Non-trivial Backgrounds in (non-perturbative) Yang-Mills Theory by the Slavnov-Taylor Identity

A Quadri
Università degli Studi di Milano and INFN, Sezione di Milano, via Celoria 16, I-20133 Milan, Italy
E-mail: andrea.quadri@mi.infn.it

Abstract. We show that in the background field method (BFM) quantization of Yang-Mills theory the dependence of the vertex functional on the background field is controlled by a canonical transformation w.r.t. the Batalin-Vilkovisky bracket, naturally associated with the BRST symmetry of the theory. Since it only relies on the Slavnov-Taylor identity of the model, this result holds both in perturbation theory and in the non-perturbative regime. It provides a general consistent framework for the systematic implementation of the BFM in non-perturbative approaches to QCD, like e.g. those based on the Schwinger-Dyson equations or the lattice, in the presence of topologically non-trivial background configurations. The analysis is carried out in an arbitrary $R_\xi$-gauge.

1. Introduction
The Background Field Method (BFM) [1, 2] has played an important role in studying the properties of non-Abelian gauge theories. In perturbative computations within the BFM, one has the main advantage that gauge invariance with respect to the background field at the quantum level, encoded in the so-called background Ward identity, can be exploited in order to obtain linear relations between 1-PI amplitudes. This is in contrast with the more complicated relations among 1-PI Green functions arising from the Slavnov-Taylor identity, which, unlike the background Ward identity, is bilinear in the vertex functional.

Since perturbatively the BFM and the usual perturbation theory based on the Gell-Mann and Low’s formula give the same results for physical gauge-invariant observables [3, 4], the BFM can be used to significantly simplify computations in several applications, ranging from perturbative calculations in Yang-Mills theories [2, 5] via the quantization of the Standard Model [6] to gravity and supergravity calculations [7].

Outside ordinary perturbation theory, the BFM has been applied as a prescription for calculating to any order the $n$-point Green functions of the pinch technique [8, 9] in the approach based on the (non-perturbative) Schwinger-Dyson equations [10, 11]. In this context one makes use of the background-quantum identities [12, 13] relating Green functions involving a given combination of quantum and background fields with the same functions where one of the background fields has been replaced by its quantum counterpart.

In the two-point sector of (pure) $SU(N)$ Yang-Mills theory these identities are important in controlling the IR dynamics of the gluon and ghost propagators. The Schwinger-Dyson equation
for the background gluon propagator can be truncated gauge invariantly by exploiting the blockwise transversality of its gluon and ghost one- and two-loop dressed contributions [11, 14]. The solution of this equation can be then related to the conventional one through the corresponding two-point background quantum identity; the result is a gauge-artifact-free propagator that can be meaningfully compared to the high quality ab-initio lattice gauge theory computations currently available [15].

The comparison between these continuum studies and lattice data provides convincing evidence that in the Landau gauge the dressing functions of the gluon and ghost propagators tend in the deep IR to a finite, non-vanishing value [16, 17], thus supporting the mechanism of a dynamically generated gluon mass [8, 21]. This is in contrast with the predictions of the IR divergent ghost dressing function of the Kugo-Ojima confinement scenario [18] and the IR divergent ghost dressing function and the IR vanishing gluon propagator typical of the Gribov-Zwanziger scenario [19]-[21].

In order to get a deeper understanding of the results arising from lattice simulations, it would be clearly interesting to obtain information on the IR behaviour of the ghost and gluon propagators in as many gauges as possible, including the background field gauge.

Indeed, the implementation of the BFM on the lattice (for whatever value of the gauge fixing parameter) would be a long awaited leap forward [22].

