Soft supersymmetry breaking of 4d $\mathcal{N} = 2$ SCFT

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Abstract: A classification of soft SUSY breaking deformation of general four dimensional $\mathcal{N} = 2$ SCFT is provided. Given the large class of newly discovered $\mathcal{N} = 2$ SCFTs and their known properties such as the central charges and full information of BPS operators, it is possible to get a huge number of new $\mathcal{N} = 1$ SCFTs and non-supersymmetric CFTs. Many properties of these new $\mathcal{N} = 1$ SCFTs such as central charges, chiral spectrum and Seiberg duality can be derived from known information of parent $\mathcal{N} = 2$ SCFT.
1 Introduction

Given a conformal field theory, it is crucial to understand its behavior under various defor-
mations such as relevant deformations, exact marginal deformations, and deformations by
turning on expectation value of some operators. For a superconformal field theory (SCFT),
one can completely classify supersymmetry (SUSY) preserving relevant or exact marginal
deformation by using the representation theory of superconformal algebra [1]. The SUSY
preserving deformation derived from turning on expectation value of operators is more
complicated and one need to determine the algebra structure involving BPS operators 1.
The operator contents of a specific model and the infrared behavior after deformations,
on the other hand, are much more difficult to study and the solution often involves the
understanding of many highly nontrivial dynamical aspects of a theory.

It is now clear that the space of four dimensional \( \mathcal{N} = 2 \) SCFTs is extremely large
[3–7]. These SCFTs are mostly strongly coupled and do not admit a conventional La-
grangian description. One can, however, learn many highly nontrivial properties about
these theories by using powerful string theory and geometric methods. In particular, a lot
of knowledge about the BPS multiplets is known so we do know the existence of many in-
teresting operators. The operator contents of strongly coupled theory are often richer and
more generic than the SCFT defined using Lagrangian description. For example, the lower
bound of Coulomb brach operator is two for a \( \mathcal{N} = 2 \) SCFT which admits a Lagrangian
description; but for Argyres-Douglas type theory [8, 9], the lower bound can be actually
arbitrarily close to one (which is the unitarity bound). So the deformation theory of these
strongly coupled SCFTs is much richer. The behavior of \( \mathcal{N} = 2 \) preserving deformations is
well studied: the solution of the Coulomb branch is captured by finding a Seiberg-Witten
geometry; and the Higgs branch solution or more generally the Schur sector can be solved
by finding an associated 2d vertex operator algebra [10, 11].

1A typical example is chiral ring structure of four dimensional \( \mathcal{N} = 1 \) SCFT [2].
The $\mathcal{N} = 1$ preserving deformation of a $\mathcal{N} = 2$ SCFT is also studied, and the main focus is on using Coulomb branch operators with scaling dimension two [12–14] and the Higgs branch operators which also has scaling dimension two [13–16]. $\mathcal{N} = 1$ preserving deformation for simplest Argyres-Douglas theories are studied in [17–19], where Coulomb branch operators with fractional scaling dimension are used. The SUSY breaking deformation is rarely studied though.

The main purpose of this note is to initiate a systematical study of $\mathcal{N} = 1$ preserving deformation of general four dimensional $\mathcal{N} = 2$ SCFT. Instead of studying arbitrary SUSY breaking deformations of a SCFT, we focus on a special class of deformation called soft SUSY breaking deformation. Such deformation has been studied in the context of SUSY breaking model building, and has many interesting features [20]. In our context, the soft SUSY breaking deformations are defined as follows: one start with a SUSY preserving relevant or marginal deformation, and then promote the coupling constant to appropriate supermultiplet.

We start with a classification of soft SUSY breaking of four dimensional $\mathcal{N} = 1$ SCFT: a): F term deformation using chiral multiplet $B_{r,(j_1,0)}$ with $r \leq 2$; b): D term deformation using conserved current multiplet $\hat{C}_{r,(0,0)}$. For $\mathcal{N} = 2$ SCFT, one can have F term soft SUSY breaking deformation by using: a): chiral multiplet $\mathcal{E}_{r,(0,0)}$ with $r \leq 2$; b): $\hat{B}_1$ multiplet. If we regard $\mathcal{N} = 2$ SCFT as a $\mathcal{N} = 1$ SCFT, we have more choices of deformations: firstly the constraint on $r$ charge on $\mathcal{N} = 2$ chiral multiplet $\mathcal{E}_{r,(0,0)}$ is relaxed to $r < 3$; secondly, $\hat{B}_1$ multiplet contains a $\mathcal{N} = 1$ conserved current multiplet $\hat{C}_{r,(0,0)}$, and one can use it to do the deformation; thirdly, $\mathcal{N} = 2$ supercurrent multiplet contains a $\mathcal{N} = 1$ conserved current $\hat{C}_{r,(0,0)}$ and one can use it to get a soft SUSY breaking deformation. Some of most interesting deformations are summarized in table. 3.

We do not attempt to study the IR behavior of general soft deformations in this paper, here we only point one interesting feature of these deformations. Some soft SUSY breaking deformation preserves certain abelian global symmetries of $\mathcal{N} = 2$ SCFT, and the known anomaly of this preserved symmetry is quite useful in determining the IR phase: a): For $\mathcal{N} = 1$ preserving deformation, one can use it to determine the IR central charges and operator spectrum; b): The deformation using bottom component of $\mathcal{N} = 2$ supercurrent multiplet preserves $SU(2)_R \times U(1)_R$ symmetry, the IR theory has to be gapless to match the anomaly, which indicates that the IR theory could be a non-supersymmetric CFT.

This paper is organized as follows: section 2 classifies soft SUSY breaking deformation of $\mathcal{N} = 1$ SCFT; section 3 classifies soft SUSY breaking deformation of $\mathcal{N} = 2$ SCFT; section 4 gave a more detailed study of $\mathcal{N} = 1$ preserving deformation of $\mathcal{N} = 2$ SCFTs; finally, a conclusion is given in section 5.

