Fibre Inflation: Observable Gravity Waves from IIB String Compactifications

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ABSTRACT: We introduce a simple string model of inflation, in which the inflaton field can take trans-Planckian values while driving a period of slow-roll inflation. This leads naturally to a realisation of large field inflation, inasmuch as the inflationary epoch is well described by the single-field scalar potential $V = V_0 \left( 3 - 4e^{-\hat{\phi}/\sqrt{3}} \right)$. Remarkably, for a broad class of vacua all adjustable parameters enter only through the overall coefficient $V_0$, and in particular do not enter into the slow-roll parameters. Consequently these are determined purely by the number of e-foldings, $N_e$, and so are not independent: $\varepsilon \simeq \frac{3}{2} \eta^2$. This implies similar relations among observables like the primordial scalar-to-tensor amplitude, $r$, and the scalar spectral tilt, $n_s$: $r \simeq 6(n_s - 1)^2$. $N_e$ is itself more model-dependent since it depends partly on the post-inflationary reheating history. In a simple reheating scenario a reheating temperature of $T_{RH} \simeq 10^9$ GeV gives $N_e \simeq 58$, corresponding to $n_s \simeq 0.970$ and $r \simeq 0.005$, within reach of future observations. The model is an example of a class that arises naturally in the context of type IIB string compactifications with large-volume moduli stabilisation, and takes advantage of the generic existence there of Kähler moduli whose dominant appearance in the scalar potential arises from string loop corrections to the Kähler potential. The inflaton field is a combination of Kähler moduli of a K3-fibered Calabi-Yau manifold. We believe there are likely to be a great number of models in this class – ‘high-fibre models’ – in which the inflaton starts off far enough up the fibre to produce observably large primordial gravity waves.

KEYWORDS: String compactifications, Inflation.
1. Introduction

Much progress has been made over the past few years towards the goal of finding cosmological inflation amongst the controlled solutions of string theory. Part of the motivation for so doing has been the hope that observable predictions might emerge that are robust...
to all (or many) realizations of inflation in string theory, but not generic to inflationary models as a whole. The amplitude of primordial gravity waves has recently emerged as a possible observable of this kind [2, 3], with unobservably small predictions being a feature of most of the known string-inflation proposals.

The prediction arises because the tensor amplitude is related (see below) to the distance traversed in field space by the inflaton during inflation, and this turns out to have upper limits in extant models, despite there being a wide variation in the nature of the candidate inflaton fields considered: including brane separation [4]; the real and imaginary parts of Kähler moduli [5, 6]; Wilson lines [7]; the volume [8, 9] and so on. Furthermore, the same prediction appears also to be shared by some of the leading proposed alternatives to inflation, such as the cyclic/ekpyrotic models.

Since the observational constraints on primordial tensor fluctuations are about to improve considerably — with sensitivity reaching down to $r \simeq 0.001$ (for $r = T/S$ the ratio of amplitudes of primordial tensor and scalar fluctuations) [10, 11] — it is important to identify precisely how fatal to string theory would be the observation of primordial gravity waves at this level. This has launched a search amongst theorists either to prove a no-go theorem for observable $r$ from string theory, or to derive explicit string-inflationary scenarios that can produce observably large values of $r$. Silverstein and Westphal [12] have taken the first steps along these lines, proposing the use of monodromies in a particular class of IIA string compactifications. In such models the inflaton field corresponds to the position of a wrapped D4 brane that can move over a potentially infinite range, thereby giving rise to observably large tensor perturbations.

In this article we provide a concrete example of large field inflation in the context of moduli stabilisation within the well studied IIB string compactifications. Working within such a framework allows us to use the well-understood properties of low-energy 4D supergravity, with the additional control this implies over the domain of validity of the inflationary calculations.

More generally, we believe the inflationary model we propose to be the simplest member of a broad new family of inflationary constructions within the rich class of IIB stabilisations known as the LARGE$^1$ Volume Scenario (LVS) [13]. Within this framework complex structure moduli are fixed semiclassically through the presence of branes and fluxes, while Kähler moduli are stabilised by an interplay between non-perturbative corrections to the low-energy superpotential, $W$, and perturbative $\alpha'$ and string-loop corrections to the Kähler potential, $K$, of the effective low-energy 4D supergravity. In particular, the LARGE volume that defines these scenarios naturally arises as an exponentially large function of the small parameters that control the calculation.

Most useful for inflationary purposes is the recent classification [14] within the LVS framework of the order in the $\alpha'$ and string-loop expansions that governs the stabilisation of the various Kähler moduli for general IIB Calabi-Yau compactifications. In particular it was found that for K3-fibred Calabi-Yaus, LVS moduli stabilisation only fixes the overall volume and blow-up modes if string loop corrections to $K$ are ignored. The fibre moduli

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$^1$The capitalisation of LARGE is a reminder that the volume is exponentially large, and not simply large enough to trust the supergravity limit.
call it $\mathcal{X}$, say – then remains with a flat potential that is only lifted once string loop corrections are also included. Consequently $\mathcal{X}$ has a flatter potential than does the overall volume modulus, making it systematically lighter, and so also an attractive candidate for an inflaton. Our proposal here is the first example of the family of inflationary models which exploits this flatness mechanism, and which we call *Fibre Inflation*.

This class of models is also attractive from the point of view of obtaining large primordial tensor fluctuations. This is because the relatively flat potential for $\mathcal{X}$ allows it to traverse a relatively large distance in field space compared with other Kähler moduli. In this note we use these LVS results to explicitly derive the inflaton potential in this scenario, where the range of field values is large enough to easily give rise to $60$ e-foldings of slow-roll inflation.

Unlike most string-inflation models (but similar to Kähler modulus inflation (KMI) [6]) slow roll is ensured by large field values rather than tuning amongst parameters in the potential. Most interestingly, within the inflationary regime all unknown potential parameters appear only in the normalization of the inflaton potential and not in its shape. Consequently, predictions for the slow-roll parameters (and for observables determined by them) are completely determined by the number of e-foldings, $N_e$, between horizon exit and inflation’s end. Elimination of $N_e$ then implies the slow-roll parameters are related by $\varepsilon \simeq \frac{3}{2} \eta^2$, implying a similar relation between $r$ and the scalar spectral tilt: $r \simeq 6(n_s - 1)^2$. [By contrast, the corresponding predictions for KMI are $\varepsilon \simeq 0$ and so $r \simeq 0$, leaving $n_s \simeq 1 + 2\eta$.]

Since the value of $N_e$ depends somewhat on the post-inflationary reheating history, the precise values of $r$ and $n_s$ are more model dependent, with larger $N_e$ implying smaller $r$. In a simple reheat model (described in more detail below) $N_e$ is correlated with the reheat temperature, $T_{rh}$, and the inflationary scale, $M_{inf}$, through the relationship:

$$N_e \simeq 62 + \ln \left( \frac{M_{inf}}{10^{16} \text{GeV}} \right) - \frac{(1 - 3w)}{3(1 + w)} \ln \left( \frac{M_{inf}}{T_{rh}} \right),$$

(1.1)

where $w = p/\rho$ parameterizes the equation of state during reheating. Numerically, choosing $M_{inf} \simeq 10^{16}$GeV and $T_{rh} \simeq 10^9$ GeV (respectively chosen to provide observably large primordial scalar fluctuations, and to solve the gravitino problem), we find that $N_e \simeq 58$, and so $n_s \simeq 0.970$ and $r \simeq 0.005$. Tensor perturbations this large would be difficult to see, but would be within reach of future cosmological observations like EPIC, BPol or CMBPol [10, 11].

Our preliminary investigations reveal several features likely to be common to the broader class of Fibre Inflation models. On one hand, as already mentioned, slow roll is ultimately controlled by the large values of the moduli rather than on the detailed tuning of parameters in the scalar potential. On the other hand, large volumes imply low string scales, $M_s$, and this drives down the inflationary scale $M_{inf}$. This is interesting because it may lead to inflation even at low string scales but could be a problem inasmuch as it makes it more difficult to obtain large enough scalar fluctuations to account for the primordial fluctuations seen in the CMB. (It also underlies the well-known tension between TeV scale supersymmetry and the scale of inflation [3, 13].) This suggests studying alternative
methods to generate density fluctuations within these models, to allow lower inflationary scales to co-exist with observably large primordial fluctuations. Although fluctuations generated in this way would not produce large tensor modes, they might be testable through their predictions for non-gaussianities.

The biggest concern for Fibre Inflation and Kähler Modulus Inflation is whether higher-loop contributions to the potential might destabilize slow roll. In KMI this problem arises already at one loop, and leads to the requirement that no branes wrap the inflationary cycle (from which the dangerous contributions arise). Fibre Inflation models do not have the same problems, and this is likely to simplify greatly the ultimate reheating picture in these models. They may yet have similar troubles once contributions from blow-up modes or higher loops can be estimated, but we find that current best estimates for these corrections are not a problem.

Finally, it is relatively simple in these models to obtain large hierarchies amongst the size of the moduli, in a way that leads to some dimensions becoming larger than others (rather than making the extra dimensions into a frothy Swiss cheese). This potentially opens up the possibility of ‘sculpting’ the extra dimensions, by having some grow relatively slowly compared to others as the observed four dimensions become exponentially large.

After a short digression, next, summarising why large $r$ has proven difficult to obtain in past constructions, the remainder of the paper is devoted to explaining Fibre Inflation, and why it is possible to obtain in it $r \simeq 0.005$. The description starts, in §2, with a review of various LVS tools which are used in subsequent sections. Aficionados of the LVS can skip this discussion, jumping right to §3 which describes both the special case of K3 fibrations, and the inflationary potentials to which they give rise. Our conclusions are briefly summarised in §4.

1.1 The Lyth bound

What is so hard about obtaining observably large primordial tensor fluctuations in string constructions? In 1996 David Lyth derived a general correlation between the ratio $r$ and the range of values through which the (canonically normalized) inflaton field, $\phi$, rolls in single-field slow-roll models:

$$r = 16 \varepsilon = \frac{8}{N_{\text{eff}}^2} \left( \frac{\Delta \phi}{M_p} \right)^2,$$

where $\varepsilon = \frac{1}{2} (V'/V)^2$ is the standard first slow-roll parameter, and

$$N_{\text{eff}} = \int_{t_{\text{he}}}^{t_{\text{end}}} \left( \frac{\xi}{\phi} \right)^{1/2} H dt.$$

Here $\xi(t) = 8(\dot{\phi}/HM_p)^2$ is the quantity whose value at horizon exit gives the observed tensor/scalar ratio, $r = \xi(t = t_{\text{he}})$, $H(t) = \dot{a}/a$ is (as usual) the Hubble parameter, and the integral runs over the $N_e \gtrsim 50$ $e$-foldings between horizon exit and inflation’s end. Notice

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2We thank Toni Riotto for numerous discussions of this point.

3We thank Markus Berg for conversations about this.
in particular that $N_{\text{eff}} = N_e$ if $\xi$ is a constant. Lyth’s observation was that the validity of the slow roll and measurements of the scalar spectral index, $n_s - 1$, constrain $N_{\text{eff}} \gtrsim 50$, and so $r \gtrsim 0.01$ requires the inflaton to roll through a transplanckian range, $\Delta \varphi \gtrsim M_p$.

This observation has proven useful because the inflaton usually has some sort of a geometrical interpretation when inflationary models are embedded into string theory, and this allows the calculation of its maximum range of variation. For instance, suppose inflation occurs due to the motion of the position, $x$, of a brane within 6 extra dimensions, each of which has length $L$. Then expressing the geometric upper limit $\Delta x < L$ in terms of the canonically normalized inflaton field, $\varphi = M_s^2 x$, gives $\Delta \varphi / M_p < M_s^2 L / M_p$, where $M_s$ is of order the string scale. But $L$ is not itself independent of $M_s$ and the 4D Planck constant, $M_p$. For instance, in the absence of warping one often has $M_p^2 \simeq M_s^5 L^6$, which allows one to write $\Delta \varphi / M_p < (M_s / M_p)^{2/3}$. Finally, consistency of calculations performed in terms of a (higher-dimensional) field theory generally require the hierarchy, $1/L \ll M_s$ which implies $M_s / M_p \simeq (M_s L)^{-3} \ll 1$, showing that $\Delta \varphi / M_p \ll 1$.

More careful estimates of brane motion within an extra-dimensional throat, with the condition that it geometrically cannot move further than the throat itself is long, lead to similar constraints [2]. It is considerations such as these that show (on a case-by-case basis) for each of the extant string-inflation constructions that the distance moved by the inflaton is too small to allow $r \gtrsim 0.01$. However, in the absence of a no-go theorem, there is strong motivation to find stringy examples which evade these kinds of constraints, and allow the inflaton to undergo large excursions.

2. The Large Volume Framework

In this section we provide a brief boilerplate review of LVS moduli stabilisation in Type IIB string compactifications, with a view to setting the stage for the K3 fibrations considered as examples in the next section. We start with a reminder of the basics of Type IIB flux compactifications themselves.

2.1 Type IIB flux compactifications

The Type IIB flux compactifications of interest are those preserving $\mathcal{N} = 1$ supersymmetry in 4D, leading to compactifications on Calabi-Yau three-folds $X$ [17]. The low-energy 4D theory obtained at low energies is described by an $\mathcal{N} = 1$ supergravity, and so is described by a Kähler potential, $K$, superpotential, $W$ and gauge kinetic function, $f_{ab}$.

2.1.1 Lowest-order expressions

To leading order in the string-loop and $\alpha'$ expansions, the resulting low-energy Kähler potential has the form:

$$K_{\text{tree}} = -2 \ln V - \ln (S + \bar{S}) - \ln \left( -i \int_X \Omega \wedge \bar{\Omega} \right).$$

In (2.1) $S$ is the axio-dilaton, $S = e^{-\phi} + iC_0$, $\Omega$ is the Calabi-Yau’s holomorphic (3,0)-form, and $V$ is its volume, measured with an Einstein frame metric $g^E_{\mu\nu} = e^{-\phi/2} g^s_{\mu\nu}$, and
expressed in units of the string length, \( l_s = 2\pi \sqrt{\alpha'} \). In general, the complex fields of the 4D theory include \( S \) and the complex moduli of the Calabi Yau geometry, including its complex structure moduli, \( U_\alpha, \alpha = 1, \ldots, h_{2,1}(X) \), and Kähler moduli, \( T_i, i = 1, \ldots, h_{1,1}(X) \). In eq. (2.1) \( \Omega \) is to be read as implicitly depending on the \( U_\alpha \)'s, and \( V \) as depending implicitly on the \( T_i \)'s.

The values of \( S \) and the complex structure moduli, \( U_\alpha \), can become fixed once background fluxes, \( G_3 = F_3 + iSH_3 \), are turned on, where \( F_3 \) and \( H_3 \) are respectively Type IIB supergravity’s RR and NSNS 3-form fluxes (for recent reviews on flux compactifications see: [18]). The potential energy for this is captured by the resulting low-energy superpotential, which is given by
\[
W_{\text{tree}} = \int_X G_3 \wedge \Omega.
\]
(2.2)

These fluxes may, but need not, break the remaining 4D \( \mathcal{N} = 1 \) supersymmetry, corresponding to whether or not the resulting scalar potential is minimized where \( D_\alpha W = \partial_\alpha W + W \partial_\alpha K \) vanishes at the minimum.

The Kähler moduli \( T_i \) do not appear in \( W_{\text{tree}} \) and so remain precisely massless at leading semiclassical order. The supergravity describing this massless sector is obtained after eliminating the heavier fields \( S \) and \( U_\alpha \) at the classical level. Provided this can be done in a supersymmetric way [19], by solving \( D_\alpha W = 0 \), the supergravity description of the remaining Kähler moduli is specified by a constant super potential, \( W = W_0 = \langle W_{\text{tree}} \rangle \), and the Kähler potential \( K = K_{cs} - 2 \ln (2/g_s) + K_0 \), with
\[
K_0 = -2 \ln V \quad \text{and} \quad e^{-K_{cs}} = \left\langle -i \int_X \Omega \wedge \bar{\Omega} \right\rangle.
\]
(2.3)

To express \( K_0 \) explicitly in terms of the fields \( T_i \) write the volume in terms of the Kähler form, \( J \), expanded in a basis \( \{\hat{D}_i\} \) of \( H^{1,1}(X,\mathbb{Z}) \) as \( J = \sum_{i=1}^{h_{1,1}} t^i \hat{D}_i \) (we focus on orientifold projections such that \( h_{-1,1} = 0 \Rightarrow h_{1,1}^+ = h_{1,1} \)). This gives
\[
V = \frac{1}{6} \int_X J \wedge J \wedge J = \frac{1}{6} k_{ijk} t^i t^j t^k.
\]
(2.4)

Here \( k_{ijk} \) are related to the triple intersection numbers of \( X \) and the \( t^i \) are 2-cycle volumes. The quantities \( t^i \) appearing here are related to the components of the chiral multiplets \( T_i \) as follows. Writing \( T_i = \tau_i + ib_i \), \( \tau_i \) turns out to be the Einstein-frame volume (in units of \( l_s \)) of the divisor \( D_i \in H_4(X,\mathbb{Z}) \), which is the Poincaré dual to \( \hat{D}_i \). Its axionic partner \( b_i \) is the component of the RR 4-form \( C_4 \) along this cycle: \( \int_{D_i} C_4 = b_i \). The 4-cycle volumes \( \tau_i \) are related to the 2-cycle volumes \( t^i \) by:
\[
\tau_i = \frac{\partial V}{\partial t^i} = \frac{1}{2} \int_X \hat{D}_i \wedge J \wedge J = \frac{1}{2} k_{ijk} t^j t^k.
\]
(2.5)

\( K_0 \) is now given as a function of \( T_i \) by solving these equations for the \( t^i \) as functions of the \( \tau_i = \frac{1}{2}(T_i + \overline{T_i}) \), and substituting the result into eq. (2.3) using eq. (2.4) to evaluate \( V \).
The $\mathcal{N} = 1$ F-term supergravity scalar potential for $T_i$ which results is given in terms of $K$ and $W$ (in 4D Planck units) by:

$$V = e^K \left\{ K^{ij} D_i W D_j \overline{W} - 3 |W|^2 \right\}, \quad (2.6)$$

where $K^{ij}$ is the inverse of the Kähler metric $K_{ij} = \partial_i \partial_j K$ and, as before, $D_i W = \partial_i W + W \partial_i K$. Notice that the above procedure ensures that $V$ is a homogeneous function of degree $3/2$ in the $\tau_i$’s, and so also ensures $K_0$ satisfies $K_0(\lambda \tau_i) \equiv K_0(\tau_i) - 3 \ln \lambda$ as an identity for all $\lambda$ and $\tau_i$. It follows from this that $K_0$ satisfies the no-scale identity: $K_0^{ij} \partial_i K_0 \partial_j K_0 \equiv 3$ for all $\tau_i$. This in turn guarantees the potential, (2.6), constructed using $K_0$ is completely flat, $V \equiv 0$, as is required for agreement with the microscopic compactification since the fluxes did not stabilize the Kähler moduli to leading order.