One could in particular explore the influence of topological non-trivial backgrounds on the Green functions of the theory in a non-perturbative setting. In the pioneering paper [23] it has been shown how to compute the quantum corrections to the classical Yang-Mills action in the presence of a background instanton configuration in the one-loop approximation. The presence of zero modes in the two-point Green function of the quantum fields propagating in the background prevents to carry out a straightforward Gaussian integration and requires a dedicated treatment. This technique has been extended to two-loop order in [24]. On the other hand, a systematic procedure for dealing with the dependence on the background field beyond perturbation theory is, to the best of our knowledge, still missing.

In [25] it was shown that the dependence of the vertex functional on the background gauge field is fixed by the Slavnov-Taylor identity. The analysis was carried out in the background Landau gauge.

In this paper we wish to extend this result to an arbitrary background $R_\xi$-gauge. This will pave the way for the systematic implementation of the BFM in a non-perturbative setting in the presence of topologically non-trivial configuration. In this way one might be able to describe what happens when topological effects are properly taken into account, e.g. by comparing with what has been observed on the lattice when center vortices are removed from the vacuum configurations [26, 27].

We will show that the dependence of the vertex functional on the background field is fixed by a canonical transformation w.r.t. the Batalin-Vilkovisky bracket naturally associated with the BRST symmetry of the model [25]. The canonical transformation provides the correct way of handling the (non-trivial) deformation of the classical background-quantum splitting, induced by quantum corrections.

We will then find that the dependence on the background field can be recovered by carrying out a suitable field redefinition, which in general involves both the gauge and the ghost fields.

Since the method relies on symmetry requirements only, and in particular on the ST identity in the presence of a background field, it can be applied in any non-perturbative computational framework which fulfills the relevant functional identities of the model, thus easing the matching of results obtained in different approaches to non-perturbative QCD.

The paper is organized as follows. In Sect. 2 we set up our notation, introduce the tree-level vertex functional and the BV bracket generated by the BRST symmetry. We also write the relevant functional identities of the theory (B-equation, antighost equation, background Ward
identity, Slavnov-Taylor identity). We work in an arbitrary background $R_\xi$-gauge. In Sect. 3 we compare the BFM in the perturbative vs. non-perturbative frameworks. In Sect. 4 we move on to the analysis of the constraints on the background field dependence of the vertex functional encoded in the ST identity. By making use of cohomological tools we will show that the full dependence on the background connection $\hat{A}_\mu$ is completely fixed by the ST identity. This is our central result. It can be summarized in a compact formula by means of homotopy techniques. This formula is the basis of further applications, which we briefly outline in the Conclusions.

2. Classical Action and Its Symmetries

We consider Yang-Mills theory based on a semisimple gauge group $G$ with generators $T_a$ in the adjoint representation satisfying

$$[T_a, T_b] = if_{abc} T_c .$$

(1)

The Yang-Mills action $S_{YM}$ is

$$S_{YM} = -\frac{1}{4g^2} \int d^4x G_{a\mu\nu} G_a^{\mu\nu}$$

(2)

where $g$ is the coupling constant and $G_{a\mu\nu}$ is the Yang-Mills field strength

$$G_{a\mu\nu} = \partial_\mu A_{a\nu} - \partial_\nu A_{a\mu} + f_{abc} A_{b\mu} A_{c\nu} .$$

(3)

We adopt a (background) $R_\xi$-gauge-fixing condition by adding to $S_{YM}$ the gauge-fixing term

$$S_{g.f.} = \int d^4x \left[ \bar{c}_a \left( \frac{\xi}{2} B_a - D_\mu [\hat{A}] (A - \hat{A})_a \right) \right]$$

$$= \int d^4x \left( \frac{\xi}{2} B_a^2 - B_a D_\mu [\hat{A}] (A - \hat{A})_a \right.$$

$$\left. - \bar{c}_a D_\mu [\hat{A}] (D^\mu [A] c)_a + (D_\mu [A] \bar{c})_a \Omega^\mu_a \right) .$$

(4)

In the above equation $\xi$ is the gauge parameter (the Landau gauge used in [25] is obtained for $\xi = 0$) and $\hat{A}_a$ denotes the background connection. $\bar{c}_a, c_a$ are the antighost and ghost fields respectively and $B_a$ is the Nakanishi-Lautrup multiplier field.

