Present limits to cosmic bubbles from the COBE-DMR three point correlation function.

P. S. Corasaniti\textsuperscript{2,1}, L. Amendola\textsuperscript{1}, F. Occhionero\textsuperscript{1}

\textsuperscript{1}Osservatorio Astronomico di Roma, Viale del Parco Mellini 84, 00136 Roma, Italy
\textsuperscript{2}Centre for Theoretical Physics, CPES, University of Sussex, Brighton BN1 9QJ, United Kingdom

ABSTRACT
The existence of large scale voids in several galaxy surveys suggests the occurrence of an inflationary first order phase transition. This process generates primordial bubbles that, before evolving into the present voids, leave an imprint on the CMB.

In this paper we evaluate an analytical expression of the collapsed three point correlation function from the bubble temperature fluctuations. Comparing the results with COBE-DMR measures, we obtain upper limits on the allowed non-Gaussianity and hence on the bubble parameters.

Key words: (Cosmology:) Cosmic microwave background

1 INTRODUCTION
In the recent past the number of papers devoted to non-Gaussian anisotropies on the Cosmic Microwave Background (CMB) has increased dramatically. This new investigation field is, in fact, a powerful tool to distinguish between the theories of structure formation based on inflation and those based on topological defects. Quantum fluctuations produced in inflationary models are scale invariant and have a Gaussian distribution. Thus we expect that the three-point correlation function of the CMB temperature vanish (Falk et al. 1993; Luo & Schramm 1993; Gangui & al. 1994).

On the contrary in models with topological defects the primordial density perturbations are scale dependent and non-Gaussian (Avelino et al. 1998): hence we expect some deviations from Gaussianity in higher order correlation functions. In this context we may also include the extended inflation model (La & Steinhardt 1989), because during the inflationary epoch we have a first order phase transition, that generates bubbles of true vacuum. These voids contribute together with ordinary quantum fluctuations to structure formation. This possibility has been investigated (Occhionero & Amendola 1994; Amendola et al. 1996); it has been shown (Occhionero et al. 1984, 1997) that primordial bubbles may be associated with the observation of large scale voids in several galaxy surveys (Kirshner et al. 1981; de Lapparent et al. 1989; da Costa et al. 1996; El Ad, Piran & da Costa 1996, 1997). Since these defects can also produce non-Gaussian anisotropies on the CMB, we may obtain some limits on the bubble parameters comparing observations with non-Gaussian predictions.

So far different statistical tests have been applied to COBE-DMR sky maps (Kogut et al. 1996) and the results have been in agreement with the Gaussian models. Although recently two groups have detected non-Gaussian signal in COBE data (Ferreira et al. 1998; Pando et al. 1998), subsequently this has been shown to derive from a systematic effect in the data (Banday et al. 1999). On the other hand, Bromley & Tegmark (1999) tried to argue that the COBE 4 year data were in fact Gaussian.

In this paper we compare the level of non-Gaussianity produced in bubble models with the COBE data. We evaluate the three point correlation function in CDM models that contain also primordial bubbles. Comparing the numerical results with the COBE-DMR measures (Kogut et al. 1996; see also Hinshaw et al. 1994, 1995) we obtain upper limits on the parameters of the voids in agreement with galaxy surveys observations.

2 METHOD
The imprints of bubbles on the CMB has been studied in several papers (Baccigalupi, Amendola & Occhionero 1997; Amendola, Baccigalupi & Occhionero 1998; Baccigalupi & Perrotta 1999). The presence of the primordial voids induces a Sachs-Wolfe effect and an acoustic perturbation propagating up to the sound horizon on the photon distribution. As a consequence, the induced temperature fluctuations of in-
individual bubbles are composed of a central spot and some concentric hotter isothermal rings; this pattern has been calculated by numerical integration of the Boltzmann equations in Amendola, Baccigalupi & Occhionero (1998). It is found that the bubble signal depends on the radius \( R \) of the void and on the central (negative) density contrast \( \delta \). We shall distribute \( N \) voids on the CMB sky and use the fraction \( X \) of the space that the voids fill today as a free parameter, where \( X = N R^3/3 \pi L_h^4 \Delta L_h \) (Amendola et al. 1998) where \( L_h \) is the horizon radius and \( \Delta L_h \) is the thickness of the last scattering surface. We consider bubbles of size \( R = 30h^{-1}\text{Mpc} \) at decoupling; due to their overcoming growth, these voids have today radii around \( 20 \sim 60h^{-1}\text{Mpc} \), like those observed in galaxy surveys (da Costa et al. 1996; El Ad et al. 1996; 1997).