### 2.2 Corrections to the leading approximation

Because the leading contributions to the potential for the $T_i$’s identically vanish, we must work to sub-leading order in $\alpha'$ and $g_s$ (string loops) in order to determine its shape. (The same is not required for $S$ and $U_{\alpha}$, whose potential is dominated by the leading order contribution.) For the present purposes there are three important corrections to track.

#### 2.2.1 Superpotential corrections

Since the superpotential receives no contributions at any finite order in $\alpha'$ and $g_s$, its first corrections arise non-perturbatively. These can be generated either by Euclidean D3 branes (ED3) wrapping 4 cycles, or by gaugino condensation by the supersymmetric gauge theories located on D7 branes also wrapping 4-cycles in the extra dimensions. The resulting superpotential is

$$W = W_0 + \sum_i A_i e^{-a_i T_i}, \quad (2.7)$$

where the sum is over the 4-cycles generating nonperturbative contributions to $W$, and as before $W_0$ is independent of $T_i$. The $A_i$ correspond to threshold effects and can depend on the complex structure moduli and positions of D3-branes. The constants $a_i$ in the exponential are given by $a_i = 2\pi$ for ED3 branes, or $a_i = 2\pi/N$ for gaugino condensation with gauge group $SU(N)$. There may additionally be higher instanton effects in (2.7), but these can be neglected so long as each $\tau_i$ is stabilised such that $a_i \tau_i \gg 1$.

The presence of such a superpotential generates a scalar potential for $T_i$, of the form

$$\delta V_{(sp)} = e^{K_0} K_0^{ij} \left[ a_j A_j a_i \bar{A}_i e^{-(a_j T_j + a_i T_i)} - \left( a_j A_j e^{-a_j T_j} \overline{W} \partial_i K_0 + a_i \bar{A}_i e^{-a_i T_i} W \partial_j K_0 \right) \right]. \quad (2.8)$$

#### 2.2.2 Leading $\alpha'$ corrections

Unlike the superpotential, the Kähler potential receives corrections order-by-order in both the $\alpha'$ and string-loop expansions. The leading $\alpha'$ corrections for the Type IIB flux compactifications of interest lead to a Kähler potential for the Kähler moduli of the form

$$K = -2 \ln \left( V + \frac{\xi}{2 g_s^{3/2}} \right), \quad (2.9)$$
up to an irrelevant $T_i$-independent constant. Here $\xi$ is given by $\xi = -\chi(X)\zeta(3)\frac{1}{2(2\pi i)^3}$, $\chi$ is the Euler number of the Calabi-Yau $X$, and the relevant value for the Riemann zeta function is $\zeta(3) \approx 1.2$. The leading contribution to the scalar potential that follows from this correction is given by (defining $\hat{\xi} \equiv \xi/g_s^{3/2}$):

$$\delta V(\alpha') = 3\hat{\xi}e^{K_0} \left( \frac{\xi^2 + 7\hat{\xi}V + V^2}{V - \hat{\xi}} \right) \frac{1}{2} |W_0|^2 \approx \frac{3\hat{\xi}}{4V^3} |W_0|^2, \quad (2.10)$$

where the validity of the $\alpha'$ and loop expansions require $V \gg \hat{\xi} \gg 1$.

### 2.2.3 String-loop corrections

$K$ also receives corrections from string loops, and although there is at present no explicit computation of string scattering amplitudes on a generic Calabi-Yau background it has proven possible to argue what the $T_i$-dependence is likely to be for the leading string loop corrections [20, 21]. This section briefly reviews these arguments.

The only explicit string computation of loop corrections to $K$ is available for $N = 1$ compactifications on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ [22] and gives:

$$\delta K(g_s) = \delta K^{KK}(g_s) + \delta K^W(g_s), \quad (2.11)$$

where $\delta K^{KK}(g_s)$ comes from the exchange between D7 and D3-branes of closed strings which carry Kaluza-Klein momentum, and reads (for vanishing open string scalars)

$$\delta K^{KK}(g_s) = -\frac{1}{128\pi^4} \sum_{i=1}^3 \frac{E^{KK}(U, \bar{U})}{\text{Re}(S)\tau_i}. \quad (2.12)$$

In the previous expression we assumed that all the three 4-cycles of the torus are wrapped by D7-branes and $\tau_i$ denotes the volume of the 4-cycle wrapped by the $i$-th D7-brane. The other correction $\delta K^W(g_s)$ is interpreted in the closed string channel as due to exchange of winding strings between intersecting stacks of D7-branes. It takes the form

$$\delta K^W(g_s) = -\frac{1}{128\pi^4} \sum_{i=1}^3 \frac{E^W(U, \bar{U})}{\tau_i \tau_j \tau_k} |_{j \neq k \neq i}, \quad (2.13)$$

where $\tau_i$ and $\tau_j$ denote the volume of the 4-cycles wrapped by the $i$-th and the $j$-th intersecting D7-branes. Note that in both cases there is a very complicated dependence of the corrections on the $U$ moduli, encoded in the functions $E_i(U, \bar{U})$, but a very simple dependence on the $T$ moduli.

These observations were used by [20] to conjecture a generalisation of these formulae to 1-loop corrections on general Calabi-Yau three-folds. Given that these corrections can be interpreted as the tree-level propagation of a closed KK string, and that a Weyl rescaling is always necessary to convert the string computation to Einstein frame, they proposed

$$\delta K^{KK}(g_s) \sim \sum_{i=1}^{h_{1,1}} \frac{C^{KK}(U, \bar{U})m_{KK}^2}{\text{Re}(S)\mathcal{V}} \sim \sum_{i=1}^{h_{1,1}} \frac{C^{KK}(U, \bar{U}) (a_{il}l^i)}{\text{Re}(S)\mathcal{V}}, \quad (2.14)$$
where $a_{il} t^l$ is a linear combination of the basis 2-cycle volumes $t_i$ that is transverse to the 4-cycle wrapped by the $i$-th D7-brane. A similar line of argument for the winding corrections gives

$$\delta K^W_{(g_s)} \sim \sum_i C^K_i (U, \bar{U}) m^{-2} \sim \sum_i C^K_i (a_{il} t^l) V,$$

with $a_{il} t^l$ now being the 2-cycle where the two D7-branes intersect. $C^K_i$ and $C^W_i$ are unknown functions of the complex structure moduli, which may be simply regarded as unknown constants for the present purposes because the complex structure moduli are already flux-stabilised by the leading-order dynamics. What is important is the leading order dependence on Kähler moduli, which the conjecture explicitly displays.

Ref. [21] used these expressions to work out the implications of eqs. (2.14)-(2.15) for the effective scalar potential. The result is a relatively simple formula that is expressible in terms of the tree-level Kähler metric $K_0 = -2 \ln V$ and the winding correction to the Kähler potential, as follows:

$$\delta V^{1\text{-loop}}_{(g_s)} = \left( \frac{C^K_i}{\text{Re}(S)^2} a_{ik} a_{ij} K_0^{k\bar{j}} - 2 \sum_i \delta K^W_{(g_s), t_i} \right) \frac{W_0^2}{V^2}. \quad (2.16)$$

For branes wrapped only around the basis 4-cycles (such as we consider below) the combination appearing in the first term degenerates to $a_{ik} a_{ij} K_0^{k\bar{j}} = K_0^{k\bar{j}}$. As emphasized in ref. [20], this contribution is subdominant in powers of $1/V$ relative to the leading $\alpha'$ correction, eq. (2.10), in the limit of current interest, where $V$ is large.

### 2.3 Exponentially large volumes

The observation of the LVS [13] is that these corrections can generate a potential for the volume modulus, $V$, with a minimum at exponentially large values. To see this it is necessary to include corrections of more than one type, and given the subdominance of the string-loop contribution, eq. (2.16), relative to the $\alpha'$ correction, eq. (2.10), it is useful to consider initially only the contributions $\delta V_{(sp)}$ and $\delta V_{(a')}$. To this end, combining (2.8) in (2.10) gives the total potential for $V$ of the form

$$V = e^{K_0} \left\{ K_0^{j\bar{i}} \left[ a_j A_j a_i \tilde{A}_i e^{-(a_j T_j + a_i T_i)} \right. \right.

$$

$$\left. \left. - \left( a_j A_j e^{-a_j T_j} W \partial_i K_0 + a_i \tilde{A}_i e^{-a_i T_i} W \partial_j K_0 \right) \right] + \frac{3 \tilde{\xi}}{4 V} |W_0|^2 \right\}. \quad (2.17)$$

Since the parameters (like $\xi$) appearing in this potential are related to the topology of the underlying Calabi-Yau space, the choices required for the existence of a minimum at large volume imply conditions on this underlying topology. These conditions are analysed in ref. [14] and can be summarised as follows:

1. The Euler number of the Calabi Yau manifold must be negative, or more precisely: $h_{12} > h_{11} > 1$. This ensures the coefficient $\tilde{\xi}$ is positive, which in turn guarantees that $V$ goes to zero from below (in a particular direction) as $V$ goes to infinity [13]. This is a both sufficient and necessary condition for the existence of a minimum.
2. To remain within the domain of approximation, it is also necessary to have a second modulus whose exponential contributions to \( W \) can balance the inverse powers of \( V \) arising from \( \alpha' \) corrections. This requires the Calabi-Yau manifold to have at least one blow-up mode corresponding to a 4-cycle modulus that resolves a point-like singularity [14]. This 4-cycle must have positive first Chern class, and so positive curvature, as a del Pezzo surface. This implies it can be shrunk to zero size, re-obtaining the singularity resolved by the blow-up.

Although these conditions ensure the stabilization of \( V \), in general the above potential is insufficient to stabilize all of the Kähler moduli. In particular, if there are \( N_{\text{small}} \) blow-up modes and \( L = (h_{11} - N_{\text{small}} - 1) \) modes which do not resolve point-like singularities or correspond to the overall volume modulus, then the above potential can stabilise all of the \( N_{\text{small}} \) blow-up moduli (at values large in string units) and the overall volume (at values that are exponentially large in the blow-up moduli). But the other \( L \) Kähler moduli are not fixed.

The lifting of these remaining flat directions occurs with the inclusion of string loop corrections, which for these modes are always dominant compared to non-perturbative effects. Since the overall volume is stabilised, the internal moduli space is compact, implying a finite range for these remaining moduli. Consequently we expect that the loop-generated potential does not simply generate a runaway for these remaining fields, but must instead generically induce a minimum. The K3 fibration example used below to derive inflation is an explicit illustration of this picture, with the inflaton being one of the moduli whose potential is loop-generated.

For instance, the original example of an exponentially large volume minimum was realised explicitly in [13] for the Calabi Yau \( \mathbb{C}P^4[1,1,1,6,9][18] \), whose volume is given by

\[
V = \frac{1}{9\sqrt{2}} \left( \tau_b^{3/2} - s^{3/2} \right) .
\] (2.18)

In the absence of fine tuning the tree-level superpotential is of order \( W_0 \sim \mathcal{O}(1) \), and so the \( \alpha' \) and non-perturbative corrections compete naturally to give an exponentially large volume (AdS) minimum that breaks SUSY, located at

\[
V \sim W_0 e^{\alpha s \sqrt{s}} \gg \tau_s \sim \xi^{2/3} \gg 1 .
\] (2.19)

Inclusion of the string loop corrections to \( V \) do not appreciably alter this minimum since due to the subleading dependence on \( V \) remarked on above [20].

For phenomenological applications it is usually necessary to up-lift this minimum from AdS to allow Minkowski (or slightly de Sitter) 4D geometries. This can be done using one of the various methods proposed in the literature (inclusion of \( \mathcal{D}3 \) branes [22], D-terms from magnetised D7 branes [33], F-terms from a hidden sector [34], etc.).

An immediate generalisation of the \( \mathbb{C}P^4[1,1,1,6,9][18] \) model that is useful for inflationary applications is the so called ‘Swiss-cheese’ Calabi-Yaus, whose volume is given by

\[
V = \alpha \left( \tau_b^{3/2} - \sum_{i=1}^{N_{\text{small}}} \lambda_i \tau_i^{3/2} \right) , \quad \alpha > 0 , \quad \lambda_i > 0 \quad \forall i = 1, \ldots, N_{\text{small}} .
\] (2.20)
Examples having this form with \( h_{1,1} = 3 \) are the Fano three-fold \( F_{11} \), the degree 15 hypersurface embedded in \( \mathbb{CP}^{1,3,3,3,5} \) and the degree 30 hypersurface in \( \mathbb{CP}^{1,1,3,10,15} \). In this case the potential stabilises the various 4-cycles, \( \tau_i \), (that control the size of the ‘holes’ of the Swiss-cheese) at comparatively small values (though still much larger than the string scale), \( \tau_i \sim \mathcal{O}(10) \), \( \forall i = 1, ..., N_{\text{small}} \). By contrast, \( \tau_b \) (which controls the overall size of the Calabi-Yau) is stabilised at the exponentially large value, \( \mathcal{V} \sim e^{a_i \tau_i} \).

2.4 Kähler modulus inflation

Finally, we briefly review the mechanism of Kähler moduli inflation \( \mathbb{H} \), since many of the features of the model presented here draw on this example. The starting point for this model is a Swiss cheese Calabi-Yau manifold, which must have at least two blow-up modes \( (N_{\text{small}} \geq 2 \) and so \( h_{1,1} \geq 3 \)), such as is true, for instance, for the \( \mathbb{CP}^{1,3,3,3,5} \) model \( [23, 14] \).

Assuming the minimal three Kähler moduli of this form, our interest is in that part of moduli space for which these satisfy \( \tau_b \gg \tau \gg \tau_s \), where \( \tau \) and \( \tau_s \) are the blow-up modes while \( \tau_b \) controls the overall volume. As a first approximation neglect string loop corrections as well as exponentials of the large moduli \( \tau_b \) and \( \tau \) in \( \mathcal{V} \). Then one finds that \( \tau_b \) and \( \tau_s \) can both be stabilised with \( \mathcal{V} \sim e^{a_s \tau_s} \) and \( \tau_s \gg 1 \).

Fixing these to their stabilised values, but now considering the subdominant dependence on \( \tau \), the potential for the remaining modulus takes the form:

\[
V = A\sqrt{\tau} e^{-2a_\tau} \frac{V}{\mathcal{V}} - B\tau e^{-a_\tau} \frac{V^2}{\mathcal{V}^2} + C\frac{\xi}{\mathcal{V}^3},
\]

where the volume \( \mathcal{V} \) should be regarded as being fixed. Varying \( \tau \) with \( \mathcal{V} \) fixed (this is the reason why \( h_{11} \geq 3 \) is needed), the potential for large \( \tau \) is dominated for the last two terms, which is naturally very flat due to its exponential form.

The above potential gives rise to slow-roll inflation, without the need for fine-tuning parameters in the potential. For the canonically normalised inflaton,

\[
\varphi = \sqrt{4\lambda/(3\mathcal{V})} \tau^{3/4},
\]

the above potential becomes

\[
V \simeq V_0 - \beta \left( \frac{\varphi}{\mathcal{V}} \right)^{4/3} e^{-d'\mathcal{V}^{2/3} \varphi^{4/3}},
\]

with \( V_0 \sim \xi/\mathcal{V}^3 \). This is very similar to textbook models of large-field inflation \( \mathbb{H} \), although with the search for observably large tensor modes in mind, one must also keep in mind an important difference. This is because although in both cases slow roll requires a large argument for the exponential in the potential, this is accomplished differently in the two cases. In the textbook examples the argument of the exponential is typically given by \( \varphi/M_p \), and so slow roll requires \( \varphi \gtrsim M_p \). In the present case, however, slow roll is typically accomplished for small \( \varphi \), due to the large factor of \( \mathcal{V}^{2/3} \) in the exponent. In this crucial way, what we have is actually a small-field model of inflation.
2.4.1 Naturalness

It is remarkable that this is one of the only string-inflation models that does not suffer from the \( \eta \) problem, inasmuch as slow roll does not require a delicate adjustment amongst the parameters in the scalar potential. However, one worries that the extreme flatness of the potential might be affected by sub-leading corrections not yet included in the scalar potential, such as string-loop corrections to the Kähler potential.

Although a definitive analysis requires performing a string loop calculation, some conclusion may be drawn using the conjectured modulus dependence \([21, 21, 14]\) discussed above. In fact, examination of the previous formulae shows that dangerous contributions can arise if D7 branes wrap the inflationary cycle, since in this case string-loop corrections take the form

\[
\delta V_{\text{1-loop}} \sim \frac{1}{\sqrt{\tau} V^3} \sim \frac{1}{\varphi^{2/3} V^{10/3}}.
\]

This is dangerous because it gives a contribution to the slow-roll parameter, \( \eta = M_p^2 V''/V \), of the form \( \delta \eta \sim M_p^2 \delta V'/V_0 \sim \varphi^{-8/3} V^{-1/3} \xi^{-1} \), which for the typical values of interest, \( \varphi \sim V^{-1/2} \ll 1 \), may be large.

One way out of this particular problem is simply not to wrap D7 branes about the inflationary cycle. In this case the remaining loop corrections discussed above do not destroy the slow roll. (Although it is not yet possible to quantitatively characterise the contributions of higher loops, see Appendix A for a related discussion of some of the issues.) Of course, if ordinary Standard Model degrees of freedom reside on a D7, not wrapping D7s on the inflationary cycle is likely to complicate the eventual reheating mechanism because it acts to decouple the inflaton from the observable sector. However we do not regard this particular objection as being too worrisome, since a proper study of reheating in these (and most other models) of string inflation remains a long way off [30].

3. Fibre Inflation

We now return to our main line of argument, and describe the simplest K3-Fibration inflationary model. We regard this model as being a representative of a larger class of constructions (Fibre Inflation), which rely on choosing the inflaton to be one of those Kähler moduli whose potential is first generated at the string-loop level.\(^4\)

3.1 K3 fibration Calabi-Yaus

To describe the model we first require an explicit example of a Calabi-Yau compactification which has a modulus that is not stabilized by nonperturbative corrections to \( W \) together with the leading \( \alpha' \) corrections to \( K \). The simplest such examples are given by Calabi Yaus which have a K3 fibration structure.

\(^4\)Even though these moduli are also Kähler moduli, their behaviour is very different from the volume and in particular the blowing-up modes that drive Kähler moduli inflation. In this sense the previous scenario might be more properly called ‘blow-up inflation’ to differentiate it from the later ‘volume’ inflation \([8]\) and ‘fibre’ inflation developed here.
For our present purposes, a K3 fibered Calabi-Yau can be regarded as one whose volume is linear in one of the 2-cycle sizes, $t_j$ \textsuperscript{[24]}. That is, when there is a $j$ such that the only non-vanishing coefficients are $k_{jia}$ and $k_{jib}$ for $k, l, m \neq j$, then the Calabi-Yau manifold is a K3 fibration having a $\mathbb{CP}^1$ base of size $t_j$, and a K3 fibre of size $\tau_j$. The simplest such K3 fibration has two Kähler moduli, with $V = \tilde{t}_1 \tilde{t}_2^2 + \frac{1}{2} \tilde{t}_2^3$. This becomes $V = \frac{1}{2} \sqrt{\tau_1} \left( \tilde{\tau}_2 - \frac{3}{2} \tilde{\tau}_1 \right)$ when written in terms of the 4-cycle volumes $\tilde{\tau}_1 = \tilde{t}_2^2$ and $\tilde{\tau}_2 = 2(\tilde{t}_1 + \tilde{t}_2)t_2$, corresponding to the geometry $\mathbb{CP}^1_{[1,1,2,2,6]}[12]$ \textsuperscript{[25]}. For later convenience we prefer to follow a slightly different basis of cycles in this geometry,

$$\tau_1 = \tilde{\tau}_1, \quad \tau_2 = \tilde{\tau}_2 - \frac{2}{3} \tilde{\tau}_1,$$  

(3.1)

with a similar change in the 2-cycle basis, $\{\tilde{t}_i\} \rightarrow \{t_i\}$. In terms of these the overall volume becomes

$$V = t_1 t_2^2 = \frac{1}{2} \sqrt{\tau_1} \tau_2 \quad \iff \quad V = t_1 \tau_1,$$  

(3.2)

where $t_1$ is the base and $\tau_1$ the K3 fiber.

