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Human wealth evolution is an accelerating expansion underpinned by a decelerating optimization process

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Optimization and expansion are two modes of staged evolution of complex systems where macroscopic observables change at a decreasing, respectively increasing, rate. The number of microscopic variables and their interactions are fixed in the first case but change dynamically in the second. A prime example of evolutionary expansion, Gross Domestic Product (GDP) time series gauge economic activities in changing societal structures, and the accelerating trend of their growth probably reflects a manyfold increase of the human interactions that drive change. Naively, one could think of cultural evolution as the result of an optimization process, and then expect the associated GDP growth to have a decelerating trend. We show how optimization and expansion can coexist by replacing ‘wall clock time’ \( t \) as independent variable with a measure of human interactions intensity \( \tau \). The latter was introduced in a previous work and is computed using the GDP time series itself.

Our analysis of eight centuries of yearly GDP data from three regions of Western Europe, corresponding to present day UK, France and Sweden is carried out in two steps. First, a Monte Carlo algorithm is used to fit the GDP data to a piecewise continuous function comprising a sequence of exponentials with different exponents. These arguably correspond to social and technological stages of societal organization. In a second step, GDP data are plotted vs. \( \tau \) and shown to display two logarithmic regimes, both decelerating, that are joined by a power-law cross-over period. We connect the end of the first regime and the beginning of the second with the dawn of the Industrial Revolution and the societal impact of new transport, communication and production technologies that became widely available after World War I. We conclude that wealth evolution in terms of \( \tau \) is a decelerating process with the hallmarks of record dynamics optimization.

I. INTRODUCTION

Optimization and expansion are two modes of evolution [1] where macroscopic observables change at a decreasing, respectively increasing, rate. In the first case, the number of microscopic variables and their interactions are fixed while in the second they change dynamically. In this work an example shows how a variable change accounting for the evolving interactions reveals an optimization process behind evolutionary expansion.

Human culture is a prime case of evolutionary expansion shaped by interactions whose intensity has changed manifold due to population growth and increasing means of communication. Its evolution is mirrored by Gross Domestic Product (GDP) pro capita [3], a measure of economic activity and, we surmise, of human activity in general.

A culture is defined by its artifacts, from cathedrals to computers, by the technical know-how needed to produce them, from masonry to electronic engineering, and by the laws and traditions that express shared values and regulate social behavior, from the Danish fiscal code to the Italian cuisine. All these things taken together are here dubbed wealth and denoted by the symbol \( w \). They are the products of preceding economic activity and shape the framework for its succeeding development.

The long time series [4, 5] that record annual GDP pro capita in present day dollars are a useful proxy for societal wealth history. The nature of the growth trend as well as the economic fluctuation around it have attracted considerable attention over the years, see e.g. [6–11] and more recently by Ref. [12] and by the present authors [2], who used GDP data to test a simple causal and strongly aggregated model.

The present work has two parts. In Section [11] a detailed GDP data analysis is presented, treating wealth growth as a socio-technical succession process [10]. The data trend is therefore modeled by a succession of exponentials having different durations and rates. Each section is interpreted as a metastable stage of a dynamical process. The transitions from one stage to the next are called ‘quakes’ [13] and allegedly express important societal transformations and/or historical events. The de-trended fluctuations appear as a stationary process characterized by a power spectrum, an autocorrelation function and an average stage length of the order of a human lifetime.

An ongoing urbanization process, see e.g. [14] intensifies the human interactions that generate innovation and wealth [15]. Interaction intensity and GDP history are linked by a previously introduced ‘interaction’ variable
τ \textsuperscript{2} that in Section \textsuperscript{III} replaces ‘wall clock’ time as independent variable in our data analysis.

Wealth evolution in terms of τ has just two different regimes connected by a rapid cross-over, both regimes featuring logarithmic growth. We find a strong indication that quakes are a log-Poisson \textsuperscript{13} process, which is the hallmark of a complex optimization process. An underlying evolutionary optimization process \textsuperscript{11} might therefore drive the evolutionary expansion of economic wealth. The final Section \textsuperscript{IV} contains a summary and a critical discussion of our findings.

