Research on the calculation model of shear for prestressed concrete T beams

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Abstract. The contribution of the prestress force on the shear capacity is existed, which is explicit on the vertical component of the prestressed reinforcement in the domestic and foreign standards, but the research on the contribution of horizontal component of prestressed reinforcement is little. Based on the analysis of prestressed concrete T beams, the proposed formula for the shear action of horizontal component of prestressed reinforcement is put forward.

1. Introduction
Concrete T beams used in modern bridge construction was more and more widely, and the research of concrete T beams is more and more deep. Zararis studied the reinforced concrete T beams and think that the shear capacity was the superposition of concrete T beams without shear reinforcement and concrete T beams, at the same time it was found that the shear bearing capacity of T beams was higher than rectangular section beams [1]. Tureyen collected the datas of 154 T beams and 370 rectangular beams and found the calculated results were conservative with different formulas. So he provided a new calculation formula of concrete T beams without stirrups [2]. Job Thomas studied nine prestressed concrete T beams with different concrete strength which reinforced by steel fiber, that showed the steel fiber could improve the shear capacity of prestressed concrete T beams [3]. Zhi Fei of Taiyuan University of Technology studied the shear strength of the T shape section beams based on the experiments of eight prestressed high strength beams and one ordinary beam with web reinforcement, and the influence and variation of concrete strength, ratio of shear span to effective depth, prestressing degree (pre compression ratio), stirrup ratio and longitudinal reinforcement ratio on the shear strength were comprehensively analysed, and a proposed formula of shear bearing capacity on the simply supported beam(T or I cross section)under concentrated load was put forward [4]. Although the system research of shear properties on prestressed concrete beams had been systematically studied in the domestic, the rectangular cross section was studied as the object in the domestic shear experiment. The research on shear bearing force of prestressed concrete T beams is rarely studied at home and abroad, and the research on the shear contribution of prestressed horizontal component in the prestressed concrete T beams is not. In lots of books related to concrete structure design, the axial pressure mainly come from prestressing force in prestressed concrete members when the shear problem discussed in concrete beams or flexural members, but the beneficial effect of the axial pressure on the shear is limited, and the upper limit value is specified in the current specification [5].

Calculation model of prestressed concrete beams was developed from early truss model, theory of plasticity to the MCFT, finally a clear distinction between ventral shear cracks and bending shear cracks of prestressed concrete beams was finished by American Illinois University, and the sum of the
shear was considered that was the total of the shear resisted to cracks and undertaken by the stirrups. The correction of classical truss model in design specification was that the shear undertaken by concrete in the calculation formula for shear bearing capacity was added, and the shear of concrete in abdominal muscle beams was assumed which was equaled to the shear bearing capacity of beams without abdominal muscle. At present in our country such a theory is used in “Code for design of concrete structures” and “Code for design of highway reinforced concrete and prestressed concrete bridge and culverts”, which is compiled in combination with some experimental datas.

Complexity of shear problem comes from the interaction of many factors. As tensile strength of concrete is low, the beams will be crazed in low tensile stress, then the cracks is developed along the tilt direction by the influence of the main tensile stress. Due to the the existence of cracks, the interaction between steel and concrete with the fracture surface is very complex. The complex concrete strength is also led by the anisotropy of concrete material and the state of multiaxial stress. In addition, the shear is rarely separately existed in structures, the shear performance of the component will be changed when the effect of bending moment and axial force, torque and the shear are worked together. Because of more test variables and complex of making the specimens, the shear experiment study on prestressed concrete beams is difficult to complete. In shear design of the national standards, that the shear failure is occurred when the shear force produced by the external load exceeded the shear bearing capacity of the critical section is assumed, the main content of the shear design is that the part of the shear is undertaken by concrete and transverse reinforcement and the corresponding shear reinforcement consumption. The truss model is mostly adopted in the design specification, the reinforced concrete beams are analogous to a truss after diagonal crack appeared. The equation is simple as the model started from the equilibrium conditions, but there are big difference with the test result [6].

Although a improvment coefficient of prestress is given for prestressed concrete flexural members in calculation formulas of the domestic bridge specification, that is provided based on the empirical datas. Once the research on shear capacity of high strength reinforced concrete rectangular beams in Hunan University was carried out, but the size of the contribution of prestress to shear capacity of T beams and the influence factors will be worth to study.

