On $Z \to \gamma\gamma$ decay and cancellation of axial anomaly in $Z \to \gamma\gamma$
transition amplitude for massive fermions

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Abstract

$Z \to \gamma\gamma$ decay amplitude is considered and proven to be zero due to properties of polarization vectors and Bose
statistics. Triangular diagrams for a pseudoscalar $\to \gamma\gamma$ and $Z \to \gamma\gamma$ processes with massive fermions in the loop
are explicitly calculated. In the Standard Model axial anomaly vanishes in the sum of these diagrams as $Z$ boson
is mixed with one of the Goldstone bosons.

Keywords: $Z$ boson; axial anomaly; Standard Model.

1 Introduction

A couple of articles has appeared recently concerning decay of a spin-1 particle into two photons in spite of the fact
that such a decay is prohibited by Landau-Yang theorem [1, 2]. Paper [3], released in two versions, one in 2011, and
the other in 2013, considers $Z \to \gamma\gamma$ decay. The first version claimed that the decay width is not zero, albeit less
than the currently established experimental boundary. The decay width was calculated from an amplitude which does
not satisfy Ward identities for photons, apparently by applying photon polarization matrix proportional to $g_{\mu \nu}$. Since
actual photon polarization matrix contains an ambiguous term proportional to photon momentum which gives zero
only as long as Ward identities are satisfied, this calculation produced an incorrect result. It has been corrected in the
second version of paper [3], by noticing that the amplitude equals zero. However, the expression for the amplitude has
not been changed, so Ward identities remained invalid. A proper expression for the amplitude both on and off shell is
provided in section 3.2 of this work.

Another paper, [4], alerts us that we should keep considering the 126 GeV Higgs boson candidate as a particle
with spin 1 in spite of the clearly observed two photons decay of it. The paper claims that Landau-Yang theorem is
inapplicable in this case. While not addressing this statement, one can demonstrate that decay of a spin-1 particle
into two photons is impossible considering only the tensor structure of the amplitude, Bose statistics, and properties
of polarization vectors of photons and the decaying particle. This demonstration is provided in section 2.

$Z \to \gamma\gamma$ transition amplitude is a textbook example of an axial anomaly appearing in a triangle diagram. A
self-consistent theory has to be free of anomalies. In the Glashow-Weinberg-Salam theory of weak interactions with
massless fermions, due to $U(1)$ hypercharge values, anomalies cancel out when a sum of all possible fermions running in
the fermion loop is accounted for [5]. However, in the case of massive fermions, straightforward calculation of $Z \to \gamma\gamma$
amplitude shows that its derivative is proportional to a term dependent on fermion mass. Since fermion masses are
free parameters of the theory, contributions of different fermions in the loop can no longer cancel out. Nevertheless,
the Standard Model features a mechanism to keep anomalies being zero which stems from the way fermion masses
are generated—spontaneous symmetry breaking. In section 4.2 it is shown that $Z$ boson is mixed with one of the
Goldstone bosons, and the latter provides the exact value to cancel out the mass-dependent term of the derivative of
$Z \to \gamma\gamma$ amplitude. Since mass-independent terms keep cancelling out in the same way as in the massless theory, it
is concluded that in the Standard Model $Z \to \gamma\gamma$ transition amplitude is free of anomalies.

2 $Z \to \gamma\gamma$ decay

Amplitude for $Z \to \gamma\gamma$ decay $M_{Z \to \gamma\gamma}$ is proportional to the following expression:

$$M_{Z \to \gamma\gamma} \sim \epsilon_\mu \epsilon'_\nu \epsilon_\lambda T^{\mu\nu\lambda}(k, k'),$$

(2.1)

where $\epsilon$ and $\epsilon'$ are polarization vectors of photons, $\tilde{\epsilon}$ is the polarization vector of the $Z$ boson, $k$ and $k'$ are photons
momenta, and $T^{\mu\nu\lambda}(k, k')$ is the tensor corresponding to the sum of diagrams [1]

$$T^{\mu\nu\lambda}(k, k') = \tilde{T}^{\mu\nu\lambda}(k, k') + \tilde{T}^{\nu\mu\lambda}(k', k),$$

(2.2)

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where $\tilde{T}^{\mu
u\lambda}(k,k')$ is the tensor corresponding to diagram [1a].

Z boson couples with fermions via both vector and axial currents of the form

$$ (g_V \bar{\psi} \gamma^\mu \psi + g_A \bar{\psi} \gamma^\mu \gamma^5 \psi) Z_{\mu}, $$

where $g_V$ and $g_A$ are coupling constants. It follows from the Furry's theorem [6, §79], that proportional to the vector coupling part of the amplitude equals zero. Thus, we need to consider only axial coupling. The most general representation of the latter is [7]

$$ \tilde{T}^{\mu\nu\lambda}(k,k') = (A k_\alpha + A' k'_\alpha) \varepsilon^{\mu\nu\lambda\alpha} + (M k^\mu + M' k'^\mu) \varepsilon^{\mu\nu\lambda\alpha} k_\alpha' k_\beta + (N k^\nu + N' k'^\nu) \varepsilon^{\mu\nu\lambda\alpha} k'_\alpha k_\beta $$

where $A, A', M, M', N, N'$ are some functions depending on $k$ and $k'$. Substitution of (2.4) into (2.2) results in the following expression:

$$ T^{\mu\nu\lambda}(k,k') = ((A - A') k_\alpha + (A' - A) k'_\alpha) \varepsilon^{\mu\nu\lambda\alpha} + ((N - M') k^\nu + (N' - M) k'^\nu) \varepsilon^{\mu\nu\lambda\alpha} k_\alpha' k_\beta + ((M - N') k^\mu + (M' - N) k'^\mu) \varepsilon^{\mu\nu\lambda\alpha} k'_\alpha k_\beta, $$

where $A^*, A'^*, \ldots$ are $A, A', \ldots$ with $k$ and $k'$ interchanged.

