Complete results for a numerical evaluation of interior point solvers for large-scale optimal power flow problems

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Abstract
Recent advances in open source interior-point optimization methods and power system related software have provided researchers and educators with the necessary platform for simulating and optimizing power networks with unprecedented convenience. Within the MatPOWER software platform a combination of several different interior point optimization methods are provided and four different optimal power flow (OPF) formulations are recently available: the Polar-Power, Polar-Current, Cartesian-Power, and Cartesian-Current. The robustness and reliability of interior-point methods for different OPF formulations for minimizing the generation cost starting from different initial guesses, for a wide range of networks provided in the MatPOWER library ranging from 1951 buses to 193,000 buses, will be investigated. Performance profiles are presented for iteration counts, overall time, and memory consumption, revealing the most reliable optimization method for the particular metric.

1 Introduction
Modern industrial developments have greatly increased electric power system complexity. As a result, modern operation tools and software have to address strong nonlinearities, in system behavior in order to guarantee reliable and economic system operation. Approximation-based optimization techniques will be less attractive to cope with stressed operation conditions. The main advantage of NLP formulations for OPF is that they accurately capture power system behavior rendering them excellent solution methods for general purpose power system software.

MatPOWER [20] a package of free, open-source Matlab-language M-files has been available for power-system researchers and educators as a simulation tool for solving power flow (PF), and extensible optimal power flow (OPF) problems. It is packaged with a library of several power networks of increasing complexity. Interfaces to multiple, high-performance nonlinear optimizers such as FMINCON, IPOPT, KNITRO, and its included default solver, MIPS, are also available for its users. Recently, several different formulations of the standard AC-OPF problem were added, including polar and Cartesian representations of complex voltage variables and both current and power versions of the nodal mismatch equations.

In this paper, we attempt to use for the first time optimization benchmarking profiles to evaluate various optimization methods and software for power grid applications. In recent years, these performance profiles have become a very popular and widely used tool for benchmarking and evaluating the performance of several optimizers when run on a large test set. Performance profiles have been introduced in [6] in 2002 and have rapidly become a standard in benchmarking of optimization algorithms. Comparative studies using performance profiles have been performed throughout the optimization literature [11], and in the evaluation of sparse linear solvers [8] also pointing out some limitations [7].
We will focus on benchmarking metrics such as the total runtime, memory requirements, or iteration count with a particular emphasis on power-grid application within the MATPOWER software framework [20], [13]. In pursuing these objectives, we focus on single-objective optimization algorithms that run in serial (i.e., that do not use parallel processing).

The reason motivating optimization benchmarking in MATPOWER is twofold: to demonstrate the value of a novel algorithm and formulation versus more classical methods, and to evaluate the performance of an optimization algorithm and the related optimization software on networks of increasing complexity and sizes. Our key contribution is a detailed performance profile study of the effects of different optimizers for large-scale single-period optimal power flow problems that will assist users in making an informed decision about how and which software should be preferred.

2 Interior point methods and related optimization software

The OPF problem is defined in terms of the conventional economic dispatch problem, aiming at determining the optimal settings for optimization variables. The standard formulation of the OPF problem takes the form of a general non-linear programming problem, with the following form:

\[
\begin{align*}
\text{minimize} \quad & f(x) \\
\text{subject to} \quad & g(x) = 0, \\
& h(x) \leq 0, \\
& x_{\min} \leq x \leq x_{\max}.
\end{align*}
\]

The objective function \( f(x) \) consists of polynomial costs of generator injections, the equality constraints \( g(x) \) are the nodal balance equations, the inequality constraints \( h(x) \) are the branch flow limits, and the \( x_{\min} \) and \( x_{\max} \) bounds include reference bus angles, voltage magnitudes and active and reactive generator injections.

2.1 Primal-dual IPMs

Primal-dual IPMs have been successfully applied to OPF problems, demonstrating high robustness and convergence, in the sense that they converge to an optimal solution from any initial point [14], and they can exploit Hessian information that is easy to compute for all OPF problems. According to standard practice, slack variables \( s \) are introduced at first to convert inequality constraints from (1c) to equality constraints, and logarithmic barrier terms are added to the objective to ensure that the slacks \( s \) will remain within their bounds as the function is minimized. A sequence of \( \mu \)-subproblems is obtained this way:

\[
\begin{align*}
\text{minimize} \quad & f(x) - \mu \sum_i \ln(s_i) \\
\text{subject to} \quad & g(x) = 0, \\
& h(x) + s = 0.
\end{align*}
\]

The solution of each \( \mu \)-subproblem are critical points of the Lagrangian,

\[
L(x, s, \lambda_g, \lambda_h) = f(x) - \mu \sum_i \ln(s_i) - \lambda_g^T g(x) - \lambda_h^T (h(x) + s),
\]

where \( \lambda_g, \lambda_h \) are the vectors representing the Lagrange multipliers for the equality and inequality constraints. The critical points for (3) satisfy the KKT conditions

\[
\begin{bmatrix}
\nabla_x L \\
\nabla_s L \\
\nabla_{\lambda_g} L \\
\nabla_{\lambda_h} L
\end{bmatrix}
= \begin{bmatrix}
\nabla_x f(x) - J_g^T \lambda_g - J_h^T \lambda_h \\
-\mu e + \Lambda_h S \\
\lambda_b^T g(x) \\
\lambda_b^T h(x) + s
\end{bmatrix} = 0,
\]

where \( \Lambda_h = \text{diag}(\lambda_h), S = \text{diag}(s), \) and \( e \) is a vector with all its entries equal to one. For convenience, we also define \( J_g = \nabla_x g(x) \) and \( J_h = \nabla_x h(x) \) to be Jacobians of the equality and inequality constraints, respectively. Also note that \( \nabla_s L \) has been postmultiplied with \( S \).
Each $\mu$-subproblem (2) is solved approximately and while $\mu$ decreases to zero, the solution of the next barrier problem is obtained using, as a starting guess, the approximate solution of the previous one [17]. The update strategy of the $\mu$ parameter influences the convergence properties of the algorithm as it is one of the factors distinguishing various optimizers. The MIPS update rule is based on scaled complementary slackness, while IPOPT uses monotone Fiacco–McCormic strategy [17] and BELTISTOS exploits adaptive Mehrotra’s probing heuristic.

The primal-dual update is obtained from the solution of the optimality conditions linearization

$$
\begin{bmatrix}
H & -J_g^T & -J_h^T \\
0 & \Lambda_h & 0 \\
-J_g & 0 & 0 \\
-J_h & I & 0
\end{bmatrix}
\begin{bmatrix}
\Delta x^k \\
\Delta s^k \\
\Delta \lambda_g^k \\
\Delta \lambda_h^k
\end{bmatrix}
=
\begin{bmatrix}
\nabla_x L \\
\nabla_s L \\
\nabla_{\lambda_g} L \\
\nabla_{\lambda_h} L
\end{bmatrix}
$$

(5)

where $H = \nabla^2 x x$. The linear system (LS) solution strategy is another factor distinguishing various optimizers. Performance of the optimizer is also greatly improved by a selection of a robust and memory efficient LS solver, since the resulting LSs are very large and highly ill-conditioned. It is also a common practice that the LS is simplified and reduced to a smaller set of equations.

### 2.2 Optimization Software

In what follows we describe several different primal-dual interior point methods used by many practitioners for OPF problems and provided in the software package MATPOWER.

**IPOPT** [17] is a software package for large-scale nonlinear optimization. It implements a primal-dual interior-point algorithm with a filter line-search method for nonlinear programming, second-order corrections, and inertia correction of the KKT matrix. It is written in C++ by Andreas Wächter and Carl Laird and it is released as open source code under the Eclipse Public License (EPL). It is distributed by the COIN-OR initiative. Pre-built MEX binaries for Windows available with OPTI Toolbox and high-performance IPOPT-PARDISO and MATPOWER pre-built MEX binaries for Mac and Linux from the PARDISO project.

**BELTISTOS** [10] contains a structure exploiting, data-compression algorithms for extreme scalability and low memory footprint for OPF problems and its multiperiod and security-constrained extensions.

**MIPS** [19], [18] is a primal-dual interior-point solver introduced by Wang for OPF problems. It is entirely implemented in MATLAB code and distributed with MATPOWER. We assume that step control is enabled (not by default), which implements additional step-size control in the MIPS algorithm.

**FMINCON** [3], [2] is a gradient-based method, the default optimization method of the MATLAB optimization toolbox, and it is designed to work on problems where the objective and constraint functions are both continuous and have continuous first derivatives. In its default setting it uses an interior point solver that can exploit the Hessian of the Lagrangian.

**KNITRO** [4] Artelys KNITRO is a commercial software package for solving large-scale mathematical optimization problems. KNITRO is specialized for nonlinear optimization. KNITRO offers four different optimization algorithms for solving optimization problems. Two algorithms are of the interior point type, and two are of the active set type. KNITRO provides both types of algorithm for greater flexibility in solving problems, and allows crossover during the solution process from one algorithm to another.

The solution of linear systems of equations is the cornerstone of a robust high-performance optimization package. Here we describe some sparse direct linear solvers. Specific results for the OPF problems are presented in section 4.

**HSL 2002** [1] is an ISO Fortran library of packages for many areas in scientific computation. It is probably best known for its codes for the direct solution of sparse linear systems, including multifrontal algorithm with approximate minimum degree ordering (MA57). IPOPT provides support for a wide

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1. [https://projects.coin-or.org/Ipopt](https://projects.coin-or.org/Ipopt)
2. [https://www.pardiso-project.org](https://www.pardiso-project.org)
3. [https://www.artelys.com/en/optimization-tools/knitro](https://www.artelys.com/en/optimization-tools/knitro)
variety of linear solvers, including HSL linear solvers MA27, MA57 and others. Evaluation of the individual solvers in terms of robustness and performance is provided in [9].

SuiteSparse [5] is a suite of sparse matrix packages, many of which are used in MATLAB. Multifrontal LU factorization from the UMPFACK appears in MATLAB as LU and backslash operators. The solver is used by default in MATLAB-based MIPS package.

PARDISO [15] is a thread-safe, high-performance, robust, memory efficient software for solving large sparse symmetric and unsymmetric linear systems of equations on shared-memory and distributed-memory multiprocessors. IPOPT and MIPS contain ready to use interfaces to the solver.

We note that in our study we consider IPOPT with PARDISO and HSL MA57 solvers and MIPS with the default backslash ‘\’ solver and PARDISO. KNITRO may utilize HSL routines MA27 or MA57 in order to solve linear systems arising at every iteration of the algorithm.

3 Performance Profiles

In order to evaluate the quality of the different optimization methods for OPF problems we will use performance profiles for compact comparison of the benchmark problems using different optimization packages. Theses profiles were first proposed in [6] for benchmarking optimization software and used in e.g. to evaluate the performance of various sparse direct linear solvers and optimizers [8, 16, 12].

The profiles are generated by running the set of optimizers $\mathcal{M}$ on a set of OPF problems $\mathcal{S}$ and recording information of interest, e.g., time to solution or memory consumption. Let us assume that a power flow optimizer $m \in \mathcal{M}$ reports a statistic $\theta_{ms} \geq 0$ for the OPF problem $s \in \mathcal{S}$; smaller statistics $\theta_{ms}$ indicates better solution strategies. We can further define $\hat{\theta}_{s} = \min_{m \in \mathcal{M}} \{ \theta_{ms} \}$, which represents the best statistic for a given OPF problem $m$. Then for $\alpha \geq 1$ and each $m \in \mathcal{M}$ and $s \in \mathcal{S}$ we define

$$k(\theta_{ms}, \hat{\theta}_{s}, \alpha) = \begin{cases} 1 & \theta_{ms} \leq \alpha \hat{\theta}_{s} \\ 0 & \theta_{ms} > \alpha \hat{\theta}_{s} \end{cases}$$

(6)

The performance profile $p_{m}(\alpha)$ of the power flow optimizer $m$ is then defined by

$$p_{m}(\alpha) = \frac{\sum_{s \in \mathcal{S}} k(\theta_{ms}, \hat{\theta}_{s}, \alpha)}{|\mathcal{S}|}.$$  

(7)

Thus, in these profiles, the values of $p_{m}(\alpha)$ indicate the fraction of all examples, which can be solved within $\alpha$ times, the time the best solver needed, e.g., $p_{m}(1)$ gives the fraction of which optimizer $m$ is the most effective package and $p_{m}^* := \lim_{\alpha \to \infty} p_{m}(\alpha)$ indicates the fraction for which the algorithm succeeded. If we are just interested in the number of wins on $\mathcal{S}$, we need only compare the values of $p_{i}(1)$ for all the solvers $i \in \mathcal{M}$, but if we are interested in optimizers with a high probability of success on the set $\mathcal{S}$, we should choose those for which $p_{i}^*$ is largest. Thereby, for a selected test set, performance profiles provide a very useful and convenient means of assessing the performance of optimizers relative to the best optimizer on each example from that set [7]. When commenting, e.g., on a performance profile presented in their paper, Dolan and Moré state that it “gives a clear indication” of the relative performance of each optimizer [6] and one can determine which optimizer has the highest probability $p_{i}(f)$ of being within a factor $f$ of the
best optimizer for \( f \) in a chosen interval. In this paper we use performance profiles to compare various aspects of problem formulation, problem setup and performance of several optimizers on sets of smooth or piecewise-smooth power flow problems. Our results provide estimates for the best configuration of the problems and identification of the optimizer with the best possible performance.

4 Performance Benchmarks

We proceed with the evaluation of various aspects for the set of benchmark cases with increasing complexity, listed in Table 2. The benchmarks are split into two groups, standard benchmarks used mainly to test robustness of optimization frameworks on wide spectrum of power grid networks and large-scale benchmarks used to test the performance. In collecting the test data we imposed only two conditions: The OPF problem be of order greater than 5’000 variables and the the data must be available to other users. The first condition was imposed because our interest in this study is in medium to large-scale scale problems. The second condition was to ensure that our tests could be repeated by other users and, furthermore, it enables other software developers to test their codes on the same set of examples and thus to make comparisons with other optimizers. Comparing algorithms for multiobjective optimization, or optimization algorithms that use parallel processing issues are outside of the scope of this paper since it would introduce another level of complexity to the benchmarking process and most of the users are using MATPOWER in default single core mode. For this reason we set the environment variable OMP_NUM_THREADS=1.

Simulations are performed on a workstation equipped with an Intel Xeon CPU E7-4880 v2 at 2.50 GHz and 1 TB RAM using latest MATPOWER release 7.0. The results are presented from four different perspectives, each being a contributing factor to complexity and behavior of the optimization procedure. These factors are (i) initial guess provided to the optimizer, (ii) OPF formulation and (iii) the optimization framework and (iv) an underlying direct sparse solver. The tolerance for the optimizers while solving the benchmarks was set to \( 10^{-4} \) and maximum number of iterations was set to 500. A CPU limit of 12 hours was imposed for each optimizers on each problem; any optimizers that had not completed after this time was recorded as having failed.
4.1 OPF Benchmark Cases and Optimization Problem Properties

Table 2 lists the number of buses, generators and lines for each MATPOWER benchmark case. Additionally, we also show properties of the corresponding optimization problem such as number of variables, equality and inequality constraints. Size of the optimization problem in terms of number of nonlinear equality and inequality constraints, depends on the formulation. For the Cartesian coordinate voltage case, voltage magnitude constraints (which are simple variable bounds for the polar case) are now nonlinear inequality constraints. Presented problem sizes in Table 2 consider the polar voltage representation.

