INEQUALITIES OF OSTROWSKI’S TYPE FOR \( m - \) AND \( (\alpha, m) - \)
LOGARITHMICALLY CONVEX FUNCTIONS VIA
RIEMANN-LIOUVILLE FRACTIONAL INTEGRALS

AHMET OCAK AKDEMIR

Abstract. In this paper, we establish some new Ostrowski’s type inequalities
for \( m - \) and \( (\alpha, m) - \) logarithmically convex functions by using the Riemann-
Liouville fractional integrals.

1. INTRODUCTION

Let \( f : I \subset [0, \infty) \to \mathbb{R} \) be a differentiable mapping on \( I^0 \), the interior of the
interval \( I \), such that \( f' \in L[a, b] \) where \( a, b \in I \) with \( a < b \). If \( |f'(x)| \leq M \), then
the following inequality holds (see [7]).

\[
\left| f(x) - \frac{1}{b-a} \int_a^b f(u) du \right| \leq \frac{M}{b-a} \left[ \frac{(x-a)^2 + (b-x)^2}{2} \right]
\]

(1.1)

This inequality is well known in the literature as the Ostrowski inequality. For
some results which generalize, improve and extend the inequality (1.1) see ([7, 8,
9, 10, 11, 18]) and the references therein.

Let us recall some known definitions and results which we will use in this paper.
The function \( f : [a, b] \to \mathbb{R} \), \( b > 0 \), is said to be convex, if we have

\[
f(tx + (1-t)y) \leq tf(x) + (1-t)f(y)
\]

for all \( x, y \in [a, b] \) and \( t \in [0, 1] \). We can define starshaped functions on \([0, b]\) which
satisfy the condition

\[
f(tx) \leq tf(x)
\]

for \( t \in [0, 1] \).

The concept of \( m - \) convexity has been introduced by Toader in [5], an interme-
diate between the ordinary convexity and starshaped property, as following:

**Definition 1.** The function \( f : [0, b] \to \mathbb{R} \), \( b > 0 \), is said to be \( m - \) convex, where
\( m \in [0, 1] \), if we have

\[
f(tx + m(1-t)y) \leq tf(x) + m(1-t)f(y)
\]

for all \( x, y \in [0, b] \) and \( t \in [0, 1] \). We say that \( f \) is \( m - \) concave if \(-f\) is \( m - \) convex.

In [3], Miheşan gave definition of \( (\alpha, m) - \) convexity as following;
Definition 2. The function $f : [0, b] \to \mathbb{R}$, $b > 0$ is said to be $(\alpha, m)$–convex, where $(\alpha, m) \in [0, 1]^2$, if we have

$$f(tx + m(1 - t)y) \leq t^\alpha f(x) + m(1 - t^\alpha)f(y)$$

for all $x, y \in [0, b]$, and $t \in [0, 1]$.

Denote by $K_m^\alpha(b)$ the class of all $(\alpha, m)$–convex functions on $[0, b]$ for which $f(0) \leq 0$. If we choose $(\alpha, m) = (1, m)$, it can be easily seen that $(\alpha, m)$–convexity reduces to $m$–convexity and for $(\alpha, m) = (1, 1)$, we have ordinary convex functions on $[0, b]$. For the recent results based on the above definitions see the papers [2]–[9].

Definition 3. ([1]) A function $f : [0, b] \to (0, \infty)$ is said to be $m$–logarithmically convex if the inequality

$$(1.2) \quad f(tx + m(1 - t)y) \leq [f(x)]^t [f(y)]^{m(1 - t)}$$

holds for all $x, y \in [0, b]$, $m \in (0, 1]$, and $t \in [0, 1]$.

Obviously, if putting $m = 1$ in Definition 3, then $f$ is just the ordinary logarithmically convex function on $[0, b]$.

Definition 4. ([1]) A function $f : [0, b] \to (0, \infty)$ is said to be $(\alpha, m)$–logarithmically convex if

$$(1.3) \quad f(tx + m(1 - t)y) \leq [f(x)]^{t^\alpha} [f(y)]^{m(1 - t^\alpha)}$$

holds for all $x, y \in [0, b]$, $(\alpha, m) \in (0, 1] \times (0, 1]$, and $t \in [0, 1]$.