The expression for the amplitude should satisfy Ward identities

$$ k_\mu T^{\mu\nu\lambda}(k,k') = 0, \quad k'_\mu T^{\mu\nu\lambda}(k,k') = 0. $$

The third Ward identity,

$$ q_\lambda T^{\mu\nu\lambda}(k,k') = 0, $$

where $q = k + k'$ is Z boson momentum, is violated, and this fact is referred to as axial anomaly. We will consider it in section 4.1. Eqs. (2.5), (2.6) provide the following relation for the coefficients:

$$ A' - A^* = (M - N^*) k^2 + (M' - N^*) k^2, $$

and another one with $k$ and $k'$ interchanged. Hence, $T^{\mu\nu\lambda}(k,k')$ has the following structure:

$$ T^{\mu\nu\lambda}(k,k') = ((N' - M^*) k_\alpha + (N - M^*) k'_\alpha) \varepsilon^{\mu\nu\lambda\alpha} + ((N - M^*) k^\nu + (N' - M^*) k'^\nu) \varepsilon^{\mu\nu\lambda\alpha} k'_{\alpha} k_{\beta} + ((M - N^*) k^\mu + (M' - N^*) k'^\mu) \varepsilon^{\mu\nu\lambda\alpha} k'_{\alpha} k_{\beta}. $$

On the mass shell $k^2 = k'^2 = 0$, and there is only one Lorentz invariant scalar which depends on one of $k$ or $k'$: $kk'$. It does not change under $k \leftrightarrow k'$ interchange, consequently, on the mass shell $M = M^*, M' = M^{*'}; \ldots$. Therefore,

$$ T^{\mu\nu\lambda}(k,k') = (N - M') (kk' \varepsilon^{\mu\nu\lambda\alpha} + (k' \varepsilon^{\mu\nu\lambda\beta} - k \varepsilon^{\mu\nu\lambda\beta}) k_{\alpha} k_{\beta}) + (N' - M) (k^\mu \varepsilon^{\mu\nu\lambda\beta} - k' \varepsilon^{\mu\nu\lambda\beta}) k'_{\alpha} k_{\beta}. $$

Let us now work in a system of coordinates where Z boson is at rest and consider relations between vectors appearing in the problem. Let the z axis be parallel to the spatial part of the photon momentum $k$. Then photon momenta can be written as follows:

$$ k = (k_0, 0, 0, k_0), \quad k' = (k_0, 0, 0, -k_0), $$

where $4k_0^2 = 2kk' = (k + k')^2 = q^2 = m_Z^2$ is Z boson mass squared. Photon polarization vectors are orthogonal to photon momenta, and can be chosen as follows:

$$ \epsilon_\mu = (0, \epsilon_1, \epsilon_2, 0), \quad \epsilon'_\mu = (0, \epsilon'_1, \epsilon'_2, 0). $$

Finally, let us take the physical polarization of Z boson:

$$ \tilde{\epsilon}_\lambda = (0, \tilde{\epsilon}_1, \tilde{\epsilon}_2, \tilde{\epsilon}_3). $$

With these equations in mind it becomes clear that substitution of (2.10) into (2.1) gives zero, because:

1 Note the absence of the $k^2 \varepsilon^{\mu\nu\lambda\beta} k_{\alpha} k_{\beta}$ term. This is due to it being linearly dependent on other terms as follows from (A.7).
1. $\epsilon_1 k^\mu = \epsilon'_1 k'^\nu = 0$,
2. $\epsilon_2 k^\mu = \epsilon'_2 k'^\nu = 0$,
3. $\epsilon_1 \epsilon'_2 \epsilon_3 \epsilon_4 (k - k')^\alpha = 0$ since the only $\alpha$ when $\epsilon_1 \epsilon'_2 \epsilon_3 \epsilon_4 (k - k')^\alpha \neq 0$ is $\alpha = 0$, but $(k - k')_0 = k_0 - k_0 = 0$.

Hence, amplitude of $Z \to \gamma \gamma$ decay is equal to zero in the $Z$ boson rest frame. Since the amplitude is Lorentz invariant, it equals zero in any other coordinate system as well, and $Z \to \gamma \gamma$ decay amplitude vanishes.

It should be stressed that the amplitude vanishes on the mass shell; it does not if one of the photons is off shell. For example, amplitude of the $Z \to \gamma e^+ e^-$ transition gives nonzero contribution to the $Z \to \gamma e^+ e^-$ decay amplitude.

### 3 Triangle diagrams

In section 4.2 we will need explicit expressions of two transition amplitudes: $Z \to \gamma \gamma$ and $\phi \to \gamma \gamma$, where $\phi$ is a pseudoscalar particle. Calculation of these amplitudes is relatively simple but tedious, and most of the necessary information is provided in any general quantum field theory course, so we will omit some intermediate steps and provide only the final results.

#### 3.1 $\phi \to \gamma \gamma$

![Triangle diagrams](image)

Figure 2: Lowest-order Feynman diagrams of $\phi \to \gamma \gamma$ transition.

The $\phi \to \gamma \gamma$ transition amplitude is equal to the sum of Feynman diagrams $\phi \to \gamma \gamma$, where $\phi$ is a pseudoscalar coupling constant between $\psi$ and fermion, $e$ and $e'$ are photon polarization vectors, $m$ is the fermion mass,

$$T^{\mu \nu}(k, k') = \tilde{T}^{\mu \nu}(k, k') + \tilde{T}^{\mu \nu}(k', k),$$

$$\tilde{T}^{\mu \nu}(k, k') = \frac{\epsilon_1 \epsilon'_2 \epsilon_3 \epsilon_4 (k - k')^\alpha}{(p - k')^2 - m^2 ((p + k')^2 - m^2)} d^4 p.$$

The trace is equal to

$$\text{tr}((p - k')^\nu (p + m) (p + k + m)^\mu) = 4im \epsilon^{\mu \alpha \beta} k'_\alpha k_\beta.$$

Calculating the integral and substituting the result into $\phi \to \gamma \gamma$, we obtain:

$$\mathcal{M}_{\phi \to \gamma \gamma} = \left[\bar{\psi} \gamma^\mu \psi\right] A \epsilon_1 \epsilon'_2 \epsilon_3 \epsilon_4 T^{\mu \nu}(k, k')$$

$$= \frac{m}{2\pi^2} \left[\bar{\psi} \gamma^\mu \psi\right] A \epsilon^{\mu \alpha \beta} \epsilon_1 \epsilon'_2 \epsilon_3 \epsilon_4 k'_\alpha k_\beta \int_0^{1-x} \frac{dz \, dx}{2x z k k' - m^2 + x (1 - x) k^2 + z (1 - z) k'^2}.$$
Let
\[ \eta_\varphi = \left( \frac{m}{m_\varphi} \right)^2, \] (3.7)
where \( m_\varphi \) is the mass of \( \varphi \). Then
\[ \mathcal{M}_{\varphi \rightarrow \gamma \gamma} = -\frac{[\bar{\psi}\gamma\psi] [\bar{\psi}\varphi\psi]}{(2\pi)^2m_\varphi} \epsilon_{\mu'\nu'\lambda'\alpha'} k'_\mu k'_\nu k'_\lambda \sqrt{\eta_\varphi} \ln^2 \left( 1 - \frac{1 + \sqrt{1 - 4\eta_\varphi}}{2\eta_\varphi} \right). \] (3.8)

