THE 1% CONCORDANCE HUBBLE CONSTANT

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Received 2014 June 6; accepted 2014 August 22; published 2014 October 1

ABSTRACT

The determination of the Hubble constant has been a central goal in observational astrophysics for nearly a hundred years. Extraordinary progress has occurred in recent years on two fronts: the cosmic distance ladder measurements at low redshift and cosmic microwave background (CMB) measurements at high redshift. The CMB is used to predict the current expansion rate through a best-fit cosmological model. Complementary progress has been made with baryon acoustic oscillation (BAO) measurements at relatively low redshifts. While BAO data do not independently determine a Hubble constant, they are important for constraints on possible solutions and checks on cosmic consistency. A precise determination of the Hubble constant is of great value, but it is more important to compare the high and low redshift measurements to test our cosmological model. Significant tension would suggest either uncertainties not accounted for in the experimental estimates or the discovery of new physics beyond the standard model of cosmology. In this paper we examine in detail the tension between the CMB, BAO, and cosmic distance ladder data sets. We find that these measurements are consistent within reasonable statistical expectations and we combine them to determine a best-fit Hubble constant of 69.6 ± 0.7 km s⁻¹ Mpc⁻¹. This value is based upon WMAP9+SPT+ACT+6dFGS+BOSS/DR11+H₀/Riess; we explore alternate data combinations in the text. The combined data constrain the Hubble constant to 1%, with no compelling evidence for new physics.

Key words: cosmological parameters – cosmology: observations – distance scale

1. INTRODUCTION

Hubble’s original attempt to establish the distance–redshift relation (Hubble 1929) highlighted the difficulty of establishing accurate distances over a sufficient volume of the universe while controlling systematic measurement errors. This lesson was reinforced in the decades-old debate between de Vaucouleurs and Sandage, who claimed Hubble constant values of ∼100 and ∼50 km s⁻¹ Mpc⁻¹, respectively. In part, this debate motivated the Hubble Space Telescope (HST) and its Key Project to determine the Hubble constant, which produced H₀ = 72 ± 8 km s⁻¹ Mpc⁻¹ (Freedman et al. 2001). Efforts over the past dozen years have produced similar values, but the uncertainties have been reduced from ∼11% to ∼3%: 73.8 ± 2.4 km s⁻¹ Mpc⁻¹ (Riess et al. 2011); see also Freedman & Madore (2010, 2013) and Livio & Riess (2013) for more detailed reviews. Perhaps not surprisingly, the current measurements of H₀ lie near the mean of the de Vaucouleurs and Sandage values.

In recent years, Hubble constant measurements based on the cosmic distance ladder have been complemented by measurements using the cosmic microwave background (CMB). By determining the energy budget of the universe from the CMB power spectra (including polarization), the Friedmann equation determines the dynamics of the universe and hence the value of the Hubble constant through the cosmological model. The nine-year Wilkinson Microwave Anisotropy Probe (WMAP) data, taken alone, but assuming a flat universe, give a 3% determination of H₀ = 70.0 ± 2.2 km s⁻¹ Mpc⁻¹ (Bennett et al. 2013; Hinshaw et al. 2013). When WMAP data are combined with smaller angular scale CMB data from the South Pole Telescope (SPT) and the Atacama Cosmology Telescope (ACT), and with distance measurements derived from baryon acoustic oscillation (BAO) observations in the range 0.1 < z < 0.6, one obtains H₀ = 68.76 ± 0.84 km s⁻¹ Mpc⁻¹, a 1.2% determination. The Planck Collaboration (2014) found a six-parameter flat ΛCDM model to be a good fit to their data and derived a best-fit Hubble constant of 67.3 ± 1.2 km s⁻¹ Mpc⁻¹. They noted a ≈2.5σ tension with the Riess et al. (2011) cosmic distance ladder measurement.

Since the two approaches to measuring H₀ sample different epochs in cosmic history, this apparent tension has led to guarded speculation that new physics beyond the standard cosmological model may be needed to reconcile them (e.g., Wyman et al. 2014; Hamann & Hasenkamp 2013; Battye & Moss 2014; Dvorkin et al. 2014). However, as noted by others (e.g., Verde et al. 2014; Gao & Gong 2013; Efstathiou 2014), the tension could simply be the result of uncharacterized systematics in the individual H₀ determinations, or a statistical fluke, and these possibilities should be explored. In this paper, we re-examine the consistency of the H₀ measurements. In Section 2 we describe the recent relevant data, we present our analysis in Section 3, and we offer conclusions in Section 4.

