Accidental Composite Dark Matter

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Motivation

In the SM, all observed global symmetries arise as accidental symmetries of the renormalizable Lagrangian. This explains why the proton is stable. It also explains flavour.

We need at least one more stable particle for Dark Matter…

New gauge theory + gap
DM is an accidentally stable “techni”-baryon
Vector-like confinement:

New confining gauge theory with fermions vectorial under SM
Vector-like confinement:

\[ SM + H \]

New confining gauge theory with fermions vectorial under SM

SM including Higgs couples to the strong sector with renormalizable couplings:

\[
\mathcal{L} = \mathcal{L}_{SM} + \bar{\Psi}_i (i\dot{\Psi} - m_i) \Psi_i - \frac{G^A_{\mu\nu}}{4g^2_{TC}} + \frac{\theta_{TC}}{32\pi^2} G^A_{\mu\nu} \tilde{G}^A_{\mu\nu} + [H \bar{\Psi}_i (y^L_{ij} P_L + y^R_{ij} P_R) \Psi_j + \text{h.c.}] 
\]
• Technicolor

\[ G \quad SM \in G/H \quad H \quad f = v \]

• Composite Higgs

\[ G \quad SM \in H \quad f > v \]
Higgs is a pseudo-Goldstone boson. Electro-weak scale determined by vacuum alignment.

\[
\text{DEVIATION SM} = \frac{1}{\text{TUNING}}
\]

- Electro-weak preserving dynamics:

\[ SM + H \]

Higgs is elementary.
Very weak bounds:

- Automatic MFV
- Precision tests ok
- LHC: $m_{\rho} > 1 - 2 \text{ TeV}$

Interesting phenomenology:

- Plausible at LHC13
- Automatic dark matter candidates
- Simple UV models
- Relaxion
• Models

$SU(N)$ gauge theory with $NF$ flavors. Techni-quarks are vectorial with respect to SM.

| Fermions | $SM$ | $SU(n)_{TC}$ | $\sum_i d[r_i] = NF$ |
|----------|------|--------------|---------------------|
| $\Psi_L$ | $\sum_i r_i$ | $n$ |                           |
| $\Psi_R$ | $\sum_i \bar{r}_i$ | $\bar{n}$ |                           |

$\langle \bar{\Psi}^i \Psi^j \rangle \sim 4\pi f^3 \delta^{ij}$

Goldstone bosons:

$$\frac{SU(N_F) \times SU(N_F)}{SU(N_F)}$$

$$\text{Adj}_{SU(N_F)} = \sum_{i=1}^{K} r_i \times \sum_{i=1}^{K} \bar{r}_i - 1$$

Vacuum does not break electro-weak symmetry.
Accidental symmetries:

• Techni-Baryon number

\[ U(1)_{TB} \quad \Psi^i \rightarrow e^{i\alpha} \Psi^i \]

• Species number

\[ U(1)_{F_i} \quad \Psi^i \rightarrow e^{i\alpha_i} \Psi^i \]

\[ \sum_{i}^{K} \alpha_i = 0 \]

• G-parity

\[ \Psi \rightarrow e^{-i\pi J_2} \Psi^c \]

Broken by hypercharge.
Automatic dark matter candidates:

- Baryons

\[ B = \epsilon_{i_1 i_1 . . . i_n} Q_{i_1}^{\alpha_1} Q_{i_2}^{\alpha_2} . . . Q_{i_n}^{\alpha_n} \]

\[ m_B \sim N m_\rho \]

Lightest multiplet has minimum spin among reps.

\[ n = 3 \quad \text{and} \quad n = 4 \]

DM candidate:

\[ Q_{TB} = T_3 + Y_{TB} = 0 \]

\[ Y_{TB} = 0 \]

\[ J = 0, 1, 2, \ldots \]
Pions can be stable due to G-parity or species number:

\[ \psi \rightarrow S \psi^C \]

\[ W_\mu^a J^a \rightarrow W_\mu^a J^a \]

\[ A^a t^a \rightarrow A^a (-t^a)^* \]

\[ S = e^{i\pi J_2} \]

\[ \Pi^J \rightarrow (-1)^J \Pi^J \]