This expression can be rewritten as follows:
\[ \mathcal{M}_{\varphi \rightarrow \gamma \gamma} = \frac{[\bar{\psi}\gamma\psi]^2 [\bar{\psi}\varphi\psi]}{(2\pi)^2m_\varphi} \epsilon_{\mu'\nu'\lambda'} \epsilon_{\mu\nu\lambda\alpha} k'_\mu k_\nu k_\lambda \sqrt{\eta_\varphi} \left\{ \left( \ln^2 \left( \frac{1 + \sqrt{1 - 4\eta_\varphi}}{2\eta_\varphi} - 1 \right) + \pi^2 \right) e^{-2i \arccot \frac{\sqrt{1 - 4\eta_\varphi}}{1 - 2\eta_\varphi}}, \right. \]
\[ \left. \left( \pi - \arctan \frac{\sqrt{4\eta_\varphi/1 - 1}}{1 - 2\eta_\varphi} \right)^2 \right\}, \]
\[ \arctan \frac{\sqrt{4\eta_\varphi/1 - 1}}{1 - 2\eta_\varphi}, \]
\[ \arctan \frac{2\sqrt{4\eta_\varphi/1 - 1}}{2\eta_\varphi - 1}. \] (3.9)

3.2 \( Z \rightarrow \gamma \gamma \)

\( Z \rightarrow \gamma \gamma \) transition amplitude is equal to the sum of Feynman diagrams 1a and 1b
\[ \mathcal{M}_{Z \rightarrow \gamma \gamma} = \int -\text{tr} \left( \frac{i}{p - k'} - i [\bar{\psi}\gamma\psi] [\bar{\psi}\varphi\psi] \right) \epsilon_{\mu'\nu'\lambda'} [\bar{\psi}\gamma\psi] A_{\mu'} [\bar{\gamma}\gamma] \left( \frac{d^4p}{(2\pi)^4} \right) + (\epsilon \leftrightarrow \epsilon', k \leftrightarrow k'), \] (3.10)

where \( [\bar{\psi}\gamma\psi] \) is the coupling constant between fermion and photon, \( [\bar{\psi}\gamma\psi] A_{\mu'} \) is the axial coupling constant between \( Z \) boson and fermion, \( \epsilon, \epsilon', \tilde{\epsilon} \) are polarization vectors of photons and \( Z \) boson, \( m \) is the fermion mass,
\[ T^{\mu\nu\lambda}(k, k') = \bar{T}^{\mu\nu\lambda}(k', k), \] (3.11)
\[ T^{\mu\nu\lambda}(k, k') = \int \text{tr} \left( \frac{(p - k' + m)\gamma'\gamma' + (p + k)\gamma\gamma}{(p - k')^2 - m^2} \right) \frac{d^4p}{(2\pi)^4}. \] (3.12)

The trace is evaluated with the help of (A.5). Unlike the case of (3.3), now the trace depends on integration variable \( p \). The resulting expression is carried through Feynman parameterization, and when the integration with respect to \( p \) is performed, it is convenient to eliminate terms proportional to \( k^\lambda \) and \( k'^\lambda \) with the help of (A.7). The final result is
\[ T^{\mu\nu\lambda}(k, k') = \frac{1}{4\pi^2} - \int_0^{1-\varepsilon} \int_0^1 \frac{d^4k}{2\pi^2} \left\{ e^{\mu\nu\lambda} \left[ (z^2(1 - x)k^2 - (1 - x)k'x + z(1 - x)k' + (1 - x)m^2)k_\lambda 
\right.ight.
\[ - \left( z^2(1 - x)k^2 - (1 - x)k'x + z(1 - x)k' - (1 - x)m^2)k_\lambda 
\right] + 2 \left\{ \varepsilon^{\nu\lambda\alpha\beta} (x(1 - x)k + z k'^\mu - \varepsilon^{\nu\lambda\alpha\beta}(xk + z k'^\mu)k'_\alpha k_\beta \right\} dz dx. \] (3.13)

Integration with respect to Feynman parameters \( x \) and \( z \) is performed over the region
\[ \{ x, z: x \geq 0, z \geq 0, x + z \leq 1 \}, \]
which is invariant with respect to interchange \( x \leftrightarrow z \). Consequently, integration variables can be interchanged independently of integration limits with no effect on the value of \( T^{\mu\nu\lambda}(k, k') \). Using this fact it is easy to see that
\[ \bar{T}^{\mu\nu\lambda}(k', k) = \bar{T}^{\mu\nu\lambda}(k', k), \] (3.14)

hence
\[ T^{\mu\nu\lambda}(k, k') = 2\bar{T}^{\mu\nu\lambda}(k', k'). \] (3.15)
Note that although (3.12) is superficially divergent, (3.13) is finite. Divergent terms disappeared when integration with respect to Feynman parameters was performed. Paper [3] argues that in this case there is no need to regularize the amplitude. However, in order to extract divergent terms a shift over the integration variable \( p \) was necessary. This shift introduced an ambiguous surface term, and now (3.13) does not satisfy Ward identities (2.6). To restore Ward identities we will follow the regularization procedure outlined in Ref. [8], and calculate the surface term explicitly.

