Universal quantum computing with nanowire double quantum dots

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Abstract

We present a method for implementing universal quantum computing using a singlet and triplets of nanowire double quantum dots coupled to a one-dimensional transmission line resonator. This method is suitable and of interest for both quantum computing and quantum control with inhibition of spontaneous emission, enhanced spin qubit lifetime, strong coupling and quantum nondemolition measurements of spin qubits. We analyze the performance and stability of all the required operations and emphasize that all techniques are feasible with current experimental technology.

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1. Introduction

A quantum computer comprised of many two-level systems (qubits) exhibits coherent superpositions and entanglement. Quantum computing, which is based on these features, enables some computational problems to be solved faster than would ever be possible with a classical computer [1] and exponentially speeds up solutions to other problems over the best-known classical algorithms [2]; it is currently attracting considerable interest. Among the promising candidates for quantum computing, solid-state implementations such as spin qubits in quantum dots [3] and bulk silicon [4], and charge qubits in bulk silicon [5] and in superconducting Josephson junctions [6], are particularly attractive because of the stability and expected scalability of solid-state systems; of these competing technologies, semiconductor double quantum dots (DQDs) are particularly important, because of the combination of spin and charge manipulations, to take advantage of the long memory times associated with spin states and at the same time to enable efficient readout and coherent manipulation of charge states.

Our goal is to develop a realizable architecture for semiconductor quantum computation. The qubit is manifested as a nanowire (NW) quantum dot pair such that each has an electron and thus the singlet and one of the triplets of two-electron states correspond to the logical state $|0\rangle$ and the orthogonal state $|1\rangle$. The resonator-assisted interaction between DQDs and a microwave transmission line resonator (TLR) is used to implement a universal set of quantum gates and readout of the qubits.

We show the advantages of our scheme over previous proposals on semiconductor quantum computation from the following two points. Firstly, for previous proposals that make use of single or double quantum dots defined by a two-dimensional electron gas (2DEG) [7–14], it would be difficult to implement a double dot in a planar resonator with lateral dots, shaped in a 2DEG by surface gates. This is because it would be difficult to prevent absorption of microwaves in the 2DEG unless one can make the electric field nonzero only in the double-dot region, which is not yet realistic experimentally. Our strategy is to use the two-electron states of DQDs inside NWs instead of 2DEG, which is more realistic for implementing quantum computing experimentally.

Secondly, there are previous proposals making use of NW DQDs [15], in which the spin–orbit interaction is used to couple the spins and the resonator. However, the weak coupling between the DQD spins and the resonator mode is a challenge in experiments. Our strategy for enhancing the interaction is to make use of the coupling between the electric dipole of charge states of DQDs and the resonator, which is a much stronger coupling compared to that in [15]. However, the decoherence of charge states is another obstacle. In this paper, combining the advantages of spin and charge states and avoiding the weak points of both, we propose a different mechanism, namely via resonator-assisted interaction, which
leads to a strong coupling between the resonator photons and effective electric dipole of the state \([0]\) and an ancillary state of DQDs, while the state \([1]\) is driven by a classical field and eventually implements quantum control on the singlet and one of the triplet spin states. Thus, we encode the quantum information in spin states and quantum control is implemented via the charge dipole transition that is driven by a TLR.

A solid-state realization of cavity QED is proposed in section 2. In sections 3.1–3.3, we discuss the case where the resonator and qubit are tuned on- and off-resonance which can be used to implement a universal set of gates including single- and two-qubit gates. The initialization of qubit states can be implemented by an adiabatic passage shown in section 3.4. The readout of qubits can be realized via microwave irradiation of the TLR by probing the transmitted or reflected photons shown in section 3.5. In section 3.6, the main decoherence processes are dissipation of the TLR, charge-based relaxation and dephasing of the NW DQDs occurring during gate operations and transportation of qubits, and spin dephasing limited by hyperfine interactions with nuclei. By numerical analysis we show that all gate operations and measurements can be implemented within the coherent lifetime of qubits. Thus, we address all DiVincenzo criteria [16] and show that all have important roles in the dynamics of the two-electron system but none represents a fundamental limit for quantum computing. A summary is presented in section 4.

2. The system: a solid-state realization of cavity QED

2.1. The Hamiltonian

We consider the system with two electrons located in adjacent quantum dots coupled via tunneling. Imagine that one of the dots is capacitively coupled to a TLR [11–14]. We assume that the left dot (L) is red-shifted with respect to the right dot (R) and that the lowest conduction level of the left dot is detuned by \(\Delta\) with respect to the right one.

