Nonlinear Interference Analysis of Probabilistic Shaping vs. 4D Geometrically Shaped Formats

Bin Chen(1), Chigo Okonkwo(2), Alex Alvarado(2)

(1) School of Computer Science and Information Engineering, Hefei University of Technology, China
bin.chen@hfut.edu.cn
(2) Department of Electrical Engineering, Eindhoven University of Technology, The Netherlands

Abstract Performance trade-offs between linear shaping and nonlinear tolerance of the recently introduced 4D orthant-symmetric 128-ary modulation format are investigated. Numerical simulations show 9.3% reach increase with respect to the 7b4D-2A8PSK format and probabilistically-shaped 16QAM with short blocklength.

Introduction In optical transmission systems, the performance of a given modulation format is determined by its tolerance to both linear amplified spontaneous emission (ASE) noise, and nonlinear interference (NLI) arising from the Kerr effect. Designing modulation formats which increase achievable information rates (AIRs) in the presence of linear and nonlinear impairments is crucial to achieve higher capacity and longer reach.

NLI depends on the average power of the transmitted signal. One of the most widely used model for NLI is the so-called Gaussian noise (GN) model. The GN model neglects most of the specific properties of the transmitted signal, including its underlying modulation format. However, in the vast majority of recent demonstrations, it has been shown that NLI also depends on the modulation format(2,3). A model considering modulation format dependency was later developed as an improvement to the GN model, known as enhanced GN (EGN) model(4,5).

Signal shaping has recently been widely investigated in optical fibre communications to improve spectral efficiency (SE), and is currently implemented in commercial products via probabilistic shaping (PS)(6) and geometric shaping (GS)(7). The performance of PS has been examined in theory, simulations, and experiments(8–12). However, PS suffers from rate loss in practical implementations with finite blocklengths(13) and also experiences increased NLI(14,15). Multidimensional (MD) geometric shaping based on constant modulus formats is known to offer NLI tolerance in the nonlinear channel(15,16). In so doing, the multidimensional modulation format “shapes out” the partial NLI at the expense of losing only partial degrees of freedom(17).

In this paper, the recently proposed four-dimensional orthant-symmetric 128-ary (4D-OS128) format(18) are studied that was designed by maximizing generalized mutual information with the benefit of significantly reducing the optimization searching space. We observe a performance trade-offs between linear shaping and nonlinear tolerance, and thus 4D-OS128 outperforms the corresponding nonlinearity-tolerant geometrically-shaped constellation 7b-4D2A8PSK and PS-16QAM with finite blocklength at the same SE.

Orthant-Symmetric MD Geometric Shaping

To solve the multi-parameter optimization challenges of MD geometric shaping and also to reduce the transceiver requirements, we proposed to impose an “orthant symmetry” constraint to the N-dimensional modulation format to be designed(19). Orthant-symmetric labeled constellations are be generated from any first-orthant constellation, where the constellation points are obtained by folding the first-orthant points to the remaining orthants(19).

In this paper, we focus on comparing modulation formats with a spectral efficiency (SE) of m = 7 bit/4D-sym, which consists of M = 2^m N = 4-dimensional points s_i, i ∈ {1,2,...,128} labeled by 7 bits b_i = [b_1,b_2,...,b_7]. For the 4D-OS128 format, each orthant contains 2^{m-N} = 8 constellation points. The 8 constellation points in the first orthant are denoted by t_j = [t_{j1},t_{j2},t_{j3},t_{j4}] ∈ \mathbb{R}_+^4 with j = {1,2,...,8}. The first orthant is labeled X-pol.