Let

\[
\tilde{T}^{\mu\nu\lambda}(k, k') = \tilde{T}^{\mu\nu\lambda}(k, k') + \tilde{\Delta}^{\mu\nu\lambda}(k, k'),
\]

where \( \tilde{\Delta}^{\mu\nu\lambda}(k, k') \) is the surface term for the integral in (3.12). It is equal to \( [8] (4.21) \)

\[
\tilde{\Delta}^{\mu\nu\lambda}(k, k') = \varepsilon^{\mu\nu\lambda\alpha} \tilde{w}_\alpha,
\]

where \( \tilde{w} \) is some vector. In the problem under consideration there are only two linearly independent vectors, \( k, k' \), so \( \tilde{w} \) has to be their linear combination. Let

\[
\tilde{w}_\alpha = \tilde{a}(k, k') \kappa_\alpha + \tilde{a}'(k, k') \kappa'_\alpha,
\]

where \( \tilde{a}(k, k') \) and \( \tilde{a}'(k, k') \) are some scalars which may depend on \( kk' \), \( k' \), and in the general case are not invariant under interchange \( k \leftrightarrow k' \). \( T^{\mu\nu\lambda}(k, k') \) is then changed in the following way:

\[
T^{\mu\nu\lambda}(k, k') \rightarrow T^{\mu\nu\lambda}(k, k') + \Delta^{\mu\nu\lambda}(k, k'),
\]

where

\[
\Delta^{\mu\nu\lambda}(k, k') = \tilde{\Delta}^{\mu\nu\lambda}(k, k') + \tilde{\Delta}^{\mu\nu\lambda}(k, k') = \varepsilon^{\mu\nu\lambda\alpha} (\tilde{a}(k, k') - \tilde{a}'(k, k')) k\alpha + (\tilde{a}'(k, k') - \tilde{a}(k, k')) k'_\alpha
\]

\[
\equiv \varepsilon^{\mu\nu\lambda\alpha} (a(k, k') \kappa_\alpha + a'(k, k') \kappa'_\alpha).
\]

Note that on shell there is only one Lorentz-invariant scalar depending on either \( k \) or \( k' \), namely \( kk' \), and in this case \( \tilde{a}(k, k') = \tilde{a}(k', k) \), \( \tilde{a}'(k, k') = \tilde{a}'(k', k) \), hence \( a(k, k') = \tilde{a}(k, k') - \tilde{a}'(k, k') = -a'(k, k') \).

Imposing Ward identities (2.6) on \( T^{\mu\nu\lambda}(k, k') \) results in the following expressions for \( a(k, k') \) and \( a'(k, k') \):

\[
a(k, k') = -a'(k, k') = -\frac{1}{2\pi^2} \int_0^1 \int_0^{1-z} \frac{[x^2(1-x)k^2 + z(1-z-x)k'^2 + 2x^2zk'k' - (1-x)m^2] d\eta d\xi}{2x^2kk' - m^2 + x(1-x)k^2 + z(1-z)k'^2}.
\]

Substitution of (3.19), (3.15), (3.13) into (3.10) provides the regularized amplitude:

\[
M_{Z \to \gamma \gamma} = \left[ \tilde{\psi} \gamma \psi \right]_A \varepsilon_\mu \psi' \varepsilon_\lambda T^{\mu\nu\lambda}(k, k')
\]

\[
= \frac{1}{2\pi} \left[ \tilde{\psi} \gamma \psi \right]_A \varepsilon_\mu \psi' \varepsilon_\lambda \left( \int_0^1 \int_0^{1-z} \frac{1}{2x^2kk' - m^2 + x(1-x)k^2 + z(1-z)k'^2} \right)
\]

\[
\cdot \left[ \varepsilon^{\nu\lambda\beta} x((1-x)k + zk')\epsilon - \varepsilon^{\mu\lambda\beta} z(xk + (1-z)k')\epsilon \right] k_\alpha k_\beta
\]

\[-\varepsilon^{\mu\nu\lambda\alpha} [zk'(xk + (1-z)k')\kappa_\alpha - z((x(1-x)k)k' + zk(k')\kappa_\alpha)] d\eta d\xi.
\]

On shell \( k^2 = k'^2 = 0 \), and

\[
M_{Z \to \gamma \gamma} = \frac{1}{2\pi} \left[ \tilde{\psi} \gamma \psi \right]_A \varepsilon_\mu \psi' \varepsilon_\lambda \left( k^\mu \varepsilon^{\nu\lambda\beta} - k_\nu \varepsilon^{\mu\lambda\beta} \right) k_\alpha k_\beta \left( \int_0^1 \int_0^{1-z} \frac{2x^2kk' - m^2}{x(1-x)dz dx} + \left( k_\nu \varepsilon^{\mu\lambda\beta} - k^\nu \varepsilon^{\mu\lambda\beta} \right) k_\alpha k_\beta - \varepsilon^{\mu\nu\lambda\alpha} kk'(k - k')\kappa_\alpha \left( \int_0^1 \int_0^{1-z} \frac{2x^2kk' - m^2}{x(1-x)dz dx} \right) \right)
\]

Comparing this expression with (2.10) we obtain:

\[
N - M' = - \frac{1}{2} \int_0^1 \int_0^{1-z} \frac{2x^2kk' - m^2}{x(1-x)dz dx} = -\frac{1}{2} \left( 1 + \eta_Z \ln (1 - \frac{1 + \sqrt{1 - 4\eta_Z}}{2\eta_Z}) \right),
\]

\[
N' - M = - \int_0^1 \int_0^{1-z} \frac{2x^2kk' - m^2}{x(1-x)dz dx} = \frac{\sqrt{1 - 4\eta_Z}}{2} \ln \left( 1 + \frac{1 + \sqrt{1 - 4\eta_Z}}{2\eta_Z} - 1 \right) - 1,
\]

where

\[
\eta_Z = \left( \frac{m}{m_Z} \right)^2,
\]

\( m \) is the fermion mass, \( m_Z \) is the Z boson mass.
4 Axial anomaly in $Z \to \gamma \gamma$ transition amplitude

4.1 The Glashow-Weinberg-Salam theory of weak interactions with massless fermions

Let us briefly remind the mechanism of cancellation of anomalies in the Glashow-Weinberg-Salam theory of weak interactions [5]. We will consider one generation of fermions in unbroken $SU(2) \times U(1)$ theory leaving consideration of spontaneously broken theory to the next subsection. The lagrangian of the theory is

$$\mathcal{L} = \left( \bar{\psi} \, i \gamma^\mu \right) \left( \frac{1 + \gamma^5}{2} \right) \left( \gamma^\mu \psi \right) + \bar{\psi} \left( \frac{1 - \gamma^5}{2} \right) \gamma^\mu \psi - \frac{1}{4} F^a_{\mu \nu} F^{a \mu \nu}$$

where

$$D_{\mu L} = \partial_{\mu} - ig A^a_{\mu} t^a - ig' B_\mu Y_L, \quad L \in \{l, q\},$$

$$D_{\mu R} = \partial_{\mu} - ig' B_\mu Y_R, \quad R \in \{e, u, d\},$$

$$Y_l = -\frac{1}{2}, \quad Y_e = -1, \quad Y_q = \frac{1}{2}, \quad Y_u = \frac{2}{3}, \quad Y_d = -\frac{1}{3},$$

$$F^a_{\mu \nu} = \partial_{\mu} A^a_{\nu} - \partial_{\nu} A^a_{\mu} + g \epsilon^{abc} A^b_{\mu} A^c_{\nu}, \quad F_{\mu \nu} = \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu}. \tag{4.2}$$

