The Internals of the Data Calculator

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ABSTRACT

Data structures are critical in any data-driven scenario, but they are notoriously hard to design due to a massive design space and the dependence of performance on workload and hardware which evolve continuously. We present a design engine, the Data Calculator, which enables interactive and semi-automated design of data structures. It brings two innovations. First, it offers a set of fine-grained design primitives that capture the first principles of data layout design: how data structure nodes lay data out, and how they are positioned relative to each other. This allows for a structured description of the universe of possible data structure designs that can be synthesized as combinations of those primitives. The second innovation is computation of performance using learned cost models. These models are trained on diverse hardware and data profiles and capture the cost properties of fundamental data access primitives (e.g., random access). With these models, we synthesize the performance cost of complex operations on arbitrary data structure designs without having to: 1) implement the data structure, 2) run the workload, or even 3) access the target hardware. We demonstrate that the Data Calculator can assist data structure designers and researchers by accurately answering rich what-if design questions on the order of a few seconds or minutes, i.e., computing how the performance (response time) of a given data structure design is impacted by variations in the: 1) design, 2) hardware, 3) data, and 4) query workloads. This makes it effortless to test numerous designs and ideas before embarking on lengthy implementation, deployment, and hardware acquisition steps. We also demonstrate that the Data Calculator can synthesize entirely new designs, auto-complete partial designs, and detect suboptimal design choices.

Let us calculate. —Gottfried Leibniz

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1 FROM MANUAL TO INTERACTIVE DESIGN

The Importance of Data Structures. Data structures are at the core of any data-driven software, from relational database systems, NoSQL key-value stores, operating systems, compilers, HCI systems, and scientific data management to any ad-hoc program that deals with increasingly growing data. Any operation in any data-driven system/program goes through a data structure whenever it touches data. Any effort to rethink the design of a specific system or to add new functionality typically includes (or even begins by) rethinking how data should be stored and accessed [1, 9, 29, 38, 58, 85, 86]. In this way, the design of data structures has been an active area of research since the onset of computer science and there is an ever-growing need for alternative designs. This is fueled by 1) the continuous advent of new applications that require tailored storage and access patterns in both industry and science, and 2) new hardware that requires specific storage and access patterns to ensure longevity and maximum utilization. Every year dozens of new data structure designs are published, e.g., more than fifty new designs appeared at ACM SIGMOD, PVLDB, EDBT and IEEE ICDE in 2017 according to data from DBLP.

A Vast and Complex Design Space. A data structure design consists of 1) a data layout to describe how data is stored, and 2) algorithms that describe how its basic functionality (search, insert, etc.) is achieved over the specific data layout. A data structure can be as simple as an array or arbitrarily complex using sophisticated combinations of hashing, range and radix partitioning, careful data placement, compression and encodings. Data structures may also be referred to as “data containers” or “access methods” (in which case the term “structure” applies only to the layout). The data layout design itself may be further broken down into the base data layout

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The name “Calculator” pays tribute to the early works that experimented with the concept of calculating complex objects from a small set of primitives [59].
and the indexing information which helps navigate the data, i.e., the leaves of a B+tree and its inner nodes, or buckets of a hash table and the hash-map. We use the term data structure design throughout the paper to refer to the overall design of the data layout, indexing, and the algorithms together as a whole.

We define “design” as the set of all decisions that characterize the layout and algorithms of a data structure, e.g., “Should data nodes be sorted?”, “Should they use pointers?”, and “How should we scan them exactly?” The number of possible valid data structure designs explodes to $10^{32}$ even if we limit the overall design to only two different kinds of nodes (e.g., as is the case for B+trees). If we allow every node to adopt different design decisions (e.g., based on access patterns), then the number of designs grows to $10^{100}$. 1 We explain how we derive these numbers in Section 2.

**The Problem: Human-Driven Design Only.** The design of data structures is a slow process, relying on the expertise and intuition of researchers and engineers who need to mentally navigate the vast design space. For example, consider the following design questions.

1. We need a data structure for a specific workload: Should we strip down an existing complex data structure? Should we build off a simpler one? Or should we design and build a new one from scratch?

2. We expect that the workload might shift (e.g., due to new application features): How will performance change? Should we redesign our core data structures?

3. We add flash drives with more bandwidth and also add more system memory: Should we change the layout of our B-tree nodes? Should we change the size ratio in our LSM-tree?

4. We want to improve throughput: How beneficial would it be to buy faster disks? more memory? or should we invest in the same budget in redesigning our core data structure?

This complexity leads to a slow design process and has severe cost side-effects [12, 21]. Time to market is of extreme importance, so new data structure design effectively stops when a design “is due” and only rarely when it “is ready”. Thus, the process of design extends beyond the initial design phase to periods of reconsidering the design given bugs or changes in the scenarios it should support. Furthermore, this complexity makes it difficult to predict the impact of design choices, workloads, and hardware on performance. We include two quotes from a systems architect with more than two decades of experience with relational systems and key-value stores.

1. “I know from experience that getting a new data structure into production takes years. Over several years, assumptions made about the workload and hardware are likely to change, and these changes threaten to reduce the benefit of a data structure. This risk of change makes it hard to commit to multi-year development efforts. We need to reduce the time it takes to get new data structures into production.”

2. “Another problem is the limited ability we have to iterate. While some changes only require an online schema change, many require a dump and reload for a data service that might be running 24x7. The budget for such changes is limited. We can overcome the limited budget with tools that help us determine the changes most likely to be useful. Decisions today are frequently based on expert opinions, and these experts are in short supply.”

Vision Step 1: Design Synthesis from First Principles. We propose a move toward the new design paradigm captured in Figure 1. Our intuition is that most designs (and even inventions) are about combining a small set of fundamental concepts in different ways or tunings. If we can describe the set of the first principles of data structure design, i.e., the core design principles out of which all data structures can be drawn, then we will have a structured way to express all possible designs we may invent, study, and employ as combinations of those principles. An analogy is the periodic table of elements in chemistry. It classifies elements based on their atomic number, electron configuration, and recurring chemical properties. The structure of the table allows one to understand the elements and how they relate to each other but crucially it also enables arguing about the possible design space; more than one hundred years since the inception of the periodic table in the 18th century, we keep discovering elements that are predicted (synthesized) by the “gaps” in the table, accelerating science.

Our vision is to build the periodic table of data structures so we can express their massive design space. We take the first step in this paper, presenting a set of first principles that can synthesize orders of magnitude more data structure designs than what has been published in the literature. It captures basic hardware conscious layouts and read operations; future work includes extending the table for additional parts of the design space, such as updates, concurrency, compression, adaptivity, and security.

Vision Step 2: Cost Synthesis from Learned Models. The second step in our vision is to accelerate and automate the design process. Key here, is being able to argue about the performance behavior of the massive number of designs so we can rank them. Even with an intuition that a given design is an excellent choice, one has to implement the design, and test it on a given data and query workload and onto specific hardware. This process can take weeks at a time and has to be repeated when any part of the environment changes. Can we accelerate this process so we can quickly test alternative designs (or different combinations of hardware, data, and queries) on the order of a few seconds? If this is possible, then we can 1) accelerate design and research of new data structures, and 2) enable new kinds of adaptive systems that can decide core parts of their design, and the right hardware.

Arguing formally about the performance of diverse designs is a notoriously hard problem [14, 66, 82, 85, 87, 88] especially as workload and hardware properties change; even if we can come up with a robust analytical model it may soon be obsolete [50]. We take a hybrid route using a combination of analytical models, benchmarks, and machine learning for a small set of fundamental access primitives. For example, all pointer based data structures need to perform random accesses as operations traverse their nodes. All data structures need to perform a write during an update operation, regardless of the exact update strategy. We synthesize the cost of complex operations out of models that describe those simpler more fundamental operations inspired by past work on generalized models [66, 85]. In addition, our models start out as analytical models since we know how these primitives will likely behave. However, they are also trained across diverse hardware profiles by running benchmarks that isolate the behavior of those primitives. This way, we learn a set of coefficients for each model that capture the subtle performance details of diverse hardware settings.

For comparison, the estimated number of stars in the universe is $10^{24}$. 

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The Data Calculator: Automated What-if Design. We present a “design engine” – the Data Calculator – that can compute the performance of arbitrary data structure designs as combinations of fundamental design primitives. It is an interactive tool that accelerates the process of design by turning it into an exploration process, improving the productivity of researchers and engineers; it is able to answer what-if data structure design questions to understand how the introduction of new design choices, workloads, and hardware affect the performance (latency) of an existing design. It currently supports read queries for basic hardware conscious layouts. It allows users to give as input a high-level specification of the layout of a data structure (as a combination of primitives), in addition to workload, and hardware specifications. The Data Calculator gives as output a calculation of the latency to run the input workload on the input hardware. The architecture and components of the Data Calculator are captured in Figure 2 (from left to right): (1) a library of fine-grained data layout primitives that can be combined in arbitrary ways to describe data structure layouts; (2) a library of data access primitives that can be combined to generate designs of operations; (3) an operation and cost synthesizer that computes the design of operations and their latency for a given data structure layout specification, a workload and a hardware profile, and (4) a search component that can traverse part of the design space to supplement a partial data structure specification or inspect an existing one with respect to both the layout and the access design choices.

Inspiration. Our work is inspired by several lines of work across many fields of computer science. John Ousterhout’s project Magic in the area of computer architecture allows for quick verification of transistor designs so that engineers can easily test multiple designs [70]. Leland Wilkinson’s “grammar of graphics” provides structure and formulation on the massive universe of possible graphics one can design [64]. Mike Franklin’s Ph.D. thesis explores the possible client-server architecture designs using caching based replication as the main design primitive and proposes a taxonomy that produced both published and unpublished (at the time) cache consistency algorithms. Joe Hellerstein’s work on Generalized Search Indexes [6, 7, 38, 54–57] makes it easy to design and test new data structures by providing templates that significantly minimize implementation time. S. Bing Yao’s work on generalized cost models [85] for database organizations, and Stefan Manegold’s work on generalized cost models tailored for the memory hierarchy [65] showed that it is possible to synthesize the costs of database operations from basic access patterns and based on hardware performance properties. Work on data representation synthesis in programming languages [18–20, 22, 35, 36, 64, 77, 79] enables selection and synthesis of representations out of small sets of (3-5) existing data structures. The Data Calculator [46] can be seen as a step toward the Automatic Programmer challenge set by Jim Gray in his Turing award lecture [33], and as a step toward the “calculus of data structures” challenge set by Turing award winner Robert Tarjan [81]: “What makes one data structure better than another for a certain application? The known results cry out for an underlying theory to explain them.”

Contributions. Our contributions are as follows:

1. We introduce a set of data layout design primitives that capture the first principles of data layouts including hardware conscious designs that dictate the relative positioning of data structure nodes (§2).

2. We show how combinations of the design primitives can describe known data structure designs, including arrays, linked-lists, skip-lists, queues, hash-tables, binary trees and (Cache-conscious) b-trees, tries, MassTree, and FAST (§2).

3. We show that in addition to known designs, the design primitives form a massive space of possible designs that has only been minimally explored in the literature (§2).

4. We show how to synthesize the latency cost of basic operations (point and range queries, and bulk loading) of arbitrary data structure designs from a small set of access primitives. Access primitives represent fundamental ways to access data and come with learned cost models which are trained on diverse hardware to capture hardware properties (§3).

5. We show how to use cost synthesis to interactively answer complex what-if design questions, i.e., the impact of changes to design, workload, and hardware (§4).

6. We introduce a design synthesis algorithm that completes partial layout specifications given a workload and hardware input; it utilizes cost synthesis to rank designs (§4).

7. We demonstrate that the Data Calculator can accurately compute the performance impact of design choices for state-of-the-art designs and diverse hardware (§5).

8. We demonstrate that the Data Calculator can accelerate the design process by answering rich design questions in a matter of seconds or minutes (§5).
2 DATA LAYOUT PRIMITIVES AND STRUCTURE SPECIFICATIONS

In this section, we discuss the library of data layout design primitives and how it enables the description of a massive number of both known and unknown data structures.

Data Layout Primitives. The Data Calculator contains a small set of design primitives that represent fundamental design choices when constructing a data structure layout. Each primitive belongs to a class of primitives depending on the high-level design concept it refers to such as node data organization, partitioning, node physical placement, and node metadata management. Within each class, individual primitives define design choices and allow for alternative tunings. The complete set of primitives we introduce in this paper is shown in Figure 11 in the appendix; they describe basic data layouts and cache conscious optimizations for reads. For example, “Key Partitioning (none|sorted|k-ary|range|temporal)” defines how data is laid out in a node. Similarly, “Key Retention (none|full|func)” defines whether and how keys are included in a node. In this way, in a B+tree all nodes use “sorted” for order maintenance, while internal nodes use “none” for key retention as they only store fences and pointers, and leaf nodes use “full” for key retention.

The logic we use to generate primitives is that each one should represent a fundamental design concept that does not break down into more useful design choices (otherwise, there will be parts of the design space we cannot express). Coming up with the set of primitives is a trial and error task to map the known space of design concepts to an as clean and elegant set of primitives as possible.

