Traveling-Wave-Based Fault Location Method for Hybrid Distribution Network Mixed with Three-Phase and Phase to Phase Supply Lines

Zhang Xinyu1*, Nie Yu2, Zhao Chuanzong3, Du Wei1, Wang Yanan1, WANG Zhigang4, LIU Yanbing4

1State Grid Liaoning Electric Power Company electric power research institute, Shenyang, Liaoning, Province, 110006, China
2State Grid Liaoning Electric Power Company Limited, Shenyang, Liaoning, Province, 110006, China
3Fushun Power Supply Company, Liaoning Electric Power Company Limited, Fushun, Liaoning, Province, 113008, China
4Beijing DHHB Power Science and Technology Co., Ltd, Haidian District, Beijing, 100085, China

*Corresponding author’s e-mail: dk_zz@126.com

Abstract. In China, a hybrid distribution network mixed with three-phase main lines and phase to phase supply lines can be found in many MV distribution networks due to lower cost of using two lines from three-phase main lines as well as lower cost of phase to phase transformer. For such a hybrid network it is a challenge for traditional fault location techniques. This paper presents a traveling wave fault location method based on double-terminal measurement techniques. The proposed method analyses the traveling wave propagation mechanism of both aerial mode and zero mode components for a fault. The method then determines which mode component is used for the fault location according to fault types. The method has been developed and assessed on a typical distribution network using ATP power network simulation tool. The tests were conducted under various fault scenarios. The results show that zero mode component is more effective for a fault on the phase of three-phase main lines which is not linked to the phase to phase supply lines. However for a phase-to-phase fault, aerial mode component is more effective. For the remaining fault types, either aerial or zero mode component can be used.

1. Introduction

Distribution networks, closely linking to users, play an important role in the power system. The accurate and fast fault location in medium-voltage (MV) distribution networks can improve the reliability of power supply and significantly reduce the loss of power consumers. In China, MV distribution networks mix with three-phase main lines and phase to phase supply lines are widely used due to using two lines (i.e. phase to phase supply lines) in rural area (normally long distance and low load) from three-phase main lines so that the cost of lines are reduced as well as a lower cost phase to phase transformer can be used [1]. However, for such a hybrid network it is a challenge for traditional fault location techniques because the network mix with three-phase main lines and phase to phase supply lines.

The fault location in MV distribution networks has been analyzed depending on different methods,
such as impedance-based fault location methods [2]-[3] and traveling wave-based fault location methods. The methods based on traveling wave have benefits for accurate fault location due to its insensitivity to high fault resistance [4]. The traveling wave-based fault location methods can be categorized into active and passive types. In the active type, a signal is injected into a network to create a traveling wave [5]-[7]. In the passive type, the traveling is produced by a fault is analyzed to determine the fault location. Methods based on single-terminal technique and wavelet transformation were proposed in [8]-[9]. The former one uses only the frequency-domain information for the fault location. The latter one uses time-frequency information to improve the identification accuracy.

In order to improve the fault location accuracy, double-terminal/multi-terminal techniques have been considered. The technique only needs to compare the first surge of traveling wave arriving time from the fault to the terminals. Hence the impact of any reflected and refracted traveling waves on lines is eliminated. In [10], a new low cost device with “LoRa Technology”, i.e. wireless technology, is proposed for fault location in power distribution system. Two devices are installed at both terminals of the MV cable lines, respectively. The system calculates the time difference of the first surge from the fault point to two terminals, respectively, to achieve the fault location calculation. In [11]-[12], a method based on multi-terminal traveling wave is proposed. When a fault occurs, the time that the first surge of traveling wave arrives at each terminal is measured and the fault distance is calculated by the network fault management system. In [13], a traveling wave-based fault location algorithm with multiple terminal techniques for hybrid transmission system consisting of one onshore overhead line and multiple offshore submarine cables is proposed. It uses the discrete wavelet transformation method to decompose the traveling wave measurement and to obtain the first surge wave arrival time to the terminal.

Since the propagation between three-phase main lines and phase to phase supply lines is different, the selection of traveling wave analysis techniques can significantly influence the fault location accuracy on such a hybrid distribution network. This paper presents a traveling wave fault location method based on double-terminal measurement techniques. The propagation mechanism of aerial mode and zero mode components for different fault types are formulated. This paper also presents detailed analysis how to calculate the fault distance based on the fault generated first surge wave arrival time difference to the terminals. The method has been tested and analyzed on a typical hybrid network under various fault scenarios. The details of the methodology, test cases and discussion are described in the following.

