Universality in a 2-component Fermi System at Finite Temperature

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Abstract

Thermodynamic properties of a fermionic system close to the unitarity limit, where the 2-body scattering length $a$ approaches $\pm \infty$ is studied at high temperature Boltzmann regime. For dilute systems the virial expansion coefficients in the Boltzmann regime are expected, from general arguments, to be universal. A model independent finite temperature $T$ calculation of the third virial coefficient $b_3(T)$ is presented. At the unitarity limit, $b_3^\infty \approx -0.48$ is an universal number. The energy density up to the third virial expansion is derived. These results are relevant for dilute neutron matter and could be tested in current atomic experiments on dilute fermionic gases near the Feshbach resonance.

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Recent experiments in atomic traps near the Feshbach resonance \([1, 2, 3, 4, 5]\) have opened opportunities to study properties of finite density systems that have not been explored before. Using an external magnetic field, it is possible to tune the 2-body \(S\)-wave scattering length \(a\) essentially at will: it can be made arbitrarily large \((a = \pm \infty)\) compared to the range of the 2-body interaction \(R\). Several groups have taken measurements near the Feshbach resonance in \(^{6}\text{Li}\) and \(^{40}\text{K}\) fermionic atomic gases \([1, 2, 3, 4, 5]\). These experiments are important checks of theoretical tools, both new and old, in previously untested territories and hold the prospect of revealing many new phenomenon. Equally important, what we learn from these atomic studies have universal application in many other subfields of physics.

At densities \(n\) that are dilute compared to the range of the interactions \((nR^3 \ll 1)\), the physics should be insensitive to the details of the interaction at the short distance scale \(R\). Therefore, near the Feshbach resonance, as \(|a| \to \infty\), where there aren’t any relevant scale left in the interaction, its properties are expected to be \textit{universal} and not just applicable to only atomic systems. Many physical systems in nature are close to this universal limit. In \(^{4}\text{He}\) atomic gases, the scattering length \(a \sim 100\text{fm}\) is much larger than the range of the interaction \(R \sim 5\text{fm}\). To take an example from nuclear physics, the neutron-neutron \(S\)-wave scattering length \(a_{nn} \sim -19\text{fm}\) is much larger than the range of the interaction set by the pion mass \(R \sim h/m_\pi \sim 1.4\text{fm}\). The universal properties learned in atomic experiments are applicable to other problems in the same universality class that are otherwise not directly accessible such as neutrino physics in supernova which depends on properties of dilute nuclear matter with a large resonating scattering length \([6]\). At the typical temperatures \(T \sim 4\text{MeV} \([7, 8]\) and densities \(n \sim 10^{-4}\text{fm}^{-3} \([6]\) of the neutrinosphere, it is possible to calculate the equation of state to a given order in \(n\lambda^3\) expansion, where \(\lambda = \sqrt{2\pi/(MT)}\) is the non-relativistic thermal wavelength of a fermion with mass \(M\). We calculate the equation of state up to the third order in \(n\lambda^3\) and it model independently describes dilute fermionic systems at finite temperature with a large scattering length even if the microscopic physics is difficult to calculate or poorly understood.

In this paper, we study the properties of a spin-\(\frac{1}{2}\) non-relativistic fermion at finite temperature \(T\) such that the thermal wavelength \(\lambda = \sqrt{2\pi/(MT)} \gg R\) and we are not sensitive to the short distance scale \(R\). However, the temperature is large enough such that the thermal wavelength is small compared to the inter-atomic distance \(n\lambda^3 \ll 1\) and scattering length \(\lambda \lesssim a\). Near the Feshbach resonance with \(|a| \to \infty\), the hierarchy of momentum scales would be then \(|a| \gg n^{-1/3} \gg \lambda \gg R\). The calculation is more general and applicable for \(n\lambda^3 \ll 1\), \(\lambda \gg R\).

