Fulde-Ferrell-Larkin-Ovchinnikov state in $d$-wave superconductors

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Abstract

The Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state can be found in layered $d$-wave superconductors such as high $T_c$ cuprates and $\kappa$-(ET)$_2$ salts in a planar magnetic field. We show the superconducting order parameter forms two dimensional crystalline lattice in the FFLO state for $d$-wave superconductors. Also the quasiparticle density of states and the thermodynamics of FFLO state are constructed. Therefore STM or NMR will provide a definitive test for the existence of the FFLO state.

Key words:
$d$-wave superconductivity, Layered materials, Fulde-Ferrell-Larkin-Ovchinnikov state

When the Pauli paramagnetism or the Zeeman term dominates the orbital effect, the superconducting state in a magnetic field enters in a new state where the superconducting order parameter varies periodically in space[1,2]. However the realization of this condition is extremely difficult in the classical s-wave superconductors. First of all the system has to be in the superclean limit where the quasiparticle mean free path $l$ is much longer than the coherence length $\xi$. Second the system has to have the Ginzberg Landau parameter $\kappa$ much larger than the unity. Indeed these two conditions appear to be met readily in $d$-wave superconductors like high $T_c$ cuprate superconductors and organic superconductors like $\kappa$-(ET)$_2$ salts and $\lambda$-(ET)$_2$ salts [3].

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Earlier we have shown that the region of the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state in $d$-wave superconductors is much more extended than the corresponding one in $s$-wave superconductors [4]. In the mean time a possible observation of the FFLO state in $\kappa$-(BEDT-TTF)$_2$Cu(NCS)$_2$ in a planar magnetic field has been reported [3]. Indeed in the pure Pauli limiting case we expect the upper critical field $H_{c2}(t, \theta)$ is independent of the direction of the magnetic field as long as $\mathbf{H}$ lies in the conducting plane [4].

The object of this paper is to extend the earlier analysis to construct the free energy associated with FFLO state. Indeed in the limit of $T \to 0$, following Larkin and Ovchinnikov we can show that the two dimensional periodic solution $\Delta(r) \sim \cos(qx) + \cos(qy)$ is the most stable. Also we obtain simple expression of the quasiparticle density of states, the specific heat, the magnetic susceptibility and $T_1^{-1}$ in NMR which should be readily accessible experimentally.

In Fig.1 we show the magnetic phase diagram of $d$-wave superconductors. As is readily seen, the direction of $\mathbf{q}$ in the most stable FFLO state switches from $\mathbf{q} || (1, 0, 0)$, (0, 1, 0), etc. (the direction of 4 maximum gaps) in the limit of $T = 0$ to $\mathbf{q} || (1, 1, 0)$, (1, $-1$, 0), etc. (the direction of 4 nodes) for $T > 0.05 T_c$. In the first region (i.e. $\mathbf{q}$ is the direction of one of 4 maximum gaps) $q = 2{\hbar \nu_p}$ at $T = 0$, where $\hbar = \mu_B H$, while in the second region (i.e. $\mathbf{q} || (1, 1, 0)$, etc.) $q \simeq 2.42{\hbar \nu_p}$. At $T = 0$ K the upper critical field is given by $h_0 = \frac{\pi}{7}T_c \exp(\frac{1}{4} \cos(4\theta))$ with $q = 2{\hbar \nu_p}$ and $\theta = 0$, where $\theta$ is the angle between $\mathbf{q}$ and the $a$-axis. Here $\gamma$ is the Euler constant.

Then following[2] we construct the equation for $\Delta(r)$, assuming that $\Delta(r) = \sum m \Delta_m e^{i\mathbf{qm} \cdot \mathbf{r}}$, as

$$
\Delta^*_n = \sum_m \{(2 - \delta_{mn})|\Delta_m|^2 \Delta_n \tilde{J}(q_n, q_m) + (1 - \delta_{mn} - \delta_{-mn}) \Delta^*_m \Delta^*_{-m} \Delta_n J(q_n, q_m) \}
$$

(1)

where $J(q_n, q_m)$ and $\tilde{J}(q_n, q_m)$ are defined in the same way as in [2]. But here we generalized their treatment to the $d$-wave superconductivity. Both $J(q_n, q_m)$ and $\tilde{J}(q_n, q_m)$ depend on only angle between $\mathbf{q}_m$ and $\mathbf{q}_n$. We have $h^2 J(0) = -15.5, h^2 J(\pi) = 3/2, h^2 J(\pi/2) = 3.0$ and $h^2 \tilde{J}(\pi/2) = 5/6$

