Sampled-Data Consensus for Networked Euler-Lagrange Systems With Differentiable Scaling Functions

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ABSTRACT

This paper is concerned with the sampled-data consensus of networked Euler-Lagrange systems. The Euler-Lagrange system has enormous advantages in analyzing and designing dynamical systems. Yet, some problems arise in the Euler-Lagrange equation-based control laws when they contain sampled-data feedback. The control law differentiates the discontinuous sampled-data signals to generate its control input. In this process, infinities in the control input are inevitable. The main goal of this work is to eliminate these infinities. To reach this goal, a class of differentiable scaling functions is designed to replace the conventional zero-order-hold and ensure a smooth transition between sampled states. The scaling functions work as multipliers on the sampled-data signals to make them differentiable, hence avoid the infinities. A consensus criterion compatible with the differentiable scaling functions is also obtained. It is shown in numerical examples that the proposed differentiable scaling functions approach both eliminates the infinities and ensures consensus under the criterion.

INDEX TERMS

Euler-lagrange system, multi-agent system, scaling function, sampled-data control.

I. INTRODUCTION

Multi-agent systems have been a focal topic of research for decades. Numerous efforts were dedicated to solving a vast range of pertinent problems. The consensus problem [1]–[11] is the most fundamental and widely studied one among them, for it remains the most common control objective of multi-agent systems. And the studies such as formation control [12]–[16] are derived from the consensus problem. A central concern of such studies is the dynamic models of the individual agents. For a theoretical formulation to properly reflect the actual physical system, the model has to be capable of representing the physical characteristics.

Euler-Lagrange system has advantages in describing a wide range of physical systems [17]–[19]. Much effort has been dedicated to the collective control of multiple Euler-Lagrange systems [1]–[9], [11]–[14]. Though the sampled-data consensus problem has been extensively covered in multi-agent systems, yet most of the studies are conducted under the assumption of continuous communication.

Modern computer-based networked systems rely exclusively on sampled-data communication. Some control schemes on Euler-Lagrange systems also adopt sampled-data communication [20], [21]. However, when sampled-data communication is applied to networked Euler-Lagrange systems [5], the calculated controller outputs go to infinity at the sampling instants, making the controller design impractical for real-world implementations. The infinities are found to be caused by the differentiation of the piece-wise constant reference velocity generated from the sampled communication information from the network. Though sampled-data communication has already been applied in Euler-Lagrange systems [20], [21], Here lies a fundamental difference between [20], [21] and the situation studied in this work: in [20], [21], the desired trajectory $q_d$ (therefore also the desired velocity $\dot{q}_d$ & acceleration $\ddot{q}_d$) are given and are bounded, but in a networked Euler-Lagrange system,
the reference velocity $\dot{q}_{r,i}$ (counterpart of the desired velocity) is calculated from the sampled distributed information, i.e., the local and neighbor agents’ states. Unsurprisingly, $\dot{q}_{r,i}$ is inevitably piece-wise constant, i.e., constant during the sampling intervals and abruptly changes to the new sampled values at the sampling instants. As a result, its derivative, i.e., the reference acceleration $\ddot{q}_{r,i}$ is 0 during the sampling intervals and infinite at the sampling instants. Therefore the control input, which contains the reference acceleration $\ddot{q}_{r,i}$, is inevitably infinite at the sampling instants.

Although discrete sampling is a fundamental nature of modern communication, some modifications can still be made to the ZOH. As is stated, the infinity problem is caused by the differentiation of the piece-wise constant function generated by the Zero Order Hold. One solution is to modify the ZOH to make it continuous.

Pulse-modulated sampled-data control is developed in [10], [22] that has advantages over the conventional sampled-data control. The control input can be time-varying during the sampling intervals. This has inspired an alternative to the ZOH: the differentiable pulse modulation. The principle of the differentiable pulse modulation is producing a differentiable reference velocity from discrete sampled data by interpreting each sampled data as a differentiable pulse. Though effective, this scheme is not efficient. The reference velocity needs to be zero at sampling instants in order to continuously transition to the next pulse, i.e., the reference trajectory is intermittent.

