The dependence of the wave mode from external periodic excitation in the harbor of port Kholmsk

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Abstract. According to observations of the waves in 2006 in the harbor of Kholmsk, chaotic vibrations were detected. Studies have shown that with an increase in the amplitude of external excitation, wind waves and swell coming to the entrance of the harbor, the energy of oscillations in the system also increases. For incoming waves higher than 3.5 m the spectrum changes, the energy of movements in the harbor increases significantly in a wide range of periods and reaches the level of energy of the harbor’s eigen oscillations, which in this case are practically not allocated. The authors suggested that at large amplitudes of waves at the entrance of the harbor the fluctuations become chaotic. Verification of this assumption was performed on the basis of numerical simulation using the Duffing equation with external periodic excitation. The simulation showed the possibility of chaotic vibrations in the harbor. The system has intermittent chaotic and periodic modes of oscillations with increasing amplitude of external excitation for a wide range of periods and increasing the energy of chaotic vibrations.

1. Introduction

The trade turnover and the state of a number of the most important sectors of the regional economy depend on the stable operation of the main ports of the Sakhalin region. For the safe operation of existing hydraulic structures, and especially in the design of new facilities, including those related to the operation of the ferry, it is necessary to have sufficiently accurate characteristics of dangerous marine phenomena in the waters of ports.

Observation of waves and meteorological conditions was conducted in the Kholmsk harbor for several years with the use of both cable and autonomous recorders of waves. In 2006, with the support of the Russian Foundation for Fundamental Research (project 06-05-96157 “Experimental study of marine dangerous phenomena in the seaport of Kholmsk, Sakhalin region”), an experiment was organized, focused primarily on the study of the wave regime of the Kholmsk harbor, that obtained a large amount of data observations of the waves.

These time series for different stats of sea waves showed the presence of stable eigen resonance oscillations of the sea level on the periods of about three and eight minutes corresponding to the seiches of the harbor. They were readily distinguished against the background of noise for any state of the sea. With the increase in the energy of sea waves, the energy of seiches (eigen oscillations) also increases and is clearly visible against the background of wave noise.

However, during a very strong storm observed on 23.11.2006, the energy peaks with periods of about three and eight minutes in the spectra practically were not allocated in this case. The authors suggested that this situation is associated with the transition of vibrations in the harbor to chaotic. Verification of
this suggestion was made using a non-linear Duffing model, which confirmed the suggestion and showed that the coming to harbor waves with periods of swell, excite the seiches and possible manifestation of the chaotic regime of vibrations.

2. Experiment
The experiment was conducted from 26 July 2006 by installing (in the seaport of Kholmsk close to ferry pier), a measuring complex consisting of quartz bottom sensor, a cable communication line, timer/counter PCI 1780, a personal computer and digital meteorological station WS 2300 with power supply. The scheme of the water area of the seaport of Kholmsk and the location of the wave sensor is shown in figure 1.

Registration of wave processes in the Kholmsk harbor was carried out with a discreteness of 2 s. Because of the large amount of incoming information, the data accumulated in the form of daily files on the hard disk of the computer. Simultaneous with the processing of the data of sea level fluctuations, data from the digital weather station were obtained with a discreteness of meteorological parameters measurement at 1 min. Measurements were carried out over a long period of time, which allowed investigation of the characteristics of wave processes both in calm weather and during strong autumn storms. Fragments of time series for different weather pattern are shown in figure 2.

Measurement 16.11.06 and 26.11.06 was performed during calm weather pattern and the amplitude of waves oscillations was small, about 2-3 cm in the harbor. 17.10.06 measurements were carried out at the wind and quite noticeable agitation (height of the waves at the entrance of the harbor according to the observations at the Hydro Meteo Service of Russia reached 2.5 m). A similar pattern was observed on 12.11.06 and 22.11.06, when the highest values of the wind speed of the southern directions were observed, although its duration was relatively small. According to visual observations, the height of wind waves on this day reached a significant value for this area – 3.5 m.

The time series relating to 23.11.06, significantly different from the others. The level oscillations in the harbor, even in comparison with cases of significant waves were, higher more than two times, and as is typical for all other cases regular oscillations with distinct periodicity was

Figure 1. The scheme of the water area of the seaport of Kholmsk and the location of the wave sensor.

Figure 2. Fragments of time series for different weather pattern. 16.11.06 and 26.11.06 – quiet weather, 17.10.06 – wave height 2.5 m, 12.11.06 and 22.11.06 – wave height 3.5 m, 23.11.06 strong storm, wave height more 3.5 m.
disrupted. As noted above, on November 22-24, a strong and steady wind of the southern direction was recorded, which maintained its strength for three days. Note that the day before there was a strong North wind, then there was a sharp change in the air flow. There was a strong storm in the port water area.

