Topological effects on non-relativistic eigenvalue solutions under AB-flux field with pseudoharmonic- and Mie-type potentials

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Abstract
In this paper, we investigate the quantum dynamics of a non-relativistic particle confined by the Aharonov–Bohm quantum flux field with pseudoharmonic-type potential in the background of topological defect produced by a point-like global monopole. We solve the radial Schrödinger equation analytically and determine the exact eigenvalue solution of the quantum system. Afterwards, we consider a Mie-type potential in the quantum system and solve the radial equation analytically and obtain the eigenvalue solution. We analyze the effects of the topological defect and the quantum flux with these potentials on the energy eigenvalue and wave function of the non-relativistic particles. In fact, it is shown that the energy levels and wave functions are influenced by the topological defect shifted the result compared to the flat space results. In addition, the quantum flux field also shifted the eigenvalue solutions and an analogue of the Aharonov–Bohm effect for bound-states is observed. Finally, we utilize these eigenvalue solutions to some known diatomic molecular potential models and presented the energy eigenvalue and wave function.

Keywords: topological defect, non-relativistic wave equation, physical potential, geometric quantum phase, special function, solutions of wave equation: bound-state

(Some figures may appear in colour only in the online journal)

1. Introduction
The searching for the exact or approximate eigenvalue solutions of the non-relativistic and relativistic wave equations have turned to be a principal part from the beginning of quantum mechanics [1]. These exact and approximate solutions of the wave equations are important in different fields, such as atomic physics, nuclear and high energy physics, quantum electrodynamics, and theory of molecular vibrations [2–10]. The exact or approximate eigenvalue solutions of the stationary Schrödinger wave equation plays an important role to examine the correctness of models and approximations in computational physics as well as in chemistry. In addition, particle interacts with potential of various kinds have used to describe importance of several physical systems since they contain all the necessary information of a quantum state under investigation. However, analytic solutions are possible only for a few simple quantum systems such as the hydrogen atom and harmonic oscillator problem that were given in many textbooks [11–15]. To obtain the exact and approximate solutions of the wave equations, various methods or techniques have employed, such as the Nikiforov-Uvarov method [16–21] and its functional analysis, supersymmetric quantum mechanics approach [22, 23], asymptotic iteration method [24, 25], Laplace’s transformation method, and special functions of various kinds.

Furthermore, studies of the quantum motions of charged particles in the presence of an uniform magnetic and the Aharonov–Bohm (AB) flux fields, which are perpendicular to the plane where the particles are confined have been done since past few years. In fact, the study of the non-relativistic and relativistic charged particles that are confined to the magnetic field has been a growing research interested because of possible applications in different fields, such as in graphene [9, 10, 26], semiconductor structures [27], chemical physics [28], biology [29], molecular vibrational and rotational spectroscopy in molecular physics [30]. The investigation of the particles with magnetic field
have been done by many authors in the literature including two-dimensional solid with Kratzer potential in the presence of screw dislocation [31] and in cosmic string space-time [32–35]. Note that the presence of the Aharonov–Bohm flux field in quantum mechanical systems shows an analogue of the Aharonov–Bohm (AB) effect [36, 37]. This AB-effect is a quantum mechanical phenomenon in which an electrically charged particle is affected by the quantum flux field despite being confined to a region in which both the magnetic and electric fields are zero. The experimental confirmation of its existence was presented in [38].

Another kind of system that has been studied in the context of quantum mechanical problems are the topological defects which were formed in the early universe through symmetry-breaking mechanism [39]. Topological defects are classified into cosmic strings [40], domain walls, and global monopoles [41–43]. The quantum mechanical systems in the background of a space-time generated by a cosmic string have been studied by several authors (see, [32–35, 44] and related references therein). The spherically symmetric static point-like global monopole space-time was first presented in [41] and have been studied both in the relativistic limit (see, [45–47]) and only a handful works in the non-relativistic limit [48–52] in quantum system. The presence of the topological defect in a quantum system changes its physical properties and modifies the energy eigenvalue and wave function of a particle compared to the flat space result.

Our aim in this work is to study the quantum motions of the non-relativistic particles under the influences of the Aharonov–Bohm flux field in the background of a point-like global monopole with different interactions potential, such as pseudoharmonic-type and Mie-type potentials. In fact, we determine the exact eigenvalue solutions analytically and analyze the effects of various factors. Afterwards, we use these eigenvalue solutions for some known molecular potential models and show the effects of the topological defect and the quantum flux field which modified the results compared to the flat space results obtained in the literature.

This paper is organized as follows: in section 2, we discuss the three-dimensional radial Schrödinger wave equation in the background of a point-like global monopole under the flux field with potential. Then, we solve this radial equation with two types of potential, namely a pseudoharmonic-type potential (section 2.1) and the Mie-type potential (section 2.2) and obtain the eigenvalue solutions. In section 3, we use these eigenvalue solutions to some some known potential models in the literature; and in section 4, we present our results. Throughout the analysis, we use the natural units \( c = 1 = h \).

2. Topological effects on radial solution under AB-flux field with potential

A static and spherically symmetric point-like global monopole space-time in the spherical coordinates \((t, r, \phi, \theta)\) is given by [43, 46–49, 51, 52]

\[
\begin{aligned}
\Psi = i \frac{d\Psi}{dr},
\end{aligned}
\]

where \( \alpha < 1 \) represents the topological defect parameter of a point-like global monopole, \( g_{ij} \) is the spatial metric tensor whose nonzero components are \( g_{rr} = \frac{1}{\sqrt{r^2 - \alpha^2}}, \quad g_{\phi\phi} = r^2 \sin^2 \theta \) with their contravariant components \( g^{rr} = \frac{1}{r^2} \), \( g^{\phi\phi} = \frac{1}{r^2 \sin^2 \theta} \). This point-like global monopole space-time have many interesting features including a naked curvature singularity on the symmetry axis \( R = R_0 = \frac{2(1 - \alpha^2)}{r} \). Other properties of this geometry are given in [43, 46].

In this section, we study the quantum motions of the non-relativistic particle under a potential in the presence of the Aharonov–Bohm flux field in this point-like global monopole background. Afterwards, we will consider the Mic-type potential and obtain the eigenvalue solution analytically and discuss the effects of topological defects as well as quantum flux on the eigenvalue solutions. Therefore, the time-dependent Schrödinger wave equation in the presence of an electromagnetic potential \( A \) is described by the following wave equation [31, 48, 49, 51–53]

\[
\begin{align}
\Psi(t, r, \theta, \phi) = e^{-iEt} Y_{l,m}(\theta, \phi) \psi(r),
\end{align}
\]

where \( E \) is the rest mass the particles, \( g \) is the determinant of the metric tensor \( g_{ij} \) with \( g^{ij} \) its inverse, and \( D_i = (\partial_i - i e A_i) \) with \( e \) is the electric charges. For the space-time (1) under consideration, the determinant of the spatial metric tensor \( g_{ij} \) is given by \( g = \frac{r^2 \sin^2 \theta}{\alpha^2} \).

