Quadrupole correlations and inertial properties of rotating nuclei

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Abstract. The contribution of quantum shape fluctuations to inertial properties of rotating nuclei has been analysed for QQ–nuclear interaction using the random phase approximation (RPA). The different recipes to treat the cranking mean field plus RPA problem are considered. The effects of the $\Delta N = 2$ quadrupole matrix elements and the role of the volume conservation condition are discussed.

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1. Introduction

Response of a many-body system to external fields provides various information about intrinsic dynamics of the system. One of the efficient approaches to get a deep insight into a nuclear structure is to study a response of a nucleus to an external Coriolis field. In particular, we can trace the interplay between single-particle and collective degrees of freedom by studying a rotational dependence of the kinematical and dynamical moments of inertia. In its turn, these quantities are benchmarks for nuclear microscopic models, that allow to understand main features of a nuclear field. For example, the importance of pairing correlations, introduced in nuclear physics by Bogoliubov [1] and Bohr, Mottelson and Pines [2] in an analogy with the correlations in superconductors, had been recognised first in the description of nuclear inertial properties [3].

The kinematical moment of inertia is described quite well within the cranking approach, for instance, in the relativistic Hartree-Bogoliubov and non-relativistic density-dependent Hartree-Fock-Bogoliubov theory with Gogny forces (see [4] and references therein). However, there is a systematic discrepancies between those theories and experiment with regard of the dynamical moment of inertia. It is quite desirable to understand the main source of discrepancies between experiment and theory, since the above approaches represent state of art of nuclear structure studies.

It is well understood that the two-body correlations are important for a correct description of the moments of inertia (see discussions in [4]). One of the systematic ways to include these correlations is the random phase approximation (RPA) based on a self-consistent solution of the cranking mean field equations [5, 6]. This enables one to also restore symmetries broken on the mean field level (see textbooks [7, 8]). Practical realisation of these aspects has been done recently in a cranking harmonic oscillator model with a residual quadrupole–quadrupole interaction [9]. In particular, it was shown that the Thouless-Valatin moment of inertia [10] (which, in general, contains Belyaev [3], Migdal [11] and other terms) calculated in the RPA is equivalent to the dynamical moment of inertia calculated in the mean field approximation

\[ J_{TV} = J_{(2)} = \frac{d}{d\Omega} \langle \Omega | \hat{J}_x | \Omega \rangle = -\frac{d^2 E_{MF}}{d\Omega^2}. \tag{1} \]

Here \( \Omega \) denotes the rotational frequency, \( |\Omega\rangle \) is a self-consistent solution of the mean field equations and \( E_{MF} \) is a total mean field energy in the rotating frame.

Quantum oscillations around the mean field solutions provide the additional contribution to the total energy. How such a quantal effect can be obtained within the RPA for the case of rotating systems is generally described in [12, 13]. However, the practical application of the RPA in rotating deformed nuclei is a demanding numerical task. Therefore, the influence of quantum fluctuations on the moment of inertia is till now scarcely studied. With regard of the easier case of pairing fluctuations more extensive investigations were performed (c.f. [14, 15]). The first attempt to incorporate the pairing and quadrupole oscillations [16] was done in a restricted configuration space and the self-consistency between the residual interaction and the mean field was not taken into account. A self-consistent analysis carried out in a simplified model [9] demonstrated that the quantum correlations modify the Thouless-Valatin moment of inertia. Therefore, there is a strong motivation to understand the role of quantum correlations in realistic calculations for inertial properties.

The aim of the present paper is to study exclusively the influence of shape vibrations on the angular momentum and on the moment of inertia. Starting from the
self-consistent cranking mean field calculation the total binding energy as a function of the rotational frequency has to be calculated up to RPA order, i.e. with inclusion of the RPA correlation energy. Our calculations are based on separable QQ-forces as an effective interaction. The mean field part of such calculations could be carried out with more advanced effective interactions like e.g. Skyrme or Gogny type forces.

However, the RPA correlation energy is not feasible for non-separable forces because of the too large configuration space needed for the cranking model. In addition, the problem of spurious solutions in the RPA calculations for the state of art effective interactions is silently avoided in the literature even for a zero rotational frequency. In other words, again the question arises about a consistency between a mean field calculations and the RPA contribution for those type of forces. On the other hand, the QQ-forces are the most important interaction term for inducing nuclear deformation and rotational excitations. The mean field plus RPA calculations with the QQ-forces based an a realistic cranking mean field potential with a spin-orbit coupling are by no means trivial. As we shall describe (see below) such calculations can be executed in different ways which enable us study specific properties of these forces and their effects in more detail.

