Large eddy simulations of confined turbulent wake flows

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Abstract.

The influence of confinement on turbulent obstacle-free wake is numerically investigated by means of large eddy simulations (LES). The numerical solver makes use of a multi-domain Fourier-Chebyshev spectral method and the LES capability is implemented through a spectral vanishing viscosity technique. The geometry is channel-like and the transverse direction is homogeneous. A top hat like velocity profile is assumed at the inlet and no slip conditions are considered at the confining walls. Prescribing the velocity ratio, defined as the ratio of the velocity gap to the mean velocity, we study the influence of confinement on such flows at the Reynolds number $Re = 5000$. Several quantities are analyzed, as one-dimensional velocity spectra, third-order structure function, turbulent kinetic energy and its dissipation rate. It turns out that for obstacle-free wakes confinement increases the intensity of turbulence and its three-dimensional feature.

1. Introduction

Following previous theoretical analysis and numerical simulations, see e.g. ???, the present study is concerned with the influence of confinement on the spatio-temporal development of wakes. More precisely, the paper is devoted to confined turbulent obstacle-free wake flows. Thus, we focus on the flow generated by a top hat velocity profile, with no slip conditions at the confining walls, as previously analyzed in the laminar case (??). With respect to the two-dimensional studies, an homogeneous direction completes the initial setting. Despite not axisymmetric, such a model is close to the situation of a real confined wake, e.g. exiting from a coaxial injector. The outline of the paper is as follows: section ?? briefly describes the models and the numerical method, while section ?? goes into the details of the influence of the confinement on the turbulent properties of the wake flow. Finally, conclusions are drawn in section ??.

2. Model and numerical method

The flow is assumed to be governed by the incompressible Navier-Stokes equations. The geometry is channel-like and the transverse direction is homogeneous. In the elongated streamwise direction, the approximation in space makes use of a domain decomposition technique. In each subdomain, the spatial approximation is based on a Galerkin Fourier-collocation Chebyshev numerical method. The pressure is only defined at the inner grid-points, so that (i) no boundary conditions are required for the pressure and (ii) the discrete problem is...
well posed (no “pressure spurious modes”). The time scheme is based on an OIF semi-Lagrangian method and a projection technique. Three steps are involved: (i) transport, (ii) diffusion and (iii) projection. Diffusion (resp. transport) terms are handled implicitly (resp. explicitly). In each subdomain, an efficient direct matrix equation solver is used and the interface values are computed using a Schur complement technique. The LES capability is implemented through the use of a spectral vanishing viscosity (SVV) technique, which consists of introducing artificial viscosity in the high frequency range of the spectral approximation. The code is vectorized and (weakly) parallelized (one subdomain / processor). Additional details may be found in several papers, see e.g. ??.

The geometry is channel like in x-streamwise and y-crossflow directions, while the z-spanwise direction is homogeneous. At the sidewalls, no slip conditions are considered. At the inlet of the channel, one imposes a top hat velocity profile, i.e. the inner flow has velocity $U_1$ and width of $2h_1$, while the outer ones have $U_2$ and $h_2$. Using as reference scales the average velocity $U_m = (U_1 + U_2)/2$ and the half-width of the inner wake $h_1$, we introduce non-dimensional quantities. The dimensionless parameters are then: the confinement ratio $h = h_2/h_1$, the velocity ratio $\Lambda = (U_1 - U_2)/(U_1 + U_2)$, and the Reynolds number $Re = (U_1 + U_2)h_1/2\nu$, where $\nu$ is the kinematic viscosity.

In the present work, the velocity ratio $\Lambda = -0.739$ and the Reynolds number $Re = 5000$ are prescribed and we analyze the influence of the confinement parameter $h$ for the three different values: $h = 3$, $h = 1$ and $h = 0.7$. The computational domain is $\Omega = (0, L) \times (-H, H) \times (-E/2, E/2)$, where $L$, $H$ and $E$ are the length, the height and the thickness of the domain, respectively. The length of the domain is settled at $L = 75$, the thickness $E$ is settled to four times the inner wake, i.e. $E = 8$, and the height $H$ varies with respect to the confinement, i.e. $H = 1 + h$. We have generally used 8 subdomains, and in each of them the polynomial approximation degree in the $x$- and $z$-directions are settled to $N_x = 100$ and $N_z = 32$ (64 grid points), respectively. Depending on the height of the domain, we have used $N_y = 90$ or $N_y = 100$ for the polynomial approximation degree in the $y$-direction.

