On massive super(bi)gravity in the constructive approach

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Abstract
In this paper we investigate the possible supersymmetric extensions for the massive (bi)gravity theories in the lowest non-trivial order. For this purpose we construct the cubic interaction vertices for massive spin-2 and one or two massive spin-3/2 fields restricting ourselves with the terms containing no more than one derivative so that such models can be considered as the smooth deformations for the usual (spontaneously broken) supergravity. Also we investigate all possible limits where one of the fields becomes massless.

Keywords: massive gravity, bigravity, supergravity

1. Introduction

A few years ago an interesting solution of the longstanding problem of massive deformation for the gravity theory was found [1, 2]. The solution appeared to be surprisingly simple: all that one has to do is to supplement the usual Lagrangian for gravity with the potential (containing terms without derivatives only) and to chooses a special form of this potential so that to avoid the appearance of the so-called Boulware–Deser ghost [3]. In some sense this theory may be considered as a smooth deformation of the usual gravity theory. Further on, the extension to the bigravity (with one massless and one massive gravitons) was also found [4–6].

Taking into account a prominent role played by supersymmetry it is natural to call for the supersymmetric extensions of these massive gravity and bigravity theories. It is strange but till now there appeared just a few papers devoted to this subject [7–10]. In this work we begin an investigation of possible supersymmetric generalizations for the massive (bi)gravity theories using the constructive approach based on the gauge invariant description for the massive spin-2 and spin-3/2 particles. It is clear that in most cases a supersymmetry must be spontaneously broken and a gravitino must be massive. So we start with the construction of the cubic
interaction vertices for the massive spin-2 and one or two massive spin-3/2. By analogy with the case of massive (bi)gravity itself, we consider the vertices containing no more then one derivative so that such models can also be considered as the smooth deformations of the usual (spontaneously broken) supergravity. Let us briefly summarize the results of our work here.

1.1. Massive spin-2 and one massive spin-3/2

The vertex exists for any non-zero values for the graviton and gravitini masses. It has a smooth limit where graviton mass goes to zero that corresponds to the case of the spontaneously broken supergravity. At the same the limit where gravitino mass goes to zero is singular and the reason is clear: massless gravitino means unbroken supersymmetry and so graviton have to be massless as well.

1.2. Massive spin-2 and massive and massless spin-3/2

The solution exists for the equal masses for the graviton and gravitino only. The reason is again quite clear: massless gravitino implies unbroken supersymmetry and so all the members of the same supermultiplet must have equal masses. At the same time, unbroken supersymmetry means that there must exists a massless spin-2 superpartner for the massless spin-3/2 so that such model must be a part of some bigravity theory similar to the ones considered in [9]. Moreover, this result agrees with the general properties of the cubic vertices for one massless and two massive fields [11] where cubic vertex with different masses requires much higher number of derivatives than for the case of equal masses. In particular, a cubic vertex for massless spin-3/2 and massive spin-2 and massive spin-3/2 with different masses does exist but requires as many as four derivatives so it is trivially gauge invariant.

1.3. Massive spin-2 and two massive spin-3/2 with different masses

The vertex exists for any three non-zero masses. The limit where the graviton becomes massless is possible for the equal masses for the two gravitini only. Again it is in agreement with the general properties of such cubic vertices [11]. Indeed the cubic vertex for the massless spin-2 and two massive spin-3/2 with different masses does exist but requires as many as four derivatives. From the other hand the limit where one of the gravitini becomes massless is possible for the equal masses for the graviton and massive gravitini only in agreement with the results of the previous case.

The paper is organized as follows. In section 2 we describe version of the constructive approach we use. Section 3 provides all necessary kinematic information on the frame-like gauge invariant description for massive spin-2 and massive spin-3/2. Sections 4–6 are devoted to the three types of cubic vertices described above. Some technical details are moved into appendix.

2. Constructive approach

We follow the constructive approach where starting with the free (quadratic) Lagrangian $L_0$ which is invariant under the non-homogeneous gauge transformations $\delta_0 \Phi$ one tries to construct interacting theory perturbatively in the number of fields:

$$L = L_0 + g L_1 + \ldots, \quad \delta \Phi = \delta_0 \Phi + g \delta_1 \Phi + O(g^2),$$
where $L_1$ contains cubic terms, while $\delta_1 \Phi$ is linear in fields and so on. In the first non-trivial approximation it requires to solve the following relation:

$$\frac{\delta L_1}{\delta \Phi} \delta_0 \Phi + \frac{\delta L_0}{\delta \Phi} \delta_1 \Phi = 0.$$  \hspace{1cm} (1)

This solution can be found in two steps. First of all one can find the cubic terms $L_1$ such that their variations vanish on the free mass shell:

$$\left. \frac{\delta L_1}{\delta \Phi} \delta_0 \Phi \right|_{\delta \Phi = 0} = 0$$

and then returning to the equation (1) find the corresponding corrections to the gauge transformations.

In the frame-like formalism one works with the pairs of physical and auxiliary fields (we denote them schematically as $\Phi$ and $\Omega$) and so in the honest first order formalism one has to solve the following relation:

$$\frac{\delta L_1}{\delta \Phi} \delta_0 \Phi + \frac{\delta L_1}{\delta \Omega} \delta_0 \Omega + \frac{\delta L_0}{\delta \Phi} \delta_1 \Phi + \frac{\delta L_0}{\delta \Omega} \delta_1 \Omega = 0.$$  

Taking into account that the equations for the auxiliary fields are purely algebraic ones, in supergravities the so-called 1 and 1/2 order formalism is very often used:

$$\left[ \frac{\delta L_1}{\delta \Phi} \delta_0 \Phi + \frac{\delta L_0}{\delta \Phi} \delta_1 \Phi \right]_{\delta \Phi = 0} = 0$$

where one takes into account variations of the physical fields only but all the calculations are made up to the terms proportional to the auxiliary field equations only. But such formalism requires to solve the complete non-linear equations for the auxiliary fields and it can be quite non-trivial task. There exists one more possibility that we called a modified 1 and 1/2 order formalism [12, 13]

$$\left[ \frac{\delta L_1}{\delta \Phi} \delta_0 \Phi + \frac{\delta L_1}{\delta \Omega} \delta_0 \Omega + \frac{\delta L_0}{\delta \Phi} \delta_1 \Phi \right]_{\delta \Phi = 0} = 0,$$  \hspace{1cm} (2)

where all that one needs are the solutions for the free auxiliary fields equations only. It is this formalism that we use in this work.

