Vortex Lattice simulations of attached and separated flows around flapping wings

Thomas Lambert¹, Norizham Abdul Razak ² and Grigorios Dimitriadis ¹,*

¹ Aerospace and Mechanical Engineering Department, University of Liège
² School of Aerospace Engineering, Universiti Sains Malaysia
* Correspondence: gdimitriadis@ulg.ac.be

Abstract: Flapping flight is an increasingly popular area of research, with applications to micro-unmanned air vehicles and animal flight biomechanics. Fast but accurate methods for predicting the aerodynamic loads acting on flapping wings are of interest for designing such air vehicles and optimising thrust production. In this work, the unsteady Vortex Lattice method is used in conjunction with three load estimation techniques in order to predict the aerodynamic lift and drag time histories produced by flapping rectangular wings. The load estimation approaches are the Katz, Joukowski and simplified Leishman-Beddoes techniques. The simulations’ predictions are compared to experimental measurements from a flapping and pitching wing presented by Razak and Dimitriadis [1]. Three types of kinematics are investigated, pitch-leading, pure flapping and pitch lagging. It is found that pitch-leading tests can be simulated quite accurately using either the Katz or Joukowski approaches as no measurable flow separation occurs. For the pure flapping tests, the Katz and Joukowski techniques are accurate as long as the static pitch angle is greater than zero. For zero or negative static pitch angles these methods underestimate the amplitude of the drag. The Leishman-Beddoes approach yields better drag amplitudes but can introduce a constant negative drag offset. Finally, for the pitch-lagging tests the Leishman-Beddoes technique is again more representative of the experimental results, as long as flow separation is not too extensive. Considering the complexity of the phenomena involved, in the vast majority of cases the lift time history is predicted with reasonable accuracy. The drag (or thrust) time history is more challenging.

Keywords: flapping flight; Vortex Lattice Method; aerodynamic loads; thrust production

1. Introduction

Flapping flight has become an increasingly popular area of research over the last couple of decades. Important experimental and numerical analyses have been carried out and flapping flight drones and aircraft have been designed, built and tested with varying degrees of success. Two major categories of activity have been differentiated:

- Birdlike flapping, which involves mainly low flapping frequencies and mainly attached flow, although flow separation can be encountered under specific circumstances
- Insect-like flapping, which involves high flapping frequencies and separated flow [2]

The first category is related to the development of ornithopter aircraft, i.e. machines that fly like birds, while the second is related to entomopters. There is also a scale difference between the two categories, since ornithopters can have sizes of the order of the meter while entomopters are measured in decimetres or even centimetres. This work will concentrate exclusively on birdlike flapping.
Many significant works have been published on birdlike flapping, starting with the 2D work of Von Karman and Burgers [3] and Garrick [4]. More recently, Young et al. [5] published an extensive review of the state of the art concerning flapping 2D airfoils. In fact, there has been a surprising number of review papers on flapping flight [6–11], considering the fact that the subject is a relatively new research area. As insect-like flight is considered to be more challenging, most researchers have concentrated on it. Nevertheless, birdlike flight is not a resolved problem and there are still many open questions concerning it.

Numerous experimental, theoretical and numerical analyses of pitching and/or plunging 2D airfoils have been performed. Such work is relevant to hydrofoils, aircraft wings and helicopter blades and therefore wide ranges of Reynolds and Mach numbers have been investigated. In contrast, work on 3D flapping has been rarer although some contributions were made in the last couple of decades [1, 12–14].

One of the important issues concerning flapping flight is the modelling approach to be used in order to estimate the aerodynamic loads. Numerous techniques from modified strip theory to Computational Fluid Dynamics (CFD) have been applied. Computational cost is quite an important consideration due to the unsteady nature of the flowfields so that lower fidelity approaches can be preferable, at least under attached flow conditions. The Vortex Lattice Method (VLM) has been proposed for modelling flapping flight by several authors [15–20]. Ames et al. [21] compared predictions from a Vortex Lattice simulation against experimental measurements for a flapping rigid rectangular wing. They found that the predictions were accurate only for the highest Reynolds numbers and lowest reduced frequencies tested, since the VLM is an inviscid approach and therefore cannot model flow separation. However, extensions for flow separation at the leading edge have been in use since the 1970s [22–25]. Roccia et al. [26] used such a modification to model flapping wings in hover conditions. Many of the VLM simulations have been coupled to structural solvers in order to simulate flexible flapping wings [27–29]. Others have used the VLM in order to optimize thrust production or power requirements for flapping flight [30,31]. Vest and Katz [32] carried out flapping wing simulations using the unsteady source and doublet method, which is also inviscid but, unlike the VLM, can model wing thickness. Another alternative is the Doublet Lattice Method studied by Isogai and Harino [33].

The objective of the present work is to investigate the accuracy of the load predictions obtained by the Vortex Lattice method when applied to the flapping of a rigid rectangular wing, in both attached and separated flow cases. Three different techniques for load estimation will be used and their predictions compared to experimental measurements.