In the (1,1) regime, with an external magnetic field \(B_z = 1\) T along the \(z\)-axis, the ground state manifold is given by the spin-aligned states

\[
|T_+\rangle = \hat{e}^{\dagger}_{L\uparrow} \hat{e}^{\dagger}_{R\uparrow} |vac\rangle = |\uparrow\uparrow\rangle,
\]

\[
|T_-\rangle = \hat{e}^{\dagger}_{L\downarrow} \hat{e}^{\dagger}_{R\downarrow} |vac\rangle = |\downarrow\downarrow\rangle,
\]

and the spin-anti-aligned states

\[
|T_0\rangle = \frac{1}{\sqrt{2}} \left( \hat{e}^{\dagger}_{L\uparrow} \hat{e}^{\dagger}_{R\downarrow} + \hat{e}^{\dagger}_{L\downarrow} \hat{e}^{\dagger}_{R\uparrow} \right) |vac\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right),
\]

\[
|(1,1)S\rangle = \frac{1}{\sqrt{2}} \left( \hat{e}^{\dagger}_{L\uparrow} \hat{e}^{\dagger}_{R\downarrow} - \hat{e}^{\dagger}_{L\downarrow} \hat{e}^{\dagger}_{R\uparrow} \right) |vac\rangle = \frac{1}{\sqrt{2}} \left( |\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle \right)
\]

with energy gaps due to the Zeeman splitting and exchange energy shown in figure 1. The notation \((n_L, n_R)\) labels the number of electrons in the left and right quantum dots. The doubly occupied state \(|(0,2)S\rangle\) is coupled via tunneling \(T\) to the singlet state \(|(1,1)S\rangle\). In the basis \([(1,1)S\rangle, |(0,2)S\rangle\], the Hamiltonian can be deduced as

\[
\hat{H}_d = -\Delta |(0,2)S\rangle \langle (0,2)S| + T |(1,1)S\rangle \langle (0,2)S| + \text{h.c.}
\]

With the energy offset \(\Delta\), degenerate perturbation theory in the tunneling \(T\) reveals an avoided crossing at this balanced point between \|(1,1)S\rangle \rangle \rangle \rangle and \|(0,2)S\rangle \rangle \rangle \rangle with an energy gap \(\omega = \sqrt{\Delta^2 + 4T^2}\), and the effective tunneling between the left and right dots with the biased energies \(\Delta\) is changed from \(T\) to \(\omega/2\).

We choose the singlet state and one of the triplet states as our qubit

\[
|0\rangle \equiv |(1,1)S\rangle, \quad |1\rangle \equiv |T_0\rangle,
\]

(2)

and the doubly occupied state as an ancillary state

\[
|\alpha\rangle \equiv |(0,2)S\rangle.
\]

(3)

The essential idea is to use an effective electric dipole moment associated with singlet states \([0]\) and \([\alpha]\) of an NW DQD coupled to the oscillating voltage associated with a TLR shown in figure 2. We consider a TLR with length \(L\), capacitance per unit length \(C_0\) and characteristic impedance \(Z_0\). A capacitive coupling \(C_c\) between the NW DQD and TLR causes the electron charge state to interact with excitations in the transmission line. We assume that the dot is much smaller than the wavelength of the resonator excitation, so the interaction strength can be derived from the electrostatic potential energy of the system

\[
\hat{H}_{\text{int}} = e\hat{V} |\alpha\rangle \langle \alpha|,
\]

(4)

where \(e\) is the electron charge,

\[
\hat{V} = \sum_n \frac{\hbar\omega_n}{LC_0} (\hat{a}_n + \hat{a}^\dagger_n)
\]

(5)

is the voltage on the TLR near the left dots, \(\hat{a}_n, \hat{a}^\dagger_n\) are the creation and annihilation operators for the mode \(k_n = [(n + 1)\pi]/L\) of the TLR, and \(C_0\) is the total capacitance
of the DQD. The fundamental mode frequency of the TLR is \( \omega_0 = \pi/LZ_0C_0 \). The TLR is coupled to a capacitor \( C_e \) for writing and reading the signals. Neglecting the higher modes of the TLR and working in the rotating frame with the rotating wave approximation, we obtain an effective Hamiltonian as

\[
\hat{H}_{\text{eff}} = \omega_0 \hat{a}^\dagger \hat{a} + \omega |a\rangle \langle a| + g (\hat{a} |a\rangle \langle 0| + \text{h.c.})
\]

with \( \hat{a} (\hat{a}^\dagger) \) being the annihilation (creation) operator of the resonator field, and the effective coupling coefficient

\[
g = \frac{1}{2} e \frac{C_c}{LC_{\text{tot}}C_0} \sqrt{\frac{\pi}{Z_0}} \sin 2\theta
\]

with \( \theta = \frac{1}{2} \tan^{-1}\left(\frac{C_c}{C_0}\right) \).

