New developments in threshold pion photo- and electroproduction

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Abstract. Photoproduction of neutral and charged pions off nucleons and deuterium has been precisely calculated in baryon chiral perturbation theory. I review the predictions in light of the accurate data that have become available over the last few years. Some progress in the description of neutral pion electroproduction off protons is also discussed.

1 Introduction

With the advent of CW machines, pion production by real and virtual photons has become a major testing ground for predictions based on nucleon chiral dynamics. In particular over the last years there has been considerable activity to precisely measure pion photo-and electroproduction in the threshold region at various laboratories, like e.g. MAMI, SAL and NIKHEF. On the other hand, refined calculations within heavy baryon chiral perturbation theory (HBCHPT) have been performed. Pion photo-and electroproduction is not only interesting per se but also allows, as we will see in the following, to get informations on other processes like for example $\pi N$ scattering. In this respect the use of CHPT, see for example Gasser et al. (1984), is particularly advantageous since it is a method for solving QCD at low energy which links different processes in a model-independent fashion thus allowing for a deeper understanding of the underlying dynamics. Another very important problem which can be addressed in dealing with these processes is isospin symmetry. It has always been one of the major goals in nuclear physics to understand isospin symmetry violation related to the light quark mass difference $m_u - m_d$ and virtual photon effects. Although the light quark mass ratio deviates strongly from unity, $m_d/m_u \sim 2$, see Gasser et al. (1975) and one could expect sizeable isospin violation, such effects are effectively masked since $m_d - m_u \ll \Lambda$, with $\Lambda$ the scale of the strong interactions (which can be chosen to be $4\pi F_\pi$ or 1 GeV or the mass of the $\rho$). To assess the isospin violation through quark mass differences, precise measurements and accurate calculations are mandatory. As pointed out by Weinberg (1977) long time ago, systems involving nucleons can exhibit such effects to leading order in contrast to the suppression in purely pionic processes due to G–parity. Thus pion photoproduction is well suited for such investigations since a large body of (precise) data exists for various isospin channels.

Here I will report on the progress made since the MIT workshop, see MIT proceedings (1994), on the calculation of charged and neutral pion photo-and
electroproduction near threshold in the framework of HBCHPT. In this framework the nucleons are treated as very heavy static sources. HBCHPT is a triple expansion in external momenta, quark masses and inverse powers of the nucleon mass (collectively denoted by the small parameter \( q \)). Calculations which will be discussed here are \( \mathcal{O}(q^n) \) with \( n=4 \) for the S-wave and \( n=3 \) for the P-waves and only take into account isospin breaking effects which are believed to be dominant, namely those through the pion mass difference in the loops.

2 Formal aspects

2.1 Effective field theory

The effective Lagrangian which will be needed here consists of the following pieces:

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(4)}_{\pi\pi} + \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N} + \mathcal{L}^{(4)}_{\pi N} + \mathcal{L}^{(0)}_{NN} + \mathcal{L}^{(1)}_{NN},
\]

where the index \( (i) \) gives the chiral dimension \( d_i \) (number of derivative and/or meson mass insertions). \( \mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(1)}_{\pi N} \) is the non-linear \( \sigma \) model Lagrangian coupled to nucleons. The terms from \( \mathcal{L}^{(3)}_{\pi N} + \mathcal{L}^{(4)}_{\pi N} \) contributing to the single–nucleon photoproduction amplitudes are given in Bernard et al. (1996(1)). \( \mathcal{L}^{(1)}_{NN} \) has been used in the nuclear force calculation of van Kolck (1994) and enters the calculation of (neutral) pion photoproduction on deuterium by Beane et al. (1997), which will be discussed below. In HBCHPT the nucleon mass term is replaced by a string of \( 1/m \) suppressed interactions so that \( \mathcal{L}^{(n)}_{\pi N} \), \( n \geq 2 \), contains \( 1/m \) terms as well as counterterms whose coefficients are the famous low–energy constants (LECs). Some of these counterterms cancel the divergences of certain loop diagrams and thus are scale dependent, \( C_i = C_i^r(\mu) + \Gamma_i L(\mu) \) where \( \mu \) is the scale of dimensional regularization (naturally the physical amplitudes are scale independent). In the following these LECs which are not fixed by chiral symmetry, will either be fitted to experimental data or will be obtained using the principle of resonance saturation. In the meson sector it can indeed be shown that the numerical values of the renormalized LECs \( L_i^r(\mu = M_\rho) \) can be understood to a high degree of accuracy from resonance saturation, i.e. they can be expressed in terms of resonance masses and coupling constants of the low-lying vector \( (V) \), axial vector \( (A) \), scalar \( (S) \) and pseudoscalar \( (P) \) multiplets, see Ecker et al. (1989). In the nucleon sector there exists no proof of this principle, however it seems to work rather well in the case of \( \mathcal{L}^{(2)}_{\pi N} \) as has been demonstrated in Bernard et al. (1997(1)) and can be seen in table 1 which gives the values of the seven finite LECs at that order. The first four have been determined (second column) from a best fit to a set of nine subthreshold and threshold \( \pi N \) observables that to one–loop order \( q^3 \) are given entirely in terms of tree graphs including insertions from these LECs and finite loop contributions, but with none from the 24 new LECs of \( \mathcal{L}^{(3)}_{\pi N} \). The other three can be determined from
the strong neutron–proton mass difference ($c_5$, which is only relevant in the case $m_u \neq m_d$) and from the anomalous magnetic moments of the proton and the neutron ($c_6, c_7$). Note that the scalar mass to coupling constant ratio $M_S/\sqrt{g_S}$ needed to saturate the LEC $c_1$ is in perfect agreement with typical ratios obtained in boson–exchange models of the NN force, where the $\sigma$–meson models the strong pionic correlations coupled to nucleons. Note also that the important $\Delta$ degree of freedom is included in the determination of the LECs through resonance exchange. I will come back to this point in the following.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
\(i\) & \(c_i\) & \(c_i^{\text{Res}}\) & \(c_i^{\text{cv}}\) & \(c_i^{\text{ranges}}\) & \(\text{Res}\) \\
\hline
1 & $-0.93 \pm 0.10$ & $-0.9^*$ & - & - & $S$ \\
2 & $3.34 \pm 0.20$ & 3.9 & 2...4 & $\Delta, R$ & \\
3 & $-5.29 \pm 0.25$ & $-5.3$ & $-4.5...-5.3$ & $\Delta, R, S$ & \\
4 & $3.63 \pm 0.10$ & 3.7 & 3.1...3.7 & $\Delta, R, \rho$ & \\
5 & $-0.09 \pm 0.01$ & - & - & - & \\
6 & 5.83 & 6.1 & - & - & $\rho$ \\
7 & $-2.98$ & $-3.0$ & - & - & $\rho, \omega$ \\
\hline
\end{tabular}
\caption{Values of the LECs $c_i$ in GeV$^{-1}$ for $i = 1, \ldots, 5$. The LECs $c_6, c_7$ are dimensionless. Also given are the central values (cv) and the ranges for the $c_i$ from resonance exchange. The $^*$ denotes an input quantity. $R$ and $S$ denote the Roper and the scalar resonances respectively.}
\end{table}

