Anisotropy in Born-Infeld brane cosmology

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The accelerated expansion of the universe together with its present day isotropy has posed an interesting challenge to the numerous model theories presented over the years to describe them. In this paper, we address the above questions in the context of a brane-world model where the universe is filled with a Born-Infeld matter. We show that in such a model, the universe evolves from a highly anisotropic state to its present isotropic form which has entered an accelerated expanding phase.

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I. INTRODUCTION

The problem of the diverging Coulomb field and self-energy of point particles in Maxwell’s theory of electrodynamics led Born and Infeld (BI) in 1934 to the construction of a nonlinear extension of classical electrodynamics [1]. In recent years, the study and use of nonlinear electrodynamics has been the focus of much attention in the context of cosmology. Such strong motivations stem from developments in string/M-theory, where it has been shown that the BI theory naturally arises in the low energy limit of the open string theory [2]. Nonlinear electrodynamics has also been applied to several branches of physics, namely, the effective theories at different levels of string/M-theory [3], cosmological models [4, 5], black holes [6, 7] and wormhole physics [8], among others.

The ubiquitous Friedmann-Robertson-Walker (FRW) model has long been considered as a standard against which other model theories are compared. In this model the universe is regarded as homogeneous and isotropic with only one dynamical feature, its rate of expansion or contraction. Its high degree of symmetry makes it easy to analyse but also makes it somewhat unrealistic. However, it does model the average properties of the observed universe, on the largest scales, quite well. The Bianchi models, on the other hand, pose a more realistic scenario. These models represent a homogeneous but anisotropic universe which are essentially indistinguishable from FRW models far from the initial singularity or, in the case of a collapsing universe, far from the future singularity. The present epoch is far from all such singularities. Bianchi models have three dynamical parameters, the expansion or contraction in the three spatial directions which are, in general, all different. The three rates of expansion or contraction asymptotically (in time) equalize, thus producing a model which looks like that of a FRW. If the universe did not begin by expanding exactly isotropically, then the Bianchi models would describe its early behavior more accurately than the FRW model.

The idea that our four-dimensional (4D) universe is a hypersurface (brane) in a 5D spacetime (bulk) [9–11] has been the dominant idea behind many model theories attempting to explain the observed universe over the past decade. One of the most successful of such higher dimensional models is that proposed by Randall and Sundrum where the bulk has the geometry of an AdS space admitting $Z_2$ symmetry [10]. They were successful in explaining what is known as the hierarchy problem; the enormous disparity between the strength of the fundamental forces. The Randall-Sundrum (RS) scenario has had a great impact on our understanding of the universe and has brought higher dimensional gravitational theories to the fore. In certain RS type models, all matter and gauge interactions reside on the brane while gravity can propagate into the bulk. In obtaining the field equations on the brane one uses the Israel junction conditions [12] and the Gauss-Codazzi equations, as employed by Shiromizu, Maeda and Sasaki (SMS) [13]. These field equations differ from the standard Einstein field equations in 4D in that they have additional terms like $\pi_{\mu\nu}$ which depend on the energy-momentum tensor on the brane and the electric part of the Weyl tensor $\mathcal{E}_{\mu\nu}$, leading to the appearance of a quadratic term in the Friedmann equation. This term was initially considered as a possible solution to the accelerated expansion of the universe. However, soon it was realized to be incompatible with the big bang nucleosynthesis, requiring additional fixes [14].

Since their introduction, the RS type models have been employed frequently in conjunction with a myriad of matter fields to study the behavior of the observed universe. What has been lacking however is such a study with nonlinear electrodynamics of the BI as the matter field. However, as we will show in the Appendix, the BI matter cannot be used in a brane-world scenario together with Israel junction conditions. The reason is that the solutions of the resulting field equations predict a universe which is incompatible with present day observations. In addition, there have been arguments on the uniqueness of the junction conditions [15] or their use when more than one non-compact extra dimension is involved. In view of the above, one is lead to consider an alternative brane-world scenario in which the Israel junction conditions become redundant. Brane-world scenarios under more general...
conditions and still compatible with the brane-world program have therefore been rather extensively studied over the recent past where it has been shown that it is possible to find a set of cosmological solutions in accordance with the current observations. Under these conditions, without using Z_{2} symmetry or postulating any junction conditions, the Friedmann equation is modified by a geometrical term which is defined in terms of the extrinsic curvature, leading to a geometrical interpretation of dark energy [16]. An example of such a scenario was presented in [17] where particles are trapped on a 4D hypersurface by the action of a confining potential. The dynamics of test particles confined to a brane by the action of such a potential at the classical and quantum levels were studied in [18]. In [19] the authors showed that the quadratic term in the energy-density can be produced in this model, suggesting that the model is identical to the SMS method at least in the isotropic universe. In this paper, we consider an anisotropic brane-world with Bianchi type I geometry filled with a nonlinear electromagnetic field of the BI type. The electromagnetic tensor has six independent components of which only two survive to construct the BI energy-momentum tensor. The anisotropy parameter and the deceleration parameter are calculated and shown to be compatible with present observations, namely that the universe is in an accelerating phase and expanding isotropically.