By the method of separation of variables, one can write the total wave function \( \Psi(t, r, \theta, \phi) \) in terms of different variables. Suppose, a possible wave function in terms of a radial wave function \( \psi(r) \) is as follows:

\[
\begin{align}
A_r = 0 = A_{\theta}, \quad A_{\phi} = \frac{\Phi}{2 \pi r \sin \theta},
\end{align}
\]

where \( \Phi = \text{const} \) is the Aharonov–Bohm flux, \( \Phi_0 \) is the quantum of magnetic flux, and \( \Phi \) is the amount of magnetic flux which is a positive integer.

Thereby, expressing the wave equation (2) in the space-time background (1) and using equations (3)–(4), we obtain the following radial and angular differential equations for \( \psi(r) \)
The radial Schrödinger equation can be written as

\[
\frac{\sin \theta}{r} \left( \frac{d}{d r} \left( r \frac{d \psi}{d r} \right) \right) + \frac{\cos \theta}{r^2} \left( \frac{d^2 \psi}{d \theta^2} - \frac{l(l+1)}{r^2} \right) \psi(r) = 0.
\]

And

\[
\frac{1}{\sin \theta} \frac{d}{d \theta} \left( \sin \theta \frac{d}{d \theta} \right) \psi(r) + \frac{1}{\sin \theta} \frac{d^2 \psi}{d \theta^2} + l(l+1) Y_{l,n}(\theta, \phi) = 0
\]

where \( l = \frac{1}{2} (1 - \Phi) \) is the effective orbital quantum number.

The radial Schrödinger equation can be written as

\[
\frac{d^2 \psi}{d r^2} + \frac{\alpha^2}{r^2} \left( \frac{d^2 \psi}{d r^2} \right) + \frac{\alpha^2}{r^2} \left( \frac{d \psi}{d r} \right) - \Lambda \psi = 0.
\]

The effective potential of the system (by changing the function \( \frac{\psi(r)}{R(r)} \) in the equation (7), one can find the one-dimensional Schrodinger-like equation) given by

\[
V_{\text{eff}}(r) = \frac{1}{\alpha^2} \left[ V(r) + \frac{(l - \Phi)(l - \Phi + 1)}{2 M r^2} \right].
\]

We can see that the effective potential of the quantum system depends on the topological defect characterized by the parameter \( \alpha \), and the quantum flux field \( \Phi_{AB} \).

In this analysis, we consider the following two different kinds of potential of physical interest and determines the exact eigenvalue solutions of the non-relativistic wave equation analytically.

2.1. Effects of pseudoharmonic-type potential

In this section, we are interested in the following potential \( V(r) \) superposition of harmonic oscillator and inverse quadratic potential given by [54–58]

\[
V(r) = V_0 + \beta \frac{r^2}{r^2} + \frac{\gamma}{r^2},
\]

where \( \beta, \gamma \) characterise the potential parameters and \( V_0 > 0 \) is a constant potential term.

Thereby, substituting potential (9) in the equation (7), we obtain the following radial equation:

\[
\frac{d^2 \psi}{d r^2} + \frac{\alpha^2}{r^2} \left( \frac{d^2 \psi}{d r^2} \right) + \Lambda \psi = 0,
\]

where we set the parameters

\[
\Lambda = \frac{2 M (E - V_0)}{\alpha^2},
\]

\[
j = \frac{(l - \Phi)(l - \Phi + 1) + 2 M \gamma}{\alpha^2},
\]

\[
\omega = \sqrt{\frac{2 M \beta}{\alpha^2}}.
\]

The effective potential of the quantum system using potential
where different parameters are defined as

\[ 2 \mu = \sqrt{2^2 + \frac{1}{4}}, \quad \nu = \frac{\Lambda}{4 \omega}. \]  

(15)

Equation (14) is the Whittaker differential equation \([59]\) and \(U(s)\) is the Whittaker function which we can write in terms of the confluent hypergeometric function of the first kind \(_1F_1(s)\) \([59–63]\) as follows

\[ U(s) = s^{-1/2} e^{-s/2} \left( \frac{\nu + 1}{2 \mu + 1} \right). \]  

(16)

Our aim is for searching the bound-state solutions of the quantum system by solving the equation (14). It is well-known that the confluent hypergeometric function \(_1F_1(s)\) becomes a finite degree polynomial in \(s\) of degree \(n\) by imposing that \((\mu - \nu + \frac{1}{2})\) is a negative integer, that is, \((\mu - \nu + \frac{1}{2}) = -n\), where \(n = 0, 1, 2, \ldots\)

After simplification of the condition \((\mu - \nu + \frac{1}{2}) = -n\) using equation (15) and then equation (11), one will find the following expression of the energy eigenvalue \(E_{n,l}\) given by

\[ E_{n,l} = V_0 + \alpha \sqrt{\frac{\beta}{M}} \times \left( 2n + \sqrt{\frac{U_{12}(1 - \Phi + 3M \gamma + 1)}{\alpha^2} + \frac{1}{2} + 1} \right). \]  

(17)

The normalized radial wave functions are given by

\[ \psi_{n,l}(s) = D_{n,l} \left( \frac{2 \sqrt{\frac{\beta}{M}}}{\alpha} \right)^{3/8} \times s^{\mu - \frac{1}{2}} e^{-\frac{s}{2}} \left[ \frac{1}{2} \right] \]  

(18)

where \(D_{n,l}\) is a constant that can be determined by the normalization condition for the radial wave function \([64]\)

\[ \frac{1}{\alpha} \int_{0}^{\infty} r^2 dr \left| \psi(r) \right|^2 = 1. \]  

(19)

To solve integral of the radial wave function, one can write the confluent hypergeometric function in terms of the associated Laguerre polynomials given by the relation \([63]\)

\[ _1F_1(-n, 2 \mu + 1; x) = \frac{n! (2 \mu)!}{(n + 2 \mu)!} L_{n}^{(2 \mu)}(x). \]  

(20)

Then, taking into account \(s = \omega r^2\), and with the help of \([61]\) to solve the integrals, the normalization constant is given by

\[ D_{n,l} = \left( \frac{1}{12 \mu n!} \right) \left( \frac{2 \alpha^{(n + 2 \mu)}}{n!} \right) \left( \frac{1}{(j^2 + \frac{1}{4})} \right)^{-\frac{2 \alpha (n + j^2 + \frac{1}{4})}{n!}}. \]  

(21)

Equation (17) is the energy levels and equations (18)–(21) are the normalized radial wave functions of a non-relativistic particle confined by the AB-flux field with pseudoharmonic-type potential under the topological defects produced by a point-like global monopole. One can see that the energy levels and the radial wave functions are influenced by the topological defect characterized by the parameter \(\alpha\), and the quantum flux field \(\Phi_{AB}\) and get them modified compared to the flat space results obtained in \([54–58]\). We have plotted a few graphs of the energy levels \(E_{n,l}\) (figure 2) and the normalized radial wave function \(\psi_{n,l}(s)\) (figure 3) for different values of various parameters.

