Neutrino masses and mixing

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Abstract. Status of determination of the neutrino masses and mixing is formulated and possible uncertainties especially due to presence of the sterile neutrinos are discussed. The data hint an existence of special “neutrino” symmetries. If not accidental these symmetries have profound implications and can substantially change the unification program. The key issue on the way to the underlying physics is relations between quarks and leptons. The approximate quark-lepton symmetry or universality can be reconciled with strongly different patterns of masses and mixings due to nearly singular character of the mass matrices or screening of the Dirac structures in the double see-saw mechanism.

1. Introduction

In the first approximations the pattern of lepton mixing has been established. The 2-3 mixing is consistent with maximal, 1-2 mixing is large but not maximal and 1-3 mixing is small and consistent with zero. The next step is determination of detailed structure of mixing, in particular, measurements of the deviations of 2-3 mixing from maximal and 1-3 mixing - from zero.

There are two key issues on the way to the underlying physics:

• possible existence of new “neutrino” symmetries behind the pattern of neutrino mass and mixing;
• relation between quarks and leptons - their possible symmetries and unification.

The two questions are related: establishing specific symmetry in the neutrino sector may substantially change the unification program.

2. Results and uncertainties

2.1. Summarizing results

Masses: The solar and the atmospheric mass differences squared give the lower bound on ratio of the second and third masses:

\[
\frac{m_2}{m_3} > \sqrt{\frac{\Delta m_{21}^2}{\Delta m_{31}^2}} = 0.15 - 0.20. \tag{1}
\]

Both cosmology and the double beta decay probe the sub-eV region which corresponds to the quasi-degenerate mass spectrum giving the upper bound \( m < 0.2 - 0.4 \text{ eV} \) \[1\]. If the Heidelberg-Moscow result \[2\] is confirmed and if it is due to exchange of the light Majorana neutrinos, the neutrino mass spectrum should be strongly degenerate.
Results on determination of the lepton mixings are summarized in fig. 1, 2.

1). 1-2 mixing: there is a very good agreement of central values and reasonable agreement of the allowed ranges at different confidence levels obtained by three different groups [3, 4, 5]. The 3ν analysis does not change the best fit value of mixing in comparison with 2 neutrino analysis but the error bars become smaller.

After the recent SNO publication [3], the 1-2 mixing has shifted to larger values by $\Delta \theta_{12} \sim 1.6^\circ$ due to increase of the CC/NC ratio and now the b.f. value equals $\theta_{12} = 33.9^\circ$. In fig. 1a we show also several theoretical benchmarks: predictions from (i) the QLC1 scenario (see sec. 4.3): $\theta_{12} = 35.4^\circ$, (ii) the QLC2 scenario: $\theta_{12} = 45^\circ - \theta_C \approx 32.2^\circ$ (see sec. 4.3), and (iii) the tribimaximal mixing $\sin \theta_{12} = 1/\sqrt{3}$, or $\theta_{12} = 35.2^\circ$. All three predictions are within 1σ. Predictions from the tri-bimaximal mixing and QLC1 almost coincide, the b.f. value is in between the QLC2 and two other predictions. To disentangle these two possibilities one needs to measure the 1-2 mixing with accuracy $\Delta \theta_{12} \sim 1^\circ$ or $\Delta \sin^2 \theta_{12} \sim 0.015$ (5%).

2). The 2-3 mixing is in agreement with maximal one (fig. 1b). A shift from maximal mixing has been found when effects of 1-2 sector have been included in the analysis [6]. According to [7] $\sin^2 \theta_{23} = 0.47$ and slightly larger shift, $\sin^2 \theta_{23} = 0.44$, follows from the analysis [5]. So, the deviation from maximal mixing can be quantified as

