Some parameters of fuzzy labeling tree using matching and perfect matching

S. Yahya Mohamed 1* and S. Suganthi 2

Abstract
A graph is said to be a complete fuzzy labeling graph if it has every pair of adjacent vertices of the fuzzy graph. A matching is a set of non-adjacent edges. If every vertex of fuzzy graph lies on the matching then the matching is said to be complete or perfect. In this paper, we introduce the new parameters in fuzzy labeling tree using matching and perfect matching like distance, length, eccentricity and center. We discuss some properties using these parameters.

Keywords
Fuzzy labelling tree, spanning subgraph, distance, eccentricity and center.

AMS Subject Classification
03E72, 05C72, 05C78.

1. Introduction
Graphs are used to represent networks of communication. Graph theory is used to find shortest path in road or a network. In Google Maps, various locations are represented as vertices or nodes and the roads are represented as edges. Graph theory is used to find the shortest path between two points. In graph theory, a tree is a connected acyclic undirected graph.

Fuzzy set is one of the main branches of modern mathematics, having more development in recent years. The concept of fuzzy set was introduced by L.A Zadeh [15] in 1965. Using this fuzzy subsets, A.Rosenfeld was introduced the fuzzy graphs in 1975.

A. Nagoorgani, D. Rajalaxmi introduced the concept of fuzzy labelling tree [4] and S.Yahya Mohamad, S.Suganthi [8] introduced matching in fuzzy labelling graph [10]. [7] Azriel Rosenfeld in 1975 developed the structure of fuzzy graphs and obtained analogs of several graph theoretical Concepts like bridges and tree.

In this paper, we introduce the new parameters in fuzzy labeling tree using matching and perfect matching like distance, eccentricity and center. We discuss some properties using these parameters. Here we consider the complete fuzzy labelling graph with even number of vertices.

2. Preliminaries

Definition 2.1. Let X and Y be two sets. Then δ is said to be a fuzzy relation from X into Y if δ is a fuzzy set of X × Y. A fuzzy graph FG = (µ, γ) is a pair of functions γ : X → [0, 1] and γ : Y × Y → [0, 1] where for all x, y ∈ X, we have

γ(x, y) ≤ min{µ(x), µ(y)}.

Definition 2.2. Let G : (µ, γ) be a fuzzy graph and H is a subset of G. If points of H is contained (or) equal to the nodes of G then H is said to be a fuzzy subgraph. It is denoted by FSG.

Definition 2.3. A FSG H of the FLG G is said to be fuzzy spanning sub graph [FSSG] of G if points of fuzzy sub graph is equal to the points of fuzzy labeling graph.
Theorem 3.3. Let $T$ be a fuzzy labelling tree with even number of vertices. Then the distance between every pair of points in $T$ is equal to the cardinality of perfect matching in $T$.

Proof. Consider a complete fuzzy labelling tree $T$ with even number of vertices.

To prove

$$d(v_i, v_j) = |M| \forall v_i, v_j \in T.$$ 

The number of perfect matching in $K_{2n}$ is $(2n)!/n!2^n$. The number of lines in the perfect matching is equal to half of the number of edges in $T$.

In a complete graph, every vertex has equal degree. So the number of lines in alternating path connecting any pair of vertices is same.

By the definition, the distance between any pair of vertices $v_i, v_j$ is defined by the number of lines in the shortest alternating path connecting these two points.

Since $T$ is complete, the shortest alternating path connecting any two points contains same number of edges.

Therefore the distance between every pair of vertices in $T$ is same and which is equal to the number of edges in the perfect matching.

Hence

$$d(v_i, v_j) = |M| \forall v_i, v_j \in T.$$

Definition 3.4. The distance between every pair of vertices in $T$ is a metric. (i.e) The distance satisfies

(i) Non-Negativity condition. $d(v_i, v_j) \geq 0$.

(ii) Symmetric condition. $d(v_i, v_j) = d(v_j, v_i)$

(iii) Triangle inequality:

$$d(v_i, v_z) \leq d(v_i, v_j) + d(v_j, v_z) \forall v_i, v_j, v_z \in T$$

Definition 3.5. The length of the fuzzy labelling tree $T$ is defined as the number of edges in the FSSG which is a tree in which every pair of vertices joined by an alternating path.

Example:

In $K_4$, there are three perfect matching exists. Each perfect matching contains two edges.

Here

$$d(v_1, v_2) = 2 : d(v_1, v_3) = 2 : d(v_1, v_4) = 2$$

$$d(v_2, v_3) = 2 : d(v_2, v_4) = 2 : d(v_3, v_4) = 2.$$
**Theorem 3.6.** Fuzzy Labeling Tree with $2n$ vertices has length $2n - 1$.

*Proof.* Let us consider $T$ be a fuzzy labeling tree with even number of vertices. Now we find the spanning subgraph of $T$.

A fuzzy labeling graph is a fuzzy tree if it contains a spanning subgraph which is a tree in which every pair of vertices joined by an alternating path.

We know that every tree with $n$ vertices has $n - 1$ edges. Hence all the spanning subgraph of $T$ contains the edges which is less than one of its vertices.

Therefore every complete fuzzy labeling tree contains a spanning subgraph with $2n$ vertices has $2n - 1$ edges. Also every pair of vertices joined by an alternating path.