In this work, we expand the notion of the pulse function and propose the differentiable scaling function, which not only avoids the infinities as the differentiable pulse does but also has a gapless transition between sampling intervals. A new consensus criterion is also obtained through rigorous and straightforward proof.

Compared with its direct precursor [5], our approach eliminates the infinities and provides a simpler yet less conservative consensus condition. Our method is uniquely compatible with sampled-data communication compared with similar works for multi-agent systems under continuous communication [6]–[8], [12]. The scaling function is also easy to realize in modern digital processors, saving the difficulties of constructing higher-order systems as in [6]–[8], [12]. It should also be noted that the scaling function approach enables very flexible dwell time assignment in a sampling interval as in [5], [23],[24]. The design of dwell time enjoys great flexibility as long as the integral-based consensus criterion is satisfied.

The rest of the paper is organized as follows. In Section II, preliminaries on the problem investigated are presented, and the problem of infinities is pointed out. In Section III, constraints on the pulse function under which the infinities are ruled out are proposed. The new consensus criterion is given through consensus analysis and is compared with that of the existing work in terms of conservativeness and applicability. Finally, numerical examples are given in Section IV to verify the theoretical results.

II. PRELIMINARIES AND PROBLEM FORMULATION

Communication links among the agents can be described by the weighted directed graph (digraph) $G = \{V, \varepsilon, A\}$, where $V = \{1, 2, \ldots, N\}$ is the set of nodes, $\varepsilon = V \times V$ is the set of edges, and $A = (a_{ij})_{N \times N}$ is the weighted adjacency matrix, $\{i, j\} \in \varepsilon$ indicates that agent $i$ receives information from agent $j$, $a_{ij} > 0$ if and only if $(j, i) \in \varepsilon$, otherwise $a_{ij} = 0$. Assume that there’s no self-loop, i.e., $a_{ii} = 0$, $i \in V$. Let $\deg(i) = \sum_{j=1}^{N} a_{ij}$, $D = diag(\deg(1), \ldots, \deg(n))$. The Laplacian matrix is $L = (\ell_{ij})_{N \times N} = D - A$. All eigenvalues of $L$ are in the open right half plane except for the one zero eigenvalue: $0 = \lambda_1 \leq Re(\lambda_2) \leq \ldots \leq Re(\lambda_r)$, where $\lambda_i \in C (i = 1, 2, \ldots, r)$ are eigenvalues of $L$ with multiplicity $N_i$.

Obviously $\sum_{i=1}^{N} N_i = N$. There exists a nonsingular matrix $U = [\frac{1}{\sqrt{N}}, u_2, \ldots, u_N]$ such that the Laplacian matrix $L$ can be transformed into a Jordan form

$$J = U^{-1}LU = diag \{0, J_2, \ldots, J_r\}$$

where $U^{-1} = [\sqrt{N}\bar{\xi}, w_2, \ldots, w_N]^{T}$ and $\xi^{T}L = 0$, $\xi$ is a vector such that $1_{N}^{T}\xi = 1$.

When the graph $G$ is undirected, the Laplacian matrix $L$ is symmetric and can be diagonalized.

A. NETWORKED EULER-LAGRANGE SYSTEMS

Consider $N$ networked Euler-Lagrange systems that are fully actuated and have the following dynamics:

$$M_i(q_i) \ddot{q}_i + C_i(q_i, \dot{q}_i) \dot{q}_i + g_i(q_i) = \tau_i, \quad i = 1, 2, \ldots, N$$

where $q_i = [q_{i1}, q_{i2}, \ldots, q_{im}]^{T} \in \mathbb{R}^m$ is the generalized position, $M_i(q_i) = M_i^{T}(q_i) \in \mathbb{R}^{m \times m}$ is the inertia matrix, $C_i(q_i, \dot{q}_i) \in \mathbb{R}^{m \times m}$ is the Coriolis and centrifugal matrix, $g_i(q_i) \in \mathbb{R}^m$ is the gravitational torque, $\tau_i \in \mathbb{R}^m$ is the control input, and the following general assumptions hold for the Euler-Lagrange system (1):

1) There exist positive-definite parameters $k_c$ and $k_d$ such that $0 < k_c I_m \leq M_i(q_i) \leq k_d I_m$.

