Abstract

This work deals with the possible solution of the solar neutrino problem in the framework of the resonant neutrino spin-flavor precession scenario. The event rate results from the solar neutrino experiments as well as the recoil electron energy spectrum from SuperKamiokande are used to constrain the free parameters of the neutrino in this model ($\Delta m^2$ and $\mu\nu$). We consider two kinds of magnetic profiles inside the sun. For both cases, a static and a twisting field are discussed.

1 Introduction

The amount of accurate solar neutrino data available at present, the numerous checks of the functioning of the solar neutrino detectors that have been and are being performed together with more precise results in the field of solar modeling which are in a very impressive agreement with high-accuracy helioseismological data, suggest strongly that the observed deficiency of the solar neutrino is one of the most convincing indications for new physics beyond the standard electroweak theory.

Two attractive solutions to this puzzle are (i) the Mikheyev - Smirnov - Wolfenstein (MSW) [1] matter-enhanced neutrino oscillations, and (ii) the spin precession via a large magnetic moment of the electron neutrino($\mu\nu$) proposed by Okun, Voloshin and Vysotsky (OVV)[2] motivated by the apparent anticorrelation of neutrino flux in the Davis experiment with the sunspot activity[3]. This precession is enhanced resonantly in the presence of matter yielding to a resonant spin flavor precession (RSFP)[4] analogous to the MSW effect.

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The present paper deals with the RSFP type solution to the solar neutrino problem. This solution has been advocated by many authors[5,6]. The magnitude and the profile of the sun magnetic field being the most uncertain variables, they have been assigned a variety of values and forms (for a review see [5,6]). It is worth noticing that the solar neutrino experimental results may be used to probe the solar interior (its magnetic field profile for instance) [7]. We explore the preliminary 708 days\(^1\) of operation results from the SuperKamiokande [8,9] experiment together with the last results of the four solar neutrino experiments (Homestake[10], Kamiokande[11], SAGE[12] and GALLEX[13]) for two variants of the solar magnetic field. The range for the RSFP neutrino parameters \(\Delta m^2\) and \(\mu_\nu\) compatible with the total measured event rates in all of the solar neutrino experiments, as well as with the SuperKamiokande electron recoil energy spectrum are investigated both for a static and a twisting magnetic field. The paper is organised as follows: In section 2 a brief review of the experimental situation is presented. In section 3 the neutrino propagation equation through solar matter is solved analytically using the Landau Zener[16] approximation. Hence we get the neutrino parameters allowed by the total event rates in neutrino experiments and the distortion of the recoil electron energy spectrum measured by SuperKamiokande. The evolution equation and the allowed regions for neutrino parameters are next considered in section 4 taking into account the twisting effect of the sun magnetic field. Our conclusions are summarised in section 5. Some details about the contour finding procedure are presented in the appendix.

2 Experimental status

Let us first briefly recall the experimental situation. Table 1 summarize the neutrino event rates that have been measured in the four pioneering solar neutrino experiments Homestake[10], Kamiokande[11], SAGE[12] and GALLEX[13] together with the \(^8B\) neutrino flux measured after 708 running days by SuperKamiokande[8,9]. The observed event rates are significantly smaller than the theoretical expectation of the BP98 model[17]. The GALLEX and SAGE experiments measure the same quantity, in what follows we consider only their weighted average rate. We also adopt the SuperKamiokande measurement as the most precise direct determination of the higher energy \(^8B\) neutrino flux.

3 Static field

3.1 Time evolution of the solar neutrino

Disregarding, for simplicity, possible neutrino mixing, the time evolution equation of the neutrino in the transverse magnetic field \(B\) is expressed in the Majorana\(^2\) weak interaction \((\nu_\nu, \bar{\nu}_\mu)\) sector as:

\[
\frac{d}{dt} \begin{bmatrix} \nu_e \\
\bar{\nu}_\mu \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2} G_F n_e}{3} & \mu B \\
\mu B & \frac{\Delta m^2}{2E} \end{bmatrix} \begin{bmatrix} \nu_e \\
\bar{\nu}_\mu \end{bmatrix},
\]

\(^1\)The present statistics of SuperKamiokande is about 825 days.
\(^2\)For Dirac neutrino, just replace \(\frac{5}{3}\) in the \(G_F n_e\) coefficient by \(\frac{11}{6}\).
where $\Delta m^2$ is the neutrino flavour mass square difference, and $G_F$ is the Fermi coupling constant. We used the approximation $n_n \simeq \frac{1}{6} n_e$ between the electron and neutron densities in the sun, valid in the convection and upper radiation zones where most of the neutrino trajectory lies\[14].

