Spatial variation of fundamental couplings and Lunar Laser Ranging

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Abstract
If the fundamental constants of nature have a cosmic spatial variation, there will in general be extra forces with a preferred direction in space which violate the equivalence principle. We show that the millimeter precision Apache Point Observatory Lunar Laser-ranging Operation provides a very sensitive probe of such variation that has the capability of detecting a cosmic gradient of the ratio between the quark masses and the strong interaction scale at the level
\[ \nabla \ln \left( \frac{m_{\text{quark}}}{\Lambda_{\text{QCD}}} \right) \sim 2.6 \times 10^{-6} \text{ Gyr}^{-1}, \]
which is comparable to the cosmic gradients suggested by the recently reported measurements of Webb et al. (2010 arXiv:1008.3907) We also point out the capability of currently planned improved equivalence principle tests, at the $\Delta g/g \lesssim 10^{-17}$ level, to probe similar cosmic gradients.

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1. Introduction
Within many extensions of the standard model, the parameters of our fundamental theory need not be universally constant but may vary in space and time. The search for such variations provides important constraints on such theories. Recently, Webb et al\textsuperscript{[1]} have reported evidence for a non-zero spatial variation of the fine structure constant $\alpha$. Parameterizing the variation of $\alpha$ by a dipole gradient
\[ \frac{\alpha(x)}{\bar{\alpha}} = 1 + B_\alpha \hat{z}_\alpha \cdot x \]
they find evidence, at the 4.2 $\sigma$ level, for a slope parameter
\[ B_\alpha = (1.10 \pm 0.25) \times 10^{-6} \text{ Gyr}^{-1} \]
relative to the unit direction $\hat{z}_\alpha$ of right ascension $\alpha = 17.4 \pm 0.6$ h and declination $\delta = -58 \pm 6^\circ$. In addition, Berengut et al\textsuperscript{[2]} found weak indications for the existence...
of a gradient of the electron to proton mass ratio $\mu \equiv m_e/m_p$ in the same direction $\hat{z}_\mu = \hat{z}_a$, with slope
\[ B_\mu = (2.6 \pm 1.3) \times 10^{-6} \text{Glyr}^{-1}. \]
Other spatial gradients are much more weakly tested. For example, Donoghue and Donoghue [3] have used the spatial constancy of the first acoustic peak in the cosmic microwave background to bound a possible variation in the cosmological constant (or generalized dark energy) at the level of an analogous slope parameter
\[ B_\Lambda < 0.91 \times 10^{-2} \text{Glyr}^{-1} \]
at the 95% confidence level.

Because the masses of all the elements depend on the parameters of the standard model, a gradient in one of these parameters will lead to a force (as noted long ago by Dicke [4]). Using the fine structure constant as an example, the dependence on $\alpha$ of the total mass–energy of the system $A$,
\[ E_A(\alpha) = c^2 M_A(\alpha), \]
implies that a spatial gradient $\nabla \alpha$ of $\alpha$ will lead to a force
\[ F = -\nabla E_A(\alpha) = -c^2 \frac{\partial M_A}{\partial \alpha} \nabla \alpha. \]
If we introduce the following dimensionless effective 'charge' associated with the $\alpha$ dependence:
\[ Q_\alpha(A) = \frac{\alpha}{M_A} \frac{\partial M_A}{\partial \alpha}, \]
and parameterize the gradient of $\alpha$ by a slope and a unit direction as in equation (1), $\nabla \alpha/\alpha = B_\alpha \hat{z}_\alpha$, the above force reads
\[ F_A = -Q_\alpha(A) M_A B_\alpha c^2 \hat{z}_\alpha. \]
If we now consider the dependence of the total mass energy $M_A c^2$ (in units of the Planck mass) of system $A$ on the various dimensionless ratios (or coupling constants) $r_i = \alpha, \mu, m_{\text{quark}}/m_p \ldots$ entering physics at energy scales $\lesssim m_p c^2$, and if we assume the existence of (fractional) spatial gradients $\nabla \ln r_i = B_{r_i} \hat{z}_{r_i}$ of the various dimensionless ratios, we see that body $A$ will be submitted to an external acceleration, $g_A$, of the form
\[ g_A = \frac{F_A}{M_A} = -\sum_i Q_{r_i}(A) B_{r_i} c^2 \hat{z}_{r_i}. \]
where $Q_{r_i} = Q_\alpha, Q_\mu, \ldots$ are the various dimensionless effective 'charges' associated with the dependence of the mass on the various ratios (or coupling constants), namely
\[ Q_{r_i}(A) \equiv \frac{r_i}{M_A} \frac{\partial M_A}{\partial r_i} = \frac{\partial \ln(M_A/M_p)}{\partial \ln r_i}. \]
In the second form of the definition of $Q_{r_i}$ we have recalled that $M_A$ is to be expressed in units of the Planck mass $M_p \equiv \sqrt{\hbar c/G}$. (This corresponds to working in the 'Einstein conformal frame', where Newton’s constant is held fixed.)

If the various effective charges $Q_{r_i}(A)$ were independent of the considered body $A$, the result would be an unobservable (gravity-like) uniform free fall with a universal acceleration $g_0 = g_A = g_B$ in a direction given by an average of the various gradients $\nabla \ln r_i = B_{r_i} \hat{z}_{r_i}$. However, composition dependence of (at least one of) the various charges, e.g. $Q_\alpha(A) - Q_\alpha(B) \neq 0$, or $Q_\mu(A) - Q_\mu(B) \neq 0$, will lead to differential accelerations
\( g_A - g_B \neq 0 \) and locally observable effects. Recently, we have studied \([5, 6]\) the composition dependence of the effective charges \( Q_{ri}(A) \) corresponding to a complete set of dimensionless ratios entering low-energy physics, namely \( r_0 = \Lambda_{QCD}/M_P \), and

\[
    r_i = \alpha, m_u/\Lambda_{QCD}, m_d/\Lambda_{QCD}, m_e/\Lambda_{QCD},
\]

where \( m_u \) and \( m_d \) are the masses of the light quarks. Here, we separated the ratio \( r_0 = \Lambda_{QCD}/M_P \), the dependence on which leads to composition-independent effects. We recall below our explicit results for the various charges \( Q_{ri}(A) (i \neq 0) \).

