Strange quark matter fragmentation in astrophysical events

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Abstract

The conjecture of Bodmer-Witten-Terazawa suggesting a form of quark matter (Strange Quark Matter) as the ground state of hadronic interactions has been studied in laboratory and astrophysical contexts by a large number of authors. If strange stars exist, some violent events involving these compact objects, such as mergers and even their formation process, might eject some strange matter into the interstellar medium that could be detected as a trace signal in the cosmic ray flux. To evaluate this possibility, it is necessary to understand how this matter in bulk would fragment in the form of strangelets (small lumps of strange quark matter in which finite effects become important). We calculate the mass distribution outcome using the statistical multifragmentation model and point out several caveats affecting it. In particular, the possibility that strangelets fragmentation will render a tiny fraction of contamination in the cosmic ray flux is discussed.

keywords: strange quark matter, strangelets, statistical multifragmentation model

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I. INTRODUCTION

Sometime after the papers of Bodmer [1] and Terazawa [2] put forward the idea of a quark ground state of strongly interacting matter, the wide and colorful discussion given by Witten [3] added considerable interest to the issue of what is now called the Strange Quark Matter (SQM) hypothesis.

The stability scenario has been systematically studied for the first time by Farhi and Jaffe [4] within the MIT bag model, where a wide parameter space for absolute stability to hold was established. More recently, it has been claimed that the preferred state would be when quarks form pairs, similarly to electrons in ordinary superconductivity, for it would allow an even lower energy per baryon number for the system due to the formation of the color condensate [5–7].

If the SQM hypothesis is valid, the low probability of a simultaneous decay of roughly a third of up and down quarks in a nuclei into strange quarks under everyday conditions would prevent the transition. However, it has been shown [8–10] that for nuclear systems at high density and moderate temperature, the transition could be favored. In this way, compact objects are naturally though as niches for the existence of SQM. Among the predicted systems and phenomena, strange stars [11–14], compact stars where the transition to SQM happens in all the stellar interior, and the possibility of strangelets [4, 15, 16], small lumps of strange quark matter, were discussed. The possibility of strangelets being a part of the cosmic ray flux, and likely involved in exotic events [17], naturally raised the question about the conditions for bulk strange quark matter to break apart, and with what mass and energy spectra the fragmentation into strangelets would ensue [18].

Starting with these early papers, some injection mechanisms for strangelets in astrophysical sites have been proposed: strange stars mergers [3, 19], phase transition during type II supernovae [18, 20, 21], and acceleration in strange pulsar environment [22]. All these process might lead to a measurable abundance of this component among the cosmic ray flux, although they have not been addressed in full detail yet. Considering this, a simple manner of testing the existence of strange matter in the interior of compact stars would be the detection by ground-based or in-orbit experiments of strangelets of astrophysical origin. In fact, some experiments claimed to have detected possible exotic components [23, 26], though a live debate has taken place without a firm confirmation of the nature of the primaries.
On the other hand, the problem of fragmentation of nuclear matter during cooling/decompression has been studied for several decades (for a review, see for example [27]), with wide applicability to laboratory experiments (e.g., nuclei collision in accelerators). A range of r-process nuclei could be produced this way [28] and the analogue situation with SQM in the place of nuclear matter appears to be justified if the Bodmer-Witten-Terazawa conjecture is true. However, the production of strangelets and subsequent acceleration still lack a detailed general analysis. In particular, the predicted fluxes of strangelets among cosmic rays are based on plausible suppositions instead of refined calculations, and in general dismiss the possible decay of these particles into ordinary nuclei [29]. Moreover, the energy spectrum of strangelets to be injected in the interstellar medium is quite uncertain. In this way, supposing the Fermi mechanism will accelerate strangelets the same way it does with ordinary cosmic rays may pose some problems [30], since only particles with a non-thermal spectrum can be accelerated by shocks. The distribution of masses and energies at the injection site are then important ingredients for all these attempts to connect some events with the possible strangelet primaries arriving to earth [31].

Here we present an analysis for the fragmentation of strange quark matter within the statistical multifragmentation model. This model can be applied to a supernova explosion driven by the conversion of ordinary nuclear matter to strange quark matter, for example, being this scenario an alternative to the neutrino-driven ones that still face difficulties in explaining the explosion in numerical simulations. The conversion during the proto-neutron star phase could provide enough energy for the expulsion of stellar material either in the form of a detonation wave or of a second neutrino wind [20, 21]. The ejected outer layers could be contaminated by strangelets due to turbulent mixing effects [32]. As a general result, we shall show that a fragmentation into mass chunks having $A \leq 100$ may be expected, although significant uncertainties in the underlying physics remain, and in fact recent calculations do not obtain ejection of SQM [33]. Since the temperatures and other parameters are quite similar, scenarios of the merging of two strange stars [31] would follow a similar fragmentation pattern.

