ν-GMSB with Type III Seesaw and Phenomenology

R. N. Mohapatra∗,  
Maryland Center for Fundamental Physics and Department of Physics,  
University of Maryland, College Park, MD 20742, USA

Nobuchika Okada†,  
Maryland Center for Fundamental Physics and Department of Physics,  
University of Maryland, College Park, MD 20742, USA and  
Theory Group, KEK, 1-1 Oho, Tsukuba, 305-0801, Japan

Hai-Bo Yu‡  
Department of Physics and Astronomy,  
University of California, Irvine, CA 92697, USA  
(Dated: July, 2008)

Abstract

We show that when the supersymmetric SU(5) model is extended to explain small neutrino masses by the type III seesaw mechanism, the new $24$-dimensional fields needed for the purpose can act as messengers for transmitting SUSY breaking from a hidden sector to the visible sector. For the three $24$ case, the constraints of grand unification and suppressed lepton flavor violation restrict the seesaw scale in this case to be in the narrow range of $10^{12} - 10^{13}$ GeV. The model predicts (i) a stable LSP gravitino with mass in the range of 1-10 MeV which can be a cold dark matter of the universe; (ii) a stau NLSP which is detectable at LHC; (iii) a lower bound on the branching ratio $BR(\mu \rightarrow e\gamma)$ larger than $10^{-14}$ testable by the ongoing MEG experiment as well as characteristic particle spectrum different from other SUSY breaking scenarios. We also discuss the case with two $24$ fields, which is the minimal case that can explain neutrino oscillation data.

∗ e-mail: rmohapat@physics.umd.edu  
† e-mail: okadan@post.kek.jp  
‡ e-mail: haiboy@uci.edu
I. INTRODUCTION

Supersymmetry (SUSY) is considered to be a main ingredient for TeV scale physics since it resolves several conceptual issues of the Standard Model (SM) such as (i) gauge hierarchy problem and (ii) electroweak symmetry breaking while providing a natural candidate for the dark matter of the universe and predicting unification of forces at a very high scale.

Supersymmetry must however be broken to be in accord with observations and understanding the origin and nature of SUSY breaking is a primary focus of research in particle physics today. One interesting proposal is the so-called gauge mediated supersymmetry breaking (GMSB) where SUSY is broken in a hidden sector and is transmitted via gauge forces to the Standard Model sector [1]. The transmission of SUSY breaking to the visible sector is carried out by SM gauge non-singlet vector like pairs of superfields (known as messenger fields) via their gauge couplings.

There are several aspects of this interesting proposal that one might like to improve to make it more appealing: (i) the messenger fields in GMSB models are generally introduced solely for the purpose of transmitting the SUSY breaking information and play no other role, which therefore allows considerable freedom in building models, making it less easy to test them; (ii) in simple GMSB scenarios, it is generally hard to understand the magnitude of $B\mu$ term [2]; (iii) typical GMSB models are specified by two arbitrary hidden sector parameters: the SUSY breaking strength $F$ and messenger mass $M$, with the soft breaking parameters in the visible sector given by $\frac{\alpha_4}{4\pi}F/M$; it would make the model more predictive if either of these parameters could be further constrained (or determined) from independent physics considerations; (iv) in simple GMSB scenarios where the messenger fields are chosen to be $5 + \bar{5}$ fields under SU(5) group, the messenger and SM matter fields can mix leading to large flavor changing effects [3]. Any new suggestion that remedies one or more of these problems is certainly worth serious investigation.

Most of the above considerations for GMSB are done in the context of Minimal Supersymmetric Standard Model (MSSM). However, since the observation of neutrino masses clearly imply extension of MSSM to include new physics, an important question is to study whether such extensions can throw any light on the above problems of GMSB. There exist several such investigations in the literature which show that neutrino mass physics can impact SUSY breaking [4, 5, 6]. We will call the subset of these models that use neutrino mass physics to implement gauge mediated SUSY breaking as $\nu$-GMSB models. One example of this is the work of Ref. [4], which uses type II seesaw mechanism [7].
In this paper we discuss an extension of MSSM embedded into SU(5) GUT where small neutrino masses are explained via the type III seesaw mechanism [8]. These models have interesting phenomenology and have been studied in several recent papers [9]. The basic idea of type III seesaw is to add SM triplet Higgs fields with zero hypercharge so that they replace the right handed neutrinos of the type I seesaw [10]. When embedded into SU(5) GUT theories, these triplets become part of new 24-dim. Higgs fields that need to be added to the minimal SU(5). We will consider supersymmetric version of this model since gauge coupling unification in these models is automatic in the presence of full SU(5) multiplets.

The new 24-dimensional fields added to minimal SUSY SU(5) [11] play a dual role in the model considered in the present paper: in addition to making neutrino masses small via type III seesaw, they also play the role of messenger fields and transmit the hidden sector SUSY breaking via GMSB. We call this model $\nu$-GMSB with type III seesaw and study its phenomenological implications. Typically current neutrino mass observations require only two 24 fields; however, these 24 fields are like matter fields and having three generations of them has a certain appeal since each 24 goes with one generation. So in the bulk of this paper, we will discuss the model with three 24s and its implications. In a subsequent section, we also discuss the two 24 case, where some of the constraints present in the three 24 case are more relaxed. For both cases, we find that the large dimensionality of messenger fields makes these models distinguishable from other types of GMSB models and testable in near future.

