Radiation in non-transparent and transparent plasma: Modified Planck distribution and zero vacuum fluctuations

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Abstract. A study of equilibrium radiation in plasma media shows that the spectral energy distribution of such radiation is different from the distribution of Planck equilibrium radiation. Using the quantum electrodynamic approach, a general relation was found for the spectral energy density of equilibrium radiation in a system of charged particles for opaque and transparent media.

1. Introduction
In the year 1901 M Planck [1] established the correct form of the spectral energy distribution of equilibrium radiation (SEDER) which corresponds to an idealized model of an absolutely black body and laid the foundation for quantum theory. This distribution of radiation depends only on temperature and exists, e.g., in a cavity filled with radiation and bounded by an absolutely absorbing substance with a temperature $T$. It is assumed that radiation is in thermodynamic equilibrium with the substance, although the effects of the interaction of photons with the substance bounding the cavity are not considered [2].

The Planck distribution

$$e_{P}(\omega) \equiv \frac{dE(\omega)}{d\omega} = \frac{V \hbar}{\pi^2 c^3} \frac{\omega^{3}}{\exp(h\omega/T) - 1}$$

is usually associated with the consideration of a macroscopic body in thermal equilibrium with the surrounding black body radiation. In (1), $V$ is the volume in which radiation is enclosed, $c$ is the speed of light in vacuum.

Last years the interest to the SEDER in a substance which is in thermodynamic equilibrium with radiation is essentially increased [3–11], in particular, in the connection of astrophysical applications (see [3–7, 11] and references therein). It is worth noting, the presence of at least a small amount of matter is necessary for the possibility of obtaining equilibrium radiation, since the direct interaction between photons is negligible [2].

The main efforts for the last two decades have been devoted for the problem of accounting of the spatial dispersion of the properties of plasma medium (along with the frequency dispersion) in the SEDER modification [3,9,10]. The solution of this problem is essential, in particular, for astrophysical applications [6,11], where the role of spatial dispersion in the dielectric permittivity
is especially essential for the relativistic case [12]. However, the general solution of this problem is still absent and various approaches lead to different results. The semi-macroscopic approach [3] in the limit of negligible spatial dispersion of the dielectric permittivity cannot reproduce the classical Brillouin result for plasma medium (see, e.g., [2, 6]). The same problem appears in the rigorous consideration on the basis of quantum electrodynamics (QED) approach [9, 13], where the peculiarities of the SEDER for the asymptotically high and low frequencies have been found. The more elaborated semi-macroscopic result is developed in [10], where the Brillouin limit is fulfilled. However, the final result in this case cannot be expressed only in terms of the transverse dielectric permittivity. It is necessary to note that detailed knowledge of electromagnetic field fluctuations in plasma (see, e.g., [8]) is not sufficient to find the SEDER.

The correct calculation of the SEDER in plasma requires the extension of the QED approach in plasma medium by accounting for the additional term in the Hamiltonian, which was previously skipped in [9, 13]. Below, for simplicity, we consider the case of non-relativistic particles.

The main result of this work is the general formula for SEDER in opaque (non-transparent) plasma. For a special case of transparent plasma, we reproduce the Brillouin formula. A modification of Planck’s law and zero fluctuations in plasma medium is proposed and discussed.

2. Average energy and spectral distribution of equilibrium radiation in plasma

As easy to show, using the Hamiltonian $H$ [14] of the system of charge particles and radiation, the average energy of equilibrium radiation $E$ in plasma contains two terms $E_1$ and $E_2$

$$E = E_1 + E_2; \quad E_1 = \sum_{k,\lambda} \hbar \omega_k \langle c_{k,\lambda}^+ c_{k,\lambda} \rangle;$$

$$E_2 = \frac{1}{c} \int d^3x \sum_a \frac{\epsilon_a^2}{m_a c} \langle \hat{a}^2(x) \rangle \langle \hat{\psi}_a^+(x) \hat{\psi}_a(x) \rangle, \quad (2)$$

where $\epsilon_a$ and $m_a$ are, respectively, charge and mass of a particle of species $a$ and integration leads over coordinates for three dimensional system. The operator $\hat{a}$ in equation (2) is the transverse inner vector potential of the plasma electromagnetic field

$$\hat{a}(x) = c \sum_{k,\lambda} \left( \frac{2\pi \hbar}{V c k} \right)^{1/2} \{ \epsilon_{k,\lambda} \hat{c}_{k,\lambda} \exp(i k x) - \epsilon_{k,\lambda}^* \hat{c}_{k,\lambda}^* \exp(-i k x) \}. \quad (3)$$

Here $\hat{c}_{k',\lambda}^+$ and $\hat{c}_{k,\lambda}$ are the creation and annihilation operators for photons (with the momentum $k$ and two polarizations $\lambda = 1$ and 2) subordinated to the Bose statistics with the commutation relations $[c_{k,\lambda}^+ c_{k',\lambda'}^-] = \delta_{k,k'} \delta_{\lambda,\lambda'}$, $\hbar \omega_k = \hbar c | k |$ is the photon energy and $k$ is the wave vector of photon.