### 3. Kinematics

The most important ingredient of the constructive approach is the presence of the gauge invariance already at the free level. That is why the constructive approach is usually associated with the theories of massless fields only. But the gauge invariant description of the massive bosonic and fermionic fields [14–17], which is possible due to the introduction of the appropriate set of Stueckelberg fields, allows one to extend such approach to any systems with massive and/or massless fields. In this section we provide all necessary information on the frame-like gauge invariant description for the massive spin-2 and spin-3/2 fields [16].
3.1. Notations and conventions

We work in the frame-like formalism where four dimensional flat Minkowski space is described (in a coordinate free way) by the (non-dynamical) frame $e^a$, its inverse $\hat{e}^a$ and covariant derivative $D$. We use the condensed notations for the products of these forms:

$$E^{ab} = e^a \wedge e^b, \quad E^{abc} = e^a \wedge e^b \wedge e^c, \quad E^{abcd} = e^a \wedge e^b \wedge e^c \wedge e^d$$

and similarly for $\hat{e}^a$, while

$$D \wedge D = 0.$$  

In what follows a wedge product sign $\wedge$ is omitted.

Completely antisymmetric products of $\gamma$-matrices are defined as follows:

$$\Gamma^{ab} = \frac{1}{2} \gamma^{[a} \gamma^b], \quad \Gamma^{abc} = \frac{1}{3!} \gamma^{[a} \gamma^b \gamma^c], \quad \Gamma^{abcd} = \frac{1}{4!} \gamma^{[a} \gamma^b \gamma^c \gamma^d].$$

We use a Majorana representation for the $\gamma$-matrices where $(\gamma^0 \gamma^a)$ and $(\gamma^0 \Gamma^{ab})$ are symmetric in their spinor indices, while $(\gamma^0), (\gamma^0 \Gamma^{abc})$ and $(\gamma^0 \Gamma^{abcd})$ are antisymmetric.

3.1.1. Massive spin-2. The frame-like gauge invariant description for the massive spin-2 field [16] requires three pairs of physical and auxiliary fields: $(\Omega^{ab}, f^a), (B^{ab}, A)$ and $(\pi^a, \sigma)$, where $\Omega^{ab}, f^a$ and $A$ are one-forms while $B^{ab}, \pi^a$ and $\sigma$—zero-forms. In the notations explained above the free Lagrangian has the form:

$$L_0 = \frac{1}{2} \hat{E}^{ab} \hat{\Omega}^{a} \hat{\Omega}^{b} - \frac{1}{2} \hat{E}^{abc} \Omega^{ab} Df^c + \frac{1}{2} B^{ab} B^{ab}$$

$$- \hat{E}_{ab} B^{ab} DA - \frac{1}{3} \hat{\pi}^a \pi^a + \frac{2}{3} \hat{\pi}^a D \sigma$$

$$+ m \hat{E}_{ab} \Omega^{ab} A + m \hat{\pi}^a B^{ab} f^b - 2m \hat{\pi}^a A$$

$$+ \frac{m^2}{2} \hat{E}_{ab} f^a f^b - m^2 \hat{\pi}^a f^a + \frac{2m^2}{3} \sigma^2.$$  

(3)

Its structure follows the general pattern for the gauge invariant Lagrangians for the massive fields. Namely, the first two lines are just the sum of the kinetic terms for the massless spin-2, spin-1 and spin-0 fields, the last line is the sum of all possible mass-like terms, while the third line contains cross-terms gluing all the fields together. The main requirement determining this structure is that the Lagrangian must still be invariant under the all (appropriately modified) gauge transformations of the initial massless fields. Indeed it is straightforward to check that this Lagrangian is invariant under the following gauge transformations:

$$\delta \Omega^{ab} = D\eta^{ab} - \frac{m^2}{2} \gamma^a \gamma^b],$$

$$\delta f^a = D\xi^a - e_b \eta^{ab} + e^b \xi^a,$$

$$\delta B^{ab} = -m \eta^{ab}, \quad \delta A = D\xi + m e_a \xi^a,$$

$$\delta \pi^a = -\frac{3m^2}{2} \xi^a, \quad \delta \sigma = 3m \xi.$$  

(4)

One of the nice features of the frame-like formalism is that for each field (both physical and auxiliary ones) one can construct the corresponding gauge invariant object. For the case at hands we obtain:
\( R^{ab} = D\Omega^{ab} + \frac{m}{2} E^{[a} B^{b]} c - \frac{m^2}{2} \epsilon^{[a} f^{b]} + \frac{m^2}{3} E^{ab} \sigma, \)
\( T^a = Df^a - e_b \Omega^{ab} + me^a A, \)
\( B^{ab} = DB^{ab} + m\Omega^{ab} - \frac{m}{3} \epsilon^{[a} \epsilon^{b]} \),
\( A = DA - \frac{1}{2} E_{ab} B^{ab} + m e_a f^a, \)
\( \Pi^a = D\pi^a - \frac{3m}{2} e^b B^{ab} + \frac{3m^2}{2} f^a - \frac{m^2}{2} \epsilon^a \sigma, \)
\( \Sigma = D\sigma - e_b \pi^b - 3mA. \) (5)

In what follows we call these objects curvatures though \( R^{ab}, T^a \) and \( A \) are the two-forms, while \( B^{ab}, \Pi^a \) and \( \Sigma \) — one-forms.

As we have explained in the previous section all the calculations are made up to the terms proportional to the free auxiliary fields equations. So in what follows ‘on-shell’ means:
\( T^a \approx 0, \quad A \approx 0, \quad \Sigma \approx 0. \) (6)

This in turn provides us with a number of algebraic and differential identities for the curvatures that do not vanish on-shell:
\( e_b R^{ab} \approx 0, \quad E_{ab} B^{ab} \approx 0, \quad e_a \Pi^a \approx 0, \) (7)
\( DR^{ab} = \frac{m}{2} E^{[a} B^{b]} c, \quad DB^{ab} = mR^{ab} + \frac{m}{3} \epsilon^{[a} \epsilon^{b]}, \quad D\Pi^a = \frac{3m}{2} e^b B^{ab}. \) (8)

One more useful fact is that variations of the free Lagrangian under any transformations for the physical fields can be conveniently expressed in terms of these curvatures:
\( \delta L_0 = -\frac{1}{2} \hat{E}_{abc} R^{ab} \delta f^c + \hat{E}_{ab} B^{ab} \delta A - \frac{2}{3} e_a \Pi^a \delta \sigma. \) (9)

3.1.2. Massive spin-3/2. For the frame-like gauge invariant description of the massive spin-3/2 field we use a one-form \( \Phi \) and a zero-form \( \phi \) (both of them are physical) with the Lagrangian
\( L_0 = -\frac{i}{2} \hat{E}_{abc} \Phi \Gamma^{abc} D\Phi + \frac{i}{2} \hat{e}_a \phi \gamma^a D\phi - \frac{3m_1}{2} \hat{E}_{ab} \Gamma^{ab} \Phi + 3im_1 \hat{e}_a \phi \gamma^a \phi - m_1 \phi \phi \) (10)
where the first line is just the sum of the kinetic terms for the massless spin-3/2 and spin-1/2 fields, while the second line contains mass-like and cross terms. This Lagrangian is invariant under the following gauge transformations:
\( \delta \Phi = D\zeta + \frac{im_1}{2} e_a \gamma^a \zeta, \quad \delta \phi = 3m_1 \zeta. \) (11)

