CORE-Collapse TIMES OF TWO-COMPONENT STAR CLUSTERS

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Abstract

We examine the corecollapse times of isolated, two-mass-component star clusters using Fokker-Planck models. With initial condition of Plummer models, we find that the core-collapse times of clusters with \( M_1/M_2 \gg 1 \) are well correlated with \((N_1/N_2)^{1/2}(m_1/m_2)^2r_{th}\), where \( M_1/M_2 \) and \( m_1/m_2 \) are the light to heavy component total and individual mass ratios, respectively, \( N_1/N_2 \) is the number ratio, and \( r_{th} \) is the initial half-mass relaxation time scale. We also find two-component cluster parameters that best match multi-component (thus more realistic) clusters with power-law mass functions.

Subject headings: celestial mechanics, stellar dynamics — globular clusters : general

1. INTRODUCTION

The course of dynamical evolution of pre- and post-collapse globular clusters is determined by many factors such as initial mass function, nature and efficiency of energy generation mechanisms, tidal cut-off, anisotropy of velocity distribution, initial population of binaries, and stellar evolution. There have been many efforts in developing more and more complex cluster models including such factors, making analysis and interpretation rather difficult. To study the dynamical evolution of globular clusters more realistically, among others, Chernoff & Weinberg (1990) included the effects of stellar evolution, Lee, Falhman, & Richer (1991) used multi-component models, Takahashi (1995) included an anisotropic velocity distribution.

However, studying simpler models could be more instructive in identifying important physical processes governing the evolution. Kim, Lee, & Goodman (1997; hereafter KLG) studied on the post-collapse evolution of cluster variables and the gravitational oscillation using two-component Fokker-Planck models. In this paper, as a supplementary study to KLG, we present a fitting formula for the core-collapse times of two-component models and compare the results of two- and multi-component models. As in KLG, here both tidal-capture binary heating and tree-body binary heating are included, and clusters are assumed to be isotropic and isolated (no tidal cutoff). For the methods that we are using here and the benefits of studying simpler models (two-component models), readers are referred to KLG and references therein.

Core-collapse times of two-component clusters were presented by Inagaki & Wiyanto (1984), Inagaki (1985), and Lee (1995) among others. These papers calculated core-collapse times as a function of \( M_2/M_1 \), the ratio of total masses of heavy component to light component, and found that the ratio of core-collapse time to initial half-mass relaxation time, \( t_{cc}/t_{rh} \), has a minimum value at \( M_2/M_1 \sim 0.1 \). However, the parameter \( M_2/M_1 \) may be divided into \( m_2/m_1 \), the individual mass ratio, and \( N_2/N_1 \), the number ratio. In the present paper, we calculate the core-collapse times of two-component models as a function of more complete two-component cluster parameters, and find a fitting formula between them. However, the clusters studied here are restricted to those with \( M_1 \gg M_2 \) as in KLG.

On the other hand, it would be helpful in interpreting the results of two-component models if the similarities and discrepancies between the results of two- and multi-component models are well known. In the present paper, we also compare two-component models to 11-component models, and thus provide a way to extrapolate to more realistic cluster the results of two-component clusters such as those in KLG.

2. CORE-Collapse TIMES

To calculate the core-collapse times of two-component star clusters, we have performed total 11 runs of direct numerical integration of the orbit-averaged Fokker-Planck equation with a local approximation. The code used here is descended from Cohn (1980). Parameters of our two-component runs are shown in Table 1. This set of parameters has been chosen such
that it provides all possible combinations of parameters $M$, $N$, $m_2/m_1$, and $N_1/N_2$, where $M$ is the cluster mass and $N$ is the total number. Note that in all our runs, the total mass of heavy component, $M_2$, is negligible compared to the total mass of light component, $M_1$, and thus $m_1 \approx M/N_1$. The initial density and velocity profiles are given by Plummer models with $v_{11}/v_{22} = 1$ and $\rho_{11}/\rho_{22} = M_1/M_2$, where $v_{11}^2$ is the three-dimensional core velocity dispersion, and $\rho_c$ is the core density.