$$D_{23} \equiv 0.5 - \sin^2 \theta_{23} \sim 0.03 - 0.06.$$  

(2)
The shift is related to the excess of e-like atmospheric neutrino events in the sub-GeV range detected by SuperKamiokande (SK). The excess is proportional to the deviation \( \Delta N_e/N_e \propto D_{23} \) \([6]\). No shift from maximal mixing has been found in the recent SK 3ν-analysis even after inclusion of the 1-2 sector (LMA oscillations) \([11]\). The difference of results may be related to treatment of uncertainties in the atmospheric neutrino flux normalization. The change of normalization competes with the effect of LMA oscillations. In the SK analysis the excess of e-like events is explained completely by the normalization. In the analysis \([7, 5]\) the excess is explained by the normalization partially, since certain distribution of the normalization factors is assumed. Still large deviation from maximal mixing is allowed:

\[
D_{23} \sin^2 \theta_{23} \sim 0.4 \quad (2\sigma).
\]

3. The 1-3 mixing is consistent with zero (fig. 2). Small non-zero best fit value from the analysis \([5]\) is related to the angular dependence of the multi-GeV e-like events measured by SuperKamiokande. The most conservative 3σ bound is \( \sin^2 \theta_{13} < 0.048 \) \([5]\). There are several benchmarks here: A very appealing possibility, \( \theta_{13} = \theta_C \), seems to be excluded at more than 3σ level. The ratio of the solar and atmospheric neutrino mass scales,

\[
\sin^2 \theta_{13} = r = \frac{\Delta m^2_{21}}{\Delta m^2_{31}} = 0.033,
\]

is allowed at about 2σ level. An additional (model dependent) factor of the order 0.3 - 2 may appear in this relation. Much smaller values of \( \sin^2 \theta_{13} \) would imply most probably certain symmetry of the mass matrix.

There are several lower bounds on the 1-3 mixing: (i) even if equality \( \sin^2 \theta_{13} = 0 \) holds at some high energy scale, \( \Lambda \) (presumably GUT or scale of flavor physics), a nonzero value of the order \( \sin^2 \theta_{13} = 0.003 \) is generated due to the renormalization group effect (unless some accidental cancellation occurs) \([12]\). (ii) Smaller values, \( \sin^2 \theta_{13} = 10^{-4} \), are expected due to possible contributions to the mass matrix from the Planck scale interactions \( \sim v^2_{EW}/M_{PL} \) \([13]\).

2.2. Uncertainties

Three types of possible effects can influence interpretation of the neutrino results.

1. Existence of new neutrino states - sterile neutrinos which are of great interest not only in connection to the LSND result. If these states are light they can directly (dynamically) influence observations. If sterile neutrinos are heavy and decouple from the low energy physics, they may substantially change implications of the results for the fundamental theory.

2. Presence of the non-standard (short range) neutrino interactions can change values of the extracted neutrino parameters.

3. Interactions with hypothetical light scalar fields produce "soft" neutrino masses which depend on properties of medium. These masses may change with time and be related to the dark energy in the universe \([14]\).

At present, however there is no well established results which could testify for deviations from the “standard” 3ν mixing scheme and the standard matter interactions.

Let us consider one aspect of possible presence of sterile neutrinos - ambiguity in interpretation of the neutrino results. Even small mixing of active neutrinos with sterile ones can substantially change the structure of active neutrino mass matrix, in particular, inducing large mixing \([15]\). Suppose the active neutrinos acquire (e.g., via seesaw) the Majorana mass matrix \( m_a \). Consider one sterile neutrino, \( S \), with Majorana mass \( M \) and mixing masses with active neutrinos \( m_{iS} \) \( (i = e, \mu, \tau) \). If \( M \gg m_{iS} \), then after decoupling of \( S \) the mass matrix of active neutrinos becomes

\[
(m_\nu)_{ij} = (m_a)_{ij} - m_{iS} m_{jS}/M,
\]
where the last term is the matrix induced by $S$. New neutrino states are irrelevant if
\[ m_{iS} m_{jS}/M \ll (m_{a})_{ij}. \]  
(6)

The smallest matrix elements are in the case of normal mass hierarchy $(m_1 \approx 0)$. The data can be well described by
\[ m_\nu = \frac{m_3}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} + \frac{m_2}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \]  
(7)

where $m_2 = \sqrt{\Delta m_{21}^2}$ and $m_3 = \sqrt{\Delta m_{31}^2}$. Assuming flavor “blindness”: $m_{iS} = m_S$, we can rewrite the condition (6) using the smallest elements in (7) as $m_S^2 / M \ll m_2 / 3$, or
\[ \sin^2 \theta_S M \ll m_2 / 3 \sim 3 \cdot 10^{-3} \text{ eV}. \]  
(8)

Here $\theta_S$ is the active-sterile mixing angle: $\sin^2 \theta_S = m_S / M$.