It is invariant under $SU(2) \times U(1)$ transformations:

$$SU(2): \quad A^a_{\mu}(x) \rightarrow A^a_{\mu}(x) + \frac{1}{2} \partial_{\mu} \alpha^a(x) + \epsilon^{abc} A^b_{\mu}(x) \alpha^c(x), \quad \psi_L(x) \rightarrow e^{-ig'\alpha^a(x)} \psi_L(x), \quad L \in \{l, q\}, \tag{4.3}$$

$$U(1): \quad B_{\mu}(x) \rightarrow B_{\mu}(x) + \frac{1}{2} \partial_{\mu} \beta(x), \quad \psi_X(x) \rightarrow e^{-ig Y_X \beta(x)} \psi_X(x), \quad X \in \{l, q, e, u, d\}, \tag{4.4}$$

where

$$\psi_L = \frac{1}{2} \left( \begin{array}{c} \nu \\ e \end{array} \right), \quad \psi_q = \frac{u}{2} \left( \begin{array}{c} \nu \\ d \end{array} \right), \quad \psi_R = \frac{1}{2} R, \quad R \in \{e, u, d\}. \tag{4.5}$$

These transformations give rise to the following Nöther currents:

$$SU(2): \quad j^a_\lambda = \left( \bar{\psi} \, i \gamma^\mu \gamma^a \frac{1 + \gamma^5}{2} \gamma^\mu \right) \psi, \quad \psi_L \rightarrow e^{-ig'\alpha^a(x)} \psi_L(x), \quad L \in \{l, q\}, \tag{4.6}$$

$$U(1): \quad j_\lambda = \left( \bar{\psi} \, i \gamma^\mu \gamma^5 \frac{1 + \gamma^5}{2} \gamma^\mu \right) \psi, \quad \psi_X \rightarrow e^{-ig Y_X \beta(x)} \psi_X(x), \quad X \in \{l, q, e, u, d\}, \tag{4.7}$$

where terms depending on boson fields were omitted.

The switch from bare gauge bosons to states with definite masses is performed with the help of the following relations:

$$W^\pm_\mu = \frac{A^1_\mu \pm i A^2_\mu}{\sqrt{2}}, \quad Z_\mu = A^3_\mu \sin \theta - B_\mu \cos \theta, \quad A_\mu = A^3_\mu \sin \theta + B_\mu \cos \theta, \tag{4.8}$$

where

$$\cos \theta = \frac{g}{\sqrt{g^2 + g'2}}, \quad \sin \theta = \frac{g'}{\sqrt{g^2 + g'^2}}. \tag{4.9}$$

$\theta$ is the Weinberg angle or electroweak mixing angle. The current coupled to Z boson is

$$j^Z_\lambda = j^a_\lambda \cos^2 \theta - j_\lambda \sin^2 \theta, \tag{4.10}$$

or, explicitly,

$$j^Z_\lambda = \frac{1}{4} \bar{\psi} \gamma_\lambda \frac{1 + \gamma^5}{2} \nu - \left( \frac{1}{4} \cos^2 \theta - \frac{1}{4} \sin^2 \theta \right) \bar{e} \gamma_\lambda e - \frac{1}{4} \bar{e} \gamma_\lambda \gamma^5 e + \left( \frac{5}{12} \cos^2 \theta - \frac{5}{12} \sin^2 \theta \right) \bar{u} \gamma_\lambda u + \frac{1}{4} \bar{u} \gamma_\lambda \gamma^5 u - \left( \frac{3}{12} \cos^2 \theta - \frac{3}{12} \sin^2 \theta \right) \bar{d} \gamma_\lambda d - \frac{1}{4} \bar{d} \gamma_\lambda \gamma^5 d. \tag{4.11}$$

According to the Nöther’s theorem, currents (4.5), (4.10) have to be conserved, therefore, $j^Z_\lambda$ has to be conserved as well:

$$\partial^\mu j^Z_\lambda = 0. \tag{4.12}$$

Let us check this statement with explicit calculation. Consider the matrix element $\langle A(k), A(k') | j^Z_\lambda(x) | 0 \rangle$. For each term in (4.10) there is a term in the leading-order approximation of this element represented by a sum of diagrams [6].
with the corresponding fermion $\psi$ running through the loops. Terms with vector coupling between $Z$ and $\psi$ zero according to Furry’s theorem. Terms with axial coupling have the following structure:

$$\int \frac{a}{\cos \theta} e^{-i\vec{x} \cdot (A(k), A(k'))} \bar{\psi} \gamma_\lambda \gamma_5 \psi(0) \, d^4x = (2\pi)^4 \delta(4)(k + k' - q) \left[ \bar{\psi} A\psi \right]^2 V^2$$

(4.12)

where square brackets denote coupling constants between the fermion and bosons; index $V$ designates vector coupling, $A$ will be used for axial. $T^{\mu\nu\lambda}(k, k')$ is the $T^{\mu\nu}(k, k')$ tensor defined by (3.22) indexed with $\psi$ to indicate its dependence on the type of fermion in the loop. However, in the theory under consideration there is no such dependence, since all fermions are massless. The derivative of the matrix element is:

$$\frac{d}{\cos \theta} \int e^{-i\vec{x} \cdot (A(k), A(k'))} \bar{\psi} \gamma_\lambda j^\mu_Z(0) \, d^4x = (2\pi)^4 \delta(4)(k + k' - q) \cdot A_Z(A(k), A(k')),$$

(4.13)

where the factor $\frac{a}{\cos \theta}$ is extracted for convenience, and

$$A_{Z}(A(k), A(k')) = \epsilon_\mu \epsilon_\nu q_\lambda T^{\mu\nu\lambda}(k, k') \sum_\psi \left[ \bar{\psi} Z\psi \right]_A \left[ \bar{\psi} A\psi \right]^2 V,$$