### 2.2 Threshold pion photo-and electroproduction

Consider the process $\gamma(k) + N(p_1) \rightarrow \pi^a(q) + N(p_2)$, with $N$ denoting the nucleon (proton or neutron), $\gamma$ a real ($k^2 = 0$) or a virtual ($k^2 < 0$) photon and $\pi^a$ a pion of isospin $a$. The polarization vector of the photon is denoted by $\epsilon_\mu$. In the threshold region, the three–momentum $q$ of the pion is small and vanishes at threshold. It is therefore advantageous to perform a multipole decomposition since at threshold only the $S$–waves survive and close to threshold one can confine oneself to $S$– and $P$–waves. The corresponding multipoles are called $(E, M, L)_{l \pm}$, where $E, M, L$ stands for electric, magnetic and longitudinal (this last one of course only comes into play for virtual photons), $l = 0, 1, 2, \ldots$ the pion orbital angular momentum and the $\pm$ refers to the total angular momentum of the pion-nucleon system, $j = l \pm 1/2$. These multipoles parametrize the structure of the nucleon as probed with low energy photons. Consequently, the $T$–matrix depends on seven (only four survive for real photons) complex multipoles and takes the following form:

\[
\frac{m}{4\pi \sqrt{s}} T \cdot \epsilon = i \sigma \cdot \epsilon \left( E_{l+} + \hat{q} \cdot \hat{k} P_1 \right) + i \sigma \cdot \hat{k} \epsilon \cdot \hat{q} P_2 + (\hat{q} \times \hat{k}) \cdot \epsilon P_3
\]

\[
+ i \sigma \cdot \hat{k} \epsilon \cdot \hat{q} \left( L_{l+} - E_{l+} + \hat{q} \cdot \hat{k} (P_4 - P_5 - P_1 - P_2) \right) + i \sigma \cdot \hat{q} \epsilon \cdot \hat{k} P_5 \quad (2)
\]
The quantities $P_{1-5}$ represent the following combinations of the five $P$-waves, $E_{1+}, M_{1+}, M_{1-}, L_{1+}, L_{1-}$,

$$
P_1 = 3E_{1+} + M_{1+} - M_{1-}, \quad P_2 = 3E_{1+} - M_{1+} + M_{1-}, \quad P_3 = 2M_{1+} + M_{1-},$$
$$P_4 = 4L_{1+} + L_{1-}, \quad P_5 = L_{1-} - 2L_{1+}. \quad (3)$$

All these amplitudes are easily calculable within CHPT. They have the conventional isospin decomposition (to first order in the electromagnetic coupling),

$$A(s,u) = A^{(+)}(s,u)\delta_{a,3} + A^{(-)}(s,u)\frac{1}{2}[\tau_a, \tau_3] + A^{(0)}(s,u)\tau_3 \quad (4)$$

There are however, four physical channels, two charged reactions and two neutral ones which will be discussed next. Thus there exists a triangle relation relating one of the physical amplitudes to the others. This relation should of course only hold if isospin is an exact symmetry. It is then clear that it is very important to have a very precise determination of the four physical channels in order to measure isospin violation.

### 3 Charged Pion Photoproduction

Charged pion photoproduction is a particularly interesting process since it allows to

- investigate the violation of isospin symmetry beyond leading order in electromagnetism. For that one of course needs to know extremely precisely the four physical photoproduction reactions as discussed previously.
- determine $\pi N$ scattering lengths.
- give a stringent constraint on the much discussed value of the pion-nucleon coupling constant $g_{\pi N}$ via the Goldberger-Miyazawa-Oehme (1955) sum rule combined with the Panofsky ratio.