The calculation is organized as an expansion in \(n\lambda^3 \ll 1\), the Boltzmann regime, with factors of \(|a|\sqrt{MT}/(2\pi)\) summed to all orders in perturbation. In the Boltzmann regime the pressure \(P\) for a 2-component fermi system can be written in terms of the so called \textit{virial} expansion \([9]\):

\[
\frac{P}{T} = \frac{2}{\lambda^3} \left[ b_1 z + b_2 z^2 + b_3 z^3 + \ldots \right],
\]

where \(z = \exp(\mu/T)\) is the fugacity for the system with chemical potential \(\mu\). In this expansion, all the density dependence is in the fugacity \(z\). This is a valid expansion for \(z \ll 1\) or equivalently for \(n\lambda^3 \ll 1\) as shown later in Eq.\([13]\).

The dimensionless virial coefficients \(b_n\) depend on the vacuum interaction and the thermal momentum \(\sqrt{MT}\). For a dilute system with thermal wavelength \(\lambda\) much larger than the range \(R\), the interaction depends only on the scattering length \(a\). Thus, as \(a \to \pm \infty\) at the unitarity limit, there are no relevant scale left in the interaction and the dimensionless coefficients \(b_n\) must be \textit{universal}. This is shown explicitly with a calculation up to the third order in the virial expansion. There would be small corrections to these universal results from effective range, higher-partial
waves, etc., that are neglected in this calculation but are otherwise straightforward to incorporate in perturbation.

The virial coefficient $b_n$ receives contribution from up to and including $n$-body physics \cite{9}. For example, $b_4$ would receive contributions from 1-body (non-interacting theory), 2-,3- and 4-body physics. $b_2$ is the first coefficient that receives contribution from the interacting theory and it is related to the 2-body scattering phase shift \cite{9,10,11}:

$$b_2^{(2)} = \frac{1}{\sqrt{2}} e^{\frac{\gamma^2}{2}} \left[ 1 + \text{Erf} \left( \frac{\gamma}{\sqrt{MT}} \right) \right],$$

(2)

where $\gamma = 1/a$. We use the following notation:

$$b_n = b_n^{(1)} + b_n^{(2)} + \cdots + b_n^{(n-1)} + b_n^{(n)},$$

$$\overline{b}_n = b_n - b_n^{(1)},$$

(3)

where $b_n^{(l)}$ is the $n$-th virial coefficient from $l$-body contribution. The thermodynamic pressure $P_k^{(l)}$, number density $n_k^{(l)}$, etc., are also denoted in a similar manner. We also define $P^{(l)}$ and $n^{(l)}$ without a subscript to denote the contribution to the pressure and number density from $l$-body physics to all orders in the $z$ expansion.

From Eq. 2, $b_2^{(2)} = 1/\sqrt{2}$ as $a \to \pm \infty$ and it is universal. A plot of $b_2^{(2)}$ and $T \partial b_2^{(2)}/\partial T$ is shown in Fig. 1 near the unitarity limit for lithium $^6$Li for temperatures around 6$\mu$K.

![FIG. 1](image)

FIG. 1: Plot of $b_2^{(2)}$, the 2-body contribution to the second virial coefficient and its derivative $T \partial_T b_2^{(2)}$ for $^6$Li as a function of the inverse scattering length $\gamma = 1/a$ in eV. 1eV $\approx 5.06 \times 10^{-4}$Å$^{-1}$ in natural units ($\hbar = 1 = c$). Solid, long-dashed, dot-dashed and short-dashed curves correspond to temperatures of 6$\mu$K, 8.4$\mu$K, 11.76$\mu$K and 16.46$\mu$K respectively (in 40% increments). The second virial coefficient is temperature independent and universal as $\gamma = 1/a \to 0^\pm$.

Traditionally the higher virial coefficients are calculated in a cluster expansion \cite{10} and there is no direct simple formula for relating it to many-body interaction. For example, it is not clear how the contribution from 3-body bound states associated with Efimov effect \cite{12} in bosons such as $^4$He near $a \to \pm \infty$ is taken into account. This is in contrast to an effective field theory calculation where the virial coefficients are directly related to many-body scattering and the $^4$He 3-body bound state
contribution can be directly taken into account. The general methodology for such an effective field theory calculation was developed earlier in Ref. [11].