(4.14)

is an anomaly. It should be zero to be consistent with (4.11). Let us check the third Ward identity (2.7). Replacing $\hat{e}_\lambda$ by $q_\lambda$ in (3.22) and noticing the similarity between the resulting expression and (4.5), we obtain:

$$q_\lambda T^{\mu\nu\lambda}(k, k') = -\frac{1}{2\pi^2} \epsilon^{\mu\nu\alpha\beta} k_\alpha k_\beta + 2m T^{\mu\nu}(k, k').$$

(4.15)

As was mentioned above, $m = 0$, so

$$q_\lambda T^{\mu\nu\lambda}(k, k') = -\frac{1}{2\pi^2} \epsilon^{\mu\nu\alpha\beta} k_\alpha k_\beta \neq 0.$$  

(4.16)

Consequently, in order for the anomaly to cancel out, the sum over fermions in (4.14) has to be zero. Using coupling constants presented in Table 1, we get:

$$\sum_\psi \left[ \bar{\psi} Z\psi \right]_A \left[ \bar{\psi} A\psi \right]^2 V = \frac{g^2}{4 \cos \theta} \cdot \left( -1 \cdot 0^2 + 1 \cdot (-1)^2 + 3 \cdot \left( -1 \cdot \left( \frac{2}{3} \right) \right)^2 + 1 \cdot \left( -\frac{1}{3} \right)^2 \right) = 0,$$

(4.17)

where the multiplier 3 takes into account the three colors of quarks.

In a similar way one can prove cancellation of anomalies for other Nöther currents as well.

| $\psi$ | $\nu$ | $e$ | $u$ | $d$ |
|-------|------|-----|-----|-----|
| $[\bar{\psi} A\psi]_V$ | 0 | $-g \sin \theta$ | $\frac{2}{3} g \sin \theta$ | $-\frac{1}{3} g \sin \theta$ |
| $[\bar{\psi} Z\psi]_A$ | $-\frac{g}{4 \cos \theta}$ | $\frac{g}{4 \cos \theta}$ | $-\frac{g}{4 \cos \theta}$ | $\frac{g}{4 \cos \theta}$ |

Table 1: Coupling constants appearing in eq. (4.14).

### 4.2 The Glashow-Weinberg-Salam theory with massive fermions

The fact that fermions are massless was crucial in the cancellation of the anomaly presented in the previous section. Let us consider now what happens when we take into account spontaneous symmetry breaking and introduce the Higgs mechanism of fermions and gauge bosons mass generation. We keep working with a single generation of fermions, but the arguments will be independent of quark mixing, and can be readily generalized to a theory with any number of generations. We also will consider only the neutral current coupled to $Z$ boson, but the same reasoning should be applicable to charged currents as well.

The Standard Model lagrangian is

$$\mathcal{L} = |D_\mu H|^2 - \frac{\lambda_1^2}{2} \left( H^\dagger H - \frac{v^2}{2} \right)^2 - \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a - \frac{1}{4} F_{\mu\nu}^\mu F_{\mu\nu}^\nu + (\bar{\nu} \cdot \bar{e}) i D_\mu \frac{1+\gamma_5}{2} \left( \frac{\mu}{\bar{e}} \right) + (\bar{\nu} \cdot \bar{e}) i D_\mu \frac{1-\gamma_5}{2} \left( \frac{\mu}{\bar{e}} \right) + (\bar{\nu} \cdot \bar{e}) i D_\mu \frac{1+\gamma_5}{2} \left( \frac{\mu}{\bar{e}} \right) + (\bar{\nu} \cdot \bar{e}) i D_\mu \frac{1-\gamma_5}{2} \left( \frac{\mu}{\bar{e}} \right) - \lambda_\epsilon (\bar{\nu} \cdot \bar{e}) H^\dagger \frac{1+\gamma_5}{2} e - \lambda_\epsilon \bar{e} H^\dagger \frac{1-\gamma_5}{2} \nu - \lambda_u (\bar{d} \cdot \bar{u}) H^\dagger \frac{1+\gamma_5}{2} u - \lambda_u \bar{u} H^\dagger \frac{1-\gamma_5}{2} \dagger - \lambda_d (\bar{d} \cdot \bar{u}) H^\dagger \frac{1+\gamma_5}{2} d - \lambda_d \bar{d} H^\dagger \frac{1-\gamma_5}{2} \dagger$$

(4.18)
where

\[ H(x) = \frac{1}{\sqrt{2}} \left( v + h(x) + i\chi(x) \right), \]  
\[ D_\mu = \partial_\mu - igA^a_\mu \gamma^a - ig'B_\mu Y, \quad Y = \frac{1}{2}. \]

This lagrangian is invariant under \( SU(2) \times U(1) \) transformations \([4.13], [4.14] \) with the following transformations for the Higgs field:

\[ SU(2): H(x) \rightarrow e^{-ig\tau^a \alpha^a(x)} H(x), \]
\[ U(1): H(x) \rightarrow e^{-igY \beta(x)} H(x). \]

Gauge currents \([4.23] \) and \([4.10] \) get additional terms which depend on \( H \):

\[ SU(2): \quad j^\mu_\mu = (\bar{\nu} \gamma \gamma^a \frac{1 + \gamma^5}{2} (\nu \gamma^a) + (\bar{u} \gamma \gamma^a \frac{1 + \gamma^5}{2} (u \gamma^a)) + i(H^\dagger \tau^a D_\mu H - (D_\mu H)^\dagger \tau^a H), \]
\[ U(1): \quad j_\mu = (\bar{\nu} \gamma Y \gamma^a \frac{1 + \gamma^5}{2} (\nu \gamma^a) + (\bar{u} \gamma Y \gamma^a \frac{1 + \gamma^5}{2} (u \gamma^a)) + \bar{u} \gamma \gamma^a \frac{1 + \gamma^5}{2} u + \bar{d} \gamma \gamma^a \frac{1 + \gamma^5}{2} d + i(H^\dagger Y D_\mu H - (D_\mu H)^\dagger Y H). \]

Explicitly, covariant derivatives have the following representation:

\[ D_{\mu L} = \left( \partial_\mu - ig\sin \theta \frac{(1 + 2Y_L)A_\mu}{2\cos \theta} - ig\cos \theta \frac{2Y_L \sin^2 \theta}{2\cos \theta} Z_\mu \right), \]
\[ D_{\mu R} = \partial_\mu - igY_R \sin \theta A_\mu + igY_R \sin^2 \theta \cos \theta Z_\mu \]

