Backreaction and the rolling tachyon – an effective action point of view

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Abstract  

We compute the decay of an unstable D9 brane in type IIA string theory including backreaction effects using an effective field theory approach. The open string tachyon on the brane is coupled consistently to the space-time metric, the dilaton and the RR 9-form. The purpose of this note is to address the fate of the open string energy density, which remains constant if no interaction with the closed string modes is included. Our computations show that taking only into account the coupling to the massless closed strings the total energy stored in the open string sector vanishes asymptotically, independently how small one chooses $g_s$. We find also the large time behaviour of the fields in the Einstein and string frames.

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1 Introduction

In recent years much progress has been made in the study of some nonperturbative aspects of string theory. The static properties of stable and unstable D-branes are by now well understood. According to Sen’s conjecture [1], an unstable brane starts rolling down the potential towards the closed string vacuum, where there are no more perturbative open string states. This has been studied in the ‘static’ context from various points of view [2,3,4].

Recently some more dynamical aspects have attracted a lot of attention, in particular the process of unstable D-brane decay in real time starting from some initial configuration. Since the unstable D-brane is described by some tachyon profile one is led to study time-dependent tachyon dynamics. Such an exact time dependent solution of open string theory at $g_s = 0$ was found by Sen [5]. It can be described as a free field BCFT with the insertion of an exact marginal operator on the boundary.

$$T_{BCFT}(t) = \tilde{\lambda} \cosh \left( \frac{t}{\sqrt{2}} \right)$$

This boundary operator identifies the time-dependent classical tachyon profile living on the brane. The calculated energy density stored in the open strings is then naturally constant with time. This leads at the end of the evolution to a pressureless tachyon matter. See [7] for a selection of time-dependent solutions in different settings.

The key question is then what are the properties of the final state of the time evolution once we allow for a nonvanishing string coupling $g_s \neq 0$. Or formulated differently: is pressureless tachyon matter purely an artefact of the ‘noninteracting’ solution (1)?

In [8,9,10] open string creation has been argued to destabilize the tachyon matter. From the boundary state perspective, the time dependent tree level couplings to the closed strings can be relatively easily computed [6,11]. Subsequently the creation of closed strings from the decaying brane has been calculated [12]. It was found, in some cases, that the total emitted energy diverges. The problem was traced to neglecting the backreaction of the emitted closed string excitations (gravitons etc.). In other words, the decaying brane emits gravitons which modify the closed string background which in turn modifies the evolution of the boundary state. However this modification of the closed string background and closed string self-interactions seem to be extremely difficult if not impossible to implement in the worldsheet
perspective. For more recent work on string production and backreaction see [13].

In this note we want to study the evolution of an unstable D9 brane in type IIA string theory. We want mainly to concentrate on the fate of the open string energy density.

The correct theoretical framework for describing such an interacting system would be an open-closed string field theory. However we lack a workable concrete formalism (but see [14]) especially in the superstring case. It may also be possible to embed closed strings in a purely open string framework [4], however they may be represented only in a rather singular form, and any description of a closed string background in this manner seems to be completely beyond our reach.

For these reasons we decided to adopt an effective action approach and couple the tachyon effective action [5, 6, 15] to the low energy supergravity action for the massless closed string modes and to study the resulting temporal evolution.

We note that the coupling of tachyon matter to gravity has already been studied, however, the emphasis was on different questions than the ones that we want to consider. On the one hand, people studied the coupling of bosonic tachyonic matter in 4D to (4D) general relativity and studied it as a possible source of inflation (in the ‘real world’) [16, 17]. On the other hand, a more related study investigated the supergravity solutions corresponding to SDp branes [18, 19, 20, 21, 23], which were introduced in [24].

As stressed before, our motivation is different. We want to determine whether in the large $t$ limit there is still open string matter or whether it has all been transformed into closed string modes or whether there is some kind of intermediate solution. As a criterion for the disappearance of open strings we will calculate the energy density of the tachyon matter (which is a source for the gravitational field) and see if it vanishes in the large $t$ limit.

Moreover, from the technical point of view, we want to consider the full system with all the relevant supergravity fields like the dilaton and the RR-form. In addition we start from the tachyon below the tip of the potential (i.e. we have static initial conditions in order for the whole evolution to come from the decay of the unstable D-brane and not from the additional initial kinetic energy of the tachyon), while in the case of SDp brane solutions the opposite conditions had to be imposed [19, 21]. We also choose to work in the Einstein frame in order to have clearer notions of energy densities.