2. Analysis for calculation model of prestressed concrete T beams

Not only the truss role, but also the arch effect is exsited in reinforced concrete members, the shear of components with web reinforcement is expressed by the superposition of the two roles. As both roles is considered in truss arch model at the same time, the model is clear and reasonable. The curve compressive bar of concrete is considered that it’s role is not only the string compressive bar but also the arch, it can be balance to the load bending moment with longitudinal steel, and can be directly transfered the oblique compressive stress to the pedestal. The stirrups is treated as the vertical rod, and the concrete between the stirrups as oblique abdominal rod, the longitudinal tensile reinforcements as tensile bottom chord, then the concrete is divided into five categories: the zero stress on the abdominal area, the common workspace of the vertical stirrups and compression concrete under the arch, one-way horizontal compressive area of arch concrete, the area of unidirectional compression curve and concentrated loading area, and the boundary conditions of beam bottom and the equilibrium conditions of micro section were used, the shear is considered to be the ultimate shear capacity when the compressive of concrete achieved the axial compressive. But the calculation formula of the model is more complex, which is not very convenient to apply [7].

Because the pre-tension is imposed beforehand in prestressed concrete structures, there are different characteristics in the stress process with the ordinary reinforced concrete structures. The occurrence and development of diagonal cracks are blocked, and the height of shear zone is increased in concrete, the shear capacity of concrete is improved. The length of the inclined cracks of prestressed concrete beams will be increased and the shear effect of stirrups will be improved too.

2.1. The stress state after diagonal cracks appeared
The failure pattern of abdominal muscle beams is similar to the beams without ventral reinforcement, that is influenced by the ratio of shear span to effective depth and the numbers of stirrups \[8\]. Fig. 1 is the stress state of prestressed concrete T beam under the concentrated load.

![Isolated body figure of prestressed concrete T beam](image)

**Fig 1.** Isolated body figure of prestressed concrete T beam

By the equilibrium condition (the friction force of oblique fracture plane and the bite force of aggregates and the pin bolt force of longitudinal reinforcement are neglected) that is:

\[\begin{align*}
\sum x &= 0 \quad D_x = T + \alpha f_p A_p \cos \theta \\
\sum y &= 0 \quad V = V_c + V_s + \beta f_p A_p \sin \theta \\
\sum M &= 0 \quad V \times x = T_x + V_s + \alpha f_p A_p \cos \theta (h - v) + \beta f_p A_p \sin \theta (c - t \tan \theta)
\end{align*}\]

The hypothesis is that the prestressed cable is perpendicular to the oblique fracture plane. Where \(\alpha\) is the horizontal direction coefficient of prestressed component; \(\beta\) is the vertical direction coefficient of prestressed component; \(h\) is the depth of beam; \(y\) is the ordinate value of prestress in the inclined cracks, \(V_c\) is the shear of stirrups.

By the above formulas, \(\alpha f_p A_p \cos \theta\) is the value that the compression \(D_x\) of shear pressure surface on the concrete of inclined section is larger than the concrete T beams without prestress in the horizontal direction because of the existence of prestressed reinforcement, that is increase in the axial pressure. And in the vertical direction, the value \(\beta f_p A_p \sin \theta\) of shear capacity is increased, \(\beta\) is 0.75 in" Specification for design of reinforced concrete and prestressed concrete highway bridges and culverts", but it is 0.8 in" Code for design of concrete structures".

The internal stress state will be changed after the prestress is imposed on beams, and the angle between the inclined cracks and the axis of beams is reduced. and the height of shear compression area is increased, so the diagonal crack can be delayed to carry out, and aggregate bite of diagonal cracks is enhanced, thus the shear of concrete is increased \[9\]. The \(V_c\) of the formula is the shear of shear-compression area, rather than the contribution of tensile stress of concrete in shear area. The shear will be influenced by the axial force when the beam is subjected to bending and shear action. The vertical cracks and inclined cracks of concrete T beams are delayed to carry out for the bending shear components beared axial compression because of the existence of compressive stress. When the prestress exists in the prestressed concrete T beams the axial pressure is increased, and the height of shear-compression area is increased, the shear of shear-compression area in concrete T beams is indirectly improved.

2.2. The theoretical analysis model

The bottom of prestressed concrete T beams starts cracking with the increase of vertical loads, and the neutral axis of cross section is moved toward the concrete compression area, at the same time the development of cracks is limited due to the constraint of stirrups, so the shear of the concrete is increased. Due to the constraint of the support the compression diaphragm pressure will be generated in the concrete beams, and the arch effect will be formed \[10\]. Because of the existence of eccentric compression \(N_p\), the upward anti arch is produced, so the arch effect is more obvious. The constraint effect of compressive membrane action of prestressed concrete T beams mainly comes from two aspects, one is from the bearing friction, on the other hand is from the axial stiffness provided by the
transverse diaphragm. Therefore, the arch effect of prestressed concrete T beams after cracking can be considered as the arch model that the arch thickness is x and the arch springing is supporting point, as shown in Fig.2. Due to the existence of the arch, the part of the vertical shear is converted to the arch axis pressure and continued to pass, that the compressive performance of concrete can be made full use to improve the shear of beams [11].

