $\nabla_M$ are covariant derivatives with superfield connections and $W_{ab}$ is a tensor superfield of the $SYM^4_3$ theory satisfying the constraint

$$\nabla_{ia}W_{bc}^L + \nabla_{ib}W_{ca}^L + \nabla_{ic}W_{ab}^L = 0.$$ (2.3)

Below we shall discuss the alternative convention for the $SYM^4_3$ constraints.

Introduce the notation $W_{ab}^L = W_{a1}^L$ and $W_{ab}^R = W_{1a}^R$. The constraint for the Abelian gauge theory produces the following relations:

$$D^k_\alpha W_{ab}^L = 0, \quad \bar{D}^k_\alpha W_{ab}^L = 0,$$

which are analogous to the constraints of $N = 2$, $D = 4$ vector multiplet.

One can also consider the on-shell constraints for different hypermultiplet superfields $q^{kb}_L$ and $q^{kb}_R$

$$\nabla_{ia}q^{kb}_L + \nabla_{ia}q^{ib}_L = 0,$$ (2.6)

$$\nabla_{ia}q^{kb}_R + \nabla_{ia}q^{ka}_R = 0.$$ (2.7)

The coset space of the automorphism group $SU(2)$ plays an important role in the harmonic description of $N = 2$, $D = 4$ superfield theory. In the harmonic approach to $N = 4$, $D = 3$ theory we can use, alternatively, the harmonic variables for cosets spanned on the generators of the corresponding $SU(2)$ groups $L$, $R$ or $(L + R)$. Let us firstly consider the harmonics $u^\pm_i$ for the group $SU_L(2)$. Using the standard harmonic methods of ref. [11] we can transform the constraints (2.2,2.6) into the following integrability ($L$-analyticity) condition:

$$\{\nabla^+_{\alpha}, \nabla^+_{\beta}\} = 0, \quad \nabla^+_{\alpha} = u^+_i \nabla^ia, \quad \nabla^+_a q^{+b} = 0,$$

$$q^{+a} = u^+_i q^{ia}.$$ (2.8)

By analogy with the $D = 4$, $N = 2$ case, one can use the $L$-analytic basis for the $SYM^4_3$ theory

$$\nabla^+_a = D^a_\alpha = \partial/\partial \theta^a_k,$$

$$\nabla^+ = D^+_a + V^+_L, \quad D^+_a V^+_L = 0,$$ (2.10, 2.11)
where $D^a_+ = (D^+_{L}, \tilde{D}^+_{R})$ and $V_{L}^{++}$ is the prepotential of $SYM^4_3$ depending on the coordinates $\zeta_L = (x^{a\beta}_L, \theta^{a\alpha}_L)$ of the $L$-analytic superspace $LSS^4_3$. In the physical gauge, it contains the components of the $D = 3$, $N = 4$ vector multiplet

$$
V_{wz}^{++} = (\theta^{a+b}\theta^m_{\alpha})\Phi_{(ab)}(x_L) + i(\theta^{a+}\gamma^m\theta^+_{a})A_m(x_L) + + (\theta^{a+b}\theta^m_{\alpha})A_m(x_L) + (\theta^+)^4u_k^i X^{k\alpha}(x_L). \quad (2.12)
$$

The solution of the constraint (2.3) has the following form in the harmonic approach:

$$
W_{L}^{a\alpha} = D^{\alpha+}D^{a+}_L V_{L}^{--}(V_{L}^{++}), \quad (2.13)
$$

where the standard solution for the 2-nd harmonic connection $V_{L}^{--}$ [13, 19] is considered.

The analytic superfield $q^{+a}$ with the infinite number of auxiliary fields is the complete analog of the corresponding $D = 4$ hypermultiplet representation. An alternative form of the analytic hypermultiplet can be obtained with the help of the harmonic duality transform $q^{+a} = u^+\omega + u^-F^{++}$ [12]. On mass shell, these hypermultiplets have the following components:

$$
q^{+a}_0 = u^+_k f^{ka}(x_L) + \theta^{+ba}\psi^{a\alpha}_b(x_L), \quad (2.14)
$$

$$
\omega_0 = f(x_L) + u^+_k u^+_l f^{k\alpha\beta}(x_L) + \theta^{+ba}u^+_k \psi^{a\beta}_b(x_L). \quad (2.15)
$$

The holomorphic effective action of the Abelian $N = 4$ gauge theory contains the chiral superfield $W = \int dv (\tilde{D}^-)^2 V_{L}^{++}$ [20]. One can consider the equivalent chiral and analytic representations of this action

$$
i \int d^3xd^4\theta \mathcal{F}(W) + c.c. = i \int d\zeta^{(-4)}_{L} dv V_{L}^{++}(D^+)^2 \frac{\mathcal{F}(W)}{W} + c.c. \quad (2.16)
$$

where $d\zeta^{(-4)}_{L}$ is the integral measure in $LSS^4_3$ and $d^4\theta$ is the spinor measure in the chiral superspace.