2. Experiments

The flapping wing experiments are described in detail in Razak and Dimitriadis [1], only a short summary is presented here. The flapping mechanism is shown in figure 1 and consisted in a tandem dual crank mechanism arrangement, connected to a single drive shaft, which was in turn driven by a brushless direct current electric motor. Root flapping was imposed by the dual crank while pitching was imposed by the tandem nature of the mechanism. The phase between the forward and aft flapping arms was constant throughout the flapping cycle. The value of the phase could be set by imposing an initial offset in the positions of the two cranks. Wing adaptors were used to join the forward and aft flapping arms. The adaptors’ function was to accommodate different orientations and positions of the flapping arms and to translate these into pitching angle without stressing the flapping arms. They also allowed the setting of a constant geometric static pitch angle $\theta_0$.

Figure 2 shows the complete apparatus in the wind tunnel, with wings installed at the maximum and minimum flap angle positions. The amplitude of the flap angle, $\gamma$, was $30^\circ$. The maximum amplitude of the pitch angle, $\theta$, was $20^\circ$ but could be set to any desired lower value. Flapping and pitching occurred at the same frequency, $f$, which was set to four values 0.79 Hz, 1 Hz, 1.23 Hz and 1.5 Hz. The frequency was kept low to avoid interference from the structural modes of vibration of
the entire assembly and to limit the inertial loads caused by the motion. Figure 2 also shows that
the flapping assembly was mounted on a beam instrumented with strain gauges that was used to
measure the lift, drag and side forces. The lift force was defined as perpendicular to the wind tunnel’s
free stream and the drag parallel to it.

![Figure 1. Flapping mechanism](image1)

![Figure 2. Maximum and minimum flap angles](image2)

Pure flapping, pitch lagging and pitch leading kinematics were imposed on the wings. Pure
flapping involved a constant pitch angle; the difference between pitch lagging and pitch leading
motions is demonstrated in figure 3. In the lagging case, the instantaneous flap angle leads the
instantaneous pitch angle by a phase difference of 90° and vice versa. Pitch leading ensures that
the kinematic angle of attack of the wing remains small and maximizes the thrust while keeping the
flow attacked. Conversely, pitch lagging causes very high kinematic angles of attack and generates
high instantaneous values of lift.

Three difference sets of wings were tested; they were all straight rectangular and untwisted and
had NACA 0012, 4412 and 6409 airfoil sections. The chord, c, and span, b, of the 6409 wing were
0.16m and 0.4m respectively while the other two wings had chord and span lengths of 0.146m and
0.46m. The distance between the wing root and the flapping axis was 0.15m for the 6409 wing and
0.19 for the other two wings. Numerous combinations of kinematics and wing speed were tested.
for each wing; the flap amplitude was always constant at 30° and the flap angle centre was always 0° so that the flapping motion was symmetric across the horizontal plane passing through the flap axis. The pitch amplitude was set to 0°, 6° and 11° and the pitch centre was varied between −6° and +14°. The airspeed, $U$, was set to 6m/s, 9.4m/s and 14.8m/s, leading to Reynolds numbers between $0.7 \times 10^5$ and $1.6 \times 10^5$ and reduced frequency values, $k = \pi f c / U$, between 0.03 and 0.13. The Strouhal number, $St = 2 z_{tip} f / U$, ranged between 0.03 and 0.17, where $z_{tip}$ is the flapping amplitude at the wingtip. Only the NACA 6409 wing will be used in the present work, as it was the only wing that was tested in pure flapping, pitch leading and pitch lagging kinematic configurations.

The loads measured by the aerodynamic balance included both aerodynamic and inertial contributions. In order to separate the two, wind-off flapping and pitching tests were carried out for each tested configuration. The force measurements obtained during these tests included only inertial contributions and were to be subtracted from the load values measured during the wind-on tests, which also included aerodynamic loads. The wind-off tests were carried out after replacing the wings with metal bars that had the same inertial characteristics in flap and generated negligible aerodynamic loads. Extension springs were also installed on top of the flapping arms in order to mimic the effect of the lift acting on the wings at wind-on conditions. Despite these precautions, the flap and pitch angle signals at wind-off and wind-on conditions were not identical, as demonstrated in figure 5 for a particular test case. All measured signals had to be cycle averaged before the subtraction of the wind-off loads from the wind-on loads could be performed. Figure 5 shows the result of the cycle-averaging process on the flap and pitch signals of figure 5. It can be seen that the wind-on and wind-off flap angle time histories are nearly indistinguishable but there are visible differences between the two pitch response. Nevertheless, it was assumed that pitching has a very small effect on the inertial loads compared to flapping and therefore the subtraction could be carried out. The cycle averaging procedure was also applied simultaneously to the measured lift and drag signals,
leading to mean waveforms and standard deviation estimates. For all test cases, cycle averaging resulted in flap signals with the same phase as the one shown in figure 5. The time history begins and ends at the middle of the upstroke, while the middle of downstroke lies near the middle of the time range. All results presented throughout this paper are calculated using the same flap phase.