II. THE SETUP

Let us start by defining the Bianchi type I universe through the metric

\[ ds^{2} = -dt \otimes dt + \sum_{i=1}^{3} a_{i}(t)^{2} dx^{i} \otimes dx^{i}, \]

where \(a_{i}\) are the scale factors. We also have the usual definition of the volume scale factor \(v\), the directional Hubble parameters \(H_{i}\) and the mean Hubble parameter \(H\), defined as

\[ v = \prod_{i=1}^{3} a_{i}, \quad H_{i} = \frac{\dot{a}_{i}}{a_{i}}, \quad 3H = \sum_{i=1}^{3} H_{i}, \]

\[ \Delta H_{i} = H_{i} - H. \quad i = 1, 2, 3. \]

The physical observables are the anisotropy parameter \(A\) and the deceleration parameter \(q\) given by

\[ 3A = \sum_{i=1}^{3} \left( \frac{\Delta H_{i}}{H} \right)^{2}, \quad q = \frac{d}{dt} H^{-1} - 1. \]

In view of the discussion above, we focus attention on a brane world scenario [18] in which the use of \(Z_{2}\) symmetry and Israel junction conditions are relaxed in favor of a confining potential \(\mathcal{V}\) whose role is to localize the gauge fields of the standard model on the brane. In this scenario a new conserved quantity, \(Q_{\mu\nu}\), appears which is totally geometric in nature and depends on the extrinsic curvature only, reflecting the effects of the extra dimension. In the SMS method one replaces this tensor with a tensor build up by the energy-momentum tensor using the Israel junction conditions. We will compute this tensor using the Codazzi equation. The field equation of the model is given by [20]

\[ G_{\mu\nu} = \alpha \tau_{\mu\nu} - \Lambda g_{\mu\nu} + Q_{\mu\nu} + E_{\mu\nu}, \]

where \(\alpha\) is the energy scale of the brane, \(\Lambda\) is the cosmological constant, \(E_{\mu\nu}\) is the electric part of the Weyl tensor and \(Q_{\mu\nu}\) is a conserved quantity, \(\nabla_{\mu} Q_{\mu\nu} = 0\), expressed in terms of the extrinsic curvature \(K_{\mu\nu}\) and its trace

\[ Q_{\mu\nu} = KK_{\mu\nu} - K_{\mu\alpha}K_{\nu}^{\alpha} + \frac{1}{2} (K_{\alpha\beta}K^{\alpha\beta} - K^{2}) g_{\mu\nu}. \]

In order to calculate \(Q_{\mu\nu}\) we first use the York’s relation

\[ K_{\mu\nu} = -\frac{1}{2} \frac{\partial g_{\mu\nu}}{\partial \xi}, \]

where \(\xi\) is a vector normal to the brane. This tells us that the extrinsic curvature is diagonal and does not depend on the spatial coordinates. After separating the spatial components, the Codazzi equation relates the diagonal elements of the extrinsic curvature tensor [20]

\[ K_{i,j,k}^{i} = K_{k,j}^{i}, \quad i, j, k = 1, 2, 3, \]

\[ K_{i,0}^{i} + \frac{\dot{a}_{i}}{a_{i}} K_{i}^{0} = \frac{\dot{a}_{i}}{a_{i}} K_{0}^{0}. \]