2) $M_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew-symmetric, i.e. for any $r \in \mathbb{R}^m$, $r^{T}(M_i(q_i) - 2C_i(q_i, \dot{q}_i)) r = 0$.

3) The dynamics (1) is linear in terms of the constant parameter $\theta_i$:

$$M_i(q_i) \ddot{\theta}_i + C_i(q_i, \dot{q}_i) \dot{\theta}_i + g_i(q_i) = Y_i(q_i, \dot{q}_i, \theta_i, \dot{\theta}_i)$$

where $Y_i(q_i, \dot{q}_i, \theta_i, \dot{\theta}_i)$ is the regressor matrix.

B. SAMPLED-DATA CONSENSUS

The networked Euler-Lagrange systems in (1) are sampled at $t_k, k = 0, 1, \ldots$, where $0 = t_0 < t_1 < \ldots < t_k < \ldots$, and $t_k \to \infty$ as $k \to \infty$. The sampling intervals can be
time-varying: $h_k = t_{k+1} - t_k$. The control objective is to design sampled-data consensus controllers that drive the networked Euler-Lagrange systems in (1) to achieve consensus, i.e., $\forall i, j \in \mathcal{V}$:

$$\lim_{t \to \infty} (q_i(t) - q_j(t)) = 0$$

The following sampled-data consensus control law is often used with continuous communication:

$$\begin{align*}
\tau_i &= -K_i s_i + Y_i (\dot{q}_i, \dot{\dot{q}}_i, \dot{\dot{q}}_r, \dot{\dot{q}}_r) \hat{\theta}_i \\
\dot{s}_i &= -\Gamma_i Y_i^T (\dot{q}_i, \dot{q}_i, \dot{q}_r, \dot{q}_r) s_i
\end{align*}$$

(2)

where $K_i$ is a positive-definite matrix. Eq. (2) may cause the problem of infinite controller output mentioned in Section I under the reference velocity with ZOH

$$\dot{q}_{r,i}(t) = -\rho \sum_{j \in \mathcal{N}_i} a_{ij} (q_i(t_k) - q_j(t_k)), t \in [t_k, t_{k+1})$$

(3)

Differentiable pulse function was designed to solve this problem, let $\alpha(t, t_k)$ be a pulse function:

$$\alpha(t, t_k) = \begin{cases} 
\tilde{\alpha}(t, t_k), & t \in [t_k, t_{k+1}] \\
0, & t \in (t_k + d_k, t_{k+1}] 
\end{cases}$$

(4)

where $\tilde{\alpha}(t, t_k)$ is the differentiable pulse function satisfying $\tilde{\alpha}(t_k, t_{k+1}) = \tilde{\alpha}(t_k + d_k, t_{k+1}) = 0$ and $\lim_{t \to t_k^+} \tilde{\alpha}(t, t_k) = 0$, and the reference velocity be:

$$\dot{q}_{r,i}(t) = -\rho \alpha(t, t_k) \sum_{j \in \mathcal{N}_i} a_{ij} (q_i(t_k) - q_j(t_k)), t \in [t_k, t_{k+1})$$

(5)

It is proved that the networked Euler-Lagrange systems (1) under the adaptive control law (2) and (5) reach consensus when a criterion on the communication graph and the pulse function $\alpha(t, t_k)$ is satisfied.

**C. PROBLEM FORMULATION**

The main disadvantage of the differentiable pulse function is that it is intermittent, i.e., the reference generalized velocities to be followed are intermittent. Apparently, this may not be desirable in certain cases like vehicles.