2 Soft SUSY breaking of $\mathcal{N} = 1$ SCFT

Let’s first consider soft SUSY breaking of four dimensional $\mathcal{N} = 1$ SCFT. We begin with a short review of representation theory of $\mathcal{N} = 1$ superconformal algebra. The bosonic symmetry group of a $\mathcal{N} = 1$ SCFT is $SO(2,4) \times U(1)_R \times G_F$, here $SO(2,4)$ is conformal group of four dimensional Minkowski space time, $U(1)_R$ is the $R$ symmetry group which

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exists for every $\mathcal{N} = 1$ SCFT, and $G_F$ are other continuous global symmetry groups. A highest weight representation is labeled as $|\Delta, r, j_1, j_2\rangle$, where $\Delta$ is the scaling dimension, $r$ is $U(1)_R$ charge, $j_1$ and $j_2$ are left and right spin. These states might also carry quantum numbers of flavor symmetry group $G_F$. Representation theory of $\mathcal{N} = 1$ SCFT has been well studied and the short representation is classified in [21, 22]. Two important short representations are chiral multiplets and multiplets for conserved currents:

\[
\mathcal{B}_{r,(j_1,0)}, \quad \Delta = \frac{3}{2}r, \\
\hat{\mathcal{C}}_{(j_1,j_2)}, \quad r = j_1 - j_2, \quad \Delta = 2 + j_1 + j_2.
\]

$\hat{\mathcal{C}}_{(0,0)}$ contains conserved currents for the global symmetry group $G_F$; $\hat{\mathcal{C}}_{(\frac{1}{2},\frac{1}{2})}$ contains other supersymmetry currents; $\hat{\mathcal{C}}_{(\frac{1}{2},\frac{1}{2})}$ contains energy-moment tensor and $U(1)_R$ current, so $\hat{\mathcal{C}}_{(\frac{1}{2},\frac{1}{2})}$ exists for any $\mathcal{N} = 1$ SCFT. The anomaly of $U(1)_R$ symmetry is related to central charge $a$ and $c$ as follows [23, 24]:

\[
a = \frac{3}{32}(3\text{tr}R^3 - \text{tr}R), \quad c = \frac{1}{32}(9\text{tr}R^3 - 5\text{tr}R).
\]  

(2.1)

Let’s now consider deformation of $\mathcal{N} = 1$ SCFT. We have following $F$ term relevant or marginal deformation [25]:

\[
\int d^2\theta Z \mathcal{B}_{r,(0,0)} + c.c, \quad r \leq 2.
\]

(2.2)

We can assign $Z$ a scaling dimension $3 - \frac{3}{2}r$ so that above deformation is dimensionless. The soft condition is that $Z$ has non-negative scaling dimension, which puts the constraint $r \leq 2$. If $Z$ is a constant, we have a supersymmetric relevant deformation for $r < 2$, and exact marginal or marginal irrelevant depending on the global symmetry charge of operator [25]. If $Z$ is promoted to a chiral superfield and all of its components are nonzero, then we can have soft supersymmetry breaking deformation. In particular, if we would just turn on the top component (with highest scaling dimension in the supermultiplet) of $Z$, then we have most relevant supersymmetry breaking deformation from this chiral multiplet:

\[
\delta S = \lambda \int d^4x \mathcal{B} + c.c.
\]

(2.3)

Here $\mathcal{B}$ is the bottom component of the chiral multiplet $\mathcal{B}_{r,(0,0)}$. This deformation breaks $U(1)_R$ symmetry, but it might preserve a combination of $U(1)_R$ symmetry and other anomaly free global symmetries. The anomaly of the preserved symmetry can be used to constrain the IR behavior. Similarly, one can use other chiral multiplet $\mathcal{B}_{r,(j_1,0)}$ with $j_1 \neq 0$ to get soft SUSY breaking deformation, and the bound on the $r$ charge is just $r \leq 2$.

If a $\mathcal{N} = 1$ SCFT has other global symmetry group $G_F$, we could have following $D$ term deformation:

\[
\delta S = \int d^2\theta d^2\bar{\theta} \Lambda \hat{\mathcal{C}}_{(0,0)}.
\]

(2.4)

\textsuperscript{2}The quantum numbers such as the scaling dimensions are the ones for the bottom component of the multiplet.
We also assign scaling dimension zero to Λ to make above deformation dimensionless. This is the only relevant or marginal deformation which can be derived from using Ĉ type operator. Now if we promote Λ to be a real superfield and assume only the top component of Λ is nonzero, we have the following relevant deformation
\[ \delta S = m^2 \int d^4x \mathcal{C}. \]  

Here C is the bottom component of Ĉ(0,0) and does not break U(1)_R symmetry. So if U(1)_R symmetry is not spontaneously broken, the IR theory should match its anomaly, and we can potentially get an interacting non-supersymmetric CFT in the IR by using above type of deformation.

3 Soft SUSY breaking of \( N = 2 \) SCFT

Let’s now discuss soft SUSY breaking of four dimensional \( N = 2 \) SCFT, and we first review some representation theory results of \( N = 2 \) superconformal algebra. The bosonic symmetry group of a general \( N = 2 \) SCFT is \( SO(2,4) \times SU(2)_R \times U(1)_R \times G_F \), here \( SO(2,4) \) is the conformal group, \( SU(2)_R \times U(1)_R \) is the R symmetry group which exists for every \( N = 2 \) SCFT, and \( G_F \) are other global symmetry groups which could be absent for some theories. A highest weight representation is labeled as \( |\Delta, R, r, j_1, j_2\rangle \), here \( \Delta \) is the scaling dimension, \( r \) is \( U(1)_R \) charge, \( R \) is \( SU(2)_R \) charge, \( j_1 \) and \( j_2 \) are left and right spin. These states could also carry quantum numbers of flavor symmetry group \( G_F \). Representation theory of \( N = 2 \) SCFT has been studied in \([26]\). and the short representation is completely classified in \([26]\). Three short representations that we are interested in are Coulomb branch operators, Higgs branch operators, and supercurrent multiplet:

- **Coulomb branch operators**: \( \mathcal{E}_{r,(0,0)}, \ R = 0, \ \Delta = r \),
- **Higgs branch operators**: \( \hat{B}_R, \ \ r = j_1 = j_2 = 0, \ \Delta = 2R \),
- **Supercurrent**: \( \hat{C}_{0,(0,0)}, \ \ r = R = 0, \ \Delta = 2 \).