Naturally, not all layout primitives can be combined. Most invalid combinations stem from the structure of the primitives, i.e., each primitive combines with every other standalone primitive. Only a few pairs of primitive tunings do not combine which generates a small set of invalidation rules. These are mentioned in Figure 11.

From Layout Primitives to Data Structures. To describe complete data structures, we introduce the concept of elements. An element is a full specification of a single data structure node; it defines the data and access methods used to access the node’s data. An element may be “terminal” or “non-terminal”. That is, an element may be describing a node that further partitions data to more nodes or not. This is done with the “fanout” primitive whose value represents the maximum number of children that would be generated when a node partitions data. Or it can be set to “terminal” in which case its value represents the capacity of a terminal node. A data structure specification contains one or more elements. It needs to have at least one terminal element, and it may have zero or more non-terminal elements. Each element has a destination element (except terminal ones) and a source element (except the root). Recursive connections are allowed to the same element.

Examples. A visualization of the primitives can be seen at the left side of Figure 3. It is a flat representation of the primitives shown in Figure 11 which creates an entry for every primitive signature. The radius depicts the domain of each primitive but different primitives may have different domains, visually depicted via the multiple inner circles in the radar plots of Figure 3. The small radar plots on the right side of Figure 3 depict descriptions of nodes of known data structures as combinations of the base primitives. Even visually it starts to become apparent that state-of-the-art designs which are meant to handle different scenarios are “synthesized from the same pool of design concepts”. For example, using the non-terminal B+tree element and the terminal sorted data page element we can construct a full B+tree specification; data is recursively broken down into internal nodes using the B+tree element until we reach the leaf level, i.e., when partitions reach the terminal node size. Figure 3 also depicts Trie and Skip-list specifications. Figure 11 provides complete specifications of Hash-table, Linked-list, B+tree, Cache-conscious B-tree, and FAST.

Elements “Without Data”. For flat data structures without an indexing layer, e.g., linked-lists and skip-lists, there need to be elements in the specification that describe the algorithm used to navigate the terminal nodes. Given that this algorithm is effectively a model, it does not rely on any data, and so such elements do not translate to actual nodes; they only affect algorithms that navigate across the terminal nodes. For example, a linked-list element in Figure 11 describes that data is divided into nodes that can only be accessed via following the links that connect terminal nodes. Similarly, one can create complex hierarchies of non-terminal elements that do not store any data but instead their job is to synthesize a collective model of how the keys should be distributed in the data structure, e.g., based on their value or other properties of the workload. These elements may lead to multiple hierarchies of both non-terminal nodes with data and terminal ones, synthesizing data structure designs that treat parts of the data differently. We will see such examples in the experimental analysis.

Recursive Design Through Blocks. A block is a logical portion of the data that we divide into smaller blocks to construct an instance of a data structure specification. The elements in a specification are the “atoms” with which we construct data structure instances by applying them recursively onto blocks. Initially, there is a single block of data, all data. Once all elements have been applied, the original block is broken down into a set of smaller blocks that correspond to the internal nodes (if any) and the terminal nodes of the data structure. Elements without data can be thought of as if they apply on a logical data block that represents part of the data with a set of specific properties (i.e., all data if this is the first element) and partitions the data with a particular logic into further logical blocks or physical nodes. This recursive construction is used when we test, cost, and search through multiple possible designs concurrently over the same data for a given workload and hardware as we will discuss in the next two sections, but it is also helpful to visualize designs as if “data is pushed through the design” based on the elements and logical blocks.

Cache-Conscious Designs. One critical aspect of data structure design is the relative positioning of its nodes, i.e., how “far” each node is positioned with respect to its predecessors and successors in a query path. This aspect is critical to the overall cost of traversing a data structure. The Data Calculator design space allows to dictate how nodes should be positioned explicitly: each non-terminal element defines how its children are positioned physically with respect to each other and with respect to the current node. For example, setting the layout primitive “Sub-block physical layout” to BFS tells the current node that its children are laid out sequentially. In addition, setting the layout primitive “Sub-blocks homogeneous” to true implies that all its children have the same layout (and therefore fixed width), and allows a parent node to access any of its children
nodes directly with a single pointer and reference number. This, in
turn, makes it possible to fit more data in internal nodes because
only one pointer is needed and thus more fences can be stored
within the same storage budget. Such primitives allow specifying
designs such as Cache Conscious B+tree [75] (Figure 11 provides
the complete specification), but also the possibility of generalizing
the optimizations made there to arbitrary structures.

Similarly, we can describe FAST [51]. First, we set “Sub-block
physical location” to inline, specifying that the children nodes are
directly after the parent node physically. Second, we set that the
children nodes are homogeneous, and finally, we set that the children
have a sub-block layout of “BFS Layer List (Page Size / Cache Line
Size)”. Here, the BFS layer list specifies that on a higher level,
we should have a BFS layout of sub-containers containing Page Size/Cache
Line Size layers; however, inside of those sub-trees pages are laid
out in BFS manner by a single level. The combination matches the
combined Page Level blocking and Cache Line level blocking of
FAST. Additionally, the Data Calculator realizes that all child node
physical locations can be calculated via offsets, and so eliminates
all pointers. Figure 11 provides the complete specification.

Size of the Design Space. To help with arguing about the possible
design space we provide formal definitions of the various constructs.

Definition 2.1 (Data Layout Primitive). A primitive \( p_i \) belongs
to a domain of values \( \mathcal{P}_i \) and describes a layout aspect of a data
structure node.

Definition 2.2 (Data Structure Element). A Data Structure Element
\( E \) is defined as a set of data layout primitives: \( E = \{ p_1, ..., p_n \} \in
\mathcal{P}_1 \times ... \times \mathcal{P}_n \), that uniquely identify it.

Given a set of \( \text{Inv}(\mathcal{P}) \) invalid combinations, the set of all possible
elements \( \mathcal{E} \), (i.e., node layouts) that can be designed as distinct
combinations of data layout primitives has the following cardinality.

\[
|\mathcal{E}| = \mathcal{P}_1 \times ... \times \mathcal{P}_n - \text{Inv}(\mathcal{P}) = \prod_{i} |\mathcal{P}_i| - \text{Inv}(\mathcal{P}) \quad (1)
\]

Definition 2.3 (Blocks). Each non-terminal element \( E \in \mathcal{E} \), ap-
plied on a set of data entries \( D \in \mathcal{D} \), uses function \( B_{\mathcal{E}}(D) = \{ D_1, ..., D_f \} \) to divide \( D \) into \( f \) blocks such that \( D_1 \cup ... \cup D_f = D \).

A polymorphic design where every block may be described by a
different element leads to the following recursive formula for the
cardinality of all possible designs.

\[
c_{\text{pol}}(D) = |\mathcal{E}| + \sum_{\forall \mathcal{E} \in \mathcal{E}} \sum_{\forall D_1 \in B_{\mathcal{E}}(D)} c_{\text{pol}}(D_1) \quad (2)
\]

Example: A Vast Space of Design Opportunities. To get insight
into the possible total designs we make a few simplifying assump-
tions. Assume the same fanout \( f \) across all nodes and terminal node
size equal to page size \( p_{\text{size}} \). Then \( N = \lceil \frac{|D|}{p_{\text{size}}} \rceil \) is the total number of
pages in which we can divide the data and \( h = \log f(N) \) is the
height of the hierarchy. We can then approximate the result of
Equation 2 by considering that we have \( |\mathcal{E}| \) possibilities for the root
element, and \( f \cdot |\mathcal{E}| \) possibilities for its resulting partitions which in
turn have \( f \cdot |\mathcal{E}| \) possibilities each up to the maximum level of
recursion \( h = \log f(N) \). This leads to the following result.

\[
c_{\text{pol}}(D) \approx |\mathcal{E}| \cdot (f \cdot |\mathcal{E}|)^{\log f(N)} \quad (3)
\]

Most sophisticated data structure designs use only two distinct
elements, each one describing all nodes across groups of levels of
the structure, e.g., B-tree designs use one element for all internal
nodes and one for all leaves. This gives the following design space
for most standard designs.

\[
c_{\text{stan}}(D) \approx |\mathcal{E}|^2 \quad (4)
\]

Using Equations 1, 3 and 4 we can get estimations of the possible
design space for different kinds of data structure designs. For
example, given the existing library of data layout primitives, and
by limiting the domain of each primitive as shown in Figure 11
appendix, then from Equation 1 we get \( |\mathcal{E}| = 10^{16} \), meaning we can
describe data structure layouts from a design space of \( 10^{16} \) possible
node elements and their combinations. This number includes only

Figure 3: The data layout primitives and examples of synthesizing node layouts of state-of-the-art data structures.
valid combinations of layout primitives, i.e., all invalid combinations as defined by the rules in Figure 11 are excluded. Thus, we have a design space of $10^{12}$ for standard two-element structures (e.g., where B-tree and Trie belong) and $10^{48}$ for three-element structures (e.g., where MassTree [67] and Bounded-Disorder [62] belong). For polymorphic structures, the number of possible designs grows more quickly, and it also depends on the size of the training data used to find a specification, e.g., it is $> 10^{100}$ for $10^{15}$ keys.

The numbers in the above example highlight that data structure design is still a wide-open space with numerous opportunities for innovative designs as data keeps growing, application workloads keep changing, and hardware keeps evolving. Even with hundreds of new data structures manually designed and published each year, this is a slow pace to test all possible designs and to be able to argue about how the numerous designs compare. The Data Calculator is a tool that accelerates this process by 1) providing guidance about what is the possible design space, and 2) allowing to quickly test how a given design fits a workload and hardware setting. A more detailed description of the primitives can be found in the appendix.

3 DATA ACCESS PRIMITIVES AND COST SYNTHESIS

We now discuss how the Data Calculator computes the cost (latency) of running a given workload on a given hardware for a particular data structure specification. Traditional cost analysis in systems and data structures happens through experiments and the development of analytical cost models. Both options are not scalable when we want to quickly test multiple different parts of the massive design space we define in this paper. They require significant expertise and time, while they are also sensitive to hardware and workload properties. Our intuition is that we can instead synthesize complex operations from their fundamental components as we do for data layouts in the previous section, and then develop a hybrid way (through both benchmarks and models) to consider the case of a Level 2 primitive and may be extended with any number of additional ones. The complete list of access primitives currently supported by the Data Calculator is shown in Table 1 in appendix.

Learned Cost Models. For every Level 2 primitive, the Data Calculator contains one or more models that describe its performance (latency) behavior. These are not static models; they are trained and fitted for combinations of data and hardware profiles as both those factors drastically affect performance. To train a model, each Level 2 primitive includes a minimal implementation that captures the behavior of the primitive, i.e., it isolates the performance effects of performing the specific action. For example, an implementation for a scan primitive simply scans an array, while an implementation for a random access primitive simply tries to access random locations in memory. These implementations are used to run a sequence of benchmarks to collect data for learning a model for the behavior of each primitive. Implementations should be in the target language/environment.

The models are simple parametric models; given the design decision to keep primitives simple (so they can be easily reused), we have domain expertise to expect how their performance behavior will look like. For example, for scans, we have a strong intuition they will be linear, for binary searches that they will be logarithmic, and for random memory accesses that they will be smoothed out step functions (based on the probability of caching). These simple models have many advantages: they are interpretable, they train quickly, and they don’t need a lot of data to converge. Through the training process, the Data Calculator learns coefficients of those models that capture hardware properties such as CPU and data movement costs.

Hardware and data profiles hold descriptive information about data and hardware respectively (e.g., data distribution for data, and CPU, Bandwidth, etc. for hardware). When an access primitive is trained on a data profile, it runs on a sample of such data, and when it is trained for a hardware profile, it runs on this exact hardware. Afterward, though, design questions can get accurate cost estimations on arbitrary access method designs without going over the data or having to have access to the specific machine. Overall, this is an offline process that is done once, and it can be repeated to include new hardware and data profiles or to include new access primitives.

Example: Binary Search Model. To give more intuition about how models are constructed let us consider the case of a Level 2 primitive of binary searching a sorted array as shown on the upper right part of Figure 4. The primitive contains a code snippet that implements the bare minimum behavior (Step 1 in Figure 4). We
observe that the benchmark results (Step 2 in Figure 4) indicate that performance is related to the size of the array by a logarithmic component. As expected there is also bias as the relationship for small array sizes (such as just 4 or 8 elements) might not fit exactly a logarithmic function. We additionally add a linear term to capture some small linear dependency on the data size. Thus, the cost of binary searching an array of \( n \) elements can be approximated as

\[
f(n) = c_1 n + c_2 \log n + y_0\]

where \( c_1, c_2, \) and \( y_0 \) are coefficients learned through linear regression. The values of these coefficients help us translate the abstract model, \( f(n) = O(\log n) \), into a realized predictive model which has taken into account factors such as CPU speed and the cost of memory accesses across the sorted array for the specific hardware. The resulting fitted model can be seen in Step 3 on the upper right part of Figure 4. The Data Calculator can then use this learned model to query for the performance of binary search within the trained range of data sizes. For example, this would be used when querying a large sorted array as well as a small node of a complex data structure that is sorted.