Each slope parameter \( B_{ri} \) defines a corresponding acceleration \( B_{ri} c^2 \), which enters the total acceleration (9), multiplied by the corresponding dimensionless effective charge \( Q_{ri}(A) \).

For instance, the \( \alpha \) gradient (2) reported by Webb et al \([1]\) corresponds (using \( c/1yr = 950 \text{ cm s}^{-2} \)) to the acceleration level

\[
    B_{\alpha} c^2 = (1.05 \pm 0.24) \times 10^{-12} \text{ cm s}^{-2},
\]

while the acceleration level corresponding to the \( \mu \) gradient suggested by Berengut et al \([2]\) is

\[
    B_{\mu} c^2 = (2.5 \pm 1.2) \times 10^{-12} \text{ cm s}^{-2}.
\]

The aim of this paper is to point out that the equivalence principle (EP)-violating effects of spatial gradients of \( \alpha \) and of the mean quark-mass ratio \( \hat{r}_m = \hat{m}/\Lambda_{QCD} \) (with \( \hat{m} = (m_u + m_d)/2 \)) at the levels (12) and (13) generate signals in the ranging to the Moon, which have a specific time structure, and an amplitude which seems large enough to be detectable by the recently started millimeter precision Apache Point Observatory Lunar Laser-ranging Operation (APOLLO) \([7, 8]\). (See \([9]\) for the results obtained from the pre-APOLLO Lunar Laser Ranging (LLR) experiments.) We also describe the weaker bounds on spatial gradients obtained by present laboratory-based experiments \([10]\), and indicate that planned EP experiments at the \( \Delta g/g \lesssim 10^{-17} \) level will probe cosmic gradients at the levels of equations (12) and (13).

Here, let us emphasize the difference in outlook between our previous work, and the present study. In \([5, 6]\), we were considering the case where the spatial or temporal variation of a dimensionless parameter indicates the existence of a field, say \( \phi \), which carries the spacetime dependence, and we were considering the violations of the ‘weak version’ of the EP, i.e. composition-dependent accelerations of body \( A \), mediated by the coupling of \( \phi \) to local matter distributions. As a consequence, the locally observable EP-violating effects depended on the product of two \( \phi \) coupling strengths, say \( \alpha_A \alpha_E \), where

\[
    \alpha_A = \partial \ln (M_A(\phi)/M_P)/\partial \ln \phi
\]

measures the coupling of \( \phi \) to body \( A \), and \( \alpha_E = \partial \ln (M_E(\phi)/M_P)/\partial \ln \phi \) its coupling to an external ‘source’ body \( E \) (which could be the Earth, the Sun or some laboratory source). However, we had normalized the definition of the fundamental couplings \( d_{ri} \) of the ‘dilaton’ field \( \phi \), as they enter the low-energy Lagrangian, so that we could write each \( \alpha_A \) in the specific form

\[
    \alpha_A = d_{r0} + \sum_{i \neq 0} d_{ri} Q_{ri}(A)
\]

exhibiting a simple factorization between the fundamental dilaton couplings \( d_{r0} \) and \( d_{ri} \), and the phenomenological effective charges defined in equation (10). Note that there is no composition-dependent charge associated with the coupling to \( r_0 = \Lambda_{QCD}/M_P \), or, said

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3 We have argued in \([5, 6]\) that the composition dependence linked to the strange quark mass \( m_s \) was subdominant.

4 Note that, from a theoretical point of view, the ratio \( r_0 = \hat{m}/\Lambda_{QCD} \) is akin to \( \mu = m_s/m_p \) in the sense that it is the ratio between a lepton mass and a hadronic one.
 differently, the charge $Q_{r_0}(A)$ associated with $r_0 = \Lambda_{QCD}/M_P$ is simply $Q_{r_0}(A) \equiv 1$ because, as the mass $M_A$ can be written as the product of the hadronic mass scale $\Lambda_{QCD}$ by a dimensionless function $f(r_\xi)$ of the dimensionless ratios, equation (11), the mass ratio $M_A/M_P$ can be identically written as $M_A/M_P = r_0 f(r_\xi)$. (Note also that the $d_r$’s entering equation (14) correspond to the differences $d_r - d_x$ in [5, 6] (with $d_x \equiv d_{x_0}$) because we defined above the ratios $r_\xi$ by equation (11) which involved a logarithmic derivative of $M_A/M_{QCD}$, while we were working there with logarithmic derivatives of $M_A/M_P$.)

When contemplating, as we do here, possible variations over cosmological distances, the field $\varphi$ must be essentially massless. However, in this work we shall not need to consider any specific model neither for the mass (or self-potential $V(\varphi)$) of $\varphi$, nor for its matter couplings $d_\varphi$ and $d_r$. Indeed, the crucial point is that the observable acceleration (9) only depends on the effective charges (10), and on the various spatial gradient parameters $\nabla \ln r_\xi = B_r \, \hat{z}_r$. This makes the present investigation model independent, as well as independent from the usual interpretation of local EP tests (which involve the bilinear products $(\alpha_A - \alpha_B) \, \alpha_E$).