Clearly, when taking $\alpha = 1$ in Definition 4, then $f$ becomes the standard $m$-logarithmically convex function on $[0, b]$.

We give some necessary definitions and mathematical preliminaries of fractional calculus theory which are used throughout this paper.

Definition 5. Let $f \in L_1[a, b]$. The Riemann-Liouville integrals $J_{a+}^\mu f$ and $J_{b-}^\mu f$ of order $\mu > 0$ with $a \geq 0$ are defined by

$$J_{a+}^\mu f(x) = \frac{1}{\Gamma(\mu)} \int_a^x (x - t)^{\mu - 1} f(t)dt, \quad x > a$$

and

$$J_{b-}^\mu f(x) = \frac{1}{\Gamma(\mu)} \int_x^b (t - x)^{\mu - 1} f(t)dt, \quad x < b$$

respectively where $\Gamma(\mu) = \int_0^\infty e^{-u}u^{\mu - 1}du$. Here is $J_{a+}^0 f(x) = J_{b-}^0 f(x) = f(x)$.

In the case of $\mu = 1$, the fractional integral reduces to the classical integral. For some recent results connected with fractional integral inequalities see [11]–[18].

The aim of this study is to establish some Ostrowski type inequalities for the class of functions whose derivatives in absolute value are $m$– and $(\alpha, m)$–geometrically convex functions via Riemann-Liouville fractional integral.
2. THE NEW RESULTS

In order to prove our results, we need the following lemma that has been obtained in [11]:

**Lemma 1.** ([11]) Let \( f : [a, b] \to \mathbb{R} \) be a differentiable mapping on \((a, b)\) with \( a < b \). If \( f' \in L[a, b] \), then for all \( x \in [a, b] \) and \( \mu > 0 \) we have:

\[
\frac{(x-a)^\mu + (b-x)^\mu}{b-a} f(x) - \frac{\Gamma(\mu + 1)}{b-a} \left[ f(x) - f(a) \right] \leq \int_0^1 t^\mu f'(tx + (1-t)a) \, dt + \int_0^1 t^\mu f'(tx + (1-t)b) \, dt
\]

(2.1)

where \( \Gamma(\mu) = \int_0^\infty e^{-u} u^{\mu-1} du \).

**Theorem 1.** Let \( f : [0, \infty) \to (0, \infty) \) be a differentiable mapping with \( a, b \in [0, \infty) \) such that \( a < b \). If \(|f'(x)|\) is \((\alpha, m)\)-logarithmically convex function with \(|f'(x)| \leq M, f' \in L[a, b] \), \((\alpha, m) \in (0, 1) \times (0, 1) \) and \( \mu > 0 \), then the following inequality for fractional integrals holds:

\[
\left| \frac{(x-a)^\mu + (b-x)^\mu}{b-a} f(x) - \frac{\Gamma(\mu + 1)}{b-a} \left[ f(x) - f(a) \right] \right| \leq \frac{1}{2\mu + 1} + K_1(\alpha, m, t) \left[ \frac{(x-a)^{\mu+1} + (b-x)^{\mu+1}}{2(b-a)} \right]
\]

(2.1)

where

\[
K_1(\alpha, m, t) = \begin{cases} 
\frac{M^{2\alpha(3 - 2\alpha)} - 1}{2(2\alpha - 2\alpha m)} & , M < 1 \\
1 & , M = 1 
\end{cases}
\]