Certain critical aspects of the training process can be automated as part of future research. For example, the data range for which we should train a primitive depends on the memory hierarchy (e.g., size of caches, memory, etc.) on the target machine and what is the target setting in the application (i.e., memory only, or also disk/flash, etc.). In turn, this also affects the length of the training process. Overall, such parameters can eventually be handled through high-level knobs, letting the system make the lower level tuning choices. Furthermore, identification of convergence can also be automated. There exist primitives that require more training than others (e.g., due to more complex code, random access or sensitivity to outliers), and so the number of benchmarks and data points we collect should not be a fixed decision.

**Synthesizing Latency Costs.** Given a data layout specification and a workload, the Data Calculator uses Level 1 access primitives to synthesize operations and subsequently each Level 1 primitive is translated to the appropriate Level 2 primitive to compute the cost of the overall operation. Figure 5 depicts this process and an example specifically for the Get operation. This is an expert system, i.e., a sequence of rules that based on a given data structure specification defines how to traverse its nodes.\(^2\) To read Figure 5 start from the top right corner. The input is a data structure specification, a test data set, and the operation we need to cost, e.g., \( \text{Get key } x \). The process simulates populating the data structure with the data to figure out how many nodes exist, the height of the structure, etc. This is because to accurately estimate the cost of an operation, the Data Calculator needs to take into account the expected state of the data structure at the particular moment in the workload. It does this by recursively dividing the data into blocks given the elements used in the specification.

In the example of Figure 5 the structure contains two elements, one for internal nodes and one for leaves. For every node, the operation synthesis process takes into account the data layout primitives used. For example, if a node is sorted it uses binary search, but if the node is unsorted, it uses a full scan. The rhombuses on the left side of Figure 5 reflect the data layout primitives that operation Get relies on, while the rounded rectangles reflect data access primitives that may be used. For each node the per-node operation synthesis procedure (starting from the left top side of Figure 5), first checks if this node is internal or not by checking whether the node contains keys or values; if not, it proceeds to determine which node it should visit next (left side of the figure) and if yes, it continues to process the data and values (right side of the figure). A non-terminal element leads to data of this block being split into \( f \) new blocks and the process follows the relevant blocks only, i.e., the blocks that this operation needs to visit to resolve.

In the end, the Data Calculator generates an abstract syntax tree with the access patterns of the path it had to go through. This is expressed in terms of Level 1 access primitives (bottom right part of Figure 5). In turn, this is translated to a more detailed abstract syntax tree where all Level 1 access primitives are translated to Level 2 access primitives along with the estimated cost for each one given the particular data size, hardware input, and any primitive specific input. The overall cost is then calculated as the sum of all those costs.

\(^2\)Figure 5 is a subset of the expert system. The complete version can be found in the appendix.
Calculating Random Accesses and Caching Effects. A crucial part in calculating the cost of most data structures is capturing random memory access costs, e.g., the cost of fetching nodes while traversing a tree, fetching nodes linked in a hash bucket, etc. If data is expected to be cold, then this is a rather straightforward case, i.e., we may assign the maximum cost a random access is expected to incur on the target machine. If data may be hot, though, it is a more involved scenario. For example, in a tree-like structure internal nodes higher in the tree are much more likely to be at higher levels of the memory hierarchy during repeated requests. We calculate such costs using the random memory access primitive, as shown in the lower right part of Figure 4. Its input is a “region size”, which is best thought of as the amount of memory we are randomly accessing into (i.e., we don’t know where in this memory region our pointer points to). The primitive is trained via benchmarking access cost of a Get operation. Every Get query will be routed through that we have 400 full data pages, and thus a tree of height 2. The records where keys and values are 8 bytes each. Overall, this means 103 records, and follow a full columnar format, where keys and values, are sorted, have a maximum page size of 250 keys or values. Leaf nodes instead are terminal; they contain both keys and values, are sorted, have a maximum page size of 250 keys or values. Leaf nodes instead are terminal; they contain both keys and values.

This primitive is used for calculating random access to any physical or logical region (e.g., a sequence of nodes that may be cached together). For example, when traversing a tree, the cost synthesis operation, costs random accesses with respect to the amount of data that may be cached up to this point. That is, for every node we need to access at Level x of a tree, we account for a region size that includes all data in all levels of the tree up to Level x. In this way, accessing a node higher in the tree costs less than a node at lower levels. The same is true when accessing buckets of a hash table. We give a detailed step by step example below.

Example: Cache-aware Cost Synthesis. Assume a B-tree -like specification as follows: two node types, one for internal nodes and one for leaf nodes. Internal nodes contain fence pointers, are sorted, balanced, have a fixed fanout of 20 and do not contain any keys or values. Leaf nodes instead are terminal; they contain both keys and values, are sorted, have a maximum page size of 250 records, and follow a full columnar format, where keys and values are stored in independent arrays. The test dataset consists of 105 records where keys and values are 8 bytes each. Overall, this means that we have 400 full data pages, and thus a tree of height 2. The Data Calculator needs two of its access primitives to calculate the cost of a Get operation. Every Get query will be routed through two internal nodes and one leaf node: within each node, it needs to binary search (through fence pointers for internal nodes and through keys in leaf nodes) and thus it will make use of the Sorted Search access primitive. In addition, as a query traverses the tree it needs to perform a random access for every hop.

Now, let us look in more detail how these two primitives are used given the exact specification of this data structure. The Sorted Search primitive takes as input the size of the area over which we

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1These numbers are in line with Intel’s VTune.
will binary search and the number of keys. The Random Access primitive takes as input the size of the path so far which allows us to take into account caching effects. Each query starts by visiting the root node. The data calculator estimates the size of the path so far to be 312 bytes. This is because the size of the path so far is in practice equal to the size of the root node which containing 20 pointers (because the fanout is 20) and 19 values sums up at root = internal node = 20 × 8 + 19 × 8 = 312 bytes. In this way, the Data Calculator logs a cost of RandomAccess(312) to access the root node. Then, it calculates the cost of binary search across 19 fences, thus logging a cost of SortedSearch(RowStore, 19 * 8). It uses the “RowStore” option as fences and pointers are stored as pairs within each internal node. Now, the access to the root node is fully accounted for, and the Data Calculator moves on to cost the access at the next tree level. Now the size of the path so far is given by accounting for the whole next level in addition to the root node. This is in total level2 = root + fanout + internal node = 312 + 20 × 312 = 6552 bytes (due to fanout being 20 we account for 20 nodes at the next level). Thus to access the next node, the Data Calculator logs a cost of RandomAccess(6552) and again a search cost of SortedSearch(RowStore, 19 * 8) to search this node. The last step is to search the leaf level. Now the size of the path so far is given by accounting for the whole size of the tree which is level2 = 400 * (250 + 16) = 1606552 bytes since we have 400 pages at the next level (20x20) and each page has 250 records of key-value pairs (8 bytes each). In this way, the Data Calculator logs a cost of RandomAccess(1606552) to access the leaf node, followed by a sorted search of SortedSearch(ColumnStore, 250 + 8) to search the keys. It uses the “ColumnStore” option as keys and values are stored separately in each leaf in different arrays. Finally, a cost of RandomAccess(2000) is incurred to access the target value in the values array (we have 8 * 250 = 2000 in each leaf).

**Sets of Operations.** The description above considers a single operation. The Data Calculator can also compute the latency for a set of operations concurrently in a single pass. This is effectively the same process as shown in Figure 5 only that in every recursion we may follow more than one path and in every step we are computing the latency for all queries that would visit a given node.

**Workload Skew and Caching Effects.** Another parameter that can influence caching effects is workload skew. For example, repeatedly accessing the same path of a data structure results in all nodes in this path being cached with higher probability than others. The Data Calculator first generates counts of how many times every node is going to be accessed for a given workload. Using these counts and the total number of nodes accessed we get a factor p = count/total that denotes the popularity of a node. Then to calculate the random access cost to a node for an operation k, a weight w = 1/(p * sid) is used, where sid is the sequence number of this operation in the workload (refreshed periodically). Frequently accessed nodes see smaller access costs and vice versa. In this way, the Data Calculator logs a cost of RandomAccess(6552) and again a search cost of SortedSearch(RowStore, 19 * 8) to search this node. The last step is to search the leaf level. Now the size of the path so far is given by accounting for the whole size of the tree which is level2 = 400 * (250 + 16) = 1606552 bytes since we have 400 pages at the next level (20x20) and each page has 250 records of key-value pairs (8 bytes each). In this way, the Data Calculator logs a cost of RandomAccess(1606552) to access the leaf node, followed by a sorted search of SortedSearch(ColumnStore, 250 + 8) to search the keys. It uses the “ColumnStore” option as keys and values are stored separately in each leaf in different arrays. Finally, a cost of RandomAccess(2000) is incurred to access the target value in the values array (we have 8 * 250 = 2000 in each leaf).

**4 WHAT-IF DESIGN AND AUTO-COMPLETION**

The ability to synthesize the performance cost of arbitrary designs allows for the development of algorithms that search the possible design space. We expect there will be numerous opportunities in this space for techniques that can use this ability to: 1) improve the productivity of engineers by quickly iterating over designs and scenarios before committing to an implementation (or hardware), 2) accelerate research by allowing researchers to easily and quickly test completely new ideas, 3) develop educational tools that allow for rapid testing of concepts, and 4) develop algorithms for offline auto-tuning and online adaptive systems that transition between designs. In this section, we provide two such opportunities for what-if design and auto-completion of partial designs.

**What-if Design.** One can form design questions by varying any one of the input parameters of the Data Calculator: 1) data structure (layout) specification, 2) hardware profile, and 3) workload (data and queries). For example, assume one already uses a B-tree-like design for a given workload and hardware scenario. The Data Calculator can answer design questions such as "What would be the performance impact if I change my B-tree design by adding a bloom..."
Auto-completion. The Data Calculator can also complete partial layout specifications given a workload, and a hardware profile. The process is shown in Algorithm 1 in the appendix: The input is a partial layout specification, data, queries, hardware, and the set of the design space that should be considered as part of the solution, i.e., a list of candidate elements. Starting from the last known point of the partial specification, the Data Calculator computes the rest of the missing subtree of the hierarchy of elements. At each step the algorithm considers a new element as candidate for one of the nodes of the missing subtree and computes the cost for the different kinds of dictionary operations present in the workload. This design is kept only if it is better than all previous ones, otherwise it is dropped before the next iteration. The algorithm uses a cache to remember specifications and their costs to avoid recomputation. This process can also be used to tell if an existing design can be improved by marking a portion of its specification as "to be tested". Solving the search problem completely is an open challenge as the design space drawn by the Calculator is massive. Here we show a first step which allows dynamic programming algorithms to select from a restricted set of elements but numerous algorithmic options are candidates for future research, e.g., genetic algorithms [44].
sequence of 10^2 Get requests, also uniformly distributed. We incrementally insert more data up to a total of 10^7 entries and at each step we repeat the query workload.

The top row of Figure 6 depicts results using a machine with 64 cores and 264 GB of RAM. It shows the average latency per query as data grows as computed by the Data Calculator and as observed when running the actual implementation on this machine. For ease of presentation results are ranked horizontally from slower to faster (left to right). The Data Calculator gives an accurate estimation of the cost across the whole range of data sizes and regardless of the complexity of the designs both in terms of the data structure. It can accurately compute the latency of both simple traversals in a plain array and the latency of more complex access patterns such as descending a tree and performing random hops in memory.

Diverse Machines and Operations. The rest of the rows in Figure 6 repeat the same experiment as above using different hardware in terms of both CPU and memory properties (Rows 2 and 3) and different operations (Rows 4 and 5). The details of the hardware are shown on the right side of each row in Figure 6. Regardless of the machine or operation, the Data Calculator can accurately cost any design. By training its Level 2 primitives on individual machines and maintaining a profile for each one of them, it can quickly test arbitrary designs over arbitrary hardware and operations. Updates here are simple updates that change the value of a key-value pair and so they are effectively the same as a point query with an additional write access. More complex updates that involve restructures are left for future work both in terms of the design space and cost synthesis. Finally, Figure 7a) depicts that the Data Calculator can accurately synthesize the bulk loading costs for all data structures.

Training Access Primitives. Figure 7b) depicts the time needed to train all Level 2 primitives on a diverse set of machines. Overall, this is an inexpensive process. It takes merely a few minutes to train multiple different combinations of data and hardware profiles.

Figure 7: Computing Bulk-loading cost (left) and Training cost across diverse hardware (right).

Figure 8: Accurately computing the latency of cache conscious designs in diverse hardware and workloads.

Cache Conscious Designs and Skew. In addition, Figure 8 repeats our base fitting experiment using a cache-conscious design, Cache Conscious B-tree (CSB). Figure 8a) depicts that the Data Calculator accurately predicts the performance behavior across a diverse set of machines, capturing caching effects of growing data sizes and design patterns where the relative position of nodes affects tree traversal costs. We use the "Full" design from Cache Conscious B-tree [75]. Furthermore, Figure 8b) tests the fitting when the workload exhibits skew. For this experiment Get queries were generated with a Zipfian distribution: $\alpha = \{0.5, 1.0, 1.5, 2.0\}$. Figure 8b) shows that for the implementation results, workload skew improves performance and in fact it improves more for the standard B+tree. This is because the same paths are more likely to be taken by queries resulting in these nodes being cached more often. Cache Conscious B+tree improves but at a much slower rate as it is already optimized for the cache hierarchy. The Data Calculator is able to synthesize these costs accurately, capturing skew and the related caching effects.

