Efficient Text Classification Using Tree-structured Multi-linear Principle Component Analysis

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Abstract—A novel text data dimension reduction technique, called the tree-structured multi-linear principle component analysis (TMPCA), is proposed in this work. Being different from traditional text dimension reduction methods that deal with the word-level representation, the TMPCA technique reduces the dimension of input sequences and sentences to simplify the following text classification tasks. It is shown mathematically and experimentally that the TMPCA tool demands much lower complexity (and, hence, less computing power) than the ordinary principle component analysis (PCA). Furthermore, it is demonstrated by experimental results that the support vector machine (SVM) method applied to the TMPCA-processed data achieves commensurable or better performance than the state-of-the-art recurrent neural network (RNN) approach.

I. INTRODUCTION

Text classification has been an active research topic for about two decades. Its implementations such as spam email detection, age/gender identification and sentiment analysis are omnipresent in our daily lives. Tasks on smaller datasets are generally regarded as simple ones, which can be typically handled by linear models such as the naive Bayes [1] classifier nowadays. Although text classification is an intensively studied topic, it still faces challenges as the tasks become more complex due to ever-increasing data amount and diversity in the Internet.

The increased complexity partly comes from diversified text patterns as a result of a larger vocabulary set and/or more sentence variations of similar meanings. Such a phenomenon is known as “the curse of dimensionality” [2]. To address the high data dimension problem, one way to reduce the data dimension lies in an efficient numericalization (or embedding) of text data. Typically, dimension reduction is conducted at the word level so that it is called the word embedding process. As the accumulated volume of language data throughout the Internet becomes higher, simple models trained on limited datasets with word embedding cannot keep up with the complexity of existing tasks [3].

In this research, we go one step further by embedding the entire input sequence and/or sentence into one vector while keeping the sequential patterns as intact as possible with an objective to facilitate the classification task that follows. However, the complexity of the whole sequence/sentence embedding is extremely high. This is only made possible by introducing some novel technique. The main contribution of this work is the proposal of a novel technique, called the tree-structured multi-linear principle component analysis (TMPCA), to reduce the dimension of input data efficiently. The TMPCA tool is applied to whole sequence/sentence embedding so that they can be effectively trained and tested using machine learning models and tools. We will show that the TMPCA tool can retain word correlations (i.e., text patterns) in the input sentence in a compact form. Furthermore, it demands much less computing power than the ordinary PCA in the training process.

The rest of this paper is organized as follows. Related previous work is reviewed in Sec. II. Then, the new TMPCA text dimension reduction technique is presented in Sec. III. Experimental results are given in Sec. IV where we compare the performance of methods using the TMPCA technique and other state-of-the-art methods. Finally, concluding remarks are drawn in Sec. V.

II. RELATED PREVIOUS WORK

The idea of representing text patterns (or numericalized sentences) with a compact vector is explored in the development of recurrent neural networks (RNN). Such a representation is stored as a hidden state of RNN’s basic computing unit known as the memory cell. There are two popular cell designs: the long short-term memory (LSTM) [4] and the gate recurrent unit (GRU) [5]. The block diagram of an LSTM system is shown in Fig. 1. As shown in this figure, each cell takes the element from a sequence as its input sequentially and computes an intermediate value that can be recurrently updated. Such a value is called the constant error carousal (CEC) in the LSTM and simply a hidden state in the GRU. Usually,
many cells are connected to form a complete RNN, and the intermediate value from each cell forms a vector called the hidden state.

It was observed in [6] that, if a hidden state is properly trained, it can represent the desired text patterns compactly and group similar semantic word level features closely. This property was further analyzed in [7]. Generally speaking, for a well designed representational vector (namely, the hidden state), the computing system (i.e., the memory cell) is powerful in exploiting the word-level dependency to facilitate the final classification task.

The representation power of a hidden state has been utilized in sequence-to-sequence (seq2seq) learning [8], [9], which is a variant of the RNN model. Its block diagram is shown in Fig. 2. The Seq2seq system reduces a higher dimensional input sequence into a lower dimensional hidden state using an encoder-decoder structure as shown in Fig. 2. Both the encoder and the decoder are implemented as RNN models. The encoder cell takes an input sequence of variable length and stores the text patterns in the hidden state. This is known as “encoding”. Given the hidden state information, the decoder learns how to correlate the input automatically and generate the desired output in the decoding process.

