Plasma emission and radiation from ultra-relativistic Brunel electrons in femtosecond laser-plasma interactions

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Abstract. A highly intense femtosecond laser pulse incident on a plasma target of supercritical density, gives rise to reflected high-order harmonics of the laser frequency. The radiation model adopted here considers Brunel electrons -those re-injected into the plasma after performing a vacuum excursion- perturbed by a localized turbulent region of an electrostatic field that is generated during the interaction, and characterized by a soliton-like structure. The observed power spectrum is characterized by a power-law decay scaled as $P_m \sim m^{-p}$, where $m$ denotes the harmonic order. In this work an appeal is made to a radiation mechanism from a single particle model that shows harmonic power decays described -as previously reported from particle-in-cell simulations- within the range $\frac{2}{3} \leq p \leq \frac{5}{3}$. Plasma emission is strongest for values of the similarity parameter $S=n_e/(n_c a_0^2)$ in the range $1 \leq S \leq 5$, and where $n_e$ and $n_c$ are the ambient electron plasma density and the critical density, respectively, and $a_0$ is the normalized parameter of the electric field of the incident light. It was found that the radiation spectra obtained from the single particle model here presented is consistent with previously reported power-law 5/3 decays from particle-in-cell simulations.

1. Introduction
High harmonic generation in relativistic interactions from femtosecond laser pulses on solid density targets has stimulated interest in a number of potential applications in plasma diagnostics and technological developments in ultrafast optics for studying dynamical processes in matter, and more recently as sources for generating ultrashort-intense pulses in the attosecond range [1].

It has long been known from experiments and particle simulations that laser-generated harmonics from the interaction of a highly intense light pulse with overcritical density plasmas are characterized by a power-law decay that can extend over hundreds or even thousands of harmonic orders. The spectral decay has been characterized in different approximations from the laser-plasma parameters [2,3]. In the mildly relativistic regime Gibbon [2] first proposed an empirical relation from PIC simulations for the harmonic efficiency $\eta$, and showed that this scaled as $\eta \sim 17.2 a_0^4 m^p$, where the decay index $p=5$, and $a_0 = 8.5 (I_{20} \lambda_{micron}^2)^{1/2}$ is the normalized quiver momentum, with $I_{20}$ denoting the light intensity in units of $10^{20}$ Wcm$^{-2}$ and $\lambda$ the laser wavelength measured in microns. As values of $a_0$ increased above $a_0=1$, relativistic effects arise in the interaction physics. In picosecond-pulse experiments, the harmonic spectra was observed to span a range of values of $p$ for increasing intensities, from $p=5.5$ at $a_0 \sim 0.64$ to $p=3.38$ at $a_0 \sim 2.85$ [3]. More recently, Baeva et al. [4] proposed a model based on a similarity model, that the underlying physics is similar when the similarity
parameter \( S \) remains constant, with an associated so-called “universal” value 8/3 to the harmonic power decay. Furthermore, the \( S \)-similarity analysis predicts a cut-off in the harmonic spectra at \( m = \sqrt{8} \) \( \alpha \), where \( \gamma \) is the relativistic factor of the plasma surface and \( \alpha \) is of order one. No clear-cut evidence has been found for the spectral cut-off experimentally, though the spectral power decay was shown to be fitted by indexes in the range \( 2.3 \leq p \leq 2.7 \), with the upper limit corresponding to the predicted “universal” value.

2. Plasma effects

In mildly relativistic interactions, Boyd and Ondarza-Rovira (BOR) found evidence of emission at the plasma line and at its harmonics [5]. In the UR regime, the ponderomotive force of the Langmuir field is strong enough to govern plasma behaviour over time scales \( t > \omega_p^{-1} \) and highly localized electrostatic fields are generated in correspondence with density structures inside the plasma. These structures were identified as sources of emission at the plasma frequency and at its harmonics. Through PIC simulations BOR explored the harmonic power decay in UR laser-plasma interactions [6] and spectral decays characterized by an index \( p=5/3 \) were found and, at higher intensities, by \( p=4/3 \) for \( p \)-polarized light. When the incident laser pulse is normal to the plasma surface, the spectrum is invariably characterized by an 8/3 power law decay, regardless of the combination \( (a_0, n_e/n_c) \).

Contrasting this, plasma effects are evident in the power harmonic decay for oblique incidence, since the excitation of plasma oscillations enable the enhancement of emission with lower harmonic power decay indexes, resulting in much flatter spectra. As an example, the plasma oscillations for the interaction \( a_0 =10, \frac{n_e}{n_c} =40 \) and a pulse length \( \tau_p=17 \) fs are shown in figure 1a around the strong electrostatic field generated, as obtained from a PIC simulation. Figure 1b shows the corresponding density contours where the high plasma density spot represents a region of strong electrostatic field generation. Also Brunel trajectories of electrons can be discerned travelling deep inside the plasma.

