I argue that perturbative scattering of quarks and gluons are incompatible with lattice and heavy ion data on QGP properties. The non-perturbative mechanisms for quasiparticle rescattering and quark production are briefly discussed, as well as experiments needed to measure matter anisotropy and quark density at early stages of the collisions.

I. EQUILIBRATION IN WEAK COUPLING

Let me start with my “naive weak coupling” approach in a quater-century-old paper [1]. It was based on the lowest order cross sections

\[
\frac{d\sigma}{dt}(gg \rightarrow gg) = \left(\frac{\pi\alpha^2}{s^2}\right)\left(\frac{9}{2} - \frac{ut}{s^2} - \frac{us}{t^2} - \frac{st}{u^2}\right)
\]

(1)

\[
\frac{d\sigma}{dt}(gg \rightarrow \bar{q}q) = \left(\frac{\pi\alpha^2}{s^2}\right)\left(\frac{u^2 + t^2}{6ut} - \frac{3u^2 + 3t^2}{8s^2}\right)
\]

(2)

I then noticed that at large angles the ratio of the two is very large \((gg \rightarrow gg)/(gg \rightarrow \bar{q}q) = 30/0.14 \sim 200\). At small angles also, the former has \(1/t^2\) term while the second has only \(1/ut\) term. I then concluded that kinetic equilibration of glue happens quicker than chemical equilibration of quarks, known as the “hot glue scenario”.

Before proceeding to more recent works, let me go directly to the point. Note that in kinetic equilibration of glue one needs to evaluate the transport cross section, which is divergent logarithmically

\[
\sigma_T = \int (1 - \cos(\theta))d\sigma.
\]

In chemical equilibration it is the total cross section which matters, \(\sigma = \int d\sigma\), which is divergent logarithmically as well.

The proper regulators of the \(t\)-channel gluon propagator are different for electric and magnetic exchanges. The former is regulated by the so called electric screening mass, which in weak coupling is [2]

\[
M_E^2 = g^2T^2(1 + \frac{N_f}{6})
\]

(3)

while, as also shown there, the magnetic fields remains unscreened in pQCD. Furthermore, the magnetic part has structure

\[
\frac{1}{Q^2} \Pi(Q) \sim Q^2
\]

because magnetic polarization tensor is \(\Pi(Q) \sim Q^2\). The combination of electric and magnetic effects leads to the following substitution in the cross section

\[
\left(\frac{1}{Q^2}\right)^2 \rightarrow Q^2(Q^2 + M_E^2)
\]

which makes the transport cross section convergent.

Modern version of the kinetics at weak coupling, with consistent IR resummation of relevant higher-order diagrams, was developed in [3], and issue of quark chemical equilibration has been recently discussed in [4]. The key role in it is played by gluon exchanges with soft “splitting” of gluons, \(g \rightarrow gg, \bar{q}q\). In order to show their role better, we show in Fig[4] the squared matrix element (without coupling in front) integrated over angle to the transport cross section, as a function of the electric screening mass (normalized to particle CM momenta \(p\)). One can see that, even after we factor out the coupling, there are basically two regimes: (i) small screening masses \(m_E/p \ll 1\), and (ii) large ones \(m_E/p \sim 1\). It is the former regime (used in the above-mentioned works) the transport cross section is larger than in the latter by about an order of magnitude.

The main conclusion of Ref.[4] is that there is no time hierarchy of processes discussed, and thus no “hot glue” scenario: the kinetic and chemical equilibration happen at the same time.

Their calculation is done for the range of the \(t\)’ Hooft coupling \(\lambda \equiv g^2 N_c = 0.1, 1, 10\) or
FIG. 1: The angle-integrated matrix element squared $\int (1 - \cos(\theta)) |M_{gg}|^2 d\cos(\theta)$ versus the electric screening mass $M_E/p$ normalized to the particle CM momenta.

$\alpha_s \approx 1/300, 1/30, 1/3$. The authors of course know that those values do not correspond to realistic coupling in experimentally produced QGP, and that at such couplings the both equilibration times are way too long to explain the data, such as collective flows. Yet they hope one can use these calculations to understand the dependence on the coupling of physical quantities and then safely extrapolate their results to "realistic" couplings.

II. CAN ONE SUCCESSFULLY EXTRAPOLATE, FROM WEAK TO STRONG COUPLING?

Let me on the onset say that my theoretical prejudice is to answer this question negatively. Let me outline two general theoretical reasons for this opinion, before plunging into details.