Triplet is stable. Behaves as minimal dark matter.

\[ m_{J=1} \sim 2.5 \text{ TeV} \]

\[ \sigma_{SI} = 0.12 \pm 0.03 \times 10^{-46} \text{ cm}^2 \]
Baryon number can be broken by dimension 6 operators

\[ \tau \sim \frac{8\pi M^4}{M_{DM}^5} \sim 10^{26} \text{ sec} \times \left( \frac{M}{M_{Pl}} \right)^4 \left( \frac{100 \text{ TeV}}{M_{DM}} \right)^5 \]

Species symmetry and G-parity can be broken by Yukawa couplings or dim 5 operators

\[ \frac{1}{M} \bar{Q}QHH, \quad \frac{1}{M} \bar{Q}\sigma^{\mu\nu}QB_{\mu\nu} \]

Within EFT baryons more likely cosmologically stable.
Flavor multiplets are split by quark masses and gauge interactions:

- **quark masses**

  \[ \delta m^2_{\pi} \sim m m_\rho \quad \delta m_B \sim m \]

- **gauge interactions**

Charged pions acquire positive mass.

\[ m^2_{\pi} = \frac{3g_i^2}{(4\pi)^2} C_2(\pi) m^2_\rho \]

After electro-weak symmetry breaking multiplets further split. Neutral component is the lightest. For triplets:

\[ m^+ - m^0 = 166 \text{ MeV} \]
\[ m_{TCb} \sim \frac{m_{TCb}^2}{m_{TCb}} \]

\[ \Delta m = \alpha_2 Q^2 m_W \sin^2 \frac{\theta_W}{2} \sim 100 \text{ MeV} \]
We take branches of unified representations

\[ R = (N, SM) \oplus (\bar{N}, S\bar{M}) \]

\[ \tilde{R} = (N, S\bar{M}) \oplus (\bar{N}, SM) \]

- SU(N) asymptotically free
- No Landau poles below the Planck scale.
- Lightest techni-baryon with \( Q=Y=0 \)
- No unwanted stable particles
Two classes of models:

- **Golden**

These models fulfil all requirements with renormalizable lagrangian

\[ N = 3 \]

\[ Q = V \quad DM = VVV \]

- **Silver**

Higher dimensional operators needed

\[ N = 3 \]

\[ Q = L + E \quad DM = LLE \]
### Renormalizable models:

