Two-loop low-energy effective actions in $\mathcal{N} = 2$ and $\mathcal{N} = 4$ three-dimensional SQED

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ABSTRACT: We study two-loop Euler-Heisenberg effective actions in three-dimensional $\mathcal{N} = 2$ and $\mathcal{N} = 4$ supersymmetric quantum electrodynamics (SQED) without Chern-Simons term. We find exact expressions for propagators of chiral superfields interacting with slowly-varying $\mathcal{N} = 2$ gauge superfield. Using these propagators we compute two-loop effective actions in the $\mathcal{N} = 2$ and $\mathcal{N} = 4$ SQED as the functionals of superfield strengths and their covariant spinor derivatives. The obtained effective actions contain new terms having no four-dimensional analogs. As an application, we find two-loop quantum corrections to the moduli space metric in the $\mathcal{N} = 2$ SQED.

KEYWORDS: Supersymmetric gauge theory, Extended Supersymmetry, Superspaces, Supersymmetry and Duality

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1 Introduction

Low-energy dynamics of three-dimensional supersymmetric gauge theories has attracted considerable attention recently (see, e.g., [1, 2] and references therein). $\mathcal{N} = 2$ and $\mathcal{N} = 4$ supersymmetric Yang-Mills-Chern-Simons theories possess many remarkable properties in classical and quantum domains such as the mirror symmetry [3–8] and Seiberg-like dualities [7–11]. A lot of information about low-energy dynamics is encoded in the structure of the moduli space which comprises both perturbative and non-perturbative effects. The perturbative quantum contributions to such moduli spaces are known only up to one-loop order [7, 12] while the higher-loop corrections are also of interest and deserve detailed investigations. This motivates study of higher-loop quantum corrections to the low-energy effective actions in three-dimensional supersymmetric gauge theories.

In this paper, we compute two-loop Euler-Heisenberg effective actions in $\mathcal{N} = 2$ and $\mathcal{N} = 4$ supersymmetric electrodynamics with vanishing Chern-Simons term. The classical
actions of these models arise as a result of dimensional reduction from the four-dimensional $\mathcal{N} = 1$ and $\mathcal{N} = 2$ SQED, respectively. The two-loop Euler-Heisenberg effective actions in the four-dimensional supersymmetric models were derived in [13, 14] using the technique of covariant perturbative multiloop computations in the $\mathcal{N} = 1$, $d = 4$ superspace [15]. The attractive features of this method are its universality, generality and a possibility to preserve manifestly the $\mathcal{N} = 1$, $d = 4$ supersymmetry and gauge invariance on all stages of loop calculations. In the present paper we extend this technique to the three-dimensional gauge theories in the $\mathcal{N} = 2$, $d = 3$ superspace. In particular, we derive exact propagators of chiral superfields on slowly-varying gauge superfield background and apply them for computing two-loop low-energy effective actions in the $\mathcal{N} = 2$ and $\mathcal{N} = 4$ SQED. As we show in the present paper, the obtained effective actions possess new terms having no four-dimensional analogs, but playing important role in the low-energy dynamics of these models.

In general, the Euler-Heisenberg superfield effective actions in three-dimensional $\mathcal{N} = 2$ and $\mathcal{N} = 4$ SQED can be represented in the following general form:

$$\Gamma = \int d^3 x d^4 \theta \mathcal{L}_{\text{eff}}(G, \Phi \bar{\Phi}, W_\alpha, \bar{W}_\alpha, D_{(\alpha} W_{\beta)}).$$  \hspace{1cm} (1.1)

Here $\mathcal{L}_{\text{eff}}$ is the effective Lagrangian which depends on the $\mathcal{N} = 2$ superfield strengths $G$, $W_\alpha$ and $\bar{W}_\alpha$ and the chiral superfield $\Phi$ which is a part of the $\mathcal{N} = 4$ gauge multiplet. The $\mathcal{N} = 2$ case corresponds to freezing the chiral superfield $\Phi$ to be equal to the $\mathcal{N} = 2$ complex mass parameter, $\Phi = m$. The superfield strengths are assumed to be slowly-varying such that we omit all their space-time derivatives and keep only the dependence on $N_{\alpha\beta} \equiv D_{(\alpha} W_{\beta)}$.\footnote{In principle, one can consider also $\bar{N}_{\alpha\beta} \equiv \bar{D}_{(\alpha} \bar{W}_{\beta)}$, but unlike the four-dimensional case this expression in not independent, $\bar{N}_{\alpha\beta} = -N_{\alpha\beta}$.} In components, the action of the form (1.1) contains all powers of the Maxwell field strength $F^{2n}$ with their supersymmetric completions. One-loop Euler-Heisenberg effective actions in the $\mathcal{N} = 2$ and $\mathcal{N} = 4$ SQED were computed in [16].

The part of the effective Lagrangian in (1.1) which depends only on $G$ and $\Phi \bar{\Phi}$ we refer to as the effective potential. In the $\mathcal{N} = 4$ gauge theory, the one-loop effective potential was derived more than a quarter of a century ago in [17] from geometrical principles,\footnote{This is a three-dimensional analog of the $\mathcal{N} = 2$, $d = 4$ improved tensor multiplet superspace action [18]. See also [19].}

$$f(G, \Phi \bar{\Phi}) = \mathcal{L}_{\text{eff}}|_{W_\alpha = \bar{W}_\alpha = 0} \propto G \ln(G + \sqrt{G^2 + \Phi \bar{\Phi}}) - \sqrt{G^2 + \Phi \bar{\Phi}}. \hspace{1cm} (1.2)$$

It is the effective potential which is responsible for the moduli space metric [7, 17]. Note that in the $\mathcal{N} = 4$ gauge theory the effective potential (1.2) is one-loop exact, but in the $\mathcal{N} = 2$ theories it can receive higher-loop quantum contributions. In the present paper we compute two-loop effective superpotential in the $\mathcal{N} = 2$ SQED and find corresponding two-loop quantum corrections to the moduli space metric. To the best of our knowledge, these two-loop corrections to the moduli space metric have not been presented before.

The rest of the paper is organized as follows. In section 2 we study the basic properties of parallel displacement propagator in $\mathcal{N} = 2$, $d = 3$ superspace and derive exact propagators of real and chiral superfields in slowly-varying gauge superfield background.
In section 3 we employ these propagators for computing the two-loop low-energy Euler-Heisenberg effective action in the $\mathcal{N} = 2$ supersymmetric electrodynamics. As an application of the obtained effective action, we find two-loop quantum corrections to the moduli space metric in the $\mathcal{N} = 2$ supergauge theory. Section 4 is devoted to computing the effective action in the $\mathcal{N} = 4$ SQED. In the last section we summarize the obtained results and discuss their possible generalizations. In appendices we collect $\mathcal{N} = 2$ superspace notations exploited throughout the text and give some technical details of computations.

2 Exact propagators on gauge superfield background

2.1 Gauge theory in $\mathcal{N} = 2$, $d = 3$ superspace

The $\mathcal{N} = 2$, $d = 3$ superspace is parametrized by the coordinates $z^A = (x^m, \theta^\alpha, \bar{\theta}^\dot{\alpha})$, $m = 0,1,2$. The corresponding supercovariant derivatives $D_A = (\partial_m, D_\alpha, \bar{D}^{\dot{\alpha}})$ are written down in (A.5).

The (Abelian) gauge superfields in the $\mathcal{N} = 2$ superspace can be introduced within the standard geometric approach based on adding gauge connections $V_A = (V_m, V_\alpha, \bar{V}^{\dot{\alpha}})$ to the “flat” superspace derivatives,

$$D_\alpha \rightarrow \nabla_\alpha = D_\alpha + V_\alpha, \quad \bar{D}_\dot{\alpha} \rightarrow \bar{\nabla}_\dot{\alpha} = \bar{D}_{\dot{\alpha}} + \bar{V}^{\dot{\alpha}}, \quad \partial_m \rightarrow \nabla_m = \partial_m + V_m, \quad (2.1)$$

and imposing the superfield constraints \cite{17,20-22},

$$\{\nabla_\alpha, \nabla_\beta\} = -2i(\gamma^m)_{\alpha\beta} \nabla_m + 2i\epsilon_{\alpha\beta}G,$$

$$[\nabla_\alpha, \nabla_m] = -(\gamma_m)_{\alpha\beta} W^\beta,$$

$$[\nabla_m, \nabla_n] = iF_{mn}. \quad (2.2)$$

The superfield strengths $G$, $W_\alpha$ and $\bar{W}_{\dot{\alpha}}$ in this algebra satisfy the following reality properties:

$$G^* = G, \quad (W_\alpha)^* = \bar{W}^{\dot{\alpha}}, \quad (F_{mn})^* = F_{mn}. \quad (2.3)$$

The algebra (2.2) possesses many Bianchi identities. In particular, the superfield strength $G$ is linear,

$$D^2 G = D^2 \bar{G} = 0, \quad (2.4)$$

while $W_\alpha$ and $\bar{W}_{\dot{\alpha}}$ can be expressed in terms of $G$,

$$W_\alpha = D_\alpha G, \quad \bar{W}_{\dot{\alpha}} = D^{\dot{\alpha}} G. \quad (2.5)$$

As a consequence, $W_\alpha$ and $\bar{W}_{\dot{\alpha}}$ are (anti)chiral,

$$\bar{D}_\dot{\alpha} W_\beta = 0, \quad D_\alpha \bar{W}_{\dot{\alpha}} = 0, \quad (2.6)$$

and obey the ‘standard’ Bianchi identity,

$$D^a W_\alpha = \bar{D}^{\dot{a}} \bar{W}_{\dot{\alpha}}. \quad (2.7)$$
The superfield strength $F_{mn}$ is also non-independent since it can be expressed in terms of other superfields,

$$F_{mn} = \frac{1}{4} \varepsilon_{mnp} (\gamma^p)^{\alpha\beta} (D_\alpha W_\beta - \bar{D}_\alpha \bar{W}_\beta).$$

(2.8)

Finally, there is one more useful identity which involves space-time derivative of $G$,

$$\partial_m G = \frac{i}{4} \gamma^m_{\alpha\beta} (D_\alpha W_\beta + \bar{D}_\alpha \bar{W}_\beta).$$

(2.9)

The algebra (2.2) is invariant under the following gauge transformations,

$$\nabla_A \rightarrow e^{i\tau(z)} \nabla_A e^{-i\tau(z)}, \quad \tau^* = \tau,$$

(2.10)

with $\tau(z)$ being arbitrary real gauge parameter.

Let us introduce a real gauge superfield, $V = V^*$, and represent the gauge connections $V_A$ in terms of it,

$$\nabla_\alpha = e^{-2V} D_\alpha e^{2V} = D_\alpha + 2D_\alpha V, \quad \bar{\nabla}_\alpha = \bar{D}_\alpha.$$

(2.11)

The algebra (2.2) leads to the following expressions for the superfield strengths,

$$G = \frac{i}{2} \bar{D}^\alpha D_\alpha V, \quad W_\alpha = -\frac{i}{4} \bar{D}^2 D_\alpha V, \quad \bar{W}_\alpha = -\frac{i}{4} D^2 \bar{D}_\alpha V.$$

(2.12)

For the problem of Euler-Heisenberg effective action it is sufficient to consider the background gauge superfield which obeys the following constraints:

(i) $\mathcal{N} = 2$ supersymmetric Maxwell equations,

$$D^\alpha W_\alpha = 0, \quad \bar{D}^\alpha W_\alpha = 0;$$

(2.13)

(ii) The superfield strengths are constant with respect to the space-time derivative,

$$\partial_\mu G = 0, \quad \partial_\mu W_\alpha = 0, \quad \partial_\mu \bar{W}_\alpha = 0.$$

(2.14)

The latter constraint means that we consider a slowly-varying gauge superfield background.

### 2.2 Parallel displacement propagator in $\mathcal{N} = 2, d = 3$ superspace

It is well known that quantization of gauge theories requires gauge fixing and, as a consequence, all off-shell quantities in gauge theories are gauge dependent. As to the effective action, it can be formulated in such a way that being gauge dependent it remains invariant under the classical gauge transformations. This formulation is called the background field method. The main idea of this method is a splitting of the gauge field into ‘background’ and ‘quantum’ parts and imposing the gauge fixing only on the quantum field. Such a gauge fixing condition is taken to be background field dependent that provides classical gauge invariance of the effective action.