The objective of this work is to design an alternative scheme to ZOH for the consensus of networked Euler-Lagrange systems that meets the following two requirements:

1) Like the differentiable pulse function, it generates differentiable reference velocity (reference velocity).

2) Unlike the differentiable pulse function, the reference velocity it generates is gapless (no forced zero at sampling instants).

**III. MAIN RESULTS**

To solve the problem formulated in Section II, we propose an improved alternative to the ZOH that generates gapless differentiable reference velocity.

**A. A GAPLESS SOLUTION**

The straightforward intuitional way is a modification to the ZOH. The difference lies in that when a new sampled information arrives, the held value gradually transitions to the newly received value rather than instantly jumps to it. This transition implies that the information from the previous sampled data is involved therein. Next, we formulate the transition process.
We borrow from the concept of dwell time in (4) for the transition phase. Let \( z(t,x) \) be the interpreted value

\[
z(t,x) = \begin{cases} \hat{z}(t,t_k) x(t_k) \\ + (1 - \hat{z}(t,t_k)) x(t_{k-1}), & t \in [t_k, t_k + d_k] \\ x(t_k), & t \in (t_k + d_k, t_{k+1}] \end{cases} ,
\]

where \( \hat{z} \in [0,1] \) is a differentiable function that satisfies the initial and terminal value condition

\[
\hat{z}(t_k, t_k) = 0, \quad \hat{z}(t_k, t_k + d_k) = 1
\]

and derivative condition

\[
\lim_{t \to t_k^+} \hat{z}(t_k) = \lim_{t \to t_k + d_k} \hat{z}(t_k) = 0
\]

According to (6), (7) and (8), \( z(t,x(t_k), x(t_{k-1})) \) is continuous and differentiable on \([t_0, \infty)\). Redefine the reference velocity using the interpretation above,

\[
\dot{q}_{r,i}(t) = z\left(t, -\rho \sum_{j \in N_i} a_{ij} (q_i(t) - q_j(t))\right)
\]

and by applying the new interpretation, the reference velocity is differentiable and no longer has the infinity problem.

Applying controller (2) with interpretation (9) to the Euler-Lagrange system (1), Choose the Lyapunov function:

\[
V(t) = \frac{1}{2} \sum_{i=1}^{N} s_i^T(t) M_i(q_i) s_i(t) + \hat{\theta}_i^T \Gamma_i \hat{\theta}_i
\]

Its derivative is

\[
\dot{V}(t) = - \sum_{i=1}^{N} s_i^T \kappa_i s_i < 0
\]

Note here that inequality (12) holds on all \( t \geq 0 \), instead of just sampling intervals. This is attributed to the controller design (2) and the subsequent closed-loop dynamics (10). Therefore, \( s_i \to 0 \) as \( t \to \infty \). Combine (3) with the auxiliary variable \( s \) yields

\[
\dot{q}_i = s_i + \dot{q}_{r,i} = s_i + z\left(t, -\rho \sum_{j \in N_i} a_{ij} (q_i(t) - q_j(t))\right)
\]

Equation (12) can be represented in stack vector form:

\[
\dot{\hat{q}} = \dot{q}_r + s = -\rho \alpha(t) (L \otimes I_m) \times (\hat{z}(t,t_k) q(t_k) + (1 - \hat{z}(t,t_k)) q(t_{k-1})) + s,
\]

\[
t \in [t_k, t_k + d_k]
\]

\[
\dot{\hat{q}} = \dot{q}_r + s = -\rho \alpha(t) (L \otimes I_m) q(t_k) + s,
\]

\[
t \in (t_k + d_k, t_{k+1}]
\]

where \( q \) and \( s \) are the stacked vectors for \( q_i \) and \( s_i \), respectively. Since \( s \) vanishes with time, the stability of the dynamic equation (13) is equivalent to that of

\[
\dot{\hat{q}} = \begin{cases} -\rho (L \otimes I_m) \times (\hat{z}(t,t_k) q(t_k) \\ + (1 - \hat{z}(t,t_k)) q(t_{k-1})) , & t \in [t_k, t_k + d_k] \\ -\rho (L \otimes I_m) q(t_k), & t \in (t_k + d_k, t_{k+1}] \end{cases}
\]

Next, the consensus of system (14) is studied, and a consensus criterion is obtained.