2. DATA

We consider three primary data sets for establishing constraints on the Hubble constant: cosmic distance scale measurements, BAO measurements, and CMB measurements, and we describe each in turn.

2.1. Cosmic Distance Scale Measurements

Riess et al. (2011) found H₀ = 73.8 ± 2.4 km s⁻¹ Mpc⁻¹ including systematic uncertainties. Their result was based on three methods for finding distances to Type Ia supernovae and therefore calibrating their luminosities: (1) a geometric distance to NGC 4258 based on megamaser measurements, (2) parallax distances to 13 Milky Way Cepheids from HST and Hipparcos data, and (3) eclipsing binary distances to 92 colocated Cepheids in the Large Magellanic Cloud. Independently, Freedman et al. (2012) recalibrated the HST Key Project distance ladder using mid-infrared observations of Cepheids to minimize some of the systematic effects present in optical observations; they obtain
$H_0 = 74.3 \pm 1.5$ (statistical) $\pm 2.1$ (systematic). Both WMAP (Bennett et al. 2013; Hinshaw et al. 2013) and Planck (Planck Collaboration 2014) adopted the Riess et al. (2011) value for their 2013 analyses which included an $H_0$ prior.

Since the Riess et al. (2011) publication, there have been proposed revisions to the underlying distance calibration and/or to the selection of calibrators. Humphreys et al. (2013) recalibrated the geometric distance to NGC 4258 and used that distance to calibrate the Cepheids in NGC 4258. That work resulted in a value of $H_0 = 72.0 \pm 3.0$ km s$^{-1}$ Mpc$^{-1}$ using only NGC 4258 as an anchor. Efstathiou (2014) makes a different Cepheid selection and uses the Humphreys et al. (2013) distance to NGC 4258 to obtain $H_0 = 70.6 \pm 3.3$ km s$^{-1}$ Mpc$^{-1}$, using the NGC 4258 recalibration alone, and $H_0 = 72.5 \pm 2.5$ km s$^{-1}$ Mpc$^{-1}$ using all three calibration methods. Riess (2014) combines the Humphreys recalibration with the other two calibration methods used in the original Riess et al. (2011) to obtain $H_0 = 73.0 \pm 2.4$ km s$^{-1}$ Mpc$^{-1}$, making use of all available calibrator data and the original Riess et al. (2011) outlier cut methodology. For our analysis we adopt $H_0$ from Riess (2014).

### 2.2. Baryon Acoustic Oscillation Data

The remnants of BAOs in the early universe imprint a characteristic scale in the two-point correlation function of large-scale structure. The BAO scale, $r_s$, is the distance a sound wave traveled in the baryon–photon plasma until decoupling at redshift $z_d$; it is common to define $r_d \equiv r_s(z_d)$. Since the fluid is relativistic, the sound speed is well determined, so the BAO scale depends primarily on the age of the universe at decoupling, which depends only weakly on the cosmic matter and radiation densities (Reid et al. 2002).

BAO measurements do not, by themselves, constrain $H_0$, but they are valuable for measuring relative distances as a function of redshift. The angular extent of the BAO scale at a given redshift determines the relative angular diameter distance, $D_A(z)/r_d$, while the radial extent, $\Delta_r$, determines the Hubble parameter $H(z)$. It is convenient to define a volume-averaged effective distance, $D_V \equiv (D_A^2 z/v_H)^{1/3}$ (Eisenstein et al. 2005), and a fiducial value of the BAO scale, $r_d/fid$, for a fiducial cosmological model. One can then quote observational results as a distance, $D_V(z) \cdot (r_d/fid)/r_d$. The fiducial distance depends on cosmological parameters (such as $H_0$), however, it is only used as a reference value and does not propagate into the BAO results that we analyze.

Table 1 summarizes the most recent BAO data for redshifts of 0.106, 0.32, 0.57, and 2.34. With the exception of the lowest redshift, these results are from the 11th Baryon Oscillation Sky Survey (BOSS) CMASS (“constant mass”) data release, DR11. There are several papers that analyze these data, and as noted in Beutler et al. (2013), they produce results within 1σ of each other. For the redshift 0.32 and 0.57 results, we adopt the “consensus” values from Anderson et al. (2013).