We will consider the following versions of the QQ-model:
(i) the QQ-forces with a full N-mixing of oscillator shells ($\Delta N = 0, \pm 2$);
(ii) the QQ-forces without N-mixing ($\Delta N = 0$) which is the commonly used a QQ-model (e.g. [7]);
(iii) the double-stretched QQ-forces with the volume conservation constraint [17, 18].

By comparing results of the various treatments (i-iii) provides new insights in the nature of both the QQ-forces and their influence onto the rotational properties. Because of the schematic character of the QQ-forces our investigations are merely an exploratory approach to the influence of quantum shape fluctuations upon rotational properties rather than a quantitative description of experimental data. The latter would require more extensive numerical efforts that are not intended in this paper.

The paper is organised as follows. In Sec. 2 we introduce our mean field and RPA models. The influence of the volume conservation constraint on the RPA correlations is discussed in Sec. 3. This is followed by a summary and discussion in Sec. 4.

2. The Model

Our mean field plus RPA calculations are based on the following Hamiltonian:

$$\hat{H} = \sum_k \epsilon_k \hat{c}^\dagger_k \hat{c}_k - \frac{\kappa}{2} \sum_{m=0,\pm 1,\pm 2} \hat{Q}^2_m = H_{\text{sph}} + H_{\text{QQ}}.$$  (2)

Here $\epsilon_k$ are the single-particle energies of the spherical oscillator Hamiltonian $\hat{h}_{\text{sph}}$

$$\hat{h}_{\text{sph}} = \frac{\hat{p}^2}{2M} + \frac{M}{2} \omega_0^2 \hat{r}^2.$$  (3)

The operators $\hat{c}^\dagger_k$ ($\hat{c}_k$) are creation (annihilation) operators with the suffix $k$ labeling a set of quantum numbers. For the sake of convenience we have chosen in equation (2) a quadratic form of the QQ-forces by taking a set of Hermitean quadrupole operators $\hat{Q}_m$ built up from $\hat{r}^2 \hat{Y}_{20}$ and the linear combinations $\hat{r}^2 (\hat{Y}_{2m} \pm \hat{Y}_{2,-m})$ for $m = (1, 2)$.

The operators $\hat{Q}_m$ read as

$$\hat{Q}_m = \sqrt{\varphi_m} \sum_{kl} q_{0,kl} (\hat{c}_k^\dagger \hat{c}_l + \hat{c}_k^\dagger \hat{c}_l^\dagger),$$  (4)
Quadrupole correlations and inertial properties of rotating nuclei

\[ \hat{Q}_{\pm 1} = \sqrt{\varphi_{\pm 1}} \sum_{kl} q_{\pm 1,kl} \left( \hat{c}_k \hat{c}_l^\dagger \pm \hat{c}_k^\dagger \hat{c}_l \right) \]  

\[ \hat{Q}_{\pm 2} = \sqrt{\varphi_{\pm 2}} \sum_{kl} q_{\pm 2,kl} \left( \hat{c}_k \hat{c}_l^\dagger \pm \hat{c}_k^\dagger \hat{c}_l \right) \]  

with

\[ \varphi_m = \begin{cases} 
1 & m = 0, -1, 2 \\
-1 & m = 1, -2 
\end{cases} \]  

where the index \( k(l) \) is labeling a complete set of quantum numbers in order to form the matrix elements \( q_{m,kl} = \langle k|\hat{Q}_m|l\rangle \).

The index \( k \) refers to the time conjugated state. The sums in equations (5-6) split in a proton and neutron part. We consider the isoscalar quadrupole interaction only, since isovector terms can be treated in a similar way. Hereafter, any isospin index is omitted.

The Hamiltonian \( \hat{H} \), equation (2) is, of course, formally the same expression for all the versions (i-iii) of the QQ-model described in the introduction. What is different concerns the particular treatment of the Q-operator (N-mixing or not and stretching transformation), the adjustment of the spherical oscillator frequency, \( \omega_0 \), to account for the volume conservation condition and the adaption of the strength parameter \( \kappa \). This is because the rotational invariance of the total Hamiltonian is required for a clean treatment of the angular momentum conservation within the RPA [12, 13].