Finally, the parasite drag due to skin friction and interference from the wing supports was also removed from the drag measurements, since the vortex lattice method cannot model such effects.
Static wind-on tests were carried out with the wings set to zero flap and pitch angles and the measured drag values were subtracted from the drag time histories obtained from the flapping wind-on tests.

3. Vortex Lattice Model

The Vortex Lattice model used for this work is similar to the one used by Dimitriadis et al. [20] with one major difference: the drag is calculated using both the Katz and Joukowski approaches, as discussed by Simpson and Palacios [34] and Lambert and Dimitriadis [35]. The VLM method is described in detail in several authoritative publications [29,36]. It consists in solving the Laplace equations for a wing undergoing a general motion using vortex rings as the elementary solution. The rings are placed on the quarter chord of geometric panels on the wing’s camber surface (bound vortex rings) as well as in the wake (wake vortex rings). Vortex rings are also placed in the wake in order to simulate the free vortex sheet. The Kutta condition that the flow separates smoothly from the trailing edge is enforced by shedding the vorticity of the bound trailing edge panels into the wake at each time step. A leading edge wake can also be shed from the bound leading edge panels in order to model leading edge flow separation [23]. This option was chosen by Roccia et al. [26] but flow separation will be represented here by means of the Leishman-Beddoes model, hence, VLM simulations are carried out with a single wake shed from the trailing edge.

The two vorticity surfaces (bound and trailing edge wake) are exemplified in figure 6. Note that $x$, $y$ and $z$ are global coordinates but $x$ is aligned with the chord and $y$ is aligned with the span when $\gamma = \theta = 0^\circ$. Once a vortex ring is shed into the wake, it is allowed to travel at the local flow velocity, so that the wakes are force-free.

According to the vortex model by Vatistas et al. [37], the velocity induced at a general field point by a straight line vortex segment, whose ends with respect to the field point are given by vectors $r_1$ and $r_2$ and whose strength is $\Gamma$, is given by

$$U_{ind} = \frac{K \Gamma}{4\pi} \frac{r_1 \times r_2}{|r_1 \times r_2|^2} (r_2 - r_1) \cdot \left( \frac{r_1}{|r_1|} - \frac{r_2}{|r_2|} \right)$$

where $K$ is defined as $K = 1$ if the vortex segment belongs to a bound vortex ring and

$$K = \frac{h^2}{r_c^2 + h^2}$$

if the segment belongs to a wake vortex ring. In this latest expression

$$h = \frac{|r_1 \times r_2|}{|r_2 - r_1|}, \quad r_c = \sqrt{0.001 + 5.1514\nu \delta t_v}$$

where $\nu$ is the kinematic viscosity of air, $\delta = 1 + 2 \times 10^{-4} \sqrt{\Gamma^2/\nu}$ and $t_v$ is the time elapsed since the vortex segment was shed into the wake.

The impermeability boundary condition is imposed by calculating the velocities induced by all the vortex rings normal to the control points of the geometric panels of the wing from equation 1 and setting the total normal velocity to zero. The boundary condition becomes

$$A \Gamma_b + B \Gamma_{TE} = U_{m} + U_{w}$$

where $\Gamma_b$ and $\Gamma_{TE}$ are vectors containing the vortex strengths of the bound and trailing edge wake vortex rings respectively, $A$, $B$ are aerodynamic influence coefficient matrices and $U_{m}$, $U_{w}$ are the components of the external velocity normal to the wing panels induced by the motion and the wake, respectively. The external velocity includes contributions due to the free stream, the flapping motion and the pitching motion. As the wing is rigid, the $A$ influence coefficient matrix is constant in time. In contrast, $B$ is time-varying as the wake deforms.
Equation 3 is solved for the unknown strengths of the bound vortices, $\Gamma_b$, at each time instance. The wake vortex strengths $\Gamma_{TE}$ are known since they were shed from bound vortex rings at previous time steps. At the start of the calculation there is no wake and $\Gamma_b = A^{-1}U_m^n$. The wake develops at the rate of one row of vortex rings per time step, so that the start of the motion is impulsive and gives rise to a starting vortex. This means that the first flapping cycle is not representative of the fully developed flapping flow and at least two cycles must be simulated.

Once the strengths of the bound vortex rings have been calculated, the forces acting on the wing can be calculated using either the Joukowski approach or the Katz approximation [34,35]. For the former, the force generated by the $i,j$th vortex ring is given by

$$F_{ij} = \sum_{m=1}^{4} F_{st}^m + F_{unst}^{ij}$$

where $i$ is the chordwise panel index, $j$ is the spanwise panel index and $F_{st}^{ij}$ is the steady contribution of the $m$th vortex segment of the ring, given by

$$F_{st}^m = \rho \Gamma_m (U_m \times l_m)$$

Here, $U_m$ is the local flow velocity evaluated at the centre of the segment, $l_m$ is the vortex segment vector and $\Gamma_m$ its vortex strength. The latter is generally equal to $\Gamma_{ij}$ for all the vortex segments of each vortex ring except for the trailing edge spanwise segments (i.e. the bound segments that are in contact with the wake segments), where $\Gamma_m = 0$. Finally, $F_{unst}^{ij}$ in equation 4 is the unsteady force contribution of the $i,j$th vortex ring, given by

$$F_{unst}^{ij} = \rho \frac{\partial \Gamma_{ij}}{\partial t} A_{ij} n_{ij}$$
where $A_{ij}$ is the area of the corresponding wing panel and $n_{ij}$ a unit vector normal to the panel. The derivative $\partial \Gamma_{ij}/\partial t$ is calculated using a first order backwards difference formula.