In a recent paper, Biswas and collaborators [34] used the statistical multifragmentation model to analyze the fragmentation of strange quark matter in a scenario of strange stars mergers, concluding that the mass spectrum results in low mass fragments and shows an exponential decay with $A$ and also presenting an estimate for the strangelet flux based on
cosmic ray diffusion properties. However, their analysis dismissed important contributions to the energy of the fragments and assumed some physical properties (discussed at length in [33]) that may significantly alter the results. These are related to the dependency of the energy of the fragment on the strange quark mass (assumed negligible) and also on the possibility of pairing between quarks, as will be described in the next section.

II. STATISTICAL MULTIFRAGMENTATION MODEL

Among the several proposals formulated to deal with the fragmentation problem, the statistical multifragmentation model (SMM) (see [27] and references therein) has provided consistent results when applied to the bulk nuclear matter → nuclei transition.

When we proposed using the SMM to treat the fragmentation of strange quark matter [36], we initially employed it to treat fragmentation in a supernova driven by the conversion of nuclear matter into SQM scenario. Recent works have shown that in the collision of two strange stars, matter achieve high temperatures [37] (of order ~ tens of MeV), which are high enough to comply with the hypotheses of the statistical multifragmentation model. Specifically, the critical condition involving the excitation energy per baryon number for the occurrence of the break-up (which must be comparable to the total binding energy, ensuring thermal and dynamical equilibrium) is satisfied. Therefore, in both scenarios the fragmentation should proceed similarly. Also, in ref. [36], we used an approximate treatment for the strangelet energy taking mean values instead of considering the full dependence of the surface and curvature energies on temperature and baryonic number, which is certainly important for the matter, and considered strange quark matter without pairing. We found in ref. [36] some inconsistencies regarding the position of the fragmentation peak, as we shall discuss bellow, the present treatment is the result of this analysis.

We based our analysis on a simplified version of the SMM [38] in which the system is studied in the grand canonical ensemble, rendering neat analytical solutions when the thermodynamical limit is taken. Generally speaking, an exponential behavior for the partition function is predicted for high masses.

We have started from the partition function of a single fragment with $A$ nucleons

$$\omega_A = V \left( \frac{mT A}{2\pi} \right)^{3/2} e^{-f_A/T}, \quad (1)$$
where $f_A$ is the internal free energy of the fragment

$$f_A = -WA + \sigma A^{2/3} + CA^{1/3}, \quad (2)$$

and $W$ represents the volume binding energy per baryon number of SQM, $\sigma$ and $C$ being the internal free energy of a fragment with baryon number $A$, rest mass $m$ and chemical potential $\mu$ corresponding to the surface and curvature contributions, respectively; and $T$ is the bulk SQM temperature.

From the definition of pressure in the grand canonical ensemble,

$$p(T, \mu) = T \lim_{V \to \infty} \frac{\ln Z(V, T, \mu)}{V}, \quad (3)$$

where $Z$ is the Laplace transform of the grand canonical partition function, the pressures for both phases are obtained from the singularities of the isobaric partition function (for details, see [38] and references therein).

The liquid and gas pressures are given by

$$p_g(T, \mu) = \nu \frac{\nu}{b}, \quad (4)$$

$$p_l(T, \mu) = \nu \frac{\nu}{b}, \quad (5)$$

where $\nu = \mu + W$ is the (shifted) chemical potential.

The fragmentation spectrum, $P_g$, can be then obtained (considering chemical equilibrium between bulk matter and the fragments) by taking the derivative of the gas pressure, $p_g$, with respect to the chemical potential of the fragments, $\mu_A$,

$$P_g(A) = \frac{\partial}{\partial \mu_A} p_g = \left( \frac{m_0 T}{2\pi} \right)^{3/2} e^{\frac{(\mu + W - b \nu) A - \sigma A^{2/3} - CA^{1/3}}{T}}, \quad (6)$$

In the model, the parameter $b$ represents the repulsive interactions in a simple Van der Waals approximation.
We have considered strange quark matter within the MIT bag model framework, in the color-flavor-locked (CFL) state \cite{5,6,39}. The energy of each fragment was calculated by employing the multiple reflexion expansion formalism as in \cite{40}, thus presenting the necessary dependence on the temperature, baryonic number, gap parameter, bag constant, and strange quark mass.