Our main results for the three 24 fields case, are: (i) the prediction of the messenger mass in a narrow range of $10^{12} - 10^{13}$ GeV, which precisely is the range to lead to small neutrino masses; (ii) the prediction of a stable lightest super-partner (LSP) gravitino with mass in the range of 1-10 MeV which can be a cold dark matter of the universe if the reheating temperature after inflation is around $10^5 - 10^6$ GeV; (iii) a lower bound of about 200 GeV for slepton masses with stau being the next to lightest super-partner (NLSP) which is detectable at the Large Hadron Collider (LHC); (iv) a lower bound on the branching ratio $BR(\mu \rightarrow e\gamma)$ larger than $10^{-14}$ testable by the MEG experiment as well as characteristic particle spectrum different from other SUSY breaking scenarios. For the two 24 case, the range of allowed messenger mass is between $10^9$ GeV to $10^{13}$ GeV. This case allows for the possibility of gravitino mass anywhere from few keV to 10 MeV. In the lower mass range, it could be a warm dark matter. This would require reheating temperature closer to a TeV.

The paper is organized as follows. In Sec. II, we present the basic structure of the model and in
Sec. III we examine the upper and lower limits on the messenger mass scale from gauge coupling unification and lepton flavor violation; in Sec. IV, we discuss phenomenology and cosmology of this model such as its predictions for particle spectrum, gravitino dark matter and in particular we emphasize an important characteristic prediction of our model that stau is the next LSP (NLSP) which may be observable at the LHC via its gravitino decay mode; in Sec. V, we also discuss the case with two 24 fields, which the minimal case that can explain neutrino oscillation data; Sec. VI is devoted to conclusions.

II. SU(5) MODEL WITH TYPE III SEESAW

Our proposed $\nu$-GMSB is an extension of supersymmetric SU(5) model to accommodate small neutrino masses via the type III seesaw mechanism. It has the following matter $\bar{F}_i(\bar{\bf 5})$, $T_i(\bf 10)$ (i=1,2,3 for generations) and Higgs fields: $\bar{F}_H(\bar{\bf 5}) \oplus F_H(\bf 5)$ and $\Phi(24)$ as in the minimal model extended by the addition of three extra 24-dimensional fields, denoted by $\Sigma_i$, (i=1,2,3). The primary role of these extra fields is to generate small neutrino masses via type III seesaw mechanism. To illustrate this we write the matter part of the superpotential as follows:

$$W_m = Y^i_{d} \bar{F}_i \bar{T}_j F_H T_j + Y^i_{u} T_i T_j F_H + Y^i_{\nu} \bar{F}_i F_H \Sigma_j + M_\Sigma \text{Tr} \Sigma_i^2$$

(1)

In the next section, we will promote $M_\Sigma$ to be the VEV of the singlet hidden sector field that also breaks supersymmetry.

SU(5) breaking and doublet-triplet splitting is achieved by the Higgs part of the superpotential given by:

$$W_H = M_\Phi \text{Tr}(\Phi^2) + \eta \text{Tr}(\Phi^3)$$

(2)

Note that the neutrino masses in this case arise via the diagram in Fig. and are given by the formula similar to type I seesaw formula [10]:

$$M_\nu = -v^2 \sin^2 \beta Y^T_\nu M_\Sigma^{-1} Y_\nu$$

(3)

with the VEV of the up-type Higgs doublet, $\langle H_u \rangle = v \sin \beta$, in MSSM. For $Y_\nu \sim 0.1 - 1$, it gives neutrino masses in the eV range if $M_\Sigma \sim 10^{13}$ or so. We will see that independent considerations such as those from gauge coupling unification and suppressed flavor violation indeed constrain this mass to be in the same range of $10^{12} - 10^{13}$ GeV giving some level of uniqueness to the model.
We will prevent couplings between $\Sigma$ and $\Phi$ fields by a $\mathbb{Z}_2$ symmetry under which all matter fields (including the $\Sigma$ field) are odd and all other fields are even. This also restores R-parity as a good symmetry of the model making the LSP of the model stable making it a possible dark matter of the Universe as we see below.

III. GAUGE MEDIATION BY 24 ($\Sigma$) FIELDS

The goal of this paper is to use the same $\Sigma$-fields used to give small neutrino mass as messengers of SUSY breaking from the hidden sector. For this purpose, first we set $M_\Sigma = 0$ in Eq. (1) and we write the following superpotential:

\[ W_{SSB} = \lambda S \text{Tr} \Sigma_i^2 \]  

We require that $\langle S \rangle \neq 0$ and $\langle F_S \rangle \neq 0$. The masses of the fermionic components of $\Sigma$ fields are given by $M = \lambda \langle S \rangle$ whereas those of the scalar components are given by $M^2_{\Sigma^\pm} = M^2 \pm F_S$ where $\Sigma^\pm = \frac{1}{\sqrt{2}}(\Sigma \pm \Sigma^\dagger)$.

