Mapping the radio to mid-infrared spectral index of Cas A: evidence for flattening and a cooling break

V. Domček,1,2 ⋆ J. Vink,1,2,3 P. Zhou1 and J. V. Hernández Santisteban4,

1 Anton Pannekoek Institute for Astronomy, University of Amsterdam, Science Park 904, 1098 XH Amsterdam, The Netherlands
2 GRAPPA, University of Amsterdam, Science Park 904, 1098 XH Amsterdam, The Netherlands
3 SRON, Netherlands Institute for Space Research, Utrecht, The Netherlands
4 SUPA, Physics and Astronomy, University of St Andrews, St. Andrews, KY16 9SS, Scotland, UK

Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT
Non-linear shock acceleration models predict a flattening of synchrotron radiation spectrum at higher frequencies. For Cassiopeia A a synchrotron spectral flattening was previously reported for a small part of the remnant in the mid-infrared regime. Here we present new measurements for spectral flattening using the archival radio (4.72 GHz) and mid-infrared (3.6 µm) data, and produce a complete spectral index map to investigate the spatial variations within the remnant. Our result shows overall spectral flattening across the remnant (α ∼−0.5 to −0.7), with the flattest values coinciding with the locations of most recent particle acceleration. In addition to overall flattening, we also detect a relatively steeper region in the Southeast of the remnant (α ∼−0.67). We argue that this could be a signature of a cooling break, and estimate the lower limit of magnetic field to be B ∼420 µG, consistent with other magnetic-field strength estimates.

Key words: acceleration of particles – ISM: individual objects: Cassiopeia A — ISM: supernova remnants — radiation mechanisms: non-thermal

1 INTRODUCTION

Cosmic rays (CRs) are highly energetic particles that have been discovered more than a century ago (Hess 1912) that span in the energy range from a few hundred MeV up to 10^{20} eV. Their spectral distribution up to 3 × 10^{15} eV is roughly a power law with a negative index of 2.7. The spectral break around 3 × 10^{15} eV is known as the “knee”. Up to the knee the CR spectrum is dominated by protons, and the CRs must be of Galactic origin (e.g. Hillas 2005). For a long time it has been suspected that the energy needed to accelerate these Galactic CRs are powered by supernovae (Baade & Zwicky 1934).

It is known that acceleration of CRs happens in supernova remnants (SNRs): the relativistic CR electrons produce radio synchrotron emission (see Dubner & Giacani 2015, for a review) and some young SNRs even produce X-ray synchrotron radiation near their shock fronts (Reynolds 2008; Held et al. 2012, for reviews). The gamma-ray detection of many SNRs have confirmed the presence of CRs. The gamma rays could either be caused by the same electron population that causes synchrotron emission, or arise from hadronic CRs, which produce gamma rays through interactions with the background gas. The latter has been positively identified in at least a few SNRs (e.g. Ackermann et al. 2013). There is not yet evidence that SNRs can accelerate up to the “knee”, but some SNRs can produce CRs close to that energy. For example, the gamma-ray bright source RX J1713.7-3946 produces gamma rays up to 100 TeV (Aharonian et al. 2006).

The CR acceleration in SNRs is thought to be caused by diffusive shock acceleration (DSA), also known as first-order Fermi acceleration (Krymskii 1977; Axford et al. 1977; Bell 1978; Blandford & Ostriker 1978). According to the DSA theory, charged particles can repeatedly scatter across the shock by the magnetic field irregularities present on both sides of the shock, thereby gaining energy. After each cycle a small fraction of the particles will not return to the shock again. The combination of these two effects will result in a power-law distribution with a (negative) spectral index of q = 2 for the expected compression ratio χ = 4 between shocked and unshocked medium.

It was soon realised that if CR acceleration is efficient, the presence of CRs ahead of the shock will influence the unshocked plasma: the CRs will push against the unshocked plasma, compressing it and setting it in motion (Eichler 1979; Ellison & Eichler 1984). Moreover, escape of the highest energy CRs far ahead of the shock, will drain energy from
the system. The net result is that the combined compression of the CR precursor and the shock compression itself will be larger than $q = 4$, whereas the shock compression itself may be reduced as the flow will arrive at the shock with a slower speed, and, therefore, lower Mach number. These processes are addressed in the theory of non-linear DSA (Malkov & Drury 2001, for a review).

