Pulsed pump in optical displacement transducer for experiments with probe bodies

Victor V. Kulagin

Sternberg Astronomical Institute, Moscow State University,
Universitetsky prospect 13, 119899, Moscow, Russia, e-mail: kul@sai.msu.ru

Abstract

The sensitivity of the displacement transducer pumped with a train of high-intensity laser pulses is estimated. Due to the multicomponent character of the pump a consideration of transformations of the signal and the noises between optical modes plays an important role in estimation of the potential sensitivity. An expression for the minimal detectable external classical force resembles those for the continuous wave pumping with substitution of the laser power by a time averaged power of pulsed laser. Possible scheme for back action noise compensation for such transducers is considered. For full suppression of back action noise the field of local oscillator has to be pulsed with the same time dependence as the pump field.

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1 Introduction

The longbase laser interferometric gravitational wave detectors are under construction at present time [1-3]. Their sensitivity to metric perturbation will be about $h \approx 10^{-21}$ that corresponds to the classical regime of operation. However for future installations with projected sensitivity $10^{-22} \div 10^{-23}$ the quantum features of the measurement process can play a significant role. At the same time there are no limits of principle on the accuracy of measurement of external classical force. Therefore the methods and schemes which give the possibility to overcome the quantum measurement limitations (or the so called standard quantum limit, SQL) is of vital importance for future generation of gravitational wave experiments.

There are several procedures which allow to achieve the sensitivity larger than the SQL [4,5]. For example in [5] an optimal filtration procedure for the simplest variant of the optical sensor - a mirror attached to a mechanical resonator and illuminated with a coherent pump field - was considered. An external force acting on the mechanical oscillator displaces its equilibrium position and thus changes the phase of the reflected field. The vacuum fluctuations of the input light act on the oscillator through the radiation pressure effect and constitute the back action noise of the measuring apparatus. For such system two quadratures of reflected wave are correlated. Using correlation (phase sensitive) processing of two quadratures one can increase a signal-to-noise ratio and overcome the SQL.

However the gain in sensitivity for the schemes overcoming the SQL is usually proportional to the square root of the ratio of laser power used for pumping the interferometer and an optimal power that corresponds to the point where the sensitivity of the interferometer achieve the SQL [4-6]. Unfortunately the optimal power is impractically large, about several dozens of kilowatts that restrains the experimental implementation of the technique.
The pumping with the ultrashort periodic laser pulses can be technically advantageous over a continuous wave pumping for practical realization of the schemes overcoming the SQL. Actually for a large power a problem of generating a train of short high-intensity laser pulses can be technically easier than a problem of cw light generation (when the averaged powers for two cases are equal) because in the first case the energy in laser resonator is spread over the large frequency band (and different spatial longitudinal modes) and high intensities can be produced relatively easily. At the same time the amplitude and frequency stability of the pulsed pump in the case of a mode locked laser can be at the same level as for the monochromatic pump [7,8]. For example in [8] the stability of intermode beats for the mode locked laser output was estimated as $5 \cdot 10^{-12}$ in 10 s.

Another consideration is that the perspectives of squeezed states generation with high nonclassicality seem more realistic for the case of short high-intensity laser pulses allowing the use of squeezed pulsed pump in displacement transducers [9].

Finally an analog to digital conversion is usually used in modern experiment during the processing of the output. Therefore it seems natural to take the pulsed pump at once so that the output will comprise a set of the values for appropriate variable at definite times.

The goals of this article are to consider a displacement transducer consisting of a mirror attached to a mechanical oscillator and illuminated with a train of high-intensity laser pulses, to reveal the algorithm of optimal signal processing for such transducer and to estimate the sensitivity of the scheme to a measurement of classical external force.

The model of displacement transducer and basic equations of motion is considered in section 2. The sensitivities for traditional measurement scheme and for correlative processing of the output quadratures in the case of time independent pump are estimated in sections 3 and 4 correspondingly. The pulsed pump for the displacement transducer is considered in section 5. The conclusions are in section 6.

