INDEPENDENT LOW-RANK MATRIX ANALYSIS BASED ON COMPLEX STUDENT’S T-DISTRIBUTION FOR BLIND AUDIO SOURCE SEPARATION

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ABSTRACT

In this paper, we generalize a source generative model in a state-of-the-art blind source separation (BSS), independent low-rank matrix analysis (ILRMA). ILRMA is a unified method of frequency-domain independent component analysis and nonnegative matrix factorization and can provide better performance for audio BSS tasks. To further improve the performance and stability of the separation, we introduce an isotropic complex Student’s t-distribution as a source generative model, which includes the isotropic complex Gaussian distribution used in conventional ILRMA. Experiments are conducted using both music and speech BSS tasks, and the results show the validity of the proposed method.

Index Terms— Blind source separation, nonnegative matrix factorization, independent component analysis, Student’s t-distribution, generative model

1. INTRODUCTION

Blind source separation (BSS) is a technique for extracting specific sources from an observed multichannel mixture signal without knowing a priori information about the mixing system. The most popular algorithm for BSS is called independent component analysis (ICA) [1], which assumes statistical independence between the sources and estimates the demixing matrix. In particular, BSS for audio signals has been well studied. For a mixture of audio signals, since the sources are convolved owing to the room reverb-eration, ICA is often applied to the time-frequency domain signal, which is called the spectrogram obtained by a short-time Fourier transform (STFT). Frequency-domain ICA (FDICA) [2, 3] independently applies ICA to the time-series signals in each frequency, then the permutation of the estimated signals is aligned on the basis of several criteria. As an elegant solution of this permutation alignment problem, independent vector analysis (IVA) [4] was proposed, which assumes higher-order dependencies among the frequency components in each source, thus avoiding the permutation problem. In [5], fast and stable optimization of IVA (AuxIVA) was derived using an auxiliary function technique that is also known as a majorization-minimization (MM) algorithm [6].

As another means of audio source separation, nonnegative matrix factorization (NMF) [7] has been a very popular approach during the last decade. NMF is a parts-based decomposition (low-rank approximation) of a nonnegative data matrix, which is typically a power or amplitude spectrogram, and the significant parts (bases and activations) can be used for source separation. Also, NMF can be statistically interpreted as a parameter estimation based on a generative model of data, and the distribution of the model defines a cost function (divergence) in NMF. For example, it was revealed that NMF based on Itakura–Saito divergence (ISNMF) assumes an isotropic complex Gaussian distribution independently defined in each time-frequency slot [8]. Recently, a new NMF based on an isotropic complex Cauchy distribution (Cauchy NMF) [9] and its generalization, NMF based on a complex Student’s t-distribution (t-NMF) [10], have been proposed. t-NMF includes both ISNMF and Cauchy NMF as special cases, and it has been reported that t-NMF provides better and more stable source separation for simple audio signals [10].

For multichannel audio source separation, NMF has been extended to multichannel NMF (MNMF) [11, 12, 13]. MNMF employs a sourcewise spatial parameter, spatial covariance, that approximates the mixing system to achieve source separation. However, the separation performance of MNMF strongly depends on the initialization of the parameters because of the difficulty of the optimization. This problem was addressed by exploiting a complex Student’s t-distribution as a source generative model in MNMF (t-MNMF) [14], which may lead to initialization-robust optimization.

NMF has been unified with the conventional ICA- or IVA-based techniques, which allows us to simultaneously model the sourcewise time-frequency structure and the statistical independence between sources. This state-of-the-art BSS is called independent low-rank matrix analysis (ILRMA) [15, 16], which is a natural extension of IVA from a vector to a low-rank matrix source model. ILRMA is equivalent to a special case of MNMF; ILRMA assumes that the mixing system is invertible and estimates the demixing system similarly to FDICA or IVA, whereas MNMF estimates the mixing system (spatial covariance) required for separation. For the optimization problem, ILRMA is much faster and more stable than MNMF. In this paper, we generalize the source generative model in ILRMA from the complex Gaussian distribution to the complex Student’s t-distribution, which is expected to further improve the performance and stability of the parameter initialization. The relationship among the conventional methods and the proposed ILRMA is depicted in Fig. 1. As shown in this figure, the proposed ILRMA can be referred to as a new extension of conventional ILRMA as well as a computationally efficient solution for the dual problem of t-MNMF under a spatially rank-1 condition.