2. The gravitational Stark effect and spatial varying couplings

The accurate monitoring of the lunar motion (most notably by LLR experiments [9]) has led to impressive tests of relativistic gravity, and notably of various aspects of the EP [11]. Here, we are interested in EP-violating effects in LLR that are linked to a fixed preferred direction in space. Such effects have been studied by Damour and Schaefer [12] in the context of binary pulsars. The analysis of such preferred-direction forces in the context of LLR has been done at leading order (LO) by Nordtvedt [13] (see also [14]), and to very high perturbative order by Damour and Vokrouhlický [15] (using the Hill–Brown lunar theory [16]). For references to analytic studies of relativistic effects in lunar motion, as well as a self-contained introduction to Hill–Brown theory, see, e.g., [17]. Lunar dynamics is a notoriously difficult problem because of the rather strong perturbation coming from the Sun’s tidal forces, which leads to badly convergent perturbation series in powers of the parameter $m = n/(n - n') \approx 1/12.3687$. (Here, $n' = 2\pi/(1\text{yr})$ denotes the mean sidereal angular velocity of the Earth around the Sun, and $n = 2\pi/(27.32\text{ days})$ [18] the mean sidereal angular velocity of the Moon around the Earth.) For some effects, a LO perturbation treatment in $m$ can be significantly inaccurate. This is, for instance, the case for the Laplace–Nordtvedt effect of polarization of the Moon’s orbit by an EP violation linked to the Sun’s gravity where higher order terms in $m$ increase the LO result by more than 62% [17, 19]. In the case of interest here of what has been called the ‘gravitational Stark effect’ [12], i.e. the perturbing influence of a differential force (with a fixed direction) acting on a gravitationally bound two-body system, the situation is similar, though with significant differences.

Let us first recall that the classical (electric or gravitational) Stark effect is an example of singular perturbation where a small perturbing force can have a large effect. If we were approximating the dynamics of the relative Earth–Moon coordinate $r = x_M - x_E$ in the presence of an external acceleration $\Delta g = g_M - g_E$ by means of the Lagrangian (after factorization of the Earth–Moon reduced mass $\mu \equiv m_E \, m_M/(m_E + m_M)$)

$$ L = \frac{1}{2} \left( \frac{dr}{dt} \right)^2 + \frac{G(m_E + m_M)}{r} + \Delta g \cdot r, \quad (15) $$

we could find the exact solution of the perturbed dynamics by separating the Hamilton–Jacobi equation corresponding to the Lagrangian (15) in parabolic coordinates ($\xi = r + z, \eta = r - z, \phi$), with a $z$ axis oriented along $\Delta g$. One then finds that the exact solution corresponding to elliptic orbits undergoes a complicated secular evolution during which the osculating elements
of the elliptic motion wander very far away from any given initial state. For instance, even if the perturbing acceleration $\Delta g$ is very small, the osculating eccentricity will not undergo small oscillations around its initial value $e_0$ but will, on time scales $na/\Delta g$, take values quite different from $e_0$. This instability of elliptic motion under a constant force can also be seen by using the averaged evolution equations of the osculating orbital elements. More precisely, if we consider the evolution of the semi-major axis $a$, of the Lagrange–Laplace–Runge–Lenz eccentricity vector $e = ea$ (directed toward the periastron), where $(a, b, c)$ are the orthonormal unit vectors with $a$ pointing toward the periastron and $c$ along the (scaled) orbital angular momentum $\ell = (1 - e^2)^{1/2}c = r \times v/\sqrt{G(m_E + m_M)a}$, one finds the averaged evolution equations of the form \[12\]

\[
\begin{align*}
\left\langle \frac{da}{dt} \right\rangle &= 0, \\
\left\langle \frac{de}{dt} \right\rangle &= f \times \ell, \\
\left\langle \frac{d\ell}{dt} \right\rangle &= f \times e,
\end{align*}
\] (16)

where

\[ f = \frac{3}{2} \frac{\Delta g}{na}, \] (17)

with $n$ denoting the sidereal angular frequency of the Moon.

We see that while $a$ stays secularly constant, the vectors $e$ and $\ell$ rotate one into another. More precisely (with $f = f^\hat{z}$), one easily sees that, while $e_x$ and $\ell_z$ stay constant, the two complex combinations $\varepsilon_x \equiv e_x + i\ell_y$ and $\varepsilon_y \equiv e_y + i\ell_x$ rotate as

\[
\varepsilon_x(t) = e^{i\omega_p t} \varepsilon_x(0); \quad \varepsilon_y(t) = e^{-i\omega_p t} \varepsilon_y(0),
\] (18)

leaving constant $|\varepsilon_x|^2 = e_x^2 + \ell_y^2$ and $|\varepsilon_y|^2 = e_y^2 + \ell_x^2$ (consistently with $e^2 + \ell^2 = 1 = \text{const}$).

This Stark instability is rooted in the well-known degeneracy of the Coulomb problem, i.e. the fact that the radial $\omega_r$ and angular frequencies $\omega_\phi$ happen to be exactly equal, $\omega_r = \omega_\phi = n$, for a $1/r$ interaction potential. As a consequence, any lifting of the Coulomb degeneracy by an additional interaction potential (causing $\omega_r$ to differ from $\omega_\phi$) will tame the Stark instability. The authors of [12] considered the case where this lifting was due to the general relativistic modifications of the $1/r$ Newtonian potential. In that case, the only modification of the secular evolution equations (equations (16)) is the appearance of an additional contribution $+ \omega_p c \times e$ on the rhs of the evolution equation of $e$, where $\omega_p = \omega_\phi - \omega_r$ is the precession frequency of the binary system, due to relativistic effects.

As a first orientation toward understanding the Stark effect in the lunar motion, let us start by assuming that, as in the case studied in [12], it is enough to replace the second secular evolution equation in equations (16) by

\[
\left\langle \frac{de}{dt} \right\rangle = f \times \ell + \omega_p c \times e,
\] (19)

where $\omega_p$ describes the precession of the orbit of the Moon, which occurs with a period of 8.85 years [18]. (Let us note in passing that the amplitude of the eccentricity evolves according to $\left\langle \frac{de}{dt} \right\rangle = (1 - e^2)^{1/2}f \cdot b_\perp$.)

In addition, as the eccentricity of the Earth–Moon system is small $e = 0.0549$, we can, in first approximation (in view of the appearance of $e$ on the rhs of the evolution equation of
the angular momentum $\ell$, neglect the small wobbling of the direction of the orbital angular
momentum $c$ and approximate equation (19) (using also $\ell = (1 - e^2)^{1/2} e \simeq c$) by
\[
\frac{de}{dt} = f \times c + \omega_p c \times e = \omega_p c \times (e - e_f),
\]
where we have introduced
\[
e_f = \frac{f}{\omega_p} = \frac{3}{2} \frac{\Delta g_1}{n a \omega_p},
\]
where $f_\perp = f - (f \cdot c) c$ is the component of the external force in the plane of the orbit.

