Unitarity and Entropy Change in Exclusive Quark Combination Models

Yi Jin,1 Shi-Yuan Li,2 Zong-Guo Si,2 and Tao Yao2

1Department of Physics, University of Jinan, 250022
2School of Physics, Shandong University, 250100

(Dated: June 25, 2010)

Unitarity of the combination model is formulated and demonstrated to play the key rôle that guarantees the non-decrease of the entropy in the exclusive combination process.

PACS numbers:

Quark Combination Model (QCM) was proposed in early seventies of 20th century [1, 2] to describe the multiproduction process in high energy collisions, based on the constituent quark model of hadrons. Various versions of QCM succeed in explaining many data (for a review, see [3], with a brief list of references therein), and recently those in central gold-gold collisions at the Relativistic Heavy Ion Collider (RHIC) [4, 5] where several ‘unexpected’ phenomena observed lay difficulties for other hadronization mechanisms [6]. Common of all the hadronization models, QCM only responds to describe the non-perturbative QCD phase: The produced ‘partons’ in various collisions turned into constituent quarks (including both quarks anti-quarks, same in the following); then these quarks combined into hadrons [14]. Especially, the combination process is just the ‘realization’ of confinement within the QCM framework. Each kind of QCM distinguishes from others by its special way of combination. Without digging into details of any special kind of QCM, one easily figures out two principles which it must respect: First of all, energy-momentum conservation is the principle law of physics, reflecting the basic symmetry space-time displacement invariance. The models must precisely (as precisely as possible, in practice) transfer the energy and momentum of the parton system into the constituent quark system and then the hadron system. Second, for a colour-singlet separated system, it is necessary to let all the constituent quarks created in the model combined into hadrons, or else there are free quarks with non-zero mass and energy, which obviously contradicts to any observations that suggest confinement. Moreover, these free quarks take away energy and momentum, hence make danger of the energy-momentum conservation. This second principle is referred as Unitarity of the relevant model. These two principles are closely connected, with the first one the natural result of the second one.

To formulate the unitarity of QCM, we formally introduce the unitary time-evolution operator $U$ to describe the combination process [8]

$$\sum_h \langle h | U | q \rangle = \langle q | U^+ | h \rangle = 1.$$  \hspace{1cm} (1)

The quark state $| q \rangle$, describes a colour-singlet quark system, and the corresponding hadron state $| h \rangle$ describes the hadron system. The matrix element $U_{hq} = \langle h | U | q \rangle$ gives the transition amplitude. The energy-momentum conservation is inherent, by the natural commutation condition $[U, H] = [U, P] = 0$, with $H, P$ the energy and momentum operator of the system. This is just the confinement which says that the total probability for the colour-singlet quark system to transit to all kinds of hadron is exactly 1, and agrees with the fact that all the quark states and the hadron states are respectively two complete sets of bases of the same Hilbert space [15], i.e., $\sum | q \rangle = \sum | h \rangle = 1$ for the colour-singlet system. Recently [8], it is observed that, in a combination model which respects unitarity, the production of exotic hadrons is naturally suppressed. In this paper, we would like to further sharpen the key rôle of unitarity in exclusive QCM, by its application to guarantee the non-decreasing of entropy in the combination process for a colour-singlet separated system.