According to the theory of non-linear DSA the CR spectrum will be steeper than $q = 2$ for the low energy particles, as a result of the lower shock compression, and it will be flatter than $q = 2$ for the highest energy particles, as these sample the total compression ratio. The total compression ratio can be in most extreme forms of the theory larger than 10 (e.g. Baring et al. 1999; Ellison et al. 2000), largely driven by very high CR acceleration efficiency of $> 50\%$. Indeed there is some evidence that the radio emission from young SNRs has the predicted deviation from power-law (e.g. concave spectrum, Reynolds & Ellison 1992). The asymptotic value for the spectral index is expected to be $q = 1.5$ (Malkov 1997).

The CRs in the precursor may also give rise to another non-linear effect: it may cause magnetic field amplification through the so-called Bell mechanism (Bell 2004). X-ray synchrotron emission provides evidence for amplified magnetic field near SNR shocks (Vink & Laming 2003; Helder & Vink 2008).

In recent years less attention has been paid to the non-linear DSA theory. First of all, although there is some evidence for enhanced compression ratios (Warren et al. 2005), the enhancement seems to be modest (see also Ferrand & Thorstensen 2001) and the brightest SNR in the sky relativley free of warm dust emission. We combine this with the secular fading of Cas A estimated to be around 0.5 mG, but in likely variations will be present (Rosenberg 1970; Atoyan et al. 2000; Vink & Laming 2003). This puts the expected cooling break in the mid- to near-infrared regime, as we will discuss later.

2 OBSERVATIONS AND DATA ANALYSIS

2.1 Radio

For the radio comparison map we used a VLA radio image (4.72 GHz) that was obtained in 2000-2001 and kindly provided by T. Delaney (Delaney 2004). Flux calibration was performed by fitting a power-law of the flux density measurements from Perley & Butler (2017) and obtaining the flux density at 4.72 GHz. For the total flux density to correspond with the year when Spitzer data were obtained (2009), this value was further corrected for the secular fading using a decline rate of 0.67% yr$^{-1}$, reported by Trotter et al. (2017). The total flux density of Cas A after these corrections is $S_{72 \text{GHz}} = 700$ Jy. Additionally we apply a low flux density mask on the data that masks out flux densities lower than $5 \times 10^{-2}$ Jy pix$^{-1}$ corresponding to the background level. We assume a 10% fractional error in the flux for each pixel as a robust estimator of the map’s uncertainty.

2.2 Mid-infrared

Mid-infrared data were observed in August 2009 (ObsID 34836224; PI: R.G.Ardent) using the Spitzer Infrared Array Camera (IRAC) (Fazio et al. 2004) and obtained from the Spitzer Heritage Archive. We used the mosaiced products of the extended pipeline (level 2) that combines individual calibrated frames with the refined telescope pointing. We restrict ourselves to the channel 1 (3.6 μm) for reasons explained at the end of this section. These data contain both the flux measurements (see Fig. 1) and their uncertainties.

The mid-infrared data require several corrections to be meaningfully compared to the radio. We correct for (a) mid-infrared environmental background, (b) point-source contribution from the stars, (c) extinction in the line of sight, and (d) contribution of other sources of emission and other sources of emission. Each correction can introduce additional uncertainties into our final result (see diagram of contributions in Fig. 3). To tackle all of these contributions, their uncertainties and inter-dependencies we employ Monte Carlo simu-
The first correction we consider is the mid-infrared background, visible in the uncorrected map of the remnant (right panel of Fig. 1). We use regions in Southwest and Northwest of Cas A to estimate the level of the background, visible in the uncorrected map of the remnant. Northwest of Cas A to estimate the level of the background, (right panel of Fig. 1). We use regions in Southwest and background, visible in the uncorrected map of the remnant a single pixel in the appendix of this paper.

The second consideration in our method is taking into account the mid-infrared extinction. Hurford & Fesen (1996) found extinction in Cas A to be varying between 4.6 < $A_v$ < 6.2 mag increasing towards the west of the remnant. Recent X-ray study measuring the hydrogen column density (Hwang & Laming 2012) further confirm these results and using the Güver & Özel (2009) $N_H$ − $A_v$ relation places a lower limit of the extinction to $A_v$ = 5 mag. The western part of the remnant shows even stronger extinction with values going up to $A_v$ = 15 mag. We use $N_H$ measurements of Hwang & Laming (2012) (see Fig. 2) and assume a 10% fractional uncertainty in each pixel as this uncertainty was not provided. The created distributions are then propagated through the $N_H$ − $A_v$ relation (Güver & Özel 2009) and extinction curve (Indebetouw et al. 2004) and applied to our image.