Rich Design Questions. In our next, experiment we provide insights about the kinds of design questions possible and how long they may take, working over a B-tree design and a workload of uniform data and queries: 1 million inserts and 100 point Gets. The hardware profile used is HW1 (defined in Figure 6). The user asks "What if we change our hardware to HW3?". It takes the Data Calculator only 20 seconds (all runs are done on HW3) to compute that the performance would drop. The user then asks: "Is there a better design for this new hardware and workload if we restrict search on a specific set of five possible elements?" (from the pool of element on right side of Figure 3). It takes only 47 seconds for the Data Calculator to compute the best choice. The user then asks: "Would it be beneficial to add a bloom filter in all B-tree leaves?" The Data Calculator computes in merely 20 seconds that such a design change would be beneficial for the current workload and hardware. The next design question is: "What if the query workload changes to have skew targeting just 0.01% of the key space?" The Data Calculator computes in 24 seconds that this new workload would hurt the original design and it computes a better design in another 47 seconds.

In two of the design phases the user asked “give me a better design if possible”. We now provide more intuition for this kind of design questions regarding the cost and scalability of computing such designs as well as the kinds of designs the Data Calculator may produce to fit a workload. We test two scenarios for a workload

Figure 9: The Data Calculator designs new hybrids of known data structures to match a given workload.

(a) Data Structure

(b) Hardware

(a) Scenario 1

(b) Scenario 2
of mixed reads and writes (uniformly distributed inserts and point reads) and hardware profile HW3. In the first scenario, all reads are point queries in 20% of the domain. In the second scenario, 50% of the reads are point reads and touch 10% of the domain, while the other half are range queries and touch a different (non-intersecting with the point reads) 10% of the domain. We do not provide the Data Calculator with an initial specification. Given the composition of the workload our intuition is that a mix of hashing, B-tree like indexing (e.g., with quantile nodes and sorted pages), and a simple log (unsorted pages) might lead to a good design, and so we instruct the Data Calculator to use those four elements to construct a design (this is done using Algorithm 1 but starting with an empty specification. Figure 9 depicts the specifications of the resulting data structures. For the first scenario (left side of Figure 9) the Data Calculator computed a design where a hashing element at the upper levels of the hierarchy allows to quickly access data but then data is split between the write and read intensive parts of the domain to simple unsorted pages (like a log) and B-tree style indexing for the read intensive part. For the second scenario (right side of Figure 9), the Data Calculator produces a design which similarly to the previous one takes care of read and writes separately but this time also distinguishes between range and point gets by allowing the part of the domain that receives point queries to be accessed with hashing and the rest via B-tree style indexing. The time needed for each design question was in the order of a few seconds up to 30 minutes depending on the size of the sample workload (the synthesis costs are embedded in Figure 9 for both scenarios). Thus, the Data Calculator quickly answers complex questions that would normally take humans days or even weeks to test fully.

6 RELATED WORK
To the best of our knowledge this is the first work to discuss the problem of interactive data structure design and to compute the impact on performance. However, there are numerous areas from where we draw inspiration and with which we share concepts. Interactive Design. Conceptually, the work on Magic for layout on integrated circuits [70] comes closest to our work. Magic uses a set of design rules to quickly verify transistor designs so they can be simulated by designers. In other words, a designer may propose a transistor design and Magic will determine if this is correct or not. Naturally, this is a huge step especially for hardware design where actual implementation is extremely costly. The Data Calculator pushes interactive design one step further to incorporate cost estimation as part of the design phase by being able to estimate the cost of adding or removing individual design options which in turn also allows us to build design algorithms for automatic discovery of good and bad designs instead of having to build and test the complete design manually. Generalized Indexes. One of the stronger connections is the work on Generalized Search Tree Indexes (GiST) [6, 7, 38, 54–57]. GiST aims to make it easy to extend data structures and tailor them to specific problems and data with minimal effort. It is a template, an abstract index definition that allows designers and developers to implement a large class of indexes. The original proposal focused on record retrieval only but later work added support for concurrency [55], a more general API [6], improved performance [54], selectivity estimation on generated indexes [7] and even visual tools that help with debugging [56, 57]. While the Data Calculator and GiST share motivation, they are fundamentally different: GiST is a template to implement tailored indexes while the Data Calculator is an engine that computes the performance of a design enabling rich design questions that compute the impact of design choices before we start coding, making these two lines of work complementary. Modular/Extensible Systems and System Synthesizers. A key part of the Data Calculator is its design library, breaking down a design space in components and then being able to use any set of those components as a solution. As such the Data Calculator shares concepts with the stream of work on modular systems, an idea that has been explored in many areas of computer science: in databases for easily adding data types [27, 28, 68, 69, 80] with minimal implementation effort, or plug and play features and whole system components with clean interfaces [11, 15, 17, 52, 60, 61], as well as in software engineering [71], computer architecture [70], and networks [53]. Since for every area the output and the components are different, there are particular challenges that have to do with defining the proper components, interfaces and algorithms. The concept of modularity is similar in the context of the Data Calculator. The goal and application of the concept is different though. Additional Topics. Appendix B discusses additional related topics such as auto-tuning systems and data representation synthesis in programming languages.

7 SUMMARY AND NEXT STEPS
Through a new paradigm of first principles of data layouts and learned cost models, the Data Calculator allows researchers and engineers to interactively and semi-automatically navigate complex design decisions when designing or re-designing data structures, considering new workloads, and hardware. The design space we presented here includes basic layout primitives and primitives that enable cache conscious designs by dictating the relative positioning of nodes, focusing on read only queries. The quest for the first principles of data structures needs to continue to find the primitives for additional significant classes of designs including updates, compression, concurrency, adaptivity, graphs, spatial data, version control management, and replication. Such steps will also require innovations for cost synthesis. For every design class added (or even for every single primitive added), the knowledge gained in terms of the possible data structures designs grows exponentially. Additional opportunities include full DSLs for data structures, compilers for code generation and eventually certified code [74, 83], new classes of adaptive systems that can change their core design on-the-fly, and machine learning algorithms that can search the whole design space.

8 ACKNOWLEDGMENTS
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Function CompleteDesign(Q, E, l, currentPath = [], H)
if blockReachedMinimunSize(H, page_size) then
    return END_SEARCH;
else
    if meaningfulPath(currentPath, Q, l) then
        return END_SEARCH;
    else
        if cacheHit = cachedSolution(Q, l, H) then
            return null;
        else
            cacheHit = true;
            bestSolution = initializeSolution(cost=\infty);
            tmpSolution = initializeSolution();
            updateCost(E, Q, tmpSolution); cost);
            if createsSubBlocks(E) then
                Q = createQueryBlocks(Q);
                currentPath = append(E);
                subSolution = completeDesign(Q, E, l + 1, currentPath);
                if subSolution.cost = \infty then
                    tmpSolution = append(subSolution);
                    if tmpSolution.cost \leq bestSolution.cost then
                        bestSolution = tmpSolution;
                        cacheSolution(Q, l, bestSolution);
return bestSolution;

Algorithm 1: Complete a partial data structure layout specification.

A APPENDIX INTRODUCTION

This appendix provides more detailed specifications for internal components and the design space of the Data Calculator. Section B discusses additional related work. Sections C and D provide detailed specifications of the layout and access primitives respectively. In Section E, we describe the complete expert system which is used for cost synthesis for the Get operation. In Section F, we provide the complete specification of all data structures used as input to the Data Calculator for the experiments and the corresponding output cost (Section G). Finally, we provide examples of the exact output which includes a dot and a JSON file that describe the design and an instance of a resulting data structure.
Auto-tuning and Adaptive Systems. Work on tuning [16, 47] and adaptive systems is also relevant as conceptually any adaptive technique tunes along a part of the design space. For example, work on hybrid data layouts and adaptive indexing automates selection of the right layout [3, 4, 8, 13, 23, 24, 26, 30–32, 34, 39–45, 48, 49, 63, 72, 73, 76, 78, 89]. Typically, in these lines of work the layout adapts to incoming requests. Similarly works on tuning via experiments [10], learning [5], and tuning via machine learning [2, 37] can adapt parts of a design using feedback from tests. While there are shared concepts with these lines of work, they are all restricted to much smaller design spaces, typically to solve a very specific systems bottleneck, e.g., incrementally building a specific index or smoothly transitioning among specific layouts. The Data Calculator, on the other hand, provides a generic framework to argue about the whole design space of data layouts. Its capability to quickly test the potential performance of a design can potentially lead to new adaptive techniques that will also leverage experience in existing adaptive systems literature to adapt across the massive space drawn by the Data Calculator.

Data Representation Synthesis. Data representation synthesis aims for programming languages that automate data structure selection. SETL [19, 20] was the first language to generate structures in the 70s as combinations of existing data structures: array, and linked hash table. A series of works kept providing further functionality, and expanding on the components used [18, 22, 35, 36, 64, 77]. Cozy [64] is the latest system; it supports complex retrieval operations such as disjunctions, negations, and inequalities and by uses a library of five data structures: array (sorted and plain), linked list, binary tree, and hash map. These works compose data structure designs out of a small set of existing data structures. This is parallel to the tuning and access path selection problem in databases. The Data Calculator introduces a new vision for what-if design and focuses on two new dimensions: 1) design out of fine-grained primitives, and 2) calculation of the performance cost given a hardware profile and a workload. The focus on fine-grained primitives enables exploration of a massive design space. For example, using the equations of Section 2 for homomorphic two-node designs, a fixed design space of 5 possible elements can generate 25 designs, while the Data Calculator can generate $10^{32}$ designs. The gap grows for polymorphic designs, i.e., $2 \times 10^5$ for a 5 element library, while the Data Calculator can generate up to $1.6 \times 10^{55}$ valid designs (for a 10M dataset and 4K pages). In addition, the focus on cost synthesis through learned models of fine-grained access primitives means that we can capture hardware and data properties for arbitrary designs. Array Mapped Tries [79] use fine-grained primitives, but the focus is only on trie-based collections and without cost synthesis.

Appendix

B ADDITIONAL RELATED AREAS

C DATA LAYOUT PRIMITIVES

In this section we list and describe in detail all 21 primitives in the design space currently supported by the Data Calculator. A summary of the primitives is also shown in Figure 11.

(1) Key retention

Domain: yes | no | func

Elements can hold data within them apart from partitioning them in sub-blocks. This data “retention” can either be full (by setting the value to yes), like in simple arrays, where the keys are stored, or partial (defined by a function), like tries where only a part of the key is stored. Alternatively, when “no” is set, they merely act as partitioners without retaining any data, like in hash-maps. Examples:

- Data pages and arrays fully store keys
- A hash map contains no data
- Tries retain only a part of the key at every level.

(2) Value retention

Domain: yes | no | func

Elements can hold data within them apart from partitioning them in sub-blocks. This data “retention” can either be full (by setting the value to yes), like in simple arrays, where the values are stored, or partial (defined by a function). Alternatively, when “no” is set, they merely act as partitioners without retaining any data, like in hash-maps. Examples:

- Data pages and arrays fully store values
- A hash map contains no data

(3) Key-value layout

Domain: columnar | row-wise | col-row-groups

Keys and values can either be stored in a columnar, a row-wise fashion, or a row groups fashion. In columnar storage all keys are stored contiguously and all values the same.
row-wise storage they are stored as key value pairs. In col-row-groups storage data are grouped in chunks by row, but within each chunk they are stored row-wise.

(4) Intra-node access
Domain: direct | head-link | tail-link | func
Description: Determines how we can access sub-blocks.
Direct access means that we can directly access each distinct sub-block. Head-link means that we can find the first sub-block through a link. Tail-link means that we can find the last sub-block through a link. Function means that we can get a link to one or more of the sub-blocks using a function.
Examples: - A linked list has head or tail links. - A B+Tree internal node allows us to directly address each sub-tree.

(5) Utilization
Domain: none | >= | func
Utilization constraints, how much empty space is allowed. It can be specified as an arbitrary function or as a "less or equal to capacity" check.
Examples: - B+Trees can have a constraint of half-empty leaves.

(6) Bloom filters
Domain: off | on(num-hashes, num-bits)
If this is set to on, bloom filters per sub-block are used to filter reads. The parameters specified as arguments to on are used for the bloom filters. Examples: - LSM trees use bloom filters per sub-block (run or level) to skip large chunks of data.

(7) Zone map filters
Domain: min | max | both | exact | off
If this is set to min, the minimum key under a sub-block is kept for filtering reads (as a fence pointer). If this is set to max, the maximum key under a sub-block is kept for filtering reads (as a fence pointer). If this is set to both, both minimum and maximum keys are kept. If this is set to exact, the exact key of only each sub-block is kept. Examples: - B+Trees trees use zone maps in internal nodes to guide search.

