Cosmology with Varying Constants

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(August 16, 2000)

I motivate and discuss some recent work on theories with varying constants, and consider some possible observational consequences and tests. Particular emphasis is given to models which can (almost) exactly mimic the predictions of standard inflationary models.

I. MOTIVATION

According to our present understanding, higher-dimensional theories, are thought to be required to provide a consistent unification of the known fundamental interactions of nature. Even though there is at present no robust ideas about how one can go from these theories to our familiar low-energy (3 + 1) spacetime, it is clear that such a process should necessarily involve two crucial mechanisms, namely dimensional reduction and compactification.

From our present purposes, the most important consequence of this process is that in such theories the effective three-dimensional constants will typically be related to the true higher-dimensional constants via the radii of the (compact) extra dimensions. Furthermore, it is well known that these radii often have a non-trivial evolution. Hence one is naturally led to the expectation of time (or even space) variations of the ‘effective’ coupling constants we can measure. This provides more than enough motivation to consider the cosmological consequences of these variations.

II. MODELLING AND MEASUREMENTS

In order to do this, one must first build ‘toy-models’ for the evolution of the effective constants.

These issues have been discussed at least since the time of Dirac, who first considered variations of the gravitational constant G. Almost half a century later, Beckenstein introduced a theory with a varying electric charge e. Finally, much more recently, there has been an extraordinary growth in the interest in theories with a varying speed of light c.

Before proceeding, one should recall that one can only measure dimensionless combinations of dimensional constants, and that such measurements are necessarily local. For example, the statement that “the speed of light here is the same as the speed of light in Andromeda” is either a definition or it’s completely meaningless. The paper by Albrecht & Magueijo in provides some further instructive examples of this point.

An important consequence of the above point is that when considering observational tests one should focus on dimensionless quantities. The most relevant example is that of the fine-structure constant,

\[ \alpha \equiv \frac{e^2}{\hbar c}. \]

One consequence of what was said above is the fact that any evidence for a variation in a dimensionfull quantity will necessarily be dependent on the choice of units in which we choose to measure it. In other words, given a theory with a varying constant (say c), one can always, by a suitable re-definition of units of measurement, transform it into another theory where another constant (say e) varies, and furthermore any two such theories will be observationally indistinguishable. Hence, deciding which system of units one adopts is, in some sense, a matter of convenience and mathematical simplicity. It seems to be the case that theories with varying speed of light c are generally easier to work with than those with a varying electric charge or Planck constant, although one could think of counter-examples of this statement.

III. OBSERVATIONAL STATUS

Tests for possible variations of the fine-structure constant \( \alpha \) have been carried out for a number of years, and the current observational status is rather exciting, but also a little bit confusing—see for a brief summary.
The best limit from laboratory experiments (using atomic clocks) is \[14\]
\[
|\dot{\alpha}/\alpha| < 3.7 \times 10^{-14} \text{yr}^{-1}.
\]  

Measurements of isotope ratios in the Oklo natural reactor provide the strongest geophysical constraints \[15\],
\[
|\dot{\alpha}/\alpha| < 0.7 \times 10^{-16} \text{yr}^{-1},
\]
although there are suggestions \[13\] that due to a number of nuclear physics uncertainties and model dependencies a more realistic bound is \[|\dot{\alpha}/\alpha| < 5 \times 10^{-15} \text{yr}^{-1}\]. Note that these measurements effectively probe timescales corresponding to a cosmological redshift of about \(z \sim 0.1\) (compare with astrophysical measurements below).

Three kinds of astrophysical tests have been used. Firstly, big bang nucleosynthesis \[17\] can in principle provide rather strong constraints at very high redshifts, but it has a strong drawback in that one is always forced to make an assumption on how the neutron to proton mass difference depends on \(\alpha\). This is needed to estimate the effect of a varying \(\alpha\) on the \(^4\)He abundance.