Stars provide an additional mid-infrared background source in the image. Their presence can increase the measured flux density, and locally change the measured spectral index. We use DAOSstarFinder utility to find and mask bright stars.

Another source of emission that usually contributes to mid-infrared wavelengths is thermal emission from the dust particles. Based on their size and chemical composition the radiation can be produced in different mid-infrared wavelengths covering different Spitzer channels. Dust in Cas A consists mostly of silicate dust (De Looze et al. 2017) and due to the Silicon grains properties we do not expect any dust emission contribution below $\lambda$ ~ 7 $\mu$m (see Figure 10 of Draine & Li (2007) and Figure 3 of Rho et al. (2008)). This is the main reason why we choose only Channel 1 ($\lambda$ = 3 – 4 $\mu$m) of the Spitzer telescope data for our analysis. This channel can further contain contribution from polycyclic aromatic hydrocarbon (PAH), but as shown in Fig.2 of Reach et al. (2006), these are not relevant for the case of Cas A.

Both Spitzer and VLA images were re-projected to common WCS header using reproject_exact tool in PYTHON which results in image pixel size of $\approx$ 1.4 ”pix$^{-1}$.

2.3 Spectral index maps

Creating spectral index maps is one of the methods to investigate the spectral evolution of extended objects like SNRs and the main method of this paper. The core of this method

---

1. https://photutils.readthedocs.io/en/stable/api/photutils.detection.DAOStarFinder.html
2. https://reproject.readthedocs.io/en/stable/api/reproject_reproject_exact.html
is in comparing flux measurements at two distinct frequencies. The further these frequencies are apart, the less dependent the measure spectral index is on systematic and statistical uncertainties of the flux measurements.

In our work we produce a spectral index map between the radio and mid-infrared (VLA and Spitzer) frequencies:

\[
\alpha = \frac{\log(F_{\nu,\text{Spitzer}}) - \log(F_{\nu,\text{VLA}})}{\log(\nu_{\text{Spitzer}}) - \log(\nu_{\text{VLA}})}.
\]

The numerator of the Equation 1 represents calibrated and a masked array of distributions of Spitzer and VLA while the denominator is the frequencies attributed to these observations. The result is an array of distributions that accounts for all the previous uncertainties of our data calibration process. We collapse these results into two maps that show the median spectral index value and its 1-σ uncertainty in Fig.4.

As we investigate the spectral evolution of the remnant, we would ideally like to have a radio-radio spectral index map for comparison. However, constructing a reliable map with sufficient resolution in radio is notoriously difficult due to calibration errors, caused, for example, by different \( u-v \) coverage of the radio observations, and the smaller frequency span between two radio frequencies. We therefore limit ourselves to comparing the measured radio to mid-infrared spectral index to the average radio spectral index (\( \alpha_{\text{radio}} = -0.77 \)) measured by Trotter et al. (2017). This provides us with a base to assess whether the radio to the mid-infrared spectral index is flattening, steepening or follows the extrapolated radio power-law spectrum.

3 RESULTS

We produced a spectral index map and its map of uncertainties between the radio (4.72 GHz) and mid-infrared (3.6 μm) energy bands (see Fig. 4). The results show overall a significant flattening of the spectral index to a mean spectral index of \( \alpha = -0.60825 \pm 0.00012 \), which is much flatter than the average spectral index measured in radio band (\( \alpha_{\text{radio}} \approx -0.77 \)).

Although a spectral flattening is required for the whole remnant, its magnitude is non-uniformly distributed across the remnant. The flattest index (with values of \( \alpha \approx -0.58 \)) is found in the regions close to the forward, and near the reverse shock in the western to southwestern parts of the remnant. The index steepens slightly to \( \alpha \approx -0.62 \) in-between the forward and reverse shocks, most visibly in the northern half of the remnant. The region that stands out most in our spectral maps is located in the southeastern part of the remnant and has a relatively steep spectral index of \( \alpha \approx -0.67 \). The lack of mid-infrared emission which gives rise to this relatively steep spectral index in this region is also noticeable in the other Spitzer channels (4.5, 5.8, 8.0 μm; see in the Appendix Fig. A4). It is also a region that appears to be devoid of non-thermal X-ray emission (Helder & Vink 2008).