It is well described by the Kroll-Ruderman term which is non-vanishing in the chiral limit,

$$E_{0+}^{thr}(\pi^+ n) = \frac{e g_{\pi N}}{4\pi \sqrt{2m (1 + \mu)^{3/2}}} = 27.6 \cdot 10^{-3}/M_\pi,$$
$$E_{0+}^{thr}(\pi^- p) = -\frac{e g_{\pi N}}{4\pi \sqrt{2m (1 + \mu)^{1/2}}} = -31.7 \cdot 10^{-3}/M_\pi, \quad (5)$$

with $\mu = M_\pi/m$ and using $g_{\pi N}^2/4\pi = 14.28, e^2/4\pi = 1/137.036, m = 928.27$ MeV and $M_{\pi^+} = 139.57$ MeV. In the limit $M_\pi = 0$, this simplifies to

$$E_{0+}^{thr}(\pi^+ n) = -E_{0+}^{thr}(\pi^- p) = 34 \cdot 10^{-3}/M_\pi. \quad (6)$$

By comparing the numbers in Eq.(5) and Eq.(6) one notices that the kinematical corrections which are suppressed by powers of the small parameter $\mu \simeq 1/7$ are quite substantial for $E_{0+}^{thr}(\pi^+ n)$. However, there are other corrections which are related to pion loop diagrams and higher dimension operators. These have
been dealt with in a systematic fashion using heavy baryon CHPT up–to–and–including order $O(\mu^3)$ in Bernard et al. (1996(2)). In this framework, one has to consider pion loop diagrams and local contact terms whose coefficients are the LECs. These were estimated by resonance exchange since not enough precise data exist to pin them all down. Frozen kaon loops contributes to $E_{0+}^{(0)}$ and $E_{0+}^{(−)}$ while $\rho$-meson exchange contributes only to $E_{0+}^{(0)}$ and the $\Delta$, the axial resonance and the $c_{1−3}$ (see previous section) to $E_{0+}^{(−)}$, where $E_{0+}(\pi^+ n) = \sqrt{2}(E_{0+}^{(0)} + E_{0+}^{(−)})$ and $E_{0+}(\pi^− p) = \sqrt{2}(E_{0+}^{(0)} - E_{0+}^{(−)})$. This of course leads to some uncertainty into the result since the resonance parameters are only known with in certain ranges. Another source of uncertainty comes from the regularization scale. Indeed within resonance saturation a spurious mild scale–dependence remains. In the calculation $\lambda$ runs in the interval $M_\rho \leq \lambda \leq m_\Delta$. To $O(\mu^3)$ one gets with $g_{\pi N} = 13.4$:

$$E_{0+}^{(0)} = (-1.6 \pm 0.1) \cdot 10^{-3}/M_\pi \ , \ E_{0+}^{(−)} = (21.5 \pm 0.4) \cdot 10^{-3}/M_\pi$$ (7)

The value of $g_{\pi N}$ will be discussed in the next section. The chiral expansion is rapidly converging in contrast to neutral pion photoproduction:

$$E_{0+}^{(0)} = (0 - 1.79 + 0.38 - 0.21) \cdot 10^{-3}/M_\pi \ ,$$
$$E_{0+}^{(−)} = (24.01 - 3.57 + 1.38 - 0.29) \cdot 10^{-3}/M_\pi$$ (8)

where the various contributions to $E_{0+}^{(0)}$ and $E_{0+}^{(−)}$ of $O(M_n^a)$ with $n=0,1,2,3$ have been collected. Translating these results into the physical channels one obtains the results shown in the second column of table 2.

|               | CHPT    | DR      | Experiment                           |
|---------------|---------|---------|--------------------------------------|
| $E_{0+}^{\pi^+ n}$ | 28.2 ± 0.6 | 28.0 ± 0.2 | 27.9 ± 0.5, 28.8 ± 0.7, 27.6 ± 0.3 |
| $E_{0+}^{\pi^- p}$ | −32.7 ± 0.6 | −31.7 ± 0.2 | −31.4 ± 1.3, −32.2 ± 1.2, −31.5 ± 0.8 |

Table 2: Predictions and data for the charged pion electric dipole amplitudes.

Also given in that table are the results of the dispersion theoretical (DR) analysis of Hanstein et al. (1997) and the experimental ones. The first two numbers in the last column corresponds to rather old data from Burq (1965), Adamovitch (1966) and Goldwasser et al. (1964) while the last ones are taken from the recent TRIUMF experiment on the inverse reaction $\pi^- p \rightarrow \gamma n$ (see Kovash et al. (1997)) and a preliminary SAL analysis of the reaction $\gamma p \rightarrow \pi^+ n$. The overall agreement is quite good though the CHPT predictions lie on the large side of the most recent data. Clearly, we need more precise data together with a better knowledge of the pion nucleon coupling constant to draw a final conclusion.