For inflationary purposes we also require a third Kähler modulus, which we can achieve by simply adding an extra blow-up mode, as is required in any case to guarantee the existence of controlled large volume solutions. We therefore begin by assuming a compactification whose volume is given in terms of its three Kähler moduli in the following way:

$$V = \lambda_1 t_1 t_2^2 + \lambda_3 t_3^3 = \alpha \left( \sqrt{\tau_1} \tau_2 - \gamma \tau_3^{3/2} \right) = t_1 \tau_1 - \alpha \gamma \tau_3^{3/2},$$  

(3.3)

where the constants $\alpha$ and $\gamma$ are given in terms of the model-dependent numbers, $\lambda_i$, by $\alpha = \frac{1}{2} \lambda_1^{-1/2}$ and $\gamma = (4\lambda_1/27\lambda_3)^{1/2}$, related to the two independent intersection numbers, $d_{122}$ and $d_{333}$, by $\lambda_1 = \frac{1}{2} d_{122}$ and $\lambda_3 = \frac{1}{9} d_{333}$. (Clearly, including more blow-up modes than we have done here is straightforward.) Given that eq. (3.3) simply expresses the addition of the blow-up mode $\tau_3$, to the geometry $\mathbb{CP}^1_{[1,1,2,2,6]}[12]$ \textsuperscript{[14]}, we do not expect there to be any obstruction to the existence of a Calabi-Yau manifold with these features.

We further assume that $h_{2,1}(X) > h_{1,1}(X) = 3$, thus satisfying the other general LVS condition. Since we seek stabilisation with $V$ large and positive, we work in the parameter regime

$$V_0 := \alpha \sqrt{\tau_1} \tau_2 \gg \alpha \gamma \tau_3^{3/2} \gg 1,$$  

(3.4)

with the constant $\gamma$ taken to be positive and order unity. This limit keeps the volume of the Calabi-Yau large, while the blow-up cycle remains comparatively small. Regarding the relative size of $\tau_1$ and $\tau_2$, we consider two situations in what follows: $\tau_2 \gg \tau_1 \gg \tau_3$ and $\tau_2 \gg \tau_1 \gg \tau_3$. (We notice in passing that the second case corresponds to $t_1 \sim \tau_2/\sqrt{\tau_1} \gg t_2 \sim \sqrt{\tau_1} \gg \tau_3$, corresponding to interesting geometries having the two dimensions of the base, spanned by the cycle $t_1$, hierarchically larger than the other four of the K3 fibre, spanned by $\tau_1$.) The similarity of eq. (3.4) with the ‘Swiss cheese’ Calabi-Yaus of previous sections,

$$V = \alpha \left( \sqrt{\tau_1} \tau_2 - \gamma \tau_3^{3/2} \right),$$  

(3.5)
leads us to expect (and our calculations below confirm) that the scalar potential has an AdS minimum at exponentially large volume, together with $(h_{1,1} - N_{\text{small}} - 1) = 1$ flat directions.

### 3.1.1 The potential without string loops

We start by considering the scalar potential computed using the leading $\alpha'$ corrections to the Kähler potential, as well as including nonperturbative corrections to the superpotential.

\[
K = K_0 + \delta K_{(\alpha')} = -2 \ln \left( \mathcal{V} + \frac{\xi}{2} \right) \quad \text{and} \quad W = W_0 + \sum_{k=1}^{3} A_k e^{-\alpha_k T_k}. \tag{3.6}
\]

Because our interest is in large volume $\mathcal{V}_0 \gg \alpha \gamma \tau_3^{3/2}$, we may to first approximation neglect the dependence of $T_{1,2}$ in $W$ and use instead

\[
W \simeq W_0 + A_3 e^{-a_3 T_3}. \tag{3.7}
\]

In the large volume limit the Kähler metric and its inverse become

\[
K^0_{ij} = \frac{1}{4\tau_2^2} \begin{pmatrix}
\frac{\tau_2^2}{\tau_1} & \gamma \left( \frac{\tau_1}{\tau_2} \right)^{3/2} & \left( -\frac{3\gamma}{2} \right) \sqrt{\frac{\tau_1}{\tau_2}} T_2 \\
\gamma \left( \frac{\tau_1}{\tau_2} \right)^{3/2} & 2 & -3\gamma \sqrt{\frac{\tau_1}{\tau_2}} \\
-\frac{3\gamma}{2} \sqrt{\frac{\tau_1}{\tau_2}} T_2 & -3\gamma \sqrt{\frac{\tau_1}{\tau_2}} & \frac{3\alpha \gamma}{2} \sqrt{\frac{\tau_1}{\tau_2}} \end{pmatrix}, \tag{3.8}
\]

and

\[
K^i_j = 4 \begin{pmatrix}
\frac{\tau_2^2}{\tau_1} & \gamma \sqrt{\frac{\tau_1}{\tau_2}} T_3 & \tau_1 T_3 \\
\gamma \sqrt{\frac{\tau_1}{\tau_2}} T_3 & \frac{1}{\tau_2} & \tau_2 T_3 \\
\tau_1 T_3 & \tau_2 T_3 & \frac{2}{3\alpha \gamma} \sqrt{\tau_1} \sqrt{\tau_2} \end{pmatrix}, \tag{3.9}
\]

where we systematically drop all terms that are suppressed relative to those shown by factors of order $\sqrt{\tau_3/\tau_2}$. In particular, here (and below), $\mathcal{V}$ now denotes $\mathcal{V}_0 = \alpha \sqrt{\tau_1 \tau_2}$ rather than the full volume, $\mathcal{V}_0 - \alpha \gamma \tau_3^{3/2}$.

We now use these expressions in eq. (2.8), adding the linearisation of $\delta \mathcal{V}_{(\alpha')}$ in $\hat{\xi}$, eq. (2.11). The following identity (to the accuracy of eqs. (3.8) and (3.9)) proves very useful when doing so:

\[
K^0_{i1} K^0_{j1} + K^0_{i2} K^0_{j2} + \text{c.c.} = -3\tau_3. \tag{3.10}
\]

The result may be explicitly minimised with respect to the $T_3$ axion direction, $b_3 = \text{Im} T_3$, with a minimum at $b_3 = 0$ if $W_0 < 0$ or at $b_3 = \pi/a_3$ if $W_0 > 0$. Once this is done, the resulting scalar potential simplifies to

\[
\mathcal{V} = \frac{8 a_3^2 A_3^2}{3\alpha \gamma} \left( \frac{\sqrt{\tau_3}}{\mathcal{V}} \right) e^{-2a_3 \tau_3} - 4W_0 a_3 A_3 \left( \frac{\tau_3}{V^2} \right) e^{-a_3 \tau_3} + \frac{3\hat{\xi} W_0^2}{4V^3}, \tag{3.11}
\]

where we take $W_0$ to be positive and neglect terms that are subdominant relative to the ones displayed by inverse powers of $\mathcal{V}$ without compensating powers of $e^{a_3 \tau_3}$.
Now comes the main point. Notice that by virtue of eq. (3.10) \( V \) depends only on two of the three moduli on which it could have depended: \( V = V(V, \tau_3) \). This occurs because we take \( a_1 \tau_1 \) to be large enough to switch off its non-perturbative dependence in \( W \). This observation has two consequences: First, it implies that within these approximations there is one modulus — any combination (call it \( \mathcal{X} \), say) of \( \tau_1 \) and \( \tau_2 \) independent of \( V \) — which describes a direction along which \( V \) is (so far) completely flat. This plays the rôle of our inflaton in subsequent sections.  

Second, the potential (3.11) completely stabilises the combinations \( \tau_3 \) and \( V \) (and, in fact, has precisely the same form as the scalar potential of the original \( \mathbb{C}P^4_{[1,1,1,6,9]} \) [18] LVS example of [13]). In particular, the only minimum satisfying \( a_3 \tau_3 \gg 1 \) is given explicitly by \( V = \langle V \rangle \) and \( \tau_3 = \langle \tau_3 \rangle \) with  

\[
\langle \tau_3 \rangle = \left( \frac{\xi}{2 \alpha \gamma} \right)^{2/3} \quad \text{and} \quad \langle V \rangle = \left( \frac{3 \alpha \gamma}{4 a_3 A_3} \right) W_0 \sqrt{\langle \tau_3 \rangle} e^{a_3 \langle \tau_3 \rangle} .
\]  

This is the minimum corresponding to exponentially large volume\(^5\).

The flat direction of the potential eq. (3.11) is manifest in Figure II, which plots this scalar potential with \( V \) fixed (using the LV parameter set discussed below), as a function of \( \tau_3 \) (on the x-axis) and \( \mathcal{X} \) — which represents any third field coordinate independent of \( \tau_3 \) and \( V \) (such as \( \tau_1 \), for instance) — (on the y-axis). In order properly to understand the potential for \( \mathcal{X} \), we must go beyond the approximations that underly eq. (3.11), in order to lift this flat direction, such as by including the leading string-loop contributions to the potential.

\(^5\)The two relations (3.12) do not take into account the shift in the volume minimum due to the up-lifting term, which are worked out explicitly in Appendix III (and incorporated in our numerics).
Sample parameter sets

In what follows it is useful to follow some concrete numerical choices for the various underly- 
ing parameters. To this end we track several sets of choices throughout the paper, 
listed in Table 1. One of these sets (call it ‘LV’) gives very large volumes, \( V \approx 10^{13} \) (and so a string scale of order \( M_s \propto V^{-1/2} \approx 10^{12} \text{ GeV} \)), and is representative of what the LARGE volume scenario likes to give for simple choices of parameters. The others (‘SV1’ and ‘SV2’, say) instead have \( V \approx 10^3 \) much smaller (and so with \( M_s \approx 10^{16} \text{ GeV} \)). While all naturally provide an inflationary regime, the LV choice has the disadvantage that the value of the classical inflationary potential turns out too small to provide observable primordial density fluctuations. The other choices are chosen to remedy this problem, and to provide illustrations of different inflationary parameter regimes. We regard all of these choices as being merely illustrative, and have not attempted to perform a systematic search through the allowed parameter space.

|     | LV   | SV1 | SV2   |
|-----|------|-----|-------|
| \( \lambda_1 \) | 1    | 15  | 21/2  |
| \( \lambda_3 \) | 1    | 1/6 | 1/6   |
| \( g_s \)     | 0.1  | 0.3 | 0.3   |
| \( \xi \)     | 0.409| 0.934| 0.755|
| \( W_0 \)     | 1    | 100 | 100   |
| \( a_3 \)     | \( \pi \) | \( \pi/5 \) | \( \pi/4 \) |
| \( A_3 \)     | 1    | 1   | 1     |
| \( \alpha \)  | 0.5  | 0.1291| 0.1543|
| \( \gamma \)  | 0.385| 3.651| 3.055|
| \( \langle \tau_3 \rangle \) | 10.46| 4.28 | 3.73  |
| \( \langle V \rangle \) | \( 2.75 \cdot 10^{13} \) | 1709.55| 1626.12|

Table 1: Some model parameters (the up-lifting to a Minkowski minimum has been taken into account).

3.1.2 Inclusion of string loops

We now specialise the string-loop corrections to the K3 fibration of interest, using expres-
sion (2.16) and working in the regime \( W_0 \gtrsim \mathcal{O}(1) \) where the perturbative corrections are important.

Consider first the contribution coming from stacks of D7 branes wrapping the blow-up cycle, \( \tau_3 \). The Kaluza-Klein loop correction to \( V \) coming from this wrapping takes the form

\[
\delta V_{(g_s),\tau_3}^{KK} = \frac{g_s^2 (C^{KK}_3)^2}{\sqrt{\tau_3} V^3},
\]

which does not depend on \( \mathcal{X} \), and is subdominant to the \( \alpha' \) correction. These features imply such a term may modify the exact locus of the potential’s minimum, but not the main
features of the model, such as the existence of the flat direction in $X$ and the minimization of $V$ at exponentially large values.

Similarly, we have seen that the winding-mode contributions to string-loop corrections arise from the exchange of closed winding strings at the intersection of stacks of D7 branes. But the form of the volume \( (3.3) \) shows that the blow-up mode, $\tau_3$, only has its triple self-intersection number non-vanishing, and so does not intersect with any other cycle. This is a typical feature of a blow-up mode which resolves a point-like singularity: due to the fact that this exceptional divisor is a local effect, it is always possible to find a suitable basis where it does not intersect with any other divisor. Hence the topological absence of the required cycle intersections implies an absence of the corresponding winding-string corrections. In the end, only three types of loop corrections turn out to arise:

$$
\delta V_{(g_s)} = \delta V_{(g_s), \tau_1}^{KK} + \delta V_{(g_s), \tau_2}^{KK} + \delta V_{(g_s), \tau_1 \tau_2}^{W},
$$

which have the form

\[
\begin{align*}
\delta V_{(g_s), \tau_1}^{KK} &= g_s^2 \left( \frac{C_1^{KK}}{\tau_1} \right)^2 \frac{W_0^2}{V^2}, \\
\delta V_{(g_s), \tau_2}^{KK} &= 2g_s^2 \left( \frac{C_2^{KK}}{\tau_2} \right)^2 \frac{W_0^2}{V^2}, \\
\delta V_{(g_s), \tau_1 \tau_2}^{W} &= - \left( \frac{2 C_1^{W}}{t^*_s} \right) \frac{W_0^2}{V^2}.
\end{align*}
\]

Here the 2-cycle $t_s$ denotes the intersection locus of the two 4-cycles whose volumes are given by $\tau_1$ and $\tau_2$. In order to work out the form of $t_s$, we need the relations:

$$
\tau_1 = \frac{\partial V}{\partial t_1} = (\lambda_1 t_2) t_2 \quad \text{and} \quad \tau_2 = \frac{\partial V}{\partial t_2} = 2t_1 (\lambda_1 t_2),
$$

and so $t_s = \lambda_1 t_2 = \sqrt{\lambda_1 \tau_1}$. Therefore the $g_s$ corrections to the scalar potential \( (3.13) \) take the general form:

$$
\delta V_{(g_s)} = \left( \frac{A}{\tau_1} - \frac{B}{V \sqrt{\tau_1}} + \frac{C \tau_1}{V^2} \right) \frac{W_0^2}{V^2}.
$$

where

\[
\begin{align*}
A &= (g_s C_1^{KK})^2 > 0, \\
B &= 2 C_1^{W} \lambda_1^{-1/2} = 4\alpha C_2^{W}, \\
C &= 2 \left( \alpha g_s C_2^{KK} \right)^2 > 0.
\end{align*}
\]

Notice that $A$ and $C$ are both positive (and suppressed by $g_s^2$) but the sign of $B$ is undetermined. The structure of $\delta V_{(g_s)}$ makes it very convenient to use $X \equiv \tau_1$ as our parameter along the flat directions at fixed $V$ and $\tau_3$.

In this way, it is also easier to have a pictorial view of the inflationary process since the K3 fiber modulus $\tau_1$ will turn out to be mostly the inflaton. Inflation will correspond to an initial situation, with the K3 fibre much larger than the base, which will dynamically evolve to a final situation with the base larger than the K3 fibre.
For generic values of \(A\), \(B\) and \(C\) we expect the potential of eq. (3.17) to lift the flat direction and so to stabilize \(X \equiv \tau_1\) at a minimum. Indeed, minimizing \(\delta V(g_s)\) with respect to \(\tau_1\) at fixed \(V\) and \(\tau_3\) gives

\[
\frac{1}{\tau_1^{3/2}} = \left( \frac{B}{8AV} \right) \left[ 1 + (\text{sign } B) \sqrt{1 + \frac{32AC}{B^2}} \right],
\]

which, when \(32AC \ll B^2\), reduces to

\[
\tau_1 \simeq \left( -\frac{BV}{2C} \right)^{2/3} \quad \text{if } B < 0 \quad \text{or} \quad \tau_1 \simeq \left( \frac{4AV}{B} \right)^{2/3} \quad \text{if } B > 0.
\]

Any meaningful minimum must lie within the Kähler cone defined by the conditions that no 2-cycle or 4-cycle shrink to zero and that the overall volume be positive, and so we must check that this is true of the above solution. Since we take \(\tau_1\) and \(\tau_2\) both much larger than \(\tau_3\), we may approximate \(V\) by \(V \simeq \alpha \sqrt{\tau_1 \tau_2} = \lambda_1 t_1 t_2^2\) where \(\lambda_1 = 1/4\alpha^2 > 0\), and this together with eq. (3.16) shows that positive \(t_1\) and \(t_2\) suffices to ensure \(\tau_1\), \(\tau_2\) and \(V\) are all positive. Consequently, the boundaries of the Kähler cone arise where one of the 2-cycle moduli, \(t_{1,2}\), degenerates to zero. Since in terms of \(V\) and \(X \equiv \tau_1\) we have

\[
t_1 = \frac{V}{\tau_1}, \quad t_2 = \left( \frac{\tau_1}{\lambda_1} \right)^{1/2} \quad \text{and} \quad \tau_2 = 2V \left( \frac{\lambda_1}{\tau_1} \right)^{1/2},
\]

the Kähler cone is given by \(0 < \tau_1 < \infty\). At its boundaries we have

\[
\tau_1 \to 0 \iff \tau_2 \to \infty \iff t_1 \to \infty \iff t_2 \to 0,
\]

while

\[
\tau_1 \to \infty \iff \tau_2 \to 0 \iff t_1 \to 0 \iff t_2 \to \infty.
\]

Comparing the solutions of eqs. (3.20) with the walls of the Kähler cone shows that when \(32AC \ll B^2\) we must require either \(C > 0\) (if \(B < 0\)) or \(A > 0\) (if \(B > 0\)), a condition that is always satisfied (see (3.18)).

In Table 1 we chose for numerical purposes several representative parameter choices, and these choices are extended to the loop-generated potential in Table 2. (The entries for \(\langle \tau_3 \rangle\) and \(\langle V \rangle\) in this table are simply carried over from Table 1 for ease of reference.) The LV case shows that loop corrections can indeed stabilise the remaining modulus, \(X \equiv \tau_1\), at hierarchically large values, \(\tau_2 \gg \tau_1 \gg \tau_3\) without requiring the fine-tuning of parameters in the potential, while the SV examples illustrate cases where \(\tau_2 \gg \tau_1 \gtrsim \tau_3\) (although \(e^{-a_1\tau_1} \ll e^{-a_3\tau_3}\)).