The plan of this paper is as follows. In section 2 we briefly recall the
effective action description of the rolling tachyon without backreaction. In
section 3 we derive the equations of motion for the supergravity+tachyon
system, and in section 4 we present the main results coming from the numer-
ical solutions and discuss the asymptotic regime. We close the paper with a
discussion.

2 The rolling tachyon without backreaction

An (approximate) effective action describing the dynamics of the open string
tachyon has been proposed by Sen [6]:

$$\int d^{p+1}x V(T) \sqrt{\det(\eta_{\mu \nu} + \partial_\mu T \partial_\nu T)}$$  \hspace{1cm} (2)

The particular choice of $V(T)$ [22]

$$V(T) = \frac{1}{\cosh \left( \frac{T}{\sqrt{2}} \right)}$$  \hspace{1cm} (3)

leads to the dynamics very similar to the one obtained from the exact BCFT
profile (1). The solution to the EOM of (2) with the initial conditions $T(0) = T_0$ and $\dot{T}(0) = 0$ is

$$T(t) = \sqrt{2} \arcsinh \left[ \sinh \left( \frac{T_0}{\sqrt{2}} \right) \cosh \left( \frac{t}{\sqrt{2}} \right) \right]$$  \hspace{1cm} (4)

For large $t$ one has $T(t) \sim t$. Note that this is quite different from the BCFT
profile (1). However there may well be some field redefinition between the
two approaches. Invariant information is encoded in the energy momentum
tensor. We can thus use the $T_{00}$ component to match the $\tilde{\lambda}$ parameter of the
BCFT profile and the effective action solution:

$$T_{00} = \frac{1 + \cos \left( 2\pi \tilde{\lambda} \right)}{2} \equiv \frac{1}{\cosh \left( \frac{T_0}{\sqrt{2}} \right)}$$  \hspace{1cm} (5)

The static initial boundary conditions thus always correspond to evolution
from below the tip of the tachyon potential.

The advantage of the specific choice of (3) is that the functional $t$-dependent
form of the $T_{ii}$ component is the same as for the exact BCFT profile (note
however that then the matching of parameters is slightly different from (5)).
3 Coupling to supergravity fields

We will now specialize to the decay of an unstable D9 brane in type IIA superstring theory. The reason for that is that we want to have a well defined bulk closed string theory (no closed string tachyon) and with the above spacefilling brane all the supergravity equations reduce just to ordinary differential equations which can be easily solved numerically.

The relevant supergravity fields will be the metric $g_{\mu \nu}$, the dilaton $\Phi$ and the RR 9-form $C_9$. The SUGRA action for these fields (in the Einstein frame) is

$$S_{\text{SUGRA}} = \frac{1}{16 \pi G_{10}} \int d^{10} x \sqrt{-\det g} \left( R - \frac{1}{2} g^{\mu \nu} \partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2} \cdot 10! e^{-\frac{2}{5} \Phi} F_{10}^2 \right)$$  \hspace{1cm} (6)

where we used massive IIA SUGRA [25] and $F_{10} = dC_9$.

The effective action for the tachyon coming from the unstable D9 brane is the curved space analogue of (2) with a Chern-Simons coupling to the RR 9-form:

$$S_T = \frac{\lambda}{16 \pi G_{10}} \left( - \int d^{10} x e^{-\Phi} V(T) \sqrt{-\det A + f(T) \int dT \wedge C_9} \right)$$  \hspace{1cm} (7)

where

$$A_{\mu \nu} = g_{\mu \nu}^{\text{str}} + \partial_\mu T \partial_\nu T = e^{\frac{T}{\sqrt{2}}} g_{\mu \nu} + \partial_\mu T \partial_\nu T$$  \hspace{1cm} (8)

and

$$\lambda = \frac{g_s}{(2 \pi \sqrt{\alpha'})^3}$$  \hspace{1cm} (9)

For the CS coupling we take, following [19], $f(T) = V(T)$. We use the potential $V(T) = 1/\cosh \left( T/\sqrt{2} \right)$.

Throughout the paper we use the Einstein frame metric in order to have a conventional interpretation for the energy density (the energy momentum tensor is obtained using variations w.r.t. the Einstein frame metric).