One can now write down the soft SUSY breaking terms. There are two contributions: gauge contribution and Yukawa contribution [12]. For Yukawa couplings involving the $\Sigma$ fields small compared to the gauge couplings (e.g. $Y_\nu \sim 0.1$ and $g_{1,2,3} \gtrsim 0.3$), the gauge coupling contributions dominate and we find sfermion masses to be given by:

\[ m^2_{\tilde{f}}(\mu) = \sum_i 2c_i \left( \frac{\alpha_i(\mu)}{4\pi} \right)^2 \left( \frac{F_S}{M} \right)^2 N_m G_i(\mu, M) , \]  

where

\[ G_i(\mu, S) = \left( \xi_i^2 + \frac{N_m}{b_i}(1 - \xi_i^2) \right) \]  

with

\[ \xi_i \equiv \frac{\alpha_i(M)}{\alpha_i(\mu)} = \left[ 1 + \frac{b_i}{2\pi} \alpha_i(\mu) \ln \left( \frac{M}{\mu} \right) \right]^{-1} . \]  

Here $b_i$ are the beta function coefficients for different groups, $c_i$ are the quadratic Casimirs, $N_m = 15$ is the Dynkin index for three 24-dimensional messenger fields, and the sum is taken corresponding to the representation of the sparticles under the SM gauge groups. For gaugino masses we have

\[ \frac{M_1(\mu)}{\alpha_1(\mu)} = \frac{M_2(\mu)}{\alpha_2(\mu)} = \frac{M_3(\mu)}{\alpha_3(\mu)} = \frac{N_m F_S}{4\pi M} . \]
Note that at the messenger scale, there is a hierarchy between the gaugino mass and sfermion mass because of the large Dynkin index, $N_m = 15$. For example, the ratio of the right-handed slepton mass and Bino is found to be $m_{\tilde{e}}^2(M)/M_1^2(M) = 1.2/N_m = 0.08$ at the messenger scale $M$. Therefore, stau is most likely to be the NLSP in our model.

The gravitino mass is given by $m_{3/2} \sim F_S/M_{Pl}$. In order to determine these masses, we need to know the values of ratio $\Lambda \equiv F_S/M$. In simple GMSB models, the value of $\Lambda$ is fixed by the requirement that squark and slepton masses must be below a TeV so that one does not require fine-tuning to understand the weak scale. This however does not fix the $F_S$ and $M$ (although avoiding having tachyonic scalar messenger fields gives $F_S \leq M^2$) individually leaving the gravitino mass pretty much a free parameter only to be constrained by phenomenology and cosmology. In the case of our $\nu$-GMSB, however, as we see below, there are very strict bounds on $M$ from gauge coupling unification and suppressed flavor violation; therefore gravitino mass is allowed only within a very narrow range.

A. Gauge coupling unification constraint on the messenger scale

It is well known that gauge coupling unification property of MSSM remains true in the presence intermediate scale multiplets as long as they are full multiplets of SU(5) group. In our case we have full 24-dim. multiplets above the weak scale so that we maintain the unification. However, while these extra multiplets leave the GUT scale unchanged, they increase the magnitude of the value of gauge coupling at the unification scale, $M_U$. Since it is necessary to maintain the validity of perturbation theory till above the GUT scale and preferably to the Planck scale, this will impose a lower bound on the mass of the new 24-fields.

Considering the evolution of the $\alpha_1$ in the presence of the new fields, we find

$$\alpha_1^{-1}(M_U) - \alpha_1^{-1}(M_S) = -\frac{33}{10\pi} \ln \frac{M}{M_S} - \frac{108}{10\pi} \ln \frac{M_U}{M},$$

where $M_S \sim 1$ TeV is a typical soft mass scale, and $M_U \simeq 2 \times 10^{16}$ GeV is the GUT scale. From this equation, we see that if we want to keep $\alpha^{-1}(M_U) \gtrsim 1$, we get $M \equiv \langle S \rangle \gtrsim 6.2 \times 10^{11}$ GeV. Similar constraints also arise from the $\alpha_{2,3}$ evolution. Thus this way of implementing gauge mediation implies that we must have $\sqrt{F_S} \gtrsim 10^{7.7} - 10^{8.2}$ GeV for a typical soft mass 100 GeV -1 TeV. This in turn implies that the gravitino mass in these models is:

$$M_{3/2} \gtrsim 1 - 10 \text{ MeV}.$$
We show in the next section under what conditions this gravitino can be a dark matter of the Universe.

B. Upper limit on messenger mass from lepton flavor violation (LFV)

In Eq. (1), the messenger fields have the Dirac Yukawa coupling \((Y_\nu)\) to \(\bar{5}\) matters, through which flavor-dependent sfermion masses are induced. For example, off-diagonal elements of left-handed slepton mass squared at the messenger scale is estimated as

\[
\Delta m_{\tilde{\ell}ij}^2 \sim m_{\tilde{\ell}}^2 \times \frac{(Y_\nu^\dagger Y_\nu)_{ij}}{g_2^2},
\]

where \(m_{\tilde{\ell}}^2\) is the flavor-diagonal soft mass squared from gauge interactions. When the Yukawa coupling is small, we can neglect their RGE evolutions.

In this subsection, we show that type III seesaw combined with present experimental constraints on lepton flavor violation e.g. branching ratio for \(\mu \to e\gamma\) gives an upper limit on the messenger mass \(M = \lambda \langle S \rangle\). This can be seen qualitatively looking at the seesaw formula for neutrino masses. Note that if the messenger scale is higher, one must increase the Dirac Yukawa coupling \(Y_\nu\) in order to get the eV range neutrino mass to fit neutrino oscillation data. Maximal neutrino mixing then suggests that off-diagonal elements of \(Y_\nu\) must also be large. This would therefore increase the off-diagonal elements of left-handed slepton mass and so the LFV branching ratios. The present experimental upper limit can therefore be used to set an upper limit on the \(Y_\nu\) elements and hence the messenger scale \(M\) so that eV neutrino masses emerge.

