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Terahertz response of gadolinium gallium garnet (GGG) and gadolinium scandium gallium garnet (SGGG)

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ABSTRACT

We report the magneto-optical response of gadolinium gallium garnet (GGG) and gadolinium scandium gallium garnet (SGGG) at frequencies ranging from 300 GHz to 1 THz and determine the material response tensor. Within this frequency window, the materials exhibit non-dispersive and low-loss optical responses. At low temperatures, significant terahertz Faraday rotations are found in the (S)GGG samples. Such a strong gyroelectric response is likely associated with the high-spin paramagnetic state of the Gd3+ ions. A model of the material response tensor is determined, together with the Verdet and magneto-optic constants.

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I. INTRODUCTION

Gadolinium gallium garnet (GGG, Gd3Ga5O12) and substituted GGG (e.g., Gd3Sc2Ga3O12) belong to the garnet material family, which are described by the general chemical formula A3B2C3O12, with A, B, and C being metal ions that are trapped inside oxygen dodecahedrals, octahedrals, and tetrahedrals, respectively.1,2

The nominal electronic configuration of the constituent ions of gadolinium scandium gallium garnet [(S)GGG] is Gd3+ ([Xe]4f75d06s0), O2− ([He]2s22p6), Sc3+ ([Ar]3d64s0), and Ga3+ ([Ar]3d104s24p10). Among them, only the Gd3+ ions have a nonzero magnetic moment.2 Under low magnetic fields, (S)GGG is paramagnetic. When the external field exceeds 1 T, however, a field-induced antiferromagnetic phase can be produced in GGG at temperatures below 1 K.3–5

Owing to the closely matched lattice structures, crystalline (S)GGG is widely used as the growth substrate for a general class of iron garnets described by X3Fe5O12 (XIG)6–15 with spintronic applications such as TIG,16–22 YIG,23–35 BIG,36–38 HIG,39 TbIG,40 and GdIG41 (X = Tm, Yb, Bi, Ho, Tb and Gd, respectively). The epitaxial strain induced by the (S)GGG substrate, tunable by the B-site substitution, can be used to effectively manipulate the magnetization and magnetic easy-axis of the XIG films. Recently, strain induced out-of-plane ferrimagnetic ordering in TIG films grown on (S)GGG substrates has been utilized to generate room-temperature magnetic proximity effects in topological insulators.30 Similar XIG/GGG heterostructures have also been used to realize spin pumping in single-layer graphene.32,43

Garnets, with the highest magneto-optical Verdet constants in bulk media, are the most common materials for Faraday rotators...
and optical isolators at visible wavelengths. In (S)GGG, the paramagnetic Gd ions with a large spin (7 unpaired 4f electrons) are effective enablers of strong magneto-optical effects. As a substrate material, (S)GGG-based heterostructures may find even more novel photonics applications. For example, alternating deposition of ultrathin XIG and GGG films has led to the realization of all-garnet magneto-optical photonic crystals (MOPCs). 

II. MATERIAL RESPONSE FORMALISM

In this section, we present the gyroelectric material response tensor and relate its elements to measured values.

According to the Onsager-Casimir symmetry relations, the permittivity and permeability tensors of a crystal should be symmetrical, i.e., for $n, m = x, y, z$,

$$\varepsilon_{nm}(\omega, k, B) = \varepsilon_{mn}(\omega, -k, -B), \quad \mu_{nm}(\omega, k, B) = \mu_{mn}(\omega, -k, -B),$$

where $\varepsilon_{nm}$ and $\mu_{nm}$ denote the generic components of the permittivity and permeability tensors, respectively. The variables $\omega$, $k$, and $B$ denote, respectively, the angular frequency, wavevector, and DC magnetic flux density vector. As a consequence, in the general case of triclinic crystal symmetry, there are 18 complex tensor components to be determined.