As in the bosonic case we can construct two gauge invariant objects (curvatures):
\( F = D\Phi + \frac{im_1}{2} e_a \gamma^a \Phi + \frac{m_1}{12} E_{ab} \Gamma^{ab} \phi, \)
\( C = D\phi - 3m_1 \Phi + \frac{im_1}{2} e_a \gamma^a \phi. \) (12)
where $\mathcal{F}$ is a two-form, while $\mathcal{C}$—one-form. These curvatures satisfy the following differential identities:

$$D\mathcal{F} = -\frac{im_1}{2} e_a \gamma^a \mathcal{F} + \frac{m_1}{12} F_{ab} \Gamma^{ab} \mathcal{C},$$

$$D\mathcal{C} = -3m_1 \mathcal{F} - \frac{im_1}{2} e_a \gamma^a \mathcal{C}. \quad (13)$$

Also the variations of the free Lagrangian under any transformations of the fields $\Phi$ and $\phi$ can be conveniently expressed in terms of these curvatures:

$$\delta \mathcal{L}_0 = -i \hat{E}_{abc} \bar{\Phi} \Gamma^{abc} \delta \Phi - i \hat{e}_a \bar{C} \gamma^a \delta \phi. \quad (14)$$

For the second spin 3/2 we use the same formulas but with the fields $\Psi$ and $\psi$, mass $m_2$ and the gauge invariant curvatures $\mathcal{H}$ and $\mathcal{D}$. Note also that in the massless limit $m_2 = 0$ the spinor field $\psi$ decouples and we obtain simply

$$\mathcal{L}_0 = -\frac{i}{2} \hat{E}_{abc} \bar{\Psi} \Gamma^{abc} D \Psi, \quad \delta \Psi = D \zeta. \quad (15)$$

4. Massive spin-2 and one massive spin-3/2

In this section we consider a cubic vertex for the massive spin-2 and one massive spin-3/2 with different masses. We follow top–down approach in the number of derivatives. Namely, we begin with the most general non-trivial (i.e. such that do not vanish and are not equivalent on-shell) terms with one derivative and require that all variations with the highest number of derivatives can be compensated by the appropriate corrections to the gauge transformations. Then we add terms without derivatives and try to achieve complete invariance introducing additional corrections if necessary.

4.1. Terms with one derivative

Complete analysis of these terms is given in the appendix while here we provide the result only:

$$\mathcal{L}_{11} = ic_1 \hat{E}_{abc} \bar{\Phi} \gamma^c \Phi + ic_2 \hat{E}_{ab} \Gamma^{abc} \Phi \eta^d$$

$$+ ic_3 \hat{e}_a \bar{\Phi} \gamma^a \Phi + ic_4 \hat{E}_{abc} \bar{\Phi} \gamma^c \Phi$$

$$+ ic_5 \hat{e}_a \bar{\Phi} \gamma^a \Phi + ic_6 \hat{e}_a \bar{\Phi} \gamma^a \Phi$$

$$+ ic_7 \hat{e}_a \bar{\Phi} \gamma^a \Phi + ic_8 \hat{e}_a \bar{\Phi} \gamma^a \Phi.$$ \quad (16)

The four lines in this expression correspond to the elementary subvertices $(2, 3/2, 3/2)$, $(2, 1/2, 1/2)$, $(1, 3/2, 1/2)$ and $(0, 3/2, 1/2)$ respectively. The possibility to compensate for the variations with the highest number of derivatives arises when

$$2c_1 = 3c_2, \quad c_4 = -4c_3, \quad c_5 = -2c_6, \quad c_8 = -c_7,$$

while the necessary corrections to the gauge transformations have the form:

$$\delta F^a = -4ic_1 \Phi \gamma^a \zeta, \quad \delta A = ic_6 \bar{e}_a \gamma^a \zeta, \quad \delta C = -\frac{3c_7}{2} \bar{e}_a \zeta, \quad$$

$$\delta \Phi = -c_1 \Gamma^{ab} \bar{\Phi} \eta_{ab} \zeta + \frac{c_1}{3} \Gamma^{ab} \eta_{ab} \Phi, \quad$$

$$\delta \phi = -2c_1 \Gamma^{ab} \eta_{ab} \phi - c_6 \Gamma^{ab} \bar{B}^{ab} \zeta + ic_7 \gamma^a \pi^a \zeta. \quad (17)$$
4.2. Terms without derivatives

Now we proceed and add the most general cubic terms without derivatives:

\[
\mathcal{L}_{10} = \frac{d_1}{6} \bar{E}_{ab} \Gamma^{ab} \Phi f^c + \frac{d_2}{6} \bar{E}_{ab} \Gamma^{ab} \phi f^c + \frac{d_3}{6} \bar{E}_{ab} \Gamma^{ab} \phi f^c + d_4 \bar{e_a} \bar{\phi} \phi f^a + d_5 \bar{E}_{ab} \Gamma^{ab} \Phi \phi \sigma + id_7 \bar{e_a} \Phi \gamma^a \phi \sigma + d_6 \phi \phi \sigma, \tag{18}
\]

This in turn requires to introduce additional corrections to the gauge transformations for the fermions:

\[
\delta_1 \Phi = im_1 c_2 \gamma^a \Phi + \frac{mc_6}{3} e^a \xi^a \phi + \frac{d_2}{6} e_a \Gamma^{ab} \xi^b \phi
\]

\[
- im_1 c_2 \gamma^a f^a \zeta - mc_2 A \zeta - \frac{id_6}{3} \alpha^a \gamma^a \zeta \sigma + mc_2 \Phi \xi,
\]

\[
\delta_1 \phi = im_1 c_4 \gamma^a \Phi \phi + d_7 \zeta \sigma + 3mc_4 \phi \zeta.
\]

The complete gauge invariance (in the linear approximation) leads to the number of relations on the parameters. Their solution looks like:

\[
c_3 = \frac{m^2 - 3m_1^2}{18m_1^2} c_1, \quad c_6 = - \frac{m}{3c_1}, \quad c_7 = \frac{4}{9} c_1,
\]

\[
d_1 = m_1 c_1, \quad d_2 = \frac{2(m^2 - 2m_1^2)}{3m_1} c_1, \quad d_3 = - \frac{m}{3} c_1, \quad d_4 = - 2m_1 c_3,
\]

\[
d_5 = 0, \quad d_6 = - \frac{3m_1}{2} c_7, \quad d_7 = - d_2, \quad d_8 = \frac{8m_1}{3} c_3.
\]

Thus all the parameters are expressed in terms of the main one \(c_1\).