\( \hat{B}_1 \) is a multiplet which contains conserved current for the flavor symmetry group \( G_F \), and transforms in adjoint representation of \( G_F \). The \( N = 1 \) subalgebra is generated by the supercharge \( Q_1 \), and the corresponding \( R \) symmetry is \( R_{N=1} = \frac{2}{3} R_{N=2} + \frac{4}{3} I_3 \). The other global symmetry group in \( N = 1 \) description is \( J = 2R_{N=2} - 2I_3 \) which commutes with the supercharge \( Q_1 \).

Supercurrent multiplet exists for every \( N = 2 \) SCFT. There are also a large class of \( N = 2 \) SCFTs whose full Coulomb branch spectrum and Higgs branch spectrum are known \([3–7]\). We also know the central charge \( c_{N=2} \) and \( c_{N=2} \) which are related to the anomalies\footnote{Here \( R_{N=2} \) is the generator for \( N = 2 U(1)_R \) symmetry, and \( I_3 \) is the Cartan subalgebra of Lie algebra associated with \( SU(2)_R \) symmetry.}

\footnote{Our normalization is that \( (Q_1, Q_2) \) are \( SU(2) \) doublet with \( I_3(Q_1) = -\frac{1}{2}, I_3(Q_2) = \frac{1}{2} \), and \( U(1)_R \) charges are \( R(Q_1) = R(Q_2) = -\frac{1}{2} \).}
Table 1: Components of $E_{r,(0,0)}$ multiplet. Here $A$, $B_{ij}$ and $C$ are scalars, $\Phi_i$ and $\Lambda_i$ are spinors. $F^{\alpha\beta} = \sigma^{\alpha\beta} F^{ab}$ with $F^{ab}$ an antisymmetric anti-selfdual tensor. $B_{ij}$ is symmetric in $i, j$ index and transform in adjoint representation of $SU(2)_R$ group, and $\Phi_i$ and $\Lambda_i$ transforms in fundamental representation of $SU(2)_R$ group.

|       | $A$  | $\Phi_i$ | $B_{ij}$ | $F^{\alpha\beta}$ | $\Lambda_i$ | $C$ |
|-------|------|----------|----------|-------------------|-------------|-----|
| $U(1)_R$ | $r$  | $r - \frac{1}{2}$ | $r - 1$  | $r - \frac{1}{2}$ | $r - 2$    |     |
| $SU(2)_R$ | $0$  | $\frac{1}{2}$    | $1$      | $0$               | $\frac{1}{2}$ | $0$ |
| $\Delta$ | $r$  | $r + \frac{1}{2}$ | $r + 1$  | $r + \frac{1}{2}$ | $r + 2$    |     |

Table 2: Components of a $\tilde{B}_1$ multiplet, here $L^{(ij)}$ satisfies a reality condition. $L^{(i)}_\alpha$ are a doublet of $SU(2)_R$ symmetry and is a spinor. $L_0$ is a complex scalar, and $L_\mu$ is a vector.

|       | $L^{(ij)}$ | $L^{(i)}_\alpha$ | $L_0$ | $L_\mu$ |
|-------|------------|------------------|-------|---------|
| $U(1)_R$ | $0$        | $-\frac{3}{2}$   | $-1$  | $0$     |
| $SU(2)_R$ | $1$        | $\frac{1}{2}$    | $0$   | $0$     |
| $\Delta$ | $2$        | $\frac{5}{2}$   | $3$   | $3$     |

of $R$ symmetries as follows [27]:

\begin{align}
\text{Tr}(R_N^{(i)}) &= 6(a_{N=2} - c_{N=2}), \quad \text{Tr}(R_N^{(c)}) = 24(a_{N=2} - c_{N=2}), \\
\text{Tr}(R_N^{2}) &= (2a_{N=2} - c_{N=2}).
\end{align}

So if a deformation preserves a subgroup $U(1)_{IR} = xR_{N=2} + yI_3$ of $N = 2 U(1)_R \times SU(2)_R$ symmetry group, and the anomaly of $U(1)_{IR}$ can be computed as follows:

\begin{align}
\text{Tr}(U(1)_{IR}) &= 6x^3(a_{N=2} - c_{N=2}) + 3xy^2(2a_{N=2} - c_{N=2}), \\
\text{Tr}(U(1)_{IR}) &= 24x(a_{N=2} - c_{N=2}).
\end{align}

If a deformation preserves $N = 1$ supersymmetry and a candidate $U(1)_R$ symmetry which is a linear combination of $N = 2$ $R$ symmetry, one can use formula 2.1 and 3.2 to compute the IR central charge:

\begin{align}
a_{N=1} &= (a_{N=2} - c_{N=2})\left[\frac{27}{16}x^3 - \frac{9}{4}x\right] + (2a_{N=2} - c_{N=2})\left[\frac{27}{32}xy^2\right], \\
c_{N=1} &= (a_{N=2} - c_{N=2})\left[\frac{27}{16}x^3 - \frac{15}{4}x\right] + (2a_{N=2} - c_{N=2})\left[\frac{27}{32}xy^2\right].
\end{align}

The $N = 2$ preserving relevant or marginal deformations have been completely classified in [1, 28], and we have:

1. Deformation using Coulomb branch operator:

\[\delta S = \int d^2 \theta_1 d^2 \theta_2 Z E_{r,(0,0)} + c.c, \quad r \leq 2\]
\[ \delta S = \lambda \int d^4 x C + c.c. \]

\[ \delta S = \lambda \int d^4 x B_{11} + c.c. \]

\[ \delta S = \lambda \int d^4 x B_{22} + c.c. \]

\[ \delta S = \lambda \int d^4 x B_{12} + c.c. \]

\[ \delta S = \lambda \int d^4 x A + c.c. \]

\[ \delta S = m^2 \int d^4 x L_0 + c.c. \]

\[ \delta S = m^2 \int d^4 x L^{(22)} + c.c. \]

\[ \delta S = m^2 \int d^4 x L^{(12)} \]

\[ \delta S = m^2 \int d^4 x J \]

**Table 3:** $\mathcal{N} = 2$ soft supersymmetry breaking deformations from $\mathcal{E}_{r,(0,0)}$, $\hat{\mathcal{B}}_1$ and $\hat{C}_{0,(0,0)}$ multiplets. We list the number of preserved SUSY, preserved global symmetry and the scaling dimension of the operator used in deformation.

