Stochastic 2-bit null detector

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Abstract. An application of the stochastic measurement approach for measurement of low AC signals, found in null detectors, is shown in the paper. The measurement structure is based on a 2-bit flash A/D converter and digital multipliers with accumulators that calculate in-phase and quadrature values for measured signal at signal frequency. Possible application of such null detector in AC bridge with two sources is presented. Simulations prove the theoretical considerations and encourage further development of a null-detector based on presented method.

1. Introduction
An alternative to a commonly used strategy in discrete digital measurements, called “measurement in a point”, where high-resolution samples of an input value are taken at chosen time instants, is called “stochastic measurement over an interval”. This alternative method has been so far researched in three challenging areas: measurements that require high accuracy and linearity [1,2], measurements of fast-changing signals [3,4] and measurements of noisy signals [5,6]. Numerous simulations, experiments and prototype instruments have proven the metrological applicability of the stochastic approach. Measurement over an interval is suitable for signals where derived quantities matter, not instantaneous values, such as in measurement of integral values (mean value, RMS etc.). Since the precision of the measurement increases with number of samples (measuring interval), this approach is also suitable for precise measurement of quasi-stationary signals.

Null detectors should measure low-level signals at a frequency of interest and to neglect the influence of other frequencies from external sources, such as power lines, radio stations etc. In automatic bridge circuits where a null detector is used to signal a need to further balance the bridge, the information that is of interest is that the bridge is still not balanced, i.e. that the signal at the null detector is above the given value – criterion for bridge balance condition.

2. Theory of operation
In general case, if, using the stochastic approach, we want to measure the average of the product of two signals \( f_1(t) \) and \( f_2(t) \) over a finite time interval \( T = t_2 - t_1 \), we can assume that for a finite sampling frequency

\[
\frac{1}{N} \sum_{i=1}^{N} \Psi_1(i) \Psi_2(i) \approx \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} f_1(t) f_2(t) \, dt
\]  

(1)
Variables $\Psi_1(i)$ and $\Psi_2(i)$ are digitized samples after the 2-bit A/D converter expressed as 
$\Psi_i(i) = (b_{i1} - b_{i2}) \cdot 2^g$, where $g$ is the A/D converter threshold, 2-bit A/D outputs $b_{i1}, b_{i2} \in \{0,1\}$ and $i = 1, 2$, and the A/D quantum is $\Delta = R = 2g$.

The variance of the mean value of the quantization error $\sigma^2_e$ is limited to [7]:

$$\sigma^2_e = \frac{\sigma^2_e}{N} \leq \frac{1}{N} \left\{ \frac{\Delta^2}{4} \left[ \int_{t_1}^{t_2} f_1^2(t) dt + \int_{t_1}^{t_2} f_2^2(t) dt \right] + \frac{\Delta^4}{16} \right\}$$  \hspace{1cm} (2)

The structure of the proposed null detector based on a stochastic approach with two-bit flash A/D converter is presented in Figure 1. The 2-bit A/D converter has variable quantum $\Delta = R = 2g$.

Using the former equations, we can develop an instrument [7] for in-phase and quadrature signals components measurement. In this instrument we have two output signals which are average values of the products of two signals: the input signal $y_1 = f_1(t) = A \sin(\omega t + \varphi) = A \sin(\omega t + \varphi)$ and one of the signals $y_2 = f_2(t) = \cos(\omega t)$ or $y_2 = f_2(t) = \sin(\omega t)$, depending on whether we want to measure cosine or sine component of an input signal at a signal frequency.

The input signal is dithered using the random number generator RNG and a D/A converter [7]. Two signals that resemble sine and cosine dithered 2-bit signals at a fundamental frequency are stored in memory and used in digital 2-bit multipliers to obtain samples of in-phase and quadrature components of an input signal. The measurement error $e$ is expressed with $\Psi = \Psi_1 \cdot \Psi_2 = y_1y_2 + e$.

In general case, the quantum of the 2-bit A/D converter, presented as two comparators, is not equal to quanta with which the sine and cosine dithered functions that are stored in memory are calculated. For different quanta of two functions we can modify the equation (2) to:

$$\sigma^2_e = \frac{\sigma^2_e}{N} \leq \frac{1}{N} \left\{ \frac{\Delta^2_1}{4} \int_{t_1}^{t_2} f_1^2(t) dt + \frac{\Delta^2_2}{4} \int_{t_1}^{t_2} f_2^2(t) dt + \frac{\Delta^4_1\Delta^2_2}{16} \right\}$$  \hspace{1cm} (3)

The special case of this equation is when the functions in memory are calculated with significantly higher resolution than the 2-bit A/D converter quantum. In that case $\Delta^2_1 \gg \Delta^2_2$, so the equation (3) becomes:

$$\sigma^2_e = \frac{\sigma^2_e}{N} \leq \frac{1}{N} \frac{\Delta^4}{4} \int_{t_1}^{t_2} f_2^2(t) dt$$  \hspace{1cm} (4)