II. STAGED EXPONENTIAL GROWTH

A. Method

Time series of GDP pro capita gauge economic activity, but, as mentioned, they are treated here as proxies for ‘wealth’, a quantity that, far beyond tradable assets, includes the underpinning social structures, culture and know-how framing human endeavors. All time series, called \( w(t) \) for wealth, are scaled by their initial value. As a first step, the logarithm of the series is fitted by a piecewise continuous linear function comprising \( n \) segments, see e.g. Fig. 1. The resulting trend \( y(t) \) is thus a continuous piecewise smooth function consisting of \( n \) exponentials. Each of these describes a metastable state, or stage, of the evolving economy.

Let \( q_l \) mark the initial time of the series, and \( q_l, l = 2, 3, \ldots, \) index the time of the quake connecting stages \( l - 1 \) and \( l \). This quake also marks a discontinuity, or ‘breakpoint’, in the derivative of \( y(t) \). The duration of the last stage is determined by the end time of the series, while all other stages have length \( p_l = q_{l+1} - q_l \).

For given \( n \), the fitting procedure described below determines the best placement of the \( n - 1 \) breakpoints of \( y(t) \) by minimizing the distance between the log of the trend and the log of the GDP data. We note that the distance \( P_T(n) \) corresponding to optimal placement mainly decreases with increasing \( n \). The improvements are initially large and diminish in stepwise fashion as \( n \) increases, see right hand column of Fig. 1. Choosing \( n \) large enough would produce a \( y(t) \) consisting of many short pieces and enable one to closely follow the details of the data. A perfect ‘fit’ can e.g. be obtained for \( n \) equal to the length of the time series.

Overly emphasizing fit quality seems beyond the point as for stages to qualify as metastable states their duration should be, at least in an average sense, of the same order as a human life time. Furthermore, details reflecting e.g. business cycles and random events should not be incorporated in a trend. A reasonable choice of \( n \) is therefore the smallest value ensuring a good fit and producing stationary de-trended fluctuations. Finally, we chose to use the same \( n \) value for all countries. Our previous investigations of these and related GDP time series support an \( n \) between 9 and 13 as we there found auto correlation functions with clear exponential decay times of 50-70 years \textsuperscript{2} as well as peaks in the power spectra at 50-70 years \textsuperscript{11 \textsuperscript{12}}. Slight variations of our choice, \( n = 11 \), have minor effects on the integral of the power-spectrum and on the number of quakes and do not qualitatively change our results. In particular, that the stage duration shown in the right hand panel of Fig. 2 has no detectable systematic dependence on wall-clock time is a property unaffected by small variations of \( n \). As a consequence, the logarithmic trend shown in the right hand panel of Fig. 3 is robust to the same variations. The above choice of \( n \) is further supported by previous investigations of the long term GDP evolution dynamics \textsuperscript{2 \textsuperscript{6} \textsuperscript{10} \textsuperscript{12}} that indicate that the duration of a metastable state, at least in an average sense, is of the order of magnitude of a human life time.

For given \( n \), the fitting procedure must determine the placement of \( y(t)’s \) \( n - 1 \) breakpoints. The \( n \) exponential functions forming \( y(t) \) have the form \( A_l e^{\alpha_l t} \), where the pre-factors, collectively termed \( A \), are determined by continuity and by the initial condition \( A_1 = 1 \). The exponents, collectively denoted by \( \alpha \), are determined by Monte Carlo (MC) optimization. For fixed \( n \), the set of all pairs \( A, \alpha \) defines the configuration space of the MC procedure. The best value of \( n \) is chosen \textit{a posteriori}, after performing separate optimizations for a range of \( n \) values \( n = 3, 4, \ldots, 14 \).