In addition to the standard MATPOWER cases, there are four larger cases, case21k – case193k, built from the case3012wp considering the largest generator outage and line contingencies. The cases are sorted in increasing order by the sum of the number of buses, number of generators, number of lines with flow limits and number of DC lines.

| MATPOWER case     | \( n_b \) | \( n_g \) | \( n_l \) | nvar | \( |g(x)| \) | \( |h(x)| \) |
|-------------------|---------|--------|--------|------|---------|---------|
| case1951rte       | 1,951   | 391    | 2,596  | 4,634| 3,902   | 4,198   |
| case2383wp        | 2,383   | 327    | 2,896  | 5,420| 4,766   | 5,792   |
| case2868rte       | 2,868   | 599    | 3,808  | 6,858| 5,736   | 4,562   |
| case_ACTIVSg2000  | 2,000   | 544    | 3,206  | 4,864| 4,000   | 6,412   |
| case2869pegase    | 2,869   | 510    | 4,582  | 6,758| 5,738   | 5,486   |
| case2737sop       | 2,737   | 399    | 3,506  | 5,912| 5,474   | 6,538   |
| case2736sp        | 2,736   | 420    | 3,504  | 6,012| 5,472   | 6,538   |
| case2746wop       | 2,746   | 514    | 3,514  | 6,354| 5,492   | 6,614   |
| case2746wp        | 2,746   | 520    | 3,514  | 6,404| 5,492   | 6,558   |
| case3012wp        | 3,012   | 502    | 3,572  | 6,794| 6,024   | 7,144   |
| case3120sp        | 3,120   | 505    | 3,693  | 6,836| 6,240   | 7,386   |
| case3375wp        | 3,374   | 596    | 4,161  | 7,706| 6,748   | 8,322   |
| case4648rte       | 6,468   | 1,295  | 9,000  | 13,734| 12,936 | 4,626   |
| case4670rte       | 6,470   | 1,330  | 9,005  | 14,462| 12,940 | 6,220   |
| case4955rte       | 6,495   | 1,372  | 9,019  | 14,350| 12,990 | 6,218   |
| case5155rte       | 6,515   | 1,388  | 9,037  | 14,398| 13,030 | 6,262   |
| case9241pegase    | 9,241   | 1,445  | 16,049 | 21,372| 18,482 | 12,590  |
| case13659pegase   | 13,659  | 4,092  | 20,467 | 35,502| 27,318 | 0       |
| case_ACTIVSg10k   | 10,000  | 2,485  | 12,706 | 23,874| 20,000 | 20,488  |

| Large-scale benchmarks                      |
|---------------------------------------------|
| case_ACTIVSg25k                             |
| case_ACTIVSg70k                             |
| case21k                                     |
| case42k                                     |
| case99k                                     |
| case193k                                    |
4.2 Initial guess for the OPF

Since we adopt gradient-based methods for our OPF benchmarks, we expect that the performance of all optimizers will be sensitive to the initial guess. In order to evaluate the influence of the initial guess, we solve the OPF problems from three different initial guesses currently provided by the MATPOWER option \texttt{opf.start} \cite{MATPOWER}. The initial guess for option 1 (flat start) is heuristically chosen to be the average of the upper and lower bounds, or close to the bound if bounded only from one side. This is the default option and does not provide any estimation of the optimal solution, nor does such guess satisfy the constraints. MATPOWER also provides two warm start options. Option 2 (MATPOWER case data, MPC) uses the values of variables specified in the input MATPOWER case and option 3 (power flow solution, PF) used the solution of the power flow equations as the initial guess. PF guarantees that the OPF constraints and variables’ bounds are satisfied. Newton’s method is used for the solution with tolerance set to $10^{-8}$ and maximum of 30 iterations. We consider the default OPF formulation with polar voltage representation and power balance equations for evaluation of the initial guess.

The most robust initial guess in our set of benchmark cases is the option “MATPOWER case data”, together with the start initialized with the power flow solution, being the two options best approximating the optimal solution. Optimizer starting from these initial points solved highest number of the benchmark cases with less iterations required and thus in lower amount of overall time. The option “MATPOWER case data” however assumes that the case is well constructed and contains high quality data, which might not be always the case. The PF solution would be more appropriate choice in such situations.

Table 3: Number of solved benchmarks for different starting points.

| Optimizer            | Flat start | MATPOWER case data | Power flow solution |
|----------------------|------------|--------------------|---------------------|
| MIPS-MATLAB'         | 16         | 23                 | 23                  |
| MIPS-PARDISO         | 16         | 23                 | 24                  |
| IPOPT-PARDISO        | 23         | 25                 | 25                  |
| IPOPT-MA57           | 21         | 22                 | 21                  |
| BELTISTOS            | 25         | 25                 | 25                  |
| FMINCON 2015b        | 18         | 19                 | 21                  |
| FMINCON 2017b        | 18         | 21                 | 20                  |
| KNITRO 10            | 20         | 23                 | 24                  |
| KNITRO 11            | 20         | 23                 | 24                  |

If the optimizer allows to set the initial value of the barrier parameter $\mu$ in the interior point method, the problems initialized by the MPC and PF should choose $\mu$ which is much closer to zero than the value used for the flat start. The reason is that MPC and PF initial points satisfy the constraints therefore the penalty for the barrier function should be very small, as opposed to the flat start where there is no guarantee of the constraints satisfaction by the initial point. The barrier parameter in this case should be very relaxed in the beginning and tightened as the current iterate approaches the solution.

We also note that since the flat start is a poor approximation of the solution nor does it satisfy the constraints, the optimizer might fail converging to solution. It thus helps if the optimizer performs a pre-solve phase, were it first tries to satisfy the constraints and only then proceeds with the regular optimization procedure.
Table 4: Overall time (s) (BELTISTOS).

| Benchmark   | Flat | MPC | PF |
|-------------|------|-----|----|
| case1951rte | 3.78 | 4.57| 2.98|
| case2383wp  | 4.41 | 6.30| 5.07|
| case2736sp  | 4.23 | 3.65| 3.71|
| case2737sop | 3.91 | 4.05| 3.22|
| case2746wop | 4.16 | 3.25| 4.28|
| case2746wp  | 4.33 | 3.60| 4.00|
| case2868rte | 6.93 | 3.01| 4.09|
| case2869pegase | 4.58 | 4.52| 3.77|
| case3012wp  | 4.69 | 7.18| 6.76|
| case3120sp  | 5.73 | 6.83| 6.40|
| case3375wp  | 5.81 | 6.70| 5.88|
| case4688rte | 9.77 | 8.01| 7.06|
| case6468rte | 7.07 | 6.49| 8.73|
| case6495rte | 8.09 | 10.72| 8.35|
| case6515rte | 7.66 | 9.42| 7.47|
| case9241pegase | 16.01 | 14.00| 9.20|
| case_ACTIVSg2000 | 4.45 | 4.20| 3.49|
| case_ACTIVSg10k | 15.31 | 13.11| 12.97|
| case13659pegase | 16.17 | 9.06| 8.37|
| case_ACTIVSg25k | 43.65 | 30.26| 29.19|
| case_ACTIVSg70k | 185.63 | 111.10| 129.16|
| case21k      | 58.87 | 81.35| 58.78|
| case42k      | 193.80 | 251.13| 184.28|
| case99k      | 540.80 | 782.43| 670.59|
| case193k     | 1,866.94 | 1,854.40| 2,557.72|

*Algorithm was forced to switch to the feasibility restoration phase in the first iteration

Table 5: Number of iterations (BELTISTOS).

| Benchmark   | Flat | MPC | PF |
|-------------|------|-----|----|
| case1951rte | 34   | 23  | 19 |
| case2383wp  | 32   | 42  | 39 |
| case2736sp  | 28   | 15  | 18 |
| case2737sop | 23   | 14  | 17 |
| case2746wop | 28   | 9   | 25 |
| case2746wp  | 30   | 14  | 23 |
| case2868rte | 65   | 12  | 22 |
| case2869pegase | 22   | 19  | 18 |
| case3012wp  | 30   | 38  | 42 |
| case3120sp  | 37   | 37  | 40 |
| case3375wp  | 38   | 32  | 34 |
| case4688rte | 52   | 30  | 31 |
| case6468rte | 26   | 16  | 39 |
| case6495rte | 40   | 35  | 35 |
| case6515rte | 39   | 36  | 31 |
| case9241pegase | 45   | 33  | 23 |
| case_ACTIVSg2000 | 37   | 21  | 22 |
| case_ACTIVSg10k | 43   | 28  | 30 |
| case13659pegase | 49   | 20  | 20 |
| case_ACTIVSg25k | 51   | 27  | 28 |
| case_ACTIVSg70k | 65   | 33  | 47 |
| case21k      | 50   | 55  | 51 |
| case42k      | 60   | 59  | 59 |
| case99k      | 65   | 71  | 71 |
| case193k     | 75   | 77  | 77 |

*Algorithm was forced to switch to the feasibility restoration phase in the first iteration
### Table 6: Overall time (s) (IPOPT-PARDISO).

| Benchmark | Flat  | MPC   | PF    |
|-----------|-------|-------|-------|
| case1951rte | 25.55 | 4.79  | 4.01  |
| case2383wp  | 9.00  | 6.89  | 26.33 |
| case2736sp  | 9.80  | 7.46  | 6.70  |
| case2737sop | 8.37  | 5.82  | 4.78  |
| case2746wop | 8.06  | 6.89  | 4.69  |
| case2746wp  | 7.40  | 5.91  | 4.29  |
| case2868rte | —     | —     | 13.07 |
| case2869pegase | —     | 9.29  | 8.18  |
| case3012wp  | 18.47 | 11.47 | 10.70 |
| case3120sp  | 22.93 | 15.47 | 11.30 |
| case3375wp  | 11.83 | 21.01 | 11.24 |
| case6468rte | 82.40 | 6.96  | 7.20  |
| case6470rte | 112.06| 29.25 | 26.05 |
| case6495rte | 27.24 | 20.85 | 125.34|
| case6515rte | 64.83 | 9.03  | 11.51 |
| case9241pegase | 184.50| 66.13 | 37.38 |
| case_ACTIVSg2000 | 15.80 | 5.22  | 6.83  |
| case_ACTIVSg10k | 95.91 | 18.18 | 20.59 |
| case_ACTIVSg25k | 341.29| 219.95| 349.49|
| case_ACTIVSg70k | 116.85| 89.49 | 88.00 |
| case21k    | 41.64 | 435.86| 222.09|
| case42k   | 692.54| 641.81| 644.92|
| case99k   | 3,136.65| 2,763.08| 1,924.00|
| case193k  | 13,236.02| 10,169.78| 9,599.37|

### Table 7: Number of iterations (IPOPT-PARDISO).

| Benchmark | Flat  | MPC   | PF    |
|-----------|-------|-------|-------|
| case1951rte | 122  | 30    | 28    |
| case2383wp  | 36   | 36    | 114   |
| case2736sp  | 39   | 43    | 26    |
| case2737sop | 27   | 25    | 36    |
| case2746wop | 38   | 24    | 30    |
| case2746wp  | 40   | 35    | 24    |
| case2868rte | —    | 45    | 46    |
| case2869pegase | —    | 31    | 33    |
| case3012wp  | 41   | 31    | 30    |
| case3120sp  | 42   | 44    | 35    |
| case3375wp  | 40   | 72    | 38    |
| case6468rte | 240  | 31    | 31    |
| case6470rte | 252  | 70    | 68    |
| case6495rte | 74   | 56    | 310   |
| case6515rte | 131  | 41    | 49    |
| case9241pegase | 119  | 56    | 38    |
| case_ACTIVSg2000 | 51   | 29    | 27    |
| case_ACTIVSg10k | 139  | 35    | 36    |
| case_ACTIVSg25k | 244  | 193   | 249   |
| case_ACTIVSg70k | 58   | 51    | 46    |
| case21k    | 141  | 126   | 50    |
| case42k   | 65   | 53    | 54    |
| case99k   | 81   | 61    | 63    |
| case193k  | 98   | 76    | 76    |

### Table 8: Overall time (s) (IPOPT-MA57).

| Benchmark | Flat  | MPC   | PF    |
|-----------|-------|-------|-------|
| case1951rte | 12.61| 2.38  | 2.47  |
| case2383wp  | 3.63 | 3.66  | 3.09  |
| case2736sp  | 4.53 | 1.81  | 2.14  |
| case_ACTIVSg10k | 95.91| 18.18 | 20.59 |
| case_ACTIVSg25k | 341.29| 219.95| 349.49|
| case_ACTIVSg70k | 116.85| 89.49 | 88.00 |
| case21k    | 461.64| 435.86| 222.09|
| case42k   | 692.54| 641.81| 644.92|
| case99k   | 3,136.65| 2,763.08| 1,924.00|
| case193k  | 13,236.02| 10,169.78| 9,599.37|

### Table 9: Number of iterations (IPOPT-MA57).

| Benchmark | Flat  | MPC   | PF    |
|-----------|-------|-------|-------|
| case1951rte | 141  | 34    | 32    |
| case2383wp  | 35   | 47    | 38    |
| case2736sp  | 36   | 42    | 45    |
| case_ACTIVSg10k | 139  | 35    | 36    |
| case_ACTIVSg25k | 244  | 193   | 249   |
| case_ACTIVSg70k | 85   | 66    | 62    |
| case21k    | 141  | 126   | 50    |
| case42k   | 65   | 53    | 54    |
| case99k   | 81   | 61    | 63    |
| case193k  | 98   | 76    | 76    |
Table 10: Overall time (s) (MIPS-MATLAB).  

| Benchmark         | Flat  | MPC  | PF   |
|-------------------|-------|------|------|
| case1951rte       | ---   | 3.38 | 3.60 |
| case2383wp        | 5.69  | 4.72 | 5.01 |
| case2736sp        | 6.47  | 5.13 | 4.85 |
| case2737sp        | 6.42  | 4.51 | 4.92 |
| case2746wp        | 7.18  | 5.25 | 5.29 |
| case2748wp        | 7.23  | 5.18 | 5.50 |
| case2868rte       | ---   | 4.54 | 4.88 |
| case2869pegase    | 9.47  | 28.74| 6.25 |
| case3012wp        | 10.26 | 5.44 | 5.54 |
| case3120sp        | 9.97  | 29.65| 7.15 |
| case3375wp        | 12.57 | 6.72 | 6.47 |
| case6468rte       | ---   | 16.71| 15.18|
| case6470rte       | ---   | 17.29| 17.72|
| case6495rte       | ---   | 26.85| 27.89|
| case6515rte       | ---   | 27.55| 20.42|
| case9241pegase    | 31.75 | 47.34| 30.25|
| case_ACTIVSg2000  | 7.86  | 4.63 | 9.50 |
| case_ACTIVSg10k   | ---   | 29.24| 67.86|
| case13659pegase   | 64.55 | 30.83| 39.65|
| case_ACTIVSg25k   | ---   | 99.76| 152.40|
| case_ACTIVSg70k   | ---   | ---  | ---  |
| case21k           | 336.77| 191.82|191.09|
| case42k           | 1,215.27|816.24 |818.85|
| case99k           | 27,695.39|24,168.30|33,890.39|
| case193k          | 129,846.69|---    |---   |

Table 12: Overall time (s) (MIPS-PARDISO).  

| Benchmark         | Flat  | MPC  | PF   |
|-------------------|-------|------|------|
| case1951rte       | ---   | 3.81 | 4.21 |
| case2383wp        | 6.70  | 5.27 | 5.58 |
| case2736sp        | 7.08  | 5.15 | 5.13 |
| case2737sp        | 6.55  | 4.61 | 5.01 |
| case2746wp        | 7.45  | 5.33 | 5.59 |
| case2748wp        | 7.55  | 5.65 | 5.76 |
| case2868rte       | ---   | 5.46 | 5.40 |
| case2869pegase    | 9.89  | 21.91| 6.80 |
| case3012wp        | 11.04 | 5.96 | 6.57 |
| case3120sp        | 10.89 | 26.63| 7.31 |
| case3375wp        | 13.22 | 7.25 | 7.20 |
| case6468rte       | ---   | 17.46| 16.69|
| case6470rte       | ---   | 19.02| 19.72|
| case6495rte       | ---   | 29.91| 31.51|
| case6515rte       | ---   | 23.00|      |
| case9241pegase    | 34.17 | 51.09| 32.38|
| case_ACTIVSg2000  | 7.18  | 4.45 | 9.34 |
| case_ACTIVSg10k   | ---   | 29.96| 66.89|
| case13659pegase   | 66.44 | 36.05| 52.77|
| case_ACTIVSg25k   | ---   | 100.54|148.73|
| case_ACTIVSg70k   | ---   | ---  | ---  |
| case21k           | 165.52| 102.68|105.03|
| case42k           | 464.73| 307.20|311.32|
| case99k           | 2,036.95|1,284.73|1,303.73|
| case193k          | 4,743.55|3,175.67|3,240.64|

Table 11: Number of iterations (MIPS-MATLAB).  

| Benchmark         | Flat  | MPC  | PF   |
|-------------------|-------|------|------|
| case1951rte       | ---   | 26   | 26   |
| case2383wp        | 31    | 29   | 31   |
| case2736sp        | 29    | 28   | 27   |
| case2737sp        | 27    | 25   | 25   |
| case2746wp        | 30    | 26   | 28   |
| case2748wp        | 30    | 28   | 28   |
| case2868rte       | ---   | 26   | 26   |
| case2869pegase    | 36    | 113  | 29   |
| case3012wp        | 43    | 28   | 29   |
| case3120sp        | 43    | 112  | 33   |
| case3375wp        | 47    | 30   | 30   |
| case6468rte       | ---   | 42   | 39   |
| case6470rte       | ---   | 43   | 44   |
| case6495rte       | ---   | 65   | 67   |
| case6515rte       | ---   | 64   | 51   |
| case9241pegase    | 40    | 64   | 41   |
| case_ACTIVSg2000  | 32    | 24   | 43   |
| case_ACTIVSg10k   | ---   | 39   | 80   |
| case13659pegase   | 73    | 38   | 50   |
| case_ACTIVSg25k   | ---   | 53   | 73   |
| case_ACTIVSg70k   | ---   | ---  | ---  |
| case21k           | 67    | 49   | 49   |
| case42k           | 78    | 59   | 59   |
| case99k           | 92    | 73   | 73   |
| case193k          | 106   | ---  | ---  |

Table 13: Number of iterations (MIPS-PARDISO).  