**Proof.** By Lemma 1 and since \(|f'|\) is \((\alpha, m)\)-logarithmically convex, we can write

\[
\left| \frac{(x-a)^\mu + (b-x)^\mu}{b-a} f(x) - \frac{\Gamma(\mu + 1)}{b-a} \left[ f(x) - f(a) \right] \right| \leq \frac{(x-a)^{\mu+1}}{b-a} \int_0^1 t^\mu |f'(tx + (1-t)a)| \, dt + \frac{(b-x)^{\mu+1}}{b-a} \int_0^1 t^\mu |f'(tx + (1-t)b)| \, dt
\]

\[
\leq \frac{(x-a)^{\mu+1}}{b-a} \int_0^1 t^\mu |f'(x)|^{\alpha} \left| f' \left( \frac{a}{m} \right) \right|^{m(1-\alpha)} \, dt + \frac{(b-x)^{\mu+1}}{b-a} \int_0^1 t^\mu |f'(x)|^{\alpha} \left| f' \left( \frac{b}{m} \right) \right|^{m(1-\alpha)} \, dt
\]

\[
\leq \frac{(x-a)^{\mu+1}}{b-a} \int_0^1 t^\mu M^{m+\alpha(1-m)} \, dt + \frac{(b-x)^{\mu+1}}{b-a} \int_0^1 t^\mu M^{m+\alpha(1-m)} \, dt
\]

By using the elementary inequality \( cd \leq \frac{c^2 + d^2}{2} \), we have

(2.2)

\[
\left| \frac{(x-a)^\mu + (b-x)^\mu}{b-a} f(x) - \frac{\Gamma(\mu + 1)}{b-a} \left[ f(x) - f(a) \right] \right| \leq \frac{(x-a)^{\mu+1}}{b-a} \int_0^1 t^{2\mu} + M^{2(m+\alpha)(1-m)} \, dt + \frac{(b-x)^{\mu+1}}{b-a} \int_0^1 t^{2\mu} + M^{2(m+\alpha)(1-m)} \, dt
\]

\[
= \left[ \frac{1}{2\mu + 1} + \int_0^1 M^{2(m+\alpha)(1-m)} \, dt \right] \left[ \frac{(x-a)^{\mu+1} + (b-x)^{\mu+1}}{2(b-a)} \right].
\]
If we choose $M = 1$, then
$$\int_0^1 M^{2(m + \alpha(1 - m))} dt = 1.$$  
If $M < 1$, then $M^{2(m + \alpha(1 - m))} \leq M^{2(m + \alpha(1 - m))}$, thus
$$\int_0^1 M^{2(m + \alpha(1 - m))} dt = \frac{M^{2m} (M^{2\alpha - 2\alpha m} - 1)}{(2\alpha - 2\alpha m) \ln M}.$$  
Which completes the proof. \qed

**Corollary 1.** Let $f : [0, \infty) \to (0, \infty)$ be differentiable mapping with $a, b \in [0, \infty)$ such that $a < b$. If $|f'(x)|$ is $m$–logarithmically convex function with $|f'(x)| \leq M$, $f' \in L[a, b]$, $m \in (0, 1]$ and $\mu > 0$, then the following inequality for fractional integrals holds:

$$\left| \frac{(x-a)^\mu + (b-x)^\mu}{b-a} f(x) - \frac{\Gamma(\mu + 1)}{b-a} [J_x^\mu f(a) + J_x^\mu f(b)] \right|$$

$$\leq \frac{1}{2\mu + 1} \left[ \frac{M^2 - M^{2m}}{2 \ln M - 2m \ln M} \right] \left[ \frac{(x-a)^{\mu+1} + (b-x)^{\mu+1}}{2(b-a)} \right].$$  

(2.3)

**Proof.** If we take $\alpha = 1$ in (2.1), we get the required result. \qed

**Corollary 2.** Let $f : [0, \infty) \to (0, \infty)$ be differentiable mapping with $a, b \in [0, \infty)$ such that $a < b$. If $|f'(x)|$ is logarithmically convex function with $|f'(x)| \leq M$, $f' \in L[a, b]$ and $\mu > 0$, then the following inequality for fractional integrals holds:

$$\left| \frac{(x-a)^\mu + (b-x)^\mu}{b-a} f(x) - \frac{\Gamma(\mu + 1)}{b-a} [J_x^\mu f(a) + J_x^\mu f(b)] \right|$$

$$\leq \frac{1}{2\mu + 1} \left[ \frac{M^2 - M^{2m}}{2 \ln M - 2m \ln M} \right] \left[ \frac{(x-a)^{\mu+1} + (b-x)^{\mu+1}}{2(b-a)} \right].$$  

