PROOF OF A SUPERCONGRUENCE CONJECTURE OF (F.3) OF SWISHER USING THE WZ-METHOD

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Abstract. For a non-negative integer \( m \), let \( S(m) \) denote the sum given by

\[
S(m) := \sum_{n=0}^{m} \left( -1 \right)^n (8n + 1) \frac{\left( \frac{1}{4} \right)_n^{3}}{n!^3}.
\]

Using the powerful WZ-method, for a prime \( p \equiv 3 \pmod{4} \) and an odd integer \( r > 1 \), we here deduce a supercongruence relation for \( S(p^r - \frac{3}{4}) \) in terms of values of \( p \)-adic gamma function. As a consequence, we prove one of the supercongruence conjectures of (F.3) posed by Swisher. This is the first attempt to prove supercongruences for a sum truncated at \( p^r - (d - 1)d \) when \( p^r \equiv -1 \pmod{d} \).

1. Introduction and statement of results

Throughout the paper, let \( p \) be an odd prime. For \( n \in \mathbb{N} \), the \( p \)-adic gamma function is defined as

\[
\Gamma_p(n) = (-1)^n \prod_{0 < j < n, p \nmid j} j.
\]

If \( \mathbb{Z}_p \) denote the ring of \( p \)-adic integers, then one can extend \( \Gamma_p \) to all \( x \in \mathbb{Z}_p \) by setting

\[
\Gamma_p(x) = \lim_{n \rightarrow x} \Gamma_p(n),
\]

where \( n \) runs through any sequence of positive integers which \( p \)-adically approaches \( x \) and \( \Gamma_p(0) = 1 \).

For a non-negative integer \( m \), let \( S(m) \) denote the sum given by

\[
S(m) := \sum_{n=0}^{m} \left( -1 \right)^n (8n + 1) \frac{\left( \frac{1}{4} \right)_n^{3}}{n!^3},
\]

where the rising factorial or the Pochhammer's symbol \((a)_n\) is defined as

\[ (a)_0 := 1 \text{ and } (a)_n := a(a+1) \cdots (a+n-1) \text{ for } n \geq 1. \]

In \[15\], VanHamme developed \( p \)-adic analogues for some of the Ramanujan's \[13\] formula for \( \frac{1}{\pi} \) series relating truncated hypergeometric series to the values of \( p \)-adic gamma function. For example, VanHamme proposed that the Ramanujan's formula

\[
S(\infty) := \sum_{n=0}^{\infty} \left( -1 \right)^n (8n + 1) \frac{\left( \frac{1}{4} \right)_n^{3}}{n!^3} \frac{2 \sqrt{2}}{\pi} = \frac{2 \sqrt{2}}{\pi}
\]

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has the nice $p$-adic analogue

\begin{equation}
S \left( \frac{p - 1}{4} \right) \equiv - \frac{p}{\Gamma_p(\frac{3}{4}) \Gamma_p(\frac{1}{4})} \pmod{p^3}
\end{equation}

if $p \equiv 1 \pmod{4}$. Using hypergeometric series identities, Swisher [14] and He [9] independently proved (1.1) together with some other supercongruences of VanHamme [15]. They also extended (1.1) to the primes $p \equiv 3 \pmod{4}$, and proved that

\begin{equation}
S \left( \frac{3p - 1}{4} \right) \equiv - \frac{3p}{\Gamma_p(\frac{3}{4}) \Gamma_p(\frac{1}{4})} \pmod{p^3}.
\end{equation}