Quantum loop calculations within the background field method assume to operate with the background field dependent propagators which, in general, cannot be written in...
an explicit form. For the problem of low-energy effective action, it is sufficient to represent these propagator as series in power of field strengths and their covariant derivatives. Such propagators are naturally obtained on the basis of proper-time technique which allows one to develop manifestly gauge invariant procedure for computing the one-loop effective action. Superfield proper-time technique and its application for finding the superfield effective actions is described e.g. in [19]. However, manifestly gauge invariant computations of multiloop contributions to effective actions require new methods in comparison with the one-loop computations. One of such efficient methods is based on the employment of the parallel displacement propagator.\footnote{The use of parallel displacement propagator for quantum field theory in curved space-time was initiated by DeWitt [23].}

The technique of multiloop quantum computations in the $\mathcal{N} = 1, d = 4$ superspace which involves the parallel displacement propagator was elaborated in [15]. The power of this method was demonstrated, in particular, in the studies of two-loop effective actions in the four-dimensional $\mathcal{N} = 1$ and $\mathcal{N} = 2$ SQED [13, 14]. Our aim is to extend this technique for the $\mathcal{N} = 2, d = 3$ superfield gauge theories. In this section we study basic properties of the parallel displacement propagator associated with the algebra (2.2). The obtained formulae will be applied in the next section for two-loop quantum computations of low-energy effective actions in the three-dimensional $\mathcal{N} = 2$ and $\mathcal{N} = 4$ supersymmetric electrodynamics.

By definition, the parallel displacement propagator $I(z, z')$ is a two-point superspace function depending on the gauge superfields with the following properties:

(i) Under the gauge transformations (2.10) it transforms as

$$I(z, z') \to e^{i\tau(z)} I(z, z') e^{-i\tau(z')} ;$$

(2.15)

(ii) It obeys the equation

$$\zeta^A \nabla_A I(z, z') = \zeta^A (D_A + V_A(z)) I(z, z') = 0 ,$$

(2.16)

where $\zeta^A = (\rho^m, \zeta^\alpha, \bar{\zeta}^\alpha)$ is the $\mathcal{N} = 2$ supersymmetric interval,

$$\zeta^\alpha = (\theta - \theta')^\alpha, \quad \bar{\zeta}^\alpha = (\bar{\theta} - \bar{\theta}')^\alpha, \quad \rho^m = (x - x')^m - i\gamma^m_{\alpha\beta} \zeta^\alpha \bar{\theta}'\beta + i\gamma^m_{\alpha\beta} \theta'\alpha \bar{\zeta}^\beta ;$$

(2.17)

(iii) For coincident superspace points $z = z'$ it reduces to the identity operator in the gauge group,

$$I(z, z) = 1 .$$

(2.18)

One can show that the properties (2.15) and (2.18) imply the important identity

$$I(z, z') I(z', z) = 1 .$$

(2.19)

The rule of Hermitian conjugation for $I(z, z')$ looks like

$$\left( I(z, z') \right)^\dagger = I(z', z) .$$

(2.20)
It is convenient to rewrite the algebra of gauge-covariant derivatives (2.2) in the following condensed form,

$$\{\nabla_A, \nabla_B\} = T_{AB}^C \nabla_C + i F_{AB}, \quad \text{(2.21)}$$

where $T_{AB}^C$ is a supertorsion and $F_{AB}$ is a supercurvature for gauge superfield connections (2.1). In [15] it was proved that, owing to (2.16), the action of the derivative $\nabla_B$ on $I(z, z')$ can be expressed in terms $T_{AB}^C$, $F_{AB}$ and their covariant derivatives,

$$\nabla_B I(z, z') = i I(z, z') \sum_{n=1}^{\infty} \frac{1}{(n+1)!} \left[ n \zeta^{A_n} \ldots \zeta^{A_1} \nabla'_{A_1} \ldots \nabla'_{A_{n-1}} F_{A_n B}(z') + \frac{(n-1)}{2} \zeta^{A_n} T_{A_n B}^C \zeta^{A_{n-1}} \ldots \zeta^{A_1} \nabla'_{A_1} \ldots \nabla'_{A_{n-2}} F_{A_{n-1} C}(z') \right], \quad \text{(2.22)}$$

There is also an equivalent form of this relation in which $I(z, z')$ appears on the right,

$$\nabla_B I(z, z') = i \sum_{n=1}^{\infty} \frac{(-1)^n}{(n+1)!} \left[ - \zeta^{A_n} \ldots \zeta^{A_1} \nabla_{A_1} \ldots \nabla_{A_{n-1}} F_{A_n B}(z) + \frac{(n-1)}{2} \zeta^{A_n} T_{A_n B}^C \zeta^{A_{n-1}} \ldots \zeta^{A_1} \nabla_{A_1} \ldots \nabla_{A_{n-2}} F_{A_{n-1} C}(z) \right] I(z, z'). \quad \text{(2.23)}$$

Recall that we consider the gauge superfield background which obeys the constraints (2.13) and (2.14). For such a background the serieses in (2.22) and (2.23) terminate and we obtain:

$$\nabla_\beta I(z, z') = \left[ - i \bar{\zeta}_\beta G + \frac{1}{2} \rho_{\alpha\beta} W^\alpha - \frac{i}{12} \bar{\zeta}^2 W_\beta + \frac{i}{6} \bar{\zeta}_\beta \zeta^\alpha W_\alpha - \frac{i}{3} \bar{\zeta}_\alpha \zeta_\alpha W_\beta \right. \quad \text{(2.24)}$$

$$\left. + \frac{1}{12} \zeta^\alpha \rho_{\alpha\beta} \bar{\nabla}_\alpha W^\gamma - \frac{1}{12} \bar{\zeta}^\alpha \rho_{\alpha\beta} \bar{\nabla}_\gamma W_\beta - \frac{i}{12} \bar{\zeta}^2 \zeta_\beta \bar{\nabla}^\alpha W_\alpha \right] I(z, z')$$

$$= I(z, z') \left[ - i \bar{\zeta}_\beta G + \frac{1}{2} \rho_{\alpha\beta} W^\alpha - \frac{7i}{12} \zeta^2 W_\beta - \frac{5i}{6} \bar{\zeta}_\beta \zeta^\alpha W_\alpha - \frac{i}{3} \zeta^\alpha \zeta_\alpha W_\beta \right.$$

$$\left. - \frac{5}{12} \zeta^\alpha \rho_{\alpha\beta} \bar{\nabla}_\alpha W^\gamma - \frac{1}{12} \bar{\zeta}^\alpha \rho_{\alpha\beta} \bar{\nabla}_\gamma W_\beta + \frac{i}{3} \bar{\zeta}^2 \zeta_\beta \bar{\nabla}_\beta W_\alpha + \frac{i}{12} \bar{\zeta}^2 \zeta_\beta \bar{\nabla}^\alpha W_\alpha \right],$$

$$\bar{\nabla}^\beta I(z, z') = \left[ - i \zeta^\beta G - \frac{1}{2} \rho^\beta W^\alpha + \frac{i}{12} \zeta^2 W^\beta - \frac{i}{6} \bar{\zeta}_\gamma \zeta^\alpha W_\alpha + \frac{i}{3} \zeta^\gamma \zeta^\alpha W^\beta \right. \quad \text{(2.25)}$$

$$\left. + \frac{1}{12} \zeta_\alpha \rho^\gamma \bar{\nabla}_\alpha W^\gamma - \frac{1}{12} \zeta_\alpha \rho^\gamma \bar{\nabla}_\gamma W_\beta - \frac{i}{12} \zeta^2 \zeta_\beta \bar{\nabla}^\alpha W_\alpha \right] I(z, z')$$

$$= I(z, z') \left[ - i \zeta^\beta G - \frac{1}{2} \rho^\beta W^\alpha + \frac{7i}{12} \zeta^2 W^\beta + \frac{5i}{6} \zeta_\gamma \zeta^\alpha W_\alpha + \frac{i}{3} \zeta^\gamma \zeta_\alpha W^\beta \right.$$

$$\left. + \frac{5}{12} \zeta_\alpha \rho^\gamma \bar{\nabla}_\alpha W^\gamma - \frac{1}{12} \zeta_\alpha \rho^\gamma \bar{\nabla}_\gamma W_\beta - \frac{i}{3} \zeta^2 \zeta_\beta \bar{\nabla}^\alpha W_\alpha + \frac{i}{12} \zeta^2 \zeta_\beta \bar{\nabla}^\alpha W_\alpha \right],$$

$$\nabla_m I(z, z') = \left[ \frac{i}{2} \rho^m F_{nm} - \frac{1}{2} \gamma_{(m)\alpha \beta} \left( \zeta^\alpha W^\beta + \bar{\zeta}^\alpha W^\beta \right. \quad \text{(2.26)}$$

$$\left. + \frac{1}{3} \zeta^\alpha \bar{\nabla}^\gamma W^\beta - \frac{1}{3} \bar{\zeta}^\alpha \bar{\nabla}^\gamma W^\beta \right) \right] I(z, z').$$
\[ I(z, z') = i \rho^n F_{nm} - (\gamma_m)_{\alpha\beta} \left( \frac{1}{2} \zeta^n W^\beta + \frac{1}{2} \tilde{\zeta}^\alpha W^\beta - \frac{1}{3} \zeta^\alpha \tilde{\nabla}_\gamma W^\beta + \frac{1}{3} \tilde{\zeta}^\alpha \zeta^\gamma \nabla^\beta \right) \]  

(2.26)

In comparison with the four-dimensional case, the expressions (2.24), (2.25), (2.26) involve the superfield \( G \) which will lead to new contributions in the effective action having no four-dimensional analogs.

### 2.3 Real superfield Green’s function and its heat kernel

There are three basic d’Alembertian-like operators which occur in covariant supergraphs \([16, 24]\): (i) the d’Alembertian \( \Box_v \) which acts in the space of real superfields; (ii) the chiral d’Alembertian \( \Box_+ \) acting on chiral superfields; and (iii) the antichiral d’Alembertian \( \Box_- \). The latter is related to the former by conjugation. Therefore we concentrate mainly on \( \Box_v \) and \( \Box_+ \).

The real superfield d’Alembertian is defined by two equivalent lines:

\[
\Box_v = \frac{1}{8} \nabla^\alpha \nabla_\alpha + \frac{1}{16} \{ \nabla^2, \bar{\nabla}^2 \} + \frac{i}{2} (\nabla^\alpha W_\alpha) + iW^\alpha \nabla_\alpha \\
= -\frac{1}{8} \nabla^\alpha \nabla_\alpha + \frac{1}{16} \{ \nabla^2, \bar{\nabla}^2 \} - \frac{i}{2} (\bar{\nabla}^\alpha W_\alpha) - i\bar{W}^\alpha \bar{\nabla}_\alpha .
\]  

(2.27)

By virtue of the algebra (2.2) it can be represented in the following form,

\[
\Box_v = \nabla^m \nabla_m + G^2 + iW^\alpha \nabla_\alpha - i\bar{W}^\alpha \bar{\nabla}_\alpha .
\]  

(2.28)

Let us consider Green’s function for this operator \( G_v(z, z') \) and the corresponding heat kernel \( K_v(z, z'|s) \),

\[
(\Box_v + m^2)G_v(z, z') = -\delta^7(z - z') , \quad G_v(z, z') = i \int_0^\infty ds K_v(z, z'|s)e^{is(m^2 + i\epsilon)} ,
\]  

(2.29)

where \( m \) is a mass parameter and \( \epsilon \to +0 \) implements standard boundary condition for the propagator. For the gauge superfield background (2.13), (2.14) the explicit expression for \( K_v \) was derived in \([16]\),

\[
K_v(z, z'|s) = \frac{1}{8(4\pi s)^{3/2}\sinh(sB)}O(s) e^{iG^2} e^{i\frac{1}{4}i(F \coth(sF))_{mn}p^n p^m \zeta^2 \tilde{\zeta}^2 I(z, z')} ,
\]  

(2.30)

where \( \zeta^2 = \zeta^\alpha \zeta_\alpha , \quad \tilde{\zeta}^2 = \tilde{\zeta}^\alpha \tilde{\zeta}_\alpha \) and \( p^m \) are the components of the supersymmetric interval (2.17) and the following notations are employed,

\[
O(s) = e^{s(W^\alpha \nabla_\alpha - \bar{W}^\alpha \bar{\nabla}_\alpha)} , \\
B^2 = \frac{1}{2} N^\alpha_\beta N^\alpha_\beta , \quad N^\alpha_\beta = D^\alpha (W_\beta) , \quad \bar{N}^\alpha_\beta = \bar{D}^\alpha (\bar{W}_\beta) .
\]  

(2.31) 

(2.32)

Note that the parallel displacement propagator \( I(z, z') \) in (2.30) provides the correct transformation properties of the heat kernel under the gauge symmetry (2.10). Note also that, owing to (2.9), for the case of constant superfield background (2.14) \( \bar{N}^\alpha_\beta \) is not
independent, but coincides with $N_{\alpha\beta}$ up to sign, $\bar{N}_{\alpha\beta} = -N_{\alpha\beta}$. Therefore we will use only $N_{\alpha\beta}$ in what follows.