For $M \sim 1 \text{ eV}$ we obtain from (8) $\sin^2 \theta_S \ll 3 \cdot 10^{-3}$. This means that if the LSND interpretation as an effect of additional neutrino states is confirmed, its impact on the neutrino mass matrix is strong and cannot be considered as perturbation. If the effect is not confirmed, the MiniBOONE sensitivity is not enough to exclude strong effect of new states. For $M \sim 1 \text{ MeV}$ we get $\sin^2 \theta_S < 10^{-9}$.

3. Neutrino symmetry

Several observations may testify for special symmetry(ies) associated to neutrinos: (i) maximal (close to maximal) 2-3 mixing; (ii) zero (very small) 1-3 mixing; (iii) special values of 1-2 mixing; (iv) degenerate mass spectrum; (v) hierarchy of mass squared differences. Some of these features can originate from the same underlying symmetry.

3.1. Schemes of mixing

In connection to the above observations the following schemes of mixing can be of relevance.

1). The bi-maximal mixing [16]:
\[ U_{\text{bm}} = U_{23}^m U_{12}^m = \frac{1}{2} \begin{pmatrix} \sqrt{2} & \sqrt{2} & 0 \\ -1 & 1 & \sqrt{2} \\ 1 & -1 & \sqrt{2} \end{pmatrix}. \]  
(9)

Identification $U_{PMNS} = U_{\text{bm}}$ is not possible due to strong (5 - 6) $\sigma$ deviation of the 1-2 mixing from maximal. However, $U_{\text{bm}}$ can play a role of dominant structure or matrix in the lowest order. Correction can originate from the charged lepton sector (mass matrix), so that $U_{PMNS} = U' U_{\text{bm}}$ and in analogy with quark mixing $U' \approx U_{12}(\theta_C)$. It generates simultaneously deviation of the 1-2 mixing from maximal and non-zero 1-3 mixing, which are related.

2). Tri-bimaximal mixing [17]
\[ U_{\text{tbm}} = U_{23}^m U_{12}(\theta_{12}) = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & \sqrt{2} & 0 \\ -1 & \sqrt{2} & \sqrt{3} \\ 1 & -\sqrt{2} & \sqrt{3} \end{pmatrix}. \]  
(10)

where $\sin^2 \theta_{12} = 1/3$. Here $\nu_2$ is tri-maximally mixed: in the middle column three flavors mix maximally, whereas $\nu_3$ (third column) is bi-maximally mixed. This matrix is in a good agreement with data, in particular, $\sin^2 \theta_{12}$ is close to the present best fit value 0.31.
3.2. $\nu_\mu - \nu_\tau$ symmetry

Maximal 2-3 mixing and zero 1-3 mixing can be consequences of the $\nu_\mu - \nu_\tau$ permutation symmetry of the neutrino mass matrix [18]. General form of such a matrix in the flavor basis is

$$M = \begin{pmatrix} A & B & -B \\ B & C & D \\ -B & D & C \end{pmatrix}. \tag{11}$$

The permutation symmetry can be a part of, e.g., discrete $S_3$ or $D_4$ groups.

The problem is that the symmetry is broken for charged leptons since $m_\mu \ll m_\tau$. So, it can not be the symmetry of complete theory. Let us discuss possible solutions of this problem.

The symmetry can be broken spontaneously, and for this the extended Higg’s sector is required. Several possibilities exist to explain why it shows up in the neutrino sector only. Apparently the difference of neutrinos and charged leptons should be related to their RH components.