Corecollapse times of our runs are shown in Table 1 in units of $10^{10}$ yr and $t_{rh}$. We empirically found that $t_{cc}$ can be fitted by the following formula:

$$t_{cc} \approx 4.2 \times 10^9 \text{yr} \left( \frac{N_1}{N_2} \right)^{1/2} \left( \frac{m_1}{m_2} \right)^2 N_3 M_5^{-1/2} \left( \frac{r_h}{5 \text{pc}} \right)^{3/2}, \tag{1}$$

where $N_5 \equiv N/10^5$, $M_5 \equiv M/10^5 M_\odot$, and $r_h$ is the initial half-mass radius. Each $t_{cc}$ value is plotted over the righthand side of the above equation in Figure 1, which shows a good X-Y correlation. Equation (1) is to be compared with the standard half-mass relaxation time scale,

$$t_{rh} \equiv \frac{v_m^2}{\langle v_\parallel^2 \rangle_{v=v_m}} = \frac{M^{1/2} r_h^{3/2}}{6.7 G^{1/2} m \ln 0.4 N}, \tag{2}$$

where $v_m$ is the root-mean-square three-dimensional velocity of the whole cluster and $\langle v_\parallel^2 \rangle_{v=v_m}$ is the average change of $v_m^2$ in parallel component to initial $v_m$ per unit time.

Isolated single-mass clusters with initial condition of Plummer models collapse at 15.4 $t_{rh}$ (Cohn 1980), where $t_{rh}$ is the half-mass relaxation time scale and does not vary much until the corecollapse takes place. However, the ratios of the time required for core-collapse $t_{cc}$ to $t_{rh}$ and core relaxation time scale $t_{rc}$ strongly depend on the density and velocity profiles. Quinlan (1996) found that for single-mass clusters, $t_{cc}$ varies much less when expressed in units of $t_{cc}$ divided by a dimensionless measure of the temperature gradient in the core. Although in single-mass clusters the velocity profile (as well as other physical parameters) evolves by the two-body relaxation, two- or multi-component clusters have another driving force: the equipartition.

Both mass segregation and equipartition are involved in determination of the time to corecollapse,

![Fig. 1.— Corecollapse times of runs in Group A. $N_5 \equiv N/10^5$ and $M_5 \equiv M/10^5 M_\odot$.](image)

and this complexity makes the theoretical interpretation of the above correlation between $t_{cc}$ and cluster parameters quite difficult. Here we suggest the following analysis as one way to explain this correlation.

The actual duration of corecollapse is very small compared to the time to corecollapse from the beginning of cluster’s evolution. Instead, clusters spend most of their precollapse phases under mass segregation process and approach to the onset of homologous phase of corecollapse. If a considerable degree of equipartition is accomplished in the precollapse phase as in all of our two-component models, the time to the onset of corecollapse will be determined by how fast the light component gains the energy from the heavy component via equipartition. Thus one may define the precollapse time scale of two-component clusters $t_{r2}$ as following:

$$t_{r2} \equiv \frac{v_{11}^2}{\langle v_{11}^2 \rangle_{v=v_m}}, \tag{3}$$

where $\langle v_{11}^2 \rangle_{v=v_m}$ is the velocity dispersion change of the light component via interactions with heavy component. Using the standard expression for the average velocity dispersion change per unit time, one has

$$\langle v_{11}^2 \rangle_{v=v_m} \propto \frac{G^2 m_2 \rho_{m2}}{v_{m2}}, \tag{4}$$
Table 1
PARAMETERS AND CORE-COLLAPSE TIMES OF TWO-COMPONENT MODELS