With the determination of the threshold value of $E_{0+}$ for the reaction $\gamma n \rightarrow \pi^- p$ one can deduce the value of the difference between the isospin $3/2$ and the
isospin 1/2 $\pi N$ scattering length through the following relation based on time reversal invariance

$$(a_3 - a_1)^2 = 9P \frac{q_{\gamma}}{q_{\pi}} E_{0+,\text{thr}}(\gamma n \rightarrow \pi^- p)$$

where $P$ is the experimentally well determined Panofsky ratio ($P = \sigma(\pi^- p \rightarrow \pi^0 n)/\sigma(\pi^- p \rightarrow \gamma n) = 1.543 \pm 0.008$) and $q_{\gamma}$ and $q_{\pi}$ are the CM momenta of the photon and neutral pion at the $p\pi^-$ threshold, respectively. In the third column of table 3, the value of the isovector scattering length $-b_1 = (a_3 - a_1)/3$ as obtained from Eq.(9) is given. It is compared with a direct CHPT calculation of the scattering process $\pi N \rightarrow \pi N$ (note that in that case the result is independent of $g_{\pi N}$ to leading order, see Bernard et al. (1995)). The two values are consistent within the error bars. These results clearly demonstrate the importance of looking at different processes at the same time. One certainly needs a better determination of the LECs, a better control on the residual scale dependence and isospin breaking effects have to be included before one can reach any definite conclusion. For comparison are given the Karlsruhe-Helsinki (KH) result, see Höhler (1983) and the value obtained by Sigg et al. (1995) from the decay width in pionic hydrogen.

| $-b_1 [10^{-3} M_{\pi}^{-1}]$ | CHPT $\pi N \rightarrow \pi N$ | CHPT $E_{0+}$ | KH | decay width |
|-----------------------------|------------------|-----|-----|-------------|
| $92 \pm 4$ | $87.3 \pm 2.4$ | $91.3 \pm 4$ | $96 \pm 7$ |

Table 3: Isovector $\pi N$ scattering length.

4 Neutral Pion Photoproduction

4.1 $\gamma p \rightarrow \pi^0 p$

Neutral pion photoproduction has been a hot topic ever since the Saclay and Mainz groups claimed a sizeable deviation from a so-called low energy theorem (LET) for the electric dipole amplitude $E_{0+}$ derived in 1970. In the MIT proceedings (1994), I reported on preliminary results which were then confirmed, from an HBCHPT calculation by Bernard et al. (1996(1)). I will briefly summarize some by now well known facts.

- $E_{0+}$: To $O(q^4)$ in HBCHPT two graphs contribute, the famous rescattering graph and the triangle graph. This last important one brings in some non-analytical pieces which invalidate the assumptions used to derive the so-called LET of the seventies, this one thus just being a LEG (low energy guess) (see Ecker and Meißner (1995)). To $O(q^4)$ one has to take into account some more loop diagrams ($1/m$ corrections to the one just discussed and loops with one vertex or with a propagator from $L_{\pi N}^{(2)}$) plus two counterterms. It was
shown that the expansion of $E_{0+}$ in powers of $\mu = M_\pi/m$ is slowly converging ($E_{0+} = -3.45(1 - 1.26 + 0.55)$ where the $M_\pi$, $M_\pi^2$ and $M_\pi^3$ contributions are given, to be compared with Eq.(8)) and therefore hard to pin down accurately. It was concluded that at threshold this multipole cannot provide the best test of chiral dynamics as was first conjectured.

- **P-waves**: LETs have been derived for $P_{1,2}$. In contrast to $E_{0+}$ these are very fastly converging functions of $\mu$. It is thus particularly important to test these directly. As we will see polarization measurements have and will be made for that purpose. It was also shown that the P-waves are of the same chiral order as $E_{0+}$, namely they are proportional to $|q|$ and not $|q||k|$ as usually postulated, where $q$ and $k$ are the pion and photon cm momenta respectively. $P_{1,2}$ have a very week energy dependence, they show no cusp effect and they have very small imaginary parts. To $O(q^3)$ $P_3$ is completely dominated by a counterterm $b_p$ which in the resonance saturation picture is essentially described by the $\Delta(1232)$ resonance.

It turned out that new data from the TAPS collaboration (see Fuchs et al. (1996)) were released and showed some discrepancies to the previously considered best data of Beck et al. (1990). They were analysed in Bernard et al. (1997(2)) using the same HBCHPT formalism as in 1996. The total and differential cross sections were fitted and $E_{0+}$ was then predicted, the result is shown in Fig.1. Note that even so the convergence for this particular observable is slow, a CHPT calculation to order $p^4$ allows to understand its energy dependence in the threshold region. Indeed nice agreement between theory and experiment is obtained as confirmed by table 4 which gives the values of $E_{0+}$ at the $\pi^0 p$ and $\pi^n n$ threshold. For comparison is also shown the result of the dispersion theoretical analysis (DR) of Hanstein et al. (1997). The reaction $\gamma p \rightarrow \pi^0 p$ was also remeasured at...
Saskatoon, see Bergstrom et al. (1996). Between $\pi^0p$ and $\pi^+n$ thresholds, the SAL data are consistent with the ones of Fuchs et al. (1996). For larger energies, however, the new SAL data agree with the older Mainz data, see Beck et al. (1990). This does not affect the threshold value of $E_{0+}$ but rather leads to a larger value of $b_p$. The experimental discrepancy remains to be clarified. It was pointed out by R. Beck at this workshop that a measurement of the so called F asymmetry in a polarized photon (circular) and polarized target ($x$-direction) $\gamma p \rightarrow \pi^0p$ experiment would allow to determine $\text{Re}E_{0+}$ with a small statistical error.

| CHPT       | DR         | Experiment          |
|------------|------------|---------------------|
| $E_{0+}(\pi^0p\text{ thr})$ | $-1.16$ | $-1.31 \pm 0.8, -1.32 \pm 0.05 \pm 0.06$ |
| $E_{0+}(\pi^+n\text{ thr})$ | $-0.44 \sim -0.4$ | $\sim -0.4$ |

Table 4: Predictions and data for the electric dipole amplitude for the neutral pion photoproduction off protons at the $\pi^0 p$ and $\pi^+ n$ threshold.