Below, few special cases of the above quantum system will be discussed.

**Case I: without topological defect and potential** \(V(r) = \left( \frac{\alpha}{r^2} + B \right)\).

In this case, we want to study the above quantum system without topological defects. In that case, for \(\alpha \rightarrow 0\), the space-time geometry (1) under consideration becomes Minkowski flat space. Furthermore, we set the constant potential term \(V_0 = 0\) in the potential expression (9).

Therefore, the energy eigenvalue in that case from (17) becomes

\[ E_{n,l} = \left( \frac{2 B}{M} \right)^{1/2} \left( 2n + 1 + \sqrt{\left( 1 - \Phi + \frac{1}{2} \right)^2 + 2 MA} \right) \]  

(22)

which is similar to the energy eigenvalue obtained in \([65]\) provided zero magnetic flux \(0 \rightarrow 0\) here. Thus, the magnetic flux considered in the quantum system shifts the eigenvalue solution of a non-relativistic particle in comparison to the known result obtained in \([65]\) with potential of the form \(V(r) = \left( \frac{\alpha}{r^2} + B r^2 \right)\). Overall, we can see that the energy eigenvalue (17) gets more modified in comparison to the result in \([65]\) due to the presence of the topological defects characterised by the parameter \(\alpha\), and the constant potential
term $V_0$ in addition to the magnetic flux field $\Phi_{AB}$ in the quantum system.

The normalized radial wave functions in that case becomes

$$\psi_{n,l}(s) = N_{n,l} \left( \frac{2MB}{\alpha^2} \right)^{3/8} \times s^{\frac{n-l}{2}} e^{-\frac{s}{2}} \, _1F_1(-n, 1 + 2 \, \mathfrak{m}; s),$$

where

$$N_{n,l} = \frac{1}{(\sqrt{i\beta + \frac{1}{2}})^n} \sqrt{\frac{2^n + \frac{1}{2} + \frac{1}{2}}{n!}},$$

$$\mathfrak{m} = \sqrt{\frac{1}{4} + \frac{1}{4}}.$$

Figure 2. The energy eigenvalue $E_{n,l}$ with topological defect parameter $\alpha$ for different values of other parameters.

Figure 3. The normalized radial wave function $\psi_{n,l}$ for different values of the parameters.
Case II: without topological defects but potential $V(r) = \frac{\alpha}{r^2} + B \cdot r^2 + C$

We discuss the above quantum mechanical system without topological defects, that is, $\alpha \to 1$. In that case, the space-time geometry (1) under consideration reduces to Minkowski flat space. In addition, the chosen potential here is of the form $V(r) = \left( \frac{\alpha}{r^2} + B \cdot r^2 + C \right)$.

Therefore, for $\alpha \to 1$ and with the chosen potential form, one will have the following energy eigenvalue expression

$$E_{n,l} = C + \frac{\gamma^2}{\alpha^2} (2 n + 1) + \sqrt{(l - \Phi + \frac{1}{2})^2 + 2 M \cdot A}$$

which is similar to the result obtained in the flat space [30] provided zero magnetic flux $\Phi \to 0$ here. Thus, the magnetic flux considered in the quantum system shifts the eigenvalue solution of a non-relativistic particle compared to the known result in [30] with this chosen potential. Hence, overall we can see that the topological defects of a point-like global monopole characterized by the parameter $\alpha$, and the quantum flux field $\Phi_{AB}$ modified the eigenvalue solution compared to the result obtained in [30] with this potential. The normalized radial wave function will be the same obtained in equations (23)–(24) with now $\lambda = 2 M (E_{n,l} - C)$.

2.2. Effects of Mie-type potential

In this section, we are interested on another kind of potential equal to repulsive Coulomb plus inverse square potential given by [54, 56, 57, 66–69]

$$V(r) = V_0 + \frac{\delta}{r} + \frac{\gamma}{r^2},$$

where $\delta, \gamma$ characterizes the potential parameter.

For this type of potential, the effective potential of the system becomes

$$V_{eff}(r) = \frac{1}{\alpha^2} \left[ V_0 + \frac{\delta}{r} + \frac{\gamma}{r^2} + \frac{(l - \Phi)(l - \Phi + 1)}{2 M \cdot r^2} \right].$$

We can see that the effective potential of the system depends on the topological defect as well as the magnetic flux. We have plotted a few graph (figure 4) showing the influences of various parameters on the effective potential.

Thereby, substituting the potential (26) into the equation (7), we have obtained the following radial equation:

$$\psi''(r) + \frac{2}{r} \psi'(r) + \left( - \Delta^2 - \frac{j^2}{r^2} - \frac{2 \kappa}{r} \right) \psi(r) = 0,$$

where $j$ is defined in equation (11) and

$$\Delta^2 = \frac{2 M (V_0 - E)}{\alpha^2}, \quad \kappa = \frac{M \cdot \delta}{\alpha^2}.$$

We perform a change of function via $\psi(r) = \frac{R(r)}{r^j}$ in the equation (28), we obtain the following radial wave equation

$$R''(r) + \frac{1}{r} R'(r) + \left( - \Delta^2 - \frac{r^2}{r^2} - \frac{2 \kappa}{r^2} \right) R(r) = 0,$$

where $\tau = \sqrt{j^2 + \frac{1}{4}} = \sqrt{(l - \Phi)(l - \Phi + 1) + 2 M \cdot \gamma} + \frac{1}{4} = 2 \mu$ (see, equation (15)).
Finally, performing a change of variables via $\xi = 2 \Delta r$ in the equation (30), we obtain

$$R^n(\xi) + \frac{1}{\xi} R'(\xi) + \left( -\frac{\tau^2}{\xi^2} - \frac{\kappa}{\Delta} \frac{1}{\xi} - \frac{1}{2} \right) R(\xi) = 0.$$  \hspace{1cm} (31)

It is well-known that the wave function $R(\xi)$ is well-behaved at the origin $\xi \to 0$, since it is a singular point of the equation (31). Suppose, a possible solution to the equation (31) is given by

$$R(\xi) = \xi^{\tau} e^{-\frac{\xi}{2}} F(\xi),$$ \hspace{1cm} (32)

where $F(\xi)$ is an unknown function.