We consider the Routhian

\[ \hat{H}' = \hat{H} - \Omega \hat{J}_z \]  

(8)

to describe the rotational properties of the system. Here \( \hat{J}_z \) is the angular momentum component about the z-axis. Note that the cranking term in \( \hat{H}' \) does not change the exact wave functions but only fixes the angular momentum about the quantisation axis (z).

The relevant mean field part of the Routhian \( \hat{H}' \) is originated from the Hamiltonian \( \hat{H} \) in equation (2). Writing the latter term in the standard form of a deformed potential (c.f. [19]), the mean field part of \( \hat{H}' \) reads as

\[ \hat{H}'_{MF} = \hat{H}_{sph} - \Omega \hat{J}_z - \sqrt{\frac{5}{4\pi}} \hbar \omega_0 \beta \left( \hat{Q}_0 \cos \gamma - \hat{Q}_2 \sin \gamma \right) \]  

(9)

where \( \beta \) and \( \gamma \) are the deformation parameters. The self-consistent solution \( \Phi_{\omega} \equiv |\Omega\rangle \) shortly denoted as \( |\rangle \) corresponds to an energy minimum of the energy surface \( E'(\beta, \gamma) = \langle \hat{H}' \rangle \).

We are aiming at the contributions of zero point quantum corrections to the moment of inertia stemming from the RPA vibrations about the above mean field solution. Accordingly, the quasi boson approximation (QBA) is applied to the Routhian \( \hat{H}' \), equation (8), which is rewritten in the form

\[ \hat{H}' = \hat{H}'_{MF} + \frac{\kappa}{2} \sum_m \left( \langle \hat{Q}_m \rangle^2 - \hat{H}_{res} \right) \]  

(10)

Here, the third term is a separable residual interaction

\[ \hat{H}_{res} = \frac{\kappa}{2} \sum_m \left( \hat{Q}_m - \langle \hat{Q}_m \rangle \right)^2 \]  

(11)

neglected in the mean field calculations.
Quadrupole correlations and inertial properties of rotating nuclei

Using the standard notation we refer to particle states (unoccupied single-particle orbitals) by subscript $p$ and to hole states (below the Fermi level) by $h$. Defining the boson-like operators

$$\hat{b}_{ph}^\dagger \equiv \hat{c}_p^\dagger \hat{c}_h, \quad \hat{b}_{ph} \equiv \hat{c}_h^\dagger \hat{c}_p$$

the QBA means to treat the above operators (12) as exact bosons obeying the commutation relations

$$\left[\hat{b}_\mu, \hat{b}_\nu\right] = \left[\hat{b}_\mu^\dagger, \hat{b}_\nu^\dagger\right] = 0, \quad \left[\hat{b}_\mu, \hat{b}_\nu^\dagger\right] = \delta_{\mu\nu}$$

where the double index $ph$ runs over all particle-hole pairs and is labelled by $\mu$ or $\nu$, respectively. In this approximation any single-particle operator $F$ can be expressed as

$$F = \langle F \rangle + F^{(1)} + F^{(2)}$$

where the second and third terms are linear and bilinear order terms in the boson expansion, respectively.

The final RPA Hamiltonian reads

$$\hat{H}'_{\text{RPA}} = \sum_\mu E_\mu \hat{b}_\mu^\dagger \hat{b}_\mu - \kappa \sum_m \left( \hat{Q}_m - \langle \hat{Q}_m \rangle \right)^2$$

where $E_\mu = e_p - e_h^\prime$ is the energy of a particle-hole excitation with respect to the mean field part $\hat{H}'_{\text{MF}}$, equation (9). We remind that in the QBA one includes all second order terms into the boson Hamiltonian such that $(F - \langle F \rangle)^2 = F^{(1)} F^{(1)}$.