1). Auxiliary symmetry, e.g. $Z_2$, can be introduced to protect neutrino sector from $\mu - \tau$ breaking. $l_R$ and $\nu_R$ should have different properties with respect to this auxiliary symmetry.

2). The symmetry basis can differ from the flavor basis. So, in the symmetry basis (one should speak about 2 - 3 permutations) the mass matrix of charged leptons is off-diagonal.

3). Among other flavor symmetries $A_4$ looks very appealing [19]. It has one triplet representation and three different singlet representations, $1$, $1'$, $1''$, which provides with enough freedom to explain data. Three leptonic doublets form the triplet of $A_4$: $L_i = (\nu_i, l_i) \sim 3$, $i = 1, 2, 3$. Required lepton mixing is generated due to different $A_4$ transformation properties of the right handed components of charged leptons and neutrinos. In some models: $l_i^c \sim 1, 1', 1''$, whereas $N_i^c \sim 3$. In other models vice versa: $l_i^c \sim 3, N_i^c \sim 1, 1', 1''$.

4). See-saw induced symmetries. In this case neither Dirac mass matrix nor Majorana mass matrix of the RH neutrinos have the required symmetry but they have certain structures (hierarchies of matrix elements). The symmetry appears as a result of see-saw mechanism.

In specific models some combinations of these mechanisms are realized [21].

3.3. Real or accidental

The main question here is whether the “neutrino” symmetries are accidental or real, that is, have some physics behind. Models proposed so far are rather complicated with a number of ad hoc assumptions. It is difficult to include quarks in these models. Further unification looks rather problematic. Asymmetries between neutrinos and leptons are embedded into theory from the beginning. This shows the price one should to pay for realization of the symmetries.

Furthermore, the facts behind the symmetries - maximal 2-3 mixing and relatively small 1-3 mixing are not yet well established. Still significant deviation of 2-3 mixing is possible and 1-3 mixing can be not so small. Structure of the neutrino mass matrix depends substantially on these deviations. So, it may happen that symmetry constructions are simply misleading.

On the other hand if symmetries are not accidental, they have consequences of the fundamental importance as the models constructed show. New structures and particles are predicted, unification path may differ substantially from what we are considering now, etc.. The symmetries may give some clue for understanding fermion masses in general.

The key question is how to test this? Obviously, we need to search for and measure deviations: of 2-3 mixing from maximal, $D_{23}$, and 1-3 mixing, $\sin \theta_{13}$, from zero. In the context of specific models the deviations (though small) are expected anyway. The facts we are discussing can originate from the same symmetry and violation of this symmetry will lead then to relations between $D_{23}$ and $\sin \theta_{13}$. 


Figure 3. Mass hierarchies of quarks and leptons. The mass of the heaviest fermion of a given type is taken to be 1.

It may happen that the symmetries are not accidental but the underlying theory has not been found yet. In this connection let us come back to the issue of active-sterile mixing. Let us assume that the couplings of $S$ with active neutrinos are universal:

$$m_{iS} = m_S(1, 1, 1) = m_2 / \sqrt{3}. \quad (12)$$

Then the induced matrix has form: $m_{ind} = m_2 D / 3$, where $D$ is the democratic matrix (the second matrix in (7)). Suppose that the original active neutrino mass matrix has structure of the first matrix in (7). Then the sum, $m_\nu = m_a + m_{ind}$, reproduces the mass matrix for the tribimaximal mixing (7). With two sterile neutrinos whole structure (7) can be obtained.

Clearly this possibility changes implications of the neutrino results. Since $S$ is beyond the SM structure (extended by RH neutrinos) it may be easier to realize “neutrino” symmetries as a consequence of certain symmetry of its couplings with active neutrinos.

4. Leptons and Quarks

4.1. Comparing leptons and quarks

There is an apparent correspondence between quarks and leptons. Each quark has its own counterpart in the leptonic sector. Leptons can be treated as the 4th color [22] following the Pati-Salam $SU(4)$ unification symmetry. Unification is possible, so that quarks and leptons form multiplets of the extended gauge group. The most appealing one is $SO(10)$ [23], where all known components of quarks and leptons (including the RH neutrinos) form unique 16-plet. It is difficult to believe that all these features are accidental. Though it is not excluded that the quark-lepton connection has some more complicated form, e.g., of the quark - lepton complementarity [24, 25].