The operators $\hat{\psi}_a^+(x)$ and $\hat{\psi}_a(x)$ are the creation and annihilation operators for particles of species $a$. Below we consider the case of isotropic system, when the external fields are absent.

The term $E_1$ was first calculated in [9]. In this paper, the essential new term $E_2$ is included and calculated to find the full radiation energy $E$. As follows from equations (2) and (3), the energy $E$ can be rewritten in the form

$$E = E_1 + E_2 = V \int \frac{d^3k}{(2\pi)^3} \left[ \hbar \omega_k + \frac{\hbar \omega_p^2}{\omega_k} \right] (2 f(k) + 1), \quad (4)$$

where the first term in square brackets corresponds after integration to $E_1$, the second one to $E_2$ and $\omega_p$ is the full plasma frequency.

Take now into account the relation between the photon distribution function $f(k) = \langle \hat{c}_k^+ \hat{c}_k \rangle$ and the photon retarded Green function in Coulomb gauge $D^R(k, \omega)$ [9, 15] for the homogeneous
Coulomb system
\[ f(k) + \frac{1}{2} = -\frac{k}{2\pi e} \int_0^\infty d\omega \coth \left(\frac{\hbar \omega}{2T}\right) \text{Im}D^R(k, \omega) \]  
(5)

and the representation for \( D^R(k, \omega) \) through the explicit transverse dielectric permittivity (TDP) of plasma subsystem \( \varepsilon^\text{tr}(k, \omega) \):
\[ D^R(k, \omega) = \frac{4\pi e^2}{\varepsilon^\text{tr}(k, \omega) \omega^2 - k^2 c^2}, \quad \text{Im}D^R(k, \omega) = -\frac{4\pi e^2 \omega^2 \text{Im}\varepsilon^\text{tr}(k, \omega)}{\varepsilon^\text{tr}(k, \omega) \omega^2 - k^2 c^2}. \]  
(6)

The convergence of the integral (5) over \( \omega \) at the pole determined by the relation \( \omega^2 \text{Re} \varepsilon^\text{tr}(k, \omega) - c^2 k^2 = 0 \) is provided by a nonzero value of the imagine part of \( \varepsilon^\text{tr}(k, \omega) \). The case of transparent plasma, when \( \text{Im}\varepsilon^\text{tr}(k, \omega) \rightarrow +0 \) is considered below in section 3.

Substituting equation (6) into (5) and the expression \( f(k) \) into equation (4) and using the definition of the SEDER \( e(\omega) \)
\[ E = \int_0^\infty d\omega e(\omega), \]  
(7)

we arrive first at the full SEDER \( e(\omega) = e^{(1)}(\omega) + e^{(2)}(\omega) \) which takes into account the spatial and frequency dispersion of the plasma medium
\[ e(\omega) = V \frac{\hbar \omega^2}{\pi^3} \coth \left(\frac{\hbar \omega}{2T}\right) \times \]
\[ \int_0^\infty dk k^2 \left[ c^2 k^2 + \omega_p^2 \right] \frac{\text{Im}\varepsilon^\text{tr}(k, \omega)}{\left(\omega^2 \text{Re}\varepsilon^\text{tr}(k, \omega) - c^2 k^2\right)^2 + \omega^4 \left(\text{Im}\varepsilon^\text{tr}(k, \omega)\right)^2}, \]  
(8)

where the second term in the square brackets is new and corresponds to \( e^{(2)}(\omega) \). The result (8) is applicable to both relativistic and non-relativistic plasma. Moreover, the TDP in equation (8) takes into account all effects of Coulomb interaction in plasma system. Naturally, the explicit analytical form of TDP is unknown. Therefore, for the concrete calculations we need to use the known models of the TDP.