The first equation simply emphasizes the result that the spatial components of the extrinsic curvature do not depend on the spatial coordinates. The choice of the ansatz \(K_{00} = d(t)\) for the temporal component of the extrinsic curvature and use of the second equation in (7) leads one to the other components of the extrinsic curvature

\[ K_{ii} = (c_{i} - e_{i} a_{i}) a_{i}, \]

where

\[ e_{i}(t) = \frac{1}{a_{i}(t)} \int d(t) \dot{a}_{i}(t) dt, \]

with \(c_{i}\)'s being constants. Using equation (5) we can obtain the components of the tensor \(Q_{\mu\nu}\)

\[ Q_{0}^{0} = -(e_{1} e_{2} + e_{2} e_{3} + e_{3} e_{1}) \]

\[ + \left( \frac{c_{1}}{a_{1}} (e_{2} + e_{3}) + \frac{c_{2}}{a_{2}} (e_{1} + e_{3}) + \frac{c_{3}}{a_{3}} (e_{2} + e_{3}) \right) \]

\[ - \left( \frac{c_{1} c_{2}}{a_{1} a_{2}} + \frac{c_{2} c_{3}}{a_{2} a_{3}} + \frac{c_{3} c_{1}}{a_{3} a_{1}} \right), \]

\[ Q_{1}^{1} = -e_{2} e_{3} - d(e_{2} + e_{3}) + \left( \frac{e_{2} c_{3}}{a_{3}} + e_{3} c_{2} a_{2} \right) \]

\[ + d \left( \frac{c_{2}}{a_{2}} + \frac{c_{3}}{a_{3}} \right) - \frac{c_{2} c_{3}}{a_{2} a_{3}}. \]
The components $Q^2_2$ and $Q^3_3$ are obtained from (11) by permutation of indices. Let us now concentrate on the case where the temporal component of the extrinsic curvature is constant, taking it as $d(t) = 1$. Such a choice can be understood in terms of the definition of $K_{\mu\nu}$ and observation that $g_{00} = -1$. In this case the components of $Q_{\mu\nu}^a$ take the simpler form

$$Q^0_0 = -3 + 2 \left( \frac{c_1}{a_1} + \frac{c_2}{a_2} + \frac{c_3}{a_3} \right) - \left( \frac{c_1 c_2}{a_1 a_2} + \frac{c_2 c_3}{a_2 a_3} + \frac{c_3 c_1}{a_3 a_1} \right),$$

$$Q^1_1 = -3 + 2 \left( \frac{c_2}{a_2} + \frac{c_3}{a_3} \right) - \frac{c_2 c_3}{a_2 a_3},$$

and again $Q^2_2$ and $Q^3_3$ are obtained by permuting the indices. If we assume that our 5D bulk has a constant curvature, $E_{\mu\nu}$ vanishes and the field equation becomes

$$G_{\mu\nu} = \alpha \tau_{\mu\nu} - \Lambda g_{\mu\nu} + Q_{\mu\nu}. \quad (13)$$

Let us now assume that the 4D universe is filled with the BI field and is described by the following action

$$S_F = \int d^4 x L(F), \quad (14)$$

where

$$L(F) = \sqrt{-g} L(F) = \frac{\beta^2}{4\pi} \left\{ \sqrt{-g} - \sqrt{\det (g_{\mu\nu} + \beta^{-1} F_{\mu\nu})} \right\}. \quad (15)$$

Here, $\beta$ is a coupling constant with the dimension of the field. In four space-time dimensions equation (15) can be expanded to give [21]

$$L(F) = \frac{\beta^2}{4\pi} (1 - \mathcal{R}), \quad (16)$$

where

$$\mathcal{R} = \sqrt{1 + \frac{1}{2\beta^2} F^{\rho\sigma} F_{\rho\sigma} - \frac{1}{16\beta^4} \left( \tilde{F}^{\rho\sigma} F_{\rho\sigma} \right)^2}. \quad (17)$$

In this equation, $F_{\mu\nu} = A_{\mu\nu} - A_{\nu\mu}$ is the usual electromagnetic tensor with 4-potential $A^\mu$ and $\tilde{F}^{\mu\nu} = \frac{i}{2} \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$. This action reduces to that of the Maxwell in the limit $\beta \to \infty$. The field equation for the BI field is obtained from the above Lagrangian

$$\nabla_\mu P^{\mu\nu} = 0, \quad (18)$$

where we have introduced the second rank tensor $P^{\mu\nu}$

$$P^{\mu\nu} = -\frac{1}{2} \frac{\partial L(F)}{\partial F_{\mu\nu}} = \frac{F^{\mu\nu} - \frac{1}{4\beta^2} \left( \tilde{F}^{\rho\sigma} F_{\rho\sigma} \right) \tilde{F}^{\mu\nu}}{\mathcal{R}}. \quad (19)$$

