Extracting infrared QCD coupling from meson spectrum

M. Baldicchi, G. M. Prosperi, C. Simolo

Dipartimento di Fisica, Università di Milano
I.N.F.N., sezione di Milano
via Celoria 16, I20133 Milano, Italy

Abstract. In the framework of the Bethe-Salpeter formalism used in previous papers to evaluate the quarkonium spectrum, here we reverse the point of view to extract an “experimental” running coupling $\alpha_{\text{exp}}(Q^2)$ in the infrared (IR) region from the data. The values so obtained agree within the errors with the Shirkov-Solovtsov analytic coupling for $200 \text{ MeV} < Q < 1.2 \text{ GeV}$, thus giving a very satisfactory unifying description of high and low energy phenomena. Below 1 GeV however $\alpha_{\text{exp}}(Q^2)$ seems to vanish as $Q \to 0$. The paper is based on a work in progress in collaboration with D. V. Shirkov.

Keywords: Running coupling, QCD, meson spectrum

PACS: 12.38.Aw, 11.10.St, 12.38.Lg, 12.39.Ki

As well known a very consistent picture of the high energy processes can be obtained by perturbative QCD if the running coupling $\alpha_s(Q^2)$, as derived from the renormalization group, is used and a good convergence is already attained at 3-loop level (see e.g. [1]). In the traditional $\overline{\text{MS}}$ renormalization scheme, however, $\alpha_s(Q^2)$ develops at any loop level unphysical singularities for $Q \sim \Lambda_{\text{QCD}}$ (Landau singularities) that make the expression useless in the IR region. This is a serious difficulty in any quark model where $Q$ should be identified with the momentum transfer taking values typically between few GeV and some hundred MeV.

Among the various attempts to eliminate Landau singularities (see e.g. [2]) we consider the proposal of Shirkov and Solovtsov, which consists in imposing analyticity on $\alpha_s(Q^2)$ [3]. At 1-loop the analytic coupling can be written explicitly

$$\alpha^{(1)}_{\text{an}}(Q^2) = \frac{1}{\beta_0} \left( \frac{1}{\ln(Q^2/\Lambda^2)} + \frac{\Lambda^2}{\Lambda^2 - Q^2} \right).$$

At 2- or 3-loop level $\alpha^{(2)}_{\text{an}}(Q^2)$ can be only numerically computed. At 3-loop an useful approximation is however given by the “1-loop-like” model

$$\alpha^{(3)}_{\text{an}}(Q^2) = \frac{4\pi}{\beta_0} \left( \frac{1}{l} + \frac{1}{1 - e^l} \right), \quad \text{with} \quad l = \ln\frac{Q^2}{\Lambda^2} + \frac{\beta_1}{\beta_0} \ln\sqrt{\ln^2\frac{Q^2}{\Lambda^2} + 2\pi^2},$$

and the proper $\overline{\text{MS}}$ value for $\Lambda$. For $500 \text{ MeV} < Q < 200 \text{ GeV}$ eq. (2) differs from the exact expression by no more than 2%. Furthermore below 1 GeV $\alpha^{(3)}_{\text{an}}(Q^2)$ with $\Lambda_{n_f=3}^{(3)} = 375 \text{ MeV}$ differs even less from eq. (1) with $\Lambda_{n_f=3}^{(1)} = 206 \text{ MeV}$.

On the other side in the last years we have developed a Bethe-Salpeter formalism like [4] that was applied with a certain success to the calculation of the meson spectrum in the light and in the heavy quark sectors. The formalism was essentially derived from QCD first principles, making only an ansatz on the Wilson loop correlator $W$, which consists in writing $i\ln W$ as the sum of a one-gluon exchange and an area term encoding
MeV
3000
2000
1000
0
q\bar{q}\ (q = u, d)
\rho
\pi
q\bar{q}\ (q = u, d)
1 S_0\ 3 S_1\ 1 P_1\ 3 P_J\ 1 D_1\ 3 D_J\ 3 F_J
\rho
\pi
q\bar{q}\ (q = u, d)
1 S_0\ 3 S_1\ 1 P_1\ 3 P_J\ 1 D_1\ 3 D_J\ 3 F_J
K^*
K
q\bar{s}\ (q = u, d)

FIGURE 1. Quarkonium spectrum, three different calculations. Diamonds refer to the truncation prescription for \(\alpha_s\), squares and circles refer to the 1-loop analytic coupling (1) and two different parametrizations for constituent masses of light quarks. Lines represent experimental data.

confinement, \(i\ln W = (i\ln W)_{\text{OGE}} + \sigma S\). The resulting reduced Salpeter equation is then in the form of the eigenvalue equation for a squared bound state mass

\[ M^2 = M_0^2 + U_{\text{OGE}} + U_{\text{CF}}, \tag{3} \]

where \(M_0 = w_1 + w_2 = \sqrt{m_{P_1}^2 + k^2} + \sqrt{m_{P_2}^2 + k^2}\) and \(U = U_{\text{OGE}} + U_{\text{CF}}\) the potential \(\text{(see [4, 5] and references therein)}\). By neglecting the spin orbit and the the tensorial term but including the hyperfine splitting term, \(U_{\text{OGE}}\) has the form

\[ \langle k|U_{\text{OGE}}|k'\rangle = \rho \frac{4}{3} \frac{\alpha_s(Q^2)}{\pi^2} \left[ -\frac{1}{Q^2} \left( q_{10}q_{20} + q^2 - \frac{(Q \cdot q)^2}{Q^2} \right) + \frac{1}{6} \sigma_1 \cdot \sigma_2 \right] \tag{4} \]

where \(\rho\) is a kinematic factor. In [5] we have computed the meson masses by the equation

\[ m_a^2 = \langle \phi_a|M_0^2|\phi_a\rangle + \langle \phi_a|U_{\text{OGE}}|\phi_a\rangle + \langle \phi_a|U_{\text{CF}}|\phi_a\rangle, \]  where \(\phi_a\) is the zero-order wave function for the state \(a\) obtained by solving the eigenvalue equation for the static limit Hamiltonian \(H_{\text{CM}} = w_1 + w_2 - \frac{4}{3} \frac{\alpha_s}{\pi} + \sigma r\) by the Rayleigh-Ritz method. To this a second order correction in the hyperfine term was added in some cases.