The RPA Hamiltonian is diagonalised by solving the equation of motion

$$\left[\hat{H}'_{\text{RPA}}, \hat{O}_\lambda^\dagger\right] = \omega_\lambda \hat{O}_\lambda^\dagger$$

for the phonon operators $\hat{O}_\lambda^\dagger$ which are linear combinations of the basic bosons $\hat{b}_\mu^\dagger$ and $\hat{b}_\mu$, equation (12). The linear part of the Q-operators in terms of the boson operators $\hat{b}^\dagger$ and $\hat{b}$ has the following form

$$\hat{Q}_m = \sqrt{\varphi_m} \sum_\mu q_{m,\mu} \left( \hat{b}_\mu^\dagger + \varphi_m \hat{b}_\mu \right)$$

Here $q_{m,\mu}$ are the single–particle matrix elements of $\hat{Q}_m$ in equations (14-15).

The equation of motion (16) leads to the following determinant of the secular equations (c.f. [13])

$$F(\omega_\lambda) = \det \left( R - \frac{1}{2\kappa} \right)$$

where the matrix elements of $R$ are given by

$$R_{km}(\omega_\lambda) = \sum_\mu \frac{q_{k,\mu} q_{m,\mu} C_{km}^{\mu}}{E_\mu^2 - \omega_\lambda^2}$$

involving the coefficients

$$C_{\mu}^{km} = \begin{cases} E_\mu & k = 0, -1, 2(1, -2); & m = 0, -1, 2(1, -2) \\ \omega_\lambda & k = 0, -1, 2(1, -2); & m = 1, -2(0, -1, 2) \end{cases}$$

The zeros of the function $F$

$$F(\omega_\lambda) = 0.$$

yield the RPA eigenfrequencies $\omega_\lambda$. 
Since the mean field violates the rotational invariance, among the RPA eigenfrequencies there exist two spurious solutions. One “spurious” solution at $\omega_\lambda = \Omega$ corresponds to a collective rotation, since $[H', J_\pm] = [H', J_x \pm i J_y] = \mp \Omega J_\pm$. The other solution with zero frequency is associated with the rotation around the $z$ axes, since $[H', J_z] = 0$. This spurious mode allows to determine the Thouless-Valatin moment of inertia, $J_{T.V.}$ [10, 20, 21], which can be calculated from the equations

\begin{equation}
[H_{RPA}, i \hat{\Phi}] = \frac{\hat{J}_z}{J_{T.V.}}.
\end{equation}

\begin{equation}
[\hat{\Phi}, \hat{J}_z] = i.
\end{equation}

Here the angle operator $\hat{\Phi}$ is the canonical partner of the angular momentum operator $\hat{J}_z$.

The total energy can be split in the mean field and the RPA contribution

\begin{equation}
E' = E'_{\text{MF}} + E'_{\text{RPA}} + \frac{\Omega^2}{2},
\end{equation}

with explicitly written the rotational “spurious” mode and the RPA contribution of non-spurious modes and of the spurious zero mode. The RPA contribution can be expressed as [12, 8]

\begin{equation}
E'_{\text{RPA}} = \frac{1}{2} \left( \sum_\lambda \omega_\lambda - \sum_\mu E_\mu \right).
\end{equation}

The exchange term [12, 8] can be neglected which means to use the Hartree approximation.

We recall that the mean field energy, the quasiparticle (particle-hole) excitations, the RPA eigenfrequencies, are calculated in the rotating frame. In other words, these quantities are functions of the rotational frequency $\Omega$ that is our external parameter. We remind that in the rotating frame the appropriate state variables are the Routhian energy $E'$ and the rotational frequency $\Omega$ in contrast to the lab system where the appropriate state variables are the energy defined by the Legendre transformation $E = E' + \Omega I$ and the angular momentum $I$. While the operator relation $dH'/d\Omega = -J_z$ holds, the corresponding relation in the RPA order

\begin{equation}
dE'/d\Omega = -\langle RPA|J_z|RPA \rangle
\end{equation}

is taken only as a reasonable approximation to the relation between the exact energy and the angular momentum. In fact, it is not only a very difficult task to calculate numerically the RPA expectation value $\langle RPA|J_z|RPA \rangle$. This calculation sensitively depends also from the validity of the QBA itself which raises another problem [15]. However, the use of the total Routhian and rotational frequency as the appropriate variables to determine the angular momentum of the system is consistent with our numerical calculations.

The proper definition of the angular momentum within RPA is a known problem which was controversially discussed by several authors. We mention below some important points of these discussions.