For the above fragments of records, the spectra of sea level fluctuations, shown in figure 3 were calculated. It is apparent that the spectra for developed sea waves with wave heights from 2.5 to 3.5 m (17.10.06, 12.11.06 and 22.11.06) have a similar character - they are distinguished an rise of energy for periods of oscillations longer 0.5 minutes, compared with the spectra for quiet weather (16.11.06 and 26.11.06). At the same time, the difference in the energy level of oscillations exceeds an order of magnitude on the periods from 2 to 15 minutes, which relate to the range of existence of resonant oscillations in the Bay [1, 2].

![Figure 3. Power spectral density of sea level oscillations for fragments of records presented in figure 1 for different heights of sea waves: 16.11.06 and 26.11.06 – quiet weather, 17.10.06 – wave height 2.5 m, 12.11.06 and 22.11.06 – wave height 3.5 m, 23.11.06 – strong storm, wave height more 3.5 m.](image)

As can be seen from figure 3, the resonant oscillations with periods of about 3 and 8 minutes are most expressed. For these fluctuations, moreover, it is characterized by a significant increase in the energy compared to calm weather, though 16 and November 26, the data peaks is also well pronounced. Note that in [3] oscillations with a period of about 3 minutes were observed both in calm weather and during the passage of the cyclone.

The spectrum of sea level fluctuations for the storm situation on November 23 is significantly different from the others. On this day, there were a highest waves that exceeding 3.5 meters, which was recorded rarely during few years of data collection. The main peaks in the spectrum of sea level fluctuations at the time were much weaker, with a common rise in the energy in a wide range of periods.

3. Model

The observed phenomenon of significant attenuation of peaks in the spectra presumably means the transition of the dynamic system - the water mass in the Kholmsk harbor, fluctuating on the resonance periods excited by external excitation – the incoming swell, to chaotic vibrations. A test of this suggestion was performed, based on diagnostic tests proposed by Moon [4]. To detect chaotic vibrations, he recommends considering the oscillation spectrum, phase portrait and Poincare section.

As it was noted above, the level oscillation spectrum for 23.11.2006 (figure 3) is broad and not contain expressed periodic wave processes, and therefore can be attributed to the spectrum of the chaotic process. Also note that the considered dynamic system is not linear, as in it there is a transformation of swell energy with periods 10-15 s in periods of lower frequency oscillations, which is possible only in nonlinear systems.

To find other diagnostic tests – phase portraits and Poincare section, the numerical simulation of oscillations in the Kholmsk harbor was carried out using the Duffing oscillator with cubic nonlinearity, that is one of the most common models of nonlinear oscillations. A special feature of the Duffing oscillator is the possibility of obtaining chaotic dynamics [5]. The equation was first found by German engineer Georg Duffing in 1918.
The nonlinear system considered here in this case can be represented by the Duffing equation, which describes a system of the 2nd order with irregular oscillations and external periodic excitation (forced vibration) [6, 5]. The system model is described as follows:

\[ \ddot{x} + k\dot{x} + \omega_0^2 x + \alpha x^3 = F \cos(\omega t), \]  

where \( F \) and \( \omega \) – amplitude and frequency of the external periodic excitation (period \( T \)); \( \omega_0 \) – frequency oscillator (\( T_0 \)); \( k \) – damping parameter, and \( \alpha \) – nonlinearity coefficient. This equation describes the motion of a classical particle in a double well potential.

Usually, in numerical modeling, equation (1) is used to reduce the order of the differential equation in accordance with the Cauchy theorem:

\[ \dot{x}_1 = x_2, \]  

\[ x_2 = -\omega_0^2 x - \alpha x^3 - kx_2 + F \cos(\omega t), \]  

(2)  

(3)

The numerical simulation of the considered system – the interaction of the seiches in the waters of the seaport of Kholmsk with incoming waves of swell was performed with using set (2, 3). Phase portraits and Poincare sections for different amplitudes of the external excitation of \( F \) and the damping parameter \( k \) are calculated for the wave periods found in the harbor and for different periods of incoming wind waves and swell. They have a similar character, and for the period of oscillations 3 minutes and the period of coming swell 12 s shown in figure 4.

The numerical solution of the nonlinear system of differential equations using the program PUAN [7] created by us in the language of Delphi was carried out in the simulation. This program allows to speed up the process of calculating the phase portrait and the Poincare section for significant density of points and calculating the bifurcation diagrams, which help to determine the dynamic behavior of the system.

The numerical investigation showed that the considered nonlinear system has a complex dynamics, including such phenomena as periodicity and chaos. The system has intermittent chaotic and periodic oscillations for a wide range of periods of external excitations - from 5 to 15 seconds and for periods of eigen oscillations of 3 and 8 minutes. At the same time, their energy increases with decreasing damping parameter \( k \) and with increasing damping decreases (figure 4, c, d) for the same periods of the seiches and incoming waves. And when the damping parameter is greater than 0.2, the energy of chaotic vibrations decreases sharply, and in this case, the oscillations in the dynamic system according to [8] and Poincare section (figure 4, f) corresponds to the multiband chaos.