The $u$-independent chiral superfield $A$ can be used for the construction of the complex analytic superfield $C^{++} = (D^+)^2 A$ satisfying the additional harmonic constraint $D^{+-}C^{++} = 0$. The superfield $A$ is treated as a dual variable with respect to the Abelian 'magnetic' gauge superfield $V_{M}^{++}$. The effective action of this system contains $V_{M}^{++}$ as a Lagrange multiplier

$$
i \int d^3xd^4\theta \mathcal{F}(A) + i \int d\zeta^{(-4)}_{L} dv V_{M}^{++}(D^+)^2 A + c.c. \quad (2.17)
$$

One can obtain the reality constraint (2.3) for $A$ and the relation

$$
\mathcal{F}'(A) = - \int dv (\tilde{D}^-)^2 V_{M}^{++} \quad (2.18)
$$

varying this action with respect to the superfields $V_{M}^{++}$ and $A$, respectively. Note that the holomorphic representation with chiral superfields breaks the $SU_R(2)$ automorphism group.

We shall not consider here the classical action and quantization of the $SYM^4_3$ theory since it can be obtained by a dimensional reduction from the known $SYM^4_3$ theory.

Consider the Abelian case of the constraint (2.7) for the $R$-hypermultiplet and define the harmonic projection $R^{-a} \equiv u_k^i g^a_k$. In the $L$-analytic basis, the basic relations for this harmonic superfield are

$$
D^a_+ R^{-b} + D^b_+ R^{-a} = 0, \quad (D^+)^2 R^{-a} = 0, \quad (2.19)
$$
where the 1-st relation is treated as the constraint and the 2-nd one as the equation of motion. Using the relation \( \{ D^+_{\alpha} D^b_{\beta} \} = -2i \varepsilon^{ab} \delta_{\alpha\beta} \) one can obtain a general covariant solution of the constraint which contains the \( L \)-analytic bosonic and fermionic superfields \( b^{-a} \) and \( f^{\alpha} \)

\[
R^{-a} = b^{-a} + D^{-a}_{\alpha} f^{\alpha} .
\]

(2.20)

The harmonic equation of motion is equivalent to the equation \( D^{-} R^{-a} = 0 \).

Thus, the hypermultiplet \( R^{-a} \) (or its derivative \( R^+ = D^+ R^{-a} \)) is reduced to the pair of \( L \)-analytic superfields and their interactions can be described in \( LSS^4 \). On-shell it has the same components as the superfield \( q^{\alpha}_{\alpha a} \).

One can identify the indices of the left and right automorphism groups and use the \( SU(2) \)-covariant spinor \( N = 4 \) coordinates and supersymmetry parameters

\[
\theta_{\alpha} = \theta_{(\alpha l)} + \varepsilon_{kl} \theta_{\alpha} , \quad \epsilon_{\alpha} = \epsilon_{(\alpha l)} + \varepsilon_{kl} \epsilon_{\alpha} ,
\]

(2.21)

where the isovector and isoscalar parts are introduced. The alternative \( C \)-form of the \( SYM^3 \) constraints contains the isovector superfield \( W_{\alpha} \)

\[
\{ \nabla_{\alpha l}, \nabla_{\beta l} \} = 2i \varepsilon^{kl} \varepsilon^{mn} \nabla_{\alpha \beta} + \frac{1}{2} \varepsilon_{\alpha \beta} (\varepsilon_{kl} W_{mn} + \varepsilon_{mn} W_{kl}) .
\]

(2.22)

This representation allows us to separate the isoscalar covariant derivative

\[
\{ \nabla_{\alpha}, \nabla_{\beta} \} = i \nabla_{\alpha \beta} , \quad \{ \nabla_{\alpha}, \nabla_{\beta}^{(kl)} \} = 0 .
\]

(2.23)

The 2-nd relation is a conventional constraint which depends on a choice of the \( SU(2) \)-frame. Below we shall discuss the commutation relations between the isovector \( N = 3 \) covariant derivatives which are frame-independent.

