On the Possibility of Large Axion Decay Constants

Tom Banks, Michael Dine, P. J. Fox, E. Gorbatov

Santa Cruz Institute for Particle Physics, Santa Cruz CA 95064

Abstract

The decay constant of the QCD axion is required by observation to be small compared to the Planck scale. In theories of “natural inflation,” and certain proposed anthropic solutions of the cosmological constant problem, it would be interesting to obtain a large decay constant for axion-like fields from microscopic physics. String theory is the only context in which one can sensibly address this question. Here we survey a number of periodic fields in string theory in a variety of string vacua. In some examples, the decay constant can be parameterically larger than the Planck scale but the effective action then contains appreciable harmonics of order $f_A/M_p$. As a result, these fields are no better inflaton candidates than Planck scale axions.
1 Introduction

It is probably fair to say that there still does not exist a compelling microscopic theory of inflation. In conventional field theory models of slow roll inflation, it is difficult to understand the required flatness of the potential. In chaotic models, one has similar issues, particularly since one is interested in fields very large compared to the Planck scale, where conventional effective field theory notions are highly suspect.

It has long seemed plausible that string moduli might play a role in inflation[1]. One of us[2] (see also [3]) has described rather detailed scenarios. In these, one supposes that there is an approximate modulus, with a potential whose magnitude is of order $\sqrt{M_{\text{gut}}^3/M_p}$, scale of variation $M_p$, and has a minimum with vanishing cosmological constant. Fields with such features are natural in the Horava-Witten limit of string theory[5], or other brane-world scenarios with higher dimensional Planck scale of order the GUT scale. In this picture, fluctuations are automatically of the correct order. One obtains slow roll and an adequate number of e-foldings provided that certain parameters which are naturally of order one are instead of order $10^{-100}$.

There is another widely studied model for inflation, “natural inflation”, in which pseudo-Goldstone bosons play the role of inflatons[4]. In this paper, we will refer to such periodic fields generically as “axions.” For this to work, it is necessary that the decay constants of these Goldstone bosons, $f_A$, be large compared to $M_p$, the four-dimensional Planck scale. This possibility has been viewed with some skepticism. As in the case of chaotic inflation, one is in a regime where effective field theory is suspect.

There is another context in which a large decay constant is possibly of interest. As discussed long ago in [13], and more recently by [14], if one has an extremely light scalar field, with mass smaller than the inverse Hubble radius today, which can vary over a very large distance in field space, one can hope to implement an anthropic solution to the cosmological constant problem. For example, if, during inflation, there was an extremely large number of e-foldings, then there will be regions with cosmological constants taking a wide range of values; some will be close to that observed today, and may be selected by anthropic considerations[12]. One of us has argued[15] that in string theory this is implausible. For scalar moduli (i.e. moduli without periodicity) such small masses (i.e. small compared to the supersymmetry breaking scale) do not arise in known string models. This paper also argued that periodic moduli typically do not have the decay constants required – many orders of magnitude larger than the Planck scale.
Arkani-Hamed et al., hoping to understand natural inflation, have recently suggested that one might obtain parameterically large $f_A/M_p$ in the context of theories with extra dimensions[6], see also [7] for a discussion of similar ideas. They focus on a Wilson line in a fifth dimension, and note that provided a certain condition on the radius of the the fifth dimension and the gauge coupling in this dimension is satisfied, the decay constant can be large.

String theory is replete with periodic moduli. In addition to Wilson lines, these arise from higher rank antisymmetric tensor fields and from inherently stringy sources. In this note, we survey periodic moduli in a variety of string models, asking whether there are regimes of moduli space where decay constants are parametrically large. One might suspect that it is difficult to make decay constants arbitrarily large. For example, in compactifications of the heterotic string, the condition of [6] requires that the compactification radius be small in string units. But then there is a $T$-dual description in which the radii are large, and in which the decay constant must be small. This behaviour, we will see, is typical of periodic moduli in toroidal string and M-theory compactifications. With enough supersymmetry, it is a theorem[16] that extreme regions in moduli space always can be mapped into large radius/weak coupling regions of some other string theory (or eleven dimensional supergravity). We will also investigate a variety of string compactifications with little or no supersymmetry, and show that even when a large $f_A/M_p$ can be obtained, the axion potential has variation of order one on the scale $M_p$, so that large numbers of e-foldings of slow roll inflation do not occur. This can be understood as due to the dominance of higher harmonics ($n \sim f_A/M_p$). In specific examples, we will explain how the appearance of additional states and/or instantons at small radius modifies the naive expectations for the axion periodicity.