To make this argument quantitative, we assume that the three \(\Sigma\) masses are degenerate and \(Y_\nu\) is a real matrix. Inverting the seesaw formula, we have

\[
Y_\nu^T Y_\nu = -\frac{M}{v^2 \sin^2 \beta} \mathcal{M}_\nu = -\frac{M}{v^2 \sin^2 \beta} U_{TB} \mathcal{D}_\nu U_{TB}^T,
\]

where \(\mathcal{D}_\nu\) is a diagonal mass eigenvalue matrix, and we have assumed that \(\mathcal{M}_\nu\) is diagonalized by the tri-bimaximal mixing matrix,

\[
U_{TB} = \begin{pmatrix}
\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\
-\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\
-\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\
\end{pmatrix}.
\]
In our analysis, we consider two typical mass spectra, the normal-hierarchical case ($D_{\nu}^{NH}$) and the inverted-hierarchical case ($D_{\nu}^{IH}$) such as

$$D_{\nu}^{NH} = \text{diag}(0, \sqrt{\Delta m^2_{12}}, \sqrt{\Delta m^2_{13}}), \quad D_{\nu}^{IH} = \text{diag}(\sqrt{\Delta m^2_{13}}, \sqrt{\Delta m^2_{12} + \Delta m^2_{13}}, 0)$$

(14)

for the neutrino oscillation data [13]

$$\Delta m^2_{12} = 7.6 \times 10^{-5} \text{eV}^2,$$
$$\Delta m^2_{13} = 2.4 \times 10^{-3} \text{eV}^2.$$

(15)

For example, one $Y_\nu$ texture for the normal-hierarchical case for $\tan \beta = 10$ is found to be

$$Y_\nu^T Y_\nu = - \begin{pmatrix} 0.0000969 & 0.0000969 & 0.0000969 \\ 0.0000969 & 0.000914 & -0.000720 \\ 0.0000969 & -0.000720 & 0.000914 \end{pmatrix} \times \left( \frac{M}{10^{12} \text{ GeV}} \right).$$

(16)

In our analysis, we adopt an approximate formula of the LFV decay rate [14] [15],

$$\Gamma(\ell_i \rightarrow \ell_j \gamma) \sim \frac{e^2}{16\pi} \frac{\alpha_2}{16\pi} \frac{|\Delta m^2_{ij}|^2}{m^8_{\tilde{\ell}}} \tan^2 \beta.$$  

(17)

Using given $Y_\nu$ textures for both the normal- and inverted-hierarchical cases and Eqs. (5) and (11), in Fig. 2 and 3 we plot the branching ratio $BR(\mu \rightarrow e\gamma)$ as a function of the $\Sigma$ mass $M$, together with the current experimental bound [16], $BR(\mu \rightarrow e\gamma) \leq 1.2 \times 10^{-11}$. In the same way, we obtain the branching ratio of LFV tau decay. Once the $Y_\nu$ texture is fixed, the following ratio is determined independently of $M$ and $\tan \beta$:

$$\frac{BR(\tau \rightarrow \mu\gamma)}{BR(\mu \rightarrow e\gamma)} \approx 10.3 \text{ and } 1.74 \times 10^3,$$

(18)

respectively, for the normal- and inverted-hierarchical cases.

We can draw two conclusions by combining the messenger mass constraints from gauge coupling unification and lepton flavor violation discussed above: (i) the seesaw scale is in a very restricted range of $10^{12} - 10^{13}$ GeV and (ii) the current MEG experiment [17] searching for the process $\mu \rightarrow e\gamma$ can test this model since for the lowest allowed value for messenger mass $M$, the prediction for the $BR(\mu \rightarrow e\gamma)$ is above $10^{-14}$ accessible to this experiment. Note that naively one might think that since the $BR(\mu \rightarrow e\gamma)$ scales like $m_{\tilde{\ell}}^{-4}$, one might reduce this by choosing a higher value of the slepton mass. However, in the GMSB model like ours, such increase can come only from an increase of $F_S/M$, which is fixed by considerations such as Higgs mass fine-tuning.
We therefore do not have much room to reduce the $\mu \to e + \gamma$ branching ratio. We also note that the $BR(\mu \to e\gamma)$ is predicted to be lower for the case of inverted mass hierarchy for neutrinos compared to the normal hierarchy case.

IV. PHENOMENOLOGY OF $\nu$-GMSB

A. sparticle and Higgs boson mass spectra

As mentioned before, one point we clearly notice from the GMSB sparticle mass formulas of Eq. (5) and (8) is that sfermion masses are lighter than their corresponding gaugino masses. This is because of the Dynkin index $N_m = 15$ and sfermion masses are suppressed by a factor $1/\sqrt{N_m}$ compared to corresponding gaugino masses. This is a similar structure to the no-scale supergravity [18]. For example, we find that gluino is the heaviest sparticle. One of the most characteristic feature in $\nu$-GMSB with type III seesaw is that the (mostly right-handed) stau is the NLSP.

In Table I and II, examples of sparticle and Higgs mass spectra are presented for $\tan \beta = 10$ and 45, respectively. In these Tables, for comparison, we also show the mass spectra in GMSB with type II seesaw (corresponding to $N_m = 7$) and minimal GMSB (mGMSB) with one pair of $5 \oplus \overline{5}$ (corresponding to $N_m = 1$). For each GMSB models, $F_S/M$ has been suitably chosen to give the same gluino mass. We can see a sharp contrast with mGMSB mass spectrum in which some of sfermions are heavier than corresponding gauginos, in particular, neutralino is the NLSP. The sparticle mass spectra of type II and III are similar, but there are sizable mass differences between the same sparticles, $\sim 50 - 70$ GeV, which will be large enough for the precision goal of the sparticle mass measurements at future colliders such as LHC and the International Linear Collider (ILC).

