Semi-Empirical Error Ellipsoid Clustering for Identifying the Second-Order Structural Features From a Laboratory AE Source Location Cloud: Method, Validation, and Application to a Hydraulic Fracturing Test

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Abstract  Point cloud of sources in the form of acoustic emission (AE) or seismicity at both field- and laboratory-scales are indicative of causative structures. The error ellipsoid clustering (EC) method, whose underlying premise is reducing the amount of discarded data, is helpful for seeking the causative structure(s) from field-scale seismic sources. However, a proper EC method does not exist for laboratory sources. Interpretation of laboratory AE sources usually involves systematic discarding of the assumed “low-quality” data. We seek the successful transfer of the field-scale EC method into the investigation of laboratory-scale sources. For that, a new semi-empirical EC method (serendipity method) is devised, and we have systematically modified the previous EC method (standard method) in (1) error ellipsoid formation, (2) iterative clustering algorithm, and (3) clustering stopping criterion. In the literature, the standard method and its limitations were validated using synthetic data. We validate the modifications by comparing its post-clustering results with the original source locations and with the in-situ and post-mortem inspections of the laboratory rock fractures; and similarly for the serendipity method. We apply the serendipity method to a laboratory hydraulic fracturing test. The serendipity method implies the existence of a 3D damage zone as the second-order feature caused by the hydraulic fracturing in addition to the primary hydraulic fracture. This complex damage zone provides a logical explanation to the previously observed reverse permeability-distance relationship for hydraulic fracture.

Plain Language Summary  Point cloud source locations of acoustic emission (AE) or seismicity at both field- and laboratory-scales are indicative of their underlying causative structures. In efforts to reduce the amount of data discarded throughout the analysis of point cloud source data, the source location errors have been used to modify/re-locate events based on their probable locations, therefore helping to identify first and second order structural features of the subsurface. Here, we transfer a field-scale method for error ellipsoid based source relocation to the laboratory-scale, and in doing so we identify modifications that must be implemented based on our validation exercises. The error ellipsoids and the statistical collapsing of source locations has successfully identified both large scale fracture structure as well as secondary fracture features that would otherwise not be recognized with acoustic emission or seismicity data.

1. Introduction

Point cloud sources from seismicity, micro-seismicity, or laboratory acoustic emissions (AEs) released from rock fractures are intuitively indicative of causative structures. The cloud spans can cross multitude orders of scale magnitudes, that is, from laboratory-scale fractures on the sub-meter-size specimens to geological scale faults in the length of tens of kilometers. The investigations at the laboratory-scale are often valuable to understand complex physical processes at geologic space and time scales (Xiong & Hampton, 2020a). Phenomena observed under well-controlled laboratory conditions can possibly provide additional information for better and clearer understanding on the field-scale physical processes, whereas the in-situ and large field-scale measurements and characterizations could sometimes be difficult or inaccessible. The analyses and the interpretation of the source location cloud from laboratory tests, however, can be similarly as challenging as its field-scale counterparts.

Traditional methods of extracting/interpreting the causative structures from the point cloud include the “abandoning” of “unqualified” or low-quality data in many ways. At the field-scale, locations of very large uncertainties that probably have displaced too far from the underlying structures would likely to be discarded ( Mori
According to the reported laboratory AE tests on different types of rocks from different research groups, the movement of all the known events in the EC algorithm. However, the EC result, however, can be fundamentally sabotaged around the known ones would possibly be detected by an array of higher sensitivity or better coverage. These following scenario: for the caldera ring fault catalog (Jones & Stewart, 1997), additional small magnitude sources around the known ones would possibly be detected by an array of higher sensitivity or better coverage. These additional sources can obey the Gutenberg-Richter law because of scale-invariance, and their number can be exponentially decaying with distance to the known ones. The EC result, however, can be fundamentally sabotaged once enough small magnitude sources surrounding the larger known ones have been collected, and “lock” the movement of all the known events in the EC algorithm.

Regardless of the success of the EC method in field-scale applications and its apparent promising application in the investigation of laboratory AE sources, we can find only a few examples of such application at the laboratory-scale; the detailed information on how to materialize a practical laboratory EC method is also very limited among them. In principle, success of the EC method application depends on a combination of factors. For instance, the combination of system sensitivity and b-value (of the Gutenberg-Richter law) can significantly alter the average density of sources taken into the EC algorithm. Greater sensitivity of the system can make the number of the locatable events increase along the exponential curve of the corresponding b-value. Imagining the following scenario: for the caldera ring fault catalog (Jones & Stewart, 1997), additional small magnitude sources around the known ones would possibly be detected by an array of higher sensitivity or better coverage. These additional sources can obey the Gutenberg-Richter law because of scale-invariance, and their number can be exponentially decaying with distance to the known ones. The EC result, however, can be fundamentally sabotaged once enough small magnitude sources surrounding the larger known ones have been collected, and “lock” the movement of all the known events in the EC algorithm.

According to the reported laboratory AE tests on different types of rocks from different research groups, the magnitude range of above-Mc (i.e., above the magnitude of completeness) laboratory AE sources can span only about one to three magnitudes (Kolář et al., 2020; Kwiatek et al., 2014; Lei, 2003; Lei et al., 2000, 2003; Petružálek et al., 2018; Petružálek et al., 2020; Wong & Xiong, 2018; Xiong & Hampton, 2020a), and the overall magnitude range would span less than 4 magnitudes (Goebel et al., 2017; Kwiatek et al., 2014; Xiong & Hampton, 2020a) with considerably fewer sources at the two ends of this span. Such a narrow magnitude span of thousands of sources within a limited space of laboratory-scale specimens makes the direct (unconditional) application of the standard EC method to laboratory AE sources awkward. Systematic investigation on, and the corresponding modifications for, the application of the field-scale EC method to the laboratory-scale, is still missing. Many issues from the error ellipsoid formation to the clustering stopping criterion must be correctly addressed before the success of this application from field-to laboratory-scales becomes possible. In this paper, we attempt to accomplish this task through a semi-empirical method. We first present the success as well as the limitations of the direct application of the standard method into the laboratory catalogs. In this part, we confirm that the parameter tuning is unavoidable in the standard method, and the determination of a proper tuning is extremely subjective. Later we will show that, some modifications in the algorithm of the standard method can achieve equivalent success of the standard method. Such modifications will be used for the serendipity method in following. Lastly, we propose a serendipity method which has no need for algorithm parameter tuning (i.e., a non-retrospective EC method).
2. Methodology: The Standard Method, Modifications, and Serendipity Method

