A novel secondary code acquisition algorithm for the BDS-3 B1C signal

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Abstract
The BDS-3 recently started broadcasting a new civil B1C signal to provide open services for global users, which brings benefits to GNSS-based applications. The BDS-3 B1C signal modulates a long secondary code on the primary code in the pilot component, and it is useful to acquire the secondary code so as to extend coherent integration time when acquiring weak BDS-3 B1C signals. However, the long secondary code of the BDS-3 B1C signal puts FFT-based and multi-hypothesis-based secondary code acquisition methods in trouble from the high computational burden. Therefore, the authors propose a novel secondary code acquisition algorithm called the partial correlation method (PCM) for the BDS-3 B1C signal. The PCM acquires the secondary code in three steps to reduce the complexity and acquisition time, and it supports up to 110 ms coherent integration and can be applied for the case of $C/N_0 \geq 25$ dB - Hz, which satisfies most cases. Further, a matched-filter-based architecture of the PCM is presented. Additionally, the characteristic length vector to determine the secondary code chip position quickly is proposed, which is better than the existing characteristic length method. Finally, experimental results based on real BDS-3 B1C signals data show that the proposed PCM is effective.

1 | INTRODUCTION

The Chinese third-generation BeiDou Navigation Satellite System (BDS-3) is a global navigation satellite system (GNSS), and the BDS-3 recently started broadcasting a new civil B1C signal to provide open services for global users. Similar to other modern GNSS signals, the BDS-3 B1C signal also adopts a tiered code architecture and modulates a long secondary code on the primary code in the pilot component. Although the long secondary code provides an increased cross-correlation property [1, 2], the longer the secondary code, the higher the complexity of secondary code acquisition methods, and the more secondary code chips will need acquiring to determine its chip position so as to conduct bit synchronisation [3]. Moreover, acquiring the secondary code is an effective method to extend the coherent integration time when acquiring weak signals [4, 5], especially the weak BDS-3 B1C signal, because the pilot component power of the BDS-3 B1C signal accounts for three-quarters of the signal power. Hence, low-complexity long secondary code acquisition algorithm is a heated topic about acquiring weak BDS-3 B1C signals.

At present, there are two main categories of secondary code acquisition algorithms. Since the secondary code chip-sign transition restricts the extension of coherent integration time, the first type of algorithm is to sequentially or simultaneously test all possible symbol combinations based on the consecutive correlated results of a primary code period [1, 6–11]. However, since the number of possible symbol combinations increases exponentially with the growth of secondary code chips, these methods are only adopted in the case of short coherent integration time.
Although some algorithms claimed to be able of reducing the theoretical number of operations when testing all possible symbol combinations [3, 12, 13], either the algorithm architecture complexity and processing time in terms of implementation in hardware have not been reduced, or only a short secondary code can adopt it and with signal-to-noise ratio (SNR) loss.

Since the secondary code is periodic and entire chips in a secondary code period are prior knowledge, the second type of algorithm is based on FFT and IFFT to perform the parallel search of all candidate secondary code phases [12–16]. In terms of the BDS-3 B1C signal, the primary code contains 10,230 chips in a period of 10 ms, and the secondary code contains 1800 chips in a period of 18 s. As a consequence, the FFT-based secondary code acquisition algorithms have an extremely heavy computational burden for both software and hardware when acquiring the weak BDS-3 B1C signal. Therefore, these FFT-based secondary code acquisition algorithms are not suitable for the BDS-3 B1C signal.

Consequently, the authors propose a novel secondary code acquisition method called the partial correlation method (PCM) to acquire the long secondary code of the BDS-3 B1C signal so as to acquire the weak BDS-3 B1C signal. The PCM acquires the secondary code in three steps. In terms of acquiring \( N \) secondary code chips, the first step is to acquire the first half of \( N \) secondary code chips, and the second step is to acquire the other half. The final step is to coherently combine two results obtained by the previous two steps to form the final decision statistic. As a result, the PCM significantly reduces the number and length of possible symbol combinations, even if extending the coherent integration time up to 110 ms (i.e. \( N = 11 \)), which significantly reduces the acquisition algorithm complexity and acquisition time and simplifies the acquisition architecture.

Additionally, bit synchronisation is dependent on the determination of secondary code chip position, and therefore the authors propose the characteristic length vector to determine the secondary code chip position quickly, which is better than the present characteristic length method [7, 17].

The BDS-3 B1C signal is briefly described in Section 2. Then, primary code acquisition principles of the BDS-3 B1C signal are introduced in Section 3. In Section 4, the proposed secondary code acquisition algorithm PCM is explained in detail. Based on the PCM, the acquisition architecture is presented in Section 5, and the performance evaluation of the PCM is conducted in Section 6. Afterwards, the characteristic length vector is calculated in Section 7. Experiments based on real BDS-3 B1C signals data are subsequently carried out to verify the PCM in Section 8. Finally, some summarising conclusions are drawn in Section 9.