The expression (2.30) contains the operator $\mathcal{O}(s)$ which acts both on the superfields and on the components of the supersymmetric interval. Let us push this operator on the right and act with it on the parallel displacement propagator,

$$K_{\nu}(z,z'|s) = \frac{1}{8(\pi s)^{3/2}} \frac{sB}{\sinh(sB)} e^{isG^2} e^{i\frac{1}{2}(F_{\nu\alpha}(sF))_{mn}^{\rho\mu}(s)\rho^{\mu}(s)\zeta_2^2(s)\zeta_2^2(s)I(z,z'|s)} ,$$

where the following notations have been introduced:

$$W^\alpha(s) = \mathcal{O}(s)W^\alpha \mathcal{O}(-s) = W^\beta(e^{-sN})_{\beta}^\alpha ,$$

$$\zeta^\alpha(s) = \mathcal{O}(s)\zeta^\alpha \mathcal{O}(-s) = \zeta_\alpha + W^\beta((e^{-sN} - 1)N^{-1})_{\beta}^\alpha ,$$

$$\bar{\zeta}^\alpha(s) = \mathcal{O}(s)\bar{\zeta}^\alpha \mathcal{O}(-s) = \bar{\zeta}_\alpha + \bar{W}^\beta((e^{-sN} - 1)N^{-1})_{\beta}^\alpha ,$$

$$\rho^m(s) = \mathcal{O}(s)\rho^m \mathcal{O}(-s) = \rho^m - i(\gamma^m)_{\alpha\beta} \int_0^s dt (W_\alpha(t)\bar{\zeta}_\beta(t) + \bar{W}_\alpha(t)\zeta_\beta(t)) ,$$

and

$$I(z,z'|s) = \mathcal{O}(s)I(z,z') .$$

Owing to (2.24) and (2.25), the expression for $I(z,z'|s)$ can be written explicitly in terms of superfield strengths and their derivatives. Indeed, by differentiating over the proper time $s$, it is easy to check the identity

$$I(z,z'|s) = \exp \left[ \int_0^s dt \Sigma(z,z'|t) \right] I(z,z') ,$$

where

$$\Sigma(z,z'|t) = \mathcal{O}(t)\Sigma(z,z') \mathcal{O}(-t) ,$$

and $\Sigma(z,z')$ solves

$$(\bar{W}^\alpha \bar{\nabla}_\alpha - W^\alpha \nabla_\alpha)I(z,z') = \Sigma(z,z')I(z,z') .$$

Applying (2.24) and (2.25) in the latter equation we immediately find $\Sigma(z,z')$,

$$\Sigma(z,z') = -i(\bar{W}^\beta \bar{\zeta}_\beta - W^\beta \zeta_\beta)G - i\frac{1}{3}\zeta_\alpha \bar{\zeta}^\beta W_\beta \bar{W}_\alpha + \frac{2i}{3} \zeta_\alpha \bar{\zeta}_\beta W^\beta \bar{W}_\alpha \bar{W}_\beta \left[ W^2 - \zeta_\alpha W_\beta D^\beta \bar{W}_\alpha \right] + \frac{i}{12} \bar{\zeta}^2 \left[ W^2 - \zeta_\alpha W_\beta D^\beta \bar{W}_\alpha \right] + \frac{1}{12} \left( \zeta_\alpha \bar{W}^\beta - \bar{\zeta}^\beta W^\alpha \right) [\rho_{\alpha\gamma} D^\gamma W_\beta + \rho_{\beta\gamma} D^\gamma \bar{W}_\alpha] .$$

The expression for $\Sigma(z,z'|s)$ appears from $\Sigma(z,z')$ by simply making all ingredients of (2.39) $s$-dependent as in (2.34).

2.4 Chiral Green’s function and its heat kernel

The d’Alembertian operators acting in the space of (anti)chiral superfields read [16, 24]

$$\Box_+ = \nabla^m \nabla_m + G^2 + \frac{i}{2} (\nabla^\alpha _m W_\alpha ) + i W^\alpha \nabla_\alpha , \quad \Box_+ \Phi = \frac{1}{16} \nabla^2 \nabla^2 \Phi , \quad \nabla_\Phi = 0 ,$$

$$\Box_- = \nabla^m \nabla_m + G^2 - \frac{i}{2} (\nabla^\alpha _m \bar{W}_\alpha ) - i \bar{W}^\alpha \nabla_\alpha , \quad \Box_- \Phi = \frac{1}{16} \nabla^2 \nabla^2 \Phi , \quad \nabla_\Phi = 0 .$$
The Green’s functions for these operators obey
\[
(\Box_+ + m^2)G_+(z, z') = -\delta_+(z, z'), \quad (\Box_- + m^2)G_-(z, z') = -\delta_-(z, z'), \tag{2.42}
\]
where \(\delta_\pm(z, z')\) are (anti)chiral delta-functions. These Green’s functions are expressed in terms of the corresponding heat kernels,
\[
G_\pm(z, z') = i \int_0^{\infty} ds\, K_\pm(z, z'|s) e^{is(m^2+i\epsilon)}, \quad \epsilon \to +0. \tag{2.43}
\]

The operators (2.40) and (2.41) are related to each other as
\[
\nabla^2\Box_+ = \Box_-\nabla^2, \quad \nabla^2\Box_- = \Box_+\nabla^2. \tag{2.44}
\]
Moreover, when the background gauge superfield obeys supersymmetric Maxwell equations (2.13), these operators are related to \(\Box_v\),
\[
\nabla^2\Box_+ = \nabla^2\Box_v = \Box_v\nabla^2, \quad \nabla^2\Box_- = \nabla^2\Box_v = \Box_v\nabla^2. \tag{2.45}
\]
As a consequence of these identities, the (anti)chiral Green’s functions can be expressed in terms of \(G_v\),
\[
G_+(z, z') = -\frac{1}{4} \nabla^2G_v(z, z'), \quad G_-(z, z') = -\frac{1}{4} \nabla^2G_v(z, z'), \tag{2.46}
\]
and similar relations hold for the corresponding heat kernels,
\[
K_+(z, z'|s) = -\frac{1}{4} \nabla^2K_v(z, z'|s), \quad K_-(z, z'|s) = -\frac{1}{4} \nabla^2K_v(z, z'|s). \tag{2.47}
\]

To compute \(K_+\) we have to differentiate (2.30) by \(\nabla^2\). Owing to the identities (2.45), the operator \(\nabla^2\) acts only on \(\bar{\zeta}^2 I(z, z')\),
\[
K_+(z, z'|s) = \frac{1}{8(i\pi s)^{3/2}} \frac{sB}{\sinh(sB)} \mathcal{O}(s) \, e^{isG_2} \, e^{\frac{i}{4}(F \coth(sF))_{\mu\nu}\rho^\mu\rho^\nu} \, \bar{\zeta}^2 \left( -\frac{1}{4} \nabla^2 \right) \, \bar{\zeta}^2 I(z, z'). \tag{2.48}
\]
The action of the derivative \(\nabla_\alpha\) on \(I(z, z')\) is given by (2.25). However, only one term from this expression survives owing to \(\zeta_\alpha\zeta_\beta\zeta_\gamma = 0\) and we get
\[
-\frac{1}{4} \bar{\zeta}^2 \nabla^2(\zeta^2 I(z, z')) = \zeta^2 e^{-\frac{1}{2} \zeta^a \rho_\alpha \rho^\alpha} W^\gamma \zeta^2 I(z, z'). \tag{2.49}
\]
Substituting (2.49) into (2.48) we find the chiral heat kernel in the following form
\[
K_+(z, z'|s) = \frac{1}{8(i\pi s)^{3/2}} \frac{sB}{\sinh(sB)} \mathcal{O}(s) \, e^{isG_2} \, e^{\frac{i}{4}(F \coth(sF))_{\mu\nu}\rho^\mu\rho^\nu} \, \bar{\zeta}^2 \left( -\frac{1}{4} \zeta^a \rho_\alpha \rho^\alpha W^\gamma \right) \zeta^2 I(z, z'). \tag{2.50}
\]
Using the properties of parallel displacement propagator (2.25) one can check that this expression for \(K_+\) is chiral with respect to both arguments.

The formula (2.50) contains the operator \(\mathcal{O}(s)\) given in (2.31). Similarly as for the heat kernel \(K_v\), we push this operator on the right,
\[
K_+(z, z'|s) = \frac{1}{8(i\pi s)^{3/2}} \frac{sB}{\sinh(sB)} \mathcal{O}(s) \, e^{isG_2} \, e^{\frac{i}{4}(F \coth(sF))_{\mu\nu}\rho^\mu\rho^\nu} \, \bar{\zeta}^2 \left( -\frac{1}{4} \zeta^a \rho_\alpha \rho^\alpha W^\gamma \right) \zeta^2 I(z, z'). \tag{2.51}
\]
All s-dependent objects in this expression are given explicitly in (2.34) and (2.36).

The computation of the antichiral heat kernel \( K_- \) goes along similar lines with the following outcome:

\[
K_-(z, z'|s) = \frac{1}{8(1+i\pi s)^{3/2}} \frac{sB}{\sinh(sB)} e^{isG^2} O(s) e^{i(F \coth(sF))_{\mu\nu\rho\sigma} - \frac{1}{2} \zeta^2 \rho_\gamma \bar{\zeta}^2 I(z, z')}.
\]  

(2.52)

Note that the expressions for (anti)chiral heat kernels (2.50) and (2.52) are very similar to the ones in the four-dimensional supersymmetric gauge theory given in [13, 15].

2.5 Green’s function \( G_{+-} \) and its heat kernel

Let \( \Phi \) be a covariantly chiral superfield, \( \bar{\nabla}_a \Phi = 0 \). The Green’s function \( G_+(z, z') \) considered in the previous section corresponds to the propagator of the covariantly chiral superfield,

\[
i\langle \Phi(z) \bar{\Phi}(z') \rangle = mG_+(z, z').
\]  

(2.53)

It is important to consider also the chiral-antichiral propagators,

\[
i\langle \Phi(z) \bar{\Phi}(z') \rangle = G_{+-}(z, z'), \quad i\langle \bar{\Phi}(z) \Phi(z') \rangle = G_{-+}(z, z').
\]  

(2.54)

By definition, these Green’s functions obey

\[
\frac{1}{4} \nabla^2 G_{+-}(z, z') + m^2 G_{-+}(z, z') = -\delta_-(z, z'),
\]

\[
\frac{1}{4} \nabla^2 G_{-+}(z, z') + m^2 G_{+-}(z, z') = -\delta_+(z, z').
\]  

(2.55)

Consider also the corresponding heat kernels,

\[
G_{+-}(z, z') = i \int_0^\infty ds K_{+-}(z, z'|s) e^{is(m^2+i\epsilon)},
\]

\[
G_{-+}(z, z') = i \int_0^\infty ds K_{-+}(z, z'|s) e^{is(m^2+i\epsilon)},
\]  

(2.56)

where the standard \( \epsilon \to +0 \) prescription is assumed.

Taking into account the definitions of the (anti)chiral d’Alembertians (2.40) and (2.41) it is easy to see that the solutions of the equations (2.55) can be expressed in terms of \( G_\pm \) as

\[
G_{+-}(z, z') = \frac{1}{4} \nabla^2 G_-(z, z'), \quad G_{-+}(z, z') = \frac{1}{4} \nabla^2 G_+(z, z'),
\]  

(2.57)

where \( G_\pm \) obey (2.42). Similar relations hold for the corresponding heat kernels,

\[
K_{+-}(z, z'|s) = \frac{1}{4} \nabla^2 K_-(z, z'|s), \quad K_{-+}(z, z'|s) = \frac{1}{4} \nabla^2 K_+(z, z'|s),
\]  

(2.58)

where \( K_+ \) and \( K_- \) are given by (2.50) and (2.52), respectively.