The abundances of the other light elements depend much less strongly on this assumption, but on the other hand these abundances are much less well known observationally. Hence one can only find the relatively weak bound
\[
|\Delta \alpha/\alpha| < 2 \times 10^{-2}, \quad z \sim 10^9 - 10^{10}.
\]  

Secondly, observations of the fine splitting of quasar doublet absorption lines probe smaller redshifts, but should be much more reliable. Unfortunately, the two groups which have been actively studying this topic report different results. Webb and collaborators \[18\] were the first to report a positive result,
\[
\Delta \alpha/\alpha = (1.9 \pm 0.5) \times 10^{-5}, \quad z \sim 1.0 - 1.6
\]  

Note that this means that \(\alpha\) was smaller in the past. Recently the same group reports two more (as yet unpublished) positive results \[19\], \(\Delta \alpha/\alpha = (-0.75 \pm 0.23) \times 10^{-5}\) for redshifts \(z \sim 0.6 - 1.6\) and \(\Delta \alpha/\alpha = (-0.74 \pm 0.28) \times 10^{-5}\) for redshifts \(z \sim 1.6 - 2.0\). On the other hand, Varshalovich and collaborators \[13\] report only a null result,
\[
\Delta \alpha/\alpha = (-4.6 \pm 4.3 \pm 1.4) \times 10^{-5}, \quad z \sim 2 - 4;
\]  

the first error bar corresponds to the statistical error while the second is the systematic one. This corresponds to the bound
\[
|\dot{\alpha}/\alpha| < 1.4 \times 10^{-14} \text{yr}^{-1}
\]  

over a timescale of about \(10^{10}\) years. It should be emphasised that the observational techniques used by both groups have significant differences, and it is presently not clear how the two compare when it comes to eliminating possible sources of systematic error. Clearly this is an issue which can only be resolved with more and better data.

Finally, a third option is the cosmic microwave background (CMB) \[20,21\]. This probes intermediate redshifts, but has the significant advantage that one has (or will soon have) highly accurate data. A varying fine-structure constant changes the Thomson scattering cross section for all interacting species, and also changes the recombination history of Hydrogen (via changes in the energy levels and binding energies of all species). The authors of \[21\] provide an analysis of these effects and conclude that future CMB experiments should be able to provide constraints on a varying \(\alpha\) at the recombination epoch (that is, at redshifts \(z \sim 1000\)) at the level of
\[
|\dot{\alpha}/\alpha| < 7 \times 10^{-13} \text{yr}^{-1},
\]  

or equivalently
\[
|\alpha^{-1} \delta \alpha/\delta z| < 9 \times 10^{-5},
\]
which seems to indicate that these constraints can only become competitive in the near future. For example, a recent analysis \[21\] which uses the BOOMERanG \[22\] determination of the position of the first Doppler peak find that this still allows for a variation of up to \(4\%\) in the speed of light after recombination. More recently \[23\], it has been shown that the BOOMERanG and MAXIMA data slightly prefer a fine-structure constant that was smaller in the past, in agreement with quasar data.

We thus see that constraints at recent times are fairly strong, and any drastic recent departures from the standard scenario are excluded. On the other hand, there are no significant constraints in the pre-nucleosynthesis era, which leaves a rather large open space for theorists to build models—in fact, too large a space, as we’ll see next.

IV. GENERALISING GENERAL RELATIVITY

Theories with varying constants require a generalisation of Einstein’s theory of relativity. However, there is no unique (or even preferred) way of doing this. Hence one will be faced with the task of choosing a set of postulates to characterise the chosen theory. An associated task is the choice of ‘fundamental units’ in which measurements are to be made in the theory—in other words, the ‘rulers and clocks’ for the theory.

In particular, one can break a number of invariance principles and conservation laws. Examples of these include covariance and Lorentz invariance, mass, particle number, energy and momentum conservation, charge conservation and various energy conditions. In some ways, this is perhaps a too drastic step (as argued in \[24\],
for example). On the other hand, it does have the rather desirable feature of allowing rather simple solutions to some outstanding cosmological enigmas, such as the horizon, flatness and cosmological constant problems.