In the case of the Higgs field \( Y = \frac{1}{2} \), and

\[ D_\mu = \left( \partial_\mu - ig\sin \theta A_\mu - ig\cos \frac{2\theta}{2\cos \theta} Z_\mu - ig\frac{2\theta}{2\cos \theta} W^\mu_\mu \right), \]

Consequently, the Higgs field introduces the following additional terms in the current \([4.10] \):

\[ j^\mu_\mu = \frac{1}{2}(\bar{\nu} \gamma \gamma^a \frac{1 + \gamma^5}{2} (\nu \gamma^a) + (\bar{u} \gamma \gamma^a \frac{1 + \gamma^5}{2} (u \gamma^a)) \]
\[ - (\frac{1}{2}\cos \theta - \frac{1}{2}\sin \theta) \bar{e} \gamma \lambda e - \frac{1}{2} \bar{e} \gamma \lambda \gamma^5 e + (\frac{1}{2}\cos \theta - \frac{1}{2}\sin \theta) \bar{u} \gamma \lambda u + \frac{1}{2} \bar{u} \gamma \lambda \gamma^5 u \]
\[ + \frac{1}{4}\cos^2 \theta \bar{Z}_\lambda \delta^\mu \delta^\nu (v + h)(W^\mu_\nu \delta^\alpha + W^\nu_\mu \delta^\alpha) + \frac{g \sin \theta \cos \theta}{2} A_\mu \delta^\alpha \delta^\nu \]
\[ + \frac{1}{2} \bar{v} + h) \partial_\alpha \chi - \frac{1}{2} \chi \partial_\alpha h + \frac{1}{4}\cos \theta Z_\lambda ((v + h)^2 + \chi^2). \]

The first term in the last line, \( \frac{1}{2}(v + h)\partial_\alpha \chi \), indicates a term in the lagrangian which mixes Z boson and the Goldstone boson \( \chi \): \( \frac{1}{2} Z_\lambda \partial^\lambda \chi \). The corresponding vertex is represented in fig. [3] It results in appearance of two extra anomalous

\[ \frac{q}{\sqrt{2}} = i [Z_\lambda] \cdot (-i(-q_\lambda)) = - [Z_\lambda] q_\lambda \]

Figure 3: Vertex \( Z_\lambda \partial^\lambda \chi \).

Diagrams \[4 \] in addition to those shown in fig. [1] Anomaly of the derivative of the matrix element \( \langle A(k), A(k') | j_\mu \gamma^\mu \rangle \) in

Figure 4: Additional lowest-order Feynman diagrams of \( Z \rightarrow \gamma \gamma \) transition appearing due to vertex in fig. [5]
this case is

\[ \mathcal{A}_{Z}(A(k), A(k')) = \epsilon_{\mu}^e \epsilon_{\nu}^e q_{\lambda} \sum_{\psi} \left\{ [\bar{\psi} Z \psi]^2 A \left[ \frac{[\bar{\psi} A \psi]_V^2}{V} T_{\psi}^{\mu\nu}(k, k') + (- [Z\chi] q^{\lambda}) \cdot \frac{i}{q^2} \cdot [\bar{\psi} \chi \psi]^A [\bar{\psi} A \psi]_V^2 T_{\psi}^{\mu\nu}(k, k') \right] \right\}, \]  

(4.28)

where the first term is a sum of derivatives of diagrams in fig. [1, 14], and the second term is a sum of derivatives of diagrams in fig. [2] expressed as a derivative of a product of vertex represented in fig. [3] a propagator of \( \chi \), and a sum of diagrams in fig. [4]. \( T_{\psi}^{\mu\nu}(k, k') \) and \( T_{\psi}^{\mu\nu}(k, k') \) are defined by (3.22) and (3.3), and do depend on fermion mass. With the help of (1.13), this expression can be rewritten as follows:

\[ \mathcal{A}_{Z}(A(k), A(k')) = - \frac{1}{2\pi^2} \epsilon_{\mu\nu\alpha\beta} \epsilon_{\rho\sigma} k_{\alpha} k_{\beta} \sum_{\psi} [\bar{\psi} Z \psi]^2 A \left[ [\bar{\psi} A \psi]_V^2 \right] + \epsilon_{\mu}^e \epsilon_{\nu}^e \sum_{\psi} \left( 2 m_{\psi} \bar{\psi} [\bar{\psi} Z \psi]^2 A - i [Z\chi] [\bar{\psi} \chi \psi]^A \right) T_{\psi}^{\mu\nu}(k, k'). \]  

(4.29)

According to the previous section, the first sum is zero. Let us consider the second sum. Axial coupling constants between \( \chi \) and fermions, as well as fermion masses, follow from the last terms of the lagrangian (4.18). Keeping only the terms with \( e \) or \( \chi \), we obtain:

\[ -\lambda_e (\bar{\psi} \bar{e}) H \frac{1 - \gamma^5}{2} e - \lambda_e \bar{e} H^\dagger \frac{1 + \gamma^5}{2} e = \cdots - \lambda_e \bar{e} e + i \lambda_e \frac{1 - \gamma^5}{2} e - \lambda_e \bar{e} \frac{1 + \gamma^5}{2} e = \cdots - m_e e \bar{e} e \]  

(4.30)

where

\[ m_e = \frac{\lambda_e v}{\sqrt{2}} \]  

(4.31)

By analogy,

\[ -\lambda_d (\bar{\bar{d}} \bar{d}) H \frac{1 - \gamma^5}{2} d - \lambda_d \bar{d} H^\dagger \frac{1 + \gamma^5}{2} d = \cdots - m_d \bar{d} d - i \lambda_d \frac{1 + \gamma^5}{2} d, \quad m_d = \frac{\lambda_d v}{\sqrt{2}}, \]  

(4.32)

\[ -\lambda_u (-\bar{u} \bar{u}) H^* \frac{1 - \gamma^5}{2} u - \lambda_u \bar{u} H \frac{1 + \gamma^5}{2} u = \cdots - m_u \bar{u} u - i \lambda_u \frac{1 + \gamma^5}{2} u, \quad m_u = \frac{\lambda_u v}{\sqrt{2}}. \]  

(4.33)