2. CONVENTIONAL METHODS

2.1. Formulation

Let N and M be the numbers of sources and channels, respectively. The complex-valued source, observed, and estimated signals are defined as \( \mathbf{s}_{ij} = (s_{i1}, \cdots, s_{iN})^T \), \( \mathbf{x}_{ij} = (x_{i1}, \cdots, x_{iM})^T \), and \( \mathbf{y}_{ij} = \)
ILRMA assumes the following time-varying distribution as the generative model of each source:

\[
\prod_{i,j} p(y_{ij} | x_{ij}) = \prod_{i,j} \frac{1}{r_{ij}} \exp \left( -\frac{|y_{ij}|^2}{r_{ij}} \right),
\]

where \( r_{ij} \) is a time-frequency-varying nonnegative variance and corresponds to the expectation of the power of \( y_{ij} \), i.e., \( r_{ij} = \mathbb{E}[|y_{ij}|^2] \). This is because \( p(y_{ij}) \) is isotropic in the complex plane. Moreover, \( t_{ij} \) and \( v_{ij} \) are the NMF parameters called basis and activation, respectively, and \( L \) is the integral index of the frequency bins, time frames, sources, and channels, respectively.

2.2. ILRMA

ILRMA assumes the following time-varying distribution as the generative model of each source:

\[
\log r_{ij} = \sum_{t} t_{ij} v_{ij},
\]

where \( r_{ij} \) is a time-frequency-varying nonnegative variance and corresponds to the expectation of the power of \( y_{ij} \), i.e., \( r_{ij} = \mathbb{E}[|y_{ij}|^2] \). This is because \( p(y_{ij}) \) is isotropic in the complex plane. Moreover, \( t_{ij} \) and \( v_{ij} \) are the NMF parameters called basis and activation, respectively, and \( L \) is the integral index of the frequency bins, time frames, sources, and channels, respectively. The generative model in \( t \)-NMF provides better and more robust source separation for simple audio signals when \( y \) is approximately narrow-band.

3. PROPOSED METHOD

3.1. ILRMA based on complex Student’s \( t \)-distribution

Motivated by the improvements in \( t \)-NMF, we propose the introduction of a complex Student’s \( t \)-distribution as a source generative model in ILRMA (t-ILRMA), which is a generalization of conventional Gaussian ILRMA based on \( t \)-distribution. The generative model in \( t \)-distribution is given by

\[
L = \text{const.} - 2J \sum_{t} \log |\mathbf{W}| + \sum_{t,j} \left( \log r_{ij} + \frac{|y_{ij}|^2}{r_{ij}} \right).
\]
ILRMA is as follows:

\[
\prod_{i,j} p(y_{ij|a}) = \prod_{i,j} \frac{1}{\alpha_{ij}^\sigma} \left(1 + \frac{2 |y_{ij|a}|^2}{\sigma_{ij}}\right)^{-\frac{1}{\sigma}}, \tag{6}
\]

where the local distribution \(p(y_{ij|a})\) is defined as an isotropic complex Student’s t-distribution, \(\sigma_{ij}\) is a time-frequency-varying non-negative scale and corresponds to an amplitude spectrum \(|y_{ij|a}|\), and \(p\) is a parameter that defines the domain of the NMF model \(\mathbf{T}_t \mathbf{V}_n\) and should satisfy \(1 \leq p \leq 2\). When \(\nu \rightarrow \infty\) and \(p = 2\), \(\sigma_{ij}\) corresponds to the generative model in ISNMF, and when \(\nu = 1\) and \(p = 1\), \(\sigma_{ij}\) corresponds to the generative model in Cauchy NMF. The negative log-likelihood function based on \(\sigma_{ij}\) can be obtained as follows by assuming independence between each source and each time frame:

\[
\mathcal{L}_i = \text{const.} - 2J \sum_{j} \log |\det \mathbf{W}| + \sum_{i,j} \left(1 + \frac{\nu}{2}\right) \log \left(1 + \frac{2 |y_{ij|a}|^2}{\nu \sigma_{ij}^2}\right) + 2 \log \sigma_{ij}. \tag{8}
\]

When \(\nu \rightarrow \infty\) and \(p = 2\), \(\mathcal{L}_i\) coincides with \(\mathcal{L}_i\).