We will sometimes use the notation $A_\mu = A_{a\mu} T_a$ and similarly for $\hat{A}_\mu, c, c, B$.

The BRST differential $s$ acts on the fields of the theory as follows

$$sA_{a\mu} = D_\mu [A] c_a \equiv \partial_\mu c_a + f_{abc} A_{b\mu} c_c ,$$

$$sc_a = -\frac{1}{2} f_{abc} \bar{c}_b c_c ,$$

$$s\bar{c}_a = B_a , \quad sB_a = 0$$

(5)

$s$ is nilpotent.

$\Omega_{a\mu}$ is an external source with ghost number +1 pairing with the background connection $\hat{A}_{a\mu}$ into a BRST doublet [28]

$$s\hat{A}_{a\mu} = \Omega_{a\mu} , \quad s\Omega_{a\mu} = 0 .$$

(6)

Since the BRST transformations of the fields $A_{a\mu}$ and $c_a$ in eq.(5) are non-linear in the quantum fields, we need a suitable set of sources, known as antifields [29, 30], in order to control
their quantum corrections. For that purpose we finally add to the classical action the following antifield-dependent term
\[
S_{a.f.} = \int d^4x \left( A_{a\mu}^* D^\mu [A] c_a - c_a^* \left( -\frac{1}{2} f_{abc} c_b c_c \right) - \bar{c}_a B_a \right).
\] (7)

Although it is not necessary for renormalization purposes, we have included in eq.(7) the antifield \( \bar{c}_a \) for \( \bar{c}_a \). This will allow us to treat on an equal footing all the fields of the theory by a single Batalin-Vilkovisky (BV) bracket \[30, 31\].

We summarize in Table 1 the ghost charge, statistics and dimension of the fields and antifields of the theory.

| Ghost charge | 0 | 1 | -1 | 0 | -1 | 0 | 1 |
|-------------|---|---|----|---|----|---|---|
| Statistics  | B | F | F  | B | F  | B | F |
| Dimension   | 1 | 0 | 2  | 2 | 3  | 4 | 2 |

Table 1. Ghost charge, statistics (B for Bose, F for Fermi), and mass dimension of both the SU\((N)\) Yang-Mills conventional fields and anti-fields as well as background fields and sources.

We finally end up with the tree-level vertex functional given by
\[
\Pi^{(0)} = S_{YM} + S_{g.f.} + S_{a.f.}.
\] (8)

\(\Pi^{(0)}\) fulfills several functional identities.

- the Slavnov-Taylor (ST) identity

The ST identity encodes in functional form the invariance under the BRST differential \( s \) in eqs.(5) and (6). In order to set up the formalism required for the consistent treatment of the quantum deformation of the background-quantum splitting, it is convenient to write the ST identity within the BV formalism.

We adopt for the BV bracket the same conventions as in [30]; then, using only left derivatives, one can write
\[
(X, Y) = \int d^4x \sum_\phi \left[ (-1)^{\epsilon_\phi (\epsilon_X + 1)} \frac{\delta X}{\delta \phi} \frac{\delta Y}{\delta \phi^*} - (-1)^{\epsilon_{\phi^*} (\epsilon_X + 1)} \frac{\delta X}{\delta \phi^*} \frac{\delta Y}{\delta \phi} \right]
\] (9)

where the sum runs over the fields \( \phi = \{ A_{a\mu}, c_a, \bar{c}_a, B_a \} \) and the antifields \( \phi^* = \{ A_{a\mu}^*, c_a^*, \bar{c}_a^*, B_a^* \} \). In the equations above, \( \epsilon_\phi, \epsilon_{\phi^*} \) and \( \epsilon_X \) represent respectively the grading of the field \( \phi \), the antifield \( \phi^* \) and the functional \( X \).