2. Methodology

2.1. Phase-mode transformation

Some text. When the fault occurs on a power network, the voltage traveling wave equation can be expressed as the second-order partial differential equation [14]:

$$\frac{\partial^2 V}{\partial x^2} = LC \frac{\partial^2 V}{\partial t^2}$$

(1)

where $V$ is the vector of the phase voltage of $n$ power supply lines. $L$ and $C$ denote $n \times n$ square matrices of the inductance and capacitance parameter of per unit length of the line. Due to the coupling phenomena between the power supply lines, $L$ and $C$ matrices are non-diagonal. In order to simplify the calculation, it is necessary to use an appropriate matrix transformation to convert the $L$ and $C$ matrix into a diagonal matrix. Karrenbauer transformation [15] is generally used to transform the phase components into mode components:

$$V_m = S^{-1}V$$

(2)

where $S$ is Karrenbauer transformation matrix and $V_m$ is the vector of the mode components. For an $n$-order matrix, the transformation matrix is
\[
S = \begin{bmatrix}
1 & \ldots & 1 & 1 \\
\ldots & 1-n & 0 & 1 \\
1 & 0 & 1-n & \ldots \\
1 & 1 & \ldots & 1-n \\
\end{bmatrix}
\]
\[
S^{-1} = \frac{1}{n} \begin{bmatrix}
1 & 1 & \ldots & 1 \\
1 & -1 & 0 & 0 \\
\ldots & 0 & -1 & 0 \\
1 & 0 & 0 & -1 \\
\end{bmatrix}
\]

Substituting (2) into (1), the voltage traveling wave equation can be expressed as
\[
\frac{\partial^2 V_n}{\partial x^2} = L'C \frac{\partial^2 V_n}{\partial t^2}
\]

where
\[
L' = S^{-1}LS \quad \text{and} \quad C' = S^{-1}CS
\]

2.1.1. Karrenbauer transformation of three-phase main lines.
Since three phases can be considered as symmetric, we obtain the following equations:
\[
V = \begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix}, \quad L = \begin{bmatrix}
l_s & l_s & l_s \\
l_m & l_m & l_m \\
l_s & l_s & l_s
\end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix}
c_s & c_m & c_m \\
c_m & c_s & c_m \\
c_m & c_m & c_s
\end{bmatrix}
\]

where \(l_s\) denotes the self inductance, and \(l_m\) denotes the mutual inductance. \(c_s\) denotes the self capacitance, and \(c_m\) denotes the mutual capacitance.

From (3), \(S\) can be obtained as
\[
S = \begin{bmatrix}
1 & 1 & 1 \\
1 & -2 & 1 \\
1 & 1 & -2
\end{bmatrix} \quad \text{and} \quad S^{-1} = \frac{1}{3} \begin{bmatrix}
1 & 1 & 1 \\
1 & -1 & 0 \\
1 & 0 & -1
\end{bmatrix}
\]

From (4), (5) and (6), the voltage traveling wave equation can be expressed as
\[
\frac{\partial^2 V_n}{\partial x^2} = \begin{bmatrix}
0 & l_s + 2l_m & 0 & 0 \\
0 & l_s - l_m & 0 & 0 \\
0 & 0 & l_s - l_m & 0
\end{bmatrix} \frac{c_s + 2c_m}{0} \frac{0}{0} \frac{0}{0} \frac{c_s - c_m}{c_s - c_m} \frac{\partial^2 V_n}{\partial t^2}
\]

where
\[
V_{\alpha(3)} \quad \text{and} \quad V_{\beta(3)} \quad \text{are aerial mode component, and} \quad V_{(3)} \quad \text{is zero mode component of three-phase main lines. From (7), the wave velocity of aerial mode and zero mode components of three-phase main lines can be calculated as}
\]
\[
v_{0(3)} = \frac{1}{\sqrt{(l_s + 2l_m)(c_s + 2c_m)}}
\]
\[
v_{\alpha(3)} = v_{\beta(3)} = \frac{1}{\sqrt{(l_s - l_m)(c_s - c_m)}}
\]

where \(v_{0(3)}\) denotes the wave velocity of zero mode component, \(v_{\alpha(3)}\) and \(v_{\beta(3)}\) denote the wave velocity of aerial mode component of three-phase main lines.