![Fig 2. Work mechanism of the concrete arch](image)

The concept of the stress path was put forward in the 1980s according to Greek scholar Kotsovos, that is the path from the pressure point of compression area in mid span section to supporting point of bearing [12]. From the experimental results of the Kotsovos, the shear capacity of T beams was significantly higher than the rectangular beams in other conditions, from another angle showed that the shear capacity of beams was decided by the tensile stress of the stress path. That the pressure of compressive area in T beams is more advantageous to transfer than the rectangular section beams with same conditions. The experiment results also showes that shear bearing capacity of T beams is higher than the rectangular section beams with same conditions.

As shown in Fig.3, \( x = \xi l \), then \( dx = ld\xi \), \( \tan \alpha = dy/dx = dy/(ld\xi) \). Near the arch springing, \( \tan \alpha = h/l \approx 1/m \), where m is the ratio of shear span to effective depth, while near the vault, \( \tan \alpha = 0 \).

The film unit of fracture plane is analyzed as shown in Fig.4. When \( \alpha \) changed from 90ºto 0, that is the film unit is slowly transferred from the arch springing to the vault in Fig.3, the axial pressure \( \sigma_d \) is increased gradually, and the tensile stress \( \sigma_r \) reduced. When the axial pressure is increased, the tensile stress of cracks is decreased, so the initiation and propagation of cracks are delayed. That is agreement with the concrete arch model in Fig.2, and in line with the stress path theory. The shear failures of beams are related to the stress which is perpendicular to the the pull pressure path, when \( \alpha \) changed from 0 to 90º in the shear-compression failure of typical failure modes, the stress is decreased gradually, the cracks are developed from the edge to the top of beam, the compression height will be left from the top of the component.

![Fig 3. Computational scheme of arch](image)  
![Fig 4. D-r coordinate system of cracking concrete](image)

3. The recommended formula of the shear capacity considering the horizontal component of prestressed reinforcement

The shear capacity of prestressed concrete T beams after cracked is composed of the following sections: the shear of shear-compression area without cracks, the shear transfered by fracture surface,
the bolt pin function of longitudinal reinforcement and the vertical force of stirrups, etc. As the interlocking effect of aggregates almost lost, Bazant\cite{13} points out that the contribution of shear force transferred by crack surface is very small. However, the bolt pin force of tensile reinforcement in cracks is existed. Taylor thinks that the shear of compressive concrete is approximately determined through measuring the strain of compressive concrete, at the same time the bolt pin force of longitudinal reinforcement and the bite of aggregates are estimated by measuring the width of diagonal cracks and the rupture along the inclined cracks. But the test method of bolt pin role is complex\cite{14}. In the national standards, the design of prestressed concrete T beams is tend to be safe without considering the bolt pin function of longitudinal reinforcement.

Recently the research put forward that the shear of reinforced concrete beams is mainly passed by the concrete of shear-compression area\cite{15}. When the ratio of shear span to effective depth is small, and the effect of arch is more obviously, the height of shear-compression area of inclined section is more higher, then the shear capacity of concrete is more bigger too. With the increase of load, the stress and strain of prestressed reinforcement are also increased, under the condition of critical damage the relative height of shear-compression area is:

\[e_a = \frac{h_0}{h_{0b}} = 0.8, \quad e_s = \frac{0.8e_a}{1 + \frac{f_{ps} - f_{ps0}}{E_s}} = \frac{0.8}{1 + \frac{f_{ps} - f_{ps0}}{0.0033E_s}}\]  \tag{4}

Under the influence of the prestressed reinforcement of the horizontal component the stress of concrete in compressive area is becoming more and more bigger with the cracks continue to expand, that can restrain the continuing extension of cracks, the formula for calculating the shear bearing capacity of inclined section in flexural members of T cross section in" Code for design of concrete structure" is:

\[V \leq V_{cs} + V_{fs}A_{sk}sina + 0.8V_{fs}A_{sk}sina_0\]  \tag{5}

However, the contribution of the vertical component of prestressed reinforcement on artifacts is only considered in the domestic concrete specification, the contribution of the horizontal component of prestressed reinforcement is not taken into account. Therefore, we select a segment of compressive concrete section for analysis:

![Stress diagram of compressive concrete](image)