4.3. Algebraic structure

All our fields are gauge or Stueckelberg ones with the non-homogeneous gauge transformations. So even in this linear approximation we can consider the commutators of the gauge transformations in the lowest order and this gives quite important information on the algebraic structure that stays behind such a model and also provides an independent check for our calculations. For the bosonic fields we can take the commutators of the two supertransformations. This gives:

\[
[\delta_1, \delta_2] f^a = D \tilde{\xi}^a - \tilde{\eta}^{ab} e_b, \quad [\delta_1, \delta_2] A = \frac{m}{2} e_a \tilde{\xi}^a, \quad [\delta_1, \delta_2] \sigma = 0.
\]

\[
\tilde{\xi}^a = 4ic_1 (\zeta_a \gamma^a \zeta_1), \quad \tilde{\eta}^{ab} = 4m_1 c_1 (\zeta_a \Gamma^{ab} \zeta_1), \quad \tilde{\zeta} = 0. \tag{20}
\]

At the same time for the fermionic fields we have non-trivial commutators for the bosonic and supertransformations:

\[
[\delta_B, \delta_1] \Phi = (D + \frac{im}{2} e_a \gamma^a) \zeta, \quad [\delta_B, \delta_1] \phi = 3m_1 \zeta,
\]

\[
\zeta = - \frac{c_1}{3} (\Gamma^{ab} \eta^{ab}) \zeta - \frac{2im_1 c_1}{3} (\gamma^a \xi^a \zeta) - \frac{2mc_1}{3} (\xi \zeta). \tag{21}
\]
4.4. Massless limits

From the solution for the parameters \( c, d \) given above one can see that the limit \( m_1 \to 0 \) where gravitino becomes massless is singular. It is quite natural because massless gravitino means unbroken supersymmetry and in this case graviton should also be massless. On the other hand, nothing prevent us to consider the limit \( m \to 0 \) where graviton becomes massless and this corresponds to the case of spontaneously broken supergravity. The cubic vertex and the corrections to the gauge transformations have the form (compare [18]):

\[
\begin{align*}
L_1 &= i c_1 \hat{E}_{abc} \Omega^{ab} \Phi^c \Psi + \frac{2 i c_1}{3} \hat{E}_{abcd} \bar{\Phi} \Gamma^{abc} \Phi f^d - \frac{i c_1}{6} \hat{e}_a \Omega^{bc} \bar{\Phi} \Gamma^{ab} \phi \\
&+ \frac{2 i c_1}{3} \hat{E}_{abc} \bar{\Phi} \gamma^c \phi^b + m_1 c_1 \hat{E}_{abc} \bar{\Phi} \Gamma^{abc} \phi^c \\
&- \frac{2 i m_1 c_1}{3} \hat{E}_{ab} \bar{\Phi} (4 \gamma^a \gamma^b\phi + \Gamma^{abc} \phi) + \frac{m_1 c_1}{3} \hat{e}_a \bar{\Phi} \phi^a, \\
\end{align*}
\]

(22)

\[
\begin{align*}
\delta_1 \Phi &= \frac{c_1}{3} \Gamma^{ab} (\gamma^a \phi - \Omega^{ab} \zeta) - \frac{2 i m_1 c_1}{9} \hat{e}_a \Gamma^{ab} \xi_b \phi, \\
\delta_1 \phi &= \frac{c_1}{3} \Gamma^{ab} \phi^b \phi + \frac{2 i m_1 c_1}{3} \gamma^a \xi_a \phi, \\
\delta_1 f^a &= -4 i c_1 \bar{\Phi} \gamma^a \zeta. \\
\end{align*}
\]

(23)

5. Massive spin-2 and massive and massless spin-3/2

In this section we consider the cubic vertex for the massive spin-2 and one massive and one massless spin-3/2. It is a special limit of the general vertex that we consider in the next section but it is worth to be considered separately.

5.1. Terms with one derivative

As in the previous case we begin with the most general non-trivial terms with one derivative. The analysis of the possible terms goes similarly to the previous case so we give here the final result only:

\[
\begin{align*}
\mathcal{L}_{11} &= 3 i c_1 \hat{E}_{abc} \Omega^{ab} \bar{\Phi} \gamma^c \Psi + i c_1 \hat{E}_{abcd} \bar{\Phi} \Gamma^{abc} \Phi f^d + i c_1 \hat{E}_{ab} \bar{\Phi} (4 \gamma^a \gamma^b \phi + \Gamma^{abc} \phi) \\
&+ c_1 \hat{e}_a \bar{\Phi} [2 B^{ab} \Phi \Psi + B^{ab} \Phi \Gamma^{abc} \phi] + i c_3 \hat{e}_a \bar{\Psi} [2 B^{ab} \gamma^c \phi + B^{ab} \Gamma^{abc} \phi] \\
&+ c_8 \hat{e}_a \bar{\Psi} [\bar{\Phi} \Gamma^{ab} \phi - \Gamma^{ab} \phi^b \phi]. \\
\end{align*}
\]

(24)

The four lines in this expression correspond to the subvertices (2, 3/2, 2/3), (1, 3/2, 3/2), (1, 3/2, 1/2) and (0, 3/2, 1/2) respectively. The corrections to the gauge transformations for the bosonic fields which are necessary to compensate for the variations with the highest number of derivatives look as follows:

\[
\begin{align*}
\delta f^a &= -6 i c_1 [\bar{\Psi} \gamma^a \zeta_1 + \Phi \gamma^a \zeta_2], \\
\delta A &= 2 c_3 [\bar{\Psi} \zeta_1 - \Phi \zeta_2] - i c_5 \bar{\Phi} \gamma^a \hat{e}_a \zeta_2, \\
\delta \sigma &= -\frac{3 c_8}{2} \bar{\Phi} \zeta_2, \\
\end{align*}
\]

(25)
while the corresponding corrections for the fermionic fields have the form:

\[
\delta \Phi = \frac{c_1}{2} [\Gamma^{ab} \eta^{cb} \Psi - \Gamma^{aa} \Omega^{bc}] - \frac{i c_3}{6} [2 B^{ab} e_a \gamma_b + \Gamma^{abc} B_{ab} e_c] \zeta_2,
\]

\[
\delta \phi = c_5 \Gamma^{ab} B_{ab} \zeta_2 + i c_8 \gamma^a \pi^a \zeta_2,
\]

\[
\delta \Psi = \frac{c_1}{2} [\Gamma^{ab} \eta^{cb} \Phi - \Gamma^{aa} \Omega^{bc}] + \frac{i c_3}{6} [2 B^{ab} e_a \gamma_b - \Gamma^{abc} B_{ab} e_c] \zeta_1.
\]

(26)