We consider a material slab of finite thickness under a $z$-directed magnetic bias whose electromagnetic (EM) response is described by permittivity and permeability tensors (denoted by $\varepsilon$ and $\mu$, respectively) of gyrotrropic form, with their components being represented by the matrices

$$\varepsilon = \begin{bmatrix} \varepsilon_x & i\varepsilon_y & 0 \\ -i\varepsilon_y & \varepsilon_x & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}, \quad \mu = \begin{bmatrix} \mu_x & i\mu_y & 0 \\ -i\mu_y & \mu_x & 0 \\ 0 & 0 & \mu_z \end{bmatrix},$$

where the coordinate system is chosen so that the $z$-axis is perpendicular to the slab. The subscripts $d$, $g$, and $a$ specify the diagonal, off-diagonal, and axial components, respectively. The cubic symmetry of the (S)GGG crystal requires the permittivity and permeability tensors of unbiased (S)GGG to be isotropic. Since the bias is perpendicular to slab, it does not break the isotropy in the $x-y$ plane, and thus the in-plane diagonal components of each of these tensors are expected to remain equal in the presence of bias. However, in general, the axial ($zz$) component of each of these tensors in the presence of the $z$-directed bias will differ from the in-plane diagonal components, i.e., $\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{dz} \neq \varepsilon_{zz} = \varepsilon_{az}$ and $\mu_{xx} = \mu_{yy} = \mu_{dz} \neq \mu_{zz} = \mu_{az}$.

A linearly-polarized normally-incident plane wave can be decomposed into left-handed circularly-polarized (LHCP) and right-handed circularly-polarized (RHCP) components of equal amplitude. These components propagate through the gyrotrropic substrate according to their corresponding refractive indices (or eigenvalues),

$$n_{RL} = \sqrt{\left(\varepsilon_x \mu_y - \varepsilon_y \mu_x\right) \pm \left(\varepsilon_x \mu_y + \varepsilon_y \mu_x\right)}.$$

As a result, the LHCP and RHCP components have different phase velocities, and the resulting phase difference causes the polarization of the plane wave to be rotated, a phenomenon known as Faraday rotation (FR). Moreover, in a lossy gyrotrropic medium, the LHCP and RHCP components are attenuated at different rates upon propagation such that the polarization state changes from linear to elliptical upon transmission. The resulting degree of ellipticity is referred to as Faraday ellipticity (FE). Excluding interference effects (which can be time-gated out), the evolution of polarization due to transmission through a gyrotrropic slab of thickness $d$ is described by

$$\frac{\mathbf{E}(\lambda)}{\mathbf{E}(\lambda)} = \sin \theta_E + i n_f \cos \theta_E$$

with $\lambda$, $n_f$, and $\theta_E$ denoting, respectively, the vacuum wavelength, the Faraday ellipticity, and the angle between the polarization of the linearly-polarized incident EM field and the major axis of the ellipse traced out by the tip of the electric field of the transmitted EM field (the FR).