In simulated COBE maps, due to the low resolution of the satellite, the signal of the individual bubbles looks like dark spots confused amidst Gaussian anisotropies; their effect appears only when calculating the correlation functions of maps containing many bubbles. The temperature fluctuation may be decoupled in two terms:

\[
\Delta (\theta, \phi) = \Delta_{\text{Gauss}} (\theta, \phi) + \Delta_V (\theta, \phi).
\]

The first term \( \Delta_{\text{Gauss}} (\theta, \phi) \) is the Gaussian temperature fluctuation field produced by the primary anisotropies; the second term is the voids signal, that vanishes in directions where there are not bubbles. In order to compare the predictions of the model with experimental data, we calculate the collapsed three-point function

\[
C_3 (\alpha) = \frac{1}{4\pi} \sum_{l_1, l_2, l_3} \sum_{m_1, m_2, m_3} P_l (\cos \alpha) a_{l_1} m_1 a_{l_2} m_2 a_{l_3} m_3 \times W_{l_1} W_{l_2} W_{l_3} \delta_{l_1 l_2 l_3}^{m_1 m_2 m_3}.
\]

where \( W^l \) is the window function of the experiment, \( P_l \) are the Legendre polynomials, \( a_l^m \) are the multipole coefficients of the spherical harmonic expansion and where

\[
\delta_{l_1 l_2 l_3}^{m_1 m_2 m_3} = (-1)^{m_1} \sqrt{2 l_1 + 1} (2 l_2 + 1) (2 l_3 + 1) \frac{\sqrt{4\pi}}{2l + 1} \times \\
\times \left( \begin{array}{ccc} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{array} \right) \left( \begin{array}{ccc} 1 & l_1 & l_2 \\ l_3 & 0 & 0 \end{array} \right) \left( \begin{array}{ccc} m_1 & m_2 & m_3 \end{array} \right).
\]

with \( \left( \begin{array}{ccc} i_1 & i_2 & i_3 \\ j_1 & j_2 & j_3 \end{array} \right) \) the Wigner 3J symbol. We compare \( C_3 (\alpha) \) to the pseudo three-point collapsed function of Kogut et al. (1996): obviously the two are identical due to the absence of noise in our case. Since the Gaussian term and the signal of the bubbles are not correlated, we may write the three-point correlation function as sum of two separate contributions:

\[
C_3 = C_{3\text{Gauss}} + C_3^V.
\]

The contribution to \( C_3 \) from gaussian fluctuations is not zero. This contribution may arise from non-linearities in the inflationary dynamics or from non-linear growth of the perturbations. However, using the analytical expression for the \( C_{3\text{Gauss}}(\alpha) \) computed in Gangui et al. (1994) it can be seen that the level of non-Gaussianity produced by the non-linearities in the inflationary dynamics is smaller than that arising from the non-linear growth (Mollerach et al. 1995), and that the latter is much smaller that produced by the bubbles. In fact, on the angular scales probed by COBE-DMR, Mollerach et al. (1995) found an amplitude \( \langle C_{R-S}^2 (\alpha) \rangle \sim 0.1 \mu K^3 \), while we find that the contribution of the voids is larger by several orders of magnitude: \( \langle C_{3}^V (\alpha) \rangle \sim 10^5 \mu K^3 \). Therefore, we neglect \( C_{3\text{Gauss}}^V \) in the following.