For a dilute system with a clear separation of scales \(1/|a| \ll n^{1/3} \ll \sqrt{MT/(2\pi)} \ll 1/R\), effective field theory is quite ideal, and provides model independent results in the region of interest. A non-relativistic system near the Feshbach resonance in the \(S\)-wave at chemical potential \(\mu\) (or densities \(n\)) and temperature \(T\) such that \(\sqrt{\mu T} \ll 1/R\) can be described by the Lagrangian density:

\[
\mathcal{L} = \psi^\dagger \left(i\partial_\tau + \frac{\nabla^2}{2M}\right)\psi + \mu \psi^\dagger \psi - \frac{g}{4} (\psi \sigma_2 \psi)^\dagger (\psi \sigma_2 \psi) + \ldots,
\]

where the “…” represents higher dimensional operators that are suppressed for dilute systems, \(\psi\) are the spin-\(\frac{1}{2}\) fermion fields and the \(\sigma_i\) matrices act in the spin space. The four-fermion coupling \(g\) is related to the 2-body scattering length \(a\) (see Refs. [13, 14] and references therein):

\[
g(\nu) = -\frac{4\pi}{M} \frac{1}{\nu - 1/a}.
\]

where \(\nu\) is the renormalization scale in dimensional regularization. One can choose to regulate using momentum cutoff \(\Lambda\) and get a \(g(\Lambda)\) similar in form to \(g(\nu)\) above [11]. The final result is independent of the regularization scheme and renormalization scale as it should be. Notice that there are no 3-body \(S\)-wave operator at leading order [14, 15]. Therefore when we calculate the third virial coefficient, it will depend only on the 2-body interaction and will be universal as the 2-body scattering length \(a\) approaches \(\pm\infty\).

The organization of the fugacity \(z\) expansion in terms of Feynman diagrams was worked out earlier in Ref. [11] and applied to bosons. For bosons, there is a 3-body interaction at leading order which can lead to a 3-body bound state that dominates the third virial coefficient \(b_3\). At the same time this prevents \(b_3\) from being universal for bosons since the 3-body binding energy is system dependent and does not necessarily approach an universal value as \(a \to \pm\infty\).

The first two virial coefficients \(b_1\) and \(b_2\) are known. Nevertheless, it is instructive to derive them in the effective field theory from Feynman diagrams before calculating the third virial coefficient \(b_3\). Feynman diagrams with a closed particle loop is order \(z\) (from the energy integral over the Matsubara frequency) and vanish in vacuum as expected. This can be seen by calculating the pressure in the free theory which is given by the loop diagram shown in Fig. 2.

\[
P(1) = 2T \sum_{n=-\infty}^{\infty} \int \frac{d^3q}{(2\pi)^3} \log \left[i(2n+1)\pi T + \mu - \frac{q^2}{2M}\right] = 2T \int \frac{d^3q}{(2\pi)^3} \log \left[1 + z \exp \left(\frac{q^2}{2MT}\right)\right]
\]

\[
= \frac{2T}{\lambda^3} \left[z - \frac{z^2}{4\sqrt{2}} + \frac{z^3}{9\sqrt{3}} - + \ldots\right],
\]

\[\Rightarrow b_1 + b_2^{(1)} + b_3^{(1)} + \ldots = 1 - \frac{1}{2^{5/2}} + \frac{1}{3^{5/2}} - + \ldots.
\]

A closed particle-particle diagram (only possible with interactions) is \(O(1)\) and does not vanish in the vacuum. However, a closed loop with “baryon number” equal to 2 i.e. a closed dimer propagator is order \(O(z^2)\). Similarly a closed trimer propagator is \(O(z^3)\). This is more easily demonstrated with the following calculation of the second-virial coefficient.
FIG. 2: The pressure $P$ from 1- and 2-body physics. Solid lines are fermion fields. Each particle-particle loop is order $g\sqrt{MT} \sim a\sqrt{MT}$ and summed to all orders to form the dressed dimer propagator, represented by the double lines.