2.1. Fundamental Principles of the Standard Method

The standard error EC method was proposed by Jones and Stewart (1997). Its fundamental notion is to use the uncertainties of the sources as the guidance of their relocation process. It assumes the joint uncertainty distribution of the three spatial errors (for the sources) is a chi-square distribution of 3 degrees of freedom ($\chi^2_3$) (Evernden, 1969), where each of the three spatial errors is normally distributed, and the variance (for the source location) is known or can be properly estimated (Jones & Stewart, 1997). Individually, a certain confidence level ($\alpha$) of encountering the “true” location of one source in the 3D space can be visualized/conceptualized as an ellipsoid centered at that raw (original) source location. Globally, the normalized 3D displacements away from the “true” locations for all raw source locations, due to the uncertainties, are following the square-root of $\chi^2_3$ distribution. Here the normalized displacement refers to the real displacement normalized by the standard deviation of the error ellipsoid for each source. The equation representing the error ellipsoid, that is, linking the source location solution, the displacement distribution, and the confidence level together, mathematically is,

$$(X^T \cdot X) = \sigma^2 Cov^{-1} \chi^2_3(\alpha)$$

where $X$ is the three-component vector of 3D source location solution. $\sigma^2$ is the data variance. $Cov^{-1}$ is the inverse of the covariance matrix, which reflects the shape and the orientation features of the ellipsoid (i.e., the ratios among the three ellipsoidal axes, and their orientations) due to the bias introduced by the array of observational stations. In the laboratory, it is the sensor array. Noticing the left side of the above equation, the three-component vector of the 3D source location solution, by this moment, is defined through the data variance and the confidence level of the distribution in the right. According to the above expression, every source could be visualized/conceptualized as an ellipsoid instead of a point.

For the optimization function solving the source location, its covariance should be, $Cov = A^T \cdot A$, where $A^T$ and $A$ depend on the first derivative of the optimization function (Flinn, 1965),

$$A = \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ \vdots & \vdots & \vdots \end{bmatrix}$$

$$V \times \left\{ \sqrt{(x_i - x_i)^2 + (y_i - y_i)^2 + (z_i - z_i)^2} \right\}$$

The $x_i$ and $y_i$ are the coordinates of source solution and observational stations in the $X$ axis, respectively. Coordinate information for the other two axes (i.e., $Y$ and $Z$ axes) are correspondingly represented. $V$ is the wave velocity of the corresponding phase.

As all sources are expressed as ellipsoids centered at their raw solutions (points), it is possible to obtain a clearer delineation on the causative structure from the source point cloud by (a) individually moving each source (as a point) within the error ellipsoid of the confidence level $\alpha$, and (b) globally ensuring the square of the normalized displacement of source relocation regains the $\chi^2_3$ distribution. These two principles plus an source-movement-iteration stopping criterion are the essential elements of the standard method. To materialize these two principles, Jones and Stewart (1997) have proposed a two-loop algorithm containing an inner loop and an outer loop. In each iteration step, a new generation of points will be produced, and the error ellipsoids (and their centers, which are the locations of the raw solutions) will remain unchanged throughout the iterations for all generations. Details of the inner loop and outer loop are provided in Supporting Information S1. The stopping criterion stops the algorithm at “either the fraction moved has been reduced 4 times”, or the reduction of misfit between the displacement distribution at a particular generation and the theoretical distribution is less than 1%. The two-loop algorithm and the stopping criterion are apt to change when the EC method is applied to the investigation of laboratory AE sources.

2.2. Theoretical Validation and Limitation of the Standard Method

Jones and Stewart (1997) had validated the above algorithm using synthetic data (Figure 1a). Synthetic points are generated from the causative structures (upper row of Figure 1a), and randomly move away from them following pre-defined uncertainty/displacement distribution (middle row of Figure 1a). The shape of the causative structures, which continue in these two examples, imbedded within the point clouds can clearly become much sharper after EC (lower row of Figure 1a), though the delineation of the causative structure extents has slightly diminished (i.e., from upper row to lower row of Figure 1). Based on the validation through synthetic data, Jones
and Stewart (1997) had argued that, the standard method “is useful when the data variance is known exactly”; the
tendency of shrinking the original structure (lower row of Figure 1a) can be viewed as secondary imperfection or
acceptable tradeoff for the recovery of first-order structure. This tendency in turn emphasizes the importance of
proper estimation on the error ellipsoid size (the variance estimation and the confidence level selection) when the
standard method is applied to real laboratory data.

We validate our code following the same procedure, along with which we will illustrate the possible limitation
of the standard method in laboratory AE source applications. One simplest example of single-point causative
structure is presented in Figure 1b, where thousands of points are generated, randomly move away (within ±3σ)
from the causative point, and recover back to the causative point after EC. The distribution for the square of the
normalized displacement also near-perfectly fits the theoretical $\chi^2$ distribution after the points move back to their
causative structure (i.e., the middle inset of Figure 1b).

The critical issue for the standard method in the application of laboratory AE data analysis is however, the
possible over-simplification of the original causative structure and the systematic overlooking of the possible
second-order structures lying within a seemingly continuous point cloud. One example of this is presented in
Figure 1c. Two point clouds generated by two close but distinctive causative structures can produce one contin-
uous artificial line structure via the EC method. This over-simplification is relatively expected as according to
Jones and Stewart (1997), “one fault slipping twice is a simpler causative model than two separate faults each
slipping once”. As a consequence, distinctive structures can be overlooked especially when their distance are less
than the sum of their variance. The overlooking of the second-order structures is, however, not acceptable for the
EC application in analyzing laboratory AE sources where the first-order structure can be apparent even without
EC. Thus, provides one of the main objectives for using EC or any modification of EC in the laboratory: identi-
fication of both first- and second-order structures.

Such unfavorable artificial recovery can also manifest as differing structural patterns. For instance, three points or
three points plus rings surrounding them can be reproduced from the cloud generated by the two-point causative

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**Figure 1.** (a) Validation of the EC method in recovering continuous structures, after Jones and Stewart (1997), where 1,000 and 2,000 points have been generated for the circle and the cross, respectively. (b) Synthetic sources from a point structure (left) and its recovery after EC (right) by our code. Distribution for the square of displacements obeys $\chi^2$ distribution (middle inset of (b)). (c) Artificial recovery of a continuous line structure from two partially overlapping point clouds where the true causative structures are two discontinuous points (indicated as two red stars).
structures under different error ellipsoid sizes (i.e., altering variance and confidence level). This trial-and-error process of changing the error ellipsoid size for achieving the optimized recovery of the causative structure(s) will later be referred as algorithm parameter tuning. Noticing that the variances for all sets of the synthetic data are exactly known in the above examples, while the variances for real laboratory AE sources are necessarily to be estimated in the application of the EC methods.