The general solution of equation (20) reads
\[
e(t) = e_f + e_p(t),
\]
where the constant vector $e_f$ describes a ‘forced eccentricity’ (or a ‘polarization’) induced by
the external EP-violating force, and where $e_p(t)$ is the usual, $\omega_p$-precessing free eccentricity,
which is allowed in the absence of the external Stark effect. Note in passing that the polarization
of the elliptic orbit by the external force is oriented in the opposite

The final observable result of this LO treatment, linked to the polarization $e_f$, is a sidereal
frequency oscillation of the Earth–Moon range, connected with the direction of the projection $f_\perp$ of the perturbing acceleration onto the orbital plane, of the form
\[
\Delta^{LO}_R(t) = \rho^{LO}_f \cos(n(t - t_0) - \phi_f),
\]
where the (algebraic) amplitude is
\[
\rho^{LO}_f = \frac{f_\perp a}{\omega_p} = \frac{3}{2} \frac{\Delta g_1}{n a \omega_p},
\]
and where $n(t - t_0)$ is the longitude of the Moon, and $\phi_f$ is the longitude of the direction $f_\perp/|f_\perp|$, both longitudes being measured within the orbital plane, from some common origin.
(Neglecting the small inclination $i \simeq 5^\circ$ of the Moon’s orbit on the ecliptic, we can consider
that both longitudes are ecliptic longitudes, counted from the vernal equinox.)

The above result was obtained as a LO approximation, under the simplifying assumption
that the main effect of the solar tide on the Earth–Moon system was to introduce a perigee
precession term in equation (19). Going beyond such a LO treatment, i.e. taking into account
the combined effect of the $G(m_E + m_M)/r$ Earth–Moon potential and of the quadrupolar
tide $\frac{1}{2} \sum_{i,j} \partial_{ij}(GmS/D)$ of the Sun, leads to slowly converging perturbation expansions in the
expansion parameter
\[
m = \frac{n'}{n - n'} \simeq 0.080849 \simeq \frac{1}{12.3687}.
\]
For instance, the perigee precession frequency of the Moon is given by a perturbation expansion of the form
\[
\frac{\omega_p}{n} \simeq \frac{3}{22} m^2 + \frac{177}{25} m^3 + \frac{1659}{27} m^4 + \frac{85205}{211} m^5 + \frac{3073531}{213} \cdot 3 m^6 + \cdots
\]
which converges so slowly that the sum of higher order terms approximately doubles the LO
analytical result $\omega_p^{LO}/n = 3m^2/4$ (see [20] for the literal computation of the perturbation expansion of $\omega_p/n$ up to the 11th power of $m$). Let us also note that the above LO result for the range perturbation due to the Stark effect contained $\omega_p/n = 3m^2/4 + \cdots$ as a small
denominator that significantly amplified the effect of the external perturbing force $f$. (The
presence of a small denominator is linked to the instability, recalled above, of elliptic motion under a constant force, because this small denominator tends to zero in the limiting case of the Lagrangian (15).

The authors of [15] worked out the lunar range perturbation \( \Delta r(t) \) induced by an external acceleration \( \Delta g \) to very high order in the powers of \( m \). This range perturbation \( \Delta r(t) \) is the sum of many different frequency components that come from the nonlinear combination of the basic sidereal frequency \( n \) of the Moon (linked to the angular distance between the Moon and the external fixed direction \( \Delta g \), or rather its projection \( \Delta g_L \) on the Moon’s orbital plane), with even multiples of the synodic frequency \( n - n' \) linked to the angular distance (seen from the Earth) between the Moon and the Sun. Among the spectrum of combined frequencies \( \pm(n + 2j(n - n')) \) \((j \in \mathbb{Z})\), two of them were found to be dominant: the basic sidereal frequency \( \pm n \) (of period 27.32 days), and the \( j = -1 \) combination \( \pm(n - 2(n - n')) = \mp(n - 2n') \) (of period 32.13 days). The result of [15] can be written5 as

\[
\Delta r(t) = \rho_f \left[ \cos[n(t - t_0) - \phi_f] + \frac{15}{8} m S_f'(m) \cos[n(t - t_0) - 2\tau(t) - \phi_f] + \ldots \right]. \tag{26}
\]

Here, \( \tau(t) \equiv (n - n')t + t_0 \) denotes the synodic phase, i.e. the angular distance between the Moon and the Sun, and the overall amplitude is given by

\[
\rho_f = -2 \frac{\Delta g_L}{n^2} S_f(m), \tag{27}
\]

where \( \Delta g_L \) and \( \phi_f \) are the magnitude and the longitude of the projection \( \Delta g_L \) of the external acceleration onto the lunar orbital plane6, respectively, and where \( S_f(m) \) and \( S_f'(m) \) are the \( m \)-perturbation series that start as \( 1 + O(m) \). For instance, the beginning of the expansion of \( S_f(m) \) reads

\[
S_f(m) = 1 - \frac{75}{8} m + \frac{235}{4} m^2 - \frac{127 637}{384} m^3 + \frac{4172 299}{2304} m^4 + O(m^5) \tag{28}
\]

and table IV of [15] gives the coefficients of this expansion to the ninth order in \( m \). Even with such a high-order expansion one finds that the last term is still of fractional order \( 10^{-3} \). This slow convergence is related to the presence of a pole in the series \( S_f(m) \) and \( S_f'(m) \) near \( m_{eq} \approx -0.18407 \). To get an accurate numerical estimate of these series, the authors of [15] used a Padé resummation, with the results (for \( m = m_{Moon} \) given by equation (25))

\[
S_f(m) \approx 0.5050
\]

and

\[
\frac{15}{8} m S_f'(m) \approx \frac{1}{5.94}
\]

for the fractional coefficient of the subleading term with frequency \( n - 2(n - n') = -(n - 2n') \).