The Katz approximation requires the calculation of three different velocities on the $ij$th panel:

- the velocity due to the wing’s motion $U_{ij}^m$,
- the velocity induced by the vorticity in the wake $U_{ij}^w$ and
- the velocity induced by only the bound chordwise vortex segments, $U_{ij}^b$.

Then the lift acting on the $i$, $j$th panel is given by

$$L_{ij} = \rho \left( (U_{ij}^m + U_{ij}^w), \tau_{ij}^\alpha \frac{\Gamma_{ij} - \Gamma_{i-1,j}}{\Delta c_{ij}} + (U_{ij}^m + U_{ij}^w), \tau_{ij}^s \frac{\Gamma_{ij} - \Gamma_{i,j-1}}{\Delta b_{ij}} + \frac{\partial \Gamma_{ij}}{\partial t} \right) A_{ij} \cos \alpha_{ij}$$

(5)

where $\tau_{ij}^\alpha$, $\tau_{ij}^s$ are chordwise and spanwise unit vectors tangential to the panel while $\Delta c_{ij}$ and $\Delta b_{ij}$ are the spanwise and chordwise lengths of the panel. The panel’s angle of attack due to the shape and motion of the wing is $\alpha_{ij} = \tan^{-1} \left( U_{ij}^m n_{ij}/U_{ij}^m \tau_{ij}^c \right)$. The drag acting on the panel is given by

$$\Delta D_{ij} = \rho \left( (U_{ij}^m + U_{ij}^w) \cdot (P_{ij} n_{ij}) (\Gamma_{ij} - \Gamma_{i-1,j}) \Delta b_{ij} + \frac{\partial \Gamma_{ij}}{\partial t} A_{ij} \sin \alpha_{ij} \right)$$

(6)

where

$$P_{ij} = I - \frac{U_{ij}^m U_{ij}^m}{|U_{ij}^m|^2}$$

and $P_{ij} n_{ij}$ is an orthogonal projection of the normal to the panel in the direction of the flow. The total aerodynamic force induced by the $i$, $j$th panel is then given by

$$F_{ij} = D_{ij} \frac{U_{ij}^m}{|U_{ij}^m|} + L_{ij} P_{ij} n_{ij}$$

(7)

Note that, for the Katz method, $L_{ij} P_{ij} n_{ij}$ is always perpendicular to the local airspeed due to the motion of the wing while $D_{ij} U_{ij}^m/|U_{ij}^m|$ is always parallel to it. Consider a static, symmetric and rectangular wing at an angle of attack $\alpha$ to a horizontal free stream. The contribution $L_{ij} P_{ij} n_{ij}$ to the total drag will be zero for any value of $\alpha$, as the lift is always perpendicular to the free stream. The $D_{ij} U_{ij}^m/|U_{ij}^m|$ contribution when the lift vector leans forward (Knoller-Betz effect). Thrust is particularly strong in the outboard part of the wing, which can see very high kinematic angles of attack during the downstroke. Nevertheless, the $D_{ij} U_{ij}^m/|U_{ij}^m|$ force component will still only produce drag. Also note that the local lift goes to zero at the wingtip therefore the part of the wing with the highest kinematic angle of attack during the downstroke does not contribute the thrust.

The load estimates of equations 4 and 7 should be similar but not identical, which is why both sets of estimates will be presented in the results sections. The advantage of the Joukowski method is simplicity and ease of implementation. The advantage of the Katz method is speed, as it requires flow velocity evaluations only on the control points of the wing panels, not on the mid-points of every bound vortex segment.

4. Estimation of separated flow aerodynamic loads using the Leishman-Beddoes model

An alternative to the Katz and Joukowski aerodynamic load estimation techniques is to use the 2D unsteady drag model presented by Leishman [38]. For a 2D airfoil oscillating in pitch in an airflow, the unsteady drag coefficient can be written as

$$c_d = c_n \sin \alpha(t) - c_c \cos \alpha(t)$$

(8)
where $\alpha(t)$ is the instantaneous pitch angle, $c_n$ the normal force coefficient and $c_c$ the chordwise force coefficient. The latter is also known as leading edge suction as it contributes thrust. For attached flow, the normal force coefficient can be estimated from

$$c_n = \eta 2\pi \alpha E \tan \alpha E$$

where $\alpha E$ is an unsteady effective angle of attack and the lift curve slope is assumed to be equal to $2\pi$. The factor $\eta$ usually takes values between 0.95 and 0.97 and represents the fact that airfoils in viscous flows cannot attain the full amount of leading edge suction. Leishman and Crouse [39] select $\eta = 0.95$. The factor $\eta$ is an overestimation, as $c_n$ contains both circulatory and non-circulatory contributions. This point will be discussed later on.

