Detecting binarity of GW150914-like lenses in gravitational microlensing events

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ABSTRACT
The recent discovery of gravitational waves (GWs) from stellar-mass binary black holes (BBHs) provided direct evidence of the existence of these systems. BBH lenses would have gravitational microlensing signatures that are distinct from single-lens signals. We apply Bayesian statistics to examine the distinguishability of BBH microlensing events from single-lens events under ideal observing conditions, using the photometric capabilities of the Korean Microlensing Telescope Network. Given one year of observations, a source star at the Galactic Centre, a GW150914-like BBH lens (total mass 65 $M_\odot$, mass ratio 0.8) at half that distance and an impact parameter of 0.4 Einstein radii, we find that binarity is detectable for BBHs with separations down to 0.0250 Einstein radii, which is nearly 3.5 times greater than the maximum separation for which such BBHs would merge within the age of the Universe. Microlensing searches are thus sensitive to more widely separated BBHs than GW searches, perhaps allowing the discovery of BBH populations produced in different channels of binary formation.

Key words: black hole physics – gravitational lensing: micro – methods: numerical.

1 INTRODUCTION
The 2015 discovery (Abbott et al. 2016a) of gravitational waves (GWs) provided the first direct proof of the existence of stellar-mass binary black holes (BBHs). This Letter investigates the feasibility of identifying such systems in our Galaxy through gravitational microlensing. BBH systems with the potential to produce detectable GWs have sufficiently small separations to complete hundreds of orbits over the course of a lensing event.

This realm of microlensing, where the orbital motion of the lens is detectable, is well explored. Zheng & Gould (2000) discussed the effects of superluminal caustics due to the rapid rotation of tight binary lenses. Dubath, Gasparini & Durrer (2007) examined the contribution of the quadrupole of compact binary lenses to microlensing signals. Penny, Mao & Kerins (2011a) examined the detection efficiency of orbital motion in binary microlensing events with varying parameters, while Penny, Kerins & Mao (2011b) extended this investigation to include binary lenses that show at least one repeating feature in their light curve. Nucita et al. (2014) studied the use of orbital motion signals in microlensing to identify the binary lens period. Sajadian (2014, 2015) explored the efficiency of detecting binary astrometric microlensing signals. Our goal is to apply Bayesian statistics to the binary microlens to examine whether BBH microlensing signals from within our Galaxy could be discernible with current telescopes under ideal observing conditions (i.e. high cadence over the course of the event and no extinction).

This work is structured as follows: In Section 2, we develop our BBH lensing model. In Section 3, we apply Bayesian statistics to constrain detectability. We present our results in Section 4 and our conclusions in Section 5.

2 METHODS
The binary lens has been explored in detail (e.g. see Schneider & Weiss 1986; Di Stefano & Scalzo 1999; Han et al. 1999). The lensing equation generalized for $N$ point lenses is (Dominik 1999)

$$y = x - \sum_{r=1}^N m_r \frac{x - x^{(r)}}{|x - x^{(r)}|^2},$$

where $y$ refers to the source position in the source plane normalized to the Einstein radius, $x$ corresponds to the normalized image position in the lens plane, $x^{(r)}$ corresponds to the normalized position of the $r$th lens and $m_r$ are the mass fractions of the lensing objects. We use (1) for the binary case, where the two black holes are assumed to travel on circular orbits about their centre of mass.

Our BBH system is described by eight parameters: total mass $M$ of the black holes, mass ratio $q$ of the black holes, binary separation $s_0$ (normalized to the Einstein radius), impact parameter $u_0$.
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Figure 1. Top left-hand panel: the BBH lens photometric light-curve magnification. Bottom left-hand panel: the residual light curve, centred at the event peak, calculated by subtracting the single lens signal from the binary lens signal. The scale of the residual is $10^{-5}$ that of the light curve above it. Top right-hand panel: the BBH lens source path (blue line), image paths (red +) and centroid position (black circles), with the BBH centre of mass at the origin. While three images are produced in the event, the third is highly de-magnified and therefore negligible for our analysis. Bottom right-hand panel: seven sample curves of the residual centroid, each corresponding to the residual centroid shift during one period of the BBH orbit. The curves are sampled approximately 50 d apart, with the fourth curve corresponding to the peak of the event. The shading moves from pale to dark as time progresses during the event. All four plots use our fiducial model: total mass $M = 65 M_{\odot}$, mass ratio $q = 0.8$, binary separation $s_0 = 0.01$ Einstein radii, proper motion $\mu_{\text{rel}} = 5.5$ mas yr$^{-1}$, impact parameter $u_0 = 0.4$ Einstein radii, inclination $\iota = \arccos 0.6$, true anomaly $f = 1$ rad and direction of proper motion $\varphi = 0.3$ rad.

