Matrix Variate RBM Model with Gaussian Distributions

Simeng Liu, Yanfeng Sun, Member, IEEE, Yongli Hu, Member, IEEE, Junbin Gao, Baocai Yin, Member, IEEE

Abstract—Restricted Boltzmann Machine (RBM) is a particular type of random neural network models modeling vector data based on the assumption of Bernoulli distribution. For multi-dimensional and non-binary data, it is necessary to vectorize and discretize the information in order to apply the conventional RBM. It is well-known that vectorization would destroy internal structure of data, and the binary units will limit the applying performance due to fickle real data. To address the issue, this paper proposes a Matrix variate Gaussian Restricted Boltzmann Machine (MVGRBM) model for matrix data whose entries follow Gaussian distributions. Compared with some other RBM algorithm, MVGRBM can model real value data better and it has good performance in image classification.

Index Terms—RBM, Gaussian distribution, Matrix Variate RBM with Gaussian Distribution.

I. INTRODUCTION

A Boltzmann Machine is a kind of stochastic recurrent neural network. It is invented by Hinton and Sejnowski [1]. Due to its fully connected variable units, this model is not suited for many applications in machine learning and pattern recognition. In order to set up a practical model, Hinton proposed a Restricted Boltzmann Machines (RBM) [2]. A RBM consists of a visible layer and a hidden layer, which are connected without any within visible layer or hidden layer connection. Visible layer is used to receive data input and hidden layer is low dimensional presentation of the input data, hence RBM can be taken as an unsupervised feature extraction model. Training a RBM is to learn the weights between visible units and hidden units from given training data.

The classic RBM can only deal with 1D data, while the data from science and technology often emerge in multi-dimensionality such as image data (2D) or video data (3D). In order to apply classic RBM to this new data structure, a typical workaround is to vectorize multi-dimensional data. The vectorization operation will destroy data’s original structure and ignore valuable information about spatial relationship among multiple arrays. To address this issue, the Tensor variate RBM (TvRBM) was first proposed by Tu et al. [3]. The TvRBM can handle tensor input data, but its hidden layer is still a vector. This results in a vectorial representation in a reduced dimension for input data. Qi et al. [4] proposed a matrix variate RBM model (MVRBM). In this model, both input and hidden variables are in matrix forms which are linearly connected, thus the structural relation among observed units can be well preserved and more spatial information can be utilized.

The classic RBM, TvRBM and MVRBM mentioned above share a similarity that all values of the input units are binary and follow Bernoulli distributions. Such a binary value assumption imposes some limitation on applications in real world tasks where data take real-values such as pixel intensities [5]. Although it is possible to regarded the real-values as the probabilities of binary variables, however whether this is a good representation for data is still questionable [6]. A better strategy to deal with this issue is to directly impose continuous density function assumption. In literature, we have seen Gaussian-Bernoulli RBM (GBRBM) [5], [7], [8]. Improved Gaussian-Bernoulli RBM (IGBRBM) [9], [10] whose visible layer follows Gaussian distribution and hidden layer still follows Bernoulli distribution. Furthermore, the RBM with Gaussian hidden units and binary visible units has also been proposed [7], [11].

In fact, Welling et al. [7] investigated a more general way of incorporating exponential family distributions. In this paper, our objective is to extend the classic RBM with Gaussian distribution to 2D input variables. The extended model is called Matrix variate Gaussian distribution RBM (MVGRBM), which is more appropriate for inputs from the continuous image data. An earlier phenomenon observed by Hinton [6] is that using Gaussian distributions for both the visible and hidden units results in an unstable parameter inference. However we note that if we make the learning rate small the model is still stable and convergent.

The main contribution of the paper lies in

1) We make an assumption of Gaussian distributions for the variables in both the visible layer and the hidden layer. Gaussian distribution can represent the continuous data well and Gaussian latent units can reserve more information in the original input data than binary units; and

2) We propose an efficient parameter inferencing algorithm for the new Gaussian-Gaussian RBM model under the matrix variate form. We have used an approximate representation of 4-mode weight tensor to largely reduce the number of parameters in order to save computational complexity in training stage.