In what follows we consider only the heat kernel \( K_{+-} \). It is obtained from \( K_- \) by acting on it with the operator \( \nabla^2 \). Owing to the identities (2.45), this operator commutes trivially with the expression \( e^{iG^2} O(s) e^{i(F \coth(sF))_{\mu\nu\rho\sigma}} \) in (2.50) and we need only
to find the action of $\nabla^2$ on the rest. This procedure is quite tedious since it involves numerous differentiation of the components of the supersymmetric interval, superfield strengths and the parallel displacement propagator. For the latter we have to apply the identity (2.25). The result can be encoded in one function $R(z, z')$ as follows

$$
-\frac{1}{4} \nabla^2 \left( e^{-\frac{1}{2} \zeta^\alpha \rho_{\alpha \beta} W^\beta} \zeta^2 I(z, z') \right) = e^{R(z, z')} I(z, z'),
$$

(2.59)

where

$$
R(z, z') = -i \zeta G + \frac{7i}{12} \tilde{\zeta}^2 W + i \frac{1}{12} \zeta^2 \tilde{\zeta} W - \frac{1}{2} \tilde{\zeta}^\alpha \rho_{\alpha \beta} W^\beta - \frac{1}{2} \zeta^\alpha \rho_{\alpha \beta} W^\beta + \frac{1}{12} \zeta^\alpha \tilde{\zeta}^\beta [\tilde{\rho}\gamma D\alpha W\gamma - 7\tilde{\rho}\alpha D\gamma W\beta].
$$

(2.60)

Here $\tilde{\rho}_{\alpha \beta} = \gamma^m_{\alpha \beta} \tilde{\rho}_m$, and $\tilde{\rho}_m$ is a chiral version of the supersymmetric interval $\rho_m$,

$$
\tilde{\rho}^m = \rho^m + i \zeta^\alpha \gamma^m_{\alpha \beta} \tilde{\zeta}^\beta, \quad D'_\alpha \tilde{\rho}^m = \tilde{D}_\alpha \tilde{\rho}^m = 0.
$$

(2.61)

Given the function $R(z, z')$, we get the following expression for the heat kernel $K_{+-}$,

$$
K_{+-}(z, z'|s) = -\frac{1}{8(i\pi)^{3/2} \sinh(sB)} sB e^{isG^2} O(s) e^{i(F \coth(sF))mn \tilde{\rho}^m \tilde{\rho}^n + R(z, z') I(z, z')},
$$

(2.62)

Finally, we have to push the operator $O(s)$ in (2.62) on the right. This makes all objects $s$-dependent,

$$
K_{+-}(z, z'|s) = -\frac{1}{8(i\pi)^{3/2} \sinh(sB)} sB e^{isG^2} e^{i(F \coth(sF))mn \tilde{\rho}^m \tilde{\rho}^n(s) + R(z, z'|s) + \int_0^s dt \Sigma(t) I(z, z'),
$$

(2.63)

where $R(z, z'|s) = O(s) R(z, z') O(-s)$ and $\Sigma(t)$ is given in (2.37), (2.39). The formula (2.63) can be identically rewritten as

$$
K_{+-}(z, z'|s) = -\frac{1}{8(i\pi)^{3/2} \sinh(sB)} sB e^{isG^2} e^{i(F \coth(sF))mn \tilde{\rho}^m(s) + R(z, z') + \int_0^s dt (R'(t) + \Sigma(t))} 
\times I(z, z').
$$

(2.64)

The expression for $R'(t)$ can be found explicitly, $R'(t) = O(t)[\tilde{W}^\alpha \tilde{D}_\alpha - W^\alpha D_\alpha, R]O(-t)$ and then combined with (2.39),

$$
R'(t) + \Sigma(t) = O(t) \left\{ 2i\zeta W G + 2i(\zeta \tilde{\zeta} W \tilde{W} - \zeta W \tilde{\zeta} \tilde{W}) + i \tilde{\zeta}^2 [W^2 - \zeta^\alpha W^\beta D_\alpha W_\beta] - \frac{1}{2} \tilde{\zeta}^2 \tilde{\zeta}^\beta [\tilde{\rho}\gamma D\alpha W\gamma - \tilde{\rho}\alpha D\gamma W\beta] \right\} O(-t).
$$

(2.65)

The form (2.64) of the heat kernel $K_{+-}$ is more useful for loop computations than (2.63) since at coincident superspace points the function $R(z, z')$ vanishes, $R(z, z')|_{z'-z} = 0$, and does not contribute.
3 Low-energy effective action in $\mathcal{N} = 2$ supersymmetric electrodynamics

3.1 Classical action and background field setup

The classical action of the $\mathcal{N} = 2$, $d = 3$ supersymmetric electrodynamics reads

$$S_{N=2} = \frac{1}{e^2} \int d^7 z \, G^2 - \int d^7 z \, (\bar{Q}_+ e^{2V} Q_+ + \bar{Q}_- e^{-2V} Q_-) - \left( m \int d^5 z \, Q_+ Q_- + c.c. \right),$$

(3.1)

where $Q_{\pm}$ are chiral superfields with opposite charges with respect to the gauge superfield $V$. This action appears by virtue of the dimensional reduction from the action of $\mathcal{N} = 1$, $d = 4$ electrodynamics [19, 25]. In principle, in three-dimensional space-time one could add the Chern-Simons term $\int d^7 z \, V G$ which does not appear from the $\mathcal{N} = 1$, $d = 4$ SQED by dimensional reduction. However, in the present work we restrict ourself to studies of the low-energy effective action in three-dimensional supersymmetric electrodynamics without the Chern-Simons term. Note that the latter does not appear as a result of the radiative corrections since the action (3.1) is parity even [8, 26–28] (see also [29] for a review).

We are interested in the part of the low-energy effective action which depends on the gauge superfield only, $\Gamma = \Gamma[V]$, while the chiral superfields $Q_{\pm}$ are integrated out. For this problem the background field method in the $\mathcal{N} = 2$, $d = 3$ superspace [30] appears to be useful. We split the gauge superfield $V$ into the background $V$ and quantum $v$ parts

$$V \to V + e \, v.$$  

(3.2)

Upon this splitting the Maxwell term in (3.1) changes as

$$\frac{1}{e^2} \int d^7 z \, G^2 \to \frac{1}{e^2} \int d^7 z \, G^2 + \frac{i}{e} \int d^7 z \, v D^\alpha W_\alpha + \frac{1}{8} \int d^7 z \, v D^\alpha \bar{D}^2 D_\alpha v.$$

(3.3)

The operator $D^\alpha \bar{D}^2 D_\alpha$ in the last term is degenerate and requires gauge fixing. In particular, the Fermi-Feynman gauge is implemented by the following gauge-fixing term

$$S_{gf} = -\frac{1}{16} \int d^7 z \, v \{ D^2, \bar{D}^2 \} v.$$

(3.4)

Adding this term to (3.1) we get

$$S_{\text{quantum}} = S_2 + S_{\text{int}},$$

$$S_2 = -\int d^7 z \left( v \mathcal{D} \mathcal{D} v + \bar{Q}_+ Q_+ + \bar{Q}_- Q_- \right) - \left( m \int d^5 z \, Q_+ Q_- + c.c. \right),$$

(3.6)

$$S_{\text{int}} = -2 \int d^7 z \left[ e \left( \bar{Q}_+ Q_+ - \bar{Q}_- Q_- \right) v + e^2 \left( \bar{Q}_+ Q_+ + \bar{Q}_- Q_- \right) v^2 \right] + O(e^3),$$

(3.7)

where $Q_{\pm}$ and $\bar{Q}_{\pm}$ are covariantly (anti)chiral superfields with respect to the background gauge superfield,

$$\bar{Q}_+ = \bar{Q}_+ e^{2V}, \quad Q_+ = Q_+, \quad \bar{Q}_- = \bar{Q}_- e^{-2V}, \quad Q_- = Q_-.$$  

(3.8)
The action $S_{\text{int}}$ specifies the interaction vertices while $S_2$ is responsible for the propagators,

$$
i\langle Q_+(z) Q_-(z') \rangle = -mG_+(z, z'),$$
$$i\langle \bar{Q}_+(z) \bar{Q}_-(z') \rangle = mG_-(z', z),$$
$$i\langle Q_+(z) \bar{Q}_+(z') \rangle = G_+(z, z') = G_{++}(z', z),$$
$$i\langle \bar{Q}_-(z) \bar{Q}_-(z') \rangle = G_-(z, z'),$$

(3.9)

where the Green’s functions $G_+$ and $G_{+-}$ are defined by the equations (2.42) and (2.55), respectively. The propagator for the real superfield $v$ reads

$$2i\langle v(z) v(z') \rangle = G_0(z, z'),$$

(3.10)

where

$$G_0(z, z') = i\int_0^\infty ds K_0(z, z'|s) e^{-s\rho^m \pi^\alpha} \zeta^{2\gamma},$$

(3.11)

Here $\rho^m$, $\zeta^\alpha$ and $\bar{\zeta}^\alpha$ are the components of supersymmetric interval (2.17).

### 3.2 Loop expansion and general structure of the effective action

Within the present considerations we restrict ourself to the two-loop effective action in the $\mathcal{N} = 2$ supersymmetric electrodynamics (3.1),

$$\Gamma_{\mathcal{N}=2}^{(1)} + \Gamma_{\mathcal{N}=2}^{(2)},$$

(3.12)

$$\Gamma_{\mathcal{N}=2}^{(1)} = i\text{Tr} \ln(\Box + m^2),$$

(3.13)

$$\Gamma_{\mathcal{N}=2}^{(2)} = -2e^2 \int d^7 z d^7 z' [G_+(z, z') G_{++}(z', z) + m^2 G_+(z, z') G_-(z, z')] G_0(z, z').$$

(3.14)

Here $\Gamma_{\mathcal{N}=2}^{(1)}$ and $\Gamma_{\mathcal{N}=2}^{(2)}$ are the one- and two-loop contributions, respectively. The covariant d’Alembertian $\Box_+$ is given in (2.40) while the Green’s functions $G_{++}$, $G_+$ and $G_0$ are expressed through the heat kernels as in (2.63), (2.51) and (3.11). The two-loop effective action $\Gamma_{\mathcal{N}=2}^{(2)}$ is represented by the Feynman graphs in figure 1. The supergraphs of Types A and B correspond to the two terms in the r.h.s. of (3.14). In principle, there could be a two-loop graph of topology ‘eight’, but it vanishes since the super-photon propagator (3.11) is equal to zero at coincident superspace points.

The one-loop effective action in three-dimensional $\mathcal{N} = 2$ SQED was calculated in [16] (see also [30]),

$$\Gamma_{\mathcal{N}=2}^{(1)} = \frac{1}{2\pi} \int d^7 z \left[ G \ln(G + \sqrt{G^2 + m^2}) - \sqrt{G^2 + m^2} \right]$$

$$+ \frac{1}{8\pi} \int d^7 z \int_0^\infty ds \frac{e^{i(s(G_2 + m^2))W_2\bar{W}_2}}{sB/2} \left( \tanh(sB/2) - 1 \right),$$

(3.15)

where $B^2$ is defined in (2.32), $B^2 = \frac{1}{2} D^2 W^2$. Recall that we consider the constant superfield background (2.14) subject to the supersymmetric Maxwell equation (2.13). Hence, the
low-energy effective action is a functional which depends on the superfield strengths $G, W_\alpha, \bar{W}_\alpha$ and only on the first Grassmann derivative of $W_\alpha$, i.e., on $N_{\alpha\beta} = D_\alpha W_\beta$. All higher derivatives of the superfield strengths either vanish or reduce to functions of $N_{\alpha\beta}$. As a consequence, the two-loop effective action have the following general superfield structure:

$$\Gamma_{N=2}^{(2)} = \frac{e^2}{16\pi^3} \int d^7 z \left[ \mathcal{L}_1(G,B) + W_\alpha \mathcal{L}_{2,\alpha} (G,N) \bar{W}_\beta + \mathcal{L}_3(G,B) W^2 \bar{W}^2 \right].$$

(3.16)

Here $\mathcal{L}_1, \mathcal{L}_{2,\alpha}$ and $\mathcal{L}_3$ are some functions of $G$ and $N_{\alpha\beta}$ to be found from direct quantum computations. Note that the contributions of the form $\mathcal{L}_1$ and $\mathcal{L}_2$ are impossible in the four-dimensional supersymmetric gauge theory. We will show that these terms do appear in the two-loop effective action in the three-dimensional supersymmetric electrodynamics.

