THE FLOW AROUND A COSMIC STRING. I. HYDRODYNAMIC SOLUTION

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ABSTRACT

Cosmic strings are linear topological defects which are hypothesized to be produced during inflation. Most searches for strings have relied on the string’s lensing of background galaxies or the cosmic microwave background. In this paper, I obtained a solution for the supersonic flow of collisional gas past the cosmic string which has two planar shocks with a shock compression ratio that depends on the angle defect of the string and its speed. The shocks result in the compression and heating of the gas and, given favorable conditions, particle acceleration. Gas heating and over-density in an unusual wedge shape can be detected by observing the HI line at high redshifts. Particle acceleration can occur in the present-day universe when the string crosses the hot gas contained in galaxy clusters and, since the consequences of such a collision persist for cosmological timescales, could be located by looking at unusual large-scale radio sources situated on a single spatial plane.

Key words: acceleration of particles – cosmology: theory – hydrodynamics – radio continuum: general – shock waves

1. INTRODUCTION

Cosmic strings are hypothetical objects generically predicted by most modern inflationary models and are expected to survive until the present day as large over-horizon kinked linear objects and smaller loops with about 10 horizon-scale strings in the observable portion of the universe (Polchinski & Rocha 2007). The velocities of strings and loops are expected to be trans-relativistic with an rms velocity \( v_s \approx 0.7c \). The main parameter of the string is the symmetry breaking scale \( \eta \) which determines the mass per unit length \( \mu = \eta^2 \) and the angle defect of the straight string \( \theta = 8\pi G \mu c^2 \). For a general introduction of cosmic strings see, e.g., Vilenkin & Shellard (1994), Hindmarsh & Kibble (1995), and Copeland et al. (2011). Cosmic strings are expected to lens background sources of light (Vilenkin 1984; Morganson et al. 2010; Sazhinna et al. 2011) and the cosmic microwave background (CMB; Vilenkin 1986). The loops and kinks will emit gravitational waves (Spergel et al. 1987) and leave behind wakes (Sornborger et al. 1997; Duplessis & Brandenberger 2013). This will affect structure formation in the early universe (Khatri & Wandelt 2008; Shlaer et al. 2012). Kinks tend to straighten themselves by emitting gravitational waves and loops tend to evaporate for the same reason.

The current upper limits on the angle defect \( \theta \) come from the lensing of background galaxies, \( \theta < 6 \times 10^{-5} \) (Morganson et al. 2010), and CMB lensing, \( \theta < 7 \times 10^{-6} \) (Wyman et al. 2005). Pulsar timing experiments (Damour & Vilenkin 2005) give tighter limits but are more model-dependent. It may potentially be interesting to look for the direct interaction of strings with ordinary collisional matter. This subject has mostly been overlooked in the literature. The straight segment of the string could produce interaction signatures that are peculiar because they lay on a single spatial plane.

The paper is organized as follows. Section 2 describes an exact hydrodynamic solution of a homogeneous flow past a linear angle defect. Section 3 estimates over-densities and the heating produced by the shocks behind the string in the early universe, and briefly discusses observational possibilities compared with previously reported dark matter wakes. Section 4 estimates the particle acceleration in the shocks and their potential observability. Section 5 contains the discussion.

2. SUPERSONIC FLOW PAST THE LINEAR ANGLE DEFECT

Ordinary matter in the form of either neutral or ionized gas is normally considered within a fluid framework due to its relatively high collisionality. In the case of atomic gas, the mean free path in hydrogen following recombination is around \( 4 \times 10^{-5} \) pc, which is tiny compared with cosmological scales or the scales of the string. Ordinary matter should therefore be considered collisional when considering a large-scale solution of a flow around the string.

Below, I will describe a supersonic flow of ordinary collisional fluid past the string (for the estimate of collisionality see Section 3). The straight string segment has no gravity of its own but is manifested by the presence of an angle defect. It is convenient to consider the flow in the string rest frame and to map the space around it onto Euclidean space. As the perpendicular cross-section of the string is a cone, the projection involves an angular cut in flat space with the sides of the cut mapped onto each other. Figure 1 shows a perpendicular cross-section of the space around the string where I have chosen to use the cut with sides, which are parallel to the fluid velocity. Such a cut ensures that the flow pattern is symmetric with respect to the cut direction. The flow changes its velocity from \( v_1 \) to \( v_2 \) at two oblique shocks that have an angle \( \beta \) with a tail direction \( x \). I will also designate the deflection angle \( \alpha = \beta + \theta/2 \) and the ratio of specific heats \( \gamma \), and I assume \( \gamma = 5/3 \) since I mostly deal with either monoatomic gas or very cold hydrogen. I will also introduce \( \beta_1 = v_1/c_s \) and the Mach number of the inflow \( M_1 = v_1/c_s \), where \( c_s \) is the sound speed. For the electron–proton plasma, \( M_1 \) can be approximated by \( 1.68 \times 10^4 \beta_1 (T/1 \text{eV})^{-1/2} \), where \( T \) is the plasma temperature.