To calculate \( C_3^V (\alpha) \) we use the same approach of texture-spot anisotropies (Magueijo 1995, Gangui & Mollerach 1996, 1997). The temperature fluctuations produced by a random distribution of bubbles, in the \( \gamma \) direction, is simply the superposition of the signal of all the bubbles, and can be written as \( \Delta_V (\gamma) = \sum_n b_n \delta_{n} \), where the signal of the n-th bubble has been decomposed as a overall amplitude \( b_n \) (corresponding to the central temperature fluctuation) and a density profile \( f_n (\gamma) \) where \( \gamma \) is the angle measured from the bubble center. The dependence on the parameters \( R_n \) and \( \delta_n \) is contained only in the amplitude; it is to be expected that this dependence is linear in \( \delta_n \) and quadratic in \( R_n \), since the central temperature fluctuation is dominated by the Sachs-Wolfe effect. The expression \( b_n = \delta_n (R_n/20H^{-1})^2 \) is indeed an accurate fit in the range we are interested (see Amendola et al. 1998). The profile \( f_n (\gamma) \) contains the full effect of the acoustic oscillations and the adiabatic fluctuations, and has been obtained numerically in Baccigalupi & Perrotta (1999).

We expand \( \Delta_V \) in spherical harmonics and obtain the multipole coefficients

\[
a_l^m = \frac{2l + 1}{4\pi} \sum_n b_n F_n^l \mathcal{W}_n(\gamma),
\]

where \( F_n^l \) is the Legendre trasform of the intensity profile,

\[
F_n^l = \frac{2l + 1}{2l + 2} \int d\Omega_n f_n (\gamma) P_l (\cos \gamma).
\]

Inserting (5) in (2) the collapsed function reduces to:

\[
C_3^V (\alpha) = 4 \pi \sum_{l_1, l_2, l_3} P_{l_1} (\cos \alpha) W_{l_1} W_{l_2} W_{l_3} J_{l_1 l_2 l_3} \times \sum_{n_1, n_2, n_3} b_{n_1} b_{n_2} b_{n_3} F_{n_1}^{l_1} F_{n_2}^{l_2} F_{n_3}^{l_3},
\]

where \( J_{l_1 l_2 l_3} \) represents

\[
J_{l_1 l_2 l_3} = \left( \begin{array}{ccc} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{array} \right)^2.
\]

We take the window function of COBE to be \( e^{-l(l+1)\sigma^2/2} \), with \( \sigma = 3.2^\circ \). We assume now that there are \( N \) identical voids on the CMB sky. Developing the sum on \( n_1, n_2 \) and \( n_3 \) we obtain three terms that represent the contribution to the \( C_3^V (\alpha) \), when the bubble signals are not correlated and when are correlated two by two or three by three and etc. We take into account the correlation at lowest order, in other words we consider just the first two terms. Then the mean value of (7) for a Poissonian bubble distribution on the sky is obtained substituting the sum on the bubble index with an
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Integral over the whole sky. In fact, the number of bubbles in a circular ring centered on a single bubble is proportional to angular extension of the ring, therefore we have:

\[
\langle C_3^V(\alpha) \rangle = 4\pi^3 \left( \frac{R}{20H^{-1}} \right)^3 N \sum_{l_1,l_2,l_3} P_1(\cos \alpha) \\
\times W^{l_1}_1 W^{l_2}_1 W^{l_3}_1 J^{l_1 l_2 l_3}_1 F^{l_1 l_2 l_3},
\]

(9)

where

\[
F^{l_1 l_2 l_3} = F^{l_1} F^{l_2} F^{l_3} + \frac{3}{2} F^{l_1} F^{l_2} \int F^{l_3}(\theta)d(\cos \theta),
\]

(10)

and

\[
F^{l_3}(\theta) = \int f(\theta + \alpha) P_3(\cos \alpha)d(\cos \alpha).
\]

(11)

Using the same approach, after a tedious calculation, we have found an analytic expression for the variance \( \sigma^2_3(\alpha) = \langle C_3^V(\alpha)^2 \rangle - \langle C_3^V(\alpha) \rangle^2 \), that we do not report for shortness.

We compare the experimental data with the behaviour of the \( \langle C_3^V(\alpha) \rangle \) for different values of the parameters \( \delta \) and \( X \). When \( \langle C_3^V(\alpha) \rangle \pm \sigma_3(\alpha) \) is larger than COBE data plus the noise and cosmic variance, we have some constraints on the parameters of our model.