This paper is organized as follows. In the next section, we discuss string theories with 32 or 16 supersymmetries. We will see explicitly for such theories how the decay constant is bounded by $M_p$, as expected from the theorem cited above. We then turn to string configurations with less supersymmetry. Here it is difficult to obtain a general result, but in a variety of examples with eight, four or zero supersymmetries we again find that the effective decay constant cannot be larger than the Planck scale. In section four, we consider similar issues in the eleven dimensional limit. We present our conclusions in section five.
2 String Configurations with 16 or 32 Supersymmetries

In this section, we consider string theories with 16 or more supersymmetries. We first study compactifications of the heterotic string at weak coupling, since they provide a simple realization of the Wilson lines discussed in [6]. We then turn to Type I theories at weak coupling, compactified on tori, and finally weakly coupled Type II theories. In all cases, we will see that the decay constants are bounded by $M_p$. We explain how a theorem due to Witten[16] allows us to understand these results in a unified way.

2.1 Weakly Coupled Heterotic Strings

We first consider a class of theories in which the decay constants cannot be arbitrarily large. These are theories which exhibit some sort of $T$ duality. In closed string theories compactified on tori, there are a variety of periodic moduli. For definiteness, consider first the weakly coupled heterotic string theory, and compactify on a six torus whose bosonic action has the form (here, and throughout this paper, since we are interested in the parametric dependence of various quantities, we will not be careful about numerical factors):

$$S_{bos} = M_s^8 \int d^{10}x \sqrt{-g} e^{-2\phi} (R - \frac{1}{2} |H_3|^2) - M_s^8 \int d^{10}x \sqrt{-g} e^{-2\phi} \frac{1}{2} |F_2|^2,$$

where $H_3 = dB_2$ and $F_2 = dA_1$. For simplicity we will take the torus to have all radii of size $R$. The generalization to asymmetric compactifications is straightforward and does not change our conclusions. Periodic moduli include:

1. Moduli from $B_{ij}$, the second rank antisymmetric tensor fields. Here the axion field is,

$$\theta = M_s^2 \int d\Sigma^{ij} B_{ij}.$$ (2)

These have periods of order 1 in string units. Dimensionally reducing the kinetic term for $B_{ij}$ one finds their decay constants are:

$$\frac{f_{a}^2}{M_p^2} = \frac{1}{(RM_s)^4}.$$ (3)

2. Moduli from $B_{\mu\nu}$ (the model-independent axion). Decay constant$^1$:

$$\frac{f_{a}^2}{M_p^2} = 1.$$ (4)

$^1$Some of the estimates we will present, may be modified by relatively large factors like $16\pi^2$. We do not know of examples where such factors help rather than hinder slow roll inflation. At any rate, factors like this could at best give of order $10^2$ e-foldings.
3. Wilson lines (the types of fields considered in [6]): $A_I$ have periods of order $1/R$ in string units (if one takes the $X^I$ coordinates to have periodicities of order $R$):

$$\frac{f_\alpha^2}{M_p^2} = \frac{1}{(RM_s)^2}.$$  

Of these, it would appear that the $B_{IJ}$ and $A_I$ moduli can have large decay constants if $R$ is small. One can check that it is precisely this small $R$ region which is the region of moduli space considered in [6]. But because of $T$-duality, one knows that there is an equivalent description in terms of a large radius theory, and from our formula, it appears that decay constants are never large in the large $R$ region.