**Figure 1**: Regions in the plasma where the emission source is correlated: (a) plasma electron density oscillations and (b) electron density contours at sites of high electrostatic field generation inside the plasma.

The Brunel electron trajectories were also observed to be correlated with the electrostatic fields in places of beam crossings, as shown in figure 2a. For \( p \)-polarization field amplification is also observed (figure 2b), being an order of magnitude greater than the \( s \)-polarized case. These reflected attosecond spikes correlate with the electron bunching where strong electrostatic fields are found, and identified as sources of emission [6].
3. Plasma and XUV harmonic emission from ultra-relativistically laser-accelerated electrons perturbed by soliton-like electrostatic plasma fields

In the following we make use of a simple dynamical model to explore the plasma effects on harmonic radiation emission from a charged particle relativistically accelerated by a strong electromagnetic pulse, and perturbed by a soliton-like electrostatic field generated in the plasma during the laser-plasma interaction as discussed in the previous section.

Radiation from electron bunching in large-amplitude Langmuir waves has been considered previously in cases where these have been excited in beam-plasma interactions. In particular, Weatherall and Hobbs [7] showed that electrostatic bunching was an effective means of generating plasma harmonics. Subsequently, Weatherall and Benford (WB) [8] modelled these fields as soliton-like structures generated inside the plasma and showed that these sources gave rise to strong nonlinear perturbations when high energy electron beams traverse such structures resulting in broadband bremsstrahlung emission. This model describes the radiation effects produced by the scattering of the electron beams that propagate inside the plasma through regions of highly intense localized electrostatic fields. These localized electrostatic structures cause the beam electrons to be scattered and to emit distinctive radiation. The WB model considers a specific shape and size of the soliton in which the characteristic turbulent density oscillation has a dipole structure within a localized region

\[ \rho = \rho_0 \frac{\hat{p} \cdot \vec{r}}{D} \exp \left( -\frac{r^2}{D^2} \right) \exp \left( -i \omega_r t \right), \]

(1)

here \( \rho_0 \) denotes the amplitude of the density oscillation localized by a Gaussian envelope of scale length \( D \), oscillating at the plasma frequency \( \omega_p \), with a dipole moment oriented in the direction of \( \hat{p} \).

It can be showed that the electric field strength near the centre of the soliton is

\[ E_y = -\frac{2}{3} D \rho_0 \exp \left( -i \omega_r t \right) \hat{p}. \]

(2)

The acceleration of a charged particle through a soliton field generates electromagnetic wave packets containing a broad spectrum of frequencies. The electrons in a uniform electron beam would radiate with no phase coherence, resulting in weak scattered emission. In contrast, an electron beam with nonuniform density can maintain a degree of phase coherence over a range of wavelengths comparable with the scale length of density fluctuations. Therefore, the superposition of coherent radiation from plasma electrons can result in strong emission if the beam is modulated or bunched. Furthermore, when the beam is ultrarelativistic the radiation could be greatly enhanced by relativistic
beaming along the direction of propagation of the laser wave. Beam radiation would be expected to dominate plasma emission when the beam density is high and the electrons highly relativistic.

Thus electron bunching results in a state of strong turbulence within the scale of the localized electrostatic field and a broad spectrum of density fluctuations develop. Beam density fluctuations produce regions of high \( E_s \) fields which would represent centres of electromagnetic radiation producing a plasmon condensate spectrum with a power-law index of \( \nu = 5/3 \). It can be demonstrated that the spectrum of beam density correlations leads to a structure function that gives the total energy radiated by the soliton, in the form

\[
\frac{dE(\omega)}{d\omega} \approx V \left( \frac{\omega - \omega_p}{v_0} \right),
\]

where \( v_0 \) is the electron beam velocity and \( V \) the beam density correlation, given by [9]

\[
V(k) \approx \frac{\Gamma(v + 1/2)}{\sqrt{\pi} \Gamma(v)} \frac{r_0}{(I + k^2 r_0^2)^{v+1/2}},
\]

where \( \Gamma \) is the gamma function, \( r_0 \) the perturbation scale length and the wave number \( k = (\omega - \omega_p)/v_0 \). The turbulent spectrum of density fluctuations gives coherence to the emission spectra when the electron beam is perturbed by scattering density centres.