| Deformation | SUSY | Global symmetry | Scaling dimension |
|-------------|------|-----------------|------------------|
| $\mathcal{E}_{r,(0,0)}$ | $\mathcal{N} = 2$ | $SU(2)_R$ | $r + 2$ |
| $\delta S = \lambda \int d^4 x C + c.c.$ | $\mathcal{N} = 1$ | $\frac{2}{3} U(1)_R + (2 - \frac{2}{r}) I_3$ | $r + 1$ |
| $\delta S = m^2 \int d^4 x L_0 + c.c.$ | $\mathcal{N} = 2$ | $SU(2)_R$ | 3 |
| $\delta S = \lambda \int d^4 x A + c.c.$ | $\mathcal{N} = 0$ | $U(1)_R$ | 2 |
| $\delta S = \lambda \int d^4 x B_{11} + c.c.$ | $\mathcal{N} = 1$ | $(2 - \frac{2}{r}) I_3$ | $r + 1$ |
| $\delta S = \lambda \int d^4 x B_{12} + c.c.$ | $\mathcal{N} = 0$ | $SU(2)_R$ | $r + 1$ |
| $\delta S = \lambda \int d^4 x A + c.c.$ | $\mathcal{N} = 0$ | $SU(2)_R$ | $r$ |
| $\delta S = m^2 \int d^4 x L^{(22)} + c.c.$ | $\mathcal{N} = 0$ | $U(1)_R$ | 2 |
| $\delta S = m^2 \int d^4 x L^{(12)}$ | $\mathcal{N} = 0$ | $SU(2)_R \times U(1)_R$ | 2 |

**Table 4:** The decomposition of a $\mathcal{N} = 2$ Coulomb branch multiplet $\mathcal{E}_{r,(0,0)}$ into three $\mathcal{N} = 1$ chiral multiplets: $\mathcal{O}$ and $S$ are of type $\mathcal{B}_{r,(0,0)}$, and $\lambda_\alpha$ is of type $\mathcal{B}_{r,(-\frac{1}{2},0)}$.

\[ \delta S = \int d^2 \theta_1 \Lambda X + c.c. \]  \hspace{1cm} (3.3)

If we regard a $\mathcal{N} = 2$ SCFT as a $\mathcal{N} = 1$ SCFT (with supercharge $Q_1$), we have more choices for soft SUSY breaking. If we promote $\Lambda$ to be a superfield, we can have non-susy deformation.

$\mathcal{E}_{r,(0,0)}$ type multiplet contains component fields $(A, \Phi_i, B_{ij}, F^{\alpha \beta}, \Lambda_i, C)$. Their quantum numbers under $R$ symmetry are listed in table. 1. Among five scalars, only $I_3(B_{11}) = -1$ and $I_3(B_{22}) = 1$ carry non-trivial $SU(2)_R$ charge. $Z$ has scaling dimension $2 - r$. If we promote $Z$ to be a $\mathcal{N} = 2$ chiral multiplet, then we can have supersymmetry breaking deformation, see table. 3 for deformations involving scalar operators in multiplet $\mathcal{E}_{r,(0,0)}$.

2. Each $\mathcal{B}_1$ operator contains fields $(L^{(ij)}, L^{(i)}, L_0, L_\mu)$, see table. 2 for their quantum numbers. There is a reality condition on fields $L^{(ij)}$. This multiplet decomposes into a $\mathcal{N} = 1$ chiral multiplet $X$ and a conserved current multiplet $L$, see table. 2. One can have a $\mathcal{N} = 2$ preserving deformation:

$\delta S = \int d^2 \theta_1 \Lambda X + c.c.$

If we promote $\Lambda$ to be a superfield, we can have non-susy deformation.

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$\mathcal{E}_{r,(0,0)}$ type multiplet contains component fields $(A, \Phi_i, B_{ij}, F^{\alpha \beta}, \Lambda_i, C)$. Their quantum numbers under $R$ symmetry are listed in table. 1. Among five scalars, only $I_3(B_{11}) = -1$ and $I_3(B_{22}) = 1$ carry non-trivial $SU(2)_R$ charge. $Z$ has scaling dimension $2 - r$. If we promote $Z$ to be a $\mathcal{N} = 2$ chiral multiplet, then we can have supersymmetry breaking deformation, see table. 3 for deformations involving scalar operators in multiplet $\mathcal{E}_{r,(0,0)}$. 5 $\mathcal{N} = 2$ chiral multiplet has the same multiplet structure as the Coulomb branch operators $\mathcal{E}_{r,(0,0)}$.
Table 5: The decomposition of a $\mathcal{N} = 2$ $\mathcal{B}_1$ multiplet into a $\mathcal{N} = 1$ chiral multiplet $X$ and a $\mathcal{N} = 1$ conserved current multiplet $L$.

|   | $L$ | $X$ |
|---|-----|-----|
| $U(1)_R$ | 0 | 0 |
| $SU(2)_R$ | 0 | $I_3(X) = 1$ |

contains three $\mathcal{N} = 1$ chiral multiplets, see table 4. The bottom component of $S$ is actually also the top component of another $\mathcal{N} = 1$ chiral if we choose a different supercharge $Q_2$. The top component of $S$ is also the top component of the whole $\mathcal{N} = 2$ chiral multiplet, so there is no new scalar SUSY breaking deformation from multiplet $S$. Thus, the new interesting SUSY breaking deformation comes from bottom component of $\mathcal{N} = 1$ chiral multiplet $O$ whose $r$ charge constraint is relaxed to be less than 3. $\mathcal{B}_1$ multiplet contains a $\mathcal{N} = 1$ chiral multiplet $X$ and a $\mathcal{N} = 1$ conserved current multiplet $L$, and one can turn on SUSY breaking deformation using the current multiplet $L$.

Moreover, the $\mathcal{N} = 2$ current multiplet contains a $\mathcal{N} = 1$ conserved current $\hat{J}$, a supersymmetry current $J_{\alpha}$, and a supercurrent multiplet $J_{\alpha \dot{\alpha}}$. One can use the conserved current $\hat{J}$ multiplet to deform our theory:

$$\delta S = m^2 \int J,$$

here $J$ is the bottom component of $\hat{C}_{(0,0)}$ multiplet and transform trivially under $U(1)_R$ and $SU(2)_R$ symmetry, and $m^2$ has scaling dimension 2.