2.3. Modifications, the Modified Method

Our final goal is to devise an EC method for the application of laboratory AE sources. To achieve that, we first attempt to modify the following three aspects of the standard method: error ellipsoid size estimation, iteration procedure, and the stopping criterion. These modifications must be validated as can achieve equivalent improvement of the standard method, and the two principles in devising the algorithm (and stopping criterion) of the standard method remain. The tuning process for this modified method remains unchanged. This modified method is an intermediate method between the standard method and the serendipity method, where the latter one for laboratory data analysis can avoid the algorithm parameter tuning process entirely. Within this modified method, we validate that the modifications of error (variance) estimation, iteration, and stopping criterion have properly addressed the features of laboratory sources, and can be used in the serendipity method.

We first modify the variance of the sources within the laboratory catalog to be equal. This exercise is performed because, unlike field-scale seismology, laboratory acquired AEs are all “close source” waveforms. The sensor array should provide good sensor coverage, where sensors are deployed at locations near signal sources, for example, the plausible rock fracture paths. The picking time of near-field waveforms should be of high accuracy and low uncertainty. The estimation of variance for the modified method does not vary as significantly as the standard method. This modification is likely not an accurate reflection of the source variations. However, we believe, according to the results of the modified method (see the following Section 3.2), it can be a reasonable modification for the application to laboratory sources as compared to the estimation used in the standard method.

The iteration and stopping criterion of the standard method makes the algorithm parameter tuning process extremely time consuming. As such we modify the inner and outer loops of the iterations and the stopping criterion. For the inner loop, we let the center of the error ellipsoid shift with the source locations of its following generations. This change can significantly accelerate the overall speed of source relocation throughout the iterations. It is also helpful for avoiding the previously observed clumpy phenomena (will be illustrated in Section 3.1), because the originally low source density area can hardly attract relocated sources; and areas of higher source density can be more attractive in the following iterations as more error ellipsoid centers will move in with the relocated sources. Upon this modification, the accumulated displacements for some sources can move them outside of their original error ellipsoids, which violates the first principle (i.e., moving the individual source point only within the error ellipsoid). As such, we adjust the outer loop to check the accumulative displacement of the sources. Once any source's accumulative displacement has moved the source outside its original error ellipsoid (centered at its raw source location), this source will be moved back to the location of its previous generation (i.e., from i+1 generation to i−1 generation), and so as the center of its error ellipsoid. These changes are consistent with the two principles that Jones and Stewart (1997) respected in designing the standard method: (a) individually moving each source (as a point) within the error ellipsoid, and (b) globally making the square of the normalized displacement of source relocation regain the $\chi^2$ distribution.

For the stopping criterion, we introduce the concept of best apparent-approximation to $\chi^2$ distribution and modify the criterion to stop at the generation whose distribution for the square of displacement has achieved the best apparent-approximation. The apparent-approximation considers the distribution for the square of displacement excluding the “unmoved” points, which are the sources where their displacements are almost zero. For instance, in the upper-left insert of Figure 2c the highest bin in the histogram for the square of displacement is always close to zero and will be considered as the excluding part. One apparent and straightforward way to judge the “best-fit” approximation is that, after excluding the unmoved sources, the highest probability/frequency of the rest is located on the likelihood peak of $\chi^2$ distribution, which is one.

Such modification appears to have violated the second principle (ensuring the displacement of source relocation regains the presumed $\chi^2$ distribution). However, the large “unmoved” population of laboratory AE events should be an inherent structural feature for laboratory AE sources. This is because, in general, laboratory AE sources can
be generated from the process including the rock fracture creation process as well as the following rock fracture evolution (Xiong et al., 2021). Events in the rock fracture creation are essentially of high spatial density. While for field-scale seismology the seismogenic structure has already existed for millions of years, and the ruptures belonging to the creation of such structure are temporally outside the observation. To note that, a large number of “unmoved” events can also exist in the standard method (i.e., the upper insert of Figure 2c). The distributed damage at the beginning of the tests, and the ultra-high concentration of AE events along the rock fracture creation path can be two sources of these unmoved AE events. The former can be unmoved for randomly occurring events at locations far away from any structure, the latter can also be unmoved as they are at the very center line of the structure inside the point cloud. Those unmoved points, however, will not deteriorate the rock structure delineation. On the contrary, the unmoved points along the center line of the structure within the laboratory AE sources would be the reason of ensuring the first-order structure delineation by the AE point cloud even before applying any EC method.
After these modifications, the iteration becomes a one-way process. The iteration will stop at the generation whose apparent-approximation (of the square of displacement) to $\chi^2$ distribution is better than its former and later generations. No adjuction on the factor of the shifting vector (i.e., the vector from its present location to the calculated centroid multiplying a factor) has been considered in this modified method. Overlooking the adjustment of that factor can on the one hand, greatly reduce the time consumed during the algorithm parameter tuning process; on the other hand, such overlooking also aligns with the goal of this paper, that is, devising a non-retrospective EC method.

### 2.4. Serendipity Method

Finally, we devise a new EC method to free the process from the algorithm parameter tuning. The basic idea behind this new method is to make further use of laboratory available information in the formation of error ellipsoids. Serendipitously, the potential second-order structural features can also be delineated by applying this method. We perturb the velocity to a value lower than that measured pre-test so that the global signal residual distribution, especially the likelihood peak, can approximate that of the $\chi^2$ distribution as much as possible. The signal residual is the residual for each observational station attributing to this particular source solution, which can be represented as the difference between the observed and the calculated time delays. At the $i$th station/sensor, the observed time delay is $t_i - t_i$, where $t_i$ is the signal’s actual arrival time at $i$th sensor. The corresponding calculated time delay is:

$$
\frac{x_i - y_i - z_i}{V}
$$

where the vector $\left( \begin{array}{ccc} x_i & y_i & z_i \end{array} \right)$ stands for the spatial distance between the source solution and the $i$th sensor, and $V$ for the wave velocity of the corresponding phase, that is, the P-phase for laboratory source location.

We assume the obtained signal residual distribution after the velocity perturbation can be globally a good approximation to the real uncertainty of the sources, so that useful information is contained for reaching better delineation on the causative (AE-genic) structure(s). The downward velocity perturbation is from the consideration of the velocity degradation caused by the rock fracture process. After the rock mass has been damaged, its global velocity will decrease and will be lower than the measured value preceding the start of the experiment. This change can be one of the primary sources for the uncertainties in laboratory AE analysis as experiments progress and damage accumulates. The estimation of variance by Jones and Stewart (1997) on the contrary, has considered only the measurement uncertainty. Following the standard method, the velocity model uncertainty “are forced to ignore them” since we “have no formal way to treat” them (Jones & Stewart, 1997). After the velocity perturbation, the error ellipsoid will be built as follows: (a) In shape, the error ellipsoid should have an ellipsoid surface that can best approximate the 3D vectors of all related signal residuals once that ellipsoid is the proper size. (b) In size, the error ellipsoid should be proportional to the magnitude of averaged (absolute) signal residuals. As such, the size of error ellipsoid is directly connected to a physical quantity, that is, the directional residual in $\mu$s multiplied by the perturbed velocity in mm/\(\mu\)s (mm/\(\mu\)s $\equiv$ km/s).