The interaction between the TLR and qubit states is switchable via tuning the electric field along the \( x \)-axis. In the case of the energy offset yielded by the electric field \( \Delta \approx 0 \), we obtain the maximum value of the coupling between the TLR and singlets in DQDs. This is the so-called optimal point. Whereas \( \Delta \gg T \), \( \theta \) tends to 0, and the interaction is switched off.

The transition from \(|1\rangle\) to \(|a\rangle\) is driven by a classical laser field with a Rabi frequency \( \Omega \). The interaction Hamiltonian is given by

\[
\hat{H}_a = \Omega e^{-i\omega_0 t} |1\rangle \langle 1| + \Omega e^{i\omega_0 t} |a\rangle \langle a|,
\]

where \( \omega_0 \) is the drive frequency.

### 2.2. Physical realization

A realization of DQDs defined using local gates to electrostatically deplete InAs NWs grown by chemical beam epitaxy has been reported in [17]. The quantum-mechanical tunneling \( T \) between the two quantum dots is about \( 0–150 \mu eV \) [17]. Thus, at the optimal point \( \Delta \approx 0 \) where the coupling is strongest, the energy gap between the singlets is about \( \omega \sim 2T \approx 0–72 \text{GHz} \). A small-diameter (\( \sim 65 \text{nm} \)), long-length (\( \sim 270 \text{nm} \)) and \( g^* \approx -13 \) \[18\] InAs NW is positioned perpendicular to the transmission line; it contains DQDs that are elongated along the NW as shown in figure 2.

The external magnetic field along the \( z \)-axis is about \( B_z = 1 \text{T} \) to make sure that the energy splitting \( E_z = g^* \mu_B B_z \) between the two triplet states \( |T_0\rangle \) is larger than \( \omega \).

The TLR can be fabricated with existing lithography techniques [19]. The dots can be placed within the TLR formed by the transmission line to strongly suppress the spontaneous emission. To prevent a current flow, the NW and transmission line need to be separated by some insulating coating material obtained, for example, by atomic layer deposition. We assume that the TLR is 3 cm long and 10 \( \mu m \) wide, \( Z_0 = 50 \Omega \), which implies for the fundamental mode \( \omega_0 \equiv \pi/LC_0Z_0 = 2\pi \times 120 \text{GHz} \). In practice, careful fabrication permits a strong coupling capacitance, with \( C_{\text{tot}} \approx 5.1C_c \) [17], so that the coupling coefficient \( g \sim 2\pi \times 10^3–10^4 \) have already been demonstrated in [20].

The effect of photon-assisted tunneling in our system is a harmful one because it destroys the qubit by lifting spin-blockade. To avoid this, our strategy is to close the tunneling barriers to the leads.

### 3. Universal quantum computing

#### 3.1. Single-qubit gate operations

Firstly, we consider the zero-detuning case when the fundamental mode frequency of the TLR is \( \omega_0 \approx \omega \). The Hamiltonian (6) has the same form as the Jaynes–Cummings Hamiltonian of a two-level system with a single-mode resonator field. In the case when the TLR is initially in the photon number state \( |n\rangle \), the time evolution of the system, governed by the Hamiltonian (6), is described by

\[
|0\rangle |n\rangle \rightarrow \cos \sqrt{n+1} \hat{g} t |0\rangle |n\rangle - \sin \sqrt{n+1} \hat{g} t |1\rangle |n+1\rangle \\
|a\rangle |n\rangle \rightarrow \frac{\Omega}{\sqrt{2}} (\cos \frac{\Omega}{\sqrt{2}} t |1\rangle + \sin \frac{\Omega}{\sqrt{2}} t |0\rangle |n\rangle - \frac{\Omega}{\sqrt{2}} t |1\rangle |n+1\rangle + \frac{\Omega}{\sqrt{2}} t |0\rangle |n-1\rangle \) \label{eq:gate}
\]

From the Hamiltonian of a drive on the DQDs shown in (8), it is straightforward to see that a pulse of duration \( t \) results in the following rotation:

\[
|1\rangle \rightarrow \cos \frac{\Omega}{2} t |1\rangle - \sin \frac{\Omega}{2} t |0\rangle \\
|a\rangle \rightarrow \frac{1}{\sqrt{2}} (\cos \frac{\Omega}{\sqrt{2}} t |1\rangle + \sin \frac{\Omega}{\sqrt{2}} t |0\rangle) \label{eq:rotation}
\]

It has been reported in [21] that the structure of the TLR and qubits tuned on-resonance can be used to implement an entangling gate on spin qubits of NW DQDs via the adiabatic evolution of the dark states.