| Run  | \(\frac{m_2}{m_1}\) | \(N_1/N_2\) | \(M\) (\(M_\odot\)) | \(N\) | \(m_2\) (\(M_\odot\)) | \(t_{cc}\) (10^6 yr) | \(t_{cc}/t_{rh}\) |
|------|------------------|-----------|-----------------|------|------------------|-----------------|-----------------|
| baab | 2                | 100       | \(10^5\)        | 1.4  | 1.27             | 12.42           |
| caab | 3                | 100       | \(10^5\)        | 1.4  | 0.93             | 6.34            |
| faab | 4                | 100       | \(277473\)      | 1.4  | 0.61             | 3.23            |
| cdab | 3                | 30        | \(10^5\)        | 1.4  | 0.56             | 3.97            |
| chab | 3                | 300       | \(10^5\)        | 1.4  | 1.62             | 10.92           |
| caab1| 3                | 100       | \(70042\)       | 3x1.4| 0.35             | 6.47            |
| caab2| 3                | 100       | \(63037\)       | \(\frac{1}{3}\)×1.4| 2.52             | 6.28            |
| baab3| 2                | 100       | \(212185\)      | \(\frac{1}{3}\)×1.4| 1.80             | 12.17           |
| faab3| 4                | 100       | \(208104\)      | \(\frac{1}{3}\)×1.4| 0.47             | 3.23            |
| caeb | 3                | 100 \(3\times10^4\) | \(63037\) | 1.4 | 0.57             | 6.34            |
| cabb | 3                | 100 \(3\times10^5\) | \(63037\) | 1.4 | 1.49             | 6.43            |

Note.—The initial half-mass radii \(r_h\) of these runs are all 5 pc.

where the heavy component mean density \(\rho_{m_2}\) is proportional to \(M_2/r_h^3\) and the Coulomb logarithm has been omitted. It is also assumed that \(v_{m_1} \sim v_{m_2}\). Spitzer (1969, 1987) showed that for a two-component cluster of polytropic index \(n\) between 3 and 5 with \(M_1 \gg M_2\) and a Maxwellian velocity distribution in a parabolic potential well, the minimum degree of the global equipartition is a function of cluster’s parameters such that

\[
\frac{m_2v_{m_2}^2}{m_1v_{m_1}^2} \bigg|_{\text{min}} \propto \left(\frac{N_2}{N_1}\right)^{2/3} \left(\frac{m_2}{m_1}\right)^{5/3}.
\]  

(5)

With the minimum value of equation (5) and assumptions that \(M_1 \gg M_2\) and \(v_{m_1}^2 \sim GM/r_h\), equation (5) now becomes

\[
t_{r_2} \propto \left(\frac{N_1}{N_2}\right)^{2/3} \left(\frac{m_1}{m_2}\right)^{5/3} N M^{-1/2} r_h^{3/2}.
\]  

(6)

The above is in the same form as equation (5) with only small discrepancies in the exponents. Although equation (6) has been used for derivation of the above equation, we find that the degrees of equipartition in the precollapse phases of our two-component runs do not directly correlate with the minimum values of equation (5). In fact, exact equipartition is usually not accomplished even when the value of equation (6) is less than unity, because as mass segregation of heavy component progresses, interactions between heavy and light components occur less. Thus equation (6) should be regarded as a degree of tendency to equipartition and it is this tendency that \(t_{r_2}\) requires in its definition.

While Quinlan (1996) introduced a temperature gradient in the core in a derivative form to explain a huge variation in \(t_{cc}\) for clusters with different initial profiles, here we introduced both density and velocity gradients of heavy component naturally into the time scale by considering global equipartition.

3. COMPARISON WITH MULTI-COMPONENT CLUSTERS

Clusters have continuous mass functions. However, mass functions are usually realized with discrete mass components in numerical calculations. Scientists found that 10 to 20 components are enough
to represent continuous mass functions for Fokker-Planck models, and such numerical representation for a given mass function is quite straightforward for these multi-component clusters: there is only a question of choice of each component’s mass bin and a representative value. However, when the number of components is reduced to 2 for the sake of analytical simplification, such choice is not so simple because dynamically important mass and corresponding number of stars may be different from simple mean mass and total number of a certain mass range. Therefore two-component cluster parameters (such as \(m_2/m_1\), \(N_1/N_2\), and \(N\)) that well represent a continuous mass function should be numerically found through comparisons of the evolution of two- and multi-component clusters.