B. CONSENSUS ANALYSIS

In the following theorem, the consensus of the system (14) is studied, and a consensus criterion compatible with the new pulse function \( \alpha \) is proposed.

Theorem 1: The multi-agent system composed of Euler–Lagrange systems in (1) with the control inputs (2) and (6) can reach consensus if the following inequality holds for \( l = 2, 3, \ldots, r, k = 0, 1, \ldots \) and \( h = 1, 2 \)

\[
|\mu_{l,h}| < 1
\]

where \( \mu_{l,h} \) is the solution of the quadratic equation about \( \mu \):

\[
\mu^2 - \mu (l - \rho (a_k + t_{k+1} - t_k - d_k) \lambda_l) + \rho (d_k - a_k) \lambda_l = 0
\]

Proof: Solve the dynamic equation (14), one has

\[
q(t) = q(t_k) + \int_{t_k}^{t} \dot{q}() d\tau
\]
and 

\[ q(t_{k+1}) = q(t_k) - \rho(L \otimes I_m) q(t_k) \]

\[ \times \left( \int_{t_k}^{t_k+d_k} \hat{z}(\tau, t_k) \, d\tau + \int_{t_k+d_k}^{t_{k+1}} d\tau \right) \]

\[ - \rho(L \otimes I_m) q(t_{k-1}) \int_{t_k}^{t_k+d_k} (1 - \hat{z}(\tau, t_k)) \, d\tau \]

(17)

Eq. (17) gives the evolution of the system, it shows that the state at one step depends on its preceding two steps. To simplify, define

\[ \int_{t_k}^{t_k+d_k} \hat{z}(\tau, t_k) \, d\tau = a_k \]

then, Eq. (17) becomes

\[ q(t_{k+1}) = q(t_k) - \rho(L \otimes I_m) [(a_k + t_{k+1} - t_k - d_k) q(t_k) + (d_k - a_k) q(t_{k-1})] \]

and this dynamic can be written as

\[
\begin{bmatrix}
q(t_k) \\
q(t_{k+1})
\end{bmatrix}
= \left[ \begin{bmatrix}
I_N \\
L (1 - I_N \xi^T) \otimes I_m
\end{bmatrix}
- \rho(a_k + t_{k+1} - t_k - d_k) L
\right]
\times \left[ \begin{bmatrix}
q(t_k) \\
q(t_{k-1})
\end{bmatrix}
\right]
\]

Let \( \hat{q}_1 = q_1 - (\xi^T \otimes I_m) q \), and \( \hat{q} = (\hat{q}_1^T, \hat{q}_2^T, \ldots, \hat{q}_r^T)^T \). Then we have \( \hat{q} = (I_N - 1_N \xi^T) \otimes I_m) q \). Applying this transformation on (17) yields

\[ \hat{q}(t_{k+1}) = \left( I - 1_N \xi^T \otimes I_m \right) \hat{q}(t_k) \]

\[ \times \left[ \begin{bmatrix}
I_N \\
L \left( I - 1_N \xi^T \right) \otimes I_m
\end{bmatrix}
- \rho(a_k + t_{k+1} - t_k - d_k) L
\right]
\times \left[ \begin{bmatrix}
q(t_k) \\
q(t_{k-1})
\end{bmatrix}
\right]
\]

\[ \hat{q}(t_k) - \rho(a_k + t_{k+1} - t_k - d_k) \left( L \otimes I_m \right) \hat{q}(t_{k-1}) \]

\[ \hat{q}(t_k) - \rho(a_k + t_{k+1} - t_k - d_k) \left( L \otimes I_m \right) \hat{q}(t_{k-1}) \]

\[ \hat{q}(t_k) - \rho(a_k + t_{k+1} - t_k - d_k) \left( L \otimes I_m \right) \hat{q}(t_{k-1}) \]

