ON THE DYNAMICS OF ULTRA COMPACT X-RAY BINARIES: 4U 1850-087, 4U 0513-40, AND M15 X-2

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ABSTRACT

In this work, we extend our dynamical study of ultra compact X-ray binaries (UCXB) 4U 1820-30 from Prodan & Murray to three more UCXBs in globular clusters: 4U 1850-087, 4U 0513-40, and M15 X-2. These three UCXBs have orbital periods ≤ 20 minutes. Two of them, 4U 1850-087 and 4U 0513-40, have suspected luminosity variations of the order of ~1 yr. There is insufficient observational data to make any statements regarding the long periodicity in the light curve of M15 X-2 at this point. The properties of these three systems are quite similar to 4U 1820-30, which prompt us to model their dynamics in the same manner. As in the case of 4U 1820-30, we interpret the suspected long periods as the period of small oscillations around a stable fixed point in the Kozai resonance. We provide a lower limit on the tidal dissipation factor $Q$, which is in agreement with results obtained for the case of 4U 1820-30.

Key words: binaries: close – celestial mechanics – stars: individual (4U1820-30, 4U 1850-087, 4U 0513-40, M15 X-2) – stars: kinematics and dynamics

1. INTRODUCTION

The possible existence of long-period (~100 days) variations in the luminosity of ultra compact X-ray binaries (UCXBs) with orbital periods ≤ 30 minutes raises the possibility that the binary is orbited by a third body. The ratio between the orbital period and the period of the luminosity variations is too large to be due to any kind of accretion disk precession or change in the viewing angle (Larwood 1998; Wijers & Pringle 1999). These long-period variations in luminosity may be due to the actual change in the mass transfer rate of the binary. The presence of a third body may induce periodic oscillations in the eccentricity of the inner binary, which in turn will cause variations of the mass transfer rate, with the same period. Following Prodan & Murray (2012), we show that this long period can be explained as the period of libration around the stable fixed point deep in a Kozai resonance. Since the expansion timescale of the inner binary is on the order of that of the accretion (~107 yr), the action is indeed an adiabatic invariant, and as we demonstrated in detail in Prodan & Murray (2012). The resonant trapping in libration around the fixed point in a Kozai resonance is a natural consequence. In contrast, tidal dissipation shrinks the semimajor axis. If tidal dissipation plays a primary role in how binary separation evolves, resonant trapping is no longer possible.

1.1. 4U 1850-087

4U 1850-087 is a UCXB located in the galactic globular cluster NGC 6712. The distance to the cluster is 6.8 kpc (Peterson & King 1975; Harris 1996). It was first detected as an X-ray burster by Swank et al. (1976), which immediately indicates that the primary is a neutron star. The cluster center and the source are separated by $6\degree \pm 6\degree$ (0.1 ± 0.1 core radii; Hertz & Grindlay 1983). This system has been observed since the very beginning of observational X-ray astronomy, with many detections. It exhibits order of magnitude flux variations (Forman et al. 1978; Hoffman et al. 1980; Warwick et al. 1981; Priedhorsky & Terrell 1984; Wood et al. 1984; Warwick et al. 1988; Kitamoto et al. 1992; Christian & Swank 1997; Juett et al. 2001). Homer et al. (1996) reported a low-amplitude periodicity for the ultraviolet counterpart of this source, where their periodogram can be equally well fitted with a sinusoidal modulation at either of the two suspected orbital periods of 20.6 or 13.2 minutes. However, as Homer et al. (1996) point out, a period as short as 13.2 minutes is very close to the 11 minutes period observed in 4U 1820-30, which is a much more luminous source. Since the larger luminosity corresponds to higher mass transfer rate, 4U 1850-087 would be underluminous by a factor of ~100 for a 13.2 minutes binary (Homer et al. 1996; Rappaport et al. 1987). Hence, 20.6 minutes is interpreted as an orbital period even though it has yet to be confirmed. Such a short period implies that a mass-losing companion has to be a degenerate and low-mass star. Following the consideration of 4U 1820-30 from Rappaport et al. (1987), Homer et al. (1996) derive the mass and the radius of the secondary to be 0.04 $M_\odot$ and 0.04 $R_\odot$, respectively, under the assumption that the secondary is a fully degenerate helium white dwarf. Additionally, assuming a low-mass white dwarf donor and mass transfer driven by gravitational radiation, Homer et al. (1996) showed that the X-ray luminosity of this system is that expected for a period of 20.6 minutes.

An interesting feature of 4U 1850-087 is a possible long period of 0.72 yr reported by Priedhorsky & Terrell (1984), at which luminosity varies by a factor of two to three. Even though, as we show further in the paper, this long period fits well in the dynamical picture given by our model, it needs to be further verified observationally.

1.2. 4U 0513-40

4U 0513-40 is a low-mass X-ray binary in the globular cluster NGC 1851. The distance to the source is 12 kpc (Harris 1996). Far ultraviolet photometry obtained by the Hubble Space Telescope (HST) revealed a 17 minute orbital modulation (Zurek et al. 2009). Observations with BeppoSAX, Chandra, XMM-Newton, and INTEGRAL confirmed the 17 minute...
periodic sinusoidal signal in the soft X-ray (Fiocchi et al. 2011). The system is known to be an X-ray burster (Galloway et al. 2008; Homer et al. 2001; Fiocchi et al. 2011), which indicates that the primary is a neutron star. The short orbital period suggests a low-mass white dwarf secondary of ~0.05 M⊙, as implied from mass–radius relation for 17 min period binaries from Deloye & Bildsten (2003).