In this section we will compare our multi-component models with the two-component models in KLG varying \(M\) and the mass function of the multi-component models. Cluster parameters of our multi-component models are given in Table 3. The initial density and velocity profiles are given by Plummer models. The initial half-mass radii of all multi- and two-component models are 5 pc. The number of component is 11 and we adopt a power-law mass function:

\[
N(m)dm \propto m^{-(x+1)}dm,
\]

where \(x\) is the mass spectral index and the Salpeter mass function has \(x = 1.35\). For a bin \(i\) with boundaries \(m_{ia}\) and \(m_{ib}\), the total mass in the bin is obtained by

\[
M_i = \int_{m_{ia}}^{m_{ib}} mN(m)dm.
\]

Then the number of stars in the bin is \(N_i = M_i/m_i\), where \(m_i\) is the representative mass of each bin. The main-sequence star mass range was selected to be 0.08 \(M_\odot\) – 0.8 \(M_\odot\). Following Sigurdsson & Phinney (1995), the stars of initial mass \(m_i\) between 0.8 \(M_\odot\) and 4.7 \(M_\odot\) were assumed to have evolved to white dwarfs of mass 0.58 + 0.22 \times (m_{MS} – 1.0) \(M_\odot\), where \(m_{MS}\) is the main-sequence mass, while stars of \(m_{MS}\) between 4.7 \(M_\odot\) and 8.0 \(M_\odot\) were assumed to disrupt completely. The stars heavier than 8.0 \(M_\odot\) but lighter than 15.0 \(M_\odot\) were assumed to become neutron stars of mass 1.4 \(M_\odot\). Neutron stars are born with a kick velocity due to an asymmetric explosion, and they are ejected from the cluster if the kick velocity is greater than the escape velocity of the cluster. However, we assumed that all neutron stars remain in the cluster, because the retention rate of neutron stars are not well known and the precise realization of real clusters is not our goal in this study. The mass range, representative mass, and number of stars of each component is shown in Table 3. Our multi-component models include both three-body binary heating and tidal-capture binary heating, but we find that the post-collapse phases of all our runs are driven by three-body binary heating.

We find a two-component model which best describes a given multi-component model by comparing the values of cluster variables \(\rho_c\), \(v_c\), \(r_h\) at \(t = 10^{11}\) yr, and \(t_{cc}\). An epoch of \(10^{11}\) yr has been selected as in KLG because by that time, our runs have reached self-similar expansion phase. With two-component models, KLG found the following numerical values:

\[
\rho_c \approx 4.5 \times 10^5 \frac{M_\odot}{pc^3} \left(\frac{m_2}{m_1}\right)^{-10/3} N_5^{10/3 - 2.0}; (9a)
\]
\[
v_c \approx 3.8 \text{ km/s} \left(\frac{m_2}{m_1}\right)^{-1/2} N_5^{1/3} M_5^{1/3} t_{11}^{-0.32}; (9b)
\]
\[
r_c \approx 0.042 \text{ pc} \left(\frac{m_2}{m_1}\right)^{7/6} N_5^{-4/3} M_5^{1/3} t_{11}^{0.65}; (9c)
\]
\[
r_h \approx 35 \text{ pc} N_5^{-2/3} M_5^{1/3} t_{11}^{0.65}; (9d)
\]

where \(N_5 \equiv N/10^5\), \(M_5 \equiv M/10^5 M_\odot\), and \(t_{11} \equiv t/10^{11}\) yr. On the other hand, the numerical values from our multi-component models are given in Table 3. There are four two-component parameters to be determined for a given multi-component model, \(m_2/m_1\), \(N_1/N_2\), \(N\), and \(M\). However, since cluster variables at a certain epoch in the postcollapse phase are independent of \(N_1/N_2\) as in equation (9). \(N_1/N_2\) has to be determined from the core-collapse time, equation (9). Then the rest three parameters, \(m_2/m_1\), \(N\), and \(M\), may be determined from equation (9). This method will be called Method A, and parameters obtained in this way are given in Table 3.