2.1.2. Karrenbauer transformation of phase-to-phase supply lines.
Similarly if phase to phase supply lines have the same parameters as three-phase main lines, we have:
\[
V = \begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix}, \quad L = \begin{bmatrix}
l_s & l_s & l_s \\
l_m & l_m & l_m \\
l_s & l_s & l_s
\end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix}
c_s & c_m \quad c_m \quad c_s \\
c_m & c_s \quad c_m \quad c_s
\end{bmatrix}
\]
From (3), $S$ can be obtained as

$$S = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad \text{and} \quad S^{-1} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$ (11)

From (4), (10) and (11), the voltage traveling wave equation can be expressed as

$$\frac{\partial^2 V_m}{\partial x^2} = \begin{pmatrix} l_m + l_a & 0 \\ 0 & l_s - l_m \end{pmatrix} \begin{pmatrix} c_m + c_m \\ c_s - c_m \end{pmatrix} \frac{\partial^2 V_m}{\partial t^2}$$ (12)

where

$$V_m = \begin{bmatrix} V_{m(2)} \\ V_{a(2)} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix}$$ (13)

$V_{a(2)}$ is aerial mode component and $V_{m(2)}$ is zero mode component of phase to phase supply lines. From (12), the wave velocity of aerial mode and zero mode components of phase to phase supply lines can be calculated as

$$v_{m(2)} = \frac{1}{\sqrt{(l_m + l_a)(c_m + c_m)}}$$

$$v_{a(2)} = \frac{1}{\sqrt{(l_s - l_m)(l_s - c_m)}}$$ (14)

where $v_{m(2)}$ denotes the wave velocity of zero mode component, and $v_{a(2)}$ denotes the wave velocity of aerial mode component of phase to phase supply lines.

Based on (9) and (14), we can highlight the following points:

(i) The wave velocity of aerial mode and zero mode components for both three phase main lines and phase to phase supply lines are not equal;

(ii) The wave velocity of aerial mode component of phase to phase supply lines is equal to that of three-phase main lines.

(iii) The wave velocity of zero mode component of phase to phase supply line is different from that of three-phase main lines.

2.2. Analysis of traveling wave propagation

For an explanation, a typical hybrid network with a three-phase main line and a phase to phase supply line as shown in Fig. 1 is used for the analysis of the traveling wave propagation. P is a primary substation which has three-phase main line feeders, Q is a secondary substation fed by a phase to phase supply line, and K is a connection point (usually a line pole or a cable junction cabinet) between three phase main lines and phase to phase supply lines, where Phase A and Phase B are linked to phase to phase supply lines. The distance between P and K is $l_1$, the distance between K and Q is $l_2$, and the distance between P and the fault point is $l_f$.

![Figure 1. Typical line voltage power supply](image-url)
Device 1 and Device 2 are installed at primary substation P and secondary substation Q, respectively. Since the fault generated traveling wave signal will propagate along the power supply line toward two terminals, the measured traveling wave propagation time signals are calculated by the device 1 and 2, and then collected by a centralized fault location server to calculate the fault distance. Note that under different circumstance the traveling wave propagation is different.

2.2.1. Single phase-to-ground fault on phase to phase supply line.
According to Fig. 1, if the fault occurs at F1 or F2 on the phase to phase supply lines, the aerial mode and zero mode components can be obtained by Karrenbauer transformation based on (13). After aerial mode and zero mode components arrive at the connection point K, the voltage of phase A and phase B at the connection point K can be obtained by Karrenbauer inverse transformation as

\[
\begin{bmatrix}
    V_a^{(2)} \\
    V_b^{(2)} \\
    V_c^{(2)}
\end{bmatrix} = 
\begin{bmatrix}
    1 & 1 & 0 \\
    1 & -1 & 0
\end{bmatrix} 
\begin{bmatrix}
    V_a^{(1)} \\
    V_b^{(1)} \\
    V_c^{(1)}
\end{bmatrix}
\]

(15)

Assuming that the three-phase main line parameters are symmetrical and the coupling coefficient between the lines is k (0<k<1) [16], the voltage of the phase C at the connection point K is

\[
V_c = kV_a + kV_b
\]

(16)

Aerial mode and zero mode components of the three-phase main line can be obtained from (8), (15) and (16).

\[
\begin{bmatrix}
    V_a^{(3)} \\
    V_b^{(3)} \\
    V_c^{(3)}
\end{bmatrix} = 
\begin{bmatrix}
    2(k+1)V_a^{(2)} \\
    2V_a^{(2)} \\
    (1-2k)V_a^{(2)} + V_a^{(2)}
\end{bmatrix}
\]

(17)

As can be seen from (17) both aerial mode and zero mode components generated exist on both phase to phase supply lines and the three-phase main lines. Therefore for this type of fault, both aerial mode and zero mode components can be used to calculate the fault distance.

