Isospin breaking, coupled-channel effects, and X(3872)

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We re-investigate the possibility of X(3872) as a D̄D* molecule with JPC = 1++ within the framework of both the one-pion-exchange (OPE) model and the one-boson-exchange (OBE) model. After careful treatment of the S-D wave mixing, the mass difference between the neutral and charged D(D*) mesons and the coupling of the D̄D* pair to D̄ D̄*, a loosely bound molecular state X(3872) emerges quite naturally with large isospin violation in its flavor wave function. For example, the isovector component is 26.24% if the binding energy is 0.30 MeV, where the isospin breaking effect is amplified by the tiny binding energy. After taking into account the phase space difference and assuming the 3π and 2π come from a virtual omega and rho meson respectively, we obtain the ratio of these two hidden-charm decay modes: B(X(3872) → π+π−π0J/ψ)/B(X(3872) → π+π−J/ψ) = 0.42 for the binding energy being 0.3 MeV, which is consistent with the experimental value.

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I. INTRODUCTION

In 2003, the Belle Collaboration observed a narrow charmonium-like state X(3872) in the exclusive decay process B+ → XK*+ followed by X → π+π−J/ψ [1]. Later, this state was confirmed by CDF [2], D0 [3] and BABAR [4]. The current value of the X(3872) mass is M_{X(3872)} = (3871.95 ± 0.48(stat) ± 0.12(syst)) MeV [5] while the updated value of the width of X(3872) is Γ_{X(3872)} < 1.2 MeV [6]. Due to its exotic properties, the X(3872) state has attracted much attention since its discovery [7]. Despite huge efforts, the nature of X(3872) is still unclear. Up to now, the proposed interpretations of the X(3872) include "hadronic molecule" [16, 21], c̅c̅g hybrid [22], tetraquark [23] and charmonium [24, 25].

Experimentally, both BABAR [26] and BELLE [6] did not find the charged partner of the X(3872), which suggests that the X(3872) is an isoscalar. However, the Belle Collaboration reported the branching fraction ratio, B(X → π+π−π0J/ψ)/B(X → π+π−J/ψ) = 1.0 ± 0.4(stat) ± 0.3(syst) [27], which indicates that there exists large isospin breaking for the hidden-charm decay of X(3872). This result was later confirmed by the BABAR collaboration, B(X → π+π−π0J/ψ)/B(X → π+π−J/ψ) = 0.8 ± 0.3 [28]. In addition, the charge parity of X(3872) is even (C = ±1) from its radiative decay X(3872) → γJ/ψ [27, 29]. The quantum numbers of X(3872) are probably JPC = 1++ or 2−+ [30, 31].

The proximity of the X(3872) to the D̄D̄ threshold strongly suggests that the X(3872) might be a weakly bound D̄D̄ molecule. If the X(3872) is really a loosely bound D̄D̄ molecule, we expect that the long-range pion exchange plays a dominant role among the exchanged mesons since the constituent hadrons of the hadronic molecule should be well-separated. We also expect that there exists strong mixing between D̄D̄ and D*+D− due to the closeness of the threshold of D̄D̄ and D*+D−. Actually, if one only considers the neutral D̄D̄ pair, the interaction strength is only one third of that of the isospin singlet. On the other hand, compared with the small binding energy of X(3872) (less than 1 MeV), the mass difference between D̄D̄ and D*+D− (∼ 8.1 MeV) is so large that the large isospin breaking may occur for the X(3872). Further more, the coupling of D̄D̄ to D̄ D̄̄ should also affect the binding of X(3872) since the mass difference is about m_{D̄D̄} − m_{X(3872)} ≈ 140 MeV only.

In the present paper, we shall take into account the S-D wave mixing which plays an important role in forming the loosely bound deuteron, the charged D̄D̄̄ pair, the mass difference between the neutral and the charged D(D̄̄) meson and the coupling of D̄D̄ to D̄ D̄̄. In order to highlight the contribution of the long-range pion exchange, we first study the system with the pion exchange alone. Then we move on and include the other light meson exchanges with the OBE framework. Since the quantum numbers of X(3872) have not been determined exactly, we investigate both the JPC = 1++ and 2−+ cases within the "hadronic molecule" framework.

The paper is organized as follows. After the introduction [1] we present the formalism including the lagrangians and the effective

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TABLE I: The different channels for Cases I, II, III and IV of X(3872) with \( J^{PC} = 1^{++} \). For simplicity, we adopt the following shorthand notations, \( \left[D^0\bar{D}^0\right]^3 S_1 > \left[D^0\bar{D}^0\right] D^+_1 > \left[D^0\bar{D}^0\right] D^0_1 > \left[D^0\bar{D}^0\right] \left[D^+D^-\right] \). “\( \cdot \)” means the corresponding channel does not exist.

| Cases | 1 | 2 | 3 | 4 | 5 | 6 |
|-------|---|---|---|---|---|---|
| I     | \( \left[D^0\bar{D}^0\right]^3 S_1 > \left[D^0\bar{D}^0\right] D^+_1 > \) | \( \left[D^0\bar{D}^0\right] D^0_1 > \left[D^0\bar{D}^0\right] \left[D^+D^-\right] \) | \( \left[D^0\bar{D}^0\right] D^0_1 > \) | \( \left[D^0\bar{D}^0\right] \left[D^+D^-\right] \) | \( \left[D^0\bar{D}^0\right] \left[D^+D^-\right] \) | \( \left[D^0\bar{D}^0\right] \left[D^+D^-\right] \) |
| II    | \( \left(D\bar{D}^*\right)^3 S_1 > \left(D\bar{D}^*\right) D^+ \) | \( \left(D\bar{D}^*\right) D^0 \) | \( \left(D\bar{D}^*\right) \left[D^+D^-\right] \) | \( \left(D\bar{D}^*\right) \left[D^+D^-\right] \) | \( \left(D\bar{D}^*\right) \left[D^+D^-\right] \) | \( \left(D\bar{D}^*\right) \left[D^+D^-\right] \) |
| III   | \( \left(D\bar{D}^*\right)^3 S_1 > \left(D\bar{D}^*\right) D^+ \) | \( \left(D\bar{D}^*\right) D^0 \) | \( \left[D^+D^-\right] \left[D^+D^-\right] \) | \( \left[D^+D^-\right] \left[D^+D^-\right] \) | \( \left[D^+D^-\right] \left[D^+D^-\right] \) | \( \left[D^+D^-\right] \left[D^+D^-\right] \) |
| IV(Phy) | \( \left[D^0\bar{D}^0\right]^3 S_1 > \left[D^0\bar{D}^0\right] D^+_1 > \left[D^0\bar{D}^0\right] \left[D^+D^-\right] \) | \( \left[D^0\bar{D}^0\right] \left[D^+D^-\right] \) | \( \left[D^0\bar{D}^0\right] \left[D^+D^-\right] \) | \( \left[D^0\bar{D}^0\right] \left[D^+D^-\right] \) | \( \left[D^0\bar{D}^0\right] \left[D^+D^-\right] \) | \( \left[D^0\bar{D}^0\right] \left[D^+D^-\right] \) |