With Method A, our multi-component model B2 is best described by a two-component model with \(m_2/m_1 = 2.3\), \(N_1/N_2 = 29\), \(N = 1.4 \times 10^5\), and \(M = 0.97 \times 10^5 M_\odot\). Note that with these parameters, \(m_2 \approx 1.6 M_\odot\), which is little higher than the mass of the heaviest component of our multi-component models, 1.4 \(M_\odot\). Neutron stars play an important role in dynamical evolution of globular clusters: a considerable fraction of dynamical binary formation (as apposed to primordial binaries) involves neutron stars. For this reason, in two-component clusters, the heavy component is often targeted for neutron stars and the light component is for main-sequence stars. Thus in finding the best matching two-component parameters, it could be more meaningful if \(m_2\) is set to
| Table 2 |
| --- |
| **PARAMETERS AND RESULTS OF MULTI-COMPONENT MODELS** |

| Run | $x$ | $M$ ($M_\odot$) | $N$ | $t_{cc}$ (10$^{10}$yr) | $\rho_c$ ($M_\odot$ pc$^{-3}$) | $v_c$ (km s$^{-1}$) | $r_h$ (pc) |
|-----|-----|------------------|-----|------------------|-----------------|----------------|-----------|
| A2  | 1.00| 10$^5$           | 402857 | 0.528            | 7.63 $\times$ 10$^4$ | 2.79           | 32.8      |
| B2  | 1.35| 10$^5$           | 509201 | 0.584            | 8.13 $\times$ 10$^4$ | 2.75           | 27.7      |
| C2  | 1.50| 10$^5$           | 552232 | 0.621            | 8.19 $\times$ 10$^4$ | 2.73           | 26.2      |
| B1  | 1.35| 3 $\times$ 10$^4$| 152760 | 0.356            | 1.71 $\times$ 10$^3$ | 1.30           | 38.5      |
| B3  | 1.35| 3 $\times$ 10$^5$| 1527603| 0.931            | 2.58 $\times$ 10$^6$ | 5.44           | 20.7      |

**Note.**—The initial half-mass radii $r_h$ of these runs are all 5 pc.

| Table 3 |
| --- |
| **MASS SPECTRA OF MULTI-COMPONENT MODELS** |

| Bin | $m_i$ ($M_\odot$) | Mass Range ($M_\odot$) | $x = 1.00$ | $x = 1.35$ | $x = 1.50$ |
|-----|-------------------|------------------------|------------|------------|------------|
|     | $N_i$             | $m_i \cdot N_i$        | $N_i$      | $m_i \cdot N_i$ | $N_i$      | $m_i \cdot N_i$ |
| 1   | 0.1               | 0.08 - 0.15            | 1.00000    | 1.00000    | 1.00000    | 1.00000    |
| 2   | 0.2               | 0.15 - 0.25            | 0.40631    | 0.81262    | 0.33263    | 0.66526    | 0.30517    | 0.61034    |
| 3   | 0.3               | 0.25 - 0.35            | 0.17842    | 0.53526    | 0.12584    | 0.37752    | 0.10826    | 0.32478    |
| 4   | 0.4               | 0.35 - 0.45            | 0.09995    | 0.39980    | 0.06359    | 0.25436    | 0.05223    | 0.20892    |
| 5   | 0.5               | 0.45 - 0.55            | 0.06385    | 0.31925    | 0.03753    | 0.18765    | 0.02985    | 0.14925    |
| 6   | 0.6               | 0.55 - 0.65            | 0.11712    | 0.70272    | 0.05766    | 0.34596    | 0.04264    | 0.25584    |
| 7   | 0.7               | 0.65 - 0.80            | 0.09010    | 0.63070    | 0.04104    | 0.28728    | 0.02945    | 0.20615    |
| 8   | 0.9               | 0.80 - 1.0             | 0.02428    | 0.21852    | 0.00822    | 0.07398    | 0.00516    | 0.04644    |
| 9   | 1.1               | 1.0 - 1.2              | 0.01286    | 0.14146    | 0.00389    | 0.04279    | 0.00232    | 0.02552    |
| 10  | 1.3               | 1.2 - 1.4              | 0.00774    | 0.10062    | 0.00215    | 0.02795    | 0.00124    | 0.01612    |
| 11  | 1.4               | 1.4 - 1.4              | 0.00913    | 0.12782    | 0.00182    | 0.02548    | 0.00091    | 0.01274    |

