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Perfect points on curves of genus 1 and consequences for supersingular K3 surfaces. (English)
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Summary: We describe a method to show that certain elliptic surfaces do not admit purely inseparable multisections (equivalently, that genus 1 curves over function fields admit no points over the perfect closure of the base field) and use it to show that any non-Jacobian elliptic structure on a very general supersingular K3 surface has no purely inseparable multisections. We also describe specific examples of genus 1 fibrations on supersingular K3 surfaces without purely inseparable multisections.

MSC:
14J28 K3 surfaces and Enriques surfaces
14G17 Positive characteristic ground fields in algebraic geometry

Keywords:
supersingular K3 surface; purely inseparable point; Weierstrass fibration; Frobenius; elliptic surface

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