Assuming that the RPA expectation value of the angular momentum in the yrast state is

\begin{equation}
\langle RPA|J_z|RPA \rangle = I,
\end{equation}

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...
Reinhardt proposed to use the equations (26) and (27) to define the quantisation condition for the angular momentum (see discussion in [22] where the rotation axis is chosen to be $x$). Since in the mean field approximation $\langle J_z \rangle = -dE_{\text{MF}}'/d\Omega$ [7, 8], we obtain from the above equations the following quantisation condition

$$\langle J_z \rangle = I + \frac{1}{2} + \frac{dE_{\text{RPA}}}{d\Omega}$$

(28)

According to Reinhardt, the smallness of the contribution of the RPA modes $dE_{\text{RPA}}'/d\Omega$ to the value of the angular momentum in the yrast state could be used as the validity criteria for the self-consistent cranking model.

It should be pointed out that Marshalek raised the question about the validity of equation (28) (see discussion in [23] where the rotation axis is also $x$). He assumes that the mean field value of the angular momentum in the yrast state is $\langle J_z \rangle = I$ which should be preserved in the RPA order too. He proposed to use the energy and the angular momentum as the appropriate variables to analyse the rotational properties. Considering the cranking Hamiltonian up to the second order of RPA, he proved that all conservation laws are restored (see Sec.4.1 in [12]). Note that to prove all conservation laws it was important to keep the second order of the cranking term $\Omega J_z^{(2)}$ in the cranking Hamiltonian as well. From this point of view, we would find the contribution of the second order term of $J_z$ operator to the expectation value $\langle \text{RPA}|J_z^{(2)}|\text{RPA} \rangle = \langle J_z \rangle + \langle \text{RPA}|J_z^{(2)}|\text{RPA} \rangle$ (see discussion in [16, 24]), which is inconsistent with Marshalek’s assumption. Furthermore, the total energy in [12] and [23] is a sum of the mean field energy $E_{\text{SCC}}$ defined in the lab frame whereas the RPA correlation energy is defined in the rotating frame. Marshalek assumed that the RPA frequencies in the lab and rotating frame are the same. This leads to an inconsistency which becomes obvious if we consider a rotation about a symmetry axis. In this case the RPA states are characterised by the projection of the angular momentum upon the symmetry axis because $[J_z, \hat{O}_\lambda^\dagger] = \lambda \hat{O}_\lambda^\dagger$ with $\lambda$ being the value of the angular momentum carried by the phonons along the $z$ axis. We thus obtain

$$[\hat{H}_{\text{RPA}}, \hat{O}_\lambda^\dagger] = [\hat{H}_{\text{RPA}} - \Omega \hat{J}_z, \hat{O}_\lambda^\dagger] = (\hat{\omega}_\lambda - \lambda \Omega) \hat{O}_\lambda^\dagger = \omega_\lambda \hat{O}_\lambda^\dagger$$

(29)

This equation explicitly demonstrates that the RPA eigenfrequencies in the lab frame $\hat{\omega}_\lambda$ are different from the RPA eigenfrequencies in the rotating frame $\omega_\lambda$ if the RPA modes carry angular momentum (see details and discussion in [25]).

Thus, considering equations (26) and (27) to be a suitable relation between the exact energy and the angular momentum and knowing the rotational dependence $E' = E' (\Omega)$ of the total energy, it enables us to calculate the angular momentum $I = I (\Omega)$ by the derivative

$$I = J_{\text{MF}} + \Delta J_{\text{RPA}} = -\frac{\Delta E_{\text{MF}}'}{\Delta \Omega} - \frac{\Delta E_{\text{RPA}}'}{\Delta \Omega} - \frac{1}{2}$$

(30)

and, subsequently, the dynamical moment of inertia as the second derivative

$$\mathcal{J}^{(2)} = \frac{\Delta I}{\Delta \Omega}.$$  

(31)

In practise the differentiations are numerically obtained with finite differences $\Delta \Omega$ as indicated in the above equations. Hereafter, we denote the angular momentum $I$ calculated in accordance with equation (30) as $J_{\text{RPA}}$.