In this paper, we show a different proposal and consider the case when the TLR and qubits are tuned off-resonant, which leads to lifetime enhancement of the qubits and implements coherent control. Assuming that the classical field and TLR are detuned from the transitions by \( \delta_1 = \omega_0 - (\omega - J) \) and \( \delta_2 = \omega_0 - \omega \), respectively, the Hamiltonian for a single
DQD coupled to the TLR and driven by a classical field is

\[ \hat{H}_{1q} = \omega_0 \hat{a}^\dagger \hat{a} + \omega |a\rangle \langle a| + g \hat{a}^\dagger \hat{a} \langle 0| + \Omega e^{-i\omega t} |a\rangle \langle 1| + \text{h.c.}. \]  

(11)

If \( \delta_1, \delta_2 \gg \Omega, g \) is satisfied, the upper level \( |a\rangle \) can be adiabatically eliminated. We then obtain the effective Hamiltonian of the system as

\[ \hat{H}_{1q}^{\text{eff}} = \frac{\Omega^2}{\delta_1} |1\rangle \langle 1| + \frac{\lambda^2}{\delta_2} \hat{a}^\dagger \hat{a} |0\rangle \langle 0| + \lambda \hat{a} |1\rangle \langle 0| + \text{h.c.}, \]  

(12)

where \( \lambda = \Omega g / 2(\delta_1 + \delta_2) \). The first two terms describe the Stark shifts for the spin states \( |0\rangle \) and \( |1\rangle \), induced by the classical field and resonator mode, respectively. The last term is the Raman coupling of the two spin states.

For single-spin qubits in \(|0\rangle, |1\rangle\) coupled, with effective strength \( \lambda \), to the TLR, driven by a classical field that is detuned from the TLR, the Hamiltonian (12) can be used to implement single-qubit rotations along the \( x \)-axis via the Rabi oscillation between the states \( |0\rangle \) and \( |1\rangle \) as shown in figure 3.

### 3.2. Two-qubit gate operations

Now we consider the case when there are two spin qubits coupled to the TLR. From the Hamiltonian (12), in the case of \( \delta_2 - \delta_1 \gg \lambda \), there is no energy exchange between the DQD system and TLR. The energy-conserving transitions are between \(|1_1, 0_2\rangle |n_1\rangle \) and \(|0_1, 1_2\rangle |n_2\rangle \). The effective Rabi frequency for the transitions between these states, mediated by \(|1_1, 0_2\rangle |n_1 + 1\rangle \) and \(|0_1, 1_2\rangle |n_2 - 1\rangle \), is given by

\[ \lambda = \frac{\{1_1, 0_2\} \hat{H}_{\text{tot}} |0_2, n_2 + 1\rangle \langle 1_1, 0_2| + \hat{H}_{\text{tot}} |1_1, 1_2\rangle |0_1, n_1 - 1\rangle - (\delta_2 - \delta_1)}{\delta_2 - \delta_1} \]

\[ = \frac{\lambda^2}{\delta_2 - \delta_1}. \]  

(13)

where \( \hat{H}_{\text{tot}} = \sum_{j=1,2} \hat{H}_{1q}^j \). The effective Hamiltonian for two qubits turns out to be

\[ \hat{H}_{2q} = \sum_{j=1,2} \frac{\Omega^2}{\delta_1} |1\rangle \langle 1| + \lambda' (\hat{a}_1^\dagger \hat{a}_2 - \hat{a}_2^\dagger \hat{a}_1) |1\rangle \langle 1| \quad - |0\rangle \langle 0| + \lambda' (|1\rangle \langle 0| \otimes |1\rangle \langle 1| + \text{h.c.}). \]  

(14)

The third and fourth terms are the photon-number-dependent Stark shifts induced by the Raman transition, and the last term is the induced dipole coupling between the two spin qubits. If the resonator mode is initially in the vacuum state, it will remain in the vacuum state throughout the process. Then the effective Hamiltonian for the two qubits is reduced to

\[ \hat{H}_{2q}^{\text{eff}} = \sum_{j=1,2} \frac{\Omega^2}{\delta_1} |1\rangle \langle 1| + \lambda' (\hat{a}_1^\dagger \hat{a}_2 - \hat{a}_2^\dagger \hat{a}_1 + \text{h.c.}). \]  

(15)

where \( \hat{a}_1 = |1\rangle \langle 0| \) and \( \hat{a}_2 = |0\rangle \langle 1| \).