Currently, BAO results at higher redshift (2.1 $\leq z < 3.5$) come from the analysis of quasar Lyα forest lines (Delubac et al. 2014; Font-Ribera et al. 2013), and are derived somewhat differently. Results are reported as separate radial and tangential determinations in the form $\alpha_R \propto c/(H(z)r_d)$ and $\alpha_T \propto D_A(z)/r_d$, respectively. The radial measurement is substantially more precise than the tangential one. We “optimally combine” the radial and tangential Lyα data as $\alpha_R^2 \alpha_T^{0.3}$ (Delubac et al. 2014). We obtain the value $\alpha_R^2 \alpha_T^{0.3} = 1.017 \pm 0.015$ from the combined contours of the cross- and auto-correlation likelihoods in Figure 13 of that paper, and we use this as our $z = 2.34$ likelihood.

### 2.3. Cosmic Microwave Background Data

#### 2.3.1. WMAP

The WMAP9 data, when used without other measurements, gives a Hubble constant of $H_0 = 70.0 \pm 2.2$ km s$^{-1}$ Mpc$^{-1}$ for a flat, six-parameter $\Lambda$CDM universe (Bennett et al. 2013; Hinshaw et al. 2013). Adding the ACT, SPT, and BAO data that were available as of 2012 mid-December, the value drops somewhat to $H_0 = 68.76 \pm 0.84$ km s$^{-1}$ Mpc$^{-1}$ (Hinshaw et al. 2013), independent of the cosmic distance scale measurements. Adding the Riess et al. (2011) $H_0$ prior of 73.8 $\pm 2.4$ km s$^{-1}$ Mpc$^{-1}$ to the WMAP9+ACT+SPT+BAO analysis gives $H_0 = 69.32 \pm 0.80$ km s$^{-1}$ Mpc$^{-1}$ (Bennett et al. 2013; Hinshaw et al. 2013). We discuss combinations of WMAP9 with more recent data sets in Section 3.

#### 2.3.2. Planck

The Planck Collaboration (2014) gives a best-fit value of $67.3 \pm 1.2$ km s$^{-1}$ Mpc$^{-1}$, derived from a six-parameter flat, $\Lambda$CDM model. Their analysis makes slightly different assumptions from those used by the WMAP team. The Planck Collaboration assumed a total neutrino mass of $\sum m_\nu = 0.06$ eV, corresponding to a physical neutrino density of $\Theta_\nu h^2 \approx 6 \times 10^{-3}$ (Planck Collaboration 2014). In addition, they vary the helium fraction, $Y_{He}$, as a function of both the physical baryon density, $\Omega_b h^2$, and the number of effective relativistic degrees of freedom in the early universe, $N_{eff}$, per the Big Bang nucleosynthesis (BBN) prediction, obtained by interpolating the PArTHEnope BBN code (Planck Collaboration 2014; Pisanti et al. 2008). Note that the $Y_{He}$ fluctuations in a Markov chain have a standard deviation less than 0.1% of the mean $Y_{He}$ value, for a simple $\Lambda$CDM model where $N_{eff} = 3.046$ is held constant. Allowing $Y_{He}$ to vary in this fashion does not provide an extra degree of freedom, because $Y_{He}$ is effectively constrained by $\Omega_b h^2$. If these BBN assumptions are applied to the WMAP9 data, the WMAP9-only $\Lambda$CDM value of $H_0$ is reduced by 0.3 to 69.7 $\pm 2.2$ km s$^{-1}$ Mpc$^{-1}$ (based on the WMAP9 chains released by the Planck Collaboration).

The results noted above are from the Planck team’s first—and at this time only—cosmological data release of 2013 March.

| Table 1 BAO Measurements |
|---------------------------|
| Quantity | $z$ | Value | Experiment | Reference |
| $D_V(r_d/fid)/D_V$ | 0.106 | 457 $\pm$ 20 Mpc | 6dFGS | Beutler et al. (2011)$^a$ |
| $D_V(r_d/fid)/D_V$ | 0.32 | 1264 $\pm$ 25 Mpc | BOSS DR11 | Anderson et al. (2013)$^b$ |
| $D_V(r_d/fid)/D_V$ | 0.57 | 2056 $\pm$ 20 Mpc | BOSS DR11 | Anderson et al. (2013) |
| $D_A/rd$ | 2.34 | 10.93 $\pm$ 0.34 | BOSS DR11 | Delubac et al. (2014)$^c$ |
| $\Delta r/rd$ | 2.34 | 9.15 $\pm$ 0.20 | BOSS DR11 | Delubac et al. (2014)$^d$ |

#### Notes.

$^a$ Beutler et al. (2011) quote their result in the form $r_d/D_V = 0.336 \pm 0.015$. This formulation assumes $r_d$ follows the fitting formula of Eisenstein & Hu (1998) rather than the CAMB version discussed by Anderson et al. (2013). We quote their result in this form for consistency with the BOSS DR11 results.