**Note.**—$N_i$ and $m_i \cdot N_i$ are normalized with bin 1 values.
1.4 \, M_\odot. With a restriction of \( m_2 = 1.4 \, M_\odot \), now
the number of variables in equation \( \ref{eq:1} \) required for
determination of two-component cluster parameters is reduced to two. Since \( \rho_c \) and \( r_h \) are two cluster variables that represent the status of the core and
envelope, respectively, we use these variables along with \( m_2 = 1.4 \, M_\odot \) and equation \( \ref{eq:1} \) for our second method
(Method B) to find the best matching two-component
model (see Table \ref{tab:4}).

With Method B, model B2 is now best described by
a two-component model with \( m_2/m_1 = 2.2 \), \( N_1/N_2 = 21 \), \( N = 1.3 \times 10^5 \), and \( M = 0.84 \times 10^5 \, M_\odot \). Note
that with these parameters, \( N_2 = 4890 \) and this value is
about the same with the number of stars in the
heaviest four bins (bins only for degenerate stars) of
model B2 (\( \sum_{i=8,11} N_i = 5176 \)). This may imply that
the epoch of corecollapse is mainly determined by the
number of stars above the turnoff mass. This fact also
holds for other runs with different \( M \) and \( x \).

For clusters with \( N \propto M \) (as for our multi-
component clusters B1, B2, and B3), equation \( \ref{eq:1} \)
may be written as \( \rho_c \propto M^{10/3} \), \( v_c \propto M^{2/3} \), and
\( r_h \propto M^{-1/3} \). From runs B1, B2, and B3 in Table \ref{tab:2},
\( \rho_c \), \( v_c \), and \( r_h \) are found to be proportional to \( M^{0.18} \),
\( M^{0.62} \), and \( M^{-0.26} \), respectively. The absolute val-
ues of these exponents are little smaller than equation \( \ref{eq:1} \).
However, since the discrepancies are not so significant, we conclude that the evolution aspects of the postcollapse multi-component clusters are still
well predictable from the numerical and analytical results
of two-component clusters. On the other hand, cluster variable values at \( t = 10^{11} \, \text{yr} \) show relatively small \( x \) dependence.

The results from Method B in Table \ref{tab:4} indicate
that multi-component clusters may be described by
two-component clusters with masses 15 to 20 \% less
and with \( m_1 \) near the turnoff mass. Of course, this
\( m_1 \) is dependent on \( x \) such that clusters with steeper
mass function are matched by two-component clusters
with smaller \( m_1 \). However, interestingly, lighter
multi-component clusters with the same \( x \) also require smaller \( m_1 \). This comes from the fact that
\( N \propto M \) holds for runs B1, B2, and B3, while not
for their matching two-component clusters by Method
B (\( N \propto M^{0.92} \)). For Method A, the best matching
two-component clusters of runs B1, B2, and B3 show
\( N \propto M^{0.98} \), which results in nearly the same \( m_1 \). Thus we conclude that the above difference in \( m_1 \) values
by Method B for clusters with the same \( x \) stems
from the restriction, \( m_2 = 1.4 \, M_\odot \).

The evolution of the two-component model with
the above parameters is plotted in Figure \ref{fig:2} as well as
that of model B2. Cluster variables \( \rho_c \), \( v_c \) and \( r_h \) of
two runs well coincide. Only the corecollapse times
show a small discrepancy. This is partly because of
the dispersion of \( t_c \) from the fitting formula, equation
\( \ref{eq:2} \), and partly because of the small \( N_1/N_2 \) value:
equation \( \ref{eq:2} \) is to be used for clusters with \( M_1 \gg M_2 \).