| $\text{SU(N)}$ techni-color. | Yukawa couplings | Allowed $N$ | Techni-pions | Techni-baryons | under |
|-----------------------------|------------------|-------------|--------------|----------------|-------|
| $N_{\text{TF}} = 3$        |                  |             | 8            | $8, 6, \ldots$ for $N = 3, 4, \ldots$ | $\text{SU(3)}_{\text{TF}}$ |
| $\Psi = V$                 | 0                | 3           | 3            | $VVV = 3$      | $\text{SU(2)}_L$ |
| $\Psi = N \oplus L$        | 1                | 3, \ldots, 14 | unstable     | $N^N \ast = 1$ | $\text{SU(2)}_L$ |
| $N_{\text{TF}} = 4$        |                  |             | 15           | $20, 20', \ldots$ | $\text{SU(4)}_{\text{TF}}$ |
| $\Psi = V \oplus N$        | 0                | 3           | $3 \times 3$ | $VVV, VNN = 3, VVN = 1$ | $\text{SU(2)}_L$ |
| $\Psi = N \oplus L \oplus \bar{E}$ | 2                | 3, 4, 5     | unstable     | $N^N \ast = 1$ | $\text{SU(2)}_L$ |
| $N_{\text{TF}} = 5$        |                  |             | 24           | $40, 50$       | $\text{SU(5)}_{\text{TF}}$ |
| $\Psi = V \oplus L$        | 1                | 3           | unstable     | $VVV = 3$      | $\text{SU(2)}_L$ |
| $\Psi = N \oplus L \oplus \bar{L}$ | 2                | 3           | unstable     | $NL\bar{L} = 1$ | $\text{SU(2)}_L$ |
| =                          | 2                | 4           | unstable     | $NNL\bar{L}, L\bar{L}L\bar{L} = 1$ | $\text{SU(2)}_L$ |
| $N_{\text{TF}} = 6$        |                  |             | 35           | $70, 105'$     | $\text{SU(6)}_{\text{TF}}$ |
| $\Psi = V \oplus L \oplus N$ | 2                | 3           | unstable     | $VVV, VNN = 3, VVN = 1$ | $\text{SU(2)}_L$ |
| $\Psi = V \oplus L \oplus \bar{E}$ | 2                | 3           | unstable     | $VVV = 3$      | $\text{SU(2)}_L$ |
| $\Psi = N \oplus L \oplus \bar{L} \oplus \bar{E}$ | 3                | 3           | unstable     | $NNL\bar{L}, L\bar{L}L\bar{L} = 1$ | $\text{SU(2)}_L$ |
| =                          | 3                | 4           | unstable     | $NNL\bar{L}, L\bar{L}L\bar{L}, N\bar{E}L\bar{L} = 1$ | $\text{SU(2)}_L$ |
| $N_{\text{TF}} = 7$        |                  |             | 48           | 112            | $\text{SU(7)}_{\text{TF}}$ |
| $\Psi = L \oplus \bar{L} \oplus E \oplus \bar{E} \oplus N$ | 4                | 3           | unstable     | $LLL, \bar{L}E\bar{E}, L\bar{L}N, E\bar{E}N = 1$ | $\text{SU(2)}_L$ |
| $\Psi = N \oplus L \oplus \bar{E} \oplus V$ | 3                | 3           | unstable     | $VVV, VNN = 3, VVN = 1$ | $\text{SU(2)}_L$ |
| $N_{\text{TF}} = 9$        |                  |             | 80           | 240            | $\text{SU(9)}_{\text{TF}}$ |
| $\Psi = Q \oplus \bar{D}$  | 1                | 3           | unstable     | $QQ\bar{D} = 1$ | $\text{SU(2)}_L$ |
| $N_{\text{TF}} = 12$       |                  |             | 143          | 572            | $\text{SU(12)}_{\text{TF}}$ |
| $\Psi = Q \oplus \bar{D} \oplus \bar{U}$ | 2                | 3           | unstable     | $QQ\bar{D}, \bar{D}D\bar{U} = 1$ | $\text{SU(2)}_L$ |
Unification

Incomplete SU(5) reps modify SM running

\[
\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_{GUT}} + \frac{b_i^{SM}}{2\pi} \log \frac{M_{GUT}}{M_Z} + \frac{\Delta b_i}{2\pi} \log \frac{M_X}{\Lambda_{TC}} + \frac{\Delta b}{2\pi} \log \frac{M_{GUT}}{M_X}
\]

\[
\ln \frac{M_X}{\Lambda_{TC}} = \frac{68}{\Delta b_{21} - 1.9\Delta b_{32}}, \quad \ln \frac{M_{GUT}}{M_X} = \frac{35.3\Delta b_{21} - 49.2\Delta b_{32}}{\Delta b_{21} - 1.9\Delta b_{32}}
\]

Giudice, Rattazzi, Strumia, '12
Example:

\[ Q + \tilde{D} \quad \text{DM} = Q\bar{Q}\tilde{D} \]

\[ \alpha_{\text{GUT}} \approx 0.06 \]

\[ M_{\text{GUT}} \approx 2 \times 10^{17} \text{ GeV} \]

\[ \Lambda_{\text{TC}} = 100 \text{ TeV} \quad M_X \approx 2 \times 10^{11} \text{ GeV} \]
Relic abundance of TB:

\[ M_{DM} \approx \begin{cases} 
100 \text{ TeV} & \text{if DM is a thermal relic,} \\
3 \text{ TeV} & \text{if DM is a complex state with a TCBr asymmetry} 
\end{cases} \]

Thermal abundance determined by annihilation into technipions

THERMAL ABUNDANCE

\[ \langle \sigma v \rangle_{BBN} \sim \frac{4\pi}{m_B^2} \]

\[ m_B \sim 50 - 100 \text{ TeV} \]
If charged TB dark matter interacts as usual WIMPS

\[ \sigma_{SI}^3 = 0.12 \times 10^{-46} \text{ cm}^2 \]
Even if TB is SM singlet can have dipole interactions

\[ \frac{e}{2m_B} \bar{\Psi} \gamma_{\mu\nu} (g_M + ig_E \gamma_5) \Psi \ F_{\mu\nu}. \]