(2.4)

**Proof.** If we take $\alpha = m = 1$ in (2.2), we get the required result. \qed

**Corollary 3.** Let $f : [0, \infty) \to (0, \infty)$ be differentiable mapping with $a, b \in [0, \infty)$ such that $a < b$. If $|f'(x)|$ is logarithmically convex function with $|f'(x)| \leq M$ and $f' \in L[a, b]$, then the following inequality holds:

$$\left| f(x) - \frac{1}{b-a} \int_a^b f(u) du \right| \leq \frac{1}{3} + M^2 \left[ \frac{(x-a)^2 + (b-x)^2}{2(b-a)} \right].$$

**Proof.** If we choose $\mu = 1$ in (2.3), we get the required result. \qed

**Theorem 2.** Let $f : [0, \infty) \to (0, \infty)$ be differentiable mapping with $a, b \in [0, \infty)$ such that $a < b$. If $|f'(x)|^q$ is $(\alpha, m)$–logarithmically convex function with $|f'(x)|^q \leq M$, $f' \in L[a, b]$, $(\alpha, m) \in (0, 1] \times (0, 1]$ and $\mu > 0$, then the following inequality for fractional integrals holds:

$$\left| \frac{(x-a)^\mu + (b-x)^\mu}{b-a} f(x) - \frac{\Gamma(\mu + 1)}{b-a} [J_x^\mu f(a) + J_x^\mu f(b)] \right|$$

$$\leq \left( \frac{q-1}{\mu(q-p) + q-1} \right)^{\frac{q-1}{q}} (K_2(\alpha, m, t))^{\frac{1}{q}} \left[ \frac{(x-a)^{\mu+1} + (b-x)^{\mu+1}}{2(b-a)} \right].$$  

(2.5)
where \( q > 1, 0 \leq p \leq q \) and
\[
K_2(\alpha, m, t) = \begin{cases} 
\frac{M^m}{(\ln M)^{\alpha(m-1))}} & , M < 1 \\
\frac{1}{\mu p + 1} & , M = 1
\end{cases}
\]