To avoiding an irregular ellipsoid, the ratio between the lengths of the longest, the intermediate, and the shortest ellipsoid axes are constrained to only vary within a range. We set the following equations as the confinement for the ratios of the three ellipsoid axes:

$$
1 \leq \frac{L_{\text{long}}}{L_{\text{short}}} \leq \frac{\max(T_{\text{residuals}})}{\min(T_{\text{residuals}})}
$$

$$
1 \leq \frac{L_{\text{intermediate}}}{L_{\text{short}}} \leq \frac{\max(T_{\text{residuals}})}{\min(T_{\text{residuals}})}
$$

where $L_{\text{long}}$, $L_{\text{intermediate}}$, and $L_{\text{short}}$, are the lengths of the longest, the intermediate, and the shortest ellipsoid axes, respectively; $\frac{\max(T_{\text{residuals}})}{\min(T_{\text{residuals}})}$ is the ratio between the related maximum and minimum signal residuals ($T_{\text{residuals}}$). Once the ratio between any pair of the ellipsoid axes is beyond this range, we will use fixed ratios to reshape the ellipsoid. For $\frac{L_{\text{long}}}{L_{\text{short}}}$, it is $\frac{L_{\text{long}}}{L_{\text{short}}} = \frac{\max(T_{\text{residuals}})}{\min(T_{\text{residuals}})}$. For $\frac{L_{\text{intermediate}}}{L_{\text{short}}}$, it is $\frac{L_{\text{intermediate}}}{L_{\text{short}}} = \frac{\max(T_{\text{residuals}})}{\min(T_{\text{residuals}})}$. We have also...
imposed the orientation of the longest ellipsoid axis parallel to the direction of the maximum signal residual. This enforcement ensures the ellipsoid function can be solvable by the minimum number of equations for the source location (i.e., a number of 4 equations for 3 spatial and 1 temporal variables of source locations). Detailed mathematical procedures for forming the error ellipsoid satisfying the first two requirements are presented in Supporting Document-I in Supporting Information S1.

3. Results

3.1. Standard Method After Tuning

We tentatively apply the standard method to one series of data sets from previously obtained laboratory AE sources (Wong & Xiong, 2018). This series of data is useful for validating the efficiency of the standard method as well as for understanding its limitations in real laboratory applications. It is because, the in-situ real time optical observation on the rock fracture process has been recorded by the camcorder simultaneously with the monitoring of AEs. Post-mortem inspection on the rock fractures has also been conducted on each post-testing specimen of this series of tests. For this series of tests, the rock fractures are induced by a central elongated single flaw within prismatic Carrara marble specimens. One example from this series of tests has been illustrated in Figure 2a. Such type of tests has been extensively conducted since the 1960s, for instance, Brace and Bombolakis (1963); Lajtai (1971). As a result, the possible paths of the induced rock fractures from the central flaw are well known. This kind of a priori knowledge is useful for the sensor array arrangement, which is crucial for achieving accurate raw AE source locations. In total, five AE data sets from five different tests are available. Such number of data sets is enough to validate if any of the observed features for applying the standard method to real laboratory data is universal instead of unique. AEs for all tests of this series are recorded via a 16-channel AE system and the AE source locations are well documented (Xiong, 2019). More experimental details about this series of tests can be found in Wong and Xiong (2018).

The raw (original) AE source locations for the example test in Figure 2a is presented in Figure 2b. The first-order feature of the rock fracture process, that is, the quasi-symmetrical extension of the two rock fractures initiating from the two tips of the central flaw, has already been clearly delineated without EC. We follow exactly the standard method by Jones and Stewart (1997), that is, setting the confidence level to four standard deviations and estimating the variance by “the sum of the squares of the residuals divided by (n−4)” (Peters & Crosson, 1972), where n is the number of the arrival time picks for the source location, and so on. The result is presented in Figure 2c. The after-EC source locations have shrunk into one continuous rock fracture. The non-AE-genic zone (i.e., central open flaw) has been filled with points of relocated AE sources. This EC result can hardly be viewed as successful for its laboratory application, and such failure of application is universal to all five sets of AE source locations (see Supporting Information S1). Such over clustering phenomenon in laboratory AE sources will be referred as clumpy phenomenon in the following. Under clumpy conditions, the first-order feature of the laboratory rock fracture is blurred and somewhat obscured, therefore resulting in a poorer representation of the first-order structure as compared to just the raw AE event locations alone.

We have systematically tuned the algorithm parameter for changing the size of error ellipsoid in the standard method. For all the values tested by Jones and Stewart (1997), we are unable to obtain a satisfactory EC result (i.e., a result which does not obscure the known first order structure). Only when we reduce the variance to a value approaching 0.1 of its originally estimated value, the clumpy phenomenon can be partially mitigated. However, along with the progressive mitigation of clumpy phenomenon, the post-EC source locations progressively move toward their raw locations—meaning as the error ellipsoid reduces to the zero-limit, the number of AE relocations approaches the zero-limit. Although “improved” source locations are attainable, the algorithm parameter tuning process is still very time consuming; one needs to delicately counter the balance between totally no improvement, that is, decreasing the size of error ellipsoids to the extreme that no point can move, and totally clumpy relocated sources. The estimation of the variance, which has been proven successful for the analysis of field-scale sources (i.e., Asanuma et al. (2005); Jones and Stewart (1997)), appears not suitable for the EC application in laboratory sources.

The iterative algorithm and the stopping criterion in the standard method appear to need modifications for its laboratory application. The square of the (normalized) displacement following the standard method (upper-right insert of Figure 2c) shows an exponentially decreasing distribution; the majority consist of small displacements,
3.2. Application of the Modified Method

We illustrate the application of the modified method to the identical example presented in Figure 2. The reason for using this example for illustration is, (a) the raw AE source locations have successfully depicted two major rock fractures as first-order structural features, and these two traces match well with the in-situ optical observation of rock fracture paths (Figures 2a and 2b); (b) bifurcations of the lower rock fracture as the second-order structural feature are observed through the in-situ optical observation as well as the post-mortem inspection (Figures 2a and 2d), however are embedded within the point cloud and can hardly be distinguished (Figure 2b). As such, it is useful for illustrating the improvement of structure delineation, especially for the potential of second-order structure delineation by applying the modified method. Examples of the applications of the modified method can be found in Supporting Information S1.