4U 0513-40 shows very interesting variability in the X-ray flux on two different timescales (Maccarone et al. 2010); a factor of ~10 variation on timescales of weeks and the variation of a factor of ~2 in the luminosity when averaged over 1 yr (Maccarone et al. 2010). This long timescale variation points toward a modulation in the mass transfer rate, even though the mean luminosity agrees with that predicted by a gravitation-radiation-driven evolutionary scenario.

1.3. M15 X-2

M15, at a distance of 10.4 kpc (Harris 1996), is the only globular cluster associated with our galaxy known to house two low-mass X-ray binaries (LMBXs). In early X-ray studies, a single source 4U 2127+119 was first identified with the optical counterpart AC 211 by Auriere et al. (1984) and further confirmed by a spectroscopic study by Charles et al. (1986) showing signatures of a low-mass X-ray binary. A modulation in the optical and the X-ray flux revealed an orbital period of 17.1 hr (Ilovaisky et al. 1993, and references within). AC 211 is among the brightest LMBXs in the optical and at the same time it has a low X-ray luminosity ~10^36 erg s^−1; the high optical-to-X-ray luminosity ratio implied that a very luminous central X-ray source is hidden behind the accretion disk (Auriere et al. 1984). However, when X-ray bursts were detected by the Ginga satellite (Dotani et al. 1990) and later on with the Rossi X-ray Timing Explorer (Smale 2001), this conclusion was highly dubious. This puzzling behavior was finally understood when Chandra observations resolved 4U 2127-119 into two X-ray sources (White & Angelini 2001). One source is, of course, already known, LMBX AC 211, while the second one, M15 X-2, which is the one producing X-ray bursts, is 2.5 times brighter in X-rays than AC 211. The source is located 3’/4 from the center of M15. The optical and the FUV counterparts of M15 X-2 were identified in HST data by White & Angelini (2001) and Dieball et al. (2005), respectively, resulting in a determination of the orbital period of 22.6 minutes by Dieball et al. (2005). The donor corresponding to such a short period is a white dwarf of mass 0.02 M⊙ ≤ M2,min ≤ 0.03 M⊙ and of radius 0.02 R⊙ ≤ R2,min ≤ 0.03 R⊙. The existence of the X-ray bursts is consistent with neutron star primary (Dieball et al. 2005).

So far no long-period luminosity variations have been reported in M15 X-2. In Section 3, we discuss a constraint on the shortest expected long period assuming the conservative mass transfer given by our model.

1.4. Plan of the Paper

In this paper, we apply the dynamical model described in Prodan & Murray (2012) on the three known UCXBs in the globular clusters just described: 4U 1850-087, 4U 0513-40, and M15 X-2. We demonstrate that the suspected long periods of 4U 1850-087 and 4U 0513-40 can be explained as libration in the Kozai resonance with the period of small oscillations around the fixed point. Our model gives a prediction for a yet undetected long period of M15 X-2. As shown in Prodan & Murray (2012), the dynamical history of these systems is a consequence of the interplay of two effects. The mass transfer via Roche lobe overflow drives the systems into a resonance, tidal dissipation tends to damp the mutual inclination close to the Kozai critical inclination. The interplay between these two effects allows us to infer a constraint on the tidal dissipation factor Q for white dwarf donors in these systems. In Section 2, we give a review of the dynamical model developed in Prodan & Murray (2012) and estimates for the systems’ parameters. The numerical results are presented in Section 3. In Section 4, we constrain the ratio of the tidal dissipation factor and the tidal Love number, Q/k2 of the white dwarf donors. We end with a discussion in Section 5.

2. OVERVIEW OF OUR DYNAMICAL MODEL

In the work of Prodan & Murray (2012) on the dynamics of 4U 1820-30, we argue that the observed long-period modulation of the luminosity (~170 day) is caused by the presence of a third body orbiting the center of mass of the binary. Variations in the eccentricity of the inner binary are associated with libration around the stable fixed point deep in the Kozai resonance. Kozai resonance is 1 : 1 resonance between the precession rate of the longitude of periastron σ and the precession rate of the longitude of the ascending node Ω of the inner binary. The condition for Kozai resonance is satisfied only in cases where the mutual inclination is above its critical value of 39°2. Taking into account the presence of additional precessions, we demonstrate that the luminosity modulation arises from the these eccentricity variations. The additional precessions are due to tidal and rotation distortion of the secondary, tidal dissipation and apsidal precession due to general relativistic effects (GR). Here we list the equations for all four precession rates:

\[ \dot{\Omega}_{\text{Kozai}} = \frac{3}{4} n \left( \frac{m_2}{m_1 + m_2} \right) \left( \frac{a_{\text{out}}}{a} \right)^3 \frac{1}{(1 - e^2)^{3/2}} \times \frac{1}{\sqrt{1 - e^2}} \times [2(1 - e^2) + 5 \sin^2 \omega(e^2 - \sin^2 i)] \] (1)

\[ \dot{\omega}_{\text{SR}} = \frac{3}{2} n \left( \frac{m_1 + m_2}{m_1} \right) \left( \frac{r_2}{a} \right) \frac{1}{(1 - e^2)} \] (2)