In summary, for a $\mathcal{N} = 2$ SCFT, we can have soft susy breaking deformation by using Coulomb branch multiplet $E_{r,(0,0)}$ with $r \leq 2$, and a $\mathcal{B}_1$ multiplet. If we regard our theory as a $\mathcal{N} = 1$ SCFT, we can have soft susy breaking by using Coulomb branch multiplet $E_{r,(0,0)}$ with $r \leq 3$, $\mathcal{B}_1$ multiplet, and $\hat{C}_{0,(0,0)}$ multiplet, see table 3.

Example: Let’s consider the deformations of the simplest Argyres-Douglas SCFT which is often called $(A_1, A_2)$ theory. This theory has following features:

- The Coulomb branch spectrum is freely generated by an operator $u = E_{r,(0,0)}$ with $r = \frac{6}{5}$. It does not have a Higgs branch, so there is no $B_1$ type multiplet. There are only two Coulomb branch operators with $r \leq 3$: $u$ and $u^2$.
- Its central charge is $a_{\mathcal{N}=2} = \frac{43}{120}$, and $c_{\mathcal{N}=2} = \frac{11}{30}$.

The $\mathcal{N} = 2$ soft SUSY breaking deformations are summarized in table 6. The $\mathcal{N} = 1$ soft SUSY breaking deformations are summarized in table 7. The $\mathcal{N} = 1$ preserving deformations were studied in [17–19]. We found three non-supersymmetric deformations with scaling dimension $\frac{6}{5}, \frac{11}{5}, \frac{12}{5}$ from using Coulomb branch operators, see table 6 and 7. These deformations break the $U(1)_R$ symmetry and preserve $SU(2)_R$ symmetry, so we can not use anomaly matching to constrain its IR behavior. The deformation using bottom component of $\hat{C}_{0,(0,0)}$ preserves $U(1)_R \times SU(2)_R$ symmetry so

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\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
Deformation & SUSY & Global symmetry & scaling dimension & $a$ & $c$ \\
\hline
$\delta S = \lambda \int d^4 x C^u + c.c.$ & $\mathcal{N} = 2$ & $SU(2)_R$ & $\frac{17}{5}$ & $\frac{17}{5}$ & $\frac{17}{5}$ \\
$\delta S = \lambda \int d^4 x B^u_{11} + c.c.$ & $\mathcal{N} = 1$ & $\frac{3}{2} U(1)_R + \frac{1}{3} I_3$ & $\frac{11}{5}$ & $\frac{11}{5}$ & $\frac{11}{23}$ \\
$\delta S = \lambda \int d^4 x B^u_{22} + c.c.$ & $\mathcal{N} = 1$ & $\frac{3}{2} U(1)_R - \frac{1}{3} I_3$ & $\frac{11}{5}$ & $\frac{11}{5}$ & $\frac{11}{23}$ \\
$\delta S = \lambda \int d^4 x B^u_{12} + c.c.$ & $\mathcal{N} = 0$ & $SU(2)_R$ & $\frac{9}{5}$ & N/A & N/A \\
$\delta S = \lambda \int d^4 x A^u + c.c.$ & $\mathcal{N} = 0$ & $SU(2)_R$ & $\frac{9}{5}$ & N/A & N/A \\
\hline
\end{tabular}
\caption{Deformations using Coulomb branch operator $u$ of $(A_1, A_2)$ AD theory, and this comes from $\mathcal{N} = 2$ soft supersymmetry breaking deformation. For generic coupling constant of $\mathcal{N} = 2$ preserving deformation, the IR theory is just a free $U(1)$ vector multiplet whose central charge is listed. For $\mathcal{N} = 1$ preserving deformation, the IR theory is conjectured to be a free $\mathcal{N} = 1$ chiral multiplet \cite{17}.}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
Deformation & SUSY & Global symmetry & scaling dimension & $a$ & $c$ \\
\hline
$\delta S = \lambda \int d^4 x B_{11}^{u^2} + c.c.$ & $\mathcal{N} = 1$ & $\frac{3}{2} U(1)_R + \frac{7}{3} I_3$ & $\frac{203}{108}$ & $\frac{203}{108}$ & $\frac{203}{108}$ \\
$\delta S = \lambda \int d^4 x A^{u^2} + c.c.$ & $\mathcal{N} = 0$ & $SU(2)_R$ & $\frac{17}{5}$ & N/A & N/A \\
$\delta S = m^2 \int d^4 x J$ & $\mathcal{N} = 0$ & $SU(2)_R \times U(1)_R$ & 2 & N/A & N/A \\
\hline
\end{tabular}
\caption{The first two rows summarizes the properties of deformations using $\mathcal{N} = 1$ chiral multiplet of Coulomb branch operator $u^2$. The third row summarizes the deformation using bottom component of $\mathcal{N} = 2$ supercurrent multiplet.}
\end{table}

Table 6: The first two rows summarizes the properties of deformations using $\mathcal{N} = 1$ chiral multiplet of Coulomb branch operator $u^2$. The third row summarizes the deformation using bottom component of $\mathcal{N} = 2$ supercurrent multiplet.

The IR theory is constrained by the anomaly matching, and it should be a gapless theory in the IR. The IR theory is most likely an interacting CFT. The corresponding operator used in the deformation has scaling dimension 2 which is smaller that the $\mathcal{N} = 1$ deformation ($[\mathcal{O}] = \frac{11}{5}$) whose IR fixed point consists of $\mathcal{N} = 1$ chiral scalar. So if the IR theory after deformation using operator $J$ is indeed an interacting CFT, its central charge should be smaller than that of a complex scalar and a Weyl fermion, which seems to be much smaller than the known four dimensional non-supersymmetric CFT.
4 $\mathcal{N} = 1$ preserving deformation of $\mathcal{N} = 2$ SCFT

Let’s now consider more details of $\mathcal{N} = 1$ preserving deformation of $\mathcal{N} = 2$ SCFT. We can consider deformations caused by Coulomb branch operators or Higgs branch operators. The $\mathcal{N} = 1$ deformations using $X$ component of $\hat{B}_1$ operator preserves $SU(2)_R$ symmetry, and the IR theory actually has $\mathcal{N} = 2$ SUSY whose behavior can be solved from Seiberg-Witten geometry. So we will mainly focus on deformation using Coulomb branch operators. A $\mathcal{N} = 2$ Coulomb branch operator consists of three $\mathcal{N} = 1$ chiral multiplets $(\mathcal{O}, \lambda_{\alpha}, S)$, and only operator $\mathcal{O}$ will give new $\mathcal{N} = 1$ preserving deformations.

