A superspace description of Chern-Simons theory in Batalin-Vilkovisky formulation

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We discuss the extended BRST and anti-BRST symmetry (including shift symmetry) in the Batalin-Vilkovisky (BV) formulation for the Chern-Simons (CS) theories in (2 + 1) spacetime dimensions. Further we develop the superspace description of BV formulation for such theories. Interestingly, the extended BRST invariant CS theories can be described in superspace in covariant manner with the help of one more (Grassmann) coordinate. However, a superspace with two Grassmann coordinates are required for a manifestly covariant formulation of the extended BRST and extended anti-BRST invariant actions for these theories.

I. INTRODUCTION

In recent past years, the CS gauge theories have been studied considerably with great interests [1–7]. The (2 + 1) dimensional CS theories with compact gauge group give natural explanations [2] for many constructions in conformal field theory and integrable lattice models that have been intensively studied [3]. The CS theories also get relevance in anti-de-Sitter (adS) supergravity theories [4]. The CS theories, quantized particularly in axial gauge, obey the topological supersymmetry which was known to hold in the Landau (covariant) gauge [6, 7]. Topological field theories are a class of gauge models with the peculiarity that their observables are of topological nature, as for instance the knot and link invariants in the case of (2 + 1)-dimensional CS theories [2]. The axial gauge has peculiar interest for the CS theories and other topological field models [2, 8, 9] since in this gauge the finiteness problem is obvious due to the complete absence of radiative corrections. The Green functions of the CS theories quantized in axial gauge are shown to be calculable as the unique exact solutions of the Ward identities which express the invariance of the theory under topological supersymmetry [7, 10]. Another important feature of topological supersymmetry, which makes it physically relevant, is its role in the construction of observables [11]. The intriguing point of the CS model is the existence of a very large algebra of symmetries.

The BV approach [12–14] is the most powerful quantization algorithm presently available which allows us to deal with very general gauge theories, including those with open or reducible gauge symmetry algebras. The BV method also provides a convenient way of analysing the possible violations of symmetries of the action by quantum effects [14]. The BV formalism, also known as field antifield formulation (previously and independently introduced by Zinn-Justin [15]) generalizes the BRST approach [16, 17]. It is usually used as a covariant method to perform the gauge-fixing in quantum field theory, but was also applies to other problems like analysing possible deformations of the action and anomalies.

A superspace description [18, 20] for the non-anomalous gauge theories in BV formulation has been studied extensively [21, 22]. It has been shown that the extended BRST and extended anti-BRST invariant
actions of these theories (including some shift symmetry) in BV formulation \cite{21, 24–27}, yield naturally the proper identification of the antifields through equations of motion. The shift symmetry plays an important role and gets relevance, for instance, in inflation particularly in supergravity \cite{28} as well as in Standard Model \cite{29}. However, in usual BV formulation the antifields are calculated from the expression of gauge-fixing fermion. Recently, this formulation has been extended for the theory of perturbative gravity \cite{30}. We extend such formulation to the topological gauge theory in \((2 + 1)\) dimensions.

In the present work we attempt to provide a superspace version of CS theory in BV formulation. For this purpose, we first consider BRST invariant CS theory in axial and Landau gauges. Then we extend the BRST symmetry of the theory by including shift symmetry. The advantage of making such analysis is that the antifields get their own identifications naturally. Further, we describe the extended BRST invariant CS theory in superspace using only one Grassmann coordinate together with \((2 + 1)\) spacetime dimensions. However, for both extended BRST and extended anti-BRST invariant CS theory we require two Grassmann coordinates.

The plan of this paper is as follows. In sec. II, we discuss the preliminaries about CS theory with its supersymmetric BRST invariance. Further, in Sec. II, we demonstrate the extended BRST invariant theory (including shift symmetry) where antifields gets their identifications naturally. The extended BRST invariant superspace formulation of the theory is discussed in Sec. IV. Sec. V is devoted to study the extended anti-BRST symmetry of the theory. In sec. VI, we analyses both extended BRST as well as extended anti-BRST invariant CS theory in superspace. We summaries our results in the last section.