An observationally important aspect of the result (26) is the appearance of a specific combination of two harmonics, with known periods and phase, and with nearly comparable magnitudes. In particular, the fact that the amplitude of the \( n - 2n' \) harmonic is only \( \approx 6 \) times smaller than the LO \( n \) harmonic is a result of the subtleties of lunar perturbation theory. This term comes from the basic solar tide perturbation which is proportional to \( m^2 \), but it has been amplified to the \( O(m) \) level by a small denominator proportional to \( m \) (with the additional factor \( 15/8 \approx 2 \), leading to \( 15m/8 \approx 1/6 \)).

5 Here we change the notation of Section IV of [15]: e.g. \( A_0 \rightarrow \Delta g_L \), \( S_{g0}(m) \rightarrow S_f(m) \), etc.

6 Even in the full Hill–Brown treatment (with neglect of the lunar inclination, and of the eccentricity of the Moon’s orbit), one finds that only the projection \( \Delta g_L \) of the external acceleration matters for the range perturbation.
For what concerns the leading term, with frequency $n$, in equation (26), it corresponds to the result (equation (23)) of the approximate treatment explained above. In both cases, $\phi_f$ is the longitude of the external acceleration projected within the orbital plane. Note that if one were using the LO analytical result $(\omega_p/n)^{\text{LO}} = (3/4)m^2$, equation (24) would read $-2\Delta g_\perp/n^2$ and would agree with the LO limit obtained by using $S_\perp^{\text{LO}}(m) = 1$ in equation (27). However, the exact result (equation (27)) is smaller than this by about a factor 2, because of the correcting factor $S_f(m) \simeq 0.5050$. As noted in [15], when using in equation (24) the actual perigee precession $\omega_p$ (which is about twice larger than its LO estimate $(\omega_p/n)^{\text{LO}} = (3/4)m^2$), one captures most of the effect of the slowly converging series $S_f(m)$.

We can finally apply the result (equation (26)) to the EP-violating acceleration $\Delta g = g_M - g_E$ where each $g_A$ is given by equation (9). Let us first note that the result only depends on the amplitude and longitude of the projection on the orbital plane of the vectorial sum of the external accelerations

$$\Delta g = g_M - g_E = -\sum_i (Q_{r_i}(M) - Q_{r_i}(E)) B_{ri} c^2 \hat{z}_{ri}. \quad (29)$$

Alternatively, one could write the range perturbation as a sum of terms of the type of the rhs of equation (26), each one having an amplitude $\rho_f(ri)$ and a phase $\phi_f(ri)$. (Note that we consider here algebraic amplitudes (that can be negative), and that the longitudes $\phi_f(ri)$ always refer to the direction of the projection $\hat{z}_{ri} \perp$ of the gradient direction $+\hat{z}_{ri}$.) Let us, as it simplifies the writing of our results, make the natural assumption that all the cosmological gradients of the coupling constants are parallel to each other, i.e. $\hat{z}_{ri} = \hat{z}$ independently of the label $r_i$, and let us denote by $\phi_f$ the longitude of the projected gradient direction $\hat{z}_\perp$. This leads to a total range perturbation of the form equation (26), with a total (algebraic) amplitude of the form (after cancellation of two minus signs)

$$\rho_f^\text{tot} = 2S_f(m) \sum_i (Q_{r_i}(M) - Q_{r_i}(E)) B_{ri} c^2 \hat{z}_{ri} \perp. \quad (30)$$

where $B_{ri} \perp$ is the magnitude of the projected gradient $B_{ri} \hat{z}_\perp$.

Note that the numerical prefactor $2S_f(m) \simeq 1.010$ is close to 1, and that the parameter combination $B_{ri} \perp c^2/n^2$, where we recall that $n' = 2\pi/1\text{yr}$, is the product of an acceleration by the square of a time, and is indeed a length. (Alternatively, we can think of it as the product of the spatial gradient $B_{ri} \perp \equiv [\text{length}]^{-1}$, by the squared length $c^2/n^2 = (1\text{yr})^2/(4\pi^2)$. As a first orientation, note that a gradient of order $B \sim 10^{-6} \text{Glyr}^{-1}$, i.e. $Bc^2 \sim 10^{-12} \text{cm s}^{-2}$ (similar to the recent suggestions, equations (2), (3), (12) and (13)), corresponds to a figure of merit $Bc^2/n^2 \simeq 25 \text{cm}$. In order to estimate the corresponding signal in LLR, we need next to estimate the numerical value of the various EP-violating charge differences $Q_{r_i}(M) - Q_{r_i}(E)$ corresponding to the difference in composition of the Moon and the Earth. This will be the focus of the next section.

Before tackling this issue, let us briefly mention that the central values of the equatorial coordinates $a = 261^\circ$, $\delta = -58^\circ$ of the cosmological gradient of the fine structure constant reported in [1] correspond to an ecliptic latitude equal to $\beta = 34.7^\circ$, and an ecliptic longitude equal to $\lambda = 95.8^\circ$. The latter ecliptic longitude predicts the value of the longitude entering the range perturbation (equation (26)), namely $\phi_f = \lambda$. On the other hand, the ecliptic latitude $\beta$ enters the observable range $\rho_f$ through the projection of the cosmological gradient
direction onto the orbital plane, i.e. (essentially) onto the ecliptic. More precisely, we have \( B_{r,\perp} = B_r \cos \beta \). Note that \( \cos \beta = 0.822 \) for the gradient reported by Webb et al so that this projection (that we will include in the estimates of the next section) reduces only by 18% the full possible observable effect of such a gradient on the lunar motion.

### 3. EP-violating charges

Let us now estimate the numerical values of the various EP-violating charge differences \( Q_r(M) - Q_r(E) \) entering the magnitude of the cosmologically induced Earth–Moon differential acceleration (9). This issue has been discussed in our previous work [5, 6] where we described the leading dependence of atomic masses on the parameters of the standard model.

