Analyze of the star formation modeling algorithm in SPH code

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Abstract. The chemical and photometric evolution of star forming disk galaxies is investigated. Numerical simulations of the complex gasdynamical flows are based on our own coding of the Chemo - Dynamical Smoothed Particle Hydrodynamical (CD - SPH) approach, which incorporates the effects of star formation. As a first application, the model is used to describe the chemical and photometric evolution of a disk galaxy like the Milky Way.

Keywords: star formation, chemical and photometric evolution, SPH code

1. Introduction

Galaxy formation is a highly complex subject requiring many different approaches of investigation. Recent advances in computer technology and numerical methods have allowed detailed modeling of baryonic matter dynamics in a universe dominated by collisionless dark matter and, therefore, the detailed gravitational and hydrodynamical description of galaxy formation and evolution. The most sophisticated models include radiative processes, star formation and supernova feedback, e.g. (Katz, 1992; Steinmetz & Müller, 1994; Friedli & Benz, 1995).

The results of numerical simulations are fundamentally affected by the star formation algorithm incorporated into modeling techniques. Yet star formation and related processes are still not well understood on either small or large spatial scales. Therefore the star formation algorithm by which gas is converted into stars can only be based on simple theoretical assumptions or on empirical observations of nearby galaxies.

Among the numerous methods developed for modeling complex three dimensional hydrodynamical phenomena, Smoothed Particle Hydrodynamics (SPH) is one of the most popular (Monaghan, 1992). Its Lagrangian nature allows easy combination with fast N - body algorithms, making possible the simultaneous description of complex gas-stellar dynamical systems (Friedli & Benz, 1995). As an example of such a combination, TREE - SPH code (Hernquist & Katz, 1989; Navarro & White, 1993) was successfully applied to the detailed
modeling of disk galaxy mergers (Mihos & Hernquist, 1996) and of
galaxy formation and evolution (Katz, 1992). A second good example
is an GRAPE - SPH code (Steinmetz & Müller, 1994; Steinmetz &
Müller, 1995) which was successfully used to model the evolution of
disk galaxy structure and kinematics.

2. The model

The hydrodynamical simulations are based on our own coding of the
Chemo - Dynamical Smoothed Particle Hydrodynamics (CD - SPH) ap-
proach, including feedback through star formation (SF). The dynamics
of the "star" component is treated in the framework of a standard N-
body approach. Thus, the galaxy consists of "gas" and "star" particles.

For a detailed description of the CD - SPH code (the star formation
algorithm, the SNII, SNIa and PN production, the chemical enrich-
ment and the initial conditions) the reader is referred to (Berczik &
Kravchuk, 1996; Berczik, 1999; Berczik, 2000). Here we briefly describe
the basic features of our algorithm.

2.1. Star formation algorithm

It is well known that SF regions are associated with giant molecular
complexes, especially with regions that are approaching dynamical in-
stability. The overall picture of star formation seems to be understood,
but the detailed physics of star formation and accompanying processes,
on either small or large scales, remains sketchy (Larson, 1969; Silk,
1987).

All the above stated as well as computer constraint cause the using
of simplified numerical algorithms of description of conversion of
the gaseous material into stars, which are based on simple theoretical
assumptions and/or on results of observations of nearby galaxies.

We modify the "standard" SPH SF algorithm (Katz, 1992; Stein-
metz & Müller, 1994; Steinmetz & Müller, 1995), taking into account
the presence of chaotic motion in the gaseous environment and the time
lag between the initial development of suitable conditions for SF, and
SF itself.

Inside a "gas" particle, the SF can start if the absolute value of
the "gas" particles gravitational energy exceeds the sum of its thermal
energy and its energy of chaotic motions:

\[ |E_{g,i}^{gr}| > E_{i}^{th} + E_{i}^{ch}. \]  (1)
Gravitational and thermal energies and the energy of random motions for the "gas" particle $i$ in model simulation are defined as:

$$
\begin{align*}
E^g_i &= -\frac{3}{5} \cdot G \cdot m_i^2 / h_i, \\
E^{th}_i &= \frac{3}{2} \cdot m_i \cdot c_i^2, \\
E^{ch}_i &= \frac{1}{2} \cdot m_i \cdot \Delta v_i^2,
\end{align*}
$$

where $c_i = \sqrt{\frac{\rho_i \cdot T_i}{\mu}}$ is the isothermal sound speed of particle $i$. We set $\mu = 1.3$ and define the random or "turbulent" square velocities near particle $i$ as:

$$
\Delta v_i^2 = \frac{1}{N_B} \sum_{j=1}^{N_B} m_j \cdot (v_j - v_c)^2 / \sum_{j=1}^{N_B} m_j,
$$

where:

$$
v_c = \frac{1}{N_B} \sum_{j=1}^{N_B} m_j \cdot v_j / \sum_{j=1}^{N_B} m_j.
$$

For practical reasons, it is useful to define a critical temperature for SF onset in particle $i$ as:

$$
T_{i}^{\text{crit}} = \frac{\mu \cdot \rho_i \cdot \Delta v_i^2}{\frac{8}{5} \cdot \pi \cdot G \cdot \rho_i \cdot h_i^2 - \Delta v_i^2}.
$$