(normalized to the Einstein radius), proper motion $\mu_{\text{rel}}$ of the source relative to the lens, inclination $\iota$, true anomaly $f$ at the closest approach of the source and angle $\varphi$ between the proper motion direction and the major axis of the BBH orbit’s projected ellipse.

The BBHs of interest here have separations much smaller than their Einstein radius and generally much smaller than the impact parameter. Given such small separations, caustic crossings – whether superluminal, such as those discussed in Zheng & Gould (2000), or otherwise – are unlikely (see Dominik 1999), and we therefore assume that the source star is a point source, as well as that three images are produced rather than five (though we exclude one of the images in our analysis due to its extreme faintness). BBHs of small binary separation will also have orbital periods much shorter than the duration of the lensing event. Thus, their microlensing light curves will consist of a quasi-periodic perturbation superimposed on the standard curve produced by a single lens of the same total mass (Guo et al. 2015). Our fiducial model corresponds to an orbital period of 8.4 d; therefore, the quasi-symmetric period of the lensing signal will be 4.2 d.

Using the eight model parameters, we find the projected lens and source positions. The partial derivatives of our modified equations comprise a Jacobian matrix; inverting the determinant of the Jacobian matrix yields the image magnifications. We iterate through the lensing event, calculating the image positions and magnifications numerically. Our model assumes exposure times that are shorter than the binary period.

The analytical image positions of a single lens with corresponding system parameter values provide initial guesses for our numerical approximation. The binary lens’s quasi-sinusoidal deviation from the standard single-lens light curve is small compared to the curve itself, so the single-lens image positions are an efficient means of initiating the solver. As expected (see Dominik 1999), the amplitudes of the binary lens perturbations are proportional to the quadrupole moment $\eta M_0 s_0^2$, where $\eta = q/(1 + q)^2$. Fig. 1 shows plots of binary and residual light curves, the binary image paths and centroid, and the residual centroid. Note that the Einstein ring crossing time-scale of the lensing event for our fiducial model parameters (see Fig. 1 caption) is $t_E \equiv \theta_E/\mu_{\text{rel}} = 1.48$ yr.

3 MODEL SELECTION

3.1 Bayesian statistics

For a binary lens to be distinguishable from a single lens, the size of the perturbations due to the binary lens must be detected with statistical certainty. Towards this end, we employ Bayesian statistics (e.g. see O’Hagan 1994). Trotta (2008) described the advantages and disadvantages of employing Bayesian rather than frequentist statistics.

The Bayesian likelihood, $L \equiv \exp(-\chi^2/2)$, measures how closely a model fits data:

$$\chi^2 = \sum_k \frac{(d_k - m_k)^2}{\sigma_k^2}. \quad (2)$$

Here, we sum over $k$ points, where $d_k$ is the observed event signal measured in number of photons, $m_k$ is the modelled result (also in
photons) and $\sigma_\xi$ represents the error associated with each measured point. In lieu of real observations, the output of the lensing code with event parameters serves as the signal. As such, for a model with parameters equal to those used to generate the event signal, all $m_{0i} = d_i$ and $L = 1$.

The Bayesian evidence $E$ is used for model selection. The evidence is the integral of the likelihood multiplied by the prior $p$ over all parameter space (e.g. O’Hagan 1994).

$$E = \int L p \, d\xi.$$ 
(3)

With our eight-parameter binary model, numerically evaluating the integral in (3) is computationally expensive. Therefore, we use the Fisher matrix to approximate the likelihood (e.g. Hobson et al. 2010).