To see how $T$ duality works it is helpful to examine the exact spectrum at tree level. For the case of a single compact dimension (the generalization to more dimensions is straightforward), the left-moving momenta in the compact dimension are:

$$K_{L5} = \frac{n}{R} + \frac{wR}{\alpha'} - q^I A_5^I - \frac{wR}{2} A_5^I A_5^I.$$  \hspace{1cm} (3)

Here the $q_I$'s lie in the momentum lattice. There are similar expressions for the other momenta. The periodicity $A_5^I \to A_5^I + \frac{1}{R}$ is manifest in this expression. For small $R$, however, eqn. 3 exhibits an additional, approximate periodicity,

$$A_5^I \to A_5^I + \frac{R}{\alpha'}.$$  \hspace{1cm} (4)

This is as expected for small radius. This symmetry appears to be only approximate. This is because the $T$-duality transformation of $A_5$ is somewhat complicated. For small $A_5$, one has simply $A_5 \to A_5 = A_5$, but there are non-linear modifications of this transformation[17]. Note that this means that if we call the periodic variable $\theta$, in the large $R$ regime, $\theta = A_5 R$, but in the small $R$, $T$-dual regime,

$$\theta = A_5^I \tilde{R} \approx \frac{A_5 \alpha'}{R}. $$  \hspace{1cm} (5)

So in the large $R$ regime, the decay constant is $\frac{1}{R^2 M_s^2}$ is Planck units, but in the small $R$ regime it is $R^2 M_s^2$.

If one breaks supersymmetry, e.g. with Scherk-Schwarz boundary conditions[9] one can also see this connection at the level of the potential. For large $R$, the potential receives its largest contribution from the momentum states; the contributions of winding states are exponentially suppressed. In the small $R$ region, the role of the two sets of states are reversed, and the periodicity of the dominant terms is different in the two cases.
It is instructive to ask how all of this looks if one works in terms of the original variables, with the $1/R$ periodicity. In this case, for small $R$, the actual periodicity of the potential (and other quantities in the effective action) is very large, but the approximate periodicity is small. This means that in the Fourier expansion in $\cos(n\theta A_5 R)$, the typical $n$’s are large, $n \sim \alpha'/R^2$. So the potential varies, not over a range $R$, but rather over $\alpha'/R$. Later we will see a similar phenomenon in examples where there are no exact dualities relating the singular limit to a large smooth manifold.

One can consider the other periodic fields similarly. For the antisymmetric tensor $B_{IJ} = \phi(x)$, with $I, J$ denoting coordinates internal to the torus, the kinetic terms have the form

$$G_N \frac{\alpha'^2}{R^4} (\partial \phi)^2.$$  

So

$$\frac{f_A^2}{M_p^2} = \frac{\alpha'^2}{R^4}.$$  

Again, this expression becomes large, formally, for small $R$. But in a T-dual picture, it has the same form, and so must be small. One can trace this back, again, to the details of the $T$-duality transformation, or to the slow convergence of the Fourier series in the original variables.

### 2.2 Type I Theories

Consider Type I string theory compactified on a six torus. We will be interested in letting the size of one dimension become large or small, so we will take the volume to be $V_s R$, where the direction $X_1$ is of size $R$. The low energy effective action is that of type I supergravity whose bosonic action contains

$$S_{bos} = M_s^8 \int d^{10}x \sqrt{-g}e^{-2\phi} R - M_s^8 \int d^{10}x \sqrt{-g}e^{-\phi} Tr |F_2|^2 + \ldots.$$  

The fields arising from the NS-NS sector behave similarly under $T$-duality to the antisymmetric tensor fields of the heterotic string we have described above and do not result in decay constants larger than $M_p$. The gauge field is more interesting. Here, the periodic variable $\theta$ will be the Wilson line of the gauge field around $X_1$,

$$\theta = M_s \int A_1.$$  

At large $R$, its decay constant, relative to the Planck scale, has an extra factor of $g$ compared to the closed string theories discussed previously:

$$\frac{f_A^2}{M_p^2} = \frac{g}{M_p^2 R^2}.$$  

On the face of it it seems possible to obtain a large $f$ in one of two regimes: $R$ comparable to the string scale and $g$ large or alternatively small $g$ and $R \ll l_s$. We will consider each of these regimes in turn but in either case we will see that there is a dual theory which provides a more appropriate description, and that $f_A$ is smaller than the Planck scale.