3 New formulation of \( N=3 \) harmonic superspace

Let us consider now the new harmonic projections of the \( N = 4 \) spinor coordinates \( \theta_{\alpha} = (x^\alpha, \theta^\alpha) \).

\[
\theta^{\alpha \pm} = u_{\alpha}^\pm u_{\alpha}^\pm \theta_{\alpha l}^{kl} , \quad \theta^{\alpha \pm} = u_{\alpha}^\pm u_{\alpha}^\pm \theta_{\alpha l}^{kl} .
\]

(3.1)

Coordinates of \( LSS^4 \) in the new representation are \( \zeta_{\alpha} = (x^\alpha_{\alpha l} , \theta^{\alpha l} , \theta^{\alpha l} , \theta^{\alpha l}) \) where

\[
x^\alpha_{\alpha l} = x^\alpha + i(\theta^{\alpha \alpha} \theta^{\beta \beta} + \theta^{\alpha \beta} \theta^{\beta \alpha}) .
\]

The infinitesimal \( N = 4 \) spinor transformations have the following form in these coordinates:

\[
\delta x^\alpha_{\alpha l} = \left\{ 2iu_{-k} u_{-l} \epsilon_{(kl)}^{(i)} \theta^{\beta \beta} + 2i[\epsilon^{\alpha} - u_{-k} u_{+l} \epsilon_{(kl)}^{(i)}] \theta^{\beta \beta} \right\} + \left\{ \alpha \leftrightarrow \beta \right\} ,
\]

(3.2)

\[
\delta \theta^{\alpha \alpha} = u_{+k} u_{+l} \epsilon_{(kl)}^{(i)} , \quad \delta \theta^{\alpha \beta} = \epsilon^{\alpha} + u_{+k} u_{-l} \epsilon_{(kl)}^{(i)} ,
\]

(3.3)

where \( \epsilon^{\alpha} \) is an isoscalar parameter of the 4-th supersymmetry. Using the subgroup with \( \epsilon^{\alpha} = 0 \) one can describe \( N = 3 \) supersymmetry in this \( L \)-analytic superspace.

A three-dimensional \( N = 3 \) supersymmetry can also be realized in the superspace \( SS^3 \) with the coordinates \( z = (x^\alpha, \theta^{(i)}_{(kl)}) \). The corresponding superfields do not depend on the
The isoscalar coordinate $\theta^\alpha$ and are not covariant with respect to the 4-th supersymmetry. The constraints of $SYM_3^4$ in this superspace are
\[
\{\nabla^{(km)}_\alpha, \nabla^{(ln)}_\beta\} = i(\varepsilon^{kl}\varepsilon^{mn} + \varepsilon^{ml}\varepsilon^{kn})\nabla_{\alpha\beta} + \frac{1}{4}\varepsilon_{\alpha\beta}(\varepsilon^{kl}W^{mn} + \varepsilon^{ml}W^{kn} + \varepsilon^{mn}W^{kl} + \varepsilon^{kn}W^{ml}),
\]
where all connections do not depend on $\theta$.

Let us introduce the alternative analytic coordinates of the $N = 3$ harmonic superspace $ASS_3^3$
\[
x_{\alpha}^\beta = x_{\alpha}^\beta + i(\theta^{\alpha\alpha}\theta^{\beta\alpha} + \theta^{\beta\alpha}\theta^{\alpha\alpha})
\]
(3.5)
\[
\theta^{\alpha\alpha} = u_k^+u_l^+\theta^{(kl)}, \quad \theta^{\alpha\alpha} = \frac{1}{2}(\theta^{\alpha\alpha} + \theta^{\alpha\alpha}) = u_k^+u_l^-\theta^{(kl)}.
\]
(3.6)

It should be stressed that there is a one-to-one correspondence between the analytic $N = 4$ and $N = 3$ superfields.