Thereby, substituting solution (32) in the equation (31), we obtain the following differential equation

$$\xi F''(\xi) + (1 + 2 \tau - \xi) F'(\xi) + \left( -\tau - \frac{\kappa}{\Delta} - \frac{1}{2} \right) F(\xi) = 0.$$ \hspace{1cm} (33)

Equation (33) is the confluent hypergeometric differential equation form \([60, 63]\) and $F(\xi)$ the confluent hypergeometric function $F(\xi) = F_1\left(\tau + \frac{n}{\Delta} + \frac{1}{2}; \tau + 1; \xi\right)$ which is well-behaved for $\xi \to \infty$. As stated earlier, for the bound-states solution of the quantum system with Mie-type potential, the function $F_1$ is a finite degree polynomial of degree $n$, and the quantity $\left(\tau + \frac{n}{\Delta} + \frac{1}{2}\right) = -n$, where $n = 0, 1, 2, \ldots$

After simplification of the condition $\left(\tau + \frac{n}{\Delta} + \frac{1}{2}\right) = -n$ using (29), one can obtain the following expression of the energy eigenvalue given by

$$E_{n,l} = V_0 - \frac{M \delta^2}{2\left(n + \frac{1}{2}\right)^2 + (l - \Phi)(l - \Phi + 1) + 2M \gamma + \frac{\gamma^2}{\tau} + l(\Phi + 1) + 2M \gamma + \frac{\gamma^2}{\tau}}.$$ \hspace{1cm} (34)

Equation (34) is the non-relativistic energy expression of particles confined by the Aharonov–Bohm flux field with Mie-type potential in a point-like global monopole background. We can see that this eigenvalue expression (34) is influenced by the topological defects of a point-like global monopole characterized by the parameter $\alpha$, and the quantum flux field $\Phi_{AB}$ and gets modified compared to the results obtained in \([54, 56, 57, 66, 67]\) in the flat space with Mie-type potential. We have plotted graphs of the energy expression $E_{n,l}$ (figure 5) and the radial wave function $\psi_{n,l}(\xi)$ (figure 6) for different radial mode with various values of parameters.

The radial wave function is therefore given by

$$R_{n,l}(\xi) = \xi^{\tau} e^{-\frac{\xi}{2}} \int F(-n, 1 + 2 \tau; \xi).$$ \hspace{1cm} (35)

That can be written as

$$\psi_{n,l}(\xi) = \sqrt{2} \Delta \xi^{\tau-1/2} e^{-\frac{\xi}{2}} \int F(-n, 1 + 2 \tau; \xi) = \sqrt{\frac{2M \delta}{\alpha(n + \tau + \frac{1}{2})}} \xi^{\tau-1/2} e^{-\frac{\xi}{2}} \times \int F(-n, 1 + 2 \tau; \xi).$$ \hspace{1cm} (36)
If we analyze the quantum system without the topological defect, that is, \( \alpha \to 1 \), the space-time geometry under this becomes Minkowski flat space. Therefore, for \( \alpha \to 1 \), the energy eigenvalue expression from equation (34) becomes

\[
E_{n,l} = V_0 \pm \frac{M \delta^2}{2 \left( n + \sqrt{(l - \Phi + \frac{1}{2})^2 + 2M \gamma + \frac{1}{4}} \right)}
\]

which is similar to the result obtained in [30, 70] provided zero magnetic flux \( \Phi \to 0 \) here. Thus, we can see that the presence of the quantum flux field modified the energy spectrum of non-relativistic particles compared to those results obtained in [30, 70]. Furthermore, the presence of the topological defect characterized by the parameter \( \alpha \) modified the eigenvalue expression (34) more in addition to the quantum flux compared to those results obtained in [30, 70].

### 3. Applications to some diatomic molecular potentials

The above results of the studied quantum system previous are now being utilized to develop solutions to some specific types of physical potential models which have application in practical problems. The pseudo-harmonic and Kratzer potentials are successfully employed [71] to study the eigenvalues spectra of a class of diatomic molecules in flat space background. Here, we will study the quantum system in curved space-time background under topological defects with these known potentials.

#### 3.1. Harmonic oscillator potential

The harmonic oscillator potential can be recovered from the potential (10) by setting the potential parameters \( \gamma = 0 \), \( V_0 = 0 \), and \( \beta = \frac{1}{2} M \omega_0^2 \), we have, \( V(r) = \frac{1}{2} M \omega_0^2 r^2 \), a harmonic oscillator potential. Here \( \omega_0 \) is the oscillator frequency. Therefore, using harmonic oscillator potential in the quantum system, one will find the following energy eigenvalue expression:

\[
E_{n,l} = 2 \alpha \omega_0 \left( n + \frac{1}{2} \sqrt{(l - \Phi)(l - \Phi + 1) + \frac{\alpha^2}{4}} \right) + \frac{1}{2}
\]

Equation (38) is the energy levels of harmonic oscillator in the presence of the Aharonov–Bohm flux field in a point-like global monopole. In absence of quantum flux field, \( \Phi_{AB} \to 0 \), the energy eigenvalues from equation (38) becomes

\[
E_{n,l} = \omega_0 \left( 2n + \sqrt{\frac{l(l + 1)}{\alpha^2}} + \frac{1}{4} \right)
\]

which is similar to those results obtained in [48, 49]. Thus, we can see that the quantum flux field shifts the eigenvalue solution of harmonic oscillator compared to those results obtained in [48, 49]. The energy eigenvalue \( E_{n,l} \) depends on the geometric quantum phase \( \Phi_B \) that shows an analogue of the Aharonov–Bohm effect [36, 37].

#### 3.2. Pseudoharmonic potential

The pseudoharmonic potential can be recovered from the potential (10) by setting the parameters \( \beta = \frac{2\omega_0}{\alpha^2}, \gamma = D_0 \sqrt{n_0^2} \).
and $V_0 = -2D_e$. Thus, we obtain the following potential form [54, 72, 73]
\[ V(r) = \frac{D_e}{r_0^2}r^2 + \frac{D_e r_0^2}{r^2} - 2D_e = D_e \left( \frac{r}{r_0} - \frac{r_0}{r} \right)^2. \] (40)

Here $D_e$ is the dissociation energy between atoms in a solid and $r_0$ is the equilibrium inter-nuclear separation. This potential is of great importance not only in physics but also in chemistry and is useful to describe the interactions of some diatomic molecules [74, 75].

Thereby, using pseudoharmonic potential in the quantum and solving the wave equation, one can find the energy eigenvalue expression as follows
\[ E_{n,l} = -2D_e + \alpha \frac{2D_e}{M \epsilon_0^2} \times \left( 2n + \frac{[(l+\Phi)(l+\Phi+1)+2M D_e \epsilon_0^2]}{\alpha^2} + \frac{1}{2} + 1 \right). \] (41)

The normalized radial wave functions are given by
\[ \psi_{n,l}(s) = B_{n,l} \left( \frac{2M D_e}{\alpha^2 \epsilon_0^2} \right)^{3/2} \text{s}^{l-\frac{1}{2}} e^{-\frac{s}{2}} |F_n|(-n, 1 + 2 \mu; s), \]
\[ B_{n,l} = \left( \frac{1}{(l+\frac{1}{2})!} \right) \sqrt{\frac{2a}{\alpha^2 \epsilon_0^2} \left( n + \sqrt{l(l+1) + 2M D_e \epsilon_0^2} + \frac{1}{2} \right)!}, \] (42)

where $\tau = 2\mu = \sqrt{2} + 1/4$, and $j = \frac{[(l+\Phi)(l+\Phi+1)+2M D_e \epsilon_0^2]}{\alpha^2}$.