$^b$ Anderson et al. (2013) quote the $z = 0.32$ results from Tojeiro et al. (2014).

$^c$ The Lyα forest results do not optimally combine to a value for $D_V$, so we quote separate radial and tangential components, derived from a combination of auto- and cross-correlation statistics. As described in the text, we use an optimal combination of these values for our $z = 2.34$ likelihood.
In version 2 of the Planck Collaboration (2014), dated 2013 December, it is shown in Appendix C.4 that noise from a 4 K cooler line produces a systematic dip in the 217 GHz power spectrum around multipole $l = 1800$. When they marginalize over a spectrum template intended to mimic this cooler line, the value of the Hubble constant derived from the full data set increases by 0.3σ. There is no plan to formally update the publicly available Markov chains to include this effect until the next data release, and the inputs required to do so are not currently available (see also Spergel et al. 2013), thus all Planck-based numbers in this paper are derived from the initially released data. Given this somewhat uncertain situation, our final estimate of $H_0$ uses WMAP9+ACT+SPT CMB data, which produces a conservative uncertainty.

2.3.3. ACT and SPT

Both WMAP and Planck use supplementary data at small angular scales from the two ground-based experiments, ACT and SPT. For Planck-based results, we use the publicly available Planck chains which incorporate ACT and SPT data available circa 2013. For WMAP-based results, we briefly quote results from the WMAP9 release (Bennett et al. 2013; Hinshaw et al. 2013), which incorporated ACT and SPT data available circa 2012; the new WMAP9-based results presented here incorporate the most recent ACT and SPT likelihoods.

The current ACT temperature and lensing likelihoods are discussed in Das et al. (2013) and Dunkley et al. (2013), while the most recent SPT likelihoods are discussed in Story et al. (2013) and van Engelen et al. (2012). A code that incorporates both ACT and SPT temperature likelihoods, called actlite, is described in Section 5 of Dunkley et al. (2013), and has been made available on LAMBDA. We use this code in the mode where it uses the ACT–E(quantorial) and SPT data, ignoring the ACT–S(southern) field to avoid double-counting the overlapping SPT and ACT–S fields. We supplement the actlite likelihood with the lensing likelihoods described in Das et al. (2013) and van Engelen et al. (2012).

3. ANALYSIS

Our analysis proceeds in two steps: first, we form various weighted averages of the CMB, BAO, and distance ladder data sets to determine the overall best value of $H_0$; these results are summarized in Table 2. All CMB+BAO results in this table assume a flat, six-parameter ΛCDM cosmology. Then, we test whether the individual measurements are consistent with one another across techniques; we find that they are.

As noted in Section 2.3.2, the released WMAP9/2013 and Planck chains have different assumptions about massive neutrinos and helium. For the WMAP9-based results in Figures 1–3, we have generated new Markov chains (denoted WMAP9/2014 in Table 2) with $\sum_m m_i = 0.06$ eV and the helium consistency relationship in Equation (8) of Sievers et al. (2013). Where noted, we include the most recent ACT and SPT data (Section 2.3.3). We adopt the 2013 March version of the Code for Anisotropies in the Microwave Background (CAMB), which is the same version used by Planck.

The weighted average results are reported in Table 2. The format of the table is as follows: the data sets we employ are listed in the first column; where that data set sufficiently

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3. http://lambda.gsfc.nasa.gov/product/act/act_cmblikelihood_get.cfm, version 2.2
4. http://camb.info