The condition for the perturbative gauge coupling unification leads to the lower bound on the messenger scale, $M \gtrsim 6.2 \times 10^{11}$ GeV. With this $M$ and $N_m = 15$, all the particle mass spectra are determined by fixing $F_S$, so that the current experimental lower bound on sparticle masses provide us the lower bound on the SUSY breaking scale $F_S$. As we will discuss more detail later, the stau NLSP is long-lived, at least, in the collider time-scale. The current lower mass bound for stable and long-lived massive charged particles was obtained by LEP2 experiments [20] as $\sim 102$ GeV.

1 We have used SOFTSUSY 2.0.11 [19] to generate sparticle and Higgs mass spectra.
When we apply this bound to the NLSP stau mass, we find $F_S/M \gtrsim 17$ TeV for $M \simeq 6.2 \times 10^{11}$ GeV and $\tan \beta = 45$ for example. It turns out that the constraint from LFV is more severe (see Fig. 3) and $F_S/M$ should be higher as $\tan \beta$ is raised.

### B. gravitino dark matter

In our $\nu$-GMSB model, the gravitino is the LSP as in all GMSB models. Since couplings of the gravitino to particles and sparticles are suppressed by the Planck mass, the gravitino cannot be in the thermal equilibrium in the early universe. In the case with stau NLSP, gravitinos are produced through scattering and decay processes of the MSSM particles in the thermal plasma, and the relic density of the gravitino LSP is evaluated as

$$\Omega h^2 \sim 0.2 \left( \frac{T_R}{10^{10}\text{GeV}} \right) \left( \frac{100\text{GeV}}{m_{3/2}} \right) \left( \frac{M_3}{1\text{TeV}} \right)^2,$$

where $T_R$ is the reheating temperature after the inflation, and $M_3$ is the running gluino mass. By appropriately fixing the gravitino mass, the reheating temperature and sparticle spectrum, the relic density suitable for the dark matter can be obtained. For our numerical examples in Table I and II, $T_R \sim 10^5$ GeV to obtain the current dark matter relic abundance $\Omega h^2 \simeq 0.11$.

### C. stau NLSP phenomenology

In the model the stau is NLSP and it decays to tau and gravitino LSP. This stau decay is of particular interests for the collider phenomenology. The lifetime of stau NLSP is estimated as

$$\tau_{\tilde{\tau}} \sim 10^{-2}\text{sec} \times \left( \frac{100\text{GeV}}{m_{\tilde{\tau}_1}} \right)^5 \left( \frac{m_{3/2}}{1\text{MeV}} \right)^2.$$

For the values given in the Table I and II, the lifetime of the stau is found to be

$$\tau_{\tilde{\tau}} \sim 2.3 \times 10^{-3}\text{sec}, \quad \tau_{\tilde{\tau}} \sim 8.5 \times 10^{-3}\text{sec}$$

for $\tan \beta = 10$ and $\tan \beta = 45$, respectively. The stau lifetime is short enough not to cause any cosmological problems, in particular, for big bang nucleosynthesis.

On the other hand, for the stau lifetime around $10^{-3}$ sec, the decay length well exceeds the detector size of the LHC and the ILC, and the NLSP decay takes place outside the detector. In this case, there have been interesting proposals for ways to trap long-lived NLSPs outside the
detector, when the NLSP is a charged particle (like the stau in our model). Detailed studies of the
NLSP decay may provide precise measurements of the gravitino mass and the four dimensional
Planck mass. Very recently, the possibility of observing long-lived NLSPs inside the detector has
been investigated [24] for lifetimes of the NLSP $\tau \lesssim 10^{-3}$ sec. The stau NLSP in our model is a
good example of this situation.

V. GMSB WITH TWO 24-DIM. MESSENGERS

In this section, we discuss implications of $\nu$-GMSB with two 24-dimensional fields for type III
seesaw as messengers. Recall that the minimum number of 24 fields for fitting neutrino oscillation
data is two. The details of the discussion for this case is very similar to the three 24 case. The
Dynkin index for two 24-dimensional messenger fields is $N_m = 10$. Due to this large Dynkin
index, at the messenger scale, there is a hierarchy between the gaugino mass and sfermion mass
as before although the hierarchy in this case is less. For example, the ratio of the right-handed
slepton mass and Bino is found to be $m_{\tilde{e}}^2(M)/M_1^2(M) = 1.2/N_m = 0.12$ at the messenger scale
$M$. Therefore, stau is still most likely to be the NLSP as in the case of three 24 type III seesaw.

To find the lower bound on the messenger scale from the gauge coupling unification, we con-
sider the evolution of the $\alpha_1$ in the presence of two 24 fields, and find

$$M \equiv \lambda\langle S\rangle \gtrsim 3.4 \times 10^9 \text{ GeV},$$

(22)
in order to keep the perturbative gauge coupling unification, say, $\alpha^{-1}(M_U) \gtrsim 1$. This implies that
the gravitino mass in this case is

$$m_{3/2} \gtrsim 1 - 10 \text{ keV}.$$  

(23)

Since the gravitino mass can be very low in this case, i.e. $m_{3/2} = \mathcal{O}(10 \text{ keV})$, it is a suitable
candidate for the warm dark matter of the Universe [26]. According to the formula Eq. (19) for
the gravitino production from thermal plasma, this requires the reheat temperature $T_R \sim 1 \text{ TeV}$ to
provide the correct relic density for the gravitino dark matter.