Hence A Fuzzy Labeling Tree with $2n$ vertices has length $2n - 1$. \qed

**Example:**

Calculation of eccentricity:

- $e_1(0.2)$
- $e_2(0.4)$
- $e_3(0.3)$
- $e_4(0.6)$
- $o_4(0.35)$

(FSSG)

Here length of FLT is 3.

**Theorem 3.7.** Let $T$ be a fuzzy labeling tree and $S$ be spanning subgraph of $T$. For any edge $e$ in $T$, $S + e$ contains a unique cycle.

*Proof.* Let $T$ be a fuzzy labeling tree and $S$ be spanning subgraph of $T$.

Consider $e$ be an edge in $T$ but it is not in $S$.

By the definition of fuzzy labelling tree, its spanning subgraph is a tree. (ie) $S$ is a connected acyclic graph.

We know that a tree with $n$ vertices has $n - 1$ edges and there exists one and only one path between every pair of vertices in a tree.

Here every spanning subgraph with $2n$ vertices has $2n - 1$ edges. Now we add an edge $e$ to $S$.

We obtain unique cycle in $S + e$.

Hence for any edge $e$ in $T$, $S + e$ contains a unique cycle. \qed

**Definition 3.8.** The eccentricity of a vertex $v$ $\rho(v)$ in a fuzzy labeling tree is defined as the distance between $v$ to the farther vertex in its spanning subgraph $S$. 


Example:

\[ T \]
\[ S \]

Here \( \rho(v_1) = 3, \rho(v_2) = 3 \rho(v_3) = 2 \rho(v_4) = 2. \)

**Definition 3.9.** The center of FLT is a vertex with least eccentricity in its spanning subgraph \( S \).

**Note:** In the above FLT centre is \( v_3 \) & \( v_4 \).

**Theorem 3.10.** A fuzzy labelling tree \( T \) has always two adjacent centers and which is a matching bridge in any one of perfect matching.

**Proof.** Consider a complete FLT \( T \) with even number of vertices and \( S \) be the spanning subgraph of \( T \).

Now we have to find the eccentricity of all vertices in \( S \). The eccentricity of a vertex \( \rho(v) \) is the distance between \( v \) to the farther vertex in its spanning subgraph \( S \).

Here exactly two vertices occur in same eccentricity because \( S \) contains all vertices of \( T \) and there exists only one alternating path between every pair of vertices.

There exists an edge between these two vertices and that edge belongs to any one of its perfect matching.

Hence \( T \) has always contains two adjacent centres.

\[ \square \]

### 4. Conclusion

In this paper, we introduced the new parameters like length of \( T \), distance, eccentricity and center in fuzzy labeling tree and its spanning sub graph. We discussed some properties using these concepts. In Future, we extend this concept in some special types of fuzzy labelling tree using matching and perfect matching.

---

**References**

1. Dheer Nial Sunil Desai and Kamal Lochan Patra, Maximizing distance between Center, centroid and sub tree core of trees, *Proc. Indian. Acad. Sci (Math. Sci)*, 129(7)(2019).

2. Frank Harary, *Graph Theory*, Indian Student Edition, Narosa/Addison Wesley, 1988.

3. S. Mathew and M. S. Sunita, Types of arcs in a fuzzy graph, *Information Sciences*, 179(11)(2009), 1760-1768.

4. A. Nagoorgani, D. Rajalaxmi, Fuzzy Labeling tree, *International Journal of Pure and Applied Mathematics*, 90(2)(2014), 131-141.

5. Narsingh Deo, *Graph Theory with Applications to Engineering and Computer Science*, India, 1999.

6. K. Ranganathan, R. Balakrishnan, *A Text Book of Graph Theory*, Springer, 2000.

7. A. Rosenfeld, Fuzzy graphs, Fuzzy Sets and their Applications to Cognitive and Decision Process, L. A. Zadeh, K. S. Fu, K. Tanaka and M. Shimura, eds., Academic Press, New York, (1975), 75-95.

8. M.S. Sunita and A. Vijayakumar, A characterization of fuzzy trees, *Information Science*, 113(1999), 293-300.

9. K. Uma, Radius, Diameterand center of directed fuzzy graphs using Algorithm, *International Journal of Scientific and Research Publications*, 7(4)(2017), 1–12.

10. S. Yahya Mohamed, S. Suganthi, Matching in Fuzzy Labeling Graph, *International Journal of Fuzzy Mathematical Archive*, 14(1)(2017),155-161.

11. S. Yahya Mohamed, S. Suganthi, Properties of Fuzzy Matching in Set Theory, *Journal of Emerging Technologies and Innovative Research*, 5(2)(2018).

12. S. Yahya Mohamed, S. Suganthi , Operations in Fuzzy Labeling Graph through Matching and Complete Matching, *International Journal of Applied Engineering Research*, 13(12)(2018), 10389-10393.

13. S. Yahya Mohamed, S. Suganthi, Energy of Complete Fuzzy Labeling Graph through Fuzzy Complete Matching, *International Journal of Mathematics Trends and Technology*, 58(3)(2018), 1–12.

14. S. Yahya Mohamad, S. Suganthi, Matching and Complete Matching Domination in Fuzzy Labelling Graph, *Journal of Applied Science and Computations*, 5(10)(2018), 12–20.

15. L. A. Zadeh, Fuzzy Sets, *Information and Control*, 8(1965), 338-353.

**********

ISSN(P):2319 – 3786
Malaya Journal of Matematik
ISSN(O):2321 – 5666
**********