It comes out that a third principle need to be checked once QCM finds its irreplaceable application in high energy central nuclear collisions [3, 6], where this model is employed to describe the hadronization (freeze-out) of a special system, a bulk of quark-gluon matter (QGM) with non-zero density and temperature. The entropies of the QGM, of the constituent quark system evolved from the QGM, as well as of the hadron system combined from the constituent quarks, can be respectively introduced. Thus the entropy change should respect the second law of thermodynamics (under this circumstance, the energy conservation is just the first law). This is crucial when one tries to track back the thermal state of the QGM from the hadron system. The combination process leads to particle number decreased (we will clarify that unitarity affirms the invariance of the degree of freedom). Some na"ive considerations often question on the entropy to be decreased. However, in our opinion, one of the key complexities of the entropy problem is as following: Each kind of QCM has its special set of inputs. These inputs are not designed as complete as to specify the concrete value of the entropy which is a state function of a set of macroscopic quantities needed to fix the thermal properties of the system [10]. Some of these quantities have no physical relation with the combination mecha-
nism, or the final state observables predicted by QCM. Therefore, they can be introduced only for the purpose to fix entropy without any other possible experimental tests. Contrary to freely tuning these ‘inputs’ to worship the second law, we find that the entropy is guaranteed to be non-decreasing by Unitarity and energy-momentum conservation of the combination process without any extra inputs. In other words, the entropy of the colour-singlet constituent quark system is not larger than that of the hadron system, provided that all of the quarks are combined into the hadrons, with the energy and momentum conservatively transferred. Any extra macroscopic quantity to fix the entropy which did not appear in the model before, will never need to appear, since the relation exposed here shows that the entropy change is not an isolated problem.

For feasibility, we discuss a colour-singlet constituent quark system. It is separated, belonging to microcanonical ensemble. All of the quarks will be exclusively combined into hadrons by some ‘combination rules’. This system is the offspring from the QGM when its interaction with the nuclear remnant in the beam fragmentation region is ignored. For QCD case, there is no free gluon radiation, contrary to the electrodynamics case. This simplification is easier to be adopted. Some inclusive calculations and discussions based on bag model (e.g., \[ \hat{\mathbf{R}} \]) treat this system as belonging to (macro)canonical ensemble, so it is open. Inspection of the entropy change for such cases does not give any special indication.

Now it is clear that combination process will never decrease the degree of freedom provided that the model respects unitarity. For an ideal gas system, this has guaranteed the entropy non-decreasing. However, only the degree of freedom is not enough to determine the entropy for a system with general case of interactions. In many kinds of QCM, the details of the interacting potential are not figured out, but do not necessarily indicate the assumption of free quark system. The ‘sQGP’ observation [11], as well as low energy scale QCD strong coupling imply the contrary. In general, the entropy \( S \) is [10]

\[
S = -\text{tr}(\rho \ln \rho). \tag{2}
\]

Here \( \rho \) is the density Matrix without referring to equilibrium. It can depend on time, as the solution of the Liouville Equation. Without specifying the Hamiltonian to solve the equation, we can formally write by definition,

\[
\rho(t) = |t, i > P_i < i, t| = U(t, 0)|0, i > P_i < i, 0|U^\dagger(t, 0) = U(t, 0)\rho(0)U^\dagger(t, 0). \tag{3}
\]

Here \( U(t, 0) \) is the time evolution operator. Taking \( \rho(0) \) as the distribution of the constituent quark system just before combination, while \( \rho(t) \) just after, of the hadron system, then \( U(t, 0) \) is exactly the operator \( U \) introduced in Eq. (1). This is a uniform unitary transformation on the Hermitian operator \( \rho \), which does not change the trace of \( \rho \ln \rho \). So the entropy holds as a constant in the combination process, same as energy and momentum. In the following, we will give an example of QCM, whose combination rules is obviously unitary.

Generally, entropy is a function of the energy of the system. By requiring maximum for equilibrium, one may get the relation of entropy and energy \[ \tag{10} \]. As unitarity preserves both energy and entropy, in practice, if the relation between them for the hadron system can be extracted from measurement, it should possibly be extrapolated to the constituent quark system.

