Wang, Haiyong
On the optimal estimates and comparison of Gegenbauer expansion coefficients. (English)
SIAM J. Numer. Anal. 54, No. 3, 1557-1581 (2016).

Expansions of given functions in Gegenbauer polynomials are of central importance in approximation theory. Especially, the asymptotic behaviour of the resulting expansion coefficients, e.g. for the expansion in Legendre coefficients is highly relevant, and in this paper, the precise growth rate of the expansion coefficients is identified (that is, optimal estimates are given). The principal tool is a new form of the contour integral representation of the said Gegenbauer coefficients. Error estimates for truncated Gegenbauer expansions are provided too, as is for example a comparison of the aforementioned Legendre coefficients and Chebyshev coefficients.

Reviewer: Martin D. Buhmann (Gießen)

MSC:
41A65 Abstract approximation theory (approximation in normed linear spaces and other abstract spaces)
41A25 Rate of convergence, degree of approximation
41A10 Approximation by polynomials
65N35 Spectral, collocation and related methods for boundary value problems involving PDEs

Keywords:
Gegenbauer coefficients; optimal estimates; error bounds; Legendre coefficients; Chebyshev coefficients

Software:
DLMF

Full Text: DOI arXiv

References:
[1] G. E. Andrews, R. Askey, and R. Roy, "Special Functions", Cambridge University Press, Cambridge, UK, 2000. - Zbl 1075.33500
[2] K. Atkinson and W. Han, "Spherical Harmonics and Approximations on the Unit Sphere: An Introduction", Lecture Notes in Math. 2014, Springer, New York, 2012. - Zbl 1254.41015
[3] S. N. Bernstein, "Sur l'ordre de la meilleure approximation des fonctions continues par les polynomes de degré donné", Mem. Cl. Sci. Acad. Roy. Belg., (1912), pp. 1–103.
[4] J. P. Boyd, "Trouble with Gegenbauer reconstruction for defeating Gibbs' phenomenon: Runge phenomenon in the diagonal limit of Gegenbauer polynomial approximations", J. Comput. Phys., 204 (2005), pp. 253–264. - Zbl 1071.65189
[5] J. P. Boyd, "Large-degree asymptotics and exponential asymptotics for Fourier, Chebyshev and Hermite coefficients and Fourier transforms", J. Engrg. Math., 63 (2009), pp. 355–399. - Zbl 1163.42001
[6] J. P. Boyd and R. Petschek, "The relationships between Chebyshev, Legendre and Jacobi polynomials: The generic superiority of Chebyshev polynomials and three important exceptions", J. Sci. Comput., 59 (2014), pp. 1–27. - Zbl 1301.41003
[7] M. J. Cantero and A. Iserles, "On rapid computation of expansions in ultraspherical polynomials", SIAM J. Numer. Anal., 50 (2012), pp. 307–327. http://dx.doi.org/10.1137/110829568. - Zbl 1241.42023
[8] D. Elliott, "The evaluation and estimation of the coefficients in the Chebyshev series expansion of a function", Math. Comp., 18 (1964), pp. 274–284. - Zbl 0119.32904
[9] D. Elliott, "Truncation errors in two Chebyshev series approximations", Math. Comp., 19 (1965), pp. 234–248. - Zbl 0127.08501
[10] D. Elliott and P. Tuan, "Asymptotic estimates of Fourier coefficients", SIAM J. Math. Anal., 5 (1974), pp. 1–10, http://dx.doi.org/10.1137/0505001. - Zbl 0238.42008
[11] L. Fox and I. B. Parker, "Chebyshev Polynomials in Numerical Analysis", Oxford University Press, London, 1968.
12. W. Gautschi and R. S. Varga, Error bounds for Gaussian quadrature of analytic functions, SIAM J. Numer. Anal., 20 (1983), pp. 1170–1186, http://dx.doi.org/10.1137/0720087.

13. D. Gottlieb and C.-W. Shu, On the Gibbs phenomenon IV: Recovering exponential accuracy in a subinterval from a Gegenbauer partial sum of a piecewise analytic function, Math. Comp., 64 (1995), pp. 1081–1095. Zbl 0852.42018

14. D. Gottlieb and C.-W. Shu, On the Gibbs phenomenon V: Recovering exponential accuracy from collocation point values of a piecewise analytic function, Numer. Math., 71 (1995), pp. 511–526. Zbl 0852.42019

15. D. Gottlieb and C.-W. Shu, On the Gibbs phenomenon III: Recovering exponential accuracy in a subinterval from a spectral partial sum of a piecewise analytic function, SIAM J. Numer. Anal., 33 (1996), pp. 280–290, http://dx.doi.org/10.1137/0733015.

16. D. Gottlieb and C.-W. Shu, On the Gibbs phenomenon and its resolution, SIAM Rev., 39 (1997), pp. 644–668, http://dx.doi.org/10.1137/S0036144596301390.

17. D. Gottlieb, C.-W. Shu, A. Solomonoff, and H. Vandeven, On the Gibbs phenomenon I: Recovering exponential accuracy from the Fourier partial sum of a nonperiodic analytic function, J. Comput. Appl. Math., 43 (1992), pp. 81–98.

18. I. S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series, and Products, Academic Press, 2007.

19. B.-Y. Guo, Gegenbauer approximation in certain Hilbert spaces and its applications to singular differential equations, SIAM J. Numer. Anal., 37 (2000), pp. 621–645, http://dx.doi.org/10.1137/S0036142998342161.

20. A. Isac, A fast and simple algorithm for the computation of Legendre coefficients, Numer. Math., 117 (2011), pp. 529–553.

21. G. J. O. Jameson, Inequalities for the perimeter of an ellipse, Math. Gazette, 98 (2014), pp. 227–234. Zbl 1383.26012

22. C. Lanczos, Tables of Chebyshev Polynomials $S_n(x)$ and $C_n(x)$, Natl. Bureau Standards Appl. Math. 9, U.S. Government Printing Office, Washington, D.C., 1952.

23. R. R. Lederman and V. Rokhlin, On the analytical and numerical properties of the truncated Laplace transform I, SIAM J. Numer. Anal., 53 (2015), pp. 1214–1235. Zbl 1317.65248

24. J. C. Mason and D. C. Handscomb, Chebyshev Polynomials, Chapman and Hall/CRC Press, Boca Raton, FL, 2003.

25. F. W. J. Olver, D. W. Lozier, R. F. Boisvert, and C. W. Clark, NIST Handbook of Mathematical Functions, Cambridge University Press, Cambridge, UK, 2010.

26. S. Olver and A. Townsend, A fast and well-conditioned spectral method, SIAM Rev., 55 (2013), pp. 462–489, http://dx.doi.org/10.1137/120865458.

27. G. Szegő, Orthogonal Polynomials, Amer. Math. Soc. Colloq. Publ. 23, AMS, Providence, RI, 1939.

28. N. M. Temme, Large parameter cases of the Gauss hypergeometric function, J. Comput. Appl. Math., 153 (2003), pp. 441–462. Zbl 1019.33005

29. L. N. Trefethen, Approximation Theory and Approximation Practice, SIAM, Philadelphia, PA, 2013.

30. H.-Y. Wang and S.-H. Xiang, On error bounds for orthogonal polynomial expansions and Gauss-type quadrature, SIAM J. Numer. Anal., 50 (2012), pp. 1443–1469. Zbl 1276.65017

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.