### 3.1.3 Canonical normalisation

To discuss dynamics and masses requires the kinetic terms in addition to the potential, which we now display in terms of the variables \(V\) and \(X \equiv \tau_1\). Neglecting the small blow-up
cycle, \( \tau_3 \), the non-canonical kinetic terms for the large moduli \( \tau_1 \) and \( \tau_2 \) are given at leading order by

\[
-\mathcal{L}_{\text{kin}} = K^0_0 \left( \partial_\mu T_i \partial^\mu T_j \right) = \frac{1}{4} \frac{\partial^2 K_0}{\partial \tau_i \partial \tau_j} \left( \partial_\mu \tau_i \partial^\mu \tau_j + \partial_\mu b_i \partial^\mu b_j \right)
\]

where the ellipses denote both higher-order terms in \( \sqrt{\tau_3/\tau_{1,2}} \), as well as axion kinetic terms. Trading \( \tau_2 \) for \( V \) with eq. (3.21), the previous expression becomes

\[
-\mathcal{L}_{\text{kin}} = \frac{3}{8 \tau_1^2} \partial_\mu \tau_1 \partial^\mu \tau_1 - \frac{1}{2 \tau_1 V} \partial_\mu \tau_1 \partial^\mu V + \frac{1}{2 V^2} \partial_\mu V \partial^\mu V + \cdots
\]

Notice that the kinetic terms in this sector can be made field independent by redefining \( \vartheta_1 = \ln \tau_1 \) and \( \vartheta_v = \ln V \), showing that this part of the target space is flat (within the approximations used). The canonically normalized fields satisfy \( -\mathcal{L}_{\text{kin}} = \frac{1}{2} \left[ (\partial \varphi_1)^2 + (\partial \varphi_2)^2 \right] \), and so may be read off from the above to be given by

\[
\begin{pmatrix}
\partial_\mu \tau_1/\tau_1 \\
\partial_\mu V/V
\end{pmatrix}
= M \cdot
\begin{pmatrix}
\partial_\mu \varphi_1 \\
\partial_\mu \varphi_2
\end{pmatrix},
\]

where the condition

\[
M^T \cdot \begin{pmatrix} 3 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{pmatrix} \cdot M = I,
\]

implies \( M^2 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \), and so if \( M = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \) then \( a_\pm = \sqrt{2 - b^2_\pm}, \ c_\pm = \frac{3}{2} - b^2_\pm \) and \( b^2_\pm = 2/(7 \pm 4 \sqrt{2}) \) (so explicitly \( a_+ \simeq 1.357, b_+ \simeq 0.398, c_+ \simeq 1.158 \) and \( a_- \simeq 0.715, b_- \simeq 1.220, c_- \simeq 0.105 \)).

Finally, we may use these results to estimate the mass of the propagation eigenstates, \( \varphi_{1,2} \), obtained at the potential’s minimum. Before diagonalizing the kinetic terms, but

|   | LV  | SV1 | SV2 |
|---|-----|-----|-----|
| \( C_1^{KK} \) | 0.1 | 0.15 | 0.18 |
| \( C_2^{KK} \) | 0.1 | 0.08 | 0.1 |
| \( C_{12}^W \) | 5 | 1 | 1.5 |
| \( A \) | \( 10^{-4} \) | \( 2 \cdot 10^{-3} \) | \( 2.9 \cdot 10^{-3} \) |
| \( B \) | 10 | 0.52 | 0.93 |
| \( C \) | \( 5 \cdot 10^{-5} \) | \( 1.9 \cdot 10^{-5} \) | \( 4.3 \cdot 10^{-5} \) |
| \( \langle \tau_3 \rangle \) | 10.46 | 4.28 | 3.73 |
| \( \langle \tau_1 \rangle \) | \( 1.07 \cdot 10^6 \) | 8.96 | 7.5 |
| \( \langle V \rangle \) | \( 2.75 \cdot 10^{13} \) | 1709.55 | 1626.12 |

Table 2: Loop-potential parameters.
writing \( \vartheta_v = \ln \mathcal{V} \) and \( \vartheta_1 = \ln \tau_1 \), we find that the derivatives of the potential at its minimum scale as \( \partial^2 V/\partial \vartheta_v^2 \sim \xi/\mathcal{V}^3 \) — since it is dominated by contributions from \( \delta V_{(\alpha')} \) and \( \delta V_{(sp)} \) — while \( \partial^2 V/\partial \vartheta_1^2 \sim \partial^2 V/\partial \vartheta_v \partial \vartheta_1 \sim 1/\mathcal{V}^{10/3} \) — since these are dominated by \( \delta V_{(gs)} \). These properties remain true for the physical mass eigenvalues after diagonalising the kinetic terms, since this mixing changes the form of the eigenvectors but not the leading scaling of the eigenvalues at large \( \mathcal{V} \). This confirms the qualitative expectation that the \( \mathcal{X} \equiv \tau_1 \) direction is systematically lighter than \( \mathcal{V} \) in the large-\( \mathcal{V} \) limit.

### 3.2 Inflationary potential

Having established the existence of a consistent LVS minimum of the potential for all fields, we now explore the inflationary possibilities that can arise when some of these fields are displaced from these minima. Since the potential for \( \mathcal{X} \equiv \tau_1 \) is systematically flat in the absence of string loop corrections, it is primarily this field that we displace in the hopes of finding it to be a good candidate for a slow-roll inflaton.

In the approximation that string-loop effects are completely turned off, we have seen that the leading large-\( \mathcal{V} \) potential stabilising both \( \mathcal{V} \) and \( \tau_3 \) is completely flat in the \( \mathcal{X} \equiv \tau_1 \) direction. We therefore perform our initial inflationary analysis within an approximation where both \( \mathcal{V} \) and \( \tau_3 \) remain fixed at their respective \( \tau_1 \)-independent minima while \( \tau_1 \) rolls towards its minimum from initially larger values. In this approximation the important evolution involves only the single field \( \tau_1 \), making it very simple to calculate. This single-field approach should be an excellent approximation for large enough \( \mathcal{V} \), and we return below to the issue of whether or not \( \mathcal{V} \) can be chosen large enough to call this approximation into question.

#### 3.2.1 The single-field inflaton

As before, we choose \( \mathcal{X} \equiv \tau_1 \) as the coordinate along the inflationary direction. When \( \tau_3 = \langle \tau_3 \rangle \) and \( \mathcal{V} = \langle \mathcal{V} \rangle \) are fixed at their \( \tau_1 \)-independent minima, so that \( \partial_\mu \tau_3 = \partial_\mu \mathcal{V} = 0 \), (3.23) shows that the relevant dynamics reduces to

\[
\mathcal{L}_{in} = -\frac{3}{8} \left( \frac{\partial_\mu \tau_1 \partial^\mu \tau_1}{\tau_1^2} \right) - V_{inf}(\tau_1),
\]

with scalar potential given by

\[
V_{inf} = V_0 + \left( \frac{A}{\tau_1^2} - \frac{B}{\sqrt{\tau_1}} + \frac{C \tau_1}{\mathcal{V}^2} \right) \frac{W_0^2}{\mathcal{V}^2}.
\]

Notice that (3.24) does not depend on the intersection numbers, \( \lambda_1 \) and \( \lambda_3 \), implying that tuning these cannot help with the search of a canonical normalisation more suitable for an inflationary roll. The \( \tau_1 \)-independent constant, \( V_0 \), of eq. (3.27) consists of

\[
V_0 = \frac{8 a_2^2 A_2^2 \sqrt{\langle \tau_3 \rangle}}{3 \alpha \gamma \langle \mathcal{V} \rangle} e^{-2a_3 \langle \tau_3 \rangle} - \frac{4 W_0 a_3 A_3 \langle \tau_3 \rangle}{\langle \mathcal{V} \rangle^2} e^{-a_3 \langle \tau_3 \rangle} + \frac{3 \xi W_0^2}{4 \langle \mathcal{V} \rangle^3} + \delta V_{up},
\]

where \( \delta V_{up} \) is an up-lifting potential, such as might be produced by the tension of an \( D3 \) brane in a warped region somewhere in the extra dimensions: \( \delta V_{up} \sim \delta_{up}/\mathcal{V}^{4/3} \). For
the present purposes, what is important about this term is that it does not depend at all on $\tau_1$ once $V$ is fixed. We imagine $\delta_{up}$ to be tuned to ensure the complete vanishing of $V$ (or a tiny positive value) at the minimum, with $\delta_{up} \sim 1/\langle V \rangle^{5/3}$ required to cancel the non-perturbative and $\alpha'$-correction parts of the potential (which together scale like $\langle V \rangle^{-3}$ at their minimum). In addition a second adjustment ($\delta_{up} \rightarrow \delta_{up} + \mu_{up}$) of order $\mu_{up}/\langle V \rangle^{4/3} = -\delta V_{(g)}(\langle V \rangle, \langle \tau_1 \rangle)$ is required to cancel the loop-generated part of $V$, for which $V_0 \sim \mathcal{O}(1/\langle V \rangle^{10/3})$.

The canonical inflaton is therefore given by

$$\varphi = \frac{\sqrt{3}}{2} \ln \tau_1, \quad \text{and so} \quad \tau_1 = e^{\kappa \varphi} \quad \text{with} \quad \kappa = \frac{2}{\sqrt{3}}. \quad (3.29)$$

In terms of this field the walls of the Kähler cone are located at

$$0 < \tau_1 < \infty \iff -\infty < \varphi < +\infty, \quad (3.30)$$

implying that any inflationary dynamics can in principle take place over an infinite range in field space. The potential $\langle V \rangle$ becomes

$$V_{inf} = V_0 + \frac{W^2}{2} \left( A e^{-2\kappa \varphi} \frac{B}{V} e^{-\kappa \varphi/2} + \frac{C}{V^2} e^{\kappa \varphi} \right) = \frac{1}{\langle V \rangle^{10/3}} \left( C_0 e^{\kappa \varphi} - C_1 e^{-\kappa \varphi/2} + C_2 e^{-2\kappa \varphi} + C_{up} \right), \quad (3.31)$$

where we shift $\varphi = \langle \varphi \rangle + \hat{\varphi}$ by its vacuum value, $\langle \hat{\varphi} \rangle = 0$. Choosing, for concreteness’ sake, $32AC \ll B^{26}$ we have $\langle \varphi \rangle = \frac{1}{\sqrt{3}} \ln (\zeta V)$, with $\zeta \simeq -B/2C$ if $B < 0$ or $\zeta \simeq 4A/B$ if $B > 0$. With these choices the coefficients $C_i$ do not depend on $\langle V \rangle$, being given by

$$C_0 = CW_0^2 \zeta^{2/3}, \quad C_1 = BW_0^2 \zeta^{-1/3}, \quad C_2 = AW_0^2 \zeta^{-4/3} \quad \text{and} \quad C_{up} = C_1 - C_0 - C_2. \quad (3.32)$$

Notice that because $A$ and $C$ are both positive, we know that $C_0$ and $C_2$ must also be. By contrast, not knowing the sign of $C_{12}^W$ precludes having similar control over the sign of $C_1$.

Table 3 gives the values for these coefficients as computed using the parameter sets of the previous tables.

Of particular interest is the case where both $A$ and $C$ are small compared with $|B|$, as might be expected by their explicit suppression by the factor $g_s^4$. For concreteness we focus in what follows on the case $B > 0$ (and so $C_1 > 0$), for which $\zeta \simeq 4A/B \ll 1$. This leads to two very useful simplifications. First, it implies that $C_0/C_1 = \zeta C/B = 4AC/B^2$ and $R := C_0/C_2 = \zeta^2 C/A = 16AC/B^2$ and so $C_0$ is systematically smaller than either $C_1$ or $C_2$. This observation allows us to neglect completely the $C_0 e^{\kappa \varphi}$ term of the potential in the vicinity of the minimum and in most of the inflationary region, as we shall see in what follows. Second, this limit implies $C_1/C_2 = \zeta B/A = 4$, showing that $C_1$ and $C_2$ are both positive, with a fixed, order-unity ratio. This observation precludes using the ratio of these parameters in the next section as a variable for tuning the inflationary potential. These choices are visible in Table 3, for which $A, C \ll B$, and so $C_0$ is small and $C_1/C_2 \simeq 4$.

Figure 4 plots the resulting scalar potential against $\varphi$.

\footnote{Notice that this is a natural choice since for $B > 0$, $CA/B^2 \sim g_s^4$.}
Table 3: Coefficients of the inflationary potential for the various parameter sets discussed in the text.

|       | LV         | SV1        | SV2        |
|-------|------------|------------|------------|
| $C_0$ | $5.8 \times 10^{-8}$ | 0.012      | 0.023      |
| $C_1$ | 292.4      | 20629.4    | 39786.9    |
| $C_2$ | 73.1       | 5157.35    | 9946.73    |
| $C_{up}$ | 219.3    | 1200.8     | 29840.2    |
| $R = C_0/C_2$ | $8 \times 10^{-10}$ | $2.3 \times 10^{-6}$ | $2.3 \times 10^{-6}$ |

3.3 Inflationary slow roll

We next ask whether the scalar potential \[(3.31)\] can support a slow roll, working in the most natural limit identified above, with $A,C \ll B$ and $B > 0$. As we have seen, this case also implies $0 < C_0 \ll C_1 = 4C_2$, leaving a potential well approximated by

\[ V \simeq \frac{C_2}{\langle V \rangle^{10/3}} \left( (3 - R) - 4 \left( 1 + \frac{1}{6} R \right) e^{-\kappa \hat{\phi}/2} + \left( 1 + \frac{2}{3} R \right) e^{-2\kappa \hat{\phi}} + R e^{\kappa \hat{\phi}} \right) \]  

which uses $C_{up} \simeq C_1 - C_0 - C_2$ and $C_1/C_2 \simeq 4$, and works to linear order in

\[ R := \frac{C_0}{C_2} = 2g_4^4 \left( \frac{C_{KK}^4 C_{KK}^2}{C_{12}^W} \right)^2 \ll 1. \]  

The normalization of the potential may instead be traded for the mass of the inflaton field at its minimum: $m_\phi^2 = V''(0) = 4 \left( 1 + \frac{2}{3} R \right) C_2/[V]^{10/3}$.

In practice the powers of $R$ can be neglected in all but the last term in the potential, where it multiplies a positive exponential which must eventually become important for...
sufficiently large \( \hat{\varphi} \). For smaller \( \hat{\varphi} \), \( R \) is completely negligible and the potential is fully determined by its overall normalisation. Furthermore, the range of \( \hat{\varphi} \) for which this is true becomes larger and larger the smaller \( R \) is, and so we start by neglecting \( R \).

We seek inflationary rolling focusing on the situation in which \( \hat{\varphi} \) rolls down to its minimum (at \( \hat{\varphi} = 0 \)) from positive values. Defining, as usual, the slow-roll parameters, \( \varepsilon \) and \( \eta \), by (recalling our use of Planck units, \( M_p = 1 \))

\[
\varepsilon = \frac{1}{2V^2} \left( \frac{\partial V}{\partial \hat{\varphi}} \right)^2, \quad \eta = \frac{1}{V} \left( \frac{\partial^2 V}{\partial \hat{\varphi}^2} \right),
\]

we find (using \( \kappa^2 = \frac{4}{3} \) and keeping \( R \) only when it comes multiplied by \( e^{\kappa \hat{\varphi}} \))

\[
\varepsilon \approx 8 \left( \frac{e^{-\kappa \hat{\varphi}/2} - e^{-2 \kappa \hat{\varphi}} + \frac{1}{2} R e^{\kappa \hat{\varphi}}}{3 - 4 e^{-\kappa \hat{\varphi}/2} + e^{-2 \kappa \hat{\varphi}} + R e^{\kappa \hat{\varphi}}} \right)^2, \quad \eta \approx -4 \left( \frac{e^{-\kappa \hat{\varphi}/2} - 4 e^{-2 \kappa \hat{\varphi}} - R e^{\kappa \hat{\varphi}}}{3 - 4 e^{-\kappa \hat{\varphi}/2} + e^{-2 \kappa \hat{\varphi}} + R e^{\kappa \hat{\varphi}}} \right),
\]

Plots of these expressions are given in Figure 3, which show three qualitatively different regimes.

**Figure 3:** Plots of the potential and the slow-roll parameters \( \varepsilon \) and \( \eta \) vs \( \hat{\varphi} \) for \( R = 10^{-8} \) (blue curve), \( R = 10^{-6} \) (green curve), and \( R = 10^{-4} \) (red curve).

**Slow-Roll Regime**

Both slow roll parameters are naturally exponentially small in the regime \( R^{1/3} \ll e^{-\hat{\varphi}/2} \ll 1 \). In this regime it is the term \( e^{-\kappa \hat{\varphi}/2} \) that dominates in (3.33), and so the dynamics is effectively governed by the approximate potential

\[
V \simeq \frac{C_2}{(V)^{10/3}} \left( 3 - 4 e^{-\kappa \hat{\varphi}/2} \right).
\]

This resembles a standard potential for large-field inflation, which drives the field to evolve towards smaller values\(^7\). The slow-roll parameters (3.36) and (3.37) in this regime simplify

\(^7\)It would be interesting to see how our inflationary mechanism fits in the general analysis of supergravity conditions for inflation performed in [29].
to
\[
\varepsilon \simeq \frac{8}{3 \left[ 3 e^{1/2} - 4 \right]^2}, 
\tag{3.39}
\]
\[
\eta \simeq -\frac{4}{3 \left[ 3 e^{1/2} - 4 \right]}, 
\tag{3.40}
\]
and for all \( \dot{\phi} \) in this regime we have the interesting relation
\[
\varepsilon \simeq \frac{3 \eta^2}{2}. 
\tag{3.41}
\]

**Small-\( \dot{\phi} \) Regime**

The slow-roll conditions break down once \( \dot{\phi} \) is small enough that the two negative exponentials are comparative in size, to produce a zero in \( \eta \). An inflection point occurs in this regime, located where
\[
\left( \frac{\partial^2 V}{\partial \dot{\phi}^2} \right)_{ip} \simeq \frac{4C_2}{3(V)^{10/3}} \left( -e^{-\kappa \dot{\phi}_{ip}/2} + 4 e^{-2\kappa \dot{\phi}_{ip}} \right) = 0, 
\tag{3.42}
\]
and so
\[
\dot{\phi}_{ip} = \frac{1}{\sqrt{3}} \ln \left( \frac{16 C_2}{C_1} \right) \simeq \frac{\ln 4}{\sqrt{3}} \simeq 0.8004... 
\tag{3.43}
\]

As Figure 3 shows, to the left of this point \( \varepsilon \) grows quickly, while at the inflection point \( \dot{\phi} = \dot{\phi}_{ip} \), we have \( \varepsilon_{ip} = 1.464 \) and \( \eta_{ip} = 0 \). Just to the right of this, at \( \dot{\phi}_{end} = 1 \) we have \( \varepsilon_{end} = 0.781 \) and \( \eta_{end} = -0.256 \), making this as good a point as any to end inflation. (In what follows we verify numerically that our results are not sensitive to precisely where we end inflation in this regime.)