(i) In weak coupling the matter is in a gas-like phase, and therefore particle interactions are adequately represented by a kinetic equation, with a mean free path as a key parameter. The famous Boltzmann’s hypothesis, that many-body distributions all factorize into a product of single-body ones, is justified. However, when coupling is large and potential energy is comparable to temperature, the matter changes to a liquid-like form, and eventually solidifies. Two- (and more) particle correlations are present permanently, there is no Boltzmann reduction and cascades. There are no in and out states, with particles not going to infinity but being near each other all the time. It is well documented even for many classical systems, studied by molecular dynamics.

(ii) The QCD-like non-Abelian theories and at $T/T_c \sim O(1)$ are known to posses strong non-perturbative phenomena, induced by gauge fields in forms of solitons – monopoles, instantons, instanton-dyons. Those are invisible in pQCD but play a significant role in dynamics (more of that at the end).

Let us for now return to Ref.[4]. For their three values of the coupling the viscosity-to-entropy-density-ratio $\eta/s$ is equal to 1900, 35 and 1, respectively. The last value is about factor 6 from the empirical value, and, taken large spread of values, one may think that extrapolation to it may be possible.

The situation is shown in Fig.[2]. We show two of the three points from [4], as the last one corresponding to $T \sim 10^{100} GeV$ is hard to fit even in the log-log plot. My point is that the key left point, with coupling $\alpha_s \sim 1/3$, should not be located as it is calculated kinetically and plotted in Fig.[2(b)] because under this conditions the perturbative expression for the screening mass (3) is invalid.

We know it from lattice studies (which do not require transition from Euclidean to Minkowski world, and are thus quite reliable.) In Fig.[3] one finds relatively recent lattice data, done by well respected Wuppertal-Budapest collaboration and carefully extrapolated to continuum limit, for $M_E/T$ ratio. Instead of being small, it is around 7.5 (!). Other lattice groups give other numbers, but none of them finds a value even close to 1, always significantly larger, 5-15. What it means, simply speaking, is that assumed dominance of small angle scaterings or “soft splittings”, is in fact in direct contradiction to lattice data. If one uses the lattice values of the $M_E/T$, one appears in the large angle regime, and therefore the transport cross section moves from the left side of Fig.[1] to the right. Then the left square point in Fig.[2(b)] needs to be moved down, by an order of magnitude or so. The second point – roughly corresponding to coupling in electroweak plasma – remains at the same place. Now, looking at them, one cannot imagine that any smooth extrapolation to the correct $s/\eta$ value may exist.
Let me give now two more general reasons why one needs to think here about magnetic monopoles, and not extrapolation of soft gluon scattering.

(i) A shortcoming of pQCD viewpoint is that it cannot explain a nonzero magnetic screening mass. But, as we see from the lattice results just shown in Fig. 3, the magnetic mass is not only nonzero, but is even comparable to the electric one!

(ii) the peak in $s/\eta$ shown in Fig. 2 is located near the deconfinement temperature $T_c$, and therefore one may suspect that one should be related to the other. The monopoles, detected on the lattice, have density peaking near $T_c$, with the magnitude comparable to that of quarks and gluons. Furthermore, this density is large enough for their Bose-Einstein condensation (BEC) to occur, at the deconfinement transition $T = T_c$. Multi-monopole Bose-clusters were observed on the lattice \cite{3} and studied in Path Integral Monte Carlo \cite{8}. The dance made by monopoles near $T_c$ is remarkably similar to that of $^4He$ atoms near the lambda point.

It was shown in \cite{10} that gluon scattering on these monopoles has interesting angular distribution, with a backward peak. Unlike $gg$ scattering, it gives the transport cross section consistent with the observed $\eta/s$.

More recently, the puzzling angular distribution of the jet quenching was explained by the monopole contribution, see \cite{11,12}.

III. THE MAGNETIC SCREENING, MONOPOLES AND VISCOSITY

FIG. 2: Both plots show the inverse $s/\eta$ ratio versus the temperature $T$ (GeV). The upper linear plot shows the empirical value (red) and the lattice result \cite{6}, with the error bars. The four points with line are from \cite{10}, representing gluon-monopole scattering. The line without points on the left corresponds to pion rescattering. The lower log-log plot includes also points from weak coupling cascades \cite{4} shown by two blue squares. The arrow and red square correspond to transition from small to large angle regime, discussed in the text.

FIG. 3: The continuum extrapolations of the electric and magnetic screening masses, normalized to the temperature, from Borsanyi et al.

We note, that our results are closest to the results from dimensionally reduced effective field theory results from Ref. \cite{42}. The continuum extrapolations of the screening masses and the ratio of the screening masses, normalized to the temperature, from Borsanyi et al. 