Following [5, 6] it is convenient to replace, in the list of ratios (11), the separate masses of the up and down quarks, by their average and difference, namely

\[
\hat{m} = \frac{(m_d + m_u)}{2}, \quad \delta m = m_d - m_u.
\]

In other words, we use as a list of dimensionless couplings the fine structure constant \( \alpha \) together with the dimensionless ratios

\[
\hat{r}_\hat{m} = \frac{\hat{m}}{\Lambda_{\text{QCD}}}, \quad r_{\delta m} = \frac{\delta m}{\Lambda_{\text{QCD}}}, \quad r_{m_e} = \frac{m_e}{\Lambda_{\text{QCD}}}.
\]

Because the fermion masses are the product of a dimensionless Yukawa coupling \( \Gamma_i \) and the Higgs vacuum expectation value (vev) \( v \), \( m_i = \Gamma_i v / \sqrt{2} \), these ratios can be considered as the product of the dimensionless \( \Gamma_i \) by the ratio \( v / \Lambda_{\text{QCD}} \) between the basic weak-interaction scale \( v \) and the basic strong-interaction scale \( \Lambda_{\text{QCD}} \). In view of the independence of the mechanisms leading to the appearance of the two basic scales \( v \) and \( \Lambda_{\text{QCD}} \), it seems a priori theoretically natural to expect that the cosmological gradients (if any) of the three mass ratios (32) will be of similar magnitudes, and therefore similar to that of the ratio \( \mu = m_e / m_p \) for which the value (3) has been recently suggested.

The authors of [5, 6] derived the following approximate estimates for the four effective charges \( Q_r \) associated with the three mass ratios (32), and with \( \alpha \):

\[
Q_{\hat{m}} = F_A \left[ 0.093 - \frac{0.036}{A^{1/3}} - 0.020 \frac{(A - 2Z)^2}{A^2} - 1.4 \times 10^{-4} \frac{Z(Z - 1)}{A^{4/3}} \right],
\]

\[
Q_{\delta m} = F_A \left[ 0.0017 \frac{A - 2Z}{A} \right],
\]

\[
Q_{m_e} = F_A \left[ 5.5 \times 10^{-4} \frac{Z}{A} \right],
\]

and

\[
Q_{\alpha} = F_A \left[ -1.4 + 8.2 \frac{Z}{A} + 7.7 \frac{Z(Z - 1)}{A^{4/3}} \right] \times 10^{-4},
\]

where \( F_A \equiv \Lambda m_{\text{amu}} / M_A \), with \( m_{\text{amu}} = 931 \text{ MeV} \) denoting the atomic mass unit. The common factor \( F_A \) is very close to 1, and we will replace it by 1 in our estimates below.

Approximating the Moon as made of silicate (i.e. essentially SiO\(_2\)), and the Earth as made of a mantle of silicate and a core of iron (representing 32% of its mass), the above formulas yield the following Moon–Earth charge differences:

\[
Q_{\hat{m}}(M) - Q_{\hat{m}}(E) = -1.1 \times 10^{-3}
\]
Table 1. Approximate EP-violating ‘effective charges’ for a sample of materials. These charges are averaged over the (isotopic or chemical, for SiO$_2$) composition.

| Material | $A$ | $Z$ | $Q_{\hat{m}}'$ | $Q_{\alpha}'$ |
|----------|-----|-----|----------------|---------------|
| Li       | 7   | 3   | $18.88 \times 10^{-3}$ | $0.345 \times 10^{-3}$ |
| Be       | 9   | 4   | $17.40 \times 10^{-3}$ | $0.494 \times 10^{-3}$ |
| Al       | 27  | 13  | $12.27 \times 10^{-3}$ | $1.48 \times 10^{-3}$ |
| Si       | 28.1| 14  | $12.11 \times 10^{-3}$ | $1.64 \times 10^{-3}$ |
| SiO$_2$  | ... | ... | $13.39 \times 10^{-3}$ | $1.34 \times 10^{-3}$ |
| Ti       | 47.9| 22  | $10.28 \times 10^{-3}$ | $2.04 \times 10^{-3}$ |
| Fe       | 56  | 26  | $9.83 \times 10^{-3}$  | $2.34 \times 10^{-3}$ |
| Cu       | 63.6| 29  | $7.67 \times 10^{-3}$  | $3.37 \times 10^{-3}$ |
| Cs       | 133 | 55  | $7.67 \times 10^{-3}$  | $3.37 \times 10^{-3}$ |
| Pt       | 195.1| 78  | $6.95 \times 10^{-3}$  | $4.09 \times 10^{-3}$ |

\[ Q_{\hat{m}}(M) - Q_{\hat{m}}(E) = -3.8 \times 10^{-5} \]  \(38\)

\[ Q_{m_a}(M) - Q_{m_a}(E) = +6.1 \times 10^{-6} \]  \(39\)

\[ Q_{\alpha}(M) - Q_{\alpha}(E) = -3.1 \times 10^{-4}. \]  \(40\)

We see that the most important charges for EP violation are $Q_{\hat{m}}$ and $Q_{\alpha}$. This dominance of $Q_{\hat{m}}$ and $Q_{\alpha}$ over the other charges is true for most values of $Z, A$. It is related both to the rather small coefficients entering $Q_{\hat{m}}$, equation (34), and $Q_{m_a}$, equation (35), and to the fact that $A \approx 2Z$ along the periodic table. If we use the facts that $F_A - 1 = O(10^{-3})$, and that $A \approx 2Z$, we can simplify the composition dependence of the main EP-violating charges $Q_{\hat{m}}$ and $Q_{\alpha}$ as

\[ Q_{\hat{m}}' = - \frac{0.036}{A^{1/3}} - 1.4 \times 10^{-4} \frac{Z(Z - 1)}{A^{4/3}} \]  \(41\)

\[ Q_{\alpha}' = +7.7 \frac{Z(Z - 1)}{A^{4/3}} \times 10^{-4}. \]  \(42\)

We list the values of $-Q_{\hat{m}}'$ and $Q_{\alpha}'$ for a sample of materials in table 1. This list shows that the maximum value of a charge difference would be the $Q_{\hat{m}}$ difference between a heavy element and a light one with $Q_{\hat{m}}$(heavy) $- Q_{\hat{m}}$(light) $\simeq +10^{-2}$. The corresponding maximum acceleration level $\Delta g_{\hat{m}}^{\text{max}} = \Delta Q_{\hat{m}}^\text{max} B_{\hat{m}} c^2 \simeq 10^{-2} B_{\hat{m}} c^2$ would numerically be

\[ \Delta g_{\hat{m}}^{\text{max}} \simeq \left( \frac{B_{\hat{m}}}{10^{-6} \text{ Gyr}^{-1}} \right) \times 10^{-14} \text{ cm s}^{-2} \]

for a cosmic gradient of order of those reported by Webb et al.