II. THE CS THEORY AND ITS BRST INVARIANCE

In this section, we discuss the preliminaries of CS theory with its BRST invariance. In this view, the CS term in \((2 + 1)\) flat spacetime dimensions is given by the following gauge invariant Lagrangian density

\[
\mathcal{L}_{CS} = -\text{Tr} \left[ \frac{k}{4\pi} \mu^{\nu\rho} \left( A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho \right) \right],
\]

(1)

where the inverse of the coupling constant \(k\) is an integer and \(A_\mu\) is a Lie algebra valued gauge field, the corresponding group being chosen to be simple and compact. The importance of the Chern-Simons action \(1\) lies in the fact that being the integral of a 3-form it does not depend on the metric which one can introduce on three dimensional manifolds. This important feature, therefore, allows to compute topological invariants of three dimensional manifolds by using perturbative field theory techniques \(2\). The topological character of the Lagrangian density \(1\) is the origin of the ultraviolet finiteness of the perturbative Feynman diagrams expansion. This Lagrangian density yields a vertex functional which obeys the Callan-Symanzik equation with vanishing \(\beta\)-function and no anomalous dimensions \(8, 31\). This Lagrangian density possesses following infinitesimal gauge invariance:

\[
\delta A_\mu = D_\mu \theta = \partial_\mu \lambda + i[\lambda, A_\mu],
\]

(2)

\(\lambda\) is a Lie algebra valued local parameter. In order to fix the redundancy of gauge freedom in the CS theory due to above gauge invariance \(1\), although there are many choices but we adopted the axial gauge

\[
n^\mu A_\mu = 0,
\]

(3)

where \(n^\mu\) is an arbitrary constant vector.

The gauge restriction can be achieved at quantum level by adding following gauge-fixing and corresponding ghost terms in the CS action \(1\):

\[
\begin{align*}
\mathcal{L}_{gf} &= \text{Tr} (B n^\mu A_\mu), \\
\mathcal{L}_{gh} &= -\text{Tr} \left( \bar{C} n^\mu D_\mu C \right),
\end{align*}
\]

(4)
where \( C \) and \( \bar{C} \) are Faddev-Popov ghost and anti-ghost fields respectively.

Now, the total Lagrangian density for CS theory in axial gauge is given by
\[
\mathcal{L} = \mathcal{L}_{CS} + \mathcal{L}_{gf} + \mathcal{L}_{gh},
\]
which is invariant under following nilpotent BRST transformations:
\[
\begin{align*}
    sA_\mu &= D_\mu C = \partial_\mu C + i[c, A_\mu], \\
    sC &= iC^2, \\
    s\bar{C} &= B, \\
    sB &= 0.
\end{align*}
\]
(6)

Where \( C^2 \) is defined as
\[ C^2 \equiv i f^{abc} C^b C^c, \]

The combination of gauge-fixing and ghost terms is BRST exact and, hence, can be written in terms of BRST variation of gauge-fixing fermion \( \Psi = (\eta^{\mu\nu} \bar{C} n_\mu A_\nu) \) as follows
\[
\mathcal{L}_{gf} + \mathcal{L}_{gh} = \text{Tr} [s(\eta^{\mu\nu} \bar{C} n_\mu A_\nu)],
\]
(7)

where \( \eta^{\mu\nu} \) is the Minkowski metric of the 3-dimensional flat spacetime.

However, for the Landau (covariant) gauge choice the gauge-fixing and ghost terms have the following expressions:
\[
\begin{align*}
    \mathcal{L}_{gf}' &= \text{Tr} (B \partial^\mu A_\mu), \\
    \mathcal{L}_{gh}' &= -\text{Tr} (\bar{C} \partial^\mu D_\mu C).
\end{align*}
\]
(8)

Employing these terms, the total Lagrangian density in Landau gauge reads
\[
\mathcal{L}' = \mathcal{L}_{CS} + \mathcal{L}_{gf}' + \mathcal{L}_{gh}' = \mathcal{L}_{CS} + \text{Tr} [s(\eta^{\mu\nu} \bar{C} \partial_\mu A_\nu)],
\]
(9)

which also remains invariant under the same set of BRST transformation (10).