The Coulomb branch chiral ring is freely generated, and the generators can often be found from the Seiberg-Witten geometry. One can turn on $\mathcal{N} = 1$ preserving relevant deformations using operators from Coulomb branch chiral ring, see row two of table. 3. Assuming we use a $\mathcal{N} = 2$ Coulomb branch operator $\mathcal{E}_{r,(0,0)}$ with $R_{\mathcal{N}=2}$ charge $r$ to deform our theory:

$$\delta S = \lambda \int d^2\theta \mathcal{O} + c.c$$  \hspace{1cm} (4.1)

Here $\mathcal{O}$ is the bottom $\mathcal{N} = 1$ chiral multiplet of $\mathcal{E}_{r,(0,0)}$. Then the candidate $U(1)_R$ symmetry for the IR theory is

$$\frac{2}{r} U(1)_R + (2 - \frac{2}{r}) I_3. \hspace{1cm} (4.2)$$

The condition of relevant deformation on $r$ is simply $r < 3$ \footnote{$r = 3$ is marginal irrelevant, and the IR $U(1)_R$ symmetry is just the $U(1)_R$ symmetry of the UV $\mathcal{N} = 2$ SCFT. Since the deformation also carries the charge for the other global symmetry $J$, one can use the argument of [25] to conclude that this deformation is marginal irrelevant.}. A necessary condition for the above symmetry to be the true IR $U(1)_R$ symmetry is that all the chiral operators obey unitarity bound, which implies that all three $\mathcal{N} = 1$ chiral multiplets contained in a $\mathcal{N} = 2$ chiral multiplet with minimal $R_{\mathcal{N}=2}$ charge $r_{\text{min}}$ should obey unitarity bound, and we have (see the quantum number of three $\mathcal{N} = 1$ chiral multiplets in table. 4.)

$$[\mathcal{O}_{\text{min}}] > 1 \rightarrow \frac{2r_{\text{min}}}{r} \times \frac{3}{2} > 1,$$
$$[(\lambda_{\alpha})_{\text{min}}] > 1 \rightarrow \left[ \frac{2(r_{\text{min}} - \frac{1}{2})}{r} + (2 - \frac{2}{r}) \frac{1}{2} \right] \times \frac{3}{2} > 1,$$
$$[S_{\text{min}}] > 1 \rightarrow \left[ \frac{2(r_{\text{min}} - 1)}{r} + (2 - \frac{2}{r})1 \right] \times \frac{3}{2} > 1,$$

and we find the following constraint:

$$r > 3 - \frac{3}{2} r_{\text{min}}. \hspace{1cm} (4.3)$$

If our theory contains Higgs branch operators $\mathcal{B}_1$, then the scaling dimension of its $\mathcal{N} = 1$ chiral $X$ would be

$$[X] > 1 \rightarrow (2 - \frac{2}{r}) \times \frac{3}{2} > 1,$$  \hspace{1cm} (4.4)

and we get the bound

$$r > \frac{3}{2}. \hspace{1cm} (4.5)$$
This bound is larger than that from using Coulomb branch operator, so we use this bound if our theory has a $\hat{B}_1$ type operator. In general, if our theory has abelian flavor symmetries, the true IR $U(1)_R$ symmetry might also has a mixing with them. The Coulomb branch operators, however, is not charged under those global symmetries, and therefore would not detect the mixings.

For the Lagrangian theory or class $S$ theory engineered using only regular punctures, the Coulomb branch operators have integral scaling dimension, and the only Coulomb branch operator that we can use to deform our theory is the one with scaling dimension two. Dimension two Coulomb branch operators give us $\mathcal{N} = 2$ exact marginal deformations, and one can often write down a weakly coupled gauge theory descriptions where the dimensional two operators are constructed from $\mathcal{N} = 2$ vector multiplet. The above $\mathcal{N} = 1$ preserving deformation is just the mass deformation for $\mathcal{N} = 1$ chiral multiplet inside a $\mathcal{N} = 2$ vector multiplet. The situation becomes a lot more interesting for more general Argyres-Douglas theories where the Coulomb branch spectrum contains operators whose scaling dimension can be arbitrary distributed above the unitarity bound which is one for four dimensional scalar.

Some properties of the IR theory can be found as follows (Assuming that the candidate IR $U(1)_R$ symmetry 4.2 is the true $U(1)_R$ symmetry of the IR SCFT, and we consider only the Coulomb type deformations):

1. **Central charges**: One can compute the IR central charge using formula 3.2, and we have $x = \frac{2}{r}$, $y = 2 - \frac{2}{r}$.

2. **Index**: The explicit form of $\mathcal{N} = 2$ Schur index of many interesting theories is known [11], and this index is actually invariant under RG flow, and can be used to get some useful information of IR theory [19].

3. **Chiral ring**: The $\mathcal{N} = 1$ chiral operator $\mathcal{O}$ which is used to deform our theory satisfying a chiral ring relation $\mathcal{O} = 0$ in the IR theory [18, 19].