**Large-\( \dot{\phi} \) Regime**

Once \( R e^{\kappa \dot{\phi}} \gg 3 \) the positive exponential dominates the potential, eq. (3.33), which becomes well-approximated by
\[
V \simeq \frac{m^2}{4} R e^{\kappa \dot{\phi}}, 
\tag{3.44}
\]
and so the slow-roll parameters plateau at constant values: \( \eta \simeq 2\varepsilon \simeq \kappa^2 = \frac{4}{3} \) (as is seen in Figure 3). This shows that the slow-roll conditions also break down for \( \kappa \dot{\phi} \simeq \ln(1/R) \), providing an upper limit to the distance over which the slow roll occurs (and so also on the number of \( e \)-foldings, \( N_e \)).

An interesting feature of transition to this large-\( \dot{\phi} \) regime is the necessity for \( \eta \) to change sign. This is interesting because, as figure 3 shows, \( \varepsilon \) is still small where it does, and so there is a slow-roll region for which \( \eta \gg \varepsilon > 0 \). This regime is unusual because it allows \( n_s > 1 \) (see Figure 4), unlike generic single-field inflationary models. In practice, in what follows we choose horizon exit to occur for \( \dot{\phi} \) smaller than this, due to the current observational preference for \( n_s < 1 \). A precise upper limit on \( \dot{\phi} \) this implies can be defined as the inflection point where \( \eta \) vanishes due to the competition between the \( e^{\kappa \dot{\phi}} \) and \( e^{-\kappa \dot{\phi}/2} \) terms of the potential. This occurs when \( e^{-\kappa \dot{\phi}/2} \simeq R e^{\kappa \dot{\phi}} \), or \( \dot{\phi}(R) \simeq \dot{\phi}_0(R) := -\ln(R)/\sqrt{3} \).
We may now ask whether the slow-roll regime is large enough to allow 60 $e$-foldings of inflation. The number of $e$-foldings $N_e$ occurring during the slow-roll regime can be computed using the approximate potential, eq. (3.38), which gives

$$N_e = \int_{\hat{\phi}_{end}}^{\hat{\phi}_*} \frac{V}{\dot{\phi}} d\hat{\phi} \simeq \frac{\sqrt{3}}{4} \int_{\hat{\phi}_{end}}^{\hat{\phi}_*} \left[ 3 e^{\kappa \hat{\phi}/2} - 4 \right] d\hat{\phi} = \left[ \frac{9}{4} e^{\kappa \hat{\phi}/2} - \sqrt{3} \hat{\phi} \right]_{\hat{\phi}_{end}}^{\hat{\phi}_*},$$

(3.45)

where $e^{\kappa \hat{\phi}_{end}} \simeq 16 C_2/C_1 \simeq 4 \ll e^{\kappa \hat{\phi}_*}$ represents the onset of the small-$\hat{\phi}$ regime, as described above, and $\hat{\phi} = \hat{\phi}_*$ denotes the value of $\hat{\phi}$ at horizon exit. Figure 4 shows how the number of $e$-foldings depends on the assumed field value during horizon exit, as well as the insensitivity of this result to the assumed point where inflation ends. This shows that interesting inflationary applications require $\hat{\phi}$ to roll through an interval of at least $O(5)$.

![Figure 4](image-url)  

**Figure 4**: Plot of the number of $e$-foldings, $N_e$, vs $\hat{\phi}_*$ (left) and $\hat{\phi}_{end}$ (right) for $R = 0$. The inflection point occurs at $\hat{\phi}_{ip} \simeq 0.8$ and $\hat{\phi}_{end} = 1$ in the left-hand plot. $\hat{\phi}_* = 5.7$ in the right-hand plot. The solid (red) curves are computed using the full potential (3.33) while the dashed (blue) curves are computed using the approximate potential (3.38).

An estimate for the upper limit to $N_e$ that can be obtained as a function of $R$ can be found by using $\hat{\phi}_* = \hat{\phi}_0(R)$ in eq. (3.45). This leads to

$$N_{e}^{\text{max}} \simeq \frac{9}{4} \left( R^{-1/3} - 2 \right) - \left[ \ln \left( \frac{1}{R} \right) - \ln 8 \right],$$

(3.46)

This result is plotted in Figure 3, and shows that more than 60 $e$-foldings of inflation requires $R \lesssim 3 \times 10^{-5}$.

The validity of the $\alpha'$ and $g_s$ expansions also set a limit to how large $\hat{\phi}_*$ can be taken, since the exponential growth of $\delta V_{(g_s)}$ for large $\hat{\phi}$ would eventually allow it to become larger than the lower-order contributions, $\delta V_{(sp)} + \delta V_{(\alpha')}$. Microscopically this arises because $\hat{\phi} \to \infty$ corresponds to $\tau_1 \to \infty$ and $\tau_2 \to 0$, leading to the failure of the expansion of $\delta V^{KK}_{(g_s),\tau_2}$ in inverse powers of $\tau_2$. However, as is argued in Appendix B, it is the slow-roll condition $\eta \ll 1$ that breaks down first as $\hat{\phi}$ increases, and so provides the most stringent upper edge to the inflationary regime. For the two sample sets SV1 and SV2 given in the
Figure 5: Plot of the maximum number of e-foldings, $N_{e}^{max}$, vs $x = \log_{10} R$, defined by the condition $\dot{\phi} = \dot{\phi}_0(R)$ as described in the text. The integration takes $\dot{\phi}_{end} = 1$, and the curves are computed using the approximate potential (3.38).

Tables, we obtain $R \simeq 2.3 \cdot 10^{-6}$, and this gives $\dot{\phi}_{max} \simeq 12.4$ (in particular allowing more than 60 e-foldings of inflation).

3.3.1 Observable footprints

We now turn to the observable predictions of the model. These divide naturally into two types: those predictions depending only on the slow roll parameters, which are insensitive to the underlying potential parameters; and those which also depend on the normalization of the inflationary potential, and so depend on more of the details of the underlying construction.

Model-independent predictions

The most robust predictions are for those observables whose values depend only on the slow roll parameters, such as the spectral index and tensor-to-scalar ratio, which are given as functions of the slow-roll parameters (evaluated at horizon exit) by

$$n_s = 1 + 2\eta_s - 6\varepsilon_s \quad \text{and} \quad r = 16 \varepsilon_s.$$

(3.47)

In general, as can be seen from (3.36) and (3.37), the two quantities $\varepsilon_s$ and $\eta_s$ are functions of two parameters, $\dot{\phi}_s$ and $R$; hence $n_s = n_s(\dot{\phi}_s, R)$ and $r = r(\dot{\phi}_s, R)$. However we have also seen that having a significant number of e-foldings requires $R \ll 1$, and so to a good approximation $n_s = n_s(\dot{\phi}_s)$ and $r = r(\dot{\phi}_s)$, unless $\dot{\phi}_s$ is large enough that $Re^{\kappa\dot{\phi}_s}$ cannot be neglected.

For small $R$ we find the robust correlation predicted amongst $r$, $n_s$ and $N_e$, as described in the introduction. The implied relation between $r$ and $n_s$ is most easily found by using the relation $\varepsilon_s = \frac{3}{2}\eta_s^2$, eq. (3.41) in eq. (3.47) and dropping $\varepsilon_s$ relative to $\eta_s$ in $n_s - 1$:

$$r \simeq 6(n_s - 1)^2,$$

(3.48)
showing that a smaller ratio of tensor-to-scalar perturbations, $r$, correlates with larger $n_s$. Figure 6 plots the predictions for $r$ and $n_s$ that are obtained in this way.

Deviations from this correlation arise for large enough $\phi^*$, for which $N_e$ approaches the maximum number of $e$-foldings possible, and this is illustrated in Figure 5, which plots $n_s$ vs $\bar{\phi}_*$ for several choices of $R$. (Notice in particular the excursion to values $n_s > 1$ shown in the figure for $\bar{\phi}_* \approx \bar{\phi}_0(R)$ when $R \neq 0$, as discussed above.) In the extreme case where $\tilde{\phi}_* = \bar{\phi}_0(R)$ we have $\eta_* \approx 0$ and $\varepsilon_* \approx \frac{2}{3} R^{2/3}$, leading to

$$r \approx \frac{32}{3} R^{2/3} \quad \text{and} \quad n_s \approx 1 - 4 R^{2/3}. \tag{3.49}$$

Recall that $N_e^{\max} \gtrsim 60$ implies $R \lesssim 3 \times 10^{-5}$, and in the extreme case $R \approx 3 \times 10^{-5}$ the above formulae lead to $r \approx 0.01$ and $n_s \approx 0.996$. Should $r \approx 0.01$ be observed and ascribed to this scenario, the close proximity of horizon exit to the beginning of inflation would likely imply other observable implications for the CMB, along the lines of those discussed in refs. [38].

Model-dependent predictions

We next turn to those predictions which depend on the normalization, $V_0$, of the inflaton potential, and so depend more sensitively on the parameters of the underlying supergravity.

**Number of e-foldings.** The first model-dependent prediction is the number of $e$-foldings itself, since this depends on the value $\tilde{\phi}_*$ taken by the scalar field at horizon exit. Indeed we have already seen that the constraint that there be enough distance between $\tilde{\phi}_*$ and $\tilde{\phi}_{\text{end}}$ to allow many $e$-foldings of inflation imposes upper limits on parameters such as $R$. The strongest such limit turned out to be the requirement that $n_s$ be low enough to agree with observed values (see the discussion surrounding eq. (3.46)). For numerical comparison of our benchmark parameter sets we formalise this by requiring $\tilde{\phi} < \bar{\phi}_{\text{max}}$, defined as the value for which $n_s < 0.974$, since this is the 68% C.L observational upper bound (for small $r$). Table 4 then lists the maximal number of $e$-foldings that are possible given the constraint $\tilde{\phi}_* < \bar{\phi}_{\text{max}}$ for the models given in Tables 1 and 2.
Figure 7: Plots of the spectral index $n_s$ vs $\hat{\phi}$ for $R = 0$ (purple curve), $R = 10^{-8}$ (red curve), $R = 10^{-6}$ (green curve), and $R = 10^{-4}$ (blue curve).

|       | LV | SV1 | SV2 |
|-------|----|-----|-----|
| $\langle \phi \rangle$ | 12.02 | 1.9 | 1.7 |
| $\hat{\phi}_{\text{max}}$ | 6.3 | 6.14 | 6.16 |
| $N_{e\text{max}}^r$ | 72 | 64 | 64 |
| $A_{\text{COBE}}$ | $2.1 \cdot 10^{-45}$ | $1.2 \cdot 10^{-7}$ | $2.8 \cdot 10^{-7}$ |
| $\mathcal{R}_{cv}$ | 1201.6 | 29.7 | 12.2 |

Table 4: Model parameters for the inflationary potential. $N_{e\text{max}}^r$ denotes the number of $e$-foldings computed when rolling from $\hat{\phi}_{\text{max}}$ to $\hat{\phi} = 1$. $A_{\text{COBE}}$ is calculated at $N_e \simeq 60$.

But how many $e$-foldings of inflation are required is itself a function of both the inflationary energy scale and the post-inflationary thermal history. For instance, suppose the inflaton energy density, $\rho_{\text{inf}} \sim M_{\text{inf}}^4 = V_{\text{end}}$, rethermalises during a re-reheating epoch during which the equation of state is $p = w \rho$, at the end of which the temperature is $T_{rh}$, and after this the radiation-dominated epoch lasts right down to the present epoch. With these assumptions, $M_{\text{inf}}, T_{rh}, w$ and $N_e$ are related by

$$N_e \simeq 62 + \ln \left( \frac{M_{\text{inf}}}{10^{16} \text{GeV}} \right) - \frac{1}{3} \ln \left( \frac{M_{\text{inf}}}{T_{rh}} \right).$$

This formula is obtained by equating the product $aH$ at horizon exit during inflation and horizon re-entry in the cosmologically recent past, $a_{he}H_{he} = a_0 H_0$, and using the intervening cosmic expansion to relate these two quantities to $N_e, T_{rh}$ and $M_{\text{inf}}$ [27]. In particular it shows (if $w < \frac{1}{3}$) that lower reheat temperatures (for fixed $M_{\text{inf}}$) require smaller $N_e$. For instance, if $M_{\text{inf}} \simeq 10^{16}$ GeV and $w = 0$ then an extremely low reheat temperature, $T_{rh} \simeq 1$ GeV, allows $N_e \simeq 50$.

\*\*We thank Daniel Baumann for identifying an error in this formula in an earlier version.
Given that $M_{inf}$ is constrained by the requirement that inflation generate the observed primordial scalar fluctuations (see below), eq. (3.50) is most usefully read as giving the post-inflationary reheat temperature that is required to have modes satisfying $k = (aH)_* \epsilon_b \eta_*$ be the right size to be re-entering the horizon at present. That is, given a measurement of $n_s$ one can invert the prediction $n_s(N_e)$ to learn $N_e$, and so also $r$ and the two slow roll parameters, $\epsilon_*$ and $\eta_*$. Then computing $M_{inf}$ from the amplitude of primordial fluctuations allows eq. (3.50) to give $T_{rh}$. In particular, eq. (3.50) represents an obstruction to using the cosmology (without assuming more complicated reheating) if $N_e$ is too low, since the required $T_{rh}$ would be so low as to be ruled out.

A few illustrative values are listed in Table 5, which assumes a matter-dominated reheating epoch ($w = 0$) and takes $M_{inf} = 4 \times 10^{15}$ GeV, to compute $N_e \simeq 57$ and $T_{rh}$ as a function of $n_s$ and $r$. These all show respectable reheat temperatures, with $10^3 \text{GeV} < T_{rh} < 10^{10}$ GeV, with the upper bound motivated by the requirement that gravitini not be overproduced during reheating [28]. Furthermore, as shown in Figure 8, these values for $n_s$ and $r$ that are predicted lie well within the observably allowed range.

| $T_{rh}$ (GeV) | $N_e$ | $n_s$ | $r$ |
|----------------|-------|-------|-----|
| $10^{10}$      | 57    | 0.9702| 0.0057|
| $5 \times 10^{7}$ | 55    | 0.9690| 0.0060|
| $10^5$         | 53    | 0.9676| 0.0064|
| $5 \times 10^3$ | 52    | 0.9669| 0.0066|

**Table 5:** Predictions for cosmological observables as a function of $T_{rh} \leq 10^{10}$ GeV fixing $M_{inf} = 5 \times 10^{15}$ GeV (for $w = 0$ and $R = 2.3 \times 10^{-6}$).
Furthermore, $r$ is large enough to allow detection by forthcoming experiments such as EPIC, BPol or CMBPol [10, 11].

**Amplitude of Scalar Perturbations:** It is not impressive to have relatively large values for the tensor-to-scalar ratio, $r$, unless the amplitude of primordial scalar perturbations are themselves observably large. Since this depends on the size of Hubble scale at horizon exit, it is sensitive to the constant $V_0 = m^2_2/4 = C_2/V_1^{10/3}$ that pre-multiplies the inflationary potential. The condition that we reproduce the COBE normalisation for primordial scalar density fluctuations, $\delta_H = 1.92 \cdot 10^{-5}$, can be expressed as:

$$A_{COBE} \equiv \left(\frac{g_s^4}{8\pi}\right) \left(\frac{V^{3/2}}{V'}\right)^2 \simeq 2.7 \cdot 10^{-7}, \quad (3.51)$$

where the prefactor $\left(\frac{g_s^4}{8\pi}\right)$ is the correct overall normalisation of the scalar potential obtained from dimensional reduction [13].

As Table 4 shows, it is possible to obtain models with many $e$-foldings and which satisfy the COBE normalisation condition, but this clearly prefers relatively large values for $g_s$ and $1/V$, and so tends to prefer models whose volumes are not inordinately large. It is then possible to evaluate the inflationary scale as

$$M_{inf} = V_{end}^{1/4} \simeq \frac{C_2}{8\pi} \left(\frac{g_s}{V^{5/6}}\right)^{1/4} M_P \sim 5 \cdot 10^{15} \text{GeV}, \quad (3.52)$$

as can be deduced from Table 6 which summarises the different inflationary scales obtained for the models SV1 and SV2 with smaller values for the overall volume. These results were used above in Table 5 to determine the correlation between observables and reheat temperature.

|       | SV1       | SV2       |
|-------|-----------|-----------|
| $C_2$ | 5157.35   | 9946.73   |
| $\langle V \rangle$ | 1709.55 | 1626.12 |
| $N_{e_{max}}$ | 64      | 64        |
| $M_{inf}$ | $5.5 \cdot 10^{15}$ | $6.8 \cdot 10^{15}$ |

**Table 6:** Inflationary scales for models with large $r$.

We have seen that although the Fibre Inflation mechanism can naturally produce inflation with detectable tensor modes if the moduli start at large enough values for $\hat{\varphi}$ (*i.e.* high-fibre models), the generic such model (*e.g.* the LV model of the Tables) predicts too small a Hubble scale during inflation to have observable fluctuations. Such models may nonetheless ultimately prove to be of interest, either by using alternative mechanisms to generate perturbations, or as a way to generate a second, shorter and relatively late epoch of inflation [37] (as might be needed to eliminate relics in the later universe).
3.4 Two-field cosmological evolution

Given that the resulting volumes, $V \gtrsim 10^3$, are not extremely large, one could wonder whether the approximations made above are fully justified or not. We pause now to re-examine in particular the assumption that $V$ and $\tau_3$ remain fixed at constant values while $\tau_1$ rolls during inflation. We first identify the combination of parameters that controls this approximation, and then re-analyse the slow roll with these fields left free to move. This more careful treatment justifies our use of the single-field approximation elsewhere.

3.4.1 Inflaton back-reaction onto $V$ and $\tau_3$

Recall that the approximation that $V$ and $\tau_3$ not move is justified to the extent that the $\tau_1$-independent stabilising forces of the potential $\delta V_{(\alpha')}^\prime$ remain much stronger than the forces in $\delta V_{(g_s)}$ that try to make $V$ and $\tau_3$ also move. And this hierarchy of forces seems guaranteed to hold because of the small factors of $g_s$ and $1/V^{1/3}$ that suppress the string-loop contribution relative to the $\alpha'$ corrections. However we also see, from (3.51) and Table 4, that observably large primordial fluctuations preclude taking $g_s^4/V^{10/3}$ to be too small – at least when they are generated by the standard mechanism. This implies a tension between the COBE normalization and the validity of our analysis at fixed $V$, whose severity we now try to estimate.