3 RESULTS

The COBE data has been taken from Kogut et al. (1996). We assume a fraction of bubbles corresponding to \( 0.31 < X < 0.54 \), consistent with da Costa et al. (1997). We have computed the \( C_3^V(\alpha) \) for \( 0.001 < \delta < 0.0026 \), without dipole and quadrupole contribution, \( l_{\text{min}} = 4 \). In the figures we report the behaviour of the \( C_3^V(\alpha) \) for two values of \( \delta = 0.002, 0.0012 \). The oscillating behaviour of the plots is due to the sum of the Legendre polynomials in (9). In the plots the errorbars are the \( \sigma(\alpha) \)'s. The level of the cosmic variance \( \sigma(\alpha) \) generated from the model is very high for \( \alpha < 40^\circ \), while it is small on the large angular scales, \( \alpha > 45^\circ \), where the contribution of the lowest multipoles is small. In figure (1) we have the model with \( \delta = 0.002 \): we may note that for \( X = 0.54 \) the signal is larger than cosmic variance and the observed data points, while \( X = 0.31 \), the plot is marginally consistent with the experimental data. In figure (2) we report the \( \langle C_3^V(\alpha) \rangle T_3^0 \) for \( \delta = 0.0012 \): it fits the COBE data very well. Notice that in the range \( \alpha > 50^\circ \) the behaviour of the collapsed function seems to follow the trend of the COBE measures. Values of \( \delta < 0.0012 \) produce a \( C_3^V(\alpha) \) within the cosmic variance band and smaller than the COBE data. In this case the observations do not impose constraints and we may obtain only an upper limit on the value of \( \delta \). We have applied a \( \chi^2 \) analysis to our models. In figure (3) we report the confidence region with a confidence level set to 99.9% (grey region) and to 99.5% (black region). We may note that all models with \( \delta \geq 0.0017 \) are ruled out by the experimental data. Then we may conclude that although the bubbles produce a non-Gaussian signal on the CMB, this is in agreement with the present observation provided that the density contrast \( \delta \leq 0.0017 \) or \( X \leq 54 \%. \) So we obtain a constraint stronger than that found in Amendola et al. (1998), where the bubble power spectrum was compared to the measures of the CAT experiment. The next high resolution experiments, like MAP and Planck, and the recent observations of Boomerang and Maxima should be able to detect the voids signal on the CMB. In fact these missions can probe the multipoles \( l > 100 \), where the contribution of

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure1}
\caption{The points are the COBE data while the thick lines are the cosmic variance of a Gaussian random field (Kogut et al., 1996). The plots are models with \( \delta = 0.002 \) and \( X = 0.54 \) (dashed line) and \( X = 0.31 \) (thin line). The errorbars are the variance of our models.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure2}
\caption{Same as in Fig. 1, but now \( \delta = 0.0012 \): agreement with observations is now obtained.}
\end{figure}
the bubbles is important, and the effects on $C_3(\alpha)$ may be large.

4 CONCLUSION

Several galaxy surveys found huge spherical voids in the matter distribution; the galaxies lying in the surrounding shells; these structures may be generated in inflationary models with first order phase transitions. These bubbles produce a non-Gaussian signal on the CMB. We analyse this signal developing an analytical expression for the three-point collapsed function of a bubble distribution, using the formalism of Magueijo (1995). Our free parameters are the density contrast and volume fraction of the bubbles, while the radius $R$ is fixed to a value consistent with the galaxy surveys. We compare the behaviour of the three point collapsed function for the bubble model with the COBE data. We obtain a constraint on the value of $\delta$: in fact, the existence of the voids at decoupling is not in contrast with the measures of the COBE three-point collapsed function, provided $\delta \leq 0.0017$ or $X \leq 0.54$. This still leaves plenty of room for the bubbles to cooperate efficiently to structure formation, both via the central voids and via the possibility of shocking on the outer shell: in fact a central density contrast of 0.001 can still evolve linearly in an empty void by today. More information will be obtained comparing the results of the future high resolution experiments.

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