4. **Chiral spectrum**: One can get some information of chiral operators from underlying $\mathcal{N} = 2$ theory. For a $\mathcal{N} = 2$ chiral with $R_{\mathcal{N}=2}$ charge $a$, the scaling dimension of its three $\mathcal{N} = 1$ chiral multiplets are

$$[O] = \frac{3a}{r}, \quad [\lambda_\alpha] = \frac{\frac{3}{2}r - 3 + 3a}{r}, \quad [S] = \frac{3(r + a - 2)}{r}. \quad (4.6)$$

If $2 < r < 3$, the minimal scaling dimension of a chiral scalar operator is $\Delta_{\text{min}} = \frac{3a}{r}$; if $1 < r \leq 2$, we have $\Delta_{\text{min}} = \frac{3(r + r_{\text{min}} - 2)}{r}$. In particular, some of the chiral operators are relevant, and one can use them to deform IR $\mathcal{N} = 1$ fixed point and flow to possibly new $\mathcal{N} = 1$ SCFT, although these deformations would break $R$ symmetry, and we have little to say about IR theory. The flavor symmetry of UV $\mathcal{N} = 2$ theory is not broken by the Coulomb branch type $\mathcal{N} = 1$ preserving deformations, so we do know the existence of $\hat{\mathcal{C}}_{(0,0)}$ type operators of IR $\mathcal{N} = 1$ SCFT from the flavor symmetry of UV theory.
Figure 1: The parent $\mathcal{N} = 2$ SCFT has an exact marginal deformation and so there are two different duality frames $T_1$ and $T_2$ where one can write down weakly coupled gauge theory descriptions. We turn on $\mathcal{N} = 1$ preserving deformations which might have different descriptions in $T_1$ and $T_2$ frames, and they actually flow to the same IR theory. The IR theory also has an exact marginal deformation, which is inherited from parent $\mathcal{N} = 2$ SCFT.

5. **Exact marginal deformations**: 1): If the UV $\mathcal{N} = 2$ theory has a multiple number of Coulomb branch operators with $R_{\mathcal{N}=2}$ charge $r$, then the IR theory would have exact marginal deformations; 2): if UV $\mathcal{N} = 2$ theory has a $\mathcal{N} = 2$ exact marginal operator, then the $S$ component of it would be exact marginal in the IR $\mathcal{N} = 1$ SCFT. 3): If $r = 2$, then the $\tilde{B}_2$ type operator might also give exact marginal deformations.

6. **Inherited $\mathcal{N} = 1$ duality**: If 4d $\mathcal{N} = 2$ has an exact marginal deformation and has different duality frames, then the IR theory would also have different duality frames. There are many $\mathcal{N} = 2$ theories whose duality frames are known [29–31], and using the $\mathcal{N} = 1$ preserving deformations, we get a large class of new type of Seiberg duality for $\mathcal{N} = 1$ SCFTs.

**Example**: Let’s consider $(A_1, G)$ theory with $G = ADE$, and these theories can be engineered by the following three-fold singularity [32]:

$$f(x, y, z, w) = f_{ADE}(x, y, z) + w^2. \quad (4.7)$$

Here $f_{ADE}$ are standard two dimensional $ADE$ singularity with following form

$$A_N : f = x^2 + y^2 + z^{N+1}, \quad D_N : f = x^2 + y^{N-1} + y^2,$$

$$E_6 : f = x^2 + x^3 + y^4, \quad E_7 : f = x^2 + x^3 + xy^3, \quad E_8 : f = x^2 + x^3 + y^5. \quad (4.8)$$

The Coulomb branch spectrum of these theories can be computed using the Jacobi algebra of the singularity [7]. Let’s review it here: the Jacobi algebra of an isolated singularity $f$ is defined as the following quotient space

$$J_f = \frac{C[x, y, z, w]}{(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}, \frac{\partial f}{\partial w})}. \quad (4.9)$$

Take a monomial basis $\phi_\alpha$ of $J_f$, then the Seiberg-Witten geometry of $f$ is

$$F(x, y, z, w) = f(x, y, z, w) + \sum \lambda_\alpha \phi_\alpha. \quad (4.10)$$
Let's denote our original \( \mathcal{N} = 1 \) SCFT with label \( D \). The above flows might be considered in two steps, and there

\[ \delta S = \lambda_1 \int d^2 \theta \mathcal{O}_1 + \lambda_2 \int d^2 \theta \mathcal{O}_2 + c.c. \]  

(4.12)

Let's denote our original \( \mathcal{N} = 2 \) SCFT as theory \( A \), and assume that the IR theory is a \( \mathcal{N} = 1 \) SCFT with label \( D \). The above flows might be considered in two steps, and there

\[ a = \frac{N(24N+19)}{24(2N+3)} \]

\[ c = \frac{N(6N+5)}{6(2N+3)} \]

\[ r_{\text{min}} = \frac{2N+4}{2N+3} \]

\[ r_{\text{max}} = \frac{6N+4}{2N+3} \]

\[ r > \frac{3N+3}{2N+3} \]

Remark 1: Other deformation and accidental symmetry: We have restricted our consideration to \( \mathcal{N} = 1 \) preserving deformation where no chiral operators violate unitarity bound under the candidate IR \( U(1)_R \) symmetry. We could also consider other deformations, and a common procedure is to assume these operators violating unitary bound to become free [33]. It would be interesting to further study these flows too.

Remark 2: New \( \mathcal{N} = 1 \) duality: We have discussed \( \mathcal{N} = 1 \) duality which is inherited from \( \mathcal{N} = 2 \) duality. Here we show that it might be possible to find new \( \mathcal{N} = 1 \) duality through following process: Let's start with a \( \mathcal{N} = 2 \) SCFT and consider two \( \mathcal{N} = 1 \) scalar chiral multiplets \( \mathcal{O}_1 \) and \( \mathcal{O}_2 \) (These are the bottom \( \mathcal{N} = 1 \) chirals inside \( \mathcal{N} = 2 \) Coulomb branch operators). The \( R_{\mathcal{N}=2} \) charges of \( \mathcal{O}_1 \) and \( \mathcal{O}_2 \) are chosen to be different. Let's now consider following flow:

\[ [\lambda_\alpha] = \frac{1 - Q_\alpha}{\sum q_i - 1}. \]  

(4.11)

here \( Q_\alpha \) is the weight of \( \phi_\alpha \) under the \( C^* \) action. The Coulomb branch chiral ring is freely generated by \( \lambda_\alpha \) with \( [\lambda_\alpha] > 1 \). The central charges \( a_{\mathcal{N}=2} \) and \( c_{\mathcal{N}=2} \) are computed using the method in [7]. We also list the value \( r > \) which is the lower bound given by formula 4.3 or 4.5 (UV theory has \( \mathcal{B}_4 \) type operators), and the maximal possible value \( r_{\text{max}} \)\(^7\) that one can use to do a \( \mathcal{N} = 1 \) preserving deformation. Using these operators, one can get a large class of interesting RG flow between a \( \mathcal{N} = 2 \) SCFT and a \( \mathcal{N} = 1 \) SCFT.