Ideas in reducing the numericalized text dimension are also present in non-neural-network based methods. Traditional text classification often uses the stop-word list to remove non-informative words such as prepositions, conjunctions, etc. The principle component analysis (PCA) was applied to matrices in [10], [11], where each entry indicates the frequency of a term occurring in a document. Text dimension reduction was also conducted in [12] by removing words of low information gain and, then, applying the PCA to the word-level feature space. Words of similar distributions were clustered and then a naive Bayes classifier was trained for text classification in [13].

All the aforementioned methods ignore the word positional information and adopt the “bag-of-words” representation in dimension reduction. Apparently, the sequential correlation of words in sentences is lost in such a treatment.

III. TREE-STRUCTURED MULTI LINEAR PCA (TMPCA)

In this section, we propose a new technique for text data dimension reduction and name it “tree-structured multi-linear principle component analysis (TMPCA)”. As compared with traditional text dimension reduction methods that focus on the word-level representation, the TMPCA technique is designed to reduce the dimension of the entire sentences or sequences while preserving the sequential order of composing words. It generates a more compact representation than the hidden state of RNNs. By eliminating the redundant information in sentences/sequences, it alleviates the overfitting problem in classifier training. We will elaborate the detailed design below.

A. Proposed TMPCA Algorithm

The TMPCA method reduces the sentence length yet keeps the word embedding size. Its block diagram is shown in Fig. 3. Suppose that each element at the top level, denoted by \( W_0 \), \( n = 1, \ldots, N \), has an embedding dimension of \( D \). At each level of the tree, every two adjacent elements are concatenated to form one vector of dimension \( 2D \) to train the PCA kernel. Multiple input vectors serve as the rows of the input matrix to the PCA. Each PCA transform two input vectors from the upper level to its corresponding lower level. The dimension of the output vector is halved from \( 2D \) to \( D \) by the PCA. For example, consider a sentence composed by four elements: \( \{ w_1^0, w_2^0, w_3^0, w_4^0 \} \), where \( w_n^0 \in \mathbb{R}^D, \forall n \in \{1,2,3,4\} \). The superscript denotes the level in the TMPCA tree and level 0 means the original numericalized sentence/word sequence.

The following data matrix is used to represent the input:

\[
\begin{bmatrix}
(w_1^0)^T & (w_2^0)^T \\
(w_3^0)^T & (w_4^0)^T
\end{bmatrix}
\]

The output of the first level MPCA can be written as

\[
\begin{bmatrix}
(w_1^1)^T \\
(w_2^1)^T
\end{bmatrix}, \quad w_1^1 = U\left( \begin{bmatrix} w_1^0 \\ w_2^0 \end{bmatrix} \right), \quad w_2^1 = U\left( \begin{bmatrix} w_3^0 \\ w_4^0 \end{bmatrix} \right)
\]

and where \( U \) is the PCA’s transform matrix defined on the sentence length dimension, and \( U \in \mathbb{R}^{D \times 2D} \). The new transformed sentence at level 1 can be expressed as \( \{ w_1^1, w_2^1 \} \). Then, it serves as the input to the next level TMPCA transform.

It is apparent that, after one-level TMPCA, the sentence length is halved while the word embedding size, \( D \), keeps the same; namely, \( w_i^1 \in \mathbb{R}^D, \forall i \in \{1,2\} \). Such a process is repeated until the whole sentence is reduced to a single-word vector.

| TABLE I: Change of dimension |
|-----------------------------|
| Input sentence | Sentence length | Embedding size |
| TMPCA Transformed sentence | 1 | D |

The dimension evolution from the initial input data to the ultimate transformed data is summarized in Table I. The original input sentence has \( N \) words and each word is
embedded with a $D$-dimensional vector. After the $\log_2 (N)$-level TMPCA transform, the sentence length becomes one word of embedding dimension $D$.

**B. Computational Complexity Analysis**

To analyze the computational complexity of the TMPCA algorithm, we consider a sentence of length $N = 2^k$. The total number of training sentences is $M$, and the word embedding size is $D$. To fit the PCA model to this training matrix of dimension $\mathbb{R}^{M \times ND}$ requires $O(MN^2D^2)$ to compute the covariance matrix of the dataset, and $O(N^3D^3)$ to compute its eigenvalues.