Since the MIT workshop no real progress was made on $\text{Im}E_{0+}$. This is still an important piece of information missing on the experimental side since it is related to the change of $\text{Re}E_{0+}$ through a dispersion relation as well as to $\pi N$ scattering length through unitarity. A recent multipole analysis of the Mainz $\gamma p \rightarrow \pi^0 p$ cross section measurements by Bernstein et al. (1997(1)) seems to indicate some sensitivity to the assumed energy dependence of the P-wave multipoles. As already emphasized in the MIT proceedings (1994) a measurement of the polarized target asymmetry $T$ ($T$ is sensitive to a linear combination of P-wave multipoles times $\text{Im}E_{0+}$) is awaited (see also Bernstein (1997(2)) for a discussion of this topic). There are actually two proposals for doing a polarized target ($y$-direction) $\gamma p \rightarrow \pi^0 p$ experiment, one at DUKE (B. Norum) and one at MAMI (A. Bernstein).

The present status of what is known from unpolarized target experiment concerning the P-waves is summarized in table 5. There are given the three P-waves $p_1$, $p_{23}$ with $p_{23}^2 = (p_2^2 + p_3^2)/2$ and $e_{1+}$ where the small letters refer to the reduced multipoles $P_i = p_i|k||q|$ where $P_i$ is defined in Eq.(2). I kept here the standard assumed behaviour of the P-waves though as pointed out before, it is not the proper one. Experimental numbers are from Fuchs et al. (1996) (first number for $p_1$ and $p_{23}$ ) and Bergstrom et al. (1997) (second number for $p_1$ and $p_{23}$ and number for $e_{1+}$). Theory and experiment agree quite nicely for $p_1$ and $p_{23}$. Note that there is no prediction for $p_{23}$ in the case of CHPT since as pointed out previously, the value of the $p_3$ multipole mainly comes from a counterterm fitted to experiment. Still it is important to notice that the value of $b_p$ is in excellent agreement with the resonance exchange estimate. In the absence of polarization data, a unique separation of the three P-wave multipoles is not possible. Thus in Bergstrom (1994) milder constraints provided by the Virginia Polytechnic Institute multipole analysis, and certain theoretical considerations have been applied to effect the separation, thus leading to a value for $e_{1+}$. The
one given in table 5 comes from a refined analysis of pion angular distributions, however the error bar is so large that it is hard to conclude anything at present. Also note that the CHPT calculation was done to $O(q^3)$ which means that only the first term in the $\mu$ expansion of $e_{1+}$ is known. It is certainly necessary to go one order higher to see how big the first correction is. Work by the BKM collaboration in this direction is in progress.

|       | CHPT  | DR            | Experiment          |
|-------|-------|---------------|---------------------|
| $p_1$ | 10.33 | 10.52         | 10.02 ± 0.15, 10.26 ± 0.10 |
| $p_{23}$ fit to exp | 10.85 | 11.44 ± 0.09, 11.62 ± 0.08 |
| $e_1$  | -0.11 | -0.15         | -0.25 ± 0.16 (from unpol.) |

Table 5: Predictions and data for the reduced p-waves multipoles in units of $|q||k|10^{-3}/M_{\pi^+}^3$

What is certainly needed is a measurement of the photon asymmetry $\Sigma$ in a polarized photon experiment. Indeed this quantity defined as

$$\Sigma(\theta) = \frac{1}{\epsilon_\gamma} \frac{d\sigma^\perp}{d\Omega} - \frac{d\sigma^\parallel}{d\Omega}$$

(10)

where $\epsilon_\gamma$ is the degree of linear photon polarization, is proportional to $P_2$:

$$\Sigma(\theta) = \frac{1}{\epsilon_\gamma} \frac{q}{k} \frac{1}{d\sigma_0/d\Omega} \frac{1}{2} (P_{3,2}^2 - P_{3,2}^2) \sin^2 \theta$$

(11)

Thus measuring $\Sigma$ allows to test the LET for $P_2$ and also knowing the multipoles $P_1$ and $P_{23}$ from unpolarized target to have an unambiguous determination of the three multipoles $M_{1+}$, $M_{1-}$ and $E_{1+}$. Data taking was completed in Mainz by Ahrens et al. (1994) in the threshold region ($E_\gamma = 145 - 190$ MeV). Preliminary results were presented by R. Beck at this workshop. The preliminary measured value of $\Sigma$ of $(8 \pm 5\%)$ for $\theta = 80^\circ - 100^\circ$ and $E_\gamma = 155$ MeV is in good agreement with the CHPT prediction of 10%.