In the context of a VLM simulation, the unsteady normal force coefficient can be obtained from either the Katz or Joukowski methods. For example, from equation 5 and for the $j$th spanwise section

$$c_{nj} = \frac{2}{U_c^2} \sum_{i=1}^{m} \left[ (U_{ij} + U_{ij}^{\infty}), \tau_{ij} - \Gamma_{ij} \pi \alpha_{ij} \Delta c_{ij} + (U_{ij}^{\infty} + U_{ij}), \tau_{ij} + \frac{\Gamma_{ij} - \Gamma_{ij-1}}{\Delta b_{ij}} \frac{\partial \Gamma_{ij}}{\partial t} \right] \Delta c_{ij} \cos \alpha_{ij}$$

This force can be used to obtain an estimate of the local unsteady angle of attack $\alpha_{Ej}$

$$\alpha_{Ej} = \frac{c_{nj}}{2\pi}$$

This expression is an overestimation, as $c_{nj}$ contains both circulatory and non-circulatory contributions. This point will be discussed later on.

The Leishman-Beddoes model can also be used to estimate the amount of local flow separation and the ensuing changes to the aerodynamic loads. The chordwise position of the separation point, $f_s$, can be estimated from Kirchoff theory. For the $j$th spanwise section, the separation point lies at

$$f_s(\alpha_{Ej}^*) = \left\{ \begin{array}{ll}
1 - 0.3e^{(\alpha_{Ej}^* - \alpha_1)/S_1} & \text{if } \alpha_{Ej}^* \leq \alpha_1 \\
0.04 + 0.66e^{(\alpha_1 - \alpha_{Ej}^*)/S_2} & \text{if } \alpha_{Ej}^* > \alpha_1
\end{array} \right.$$  

where $\alpha_1$ is the value of the static angle of attack when $f_s = 0.7$, i.e. the separation point lies at 0.7c and $S_1, S_2$ are constants to be determined for each airfoil and Reynolds number. Furthermore, $\alpha_{Ej}^* = (c_{nj} - c_{n0})/2\pi$, were $c_{n0}$ is the normal coefficient offset required to make the airfoil’s lift curve symmetrical (see next paragraph). Consequently, the normal and chordwise force coefficients at the $j$th spanwise station are given by

$$c_{nj} = 2\pi \alpha E \left( 1 + \frac{f_s(\alpha_{Ej}^*)}{2} \right)$$

$$c_{cj} = \eta 2\pi \alpha E \sqrt{f_s(\alpha_{Ej})} \tan \alpha_{Ej}$$

Finally, the total lift and drag can be approximated from

$$C_L = \frac{\cos \gamma(t)}{b} \sum_{j=1}^{n} \left( c_{nj} \sin \theta(t) + c_{cj} \cos \theta(t) \right) \Delta b_j$$

$$C_D = \frac{\cos \gamma(t)}{b} \sum_{j=1}^{n} \left( c_{nj} \sin \theta(t) - c_{cj} \cos \theta(t) \right) \Delta b_j$$
The aerodynamic load coefficients of equations 12 and 13 were calculated at each time instance using the current estimate of $c_{n_i}$. The values of $c_{n_0}$, $\alpha_1$, $S_1$ and $S_2$ were obtained from the experimental measurements at low Reynolds numbers presented by Durgesh et al. [40]. Figure 7 plots the results of the Kirchoff theory curve fits, compared to the experimental data. Note that there is a discontinuity in the experimental lift curve at $Re = 5 \times 10^6$ that cannot be represented by the Kirchoff exponential functions. The minimum Reynolds number tested in flapping was 66,000 so that the curve fit of figure 7(b) was used for setting the Kirchoff model’s parameters, leading to $\alpha_1 = 10.31^\circ$, $S_1 = 0.02$, $S_2 = 0.043$ and $c_{n_0} = 0.5709$.

![Figure 7. Lift measurements and Kirchoff theory curve fits for NACA 6409 airfoil](image)

In the complete Leishman-Beddoes model, both $c_n$ and $f_s(\alpha_E)$ are delayed by time lags $T_p$ and $T_f$, respectively, before calculating the normal and chordwise load coefficients. The time lags represent the stall delay phenomenon and are estimated from dynamic experiments on the airfoil section undergoing sinusoidal pitching; such experiments were not carried out as part of the present work and values for $T_p$ and $T_f$ were not found in the literature for the NACA 6409 at the Reynolds number range of interest. Furthermore, the Reynolds numbers and reduced frequencies of the experiments were quite low, suggesting that stall delay may not be very important. Consequently, the time delays were not applied in this work. The Leishman-Beddoes-based technique presented here also neglects the modelling of the Dynamic Stall Vortex (DSV).