But are these drastic steps really necessary, and what is physically the role of a varying speed of light (or other constants)? In \[24\] it was conjectured that *any theory that reduces to General Relativity in some appropriate limit and solves the horizon and flatness problems must necessarily violate at least one of the following*: the strong energy condition, Lorentz invariance or covariance. Note that inflation is of course an example of a theory which solves the horizon and flatness problems through a violation of the strong energy condition. The above statement is a conjecture in the sense that no rigorous ‘mathematical’ proof was provided, although physical arguments were discussed in \[24\] which make it (we think) rather plausible.

What one can prove is, in some sense, the opposite result: No theory that reduces to General Relativity and obeys the above three principles will be able to solve the horizon and flatness problems, no matter what varying constants are included in it. Physically this is because for any theory that is subject to these four requirements one can always find a particular choice of fundamental units that will transform it into the ‘standard’ theory, where there are no varying constants and the standard cosmological problems can not be solved.

Nevertheless, if one considers that breaking Lorentz invariance in this way is too drastic, there are alternatives. Most notably, Bassett et al. \[24\] have extensively discussed the consequences of soft Lorentz invariance breaking. In particular, this will still solve the horizon problem, but not the flatness problem (at least by itself). As discussed in detail in \[24\], there are numerous examples of this kind of ‘effective’ theories in other areas of physics.

Physically, theories with soft Lorentz invariance breaking will have two (or more) natural speed parameters, and hence to the realm of two-metric theories. For example, one can have a characteristic speed for photons and another one for the rest of particles, or one for gravitons and another one for the rest, or one for bosons and another one for fermions, or even (as will be the case in the example that we will consider in some detail in the next section) one characteristic speed for the Higgs sector and another one for all the standard model (including gravity) particles.

V. PRIMORDIAL ADIABATIC FLUCTUATIONS FROM DEFECTS

There are currently two basic classes of models that could be responsible for producing “seeds” for structure formation. In the first \[25\], it is assumed that the universe was smooth at the start of its standard evolution, and defects were produced at one or more symmetry-breaking phase transitions which then continuously seeded structures on a specific set of comoving scales. In the second \[26\], an inflationary epoch is assumed to have happened before the standard evolution of the universe began, and the corresponding primordial fluctuations were laid out at this earlier epoch.

The main difference between these two scenarios is related to causality. In the first case, the initial conditions for the defect network that will be responsible for the primordial seeds are set up on a Cauchy surface that is part of the standard history of the universe. Hence, there will not be any correlations between quantities defined at any two spacetime points whose backward light cones do not intersect on that surface. On the other hand, inflation effectively pushes this surface to much earlier times, and if the inflationary epoch lasts long enough to solve the well-known set of “cosmological enigmas” then there will be essentially no causality constraints.

This can be seen in an alternative way by noting that inflation can be physically defined as an epoch when the comoving Hubble length decreases. Hence this length starts out very large, and perturbations can therefore be generated causally. Then inflation forces this length to decrease enough so that, even though it grows again after inflation ends, it’s never as large (by today, say) as the pre-inflationary era value. Note that once the primordial fluctuations are produced they can simply freeze in comoving coordinates and let the Hubble length shrink and then (for small enough scales) grow past them later.

As a first step towards identifying the specific model that operated in the early universe, one would like to be able to determine which of the two mechanisms above (if any) was involved. Features like super-horizon perturbations or the so-called ‘Doppler peaks’ \[27\] on small angular scales in the CMB angular power spectrum, however, do not provide good discriminators (at least on their own) \[28\].