A. The Lagrangians and The Coupling Constants

The lagrangians with the heavy quark symmetry and the chiral symmetry read [33–40]

\[
\mathcal{L}_{P^0} = -i \frac{\sqrt{2}}{f} \bar{g}_V P^0_{ab} \partial^0 M_{ab} P^0_{ab} + \frac{\sqrt{2}}{f} \bar{g}_V P^0_{ab} \partial^0 \bar{P}^0_{ab}
\]

\[
\mathcal{L}_{P^0} = -i \frac{\sqrt{2}}{f} \bar{g}_V P^0_{ab} \partial^0 M_{ab} P^0_{ab} - \frac{\sqrt{2}}{f} \bar{g}_V P^0_{ab} \partial^0 \bar{P}^0_{ab}
\]

\[
\mathcal{L}_{P^0} = -i \frac{\sqrt{2}}{f} \bar{g}_V P^0_{ab} \partial^0 M_{ab} P^0_{ab} - \frac{\sqrt{2}}{f} \bar{g}_V P^0_{ab} \partial^0 \bar{P}^0_{ab}
\]

\[
\mathcal{L}_{P^0} = -i \frac{\sqrt{2}}{f} \bar{g}_V P^0_{ab} \partial^0 M_{ab} P^0_{ab} - \frac{\sqrt{2}}{f} \bar{g}_V P^0_{ab} \partial^0 \bar{P}^0_{ab}
\]
The exchanged pseudoscalar meson and vector meson matrices \( M \) and \( \bar{\rho}^\mu \) are defined as

\[
M = \begin{pmatrix}
\frac{x^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \frac{\pi^-}{\sqrt{2}} & K^+ \\
\frac{\eta}{\sqrt{6}} & -\frac{\pi^0}{\sqrt{2}} & K^0 \\
-K^- & K^0 & \frac{\sqrt{6}}{2}\eta
\end{pmatrix}, \quad \bar{\rho}^\mu = \begin{pmatrix}
\rho^+ \\
\rho^- \\
K^{*+} \\
K^{*-}
\end{pmatrix}.
\]

In the OPE model, there are two coupling constants \( f_\pi \) and \( g \). \( f_\pi = 132 \text{ MeV} \) is the pion decay constant. The coupling constants \( g \) was studied by many theoretical approaches, such as quark model [40] and QCD sum rule [41,42]. Here, we take the experimental result of the CLEO Collaboration, \( g = 0.59 \pm 0.07 \pm 0.01 \), which was extracted from the full width of \( D^{*+} \) [43]. Following [44,45], the parameters related to the vector meson exchange are \( g_v = 5.8 \) and \( \beta = 0.9 \) determined by the vector meson dominance mechanism, and \( \lambda = 0.56 \text{ GeV}^{-1} \) obtained by matching the form factor predicted by the effective theory approach with that obtained by the light cone sum rule and the lattice QCD. The coupling constant for the scalar meson exchange is \( g_s = g_\pi/(2 \sqrt{6}) \) [46] with \( g_\pi = 3.73 \). We summarize the parameters used in our calculation in Table II.

### Table II: The coupling constants of the heavy mesons and the exchanged light mesons used in our calculation.

| Pseudoscalar | Coupling Constants | Scalar | Heavy Mesons | Exchanged Mesons |
|--------------|--------------------|--------|--------------|-----------------|
| \( g = 0.59 \) | \( g_v = 5.8 \) | \( g_s = \frac{g_\pi}{2 \sqrt{6}} \) with \( g_\pi = 3.73 \) | \( m_{D^0} = 1869.60 \) | \( m_{\pi^+} = 139.57 \) |
| \( f_\pi = 132 \text{ MeV} \) | \( \beta = 0.9 \) | \( \lambda = 0.56 \text{ GeV}^{-1} \) | \( m_{\rho^0} = 1864.83 \) | \( m_{\rho^0} = 134.98 \) |
| \( m_{D^{*+}} = 2010.25 \) | \( m_{K^{*0}} = 547.85 \) | \( m_{K^{*0}} = 2006.96 \) | \( m_{K^{*0}} = 775.49 \) |
| \( m_{D^{*+}} = 782.65 \) | \( m_{D^{*+}} = 600 \) |

## B. The Effective Potentials

The are four types of feynman diagrams at the tree level which are shown in Fig. I. With the feynman diagrams and the lagrangians given in Eqs. (1-3), we derive the effective potentials with the help of the relation between the effective potential \( V(q) \) and the scattering amplitude \( M(q) \)

\[
V(q) = -\frac{M(q)}{qM_i},
\]

where \( M_i \) is the mass of the heavy meson. After the Fourier transformation, we get the effective potentials in the coordinate space

\[
V(r) = \frac{1}{(2\pi)^3} \int d^3q e^{iqr}V(q)F^2(q)
\]

where \( F(q) \) is the monpole form factor defined as \( F(q) = (\Lambda^2 - m_{\rho^0}^2)/(\Lambda^2 - q^2) = (\Lambda^2 - m_{\rho^0}^2)/(\chi^2 + q^2) \) with \( \chi^2 = \Lambda^2 - q_0^2 \). The role of the form factor is to remove or suppress the contribution from the ultraviolet region of the exchanged momentum since the light mesons “see” the heavy mesons as a whole and do not probe their inner structure.