1. For a given placement \( q \) of the break-points on the time axis, calculate \( \alpha \) by minimizing the norm of the logarithmic difference \( \ln(y(t)/w(t)) \). The Matlab \textit{fminsearch} function is used for this step. Subsequently, the mean distance between consecutive breakpoints is calculated, and to each section of the fit a small penalty is assigned proportional to the difference between its length and the mean distance. This slightly favors equidistant points. The error \( E(q, \alpha) \) corresponding to the best fit is saved.

2. Optimize the placement of breakpoints \( q \); candidate placements are generated by randomly picking a breakpoint and randomly changing its position by \( \pm 1 \) year. The error \( E(q, \alpha) \) for the candidate configuration and its difference to the error of the current configuration is calculated \( E_i - E_{i-1} = \delta \). The Metropolis acceptance criterion is used, i.e. the move is accepted with probability \( P = \min(\exp(-\beta\delta), 1) \). The inverse temperature \( \beta = 200 \) was used in all simulations.

For each \( n \) the procedure yields a trend \( y(t) \), a power spectrum of the de-trended data, and the square root of the integrated power spectrum. The latter is a measure of the intensity of the fluctuations.
Figure 1: Top to bottom: data analysis of the logarithm of GDP pro capita for regions corresponding to modern UK, Sweden and France. Left column: Black lines depict a piecewise continuous fit, \( y(t) \), composed by eleven line segments, each corresponding to an exponential function. The yellow circles indicate the start of a new segment or stage in the GDP evolution process. The red jagged line depicts the GDP time series normalized by its initial value. Insert: the logarithm of the ratio of the GDP time series to its fit is plotted vs. time. Right column: Power spectrum as a function of inverse frequency, for \( n = 11 \). Insert: The 'goodness of fit' \( P_T(n) \) vs. the number \( n \) of stages depicted as blue circles. Each point is the square root of the integral of the power spectrum. Lower values mean better fit. The \( P_T(n) \) value for \( n = 11 \), the number used in the data analysis presented, is highlighted by a larger blue circle enclosed in a red annulus. Data are from [1].

B. Results

Our fitting procedure is applied to time series [4] of GDP per capita in a geographical regions corresponding to our modern UK, Sweden and France. The GDP data scaled to initial value one are shown as \( w(t) \) by red dots in the left hand column of Fig. 1 together with their respective trends \( y(t) \). The trends are plotted us-
ing black lines, and the quakes are marked by yellow circles. Note the logarithmic ordinate. Fluctuations, dubbed Δw, are shown in the inserts. They are calculated as differences between the logarithms of trends and data and their power spectra are shown in the right hand column of Fig. 1. The square root of the integral of the power spectra, i.e. the $L^2$ norm of the fluctuations, are shown vs. n as inserts of the power spectra plots. The chosen value $n = 11$ is highlighted with a larger red symbol. The choice is good for the UK data shown in the top row, while $n = 9$ and $n = 13$ would be better for Sweden and France respectively.

The most significant features of the power spectra in Fig. 1 are: Higher power for low frequencies ($1/f > 20$ years) than for high frequencies ($1/f < 20$ years). Low frequency part of the spectra are at a close to constant level, which indicates a lack of significant correlations. A decreasing slope for high frequencies indicates the presence of some correlations in that region. When comparing the spectra in Fig. 1 with those obtained by subtracting a single globally estimated trend from the same time series [2], we see that the staged trend removal process reduces the low frequency power by three orders of magnitude. Furthermore, the stage lengths identify the ‘long waves’ in the GDP data.

Oscillatory behavior of shorter GDP time series is widely discussed, see e.g. 17, 18. Cycles with a period of 7 - 11 years are usually identified as business cycles, those with periods of 15 - 25 years are associated with Simon Kuznets [19], and long cycles of 50 years or more with Nikolai Kondratiev [6]. These oscillations seem to be present in our power spectra although long cycles are virtually gone compared to Figs. 5 and 6 in [2], being replaced by different stages, as explained further below.