### 3.2. Derivation of update rules for demixing matrix

Similar to the derivation described in [15], we apply an MM algorithm and iterative projection (IP) [5] to derive the update rules for the demixing matrix \(\mathbf{W}\) with a full guarantee of the monotonic convergence. IP was the method originally used to solve the simultaneous vector equations in AuxIVA, which are equivalent to the HEAD problem [17]. Unlike the conventional MNMF methods such as that in [14] that estimate the mixing matrix \(\mathbf{A}\) (not the demixing matrix \(\mathbf{W}\)), IP can lead to much faster and more stable estimation of \(\mathbf{W}\) in BSS, as reported in [15][5]. However, the major drawback of IP is the limited number of applicable functions; i.e., generally the term \(|y_{ij|a}|^2 = |\mathbf{w}_i^H \mathbf{x}_j|^2\) should appear as is in the objective function, e.g., in [5] (should not appear as a part of variable inside a nonlinear function).

For the \(\ell_1\)-ILRMA's cost function \(\mathcal{L}_i\), whose \(|y_{ij|a}|^2\) term is intrinsic, as a trick to enable the introduction of IP, we apply a tangent line inequality to the logarithmic terms in \(\mathcal{L}_i\). The tangent line inequality can be represented as

\[
\log \left(\sum_{q} z_q \right) \leq \frac{1}{4} \left(\sum_{q} z_q - \lambda\right) + \log \lambda, \tag{9}
\]

where \(z_q\) is the original variable and \(\lambda > 0\) is an auxiliary variable. The equality of \(\mathcal{L}_i\) holds if and only if \(\lambda = \sum_{q} z_q\). By applying \(\mathcal{L}_i\) to the second and third logarithm terms in \(\mathcal{L}_i\), the following majorization function can be designed:

\[
\mathcal{L}_i^+ = \text{const.} - 2J \sum_{j} \log |\det \mathbf{W}| + \sum_{i,j} \left(1 + \frac{\nu}{2}\right) \log \left(1 + \frac{2 |y_{ij|a}|^2}{\nu \sigma_{ij}^2}\right) + 2 \log \sigma_{ij} + \frac{2}{p} \log \beta_{ij}, \tag{10}
\]

where \(\sigma_{ij} = (\sum_i t_{ij} v_{ij})^{1/p}\) is partly substituted, \(\alpha_{ij}, \beta_{ij} > 0\) are auxiliary variables, and \(\mathcal{L}_i^+\) and \(\mathcal{L}_i^+\) become equal only when

\[
\alpha_{ij} = 1 + \frac{2 |y_{ij|a}|^2}{\nu \sigma_{ij}^2}, \tag{11}
\]

\[
\beta_{ij} = \sum_i t_{ij} v_{ij}. \tag{12}
\]

Because \(|y_{ij|a}|^2 = |\mathbf{w}_i^H \mathbf{x}_j|^2\) in \(\mathcal{L}_i^+\) exists outside the logarithmic function, we can apply IP in analogy with the derivation in conventional ILRMA using \(\sigma_{ij}\). The majorization function \(\mathcal{L}_i^+\) can be reformulated as

\[
\mathcal{L}_i^+ = \text{const.} - 2J \sum_{j} \log |\det \mathbf{W}| + J \sum_i \mathbf{w}_{ij}^H \mathbf{U}_i \mathbf{w}_{ij} + \sum_{i,j} \left(1 + \frac{\nu}{2}\right) \left(\alpha_{ij} - 1 + \log \alpha_{ij}\right) + \frac{2}{p} \log \beta_{ij} \left(\sum_i t_{ij} v_{ij} - \beta_{ij}\right), \tag{13}
\]

\[
\mathbf{U}_j = \frac{1}{J} \left(\frac{2}{\nu} + 1\right) \sum_i \frac{1}{\alpha_{ij} \sigma_{ij}^2} \mathbf{x}_j \mathbf{x}_j^H. \tag{14}
\]

Since the majorization function \(\mathcal{L}_i^+\) is the same form as that of AuxIVA with respect to \(\mathbf{w}_{ij}\), the following simultaneous equations are obtained:

\[
\mathbf{w}_{ij}^H \mathbf{U}_i \mathbf{w}_{ij} = \delta_{ij}, \tag{15}
\]

where \(\delta_{ij} = 1\) when \(k = n\) and \(\delta_{ij} = 0\) when \(k \neq n\). By applying IP to \(\mathcal{L}_i^+\), we can obtain the update rules for the demixing matrix as

\[
\mathbf{w}_{ij} \leftarrow (\mathbf{W} \mathbf{U}_j)^{-1} \mathbf{e}_n, \tag{16}
\]

\[
\mathbf{w}_{ij} \leftarrow \frac{\mathbf{w}_{ij}}{\sqrt{\mathbf{w}_{ij}^H \mathbf{U}_j \mathbf{w}_{ij}}}, \tag{17}
\]

where \(\mathbf{e}_n\) denotes the unit vector with the \(n\)th element equal to unity. After the update of \(\mathbf{W}_i\), the separated signal \(\mathbf{y}_j\) should be updated as \(\mathbf{y}_{ij} \leftarrow \mathbf{w}_{ij}^H \mathbf{x}_j\).

