Design considerations for polarization-sensitive optical coherence tomography with a single input polarization state

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Abstract: Using a generalized design for a polarization-sensitive optical coherence tomography (PS-OCT) system with a single input polarization state (SIPS), we prove the existence of an infinitely large design space over which it is possible to develop simple PS-OCT systems that yield closed form expressions for birefringence. Through simulation and experiment, we validate this analysis by demonstrating new configurations for PS-OCT systems, and present guidelines for the general design of such systems in light of their inherent inaccuracies. After accounting for systemic errors, alternative designs exhibit similar performance on average to the traditional SIPS PS-OCT system. This analysis could be extended to systems with multiple input polarization states and could usher in a new generation of PS-OCT systems optimally designed to probe specific birefringent samples with high accuracy.

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References

1. B. Cense, T. C. Chen, B. H. Park, M. C. Pierce, and J. F. de Boer, “In vivo depth-resolved birefringence measurements of the human retinal nerve fiber layer by polarization-sensitive optical coherence tomography,” Opt. Lett. 27, 1610–1612 (2002).

2. J. Strasswimmer, M. Pierce, B. Park, and V. Neel, “Polarization-sensitive optical coherence tomography of invasive basal cell carcinoma,” J. Biomed. Opt. 9, 292–298 (2004).

3. M. Hee, D. Huang, E. Swanson, and J. Fujimoto, “Polarization-sensitive low-coherence reflectometer for birefringence characterization and ranging,” J. Opt. Soc. Am. B 9, 903–908 (1992).

4. M. Pircher, C. K. Hitzenberger, and U. Schmidt-Erfurth, “Polarization sensitive optical coherence tomography in the human eye,” Prog. Retin. Eye Res. 30, 431–451 (2011).

5. E. Götzinger, M. Pircher, and C. K. Hitzenberger, “High speed spectral domain polarization sensitive optical coherence tomography of the human retina,” Opt. Express 13, 10217–10229 (2005).

6. M. K. Al-Qaisi and T. Akkin, “Polarization-sensitive optical coherence tomography based on polarization-maintaining fibers and frequency multiplexing,” Opt. Express 16, 13032–13041 (2008).

7. E. Götzinger, B. Baumann, M. Pircher, and C. K. Hitzenberger, “Polarization maintaining fiber based ultra-high resolution spectral domain polarization sensitive optical coherence tomography,” Opt. Express 17, 22704–22717 (2009).

8. S.-W. Lee, J.-Y. Yoo, J.-H. Kang, M.-S. Kang, S.-H. Jung, Y. Chong, D.-S. Cha, K.-H. Han, and B.-M. Kim, “Optical diagnosis of cervical intraepithelial neoplasm (CIN) using polarization-sensitive optical coherence tomography,” Opt. Express 16, 2709–2719 (2008).
1. Introduction

Optical coherence tomography (OCT) is a non-invasive, high-resolution technique capable of generating three-dimensional images of the structure of biological samples. Polarization-sensitive OCT (PS-OCT) is a functional extension of OCT that additionally measures depth-resolved birefringence. PS-OCT has been implicated in the study of disease in major organs like the skin and the eye, as the birefringence provides important contrast [1, 2].

One of the most prevalent system designs for PS-OCT in the literature [3, 4] is laudably simple, both in terms of the choice of components and the algorithm for processing birefringence data. This design uses a single input polarization state (SIPS) to probe the birefringence of a non-diattenuating sample. While PS-OCT itself has undergone significant improvements in speed, sensitivity [5], flexibility [6, 7] and processing [8, 9] during its 20-year history, the underlying combination of polarization controlling components (PCCs) used in SIPS schemes has not. To the best of our knowledge, among all demonstrated systems that use a SIPS, all but one [10] send circularly polarized light to the sample arm and interfere it with 45-degree linearly polarized light from the reference arm. The persistence of this particular design speaks to its inherent advantages over other schemes for PS-OCT: systems that probe sample birefringence using multiple polarization states multiplexed in frequency [11–13] or in time [14–18] are more complex and suffer noise from motion artifacts inherent to non-concurrent measurements, respectively; systems implemented in single-mode fiber are restricted to measuring relative rather than absolute optic axis without significant calibration [19, 20].