Extending the Ramanujan-type supercongruences of VanHamme, Swisher [14] further listed a number of general VanHamme-type supercongruence conjectures supported by computational evidence computed using Sage, some particular cases of which have been proved by He [7, 8, 10], Chetry and the second author [1], and the authors [12]. The general VanHamme-type supercongruence conjecture of Swisher corresponding to (F.2) states that

$$
\begin{cases}
S \left( \frac{p - 1}{4} \right) \equiv (-1)^{\frac{p-1}{4}} pS \left( \frac{p - 1}{4} \right) \pmod{p^{3r}} & p \equiv 1 \pmod{4}, \\
S \left( \frac{p - 1}{4} \right) \equiv p^2 S \left( \frac{p - 1}{4} \right) \pmod{p^{3r-2}} & p \equiv 3 \pmod{4}, r \geq 2 \text{ even}, \\
S \left( \frac{p - 3}{4} \right) \equiv p^2 S \left( \frac{p - 3}{4} \right) \pmod{p^r} & p \equiv 3 \pmod{4}, r \geq 3 \text{ odd.}
\end{cases}
$$

Using the powerful $q$-WZ method and properties of cyclotomic polynomials, Guo [6] gave $q$-analogous for (1.1) and (1.2), and proved the generalizations, for positive integers $d,r$ and a prime $p$,

\begin{equation}
\sum_{n=0}^{\frac{p - 1}{d}} (-1)^n (2dn + 1) \left( \frac{1}{d} \right)_n^3 \equiv p^r (-1)^{\frac{p - 1}{d}} \pmod{p^{r+2}}
\end{equation}

if $p^r \equiv 1 \pmod{d}$, and

\begin{equation}
\sum_{n=0}^{\frac{d-1}{d}p^r - 1} (-1)^n (2dn + 1) \left( \frac{1}{d} \right)_n^3 \equiv (d - 1)p^r (-1)^{\frac{d-1}{d}p^r - 1} \pmod{p^{r+2}}
\end{equation}

if $p^r \equiv -1 \pmod{d}$. Using hypergeometric series identities and evaluations, the authors [11] have also given proofs for (1.3) and (1.4).

It has been noticed that in all supercongruences related to VanHamme’s supercongruences proved so far, the sum is truncated at $\frac{p - 1}{d}$ if $p^r \equiv 1 \pmod{d}$ or at $\frac{(d-1)p^r - 1}{d}$ if $p^r \equiv -1 \pmod{d}$. Thus, in literature, we have not found any result similar to the third supercongruence of (F.3) where the sum truncates at $\frac{p - (d-1)}{d}$ if $p^r \equiv -1 \pmod{d}$. Thus we here make the first attempt to prove a supercongruence for $S \left( \frac{p - 3}{4} \right)$. Following the work of [17], we use of the powerful WZ-method to prove the supercongruence.

**Theorem 1.1.** Let $p$ be an odd prime such that $p \equiv 3 \pmod{4}$. If $r > 1$ is any odd integer, then

$$S \left( \frac{p^r - 3}{4} \right) \equiv - \frac{16 \Gamma_p \left( \frac{3}{4} \right)}{\Gamma_p \left( \frac{1}{4} \right)^3} p^{r-1} \pmod{p^r}.$$
We further prove the third supercongruence conjecture of (F.3) for odd integers $r > 3$.

**Theorem 1.2.** Let $p \equiv 3 \pmod{4}$ and $r > 3$ an odd integer, then

$$S\left(\frac{p^r - 3}{4}\right) \equiv p^2 S\left(\frac{p^{r-2} - 3}{4}\right) \pmod{p^r}.$$ 

2. Preliminaries

In this section, we recall and prove some results concerning $p$-adic gamma function and rising factorials. Let $\mathbb{Q}_p$, and $\nu_p(.)$ denote the field of $p$-adic numbers and the $p$-adic valuation on $\mathbb{Q}_p$, respectively. In the following lemma, we list some basic properties of $p$-adic gamma function which are easy consequences of its definition.

**Lemma 2.1.** [2] Section 11.6] Let $p$ be an odd prime and $x, y \in \mathbb{Z}_p$. Then

(i) $\Gamma_p(1) = -1$.