At large scattering length $|a|\sqrt{MT}/(2\pi) \gtrsim 1$, the dimer propagator is given by an infinite geometric sum shown in Fig. 2:

$$D(p_0, \vec{p}) \approx \frac{4\pi}{M} \frac{1}{\frac{1}{a} + \sqrt{\frac{p_0^*}{4} - Mp_0 - 2M\mu}} + O(z^2),$$

where $p = |\vec{p}|$, $q = |\vec{q}|$ and $p_0$ is an odd multiple of $i\pi T$. Some care is needed in evaluating $D(p_0, \vec{p})$ where $p_0$ is to be identified with an “final” loop integral variable in Eqs. 9 and 11. In the center of mass frame $\vec{p} = \vec{k} - \vec{k} = 0$, analytically continuing $p_0$ to the center of mass energy in vacuum $p_0 \to k^2/M - 2\mu + i0^+$, the dimer propagator $D(k^2/M - 2\mu, 0)$ reproduces the correct large scattering length 2-body amplitude:

$$A(k) = \frac{4\pi}{M} \frac{1}{\frac{1}{a} - ik}.$$

The contribution from the dimer propagator to the pressure is

$$P^{(2)} = -T \sum_{n=-\infty}^{\infty} \int \frac{d^3 q}{(2\pi)^3} \log[D(i(2n+1)\pi T, \vec{q})]$$

$$\approx -\frac{1}{2\pi i} \oint \frac{d\eta}{\exp(\eta/T) + 1} \int \frac{d^3 q}{(2\pi)^3} \left[ \log \left( -\frac{1}{a} + \sqrt{\frac{q^2}{4} - M\eta - 2M\mu} \right) + O(z^2) \right].$$

The contour integral in $\eta$ is in an anti-clockwise sense over all the odd integer multiples of $i\pi T$. Looking at Eqs. 7 and 9, one can see how the energy integral over $\eta$ for a dimer propagator are $O(z^2)$ because a dimer carries chemical potential $2\mu$. This is similar to the energy $\eta$ integral for a closed single particle propagator that carries chemical potential $\mu$ and $1/\{\exp(\beta\eta) + 1\} \sim O(z)$ for
\( \eta \sim -\mu \). \( P^{(2)} \) is \( O(z^2) \) and an explicit calculation shows:

\[
P^{(2)} \approx P_2^{(2)} + P_3^{(2)} + O(z^4),
\]

\[
P_2^{(2)} = \frac{\sqrt{2} T}{\lambda^3} z^2 e^{\frac{\gamma^2}{MT}} \left[ 1 + \text{Erf} \left( \frac{\gamma}{\sqrt{MT}} \right) \right],
\]

\[
P_3^{(2)} = 0 \Rightarrow b_3^{(2)} = 0.
\]

Comparing Eqs. 1, 2, 6 and 10, we see that the known results up to the second order in the virial expansion are exactly reproduced in the effective field theory calculation.

\[n_3^{(3)} = \ldots\]

![Diagram](attachment:image.png)

**FIG. 3:** 3-body contribution to the number density \( n_3^{(3)} \).

The calculation of the third virial coefficient \( b_3^{(3)} \) is technically more challenging but it follows the same procedure. First, the leading order trimer propagator in the fugacity \( z \) expansion is calculated, and then the energy integral over imaginary odd multiples of \( i\pi T \) is carried out. The trimer propagator is \( O(1) \) in the \( z \) expansion and carries chemical potential \( 3\mu \). Thus an energy integration over a closed trimer loop contributes at \( O(z^3) \). Like the dimer, the trimer propagator requires summation of an infinite series of Feynman diagrams for \( |a| \sqrt{MT/(2\pi)} \gtrsim 1 \). However, unlike the dimer, the 3-body contributions do not form a geometric series and this makes the trimer more challenging. The solution to the trimer problem has been known \([14, 15]\) and we will not repeat it here. The relevant Feynman diagrams are shown in Fig. 3. It is easier to calculate the number density \( n = \partial_\mu P \) instead of the pressure \( P \) for the trimer. We find:

\[
n_3^{(3)} = -\frac{9\sqrt{3} M}{4\pi^2 \lambda^3} z^3 \int d\eta \int_0^\infty dk \, k^2 \frac{\gamma + \sqrt{\frac{3}{4} k^2 - M\eta}}{\sqrt{\frac{3}{4} k^2 - M\eta (\frac{3}{4} k^2 - M\eta - \gamma^2)^2}} e^{-\eta/T} a(\eta, k, k)
\]