In this work, we present an expanded design space for the historic SIPS PS-OCT scheme,
Fig. 1. The generalized PS-OCT system. The detector collects two interferograms with orthogonal polarizations from interfered reference ($E_r$) and sample ($E_s$) arm light; inset shows the PS-detector used in this study. $J_s$ or $J_r$: Jones matrix of the sample or polarization-controlling components in interferometer arm $x = \{\text{src, ref, samp, det}\}$, $E_y$: normalized Jones vector at $y = \{\text{src, in, out}\}$, src: source, det: detector, ref: reference, samp: sample, BS: non-polarizing beam splitter, Pol: linear polarizer, HWP: half-wave plate, L: lens, H: horizontal, V: vertical.

with particular emphasis on identifying novel combinations of PCCs that enable quantitative measurement of birefringence from closed form expressions and on assessing the accuracy with which different SIPS systems can measure optic axis and retardance. Using both simulation and experimental methods, we validate the feasibility of a generalized model to inform the design of SIPS systems. We believe that presentation of this design space may serve as a useful guide to the implementation of PS-OCT systems with convenient and robust probe designs tailored to new applications or birefringent samples of interest.

2. Theory for the generalized SIPS PS-OCT system

All SIPS PS-OCT systems may be generalized in the form of a Michelson interferometer with arbitrary PCCs that are described by Jones matrices (Fig. 1). As a result, a generalized equation for the A-scan encoding the depth-dependent reflectivity and birefringence can be derived from the measured intensity for all SIPS PS-OCT systems.

2.1. Derivation of the A-scan vector

Source light is made fully polarized by the PCCs in the source arm ($J_{\text{src}}$) and enters the interferometer via a non-polarizing beam splitter. We place the material whose properties we wish to measure in the sample arm of the interferometer. For example, most biological samples may be modeled as a non-diattenuating, uniaxial birefringent crystal whose Jones matrix is given below and characterized by two parameters: retardance, $\eta$, and optic axis, $\theta$ [16]:

$$J_s(\eta, \theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} \exp(i\eta) & 0 \\ 0 & \exp(-i\eta) \end{bmatrix} \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}. \quad (1)$$
The depth-dependent values of these birefringent parameters are encoded in the polarization state of the interferogram when measured using a PS-detector capable of resolving two linearly independent polarization states (e.g., linear horizontal and vertical polarizations).

For an isolated, birefringent reflector positioned at optical pathlength $z_s$ (assumed positive) from the location of zero path length delay (ZPD), the interferometric component of the measured interferogram resembles that of a non-birefringent sample located at $z_s$ but includes additional terms associated with the polarization state of the returning light. After taking the Fourier transform, one arrives the following expression for the interferometric component of the complex A-scan:

$$\mathbf{I}(z) = \mathbf{E}_s \odot \mathbf{E}_r^* \sqrt{R_s} \mathbf{R}_s(z) \exp(i\psi(z))$$

(2)

$$\mathbf{E}_r = \mathbf{J}_{\text{det}} \mathbf{J}_{\text{ref}}^T \mathbf{J}_{\text{ref}} \mathbf{E}_{\text{src}}$$

(3)

$$\mathbf{E}_s = \mathbf{J}_{\text{det}} \mathbf{J}_{\text{samp}}^T \mathbf{J}_{\text{samp}} \mathbf{E}_{\text{src}}$$

(4)

The symbol $\odot$ represents element-wise multiplication. $\mathbf{J}_s$ is the Jones matrix of the PCCs in the reference (ref), source (src), sample (samp) or detector (det) arms of the interferometer, and $\mathbf{J}_{i}(\eta, \theta)$ is the Jones matrix of a birefringent sample with double-pass retardance $\eta$ and optic axis $\theta$. $\mathbf{E}_{\text{src}}$, $\mathbf{E}_s$ and $\mathbf{E}_r$ are the normalized Jones vectors at various locations in the system (Fig. 1). $R(z) = \sqrt{R_s R_r(z)}$ and $\psi(z)$ are the magnitude and phase, respectively, that would be associated with the reflectivity and optical pathlength of a non-birefringent reflector located at $z_s$.

By replacing $\mathbf{E}_s$ with its constituent Jones matrices (Eq. 4), and the element-wise multiplication between $\mathbf{E}_s$ and $\mathbf{E}_r$ with matrix multiplication by defining $\mathbf{J}_s = [\mathbf{E}_{\text{rH}}^* \quad 0 \quad \mathbf{E}_{\text{rV}}^*]$ (the conjugate terms result from detecting the intensity), Eq. 2 becomes

$$\mathbf{I}(z) = \begin{bmatrix} I_H(z) \\ I_V(z) \end{bmatrix} = \mathbf{J}_s \mathbf{J}_{\text{det}}^T \mathbf{J}_{\text{samp}} \mathbf{J}_{\text{samp}}^T \mathbf{J}_{\text{src}} \sqrt{R(z)} \exp(i\psi(z)).$$

(5)

Still further simplification is possible. As we will show, the alternative representation of Eq. 5 given below emphasizes the effect of the system design (i.e., the choice of PCCs) on the ability to measure birefringence.