Figure 2 left hand panel, shows the GDP fitted trends $y(t)$, with different colors and symbols used for the three regions considered. The corresponding stage lengths form an ordered series, whose sample autocorrelation $C_{st}$ is plotted as a function of the lag variable $k$ in the insert of Fig. 2. All lines are just guides to the eye. It is clear that Sweden and France follow each other closely and trail the UK for a couple of centuries. The sample autocorrelation for the UK data shows weak correlations. This is not the case for France, where several short stages are the effect of very large GDP fluctuations. Sweden is in this respect in-between the other two countries.

The right hand panel of Fig. 2 is a scatterplot of stage durations vs. stage initial times. The full line indicates the average stage length and the two dotted lines the average shifted by one standard deviation. Note that stage lengths do not show a systematic change over time and that the average stage length, just below 70 years, is close to a human lifetime. This provides an additional time scale that is connected to regime changes rather than to oscillatory behavior.

### III. WEALTH EVOLUTION VS INTERACTION INTENSITY

#### A. Method

Cultural evolution is brought about by human interactions that have increased manifolds over time, thanks to population growth, a steady process of urbanization [14] and ever faster and more efficient means of communication [29]. The rapid and substantial change over time of the interaction intensity suggests an alternative to ‘wall clock time’ t as independent variable of the evolution process.

A variable $\tau$ describing the frequency of human interactions was introduced in [2]. There we argued that a fraction $\gamma w(t)$ of the current wealth is used to improve communication intensity, where $\gamma$ is a positive constant. This means $\frac{d\tau}{dt} = \gamma w(t)$ and, by integration,

$$\tau(t) = \gamma \int_0^t w(z) dz \quad (1)$$

The rapid fluctuations of the data are smoothened out by the integration, leading to the monotonously increasing form of $\tau(t)$ seen in Fig. 3. On sufficiently short time scales, w’s trend is nearly constant, and $\tau(t) \propto t$. On longer time scales the increasing interaction intensity leads to $\tau$ growing faster than $t$. Importantly, $\tau$ depends on the cumulated wealth and retains a permanent memory of past events. Permanent memory seems reasonable since forgetting past achievements, e.g. the printing press, the transistor, or women’s rights to vote, requires a near total destruction of human society. Finally, varying the value of $\gamma$ rigidly shifts the wealth vs. $\tau$ dependencies shown in Fig. 1 but does not affect the ‘wall-clock’ time values of the transitions see left hand panel of Fig. 3 and Table I. This was numerically verified using ten different values of $\gamma$ in the unit interval. The value $\gamma = 1$ is used for simplicity.

We are interested in the $\tau$ dependence of the wealth $w(t) \propto \tau$ and in the quake distribution on the $\tau$ axis. Since both quantities have a general form independent of the value of $\gamma$, the latter is set to one for convenience. As shown in Fig. 3, our ‘interaction’ variable $\tau(t)$, obtained by numerical evaluation of Eq. (1), is close to a linear function of time for the first part of the time series. Once $\tau(t)$ is determined $w$ can be plotted vs. $\tau$ without further assumptions.

#### B. Results

The interaction variable $\tau$ is calculated by replacing the integral in Eq. (1) with a sum. Figure 3 shows how $\tau$ depends on the wall clock time $t$, with the UK, Sweden and France in green, red and blue, respectively. In an
The continuity of the fitting function. All parameters are given in the table below:

|     | \(a_1\) | \(a_2\) | \(a_3\) | \(a_4\) | \(t_1(\text{AD})\) | \(t_2(\text{AD})\) |
|-----|---------|---------|---------|---------|-----------------|-----------------|
| UK  | 0.13    | 0.63    | 1.19    | 39.2    | 1600            | 1920            |
| Sweden | 0.079  | 0.97    | 3.29    | 39.5    | 1829            | 1925            |
| France | 0.092  | 0.84    | 2.85    | 34.6    | 1829            | 1925            |

Since wealth vs. \(\tau\) grows at a decelerating pace, except in the cross over region connecting the slow to the fast logarithmic phase, it behaves as an evolutionary optimization process \([1]\) in most of its range.