When obtaining the mass number for which the fragment distribution reaches its maximum in the coexistence region, we have checked that the peak is always obtained for strangelets with mass numbers \( A \ll 1 \), leaving only the exponential behavior being apparent. It is not clear what is the meaning of this result. One possibility is that the description is actually incomplete, or one cannot use the grand canonical approach either. An alternative interpretation is that the system would not fragment at all, remaining in the bulk SQM state in spite of the mechanical and thermal perturbations to which it is subject in the outbreak and further expansion. In particular, thermal equilibrium between the bulk and the fragments throughout the whole process had to be assumed in the calculation and may not be valid. If the system is to fragment in few large chunks of matter, then an statistical approach would not be adequate.

However, one factor which can be important is related to the presence of the vacuum, represented by the bag constant \( B \). This term is naturally absent when considering nuclear matter fragmentation since the parameters obtained for describing it already consider the influence of the vacuum. But here it is possible to look at the fragmentation of SQM as a process in which a fraction of the vacuum energy is used to provide strangelets with surface and curvature energies among other finite size effects. Therefore, there is a difference of energy density per baryon number between the liquid and gaseous phases which should be taken into account.

In this way, we have introduced the bag constant directly into the energy density of the gas and liquid (bulk) phases by substituting the volume internal free energy per baryon number for \( W = W_0 + Bv \) and we shall continue to use this approach throughout the rest of our analysis. This last term is not the same for both phases due to the dependency with the proper volume associated with each system:
\[ W_l = W_0 + Bv_{liq}, \]
\[ W_g = W_0 + Bv_{gas}. \]

The density of fragments with baryon number \( A \) is given by Eq. 6. The argument of the exponential in the mass distribution \( [(\nu - b p_g)A] \) in the coexistence phase \( (p_g^* = p_l) \) is now

\[
[\nu - b p_g^*]A = [\nu_g - \nu_l^*]A = [\mu_g + W_g - \mu_l - W_l]A
= [B (v_{gas} - v_{liq})]A,
\]

where \( v \) is the volume per baryon number.

Following the approach for deriving the temperature dependent internal free energy presented in [40], we have obtained the normalized mass distribution function shown in Figure 1.

We see that the whole fragment distribution is now shifted to higher values of \( A \), although the peak is still not in a physical position. Also, we notice that although increasing the system’s temperature leads to a less stable system, it also decreases the values of the surface and curvature’s terms [40], thus favoring strangelets with higher \( A \). It must be pointed out, nevertheless, that high temperature strangelets would be more prone to evaporation [3] and would have to cool down in order to survive.

Figure 2 considers a strangelet injection scenario with the possible ejection of \( 10^{-4} M_\odot \) so we can compare our results with the one presented in reference [34]. For the values presented of strange quark mass, bag constant and gap parameter, strange quark matter would be stable for almost all masses (the minimum baryon number for stability is \( A = 4 \)). Nevertheless, we stress that this form of SQM is obtained using almost “optimal” values for these unknown parameters, and therefore may not be very realistic. Any increase in \( B \) or \( m_s \) or a decrease in \( \Delta \) would lead to a much less stable strange matter, this instability being greater as the temperature increases, as can be seen in Figure 3.

It is also important to remark that if the superconducting phase is not considered, strangelets would be even more unstable. For example, for \( B^{1/4} = 145 \text{ MeV} \) and \( m_s = 150 \text{ MeV} \), at zero temperature, strangelets with \( A \lesssim 15 \) would be unstable but at 30 MeV,
FIG. 1: Normalized distribution function of fragments for CFL strangelets with \( B^{1/4} = 145 \text{ MeV}, m_s = 100 \text{ MeV}, \) and \( \Delta = 50 \text{ MeV} \). On the left panel, we show its dependence on the temperature considering the SMM with direct introduction of the bag constant. On the right panel, a comparison of the results obtained for a given temperature with (full line) and without (dashed line) the introduction of the vacuum energy.

only those with \( A \gtrsim 2700 \) would not decay to normal nuclear matter as is exemplified in Figure 4. In this way, in this scenario of bulk strange quark matter fragmentation driven by expansion, the existence of a large fraction of strangelets in the cosmic ray flux is highly unlikely and would certainly be negligible if color superconductivity is not considered, as already proposed in reference [30]. Also, if all quarks were assumed to have zero mass, the stability of this system would be artificially enhanced since the surface tension is associated with a non-zero strange quark mass. Both simplified features were employed in reference [34]. These remarks explain why in the work of Biswas and collaborators is claimed that the amount of light fragments is increased with temperature with the suppression of heavy fragments, in opposition of what is seeing in this work.

It has been suggested [41], however, that strange stars may present a strangelet crust embedded in an electron background. If this is the case, then during a merger event strangelets would already be ejected with a mass spectrum with baryonic number of a few hundreds and one should expect a considerable amount of strangelets in the cosmic ray flux, an idea so far not favored by experiments.
FIG. 2: Distribution function of fragments for CFL strangelets with $B^{1/4} = 145$ MeV, $m_s = 100$ MeV, and $\Delta = 50$ MeV considering the ejection of $10^{-4}M_\odot$ as a function of the temperature.