$$\mathbf{I}(z) = \mathbf{J}_{\text{out}} \mathbf{J}_{\text{samp}} \mathbf{E}_{\text{in}} \mathbf{J}_{\text{samp}}^T \sqrt{R(z)} \exp(i\psi(z))$$

(6)

Here, $\mathbf{E}_{\text{in}} = \mathbf{J}_{\text{samp}} \mathbf{E}_{\text{src}} / ||\mathbf{J}_{\text{samp}} \mathbf{E}_{\text{src}}||$ is the polarization state incident upon the sample and $\mathbf{J}_{\text{out}} = \mathbf{J}_s \mathbf{J}_{\text{det}} \mathbf{J}_{\text{samp}}^T$ converts the polarization state of light returning from the sample, $\mathbf{E}_{\text{out}} = \mathbf{J}_s(\eta, \theta) \mathbf{E}_{\text{in}}$, into the polarization-dependent intensity measured at the detector.

From Eq. 6 it is clear that the ability to measure birefringence breaks down when $\mathbf{J}_{\text{out}}$ is not full rank (e.g., the columns of the matrix are linearly dependent), which stipulates that $\mathbf{J}_{\text{samp}}$, $\mathbf{J}_{\text{det}}$, and $\mathbf{J}_s$ all be full rank. This requires that neither the sample nor detector arm include linear polarizers, and that the Jones vector of the light returning from the reference arm have non-zero components along both axes of detection (e.g., $\mathbf{E}_r$ must not be horizontally or vertically polarized). This notwithstanding, the design space associated with the range of combinations of PCCs yielding valid expressions for $\mathbf{I}(z)$ is infinitely large.

2.2. Derivation of equations for sample birefringence

One can define a set of closed form equations that describe the reflectance, retardance and optic axis of the sample for an arbitrary system in the design space. Total reflectance ($R$) of the
sample at \( z_s \) can be calculated from the measured interferogram through the A-scan vector (Eq. 5) and is given by

\[
R = |J^{-1}_{out}I(z_s)|^2. \tag{7}
\]

The general effect of birefringence is to transform the vector \( E_{in} \), that is incident on the sample vector \( E_{out} \) (Eq. 6), which travels to the detector through the beamsplitter and is transformed by Jones matrix \( J_{out} \) along the way. \( E_{out} \) can be determined directly from knowledge of the PCC matrices and the A-scan vector. The full rank requirement for \( J_{out} \) ensures its invertibility.

\[
E_{out} = \frac{J^{-1}_{out}I(z_s)}{||J^{-1}_{out}I(z_s)||} \tag{8}
\]

The nature of the transformation of \( E_{in} \) to \( E_{out} \), as well as the derivation of the closed form expressions to calculate the birefringence parameters of the sample, are better viewed on the Poincaré sphere. As such, we first transform all Jones vectors to Stokes vectors. Let \( S_{in} = [I_{in}, Q_{in}, U_{in}, V_{in}] \) and \( S_{out} = [I_{out}, Q_{out}, U_{out}, V_{out}] \) be the Stokes vector representations of the normalized Jones vectors \( E_{in} \) and \( E_{out} \), respectively. For an arbitrary \( S_{in} \), which is established by the PCCs of the system, it can be shown that the transformation to \( S_{out} \) is constrained on a Poincaré sphere with unity radius by rotation about the vector \([\cos(2\theta), \sin(2\theta), 0]\). This vector is defined by the optic axis of the birefringent sample and is constrained to the QU plane. Rotation from \( S_{in} \) to \( S_{out} \) sweeps an arc of a circle with arc length proportional to the sample retardance, \( \eta \) (Fig. 2(b)). From this description, closed form equations for optic axis and retardance easily emerge.

1. The optic axis is proportional to the angle between the +U axis and the projection of the difference vector \( (S_{out} - S_{in}) \) into the QU plane (Fig. 2(c)):

\[
\theta = -\frac{1}{2}\arctan \left( \frac{Q_{out} - Q_{in}}{U_{out} - U_{in}} \right). \tag{9}
\]

2. The retardance is proportional to the angle between the projections of \( S_{in} \) and \( S_{out} \) onto the plane where the optic axis vector is the unit norm and its coordinates are the origin (Fig. 2(d)), as described in Eq. 10. Note that \( \arctan2 \) is similar to the \( \arctan \) function but accounts for the quadrant of the vector and is therefore defined over \([0^\circ, 360^\circ]\).

\[
\eta = \frac{1}{2}\arctan2(U_{out} \cos(2\theta) - Q_{out} \sin(2\theta), V_{out}) - \frac{1}{2}\arctan2(U_{in} \cos(2\theta) - Q_{in} \sin(2\theta), V_{in}) \tag{10}
\]

