QUARK-GLUON EVOLUTION IN EARLY STAGE OF ULTRA-RELATIVISTIC HEAVY-ION COLLISIONS

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Abstract
A set of coupled kinetic equations describing in the Abelian approximation a mixture of quarks and self-interacting gluons is formulated and solved numerically. The model includes the Schwinger-like mechanism for particle creation in a strong field as well as two-particle elastic collisions between all mixture components in the Landau approximation of small-angle scattering. The process of equilibration at the initial energy density exhibits a dominant quark creation in the very early time of interaction. It is shown that damping of energy density oscillations due to elastic scattering of perturbative quarks and gluons is not strong enough to reach thermodynamic equilibrium in a reasonable relaxation time. A possible account for such behavior is discussed.

Key-words: QGP, kinetic theory, heavy-ion collision, equilibrium, Schwinger’s mechanism, flux tube model.

1 Effective Lagrangian
Quark-gluon plasma (QGP) evolution is considered in a following physics scenario. In early stage of heavy-ion collisions at ultra-relativistic energy a chromo-electric flux tube may be stretched between nuclear residuals and this strong field results in a spontaneous vacuum pair creation (Schwinger mechanism). In its turn, the created particles can influence on this background field (back reaction process) and also suffer subsequent rescatterings relaxing to some equilibrium state, in general. This picture is treated in the Abelian approximation based on the kinetic equations (KE) derived in [1, 2]. In our paper we focus on the relative role of components of quark-gluon mixture in the pre-equilibrium stage as well as on the question how fast the relaxation process is.

To describe QGP we use the following effective model Lagrangian [3]:

\[
L(x) = \partial_\mu \Phi^*(x) \partial^\mu \Phi(x) - m_+^2 |\Phi(x)|^2 - e_+^2 |\Phi(x)|^4 \\
+ i e_+ \left\{ [\nabla_n \Phi^*(x)] \Phi^2(x) - [\Phi^*(x)]^2 \nabla_n \Phi(x) \right\} \\
+ \frac{i}{2} \bar{\psi}(x) \gamma^\mu \partial_\mu \psi(x) - m_\psi \bar{\psi}(x) \psi(x) \\
+ \frac{e_-}{2} (\Phi(x) + \Phi^*(x)) \bar{\psi}(x) (\gamma_n) \psi(x),
\]

where indexes ‘+’ and ‘−’ are used for bosons (gluons) and fermions (quarks), respectively, and \( \nabla_n = n^\mu \partial_\mu \).
We suppose that the total bosonic field can be decomposed into a mean-field contribution $\Phi_0(t)$ and its fluctuations
\[ \Phi(x) = \Phi_0(t) + \varphi(x). \] (2)

We consider $\Phi_0$ as a neutral space homogeneous background field. The field of fluctuations $\varphi(x)$ is complex and corresponds to charged field with vanishing mean value. The $\phi^4$ term in our Lagrangian (1) simulates the gluon self-interaction. Keeping only the second order terms in fluctuations, the corresponding equations of motion read
\[ i\gamma^\mu \partial_\mu - e^- e_n^\mu \Phi_0(t) - m_- \psi(x) = 0, \]
\[ [D^*_\mu D^\mu + m^2_+] \varphi(x) = 0, \] (4)
\[ \partial_0^2 \Phi_0(t) + m^2_+ \Phi_0(t) + 4e^2_+ \Phi^3_0(t) - j_-(t) - j_+(t) = 0, \] (5)

where the covariant derivative $D_\mu = \partial_\mu + ie^- e_n^\mu \Phi_0(t)$.

Alongside with the Dirac equation (3) and the Klein-Gordon equation (4) we get eq.(5) for evolution of the background mean field. One sees that equations of motion are self-consistently coupled: the created particles generate the currents $j_{\pm}(t)$ which form the background field according to (5). In the mean-field approximation these currents are given as
\[ j_-(t) = -e_- < \bar{\psi}(x)(\gamma n)\psi(x) >, \] (6)
\[ j_+(t) = -e_+ < i\varphi^*(x)\nabla_n \varphi(x) - i(\nabla_n \varphi^*)(x)\varphi(x) \]
\[ -2e_+ \Phi_0(t)\varphi^*(x)\varphi(x) >. \] (7)

## 2 Kinetic equations

Starting from these equations of motion one can introduce quasi-particles and obtain the KE for the single particle distribution function on the basis of diagonalization of Hamiltonian by the time-dependent Bogoliubov transformation. This result is exact in the mean-field approximation for a space homogeneous field. In our model, the vector $n_\mu$ corresponds to Flux Tube Model (FTM) geometry and is chosen to be (0,0,0,1). So, we arrive at the following KE:
\[ \frac{\partial f_\pm(\bar{p}, t)}{\partial t} + e_\pm \sigma(t) \frac{\partial f_\pm(\bar{p}, t)}{\partial p_\parallel} = S_\pm(\bar{p}, t) + C_\pm(\bar{p}, t). \] (8)