\[ \dot{\omega}_{\text{TB}} = \frac{15}{16} n k_2 \left( \frac{m_2}{m_1} \right) \left( \frac{R_2}{a} \right)^5 \times \frac{1}{(1 - e^2)^2} \times [2\Omega_e^2 - \dot{\Omega}_e^2 - \dot{\Omega}_q^2 + 2\dot{\Omega}_e \cot i(\dot{\Omega}_e \sin \omega + \dot{\Omega}_q \cos \omega)]] \] (3)

\[ \dot{\omega}_{\text{RB}} = \frac{n k_2}{4} \left( \frac{m_1 + m_2}{m_2} \right) \left( \frac{R_2}{a} \right)^5 \times \frac{1}{(1 - e^2)^2} \times [2\Omega_e^2 - \dot{\Omega}_e^2 - \dot{\Omega}_q^2 + 2\dot{\Omega}_e \cot (\dot{\Omega}_e \sin \omega + \dot{\Omega}_q \cos \omega)] \] (4)

Masses of the primary, secondary, and tertiary are m1, m2, and m3. The orbital elements of the inner binary are as follows: eccentricity e, semimajor axis a, mutual inclination between the inner binary and the outer binary orbit i, the argument of periastron ω, and the longitude of the ascending node Ω. G is Newton’s constant and c is the speed of light. n = 2π/P = [GM/a^3]^{1/2} denotes the mean motion of the inner binary. k2 is the tidal Love number and R2 is the radius of the white dwarf. The quantity r2 = 2GM1/c^2 in Equation (2) is the Schwarzschild radius of the neutron star. ωout and eout are the semimajor axis and eccentricity of the outer binary. Ωe, Ωq, and Ωh are spin projections onto the triad defined by the Laplace–Runge vector e, the total angular momentum vector h, and their cross product q = e × h.
The sign of the Kozai term depends on \( \sin i \), while the contribution from the tidal bulge of the secondary and the GR term are always positive. The contribution from the rotational bulge of the secondary is positive if the secondary is tidally locked and if the spins are aligned; in the opposite case it is negative. Throughout the paper, we adopt \( k_2 = 0.01 \) for helium white dwarf. Such a value of \( k_2 \) is obtained for the helium white dwarf in 4U 1820-30, assuming that it is a fluid object, as the ratio of the potential due to the perturbed mass distribution, to the external potential causing the perturbed mass \( (\text{Arras 2010, private communication}) \). Assuming that the system is tidally locked, to produce these observed long-period modulations, we are looking toward cancellation better than 10% between the Kozai term and all other terms. It turns out that the dominant contribution, other than the Kozai precession, comes from the tidal bulge for our fiducial value of \( k_2 \). The fine tuning of inclination, required in such a situation, can be avoided if the system librates around the stable fixed point. In this case, the precession rates sum to zero. Therefore, these long periods of libration are associated with a period of small oscillations around the fixed point.

The maximum allowed eccentricities for the system in consideration, listed in Table 1, are of the order of \( \sim 0.05 \). These values are given by Regős et al. (2005), assuming the conservative mass transfer and lack of mass loss via the L2 Lagrangian point at the mass ratios of these three systems. Prodan & Murray (2012) demonstrate that the periods of small oscillation of the order of several hundred days require such small eccentricities. When the mutual inclination is close to the Kozai critical value, such small eccentricity oscillations arise naturally.

As shown in Prodan & Murray (2012), via both analytical and numerical calculations, in order to trap the system in the resonance and then for it to remain trapped for at least \( 10^5 \) yr (which is about \( \sim 1% \) of the lifetime for such systems), the semimajor axis has to expand. In another words, the mass transfer has to dominate the evolution of the semimajor axis. Since the semimajor axis expands on a timescale much larger than any orbital or precession timescale in the system, the action of the Hamiltonian that describes the dynamics of such systems is an adiabatic invariant. For a detailed discussion on the adiabatic invariant, we refer reader to Prodan & Murray (2012). The semimajor axis expansion of the inner binary leads to the increase of the torque between the two orbits, which is equivalent to deepening the Kozai potential. The fact that the action is an adiabatic invariant means that the action of all orbits other than the separatrix remain constant, while the action of the separatrix increases. As the action of the separatrix grows, eventually it will exceed the action of the initially circulation orbit. At that point, the orbit in question is captured in the resonance and starts to librate. The semimajor axis expansion eventually leads the orbit close to the fixed point, where is librates with the period of small oscillations.

On the other hand, if tidal dissipation, which has a tendency to shrink the semimajor axis, plays dominant role in the evolution of the semimajor axis, the system does not remain captured in the resonance. Intense tidal dissipation reduces the eccentricity of the inner pair within the \( 10^{-3} \) of the system’s lifetime, leading toward the loss of the long-period modulation of luminosity. Therefore, we would have to be extremely fortunate to observe the system during such a short phase when the eccentricity modulation induced by the tertiary is important. Hence, we assert that the effect causing the trapping of the system deep in the Kozai resonance is the expansion of the semimajor axis driven by Roche lobe overflow. While in the resonance, angular momentum is transferred back and forth periodically between the inner binary and the third star. Such an exchange of angular momentum does not by any means affect the semimajor axis of both binaries. However, when the forced eccentricity is at its maximum and the mutual inclination at its minimum, strong tidal dissipation reaches its maximum in removing the energy from the inner orbit. Such coupled Kozai-tidal evolution brings