Calculations have been performed by using both a truncation prescription for \(\alpha_s(Q^2)\) and the 1-loop analytic coupling (1). The results of three sets of calculations are graphically reported in Fig. 1 for the light-light and light-strange sectors as an example. The key point is that, while the two different assumptions on \(\alpha_s(Q^2)\) give similar results for the heavy-heavy quark states a correct reproduction of the \(\pi\) and \(K\) masses can be obtained, as it can be seen, only with the analytic coupling.

In this paper we focus our attention on the reversed point of view. \(\Lambda_{n_f=3}^{(1)}\) and quark masses have been fixed by fitting \(\pi, \rho, \phi, J/\psi\) and \(\Upsilon\) mesons, while the string tension has been fixed a priori to the value \(\sigma = 0.18\text{ GeV}^2\) (see Fig. 2). For each state \(a\) we then define a theoretical fixed coupling \(\alpha_{s,a}^{\text{th}}\) which leads to the same theoretical mass as by using \(\alpha_{s,a}^{(1)}(Q^2)\). Thus an effective momentum transfer \(Q_a\) is assigned to each state by the equation \(\alpha_{s,a}^{(1)}(Q_a^2) = \alpha_{s,a}^{\text{th}}\). We finally define \(\alpha_{s,a}^{\text{exp}}(Q_a^2)\) as the value of the coupling to be inserted in (4) in order to exactly reproduce the experimental mass:
\[ \langle \phi_a | M_0^2 | \phi_a \rangle + \alpha_s^{\text{exp}}(Q_a^2)\langle \phi_a | \mathcal{O}(q; Q) | \phi_a \rangle + \langle \phi_a | U_{\text{CF}} | \phi_a \rangle = m_{\text{exp}}^2 \mathcal{O}(q; Q) \text{ given by (4)).} \]

\[ \sigma = 0.18 \text{ GeV}^2 \]
\[ \Lambda_{n_f=3}^{(1)} = 206 \text{ MeV} \]
\[ m_u = m_d = 211 \text{ MeV} \]
\[ m_s = 306 \text{ MeV} \]
\[ m_c = 1.524 \text{ GeV} \]
\[ m_b = 4.865 \text{ GeV} \]

**FIGURE 2.** 3-loop analytic coupling with \( \Lambda_{n_f=3}^{(3)} = 375 \text{ MeV} \) and \( \alpha_s^{\text{exp}} \). Circles, stars and squares refer respectively to \( q\bar{q}, s\bar{s} \) and \( q\bar{s} \) with \( q = u, d \); diamonds and crosses stay for \( c\bar{c} \) and \( b\bar{b} \), plus signes for \( q\bar{c} \) and \( q\bar{b} \), while asterisks for \( s\bar{c} \) and \( s\bar{b} \). Error bars are drawn only if relevant.

The results are given pictorially on Fig. 2; points representing \( \alpha_s^{\text{exp}}(Q_a^2) \) are compared with the analytic curve (2) for \( \Lambda_{n_f=3}^{(3)} = 375 \text{ MeV} \). Error bars take into account the theoretical errors in the determination of the spectrum as well as the experimental ones when relevant. The theoretical incertitude expected in our procedure, that does not include coupling among different channels, is assumed to be roughly expressed by the half width of the state. As it can been seen the \( \alpha_s^{\text{exp}}(Q_a^2) \) values agree rather well with the analytic coupling expression within the quoted errors for \( 200 \text{ MeV} < Q < 1.2 \text{ GeV} \). Below 200 MeV, however, there seems to exist a consistent tendency of \( \alpha_s^{\text{exp}}(Q_a^2) \) to vanish rather than to approach a finite limit.

The present paper is based on a work in collaboration with D. V. Shirkov.

**REFERENCES**

1. W. M. Yao *et al.* [Particle Data Group], J. Phys. G 33 (2006) 1; S. Bethke, arXiv:hep-ex/0606035.
2. G. M. Prosperi, M. Raciti and C. Simolo, arXiv:hep-ph/0607209.
3. D. V. Shirkov and A. V. Zayakin, arXiv:hep-ph/0512325; D. V. Shirkov, Eur. Phys. J. C 22 (2001) 331; D. V. Shirkov and I. L. Solovtsov, Phys. Rev. Lett. 79 (1997) 1209.
4. N. Brambilla, E. Montaldi, G. M. Prosperi, Phys. Rev. D 54 (1996) 3506; G.M. Prosperi, Problems of Quantum Theory of Fields, Pag. 381, B.M. Barbashov, G.V. Efimov, A.V. Efremov Eds. JINR Dubna 1999, hep-ph/9906237.
5. M. Baldicchi and G. M. Prosperi, AIP Conf. Proc. 756 (2005) 152; Color confinement and hadrons Quantum Chromodynamics, Page. 183, H. Suganuma, *et al.* eds, World Scientific 2004, hep-ph/0310213; Phys. Rev. D 66 (2002) 074008; Phys. Rev. D 62 (2000) 114024; Fizika B 8 (1999) 2, 251; Phys. Lett. B 436 (1998) 145.