The $SO(9)$ symmetry of the unstable D9 brane decay allows us to make the ansatz:

$$ds^2 = -dt^2 + a^2(t)((dx^1)^2 + \ldots)$$  \hspace{1cm} (10)

$$C_9 = C(t) dx^1 \wedge \ldots dx^9$$  \hspace{1cm} (11)
and, of course, $T = T(t)$ and $\Phi = \Phi(t)$. Then the Einstein tensor is

\begin{align}
G_{00} &= 36\frac{\dot{a}^2}{a^2} \\
G_{ii} &= -28\dot{a}^2 - 8\ddot{a}
\end{align}

and the Einstein equations are $G_{\mu
u} = T_{\mu\nu}$. The energy momentum tensors for the relevant fields are

\begin{align}
T_{00}[\Phi] &= \frac{1}{4}\dot{\Phi}^2 \\
T_{ii}[\Phi] &= \frac{1}{4}a^2\dot{\Phi}^2 \\
T_{00}[C_9] &= \frac{1}{4}e^{-\frac{5\Phi}{2}}\dot{C}^2a^{-18} \\
T_{ii}[C_9] &= -\frac{1}{4}e^{-\frac{5\Phi}{2}}\dot{C}^2a^{-16}
\end{align}

and

\begin{equation}
T_{\mu\nu}[T] = \frac{1}{\sqrt{-\det g}}\lambda e^{-\frac{1}{2}\Phi} \frac{1}{2} V(T) \sqrt{-\det A(A^{-1})_{\mu\nu}}
\end{equation}

Hence

\begin{align}
T_{00}[T] &= \frac{\lambda}{2} e^{\frac{3\Phi}{2}} V(T) \sqrt{\Delta} \\
T_{ii}[T] &= -\frac{\lambda}{2} e^{\frac{3\Phi}{2}} V(T) \sqrt{\Delta} a^2
\end{align}

where\footnote{Note that due to the fact that we are using the Einstein frame, $\Delta$ is different from the one in e.g. \cite{19}.}

\begin{equation}
\Delta \equiv 1 - e^{-\frac{1}{2}\Phi}\dot{T}^2
\end{equation}

In addition to the Einstein equations we have EOM for the matter fields:

\begin{align}
\ddot{T} + \frac{\dot{\Delta}}{2\Delta} + \dot{\Phi} \dot{T} + 9\frac{\dot{a}}{a} \dot{T} + e^{\frac{3\Phi}{2}} \frac{dV}{dT} &= -a^{-9} \dot{C} \sqrt{\Delta} e^{-\Phi} \\
\frac{d}{dt} \left( e^{-\frac{5\Phi}{2}} a^{-9} \dot{C} \right) &= \lambda V(T) \dot{T} \\
\frac{d}{dt} \left( a^9 \dot{\Phi} \right) &= -\frac{5}{4} e^{-\frac{5\Phi}{2}} a^{-9} \dot{C}^2 - \lambda a^9 e^{\frac{3\Phi}{2}} V(T) \left( \frac{3}{2} \sqrt{\Delta} + \frac{e^{-\frac{1}{2}\Phi}\dot{T}^2}{4\sqrt{\Delta}} \right)
\end{align}

4 Numerical results

We solve numerically the equations \cite{19}-(21) and the first order equation for $a(t)$:

\begin{equation}
36\frac{\dot{a}^2}{a^2} = T_{00}[\Phi] + T_{00}[C_9] + T_{00}[T]
\end{equation}
The second Einstein equation \( G_{ii} = T_{ii} \) is not independent and as a cross-check we verified numerically that it is indeed satisfied. We also checked explicitly that the total energy momentum tensor is covariantly conserved.

We choose the initial conditions \( T(0) = T_0, \dot{T}(0) = \Phi(0) = \dot{\Phi}(0) = C'(0) = \dot{C}(0) = 0 \) and \( a(0) = 1 \) i.e. initially at \( t = 0 \) we have the D9 brane in ordinary flat Minkowski space. The initial condition for \( \dot{a} \) is not a free parameter but is determined by the einstein equation:

\[
\frac{\dot{a}}{a}(0) = \frac{1}{6} \sqrt{\frac{\lambda}{2 \cosh \left( T_0 / \sqrt{2} \right)}}
\]  

(23)

Note that we always choose explicitly a positive initial Hubble parameter.

Numerically the system of equations is difficult to solve and we had to use high precision calculations. Nevertheless still we could not reach asymptotic values of \( t \) (e.g. \( t < 60 \) for \( T(0) = 0.5 \)). The reason for the numerical instability is the expression for the tachyon energy

\[
\frac{V(T)}{\sqrt{\Delta}}
\]

(24)

The numerator is exponentially suppressed, but \( \Delta \) also exponentially aproaches zero. In order to circumvent the problem we derived an approximate expression for \( \Delta \) and used it for evolving the system to large \( t \) with initial conditions obtained at some intermediate time \( t_0 \) from the exact evolution. In this way we can reach very long times (beyond \( t = 100000 \) for \( T(0) = 0.5 \)). We checked that the solutions of the asymptotic set of equations coincide almost exactly with the exact solutions in the common domain of validity.