2 Model for displacement transducer and transformation of quadrature components

Let consider the most simple case of optical displacement transducer - a mirror attached to a mass of a mechanical oscillator and illuminated with a train of high-intensity laser pulses. An external force displaces an equilibrium position of mechanical oscillator changing the phase of reflected wave. The variation of the reflected field phase is measured by a readout system. This model is easy to calculate and it contains at the same time all features of displacement transducers with pulsed pump. For the incident $E_i$ and reflected $E_r$ waves one can use the quasimonochromatic approximation

$$
E_i = (A(t-x/c) + a_1) \cdot \cos \omega_p(t-x/c) - a_2 \cdot \sin \omega_p(t-x/c)
$$

$$
E_r = (B(t+x/c) + b_1) \cdot \cos \omega_p(t+x/c) - b_2 \cdot \sin \omega_p(t+x/c)
$$

where $A(t-x/c)$ and $\omega_p$ are an amplitude (mean value) and a frequency of the pump wave, $a_1$ and $a_2$ are the operators of the quadrature components (fluctuations) of the pump wave (vacuum for coherent state), $B(t+x/c)$ is an amplitude (mean value) of the reflected wave, $b_1$
and $b_2$ are the operators of the quadrature components (fluctuations) of the reflected wave. The periodic envelope function $A(t - x/c)$ consists of a train of equally spaced pulses and the duration of each pulse is considerably larger than the period of light wave but considerably smaller than the period of the mechanical oscillator.

To obtain the equation coupling the amplitudes of the incident and reflected waves for the moving mirror one can use a transformation of electromagnetic field for moving reference frame [10]. For a constant velocity of the mirror $V$ one has

$$E_r = -[(1 - V/c)/(1 + V/c)] \cdot E_i \exp(-2i\omega_p X/c)$$  \hspace{1cm} (2)

where for simplicity the reflection coefficient of the mirror is taken to be $r \approx -1$ and $X$ is the position of the mirror. Let suppose that this expression is valid also for the slowly varying velocity $V(t)$ and position $X(t)$ of the mirror and $|V(t)| \ll c$ (the validity of equation (2) has been proved for the mirror consisting of free electrons for the general case of relativistic velocity $V(t)$ in [11]). Then in linear approximation in $V/c$ one can obtain from equation (2) the following expression

$$E_r = -(1 - 2V(t)/c - 2i\omega_p X(t)/c) \cdot E_i$$  \hspace{1cm} (3)

The first term in (3) is an amplitude modulation of the reflected wave due to the mirror movement and the second is a phase modulation. For slow motion of the mirror $V \approx \omega_\mu X$ ($\omega_\mu$ is a frequency of mechanical oscillator) and the second term in brackets is considerably smaller than the third term. Therefore for the transformation of the quadrature components of the field one can obtain

$$b_1(t) = -a_1(t)$$ $$b_2(t) = -a_2(t) + 2\omega_p A(t)X(t)/c$$ \hspace{1cm} (4)

For the equation of mirror motion one has

$$\ddot{X}(t) + 2\delta_\mu \dot{X}(t) + \omega_\mu^2 X(t) = M^{-1}(F_s(t) + F_p(t) + F_{th}(t))$$ \hspace{1cm} (5)

where $M$ and $\delta_\mu$ are the mass and the damping coefficient of mechanical oscillator, $F_s(t)$ is a signal force, $F_p(t)$ is radiation pressure force and $F_{th}(t)$ is a force associated with the damping of the oscillator. Let suppose for simplicity that $\delta_\mu$ tends to zero. Then the displacement $X(t)$ of the mirror will consist of two parts - a signal displacement $X_s(t)$ and a radiation pressure displacement $X_p(t)$. For $F_p(t)$ one has

$$F_p(t) = SA(t) \cdot a_1(t)/(4\pi)$$ \hspace{1cm} (6)

where $S$ is a cross section of the laser beam. Therefore the equations of motion for the displacement transducer are

$$b_1(t) = -a_1(t)$$ $$b_2(t) = -a_2(t) + 2\omega_p A(t)X(t)/c$$ $$\ddot{X}(t) + 2\delta_\mu \dot{X}(t) + \omega_\mu^2 X(t) = M^{-1}(F_s(t) + SA(t) \cdot a_1(t)/(4\pi))$$ \hspace{1cm} (7)
3 Sensitivity for a traditional measurement scheme

For traditional measurement scheme [4,6] the amplitude of the pump is constant. Therefore one can easily obtain the transformation relations for the quadratures $b_1$ and $b_2$ from equations (7)

\begin{align*}
    b_1(\omega) &= -a_1(\omega) \\
    b_2(\omega) &= -a_2(\omega) + \lambda \xi(\omega) A^2 a_1(\omega) + A \xi(\omega) F_s(\omega)
\end{align*} \tag{8}

where $\xi(\omega) = 2 \omega_p G(\omega)/c$, $G(\omega) = \left[M(-\omega^2 - 2 \delta \mu \omega + \omega^2)\right]^{-1}$ is mechanical oscillator transfer function and $\lambda = S/(4\pi)$.