Equations 9 and 10 define optic axis and retardance for \( \theta \in [0^\circ, 90^\circ] \) and \( \eta \in [0^\circ, 180^\circ] \), respectively; however, the standard ranges for optic axis and retardance are \([0^\circ, 180^\circ]\) and \([0^\circ, 90^\circ]\) [5]. The calculated birefringence values in these equations can be adjusted to suit the standard range by defining new standardized values for optic axis and retardance, which are used throughout in the remainder of this manuscript.

\[
\theta_{std} = \begin{cases} 
\theta & \eta \leq 90^\circ \\
\theta + 90^\circ & \eta > 90^\circ
\end{cases} \tag{11}
\]

\[
\eta_{std} = \begin{cases} 
\eta & \eta \leq 90^\circ \\
180^\circ - \eta & \eta > 90^\circ
\end{cases} \tag{12}
\]
3. Methods

3.1. Description of system designs

To demonstrate the range of realizable SIPS PS-OCT designs, we constructed a free-space spectral-domain (SD) OCT system with interchangeable PCCs (Fig. 1). Common to all systems were a superluminescent diode (SLD) source centered at 840 nm with a 3-dB bandwidth of 45 nm (IPSDD0808, Inphenix), a stationary silvered mirror as the reference surface, and a fiber-coupled spectrometer (CCS175, ThorLabs) as the detector. The combination of a linear polarizer (Pol) and retractable achromatic half-wave plate (HWP) permitted sequential measurement of the horizontal and vertical components of the polarized interferograms with one spectrometer. Although this detector introduced phase-based noise due to the sequential acquisition, the phase stability was modest (10 mrad over 5 ms) and was approximately constant between systems, permitting comparisons between systems and to simulations.

To enable comparisons across systems, the power incident on the polarizer in detector arm was fixed to 1.2 mW by adjusting the power from the source for each system, and the integration time of the spectrometer was fixed at 20 µs for each A-scan. The sensitivity was 60 dB because the high readout noise of the commercial spectrometer prevented shot noise-limited operation. The signal to noise ratio (SNR) of the sample reflector was 45 dB.

We constructed four distinct systems (Table 1) to demonstrate the effects of varied input polarization state to the sample (circular, vs. linear, vs. elliptical) as well as the multiplicity of design options for a given input polarization state (two alternatives for circular polarization). The PCCs in each system were simple: only combinations of linear polarizers and achromatic quarter waveplates were used and no more than one PCC was used in each arm. Note that the first row of Table 1 describes the traditional (Trad) PS-OCT system mentioned earlier [5], which served as a comparison for the other systems. The major axes of the linearly polarized (LP) input light (Row 3) and elliptically polarized (EP) input light (Row 4) were both 22.5°. The phase difference between the H and V polarization states of the EP light was 55°.

3.2. Assessment of system performance

We compared the performance of different systems for measuring birefringence over a range of optic axis and retardance values via simulation and experiment. The simulation, carried
Table 1. Description of the experimental systems we constructed in terms of their constituent combinations of physical components (Polarization Components) or induced changes in polarization state (Polarization Properties) as described in Fig. 1a

| System | Polarization Components | Polarization Properties |
|--------|-------------------------|-------------------------|
|        | \( J_{src} \)            | \( J_{det} \)           |
| Trad   | Pol 90°                  | Left \( E_{r} \)        |
|        | QWP 22.5°                | LP 45°                  |
| CP     | Pol 45°                  | Left \( E_{r} \)        |
|        | QWP 90°                  | LP -45°                 |
| LP     | Pol 22.5°                | LP 22.5°                |
|        | QWP 33.75°               | LP identity matrix      |
| EP     | Pol 45°                  | EP 22.5°                |
|        | QWP 112.5°               | EP -45°                 |

aTraditional system is abbreviated as Trad; the other systems are named by their input polarization state - CP: circularly polarized, LP: linearly polarized and EP: elliptically polarized. \( J_{d-s} = J_{det} J_{samp} \) gives the Jones matrix that transforms light from the sample prior to reaching the detector. Pol: linear polarizer QWP: quarter waveplate.

out in MATLAB, used the equations in Section 2.2 to calculate the optic axis and retardance one would expect to measure with a particular system in the presence of noise. We generated a noisy A-scan vector \( I'(z_s) \) from the addition of a noiseless A-scan vector, \( I(z_s) \), and noise vector \( \mathbf{N} = [N_H, N_V]^T \), which accounts for system-dependent noise. We assumed all systems had a read noise-limited dynamic range of 100 dB and a shot noise-limited sensitivity of 60 dB, since ideal OCT systems should be shot-noise limited. The noise vectors were computed by assuming that the real and imaginary parts of each element of the vector are independent and identically distributed Gaussians with a standard deviation proportional to the expected signal intensity [21].