The evolution of the effective two-qubit Hamiltonian can be used to implement an entangling two-qubit gate—\( \sqrt{\text{SWAP}} \). In a frame rotating at the qubit’s frequency, the Hamiltonian (15) generates the evolution

\[ U_{2q}(t) = \prod_{j=1,2} \exp \left[ -i \frac{\Omega^2}{\delta_1} t |1\rangle \langle 1| \right] \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \lambda' t & \sin \lambda' t & 0 \\ 0 & \sin \lambda' t & \cos \lambda' t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \]  

(16)

Up to the phase factor, it corresponds at \( t_{2q} = \pi/4\lambda' \) to a \( \sqrt{\text{SWAP}} \) logical operation. Up to single-qubit gates, the above operation is equivalent to the controlled-NOT gate. Together with single-qubit gates, the interaction \( \hat{H}_{2q}^{\text{eff}} \) is therefore sufficient for universal quantum computing.

When the qubits are detuned from each other, the off-diagonal coupling provided by \( \hat{H}_{2q}^{\text{eff}} \) is only weakly effective and the coupling is, for all practical purposes, turned off. Two-qubit logical gates in this setup can therefore be controlled by individually tuning the qubits. Moreover, single- and two-qubit logical operations on different qubits and pairs of qubits can both be realized simultaneously, a requirement to reach currently known thresholds for fault-tolerant quantum computation [22].

Hence, we have built a universal set of gates for quantum computing with semiconductor DQDs coupled to a resonator field. The feasibility of single-qubit gates has already been proved in [6] experimentally. For the two-qubit gate, we realize it with the off-resonant interaction between both qubits and TLR. With the experimental parameters \( \{T, J, \omega_0, \omega, \omega_d, g, \Omega\}/2\pi = \{2.5, 0.25, 10, 5, 9.74, 0.12, 1\} \) GHz, the detunings \( \delta_1/2\pi = 4.99 \) GHz and \( \delta_2/2\pi = 5 \) GHz, and the efficient coupling coefficients \( \lambda = 3 \) MHz and \( \lambda' = 0.9 \) MHz, we can estimate the time scaling for quantum computing. The operating time for the single-qubit rotation along the \( x \)-axis is \( t_x \sim 1/\lambda \approx 300 \) ns with the above parameters. The single-qubit rotation along the \( z \)-axis takes the same time scaling as the two-qubit gate in (15) \( t_{2q} \), which satisfies \( \lambda' t_{2q} = \pi/4 \) and is calculated as \( t_{2q} \approx 1 \mu s \).
Figure 4. Fidelity \( F \) of the two-qubit \( \sqrt{\text{SWAP}} \) gate versus the resonator decay rate \( \kappa \) with the experimental parameters \( \{ \omega_0, \omega, \omega_{\text{res}}, \gamma, \Omega \}/2\pi = \{10, 5, 9.74, 0.12, 1 \} \) GHz. The lines with triangles, stars and boxes describe the cases when the spontaneous emission rate of the singlet states is \( 1/T_1 = 0.01 \), 0.05 and 0.1, respectively.

3.3. Fidelity of two-qubit gates

Now we analyze the effect on gate operations due to noise and derive the fidelity of two-qubit gates. We use the two-qubit gate equation (16) as an example. With the time-dependent fluctuations \( \delta \lambda(t) \) of the effective coupling coefficient \( \lambda' \), the evolution operator of the system becomes

\[
U_{2q}' = U_{2q} \exp \left[ -i \int_0^{t_2} \delta \lambda(t) \big( \hat{\sigma}_z^1 \hat{\sigma}_z^2 + \hat{\sigma}_0^1 \hat{\sigma}_0^2 \big) \right], \tag{17}
\]

where the unwanted phase \( \phi = \int_0^{t_2} \delta \lambda(t) \). The distribution of the unwanted phase becomes a Gaussian distribution because \( \lambda' \) is in Gaussian distribution. With the parameters above, we numerically calculate the variance of the unwanted phase \( \text{Var}(\phi) \sim 2 \times 10^{-3} \pi \).