The expressions of the effective potentials are

\[
V_{\rho}(r) = -C_{\rho}(i, j)\frac{\beta^2 g_s^2}{4\pi} \frac{u}{2} \frac{H_0(\Lambda, m_{\rho/\omega}, r)}{S(\epsilon_1, \epsilon_2)},
\]

\[
V_{\sigma}(r) = -C_{\sigma}(i, j)\frac{g_s^2}{4\pi} \frac{u}{2} \frac{H_0(\Lambda, m_{\rho/\omega}, r)}{S(\epsilon_1, \epsilon_2)}.
\]
FIG. 1: The Feynman diagrams at the tree level. The thick line denotes the heavy vector meson (or antimeson) while the thin line stands for the heavy pseudoscalar meson (or antimeson).

for $D\bar{D}' \leftrightarrow D\bar{D}'$,

$$
V_{\pi}(r) = -C_\pi(i,j)\frac{g^2}{f_\pi^2}\frac{\theta^0}{12\pi} \left[ M_3(\Lambda,m_\pi,r)T(\epsilon_3^i,\epsilon_2) + M_t(\Lambda,m_\pi,r)S(\epsilon_3^i,\epsilon_2) \right],
$$

$$
V_{\rho}(r) = -C_\rho(i,j)\frac{g^2}{f_\pi^2}\frac{\theta^0}{12\pi} \left[ H_3(\Lambda,m_\rho,r)T(\epsilon_3^i,\epsilon_2) + H_1(\Lambda,m_\rho,r)S(\epsilon_3^i,\epsilon_2) \right],
$$

$$
V_{\rho/\omega}(r) = -C_{\rho/\omega}(i,j)\lambda^2 g^2 \frac{\theta^0}{6\pi} \left[ H_3(\Lambda,m_{\rho/\omega},r)T(\epsilon_3^i,\epsilon_2) - 2H_1(\Lambda,m_{\rho/\omega},r)S(\epsilon_3^i,\epsilon_2) \right],
$$

for $D\bar{D}' \leftrightarrow D'\bar{D}$,

$$
V_{\pi}(r) = C_{\pi}(i,j)\frac{g^2}{f_\pi^2}\frac{\theta^0}{12\pi} \left[ H_3(\Lambda,m_{\pi/\eta},r)T(\epsilon_3^i,\epsilon_1,\epsilon_4^i,\epsilon_2) + H_1(\Lambda,m_{\pi/\eta},r)S(\epsilon_3^i,\epsilon_1,\epsilon_4^i,\epsilon_2) \right],
$$

$$
V_{\rho}(r) = -C_{\rho}(i,j)\lambda^2 g^2 \frac{\theta^0}{2}\frac{u^3}{6\pi} H_0(\Lambda,m_{\rho/\omega},r)C(\epsilon_3^i,\epsilon_1,\epsilon_4^i,\epsilon_2)
$$

$$
+ C_{\rho}(i,j)\lambda^2 g^2 \frac{\theta^0}{6\pi} \left[ H_3(\Lambda,m_{\rho/\omega},r)T(\epsilon_3^i,\epsilon_1,\epsilon_4^i,\epsilon_2) - 2H_1(\Lambda,m_{\rho/\omega},r)S(\epsilon_3^i,\epsilon_1,\epsilon_4^i,\epsilon_2) \right],
$$

$$
V_{\omega}(r) = -C_{\omega}(i,j)\frac{g^2}{f_\pi^2}\frac{\theta^0}{12\pi} \left[ H_3(\Lambda,m_{\omega},r)T(\epsilon_3^i,\epsilon_1,\epsilon_4^i,\epsilon_2) \right],
$$

for $D'\bar{D}' \leftrightarrow D'\bar{D}'$. In the above equations, $C(A,B) = AB, S(A,B) = A \cdot B$ and $T(A,B) = 3A \cdot \hat{B} \cdot \hat{r} - A \cdot B$, which are the generalized central, spin-spin and tensor operators, respectively. Their matrix elements are given in Table III. $C_{\pi}(i,j)$ is the channel-dependent coefficient, and its numerical value is given in Table IV. The functions $H_0(\Lambda,m_{\rho/\omega},r), H_1(\Lambda,m_{\rho/\omega},r), H_3(\Lambda,m_{\rho/\omega},r), M_1(\Lambda,m_{\rho/\omega},r)$ and $M_3(\Lambda,m_{\rho/\omega},r)$ are given in the APPENDIX. $u^2 = m_{\rho/\omega}^2 - q_0^2$ and $\theta^0 = -(m_{\rho/\omega}^2 - q_0^2)$ with $q_0$ adopted as,

$$
D^0\bar{D}'^0 \leftrightarrow D^0\bar{D}'^0, \quad q_0 = 0, \quad D^0\bar{D}'^0 \leftrightarrow D^0\bar{D}'^0, \quad q_0 = m_{\rho/\omega} - m_{\rho/\omega}.$$
\[D^0 \bar{D}^0 \leftrightarrow D^+ D^-, \quad q_0 = 0, \quad D^0 \bar{D}^0 \leftrightarrow D^+ D^-, \quad q_0 = m_{D^0} - m_{D^0},\]
\[D^+ D^- \leftrightarrow D^+ D^-, \quad q_0 = 0, \quad D^+ D^- \leftrightarrow D^+ D^-, \quad q_0 = m_{D^+} - m_{D^-},\]
\[D^0 \bar{D}^{0*} \leftrightarrow D^0 \bar{D}^{0*}, \quad q_0 = \frac{m_{D^0} - m_{D^0}}{2}, \quad D^0 \bar{D}^{0*} \leftrightarrow D^+ D^{-*}, \quad q_0 = \frac{m_{D^0} - m_{D^{0*}}}{2},\]
\[D^+ D^{*-} \leftrightarrow D^+ D^{*-}, \quad q_0 = \frac{m_{D^+} - m_{D^{*-}}}{2}, \quad D^+ D^{*-} \leftrightarrow D^+ D^{*-}, \quad q_0 = \frac{m_{D^+} - m_{D^{*-}}}{2}.\]

(11)

For the pion exchange in the transition process \(D \bar{D}^* \leftrightarrow D^* \bar{D}, m_{D^*} - m_D > m_\pi\), which leads to the complex effective potential. Here, we take its real part [48], which has an oscillation form as shown in Eq. (8).