The rest of the paper is organized as follows. In Section II, we introduce our model based on the classic model. In Section III, we formulate the objective function to optimize, propose
update rules for model parameters in a summarized algorithm. The experimental results are reported and analyzed in Section \[\text{V}\]. Finally, conclusions and suggestions for future work are provided in Section \[\text{V}\].

II. MATRIX VARIATE GAUSSIAN RESTRICTED BOLTZMANN MACHINE MODEL.

The classic Restricted Boltzmann Machines are undirected graph models which have the layer of visible units and hidden units connected through weights. The energy function is as follows

$$E(\mathbf{v}, \mathbf{h}; \Theta) = - \mathbf{v}^T \mathbf{W} \mathbf{h} - \mathbf{h}^T \mathbf{v} - \mathbf{c}^T \mathbf{h},$$

(1)

where \( \mathbf{v} \in \mathbb{R}^I \) and \( \mathbf{h} \in \mathbb{R}^K \) are visible units and hidden units, respectively. All the units take binary values which follow Bernoulli distributions. The parameters \( \mathbf{b} \in \mathbb{R}^I \) and \( \mathbf{c} \in \mathbb{R}^K \) are the biases of visible layer and hidden layer. And \( \mathbf{W} \in \mathbb{R}^{I \times K} \) is the connection weight matrix between the visible layer and hidden layer. We collect all the model parameters in \( \Theta = \{ \mathbf{b}, \mathbf{c}, \mathbf{W} \} \). So the mission of training the model is to determine the parameter values from training data.

Qi et al. [4] proposed matrix variate RBM model which extends the classic RBM model to 2D input data. They use the following energy function

$$E(\mathbf{X}, \mathbf{Y}; \Theta) = - \text{tr}(\mathbf{U}^T \mathbf{YXV}^T) - \text{tr}(\mathbf{X}^T \mathbf{B}) - \text{tr}(\mathbf{Y}^T \mathbf{C})$$

(2)

where \( \mathbf{X} \in \mathbb{R}^{I \times J} \) and \( \mathbf{Y} \in \mathbb{R}^{K \times L} \) represent the binary visible and hidden matrices, respectively, and every element in the matrices follows a Bernoulli distribution. \( \mathbf{B} \in \mathbb{R}^{I \times J} \) and \( \mathbf{C} \in \mathbb{R}^{K \times L} \) are the visible and hidden layers’ bias matrices, and \( \mathbf{U} \in \mathbb{R}^{I \times I}, \mathbf{V} \in \mathbb{R}^{L \times J} \) are decomposition matrices of the 4-order connection weight tensor \( \mathcal{W} \in \mathbb{R}^{I \times J \times K \times L} \). In this model, the distributions of visible units and hidden units are the same as that in the classic RBM, but the new model is an extension of the classic model to higher dimensional spaces. And it largely reduces the number of parameters in weights by introducing Kronecker structures in the weight tensor so as to decrease computation complexity.

In this study, we extend the MVGRBM model for matrix variates to incorporate Gaussian distributions for both the visible layer and hidden layer units, termed as MVGRBM. The energy function we are considering is defined as follows,

$$E(\mathbf{X}, \mathbf{Y}) = \sum_{i=1}^{I} \sum_{j=1}^{J} \frac{(x_{ij} - b_{ij})^2}{2\sigma_{ij}^2} + \sum_{k=1}^{K} \sum_{l=1}^{L} \frac{(y_{kl} - c_{kl})^2}{2\gamma_{kl}^2}$$