The RPA solution $\omega_\lambda = \Omega$ that contributes to the total angular momentum by $-1/2$, is very sensitive to numerical inaccuracies and to the limits of the configuration space in the calculation. In fact, at small but nonzero rotational frequencies the
Quadrupole correlations and inertial properties of rotating nuclei

spurious solution at zero and the solution at the rotational frequency get mixed up because of numerical inaccuracies. The rotational solution will then no longer be exactly at $\Omega$. Since the angular momentum is calculated as the derivative, these errors can lead to large erratic contributions to the angular momentum. We avoid such errors by explicitly finding the rotational solution and subtracting from the RPA energy. The missing term is then replaced by $\frac{\hbar \Omega^2}{2}$. The rotational solution can be identified by its collective nature and large overlap with the $\hat{J}_x \sim \hat{J}_x + i \hat{J}_y$ operator to which it should be identical if the solution was found exactly. This method is applicable when the level density is low, which is satisfied when $\hbar \Omega \leq 0.3$ MeV. At larger $\hbar \Omega$ the rotational and the spurious solution decouple in a numerically stable way and the solution at the rotational frequency is found with a high accuracy. To calculate the RPA energy [25] it was crucial to apply the contour integral method developed in [26].

It is important to use a large enough configuration space in the calculations which include mixing of different $N$-shells. To reduce the number of $N$-shells needed in the calculation we always transform the matrix elements into the stretched basis.

3. The RPA quadrupole correlations

In this section we focus our analysis on the contribution of the RPA correlations to the self-consistent mean field solution. Parts of these results were presented in [27, 9].

The self-consistent mean field Hamiltonian agrees with that of a rotating three-dimensionally deformed oscillator

$$\hat{H}'_{MF} = \sum_i \left[ \frac{p_i^2}{2M} + \frac{M}{2} \left( \omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2 \right) \right]_i - \hbar \Omega \hat{J}_z$$  \hspace{1cm} (32)

where $\omega_{x,y,z}$ are the three oscillator frequencies [19].

We start with the version (i) of the QQ-model ($\Delta N = 0, \pm 2$ shell mixing is included) and without the volume conservation constraint. The self-consistent mean field solutions of the Routhian, equation (8), are found from the relations

$$\kappa \langle \hat{Q}_0 \rangle = \sqrt{\frac{5}{4\pi}} \hbar \omega_0 \beta \cos \gamma,$$

$$\kappa \langle \hat{Q}_2 \rangle = -\sqrt{\frac{5}{4\pi}} \hbar \omega_0 \beta \sin \gamma,$$

$$\langle \hat{Q}_{\pm1,-2} \rangle = 0$$  \hspace{1cm} (33)

where, as usual, the contributions of exchange terms are omitted.

Two systems are considered: (a) the lighter nucleus $N=Z=14$ with the self-consistent deformation ($\varepsilon_2 = 0.3, \gamma = 0$, at $\Omega = 0$) and (b) the nucleus $N=Z=76$ defined with the corresponding set ($\varepsilon_2 = 0.3, \gamma = 0$, at $\Omega = 0$) by choosing the appropriate strength $\kappa$ for each system. The largest $N$-shell included in the calculation is $N=6$ in (a) and $N=8$ in (b).

It will be seen that both systems behave qualitatively similar with respect to the rotation. By searching for the energy minimum for finite rotational frequencies the self-consistent mean field solutions are found. This minimisation makes also sure that the RPA will restore the rotational invariance. The mean field angular momentum for these systems calculated by means of equation [26] is shown at Figs. 1, 2. It is
Figure 1. The angular momentum, in units of $\hbar$, for a rotating deformed harmonic oscillator with (solid line) and without (dashed line) QQ ($\Delta N = 2$) isoscalar RPA correlations as a function of $\Omega$. The self-consistent deformed oscillator is filled with 14 protons and 14 neutrons.

Figure 2. The angular momentum, in units of $\hbar$, for a rotating deformed harmonic oscillator with (solid line) and without (dashed line) QQ ($\Delta N = 2$) isoscalar RPA correlations as a function of $\Omega$. The self-consistent deformed oscillator is filled with 76 protons and 76 neutrons.
proportional to the rotational frequency $\Omega$. This means that our system even if the deformation is changing with the rotation gets an approximately constant moment of inertia [28].