The extended ST identity arising from the invariance of \( \Pi^{(0)} \) under the BRST differential in eq.(5) and eq.(6), in the presence of a background field, can now be written as
\[
\int d^4x \Omega^\mu_{a}(x) \frac{\delta \Pi^{(0)}}{\delta A_{a\mu}^a(x)} = -\frac{1}{2} (\Pi^{(0)}, \Pi^{(0)}).
\] (10)

- the B-equation

\[
\frac{\delta \Pi^{(0)}}{\delta B_a} = \xi B_a - D_\mu [\hat{A}] (A - \hat{A})_a - \bar{c}_a^*.
\] (11)
The B-equation guarantees the stability of the gauge-fixing condition under radiative corrections. Notice that the r.h.s. of the above equation is linear in the quantum fields and thus no new external source is needed in order to define it. It does not receive any quantum corrections.

- the antighost equation

\[ \frac{\delta \Pi^{(0)}}{\delta \bar{c}_a} = -D\hat{A}_\mu \frac{\delta \Pi^{(0)}}{\delta A^\mu_a} - D_\mu [\hat{A}] \Omega_{a \mu} . \]  

- the background Ward identity

By using the background gauge-fixing condition in eq.(4), the vertex functional \( \Pi^{(0)} \) becomes invariant under a simultaneous gauge transformation of the quantum fields, external sources and the background connection, i.e.

\[ W_{\alpha} \Pi^{(0)} = -\partial_\mu \frac{\delta \Pi^{(0)}}{\delta \hat{A}_{\alpha\mu}} + f_{acb} \hat{A}_{\alpha\mu} \frac{\delta \Pi^{(0)}}{\delta \hat{A}_{\beta\mu}} - \partial_\nu \frac{\delta \Pi^{(0)}}{\delta \hat{A}_{\nu\mu}} + f_{acb} \hat{A}_{\nu\mu} \frac{\delta \Pi^{(0)}}{\delta \hat{A}_{\beta\mu}} + \sum_{\Phi \in \{B,c\}} f_{acb} \Phi_b \frac{\delta \Pi^{(0)}}{\delta \Phi_c} + f_{acb} \Phi_b \frac{\delta \Pi^{(0)}}{\delta \Phi_c} + f_{acb} \bar{\bar{c}}_b \frac{\delta \Pi^{(0)}}{\delta \bar{c}_c} + f_{acb} \bar{\bar{c}}_b \frac{\delta \Pi^{(0)}}{\delta \bar{c}_c} = 0 . \]  

Several comments are in order here. First we remark that the ST identity (10) is bilinear in the vertex functional, unlike the background Ward identity (13). Thus the relations between 1-PI amplitudes, derived by functional differentiation of the ST identity in eq.(10), are bilinear, in contrast with the linear ones generated by functional differentiation of the background Ward identity (13). One should notice that the background Ward identity is no substitute of the ST identity: physical unitarity stems from the validity of the ST identity and does not follow from the background Ward identity alone [4].

Since the theory is non-anomalous, in perturbation theory all the functional identities in eqs. (10), (11), (12) and (13) are fulfilled also for the full vertex functional \( \Gamma \). This can be proven in a regularization-independent way by standard methods in Algebraic Renormalization [4]. In what follows we assume that the same identities hold true for the vertex functional of the theory in the non-perturbative regime.

### 3. Perturbative vs. Non-Perturbative Background Field Method

The source \( \Omega_{a \mu} \) was used in [4, 32] in order to control the dependence of the local cohomology \( H(s|d) \) of the BRST differential on the background field \( \hat{A}_\mu \). It guarantees that \( H(s|d) \) in the presence of the background is isomorphic to the cohomology \( H(s|d) \) when \( \hat{A}_\mu \) is set equal to zero [4]. This implies that the set of local observables of the theory is not altered by the introduction of the background field and is given by gauge-invariant operators built out from the Yang-Mills field strength and covariant derivatives thereof [33].