(3)

$$- \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{l=1}^{L} w_{ijkl} \frac{y_{kl} x_{ij}}{\gamma_{kl}^2 \sigma_{ij}}.$$ 

where \( \mathbf{X} = [x_{ij}] \in \mathbb{R}^{I \times J} \) and \( \mathbf{Y} = [y_{kl}] \in \mathbb{R}^{K \times L} \) represent the visible units and the hidden units, respectively. Given the parameter of a 4-order tensor \( \mathcal{W} = [w_{ijkl}] \in \mathbb{R}^{I \times J \times K \times L} \), visible units bias \( \mathbf{B} = [b_{ij}] \in \mathbb{R}^{I \times J} \) and the hidden units bias \( \mathbf{C} = [c_{kl}] \in \mathbb{R}^{K \times L} \), we assume that the independent random variables \( x_{ij} \) and \( y_{kl} \) follow Gaussian distributions with variance \( \sigma_{ij} \) and \( \gamma_{kl} \), respectively. Denote by \( \Sigma = \{\sigma_{ij}\} \in \mathbb{R}^{I \times J} \) and \( \Gamma = \{\gamma_{kl}\} \in \mathbb{R}^{K \times L} \) the variance matrices, respectively.

Denote by \( \Theta = \{\mathbf{W}, \mathbf{B}, \mathbf{C}, \Sigma, \Gamma\} \) all the parameters in the new model. There are a total number of \( I \times J \times K \times L + 2 \times I \times J + 2 \times K \times L \) free parameters in \( \Theta \). So it requires a large amount of training samples and too much time for training this model. In order to reduce the number of free parameters and save computational complexity in training and inference, we intend to specify a multiplicative interaction between visible units and hidden units by taking \( w_{ijkl} = u_{ij} v_{kl} \). Thus we can re-write the energy function (3) into the following form,

$$E(\mathbf{X}, \mathbf{Y}; \Theta) = \sum_{i=1}^{I} \sum_{j=1}^{J} \frac{(x_{ij} - b_{ij})^2}{2\sigma_{ij}^2} + \sum_{k=1}^{K} \sum_{l=1}^{L} \frac{(y_{kl} - c_{kl})^2}{2\gamma_{kl}^2}$$

$$- \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{l=1}^{L} u_{ij} v_{kl} \frac{y_{kl} x_{ij}}{\gamma_{kl}^2 \sigma_{ij}}.$$ 

(4)

Both matrices \( \mathbf{U} \) and \( \mathbf{V} \) jointly define the interaction between input matrix \( \mathbf{X} \) and hidden matrix \( \mathbf{Y} \). The total number of free parameters in (3) has been reduced to \( I \times K + L \times J + 2 \times I \times J + 2 \times K \times L \) in (4). Now, we can get the rewrite the energy function (4) into the following form,

$$E(\mathbf{X}, \mathbf{Y}; \Theta) = \text{tr}(\mathbf{X}^T \mathbf{B}) - \text{tr}(\mathbf{Y}^T \mathbf{C})$$

$$- \text{tr}(\mathbf{U}^T \mathbf{YXV}^T).$$ 

(5)

where the \( \mathbf{X} = \mathbf{X}/\Sigma^2 \) and \( \mathbf{Y} = \mathbf{Y}/\Gamma^2 \), respectively, and the operator \( ./ \) means element-wise division.

Given the energy function (5), we can derive the conditional probability of hidden layer’s over the visible layer

$$p(\mathbf{Y} | \mathbf{X}; \Theta) = \mathcal{N}(\mathbf{C} + \mathbf{U} \mathbf{XV}^T, \mathbf{\Sigma}^2).$$ 

(6)

So the conditional hidden layer follows a Gaussian distribution, whose mean value is \( \mathbf{C} + \mathbf{U} \mathbf{XV}^T \) and covariance is \( \mathbf{\Sigma}^2 \).

Similarly we can derive the conditional probability of the visible layer over the hidden layer

$$p(\mathbf{X} | \mathbf{Y}; \Theta) = \mathcal{N}(\mathbf{B} + \mathbf{U}^T \mathbf{YV}, \mathbf{\Sigma}^2),$$ 

(7)

which is a Gaussian distribution with the mean \( \mathbf{B} + \mathbf{U}^T \mathbf{YV} \) and the covariance \( \mathbf{\Sigma}^2 \).