- Magnetic

\[ g_M = \mathcal{O}(1) \]

- Electric

Need CP violation:

\[ g_E \sim \frac{e \theta_{TC} \min|m_Q|}{M_{DM}} \]
Dipole interactions:

\[ \frac{d\sigma}{dE_R} \approx \frac{e^2 Z^2}{16\pi m_B^2 E_R} \left( g_M^2 + \frac{g_E^2}{v^2} \right) \]

\[ g_M^2 + 10^7 g_E^2 < \left( \frac{m_B}{5 \text{ TeV}} \right)^3 \]
\[ N = N_F = 3 \]

Pions and lightest baryons are adjoint of SU(3).

Rescale QCD:

\[ \frac{m_{\rho}}{f} \approx 7 \quad \frac{m_B}{m_{\rho}} \approx 1.3 \quad \Delta^g m^2_\pi \approx \alpha_2 J(J+1)m_{\rho}^2 \]

\[ b_2 \sim -2b'_1 \sim -0.3m_B^{-1} \quad D = 0.6 \quad F = 0.4 \]

Technibaryon thermal abundance:

\[ \sigma_{pp}^{QCD} \sim 100 \text{ GeV}^{-2} \quad \frac{\Omega_{DM}}{\Omega_{DM}^c} \sim \left( \frac{M_B}{200 \text{ TeV}} \right)^2 \]
$$\Delta m_N \approx 4m_\rho (b_1 m_L + b_2 m_E) \quad \Delta m_\Xi \approx 4m_\rho (b_2 m_L + b_1 m_E)$$

$$\Delta m_\Sigma \approx 4m_\rho (b_1 + b_2)m_L \quad \Delta m_\Lambda \approx \frac{4}{3} m_\rho (b_1 + b_2)(m_L + 2m_E)$$

Triplet is never lightest. Singlet can be DM:

$$m_L \sim m_E$$

Dipole interactions:

$$\frac{1}{4m_B} \bar{B} \gamma^{\mu\nu} (g_M + ig_E \gamma_5) \partial_\nu \partial_\mu$$

$$g_M \sim O(1)$$

$$g_E^{B^1} \sim -0.15 \frac{m_\pi^2}{f^2} \log \frac{m_B^2}{m_\pi^2} \times \theta_{TC}.$$
• SO(N) models

With NF fundamental flavors:

\[ \langle 0| q_i^a q_i^b |0 \rangle \sim 4\pi f^3 \delta^{ab} \rightarrow \frac{SU(N_F)}{SO(N_F)} \]

Fermions are in a real TC representation:

- No difference between baryons and anti-baryons. Two baryons can annihilate into N pions

\[ \varepsilon^{i_1 i_2 \ldots i_N} \varepsilon^{j_1 j_2 \ldots j_N} = (\delta_{i_1 j_1} \delta_{i_2 j_2} \ldots \delta_{i_N j_N} \pm \text{permutations}) \]

- Majorana masses are possible for real SM reps.
After electro-weak symmetry breaking neutral baryons with hyper charge mix with Majorana ones. Mass eigenstates are real. Similar to neutralinos.

SO(N) baryons are Majorana fermions or real scalars:

- **production**
  Cannot be produced through an asymmetry.
  Thermal abundance:

  \[ m_{DM} \sim 100 \text{ TeV} \]

- **detection**

  There are no vector couplings with Z and no dipole interactions. This avoids spin independent bounds.
### Renormalizable models:

| $SO(N)$ techni-color. | Techni-quarks | Yukawa couplings | Allowed $N$ | Techni-pions | Techni-baryons |
|-----------------------|---------------|------------------|-------------|--------------|----------------|
| $N_{TF} = 3$          | $\Psi = V$    | 0                | 3, 4, .., 7 | 5            | $V^N = 3, 1, ...$ for $N = 3, 4, ...$ |
|                       |               |                  |             | unstable     | $V^N = 3, 1, ...$               |
| $N_{TF} = 4$          | $\Psi = N \oplus V$ | 0               | 3, 4, .., 7 | 9            | $4, 1, ...$ |
|                       |               |                  |             | unstable     | $V^N = 1, VVV + NN = 3,$ |
|                       |               |                  |             |              | $VV(VV + NN) = 1, ...$          |
| $N_{TF} = 5$          | $\Psi = L \oplus N$  | 1               | 3, 4, .., 14| 14           | 5, 1... |
|                       |               |                  |             | unstable     | $L\bar{L}N = 1,$ |
|                       |               |                  |             |              | $L\bar{L}(L\bar{L} + NN) = 1, ...$ |
| $N_{TF} = 7$          | $\Psi = L \oplus V$  | 1               | 4            | 27           | 1, ... |
|                       |               |                  |             | unstable     | $(L\bar{L} + VV)^2 = 1$ |
| $N_{TF} = 8$          | $\Psi = L \oplus E \oplus N$ | 2               | 4, 5         | 35           | 1 |
|                       |               |                  |             | unstable     | $(E\bar{E} + L\bar{L})^2 + NN(L\bar{L} + E\bar{E}) = 1$ |
| $N_{TF} = 9$          | $\Psi = G$    | 0                | 4            | 44           | 1 |
|                       |               |                  |             | unstable     | $GGGG = 1$ |
| $N_{TF} = 10$         | $\Psi = L \oplus N \oplus V$ | 2               | 4            | 54           | 1 |
|                       |               |                  |             | unstable     | $L \oplus E \oplus V + NN(L\bar{L} + E\bar{E} + VV) = 1$ |
|                       |               |                  |             |              | $SU(2)_L$ |


Lightest baryons are likely a quintuplet of SO(5): “Higgsino” + “bino”

\[
N = 3 : \quad \begin{pmatrix}
1_0 \\
1_0 \\
2_{1/2} \\
2_{-1/2} \\
\vdots
\end{pmatrix}_{\text{SU}(N_{\text{TF}})} = \begin{pmatrix}
1_0 & 2_{1/2} & 2_{-1/2} & \cdots \\
m_{10} & y_L v & y_R v & \cdots \\
y_L^* v & 0 & m_{2_{1/2}} & \cdots \\
y_R^* v & m_{2_{1/2}} & 0 & \cdots \\
\vdots & \vdots & \vdots & \vdots
\end{pmatrix}_{\text{SO}(N_{\text{TF}})}
\]

- **Higgsino DM** \(m_{2_{1/2}} < m_{10}\)

\[
\Delta m_M \sim \frac{y^2 v^2}{\Delta m} \quad \alpha \sim \frac{y v}{\Delta m} \quad \Delta m \equiv m_{2_{1/2}} - m_{10}
\]

\(\Delta m > \frac{\alpha^2}{4\pi} \sim 0.03 \times m_B\)
- spin-dependent $x$-sec

$$g_A Z_\mu \frac{g_2 \bar{X} \gamma^\mu \gamma^5 X}{\cos \theta_W} \frac{1}{2} |g_A| < 1.2 \frac{M_{DM}}{\text{TeV}}$$

$$g_A \sim \frac{y^2 v^2}{(\Delta m)^2} \ll 1$$

- inelastic dark matter

$$\sim \frac{g_2 Y}{\cos \theta_W} \bar{\Psi}^+_M \gamma^\mu \Psi^-_M Z_\mu \quad \Delta m_M > 100 \text{ KeV} \quad \Rightarrow \quad y > 10^{-2} \times \left( \frac{\Delta m}{100 \text{ TeV}} \right)^{\frac{1}{2}}$$

- spin-independent $x$-sec

$$\sigma_{SI} = \frac{g_{DM}^2 m_N^4 f_N^2}{2\pi v^2 M_h^4} \quad g_{DM} < \sqrt{M_{DM}/75\text{TeV}} \quad g_{DM} = \frac{\partial M_{DM}}{\partial h} \sim y$$
OTHER PHENO

