Fractal analysis of radioactive contamination as a result of accidents at radiation-hazardous facilities

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Abstract. We estimate the fractal dimension and the associated Hurst index for radioactive contamination of the territory resulting from the accidents at the Chernobyl and Fukushima-1 nuclear power plants. The fractal dimension (Hurst index) of the considered surfaces of significantly different areas of contamination density is estimated as they change over time. As a result of the study, the opinions of Hurst indicators for the considered surface type are given, and it is established that they remain almost constant in the considered time interval. The results of this work allow us to choose the value of the fractal dimension when modelling the zone of radiation pollution that occurred as a result of accidents at radiation-hazardous facilities, including nuclear power facilities, with a through release of radioactive substances outside the sanitary protection zones, and their distribution in conditions of high classes of atmospheric instability.

1. Introduction

Investigation of the spread of various impurities in the atmosphere resulting from various accidents at radiation-hazardous facilities is an important task. At the same time, the spread of radioactive impurities, provided they are located in the surface layer of a volumetric infection centre, is usually carried out in conditions of significant turbulence, uncertainty of wind direction and speed, which makes it possible to model the consequences of such accidents using fractal geometry methods.

Loss of radioactive material to the area of volumetric source of contamination in the atmosphere in the accident at radiation hazardous objects of nuclear energy leads to the formation of radioactive contamination, examples of which are shown in Figure 1 to the accidents at Fukushima-1 (a) and the Chernobyl nuclear power plant (NPP) (b) [1].

![Figure 1](image1.jpg)

**Figure 1.** Fragments of maps of Cs-137 isotope contamination density in the area after the accident at Fukushima-1 (a) and Chernobyl (b).
Figure 2. Maps of the density of Cs-137 isotope contamination in the Bryansk region after the Chernobyl accident as of 1986 – 2016 and 2026-2056 (forecast) [1].
As a result, a certain field of distribution of the density of contamination by one or another isotope is formed on the ground, which subsequently changes due to the processes of migration of this radionuclide in the medium and due to its decay. Currently, data on the results of measuring and forecasting these fields for various regions of Russia and Belarus (atlas of current and forecast aspects of the Chernobyl accident) are published, which reflects data on the dynamics of changes in this field in the period from 1986 to the present, and its forecast until 1956. As an example, maps of the Cs-137 isotope contamination density in the Bryansk region are shown (Fig. 2).

2. Methods
The formation of fields of this type is influenced by many different factors, the exact value and characteristics of which cause difficulties due to the significant nature of their uncertainty. First of all, it concerns the uncertainty in the direction and speed of the wind in the surface layer [2-7]. Features of the spread of radioactive substances at a relatively low altitude from the earth's surface allow us to make assumptions that the formed field may be fractal (self-similar), which is indirectly confirmed by the similarity of the fields under consideration with known fractal structures (terrain, etc.) [8-12].

The fractal surface is characterized by a fractal dimension, which is determined in accordance with the Equation 1:

$$D = \lim_{\varepsilon \to 0} \frac{\ln N_\varepsilon}{\ln \varepsilon}$$

where $\varepsilon$ – the scale factor, $N_\varepsilon$ – the number of elements (parts corresponding to this scale).

For ordinary objects (straight line, plane, and space), the dimension defined in this way coincides with the topological dimension and is equal to 1, 2, or 3, respectively. The fractal dimension itself can take non-integer values and may exceed the topological dimension of a particular element.

The Hurst exponent is associated with the fractal dimension $H$, which can be calculated using the ratio $D = 2 - H$.

Methods for estimating the fractal dimension of various objects are known. Their general idea boils down to the choice of a certain scale factor $\varepsilon$ and counting the number of elements of the curve (surface or volume) corresponding to this scale in the analysed surface (according to Eq. 1). Then the fractal dimension itself can be calculated using this formula or, as a rule, it is found through the tangent of the slope of the corresponding approximating line in double logarithmic coordinates (Fig. 3). The tangent of the slope angle, considering the sign, coincides with the fractal dimension.

![Figure 3. Estimating the fractal dimension of surfaces by the coating method based on the angle of inclination of the corresponding approximating line.](image-url)
Considering the cross-section of this two-dimensional field (on the plane), we obtain a certain curve, for which we also determine the fractal dimension, similar to the one under consideration (Fig. 4). The figure shows a field obtained by random addition for the Hurst exponent equal to 0.9. The cross-section of this field with the normal plane to the surface along lines 1, 2, and 3 gives the corresponding fractal lines, the dimension of which can be determined in the same way.

![Figure 4. Modelling a two-dimensional surface by random addition with the Hurst index $H = 0.9$ (a) and sections of this surface along some straight lines 1, 2, and 3 (b). The colour shows different values of the simulated value at any scale. A fractal curve obtained on the plane when considering the line of this level (c).](image)

For the analysed fields, the curves obtained as a result of similar cross-sections are usually unknown, but in this case, we can consider the dimension of the curve formed on the plane by a given level (Fig. 4c). Curves similar to the given ones represent the boundaries of zones with different values of pollution density and are visually easy to trace on maps. Analysis of the fractal dimension of such conditionally "one-dimensional" curves consists in counting the number of cells $N_ε$, where such a curve falls when the cell scale changes $ε$ and further construction of approximating curves. This is explained in Figure 5 for some arbitrary scale $ε$, coinciding in this case with the side of the square.

Similarly, you can build dependences for a two-dimensional surface. Note that we can consider similar partitions and calculate the corresponding polyline lengths for the one-dimensional case or the area of zones for the two-dimensional case.

Based on the available data on contamination of the area as a result of the accident at the Chernobyl nuclear power plant, it is possible to estimate the resulting fractal dimensions of the corresponding fields on the examples of various regions and their time dynamics.

The calculation was performed using a software product that works according to the algorithm described above.
3. Results

The calculation results for the Bryansk region by year are shown in Table 1. The value of the Hurst index for the analysed 10 regions from the Atlas of current and forecast consequences of the Chernobyl nuclear power plant accident is in the range of 0.90-0.92. For Fukushima-1, the obtained value of the Hurst index is close to the values of the Chernobyl accident and is 0.88.

Table 1. Parameters of calculated surfaces with different numbers of control points, relative units.

| Year | 1986 | 1996 | 2006 | 2016 | 2026 | 2036 | 2046 | 2056 |
|------|------|------|------|------|------|------|------|------|
| Estimation of the average value of the Hurst index | 0.92 | 0.94 | 0.89 | 0.90 | 0.95 | 0.92 | 0.93 | 0.90 |

The Hurst index corresponding to the considered pollution density fields for the analysed areas is close to 0.9. There was no significant correlation of the indicator over time.

The obtained estimate of the Hurst index can be used for modelling areas of radioactive contamination by known methods of fractal geometry. For example, when using the random addition method for this indicator, you can get pictures of fields that are qualitatively well consistent with the observed ones (Fig. 6).

![Figure 6](image_url)
4. Conclusion
The resulting value can be used to create a universal, but coarser model. This allows us to solve the problem of operational modelling of radioactive contamination of the territory after accidents at radiation-hazardous facilities in unstable, non-standard climatic conditions. In relation to this problem, the methods of fractal geometry have a number of advantages, such as: the efficiency of modelling and versatility, which play an important role in the condition of a radiation accident [13-15].

For a more accurate assessment of the characteristics of the fractal dimension of the fields under consideration, it is necessary to analyse the relationship between the obtained dimension and the resulting dimension of the curves of the corresponding pollution level.

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