3. Influence of the confinement

We investigate the influence of the confinement on several properties of the turbulence at $Re = 5000$, using the inflow profile described in section ???. The statistical quantities as mean velocity profiles, turbulent kinetic energy and its dissipation rate have been calculated once the transient state due to the initial random perturbation has left the domain and the turbulent flow is fully developed, i.e. in practice for $100 < t \leq 400$. Simulations have been carried out on the NEC SX8 computer of the IDRIS center or on the cluster of the Mésocentre SIGAMM, located at the “Observatoire de la Côte Azur”. On a NEC SX8 parallel-vectorized super computer, each simulation requires about 500 CPU hours.

3.1. Instantaneous flow field

We show in figure ?? the instantaneous modulus of the vorticity $|\omega|$ in the $(x,y)$ plane $z = 0$ at time $t = 400$ for different confinement configurations: (a) $h = 3$, (b) $h = 1$ and (c) $h = 0.7$. One observes that the position of the turbulent front depends on the confinement parameter, since it is located upstream when the confinement is strengthened. While for $h = 1$ and $h = 0.7$ the flow seems to be fully 3D turbulent, for $h = 3$ the turbulence seems to be characterized by a peculiar two-/three-dimensional behavior. This hypothesis is confirmed by figure ??, where we show the instantaneous modulus $|\omega|$ of the vorticity in the $(x,z)$ plane $y = 0$ at the same time $t = 400$. Looking at figure ??a it is clear that in the first part of the domain, i.e. for $x \lesssim 45$, the effect of the third dimension is negligible, which enforces the conjecture of a two-dimensional turbulence in the considered region.
Figure 1. Instantaneous modulus $|\omega(x, y, 0)|$ of the vorticity at $t = 400$ for (a) $h = 3$, (b) $h = 1$ and (c) $h = 0.7$; $Re = 5000$.

Figure 2. Instantaneous modulus $|\omega(x, 0, z)|$ of the vorticity at $t = 400$ for (a) $h = 3$, (b) $h = 1$ and (c) $h = 0.7$; $Re = 5000$. 
### Table 1. Reynolds and Strouhal numbers observed in the present study, using two different sets of reference quantities.

| $h$ | $Re$ | $St$ | $Re_c$ | $St_c$ |
|-----|------|------|--------|--------|
| 3   | 5000 | 0.280| 13695  | 0.409  |
| 1   | 5000 | 0.240| 10000  | 0.480  |
| 0.7 | 5000 | 0.210| 8696   | 0.483  |

Furthermore, in figure ??a the structures around the vortices upstream of the turbulent front suggest a direct cascade of enstrophy, which is peculiar to a two-dimensional turbulence. Such structures are not detected for stronger confinements. One may then think that confinement promotes fully 3D turbulence. This hypothesis has however to be confirmed by more detailed analyses.

### 3.2. Strouhal number

The Fourier analysis of temporal data at some measurement points allows the determination of the Strouhal number, in order to estimate the influence of the confinement on the dominant frequency. For the wake of a cylinder the influence of confinement on the shedding frequency has already been investigated. Particularly, in experiments with a confined circular cylinder at Reynolds number such that $10^4 < Re < 10^7$, it is found that the Strouhal number increases when enforcing the confinement until $h = 1$ (??). A similar tendency was found by ?, who compared $h = 4$ and $h = \infty$ results for LES of confined turbulent wakes past a square cylinder at $Re = 3000$. However, cylinder wakes are quite different from the obstacle-free wakes considered in our simulations, for which, up to our knowledge, no previous experiments or numerical simulations in the turbulent regime have been carried out. Moreover, as discussed hereafter, the usual choices of the relevant reference quantities differ for these two different kinds of wake flows.

Table ?? shows the values of the Strouhal number, $St$, using as reference frequency $U_m/h_1$. When strengthening the confinement, it appears that the Strouhal number decreases. Hence, it seems that the dependence of the Strouhal number on the confinement is opposite of that one discerned for confined cylinder wakes. Indeed, as noted in ?, for laminar flows at lower Reynolds number, i.e. $Re = 100$, the Strouhal number seems to slightly increase with the confinement.

For confined cylinder wakes, $Re$ and $St$ are generally based on the flow-rate velocity $\overline{U}$ and the cylinder diameter $2h_1$, in contrast with our definition. To allow comparisons, the Reynolds and Strouhal number rescaled by means of these reference scales are named $Re_c$ and $St_c$, respectively. Table ?? also provides the values of $Re_c$ and $St_c$ for the three different confinements. We notice that the variations of $St$ and $St_c$ are opposite, since $St_c$ is decreasing with $h$ whereas $St$ is increasing. Despite the fact that $Re_c$ is decreased, so that one could imagine a decrease of $St_c$, see ?? for a wake of a cylinder, the increase of confinement induces a growth of the $St_c$ similar to cylinder wakes (??). This tendency on $St_c$ was already encountered by ? for $Re = 100$, for the same value of the velocity ratio $\Lambda = -0.739$.