The type III seesaw mechanism with two 24-dim. messengers predicts one massless light
neutrino eigenstate. Since our numerical fitting for neutrinos given in Eq. (14) also assumes one
massless neutrino, the same analysis is applicable to the present case, except we use $N_m = 10$ and
also different inputs for $F_S/M$. 

11
Using the $Y_\nu$ textures for both normal- and inverted-hierarchical cases (normal hierarchy case given in Eq. (16)), we plot in Fig. 3 and 4 the branching ratio $BR(\mu \rightarrow e\gamma)$ as a function of the $\Sigma$ mass $M$ and compare it with the current experimental bound, $BR(\mu \rightarrow e\gamma) \leq 1.2 \times 10^{-11}$. In the same way, we obtain the branching ratio for LFV tau decay $\tau \rightarrow e + \gamma$. Since the $Y_\nu$ texture we have used here is the same as in the three 24 model, we arrive at the same result of Eq. (18) for the ratio $BR(\tau \rightarrow \mu\gamma)/BR(\mu \rightarrow e\gamma)$.

As far as the sparticle spectrum goes, in Table III, we present examples of sparticle and Higgs mass spectra for $\tan \beta = 10$ and 45. Here we have chosen suitable inputs for $F_S/M$ to be consistent with the current sparticle and Higgs mass bounds. In $\tan \beta = 10$ case, the resultant lightest Higgs boson mass is at the current lower bound $m_h = 114$ GeV [25], while the stau NLSP mass is at the current lower bound $m_{\tilde{\tau}_1} \simeq 102$ GeV [20] in $\tan \beta = 45$ case. As Higgs mass increases, we need to increase the $F_S/M$ value although one needs to do more fine-tuning to get the right $Z$-mass.

Turning now to the stau NLSP, for the parameter values given in the Table III, the lifetime of the stau is found to be

$$\tau_{\tilde{\tau}} \sim 1.8 \times 10^{-7} \text{ sec, } \tau_{\tilde{\tau}} \sim 4 \times 10^{-6} \text{ sec}$$

for $\tan \beta = 10$ and $\tan \beta = 45$, respectively. Note that the stau life is much shorter than the three 24 case due to the fact that the SUSY breaking parameter $F_S$ can be much lower for the two 24 case. It therefore presents a more favorable possibility for detection inside the LHC detector [24].

VI. CONCLUSIONS

In conclusion, we have studied the implications of the type III seesaw mechanism for neutrino masses in an extension of SUSY SU(5) for supersymmetry breaking phenomenology. We use the extra 24 dim. fields needed for implementing type III seesaw as messengers that transmit SUSY breaking from the hidden to visible sector via the gauge forces. We find that this kind of GMSB models, specially with three type III seesaw 24 fields, are unique in the sense that they considerably narrow the messenger scale and hence the SUSY breaking parameter $F_S$ compared to other GMSB models. This allows us to make two important predictions: the cold dark matter is a stable gravitino LSP with mass in the range of 1-10 MeV and a light stau as the NLSP with mass in the range of 200 GeV. The stau NLSP can decay to gravitino with lifetime longer than typical collider time scales and can be detectable at LHC. We also find that the characteristic sparticle
spectrum of this model is different from other GMSB models which in principle can be used to test the model. We repeat the same discussion for the case of two 24 messengers case. The limits on the messenger mass is clearly less restrictive in this case. As a result, the gravitino mass can be lower and hence it can be a warm dark matter. The stau NLSP life time is in the much more favorable range for detection at LHC than the three 24 case.

Acknowledgments

The work of R.N.M. is supported by the National Science Foundation Grant No. PHY-0652363. The work of N.O. is supported in part by the Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture of Japan (No. 18740170). The work of H.B.Y. is supported by the National Science Foundation under Grant No. PHY-0709742. H.B.Y acknowledges the Maryland Center for Fundamental Physics for its hospitality during the completion of this work.

[1] M. Dine, A. E. Nelson and Y. Shirman, Phys. Rev. D 51, 1362 (1995); M. Dine, A. E. Nelson, Y. Nir and Y. Shirman, Phys. Rev. D 53, 2658 (1996); For earlier works, M. Dine, W. Fischler and M. Srednicki, Nucl. Phys. B 189, 353 (1981); S. Dimopoulos and S. Raby, Nucl. Phys. B 192, 353 (1982); C. Nappi and B. Ovrut, Phys. Lett. B 113, 175 (1982); L. Alvarez-Gaume, M. Claudson and M. Wise, Nucl. Phys. B 207, 96 (1982); for a review, see G. F. Giudice and R. Rattazzi, Phys. Rept. 322, 419 (1999).

[2] G. R. Dvali, G. F. Giudice and A. Pomarol, Nucl. Phys. B 478, 31 (1996).

[3] M. Dine, Y. Nir and Y. Shirman, Phys. Rev. D 55, 1501 (1997).

[4] F. R. Joaquim and A. Rossi, Phys. Rev. Lett. 97, 181801 (2006); F. R. Joaquim and A. Rossi, Nucl. Phys. B 765, 71 (2007).

[5] R. N. Mohapatra, N. Setzer and S. Spinner, JHEP 0804, 091 (2008).

[6] R. N. Mohapatra, N. Okada and H. B. Yu, Phys. Rev. D 77, 115017 (2008).