Let us briefly summarize the key features of the above simplifications.

Firstly, the equation for the RR form can be solved exactly:

\[
e^{-\frac{5}{2} \Phi} a^{-9} \dot{C} = \lambda \int_{T_0}^{T(t)} V(T) dT
\]

(25)

For large \( t \), since \( T(t) \to \infty \) the above quantity reaches a constant \( \tilde{C} \):

\[
e^{-\frac{5}{2} \Phi} a^{-9} \dot{C} \to \tilde{C} \equiv \frac{\lambda \pi}{\sqrt{2}} - 2\sqrt{2} \lambda \arctan \left[ \tanh \left( \frac{T(0)}{2\sqrt{2}} \right) \right]
\]

(26)

Secondly, since \( \Delta \) approaches exponentially 0 we may identify (up to exponentially supressed terms)

\[
\dot{T} = e^{\frac{5}{2} \Phi}.
\]

(27)
Neglecting the terms in the tachyon EOM \[ (19) \] which are proportional to \( \sqrt{\Delta} \), approximating \( \left(1/V\right)dV/dT \sim -1/\sqrt{2} \) and using (27) we obtain

\[
\frac{d}{dt}(\log \Delta) = -\sqrt{2} \dot{T} + \frac{5}{2} \dot{\Phi} + 18 \frac{\dot{a}}{a} \tag{28}
\]

which yields

\[
\sqrt{\Delta} = \text{const}^{-1} e^{-\frac{T}{\sqrt{2}}} e^{\frac{5}{2} \Phi} a^9 \tag{29}
\]

The resulting tachyon energy density behaves asymptotically as

\[
T_{00}[T] \sim \lambda \cdot \text{const} \cdot a^{-9} e^{\frac{5}{2} \Phi} \tag{30}
\]

The constant is fixed from the numerical solution of the exact equations.

**Tachyon energy density**

The main motivation for this paper was to study the influence of the backreaction of the emitted closed string fields on the rolling tachyon dynamics. In particular we study the behaviour of the energy contained in the open string sector, which would remain constant without backreaction taken into account. In figure 1 we plot the time evolution of the tachyon energy density

\[
T_{00}[T] = \frac{\lambda}{2} e^{\frac{5}{2} \Phi} V(T) \frac{1}{\sqrt{\Delta}} \tag{31}
\]

We see that it goes to zero. Moreover this is not due just to the vanishing of the dilaton prefactor as can be verified using the asymptotic behaviours.
derived in the following section. The same asymptotic vanishing can be seen to hold also for the combination $\sqrt{-\det g T_{00}}[T]$. In figure 2 we also plot the coupling to the RR 9-form $\lambda V(T) \dot{T}$ which eventually also vanishes.

The above results indeed support the hypothesis that the whole energy initially concentrated in the open string modes gets transferred into the closed string sector, no matter how small one chooses $g_s$. However as we find below, the asymptotic geometry is not flat static Minkowski space but rather a weakly expanding background with nontrivial dilaton and RR 9-form fields.

**Asymptotic region**

In the asymptotic region we obtain the set of equations

$$\ddot{\Phi} + \frac{9}{a} \dot{\Phi} + \frac{5}{4} \tilde{C}^2 e^{2\Phi} = -\frac{1}{2} T_{00}[T]$$

(32)
\[ 18 \frac{a'^2}{a^2} - \frac{1}{8} \Phi'^2 - \frac{1}{8} \tilde{C}^2 e^{\frac{3}{2} \Phi} = \frac{1}{2} T_{00}[T] \]  
(33)

where \( T_{00}[T] \) is substituted by (30) and \( \tilde{C} \) is defined in (26).

We will now heuristically determine the asymptotic scaling dependence of the fields. Assuming a power law dependence \( e^{\Phi} \sim t^\alpha \), \( a \sim t^\beta \) and requiring that all the terms in the above asymptotic expressions are of the same order of magnitude (i.e. \( \sim t^{-2} \)) we obtain
\[
e^{\Phi(t)} \sim t^{-\frac{4}{5}} \quad (34)
\]
\[
a(t) \sim t^{\frac{1}{5}} \quad (35)
\]
and using \( \dot{T} \sim e^{\Phi/4} \) we get
\[
T(t) \sim t^{\frac{4}{5}} \quad (36)
\]
We verified numerically that the above scalings indeed do set in, but only at very large times (see figure 3). Indeed the approach to asymptotics is quite slow and due to the complexity of the equations we were unable to quantify it further.