As one pursues the above formal discussions to the concrete models, i.e., to employ a concrete QCM to investigate the entropy change before and after the combination process, unitarity is again one of the pillars. Two systems (states) need to be set more definite, to ‘squeeze’ the combination process itself: \( A \), constituent quark system which is to be combined; \( B \), the hadron system, resulting just from the combination. This is to eliminate other underlying processes before/after combination such as expansion of the system or hadron interaction and decay to affect the entropy. For generality, we need to justify that the entropy can be defined for states \( A \) and \( B \) without the à priori assumption of, or waiting long time for, equilibrium of them. Though one can expect and has many arguments to support local equilibrium of QGM, it is not the state we are treating. The constituent quark system bursts out from QGM, according to the unsolved non perturbative dynamics. Assumptions like adequate long time for their thermalization and equilibrium before combination can not straightforwardly result from (possible) equilibrium of QGM, neither inherent in the combination rules/methods. Same case is for state \( B \).

The density matrix \( \rho(t) \) describes expectation of the probability of a system to take a certain state from a certain Hilbert space \( \mathbf{H} \) at time \( t \). However, given \( \rho(t) \), for each time we can construct a ‘tangent’ Hilbert space \( \mathbf{H}_t \) in which \( \rho(t) \) is the most probable distribution and gives maximum. Hence it effectively describes an equilibrium state with respect to \( \mathbf{H}_t \). On \( \mathbf{H}_t \), the temporal states \( A \) and \( B \) are definite thermal states with definite entropies. Unitarity guarantees the \( \mathbf{H}_t \) for \( A \) or \( B \) can be taken as the same. Thus \( A \) and \( B \) can be considered as two states of a specific system. It is not necessary to think of extra inputs to describe the details of them in one model, which will make the proof of entropy non-decreasing in this way a vicious circle. Any reversal process integral

\[
\Delta \mathbf{S} = \int_A^B \frac{dQ}{T}. \tag{4}
\]

is just the entropy change. So we can introduce any quasi-static, reversal process to combine these two states, which will always give the unique result.

Such ideal processes in fact embeds in many Monte-Carlo programmes to realizing the QCM. Though the
combination process could happen in a very short time interval in reality, in many models it is described and realized step by step. Each step only corresponds to the combination of a quark-antiquark pair to meson or three (anti)quarks to (anti)baryon. When the number of quarks is large \((\to \infty)\), as encountered in the central heavy ion collisions, each step leads to minor (infinitesimal) change of the quark system. So the programme is the discrete realization of a quasi-static process. When the programme completes running to combine the quark system to the hadron system, let the programme run inversely, we come back to the initial state without any effect residual. So it is reversal. Hence the integral of eq. \[\text{Eq. 1}\] can be calculated by step \((i)\) summation \(\sum\delta Q_i/T_i\) employing such programmes. For the programmes respect unitarity, no free quarks going away and no free gluon radiation, but all quarks are exclusively combined into hadrons. Ignoring QED, the whole process is adiabatic. \(\delta Q_i \equiv 0, \forall i\), then \(\Delta S = 0\). One gets the same conclusion as above.

However, there is one complexity for the concrete programmes in practice. The fundamental equation of thermodynamics, for each combination step of the programmes, is \(\Delta U_i = \delta Q_i + \delta W_i\), where \(\delta W_i\) is the work done to the system, and \(\Delta U_i\) is the energy change. Since the system is separated while ignoring interactions with nuclear remnant, in this quasi-static process \(\delta W_i \equiv 0, \forall i\), we get
\[
\Delta S = \sum_i \Delta U_i/T_i. \tag{5}
\]

One should check that the energy of the system keeps unchanged in each step.

At first sight, it seems to encounter the crucial ‘difficulty’ in combination models. Most of them adopt the constituent quark model with fixed quark mass. Since hadron mass is definite, the \(2 \to 1\) and \(3 \to 1\) processes can not keep energy-momentum conservation at the same time for all particles on-shell. Here we employ an example QCM proposed by Shandong Group (SDQCM) to discuss this problem. In the calculation, we adopt the scenario as fixed but tunable quark mass with only three-momentum conservation, leave open the energy for each step. This is a simplified way to realize the model, without inputs of the potentials among the quark system. The energy conservation can be considered in two aspects:

1. In the simulation of a definite event, each special combination of quarks (e.g., \(ud\)) can correspond to several kinds of hadron states \((\pi^+, \rho^+\ldots)\) with various masses. \(\Delta U_i\) can be either negative or positive. This variety leads \(\sum \Delta U_i (\forall i)\) to be vanishing, by cancelation of \(\Delta U_i\) with different signs. In this case the final result of the combination can conserve energy, which is required to give a correct description of data, as mentioned in the beginning of this letter. For a constant temperature in the combination process, \(\Delta S = 0\). The constant temperature is not an assumption but inherent in the static, reversal and ideal process simulated by the programme. For all the intermediate states linking state \(A\) and \(B\), there are both quarks and hadrons. These states are considered as in a mixed phase, with constant temperature.

2. In the fundamental equation of thermodynamics, the thermal quantities must be the ensemble-averaged one, which is calculated by averaging among infinitely many events employing the Monte-Carlo programme. Same as the arguments in \(1\), \(\Delta U_i\) can be positive or negative for each step in various events. The event-averaged \(\Delta U_i\) is hoped to be vanishing. From such an observation, the temperature behaviour needs no care.

Now we demonstrate the above two points with SDQCM. SDQCM has been applied to various high energy collisions including nuclear collision \([3, 5, 12]\). In this model, all (anti)quarks are arranged into a chain along the rapidity axis. The combination starts from one end to the other step by step. The nearest quarks are combined into meson or baryon. This process is exclusive and obviously respects unitarity. It can be modified to accommodate exotic hadrons, without breaking unitarity \([8]\). This model employs a non-saturation potential \([13]\) to explain the constituent quark creation and to compensate for energy-momentum conservation during the combination process. In programmes, the parameters such as constituent quark masses are introduced. The details of the potential are ignored. For the most general case, three light quark masses, and three kinds of \(p_T\) distributions of these quarks, can be treated as parameters. Only the \(p_T\) and rapidity distribution of hadrons from experiments can be taken as constrains. The set of the parameter values is not unique for one set of data. Here in calculation, we take the special case that up and down quarks have the similar mass and \(p_T\) distribution.

To investigate the energy change of the colour-singlet system in SDQCM before and after each combination step, we define an ensemble averaged variable,
\[
R_i = 1 - E_{i-1}/E_i. \tag{6}
\]

Here \(E_i\) denotes energy of the colour-singlet system after the \(i\)th combination step, \(E_{i<0} = E_0\). The ensemble average is obtained via event average. For the exact cancelation among events and energy conservation of a certain step, \(R_i = 0\). The system in numerical calculation corresponds to the most central gold-gold collision at center of mass frame energy \((\text{per nucleon})\) of \(200\ \text{GeV}\). The results averaged over around \(10^8\) events at various precision of total energy conservation with various quark mass values are given in Fig.\[\text{Fig 1}\]. Note that in center of mass frame of the collision, the half of the total step-number approximately corresponds to the mid-rapidity region. Fig.\[\text{Fig 1}\] shows the \(R_i\) as a function of combination step. It is clear that one can gain high precision of the total energy conservation, and a vanishing \(\Delta U_i\) as well. Within the corresponding precision, Eq. \[\text{Eq. 6}\] gives
\[ \Delta S = 0. \] The effect on \( R_i \) from modifying the invariant mass of the quark cluster to be the corresponding hadron mass is most significant in mid-rapidity region. This comes from the fact that the three-momentum for each combination step is exactly conserved. Only the error (variation) of invariant mass of the quark cluster leads to the error (variation) of the energy. This error is proportional to \( 1/\cosh y \).

In summary, the entropy ‘problem’ is not isolated. Unitarity and energy conservation guarantee the entropy non-decreasing for the colour-singlet separated system of constituent quarks combined into hadrons. Any extra inputs are unnecessary. Unitarity and energy conservation is inherent in any QCM which is applied to get a correct exclusive description for the hadron production data and consistent with confinement. The relevant discussions can be applied to other processes which respect unitarity and energy conservation.