The electron density $n_e$ decreases exponentially along the neutrino trajectory and is well approximated by\[14].

$$G_F n_e = 2.11 \times 10^{-11} \exp \left( -\frac{r}{0.09 R_\odot} \right) \text{ eV},$$

(2)

$r$ being the distance from the center of the sun and $R_\odot$ being the solar radius.

We use the analysis of Parke\[15] to determine the average $\nu_e$ survival probability:

$$P_{\nu_e \rightarrow \nu_e} = \frac{1}{2} + \left( \frac{1}{2} - P_{LZ} \right) \cos 2\hat{\theta}_i \cos 2\hat{\theta}_f,$$

(3)

where $\hat{\theta}_i(f)$ denotes the initial(final) value of the mixing angle $\hat{\theta}$ such that,

$$\sin^2 \hat{\theta} = \frac{\mu^2 B^2}{\mu^2 B^2 + \frac{1}{4} \left\{ \frac{5G_F}{3\sqrt{2}} n_e - \frac{\Delta m^2}{2E} + \sqrt{\left(\frac{5G_F}{3\sqrt{2}} n_e - \frac{\Delta m^2}{2E}\right)^2 + 4\mu^2 B^2} \right\}^2},$$

(4)

and the jump probability $P_{LZ}$ is taken in the Landau Zener\[16] approximation

$$P_{LZ} = \exp \left[ -2\pi \frac{(\mu B)^2}{2E} \frac{1}{n_e} \frac{dn_e}{dr} \right],$$

(5)

calculated at the resonance point obtained by requiring,

$$\frac{5G_F}{3\sqrt{2}} n_e - \frac{\Delta m^2}{2E} = 0.$$  

(6)

The generalisation of eq.3 to include the neutrino energy and production range distributions is thus:

$$P_{\nu_e \rightarrow \nu_e} = \int w_E w_r \left( \frac{1}{2} + \left( \frac{1}{2} - P_{LZ} \right) \cos 2\hat{\theta}_i \cos 2\hat{\theta}_f \right) dE dr_i,$$

(7)

where the function $w_E$, $w_r$ represent respectively the probability density of neutrino production per unit energy and per unit length.

Unfortunately, the magnetic field inside the sun is not accessible to direct observation. At the moment there is no model for the solar magnetic field and very little is known about it: not only its profile is unknown, but even its strength is very uncertain. An upper limit on the strength of the solar magnetic field comes from the requirement that the field pressure must be smaller than that of matter. This is a rather weak limit (for the convective zone $B < 10^7 G$) and all other more stringent bounds are highly model dependent. Several plausible profiles have been proposed and investigated in literature.

In the following we consider two distributions of the solar magnetic field:

---

\[3\] The Landau-Zener approximation which assumes a linearly decreasing density in the vicinity of the critical point works rather well in the sun\[18\].
1. Linear distribution:

\[
B = \begin{cases} 
10^5 G & r \leq 0.7 R_{\odot} \\
10^5 G(1 - 3.33(r/R_{\odot} - 0.7)) & r > 0.7 R_{\odot}.
\end{cases}
\]  

(8)

2. Wood-Saxon (WS) distribution:

\[
B(r) = \frac{10^5 G}{1 + \exp \frac{10(r - R_{\odot})}{R_{\odot}}},
\]

(9)