### TABLE III: The matrix elements of the operators appearing in Eqs. (7-10).

| \(\Delta\) | \(S(e^i_1 \times e^i_2)\) | \(T(e^i_3 \times e^i_4 \times e^i_5)\) | \(S(e^i_4 \times e^i_5 \times e^i_6)\) | \(C(e^i_2 \cdot e^i_3 \times e^i_4)\) | \(T(i e^i_3 \times e^i_4 \times e^i_5)\) | \(S(i e^i_4 \times e^i_5 \times e^i_6)\) |
|---|---|---|---|---|---|---|
| \(<^3 S^1 | \Delta | ^3 S^1>\) | 1 | 0 | 1 | 1 | 0 | -1 |
| \(<^3 S^1 | \Delta | ^3 D^1>\) | 0 | -√2 | 0 | 0 | √2 | 0 |
| \(<^3 D^1 | \Delta | ^3 S^1>\) | 0 | -√2 | 0 | 0 | √2 | 0 |
| \(<^3 D^1 | \Delta | ^3 D^1>\) | 1 | 0 | 1 | 1 | -1 | -1 |

| \(\Delta\) | \(T(e^i_3 \times e^i_4 \times e^i_5)\) | \(S(e^i_1 \times e^i_2 \times e^i_3)\) | \(T(i e^i_4 \times e^i_5 \times e^i_6)\) | \(S(i e^i_2 \times e^i_3 \times e^i_4)\) | \(T(i e^i_3 \times e^i_4 \times e^i_5)\) | \(S(i e^i_2 \times e^i_3 \times e^i_4)\) |
|---|---|---|---|---|---|---|
| \(<^3 S^1 | \Delta | ^3 S^1>\) | 0 | √2 | 0 | √2 | 0 | -√2 |
| \(<^3 S^1 | \Delta | ^3 D^1>\) | 1 | 0 | -2 | 0 | -1 | 0 |
| \(<^3 D^1 | \Delta | ^3 S^1>\) | 1 | 0 | -2 | 0 | -1 | 0 |
| \(<^3 D^1 | \Delta | ^3 D^1>\) | -√2 | √2 | √2 | -√2 | -√2 | -√2 |

### TABLE IV: The numerical values of the channel-dependent coefficients in Eqs. (7-10).

\[D^0 \bar{D}^0 \equiv \frac{1}{\sqrt{2}} \left( D^0 \bar{D}^0 - D^0 \bar{D}^0 \right), \quad [D^0 D^0] \equiv \frac{1}{\sqrt{2}} \left( D^0 D^0 - D^0 D^0 \right) \text{ and } [D^0 D^0] \equiv \frac{1}{\sqrt{2}} \left( D^0 D^0 + D^0 D^0 \right) \text{ and } [D^0 D^0] \equiv \frac{1}{\sqrt{2}} \left( D^0 D^0 + D^0 D^0 \right). \] For simplicity, we denote the channel with the form \(D \bar{D}^* \leftrightarrow D \bar{D}^*\) as the “Direct” channel and the channel with the form \(D \bar{D}^* \leftrightarrow D \bar{D}^*\) as the “Cross” channel.

| Channels | \(C_{\alpha^0}\) | \(C_{\alpha^*}\) | \(C_{\sigma}\) | \(C_{\rho^0}\) | \(C_{\rho^*}\) | \(C_{\sigma^*}\) | \(C_{\sigma}\) |
|---|---|---|---|---|---|---|---|
| \([D^0 \bar{D}^0] \leftrightarrow [D^0 \bar{D}^0]\) | Direct | \(\frac{1}{2}\) | \(\frac{1}{2}\) | 1 |
| \(\text{Cross}\) | \(-\frac{1}{2}\) | \(-\frac{1}{2}\) | \(-\frac{1}{2}\) |
| \([D^0 D^0] \leftrightarrow [D^0 D^0]\) | \(\text{Direct}\) | 1 |
| \(\text{Cross}\) | \(-1\) |
| \([D^0 D^0] \leftrightarrow [D^0 D^-]\) | \(\text{Direct}\) | \(\frac{1}{2}\) | \(\frac{1}{2}\) | 1 |
| \(\text{Cross}\) | \(-\frac{1}{2}\) | \(-\frac{1}{2}\) | \(-\frac{1}{2}\) |
| \([D^0 \bar{D}^0] \leftrightarrow [D^0 \bar{D}^-]\) | \(-\frac{1}{2}\) | \(-1\) | \(-\frac{1}{2}\) | \(-1\) | \(-\frac{1}{2}\) |
| \([D^0 D^0] \leftrightarrow [D^0 D^0]\) | \(-\frac{1}{2}\) | \(-1\) | \(-\frac{1}{2}\) | \(-1\) | \(-\frac{1}{2}\) |
| \([D^0 D^0] \leftrightarrow [D^0 D^0]\) | \(-\frac{1}{2}\) | \(-1\) | \(-\frac{1}{2}\) | \(-1\) | \(-\frac{1}{2}\) |

### III. NUMERICAL RESULTS

With the effective potentials given in the Subsection [11] we use the FORTRAN program FESSDE [49] to solve the coupled channel Schrödinger equation.
A. The Results With The OPE Potential

Since the hadronic molecule is a loosely bound state composed of hadrons, one expects that the long-range pion exchange plays a dominant role among the exchanged mesons. To highlight the contribution of the pion exchange, we first present the numerical results in the pion exchange model. Now we have only one free parameter: the cutoff value. In the deuteron case, the cutoff is fixed around 1 GeV in order to reproduce the properties of the deuteron within the same one-boson-exchange model.

We collect the numerical results, which include the binding energy (B.E.), the root-mean-square radius ($r_{rms}$) and the probability of the individual channel ($P_i$) with the pion exchange potential alone for the Cases I, II, III and IV in Table \[\text{V}\].

For Case I, we find no binding solutions with the cutoff parameter around 0.8 ~ 2.0 GeV. After adding the charged $D\bar{D}$ mode and assuming they are degenerate with the neutral mode, we obtain a loosely bound state with binding energy 0.32 MeV for the cutoff parameter being 1.55 GeV. The root-mean-square radius is 4.97 fm. The S wave is dominant, with a probability of 98.81% while the contribution of the D wave is 1.19%. When we increase the cutoff parameter to 1.80 GeV, the binding energy increases to 7.70 MeV, and the root-mean-square radius decreases to 1.36 fm. Comparison of the results of Case I with those of Case II indicates that the charged mode of $D\bar{D}$ strengthens the attraction significantly. This can be easily seen from the following simple derivation. If we only consider the neutral $D^0\bar{D}^0$, we assume the interaction strength with the pion exchange is

$$V_{[D^0\bar{D}^0\rightarrow D^0\bar{D}^0]} = V.$$ 

After adding the charged mode of $D\bar{D}$, the interaction strength with the exact isospin limit of $I = 0$ changes into

$$V_{[D^+\bar{D}^-\rightarrow D^+\bar{D}^-]} = \frac{V}{2}, \quad V_{[D^0\bar{D}^\mp\rightarrow D^0\bar{D}^\mp]} = \frac{V}{2}, \quad V_{[D^0\bar{D}^\mp\rightarrow D^0\bar{D}^\mp]} = V, \quad V_{[D^0\bar{D}^\mp\rightarrow D^0\bar{D}^\mp]} = V.$$ 