The arrival time of aerial mode and zero mode components from the fault point to P are

\[
t_{Pa} = \frac{l_1 + l_2 - l_f}{v_a^{(2)}} \quad \text{and} \quad t_{P0} = \frac{l_1 + l_2 - l_f}{v_c^{(2)}}
\]

(18)

Similarly the arrival time of aerial and zero mode components from the fault point to Q are

\[
t_{Qa} = \frac{l_1 + l_2 - l_f}{v_a^{(2)}} \quad \text{and} \quad t_{Q0} = \frac{l_1 + l_2 - l_f}{v_c^{(2)}}
\]

(19)

The time difference of aerial mode and zero mode between P and Q are

\[\Delta t_a = t_{Pa} - t_{Qa} \quad \text{and} \quad \Delta t_0 = t_{P0} - t_{Q0}\]

(20)

Based on (18), (19), (20) and \(v_a^{(2)} = v_a^{(1)}\), the fault distance calculation equation using aerial mode is

\[
l_f = \frac{\Delta t_a \cdot v_a^{(1)} + l_1 + l_2}{2}
\]

(21)

The fault distance calculation equation using zero mode is

\[
l_f = \frac{(\Delta t_0 - l_1/v_a^{(3)}) \cdot v_a^{(2)} + 2l_1 + l_2}{2}
\]

(22)

Principally the fault distance calculation using either (21) or (22) is the same or very close to each other.

2.2.2. Single phase-to-ground fault on three-phase main line.
In this scenario, there are two possible fault conditions. One is the fault on the phase which is linked to phase to phase supply lines. The other is the fault on the phase which is not linked to phase to phase
supply line.

2.2.2.1. The fault on the phase linked to the phase to phase supply line
As shown in Fig. 1, if the fault occurs at F3 or F4 the aerial mode and zero mode components can be obtained by Karrenbauer transformation based on (8). After aerial mode and zero mode components arrives at the connection point K, the voltage of phase A, phase B and phase C at the connection point K can be obtained by Karrenbauer inverse transformation as

\[
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 \\
1 & -2 & 1 \\
1 & 1 & -2
\end{bmatrix}
\begin{bmatrix}
V_{0(3)} \\
V_{a(3)} \\
V_{\beta(3)}
\end{bmatrix}
\tag{23}
\]

Based on (13) and (23), aerial mode and zero mode components of the phase-to-phase line can be obtained by

\[
\begin{bmatrix}
V_{a(2)} \\
V_{\alpha(2)}
\end{bmatrix} = \frac{1}{2}
\begin{bmatrix}
2V_{0(3)} - V_{a(3)} + 2V_{\beta(3)} \\
3V_{a(3)}
\end{bmatrix}
\tag{24}
\]

As can be seen from (24) both aerial mode and zero mode components generated exist on both phase to phase supply lines and the three-phase main lines. Therefore for this type of fault, both aerial mode and zero mode components can be used to calculate the fault distance.

The arrival time of aerial mode and zero mode components from the fault point to P are

\[
t_{P\alpha} = \frac{l_f}{v_{a(3)}} \quad \text{and} \quad t_{P\beta} = \frac{l_f}{v_{0(3)}}
\tag{25}
\]

Similarly the arrival time of aerial mode and zero mode components from the fault point to Q are

\[
t_{Q\alpha} = \frac{L_2 - l_f}{v_{a(3)}} \quad \text{and} \quad t_{Q\beta} = \frac{L_2 + l_f}{v_{0(3)}}
\tag{26}
\]

Based on (20), (25), (26) and \(v_{a(2)} = v_{a(3)}\), aerial mode component equation (21) can be used to calculate the fault distance. However for zero mode component, the fault distance can be calculated by

\[
l_f = \frac{(\Delta t_\alpha + L_2 / v_{a(3)}) \cdot v_{0(3)} + L_1}{2}
\tag{27}
\]

2.2.2.2. The fault on the phase not linked to phase to phase supply line
As shown in Fig. 1, if the fault occurs at F5 the aerial mode and zero mode components propagate along three-phase main line. Since the coupling between phase C to phase A and phase C to phase B are the same, based on (8) the aerial mode component of three-phase main line is

\[
V_{a(3)} = 0
\]

Based on (24) the aerial mode component of phase to phase supply line is

\[
V_{a(2)} = 0
\]

As can be seen that the aerial mode component of phase to phase supply line is zero, hence only zero mode component can be used to calculate the fault distance by (27).