Since the crucial issue is the relative size of the forces on $V$ due to $\delta V_{(\alpha')}$, $\delta V_{(sp)}$ and $\delta V_{(g_s)}$, we first compare the derivatives of these potentials. Keeping in mind that it is the variable $\vartheta_v \sim \ln V$ that satisfies the slow-roll condition, we see that the relevant derivative to be compared is $V \partial/\partial V$. Furthermore, since it is competition between derivatives of $\delta V_{(sp)}$ and $\delta V_{(\alpha')}$ in eq. (3.11) that determines $V$ in the leading approximation, it suffices to compare the string-loop potential with only the $\alpha'$ corrections, say. We therefore ask when

$$\left| V \frac{\partial \delta V_{(g_s)}}{\partial V} \right| \ll \left| V \frac{\partial \delta V_{(\alpha')}}{\partial V} \right| ,$$

or when

$$\frac{10C_2}{V^{10/3}} \ll \frac{9 \xi W_0^2}{4 g_s^{3/2} V^3} ,$$

where we take $3 \gg 4 e^{-\kappa \hat{\phi}/2}$ during inflation when simplifying the left-hand side. Grouping terms we find the condition

$$R_{cv} := \left( \frac{9 \xi W_0^2}{4 g_s^{3/2}} \right) \frac{V^{1/3}}{C_2} \simeq \left( \frac{9 \xi \zeta^{4/3}}{4 g_s^{3/2}} \right) \frac{V^{1/3}}{A} \gg 1 ,$$

which is clearly satisfied if we can choose $g_s$ and $1/V^{1/3}$ to be sufficiently small. The value for $R_{cv}$ predicted by the benchmark models of Tables 1 and 2 is given in Table 4. This Table shows that large $R_{cv}$ is much larger in large-$V$ models, as expected, with $R_{cv} > 10^3$ in the LV model. By contrast, $R_{cv} \geq 10$ for inflationary parameter choices (SV1 and SV2) that satisfy the COBE normalisation. Although these are large, the incredible finickiness of inflationary constructions leads us, in the next section, to study the multi-field problem where the volume modulus is free to roll in addition to the inflaton. Be doing so we hope to widen the parameter space of acceptable inflationary models.
3.4.2 Relaxing the Single-Field Approximation

In this section we redo the inflationary analysis without making the single-field approximation. We start from the very general scalar potential, whose form is displayed in Figure 9:

\[ V = \mu_1 \sqrt[3]{\tau_3} e^{-2a_3\tau_3} - \mu_2 W_0 \frac{\tau_3 e^{-a_3\tau_3}}{\sqrt{2}} + \mu_3 \frac{W_0^2}{\sqrt{3}} + \delta_{up} + \frac{D}{\sqrt{\tau_3}} \left( \frac{A}{\sqrt{\tau_1}} - \frac{B}{\sqrt{\tau_1}} + \frac{C \tau_1}{\sqrt{\tau_1}} \right) \frac{W_0^2}{\sqrt{2}}. \]

(3.56)

Here

\[ \mu_1 \equiv \frac{8a_3^2 A_3^2}{3\alpha\gamma}, \quad \mu_2 \equiv 4a_3 A_3, \quad \mu_3 \equiv \frac{3\xi}{2g_s^{3/2}}. \]

(3.57)

Recall that the correction proportional to \( D \) does not depend on \( \tau_1 \) which is mostly the inflaton, but it can change the numerical value obtained for \( \tau_3 \) and \( V \) at the minimum. However, for \( D = g_s^2 \left( L_3^{KK} \right)^2 \ll 1 \) this modification is negligible. Thus we set \( D = 0 \) from now on.

![Figure 9: Two views of the inflationary trough representing the potential as a function of the volume and \( \tau_1 \) for \( R = 0 \). The rolling is mostly in the \( \tau_1 \) direction (‘north-west’ direction in the left-hand figure and ‘south-west’ direction in the right-hand figure).](image)

The result for \( V \) obtained by solving \( \partial V / \partial \tau_3 = 0 \), in the limit \( a_3 \tau_3 \gg 1 \), reads

\[ V = \frac{2\mu_2 W_0}{\mu_1} \sqrt[3]{\tau_3} \left( \frac{1 - a_3\tau_3}{1 - 4a_3\tau_3} \right) e^{a_3\tau_3} \simeq \frac{\mu_2 W_0}{2\mu_1} \sqrt[3]{\tau_3} e^{a_3\tau_3}. \]

(3.58)

Solving eq. (3.58) for \( \tau_3 \), we obtain the result

\[ a_3 \tau_3 \simeq a_3 \tau_3 + \ln \left( \frac{\sqrt{\tau_3}}{2} \right) := \ln (eV), \]

(3.59)

where \( c = 2a_3 A_3 / (3\alpha \gamma W_0) \). Here the first approximate equality neglects the slowly-varying logarithmic factor, bearing in mind that in most of our applications we find \( \sqrt{\tau_3} \simeq 2 \). Using
this to eliminate $\tau_3$ in (3.56) then gives the following approximate potential for $\mathcal{V}$ and $\tau_1$

$$V = \left[ -\mu_4 (\ln (c \mathcal{V}))^{3/2} + \mu_3 \right] \frac{W_0^2}{\mathcal{V}^3} + \delta_{up} \frac{\mathcal{W}_a}{\mathcal{V}^{3/2}} + \left( \frac{A}{\tau_1^2} - \frac{B}{\mathcal{V}^{1/2}} + \frac{C \tau_1}{\mathcal{V}^2} \right) \frac{W_0^2}{\mathcal{V}^2},$$

(3.60)

where $\mu_4 = \frac{3}{2} \alpha a^{-3/2}$.

Given that we set $\tau_3$ at its minimum, $\partial_\mu \tau_3 = 0$, and so the non canonical kinetic terms look like (3.23). In order now to study inflation, we let the two fields $\mathcal{V}$ and $\tau_1$ evolve according to the cosmological evolution equations for non-canonically normalised scalar fields:

$$\left\{ \begin{array}{l}
\ddot{\varphi}^i + 3H \dot{\varphi}^i + \Gamma^i_{jk} \dot{\varphi}^j \varphi^k + g^{ij} \frac{\partial V}{\partial \varphi^j} = 0, \\
H^2 = \left( \frac{a}{\mathcal{V}} \right)^2 = \frac{1}{3} \left( \frac{1}{2} g_{ij} \dot{\varphi}^i \dot{\varphi}^j + V \right),
\end{array} \right.$$  

(3.61)

where $\varphi_i$ represents the scalar fields ($\mathcal{V}$ and $\tau_1$ in our case), $a$ is the scalar factor, and $\Gamma^i_{jk}$ are the target space Christoffel symbols using the metric $g_{ij}$ for the set of real scalar fields $\varphi^i$ such that $\frac{\partial^2 K}{\partial \Phi^{ij} \partial \Phi^{kl} \partial \Phi^{rs}} = \frac{1}{2} g_{ij} \partial^r \varphi^l \partial^s \varphi$. For numerical purposes it is more convenient to write down the evolution of the fields as a function of the number $N_e$ of $e$-foldings rather than time. Using

$$a(t) = e^{N_e}, \quad \frac{d}{dt} = H \frac{d}{dN_e},$$

(3.62)

we avoid having to solve for the scale factor, instead directly obtaining $\mathcal{V}(N_e)$ and $\tau_1(N_e)$. The equations of motion are (with ' denoting a derivative with respect to $N_e$):

$$\tau_1'' = - (\mathcal{L}_{kin} + 3) \left( \tau_1' + 2 \tau_1 \frac{V_{\tau_1}}{V} + \tau_1 V_{\mathcal{V}} \frac{V_{\mathcal{V}}}{V} \right) + \frac{\tau_{1}^2}{\tau_1},$$

$$\mathcal{V}'' = - (\mathcal{L}_{kin} + 3) \left( V' + \tau_1 V_{\tau_1} \frac{V_{\tau_1}}{V} + 3 \frac{V_{\mathcal{V}}^2}{2} \frac{V_{\mathcal{V}}}{V} \right) + \frac{V''}{V},$$

(3.63)

We shall focus on the parameter case SV2, for which a numerical analysis of the full potential gives:

$$\langle \mathcal{V} \rangle = 1413.26, \quad \langle \tau_1 \rangle = 6.77325, \quad \delta_{up} = 0.082.$$  

(3.64)

To evaluate the initial conditions, we fix $\tau_1 \gg \langle \tau_1 \rangle$ and then we work out numerically the minimum in the volume direction $\langle \mathcal{V} \rangle = \langle \mathcal{V} \rangle(\tau_1)$. Notice that, in general, in the case of unwarped up-lifting $\frac{\delta_{up}}{V^{1/2}}$, the volume direction develops a run-away for large $\tau_1$, whereas the potential is well behaved for the case with warped up-lifting $\frac{\delta_{up}}{V^{1/2}}$. Thus we set the following initial conditions:

$$\tau_1(0) = 5000 \Rightarrow \mathcal{V}(0) = \langle \mathcal{V} \rangle(\tau_1 = 5000) = 1841.25, \quad \tau_1'(0) = 0, \quad \mathcal{V}'(0) = 0.$$  

(3.65)

We need to check now that for this initial point we both get enough $e$-foldings and the spectral index is within the allowed range. In order to do this, we start by recalling the generalisation of the slow-roll parameter $\varepsilon$ in the two-field case:

$$\varepsilon = - \frac{(V_{\tau_1} \tau_1 + V_\mathcal{V} \mathcal{V})^2}{4 \mathcal{L}_{kin} V^2},$$

(3.66)
and so it becomes a function of the number of $e$-foldings. In the case SV2, $\varepsilon \ll 1$ for the first 65 $e$-foldings as it is shown in figure 10 below.

However $\varepsilon$ grows faster during the last 5 $e$-foldings until it reaches the value $\varepsilon \simeq 0.765$ at $N = 70$ at which point the slow-roll approximation ceases to be valid and inflation ends. This can be seen in figure 10. (From here on we save $N_e$ to refer to the physical number of $e$-foldings, and denote by $N$ the variable that parameterises the cosmological evolution of the fields).

Therefore focusing on horizon exit at 58 $e$-foldings before the end of inflation, we need to start at $N = 12$. We also find numerically that at horizon exit $\varepsilon(N = 12) = 0.0002844$ which corresponds to a tensor-to-scalar ratio $r = 4.6 \cdot 10^{-3}$. Figure 11 shows the cosmological evolution of the two fields during the last 58 $e$-foldings of inflation before the fields start oscillating around the minimum. It is clear how the motion is mostly along the $\tau_1$ direction, as expected.

Figure 10: $\varepsilon$ versus $N$ for (left) the first 65 $e$-foldings of inflation and (right) the last 5 $e$-foldings.

Figure 11: $\tau_1$ (red curve) and $\mathcal{V}$ (blue curve) versus $N$ for the last 58 $e$-foldings of inflation.
Figure 12 gives a blow-up of the $\tau_1$ and $V$ trajectory close to the minimum for the last 2 $e$-foldings of inflation, where it is evident how the fields oscillate before sitting at the minimum.

![Figure 12: Plot of the $\tau_1$ (red curve on the left) and $V$ (blue curve on the right) vs $N$ for the last 2 $e$-foldings of inflation.](image)

Finally, figure 13 illustrates the path of the inflation trajectory in the $\tau_1$-$V$ space.

![Figure 13: Path of the inflation trajectory in the $\tau_1$-$V$ space for the last 58 (left) and 12 (right) $e$-foldings of inflation.](image)

To consider the experimental predictions of Fibre Inflation we need to make sure that the inflaton is able to generate the correct amplitude of density fluctuations. After multiplying the scalar potential (3.60) by the proper normalisation factor $g_s^4/(8\pi)$, the COBE normalisation on the power spectrum of scalar density perturbations is given by

$$\sqrt{P} \equiv \frac{g_s^4}{20\sqrt{3}\pi^{3/2}} \sqrt{\frac{V}{\varepsilon}} = 2 \cdot 10^{-5},$$

(3.67)
where both $V$ and $\varepsilon$ have to be evaluated at horizon exit for $N = 12$ corresponding to $N_e = 58$. We find numerically that the COBE normalisation is perfectly matched:

at $N = 12$: $\tau_1 = 3710.5$, $V = 1832.74$, $\Rightarrow V = 6.1 \cdot 10^{-7} \Rightarrow \sqrt{P} = 2.15 \cdot 10^{-5}$.

We need also to evaluate the spectral index which is defined as

$$n_s = 1 + \frac{d \ln P(k)}{d \ln k} \simeq 1 + \frac{d \ln P(N)}{d N},$$

(3.68)

where the latter approximation follows from the fact that $k = aH \simeq H e^H$ at horizon exit, so $d \ln k \simeq dN$. In figure 14 we plot the spectral index versus $N$ around horizon exit, namely between 65 and 44 $e$-foldings before the end of inflation. It turns out that $n_s(N = 12) = 0.96993$, and so our starting point is within the experimentally allowed region for the spectral index.

![Figure 14](image_url)

*Figure 14:* Left: $n_s$ versus $N$ between 65 and 44 $e$-foldings before the end of inflation. Right: $\eta$ versus $N$ during the last 58 $e$-foldings of inflation.

We also checked that the second slow-roll parameter $\eta$, obtainable from $\eta = (n_s + 6\varepsilon - 1)/2$, is always less than unity during the last 58 $e$-foldings as shown in figure 14 below. It is interesting to notice that $\eta$ vanishes very close to the end of inflation for $N = 69.88$. This is perfect agreement with the presence of the inflection point previously found in the fixed-volume approximation.

The inflationary scale evaluated at the end of inflation turns out to be

$$M_{inf} = \frac{V_{end}}{\sqrt{M_P}} = V(N = 70)^{1/4}M_P = 5.2 \cdot 10^{16} GeV,$$

(3.69)

and so, using (3.50) for $w = 0$, we deduce that we can obtain $N_e = 58$ if $T_{rh} = 2.27 \cdot 10^9 GeV$ which is correctly below $10^{10} GeV$ to solve the gravitino problem. Finally we conclude that we end up with the following experimental predictions:

$$n_s \simeq 0.970, \quad r \simeq 4.6 \cdot 10^{-3},$$

(3.70)

in agreement with our earlier single-field results.
3.5 Naturalness

Finally, we return to the issue of the stability of the inflationary scenario presented here to various kinds of perturbations, and argue that it is much more robust than are generic inflationary mechanisms because of the control afforded by the LARGE volume approximation.\footnote{We thank Liam McAllister for several helpful conversations on this point.}

There are several reasons why inflationary models are generically sensitive to perturbations of various kinds, of which we list several.

*Dimension-six Operators and the $\eta$ problem:*

A generic objection to the stability of an inflationary scenario rests on the absence of symmetries protecting scalar masses. This line of argument\footnote{We thank Liam McAllister for several helpful conversations on this point.} grants that it is possible to arrange a regime where the scalar potential is to a good approximation constant, $V = V_0$, chosen to give the desired inflationary Hubble scale, $3M_p^2 H^2 = V_0$. It then asks whether there are dangerous higher-dimension interactions in the effective theory that are small enough to allow an effective field theory description, but large enough to compete with the extraordinarily flat inflationary potential.

In particular, since $V_0$ is known (by assumption) not to be precluded by symmetries of the problem, and since scalar masses are notoriously difficult to rule out by symmetries, one worries about the possibility of the following dimension-six combination of the two:

$$\mathcal{L}_{\text{eff}} = \frac{1}{M^2} V_0 \varphi^2,$$

where $\varphi$ is the canonically normalised inflaton and $M$ is a suitable heavy scale appropriate to any heavy modes that have been integrated out. Such a term is dangerous, even with $M \sim M_p$, because it contributes an amount $V_0/M_p^2 \sim H^2$ to the inflaton mass, corresponding to $\delta \eta \sim \mathcal{O}(1)$. Supersymmetric versions of this argument use the specific form $V_F = e^K U$, where $U$ is constructed from the superpotential, and argue that inflation built on regions of approximately constant $U$ get destabilised by generic $\delta K \sim \varphi^* \varphi$ corrections to the Kähler potential.

A related question that is specific to the large-field models required for large tensor fluctuations asks what controls the expansion of the effective theory in powers of $\varphi$ if fields run over a range as large as $M_p$.

We believe that neither of these problems arise in the Fibre Inflation models considered here. First, both arguments rely on generic properties of an expansion in powers of $\varphi$, which is strictly a valid approximation only for small excursions about a fixed point in field space. As the previous paragraph points out, such an expansion cannot be used for large field excursions and one must instead identify a different small parameter with which to control calculations. In the present instance this small parameter is given both by powers of $1/V$ and by powers of $g_s$, since these control the underlying string perturbation theory and low-energy approximations. In particular, as we find explicitly in Appendix \ref{appendix}, in the supersymmetric context these ensure that perturbations to $K$ have the form
\[ \delta e^K \simeq \delta(1/V^2) \simeq -2\delta V/V^3, \] and it is the suppression by additional powers of the LARGE volume that makes such corrections less dangerous than they would generically be.

Normally when dangerous corrections are suppressed by a small expansion parameter, the suppression can be traced to additional symmetries that emerge in the limit that the parameter vanishes. But for large-volume expansions, the corrections vanish strictly in the de-compactification limit, \( V \to \infty \), which does enjoy many symmetries (like higher-dimensional general covariance) that are not evident in the effective lower-dimensional theory. It would be worth understanding in more detail whether the natural properties of the large-volume expansion can be traced to these additional symmetries of higher dimensions.

**Integrating out sub-Planckian modes:**

There is a more specific objection, related to the above. Given the potential sensitivity of inflation to higher-dimension operators, this objection asks why the inflaton potential is not destabilised by integrating out the many heavy particles that are likely to live above the inflationary scale, \( M_I \simeq V^{1/4} \) and below the Planck scale? (See, for instance, ref. [38] for more specific variants of this question.)

In particular, for string inflation models one worries about the potential influence of virtual KK modes, since these must be lighter than the string and 4D Planck scales, and generically couple to any inflaton field. For Fibre Inflation this question can be addressed fairly precisely, since virtual KK modes are included in the string loop corrections that generate the inflationary potential in the first place.

There are generically two ways through which loops can introduce the KK scale into the low-energy theory. First, the lightest KK masses enter as a cutoff for the virtual contribution of the very light states that can be studied purely within the 4D effective theory. These states contribute following generic contributions to the inflaton potential

\[
\delta V_{inf}^{4D} \simeq c_1 \text{STr} M^4 + c_2 m_{3/2}^2 \text{STr} M^2 + \cdots, \tag{3.72}
\]

where \( c_1 \) and \( c_2 \) are dimensionless constants, the gravitino mass, \( m_{3/2} \), measures the strength of supersymmetry breaking in the low-energy theory, and the super-traces are over powers of the generic 4D mass matrix, \( M \), whose largest elements are of order the KK scale, \( M_{KK} \). In general low-energy supersymmetry ensures \( c_1 = 0 \), making the second term the leading contribution.

Now comes the important point. In the LARGE volume models of interest, we know that \( m_{3/2} \sim V^{-1} \), and we know that \( M_{KK} \) is suppressed relative to the string scale by \( V^{-1/6} \), and so in Planck units \( M_{KK} \sim \sqrt{V}^{-2/3} \). These together imply that \( \delta V_{inf}^{4D} \sim V^{-10/3} \), in agreement with the volume-dependence of the loop-generated inflationary potential discussed above.

But \( \delta V_{inf} \) also potentially receives contributions from scales larger than \( M_{KK} \) and these cannot be described by the 4D loop formula, eq. (3.72). These must instead be computed using the full higher-dimensional (string) theory, potentially leading to the dangerous effective interactions in the low-energy theory. This calculation is the one that is explicitly performed for torii in [22] and whose properties were estimated more generally in [20, 21].
Their conclusion is that such effective contributions do arise in the effective 4D theory, appearing there as contributions to the low-energy Kähler potential. The contributions from open-string loops wrapped on a cycle whose volume is \( \tau \) have the generic form

\[
\delta K \simeq \frac{1}{V} \left[ a_1 \sqrt{\tau} + \frac{a_2}{\sqrt{\tau}} + \cdots \right],
\]  

(3.73)

where it is \( 1/\tau \) that counts the loop expansion.