First consider the case of small $g$. Since $R$ is smaller than the string scale we should study the T-dual Type I′ theory[18]. Type I′ is equivalent to Type IIA string theory on $S^1/\Omega \circ Z_2$ with radius $R' = \alpha'/R$. At the two fixed points of $Z_2$ there are two orientifold O8 planes, with 16 D8 branes in between. The locations of the D8 branes correspond to the Wilson line of the type I theory. To compute $f_A$ in this situation we need to recall that the low energy SUGRA action is that of massive IIA supergravity with the usual form of the Einstein-Hilbert action. In addition to the SUGRA contribution to the action we need to include the worldvolume action coming from the D8-branes. This is given by SYM theory in nine dimensions. The only part of the action that is relevant to us is that of the transverse fluctuations of the D8-branes. Let $X_9$ be the coordinate describing these fluctuations. Then the Wilson line axion is $X_9/R'$. The coordinate $X_9$ has periodicity $2\pi R'$ and the axion now has kinetic term,

$$M_s^7 \int d^9 x \sqrt{-g} e^{-\phi} (\partial \theta)^2 \left( \frac{R'}{l_s} \right)^2,$$

thus in the type I′ theory,

$$\frac{f_A^2}{M_p^2} = g' M_s R'.$$

The coupling in the $T$-dual theory is larger than the original coupling by a factor of $(R'/R)^{1/2}$. So we recover the result of equation 8 in the Type I′ theory. It would then appear that if we take $R'$ large, we can obtain a large $f_A$. We simply require that $g' > \frac{1}{\sqrt{M_s}}$.

But Polchinski and Witten, in [18] noted that this region of the Type I′ theory is not weakly coupled; in fact, $R' = \frac{M_s}{g'}$ is a limiting length in the theory. This follows from considering Type I-O(32) duality (Polchinski and Witten actually viewed this as a test of this duality). In terms of the original Type I variables, $g_h = \frac{1}{g'}$, while the radius goes over to $R_h^2 = \frac{1}{g_l} R_i^2$. So in the dual heterotic theory, $R_h \sim M_s^{-1}$. But this is the limiting, self-dual radius of the O(32) theory. Thus $f_A$ cannot be parameterically larger than $M_p$.

Polchinski and Witten explained how this limiting radius shows up in the Type I′ theory: at these values of $g' R'$ perturbation theory of type I′ string theory breaks down. The dilaton has a non-trivial profile. This can be understood as arising from the non-local cancellation of tadpoles. This profile is linear to leading order in $g' R'$, as expected in this quasi-one dimensional geometry; perturbation theory breaks down when $g' R'$ is of order 1.
Now consider the other possibility that the compactification radius is of order the string scale and the string coupling is large. Here the theory is described by heterotic $SO(32)$ theory, at weak coupling. Under this transformation, $g_h = \frac{1}{g_t}$, while the radius goes over to $R_h^2 = \frac{1}{g_t} R_l^2$. The radius $R_h$ is small and we must $T$-dualize to weakly coupled heterotic $E_8 \times E_8$ with large radius. Following this path of dualities and taking into account their effect on $\theta$ as defined in (7) we again find that $f$ is bounded above by $M_p$.

2.3 Type IIA

It is instructive to first consider the case of Type IIA compactified on a 6 torus. Because of the high degree of supersymmetry, all extreme regions of the theory are equivalent to some large radius, weak coupling description. The way in which this works can be rather intricate. The Type IIA string has a Ramond-Ramond one form gauge potential, $C_1$ and $\theta$ is the Wilson line of this gauge field around one of the compact toroidal directions, $\Sigma_1$

$$\theta = M_s \int_{\Sigma_1} C_1.$$  \hspace{1cm} (11)

To obtain the kinetic term for $\theta$ recall that the Ramond-Ramond field enters the 10 dimensional supergravity action as,

$$S = M_s^8 \int d^{10}x \sqrt{-g} |F_2|^2,$$ \hspace{1cm} (12)

where $F_2$ is the field strength of $C_1$. Compactifying on a torus of volume $V_5 R$ where $R$ is the size of $\Sigma_1$ results in a decay constant for $\theta$ of,

$$f_A^2 = \frac{M_s^6 V_5}{R}.$$ \hspace{1cm} (13)

In order to compare to the four dimensional Planck scale recall the form of the Einstein-Hilbert action in string frame,

$$S = M_s^8 \int d^{10}x \sqrt{-g} e^{-2\phi} R.$$ \hspace{1cm} (14)