| Benchmark         | Flat  | MPC  | PF   |
|-------------------|-------|------|------|
| case1951rte       | ---   | 26   | 26   |
| case2383wp        | 31    | 29   | 31   |
| case2736sp        | 29    | 28   | 27   |
| case2737sp        | 27    | 25   | 25   |
| case2746wp        | 30    | 26   | 28   |
| case2748wp        | 30    | 28   | 28   |
| case2868rte       | ---   | 26   | 26   |
| case2869pegase    | 36    | 82   | 29   |
| case3012wp        | 43    | 28   | 29   |
| case3120sp        | 43    | 101  | 33   |
| case3375wp        | 47    | 30   | 30   |
| case6468rte       | ---   | 42   | 39   |
| case6470rte       | ---   | 43   | 44   |
| case6495rte       | ---   | 65   | 67   |
| case6515rte       | ---   | 64   | 51   |
| case9241pegase    | 40    | 64   | 41   |
| case_ACTIVSg2000  | 32    | 24   | 43   |
| case_ACTIVSg10k   | ---   | 39   | 79   |
| case13659pegase   | 63    | 38   | 55   |
| case_ACTIVSg25k   | ---   | 53   | 73   |
| case_ACTIVSg70k   | ---   | ---  | ---  |
| case21k           | 67    | 49   | 49   |
| case42k           | 78    | 59   | 60   |
| case99k           | 92    | 73   | 73   |
| case193k          | 106   | 87   | 87   |
Table 14: Overall time (s) (FMINCON 2015b).

| Benchmark       | Flat  | MPC   | PF    |
|-----------------|-------|-------|-------|
| case1951rte     | —     | 37.82 | 19.27 |
| case2383wp      | 30.39 | 60.74 | 64.53 |
| case2736sp      | 14.39 | —     | 13.49 |
| case2737sop     | 10.70 | 12.01 | 9.02  |
| case2746wop     | 11.42 | 19.94 | 12.35 |
| case2746wvp     | 13.08 | 140.85| 12.61 |
| case2868rte     | —     | 49.16 | 37.26 |
| case2869pegase  | 10.40 | 22.05 | 9.40  |
| case3012wp      | 33.27 | 404.88| —     |
| case3120sp      | 31.65 | —     | 17.85 |
| case3375wp      | 35.74 | —     | —     |
| case6468rte     | —     | 92.66 | 87.63 |
| case6495rte     | —     | 31.31 | 34.66 |
| case6515rte     | —     | 108.86| 79.90 |
| case9241pegase  | 149.04| 304.81| 191.83|
| case_ACTIVSg2000| 56.89 | 10.62 | 12.38 |
| case_ACTIVSg10k | 19.30 | —     | 58.32 |
| case13659pegase | 1,259.17| 2,601.37| 2,960.30|
| case_ACTIVSg25k | 481.70 | 214.44| 272.03 |
| case_ACTIVSg50k | 1,359.42| 836.19 | 923.42|
| case21k         | 669.99| 2,742.73| 2,228.12|
| case42k         | 1,834.85| —     | 6,163.04|
| case699k        | 26,188.50| 56,236.69| 62,359.75|
| case193k        | 49,073.79| —     | —     |

Table 15: Number of iterations (FMINCON 2015b).

| Benchmark       | Flat | MPC | PF |
|-----------------|-----|-----|----|
| case1951rte     | —   | 77  | 62 |
| case2383wp      | 111 | 184 | 85 |
| case2736sp      | 46  | —   | 44 |
| case2737sop     | 33  | 42  | 31 |
| case2746wop     | 33  | 61  | 39 |
| case2746wvp     | 39  | 333 | 41 |
| case2868rte     | —   | 58  | 61 |
| case2869pegase  | 27  | 50  | 28 |
| case3012wp      | 91  | 434 | —  |
| case3120sp      | 90  | —   | 52 |
| case3375wp      | 82  | —   | —  |
| case6468rte     | —   | 57  | 45 |
| case6495rte     | —   | 45  | 47 |
| case6515rte     | —   | 108 | 74 |
| case9241pegase  | 39  | 81  | 44 |
| case_ACTIVSg2000| 67  | 35  | 40 |
| case_ACTIVSg10k | —   | 18  | 52 |
| case13659pegase | 65  | 63  | 63 |
| case_ACTIVSg25k | 116 | 48  | 48 |
| case_ACTIVSg70k | 114 | 79  | 82 |
| case21k         | 128 | 437 | 396|
| case42k         | 135 | —   | 386|
| case999k        | 141 | 297 | 354|
| case193k        | 153 | —   | —  |

Table 16: Overall time (s) (FMINCON 2017b).

| Benchmark       | Flat  | MPC   | PF    |
|-----------------|-------|-------|-------|
| case1951rte     | —     | 42.13 | 20.69 |
| case2383wp      | 33.49 | 69.35 | 66.65 |
| case2736sp      | 15.75 | 324.89| 16.20 |
| case2737sop     | 11.84 | 14.92 | 11.93 |
| case2746wop     | 12.68 | 24.39 | 15.46 |
| case2746wvp     | 14.30 | 165.37| 15.86 |
| case2868rte     | —     | 54.29 | 41.41 |
| case2869pegase  | 11.18 | 25.07 | 12.53 |
| case3012wp      | 36.38 | —     | 21.58 |
| case3375wp      | 36.06 | —     | 21.58 |
| case6468rte     | 40.65 | 183.98| —     |
| case6470rte     | —     | 92.65 | 97.92 |
| case6495rte     | —     | 37.76 | 38.41 |
| case6515rte     | —     | 123.73| 80.07 |
| case6241pegase  | 141.15| 281.36| 186.36|
| case_ACTIVSg2000| 52.67 | 12.81 | 14.06 |
| case_ACTIVSg10k | —     | 26.88 | 61.70 |
| case13659pegase | 1,848.93| 1,866.80| 2,269.08|
| case_ACTIVSg25k | 442.05| 210.12| 254.40|
| case_ACTIVSg70k | 1,280.10| 786.05| 874.72|
| case21k         | 590.86| —     | 2,311.17|
| case42k         | 1,624.20| 8,428.31| —     |
| case999k        | 8,330.90| 20,554.62| 24,267.84|
| case193k        | 16,680.00| 70,150.87| —     |

Table 17: Number of iterations (FMINCON 2017b).

| Benchmark       | Flat | MPC | PF |
|-----------------|-----|-----|----|
| case1951rte     | —   | 77  | 62 |
| case2383wp      | 111 | 184 | 85 |
| case2736sp      | 46  | —   | 44 |
| case2737sop     | 33  | 42  | 31 |
| case2746wop     | 33  | 61  | 39 |
| case2746wvp     | 39  | 333 | 41 |
| case2868rte     | —   | 58  | 61 |
| case2869pegase  | 27  | 50  | 28 |
| case3012wp      | 91  | 434 | —  |
| case3120sp      | 90  | —   | 52 |
| case3375wp      | 82  | —   | —  |
| case6468rte     | —   | 57  | 45 |
| case6495rte     | —   | 45  | 47 |
| case6515rte     | —   | 108 | 74 |
| case9241pegase  | 39  | 81  | 44 |
| case_ACTIVSg2000| 67  | 35  | 40 |
| case_ACTIVSg10k | —   | 18  | 52 |
| case13659pegase | 62  | 81  | 99 |
| case_ACTIVSg25k | 116 | 48  | 48 |
| case_ACTIVSg70k | 114 | 79  | 82 |
| case21k         | 128 | —   | 383|
| case42k         | 135 | 435 | —  |
| case999k        | 141 | 324 | 359|
| case193k        | 153 | 482 | —  |
### Table 18: Overall time (s) (KNITRO 10).

| Benchmark       | Flat | MPC | PF  |
|-----------------|------|-----|-----|
| case1951te      | —    | 10.22 | 11.44 |
| case2383wp      | 5.09 | 4.92 | 4.84 |
| case2736wp      | 4.05 | —   | 4.36 |
| case2737wp      | 4.01 | 3.97 | 4.40 |
| case2746wop     | 3.94 | 45.41 | 4.56 |
| case2746wp      | 4.11 | 13.07 | 4.65 |
| case2868rte     | —    | —   | 23.06 |
| case2869pegase  | 4.53 | 4.74 | 4.95 |
| case3012wp      | 5.15 | 7.02 | 7.86 |
| case3120wp      | 5.15 | 5.35 | 5.06 |
| case3375wp      | 5.48 | 6.73 | 7.38 |
| case6468rte     | 94.71 | 42.25 | 45.22 |
| case6470rte     | —    | 10.17 | 9.91 |
| case6495rte     | —    | 9.33 | 9.50 |
| case6515rte     | 114.61 | 12.58 | 13.54 |
| case9241pegase  | 101.92 | 16.87 | 14.42 |
| case_ACTIVSg2000 | 4.16 | 4.37 | 4.70 |
| case_ACTIVSg10k  | 12.47 | 12.37 |
| case13609pegase | 166.12 | 93.37 | 160.42 |
| case_ACTIVSg25k  | 49.66 | 46.64 | 48.47 |
| case_ACTIVSg70k  | 163.39 | 162.04 | 183.33 |
| case21k         | 51.71 | 206.98 | 224.30 |
| case42k         | 164.11 | 1,147.66 | — |
| case99k         | 785.26 | 2,398.32 | 7,541.11 |
| case193k        | 1,038.66 | 2,778.65 | 2,914.72 |

### Table 19: Number of iterations (KNITRO 10).

| Benchmark       | Flat | MPC | PF  |
|-----------------|------|-----|-----|
| case1951te      | —    | 97    | 107   |
| case2383wp      | 34   | 32    | 29    |
| case2736wp      | 20   | —    | 20    |
| case2737wp      | 20   | 20    | 20    |
| case2746wop     | 20   | 440   | 22    |
| case2746wp      | 20   | 117   | 20    |
| case2868rte     | —    | —    | 169   |
| case2869pegase  | 22   | 25    | 23    |
| case3012wp      | 28   | 48    | 55    |
| case3120wp      | 27   | 28    | 24    |
| case3375wp      | 28   | 39    | 42    |
| case6468rte     | 360  | 185   | 194   |
| case6470rte     | —    | 33    | 33    |
| case6495rte     | —    | 36    | 36    |
| case6515rte     | 469  | 46    | 46    |
| case9241pegase  | 188  | 32    | 28    |
| case_ACTIVSg2000 | 18  | 21    | 20    |
| case_ACTIVSg10k  | 23   | 25    | —     |
| case13609pegase | 363  | 203   | 355   |
| case_ACTIVSg25k  | 46   | 45    | 45    |
| case_ACTIVSg70k  | 52   | 52    | 52    |
| case21k         | 43   | 181   | 163   |
| case42k         | 50   | 325   | —     |
| case99k         | 60   | 150   | 437   |
| case193k        | 59   | 145   | 156   |

### Table 20: Overall time (s) (KNITRO 11).

| Benchmark       | Flat | MPC | PF  |
|-----------------|------|-----|-----|
| case1951te      | —    | 11.39 | 13.09 |
| case2383wp      | 5.61 | 5.28 | 5.40 |
| case2736wp      | 4.33 | —    | 5.15 |
| case2737wp      | 4.27 | 4.48 | 4.87 |
| case2746wop     | 4.23 | 51.25 | 4.76 |
| case2746wp      | 4.30 | 14.48 | 4.71 |
| case2868rte     | —    | —    | 25.16 |
| case2869pegase  | 4.77 | 4.92 | 5.05 |
| case3012wp      | 5.34 | 8.09 | 9.11 |
| case3120wp      | 5.37 | 5.50 | 5.64 |
| case3375wp      | 6.01 | 6.92 | 7.75 |
| case6468rte     | 98.29 | 43.70 | 45.99 |
| case6470rte     | —    | 11.30 | 10.80 |
| case6495rte     | —    | 9.70 | 11.30 |
| case6515rte     | 123.62 | 13.28 | 13.29 |
| case9241pegase  | 97.82 | 17.22 | 14.87 |
| case_ACTIVSg2000 | 4.34 | 4.55 | 5.04 |
| case_ACTIVSg10k  | 12.17 | 13.06 |
| case13609pegase | 172.98 | 100.72 | 184.77 |
| case_ACTIVSg25k  | 53.18 | 50.18 | 50.18 |
| case_ACTIVSg70k  | 169.53 | 167.55 | 195.43 |
| case21k         | 52.13 | 219.83 | 229.28 |
| case42k         | 167.34 | 1,162.01 | — |
| case99k         | 792.86 | 2,558.94 | 7,209.15 |
| case193k        | 1,052.30 | 2,654.42 | 2,919.41 |

### Table 21: Number of iterations (KNITRO 11).

| Benchmark       | Flat | MPC | PF  |
|-----------------|------|-----|-----|
| case1951te      | —    | 97    | 107   |
| case2383wp      | 34   | 32    | 29    |
| case2736wp      | 20   | —    | 20    |
| case2737wp      | 20   | 20    | 20    |
| case2746wop     | 20   | 440   | 22    |
| case2746wp      | 20   | 117   | 20    |
| case2868rte     | —    | —    | 169   |
| case2869pegase  | 22   | 25    | 23    |
| case3012wp      | 28   | 48    | 55    |
| case3120wp      | 27   | 28    | 24    |
| case3375wp      | 28   | 39    | 42    |
| case6468rte     | 360  | 185   | 194   |
| case6470rte     | —    | 33    | 33    |
| case6495rte     | —    | 36    | 36    |
| case6515rte     | 469  | 46    | 46    |
| case9241pegase  | 188  | 32    | 28    |
| case_ACTIVSg2000 | 18  | 21    | 20    |
| case_ACTIVSg10k  | 23   | 25    | —     |
| case13609pegase | 363  | 203   | 355   |
| case_ACTIVSg25k  | 46   | 45    | 45    |
| case_ACTIVSg70k  | 52   | 52    | 52    |
| case21k         | 43   | 181   | 163   |
| case42k         | 50   | 325   | —     |
| case99k         | 60   | 150   | 437   |
| case193k        | 59   | 145   | 156   |
Figure 2: Initial guess performance profiles using IPOPT-PARDISO considering small and medium sized benchmarks.

Figure 3: Performance profiles for the initial guess using IPOPT-PARDISO considering large-scale benchmarks.

Figure 4: Performance profiles for the initial guess using BELTISTOS considering large-scale benchmarks.

Figure 5: Performance profiles for the initial guess using BELTISTOS considering all benchmarks.
4.3 OPF variants

The bus voltages in the standard AC OPF problem can be represented either in Cartesian, or polar coordinates. Another variation of the standard AC OPF problem uses current balance constraints in place of the power balance constraints. Different representations of the complex voltage variables and formulation of the nodal balance equations lead to a different number of constraints and sparsity structure of the problem, which, in turn, influences the numerical behavior of the optimizer. The corresponding MATPOWER options are `opf.v_cartesian`, specifying whether to use polar or Cartesian voltage coordinates, and option `opf.current_balance`, which selects using either a current or power balance formulation for AC OPF.

The results presented in Table 22 suggest that robust solver such as BELTISTOS is able to solve all OPF formulations, while for the rest of the solvers the choice of formulation can significantly influence whether the case can be successfully solved. MIPS (Section 4.3.2), and IPOPT-PARDISO optimizers are more robust with polar voltage coordinates and nodal power balance while the opposite is true for FMINCON and KNITRO, which is more robust with Cartesian voltage coordinates (see Section 4.3.4). Less robust optimizers are also not able to solve the large-scale cases due to the extensive time requirements of the linear solver or insufficient precision of the solution. The performance of various formulations is discussed in detail for BELTISTOS and IPOPT in Section 4.3.1.

Table 22: Number of solved benchmarks for different OPF formulations.

| Optimizer       | Polar Power | Polar Current | Cartesian Power | Cartesian Current |
|-----------------|-------------|---------------|-----------------|-------------------|
| MIPS-MATLAB'    | 23          | 21            | 21              | 20                |
| MIPS-PARDISO    | 23          | 20            | 23              | 17                |
| IPOPT-PARDISO   | 25          | 20            | 25              | 22                |
| IPOPT-MA57      | 22          | 21            | 21              | 21                |
| BELTISTOS       | 25          | 25            | 25              | 25                |
| FMINCON 2015b   | 19          | 18            | 24              | 23                |
| FMINCON 2017b   | 21          | 18            | 25              | 25                |
| KNITRO 10       | 23          | 24            | 25              | 24                |
| KNITRO 11       | 23          | 24            | 25              | 24                |

4.3.1 BELTISTOS and IPOPT

The performance profiles for various OPF formulations presented in Figure 9 and 10 were obtained using the BELTISTOS optimizer, which successfully solved all benchmark cases with all possible OPF formulations (Figure 6). The performance profiles reveal the gap between the polar and Cartesian voltage formulations. The polar formulations were observed to lead up to twofold speedup in the solution times when compared to the Cartesian voltage formulations.

When it comes to other optimizers, IPOPT-MA57 also displays slightly slower convergence with Cartesian voltage formulation for small and medium sized benchmarks. Neither OPF formulation was solved for large-scale benchmarks case21k–case193k due to prohibitive time requirements (see Figure 8). IPOPT-PARDISO does not seem to be influenced by voltage formulation, although it fails for large-scale benchmarks using current nodal balance equations, as can be observed in Figure 7. The MIPS solver performs better with polar voltage coordinates, while the opposite is true for FMINCON, which successfully converges for more benchmarks with Cartesian voltage coordinates, as presented in Table 22. There is also non-negligible influence of the nodal balance formulation. All optimizers prefer power based formulation of the nodal balance equations. The power balance was observed to be more robust and exhibit faster solution times in conjunction with both polar and Cartesian voltage formulations.
Figure 6: Iterations until convergence for various OPF formulations. Results obtained by BELTISTOS using MATPOWER case data as an initial guess.