Furthermore, the decoherence processes such as dephasing reduce the fidelity of gate operations as well. We analyze the dephasing rate due to the variation of the ac Stark shift \( g^2/\delta \lambda \hat{a}^\dagger \hat{a} \hat{\sigma}_z \) (where \( \hat{\sigma}_z = |0\rangle \langle 0| - |1\rangle \langle 1| \)) caused by quantum fluctuations in the number of photons \( \bar{n} \) within the resonator. To determine the dephasing rate, we assume that the resonator is driven at the bare resonator frequency \( \omega_0 \) and the pull of the resonance is small compared to the linewidth \( \kappa \). The relative phase accumulated between the two singlet states \( |0\rangle \) and \( |a\rangle \) is \( \theta(t) = 2g^2 \int_0^t dt' \langle a | \hat{a}^\dagger \hat{a} \rangle \), which yields a mean phase advance \( \langle \theta \rangle = 2g^2 \bar{n} \kappa / 2 \). Defocusing can be evaluated by the decay of the correlator \( \langle a | \hat{a}^\dagger \hat{a} \rangle \) if the resonator is not driven, the photon number correlator rather decays at a rate \( \kappa \), and the rate of transmission on-resonance is \( \gamma_0 = \kappa \bar{n} / 2 \). In the dispersive regime, the dephasing rate is reduced to \( \gamma_0 = 8\pi (g^4/\kappa^2)^{1/2} \).

From the analysis, we show that even if decoherence and noise occur over the gate operation, we can still implement a universal set of gates with high fidelities. Figure 4 shows the fidelity \( F = \text{int} \langle \psi | U_{2q} \rho U_{2q}^\dagger |\psi\rangle \text{int} \) of the two-qubit gate \( \sqrt{\text{SWAP}} \) as a function of the resonator decay \( \kappa \) and spontaneous emission of the DQD singlet state \( 1/T_1 \), where \( \rho \) is the reduced density matrix calculated by solving the master equation with decoherence and tracing out the resonator photon and \( |\psi\rangle \text{int} \) is the initial state of the spin states. With the experimental parameters \( \{ \omega_0, \omega, \omega_{\text{res}}, \gamma, \Omega, \kappa \}/2\pi = \{10, 5, 9.74, 0.12, 1, 0.001, 0.001 \} \) GHz, a fidelity as high as 0.991 is achieved.

For the single-qubit \( \hat{\sigma}_x \) gate, the unwanted phase is \( \int_0^{t_2} \delta t \delta \lambda(t) \). With the same method, we can calculate the variance of the phases.

3.4. Initialization and transportation

Initialization of qubit states can be implemented by an adiabatic passage between the two singlet states \( |0\rangle \) and \( |a\rangle \) [10]. Controllably changing \( \Delta \) allows for adiabatic passage to beyond the charge transition, with \( |a\rangle \) as the ground state if \( \Delta > T \) is achieved. First we turn on the external electric field along the \( x \)-axis and prepare the two electrons of NW DQDs in the state \( |a\rangle \) by a large energy offset \( \Delta \). We change \( \theta \) in equation (7) adiabatically to \( \pi/4 \) by tuning the electric field and then initialize the qubits in the state \( |0\rangle \).

The SWAP operation [12], where a qubit state is swapped with a photonic state of the TLR, can be used to implement the transmission of qubits. If there is no photon in the TLR, with the evolution time \( \pi/\lambda \), a qubit is mapped to the photonic state in the TLR

\[
|\alpha \rangle |0\rangle + |\beta \rangle |1\rangle \rightarrow |\alpha \rangle |0\rangle + |\beta \rangle |1\rangle. \tag{18}
\]

Then we switch off the coupling between this qubit and TLR and switch on that between the desired qubit and TLR via the local electric fields along the \( x \)-axis. After the same evolution time, the previous qubit state is transmitted to the desired qubit via the interaction with the TLR. The time for transmitting a qubit to a photonic qubit in the TLR is about \( t_\text{tr} = \pi/\lambda \approx 900 \) ns with the experimental parameters shown in section 3.