5.2. Terms without derivatives

Now we proceed and add the most general cubic terms without derivatives\(^1\):

\[
L_{10} = \hat{E}_{abc} (d_1 \Phi \Gamma^{ab} \Psi f^c + d_2 \Phi \Gamma^{abcd} \Psi f^d) + i \hat{E}_{ab} (d_4 \Psi \gamma^a \phi^b + d_6 \Psi \Gamma^{abc} \phi^c)
+ i d_9 \hat{E}_{abc} \Gamma^{abc} \Psi A + d_{13} \hat{E}_{ab} \Phi \Gamma^{ab} \Psi \sigma + i d_{15} \hat{e}_a \Psi \gamma^a \phi \sigma.
\]

(27)

First of all to cancel all the remaining variations we have to put \(m_1 = m\), i.e. solution exists for the equal masses for the graviton and gravitino only. The reason is quite clear: massless gravitino implies unbroken supersymmetry and so all the members of the same supermultiplet must have equal masses. At the same time, unbroken supersymmetry means that there must exists a massless spin-2 superpartner for the massless spin-3/2 so that such model must be a part of some bigravity theory similar to the ones considered in [9]. Moreover, this result agrees with the general properties of the cubic vertices for one massless and two massive fields [11] where cubic vertex with different masses requires much higher number of derivatives than for the case of equal masses. In particular, a cubic vertex for massless spin-3/2 and massive spin-2 and massive spin-3/2 with different masses does exist but requires as many as four derivatives so it is trivially gauge invariant.

As in the previous case all the parameters are expressed in terms of the main one \(c_1\):

\[
c_3 = c_5 = \frac{3 c_1}{2}, \quad c_8 = 2 c_1,
\]

\[
d_1 = \frac{3 m c_1}{2}, \quad d_2 = \frac{m c_1}{2}, \quad d_4 = m c_1, \quad d_6 = - \frac{m c_1}{2},
\]

\[
d_9 = 2 m c_1, \quad d_{13} = -3 m c_1, \quad d_{15} = m c_1,
\]

with the additional corrections for the fermionic fields:

\[
\delta \Phi = i m c_1 \gamma^a \xi^a \Psi - m c_1 \Psi \xi = i m c_1 \gamma^a f^a \zeta_2 + m c_1 A \zeta_2 + \frac{i m c_1}{2} \epsilon_a \gamma^a \sigma \zeta_2,
\]

\[
\delta \phi = m c_1 \sigma \zeta_2,
\]

\[
\delta \Psi = - \frac{m c_1}{6} [3 e_a \epsilon^a - \Gamma^{ab} e_b \xi_b] \phi + 3 m c_1 \Phi \xi - 3 m c_1 A \xi_1 + \frac{i m c_1}{2} \epsilon_a \gamma^a \sigma \xi_1.
\]

(28)

5.3. Algebraic structure

Again it is instructive to consider the commutators of the gauge transformations in the lowest order. For the bosonic fields the commutators of the two supertransformations have the form:

\(^1\)The coefficients are chosen so that they correspond to the similar coefficients in the general case considered in the next section.
while the commutators of the bosonic and supertransformations look like:

\[ [\delta_B, \delta_c] \Phi = (D + \frac{im}{2} e_a \gamma^a) \tilde{\zeta}_1, \quad [\delta_B, \delta_c] \phi = 3m \tilde{\zeta}_1, \]

\[ \tilde{\zeta}_1 = -\frac{c_1}{2} (\Gamma^{ab} \eta^{ab} \zeta_2) - imc_1 (\gamma^a \xi^a \zeta_2) + m c_1 (\xi_2), \]

\[ [\delta_B, \delta_c] \Psi = D \tilde{\zeta}_2, \quad \tilde{\zeta}_2 = -\frac{c_1}{3} (\Gamma^{ab} \eta^{ab} \zeta_1) - 3m c_1 (\xi_1). \]

6. Massive spin-2 and two spin-3/2 with different masses

At last we consider the most general case—massive spin-2 and two massive spin-3/2 with different masses.

6.1. Terms with one derivative

In this case we have quite a lot of possible terms with one derivative:

\[ L_{11} = 3i c_1 \tilde{E}_{ab \gamma} \Gamma^{ab} \phi \gamma^\gamma \Psi + ic_1 \tilde{E}_{abcd} \tilde{F}^{abc \phi} \phi \phi^d + ic_1 \tilde{E}_{abc} \tilde{H} \Gamma^{abc} \phi \phi^d \]

\[ + 4ic_2 \tilde{E}_{abc} \tilde{C}^{abc \phi \delta^b} + 4ic_2 \tilde{E}_{abc} \tilde{D} \Gamma^{abc \phi \delta^b} + ic_2 \tilde{E}_{abc} \tilde{C} \Gamma^{abc \phi \delta^b} + ic_2 \tilde{E}_{abc} \tilde{D} \Gamma^{abc \phi \delta^b} \]

\[ + 2ic_3 \tilde{E}_{abc} \bar{B}^{abc} \phi \phi + c_3 \tilde{E}_{abc} \bar{B}^{abc} \phi \phi + 2ic_3 \tilde{E}_{abc} \bar{B}^{abc \phi \delta^b} - ic_3 \tilde{E}_{abc} \bar{B}^{abc \phi \delta^b} \phi \]

\[ + c_3 \bar{B}^{abc} \phi \Gamma^{abc} \phi + c_7 \bar{B}^{abc} \phi \Gamma^{abc \phi \delta^b} + c_8 \bar{B}^{abc} \phi \Gamma^{abc \phi \delta^b} \phi \phi \phi, \]

where separate lines correspond to the subvertices (2, 3/2, 3/2), (2, 1/2, 1), (2, 3/2, 3/2), (1, 3/2, 1/2, 1/2), (1, 3/2, 1/2) and (0, 3/2, 1/2). The required corrections to the gauge transformations for the bosonic fields look like:

\[ \delta f^a = -6ic_1 \bar{\psi}_1 \gamma^a \zeta_1 - 6ic_1 \phi \gamma^a \zeta_2, \]

\[ \delta A = 2ic_3 \bar{\psi}_1 - 2ic_3 \bar{\psi}_2 - ic_4 \bar{\psi}_1 e_a \zeta_1 - ic_5 \bar{\psi}_1 e_a \zeta_2, \]

\[ \delta \sigma = -\frac{3c_7}{2} \bar{\psi}_1 - \frac{3c_8}{2} \bar{\psi}_2, \]

(33)

while the corresponding corrections for the fermions have the form:

\[ \delta \Phi = \frac{c_1}{2} \Gamma^{ab} \eta^{ab} \phi \Psi - \frac{c_1}{2} \Gamma^{ab} \phi \eta^{ab} \zeta_1 - \frac{ic_3}{3} \bar{B}^{abc} e_a \gamma_b \zeta_2 + \frac{ic_3}{6} \Gamma^{abc} B^{abc} e_a \zeta_2, \]

\[ \delta \phi = 2ic_2 \Gamma^{ab} \eta^{ab} \psi + c_3 \Gamma^{ab} \eta^{ab} \zeta_2 + ic_8 \gamma^{a \delta} \xi^a \zeta_2, \]