In order to compare these interesting regions with the results for the whole remnant, we produce separate histograms for some regions and calculate their mean spectral index. For easier communication of the results, we specify the regions as: i) forward shock (pink colour), ii) northern arc (blue colour), iii) dark spot (purple colour). We show their locations in Fig. 5 and list the mean values in Table 1, and the respective histograms in Fig. 6 (left).

We tested whether there is a correlation between low or high mid-infrared flux and mean spectral index value. We perform this as a test of the robustness of our results and its dependency on the mid-infrared background subtraction. If, for example, we had been over-subtracting the background, it would mainly have affected the histogram of the low-flux region and offset it towards the steeper spectral indices. The mid-infrared flux of \( 4 \times 10^{-5} \) Jy was chosen to be the separating value between the low- and high-flux regions. We find that low-flux regions have only marginally flatter spectra than the high-flux regions (see Fig. 6 (right) and Tab. 1). This confirms that our method for background subtraction is robust and does not significantly skew our results.

We further tested the effect of increasing the uncertainty of the background to \( 3 \times 10^{-5} \) Jy. Although it does not change the spectral index values by itself, it can in the most affected low mid-infrared flux regions increase the uncertainty up to \( \sigma \sim 0.06 \). This however still points towards significant enough spectral flattening.

| Region                  | Mean spectral index | Distribution width |
|-------------------------|---------------------|--------------------|
| Whole remnant           | -0.6083 ± 0.0001    | 0.031              |
| Forward shock           | -0.5957 ± 0.0003    | 0.034              |
| Northern arc            | -0.6325 ± 0.0003    | 0.024              |
| Dark spot               | -0.6514 ± 0.0005    | 0.020              |
| Low mid-IR flux         | -0.6090 ± 0.0001    |                    |
| High mid-IR flux        | -0.6069 ± 0.0002    |                    |

Table 1. Mean spectral index and width of the distributions for regions of interest shown in Fig. 6.
Mapping the spectral index of Cas A

4 DISCUSSION

Our study shows that there is an overall flattening of the synchrotron spectrum of Cas A between the radio and mid-infrared. Here we will compare our results to previous studies on the broad band spectral properties of the synchrotron emission, and discuss the results in the context of non-linear shock acceleration models. For the steepest spectrum region we examine whether the steepness could be due to a cooling break, and discuss the implications for the magnetic-field strength in this part of the SNR shell.

4.1 Comparison with other studies

Evidence for spectral flattening of the synchrotron spectrum has been found before. High frequency radio observations above 30 GHz already showed that the overall spectrum of Cas A flattens at high frequencies (Mezger et al. 1986; Onič & Urošević 2015). This is also supported by infrared studies in the 2 µm range Jones et al. (2003); Rho et al. (2003). The most important difference between these studies and what is presented here is that the radio studies concerned the spectral flattening of the whole remnant, without the ability to detect spatial variation. The infrared study by Jones et al. (2003) was restricted to small regions in the

Figure 3. Procedure towards obtaining the spatially-dependent spectral index $\alpha$. Each step introduces a new uncertainty which is propagated towards the final result shown in Fig. 4.

Figure 4. Radio to mid-infrared spectral index map (left) with its uncertainties (right).
northwest part of the remnants, and the study by Rho et al. (2003) was limited mainly to the bright shell, because of the low flux of diffuse emission in the K-band, and the relatively high background.

Moreover, we find, on average, flatter spectral indices \( \alpha \sim -0.6 \) compared to those reported at 2 \( \mu \)m \( (\alpha_{\text{Radio}} \sim -0.7) \), even for comparable locations. The discrepancy could be caused by either a general steepening at shorter wavelengths, or by differences in the employed methodology, for example by our more detailed treatment of extinction and uncertainties (see Fig. 3).

As there are some systematic uncertainties regarding the measurements of \( N_{\gamma} \) of the order of 25% (see for example Zhou & Vink 2018), we tested what the effect is of a 25% overestimation of \( N_{\gamma} \) on our final spectral index, and found it to be \( \Delta \alpha \approx 0.02 \) at most. So this cannot account for the discrepancy between our measurements and those using the emission at 2 \( \mu \)m.

A general steepening at shorter wavelength would be an interesting possibility; as it may hint at a synchrotron emission at 2 \( \mu \)m, \( \sim \)it to be a high background.