We note that in the weak field limit $P^{\mu\nu} \approx F^{\mu\nu}$. The energy-momentum tensor can be written as

$$\tau_{\mu\nu} = \frac{1}{4\pi} \left[ \frac{F_\mu^\lambda F_\nu^\lambda + \beta^2 \left( \mathcal{R} - 1 - \frac{1}{2\beta^2} F^{\rho\sigma} F_{\rho\sigma} \right) g_{\mu\nu}}{\mathcal{R}} \right]. \quad (20)$$

In general, the anti-symmetric tensor $F_{\mu\nu}$ has six independent components. However, since the Einstein and $Q_{\mu\nu}$ tensors are diagonal, the six off-diagonal components of (13) result in six algebraic equations with solutions where only two non-zero components, $F^{\mu x}$ and $F^{\nu y}$, survive. Using the field equation for the BI matter (18) and conservation equation for the BI energy-momentum tensor, $\nabla^\mu \tau_{\mu\nu} = 0$, one can find the two remaining components

$$F^{\mu x} = \frac{\beta f_1}{\sqrt{16\beta^2 v^2 + a^2}}, \quad F^{\nu y} = -\frac{f_2}{4} \left( \frac{a_1}{v} \right)^2, \quad (21)$$

where $f_1$ are constants and

$$f_1^2 + f_2^2 = 1. \quad (22)$$

This relation ensures that the conservation equation and the field equation of the BI matter are satisfied and eliminates the constants from the energy-momentum tensor which can be written using equation (20) as

$$\tau^0_0 = \tau^1_1 = \frac{\beta^2}{4\pi} (1 - \Delta), \quad \tau^2_2 = \tau^3_3 = \frac{\beta^2}{4\pi} \left( 1 - \frac{1}{\Delta} \right), \quad (23)$$

where

$$\Delta = \sqrt{1 + \frac{1}{16\beta^2} \left( \frac{a_1}{v} \right)^2}. \quad (24)$$

The energy-momentum tensor of the BI field can be written in the form of a perfect fluid $\tau^\mu_\nu = \text{diag}(\rho, p, p, p)$ with the equation of state of the form $p^\parallel = -\rho$. Another equation of state relating the transverse pressure can be obtained from (23)

$$p^\perp = \frac{\rho}{1 - \frac{4\pi\rho}{3}}. \quad (25)$$

In the limit $\beta \to \infty$ one can see from equation (25) that the energy-momentum tensor of the BI field takes the form $\text{diag}(-\rho, -\rho, p, p)$. As can be expected, this is the energy-momentum of the Maxwell’s field with

$$\rho_{\text{Maxwell}} \propto \left( \frac{a_1}{v} \right)^2, \quad (26)$$

where $v$ is defined in (2). Equations (12), (23) and the Einstein tensor for the Bianchi I metric suggest that the
scale factors in the \((yy)\) and \((zz)\) directions are proportional. Solving the \((yy)\) and \((zz)\) components of the field equation \((13)\) one obtains

\[
\frac{a_3(t)}{c_3} = \frac{a_2(t)}{c_2},
\]  

(27)

This means that the second and third components of the Einstein equation are identical. Equation \((27)\) also implies that the constants \(c_2\) and \(c_3\) must have the same sign. It only remains to solve the field equations \((13)\) which, using equation \((2)\) can be written as

\[
H_2^2 + 2H_1H_2 - \Lambda - 3 + 2\frac{c_1}{a_1} + 4\frac{c_2}{a_2} - 2\frac{c_1}{a_1}c_2 - \left(\frac{c_2}{a_2}\right)^2 = \frac{\alpha\beta^2}{4\pi}(\Delta - 1),
\]  

(28)

\[
3H_2^2 + 2H_2 - \Lambda - 3 + 4\frac{c_2}{a_2} = \frac{\alpha\beta^2}{4\pi}(\Delta - 1),
\]  

(29)

\[
H_1^2 + H_2 + H_1^2 + H_2 = \Lambda - 3 + 2\frac{c_1}{a_1} + 2\frac{c_2}{a_2} - \frac{c_1}{a_1}c_2 = \frac{\alpha\beta^2}{4\pi}\left(\frac{1}{\Delta} - 1\right),
\]  

(30)

where \(\Delta\) takes the form

\[
\Delta = \sqrt{1 + \left(\frac{c_2}{4\beta c_3 a_2^2}\right)^2}.
\]  

(31)