Equation (41) is the energy eigenvalue expression and equation (42) is the normalized radial wave function of non-relativistic particles confined by the quantum flux field with pseudoharmonic potential in a point-like global monopole background. One can see that the eigenvalue solution equations (41)–(42) get modified by the topological defect characterized by the parameter $\alpha$, and the quantum flux field compared to those results obtained in [54, 72, 73].

For zero quantum flux field, $\Phi_{AB} \rightarrow 0$, the energy eigenvalue of the non-relativistic particles from equation (41) becomes
\[ E_{n,l} = -2D_e + \alpha \frac{2D_e}{M \epsilon_0^2} \times \left( 2n + \frac{[(l+\Phi)(l+\Phi+1)+2M D_e \epsilon_0^2]}{\alpha^2} + \frac{1}{2} + 1 \right). \] (43)

And that the corresponding normalized wave function will be
\[ \psi_{n,l}(s) = B_{n,l} \left( \frac{2M D_e}{\alpha^2 \epsilon_0^2} \right)^{3/2} \text{s}^{l-\frac{1}{2}} e^{-\frac{s}{2}} |F_n|(-n, 1 + 2 \mu; s), \]
\[ B_{n,l} = \left( \frac{1}{(l+\frac{1}{2})!} \right) \sqrt{\frac{2a}{\alpha^2 \epsilon_0^2} \left( n + \sqrt{l(l+1) + 2M D_e \epsilon_0^2} + \frac{1}{2} \right)!}, \] (44)

We can see that the global effects of the geometry characterized by the parameter $\alpha$ of a point-like global monopole modified the result compared to the flat space with pseudoharmonic potential obtained in [54, 72, 73].

3.3. The Kratzer-Fues or modified Kratzer potential

The modified Kratzer potential can easily be recovered from the potential (26) by setting the potential parameters $\delta = -2D_e r_0$, $\gamma = D_e r_0^2$, and $V_0 = D_e$, we have [76]
\[ V(r) = D_e + \frac{D_e r_0^2}{r^2} - 2D_e \frac{r_0}{r} = D_e \left( \frac{r-r_0}{r} \right)^2. \] (45)

This potential also known as Kratzer-Fues potential and has been used by several authors [56, 58, 66, 67, 72, 73] to describe the molecular structures and interactions.

Thereby, using this potential in the quantum system and solving the wave equation, one will find the energy eigenvalue expression as follows
\[ E_{n,l} = D_e \frac{2M D_e \epsilon_0^2}{\alpha^2 \epsilon_0^2} \left( n + \frac{[(l-\Phi)(l-\Phi+1)+2M D_e \epsilon_0^2]}{\alpha^2} + \frac{1}{2} \right). \] (46)

The radial wave function is therefore given by
\[ R_{n,l}(\xi) = \xi^\tau e^{-\frac{\xi}{2}} |F_n|(-n, 1 + 2 \tau; \xi), \] (47)

where $\tau = \sqrt{\frac{[(l-\Phi)(l-\Phi+1)+2M D_e \epsilon_0^2]}{\alpha^2} + \frac{1}{2}}$.

Equation (46) is the non-relativistic energy expression and equation (47) is the radial wave function of non-relativistic particles confined by the quantum flux field with modified Kratzer potential in a point-like defect. One can see that the eigenvalue solution gets modified by the topological defect of the geometry characterized by the parameter $\alpha$, and the flux field $\Phi_{AB}$ compared to those results obtained in [54, 56–58, 66, 67, 72, 73, 76] with this potential.

If we analyze the quantum system without the topological defect, that is, $\alpha \rightarrow 1$, the space-time geometry becomes flat space. Therefore, $\alpha \rightarrow 1$. the energy eigenvalue expression from equation (46) becomes
\[ E_{n,l} = D_e \frac{2M D_e \epsilon_0^2}{\alpha^2 \epsilon_0^2} \left( n + \frac{[(l-\Phi)(l-\Phi+1)+2M D_e \epsilon_0^2]}{\alpha^2} + \frac{1}{2} \right). \] (48)

The radial wave functions are given by
\[ R_{n,l}(\xi) = \xi^\tau e^{-\frac{\xi}{2}} |F_n|(-n, 1 + 2 \tau; \xi), \] (49)

where $\tau_0 = \sqrt{\frac{[(l-\Phi)(l-\Phi+1)+2M D_e \epsilon_0^2]}{\alpha^2} + \frac{1}{2}}$.

Equation (48) is the energy spectra and equation (49) is the radial wave function of non-relativistic particles confined by the quantum flux field with modified Kratzer potential in flat space background.

3.4. Kratzer potential

The Kratzer potential can be recovered from potential (26) by setting the potential parameters $\delta = -2D_e r_0$, $\gamma = D_e r_0^2$, and
$V_0 = 0$, we obtain [73, 77–82]
\[
V(r) = \frac{D_e \alpha^2}{2} - \frac{2 D_e \alpha}{r} = 2 D_e \left[ \frac{1}{2} \left( \frac{\alpha}{r} \right)^2 - \left( \frac{\alpha}{r} \right) \right].
\] (50)

This potential has extensively been used to describe the molecular structures and interactions in chemistry.

Thereby, using Kratzer potential in the quantum system and solving the wave equation, one can obtain bound-states energy eigenvalue expression as follows
\[
E_{n,l} = -\frac{2 M D_e^2 \alpha^2}{\left( (l + \frac{1}{2})^2 + \sqrt{(l - \Phi)(l - \Phi + 1) + 2 M D_e \alpha^2 + \frac{\alpha^2}{r}} \right)^2}.
\] (51)

The radial wave functions are given by
\[
R_{n,l}(\xi) = \xi^{\frac{1}{4}} \xi^{-\frac{1}{4}} \phi(-n, 1 + 2 \tau; \xi),
\] (52)
where $\tau = \sqrt{(l - \Phi)(1 - \Phi + 1) + 2 M D_e \alpha^2 + \frac{\alpha^2}{r}} + \frac{1}{4}$.

Equation (51) is the energy expression and equation (52) is the radial wave function of non-relativistic particles confined by the quantum flux field with Kratzer potential in a point-like defect. One can see that the eigenvalue solution are influenced by the topological defect characterized by the parameter $\alpha$, and the quantum flux field $\Phi_{AB}$ and gets them modified compared to those results obtained in [73, 79–82].

4. Conclusions

In this analysis, we have shown that in the presence of different kinds of interaction potential models, the topological defect changes the physical properties of the quantum system under investigation. Furthermore, the non-relativistic particles under the influences of the quantum flux field but experiencing no direct interactions with a magnetic field also modified the properties of the system. Throughout the analysis, we have observed a shifting in the orbital quantum number $l$, that is, $l \rightarrow l' = (l - \frac{\alpha^2}{2})$ called an effective orbital quantum number that depends on the flux field. This effective quantum number appeared in the energy expressions and thus, the eigenvalue depends on the geometric quantum phase $\Phi_{AB}$ and is a periodic function with a periodicity $\Phi_0$, that is, $E_{n,l}(\Phi_{AB} \pm \Phi_0 \ell) = E_{n,l(\Phi_{AB})}$, where $\ell = 0, 1, 2, \ldots$. This dependence of the eigenvalue on the geometric phase shows an analogue of the Aharonov–Bohm (AB) effect [36, 37] for the bound-states. It is well-known in condensed matter physics that the dependence of the eigenvalue on geometric phase gives rise to a persistent currents which we will discuss in the future work.