Table 8: The central charges for \( (A_1, G) \) type Argyres-Douglas theories. We listed the minimal scaling dimension \( r_{\text{min}} \) of Coulomb branch operators, lower bound \( r > \) and upper bound \( r_{\text{max}} \) so that the corresponding \( \mathcal{N} = 1 \) preserving deformations satisfies the unitarity constraint.

| \( \mathcal{T} \)          | \( a \)          | \( c \)          | \( r_{\text{min}} \) | \( r_{\text{max}} \) | \( r > \) |
|--------------------------|------------------|------------------|-----------------------|-----------------------|-----------|
| \( (A_1, A_{2N}) \)      | \( \frac{N(24N+19)}{24(2N+3)} \) | \( \frac{N(6N+5)}{6(2N+3)} \) | \( \frac{2N+4}{2N+3} \) | \( \frac{6N+4}{2N+3} \) | \( \frac{3N+3}{2N+3} \) |
| \( (A_1, A_{2N-1}) \)    | \( \frac{12N^2-N^2-5}{24(N+1)} \) | \( \frac{3N^2-N-1}{6(N+1)} \) | \( \frac{2N+4}{2N+2} \) | \( \frac{6N+4}{2N+2} \) | \( \frac{3}{2} \) |
| \( (A_1, D_{2N+1}) \)    | \( \frac{N(8N+3)}{24(N+1)} \) | \( \frac{N}{2} \) | \( \frac{2N+2}{2N+1} \) | \( \frac{6N+2}{2N+1} \) | \( \frac{2}{2} \) |
| \( (A_1, D_{2N}) \)      | \( \frac{N}{2} - \frac{5}{12} \) | \( \frac{N}{2} - \frac{1}{4} \) | \( \frac{N+1}{N} \) | \( \frac{3N-1}{2N} \) | \( \frac{3}{2} \) |
| \( (A_1, E_6) \)         | \( \frac{9}{26} \) | \( \frac{10}{17} \) | \( \frac{7}{7} \) | \( \frac{7}{7} \) | \( \frac{1}{1} \) |
| \( (A_1, E_7) \)         | \( \frac{15}{24} \) | \( \frac{23}{24} \) | \( \frac{15}{14} \) | \( \frac{33}{32} \) | \( \frac{3}{3} \) |
| \( (A_1, E_8) \)         | \( \frac{1}{1} \) | \( \frac{1}{1} \) | \( \frac{3}{2} \) | \( \frac{21}{21} \) | \( \frac{1}{1} \) |

\(^7\)The lower and maximal bound might not be realized in these theories.
are two choices in first step: we can first use operator $O_1$ to flow to theory $B$ and then use operator $O_2$ to flow to theory $D$; or we can first use operator $O_2$ to flow to a theory $C$, and then use operator $O_1$ to flow to theory $D$. So we have established a $\mathcal{N} = 1$ duality for theory $B$ and $C$, and this is quite similar to Seiberg duality [34], and is manifest using the parent $\mathcal{N} = 2$ description. See figure. 2.

**Remark 3: Turning on expectation values:** Up to this point, we focus on the IR behavior of the origin of $\mathcal{N} = 2$ moduli space ($\mathcal{N} = 2$ SCFT point) after the relevant $\mathcal{N} = 1$ preserving deformation, and it is interesting to consider the IR behavior of other points on the moduli space after the deformation. We leave the general study of this question to the future.

![Diagram](image)

**Figure 2:** A $\mathcal{N} = 2$ theory $A$ is deformed by two $\mathcal{N} = 1$ chiral multiplets $O_1$ and $O_2$ to get a theory $D$. There are two paths to interpret this flow: a) We first use operator $O_1$ to get a $\mathcal{N} = 1$ theory $B$ and then use operator $O_2$ to get theory $D$; b) We first use operator $O_2$ to get a $\mathcal{N} = 1$ theory $C$ and then use operator $O_2$ to get theory $D$. Theory $B$ and $C$ are very distinct $\mathcal{N} = 1$ SCFT, so from $\mathcal{N} = 1$ point of view, we find a Seiberg-duality: theory $B$ with a relevant deformation flows to the same theory as theory $C$ deformed by a different relevant deformation.

5 Discussion

The space of $\mathcal{N} = 2$ SCFT is increased dramatically in last few years, and lots of important properties about these theories such as the space of BPS operators, central charges, weakly coupled gauge theory duality frames, etc are known. Most of previous studies focuses on the properties of $\mathcal{N} = 2$ preserving deformations such as Seiberg-Witten geometry of Coulomb branch and Higgs branch chiral ring. Given the vast amount of knowledge of these $\mathcal{N} = 2$ theories, it is time to study more about supersymmetry breaking deformations. In this paper, We use the classification of $\mathcal{N} = 2$ and $\mathcal{N} = 1$ preserving relevant or marginal deformations to classify soft SUSY breaking of general $\mathcal{N} = 2$ SCFT.

Given the rich spectrum of Coulomb branch operators of a general $\mathcal{N} = 2$ SCFT, it is now possible to construct a large class of new four dimensional $\mathcal{N} = 1$ SCFT. Many
interesting properties about these theories such as central charges, chiral spectrum, Seiberg duality can be derived from the parent $\mathcal{N} = 2$ SCFT.

We can also consider many interesting non-supersymmetric (non-SUSY) deformations, but we do not know too much about the phase structure of the IR theory. The hope is that since the deformations are soft, one can use the information of $\mathcal{N} = 2$ SCFT like Seiberg-Witten solution to constrain the behavior of IR theory, and we would like to further study this question in the future. For the deformation that preserves some global symmetries whose anomalies are known and nontrivial, one can use it to constrain the IR theory, i.e. the IR theory should be gapless to match the anomaly. So simply from anomaly matching, we have found a large class of new non-SUSY CFT. Our non-SUSY CFT seems quite different from those found from non-abelian gauge theory 8, and it is definitely interesting to further study them.

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