Equation \((29)\) can be integrated to find the scale factor \(a_2\) implicitly

\[
t = \int \frac{da_2}{\sqrt{(1 + \frac{4}{\beta}) a_2^2 - \left(\frac{\alpha\beta^2}{12\pi} + 2c_2\right) a_2 + c_2^2 + \frac{c_4}{a_2} - \frac{\alpha\beta^2}{4\pi} \frac{1}{a_2} \int a_2 \sqrt{f^4 + \left(\frac{c_5}{a_2}\right)^2} df}}.
\]  

(32)

One can also find a relation between the scale factors \(a_1(t)\) and \(a_2(t)\)

\[
a_1 = a_2\left(c_1\int \frac{c_2 - a_2 dt}{a_2^2} - c_3\right),
\]  

(33)

where \(c_5\) is an integration constant.

### III. LATE TIME BEHAVIOR

An explicit form for the solution of the above equations seems to be unavailable. It is therefore desirable to analyze the asymptotic behavior of the observable quantities at late times. Assuming \(\beta < \infty\), the behavior of the energy-density and pressure of the BI field is found to be

\[
\rho = \frac{c_2^2}{128\pi c_3^2 a_2^4} - \frac{c_4^2}{8192\pi c_3^2 a_2^8} + \mathcal{O}\left(\frac{1}{a_2^2}\right),
\]

\[
p^+ = \frac{c_2^2}{128\pi c_3^2 a_2^4} - \frac{3c_4^2}{8192\pi c_3^2 a_2^8} + \mathcal{O}\left(\frac{1}{a_2^2}\right).
\]  

(34)

Equations \((34)\) are obtained by writing \(H_2\) in terms of \(a_2\) from equation \((32)\) followed by writing \(H_1\) in terms of \(a_2\) using equation \((33)\). These equations show that at late times the BI field differ from that of the Maxwell in the second terms which is of order \(a_2^{-8}\). Consequently, the BI field very rapidly turns into the Maxwell field unless the constant \(\frac{c_4}{a_2}\) is very large. One can also see that the anisotropy and deceleration parameters behave according to

\[
A = \frac{A_2}{a_2^2} + \mathcal{O}\left(\frac{1}{a_2^2}\right),
\]  

(35)

\[
q = -1 + \frac{q_1}{a_2} + \frac{q_2}{a_2^2} + \mathcal{O}\left(\frac{1}{a_2^2}\right),
\]  

(36)

where we have defined

\[
A_2 = \frac{6}{c_5^2}\left(c_1 - \frac{1}{3} c_2 c_5 \sqrt{9 + 3\Lambda}\right)^2
\]

\[
q_1 = \frac{2c_2 c_5 (\Lambda + 3) - c_4 \sqrt{3\Lambda + 9}}{c_5 (\Lambda + 3)^3},
\]

\[
q_2 = \frac{17\sqrt{9 - 3c_1 c_2 c_5 \Lambda + c_2^2 c_5 \Lambda (\Lambda + 3)(\Lambda - 2) + 12c_2^2}}{c_5^2 (\Lambda + 3)^3}.
\]  

(37)
The asymptotic behavior of the scale factor is

\[ a_1 \sim \frac{c_5}{3} \sqrt[3]{3 \Lambda + 9a_2}. \]  

(38)

One can see from equations (27) and (38) that the Hubble parameter of the three scale factors become equal at late times, predicting an isotropic universe. In figures (1), (2) and (3) we have plotted the anisotropy and deceleration parameters for three different values of \( \beta \), including the Maxwell’s regime \( \beta = \infty \) and constants \( c_1 = -100, c_2 = -1, c_3 = -60 \). These figures show that the behavior of the anisotropy parameter and the deceleration parameter are largely dependent on the Born-Infeld coupling. As the constant \( \beta \) decreases, the early time anisotropy of the BI universe becomes less pronounced. From the first figure we see that the slope of the anisotropy parameter decreases, leading to a less anisotropic universe which becomes isotropic with a smaller rate than the Maxwell universe. For \( \beta = 10^{-5} \), the universe reaches an isotropic stage at early times, becoming less anisotropic again and finally reaches the present isotropic state. The deceleration parameter diagram shows the same behavior; the BI universe starts from a smaller amount of deceleration at early times and becomes accelerating considerably slower than the Maxwell case at late times.