Only quadrature $b_2$ contains the signal and it is this quadrature that is measured in traditional measurement scheme [4,6]. This corresponds to the measurement of the phase of the reflected wave. The first term in the right hand side of equation (8) for $b_2$ can be treated as an additive noise and the second term as a back action noise. For small pump amplitudes the sensitivity is increasing with the increase of $A$ because the signal is proportional to $A$. However for large pump amplitudes the second term in r.h.s. of (8) becomes dominant and the sensitivity is decreasing with the increase of $A$. Therefore there is an optimal value of the pump amplitude and the sensitivity to external force at this pump amplitude is just the SQL [6].

4 Correlative processing of quadratures for time independent amplitude of the pump

Two quadratures of the reflected field according to equation (8) have the dependence on the amplitude fluctuations of the incident field $a_1$. Therefore one can expect that the sensitivity can be increased for the correlative processing of the output [5,12]. Actually if one combine with appropriate weight coefficients the quadratures $b_1$ and $b_2$ of the output wave then in this combination the noise term depending on $a_1$ can be cancelled. This weighting can be done by a homodyne detector with appropriate choise of a local oscillator phase $\phi$.

Let the field of the local oscillator have the form

\[ E_L(t) = A_L \cos(\omega_p t + \phi) \tag{9} \]

Then the photodetector output is proportional to the following expression according to (1), (9)

\[ I_{pd} \propto A_L(b_1 \cos \phi + b_2 \sin \phi) \tag{10} \]

and at certain frequency $\omega_f$ one can obtain

\[ I_{pd} \propto A_L[a_1(\omega_f)(- \cos \phi + \lambda \xi(\omega_f) A^2 \sin \phi) - a_2(\omega_f) \sin \phi + A \xi(\omega_f) F_s(\omega_f) \sin \phi] \tag{11} \]

Therefore choosing the phase $\phi$ according to the equation ($\xi(\omega_f)$ is real for $\delta \mu = 0$)
\[ -\cos \phi + \lambda \xi(\omega_f) A^2 \sin \phi = 0 \] 

(12)

one can make the photocurrent insensitive to the amplitude fluctuations \(a_1\) of the input field at certain frequency \(\omega_f\) of the signal. In this case the increase of the pump amplitude \(A\) results in the relative increase of the output signal at frequency \(\omega_f\) according to the equation (11) with respect to the noise level defined by \(a_2\).

For compensation of the back action noise inside definite frequency band one has to use the time dependent local oscillator phase \(\phi(t)\) [12,13]. In this case the optimal dependence of \(\phi\) on \(t\) is defined by the displacement transducer transfer function \(\xi(\omega_f)\) and by the spectrum of the external force \(F_s(\omega)\) [12].

So a signal-to-noise ratio is proportional to \(A^2\) (there is no optimal power) and in principle there is no sensitivity limitation by the SQL. In real experiment when the pump power gets larger the output signal and noises get smaller according to equation (11) if the condition (12) is kept valid therefore when \(A\) becomes greater than a certain value then the noises of photodetector electronics can limit the sensitivity. However this noises have technical character and will be neglected in the following.

Another sensitivity restriction can arise due to the damping in mechanical oscillator (mirror) [14,15]. This problem is general for all supersensitive measurements. At the same time an intrinsic dissipation obtained in modern experiments for mechanical oscillator is far larger (by several orders of magnitude) than the value expected from the first principles [16] therefore it can be treated also as a technical problem now and will not be addressed below.

It is worth to mention that the increase in sensitivity over the usual measurement scheme occurs here due to utilization of the internal squeezing (self-squeezing) of the reflected beam because of the nonlinear (quadratic) interaction of the incident light and the mirror [17,18]. Actually two quadratures of the reflected beam are correlated and it is this fact that allow to use the correlative processing of the output. On the other hand the correlation of the quadratures according to equations (8) means the squeezing of the beam and the larger the correlation coefficient \(\lambda \xi(\omega) A^2\) the larger the internal squeezing [17].