3.2 Total event rates

Using only the total event rates measured at the Homestake, Gallium and Super-Kamiokande experiments, figure 1(2) shows the allowed region in the \((\mu_\nu, \Delta m^2)\) parameters space for the linear (Wood-Saxon) profile. The black dot within each allowed region indicates the position of the best fit point in the parameters space. The best fit for the linear profile is obtained for:

\[
\Delta m^2 = 1.8 \times 10^{-8} eV^2,
\]

\[
\mu = 3.9 \times 10^{-12} \mu_B,
\]

(10)

for which \(\chi^2_{min} = 1.57\). For the Wood-Saxon profile, the best fit occurs at:

\[
\Delta m^2 = 1.7 \times 10^{-8} eV^2,
\]

\[
\mu = 7.7 \times 10^{-11} \mu_B,
\]

(11)

with \(\chi^2_{min} = 0.94\).

3.3 Recoil electron energy spectrum

Unlike the neutrino event rate deficit, the energy spectrum of recoil electron observed at SuperKamiokande is one of the most important model independent solar observables. Therefore, a deviation of the observed electron recoil energy spectrum shape from what is predicted by standard electroweak theory would be an indication of new physics (such as neutrino oscillations).

The SuperKamiokande experiment[9] has measured the energy spectrum of recoil electrons from the neutrino-electron elastic scattering in water above 5.5 MeV. In what follows we will use the SuperKamiokande data relative to 708 days of operation. The data are given in an 18-bin energy histogram, where for each bin we have the ratio between the experimental event rate and the theoretical one. The first 17 bins have a width of 0.5 MeV starting from 5.5 MeV while the last bin include events with energies from 14 MeV to 20 MeV.

Fig.3 and fig.4 show (respectively for linear and W-S magnetic field profiles) the RSFP neutrino parameters regions that are allowed when we take into account the information from the SuperKamiokande recoil electron energy spectrum alone. For both profiles, a

\[\text{\footnotesize \textsuperscript{4}}\text{Since the quoted uncertainty in the Kamiokande rate is much larger than the uncertainty in the SuperKamiokande, the results are essentially unchanged if the rate from kamiokande is also considered.}\]
large region of RSFP parameters space is consistent with the data. The best fit is obtained with the linear magnetic field at:
\begin{align}
\Delta m^2 &= 3.0 \times 10^{-8} eV^2, \\
\mu &= 3.1 \times 10^{-12} \mu_B,
\end{align}
with $\chi^2_{min} = 0.90$.

For the Wood-Saxon profile the best fit occurs at:
\begin{align}
\Delta m^2 &= 1.0 \times 10^{-8} eV^2, \\
\mu &= 3.7 \times 10^{-11} \mu_B,
\end{align}
and $\chi^2_{min} = 1.08$.

Fig. 5 and fig.6 show the regions compatible with both the event rates and the information from SuperKamiokande spectrum data. Even though the best fit solution considering only the spectrum information (the dark point in figures 3 and 4) do not lie within the allowed regions by the analysis taking into account the event rate information, a large region in the neutrino parameters space ($\Delta m^2, \frac{\mu}{\mu_B}$) are consistent with constraints from the total measured event rates as well as with those from the SuperKamiokande recoil electron energy spectrum. In fig.7 we plot the recoil electron energy distribution divided by the standard prediction expected to be observed in the SuperKamiokande detector, using the best fit parameters found (eqs.10 and 11 respectively for the linear and the W-S profiles) together with the data from SuperKamiokande[9].

### 4 Twisting magnetic field

Several works proposed that the transverse component of the solar magnetic field may change its direction along the neutrino trajectory[19]. This can lead to new interesting phenomena in neutrino physics[20]. In this section we report on the observed deficit interpreted in terms of RSFP for a twisting magnetic field. We also discuss how such rotation would affect the recoil electron energy spectrum observed by the SuperKamiokande experiment.