| Symbol | Definition                  | Value            | Citation            |
|--------|-----------------------------|------------------|--------------------|
| \( m_2 \) | White dwarf (secondary) mass             | 0.04 \( M_\odot \) | Homer et al. (2001) |
| \( R_2 \) | White dwarf (secondary) radius             | 2.78 \( \times 10^3 \) cm |                   |
| \( L_X \) | X-ray luminosity             | 1 \( \times 10^{36} \) erg s\(^{-1} \) | Kitamoto et al. (1992) |
| \( e_{\text{max}} \) | Maximum inner binary eccentricity             | 0.05 | Regős et al. (2005) |
| \( P_0 \) | Long period                 | 0.72 yr          | Priedhorsky (1986) |

| Symbol | Definition                  | Value            | Citation            |
|--------|-----------------------------|------------------|--------------------|
| \( m_2 \) | White dwarf (secondary) mass             | 0.045 \( M_\odot \) |                      |
| \( R_2 \) | White dwarf (secondary) radius             | 2.5 \( \times 10^3 \) cm |                      |
| \( L_X \) | X-ray luminosity             | 3 \( \times 10^{36} \) erg s\(^{-1} \) | Callanan et al. (1995) |
| \( e_{\text{max}} \) | Maximum inner binary eccentricity             | 0.05 | Regős et al. (2005) |
| \( P_0 \) | Long period                 | 1 yr             | Maccarone et al. (2010) |

| Symbol | Definition                  | Value            | Citation            |
|--------|-----------------------------|------------------|--------------------|
| \( m_2 \) | White dwarf (secondary) mass             | 0.034 \( M_\odot \) |                      |
| \( R_2 \) | White dwarf (secondary) radius             | 2.75 \( \times 10^3 \) cm |                      |
| \( L_X \) | X-ray luminosity             | 0.74 \( \times 10^{36} \) erg s\(^{-1} \) | Homer et al. (1996) |
| \( e_{\text{max}} \) | Maximum inner binary eccentricity             | 0.04 | Regős et al. (2005) |
| \( P_{0,\text{min}} \) | Minimum long period             | 1 yr             |                     |
the mutual inclination toward its critical value (~40°; Wu et al. 2007; Fabrycky & Tremaine 2007).

For additional discussion on the Kozai mechanism in the presence of external forces and its application to different systems, we refer reader to: Eggleton & Kiseleva-Eggleton (2001), Blaes et al. (2002), Miller & Hamilton (2002), Wu & Murray (2003), Wu et al. (2007), Fabrycky & Tremaine (2007), Antonini et al. (2010), Naoz et al. (2011), Antonini & Perets (2012), Naoz et al. (2013a, 2013b), Katz & Dong (2012), Prodan et al. (2013), Hamers et al. (2013), Shappee & Thompson (2013), Antonini et al. (2014), and Prodan et al. (2014).

2.1. Estimating the Mass, the Radius, and the Mass Transfer Rate of the White Dwarf Secondary

To model the evolution of the system due to the mass transfer, we follow the prescription of Rappaport et al. (1987). Assuming that the secondary is a polytrope with index \( n = 3/2 \), completely degenerate, and hydrogen-depleted, its mass-radius relation is given by (Rappaport et al. 1987)

\[
\frac{R_2}{R_\odot} = 0.0128 \left( \frac{m_2}{M_\odot} \right)^{-\frac{1}{3}}. \tag{5}
\]

Since the secondary fills its Roche lobe, we have

\[
P = \frac{9\pi}{2\sqrt{2}} (Gm_2)^{1/2} R_2^{3/2}. \tag{6}
\]

Combining Equations (5) and (6), we obtain the mass-period relation (Rappaport et al. 1987)

\[
m_2 = 0.769 \left( \frac{P}{\text{min}} \right)^{-1} M_\odot. \tag{7}
\]

Equation (7) puts a constraint on the mass of the donor star and, knowing its mass, we constrain the radius of the donor using Equation (5). For each binary, we list the mass and the radius of the donor star, as well as the X-ray luminosity, maximum eccentricity, and the long period, in Table 1. The estimated masses and radii of the secondary for each system listed in Table 1 are in agreement with the approximate values quoted in the Introduction for the masses of the secondary given by the mass-radius relation for appropriate orbital period by Deloye & Bildsten (2003). For the mass of the neutron star primary, we adopt a canonical mass of 1.4 \( M_\odot \).

The mass transfer rate is given by (for detailed derivation, see Rappaport et al. 1987)

\[
\dot{m}_2 = 6.21 \times 10^{-4} \left( \frac{m_1}{M_\odot} \right) \left( \frac{P}{\text{min}} \right)^{-3/2} \frac{M_\odot}{\text{yr}}.
\]

\[
\dot{m}_2 = 1.23 \times 10^{-30} \left( \frac{\text{AU}}{a} \right)^{-7} \left( \frac{m_1}{M_\odot} \right)^3 \frac{M_\odot}{\text{yr}}. \tag{8}
\]

Figure 1. X-ray luminosity, \( L_x \), vs. the orbital period of the binary. The solid line represents Equation (9) while the dots are the averaged observed X-ray luminosities. As shown, Equation (9) is not in contradiction with the X-ray luminosity of 4U 1850-087, 4U 0513-40, and M15 X-2.