Note the appearance of the string coupling in the action in contrast to (12). Consequently $M_p^2 = M_s^8 g^{-2} V_5 R$. Measured in units of the four dimensional Planck scale,

$$\frac{f_A^2}{M_p^2} = \frac{g^2}{(M_s R)^2}.$$ \hspace{1cm} (15)

We can attempt to obtain a large decay constant by taking $R \sim l_s$ and $g \gg 1$. For simplicity we will assume that $V_5$ is large compared to the string scale. When $g \gg 1$ one might try to
obtain a description of the physics in the 11 dimensional supergravity dual. Rewriting (15) in
11 dimensional units one sees that \((f_A/M_p)^2 = (R_{11}/R)^2\) where \(R_{11} = g^{2/3}l_{11}\) is the (large)
radius of the 11 dimension. Naively, it would seem possible to realize large \(f_A\). However,
\[
R = l_{11}g^{-1/3} = l_{11}\frac{l_{11}}{R_{11}} \ll l_{11}.
\]
so a better description of this limit is in fact weakly coupled type IIA string theory on a torus
whose sizes are given by \(V_5\) and \(R_{11}\) both of which are large in string units, with small string
coupling, \(g_s' = R/\ell_{11}\). In this description, the coupling is weak and all radii large, and \(f_A\) is
small. In the more general case where one or more of the radii of \(V_5\) is small the conclusion
applies although the details vary.

Using the results of [16] we can understand the results for toroidal compactification in
a much more general way. Witten has shown that the space of all compactifications to four
dimensions with 32 supercharges is a connected space. Furthermore, every asymptotic direction
of this space can be mapped by duality transformations into either Type II string theory with
weak coupling and radii larger than the string scale, or 11D SUGRA with radii larger than the
Planck scale. In these regimes it is easy to see that one never gets axion decay constants larger
than the four dimensional Planck scale. The case of 16 supercharges is more subtle and we
have not studied it completely. The moduli space has disconnected components, only one of
which is connected to weakly coupled heterotic strings. The results of [11] show that a result
similar to that of [16] is obtained, but there are a variety of asymptotic regions where fully
controlled calculations are not possible\(^2\). The simplest of these is F-theory on K3 manifolds.
They are described by string theory with weak coupling on a large compact space where the
coupling varies and sometimes becomes strong. We have seen no evidence for large axion decay
constants in these regimes, but have no definitive proof that they cannot be found.

3 Theories with Less Supersymmetry

With less than 16 supersymmetries, the structure of the moduli space is not well understood.
There is no simple argument that all extreme regions are equivalent to large radius, weakly
coupled theories. So for such theories one might hope to find parameterically large decay
constants. In this section we consider several examples, and find that in an appropriate sense,
decay constants cannot be larger than \(M_p\). Our search cannot be considered exhaustive, but
\(^2\)These were dubbed **sprantiloid** regions by L. Motl, to emphasize that we could name them, but not understand
them.
we suspect it is indicative of the general situation.

3.1 N=0 Supersymmetry

The space of (classical) string states with zero supersymmetry is not well explored. Toroidal compactifications of Type II strings yield theories with 32 supercharges; of the heterotic string, 16 supercharges. We can obtain a theory with no supersymmetries by compactifying the Type II theory on a torus with Scherk-Schwarz boundary conditions[9]. These theories have a one loop instability, and may not really exist, but we can examine them using weak coupling string theory. In this case a naive treatment yields large $f_A/M_p$ for small radii. But T-duality again implies that the correct description of the theory has a small $f_A$ (in general, the small radius theory is equivalent to some other non-supersymmetric string theory). Little is known about the strong coupling limits of these theories, so it is difficult to make definitive statements as in the supersymmetric case, but most likely the strong coupling limit, if it makes sense, is again equivalent to some weakly coupled theory. In any case, one cannot obtain a large decay constant in any controlled approximation.