### 3.3. Derivation of update rules for NMF parameters

The update rules for \(t_{ij|a}\) and \(v_{ij|a}\) can be derived by the MM algorithm, which is a popular approach for NMF. To obtain the differentiable majorization function for NMF parameters in \(\mathcal{L}_i^+\), we apply Jensen’s inequality to \(\sigma_{ij} = (\sum_i t_{ij} v_{ij})^{1/p}\). Jensen’s inequality can be represented as

\[
\left(\sum_q z_q \right)^{2/p} \leq \frac{1}{q} \sum_q \mu_q z_q \leq \sum_q \mu_q \left(z_q \right)^{2/p} = \sum_q \mu_q z_q^{2/p}, \tag{18}
\]

where \(\mu_q > 0\) is an auxiliary variable that satisfies \(\sum_q \mu_q = 1\). Note that the left-hand side of \(\mathcal{L}_i^+\) is a convex function for the variable \(z_q\) because we consider \(1 \leq p \leq 2\). The equality of \(\mathcal{L}_i^+\) holds if and only if \(\mu_q = z_q \sum_q z_q^2\). By applying \(\mathcal{L}_i^+\) to \(\sigma_{ij} = (\sum_i t_{ij} v_{ij})^{1/p}\) in \(\mathcal{L}_i^+\).
the following majorization function can be designed:

\[ \mathcal{L}_t^* \leq \text{const.} - 2T \sum_{t} \log |\text{det} W_{t}| \]

\[ + \sum_{i,j} \left( \frac{1}{2} \alpha_{i,j} + \frac{1}{2} \left( \sum_{i,j} \gamma_{i,j}^2 |\text{det} W_{i,j}| \right)^{\frac{1}{2}} \right)^2 \]

\[ + \left( \sum_{i,j} \left( \frac{1}{2} \log \alpha_{i,j} + \frac{2}{p} \log \beta_{i,j} \right) \right) \]

\[ \equiv \mathcal{L}_t^{* *}, \] (19)

where \( \gamma_{i,j} > 0 \) is an auxiliary variable and \( \mathcal{L}_t^* \) and \( \mathcal{L}_t^{* *} \) become equal only when

\[ \gamma_{i,j} = \frac{\tau_{i,j}}{\sum_{t} \tau_{i,j} \sigma_{i,j}}. \] (20)

From \( \partial \mathcal{L}_t^{* *}/\partial \tau_{i,j} = 0 \), we obtain

\[ \tau_{i,j} = \left( \frac{\gamma_{i,j}^2 + 1}{\sum_{i,j} \gamma_{i,j}^2} \right)^{\frac{1}{2}}. \] (21)

By substituting (12) and (20) into (21), we have the following update rule for \( \tau_{i,j} \):

\[ \tau_{i,j} \leftarrow \frac{\mathcal{S}_{i,j} \sigma_{i,j}^{\text{new}}}{\sum_{t} \mathcal{S}_{i,j} \sigma_{i,j}^{\text{new}}}. \] (22)

Similarly to (23), the update rule for \( \sigma_{i,j} \) can be obtained as

\[ \sigma_{i,j} \leftarrow \left[ \frac{\sum_{t} \mathcal{S}_{i,j} \sigma_{i,j}^{\text{new}}}{\sum_{t} \mathcal{S}_{i,j} \sigma_{i,j}^{\text{new}} \sigma_{i,j}^{\text{old}}} \right]^{\frac{1}{2}}. \] (23)

These update rules are similar to those in \( t \)-NMF, but they include the new domain parameter \( p \). After we update the parameters \( \tau_{i,j} \) and \( \sigma_{i,j} \), the model \( \sigma_{i,j} \) should be updated by (7).