The gyrotrropic response described by Eq. (3) can be induced by an external bias, e.g., a perpendicularly-applied static magnetic flux intensity $B_z$. In the limit of weak gyrotropy, where the bias is small, i.e., $|\varepsilon_d| \ll |\varepsilon_z|$ and $|\mu_d| \ll |\mu_z|$, the LHCP and RHCP refractive indices differ by a small amount, and the FR is not
expected to be large. As a result, Eq. (5) can be simplified to
\[
\frac{\theta_\parallel}{E_\parallel(\lambda)} \cong \frac{\pi d}{\lambda} \ln \left| n_k - n_l \right|,
\]
(6)
and the combination of Eqs. (4) and (6) results in
\[
\theta_\parallel + i n_h \cong \frac{\pi d}{\lambda} \left[ \frac{\varepsilon_d \mu_d + \varepsilon_d \mu_d}{\sqrt{\varepsilon_d \mu_d}} \right].
\]
(7)
Assuming the medium to exhibit gyroelectric response, the magnetic bias results in a nondiagonal permittivity tensor, while the permeability tensor remains diagonal, i.e., \( \varepsilon_d \neq 0 \) and \( \mu_d = 0 \). In this case, Eq. (7) reduces to
\[
\theta_\parallel + i n_h \cong \frac{\pi d}{\lambda} \sqrt{n_\parallel^2 - \chi},
\]
(8)
Therefore, the off-diagonal elements of the permittivity tensor of a gyroelectric medium can be studied through FR and FE measurements. Throughout this work, the weak gyrotricity limit is assumed, in which only the terms linear in bias are retained. Since the bias-dependence of \( \varepsilon_d \) in Eq. (8) manifests itself through terms quadratic in bias and higher, it can be replaced with the permittivity of unbiased substrate, \( \varepsilon_d,0 \).
\[
\varepsilon_d \cong \varepsilon_d,0 = (n - i \kappa)^2,
\]
(9)
with \( n > 0 \) being the refractive index and extinction constant of the unbiased substrate. Therefore, Eq. (8) reduces to
\[
\theta_\parallel + i n_h \cong \frac{\pi d}{\lambda} \sqrt{n_\parallel^2 - \chi},
\]
(10)
where the relative permeability \( \mu_d \) is expressed in terms of the magnetic susceptibility of the substrate, assumed to be real, and defined as \( \chi = \mu_d - 1 \). At frequencies close to electron paramagnetic resonance (EPR), the magnetic susceptibility of (S)GGG, in its paramagnetic phase, has an imaginary part, which manifests itself through microwave loss.\(^{23,24,66}\) The EPR resonance frequency is proportional to the DC magnetic bias, and for a magnetic bias as high as 0.4 T, the EPR peak occurs at around 100 GHz. In this work, we focus on the low frequency range between 0.3 and 1 THz, in which the samples measured exhibit almost negligible loss. The higher frequency properties, involving strong coupling to the phonon modes in (S)GGG, will be discussed elsewhere. Within this frequency range, the imaginary part of susceptibility can be approximated to be proportional to the bias, and its contribution to the left-hand side of Eq. (9) is through terms which are quadratic in bias or higher. As a result, within the 0.3 – 1 THz frequency range, and for magnetic biases up to 0.4 T, the magnetic susceptibility of (S)GGG can be approximated by its purely real DC value (i.e., we can neglect dispersion), and the real and imaginary parts of the off-diagonal component of the permittivity tensor can be obtained from experimental data as
\[
\text{Re}[\varepsilon_d] \cong \frac{\lambda}{\pi d} \left[ n \theta_\parallel + \kappa n_h \right],
\]
(11)
\[
\text{Im}[\varepsilon_d] \cong \frac{\lambda}{\pi d} \left[ n n_h - \kappa \theta_\parallel \right].
\]
(12)
Therefore, in order to determine the permittivity tensor elements, it is necessary to measure the refractive index, \( n \), extinction coefficient, \( \kappa \), Faraday rotation, \( \theta_\parallel \), Faraday ellipticity, \( \eta_h \), and magnetic susceptibility, \( \chi \). We do not obtain \( \varepsilon_d \) or \( \mu_d \) in this work.

III. TRANSMISSION MEASUREMENTS OF (S)GGG SUBSTRATES

A. Terahertz time-domain spectroscopy

Samples used in this experiment are (111)-oriented single crystal GGG and SGGG with a nominal thickness of \( d = 0.5 \) mm. Variable-temperature terahertz transmission and Faraday rotation measurements are performed in a cryostat using terahertz time-domain spectroscopy (THz-TDS). Figure 1 shows the typical transient terahertz waveform transmitted through the sample. The transmission spectrum is obtained by comparing the Fourier transforms of the time-domain signals measured with and without the sample. The time-domain sample signal is truncated to remove the interference effects associated with the echo pulses.\(^{70}\) The Faraday rotation is characterized by comparing the transmitted signals measured with two different detector polarizer angles (\( \pm 30^\circ \)).

B. Transmission measurement of unbiased substrates: Refractive index and absorption

For weak magnetic bias, the diagonal elements of the permittivity tensor can be obtained from the complex refractive index, \( n = n - i \kappa \), of the unbiased sample. The transmission-mode THz-TDS measurements make it possible to extract \( n \) from the following equation:71
\[
\frac{E_{\text{sam}}(\lambda)}{E_{\text{ref}}(\lambda)} \cong \frac{4 n}{(1 + n)^2} e^{-\kappa(1 - n)},
\]
(13)
where \( E_{\text{sam}}(\lambda) \) and \( E_{\text{ref}}(\lambda) \) are, respectively, the discrete Fourier transforms of the first-pass and reference pulses shown in Fig. 1(a). The amplitude and phase of these signals are presented in the inset of Fig. 1(a). Within the frequency range of 0.3 – 1 THz, the samples measured exhibit almost negligible loss. Instead of reporting the small \( \kappa \) values that are subject to measurement noise, upper bounds for the extinction coefficient are listed in Table I, which are calculated assuming zero reflection of the terahertz beam off the sample, i.e.,
\[
n = 1 \Rightarrow \kappa_{\text{max}} = \frac{\lambda}{2 \pi d} \ln \left| \frac{E_{\text{sam}}(\lambda)}{E_{\text{ref}}(\lambda)} \right|.
\]
(14)
The refractive index and attenuation constant extracted via Eqs. (13)–(15) do not exhibit any considerable temperature

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dependence within 5 – 295 K. Moreover, the extracted results do not show any strong frequency dependence within 0.3 – 1 THz, and therefore, the spectrally-averaged results are presented in Table I.