The RR 9-form behaves asymptotically as \( C(t) \sim t^{4/5} \), which is a direct consequence of (26) and the above results. With the above asymptotics, the energy momentum tensors \( T_{00}[\ldots] \) behave like \( 1/t^2 \), and although they vanish asymptotically they are still able to drive a weak power-law expansion of the space-like geometry.

**Asymptotic region in the string frame**

It is interesting to see how the asymptotic region looks like in string frame since it is the string frame metric which appears in the (closed) string sigma-model. Using the relation \( g_{\mu
u}^{\text{string}} = e^{\Phi/2} g_{\mu
u}^{E} \) we get for our ansatz:
\[
ds^{2}_{\text{string}} = -e^{\frac{4}{5} \Phi} dt^2 + e^{\frac{1}{5} \Phi} a^2(t) dx^2 \equiv -dt'^2 + a_{\text{string}}^2(t') dx^2 \quad (37)
\]
where we introduced natural string-frame time coordinate \( t' \). It is easy to check that the new time is related asymptotically to the Einstein-frame coordinate through
\[
t' \sim t^{\frac{4}{5}} \quad (38)
\]
The string frame scale factor then reaches asymptotically a constant:
\[
a_{\text{string}}^2 = e^{\frac{1}{5} \Phi} a^2(t) \sim \text{const.} \quad (39)
\]
Therefore the asymptotic metric seen by the strings is just flat Minkowski space \(-dt'^2 + d\vec{x}'^2\). Yet this is not the background of the classical flat space as the dilaton and the RR 9-form still have nontrivial \(t'\) dependence:

\[
e^\Phi \sim \frac{1}{t'} \quad C \sim t'
\] (40)

thus the effective string coupling constant vanishes for large times.

Note that (39) has a different behaviour than the one discussed in [19]. In that paper the authors found that the Einstein metric saturates while the string frame metric collapses. One might think that this has to do with the different initial condition they use: in the SDp brane context the natural initial conditions are of the type \(\dot{T}(0) \neq 0\) and \(T(0) = 0\) which correspond to an initial energy density above the tip of the potential. One could thus expect qualitatively that the resulting additional energy density may be enough to cause string-frame gravitational collapse (or stop the Einstein frame expansion that we observe). However we checked explicitly that these initial conditions \(T(0) = 0\) and \(\dot{T}(0) \neq 0\) lead qualitatively to the same asymptotic behaviour that we obtained.

We also verified that if one where to continue these solutions into the past one would encounter singular behaviour. This however is beyond the scope of this note as we are mainly interested in the dynamics of the time evolution from some initial configuration and so we do not care how this initial configuration was prepared in the first place. See [20, 21] for a discussion of singularity theorems in the tachyon matter context.

5 Discussion

In this note we found a solution corresponding to a decaying unstable D9 brane in type IIA string theory. The decaying brane is described by a time dependent tachyon profile and is coupled consistently to the graviton, the RR 9-form and the dilaton. Note once more that we were interested in static initial conditions with positive initial Hubble parameter.

The resulting asymptotic spacetime dynamics is described by a weak power-law expanding FRW metric. Our computation shows that the energy density stored in the tachyonic open string sector is transferred completely into the closed string sector. The large time behaviour is described by a Minkowskian string-frame metric supplemented by a time-dependent RR9 form \(C \sim t'\) and a decreasing string coupling \(e^\Phi \sim 1/t'\).
It would be very interesting to see how the inclusion of massive closed strings modifies the advocated picture. There is no a priori reason to neglect the massive closed string states, only then we do not have an analogous effective action description. However, we have shown that it is enough to include just the massless closed string modes to get $T_{00}[T] \to 0$. We believe that it would be very improbable that the inclusion of massive closed string states would undo this qualitative behaviour.

Note that the limit when we approach the tip of the potential is somewhat singular. With the static initial conditions $T'(0) = 0$ this corresponds to $T(0) = 0$ which leads, since the ODE’s are 2nd order, just to a constant vanishing tachyon $T(t) = 0$. The precise behaviour thus depends on the detailed form of the action for small $T$ (and possibly multiple derivative extensions) – therefore perhaps different treatment is needed. We leave this case as an interesting open problem.

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14