III. THE MAXIMUM LIKELIHOOD AND CD ALGORITHM FOR MVGRBM

Let \( \mathcal{X} = \{\mathbf{X}_1, \ldots, \mathbf{X}_N\} \) be an observed data set. We use maximum likelihood method to train the MVGRBM model. The log likelihood of \( \mathcal{X} \) is defined by

$$\ell = \frac{1}{N} \sum_{n=1}^{N} \log p(\mathbf{X}_n; \Theta)$$

$$= \frac{1}{N} \sum_{n=1}^{N} \log \left( \sum_{\mathbf{Y}_n \in \mathcal{Y}} e^{-E(\mathbf{X}_n, \mathbf{Y}_n)} \right)$$

(8)

$$- \log \left( \sum_{\mathbf{X}_n \in \mathcal{X}} \sum_{\mathbf{Y}_n \in \mathcal{Y}} e^{-E(\mathbf{X}_n', \mathbf{Y}_n')} \right).$$ 

The likelihood \( \ell \) is a nonlinear function of a large number of parameters and it is a common practice to adopt a numerical approximation method such as the gradient ascent algorithm.
[12], [13] to find the maximum. For any component \( \theta \) of \( \Theta \), we can prove that
\[
\frac{\partial \ell}{\partial \theta} = -\frac{1}{N} \sum_{n=1}^{N} \sum_{y \in Y} p(Y_n | X_n; \Theta) \frac{\partial E(X_n, Y_n; \Theta)}{\partial \theta}
\]
\[
+ \sum_{x' \in X, y' \in Y} p(x', y'; \Theta) \frac{\partial E(x', y'; \Theta)}{\partial \theta} \quad \text{(9)}
\]
We call the first term of the right side of (9) the data expectation and the second term the model expectation.

The main idea in the CD algorithm is as follows: a Gibbs training set, then alternatively use (7) and (6) to produce the expectation and the second term the model expectation. However, the Contrast Divergence (CD) \([14]–[16]\) procedure allows fast approximation using short Markov chains. The main idea in the CD algorithm is as follows: a Gibbs chain is initialized with a training example \( x^{(0)}_n = x_n \) in the training set, then alternatively use (7) and (6) to produce the chain \( \{ (x^{(0)}_n, y^{(0)}_n), (x^{(1)}_n, y^{(1)}_n), \ldots, (x^{(k)}_n, y^{(k)}_n), \ldots \} \). The CD-k algorithm takes the samples \( \{ x^{(k)}_n \}_{n=1}^{N} \) at step \( k \) to approximate the model expectation, that is,
\[
E(x^{(k)}_n, y^{(k)}_n; \Theta) \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{y \in Y} p(y | x^{(k)}_n; \Theta) \frac{\partial E(x^{(k)}_n, y^{(k)}_n; \Theta)}{\partial \theta}. \quad \text{(10)}
\]
(10) is actually the data expectation over the sampled data in the \( k \)-th step of all the Gibbs chains from training samples, which is similar to the data expectation in (9). Finally the CD algorithm is implemented by
\[
\frac{\partial \ell}{\partial \theta} \approx -\frac{1}{N} \sum_{n=1}^{N} \sum_{y \in Y} p(Y_n | X_n; \Theta) \frac{\partial E(X_n, Y_n; \Theta)}{\partial \theta}
\]
\[
+ \frac{1}{N} \sum_{n=1}^{N} \sum_{y' \in Y} p(Y' | x^{(k)}_n; \Theta) \frac{\partial E(x^{(k)}_n, y'; \Theta)}{\partial \theta}. \quad \text{(11)}
\]
In order to achieve the result close to the reality, we need to update \( \Sigma \) and \( \Gamma \).