The total interaction is 3V, three times of that with only the neutral $D^0\bar{D}^0$. Actually, this has been pointed out by Close et al previously [19]. Therefore the charged mode of $D\bar{D}$ is important in the formation of a bound state, although the required cutoff parameter is larger than 1.5 GeV. This is consistent with the conclusion of Ref. [51], but somewhat different from that of Refs. [52] [53]. In [52,53], Braaten et al studied the line sharp of $X(3872)$. They conclude that at energies within only a few MeV of the $D^0\bar{D}^0$ threshold, the results with only the neutral $D^0\bar{D}^0$ is accurate but generalized to the entire $D\bar{D}$ threshold the charged $D^0\bar{D}^\pm$ plays a significant role.

In Case II, we see the role of the coupled-channel effects after we turn on the coupling of $D\bar{D}$ to $D^*\bar{D}^*$. In fact, the binding energy increases by several tens of MeV compared with Case II with the same cutoff parameter as shown in Table \[\text{V}\]. The binding energy and the root-mean-square radius of the bound state are 0.76 MeV and 3.79 fm respectively with the cutoff parameter around 1.10 GeV, which is a reasonable value. The dominant channel is still $\frac{1}{2}\left[(D^0\bar{D}^0 - D^0\bar{D}^0) + (D^0\bar{D}^0 - D^0\bar{D}^0)\right]|S_1 >$, with a probability of 97.82%. The probability of $D^0\bar{D}^0$ is small, about (1.24 + 0.20)% = 1.44%.

Since the state in Case I only contains the neutral $D\bar{D}$ mode, it is an equal superposition state of the isoscalar and isovector state. The states in Cases II and III are definitely isoscalar. Actually, none of the states in Cases I, II and III correspond to the physical state of $X(3872)$. As mentioned before, the hidden-charm di-pion decay mode of $X(3872)$ violates isospin symmetry.

In order to reproduce the physical $X(3872)$ state, we move on to Case IV and explicitly consider the mass splitting of the neutral and charged $D(D^*)$ mesons. Now the binding energy decreases by roughly 2.5 ~ 3 MeV compared to Case III with the same cutoff parameter as shown in Table \[\text{V}\]. which is an expected result since the charged $D^*\bar{D}^\mp$ pair is almost 8 MeV heavier than the $D^0\bar{D}^0$ pair. For example, the binding energy is 0.26 MeV when the cutoff parameter is 1.15 MeV. For comparison, the binding energy is 2.72 MeV in Case III with $\Lambda = 1.15$ MeV. We will show below that the flavor wave function of this very loosely bound molecular state contains a large isovector component, which decays into the $J\psi\rho$ mode. In other words, this molecular state can be interpreted as $X(3872)$.

B. The Results With The OBE Potential

Taking into account the heavier $\eta$, $\sigma$, $\omega$ and $\rho$ exchanges as well as the pion exchange, we collect the numerical results for Cases I, II, III and IV with the OBE potential in Table \[\text{V}\]. To make a rough estimation of the specific role of the exchanged meson, we plot the effective potential for Case IV when the cutoff parameter is fixed at 1.05 GeV in Figs. \[\text{2}\] and \[\text{3}\]. From Figs. \[\text{2}\] and \[\text{3}\] we know that the heavier $\eta$, $\sigma$, $\omega$ and $\rho$ exchange cancel each other to a large extent. Therefore, the pion exchange plays a dominant role in forming the loosely bound state. Although the potentials of the heavier scalar and vector meson exchange cancel each other greatly, the residual effect of the heavier meson exchange can still modify the binding solution.

Different from the OPE case, we obtain a loosely bound state with binding energy 0.21 MeV and root-mean-square radius 5.36 fm with the cutoff parameter around 1.85 GeV in Case I. In other words, the heavier scalar and vector meson exchange plays some role in the formation of the bound state. In Case II, if the cutoff parameter is fixed at 1.10 GeV, the binding energy and the root-mean-square radius of the bound state obtained are 0.61 MeV and 4.21 fm, respectively. With the OBE potential,
TABLE V: The binding solutions of X(3872) with the OPE potential. $A$ is the cutoff parameter. **“B.E.”** is the binding energy while **“Mass”** is the calculated mass of X(3872). $r_{rms}$ and $P_i$ are the root-mean-square radius and the probability of the $i$th channel, respectively. **“×”** means no binding solutions, and **“−”** denotes that the corresponding component does not exist.

| Cases | $A$(GeV) | B.E. (MeV) | Mass (MeV) | $r_{rms}$ (fm) | $P_1(\%)$ | $P_2(\%)$ | $P_3(\%)$ | $P_4(\%)$ | $P_5(\%)$ | $P_6(\%)$ |
|-------|---------|----------|-----------|---------------|-----------|-----------|-----------|-----------|-----------|-----------|
| I     | 0.80 ~ 2.0 | ×        | ×         | ×             | ×         | ×         | ×         | ×         | ×         | ×         |
|       | 1.55   | 0.32     | 3871.49   | 4.97         | 98.81     | 1.19      | -         | -         | -         | -         |
|       | 1.60   | 0.92     | 3870.89   | 3.51         | 98.39     | 1.61      | -         | -         | -         | -         |
| II    | 1.65   | 1.90     | 3869.91   | 2.56         | 98.01     | 1.99      | -         | -         | -         | -         |
|       | 1.70   | 3.31     | 3868.50   | 1.99         | 97.69     | 2.31      | -         | -         | -         | -         |
|       | 1.80   | 7.70     | 3864.11   | 1.36         | 97.18     | 2.82      | -         | -         | -         | -         |
|       | 1.10   | 0.76     | 3871.05   | 3.79         | 97.82     | 0.73      | -         | -         | 1.24      | 0.20      |
|       | 1.15   | 2.72     | 3869.09   | 2.17         | 96.15     | 0.82      | -         | -         | 2.64      | 0.40      |
| III   | 1.20   | 6.25     | 3865.56   | 1.49         | 94.26     | 0.77      | -         | -         | 4.37      | 0.60      |
|       | 1.25   | 11.66    | 3860.15   | 1.13         | 92.20     | 0.67      | -         | -         | 6.32      | 0.81      |
|       | 1.30   | 19.21    | 3852.60   | 0.91         | 90.05     | 0.55      | -         | -         | 8.38      | 1.02      |
|       | 1.55   | 95.79    | 3776.02   | 0.47         | 80.68     | 0.16      | -         | -         | 17.37     | 1.80      |
|       | 1.15   | 0.26     | 3871.55   | 4.79         | 85.68     | 0.22      | 12.29     | 0.24      | 0.36      | 0.21      |
|       | 1.17   | 1.03     | 3870.78   | 2.99         | 76.37     | 0.30      | 20.27     | 0.33      | 2.39      | 0.35      |
|       | 1.25   | 7.99     | 3863.82   | 1.20         | 56.72     | 0.32      | 35.76     | 0.34      | 6.08      | 0.79      |
|       | 1.30   | 15.36    | 3856.45   | 0.93         | 51.59     | 0.27      | 38.61     | 0.28      | 8.25      | 1.01      |