We derived the radial wave equation of the Schrödinger equation with the quantum flux field and potential in a topological defect space-time produced by a point-like global monopole. Then, in section 2.1, we have considered pseudoharmonic-type potential and solved the radial equation analytically. The exact eigenvalue expression of the particles are given by the equation (17) and the normalized radial wave function by equations (18)–(21). We have shown that the topological defect characterized by the parameter $\alpha$ and the quantum flux field influences the eigenvalue solution and gets them modified compared to those results obtained in [54–58] in the flat space. As special cases, we have analyzed the eigenvalue solution without topological effects, and have shown that only the flux field shifted the energy levels and wave function compared to those results obtained in [30, 65].

In section 2.2, we have considered the Mie-type potential and solved the quantum mechanical problem in the background of a point-like defect analytically. The energy eigenvalues expression are given by the equation (34) and radial wave function by (36) of the particles. Here also, we have shown that the topological defect of a point-like defect characterized by the parameter $\alpha$, and the quantum flux field influences the eigenvalue solution and modified the result compared to the flat space results obtained in [54, 56, 57, 66, 67]. Also, the presented eigenvalue solution is analyzed without topological effects and have shown that the quantum flux field shifted the result compared to those obtained in the flat space in [30, 70].

In section 3, we have utilized these eigenvalue solutions of the quantum system obtained in section 2.1 and section 2.2 for some known interaction potential models which have been used for diatomic molecular structure in physics and chemistry. These potential models include harmonic oscillator potential (section 3.1), pseudoharmonic potential (section 3.2), modified Kratzer or Kratzer-Fues potential (section 3.3), and Kratzer potential (section 3.4). With these potential models, we have presented the energy eigenvalue expressions and radial wave functions of the non-relativistic particles under the influences of the quantum flux field in a point-like global monopole. We have shown that both the topological defect of a point-like global monopole space-time characterized by the parameter $\alpha$ and the quantum flux field $\Phi_{AB}$ modified the eigenvalue solutions compared to the known results obtained in the flat space in the literature.

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Conflict of interest

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Appendix. The solutions of angular equations

Expressing the wave equation (2) in the space-time background (1) and using equations (3)-(4), we obtain the following equation

\[
\frac{1}{\sin \theta} \left[ \frac{d}{d \theta} \left( \sin \theta \frac{d \psi}{d \theta} \right) \right] + 2 M (E - V(r)) \sin^2 \psi (r) = - \frac{1}{\sin \theta} \left[ \frac{d}{d \theta} \left( \sin \theta \frac{d \psi}{d \theta} \right) \right] = - \frac{1}{\sin \theta} \left[ \frac{d}{d \phi} (\sin \theta \frac{d \psi}{d \phi}) \right] + \sin \theta \left( \frac{d}{d \phi} - i \Phi \right)^2 Y(\theta, \phi) \]

Writing both side equal to \( l'(l' + 1) \), we obtain the radial equation (5). The angular equation is given by

\[
- \frac{d}{d \theta} \left( \sin \theta \frac{d}{d \theta} \right) \left( \sin \theta \frac{d}{d \theta} \right) + \left( \frac{d}{d \phi} - i \Phi \right)^2 Y(\theta, \phi) = 0. \tag{A2}
\]

Assuming a separable solution to the equation (A3), substitution of \( Y(\theta, \phi) = A(\theta)B(\phi) \) yields

\[
\frac{1}{A(\theta)} \left[ \sin \theta \frac{d}{d \theta} \left( \sin \theta \frac{d A(\theta)}{d \theta} \right) \right] + \frac{1}{B(\phi)} \left( \frac{d}{d \phi} - i \Phi \right)^2 B(\phi) = 0. \tag{A4}
\]

Considering a separation constant equal to \( b^2 \), we have the azimuthal equation

\[
- \frac{1}{B(\phi)} \left( \frac{d}{d \phi} - i \Phi \right)^2 B(\phi) + b^2 B(\phi) = 0. \tag{A5}
\]

Suppose, a possible solution to the equation (A5) is given by

\[
B(\phi) = \frac{1}{\sqrt{2 \pi}} e^{i m \phi}. \tag{A6}
\]

Substituting the solution (A6) in equation (A5), we obtain

\[
(-m^2 + 2m \Phi - \Phi^2)B(\phi) + b^2 B(\phi) = 0
\]

\[
\Rightarrow b^2 = (m - \Phi)^2 = m^2, \tag{A7}
\]

where \( m' = (m - \Phi) \) and \( m \) is the magnetic quantum number.

And the polar \( A(\theta) \) equation is

\[
\frac{1}{A(\theta)} \left[ \sin \theta \frac{d}{d \theta} \left( \sin \theta \frac{d A(\theta)}{d \theta} \right) \right] + \frac{1}{\sin \theta} \left( \frac{d}{d \phi} \right) \left( \sin \theta \frac{d A(\theta)}{d \theta} \right) + \left[ l'(l' + 1) - \frac{m^2}{\sin^2 \theta} \right] A(\theta) = 0.
\]

Let us change a variable by \( x = \cos \theta \). The equation with the function \( A(\theta) \) becomes

\[
\frac{d}{dx} \left[ (1 - x^2) \frac{dA(x)}{dx} \right] + \left[ l'(l' + 1) - \frac{m^2}{1 - x^2} \right] A(x) = 0. \tag{A8}
\]

which is the associated Legendre polynomials equation whose solutions are given by

\[
A(x) = P^m_l(x) = (-1)^m \times (1 - x^2)^{m/2} \frac{d^m}{dx^m} (P_l^m(x)), \tag{A9}
\]

where superscript \( m' \) indicates the order and \( P^m_l(x) \) are polynomials of degree \( l' \) given by

\[
P_l^m(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^m. \tag{A10}
\]

Noted that the Legendre polynomials \( P_l^m(x) \) are polynomials of order \( l' \) provided the magnitude of \( m' \) must have values less than or equal to \( l' \). That is

\[
l' \geq |m'| \quad \text{or} \quad l' = |m'| + \kappa, \quad \kappa = 1, 2, \ldots. \tag{A11}
\]

Throughout the analysis we have written \( m' = (m - \Phi) > 0 \) always positive, and hence, \( l' = (l - \Phi) \) due to the quantum flux field \( \Phi_{ab} \) present in the quantum system. For zero magnetic flux \( \Phi \rightarrow 0 \), one will get back the standard azimuthal and polar equations which were given in many textbooks. In that case, the relation (A11) can be written as \( l = (|m| + \kappa) \). The shifting in the quantum numbers \( l \rightarrow l' \) and \( m \rightarrow m' \) are respectively called the effective orbital and magnetic quantum numbers of the quantum syste.