We tune the algorithm parameter, that is the multiplying factor for the unit variance, from one to five for the error ellipsoid. The incremental interval between two successive tunings is one, and a total five tunings for each of the five AE data sets. This corresponds to $\sqrt{1}$ to $\sqrt{5}$ for the scale of the error ellipsoid (scaling factor). The tunings from 1 to 5 with an incremental interval of one will have 25 trials for five different data sets. The results for the remaining four AE data sets are presented in Supporting Information S1. This tuning process is inherently necessary for the modified method because the variance estimation has been modified in this method, and under this modification the proper range for the variance of laboratory data for creating a suitable size for the error ellipsoids is in principle unknown. Better results for each test/data set may be achievable once the incremental interval between tunings decrease. However, this decrease will make the number for tuning trials and the time consumed on them increase geometrically.

Clear improvement over the original source locations can be observed on the best tuning result (Figure 3a). This “best” tuning is achieved under scaling factor of $\sqrt{3}$ for this example. The bifurcations, as the second-order feature, have been clearly identified as compared to the original source locations. Cases of under and over clustering exist, although the changes on the ellipsoid scale from 1 to $\sqrt{3}$ is very small (i.e., comparing with Jones and Stewart (1997), they have attempted scaling factors from 0.25 to 4.0). For under clustering, the improvement is non-significant (Figure 3b, when scaling factor is one). The bifurcation begins to emerge within the point cloud, and the improvement is not as apparent as the best tuning. For over clustering, the bifurcations have been over simplified (i.e., three bifurcations reduce to two), and the length of the rock fracture shortens significantly (Figure 3c, when scaling factor is $\sqrt{5}$).

The post-mortem check indicates, two of the bifurcations are connected (labeled as 1, and two in Figure 2d, bifurcated at the back while connected at the front). This result may question if the best tuning trial (Figure 3a) is
actually better than the over tuning trial (Figure 3c). The Z axis distributions of the original and after-EC sources, however, show a clear over clustering tendency for the over tuning one (see Figure 4c lower row, blue histograms for original sources vs. orange histograms for after-EC sources). The displacement distributions of these trials suggest, the stopping criterion must use the modified one. Although the majority of the points will be moved, the laboratory sources can produce an outstanding “unmoved” bin with small movement in the histogram (see the displacement distribution for the under tuning trial, in Figure 4b upper row), which can easily sabotage the Kolmogorov test. Following the stopping criterion of the standard method, the iteration would have to be stopped with an infinitely large remaining misfit, or infinitely continuous. This phenomenon can be observed throughout all the five laboratory AE data sets.

For all five laboratory AE data sets, improved delineation of the rock fractures can be achieved via proper tuning. One important feature from the tuning of five different AE data sets should be noted. The best tuning results for different data sets can be achieved at different scaling factors (see Supporting Information S1), even if we only have used a comparatively coarse incremental interval, that makes only five tuning trials for each data set. This reality imposes a critical requirement on the successful application of EC method to laboratory AE data analysis: the algorithm parameter tuning process must not be included in an EC method suitable for the general application of laboratory AE data analysis, where the real-time cross verification for the best tuning trial is unavailable. This requirement is important for the analysis of tests having a rapid spatiotemporally evolving rock fracture process, like hydraulic fracture tests.

3.3. Application of the Serendipity Method

We apply the serendipity method to the five AE data sets. The modifications validated in the modified method, that is, the modified actions within the inner and outer loops of the iterations, the equal variance for the normalization of displacement, the apparent-approximation to $\chi^2$ distribution as the stopping criterion, are also used in the serendipity method. Only the example test will be presented for illustration, and the validation for the seren-
The serendipity method has again passed through all five tests (see Supporting Information S1). The algorithm parameter in the serendipity method is the perturbed velocity, which is the value inducing the best approximation on its likelihood peak to that of the $\chi^2$ distribution. As such, the entire process, from the algorithm parameter determination to reaching the final after-EC result of the serendipity method is non-retrospective—no retrospective algorithm parameter tuning process is involved.

We obtain the best perturbation for the example AE data set at $V = 4.6$ km/s with 0.1 km/s (km/s $\equiv$ mm/μs) resolution from the original velocity of 5.4792 km/s, where the likelihood peak of its signal residual distribution best approximates that of the $\chi^2$ distribution (the upper plot in Figure 5c). After applying the serendipity method, clearer delineation of the original source locations can be achieved (Figures 5a and 5b). Bifurcations can be observed in the result of serendipity method. This result subtly indicates a bifurcation condition lies between three bifurcations and two bifurcations (Figure 5b). This subtle condition appears to be a reasonable delineation to the real rock fracture bifurcation condition, which as we have mentioned previously, two of the three bifurcations are spatially connected. It is fair to argue that the best tuning result from the modified method can achieve better delineation of the bifurcation (Figure 5d left, or Figure 3a). However, the source location distribution along the thickness direction shows that, the serendipity method produces better resistance to the over clustering tendency (compare the lower-right plot in Figure 5c and the upper-right plot of Figure 5d). The best tuning result of the modified method shows clear tendency that its source location distribution along the thickness direction moves toward the center line of the specimen. The result of the serendipity method even shows a slightly lower hint of over clustering in its source location distribution along the thickness direction than the result of modified method under scaling factor of one (compare the lower-right plot in Figure 5c and the lower-right plot of Figure 5d).

Spatially, the length of the bifurcated fractures from the result of the serendipity method is also longer than that from the best tuning result of modified method. While the structural delineation of the serendipity method (Figure 5b) is better than that achieved in the modified method using a scaling factor of one (middle plot in Figure 5d, or Figure 3b). This result suggests, the serendipity method can be non-retrospectively applied for laboratory AE data sets while its achievable improvement on the AE-genic structure delineation will not be significantly weaker than that of the best tuning result from the modified method. This statement holds true for all the five different AE data sets; and for several of them, the serendipity method has better performance in catching the correct length of rock fractures than the modified method (see Supporting Information S1).