Equation (8) gives the following expression for X-ray luminosity (Rappaport et al. 1987; Homer et al. 2001)

\[
L_x = 1.06 \times 10^{38} \left( \frac{m_1}{1.4 M_\odot} \right)^{4/3} (R_1/10 \text{km})^{-1} (P/11.4 \text{ min})^{-\frac{14}{5}} \text{ erg s}^{-1}. \tag{9}
\]

Both Equation (8) and Equation (9) are scaled with respect to the parameters of 4U 1820-30. We plot Equation (9) in Figure 1 and we demonstrate that this equation, originally derived to describe the evolution of 4U 1820-30, is not in contradiction with the X-ray luminosity of all three UCXBs: 4U 1850-087, 4U 0513-40, and M15 X-2.

In order to account for the small eccentricity of the inner orbit when calculating \( \dot{m}_2 \) in our numerical model instead of semimajor axis \( a \), we use the periapse distance \( a(1-e) \). Therefore, Equation (8) becomes

\[
\dot{m}_2 = 1.23 \times 10^{-30} \left( \frac{\text{AU}}{a(1-e)} \right)^{-7} \left( \frac{m_1}{M_\odot} \right)^3 \frac{M_\odot}{\text{yr}} \tag{10}
\]

2.2. The Eccentricity and the Period of Small Oscillations of the Inner Binary

We use Delaunay variables to characterize the motion of the inner pair: the argument of periastron \( \omega \), the longitude of the ascending node \( \Omega \), and the mean anomaly \( l \). In the Hamiltonian averaged over \( l \) and \( l_{out} \), only \( \omega \) appears. Their respective conjugate momenta are

\[
\mathcal{L} = m_1 m_2 \sqrt{\frac{Ga}{m_1 + m_2}} \tag{11}
\]

\[
\mathcal{G} = \mathcal{L} \sqrt{1 - e^2} \tag{12}
\]

\[
\mathcal{H} = \mathcal{G} \cos i. \tag{13}
\]
The averaged Hamiltonian is given by (Innanen et al. 1997; Ford et al. 2000; Fabrycky & Tremaine 2007; Prodan & Murray 2012)

\[
H = -\frac{3A}{2}\left[ -5 - \frac{3H^2}{L^2} + \frac{G^2}{L^2} + 5\frac{\mathcal{H}^2}{G^2} + 5\cos 2\omega \right. \\
\left. \times \left( 1 - \frac{G^2}{L^2} \right) \right] \\
- B\frac{L}{G} - k_2C\left( 35\frac{L^9}{G^9} - 30\frac{L^7}{G^7} + 3\frac{L^5}{G^5} \right) - k_2D\frac{L^3}{G^3},
\]

(14)

where the term proportional to \(A\) is the Kozai term and the term proportional to \(B\) is the GR apsidal precession. The terms proportional to \(C\) and \(D\) correspond to the tidal and rotational bulges. The expressions for the constants are

\[
A = \frac{1}{8}\Phi \frac{m_1m_2}{(m_1 + m_2)^2} \left( \frac{a}{a_{\text{out}}} \right)^3 \frac{1}{(1 - e_{\text{out}}^2)^{3/2}},
\]

(15)

\[
B = \frac{3}{2}\Phi \frac{m_2}{m_1} \frac{r_s}{a},
\]

(16)

\[
C = \frac{1}{16}\Phi \frac{m_1}{m_1 + m_2} \left( \frac{R_2}{a} \right)^5.
\]

(17)

\[
D = \frac{1}{12}\Phi \left( \frac{R_2}{a} \right)^5 f(\tilde{\Omega}_{\text{spin}}),
\]

(18)

where

\[
\Phi \equiv \frac{G(m_1 + m_2)m_1}{a}.
\]

(19)

Eccentricity and the semimajor axis of the outer body’s orbit are \(e_{\text{out}}\) and \(a_{\text{out}}\). We denote the Schwarzschild radius of the neutron star by \(r_s = 2Gm_1/c^2\).

To obtain the frequency or the period of the small oscillation, we evaluate the second derivative of the Hamiltonian given by Equation (14) at the fixed point

\[
\omega_0 = \omega_A \left[ \left( 18 + 90\frac{H^2}{G^2} \right) + 2\frac{B}{A} \frac{L^3}{G^3} + k_2\frac{C}{A} \right] \\
\times \left( \frac{3150}{G^2} - 1680\frac{L^9}{G^9} + 90\frac{L^7}{G^7} \right)^{1/2} \\
+ 12k_2\frac{D}{A} \frac{L^5}{G^5} \left( 1 + \sqrt{1 - e_f^2} \right),
\]

(20)

where \(e_f\) and \(t_{\text{crit}}\) are the eccentricity of the fixed point and the critical mutual inclination given by (Prodan & Murray 2012)

\[
e_f = \sqrt{\frac{30[\cos^2 t_{\text{crit}} - \cos^2 i]}{60\frac{H^2}{G^2} + 3\frac{H^2}{L^2} + 840\frac{C}{G^2} + 15\frac{k_2}{2} D}}
\]

(21)

\[
\cos^2 t_{\text{crit}} = \frac{3}{5} - \frac{1}{30} \frac{B}{A} - 4k_2 \frac{C}{A} - \frac{1}{10} k_2 \frac{D}{A}.
\]

(22)

The argument of periastron of the fixed point is \(\omega_f = 90^\circ, 270^\circ\).