Experimental validation of the various systems was carried out by measuring the birefringent properties of a calibrated sample. The sample consisted of an electrically controllable liquid crystal variable retarder (VR) (LCR-1-IR1, ThorLabs) followed by a silvered mirror located 101 ± 2 \( \mu \)m from ZPD. The optic axis and retardance of the VR could be independently controlled by rotating the waveplate and changing the voltage across the liquid crystal, respectively. We varied the optic axis from 0° to 180° (in 15° increments) and retardance from 0° to 90° (in 7.5° increments). For each permutation of optic axis and retardance, we collected 50 interferograms for each the horizontal and vertical polarization detection axes. Custom software (MATLAB) was used for data processing, which included resampling, cubic spline interpolation, Fourier transform computation, and calculation of birefringence from the interferometric data.

4. Results and discussion

4.1. Theoretical analysis of system performance

Figure 3(a) plots the absolute error between ideal (noiseless) and simulated (noisy) birefringence measurements for the four SIPS systems we implemented. Darker regions correlate with higher expected errors. These images simulate the output for a representative snapshot in time, which accounts for their speckled appearance. Optic axis and retardance values are distributed linearly from 0° to 180° and 0° to 90°, respectively. Although the range of the optic axis is 180°, the maximum error was 90° because the phase associated with the optic axis wraps around 180° (e.g., an optic axis of 0° is equal to an optic axis of 180°); retardance does not wrap in this way. A comparison across the columns of Fig. 3(a) suggests that: (1) the sensitivity to
Fig. 3. Effects of polarization properties of the system on the accuracy of the birefringence measurement. (a) Simulated absolute retardance and optic axis error for the systems described in Table 1. Error is associated with a noisy version of the A-scan vector from Eq. 6 and locations of large error depend on $E_{in}$ (Media 1 and Media 2). (b) Location of nodes (black, associated with H: horizontal and V: vertical polarization states) and convergence loci (dashed lines) in the retardance-optic axis space. Nodes typically depress error; convergence loci lead to increased error. (c) Convergence points on Poincaré sphere (arrows) associated with the convergence loci in (b), derived from the intersections of contour lines for optic axis (black) and/or retardance (gray); the location of the H node (red circle) is independent of the system design. (d) Average absolute error (mean of logarithm) across the whole retardance-optic axis space as a function of the polarization state of $E_r$, modeled as a linear polarizer (LP) (Media 3). (e) Average absolute error as a function of the rotation applied by the sample-to-detector Jones matrix ($J_{d-s} = J_{det} J_{samp}$), modeled as a QWP (Media 4).
error is not uniformly distributed for a given system, (2) the systems have different sensitivities at particular values of optic axis or retardance - including systems that have higher sensitivity than the traditional system in specific regions of the parameter space, and (3) some systems appear to perform better over the entire parameter space than others (e.g. Trad and CP versus LP and EP); however, the average optic axis and retardance error over the birefringence parameter space differed for the four systems by no more than 0.25°, a value which we will demonstrate is much less than the experimental error for all systems.

A more detailed assessment reveals that the errors associated with each system align with the “convergence loci” associated with that design. Convergence loci are multi-valued functions that map many optic axis-retardance pairs to a single output polarization state, and can be visualized easily on the Poincaré sphere. Figure 3(c) shows lines of constant optic axis and retardance for the four implemented systems as contours on the Poincaré sphere. The convergence loci correspond to locations where multiple contour lines meet (arrows). The high density of contour lines around these convergence points implies that large differences in birefringence produce only small changes in the measured polarization state. Thus, such values are very sensitive to noise, leading to high error. Figure 3(b) maps the convergence points on the Poincaré sphere to loci in the optic axis-retardance space (dashed lines). As expected, all systems have convergence loci for zero-degree retardance. This makes sense because $J = I$ when the retardance is identically zero and becomes independent of the optic axis. Similarly, the two additional convergence loci associated with the LP system ($E_{in}$ is linearly polarized) occur because the sample Jones matrix also collapses to the identity matrix when the optic axis of the polarized input is parallel or perpendicular to the optic axis of the sample, which makes retardance impossible to measure. The two systems for which $E_{in}$ is circularly polarized (traditional and CP) exhibit an additional convergence locus for retardances of 90°. The second convergence locus for the EP system ($E_{in}$ is elliptically polarized) is parabolic. The parabolic shape can be viewed as a transition between the horizontally and vertically oriented convergence loci for LP and CP systems, respectively.