LSY thanks the help of Prof. V. A. De Lorenci and Dr. W. Han at the early stage of this work. This part of work was done when LSY visited UNIFEI, Brasil, supported by FAPMIG. The hospitality of UNIFEI and especially of the De Lorenci family are greatly thanked. Discussions with former and present members of the Shandong Group and Dr. J. Deng are thanked. This work is partially supported by NSFC with grant Nos. 10775090, 10935012, and Natural Science Foundation of Shandong Province, China, with grant nos. ZR2009AM001, JQ200902. YT is supported by Independent Innovation Foundation of Shandong University.

[1] V. V. Anisovich and V. M. Shekhter, Nucl. Phys. B 55 (1973) 455.
[2] J. D. Bjorken and G. R. Rarrar, Phys. Rev. D 9, 1449 (1974).
[3] Y. Jin, Z. G. Si, Q. B. xie and T. Yao, arXiv:1008.1427 [hep-ph].
[4] R. C. Hwa and C. B. Yang, Phys. Rev. C 67, 034902 (2003); R. J. Fries, B. Muller, C. Nonaka and S. A. Bass, Phys. Rev. Lett. 90, 202303 (2003); V. Greco, C. M. Ko and P. Levai, Phys. Rev. Lett. 90, 202302 (2003).
[5] F. L. Shao, T. Yao and Q. B. Xie, Phys. Rev. C 75, 034904 (2007).
[6] Adler SS et al. [PHENIX Collaboration], Phys. Rev. Lett. 91:172301 (2003); S. S. Adler et al. [PHENIX Collaboration], Phys. Rev. Lett. 91, 182301 (2003); J. Adams et al. [STAR Collaboration], Phys. Rev. Lett. 92, 052302 (2004).
[7] B. Andersson, G. Gustafson, G. Ingelman and T. Sjostrand, Phys. Rept. 97 (1983) 31.
[8] W. Han, S. Y. Li, Y. H. Shang, F. L. Shao and T. Yao, Phys. Rev. C 80 (2009) 035202.
[9] J. Song, Z. T. Liang, Y. X. Liu, F. L. Shao and Q. Wang, Phys. Rev. C 81 (2010) 057901.
[10] L. D. Landau and E. M. Lifshitz, Statistical Physics Part I, Pergamon Press 1980.
[11] T. D. Lee, Proceeding of RIKEN BNL Research Center Workshop, Volume 62, p.1-14, May 14-15,2004 (BNL-30 2004); E. Shuryak, J. Phys. G 30 (2004) S1221.
[12] F. L. Shao, Q. B. Xie and Q. Wang, Phys. Rev. C 71, 044903 (2005); T. Yao, W. Zhou and Q. B. Xie, Phys. Rev. C 78, 064911 (2008).
[13] Q. B. Xie and X. M. Liu, Phys. Rev. D 38, 2169 (1988).
[14] Other hadronization models have corresponding issues in different languages, e.g., Lund string [7].
[15] This is very natural, if one adopts that QCD is really the uniquely correct theory for the hadron physics, with its effective Hamiltonian \( H_{QCD} \). Then all the hadron states with definite energy-momentum should be its eigenstates and expand the Hilbert space of states (though we do not know how to solve \( H_{QCD} \) mathematically). While a model is proposed in language of constituent quarks which composite the hadrons, all of the quark states with definite energy-momentum should be eigenstates of the same \( H_{QCD} \) (Here we consider constituent quark model, and ignore the rare probability of exotic hadrons like glueball, hence need not consider gluon states). So these two sets of bases are of different representations, as is more easy to be recognized if one imagines that all the wave functions of hadrons are written in terms of quark states in some special framework of quark models and notices that the planer wave function as well as other special functions (bound state wave functions) are all possible to be complete bases for a definite functional space, mathematically.