[7] G. Lazarides, Q. Shafi and C. Wetterich, Nucl.Phys. B181, 287 (1981); R. N. Mohapatra and G. Senjanović, Phys. Rev. D 23, 165 (1981). For Higgs triplet induced neutrino masses, see also R. E. Marshak and R. N. Mohapatra, VPI-HEP-80/02 invited talk given at Orbis Scientiae, Coral Gables, Fla., Jan 14-17, 1980, published in Orbis Scientiae 1980; P277; J. Schechter and J. W. F. Valle, Phys. Rev.
[8] R. Foot, H. Lew, X. G. He and G. C. Joshi, Z. Phys. C 44, 441 (1989).

[9] E. Ma, Phys. Rev. Lett. 81, 1171 (1998); B. Bajc, M. Nemevsek and G. Senjanovic, Phys. Rev. D 76, 055011 (2007); P. Fileviez Perez, Phys. Rev. D 76, 071701 (2007); Phys.Lett. B654,189 (2007); I. Dorsner and P. F. Perez, JHEP 0706, 029 (2007); I. Gogoladze, N. Okada and Q. Shafi, arXiv:0805.2129 [hep-ph]; W. Fischler and R. Flauger, arXiv:0805.3000 [hep-ph]; S. Blanchet and P. F. Perez, arXiv:0807.3740 [hep-ph].

[10] P. Minkowski, Phys. Lett. B 67, 421 (1977); M. Gell-Mann, P. Ramond, and R. Slansky, *Supergravity* (P. van Nieuwenhuizen et al. eds.), North Holland, Amsterdam, 1979, p. 315; T. Yanagida, in *Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe* (O. Sawada and A. Sugamoto, eds.), KEK, Tsukuba, Japan, 1979, p. 95; S. L. Glashow, *The future of elementary particle physics*, in *Proceedings of the 1979 Cargèse Summer Institute on Quarks and Leptons* (M. Lévy et al. eds.), Plenum Press, New York, 1980, p. 687; R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. 44, 912 (1980).

[11] S. Dimopoulos and H. Georgi, Nucl. Phys. B 193, 150 (1981).

[12] Z. Chacko and E. Ponton, Phys. Rev. D 66, 095004 (2002).

[13] W. M. Yao et al. [Particle Data Group], J. Phys. G 33 (2006) 1.

[14] J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. D 53, 2442 (1996).

[15] For the early work, see F. Borzumati and A. Masiero, Phys. Rev. Lett. 57, 961 (1986).

[16] M. L. Brooks et al. [MEGA Collaboration], Phys. Rev. Lett. 83, 1521 (1999).

[17] A. Maki, AIP Conf. Proc. 981, 363 (2008).

[18] J. R. Ellis, K. Enqvist and D. V. Nanopoulos, Phys. Lett. B 147, 99 (1984); J. R. Ellis, C. Kounnas and D. V. Nanopoulos, Nucl. Phys. B 241, 406 (1984). For a review, see A. B. Lahanas and D. V. Nanopoulos, Phys. Rept. 145, 1 (1987).

[19] B. C. Allanach, Comput. Phys. Commun. 143, 305 (2002).

[20] G. Abbiendi et al. [OPAL Collaboration], Phys. Lett. B 572, 8 (2003).

[21] J. R. Ellis, J. E. Kim and D. V. Nanopoulos, Phys. Lett. B 145, 181 (1984). M. Bolz, W. Buchmuller and M. Plumacher, Phys. Lett. B 443, 209 (1998); M. Bolz, A. Brandenburg and W. Buchmuller, Nucl. Phys. B 606, 518 (2001).

[22] G. Hinshaw et al. [WMAP Collaboration], arXiv:0803.0732 [astro-ph].

[23] W. Buchmuller, K. Hamaguchi, M. Ratz and T. Yanagida, Phys. Lett. B 588, 90 (2004); J. L. Feng,
A. Rajaraman and F. Takayama, Int. J. Mod. Phys. D 13, 2355 (2004); K. Hamaguchi, Y. Kuno, T. Nakaya and M. M. Nojiri, Phys. Rev. D 70, 115007 (2004); J. L. Feng and B. T. Smith, Phys. Rev. D 71, 015004 (2005) [Erratum-ibid. D 71, 0109904 (2005)].

[24] K. Ishiwata, T. Ito and T. Moroi, arXiv:0807.0975 [hep-ph].

[25] R. Barate et al. [LEP Working Group for Higgs boson searches], Phys. Lett. B 565, 61 (2003).

[26] D. Gorbunov, A. Khmelnitsky and V. Rubakov, arXiv:0805.2836 [hep-ph].
| Mediation | Type III | Type II | mGMSB |
|-----------|----------|---------|-------|
| $M$       | $10^{12}$| $10^{12}$| $10^{12}$ |
| $F_S/M$   | 15 TeV  | 31.93 TeV | 214.2 TeV |
| $m_h$     | 116      | 116     | 117   |
| $m_A$     | 877      | 933     | 1394  |
| $m_{H^0}$ | 877      | 933     | 1395  |
| $m_{H^±}$ | 880      | 937     | 1397  |
| $m_{\tilde{\chi}^±_{1,2}}$ | 568, 784 | 567, 813 | 557, 1090 |
| $m_{\tilde{\chi}^0}$ | 299, 563, 769, 786 | 298, 564, 801, 816 | 288, 567, 1085, 1091 |
| $m_{\tilde{\nu}}$ | 1578 | 1578 | 1578 |
| $m_{\tilde{\tilde{\nu}}_{1,2,3}}$ | 429,429,429 | 487,487,486 | 883,883,881 |
| $m_{\tilde{\tilde{\nu}}_{1,2}}$ | 228, 439 | 273, 496 | 551, 890 |
| $m_{\tilde{\tilde{\nu}}_{1,2}}$ | 222, 440 | 267, 496 | 543, 888 |
| $m_{\tilde{\tilde{\nu}}_{1,2}}$ | 3.55 MeV | 7.57 MeV | 50.8 MeV |
| NLSP      | stau     | stau    | neutralino |