Media 1 and Media 2 depict the relationship between the locations of the convergence loci and the value of $E_{in}$ (Fig. 3). It is clear from the videos that for EP light, the location of the convergence loci correspond to the major and minor axis of the polarization ellipse (i.e., the parabola intersects the $\eta = 90^\circ$ line at the angles corresponding to the two axes). The shape of the parabola relates to the eccentricity of the elliptical polarization state: it is elongated toward the $\eta = 0^\circ$ line for a more LP-like state, or flattened toward the $\eta = 90^\circ$ line for a more CP-like state. It is also clear from the video that the convergence loci of the LP system correspond with the angles parallel and perpendicular to the optic axis of the incoming light.

The convergence loci only partially describe our simulation results. To understand differences between the Trad and CP systems we implemented, we also investigated what happens to the polarization state of light after it emerges from the sample. We consider two effects: (1) the transformation of $E_{out}$ into $E_s$ (through matrix $J_{d-s} = J_{det} J_{samp}$) by downstream PCCs prior to reaching the detector and (2) the interference between $E_r$ with light returning from the reference arm ($E_r$). Table 2 summarizes the additional systems we simulated for this investigation. Varying the LP state of $E_r$ is equivalent to varying the ratio of reference light power at the two detectors, while modeling $J_{d-s}$ as a QWP with varying orientation is equivalent to rotating the $S_{out}$ vector in Stokes space. These two effects are simulated in Media 3 and Media 4, respectively (Fig. 3(c-d)). We chose to model $J_{d-s}$ as a QWP because this corresponds to the primary distinction between the Trad and CP systems we constructed. We assumed the axes of the PS-detector measure H and V LP states.

Figure 3(d) shows the change in the average of the absolute retardance and optic axis error as a function of the LP axis of $E_r$. We report the mean of the logarithm of the data to best
Table 2. Summary of the parameters varied to test the contribution of different polarization properties by simulating modification of the traditional system from Table 1

| Simulation Type | System Polarization Properties |
|-----------------|--------------------------------|
| \( E_{in} \)   | vary from CP to LP               |
| \( E_r \)      | Left CP                         |
| \( J_{d-s} \)  | Left CP                         |
| \( E_{in} \)   | LP 45°                          |
| \( E_r \)      | vary LP orientation              |
| \( J_{d-s} \)  | LP 45°                          |

capture the average magnitude of error. In this simulation, the maximum intensity measurable by a given detector was constant across all LP states: that is, if the LP 45° state was \([1 1]\), a near horizontal state was \([1 0 0.01]\). This is realistic given the finite full well capacity of real detectors. Thus, fewer photons are detected as the power between the H and V components of \( E \) becomes unbalanced, which in turn leads to lower measurement accuracy overall for that system. Hence, as expected, the minimum error occurs for \( E_r = LP(45°) \). At the extreme, horizontally or vertically polarized \( E_r \) will yield the poorest accuracy, as one of the detection channels will have no reference light to contribute to interference. In fact the error becomes infinite (not shown), which makes sense given our previous discussion about the impossibility of measuring birefringence for these cases (Section 2.1).

Figure 3(e) depicts the effect of \( J_{d-s} \) (modeled as a QWP) on the average absolute error. The periodicity mirrors the periodicity of locations where the birefringent parameters map to a perfectly horizontal or vertical polarization state at the detector: that is, where \( I(z_s) \) is identically H or V. These locations are generally associated with lower error, as measurements about these polarization states are more tolerant to noise. Figure 3(b) denotes the locations of these H and V nodes with solid lines or dots, as appropriate to each system. As seen in Media 4, rotating the QWP amounts to moving the position of these nodes (Fig. 3(d)). Comparing the Trad and CP systems, which essentially differ only by a change in \( J_{d-s} \) (i.e., orientation of the QWP), we find that lower average error is associated with close proximity between the H and V nodes and the convergence loci.

The latter simulations separately consider the effects of \( E_r \) and \( J_{d-s} \). Keep in mind, however, that it is the combination \( E_r \) and \( J_{d-s} \) that transform the vector leaving the sample (\( E_{out} \)) into \( I(z_s) \). Thus, one may alternatively consider the effect of \( J_{out} \), the Jones matrix that transforms \( E_{out} \) to \( I(z_s) \), as the product of a scaling matrix (with real components) and a rotation matrix (with unit norm). These two transformations do not affect the positions of convergence loci (which are determined purely by \( E_{in} \)), but do affect the accuracy of the birefringence measurement. This alternative representation can accommodate non-linearly polarized \( E_r \) or a \( J_{d-s} \) that contains a partial polarizer. The simulations we present correspond only with a special case where \( J_r \) and \( J_{d-s} \) serve as proxies for a more general scaling and rotation matrix pair.

4.2. Guidelines for the design of novel PS-OCT systems

In light of these observations, we present the following guidelines for consideration in the design of PS-OCT systems.