(ii) $\Gamma_p(x+1) \Gamma_p(x) = \{ -x, \; \text{if } \nu_p(x) = 0; \}
\{ -1, \; \text{if } \nu_p(x) > 0. \}$

(iii) $\Gamma_p(x)\Gamma_p(1-x) = (-1)^{a_0(x)}$, where $a_0(x) \in \{1, 2, \ldots, p\}$ satisfies $a_0(x) \equiv x \pmod{p}$.

(iv) if $x \equiv y \pmod{p}$, then $\Gamma_p(x) \equiv \Gamma_p(y) \pmod{p}$.

**Lemma 2.2.** [8] Lemma 2.2] Let $p$ be an odd prime, $m \geq 3$ an integer and $\zeta$ a $m$-th primitive root of unity. Suppose $a \in \mathbb{Z}_p[\zeta], n \in \mathbb{N}$ and $A(p)$ is the product of the elements in $\{a + k \mid k = 0, 1, \ldots, n-1\}$ which are divisible by $p$. Then

$$(a)_n = (-1)^n A(p) \frac{\Gamma_p(a + n)}{\Gamma_p(a)}.$$

Using this, we now deduce some recurrence relations for certain rising factorials which will be used in the proofs of our main results.

**Lemma 2.3.** Let $p$ be a prime with $p \equiv 3 \pmod{4}$. If $r > 1$ is any odd positive integer, then

(a) $\left(\frac{1}{4}\right)_{\frac{p^r - 3}{2}} = \frac{p^{\frac{r-1}{2}} - \frac{p^{r-2}}{2}}{\Gamma_p \left(\frac{p^r - 3}{4}\right)} \frac{\Gamma_p \left(\frac{p^r - 1}{2} + \frac{1}{4}\right)}{\Gamma_p \left(\frac{1}{4}\right)} \frac{\Gamma_p \left(\frac{p^r - 2}{2} - \frac{5}{4}\right)}{\Gamma_p \left(\frac{p^r - 2}{2} - \frac{1}{4}\right)}$.

(b) $\left(\frac{1}{1}\right)_{\frac{p^r - 3}{2}} = \frac{p^{\frac{r-1}{2}} - \frac{p^{r-2}}{2}}{\Gamma_p \left(\frac{p^r - 1}{4}\right)} \frac{\Gamma_p \left(\frac{p^r - 1}{4} + \frac{1}{4}\right)}{\Gamma_p \left(\frac{1}{4}\right)}$.

(c) $\left(\frac{1}{4}\right)_{\frac{p^r - 3}{2}} = \frac{p^{\frac{r-1}{2}} - \frac{p^{r-2}}{2}}{\Gamma_p \left(\frac{p^r - 1}{4}\right)} \frac{\Gamma_p \left(\frac{p^r - 1}{4} + \frac{1}{4}\right)}{\Gamma_p \left(\frac{1}{4}\right)} \frac{\Gamma_p \left(\frac{p^r - 2}{2} - \frac{1}{2}\right)}{\Gamma_p \left(\frac{p^r - 2}{2}\right)}$.

**Proof.** For any complex number $a$, let $f_p[(a_n)]$ denote the product of $p$-factors present in the rising factorial $(a)_n$.