Proof. From Lemma 1 and by using the properties of modulus, we have
\[
\left| \frac{(x-a)^\mu + (b-x)^\mu}{b-a} f(x) - \frac{\Gamma(\mu+1)}{b-a} [J_\mu f(a) + J_\mu f(b)] \right|
\leq \frac{(x-a)^{\mu+1}}{b-a} \int_0^1 t^\mu |f'(tx + (1-t)a)dt| + \frac{(b-x)^{\mu+1}}{b-a} \int_0^1 t^\mu |f'(tx + (1-t)b)|dt.
\]
By applying the Hölder inequality for \( q > 1, 0 \leq p \leq q \), we get
\[
\left| \frac{(x-a)^\mu + (b-x)^\mu}{b-a} f(x) - \frac{\Gamma(\mu+1)}{b-a} [J_\mu f(a) + J_\mu f(b)] \right|
\leq \frac{(x-a)^{\mu+1}}{b-a} \left[ \left( \int_0^1 t^{\mu\left(\frac{q}{p} - 1\right)}dt \right)^{\frac{q-1}{q}} \left( \int_0^1 t^{\mu p} |f'(tx + (1-t)a)|^q dt \right)^{\frac{1}{q}} \right]
+ \frac{(b-x)^{\mu+1}}{b-a} \left[ \left( \int_0^1 t^{\mu\left(\frac{q}{p} - 1\right)}dt \right)^{\frac{q-1}{q}} \left( \int_0^1 t^{\mu p} |f'(tx + (1-t)b)|^q dt \right)^{\frac{1}{q}} \right].
\]
It is easy to see that
\[
\int_0^1 t^{\mu\left(\frac{q}{p} - 1\right)}dt = \frac{q-1}{\mu (q-p) + q-1}.
\]
Hence, by \((\alpha, m)\) -logarithmically convexity of \(|f'|^q\), we have
\[
\left(2.6\right) \quad \left| \frac{(x-a)^\mu + (b-x)^\mu}{b-a} f(x) - \frac{\Gamma(\mu+1)}{b-a} [J_\mu f(a) + J_\mu f(b)] \right|
\leq \frac{(x-a)^{\mu+1}}{b-a} \left( \frac{q-1}{\mu (q-p) + q-1} \right)^{\frac{q-1}{q}} \left( \int_0^1 t^{\mu p} |f'(x)|^\alpha \left| f' \left( \frac{a}{m} \right) \right|^m dt \right)^{\frac{1}{q}}
+ \frac{(b-x)^{\mu+1}}{b-a} \left( \frac{q-1}{\mu (q-p) + q-1} \right)^{\frac{q-1}{q}} \left( \int_0^1 t^{\mu p} |f'(x)|^\alpha \left| f' \left( \frac{b}{m} \right) \right|^m dt \right)^{\frac{1}{q}}
= \frac{(x-a)^{\mu+1}}{b-a} \left( \frac{q-1}{\mu (q-p) + q-1} \right)^{\frac{q-1}{q}} \left( \int_0^1 t^{\mu p} M^m \left| f'(x) \right|^m dt \right)^{\frac{1}{q}}
+ \frac{(b-x)^{\mu+1}}{b-a} \left( \frac{q-1}{\mu (q-p) + q-1} \right)^{\frac{q-1}{q}} \left( \int_0^1 t^{\mu p} M^m \left| f'(x) \right|^m dt \right)^{\frac{1}{q}}.
\]
If we choose $M = 1$, then
\[
\int_0^1 t^{\mu p} dt = \frac{1}{\mu p + 1}.
\]
If $M < 1$, then $M^{m+t\alpha(1-m)} \leq M^{m+\alpha t(1-m)}$, thus
\[
\int_0^1 t^{\mu p} M^{m+\alpha t(1-m)} dt = \frac{M^n (\Gamma(\mu p + 1) - \Gamma(\mu p + 1, \ln M^{\alpha(m-1)}))}{(\ln M^{\alpha(m-1)})^{\mu p + 1}}
\]
Which completes the proof.

Corollary 4. Let $f : [0, \infty) \to (0, \infty)$ be differentiable mapping with $a, b \in [0, \infty)$ such that $a < b$. If $|f'(x)|^q$ is $m$-logarithmically convex function with $|f'(x)|^q \leq M, f' \in L[a, b]$, $m \in (0, 1]$ and $\mu > 0$, then the following inequality for fractional integrals holds:
\[
\left| \frac{(x - a)^\mu + (b - x)^\mu}{b - a} f(x) - \frac{\Gamma(\mu + 1)}{b - a} \left[ J_{x-}^\mu f(a) + J_{x+}^\mu f(b) \right] \right| 
\leq \left( \frac{q - 1}{\mu (q - p) + q - 1} \right)^{\frac{1}{\mu p + 1}} \left( K_2(1, m, t) \right)^{\frac{1}{\mu p + 1}} \left[ \frac{(x - a)^{\mu + 1} + (b - x)^{\mu + 1}}{(b - a)} \right]
\]
where $q > 1$, $0 \leq p \leq q$ and
\[
K_2(1, m, t) = \begin{cases} 
\frac{M^n (\Gamma(\mu p + 1, \ln M^{\alpha(m-1)}))}{(\ln M^{\alpha(m-1)})^{\mu p + 1}} & , M < 1 \\
\frac{1}{\mu p + 1} & , M = 1
\end{cases}
\]
Proof. If we set $\alpha = 1$ in 2.3, the proof is completed.