### 3.3. Velocity spectra and structure function

One-dimensional velocity spectra, based through the Taylor hypothesis on the Fourier analysis of temporal data at some measurement points, allow a comparison with the Kolmogorov spectrum typical of isotropic turbulent flows. In figure ??a the one dimensional crossflow velocity spectra are presented. The measurement point is located at $x = 51.85$, $y = 0$. The most confined cases, i.e. $h = 1$ and $h = 0.7$, present a classical inertial subrange, where the Kolmogorov direct energy
Figure 3. (a) Fourier spectra of the velocity crossflow component \( v(x = 51.85, y = 0) \). Thin lines represent the slopes of -5/3 (continuous) and -3 (dashed). (b) Absolute value of the third order structure function \( S_3 \) versus time shift \( \Delta t \) for \( u(x = 51.85, y = 0) \). The continuous lines depict the negative values of \( S_3 \) and the dashed ones depict the positive values. Thin lines represent the slopes of 1 (continuous) and 3 (dashed). In both figures, the black bold line depicts \( h = 3 \), the dark gray bold one \( h = 1 \), and the light gray bold one \( h = 0.7 \).

cascade takes place peculiar to a fully 3D turbulent flow \( ? \). In the inertial subranges, which however are not very large, the slopes of the spectra fit satisfactorily with the expected \(-5/3\) value. Towards small scales, the two curves show a subgrid regime analogous to the dissipative regime, predicted by Kolmogorov’s hypotheses \( ? \). In contrast to this behavior, the spectrum of the less confined case, i.e. \( h = 3 \), presents two different tendencies in the inertial subrange, which is a reminiscence of the spectra of a two-dimensional turbulent flow \( ? \). In the large scale range, the spectrum has a slope close to \(-5/3\), which means that a cascade of energy with a constant flux takes place. For smaller scales, the slope of the spectrum rather approximates the value of \(-3\), see e.g. \( ?? \). Following \( ? \), this behavior of the spectrum is typical of an inverse cascade of enstrophy, further hint of the 2D nature of the turbulence for \( h = 3 \).

In order to confirm this intuition, one can compute the third order velocity structure function of the velocity \( S_3(l) = \langle |u(x + l, t) - u(x, t)| \cdot l/|l| \rangle^3 \rangle \), which can distinguish between 2D and 3D turbulent flows \( ? \). In streamwise direction, by means of the Taylor’s hypothesis the third order structure function may be computed as a function of the time shift \( \Delta t \), \( S_3(\Delta t) = \langle |u(x, t + \Delta t) - u(x, t)| \rangle^3 \rangle \), at the same point \( (x = 51.85 \text{ and } y = 0) \) and still averaging between several \( z \)-points. In an homogeneous and isotropic turbulent flow, the sign of the third order-velocity structure function can determine the direction of the energy cascade. If it is negative, there is a transfer of turbulent kinetic energy from the large scales towards the small scales, sign of a fully 3D turbulence. Otherwise, if it is positive the energy is transferred from the small scales towards the large ones, peculiar to a two-dimensional turbulent flow. Both inverse and direct cascade of energy have a slope of \( S_3(\Delta t) \) in log-scale equal to 1 \( ? \). For 2D turbulence, in \( ? \) it is also shown that in the range where the direct enstrophy cascade takes place, the third order-velocity structure function \( S_3(\Delta t) \) is still positive and its slope in log-scale fits 3. This behavior was confirmed by \( ? \) in atmospheric turbulence and recently by \( ? \) for forced 2D Navier-Stokes equations. Furthermore, in the subgrid regime the behavior of \( |S_3| \) approximates
the theoretical slope of 3 for both 2D- and 3D-turbulence.

In figure ??b, the behavior of the absolute value $|S_3(\Delta t)|$ is reported in log-scale for the three confinement configurations. Continuous lines mean negative values of $S_3$, while dashed lines mean positive ones. The slopes of 1 and 3 are depicted by continuous and dashed lines, respectively. One can note that for $h = 1$ and $h = 0.7$ the third order structure function is still negative, according to the hypothesis of a fully 3D turbulent flow, while for $h = 3$, a region for which $S_3$ is positive is encountered. Considering that our turbulent wakes do not respect the conditions of homogeneity and isotropy, we can conclude that the theoretical slopes are well recovered by the variations of $S_3$. In particular, while in the subgrid ranges the three configurations have a similar slope of 3, in the inertial regime the behaviors differ. The fully 3D turbulent cases present a clear slope of 1 which represents the energy cascade, while for $h = 3$, the slope of $S_3$ is close to 3, sign of an enstrophy cascade. However, for $h = 3$ the presence of a range with positive values of $S_3(\Delta t)$ confirms the hypothesis of 2D turbulence. Furthermore, if one associates $f$ to $1/\Delta t$, the positive range of the third order velocity structure function corresponds roughly to the inertial subrange where the direct cascade of enstrophy takes place, see figure ??a. Finally, the behaviors of both the spectra and the third order velocity structure function suggest that a stronger confinement favors the formation of fully 3D turbulent flow.