(a) Using Lemma 2.2 we have

$$\left(\frac{1}{4}\right)_{\frac{p^r - 3}{2}} = (-1)^{\frac{p^r - 3}{2}} \frac{\Gamma_p \left(\frac{p^r - 5}{4}\right)}{\Gamma_p \left(\frac{1}{4}\right)} f_p \left[\left(\frac{1}{4}\right)_{\frac{p^r - 3}{2}}\right].$$
Noting that the $p$-factors present in $(\frac{3}{4})^{pr-3}$ are
\[ \left\{ \left( k + \frac{3}{4} \right) p \mid 0 \leq k \leq \frac{pr-1-3}{2} \right\}, \]
we obtain
\[
\left( \frac{1}{4} \right)^{pr-3} = (-1)^{pr-3} \frac{\Gamma_p \left( \frac{pr}{4} - \frac{5}{4} \right)}{\Gamma_p \left( \frac{3}{4} \right)} \prod_{k=0}^{pr-3} \left(kp + \frac{3p}{4}\right)
\]
\[
= (-1)^{pr+pr-1-4} \frac{\Gamma_p \left( \frac{pr}{4} - \frac{5}{4} \right)}{\Gamma_p \left( \frac{3}{4} \right)} \frac{\Gamma_p \left( \frac{pr-1}{4} + \frac{1}{4} \right)}{\Gamma_p \left( \frac{3}{4} \right)} f_p \left[ \left( \frac{3}{4} \right)^{pr-1-1} \right],
\]
where the last equality follows from Lemma 2.2. Since $(\frac{3}{4})^{pr-1-1}$ contains the $p$-factors
\[ \left\{ \left( k + \frac{1}{4} \right) p \mid 0 \leq k \leq \frac{pr-2-1}{2} \right\}, \]
we have
\[
f_p \left[ \left( \frac{3}{4} \right)^{pr-1-1} \right] = \prod_{k=0}^{pr-2-1} \left\{ \left( k + \frac{1}{4} \right) p \right\}
\]
\[
= p^{pr-2-1} \left( \frac{1}{4} \right)^{pr-2-1} \left( \frac{pr-2}{2} - \frac{5}{4} \right) \left( \frac{pr-2}{2} - \frac{1}{4} \right),
\]
and hence the result follows.

(b) In view of Lemma 2.2 we have
\[ (1)^{pr-3} = (-1)^{pr+1} \frac{\Gamma_p \left( \frac{1+p^r}{4} \right)}{\Gamma_p \left( \frac{3+p^r}{4} \right)} f_p \left[ (1)^{pr-1} \right]. \]
The $p$-factors present in $(1)^{pr-3}$ are
\[ \left\{ kp \mid 1 \leq k \leq \frac{pr-1-1}{4} \right\}. \]
As a result,
\[ f_p \left[ (1)^{pr-3} \right] = p^{pr-1-1} (1)^{pr-1-1} = \frac{p^{pr-1-1}}{(-1)^{pr-1-1}} \Gamma_p \left( \frac{3+p^r-1}{4} \right) f_p \left[ (1)^{pr-1-1} \right]. \]
Again,
\[ f_p \left[ (1)^{pr-1-1} \right] = p^{pr-2-3} (1)^{pr-2-3} \]
because of the fact that the $p$-factors present in $(1)^{pr-1-1}$ are
\[ \left\{ kp \mid 1 \leq k \leq \frac{pr-2-3}{4} \right\}. \]
Combining all these together, we obtain the desired result.

(c) It is clear that the \( p \)-factors present in \( \left( \frac{3}{4} \right)^{p-3} \) and \( \left( \frac{3}{4} \right)^{p-1} \) are

\[
\left\{ \left( k + \frac{3}{4} \right) p \mid 0 \leq k \leq \frac{p^r-1-5}{4} \right\}
\]

and

\[
\left\{ \left( k + \frac{1}{4} \right) p \mid 0 \leq k \leq \frac{p^r-1-3}{4} \right\},
\]

respectively. Thus following the proof of (a) and (b), we complete the proof. \( \square \)

3. Proof of main results

We prove our results using the powerful WZ-method designed by Wilf and Zeilberger [16]. The method is based on finding a Wilf-Zeilberger pair (WZ pair) for which one needs to have a preliminary human guess. Our motivation of finding the WZ-pair is based on the work of [3, 4, 5, 17].