(O. Antipin, MR, arxiv:1508.01112)
Framework predicts Goldstone bosons and vector bosons with SM charges:

$$\langle 0 | \bar{\Psi} \gamma^\mu T^a \Psi | \rho^b \rangle = -\delta^{ab} m_\rho f_\rho \epsilon^\mu$$

$$\langle 0 | \bar{\Psi} \gamma^\mu \gamma^5 T^a \Psi | \pi^b \rangle = -i\delta^{ab} f p^\mu$$

Heavy vectors mix with SM gauge bosons

$$m_\rho \sim g_\rho f$$

Differently from composite Higgs fermions are elementary.
Decay to hidden pions and back to SM gauge bosons through anomalies or $qq\bar{q}$

\[ \sigma(pp \rightarrow \rho^+ + X) \sim \frac{g_\rho^4}{g_\rho^2} \sigma_{u\bar{d}} \]
\[ \sigma(pp \rightarrow \rho^- + X) \sim \frac{g_\rho^4}{g_\rho^2} \sigma_{d\bar{u}} \]
\[ \sigma(pp \rightarrow \rho^0 + X) \sim \frac{g_\rho^4}{g_\rho^2} (\sigma_{u\bar{u}} + \sigma_{d\bar{d}}) \]

Pions can also be collider stable or long lived.

Greco, Liu ’14

\[ \text{Br}(\rho \rightarrow q\bar{q}) \propto \frac{g_\rho^4}{g_\rho^4} \]
Pions can also be produced through SM interactions

\[ pp \rightarrow W^{\pm} \rightarrow \pi_3^\pm \pi_3^0 \rightarrow 3\gamma + W^{\pm} \]

Only search from CDF!

\[ m_{\pi_3} > 230 \text{ GeV} \]
New features arise with Yukawa couplings

\[ H \bar{Q}_i (y^L_{ij} P_L + y^R_{ij} P_R) Q_j \rightarrow y m_\rho f H \pi_2 + \ldots \]

**MIXING :**

\[ \epsilon \sim y \frac{m_\rho f}{m^2_{\pi_2}} \]

- Pions with species number decay through Higgs:

\[ \pi_{2_{1/2}} \rightarrow H \pi_{10}, \quad \pi_{1_1} \rightarrow H H \pi_{10} \]

- Small effects in precision tests, Higgs couplings etc…

\[ \delta \hat{T} \sim \frac{v^2}{f^2} \epsilon^4 \quad \delta \hat{S} \sim \frac{m^2_W}{m^2_\rho} \epsilon^2 \quad \frac{h_{WW}}{h^\text{SM}_{WW}} \sim \epsilon^3 \frac{v^2}{f^2} \]
Electro-weak VEV

\[ M^2 = \begin{pmatrix} m_0^2 & \epsilon m_{\pi_2}^2 \\ \epsilon m_{\pi_2}^2 & m_{\pi_2}^2 \end{pmatrix} \]

 Electro-weak VEV

\[ m_0^2 - \epsilon^2 m_{\pi_2}^2 \approx 0 \]

- \( \epsilon < 1 \)  \hspace{1cm} \text{Elementary Higgs}
- \( \epsilon > 1 \)  \hspace{1cm} \text{Composite Higgs}

Elementary Higgs generates vacuum misalignment of composite Higgs!

\[ \lambda H \bar{q}_L q_R \]

\[ y^{SM} \approx \frac{\lambda}{\epsilon} \]

Viable UV completion of composite Higgs. Not natural... supersymmetry? relaxion?
EDM for SM particles generated with complex Yukawas:

\[ Q = L + V \]

\[ m_L \bar{L}L + \frac{m_V}{2} VV + y_L H^\dagger VL + y_R^* HV \bar{L} + \text{h.c.} \]

\[ d_e \sim 10^{-27} \text{e cm} \times \text{Im}[y_L y_R] \times \frac{N}{3} \times \left( \frac{\text{TeV}}{m_\pi} \right)^4 \times \left( \frac{m_\rho}{\text{TeV}} \right)^2 \]

\[ d_e < 8.7 \times 10^{-29} \text{ e cm} \quad \text{@ 90\% C.L.} \]
CONCLUSIONS

• Stability of DM suggests the existence of accidental symmetries beyond the SM. DM can be naturally realised within “Vector-like confinement” models as techni-baryons.