These two terms can potentially give contributions to the scalar potential, and if so these would scale with \( V \) in the following way,

\[
\delta V_{\text{inf}} \sim a_1 \frac{1}{V^{8/3}} + a_2 \frac{1}{V^{10/3}} + \cdots.
\]  

(3.74)

Notice that the first term is therefore potentially dangerous, scaling as it does like \( M_{KK}^4 \). However a simple calculation shows that the contribution of a term \( \delta K \propto \tau^{\omega}/V \) gives a contribution to \( V_{\text{inf}} \) of the form \( \delta V_{\text{inf}} \propto (\omega - \frac{1}{2}) V^{-8/3} \) [21], implying that the leading correction to \( K \) happens to drop out of the scalar potential (although it does contribute elsewhere in the action).

These calculations show how LARGE volume and 4D supersymmetry can combine to keep the potentially dangerous loop contributions of KK and string modes from destabilising the inflaton potential. We regard the study of how broadly this mechanism might apply elsewhere in string theory as being well worthwhile.

4. Conclusions

This year the Planck satellite is expected to start a new era of CMB observations, and to be joined over the next few years by other experiments aiming to measure the polarization of the cosmic microwave background and to search for gravitational waves. We have presented a new class of explicit string models, with moduli stabilisation, that both agrees with current observations and can predict observable gravitational waves, most probable not at Planck but at future experiments. Many of the models’ inflationary predictions are also very robust against changes to the underlying string/supergravity parameters, and in particular predict a definite correlation between the scalar spectral index, \( n_s \), and tensor-to-scalar ratio, \( r \). It is also encouraging that these models realise inflation in a comparatively natural way, inasmuch as a slow roll does not rely on fine-tuning parameters of the potential against one another.

Other important features of the model are:

- The comparative flatness of the inflaton direction, \( \chi \), is guaranteed by general features of the modulus potential that underly the LARGE volume constructions. These ultimately rely on the no-scale structure of the lowest-order Kähler potential and the fact that the leading \( \alpha' \) corrections depend on the Kähler moduli only through the Calabi-Yau volume.
• The usual $\eta$ problem of generic supergravity theories is also avoided because of the special features of the no-scale LARGE-volume structure. In particular, the expansion of the generic $e^K = \mathcal{V}^{-2}$ factor of the $F$-term potential are always punished by the additional powers of $1/\mathcal{V}$, which they bring along: $\delta e^K = -2\mathcal{V}^{-3}\delta\mathcal{V}$. This result is explicitly derived in Appendix A.

• The exponential form of the inflationary potential is a consequence of two things. First, the loop corrections to $K$ and $V$ depend on powers of $X$ and the volume. And second, the leading-order Kähler potential gives a kinetic term for $X$ of the form $(\partial \ln X)^2$, leading to the canonically normalised quantity $\varphi$, with $X = e^{\kappa \hat{\varphi}}$, with $\kappa = 2/\sqrt{3}$. So we know the potential can have a typical large-field inflationary form, $V = K_1 - K_2 e^{-\kappa_1 \hat{\varphi}} + K_3 e^{-\kappa_2 \hat{\varphi}} + \cdots$, without knowing any details about the loop corrections.

• The robustness of some of the predictions then follows because the coefficients $K_i$ turn out to be proportional to one another. They are proportional because of our freedom to shift $\varphi$ so that $\hat{\varphi} = 0$ is the minimum of $V$, and our choice to uplift this potential so that it vanishes at this minimum. The two conditions $V(0) = V'(0) = 0$ impose two conditions amongst the three coefficients $K_1$, $K_2$ and $K_3$ (where three terms in the potential are needed to have a minimum). The remaining normalisation of the potential can then be expressed without loss of generality in terms of the squared mass, $m^2_{\varphi} = V''(0)$.

• The exact range of the field $\hat{\varphi}$ depends only on the ratio of two parameters ($B/A$) of the underlying supergravity. This quantity is typically much greater than one due to the string coupling dependence of this ratio, leading to ‘high-fiber’ models for which $\hat{\varphi}$ can naturally run through trans-Planckian values. $B/A \gg 1$ also suffices to ensure that the minimum $\langle \varphi \rangle$ lies inside the Kähler cone. But the range of $\hat{\varphi}$ also cannot be too large, since it depends only logarithmically on $B/A$. This implies that $\hat{\varphi}$ at most rolls through a few Planck scales, which can allow $50 - 60$ e-foldings, or even a bit more. This makes the models potentially sensitive to details of the modulus dynamics at horizon exit, along the lines of [38], since this need not be deep in an inflationary regime.

• The COBE normalisation is the most constraining restriction to the underlying string/supergravity parameters. In particular, as usual, it forbids the volume from being very large because it restricts the string scale to be of the order of the GUT scale. This leads to the well known tension between the scale of inflation and low-energy supersymmetry [4]. Of course, this conclusion assumes the standard production mechanism for primordial density fluctuations, and it remains an interesting open question whether alternative mechanisms might allow a broader selection of inflationary models in this class. In particular, this makes the development of a reheating mechanism particularly pressing for this scenario.
The model is extremely predictive since the requirement of generating the correct amplitude of scalar perturbations fixes the inflationary scale of the order the GUT scale, which, in turn, fixes the numbers of $e$-foldings. Lastly the number of $e$-foldings is correlated with the cosmological observables and we end up with the general prediction: $n_s \simeq 0.970$ and $r \simeq 0.005$. We find examples with $r \simeq 0.01$ and $n_s \simeq 1$ also to be possible, but only if horizon exit occurs very soon after the onset of inflation.

For these reasons, even though the string-loop corrections to the Kähler potential are not fully known for general Calabi-Yau manifolds, because they come as inverse powers of Kähler moduli and the dilaton we believe the results we find here are likely to be quite generic. Of course, it would in any case be very interesting to have more explicit calculations of the loop corrections to Kähler potentials in order to better understand this scenario. Furthermore even though blow-up modes are very common for Calabi-Yau manifolds, it would be useful to have explicit examples of K3 fibration Calabi-Yau manifolds with the required intersection numbers.

During Fibre Inflation an initially large K3 fiber modulus $\tau_1$ shrinks, with the volume $V = t_1 \tau_1$ approximately constant. Consequently, the value of the 2-cycle modulus $t_1$, corresponding to the base of the fibration, must grow during inflation. This forces us to check that $t_1$ is not too small at the start of inflation, in particular not being too close to the singular limit $t_1 \to 0$ where perturbation theory breaks down. We show in appendix B that the inflationary region can start sufficiently far away from this singular limit. The more restrictive limit on the range of the inflationary regime is the breakdown of the slow-roll conditions as $t_1$ gets smaller, arising due to the growth of a positive exponentials in the potential when expressed using canonical variables. One can nonetheless show that natural choices of the underlying parameters can guarantee that enough $e$-foldings of inflation are achieved before reaching this region of field space.

It is worth emphasising that, independent of inflation and as mentioned in section 3.2, we have also shown that our scenario allows for the LARGE volume to be realised in such a way that there is a hierarchy of scales in the Kähler moduli, allowing the interesting possibility of having two dimensions much larger than the rest and making contact with the potential phenomenological and cosmological implications of two large extra dimensions scenarios.[40, 41]

We do not address the issues of initial conditions, which in our case ask why the other fields start initially near their minimum, and why inflationary modulus should start out high up a fiber. As for Kähler modulus inflation, one argument is that any initial modulus configuration must evolve towards its stabilised value, and so if the last modulus to reach its minimum happens to be a fibre modulus we expect this inflationary mechanism to be naturally at work.
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A. Higher order corrections to the inflationary potential

In this appendix we derive explicitly the leading corrections to the fixed-volume approximation, which give rise to higher order operators. We show that these operators do not introduce an $\eta$ problem since they are suppressed by inverse powers of the overall volume.

A.1 Derivation of the $\tau_1$ dependent shift of $\langle \mathcal{V} \rangle$

We start from the very general scalar potential (3.60):

$$V = \left[ -\mu_4 (\ln (c \mathcal{V}))^{3/2} + \mu_3 \right] \frac{W_0^2}{\mathcal{V}^3} + \delta_{up} \frac{V^2}{\mathcal{V}^2} + \left( \frac{A}{\tau_1^2} - \frac{B}{\mathcal{V} \sqrt{\tau_1}} \right) \frac{W_0^2}{\mathcal{V}^2}, \quad (A.1)$$

where we have set $C = 0$ since the loop corrections proportional to $C$ turn out to be numerically small in the cases of interest, both to finding the minimum in $\tau_1$ and to the inflationary region. We now minimise this potential to obtain $\langle \mathcal{V} \rangle$, first turning off the loop potential to investigate how the uplifting term changes the minimum for $\mathcal{V}$. We follow this by a perturbative study of the additional $\tau_1$-dependence generated by the loop corrections: $\langle \mathcal{V} \rangle = \mathcal{V}_0 + \delta \mathcal{V}(\tau_1)$.

Uplifting only

In the absence of loop corrections the potential reads

$$V = \left[ -\mu_4 (\ln (c \mathcal{V}))^{3/2} + \mu_3 \right] \frac{W_0^2}{\mathcal{V}^3} + \frac{\delta_{up}}{\mathcal{V}^2}, \quad (A.2)$$

where the up-lifting term is chosen to ensure

$$\langle \mathcal{V} \rangle = \left[ -\mu_4 (\ln (c \mathcal{V}_0))^{3/2} + \mu_3 \right] \frac{W_0^2}{\mathcal{V}_0^3} + \frac{\delta_{up}}{\mathcal{V}_0^2} = 0, \quad (A.3)$$

and so

$$\delta_{up} = \left[ \mu_4 (\ln (c \mathcal{V}_0))^{3/2} - \mu_3 \right] \frac{W_0^2}{\mathcal{V}_0^2}. \quad (A.4)$$
Here $V_0$ satisfies $\partial V/\partial V|_{V_0} = 0$, and so must solve

$$\frac{4\delta_{up} V_0}{\mu_4 W_0^2} + \frac{6\mu_3}{\mu_4} + 3(\ln (cV_0))^{1/2} - 6(\ln (cV_0))^{3/2} = 0. \quad (A.5)$$

This is most simply analysed once it is rewritten as

$$\psi + p (\ln (cV_0))^{1/2} - (\ln (cV_0))^{3/2} = 0, \quad (A.6)$$

with

$$\psi := \frac{\mu_3}{\mu_4} = \frac{\xi}{2\alpha\gamma} \left( \frac{a_3}{g_s} \right)^{3/2}, \quad (A.7)$$

and the parameter $p$ takes the value $p = \frac{3}{2}$ if we evaluate $\delta_{up}$ using $(A.4)$, or $p = \frac{1}{2}$ if we take $\delta_{up} = 0$. Tracking the dependence on $p$ therefore allows us to understand the sensitivity of the result to the presence of the uplifting term.

The exact solution of $(A.6)$ is

$$\ln (cV_0) = \frac{12p + \left( 108\psi + 12\sqrt{81\psi^2 - 12p^2} \right)^{2/3}}{36 \left( 108\psi + 12\sqrt{81\psi^2 - 12p^2} \right)^{2/3}}, \quad (A.8)$$

which approaches the $p$-independent result

$$\ln (cV_0) \simeq \psi^{2/3} = a_3 \left( \frac{\hat{\xi}}{2\alpha\gamma} \right)^{2/3}, \quad (A.9)$$

when $\psi \gg 1$, in agreement with eq. (3.59) together with expression (3.12) for $\tau_3$. This shows that we may expect the uplifting corrections to $V_0$ to be small when $\psi \gg 1$.

**Including loop corrections**

The potential now is given by $(A.1)$ and so the presence of the loops will generate a $\tau_1$ dependent shift of $V$ such that

$$V = V_0 + \delta V(\tau_1), \quad \text{with} \quad \delta V(\tau_1) \ll V_0 \forall \tau_1. \quad (A.10)$$

In order to calculate $\delta V(\tau_1)$ at leading order, let us solve the minimisation equation for the volume taking into account that now that we have turned on the string loops, we need to replace $\delta_{up}$ by $\delta'_{up} = \delta_{up} + \mu_{up}$, where $\mu_{up}$ is the constant needed to cancel the contribution of the loops to the cosmological constant.

$$\frac{\partial V}{\partial V} = 0 \iff \frac{4AV}{\mu_4 \tau_1^2} - \frac{6B}{\mu_4 \sqrt{\tau_1}} + 4 \frac{\delta'_{up} V}{\mu_4 W_0^2} + 6\frac{\mu_3}{\mu_4} + 3(\ln (cV))^{1/2} - 6(\ln (cV))^{3/2} = 0. \quad (A.11)$$

We notice that the logarithm in the previous expression can be expanded as follows

$$\ln (cV) = \ln (cV_0) + \frac{\delta V(\tau_1)}{V_0}. \quad (A.12)$$
and by means of another Taylor series and the result (A.3), we are left with
\[
\left(4 \frac{\delta_{up} V_0}{\mu_4 W_0^2} + 4 \frac{\mu_{up} V_0}{\mu_4 W_0^2} + 4 A V_0 + 3 \frac{1}{2} \frac{1}{(\ln (c V_0))^{-1/2} - 9 (\ln (c V_0))^{1/2}}\right) \frac{\delta V(\tau_1)}{V_0} = \\
- 4 A V_0 \frac{1}{\mu_4 W_0^2} + \frac{6 B}{\mu_4 \sqrt{\tau_1}} - 4 \frac{\mu_{up} V_0}{\mu_4 W_0^2}.
\] (A.13)

Now recalling the expression (A.4) for \(\delta_{up}\) combined with (A.6), we obtain
\[
\frac{\delta V(\tau_1)}{V_0} = \frac{\left(4 A V_0 - \frac{6 B}{\mu_4 \sqrt{\tau_1}} + 4 \frac{\mu_{up} V_0}{\mu_4 W_0^2}\right)}{\left(3 (\ln (c V_0))^{1/2} - \frac{3}{2} (\ln (c V_0))^{-1/2} - 4 A V_0 \frac{1}{\mu_4 W_0^2} - 4 \frac{\mu_{up} V_0}{\mu_4 W_0^2}\right)}.
\] (A.14)

We can still expand the denominator in (A.14) and working at leading order we end up with
\[
\frac{\delta V(\tau_1)}{V_0} = \frac{\left(4 A V_0 - \frac{6 B}{\mu_4 \sqrt{\tau_1}} + 4 \frac{\mu_{up} V_0}{\mu_4 W_0^2}\right)}{\left(3 (\ln (c V_0))^{1/2} - \frac{3}{2} (\ln (c V_0))^{-1/2}\right)}.
\] (A.15)

We have now all the ingredients to work out the canonical normalisation.

### A.2 Canonical normalisation

As we have seen in the previous subsection of this appendix, \(\mathcal{V}\) and \(\tau_3\) will both have a \(\tau_1\) dependent shift of the form
\[
\mathcal{V} = V_0 + \delta \mathcal{V}(\tau_1),
\] (A.16)
\[
\tau_3 = \frac{\ln (c V_0)}{a_3} + \frac{\delta \mathcal{V}(\tau_1)}{a_3 V_0},
\] (A.17)

which will cause \(\partial_\mu \mathcal{V}\) and \(\partial_\mu \tau_3\) not to vanish when we study the canonical normalisation of the inflaton field \(\tau_1\) setting both \(\mathcal{V}\) and \(\tau_3\) at its \(\tau_1\) dependent minimum. Thus we have
\[
\partial_\mu \mathcal{V} = \frac{\partial (\delta \mathcal{V}(\tau_1))}{\partial \tau_1} \partial_\mu \tau_1,
\] (A.18)
\[
\partial_\mu \tau_3 = \frac{1}{a_3 V_0} \frac{\partial (\delta \mathcal{V}(\tau_1))}{\partial \tau_1} \partial_\mu \tau_1.
\] (A.19)

The non canonical kinetic terms look like
\[
- \mathcal{L}_{\text{kin}} = \frac{1}{4} \frac{\partial^2 K}{\partial \tau_i \partial \tau_j} \partial_\mu \tau_i \partial_\mu \tau_j
\]
\[
= \frac{3}{8 \tau_1^3} \left(1 - \frac{2 \alpha \gamma \tau_3^{3/2}}{3} \right) \partial_\mu \tau_1 \partial_\mu \tau_1 - \frac{1}{2 V \tau_1} \left(1 - \alpha \gamma \tau_3^{3/2} \right) \partial_\mu \tau_1 \partial_\mu \mathcal{V}
\]
\[
+ \frac{1}{2 V \tau_1} \partial_\mu \mathcal{V} \partial_\mu \mathcal{V} - \frac{3 \alpha \gamma \tau_3}{2 V^2} \partial_\mu \tau_3 \partial_\mu \mathcal{V} + \frac{3 \alpha \gamma}{8 V \sqrt{\tau_3}} \partial_\mu \tau_3 \partial_\mu \tau_3
\]
\[
\simeq \frac{3}{8 \tau_1^3} \partial_\mu \tau_1 \partial_\mu \tau_1 - \frac{1}{2 V \tau_1} \partial_\mu \tau_1 \partial_\mu \mathcal{V} + \frac{1}{2 V \tau_1} \partial_\mu \mathcal{V} \partial_\mu \mathcal{V}
\]
\[
- \frac{3 \alpha \gamma \sqrt{\tau_3}}{2 V^2} \partial_\mu \tau_3 \partial_\mu \tau_3 + \frac{3 \alpha \gamma}{8 V \sqrt{\tau_3}} \partial_\mu \tau_3 \partial_\mu \tau_3.
\] (A.20)
Now using (A.18) and (A.19), we can derive the leading order correction to the canonical normalisation in the constant volume approximation:

\[-L_{\text{kin}} = \frac{3}{8\tau_1^2} \left[ 1 - \frac{4\tau_1}{3} \frac{\partial}{\partial \tau_1} \left( \frac{\delta V(\tau_1)}{V_0} \right) \right] \frac{\partial_\mu \tau_1 \partial^\mu \tau_1}{2} = \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi, \tag{A.21}\]

where \(\varphi\) is the canonically normalised inflaton. Writing \(\varphi = g(\tau_1)\) we deduce the following differential equation

\[\frac{\partial g(\tau_1)}{\partial \tau_1} = \frac{\sqrt{3}}{2\tau_1} \sqrt{1 - \frac{4\tau_1}{3} \frac{\partial}{\partial \tau_1} \left( \frac{\delta V(\tau_1)}{V_0} \right)}, \tag{A.22}\]

which, after expanding the square root, admits the straightforward solution

\[\varphi = \frac{\sqrt{3}}{2} \ln \tau_1 - \frac{1}{\sqrt{3}} \left( \frac{\delta V(\tau_1)}{V_0} \right) = \frac{\sqrt{3}}{2} \ln \tau_1 \left[ 1 - \frac{2}{3 \ln \tau_1} \left( \frac{\delta V(\tau_1)}{V_0} \right) \right], \tag{A.23}\]

where the leading order term reproduces what we had in the main text. We still need to invert this relation to get \(\tau_1\) as a function of \(\varphi\) and then plug this result back in the potential. We can write this function as

\[\tau_1 = e^{2\varphi/\sqrt{3}} \left( 1 + h(\varphi) \right), \tag{A.24}\]

where \(h(\varphi) \ll 1\).