In the quark sector we have rather complete information about masses and mixings and still no explanation has been found. It seems, neutrinos have not helped yet, and on the contrary, made a situation even more complicated. Comparison of masses and mixing in the quark and lepton sectors and establishing certain relations between them may give some insight.

Apparently the mixing patterns of leptons and quarks is strongly different: The only common feature is that the 1-3 mixing (between the “remote” generations) is small in both cases. Two other angles are not equal but complementary in a sense that they sum up to maximal mixing:

$$\theta_{12} + \theta_C = \frac{\pi}{4}. \quad (13)$$
and similar approximate relation can be written for the 2-3 mixings. For various reasons it is
difficult to expect precise relation but qualitatively one can say that, the 2-3 mixing in the
lepton sector is close to maximal because the corresponding quark mixing is small, the 1-2 mixing
deviates from maximal substantially because the 1-2 (Cabibbo) quark mixing is relatively large.
It seems that for the third angle we do not expect simple relation and apparently the quark
relation $\theta_{13} \sim \theta_{12} \times \theta_{23}$ does not work in the lepton sector.

The ratio of neutrino masses (1) can be compared with ratios for charged leptons and quarks
(at $m_Z$ scale): $m_\mu/m_\tau = 0.06$, $m_s/m_b = 0.02 - 0.03$, $m_c/m_t = 0.005$. The neutrino hierarchy
(if exists at all) is the weakest one. This is consistent with possible mass-mixing relation: large
mixings are associated to weak mass hierarchy.

In fig. 3 we show the mass ratios for three generations. The strongest hierarchy and geometric
relation $m_u \times m_t \sim m_2^2$ exist for the upper quarks. Apart from that no simple relations show
up. What is behind this picture? Symmetry, regularities, relation? In the quark sector we can
speak about fermion families with weak interfamily connection (mixing) which means strong
flavor alignment. Peculiar situation here is that spectra have small number of states (levels) - 3
and on the other hand there is no simple relations between parameters of spectra. It looks like
the observed pattern is an interplay of some regularities and randomness (“anarchy”).

4.2. Quark-lepton symmetry and Quark-lepton universality
The picture described in the previous section is still consistent with the approximate quark-
lepton symmetry or universality. The symmetry is realized in terms of mass matrices (matrices
of the Yukawa couplings) and not in terms of observables - mass ratios and mixing angles.

The key point is that similar mass matrices can lead to substantially different mixing angles
and masses (eigenvalues) if the matrices are nearly singular (rank-1) [26, 27]. The singular
matrices are “unstable” in a sense that small perturbations can lead to strong variations of mass
ratios and mixing angles in the context of seesaw.

Let us consider the universal structure for the mass matrices of all quarks and leptons [27]:

$$Y_u \sim Y_d \sim Y_D \sim Y_M \sim Y_L \sim Y_0,$$

where $Y_D$ is the Dirac type neutrino Yukawa matrix, $Y_M$ is the Majorana type matrix for the
RH neutrinos and $Y_0$ is the singular matrix. As an important example we can take

$$Y_0 = \begin{pmatrix}
\lambda^4 & \lambda^3 & \lambda^2 \\
\lambda^3 & \lambda^2 & \lambda \\
\lambda^2 & \lambda & 1
\end{pmatrix}, \quad \lambda \sim 0.2 - 0.3. \quad (15)$$

This matrix has one non-zero eigenvalue and determines mixing for the third generation.

Let us introduce perturbations $\epsilon$ in the following form

$$Y^f_{ij} = Y^0_{ij}(1 + \epsilon^f_{ij}), \quad f = u, d, e, \nu, N, \quad (16)$$

where $Y^0_{ij}$ is the element of the original singular matrix. This form can be justified, e.g. in the
context of the Froggatt-Nielsen mechanism [28]. (The key element is the form of perturbations
(16).) It has been shown that small perturbations $\epsilon \leq 0.25$ are enough to explain large difference
in mass hierarchies and mixings of quarks and leptons [27].