5 Sensitivity for the pulsed pump

Let consider the periodic envelope \(A(t)\) which consists of a train of equally spaced pulses with duration \(\tau\) and period \(T\). The spectrum of this pump has also the form of a train of pulses in frequency domain with the distance between neighbour pulses

\[ \omega_q = 2\pi T^{-1} \] 

(13)

For the amplitude of the pump \(A(t)\) one can use now the expansion into the Fourier series

\[ A(t) = \sum_{n=-\infty}^{\infty} g_n \exp(-im\omega_q t) \] 

(14)

and the particular form of \(A(t)\) is defined by the set of Fourier amplitudes \(g_n\).

The response of the displacement transducer now have many frequency components at \(\omega = n\omega_q, n = 0, 1 \ldots\) according to the equations (4) and each frequency component contains
the signal part besides the radiation pressure force $F_p(t)$ have also wide spectrum (cf. (6)). So there are two problems: how to collect the signal parts from the whole spectral band of the output and how to achieve the compensation of the radiation pressure noise in the output. It is clear that the monochromatic local oscillator is inappropriate for the homodyning because quadrature $b_1(t)$ of the output signal contains in this case the quadrature $a_1(t)$ of the input noises only from one frequency and the radiation pressure force $F_p(t)$ in expression for $b_2(t)$ (cf. (7)) contains $a_1(t)$ from all frequencies $n\omega_q$ therefore the full compensation is impossible.

Fortunately two problems can be overcome by the use of the pulsed local oscillator with the amplitude time dependence resembling that for the pump.

For the radiation pressure displacement $X_p$ of the mechanical oscillator one has from equations (5), (6) and (14) the following expression

$$X_p(\omega) = G(\omega) F_p(\omega) = \lambda G(\omega) \sum_{n=-\infty}^{\infty} g_n a_1(\omega - n\omega_q)$$  \hspace{1cm} (15)

For the quadrature transformation one can obtain instead of (8) the following equations from (4) and (14)

$$b_1(\omega) = -a_1(\omega)$$
$$b_2(\omega) = -a_2(\omega) + 2\omega_p c^{-1} \sum_{k=-\infty}^{\infty} g_k (X_p(\omega - k\omega_q) + X_s(\omega - k\omega_q))$$  \hspace{1cm} (16)

Let suppose the local oscillator field in the form of

$$E_L(t) = A_L(t) \cos(\omega_p t + \phi)$$  \hspace{1cm} (17)

where the dependence of the amplitude $A_L(t)$ on $t$ is much slower than $\cos \omega_p t$. Then for the envelope of the local oscillator field $A_L(t)$ the Fourier expansion similar to (14) is valid

$$A_L(t) = \sum_{n=-\infty}^{\infty} e_n \exp(-in\omega_q t)$$  \hspace{1cm} (18)

The photodetector current has now the following form

$$I_{pd} \propto A_L(t)(b_1(t) \cos \phi + b_2(t) \sin \phi)$$  \hspace{1cm} (19)

and in the frequency domain one has

$$I_{pd}(\omega) \propto \cos \phi \cdot \sum_{n=-\infty}^{\infty} e_n b_1(\omega - n\omega_q) + \sin \phi \cdot \sum_{n=-\infty}^{\infty} e_n b_2(\omega - n\omega_q)$$  \hspace{1cm} (20)

Let consider different parts in the photodetector output. The first term in equation (20) depends only on the amplitude fluctuations of the input field according to (16)

$$\cos \phi \cdot \sum_{n=-\infty}^{\infty} e_n b_1(\omega - n\omega_q) = - \cos \phi \cdot \sum_{n=-\infty}^{\infty} e_n a_1(\omega - n\omega_q)$$  \hspace{1cm} (21)
The second term in equation (20) contains the signal and the noise parts. The noise part consists of the additive noise and the back action noise and has the following expression according to (15) and (16)

\[
\left[ \sin \phi \cdot \sum_{n=-\infty}^{\infty} e_n b_2(\omega - n\omega_q) \right]_{\text{noise}} = -\sin \phi \cdot \sum_{n=-\infty}^{\infty} e_n a_2(\omega - n\omega_q) + 2\omega_p c^{-1} \sin \phi \cdot \lambda \cdot \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} e_n g_k G(\omega - k\omega_q - n\omega_q) \left\{ \sum_{m=-\infty}^{\infty} g_m a_1(\omega - k\omega_q - m\omega_q) \right\} \tag{22}
\]