FIG. 3: On the left, distribution function of fragments for CFL strangelets with $T = 30$ MeV, $m_s = 150$ MeV, and $\Delta = 50$ MeV considering the ejection of $10^{-4}M_\odot$ as a function of value of the bag constant. The stability limit, i. e., the value of $A$ below which strangelets are not stable, is represented by the vertical lines. On the right, the same but fixing the bag constant at $B^{1/4} = 145$ MeV and varying $\Delta$.

III. CONCLUSIONS

We have presented an analysis of the fragmentation of SQM into strangelets in high-temperature astrophysical settings. We have tried to make explicit the assumptions made to calculate the distribution of fragments and how the uncertainties could affect the final result.

We note that previous studies have argued that SQM must fragment into very large
FIG. 4: Energy of strangelets without pairing with $B^{1/4} = 145$ MeV, $m_s = 150$ MeV as a function of $A$ for a fixed temperature, as indicated. The horizontal line indicates the threshold for strangelets to decay to normal nuclear matter.

pieces [16], comparable to asteroid sizes, when mechanically stressed by external fields. The physical argument was that, in opposition to nuclear matter, the energy per baryon number always decreases with an increase in $A$ for SQM. This indicates that it is necessary to introduce a large amount of energy external to the system to break SQM into smaller (but very macroscopic) fragments. This conclusion is not based on the application of a fragmentation framework, and even if true it is clear that it would not emerge from an analysis within the grand canonical ensemble.

We have shown that the simplest fragmentation of bulk SQM into strangelets yield an odd result within the SMM, since the fragment peak would fall in an unphysical region, perhaps in agreement with the previous expectations [16]: matter would remain in the bulk phase provided the SMM model consistently describes the process. Alternatively, one can question the very validity of the latter for this description. It has also been shown that the explicit consideration of the vacuum energy density shifts the fragmentation spectra towards higher values of $A$, although the position of the peak is still for $A \ll 1$. This tentative mode for SQM fragmentation results in strangelets with a characteristic mass scale as shown in Fig. 2. This points to a possible negligible presence of this exotic particles in the cosmic ray flux.

The work of Biswas and collaborators [34] disregarded an important feature in the SQM energy derivation, the fact that the strange quark mass is of order of 100 MeV [42], giving
rise to a significant surface energy. It will certainly increase the total energy of the system with obvious implications to the minimum baryon number for absolute stability to hold. Also, the inclusion of an extra term in the exponential in Eq. \(6\) will make the fragment distribution decrease with a faster pace with increasing \(A\). In this way, most strangelets fragmenting in their scenario would decay into normal nuclear matter as soon as formed.

Our own results can be interpreted as indicating that most of the SQM will not fragment and that mostly unstable fragments (those subject to evaporation at a given temperature) will dominate the process, leading to the production of ordinary clusters and nucleons. Moreover, when considering higher temperatures, the spectrum is shown to extend to higher masses (Fig. 3), but the strangelets are more vulnerable to evaporation and the net outcome is no strangelets at all. Therefore, we conclude that a non-negligible amount of strangelets could only result from milder temperatures and paired quarks. Even if a huge fraction of SQM remains in bulk it will be subject to evaporation and should be affected, possibly surviving if the pairing is strong enough [43].

The attempts made by us to confirm/validate the mass fragmentation spectra of SQM have rendered ambiguous and/or inconsistent results. For instance, the minimization of information entropy of Aichelin & Huefner [44] which, in principle, could be adequate for the merging of strange stars and the supernova ejection alike, predicts, in general, a peak in the mass distribution, but its extension to large macroscopic masses is difficult and the results obtained inconclusive. The same is true when one tries a very simple approach of constructing the phase region à la Gibbs, that is, using the conditions \(T_{gas} = T_{liq}\), \(\mu_{gas} = \mu_{liq}\), and \(P_{gas} = P_{liq}\). It is precisely the question of whether or not there would be a large number of fragments that could be treated in an statistical manner that is behind these ambiguities.

If the process is such that one can use the grand canonical formalism for its study and take the solution to the thermodynamic limit, or if a random sampling directly dealing with the microcanonical ensemble of all the decay channels is necessary, or if this process can happen out of equilibrium (violating the Gibbs criteria) are still open questions. There is a final overall caveat concerning the role of the residual strong interaction between nucleons, since it has a different behavior at large distances than gluon-exchange forces mediating interactions between quarks. The crude approximations built-in in any of these formalisms may hide its true importance and reliability of the results themselves. This is an important subject deserving a deeper understanding in order to provide better predictions of the possible
contamination of strangelets in the cosmic ray flux.

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