2.2.3. Phase-to-Phase fault on either three-phase or phase to phase supply line.
If phase-to-phase fault occurs on both three-phase main line (e.g. F6 and F7) and phase to phase supply line (e.g. F8), the zero mode component does not exist because fault current does not flow to the earth. Therefore for this type of fault, only aerial mode component can be used to calculate the fault distance by (21).

To sum up, the traveling wave propagation is complex in a distribution network mixed with three-phase main lines and phase to phase supply lines. Specific aerial mode or zero mode component should be chosen according to the different fault types.
2.2.4. Determination of the fault sections.
In order to determine a fault on either three phase main lines or phase to phase supply line, a boundary or critical fault condition at K needs to be determined. Assuming a fault generated traveling wave occurs at the connection point K between three-phase main line and phase to phase supply line, we can use (18)-(20) and (25), (26) to calculate the time differences of the first surge of aerial mode and zero mode components respectively as

\[
\Delta t_{a\text{-critical}} = \frac{l_1}{v_a(3)} - \frac{l_2}{v_a(2)}
\]

and

\[
\Delta t_{0\text{-critical}} = \frac{l_1}{v_0(3)} - \frac{l_2}{v_0(2)}
\]

where \(\Delta t_{a\text{-critical}}\) is the time difference of the first surge of aerial mode component, and \(\Delta t_{0\text{-critical}}\) is the time difference of the first surge of zero mode component.

If a fault generated traveling wave occurs on the three-phase main line, we will have \(\Delta t_{a\text{-critical}} < \Delta t_{a\text{-critical}}\) and \(\Delta t_{0\text{-critical}} < \Delta t_{0\text{-critical}}\). If a fault generated traveling wave occurs on the phase to phase supply line, we will have \(\Delta t_{a\text{-critical}} > \Delta t_{a\text{-critical}}\) and \(\Delta t_{0\text{-critical}} > \Delta t_{0\text{-critical}}\).

2.3. The procedure of fault location
Based on the analysis in section 2.2, the location procedure is given as follows.

(i) The voltage measurement signals monitored by the devices at different terminals are synchronized using GPS technology [17].

(ii) If a fault occurs, the faulted generated traveling wave signals are analyzed to obtain aerial mode or zero mode component according to specific fault types and the time mark of their first surges.

(iii) The first surge time marks calculated from different terminals are send by the devices to the server.

(iv) According to 2.2.5 the fault either on the three phase main line or on the phase to phase supply line can be determined.

(v) The fault location can now be calculated as follows:

* If a fault occurs on phase to phase supply lines, either (21) of aerial mode component or (22) of zero mode component is used to calculate the fault distance
  * If a fault occurs on three-phase main lines, either (21) of aerial mode component or (27) of zero mode component is used to calculate the fault distance

3. Performance evaluation

3.1. Network modelling
As shown in Fig. 2, a typical 10kV distribution network is used for the studies. Four feeder lines are connected to the substation busbar and Line 4 has a section of phase to phase supply line connected to the three-phase main lines. P is primary substation, Q is secondary substation, and K is connection point. The distance between P and K is \(l_1=5000\text{m}\), and the distance between K and Q is \(l_2=5000\text{m}\). The neutral point is grounded via 10Ω resistance.
Figure 2. ATP model of typical phase to phase supply line

The inductance and capacitance parameters of the three-phase and phase to phase supply line are given in Table 1.

|               | \(L_1\) (mH/km) | \(L_2\) (mH/km) | \(C_1\) (µF/km) | \(C_2\) (µF/km) |
|---------------|-----------------|-----------------|-----------------|-----------------|
| Phase to phase | 1.058           | 3.698           | 10.678          | 5.733           |
| Three-phase   | 1.058           | 5.018           | 10.678          | 2.871           |

From Table 1, the aerial mode velocity of three-phase and phase to phase supply line can be calculated as

\[ v_{a(3)} = 2.9752 \times 10^5 \text{ km/s} \]