3.2 Eight or Four Supersymmetries

With four or eight supersymmetries, much more is known about the theories, but there are richer possibilities than in the more supersymmetric cases. The previous examples illustrated that in compactifications in which small radius is equivalent to another theory at large radius, one cannot obtain parametrically large decay constants. So it is necessary to consider models where such an equivalence does not hold in such a simple way. An example is provided by IIA theory compactified on a Calabi-Yau space near a conifold singularity. Near the conifold point in moduli space, there is a singular region in the geometry which is topologically $S^2 \times S^3$, with each sphere shrinking to zero size.

Consider the (pseudo)-scalar field arising from the three form Ramond-Ramond field $C_{MNO}$:

$$\theta = M_s^3 \int_{S^3} C_{IJK} d\Sigma^{IJK}$$

where $\Sigma_{IJK}$ is the volume form of the three sphere. This has unit periodicity in string units, but its decay constant is of order:

$$\frac{f_A^2}{M_p^2} = g^2 \frac{1}{(M_s^3 V_{S^3})^2}.$$
The extra factor of $g$ arises because $C$ is a Ramond-Ramond field. Here there is no simple dual description of the theory for small $V_{S^3}$, so one might hope to obtain a large $f_A$.

New light states, however, do appear in the theory as one shrinks $V_{S^3}$. These arise from $D4$ branes wrapped on the $S^3$. These states couple magnetically to the $C$ field, and alter the axion effective action. These are strings of tension

$$T_{D4} = \frac{1}{g} M_s^5 V_{S^3}. \quad (19)$$

In the limit that $f_A/M_p$ is large, these states become light compared to the string scale. Thus it is not possible to simultaneously keep the wrapped $D4$ brane states heavy and have a decay constant larger than the Planck scale.

In addition, there are $D2$ brane instantons, whose action is precisely $M_p/f_A$ in the region where $f_A/M_p$ is large. An $n$ instanton contribution to the action would be of order $e^{-n M_p f_A} e^{i n \theta}$, where $\theta$ is the dimensionless axion field with period $2\pi$. Thus, the instanton series would effectively terminate only at $n \sim f_A/M_P$ and the action for canonical fields is rapidly varying. In this highly supersymmetric case, there is no axion potential, but we will see below that a similar mechanism produces rapidly varying potentials in models with only four supercharges.

### 3.3 Type IIB

Now we consider the Type IIB theory in singular Calabi-Yau compactifications. In particular we will consider geometric singularities due to a collapsing 2 cycle, $S^2$. Examples include a conifold point, and an $A-D-E$ type singularity in the case of a $K3$ fibration.

Recall that the bosonic part of the type IIB supergravity action includes the terms:

$$S_{bos} = \int d^{10}x \sqrt{-g} e^{-2\phi} \left( R - \frac{1}{2} |H_3|^2 \right) - M_s^8 \int \sqrt{-g} \frac{1}{2} |F_3|^2 + \ldots, \quad (20)$$

where $H_3 = dB_2$ and $F_3 = dC_2$ and $C_0$ has been set to zero. Now one can hope to obtain a large $f_A$ as $V_{S^2} \to 0$. There are two 2 form gauge fields from which one can obtain an axion: the RR sector two-form field $C_2$ and the NS-NS sector, $B_2$. First consider $B_2$,

$$\theta = M_s^2 \int_{S^2} B_1 d\Sigma^{1J}. \quad (21)$$

There are additional light states in this limit coming from wrapping three branes on $S^2$. But they do not couple directly to the two form field, so one might hope that they would not interfere with obtaining a small $f_A$. 

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Dimensional reduction of the kinetic term for $B_2$ results in a decay constant for $\theta$ of,

$$f_A^2 = \frac{1}{M_p^2(V_{S^2}M_S^2)^2}. \tag{22}$$

Again, it seems that it is possible to obtain a large $f_A$ provided the $S^2$ is small in string units. In string perturbation theory, the action is independent of $\theta$; the leading effects which violate the continuous shift symmetry are worldsheet instantons. Their contribution to the effective action is a function of $u = \exp(M_s^2 \int_{S^2} \sqrt{g} + i\theta)$. In the limit required for a large $f_A$ the large $n$ sector instantons are not suppressed. The effective action for the canonical axion field varies on scales $f_A/n \sim M_p$.