(34)

\[ \delta \Psi = \frac{c_1}{2} \Gamma^{ab} \eta^{ab} \Phi - \frac{c_1}{2} \Gamma^{ab} \phi \eta^{ab} \zeta_1 + \frac{ic_3}{3} \bar{B}^{abc} e_a \gamma_b \zeta_1 - \frac{ic_3}{6} \Gamma^{abc} B^{abc} e_a \zeta_1, \]

\[ \delta \psi = 2ic_2 \Gamma^{ab} \eta^{ab} \phi + c_4 \Gamma^{ab} \phi B^{abc} \zeta_1 + ic_7 \gamma^{a \delta} \xi^a \zeta_1. \]

(35)
6.2. Terms without derivatives

The most general cubic terms without derivatives can be written as follows:

\[
\mathcal{L}_{01} = d_1 \bar{E}_{abc} \Phi_{\Gamma}^{ab} \Psi_{\Gamma}^{c} + d_2 \bar{E}_{abc} \Phi_{\Gamma}^{ab} \psi f^c + id_3 \bar{E}_{abc} \Phi^{a} \psi f^b + id_4 \bar{E}_{abc} \Psi^{a} \phi f^b \\
+ id_5 \bar{E}_{abc} \Phi_{\Gamma}^{ab} \psi f^c + id_6 \bar{E}_{abc} \Phi_{\Gamma}^{abc} \psi f^d + d_7 \bar{E}_{abc} \Phi_{\Gamma}^{ab} \psi f^b + d_8 \bar{E}_{abc} \Phi_{\Gamma}^{abc} \psi f^b \\
+ id_9 \bar{E}_{abc} \Phi_{\Gamma}^{ab} \Psi \sigma A + d_{10} \bar{E}_{abc} \Phi_{\Gamma}^{ab} \psi A + d_{11} \bar{E}_{abc} \Phi_{\Gamma}^{ab} \psi A + id_{12} \bar{E}_{abc} \Phi_{\Gamma}^{ab} \psi A \\
+ d_{13} \bar{E}_{abc} \Phi_{\Gamma}^{ab} \Psi \sigma + id_{14} \bar{E}_{abc} \Phi_{\Gamma}^{ab} \psi \sigma + id_{15} \bar{E}_{abc} \Phi_{\Gamma}^{ab} \psi A + d_{16} \bar{E}_{abc} \Phi_{\Gamma}^{ab} \psi \sigma,
\]

while the additional corrections for the fermions look like:

\[
\delta \Phi = im_1 c_1 (\gamma^a \xi^a \psi - \gamma^a f^a \xi_2) + \beta_2 e_a \xi^a \psi + \beta_3 \Gamma^{ab} e_a \xi_b \psi \\
+ (mc_1 - d_9)(\Psi \xi - A \xi_2) - \frac{id_{13}}{6} e_a \gamma^a \sigma \xi_2.
\]

\[
\delta \phi = i\beta_4 \gamma^a \xi^a \psi + (12mc_2 - d_{12}) \Psi \xi + d_{15} \sigma \xi_2.
\]

where

\[
\beta_2 = -\frac{M + 3m_2^2}{6m_2} c_1, \quad \beta_3 = \frac{M + m_2^2}{6m_1} c_1, \quad \beta_4 = -\frac{(2m_1 - m_2) M}{3m_1 m_2} c_1
\]

and

\[
\delta \Psi = im_2 c_1 (\gamma^a \xi^a \Phi - \gamma^a f^a \xi_1) + \tilde{\beta}_2 e_a \xi^a \phi + \tilde{\beta}_3 \Gamma^{ab} e_a \xi_b \phi \\
+ (mc_1 + d_9)(\Phi \xi - A \xi_1) - \frac{id_{13}}{6} e_a \gamma^a \sigma \xi_1.
\]

\[
\delta \psi = i\tilde{\beta}_4 \gamma^a \xi^a \phi + (12mc_2 + d_{12}) \Phi \xi + d_{14} \sigma \xi_1.
\]

where

\[
\tilde{\beta}_2 = -\frac{M + 3m_1^2}{6m_1} c_1, \quad \tilde{\beta}_3 = \frac{M + m_1^2}{6m_1} c_1, \quad \tilde{\beta}_4 = \frac{(m_1^2 - m_2^2) M}{3m_1 m_2} c_1.
\]

Here to simplify presentation we introduced a combination

\[
M = m^2 - (m_1^2 + m_1 m_2 + m_2^2).
\]

As in the both previous cases all the coefficients can be expressed in term of the one main coefficient \(c_1\), but to simplify formulas we give here their expressions in terms of the \(c_1\) and \(c_2\) that are not independent but satisfy the relation

\[
12m_1 m_2 c_2 = -Mc_1.
\]

All other coefficients then look like:
\[2mc_3 = 3(m_1 - m_2)c_1, \quad 2mc_4 = 3m_2c_1 - 12m_1c_2,\]
\[2mc_5 = 3m_1c_1 - 12m_2c_2, \quad mc_6 = 4(m_1 - m_2)c_2,\]
\[m^2c_7 = 2m_2^2c_1 + 8m_1(m_1 - 2m_2)c_2,\]
\[m^2c_8 = 2m_1^2c_1 - 8m_2(2m_1 - m_2)c_2,\]
\[2d_1 = 3(m_1 + m_2)c_1, \quad 2d_2 = (m_1 - m_2)c_1,\]
\[d_3 = m_2c_1 - 12m_1c_2, \quad d_4 = m_1c_1 - 12m_2c_2,\]
\[d_5 = -m_2c_1/2 + 3m_1c_2, \quad d_6 = -m_1c_1/2 + 3m_2c_2,\]
\[d_7 = 2(m_1 + m_2)c_2, \quad d_8 = 4(m_1 - m_2)c_2,\]
\[md_9 = 2(m_1^2 - m_2^2)c_1, \quad d_{10} = d_{11} = 0, \quad md_{12} = 8(m_1^2 - m_2^2)c_2,\]
\[m^2d_{13} = -3(m_1^3 + m_2^3)c_1 + 12m_1m_2(m_1 + m_2)c_2,\]
\[m^2d_{14} = -m_2(3m_1^2 - 2m_1m_2 - 4m_2^2)c_1 - 4m_1(-3m^2 - 2m_1^2 + 8m_2^2)c_2,\]
\[m^2d_{15} = -m_1(3m_2^2 - 4m_1^2 - 2m_1m_2)c_1 - 4m_2(-3m^2 + 8m_1^2 - 2m_2^2)c_2,\]
\[3m^2d_{16} = -8(m_1 + m_1)(m^2 + 2(m_1 - m_2)^2)c_2.\]

### 6.3. Massless limits

From the expressions above it follows that the limit of massless graviton \(m \to 0\) is possible for the equal masses for the two gravitini \(m_1 = m_2\) only. Again it is in agreement with the general properties of such cubic vertices [11]. Indeed the cubic vertex for the massless spin-2 and two massive spin-3/2 does exist but requires as many as four derivatives. From the other hand the limit \(m_2 \to 0\) then one of the gravitini becomes massless is possible for the equal masses \(m = m_1\) for the graviton and massive gravitini only in agreement with the results of the previous section.