4. Observational signals linked to possible cosmological gradients

Putting together our results, keeping only the dominant terms linked to $Q_{\hat{m}}$ and $Q_{\alpha}$, and scaling the possible cosmological gradients of $\hat{m}$ and $\alpha$ by the recently reported values\(^8\)

\(^8\) As mentioned above, it is natural to expect that a cosmological gradient of $\mu = m_e/m_p$ implies a similar gradient in the weak-interaction/strong-interaction ratios $r_i$, and notably in $\hat{m}/\Lambda_{\text{QCD}}$. 10
(equations (3), (2), (13) and (12), we conclude that those cosmological gradients entail EP-violating differential accelerations on the Moon directed along the unit vector \( \hat{z}_\perp \) (with ecliptic longitude equal to \( \phi_f \simeq \lambda \simeq -95.8^\circ \)), and with algebraic magnitudes

\[
\Delta g_{\perp} = \Delta g_{\theta \perp} + \Delta g_{\alpha \perp},
\]

where

\[
\Delta g_{\theta \perp} = - (Q_{\theta}(M) - Q_{\theta}(E)) B_{\theta \perp} c^2 = +2.2 \left( \frac{B_{\theta}}{2.6 \times 10^{-6} \text{ Glyr}^{-1}} \right) \times 10^{-15} \text{ cm s}^{-2}
\]

and

\[
\Delta g_{\alpha \perp} = - (Q_{\alpha}(M) - Q_{\alpha}(E)) B_{\alpha \perp} c^2 = +2.7 \left( \frac{B_{\alpha}}{1.1 \times 10^{-6} \text{ Glyr}^{-1}} \right) \times 10^{-16} \text{ cm s}^{-2}.
\]

These differential accelerations then induce a perturbation in the Earth–Moon range which has the specific time signature (26) with an algebraic magnitude

\[
\rho_f = \rho_{\theta} + \rho_{\alpha},
\]

where

\[
\rho_{\theta} = - \left( \frac{B_{\theta}}{2.6 \times 10^{-6} \text{ Glyr}^{-1}} \right) 0.59 \text{ mm}
\]

and

\[
\rho_{\alpha} = - \left( \frac{B_{\alpha}}{1.1 \times 10^{-6} \text{ Glyr}^{-1}} \right) 0.068 \text{ mm}.
\]

As we see, the dominant effect is expected to be linked to the cosmological gradient of \( \hat{m}/\Lambda_{\text{QCD}} \), so that the recent findings of Webb and collaborators [1, 2] suggest the presence of millimeter-level sidereal fluctuations in the Earth–Moon range.

Such fluctuations, if they exist, might be detectable by the APOLLO experiment. Indeed, this experiment has shown its capability of obtaining ‘normal-point’ range measurements with nightly median uncertainty of 1.8 mm for their entire data set, and 1.1 mm for their recent data [8]. Though it is beyond the scope of this paper to attempt a full covariance analysis of the capability of the APOLLO experiment to measure the specific signal (26) on top of many other range effects, we expect that the accumulation, in sufficient number \( N \), of normal-point data with individual millimeter-level range uncertainty \( \delta \), over a sufficiently long time (comparable to the perigee period \( \simeq 8.85 \) years), will allow one to decorrelate the specific sidereal signal (26) from the many other model parameters, and to measure its amplitude \( \rho_f \) to a fraction of a millimeter. We recall that the statistical probable error on \( \rho_f \) can be written (assuming the statistical independence of individual errors) as

\[
\frac{\delta}{\sqrt{N(1-c^2)}},
\]

where \( c \) denotes the global correlation coefficient of \( \rho_f \) with all the other model parameters. The quite specific sidereal frequency of the signal (26) can, in principle, distinguish it from the other model signals, though at the cost of having a time span large enough to separate the sidereal frequency \( n \) from the nearby synodic, \( n - n' \), and anomalous, \( \omega_r = n - \omega_p \), frequencies, hence the need of an observational time comparable to the perigee period. Then, even if the global correlation coefficient \( |c| \) is close to 1, one might hope that a number \( N \sim 10^3 \) of individual normal points will allow one to end up with a final error on \( \rho_f \) at the needed level \( \lesssim \delta \). An indication that

\footnote{Our present remarks assume that one will be able to develop a model of APOLLO range data whose remaining systematic errors do not mimic a millimeter-level sidereal effect of the form (26).}
this might happen comes from the results of a similar analysis of the pre-APOLLO LLR data. Indeed, a sidereal range perturbation of the approximate form (23), with a phase \(\phi_f\) linked to the center of the Galaxy, has been searched for in the pre-APOLLO, few-centimeter-level LLR data after the suggestion given in [13]. Nordtvedt et al [21] published an upper limit of
\[
\Delta g_{\text{gal}} < 3 \times 10^{-14} \text{ cm s}^{-2}
\]
(49) on a possible perturbing differential acceleration linked to the Galactic center, while a further analysis of Müller (cited in [22]) obtained
\[
\Delta g_{\text{gal}} = (4 \pm 4) \times 10^{-14} \text{ cm s}^{-2}.
\]
(50) Note that, in view of equation (27), such acceleration levels correspond to an error level on \(\rho_f\) of order of 1 cm (while most of the pre-APOLLO range data had uncertainties of a few centimeters). The gain in sensitivity of the APOLLO experiment, by more than a factor 10, and the richer time signature of the more complete signal (26) make us expect that it will be possible to probe the acceleration levels (44) and (45) above. Depending on the result of such an analysis, the APOLLO experiment could either establish the reality of a cosmological gradient of coupling constants, or set upper bounds on the gradients \(B_{\text{gal}}\) and \(B_{\text{m}}\) (or more precisely on the combination \(B_{\text{gal}} + 0.28 B_{\text{m}}\) entering \(\Delta g_{\perp}\)) at levels smaller than the levels (3) and (2) (and (13) and (12)).