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**References**

[1] Flugge S 1974 *Practical Quantum Mechanics* (New York: Springer)

[2] Eshghi M and Mehrabian H 2016 Non-relativistic continuous states in arbitrary dimension for a ring-shaped pseudo-Coulomb and energy-dependent potentials *Math. Meth. Appl. Sci.* **39** 1599

[3] Aydogdu O and Sever R 2010 Exact solution of the Dirac equation with the Mie-type potential under the pseudospin and spin symmetry limit *Ann. Phys. (N. Y.)* **325** 373

[4] Arda A and Sever R 2012 Exact solutions of the Morse-like potential, step-up and step-down operators via Laplace transform approach *Commun. Theor. Phys.* **58** 27
[5] Zhang M C, Sun G H and Dong S H 2010 Exactly complete solutions of the Schrödinger equation with a spherically harmonic oscillatory ring-shaped potential Phys. Lett. A 374 704

[6] Ortkaya S 2012 Exact solutions of the Klein–Gordon equation with ring-shaped oscillator potential by using the Laplace integral transform Chin. Phys. B 21 070303

[7] Ishkhanyan A 2016 The third exactly solvable hypergeometric quantum-mechanical potential EPL 115 20002

[8] Eshghi M and Mehrabian H 2012 Eigen spectra in the Dirac-hyperbolic problem with tensor coupling Chin. J. Phys. 50 533

[9] Eshghi M and Mehrabian H 2016 Effective of the q-deformed pseudoscalar magnetic field on the charge carriers in graphene J. Math. Phys. 57 082105

[10] Eshghi M and Mehrabian H 2017 Exact solution of the Dirac-Weyl equation in graphene under electric and magnetic fields C. R. Physique. 18 47

[11] Schiff L I 1995 Quantum Mechanics (New York: McGraw-Hill)

[12] Landau L D and Lifshitz E M 1977 Quantum Mechanics, Non-Relativistic Theory (USA: Pergamon)

[13] Dong S H 2007 Factorization Method in Quantum Mechanics (Amsterdam: Springer)

[14] Greiner W 2001 Quantum Mechanics: An Introduction (Berlin: Springer)

[15] Griffiths D J 1995 Introduction to Quantum Mechanics (Cambridge: Cambridge University Press)

[16] Ahmed F 2019 The Klein-Gordon oscillator in (1+2)-dimensions Gurses space-time backgounds Ann. Phys. (N. Y.) 404 1

[17] Ahmed F 2019 The generalized Klein–Gordon oscillator with Coulomb-type potential in (1+2)-dimensions Gurses space-time Gen. Relativ. Gravit. 51 69

[18] de Montigny M, Zare S and Hassanabadi H 2018 Fermi field and Dirac oscillator in a Som-Raychaudhuri space-time Gen. Relativ. Gravit. 50 47

[19] de Montigny M, Hassanabadi H, Pinfold J and Zare S 2021 Exact solutions of the generalized Klein-Gordon oscillator in a global monopole space-time Eur. Phys. J. Plus 136 788

[20] de Montigny M, Pinfold J, Zare S and Hassanabadi H 2022 Klein-Gordon oscillator in a global monopole space-time with rainbow gravity Eur. Phys. J. Plus 137 54

[21] Ahmed F 2022 Effects of Lorentz symmetry breaking by a tensor field subject to Coulomb-type potential on relativistic quantum oscillator Proc. R. Soc. A 476 20220091

[22] Hassanabadi H, Maghsoodi E and Zarrinkamar S 2012 Relativistic symmetries of Dirac equation and the Tietz potential Eur. Phys. J. Plus 127 31

[23] Oyewumi K J and Akosihle C O 2010 Bound-state solutions of the Dirac-Rosen-Morse potential with spin and pseudospin symmetry Eur. Phys. J. A 45 311

[24] Oyewumi K J, Falaye B J, Onate C A, Oluwadare O J and Yahaya W A 2014 Thermodynamic properties and the approximate solutions of the Schrödinger equation with the shifted Deng-Fan potential model Mol. Phys. 112 127

[25] Oyewumi K J, Falaye B J, Onate C A, Oluwadare O J and Yahaya W A 2014 κ-state solutions for the fermionic massive spin-1/2 particles interacting with double ring-shaped Kratzer and oscillator potentials Int. J. Mod. Phys. E 23 1450005

[26] Kryuchkov S V and Kukhar E I 2014 Effect of high-frequency electric field on the electron magnetotransport in graphene Physica B 445 93

[27] Weishbuch C and Vinter B 1993 Quantum Semiconductor Heterostructure (New York: Academic)

[28] Baura A, Sen M K and Chandra B B 2013 Effect of non Markovian dynamics on Barrier crossing dynamics of a charged particle in presence of a magnetic field Chem. Phys. 417 30

[29] Haken H and Wolf H C 1995 Molecular Physics and Elements of Quantum Chemistry: Introduction to Experiments and Theory (Berlin: Springer)

[30] Arda A and Sever R 2012 Exact solutions of the Schrödinger equation via Laplace transform approach: pseudoharmonic potential and Mie-type potentials J. Math. Chem. 50 971

[31] Soheibi N, Hamzavi M, Eshghi M and Ikhdair S M 2017 Screw dislocation and external fields effects on the Kratzer pseudopot EPL. Eur. Phys. J. B 90 212

[32] Ahmed F 2020 Aharonov-Bohm effect on a generalized Klein–Gordon oscillator with uniform magnetic field in a spinning cosmic string space-time EPL 130 40003

[33] Ahmed F 2020 Relativistic scalar charged particle in a rotating cosmic string space-time with Cornell-type potential and Aharonov–Bohm effect EPL 131 30002

[34] Ahmed F 2020 Quantum dynamics of spin-0 scalar particle with a uniform magnetic field and scalar potential in spinning cosmic string space-time and AB-effect EPL 132 20009

[35] Ahmed F 2021 Effects of Coulomb- and Cornell-types potential on a spin-0 scalar particle under a magnetic field and quantum flux in topological defects space-time EPL 133 50002

[36] Aharonov Y and Bohm D 1959 Significance of electromagnetic potentials in the quantum theory Phys. Rev. 115 485

[37] Peshkin M and Tomonour A 1989 The Aharonov–Bohm Effect, Lecture Notes Phys vol 340 (Berlin: Springer)

[38] Osakabe N, Matsuura T, Kawasaki T, Endo J, Tomonura A, Yano S and Yamada H 1986 Experimental confirmation of Aharonov-Bohm effect using a toroidal magnetic field confined by a superconductor Phys. Rev. A 34 815

[39] Kibble T W B 1980 Some implications of a cosmological phase transition Phys. Rep. 67 183

[40] Zare S, Hassanabadi H and de Montigny M 2020 Non-inertial effects on a generalized DKP oscillator in a cosmic string space-time Gen. Relativ. Grav. 52 25

[41] Barriola M and Vilenkin A 1989 Gravitational field of a global monopole Phys. Rev. Lett. 63 541