\[
= \frac{-9\sqrt{3} M}{2\pi^2 \lambda^3} z^3 \int d\eta \int_0^\infty dk \, k^2 \frac{\gamma^2}{\frac{3}{4} k^2 - M\eta - \gamma^2} e^{-\eta/T} b(\eta, k, k)
\]

\[= \partial_\mu P_3^{(3)} = \frac{6}{\lambda^3} b_3^{(3)} z^3,
\]
where the contribution from the trimer propagator, after projecting onto the $S$-wave, is defined through the integral equations

$$
a(\eta, k, k') = -\frac{1}{2} K(\eta, k, k') - \frac{1}{\pi} \int_0^\infty dl \frac{l^2 a(\eta, k, l)}{l^2 - \frac{4}{3}(M\eta + \gamma^2)} K(\eta, l, k'),
$$

$$
b(\eta, k, k') = -\frac{8}{3} \frac{\gamma + \sqrt{\frac{3}{2}k^2 - M\eta}}{(k^2 + k'^2 + kk' - M\eta)(k^2 + k'^2 - kk' - M\eta)}
- \frac{2}{\pi} \int_0^\infty dl \frac{l^2 b(\eta, k, l)}{l^2 - \frac{4}{3}(M\eta + \gamma^2)} K(\eta, l, k'),
$$

$$
K(\eta, k, k') = \frac{4}{3} \frac{\gamma + \sqrt{\frac{3}{2}k^2 - M\eta}}{kk'} \log \left[ \frac{k^2 + k'^2 + kk' - M\eta}{k^2 + k'^2 - kk' - M\eta} \right].
$$

![Plot of $b_3$ and its derivative $T\partial_T b_3$ for $^6\text{Li}$ as a function of $\gamma = 1/a$ in eV. The different curves are for the same set of temperatures as in Fig. 1 and uses the same notations. $b_3$ shows universal behavior.](image)

In Fig. 4 the third virial coefficient $b_3 = b_3 - b_3^{(1)} = b_3^{(2)} + b_3^{(3)} = b_3^{(3)}$ and its derivative $T\partial_T b_3$ are shown for $^6\text{Li}$ at temperatures around $6\mu\text{K}$. As expected from physical arguments presented earlier, the third virial coefficient is universal. This can be made more explicit in Eqs. 10 and 11 for $|a| \to \infty$, and write, for example, the coefficient $b_3^{(3)}$ in terms of dimensionless integrals. We find $b_3^{\infty} \approx -0.54$.

From this calculation, we see that the virial coefficients up to $O(z^3)$ are not un-naturally large [numbers of $O(1)$] and for $z \ll 1$, the virial expansion seems consistent. Formally, the expansion in $z$ is equivalent to an expansion in diluteness parameter in perturbation [9, 16] for $n\lambda^3 \ll 1$:

$$
n = \frac{\partial P}{\partial \mu} \approx \frac{2}{\lambda^3} \left[ b_1 z + 2b_2 z^2 + 3b_3 z^3 \right] + \ldots,
$$

$$
\Rightarrow z \approx -\frac{n\lambda^3}{2} - 2b_2 \left( \frac{n\lambda^3}{2} \right)^2 + (8b_2^2 - 3b_3) \left( \frac{n\lambda^3}{2} \right)^3 + \ldots,
$$

where we used $b_1 = 1$. Defining the density $n$ in terms of the Fermi temperature $T_F$, $n\lambda^3/2 = 4/(3\sqrt{\pi})(T_F/T)^{3/2}$ is the small expansion parameter for $T \gg T_F$. The energy density can be
obtained from the pressure using standard thermodynamic relations \( \varepsilon = -P + \mu n + T s \), where \( s = \partial_T P \) is the entropy density.