3.3 Computing two-loop diagrams

The two-loop diagrams of Type A in figure 1 correspond to the following contribution to the effective action:

$$\Gamma_A = -2e^2 \int d^7z d^7z' G_{+-}(z,z')G_{+-}(z',z)G_0(z,z').$$

(3.17)

The propagators $G_{+-}$ and $G_0$ are expressed in terms of the heat kernels as in (2.56) and (3.11). Hence,

$$\Gamma_A = 2ie^2 \int d^7z d^7z' \int_0^\infty ds dt du K_0(z,z'|u)K_{+-}(z,z'|s)K_{+-}(z',z|t)e^{im^2(s+t)}$$

$$= 2ie^2 \int d^7z d^3\rho \int_0^\infty ds dt du \frac{e^i e^{m^2m}}{4i\pi u} e^{i m^2(s+t)}K_{+-}(z,z'|s)K_{+-}(z',z|t)\bigg|_{\zeta=0}. \quad (3.18)$$

Here we have taken into account that the heat kernel $K_0$ given by (3.11) contains $\zeta^2\xi^2$ which is nothing but the delta-function over the Grassmann variables. Hence, the expression (3.18) involves the integration over only one set of Grassmann variables, but we need to evaluate the heat kernel $K_{+-}$ at coincident points, $\theta = \theta'$. Evaluating this limit is a
very straightforward, but tedious procedure. Some details of this procedure are collected in appendix B. Here we present the result:

\[
K_{+-}(z, z'|s) = \frac{1}{8i\pi s^{3/2}} \frac{sB}{\sinh(sB)} e^{isG^2} \exp \left\{ \frac{i}{4} \left( F \coth(sF) \right)_{mn} \rho^m \rho^n \right. \\
+ iGW^\alpha f_{\alpha}^\beta(s) W_\beta + W^\alpha \rho_m f_{\alpha\beta}(s) W_\beta + \frac{i}{2} W^2 W^2 f(s) \right\},
\]

where the following functions have been introduced:

\[
f_{\alpha}^\beta(s) = 2B^{-2}(1 - sN - e^{-sN})_{\alpha}^\beta,
\]

\[
f(s) = \frac{1}{sB^4} \left[ (sB)^2 - 4\sinh^2(sB/2)(1 + sB \tanh(sB/2)) \right],
\]

\[
f_{\alpha\beta}^m(s) = \frac{1}{2} B^{-2} (\cosh(sB) - 1) \left[ (e^{-sN})_{\beta}^\gamma N_{\alpha}^\delta (\gamma^m)_{\gamma\delta} + \epsilon_{\alpha\beta} (\gamma^m)_{\alpha\delta} \right] - \frac{1}{2} \left( F \coth(sF) \right)_{mn} \gamma_{m}^\gamma \left[ (e^{-sN} - 1)^N \gamma_{m}^\gamma - \frac{\epsilon_{\alpha\beta} N^\delta}{B^3} (sB - \sinh(sB)) \right].
\]

Note that (3.18) includes also \(K_{+-}(z', z|s)\) which has the same form as (3.19), but the superspace points should be swapped, \(z \leftrightarrow z'\), or \(\rho_m \to -\rho_m\). Hence, substituting (3.19) into (3.18) we find

\[
\Gamma_A = \frac{4ie^2}{(4\pi)^{3/2}} \int d^7 z d^3 \rho \int_0^\infty ds dt du \exp \left\{ i(s+t)(G^2+m^2) \frac{stB^2}{\sinh(sB) \sinh(tB)} e^{iW^2W^2(f(s)+f(t))} \right\} \\
\times \exp \left\{ \frac{i}{4} \left( F \coth(sF) \right)_{mn} + \frac{\eta_{mn}}{u} \right\},
\]

is a symmetric 3 \times 3 matrix with Lorentz indices. It is convenient to express this matrix in terms of Lorentz projectors \(P^+\) and \(P^-\),

\[
A_{mn} = P_{mn}^+(a + u^{-1}) + P_{mn}^-(b + u^{-1}),
\]

where

\[
a(s, t) = B \coth(sB) + B \coth(tB), \quad b(s, t) = s^{-1} + t^{-1}
\]

and

\[
P_{mn}^+ = \eta_{mn} + \frac{1}{4B^2} (N_{\alpha\beta} \bar{\gamma}_m^{\alpha\beta}) (N_{\gamma\delta} \gamma_n^{\gamma\delta}), \quad P_{mn}^- = -\frac{1}{4B^2} (N_{\alpha\beta} \bar{\gamma}_m^{\alpha\beta}) (N_{\gamma\delta} \gamma_n^{\gamma\delta}).
\]

These matrices obey standard properties of projection operators,

\[
(P^+)^2 = P^+ \quad (P^-)^2 = P^- \quad P^+ P^- = 0 \quad P^+_m + P^-_m = \eta_{mn}.
\]

\footnote{Each of the two heat kernels \(K_{+-}^\pm\) in (3.18) contains the parallel displacement propagator \(I(z, z')\). These propagators cancel each other owing to the identity (2.19) and the rest of (3.18) depends only on superfield strengths and their derivatives. Therefore we omit \(I(z, z')\) further and do not write it explicitly in the heat kernel.}
The integration over $d^3 \rho$ in (3.21) is simply Gaussian,

$$
\int d^3 \rho e^{i P_m A_{mn} \rho_n + \rho_m W^\alpha_m (f_{\alpha \beta}^m(s) - f_{\alpha \beta}^m(t)) \tilde W^\beta} = -\frac{(4\pi i)^{3/2}}{\sqrt{\det A}} e^{i^2 W^2 \bar W^2 \mathcal{F}(s,t,u)},
$$

(3.27)

where

$$
\mathcal{F}(s, t, u) = -\frac{1}{2} \left( f_{\alpha \beta}^m(s) - f_{\alpha \beta}^m(t) \right) (A^{-1})_{mn} \left( f_{\alpha \beta}^n(s) - f_{\alpha \beta}^n(t) \right).
$$

(3.28)

Owing to the representation of the matrix $A_{mn}$ in terms of projectors (3.23), it is easy to find its determinant and the inverse,

$$
\frac{1}{\sqrt{\det A}} = \frac{1}{(a + u^{-1})(b + u^{-1})^{1/2}}, \quad (A^{-1})_{mn} = \frac{P^m_{+}}{a + u^{-1}} + \frac{P^m_{-}}{b + u^{-1}}.
$$

(3.29)

Now, using the explicit expression for the function $f_{\alpha \beta}^m$ (3.20) and the projectors $P^{\pm}$ (3.25) we compute the contractions of all indices in (3.28),

$$
\mathcal{F}(s, t, u) = \frac{F^+(s,t)}{a + u^{-1}} + \frac{F^-(s,t)}{b + u^{-1}},
$$

(3.30)

where

$$
F^-(s,t) = \frac{1}{B^n} \left[ \left( \frac{\sinh(sB)}{s} - \frac{\sinh(tB)}{t} \right)^2 - \left( \frac{\cosh(sB) - 1}{s} - \frac{\cosh(tB) - 1}{t} \right)^2 \right],
$$

$$
F^+(s,t) = -\frac{2}{B^n} \left[ (\cosh(2Bs) + 2) \tanh^2 \left( \frac{Bs}{2} \right) - 2(\cosh(B(s-t)) + \cosh(2B(s-t))) \right]
$$

$$
+ \cosh(B(s+t)) \tanh \left( \frac{Bt}{2} \right) \tanh \left( \frac{Bs}{2} \right) + (\cosh(2Bt) + 2) \tanh^2 \left( \frac{Bt}{2} \right).\tag{3.31}
$$

As the final step, we expand the exponent in the second line of (3.21) in a series\(^5\) and compute the integrals over $du$:

$$
\int_0^\infty \frac{du}{u^{3/2}(a + u^{-1})(b + u^{-1})^{1/2}} = 2 \arccosh \sqrt{\frac{a}{b}} \sqrt{\frac{a}{a-b}}, \quad (a > b),
$$

(3.32)

$$
\int_0^\infty \frac{du}{u^{3/2}(a + u^{-1})(b + u^{-1})^{1/2}} \left[ \frac{F^+}{a + u^{-1}} + \frac{F^-}{b + u^{-1}} \right]
$$

$$
= \frac{1}{a-b} \left( \frac{2 F^-}{b} - \frac{F^+}{a} \right) + \frac{2a(F^+ - F^-) - bF^+}{(a-b)^{3/2}} \arccosh \sqrt{\frac{a}{b}}.
$$

(3.33)

As a result, the contribution to the low-energy effective action from the diagrams of Type A in figure 1 matches previously discussed superfield structure (3.16),

$$
\Gamma_A = \frac{e^2}{16\pi^3} \int d^7 z \left( \mathcal{L}_1 + W^\alpha \mathcal{L}_2 + \bar W^\beta \mathcal{L}_3^{(A)} \right),
$$

(3.34)

with $\mathcal{L}_1$, $\mathcal{L}_2$ and $\mathcal{L}_3$ given by

$$
\mathcal{L}_1 = \int_0^\infty ds dt e^{i(s+t)(G^2+m^2)} \frac{B^2}{\sinh(sB) \sinh(tB)} \frac{2 \arccosh \sqrt{\frac{a}{b}}}{\sqrt{\frac{a}{a-b}}},
$$

\(^5\)Actually, the series terminates owing to the Grassmann nature of superfield strengths, $W_a W_b W_c \equiv 0$.\footnote{Actually, the series terminates owing to the Grassmann nature of superfield strengths, $W_a W_b W_c \equiv 0$.}
\[
\mathcal{L}_{20}^{\alpha} = -\frac{2iG}{B^2} \int_0^\infty \frac{ds dt}{\sqrt{st}} e^{i(s+t)(G^2+m^2)} \frac{B^2}{\sinh(sB) \sinh(tB)} \frac{2 \arccosh \sqrt{a/b}}{\sqrt{a(a-b)}} \times (e^{-sN} - 1 + sN + e^{-tN} - 1 + tN)^\alpha \beta, \\
\mathcal{L}_3^{(A)} = \int_0^\infty \frac{ds dt}{\sqrt{st}} \frac{e^{i(s+t)(G^2+m^2)} B^2}{\sinh(sB) \sinh(tB)} \frac{1}{a-b} \left( \frac{2F^- - F^+}{b} \right) \\
+ \frac{G^2}{B^2} ((sB - \sinh sB + tB - \sinh tB)^2 - (\cosh sB + \cosh tB - 2)^2) 2 \arccosh \sqrt{a/b} \sqrt{a(a-b)}.
\]

(3.36)

Consider now the diagram of Type B in figure 1,

\[
\Gamma_B = -2e^2m^2 \int d^7z d^7z' G_+ (z, z') G_- (z, z') G_0 (z, z') \\
= 2ie^2m^2 \int d^7z d^7z' \int_0^\infty ds dt du K_+ (z, z'|s) K_- (z, z'|t) K_0 (z, z'|u) e^{im^2(s+t)}.
\]

(3.38)

Recall that the heat kernel \( K_0 \), given by (3.11), contains \( \zeta^2 \xi^2 \) which is nothing but the delta-function over the Grassmann variables. Hence, the integration over only one set of Grassmann variables remains,

\[
\Gamma_B = 2ie^2m^2 \int d^7z d^7z' \int_0^\infty ds dt du |\frac{e^{im^2(s+t)} K_+ (z, z'|s) K_- (z, z'|t) K_0 (z, z'|u)}{4i\pi u}|_{\zeta \to 0}.
\]

(3.39)

The heat kernel \( K_+ \) is given explicitly by (2.51). Owing to the identity

\[
\zeta^2 (s)|_{\zeta \to 0} = s^2 W^2 \frac{\sinh^2 \left( \frac{sB}{2} \right)}{\left( \frac{sB}{2} \right)^2},
\]

(3.40)

it is easy to find the limit \( \zeta \to 0 \) in (2.51),

\[
K_+ (z, z'|s)|_{\zeta \to 0} = \frac{1}{4(4\pi s)^{3/2}} \frac{sW^2}{B} \tanh \frac{sB}{2} e^{iG^2 e^{i/2} (F \coth (sF))_{mn} \rho^m \rho^n}.
\]

(3.41)

Indeed, the expression (3.40) contains \( W^2 \) and this prevents appearing of other superfield contributions which could come from the exponent in (2.51). The antichiral heat kernel \( K_- (z, z'|s) \) in the limit \( \zeta \to 0 \) has the same structure as (3.41), but one should replace \( W^2 \to \bar{W}^2 \).

Substituting (3.41) into (3.39), we get

\[
\Gamma_B = \frac{ie^2m^2}{64(4\pi)^{9/2}} \int d^7z \frac{W^2 \bar{W}^2}{B^2} \int_0^\infty ds \frac{du}{\sqrt{s}u^{5/2}} e^{i(s+t)(G^2+m^2)} \\
\times \tanh \frac{sB}{2} \tanh \frac{tB}{2} \int d^3\rho e^{i/2} \rho^m A_{mn} \rho^n,
\]

(3.42)

\footnote{Here we omit the expressions for parallel displacement propagators since they cancel in (3.39) due to (2.19).}
where the matrix $A_{mn}$ is given by (3.22). The Gaussian integral over $d^3\rho$ in (3.42) is computed according to (3.27),

$$
\int d^3 \rho e^{i \rho_m A_{mn} \rho_n} = -\frac{(4\pi i)^{3/2}}{(a+u^{-1})(b+u^{-1})^{1/2}}.
$$

The integral over $du$ is evaluated in (3.32). As a result, we find the contribution to the low-energy effective action from the diagram of Type B in figure 1 in the form

$$
\Gamma_B = \frac{e^2}{16\pi^3} \int d^7 z W^2 \bar{W}^2 \mathcal{L}_3^{(B)},
$$

with

$$
\mathcal{L}_3^{(B)} = \frac{4m^2}{B^2} \int_0^\infty \frac{ds dt}{\sqrt{st}} e^{i(s+t)(G^2+m^2)} \tanh \frac{sB}{2} \tanh \frac{tB}{2} \arccosh \sqrt{a/b} \sqrt{a(a-b)}.
$$

To summarize, the two-loop effective action in the $\mathcal{N} = 2$ supersymmetric electrodynamics is given by

$$
\Gamma^{(2)}_{\mathcal{N}=2} = \Gamma_A + \Gamma_B = \frac{e^2}{16\pi^3} \int d^7 z \left[ \mathcal{L}_1 + W^\alpha \mathcal{L}_2^\alpha \beta \bar{W}_\beta + (\mathcal{L}_3^{(A)} + \mathcal{L}_3^{(B)}) W^2 \bar{W}^{-2} \right],
$$

where the functions $\mathcal{L}_1$, $\mathcal{L}_2$, $\mathcal{L}_3^{(A)}$ and $\mathcal{L}_3^{(B)}$ are given by (3.35), (3.36), (3.37) and (3.45), respectively. It is important to note that all these functions are free of UV quantum divergences because the integrations over $s$ and $t$ are regular. This is not surprising since the three-dimensional electrodynamics without the Chern-Simons term is superrenormalizable because the gauge coupling is dimensionful, $|e^2| = 1$. Quantum divergences may appear in the sector of effective Kähler potential, but the Euler-Heisenberg effective action for the gauge superfield is UV-finite.