Under the assumption that there is no mass loss through the Lagrangian point \(L_2\), we list in Table 1 the maximum possible eccentricities in the inner binaries given by Regős et al. (2005) for the mass ratio of the binary. 4U 1850-087 and 4U 0513-40 have suspected but yet not confirmed long periods of 0.72 yr (Priedhorsky 1986) and 1 yr (Maccarone et al. 2010), respectively. Their eccentricities are well below the maximum possible eccentricities given in Table 1. Adopting the listed value for the maximum eccentricity, we estimate using Equations (21) and (20) the value for the minimum possible period of small oscillations of M15 X-2 to be of the order of 1 yr. This estimate provides values on the verge of the overflowing \(L_2\), point; hence, in reality we would expect this period to be longer and the eccentricity to be smaller.

3. NUMERICAL RESULTS

In our numerical calculation, the gravitational impact due the third star is calculated in the quadrupole approximation that includes the Kozai resonance described before. We demonstrated in Prodan & Murray (2012) that the results do not change qualitatively due to the introduction of the octupole approximation. We derive equations of motion from the Hamiltonian averaged over the orbital periods of the inner and the outer binary. The equations of motion, given in Appendix A of Prodan & Murray (2012), incorporate the following.

1. Periastron advance due to general relativity.
2. Periastron advance caused by quadrupole distortions of the white dwarf secondary due to tides and rotation.
3. Orbital shrinkage of the inner binary orbit due to tidal dissipation in the white dwarf secondary.
4. Orbital angular momentum loss due to gravitational radiation.
5. Conservative mass transfer from the white dwarf secondary to the neutron star primary driven by the emission of gravitational radiation.

The initial conditions that give us appropriately long periods for each binary are listed in Tables 2–4. These parameters are used throughout this chapter unless otherwise stated.

Figures 2–4 show the eccentricity oscillations with corresponding long periods for UCXB 4U 1850-087, 4U 0513-40, and M15 X-2. The amplitude of the eccentricity oscillations is \(\sim 3 \times 10^{-3}\) in the case of 4U 1850-087 and 4U 0513-40, while in the case of M15 X-2 the amplitude of eccentricity oscillations is \(\sim 3 \times 10^{-3}\). The observed factor \(\sim 3\) luminosity variations are explained by these eccentricity oscillations (Zdziarski et al. 2007). Trapping the system in libration around the fixed point provides a natural explanation for the origin of the long periods and the small eccentricity oscillations that lead to observed luminosity variations.

In order to keep the fiducial eccentricity of M15 X-2 below the maximum eccentricity given in Table 1 at all times and to have sufficiently large eccentricity oscillation to produce luminosity variations of a factor of 2–3, we require in our numerical calculations a long period of 3.4 yr, which is a factor of \(\sim 3\) larger than the estimated minimum long period listed in Table 1. Note that any long-period variations in luminosity for M15 X-2 remain undetected at this time.

3.1. Resonant Trapping and Detrapping

As argued in Prodan & Murray (2012), the semimajor axis of the inner binary has to expand. This expansion is expected in the standard evolutionary scenario (see the Introduction). As the orbit expands due to the mass transfer, the action of the separatrix
increases adiabatically on the accretion timescale, leading to trapping in a resonance around the fixed point. One might expect that tidal effects could be dominant at such a small separation. There are two arguments against a shrinking semimajor axis. The first argument is that the phase where the semimajor axis shrinks and the eccentricity decays due to tidal dissipation is short-lived; there are only a few thousand years until the mass transfer driven expansion dominates the evolution. The second argument is that the shrinking orbit drives the initially librating orbit out of resonance into circulation which would dramatically change the period of luminosity variations. Thus, as shown previously for the case of 4U 1820-30, the evolution of the system is dominated by mass transfer but the rate of expansion of the orbit is decreased due to tidal dissipation.

In this section, we demonstrate that just this physical picture applies in the case of these three binaries as well and it can indeed explain the origin of their long periods. Figures 5, 6, and 7 show the systems initially put on circular orbit with a choice of $Q$ such that the orbit expands. As integration proceeds, the separatrix continues to expand and at some point captures the initially circulating orbit in libration around the fixed point. Once captured, the system librates for at least $10^5$ yr. We consider $10^5$ yr to be a reasonable fraction of the system's lifetime during which it can be observed in such a state (see Section 4).

### Table 2
4U 1850-087: System Parameters

| Symbol | Definition | Value | Citation |
|--------|------------|-------|----------|
| $m_1$  | Neutron star (primary mass) | $1.4 M_\odot$ |  |
| $m_2$  | White dwarf (secondary mass) | $0.04 M_\odot$ | Homer et al. (2001) |
| $m_3$  | Third companion mass | $0.55 M_\odot$ |  |
| $a_1$  | Inner binary semimajor axis | $1.95 \times 10^{10}$ cm | Homer et al. (1996) |
| $a_{out}$ | Outer binary semimajor axis | $8.78 a_1$ |  |
| $e_{in,0}$ | Inner binary initial eccentricity | $0.018$ |  |
| $e_{out,0}$ | Outer binary eccentricity | $10^{-4}$ |  |
| $i_{init}$ | Initial mutual inclination | $44/657$ |  |
| $\omega_{in,0}$ | Initial argument of periastron | $90^\circ$ |  |
| $\Omega_{in}$ | Longitude of ascending node | $0$ |  |
| $R_2$ | White dwarf radius | $2.78 \times 10^8$ cm | see Section 2.1 |
| $k_2$ | Tidal Love number | $0.01$ | Arras (2010, private communication) |
| $Q$ | Tidal dissipation factor | $6 \times 10^7$ |  |