The permittivity corresponding to the measured values of \( n \) for all four substrates ranges from 12 to 15. This is consistent with the dielectric constant of 12.11 measured along the (111) direction of crystalline GGG and the measured polycrystalline dielectric constant of 11.9 ± 1.9. In contradistinction to the terahertz results, at a wavelength of 632.8 nm (474 THz), the refractive index of crystalline GGG measured using the ellipsometry technique is reported to be 1.98 ± 0.001, independent of crystal orientation.

### C. Transmission measurement of biased substrates: Faraday rotation, Faraday ellipticity, and Verdet constant

In Refs. 25, 75, and 76, wherein GGG is used as a substrate for substituted YIG films, the contribution of GGG to the overall FR of the composite system has been observed at \( \lambda = 690 \text{ nm}, 530 \text{ nm}, \) and \( 532 \text{ nm} \), respectively. Here, we present the FR measured within 0.3 – 1 THz for a bare (S)GGG substrate. The detected signals shown in Figs. 1(b) and 1(c) are measured when the sample is biased with \( \mu_0 H_z = \pm 400 \text{ mT} \) for wire-grid polarizer (WGP) angles of \( \pm 30^\circ \), respectively. Unlike the spectra shown in Fig. 1(a) that are obtained in plain transmission measurement without magnet, the spectra in Figs. 1(b) and 1(c) are from the Faraday rotation setup with magnet. The presence of the magnet in the input and output path causes significant damping to the terahertz light intensity, and that is why the pulse profile in Figs. 1(b) and 1(c) becomes much more broadened in comparison to the ones shown in Fig. 1(a). At a fixed external magnetic field, the polarization rotation of the transmitted terahertz pulse is obtained from the difference in signals detected at the two WGP angles

\[
\theta_F = \arcsin \left( \frac{E(30^\circ) - E(-30^\circ)}{2 \sin(30^\circ) E(0^\circ)} \right),
\]  

where \( E(\pm 30^\circ) \) and \( E(0^\circ) \) are the transmission field strength measured at WPG angles of \( \pm 30^\circ \) and \( 0^\circ \), respectively. The pair of polarization rotation angles measured at magnetic fields with the same strength but opposite directions are compared and symmetrized to extract the components that are odd or even functions of the field. The odd component is attributed to the FR effect (Fig. 2), while the even component may originate from alternative field-induced light modulation, such as the quadratic magneto-optical effect. Our measurement results are dominated by the FR field-induced light modulation, such as the quadratic magneto-optical effect. Our measurement results are dominated by the FR

| Sample       | \( n \)  | \( \kappa_{\text{max}} \) |
|--------------|--------|------------------|
| Annealed GGG | 3.46   | 0.062            |
| Annealed SGGG| 3.79   | 0.066            |
| Untreated GGG| 3.49   | 0.059            |
| Untreated SGGG| 3.80  | 0.066            |

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leads to the conclusion that the imaginary part of $\varepsilon_g$ should be negligible for weak amounts of magnetic bias. Implementing this assumption in Eq. (12) yields $\eta_F \approx \frac{\theta_F}{C}$, which implies that the FE is expected to be much smaller than FR. As a result, the FE and attenuation constant are assumed negligible in our calculations.

As a result, the Verdet constant is defined without the incorporation of FE, i.e.,

$$V = \frac{\theta_F}{B_z d}. \quad (16)$$

Since the magnetic fields (<400 mT) used in our experiments are much smaller than the typical saturation values (∼10 T) found in garnets, it is reasonable to treat the FR signal as a linear function of the field strength. This assumption is consistent with our field-dependent measurement results [Figs. 2(b) and 2(c)]. Furthermore, the thermal expansion coefficient of the GGG crystal, i.e., $\alpha_T = 1.4 \times 10^{-5} K^{-1}$, is of the order of $7 \times 10^{-7} K^{-1}$ within the temperature range of 6 – 310 K, with $a$ being the lattice constant. Therefore, the sample thickness is not expected to show a considerable temperature dependence, and the FR and Verdet constant are expected to exhibit the same temperature dependence.