3.2 Fisher matrix approximation

Using exposure times shorter than the binary period, the appropriate Fisher matrix is

$$F_{ij} = \sum_k \frac{1}{\sigma_k^2} \frac{\partial m_k}{\partial \xi_i} \frac{\partial m_k}{\partial \xi_j},$$
(4)

where $\xi$ is the set of parameters, $\sigma_k$ are the photometric errors associated with the 4th point and $\frac{\partial m}{\partial \xi}$ are numerical derivatives of the photometric lensing curve.

The resulting real symmetric $8 \times 8$ Fisher matrix is used to approximate the likelihood. Taking $m_i(\xi) \simeq m_i(\xi_0) + \sum_j \frac{\partial m_i}{\partial \xi_j}(\xi_j - \xi_{0j})$, we find

$$\chi^2(\xi) = \sum_i \sum_j \sum_k \frac{1}{\sigma_k^2} \frac{\partial m_k}{\partial \xi_i}(\xi_j - \xi_{0j})(\xi_j - \xi_{0j}),$$
(5)

$$= \sum_i \sum_j F_{ij}(\xi_j - \xi_{0j})(\xi_j - \xi_{0j}),$$
(6)

where $\xi_j$ is the fiducial parameter set. Comparing the numerical likelihood with the Fisher matrix approximation, we confirm that the approximation is valid near the peak, which will dominate the evidence integral in (3).

We consider hypothetical observations with the photometric accuracy and cadence of the Korean Microlensing Telescope Network (KMTNet). KMTNet’s 10-min cadence provides many samples of the light curve over the course of one binary period. From equations (43) through (47) of Henderson et al. (2014),

$$\sigma_k = \sqrt{N_{obj} + \left( \frac{N_{obj} \sigma_{sys} \ln 10}{2.5} \right)^2},$$
(7)

where $\sigma_{sys} = 0.004$ mag is the systematic fractional error floor and $N_{obj} = 4.91 \times 10^2 \times 10^{-0.4(1 - 22.0)}$ is the number of photons measured from the lensed source star. $t_{exp}$ is the 120-s exposure time and $I$ is the lensed star’s apparent magnitude in the $I$ band. We assume that the sky background is negligible.

We do not consider astrometric information in our analysis, as there are no current astrometric surveys with high enough cadence to effectively detect the small changes in centroid position. For example, Gaia only expects $\sim 14$ measurements per year (see European Space Agency 2014).

3.3 Priors and evidence

To calculate the Bayesian evidence, we set a prior on each of our parameters. The prior for $M$ is flat from 2 to 200 $M_\odot$. Note that the black hole masses identified in the first two detected BBH mergers are 36 and 29 $M_\odot$ (Abbott et al. 2016a), and 14 and 8 $M_\odot$ (Abbott et al. 2016b). The prior for $q$ is flat from 0.1 to 1. Our BBH separation prior is flat in log $s_0$ from $-4$ to $-1$, with $s_0$ in units of Einstein radii. Uncertainties in the stellar evolution of BBH progenitors yield poor constraints on BBH masses and separations making the choice of flat priors a reasonable one (see Abbott et al. 2016c).

For $w_0$, the prior is flat from 0.3 to 2 Einstein radii. Since the size of a tight binary lens’s central caustic is proportional to $s_0^2$ (see Dominik 1999), the chances of a caustic crossing, given our small BBH separations and allowed values of $w_0$, are low. For $\mu_{rel}$, our prior is flat between 0 and 10 mas yr$^{-1}$ (see Henderson 2015). The priors for $\varphi$ and $f$, respectively, are flat from 0 to $\pi$ and 0 to $2\pi$ rad.

For inclination, the prior is flat in cos $i$ from $-1$ to 1. Note that while flat priors for some of our parameters are unrealistic, the assumptions are reasonable, given poor constraints and the resulting ease of computation.