The equation for the flavor neutrino wave function describing the propagation of neutrino in matter with twisting transverse magnetic field can be written -for the case of interest- as:
\begin{align}
\frac{i}{\hbar} \frac{d}{dt} \begin{bmatrix} \nu_e \\ \bar{\nu}_\mu \end{bmatrix} &= \begin{bmatrix} \frac{5}{3} G_F n_e + \dot{\phi} \frac{\mu B_T}{\mu B_T} \frac{\Delta m^2}{2E} \end{bmatrix} \begin{bmatrix} \nu_e \\ \bar{\nu}_\mu \end{bmatrix},
\end{align}
The angle $\phi(t)$ defines the direction of the magnetic field $B_T(t)$ in the orthogonal plane to the neutrino momentum and $B_T = |\vec{B}_T(t)|$.

The evolution equation looks like the one obtained in the static case (no twisting) with the addition of a quantity proportional to $\dot{\phi}$ to the effective matter density. The formalism sketched above for the calculus of the neutrino survival probability remains valid.

Let us assume that $\dot{\phi} \sim \frac{1}{r_0}$ where $r_0$ is the curvature radius of the magnetic field lines, then it is easy to see that the effect of the twisting field becomes significant when:
\begin{align}
\frac{1}{r_0} \sim \frac{5}{3} \sqrt{2} n_e.
\end{align}
For matter density values taken at the bottom of the convective zone of the sun, this gives:

$$r_0 \sim 0.1 R_\odot.$$  \hfill (16)

We did the same calculation as before assuming both linear and W-S profiles for $B_T(r)$ and taking for $k = R_\odot/r_0$ (a dimensionless factor which characterise the twisting velocity) the values $\pm 10$ according to the interesting feature expected from eq.16. Fig.8 shows the allowed region obtained using the linear profile while fig.9 gives the analogous result for the W-S profile.

For $k=+10$, the two field configurations used in our calculations give poor fits to the total event rates. ($\chi^2_{min} = 8.77$ and $\chi^2_{min} = 8.15$ for the W-S and the linear profiles respectively)

In contrast, when the field twists in the opposite side ($k=-10$), good fits to the total event rates are obtained with both configurations of the magnetic field.

Using the linear field distribution, the best fit is found for:

$$\Delta m^2 = 9.9 \times 10^{-11} eV^2,$$
$$\mu = 2.3 \times 10^{-12} \mu_B,$$

with a shallow $\chi^2_{min} = 0.18$. In an analogous way, we find the best fit to the data for the W-S field at:

$$\Delta m^2 = 8.4 \times 10^{-11} eV^2,$$
$$\mu = 3.4 \times 10^{-11} \mu_B,$$

with $\chi^2_{min} = 0.2$.

The most important change in the allowed range of neutrino parameters compared to the standard case (no twisting) is the disappearance of the area at the lower right corner of figs.1-2 compatible with the measured experimental event rates at the 99\%CL. The allowed regions are also extended to somewhat smaller values of $\Delta m^2$. The main reason for this is the fact that the resonant density depends on the magnitude of $\Delta m^2$ as well as on the velocity $\dot{\phi}$ which has a tendency to move it inward the sun, so for $\Delta m^2$ small enough, different solar neutrinos types can also undergo resonant transitions in both convective and radiative zones.

Fig.10 and fig.11 show the regions of neutrino parameters compatible with the SuperKamiokande spectrum of the recoil electron energy for the field distributions given by (8) and (9) and the two twisting magnitudes ($k=\pm 10$).

In the case $k=10$, better fits to the SuperKamiokande electron spectrum are obtained compared to the total event rates case. For a linear distribution we have:

$$\Delta m^2 = 1.0 \times 10^{-7} eV^2,$$
$$\mu = 2.9 \times 10^{-12} \mu_B,$$
$$\chi^2_{min} = 0.68,$$  \hfill (19)

while for a W-S distribution, the result is:

$$\Delta m^2 = 8.0 \times 10^{-8} eV^2,$$
$$\mu = 3.8 \times 10^{-11} \mu_B,$$
$$\chi^2_{min} = 0.81,$$  \hfill (20)
Good fits are also obtained for the case \((k=-10)\) and they give:

\[
\begin{align*}
\Delta m^2 &= 7.6 \times 10^{-9} eV^2, \\
\mu &= 2.7 \times 10^{-12} \mu_B, \\
\lambda_{min}^2 &= 1.45, \\
\end{align*}
\]

(21)

for the linear field and:

\[
\begin{align*}
\Delta m^2 &= 3.1 \times 10^{-9} eV^2, \\
\mu &= 4.3 \times 10^{-11} \mu_B, \\
\lambda_{min}^2 &= 1.52, \\
\end{align*}
\]

(22)

for the W-S field.