## 2 BDS-3 B1C SIGNAL

The BDS-3 B1C signal is a modern GNSS signal, which contains two components, the data component and pilot component, respectively. The data component modulates data bits on the primary code, and the pilot component modulates a secondary code on the primary code. The secondary code of the BDS-3 B1C signal is the Weil code, which contains 1800 chips in a period of 18 s. Each secondary code chip strictly aligns with one period of primary code, and the primary code contains 10,230 chips in a period of 10 ms. Moreover, every pseudo random noise (PRN) number corresponds to a unique secondary code. In comparison to other secondary codes, the secondary code of the BDS-3 B1C signal is really a long secondary code as Table 1 shows.

As Table 2 shows, the pilot component of the BDS-3 B1C signal adopts the QMBOC(6,1,4/33) modulation mode that consists of two parts, BOC(1,1) and BOC(6,1), respectively. On one hand, the power of the BOC(1,1) accounts for 29/44 of the total power of the BDS-3 B1C signal. On the other hand, the sub-carrier frequency of the BOC(1,1) is 1.023 MHz, and it is less than that of the BOC(6,1), which indicates the acquisition engine designed for the BOC(1,1) generally has lower complexity than that designed for the BOC(6,1). Therefore, we merely consider the BOC(1,1) of the pilot component herein.

### 3 PRIMARY CODE ACQUISITION

First of all, attention here is mainly paid to the secondary code acquisition algorithm for the BDS-3 B1C signal. Hence, detailed information about the primary code acquisition method is not the focus. To illustrate the proposed PCM, the two-dimension search method based on short-time coherent integration plus a FFT is considered [18–20], as Figure 1 shows. It should be noted, that other search methods such as serial search [21], parallel frequency search [22], and parallel

| TABLE 1 | Secondary code characteristics |
|----------|-------------------------------|
| Signal   | Primary Code | Secondary Code |
|          | Length (Chip) | Period (ms) | Length (Chip) | Period (ms) |
| GPS L5   | 10,230       | 1           | 20            | 20           |
| Galileo E1 | 4092      | 4           | 25            | 100          |
| Galileo E5 | 10,230   | 1           | 100           | 100          |
| BDS-3 B1C | 10,230   | 10          | 1800          | 18,000       |

| TABLE 2 | Characteristics of BDS-3 B1C signal |
|----------|-------------------------------|
| Signal Component | BDS-3 B1C |
| Modulation mode | BOC(1,1) | QMBOC(6,1,4/33) |
|                | BOC(1,1) | BOC(6,1) |
| Phase          | 0       | 90      | 0       |
| Power ratio    | One-quarter | 29/44  | 1/11   |

Abbreviation: BDS, beidou navigation satellite system; BOC, binary offset carrier; QMBOC, quadrature multiplexed binary offset carrier.
code search [23], are also suitable for primary code acquisition
and cooperation with the PCM. Additionally, the BPSK-like
method [24] is applied to achieve unambiguous acquisition
of the BOC(1,1).

Generally, the discrete-time BOC(1,1) signal of the BDS-3
B1C signal pilot component at the output of the radio fre-
quency front-end is given by:

\[
\begin{align*}
    r[n] &= \sqrt{2P} s[n - \tau] \cdot c[n - \tau] \cdot \text{sign}(2\pi f_{\text{IF}} n T_s) \\
    &\quad \cdot \sin[2\pi (f_{\text{IF}} + f_d) n T_s + \phi_0] + \eta[n]
\end{align*}
\]

where \( P \) represents signal power, \( s[n] \) denotes the secondary
code, \( c[n] \) is the primary code, \( \text{sign}(2\pi f_{\text{IF}} n T_s) \) is the sub-
carrier, \( f_{\text{IF}} = 1.023 \text{ MHz} \), \( \tau \) represents the path delay,
\( f_{\text{IF}} \) is the nominal intermediate frequency (IF), \( f_d \) represents the
Doppler frequency, \( \phi_0 \) is the initial phase, \( \eta[n] \) denotes the
additive white Gaussian noise with two-side noise power
spectrum density (PSD) of \( N_0/2 W/Hz \), and \( T_s \) is the sam-
ping interval.

Subsequently, the local generated complex exponential
carrier \( \exp[-j2\pi(f_{\text{IF}} + f_d) n T_s] \) and primary code replica
\( c[n - \hat{\tau}] \) are generated to correlate with \( r[n] \). In addition,
the short-time correlation time is \( T_{\sigma} \), and \( T_{\sigma} = N_p T_s \). Taking \( T_p \)
the period of the primary code into account, \( T_p = LT_{\sigma} = 10 \text{ ms} \). In order to simplify the expression,
the low pass filter (LPF) designed for the BPSK-like method is
assumed as an ideal finite impulse response (FIR) filter. After
filtering operation of the LPF and the short-time coherent
integration, the \( l \)-th result is obtained as:

\[
\begin{align*}
    G(\hat{\tau}, l) &= \sum_{n=0}^{(l+1)N_p-1} LPF\{r[n] \cdot \exp[-j2\pi(f_{\text{IF}} + f_d) n T_s] \} \\
    &\quad \cdot c[n - \hat{\tau}]
\end{align*}
\]

\[
\begin{align*}
    &= \sqrt{2P} \cdot R(\Delta\tau) \\
    &= \frac{2}{\sin(\pi f_{\text{IF}} T_s)} \cdot \sin(\pi f_d T_s) \\
    &\quad \cdot \exp\left\{ j\left(\pi f_d (2N_p + N_p - 1) T_s + \phi_0 \right) \right\} + \xi[l]
\end{align*}
\]

where \( \Delta\tau = \tau - \hat{\tau} \), \( R(\Delta\tau) \) represents the auto-
correlation function (ACF) of the primary code, and \( \xi[l] \) is the present
complex noise item.

