A Lorentz-Violating Origin of Neutrino Mass?

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Abstract

We explore implications for neutrino physics of Very Special Relativity (VSR), wherein the symmetry group of nature includes only a 4-parameter subgroup of the Lorentz group. VSR can provide a natural origin to lepton-number conserving neutrino masses without need for sterile (right-handed) states. Neutrinoless double beta decay is forbidden if VSR is solely responsible for neutrino masses. For ultra-relativistic neutrinos, such as are ordinarily studied, VSR and conventional neutrino masses are indistinguishable. However, we show that VSR effects can be significant near the beta decay endpoint where neutrinos are not ultra-relativistic.

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Very Special Relativity (VSR) is based on the hypothesis that the space-time symmetry group of nature is smaller than the Poincaré group, consisting of space-time translations and one of certain subgroups of the Lorentz group [1]. Here we focus on implications of VSR for neutrino physics with the VSR subgroup chosen to be the 4-parameter group SIM(2).

One might expect detectable consequences of a theory lacking Lorentz invariance. As noted in [1], most of the implications of special relativity (Lorentz contraction, time dilation, isotropic propagation of light in all inertial frames, etc.) are preserved by SIM(2) invariance. Furthermore, the additional hypothesis of CP invariance promotes SIM(2) invariance to full Lorentz symmetry. Because the Lorentz invariant standard model works well, and because violations of CP are small and have not been detected in the flavor-diagonal sector, we expect VSR departures from Lorentz invariance to be tiny. In this paper, we ask whether the relevant parameter governing these effects may be related to the scale of neutrino masses.

Consider the Lorentz invariant propagation of a free massive neutrino. If its 4-component wave function $\nu$ satisfies the equation $(\slashed{p} - m_\nu)\nu = 0$, its Dirac mass is lepton-number conserving, with $\nu_L \equiv \frac{1}{2}(1 - \gamma_5)\nu$ a member of a weak doublet and the additional state $\nu_R \equiv \frac{1}{2}(1 + \gamma_5)\nu$ a sterile singlet. Alternatively, if the neutrino were to satisfy the equation $\slashed{p}\nu_L - m_\nu\nu_L^c = 0$ (where the superscript $c$ denotes charge conjugation) its Majorana mass would be lepton-number violating.

VSR admits the unconventional possibility of neutrino masses that neither violate lepton number nor require additional sterile states. As discussed in [1], the representations of the little group of SIM(2) for massive states are one dimensional. Consequently, VSR requires only two degrees of freedom for a particle carrying lepton number: one for the neutrino with lepton number 1, and one for the antineutrino with lepton number $-1$. The VSR neutrino at rest is necessarily an eigenstate of angular momentum in the preferred direction with eigenvalue $+1/2$. States with any non-zero spatial momentum may be obtained from this state by a VSR transformation. The following equation captures these features:

$$
\left( \slashed{p} - \frac{m_\nu^2}{2} \frac{\slashed{n}}{p \cdot n} \right) \nu_L = 0,
$$

where $n$ is the light-like 4-vector $(1, 0, 0, 1)$. This equation is manifestly not Lorentz invariant, but is invariant under SIM(2). To see this, recall that SIM(2), with its preferred direction chosen along the $z$ axis, is generated by $K_x + J_y$ and $K_y - J_x$, along with rotations and boosts along the $z$ direction ($J_z$ and $K_z$), where $\mathbf{J}$ and $\mathbf{K}$ are rotations and boosts, respectively. The
preferred 4-vector $n$ is changed in scale by $K_z$, but is unchanged by the other generators of SIM(2). Thus the term in (1) proportional to $m_\nu^2$ and homogeneous in $n$ is a SIM(2) invariant.

States in a massive unitary representation of SIM(2) (along with space-time translations) are labeled by the invariant length of the 4-momentum, $m$, as in the Lorentz invariant case. This implies that all massive particles, including the VSR neutrino, have a conventional dispersion relation $p^2 = m^2$. For the case at hand, this may be verified by squaring the parenthesized operator in (1) and using the fact that $n \cdot n = 0$. Thus the neutrino acquires a lepton-number conserving mass by construction, a result that cannot be obtained in a Lorentz invariant context without introducing sterile neutrinos.

We have described a single neutrino state, but we may just as well consider the physically relevant case of three active neutrino species, whereupon the parameter $m_\nu^2$ becomes an arbitrary non-negative Hermitean $3 \times 3$ matrix. Neutrino oscillation phenomena remain entirely conventional in the VSR scenario: they are described by the usual three mixing angles, one CP violating parameter, two squared-mass differences (and two essentially unobservable ‘Majorana phases’). While VSR can provide a novel origin of neutrino mass, other mechanisms may contribute as well, such as a see-saw involving heavy unobserved states. However, guided by the principle of simplicity, we adopt the tentative hypothesis that VSR is the sole or dominant origin of neutrino mass.

Equation (1) is not Lorentz invariant, implying (as we have noted) that neutrinos at rest have spins pointing in the preferred $z$ direction. However, for ultra-relativistic neutrinos ($E/m \equiv \gamma \gg 1$), the consequent Lorentz-violating effects are tiny ($\sim 1/\gamma^2$) except within a narrow cone about the $z$ axis with opening angle $\theta \sim 1/\gamma$. Under practically all realizable circumstances, no observable effects ensue. E.g. a VSR mass of 1 eV would modify the total cross-section of MeV neutrinos off electrons by mere parts per trillion. Fortunately, there is an experimentally accessible venue in which neutrinos are not ultra-relativistic: near the endpoint of the electron spectrum of beta decay, where VSR neutrinos produce effects quite different from those due to Lorentz invariant massive neutrinos.