The coupling of $D\bar{D}^*$ to $D^*\bar{D}^*$ increases the binding energy by about 5 MeV. For example, if the $D^*\bar{D}^*$ component is not included, the binding energy is 0.61 MeV for the cutoff parameter fixed at 1.10 GeV. In contrast, after turning on the coupling of $D\bar{D}^*$ to $D^*\bar{D}^*$, the binding energy increases to 5.69 MeV with the same cutoff, see Cases II and III in Table V.

If we further consider the isospin breaking, we obtain a loosely bound state. When the cutoff parameter is fixed at 1.05 GeV, its mass is 3871.51 MeV, which corresponds to the experimental value of the mass of X(3872) [3, 4, 5]. The root-mean-square radius is 4.76 fm which is larger than that of the deuteron (about 2.0 fm). The dominant channel is $[D^0\bar{D}^0 - D^*\bar{D}^*] |1\bar{S}_1 >$, with a probability of 86.80%. The second dominant channel is $[D^+D^- - D^+\bar{D}^-] |2\bar{S}_1 >$, the probability of which is 11.77%. And, the total probabilities of the other channels is less than 1.5%. We plot the radial wave functions of the individual channels in Fig. 3. When we increase the cutoff parameter to 1.10 GeV, the mass of the bound state decreases to 3869.28 MeV, and the root-mean-square radius is 2.09 fm. The probability of the dominant channel decreases to 70.44% while that of the second dominant one increases to 26.46%. In order to make clear the dependence of the binding solution on the cutoff, we plot the variations of the mass and the root-mean-square radius with the cutoff in Fig. 4.

### IV. ISOSPIN BREAKING IN THE HIDDEN-CHARM DECAYS OF X(3872)

We focus on the isospin breaking in the wave function of X(3872). For simplicity, we analyze the numerical results in the OBE model for illustration. Again, we adopt the following short-hand notations, $[D^0\bar{D}^0] \equiv \frac{1}{\sqrt{2}} (D^0\bar{D}^0)$, $[D^+D^-] \equiv \frac{1}{\sqrt{2}} (D^+D^- - D^+\bar{D}^-)$, $[D^*\bar{D}^*] \equiv \frac{1}{\sqrt{2}} (D^0\bar{D}^0 + D^+\bar{D}^*)$. The flavor wave function of the $I = 1, I_z = 0$ state is $|10 > = \frac{1}{\sqrt{2}} ([D^0\bar{D}^0] + [D^+D^-])$ while that of the isoscalar state is $|00 > = \frac{1}{\sqrt{2}} ([D^0\bar{D}^0] + [D^+D^-])$.

The flavor wave function of the X(3872) can be expanded as

$$X(3872) = \frac{\chi_1(r)}{r} [D^0\bar{D}^0] |1\bar{S}_1 > + \frac{\chi_2(r)}{r} [D^0\bar{D}^0] |2\bar{D}_1 > + \frac{\chi_3(r)}{r} [D^0\bar{D}^0] |3\bar{S}_1 >$$
Finally we obtain the branching fraction ratio

\[ \text{branching fraction ratio} = \frac{B(X(3872) \rightarrow J/\psi \pi^+ \pi^- 0)}{B(X(3872) \rightarrow \pi^+ \pi^- J/\psi)} = 0.42 \]

with the binding energy being 0.3 MeV.

Again, this ratio depends very sensitively on the binding energy since the isospin breaking effect is very sensitive to the binding energy. We provide several groups of the values of \( R \) when the binding energy varies from 0.1 MeV to 1.0 MeV in Table VII.

Given the uncertainty of experimental value of the mass of \( X(3872) \), this ratio is consistent with the experimental value, 1.0 ± 0.4(stat) ± 0.3(syst) from Belle Collaboration [27] and 0.8 ± 0.3 from BABAR Collaboration [28].
TABLE VI: The binding solutions of \(X(3872)\) with the OBE potential. \(\Lambda\) is the cutoff parameter. “B.E.” is the binding energy while “Mass” is the calculated mass of \(X(3872)\). \(r_{\text{rms}}\) and “\(P_i\)” are the root-mean-square radius and the probability of the \(i\)th channel, respectively. “\(\times\)” means no binding solutions, and “\(-\)” denotes that the corresponding component does not exist.