(8) Filters memory layout
Domain: consolidate | scatter
Filters like zone maps or bloom filters can either be stored contiguously for an element or scattered and stored per sub-block. For example all minimum values could be consolidated in a single array thus allowing binary search. Alternatively, filters can be scattered per sub-block and stored with it. Examples: - B+Trees consolidate zone maps - Linked list of very large pages could include per-page headers that contain scattered zone maps.

(9) Fanout/Radix
Domain: fixed(size) | func | unlimited | terminal(capacity)
Description: This primitive allows us to specify the type and number of sub-blocks for this element. Unlimited means any number of sub-blocks is allowed. For fixed, the number of sub-blocks is static. If it is set to function, then the number of sub-blocks is determined by the result of a function, while terminal means that there are no sub-blocks and this element absorbs all data up to a certain capacity.
Examples: - Linked-lists have an infinite number of sub-blocks, as new sub-blocks can always be appended to the structure.
- Traditional B+Tree nodes have a fixed number of sub-blocks, as the fanout of each node is fixed a priori.
- Partitioned arrays with fixed partition size can have a functional fanout determined by the maximum number of data to be inserted.

(10) Key/fence partitioning
Domain: append(fw|bw) | data-dependent(sorted | k-ary |
Sub-block capacity

Domain: fixed | balanced | unrestricted | func

**Description:** This is used to define the capacity of each sub-block. This is the amount of tuples that can (at maximum) end up under each one of an element’s sub-blocks. One can have functional capacity, fixed capacity, unrestricted capacity (where capacity can arbitrarily grow in each sub-block) and balanced capacity where the total number of records is equally divided across all sub-blocks.

**Examples:**
- A hash-map uses unrestricted capacity as the amount of elements in each bucket is not known a-priori and buckets should arbitrarily grow
- A linked list has a fixed capacity, as each sub-block can hold 1 or more (in the case of linked lists of pages) elements. When one sub-block overflows a new sub-block should be created.
- A B+Tree node has balanced capacity as under the capacity of each sub-tree of an internal node is equal to its siblings.

(11) **Sub-block capacity**

(12) **Immediate node links**

**Domain:** next | previous | both | none

**Description:** This indicates that each sub-block includes a pointer to its immediate sibling in the “forward direction” (defined by the structure), in the “backward direction”, or both.

**Examples:**
- A linked list contains sub-blocks that are linked to each other.

(13) **Skip node links**

**Domain:** perfect | randomized | func | none

This is set to none when no skip links are used. Skip links can navigate from one sub-block to a sub-block further away from it. When set to perfect, all links that allow for binary-search style navigation are materialized. When set to randomized, only a probabilistic set of skip links is included, enough to provide some performance guarantees. When set to functional, an arbitrary function defines the links that are going to be materialized.

**Examples:**
- Classic perfect skip-lists contain perfect skip-links.
- Randomized skip-lists contain randomized skip-lins.

(14) **Area links**

**Domain:** forward | backward | both | none

Each sub-tree can be connected with another sub-tree at the leaf level through area links. This can happen in a forward or backward way or both.

**Examples:**
- The linked leaves of a B+Tree are connected to each other for efficient range queries.
(15) **Sub-block physical location**  
**Domain:** inline | pointed | double pointed | none  
**Description:** This is used to define the physical location of sub-blocks, are they inlined (i.e., physically contained within the parent element) or pointed to using pointers to arbitrary locations in memory. Double pointed means that the sub-block points back to the parent as well. None means that sub-blocks are either nonexistent (i.e., the element is terminal) or not stored.  
**Examples:**  
- A classic B+Tree uses pointed sub-blocks where sub-trees are pointed to by their parents. - A hash-map followed by per bucket linked-lists can contain the linked list head per bucket inlined.

(16) **Sub-block physical layout**  
**Domain:** BFS | BFS layer grouping | Scatter  
**Description:** This represents the physical layout of sub-blocks. Scatter: random placement in memory. BFS: laid out in a breadth-first layout. BFS layer grouping: hierarchical level nesting of BFS layouts.  
**Examples:**  
- A cache conscious B+Tree uses BFS to layout nodes. - A linked list can have blocks that are randomly scattered in main memory.

(17) **Sub-blocks homogeneous**  
**Domain:** boolean  
This is set to true in the case where the sub-blocks have the same definition.  
**Examples:**  
- Data structures like bounded disorder and ART are not homogeneous, as sibling nodes can be of different types. - A classic B+Tree is homogeneous as all children of a node are of the same type.

(18) **Sub-block consolidation**  
**Domain:** boolean  
If the node has only one sub-block, this node is suppressed.

(19) **Sub-block instantiation**  
**Domain:** lazy | eager  
If a sub-block is empty, it is represented with a null pointer when set to lazy, or instantiated in any case when set to eager.

(20) **Links location**  
**Domain:** consolidate | scatter  
In case links are used this tells us how they are stored. Consolidate means that they are stored as a contiguous array, scatter means that they are stored per partition.  
**Examples:**  
- Linked lists have scattered links, where each sub-block contains the links. - Unix inodes consolidate within the current inode links to blocks or indirect blocks.

(21) **Recursion**  
**Domain:** yes(func) | no  
If yes, sub-blocks will be subsequently inserted into a node.
of the same type until a maximum depth (expressed as a function) is reached. Then the terminal node type of this data structure will be used.

D DATA ACCESS PRIMITIVES AND LEARNED COST MODELS

We now present in detail the Level 2 access primitives and discuss how to generate learned cost models to perform cost synthesis. Each data access primitive has three stages.

Stage 1: In the first stage, C++ code implementing the Level 2 access primitive is run. This implementation usually consists of doing \( n \) iterations of the required action in a tight loop; as an example, we might perform \( n = 10000 \) binary searches an array for some set of 10000 chosen values sampled with replacement from the values within the array. After doing the required action \( n \) times, the benchmark latency is observed and normalized (i.e. latency = total latency / \( n \)).

This implementation benchmark is run multiple times with controlled values for various input parameters such as data size. The benchmark latency is observed and recorded for each set of input parameters. Afterwards, these values are collected into 1) a matrix of the parameter values \( X \) and 2) a corresponding array of the performance (latency) \( Y \) observed in each benchmark. In the joined matrix \( (X|Y) \), each row contains a single configuration of parameters and a single benchmark latency. A column in \( X \) contains all the configuration values of a parameter, and the \( Y \) column is the benchmark latencies. Below are two examples of benchmarks and their corresponding data matrices:

Example 1 (Binary Search): Matrix of parameter values is \((d \times 1)\) vector, with \( d \) = the number of input sizes. A concrete example might be sizes = \{128,256,512,1024\}. Additionally, a \( d \times 1 \) vector of benchmark latencies is passed.

Example 2 (Bloom Filter): The matrix of parameter values in this case is \( d \times 2 \) (recall that variables such as hash function are chosen via the Level 1 Benchmark), with the two variables being data size and number of hash functions. A concrete example might be

\[
\begin{pmatrix}
128 & 1 \\
128 & 2 \\
256 & 1 \\
256 & 2 \\
\end{pmatrix}
\]

The set of benchmark latencies is still a \( d \times 1 \) vector, with the recorded latencies passed in the same order as the parameter of matrix values (i.e., the first benchmark latency corresponds to the first row of parameters in the matrix of parameter values).

Stage 2: A model-specific Python function is then called to train the model. The matrix of parameter values and corresponding array of latencies is passed to the function. Depending on the model, some combination of NumPy, SciPy, and PyTorch is used to train the models. For simple models this can be as simple as a call to the least squares regression function; for others, this requires some pre-processing of data or training via gradient descent.

Example 1 (Interpolation Search): As input, Python is given the required input of data sizes and benchmark latencies. It performs two basic actions, which essentially provide new features. These two actions are to compute log data size and \( \log(\log \text{data size}) \). Thus each row in \( X \) now contains three input features, data size, log data size, and \( \log(\log \text{data size}) \). At this point we run regular linear regression via NumPy. We record the optimal coefficients and return them as the return values of our train function back to the main cost synthesis routine of the Data Calculator in C++.

Example 2 (Random Memory Access): The benchmark for a random memory access is fit by a sum of sigmoids model: 

\[
f(x) = \sum_{i=1}^{k} \left[ \frac{c_i}{1 + e^{-x_i \log x - x_i}} \right] + y_0
\]

with all coefficients learned as usual. This model is no longer convex in its L2 loss and cannot be learned via off the shelf techniques such as least squares regression. Instead, we first pre-process the data by calculating rates of change for intervals of some size \( z \). We find the \( k \) highest local maxima for this graph of rates of change and set these as our initial guesses for the \( x_i \) values. For the values of the \( k \), \( c_i \) we initialize them to random positive numbers between 0 and 1. Our initial guess for \( y_0 \) is just the first point. At this point, we use SciPy’s curve fit package to train the parameters and observe the results.

The learning of the coefficients for the sum of sigmoid model requires good initial conditions for the guesses of the \( x_i \). It is not sensitive to initial guess for the parameters of \( k_i \) or \( c_i \). We found that for each of \( z = 0.1, 0.5, \) and 1, the initial guesses for the \( x_i \) were good enough so that the fitted model produces accurate results.

Stage 3: These coefficients are then stored in a model specific class such as “SimpleLinearModel". The class then has a prediction function which uses the coefficients to calculate costs given input parameters. Thus, even though the training is in Python, the prediction is made in C++ classes.

Example 1 (Interpolation Search): In the case of interpolation search, the corresponding model is LogLogLinear model. The model is defined by 

\[
f(x) = c_1 x + c_2 \log x + c_3 \log \log x + c_6
\]

It’s class constructor takes in 4 coefficients, and stores them. Then for prediction, it takes as input \( x \), and computes \( f(x) \) using the given coefficients. In the case of Interpolation Search, the input variable \( x \) is data size and so the model is producing a prediction for search cost given the data size.

Example 2 (Weighted Nearest Neighbors): Nearest neighbors is our only non-parametric model at the moment, although we expect to add more in the future. Unlike the previous example, which is a parametric model (defined by the coefficients \( c_1, c_2, c_3, c_6 \)), the non-parametric model needs to keep track of its data. Thus in weighted
### Node organization

- **New node.** No node contains no real key data, e.g., intermediate nodes of b-trees and linked lists. Yes: contains complete key data, e.g., nodes of b-trees, and arrays. *Function:* contains only a subset of the values.

### Node filters

- **New node.** No node contains no real value data, e.g., intermediate nodes of b-trees, and linked lists. Yes: contains complete value data, e.g., nodes of b-trees, and arrays. *Function:* contains only a subset of the values.

### Partitioning

- **New node.** No node contains no real data. e.g., intermediate nodes of b-trees, and arrays. *Function:* contains only a subset of the values.

### Fast rules

- **Rules:** requires fanout/radix != terminal.

---

**Figure 11:** Data layout primitives and synthesis examples of data structures.
nearest neighbors, its constructor takes in the matrix $X$ and $Y$ and predictions are made using these values explicitly via the function:

$$f(x) = \frac{1}{\sum_{i=1}^{k} d(x, x_i)} \sum_{i=1}^{k} \frac{1}{d(x, x_i)} y_k.$$  

### D.1 Notes for Presentation

For the example graphs presented next, each fitted model is from the same machine and used 8 byte key and value sizes. The graphs are given with Data Size as the dependent variable for various graphs, although the input into the Level 2 primitives can be either the number of elements or data size (for fixed size elements). For fixed size elements, the graphs are more easily read as number of elements. Finally, in the following models, there is some small amount of terminology that makes naming of the benchmarks easier. We provide the terms below.

**RowStore vs. ColumnStore:** This refers to how the data is laid out. In general RowStore means that we have arrays of key-value pairs, whereas a column store means we have separate arrays, one for keys and one for values. This is the input of the Data Layout option that comes into higher level benchmarks. (i.e. to cost a scan over data that is stored as a "row store", you pass Data Layout = "row store" into the higher level benchmark).

**Hash Family:** For the purpose of benchmarking and cost modeling, we take the general form of the hash function and randomly generate the coefficient values during execution. That is, no particular value of the coefficients is assumed and we do not attempt (currently) to learn optimal values of these coefficients. As an example, for the multiply-shift scheme, the form is assumed to be $h(x) = (a \cdot x) >> (w - M)$, where $b = 2^M$ is the number of bins and $w$ is the word size (both fixed). During benchmarking, we conduct multiple runs and generate the value of $a$ randomly during each run.

### D.2 Learned Cost Models

1. **Scalar Scan (RowStore, Equal)**

   **Description:** This benchmark performs a FOR loop over an array of key-value pairs. At each array index, we check whether the key at that position matches the desired key.

   **Model:** The benchmark is fit by a linear model: $f(x) = ax + b$ with $a$ and $b$ being learned coefficients. Both coefficients are constrained to be non-negative.

   **Benchmarking PseudoCode**

   ```python
   Require: array of n key-value pairs, search val x
   1: for i in 1, ..., n do
   2:     if array[i].key == x then
   3:         return array[i].val
   4:     end if
   5: end for
   ```
The graphs for the scans tend to look very similar, even down to what initially appears to be noise. Each graph is in fact different. For instance, you can see that the upper right two points for this graph are different than the upper two right points in the prior graph. As well, it makes sense that the two graphs are very similar, they have almost identical code and the process is computationally bound.