To characterize statistically the quakes identified in the time series analysis, 1000 logarithmically equidistant points are placed on the \(\tau\) axis. For each country, \(n_q(\tau)\) is the number of quakes occurring prior to \(\tau\). The function comprises a number of plateaus connected by unit ‘jumps’.

The left hand panel of Fig. 4 depicts wealth \(w\) vs. \(\tau\) using grey dots for all raw data and green, red and blue lines for the fitted trends of the UK, Sweden and France. The two turning points of the trend lines are highlighted by symbols of the same color.

Note that the symbols for Sweden and France overlap.

In the right hand panel of the same figure, \(n_q(\tau)\), the number of quakes occurring prior to \(\tau\), is plotted using a logarithmic abscissa and green, red and blue dotted lines for the UK, Sweden and France, respectively. The symbols specified in the legend highlight the endpoints of the plateaus of the staircase shaped form of \(n_q(\tau)\). The logarithmic fit plotted as a black line and described...
in the legend, is based on all the endpoints falling in the region $\tau > 240$. That the $n_q(\tau)$ trend grows at a constant logarithmic rate, except at the very beginning of the quaking process, is strongly reminiscent of a Record Dynamics [12] description of complex optimization.

**IV. DISCUSSION AND OUTLOOK**

Eight centuries of GDP time series for the UK, Sweden and France are first analyzed as staged processes [2, 10–12] and then described in terms of the ‘interaction’ variable $\tau$ introduced in [2]. Transitions between one stage and the next are salient events termed ‘quakes’. Using Monte Carlo techniques, we first identify the best placement on the time axis of $n = 11$ quakes and then argue that GDP evolution is a decelerating process when described in terms of $\tau$. The last step requires simple numerical operations on raw data and is robust to small variations of $n$.

Eleven stages of exponential growth with mainly positive rates give a good description of the GDP series, with quake positions that differ across the three countries, see the left hand side of Figs. [1] and [2]. For our choice of $n$, the mean stage duration lies slightly below 70 years and the de-trended fluctuations have near zero mean. A brief comparison with a deterministic trend model [2] shows that our analysis based on an eleven stage trend qualitatively concurs with the corresponding results of that model. Having only three free parameters, the latter produces de-trended data with larger variations than in the present case. Hence, compared to the present case, the power spectra of the de-trended time series have three orders of magnitude higher power for low frequencies with strong peaks for $1/f$ between 50 and 70 years. Further, a clear $f^{-2}$ background signal is present that is attributed to the exponential decay of the fluctuation autocorrelation function with decay times between 50 and 70 years.

The power spectra in the present analysis have higher power for low frequencies ($1/f > 20$ years) than for high frequencies ($1/f < 20$ years) and a close to constant low frequency trend, see Fig. [1]. This indicates a lack of significant correlations between low frequency de-trended fluctuations. A distinct $f^{-2}$ background is missing and the mean duration of the identified stages lies between 60 and 80 years for $n$ between 9 and 13 and slightly below 70 years for the $n = 11$ case. Furthermore, the autocorrelation function of the stage duration is nearly structureless, as seen the insert of Fig. [2]. The three observations combined suggest that stages qualitatively correspond to the ‘long waves’ in the GDP data appearing as strong low-frequency peaks in the power-spectra of Ref. [2]. Interestingly, this means that our current analysis also suggests that ‘long waves’, or more correctly, long term rhythmic growth patterns, also are detectable in pre-industrial times, while they originally were thought as being a feature of the industrial economy, see Ref. [5]. A decreasing trend for higher frequencies only spans less than a decade and is likely insignificant.