Spinor derivatives have the following form in this $N = 3$ superspace:
\[
D^\alpha = \partial^\alpha + \frac{1}{2}\partial^\alpha - i\theta^{\alpha\beta}\partial^\beta
\]
(3.7)
\[
D^\alpha = -\frac{1}{2}\partial^\alpha + i\theta^{\alpha\beta}\partial^\beta, \quad \partial^\alpha + \theta^{\alpha\beta} = \partial^\beta = \delta^\beta.
\]
(3.8)

The corresponding covariant harmonic derivatives are
\[
D^{++} = \partial^{++} - 2i\theta^{\alpha\beta}\partial^{\alpha\beta} + \theta^{\alpha\beta}\partial^\alpha + 2\theta^{\alpha\beta}\partial^\alpha
\]
(3.9)
\[
D^{--} = \partial^{--} + 2i\theta^{\alpha\beta}\partial^{\alpha\beta} + \theta^{\alpha\beta}\partial^\alpha + 2\theta^{\alpha\beta}\partial^\alpha
\]
(3.10)
\[
[D^{--}, D^{++}] = [D^{++}, D^{--}] = 2D^\alpha, \quad [D^{\pm\pm}, D^\alpha] = D^{\pm\pm}.
\]
(3.11)

Analytic $N = 3$ superfields do not depend on $\theta$ and are unconstrained objects in $ASS_3^3$.

The $N = 3$ covariant derivatives $\nabla^{(kl)}_\alpha$ in the central basis can be transformed to the harmonized covariant derivatives of the $SYM_3^4$ theory in the basis with the analytic gauge group
\[
\nabla^{++} = D^{++}, \quad \nabla^{--} = D^{--} + V^{++},
\]
(3.12)
\[
\nabla^{--} = D^{--} + V^{--}(V^{++}), \quad \nabla^\alpha = D^\alpha - \frac{1}{2}D^\alpha V^{--},
\]
(3.13)
\[
\nabla^{--} = \left[\nabla^{--}, D^\alpha\right],
\]
(3.14)

where $V^{++}$ is the analytic gauge prepotential in the adjoint representation of the gauge group, and $V^{--}(V^{++})$ is the solution of the zero-curvature equation for harmonic connections [13, 19].

In the physical WZ-gauge the prepotential contains the components of the $N = 3$ vector supermultiplet
\[
V_{wz}^{WZ} = (\theta^{++})^2u_k^-u_l^-\Phi^{(kl)}(x_\alpha) + i(\theta^{++}\gamma^m\theta^\alpha)A_m(x_\alpha) + (\theta^\alpha)^2\theta^{\alpha\alpha}\lambda_\alpha(x_\alpha) +
\]
\[
(\theta^{++})^2\theta^{\alpha\alpha}u_k^-u_l^-\lambda_\alpha^{(kl)}(x_\alpha) + (\theta^{++})^2(\theta^\alpha)^2u_k^-u_l^-X^{(kl)}(x_\alpha),
\]
(3.15)

\footnote{In ref. [14], we have used the isovector $N = 3$ spinor coordinates $\theta^\alpha = (1/2)(\tau_3)\theta^{(kl)}$.}
which are analogous to the $N = 4$ components (2.12) with identified $L$ and $R$ isospinor indices.

The basic superfield tensor of $SYM^3_3$ is analytic

$$W^{++} = \frac{1}{2} D^{\alpha+\beta}D^{\alpha+\beta}V^{--}$$ (3.16)

and satisfies the additional $H$-constraint (Bianchi identity)

$$\nabla^{++}W^{++} \equiv 0 .$$ (3.17)

It is a specific feature of the $SYM^3_3$ theory that the prepotential $V^{++}$ and its superfield-strength $W^{++}$ belong to the same analytic superspace $ASS^3_3$.

Let us define integral measures in the full and analytic $N = 3$ harmonic superspaces

$$d^9z_A = \frac{1}{64}d^3x_A(D^{\alpha++}D^{\alpha+})(D^{\alpha--}D^{\alpha-})(D^{\alpha0}D^{\alpha0}) ,$$ (3.18)

$$d\zeta^{(-4)} = \frac{1}{16}d^3x_A(D^{\alpha--}D^{\alpha-})(D^{\alpha0}D^{\alpha0}) .$$ (3.19)

Note that these measures have dimensions $d = 0$ and 1, respectively.

The standard kinetic term of the $SYM^3_3$ action is

$$S_k = \frac{1}{g^2}\int d\zeta^{(-4)}du \text{Tr} W^{++}W^{++} ,$$ (3.20)

where $g$ is the coupling constant with dimension $d = -1/2$.