In ordinary perturbation theory, i.e. when the path-integral is carried out by expanding around the trivial vacuum \( \hat{A}_\mu = 0 \), this result is the starting point for establishing the Background Equivalence Theorem (BET) [3, 4]. We denote by \( Q_\mu \) the quantum fluctuation around the background \( \hat{A}_\mu \), i.e. we set

\[ A_\mu = \hat{A}_\mu + Q_\mu . \]  

According to the BET, the connected Green functions of gauge-invariant operators can be equivalently computed by using either of the following two connected generating functionals:

\[ W = \Gamma[Q, \hat{A}, \Phi, \xi]_{\hat{A}=0} + \int d^4x J_Q Q + \int d^4x J_\Phi \Phi , \]

\[ W_{bg} = \Gamma[Q, \hat{A}, \Phi, \xi]_{Q=0} + \int d^4x J_\hat{A} \hat{A} + \int d^4x J_\Phi \Phi . \]
In eq.(15) $\Gamma$ denotes the 1-PI generating functional, $\Phi$ is a collective notation for the quantum fields of the theory different than the gauge field $Q$ and $\zeta$ is a collective notation for the external sources coupled to local composite operators in the 1-PI generating functional. $J_Q$ is the conjugate variable of the quantum gauge field $Q$ under the Legendre transform in eq.(15), $J_{\hat{A}}$ is the conjugate variable of $\hat{A}$ and finally $J_\Phi$ is the conjugate variable of $\Phi$.

The BET states that
\[ \delta^{(n)}W = \frac{\delta^{(n)}W_{bkg}}{\delta \beta_1(x_1) \ldots \delta \beta_n(x_n)} \bigg|_{J_Q=J_\Phi=\zeta=0} = \frac{\delta^{(n)}W_{bkg}}{\delta \beta_1(x_1) \ldots \delta \beta_n(x_n)} \bigg|_{J_{\hat{A}}=J_\Phi=\zeta=0} \]  

for any set of sources $\beta_1(x_1), \ldots, \beta_n(x_n)$ coupled to gauge-invariant operators $\mathcal{O}_1(x_1), \ldots, \mathcal{O}_n(x_n)$ [4]. Combinatorially, according to eq.(16) the connected correlator $\langle T \mathcal{O}_1(x_1) \ldots \mathcal{O}_n(x_n) \rangle$ can be equivalently computed by joining with the $Q$-propagator 1-PI amplitudes involving gauge quantum legs, evaluated at zero background (in this case one uses the functional $W$), or by joining with background propagator $\hat{A}$ 1-PI amplitudes, involving background gauge legs and evaluated at zero quantum gauge field (as prescribed by the functional $W_{bkg}$).

This is a very powerful result, since it allows to evaluate physical amplitudes from 1-PI Green functions with background external legs, which are in several cases significantly simpler to compute.

In a non-perturbative framework, e.g. on the lattice or in the approach based on the Schwinger-Dyson equations, one wishes to compute physical observables in the presence of a topologically non-trivial background, i.e. the path-integral is carried out by expanding around a non-trivial background connection $A_\mu$.

Our aim is to provide a systematic procedure for the determination of the dependence of the vertex functional $\Pi$ on the background field by exploiting the functional identities of the theory only and in particular the extended ST identity.

Since the method is based on the symmetries of the theory, it holds independently of the particular technique used for the non-perturbative evaluation of Green functions. In this formalism, the relations between background and quantum 1-PI amplitudes, arising from the ST identity, yield powerful consistency conditions that can be used as a check of computations carried out in different non-perturbative approaches.