### III. Extended BRST Invariant Lagrangian Density

In this section, we analyses the extended BRST transformations for CS theory in BV formulation. The advantage of doing so is that antifields get identification naturally. We start the analysis by shifting all the fields from their original value as follows
\[
A_\mu \rightarrow A_\mu - \tilde{A}_\mu \quad C \rightarrow C - \tilde{C} \quad \bar{C} \rightarrow \bar{C} - \tilde{C} \quad B \rightarrow B - \tilde{B}.
\]
(10)

Under such shifting of fields the Lagrangian densities (5) and (9) also get shifted respectively as follows:
\[
\begin{align*}
    \tilde{\mathcal{L}} &= \mathcal{L}(A_\mu - \tilde{A}_\mu, C - \tilde{C}, \bar{C} - \tilde{C}, B - \tilde{B}), \\
    \tilde{\mathcal{L}}' &= \mathcal{L}'(A_\mu - \tilde{A}_\mu, C - \tilde{C}, \bar{C} - \tilde{C}, B - \tilde{B}).
\end{align*}
\]
(11)

These shifted Lagrangian densities remain invariant under BRST transformation in tandem with shift symmetry transformation, commonly known as extended BRST transformation. The Lagrangian densities (11) and (12) admit the following extended BRST symmetry transformations:
\[
\begin{align*}
    sA_\mu &= \psi_\mu, \\
    s\tilde{A}_\mu &= \psi_\mu - D_\mu (C - \tilde{C}),
\end{align*}
\]
\[ sC = \epsilon, \]
\[ s\tilde{C} = \epsilon - i(C - \tilde{C})^2, \]
\[ s\bar{C} = \bar{\epsilon}, \]
\[ s\tilde{\bar{C}} = \bar{\epsilon} - (B - \tilde{B}), \]
\[ sB = \rho, \]
\[ s\tilde{B} = \rho, \]
\[ (13) \]

where \( \psi_\mu, \epsilon, \bar{\epsilon} \) and \( \rho \) are the ghost fields associated with shift symmetry for \( A_\mu, C, \tilde{C} \) and \( B \) respectively. To preserve the nilpotency of extended BRST symmetry (13) the ghost fields are required to have following BRST transformation

\[ s\psi = 0, \]
\[ s\epsilon = 0, \]
\[ s\bar{\epsilon} = 0, \]
\[ s\rho = 0. \]
\[ (14) \]

We further need to introduce the anti-fields for the ghost fields \( A^*_\mu, C^*, \tilde{C}^* \) and \( B^* \) in the theory to make it ghost free. The BRST variation of anti-ghost fields are given by

\[ sA^*_\mu = -\zeta_\mu, \]
\[ sC^* = -\sigma, \]
\[ s\tilde{C}^* = -\bar{\sigma}, \]
\[ sB^* = -\bar{\upsilon}, \]
\[ (15) \]

where \( \zeta_\mu, \sigma, \bar{\sigma} \) and \( \bar{\upsilon} \) are the Nakanishi-Lautrup type auxiliary fields corresponding to shifted fields \( \tilde{A}_\mu, \tilde{C}, \tilde{\bar{C}} \) and \( \tilde{B} \) with vanishing BRST variations

\[ s\zeta_\mu = 0, \]
\[ s\sigma = 0, \]
\[ s\bar{\sigma} = 0, \]
\[ s\bar{\upsilon} = 0. \]
\[ (16) \]

Now, our original theory can be recovered by fixing the gauge of shift symmetry properly such that all the tilde fields vanish. We achieve this by adding following gauge-fixed Lagrangian density in the shifted Lagrangian densities (11) and (12):

\[ \tilde{\mathcal{L}}_{gf+gh} = \text{Tr} \left[ -\zeta^\mu \tilde{A}_\mu - A^*_\mu [\psi^\mu - D^\mu (C - \tilde{C})] - \sigma \tilde{C} + C^* [\epsilon - (B - \tilde{B})] \right. \]
\[ \left. - \bar{\sigma} \tilde{\bar{C}} + \bar{\sigma}^* [\epsilon - i(C - \tilde{C})^2] - \bar{\upsilon} \tilde{B} - B^* \rho \right]. \]
\[ (17) \]

The gauge-fixing and ghost terms of Lagrangian density \( \tilde{\mathcal{L}}_{gf+gh} \) is also invariant under the extended BRST symmetry transformations mentioned above. Now, performing equations of motion of auxiliary fields in the above expression we obtain

\[ \tilde{\mathcal{L}}_{gf+gh} = \text{Tr} \left[ -A^*_\mu (\psi^\mu - D^\mu C) + C^* (\epsilon - B) + \tilde{C}^* (\epsilon - iC^2) - B^* \rho \right]. \]
\[ (18) \]