\[
\frac{\partial \ell}{\partial \Sigma} = \frac{1}{N} \sum_{n=1}^{N} (G^{(0)}_n - 2(U^T Y^{(0)}_n) \circ X^{(k)}_n)
\]
\[
- \frac{1}{N} \sum_{n=1}^{N} (G^{(k)}_n - 2(U^T Y^{(k)}_n) \circ X^{(k)}_n) \quad \text{(16)}
\]
\[
\frac{\partial \ell}{\partial \Gamma} = \frac{1}{N} \sum_{n=1}^{N} (H^{(0)}_n - 2(UX^{(0)}_n V^T) \circ \hat{Y}^{(0)}_n)
\]
\[
- \frac{1}{N} \sum_{n=1}^{N} (H^{(k)}_n - 2(UX^{(k)}_n V^T) \circ \hat{Y}^{(k)}_n) \quad \text{(17)}
\]
where \( G_n = \left[ g_{ij}^{(n)} = \frac{(x_{ij}^{(n)} - \tilde{y}_{ij})^2}{\sigma_{ij}^2} \right] \) and \( X_n = \left[ \tilde{x}_{ij}^{(n)} = \frac{x_{ij}^{(n)}}{\sigma_{ij}} \right] \).

\[ H_n = \left[ h_{kl}^{(n)} = \frac{(\tilde{x}_{ki}^{(n)} - \tilde{x}_{kj}^{(n)})^2}{\gamma_{ki}^2} \right] \text{ and } \hat{Y}_n = \left[ \tilde{y}_{kl}^{(n)} = \frac{y_{kl}^{(n)}}{\gamma_{kl}} \right]. \]

As the model only depends on the product of parameters \( U \) and \( V \), one parameter may go up in any scale \( s \) while the other goes down to \( 1/s \). To avoid the issue of un-identifying model parameters, we add a penalty of \( \frac{s}{2} (||U||_F^2 + ||V||_F^2) \) to the log likelihood objective \( \ell \). Using (12) to (17), the gradient values which are used to update the parameters of the model are as follows:
\[
\Delta U = \lambda \Delta U + \alpha \left( \frac{\partial \ell}{\partial U} - \beta U \right);
\]
\[
\Delta V = \lambda \Delta V + \alpha \left( \frac{\partial \ell}{\partial V} - \beta V \right);
\]
\[
\Delta B = \lambda \Delta B + \alpha \left( \frac{\partial \ell}{\partial B} \right);
\]
\[
\Delta C = \lambda \Delta C + \alpha \left( \frac{\partial \ell}{\partial C} \right);
\]
\[
\Delta \Sigma = \lambda \Delta \Sigma + \alpha \left( \frac{\partial \ell}{\partial \Sigma} \right);
\]
\[
\Delta \Gamma = \lambda \Delta \Gamma + \alpha \left( \frac{\partial \ell}{\partial \Gamma} \right);
\]
where \( \lambda \) is a momentum term and \( \alpha \) is learning rate to control the convergence of the parameters.

We summarize the overall CD procedure for Matrix variate Gaussian RBM in Algorithm [1]. In all our experiments, we use the special CD-1 algorithm for training.

### A. Multimodal MVGRBM

Real world information comes with multiple channels. For example in the image super-resolution, the lower resolution images are usually associate with many types of features. The classic RBM model has been engineered to handle multimodal data [17]. We can extend our MVGRBM to cope with multimodal data. We assume that the visible layer consists of two separate matrices \( X \in \mathbb{R}^{I \times J} \) and \( Z \in \mathbb{R}^{H \times W} \). Both \( X \) and \( Z \) could be in the same dimension and will be connected to the same set of hidden layer units given by a matrix variate \( Y \in \mathbb{R}^{K \times L} \). We assume the standard covariance of variable
Algorithm 1 CD-K algorithm for MVGRBM

Input: A set of training data of $N$ matrices $X = \{X_1, ..., X_N\}$, the maximum iteration number $T$ (default value = 1,000), the learning rate $\alpha$ (default value = $10^{-4}$), the weight regularizer $\beta$ (default value = 0.01), the momentum $\gamma$ (default value = 0.5), the batch size $b$ (default value = 100) and the CD step $K$ (default value = 1).

Output: Model parameters $\Theta = \{U, V, B, C, \Sigma, \Gamma\}$.

1: **Initialization**: Randomly initialize values for $U$ and $V$, set the bias $B = 0$ and $C = 0$ and the gradient increments $\Delta U = \Delta V = \Delta B = \Delta C = 0$.

