Calibration interval of standard platinum resistance thermometers in a range between 0 °C and 419 °C

Seung-Nam Park1,2,3, Inseok Yang1,2

1Division of Physical Metrology, Korea Research Institute of Standards and Science (KRISS), 267 Gajeong-Ro, Yuseong-Gu, Daejeon, 34113, Korea
2Major on Measurement Science, UST-KRISS Campus, University of Science and Technology (UST), 217 Gajeong-Ro, Yuseong-Gu, Daejeon 34113, Korea

Abstract. The calibration interval of standard platinum resistance thermometers (SPRTs) was studied using the fixed-point calibration data accumulated for eleven years. A model was proposed to predict the long-term drift of the change rate of resistance $R_{TPW}$ at the triple point water and the resistance ratio at the freezing point of Sn ($W_{Sn}$) and Zn ($W_{Zn}$). A Monte Carlo simulation was carried out to present the calibration interval in terms of the measurement reliability of SPRT over elapsed times after a calibration based on the model with various target tolerance limit of the drifts.

1. Introduction

The standard platinum resistance thermometer (SPRT) is the most important to interpolate a wide temperature range between 13.8033 K and 1234.93 K according to the International Temperature Scale of 1990 [1]. Because of its high precision temperature measurement capability with an uncertainty of a few mK or less, SPRTs have been used as reference thermometers in calibration laboratories including national metrology institutes (NMIs).

When calibration certificates of SPRTs are issued at NMIs, the long-term stability has not been included in the uncertainty budget, which is compliant with ISO Guide 17025. The calibration laboratories using the SPRTs as the reference thermometers should include a specific long-term stability of each SPRT or a general value of all SPRTs in the certificates depending on a calibration interval policy of each calibration laboratory. Because considering the long-term stability requires the more technical experiences, the less experienced laboratories prefer to apply a regulated calibration interval policy recommended by their accreditation bodies. For the sake of safety, the general intervals tend to be more frequent, which results in an additional cost of the calibration laboratories. Despite that NSCLI has published a comprehensive document about calibration interval [2], it is not easy to understand all theoretical aspects underpinning the document and apply it to practical calibration tasks. In this paper, we carried out a Monte Carlo simulation to study the calibration interval of SPRTs in terms of the measurement reliability over the elapsed time after a calibration of SPRT based on a long-term drift model. This methodology and its demonstration in this paper are expected useful to the calibration laboratories traceable to the NMIs.

3 To whom any correspondence should be addressed.
2. A long-term drift model of SPRT

The long-term stabilities of SPRTs were intensively studied using the calibration histories accumulated for more than 10 years at KRISS [3] and then at NPL [4]. The stabilities were described in the rate of change of resistance $R_{TPW}$ at the triple point of water (TPW), $r(R_{TPW})$ in mK/year, and the rate of the resistance ratio at the freezing point of Sn ($W_{Sn}$), $r(W_{Sn})$ in mK/year and Zn ($W_{Zn}$), $r(W_{Zn})$ in mK/year. Their findings were expressed by histograms and cumulative functions for the fixed-points and by correlations between the fixed-points. The histograms at the NPL data show much less scatters than that of KRISS. It is expected to originate from any difference of technical knowledge and handling of SPRTs between customers of the two NMIs. Because this paper uses the same data as described in [3], it should be noted that this calibration interval analysis is applicable to only the group of SPRTs calibrated at KRISS.

A long-term drift model of SPRTs is proposed by regarding the sets of data \{\(r(R_{TPW}), r(W_{Sn}), r(W_{Zn})\)\} obtained by regular calibrations at intervals of two years as ‘independent samples’ of the measurement of the rate of change in \(R_{TPW}, W_{Sn}\) and \(W_{Zn}\). This can be justified because this analysis did not find any significant correlations between the successive calibrations of the same SPRT performed more than once. As far as the sign of $r(R_{TPW})$ is concerned, we could consider the sampling as a biased coin toss of a probability of about 7:3 for the positive and negative sign, respectively. Most of the correlations among the signs of $r(R_{TPW})$ for successive $r(R_{TPW})$ data can be explained by a binomial distribution probability of the biased coin.

3. Numerical experiment and Discussion

The long-term drift model enables us to form the cumulative distribution functions from the histograms of \{\(r(R_{TPW}), r(W_{Sn}), r(W_{Zn})\)\}. We conducted a numerical experiment using a Monte Carlo method for taking time-series data of virtual calibrations of 10 000 SPRT samples with a regular interval of a month. We used the uniform random generator included in Visual Basic for Applications (VBA) of Microsoft Excel. The simulation records the elapsed time in month when an accumulated drift exceeds a TTL (target tolerance limit) of temperature drift and then plots the histogram of the counts with bins of elapsed times since calibration. Finally, for various TTLs, the histogram is converted to the measurement reliability that is defined as: the probability that an attribute of an item of equipment conforms to performance specifications [2]. In this simulation, the measurement reliability is given by fractions of the number of instruments remaining within a reliability target of 95 % for various elapsed time. We can have the measurement reliability smooth with a number of SPRTs more than 10 000. The execution of all the programs written in the VBA takes less than 2 minutes.