The relativistic electron population responsible for the synchrotron emission will also produce gamma-ray emission through inverse Compton scattering, with a power law slope that should be \( \Gamma = \alpha + 1 \). However, for Cas A the gamma-ray emission is suspected to be dominated by hadronic gamma-ray radiation: protons and other atomic nuclei colliding with background plasma, thereby producing neutrals pions, which immediately decay into photons. If we denote the particle spectral index with \( p \), the gamma-ray spectral index for pion decay should be \( \Gamma \approx p \), whereas for inverse Compton scattering it should be \( \Gamma = (p+1)/2 \) (see Hinton & Hofmann (2009) for a review on the gamma-ray emission from SNRs).

The latest interpretation of the gamma-ray emission from Cas A by MAGIC collaboration, which analysed 168h of MAGIC and 8 years of Fermi data Ahnen et al. (2017), points towards a required hadronic component with the proton index \( p = 2.21 \). The most recent paper by VERITAS collaboration (Abeysekara et al. 2020) is in agreement with Ahnen et al. (2017) results, although with slightly different spectral modeling results. According to the theory of DSA, both electrons and hadrons are accelerated with the same spectral index in energy, provided the hadrons are relativistic. So if the gamma-ray emission from Cas A is hadronic, the accelerated electrons have the same slope, we expect a radio spectral index of \( \alpha = -(p-1)/2 \approx -0.61 \). This is indeed consistent with our results, and inconsistent with the spectral index below 10 GHz.

### 4.2 Consequences for the shock acceleration models

The observed flattening of the spectrum from radio to infrared synchrotron radiation is in line with the predictions of non-linear DSA. This theory predicts that the flattening is expected to occur in those electron populations accelerated by the most efficiently accelerating shocks, i.e. those shock capable of converting a large fraction of the shock energy into accelerated particles. Our results indicate that the flattest spectra are located in the southwestern quadrant. This is also the location of stronger non-thermal X-ray emission (see Fig.6 of Helder & Vink (2008)), where the forward and reverse shock are closest to each other. It is therefore probably a place of more efficient particle acceleration, which could exhibit stronger flattening effects as well. As discussed by Helder & Vink (2008), the western part is also the region where the reverse shock has the highest shock velocity in the frame of the ejecta.

Going from the forward shock inwards into the radio ‘plateau’ (northern arc region in Fig. 5) a relative gradual steepening of indices is observed. The steepening could indicate that in the past the broad-band spectra were steeper, and less affected by non-linear effects. However, since the shock velocity was higher in the past, a more likely explanation is that the steeper spectra have been affected by synchrotron cooling. It explains why the further away from either the reverse or forward shock fronts we look, the steeper spectral indices we get. We come back to this explanation in the next subsection, in light of the steepest spectra found in the southeastern region.

The flattest spectral indices that we measure near the shock regions are, however, not supporting the case for the extreme non-linear acceleration, which predicts particle spectral indices \( p < 2 \) (Malkov 1997), and hence synchrotron spectral indices \( \alpha < 0.5 \). In more recent updates of the non-linear DSA models (e.g. Vladimirov et al. 2008; Caprioli et al. 2009), more emphasise is placed on the effects of magnetic-field amplification by the Bell mechanism (Bell 2004) on the energy distribution of the accelerated particles. In a recent paper Bell et al. (2019) shows that the energy...
transferred from the accelerating particles to the magnetic field will also lead to steeper particle spectra. Another way in which magnetic fields may affect the particle spectra is if the particles scatter off Alfvén wave that have an average velocity with respect to the plasma velocity. This can reduce the effective contrast in velocities experienced by the accelerating particles, resulting in steeper spectra (Zirakashvili et al. 2008). Although modification of the non-linear DSA model indeed predict steeper spectra, the idea that particles of different energy have experienced different shock compression ratio still stands. Thus, the expected result is still a spectrum that is steeper at low energies than at high energies, consistent with our findings.

Note that an altogether different explanation for spectral variation was presented by Atoyan et al. (2000), who suggested a two-zone model trying to explain the observed flattening in the averaged radio spectra (Mezger et al. 1986). They argued that the flattening is caused by accelerated particles escaping from bright steep-spectrum radio structures (knots, ring) into the diffuse ‘plateau’. However, this model predicts flatter spectra in the plateau and steeper at the acceleration sites. Our spatially resolved results are in contradiction with this model, and show the opposite to be true.