• SU(N) models generate complex DM with sizeable electric and magnetic dipoles. SO(N) models give Majorana DM giving rise for example to inelastic DM.

• The models automatically realize MFV and predict very small deviations from SM, yet they could be discovered at colliders if the dynamical scale is around TeV. Interesting effects include: resonances production, electron EDMs, gravitational waves, unification...
OBRIGADO!!
Gravitational waves (GW)

$SU(N)$ confining theories with $N_F$ massless flavours give rise to a 1st order P.T. for

$$3 \leq N_F \leq 4N \quad \text{and} \quad N > 3$$

P.T. occurs at: $T \sim \Lambda_{TC} \sim \mathcal{O}(10 \ \text{TeV})$

Peak frequency of the GW signal:

$$f_{\text{peak}} = 3.3 \times 10^{-3} \ \text{Hz} \times \left(\frac{T}{10 \ \text{TeV}}\right) \times \left(\frac{\beta}{10H}\right)$$

Amplitude of the GW signal:

$$h^2\Omega_{GW} \sim 10^{-9}$$

P. Schwaller 15'
**Electron EDM**

\[ Q = L + N \]

\[
\mathcal{L}_M = m_L LL^c + m_N NN^c + yHLN^c + \tilde{y}H^\dagger L^cN + h.c.
\]

**Anomalies:**

\[
L^{\text{EDM}} = -\frac{m_{\pi_3}^2}{2} (\pi_3^a)^2 - \frac{m_\eta^2}{2} \eta^2 + \frac{4\text{Im}(y\tilde{y}) g_\rho f_\pi^3}{m_{K_2}^2} \left( H^\dagger \sigma^a H \pi_3^a - \frac{1}{\sqrt{3}} \eta H^\dagger H \right)
+ \frac{g_1 g_2 N}{64\pi^2 f_\pi} \pi_3^a W_{\mu\nu} \tilde{B}^\mu\nu - \frac{N}{64\sqrt{3}\pi^2 f_\pi} \eta \left[ g_2^2 \tilde{W}\nu^{i\mu\nu} W_i^{\mu\nu} + g_1^2 \tilde{B}^\mu\nu B^{\mu\nu} \right]
\]

**Integrate out GBs:**

\[
L^{\text{EDM}} = \frac{-m_{\pi_3}^2}{2} L_{\text{EDM}}^{\text{eff}} \subset \frac{-e^2 N}{48\pi^2} \frac{\text{Im}(y\tilde{y})(3m_\eta^2 - 2m_{\pi_3}^2)m_\rho^2}{m_{\pi_3}^2 m_\eta^2 m_{K_2}^2} F\tilde{F} h_0^\dagger h_0 \equiv -\frac{cH}{\Lambda^2} F\tilde{F} h_0^\dagger h_0
\]

\[
d_e \approx \frac{e m_e c_H}{4\pi^2 \Lambda^2} \log \frac{\Lambda^2}{m_h^2}
\]
• $\Theta$-angle

$$- \sum_i m_i \bar{Q}^i Q^i + \frac{\theta_{TC}}{32\pi^2} g_{\mu\nu} \tilde{g}^{\mu\nu}$$

Vacuum:

$$V(U) = -\frac{f_\pi^2}{2} \left( \text{Tr}[MU + M^\dagger U^\dagger] - \frac{a}{N} (-i \log \det U - \theta)^2 \right)$$

$$U = U_0 V \quad U_0 = \text{Diag}[e^{\phi_1}, e^{\phi_2}, \ldots, e^{i\phi_{NF}}]$$

Dashen equations:

$$\mu_i^2 \sin \phi_i = \frac{a}{N} (\theta - \sum \phi_i) \quad i = 1, \ldots, N_F$$

$$\begin{pmatrix}
  m_{n'}^2 \sim \frac{3a}{N} \\
  \mu_i^2 \sim \frac{4\pi}{\sqrt{N}} f m_i \quad \theta - \sum \phi_i \equiv \frac{a}{N} \bar{\theta}
\end{pmatrix}$$
Baryons action depends on $\theta$

$$\mathcal{L}_B \sim \bar{B}(i \gamma^\mu D_\mu - m_B)B - b \text{Tr}\bar{B}\tilde{\chi}B + \frac{c}{f} D_\mu \Pi \bar{B}\gamma^\mu \gamma^5 B$$

