Optimal Tree Hash Modes: the Case of Trees Having their Leaves at All the Levels

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Abstract. A recent work shows how we can optimize a tree based mode of operation for a rate-1 hash function where the sizes of input message blocks and digest are the same, subject to the constraint that the involved tree structure has all its leaves at the same depth. In this note, we show that we can further optimize the running time of such a mode by using a tree having leaves at all its levels.

1 Introduction

In the cryptographic hashing context, we are interested in a method of finding a tree structured circuit topology to optimize both the running time and the number of involved processors. We consider hash tree modes using a rate-1 hash function (or variable input length compression function), denoted \( f \), where the digest and the message block sizes coincide. A rate-1 hash (or compression) function needs only \( l \) invocations of the underlying primitive to process a \( l \)-block message. Let us assume a hash tree of height \( h \) having all its leaves (i.e. message blocks) at the same depth. If we denote by \( a_i \) the arity of level \( i \) (for \( i = 1\ldots h \)), then the parallel running time to obtain the root node value is \( \sum_{i=1}^{h} a_i \). A recent work \(^1\) shows that we can select the good parameters to construct such trees that minimize both the running time and the number of processors. The aim of the present note is to show that we can further decrease the parallel running time of a tree-based hash function by removing this structural constraint on the tree. We then remark that the allocation of tasks to the processors is a bit more subtle, and that the parallel running time is no longer the sum of the level arities.

This note is organized in the following way. We give some definitions about trees and hash functions in Section 2 and describe a new tree construction suitable for parallel hashing in Section 3.

2 Terminology

Throughout this note we use the convention\(^1\) that a node is the result of an inner function \( f \) called on a data composed of the node’s children. A node value

\(^1\) This corresponds to the convention used to describe Merkle trees. The other (less frequent) convention is to define a node as being a \( f \)-input.
then corresponds to an image by such a function and a child of this node can be either an other image or a message block. In this note, a k-ary tree of height h is a tree having the following properties:

- The root node (at level h) can be of arity a, with 1 < a ≤ k.
- A level i (≠ h) has all its nodes of arity k, except the rightmost one that can be of smaller arity.

We define the arity of a level in the tree as being the greatest node arity in this level.

We assume that a node is computed using a rate-1 inner function that iteratively processes message blocks of size N bits and produces a digest of N bits. Some existing inner functions are of this type, such as the UBI (Unique Block Iteration) function of Skein [3] and the single-block-length hash functions [4,5].

3 Optimal trees having their leaves at all the levels

The idea of processing both message blocks and chaining values (non-leaf nodes, i.e. digests) using a single inner function evaluation was suggested in [2] (under the name of kangaroo hopping) in order to avoid certain computation overheads. With our assumptions, applying this idea for all nodes in a tree leads to a hash tree having an interesting parallel running time.

**Theorem 1.** Let a message of length l blocks. We can construct a hash tree allowing a parallel running time of exactly ⌈log₂ l⌉ + 1, using ⌈l/2⌉ processors.

**Proof.** We first give the construction of a tree structure. Then, we consider a hash function based on it, and we give a scheduling strategy to perform all the computations in parallel. Let us consider a binary tree of height h = ⌈log₂ l⌉. We denote by lᵢ the number of nodes of level i ≥ 1. Remark that this binary tree can be such that all its leaves are at the same depth and lᵢ = ⌈l/2ⁱ⌉. The lᵢ nodes of the level i are indexed. For j = 1...lᵢ, one node of this level is denoted Nᵢj and, in particular, its leftmost child is denoted Nᵢⱼ,LC. Note that if Nᵢⱼ has a single child, this latter is still considered as its leftmost child. At each level i of this tree, starting from level 2 up to level h, we transform the nodes in the following way: for j = 1...lᵢ, the node Nᵢⱼ,LC is discarded and its children become the children of Nᵢⱼ. We notice that once this operation is performed, a node Nᵢⱼ can have a higher number of children. The result is a transformed tree which is no longer a binary tree and where leaves are located at all the levels. An example of execution of this algorithm is depicted in Figure 1.

We now consider a hash function based on this tree structure. The computations are done in parallel in the following way: in a same parallel step, each processor starts the computation of one of the ⌈l/2⌉ nodes that has leaves. This parallel step requires 2 units of time. Hence, the computations of these nodes (or of their parent nodes) can progress in a parallel step of one unit of time. We need to repeat such a parallel step as many times as necessary to complete the
processing of this hash tree, i.e., $\lceil \log_2 l \rceil - 1$ times. We then deduce a parallel running time of $\lceil \log_2 l \rceil - 1 + 2$ units of time. The number of involved processors corresponds to the number of nodes having leaves, i.e. $\lceil l/2 \rceil$. An example of parallel hash computation is depicted in Figure 2.

Fig. 1: Derivation of a tree structure from a binary tree that processes a message of 7 blocks

Fig. 2: Example of the processing of 8 blocks $m_1, m_2, ..., m_8$, using another tree representation. We have 4 processors denoted $P_1, P_2, P_3$ and $P_4$. A dotted line represents a serial computation using the same hash context, while a solid line indicates that a hash state is used by another hash context. For instance, the processor $P_2$ computes the hash of $m_3 \parallel m_4$, denoted $c_1$. The chaining values $c_1$ and $c_3$ are used by the hash context of the processor $P_1$. The parallel running time to compute the root node is equal to the running time required for computing the hash of $m_1 \parallel m_2 \parallel c_1 \parallel c_3$, i.e., 4 units of time.
**Performances improvements.** The (parallel) running time of such a tree is to be compared with the running time of an optimal tree having its leaves at the same depth $\Pi$, i.e. approximately $3\lceil\log_3 l\rceil$. This represents, approximately, a $2x$ speedup.

What if we apply the algorithm above on a ternary tree to construct another tree? The transformed tree would lead to a parallel running time of at most $(\lceil\log_3 l\rceil - 1) \cdot 2 + 3 = 2\lceil\log_3 l\rceil + 1$. More precisely, if the ternary tree has a root node of arity 3, then the hash function based on the transformed tree has a parallel running time of exactly $2\lceil\log_3 l\rceil + 1$. Otherwise, if it is of arity 2, the transformed tree leads to a parallel running time of exactly $2\lceil\log_3 l\rceil$.

For a large message length $l$, we have $\lceil\log_3 l\rceil + 1 < 2\lceil\log_3 l\rceil$. It is thus more interesting to use the topology derived from a binary tree. For a finite and small number of $l$, the tree topology derived from a ternary tree gives the same running time. For these message lengths, such a topology is preferable since it decreases the number of involved processors. For the reasons outlined below, deriving a topology from a quaternary tree or any tree of arity greater than 4 worsen the parallel running time.

Can we further decrease the number of processors while conserving the running time stated in the theorem above? To do so, we should be able to increase the number of nodes or message blocks processed by one processor during one parallel step. Let us see a counter-example. Suppose that we have a hash tree that can be processed in a parallel running time of $\lceil\log_2 x\rceil + 1$ and that one node in this tree has more than 3 leaves, say $x$ leaves. We have $\lceil\log_2 x\rceil + 1 < x$ when $x > 3$, meaning that we can transform this node in order to improve the overall running time.

**References**

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