Corollary 5. Let $f : [0, \infty) \to (0, \infty)$ be differentiable mapping with $a, b \in [0, \infty)$ such that $a < b$. If $|f'(x)|^q$ is logarithmically convex function with $|f'(x)|^q \leq M, f' \in L[a, b]$ and $\mu > 0$, then the following inequality for fractional integrals holds:
\[
\left| \frac{(x - a)^\mu + (b - x)^\mu}{b - a} f(x) - \frac{\Gamma(\mu + 1)}{b - a} \left[ J_{x-}^\mu f(a) + J_{x+}^\mu f(b) \right] \right| 
= \left( \frac{q - 1}{\mu (q - p) + q - 1} \right)^{\frac{1}{\mu p + 1}} \left( \frac{1}{\mu p + 1} \right)^{\frac{1}{\mu p + 1}} \left[ \frac{(x - a)^2 + (b - x)^2}{(b - a)} \right]
\]
where $q > 1$, $0 \leq p \leq q$.

Proof. If we set $\alpha = m = 1$ in 2.6, the proof is completed.

Corollary 6. Let $f : [0, \infty) \to (0, \infty)$ be differentiable mapping with $a, b \in [0, \infty)$ such that $a < b$. If $|f'(x)|^q$ is $(\alpha, m)$-logarithmically convex function with $|f'(x)|^q \leq M, f' \in L[a, b]$ and $(\alpha, m) \in (0, 1] \times (0, 1]$, then the following inequality

\[
\left| \frac{(x - a)^\mu + (b - x)^\mu}{b - a} f(x) - \frac{\Gamma(\mu + 1)}{b - a} \left[ J_{x-}^\mu f(a) + J_{x+}^\mu f(b) \right] \right| 
\leq \left( \frac{q - 1}{\mu (q - p) + q - 1} \right)^{\frac{1}{\mu p + 1}} \left( \frac{1}{\mu p + 1} \right)^{\frac{1}{\mu p + 1}} \left[ \frac{(x - a)^2 + (b - x)^2}{(b - a)} \right]
\]

where $q > 1$, $0 \leq p \leq q$. 

Proof. If we set $\alpha = m = 1$ in 2.6, the proof is completed.
holds:

\[
(2.7) \quad \left| f(x) - \frac{1}{b-a} \int_a^b f(u) \, du \right| \leq \left( \frac{q-1}{2q-p-1} \right)^\frac{q-1}{q} \left( K_1(\alpha, m, t) \right)^\frac{1}{q} \left[ \frac{(x-a)^2 + (b-x)^2}{(b-a)} \right]
\]

where \( q > 1, \ 0 \leq p \leq q \) and

\[
K_3(\alpha, m, t) = \begin{cases} 
\frac{M^m \left( \Gamma(p+1) - \Gamma(p+1, \ln M^{m-1}) \right)}{(\ln M^{m-1})^{\frac{p+1}{p+1}}} , & M < 1 \\
\frac{1}{p+1} & , M = 1
\end{cases}
\]

**Proof.** If we set \( \mu = 1 \) in (2.6), the proof is completed. \( \square \)

**Corollary 7.** Let \( f : [0, \infty) \to (0, \infty) \) be differentiable mapping with \( a, b \in \mathbb{R} \) such that \( a < b \). If \([f'(x)]^q \) is \((\alpha, m)\)-logarithmically convex function with \([f'(x)]^q \leq M, f' \in L[a, b] \) and \((\alpha, m) \in (0, 1] \times (0, 1] \), then the following inequality holds:

\[
(2.8) \quad \left| f(x) - \frac{1}{b-a} \int_a^b f(u) \, du \right| \leq \left( \frac{1}{2} \right)^{\frac{q-1}{q}} \left( K_4(\alpha, m, t) \right)^\frac{1}{q} \left[ \frac{(x-a)^2 + (b-x)^2}{(b-a)} \right]
\]

where \( q > 1, \ 0 \leq p \leq q \) and

\[
K_4(\alpha, m, t) = \begin{cases} 
\frac{M^m \left( \Gamma(2) - \Gamma(2, \ln M^{m-1}) \right)}{(\ln M^{m-1})^2} , & M < 1 \\
\frac{1}{2} & , M = 1
\end{cases}
\]

**Proof.** If we set \( p = 1 \) in (2.7), the proof is completed. \( \square \)

**References**

[1] R.-F. Bai, F. Qi and B.-Y. Xi, Hermite-Hadamard type inequalities for the \( m \)- and \((\alpha, m)\)-logarithmically convex functions, Filomat 27 (2013), 1-7.