### Table 3
4U 0513-40: System Parameters

| Symbol | Definition | Value | Citation |
|--------|------------|-------|----------|
| $m_1$  | Neutron star (primary mass) | $1.4 M_\odot$ |  |
| $m_2$  | White dwarf (secondary mass) | $0.045 M_\odot$ | see Section 2.1 |
| $m_3$  | Third companion mass | $0.55 M_\odot$ |  |
| $a_1$  | Inner binary semimajor axis | $1.65 \times 10^{10}$ cm | Zurek et al. (2009) |
| $a_{out}$ | Outer binary semimajor axis | $8.36 a_1$ |  |
| $e_{in,0}$ | Inner binary initial eccentricity | $0.02$ |  |
| $e_{out,0}$ | Outer binary eccentricity | $10^{-4}$ |  |
| $i_{init}$ | Initial mutual inclination | $46/377$ |  |
| $\omega_{in,0}$ | Initial argument of periastron | $90^\circ$ |  |
| $\Omega_{in}$ | Longitude of ascending node | $0$ |  |
| $R_2$ | White dwarf radius | $2.64 \times 10^8$ cm | see Section 2.1 |
| $k_2$ | Tidal Love number | $0.01$ | Arras (2010, private communication) |
| $Q$ | Tidal dissipation factor | $5 \times 10^7$ |  |

### Table 4
M15 X-2: System Parameters

| Symbol | Definition | Value | Citation |
|--------|------------|-------|----------|
| $m_1$  | Neutron star (primary mass) | $1.4 M_\odot$ |  |
| $m_2$  | White dwarf (secondary mass) | $0.034 M_\odot$ | see Section 2.1 |
| $m_3$  | Third companion mass | $0.55 M_\odot$ |  |
| $a_1$  | Inner binary semimajor axis | $2.1 \times 10^{10}$ cm | Dieball et al. (2005) |
| $a_{out}$ | Outer binary semimajor axis | $9.5 a_1$ |  |
| $e_{in,0}$ | Inner binary initial eccentricity | $0.015$ |  |
| $e_{out,0}$ | Outer binary eccentricity | $10^{-4}$ |  |
| $i_{init}$ | Initial mutual inclination | $44/643$ |  |
| $\omega_{in,0}$ | Initial argument of periastron | $90^\circ$ |  |
| $\Omega_{in}$ | Longitude of ascending node | $0$ |  |
| $R_2$ | White dwarf radius | $2.75 \times 10^8$ cm | see Section 2.1 |
| $k_2$ | Tidal Love number | $0.01$ | Arras (2010, private communication) |
| $Q$ | Tidal dissipation factor | $6 \times 10^7$ |  |
Figures 2, 4U1850-087: upper panel shows the eccentricity as a function of time while the lower panel shows the phase space, $e$ vs. $\omega$, for the fiducial model of the system. The long period—i.e., the period of the eccentricity oscillations—is 0.72 yr. The amplitude of the eccentricity oscillation is large enough to give rise to the observed factor of two to three variation in luminosity.

Figures 8, 9, and 10, on the other hand, show the case where the system is initially on librating orbit with $Q$ such that semimajor axis shrinks. In all three cases, detrapping from the resonance occurs fairly quickly, making observation of the system in such a state highly unlikely.

4. CONSTRAINING THE TIDAL DISSIPATION FACTOR $Q$

For the White Dwarf Companions

In this section, we constrain the tidal dissipation factor $Q$ for the white dwarf companions simply by asking that the system remain in the resonance for more than $10^5$ yr. Estimated mass transfer rates for these systems are $(5 \times 10^{-10} - 10^{-9}) M_\odot \, yr^{-1}$ (see Equation (8)), giving a lifetime of $\sim 7 \times 10^7$ yr during which these systems can sustain the observed luminosity. Therefore, a reasonable fraction of time to remain in the resonance is indeed at least $10^5$ yr. Figures 11–13 demonstrate that for the fiducial values of $Q$, or, more precisely, $e^2 Q/k$ listed in Table 5, these systems remain trapped in the resonance for a reasonable fraction of their lifetime during which the maximum eccentricity does not exceed values given in Table 1.

5. DISCUSSION

A long-term luminosity periodicity in UCXBs has been suspected for a while now, but the observations are not sufficiently
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Figure 4. M15 X-2: same as Figure 2. The long period—i.e., period of the eccentricity oscillations—is 3.4 yr.

Table 5
Tidal Dissipation Factor $Q$

| Symbol | Value |
|--------|-------|
| 4U 1850-087 | $(\omega_0^2 \Omega \ell_2) 6 \times 10^9$ |
| 4U 0513-40 | $(\epsilon_0^2 \Omega \ell_2) 5 \times 10^9$ |
| M15 X-2 | $(\epsilon_0^2 \Omega \ell_2) 6 \times 10^9$ |

good to confirm it with reasonable certainty. The only system for which the long periodicity is certain is the 11 minutes binary 4U 1820-30. The detection of X-ray bursts in this object (Grindlay et al. 1976) led to extensive observations and hence a very well sampled light curve. Several authors suggested that this long periodicity may be caused by a third body (Grindlay 1988; Chou & Grindlay 2001; Zdziarski et al. 2007). In Prodan & Murray (2012), we attribute the origin of the long-period variations in the luminosity to the libration in Kozai resonance with the frequency of small oscillations around the fixed point. In addition to the perturbations from a third body, we consider tidal
Trapping in libration occurs after ≈2 × 10^4 yr. The eccentricity does not significantly exceed the estimated maximum value of 0.05 for at least 3 × 10^5 yr, a time it spends trapped in libration.