4. Canonical Transformation for the Background Dependence

In order to control the dependence on the background connection we start from eq.(10) for the full vertex functional $\Pi$:
\[ \int d^4x \Omega_{a\mu}(x) \frac{\delta \Pi}{\delta A_{a\mu}(x)} = -\frac{1}{2} \langle \Pi', \Pi \rangle. \]  

By taking a derivative w.r.t. $\Omega_{a\mu}(x)$ and then setting $\Omega_{a\mu} = 0$ we get
\[ \frac{\delta \Pi}{\delta A_{a\mu}(x)} \bigg|_{\Omega_{a\mu}=0} = -\left( \frac{\delta \Pi}{\delta \Omega_{a\mu}(x)} \bigg)_{\Omega_{a\mu}=0} \right. \bigg|_{\Omega_{a\mu}=0}. \]  

This equation states that the derivative of the full vertex functional $\Pi$ w.r.t. $A_{a\mu}$ at $\Omega_{a\mu} = 0$ equals the variation of $\Pi$ w.r.t. to a canonical transformation generated by the fermionic functional $\Phi_{a\mu}(x)$.

This a crucial observation. First of all it shows that the source $\Omega_{a\mu}$ has a clear geometrical interpretation, being the source of the fermionic functional which governs the canonical
transformation [30] giving rise to the background field dependence. Moreover, the dependence of the vertex functional on the background field is designed in such a way to preserve the validity of the ST identity (since the transformation is canonical).

In a non-perturbative setting, we can use eq. (18) in order to control the background-dependent amplitudes. For that purpose one needs a method for solving eq. (18). An effective recursive procedure is based on cohomological techniques. Let us introduce the auxiliary BRST differential $\omega$ given by [25]

$$\omega \hat{A}_{\mu} = \Omega_{\mu}, \quad \omega \Omega_{\mu} = 0,$$

while $\omega$ does not act on the other variables of the theory. Clearly $\omega^2 = 0$ and, since the pair $(\hat{A}_{\mu}, \Omega_{\mu})$ forms a BRST doublet [28] under $\omega$, the cohomology of $\omega$ in the space of local functionals spanned by $\hat{A}_{\mu}, \Omega_{\mu}$ is trivial.

This allows us to introduce the homotopy operator $\kappa$ according to

$$\kappa = \int d^4 x \int_0^1 dt \hat{A}_{\mu}(x) \lambda_t \frac{\delta}{\delta \Omega_{\mu}(x)},$$

where the operator $\lambda_t$ acts as follows on a functional $X(\hat{A}_{\mu}, \Omega_{\mu}; \Xi)$ depending on $\hat{A}_{\mu}, \Omega_{\mu}$ and on other variables collectively denoted by $\Xi$:

$$\lambda_t X(\hat{A}_{\mu}, \Omega_{\mu}; \Xi) = X(t \hat{A}_{\mu}, t \Omega_{\mu}; \Xi)$$

The operator $\kappa$ obeys the relation

$$\{\omega, \kappa\} = 1_{\hat{A}, \Omega}$$

where $1_{\hat{A}, \Omega}$ denotes the identity in the space of functionals containing at least one $\hat{A}_\mu$ or $\Omega_\mu$.

Then we can rewrite the ST identity (17) as

$$\omega \Pi = \Upsilon,$$

where

$$\Upsilon = -\left(\Pi, \Pi'\right).$$

By the nilpotency of $\omega$

$$\omega \Upsilon = 0.$$

Since $\Upsilon|_{\Omega=0} = 0$, we have from eq. (22)

$$\Upsilon = \{\omega, \kappa\} \Upsilon = \omega \kappa \Upsilon$$

Thus from eq. (23) we have the identity

$$\omega(\Pi' - \kappa \Upsilon) = 0,$$

which has the general solution

$$\Pi' = \Pi_0 + \omega \Xi + \kappa \Upsilon$$
with \( \Xi \) an arbitrary functional with ghost number \(-1\). In the above equation \( \Pi_0 \) denotes the vertex functional evaluated at \( \hat{A}_\mu = \Omega_\mu = 0 \) (i.e. the set of 1-PI amplitudes with no background insertions and no \( \Omega_\mu \)-legs). The second term vanishes at \( \Omega_\mu = 0 \) but is otherwise unconstrained. I.e. the extended ST identity is unable to fix the sector where \( \Omega_\mu \neq 0 \). However this ambiguity is irrelevant if one is interested in the 1-PI amplitudes with no \( \Omega_\mu \)-legs, which are those needed for physical computations.