The gauge-fixing and ghost terms of Lagrangian density are BRST exact and can be expressed in terms of a general gauge-fixing fermion \( \Psi \) as

\[ \text{Tr}(s\Psi) = \text{Tr} \left[ sA^*_\mu \frac{\delta \Psi}{\delta A^*_\mu} + sC^* \frac{\delta \Psi}{\delta C} + s\tilde{C}^* \frac{\delta \Psi}{\delta \tilde{C}} + sB^* \frac{\delta \Psi}{\delta B} \right], \]
\[ = \text{Tr} \left[ -\frac{\delta \Psi}{\delta A^*_\mu} \psi_\mu + \frac{\delta \Psi}{\delta C} \epsilon + \frac{\delta \Psi}{\delta \tilde{C}} \bar{\epsilon} - \frac{\delta \Psi}{\delta B} \rho \right], \]
\[ (19) \]
The Lagrangian densities in equations (11), (18) and (19) together describes the complete effective action for CS theory in axial gauge possessing extended BRST symmetry as

$$\mathcal{L}_{\text{eff}} = \tilde{\mathcal{L}} + \mathcal{L}_{gf} + \mathcal{L}_{gh} + \tilde{\mathcal{L}}_{gf+gh},$$

$$= \tilde{\mathcal{L}} + \text{Tr}\left[\left(-A^*_\mu - \frac{\delta \Psi}{\delta A^\mu}\right) \psi^\mu + \left(C^* + \frac{\delta \Psi}{\delta C}\right) \epsilon + \left(C^* + \frac{\delta \Psi}{\delta C}\right) \tilde{\epsilon}ight] + \left(-B^* - \frac{\delta \Psi}{\delta B}\right) \rho + A^*_\mu D^\mu C - C^* B - i\tilde{C}^* C^2\right], \quad (20)$$

where $\Psi$ refers here the gauge-fixing fermion corresponding to the axial gauge. Using equations of motion of the ghost fields associated with shift symmetry, we obtain

$$A^*_\mu = \frac{\delta \Psi}{\delta A^\mu},$$
$$\tilde{C}^* = -\frac{\delta \Psi}{\delta C},$$
$$C^* = -\frac{\delta \Psi}{\delta C},$$
$$B^* = \frac{\delta \Psi}{\delta B}. \quad (21)$$

For the gauge-fixing fermion $\Psi$ corresponding to axial gauge given in (7), the above expressions of anti-ghost fields yield

$$A^*_\mu = \eta_{\mu\nu} \tilde{C} n^\nu,$$
$$\tilde{C}^* = 0,$$
$$C^* = \eta^\mu_{\nu} \eta_{\mu} A^\nu,$$
$$B^* = 0. \quad (22)$$

Plugging these expression of anti-ghost fields in (20), we recover the Lagrangian density of our original CS theory in axial gauge.

However, for the gauge-fixing fermion corresponding to the Landau gauge given in (9), the anti-ghost fields

$$A^*_\mu = -\eta_{\mu\nu} \tilde{\theta}^\nu \tilde{C},$$
$$\tilde{C}^* = 0,$$
$$C^* = \eta^\mu_{\nu} \partial_\mu A^\nu,$$
$$B^* = 0. \quad (23)$$

With these values of anti-ghost fields, we can recover the Lagrangian density of CS theory in Landau gauge.

### IV. EXTENDED BRST INVARIANT SUPERSPACE DESCRIPTION

In this section, we study the extended BRST invariant CS theory in a superspace labelled by the coordinates $(x, \theta)$ where $\theta$ is Grassmann in nature and $x_\mu$ is space time in 2+1 dimension. Superspace description for the extended BRST invariant theory is obtained by defining the superfields of the form:

$$A_\mu(x, \theta) = A_\mu + \theta \psi_\mu,$$
$$\tilde{A}_\mu(x, \theta) = \tilde{A}_\mu + \theta [\psi_\mu - D_\mu (C - \tilde{C})],$$
$$\chi(x, \theta) = C + \theta \epsilon,$$
\[ \tilde{\chi}(x, \theta) = \tilde{C} + \theta[\epsilon - i(C - \tilde{C})^2], \]
\[ \bar{\chi}(x, \theta) = C + \theta \bar{\epsilon}, \]
\[ \tilde{\bar{\chi}}(x, \theta) = \tilde{\bar{C}} + \theta[\bar{\epsilon} - (B - \tilde{B})], \]
\[ B(x, \theta) = B + \theta \rho, \]
\[ \tilde{B}(x, \theta) = \tilde{B} + \theta \tilde{\rho}. \] (24)