Afterwards, a \( N_{\text{FFT}} \)-points zero-padding FFT is used to
conduct parallel Doppler frequency search, and \( N_{\text{FFT}} = 2^m \)
\( (m \in N^*) \). To avoid the scolding loss, \( N_{\text{FFT}} \geq 2L \), and the
number of padding zeros is \( N_{\text{FFT}} - L \). Consequently, the output of \( i \)-th FFT operation is calculated as:

\[
F_i(\hat{\tau}, \hat{f}_d) = \sum_{l=0}^{L-1} G(\hat{\tau}, l) \cdot exp\left(-j\frac{2\pi}{N_{\text{FFT}}} kl\right)
\]

\[
= \frac{\sqrt{2P} \cdot R(\Delta\tau)}{2} \cdot \sin(\pi f_{\text{IF}} T_s) \cdot \sin(\pi f_d T_s) \cdot \sin(\pi \Delta f_d T_s)
\]

\[
= \sum_{l=0}^{L-1} \exp(j\phi_l) + \zeta_i
\]

where \( \Delta f_d = f_d - \hat{f}_d \), and \( \hat{f}_d = k/(N_{\text{FFT}} \cdot T_c) \), \( \phi_l = \phi_0 + \pi f_{\text{IF}} (N_p - 1) T_s + \pi \Delta f_d (2L + L - 1) T_s \), \( \zeta_i \) is the present
noise item. Considering the long-time coherent integration up
to 110 ms, the Doppler frequency estimation deviation \( \Delta f_d \)
should be generally within a small range of \(-2.5 \text{ Hz} \sim 2.5 \text{ Hz} \).
Moreover, it should be noted that Doppler compensation
may be necessary for long-time coherent integration. As the
main focus here is on the secondary code acquisition, detailed
information about Doppler compensation can be found in [25].

According to the central limit theorem, \( \zeta_i \) in Equation (3) is
a complex Gaussian random variable with zero mean and variance
\( 2a^2 \). The real and imaginary parts of the \( \zeta_i \) are independent
and have zero mean and equal variance \( \sigma^2 \). Therefore, the carrier-to-
noise ratio \( (C/N_0) \) in dB - Hz (dB*Hz) is given by:

\[
C/N_0 = 10 \cdot \log \left( \frac{A_0^2}{2a^2} \cdot \frac{1}{T_p} \right) = 10 \cdot \log \left( \frac{A_0^2}{2a^2} \right) + 20
\]

where \( C/N_0 \) is a key parameter to evaluate the performance of
the PCM.
4 | SECONDARY CODE ACQUISITION

As previously discussed, the secondary code of the BDS-3 B1C signal contains 1800 chips in a period of 18 s, that is, \( N_t = 1800 \). In terms of acquiring \( N \) secondary code chips, considering the ambiguity of the secondary code chip sign, the number of possible symbol combinations is given by:

\[
M = \min \{ 2^{N-1}, N_t \}
\]

(5)

where \( N \leq 11 \), \( M = 2^{N-1} \). The case of \( N \leq 11 \) means the coherent integration time can reach up to 110 ms, which satisfies most of the weak signal situations when considering the optional non-coherent integration method after coherent integration. Hence, the focus here is only on the case of \( N \leq 11 \).

As previously discussed, the PCM acquires the secondary code in three steps, and then it is given by:

\[
\begin{align*}
M_1 &= 2^{N_t - 1}, \quad N_t = \text{ceil}(N/2) \\
M_2 &= 2^{N_t - 1}, \quad N_t = N - N_t
\end{align*}
\]

(6)

According to Equation (3), \( N \) consecutive realisations of \( F_i(\hat{\tau}, f_d) \) are obtained and expressed in vector:

\[
F = \left[ F_0(\hat{\tau}, f_d), F_1(\hat{\tau}, f_d), \ldots, F_{N-1}(\hat{\tau}, f_d) \right]^T
\]

(7)

where the input secondary code sequence is \( S_g \) and \( S_g = [s_{0,g}, s_{1,g}, \ldots, s_{N_t-1,g}]^T \). It is clear that \( S_g \) has \( 2^{N_t} \) possibilities when considering the ambiguity of chip sign, that is, \( 1 \leq g \leq 2^{N_t} \).

\[
F = \left[ F_1^T, F_2^T \right]^T
\]

(8)

where \( F_1 = [F_0(\hat{\tau}, f_d), \ldots, F_{N_t-1}(\hat{\tau}, f_d)]^T \), and \( F_2 = [F_{N_t}(\hat{\tau}, f_d), \ldots, F_{N-1}(\hat{\tau}, f_d)]^T \). At this point, \( S_g = [S_u^T, S_v^T]^T \), or \( S_g = [S_u^T, -S_v^T]^T \). \( S_u = [s_{0,u}, s_{1,u}, \ldots, s_{N_t-1,u}]^T \) \((1 \leq u \leq M_1)\) and \( S_v = [s_{0,v}, s_{1,v}, \ldots, s_{N_t-1,v}]^T \) \((1 \leq v \leq M_2)\). It should be noted that the logic values \( \{0', 1'\} \) of secondary code chips are separately transformed into digital values \( \{1, -1\} \) during acquisition. Without loss of generality, \( S_v = [1, 1, 1, \ldots] \) \((S_v = [0', 0', 0', \ldots])\), and then \( -S_v = [-1, -1, -1, \ldots] \) \((-S_v = [1', 1', 1', \ldots])\). Therefore, \( -S_v \) means the transition of polarity of the secondary code sequence \( S_v \).