Figure 1. WMAP9-based (top, middle) and Planck-based (bottom) CMB data sets for a flat ΛCDM cosmological model, compared to local $H_0$ measurements and BAO data at $z = 0.57$. Gray points in the top panel are from this paper’s WMAP9/2014 chain; those in the middle are from (WMAP9+ACT+SPT)/2014. Gray points in the bottom panel are from the Planck (Planck Collaboration 2014) which include the (ACT+SPT)/2013 data combination (see text and Table 2). The peak of each 2D-marginalized CMB likelihood is indicated with a black filled circle and the contours for the chain points enclose 68% and 95% of the probability, as computed from a ten-component 2D Gaussian mixture fit (see text). The BAO and $H_0$ measurements are indicated by vertical and horizontal lines, respectively, with dashed lines showing 1σ error bars. The gray ellipses show 68% and 95% contours for the product of the BAO and $H_0$ likelihoods. The peak of the combined CMB+BAO+$H_0$ likelihoods is indicated with an open square, and its value is noted in each panel.
constrains $H_0$ by itself, we quote that value in the second column. Subsequent columns, labeled A through H, indicate different combinations of data, and their weighted average is given in the bottom row of each column. Columns A and B contain Planck-based data sets (Planck Collaboration 2014), both without (Column A) and re-weighted with (Column B) the local $H_0$ value included. Column C gives the WMAP9+ACT+SPT+BAO result reported by Hinshaw et al. (2013), with no $H_0$ prior. Column D adds an $H_0$ prior, as reported by Bennett et al. (2013), and Column E gives the corresponding result re-weighted with the revised $H_0$ value in Riess (2014). Column F gives our new result based on (WMAP9+BAO+H0)/2014. Column G is the same as Column F, with the addition of ACT and SPT data, and the removal of the local $H_0$ prior. Column H is the same as Column F, with the addition of ACT and SPT data. As discussed below, Column H represents our current best-estimate concordance value of $H_0$, 69.6 $\pm$ 0.7 km s$^{-1}$ Mpc$^{-1}$.

We test consistency of the data in Figure 1. To compare $H_0$, BAO, and CMB measurements, we present constraints in the space $\{H_0, D_V r_d, \Omega_M, n_s, \Omega_K\}$ since this allows us to place $H_0$ measurements on the horizontal axis and BAO measurements orthogonally on the vertical axis. Since the $z = 0.57$ BAO measurements have the smallest errors (representing a 1% measurement of the distance scale), and since the other BAO measurements are largely consistent with the $z = 0.57$ measurement, we confine our plotted consistency checks to this BAO redshift only. We add CMB data by projecting the Monte Carlo Markov Chain points into this space.

Figure 1 contains three panels with CMB constraints derived assuming ΛCDM. The top panel shows WMAP9/2014 constraints, the middle panel shows (WMAP9+ACT+SPT)/2014, and the bottom panel shows (Planck+ACT+SPT)/2013, on the same scale for ease of comparison. To quantitatively assess consistency, we fit a ten-component two-dimensional (2D) Gaussian mixture function to the CMB chain points. This function is evaluated numerically on a large 2D grid and contours that enclose 68.27% and 95.45% of the probability are computed and plotted. (The accuracy of the Gaussian mixture can be checked by eye in Figures 1–3, where we plot the fitted contours over the chain points) The Gaussian mixture provides an analytic 2D likelihood which we can combine with the 2D BAO+$H_0$ Gaussian likelihood (see below) to evaluate the best 2D fit.

We note that neither the BAO nor the local Hubble constant measurements have extra variables in their likelihoods that are correlated with CMB measurements in other dimensions. For example, neither the BAO measurement $D_V r_d, \Omega_M, \Omega_K$ nor the local $H_0$ measurement depend on $\Omega_b h^2$, the scalar spectral index $n_s$, etc. While it is true that $r_d = r_d(z_f)$ does depend on $\Omega_b h^2$, the quantity measured by the BAO, $D_V r_d$, is independent of $r_d$. Given this lack of correlated hidden variables, we can determine the consistency of these likelihoods directly in this 2D space.

Since the BAO and $H_0$ data each constrain one dimension of our parameter space, we can combine them into a single likelihood to get a 2D Gaussian likelihood that constrains both $D_V r_d$ and $H_0$. We take $H_0 = 73.0 \pm 2.4$ km s$^{-1}$ Mpc$^{-1}$ (Riess 2014) and $D_V r_d h = 2056 \pm 20$ Mpc at $z = 0.57$ (Anderson et al. 2013) and plot the combined likelihood as grey ellipses in Figures 1–3. We further note that the CMB likelihood for flat ΛCDM is well approximated by a single Gaussian in this space, producing contours that are virtually indistinguishable in Figure 1. Under the assumption that both