Coupling constants are summarized in Table [2]. With its help we find out that the following equation holds true:

\[ 2 m_{\psi} \left[ \bar{\psi} Z \psi \right]^A_A - i [Z\chi] \left[ \bar{\psi} \chi \psi \right]^A_A = 0, \quad \psi \in \{\nu, e, u, d\}. \]  

(4.34)

Consequently, (4.29) is equal to zero, and the Z boson current is conserved.

| \( \bar{\psi} \psi \) | \( \bar{\psi} \chi \psi \) | \( \bar{\chi} \psi \) | \( \bar{\chi} \chi \psi \) |
|---|---|---|---|
| \( [\bar{\psi} Z \psi]^A_A \) | \( - \frac{g}{4 \cos \theta} \) | \( \frac{g}{4 \cos \theta} \) | \( - \frac{g}{4 \cos \theta} \) |
| \( [\bar{\psi} \chi \psi]^A_A \) | \( 0 \) | \( - \frac{m_e}{\sqrt{2} \cos \theta} \) | \( \frac{m_u}{\sqrt{2} \cos \theta} \) |
| \( [Z\chi] \) | \( \frac{g u}{4 \cos \theta} \) |

Table 2: Coupling constants appearing in eq. (4.29).

5 Conclusions

Z \( \rightarrow \gamma \gamma \) decay is once again proven to be impossible due to properties of polarization vectors of Z boson and photons and Bose statistics. Of course, it does not necessarily mean that we should stop looking for it. On the contrary, discovery of such a decay would provide evidence for physics beyond our current understanding of the quantum field theory. However, the hope is quite small, and we probably should not waste too much resources on it. In this aspect its investigation is very similar to the search of faster-than-light neutrinos or violations of CPT theorem.

To be renormalizable, a theory has to be free of axial anomalies in gauge (Nöther) currents. The mechanism of cancellation of anomalies in the Standard Model has been explicitly demonstrated. Its key features are:

- Anomaly gives two terms in an amplitude, one depends on fermion masses, and the other does not.
- Mass-independent term vanishes in the same way as in the theory with massless fermions (hypercharge values are such that the corresponding sum of coupling constants vanishes).
• Spontaneous symmetry breaking not only generates fermion masses, but also mixes Z boson with Goldstone boson. Thus fermion masses and Higgs boson vacuum expectation value appear in Goldstone boson coupling constants.

• Goldstone boson provides an additional term in the amplitude which exactly cancel out the mass-dependent term of the anomaly.

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A Trace of six Dirac matrices times $\gamma^5$

An expression for $\text{tr} \gamma^a \gamma^b \gamma^c \gamma^d \gamma^e \gamma^f \gamma^5$ can be easily derived. In four dimensions there are four different Dirac matrices, however in this expression there are six factors. Therefore, some factors have to be the same. There cannot be only two equal factors since it would require five different matrices. Let there be two pairs of equal factors, for instance, $a = b$ and $c = d$. Then

$$\text{tr} \gamma^a \gamma^b \gamma^c \gamma^d \gamma^e \gamma^f \gamma^5 = \frac{1}{4} \text{tr} \left( \{ \gamma^a, \gamma^b \} \{ \gamma^c, \gamma^d \} \gamma^e \gamma^f \gamma^5 \right) = g^{ab} g^{cd} \text{tr} \gamma^e \gamma^f \gamma^5 = 0. \quad (A.1)$$

Choice of other pairs of equal factors results only in change of the sign of the resulting expression due to swapping of gamma matrices, so the trace keeps being equal to zero. Obviously, if there are three pairs of equal factors, the result equals zero again.

Consider the case of three equal factors. First, let $d$, $e$ and $f$ be different. Then among $a$, $b$ and $c$ there are two or three equal indices. Let $a = b \neq c$. Then

$$\text{tr} \gamma^a \gamma^b \gamma^c \gamma^d \gamma^e \gamma^f \gamma^5 = g^{ab} \text{tr} \gamma^c \gamma^d \gamma^e \gamma^f \gamma^5 = 4i g^{ab} c^{def}. \quad (A.2)$$

Now, let $a = c \neq b$. Then the right hand side of $(A.2)$ is equal to zero. However, the following expression is valid in both cases:

$$\text{tr} \gamma^a \gamma^b \gamma^c \gamma^d \gamma^e \gamma^f \gamma^5 = 4i \left( g^{ab} c^{def} - g^{ac} c^{bde} \right). \quad (A.3)$$

Let $b = c \neq a$. Then, once again, right hand sides of both $(A.2)$ and $(A.3)$ equals zero, but the following equation is valid in all three cases:

$$\text{tr} \gamma^a \gamma^b \gamma^c \gamma^d \gamma^e \gamma^f \gamma^5 = 4i \left( g^{ab} c^{def} - g^{ac} c^{bde} + g^{bc} c^{ade} \right). \quad (A.4)$$

It is also valid when $a = b = c$.

Finally, let $a$, $b$ and $c$ be different. Applying the same reasoning to $d$, $e$ and $f$, and noting that the right hand side of $(A.3)$ equals zero, we get the final result

$$\text{tr} \gamma^a \gamma^b \gamma^c \gamma^d \gamma^e \gamma^f \gamma^5 = 4i \left( g^{ab} c^{def} - g^{ae} c^{bde} + g^{be} c^{ade} - g^{de} c^{abe} + g^{ef} c^{abc} \right), \quad (A.5)$$

which is valid for any combination of $\gamma$ matrices.

Moving the $\gamma^a$ matrix through the trace, a useful identity can be obtained:

$$\text{tr} \gamma^a \gamma^b \gamma^c \gamma^d \gamma^e \gamma^f \gamma^5 = 2g^{ab} \text{tr} \gamma^c \gamma^d \gamma^e \gamma^f \gamma^5 - \text{tr} \gamma^b \gamma^a \gamma^c \gamma^d \gamma^e \gamma^f \gamma^5 = \ldots$$

$$= 8i \left( g^{ab} c^{def} - g^{ae} c^{bde} + g^{be} c^{ade} - g^{de} c^{abe} + g^{ef} c^{abc} \right) + \text{tr} \gamma^b \gamma^c \gamma^d \gamma^e \gamma^f \gamma^5 \gamma^a. \quad (A.6)$$

Since the last trace is equal to the first one, the expression in the parentheses equals zero:

$$g^{ab} c^{def} - g^{ae} c^{bde} + g^{be} c^{ade} - g^{de} c^{abe} + g^{ef} c^{abc} = 0. \quad (A.7)$$

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