On the other hand, the super-antifields utilizing the extended BRST transformation for antifields are defined by
\[ \tilde{A}^*_\mu(x, \theta) = A^*_\mu - \theta \zeta_\mu, \]
\[ \tilde{\chi}^*(x, \theta) = C^* - \theta \sigma, \]
\[ \tilde{\bar{\chi}}^*(x, \theta) = \tilde{\bar{C}}^* - \theta \bar{\sigma}, \]
\[ \tilde{B}^*(x, \theta) = B^* - \theta \bar{\upsilon}. \] (25)

With the help of these superfields and super-antifields, we calculate
\[ \frac{\delta (\tilde{A}^*_\mu \tilde{A}^\mu)}{\delta \theta} = -A^*_\mu[\psi^\mu - D^\mu(C - \tilde{C})] - \zeta_\mu \tilde{A}^\mu, \]
\[ \frac{\delta (\tilde{\chi}^* \tilde{\chi})}{\delta \theta} = \tilde{C}^*[\epsilon - i(C - \tilde{C})^2] - \sigma \tilde{C}, \]
\[ \frac{\delta (\tilde{\bar{\chi}}^* \tilde{\bar{\chi}})}{\delta \theta} = -\bar{\sigma} \tilde{C} + C^*[\bar{\epsilon} - (B - \tilde{B})], \]
\[ \frac{\delta (\tilde{B}^* \tilde{B})}{\delta \theta} = -B^* \rho - \bar{\upsilon} \tilde{B}. \] (26)

Combining all the equations of (26), we find that
\[ \text{Tr} \left[ \frac{\delta}{\delta \theta} (\tilde{A}^*_\mu \tilde{A}^\mu + \tilde{\chi}^* \tilde{\chi} + \tilde{\bar{\chi}}^* \tilde{\bar{\chi}} + \tilde{B}^* \tilde{B}) \right] = \text{Tr} \left[ -A^*_\mu[\psi^\mu - D^\mu(C - \tilde{C})] - \zeta_\mu \tilde{A}^\mu + \tilde{C}^*[\epsilon - i(C - \tilde{C})^2] \right. \]
\[ - \bar{\sigma} \tilde{C} + C^*[\bar{\epsilon} - (B - \tilde{B})] - B^* \rho - \bar{\upsilon} \tilde{B}, \] (27)

which is nothing but the shifted gauge-fixed Lagrangian density \( \hat{L}_{gf + gh} \) given in (18). Now, we define the general super-gauge-fixing fermion written in superspace formulation as follows
\[ \Phi(x, \theta) = \Psi(x) + \theta(s \Psi), \] (28)

which can further be expressed as
\[ \Phi(x, \theta) = \Psi(x) + \theta \left( -\frac{\delta \Psi}{\delta A_{\mu}} \psi_{\mu} + \frac{\delta \Psi}{\delta C} \epsilon + \frac{\delta \Psi}{\delta \bar{C}} \bar{\epsilon} - \frac{\delta \Psi}{\delta B_{\rho}} \right). \] (29)

So, the original gauge-fixing Lagrangian density in the superspace can be defined as the left derivation of super-gauge-fixing fermion with respect to \( \theta \) as \( \text{Tr} \left[ \frac{\delta \Phi(x, \theta)}{\delta \theta} \right] \).