To answer the question of what value of \( \beta \) is the most suitable, one requires comparison against the observational data which may become a realistic possibility in not too distant a future. In figure (2) we have plotted the early time behavior of the deceleration parameter. One can see from the figure that the BI universe with the coupling \( \beta = 10^{-5} \) has an early accelerating stage which alternately becomes decelerating and accelerating at late times. Finally we note that the late time behavior of the theory discussed in this section does not depend on the values of the constants of the model. Indeed at late times the model asymptotes the Maxwell’s theory, which can be seen from the figures. However the detailed early time behavior of the model depends on the values of the constants of the model, but its qualitative behavior is the same as the constants assume different values.

IV. CONCLUSIONS AND REMARKS

The observation of the present acceleration of the universe is one of the most intriguing problems in the present-day cosmology. Explanations based on the existence of dark energy or modified theories of gravity are common approaches to describe it.

In this paper we have dealt with this problem in the context of a brane-world model in which the assumption of the \( \mathbb{Z}_2 \) symmetry and use of the Israel junction conditions are relaxed and the matter content of the universe is assumed to be of the BI type. To calculate the conserved quantity \( Q_{\mu \nu} \) appearing in the field equations, one needs to calculate the components of the extrinsic curvature.

FIG. 1: The Anisotropy parameter in the cases: \( \beta = \infty \) (dashed-dotted line), \( \beta = 1 \) (dashed line), \( \beta = 10^{-5} \) (solid line)

FIG. 2: The early time behavior of the deceleration parameter in the cases: \( \beta = \infty \) (dashed-dotted line), \( \beta = 1 \) (dashed line), \( \beta = 10^{-5} \) (solid)

FIG. 3: The late time behavior of the deceleration parameter in the cases: \( \beta = \infty \) (dashed-dotted line), \( \beta = 1 \) (dashed line), \( \beta = 10^{-5} \) (solid)
We have used the Codazzi equation to achieve this. The components of the extrinsic curvature are obtained by the choice of an ansatz for its (00) component. However, as long as the bulk metric is not specified, this component can be chosen in such a way as to make the tensor $Q_{μν}$ taking as simple a form as possible. In the SMS method, this is achieved by the form of the energy-momentum tensor. However, as is discussed in the Appendix, the form of the BI energy-momentum tensor is not arbitrary and is calculated from the BI field equations, indicating that the SMS procedure may not provide solutions compatible with the present observations when confining the BI field as the matter source to the brane. In other words, in the SMS method one restricts the confinement of the BI matter on the brane a possibility. In contrast, the method used in this paper provides more freedom in choosing the extrinsic curvature, making the confinement of the BI matter on the brane a possibility. In other words, in the SMS method one restricts the bulk geometry to be satisfied by the Israel junction conditions, leaving the energy-momentum tensor arbitrary. In our model however, the procedure does not have such a restriction.

The asymptotic behavior of the BI matter was also studied. As can be seen from equation (34), its behavior mimics that of the Maxwell field in a certain limit; the BI field makes the transition of the universe from a highly anisotropic early state to its present isotropic form slower than the Maxwell field and this depends on the parameter $β$, as is seen from figure (1).

V. APPENDIX

Let us now discuss the consequences of using the BI field in brane world scenarios where the confinement of gauge fields is achieved through the SMS method. As is well known, in this method one uses the Israel junction conditions to replace the extrinsic curvature tensor with the energy-momentum tensor. It follows that a new tensor, $π^{μν}$, emerges which is quadratic in the energy-momentum tensor. This tensor can be written as

$$π^{μν} = \frac{1}{4} τ_{ασ} τ_ν^α + \frac{1}{12} ττ_{μν} + \frac{1}{8} g_{μν} τ_α τ^α_β τ^α_β - \frac{1}{24} g_{μν} τ^2,$$

where $τ = τ^α_α$, and $g_{μν}$ is the brane metric. One of the consequences of the SMS field equations is that for the constant curvature bulk which we have considered in this paper, the above tensor must be conserved $∇_μ π^{μν} = 0$ [13]. If we consider the Bianchi I metric and note that the energy-momentum tensor for this space-time takes the form $τ^μ_μ = \text{diag}(−ρ, −ρ, p, p)$, equation (23), the conservation equation for $π^{μν}$ reduces to

$$(p + p)(ρ + 2p) = 0.$$  

The first term cannot be zero as one can see easily from definitions of $ρ = τ^0_0$ and $p = τ^2_2$ from equations (23). So the only equation of state for the BI matter, consistent with the above constraint is $p = 2p = \text{const}$. This equation can be solved for the scale factors, with solution

$$a_2 \propto \frac{1}{a_3}.$$  

This solution is obviously ruled out by present observations.

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