Figure 7: Iterations until convergence for various OPF formulations. Results obtained by IPOPT-PARDISO using MATPOWER case data as an initial guess.

Figure 8: Iterations until convergence for various OPF formulations. Results obtained by IPOPT-MA57 using MATPOWER case data as an initial guess.
Table 23: Overall time (s) (BELTISTOS).

| Benchmark       | Polar-Power | Polar-Current | Cartesian-Power | Cartesian-Current |
|-----------------|-------------|---------------|-----------------|-------------------|
| case1951rte     | 4.57        | 3.53          | 3.60            | 3.71              |
| case2383wp      | 6.30        | 6.80          | 7.91            | 6.71              |
| case2736sp      | 3.65        | 4.33          | 5.08            | 4.77              |
| case2737sop     | 4.05        | 3.83          | 4.14            | 4.77              |
| case2746wp      | 3.25        | 2.99          | 3.89            | 4.89              |
| case2868rte     | 3.01        | 4.05          | 4.37            | 3.74              |
| case2869pegase  | 4.52        | 4.11          | 5.31            | 4.90              |
| case3012wp      | 7.18        | 6.95          | 6.91            | 6.52              |
| case3120sp      | 6.83        | 6.53          | 8.70            | 8.53              |
| case3375wp      | 6.70        | 6.74          | 6.61            | 6.51              |
| case6470rte     | 8.01        | 8.29          | 9.22            | 9.17              |
| case6470rte     | 6.49        | 5.28          | 8.78            | 8.02              |
| case6495rte     | 10.72       | 9.29          | 11.41           | 12.21             |
| case6515rte     | 9.42        | 10.61         | 14.87           | 10.45             |
| case9241pegase  | 14.00       | 11.62         | 16.36           | 20.98             |
| case_ACTIVSg2000| 4.20        | 3.92          | 3.86            | 3.62              |
| case_ACTIVSg10k | 13.11       | 12.98         | 9.71            | 9.50              |
| case13659pegase | 9.06        | 11.11         | 12.46           | 17.75             |
| case_ACTIVSg25k | 30.26       | 28.45         | 45.94           | 38.78             |
| case_ACTIVSg70k | 111.10      | 146.02        | 161.20          | 116.57            |
| case21k         | 81.35       | 76.88         | 89.55           | 80.70             |
| case42k         | 251.13      | 412.33        | 327.75          | 326.30            |
| case99k         | 782.43      | 1,458.81      | 854.51          | 1,393.17          |
| case193k        | 1,854.40    | 2,809.14      | 2,172.67        | 3,671.13          |

Table 24: Number of iterations (BELTISTOS).

| Benchmark       | Polar-Power | Polar-Current | Cartesian-Power | Cartesian-Current |
|-----------------|-------------|---------------|-----------------|-------------------|
| case1951rte     | 23          | 20            | 16              | 18                |
| case2383wp      | 42          | 44            | 45              | 42                |
| case2736sp      | 15          | 21            | 22              | 19                |
| case2737sop     | 14          | 18            | 15              | 16                |
| case2746wp      | 9           | 10            | 17              | 24                |
| case2868rte     | 14          | 15            | 27              | 27                |
| case2869pegase  | 12          | 22            | 20              | 16                |
| case3012wp      | 38          | 38            | 34              | 35                |
| case3120sp      | 37          | 35            | 49              | 43                |
| case3375wp      | 32          | 39            | 31              | 29                |
| case6468rte     | 30          | 34            | 33              | 35                |
| case6470rte     | 16          | 17            | 27              | 25                |
| case6495rte     | 35          | 37            | 46              | 50                |
| case6515rte     | 36          | 51            | 56              | 42                |
| case9241pegase  | 33          | 27            | 30              | 46                |
| case_ACTIVSg2000| 21          | 21            | 22              | 15                |
| case_ACTIVSg10k | 28          | 28            | 15              | 15                |
| case13659pegase | 20          | 30            | 26              | 43                |
| case_ACTIVSg25k | 27          | 27            | 36              | 34                |
| case_ACTIVSg_70k| 33          | 46            | 43              | 27                |
| case21k         | 55          | 54            | 52              | 52                |
| case42k         | 59          | 58            | 62              | 62                |
| case99k         | 71          | 79            | 72              | 72                |
| case193k        | 77          | 81            | 86              | 86                |
Table 25: Overall time (s) (IPOPT-PARDISO).

| Benchmark   | Polar-Power | Polar-Current | Cartesian-Power | Cartesian-Current |
|-------------|-------------|---------------|-----------------|-------------------|
| case1951rte | 7.16        | 3.62          | 5.31            | 3.60              |
| case2383wp  | 9.66        | 5.97          | 12.81           | 13.95             |
| case2736sp  | 6.54        | 4.00          | 3.61            | 3.75              |
| case2737sp  | 6.09        | 3.94          | 3.77            | 4.03              |
| case2746wp  | 5.43        | 3.32          | 2.89            | 3.16              |
| case2868rte | 7.46        | 3.59          | 4.20            | 3.70              |
| case2869rte | 9.21        | 16.97         | 12.08           | 13.62             |
| case3012wp  | 10.37       | 9.10          | 12.65           | 32.17             |
| case3120sp  | 12.95       | 6.57          | 15.04           | 8.92              |
| case3375wp  | 13.06       | —             | 12.42           | 205.81            |
| case6468rte | 12.23       | 8.91          | 11.52           | 10.36             |
| case6470rte | 41.56       | 50.25         | 36.67           | 54.12             |
| case6495rte | 21.73       | 10.39         | 15.52           | 15.51             |
| case6515rte | 16.85       | 11.67         | 12.15           | 11.39             |
| case9241pegase | 45.79 | 98.38 | 49.23 | 179.06 |
| case_ACTIVSg2000 | 7.45 | 4.66 | 5.69 | 5.22 |
| case_ACTIVSg10k | 26.64 | 19.06 | 18.60 | 17.46 |
| case_ACTIVSg25k | 266.01 | 259.19 | 400.67 | 559.03 |
| case_ACTIVSg70k | 83.68 | 63.04 | 67.87 | 94.37 |
| case21k | 262.59 | — | 166.43 | — |
| case42k | 747.87 | — | 621.89 | — |
| case99k | 3,101.47 | — | 3,080.08 | 47,944.78 |
| case193k | 10,105.87 | — | 12,489.93 | — |

Table 26: Number of iterations (IPOPT-PARDISO).

| Benchmark   | Polar-Power | Polar-Current | Cartesian-Power | Cartesian-Current |
|-------------|-------------|---------------|-----------------|-------------------|
| case1951rte | 33          | 34            | 33              | 31                |
| case2383wp  | 33          | 35            | 40              | 40                |
| case2736sp  | 40          | 23            | 27              | 25                |
| case2737sp  | 26          | 24            | 27              | 25                |
| case2746wp  | 24          | 17            | 18              | 21                |
| case2746wp  | 37          | 20            | 26              | 25                |
| case2868rte | 54          | 59            | 44              | 46                |
| case2869rte | 31          | 45            | 36              | 57                |
| case3012wp  | 31          | 31            | 34              | 31                |
| case3120sp  | 43          | 47            | 43              | 41                |
| case3375wp  | 38          | —             | 30              | 262               |
| case6468rte | 31          | 34            | 34              | 38                |
| case6470rte | 70          | 81            | 66              | 72                |
| case6495rte | 59          | 52            | 57              | 56                |
| case6515rte | 52          | 46            | 42              | 44                |
| case9241pegase | 44 | 71 | 41 | 79 |
| case_ACTIVSg2000 | 29 | 28 | 27 | 29 |
| case_ACTIVSg10k | 35 | 38 | 34 | 34 |
| case_ACTIVSg25k | 214 | 193 | 197 | 241 |
| case_ACTIVSg70k | 83 | 69 | 72 | 76 |
| case21k | 48          | —             | 49              | —                 |
| case42k | 52          | —             | 54              | —                 |
| case99k | 62          | —             | 63              | 312               |
| case193k | 77          | —             | 77              | —                 |
Table 27: Overall time (s) (IPOPT-MA57).

| Benchmark     | Polar-Power | Polar-Current | Cartesian-Power | Cartesian-Current |
|---------------|-------------|---------------|-----------------|-------------------|
| case1951rte   | 4.77        | 2.61          | 2.56            | 2.58              |
| case2383wp    | 5.20        | 3.90          | 4.21            | 3.95              |
| case2736sp    | 3.28        | 2.15          | 2.54            | 2.28              |
| case2737sop   | 3.84        | 1.80          | 2.25            | 2.20              |
| case2746wop   | 3.08        | 1.50          | 2.08            | 2.96              |
| case2868rte   | 3.21        | 1.67          | 2.90            | 2.87              |
| case2869pegase| 4.48        | 4.38          | 4.21            | 4.05              |
| case3012wp    | 4.94        | 3.35          | 4.76            | 3.83              |
| case3120sp    | 5.67        | 3.97          | 4.57            | 4.66              |
| case3375wp    | 5.53        | 4.14          | 4.96            | 4.92              |
| case3630rte   | 6.28        | 4.39          | 4.67            | 4.52              |
| case6470rte   | 7.91        | 5.26          | 9.06            | 11.37             |
| case6495rte   | 10.96       | 6.92          | 10.66           | 11.42             |
| case6515rte   | 12.41       | 8.75          | 184.76          | 162.49            |
| case9241pegase| 11.16       | 8.10          | 118.26          | 51.35             |
| case_ACTIVSg2000 | 21.78     | 14.05        | 21.35           | 16.33             |
| case_ACTIVSg10k | 3.70       | 2.60          | 3.25            | 3.05              |
| case_ACTIVSg10k | 16.21     | 11.60        | 12.71           | 14.63             |
| case_ACTIVSg25k | 30.32     | 15.69        | 27.97           | 22.44             |
| case_ACTIVSg70k | 62.12     | 33.39        | 49.59           | 51.31             |
| case_ACTIVSg2000 | 198.12    | 149.67       | 330.07          | 292.97            |
| case_ACTIVSg2000 | —         | —            | —               | —                 |
| case_ACTIVSg10k | 75,386.85 | —            | —               | —                 |
| case_ACTIVSg70k | 32         | 32           | 38              | 34                |
| case_ACTIVSg2000 | 42         | 39           | 40              | 40                |
| case_ACTIVSg10k | 41         | 38           | 55              | 42                |
| case_ACTIVSg70k | 42         | 42           | 39              | 40                |
| case_ACTIVSg2000 | 35         | 37           | 48              | 55                |
| case_ACTIVSg25k | 50         | 43           | 50              | 55                |
| case_ACTIVSg70k | 56         | 55           | 71              | 70                |
| case_ACTIVSg25k | 54         | 55           | 64              | 62                |
| case_ACTIVSg70k | 47         | 42           | 51              | 46                |
| case_ACTIVSg25k | 30         | 32           | 32              | 36                |
| case_ACTIVSg70k | 66         | 67           | 62              | 70                |
| case_ACTIVSg25k | 46         | 41           | 47              | 50                |
| case_ACTIVSg70k | 57         | 55           | 66              | 65                |
| case_ACTIVSg25k | —          | —            | —               | —                 |
| case_ACTIVSg70k | —          | —            | —               | —                 |
| case_ACTIVSg2000 | —          | —            | —               | —                 |
| case_ACTIVSg25k | —          | —            | —               | —                 |
| case_ACTIVSg70k | —          | —            | —               | —                 |

Table 28: Number of iterations (IPOPT-MA57).

| Benchmark     | Polar-Power | Polar-Current | Cartesian-Power | Cartesian-Current |
|---------------|-------------|---------------|-----------------|-------------------|
| case1951rte   | 34          | 33            | 30              | 32                |
| case2383wp    | 47          | 46            | 46              | 45                |
| case2736sp    | 19          | 22            | 25              | 22                |
| case2737sop   | 23          | 18            | 22              | 21                |
| case2746wop   | 14          | 14            | 29              | 30                |
| case2746wp    | 16          | 16            | 29              | 27                |
| case2868rte   | 33          | 43            | 39              | 38                |
| case2869pegase| 32          | 32            | 38              | 34                |
| case3012wp    | 42          | 39            | 40              | 40                |
| case3120sp    | 41          | 38            | 55              | 42                |
| case3375wp    | 42          | 42            | 39              | 40                |
| case6468rte   | 35          | 37            | 48              | 55                |
| case6470rte   | 50          | 43            | 50              | 55                |
| case6495rte   | 56          | 55            | 71              | 70                |
| case6515rte   | 54          | 55            | 64              | 62                |
| case9241pegase| 47          | 42            | 51              | 46                |
| case_ACTIVSg2000 | 24         | 26            | 33              | 31                |
| case_ACTIVSg10k | 30         | 32            | 32              | 36                |
| case_ACTIVSg25k | 66         | 67            | 62              | 70                |
| case_ACTIVSg70k | 46         | 41            | 47              | 50                |
| case_ACTIVSg25k | 57         | 55            | 66              | 65                |
| case_ACTIVSg70k | —          | —            | —               | —                 |
| case_ACTIVSg25k | —          | —            | —               | —                 |
| case_ACTIVSg70k | —          | —            | —               | —                 |
| case_ACTIVSg2000 | —          | —            | —               | —                 |
| case_ACTIVSg25k | —          | —            | —               | —                 |
| case_ACTIVSg70k | —          | —            | —               | —                 |
| case_ACTIVSg25k | —          | —            | —               | —                 |
| case_ACTIVSg70k | —          | —            | —               | —                 |

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Figure 9: Performance profiles for the OPF formulations using BELTISTOS considering large-scale benchmarks.

Figure 10: Performance profiles for the OPF formulations using BELTISTOS considering all benchmarks.

Figure 11: Performance profiles for the OPF formulations using IPOPT-PARDISO considering small and medium sized benchmarks.

Figure 12: Performance profiles for the OPF formulations using IPOPT-PARDISO considering large-scale benchmarks.
4.3.2 MIPS

MIPS with default MATLAB \backslash\ LS solver performs best with the Polar-Power formulation, failing only for 2 large-scale cases altogether. All large scale benchmarks were solved using the Cartesian-power formulation, while 4 small sized benchmarks failed. Overall time is very large compared to MIPS-PARDISO, where the majority of the time is spent by solving the KKT linear systems. Contribution of the LS solver to the overall performance is discussed in detail in Section 4.4.

Table 29: Overall time (s) (MIPS).

| Benchmark  | Polar-Power | Polar-Current | Cartesian-Power | Cartesian-Current |
|------------|-------------|---------------|-----------------|------------------|
| case1951rte | 5.57        | 2.99          | 11.71           | 41.08            |
| case2383wp  | 4.69        | 4.29          | 4.61            | 6.70             |
| case2736sp  | 5.19        | 3.78          | 4.86            | 7.71             |
| case2737op  | 5.04        | 4.04          | 4.20            | 4.78             |
| case2746wop | 5.46        | 3.89          | 4.15            | 4.65             |
| case2746wp  | 5.88        | 4.27          | 4.84            | 4.91             |
| case2868rte | 4.84        | —             | 5.00            | 29.21            |
| case2869pegase | 26.87     | 14.32         | 17.05           | —                |
| case3012wp  | 6.33        | 5.52          | 6.35            | 6.63             |
| case3120op  | 29.96       | 9.30          | 11.35           | 10.34            |
| case3375wp  | 7.62        | 7.67          | 7.99            | 9.01             |
| case6460rte | 17.48       | 88.08         | —               | 99.81            |
| case6470rte | 19.51       | —             | —               | —                |
| case6495rte | 29.85       | 43.03         | 73.43           | 73.41            |
| case6515rte | 30.03       | 20.55         | —               | 68.62            |
| case9241pegase | 50.50     | 104.00        | 106.71          | —                |
| case_ACTIVSg2000 | 6.03     | 16.43         | 4.87            | 16.20            |
| case_ACTIVSg10k | 32.08    | 53.12         | 65.39           | 68.48            |
| case_ACTIVSg25kg | 37.24 | —             | —               | —                |
| case_ACTIVSg50kg | 109.44 | 104.71        | 156.68          | 192.00           |
| case_ACTIVSg70kg | —        | 1,075.49      | 1,079.36        | 1,322.40         |
| case21k     | 171.01      | 142.27        | 183.87          | 171.01           |
| case42k     | 2,639.47    | 2,672.00      | 2,897.26        | 2,601.01         |
| case99k     | 25,936.42   | 18,011.52     | 19,458.39       | 18,472.06        |
| case193k    | —           | —             | 81,198.04       | —                |

Table 30: Number of iterations (MIPS).

| Benchmark  | Polar-Power | Polar-Current | Cartesian-Power | Cartesian-Current |
|------------|-------------|---------------|-----------------|------------------|
| case1951rte | 26          | 27            | 73              | 219              |
| case2383wp  | 29          | 33            | 35              | 46               |
| case2736sp  | 28          | 27            | 34              | 47               |
| case2737op  | 25          | 28            | 30              | 32               |
| case2746wop | 26          | 25            | 28              | 31               |
| case2746wp  | 28          | 29            | 34              | 35               |
| case2868rte | 26          | —             | 34              | 142              |
| case2869pegase | 113       | 75            | 76              | —                |
| case3012wp  | 28          | 29            | 34              | 35               |
| case3120wp  | 112         | 47            | 54              | 54               |
| case3375wp  | 30          | 36            | 37              | 40               |
| case6468rte | 42          | 196           | —               | 214              |
| case6470rte | 43          | —             | —               | —                |
| case6495rte | 65          | 98            | 152             | 150              |
| case6515rte | 64          | 54            | —               | 141              |
| case9241pegase | 64       | 132           | 120             | —                |
| case_ACTIVSg2000 | 24       | 69            | 27              | 68               |
| case_ACTIVSg10k | 39       | 66            | 77              | 86               |
| case_ACTIVSg25k | 53       | 59            | 80              | 94               |
| case_ACTIVSg70k | —        | 157           | 154             | 199              |
| case21k     | 49          | 49            | 57              | 57               |
| case42k     | 59          | 59            | 67              | 68               |
| case99k     | 73          | 73            | 79              | 79               |
| case193k    | —           | —             | 93              | —                |
MIPS with PARDISO LS solver displays preference for solving formulation with nodal power balance equations, while current formulation fails for larger number of cases, especially in combination with Cartesian voltage. Note the significantly lower solution times as compared with the default MATLAB’s LS solver.