This results can be easily understood if we consider the dependence of neutrino survival probability on the twisting magnitude \(\dot{\phi}\) (see fig.12). For a given \(\Delta m^2\), at energies of the solar neutrino spectrum low enough, \(\dot{\phi}\) is very small with respect to the neutrino oscillation coefficient \(\frac{\Delta m^2}{2E}\). It follows that the twisting effect is absent at this energy scale. The twisting effect becomes significant for the highest energy part of the spectrum and depends on the sign of \(\dot{\phi}\). RSFP is amplified when \((k=-10)\) and suppressed for the opposite sign. This dependence is in favour of a positive twisting \((k=10)\) to fit the electron spectrum measured by the SuperKamiokande experiment.

Comparing the regions allowed by the total event rates shown in figs.8 and 9 with those allowed by the energy spectrum shown in Figs.10 and 11, it follows (Fig.13-14) that for \(k=+10\), none of the field distribution is able to explain the whole data from underground experiments. For the case \(k=-10\), there is a large region in \((\Delta m^2, \frac{\mu}{\mu_B})\) compatible with both constraints for the two fields. For this later case, and using the best fit parameters found (eqs.21-22), we plot in figure 15 the expected RSFP distortion of the SuperKamiokande recoil energy spectrum.

We stress that we have also studied two extreme cases corresponding to slow \((k \leq 1)\) and fast \((k \geq 100)\) twisting. While no appreciable change from the standard case (no twisting) has been found for the case of slow twisting, the highest values of \(k\) decouples the \((\nu_e, \bar{\nu}_\mu)\)system and consequently the fast twisting scenario is surely unable to explain the deficit found by the underground experiments.

5 Conclusion:

The results from the SuperKamiokande experiment opens a new area in the solar neutrino studies. Using the Landau-Zener formalism, we have investigated the RSFP way for a solution to the solar puzzle in the light of the latest experimental results as well as the theoretical predictions.

We have identified the allowed regions of neutrino parameters for either static and twisting magnetic field in the sun. This was done first by considering the constraints from total event rate results. The obtained subset of the parameters that are consistent with the total rates was then confronted to the SuperKamiokande electron recoil energy spectrum to extract possible neutrino parameter regions compatible with the whole data set. We found that the RSFP scenario can account for both the observed deficiency of solar neutrino flux and the measured SuperKamiokande spectrum. This is the case of the
static field and of the rotating one with negative $k$. Although it provides a good fit to the spectrum data, solar twisting field with positive $k$ seems to be disfavoured to explain the whole data available from underground experiments.

We should stress that the quality of the fits depends also on the chosen field configuration. Differences between the two cases as large as one order of magnitude in the $\mu$ values are found.

We emphasize that the somewhat vagueness of the conclusions drawn by this work are related to the poor knowledge we have of the field distribution and -in the case of a rotating field- of the magnitude and the sign of the rotation velocity $\dot{\phi}$.

It is crucial to have a good enough knowledge of the sun magnetic field parameters in order to draw an ultimate conclusion to the RSFP contribution to the solution of the solar neutrino puzzle.

It is also of great importance to compare the RSFP with other oscillation scenarios (i.e. the MSW, the just so and the vacuum solutions) to set the more likely solution. The present energy spectrum data are uncertain enough (especially in high energy part of the spectrum) to inhibit any clear statement. However, the growing experimental accuracy will - in a near future- allow to validate/exclude more easily the various proposed models. The possible time variation (anticorrelation with the Wolf cycle of the sun activity) that may be observed in the SK signal should also help making the comparison.