Since all of the priors are flat, the prior term can be pulled out of the integral in (3) as a factor of $p \equiv V^{-1}$, where $V \equiv \Pi_i (\xi_i^+ - \xi_i^-)$ is the volume of the priors. $\xi_i^+$ is the upper boundary of the prior for parameter $\xi_i$, and $\xi_i^-$ is the lower boundary. Using the Fisher matrix approximation, the evidence is then

$$E = \frac{1}{V} \int \exp \left( -\frac{1}{2} \sum_i \sum_j F_{ij}(\xi - \xi_{0i})(\xi - \xi_{0j}) \right) \, d\xi.$$ 
(8)

We use Monte Carlo integration to compute the evidence integrals.

4 RESULTS

The test in Bayesian statistics of support for one model over another is the Bayes factor, which is the ratio of the binary-lens evidence to the single-lens evidence: $B \equiv E_B/E_s$. We apply the scale from Jeffreys (1961) to determine support, where $|\ln B| \geq 5$ is strong evidence, $2.5 \leq |\ln B| < 5$ is moderate evidence, $1 \leq |\ln B| < 2.5$ is weak evidence and $|\ln B| < 1$ is inconclusive. In the limits that $q \rightarrow 0$ and $s_0 \rightarrow 0$, the peak likelihood ratio $\rightarrow 1$, while $\ln B$ becomes negative, indicating that the single-lens model is favoured.

We compute whether a BBH system with mass parameters similar to those of the first detected LIGO event, GW150914, would be detectable by microlensing in our own Galaxy. Our fiducial model is $M = 65 M_\odot$, $q = 0.8$, $\mu_{rel} = 5.5$ mas yr$^{-1}$ and $w_0 = 0.4$ Einstein radii. We choose a lens distance $D_L = 4$ kpc and a source star in the Galactic Bulge at $D_S = 8$ kpc with apparent magnitude $I = 15$, on the bright end of typical source magnitudes in current microlensing surveys. For this model, we find that BBH separations down to approximately $s_0 = 0.0250$ Einstein radii are detectable with strong evidence, given ideal observing conditions for one year around the peak of the event. For a dimmer source star with $I = 17$, separations down to only $s_0 = 0.0270$ Einstein radii are detectable.

We also calculate the $\Delta \chi^2$ between the binary and single lenses, both for our fiducial separation $s_0 = 0.01$ Einstein radii and the minimum detectable separation $s_0 = 0.0250$ Einstein radii. For the fiducial case, $\ln B = -7.65$ corresponds to $\Delta \chi^2 = 1.36$; for the minimum detectable $s_0 = 0.0250$ Einstein radii, $\ln B \approx 5$ and $\Delta \chi^2 = 53.4$.

The approximate merger time of BBHs is (see Peters & Mathews 1963) as follows:

$$t_0 = \frac{5s_0^2c^5}{256G \eta M^2}.$$ 
(9)

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where $c$ is the speed of light and $G$ is the gravitational constant. Therefore, BBHs that fit our fiducial model with a separation of $s_0 = 0.0250$ Einstein radii will merge in about $2.08 \times 10^7$ Gyr, which is much greater than the 13.8 Gyr age of the Universe. BBHs formed with these parameters in the early Universe can serve only as present-day GW sources if their initial separation was $s_0 \leq 0.00716$ Einstein radii, $\sim 3.5$ times smaller than the minimum separation detectable by KMTNet. The dependence of this minimum separation on the binary parameters is explored in Fig. 2, which displays Bayes factor contours for one year of ideal observations for our fiducial model in the $q$–$s_0$, $u_0$–$s_0$, $M$–$s_0$ and $\mu_0$–$s_0$ planes along with curves of constant merger time.