Fig 5. Stress diagram of compressive concrete

According to the calculation method of static friction force: \(f = \mu N_{p} \cos \alpha_0\), where the vertical shear is \(2f\), \(\mu\) is the factor of maximum static friction, it is related to the bite between aggregates in concrete, and be related to cohesive force between aggregates and stirrups, so the suggestion formula for the shear which considering the horizontal component of the prestressed reinforcement is:

\[V_{ph} = 2f = 2k_1k_2N_{p} \cos \alpha_0\]  \tag{6}

where \(V_{ph}\) is the shear contribution of the horizontal component of prestressed reinforcement; \(k_1\) is the coefficient of bite between aggregates in concrete; \(k_2\) is the coefficient of cohesive force between aggregates and stirrups.

After the suggestion formula of the shear which considering the horizontal component of the prestressed reinforcement is established, the value and the reliability and accuracy of the formula must be verified with experiment, then the experiment is magined based on large amount of literatures.

The shear of the horizontal component of prestressed reinforcement is considered, it is necessary to contrast the tests of two kinds of components whether or not having prestressed reinforcement, the second is the different bend-up angle of the prestressed steel. So the above two factors is considered,
the contribution of prestressed reinforcement to shear in concrete T beams will be eventually calculated and analyzed.

4. Conclusions
Through the analysis of the theoretical model of prestressed concrete T beams, the effect of prestress on shear for T beams is not limited to vertical prestress or the vertical component of curve prestressed reinforcement, and the shear function of level prestress or the horizontal component of prestressed reinforcement can not be ignored. The shear recommended formula on the contribution of the horizontal component of the prestressed reinforcement is put forward, and the design of the test components for determining the value of the contribution is imagined. Finally through using the research results the formula of the shear in the specification will be improved and corrected.

References
[1] Ioannis P.Zararis, Maria K.Karavezirogolou, and Prodromos D.Zararis, “Shear Strength of Reinforced Concrete T-Beams”, ACI Structural Journal, 103-S71,2006.
[2] A.Koray Tureyen, Tyler S.Wolf, and Robert J.Frosch, “Shear Strength of Reinforced Concrete T-Beams without Transverse Reinforcement”, ACI Structural Journal, 103-S67,2012.
[3] Job Thomas, Ananth Ramaswamy, “Shear Strength of Prestressed Concrete T-Beams with Steel Fibers Over Partial/Full Depth”, ACI Structural Journal, 103-S45,2007.
[4] Zhi Fei, Ye Zhi-man, “Experimental investigation on shear capacity of simply supported and pre-stressed high strength concrete beam(T-section) with web reinforcement”, Industrial Construction, vol.37, pp.321-324, 2007.
[5] Yi Wei-jian, Research on Concrete Structure Experiment and Theoretical, Beijing: Science Press, 2012.
[6] Lv Yan-mei, “Theoretical and Experimental Research on Shear Capacity of High Strength Concrete Beams with High Strength Stirrups”, Ph. D. thesis, Hunan University, CHINA, April, 2009.
[7] Pan Bai-rong, “Research on Shear Capacity of High Strength Concrete Beams”, Master's thesis, Hunan University, CHINA, May, 2009.
[8] Ye Jian-shu, Principles of Structure Design, Beijing: China Communications Press, 2005.
[9] Wang You-zhi, Xue Yun-hu, Zhang Qi-hai, Gao xi-qun. Prestressed concrete structures, Beijing: China WaterPower Press, 2001.
[10] Zheng Yu, Su E. Taylor, Des. Robinson,”Prediction methods of loading carrying capacities in concrete bridge deck slabs with consideration of compressive membrane action”, Journal of Harbin institute of technology university, vol. 42, pp. 644-651, April 2010.
[11] Zhou Xian-yun, Wang Jie-jun, Bridge Engineering, Beijing: Peking University Press, 2012.
[12] Kotsovos M D,”Mechanisms of “shear” failure”, Magazine of Concrete Research, vol. 123(5), pp. 99-106, 1983.
[13] Bazant Z P, ”Fracturing truss model: size effect in shear failure of reinforced concrete”, Journal of Engineering Mechanics, vol. 123(12), pp.: 1276-1288, 1997.
[14] Taylor H P J, ”The Fundamental Behavior of Shear Resistance of Concrete Beams in Bending and Shear”, ACI Structural Journal, 43-77, 1974.
[15] Choi K, Park H, Wight J K, ”Unified shear strength model for reinforced concrete beams-Part 1: development”, ACI Structural Journal, vol. 104(2), pp. 142-152, 2007.