![Figure 1. Stochastic null detector.](image)

The special case of this equation is when the functions in memory are calculated with significantly higher resolution than the 2-bit A/D converter quantum. In that case $\Delta^2_1 \gg \Delta^2_2$, so the equation (3) becomes:
If $y_2 = f_2(t)$ is a function from Fourier orthonormal basis, then the measurement uncertainty is limited by a constant when the time interval $T = t_2 - t_1$ equals one or more periods of the signal $y_2 = f_2(t)$.

$$\sigma_x^2 = \frac{\sigma_x^2}{N} \leq \frac{1}{N} \frac{\Delta_1^2}{4} \frac{1}{2}$$  \hspace{1cm} (5)

In the case of a null detector, the input signal amplitude is low, so we can assume that $|y_1|_{\text{max}} \approx 0$ or $|y_1|_{\text{max}} \ll \Delta_1$. In that case $\sigma_x^2(y_1) \leq \sigma_x^2(A) \cdot A$, so the equation (5) becomes:

$$\sigma_x^2 \leq \frac{1}{N} \frac{\Delta_1 \cdot A}{2} = \varepsilon^2$$  \hspace{1cm} (6)

If we want to measure the amplitude of a signal using the stochastic digital method,

$$A_n = \sqrt{a^2 + b^2} \leq \sqrt{(a + \varepsilon)^2 + (b + \varepsilon)^2}$$  \hspace{1cm} (7)

where $a_m = \frac{2}{N} \sum_{i=1}^{N} \Psi_2(i) \Psi_{1c}(i) \approx a$ and $b_m = \frac{2}{N} \sum_{i=1}^{N} \Psi_1(i) \Psi_{2s}(i) \approx b$ are the calculated values of cosine and sine components over an interval with $N$ samples, which are calculated from the contents of two accumulators and a counter shown in Figure 1. When $A \to 0$, then also $a^2 \to 0$ and $b^2 \to 0$, or $|a| \to 0$ and $|b| \to 0$, so the equation (7) becomes:

$$A_m \leq \sqrt{2\varepsilon^2} = \frac{\Delta_1 \cdot A}{\sqrt{N}}$$  \hspace{1cm} (8)

The measurement uncertainty boundary is a function of measured amplitude. From equation (8) we can conclude that the probability that $A$ is in boundaries expressed with (8) (at the boundary the value is $A_s$, so from $A_m = A_s = k \sqrt{\frac{\Delta_1 \cdot A}{N}}$ we obtain that the boundary value is $A_s = \frac{k^2 \Delta_1}{N}$ where $k$ is the coverage factor), is 66 % because the coverage factor is 1. For coverage factor 2, the probability that the amplitude of measured signal is bounded by $A_s = \frac{4\Delta_1}{N}$ is 95 % and so on.

When the null detector is used to balance the measurement bridge, the information when the amplitude is outside the defined boundaries is of importance: that information says that the bridge is not yet balanced.

![Figure 2. Simulation results for different coverage factors.](image-url)
3. Simulation results

The simulation for theoretical hardware model of the null detector, shown in Figure 1, with parameters $\Delta_i = 1 \text{ mV}$ - A/D converter quantum, $f_s = 500 \text{ kHz}$ - sampling frequency, $t_2 - t_1 = 50 \text{ s}$ - interval for AC amplitude measurement and $a = b = A/\sqrt{2}$ - proposed values for measured cosine and sine components $a$ and $b$ are shown in Figure 2. The simulation was run in 10 points, starting from 0.105 nV to 2066.715 nV in geometry progression with factor of 3. In each point the simulation was run 256 times.

Table 1 shows comparison between theoretical and simulated boundaries for null detection for coverage factors of 3, 4 and 5 (respective confidence intervals of 99.7 %, 99.99 % and 99.9999 %).

| Coverage factor | Theoretical boundary for $A_g (\text{nV})$ | Simulated boundary for $A_g (\text{nV})$ | Relative difference (%) | Absolute difference (pV) |
|----------------|------------------------------------------|----------------------------------------|-------------------------|-------------------------|
| 3              | 0.36                                     | 0.374                                  | 3.9                     | 14                      |
| 4              | 0.64                                     | 0.619                                  | -3.3                    | -21                     |
| 5              | 1.00                                     | 0.952                                  | -4.8                    | -48                     |

4. Conclusion

A stochastic null detector based on 2-bit A/D converter is presented, with simulation results which are encouraging. A two-bit A/D converter is a simple device with low number of systematic error sources, suitable for measurements over an interval.

The main conclusion is that the null detector for AC impedance measuring bridge can be built based on stochastic approach. The performances could be improved by lowering the A/D converter quantum, increasing the measurement time and by amplifying the input signal.

5. References

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