### 3.4 Two-loop moduli space metric

A lot of information about low-energy dynamics of supersymmetric gauge theories is encoded in the structure of moduli space. The analysis of moduli spaces in three-dimensional supersymmetric gauge theories plays important role in studying the aspects of mirror symmetry [3–8] and Seiberg-like dualities [7–11] (see also [1, 2] for very recent discussions of these problems). The perturbative quantum corrections to the moduli space metric in the $\mathcal{N} = 2$, $d = 3$ gauge theories are known only up the one-loop order [3, 4, 7]. In the present section we derive two-loop quantum corrections to this metric which are stipulated by the effective action (3.46).

The moduli space in $\mathcal{N} = 2$, $d = 3$ supersymmetric gauge theories is a Kähler manifold which is two-dimensional in our case. It can be parametrized by two real coordinates $r$ and $\sigma$. The coordinate $r$ is naturally identified with the vev of the scalar field $\phi$ which is a part of the $\mathcal{N} = 2$, $d = 3$ gauge multiplet, $r = \langle \phi \rangle$. This scalar is the lowest component of the superfield strength $G$,

$$
G|_{\theta \to 0} = \phi.
$$
Another scalar field \( a \) appears upon dualizing the Abelian vector \( A_m \),
\[
\partial_m a \propto \varepsilon_{mnp} F^{np},
\]
where \( F_{mn} \) is the Maxwell field strength corresponding to the Abelian vector field \( A_m \).

The coordinate \( \sigma \) corresponds to the vev of this scalar, \( \sigma = \langle a \rangle \). In the present section we find the metric on the moduli space parametrized by \( r \) and \( \sigma \),
\[
ds^2 = g_{rr}(r, \sigma) dr^2 + g_{\sigma\sigma}(r, \sigma) d\sigma^2.
\]

The procedure of deriving the metric (3.49) from the low-energy effective action is well described in [7]. The moduli space metric is defined by the part of low-energy effective action which is given by the full superspace Lagrangian of the superfield strength \( G \) without derivatives,
\[
S_{\text{low-energy}} = \int d^7 z f(G).
\]

The classical action (3.1) and the one-loop effective action (3.15) contribute to \( f(G) \) as
\[
\begin{align*}
f^{(0)}(G) &= \frac{1}{e^2} G^2, \\
f^{(1)}(G) &= \frac{1}{2\pi} \left[ G \ln(G + \sqrt{G^2 + m^2}) - \sqrt{G^2 + m^2} \right].
\end{align*}
\]

To obtain the two-loop contribution to \( f(G) \) we need to evaluate the limit \( B \to 0 \) in the part of the effective action (3.35). Taking into account the explicit form of the functions \( a(s, t) \) and \( b(s, t) \) given in (3.24) we find
\[
\lim_{B \to 0} \frac{\text{arccosh} \sqrt{a/b}}{\sqrt{a(a - b)}} = \frac{s t}{s + t}.
\]

Substituting this expression into (3.35) and computing the integrals over the parameters \( s \) and \( t \) we get
\[
\lim_{B \to 0} \mathcal{L}_1 = -2\pi \ln(G^2 + m^2).
\]

Hence, the two-loop contribution to \( f(G) \) reads
\[
f^{(2)}(G) = -\frac{e^2}{8\pi^2} \ln(G^2 + m^2).
\]

Summarizing (3.51), (3.52) and (3.55) we conclude
\[
f(G) = \frac{1}{e^2} G^2 + \frac{1}{2\pi} \left[ G \ln(G + \sqrt{G^2 + m^2}) - \sqrt{G^2 + m^2} - \frac{e^2}{4\pi} \ln(G^2 + m^2) \right].
\]

Given the function \( f(G) \) one dualizes the linear superfield \( G \) into a chiral superfield \( \Phi \) as is described in [17]. The chiral superfield serves as the Lagrange multiplier for the linearity constraint (2.4),
\[
S_{\text{low-energy}} = \int d^7 z \left[ f(G) - G(\Phi + \bar{\Phi}) \right].
\]

\footnote{Here we use the same notation for the usual Maxwell field strength as for the superfield \( F_{mn} \) introduced in (2.2). The former appears as the lowest component of the latter. We hope that this does not lead to any confusions.}
The superfield $G$ is treated now as unconstrained. Varying \((3.57)\) with respect to $G$ we get
\[
\Phi + \bar{\Phi} = f'(G) = \frac{2}{e^2} G + \frac{1}{2\pi} \ln(G + \sqrt{G^2 + m^2}) - \frac{e^2}{4\pi^2} \frac{G}{G^2 + m^2}. \tag{3.58}
\]
From this equation the superfield $G$ should be expressed in terms of $\Phi + \bar{\Phi}$ and substituted back to \((3.57)\). This yields a sigma-model action,
\[
S_{\text{low-energy}} = \int d^7 z K(\Phi + \bar{\Phi}), \tag{3.59}
\]
with some function $K(\Phi + \bar{\Phi})$ which is hard to write down explicitly. However, we do not need the manifest expression for $K$ since the Kähler metric is defined rather by its second derivative,
\[
d s^2 = K''(\Phi + \bar{\Phi}) d\Phi d\bar{\Phi}. \tag{3.60}
\]
This metric should be expressed in terms of $r$ and $\sigma$ where $r = \langle G \rangle$ and $\sigma$ can be identified with the imaginary part of $\Phi$, $\sigma = \text{Im}\Phi$. Using the fact that the inverse Legendre transform is a Legendre transform, we have
\[
K'(\Phi + \bar{\Phi}) = r. \tag{3.61}
\]
From this equation and from \((3.58)\) we conclude
\[
K''(\Phi + \bar{\Phi}) = \left( \frac{\partial(\Phi + \bar{\Phi})}{\partial r} \right)^{-1} = \frac{1}{2 g(r)}, \tag{3.62}
\]
where
\[
g(r) = \frac{1}{e^2} + \frac{1}{4\pi} \frac{1}{\sqrt{r^2 + m^2}} + \frac{e^2}{8\pi^2} \frac{r^2 - m^2}{(r^2 + m^2)^2}. \tag{3.63}
\]
Finally, we note that \((3.58)\) implies that
\[
d\Phi = g(r)dr + id\sigma, \quad d\bar{\Phi} = g(r)dr - id\sigma. \tag{3.64}
\]
Substituting now \((3.62)\) and the latter identities into \((3.60)\) we find the moduli space metric in the form
\[
ds^2 = \frac{1}{2} g(r)dr^2 + \frac{1}{2 g(r)} d\sigma^2. \tag{3.65}
\]
In the massless limit the function $g(r)$ in \((3.63)\) simplifies such that
\[
ds^2|_{m=0} = \frac{1}{2} \left( \frac{1}{e^2} + \frac{1}{4\pi r} + \frac{e^2}{8\pi^2 r^2} \right) dr^2 + \frac{1}{2} \left( \frac{1}{e^2} + \frac{1}{4\pi r} + \frac{e^2}{8\pi^2 r^2} \right)^{-1} d\sigma^2. \tag{3.66}
\]
Equation \((3.66)\) shows that the one-loop metric is corrected by the two-loop contribution $\frac{e^2}{8\pi^2 r^2}$. It is naturally to expect that the $n$-loop correction could be of the form $c_n \frac{1}{2} \left( \frac{e^2}{r} \right)^n$, with some coefficient $c_n$. It is very tempting to compute such higher-loop coefficients $c_n$ and to find a closed expression for all-loop moduli space metric both for the Abelian and non-Abelian $\mathcal{N} = 2$, $d = 3$ gauge theories. In principle, it could resolve the singularity of the moduli space metric at small $r$.\(^8\)

\(^8\)An alternative mechanism for resolving the singularity of the moduli space metric was proposed recently in \cite{31}. 

4 Low-energy effective action in $\mathcal{N} = 4$ supersymmetric electrodynamics

4.1 Classical action and structure of two-loop effective action

The classical action of the $\mathcal{N} = 4$ supersymmetric electrodynamics reads

$$S_{\mathcal{N}=4} = \frac{1}{e^2} \int d^7 z (G^2 - \frac{1}{2} \bar{\Phi} \Phi) + S_Q,$$

where the covariantly chiral superfields $Q_{\pm}$ are related to the standard chiral superfields $\tilde{Q}_{\pm}$ as in (3.8). The action (4.1) is invariant under the following ‘hidden’ $\mathcal{N} = 2$ supersymmetry,

$$\delta V = \frac{1}{2} (\bar{\epsilon}^a \tilde{\theta}_a \Phi - \epsilon^a \theta_a \bar{\Phi}),$$
$$\delta \Phi = i \epsilon^a W_\alpha,$$
$$\delta Q_+ = -\frac{1}{4} \nabla^2 (\epsilon^a \tilde{\theta}_a \bar{Q}_+),$$
$$\delta Q_- = \frac{1}{4} \nabla^2 (\epsilon^a \theta_a \bar{Q}_-),$$

where $\epsilon^a$ and $\bar{\epsilon}^a$ are the supersymmetry parameters. Note that the action (4.1) appears from the $\mathcal{N} = 2$, $d = 4$ electrodynamics by means of the dimensional reduction. Two-loop Euler-Heisenberg effective action in the latter was studied in [13].

The $\mathcal{N} = 4$ gauge multiplet is described by the pair $(V, \Phi)$. We make the background-quantum splitting for both these superfields,

$$V \rightarrow V + e v, \quad \Phi \rightarrow \Phi + e \phi,$$

where the hypermultiplet $(Q_+, Q_-)$ is considered as the ‘quantum’ superfield which should be integrated out in the path integral. The background gauge superfield $V$ obeys the constraints (2.13) and (2.14) while $\Phi$ is simply constant,

$$D_\alpha \Phi = 0.$$

Upon quantization in the Fermi-Feynman gauge (3.4), we end up with the following action for ‘quantum’ superfields,

$$S_{\text{quantum}} = S_2 + S_{\text{int}},$$
$$S_2 = -\int d^7 z (v \Box v + \frac{1}{2} \bar{\phi} \phi + \bar{Q}_+ Q_+ + \bar{Q}_- Q_-) - \left( \int d^5 z \bar{Q}_+ \Phi Q_- + \text{c.c.} \right),$$
$$S_{\text{int}} = -2 \int d^7 z \left[ e (\bar{Q}_+ Q_+ - \bar{Q}_- Q_-) v + e^2 (\bar{Q}_+ Q_+ + \bar{Q}_- Q_-) v^2 \right]$$
$$-e \int d^5 z \bar{Q}_+ \phi Q_- + e \int d^5 z \bar{Q}_+ \bar{\phi} Q_+ + O(e^3).$$

The propagators for the hypermultiplets and for the gauge superfield $V$ are the same as in the $\mathcal{N} = 2$ electrodynamics, (3.9) and (3.10), but the mass parameter $m$ is now
promoted to the background superfield $\Phi$. Additionally, there is the propagator for the chiral superfield $\phi$,

$$\langle \phi(z)\bar{\phi}(z') \rangle = -\frac{i}{8} D^2 D^2 G_0(z, z').$$  (4.8)

There are also vertices with the chiral superfield $\phi$ represented in the last line in (4.7). Owing to these propagators and vertices with the chiral superfield $\phi$ the two-loop effective action in the $\mathcal{N} = 4$ electrodynamics gets additional contribution $\Gamma_C$ as compared with the $\mathcal{N} = 2$ case (3.14),

$$\Gamma_C^{(2)} = \Gamma_A + \Gamma_B + \Gamma_C,$$  (4.9)

$$\Gamma_A = -2e^2 \int d^7z d^7z' G_{+-}(z, z') G_{+-}(z', z) G_0(z, z'),$$  (4.10)

$$\Gamma_B = -2e^2 \int d^7z d^7z' \Phi \bar{\Phi} G_{+(z, z')} G_{-(z, z')} G_0(z, z'),$$  (4.11)

$$\Gamma_C = 2e^2 \int d^7z d^7z' G_{+-}(z, z') G_{+-}(z, z') G_0(z, z').$$  (4.12)

The part of the effective action $\Gamma_A$ takes into account the graphs of Type A in figure 1 which are exactly the same as in the $\mathcal{N} = 2$ electrodynamics. Therefore we can borrow the result (3.34) for $\Gamma_A$ from the effective action of the $\mathcal{N} = 2$ electrodynamics.