Taking into account the RPA correlations, we obtain a substantial decrease of the total energy $E'$ and also a dependence of the RPA angular momentum $J_{\text{RPA}}$ on $\Omega$. It turns out that $J_{\text{RPA}} \propto \Omega$. The dynamical moment of inertia is approximately constant. However, it is reduced (the reduction is 16% and 4% in $N=Z=14$ and 76, respectively) due to the RPA correlations, as seen in figure 1. The RPA correlations cause relatively smaller effect on the moment of inertia in the heavier system than in the lighter one. This is because of the absolute $\Omega$-dependence of the $E_{\text{RPA}}$ becomes slightly smaller and the mean field moment of inertia becomes much larger in the heavier system. The spherical oscillator part in our Hamiltonian, equation (2), can in these cases be replaced by any set of spherical single-particle energies. This does not affect the general properties of the RPA correlations. The result would not qualitatively change when including a spin-orbit splitting.

Now the above systems are studied for the version (ii), i.e. by dropping the $\Delta N = \pm 2$ matrix elements of the quadrupole operators $Q_m$, equations (4)-(6). We recall that the quadrupole Hamiltonian with $\Delta N = 0$ is commonly used for obtaining the self-consistent solutions which includes both non-zero spin-orbit terms as well as the QQ-forces [7]. This task needs much less efforts, since solutions can be found in a relatively small configuration space. Having in mind that the $\Delta N = 2$ particle-hole excitations are energetically unfavorable relative to the $\Delta N = 0$, one would expect a reliable results within the model. In addition, the effect of the higher lying $\Delta N = 2$ vibrations could be studied. The self-consistent mean field solutions of the Routhian in equation (5) are again found from the relations (33) where this time the quadrupole operators are written in the stretched representation [29, 19].

In this model the angular momentum is as above proportional to $\Omega$ but the RPA correlation energy does not any more influence the moment of inertia as can be seen in figure 3. We conclude that an important part of the RPA correlations originate from the $\Delta N = 2$ part of the QQ-forces. It is interesting to mention that the simple QQ-forces with $\Delta N = 0$ provide quite reasonable results with regard equilibrium deformations in the mean field calculations.

An explicit calculation of the commutation relation
\[
[\hat{H}_{\text{RPA}}, \hat{J}_z] = 0, \quad (34)
\]
gives a stringent check of the RPA restoration of the rotational symmetry. In the numerical calculation of the $N=Z=14$ case above we found that equation (34) was fulfilled with an accuracy of $10^{-3}$ when the deformation parameters $\varepsilon_2$ and $\gamma$ were determined with the accuracy of 4 and 2 decimal figures, respectively.

Next, we apply the volume conservation constraint and the double-stretched representation of quadrupole operators and compare the results with those obtained with the (normal) QQ-forces. We remind that the double-stretched representation is based on the condition such that the change in the density distribution must be accompanied with a change in the potential [30].

For the triaxially deformed oscillator the self-consistency condition [31]
\[
\omega_x^2 \langle x^2 \rangle = \omega_y^2 \langle y^2 \rangle = \omega_z^2 \langle z^2 \rangle
\]
and the volume conservation constraint $\omega_x \omega_y \omega_z = \omega_0^3$ yield a self-consistent residual
The quadrupole interaction \cite{17,18}
\[ \hat{H}' = \hat{H}'_{MF} - \hat{H}_{res} \] (36)
where \( \hat{H}'_{MF} \) is defined by equation (32) and
\[ \hat{H}_{res} = \overline{\kappa} \sum_{m=0,1,2} Q_m \overline{Q}_m. \] (37)

Here \( \overline{Q} \) is the quadrupole operator written in doubly stretched coordinates
\[ x_i = x_i \frac{\omega_i}{\omega_0}, \quad i = 1, 2, 3, \] (38)
where \( \omega_i \) is one of the oscillator frequencies. The self-consistent \( \overline{\kappa} \) can be found as \cite{18}
\[ \overline{\kappa} = \frac{4\pi}{5} \frac{M\omega_0^2}{A \langle \overline{\Omega}^2 \rangle}. \] (39)

From the self-consistency condition, equation (35), follows that
\[ \langle \overline{Q}_m \rangle = 0, \quad m = 0, \pm 1, \pm 2. \] (40)

The \( \overline{Q} \) operators do therefore not affect the mean field energy and can be used directly in the RPA formalism.