Then we can solve for different forms for the lattice. Here are;

(a) $\Delta(r) \sim e^{i\mathbf{q} \cdot \mathbf{r}}$, we have $|\Delta|^2 = -0.065\hbar_0(\hbar_0 - \hbar)$

(b) $\Delta(r) \sim \cos(qx)$, we have $|\Delta|^2 = -0.08\hbar_0(\hbar_0 - \hbar)$

(c) $\Delta(r) \sim \cos(qx) + \cos(qy)$, we have $|\Delta|^2 = \frac{5}{7}\hbar_0(\hbar_0 - \hbar)$

Therefore the solution (c) is the only one which is stable in the layered $d$-wave superconductors. Limiting ourselves to $d$-wave superconductors in a planar magnetic field, we conclude the two dimensional periodic solution, i.e. 2 di-
dimensional square lattice, is the most stable one in the limit $T \to 0$ K. This is in a sharp contrast to the case of 3D $s$-wave superconductors where a stripe-like state is favored [2].

The quasiparticle density of states in the vicinity of $h = h_0$ is given by

$$N(E)/N_0 \simeq 1 + \frac{\Delta^2}{4} \sum_{\pm} \left( \frac{\cos^2(2\phi)}{|E \pm h + \frac{vq}{2} \cos \phi|^2} \right) = 1 + \frac{\Delta^2}{2\hbar^2} J \left( \frac{E}{\hbar}, \frac{vq}{2\hbar} \right)$$  \hspace{1cm} (2)

and

$$J(\varepsilon, p) = \frac{1}{2} \sum_{\pm} \text{Re} \{ \frac{|\varepsilon \pm 1|}{(|\varepsilon \pm 1|^2 - p^2)^{3/2}} + \frac{4}{p^1} [(3(\varepsilon \pm 1)^2 - p^2)]$$

$$- (3(\varepsilon \pm 1)^2 - 2p^2) \frac{|\varepsilon \pm 1|}{\sqrt{(\varepsilon \pm 1)^2 - p^2}} \} \} \}$$  \hspace{1cm} (3)

and

$$J(0, p) = \text{Re} \frac{1}{(1 - p^2)^{3/2}} + \frac{4}{p^1} \frac{3 - p^2}{2} \frac{1}{\sqrt{1 - p^2}} (3 - 2p^2) \text{Re} \frac{1}{\sqrt{1 - p^2}}$$  \hspace{1cm} (4)

In Fig.2 we show the density of state for $p = \frac{vq}{2\hbar} = 1, 1.1$ and 1.21. As readily seen from Eq.(2) $N(E)$ diverges for $E = (p-1)h$ and $E = (p+1)h$ where $p = \frac{vq}{2\hbar}$ for $p \geq 1$. When $p = 1$, $N(E)$ diverges at $E = 0$ and $E = \pm 2h$. On the other hand when $p > 1$, the density of states at Fermi energy is given by

$$N(0)/N_0 = 1 + 2\left( \frac{\Delta}{\hbar} \right)^2 p^{-4} (3 - \frac{p^2}{2})$$  \hspace{1cm} (5)

$N(0)$ can be accessible through the spin susceptibility at $T = 0$ K, $\chi/\chi_n = N(0)/N_0$, or the nuclear spin lattice relaxation rate at $T = 0$ K, $T^{-1}_1/T_{1n}^{-1} = (N(0)/N_0)^2$ etc. It is shown in the inset of Fig.2. Also for $p > 1$, $N(E)$ has two peak at $E = (p-1)h$ and $E = (p+1)h$. These peaks should be detectable by STM for example. Therefore the presence of 2 peaks provides a clear sign of FFLO in $d$-wave superconductors.

References

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Fig. 1. The phase diagram of FFLO state in $d$-wave superconductor. The solid line describes the critical magnetic field for $q \parallel (1,0,0)$, while the broken line for $q \parallel (1,1,0)$. The dotted line is the 1st order transition line for the uniform state. $\Delta_{0d}$ is the $d$-wave superconducting order parameter at $T = 0$.

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Fig. 2. The density of state in Eq. (2) for $vq = 1, 1.1$ and $1.21$ is shown. $J(0, p) = \frac{2\hbar}{\Delta^2} (N(0)/N_0 - 1)$ is shown in the inset.