Applying the conservation of matter, momentum, and energy to the flow depicted on Figure 1, and excluding most variables, I arrive at the oblique shock relation, see, e.g., Landau &
Lifshitz (1959), where the deflection angle and the shock angle are related by the angle defect of the string:

$$\cot \frac{\alpha}{2} = \tan \alpha \left[ \frac{(\gamma + 1)M_i^2}{2(M_i^2 \sin^2 \alpha - 1)} - 1 \right].$$

Equation (1) can be solved for $\alpha$ and has two branches of solutions, one of which provides larger $\alpha$ and is only realized in confined geometries, while the solution with smaller $\alpha$ is realized in open boundary flows (Landau & Lifshitz 1959). The expected values for the angle defect $\theta$ for astrophysical strings are fairly small, $< 10^{-4}$, see Section 1; in the second branch of the solution of Equation (1), this means that $\alpha, \beta \ll 1$ as well. Assuming that $\alpha, \beta, \theta \ll 1$ but $M_i \alpha$ and $M_i \beta$ are not necessarily small, the second branch will give the following equation for $\alpha$:

$$4M_i^2 \sigma^2 - \alpha M_i^2 \theta (\gamma + 1) - 4 = 0.$$  

If $\theta \gg 1/M_i \approx 10^{-4} \beta_i^{-1} T^{1/2}$, then the shocks are strong and solving Equation (2) gives $\beta = \theta (\gamma - 1)/4$. In the opposite limit, $\theta \ll 10^{-4} \beta_i^{-1} T^{1/2}$, the shock is weak and the solution is $\beta = 1/M_i$, realizing the “Mach cone.” In the case of a weak shock, the effective Mach number in the frame of the shock is $M = 1 + \theta (\gamma - 1) M_i/2$ and the temperature ratio is $\rho_2/\rho_1 = 1 + \theta M_i/2$, while in the case of a strong shock it is $M = \theta (\gamma + 1) M_i/4$ and the compression ratio approaches $(\gamma + 1)/(\gamma - 1)$.

It is potentially interesting to look for flow solutions in the magnetized media, i.e., to consider the magnetohydrodynamic (MHD) problem. The general orientation of the field will break the symmetry of the flow that I have used to derive its structure in Figure 1, and so such a flow will be more complex. Two special MHD cases can be treated relatively easily, however. If the magnetic field is parallel to the string, then both shocks will be perpendicular shocks and the shock condition is the same, except that magnetic pressure is added to the plasma pressure (Landau & Lifshitz 1960). If the magnetic field is perpendicular to both the string and the inflow velocity and $\theta$ is small, then the solution will be similar to the hydrodynamic solution. The general MHD case will be considered elsewhere.

3. DETECTION IN THE EARLY UNIVERSE BY THE 21 cm LINE

As I demonstrated in Section 2, strings will leave behind a wake of compressed and heated material which has a well-defined shape, over-density, and dimensions, depending on the Mach number of the flow and the angle defect of the string. So far, cosmic string wakes have been considered primarily in a collisionless medium where they produce the wedge-shaped wake with angle $\theta/2$ and an over-density of two (see, e.g., Silk & Vilenkin 1984; Brandenberger et al. 2010; Tashiro 2013; Brandenberger 2014). The gas will subsequently accrete onto the dark matter to produce features in HI. This subsequent accretion will happen at much later times, however. In this section, I will neglect the self-gravitational effect of the wake and concentrate on the direct hydrodynamic interaction between the gas and the string.

I will assume that $M \gg 1$, $\gamma = 5/3$, which should be the case for molecular hydrogen with $T < 70$ K. The over-density and temperature of such a supersonic trail right after interaction with the string could be expressed as

$$\rho_2/\rho_1 = 4/(1 + 3M^{-2}) \approx 4,$$  

$$T_2/T_1 \approx \frac{5}{16} M^2.$$  

Assuming that the hydrogen temperature scales adiabatically as $(1 + z)^{(\gamma - 1)} = (1 + z)^2$ after $z = 500$, the effective Mach number will be

$$M = \frac{2}{3} \theta M_i = 120 \left( \frac{\theta}{10^{-5}} \right) \left( \frac{\beta_i}{0.5} \right) \left( \frac{1}{1 + z} \right),$$  

that is, I expect the shocks from the string to become strong starting from $(1 + z) \approx 120$ and at later times before re-ionization, and produce heating and compression resulting in excess 21 cm emission due to both higher temperature and high density.

