Geisser, Thomas H.; Morin, Baptiste  
On the kernel of the Brauer-Manin pairing.  
J. Number Theory 238, 444-463 (2022)  

Summary: Let $X$ be a regular scheme, flat and proper over the ring of integers of a $p$-adic field, with generic fiber $X$ and special fiber $X_s$. We study the left kernel $\text{Br}(X)$ of the Brauer-Manin pairing $\text{Br}(X) \times \text{CH}_0(X) \to \mathbb{Q}/\mathbb{Z}$. Our main result is that the kernel of the reduction map $\text{Br}(X) \to \text{Br}(X_s)$ is the direct sum of $(\mathbb{Q}/\mathbb{Z})_s \oplus (\mathbb{Q}/\mathbb{Z})^t$ and a finite $p$-group, where $s + t = \rho_X - \rho_X - I + 1$, for $\rho_X$ and $\rho_X$ the Picard numbers of $X_s$ and $X$, and $I$ the number of irreducible components of $X_s$. Moreover, we show that $t > 0$ implies $s > 0$.

MSC:
- 14F22 Brauer groups of schemes
- 14F40 de Rham cohomology and algebraic geometry
- 14F42 Motivic cohomology; motivic homotopy theory
- 11G40 $L$-functions of varieties over global fields; Birch-Swinnerton-Dyer conjecture

Keywords:
- Brauer group
- Brauer Manin pairing
- local fields
- Picard number

Full Text: DOI

References:
[1] Berthelot, P.; Ogus, A., F-isocrystals and de Rham cohomology. I, Invent. Math., 72, 2, 159-199 (1983)
[2] Colliot-Thélène, J. L.; Saito, S., Zéro-cycles sur les variétés p-adiques et groupe de Brauer, Int. Math. Res. Not., 4, 151-160 (1996)
[3] M. Flach, D. Siebel, Special values of the zeta function of an arithmetic scheme, Preprint 2019.
[4] de Jong, A. J., A result of Gabber
[5] Deligne, P., Théorème de Lefschetz et critères de dégénérescence de suites spectrales, Publ. Math. I.H.E.S., 35, 1, 107-126 (1968)
[6] Grothendieck, A., Éléments de géométrie algébrique. III. Étude cohomologique des faisceaux cohérents. I, Publ. Math. Inst. Hautes Études Sci., 11 (1961), 167 pp
[7] Grothendieck, A., Le groupe de Brauer. III. Exemples et compléments, (Dix Exposés sur la Cohomologie des Schémas. Dix Exposés sur la Cohomologie des Schémas, Adv. Stud. Pure Math., vol. 3 (1968), North-Holland; North-Holland Amsterdam), 88-188
[8] Jannsen, U., Continuous étale cohomology, Math. Ann., 280, 2, 207-245 (1988)
[9] Jannsen, U., Continuous étale cohomology, Math. Ann., 280, 2, 207-245 (1988)
[10] Jensen, C. U., Les foncteurs dérivés de lim et leurs applications en théorie des modules, Lecture Notes in Mathematics, vol. 254 (1972), Springer-Verlag; Springer-Verlag Berlin-New York, iv + 103 pp
[11] Kim, W.; Madapusi Pera, K., 2-adic integral canonical models, Forum Math. Sigma, 4, Article e28 pp. (2016)
[12] Lichtenbaum, S., Duality theorems for curves over $p$-adic fields, Invent. Math., 7, 120-136 (1969)
[13] Madapusi Pera, K., The Tate conjecture for K3 surfaces in odd characteristic, Invent. Math., 201, 2, 625-668 (2015)
[14] Mattuck, A., Abelian varieties over $p$-adic ground fields, Ann. Math. (2), 62, 92-119 (1955)
[15] Roquette, P., Einheiten und Divisorsklassen in Endlich Erzeugbaren Körperrn, Jber. Deutsch. Math.-Verein., 60, Abt. 1, 1-21 (1957)
[16] Rotman, J., An Introduction to the Theory of Groups, Graduate Texts in Mathematics, vol. 148 (1995), Springer-Verlag; Springer-Verlag New York
[17] Saito, S.; Sato, K., Zero-cycles on varieties over $p$-adic fields and Brauer groups, Ann. Sci. Éc. Norm. Supér. (4), 47, 3, 505-537 (2014)
[18] Tate, J., Endomorphisms of Abelian varieties over finite fields, Invent. Math., 2, 134-144 (1966)
[19] The L-functions and modular forms database

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.