**Example of Learned Cost Model:** Figure 13

(3) **Scalar Scan** (RowStore, Range)

**Description:** This benchmark performs a for loop over an array of key-value pairs. At each array index, we check whether the key at that position is less than the desired key. If the key satisfies the predicate, the corresponding value is added to a set of values to be returned. After looping through the whole array, we return the list of values that had keys less than the desired predicate. Similar benchmarks exist for between predicates.

**Model:** The benchmark is fit by a linear model: \( f(x) = ax + b \) with \( a \) and \( b \) being learned coefficients. Both coefficients are constrained to be non-negative.

**Benchmarking PseudoCode**

```
Require: array of \( n \) key-value pairs, search \( x \)
Ensure: output array of values
1: count = 0
2: for \( i \) in 1, \( \ldots \), \( n \) do
3:   if \( \text{array}[i].\text{key} < x \) then
4:     output[count] = \( \text{array}[i].\text{val} \)
5:     count++
6:   end if
7: end for
8: return output
```

**Example of Learned Cost Model:** Figure 14

(4) **Scalar Scan** (ColumnStore, Range)

**Description:** This benchmark performs a for loop over an array of keys. At each array index, we check whether the key at that position is less than the desired key. If the key is less than the desired key, we add the corresponding value in the values array to a set of values to be returned. After looping through all the values, the set of values is returned. Similar benchmarks exist for between predicates.

**Model:** The benchmark is fit by a simple linear model: \( f(x) = ax + b \) with \( a \) and \( b \) being learned coefficients. Both coefficients are constrained to be non-negative.

**Benchmarking PseudoCode**

```
Require: array of \( n \) keys, array of \( n \) values, search \( x \)
Ensure: output array of values
1: count = 0
2: for \( i \) in 1, \( \ldots \), \( n \) do
3:   if keys[i] < x then
4:     output[count] = values[i]
5:     count++
6:   end if
7: end for
8: return output
```

**Example of Learned Cost Model:** Figure 15

(5) **Simd-AVX Scan** (ColumnStore, Equal)

**Description:** This benchmark performs a SIMDified loop over an array of keys. At each point, we compare a SIMD bank of keys to the desired value. If a matching key is found,
to break the loop and return the desired value at the corresponding position.

**Model:** The benchmark is fit by a linear model: \( f(x) = ax + b \) with \( a \) and \( b \) being learned coefficients. Both coefficients are constrained to be non-negative.

---

**Benchmarking PseudoCode**

**Require:** array of \( n \) keys, array of \( n \) values, search val \( x \).

1. [SIMD bank is of width \( b = 128 \) / element size.]
2. for each SIMD bank do
3. load memory into SIMD register
4. comp_bank = SIMD compare equal
5. if test_not_all_zeros(comp_bank) then
6. get mask from comp_bank and use this to pull value from values array
7. return pulled value
8. end if
9. end for
10. return output

---

**Example of Learned Cost Model:** Figure 16

(6) **Simd-AVX Scan** (ColumnStore, Range)

**Description:** This benchmark performs a SIMDified loop over an array of keys. We loop through the array, performing actions one bank of SIMD values at a time. For each bank of SIMD values, we perform a less than predicate on the bank of keys, comparing it with the predicate value. After finding which keys satisfy the predicate, we perform a swizzle instruction to put the corresponding values to the matching keys at the beginning of a separate SIMD bank. We then store the matching values. Similar benchmarks exist for between predicates. More than the other benchmarks, this benchmark depends heavily on the input key size as the code changes with each key size. In particular, the way in which we perform the swizzle instructions depends on the width of the key.

**Model:** The benchmark is fit by a linear model: \( f(x) = ax + b \) with \( a \) and \( b \) being learned coefficients. Both coefficients are constrained to be non-negative.

---

**Benchmarking PseudoCode**

**Require:** array of \( n \) keys, array of \( n \) values, search val \( x \).

1. {SIMD bank is of width \( b = 128 \) / element size.}
2. count = 0
3. for each SIMD bank do
4. load memory into SIMD register
5. comp_bank = SIMD comp. less than
6. pull mask from comp_bank
7. use mask to swizzle around values
8. store values into output[count]
9. count += popcnt mask
10. end for
11. return output

---

**Example of Learned Cost Model:** Figure 17

(7) **Binary Search** (RowStore)

**Description:** This benchmark performs standard binary search over an array of key-value pairs.

**Model:** The benchmark is fit by a log-linear model: \( f(x) = ax + b \log x + c \) with \( a, b, c \) being learned coefficients. \( a, b, c \) are all constrained to non-negative.

---

**Benchmarking PseudoCode**

**Require:** array of \( n \) key-value pairs, search val \( x \)

1. low = 0, high = array.size - 1
Example of Learned Cost Model: Figure 18

**Binary Search (ColumnStore)**

**Description:** This benchmark performs standard binary search over an array of keys. After, it returns the corresponding value.

**Model:** The benchmark is fit by a log-linear model: \( f(x) = ax + b \log x + c \) with \( a, b, \) and \( c \) being learned coefficients. \( a, b, c \) are all constrained to non-negative.

**Benchmarking PseudoCode**

```plaintext
Require: array of \( n \) keys, array of \( n \) values, search val \( x \).
1: low = 0, high = keys.size - 1
2: middle = (low + high) / 2
3: while low < high do
4:   if keys[middle].key < x then
5:     low = middle + 1
6:   else
7:     high = middle
8:   end if
9:   middle = (low + high) / 2
end while
10: if keys[middle].key == x then
11:   return array[middle].val
12: else
13:   return NOT_FOUND
end if
```

Example of Learned Cost Model: Figure 19

(9) **Interpolation Search (RowStore)**

**Description:** This benchmark performs interpolation search over an array of key-value pairs, searching by key.

**Model:** The benchmark is fit by a loglog-linear model: \( f(x) = ax + b \log x + c \log \log x + d \) with \( a, b, c, \) and \( d \) being learned coefficients.

**Benchmarking PseudoCode**

```plaintext
Require: array of \( n \) key-value pairs, search val \( x \).
1: low = 0, high = array.size - 1
2: key_low = array[low].key, key_high = array[high].key
3: diff = key_high - key_low
4: while low < high do
5:   si = (high - low) * ((x - val_low) / diff)
6:   if array[si].key < x then
7:     low = si + 1
8:   else if array[si].key == x then
9:     return array[si].val
10: else
11:   high = si
12: end if
13: end while
14: return NOT_FOUND
```
Interpolation Search (ColumnStore)

Description: This benchmark performs interpolation search over an array of keys. After, it returns the corresponding value.

Model: The benchmark is fit by a loglog-linear model: \( f(x) = ax + b \log x + c \log \log x + d \) with \( a, b, c \) and \( d \) being learned coefficients.

Benchmarking PseudoCode

```
Require: array of \( n \) keys, array of \( n \) values, search val \( x \).
1: low = 0, high = keys.size - 1
2: key_low = keys[low], key_high = keys[high]
3: diff = key_high - key_low
4: while low < high do
5:  si = (high - low) * ((x - val_low) / diff)
6:  if keys[si] < x then
7:    low = si + 1
8:  else if keys[si] == x then
9:    return vals[si]
10:  else
11:    high = si
12:  end if
13: end while
14: return NOT_FOUND
```

Hash Probe (Multiply Shift)

Description: This benchmark computes a hash value and probes the bucket of that hash value. This is used, for example, in hash partitioning. We note that in this model there are no collisions, which is why the benchmark is a hash probe and not a model of a hash table.

We generate an array of length \( k \), which is the number of elements needed to make a structure size of input parameter Structure Size. We call this array the probe array. Each element in the array stores a random number. A second array of length \( n \), which is the number of trials we will perform, is created as well, with each element in that array also random. This array is called the "scan array". A full justification of why two arrays are needed can be seen in the description of the Random Memory Access Benchmark.

To perform the benchmark we compute the necessary hash value for each trial. The hash value is dependent on 1) the value stored at the previously probed hash bucket and 2) the value in the scan array for the current trial. 1) is required so that the request for memory at trial \( n \) requires the value for trial \( n-1 \), so that the cpu can’t issue multiple memory requests in parallel. 2) is required so that cycles don’t occur in the access patterns for hash probing (which would throw off the expected cache latencies for large arrays of hash buckets).

The previous paragraphs are true for all hash probe benchmarks. The following is true only for the multiply shift hash benchmark. 1) The input size must create a number of buckets which is a power of 2. 2) If \( S \) is the size of the created hash array, let \( s = \log_2 S \). The equation for the multiply shift hash function is

\[ h(x) = a \times x >> (64 - s) \]

where it is assumed that the processor uses 64 bit words. In the equation, \( a \) is a randomly drawn odd integer.

Model: The model for the hash probe can be a sum of sigmoids or the weighted nearest neighbor model. See the random memory access benchmark for more details on training these models.

Benchmarking PseudoCode
**Region Size (bytes)**

- 0.0
- 0.2
- 0.4
- 0.6
- 0.8
- 1.0
- 1.2
- 1.4

**Time (s)**

- $10^4$
- $10^5$
- $10^6$
- $10^7$
- $10^8$
- $10^9$

Figure 22: Example of Benchmark (11)

**Require:** pa: array of $k$ longs, sa: array of $k$ longs, i: numAccesses

1. fill each slot of pa with random $x$ = Uniform(0, $k-20$)
2. fill each slot of sa with random $x$ = Uniform(0, 20)
3. $x = 0$, $s = \log_2 k$
4. for $i = 0, \ldots, n - 1$ do
5.   $x = [a \times (pa[p] + sa[i]) + b] \mod p$
6. end for
7. print $x$

**Example of Learned Cost Model:** Figure 22

(12) Hash Probe (k-wise independent)

**Description:** For a description of the common setup of all hash benchmarks, see the benchmark above. The following is true only for the 2-wise independent hash probe benchmark. 1) The input size must create a number of buckets which is a prime number. We will denote this number by $p$. 2) The equation for the multiply shift hash function is $h(x) = (a \times x + b) \mod p$

where it is assumed that the processor uses 64 bit words. In the equation, $a$ and $b$ are randomly drawn integers between 0 and $p$.

**Model:** The model for the hash probe can be a sum of sigmoids or the weighted nearest neighbor model. The model is trained similarly to the sum of sigmoids model. More details on model training can be found in the description for the random memory access benchmark.

**Benchmarking PseudoCode**

**Require:** pa: array of $k$ longs, sa: array of $k$ longs, i: numAccesses

1. fill each slot of pa with random $x$ = Uniform(0, $k-20$)
2. fill each slot of sa with random $x$ = Uniform(0, 20)
3. $x = 0$
4. for $i = 0, \ldots, n - 1$ do
5.   possiblyInSet = true
6. end for
7. print $x$

Example of Learned Cost Model: Figure 23

(13) Bloom Filter Probe (Multiply Shift)

**Description:** This benchmark first builds a bloom filter by computing $k$ hash values for each entry and setting the bits at the computed $k$ hash values to 1. It then probes the bloom filter using the same $k$ hash functions. In this benchmark, each of the $k$ hash functions is from the multiply shift family. There are two inputs the bloom filter benchmark, the size of the filter and the number of hash functions. For each of the $n$ trials, we randomly generated a value to probe for.

The following describes the multiply shift hash function. 1) The bloom filter size must be a power of 2. 2) If $S$ is the size of the created hash array, let $s = \log_2 S$. The equation for the multiply shift hash function is $h(x) = a \times x \gg (64 - s)$

where it is assumed that the processor uses 64 bit words. In the equation, $a$ is a randomly drawn odd integer.

**Model:** The model for the bloom filter is the sum of sum of sigmoids model or the weighted nearest neighbor model. The model is trained similarly to the sum of sigmoids model. More details on the model training can be found in the description for the random memory access benchmark.

**Benchmarking PseudoCode**

**Require:** pa: array of $k$ longs, n: numAccesses, p: size

1. build bloom filter BF of size p, with p prime (build is similar to probe shown below)
2. fill each slot of pa with random $x$.
3. count = 0
4. for $i = 0, \ldots, n - 1$ do
5.   possiblyInSet = true
6.  end for
(14) Bloom Filter Probe (k-wise independent)

**Description:** This benchmark first builds a bloom filter by computing $k$ hash values for each entry and setting the bits at the computed $k$ hash values to 1. It then probes the bloom filter using the same $k$ hash functions. In this benchmark, each of the $k$ hash functions is from the multiply shift family. There are two inputs to the bloom filter benchmark, the size of the filter and the number of hash functions. For each of the $n$ trials, we randomly generated a value to probe for.

The following describes the multiply shift hash function. 1) The bloom filter size must be a power of 2. 2) If $k$ hash functions is kept constant.

The equation for the multiply shift hash function is

$$h(x) = a + x \gg (64 - s)$$

where it is assumed that the processor uses 64 bit words. In the equation, $a$ is a randomly drawn odd integer.