(18)

The facts that \( L1N = 0 \) and \( \xi^TL = 0 \) are exploited to obtain (18). Therefore,

\[
\begin{bmatrix}
\hat{q}(t_k) \\
\hat{q}(t_{k+1})
\end{bmatrix}
= \left[ \begin{bmatrix}
I \\
0
\end{bmatrix}
- \rho(a_k + t_{k+1} - t_k - d_k) \left( L \otimes I_m \right)
\right]
\times \left[ \begin{bmatrix}
q(t_k) \\
q(t_{k-1})
\end{bmatrix}
\right]
\]

\[ \times \left( \begin{bmatrix}
I \\
0
\end{bmatrix}
- \rho(a_k + t_{k+1} - t_k - d_k) L
\right)
\otimes I_m
\]

\[ \hat{y}(t_{k+1}) = \hat{y}(t_k) - \rho(a_k + t_{k+1} - t_k - d_k) \left( J \otimes I_m \right) \hat{y}(t_{k-1}) \]

\[ \hat{y}(t_k) - \rho(a_k + t_{k+1} - t_k - d_k) \left( J \otimes I_m \right) \hat{y}(t_{k-1}) \]

(18)

Dividing the \( n - 1 \) components of \( \hat{y} \) into \( \hat{y} = \left( \hat{y}_2, \hat{y}_3, \ldots, \hat{y}_r \right)^T \) according to the Jordan blocks, (18) gets decoupled into

\[ \left[ \begin{array}{c}
\hat{y}_1(t_{k+1}) \\
\hat{y}_2(t_{k+1}) \\
\vdots \\
\hat{y}_r(t_{k+1})
\end{array} \right]
= \left( \begin{bmatrix}
I \\
0
\end{bmatrix}
- \rho(a_k + t_{k+1} - t_k - d_k) \left( J \otimes I_m \right)
\right)
\times \left[ \begin{array}{c}
\hat{y}_1(t_k) \\
\hat{y}_2(t_k) \\
\vdots \\
\hat{y}_r(t_k)
\end{array} \right]
\]

\[ \times \left( \begin{bmatrix}
I \\
0
\end{bmatrix}
- \rho(a_k + t_{k+1} - t_k - d_k) L
\right)
\otimes I_m
\]

\[ \times \left[ \begin{bmatrix}
I \\
0
\end{bmatrix}
- \rho(a_k + t_{k+1} - t_k - d_k) \left( J \otimes I_m \right)
\right]
\times \left[ \begin{array}{c}
\hat{y}_1(t_k) \\
\hat{y}_2(t_k) \\
\vdots \\
\hat{y}_r(t_k)
\end{array} \right]
\]

\[ \times \left( \begin{bmatrix}
I \\
0
\end{bmatrix}
- \rho(a_k + t_{k+1} - t_k - d_k) L
\right)
\otimes I_m
\]

\[ \times \left[ \begin{bmatrix}
I \\
0
\end{bmatrix}
- \rho(a_k + t_{k+1} - t_k - d_k) \left( J \otimes I_m \right)
\right]
\times \left[ \begin{array}{c}
\hat{y}_1(t_k) \\
\hat{y}_2(t_k) \\
\vdots \\
\hat{y}_r(t_k)
\end{array} \right]
\]

Let \( M = \left[ \begin{bmatrix}
I \\
0
\end{bmatrix}
- \rho(a_k + t_{k+1} - t_k - d_k) \left( J \otimes I_m \right)
\right] \), then its eigenvalues can be obtained by solving for \( \mu \) in

\[ \mu I - \left( I - \rho(a_k + t_{k+1} - t_k - d_k) \left( J \otimes I_m \right) \right) \hat{J} \hat{J} \hat{J} \hat{J} = 0 \]
By Schur’s complement, that’s equivalent to
\[ |\mu I| \begin{vmatrix} \mu I - I + \rho (a_k + t_{k+1} - t_k - d_k) \hat{J}_i \\ - \rho (d_k - a_k) \hat{J}_i (\mu I)^{-1} (-I) \end{vmatrix} = 0 \]