Smallness of neutrino mass is explained by the seesaw mechanism. Furthermore, nearly sin-
gular matrix of the RH neutrinos leads to enhancement of lepton mixing [29] and flip of sign
of mixing angle which comes from diagonalization of the neutrino mass matrix. So, the angles
from the charged leptons and neutrinos sum up, whereas in quark sector mixing angles from up
and down quark mass matrices subtract.

Keeping this in mind one can consider the following “working” Hypothesis:

1. No particular “neutrino” symmetry exists, and in general one expects some deviation of the 2-3 mixing from maximal as well as non-zero 1-3 mixing. Nearly maximal 2-3 mixing would be accidental in this case.

2. Seesaw mechanism with the scale of RH neutrino masses $M \sim 10^7 - 10^{15}$ GeV explains smallness of neutrino mass.

3. The quark-lepton unification or Grand Unification are realized in some form, e.g. $SO(10)$.

4. The quark-lepton symmetry is (weakly) broken and there are some observable consequences like $m_b = m_\tau$.

5. Large lepton mixing is a consequence of the seesaw mechanism - seesaw enhancement of lepton mixing due to special structure of the RH neutrino mass matrix, (or/and of the contribution from the type II seesaw).

6. Flavor symmetry or/and physics of extra dimensions determine this special structure.

4.3. Quark-lepton complementarity (QLC)

Being confirmed the complementarity (13) would require certain modification of the picture described above [24]. The latest determination of the solar mixing angle gives $\theta_{12} + \theta_C = 46.7^\circ \pm 2.4^\circ$ ($1\sigma$) which is consistent with maximal mixing angle within $1\sigma$. Is the QLC-relation accidental or there is some physics behind, that should include non-trivial quark-lepton connection? The fact that for the 2-3 mixings the approximate complementarity is also fulfilled hints some more serious reasons than just numerical coincidence.

A general scheme is that “lepton mixing = bimaximal mixing − CKM”. There is a number of non-trivial conditions for the exact QLC relation to be realized.

(i) Order of rotations: apparently $U_{12}^{\nu}$ and $U_{12}^{CKM\dagger}$ should be attached

$$U_{PMNS} \equiv U_L^{\dagger}U_{\nu} = ...U_{23}^{\nu}...U_{12}^{\nu}U_{12}^{CKM\dagger}$$

(17)

(two last rotations can be permuted). Different order leads to corrections to the exact QLC relation; (ii) Matrix with CP violating phases should not appear between $U_{12}^{CKM\dagger}$ and $U_{12}^{\nu}$; (iii) Presumably the quark-lepton symmetry which leads to the QLC relation is realized at high mass scales. Therefore the renormalization group effects should be small enough, etc..

Let us describe two possible scenarios which differ by origin of the bi-maximal mixing and lead to different predictions.

1). QLC1: The bi-maximal mixing is generated by the neutrino mass matrix, presumably due to seesaw. The charged lepton mass matrix produces the CKM mixing as a consequence of the q-l symmetry: $m_l = m_d$. In this case the order of matrices (17) is not realized ($U_{12}^{CKM\dagger}$ should be permuted with $U_{23}^{\nu}$), and consequently the QLC relation is modified:

$$\sin \theta_{12} = \sin(\pi/4 - \theta_C) + 0.5 \sin \theta_C(\sqrt{2} - 1).$$

(18)

Numerically we find $\tan^2 \theta_{12} = 0.495$.

2). QLC2: Maximal mixing comes from the charged lepton mass matrix and the CKM mixing originates from the neutrino mass matrix due to the q-l symmetry: $m_D \sim m_\nu$ (assuming also that in the context of seesaw the RH neutrino mass matrix does not influence mixing). In this case the QLC relation is satisfied precisely: $\sin \theta_{12} = \sin(\pi/4 - \theta_C)$.