2: for iteration step $t = 1 \rightarrow T$ do

3: Randomly divide $X$ into $M$ batches $X_1, ..., X_M$ of size $b$, then

4: for batch $m = 1 \rightarrow M$ do

5: For all the data $X^{(0)} = X \in X_m$ run the Gibbs sampling at the current model parameters $\Theta$:

6: for $k = 0 \rightarrow K - 1$ do

7: sample $Y^{(k)}$ according to (6) with the current $X^{(k)}$;

8: sample $X^{(k+1)}$ according to (7) with $Y^{(k)}$;

9: end for

10: Update the gradient increment with $\lambda_{\theta_{m}}$ and $\lambda_{\theta_{m}}^{(K)}(X^{(K)})$ by using (18);

11: Update model parameters $\theta \leftarrow \theta + \Delta \theta$;

12: end for

13: end for

$X, Z, Y$ is $\Sigma = [\sigma], \Psi = [\psi]$ and $\Gamma = [\gamma]$. This prompts the following energy function

$$E(X, Z, Y; \Theta) = \text{tr}(X - B)^2/\Sigma^2 + \text{tr}(Y - C)^2/\Gamma^2$$

$$- \text{tr}(Q^T Y R Z^T) + \text{tr}(Z - A)^2/\Sigma^2 - \text{tr}(U^T Y V X^T)$$

(19)

where $\bar{X} = X/\Sigma^2, \bar{Z} = Z/\Psi^2$, and $\bar{Y} = Y/\Gamma^2$.

IV. Experimental Results and Analysis

In this section, we conduct several experiments on some public databases to verify the effectiveness of the proposed model. These experiments are designed to demonstrate the performance of MVGRBM in feature extraction and reconstruction compared with other RBM models. The algorithm is coded by Matlab, and the codes run on the computer with 3.40GHz Intel Core i7-2600 CPU and 16.0GB internal storage capacity.

A. Reconstruction Performance

In this section, we test the reconstruction errors of the proposed MVGRBM model on MNIST [18] handwritten digit database. This database consists of a train set and a test set. The train set contains 60000 images, and the test set 10000 images. Every image is a digit number from 0 to 9 and the size is 28 × 28 gray scale. First, we would like to demonstrate that the proposed model learn the information from input data samples. We set the hidden layer as 15 × 15, use most of default parameter values in Algorithm 1 and let all the training set as the input data. The original digit number images and the reconstructed images is shown in Figure 1.

We calculate the relative reconstruction error which is defined as $D = \|X - X^{'}\|_F$, where the $X$ is the original input image, and $X^{'}$ is the reconstructed image. So the overall reconstruction error of the whole image set is defined as $\frac{1}{N} \sum_{n=1}^{N} D_n$. Figure 2 shows the reconstruction errors for our method and other methods on the MNIST database. Compared to MVRBM and IGBRBM, our method shows much faster convergence speed and the value of reconstruction error is much smaller than other two algorithms at the same iteration steps.

B. Recognition Performance

In this section, we compare the recognition performances of traditional RBM, IGBRBM, MVRBM and MVGRBM on ETH80 and Ballet database.

1) Recognition on ETH80: The ETH80 database consists of 8 ground truth object categories, each consisting of 10 different instances from 41 different views, for a total of 3280 images. These objects undergo severe view point change, posing significant challenges to recognition due to the high intra-class diversity. More details about the database can be found in [19]. We randomly partitioned 41 images of different
views into two sets: 21 images of all objects for training (21 × 10 × 8) and 20 images for testing (20 × 10 × 8). We train the parameters with training set and calculate the hidden units of test set using the trained parameters. Then we use the hidden units as inputs to the nearest neighbor classifier (1-NN). Table I shows the classification results of traditional RBM, IGBRBM, MVRBM and MGGRBM.

2) Recognition on Ballet: The Ballet dataset contains 44 video clips collected from an instructional ballet DVD. Each clip has 107 to 506 frames. The dataset consists of 8 complex action patterns performed by 3 subjects. These actions include: left-to-right hand opening, right-to-left hand opening, standing hand opening, leg swinging, jumping, turning, hopping and standing still. We randomly extracted 200 images from each action as the training sample. Every image is down-sampled to size 32 × 32 gray image. We test the recognition performance with the same way as on ETH80. Table II shows the classification result of traditional RBM, IGBRBM, MVRBM and MGGRBM. The best recognition accuracy rate achieved 0.9357 that is much better than other three algorithms.