Lemma 3.1. Let \( n \) and \( k \) be non-negative integers. Suppose

\[
F(n, k) = (-1)^{n+k}(8n+1)\left(\frac{1}{4} \right)^{n+k} \frac{(\frac{1}{4})^2}{\binom{n}{k} \binom{n-k}{\frac{k+1}{4}}}.
\]

and

\[
G(n, k) = (-1)^{n+k+1} \frac{\binom{n}{k} \binom{n-k}{\frac{k+1}{4}}}{n-k+1}.
\]

where \( 1/(1)_m = 0 \) for \( m = -1, -2, \ldots \). Then

\[
F(n, k-1) - F(n, k) = G(n+1, k) - G(n, k).
\]

Proof. We first note that

\[
\frac{F(n, k-1)}{F(n, k)} = -\frac{(4n-3)^2}{(4n+4k-3)(n-k+1)}
\]

\[
G(n+1, k) = -\frac{4 \cdot (4n+1)^2}{(8n+1)(n-k+1)},
\]

and

\[
\frac{G(n, k)}{F(n, k)} = \frac{4n^2}{(4n+4k-3)(8n+1)}.
\]

As a result, it is clear that

\[
\frac{F(n, k-1)}{F(n, k)} - 1 = \frac{G(n+1, k)}{F(n, k)} - \frac{G(n, k)}{F(n, k)},
\]

and hence the result follows. \( \square \)

Thus \((F(n, k), G(n, k))\) form a WZ-pair, and we shall use this pair to prove our results.

Lemma 3.2. Let \( r \geq 3 \) be an odd positive integer and \( p \equiv 3 \pmod{4} \). For \( k = 1, 2, \ldots, \frac{p^r-3}{4} \), we have

\[
G\left( \frac{p^r+1}{4}, k \right) \equiv 0 \pmod{p^{\frac{3r-1}{2}}}.
\]
Proof. Noting that \((a)_{n+1} = (a+n)(a)_n, (a)_{n+k} = (a)_n(a+n)_k\), and \((a)_{n-k} = (-1)^{k-2}(a)_{n-2(1-a-n)}(1-a-n)_k\), we have

\[
G\left(\frac{p^r+1}{4} , k\right) = (-1)^{p^r+1+k}4 \frac{\left(\frac{1}{4}\right)^{p^r-3} \left(\frac{1}{4}\right)^{-1}+k}{\left(\frac{1}{4}\right)^{p^r-3} \left(\frac{1}{4}\right)^{-1-k}} \\
= (-1)^{p^r+1}4 \left(\frac{p^r}{4} - \frac{1}{2}\right)^2 \left\{ \left(\frac{1}{4}\right)^{p^r+1} \right\}^3 \left(\frac{p^r-2}{4} \right) \left(\frac{p^r}{4} \right) \left(\frac{1}{4}\right)^{2k}.
\]

Using Lemma 2.3 (b), (c) repeatedly, we obtain

\[
\nu_p \left\{ \left(\frac{1}{4}\right)^{p^r-3} \left(\frac{1}{4}\right)^{-1} \right\} = 3 \left( \frac{r-1}{2} \right) .
\]

It is easy to see that \(j_1 = \frac{p+3}{2}, j_2 = \frac{p+5}{2}, \) and \(j_3 = \frac{3p+3}{4}\) are the least positive integers for which \(\left(\frac{p^r}{4}\right)_{j_1}, \left(\frac{-1-p^r}{4}\right)_{j_2},\) and \(\left(\frac{1}{4}\right)_{j_3}\) have one \(p\)-factor each. Since \(\frac{p+3}{2} < \frac{3p+3}{4}\) and \(\frac{p+5}{2} < \frac{3p+3}{4}\), we must have

\[
\nu_p \left\{ \left(\frac{p^r-2}{4} \right) \left(\frac{p^r}{4} \right) \left(\frac{1}{4}\right)^{2k} \right\} \geq 0
\]

for any non-negative integer \(k\). Combining all these together, we complete the proof of the lemma. \(\square\)

Lemma 3.3. Let \(r \geq 3\) be an odd positive integers. If \(p \equiv 3 \mod 4\), then

\[
F\left(\frac{p^r-3}{4}, \frac{p^r-3}{4}\right) = -16 \frac{\Gamma_p\left(\frac{3}{4}\right)}{\Gamma_p\left(\frac{1}{4}\right)} \left(\frac{1}{4}\right)^{\nu_p-3} (\mod p^r).
\]