$T_c = 150$ MeV, no continuum extrapolation $m/T_c$=7

$T_c = 200$ MeV, with the error bars. The four points with line are from \cite{41}, representing gluon-monopole scattering. The line without points on the left corresponds to pion rescattering. The lower log-log plot includes also points from weak coupling cascades \cite{4} shown by two blue squares. The arrow and red square correspond to transition from small to large angle regime, discussed in the text.
IV. QUARK PRODUCTION VIA THE INSTANTON/SFHALERON MECHANISM

Perturbative production of quark pairs is not the only way chemical equilibration of QGP can proceed. Instanton contribution to inelastic hadronic collisions were discussed in [13]. It was then realized that this process leads to instanton-sphaleron conversion, with subsequent (over-the barrier or Minkowskian) sphaleron explosion [18]. Recently, in Glasma model framework, the sphaleron quark production has been studied in [15], which concluded that this mechanism is quite effective.

There is a very important distinction between the pQCD (both soft or hard) production of quarks, and the sphaleron mechanism. The former produce left–left or right–right polarized pairs, and thus does not produce any chiral imbalance in the QGP. On the contrary, the sphaleron mechanism produces final states like $(\bar{u}_R u_L)(\bar{d}_R d_L)(\bar{s}_R s_L)$, with 6 units of axial charge per event. Although average is still zero, fluctuations can create chiral imbalance.

V. SUMMARY AND DISCUSSION

The main point of these comments is that not only a weak coupling regime is not occurring in realistic QGP conditions, but it is also impossible to get its kinetic properties by extrapolation from weak coupling.

Soft kicks following parton splitting works well in jet quenching, the BDMPS theory etc, because the momentum is large

$$|\vec{p}| \sim 10 - 1000 \text{GeV} \gg Q \sim 1 \text{GeV}$$

The QGP constituents with the typical gluon momenta $p \sim 3T \sim 1 \text{GeV}$ are not in this regime. They cannot be softly split into two, just because their effective masses are also $\sim M_E \sim 1 \text{GeV}$. Furthermore, there are no soft gluon exchanges because the screening masses are that large.

The debate between weak and strong coupling scenarios of heavy ion collisions is in fact rather old. It was intense around the year 2000, before the RHIC era. The soft gluon exchanges were the basis of the so called bottom-up scenario [5]. Through 1990s nearly all high energy physicists were telling us that there is no hope to produce new form of matter in heavy ion collisions, and all we will see would be a fireworks of minijets, without any collective effects. The predictions of Molnar (in [16]), based on exactly this soft gluon cascade, was that $v_2$ should drop down at RHIC.

Fortunately, this pessimistic point of view was spectacularly overthrown in the first years of RHIC operation. The data confirmed instead the robust hydro explosion, with hydrodynamics describing it quite accurately. The observed elliptic flow growth with $p_T$ to large values was never reproduced by gluon cascades, even with huge assumed cross sections (completely incompatible with screening masses).

Further observation of elliptic flow and higher harmonics has lead to viscosity measurement, giving the value we discussed above. The notion of strongly-coupled QGP has prevailed. Using AdS/CFT correspondence one found good description of rapid convergence to hydro regime, in a time of fraction of fm/c. The equilibration mechanism was found to proceed in the opposite direction, from UV to IR (top-down scenario) [17].

Apparent resurgence of weak coupling methods in the last few years looks quite surprising. I even heard statements at some meetings of “hydrodynamics without a fluid”. So, let me state it again: the bottom-up scenario and weakly coupled cascades in general were before and still are completely incompatible with the data on elliptic flow. It is of course especially obvious for “small system”, $pA,pp$ in which flows were discovered lately.

While the issue of weak-versus-strong-coupling-equilibration was, in fact, resolved some time ago, direct measurements of anisotropy of matter at early time would be desirable. The proposal how to do so by dilepton polarization has been made in [19].

The issue of quark production/equilibration however still requires a lot of work. Can we check experimentally which mechanism of QGP chemical equilibration is in place in real-world heavy ion collisions?

One way proposed is to use dileptons [18], more specifically the so called “intermediate mass dileptons” (IMD) (between $\phi$ and $\psi$ peaks) produced early in the collision. If the quark production is delayed, one expects a
deficit of such dileptons in respect to standard calculations assuming fully equilibrated QGP. To my knowledge no such deficit has been reported in all comparisons made so far, although the accuracy of that needs to be further investigated.

A specific consequence of sphaleron mechanism is chiral imbalance, on event-by-event basis. This was an important assumption in well known proposal to observe the chiral magnetic effect (CME). Hopefully, recent RHIC run with two isotopes of $A = 96$ will clarify the effect of magnetic field and magnitude of the CME. As a consequence, it should be able to establish the magnitude of the sphaleron production rate.

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