3. SEDER for transparent plasma

As a simple example, let us consider the case of transparent non-relativistic plasma. In this case \( \text{Im}\varepsilon^\text{tr}(k, \omega) \rightarrow +0 \) and the real part of the TDP equals \( \text{Re} \varepsilon^\text{tr}(k, \omega) \equiv \varepsilon_0(\omega) = 1 - \omega_p^2/\omega^2 \). In this case integration can be fulfilled analytically
\[ \left[ (\omega^2 \text{Re}\varepsilon^\text{tr}(k, \omega) - c^2 k^2)^2 + \left(\text{Im}\varepsilon^\text{tr}(k, \omega)\right)^2 \right]_{k \rightarrow 0} \rightarrow \pi \delta(\omega^2 - \omega_p^2 - c^2 k^2), \]  
(9)

according to the known Dirac \( \delta \)-function determination \( \lim_{x \rightarrow 0} [x/(a^2 + x^2)] = \pi \delta(a) \). After the simple calculation we find the answer for the full SEDER in transparent plasma \( e_{\text{trans}}(\omega) \)
\[ e_{\text{trans}}(\omega) = \frac{V \hbar \omega^3}{2\pi^2 c^3} \sqrt{\frac{1 - \omega_p^2}{\omega^2}} \theta(\omega - \omega_p) + \frac{V \hbar \omega^3}{\pi^2 c^3} \frac{1}{\exp(h\omega/T) - 1} \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \theta(\omega - \omega_p), \]  
(10)

where \( \theta(\omega) \) is the Heaviside step function. In equation (10), the square root of the dielectric permittivity \( \varepsilon_0(\omega) \) appears, instead \( [\varepsilon_0(\omega)]^{1/2} \), which is present for the accounting only \( e^{(1)}(\omega) \) [9]. The result (10) coincides with the Brillouin one for transparent plasma (see, e.g., [16]). The first term in the right side of equation (10) defines the modified zero fluctuations. They are non-negative and exist only for \( \omega > \omega_p \), smoothly decreasing till zero when \( \omega \) decreases to \( \omega_p \). The second term in equation (10) is the modified Planck distribution for the case under consideration.
As follows from equation (10) the renormalization of zero fluctuations is necessary. To see that, let us rewrite (10) separating zero fluctuations in the standard vacuum form $e_{0,\text{vac}} = \frac{V \hbar \omega^3}{(2\pi^2 c^3)}$ which follows from equation (10) in the limit $\omega_p = 0$ and after subtraction of the Planck distribution (in details see, e.g., [16]). Then, as easy to see, the remaining after subtraction of $e_{0,\text{vac}}$ part of the SEDER (10) in plasma, when $\omega_p \neq 0$, becomes negative and proportional to $-V \hbar \omega_p^2 / (4\pi^2 c^3)$ for large $\omega \gg \omega_p$. This statement is evident after expansion of the first term in (10) on the small ratio $\omega_p / \omega$ and the omitting of the exponentially small second term in (10). Since this remaining part represents the modified Planck distribution in plasma (if we suppose that the vacuum fluctuations have a standard form $e_{0,\text{vac}}$) we should conclude that this standard form of zero fluctuations separation is impossible for physical reasons. Therefore, the zero fluctuations in plasma necessary to modify as is in equation (10), when they depends on plasma density. The problem of the zero fluctuations separation for the SEDER in electron gas has been recently discussed in [17].

4. Conclusions

Thus, a new and essential term for the energy of a system of photons and charged particles has been identified and calculated. The complete spectral energy distribution of the equilibrium radiation in the plasma is found. The obtained result for the SEDER depends only on the explicit transverse dielectric permittivity which includes frequency and spatial dispersion as well as all effects of correlations and damping in plasma medium. For simplest case of the SEDER in transparent plasma the necessity of renormalization of zero vacuum fluctuations is established in parallel with the modification of the Planck law in plasma medium. The necessity of a separation of zero vacuum fluctuations in plasma and the modified Planck distribution in general case of the explicit TDP will be discussed in a separate publication. It is not out of place to note, that for an explicit but unknown TDP, as well as for some special models of TDP, the need to modify zero fluctuations is not obvious. It is the problem for future investigations.

Moreover, on the basis of the existing experiments on the Casimir effect [18], we can suppose, generally speaking, that the full SEDER manifests itself in the experiments. However, the separation of the full SEDER on zero fluctuations and Planck distribution is very useful, since the experimental parameters for simultaneous observations of these parts of the full SEDER is very different and till now incompatible. Other wards, the experimental conditions for observations of the Planck part of the full SEDER crucially different from the experimental observations of zero fluctuations.

The obtained results can be applied to the early stage of Universe evolution when the density of charged particles and their temperature were extremely high, as well as to the laboratory plasma and radiation (see, e.g., [19, 20]), to consideration internal processes in the planets and to other Space objects.

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