A key observation from the above two sets of experiments is that incorporating Gaussian units in the model significantly improve the classification accuracy for image recognition. With iteration times increased, the classification accuracy goes on but it will reach peak at a certain iteration time. In the recognition experiment on ETH80, the accuracy of 2000 iteration times smaller than the 1000 times.

C. Image Super-resolution

In this experiment we apply our Multimodal MVGRBM for image super-resolution. We follow the same setting used in [20] to prepare training data. The training patches are randomly taken from 69 Miscellaneous color images [21]. The testing image data set named Miscellaneous which includes 64 images and each one is 512 × 512 pixel.

In this experiment, we randomly sample N=10000 training patches. The patch size is 10×10 and the hidden size is 20×20. Figure 3(a) shows sample patches of size 10 and Figure 3(b) shows the learned U and V in terms of $U^T \otimes V^T$.

We tested the image super-resolution performance with Bicubic Interpolation, SR, MVRBM and MVGRBM. At first the images are down-sampled to 256 × 256. Then get the Super-resolution restoration images through the algorithms mentioned above. And then calculate the PSNR value using the original high resolution image and the restoration image. The assessment criteria is reconstruction Peak signal to noise ratio (PSNR). We give the PSNR values of different methods which are shown in Table III and the reconstructed images are shown in Figure 4. We selected 6 PSNR result of image super-resolution. It can be seen that the ‘avion’, ‘house’, ‘safari04’ and ‘safari16’ results generated by our algorithm are the most optimal. The other two are suboptimal but not differ a lot with the optimal result. It proves our algorithm has superiority on the super-resolution applications.

V. CONCLUSION

In this paper, we proposed the matrix variate RBM with Gaussian units named MVGRBM. The proposed model assumes both input and hidden units are in matrix format without vectorizing data as done in traditional RBM models and it also assumes Gaussian units on both the visible layer and the hidden layer. The model is more appropriate to adapt to the real-values data from reality world as demonstrated by our experimental results. This model can be used in many applications such as image reconstruction, image classification and image super-resolution. The experimental results proved effectiveness of the model.

Despite of these demonstrated applications of MVGRBM, it is very challenging to select the learning rate and adjust the other hyper-parameters in the training process. It is worthwhile to further research towards improving and easing the parameter adjustment in training.

REFERENCES

[1] G. E. Hinton and T. J. Sejnowski, “Optimal perceptual inference,” in Proceedings of the First IEEE conference on Computer Vision and Pattern Recognition. Washington: IEEE, 1983, pp. 448–453.
[2] G. E. Hinton, “Training products of experts by minimizing contrastive divergence,” Neural Computation, vol. 14, no. 8, pp. 1771–800, 2002.
[3] T. D. Nguyen, T. Tran, D. Phung, and S. Venkatesh, “Tensor-variate restricted Boltzmann machines,” in Proceedings of the Twenty-Ninth National Conference on Artificial Intelligence. Austin, Texas: AAAI Press, 2015, pp. 2887–2893.
[4] G. Qi, Y. Sun, J. Gao, Y. Hu, and J. Li, “Matrix variate RBM and its applications,” Preprint: [arXiv:1601.00722], Tech. Rep., 2016.
[5] A. Krizhevsky, “Learning multiple layers of features from tiny images,” Master’s thesis, University of Toronto, 2009.
| Image Name     | Bicubic Interpolation | SR | MVRBM | MVGRBM |
|---------------|-----------------------|----|-------|--------|
| 'avion.ppm'   | 32.5152               | 34.0530 | 33.4382 | 34.5163 |
| 'house.ppm'   | 30.3939               | 31.5096 | 31.1922 | 31.8053 |
| 'safari04.ppm'| 34.7176               | 36.5163 | 35.7185 | 36.3406 |
| 'safari16.ppm'| 36.5781               | 38.3812 | 37.5188 | 38.5799 |
| 'lena.ppm'    | 35.4241               | 37.3995 | 36.2041 | 36.9283 |
| 'ivytree.ppm' | 30.1023               | 31.5747 | 31.3910 | 31.5607 |

TABLE III
IMAGE SUPER-RESOLUTION RECONSTRUCTION BY DIFFERENT METHOD LIKE BICUBIC INTERPOLATION, MVRBM, SR AND OUR METHOD.