**Model:** The model for the bloom filter is the sum of sum of sigmoids model or the weighted nearest neighbor model. The model is trained similarly to the sum of sigmoids model. More details on the model training can be found in the description for the random memory access benchmark.

**Benchmarking PseudoCode**

**Require:** array of $k$ longs, $n$: numAccesses, $s$: size

---

(15) QuickSort

**Description:** This benchmark performs QuickSort to sort a list of keys.

**Model:** The benchmark is fit by a NLogN model. $f(x) = ax \log x + bx + c$ with $a$, $b$, and $c$ being learned coefficients.

**Benchmarking PseudoCode**

**QUICKSORT:**

**Require:** array of $n$ key-value pairs

1. if $low < high$ then
2. $p = \text{partition}(A, low, high)$
3. quicksort $(A, low, p - 1)$
4. quicksort $(A, p + 1, high)$
5. end if

**PARTITION:**

**Require:** array of $n$ key-value pairs, integers low, high

1. $pivot = A[high]$
2. $i = low - 1$
3. for $j = low, \ldots, high - 1$ do
4. if $A[j] < pivot$ then
5. $i = i + 1$
Example of Learned Cost Model: Figure 26

(16) Random Memory Access

Description: This benchmark performs random memory accesses within a region.

To do this, we first generate an array of values of length $k = \text{region size} / \text{sizeof(long)}$. This array will be the “probe array”, or “pa” for short. Next, we generate a random number between 0 and $k - 20$ for each slot in the array. Additionally, we create a “scan array”, or “sa”, of $n$ longs which contains random numbers between 0 and 20. We will be performing exactly $i$ random memory accesses, and so this second array is generally quite large.

We will initialize a starting pointer value $p$ to be 0. At each round $i = 0, \ldots, n - 1$, we will jump to $pa[p] + sa[i]$. At the end of all rounds, we will print $p$. This makes the compiler unable to optimize out the memory accesses, and is a negligible overhead to the benchmark. The combination of two random numbers, one between $[0,k-20]$ and the other between $[0,20]$, creates a nearly uniform distribution over $[0,k]$ (the first and last 20 slots have slightly lower chances of being picked via a combinatorial argument similar to why 7s are the most common dice roll). The embedding of one part of the next slot to go to into the probe array $pa$ prevents the processor from being able to fetch the next memory location before the data arrives. The embedding of a second random part into the scan array $sa$ is to prevent cycles in the probing pattern, which would occur if we just used the probe array for the next slot.

Model: The benchmark is fit by a sum of sigmoids model:

$$f(x) = \sum_{i=1}^{k} \frac{c_i}{1 + e^{-x_i + s_i}} + y_0$$

with all coefficients learned as usual. This model is no longer convex in its L2 loss and cannot be learned via off the shelf techniques such as least squares regression. Instead, we first preprocess the data by calculating rates of change for intervals of some size $z$. We find the $k$ highest local maxima for this graph of rates of change and set these as our initial guesses for the $x_i$ values. For the values of the $k_i$, $c_i$ we initialize them to random positive numbers between 0 and 1. Our initial guess for $y_0$ is just the first point. At this point, we use scipy’s curve fit package to train the parameters and observe the results.

The learning of the coefficients for the sum of sigmoid model requires good initial conditions for the guesses of the $x_i$. It is not sensitive to initial guess for the parameters of $k_i$ or $c_i$. We found that for each of $z = 0.1, 0.5, 1$, and 1, the initial guesses for the $x_i$ were good enough so that the fitted model produces accurate results.

Additionally, the user can select a different model for the Random Memory Access Benchmark should they so choose. We currently support k nearest neighbor regression. In the future, we plan to add support for both locally weighted linear regression and Gaussian Process Regression. All these techniques are non-parametric and so don’t need training, and all three techniques produce good predictions given enough data. As a tradeoff, these models are significantly slower in producing predictions and are not interpretable in any way. This contrasts with the sum of sigmoids model, whose values of the $x_i$ tell you the cache boundaries and whose values $c_i$ tell you the difference in memory access latency between each subsequent level of cache.

Benchmarking PseudoCode

Require: $pa$: array of $k$ longs, $sa$: array of $k$ longs, $i$: numAccesses

1: fill each slot of $pa$ with random $x=\text{Uniform}(0,k-20)$
2: fill each slot of $sa$ with random $x=\text{Uniform}(0,20)$
3: $p = 0$
4: for $i = 0, \ldots, n - 1$ do
5:   $p = pa[p] + sa[i]$
6: end for
7: print $p$

Example of Learned Cost Model: Figure 27

(17) Batched Random Memory Access

Description: The batched random memory access benchmark is similar to the random memory access benchmark, except that now there is nothing to prevent the test from...
Model: The batched random memory access benchmark is fit by the weighted nearest neighbors model and the sum of sigmoids model. The training is similar to the random memory access benchmark.

Benchmarking PseudoCode

Require: pa: array of k longs, sa: array of k longs, i: number of accesses
1. fill each slot of pa with random \( x = \text{Uniform}(0,k) \)
2. fill each slot of sa with random \( x = \text{Uniform}(0,k) \)
3. \( p = 0 \)
4. for \( i = 0, \ldots, n-1 \) do
5. \( p += pa[sa[i]] \)
6. end for
7. print \( p \)

Example of Learned Cost Model: Figure 28

Final Notes: Currently the technical report contains 17 out of the 24 Level 2 Access Primitives. We will be adding the remaining 7.

E COST SYNTHESIS

Given a data layout specification and a workload, the Data Calculator uses Level 1 access primitives to synthesize operations and subsequently each Level 1 primitive is translated to the appropriate Level 2 primitive to compute the cost of the overall operation. This process relies on an expert system which captures fundamental properties of how basic layout principles are expected to behave and which access (high-level) algorithms are expected to best utilize each layout. In the main part of the paper, we presented a more high level view of the expert system. Figure 29 now provides a more detailed look into the internals of the expert system. Each empty rounded shape in Figure 29 represents a Level 1 access primitive call, each diamond a decision, and each dark round shape a data access primitive. Finally, blue star shaped (marked P1-P8) represent additional processes described at the bottom of Figure 29. Together they form the complete expert system for algorithm and cost synthesis for the Get operation.

F INPUT EXAMPLES

Here we provide more examples of data structure input specifications. The complete set of elements used in the experiments are defined in Table 30. Specifically, we list the definition of all 8 elements used in the paper: Unordered Data Page (UDP), Ordered Data Page (ODP), Hash partitioning (Hash), Range partitioning (Range), Trie node, B+Tree internal node (B+), Linked List (LL), Skip List (SL).

Then full data structures are defined as hierarchies of those elements. We use the following notation: \( A \rightarrow B \), which means that all sub-blocks of element A are of type B. Dots represent multiple levels in the hierarchy. Below we describe all full data structures used in the experiments using this notation.

- Linked List: \( LL \rightarrow UDP \)
- Array: UDP with fixed capacity value equal to the number of puts.
- Range Partitioned Linked List: \( \text{Range} \rightarrow LL \rightarrow UDP \)
- Skip-List: \( SL \rightarrow UDP \)
- Trie: \( \text{Trie} \rightarrow \ldots \rightarrow \text{Trie} \rightarrow UDP \)
- B+Tree: \( \text{B+} \rightarrow \ldots \rightarrow \text{B+} \rightarrow ODP \)
# Data Access Primitives and Fitted Models

| Data Access Primitives Level 1 | Model Parameters | Data Access Primitives Layer 2 | Fitted Models |
|--------------------------------|------------------|-------------------------------|---------------|
| 1. Scan                        | Data Size        | Linear Model (1)              |               |
| (Element Size, Comparison,    |                  |                              |               |
| Data Layout, None)            |                  |                              |               |
| 2. Weighted Nearest           | Scalar Scan      | Linear Model (1)              |               |
| Batch Ordered Write (RowStore)|                  |                              |               |
| 3. Linear Model (1)           | Scalar Scan      | Linear Model (1)              |               |
| (Element Size, Algorithm)     | (ColumnStore, Range) | Linear Model (1) |               |
| 4. Linear Probing (Multiply-shift [25]) |                  |                              |               |
| 5. Linear Probing (k-wise independent, k=2,3,4,5) |                  |                              |               |
| 6. Hash Probe (Hash Family)   | Structure Size   | Linear Probing (k-wise independent, k=2,3,4,5) |               |
| 7. Binary Search (RowStore)   | Data Size        | Sum of Sigmoid (5), Weighted Nearest Neighbors (7) |               |
| 8. Interpolation Search       | Binary Search    | Sum of Sigmoid (5), Weighted Nearest Neighbors (7) |               |
| (RowStore)                    | (ColumnStore)    |                              |               |
| 9. Region Size                | Interpolation    | Log + Log-Log Model (3)       |               |
| 10. Search (ColumnStore)      | Search           |                              |               |
| 11. External Merge Sort       | Region Size      | Sum of Sigmoid (5), Weighted Nearest Neighbors (7) |               |
| 12. Bloom Filter Probe        | Structure Size   | Sum of Sigmoid (5), Weighted Nearest Neighbors (7) |               |
| (Hash Family)                 | Number of Hash Functions | Sum of Sigmoid (5), Weighted Nearest Neighbors (7) |               |
| 13. Bloom Filter Probe (k-wise independent, k=2,3,4,5) |                  |                              |               |
| 14. Sort                      | Data Size        | NLogN Model (4)               |               |
| (Element Size, Algorithm)     | QuickSort        |                              |               |
| 15. Random Memory Access      | Region Size      | NLogN Model (4)               |               |
| 16. Batched Random Memory     | Region Size      | NLogN Model (4)               |               |
| Access                        | Batched Random Memory Access | Sum of Sigmoid (5), Weighted Nearest Neighbors (7) |               |
| 17. Unordered Batch Write     | Write Data Size  | Linear Model (1)              |               |
| (Layout, )                    | Contiguous Write (RowStore) | Linear Model (1) |               |
| 18. Batched Random Memory     |                  |                              |               |
| Access                        |                  |                              |               |
| 19. Ordered Batch Write       | Write Data Size  | Linear Model (1)              |               |
| (Layout, )                    | Batch Ordered Write (RowStore) | Linear Model (1) |               |
| 20. Batched Random Memory     |                  |                              |               |
| Access                        |                  |                              |               |
| 21. Scatter Batch Write       | Number of Elements, Region Size | Linear Model (1) |               |
| 22. Intertransition Write     |                  |                              |               |
| (ColumnStore)                 |                  |                              |               |
| 23. Scatter Batch Write       |                  |                              |               |

## Models used for fitting data access primitives

| Model               | Description                                                                 | Formula |
|---------------------|-----------------------------------------------------------------------------|---------|
| 1. Linear           | Fits a simple line through data                                             | $f(v) = w_1 \phi(v) + \phi(v) = (v)$ |
| 2. Log-Linear       | Fits a linear model with a bias composed of the identity and logarithmic    | $f(v) = w_1 \phi(v) + y_1 \phi(v) = \frac{v}{\log v}$ |
|                     | functions plus a bias                                                      |         |
| 3. Log + LogLog     | Fits a model with log, log log, and linear components                      | $f(v) = w_1 \phi(v) + y_1 \phi(v) = \frac{v}{\log \log v}$ |
| 4. NLogN            | Fits a model with primarily an NLogN and linear component                  | $f(v) = w_1 \phi(v) + y_1 \phi(v) = \frac{v}{\log \log v}$ |
| 5. Sum of Sigmoid   | Fits a model that has $k$ approximate steps.                               | $f(x) = \sum_{i=1}^{k} \phi(x_i)$ |
| 6. Sum of Sum of   | Fits a model which has two cost components, both of which have $k$         | $f(x, m) = \sum_{i=1}^{k} \phi(x_i) + \sum_{i=1}^{m} \phi(x_i) + y_i$ |
| Sigmoids            | approximate steps occurring at the same locations.                        |         |
| 7. Weighted Nearest | Takes the $k$ nearest neighbors under the $l_2$ norm and computes a        | $f(x) = \sum_{i=1}^{k} \frac{w_i}{\|x_i - x\|_2} y_i$ |
| Neighbors           | weighted average of their outputs.                                         |         |

Note: $f$ is a function, $v$ is a vector, and $x, m, a, c, l, c$ are scalars. $\log(v)$ returns a vector with log applied on an element by element basis to $v$; i.e. if $v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ then $\log(v) = \begin{pmatrix} \log v_1 \\ \log v_2 \end{pmatrix}$. Finally, if we have vectors $v^{(1)}, v^{(2)}$ of lengths $n, m$ stacked on each other as $\begin{pmatrix} v^{(1)} \\ v^{(2)} \end{pmatrix}$, then this signifies the $n + m$ length vector produced by stacking the entries of $v^{(1)}$ on top of the entries of $v^{(2)}$, i.e. $\begin{pmatrix} v^{(1)} \\ v^{(2)} \end{pmatrix} = \begin{pmatrix} v^{(1)}_1, \ldots, v^{(1)}_n, v^{(2)}_1, \ldots, v^{(2)}_m \end{pmatrix}$. |

Table 1: Data access primitives and models used for operation cost synthesis.
• **Sorted Array**: ODP with fixed capacity value equal to the number of puts.
• **Hash Table**: Hash → LL → UDP (with fixed capacity value 5)

The number of levels an actual instance of a data structure would have depends on the size of the dataset for the case of the B+Tree, and on the size of the keys for the Trie.