One thing should be noted: a wide range of downwardly perturbed velocity values has been attempted for the serendipity method, and this method has proven to have a great tolerance for the downward perturbation range. For instance, for this example AE data set, apparent improvement in the delineation of AE-genic structure can be achieved from a velocity of 5.0 km/s to a velocity of 3.7 km/s, with 0.1 km/s resolution. Among this range, the likelihood peak of the signal residual distribution from the perturbed velocity deviates only slightly from that of the $\chi^2$ distribution. However, failures or difficulties can arise from the extremes of under and over downward velocity perturbation (see Figure S1 in Supporting Information S1). For instance, using the un-perturbed (original) velocity for the error ellipsoid formation procedure of the serendipity method can lead to the clumpy phenomenon (Figure S1a in Supporting Information S1). While using the over-perturbed velocity for that error ellipsoid formation procedure, for instance when the velocity has perturbed down to 2.8 km/s for this example AE data set, can lead to another type of clumpy phenomenon (Figure S1b in Supporting Information S1).

From these observations, we suggest that the serendipity method using velocity perturbations is a safe way to non-retrospectively reach an improved AE-genic structure delineation for laboratory AE data sets as well as serendipitously identifying second-order structural features.

4. Application to a Hydraulic Fracturing Test

4.1. Previous Hydraulic Fracturing Data Analysis, and Un-Solved Problems

The hydraulic fracturing test was conducted on a prismatic (150 $\times$ 150 $\times$ 250 mm) granite specimen from South Dakota, USA (Hampton et al., 2019). This type of laboratory test is the analogue of the common hydraulic fracturing practice in the energy industry for enhancing the productivity of oil/gas and geothermal reservoirs. The specimen-equipment assembly of the hydraulic fracturing test is illustrated in Figure 6a. The AEs from the hydraulic fracturing process are recorded through a 12 channel AE system. They are temporally well separated into reverse Omori-Utsu (OU) law AE releases and regular OU law AE releases (Xiong and Hampton, 2020b,
An artificial fault was pre-cut into the specimen to investigate the fault and hydraulic fracture interactions (thick red line in Figure 6c). Although with limited signal waveform sampling rate (i.e., 2 μs/point), with the massive number of located sources (i.e., 8,357 sources), the evidence of hydraulic fracture reactivation due to the influence of the pre-fault can still be discovered (Xiong & Hampton 2021a, see Figure 6c). The clear downward growth and re-orientation of the AE point cloud toward the pre-fault is identified, and this re-orientation occurring at the period of regular OU law AE releases (Figure 6c right) is during a greatly reduced...
hydraulic fracturing pressure (Figure 6b). The previous research (Xiong & Hampton, 2021a) has also proposed that, the primary rock fracture process in the periods of reverse OU law AE releases (Figure 6b) was hydraulic fracture creation, while its following period of regular OU law AE releases is primarily for the further hydraulic fracture reactivation under greatly reduced wellbore hydraulic pressure (Figure 6b).

Permeability on two series of parallel sub-cores obtained post-mortem (left depiction in Figure 7a) are measured (Hampton et al., 2019). An inverse permeability-distance relationship (i.e., permeability decreases as distance from the specimen boundary to the hydraulic fracture increases) has been observed by combining the post-mortem permeability measurements (Figure 7a) and the in-situ AE point cloud, especially the AE point cloud for the reverse OU law period (Figure 7b). This inverse permeability-distance relationship states that, the further orthogonal distance of one post-mortem sub-core to the center line of AE cluster, the higher the permeability of that sub-core can be measured (Hampton et al., 2019; Xiong & Hampton, 2021a). This relationship is actually in agreement with the observation of permeability damage on core sample tests (Zhu & Wong, 1997; Zoback & Byerlee, 1975), which suggest the permeability of the rock material can actually decrease instead of increase after loading before the “brittle faulting” to “cataclastic flow” stage(s) (Wong et al., 1997). As such, to constitute that inverse permeability-distance relationship, the damage decreasing the permeability would be most likely from the hydraulic fracture induced damage zone instead of the actual macroscale hydraulic fracture. The post-mortem observation on the two sub-core series has also indicated that the actual hydraulic fracture was passing through the center of these two series of sub-cores (i.e., between sub-core number 4 and 5, of 8), which is different with the places where the lowest permeability has been measured within these cores.

We have also tried the traditional low-quality data filtering in previous analysis (Figures 7b and Xiong and Hampton (2021a)). The possible damage zones which are supportive to the inverse permeability-distance relationship (in XY planar projection) can be observed by abandoning the small magnitude events (Figure 7b right). However, second-order imperfection still exists (blue-circle in the middle plot of Figure 7a), and no comprehensive explanation for this imperfection can be achieved by merely systematically abandoning the “low-quality” data.

More importantly, the hydraulic fracturing process (including the actual hydraulic fracture creation and its following re-activation toward the pre-fault) are temporal and transient. Exact causative structure of that perme-
ability change cannot be directly proven through the post-mortem inspection on the hydraulic fractures, as such fractures are the cumulative consequence of a series of fracture evolution/processes. Further investigations of the interesting but transient phenomena in hydraulic fractures can primarily rely on the in-situ released energy from the hydraulic fracturing process, that is, the AE releases throughout the test. In other words, information on the transient behavior of fracture systems cannot be extracted from traditional post-mortem assessments.

4.2. Applying Serendipity Method to the Previous Hydraulic Fracturing AE Data Set

We apply the serendipity method to the data set of the hydraulic fracturing test (Hampton et al., 2019). To note, the delineation of any structure caused by hydraulic fracturing is in principle in 3D space, which introduces new challenges in the AE data analysis and interpretation. The original, and the after applying serendipity method source locations are presented through the three planar projections of these source locations. The serendipity method is separately applied to the AE sources of hydraulic fracturing from the reverse OU law period and from its following regular OU law period because they are two separate stages of the observed hydraulic fracturing (Figure 6b, also see Figure S2a in Supporting Information S1, Xiong and Hampton (2021a)). The best velocity perturbation (0.1 km/s resolution) is 5.5 km/s for the reverse OU law period, and 5.4 km/s for the regular OU law period. These perturbations coincidentally fit the physical understanding on the rock fracture process where the later stage of rock fracture(s) should induce a globally slower velocity.
In the YZ planar projections, the point cloud from the reverse OU law period after applying serendipity method (Figures 8a and 8b, also in Figure S2d in Supporting Information S1) clearly indicates a possible curved damage zone surrounding/circling the perimeter of core 1 (i.e., highlighted with red box in Figure 8b, also see Figure S2d in Supporting Information S1). This circling path of damage zone can be much more clearly indicated by the projection of original AE sources for the reverse OU law period (highlighted with red box in Figure 8a, also see Figure S2b in Supporting Information S1). For the regular OU law period, the three downward protrusions from the AE cloud (Figure 8d, and indicated with black arrows in Figure S2e in Supporting Information S1, toward the pre-fault) have clearly become much sharper after applying the serendipity method (Figure 8d, also see Figure S2e in Supporting Information S1) than their original shape (Figure 8c, also see Figure S2c in Supporting Information S1). Combining the delineations from the after-applying-serendipity-method projections for the reverse and the regular OU law periods, a circling 3D path of the damage zone becomes plausible (Figure 8e, also see Figure S2f in Supporting Information S1).