3.5. Readout

To carry out a measurement of qubits, the classical field and TLR are tuned from the respective transitions modeled by equation (12). In the dispersive regime \( (\delta_1, \delta_2 > \Omega, g) \), the energy gap between the dressed states \( |0\rangle \) and \( |1\rangle \), is \( \omega_0 = g^2/\delta_2 \) for the qubit in the state \( |0\rangle \), while the energy gap \( \omega_0 \) for the state \( |1\rangle \) remains unchanged. The operator being probed is \( \hat{\sigma}_z \) and the qubit-measurement apparatus interaction Hamiltonian is \( g^2/\delta \hat{a}^\dagger \hat{a} \hat{\sigma}_z \), such that \( [\hat{\sigma}_z, g^2/\delta \hat{a}^\dagger \hat{a} \hat{\sigma}_z] = 0 \). Depending on the qubit being in the state \( |0\rangle \) or \( |1\rangle \), the transmission spectrum presents a peak of width \( \kappa \) (the resonator decay rate) at \( \omega_0 = g^2/\delta_2 \) or \( \omega_0 \). The dispersive pull of the resonator frequency is \( 0 \sim g^2/\kappa \delta_2 \), and the pull is power dependent and decreases in magnitude for photon numbers inside the TLR [23]. Through microwave irradiation of the TLR by probing the transmitted or reflected photons, readout of qubits can be realized and completed on a time scaling \( t_\text{rd} = 1/\gamma_0 \), where \( \gamma_0 \) is the dephasing rate due to quantum fluctuations in the number of photons \( \bar{n} \) within the TLR as shown in section 3.3. Compared to the dephasing rate of transmission on resonance \( \gamma_0 = \kappa \bar{n} / 2 \), in the dispersive regime the phase noise induced by the ac Stark shift \( g^2/\delta \hat{a}^\dagger \hat{a} \hat{\sigma}_z \), results in the dephasing rate \( \gamma_0 = 8\pi (g^4/\kappa^2)^{1/2} \) and an enhanced lifetime of spin qubits. This approach can serve as a high-efficiency quantum nondemolition dispersive readout of the qubit states: \( P_2 = |0\rangle \langle 0| \) and \( P_2 = |1\rangle \langle 1| \). Readout of qubits takes time \( t_\text{rd} \approx 1.5 \) ns for the case \( \bar{n} = 100 \) with the experimental parameters shown above.
3.6. Decoherence

In section 3.3, we showed the decoherence occurring over the two-qubit gate operation; now we analyze the dominant noise sources of the system existing during all preprocessings, including the spin phase noise due to hyperfine coupling, the charge-based dephasing and relaxation occurring during gate operations and transportation and the photon loss due to resonator decay. The characteristic charge dephasing is with a rate $T^{-1}_\text{c}$. The time-ensemble-averaged dephasing time $T^*_\text{c}$ is limited by hyperfine interactions with nuclear spins. Coupling to a phonon bath causes relaxation of the charge system in a time $T_1$. The decay of the TLR $\kappa$ is considered to be another dominant source of decoherence.

Hyperfine interactions with gallium arsenide host nuclei cause nuclear spin-related dephasing $T^*_\text{c}$. The hyperfine field can be treated as a static quantity, because the evolution of the random hyperfine field is several orders slower than the electron spin dephasing. From the operational point of view, the most important decoherence due to the hyperfine field is the dephasing between the singlet state $|\{1,1\rangle\rangle S\rangle$ and one of the triplet states $|T_i\rangle$. By suppressing nuclear spin fluctuation, the dephasing time can be obtained by quasi-static approximation as $T^*_c = 1/g\mu_B|\Delta B_z^{*}\rangle_{\text{rms}}$, where $\Delta B_z^{*}$ is the nuclear hyperfine gradient field between two coupled dots and rms means a root-mean-square time-ensemble average. A measurement of the dephasing time $T^*_c = 100$ ns has been demonstrated in [15] and we expect that coherently driving the qubit will prolong the $T^*_c$ time up to 1 $\mu$s with echo up to 10 $\mu$s [8].

For the charge relaxation time $T_1$, the decay is caused by coupling qubits to a phonon bath. With the spin–boson model, perturbation theory gives an overall error rate from the relaxation and the incoherent excitation, with which one can estimate the relaxation time $T_1 = 10\mu$s [12] that is studied in great detail for the GaAs quantum dot in 2DEG and a similar rate is expected for NW quantum dots.

The charge dephasing $T_2$ arises from the variation of the energy offset $\Delta(t) = \Delta + \epsilon(t)$ with $\langle \epsilon(t)\epsilon(t') \rangle = \int d\omega S(\omega)e^{i\omega t}\delta(t-t')$, which is caused by a low-frequency fluctuation of the electric field. The gate bias of the qubit drifts randomly when an electron tunnels between the metallic electrodes. Due to the low frequency property, the effect of $1/f$ noise on the qubit is dephasing rather than relaxation. At the zero derivative point, compared to a bare dephasing time $T_0 = 1/\sqrt{\int d\omega S(\omega)}$, the charge dephasing is $T_2 \sim \omega T_0^2$ near the optimal point $\Delta = 0$. The bare dephasing time $T_0 \sim 1$ ns was observed in [24]. Then the charge dephasing is estimated as $T_2 \sim 10-100$ ns. Using quantum control techniques, such as better high- and low-frequency filtering of electronic noise, $T_0$ exceeding $1\mu$s was observed in 2DEG [8] (we assume a similar result for the present case), which suppresses the charge dephasing.