At level $s$, the dimension of the training matrix is equal to $M \frac{N^{2^s}}{2} \times 2D$. Thus, the total computational complexity of the TMPCA algorithm can be derived as

$$O(f_{TMPCA}) = O\left(\sum_{s=1}^{\log_2 N} (2D)^3 + \frac{M N}{2^s}(2D)^2\right)$$

$$= O\left(8LD^3 + 4M(N - 1)D^2\right)$$

$$= O\left(2LD^3 + MN D^2\right).$$

The complexity of the traditional PCA can be written as

$$O(f_{PCA}) = O\left(N^3 D^3 + MN^2 D^2\right).$$

By comparing (3) and (4), we see that the time complexity of the TMPCA algorithm grows at most linearly with sentence length $N$. Furthermore, if $N \ll D$, $O(f_{TMPCA})$ grows logarithmically with $N$. In contrast, the traditional PCA grows at least quadratically with $N$. Thus, $f_{PCA}$ grows much faster than $f_{TMPCA}$, or $f_{TMPCA} = o(f_{PCA})$

If we concatenate $P$ non-overlapping elements at each tree-level, the time complexity is then:

$$O(f_{TMPCA}) = O\left((P^3 \log_2 N)D^3 + MPND^2\right).$$

As shown in Eq. (5), the time complexity increases with $P \in \{2, \cdots, N\}$. The worst case is $P = N$, which is the traditional PCA applied to the entire sentence.

One reason to combine two non-overlapping elements at each tree-level is its computational efficiency. Another is that it can preserve the sentence structure well. We set all sentences to be of the same length $N$. Any sentence of length shorter than $N$ is padded by one or more special symbols to length $N$. Sentences of length longer than $N$ will be truncated to $N$. The sentences are tokenized, and each token corresponds to an embedded/numericalized word vector. The embedding can be done by either one-hot vector embedding or word2vec embedding as reviewed in Sec. [IV]

**IV. EXPERIMENTAL RESULTS**

**A. Experimental Setup**

We conducted experiments on the following four datasets.

1) SMS Spam dataset (SMS SPAM). It has “Spam” and “Ham” as two target classes.
2) Standford Sentiment Treebank (SST). It has “positive” and “negative” as two target classes. The labels are generated using the Stanford CoreNLP toolkit [14]. The sentences labeled as very negative or negative are grouped into one negative class. Sentences labeled as very positive or positive are grouped into one positive class. We keep only the positive and negative sentences for training and testing.
3) Semantic evaluation 2013 (SEMEVAL). We focus on Sentiment task-A with positive/negative two target classes. Sentences labeled as neutral are removed.
4) Cornell Movie review (IMDB). It contains a collection of movie review documents with their sentiment polarity - positive or negative.

More details of these four datasets are given in Table III.

The sentence length is fixed for each dataset for the four benchmarking methods; namely, TMPCA with SVM, PCA with SVM, SVM only and RNN. The sentence length values are shown in Table III. To numericallyize the text data, we remove stop words from sentences according to the stop-word list, tokenize sentences and, then, stem tokens using the python natural language toolkit (NLTK). Afterwards, we use the Wiki2vec embedding [13] to embed stemmed tokens into vectors. The embedding size is 1000. We used the SVM as the classifier and applied it on these embedded vectors in their raw forms or processed by TMPCA and PCA, respectively.

We compare the performance of the following four methods:
- SVM: Raw embedded features followed by SVM;
- TMPCA+SVM: TMPCA-processed features followed by SVM;
- PCA+SVM: PCA-processed features followed by SVM;
- RNN.

The input to these methods is a single long vector by concatenating all embedded word vectors from a sentence in order. The first three were trained on Intel Core i7-5930K CPU while the RNN was trained on the GeForce GTX TITAN X GPU. The setup of the RNN is given in Table IV.

B. Experimental Results

The error rates of four benchmarking methods for four datasets are shown in Table V where the best figures are highlighted in boldface. We see from Table V that, although the data dimension is reduced to approximately a 32th or 64th of the original size, the performance of the TMPCA+SVM method does not degrade as much as that of the PCA+SVM method. This substantiates the claim that TMPCA is better in preserving the structure of input sentences. In addition, the fact that SVM performs better on reduced datasets demonstrates that the TMPCA is able to remove the weakly or non-correlated information from the dataset while preserving the principle ones. This helps alleviate the overfitting problem.