Then, if the temperature of the "gas" particle $i$, drops below the critical one, SF can proceed.

$$
T_i < T_i^{\text{crit}}.
$$

The chosen "gas" particle produces stars only if the above condition holds over the time interval exceeding its free - fall time:

$$
t_{ff} = \sqrt{\frac{3 \cdot \pi}{32 \cdot G \cdot \rho}}.
$$

We also check that the "gas" particles remain cool, i.e. $t_{cool} < t_{ff}$. We rewrite these conditions following (Navarro & White, 1993):

$$
\rho_i > \rho_{\text{crit}}.
$$

We set the value of $\rho_{\text{crit}} = 0.03$ cm$^{-3}$.

When the collapsing particle $i$ is defined, we create the new "star" particle with mass $m^{\text{star}}$ and update the "gas" particle $m_i$ using these simple equations:

$$
\begin{align*}
\begin{cases}
m^{\text{star}} = \epsilon \cdot m_i, \\
m_i = (1 - \epsilon) \cdot m_i.
\end{cases}
\end{align*}
$$
In the Galaxy, on the scale of giant molecular clouds, the typical values for SF efficiency are in the range $\epsilon \approx 0.01 \div 0.4$ (Duerr et al., 1982; Wilking & Lada, 1983).

We did not fix this value but rather also derived $\epsilon$ from the "energetic" condition:

$$\epsilon = 1 - \frac{E_{th}^i + E_{ch}^i}{|E_{gr}^i|}.$$  \hfill (10)

At the moment of birth, the positions and velocities of new "star" particles are set equal to those of parent "gas" particles. Thereafter these "star" particles interact with other "gas" and "star" or "dark matter" particles only by gravity.

2.2. THERMAL SNII FEED-BACK

For the thermal budget of the ISM, SNIIs play the main role. Following to (Katz, 1992; Friedli & Benz, 1995), we assume that the energy from the explosion is converted totally to thermal energy. The total energy released by SNII explosions ($10^{44}$ J per SNII) within "star" particles is calculated at each time step and distributed uniformly between the surrounding "gas" particles (Raiteri et al., 1996).

2.3. CHEMICAL ENRICHMENT OF GAS

In our SF scheme, every new "star" particle represents a separate, gravitationally bound, star formation macro region (like a globular cluster). The "star" particle has its own time of birth $t_{begSF}$ which is set equal to the moment the particle is formed. After the formation, these particles return the chemically enriched gas into surrounding "gas" particles due to SNII, SNIa and PN events.

We concentrate our treatment only on the production of $^{16}$O and $^{56}$Fe, yet attempt to describe the full galactic time evolution of these elements, from the beginning up to present time (i.e. $t_{evol} \approx 15.0$ Gyr).

2.4. PHOTOMETRIC EVOLUTION OF STAR COMPONENT

The code also includes the photometric evolution of each "star" particle, based on the idea of the Single Stellar Population (SSP) (Bressan et al., 1994; Tantalo et al., 1996).

At each time - step, absolute magnitudes: $M_U$, $M_B$, $M_V$, $M_R$, $M_J$, $M_K$, $M_M$ and $M_{bol}$ are defined separately for each "star" particle. The SSP integrated colours (UBVRIKM) are taken from Tantalo et al. (1996). The spectro - photometric evolution of the overall ensemble of "star" particles forms the Spectral Energy Distribution (SED) of the galaxy.
We do not model the energy distribution in spectral lines nor the scattered light by dust. However according to Tantalo et al. (1996) our approximation is reasonable, especially in the UBV spectral band.

2.5. Initial conditions

As the initial model (relevant for CDM - scenario) we took a constant-density homogeneous gaseous triaxial configuration \( (M_{\text{gas}} = 10^{11} \, M_\odot) \) within the rigid Plummer-type dark matter halo \( (M_{\text{halo}} = 10^{12} \, M_\odot) \). We set the scale length of dark matter halo: \( b_{\text{halo}} = 25 \, \text{kpc} \). These values of \( M_{\text{halo}} \) and \( b_{\text{halo}} \) are typical for dark matter halo in disk galaxies (Navarro et al., 1996; Navarro et al., 1997; Burkert, 1995). We set \( A = 100 \, \text{kpc}, \, B = 75 \, \text{kpc} \) and \( C = 50 \, \text{kpc} \) for semiaxes of system. Such triaxial configurations are reported in cosmological simulations of the dark matter halo formation (Eisenstein & Loeb, 1995; Frenk et al., 1998; Warren et al., 1992).