A close correspondence between quakes and identifiable historical event cannot be clearly identified. For example, the plague outbreak that reached Messina in 1348 AD [21] falls between two quakes 1328 and 1398 AD in the UK data. After the last event, the UK GDP enters a period of modest and even negative growth that ends in 1648 AD, a century earlier than the conventional beginning of the Industrial Revolution [22].

In terms of the interaction variable $\tau$, all wealth trends

![Figure 4: Left hand panel: UK, Sweden, and France GDP per capita are plotted vs. $\tau$ with a logarithmic abscissa using grey dots. Green, red and blue lines depict the fits described in the main text. The symbols, two for each line, mark the transition from the first logarithmic growth stage to a power law stage and from the latter to a second stage of logarithmic growth. The transitions occur at $\tau$ values corresponding to $t = 1829$ and 1929 AD for France and Sweden and to $t = 1600$ and 1920 AD for the UK. Right hand panel: The number of quakes $n_q$ occurring prior to interaction time $\tau$ is plotted vs. $\tau$ using a dotted line and a logarithmic abscissa for the UK, Sweden and France. Symbols are placed at the beginning and end of each of $n_q$'s plateau. Note that some of the symbols overlap. The black line depicts the linear function specified in the legend.](image-url)
have two periods of logarithmic growth, the first at a lesser rate than the second. In the cross over between the two $w$ is well fitted by a power-law with exponent larger than one. See Table I. In the UK, the transition start- and endpoints fall more than two centuries earlier and more than five years earlier, respectively, than in Sweden and France. That France and Sweden trail the UK’s longer and more gradual transition broadly concurs with the fact that the Industrial Revolution commenced in the UK. As mentioned, the UK transition starts more than one century before the ‘official’ beginning of the Industrial Revolution [22]. Note that the UK experienced a significant GDP growth in the 16’th and 17’th centuries due to the colonial activities and trade. All three transition endpoints fall shortly after World War I.

We note that the fluctuations around the trend line are small and far apart, and not always temporally close to memorable historical events. Thus, these turning points could likely be a (time-delayed) consequence of innovations that (i) changed human communication and interaction and (ii) created the technical basis for the intensive farming needed to support growing urban populations. These mainly occurred during the economic laissez-faire period ending with WWI [23]. To the first category belong the electrification of cities, including urban tram and subway systems, the building of railway and telephone line networks and, last but not least, the invention of the radio [24]. To the second belong the production of ammonia on an industrial scale, achieved in 1913 by the Haber-Bosch process [25] and the use of tractors. The first gasoline powered tractors were built in Illinois, by John Charter in 1889, and the Fordson, a popular mass-produced tractor was introduced in 1917 [26].

Since the cumulated number of quakes occurring prior to a given $\tau$, see the right hand panel of Fig. 4, has a clear logarithmic trend, GDP evolution expressed in terms of $\tau$ is reminiscent of the decelerating optimization processes described in Record Dynamics (RD) [13]. In essence, RD links a decreasing rate of change of macroscopic variables to a hierarchy of dynamical barriers separating increasingly robust metastable states. Once the effect of the ever increasing interaction intensity is removed by replacing $t$ with $\tau$, the structures regulating human behavior become indeed progressively harder to change. For example, modifying existing scientific knowledge requires progressively more advanced instruments and manpower, and the growing body of rules which regulate human interactions, e.g. legislation, must be tended by an increasing bureaucracy.

The importance of hierarchies in complex dynamics has long been appreciated [27], see [28] for a recent application to hierarchical organization of urban space. In our case, the barrier hierarchy behind evolutionary expansion could be associated with how growing social structures develop their interactions. In conclusion, GDP growth, a manifestation of cultural expansion, is an accelerating process as function of ‘wall clock’ time $t$ but appears as a sequence of two decelerating optimization processes each underpinned by a search within a hierarchy, when studied as function of the ‘interaction’ variable $\tau$. Open problems are the driving force behind the searches, i.e. the target of the optimization process, and the structure of the corresponding dynamical hierarchies.

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