Hence, the complete effective action for the CS theory in general gauge in the superspace is now given by
\[ \mathcal{L}_{gen} = \hat{\mathcal{L}} + \text{Tr} \left[ \frac{\delta}{\delta \theta} (\tilde{A}^*_\mu \tilde{A}^\mu + \tilde{\chi}^* \tilde{\chi} + \tilde{\bar{\chi}}^* \tilde{\bar{\chi}} + \tilde{B}^* \tilde{B} + \Phi) \right]. \] (30)

This compact expression indicates that the BV action of the extended CS theory in superspace is invariant under extended BRST transformations.
V. EXTENDED ANTI-BRST LAGRANGIAN DENSITY

In this section, we construct the extended anti-BRST transformation under which the extended Lagrangian density remains invariant as follows

\[
\begin{align*}
\bar{s}A_\mu &= A_\mu^* + D_\mu(\bar{C} - \tilde{\bar{C}}), \\
\bar{s}\tilde{A}_\mu &= A_\mu^*, \\
\bar{s}C &= C^* + (B - \tilde{B}), \\
\bar{s}\tilde{C} &= C^*, \\
\bar{s}\tilde{C} &= (\tilde{C}^* - \bar{C}^*) + i(\bar{C} - \tilde{C})^2, \\
\bar{s}\tilde{B} &= B^*, \\
\bar{s}B &= B^*. 
\end{align*}
\] (31)

The ghost fields associated with the shift symmetry under extended anti-BRST symmetry transforms as

\[
\begin{align*}
\bar{s}\psi_\mu &= \zeta_\mu, \\
\bar{s}\epsilon &= \sigma, \\
\bar{s}\tilde{\epsilon} &= \tilde{\sigma}, \\
\bar{s}\rho &= \tilde{\upsilon}. 
\end{align*}
\] (32)

However, under the extended anti-BRST transformation the anti-fields of the auxiliary fields associated with the shift symmetry do not change

\[
\begin{align*}
\bar{s}\zeta_\mu &= 0, \\
\bar{s}A_\mu^* &= 0, \\
\bar{s}\sigma &= 0, \\
\bar{s}C &= 0, \\
\bar{s}C^* &= 0, \\
\bar{s}\tilde{\sigma} &= 0, \\
\bar{s}\tilde{\epsilon} &= 0, \\
\bar{s}\tilde{\sigma} &= 0, \\
\bar{s}\tilde{B} &= 0. 
\end{align*}
\] (33)

The anti-gauge-fixing fermions for the CS theory in axial gauge ($\bar{\Psi}$) and in Landau gauge ($\bar{\Psi}'$) are respectively defined by

\[
\begin{align*}
\bar{\Psi} &= \eta\mu\nu C_{\mu\nu}A_\nu, \\
\bar{\Psi}' &= \eta\mu\nu C\partial_\mu A_\nu. 
\end{align*}
\] (34)

The anti-BRST variation of these gauge-fixing fermions give the corresponding gauge-fixing and ghost parts of the complete Lagrangian density.

VI. EXTENDED BRST AND ANTI-BRST INvariant SUPERSPACE

The extended BRST and anti-BRST invariant Lagrangian density is written in superspace with the help of two Grassmannian coordinates $\theta$ and $\bar{\theta}$. Requiring the field strength to vanish along unphysical directions $\theta$ and $\bar{\theta}$ direction we determine the superfields in the following forms

\[
\begin{align*}
A_\mu(x, \theta, \bar{\theta}) &= A_\mu(x) + \theta \psi_\mu + \theta[A_\mu^* + D_\mu C] + \theta\bar{\theta} [\zeta_\mu + \partial_\mu \bar{\epsilon}], \\
\bar{A}_\mu(x, \theta, \bar{\theta}) &= \bar{A}_\mu(x) + \theta [\bar{\psi}_\mu - D_\mu(\bar{C} - \tilde{\bar{C}})] + \theta A_\mu^* + \theta\bar{\theta} \zeta_\mu, \\
\chi(x, \theta, \bar{\theta}) &= C(x) + \theta \epsilon + \theta\bar{\epsilon}[C^* + (B - \tilde{B})] + \theta\bar{\theta} \sigma, \\
\tilde{\chi}(x, \theta, \bar{\theta}) &= \tilde{C}(x) + \theta [\bar{\epsilon} - iCC] + \bar{\theta} C^* + \theta\bar{\theta} \sigma, \\
\bar{\chi}(x, \theta, \bar{\theta}) &= \bar{C}(x) + \theta \bar{\epsilon} + \bar{\theta}[\bar{C}^* + i \bar{C}\bar{C}] + \theta\bar{\theta} \bar{\sigma}. 
\end{align*}
\] (34)
\[ \tilde{\chi}(x, \theta, \bar{\theta}) = \tilde{C}(x) + \theta[\bar{\epsilon} - B] + \bar{\theta} \tilde{C}^* + \theta \bar{\theta} \bar{\sigma}, \]
\[ B(x, \theta, \bar{\theta}) = B(x) + \theta \rho + \theta B^* + \theta \bar{\theta} \bar{\rho}, \]
\[ \tilde{B}(x, \theta, \bar{\theta}) = \tilde{B}(x) + \theta \rho + \bar{\theta} B^* + \theta \bar{\theta} \bar{\rho}. \]  
(35)