Let consider only the photocurrent at small frequencies \( \omega \approx \omega_\mu \). Then the main input into the photocurrent will be given by the resonant terms for which \( k + n = 0 \). With this supposition one has from equation (22)

\[
\left[ \sin \phi \cdot \sum_{n=-\infty}^{\infty} e_n b_2(\omega - n\omega_q) \right]_{\text{noise}} = -\sin \phi \cdot \sum_{n=-\infty}^{\infty} e_n a_2(\omega - n\omega_q) + \sin \phi \cdot \lambda \xi(\omega) \cdot \sum_{m=-\infty}^{\infty} e_m g_{-m} \cdot \sum_{n=-\infty}^{\infty} g_n a_1(\omega - n\omega_q) \tag{23}
\]

Comparing equations (21) and (23) one can conclude that full compensation of back action noise is possible only for

\[
e_n = \alpha g_n \tag{24}
\]

where \( \alpha \) is the same for all numbers \( n \) so the forms of pump and local oscillator fields have to be the same (apart from the scale factor \( \alpha \)).

Let now consider the signal part of the second term in the r.h.s. of equation (20). From equations (7), (16) and (20) one has

\[
\left[ \sin \phi \cdot \sum_{n=-\infty}^{\infty} e_n b_2(\omega - n\omega_q) \right]_{\text{signal}} = \\
\sin \phi \cdot \sum_{n=-\infty}^{\infty} e_n \left\{ \sum_{k=-\infty}^{\infty} g_k \xi(\omega - k\omega_q - n\omega_q) F_s(\omega - k\omega_q - n\omega_q) \right\} \tag{25}
\]

Evaluation of this expression for the condition \( k + n = 0 \) gives

\[
\left[ \sin \phi \cdot \sum_{n=-\infty}^{\infty} e_n b_2(\omega - n\omega_q) \right]_{\text{signal}} = \sin \phi \cdot \xi(\omega) F_s(\omega) \sum_{n=-\infty}^{\infty} e_n g_{-n} \tag{26}
\]

Combining equations (20), (23), (24) and (26) and supposing that the back action noise is compensated in the output of the photodetector one can obtain for the spectral density of noises in the photocurrent the following expression

\[
N(\omega) \propto \sin \phi \cdot N_0 \cdot \sum_{n=-\infty}^{\infty} g_n g_{-n} = \sin \phi \cdot N_0 P \tag{27}
\]
where it is supposed that fluctuations at frequencies $\omega - n\omega_q$, $n = 0, 1 \ldots$ are uncorrelated and have the same spectral density $N_0$ (this assumption is valid for not very small duration of pump pulses), $P$ is proportional to the time averaged power of the pulsed pump. Then for the signal-to-noise ratio $\mu$ one has from equations (26) and (27) the following expression

$$\mu \propto N_0^{-1} P \int_{-\infty}^{\infty} |\xi(\omega)F_s(\omega)|^2 d\omega$$

(28)

This value is just equal to the signal-to-noise ratio for continuous wave pump with a power $P$ and correlative processing of the output (cf. equation (11)). Note that the sensitivity here is not limited by the SQL like in the case of correlative processing of quadratures for the monochromatic pump and is increasing with the increase of $P$.

It is worth to mention that the condition for the back action noise compensation for the pulsed pump is just the same as for the monochromatic pump (cf. equation (12)) with substitution of the $A^2$ with the time averaged value $P$. Therefore the compensation of the back action noises for the finite frequency band can be possible for the time varying phase of the local oscillator [12, 13].

6 Conclusion

The pumping of the displacement transducer with a train of the short high-intensity laser pulses is considered. The algorithm of optimal signal processing for such transducer is revealed. It consists of the correlative processing of the output using the pulsed local oscillator with the same envelope as for the pump field (apart from the scale factor). In this case the back action noise due to the radiation pressure force can be fully compensated and the sensitivity of the scheme to a detection of a classical external force is not limited by the SQL (as for the case of correlative quadrature processing and monochromatic pump field).

The pulsed pump can be advantageous over the single frequency pumping when the non-linear optical elements are used inside the system. Thus considerable increase in sensitivity can be achieved for a gravitational interferometric Fabry-Perot type detector with a non-linear optical element placed in a waist of the beam [19]. The use of the phase-conjugate mirrors in a gravitational detector of the Michelson type allows to construct the system with the parallel arms [20]. For such systems an efficiency depends on the instant power of the light beam and can be high for the short intensive pulses.

In this article only the problem of the force detection with known spectrum is considered. The reconstruction of unknown external force acting on the displacement transducer with the pulsed pump below the standard quantum limit will be considered elsewhere.

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