Gaussianity tests \[29\] are in principle a better discriminator, though it is possible to build inflationary models that produce some forms of non-Gaussianity. Notably, it is easy to obtain non-Gaussianity with a chi-squared distribution—an example are the so called iso-curvature inflation models \[37\]. On the other hand, if one

\[^{3}\text{Stronger requirements will be needed if one wants to solve the cosmological constant problem as well—which is something that inflation can not do.}\]

\[^{4}\text{On the other hand, it should be said that a number of other people, most notably J. Magueijo, came quite close to providing counter-examples.}\]

\[^{5}\text{This is in some sense analogous to the concept of spontaneous symmetry breaking in particle physics. In this analogy, the Lorentz invariance breaking process described previously would correspond to explicit symmetry breaking.}\]
found non-Gaussianity in the form of line discontinuities, then it is hard to see how cosmic strings could fail to be involved.

The above discussion shows that even though defect and inflationary models have of course a number of distinguishing characteristics, there is a greater overlap between them than most people would care to admit. Moreover, it is also quite easy to obtain models where both defects and inflation generate density fluctuations \(^{38,39}\). We discuss an explicit example \([11]\) of a model where the primordial fluctuations are generated by a network of cosmic defects, but are nevertheless very similar to a standard inflationary model; the only difference between these models and the standard inflationary scenario will be a small non-Gaussian component.

We consider a theory that contains two different speed parameters, say \(c_\phi\) and \(c\); the first is relevant for the dynamics of the scalar field which will produce topological defects, while the second is the ordinary speed of light that is relevant for gravity and all standard model interactions.

The basic idea should now be clear. We assume that \(c_\phi \gg c\) so that the correlation length of the network of topological defects will be much greater than the horizon size (which is of course defined with respect to \(c\)). We could, in analogy with \([24]\), explicitly define our effective theory by means of an action, and postulate a relation between the ‘standard’ metric and the one describing the propagation of our scalar field. However, this is not needed for the basic point we’re discussing here. Also, we concentrate on the case of cosmic strings, whose dynamics and evolution are much better known than those of other defects \([23,12]\) although much of what we will discuss will apply to other defects as well.

Note that \(c_\phi\) could either be a constant, with say

\[ [g_\phi]_{00} = \frac{c_\phi^2}{c^2} g_{00}, \tag{10} \]

or, as discussed in \([24]\), one could set up a model such that the two speeds are equal at very early and at recent times, and between these two epochs there is a period, limited by two phase transitions, where \(c_\phi \gg c\). As will become clear below, the basic mechanism will work in both cases, although the observational constraints on it will of course be different for each specific realization.

The evolution of the string network will be qualitatively analogous to the standard case \([23,10,12]\), and in particular a “scaling” solution will be reached after a relatively short transient period. Thus the long-string characteristic length scale (or “correlation length”) \(L\) will evolve as

\[ L = \gamma c_\phi t, \tag{11} \]

with \(\gamma = \mathcal{O}(1)\), while the string RMS velocity will obey

\[ v_\phi = \beta c_\phi, \tag{12} \]

with \(\beta < 1\).

Note, however, that there are a small number of important differences relative to the standard scenario. The first one is the most obvious: if \(c_\phi \gg c\), then the string network will be outside the horizon, measured in the usual way. Hence these defects will induce fluctuations when they are well outside the horizon, thus avoiding causality constraints.

On the other hand, we also expect the effect of gravitational back-reaction to be much stronger than in the standard case \([33,23]\). The general effect of the back-reaction is to reduce the scaling density and velocity of the network relative to the standard value, as has been discussed elsewhere \([43]\). Thus we should expect fewer defects per “\(c_\phi\)-horizon”, than in the standard GUT-scale case. Having said that, it is also important to note that despite this strong back-reaction, strings will still move relativistically. Indeed, it can be shown \([43]\) that although back-reaction can slow strings down by a measurable amount, only friction forces \([41,42]\) can force the network into a strong non-relativistic regime. Thus we expect \(v_\phi\) to be somewhat lower than \(c_\phi\), but still larger than \(c\).