**TABLE I**: Sparticle and Higgs boson mass spectra (in units of GeV) in the type III, type II and the mGMSB, for $\tan \beta = 10$. 
| Mediation | Type III | Type II | mGMSB |
|-----------|----------|---------|-------|
| $M$       | $10^{12}$ | $10^{12}$ | $10^{12}$ |
| $F_S/M$   | 24 TeV   | 51.1 TeV | 342.6 TeV |
| $m_h$     | 119      | 119     | 120   |
| $m_A$     | 941      | 1009    | 1556  |
| $m_H$     | 941      | 1009    | 1556  |
| $m_{H^\pm}$ | 945     | 1012    | 1558  |
| $m_{\tilde{\chi}_{1,2}^\pm}$ | 914, 1134 | 913, 1181 | 892, 1607 |
| $m_{\tilde{\chi}_{1,0}^0}$ | 484, 908, 1126, 1139 | 482, 908, 1175, 1186 | 465, 891, 1610, 1613 |
| $m_{\tilde{g}}$ | 2419     | 2419    | 2419  |
| $m_{\tilde{u},\tilde{c}_{1,2}}$ | 2064, 2128 | 2143, 2223 | 2843, 3051 |
| $m_{\tilde{t}_{1,2}}$ | 1748, 1965 | 1799, 2040 | 2262, 2720 |
| $m_{\tilde{d},\tilde{s}_{1,2}}$ | 2043, 2138 | 2119, 2233 | 2791, 3062 |
| $m_{\tilde{b}_{1,2}}$ | 1888, 1956 | 1954, 2031 | 2543, 2714 |
| $m_{\tilde{\nu}_{1,2,3}}$ | 679, 679, 656 | 770, 770, 741 | 1397, 1397, 1328 |
| $m_{\tilde{e},\tilde{\mu}_{1,2}}$ | 358, 686 | 430, 777 | 875, 1403 |
| $m_{\tilde{\tau}_{1,2}}$ | 207, 677 | 269, 757 | 609, 1336 |
| $m_{3/2}$ | 5.69 MeV | 12.1 MeV | 81.2 MeV |
| NLSP      | stau     | stau    | neutralino |

TABLE II: Sparticle and Higgs boson mass spectra (in units of GeV) in the type III, type II and the mGMSB, for $\tan \beta = 45$. 
| $\tan \beta$ | 10             | 45             |
|--------------|----------------|----------------|
| $M$          | $3.4 \times 10^9$ | $3.4 \times 10^9$ |
| $F_S/M$      | 18.4 TeV       | 25.93 TeV      |
| $m_h$        | 114            | 116            |
| $m_A$        | 669            | 646            |
| $m_H$        | 669            | 646            |
| $m_{H^\pm}$  | 674            | 652            |
| $m_{\tilde{\chi}_1^\pm}$ | 455, 613 | 649, 793 |
| $m_{\tilde{\chi}_1^0}$ | 244, 453, 588, 614 | 348, 647, 772, 795 |
| $m_{\tilde{g}}$ | 1314          | 1794           |
| $m_{\tilde{u}, \tilde{c}_{1,2}}$ | 1111, 1137 | 1507, 1546 |
| $m_{\tilde{t}_{1,2}}$ | 946, 1113 | 1298, 1458 |
| $m_{\tilde{d}_{1,2}}$ | 1101, 1145 | 1493, 1555 |
| $m_{\tilde{b}_{1,2}}$ | 1075, 1097 | 1386, 1446 |
| $m_{\tilde{\mu}_{1,2,3}}$ | 327, 327, 327 | 459, 459, 447 |
| $m_{\tilde{e}_{1,2}}$ | 170, 339 | 235, 469 |
| $m_{\tilde{\tau}_{1,2}}$ | 164, 340 | 102, 477 |
| $m_{3/2}$ | 14.8 keV | 20.9 keV |
| NLSP         | stau           | stau           |

**TABLE III:** Sparticle and Higgs boson mass spectra (in units of GeV) of the model with two 24 messengers for $\tan \beta = 10$ and 45.
FIG. 1: The diagram of type III seesaw mechanism.

FIG. 2: The branching ratio of the muon LFV decay as a function of $M$, for $\tan \beta = 10$ of three $24$ model. Here we have fixed $F_S/M = 15$ TeV (see Table I). The solid and dashed lines correspond to the normal- and inverted-hierarchical cases, respectively, while the dotted horizontal line is the current experimental upper bound $BR(\mu \rightarrow e\gamma) \leq 1.2 \times 10^{-11}$.
FIG. 3: Same as Fig. 2 but for $\tan \beta = 45$. Here we have fixed $F_S/M = 24$ TeV (see Table II).

FIG. 4: The branching ratio of the muon LFV decay as a function of $M$, for $\tan \beta = 10$ of two-$24$ model. Here we have fixed $F_S/M = 18.4$ TeV (see Table III). The solid and dashed lines correspond to the normal- and inverted-hierarchical cases, respectively, while the dotted horizontal line is the current experimental upper bound $BR(\mu \rightarrow e\gamma) \leq 1.2 \times 10^{-11}$. 


FIG. 5: Same as Fig. 4 but for $\tan \beta = 45$. Here we have fixed $F_S/M = 25.93$ TeV (see Table III).