Then, based on Equation (6), Equation (7) can be rewritten as:

\[
Mat_1 = \begin{bmatrix} \hat{S}_1, \hat{S}_2, \ldots, \hat{S}_{M_1} \end{bmatrix}^T
\]

\[
= \begin{bmatrix} \hat{s}_{0,1}, \hat{s}_{1,1}, \ldots, \hat{s}_{N_t-1,1} \\ \hat{s}_{0,2}, \hat{s}_{1,2}, \ldots, \hat{s}_{N_t-1,2} \\ \vdots \quad \vdots \quad \vdots \\ \hat{s}_{0,M_1}, \hat{s}_{1,M_1}, \ldots, \hat{s}_{N_t-1,M_1} \end{bmatrix}
\]

(9)

where \( \hat{S}_j (1 \leq j \leq M_1) \) is the \( j \)-th candidate symbol combination, and \( \hat{S}_j = [\hat{s}_{0,j}, \hat{s}_{1,j}, \ldots, \hat{s}_{N_t-1,j}]^T \). Subsequently, secondary code correlation results are calculated as:

\[
X = \begin{bmatrix} x_1(\hat{\tau}, f_d) \\ x_2(\hat{\tau}, f_d) \\ \vdots \\ x_{M_1}(\hat{\tau}, f_d) \end{bmatrix} = Mat_1 \cdot F_1
\]

(10)

where

\[
x_j(\hat{\tau}, f_d) = \sum_{i=0}^{N_t-1} \hat{s}_{ij} \cdot F_i(\hat{\tau}, f_d)
\]

(11)

and

\[
R_{j,u} = \hat{S}_j^T \cdot S_u = \sum_{i=0}^{N_t-1} \hat{s}_{ij} \cdot s_{iu}
\]

(12)

Also in Equation (11), \( A = A_0 \cdot \frac{\sin(\pi f_d T_p N_t)}{\pi f_d T_p N_t} \), \( R_{j,u} \) represents the cross-correlation results between the local \( \hat{S}_j \) and the input secondary code sequence \( S_u \), \( \phi \) is the residual carrier phase, and \( \zeta_j \) is present noise item. Finally, the optimal estimate of the first step is generally given by [26]:

\[
\left| x_u(\hat{\tau}, f_d) \right|^2 = \max_j \left\{ \left| x_j(\hat{\tau}, f_d) \right|^2 \right\}
\]

(13)

Hence, \( \hat{S}_u \) is taken as the optimal estimate of the first step.

4.1 | First step of the PCM

To acquire first \( N_t \) secondary code chips, a total of \( M_1 \) possible symbol combinations are generated:

4.2 | Second step of the PCM

To acquire the rest of the \( N_t \) secondary code chips, a total of \( M_2 \) possible symbol combinations are generated:
\[
\text{Mat}_2 = \begin{bmatrix}
\hat{S}_{1,1} & \hat{S}_{2,1} & \cdots & \hat{S}_{N_2-1,1} \\
\hat{S}_{1,2} & \hat{S}_{2,2} & \cdots & \hat{S}_{N_2-1,2} \\
\vdots & \vdots & \ddots & \vdots \\
\hat{S}_{1,M_2} & \hat{S}_{2,M_2} & \cdots & \hat{S}_{N_2-1,M_2}
\end{bmatrix}^T
\]

(14)

where \( \hat{S}_b \) \((1 \leq b \leq M_2)\) is the \( b \)-th candidate symbol combination, and \( \hat{S}_b = [\hat{s}_{0,1}, \hat{s}_{1,1}, \ldots, \hat{s}_{N_2-1,1}]^T \). Similar to the first step, secondary code correlation results are calculated as:

\[
Y = \begin{bmatrix}
y_1(\hat{r}, \hat{f}_d) \\
y_2(\hat{r}, \hat{f}_d) \\
\vdots \\
y_{M_2}(\hat{r}, \hat{f}_d)
\end{bmatrix} = \text{Mat}_2 \cdot F_2
\]

(15)

Finally, the optimal estimate of the second step is generally given by [26]:

\[
|y_w(\hat{r}, \hat{f}_d)|^2 = \max_b \left\{ |y_b(\hat{r}, \hat{f}_d)|^2 \right\}
\]

(16)

Hence, \( \hat{S}_c \) is taken as the optimal estimate of the second step.

