B \rightarrow \pi l \bar{\nu} Form Factors with NRQCD Heavy Quark and Clover Light Quark Actions

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We report results on semileptonic B \rightarrow \pi l \bar{\nu} decay form factors near q_{\text{max}}^2 using NRQCD heavy quark and clover light quark actions and currents improved through O(\alpha_s a) with the clover coefficient \kappa_{\text{sw}} determined by mean field improved perturbation theory at one-loop order. The heavy-light vector current involved in the matrix element is renormalized to O(\alpha_s a) using the one-loop calculation of Morningstar and Shigemitsu\textsuperscript{1}, and of Ishikawa\textsuperscript{4} including mixings with higher dimensional operators.

The calculation of f^0(q_{\text{max}}^2) was performed at \beta=5.7 and 5.9 on 12^3 \times 32 and 16^3 \times 40 lattices with 232 and 222 gauge configurations, respectively. We took five different heavy quark masses covering m_b and four different light quark masses ranging from m_s to m_s/2. For f^+(q^2) we only analyzed the \beta = 5.7 data, so far. The momentum combinations \rho_B=(0,0,0), k=0, (1,0,0), (1,1,0), and \rho_B=(1,0,0), k=(0,0,0) in units of 2\pi/(12a) are considered at \kappa_{\text{f}} = 0.1369, which is around m_s.

3. Results for f^0(q_{\text{max}}^2)

Let us first present the results for f^0(q_{\text{max}}^2), for which the soft pion relation f^0(q_{\text{max}}^2) = f_B/f_\pi in the chiral limit provides an important check of the lattice calculation. The 1/M_B dependence of \sqrt{M_B} f^0(q_{\text{max}}^2) (\alpha_s(M_B)/\alpha_s(M_B^{\text{phys}}))^{2/\beta_0} and a comparison with \sqrt{M_B} f_B/f_\pi (\alpha_s(M_B)/\alpha_s(M_B^{\text{phys}}))^{2/\beta_0} is shown in

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Figure 1. Comparison of \( \sqrt{M_Bf^0(q_{\text{max}}^2)}\eta \) with \( \sqrt{M_Bf_B/f_\pi}\eta \), where \( \eta \) in defined as \( (\alpha_s(M_B)/\alpha_s(M_B^{\text{phys}}))^{2/\beta_0} \). \( f^0(q_{\text{max}}^2) \) with heaviest and with lightest light quark mass is plotted at \( \beta=5.7 \) and 5.9. Data for \( f_B/f_\pi \) are taken from our previous works: heavy clover \( \frac{\text{V}}{\text{HL}} \) and NRQCD \( \frac{\text{V}}{\text{HL}} \).

The chiral (soft pion) limit is not taken for the points given in the plot, which is a possible reason of the violation of the soft pion relation. We find, however, that the light quark mass dependence of \( f^0(q_{\text{max}}^2) \) is consistent with a constant within statistical error, and a polynomial chiral extrapolation in \( m_q \) and \( m_q^2 \) gives a consistent result. With the present statistics, we are not able to fit our data with both \( \sqrt{m_q^2} \) and \( m_q^2 \) in the fitting function.

Another possible reason for the disagreement with the soft pion relation is the large uncertainty in the matching constants. Since the vector and axial vector heavy-light currents are involved on the two sides of the equality, perturbative errors in the matching between the continuum and lattice operators could be important. Naively this error is \( O(\alpha_s^2) \), and hence should be small at \( \beta \sim 6.0 \). Nonetheless, the large one-loop correction in the renormalization constant \( Z_A^{\text{HL}} \) suggests that there could be large higher order corrections.

In order to see how such higher order effect contributes, we computed the ratio of the renormalization constants \( Z_A^{\text{HL}}/Z_V^{\text{HL}} \) nonperturbatively in the static limit, using the chiral Ward-Takahashi identity

\[
Z_A^{\text{HL}} \int d^4y (\bar{\psi}A_\mu - 2m_qP)(y)V_0^{\text{HL}}(x)\mathcal{O}) = -Z_A^{\text{HL}}(A_0^{\text{HL}}(x)\mathcal{O}),
\]

where \( A_\mu \) and \( P \) denote light-light axial-current and pseudoscalar density, with \( Z_A \) the renormalization factor for \( A_\mu \), and \( V_0^{\text{HL}} \) and \( A_0^{\text{HL}} \) are the heavy-light (static-light in this particular case) currents. We performed simulations on a \( 12^3 \times 32 \) lattice at \( \beta=6.0 \) following the methods of Maiani-Martinelli \( \text{[8]} \). For the operator \( \mathcal{O} \) we took a heavy-light meson interpolation operator with wall source. The clover coefficient \( c_{\text{sw}} \) for the light quark was chosen to be the nonperturbative value from \( \text{[8]} \).

Combining our result for \( Z_A^{\text{HL}}/(Z_V^{\text{HL}}Z_A) \) from (1) with the nonperturbative value of \( Z_A^{\text{HL}}/Z_V^{\text{HL}} \) we obtained \( Z_A^{\text{HL}}/Z_V^{\text{HL}}=0.72(2) \) to be compared with the perturbative result 0.87(3). We find that the nonperturbative value for \( Z_A^{\text{HL}}/Z_V^{\text{HL}} \) is about 20% smaller than the corresponding one-loop result. While this explains part of the discrepancy between \( f_B/f_\pi \) and \( f^0(q_{\text{max}}^2) \), the reduction is not sufficient to remove the disagreement seen in Fig. 1.

In our study of the renormalization constant, the light-light and static-light current did not include corrections from higher dimensional operators. Since these corrections are known to give a large contribution in the calculation of \( f_B \), it would be important to perform a study of renormalization constant with improved currents.

4. Results for \( f^+(q^2) \)

We next study the \( q^2 \) and \( 1/M_B \) dependence of the form factor \( f^+(q^2) \). For the large \( q^2 \) region, the form factor \( f^+(q^2) \) should be well approximated by the pole dominance model with the \( B^* \) pole, since \( B^* \) is almost degenerate with \( B \) in the heavy quark limit and the pole is very close to \( q_{\text{max}}^2 \). The pole model predicts

\[
1/f^+(q^2) = -c_1(q^2 - M_{B^*}^2) + O((q^2 - M_{B^*}^2)^2),
\]

where the coefficient \( c_1 \) is written in terms of the dimensionless \( B^*B\pi \) coupling \( g \) and the \( B^* \) me-
Figure 2. Pole fit of $1/f_+ (q^2)$. Lattice results at five different heavy quark mass are shown at $\beta=5.7$. The $B^*$ pole is almost degenerated with $B$, and located at the origin in the plot. $aM_0$ is the bare heavy quark mass.

Figure 3. $1/M_B$ scaling of the pole fit coefficient.

diction of partial decay rate. We have found that the pole dominance model provides an excellent way to fit the observed shape of $f_+ (q^2)$ in the large $q^2$ region. The $1/M$ scaling is also consistent with the pole model prediction.

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