In practical applications it is convenient to expand the term \( \kappa \Upsilon \) in eq.(28) according to the number of background legs in order to write a tower of equations allowing to solve for the dependence on \( \hat{A}_\mu \) recursively down to the boundary condition (i.e. the vertex functional at zero background) \( \Pi_0 \). We will discuss this point elsewhere.

In the zero background ghost sector \( \Omega_\mu = 0 \), the \( \omega \Xi \) term in Eq. (28) drops out, and one is left with the result

\[
\Pi|_{\Omega=0} = \kappa \Upsilon + \Pi_0
\]

\[
= - \int d^4x \, \hat{A}_\mu^a(x) \int_0^1 dt \, \lambda_t \frac{\delta}{\delta \Omega_\mu^a(x)} \int d^4y \left[ \Pi_{\omega^a_b(y)} \Pi_{\omega^b_c(y)} + \Pi_{\omega^a_b(y)} \Pi_{\omega^b_e(y)} \right] |_{\Omega_\mu=0}
\]

\[
+ \Pi_0.
\]

In the above equation we have used the short-hand notation \( \Pi_{\omega^a_b} = \frac{\delta \Pi}{\delta \omega^a_b} \). Finally, if one is interested in the sector where ghosts are absent, the formula above further simplifies to

\[
\Pi|_{c=0} = - \int d^4x \, \hat{A}_\mu^a(x) \int_0^1 dt \, \lambda_t \int d^4y \left[ \Pi_{\omega^a_b(y)} \Pi_{\omega^b_c(y)} + b\frac{\delta}{\delta x^a_b(y)} \Pi_{\omega^a_b(x,y)} \right] |_{\Omega_\mu,c=0}
\]

\[
+ \Pi_0|_{c=0}.
\]

The equation above is quite remarkable, for it provides a representation of the vertex functional in the ghost-free sector that isolates the dependence on the background gauge field \( \hat{A}_\mu \).

Both eqs.(29) and (30) hold in an arbitrary background \( R_\xi \)-gauge. Their validity can in fact be further extended, e.g. to nonlinear gauges, since in the derivation of eq.(28) the only condition on the gauge-fixing functional is that it should be BRST-exact (compare with eq.(4)).

5. Conclusions

In the present paper we have shown that the dependence of the vertex functional on the background field is completely fixed by the extended ST identity. This result is general and only relies on the validity of the relevant functional symmetries of the theory, thus it can be applied in any symmetric non-perturbative setting. One recovers the dependence on the background gauge field by carrying out the field redefinition associated with the canonical transformation (18). This entails that the classical background configuration gets deformed by quantum corrections.

One could then try to evaluate explicitly these deformations, e.g. for the instanton profile in SU(2) Yang-Mills theory. Moreover, the present formalism could be applied in order to analyze the dependence on the subtraction scale \( \mu \) of physical observables in the presence of a background field configuration. It is known that such a dependence is non-trivial: at two-loop level renormalization group-invariance of physical quantities (like e.g. the ratio of the vacuum expectation value in the presence of an instanton configuration over the vacuum expectation value around the trivial solution \( \hat{A}_\mu = 0 \)) is only achieved by a proper treatment of the anomalous dimensions and the integration over the collective coordinates of the instanton [24].

The techniques discussed in this paper could be used in order to derive the appropriate formulation of the Callan-Symanzik and of the renormalization group equations in the presence of a non-trivial background.
Another important problem which awaits to be discussed is to see whether the present approach can be applied in order to implement the BFM for the well-known Cornwall-Jackiw-Tomboulis 2PI effective action [34].

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