As \( a \to \pm \infty \), the energy density can be written as \([16]\):

\[
\varepsilon^\infty = \frac{3}{2} T n \left[ 1 + \frac{1}{2^{5/2}} + \frac{n\lambda^3}{2} + \frac{n^2\lambda^6}{4} \left( \frac{1}{8} - \frac{2}{9\sqrt{3}} \right) \right] + \varepsilon^\infty_{\text{int}} \equiv \varepsilon_{\text{kin}} + \varepsilon^\infty_{\text{int}},
\]

\( \varepsilon^\infty_{\text{int}} \) is the contribution from the interacting theory in Eq. \(14\) in the unitarity limit \( a \to \pm \infty \).

In Fig. 5 we have plotted the contributions from the third virial coefficient to the energy density. In the left subplot in Fig. 5, the solid curve shows the contribution up to \( \mathcal{O}(n\lambda^3) \) normalized to the leading order contribution as a function of \( T_F/T \). The dashed curve is the \( \mathcal{O}(n^2\lambda^6) \) contribution which is a smaller correction for \( T_F/T \lesssim 0.49 \). This is shown more clearly in the right subplot in Fig. 5, the convergence (error) plot \( [17] \). It is deduced that the range of convergence is \( T_F/T \lesssim 0.49 \).

The ratio of the interaction energy \( \varepsilon_{\text{int}} \) to the contribution from the free Lagrangian density \( \varepsilon_{\text{kin}} \) is shown in Fig. 6 for a range of \( T_F/T \) where the virial expansion converges at this order of the calculation. For the experiments on \(^6\text{Li}\), at temperatures around \( T \sim 3T_F \) we expect the virial expansion to be valid. At these temperatures, the effect of the third virial coefficient should be detectable in experimental data for dilute systems satisfying the constraint \( n^{1/3} \ll \sqrt{MT/(2\pi)} \ll 1/R \).

![FIG. 5: Left subplot: the second (solid curve) and third order (dashed curve) energy density \( \varepsilon^\infty \) in the \( n\lambda^3/2 \) expansion in Eq. \(14\) as a fraction of the leading order result \( 3Tn/2 \) vs. \( T_F/T \). Right subplot: Convergence (error) plots \( [17] \). Absolute value of the ratio of only the \( \mathcal{O}(n\lambda^3) \) to the leading order contribution (solid curve) and only the \( \mathcal{O}(n^2\lambda^6) \) to the leading order contribution (dashed curve) to the energy density as a function of \( T_F/T \). For small values of \( T_F/T \), the \( \mathcal{O}(n^2\lambda^6) \) effects are smaller than \( \mathcal{O}(n\lambda^3) \). These effects are equal at around \( T \sim 2T_F \) and the perturbation breaks down. Note the \( \mathcal{O}(n^2\lambda^6) \) effects equal the leading order result (ratio equals 1) at a smaller temperature \( T \sim 1T_F \), at fixed \( T_F \). The \( \mathcal{O}(n\lambda^3) \) results are smaller than the leading order results to even smaller temperatures (not shown).](image)

In conclusion, we considered a spin-\( \frac{1}{2} \) dilute fermionic system near the unitarity limit \(|a| \to \infty \). At finite temperature the thermodynamic pressure was calculated in a model independent way using effective field theory up to the third order in the virial expansion. At this order, universality was demonstrated and a range of temperature \( T \gtrsim 3T_F \) for a fixed density (conversely a range of densities for a fixed temperature) was identified for a self-consistent application of the results.
FIG. 6: Ratio $\frac{\epsilon_{\text{int}}}{\epsilon_{\text{kin}}}$ of interaction to kinetic energy as a function of $T_F/T$ in the range of convergence $T_F/T \lesssim 0.49$ deduced in Fig. 5. Solid curve: leading order $[O(n^3\lambda^3)]$ result, dashed curve: next-to-leading order $[O(n^2\lambda^6)]$ result.

The virial expansion is an useful tool for dilute systems where the microscopic physics is poorly known or difficult to calculate. The universal forms for the equation of state derived here are applicable to a wide class of physical problems in nuclear and atomic physics. These results are relevant for understanding neutrino physics in hot dilute nuclear matter $n \sim 10^{-4}$ fm$^3$, and the same equations for energy density, etc., are testable in atomic physics experiments. At lower temperatures, numerical lattice calculations are more appropriate [18, 19, 20]. However, at intermediate temperatures, our results could be used as checks for the lattice calculations.

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