For decades experiments have attempted to determine the electron neutrino mass by means of precision studies of the electron spectrum in beta decay near its endpoint. The most sensitive experiments yet performed provide only upper bounds to $m_\nu$, which at 95%
The proposed KATRIN experiment should be capable of detecting and measuring a conventional (Lorentz invariant) electron neutrino mass if it exceeds 0.2 eV. In view of what has been learned about neutrino masses from oscillation experiments, a positive result to the KATRIN experiment would imply that the three neutrino species are quasi-degenerate. Thus neutrino mixing effects may be neglected in the analysis of the data. Here we examine the consequences for this experiment were neutrino masses to have a purely or dominantly VSR origin.

For Lorentz invariant neutrinos, the effect of their mass on the electron spectrum is due exclusively to the kinematic restriction of phase space:

\[
\frac{dN/dE|_{\text{conv}}}{dN/dE|_{\mu} = 0} = \frac{p_\nu}{E_\nu} \tag{3}
\]

where the neutrino energy \( E_\nu \) and momentum \( p_\nu \) are determined by the electron kinetic energy \( E \):

\[
E_\nu = E_0 - E \quad \text{and} \quad p_\nu = \sqrt{E_\nu^2 - m_\nu^2}, \tag{4}
\]

with \( E_0 \approx 18.574 \text{ keV} \), the nominal endpoint, i.e. the maximum kinetic energy of the electron for a massless neutrino.

For VSR neutrinos, however, the phase space restriction is augmented by a change in the relevant matrix element. From we see that the weak leptonic charged current \( j^\mu \) must be modified to ensure its conservation:

\[
j^\mu = \bar{e}\gamma^\mu\nu_L + \frac{m_\nu^2}{2} \frac{n^\mu \hat{\psi}}{n \cdot p_e n \cdot p_\nu} \nu_L \tag{5}
\]

The unconventional second term yields a correction of order \( E_\nu/m_\nu \), which is an entirely negligible effect near the endpoint. More significantly, the first term, although not modified in form, differs from the conventional current because the neutrino spinor \( \nu_L \) is different. The form for this spinor may be found by solving \( \Pi \). The square of the matrix element for beta decay involves the VSR neutrino spinor times its conjugate:

\[
\nu_L \bar{\nu}_L = P_L \left( \phi - \frac{m_\nu^2}{2} \frac{\hat{\psi}}{p \cdot n} \right), \tag{6}
\]

in contrast to its conventional expression \( P_L \phi \). Averaging the square of the matrix element over electron spins, multiplying by the phase space, and integrating over final state angles,
we obtain

\[
\frac{dN/dE}{dN/dE|_{m_\nu=0}} = \frac{p_\nu}{E_\nu} \left( 1 - \frac{1}{4} \frac{m_\nu^2}{E_\nu p_\nu} \ln \frac{E_\nu + p_\nu}{E_\nu - p_\nu} \right) = \frac{p_\nu}{E_\nu} \left( 1 - \frac{1}{2} \frac{m_\nu^2}{E_\nu p_\nu} \phi \right),
\]

where \( \phi \) denotes the neutrino rapidity, \( \cosh \phi = E_\nu / m_\nu \). The factor in parentheses—the VSR modification of (3)—increases monotonically from 1/2 at the endpoint toward one.

Our result for \( dN/dE \) near the tritium endpoint is shown in Figure 1. The upper curve describes a massless neutrino; the middle curve a conventional 2.3 eV neutrino; and the lower curve a VSR neutrino of the same mass. Both the magnitude of the neutrino mass effect and its energy dependence are different for the two cases. In particular the present bounds on \( m_\nu \) from endpoint experiments, (2), are likely to be more stringent for VSR neutrinos.

The KATRIN experiment and its predecessors measure the \textit{integrated} energy spectrum from the endpoint downward:

\[
R(E) = \int_{E}^{E_0 - m_\nu} \frac{dN}{dE'} dE'.
\]

The effect of neutrino mass on \( R(E) \) is conveniently expressed as the difference from the
FIG. 2: The integrated endpoint difference $R_{m_\nu=0}(E) - R(E)$ for a neutrino with conventional mass 2.3 eV (lower curve), one with VSR mass of 2.3 eV (dashed curve), and one with VSR mass of 1.08 eV (middle curve).

massless case. Figure 2 shows this difference for three cases. The central curve corresponds to a conventional neutrino of mass 2.3 eV. The upper curve (truncated), corresponding to a VSR neutrino of the same mass, shows that the VSR effect on the difference is considerable. For the lower curve, we have adjusted the VSR mass to 1.08 eV so as to give an integrated difference at 50 eV indistinguishable from that for a conventional 2.3 eV neutrino.

We see from Figure 2 that the effect on the electron spectrum of a Lorentz invariant massive neutrino is similar to (but not identical with) that of a VSR neutrino with about half the mass. A future endpoint experiment that succeeds in detecting and measuring the effects of neutrino mass may also be capable of distinguishing between the conventional and VSR cases.

We have proposed an unconventional explanation for the origin of neutrino mass in terms of Very Special Relativity. In contrast to the usual Lorentz invariant scenario, VSR masses need not result from Yukawa couplings. Furthermore no neutral states must be added, such as the heavy neutrinos of see-saw models or the sterile partners of neutrinos in the
Dirac scheme. VSR neutrinos have two observable implications for neutrino physics: the absence of neutrinoless double beta decay, and a characteristic modification of the tritium endpoint spectrum that may be detectable by the next generation of endpoint experiments. Weak $SU(2)$ symmetry suggests that VSR may also have observable implications for charged leptons. The anticipated size of such effects for electrons, $m^2_\nu/m^2_e$, suggests that they may be accessible to sensitive atomic physics experiments. It would be truly remarkable if experiments at very high precision and very low energy were to provide the first indication of a failure of Lorentz invariance.

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