### 4.3 Third step of the PCM

The third step is to coherently combine two results obtained by the previous two steps to form the final decision statistic. First, two candidate estimates of the input secondary code sequence separately corresponding to hypothesis \( H_1 \) and \( H_2 \) are given by:

\[
\begin{align*}
S_g &= \hat{S}_1 = \left[ \hat{S}_u, \hat{S}_c \right]^T, \quad H_1 \\
S_g &= \hat{S}_2 = \left[ \hat{S}_u, -\hat{S}_c \right]^T, \quad H_2
\end{align*}
\]

(17)

Based on Equation (17), two candidate decision statistics \( z_1 \) and \( z_2 \) are obtained as:

\[
\begin{align*}
z_1(\hat{r}, \hat{f}_d) &= y_u(\hat{r}, \hat{f}_d) + y_c(\hat{r}, \hat{f}_d), \quad H_1 \\
z_2(\hat{r}, \hat{f}_d) &= y_u(\hat{r}, \hat{f}_d) - y_c(\hat{r}, \hat{f}_d), \quad H_2
\end{align*}
\]

(18)

Afterwards, the final optimal estimate is generally given by [26]:

\[
|z_w(\hat{r}, \hat{f}_d)|^2 = \max_i \left\{ |z_i(\hat{r}, \hat{f}_d)|^2 \right\}
\]

(19)

Eventually, the optimal estimate of the input secondary code sequence is \( \hat{S}_w \) \((w \in \{1, 2\})\). The \( |z_w(\hat{r}, \hat{f}_d)|^2 \) is compared with the predefined threshold \( \beta \) to make acquisition decisions. The predefined threshold \( \beta \) of the PCM is unquestionably the same as that of the multi-hypothesis based method (MHM) in terms of the same false-alarm probability.

### 5 Architecture of the PCM

To reduce the architecture complexity and hardware resources consumption, the architecture of the PCM is based on an inverse-structure matched filter. According to the method of constructing the m-sequence proposed in paper [1], as Figure 2 shows, two m-sequences \( S_{m,1} \) and \( S_{m,2} \) are first constructed that separately correspond to \( \text{Mat}_1 \) and \( \text{Mat}_2 \). The \( S_{m,1} \) contains \( 2^{N_1-1} \) elements, and the \( S_{m,2} \) contains \( 2^{N_2-1} \) elements. Based on \( S_{m,1} \) and \( S_{m,2} \), two extended combining sequences \( S_{ext,1} \) and \( S_{ext,2} \) are established respectively, as shown in Figure 3.

Finally, the architecture of the PCM is as shown in Figure 4. The circular correlation between \( S_{ext,1} \) and \( F_1 \), and the circular correlation between \( S_{ext,2} \) and \( F_2 \) are carried out step by step so as to test all possible symbol combinations.

### 6 Performance of the PCM

Performance analyses of the PCM mainly include three aspects, hardware resources consumption, acquisition time, and detection probability, which are based on comparisons between the PCM and the MHM. As far as the MHM, m-sequence \( S_m \) and extended combining sequence \( S_{ext} \) are also established by above-mentioned methods, and then, similar to the

| For example, \( N = 4 \) |
|---|
| \( S_I \) | 1 | 1 | 1 | 1 |
| \( S_2 \) | 1 | 1 | 1 | -1 |
| \( S_3 \) | 1 | 1 | -1 | -1 |
| \( S_4 \) | 1 | -1 | -1 | 1 |
| \( S_5 \) | 1 | -1 | 1 | -1 |
| \( S_6 \) | 1 | -1 | -1 | -1 |
| \( S_7 \) | 1 | 1 | 1 | -1 |
| \( S_8 \) | 1 | 1 | -1 | -1 |

**FIGURE 2** Figure showing the method of constructing the m-sequence.

\[
S_{ext,1} = [1, 2, \ldots, 2^{N_1-1}, \ldots, 2^{N_1-1} + N_I - 1]
\]

\[
S_{ext,2} = [1, 2, \ldots, 2^{N_2-1}, \ldots, 2^{N_2-1} + N_I - 1]
\]

**FIGURE 3** Drawing showing establishment of the extended combining sequence based on m-sequence.
architecture of the PCM, the architecture of the MHM is as shown in Figure 5.

### 6.1 Hardware resources consumption and acquisition time

Hardware resources consumption is one of the most important aspects in terms of a highly complex weak-signal acquisition engine [27]. As Figure 4 shows, the matched filter in the architecture of the PCM contains \( N_1 \) correlators, and the matched filter in the architecture of the MHM contains \( N \) correlators, as shown in Figure 5. On one hand, every correlator in the matched filter is usually implemented by an accumulator in practice, which is the main source of matched-filter complexity. On the other hand, as Equation (6) shows, \( N_1 \) is approximately half of \( N \). As a consequence, the PCM consumes less hardware resources than the MHM.