Only in the case of monopoles, which are point-like, one would expect the defect velocities to drop below \(c\) due to graviton radiation \([24]\). This does not happen for extended objects, since their tension naturally tends to make the dynamics take place with a characteristic speed \(c_\phi\) \([44]\). This point is actually crucial, since if the network was completely frozen while it was outside the horizon (as it happens in more standard scenarios \([38]\)) then no significant perturbations would be generated.

A third important aspect, to which we shall return below, is that the scale of the symmetry breaking, say \(\Sigma\), which produces the defects can be significantly lower than the GUT scale, since density perturbations can grow for a longer time than usual. Indeed, the earlier the defects are formed, the lighter they could be. Proper normalisation of the model will produce a further constraint on \(\Sigma\).

Finally, we also point out that in the scenario we have outlined above where \(c_\phi\) is a time-varying quantity which only departs from \(c\) for a limited period (which is started and ended by two phase transitions), the defects will become frozen and start to fall inside the horizon after the second phase transition. In this case what we require is that the defects are sufficiently outside the horizon and are relativistic when density fluctuations in the observable scales are generated. This will introduce additional constraints on the parameters of the model, and in particular on the epochs at which the phase transitions take place.

The evolution of the primordial fluctuations in our model is detailed in \([13]\). One obtains a model with primordial, adiabatic (\(\delta_i = 46m_i/3\)), nearly Gaussian fluctuations whose primordial spectrum is of the Harrison-Zel’dovich form. This model is almost indistinguishable
from the simplest inflationary models (as far as structure formation is concerned) except for the small non-Gaussian component which could be detected with future CMB experiments. The $C_l$ spectrum and the polarisation curves of the CMBR predicted by this model should also be identical to the ones predicted in the simplest inflationary models as the perturbations in the CMB are not generated directly by the defects.

Note that the key ingredient consists of having the speed characterising the defect-producing scalar field much larger than the speed characterising gravity and all standard model particles. This provides a ‘violation of causality’, as required in the criterion provided by Liddle [24]. The only distinguishing characteristic of this model, by comparison with the simplest inflationary models, will be a small non-Gaussian signal which could be detected by future experiments.

In fact, one can even think of an alternative model arising in the same context, but where the defect-induced primordial fluctuations are also Gaussian. In this case we require that the characteristic speed of the decay products of the defect network is much larger than the speed characterising gravity and all the standard model particles. Thus this model will exactly reproduce the CMB and large-scale structure predictions of the standard inflationary models, and the only way to identify it would be through the decay products of the defect network involved.

We emphasise again that in open or hybrid models of inflation defects can also be stretched outside the horizon but in this case they are frozen in comoving coordinates so that the perturbations they induce while being outside the horizon are negligible.

Admittedly, these models might admittedly seem somewhat “unnatural” in the context of our present theoretical prejudices, though they are certainly not the only ones to fit in this category [25-28]. However, if one keeps in mind that any fully consistent cosmological structure formation model candidate should eventually be derivable from fundamental physics, one could argue that at this stage they are, caeteris paribus, on the same footing as inflation. Certainly no single fully consistent realization of an inflationary model is known at present.

Be that as it may, however, the fact that these examples can be constructed (and one wonders how many more are possible) highlights the fact that extracting robust predictions from cosmological observations is a much more difficult and subtle task than many experimentalists (and theorists) believe.

ACKNOWLEDGMENTS

The work presented here was done in collaboration with Pedro Avelino, and is part of an ongoing collaboration which also involves Graca Rocha and Pedro Viana.

I thank Bruce Bassett, João Magueijo, Anupam Mazumdar and Paul Shellard for useful discussions and comments.

This work has been supported by FCT (Portugal) under ‘Programa PRAXIS XXI’, grant no. PRAXIS XXI/BPD/11769/97, and by FCT/ESO, under project ESO/PRO/1258/98.

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