It is instructive to compare the (potential) sensitivity of LLR experiments to external EP-violating accelerations with the sensitivity of other EP experiments. There have been constraints on anomalous cosmic accelerations by laboratory-based EP tests. The most precise is quoted as a differential acceleration in any direction of the sky [10]:
\[
|g(\text{Be}) - g(\text{Ti})| < 8.8 \times 10^{-13} \text{ cm s}^{-2} (95\% \text{ C.L.}).
\]
The charge differences \(Q_{\text{Be}}(\text{Be}) - Q_{\text{Be}}(\text{Ti})\) are again dominated by \(Q_{\text{Be}}(\text{Be}) - Q_{\text{Be}}(\text{Ti}) = -7.23 \times 10^{-3}\) and \(Q_{\text{Be}}(\text{Be}) - Q_{\text{Be}}(\text{Ti}) = -1.56 \times 10^{-3}\). Assuming, as above, that the effect of the gradient of \(\hat{m}\) (which couples to the dominant charge difference) dominates, the above upper bound on \(|g(\text{Be}) - g(\text{Ti})|\) can be readily converted to a bound on the corresponding cosmological gradient \(B_{\text{Be}}\), namely
\[
|B_{\text{Be}}| < 1.3 \times 10^{-4} \text{ Glyr}^{-1}.
\]
(51) This is weaker than the recently suggested (theoretically similar) gradient (equation (3)) by a factor \(\approx 50\). Such a difference in acceleration sensitivity between Earth-based EP experiments and LLR ones might seem surprising in view of the fact that both types of experiments currently lead to comparable limits on the (Eötvös) EP-violation parameter \(\eta = \Delta g/g\), namely \(\eta_{\text{EarthMoon}} = (-1.0 \pm 1.4) \times 10^{-13}\) [9], versus \(\eta_{\text{EoTi}} = (0.3 \pm 1.8) \times 10^{-13}\) [10], and that both types of experiments use comparable background accelerations \(g\) in the ratio \(\Delta g/g\). Indeed, the \(g\) due to the Sun at Earth is \(g_{\odot} \approx 0.6 \text{ cm s}^{-2}\), while torsion balance experiments use only the horizontal component of the Earth gravity, namely \(g_{\perp} \approx 1.7 \text{ cm s}^{-2}\) at a latitude of 45\(^\circ\). We note that the greater sensitivity of LLR experiments to external (especially fixed-direction) accelerations is essentially rooted in the specific Stark instability mentioned above. Indeed, generally speaking, a differential acceleration \(\Delta g\) acting during a characteristic time \(t_c\) (which is \(t_c \sim \omega^{-1} \approx T/(2\pi)\) for a periodic phenomenon of the angular frequency \(\omega\) and period \(T\) ) corresponds to a measurable displacement of order \(\Delta r \sim \Delta g t_c^2 = \Delta g/\omega^2\). In the LLR case, we saw above that the range perturbation is \(\Delta r \sim \Delta g/n^2\) which is larger than the expected perturbation \(\sim \Delta g/n^2\) associated with the lunar frequency \(n\) by a factor
\[
\left(\frac{n}{n_1}\right)^2 = \left(\frac{1 \text{ year}}{1 \text{ sidereal month}}\right)^2 = (13.37)^2 = 178.7.
\]
This amplification factor lies at the root of the increased sensitivity of LLR experiments to external EP-violating accelerations having a fixed direction. We note in passing that the LLR sensitivity to the usually considered Laplace–Nordtvedt solar-rooted EP-violating acceleration is only amplified, w.r.t. $\Delta g/n^2$, by a parametrically smaller factor $\sim (3/2)(n/n') \sim 20$, i.e. about ten times less than that in the ‘Stark’, fixed-direction case. This difference is due to the difference in the corresponding small denominators, namely $O(m^2)$ in the sidereal-frequency (Stark) case, versus $O(m)$ in the synodic-frequency (Laplace–Nordtvedt) one.

Let us finally note that presently planned improved EP tests such as the Satellite Test of the Equivalence Principle (STEP) [23] ($\eta \sim 10^{-18}$) or proposed cold-atom-technology tests ($\eta \sim 10^{-17}$) [24], which will make use of the full strength of the Earth gravity, $g_E \simeq 980 \text{ cm s}^{-2}$, will be able to probe the cosmological-gradient-induced differential accelerations discussed above. Indeed, the acceleration (44) linked to a cosmic gradient of $\hat{m}$ can be rewritten (modulo a cosine factor, with a specific time dependence linked to the projection onto the sensitive direction of the considered EP experiment) as

$$\eta = \frac{\Delta g}{g} = 2.5 \frac{g_E}{g} \left( \frac{\Delta Q_{\hat{m}}}{10^{-2}} \right) \left( \frac{B_{\hat{m}}}{2.6 \times 10^{-6} \text{ Gyr}^{-1}} \right) \times 10^{-17},$$

where we allowed the considered man-made EP test to optimize the choice of materials by having a $\Delta Q_{\hat{m}} \sim 10^{-2}$, i.e. ten times better than for the Earth–Moon case (see table 1). This result shows that the LLR test of the cosmological acceleration (44), that should be doable by the APOLO experiment, corresponds (from the point of view of the sensitivity to a cosmic gradient) to an Earth-based EP test at the $\eta \sim 10^{-17}$ level.

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