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**Table 2**

| Data Set | $H_0$ (km s$^{-1}$ Mpc$^{-1}$) | Combined Results |
|----------|-------------------------------|------------------|
| WMAP9/2013$^b$ | 70.0 $\pm$ 2.2 | ✓ ✓ ✓ |
| WMAP9/2014$^c$ | 69.4 $\pm$ 2.2 | ✓ ✓ ✓ |
| Planck+WMAPPol/2013$^d$ | 67.3 $\pm$ 1.2 | ✓ ✓ ✓ ✓ |
| ACT+SPT/2012$^e$ | ... | ✓ ✓ ✓ ✓ |
| ACT+SPT/2013$^f$ | ... | ✓ ✓ ✓ ✓ |
| ACT+SPT/2014$^g$ | ... | ✓ ✓ ✓ |
| BAO/2012$^h$ | ... | ✓ ✓ ✓ |
| BAO/2013$^i$ | ... | ✓ ✓ ✓ |
| BAO/2014$^j$ | ... | ✓ ✓ ✓ |
| $H_0$, Riess et al. (2011) | 73.8 $\pm$ 2.4 | ✓ |
| $H_0$, Riess (2014) | 73.0 $\pm$ 2.4 | ✓ ✓ ✓ ✓ |
| Combined $H_0$ (km s$^{-1}$ Mpc$^{-1}$) | 67.8 | 68.3 68.8 69.3 69.2 69.3 69.3 69.6 |
| Combined $\sigma$ (km s$^{-1}$ Mpc$^{-1}$) | 0.8 | 0.7 0.8 0.8 0.8 0.8 0.7 0.7 |

**Notes.**

$^a$ Column H represents our current best-estimate concordance value of $H_0$, 69.6 $\pm$ 0.7 km s$^{-1}$ Mpc$^{-1}$, assuming flat ΛCDM.

$^b$ WMAP9/2013: From Hinshaw et al. (2013). Assumes flat ΛCDM, with $\sum m_i = 0$, $Y_{le} = 0.24$, $N_{eff} = 3.04$ (instead of 3.046), and the 2012 January version of CAMB.

$^c$ WMAP9/2014: this paper. Assumes flat ΛCDM, with $\sum m_i = 0.06$ eV, $Y_{le}$ co-varies with $\Omega_b h^2$ per BBN, $N_{eff} = 3.046$, and the 2013 March version of CAMB.

$^d$ Planck: from Planck Collaboration (2014). Assumes flat ΛCDM. Likelihood (2 $\leq l \leq 2500$)+lensing+low-l pixel-space polarization from the WMAP9 likelihood.

$^e$ ACT+SPT/2012: uses ACT and SPT data that were available as of the initial WMAP9 release. ACT+SPT/2013: uses data that were available as of the initial Planck release. ACT+SPT/2014: uses data currently available for use in this paper.

$^f$ BAO/2012: from Table 1 of Hinshaw et al. (2013). BAO/2013: from Section 5.2 of Planck Collaboration (2014).

$^g$ BAO/2014: from Table 1 of this paper.
experiments are measuring the same cosmological quantities with different noise, the covariance matrix of the difference between the two measurements is the sum of their individual covariance matrices.

For the WMAP9/2014 data in the top panel of Figure 1, we find the mean value of \( H_0 \) for the CMB-only likelihood to be 69.4 km s\(^{-1}\) Mpc\(^{-1}\). The WMAP/2014 \( H_0 \) and \( D_V \) values are entirely consistent with the BAO+\( H_0 \) likelihood; the difference has a \( \chi^2 \) of 2.5 for 2 degrees of freedom, with a probability to exceed (PTE) of 28%. Combining the three data sets gives a weighted mean of \( H_0 = 69.5 \) km s\(^{-1}\) Mpc\(^{-1}\), which is also clearly consistent with the individual data sets. The maxima of the separate CMB and BAO+\( H_0 \) likelihoods in this space are indicated with black filled circles, and the best fit to both distributions is indicated with an open square. The best-fit \( H_0 \) value for the CMB, local \( H_0 \), and BAO\(_{\odot} = 0.57 \) likelihoods is also given in the figure. Because the more sophisticated Gaussian mixture fit changes the results only very slightly, we consider the single Gaussian approximation to be acceptable for checking consistency of the fit.

Using an identical analysis for the (WMAP9+ACT+SPT)/2014 data, we find the mean of the CMB-only likelihood to be 71.1 km s\(^{-1}\) Mpc\(^{-1}\), with a \( \chi^2 \) for the difference of 3.8, with a PTE of 15%. For the combined CMB+\( H_0 \)+BAO\(_{\odot} = 0.57 \) data, we find \( H_0 = 69.8 \) km s\(^{-1}\) Mpc\(^{-1}\). Again, this is quite close to the Gaussian mixture result in the middle panel of Figure 1.