The analysis of energy of chaotic fluctuations versus external influence in a system showed that increase in amplitude \( F \) causes periodic occurrence of chaotic vibrations and change of a form of Poincare sections (figure 4, a, b and c, d). The obtained Poincare section (figure 4, d) is close to the section obtained by Ueda [9]. At the same time, for the values of the amplitudes of the external excitation corresponding to the weak energy of chaotic vibrations, a decrease in the damping parameter leads to the intensification of chaos, that is obvious.

The same occurs with an increase in the nonlinearity coefficient, with a significant decrease in which the system transitions to periodic oscillations. Also, with an increase in the amplitude of the external excitation, an overall trend is observed, corresponding to an increase in the energy of chaotic vibrations. The same conclusion is made on the basis of studies of transition to chaos in a non-linear dynamic system, given in paper [10].

The bifurcation diagrams were calculated for different values of the parameters included in the set of differential equations (2, 3), one of which is shown (figure 5). Since the nature of the diagram as a whole does not change, the fragments for \( F \) from 0 to 1.5 and for \( F \) from 17 to 20 are given for a more detailed display. They confirmed the intermittency of the chaotic vibrations varies with the amplitude of the external excitation.
The bifurcation diagram clearly shows that for some values of the parameter \( F \), the variable \( x \) takes a single value, it means the process does not change over time. Then the transition is occurred to doubling of period when there is a cyclic change of the variable, taking turns two values, for example, for the values of \( F \) about 0.5, 17.3, 18.2. The observed bifurcations are called period doubling.

**Figure 4.** Phase portraits (a, c, e) and Poincare sections (b, d, f) for different amplitudes of external excitation \( F \) and damping parameter \( k \) for periods of external excitation 12 s and periods of eigen oscillations 3 min.

Note that the doubling points accumulate to some critical point according to the law of geometric progression. This is the law of the nonlinear world, the universal character of which was realized and justified by M. Feigenbaum [11]. For the critical point of possible chaos, which is a bifurcation tree
looks like “smeared” areas of the crown. In this case, the dynamic variable performs an irregular motion that looks like noise, although it is a “product” of the simplest iterative equation.

Figure 5. Bifurcation diagrams of $x$ versus the external excitation amplitude for fragments with $F$ from 0 to 1.5 and for $F$ from 17 to 20.

4. Conclusion
The analysis of the wave modes in the Kholmsk harbor (Sakhalin Island) using of observational data of 2006 was performed. Calculated from the measured time series the spectra of the sea level fluctuations for various weather patterns showed the presence of peaks at periods of approximately three and eight minutes, which correspond to resonant oscillations of sea level – seiches that were well distinguished against the background of noise. The energy of these oscillations increases with the increase of wind waves and swell coming to the harbor.

For wind waves and swell higher than 3.5 m, coming to the entrance of the harbor, the spectrum has a different character – the energy of oscillations in the harbor increases significantly in a wide range of periods and reaches the level of energy of the seiches, which in this case are practically not allocated. The authors suggest that at large amplitudes of the coming waves the oscillations in the harbor become chaotic. Verification of this suggestion was made using a non-linear Duffing equation with external periodic excitation.

The numerical study of the oscillations in the dynamic system – the water mass in the Kholmsk harbor, excited by external excitation such as wind waves and swell, showes that the considered nonlinear system has a complex dynamics, including such phenomena as periodicity and chaos. In this case, the system is observed intermittency of chaotic and periodic modes of oscillation with an increase in the amplitude of the external excitation for a wide range of its periods – from 5 to 15 s and for periods of eigen oscillations of 3 and 8 minutes. With the increase in the amplitude of the waves coming to the entrance of the harbor, the energy of chaotic vibrations in the harbor grows. At very large amplitudes of the external excitation the energy of the chaotic vibrations is comparable to periodic oscillation.

A model analysis of the versus of the energy of chaotic vibrations in the system on the amplitude of the external excitation showed that its increase causes an increase in the energy of chaotic vibrations in the system and a change in the form of the Poincare section. At the same time, for the values of the amplitude of the external excitation corresponding to the weak energy of chaotic vibrations, a decrease in the damping parameter leads to the intensification of chaos. The same happens with an increase in the nonlinearity coefficient, with a significant decrease in which the system transitions to periodic oscillations. The increase in the amplitude of the external excitation causes a general constant trend corresponding to an increase in the energy of chaotic vibrations.
Bifurcation diagrams calculated in the course of numerical simulation confirm the intermittency of chaotic and periodic modes of system oscillations. The diagrams also show that for some values of the amplitude of the external excitation, the dynamics of the system does not change over time, but when this parameter changes, the period of oscillations of the system doubles, after which chaos is possible, which manifests itself on the bifurcation tree in the form of “smeared” sections of the crown.

The study of the behavior of marine dynamic systems with chaos that is considered here, is necessary for practical purposes and to take into account the consequences that can lead to the emergence of complex dynamics. For example, to solve the problem of the dynamics of a ship or an oil platform in the presence of waves [12]. Only non-linear analysis provides a comprehensive understanding of the situation and the development of conditions to avoid disaster.

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