The term $\Gamma_B$ in (4.9) corresponds to the supergraph of Type B in figure 1. The expression (4.11) has the same form as (3.38), but the mass parameter $m$ should now be replaced with the chiral superfield $\Phi$. Since we consider the background with constant chiral superfield (4.4), the result of computing this diagram is given by (3.44), where one should replace $m^2 \rightarrow \Phi \bar{\Phi}$.

The term $\Gamma_C$ in (4.9) is new as compared with the $\mathcal{N} = 2$ supersymmetric electrodynamics since it involves the propagator of the chiral superfield (4.8). It is represented by the supergraph of Type C in figure 2.

The one-loop effective action in the $\mathcal{N} = 4$ electrodynamics was computed in [16]. It has the same form as (3.15), but the mass parameter $m$ should now be promoted to the
the background superfield $\Phi$,

$$
\Gamma_{N=4}^{(1)} = \frac{1}{2\pi} \int d^7 z \left[ G \ln(G + \sqrt{G^2 + \hat{\Phi}^2}) - \sqrt{G^2 + \hat{\Phi}^2} \right] + \frac{1}{8\pi} \int d^7 z \int_0^\infty \frac{ds}{\sqrt{1 + s^2}} \frac{e^{is(G^2 + \hat{\Phi}^2)}}{B^2} \left( \tanh(sB/2) - 1 \right). \tag{4.13}
$$

In what follows, we concentrate on computing two-loop contributions to the effective action.

### 4.2 Computing two-loop effective action

Consider the diagram of Type C in figure 2. Its analytic expression (4.12) is very similar to (3.17), but the arguments of one of the Green’s function are swapped. Hence, the algorithm of computing this graph is the same as in section 3.3.

Using the super-photon propagator (3.11) and the definition of the heat kernel $K_{+-}$ (2.56) we get

$$
\Gamma_C = -2ie^2 \int d^7 z d^3 \rho \int_0^\infty ds dt du \frac{e^{i\rho m mn}}{4(\pi i)^3/2} e^{i\Phi^2 + \Phi} \left. K_{+-}(z, z'|s)K_{+-}(z, z'|t) \right|_{z \to 0}. \tag{4.14}
$$

The propagator $K_{+-}$ at coincident Grassmann points is given by (3.19). Substituting this expression into (4.14) we find

$$
\Gamma_C = -\frac{4ie^2}{(4\pi i)^3/2} \int d^7 z d^3 \rho \int_0^\infty ds dt du \frac{e^{i\rho m mn}}{4(\pi i)^3/2} e^{i\Phi^2 + \Phi} \left. K_{+-}(z, z'|s)K_{+-}(z, z'|t) \right|_{z \to 0}, \tag{4.15}
$$

where the matrix $A$ is given in (3.22) and the functions $f$ are written down in (3.20). Note that (4.15) differs from (3.21) only in one sign in the last line. Hence, many cancellations occur among (4.15) and (3.21).

Let us compute the Gaussian integral over $d^3 \rho$ in (4.15),

$$
\int d^3 \rho e^{i\rho m mn + \rho m W^\alpha (f_{\alpha \beta}(s) + f_{\alpha \beta}(t))W_\beta - 2GW^\alpha (f_{\alpha \beta}(s) + f_{\alpha \beta}(t))W_{\beta}} = \frac{(4\pi i)^{3/2}}{\sqrt{\det A}} e^{4\pi i^2 W^2 \tilde{F}(s, t, u)}, \tag{4.16}
$$

where

$$
\tilde{F}(s, t, u) = -\frac{1}{2} \left( f_{\alpha \beta}^m(s) + f_{\alpha \beta}^m(t) \right) \left( A^{-1} \right)^m_n \left( f_{\alpha \beta}^n(s) + f_{\alpha \beta}^n(t) \right). \tag{4.17}
$$

In the part of the effective action $\Gamma_A + \Gamma_C$ the function (4.17) appears in the following combination with (3.28),

$$
\mathcal{F}(s, t, u) - \tilde{F}(s, t, u) = 2\left( f_{\alpha \beta}^m(s) + f_{\alpha \beta}^m(t) \right) \left( A^{-1} \right)^m_n \left( f_{\alpha \beta}^n(s) + f_{\alpha \beta}^n(t) \right). \tag{4.18}
$$

Given the function $f_{\alpha \beta}^m$ in (3.20) and the inverse matrix $A^{-1}$ in (3.29) we compute the contractions in (4.18),

$$
\mathcal{F}(s, t, u) - \tilde{F}(s, t, u) = \frac{\mathcal{F}^+(s, t)}{a + u^{-1}} + \frac{\mathcal{F}^-(s, t)}{b + u^{-1}}, \tag{4.19}
$$

where

$$
\mathcal{F}^+(s, t) = \frac{\mathcal{F}^+(s, t)}{a + u^{-1}} + \frac{\mathcal{F}^-(s, t)}{b + u^{-1}}.
$$
where \(a\) and \(b\) are given in (3.24) and
\[
\mathcal{F}^+(s, t) = \frac{8}{B^2} \tanh \frac{sB}{2} \tanh \frac{tB}{2} \left[ \cosh(B(s-t)) + \cosh(2B(s-t)) + \cosh(B(s+t)) \right],
\]
\[
\mathcal{F}^-(s, t) = \frac{4}{stB^2} \left[ (\cosh sB - 1)(\cosh tB - 1) - (\sinh sB - sB)(\sinh tB - tB) \right].
\]

Hence, for the sum of \(\Gamma_A\) and \(\Gamma_C\) we find
\[
\Gamma_A + \Gamma_C = \frac{i e^2}{32 \pi^3} \int d^7 z \ W^2 \bar{W}^2 \int_0^\infty ds \ dt \ du \frac{e^{i(s-t)(G^2 + \bar{\Phi} \Phi)}}{(st)^{3/2}} \left[ \mathcal{F}^+(s, t) + \mathcal{F}^-(s, t) \right].
\]

Finally, we perform the integration over \(du\) using (3.33),
\[
\Gamma_A + \Gamma_C = \frac{i e^2}{32 \pi^3} \int d^2 z \ W^2 \bar{W}^2 \int_0^\infty ds \ dt \ \frac{1}{\sinh sB \sinh tB (a + u^{-1})(b + u^{-1})} \left[ \mathcal{F}^+(s, t) + \mathcal{F}^-(s, t) \right].
\]

The contribution from the diagram of Type B in figure 1 can be easily obtained from (3.44), (3.45),
\[
\Gamma_B = \frac{e^2}{4 \pi^3} \int d^7 z \ W^2 \bar{W}^2 \Phi \bar{\Phi} \int_0^\infty ds \ dt \ e^{i(s+t)(G^2 + \bar{\Phi} \Phi)} \tan \frac{sB}{2} \tanh \frac{tB}{2} \arccosh \sqrt{\frac{a}{b}} \sqrt{a(a-b)}.\]

We conclude that the two-loop low-energy effective action in the \(\mathcal{N} = 4\) SQED is given by (4.9) with \(\Gamma_A + \Gamma_C\) and \(\Gamma_B\) written down explicitly in (4.22) and (4.23), respectively. As is seen from these expressions, all contributions to the two-loop effective action \(\Gamma^{(2)}_{\mathcal{N}=4}\) contain \(W^2 \bar{W}^2\) and the terms without \(W\)'s, like (3.35), do not appear. Hence, in the \(\mathcal{N} = 4\) SQED there are no two-loop quantum corrections to the low-energy effective action of the form (3.50) and the moduli space metric is one-loop exact.\(^9\) This is in agreement with the conclusions of [7].

5 Summary and discussion

We have developed a manifestly \(\mathcal{N} = 2\) supersymmetric and gauge-covariant technique for studying contributions to low-energy effective actions in three-dimensional supersymmetric gauge theories beyond one-loop order. As an application, we computed two-loop Euler-Heisenberg effective actions in \(\mathcal{N} = 2\) and \(\mathcal{N} = 4\) supersymmetric electrodynamics in the \(\mathcal{N} = 2, d = 3\) superspace.

One of the features of the three-dimensional \(\mathcal{N} = 2\) supersymmetric gauge theory in comparison with the four-dimensional \(\mathcal{N} = 1\) supersymmetric gauge theory is that the gauge superfield has not only ‘spinorial’ superfield strengths \(W_\alpha\) and \(\bar{W}_\alpha\), but also the ‘scalar’ superfield strength \(G\). As a consequence, in the three-dimensional SQED, there

\(^9\)This is the well-known Taub-NUT metric derived in [17] from geometrical principles.
are completely new superfield contributions to the two-loop effective actions stipulated by this superfield $G$ having no analogs in four dimensions (see the two-loop effective action for $\mathcal{N} = 1, d = 4$ gauge theories in [13, 14]). In particular, in the $\mathcal{N} = 2, d = 3$ SQED these new terms are found in the form (3.35) and (3.36). We argue that these expressions play important role in the low-energy dynamics since they are responsible, in particular, for the two-loop quantum corrections to the moduli space metric. We explicitly computed the moduli space metric (3.66) in the $\mathcal{N} = 2$ SQED which takes into account the two-loop corrections. To the best of our knowledge, only one-loop perturbative corrections to the moduli space have been known so far [7, 8].

Two-loop effective action in the $\mathcal{N} = 4, d = 3$ SQED receives additional contributions represented by the graph in figure 2 as compared with the two-loop effective action of the $\mathcal{N} = 2$ SQED described by the background field dependent Feynman diagrams in figure 1. Indeed, the chiral superfield $\Phi$ becomes dynamical in the $\mathcal{N} = 4$ theory and propagates inside the two-loop diagram. This leads to many cancellations among the diagrams of Type A in figure 1 and Type C in figure 2. In particular, the terms of the form (3.35) and (3.36) are completely cancelled in the $\mathcal{N} = 4$ SQED two-loop effective action. As a consequence, the moduli space in the $\mathcal{N} = 4$ electrodynamics is not renormalized by two-loop corrections and remains one-loop exact. This is in agreement with conclusions made in [3, 7].

Concerning technical details of two-loop computations performed in the present work, we obtained exact propagators of chiral superfields interacting with slowly-varying background gauge superfield. The propagators involve the so-called parallel displacement propagator $I(z, z')$ which provides the gauge covariance on all stages of quantum loop computations. This technique is a three-dimensional analog of the methods of covariant perturbative computations in the $\mathcal{N} = 1, d = 4$ superspace [15]. We believe that the properties of parallel displacement propagator $I(z, z')$ and the exact propagators for chiral superfields in three-dimensional gauge theories explored in the present paper will be useful for studying low-energy effective actions in other three-dimensional gauge theories including non-Abelian ones.

An important extension of the results of the present paper is the inclusion of the Chern-Simons term into considerations. With the non-trivial Chern-Simons term the form of super-photon propagator (3.11) changes [32] acquiring extra spinorial derivatives on the full superspace delta-function. Careful accounting of these derivatives in the two-loop computations requires separate studies. However, the two-loop low-energy effective actions in the three-dimensional supergauge models with Chern-Simons terms are of high interest in the light of recent discussions [1, 2].\(^{10}\) Next, it is important to study the low-energy effective actions in the BLG [35–40] and ABJM [41] models which should describe the low-energy dynamics of multiple M2 branes.

Another important extension of the present considerations is the study of two-loop effective actions in non-Abelian three-dimensional supersymmetric gauge theories. The one-loop effective action in various three-dimensional super Yang-Mills models were found in [24], but the two-loop extension of these results remains an open problem.

\(^{10}\)One-loop effective action in $\mathcal{N} = 2$ Chern-Simons gauge theory coupled to matter is studied in [33, 34].
Finally, it is tempting to study two-loop quantum corrections to the Kähler potential in three-dimensional $\mathcal{N} = 2$ gauge theories (the two-loop Kähler potential in the $\mathcal{N} = 2, d = 3$ sigma-models was obtained in [42].)