In the calculations presented here the $\Delta(1232)$ resonance is not treated as an explicit degree of freedom. It enters in the counterterms through the principle of resonance saturation. It was argued by Jenkins and Manohar (1991) that this resonance should be explicitly taken into account since its mass is very close to the nucleon mass ($\Delta = m_\Delta - m_N \sim 3F_\pi$) and the couplings of the $N\Delta$ system to pions and photons are very strong, e.g. $g_{\pi N \Delta} \sim 2g_{N \pi N}$. Recently, Hemmert et al. (1997(1)) proposed a systematic way of including the $\Delta(1232)$ based on an effective Lagrangian of the type $L_{\text{eff}}[U,N,\Delta]$ which has a systematic "small scale expansion" in terms of three small parameters (collectively denoted as $\epsilon$), $E_\pi/\Lambda$, $M_\pi/\Lambda$, $\Delta/\Lambda$, with $\Lambda \in [M_\rho, m_N, 4\pi F_\pi]$. The method has been applied in particular to the calculation of $E_{0+}$ in neutral pion photoproduction off protons at threshold, see Hemmert et al. (1997(2)). The result obtained is quite similar to the one of Bernard et al. (1997(2)), meaning that in that case no explicit
\( \Delta \) is needed which is quite reasonable. The situation might be different for the P-waves. There one indeed expect larger sensitivity to the \( \Delta \). The inclusion of the \( \Delta \) as an explicit degree of freedom for these multipoles is in progress, see Bernard et al. (1997(3)).

In all the calculations done so far the pion coupling constant \( g_{\pi NN} \) was fixed to the KH value of 13.4. As pointed out by various persons, the chiral predictions thus obtained for \( E_{0+}^{\pi^+n} \), \( E_{0+}^{\pi^+n} \) and \( P_{1}^{\pi^0p} \) are consistently somewhat too big as compared to experiment. Decreasing \( g_{\pi NN} \) to 13.06 \( \pm \) 0.15 one gets very good agreement with experiment for this set of observables. This value turns out to be consistent with various recent determinations, see the proceedings of the Seventh International Symposium on Meson-Nucleon Physics and the Structure of the Nucleon, Vancouver (1997). Of course, one has to check what happens for all quantities. Furthermore, all isospin breaking effects have to be included before one can draw any conclusion. This is one of the major directions which remains to be explored on the theoretical side. Work in this direction has started since the effective Lagrangian including virtual photons has already been constructed by Meißner and Steininger (1997).

4.2 \( \gamma n \to \pi^0n \)

As already discussed, it is very important to have predictions for the reaction \( \gamma n \to \pi^0n \) which is experimentally very difficult to assess. Having fitted the LECs to the proton case Bernard et al. (1996(1)) have determined \( E_{0+}(\pi^0n) \). A much better convergence is obtained due to the fact that the famous triangle diagram already appears to first order which is already \( O(M_\pi^2) \). The first corrections are \( \sim 30\% \). The result is shown on the right panel of Fig.1. The amplitude clearly exhibits the unitary cusp due to the opening of the secondary threshold \( \gamma n \to \pi^-p \to \pi^0n \). It is sizeably larger in magnitude than for the proton (compare with the left panel). This is however not the case at threshold within dispersion relations. These indeed tend to give values of the same size, \( E_{0+}(\pi^0n) = 1.19 \) to be compared with the CHPT result \( E_{0+}(\pi^0n) = 2.13 \) (note that the DR treatment for the neutral channels is less stables than for the charged ones, see discussion in Hanstein et al. (1997)).

5 Neutral pion photoproduction off the deuteron

The elementary neutron amplitude \( \gamma n \to \pi^0n \) can only be inferred indirectly from reactions involving few-nucleon systems like the deuteron or \( ^3\text{He} \). Beane et al. (1997) concentrated on coherent neutral pion production off deuterium in the threshold region and studied the sensitivity of the deuteron electric dipole amplitude \( E_d \) to the elementary neutron amplitude \( E_{0+}^{\pi^0n} \). To tackle this problem it is necessary to use a framework which allows one to systematically include and order the various contributions arising from single and multiple scattering processes. Following the idea of Weinberg (1992) one calculates matrix element
of the type $<\Psi_d|K|\Psi_d>$ by using deuteron wave functions $\Psi_d$ obtained from accurate phenomenological NN potentials and chirally expands the kernel $K$.

Using a large variety of these potentials allows one to assess to which degree of accuracy one is sensitive to the chiral symmetry constraints used in determining the irreducible scattering kernel.

The deuteron S-wave multipole is defined in a similar way as the nucleon one: $T = 2i\mathbf{J} \cdot \mathbf{E}_d$ with $\mathbf{J} = \mathbf{L} + \mathbf{S}$ the deuteron total angular momentum. The slope of the differential cross section at threshold takes the form: $|\mathbf{k}|/|\mathbf{q}|d\sigma/d\Omega|_{\text{thr}} = 8/3|\mathbf{E}_d|^2$. There are two types of contributions to $E_d$:

- **single scattering contribution**

  The single scattering contribution is given by all diagrams where the photon and the pion are absorbed and emitted, respectively, from one nucleon with the second nucleon acting as a spectator (the so-called impulse approximation), leading to

  $$E_{ss}^d = \frac{1 + M_{\pi}/m}{1 + M_{\pi}/m_d} \left\{ \frac{1}{2} \left( E_{0+}^{\pi^0p} + E_{0+}^{\pi^0n} \right) \int d^3p \ \hat{\phi}_f^*(\mathbf{p}) \ \mathbf{e} \cdot \mathbf{J} \ \phi_i(\mathbf{p} - \mathbf{k}/2) \right. $$

  $$- \frac{k}{m} \hat{\mathbf{p}} \cdot \int d^3p \ \hat{\phi}_f(\mathbf{p}) \ P_{1}^{\pi^0p} + P_{1}^{\pi^0n}) \ \phi_i(\mathbf{p} - \mathbf{k}/2) \} , \tag{12}$$