### Table 31: Overall time (s) (MIPS-PARDISO).

| Benchmark     | Polar-Power | Polar-Current | Cartesian-Power | Cartesian-Current |
|---------------|-------------|---------------|-----------------|------------------|
| case1951rte   | 6.55        | 4.06          | 8.35            | 58.03            |
| case2383wp    | 5.89        | 5.44          | 6.05            | 8.25             |
| case2736sp    | 6.69        | 5.25          | 6.43            | 9.81             |
| case2737sop   | 6.07        | 5.28          | 5.46            | 5.96             |
| case2746wop   | 6.76        | 4.95          | 5.37            | 6.14             |
| case2746wp    | 7.01        | 5.87          | 6.74            | 7.02             |
| case2868rte   | 6.32        | —             | 6.91            | —                |
| case2869pegase| 22.90       | 17.17         | 18.82           | —                |
| case3012wp    | 7.15        | 6.14          | 7.19            | 7.55             |
| case3120sp    | 27.89       | 10.25         | 12.42           | 11.84            |
| case3375wp    | 8.29        | 8.61          | 8.61            | 9.03             |
| case6468rte   | 19.20       | 27.73         | —               | 102.66           |
| case6470rte   | 20.83       | —             | 107.49          | —                |
| case6495rte   | 32.97       | 42.32         | 95.03           | —                |
| case6515rte   | —           | 24.12         | —               | 80.58            |
| case9241pegase| 57.58       | 89.40         | 88.03           | —                |
| case_AACTIVSg2000 | 6.09    | 16.60         | 4.87            | 15.45            |
| case_AACTIVSg10k | 36.11    | 58.64         | 59.41           | 76.56            |
| case13659pegase| 42.16    | —             | 115.57          | —                |
| case_AACTIVSg25k | 118.95 | 118.36        | 164.44          | 208.94           |
| case_AACTIVSg70k | —       | 627.72        | 879.13          | 1,475.98         |
| case21k       | 140.35      | 126.71        | 141.39          | 142.27           |
| case42k       | 518.21      | —             | 438.26          | 541.52           |
| case99k       | 1,612.29    | 4,077.31      | 1,524.19        | —                |
| case193k      | 4,484.02    | —             | 3,530.59        | —                |

### Table 32: Number of iterations (MIPS-PARDISO).

| Benchmark     | Polar-Power | Polar-Current | Cartesian-Power | Cartesian-Current |
|---------------|-------------|---------------|-----------------|------------------|
| case1951rte   | 26          | 27            | 48              | 242              |
| case2383wp    | 29          | 33            | 35              | 46               |
| case2736sp    | 28          | 27            | 34              | 47               |
| case2737sop   | 25          | 28            | 30              | 32               |
| case2746wop   | 26          | 25            | 28              | 31               |
| case2746wp    | 28          | 29            | 34              | 35               |
| case2868rte   | 26          | —             | 34              | —                |
| case2869pegase| 82          | 72            | 73              | —                |
| case3012wp    | 28          | 29            | 34              | 35               |
| case3120sp    | 101         | 47            | 54              | 54               |
| case3375wp    | 30          | 36            | 37              | 40               |
| case6468rte   | 42          | 67            | —               | 200              |
| case6470rte   | 43          | —             | 191             | —                |
| case6495rte   | 65          | 88            | 174             | —                |
| case6515rte   | —           | 54            | —               | 146              |
| case9241pegase| 64          | 103           | 94              | —                |
| case_AACTIVSg2000 | 24     | 69            | 27              | 68               |
| case_AACTIVSg10k | 39       | 66            | 67              | 82               |
| case13659pegase| 38        | —             | 110             | —                |
| case_AACTIVSg25k | 53       | 59            | 78              | 94               |
| case_AACTIVSg70k | —        | 91            | 118             | 186              |
| case21k       | 49          | 53            | 56              | 59               |
| case42k       | 59          | —             | 67              | 78               |
| case99k       | 73          | 207           | 80              | —                |
| case193k      | 87          | —             | 101             | —                |
4.3.4 FMINCON

FMINCON is exceptionally robust with Cartesian voltage formulation, solving all the small benchmarks and all of the large-scale cases up to 42k buses. The number of iterations until convergence is also significantly improved with the Cartesian formulations. The overall time starts to rapidly grow for the large-scale cases above 42k buses, as compared to the performance of IPOPT or BELTISTOS.

Table 33: Time to solution in seconds (FMINCON 2015b).

| Benchmark  | Polar-Power | Polar-Current | Cartesian-Power | Cartesian-Current |
|------------|-------------|---------------|-----------------|------------------|
| case1951rte| 41.30       | 172.36        | 10.71           | 26.93            |
| case2383wp | 60.49       | 186.78        | 92.97           | 25.56            |
| case2736sp | —           | 53.72         | 15.83           | 19.31            |
| case2737sop| 12.81       | 10.81         | 11.92           | 11.78            |
| case2746wop| 20.92       | 386.63        | 9.81            | 14.69            |
| case2746wp | 138.32      | —             | 13.75           | 13.36            |
| case2868rte| 47.87       | 52.78         | 35.28           | 20.59            |
| case2869pegase| 21.91   | 28.90         | 24.45           | 92.40            |
| case3012wp | 390.19      | —             | 22.30           | 22.46            |
| case3120sp | —           | —             | 40.37           | 51.10            |
| case3375wp | —           | —             | 30.87           | 57.18            |
| case6468rte| —           | 64.31         | 46.83           | 128.46           |
| case6470rte| 89.38       | 72.50         | 36.29           | 442.63           |
| case6495rte| 31.74       | 174.01        | 42.59           | 474.82           |
| case6515rte| 105.27      | 191.54        | 81.70           | 149.07           |
| case9241pegase| 292.17  | 386.13        | 226.68          | 1,137.05         |
| case_ACTIVSg2000 | 11.41 | 11.22 | 12.03 | 15.07 |
| case_ACTIVSg10k| 25.48 | 43.15 | 39.23 | 235.09 |
| case13659pegase| 2,510.03 | 5,061.67 | 6,468.63 | 1,850.49 |
| case_ACTIVSg25k| 214.47 | 217.22 | 523.42 | 381.75 |
| case_ACTIVSg70k| 798.08 | 1,440.00 | 15,290.34 | 2,961.66 |
| case21k | 2,460.14 | 5,140.34 | 367.37 | 406.98 |
| case42k | — | — | 4,602.00 | 5,093.77 |
| case99k | 53,656.32 | — | 42,371.89 | — |
| case193k | — | — | — | — |

Table 34: Iterations until convergence (FMINCON 2015b).

| Benchmark  | Polar-Power | Polar-Current | Cartesian-Power | Cartesian-Current |
|------------|-------------|---------------|-----------------|------------------|
| case1951rte| 77          | 90            | 39              | 45               |
| case2383wp | 184         | 465           | 60              | 70               |
| case2736sp | —           | 162           | 42              | 50               |
| case2737sop| 42          | 40            | 32              | 32               |
| case2746wop| 61          | 342           | 25              | 38               |
| case2746wp | 333         | —             | 35              | 35               |
| case2868rte| 58          | 51            | 31              | 29               |
| case2869pegase| 50       | 61            | 52              | 96               |
| case3012wp | 434         | —             | 51              | 53               |
| case3120sp | —           | —             | 72              | 106              |
| case3375wp | —           | —             | 65              | 19               |
| case6468rte| —           | 50            | 51              | 71               |
| case6470rte| 57          | 73            | 35              | 54               |
| case6495rte| 45          | 141           | 55              | 68               |
| case6515rte| 108         | 94            | 65              | 58               |
| case9241pegase| 81      | 92            | 75              | 94               |
| case_ACTIVSg2000 | 35     | 36            | 35              | 40               |
| case_ACTIVSg10k| 18      | 35            | 27              | 50               |
| case13659pegase| 63     | 98            | 96              | 62               |
| case_ACTIVSg25k| 48      | 51            | 100             | 65               |
| case_ACTIVSg70k| 79      | 73            | 129             | 160              |
| case21k | 437         | 410           | 59              | 68               |
| case42k | —           | —             | 64              | 64               |
| case99k | 297         | —             | 83              | —                |
| case193k | —           | —             | —               | —                |
Table 35: Time to solution in seconds (FMINCON 2017b).

| Benchmark         | Polar-Power | Polar-Current | Cartesian-Power | Cartesian-Current |
|-------------------|-------------|---------------|-----------------|------------------|
| case1951rte       | 42.13       | 46.21         | 14.09           | 32.63            |
| case2383wp        | 69.35       | 163.32        | 102.23          | 30.05            |
| case2736wp        | 324.89      | 60.62         | 19.03           | 22.62            |
| case2737sop       | 14.92       | 13.87         | 15.33           | 14.60            |
| case2746wp        | 24.39       | 511.82        | 13.01           | 17.44            |
| case2868rte       | 165.37      | —             | 17.27           | 16.61            |
| case2869pegase    | 54.29       | 284.08        | 40.68           | 24.92            |
| case3012wp        | 25.07       | 32.82         | 30.08           | 103.72           |
| case3120sp        | —           | 230.33        | 25.37           | 27.22            |
| case3375wp        | 183.98      | —             | 45.17           | 56.38            |
| case6470rte       | —           | 69.92         | 51.51           | 280.19           |
| case6495rte       | 92.65       | 86.24         | 42.04           | 294.48           |
| case6515rte       | 37.76       | 538.31        | 48.09           | 701.07           |
| case9241pegase    | 123.73      | 183.28        | 93.03           | 157.91           |
| case_ACTIVSg2000  | 281.36      | 311.00        | 247.35          | 1,126.93         |
| case_ACTIVSg10k   | 12.81       | 14.73         | 15.30           | 19.94            |
| case_ACTIVSg10k   | 26.88       | 56.26         | 52.87           | 274.61           |
| case_ACTIVSg25k   | 1,866.80    | 1,140.71      | 6,085.81        | 4,190.40         |
| case_ACTIVSg70k   | 210.12      | 224.92        | 549.13          | 397.45           |
| case_ACTIVSg70k   | 786.05      | 1,464.73      | 15,831.40       | 10,398.00        |
| case21k           | —           | —             | 434.96          | 486.57           |
| case42k           | 8,428.31    | —             | 1,322.59        | 1,461.98         |
| case99k           | 20,554.62   | —             | 16,603.51       | 20,352.13        |
| case193k          | 70,150.87   | —             | 14,281.20       | 45,494.82        |

Table 36: Iterations until convergence (FMINCON 2017b).

| Benchmark         | Polar-Power | Polar-Current | Cartesian-Power | Cartesian-Current |
|-------------------|-------------|---------------|-----------------|------------------|
| case1951rte       | 77          | 78            | 39              | 45               |
| case2383wp        | 184         | 462           | 60              | 70               |
| case2736wp        | 455         | 159           | 42              | 50               |
| case2737sop       | 42          | 40            | 32              | 32               |
| case2746wp        | 61          | 407           | 25              | 38               |
| case2868rte       | 343         | —             | 35              | 35               |
| case2869pegase    | 58          | 66            | 31              | 29               |
| case3012wp        | 50          | 61            | 52              | 87               |
| case3120sp        | —           | 495           | 51              | 53               |
| case3375wp        | 383         | —             | 65              | 19               |
| case6468rte       | —           | 50            | 51              | 71               |
| case6470rte       | 57          | 73            | 35              | 45               |
| case6495rte       | 45          | 135           | 55              | 74               |
| case6515rte       | 108         | 88            | 65              | 58               |
| case9241pegase    | 81          | 94            | 75              | 81               |
| case_ACTIVSg2000  | 35          | 36            | 35              | 40               |
| case_ACTIVSg10k   | 18          | 35            | 27              | 50               |
| case_ACTIVSg25k   | 81          | 79            | 75              | 126              |
| case_ACTIVSg70k   | 79          | 78            | 88              | 91               |
| case21k           | —           | 59            | 68              | 68               |
| case42k           | 435         | —             | 64              | 64               |
| case99k           | 324         | —             | 83              | 82               |
| case193k          | 482         | —             | 106             | 108              |
4.3.5 KNITRO

For KNITRO, the default OPF formulation with polar voltage and power balance displays the weakest performance, failing to solve two cases and more than one third of the benchmarks require extensive number of iterations, and thus time, in order to converge when compared to other formulations. In general, Cartesian voltage coordinates should be the preferred choice.

| Benchmark     | Polar-Power | Polar-Current | Cartesian-Power | Cartesian-Current |
|---------------|-------------|---------------|-----------------|-------------------|
| case1951rte   | 10.22       | 5.44          | 4.40            | 8.62              |
| case2383wp    | 4.92        | 4.84          | 4.77            | 4.91              |
| case2736sp    | —           | 4.85          | 4.39            | 4.53              |
| case2737sop   | 3.97        | 4.21          | 4.55            | 4.21              |
| case2746wop   | 45.41       | 3.94          | 5.27            | 4.13              |
| case2746wp    | 13.07       | 4.61          | 4.51            | 4.67              |
| case2868rte   | —           | 9.32          | 5.48            | 10.41             |
| case2869pegase| 4.74        | 5.69          | 5.63            | 6.46              |
| case3012wp    | 7.02        | 5.26          | 5.61            | 5.36              |
| case3120sp    | 5.35        | 5.31          | 5.12            | 5.82              |
| case3375wp    | 6.73        | 5.89          | 5.79            | 6.07              |
| case6468rte   | 42.25       | 9.35          | 8.94            | 14.71             |
| case6470rte   | 10.17       | 12.52         | 10.58           | 14.80             |
| case6495rte   | 9.33        | 13.31         | 11.81           | 21.10             |
| case6515rte   | 12.58       | 13.80         | 10.01           | 17.39             |
| case9241pegase| 16.87       | 21.66         | 18.16           | 119.23            |
| case_ACTIVSg2000 | 4.37    | 4.30          | 4.41            | 4.77              |
| case_ACTIVSg10k | 12.47     | 11.98         | 14.02           | 15.82             |
| case13659pegase | 93.37  | —             | 133.52          | 51.77             |
| case_ACTIVSg25k | 46.64    | 47.32         | 55.67           | 51.77             |
| case_ACTIVSg70k | 162.04   | 150.89        | 200.42          | 216.53            |
| case21k       | 206.98      | 57.54         | 66.96           | 68.14             |
| case42k       | 1,147.66    | 160.24        | 157.81          | 156.70            |
| case99k       | 2,396.32    | 491.46        | 531.30          | 517.16            |
| case193k      | 2,778.65    | 949.67        | 1,085.04        | 987.93            |

Table 37: Overall time (s) (KNITRO 10).

| Benchmark     | Polar-Power | Polar-Current | Cartesian-Power | Cartesian-Current |
|---------------|-------------|---------------|-----------------|-------------------|
| case1951rte   | 97          | 42            | 26              | 68               |
| case2383wp    | 32          | 32            | 29              | 29               |
| case2736sp    | —           | 25            | 21              | 22               |
| case2737sop   | 20          | 23            | 21              | 21               |
| case2746wop   | 440         | 18            | 19              | 20               |
| case2746wp    | 117         | 25            | 22              | 22               |
| case2868rte   | —           | 65            | 24              | 66               |
| case2869pegase| 25          | 34            | 28              | 33               |
| case3012wp    | 48          | 28            | 25              | 24               |
| case3120sp    | 28          | 30            | 28              | 30               |
| case3375wp    | 39          | 27            | 26              | 26               |
| case6468rte   | 185         | 39            | 31              | 51               |
| case6470rte   | 33          | 45            | 35              | 46               |
| case6495rte   | 36          | 54            | 40              | 68               |
| case6515rte   | 46          | 52            | 35              | 59               |
| case9241pegase| 32          | 47            | 37              | 240              |
| case_ACTIVSg2000 | 21       | 20            | 22              | 23               |
| case_ACTIVSg10k | 23        | 26            | 28              | 32               |
| case13659pegase | 203       | —             | 234             | —                |
| case_ACTIVSg25k | 45        | 48            | 47              | 48               |
| case_ACTIVSg70k | 52        | 53            | 55              | 57               |
| case21k       | 181         | 43            | 42              | 42               |
| case42k       | 325         | 48            | 48              | 47               |
| case99k       | 150         | 48            | 54              | 54               |
| case193k      | 145         | 57            | 54              | 53               |