As shown in Fig. 3(b), the measured Verdet constant is highly temperature dependent. Below 100 K, the Verdet constant within 0.3 – 1 THz significantly exceeds its reported values at visible-MIR (mid-infrared) frequencies, which range from 1.25 to 22.3 (rad T⁻¹ m⁻¹). Therefore, the absence of an electron gas eliminates the possibility of a plasmalike gyrotropic response. On the other hand, our measurements indicate that the inverse of the Verdet constant increases linearly with temperature, as shown in Fig. 3(c), with linear fitting parameters provided in Table II.

### IV. Paramagnetic susceptibility and magneto-optic constant

Electrically-conductive and/or ferromagnetic material are known to exhibit gyrotropic response under magnetic bias. However, (S)GGG does not fall in either of these categories; optical measurements of GGG indicate a bandgap of 5.66 eV. Therefore, the presence of an electron gas eliminates the possibility of a plasmalike gyrotropic response. On the other hand, our measurements indicate that the inverse of the Verdet constant increases linearly with temperature, as shown in Fig. 3(c), with linear fitting parameters provided in Table II.

### TABLE II. Linear fit parameters for annealed/untreated monocrystalline (S)GGG substrates; the inverse of the real part of the Verdet constant is a linear function of temperature, i.e., $V = \frac{\beta}{T_0} \frac{\theta_F}{B_z d}$ [see Fig. 3(c)].

| Sample         | $\beta$(K) | $T_0$(K) |
|----------------|------------|----------|
| Annealed GGG   | $36 \times 10^2$ | $-8.2$  |
| Annealed SGGG  | $34 \times 10^2$ | $-1.4$  |
| Untreated GGG  | $38 \times 10^2$ | $-13$   |
| Untreated SGGG | $39 \times 10^2$ | $-9.0$  |
This behavior is in agreement with the observation made in Refs. 82–84. Moreover, the magnetic susceptibility of the (S)GGG substrates was measured using the vibrating-sample magnetometry (VSM) technique even at \( T = 5 \) K, the sample magnetization did not show any hysteresis under sweeping the magnetic field \( H_z \), and the positive slope of the \( M - H \) linear curves indicated a paramagnetic response. This result is in agreement with the paramagnetic contribution of GGG to the perpendicular component of magnetization, which has been observed as a linear background in the \( M - H \) hystereses curves of composite XIG/GGG layered systems. As suggested by Fig. 4, the temperature dependence of the measured susceptibility data is described with the Curie-Weiss law,

\[
\chi = \frac{C}{T - \Theta_{CW}},
\]

where \( \Theta_{CW} \) is the Curie-Weiss temperature and \( C \) is the Curie-Weiss constant, which, by definition, is independent of temperature. The constants \( \Theta_{CW} \) and \( C \) have been obtained through linear fitting of the inversely susceptibility data and presented in Table III. As shown in Fig. 4, inverse susceptibility also increases linearly with temperature. However, \( 1/\chi \) is temperature dependent, which has been reported for the visible-frequency Verdet constant of the paramagnetic insulators such as NdF\(_3\), PrF\(_3\), and CeF\(_3\).82,83,89,90

### Table III. Curie-Weiss parameters obtained through fitting the measured magnetic susceptibility for annealed/untreated monocrystalline (S)GGG substrates with the Curie-Weiss law given by Eq. (17). Comparison with the Curie-Weiss parameters obtained for a polycrystalline GGG sample in Ref. 13 shows reasonable agreement.

| Sample            | \( C(K) \) | \( \Theta_{CW}(K) \) |
|-------------------|------------|----------------------|
| Annealed GGG      | 2.2        | −3.9                 |
| Annealed SGGG     | 2.1        | −3.7                 |
| Untreated GGG     | 2.0        | −3.9                 |
| Untreated SGGG    | 2.2        | −3.0                 |
| GGG (Ref. 13)     | 2.0        | −2.1                 |