We compare the total training time for the TMPCA+SVM, PCA+SVM and RNN methods against the four datasets in Fig. 4(a)-(d). Clearly, the total training time taken by TMPCA+SVM is shorter than PCA+SVM and RNN. It is also worthwhile to mention that the computation of TMPCA and SVM was done on the CPU while the RNN was run on the GPU. The latter is known to be more efficient in large scale data computation.

As a result of the reduced dimension, the SVM training time on the TMPCA processed data is only a fraction of time used by the SVM on the original data. Their SVM training time is compared in Fig. 5(a)-(d). We can see the advantage of data reduction clearly in training time saving.

Since the N-gram technique is popular in text classification, we would like to see whether it can bring any benefit to the proposed TMPCA+SVM method. We preprocessed sentences into different gram forms. For example, the bigram form of a sentence consisting of 4 words denoted by “1, 2, 3, 4” is “12, 23, 34”. Its 3-gram is in form of “123, 234”, etc. Then, we train the TMPCA+SVM model on the N-gram preprocessed sentences, and test the model on the original test sentences, which are not N-gram preprocessed, to check the robustness of the TMPCA method. The value of $N$ chosen in our experiments were 1, 2, 4 and 8.

The error rate results are shown in Fig. 6. We see that the TMPCA method is robust with respect to the test dataset although it was trained on the N-gram preprocessed data. This can be explained as follows. The TMPCA only examines the local property of grams in the first tree-level. In the following levels, the redundant information between grams are removed, making the final reduced output contains only semantic patterns. These patterns are also present in sentences which are not preprocessed by the N-gram approach. The TMPCA performs the best with the unigram in the SPAM error rate, with the bigram in the IMBD and SST datasets, with the 4-gram in the SemEval dataset.

Overall, the TMPCA+SVM method achieves the state-of-the-art performance that is commensurable or better than the RNN method. Such a dimension reduction technique is very attractive by considering its less computing power, shorter processing time and lower error rates. The same technique could be beneficial to other NLP problems as well.

V. Conclusion

A novel data processing technique called the TMPCA was proposed for text classification problems in this work. The TMPCA can efficiently reduce the dimension of the entire sentence data to facilitate the machine learning task that follows. The complexity of the TMPCA was analyzed mathematically to demonstrate its computational efficiency over the traditional PCA method. Furthermore, the classifiers need less training time to fit the TMPCA processed data due to dimension reduction. Finally, the TMPCA method achieves the lowest error rates in all four datasets among benchmarking methods. We would like to apply the TMPCA technique to other challenging tasks and datasets as an extension of our current research efforts.

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TABLE II: Dataset

| Dataset   | Target Classes | Train Sentences | Development Sentences | Test Sentences | # of Tokens |
|-----------|----------------|-----------------|-----------------------|----------------|-------------|
| SMS SPAM  | 2              | 5,574           | 500                   | 18,657         | 14,657      |
| SST       | 2              | 5,409           | 500                   | 18,03          | 18,519      |
| SEMEVAL   | 2              | 5,098           | 915                   | 20,34          | 25,167      |
| IMDB      | 2              | 10,162          | 500                   | 20,892         | 20,892      |

TABLE III: Sentence Length

|               | SMS SPAM | SST | SEMEVAL | IMDB |
|---------------|----------|-----|---------|------|
| Sentence Length (N) | 64       | 64  | 32      | 64   |

Fig. 4: Comparison of the total training time of three methods against four datasets.

TABLE IV: RNN setup details.

| RNN model          | seq2seq with attention [9] |
|--------------------|----------------------------|
| Cell               | LSTM [4]                   |
| Number of layers   | 1                          |
| Embedding size     | 512                        |
| Number of cell     | 512                        |
| Training steps     | 10 epochs                  |
| Learning rate      | 0.5                        |
| Training optimizer | AdaGrad [16]               |

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## TABLE V: Error Rate (%)

|       | TMPCA+SVM | PCA+SVM | SVM | RNN |
|-------|-----------|---------|-----|-----|
| SMS SPAM | 2.33     | 2.87    | 2.69 | 2.51 |
| SST    | 21.24    | 27.18   | 27.07 | 24.57 |
| SEMEVAL | 24.09   | 24.49   | 24.78 | 25.17 |
| IMDB   | 24.40    | 25.60   | 29.20 | 30.20 |

Fig. 5: Comparison of the SVM training time based on the TMPCA processed data and the raw input data.

Fig. 6: Comparison of error rates for the TMPCA method with N-gram preprocessed data where \( N = 1, 2, 4, 8 \).