For the Planck-based CMB data in the bottom panel of Figure 1, we find the mean value of \( H_0 \) for the CMB-only likelihood to be 67.3 km s\(^{-1}\) Mpc\(^{-1}\). This is only slightly less consistent with the BAO+\( H_0 \) likelihood; the difference has a \( \chi^2 \) of 4.6 for 2 degrees of freedom and a PTE of 10%. The weighted mean of CMB+\( H_0 \)+BAO\(_{\odot} = 0.57 \) is 68.6 km s\(^{-1}\) Mpc\(^{-1}\). This is 1.8\( \sigma \) from the central value of the local Hubble prior, and so is not significantly different.
with caution. The Gaussian mixture approach used is not ideal so the contours should be viewed as space curvature and/or an equation of state parameter $\omega \neq -1$, and/or massive neutrinos. For models only weakly constrained by CMB data, the 2D likelihoods used (CMB, BAO) can be converted to a number of standard deviations. None of these values are above 2σ, so the combined parameters are consistent with each individual likelihood.

As shown in Figure 1, both Planck and WMAP data fall along the same degeneracy line. This is primarily because they provide consistent measurements of the angular scale of the CMB peaks (see Page et al. 2003 and Percival et al. 2002 for a discussion of these angular scales). The Planck result itself prefers slightly lower $H_0$ values than WMAP, but when combined with BAO data we see that the BAO data tend to pull Planck upward and WMAP downward, yielding closer agreement between combined $H_0$ values for CMB+BAO. Therefore, it does not matter greatly whether we use the WMAP9+ACT+SPT or Planck+ACT+SPT data combination since they both constrain the ultimate $H_0$ fit value by virtue of following the same CMB degeneracy line. Since the Planck team has only had one data release so far and has not yet released revised chains reflecting the correction for the 4 K cooler line effect mentioned in Section 2.3.2, we consider WMAP9+ACT+SPT CMB data in the remainder of this analysis. We look forward to future releases of Planck data, and in particular the updated chains and accompanying likelihood which will permit further independent analyses.

In Figures 2 and 3 we plot constraints on $H_0$ and a BAO-measured distance to $z = 0.57$ for various cosmological models, to investigate the dependence on model. We find that the constraint from the CMB on all the models in Figure 2 is largely the same; all these models predict the same expansion history.
of the universe. This includes a tensor to scalar ratio $r$, running of the spectral index with log wave number $k$ in Mpc$^{-1}$ given by $dn_s/\ln k$, both running and $r$, and correlated and uncorrelated isocurvature modes.

Figure 3 shows models with the following additional parameters: curvature, equation of state $w \neq -1$, curvature along with $w \neq -1$, sum of the neutrino masses, sum of the neutrino masses along with $w \neq -1$, a varying primordial helium fraction $Y_H$, and number of effective degrees of freedom in the early universe ($N_{\text{eff}}$). While the models in Figure 2 all give substantially similar results, the models that vary $N_{\text{eff}}$ or the equation of state in Figure 3 allow more freedom in the $D_V$, $H_0$ plane. In some cases, this freedom provides very little CMB constraint at all, thus allowing $\Delta H_0 H_0$ to dominate the combined result. In the case of the $w$CDM plot with massive neutrinos, all that can really be concluded is that the CMB likelihood is relatively flat over the BAO and $H_0$ best-fit region, and still consistent with the WMAP9 cosmological priors.

In practice, WMAP and Planck chains that include BAO data (such as those presented in Table 2) utilize BAO measurements over a range of redshifts. None of the currently released chains have included BAO data for $z > 1$. Representative BAO measurements at various redshifts (see Table 1) are plotted in Figure 4, for comparison purposes. Because these data mostly constrain the expansion history, we plot these as constraints on $\Omega_m$ and $H_0$. If we assume zero curvature, then $\Omega_m + \Omega_\Lambda = 1$ (neglecting tiny amounts of radiation and neutrinos), and so $\Omega_m$ and $H_0$ determine nearly the entire expansion history of the universe, and certainly the region measured by BAO. To obtain the drag radius that is directly measured by BAO, we assume the standard effective number of neutrinos ($N_{\text{eff}} = 3.046$), and set $\Omega_b h^2 = 0.02223$ as measured by Bennett et al. (2013). Modification of these numbers slightly changes $r_d$ and therefore the contours on this plot, but not substantially. In this parameter space, it is clear that BAO measurements at different redshifts have different degeneracy directions, so that the inclusion of $z = 2.34$ along with the lower redshifts provides a fairly strong BAO-only constraint (see also Addison et al. 2013). In addition, we plot the 1$\sigma$ and 2$\sigma$ likelihood contours for the combination of BAO at all four redshifts, the local $H_0$ measurement, and (WMAP9+ACT+SPT)/2014. There is no significant tension between our mean value and these measurements. The $\chi^2$ of our best-fit point with the four BAO likelihoods is 3.5 for 4 degrees of freedom. The best-fit $H_0 = 69.6 \pm 0.7$ km s$^{-1}$ Mpc$^{-1}$ is 1.4$\sigma$ away from the local $H_0$ measurement. Finally, the best-fit point is within the 1$\sigma$ contour of the CMB points in this projection, so it is also a good fit to the CMB data.