Acquisition time is another key factor used to evaluate the performance of an acquisition algorithm [28]. At this point, acquisition time is mainly concerned with secondary code correlation operations. The time to load new data and the latency in the processing are usually not considered. In terms of the PCM, the length of \( S_{\text{ext,1}} \) is \( 2^{N_1 - 1} + N_1 - 1 \), and the length of \( S_{\text{ext,2}} \) is \( 2^{N_1 - 1} + N_2 - 1 \). Hence, the acquisition time of the PCM is given by:

\[
T_{\text{PCM}} = \frac{(2^{N_1 - 1} + N_1 - 1) + (2^{N_1 - 1} + N_2 - 1)}{f_{\text{clk}}}
\]

(20)

where \( f_{\text{clk}} \) represents system clock frequency. In terms of the MHM, similarly, acquisition time is given by:

\[
T_{\text{MHM}} = \frac{2^{N-1} + N - 1}{f_{\text{clk}}}
\]

(21)

Then, the acquisition time ratio is given by:

\[
\frac{T_{\text{PCM}}}{T_{\text{MHM}}} = \frac{2^{N_1 - 1} + 2^{N_1 - 1} + N - 2}{2^{N-1} + N - 1}
\]

(22)

As Figure 6 shows, the acquisition time of the PCM is less than that of the MHM. With the growth of \( N \), the acquisition time ratio decreases, and the advantage of the PCM over the MHM increases.

### 6.2 Detection probability

Firstly, both false-alarm probability and detection probability are both challenging issues concerned with secondary code acquisition, because different local symbol combinations are not independent when \( N \geq 3 \). Even so, some conclusions about the false-alarm probability are given by [1], and then the predefined threshold \( \beta \) can be obtained with given false-alarm probability.

Subsequently, the focus now is on the detection probability. Hypothesis \( H_g \) holds when the input secondary code sequence

\[
\text{Figure 4} \quad \text{Diagram block of the proposed partial correlation method}
\]

\[
\text{Figure 5} \quad \text{Diagram block of the existing multi-hypothesis-based method (MHM)}
\]
is \( S_k \). It is generally assumed that every hypothesis has the same occurrence probability:

\[
Pr\{H_{g,k}\} = \frac{1}{2^{N-1}} \quad (23)
\]

As a consequence, the detection probability of the MHM is given by Equation (25). Also in Equation (25), \( z_k(\hat{r}, \hat{f}_d) \) corresponds to \( S_k \). It is obvious that \( Pr\{z_k(\hat{r}, \hat{f}_d)^2 \geq \beta|H_g, N\} = Pr\{|z_k(\hat{r}, \hat{f}_d)|^2 \geq \beta|H_g, N\} \ (a \neq b) \) when based on the same acquisition architecture.

As far as the PCM is concerned, based on previous discussions, the detection probability of the PCM is given by Equation (26). Also in Equation (26), \( Pr\{H_u\} = 1/2^{N-1} \), and \( Pr\{H_v\} = 1/2^{N-1} \).

To simplify Equations (25) and (26), based on Equation (12), the coefficient of association between the local \( S_j \) and the input secondary code sequence \( S_u \) is given by:

\[
\rho_{j,u} = \frac{1}{N_1} R_{j,u} = \frac{1}{N_1}\sum_{i=0}^{N_1-1} z_{ij} \cdot s_{i,u} \quad (24)
\]

\[
P_{d,MHM} = \sum_{g=1}^{2^{N_1-1}} Pr\{z_g(\hat{r}, \hat{f}_d)^2 \geq \beta|H_g, N\} \\
= \max_i \left\{z_i(\hat{r}, \hat{f}_d)^2\right\} \cdot Pr\{H_g\} \\
= \max \left\{\sum_{a=1}^{2^{N_1-1}} Pr\{z_a(\hat{r}, \hat{f}_d)^2 \geq \beta|H_a, N\}\right\} \cdot Pr\{H_g\} \quad (25)
\]

\[
P_{d,PCM} = Pr\{z\hat{r}(\hat{r}, \hat{f}_d)^2 \geq \beta|H_u, H_v, N\} \\
= \max \left\{z_i(\hat{r}, \hat{f}_d)^2\right\} \cdot Pr\{H_u, H_v, N\} \\
= \max \left\{\sum_{a=1}^{2^{N_1-1}} Pr\{z_a(\hat{r}, \hat{f}_d)^2 \geq \beta|H_a, H_v, N\}\right\} \cdot Pr\{H_u, H_v, N\}
\]

Then, according to Equation (24), the statistical distribution of \( |\rho_{j,u}| \) is proposed, which comprises the number of different values of \( |\rho_{j,u}| \). For example, without loss of generality, \( N_1 = 4 \) and \( S_u = [1, 1, 1, 1]^T \). The statistical distribution of \( |\rho_{j,u}| \) is as shown in Table 3. It is easy to verify that all possibilities of \( S_u \) have the same statistical distribution of \( |\rho_{j,u}| \) under the case of \( N_1 \) secondary code chips. Furthermore, \( \rho_{j,u} \) and \( -\rho_{j,u} \) have the same effect on the final acquisition decision according to Equations (13), (16), and (19). Consequently, as Figure 7 shows, it is given by:

\[
Pr\{z_i(\hat{r}, \hat{f}_d)^2 \geq \beta|H_u, H_v, N\} = \max \left\{z_i(\hat{r}, \hat{f}_d)^2\right\} \cdot Pr\{H_u, H_v, N\}
\]

\[
\begin{array}{|c|c|c|}
\hline
S_u & |\rho_{j,u}| & S_j & |\rho_{j,u}| \\
\hline
S_1 & [1, 1, 1, 1]^T & 1 & S_1 & [1, -1, 1, 1]^T & 0.5 \\
S_2 & [1, 1, 1, -1]^T & 0.5 & S_2 & [1, -1, -1, 1]^T & 0 \\
S_3 & [1, 1, -1, 1]^T & 0.5 & S_3 & [1, -1, -1, 1]^T & 0 \\
S_4 & [1, 1, -1, -1]^T & 0 & S_4 & [1, -1, -1, 1]^T & 0.5 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|}
\hline
\text{Value} & |\rho_{j,u}| \\
\hline
1 & 1 \\
0.5 & 4 \\
0 & 3 \\
\hline
\end{array}
\]

where \( c \neq d, 1 \leq c \leq M_1, \) and \( 1 \leq d \leq M_1 \).