  evaluated at the threshold value $|\mathbf{k}| = k_{\text{thr}} = M_{\pi^0} - M_{d}^2/(2m_d) = 130.1$ MeV and with $\mathbf{J} = (\sigma_1 + \sigma_2)/2$. A number of remarks concerning Eq.(12) are in order. It is important to differentiate between the $\pi^0d$ and the $\pi^0N$ ($N = p, n$) center–of–mass (COM) systems. At threshold in the former, the pion is produced at rest, it has, however, a small three–momentum in the latter, see Koch et al. (1977). Consequently, one has a single–nucleon P–wave contribution proportional to the elementary amplitudes $P_{1}^{\pi^0p}$ and $P_{1}^{\pi^0n}$ as defined in Eq.(2). Their values, $P_{1}^{\pi^0p} = 0.480 |\mathbf{q}| \text{GeV}^{-2}$ and $P_{1}^{\pi^0n} = 0.344 |\mathbf{q}| \text{GeV}^{-2}$ have been taken from the P–wave low–energy theorems found in Bernard et al. (1996(1)) with $\mathbf{q} = \mu (1 - \mu) \mathbf{p} - \mu^2 m (1 - 5\mu/4) \hat{k}/2$ ($\mu = M_{\pi}/m$ and $\mathbf{p}$ is the nucleon three–momentum in the $\pi^0d$ COM system). $E_{ss}^d$ has been evaluated using the Argonne V18, the Reid soft core (RSC), the Nijmegen and the Paris potential, $E_{ss}^d = 0.36 \times 10^{-3}/M_{\pi^+}$ with an uncertainty of $\delta E_{ss}^d = 0.05 \times 10^{-3}/M_{\pi^+}$ due to the various potentials used. The P–wave contribution amounts to a 3% correction to the one from the S–wave, i.e. it amounts to a minor correction. The sensitivity of the single–scattering contribution $E_{ss}^d$ to the elementary neutron–$\pi^0$ amplitude is given by

  $$E_{ss}^d = 0.36 - 0.38 \times (2.13 - E_{0+}^{\pi^0n}) , \tag{13}$$

  all in units of $10^{-3}/M_{\pi^+}$. Consequently, for $E_{0+}^{\pi^0n} = 0$, one has $E_{ss}^d = -0.45$ which is of opposite sign to the value based on the chiral perturbation theory prediction for $E_{0+}^{\pi^0n}$. If one were to use the empirical value for the proton amplitude, the single–scattering contribution would be somewhat reduced.

- **three body contribution**
Fig. 2. Graphs contributing at order $q^4$ to neutral pion photoproduction off deuteron. The circles denote an insertion from $\mathcal{L}^{(2)}_{\pi N}$.

To $\mathcal{O}(q^3)$ only two diagrams contribute at threshold, see Beane et al. (1995). In one the photon couples to the pion in flight, the other is a seagull term which involves the charge exchange amplitude which is expected to dominate the single scattering contribution. To $\mathcal{O}(q^4)$ few more graphs have to be added, these are shown in Fig. 2. These graphs do not involve any new unknown LEC. Furthermore there is no contribution from a possible four-fermion contact terms. One can therefore calculate $E_d$ in a parameter-free manner. One finds:

$$E_d = E_{d}^{ss} + E_{d}^{tb,3} + E_{d}^{tb,4} = 0.36 - 1.90 - 0.25 = -1.8 \pm 0.2 \quad (14)$$

in units of $10^{-3}/M_{\pi^+}$. The theoretical error is an educated guess. As expected $E_{d}^{tb,3}$ is rather large. Note that $tb, 4$ is much smaller than $tb, 3$ which signals a good convergence. To see the sensitivity to the elementary neutron $\pi^0$ amplitude, one sets the latter to zero and find $E_d = -2.6 \times 10^{-3}/M_{\pi^+}$ which is considerably different from the chiral theory prediction, Eq.(14). For other values of $E_{d}^{n,n}$, $E_d$ can be calculated from Eq.(13). Obviously, the sensitivity to the neutron amplitude is sizeable and is not completely masked by the larger charge–exchange amplitude as it is often stated. On the experimental side, there exist one determination of $E_d$, which is a reanalysis of older Saclay data by Argan et al. (1988) giving $E_d = -1.7 \pm 0.2 \times 10^{-3}/M_{\pi^+}$ in good agreement with the CHPT result. This number however has to be taken with care since the extraction of the empirical number relies on the input from the elementary proton amplitude to fix a normalization constant. A more precise experimental determination of $E_d$ has just began. Indeed recent data taking have been performed in Saskatoon with the preliminary result $E_d = -1.45 \pm 0.09 \times 10^{-3}/M_{\pi^+}$, see these proceedings. This is somewhat smaller in magnitude than the CHPT value. The possible role
of higher order unitarity corrections has been discussed by Wilhelm at this workshop. Also isospin breaking effects have to be included before one can draw any definite conclusion. Another source of information will come from an approved experiment at the Mainz Microton to measure the threshold cross section for coherent neutral pion electroproduction off deuterium at a photon virtuality of \( k^2 = -0.075 GeV^2 \). It will certainly be necessary to have a CHPT calculation to compare with.