Simplify the above equation:
\[ \mu^2 I - \mu \left( I - \rho (a_k + t_{k+1} - t_k - d_k) \hat{J}_i \right) + \rho (d_k - a_k) \hat{J}_i = 0 \]

Note that \( \mu^2 I - \mu \left( I - \rho (a_k + t_{k+1} - t_k - d_k) \hat{J}_i \right) + \rho (d_k - a_k) \hat{J}_i \) is upper-diagonal, it further reduces to
\[ \mu^2 - \mu (I - \rho (a_k + t_{k+1} - t_k - d_k) \lambda_i) + \rho (d_k - a_k) \lambda_i = 0 \]

Since matrix \( U \) is nonsingular, it is well established that \( \dot{q}(k) \) converges to zero if and only if \( \dot{y} \) converges to zero, i.e. the solutions of (20) for \( l = 2, \ldots, r \) and \( h = 1, 2 \) satisfy \( |\mu|<1 \). So it can be concluded that the closed-loop Euler–Lagrange systems can reach consensus if (16) holds.

Remark 1: With the consensus settled, there’s another detail worth mentioning. The reference acceleration \( \ddot{q}_{r,i} \) in the control input \( \tau_1 \) in (2, 5) is obtained through differentiation of \( \dot{q}_{r,i} \). Such an operation in real-world applications is usually carried out by a practical differentiator since ideal differentiators don’t exist physically. With the scaling function present, there’s no need for a differentiator anymore. Since in (5) the \( \sum_{j\in N_i} a_{ij} (q_i (t_k) - q_j (t_k)) \) part is constant so that we can use \( \dot{q}_{r,i} = -\rho \dot{a} (t, t_k) \sum_{j\in N_i} a_{ij} (q_i (t_k) - q_j (t_k)) \) instead with \( \dot{a} (t, t_k) \) being pre-determined.

Remark 2: Although the high-order filtering method can be modified and adjusted for the sampled-data communication studied here, they have to rely on dedicated analog components to achieve the previously mentioned filtering and differentiation. In contrast, the scaling function method needs no analog components and fits well with digital systems, and therefore is easier to implement.

Remark 3: Usually, gravitational torques and friction forces are considered in the study of Euler-Lagrange systems [21], [23] to better reflect real physical systems. They are omitted in this work since the focus here is to solve the infinity problem.

IV. NUMERICAL EXAMPLES

In this section, we first illustrate the infinities in the control inputs under conventional sampled-data control, and the intermittent nature of generalized velocity under the pulse modulated control. Then present a simulation for the differentiable scaling function that eliminates both the infinities and the gaps of zero generalized velocity.

To clearly show how the infinities occur in networked Euler-Lagrange systems and the reason behind it, we choose a very simple example of 2 agents, each with 2 degrees of freedom and a large sampling period of 0.5s. According to the control law (2):
\[ \tau_i = -K_i s_i + M_i (q_i) \ddot{q}_{r,i} + C_i (q_i, \dot{q}_i) \dot{q}_{r,i} \]
the local consensus error \( \sum_{j\in N_i} a_{ij} (q_i (t_k) - q_j (t_k)) \) widely used in consensus algorithms appears in \( \dot{q}_{r,i} \), i.e., the reference velocity:
\[ \dot{q}_{r,i} = -\rho \sum_{j\in N_i} a_{ij} (q_i (t_k) - q_j (t_k)), \quad t \in [t_k, t_{k+1}) \]

Note here the local and neighbor states \( q_i \) and \( q_j \) are sampled at the sampling instants \( t_k \), meaning their values only change at \( t_k \), and remains constant during the sampling intervals. The problem arises in the calculation of \( \ddot{q}_{r,i} \) which is instrumental for the control of Euler-Lagrange systems. For \( \ddot{q}_{r,i} \), the time derivative of the piece-wise constant \( \ddot{q}_{r,i} \) is zero during the sampling intervals and infinite at the sampling instants \( t_k \).