By iteratively calculating the update rules (16), (17), (22), and (23), the cost function \( \mathcal{L}_t \) monotonically decreases, and the convergence is theoretically guaranteed. However, a scale ambiguity exists in the estimated signal \( y_{i,j} \) in ILRMA, and \( W_{t,i} \), \( \tau_{i,j} \), and \( \sigma_{i,j} \) should be normalized in each iteration as

\[ w_{i,j} \leftarrow w_{i,j} \eta_{i,j}^{-1}, \] (24)

\[ y_{i,j} \leftarrow y_{i,j} \eta_{i,j}^{-1}, \] (25)

\[ \sigma_{i,j}^{\text{new}} \leftarrow \sigma_{i,j}^{\text{new}} \eta_{i,j}^{p}, \] (26)

\[ \tau_{i,j} \leftarrow \tau_{i,j} \eta_{i,j}^{p}, \] (27)

where \( \eta_{i,j} \) is an arbitrary source-wise normalization coefficient, such as the source-wise average power

\[ \eta_{i,j} = \sqrt{\frac{1}{IJ} \sum_{i,j} |y_{i,j}|^2}. \] (28)

The signal scale of \( y_{i,j} \) can easily be restored by applying a back-projection technique after the cost function has converged, as

\[ \tilde{y}_{i,j} \leftarrow W_{i,j}^{-1} (e_{i} \circ y_{i,j}), \] (29)

where \( \tilde{y}_{i,j} \) is a scale-restored estimated source image and \( \circ \) denotes element-wise multiplication. The algorithm for \( t \)-ILRMA is summarized in Algorithm 1.

### 4. EXPERIMENTS

#### 4.1. Conditions

We confirmed the validity of the proposed generalization of ILRMA by conducting a BSS experiment using music and speech mixtures.
4.2. Computational times compared with those of t-MNMF

To clarify the advantage of ILRMA-based BSS in a determined situation ($N = M$), we compared the computational times of t-ILRMA and t-MNMF. The update calculation for the NMF parameters in each algorithm is almost the same, but the estimation of the spatial parameter ($W_i$ for t-ILRMA and the spatial covariance for t-MNMF) is different. Although t-ILRMA requires one inverse of $W_iU_i$, for each $i$ and $n$, t-MNMF requires $J$ inverses and two eigenvalue decompositions of the $M \times M$ matrix. Table 2 shows an example of relative computational times normalized by that of AuxIVA [5], where we used MATLAB 9.2 (64-bit) with an AMD Ryzen 7 1800X (8 cores and 3.6 GHz) CPU. From this table, we can confirm that the computational time of t-ILRMA does not increase significantly compared with that of IVA, whereas that of t-MNMF markedly increases. t-ILRMA was about six times faster than t-MNMF in the two-source case and about 53 times faster in the three-source case.

4.3. Results with random initialization

Fig. 4 shows an example of average SDR improvements and their standard deviations for various values of $\nu$, where the separation was performed 10 times with different random initializations for the parameters. Note that the result for $\nu \rightarrow \infty$ and $p = 2$ corresponds to that

Table 3. Average SDR improvements [dB] of t-ILRMA for music and speech signals

| $\nu$ | Music ($p = 1$) | Music ($p = 2$) | Speech ($p = 1$) | Speech ($p = 2$) |
|-------|----------------|----------------|-----------------|-----------------|
| 1     | 3.48           | 3.32           | -0.18           | -0.15           |
| 2     | 3.56           | 3.44           | -0.24           | -0.22           |
| 3     | 3.58           | 3.50           | -0.26           | -0.22           |
| 4     | 3.62           | 3.61           | -0.29           | -0.30           |
| 5     | 3.82           | 3.99           | -0.30           | -0.30           |
| 10    | 5.65           | 6.16           | -0.13           | -0.19           |
| 30    | 11.57          | 11.02          | 3.71            | 3.44            |
| 100   | 13.09          | 12.66          | 7.58            | 6.87            |
| 300   | 13.22          | 12.78          | 7.55            | 7.18            |
| 1000  | 13.27          | 12.76          | 7.74            | 7.09            |
| $\infty$ | -               | 12.89          | -               | 6.72            |