In addition, according to Equation (17), the coefficient of association between \( S_1 \) and \( S_2 \) is given by:
\[ \rho_{\hat{S}_1, \hat{S}_2} = \begin{cases} 0, & \text{N is even} \\ \frac{1}{N}, & \text{N is odd} \end{cases} \quad (28) \]

As can be seen, \( \hat{S}_1 \) and \( \hat{S}_2 \) are approximately orthogonal. Thus, based on Equations (18) and (19), it is given by:

\[ P \left\{ \left| z_{\omega} (\hat{t}, \hat{f}_d) \right|^2 = \max_i \left| z_i (\hat{t}, \hat{f}_d) \right|^2 \right\} |H_v, H_s, N_\tau| \approx 1 \quad (29) \]

It should be noted that Equation (29) is also based on the fact that \( C/N_0 \geq 25 \text{ dB} \cdot \text{Hz} \), as Figure 8 shows.

In summary, without loss of generality, Equation (25), based on Equation (27), can be rewritten as Equation (31). Also in Equation (31), \( H_1 \) corresponds to \( S_{\alpha \beta \gamma \delta} = [1, 1, \ldots, 1]^T \). All elements in vector \( S_{\alpha \beta \gamma \delta} = 1 \). Similarly, Equation (26), based on Equations (27) and (29), can be rewritten as Equation (32). Also in Equation (32), \( H_{\alpha \beta \gamma \delta} = [1, 1, \ldots, 1]^T \), and \( H_{\alpha \beta \gamma \delta} = [1, 1, \ldots, 1]^T \). All elements in \( S_{\alpha \beta \gamma \delta} = 1 \).

The detection probability ratio between the PCM and the MHM is given by Equation (33). It should be noted that the MHM is the optimal algorithm in comparison to the PCM only when the coherent integration operation brings the same SNR gain to the MHM and the PCM, and the Doppler frequency estimation deviation \( \Delta f_d \) leads to a loss of the SNR gain of the coherent integration operation as Equation (11) shows. When \( \Delta f_d = 0 \text{ Hz} \), there is no loss to the SNR gain of the coherent integration operation, and the coherent integration operation brings about the same SNR gain to the MHM and the PCM. Therefore, the MHM is better than the PCM, and the \( P_d, \text{PCM} \) is very close to the \( P_d, \text{MHM} \) as the left panel in Figure 9 shows. When \( \Delta f_d = 2.5 \text{ Hz} \), however, as the right panel in Figure 9 shows, the \( P_d, \text{PCM} \) gradually becomes greater than the \( P_d, \text{MHM} \) with the growth of the number of secondary code chips, because the coherent integration time of the PCM gradually becomes much shorter than that of the MHM, and therefore the PCM suffers less losses of the SNR gain of the coherent integration operation than the MHM.

7 | CHARACTERISTIC LENGTH VECTOR

Firstly, the secondary code Galileo E1C CS251 ‘0011100000001011101100010’ is used as an example to point out that the existing characteristic length method
ignores the ambiguity problem of the secondary code chip sign. According to [7], the characteristic length of the CS25 is 7. Then, the characteristic sequence corresponding to the second secondary code chip is ‘0111000’, and another characteristic sequence corresponding to the 24th secondary code chip is ‘1000111’. It is obvious that the sequence ‘0111000’ (1, −1, −1, −1, 1, 1, 1) cannot be distinguished from the sequence ‘1000111’ (−1, 1, 1, 1, −1, −1, −1) to identify the chip position exactly, because there is an ambiguity problem of the secondary code chip sign before bit synchronisation. Consequently, the characteristic length vector should solve this problem.

The characteristic length vector $\mathbf{L}$ is expressed as:

$$
\mathbf{L} = [L_1, L_2, \ldots, L_{N_c}]^T
$$

(30)

where $N_c$ is the number of secondary code chips in a period, and the $L_i$ ($i \in [1, N_i]$) is the $i$-th characteristic length element.