### 4.3 Steepening in the southeast of the remnant

The spectral map (Fig. 4) shows that the plateau in the northern and southeastern part of the remnant have in general steeper spectra. This is particular true for the southeastern part, roughly the dark spot region indicated in Fig. 5, whose spectral index histogram is displayed in Fig. 6.

There are several possible reasons why in this region the spectral index is steeper than in the rest of the remnant: i) the steep spectrum could be intrinsic to the particle acceleration properties that shaped the spectrum of that region, for example stronger magnetic-field amplification in that region (Bell et al. 2019) as compared to other regions, or ii) it could be caused by synchrotron cooling, which has steepened the spectrum since the acceleration took place. The first option seems less likely, because if anything, all indications points towards a more efficient acceleration is taking place in the western part, which has the flattest spectra (in agreement with non-linear shock acceleration models, and where the X-ray synchrotron emission is brightest. However, we cannot exclude the possibility that the situation was different one or two centuries ago, when the electrons in the southern region were accelerated.

The second option, therefore, is more in line with the observations. Nevertheless, here we have to address why in particular the southeastern region would show the strongest synchrotron cooling effects. Given that the cooling effects are a function of time since the electrons have been accelerated and the magnetic-field strength, the implications are that this region has been shocked earlier, or that the magnetic-field strength is higher in the southeastern part of the bright radio shell. We explore the second possibility below.

A relativistic electron gyrating around the magnetic field line radiates away its energy within a cooling time scale, \( \tau_{\text{syn}} \), (Ginzburg & Syrovatskii 1965)

\[
\tau_{\text{syn}} = \frac{E}{dE/dt} = \frac{9 (m_e c^2)^4}{4 \varepsilon^2 c^2 B^2 E} \approx 1250 \left( \frac{E_{\text{eV}}}{1 \text{TeV}} \right)^{-1} \left( \frac{B}{100 \mu \text{G}} \right)^{-2} \text{yr},
\]

where \( m_e \) and \( e \) are the mass and mass energy charge of the electron, \( E_e \) is its energy and \( B \) is the magnetic field strength.

Equating synchrotron loss timescale \( \tau_{\text{syn}} \) with the age of the electron population \( t_{\text{age}} \) we obtain the time-dependent energy that represents a turnover in the particle spectra. Above this energy, we do not expect to detect any more particles due to synchrotron losses.

For a given electron energy, the typical synchrotron radiation frequency, \( \nu_{\text{syn}} \), (Ginzburg & Syrovatskii 1965)

\[
\nu_{\text{syn}} = 4.6 \times 10^{13} \left( \frac{B}{100 \mu \text{G}} \right) \left( \frac{E_{\text{eV}}}{1 \text{TeV}} \right)^2 \text{Hz}.
\]

---

**Figure 6.** Left: histogram of median spectral index values for regions of interest. Forward shock region appears to significantly flatter than the Northern arc and Dark spot. Right: histogram of Low and High infrared flux. Distributions have similar shapes and mean value pointing to correctly subtracted background. Dashed lines in both sub-figures represent mean values of individual distributions.
Combining Eq. 3 with Eq. 2 gives us the typical frequency above which the synchrotron spectrum should be affected by radiative losses (i.e. the cooling-break frequency):

\[
\nu_{\text{age}} \approx 7.2 \times 10^{16} \left( \frac{B}{100 \mu G} \right)^{-3} \left( \frac{\nu_{\text{break}}}{100 \text{yr}} \right)^{-2} \text{Hz}.
\] (4)

In a SNR, the total emission arises from a combination of synchrotron cooled spectra with different ages for the electron populations. So whereas each population will have its own sharp cut-off in frequency, the combination of different populations leads to a broken power-law spectrum, with the break frequency corresponding to the age of the oldest electron population (e.g. Longair 2011), and the break frequency given by Eq. 4.