At this point we can substitute (A.24) in (A.23) and by means of a Taylor expansion, derive an equation for \(h(\varphi)\):

\[\varphi = \varphi - \varphi \left[ \frac{2}{3 \ln \tau_1} \left( \frac{\delta V(\tau_1)}{V_0} \right) \right]_{\tau_1 = e^{2\varphi/\sqrt{3}}} + \frac{\sqrt{3}}{2} h(\varphi) + \ldots
\]

\[\Rightarrow h(\varphi) = \frac{2}{3} \left( \frac{\delta V(\tau_1)}{V_0} \right)_{\tau_1 = e^{2\varphi/\sqrt{3}}}, \tag{A.25}\]

where we have imposed that the two first order corrections cancel in order to get the correct inverse function. Therefore the final canonical normalisation of \(\tau_1\) which goes beyond the constant volume approximation reads

\[\tau_1 = e^{2\varphi/\sqrt{3}} \left[ 1 + \frac{2}{3} \left( \frac{\delta V(\tau_1)}{V_0} \right) \right]_{\tau_1 = e^{2\varphi/\sqrt{3}}}. \tag{A.26}\]

**A.3 Leading correction to the inflationary slow roll**

In order to derive the full final inflationary potential at leading order, we have now to substitute \(V = V_0 + \delta V(\tau_1)\) in (A.13) to obtain a function of just \(\tau_1\). After two subsequent Taylor expansions, the potential reads

\[V = \left[ -\mu_4 (\ln (c V_0))^{3/2} \left( 1 + \frac{3\delta V(\tau_1)}{2V_0 \ln (c V_0)} \right) + \mu_3 + \frac{\delta_{up} (V_0 + \delta V(\tau_1))}{W_0^2} + \frac{AV}{\tau_1} - \frac{B}{\sqrt{\tau_1}} \right] \frac{W_0^2}{V^3}.\]
Recalling the expression (A.4) for $\delta_{up}$, the leading contribution of the non-perturbative and $\alpha'$ bit of the scalar potential cancels against the up-lifting term and we are left with the expansion of $V_{(np)} + V_{(\alpha')} + V_{(up)}$ plus the loops:

$$V = \left[ -\frac{3\mu_4}{2}(\ln(c\mathcal{V}_0))^{1/2} \frac{\delta \mathcal{V}(\tau_1)}{\mathcal{V}_0} + \frac{\delta_{up} \delta \mathcal{V}(\tau_1)}{W_0^2} + \frac{\mu_{up} \mathcal{V}}{W_0^2} + \frac{A\mathcal{V}}{\tau_1^{1/2}} - \frac{B}{\sqrt{\tau_1}} \right] \frac{W_0^2}{\mathcal{V}^3}. \quad (A.27)$$

It is now very interesting to notice in the previous expression that the leading order expansion of the non-perturbative and $\alpha'$ bit of the potential cancels against the expansion of the up-lifting term. In fact from (A.27), we have that

$$\delta V_{(np)} + \delta V_{(\alpha')} = -\frac{3\mu_4}{2}(\ln(c\mathcal{V}_0))^{1/2} \frac{\delta \mathcal{V}(\tau_1)}{\mathcal{V}_0} \frac{W_0^2}{\mathcal{V}^3}, \quad (A.28)$$

along with

$$\delta V_{(up)} = \frac{\delta_{up} \delta \mathcal{V}(\tau_1)}{\mathcal{V}_0^3} = \frac{3\mu_4}{2}(\ln(c\mathcal{V}_0))^{1/2} \frac{\delta \mathcal{V}(\tau_1)}{\mathcal{V}_0} \frac{W_0^2}{\mathcal{V}^3}, \quad (A.29)$$

where the last equality follows from (A.4) and (A.6). This result was expected since we fine tuned $V_{(up)}$ to cancel $V_{(np)} + V_{(\alpha')}$ at $\mathcal{V} = \mathcal{V}_0$ and then we have applied the same shift $\mathcal{V} = \mathcal{V}_0 + \delta \mathcal{V}(\tau_1)$ to both of them, so clearly still obtaining a cancellation.

Thus we get the following exact result for the inflationary potential

$$V_{inf} = \left[ \frac{\mu_{up} \mathcal{V}}{W_0^2} + \frac{A\mathcal{V}}{\tau_1^{1/2}} - \frac{B}{\sqrt{\tau_1}} \right] \frac{W_0^2}{\mathcal{V}^3}. \quad (A.30)$$

It is now possible to work out the form of $\mu_{up}$. The minimum for $\tau_1$ lies at

$$\langle \tau_1 \rangle = \left( \frac{4A}{B} \mathcal{V} \right)^{2/3}, \quad (A.31)$$

and so by imposing $\langle V_{inf} \rangle = 0$ we find

$$\mu_{up} = \frac{3}{A^{1/3}} \left( \frac{B}{4} \right)^{4/3} \frac{W_0^2}{\mathcal{V}^{4/3}}. \quad (A.32)$$

We can now expand again $\mathcal{V}$ around $\mathcal{V}_0$ and obtain:

$$\mu_{up} = \frac{3}{A^{1/3}} \left( \frac{B}{4} \right)^{4/3} \frac{W_0^2}{\mathcal{V}^{4/3}} \left( 1 - \frac{4}{3} \frac{\delta \mathcal{V}(\tau_1)}{\mathcal{V}_0} \right), \quad (A.33)$$

along with

$$V_{inf} = V^{(0)} + \delta V, \quad (A.34)$$

where

$$V^{(0)} = \left[ \frac{3}{A^{1/3}} \left( \frac{B}{4} \right)^{4/3} \frac{1}{\mathcal{V}_0^{1/3}} + \frac{A\mathcal{V}_0}{\tau_1^{1/2}} - \frac{B}{\sqrt{\tau_1}} \right] \frac{W_0^2}{\mathcal{V}^3}, \quad (A.35)$$

is the inflationary potential derived in the main text in the approximation that the volume is $\tau_1$-independent during the inflationary slow roll, and

$$\delta V = \left( \frac{\delta \mathcal{V}(\tau_1)}{\mathcal{V}_0} \right) \left[ -10 \frac{A}{A^{1/3}} \left( \frac{B}{4} \right)^{4/3} \frac{1}{\mathcal{V}_0^{1/3}} - 2 \frac{A\mathcal{V}_0}{\tau_1^{1/2}} + 3 \frac{B}{\sqrt{\tau_1}} \right] \frac{W_0^2}{\mathcal{V}^3}, \quad (A.36)$$
is the leading order correction to that approximation.

Now that we have an expression for the up-lifting term \( \mu_{up} \) given by (A.33), we are able to write down explicitly the form of the shift of \( V \) due to \( \tau_1 \) (A.13) at leading order:

\[
\frac{\delta V(\tau_1)}{V_0} = \left( \frac{4A V_0}{\mu_4 \tau_1^4} - \frac{6 \delta V}{\mu_4 \sqrt{\tau_1}} + \frac{3B^{4/3}}{\mu_4 (4A)^{1/3}} \frac{1}{\sqrt[3]{V_0}} \right) \left( 3 (\ln (c V_0))^{1/2} - \frac{3}{2} (\ln (c V_0))^{-1/2} \right).
\] (A.37)

Notice that the other possible source of correction to \( V^{(0)} \) is the modification of the canonical normalisation of \( \tau_1 \) due to \( \delta V(\tau_1) \) given by (A.26). Let us therefore evaluate now the contribution coming from this further correction. Working just at leading order, we have to substitute (A.26) in \( V^{(0)} \) and then expand obtaining

\[
V^{(0)} = V^{(0)}_{inf} + \delta V^{(0)},
\] (A.38)

whereas we can just substitute \( \tau_1 = e^{2\phi/\sqrt{3}} \) in \( \delta V \) since an expansion of this term would be subdominant. At the end, we find that

\[
V_{inf} = V^{(0)}_{inf} + \delta V_{inf},
\] (A.39)

where

\[
V^{(0)}_{inf} = \left[ \frac{3}{4A^{1/3}} \left( \frac{B}{4} \right)^{4/3} \frac{1}{\sqrt[3]{V_0}} + A V_0 e^{-4\phi/\sqrt{3}} - B e^{-\phi/\sqrt{3}} \right] \frac{W_0^2}{V_0^3},
\] (A.40)

is the canonically normalised inflationary potential used in the main text in the constant volume approximation. Moreover, \( \delta V^{(0)} \) and \( \delta V \) turn out to have the same volume scaling and so their sum will give the full final leading order correction to \( V^{(0)}_{inf} \):

\[
\delta V_{inf} = \delta V^{(0)} + \delta V,
\]

\[
\delta V_{inf} = -\frac{10}{3} \left( \frac{\delta V(\tau_1)}{V_0} \right) \bigg|_{\tau_1 = e^{2\phi/\sqrt{3}}} V^{(0)}_{inf},
\] (A.41)

Comparing (A.40) with (A.41), we notice the interesting relation

\[
\delta V_{inf} = -\frac{10}{3} \left( \frac{\delta V(\tau_1)}{V_0} \right) \bigg|_{\tau_1 = e^{2\phi/\sqrt{3}}} V^{(0)}_{inf},
\] (A.42)

which implies

\[
V_{inf} = V^{(0)}_{inf} \left[ 1 - \frac{10}{3} \left( \frac{\delta V(\tau_1)}{V_0} \right) \bigg|_{\tau_1 = e^{2\phi/\sqrt{3}}} \right].
\] (A.43)

This last relation shows a special instance of the general mechanism discussed in the main text of how this model avoids the \( \eta \)-problems that normally plague inflationary potentials. In particular, the corrections from the one loop potential is seen to enter in the volume-suppressed combination \( \delta V/V_0 \ll 1 \), ensuring that their contribution to the inflationary parameters \( \varepsilon \) and \( \eta \) is negligible.
B. Loop Effects at High Fibre

In this section we investigate in more detail what happens at the string loop corrections when the K3 fibre gets larger and larger and simultaneously the $CP^1$ base approaches the singular limit $t_1 \to 0$. One’s physical intuition is that loop corrections should signal the approach to this singular point. In fact, we show here that the Kaluza-Klein loop correction in $\tau_2$ is an expansion in inverse powers of $\tau_2$ which goes to zero when $\tau_1 \to \infty \iff t_1 \to 0$, as can be deduced from (3.21). Therefore the presence of the singularity is signaled by the blowing-up of these corrections. We then estimate the value $\tau_1^*$ below which perturbation theory still makes sense and so we can trust our approximation in which we consider only the first term in the 1-loop expansion of $\delta V_{KK}^{1-\text{loop}}$ and we neglect all the other terms of the expansion along with higher loop effects. However it will turn out that, still in a region where $\tau_1 < \tau_1^*$, $\delta V_{KK}^{1-\text{loop}}$, corresponding to the positive exponential in $V$, already starts to dominate the potential and stops inflation.

Let us now explain the previous claims more in detail. Looking at the expressions (3.15) for all the possible 1-loop corrections to $V$, we immediately realise that both $\delta V_{KK}^{1-\text{loop}}$ and $\delta V_{W}^{1-\text{loop}}$ go to zero when the K3 fibre diverges since $t^* = \sqrt{\lambda_1 \tau_1}$. Therefore these terms are not dangerous at all. Notice that there is no correction at 1-loop of the form $1/ (t_1^3 V^3)$ because, just looking at the scaling behaviour of that term, we realise that it should be a correction due to the exchange of winding strings at the intersection of two stacks of $D7$ branes given by $t_1$, but the topology of the K3 fibration is such that there are no 4-cycles which intersect in $t_1$, and so these corrections are absent.

However the sign at 1-loop that there is a singularity when $\tau_1 \to \infty \iff t_1 \to 0$, is that $\delta V_{KK}^{1-\text{loop}}$ blows-up. In fact, following our previous analysis [21], the contribution of $\delta K_{1-\text{loop}}^{KK}$ at the level of the scalar potential is given by the following expansion:

$$\delta V_{KK}^{1-\text{loop}} = \sum_{p=1}^{\infty} \left( \alpha_p g_s^{p} (C_{KK}^2)^p \frac{\partial^p (K_0)}{\partial \tau_2^p} \right) \frac{W_0^2}{V^2}$$

with $\alpha_p = 0 \iff p = 1$. (B.1)

The vanishing coefficients of the first contribution is the ‘extended no-scale structure.’ Hence we obtain an expansion in inverse powers of $\tau_2$:

$$\delta V_{KK}^{1-\text{loop}} = \left[ \alpha_2 \left( \frac{\rho}{\tau_2} \right)^2 + \alpha_3 \left( \frac{\rho}{\tau_2} \right)^3 + \ldots \right] \frac{W_0^2}{V^2}$$

with $\rho \equiv g_s C_{KK}^2 \ll 1$ and $\alpha_i \sim \mathcal{O}(1)$ $\forall i$. (B.2)

We can then see that, since from (B.21) when $\tau_1 \to \infty \iff t_1 \to 0$, $\tau_2 \to 0$, all the terms in the expansion (B.2) diverge and perturbation theory breaks down. Thus the region where the expansion (B.2) is under control is given by

$$\frac{\rho}{\tau_2} \leq 2 \cdot 10^{-2} \iff \frac{V}{\alpha \sqrt{\tau_1}} \geq 50 g_s C_{KK}^2 \iff \tau_1 \leq \sigma_1 V^2 \text{ with } \sigma_1 \equiv (50 \alpha g_s C_{KK}^2)^{-2}.$$ (B.3)
We need still to evaluate what happens at two and higher loop level. The behaviour of the 1-loop corrections was under rather good control since it was conjectured from a generalisation of an exact toroidal calculation \[20\] and it was tested by a low energy interpretation in \[21\]. However there is no exact 2-loop calculation for the toroidal case which we could try to generalise to an arbitrary Calabi-Yau. Thus the best we can do, is to constrain the scaling behaviour of the 2-loop corrections from a low energy interpretation.

A naive scaling analysis following the lines of \[21\], suggests that

\[
\frac{\partial^2 \left( \delta K_{K,K}^{2\text{-loops}} \right)}{\partial \tau_2^2} \sim \frac{g_s}{16\pi^2} \frac{1}{\tau_2^2} \frac{\partial^2 \left( \delta K_{K,K}^{1\text{-loop}} \right)}{\partial \tau_2^2},
\]

and so \(\delta K_{K,K}^{2\text{-loops}}\) is an homogeneous function of degree \(n = -4\) in the 2-cycle moduli, exactly as \(\delta K_{W}^{1\text{-loop}}\). Given that

\[
\frac{\partial \left( \delta K_{K,K}^{1\text{-loop}} \right)}{\partial \tau_2} = -g_s C_2^{KK} \frac{\partial^2 \left( K_{\text{tree}} \right)}{\partial \tau_2^2},
\]

equation \((B.4)\) takes the form

\[
\frac{\partial^2 \left( \delta K_{K,K}^{2\text{-loops}} \right)}{\partial \tau_2^2} \sim -g_s^2 C_2^{KK} \frac{1}{16\pi^2} \frac{\partial^3 \left( K_{\text{tree}} \right)}{\partial \tau_2^3}.
\]

The previous relation and the homogeneity of the Kähler metric, produce then the following guess for the Kaluza-Klein corrections at 2 loops

\[
\delta K_{K,K}^{2\text{-loops}} \sim -\frac{g_s^2 C_2^{KK}}{16\pi^2} \frac{\partial^2 \left( K_{\text{tree}} \right)}{\partial \tau_2^2},
\]

that at the level of the scalar potential would translate into

\[
\delta V_{K,K}^{2\text{-loops}} = \frac{g_s^2 C_2^{KK}}{8\pi^2} \left[ \frac{1}{\tau_2^2} + \mathcal{O} \left( \frac{1}{\tau_2^2} \right) \right] \frac{W_0^2}{\sqrt{2}}.
\]

We notice that \((B.8)\) has the same behaviour of \((B.2)\) apart from the suppression factor \((8\pi^2 C_2^{KK})^{-1} \sim \mathcal{O}(10^{-2})\). This is not surprising since the leading contribution of \(\delta K_{K,K}^{1\text{-loop}}\) in \(V\) is zero due to the extended no-scale but the leading contribution of \(\delta K_{K,K}^{2\text{-loops}}\) in \(V\) is non-vanishing. Thus we conclude that in the region \(\tau_1 \ll \sigma_1 V^2\) both higher terms in the 1-loop expansion \((B.2)\) and higher loop corrections \((B.8)\) are subleading with respect to the first term in \((B.2)\) which we considered in the study of the inflationary potential.

However, writing everything in terms of the canonically normalised inflaton field \(\hat{\phi}\) expanded around the minimum, the first term in \((B.2)\) turns into the positive exponential which, as we have seen in section \[3.3\], destroys the slow-roll conditions when it starts to dominate the potential at \(\hat{\phi}_{\text{max}} = 12.4\) for \(R = 2.3 \cdot 10^{-6}\). In this point \(\delta V_{(g_s),\tau_2}^{W}\) is not yet completely subleading with respect to \(\delta V_{(g_s),\tau_2}^{K,K}\) and so the slow-roll conditions are still satisfied. The form of this bound in terms of \(\tau_1\) can be estimated as follows:

\[
\tau_1 = \langle \tau_1 \rangle e^{2\hat{\phi}_{\text{max}}/\sqrt{3}} = \left( \frac{4A}{B} \right)^{2/3} y_{\text{max}}^{2/3} V^{2/3} \Leftrightarrow \tau_1 \leq \sigma_2 V^{2/3} \text{ with } \sigma_2 \equiv 4.2 \cdot 10^6 \left( \frac{A}{B} \right)^{2/3}.
\]

\[
(B.9)
\]
Figure 15: Plots of the constraints $\tau_{1}^{\text{max}} = \sigma_1 V^2$ (red curve) and $\tau_{1}^{\text{max}} = \sigma_2 V^{2/3}$ (blue curve) in the $(\tau_1, V)$ plane for the case SV2.

Let us now compare the bound (B.3) with (B.9) to check which is the most stringent one that constrains the field region available for inflation. The value of the volume at which the two bounds are equal is

$$V^* = \left(\frac{\sigma_2}{\sigma_1}\right)^{3/4} = 1.65 \cdot 10^7 \alpha g_s^{5/2} \frac{C_{KK}^K (C_{KK}^K)^{3/2}}{(C_{12}^W)^{1/2}}.$$  (B.10)

Using the natural choice of parameter values made in the main text (for SV2 for example), $V^* = 582$, and so, since we always deal with much larger values of the overall volume, we conclude that the most stringent constraint is (B.9) as can be seen from figure 15.

Therefore the final situation is that, when the K3 fibre gets larger, $\delta V_{(g_s), \tau_2}$ starts dominating the potential and ruining inflation well before one approaches the singular limit in which the perturbative expansion breaks down and these corrections blow-up to infinity.
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