\[
\begin{align*}
\frac{P_{d,PCM}}{P_{d,MHM}} &= \frac{\Pr\left\{\left|x_1(\hat{\tau}, \hat{f}_d)\right|^2 \leq \beta[H_1, N]\cdot \Pr\left\{\left|x_i(\hat{\tau}, \hat{f}_d)\right|^2 \leq \beta[H_1, N]\right\} \cdot \Pr\left\{\left|y_1(\hat{\tau}, \hat{f}_d)\right|^2 \leq \max_k\left|y_k(\hat{\tau}, \hat{f}_d)\right|^2\right\}\left|H_{\omega|e=1, N_2}\right\}\right\} \\
&= \frac{\Pr\left\{\left|x_1(\hat{\tau}, \hat{f}_d)\right|^2 \leq \max_i\left|z_i(\hat{\tau}, \hat{f}_d)\right|^2\right\}\left|H_1, N\right\}\right\} \\
&= \frac{\Pr\left\{\left|x_1(\hat{\tau}, \hat{f}_d)\right|^2 \leq \max_i\left|z_i(\hat{\tau}, \hat{f}_d)\right|^2\right\}\left|H_1, N\right\}\right\} \\
\end{align*}
\]

(33)
$L_i$ is the minimum sequence length required to identify the $i$-th secondary code chip position. In terms of the BDS-3 B1C signal, $N_i = 1800$ and $L_i < N_i$ according to the authors’ research. Hence, there is no need to demodulate all $N_i$ secondary code chips to determine the secondary code chip position, and the characteristic length vector helps in identifying the secondary code chip position quickly.

For the convenience of introducing the method of obtaining the characteristic length vector, without loss of generality, the authors still take the secondary code Galileo E1C CS251 ‘0011100000001010110110010’ as an example and follow these steps.

(1) The secondary code matrix $M_s$ is established as:

$$
M_s = \begin{bmatrix}
S_1 & S_2 & S_3 & \cdots & S_{N_i} \\
S_2 & S_3 & S_4 & \cdots & S_1 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
S_{N_i} & S_1 & S_2 & \cdots & S_{N_i-1}
\end{bmatrix}
$$

(34)

where $S_i$ denotes the $i$-th secondary code chip.

(2) Based on $M_s$, the polarity reversal of all the rows whose first chip is logic '0' is conducted, and other rows remain constant. In this way, another matrix $M_r$ is obtained as:

$$
M_r = \begin{bmatrix}
1 & 1 & 0 & \cdots & 1 \\
1 & 0 & 0 & \cdots & 1 \\
1 & 1 & 1 & \cdots & 1
\end{bmatrix}
$$

(35)

where the elements in the first column of $M_r$ are all logic '1'.

(3) Positive integer $l$ is initialised to 2.

(4) The matrix $M_r$ is divided into two sub-matrices: an $N_i$ by $l$-dimensional matrix $M_{r,1}$ and an $N_i$ by $N_i - l$-dimensional matrix $M_{r,2}$, respectively, that is, $M_r = [M_{r,1}, M_{r,2}]$.

(5) Examine the matrix $M_{r,1}$ for the unique row that is different from other rows. If the $i$-th row of the matrix $M_{r,1}$ is the unique row, the present value of $l$ will be assigned to $L_i$ in $L$, and the $i$-th row will never be examined again. If any two rows are identical, then increment $l$ and repeat the step (4) until no two rows of the matrix $M_{r,1}$ are identical.

For the convenience of conducting the comparison between the proposed characteristic length vector and the existing characteristic length method, the improved characteristic length based on the characteristic length is to overcome the ambiguity problem of the secondary code chip sign. Even if the improved characteristic length overcomes the ambiguity problem of the secondary code chip sign, the proposed characteristic length vector is still significantly better than it, as shown in Figure 10, because the value of the improved characteristic length is equal to the maximal value in the characteristic length vector.
Eight consecutive realisations of the $|F_i(\hat{x}, \hat{f}_d)|^2$ are shown in Figure 11, and $S_{\text{ext},1} = S_{\text{ext},2} = [1, 1, 1, 1, -1, -1, -1, 1, 1, 1, 1]$. Based on the acquisition architecture shown in Figure 4, an experiment is conducted. The final acquisition results of the proposed PCM are as shown in Figure 12. All eight secondary code chips were demodulated correctly.

9 | CONCLUSIONS

The proposed PCM acquires secondary codes in three steps, which brings about low algorithm complexity. Hence, the proposed PCM overcomes the drawback of high computational burden of the multi-hypothesis-based algorithm when acquiring a long secondary code. In comparison to the multi-hypothesis-based algorithm, the proposed PCM consumes less hardware resources and acquisition time.

In addition, the concept of statistical distribution of the $|\rho_{j,n}|$ is proposed here, which reveals that the overall correlation relationship between all possible symbol combinations is steady. Based on this concept, the detection probability formulas of the multi-hypothesis-based algorithm and the proposed PCM are significantly simplified, which further contributes to the analyses of the detection probability. This is an important innovation and contribution described herein. The results of Monte Carlo simulations show that the detection probability of the proposed PCM is very close to or slightly greater than that of the multi-hypothesis-based algorithm when $C/N_0 \geq 25 \text{ dB} - \text{Hz}$, which is dependent on the Doppler frequency estimation deviation.

Furthermore, the proposed characteristic length vector not only overcomes the secondary code chip sign ambiguity ignored by the existing characteristic length method, but also determines the secondary code chip position much faster than the characteristic length method, in statistic. Hence, the proposed characteristic length vector is better than the existing characteristic length method.

Consequently, the proposed PCM and characteristic length vector are very suitable for the acquisition of a long secondary code, which brings benefits to the acquisition of weak signals with a long secondary code like weak BDS-3 B1C signals.

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