### 6.4. Algebraic structure

Again it is instructive to consider the commutators in the lowest non-trivial order. For the bosonic fields we may take the commutators of the two supertransformations and obtain:

\[\left[\delta_1, \delta_2\right] f^a = D\tilde{\xi}^a - \tilde{\eta}^{ab} e_b + me^a\tilde{\xi}, \quad \left[\delta_1, \delta_2\right] A = D\tilde{\xi} + \frac{m}{2} e_a\tilde{\xi}^a, \quad \left[\delta_1, \delta_2\right] \sigma = 3m\tilde{\xi},\]

\[\tilde{\xi}^a = 6ic_1(\tilde{\zeta}_2\gamma^a\zeta_1), \quad \tilde{\eta}^{ab} = 3(m_1 + m_2)c_1(\tilde{\zeta}_2\Gamma^{ab}\zeta_1), \quad \tilde{\xi} = -\frac{3(m_1 - m_2)c_1}{m}(\tilde{\zeta}_2\zeta_1).\]  

(39)

For the first gravitino we may take the commutators of the bosonic and second supertransformations:

\[\left[\delta_\theta, \delta_\zeta\right] \Phi = (D + \frac{im}{2} e_a\gamma^a)\tilde{\zeta}_1, \quad \left[\delta_\theta, \delta_\zeta\right] \phi = 3m\tilde{\zeta}_1,\]

\[\tilde{\zeta}_1 = -\frac{c_1}{2}(\Gamma^{ab}\tilde{\eta}^{ab}\zeta_2) - imc_1(\gamma^a\zeta_2) - \frac{(m^2 - 2m_1^2 + 2m_2^2)c_1}{m}(\zeta_2)\zeta_2,\]

(40)

while for the second gravitino—the the commutators of the bosonic and first supertransformations:
\[ [\delta_B, \delta_1] \Psi = (D + \frac{im_2}{2}e_a \gamma^a) \tilde{\zeta}_2, \quad [\delta_B, \delta_1] \psi = 3m_2 \zeta_2. \]

\[ \tilde{\zeta}_2 = -\frac{c_1}{2} (\Gamma^{ab} \eta^{ab} \zeta_1) - im_2 c_1 (\gamma^a \xi^a \zeta_1) - \frac{(m^2 + 2m_1^2 - 2m_2^2) c_1}{m} (\zeta'_1 \zeta_1). \quad (41) \]

It is easy to check that for the case \( m_1 = m_2 \) these expressions correctly reproduce the results of section 4, while for the case \( m_2 = 0 \)—the results of section 5.

**Appendix. One derivative vertices**

In this appendix we analyze all possible cubic terms for the bosonic spin-2,1,0 and fermionic spin-3/2,1/2 components with one derivative. Following our general formalism we consider terms that do not vanish (or are not equivalent) on-shell and the main requirement is that all variations with the highest number of derivatives can be compensated by the appropriate corrections to the gauge transformations. As for the less derivative variations we take them into account in the main text together with the variations of the terms without derivatives.

**A.1. Subvertex 2 \(-\,3/2 \,-\,3/2\)**

In this case the only possibility is:

\[ L_{1a} = i c_1 \bar{E}_{a}^{bc} \Omega^{ab} \phi \gamma^b \phi + ic_2 \bar{E}_{abcd} \Omega^{abc} \bar{\phi} \gamma^d \phi. \quad (A.1) \]

Let us consider \( \zeta \)-transformations first. They produce the following variations:

\[ \delta \zeta L_{1a} = -2ic_1 \bar{E}_{a}^{bc} \Omega^{ab} \phi \gamma^b \phi + 2ic_1 \bar{E}_{abc} \Omega^{ab} \bar{\phi} \gamma^c \zeta - 3ic_2 \bar{E}_{abcd} \Omega^{abc} \bar{\phi} \gamma^d \zeta \\
+ 4m_1 c_1 \bar{E}_{abc} \Psi \phi \gamma^b \phi + \frac{4im_1 c_1}{3} \bar{\phi} \Omega^{bc} \bar{\phi} \gamma^b \zeta + \frac{im_1 c_1}{3} \bar{\phi} \Omega^{bc} \bar{\phi} \gamma^b \zeta \\
+ m_1 c_2 \bar{E}_{abc} \Omega^{abc} \bar{\phi} \gamma^c \zeta - im_2 c_2 \bar{E}_{abc} \Omega^{abc} \bar{\phi} \gamma^c \zeta \\
- 4im_2 c_1 \bar{E}_{abc} \Omega^{abc} \bar{\phi} \gamma^c \zeta \sigma. \]

To compensate the terms with the highest number of derivatives (the first line) we introduce the following corrections:

\[ \delta_1 \phi = i \alpha_1 \bar{\Phi} \gamma^a \phi, \quad \delta_1 \Phi = \alpha_2 \Gamma^{ab} \Omega^{ab} \zeta. \quad (A.2) \]

Their contribution looks like:

\[ \delta_1 L_0 = -\frac{i \alpha_1}{2} \bar{\Phi} R^{ab} \phi \gamma^c \zeta + 6i \alpha_2 \bar{\Phi} (\Omega^{ab} \gamma^c - \Gamma^{ab} \Omega^{c}) \zeta. \]

Thus we have to put:

\[ \alpha_1 = -4c_1, \quad 6 \alpha_2 = -2c_1 = -3c_2. \]

Let us turn to the \( \eta^{ab} \)-transformations. They produce:
\[ \delta_{\eta} L_{1a} = -2i c_1 \hat{E}_{abc} \eta^{ac} \bar{F}^{\gamma c} \Phi + 3i c_2 \hat{E}_{abc} \eta^{ad} \bar{F}^{bcd} \Phi \]
\[ + 2M_1 c_1 \hat{E}_{ab} \eta^{ab} \bar{F}^{bc} \Phi + \frac{4im_1 c_1}{3} \varepsilon_a \eta^{ab} \bar{F}^{\gamma b} \phi - \frac{im_1 c_1}{3} \varepsilon_a \eta^{bd} \bar{F}^{\Gamma abc} \phi. \]

To compensate for the terms in the first line we introduce:

\[ \delta_1 \Phi = \alpha_3 \Gamma^{ef} \eta^{ef} \Phi \]

and this gives us

\[ \delta_1 L_0 = 6i \alpha_3 \hat{E}_{abc} \bar{F} (\eta^{ab} \gamma^c - \eta^{ad} \Gamma^{bcd}) \Phi. \]

So we obtain (in agreement with the previous relation on \(c_{1,2}\)):

\[ 6 \alpha_3 = 2c_1 = 3c_2. \]

A.2. Subvertex 2 \(-\frac{1}{2} - \frac{1}{2}\)

In this case we consider the terms:

\[ L_{1b} = ic_3 \varepsilon_a \bar{C}^{bc} \Phi \gamma^b \phi + ic_4 \hat{E}_{abc} \bar{C}^{\gamma b} \phi \]

(A.4)

Note that there is one more possible term \( \hat{E}_{abc} \bar{C}^{\gamma b} \phi \) but (up to the terms without derivatives) it is equivalent to the term with coefficient \(c_3\).