Here, the chromo-electric field strength is $\sigma(t) = -d\Phi_0/dt$, $C_\pm(\bar{p}, t)$ is the collision integral and $S_\pm(\bar{p}, t)$ is the source term:
\[ S_\pm(\bar{p}, t) = \frac{1}{2} W_\pm(\bar{p}, t) \int_{-\infty}^{t} dt' W_\pm(\bar{p}, t, t') \times \]
\[ [1 \pm 2f_\pm(P_\pm(t, t'), t')] \cos \left\{ \begin{array}{l} 2 \int_{\tau'}^{t} d\tau \omega_\pm(\bar{p}; t, \tau) \end{array} \right\} \]
\[ 2s_\pm \varepsilon_\pm(\bar{p}; t, t') P_\pm(t, t') \] (9)

with the transition amplitude
\[ W_\pm(\bar{p}; t, t') = \frac{e_\pm \sigma(t') P_\pm(t, t')}{\omega_\pm^2(\bar{p}; t, t')} \left[ \frac{\varepsilon_\pm}{P_\pm(t, t')} \right]^{2s_\pm}. \] (10)
where \( s_+ = 0 \) for gluons and \( s_- = 1/2 \) for quarks. The quasi-particle energy entering this equation is

\[
\omega_{\pm}^2(\vec{p}, t, t') = \varepsilon_{\pm \pm}^2 + P_{\pm}(t, t') \tag{11}
\]

where \( P_{\pm}(t, t') = p_{\pm}^0 - e_{\pm} \int_{t'}^{t} d\tau \sigma(\tau) \) and \( \varepsilon_{\pm \pm}^2 = m_{\pm}^2 + p_{\perp}^2 \). General features of this source term are described in [2, 4].

Kinetic equation (8) may be rewritten in the form of a system of ordinary differential equations [4, 5]:

\[
\frac{df_{\pm}(\vec{p}, t)}{dt} = \frac{1}{2} W_{\pm}(\vec{p}, t) v_{\pm}(\vec{p}, t) + C_{\pm}(\vec{p}, t), \tag{12}
\]

\[
\frac{dv_{\pm}(\vec{p}, t)}{dt} = W_{\pm}(\vec{p}, t) \left(1 \pm 2 f_{\pm}(\vec{p}, t)\right) - 2 \omega_{\pm}(\vec{p}) u_{\pm}(\vec{p}, t), \tag{13}
\]

\[
\frac{du_{\pm}(\vec{p}, t)}{dt} = 2 \omega_{\pm}(\vec{p}) v_{\pm}(\vec{p}, t), \tag{14}
\]

where two new functions have been introduced:

\[
u_{\pm}(\vec{p}, t) = \int_0^t dt' W_{\pm}(\vec{p}, t, t') \left(1 \pm 2 f_{\pm}(\vec{p}(t, t'), t')\right) \sin[2 \int_t^{t'} d\tau \omega_{\pm}(t, \tau)], \tag{15}
\]

\[
u_{\pm}(\vec{p}, t) = \int_0^t dt' W_{\pm}(\vec{p}, t, t') \left(1 \pm 2 f_{\pm}(\vec{p}(t, t'), t')\right) \cos[2 \int_t^{t'} d\tau \omega_{\pm}(t, \tau)]. \tag{16}
\]

### 3 Collision integral

Neglecting particle-particle collisions does not result in complete picture of a collision at large time moments when the interaction force between particles is getting greater than the mean-field interaction. However, the direct evaluation of the CI gives rise to a huge numerical problem. In earlier papers the CI was introduced in the relaxation time approximation with time- and momentum-independent relaxation time what does not allow to restore true dynamics of the relaxation process and to estimate properly the relaxation time [2, 4]. Here, the CI will be obtained on a dynamical basis but in a simplified manner by making use of the Landau approximation, i.e. assuming small momentum transfer in elastic \( qq \)-, \( qg \)- and \( gg \)-collisions. The appropriate cross sections are calculated in the perturbative approximation:\n
\[
\frac{d\sigma_{qq}}{dt} = \frac{\pi N_c^2 - 1}{2} \frac{\alpha_s^2}{N_c^2 s^2} \left[s^2 + u^2 \frac{t^2}{t^2 + u^2} + \frac{s^2 + t^2}{u^2} - 2 \frac{s^2}{3 tu}\right], \tag{17}
\]

\[
\frac{d\sigma_{qg}}{dt} = 4\pi \frac{N_c^2}{N_c^2 - 1} \frac{\alpha_s^2}{s^2} \left[3 - \frac{ut}{s^2} - \frac{us}{t^2} - \frac{st}{u^2}\right], \tag{18}
\]

\[
\frac{d\sigma_{gg}}{dt} = 2\pi \frac{\alpha_s^2}{s^2} \left(s^2 + u^2\right) \left[\frac{1}{t^2} - \frac{N_c^2 - 1}{2 N_c^2 su}\right], \tag{19}
\]

where \( \alpha_s \) is the QCD coupling constant and \( s, t, u \) are Mandelstam’s variables.