$$\tilde{\chi} = v_0 \xi^\dagger M \xi \quad \quad U = \xi^2$$

CP violating interactions from mass terms.
As for the neutron EDM is generated:

$$\mathcal{L}_\theta \sim i \frac{a}{Nf} \bar{\theta} b \bar{B}\pi B$$

$$i \frac{d_{DM}}{2} \bar{B}\sigma_{\mu\nu} \gamma_5 B \ F_{\mu\nu}$$

$$d_{DM} \sim \frac{e}{16\pi^2} \frac{abc}{Nf^2} \bar{\theta} \log \frac{m_B^2}{m_\pi^2}$$
Baryons in SO(N) models

Start from the SU(NF) HB and decompose under SO(NF)

\[
\begin{align*}
N = 3 &: \quad \left( \begin{array}{c}
\text{\textbullet} \\
\text{\textbullet} \\
\text{\textbullet}
\end{array} \right)_{\text{SU}(N_{\text{TF}})} &= \left( \begin{array}{c}
\text{\textbullet} \\
\text{\textbullet} \\
\text{\textbullet} \\
\text{\textbullet}
\end{array} \right)_{\text{SO}(N_{\text{TF}})} \\
N = 4 &: \quad \left( \begin{array}{c}
\text{\textbullet} \\
\text{\textbullet} \\
\text{\textbullet} \\
\text{\textbullet}
\end{array} \right)_{\text{SU}(N_{\text{TF}})} &= \left( \begin{array}{c}
\text{\textbullet} \\
\text{\textbullet} \\
\text{\textbullet} \\
\text{\textbullet} \\
\text{\textbullet}
\end{array} \right)_{\text{SO}(N_{\text{TF}})} \\
N = 5 &: \quad \left( \begin{array}{c}
\text{\textbullet} \\
\text{\textbullet} \\
\text{\textbullet} \\
\text{\textbullet} \\
\text{\textbullet}
\end{array} \right)_{\text{SU}(N_{\text{TF}})} &= \left( \begin{array}{c}
\text{\textbullet} \\
\text{\textbullet} \\
\text{\textbullet} \\
\text{\textbullet} \\
\text{\textbullet} \\
\text{\textbullet}
\end{array} \right)_{\text{SO}(N_{\text{TF}})}
\end{align*}
\]

Example: QCD “eightfold way” splits spin-1/2 HB

\[
8 = \left( \begin{array}{c}
\text{\textbullet} \\
\text{\textbullet} \\
\text{\textbullet}
\end{array} \right)_{\text{SU}(3)} = \left( \begin{array}{c}
\text{\textbullet} \\
\text{\textbullet} \\
\text{\textbullet} \\
\text{\textbullet}
\end{array} \right)_{\text{SO}(3)} = 5 \oplus 3
\]

similarly for the heavier spin-3/2 HB:

\[
10 = \left( \begin{array}{c}
\text{\textbullet} \\
\text{\textbullet} \\
\text{\textbullet}
\end{array} \right)_{\text{SU}(3)} = \left( \begin{array}{c}
\text{\textbullet} \\
\text{\textbullet} \\
\text{\textbullet} \\
\text{\textbullet} \\
\text{\textbullet}
\end{array} \right)_{\text{SO}(3)} = 7 \oplus 3
\]
Heavier baryons

\[ B_3^{\text{heavy}} = \begin{array}{cccc} \hline \hline \hline \end{array} \quad \text{or} \quad B_4^{\text{heavy}} = \begin{array}{cccc} \hline \hline \hline \end{array} \oplus \begin{array}{cccc} \hline \hline \hline \end{array} \]

\[ s = \frac{3}{2} \quad s = 1 \quad s = 2 \]

\[ M_B \sim N m_0 + m_1 + \frac{J(J+1)}{N} m_2 + \mathcal{O} \left( \frac{1}{N^2} \right) \]

Baryons with species number can also be stable

Ex:

\[ \Omega^- (sss) \rightarrow \Xi^0 (uss) + K^- (\bar{us}) \]

Forbidden by phase space in QCD.