In the YZ planar projections, the point cloud from the reverse OU law period after applying serendipity method (Figures 8a and 8b, also in Figure S2d in Supporting Information S1) clearly indicates a possible curved damage zone surrounding/circling the perimeter of core 1 (i.e., highlighted with red box in Figure 8b, also see Figure S2d in Supporting Information S1). This circling path of damage zone can be much more clearly indicated by the projection of original AE sources for the reverse OU law period (highlighted with red box in Figure 8a, also see Figure S2b in Supporting Information S1). For the regular OU law period, the three downward protrusions from the AE cloud (Figure 8d, and indicated with black arrows in Figure S2e in Supporting Information S1, toward the pre-fault) have clearly become much sharper after applying the serendipity method (Figure 8d, also see Figure S2e in Supporting Information S1) than their original shape (Figure 8c, also see Figure S2c in Supporting Information S1). Combining the delineations from the after-applying-serendipity-method projections for the reverse and the regular OU law periods, a circling 3D path of the damage zone becomes plausible (Figure 8e, also see Figure S2f in Supporting Information S1).

We further check the XY planar projections for the original source location solutions of (a) the reverse OU law AE releases, and (b) the regular OU law AE releases; XY planar projections for the after-serendipity EC source locations of (c) the reverse OU law AE releases, and (d) the regular OU law AE releases. Figure 9. XY planar projections for the original source location solutions of (a) the reverse OU law AE releases, and (b) the regular OU law AE releases; XY planar projections for the after-serendipity EC source locations of (c) the reverse OU law AE releases, and (d) the regular OU law AE releases.
and four of core one; the other (the right one) coincides with sub-core number six of core 1. At this sub-core (number 6), permeability lower than its neighboring two sub-cores has been measured (see Figure 7a middle). As such, it is possible that the permeability damage measured on the post-mortem sub-cores of core one is correlated with two damage zones: one is toward the lowest permeability sub-cores (sub-cores numbers 3 and 4), and the other constitutes a 3D circling path around the spatial perimeter of the sub-core number 6 (Figures 7b and 9e). For the later regular OU law period, we also observe plausible structure on the after-applying-serendipity-method sources (highlighted with black box in Figure 9d). Comparing the AE clouds of the original sources (Figure 9b) and that of the after-applying-serendipity-method sources (Figure 9d), the latter become sharper and elongated in the core 2 direction (Figure 9d), and it points clearly toward the major permeability decrease within core 2.

In the XZ planar projections, we observe the AE cloud first moves upward from the open wellbore section in the reverse OU law period (Figures 10a and 10c); later, moves downward toward the fault in the following normal OU law period (Figures 10b and 10d). The delineation on the plausible downward damage growth from the after-applying-serendipity-method sources (Figure 10d) again becomes significantly sharper and more elongated than that of the original ones (Figure 10b). Visualizing the 3D damage growth path from upward (Figure 10b) to downward (Figure 10d), it happens to be very close to the sub-core having lower permeability than its neighboring two sub-cores (i.e., sub-core number six of core 1). As it is known that EC can possibly shrink the original causative structure, such spatial features for the after-applying-serendipity-method sources would be indicative of the plausible 3D circling damage zone.

Combining the evidence of the projections from three planes, we conceptualize/visualize a plausible 3D circling path of the damage zone caused by hydraulic fracturing (Figures 11a and 11b). Due to its 3D curvature, one can reasonably postulate that: the AE cloud of higher density in XY planar projection is at the orientation toward sub-core number six in core 1. This higher density cloud is caused by the overlapping contribution of the 3D circling path. As such, its density can even be higher than the AE cloud pointing toward the major permeability decrease within core 1 (in XY planar projection). The 3D circling path of the damage zone around the sub-core number six of core 1 however, can simultaneously weaken its influence on the permeability damage on the sub-core it encircles. Consequently, the lowest permeability will still be measured at the two sub-cores number 3 and 4 in core 1, while those two sub-cores can be observed to have relatively lower AE event density than sub-core 6 in core 1. The existence of this circling path has provided auxiliary explanation to the mismatch between the AE density and the post-mortem permeability measurements (blue-circled in the middle plot of Figure 7a).

4.3. Serendipity Method Versus Modified Method: From the Dilemma of Algorithm Parameter Tuning Trials for the Hydraulic Fracturing Data Set

We have further compared the result from serendipity method with the results from modified method in this hydraulic fracture test. Again, the algorithm parameter tuning covers the range from 1 to 5, and the reverse OU law period and the regular OU law period are analyzed separately. A variety of indicative evidence for the existence of the circling damage zone can be observed under different tuning trials. In YZ planar projections for instance, when the algorithm parameter = 2 (i.e., C2G1 in Figure S3a and b in Supporting Information S1). This evidence further supports the existence of that circling damage zone. However, one cannot non-retrospectively determine the proper tuning, and clumpy (or over clustering) can easily occur for a slight increase on the algorithm parameter (Figure S3 in Supporting Information S1). For over clustering, all AE sources are clumping.
toward the open borehole section (i.e., cases of C4G3 and C3G4 in Figure S3b in Supporting Information S1), and the first-order structural features begin to disappear.

The separation in the XY planar projection can also occur in some tuning trials of modified method (i.e., C3G2 and C4G3 in Figure S4a in Supporting Information S1). This observation is again positive for the existence of the circling damage zone. One critical issue for the algorithm parameter tuning process is, the algorithm parameter for the tuning trial making the circling path occur in YZ planar projection (Figure S3, c2 in Supporting Information S1) is different with that for the tuning trial making the separation in XY planar projection occur (Figure S4a, c3 in Supporting Information S1). Considering the limited tuning resolution, a deliberately selected algorithm parameter between 2 and 3 may possibly make the above-mentioned two phenomena in YZ and XY planar projections occur simultaneously. Unfortunately, however, one would have no a priori knowledge for when this deliberate selection has been achieved. Nonetheless, the time consumed for the higher resolution tuning will geometrically increase.