[2] M.K. Bakula, J. Pečarić and M. Ribibić, Companion inequalities to Jensen’s inequality for \( m \)-convex and \((\alpha, m)\)-convex functions, J. Inequal. Pure and Appl. Math., 7 (5) (2006), Article 194.

[3] S.S. Dragomir and G. Toader, Some inequalities for \( m \)-convex functions, Studia University Babes Bolyai, Mathematica, 38 (1) (1993), 21-28.

[4] V.G. Mihešan, A generalization of the convexity, Seminar of Functional Equations, Approx. and Convex, Cluj-Napoca (Romania) (1993).

[5] G. Toader, Some generalization of the convexity, Proc. Colloq. Approx. Opt., Cluj-Napoca, (1984), 329-338.

[6] G. Toader, On a generalization of the convexity, Mathematica, 30 (53) (1988), 83-87.

[7] A. Ostrowski, Über die Absolutabweichung einer differentierbaren Funktion von ihren Integraalmittelwert, Comment. Math. Helv., 10, 226-227, (1938).

[8] M.E. Özdemir, H. Kavurmaci, E. Set, Ostrowski’s type inequalities for \((\alpha, m)\)-convex functions, KYUNGPOOK Math. J. 50 (2010) 371–378.

[9] H. Kavurmaci, M. Avci and M.E. Özdemir, New Ostrowski type inequalities for \( m \)-convex functions and applications, Hacettepe Journal of Mathematics and Statistics, Volume 40 (2) (2011), 135 – 145.
[10] M. Alomari and M. Darus, *Some Ostrowski type inequalities for convex functions with applications*, RGMIA Res. Rep. Coll., (2010) 13, 2, Article 3. [ONLINE: http://ajmaa.org/RGMIA/v13n2.php].

[11] E. Set, *New inequalities of Ostrowski type for mappings whose derivatives are $s$-convex in the second sense via fractional integrals*, Comput. Math. Appl., 63 (2012) 1147-1154.

[12] S. Belarbi and Z. Dahmani, *On some new fractional integral inequalities*, J. Ineq. Pure and Appl. Math., 10(3), Art. 86 (2009).

[13] Z. Dahmani, *New inequalities in fractional integrals*, International Journal of Nonlinear Science, 9(4), 493-497 (2010).

[14] Z. Dahmani, *On Minkowski and Hermite-Hadamard integral inequalities via fractional integration*, Ann. Funct. Anal. 1(1), 51-58 (2010).

[15] Z. Dahmani, L. Tabharit and S. Taf, *Some fractional integral inequalities*, Nonl. Sci. Lett. A., 1(2), 155-160 (2010).

[16] M.Z. Sarıkaya, E. Set, H. Yaldız and N. Başak, *Hermite-Hadamard’s inequalities for fractional integrals and related fractional inequalities*, Mathematical and Computer Modelling, In Press.

[17] Z. Dahmani, L. Tabharit and S. Taf, *New generalizations of Grüss inequality using Riemann-Liouville fractional integrals*, Bull. Math. Anal. Appl., 2(3), 93-99 (2010).

[18] M.E. Özdemir, H. Kavurmacı and M. Avci, *New inequalities of Ostrowski type for mappings whose derivatives are $(\alpha, m)$-convex via fractional integrals*, RGMIA Research Report Collection, 15, Article 10, 8 pp (2012).

AĞRI İBRAHİM ÇEÇEN UNIVERSITY, FACULTY OF SCIENCE AND LETTERS, DEPARTMENT OF MATHEMATICS, AĞRI, 04100, TURKEY.

E-mail address: ahmetakdemir@agri.edu.tr