Similarly one can carry out the analysis for $C_2$, defining

$$\theta = M_s^2 \int_{S^2} C_{IJ} d\Sigma^{IJ}. \tag{23}$$

This field has a decay constant

$$f_A^2 = \frac{g^2}{(V_{S^2}M_S^2)^2}. \tag{24}$$

Now it seems possible to obtain a large $f_A$ whilst keeping $V_{S^2}$ large in string units, by going to strong coupling. Alternatively we can keep $g$ perturbative and consider shrinking the $S^2$ to below the string scale. Either case suffers from the same problem of instanton corrections mentioned above. This time the instanton corrections come from D-string worldsheet instantons and are a function of $u = \exp(M_s^2/g \int_{S^2} \sqrt{-g} + i\theta)$. The extra factor of $1/g$ comes from the fact that the D-string couples to the RR sector. Again, the large $n$ instantons contribute and result in the decay constant being no larger than $M_p$. The fact that the analyses for $B_2$ and $C_2$ result in the same conclusion is to be expected due to the $SL(2,\mathbb{Z})$ symmetry of type IIB string theory.

4 M Theory

We now wish to consider the strong coupling limit of some of the string theories considered earlier. We will start with the $E_8 \times E_8$ heterotic string. The low energy limit of this string theory at large string coupling is given by supergravity on $R^{1,9} \times S^1/\mathbb{Z}^2$ where the size of the $S^1$ is given by the string coupling. The $\mathbb{Z}_2$ action has two fixed points. At each of these there is a 10 dimensional plane with an $E_8$ gauge theory living on it. The $\mathbb{Z}_2$ projection breaks half of the original 32 supercharges of 11 dimensional supergravity.
The supergravity sector of the low energy effective action of heterotic M-theory is that of 11 dimensional supergravity whose bosonic sector is given by

\[ S_{11} = M_{11}^9 \int d^{11}x \sqrt{-g} \left( R - \frac{1}{2} F_4^2 \right) - \frac{1}{6} C_3 \wedge F_4 \wedge F_4. \]  

(25)

The final, Chern-Simons, term is necessary for anomaly cancellation and SUSY. For the action to be \( \mathbb{Z}_2 \) invariant the gauge field \( C_3 \) must be odd under the \( \mathbb{Z}_2 \) projection. Consequently, the \( C_3 \) field must have a component along the 11 direction.

We will consider heterotic M-theory on a Calabi-Yau in order to get a four dimensional minimally supersymmetric theory. In particular we will consider the point in the moduli space of the Calabi-Yau where a two cycle, \( \Sigma_2 \), shrinks to zero size. The axion is the \( C_3 \) field with two directions on \( \Sigma_2 \) and the other along the 11 direction,

\[ \theta = M_{11}^3 \int_{\Sigma_2 \times S^1/\mathbb{Z}_2} C_{ij11}. \]

(26)

At length scales larger than \( R_{11} \) we have an effective four dimensional theory containing 4 dimensional supergravity, non-abelian gauge theory and the axion.

Dimensionally reducing the kinetic term for \( C_3 \), the four dimensional effective action has a kinetic term for \( \theta \) given by,

\[ S = \frac{V_{CY} R_{11}}{M_{11}^3 R_{\Sigma_2}^3} \int d^4x \sqrt{-g_4} (\partial \theta)^2. \]

(27)

In units of the four dimensional Planck scale, the axion’s decay constant is,

\[ \frac{f_A}{M_P} = \frac{1}{M_{11}^3 R_{11}^2 R_{\Sigma_2}^2}. \]

(28)

Naively it seems plausible that by shrinking the two cycle one may reach the regime where \( f_A \gg M_P \). In fact, one needs \( R_{\Sigma_2} \ll l_{11} \) while keeping \( R_{11} \) large in 11 dimensional units as required by the large expectation value of the heterotic dilaton.

However, in this limit the classical supergravity analysis is incomplete since the shrinking two cycle leads to a geometric singularity. One must worry about what physics resolves this singularity. In particular, one may worry about new light states coupled to \( \theta \) appearing at the singularity. This would alter the form of the \( \theta \) potential.

Even without worrying about such states, however, we can see that, once again, high harmonics are likely to be important. Consider the effect of membrane instantons. These give contributions to the action governed by powers of \( u \), where

\[ u = e^{-M_{11}^3 \int_{\Sigma_2 \times S^1/\mathbb{Z}_2} \sqrt{-\theta + i \int_{\Sigma_2 \times S^1/\mathbb{Z}_2} C_{ij11}}}. \]

(29)
In particular, they are not small in the limit where $f_A$ is large.