In figure 4 results for the harmonic oscillator with 10 protons and 10 neutrons are presented. As for the cases studied above the QQ forces give a reduced angular momentum and lead to a delayed start of the rotational band at \( \hbar \Omega = 1.1 \) MeV (where the calculated angular momentum becomes positive). The RPA correlations obtained with the use of the double-stretched residual interaction results in an increased dynamical moment of inertia in contrast to the results with the QQ-forces. The rotational dependence of the equilibrium deformations is presented in figure 5. The
Quadrupole correlations and inertial properties of rotating nuclei

Figure 4. The angular momentum, in units of $\hbar$, for a rotating deformed harmonic oscillator with (solid line) and without (dashed line) isoscalar RPA correlations as a function of $\Omega$. Upper part: volume conservation, lower part: QQ-forces. The self-consistent deformed oscillator is filled with 10 protons and 10 neutrons. Both systems have the same deformation at $\Omega = 0$.

minimum calculated with the volume conservation constraint seems to be changing more slowly then with the QQ-forces.

From figure 6, which shows the mean field energy as a function of the deformation at different rotational frequencies with and without the volume conservation constraint, one can qualitatively understand the different results. The RPA contribution to the angular momentum is obtained as the negative derivative of the correlation energy. The QQ-forces give a very flat minimum that is changing relatively fast with the rotational frequency. This causes a large correlation energy that also is changing relatively fast with the rotational frequency. As a consequence, there is the large contribution from the RPA correlations to the angular momentum seen in figure 4. A small contribution to the total angular momentum could be explained
as the result of the increase of the rigidity of the potential with the increase of the rotational frequency $\Omega$. The flat potential energy surface is also the reason why we obtain a delayed start of the rotational band using the QQ-forces. With the volume conservation constraint one observes an opposite behavior. The minimum is much more distinct and is less affected by the rotation. In figure 4 a small increase in the dynamical moment of inertia can be seen which is consistent with the fact that the potential gets flatter with increasing $\Omega$ when using the volume conservation constraint.

In figure 7 the mass dependence of the RPA contribution to the moment of
inertia, as calculated with the volume conservation constraint and the double-stretched quadrupole residual interaction, is plotted. Even though the RPA contribution to $J^{(2)}$ is increasing with increasing mass number $A$ the relative contribution is decreasing. This is consistent with the simple picture that the mean field approximation works better in heavier systems.

4. Summary

The value of the angular momentum and the moment of inertia are generally dependent on the correlations induced by the shape vibrations. The size of the RPA correlations depends on the curvature of the mean field potential. Their influence on the angular momentum and on the moment of inertia is determined on how the potential curvature depends on the rotational frequency. This dependence is different in different mass regions and is determined by the degree of filling of the shell. The scale of the variation of the correlations is sensitive to the accuracy of the mean field approximation and to the size of the configuration space.

The very flat potential of the QQ-forces leads to large effects of the quadrupole correlations upon the angular momentum and the moment of inertia. In fact, the large RPA contributions for the QQ-forces cause the delay of the rotation in contrast to the results obtained with the volume conservation constraint. In addition, the QQ-forces, without the volume conservation condition, may not produce any stable minimum above a certain deformation and it seems that the deformation in the considered cases is close to that limit. When comparing the results with and without the $\Delta N = 2$ mixing one notices that even when the two calculations provide similar mean field solutions, which is expected [29], the RPA does yield significant differences. We conclude that these forces are not very realistic in describing RPA correlations in nuclear structure.
The RPA calculations with the volume conservation constraint and the double-stretched quadrupole residual interaction give a smaller contribution to the angular momentum. The RPA contributions to the moment of inertia is almost constant as a function of the rotational frequency but show a strong dependence on the mass number. Even though the absolute contribution is increasing with increasing the mass number, the relative contribution is decreasing from 11% at A=20 to less then 2% at A=80. The RPA based on the double-stretched quadrupole residual interaction provides a more realistic result in contrast to the cases of the QQ-forces discussed above.

Finally, we conclude that the volume conservation constraint is very important requirement when considering effective forces. Further analysis of the self-consistency between a mean field approximation and a treatment of the quadrupole shape vibrations for various realistic forces is needed to make reliable comparisons with experimental results. Another important question which has to be addressed in further investigations is how other multipoles such as monopole and octupole ones would affect the present results.

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Quadrupole correlations and inertial properties of rotating nuclei

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