**Figure 2.** Contours of the log Bayes factor $\ln B$ for our fiducial model: total mass $M = 65 M_\odot$, mass ratio $q = 0.8$, proper motion $\mu_{\text{rel}} = 5.5$ mas yr$^{-1}$ and impact parameter $u_0 = 0.4$ Einstein radii. The $x$-axis in each panel is log $s_0$, where $s_0$ is the binary separation in units of the Einstein radius. Top left-hand panel: $q$ is varied on the $y$-axis. Bottom left-hand panel: $u_0$ is varied. Top right-hand panel: $M$ is varied. Note that since the Einstein radius is proportional to $\sqrt{M}$, the physical BBH separation will increase for increasing $M$ and constant $\log s_0$. Bottom right-hand panel: $\mu_{\text{rel}}$ is varied. Dotted curves correspond to weak (1.0), moderate (2.5) and strong (5.0) evidence levels for Bayes factor values. Solid colour lines correspond to BBH merger time in $10^7$ Gyr. Points in the plane with $\ln B \geq 5$ correspond to BBH systems that could theoretically be detected with high certainty, given ideal observations with KMTNet. Points in the plane with $\ln B \leq -5$ correspond to BBH systems that would appear to be single lenses with high certainty. Red stars in each panel correspond to the parameters of our fiducial model at the binary separation $s_0 = 0.0250$ Einstein radii, marking the threshold of strong evidence in support of binarity ($\ln B = 5$).

**5 DISCUSSION**

The Bayes factor values we calculate are for ideal observing conditions over the course of one year, centred on the event peak. We do not explore the effects of finite-source size or other potential issues such as interstellar extinction or the difficulties of observing through Earth’s atmosphere.

Our results indicate that KMTNet cannot detect binary modulation in the microlensing light curves of BBHs with separations small enough to merge through GW emission within the age of the Universe. The BBHs that can be identified have wider separations than these GW sources, potentially corresponding to systems that failed to experience a decrease in orbital separation during common-envelope evolution between the core collapses of the BBH stellar progenitors (see Webbink 1984; Dominik et al. 2012; Gerosa et al. 2013). Microlensing BBH searches thus complement GW-based searches through their sensitivity to more widely separated BBHs.

The minimum separation for which the binary-lens model is favoured can be reduced by decreasing the source distance $D_\text{s}$, decreasing the ratio $D_\text{s}/D_\text{c}$, decreasing the impact parameter $u_0$, decreasing proper motion $\mu_{\text{rel}}$, or increasing $q$. Larger total masses $M$ can also slightly improve detectability. To reduce this minimum separation from 0.0250 to 0.00716 Einstein radii (small enough to merge within the age of the Universe), a future experiment must increase the $\Delta \chi^2$ beyond that obtainable with KMTNet by a factor of $(0.025/0.00716)^2 \approx 150$, since the binary signal $d_\text{c} - m_\text{c}$ is proportional to the binary quadrupole moment $(\alpha_s \lambda^3)$ and $\chi^2 \propto \left( d_\text{c} - m_\text{c} \right)^2$. This could be achieved, for example, with continuous observations (10-min cadence/120-s exposure $= 5$) over 10 yr of the event by a network with three times as many telescopes as KMTNet ($5 \times 10 \times 3 = 150$).

As discussed in Penny et al. (2011a,b), there are some degeneracies of note with binary microlensing signals. For example, a binary stellar source with a single lens could mimic the astrometric signal of a binary lens, though the photometric signal would be discernibly different. We expect the centroid shifts due to binary sources to be much smaller than those due to binary lenses – if the source’s apparent magnitude is bright enough, Gaia’s 24-µarcsec precision
Calculating the rates of BBH microlensing events would require astrophysical modelling of BBH formation beyond the scope of this Letter (see Belczynski et al. 2016). Di Stefano (2008) estimated that current microlensing surveys can detect 0.38 black hole lenses per decade per deg². The fraction of these black holes in binary systems is highly uncertain but could be of order unity. KMTNet has a 4 deg² field of view and can look at four fields with a 10-min cadence (Henderson et al. 2014), yielding approximately six black hole microlensing events per decade. In addition, future space-based surveys like WFIRST will likely achieve increased photometric capability over KMTNet while still preserving the 10-min cadence and ~1-yr survey duration, likely leading to more detections of black hole and BBH microlensing events.

KMTNet is capable of identifying BBHs with high-cadence sampling of a microlensing event. However, as demonstrated in this Letter, it does not possess the photometric precision necessary to observe microlensing events by BBHs with merger times within the age of the Universe. We encourage analyses of current and future microlensing survey data to search for similar modulation in all long-duration events, providing a new channel for the discovery of short-period BBHs in our Galaxy.

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