Table 38: Number of iterations (KNITRO 10).
Table 39: Overall time (s) (KNITRO 11).

| Benchmark      | Polar-Power | Polar-Current | Cartesian-Power | Cartesian-Current |
|----------------|-------------|---------------|-----------------|-------------------|
| case1951rte    | 11.39       | 5.70          | 4.68            | 9.24              |
| case2383wp     | 5.28        | 5.01          | 5.19            | 4.87              |
| case2736wp     | —           | 5.62          | 4.46            | 5.20              |
| case2737wp     | 4.48        | 4.67          | 4.61            | 4.67              |
| case2746wp     | 51.25       | 4.20          | 4.39            | 4.74              |
| case2868rte    | 14.48       | 4.92          | 4.82            | 4.65              |
| case2869rte    | —           | 10.21         | 5.38            | 10.38             |
| case2869rte    | 4.92        | 6.06          | 6.08            | 7.06              |
| case2869rte    | 8.09        | 5.44          | 5.55            | 5.62              |
| case3012wp     | 5.50        | 5.68          | 6.15            | 6.59              |
| case3012wp     | 6.92        | 5.79          | 6.18            | 7.05              |
| case6468rte    | 43.70       | 10.10         | 8.96            | 14.69             |
| case6470rte    | 11.30       | 12.58         | 11.11           | 14.02             |
| case6495rte    | 9.70        | 15.02         | 11.31           | 21.06             |
| case6515rte    | 13.28       | 14.40         | 11.04           | 18.88             |
| case9241rte    | 17.22       | 20.67         | 19.75           | 125.74            |
| case9241rte    | 4.55        | 4.38          | 4.79            | 5.34              |
| case9241rte    | 12.17       | 12.84         | 15.96           | 16.13             |
| case13659rte   | 100.72      | —             | 141.17          | —                 |
| case13659rte   | 50.18       | 49.10         | 56.12           | 54.33             |
| case13659rte   | 167.55      | 157.18        | 196.43          | 213.33            |
| case21k        | 219.83      | 57.65         | 67.32           | 70.69             |
| case24k        | 1,162.01    | 166.12        | 163.23          | 156.66            |
| case24k        | 2,558.94    | 528.73        | 523.98          | 559.30            |
| case24k        | 2,654.42    | 960.44        | 1,058.22        | 1,118.77          |

Table 40: Number of iterations (KNITRO 11).

| Benchmark      | Polar-Power | Polar-Current | Cartesian-Power | Cartesian-Current |
|----------------|-------------|---------------|-----------------|-------------------|
| case1951rte    | 97          | 42            | 26              | 68                |
| case2383wp     | 32          | 32            | 29              | 29                |
| case2736wp     | 20          | 23            | 21              | 22                |
| case2737wp     | 440         | 18            | 19              | 20                |
| case2746wp     | 117         | 25            | 22              | 22                |
| case2868rte    | —           | 65            | 24              | 66                |
| case2869rte    | 25          | 34            | 28              | 33                |
| case3012wp     | 48          | 28            | 25              | 24                |
| case3012wp     | 28          | 30            | 28              | 30                |
| case3375wp     | 39          | 27            | 26              | 26                |
| case6468rte    | 185         | 39            | 31              | 51                |
| case6470rte    | 33          | 45            | 35              | 46                |
| case6495rte    | 36          | 54            | 40              | 68                |
| case6515rte    | 46          | 52            | 35              | 59                |
| case9241rte    | 32          | 47            | 37              | 240               |
| case9241rte    | 21          | 20            | 22              | 23                |
| case9241rte    | 23          | 26            | 28              | 32                |
| case13659rte   | 203         | —             | 234             | —                 |
| case13659rte   | 45          | 48            | 47              | 48                |
| case13659rte   | 52          | 53            | 55              | 57                |
| case21k        | 181         | 43            | 42              | 42                |
| case24k        | 325         | 48            | 48              | 47                |
| case99k        | 150         | 48            | 54              | 54                |
| case193k       | 145         | 57            | 54              | 53                |
4.4 Linear solvers

The robustness and performance of the optimization package heavily depends on used sparse direct solver used to compute update of the solution in each iteration. Linear systems resulting from the IPM methods are known to be very ill-conditioned, especially in the last iterations before convergence and depending on the benchmark case, can be also very large.

Tables 41, 42, 43 and 44 demonstrate the difference between build-in Matlab linear solver (SuiteSparse UMFPACK, also known as the ‘\’ operator) and PARDISO, which are currently the two linear solvers supported by MIPS framework. The difference is particularly visible in Table 42 and 44 for the large-scale cases, where PARDISO outperforms MATLAB’s backslash operator by a factor up to 26. PARDISO also reduces the number of iterations until convergence for some cases due to more accurate solution and thus better descent search direction provided to the optimizer.

Table 41: Performance of MIPS with various linear solvers - number of iterations.

| Benchmark       | Flat         |          |          |          |          |          |
|-----------------|--------------|----------|----------|----------|----------|----------|
|                 | PARDISO      | Backslash | PARDISO  | Backslash | PARDISO  | Backslash |
| case1951rte     | —            | —        | 26       | 26       | 26       | 26       |
| case2363wp      | 31           | 31       | 29       | 29       | 31       | 31       |
| case2736sp      | 29           | 29       | 28       | 28       | 27       | 27       |
| case2737sop     | 27           | 27       | 25       | 25       | 25       | 25       |
| case2746wop     | 30           | 30       | 26       | 26       | 28       | 28       |
| case2746wp      | 30           | 30       | 28       | 28       | 28       | 28       |
| case2868rte     | —            | —        | 26       | 26       | 26       | 26       |
| case2869pegase  | 36           | 36       | 82       | 113      | 29       | 29       |
| case3012wp      | 43           | 43       | 26       | 26       | 29       | 29       |
| case3120sp      | 43           | 43       | 101      | 112      | 33       | 33       |
| case3375wp      | 47           | 47       | 30       | 30       | 30       | 30       |
| case6468rte     | —            | —        | 42       | 42       | 39       | 39       |
| case6470rte     | —            | —        | 43       | 43       | 44       | 44       |
| case6495rte     | —            | —        | 65       | 65       | 67       | 67       |
| case6515rte     | —            | —        | 64       | 51       | 51       | 51       |
| case9241pegase  | 40           | 40       | 64       | 64       | 41       | 41       |
| case_ACTIVSg2000| 32           | 32       | 24       | 24       | 43       | 43       |
| case_ACTIVSg10k | —            | —        | 39       | 39       | 79       | 80       |
| case_ACTIVSg25k | —            | —        | 53       | 53       | 73       | 73       |
| case_ACTIVSg70k | 63           | 73       | 38       | 38       | 55       | 50       |
| case21k         | 67           | 67       | 49       | 49       | 49       | 49       |
| case42k         | 78           | 78       | 59       | 59       | 60       | 59       |
| case99k         | 92           | 92       | 73       | 73       | 73       | 73       |
| case193k        | 106          | —        | 87       | —        | 87       | —        |
Table 42: Performance of MIPS with various linear solvers - overall time (s).

| Benchmark       | Flat | MPC | PF |
|-----------------|------|-----|----|
|                 | PARDISO | Backslash | PARDISO | Backslash | PARDISO | Backslash |
| case1951rte     | —     | —   | 3.81 | 3.38 | 4.21 | 3.60 |
| case2383wp      | 6.70  | 5.69 | 5.27 | 4.72 | 5.58 | 5.01 |
| case2736wp      | 7.08  | 6.47 | 5.15 | 5.13 | 5.13 | 4.85 |
| case2737sop     | 6.55  | 6.42 | 4.61 | 4.51 | 5.01 | 4.92 |
| case2746wop     | 7.45  | 7.18 | 5.33 | 5.25 | 5.59 | 5.29 |
| case2746wp      | 7.55  | 7.23 | 5.65 | 5.18 | 5.76 | 5.50 |
| case2868rte     | —     | —   | 5.46 | 4.54 | 5.40 | 4.88 |
| case2869pegase  | 9.89  | 9.47 | 21.91 | 28.74 | 6.80 | 6.25 |
| case3012wp      | 11.04 | 10.26 | 5.96 | 5.44 | 6.57 | 5.54 |
| case3120sp      | 10.89 | 9.97 | 26.65 | 29.65 | 7.31 | 7.15 |
| case3375wp      | 13.22 | 12.57 | 7.25 | 6.72 | 7.20 | 6.47 |
| case6468rte     | —     | —   | 17.46 | 16.71 | 16.60 | 15.18 |
| case6470rte     | —     | —   | 19.02 | 17.29 | 19.72 | 17.72 |
| case6495rte     | —     | —   | 29.91 | 26.85 | 31.51 | 27.89 |
| case6515rte     | —     | —   | 27.55 | 23.00 | —     | —     |
| case9241pegase  | 34.17 | 31.75 | 51.09 | 47.34 | 32.38 | 30.25 |
| case_ACTIVSg2000| 7.18  | 7.86 | 4.45 | 4.63 | 9.34 | 9.50 |
| case_ACTIVSg10k | —     | —   | 29.96 | 28.24 | 66.89 | 67.86 |
| case_ACTIVSg25k | —     | —   | 100.54 | 99.76 | 148.73 | 152.40 |
| case13659pegase | 66.44 | 64.55 | 36.05 | 34.34 | 35.34 | 35.35 |
| case13659rte    | 26    | 26   | 29.11 | 26.85 | 31.51 | 27.89 |
| case21k         | —     | 101  | 336.77 | 102.68 | 191.82 | 105.03 |
| case42k         | 464.73 | 1215.27 | 367.20 | 311.24 | 613.85 | 613.85 |
| case99k         | 2036.95 | 27695.39 | 1284.73 | 1284.73 | 33890.39 | 33890.39 |
| case193k        | 4743.55 | 3175.67 | 73.17 | 61.69 | 1518.40 | 1518.40 |

Table 43: Performance of MIPS with various linear solvers - number of iterations.

| Benchmark       | Polar-Power | Polar-Current | Cartesian-Power | Cartesian-Current |
|-----------------|-------------|---------------|-----------------|-------------------|
|                 | PARDISO | Backslash | PARDISO | Backslash | PARDISO | Backslash | PARDISO | Backslash |
| case1951rte     | 24     | 55     | 27     | 55     | —     | —     | 24     | 55     |
| case2383wp      | 29     | 29     | 29     | 55     | 27     | 27     | 27     | 27     |
| case2736wp      | 28     | 28     | 28     | 28     | 28     | 28     | 28     | 28     |
| case2737sop     | 25     | 25     | 25     | 25     | 25     | 25     | 25     | 25     |
| case2746wop     | 28     | 28     | 28     | 28     | 28     | 28     | 28     | 28     |
| case2746wp      | 26     | 26     | 26     | 26     | 26     | 26     | 26     | 26     |
| case2868rte     | 26     | 26     | 26     | 26     | 26     | 26     | 26     | 26     |
| case2869pegase  | 82     | 113    | 72     | 72     | 73     | 73     | 73     | 73     |
| case3012wp      | 28     | 28     | 28     | 28     | 28     | 28     | 28     | 28     |
| case3120sp      | 101    | 112    | 47     | 47     | 54     | 54     | 54     | 54     |
| case3375wp      | 30     | 30     | 30     | 30     | 30     | 30     | 30     | 30     |
| case6468rte     | 42     | 42     | 42     | 42     | 42     | 42     | 42     | 42     |
| case6470rte     | 43     | 43     | 43     | 43     | 43     | 43     | 43     | 43     |
| case6495rte     | 65     | 65     | 65     | 65     | 65     | 65     | 65     | 65     |
| case6515rte     | 64     | 64     | 64     | 64     | 64     | 64     | 64     | 64     |
| case9241pegase  | 64     | 64     | 64     | 64     | 64     | 64     | 64     | 64     |
| case_ACTIVSg2000| 24     | 24     | 24     | 24     | 24     | 24     | 24     | 24     |
| case_ACTIVSg10k | 39     | 39     | 39     | 39     | 39     | 39     | 39     | 39     |
| case_ACTIVSg25k | 53     | 53     | 53     | 53     | 53     | 53     | 53     | 53     |
| case13659pegase | 38     | 38     | 38     | 38     | 38     | 38     | 38     | 38     |
| case_ACTIVSg70k | —     | —     | —     | —     | —     | —     | —     | —     |
| case21k         | 49     | 49     | 49     | 49     | 49     | 49     | 49     | 49     |
| case42k         | 59     | 59     | 59     | 59     | 59     | 59     | 59     | 59     |
| case99k         | 73     | 73     | 73     | 73     | 73     | 73     | 73     | 73     |
| case193k        | 87     | 87     | 87     | 87     | 87     | 87     | 87     | 87     |
| Benchmark   | Polar-Power | Polar-Current | Cartesian-Power | Cartesian-Current |
|-------------|-------------|---------------|-----------------|-------------------|
|             | PARDISO     | Backslash     | PARDISO         | Backslash         |
| case1951rte | 6.55        | 5.57          | 4.06            | 2.99              |
| case2383wp  | 5.89        | 4.69          | 5.44            | 4.29              |
| case2736sp  | 6.69        | 5.19          | 5.25            | 3.78              |
| case2737sop | 6.07        | 5.04          | 5.28            | 4.04              |
| case2746wop | 6.76        | 5.46          | 4.95            | 3.89              |
| case2746wp  | 7.01        | 5.88          | 5.87            | 4.27              |
| case2868rte | 6.32        | 4.84          | —               | —                 |
| case2869pegase | 22.90   | 26.87          | 17.17           | 14.32             |
| case3012wp  | 7.15        | 6.33          | 6.14            | 5.52              |
| case3120sp  | 27.89       | 29.96         | 10.25           | 9.30              |
| case3375wp  | 8.29        | 7.62          | 8.61            | 7.67              |
| case6468rte | 19.28       | 17.48         | 27.73           | 28.08             |
| case6470rte | 20.83       | 19.51         | —               | 107.49            |
| case6495rte | 32.97       | 29.85         | 42.32           | 43.03             |
| case6515rte | —           | 30.03         | 24.12           | 20.55             |
| case9241pegase | 57.58   | 50.50          | 89.40           | 104.00            |
| case_ACTIVSg2000 | 6.09     | 6.03          | 16.60           | 16.43             |
| case_ACTIVSg10k | 36.11     | 32.08         | 58.64           | 53.12             |
| case_ACTIVSg25k | 118.95  | 109.44        | 118.36          | 104.71            |
| case_ACTIVSg70k | 42.16    | 37.24         | —               | 115.57            |
| case13659pegase | —       | —             | 627.22          | 1,075.49          |
| case21k     | 140.55      | 171.01        | 126.71          | 142.27            |
| case42k     | 518.21      | 2,639.47      | —               | 2,672.00          |
| case99k     | 1,612.28    | 25,936.42     | 4,077.31        | 18,011.52         |
| case193k    | 4,484.02    | —             | —               | 3,530.59          |
Tables 45, 46, 47, and 48 demonstrate performance of the IPOPT with two different linear solvers, PARDISO and HSL MA57. MA57 is a robust solver but as the problem size increases it requires significantly higher computational resources than PARDISO. On the other hand, computational times using IPOPT with PARDISO remain feasible also for the large-scale networks and it is thus possible to solve more benchmarks.