The gas was assumed to be involved in the Hubble flow \( (H_0 = 65 \, \text{km/s/Mpc}) \) and the solid - body rotation around \( z \)-axis. The spin parameter in our numerical simulations is \( \lambda \approx 0.08 \), defined in Peebles (1969) as:

\[
\lambda = \frac{|L_0| \cdot \sqrt{|E_{\text{gr}}^0|}}{G \cdot (M_{\text{gas}} + M_{\text{halo}})^{5/2}},
\tag{11}
\]

\( L_0 \) is the total initial angular momentum and \( E_{\text{gr}}^0 \) is the total initial gravitational energy of a protogalaxy. It is to be noted that for a system in which angular momentum is acquired through the tidal torque of the surrounding matter, the standard spin parameter does not exceed \( \lambda \approx 0.11 \) (Steinmetz & Bartelmann, 1995). Moreover, its typical values range between \( 0.07^{+0.04}_{-0.03} \), e.g. \( 0.02 \leq \lambda \leq 0.11 \).

3. Results and discussion

Our SPH calculations were carried out for the three different number of "gas" particles \( N_{\text{gas}} = 2109; \, 4109; \, 11513 \). As a "base" model we consider the case with \( N_{\text{gas}} = 2109 \). For this case we calculate the full evolution of galactic system (i.e. \( t_{\text{evol}} \approx 15.0 \, \text{Gyr} \)). According to Navarro & White (1993) and Raiteri et al. (1996), such a number seems adequate for a qualitatively correct description of the systems behaviour. Even this small a number of "gas" particles produces \( N_{\text{star}} = 31631 \) "star" particles at the end of the calculation. The other two cases: \( N_{\text{gas}} = 4109 \) and 11513 we need for the more extensive study of our SF scheme. In the case of \( N_{\text{gas}} = 4109 \) we calculate the evolution up to
\( t_{\text{evol}} \approx 5.0 \) Gyr and in the case \( N_{\text{gas}} = 11513 \) up to \( t_{\text{evol}} \approx 2.5 \) Gyr. In all this two cases within these time scale the total number of particles access the \( N_{\text{max}} = 65535 \) and after this we, stop any SF activity.

The more extended discussion of the dynamical and chemical data can be found in the papers Berczik (1999) and Berczik (2000). Here we mainly concentrate our attention to the extensive check of our SF scheme and the photometric behaviour of our "base" galaxy model.

In Figure 1 we present the column density distribution -(a) and the rotational velocity distribution - (b) for our "base" galaxy model at the last time step. The total density distribution \( \sigma_{\text{tot}} = \sigma_{\text{gas}} + \sigma_{\text{star}} \) is well approximated by an exponential disk with radial scale length \( \sim 3.5 \) kpc. The rotational velocity \( (V_{\text{rot}}) \) distribution also well coincides with the data for our own galaxy (Vallee, 1994).

In Figure 2a we present the time evolution of "star" particles number \( N_{\text{star}} \). In Figure 2b we present the time evolution of the total mass inside "star" particles \( M_{\text{star}} \). The number of "star" particle is very different in all three cases, but the total mass inside the "star" component is very similar. Some differences are found only after the SF artificially stops in the models with high "gas" particle numbers. After stop of SF the mass inside "star" component is reduced, according to returning the mass from "star" to surrounding "gas" particles due to SNII, SNIa and PN events.

The star formation history (SFH - \( dM_{\text{star}}/dt \)) for all three models is presented in Figure 3. In this figure we clearly see that the SFH practically does not depend on the "gas" or "star" particle number. The star formation efficiency (SFE - \( \epsilon \)) in each act of SF is presented in the Figure 4. The averaged SFE is about \( \sim 10\% \). However we have a wide spread from some percent up to the 50% at the late stage of evolution. In all three models we have a similar SFE trend.

In Figure 5 and Figure 6 we present the chemical evolution of our models. All three models have a very similar chemical history and metallicity distribution.

The total photometric (in UBVK spectral brand) and color ((U-B), (B-V), (V-K)) evolution of our "base" model is presented in the Figure 7a and Figure 7b. In case of our own Milky Way galaxy at the present day we have a value for (B-V) \( \approx 0.8 \) (van der Kruit, 1986), what is close to our value for model galaxy \( \approx 0.7 \).

The color vs. color evolution of the model galaxy presented in Figure 8.
4. Conclusion

The presented model describes well the time evolution of the basic dynamical, chemical and photometric parameters of a disk galaxy similar to the Milky Way. The metallicity, luminosity and colors obtained are typical for such disk galaxies. During the calculations we made an extended test of the proposed new SF criteria. We find that the obtained results with different "gas" and "star" particle numbers are not only qualitatively but also quantitatively similar.

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Figure 1. Column density & rotational velocity distribution in the final step for the "basic" model.

Figure 2. N_{star} & M_{star} evolution of the model galaxy with different "gas" particle number.
Figure 3. $dM_{\text{star}}/dt$ evolution of the model galaxy with different "gas" particle number.

Figure 4. $\epsilon$ evolution of the model galaxy with different "gas" particle number.
Figure 5. [Fe/H] evolution of the model galaxy with different "gas" particle number.

Figure 6. [O/Fe] vs. [Fe/H] of the model galaxy with different "gas" particle number.
Figure 7. Photometric & color evolution of the galaxy for the "basic" model.

a. U, B, V, K evolution.

b. (U-B), (B-V), (V-K) evolution.

Figure 8. Color vs. color evolution of the galaxy for the "basic" model.

a. (U-B) vs. (V-K).

b. (U-B) vs. (B-V).