By integrating (4) we found that the power-law index of \( \nu = 5/3 \) that governs the spectrum decay is retrieved when the parameter \( \nu = 1/3 \). Figure 3a shows the function \( V(k) \) representing the beam radiation power characterized by a prominent line at the plasma frequency, a power spectral decay 5/3 over high harmonic orders and an sudden cut-off representing an upper bound at the highest frequencies emitted by the electrons that are scattered by the solitonic structure.

Figure 3: (a) Beam radiation power spectrum from density correlations \( V(k) \) as given by equation (4), and (b) single particle radiation from a laser-accelerated electron perturbed by a soliton field (equation (2)), for \( a_0 = 40, n_e/n_c = 167 \), both having power-law index decays \( \nu = 5/3 \). The electron dynamics was integrated from equation (5).

Thus, the XUV spectral decay is described by a power law \( P(\omega) \sim \omega^{-5/3} \), below an abrupt cut-off at a critical frequency \( \omega \leq \omega_c = 2 \gamma^2 c/D \), where \( \gamma \) is the relativistic factor for the electron beam [8].

By integrating the Lorentz equation we determine the motion of an electron plasma relativistically accelerated by a short highly intense field of a laser pulse. The dynamics was obtained from the relativistic equations for the energy and momentum [10].
\[ m c \gamma = \beta \cdot p, \]
\[ \beta = \frac{e}{m c \gamma} \left[ E - \beta \left( E \cdot \beta \right) + \beta \times B \right] + E_x, \]  

where \( \gamma = (1 - \beta^2)^{-1/2} \) is the relativistic factor, with \( \beta = v/c \) the normalized electron velocity and \( c \) the speed of light in vacuum. The electric and magnetic fields of the monochromatic plane-polarized wave are given by \( E = \frac{\partial A}{\partial t} \) and \( B = \nabla \times A \), respectively, where \( A(\eta) = A_0 P(\eta) \left[ \hat{x} \delta \cos \eta + \hat{y} (1 - \delta^2)^{1/2} \sin \eta \right] \) is the electromagnetic potential and \( \eta = \omega_L t - k_L x \), the phase of the wave with propagation along the \( x \) direction. \( A_0, P(\eta) \) and \( \delta \) are the amplitude, the shape pulse and the polarization parameter, respectively. The relativistic dynamics of an electron is characterized by strong nonlinearities that transform the periodic motion of a driven harmonic oscillator into irregular motion, giving rise to very high order harmonics in the radiation spectra. A typical spectrum is shown in figure 3b for the combination \((a_0, n_e/n_c)=(40, 167)\). The spectrum is characterized not only by a prominent emission at the plasma line but also by a spectral 5/3-decay.

Figures 4 a-d show spectra obtained from different combinations of the parameters \((a_0, n_e/n_c)\), and where emission at the plasma line, and its harmonics, are clearly observed together with a harmonic power decay index in the range \( 2/3-5/3 \) at high orders. Some modulation around the plasma emission is also discerned in these simulations with a cut-off at a high harmonic order as predicted by the parameter \( \omega_c \).

4. Conclusions

The formulated single particle radiation model is not inconsistent with our previous results obtained from PIC simulations, with the observed highly localized electrostatic fields generated inside the plasma during the interaction, that correlate with the electron trajectory crossings produced when strongly accelerated beam-plasma electrons -the so-called Brunel electrons- are re-injected into the target by the laser electric field after performing vacuum excursions outside the plasma. The radiation
from perturbed plasma electrons by a soliton field representing the electrostatic fields in the plasma is reminiscent of the 5/3 UR power law decay observed in PIC emission spectra.

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References
[1] Teubner U and Gibbon P 2009 Rev. Mod. Phys. 81 445
[2] Norreys PA et al. 1996 Phys. Rev. Lett. 76 1832, Gibbon P 1996 Phys. Rev. Lett. 76 50
[3] Dromey B et al. 2006 Nat. Phys. 2 456
[4] Baeva T, Gordienko S and Pukhov A 2006 Phys. Rev. E 74 046404
[5] Boyd TJM and Ondarza-Rovira R 2008 Phys. Rev. Lett. 101 125004; 2008 Laser-Driven Relativistic Plasmas, AIP Conf. Proc. 1024 233
[6] Boyd TJM and Ondarza-Rovira R 2000 Phys. Rev. Lett. 85 1440, idem 2010 Phys. Lett. A 374 1517, idem 2012 Phys. Rev. E 86 026407
[7] Weatherall JC and Hobbs WE 1986 Phys. Fluids 29 2292
[8] Weatherall JC and Benford G 1991 Astrophys. J. 378 543
[9] Tatarski VI 1967 Wave Propagation in a Turbulent Media (New York: Dover)
[10] Boyd TJM and Sanderson JJ 2003 The Physics of Plasmas (UK: Cambridge University Press)