It should be noted that Equations (3)–(5) describe over-density and heating only as a function of redshift and the angle defect. This makes them distinctly different from the expressions obtained for gravitational accretion on wakes with an extra unknown, which is the time allowed for accretion. Our expression will therefore be easier to use for direct estimation of $\theta$, given the redshift and the excess 21 cm emission, however, special care should be taken to not confuse the two effects.

![Figure 1. Flow around a linear topological angle defect of $\theta$. I use a Euclidean 2D plane with a dashed area representing an angular cut $\theta$, which is parallel to the flow velocity.](image-url)
4. DETECTION BY RADIO EMISSION

Shocks propagating through magnetized plasma tend to accelerate particles and produce radio emission by synchrotron mechanism and γ-ray emission through inverse Compton mechanisms. I will consider string propagating through the present-day, well-ionized intergalactic or intracluster medium (ICM) and estimate the effects of shock acceleration. Using the effective Mach number for oblique shocks derived in Section 2, the change in enthalpy of the gas per unit time per unit area—the power, in principle, available for acceleration—on both shocks can be estimated as

\[
P_s = 3n_m c_s^2 c \beta_s (\gamma + 1)/4
\]

\[
= 3.2 \times 10^{-6} \frac{\text{erg}}{\text{cm}^2 \text{s}} \frac{n}{10^{-3}} \frac{T}{1 \text{ keV}} \beta_s \theta 10^{-5},
\]

for \( \theta \ll 10^{-4} \beta_s^{-1} T^{1/2}, \) or

\[
P_s = 3n_m c_s^2 c \beta_s (\gamma + 1)/4 \theta^3
\]

\[
= 1.3 \times 10^{-6} \frac{\text{erg}}{\text{cm}^2 \text{s}} \frac{n}{10^{-3}} \beta_s^3 \left( \frac{\theta}{10^{-3}} \right)^3
\]

for \( \theta \gg 10^{-4} \beta_s^{-1} T^{1/2}. \)

The radiation efficiencies of the shocks are fairly uncertain for several reasons. First-principle calculations of acceleration efficiencies are still not available due to the complex nature of the acceleration process (Malkov & O’C Drury 2001), however, some results based on a phenomenological model for particle scattering is available (Kang et al. 2012). For the same reason, the injection process is not fully understood and the electron/proton ratio is not exactly known. While in supernova shocks, given typical densities and shock Mach numbers, the amplified magnetic field at the shock dominates over the radiation field and synchrotron losses dominate over inverse Compton losses, in the tenuous ICM and intergalactic medium, the opposite could be true. The conventional approach to deal with these uncertainties is to introduce parameters such as acceleration efficiency and the magnetic field amplification efficiency and make an educated guess based on available theory studies as well as observations (see, e.g., Keshet et al. 2004a). I will further simplify the above approach, introducing the radio emission efficiency of \( \eta \), keeping in mind that it depends on the gas density and the Mach number in a fairly complex way. We would expect the radiation efficiency to be in the range \( 10^{-2} - 10^{-6} \). The radio spectrum is fairly uncertain for the same reasons as above. I can estimate the spectral brightness near the peak of the emission using the total emitted power as \( \nu T_\text{B} \nu_s \nu_m \approx P \eta \nu_s/4 \pi \sin i \), where \( i \) is an angle between the line of sight and the velocity of the string, provided that it is not much smaller than \( 1/M \). I obtain the following expression for the surface brightness temperature \( T_\text{B} = \nu_s c^2/2k\theta^2 \):

\[
T_\text{B} = \frac{83 \text{ mK} (\sin i)^{-1} \eta_c \nu_m}{10^{-4} \left( \frac{\nu_m}{1 \text{ GHz}} \right)} \left( \frac{T}{1 \text{ keV}} \right) \beta_s \theta 10^{-5},
\]

where I used the weak shock case.