1. Through placement of the convergence loci, the input polarization state to the sample (\( E_{in} \)) determines the regions where birefringence may be difficult to measure. Thus, in the presence of noise it is advised to choose an input polarization state that maximizes the Stokes distance between the convergence loci and the expected range of birefringence values for the sample of interest. Alternatively, systems based on circularly polarized input light do fairly well across most of the birefringence parameter space. Knowledge of those polarization states corresponding to poor birefringence measurements may also be
of value in assessing confidence in the accuracy of a measurement, as may be relevant for samples of interest with birefringent properties falling in particular range. As an obvious example, retardance measurements of $0^\circ$ or $90^\circ$ obtained with the traditional system suggest poor accuracy in measurement of the optic axis.

2. The output matrix $J_{out}$, when decomposed as the product of a scaling and rotation matrix, will also affect system accuracy. The scaling matrix will affect the maximum allowable intensity for a particular detector (i.e., the number of photons that are collected). For best accuracy across the whole birefringence space, the scaling matrix should have equal elements on its diagonal. Complementarily, the rotation matrix will determine which birefringent parameters map to the polarization vectors that characterize the axes of detector (e.g., H and V). The rotation matrix should be selected to minimize error near the parameter space of interest, potentially by intersecting the those vectors with the convergence loci.

While the choice of PCCs does affect the overall accuracy as presented in the above guidelines, the relative differences between two different designs should not be exaggerated. Note that changes to the output matrix, $J_{out}$, lead to differences in average error of less than $0.01^\circ$ in most cases, and the choice of input polarization state, $E_{in}$, less than $0.25^\circ$. Further, if the sample has particular birefringence properties, one may benefit from selection of a system whose performance in a specific region of the optic axis-retardance space is optimal, though the system itself may manifest a less desirable error when averaged over the full optic axis-retardance space.

These design considerations have been framed in terms of the Jones vectors and matrices present in the system. Note that there are numerous physical combinations of PCCs enabling multiple designs for a given selection of $E_{in}$ and $J_{out}$. Some designs are likely to be better suited for particular applications or samples; at times it may be necessary to balance noise characteristics with facility of implementation. Also note that our investigation of the effects of $E_{in}$ on the measurement accuracy permit our results and guidelines to extend beyond SIPS systems to include those that use multiple input polarization states. This will be true to the extent that the input polarization state affects the accuracy of measurement, as we have shown is true. The analysis can be further extended to consider all systems using multiple input polarization states by redefining the relationships between $E_{in}$ and $J_{out}$.

The specific performance of a given system is also a function of dominant noise sources present. Given the tendency of the OCT community to prefer shot-noise-limited systems, our simulations reflect the performance of shot noise-limited systems. The systems we constructed, however, were read-noise limited. As we discuss next, however, this distinction between read noise and shot noise-limits does not significantly affect our measured results.

4.3. Experimental analysis of system performance

Figure 4(a) shows the measured birefringence parameters of the calibrated VR plate. A comparison between columns of the results for each system with the expected values (ideal) shows generally good agreement. This validates the assertion that there are several designs capable of quantitatively measuring birefringence parameters. In fact, for all but the traditional system, our work is the first demonstration of quantitative birefringence measurements with the specific configurations and combinations of PCCs we used, including the first with EP source light and unique systems based on CP and LP light.

Some error in our measurements is expected because it is inherent to the systems themselves, as revealed through the simulation. Nonetheless, system non-idealities, such as birefringence in the beam splitter and the bandwidth-limited performance of achromatic components, introduce additional error. This is more clearly seen by comparing Fig. 4(a) to Fig. 3(a) and noting the
Fig. 4. Measured birefringence parameters for the four experimental systems from Table 1. (a) Raw data. (b) Revised data after compensating for systemic non-idealities. The ideal behavior of these graphs is shown in the first column for comparison.

discrepancy between the appearance of the expected and actual positions of convergence loci. This inconsistency suggests, for example, that the expected input polarization state based on our choice of PCCs is not the actual polarization state of the implemented system, despite rigorous alignment. We believe these errors are due to the non-idealities of components in the system.