G OUTPUT EXAMPLES

In this section, we give examples of the raw output of the Data Calculator. This includes both cases where the output is a calculation of the response time and cases where the output is the design of a new data structure.

G.1 Output Performance Break Down

We first present a representative cost breakdown for performing 1 Get query in 100,000 inserts. These numbers correspond to Figure 6. The cost for each operation is broken down in Level 1 access primitives: \( P \) stands for random probe, \( S \) for serial scan, and \( B \) for binary search. The input in each function is the size of the data to be accessed.

• **Linked List**: \( P(782) + 6P(200974) + 5S(256) + S(141) + P(256) \)

1 probe for getting the head of the linked list, 6 random probes for fetching the first 6 pages (since the answer is found on the 6-th page), 5 full scans for the first 5 pages, 1 scan until 141 (the answer) and 1 probe for getting the value (data is stored in a columnar format).

• **Array**: \( S(1421) + P(100000) \)

1 serial scan of the keys until the record is found (position 1421), and a random probe in the whole array of values (data is stored in a columnar format) to find the values.

• **Range Partitioned Linked List**: \( P(842) + P(216394) + S(185) + P(256) \)

1 probe for the head of the data structure, 1 probe for the list head of the given range, one scan on the first page of the linked list and one probe for the value in the first page of the linked list (data is stored in a columnar format).

• **Skip-List**: \( P(1181) + 201P(391) + P(201373) + B(256) + P(256) \)

Various probes as we navigate in the skip-list checking the zone maps, one binary search when the correct page is found, followed by a probe for the value.

• **Trie**: \( P(256) + P(512) + P(768) + P(1024) + P(1280) + P(1536) + P(2048) + P(102144) + P(228628) + P(32608532) + S(185) + P(256) \)

Multiple probes as we navigate the trie, a serial scan on the target page, followed by a probe for getting the value.

• **B+Tree**: \( 2B(1400) + P(205640) + P(40) + P(840) + B(256) + P(256) \)

Multiple probes as we navigate the tree, a serial scan on the target page, followed by a probe for getting the key, and a probe for getting the value.

• **Sorted Array**: \( B(100000) + P(100000) \)

Binary search for finding the key, probe for finding the value.

• **Hash Table**: \( P(39718) + P(10207526) + S(2) + P(5) \)

Probe for the head, probe for the linked list, serial scan first page of linked list, probe for value.

G.2 Output Design Example

We now discuss design output. In the experiment of Figure 9, the Data Calculator designed new data structures given a workload. For the first scenario (Figure 9 left side) the Data Calculator computed a design where a hashing element at the upper levels of the hierarchy allows to quickly access data but then data is split between the write and read intensive parts of the domain to simple unsorted pages (like a log) and B+tree-style indexing for the read intensive part.

For the second scenario (right side of Figure 9), the Data Calculator produces a design which similarly to the previous one takes care of read and writes separately but this time also distinguishes between range and point gets by allowing the partial values that receives point queries to be accessed with hashing and the rest via B+tree style indexing.

In both scenarios the following elements were chosen by the Data Calculator: Hash Partitioning, B+Tree Internal, and Ordered Data Page. Below, we list all specifications as generated by the system in JSON format and blended into a full designs as shown in Figure 9.

G.2.1 Hash Partitioning Element

```json
{
    "external.links.next": false,
    "external.links.prev": false,
    "inter-block.blockAccess.direct": true,
    "inter-block.blockAccess.headLink": false,
    "inter-block.blockAccess.tailLink": false,
    "inter-block.fanout.defaultValue": 100,
    "inter-block.fanout.function": "fixed",
    "inter-block.fanout.type": "fixed",
    "inter-block.orderMaintenance.type": "none",
    "inter-block partitioning.function": "KeyMod(100)",
    "inter-block partitioning.logStructured.filterPerLevel": false,
    "inter-block partitioning.logStructured.filterPerRun": false,
    "inter-block partitioning.logStructured.initialRunSize": 0,
    "inter-block partitioning.logStructured.maxRunPerLevel": 0,
    "inter-block partitioning.logStructured.mergeFactor": 0,
    "inter-block.partitioning.type": "function",
    "intra-block.blockProperties.location": "inline",
    "intra-block.blockProperties.layout": "inline",
    "intra-block.blockProperties.homogeneous": true,
    "intra-block.capacity.function": "variable",
    "intra-block.capacity.type": "variable",
    "intra-block.capacity.value": 0,
    "intra-block.dataRetention.keyRetention.compression": true,
    "intra-block.dataRetention.keyRetention.function": "variable",
    "intra-block.dataRetention.keyRetention.type": "none",
    "intra-block.dataRetention.retainedDataLayout": false,
    "intra-block.dataRetention.valueRetention.compression": true,
    "intra-block.dataRetention.valueRetention.function": "variable",
    "intra-block.dataRetention.valueRetention.type": "none",
    "intra-block.filters.bloomFilter.active": false,
    "intra-block.filters.bloomFilter.hashFunctionsNumber": 0,
    "intra-block.filters.bloomFilter.hashFunctionsNumber": 0,
    "intra-block.filters.bloomFilter.numberOfBits": 0,
    "intra-block.filters.filterMemoryLayout": "scatter",
    "intra-block.filters.zoneMaps.max": false,
    "intra-block.filters.zoneMaps.min": false,
    "intra-block.filters.zoneMaps.path": "scatter",
    "intra-block.links.linkMemoryLayout": "scatter",
    "intra-block.links.next": false,
    "intra-block.links.prev": false,
    "intra-block.links.probability": 0,
    "intra-block.useUtilization.constraint": "none",
    "intra-block.useUtilization.function": "variable",
    "layout.oneChildCompression": true,
    "layout.zerosElementNull": false,
    "layout.undirectedPointers": false
}
```

G.2.2 Ordered Data Page

```json
{
    "external.links.next": true,
    "external.links.prev": false,
    "inter-block.blockAccess.direct": true,
    "inter-block.blockAccess.headLink": false,
    "inter-block.blockAccess.tailLink": false,
    "inter-block.fanout.defaultValue": 256,
    "inter-block.fanout.function": "variable",
    "intra-block.blockProperties.location": "inline",
    "intra-block.blockProperties.layout": "inline",
    "intra-block.capacity.function": "variable",
    "intra-block.capacity.type": "variable",
    "intra-block.capacity.value": 0,
    "intra-block.dataRetention.keyRetention.compression": true,
    "intra-block.dataRetention.keyRetention.function": "variable",
    "intra-block.dataRetention.keyRetention.type": "none",
    "intra-block.dataRetention.keyRetention.type": "none",
    "intra-block.dataRetention.valueRetention.compression": true,
    "intra-block.dataRetention.valueRetention.function": "variable",
    "intra-block.dataRetention.valueRetention.type": "none",
    "intra-block.filters.bloomFilter.active": false,
    "intra-block.filters.bloomFilter.hashFunctionsNumber": 0,
    "intra-block.filters.bloomFilter.numberOfBits": 0,
    "intra-block.filters.filterMemoryLayout": "scatter",
    "intra-block.filters.zoneMaps.max": false,
    "intra-block.filters.zoneMaps.min": false,
    "intra-block.filters.zoneMaps.path": "scatter",
    "intra-block.links.linkMemoryLayout": "scatter",
    "intra-block.links.next": false,
    "intra-block.links.prev": false,
    "intra-block.links.probability": 0,
    "intra-block.useUtilization.constraint": "none",
    "intra-block.useUtilization.function": "variable",
    "layout.oneChildCompression": true,
    "layout.zerosElementNull": false,
    "layout.undirectedPointers": false
}
```
G.3 Data Structure Instances

Finally, we present the raw output of the optimal data structure instances for both scenarios in Figure 9. For demonstration purposes we converted the raw data structures to graph structures. Each graph node represents an element and each line represents a connection from a parent element to its sub-blocks. Since we are plotting an instance of the data structure there is a lot of information. Zooming in the plots allows readers to read the type of each node. However, intuitively, very high fanout elements represent hash nodes, low fanout elements represent B+Tree internal nodes and no-fanout (terminal) elements represent data pages.

**Scenario 1.** We have a hash-map followed by either shallow B+Trees or large log-style data pages for write intensive parts of the domain (corresponding to the design at the left of Figure 9).

**Scenario 2.** We have a B+Tree followed by either hash-maps for point-get intensive parts of the domain or B+Trees for range intensive parts, and log-style big data pages for write intensive parts of the domain (corresponding to the design at the right of Figure 9).
Figure 29: Expert System for get cost synthesis
| Primitive                  | UDP | ODP | H | R | T | B+ | LL | SL |
|---------------------------|-----|-----|---|---|---|----|----|----|
| Key retention. No: node contains no real key data, e.g., intermediate nodes of b-trees and linked lists. Yes: contains complete key data, e.g., nodes of b-trees and arrays. | yes | yes | no | no | func | no | no | no |
| Value retention. No: node contains no real value data, e.g., intermediate nodes of b+trees, and linked lists. Yes: contains complete value data, e.g., nodes of b-trees, and arrays. | yes | yes | no | no | func | no | no | no |
| Key-value access. Determines how sub-blocks (one or more keys of this node) can be addressed and retrieved within a node, e.g., with direct links, a link only to the first or last block, etc. | col | col | col | col | col | col | col | col |
| Utilization. Utilization constraints in regards to capacity. For example, >= 50% denotes that utilization has to be greater than or equal to half the capacity. | none | none | none | none | none | none | none | min |
| Bloom filters. A node’s sub-block can be filtered using bloom filters. Bloom filters get as parameters the number of hash functions and number of bits. | off | off | off | off | off | off | off | off |
| Zone map filters. A node’s sub-block can be filtered using zone maps, e.g., they can filter based on mix/max keys in each sub-block. | off | off | off | off | off | off | off | off |
| Rules: requires key retention != no or value retention != no. | | | | | | | | |
| Node filters                      |     |     |     |     |     |     |     |     |
| Fanout/Radix. Fanout of current node in terms of sub-blocks. This can either be unlimited (i.e., no restriction on the number of sub-blocks), fixed to a number, decided by a function or the node is terminal and thus has a fixed capacity. | none | none | none | none | none | none | none | none |
| Key/fence partitioning. Set if there is a pre-defined key partitioning imposed. e.g. the sub-block where a key is located can be dictated by a radix or range partitioning function. Alternatively, keys can be temporarily partitioned. If partitioning is set to none, then keys can be forward or backwards appended. | none | none | none | none | none | none | none | none |
| Sub-block capacity. Capacity of each sub-block. It can either be fixed to a value, or balanced (i.e., sub-blocks of same size), unrestricted or functional. | none | none | none | none | none | none | none | none |
| Rules: requires fanout/radix != terminal. | | | | | | | | |
| Immediate node links. Whether and how sub-blocks are connected. | none | none | none | none | none | none | none | none |
| Skip node links. Each sub-block can be connected to another sub-block (not only the next or previous) with perfect, randomized or custom skip-links. | none | none | none | none | none | none | none | none |
| Area-links. Each sub-tree can be connected with another sub-tree at the leaf level through area links. Examples include the linked leaves of a B+Tree. | fore | none | none | none | none | none | none | none |
| Sub-block physical location. This represents the physical location of the sub-blocks. Pointed: in heap, Inline: block physically contained in parent. Double-pointed: in heap but with pointers back to the parent. | pointed | pointed | pointed | pointed | pointed | pointed | pointed | pointed |
| Rules: requires fanout/radix != terminal. | | | | | | | | |
| Sub-block physical layout. This represents the physical layout of sub-blocks. Scatter: random placement in memory. BFS: laid out in a breadth-first layout. BFS layer list: hierarchical level nesting of BFS layouts. | scatter | scatter | scatter | scatter | scatter | scatter | scatter | scatter |
| Rules: requires fanout/radix != terminal. | | | | | | | | |
| Sub-blocks homogeneous. Set to true if all sub-blocks are of the same type. | true | true | true | true | true | true | true | true |
| Rules: requires fanout/radix != terminal. | | | | | | | | |
| Sub-block consolidation. Single children are merged with their parents. | lazy | lazy | lazy | lazy | lazy | lazy | lazy | lazy |
| Rules: requires fanout/radix != terminal. | | | | | | | | |
| Sub-block links layout. If there exist links, are they all stored in a single array (consolidate) or spread at a per partition level (scatter). | lazy | lazy | lazy | lazy | lazy | lazy | lazy | lazy |
| Rules: requires immediate node links != none or skip links != none. | | | | | | | | |
| Recursion allowed. If yes, sub-blocks will be subsequently inserted into a node of the same type until a maximum depth (expressed as a function) is reached. Then the terminal node type of this data structure will be used. | no | no | yes | (logn) | no | no | no | no |

Figure 30: Data Layout Specifications
Figure 31: Real output instance (Scenario 1 of Figure 1).
Figure 32: Real output instance (Scenario 2 of Figure 9).