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A $\mathcal{N} = 2$ superspace conventions

In the present paper we use $\mathcal{N} = 2$, $d = 3$ superspace conventions following previous works [16, 24]. In particular, the gamma matrices $(\gamma^0)_{\alpha}^\beta = -i\sigma_2$, $(\gamma^1)_{\alpha}^\beta = \sigma_3$, $(\gamma^2)_{\alpha}^\beta = \sigma_1$ obey the Clifford algebra

$$\{\gamma^m, \gamma^n\} = -2\eta^{mn}, \quad \eta^{mn} = \text{diag}(1, -1, -1), \quad (A.1)$$

and the following orthogonality and completeness relations

$$(\gamma^m)_{\alpha\beta}(\gamma^n)_{\alpha\beta} = 2\eta^{mn}, \quad (\gamma^m)_{\alpha\beta}(\gamma^m)_{\rho\sigma} = (\delta^\rho_\alpha \delta^\sigma_\beta + \delta^\rho_\beta \delta^\sigma_\alpha). \quad (A.2)$$

We raise and lower the spinor indices with the $\varepsilon$-tensor, e.g., $(\gamma_m)_{\alpha\beta} = \varepsilon_{\alpha\sigma}(\gamma_m)^{\sigma\beta}, \varepsilon_{12} = 1$.

Any vector index is converted into a pair of spinor ones according to the following rules

$$x^{\alpha\beta} = (\gamma_m)^{\alpha\beta} x^m, \quad x^m = \frac{1}{2}(\gamma_m)_{\alpha\beta} x^{\alpha\beta}, \quad (\partial_{\alpha\beta} = (\gamma_m)_{\alpha\beta} \partial_m, \quad \partial_m = \frac{1}{2}(\gamma_m)_{\alpha\beta} \partial_{\alpha\beta}, \quad (A.3)$$

so that

$$\partial_m x^n = \delta^n_m, \quad \partial_{\alpha\beta} x^{\rho\sigma} = \delta^{\rho}_{\alpha}\delta^{\sigma}_{\beta} + \delta^{\rho}_{\beta}\delta^{\sigma}_{\alpha} = 2\delta^{(\rho\sigma)}_{\alpha\beta}. \quad (A.4)$$

The covariant spinor derivatives

$$D_{\alpha} = \frac{\partial}{\partial \theta^\alpha} + i\bar{\theta}^\beta \partial_{\alpha\beta}, \quad \bar{D}_{\alpha} = -\frac{\partial}{\partial \bar{\theta}^\alpha} - i\theta^\beta \partial_{\alpha\beta} \quad (A.5)$$

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obey the standard anticommutation relation
\[ \{D_\alpha, \bar{D}_\beta\} = -2i\partial_{\alpha\beta}. \] (A.6)

The integration measure in the full \( N = 2, d = 3 \) superspace is defined as
\[ d^7z \equiv d^3x d^4\theta = \frac{1}{16} d^3x D^2 \bar{D}^2, \] so that
\[ \int d^3x f(x) = \int d^7z \theta^2 \bar{\theta}^2 f(x), \] (A.7)
for some field \( f(x) \). Here we use the following conventions for contractions of the spinor indices
\[ D^2 = D^a D_a, \quad \bar{D}^2 = \bar{D}^\alpha \bar{D}_\alpha, \quad \theta^2 = \theta^a \theta_a, \quad \bar{\theta}^2 = \bar{\theta}^\alpha \bar{\theta}_\alpha. \] (A.8)

The chiral subspace is parametrized by \( z_+ = (x_+^m, \theta_\alpha) \), where \( x_\pm^m = x^m \pm i\gamma^m_\alpha \theta^\alpha \bar{\theta}^\beta \). The integration measure in the chiral superspace \( d^5z \equiv d^3x d^2\theta \) is related to the full superspace measure (A.7) as
\[ d^7z = -\frac{1}{4} d^5\bar{z} \bar{D}^2 = -\frac{1}{4} d^5z D^2. \] (A.9)

B Heat kernel \( K_{+-} \) at coincident points

Consider the heat kernel \( K_{+-} \) given by the expression (2.63),
\[ K_{+-}(z, z'|s) = \frac{1}{8(i\pi s)^{3/2}} \sinh(sB) e^{isG^2 e^{iX(s)}}, \] (B.1)
where
\[ X(s) = \frac{i}{4} (F \coth(sF))_{mn} \tilde{\rho}^m(s) \tilde{\rho}^n(s) + R(z, z') + \int_0^s dt (R'(t) + \Sigma(t)), \] (B.2)
and \( R'(t) + \Sigma(t) \) is given in (2.65),
\[ R'(t) + \Sigma(t) = 2i\tilde{\zeta}^\alpha(t) W_\alpha(t) G + 2i[\zeta^\alpha(t) \tilde{\zeta}_\alpha(t) W^\beta(t) \bar{W}_\beta(t) - \zeta^\alpha(t) W_\alpha(t) \tilde{\zeta}^\beta(t) \bar{W}_\beta(t)] + i\tilde{\zeta}^\gamma(t) W^\gamma(t) - \frac{1}{2} \zeta^\alpha(t) W^\alpha(t) [\rho_{\beta\gamma}(t) D^\gamma \bar{W}_\beta - \rho_{\alpha\gamma}(t) D^\gamma W_\beta]. \] (B.3)

The \( t \)-dependent objects in the r.h.s. of (B.3) are given in (2.34). Note that here we use the bosonic interval \( \rho_{\alpha\beta} \) rather than its chiral version \( \tilde{\rho}_{\alpha\beta} \) given in (2.61). It is clear that the problem of computing the heat kernel \( K_{+-} \) at coincident points is reduced to finding \( X(s)|_{\zeta \to 0} \).

First of all, we point out that the function \( R(z, z') \) given in (2.60) vanishes in this limit,
\[ R(z, z')|_{\zeta \to 0} = 0. \] (B.4)

Hence, we need to compute
\[ X(s)|_{\zeta \to 0} = \left( \frac{i}{4} (F \coth(sF))_{mn} \tilde{\rho}^m(s) \tilde{\rho}^n(s) + \int_0^s dt (R'(t) + \Sigma(t)) \right)|_{\zeta \to 0}. \] (B.5)

\[ \text{We omit the parallel displacement propagator } I(z, z'). \]
Consider $\tilde{\rho}^m(s)$. Using (2.61) and (2.34), it can be rewritten as

$$\tilde{\rho}^m(s) = \tilde{\rho}^m - 2i\gamma^m_{\alpha\beta} \int_0^s W^\alpha(t) \tilde{\zeta}^\beta(t) dt.$$  \hspace{1cm} (B.6)

We substitute here the expressions (2.34) for $W^\alpha(s)$ and $\tilde{\zeta}^\beta(s)$ and compute the integral over $dt$,

$$\tilde{\rho}^m(s) = \rho^m + i(\gamma^m_{\alpha\beta} N^\alpha_{\beta\gamma}) \frac{W^\gamma W^\beta}{B^3} (sB - \sinh sB)$$

$$+ i\gamma^m_{\alpha\beta} W^\gamma W^\beta \left( \frac{e^{-sN} - 1}{N} \right) \alpha \left( \frac{e^{-sN} - 1}{N} \right) \beta.$$

Here the following identities have been used

$$(N^{2n})_{\alpha} = \delta^\beta_{\alpha} B^{2n}, \quad (N^{2n+1})_{\alpha} = N_{\alpha} B^{2n}, \quad B^2 = \frac{1}{2} N_{\alpha} N_{\beta}.$$

Hence, for the first term in r.h.s. in (B.5) we find

$$\frac{i}{4} (F \coth(sF))_{mn} \tilde{\rho}^m(s) \tilde{\rho}^n(s) = \frac{i}{4} (F \coth(sF))_{mn} \rho^m \rho^n$$

$$- \frac{1}{2} (F \coth(sF))_{mn} \rho^m \gamma_{\alpha\beta} W \gamma W^\beta (sB - \sinh sB)$$

$$+ W^\gamma W^\beta \left( \frac{e^{-sN} - 1}{N} \right) \alpha \left( \frac{e^{-sN} - 1}{N} \right) \beta$$

$$+ \frac{i}{2} \frac{W^2 \tilde{W}^2}{B^4} \left[ (2B \coth sB + \frac{1}{s})(\cosh sB - 1)^2 - \frac{1}{s}(\sinh sB - sB)^2 \right].$$

In deriving this expression the following identities could be useful

$$(F \coth(sF))_{mn} = 2B \coth sB + \frac{1}{s}, \quad (F \coth(sF))_{mn} (N_{\gamma}^m)(N_{\gamma}^n) = -4 \frac{B^2}{s}.$$  \hspace{1cm} (B.10)

Consider now the last term in (B.5) which is given by $\int_0^s dt (R^\prime(t) + \Sigma(t))$. For this purpose we compute the limit of coincident Grassmann points of various terms in (B.3),

$$W^\alpha(s) W_\alpha(s) = W^2, \quad W^\alpha(s) W_\alpha(s) = W^\alpha W_\alpha.$$  \hspace{1cm} (B.11)

$$\zeta^2(s) = \frac{4W^2}{B^2} \sinh^2 \left( \frac{sB}{2} \right), \quad \tilde{\zeta}^2(s) = \frac{4\tilde{W}^2}{B^2} \sinh^2 \left( \frac{sB}{2} \right).$$

$$\zeta^\alpha(s) W_\alpha(s) = -W^\alpha W_\alpha \frac{\sinh(sB)}{B} + \frac{W^\alpha \tilde{W} \beta N_{\alpha\beta} 2 \sinh^2 \frac{sB}{2}}{B^2},$$

$$\tilde{\zeta}^\alpha(s) W_\alpha(s) = -W^\alpha W_\alpha \frac{\sinh(sB)}{B} - \frac{W^\alpha \tilde{W} \beta N_{\alpha\beta} 2 \sinh^2 \frac{sB}{2}}{B^2},$$

$$\zeta^\alpha(s) W_\alpha(s) = -W^2 \sinh(sB) \frac{B}{2}, \quad \tilde{\zeta}^\alpha(s) W_\alpha(s) = -W^2 \sinh(sB) \frac{B}{2},$$

$$\langle \zeta W \rangle = -W^2 \tilde{W}^2 \frac{B}{2}(\sinh^2 sB - \cosh sB + 1),$$

$$\langle \zeta W \rangle = -W^2 \tilde{W}^2 \frac{B}{2}(\sinh^2 sB - \cosh sB + 1).$$

$$\langle \zeta W \rangle = -W^2 \tilde{W}^2 \frac{B}{2}(\sinh^2 sB - \cosh sB + 1).$$
\[ \zeta \bar{W} W - \zeta W \bar{W} = -2\zeta W \bar{W} - (\zeta \bar{W})(\zeta W) \]
\[ = \frac{W^2 \bar{W}^2}{B^2} (1 - \cosh sB - \sinh^2 sB), \quad (B.17) \]
\[ 2i(\zeta \bar{W} W - \zeta W \bar{W}) + i\zeta^2 \bar{W}^2 = -2i \frac{W^2 \bar{W}^2}{B^2} \sinh^2 sB. \quad (B.18) \]

Substituting these expressions into (B.3) and integrating over the parameter \( t \) we obtain
\[ \int_0^s dt (R'(t) + \Sigma(t)) = \frac{2iG}{B} W^\alpha (e^{-sN} - 1 + sN)_{\alpha}^\beta \bar{W}_\beta \]
\[ - i \frac{W^2 \bar{W}^2}{B} (\sinh sB \cosh sB - sB) \]
\[ - \frac{1}{2} \rho^m (\cosh sB - 1) \bar{W}^\alpha (e^{-sN})_{\alpha}^\beta ((\gamma_m)_{\beta}^\gamma N_{\gamma}^\delta - N_{\beta}^\gamma (\gamma_m)_{\gamma}^\delta) W_\delta. \quad (B.19) \]

Putting (B.9) and (B.19) together, we find
\[ X(s) = \left| i \frac{4}{F \coth(sF)} m_\rho \rho^m \rho^N + \rho_m f_{\alpha\beta}(s) W^\alpha W^\beta + \frac{i}{2} W^2 \bar{W}^2 f(s) - iGW^\alpha f_{\alpha\beta}(s) \bar{W}_\beta \right|. \quad (B.20) \]

where
\[ f_{\alpha\beta}(s) = 2B^{-2} (1 - sN - e^{-sN})_{\alpha}^\beta, \quad (B.21) \]
\[ f(s) = \frac{1}{B^4} \left[ (sB)^2 - 4 \sinh^2(sB/2) (1 + sB \tanh(sB/2)) \right], \]
\[ f_m^\alpha(s) = \frac{1}{2} B^{-2} (\cosh(sB) - 1) \left[ (e^{-sN})_{\beta}^\gamma N_{\alpha}^\delta (\gamma_m)_{\gamma}^\delta + (N(e^{-sN}))_{\beta}^\gamma (\gamma_m)_{\gamma}^\delta \right] - \frac{1}{2} F \coth(sF) \right] m_\gamma \gamma_{\alpha\beta} \left[ \left( \frac{e^{-sN} - 1}{N} \right)_{\alpha}^\gamma \left( \frac{e^{-sN} - 1}{N} \right)_{\beta}^\gamma + \frac{\varepsilon_{\alpha\beta} N_{\gamma}^\delta}{B^3} (sB - \sinh(sB)) \right]. \]

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