The zero mode wave velocity of three-phase main line is

\[ v_{0(3)} = 2.6348 \times 10^5 \text{ km/s} \]

and the zero mode wave velocity of phase to phase supply line is

\[ v_{0(2)} = 2.1718 \times 10^5 \text{ km/s} \]

For the critical fault condition at K based on (28), the time difference of the first surge of aerial mode component is given as

\[ \Delta t_{a\text{-critical}} = \frac{5}{2.9752 \times 10^5} - \frac{5}{2.9752 \times 10^5} = 0 \mu s \]

Based on (28), the time difference of the first surge of zero mode component is given as

\[ \Delta t_{0\text{-critical}} = \frac{5}{2.6348 \times 10^5} - \frac{5}{2.1718 \times 10^5} \approx 4.046 \mu s \]

3.2. Case studies

Different cases under various fault conditions were considered and analyzed by using ATP simulation software. Among them, different types of faults are selected and discussed in detail as flows.

3.2.1. Single phase-to-ground fault on phase to phase supply line.

Assume that Phase A-to-ground fault occurs at the distance of 7540m from node P, and fault inception
angle is 30°. Both the aerial and zero mode components can be used to calculate the fault distance. The Phase A to Phase B voltage aerial mode components at P and Q are shown in Fig. 3. The voltage zero mode components at P and Q are shown in Fig. 4.

As can be seen from Fig. 3, the arriving time of aerial mode component at P is 45.026ms and at Q is 45.008ms. This gives the time difference \( t_\Delta = 18 \mu s \). Similarly, the time difference \( t_0 = 19 \mu s \) of zero mode component between P and Q is 19\( \mu s \).

Based on the critical fault condition calculation in section 3.1, \( t_\Delta > t_\Delta - \text{critical} = 18 \mu s > 4.046 \mu s \). Therefore, the fault occurs at the phase to phase supply line, i.e. between node K and node Q. Therefore, the fault distance can be calculated respectively as

\[
\begin{align*}
I &= \frac{v_0(3) + v_1 + v_2}{2} = 7677.68 m \quad \text{and} \quad I = \frac{\left(\frac{v_0 - l}{v_0(3)}\right) + v_2 + 2l_1 + l_2}{2} = 7502.52 m
\end{align*}
\]

The fault distance under different fault distance were also simulated and analysed. The results and error are listed in table 2 and table 3, respectively.

### Table 2. Result of fault location by aerial mode components

| Fault distance (m) | Calculation result (m) | Absolute error (m) | Relative error |
|-------------------|------------------------|--------------------|---------------|
| 5720              | 5892.56                | 172.56             | 3.0%          |
| 6340              | 6487.60                | 147.60             | 2.3%          |
| 7540              | 7677.68                | 137.68             | 1.8%          |
| 8100              | 8123.96                | 23.96              | 0.3%          |
| 9200              | 9165.28                | 34.72              | 0.4%          |

### Table 3. Result of fault location by zero mode components

| Fault distance (m) | Calculation result (m) | Absolute error (m) | Relative error |
|-------------------|------------------------|--------------------|---------------|
| 5720              | 5656.49                | 63.51              | 1.1%          |
| 6340              | 6308.03                | 31.97              | 0.5%          |
| 7540              | 7502.52                | 37.48              | 0.5%          |
| 8100              | 7936.88                | 163.12             | 2.0%          |
| 9200              | 9022.78                | 177.22             | 1.9%          |
3.2.2. Single phase-to-ground fault on three-phase main line.

In this scenario, there are two possible fault conditions. One is the fault on the phase which is linked to phase to phase supply lines. The other is the fault on the phase which is not linked to the phase to phase supply line. For the former fault condition, the single phase fault location can be done as by using both aerial mode and zero mode components which is the same as section A. For the latter fault condition, further analysis is discussed here in details.

Assume that Phase C-to-ground (i.e. Phase C is not linked to the phase to phase supply line) via 10Ω resistance fault occurs at the distance of 2850m from node P, and fault inception angle is 30°. The Phase A to Phase B voltage aerial mode components and zero mode components at P and Q are obtained as shown in Fig. 5 and Fig. 6, respectively. As can been seen from Fig. 5, there is no sudden change in the aerial mode components at both terminals, hence the Phase A to Phase B aerial mode component cannot be used to locate the fault.