As is seen, the cross sections are divergent at small momentum transfer. So, only leading terms of the \( 1/t^2 \) order are kept in [17]-[19] which dominate for gluon/quark collisions.
within the Landau CI approximation. In this approximation one can obtain the Landau-like CI:

\[
C_a(\vec{p}_a, t) = \frac{\partial}{\partial p_a} \sum_b \int d^3 p_B B_{a\beta}(\vec{p}_a, \vec{p}_b) \times 
\left[ \frac{\partial f_a}{\partial p_{a\beta}} f_b(1 \pm f_b) - \frac{\partial f_b}{\partial p_{b\alpha}} f_a(1 \pm f_a) \right],
\]

where \(B\) is a kernel. This kernel is further simplified for massless particles [5]:

\[
B_{a\beta}(\vec{p}_a, \vec{p}_b) = \xi_{ab} \left[ (1 - v_a v_b) \delta_{a\beta} + v_{aa} \delta_{b\beta} + v_{ba} v_{a\beta} \right],
\]

where \(v_{aa} = p_{aa}/\omega(p_a, t), \xi_{ab} = 2\pi\alpha^2 G_{ab} L\). Coefficients \(G_{ab}\) are defined by corresponding cross sections:

\[
G_{gg} = \frac{N_c^2}{N_c^2 - 1}, \quad G_{qg} = \frac{1}{4}, \quad G_{qq} = \frac{N_c^2 - 1}{8N_c^2}.
\]

\section{4 Calculation results}

The system of three differential equations (12)-(14), describing self-consistently the evolution of distribution functions and fields, is closed and can be solved numerically as the initial Cauchy problem. We choose the zero initial conditions for distribution functions of bosons and fermions and for a non-zero initial value of the chromo-electric field strength \(e\sigma(t = 0)\). These conditions correspond to the FTM, where colliding ions generate a large-amplitude field at passing ions through each other. This field is decaying due to creation of quarks and gluons.

The analysis of abundance evolution within the pQCD model shows the dominant role of gluons [7]. But as is seen from Fig.1a, it is not the case for the model considered: Quark production dominates over gluons during few first \(fm/c\) in the very early stage of interaction. This ”fermion dominance” originates from spin effect in the source term (1) suppressing low \(P_{\parallel}\) gluons [3]. In the early stage of a collision when the created particle density is still small, in the phase space there are many free states for the final state of created particles and the influence of the Pauli blocking is not so essential. However, with subsequent density increase, the fermion creation is suppressed by the factor \((1 - 2f_-)\) due to occupied states and bosons are enhanced by the \((1 + 2f_+ f_-)\) factor. Therefore, only at later time the system evolution resembles that in the pQCD model exhibiting the dominant gluon production.

The mass ratio \(m_+/m_-\) dependence of the fermion dominance time \(\tau_f\) is of great interest. As shown in Fig.1b, \(\tau_f \approx \text{const}\) in the range of \(m_+/m_- \lesssim 0.5\) corresponding to non-zero fermion dominance time even in the limit of zero boson mass. However, \(\tau_f \rightarrow \infty\) when the boson mass equals to or exceeds the fermion mass.

Unfortunately, the CI (20), taking into account mainly hard partons, is of minor importance and can not result effectively in quantum oscillation damping of the mean field. The mean field causes a rippling excitation of the distribution function and makes impossible to achieve equilibrium in QGP in the small momentum-transfer approximation used. Fig.2 illustrates this fact for pure gluon plasma and for QGP. In the first case considered the system is close to equilibrium, but taking into account additionally the quarks degrees of freedom, we obtain far-of-equilibrium dynamics. The latest work [7] shows that this
problem is not conditioned by using the Landau CI and can not be solved by including higher terms in evaluation of corresponding cross-sections. Due to the fermion dominance at the early stage, quarks cause significant changes in the gluon distribution function and the system evolves extremely slowly towards a quasi-equilibrium state.

Figure 2: Time dependence of longitudinal (solid line) and transverse (dotted line) pressure for pure gluon system (a) and that for different parton components in quark-gluon plasma (b).

5 Conclusions

The self-consistent system of equations for describing early stage of heavy ions collisions has been derived. Being solved numerically, the set of equations does not result in a fast attainment of a quasi-equilibrium state for the system under discussion. To get a reasonable estimate for the relaxation time of quark-gluon plasma one needs to increase the cross sections in few times what effectively would correspond to accounting for radiative pertubative processes and scatterings of nonperturbative partons, as well. Instead of the gluons dominance predicted by pQCD models, we observe the quark dominance in the very
early stage of particle production at the time scale of few \( fm/c \) corresponding to QGP evolution. If expansion of excited matter is included into consideration, this time \( \tau_f \) should be even longer.

One should stress that the KE method used is rather simplified with respect to the CI \((20)\) in KE \((8)\). This Boltzmann-like integral is obtained in the nearest order approximation of the gradient expansion for the distribution function, neglecting all coherence effect to be related to the mean field. As expected, the consecutive dynamical approach to the CI problem can lead to an important correction of the result discussed: Strong quasiclassical fields have to influence on quantum fluctuations of quark and gluon fields. This problem is a real challenge to the kinetic theory of a particle-antiparticle plasma in a strong field.

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