Table 45: Performance of IPOPT with various linear solvers - number of iterations.

| Benchmark | Flat | MPC | PF |
|-----------|------|-----|----|
|           |      |     |    |
| case1951rte | 122  | 141 | 30 |
| case2383wp | 38   | 32  | 34 |
| case2736sp | 27   | 28  | 23 |
| case2746wop | 38   | 32  | 24 |
| case2746wp | 40   | 33  | 35 |
| case2868rte | —    | 90  | 45 |
| case3012wp | 41   | 36  | 31 |
| case3120sp | 42   | 39  | 44 |
| case3375wp | 40   | 40  | 72 |
| case4688rte | 240  | 149 | 31 |
| case6495rte | 74   | 177 | 56 |
| case6515rte | 131  | 133 | 41 |
| case9241pveage | 119  | 41  | 56 |
| case_ACTIVSg2000 | 51   | 94  | 29 |
| case_ACTIVSg10k | 139  | 47  | 35 |
| case_ACTIVSg25k | 244  | 60  | 193|
| case_ACTIVSg70k | 85   | 75  | 66 |
| case21k | 141  | —   | 126|
| case42k | 65   | —   | 53 |
| case99k | 81   | —   | 61 |
| case193k | 98   | —   | 76 |

Table 46: Performance of IPOPT with various linear solvers - overall time (s).

| Benchmark | Flat | MPC | PF |
|-----------|------|-----|----|
|           |      |     |    |
| case1951rte | 25.55 | 12.61 | 4.79 |
| case2383wp | 9.00 | 3.63 | 6.89 |
| case2736wp | 9.80 | 4.53 | 7.46 |
| case2737wp | 8.37 | 3.93 | 5.82 |
| case2746wp | 8.06 | 4.41 | 4.60 |
| case2746wp | 7.40 | 4.40 | 5.91 |
| case2868rte | —    | 10.30 | 13.07|
| case3012wp | 18.47 | 4.92 | 11.47|
| case3120wp | 22.93 | 5.79 | 15.47|
| case3375wp | 11.83 | 5.82 | 21.01|
| case6468rt | 82.40 | 29.45 | 6.96|
| case6470rte | 112.06 | 22.02 | 29.25|
| case6495rte | 27.24 | 35.88 | 20.85|
| case6515rte | 64.83 | 25.45 | 9.03|
| case9241pveage | 184.50 | 21.99 | 66.13|
| case_ACTIVSg2000 | 15.80 | 15.52 | 5.22|
| case_ACTIVSg10k | 95.91 | 22.63 | 18.18|
| case_ACTIVSg25k | 341.29 | 33.40 | 219.95|
| case_ACTIVSg70k | 116.85 | 81.03 | 89.49|
| case_ACTIVSg25k | 419.38 | 273.04 | 301.24|
| case_activs_g2000 | 341.29 | 33.40 | 219.95|
| case_activs_g10k | 116.85 | 81.03 | 89.49|
| case_activs_g70k | 419.38 | 273.04 | 301.24|
| case21k | 131.46 | — | 343.86|
| case42k | 692.54 | — | 641.81|
| case99k | 3,136.65 | — | 2,763.08|
| case193k | 13,236.02 | — | 10,169.78|

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Table 47: Performance of IPOPT with various linear solvers - number of iterations.

| Benchmark   | Polar-Power | Polar-Current | Cartesian-Power | Cartesian-Current |
|-------------|-------------|---------------|-----------------|-------------------|
|             | PARDISO     | MA57          | PARDISO         | MA57              |
| case1951rte | 33          | 34            | 34              | 33                |
| case2383wp  | 33          | 47            | 33              | 46                |
| case2736sp  | 40          | 19            | 23              | 22                |
| case2737sop | 26          | 23            | 24              | 18                |
| case2746wp  | 37          | 16            | 20              | 16                |
| case2868rte | 54          | 33            | 59              | 43                |
| case2869pegase | 31        | 32            | 45              | 32                |
| case3012wp  | 31          | 42            | 31              | 39                |
| case3120sp  | 43          | 41            | 47              | 38                |
| case3375wp  | 38          | 42            | 42              | 39                |
| case6468rte | 31          | 35            | 34              | 37                |
| case6470rt  | 70          | 50            | 81              | 43                |
| case6495rte | 59          | 56            | 52              | 55                |
| case6515rte | 52          | 54            | 46              | 55                |
| case9241pegase | 44        | 47            | 71              | 42                |
| case_ACTIVSg2000 | 29       | 24            | 28              | 26                |
| case_ACTIVSg10k | 35        | 30            | 38              | 32                |
| case13659pegase | 214       | 66            | 193             | 67                |
| case_ACTIVSg25k | 44        | 46            | 48              | 41                |
| case_ACTIVSg70k | 83        | 57            | 69              | 55                |
| case21k     | 48          | —             | —               | —                 |
| case42k     | 52          | 370           | —               | —                 |
| case_ACTIVSg20000 | 29       | 24            | 28              | 26                |
| case_ACTIVSg10k | 35        | 30            | 38              | 32                |
| case13659pegase | 214       | 66            | 193             | 67                |
| case_ACTIVSg25k | 44        | 46            | 48              | 41                |
| case_ACTIVSg70k | 83        | 57            | 69              | 55                |
| case21k     | 48          | —             | —               | —                 |
| case42k     | 52          | 370           | —               | —                 |
| case_ACTIVSg20000 | 29       | 24            | 28              | 26                |
| case_ACTIVSg10k | 35        | 30            | 38              | 32                |
| case13659pegase | 214       | 66            | 193             | 67                |
| case_ACTIVSg25k | 44        | 46            | 48              | 41                |
| case_ACTIVSg70k | 83        | 57            | 69              | 55                |
| case21k     | 48          | —             | —               | —                 |
| case42k     | 52          | 370           | —               | —                 |
| case99k     | 62          | —             | —               | —                 |
| case193k    | 77          | —             | —               | —                 |

Table 48: Performance of IPOPT with various linear solvers - overall time (s).

| Benchmark   | Polar-Power | Polar-Current | Cartesian-Power | Cartesian-Current |
|-------------|-------------|---------------|-----------------|-------------------|
|             | PARDISO     | MA57          | PARDISO         | MA57              |
| case1951rte | 7.16        | 4.77          | 3.62            | 2.61              |
| case2383wp  | 9.66        | 5.20          | 5.97            | 3.90              |
| case2736sp  | 6.54        | 3.28          | 4.00            | 2.15              |
| case2737sop | 6.09        | 3.84          | 3.94            | 1.80              |
| case2746wp  | 5.43        | 3.08          | 3.32            | 1.50              |
| case2746wp  | 7.46        | 3.21          | 3.59            | 1.67              |
| case2868rt  | 9.21        | 4.48          | 16.97           | 4.38              |
| case2869pegase | 10.37      | 4.94          | 9.10            | 3.35              |
| case3012wp  | 12.95       | 5.67          | 6.57            | 3.97              |
| case3120sp  | 10.83       | 5.53          | 9.78            | 4.14              |
| case3375wp  | 13.06       | 6.28          | —               | 4.39              |
| case42k     | 12.23       | 7.91          | 8.91            | 5.26              |
| case_ACTIVSg2000 | 29       | 24            | 28              | 26                |
| case_ACTIVSg10k | 35        | 30            | 38              | 32                |
| case13659pegase | 214       | 66            | 193             | 67                |
| case_ACTIVSg25k | 44        | 46            | 48              | 41                |
| case_ACTIVSg70k | 83        | 57            | 69              | 55                |
| case21k     | 48          | —             | —               | —                 |
| case42k     | 52          | 370           | —               | —                 |
| case99k     | 62          | —             | —               | —                 |
| case193k    | 77          | —             | —               | —                 |

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4.5 Optimization frameworks performance

In this section we evaluate high-performance nonlinear optimizers that are supported by MATPOWER. These include BELTISTOS, IPOPT, FMINCON 2017b, KNITRO 11, and MATPOWER’s included default solver, MIPS. For the performance benchmark, we consider MATPOWER case data as a starting point and the default polar-power OPF formulation, as their superior performance was demonstrated in the previous sections.

The summary of the results is presented in tables 49, 50 and 51 for the metrics including overall time, number of iterations and memory requirements for the large-scale benchmarks, respectively. The performance profiles for each of the three aspects for the large-scale benchmarks are shown in Figures 13, 14, and 15, respectively.

The results reveal that only BELTISTOS and IPOPT-PARDISO optimizers converged to the optimal solution for all benchmark cases, followed by KNITRO and MIPS-PARDISO which numerically failed for two benchmark cases. We report that other optimizers were not competitive, both in terms of robustness and performance, failing for the large-scale cases and being slower up to a factor of 300. It is important to state that since the FMINCON and KNITRO perform better with the Cartesian formulations, this benchmark setup undermines their robustness and performance. Figure 13 reveals that BELTISTOS was the fastest optimizer for all large-scale cases. MIPS-PARDISO was slower by a factor of 4 when compared to the best optimizer, while KNITRO and IPOPT-PARDISO were up to 5.5 times slower. When it comes to the number of iterations, BELTISTOS is also the best optimizer, while MIPS-PARDISO and IPOPT-PARDISO perform up to 2, or 3 times more iterations, respectively. Considering the maximum memory requirements, MIPS-PARDISO is the most efficient optimizer for roughly 50% of the benchmark cases, very closely followed by BELTISTOS and IPOPT-PARDISO. KNITRO needed up to 50% more memory, while MIPS with the default LS solver required up to 7 times more memory while solving the largest benchmark.

Table 49: Time to solution (s).

| Benchmark | MIPS-PARDISO | IPOPT-PARDISO | IPOPT-MAT7 | BELTISTOS | FMINCON | KNITRO |
|-----------|--------------|---------------|------------|-----------|---------|--------|
| case1951rte | 5.57 | 6.55 | 7.16 | 4.77 | 4.57 | 42.13 | 11.39 |
| case2383wp | 4.69 | 5.89 | 9.66 | 5.20 | 6.30 | 69.35 | 5.28 |
| case2736sp | 5.19 | 6.69 | 6.54 | 3.28 | 3.65 | 324.89 | — |
| case2737sop | 5.04 | 6.07 | 6.09 | 3.84 | 4.65 | 14.92 | 4.48 |
| case2746wop | 5.46 | 6.76 | 5.43 | 3.08 | 3.25 | 24.39 | 51.25 |
| case2746wpe | 5.88 | 7.01 | 7.46 | 3.21 | 3.60 | 165.37 | 14.48 |
| case2868rte | 4.84 | 6.32 | 9.21 | 4.48 | 3.01 | 54.29 | — |
| case2869wpe | 26.87 | 22.90 | 10.37 | 4.94 | 4.52 | 25.07 | 4.92 |
| case3012wpe | 6.33 | 7.15 | 12.95 | 5.67 | 7.18 | — | 8.09 |
| case3120sp | 29.96 | 27.89 | 10.83 | 5.53 | 6.83 | 183.98 | 6.92 |
| case3375wp | 7.62 | 8.29 | 13.06 | 6.28 | 6.70 | 43.70 | — |
| case4646rte | 17.48 | 19.29 | 12.23 | 7.91 | 8.01 | — | 43.70 |
| case4647rte | 19.51 | 20.83 | 41.56 | 10.96 | 6.49 | 92.65 | 11.30 |
| case4648rte | 29.85 | 32.97 | 21.73 | 12.41 | 10.72 | 37.76 | 9.70 |
| case6515rte | 30.03 | — | 16.85 | 11.16 | 9.42 | 123.73 | 13.28 |
| case9241pegase | 50.30 | 57.58 | 45.79 | 21.78 | 14.00 | 281.36 | 17.22 |
| case13659pegase | 6.03 | 6.09 | 7.45 | 3.70 | 4.20 | 12.81 | 4.55 |
| case_ACTIVSg10k | 32.08 | 36.11 | 26.64 | 16.21 | 13.11 | 26.88 | 12.17 |
| case_ACTIVSg25k | 37.24 | 42.16 | 266.01 | 30.32 | 9.06 | 1,866.80 | 100.72 |
| case_ACTIVSg70k | 109.44 | 118.95 | 83.68 | 62.12 | 30.26 | 210.12 | 50.18 |
| case_ACTIVSg70k | — | — | 491.83 | 192.12 | 111.10 | 786.05 | 167.55 |
| case21k | 171.01 | 140.55 | 262.59 | — | 81.35 | — | 219.83 |
| case42k | 2,639.47 | 518.21 | 747.87 | 75,386.85 | 251.13 | 8,428.31 | 1,162.01 |
| case99k | 25,936.42 | 1,612.29 | 1,011.47 | — | 782.43 | 20,554.62 | 2,558.94 |
| case193k | 4,484.02 | 10,105.87 | — | 1,854.40 | 70,150.87 | 2,654.42 |
Table 50: Number of iterations.

| Benchmark       | MIPS-\`\` | MIPS-PARDISO | IPOPT-PARDISO | IPOPT-MA57 | BELTISTOS | FMINCON | KNITRO |
|-----------------|-----------|--------------|---------------|------------|-----------|---------|--------|
| case1951rte     | 26        | 26           | 33            | 34         | 23        | 77      | 97     |
| case2383wp      | 29        | 29           | 33            | 47         | 42        | 184     | 32     |
| case2736x       | 28        | 28           | 40            | 19         | 15        | 455     | —      |
| case2737sop     | 25        | 25           | 26            | 23         | 14        | 42      | 20     |
| case2746wp      | 26        | 26           | 24            | 14         | 9         | 61      | 440    |
| case2746wp      | 28        | 28           | 37            | 16         | 14        | 343     | 117    |
| case2868rte     | 26        | 26           | 54            | 33         | 12        | 58      | —      |
| case2869pegase  | 113       | 82           | 31            | 32         | 19        | 50      | 25     |
| case3012wp      | 28        | 28           | 31            | 42         | 38        | —       | 48     |
| case3120sp      | 112       | 101          | 43            | 41         | 37        | —       | 28     |
| case3375wp      | 30        | 30           | 38            | 42         | 32        | 383     | 39     |
| case6469rte     | 42        | 42           | 31            | 35         | 30        | —       | 185    |
| case6470rte     | 43        | 43           | 70            | 50         | 16        | 57      | 33     |
| case6495rte     | 65        | 65           | 59            | 56         | 35        | 47      | 36     |
| case6515rte     | 64        | —            | 52            | 54         | 36        | 108     | 46     |
| case9241pegase  | 64        | 64           | 44            | 47         | 33        | 81      | 32     |
| case_ACTIVSg2000| 24        | 24           | 29            | 24         | 21        | 35      | 21     |
| case_ACTIVSg10k | 39        | 39           | 35            | 30         | 28        | 18      | 23     |
| case13659pegase | 38        | 38           | 214           | 66         | 20        | 81      | 203    |
| case_ACTIVSg23k | 53        | 53           | 44            | 46         | 27        | 48      | 45     |
| case_ACTIVSg70k | —         | —            | 83            | 57         | 33        | 79      | 52     |
| case21k         | 49        | 49           | 48            | —          | 55        | —       | 181    |
| case42k         | 59        | 59           | 52            | 370        | 59        | 435     | 325    |
| case99k         | 73        | 73           | 62            | —          | 71        | 324     | 150    |
| case193k        | —         | 87           | 77            | —          | 77        | 482     | 145    |

Table 51: Maximum memory requirements (MB).

| Benchmark       | MIPS-\`\` | MIPS-PARDISO | IPOPT-PARDISO | IPOPT-MA57 | BELTISTOS | FMINCON | KNITRO |
|-----------------|-----------|--------------|---------------|------------|-----------|---------|--------|
| case_ACTIVSg25k | 1,029.90  | 931.84       | 938.35        | 947.64     | 920.61    | 1,658.48| 1,183.79|
| case_ACTIVSg70k | —         | —            | 1,875.51      | 1,840.02   | 1,890.28  | 3,029.68| 2,549.75|
| case21k         | 1,681.94  | 1,201.12     | 1,188.38      | —          | 1,210.98  | —       | 1,482.34|
| case42k         | 3,777.44  | 1,974.29     | 2,050.20      | 2,117.25   | 2,332.96  | 3,350.01| 2,663.84|
| case99k         | 15,211.34 | 4,107.19     | 4,270.91      | —          | 4,245.63  | 8,362.46| 6,288.27|
| case193k        | —         | 7,478.32     | 7,942.22      | —          | 8,007.13  | 14,646.67| 11,065.25|

Table 52: Ratio of the solved benchmark cases.

| Optimizer   | All benchmarks | Large-scale benchmarks |
|-------------|----------------|------------------------|
| MIPS-\`\`   | 92%            | 67%                    |
| MIPS-PARDISO| 92%            | 83%                    |
| IPOPT-PARDISO| 100%          | 100%                   |
| IPOPT-MA57  | 88%            | 50%                    |
| BELTISTOS   | 100%           | 100%                   |
| FMINCON 2017b| 84%           | 83%                    |
| KNITRO 11   | 92%            | 100%                   |
Figure 13: Overall time profile, large-scale benchmarks.

Figure 14: Profile for iterations until convergence, large-scale benchmarks.

Figure 15: Memory efficiency profile, large-scale benchmarks.
4.6 Validation of the optimization results

We conclude our study by reporting the optimal solutions found by all the optimization frameworks and for all initial guesses. We consider the different solutions with the same objective function value to be equivalent. We report that all optimizers found the same solution up to a relative difference 10⁻⁷.

The OPF problems are non-convex and thus different local minima can be reached from different starting points. We report that the relative difference between the solutions for any given optimizer using different initial guesses is up to 10⁻³. The optimizers thus converged to the same solution, no matter which starting point was used (but it is the case that for poor initial guess the optimizer might not converge at all as can be seen in Tables 53, 54, 55, 56, 57, 58 and 59).