5. Discussion, Possible Mechanism for the Success of the Serendipity Method

The rock fracture process can decrease the velocity of wave propagation. The width of the rock fracture process zone (FPZ) can be significantly larger than that in metals and fine grain ceramics (Nolen-Hoeksema & Gordon, 1987), creating a gradient of decreased velocity around the actual fracture. In the laboratory, the velocity change due to the rock fracture process can only be updated with limited spatial and temporal resolutions, for instance, Lei et al. (2016); Li et al. (2016); Li et al. (2017); Zang et al. (2000), and so on, and those experiments having a velocity update are already of outstanding high-quality among the literature. Delineation of first-order

Figure 11. Depiction of the circling damage zone according to the evidence from a series of multiplanar projections.
feature(s) for the rock fracture process by AE sources, in most cases, can be achieved by the source locations without EC and without a velocity update. The second-order feature(s) however, can easily be overlooked, but are still of high importance.

Due to the gradient of velocity decrease in the rock FPZ, sources closer to the center path of the FPZ can have higher residual once its location is calculated under the unperturbed velocity (see Figure S5 in Supporting Information S1). For a source at the center path, the velocity in all directions will decrease. For a source at the boundary portion of that gradient, only the velocity toward the rock FPZ will significantly decrease and the velocity oriented outwardly will be less affected—that is, heterogeneous distribution of velocity changes due to microcracking at the specimen-scale. After the downward velocity perturbation, sources closer to the center path of the rock FPZ, at this moment, would likely have smaller residual. Consequently, smaller error ellipsoids will be attached with these close-to-center sources and will fix these sources with limited movable/displaceable space. Simultaneously, sources at the boundary portion would likely have larger residual and will be assigned larger error ellipsoids.

One serendipity benefit from the error ellipsoid formation process of the serendipity method may be that, after velocity perturbation, the longest axis of the error ellipsoids for the sources at the boundary portion would most likely point toward the center path of FPZ. It is because for those sources, the residuals for all outward source-to-sensor paths will increase after velocity perturbation (see Figure S5 in Supporting Information S1). This may serendipitously force the sources to move toward the real center path of the FPZ, achieving better delineation of even the second-order structures on rock fracture.

Another critical issue may affect the result of applying the serendipity method is the inherent source location error of the laboratory data set. The data sets of Wong and Xiong (2018) for the validations of methods are sampled at a higher resolution than the hydraulic fracturing data set (Hampton et al., 2019). As a result, one can clearly see that the improvements on the data sets of Wong and Xiong (2018) through modified or serendipity methods are significant. The analysis on the hydraulic fracturing data set has taken the advantage that a much larger number of AE events has been recorded in that test (Xiong & Hampton, 2021a). As such indicative changes on the density contours after applying the serendipity method can still be achieved. Some detailed settings in the source location algorithm will also influence the source location result. For example, the strategy of perceiving an algorithm output as physically real energy release can affect the number of located sources (Xiong & Hampton, 2020b), and the b-value of the Gutenberg-Richter law of a scale-invariant system can determine the range of signal-to-source ratio (Xiong & Hampton, 2021b). Such may indirectly affect the EC results.

One additional fact for the investigations on the laboratory AE sources should also be noticed. The AE analysis without post-mortem checks, or in situ optical observations as cross validation(s) can usually be perceived as a type of “soft” evidence. Out of this standpoint, the AE source location results as well as the projections after applying the serendipity method should not be accepted as stand-alone evidence to confirm or to reject the existence of a hypothesized structure. However, the results from the serendipity method as well as the results from the traditional low-quality data filtering (Figure 7b) have both provided suggestive information for the existence of a possible circling damage zone. Certain limitations have been encountered by merely applying the low-quality data filter. Extensive and positive observations of the existence of that circling damage zone has been obtained as the serendipity method is applied. The combined use of the traditional and the serendipity methods in AE data analysis can serve as the information provider for the following post-mortem exams. This type of information can be a great time saving indicator for the following post-mortem exams for a systematic laboratory-scale study.

6. Conclusion

We have investigated the translation of the field-scale EC method to the application of improving the causative structure delineation for laboratory AE sources. We first have distinguished the difficulties and problems of the direct application of the standard method to analyzing laboratory sources. Due to the features of laboratory AE sources, we designed a modified method where the following aspects for the standard method has been modified: (a) the stopping criterion for the algorithm iteration (i.e., using the apparent-approximation to \( \chi^2 \) distribution as the stopping criterion), (b) the estimation of source data variance (i.e., using unified variance for all sources), and (c) the actions completed by the inner and outer loops. We validate the modified method using synthetic data and the AE data sets where real-time and post-mortem inspections on the loading induced rock fractures is available.
(i.e., the data sets from Wong and Xiong (2018)). The modified method has proven its potential in recovering the second-order structures from the source point cloud, although a retrospective tuning process is still necessary for the successful application of the modified method.

We further devise a serendipity method for laboratory application of EC. The modifications validated in the modified method have been used in this serendipity method. The critical feature of the serendipity method is that it can reach the improvement of point cloud delineation in a non-retrospective manner (without the need for algorithm parameter tuning). We validate the ability of the serendipity method in improving the causative structure delineation, including its potential capacity for delineating the second-order structural features, using the same data sets that used for validating the modified method (i.e., from Wong and Xiong (2018)). After that, we apply the serendipity method in the analysis of laboratory AEs released from a hydraulic fracturing test (from Hampton et al. (2019)). We have observed indicative information for a circling damage zone caused by the hydraulic fracture, which is at the spatial perimeter of the sub-core number 6 in the post-mortem core 1. The existence of the circling damage zone further details the previously observed reverse permeability-distance relationship and has resolved the second-order deficiency in this relationship: permeability increases with the increase of the distance from the center line of AE cluster (Hampton et al., 2019; Xiong & Hampton, 2021a). This discovery makes the permeability-distance relationship comprehensively explained by the actual path of formed damage zone caused by the hydraulic fracturing.

While for the modified method there is no way to determine at what moment a proper parameter tuning has been achieved once no direct optical observation on the rock fracture process is available. Further, even if post-mortem assessments are available, the temporal nature of the rock fracture process can hide second order features. Consequently, the application of the modified method to a hydraulic fracturing test can only show a series of provocative evidence on every independent series of planar projections (i.e., the projections on XY, YZ, and XZ planes), and no universal value in the scaling factor (the tuning parameter) can successfully identify the detoured path in all independent three planar projections in the modified method. This newly proposed serendipity method should be useful for analyzing the AE data from a variety of other types of laboratory rock fracture tests, where direct optical observations are not realistic (volumetric rock fracture experiments).

**Data Availability Statement**

The test data of 5 single flaw tests, which are used for validating the modified and the serendipity methods, are available through previous publication: Wong and Xiong (2018). The test data for the hydraulic fracturing test has partially been available through previous publications: Hampton et al. (2019), and Xiong and Hampton (2021), and an organized data set is available through Zenodo (Xiong & Hampton, 2022). The coding and code manual for the algorithms are on MATLAB platform and can be available through reasonable contact to the authors.

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