[42] Bezerra de Mello E R and Furtado C 1997 Nonrelativistic scattering problem by a global monopole Phys. Rev. D 56 1345

[43] de Mello E R B 2001 Physics in the global monopole spacetime Braz. J. Phys. 31 211

[44] Eshghi M and Hamzavi M 2018 Yukawa-like confinement potential of a scalar particle in a Gödel-type spacetime with any 1 Eur. Phys. J. C 78 522

[45] de Oliveira A L C and Bezerra de Mello E R 2003 Nonrelativistic charged particle-magnetic monopole scattering in the global monopole background Int. J. Mod. Phys. A 18 3175

[46] de Oliveira A L C and de Mello E R B 2006 Exact solutions of the Klein-Gordon equation in the presence of a dyon, magnetic flux and scalar potential in the spacetime of gravitational defects Class. Quantum Grav. 23 5249

[47] Ahmed F 2022 Relativistic motions of spin-zero quantum oscillator field in a global monopole space-time with external potential and AB-effect Sci. Rep. 12 8794

[48] Furtado C and Moraes F 2000 Harmonic oscillator interacting with conical singularities J. Phys. A: Math. Gen. 33 5513

[49] Vitoria R L L V and Belich H 2019 Harmonic oscillator in an environment with a pointlike defect Phys. Scr. 94 125301

[50] de Marques G A and Bezerra V B 2022 Non-relativistic quantum systems on topological defects spacetimes Class. Quantum Grav. 19 985

[51] Nwabuzor P, Edet C, Ikot A N, Okorie U, Ramantswana M, Horchani R, Atty A H A and Rampho G 2021 Analyzing the effects of topological defect (TD) on the energy spectra and
thermal properties of LiH, TiC and I₂ diatomic molecules

[52] Ahmed F 2022 Approximate eigenvalue solutions with diatomic molecular potential under topological defects and Aharonov-Bohm flux field: application for some known potentials Mol. Phys. 120 e2124935

[53] Eshghi M, Azar I A and Soudi S 2021 Schrödinger equation based analytic assessment of thermal properties of confined electrons under vector and scalar fields Math. Meth. Appl. Sci. 44 12774

[54] Ikhdair S M and Sever R 2007 Exact polynomial eigensolutions of the Schrödinger equation for the pseudoharmonic potential J. Mol. Struc.(Theochem) 806 155

[55] Dong S H, Morales D and Ravelo J G 2007 Exact quantization rule and its applications to physical potentials Int. J. Mod. Phys. E 16 189

[56] Oyewumi K J and Bangudu E A 2003 Isotropic harmonic oscillator plus inverse quadratic potential in N-dimensional spaces Arab. J. Sci. Eng. 28 173

[57] Ikhdair S M and Sever R 2007 Exact solutions of the radial Schrödinger equation for some physical potentials Cent. Eur. J. Phys. 5 516

[58] Kumar V, Singh R M, Bhardwaj S B, Rani R and Chand F 2022 Analytical solutions to the Schrödinger equation for a generalized Cornell potential and its applications to diatomic molecules and heavy mesons Mod. Phys. Lett. A 37 2250010

[59] Machado K D 2012 Equacoes Diferenciais Aplicadas vol 1 (Parana: Toda Palavra)

[60] Abramowitz M and Stegun I A 1964 Hand Book of Mathematical Functions with Formulas, Graphs, and Mathematical Tables (New York: Dover)

[61] Prudnikov A, Brychkov Y and Marichev O 1986 Integrals and Series: Special Functions vol 02 (Harwood: Gordon and Breach Science Publishers)

[62] Slater L J 1960 Confluent Hypergeometric Functions (Cambridge: Cambridge University Press)

[63] Arfken G B and Weber H J 2005 Mathematical Methods For Physicists (London: Elsevier)

[64] Braganca E A F, Vitoria R L L, Belich H and de Mello E R B 2020 Relativistic quantum oscillators in the global monopole spacetime Eur. Phys. J. C 80 206

[65] Constantinescu F and Magyari E 1971 Problems in Quantum Mechanics (Oxford: Pergamon)

[66] Ikhdair S M and Sever R 2008 On solutions of the Schrödinger equation for some molecular potentials : wave function ansatz Cent. Eur. J. Phys. 6 697

[67] Ikhdair S M and Sever R 2008 Polynomial solutions of the Mie-type potential in the D-dimensional Schrödinger equation J. Mol. Struct. (THEOCHEM) 855 13

[68] Eshghi M and Ikhdair S M 2014 Relativistic effect of pseudospin symmetry and tensor coupling on the Mie-type potential via Laplace transformation method Chinese Phys. B 23 120304

[69] Eshghi M and Ikhdair S M 2014 Laplace transformation approach to the spin symmetry of the Mie-type potential with a coulomb tensor interaction Z. Naturforsch. A 69 111

[70] Sever R and Tezcan C 2008 Exact solution of the Schrödinger equation for the modified Kratzer’s molecular potential with position-dependent mass Int. J. Mod. Phys. E 17 1327

[71] Liu G, Guo K, Hassanahadi H, Lu L and Yazarloo B H 2013 A theoretical study of nonlinear optical absorption and refractive index changes with the three-dimensional ring-shaped pseudoharmonic potential Physica B 415 92

[72] Sever R, Tezcan C, Aktas M and Yesiltas O 2008 Exact solution of Schrödinger equation for Pseudoharmonic potential J. Math. Chem. 43 845

[73] Rani R, Bhardwaj S B and Chand F 2018 Bound state solutions to the Schrödinger equation for some diatomic molecules Pramana-J. Phys. 91 46

[74] Cetin A 2008 A quantum pseudodot system with a two-dimensional pseudoharmonic potential Phys. Lett. A 372 3852

[75] Ikhdair S M and Hamzavi M 2012 A quantum pseudodot system with two-dimensional pseudoharmonic oscillator in external magnetic and Aharonov-Bohm fields Physica B 407 4198

[76] Berkdemir C, Berkdemir A and Han J 2006 Bound state solutions of the Schrödinger equation for modified Kratzer’s molecular potential Chem. Phys. Lett. 417 326

[77] Kratzer A 1920 Die ulratonen Rotationsspektren der Halogenwasserstoffe Z. Phys. 3 289

[78] Fues E 1926 The natural vibrational spectrum of diatomic molecules in undulation mechanics Ann. Phys. (Paris) 385 367

[79] Akcey H and Sever R 2012 Analytical solutions of Schrödinger equation for the diatomic molecular potentials with any angular momentum J. Math. Chem. 50 1973

[80] Dutta H, Gunerhat K K, Ali K K and Yilmazer R 2020 Exact soliton solutions to the cubic-quartic non-linear Schrödinger equation with conformable derivative Front. Phys. 8 62

[81] Rai A K and Rathaud D P 2015 The mass spectra and decay properties of dimesonic states, using the Hellmann potential Eur. Phys. J. C 75 462

[82] Oyewumi K J 2005 Analytical solutions of the kratzer-fues potential in an arbitrary number of dimensions Found. Phys. Lett. 18 75