| Cases  | \(\Lambda\) (GeV) | B.E. (MeV) | Mass (MeV) | \(r_{\text{rms}}\) (fm) | \(P_1\) (%) | \(P_2\) (%) | \(P_3\) (%) | \(P_4\) (%) | \(P_5\) (%) |
|--------|------------------|------------|------------|-----------------|-------------|-------------|-------------|-------------|-------------|
| I      | 1.85             | 0.21       | 3871.60    | 5.36            | 99.54       | 0.46        | -           | -           | -           |
|        | 1.95             | 0.53       | 3870.85    | 3.48            | 99.18       | 0.82        | -           | -           | -           |
|        | 2.00             | 1.51       | 3870.30    | 2.88            | 98.99       | 1.10        | -           | -           | -           |
| II     | 1.10             | 0.61       | 3871.20    | 4.21            | 98.82       | 1.18        | -           | -           | -           |
|        | 1.15             | 2.15       | 3869.66    | 2.54            | 98.27       | 1.73        | -           | -           | -           |
|        | 1.20             | 4.58       | 3867.23    | 1.84            | 97.28       | 2.18        | -           | -           | -           |
|        | 1.25             | 7.84       | 3863.97    | 1.48            | 97.40       | 2.60        | -           | -           | -           |
|        | 1.30             | 11.87      | 3859.94    | 1.26            | 97.01       | 2.99        | -           | -           | -           |
| III    | 1.00             | 0.74       | 3871.07    | 3.92            | 98.38       | 0.79        | -           | -           | 0.66        |
|        | 1.10             | 5.69       | 3866.12    | 1.66            | 96.39       | 1.07        | -           | -           | 1.91        |
|        | 1.15             | 9.67       | 3862.14    | 1.34            | 95.51       | 1.12        | -           | -           | 2.46        |
|        | 1.20             | 14.51      | 3857.30    | 1.15            | 94.65       | 1.15        | -           | -           | 2.94        |
|        | 1.25             | 20.18      | 3851.63    | 1.02            | 93.82       | 1.17        | -           | -           | 3.35        |
|        | 1.30             | 26.68      | 3845.13    | 0.92            | 92.98       | 1.18        | -           | -           | 3.71        |
| IV(Phy)| 1.05             | 0.30       | 3871.51    | 4.76            | 86.80       | 0.27        | 11.77       | 0.28        | 0.67        |
|        | 1.06             | 0.60       | 3871.21    | 3.85            | 82.83       | 0.33        | 15.35       | 0.34        | 0.88        |
|        | 1.08             | 1.43       | 3870.38    | 2.69            | 75.80       | 0.41        | 21.68       | 0.42        | 1.28        |
|        | 1.10             | 2.53       | 3869.28    | 2.09            | 70.44       | 0.46        | 26.46       | 0.47        | 1.62        |
|        | 1.12             | 3.84       | 3867.97    | 1.75            | 66.40       | 0.50        | 30.00       | 0.51        | 1.92        |
|        | 1.15             | 6.16       | 3865.65    | 1.46            | 62.03       | 0.53        | 33.72       | 0.54        | 2.31        |
|        | 1.20             | 10.83      | 3860.98    | 1.19            | 57.38       | 0.56        | 37.42       | 0.56        | 2.85        |

TABLE VII: The variation of the branching fraction ratio, \(R = \mathcal{B}(X(3872) \rightarrow \pi^+ \pi^- \rho^0 J/\psi)/\mathcal{B}(X(3872) \rightarrow \pi^+ \pi^- J/\psi)\), with the binding energy. “\(R_{\text{Phase}}\)” is the ratio of the phase space between \(J/\psi \omega\) and \(J/\psi \rho\). \(R_f = \rho(I = 0)/\rho(I = 1)\) is the ratio of the isoscalar and isovector component.

| B.E. (MeV) | \(R_{\text{Phase}}\) | \(R_f\) | \(R\) |
|------------|------------------|--------|------|
| 0.10       | 0.154            | 65.76/34.04 | 0.30 |
| 0.20       | 0.153            | 70.05/29.95 | 0.36 |
| 0.30       | 0.152            | 73.76/26.24 | 0.42 |
| 0.60       | 0.150            | 79.81/20.19 | 0.59 |
| 1.00       | 0.147            | 84.32/15.68 | 0.79 |

V. CONCLUSIONS

In the present work, we have performed an extensive study of the possibility of \(X(3872)\) as a \(J^{PC} = 1^{++}\) “hadronic molecule” with both the OPE and OBE potential. We have considered the contribution from the light meson exchange including \(\pi, \eta, \sigma, \rho\) and \(\omega\). It is important to note that the contribution from the heavier \(\eta, \sigma, \rho\) and \(\omega\) exchanges cancels each other to a very large extent. As a consequence, the long-range pion exchange plays a dominant role in forming the loosely bound \(X(3872)\) state.

We have considered the S-D wave mixing which plays an important role in the deuteron case, the charged \(DD^*\) mode, the coupling of \(DD^*\) to \(D^* D^*\) and the isospin breaking coming from the mass difference between the neutral and charged \(D(D^*)\) meson. All these factors play an important role in forming the loosely bound \(X(3872)\).

The inclusion of the charged \(DD^*\) mode enhances the attraction. Now there exists one loosely bound isoscalar state with a reasonable cutoff around 1.1 GeV within the OBE model. The coupling of \(DD^*\) to \(D^* D^*\) will further enhance the binding force and increase the binding energy by around 5 MeV with the same parameter. However the resulting state is still an isoscalar. If we take into account the 8 MeV mass difference between the charged and neutral \(DD^*\) pairs, the binding energy decreases by about 3 MeV. Our numerical analysis indicates that the hadronic molecule with \(J^{PC} = 1^{++}\) in Case IV can be identified as the physical \(X(3872)\) state.

If we take the binding energy of \(X(3872)\) as 0.3 MeV, the channel \(\frac{1}{2}\sum_j [D^0 \bar{D}^{*0} - D^{*0} \bar{D}^0] |^3S_1\rangle\) is dominant with a largest
probability 86.80%, while that of the channel $\frac{1}{\sqrt{2}}[D^+D^- - D^{*+}D^-]|^2 S_1$ is 11.77%. Moreover, our results indicate that there exists large isospin breaking in the flavor wave function of $X(3872)$. The isospin breaking depends strongly on the binding energy. The deeper the binding is, the smaller the isospin breaking effect becomes. When the binding energy is 0.30 MeV, the probabilities of the isoscalar and isovector components are around 73.76% and 26.24% respectively. However, they change to about 98.51% and 1.49% respectively when the binding energy increases to 10.83 MeV. The extreme sensitivity of the physical observables to the binding energy is characteristic of the very loosely bound system.

Taking into account the phase space difference as well as the isospin breaking, we obtain the branching fraction ratio $R = \mathcal{B}(X(3872) \rightarrow \pi^+\pi^-\pi^0\psi/\mathcal{B}(X(3872) \rightarrow \pi^+\pi^- J/\psi) = 0.42$ for the binding energy being around 0.3 MeV, which is in rough agreement with the current experimental measurement from Belle and Babar Collaborations.