4. CONCLUSIONS

1. Our combined data set with CMB, BAO, and distance ladder $H_0$ determinations results in a best-estimate concordance Hubble constant value of $69.6 \pm 0.7$ km s$^{-1}$ Mpc$^{-1}$. Given the currently available data, we argue that there is no compelling need for physics beyond standard flat ΛCDM to establish consistency between the $H_0$ values derived using three separate but complementary methods.

2. Following a similar analysis, the concordance value of $\Omega_m = 0.286 \pm 0.008$.

3. The previous ≈2.5$\sigma$ tension between the $H_0$ values determined by Planck Collaboration (2014) and Riess et al.
(2011) has been recently mitigated somewhat (to < 2σ) by a decrease in the distance ladder value by roughly 0.3σ (Riess, 2014), based on a recalibrated megamaser distance) and a slight increase (also by ≈0.3σ) in the Planck value derived assuming ΛCDM after correction for a 4 K cooler line instrumental effect (Planck Collaboration 2014, Appendix C.4). Since the data incorporating the cooling line correction are not yet public, we eagerly await the further insight into the Hubble constant that will become available with the next Planck data release.

4. Within the 2D $D_L$ versus $H_0$ parameter space, CMB constraints tend to be confined along a thin diagonal which is defined primarily by the acoustic scale of the CMB power spectrum. As shown in Figure 1, both Planck and WMAP data provide very consistent measurements of the angular scale of the CMB peaks. Planck data occupy a slightly different location along the degeneracy line than do WMAP data. While the Planck result prefers a lower $H_0$ value than WMAP, the BAO results pull Planck upward and WMAP downward, yielding CMB+BAO+$H_0$ results that differ by roughly 1 km s$^{-1}$ Mpc$^{-1}$. If the Planck solution for $H_0$ in the next formal parameter release is significantly lower than the WMAP+ACT+SPT fit, this would primarily indicate tension among the CMB data sets, which would require resolution before concluding that evidence for new physics is present.

5. We have shown that additional parameters are not necessary to have consistency among the CMB, BAO, and $H_0$ results. Should future measurements substantially increase the tension, this could be resolved with additional model parameters. Figure 3 shows that this is most easily done with additional relativistic degrees of freedom in the early universe ($N_{	ext{eff}}$), or an equation of state not equal to $-1$, but we again emphasize that current data do not require nonstandard values for these parameters.

6. It is vital to continue to improve the various measurements of $H_0$ in order to search for new physics. Of particular interest at this time would be a local distance ladder. Future measurements that improve our knowledge of the CMB scale of the CMB peaks (e.g., Sloan Digital Sky Survey with a larger survey volume, Gaia and WFC3 spatial scanning parallax results for better calibrator distances) and look forward to new contributions and constraints that these may provide.

We thank Adam Riess for useful comments. We also thank Daniel Eisenstein for suggesting the use of a Gaussian mixture model to fit the Markov chain points, and an anonymous referee for suggesting helpful clarifications. We acknowledge use of the scikit-learn$^5$ code (Pedregosa et al. 2011) for testing Gaussian mixture models. This research was supported in part by NASA grant NNX14AF64G and by the Canadian Institute for Advanced Research (CIFAR). We acknowledge use of the HEALPix (Gorski et al. 2005) and CAMB (Lewis et al. 2000) packages. This research has made use of NASA’s Astrophysics Data System Bibliographic Services. We acknowledge the use of the Legacy Archive for Microwave Background Data Analysis (LAMBDA). Support for LAMBDA is provided by NASA Headquarters.

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$^5$ http://scikit-learn.org/stable/index.html