We thus developed a new algorithm to compensate for such systemic errors and improve the measurement accuracy. For each system, the algorithm involved selecting the input polarization state \( \mathbf{E}_{\text{in}}^{\text{comp}} \) and output Jones matrix \( \mathbf{J}_{\text{out}}^{\text{comp}} \) that best fit the entire parameter space of the measured data. To find these values, we iterated over many choices of \( \mathbf{E}_{\text{in}}^{\text{comp}} \) around the neighborhood of the expected \( \mathbf{E}_{\text{in}} \). For each, \( \mathbf{J}_{\text{out}}^{\text{comp}} \) was computed through least-squares minimization of the measured polarization states and the expected polarization states for a hypothetical system with input state \( \mathbf{E}_{\text{in}}^{\text{comp}} \) and \( \mathbf{J}_{\text{out}} \) equal to the identity matrix. We then recomputed the birefringence values using the \( \mathbf{E}_{\text{in}}^{\text{comp}} \) and \( \mathbf{J}_{\text{out}}^{\text{comp}} \) that gave the lowest average errors for optic axis and retardance. Table 3 lists the improvements we obtained for each system and Fig. 4(b) shows the improved results. Compensation reduced the errors in optic axis and retardance measurements for all systems (including the traditional). After compensation, comparable performance is achieved for all four of the systems, again suggesting that similar performance can be achieved for a large range of PS-OCT designs. In particular, as seen in Table 3, the errors in the Trad and CP systems are not significantly different. Further, the magnitude of the error is commensurate with the demonstrated accuracy of other PS-OCT systems [17, 22].
Table 3. Median of the absolute error before (pre) and after (post) compensation for the four implemented systems (rows). Data given are median values taken over all errors in the retardance-optic axis space.

| System | Retardance Error | Optic Axis Error |
|--------|------------------|------------------|
|        | Pre | Post | Pre | Post |
| Trad   | 1.7° | 1.2° | 3.4° | 2.0° |
| CP     | 4.7° | 1.2° | 7.1° | 2.1° |
| LP     | 8.4° | 1.7° | 3.4° | 1.5° |
| EP     | 5.0° | 2.2° | 8.6° | 2.1° |

Figure 5 elucidates the improved agreement between the locations of large errors in the measured data and convergence loci of the compensated systems. Although the patterns of error match well between the simulations and experiments, the magnitude of error is quite disparate (as the reader should note by the different scale bars in Fig. 5). These differences may be attributed to factors not accounted for in the simulation including phase jitter, speckle noise, or non-ideal components as well as lower SNR for experimental systems than simulated systems. The simulations of expected error for the compensated systems (Fig. 5(a)) better match the actual errors in the measured systems (Fig. 5(b)) and significantly improve the errors obtained in the measured data after applying the compensation algorithm (Fig. 5(c)). This improvement is seen both close to and far from the convergence loci. Another prominent feature of this dataset is large optic axis error of nearly 90° in some places in Fig. 5(b) and (c). This error arises in the presence of noise when the retardance measurements are near 0° or 90°, and it is due to a discontinuity in the function that converts the optic axis to the standard optic axis range (Eq. 11). These errors are not easily corrected, but would be eliminated if the non-standard domains for optic axis and retardance were used instead.

We also note that our choice of spectrometer affects the accuracy of measured data by limiting the dynamic range. Additionally, the sequential acquisition scheme of our PS-detector subjects us to larger errors in the measurement of phase as a result of phase drift. These errors, however, are present for all systems and do not affect our results comparing the performance of the different designs. The theory we have outlined, moreover, is also relevant to systems where two linearly independent polarization states are collected simultaneously by the PS-detector.

5. Conclusion

We have explored the full design space for SIPS PS-OCT systems and characterized their expected and actual performance for quantitative measurement of birefringence. The design space we identify is large – infinitely so. In all cases, birefringence parameters can be calculated with simple closed form expressions, as in the traditional PS-OCT system. In the course of our investigation, we have implemented new system designs and developed general guidelines for the design of PS-OCT systems. In particular, we demonstrate the first working SIPS PS-OCT system based on the use of EP light incident to the sample, and suggest guidelines for design that are relevant to all PS-OCT systems. We demonstrate through simulations and experiments that there are many alternative designs with comparable performance to the traditional SIPS PS-OCT design. Though some aspects of the analysis directly translate only to single input polarization state systems based on free-space optics or polarization-maintaining fiber, the overall analysis may also help explain the poor accuracy of some non-PM-fiber-based systems in light of the time-varying polarization state input to the sample. The framework of the analysis may also extend to systems with multiple input polarization states, and the proposed guidelines may offer a route to designing better, more flexible systems. Additionally, we introduce a compen-
Fig. 5. Absolute error between ideal (set) and actual retardance and optic axis measurements for (a) simulations of compensated systems and (b) experimental data both before and (c) after compensation. Convergence loci for the compensated systems are overlaid as thin black lines in (b) and (c) as a visual aid.
sation algorithm that improves the measurement accuracy of the systems we implemented by accounting for systemic errors.

This work suggests new options for PS-OCT systems to optimize around novel design constraints. For example, the design space we identify does not preclude common-path PS-OCT systems or systems with no PCCs in the sample arm, as might be useful for needle-based or endoscopic PS-OCT. We propose as future work to extend the theory to include a more generalized model of the sample that can account for factors such as diattenuation. Additional work may include 2D and 3D imaging of biological samples using alternative PS-OCT designs that are optimal for probing the birefringent properties of particular sample or that can satisfy stringent design requirements.

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