The real part of \( \varepsilon_g \) is computed for \( \mu_0 H_z = 400 \) mT via Eq. (11), and the results normalized by wavelength are presented in Fig. 5(a). Since (S)GGG is an insulator, its magnetic response leads to the assumption of \( \varepsilon_g \) being proportional to the DC magnetization: the magneto-optical (MO) response model, i.e.,

\[
\varepsilon_g = \mu_0 \gamma_{MO} M_z \tag{18}
\]

with \( \gamma_{MO} \) being the MO constant of the medium, \( M_z \) denoting the projection of the DC magnetization vector onto the direction of

![Fig. 5](image-url)
propagation ($z$), and $\mu_0$ being the permeability of free space. In Ref. 93, the assumption of MO response has been applied to TGG ($Th_5Ga_3O_{12}$), which has a similar chemical and crystallographic structure to GGG.

The microscopic origin of the MO response is explained by the rotation of the excited dipolar currents as a result of the asymmetry of the electronic wave functions induced by the spin–orbit interaction.29 Combining Eqs. (11), (12), (16), and (18), and assuming linear response between the magnetization and applied field, yields

$$\gamma_{\text{MO}} \approx \frac{\lambda}{\pi \sqrt{1 + \chi}} \frac{n}{V}. \quad (19)$$

The $\sqrt{1 + \chi}$ factor in Eq. (19) can be traced back to the LHCP and RHCP refractive indices given by Eq. (4). Since $M_z$ in Eq. (18) is purely real, $\gamma_{\text{MO}}$ is required to be complex. However, as mentioned in Sec. III C, the imaginary part of $\gamma_{\text{MO}}$ is negligible within 0.3 – 1 THz. The real part of $\gamma_{\text{MO}}$, normalized by wavelength, is presented in Fig. 5(b) and appears to be nearly temperature-independent, except at low temperatures. The mechanism behind this is unclear to us.

It is worthwhile to mention that for magnetic biases as strong as 50 T, the Faraday rotation of (S)GGG substrates is expected to saturate with respect to magnetic bias due to the diamagnetic contribution of the oxygen-gallium bonds to the overall magnetization; a response which has been experimentally reported in Ref. 95 for the case of TGG. As a result, the relation given by Eq. (19) may not be applicable to the case wherein the sample is subject to strong magnetic bias.

V. SUMMARY AND CONCLUSIONS

The gyroelectric permittivity tensor of annealed/untreated (S)GGG substrates is determined in the frequency range 0.3 – 1 THz and the temperature range 5 – 295 K using FR, magnetic susceptibility, and refractive index measurements, whereas the ellipticity and absorption were found to be negligible. The Verdet and magneto-optic constants have been determined, and it was found that the diagonal elements do not exhibit any frequency dependence, and the off-diagonal elements are proportional to wavelength. The latter comment follows from Eqs. (11) to (12), and the observation that both the refractive index and the Faraday rotation are frequency-insensitive within the considered frequency range.

Large Verdet constants approaching 300 rad/(T m) are found in these paramagnetic materials at low temperatures. Such an effect likely originates from the large magnetic permeability associated with high-spin state of the Gd$^{3+}$ ions and the sizable magneto-optic constant. Future first-principle calculation and material modeling are called for to elucidate the detailed microscopic mechanism that gives rise to strong magneto-optic responses. The large Faraday rotation observed is insensitive to cation substitute and thermal treatment. The robustness of the strong magneto-optical effect, in conjunction with its broadband characteristics and the negligible loss found in the material, makes (S)GGG wonderful candidates for making cryogenic terahertz isolators and circulators. As important substrate materials used for magnetic garnet thin film growth, the terahertz properties of (S)GGG systematically characterized in this work will also provide important information that is critical for the future development of garnet heterostructures based spintronic and magneto-optic devices.

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states, which again argues for \( \mu_g \approx 0 \). The local exchange fields due to the Gd
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\[ \frac{\epsilon}{\mu} \approx \frac{\epsilon}{\mu} \]

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Our DFT calculations for GGG support these conclusions: The weak interactions result from both the relatively large Gd-Gd separations and, more importantly, from the fact that the Gd 4f states are effectively corelike, with little hybridization with the GGG valence states, which again argues for \( \mu_g \approx 0 \). The local exchange fields due to the Gd 4f moments cause a polarization of the Gd valence and conduction states, but the quantization direction is determined by the external field.

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