![Fig. 1: First orthant of 4D-OS128 modulation formats and associated bit mapping |b_{j5},b_{j6},b_{j7}|, j ∈ {1,2,...,8}.

\[ Q \]
\[ I \]
\[ X-pol \]
\[ Y-pol \]
More nonlinearly shaped

More linearly shaped

More linearly shaped

More nonlinearly shaped

Probabilistic Amplitude Shaping

In addition to the orthant-symmetric MD modulation format, we also consider probabilistic amplitude shaping (PAS) with quadrature amplitude modulation (QAM) to achieve the same SE of 7 bit/4D-sym in 4D-OS128 format. The five amplitude values of the 4D-OS128 format in Fig. 1 optimized for SNR of 9.5 dB are $(a_1, a_2, a_3, a_4, a_5) = (0.2875, 0.3834, 0.4730, 1.1501, 1.2460)$. The nonlinear shaping is in contradiction with Gaussian shaped constellations choosing symbols within the multidimensional balls, which is referred as linear shaping.

Fig. 2 shows the example of linear shaping and nonlinear shaping by comparing the probability distribution of 4D symbol’s energy for three different formats (all normalized to unit energy per polarization): 4D constant-modulus (CM) constellations, 4D-OS128 and PS-16QAM with block length of $n = 64$ and $n = 128$. We can observe that nonlinear shaping is in contradiction with linear shaping, which moves the constellation symbols away from the average energy $E_s$ (also see 2D geometric representation of energy per 4D symbol as inset in Fig. 2). In addition, the 4D constellation symbols’ energy spread out from the average normalized energy $E_s = 2$, which are divided as three groups: low energy symbols, medium energy symbols and high energy symbols. We will show in following section that this 4D energy distribution can induce NLI fluctuation for the nonlinear fibre channel.

Numerical Simulations

Split-step Fourier method simulations with a step size of 0.1 km were performed to compare the modulation formats and predict system performance. The simulation parameters are given in Table 1 for the optical multi-span fibre link under consideration, which comprises multiple standard single-mode fibre spans of 75 km, amplified at the

| Parameter name                  | Value                  |
|---------------------------------|------------------------|
| Symbol rate                     | 41.79 Gbd              |
| Root-raised-cosine roll-off factor | 1%                    |
| Channel frequency spacing       | 50 GHz                 |
| Center wavelength               | 1550 nm                |
| Attenuation                     | 0.21 dB/km             |
| Dispersion parameter            | 16.9 ps nm$^{-1}$km$^{-1}$ |
| Nonlinearity parameter          | 1.31 W$^{-1}$ km$^{-1}$ |

Table 1: Simulation parameters.

NLI, which can be considered as nonlinear shaping. This is in contradiction with Gaussian shaped constellations choosing symbols within the multidimensional balls, which is referred as linear shaping.
In Fig. 4 we show the SNR and AIR of 7b4D-2A8PSK, 4D-OS128 and PS-16QAM with DM blocklengths $n = 32, 64, 128$. We observe that PS with $n = 128$ gives slightly higher AIR, but the resulting increased rate loss diminishes the efficiency of DM as the blocklength decreases. With $n = 32$ and $n = 64$, PS-16QAM has even worse performance than 4D-OS128. These losses of PS-16QAM are shown in Fig. 4(a) and (b), and are particularly visible in the highly nonlinear regime. For the considered optical fiber transmission setup, 4D-OS128 can achieve approximately 9.3% (700 km) of reach increase with respect to 7b4D-2A8PSK and PS-16QAM with blocklength $n = 32$ at the same transmission rate. In Fig. 4(a), we also observe that PS with short blocklengths can also slightly increase the nonlinear tolerance, and thus, the SNR$_{eff}$. The phenomena has been reported in [23] and very recently in [33].

Conclusions

We have studied the performance of various signal shaping schemes in the presence of fibre nonlinearities. All formats have with the same spectral efficiency (7 bit/4D-sym), however, they differ greatly in the distribution of their symbol energies. 4D symbol energy considerations showed that constant-modulus constellations reduce the NLI and that probabilistic shaping exhibits large SNR variations across symbols with different energies. The newly introduced 4D-OS128 format was shown to be able to trade-off linear and nonlinear tolerance giving SNR improvements with respect to PS-16QAM. This is achieved by introducing less 4D symbol energy variations in the transmit sequences that effectively mitigates fiber nonlinearities.

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