As mentioned earlier, equation 9 is an overestimation, since the Katz approach does not differentiate between circulatory and non-circulatory aerodynamic loads. A simple and approximate solution to this problem is adopted in the present work. Equation 10 is re-written as

$$c_{n_j}^s = \eta 2\pi \alpha E_j \left( \frac{1 + \sqrt{f_s(\alpha_j^*)}}{2} \right)^2$$

(14)

and the correction factor in both equations 14 and 11 is set to $\eta = 0.75$. This modification implies that around 20% of the inviscid aerodynamic normal force over the entire span is non-circulatory in origin. This statement is justified only because the reduced frequencies investigated in the present work are low. The value of 0.75 was set by trail-and-error and is constant over the entire span.

5. Pitch leading

The pitch leading test cases were simulated using only the Katz and Joukowski load estimation techniques, without any stall modelling. The kinematic angle of attack for these cases is always low...
throughout the wing and therefore no visible flow separation is occurring. The simulations were carried out with $m = 14$ chordwise panels and $n = 12$ spanwise panels. The spacing of the panels was sinusoidal in the spanwise direction so that the grid was denser near the wingtips. The time step was set to $\Delta t = 5c/Um$ and each simulation was run for two full flapping cycles. Both the Katz and Joukowski estimates of the drag were calculated for all cases. The chosen values of $m$, $n$ and $\Delta t$ led to converged aerodynamic load estimates and refining the spatial and temporal grids produced negligible changes in the predicted lift and drag. For each test case, the cycle averaged experimental flap and pitch angle time histories were used as kinematic input to the VLM. This means that each flapping cycle always started and finished at mid-upstroke. The experimental signals were re-sampled at the simulation time step and repeated in order to simulate two full cycles.

![Graphs showing lift and drag coefficients](image)

Figure 8. Pitch leading, $U = 6$ m/s, $f = 0.79$ Hz, $\theta_{\text{min}} = -4^\circ$, $\theta_{\text{max}} = 8^\circ$

The agreement between the experimental and simulated lift and drag coefficients was generally good for most test cases. Figure 8 plots the lift and drag coefficients for the test case $U = 6$ m/s, $f = 0.79$ Hz and a pitch oscillation between $-4^\circ$ and $8^\circ$. The reduced frequency is $k = 0.07$ and the experimental results are plotted with error bars that represent the standard deviation of the cycle averaging procedure. Note that the error bars of the lift coefficient are much higher than those of the drag; pitch leading oscillations at this low airspeed produce small amounts of lift, such that the difference between aerodynamic and inertial loads is quite low. The inertial loads act mainly in the lift direction and have a limited effect on the drag, whose standard deviation is much lower than that of the lift.

Figure 8(a) shows that the VLM predictions for the lift obtained by the Katz and Joukowski methods are very similar. They are both lower than the cycle averaged experimental lift signal but lie nearly always within the error bars. The phases of the simulated and experimental lift coefficients are nearly identical. Figure 8(b) plots the predicted and experimental drag coefficients. The estimates obtained by the Katz and Joukowski techniques lie very close to each other, except at the very start and end of the cycle. Note that the drag is negative through part of the cycle, i.e. the wing is producing thrust. The maximum thrust is slightly overestimated by the VLM approach but the experimental and simulated waveforms are actually very similar.

Figure 9 plots the lift and drag coefficients for another test case, defined by $U = 6$ m/s, $f = 1.23$ Hz and a pitch oscillation between $-10^\circ$ and $2^\circ$, with $k = 0.10$. In this case there is a noticeable phase delay between the simulated and experimental lift signals. Furthermore, the maximum lift coefficient is predicted accurately by the VLM but not the minimum, which is in fact a downforce in the experimental results. Nevertheless, the simulated waveforms lie within the error bars and are therefore considered reasonable predictions. Once more, the Katz and Joukowski method estimates
are quite similar. The simulated drag waveform lies again lower than the experimental average but still within the error bars. The simulation misses the positive drag peak occurring at the end of the upstroke but provides a satisfactory prediction of the thrust.

Finally, figure 10 plots the lift and drag coefficients for the test case $U = 14.8\text{ m/s}$, $f = 1.23\text{ Hz}$ and a pitch oscillation between $-8^\circ$ and $4^\circ$, so that the reduced frequency is $k = 0.04$. The mean of the predicted lift signal is accurate but the amplitude is slightly lower than the experimental amplitude and there is also a noticeable phase delay. Note that at this airspeed the aerodynamic loads become significant and the standard deviation of the cycle averaging procedure is smaller. The simulated lift would still lie inside the error bars if it were not for the phase shift. The VLM drag predictions again overestimate the thrust; in the experiment very little thrust is produced. During the upstroke the simulated drag signals are quite similar to the experimental measurements.