6 Neutral pion electroproduction off protons

Producing the pion with virtual photons offers further insight since one can extract the longitudinal S-wave multipole \( L_{0+} \) and also novel P-wave multipoles. The CHPT calculation proceeds in exactly the same way as for photoproduction, see Bernard et al. (1996(3)). One has, however, some new counterterms. In addition to the expected form factor corrections, \( E(k^2) = L(k^2) \propto M_\pi k^2 \delta r_{1p} \) (where \( \delta r_{1p} \) is fixed from the knowledge of the isovector nucleon radius) one has two new LECs, \( a_3 \) and \( a_4 \) for the S-wave multipoles which are such that by definition \( L - E \sim 1 + \rho (\rho = -k^2/M_\pi^2) \),

\[
\begin{align*}
E^{a_4}(k^2) &= e M_\pi \left\{ (a_1 + a_2) M_\pi^2 - a_3 k^2 \right\}, \\
L^{a_4}(k^2) &= e M_\pi \left\{ (a_1 + a_2) M_\pi^2 - a_3 k^2 + a_4 (M_\pi^2 - k^2) \right\}.
\end{align*}
\tag{15}
\]

It turns out that these are strongly constrained by a soft-pion theorem, see Bernard et al. (1994). It implies that there is a strong correlation between the counterterms of \( E \) and \( L \) to order \( q^4 \) and furthermore that \( L^{a_4} \) is \( k^2 \)-independent. To \( O(q^4) \) with the \( k^2 \)-dependence of \( L(k^2) \) coming solely from the Born and loop graphs one is unable to fit the existing data. The first corrections to the soft-pion constraint \( (a_3 + a_4 = 0) \) away from the chiral limit have to be included. This induces terms of the type \( E_{0+}^{a_1}, L_{0+}^{a_1} \sim a_5 M_\pi^2 k^2 \) which are arising from terms in the Lagrangian \( \mathcal{L}_{\pi N}^{a_1} \) and are thus of higher order. These are the minimal terms one has to take to be able to describe the data at \( k^2 = -0.1 \ GeV^2 \). Of course, there are other counterterms at this order. These, however, merely amount to quark mass renormalizations of the already considered \( k^2 \)-independent counter terms and have therefore been set to zero. In Bernard et al. (1996(3)) a combined fit to the NIKEHF (\( \epsilon = 0.67 \)), see van den Brink et al. (1995) and the MAMI (\( \epsilon = 0.582 \) and 0.885), see Distler (1997), data at \( k^2 = -0.1 \ GeV^2 \) has been performed. Re \( E_{0+} \), Re \( L_{0+} \) and the S-wave cross section \( a_0 = |E_{0+}|^2 + \epsilon L |L_{0+}|^2 \) have been then predicted. As can be seen from Fig. 3 there is a very nice agreement between theory and experiment. One notices that Re \( E_{0+} \) has changed sign as compared to the photoproduction case, it shows the typical cups effect at the opening of the \( \pi^+ \) threshold. In contrast, Re \( L_{0+} \) is essentially energy-independent with a very small cusp. Note that \( a_0 \) is completely dominated by the \( L_{0+} \) multipole (dot-dashed line) since \( E_{0+} \) passes through zero at \( k^2 \approx -0.04 \ GeV^2 \). It should be however kept in mind that as one reaches values of \( k^2 \) of the order of \( -0.1 \ GeV^2 \) the one loop corrections are large so one should better
Fig. 3. Upper left panel: $\text{Re } E_{0^+}$, upper right panel: $\text{Re } L_{0^+}$ at $k^2 = -0.1 \text{ GeV}^2$. The diamonds are the NIKHEF data and the boxes the Mainz data. Lower part: S-wave cross section $a_0$ for $\epsilon = 0.67$ (solid line) in comparison to the data from Welch (boxes) and from van den Brink (cross). The dot-dashed line is the contribution from $\epsilon L_{0^+}$. 

compare at lower virtualities. In Bernard et al. (1996(3)), many predictions for $k^2 \approx -0.05 \text{ GeV}^2$ are given. At MAMI, data have been taken at $k^2 = -0.05 \text{ GeV}^2$. The analysis is underway, see H. Merkel, these proceedings. Concerning the P-wave multipoles novel low−energy theorems have been derived to $O(q^3)$, see Bernard et al. (1995). The combination $M_{1^+} - M_{1^-}$ shows a rather good convergence. In the case of $L_{1^+}, L_{1^-}$ and $E_{1^+}$, an $O(q^4)$ calculation is mandatory to have an idea of the next to leading order contributions. Note that these P−wave LETs have indeed been used in the analysis of the NIKHEF data, see van den Brink et al. (1995).

7 Conclusions

Since the MIT workshop considerable progress has been made:

- charged pion photoproduction has been calculated to $O(q^4)$,
- the energy dependence of $E_{0^+}$ in the reaction $\gamma p \rightarrow \pi^0 p$ in the threshold region has been understood,
New developments in threshold pion photo- and electroproduction

- A study of the sensitivity of $\pi^0$ photoproduction off deuteron to the neutron amplitude has been performed showing a rather large one,

- A fit to the data for $\pi^0$ electroproduction off protons at $k^2 = -0.1$ GeV$^2$ has been performed allowing to make predictions at smaller virtualities,

- another interesting process, which I had no time to discuss here, is double neutral pion photoproduction. It is found within an HBCHPT calculation to $O(q^4)$, see Bernard et al. (1996(4)), that this photoproduction channel is significantly enhanced close to threshold due to pion loops. The experimental analysis of the TAPS data is underway, see Ströher (1997).

More precise low–energy data as well as more refined calculations are still needed to further test the chiral dynamics of QCD, one of the most important question now being related to isospin violation in the pion-nucleon interaction and the role of electromagnetic corrections. Work along these lines has recently started, see Meißner and Steininger (1997).

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