A quality factor $Q$ $10^6$ for the TLR in the microwave domain can be achieved [19]. In practice, the local external magnetic field of 1 T reduces the limit of the quality factor to $Q$ $\sim 10^4$–$10^6$ [20]. The dissipation of the TLR $\kappa = \omega_0 / Q$ leads to a decay time of about 100 ns–1 $\mu$s with the parameter $\omega_0 = 2\pi \times 10$ GHz. Thus, the operating times of all these gates $\{t_s, t_{2Q}, t_\mu, t_B\} = \{300$ ns, $1\mu$s, $900$ ns, $1.5$ ns$\}$ are less than the minimum decoherence time.

4. Summary

Advances in fabrication have led to the development of solid-state systems, with an obvious potential for quantum computing. The Heisenberg exchange coupling, optical dipole–dipole interactions, capacitive coupling and optical cavity-mediated interactions between spin and charge states can be used to realize controlled quantum state operations. In this paper, we focus on the NW DQD quantum computer, which would capitalize on chip fabrication technology and could be hybridized with existing computers. We propose the realization of cavity QED via electrically controlled semiconductor spins of NW DQDs coupled to a microwave TLR on a chip. Combining the advantages of spin and charge states and avoiding the weak points of both, we propose a mechanism to achieve a scalable architecture for quantum computing with NW DQDs inside a TLR, namely via resonator-assisted interaction that leads to an efficient, strong coupling between the resonator photon and the effective electric dipole of DQDs. Thus, we encode the quantum information in spin states and the gate operation is implemented via the charge dipole transition which is driven by a resonator. The initialization of qubits can be realized with an adiabatic passage. With the switchable coupling to the TLR, we can implement a universal set of quantum gates on any qubit. Because of the switchable coupling between the double-dot pairs and TLR, we can apply this entangling gate to any two qubits without affecting others, which is not a trivial result for implementing scalable quantum computing and generating a large entangled state. The fidelities of the gates in our protocol are studied, including all kinds of major decoherence, with promising results for reasonably achievable experimental parameters, and these results demonstrate the practicality by way of current experimental technologies. This work shows how an experiment can be performed under existing conditions to demonstrate the first architecture for quantum computing for spin qubits in quantum dots in the laboratory.

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References

[1] Grover L 1997 Phys. Rev. Lett. 79 325
[2] Shor P W 1994 Proc. 35th Annua. Symp. on Foundations of Computer Science (Los Alamitos, CA: IEEE Computer Society Press) p 124
[3] Loss D and DiVincenzo D P 1998 Phys. Rev. A 57 120
[4] Kane B E 1998 Nature 393 133
[5] Andresen S E S et al 2007 Nano Lett. 7 2000
[6] Wallraff A et al 2005 Phys. Rev. Lett. 95 060501
[7] Imamoglu A et al 1999 Phys. Rev. Lett. 83 4204
[8] Petta J R et al 2005 Science 309 2180
[9] Johnson A C 2005 Nature 435 925
[10] Taylor J M et al 2007 Phys. Rev. B 76 035315
[11] Burkard G and Imamoglu A 2006 Phys. Rev. B 74 041307
[12] Taylor J M and Lukin M D 2006 arXiv:cond-mat/0605144
[13] Lin Z R et al 2008 Phys. Rev. Lett. 101 230501
[14] Xue P 2010 Phys. Lett. A 374 2601
[15] Trif M, Golovach V N and Loss D 2008 Phys. Rev. B 77 045434
[16] DiVincenzo D P 2000 Fortschr. Phys. 48 771
[17] Fash C, Fuhrer A, Björk M T and Samuelson L 2005 Nano Lett. 5 1487
[18] Björk M T et al 2005 Phys. Rev. B 72 201307
[19] Wallraff A et al 2004 Nature 431 162
[20] Frunzio L et al 2005 Appl. Supercond. IEEE Trans. 15 860
[21] Zheng S B 2005 Phys. Rev. Lett. 95 080502
Xue P 2010 Chin. Phys. Lett. 27 060301
[22] Aharonov D and Ben-Or M M 1996 Proc. 37th Annu. Symp.
on Foundations of Computer Science (Los Alamitos, CA: IEEE Computer Society Press) p 46
[23] Blais A et al 2004 Phys. Rev. A 69 062320
Gambetta J et al 2006 Phys. Rev. A 74 042318
Blais A et al 2007 Phys. Rev. A 75 032329
[24] Hayashi T et al 2003 Phys. Rev. Lett. 91 226804
Petta J R et al 2004 Phys. Rev. Lett. 93 186802