The results presented in this section show that the predictions obtained by the VLM simulations are acceptable for pitch leading test cases. The overestimation of the thrust is expected, as the VLM does not model airfoil thickness or viscous effects. The phase shift between the simulated and experimental lift signals may be due to the cycle averaging procedure; the cycle averaged pitch signal
may not have the correct phase, resulting in a phase delay of the predicted lift coefficient. In fact, the lift coefficient is quite sensitive to the pitch phase; the drag predictions are less sensitive to pitch phase, although still affected by it.

![Figure 11. Pure flapping, $U = 6 \text{ m/s}, f = 0.79 \text{ Hz}, \theta_0 = 4^\circ$](image)

### 6. Pure flapping

The pure flapping tests were run for different constant pitch angle values, $\theta_0$. They involved higher kinematic angles of attack than the pitch leading cases and therefore results from the Leishman-Beddoes approach of equations 12 and 13 will also be presented in this section. Figure 11 plots the experimental and simulated lift and drag coefficients for the test case $U = 6 \text{ m/s}$, $f = 0.79 \text{ Hz}$ and $\theta_0 = 4^\circ$. The three load estimation techniques predict nearly identical lift time histories but underestimate the maximum experimental lift by a significant amount. The three drag predictions are also quite similar, except for the fact that the Leishman-Beddoes time history features a constant negative offset of around -0.03. The Katz and Joukowski drag signals are quite close to the experimental measurement, lying inside the error bars except during the second half of the downstroke.

Figure 12 plots results from a different test case, whereby $U = 6 \text{ m/s}$ and $f = 1.23 \text{ Hz}$, $\theta_0 = -4^\circ$. Again, the three lift predictions are quite similar but underestimate the maximum experimental lift. Furthermore, they overestimate the drop in lift during the upstroke, at the start of the time history. The simulated drag results all miss the increase in drag during the upstroke. In fact, the Leishman-Beddoes estimate would be quite similar to the experimental drag time history if it were not offset by about -0.05. In contrast, the peak-to-trough amplitude of the Katz and Joukowski signals is significantly lower than that of the experimental measurement. If these two signals were offset upwards they would underestimate the thrust during the downstroke by quite a margin.

Throughout the flapping tests, it was observed that the Katz and Joukowski approaches systematically under-predict the drag amplitude when $\theta_0 \leq 0$. Figure 13 plots the experimental and simulated drag coefficients for the case $U = 9.4 \text{ m/s}$, $f = 1.5 \text{ Hz}$ and for four different values of $\theta_0$. In all these plots each of the drag time histories were centred around $C_D = 0$ at the upstroke by subtracting from each signal its maximum value. In this way, only differences in amplitude are assessed.

For $\theta_0 = -8^\circ$ and $-4^\circ$ the Katz and Joukowski drag predictions clearly have much lower amplitude than the experimental measurements. The Leishman-Beddoes drag estimates are significantly better for these cases. In contrast, for the $\theta_0 = 4^\circ$ and $8^\circ$ all three techniques give
Figure 12. Pure flapping, $U = 6 \text{ m/s}, f = 1.23 \text{ Hz}, \theta_0 = -4^\circ$

Figure 13. Drag coefficients for pure flapping, $U = 9.4 \text{ m/s}, f = 1.5 \text{ Hz}$
acceptable solutions. In fact, the Leishman-Beddoes approach overestimates the thrust by a small amount for these cases.

It is not clear why negative pitch angles cause this problem with the Katz and Joukowski drag estimate. The reason might be stall, as flow visualisation showed that extensive flow separation can take place even at midspan during pure flapping test cases [1]. Nevertheless, the Leishman-Beddoes technique does not identify any significant flow separation. In the example of figure 13(a), the kinematic angle of attack at the tip oscillates between $-20^\circ$ and $10^\circ$ but the effective angle of attack, $\alpha_E$, only varies between $-9^\circ$ and $1^\circ$, so that the furthest upstream instantaneous position of the separation point lies at $0.87c$. The effective angle of attack is much smaller than the kinematic one because of the effect of the downwash, which decreases significantly the normal force over the outboard part of the wing.

Figure 14 presents one final pure flapping example that is of particular interest. The test case is defined by $U = 6 \text{ m/s}, f = 1.23 \text{ Hz}, \theta_0 = 10^\circ$. The flow visualisations presented by Razak and Dimitriadis [1] show that, for this case, the flow is fully separated over the mid-span of the upper side of the wing at the middle of the downstroke. The experimental lift time history of figure 14(a) features a double peak, with a significant dip at the middle of the downstroke. This behaviour can clearly not be represented by the simulations carried out in this work. All three load estimation methods fail to predict the existence of the double peak. The Leishman-Beddoes technique predicts significant flow separation at the middle of the downstroke but it is trailing edge separation, extending upstream to a maximum of $0.37c$. This phenomenon results in a plateau rather than a double peak. The Katz and Joukowski drag predictions are also quite inaccurate, as they extensively overestimate the drag at the upstroke. The Leishman-Beddoes technique is more successful in predicting the drag for this case yet it still overestimates the thrust at the downstroke.