The median spectral index in the southeast (\(\alpha = -0.65\)) is steeper than for the rest of the remnant, but not extremely steep, certainly not when compared to the radio spectral of Cas A of \(\alpha \approx -0.77\). So we expect that if the southeastern spectrum in the mid-infrared has been affected by synchrotron cooling, the cooling break should be just below the frequency of Spitzer band used (\(8.3 \times 10^{13}\) Hz). If it would be much lower we expect a much steeper of \(\alpha \approx -1\), and for \(\nu_{\text{age}} > 8.3 \times 10^{13}\) Hz nothing would be observed at all. Keeping this in mind, we can rewrite Eq. 4 to give an estimate of the average magnetic-field strength in the southeastern region:

\[
B \approx 0.96 \left( \frac{\nu_{\text{break}}}{8.3 \cdot 10^{13} \text{Hz}} \right)^{\frac{1}{3}} \left( \frac{\nu_{\text{age}}}{100 \text{yr}} \right)^{-\frac{2}{3}} \text{mG}.
\] (5)

The true age of the plasma, however, depends on the shock acceleration history Cas A. Was the bulk of southeastern shell formed 100 yr ago? Or did the formation of the shell start immediately after the supernova explosion. In the latter case we should use the age of Cas A, which is about \(\sim 340\) years. This would translate into a lower limit of the magnetic-field strength of \(\sim 420\) \(\mu\)G. The southeastern shell is already seen in the earliest radio-synthesis maps of Cas A, based on observations in the early 1960s (Ryle et al. 1965), suggesting that the age of the plasma is at least 50 yr old.

These magnetic-field estimates, based on the assumption of the presence of a cooling break below \(8.3 \times 10^{13}\) Hz, are consistent with other estimates of the magnetic-field strength for Cas A. Near the shock fronts, X-ray synchrotron observations provide estimates of the magnetic-field strengths of \(100 - 500\) \(\mu\)G (Vink & Laming 2003; Berezhko & Völk 2004; Bamba et al. 2005; Ballet 2006; Heider et al. 2012). Joint modelling of the broad band spectral energy distribution of the non-thermal radiation (radio, X-rays and gamma-rays) also suggests magnetic fields of \(\geq 150\) \(\mu\)G (Atoyan et al. 2000; Ahnen et al. 2017; Abeysekara et al. 2020). Finally, early estimates of the overall magnetic-field strength based on the minimum energy argument were \(\sim 500\) \(\mu\)G (Rosenberg 1970).

Given the all the previous estimates are either for the regions around the shocks, or for the remnant as a whole, a somewhat higher field of around 1 mG in the southeastern region is very plausible. Even if in this region the magnetic field is \(B \sim 500\) \(\mu\)G, in line with most other estimates, this would still be consistent with a cooling break around \(8.3 \times 10^{13}\) Hz in the southeastern region, if the plasma has been built up over the last 340 yr.

Although we cannot measure the exact frequency of the cooling break, our measurements at \(8.3 \times 10^{13}\) Hz suggests that this frequency is close to the cooling break. It will be difficult to map synchrotron radiation at lower frequencies, due the thermal emission from dust grains. So the best option would be to obtain deep near-infrared imaging. Indeed, the 2 \(\mu\)m measurements by Rho et al. (2003) provides a hint that the synchrotron spectrum is generally steepening with respect to our measurements at \(8.3 \times 10^{13}\) Hz.

5 CONCLUSIONS

We investigated the spatially-dependent spectral changes in synchrotron emission from Cas A by comparing a high spatial resolution radio (4.7 GHz and mid-infrared (3.6 \(\mu\)m) maps of Cas A, and for the first time obtain spectral index maps between these two bands. The aim was to verify earlier reports that the the non-thermal spectrum of Cas A is flattening, consistent with predictions from non-linear DSA theory. Unlike previous studies, we obtain a spatial map of spectral indices, after correcting for interstellar extinction, and checking for potential systematic errors.

A summary of our findings are:

- We see flattening of the non-thermal spectrum all across the remnant with average radio to mid-infrared spectral index \(\alpha = -0.61\). This is 0.16 flatter than the measured average spectral index in radio of \(\text{radio} \sim -0.77\).
- The flattest indices are obtained at the locations of assumed active particle acceleration, such as near the forward shock and the reverse shock regions. In particular, the western part of Cas A has the flattest spectral index, which coincides with the regions that are brightest in non-thermal X-ray emission.
- The flattening of the spectrum at high frequencies is in qualitative agreement with non-linear DSA theory. The spectral index is however still steeper than the predictions of the early models (Malkov & Drury 2001).
- The data also show a gradient of relatively steeper indices further downstream of the forward shock, roughly coinciding with the radio plateau. One possible explanation is that we observe a combination of spectral flattening, but modified by synchrotron cooling affecting the highest energy electrons, after having been accelerated decades to centuries ago.
- In particular, the steepest spectra we find in the southeastern part of the remnant. Assuming that this is a result of a synchrotron cooling break that happens below the Spitzer observational frequency of the 1st channel (3.6 \(\mu\m\), \(8.3 \times 10^{13}\) Hz), we estimate a lower limit to the magnetic-field strength of \(420\) \(\mu\)G. If the shell was formed only a hundred years ago, the magnetic-field strength could be of the order \(\sim 1\) mG.