3.4. Mean flow and turbulence statistics
In this section statistical quantities characteristic of turbulence are analyzed. These quantities are averaged in time but also over the periodic spanwise direction, except the dissipation rates which are computed at $z = 0$.

3.4.1. Mean flow profiles Figure ?? provides the mean streamwise velocity $u_m(x, 0)$ along the centerline. Despite the inlet profiles are co-flow wakes, for $h = 1$ and $h = 0.7$ the velocity reaches negative values at the centerline, while for $h = 3$ no recirculation zone is present. Moreover, the largest recirculation region is obtained for $h = 0.7$. Thus, the confinement seems to promote the formation of a recirculation bubble in the domain.
3.4.2. Turbulent kinetic energy  The behavior of the turbulent front observed in figures ?? and ?? may be interpreted as a destabilizing effect due to confinement. In order to confirm this intuition, the turbulent kinetic energy profiles should be analyzed.

Figure ?? illustrates the behavior of the turbulent kinetic energy $k$ along the streamwise direction, for (a) $y = 0$, (b) $y = 1.04$, (c) $y = 1.34$, and (d) $y = 1.52$. At the centerline $y = 0$, see figure ??a, the case with $h = 3$ presents the highest turbulent kinetic energy. However, $k$ at the centerline is negligible compared to its values closer to the boundaries, see figure ???. Furthermore, for $y = 1.34$ and $y = 1.52$ the configurations with the strongest confinement present the highest maxima, which means that the confinement enhances the turbulent character of the flow. However, this destabilizing influence is only noticed close to the entry.

One may think that the boundary layers dissipate rapidly the turbulent kinetic energy, so that the least confined wake flow is the most turbulent in the final part of the domain. One may also conjecture that (i) once the turbulent wake flow has become similar to a standard turbulent channel flow, the velocity fluctuations are much less important and (ii) that this occurs farther...
downstream for the least confined flow. Finally, it is clear that the mean flow velocity and the width of the channel increase with $h$, so that a Reynolds number based on these quantities is higher for the least confined flow.

3.4.3. Dissipation rates

By means of formula proposed in ?, the dissipation rates $\varepsilon$ of the turbulent kinetic energy can be computed and the connection of this quantity with the confinement observed. Figure 6 illustrates the behavior of $Re\varepsilon$ along the centerline $y = z = 0$. One observes that the maximum of dissipation rate $\varepsilon$ is enhanced when the confinement is increased. Another effect should be noticed: while for the two strongest confinements a large part of the dissipation occurs close to the inlet, for $h = 3$ the dissipation persists along the domain. It is clear that the explanations provided for the turbulent kinetic energy also hold for the dissipation rates.

4. Conclusions and perspectives

By means of a high order LES based on a SVV technique (SVVLES Chebyshev-Fourier spectral code) we have studied the influence of confinement on turbulent obstacle-free wakes. Considering several quantities as one-dimensional spectra, third order velocity structure function, turbulent kinetic energy and dissipation rate, we have shown that confinement enhances the turbulent character of the flow. This is especially pointed out by the increase of turbulent kinetic energy and dissipation rate when the confinement is increased.

Also, the influence of the confinement on the Strouhal number was examined in the turbulent regime, pointing out a difference of behavior with respect to confined laminar wakes.

Depending on the confinement parameter, the turbulence has shown either 3D or 2D features. Although the confinement seems to favor the formation of a fully-developed 3D turbulence, it would be interesting to study the role of the confinement in a fully 2D domain in order to exclude artificially the development of a 3D flow. Then it would be possible to study if the intensity of two-dimensional turbulence is favored by confinement.

Finally, since the results of the present paper are obtained in a channel like geometry, it would be interesting to understand if the confinement has the same influence on turbulent wakes also
in an axisymmetric geometry, e.g. closer to a real coaxial injector. Furthermore it would be of interest to achieve more detailed numerical or experimental studies, devoted to confined laminar / turbulent obstacle-free wakes, in order to unveil the role of confinement on the appearance of laminar instabilities and 3D coherent structures and, at the end, to understand the transition to turbulence of confined wakes.

Further analyses on the same topic can be found in ?.

Acknowledgements

For the numerical simulations carried out in this work, we have used the support of the IDRIS (Institut du Développement et des Ressources en Informatique Scientifique), as well as the Mésocentre SIGAMM of the Observatory of Côte Azur. Stefano Musacchio is warmly acknowledged for stimulating discussions. The authors would like to thank Jean-Marc Lacroix for his kindness and technical support.

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