7. Pitch lagging

In pitch lagging motion, the pitch angle lags the flap and therefore massive flow separation can occur. The first test case to be considered is defined by $U = 9.4 \text{ m/s}, f = 1.23 \text{ Hz}$ and a pitch oscillation between $-5^\circ$ and $7^\circ$. The reduced frequency is $k = 0.07$ and the Strouhal number $St = 0.07$, while the kinematic angle of attack at the wingtip oscillates between $-19^\circ$ and $21^\circ$. Figure 15 plots the lift and drag coefficient time histories for this case. It can be seen that all three load estimation techniques predict the lift successfully, despite the very high instantaneous values of kinematic angle of attack at the wingtip. In contrast, the drag estimated by both the Katz and Joukowski methods is quite inaccurate. In particular, more thrust is predicted during the upstroke than the downstroke,
which is not the case in the experimental drag measurement. In contrast, the Leishman-Beddoes technique yields a drag time history that is quite similar to the experimental signal, except for a slight phase shift.

Figure 15. Inviscid VLM results, pitch lagging, \( U = 9.4 \text{ m/s}, f = 1.23 \text{ Hz}, \theta_{\text{min}} = -5^\circ, \theta_{\text{max}} = 7^\circ \)

Figure 16 plots the lift and drag coefficient estimates obtained for the case \( U = 6 \text{ m/s}, f = 1.5 \text{ Hz}, \theta_{\text{min}} = -12^\circ \) and \( \theta_{\text{max}} = 0^\circ \). The reduced frequency is \( k = 0.13 \) and the kinematic angle of attack at the tip oscillates between \(-39^\circ\) and \(24^\circ\). The Katz and Joukowski techniques match the experimental lift curve with good accuracy. The Leishman-Beddoes lift prediction is also quite good, except at the upstroke, where the amount of flow separation is exaggerated, leading to lower than expected values of the downforce. The Katz and Joukowski drag estimates are again quite inaccurate, as they underestimate the thrust during the downstroke and overestimate it during the upstroke. The Leishman-Beddoes approach yields a drag time history that is closer to the experimental curve but still underestimates the thrust.

Figure 16. VLM results with separated flow contributions, pitch lagging, \( U = 6 \text{ m/s}, f = 1.5 \text{ Hz}, \theta_{\text{min}} = -12^\circ, \theta_{\text{max}} = 0^\circ \)

The test case of figure 16 is one of the most challenging, as it combines the highest reduced frequency and a very large variation in kinematic angle of attack. Despite this fact, the simulation
results are still acceptable. However, increasing the pitch angle limits to $\theta_{\text{min}} = 4^\circ$, $\theta_{\text{max}} = 16^\circ$, while keeping all other test parameters constant, leads to unsatisfactory simulation predictions, particularly for the drag. This situation is demonstrated in Figure 17. The Katz and Joukowski lift estimates are quite good but the Leishman-Beddoes technique predicts massive separation during the downstroke and features a plateau in the lift time history. Furthermore, none of the three approaches produce good predictions for the drag.

![Graphs showing lift and drag coefficients](image)

**Figure 17.** VLM results with separated flow contributions, pitch lagging, $U = 6$ m/s, $f = 1.5$ Hz, $\theta_{\text{min}} = 4^\circ$, $\theta_{\text{max}} = 16^\circ$

### 8. Conclusions

Considering the complexity of the experiments and of the phenomena involved in flapping, the simulation results presented in this work can be considered mostly successful. However, it is clear that none of the load estimation methods investigated here are successful in all cases. The lift is predicted generally well, except in cases where significant stall is occurring. Even the Leishman-Beddoes model does not represent the full physics of some of the flows, as seen in figures 14 and 17.

Admittedly, the Leishman-Beddoes model used here is simplified, as it does not feature any of the time delay effects of the full model and does not calculate the effects of the Dynamic Stall Vortex. The lack of time lagging does not seem to reduced the accuracy of the method for the low reduced frequencies tested here. Furthermore, there are no convincing dynamic stall models for highly three-dimensional flows. While significant amount of research has been carried out on 2D dynamic stall, very few studies have addressed the 3D phenomenon.

The general conclusion is that the VLM is suitable for flapping flight simulations for cases that do not involve significant flow separation. The two most popular load estimation methods, i.e. the Katz and Joukowski techniques, produce very similar predictions during the downstroke, although some differences can occur over the upstroke. These differences are mostly visible in the drag time histories and concern cases where neither method is very successful (i.e. pure flapping with negative $\theta_0$ and pitch lagging). The Leishman-Beddoes approach can provide better drag predictions than the other two methods for specific cases but can also feature a constant negative offset in its time histories, that sometimes can be significant.

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Lambert also coded the Joukowski load estimation method and carried out part of the comparison to the experimental data. Grigorios Dimitriadis wrote the Leishman-Beddoes code, carried out additional comparisons and wrote the paper.

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**Abbreviations**

The following abbreviations are used in this manuscript:

- **VLM** Vortex Lattice Method
- **CFD** Computational Fluid Dynamics
- **DSV** Dynamic Stall Vortex

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