ACKNOWLEDGEMENTS

The work of VD is supported by a grant from NWO graduate program/GRAPPA-PhD program. JVHS acknowledges support from STFC grant ST/R000821/1. We would like to thank Una Hwang for providing the \(N_H\) maps and Tracy Delaney for providing the VLA data and helpful discussion. We
APPENDIX A: EXAMPLE OF OBTAINING SPECTRAL INDICES PER PIXEL

In order to illustrate our procedure for measuring a spectral index for a single pixel and to prove the overall robustness of our results given the statistical and systematic errors, we show here our procedure for a given pixel, and how it leads to a likelihood range in spectral index.

We chose a pixel in the southeast location at coordinates (Ra 23h 23m 39.1s, Dec 58° 47' 23.8'') as our example pixel to provide a more complete picture of our data handling process. The location is ideal to showcase the effect of the strongest effect of background subtraction as this region provides the lowest mid-infrared flux.

A0.1 Background subtraction

The flux in the chosen pixel is 190.1×10^{-7} Jy with an uncertainty 9.1×10^{-7} Jy. Using the Monte Carlo technique we produce a normal distribution of both mid-infrared flux (Fig. A1 left) and estimated background (Fig. A1 centre).

The background-subtracted distribution (Fig. A1 right) has a Gaussian shape. This is a result that should be expected if the background correction was applied correctly. If we were over-subtracting background, the distribution would be cut from the left side and introduce inaccuracy and bias into our method. Since we chose a pixel in the low-flux southeastern region, this is additional proof that the steeper values we see there are not caused by over-subtraction.

A0.2 Extinction calculation

We obtain an extinction value from the X-ray measurements of hydrogen column density \(N_H\) provided by Hwang & Laming (2012). For our chosen pixel, \(N_H = 1.54 \times 10^{22} \text{ cm}^{-3}\). Since the uncertainty is not available we assume it to be a 10% of the \(N_H\) value. The simulated distribution (Fig. A2 left) is converted into visual extinction \(A_V\) (Fig. A2 centre) using \(N_H - A_V\) relation of Güver & Özel (2009) and further transformed to extinction at 3.6 \(\mu m\) (Fig. A2 right) using the extinction curve of Indebetouw et al. (2004). For this conversion we also accounted for the uncertainties for relations between \(N_H\), \(A_V\) and the final extinction at 3.6 \(\mu m\). The resulting extinction distribution has a longer tail in higher extinction values. This is a product of unequal uncertainties in the relation of the extinction curve and would be much harder to account for analytically. The distribution’s shape is however still approximately Gaussian.

A0.3 Spectral index

The final step is combining the background and extinction corrected distribution of mid-infrared flux (Fig. A3 left) with the radio data (Fig. A3 centre). The radio flux for the given pixel is 1.05 × 10^{-2} Jy. As the uncertainty was not available we assumed it to be 10% of the flux. The final result has also Gaussian shape and is shown in Fig. 1 right.

This paper has been typeset from a \TeX/L\TEX file prepared by the author.
Figure A1. Spitzer background correction process. Left and centre images show simulated normal distributions for Spitzer data for given pixel and estimated global mid-infrared background respectively. Right: final background-subtracted distribution.

Figure A2. Extinction conversion process from hydrogen column density measured in X-rays into extinction at 3.6 µm. Left: distribution simulated from X-ray NH data. Centre: distribution after conversion to visual magnitude Av (using Güver & Özel 2009). Right: distribution adapted to 3.6 µm extinction (using Indebetouw et al. 2004).

Figure A3. Obtaining the final power-law index. Left: background and extinction corrected distribution. Centre: simulated distribution based on radio data. Right: final spectral index distribution for the given pixel.
Figure A4. Spitzer data maps in all 4 channels. Top left 3.6 µm, top right 4.5 µm, bottom left 5.8 µm and bottom right 8 µm. The southeastern region that we dubbed with name “dark spot” has noticeably less flux in all 4 channels. See Fig. 5 for details of the analysis region.