**Figure 1.** Histogram of samples binned with the rate of change of $R_{TPW}$, $r(R_{TPW})$.

**Figure 2.** Normalized cumulative function for the rate of change of $R_{TPW}$, $r(R_{TPW})$.
numerical solution is required to get a cumulative function. Figure 2 shows the normalized cumulative function for the rate of change of resistance $R_{TPW}$ at the TPW. A value of $r(R_{TPW})$ is obtained from the x-coordinate by assigning a point on the plot of which the y-coordinate is generated by a uniform random number generator of VBA between 0 and 1.

**Figure 3.** Histogram of number of SPRT for a target drift tolerance temperature of 7 mK binned with the elapsed times.

Figure 3 plots the histogram with bins of the elapsed time. The number of SPRT was counted up when the accumulated drift of $R_{TPW}$ exceeds the TTL of 7 mK. Converting the histograms for various TTL yields the measurement reliability functions as shown in Figure 4. These plots resemble Weibull model that is one of theoretical models for explaining the measurement uncertainty growth mechanism [2]. The Weibull model can address either a “burn-in” or a “wear-out mechanism and converges to the exponential model in a certain condition of limit [2]. We have known that it converges to the exponential model as the TTL reduces. The calibration interval for maintaining the reliability of 95 % ranges from 6 months to 29 months for the TTL from 5 mK to 10 mK. However, many calibration laboratories are using the TPW to check any drift of the resistance of SPRT. In this case, these calibration intervals are not limited by the drifts at the TPW. Nevertheless, the information on the calibration interval at the TPW would be useful for the calibration laboratories that does not frequently check the drift.

**Figure 4.** Measurement reliability at the triple of water for various TTLs from 5 mK to 10 mK.

We continued similar simulations with the rates of change of the resistance ratio at the freezing point of tin and zinc. Because each of the rate is related to the resistance of SPRTs at the TPW, both of the rates are closely correlated, of which the correlation originates from an intrinsic property of platinum. In practice, the calibration experiment shows that the correlation coefficient is similar as that derived

**Figure 5.** Histogram of samples binned with the rate of change of $r(W_{Zn})$.

**Figure 6.** Normalized cumulative function for $r(W_{Zn})$. 
from the interpolation equation recommended by the ITS-90. Both simulation results at the freezing point of tin and zinc are similar. Because the calibration interval is constrained by the higher temperature, we presented only a whole set of results for the zinc.

Figure 5 shows a histogram of samples binned with the rate of change of resistance ratio at the freezing point of zinc. It shows a strong asymmetric shape and a significant bias to the negative direction. From Figure 5, we obtain the respective normalized cumulative function for the rate of change of resistance ratio at the freezing point of zinc as shown in Figure 6. Figure 7 plots the histogram with bins of the elapsed time that count the numbers of SPRTs of which accumulated drifts exceed the TTL of 4 mK. Converting the histograms for various TTLs yields the measurement reliability as shown in Figure 8. These plots resemble a warranty model that is described by Fermi-Dirac statistics in the reliability models [2]. The calibration interval for maintaining the target reliability of 95 % is 4 months, 10 months, 16 months, 21 months, 27 months and 33 months for the TTL of 2 mK, 3 mK, 4 mK, 5 mK, 6 mK and 7 mK, respectively.

Figure 7. Histogram of number of SPRT at zinc point for a target tolerance drift limit of 4 mK binned with the elapsed time.

Figure 8. Measurement reliability at zinc point for various TTLs from 2 mK to 7 mK.

4. Conclusion
The calibration interval of SPRTs was studied using the fixed-point calibration data accumulated for eleven years. A model was proposed to predict the long-term drift of the change rate of resistance $R_{TPW}$ at the triple point water and the resistance ratio at the freezing point of Sn ($W_{Sn}$) and Zn ($W_{Zn}$). A numerical experiment was carried out using the Monte Carlo simulation that presents the measurement reliability of SPRT over elapsed times after a calibration. The calibration interval with the reliability of 95 % at the TPW ranges from 6 months to 29 months for the TTL from 5 mK to 10 mK. However, the calibration interval with the same reliability target at the freezing points of zinc is 4 months, 10 months, 16 months, 21 months, 27 months and 33 months for the TTL of 2 mK, 3 mK, 4 mK, 5 mK, 6 mK and 7 mK, respectively. It suggests that the calibration laboratories have to perform or send out for regular calibrations at the TPW with an interval of 6 months and at the freezing point of zinc with an interval of 21 months to maintain the target reliability of 95 % with a target tolerance temperature limit of 5 mK. This study indicates that Weibull model is suitable for explaining the resistance change of SPRT at the TPW. In the other hand, warranty model seems more reasonable for the change of resistance ratio at tin and zinc points. The further study on the most suitable model is expected to help understanding a mechanism of the uncertainty growth and the failure in SPRTs.

REFERENCES
[1] Preston-Thomas H 1990 Metrologia 27 3-10
[2] NCSLI Calibration Interval Committee 2010 Recommended Practice-1: Establishment and Adjustment of Calibration Intervals (Boulder, CO: NCSL International)
[3] Yang I, Song C H, Gam K S and Kim Y-G 2012 Metrologia 49 803-808
[4] Pearce J V, Crabb J, Elliott C J and Rusby R L 2016, Int. J. Thermophys. 37 124-135