Proof. Clearly,

\[
F\left(\frac{p^r-3}{4}, \frac{p^r-3}{4}\right) = \left(2p^r - 5\right) \left(\frac{1}{4}\right)^{\nu_p-3}.
\]

We use Lemma 2.3 (a), (b) repeatedly to obtain

\[
\frac{\left(\frac{1}{4}\right)^{\nu_p-3}}{\left(\frac{1}{4}\right)^{\nu_p-3}} = p^{r-1} \prod_{j=1}^{\frac{p+3}{2}} \left\{ \Gamma_p\left(\frac{p^r-1}{2} - \frac{3}{4}\right) \Gamma_p\left(\frac{p^{r+1}-1}{2} - \frac{5}{4}\right) \Gamma_p\left(\frac{p^{r+1}}{2} - \frac{5}{4}\right) \Gamma_p\left(\frac{p^{r+1}}{2} + \frac{3}{4}\right) \right\} \left(\frac{1}{4}\right)^{\nu_p-3}.
\]

Noting that \(\left(\frac{1}{4}\right)^{\nu_p-3} = (-1)^{\nu_p-3} \Gamma_p\left(\frac{3-4}{4}\right)\) and \(\left(\frac{1}{4}\right)^{\nu_p-3} = (-1)^{\nu_p-3} \Gamma_p\left(\frac{3+4}{4}\right)\), we have

\[
F\left(\frac{p^r-3}{4}, \frac{p^r-3}{4}\right) \equiv -5p^{r-1} \Gamma_p\left(\frac{5}{4}\right) \left\{ 5 \cdot \Gamma_p\left(\frac{5}{4}\right) \Gamma_p\left(\frac{3}{4}\right) \right\} \left(\mod p^r\right)
\]

because of Lemma 2.1 (i), (iv). Using Lemma 2.1 (ii), we have \(\Gamma_p\left(\frac{5}{4}\right) = \frac{\Gamma_p\left(\frac{3}{4}\right)}{\Gamma_p\left(\frac{1}{4}\right)}\). Hence Lemma 2.1 (iii) completes the proof of the lemma. \(\square\)

Proof of Theorem 1.1. Since

\[
F(n, k-1) - F(n, k) = G(n + 1, k) - G(n, k),
\]
we have
\[ \sum_{n=0}^{p^r-3} F(n, k - 1) - \sum_{n=0}^{p^r-3} F(n, k) = G \left( \frac{p^r + 1}{4}, k \right) - G(0, k). \]

As a result, Lemma 3.1 and Lemma 3.2 yield
\[ \sum_{n=0}^{p^r-3} F(n, k - 1) \equiv \sum_{n=0}^{p^r-3} F(n, k) \pmod{p^{3(r-1)}}. \]

It is easy to see that
\[ S \left( \frac{p^r - 3}{4} \right) = \sum_{n=0}^{p^r-3} F(n, 0), \]

and hence
\[ S \left( \frac{p^r - 3}{4} \right) \equiv \sum_{n=0}^{p^r-3} F \left( n, \frac{p^r - 3}{4} \right) = F \left( \frac{p^r - 3}{4}, \frac{p^r - 3}{4} \right) \pmod{p^{3(r-1)}}. \]

Using Lemma 3.3 we complete the proof of the theorem.

\[ \square \]

**Proof of Theorem 1.2.** For \( r > 3 \), we have from Theorem 1.1 that
\[ S \left( \frac{p^r - 3}{4} \right) \equiv -16 \Gamma_p (\frac{3}{4}) p^{r-3} (\mod p^{r-2}). \]

Therefore,
\[ p^2 S \left( \frac{p^r - 3}{4} \right) \equiv -16 \Gamma_p (\frac{3}{4}) p^{r-1} (\mod p^r). \]

Thus we complete the proof of the theorem because of Theorem 1.1.

\[ \square \]

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