Table 4. Average SDR improvements [dB] of t-ILRMA with initialization for music and speech signals

| $\nu$ | Music ($p = 1$) | Music ($p = 2$) | Speech ($p = 1$) | Speech ($p = 2$) |
|-------|----------------|----------------|-----------------|-----------------|
| 1     | 13.15          | 13.09          | 6.93            | 7.11            |
| 2     | 13.31          | 13.27          | 6.97            | 7.01            |
| 3     | 13.34          | 13.39          | 6.89            | 6.97            |
| 4     | 13.27          | 13.26          | 6.99            | 6.85            |
| 5     | 13.25          | 13.32          | 6.94            | 6.89            |
| 10    | 13.41          | 13.38          | 7.04            | 6.87            |
| 30    | 13.39          | 13.33          | 7.25            | 6.78            |
| 100   | 13.39          | 13.35          | 7.29            | 6.82            |
| 300   | 13.39          | 13.27          | 7.21            | 6.83            |
| 1000  | 13.38          | 13.32          | 7.19            | 6.84            |
| $\infty$ | -               | 12.89          | -               | 6.72            |

The dry sources were obtained from SiSEC2011 [18] and are shown in Table 1. To simulate a reverberant mixture, the mixture signals were produced by convoluting the impulse response E2A ($T_{60} = 300$ ms), which was obtained from the RWCP database [19], with each source. The recording conditions of the impulse responses are shown in Fig. 3. The initial demixing matrix $W_i$ was always set to the identity matrix, and the NMF parameters $t_{il}$, $n_{il}$, and $v_{lj}$, $n_{lj}$ were initialized by random values. An STFT was performed using a 512-ms-long Hamming window with a 128 ms shift. The number of bases $L$ was set to five for music signals and two for speech signals, and the update rules were iterated 200 times. As the evaluation core, we used the signal-to-distortion ratio (SDR) [20], which indicates the overall separation quality.

Fig. 4 shows an example of average SDR improvements and their standard deviations for various values of $\nu$, where the separation was performed 10 times with different random initializations for the parameters. Note that the result for $\nu \rightarrow \infty$ and $p = 2$ corresponds to that...
for the conventional ILRMA assuming a complex Gaussian source generative model. From this result, we can confirm that the separation performance becomes stable and robust for the random initialization when \( \nu \) is set to a small value. However, the performance is degraded for both music and speech signals when \( \nu \leq 10 \). Only for the signals in Fig. 4, does the proposed method with \( (\nu, p) = (100, 1) \) for the music signal and \( (\nu, p) = (1000, 1) \) for the speech signal provide the best separation score. However, this tendency can vary with the dataset, namely, the optimal value of \( \nu \) depends on the instrument or the speaker in the mixture signal. Table 3 shows the average scores of all music or speech signals. We can confirm that a higher value of \( \nu \) and \( p = 1 \) are always preferable for the separation.

4.4. Results with conventional ILRMA initialization

When \( \nu \) is small, the Student’s \( t \)-distribution approaches the Cauchy distribution, where the latter can ignore outlier components. In particular, Cauchy NMF is suitable for extracting significant bases from a truly low-rank data matrix contaminated by outlier noise [9]. In the early stage of \( t \)-ILRMA iterations, the estimated spectrogram \( S_n \) includes almost all the source components because the initial demixing matrix is set to the identity matrix, and it is not a low-rank matrix even though each source spectrogram \( S_n \) is truly low-rank. In such a case, \( t \)-ILRMA with a small value of \( \nu \), such as Cauchy NMF, may not extract the useful bases for BSS, and the optimization will be trapped at a poor solution.

To solve this problem, in this experiment, we apply conventional ILRMA \((t \to \infty)\) in the early stage of iterations, then \( t \)-ILRMA with an arbitrary \( \nu \) is applied in the late stage, where for \( t \)-ILRMA, the bases and activations are pretrained using the outputs of conventional ILRMA via \( t \)-NMF. Fig. 5 and Table 4 show the average results of this approach, where conventional ILRMA is performed for the first 100 iterations and \( t \)-ILRMA is applied for the last 100 iterations. Compared with the previous results, a smaller value of \( \nu \) tends to provide better results and to outperform conventional ILRMA, although the stability is not improved because of the conventional-ILRMA-based initialization.

5. CONCLUSION

In this paper, we generalized the source distribution assumed in ILRMA from a complex Gaussian distribution to a complex Student’s \( t \)-distribution, which allows us to control the robustness to outlier components and includes the Cauchy distribution when \( \nu = 1 \). The proposed \( t \)-ILRMA can outperform conventional ILRMA with an appropriate value of \( \nu \). Also, initialization with a Gaussian assumption leads to further improvement for both music and speech BSS tasks.

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