(a) The original image  
(b) Bicubic Interpolation recovery result  
(c) The Sparse Representation recovery  
(d) The MVRBM recovery  
(e) The MVGRBM recovery

Fig. 4. Super-resolution result of the house image.

[6] G. Hinton, “A practical guide to training restricted Boltzmann machines,” *Momentum*, vol. 9, no. 1, p. 926, 2010.
[7] M. Welling, M. Rosen-Zvi, and G. E. Hinton, “Exponential family harmoniums with an application to information retrieval,” in *Proceedings of the Seventeenth Conference on Neural Information Processing Systems*. Vancouver, Canada: MIT Press, 2004, pp. 1481–1488.
[8] G. E. H. Vinod Nair, “Rectified linear units improve restricted Boltzmann machines,” in *Proceedings of the 27th International Conference on Machine Learning*, 2010, pp. 807–814.
[9] K. Cho, A. Ilin, and T. Raiko, “Improved learning of Gaussian-Bernoulli restricted Boltzmann machines,” in *Proceedings of the Twenty-first International Conference on Artificial Neural Networks*. Springer, 2011, pp. 10–17.
[10] K. Cho, “Improved learning algorithms for restricted Boltzmann machines.” Master’s thesis, Aalto University School of Science, 2011.
[11] R. Salakhutdinov, A. Mnih, and G. Hinton, “Restricted Boltzmann machines for collaborative filtering,” in *Proceedings of the Twenty-fourth International Conference on Machine Learning*, ACM. Corvallis, Oregon, United States: ACM, 2007, pp. 791–798.
[12] M. Zinkevich, “Online convex programming and generalized infinitesimal gradient ascent,” in *Proceedings of the Twentieth International Conference Machine Learning*, pp. 928–936, 2003.
[13] G. Montavon, G. Orr, and K. R. Miller, *Neural networks: tricks of the trade*, 2nd ed. Springer, 2003.
[14] A. Fischer and C. Igel, *An Introduction to Restricted Boltzmann Machines*. Springer Berlin Heidelberg, 2012.
[15] Y. Bengio and O. Delalleau, “Justifying and generalizing contrastive divergence,” *Neural Computation*, vol. 21, no. 6, pp. 1601–21, 2008.
[16] M. A. Carreira-Perpinan and G. E. Hinton, “On contrastive divergence learning,” *Artificial Intelligence & Statistics*, vol. 10, pp. 33–40, 2005.
[17] R. Salakhutdinov and G. E. Hinton, “Replicated softmax: an undirected topic model,” in *Proceedings of the Twenty-second Conference on Neural Information Processing Systems*. Vancouver, Canada: Curran Associates, 2009, pp. 1607–1614.
[18] Y. LeCun and C. Cortes, “The MNIST database of handwritten digits, available: http://yann.lecun.com/exdb/mnist/index.html,” 2015.
[19] B. Leibe and B. Schiele, “Analyzing appearance and contour based methods for object categorization,” in *Proceedings of the Sixteenth IEEE Conference on Computer Vision and Pattern Recognition*. Madison, Wisconsin, USA: IEEE, 2003, pp. 409–415.
[20] J. Yang, J. Wright, T. S. Huang, and Y. Ma, “Image super-resolution via sparse representation,” *IEEE Transactions on Image Processing*, vol. 19, no. 11, pp. 2861–2873, 2010.
[21] U. o. G. Computer Vision Group, “Miscellaneous color images. available: http://decsai.ugr.es/cvg/dbimagenes/c512.php.”