The model developed for 4U 1820-30 predicts long periodicities in the light curve of 4U 1850-087 and 4U 0513-40 as well as that of M15 X-2. Requiring the systems to remain trapped in a resonance for a reasonable fraction of their lifetime allows us to put the upper limit on the tidal dissipation factor for the white dwarf donors. The actual values are listed in Table 5. Obtaining the lower limit for tidal dissipation factor Q of the order of few × 10^7 is in agreement with the results of Piro (2011), Fuller & Lai (2011), and Prodan & Murray (2012).

The three binaries examined in this paper have similar properties to 4U 1820-30. For two of them, 4U 0513-40 and 4U 1850-087, long-period variations have been suggested (Priedhorsky & Terrell 1984; Maccarone et al. 2010). Therefore, we suggest that these three systems may be triples as well and furthermore we would anticipate that a majority of UCXBs in globular clusters are actually triples. Short-period binaries like these are quite likely to acquire a third body in the dense environment of globular clusters. Comparing the confirmed orbital periods in the field to those in globular clusters, the trend seems to indicate that field UCXBs have orbital periods of the order of 40 minutes, while those in globular clusters have periods ≤20 minutes. Such a trend hints at different formation scenarios operating in these two environments. Very long-period...
variations, which cannot be due to accretion disk precession or a change in the viewing angle, seem to be characteristic of UCXBs in globular clusters. To check this speculation, it is necessary to determine orbital periods for more of these systems and obtain a better statistical sample.

Several explanations about why the accretion rates differ from those expected based on the binary parameters has been suggested. Other than the presence of the third body, which we discussed in great detail, two viable candidates two are tidal disk instabilities (Osaki 1995) and the irradiation of the donor responsible for the modulation of the mass transfer rate (Hameury et al. 1986). Their common characteristic is that both cause more stochastic variations than those expected from the presence of a third body. The light curves of these binaries, as well as the light curve of 4U 1820-30, are not sampled. Even though the data may indicate the potential presence of a regular modulation, there are many irregularities that may be a consequence of a combination of regular modulation due to the third body and other mechanisms causing aperiodic variations.

The basic idea of the tidal disk instability model relies on the assumption that the mass transfer rate is constant and that all outbursts of accretion onto the primary are caused by intrinsic instabilities in the accretion disk. The disk is compact during the minimum luminosity phase of the long-period cycle. The thermal instability generates only quasi-periodic episodes of accretion that are observed as normal outbursts. In normal outbursts the accreted mass is less than the mass transferred during the quiescent phase. The reason for this lies in the inefficiency of tidal removal of angular momentum from the disk. Continuous build up of the mass and the angular momentum of the disk forces the disk to expand further and

Figure 8. 4U 1850-087: (a) $\omega$ vs. $t$. Initially the system is placed on a librating orbit. We set $Q = 2 \times 10^7$, which leads to shrinking of the semimajor axis and the action of the separatrix is decreasing. Consequently, the system is driven out from the resonance after $\approx 35,000$ yr. (b) The eccentricity as a function of time.

Figure 9. 4U 0513-40: same as Figure 8. Resonance detrapping occurs after $\approx 1800$ yr.
further with each consecutive outburst. The expansion continues until the disk reaches a critical radius for tidal instability. At this point, the final normal outburst sets off the tidal instability, creating a precessing eccentric disk. Such a precession eccentric disk is observed as a super hump. It greatly enhances the tidal torque, resulting in the super outburst that significantly clears out the disk mass. At the end of the described super outburst, the disk goes back to the initial compact state.

The second model considers a mass-loss instability in the donor star as a consequence of illumination of its atmosphere by the X-ray flux from the compact object. During the quiescent phase, the donor does not completely fill in its Roche lobe, which leads to a lower accretion rate but is still sufficient to heat up the external layers of the donor’s atmosphere. As these layers are heated up slowly by an X-ray flux comparable to the stellar flux at the vicinity of the $L_1$ point, they expand. Ultimately, the heating brings the atmosphere in the unstable regime where matter flows at a high rate through the $L_1$ point. Eventually, the shielding by the accretion disk may prevent X-ray flux from reaching the $L_1$ region, which will cause the outburst to cease. By this time, the heated layers have been transported to the disk. The outburst stops when the entire disk is accreted onto the compact object.

Tidal instabilities in the accretion disk may explain the variations in the accretion rate on a timescale of weeks, such as those seen in 4U 0513-40, but most likely not the long-period...
Figure 12. 4U 0513-40: same as Figure 11 with $Q = 5 \times 10^7$.

Figure 13. M15 X-2: same as Figure 11 with $Q = 6 \times 10^7$.

There are no studies of the irradiation-induced mass transfer in the context of white dwarfs and hence it is very difficult to make any conclusive statements. The observations clearly show that irradiation of the white dwarf donor in 4U 0513-40 is significant (Maccarone et al. 2010), but the same is not observed in 4U 1820-30, which is a brighter system. Provided that one finds a reasonable explanation for this discrepancy, the irradiation-induced mass transfer model could be feasible. Unquestionably, to understand the details of the dynamics and evolution of UCXBs, more observations are required.

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