Appendix

Given the experimental rates $R_i$ and their uncertainties $\sigma_i$, the $\chi^2$ is defined as:

$$\chi^2_{\text{rates}} = \sum_{i,j} [R_{i,\text{th}} - R_{i,\text{exp}}][\sigma_{ij,\text{tot}}^2]^{-1}[R_{j,\text{th}} - R_{j,\text{exp}}],$$

(23)

where i,j label the experiment type (Ga, Cl and SK). The expected rates and the error matrix are derived following the method of [21] and

$$\sigma_{ij,\text{tot}}^2 = \sigma_{ij,\text{exp}}^2 + \sigma_{ij,\text{th}}^2,$$

(24)

$$\sigma_{ij,\text{exp}}^2 = \delta_{ij}\sigma_{i,\text{exp}}\sigma_{j,\text{exp}},$$

(25)

denoting by $\sigma_{i,\text{exp}}$, the experimental error given in table 1.

$\sigma_{ij,\text{th}}^2$ is the sum of two contributions: the one coming from uncertainties on cross sections (CS) and the one from the uncertainties on astrophysical parameters (AP)

$$\sigma_{ij,\text{th}}^2 = \sigma_{ij,\text{CS}}^2 + \sigma_{ij,\text{AP}}^2.$$  

(26)

If we write a particular rate event as: $R_i = \sum C_{ij}\phi_j$

then we have

$$\sigma_{ij,\text{CS}}^2 = \delta_{ij} \sum_{k,l=1}^8 \frac{\partial R_i}{\partial \ln C_{kj}} \frac{\partial R_i}{\partial \ln C_{lj}} \Delta \ln C_{kj}\Delta \ln C_{lj} = \delta_{ij} \sum_{k=1}^8 (R_{ik}\Delta \ln C_{ik})^2,$$

(27)

where $R_{ik} = C_{ik}\phi_k$ are the partial rates and the sum runs over the eight relevant neutrino fluxes (i.e. $p\bar{p}, p\bar{e}, n\bar{e}, 7\text{Be}, 8\text{B}, 13\text{N}, 15\text{O}$ and $17\text{F}$).
Similarly we obtain:

$$\sigma_{ij,AP}^2 = \delta_{ij} \sum_{k,l=1}^{8} R_{ik} R_{jl} \Delta \ln \phi_k \Delta \ln \phi_l.$$  \hfill (28)

The $\Delta \ln \phi_i$ are calculated using the Bahcall’s code exportrates.f [22] with some minor modifications.

The correlation matrix used in this work is:

| Experiment         | Correlation matrix |
|--------------------|--------------------|
| Ga.                | 1.000              |
| Cl.                | 0.671 1.000        |
| SuperKamiokande    | 0.687 0.964 1.000  |

On the other hand, for the study of the observed SuperKamiokande energy spectrum, we use the following $\chi^2$:

$$\chi^2_{\text{spec}} = \sum_{i,j} [\beta S_{i,th} - S_{i,exp}] [W_{ij}^2]^{-1} [\beta S_{j,th} - S_{j,exp}],$$  \hfill (29)

where $S_{i,th}$ is the predicted event rate for the $i$-th energy bin, $S_{i,exp}$ is the measured rate and $\beta$ is a free parameter which normalizes the predicted $^8B$ solar neutrino flux to the measured flux.

The input neutrino fluxes have been taken from[17]. Asymmetric errors have been conservatively symmetrized to the largest one. We use the improved neutrino cross section for each detector given in [23] and the neutrino spectra given in [24]. Allowed regions in the ($\mu$, $\Delta m^2$) plane are obtained by finding the minimum $\chi^2$ and plotting contours of constant $\chi^2 = \chi^2_{\text{min}} + \Delta \chi^2$ where $\Delta \chi^2 = 4.61$ for 90\% C.L., 5.99 for 95\% C.L. and 9.21 for 99\% C.L. All results are given as function of the reduced $\chi^2$, (i.e. $\chi^2 = \chi^2/\text{ndof}$.)