From Fig. 6, the time difference $\Delta t_0$ between P and Q is $-20\mu$s (i.e. $\Delta t_0 = -20\mu$s). Based on the critical fault condition calculation in section 3.1, $\Delta t_0$ of $-20\mu$s < $\Delta t_{0\text{-critical}}$ of $-4.046\mu$s. Therefore, the fault occurs at the three-phase main line, i.e. between node P and node K. The fault distance can thus be calculated as

$$l_f = \frac{(\Delta t_0 + l_2/v_{(02)}) \cdot v_{(01)} + l_1}{2} = 2898.19$$

The fault distance under different fault distance were also simulated and analysed. The results and error are listed in table 4.

| Fault distance (m) | Calculation result (m) | Absolute error (m) | Relative error |
|-------------------|------------------------|-------------------|---------------|
| 1100              | 1070.60                | 29.40             | 2.7%          |
| 1900              | 1844.24                | 55.75             | 2.9%          |
| 2850              | 2898.19                | 49.19             | 1.7%          |
| 3910              | 3952.09                | 42.09             | 1.1%          |
| 4570              | 4610.79                | 40.79             | 0.9%          |

3.2.3. Phase-to-Phase faults.

In this scenario, there are two possible fault conditions. One is the phase-to-phase fault on the three phase main line. The other is the phase-to-phase fault on the phase to phase supply line. For both conditions, only aerial mode component is needed to locate the fault distance.

Assume that Phase A-to-phase B fault occurs at the distance of 7540m from node P on the phase to
phase supply line, and fault inception angle is 30°. The voltage zero mode component at Q is shown in Fig. 7. As can been seen from Fig. 7, there is no sudden change in the zero mode component at Q, hence the zero mode component cannot be used to locate the fault. The Phase A to Phase B voltage aerial mode components at P and Q are shown in Fig. 8.

From Fig. 8, the time difference \( t_\Delta \) between P and Q is 18\( \mu \)s (i.e. \( t_\Delta = 18 \mu s \)). Based on the critical fault condition calculation in section 3.1, \( t_\Delta \) of 18\( \mu \)s > \( t_{\Delta \text{critical}} \) of 0\( \mu \)s. Therefore, the fault occurs at the phase to phase supply line, i.e. between node K and node Q. Therefore, the fault distance is calculated as

\[
I_f = \frac{\Delta t_a \cdot v_{ao(3)} + I_1 + I_2}{2} = 7677.68 \text{m}
\]

The fault distance under different fault distance were also simulated and analysed. The results and error are listed in table 5.

| Fault distance (m) | Calculation result (m) | Absolute error (m) | Relative error |
|-------------------|------------------------|--------------------|---------------|
| 5720              | 5892.56                | 172.56             | 3.0%          |
| 6340              | 6487.60                | 147.60             | 2.3%          |
| 7540              | 7677.68                | 137.68             | 1.8%          |
| 8100              | 8123.96                | 23.96              | 0.3%          |
| 9200              | 9314.04                | 114.04             | 1.2%          |

Similarly phase-to-phase faults on three phase main lines under various fault distance were studied and analysed. The results confirm the method correctly.

Based on the results from table 2, 3, 4 and 5, the errors between the calculated fault distance and the actual fault distance for typical faults are small. This confirms the proposed method can locate the different types of faults effectively.

4. Conclusion

This paper proposes a traveling-wave-based fault location method for hybrid distribution networks mixed with three-phase main line and phase to phase supply lines. It measures a fault generated traveling wave propagation time at double terminals to obtain the appropriate aerial mode and zero mode components. The fault distance is then calculated in accordance with the traveling arriving time difference of these mode components between two terminals.

Studies have been carried out by deploying this method on a typical 10kV distribution network model.
Different cases under various fault conditions were considered and analysed by using ATP simulation software. From them, different types of faults, the influence of fault resistance and inception angels are selected and discussed. The results show that zero mode component is more effective for a fault on the phase of three phase main line which is not linked to the phase to phase supply line. However for a phase-to-phase fault, aerial mode component is more effective on both three phase main lines and phase to phase supply lines. For the remaining fault types, either zero or aerial mode component can be used. The studies confirm that the proposed method can be used to locate any types of faults on the network mix with three-phase and phase to phase supply lines. The studies also show the method is not affected by the variation of high fault resistance and inception angle.

References

[1] Wang, W. (2010) Technology research and techno-economic analysis on single-phase power supply mode in distribution network. In: CICED 2010 Proceedings. Nanjing, pp. 1-5.