### Table 4

| Run | \( m_2/m_1 \) | \( N_1/N_2 \) | \( N_5 \) | \( M_5 \) | \( m_2/m_1 \) | \( N_1/N_2 \) | \( N_5 \) | \( M_5 \) | \( N_2 \) |
|-----|--------------|---------------|----------|----------|--------------|---------------|----------|----------|----------|
| A2  | 1.78         | 13.1          | 1.05     | 0.90     | 1.70         | 10.9          | 1.00     | 0.82     | 8400     |
| B2  | 2.33         | 28.5          | 1.40     | 0.97     | 2.17         | 21.3          | 1.30     | 0.84     | 5800     |
| C2  | 2.56         | 39.4          | 1.54     | 0.99     | 2.36         | 28.4          | 1.41     | 0.84     | 4800     |
| B1  | 2.74         | 53.9          | 0.51     | 0.35     | 2.37         | 30.0          | 0.44     | 0.26     | 1400     |
| B3  | 2.05         | 18.1          | 3.47     | 2.49     | 2.00         | 16.3          | 3.38     | 2.36     | 20000    |

Note.—\( N_5 \equiv N/10^5 \) and \( M_5 \equiv M/10^5 \, M_\odot \). \( N_2 \) has been approximately
calculated by \( N/(N_1/N_2 + 1) \) and has only two significant digits.
Fig. 2.— Comparison of the evolution of multi-component model (run B2; thick lines) and best-matching two-component model by Method B ($m_2/m_1 = 2.2, N_1/N_2 = 21, N = 1.3 \times 10^5$, and $M = 0.84 \times 10^5 M_\odot$; thin lines). The units of $\rho_c$, $v_c$, and $r_h$ are $M_\odot$ pc$^{-3}$, km s$^{-1}$, and pc, respectively.

4. SUMMARY

We have investigated the evolution of isolated two-component clusters with initial condition of Plummer models. The corecollapse time $t_{cc}$ showed a good correlation with a parameter $(N_1/N_2)^{1/2}(m_1/m_2)^2 t_{rh}$. To explain this correlation, a new time scale for the precollapse evolution of two-component clusters, $t_{r2} \equiv (N_1/N_2)^{2/3}(m_1/m_2)^{5/3} t_{rh}$ have been introduced using Spitzer’s (1969, 1987) global equipartition analysis.

We also found two-component clusters which best match with our multi-component clusters with power-law mass functions. For example, the evolution of 11-component cluster with a Salpeter mass function and $M = 10^5 M_\odot$ was well described by a two-component cluster with $m_2/m_1 = 2.2, N_1/N_2 = 21, N = 1.3 \times 10^5$, and $M = 0.84 \times 10^5 M_\odot$. Furthermore, it has been found that the best matching two-component cluster has $N_2$ very close to the number of stars heavier than turnoff mass of the multi-component cluster.

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REFERENCES

Chernoff, D. F. & Weinberg, M. 1990, ApJ, 351, 121
Cohn, H. N. 1980, ApJ, 242, 765
Inagaki, S., & Wiyanto, P. 1984, PASJ, 36, 391
Inagaki, S. 1985, in IAU Symposium 113, Dynamics of star clusters, ed. J. Goodman and P. Hut (Dordrecht: Reidel), p. 189
Kim, S. S., Lee, H. M., & Goodman, J. 1997, submitted to ApJ
Lee, H. M. 1995, MNRAS, 272, 605
Lee, H. M., Fahlman, G. G., & Richer, H. B. 1991, ApJ, 366, 455
Quinlan, G. D. 1996, New Astronomy, 1, 255
Sigurdsson, S., & Phinney, E. S. 1995, ApJS, 99, 609
Spitzer, L. Jr. 1969, ApJ, 158, L139
Spitzer, L. Jr. 1987, Dynamical Evolution of Globular Clusters (Princeton: Princeton University Press)
Takahashi, K. 1995, PASJ, 47, 561