From the networked Euler-Lagrange system’s perspective in real-time, during the sampling interval, the reference velocity \( \ddot{q}_{r,i} = -\rho \sum_{j\in N_i} a_{ij} (q_i (t_k) - q_j (t_k)) \) uses the states at the newest sampling instant \( q_i (t_k) \) and \( q_j (t_k) \)’s, thus \( \ddot{q}_{r,i} \) is held constant, and its derivative \( \ddot{q}_{r,i} \) is zero. But when the time arrives at a sampling instant, namely \( t_{k+1} \), the agent receives the new states \( q_i (t_{k+1}) \) and \( q_j (t_{k+1}) \)’s and update \( \ddot{q}_{r,i} \) accordingly, making \( \ddot{q}_{r,i} \) discontinuous at this moment. Consequently, \( \ddot{q}_{r,i} \) becomes infinite by differentiating this discontinuity.

The simulation step size is chosen to be 0.00001 for finer differentiation to show the infinities. As predicted in the above analysis, the reference acceleration \( \ddot{q}_{r,i} \) approaches infinity at the sampling instants, and the control input \( \tau_1 \) inherits this property for including \( \ddot{q}_{r,i} \) in its composition.

Consider a network of five 2-DOF robots, and the interaction graph among the five robots is shown in the following figure.

The sampling period is chosen as 1s. The initial generalized positions of the robots are chosen as \( q_1 (0) = \left[ \frac{\pi}{3}, \frac{\pi}{3} \right]^T \), \( q_1 (0) = \left[ -\frac{\pi}{3}, \frac{\pi}{3} \right]^T \), \( q_1 (0) = \left[ -\frac{\pi}{3}, -\frac{\pi}{3} \right]^T \), \( q_1 (0) = \left[ \frac{\pi}{3}, -\frac{\pi}{3} \right]^T \), \( q_1 (0) = \left[ -\frac{2\pi}{3}, -\frac{2\pi}{3} \right]^T \), and the initial velocities
are all set as 0. As is shown in the figures, the control input $\tau_1$ becomes impulses at sampling instants due to the derivation of the reference velocity $\dot{q}_{r,1}$.

The differentiable pulse function eliminates impulses in control inputs under sampled-data control, see Fig. 10(a), yet it causes another difficulty: intermittent generalized velocities, see Fig. 10(b). Consequently, the change of positions happen only on dwell times as shown in Fig. 8(c), which is obviously undesirable in applications that require high speed.

Next, in the differentiable scaling function, the length of the transition phase is chosen as $d_k = 0.3, k = 1, 2, \ldots$. The transition function is designed as

$$ z = \tilde{z}(t, t_k) x(t_k) + \left(1 - \tilde{z}(t, t_k)\right) x(t_{k-1}), \quad t \in [t_k, t_{k} + d_k] $$

where $\tilde{z}(t, t_k) = \frac{1}{2} + \cos \left(\frac{t - t_k}{d_k} \pi\right)$.
Sub-figures (a) of Fig. 11 show that the control torques are larger during the transition phases, i.e., exactly when the reference velocities change, and are smaller during the non-transition time. Accordingly, the referenced velocities change during the transition phases and keep relatively unaffected out of the transition phases. The proposed differentiable scaling function not only eliminates infinities in the control input but also avoids gaps of zero generalized velocity, thus is able to achieve faster convergence.

V. CONCLUSION

In this paper, we have investigated the consensus of networked Euler-Lagrange systems over sampled-data communication. The infinity problem that hinders the implementation of sampled-data Euler-Lagrange systems is reviewed, and the cause for it is analyzed and verified with simulation. To solve the infinity problem and avoid intermittent velocity for smoother and faster consensus, a new controller with specially designed differentiable scaling functions is designed. It is proved that when a criterion on the scaling function is satisfied, the networked Euler-Lagrange systems can reach consensus. Finally, numerical simulations are given to verify the theoretical results.

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