The only transformations that produce variations with the highest number of derivatives are \(\eta^{ab}\)-transformations:

\[ \delta_{\eta} L_{1b} = ic_4 \varepsilon_a \eta^{ab} \bar{C}^{\gamma b} \phi - 2ic_3 \hat{E}_{abc} \bar{C}^{\gamma b} \phi \]
\[ - 6im_1 c_3 \varepsilon_a \eta^{bd} \bar{F}^{\Gamma abc} \phi. \]

To compensate for the terms in the first line we introduce

\[ \delta_1 \phi = \alpha_4 \Gamma^{ef} \eta^{ef} \phi, \]

which gives:

\[ \delta_1 L_0 = -i \alpha_4 \varepsilon_a \bar{C} (2\eta^{ab} \gamma^c + \Gamma^{abc} \eta^{bc}) \phi. \]

So we obtain:

\[ 2\alpha_4 = -4c_3 = c_4. \]

A.3. Subvertex 1 \(-\frac{3}{2} - \frac{1}{2}\)

For this case we consider:

\[ L_{1c} = ic_5 \varepsilon_a B^{ab} \Phi \gamma^b \phi + ic_6 \varepsilon_a B^{bc} \Phi \bar{F}^{\Gamma abc} \phi. \]

(A.6)

Note that there are two more possible terms: \( \hat{E}_{abc} \bar{F} \bar{C} \Phi \) and \( \hat{E}_{abc} \Phi \Gamma^{abc} \bar{C} \). However, one their combination is equivalent (up to the terms without derivatives) to the term with coefficient \(c_6\). Besides, there exists a field redefinition \( \Phi \rightarrow \Phi + \kappa_3 A \phi \).

The only transformations we have to consider here are \(\zeta\)-transformations which produce the following variations:
\[ \delta_L L_{1c} = i c_5 \tilde{e}_a B^{ab} \tilde{\phi} \tilde{\gamma}^b \zeta + i c_5 \tilde{e}_a B^{ab} \tilde{C} \gamma^b \zeta - i c_6 \tilde{e}_a B^{bc} \tilde{C} T^{abc} \zeta \\
- imc_5 \tilde{e}_a \Omega^{ab} \tilde{\phi} \gamma^b \zeta + imc_5 \tilde{e}_a \Omega^{bc} \tilde{\phi} T^{abc} \zeta \\
+ 6 im_1 c_5 \tilde{e}_a B^{ab} \tilde{\phi} \gamma^b \zeta - m_1 c_5 B^{ab} \tilde{\phi} T^{abc} \zeta \\
+ imc_5 \tilde{\phi} \gamma^a \zeta \pi^a. \]

To compensate for the terms in the first line we introduce:

\[ \delta_1 A = i \alpha_5 \tilde{\phi} \gamma^a \zeta, \quad \delta_1 \phi = \alpha_6 \Gamma^{ef} B^{ef} \zeta \] (A.7)

and this gives us

\[ \delta_1 L_0 = 2i \alpha_5 \tilde{e}_a B^{ab} \tilde{\phi} \gamma^b \zeta - i \alpha_6 \tilde{e}_a \tilde{C} (2 B^{ab} \gamma^b + B^{bc} \Gamma^{abc}) \zeta. \]

Thus we have to put:

\[ 2 \alpha_5 = -c_5, \quad 2 \alpha_6 = c_5 = -2c_6. \]

A.4. Subvertex 0 \(-\frac{3}{2} - \frac{1}{2}\)

For this case we choose:

\[ L_{1d} = c_7 \tilde{e}_a \tilde{\phi} \gamma^a \pi + c_8 \tilde{e}_a \tilde{\Phi} \Gamma^{ab} \phi \pi. \] (A.8)

Note that there are two more possible terms: \( \tilde{E}_{ab} \tilde{\Phi} \Gamma^{ab} \phi \pi \) and \( \tilde{E}_{ab} \Phi \Gamma^{ab} C \phi \). However, one their combination is equivalent (up to the terms without derivatives) to the term with coefficient \( c_8 \). Besides, there exists a field redefinition \( \Phi \Rightarrow \Phi + \nu \sigma a^{\gamma^a} \phi \).

In this case we have to consider \( \zeta \)-transformations only:

\[ \delta_\zeta L_{1d} = -c_7 \tilde{e}_a \tilde{\phi} \zeta \Pi^a - c_7 \tilde{e}_a \tilde{C} \zeta \pi^a + c_8 \tilde{e}_a \tilde{C} \Gamma^{ab} \zeta \pi^b \\
- \frac{3 mc_8}{2} B^{ab} \tilde{\phi} \Gamma^{ab} \zeta + 6 m_1 c_8 \tilde{e}_a \tilde{\Phi} \Gamma^{ab} \zeta \pi^b + 3 im_1 c_8 \tilde{\phi} \gamma^a \zeta \pi^a \\
+ \frac{3 mc_7}{2} \tilde{e}_a \tilde{\phi} \zeta \pi^a - \frac{3 mc_7}{2} \tilde{e}_a \tilde{\Phi} \Gamma^{ab} \zeta \pi^f - 2 m^2 c_7 \tilde{\phi} \zeta \sigma. \]

To compensate for the terms in the first line we introduce:

\[ \delta_1 \sigma = \alpha_7 \tilde{\phi} \zeta, \quad \delta_1 \phi = i \alpha_8 \gamma^b \pi^b \zeta. \] (A.9)

Taking into account their contributions:

\[ \delta_1 L_0 = -\frac{2 \alpha_7}{3} \tilde{e}_a \tilde{\phi} \zeta \Pi^a + \alpha_8 \tilde{e}_a \tilde{C} (\pi^a + \Gamma^{ab} \pi^b) \zeta, \]

we obtain:

\[ \alpha_7 = -\frac{3 c_7}{2}, \quad \alpha_8 = c_7 = -c_8. \]

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