There are two main issues related to the QLC relation: (1) origin of the bi-maximal mixing; (2) mechanism of propagation of the CKM mixing from the quark to the lepton sector. The problem here is large difference of mass ratios in the quark and lepton sectors: $m_c/m_\mu = 0.0047,$
\[ m_d/m_s = 0.4 - 0.06, \text{ as well as difference of masses of muon and s-quark at the GU scale. This means that mixing should weakly depend or be independent on masses.} \]

Mass matrices are different for quarks and leptons and “propagation” of the CKM mixing leads to corrections to the QLC relation of the order [25]

\[ \Delta \theta_{12} \sim \theta_C m_d/m_s \sim 0.5 - 1.0^\circ. \] (19)

The Cabibbo mixing can be transmitted to the lepton sector in more complicated way (than via the q-l symmetry). In fact, \( \sin \theta_C \) may turn out to be the generic parameter of theory of fermion masses and therefore to appear in various places: mass ratios, mixing angles. The relation: \( \sin \theta_C \approx \sqrt{m_\mu/m_\tau} \) is in favor of this possibility.

So, if not accidental the QLC relation may have two different implications: One includes the quark-lepton symmetry, existence of some additional structure which produces the bi-maximal mixing, weak dependence of the mixings on mass eigenvalues. Alternatively, it may imply certain flavor physics with \( \sin \theta_C \) being the “quantum” of this physics.

4.4. Screening of Dirac structure

The quark-lepton symmetry manifests as certain relation (similarity) between the Dirac mass matrices of quarks and leptons, and it is this creates problem for explanation of strongly different mixings and possible existence of the “neutrino” symmetries. We consider an extreme case when in spite of the q-l unification, the Dirac structure in the lepton sector is completely eliminated - “screened” [30].

Let us introduce one heavy neutral state \( S \) for each generation and consider mass matrix in the basis \((\nu, N^c, S)\) of the following form

\[
m = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_S^T \\ 0 & M_D & M_S \end{pmatrix}.
\] (20)

Here \( M_S \) is the Majorana mass matrix of new fermions. For \( m_D \ll M_D \ll M_S \) it leads to the double (cascade) seesaw mechanism [31]:

\[
m_\nu = m_D^T M_D^{-1} M_S M_D^{-1} m_D,
\] (21)

and \( M_R = -M_D M_S^{-1} M_D^T \). If two Dirac mass matrices are proportional each other

\[
M_D = A^{-1} m_D, \quad A \equiv v_{EW}/V_{GU},
\] (22)

they cancel in (20) and we obtain

\[
m_\nu = A^2 M_S.
\] (23)

That is, the structure of light neutrino mass matrix is determined by \( M_S \) immediately and does not depend on the Dirac mass matrix. The seesaw mechanism provides scale of neutrino masses but not the flavor structure of the mass matrix. It can be shown that at least in SUSY version the radiative corrections do not destroy screening [30]. The relation (22) can be a consequence of Grand Unification with extended gauge group or/and certain flavor symmetry [30].

Structure of the light neutrino mass matrix depends on \( M_S \) which can be related to some physics at the Planck scale, and consequently lead to usual neutrino properties. In particular, (i) \( M_S \) can be the origin of “neutrino” symmetry; (ii) the matrix \( M_S \propto I \) leads to the quasi-degenerate spectrum; (iii) \( M_S \) can be the origin of bi-maximal or maximal mixing thus leading to the QLC relation (QLC1) if the charged lepton mass matrix generates the CKM rotation.
5. Conclusions

It may happen that neutrinos prepare new surprise for us on the top of existence of large mixings. It is difficult to construct complete theory of quark and lepton masses. Still we can try to answer some generic questions:

- Is “neutrino symmetry” accidental or not?
- What are relations between quarks and leptons (universality, symmetry, complementarity)?
- Do new neutrino states and their mixing with active neutrinos exist?

In a sense we are on the cross-roads and our further advance may depend on how we will answer these questions. The way to answer is precision measurements of neutrino parameters (some benchmarks are identified), study of test equalities, searches for new (sterile) neutrinos. It may happen, that something important (in principles or context) is still missed.

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