With these expressions of superfields we are able to establish the following relation
\[ -\frac{1}{2} \text{Tr} \left[ \frac{\partial}{\partial \theta} \frac{\partial}{\partial \bar{\theta}} (\tilde{A}_\mu \tilde{A}^\mu + \tilde{\chi} \tilde{\chi} + \tilde{B} \tilde{B}) \right] = \text{Tr} \left[ -\zeta^\mu \tilde{A}_\mu - A^\mu [\psi^\mu - D^\mu (C - \tilde{C})] - \sigma \tilde{C} + C^* [\bar{\epsilon} - (B - \tilde{B})] \right. \]
\[ - \left. \bar{\sigma} \tilde{C} + \tilde{C}^* [\epsilon - i(C - \tilde{C})^2] - \bar{\sigma} \tilde{B} - B^* \rho \right], \]
(36)

which is nothing but the shifted gauge-fixed Lagrangian density. Being the \( \theta \bar{\theta} \) component of a superfield, this gauge-fixed Lagrangian density is manifestly invariant under the extended BRST and the anti-BRST transformations.

Now, we define the general super-gauge-fixing fermion for the extended BRST and the anti-BRST invariant theory as follows
\[ \Phi(x, \theta, \bar{\theta}) = \Psi(x) + \theta(s \Psi) + \bar{\theta}(\bar{s} \Psi) + \bar{\theta} \bar{\theta} (s \bar{s} \Psi), \]  
(37)

which yields the original gauge-fixing and ghost part of the complete effective Lagrangian density as
\[ \text{Tr} \left[ \frac{\partial}{\partial \bar{\theta}} \left[ s(\bar{\theta}) \Phi(x, \theta, \bar{\theta}) \right] \right]. \]

Therefore, the complete Lagrangian density for the extended BRST and anti-BRST invariant CS theory in the general gauge can now be given by
\[ \mathcal{L}_{gen} = \mathcal{L} - \frac{1}{2} \text{Tr} \left[ \frac{\partial}{\partial \theta} \frac{\partial}{\partial \bar{\theta}} (\tilde{A}_\mu \tilde{A}^\mu + \tilde{\chi} \tilde{\chi} + \tilde{B} \tilde{B}) \right] + \text{Tr} \left[ \frac{\partial}{\partial \bar{\theta}} \left[ s(\bar{\theta}) \Phi(x, \theta, \bar{\theta}) \right] \right]. \]  
(38)

Performing equations of motion of auxiliary fields the shift fields can be removed from the above expression and by integrating out the ghost fields for the shift symmetry we obtain the exact expressions of antifields.

**VII. CONCLUSION**

The (2+1) dimensional CS theory is subject of current interest because of its some intriguing properties. For example, the Green functions for the the model in axial gauge are shown the unique and exact solution of the Ward identities without reference to any action principle [32]. It is also well-known that in axial gauge the the Faddeev-Popov determinant of this gauge-fixing procedure is a constant function [3].

In this work we have considered (2 + 1) dimensional CS theory in both the axial and the Landau gauges and have attempted to describe the extended BRST and anti-BRST invariant (including some shift symmetry) CS theory in BV formulation. We show that antifields arises naturally in such formulation. We have further provided superspace and superfield description of such CS theory. We have shown that the BV action for such CS theory can be written in a manifestly extended BRST invariant manner in a superspace with one fermionic coordinate. However, a superspace with two Grassmann coordinates are required for a manifestly covariant formulation of the extended BRST and extended anti-BRST invariant BV actions for CS gauge theory in any arbitrary gauge. It will be interesting to extend this formulation for anomalous gauge theories.
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