The effective action of the Abelian $N = 3$ theory contains an arbitrary function of the $H$-constrained superfield $W^{++}$

$$\int d\zeta^{(-4)}du \ G^{(+4)}(W^{++}, u) = \int d\zeta^{(-4)}du \left[ \tau(W^{++})^2 + \sum_{p=1}^{\infty} c_{l_1\ldots l_{2p}} u_{l_1}^{-1}\ldots u_{l_{2p}}^{-1}(W^{++})^{p+2} \right] ,$$ (3.21)

where $\tau, c_{l_1\ldots l_{2p}}$ are some constants. It is clear that only quadratic term of the general action conserves the $SU_C(2)$ symmetry. This analytic representation of the low-energy effective action is alternative to the holomorphic $N = 4$ representation (2.16).

The interaction of the gauge superfield $W^{++}(V^{++})$ is dual to the following interaction of the unconstrained real analytic superfields $\omega$ and $A^{++}$:

$$\int d\zeta^{(-4)}du \ G^{(+4)}(A^{++}, u) + A^{++}D^{++}\omega] .$$ (3.22)

Varying $\omega$ yields the constraint $D^{++}A^{++} = 0$. This action is the first-order form of the special interaction of $\omega$ and $D^{++}\omega$, although the elimination of the superfield $A^{++}$ is a non-trivial algebraic problem for the general function $G^{(+4)}$.

An important feature of the $SYM^3_3$ theory is the existence of a topological mass (Chern-Simons) term [14]. In the improved $N = 3$ harmonic formalism this term can be constructed by the analogy with the action of $SYM^2_4$ [19]

$$S_m = \frac{m}{g^2} \sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{n} \int d^9z du_1\ldots du_n \frac{\text{Tr} [V^{++}(z, u_1)\ldots V^{++}(z, u_n)]}{(u_1^+u_2^+)\ldots(u_n^+u_1^+)} .$$ (3.23)
where \((u_1^+ u_2^+)^{-1}\) is the harmonic distribution [12]. Note that the measure \(d^2\zeta\) in this term is not covariant with respect to the 4-th supersymmetry. The analytic version of the topological mass term and the Fayet-Iliopoulos term for the Abelian theory has the following form:

\[
S_m + S_{FI} = \frac{1}{g^2} \int d\zeta (mV^{++}W^{++} + \xi^{++}V^{++}) .
\] (3.24)

The action \(S_k + S_m + S_{FI}\) yields the free equation of motion for the \(N = 3\) Abelian gauge theory

\[
[(D^a\partial_D^b) + m] W^{++} + \xi^{++} = 0 .
\] (3.25)

This equation has the following vacuum solutions:

\[
W^{++} = -\frac{1}{m} \xi^{++} \quad \text{for} \quad m \neq 0 ,
\] (3.26)

\[
W^{++} = a^{(kl)} u_k^+ u_l^+ - 2[(\theta^o)^2 \xi^{++} - 2(\theta^{+++} \theta^o)\xi^o + (\theta^{++})^2 \xi^{--}] \quad \text{for} \quad m = 0 ,
\] (3.27)

where \(\xi^{\pm \pm} = \xi^{(kl)} u_k^\pm u_l^\pm\), \(\xi^o = (1/2)D^{++}\xi^{--}\) and \(a^{(kl)}\) and \(\xi^{(kl)}\) are arbitrary constants. Note that the first solution does not break supersymmetry.

For the case \(m = 0\), \(\xi^{(kl)} = 0\) we can study the background Abelian prepotential

\[
V^{++} = \frac{1}{2} a^{(kl)} [(\theta^o)^2 u_k^+ u_l^+ - 2(\theta^{+++} \theta^o)u_k^+ u_l^- + (\theta^{++})^2 u_k^- u_l^-] ,
\] (3.28)

which introduces the \(N = 3\) central charges and produces masses of charged superfields by analogy with ref. [21].

The minimal gauge interaction of the \(q^+\) hypermultiplet has the following form:

\[
\int d\zeta (D^{++} + V^{++}) q^+ .
\] (3.29)

The free hypermultiplet satisfies the equation \(D^{++} q^+ = 0\) and contains a finite number of complex on-shell components

\[
q_0^+ = u_k^+ f^k(x_A) + (\theta^{+++} u_k^- - \theta^{+++} u_k^+)\psi^k_A (x_A) .
\] (3.30)

The real \(N = 3\) \(\omega\)-hypermultiplet has been described in ref. [14].

Note that the similar harmonic methods can be used for description of two-dimensional models with \((3,3)\) supersymmetry.

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