Majorana neutrinos with ($\nu_e \to \bar{\nu}_\mu$) have been used through this work. The results for Dirac neutrinos being practically identical.

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| Experiment   | Ref. | Data       | Theory     | units       |
|--------------|------|------------|------------|-------------|
| Homestake    | [10] | 2.56 ± 0.16 ± 0.16 | 7.7±1.4   | SNU         |
| Kamiokande   | [11] | 2.80±0.19±0.33    | 5.15±1.0  | 10^6 cm^-2 s^-1 |
| SAGE         | [12] | 66.6±7.8        | 129±8     | SNU         |
| GALEX        | [13] | 77.5±6.2±4.3    | 129±8     | SNU         |
| SuperKamiokande | [8,9] | 2.44±0.05±0.09 | 5.15±1.0  | 10^6 cm^-2 s^-1 |

Table 1: Neutrino event rates measured by solar neutrino experiments, and corresponding predictions from the BP98 solar model[17]. The quoted errors are at 1σ.
Figure 1: For the linear field we show the regions of parameters $\Delta m^2$ and $\frac{\mu}{\mu_B}$, obtained by a fit to the total event rates only both at 95\%C.L.(doted erea) and 99\%C.L.(solide line). The best fit is indicated by dark filled circle.
Figure 2: Same as figure 1 using the Wood Saxon profile for the solar magnetic field.
Figure 3: Considering only the SuperKamiokande energy spectrum, allowed region for neutrino parameters at 99% C.L. (solid line) and 95% C.L. (dashed line) are shown for the linear profile. The best fit is indicated by a filled circle.
Figure 4: Same as figure 3 using the Wood Saxon field.
Figure 5: For the linear field (eq. 8) we compare the allowed neutrino parameters regions obtained separately by the total event rates (solid line) and the SuperKamiokande energy spectrum (dashed line). The filled circle is the best fit to the SuperKamiokande spectrum (eq. 12) while the open circle is the best fit to the total event rates (eq. 10). The comparison is done at 99% C.L.
Figure 6: Same as in figure 5 for the Wood Saxon filed.
Figure 7: We plot the recoil energy spectra expected from RSFP scenario using the best fit parameters (eqs. 10-11) for the linear field (solid line) and Wood Saxon field (dotted line) divided by the SSM expectation. SuperKamiokande data are also shown with error bars representing the statistical and systematical errors added in quadrature. The data were directly read from M.B. Smy in [9].
Figure 8: Regions of the parameters $\Delta m^2$ and $\frac{\mu}{\mu_B}$, obtained by a fit to the total event rates only, using the field distribution given by eq.8 and the two values of k(see the text for definition). The best fit is indicated by dark circles, results are at 99%C.L. and 95%C.L.(doted area).
Figure 9: Same as in figure 8 for the solar field given by eq.9
Figure 10: Considering only the SuperKamiokande data spectrum, the allowed regions for neutrino parameters at 99\%C.L. (solid line) and 95\%C.L.(dashed line) are shown for the linear profile and two values of k.
Figure 11: Same as in figure 10 using the Wood Saxon field.
Figure 12: Neutrino survival probability computed using the linear(a) and the Wood Saxon(b). The neutrino parameters are those of the static best fit found in eqs.10-11. The solid lines correspond to the static case. Doted and dash-doted lines refer to the rotated case respectively with \( k = -10 \) and \( k = +10 \).
Figure 13: For the linear field (eq. 8) and the two values of $k$, we compare the allowed neutrino parameters regions obtained by considering only constraints coming from the total event rates (figure 8) with those coming from the SuperKamiokande data spectrum (figure 10).
Figure 14: For the Wood Saxon field (eq.9) and the two values of $k$, we compare the allowed neutrino parameters regions obtained by considering only constraints coming from the total event rates (figure 9) with those coming from the SuperKamiokande data spectrum (figure 11).
Figure 15: As in figure 7, for the linear field (solid line) and Wood Saxon field (dotted line) and for k=-10, we plot the recoil electron energy spectra from RSFP scenario using the best fit parameters (eq.21 and eq.22) divided by the SSM prediction.