[2] Rose, C., Thomas, D., Sumner, M., Christopher, E., Arevalo, S. L. (2014) Intelligent impedance based fault-location for zonal power systems. In: 12th IET International Conference on Developments in Power System Protection. Copenhagen. pp. 1-5.

[3] Bi, T., Yao, L. (2012) A novel approach to impedance-based fault location for high voltage cables. In: 2012 IEEE Industry Applications Society Annual Meeting. Las Vegas. pp. 1-4.

[4] Zimath, S. L., Ramos, M. A., Filho, J. S., Beck, J. M., Mueller, N. (2010) Traveling wave-based fault location experiences. In: Proc. Protect. Relay Eng. College Station, TX, USA, pp. 1-7.

[5] Ghaderi, A., Mohammadpour, H. A., Ginn, H. (2015) Active fault location in distribution network using time-frequency reflectometry. In: 2015 IEEE Power and Energy Conference at Illinois (PECI). Champaign. pp. 1-7.

[6] Elkalashy, Nagy I., Sabiha, N. A., Lehtonen, M. (2015) Earth Fault Distance Estimation Using Active Traveling Waves in Energized-Compensated MV Networks. IEEE Transactions on Power Delivery., 30(02): 836-843.

[7] Yan, F., Liu, W.X., Tian, L. (2011) Fault location for 10kV distribution line based on traveling wave-ANN theory. In: 2011 IEEE Power Engineering and Automation Conference. Wuhan. pp. 437-440.

[8] Borghetti, A., Bosetti, M., Di Silvestro, M., Nucci, C. A., Paolone, M. (2008)) Continuous-Wavelet Transform for Fault Location in Distribution Power Networks: Definition of Mother Wavelets Inferred From Fault Originated Transients. IEEE Transactions on Power Systems., 23(02): 380-388.

[9] Borghetti, A., Bosetti, M., Nucci, C. A., Paolone, M., Abur, A. (2010) Integrated Use of Time-Frequency Wavelet Decompositions for Fault Location in Distribution Networks: Theory and Experimental Validation. IEEE Transactions on Power Delivery.,25(04): 3139-3146.

[10] Vigni, V. L., Di Stefano, A., Candela, R., Sanseverino, E. R., (2017) A two-end traveling wave fault location system for MV cables based on LoRa technology. In: 2017 IEEE International Conference on Environment and Electrical Engineering and 2017 IEEE Industrial and Commercial Power Systems Europe (EEEIC / I&CPES Europe). Milan, pp. 1-6.

[11] Jia, H.B., Qian, H.B., Li, F. (2012) A fault location method in distribution network with use of wavelet-based traveling-wave. In: 2012 International Conference on Wavelet Analysis and Pattern Recognition. Xian. pp. 307-312.

[12] Liu, H., Zeng, X.J., Wang, Y., Peng, F. (2012) Research on traveling wave transmission characteristics in distribution network. In: 2012 China International Conference on Electricity Distribution. Shanghai. pp. 1-5.

[13] Hamidi, R.J., Livani, H. (2017) Traveling-Wave-Based Fault-Location Algorithm for Hybrid Multiterminal Circuits. IEEE Transactions on Power Delivery., 32(01): 135-144.

[14] Napolitano, F., Tossani, F., Nucci, C.A., Rachidi, F. (2015) On the Transmission-Line Approach for the Evaluation of LEMP Coupling to Multiconductor Lines. IEEE Transactions on Power Delivery.,30(02): 861-869.
[15] Zhang, B., He, J.H., Bo, Z.Q., Caunce, B.R.J., Klimek, A. (2006) Transient Directional Protection Based on the Transformation of Positive Sequence Component. In: Proceedings of the 41st International Universities Power Engineering Conference. Newcastle-upon-Tyne, pp. 828-831.
[16] Zhang, Q., Zhang, L., Tang, X., Gao, J. (2014) An Approximate Formula for Estimating the Peak Value of Lightning-Induced Overvoltage Considering the Stratified Conducting Ground. IEEE Transactions on Power Delivery., 29(02): 884-889.
[17] Korkali, M., Lev-Ari, H., Abur, A. (2012) Traveling-Wave-Based Fault-Location Technique for Transmission Grids Via Wide-Area Synchronized Voltage Measurement. IEEE Transactions on Power Systems., 27(02): 1003-1011.