The new generation of radio telescopes, such as SKA, should be able to detect such low surface brightness objects, e.g., 50% of the SKA will detect the 5 mK surface brightness at the 3σ level with a beam of 8″ in 1 hr (Feretti et al. 2004), however, the problem of confusion with other objects and dealing with the galactic background is indeed quite challenging. Several morphological features specific to the remnants of string activity can help to differentiate these objects, however. Speaking of the collision of the string with the galaxy cluster, other large-scale (>1 Mpc), low surface brightness objects expected to be detected with the new generation of radio telescopes include intergalactic shocks (Keshet et al. 2004a, 2004b); accretion shocks, currently detected only in some clusters as the so-called radio relics (van Weeren et al. 2010); and diffuse radio halos (Carilli & Taylor 2002; Cassano et al. 2010), currently detected only in merging clusters but thought to be present universally. Out of these three, all have different morphological and/or radiative features compared to the remnants of string activity. Intergalactic and accretion shocks are expected to be detected on the outskirts of the cluster, where the surface brightness is enhanced due to the projection effect. For the string shocks, the projection factor \((\sin i)^{-1} \) is basically a constant, while the surface brightness should strongly increase with higher density toward the center of the cluster. Both types of objects are expected to emit significantly polarized radio emission. Comparing cluster halos and string trails, the former are unpolarized and rather spherical, mimicking the shape of the cluster, while the latter will be polarized and, in general, rather elliptic as they cross the cluster at some angle with respect to the field of view. In fact, due to the selection effect, the trails with higher projection factors will be much more likely to be observed, while their morphology will be the most unusual—basically a thin stripe across the cluster, see Figure 2.

Finally, the surface of a past interaction of the string with dense matter over a Hubble time will be very large and it is likely to have patches where the shocks will be amplified when propagating down the density gradients (Ostriker & McKee 1988). Also, the acceleration efficiency of the twin shock is higher than that of a single shock, as the downstream particle traveling from one shock could diffuse to the upstream of the other (Melrose & Pope 1993).

Another method to differentiate string trails and other objects can rely on the strings themselves and their trajectories being fairly straight on sub-horizon scales. This means that the straight segments of the string will leave behind relics that lay on the single spatial plane. Searching for spatial planes that contain a significant number of large-scale (>1 Mpc) radio sources could be another viable method which will help to avoid confusion with accretion/intergalactic shocks.

5. DISCUSSION

In this paper I presented the solution of the flow of collisional matter around the cosmic string for the first time. Aside from shocks in the collisional medium, strings can also produce wakes in the dark matter (Silk & Vilenkin 1984; Brandenberger et al. 2010), which also has a wedge shape but with a constant angle of \( \theta/2 \) and a constant dark matter compression ratio of two, independent of \( \theta \). The subsequent self-gravitational contraction of such wakes will also draw in ordinary matter, possibly resulting in secondary shocks.
and various observational effects for the entrained hydrogen, e.g., enhanced 21 cm emission (Brandenberger et al. 2013; Tashiro 2013; Brandenberger 2014). As the wake gravitationally contracts, and entrains and heats hydrogen, it no longer presents such a clear and well-defined angular shape. In contrast, the trail in collisional matter described in this paper momentarily heats the gas. The relative importance of the trails considered here and the trails produced by the collapse of dark matter wakes for the HI structures in the early universe will be considered in a future publication.

Given that the typical string segment length as well as the distance between each other is of the order of 1 Gpc, the radio searches for strings should survey large-scale distant objects, such as clusters, and focus on extended sources of at least 1 Mpc in physical size. The cluster radio halos could be confused with the string trails, but they are associated with turbulent acceleration in clusters (see, e.g., Brunetti & Lazarian 2007; Beresnyak et al. 2013) that underwent a recent merger, so surveying smaller and quieter clusters is more advantageous. Also, as I pointed out above, they should have different polarization properties and morphologies. The collision of the string with giant molecular clouds (GMCs) could in principle produce a much stronger signal, e.g., for $\theta = 10^{-5}$ and $T = 10$ K the shocks will have an effective Mach number of 3, and assuming a density of $10^3$ cm$^{-3}$, $\beta = 0.5$ and an acceleration efficiency of $10^{-2}$, the surface brightness temperature is around 4 K. Given the small volume fraction of GMCs in the universe, such a collision is fairly unlikely, however.

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Figure 2. Expected radio emission from the string with $\beta = 0.5$ and $\theta = 7 \times 10^{-6}$, crossing a simulated galaxy cluster at an angle of 15° with the line of sight and with a closest approach of 0.5 Mpc to the cluster center (contours). I assumed shock radiation efficiency $\eta_s = 10^{-4}$. Contours are labeled by brightness temperature in nK at 1 GHz. Red represents cluster bremsstrahlung X-ray emission in log-scale and arbitrary units, the image is 3 Mpc across. The cluster density and temperature, courtesy of Hao Xu, were obtained by the cosmological simulation of structure formation described in Xu et al. (2010).
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