Recently Faccini et al. have performed a study of the spin of $X(3872)$ by fitting the experimental data. In their combined fit, they excluded the $2^{++}$ hypothesis at 99.9% C.L., but return a probability of only 5.5% of the $1^{++}$ hypothesis being correct. However, in their separate fit they obtained a preference for the $1^{++}$ hypothesis in the $J/\psi\rho$ channel with a probability of 23% and an 81% preference for the $2^{--}$ assignment in the $J/\psi\omega$ channel [55].

We have also studied the possibility of the $2^{--}$ assignment of the $X(3872)$. We considered the charged mode of $DD^*$, the isospin-breaking and the coupling to $D^{*+}D^-$. But, we find no binding solutions with a reasonable cutoff parameter less than 2.0 GeV. In the $1^{++}$ case, the coupling to $D^*D^-$ increases the binding energy by a few MeV. One may also wonder about the possibility of $X(3872)$ being a deeply bound P-wave $D^*D^*$ state. Therefore we have also investigated the $D^*D^*$ channel with explicit isospin-breaking and the $P$-wave and $F$-wave mixing. We obtained a deeply bound state with mass 3871.61 MeV when we tuned the cutoff parameter to be as high as 2.642 GeV. With so large a binding energy (142.33 MeV), the isospin breaking effect almost disappears completely. Its flavor wave function is an isoscalar, which is in conflict with the experimental result. It seems that the $2^{--}$ assignment of $X(3872)$ is not favored within the present meson exchange model. Certainly the investigation of the $2^{--}$ possibility with other theoretical approaches will be very helpful to settle this issue.

In short summary, the existence of the loosely bound $X(3872)$ state and the large isospin symmetry breaking in its hidden-charm decay arises from the combined and very delicate efforts of the several driving forces including the long-range one-pion exchange, the S-D wave mixing, the mass splitting between the charged and neutral $D(D^*)$ mesons, and the coupled-channel effects.

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FIG. 3: The potentials of the different channels of $X(3872)$ with $J^{PC} = 1^{++}$ when the cutoff parameter is fixed at 1.05 GeV.
FIG. 4: (Color online) The radial wave functions of the different channels of $X(3872)$ when the binding energy is 0.30 MeV.

FIG. 5: The dependence of the binding solutions of $X(3872)$ with $J^{PC} = 1^{++}$ on the cutoff parameter. The left panel is for the mass while the right panel is for the root-mean-square radius.

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where,

\[ \beta^2 = \Lambda^2 - m^2, \quad u^2 = m^2 - Q_0^2, \quad \theta^2 = -(m^2 - q_0^2), \quad \chi^2 = \Lambda^2 - q_0^2, \]

and

\[ Y(x) = \frac{e^{-x}}{x}, \quad Z(x) = \left(1 + \frac{3}{x} + \frac{3}{x^2}\right)Y(x), \quad Z_1(x) = \left(\frac{1}{x} + \frac{1}{x^2}\right)Y(x), \quad Z_2(x) = (1 + x)Y(x). \]
Fourier transformation formulae read:
\[
\frac{1}{u^2 + q^2} \rightarrow \frac{u}{4\pi} H_0(\Lambda, m, r), \quad \frac{q^2}{u^2 + q^2} \rightarrow -\frac{u^3}{4\pi} H_1(\Lambda, m, r),
\]
\[
\frac{q}{u^2 + q^2} \rightarrow \frac{i u^3 r}{4\pi} H_2(\Lambda, m, r), \quad \frac{q_i q_j}{u^2 + q^2} \rightarrow -\frac{u^4}{12\pi} \left[ H_3(\Lambda, m, r) k_{ij} + H_1(\Lambda, m, r) \delta_{ij} \right],
\]
where, \(k_{ij} = 3 \frac{r_i r_j}{r^2} - \delta_{ij}\).

B. The Possible \(D\bar{D}^*\) molecular State With \(J^{PC} = 1^{++}\)

As a byproduct, we extend our formalism to the \(J^{PC} = 1^{++}\) case and collect the numerical results in Table VIII. The flavor wave function of the state with \(J^{PC} = 1^{++}\) (denoted as \(X'\)) is
\[
X' = \chi_1(r) \frac{1}{\sqrt{2}} (D^0 \bar{D}^0 - D^0 \bar{D}^0) |^3 P_1 > + \chi_2(r) \frac{1}{\sqrt{2}} (D^+ D^- - D^+ D^-) |^3 P_1 > + \chi_3(r) \frac{1}{\sqrt{2}} (D^0 \bar{D}^0 + D^+ D^-) |^3 P_1 >
\]
(22)
The other three channels, \(\frac{1}{\sqrt{2}} (D^0 \bar{D}^0 + D^+ D^-) |^3 P_1 >\) and \(\frac{1}{\sqrt{2}} (D^0 \bar{D}^0 + D^+ D^-) |^5 F_1 >\) have been omitted with the same reason as for the \(J^{PC} = 1^{++}\) case.

We obtain a loosely bound state with binding energy 1.60 MeV and root-mean-square radius 1.49 fm when the cutoff parameter is fixed at 1.80 GeV. The probabilities of the channels \(\frac{1}{\sqrt{2}} (D^0 \bar{D}^0 - D^0 \bar{D}^0) |^3 P_1 >\) and \(\frac{1}{\sqrt{2}} (D^+ D^- - D^+ D^-) |^3 P_1 >\) are 56.66\% and 41.06\%, respectively. However, when we increase the cutoff parameter to 1.90 GeV, the binding energy increases sharply to 23.66 MeV, and the root-mean-square radius decreases to 0.83 fm, which suggests that the results depend sensitively on the cutoff parameter.

| \(\Lambda\) (GeV) | B.E. (MeV) | Mass | \(r_{rms}\) (fm) | \(P_1(%)\) | \(P_2(%)\) | \(P_3(%)\) |
|------------------|-----------|------|----------------|----------|----------|----------|
| 1.80             | 1.60      | 3870.21 | 1.49          | 56.66    | 41.06    | 2.28     |
| 1.82             | 5.04      | 3866.77 | 1.17          | 54.05    | 43.26    | 2.69     |
| 1.84             | 8.98      | 3862.83 | 1.04          | 52.66    | 44.28    | 3.06     |
| 1.86             | 13.41     | 3858.40 | 0.95          | 51.71    | 44.87    | 3.42     |
| 1.88             | 18.30     | 3853.51 | 0.88          | 50.98    | 45.24    | 3.78     |
| 1.90             | 23.66     | 3848.15 | 0.83          | 50.39    | 45.46    | 4.14     |