In the weak coupling limit, the resolution of the singularities is understood. The lowest order, classical supergravity analysis is modified by perturbative corrections in $\alpha'$ as well as worldsheet instantons. Witten showed [8] that the only non-zero contributions come from lowest order in $\alpha'$ and from worldsheet instantons. These effects alone resolve the singularity with no new massless states in the low energy description. However the inclusion of the worldsheet instanton effects modify the axion effective action. In the weak coupling limit, the membrane instantons are just the world sheet instantons. World sheet instanton effects are functions of $u = \exp(-T \int_{\Sigma^2} \sqrt{-g} + i \int_{\Sigma^2} B)$. This is precisely the same quantity as in eqn. 29, translated into the weak coupling language.

So, once again, we see that the dominant harmonics have $n$ up to $n \sim f_A/M_p$. So the Fourier series for the effective action will converge slowly, and will be a rapidly varying function of $\theta$; the action for the canonical field will vary on the scale $M_p$.

### 4.1 $G_2$ Example

Finally, we will consider an example of 11D SUGRA compactification on a manifold with $G_2$ holonomy. An example of interest, closely related to the conifold of string theory, is that of a $G_2$ manifold which is asymptotically a cone over a six dimensional manifold which is $S^3 \times S^3$. Topologically the space is $R^4 \times S^3$. There is a vanishing supersymmetric cycle which is a three sphere, $X$. The axion is now,

$$\theta = M_{11}^3 \int_X C_3,$$

where $C_3$ is again the three form of 11 dimensional supergravity.

$C_3$ appears in the supergravity action as,

$$M_{11}^9 \int d^{11}x \sqrt{-g} |F_4|^2,$$

from here one sees that the decay constant in Planck units is,

$$\frac{f_A^2}{M_p^2} = \frac{1}{(V_X M_{11}^3)^2}.$$

In order to get large $f_A$ we need the shrinking three cycle to be small compared to the 11 dimensional scale, $l_{11}$. As before, the supergravity analysis is corrected by instanton effects. In this case the instantons come from M2 branes wrapping $X$. Their contribution to the action
will be a rational function of $u$ where,

$$u = e^{-M_{11}^3 \int_X \sqrt{-g} + i \int_X C}.$$  

(33)

These membrane instanton effects are not suppressed in the limit of small volume of $X$.

The large $n$ instanton contributions are no longer suppressed and the axion decay constant is restricted to be $f_A/n$. The instanton effects mean that the effective action is once again dominated by high harmonics, so that the potential varies on the Planck scale.

A similar analysis applies to any $G2$ singularity where some number of three cycles shrinks to zero.

5 Conclusions

Axion like fields, with decay constants larger than the Planck scale, could give rise to models of natural inflation, and if the decay constant were extremely large, might form the basis for an anthropic explanation of the value of the cosmological constant. One can only study the plausibility of such scenarios in a framework where Planck scale physics is under control. This is the case in certain extreme regions of the moduli space of string/M theory, and, as far as we are aware, only in this arena.

We have examined a variety of regions in moduli space where a large decay constant could arise, but have found no consistent scenario of this type. In each case the required axion field arises from the components of a p-form gauge potential along a p-cycle in the limit that the cycle shrinks to zero size. There are always light states and/or low action instantons, which arise in this limit, and give rise to rapidly varying contributions to the axion potential (in those cases where a potential is allowed by supersymmetry) which effectively rescale the decay constant to the Planck scale.

Another way that one might imagine that string moduli might serve the purposes of natural inflation is in the noncompact regions of moduli space. However, in those regions, general arguments show that the potential falls so rapidly that no solutions with accelerated expansion can be found. The motion of the moduli in such noncompact regions is dominated by kinetic energy, rather than friction.

Although our arguments sound rather general, we have certainly not explored all possible singular limits of M-theory, so it would be premature to claim that our results are a no-go
In our opinion they seem suggestive of such a theorem, and it seems worthwhile to explore further, in search of a counterexample, or a proof.

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