### Table 53: Final objective function value ($/h$) for IPOPT-PARDISO.

| Benchmark      | Flat start | MATPOWER case data | Power flow solution | Rel. error |
|----------------|------------|--------------------|---------------------|------------|
| case1951trte   | 81,737.68  | 81,737.68          | 81,737.68           | 1.22 × 10⁻⁹|
| case2383wp    | 1,868,511.76 | 1,868,511.76      | 1,868,511.76        | 5.35 × 10⁻¹⁰|
| case2736sp    | 1,307,883.09 | 1,307,883.10      | 1,307,883.10        | 1.68 × 10⁻⁹|
| case2737sp    | 777,629.28  | 777,629.28        | 777,629.28          | 1.29 × 10⁻⁹|
| case2746wop   | 1,208,279.78 | 1,208,279.78      | 1,208,279.78        | 1.66 × 10⁻⁹|
| case2746wp    | 1,631,775.05 | 1,631,775.05      | 1,631,775.05        | 1.47 × 10⁻⁹|
| case2868trte  | —          | 79,794.68         | 79,794.68           | 0.10 × 10⁻⁹|
| case2869pegase| —          | 133,999.29        | 133,999.29          | 0.10 × 10⁻⁹|
| case3012vp    | 2,591,706.50 | 2,591,706.50      | 2,591,706.50        | 1.54 × 10⁻³|
| case3126sp    | 2,142,703.72 | 2,142,703.72      | 2,142,703.72        | 6.53 × 10⁻³|
| case3375wp    | 7,412,030.65 | 7,412,030.65      | 7,412,030.65        | 7.69 × 10⁻⁴|
| case6460trte  | 86,829.02   | 86,829.02         | 86,829.02           | 2.3 × 10⁻⁹|
| case6470trte  | 98,345.49   | 98,345.49         | 98,345.49           | 1.02 × 10⁻⁹|
| case6495trte  | 106,283.38  | 106,283.37        | 106,283.37          | 2.82 × 10⁻⁹|
| case6515trte  | 109,804.24  | 109,804.23        | 109,804.23          | 9.11 × 10⁻⁹|
| case9241pegase| 315,912.02  | 315,911.56        | 315,911.56          | 1.47 × 10⁻⁶|
| case_ACTIVSg2000 | 1,228,487.05 | 1,228,487.05      | 1,228,487.05        | 4.56 × 10⁻⁹|
| case_ACTIVSg10k | 2,485,898.72 | 2,485,898.71      | 2,485,898.71        | 7.84 × 10⁻⁹|
| case13659pegase | 386,106.52  | 386,106.52        | 386,106.52          | 1.01 × 10⁻⁷|

### Table 54: Final objective function value ($/h$) for BELTISTOS.

| Benchmark      | Flat start | MATPOWER case data | Power flow solution | Rel. error |
|----------------|------------|--------------------|---------------------|------------|
| case1951trte   | 81,737.68  | 81,739.99          | 81,743.35           | 6.94 × 10⁻⁵|
| case2383wp    | 1,868,511.96 | 1,868,511.76      | 1,868,511.76        | 1.04 × 10⁻⁷|
| case2736sp    | 1,307,883.13 | 1,307,883.13      | 1,307,883.13        | 1.24 × 10⁻⁸|
| case2737sp    | 777,632.48  | 777,629.30        | 777,632.28          | 4.09 × 10⁻⁸|
| case2746wop   | 1,208,279.97 | 1,208,280.38      | 1,208,279.85        | 4.41 × 10⁻⁷|
| case2746wp    | 1,631,775.17 | 1,631,775.09      | 1,631,775.13        | 4.67 × 10⁻⁸|
| case2868trte  | 79,795.03   | 79,834.74         | 79,795.12           | 4.98 × 10⁻⁴|
| case2869pegase| 134,000.51  | 134,000.25        | 134,000.25          | 3.45 × 10⁻⁶|
| case3012wp    | 2,591,707.55 | 2,591,707.71      | 2,591,707.64        | 4.11 × 10⁻⁷|
| case3126sp    | 2,142,703.72 | 2,142,703.72      | 2,142,703.72        | 0 × 10⁻⁹|
| case3375wp    | 7,412,043.31 | 7,412,043.70      | 7,412,041.01        | 3.63 × 10⁻⁷|
| case6460trte  | 86,829.02   | 86,829.05         | 86,829.05           | 3.75 × 10⁻⁷|
| case6470trte  | 98,405.22   | 98,397.88         | 98,347.19           | 5.9 × 10⁻⁴|
| case6495trte  | 106,321.55  | 106,316.60        | 106,325.79          | 8.64 × 10⁻⁵|
| case6515trte  | 109,823.73  | 109,823.82        | 109,824.59          | 7.81 × 10⁻⁶|
| case9241pegase| 315,911.57  | 315,912.64        | 315,914.93          | 1.06 × 10⁻⁵|
| case_ACTIVSg2000 | 1,228,487.06 | 1,228,487.97      | 1,228,498.32        | 3.67 × 10⁻⁷|
| case_ACTIVSg10k | 2,485,898.74 | 2,485,898.74      | 2,485,898.74        | 4.83 × 10⁻¹⁰|
| case13659pegase | 386,106.62  | 386,260.62        | 386,205.20          | 3.99 × 10⁻⁴|
### Table 55: Final objective function value ($/h$) for IPOPT-MA57.

| Benchmark       | Flat start | MATPOWER case data Power flow solution | Rel. error |
|-----------------|------------|----------------------------------------|------------|
| case1951rte     | 81,737.68  | 81,737.68                              | 81,737.68  |
| case2383wp      | 1,868,511.76 | 1,868,511.76                           | 1,868,511.76 | 2.68·10^-10 |
| case2736sp      | 1,307,883.10 | 1,307,883.10                           | 1,307,883.10 | 4.59·10^-10 |
| case2737sop     | 777,629.28  | 777,629.28                              | 777,629.28  | 1.29·10^-10 |
| case2746wp      | 1,208,279.78 | 1,208,279.78                           | 1,208,279.78 | 2.48·10^-10 |
| case2746sp      | 1,631,775.05 | 1,631,775.05                           | 1,631,775.05 | 3.68·10^-10 |
| case2868rte     | 79,794.68   | 79,794.68                              | 79,794.68   | 2.51·10^-9  |
| case2869pegase  | 133,999.29  | 133,999.29                             | 133,999.29  | 0·10^0      |
| case3012wp      | 2,591,706.50 | 2,591,706.50                           | 2,591,706.50 | 0·10^0      |
| case3126sp      | 2,142,703.72 | 2,142,703.72                           | 2,142,703.72 | 5.13·10^-10 |
| case3357wp      | 7,412,030.65 | 7,412,030.65                           | 7,412,030.65 | 9.98·10^-9  |
| case4646rte     | 86,829.02   | 86,829.02                              | 86,829.02   | 5.76·10^-9  |
| case6470rte     | 98,345.49   | 98,345.49                              | 98,345.49   | 1.02·10^-8  |
| case6495rte     | 106,283.38  | 106,283.38                             | 106,283.38  | 4.7·10^-9   |
| case6515rte     | 109,804.23  | 109,804.23                             | 109,804.23  | 4.55·10^-9  |
| case9241pegase  | 315,911.56  | 315,911.56                             | 315,911.56  | 1.47·10^-6  |
| case_ACTIVSg2000| 1,228,487.06 | 1,228,487.06                         | 1,228,487.06 | 1.23·10^-8  |
| case_ACTIVSg10k | 2,485,898.73 | 2,485,898.71                           | 2,485,898.73 | 8.25·10^-9  |
| case13659pegase | 386,106.56  | 386,106.55                             | 386,106.55  | 1.17·10^-7  |

### Table 56: Final objective function value ($/h$) for MIPS-MATLAB ‘.’

| Benchmark       | Flat start | MATPOWER case data Power flow solution | Rel. error |
|-----------------|------------|----------------------------------------|------------|
| case1951rte     | —          | 81,737.68                              | 81,737.68  | 2.45·10^-9  |
| case2383wp      | 1,868,512.99 | 1,868,512.12                           | 1,868,511.97 | 7.82·10^-9  |
| case2736sp      | 1,307,883.13 | 1,307,883.13                           | 1,307,883.13 | 7.63·10^-9  |
| case2737sop     | 777,629.30  | 777,629.30                              | 777,629.30  | 5.73·10^-9  |
| case2746wp      | 1,208,279.81 | 1,208,279.81                           | 1,208,279.81 | 2.98·10^-9  |
| case2746sp      | 1,631,775.10 | 1,631,775.10                           | 1,631,775.10 | 6.13·10^-11 |
| case2868rte     | —          | 79,794.69                              | 79,794.69   | 8.77·10^-9  |
| case2869pegase  | 133,999.29  | 133,999.29                             | 133,999.29  | 7.46·10^-10 |
| case3012wp      | 2,591,706.57 | 2,591,706.57                           | 2,591,706.57 | 1.97·10^-9  |
| case3126sp      | 2,142,703.76 | 2,142,703.76                           | 2,142,703.77 | 1.03·10^-9  |
| case3357wp      | 7,412,030.69 | 7,412,030.69                           | 7,412,030.69 | 4.72·10^-10 |
| case4646rte     | —          | 86,829.02                              | 86,829.02   | 4.95·10^-8  |
| case6470rte     | —          | 98,345.49                              | 98,345.49   | 4.47·10^-8  |
| case6495rte     | —          | 106,283.40                             | 106,283.40  | 4.7·10^-9   |
| case6515rte     | —          | 109,804.26                             | 109,804.26  | 1.55·10^-9  |
| case9241pegase  | 315,911.58  | 315,911.58                             | 315,911.58  | 2.74·10^-9  |
| case_ACTIVSg2000| 1,228,487.33 | 1,228,487.24                         | 1,228,487.41 | 1.34·10^-7  |
| case_ACTIVSg10k | 2,485,898.79 | 2,485,898.76                           | 2,485,898.76 | 1.04·10^-8  |
| case13659pegase | 386,114.08  | 386,117.51                             | 386,113.57  | 1.02·10^-5  |
### Table 57: Final objective function value ($/h$) for MIPS-PARDISO.

| Benchmark         | Flat start | MATPOWER case data | Power flow solution | Rel. error |
|-------------------|------------|--------------------|---------------------|------------|
| case1951rte       | —          | 81,737.68          | 81,737.68           | 2.45×10⁻⁶  |
| case2383wp        | 1,868,512.09 | 1,868,512.12     | 1,868,511.97        | 7.82×10⁻⁸  |
| case2736sp        | 1,307,883.13 | 1,307,883.13     | 1,307,883.13        | 7.65×10⁻¹¹ |
| case2737sop       | 777,629.30  | 777,629.30        | 777,629.30          | 3.73×10⁻⁹  |
| case2746wop       | 1,208,279.81 | 1,208,279.81     | 1,208,279.81        | 2.98×10⁻⁹  |
| case2748wp        | 1,631,775.10 | 1,631,775.10     | 1,631,775.10        | 6.13×10⁻¹³ |
| case2868rte       | —          | 79,794.69         | 79,794.69           | 8.77×10⁻⁹  |
| case2869ppegase   | 133,999.29  | 133,999.29        | 133,999.29          | 2.24×10⁻⁹  |
| case3012wp        | 2,591,706.57 | 2,591,706.57     | 2,591,706.57        | 1.97×10⁻⁹  |
| case3126sp        | 2,142,703.76 | 2,142,703.76     | 2,142,703.77        | 1.03×10⁻⁹  |
| case3575wp        | 7,412,030.69 | 7,412,030.69     | 7,412,030.69        | 4.72×10⁻¹⁰ |
| case6468rte       | —          | 86,829.02         | 86,829.02           | 4.95×10⁻⁸  |
| case6470rte       | —          | 98,345.50         | 98,345.49           | 4.47×10⁻⁸  |
| case6495rte       | —          | 106,283.40        | 106,283.40          | 4.7×10⁻⁸   |
| case6515rte       | —          | —                 | 109,804.26          | 0.10       |
| case9241ppegase   | 315,912.45  | 315,912.18        | 315,911.59          | 2.72×10⁻⁹  |
| case_ACTIVSg2000  | 1,228,487.33 | 1,228,487.24     | 1,228,487.35        | 8.09×10⁻⁹  |
| case_ACTIVSg10k   | —          | 2,485,898.79      | 2,485,898.78        | 1.29×10⁻⁹  |
| case13659ppegase  | 386,114.09  | 386,117.51        | 386,114.37          | 8.86×10⁻⁶  |

### Table 58: Final objective function value ($/h$) for FMINCON 2017b.

| Benchmark         | Flat start | MATPOWER case data | Power flow solution | Rel. error |
|-------------------|------------|--------------------|---------------------|------------|
| case1951rte       | —          | 81,737.76          | 81,737.69           | 8.52×10⁻⁷  |
| case2383wp        | 1,868,511.08 | 1,868,511.92     | 1,868,511.85        | 4.49×10⁻⁷  |
| case2736sp        | 1,307,883.17 | 1,307,883.18     | 1,307,883.10        | 6.25×10⁻⁹  |
| case2737sop       | 777,629.44  | 777,629.30        | 777,629.33          | 1.76×10⁻⁷  |
| case2746wop       | 1,208,281.12 | 1,208,279.88     | 1,208,279.88        | 1.03×10⁻⁶  |
| case2748wp        | 1,631,775.12 | 1,631,819.03     | 1,631,775.17        | 2.69×10⁻⁵  |
| case2868rte       | —          | 79,794.79         | 79,794.79           | 1.06×10⁻⁶  |
| case2869ppegase   | 133,999.37  | 133,999.29        | 133,999.40          | 8.06×10⁻⁷  |
| case3012wp        | 2,591,706.66 | 2,591,706.66     | 2,591,706.66        | 0.10       |
| case3126sp        | 2,142,703.84 | 2,142,711.45     | 2,142,711.45        | 3.56×10⁻⁸  |
| case3375wp        | 7,412,030.80 | 7,412,030.80     | 7,412,030.80        | 2.43×10⁻⁸  |
| case6468rte       | —          | —                 | —                   | —          |
| case6470rte       | —          | —                 | —                   | —          |
| case6495rte       | —          | 106,287.86        | 106,287.86          | 4.7×10⁻⁹   |
| case6515rte       | —          | 109,804.41        | 109,804.39          | 1.27×10⁻⁷  |
| case9241ppegase   | 315,912.34  | 315,911.85        | 315,911.62          | 2.29×10⁻⁹  |
| case_ACTIVSg2000  | 1,228,487.08 | 1,228,486.37     | 1,228,487.15        | 7.57×10⁻⁶  |
| case_ACTIVSg10k   | —          | 2,485,940.14      | 2,485,899.14        | 1.65×10⁻⁵  |
| case13659ppegase  | 386,107.83  | 386,106.64        | 386,106.64          | 3.09×10⁻⁶  |
Table 59: Final objective function value ($/h$) for KNITRO 11.

| Benchmark    | Flat start | MATPOWER case data | Power flow solution | Rel. error |
|--------------|------------|--------------------|---------------------|------------|
| case1951rte  | —          | 81,737.68          | 81,737.68           | 1.22 \cdot 10^{-9} |
| case2383wp   | 1,868,511.83 | 1,868,511.83       | 1,868,511.83        | 1.61 \cdot 10^{-10} |
| case2736sp   | 1,831,775.10 | 1,831,775.10       | 1,831,775.10        | 6.19 \cdot 10^{-9} |
| case2737sop  | 1,307,883.13 | —                  | 1,307,883.13        | 0          |
| case2746wop  | 1,208,279.82 | 1,208,279.82       | 1,208,279.82        | 5.05 \cdot 10^{-9} |
| case2746wp   | 1,631,775.10 | 1,631,775.10       | 1,631,775.10        | 6.19 \cdot 10^{-9} |
| case2868rte  | —          | —                  | 79,794.68           | 0          |
| case2869pegase | 133,999.29 | 133,999.29         | 133,999.29          | 0          |
| case3012wp   | 2,591,706.57 | 2,591,706.57       | 2,591,706.57        | 2.32 \cdot 10^{-10} |
| case3120sp   | 2,142,703.77 | 2,142,703.77       | 2,142,703.77        | 4.67 \cdot 10^{-11} |
| case3375wp   | 7,412,030.68 | 7,412,030.68       | 7,412,030.68        | 1.62 \cdot 10^{-10} |
| case_ACTIVSg2000 | 1,228,487.06 | 1,228,487.06       | 1,228,487.06        | 2.44 \cdot 10^{-9} |
| case_ACTIVSg10k | —          | 2,485,898.75       | 2,485,898.75        | 8.05 \cdot 10^{-11} |
| case9241pegase | 315,912.41 | 315,912.41         | 315,911.56          | 2.69 \cdot 10^{-6} |
| case13659pegase | 386,107.53 | 386,106.56         | 386,106.56          | 2.52 \cdot 10^{-6} |

5 Conclusions

We investigated several OPF solution strategies combining different optimizers with four OPF formulations. All optimization frameworks demonstrate similar performance for small and medium size networks. Significant performance differences between optimizers tend to appear for sufficiently large cases. The choice of the initial guess was demonstrated to be crucial since it may either speed up convergence to the optimal solution, or delay it significantly. Reliable optimizers such as BELTISTOS seem to be marginally influenced by the choice of the OPF formulation. On the contrary all other optimizers seem to prefer one formulation over another. Although failures to converge may be attributed to the LS employed for the solution of the KKT systems, we observed that all methods favor power-based formulations with either polar or Cartesian voltage coordinates.

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