Artificial Ground Motion Simulation with Multi Control Points and Automatic Recognition in Iterative Process

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Abstract. Artificial ground motion simulation with target response spectrum being compatible is a significant challenge for structural seismic design. In the presented scheme, two major modifications are implied on conventional frequency domain methodology as to realize automatic recognition of stubborn points in iterative process and eliminate them by handling multiple control points in frequency domain. The proposed methodology is evaluated by numerical simulation with the target response spectrum from Chinese Specification for Seismic Design of Highway Bridges compared with conventional Stochastic Method. The result shows that the presented scheme enables to recognize stubborn points and eliminate them in iterative process as it is designed, and the fitting accuracy is improved considerably. The scheme is verified reliable to generate input ground motion candidates as a reference for engineers to conduct seismic dynamic analysis.

1. Introduction
Considering the conception of performance-based earthquake engineering[2], dynamic analysis is essential for structural seismic design and input ground motion is required to conduct such a process. Due to the issue that the number of previous acceleration time histories of natural intense earthquakes is limited, it is widely accepted to generate response spectrum-compatible artificial ground motion as the input ground motion for dynamic analysis in structural seismic design[3,4,8]. Conventionally, frequency domain methodology is often utilized to synthesize response spectrum compatible artificial ground motion based on Fourier transform[5,10]. However, spectral acceleration correlated to some certain points in frequency domain does not converge in iterative process, which is called ‘stubborn point’[9].

In order to solve such a problem, a modified methodology in frequency domain is proposed, in which Fourier amplitudes of multiple control points in frequency domain have fluctuated in single iterative step and the stubborn point is automatically recognized in the algorithm as well.

2. Stochastic Method
Conventional frequency domain methodology for response spectrum compatible artificial ground motion simulation is originally proposed around 1980s as Stochastic Method[1]. In the method, an initial power spectrum density (PSD) is accessed from target response spectrum[6]:

...
\[ S_x(\omega) = \frac{\xi}{\pi \omega} \cdot S_a^2(\omega) \cdot \left\{ -\ln \left( \frac{-\pi \ln(1 - \gamma)}{\omega} \right) \right\}^{-1} \]  

where \( S_a(\omega) \) denotes PSD transformed from target response spectrum \( S_a(\omega) \), and \( \xi \) denotes damping ratio with \( \gamma \) representing exceedance probability.

Then, inverse Fast Fourier Transform is conducted with random Fourier phase spectrum and Fourier amplitude spectrum generated from PSD in equation (1):

\[ x(t) = I(t) \cdot \sum \omega A(\omega) \sin(\omega t + \theta(\omega)) \]  

where \( A(\omega) \) and \( \theta(\omega) \) represent Fourier amplitude spectrum and Fourier phase spectrum respectively, with \( I(t) \) denoting an enveloping function and \( x(t) \) denoting the artificial acceleration time-series.

The response spectrum of acceleration time history in equation (2) is accessed by utilizing Duhamel's integral and the Fourier amplitude is iterated based on the ratio of response spectrum of artificial signal and the target response spectrum:

\[ A_{n+1}(\omega) = A_n(\omega) \cdot \left( \frac{S_n(\omega)}{S_T(\omega)} \right)^2 \]  

where \( A_n(\omega) \) denotes Fourier amplitude spectrum in \( n \)th iterative step and \( S_n(\omega) \) denotes response spectrum of the artificial ground motion in \( n \)th iterative step, with \( S_T(\omega) \) representing target response spectrum. The general procedure is plotted in the following flow chart.

**Figure 1. Flow chart of the procedure of Stochastic Method**

Basically, Stochastic Method is considered as a simple and fast tool of signal processing to simulate artificial ground motions which are compatible with target response spectrum. However, stubborn points in frequency domain occur as the impact of iterative correlation and phase-angle spectrum is ignored in process of the certain methodology.

### 3. Modifications on conventional method

The objective of present paper is to automatically recognize the stubborn points in frequency domain in the iterative process of conventional frequency method and manage to eliminate them. Two major modifications are introduced in the scheme and the first one aims to reveal the conditions of automatic recognition of stubborn points:

\[ \frac{S_T(\omega)}{S_n(\omega)} > \frac{S_T(\omega)}{S_n(\omega)} > 1 \quad \vee \quad \frac{S_T(\omega)}{S_n(\omega)} < \frac{S_T(\omega)}{S_n(\omega)} < 1 \]  

Equation (4) assumes the condition that the \( n \)th step of iteration does not help the response spectrum of the artificial ground motion converge to the target response spectrum, such that for the certain control frequencies, further divergence of response spectrum can be prevented based on this modification.

Fourier amplitude of the exact frequency-domain point is altered in conventional method, possibly
leading to some extreme situation which dramatically amplifies frequency spectrum. In order to address the problem, a modification is introduced to equation (3):

$$ A_{n+1}(\omega) = A_n(\omega) \cdot \left( \frac{S_m(\omega)}{S_f(\omega)} \right) \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{(\omega - \omega_k)^2}{2}} $$

(5)

$$ \omega = \omega_{k-3}, \omega_{k-2}, \omega_{k-1}, \omega_k, \omega_{k+1}, \omega_{k+2}, \omega_{k+3} $$

(6)

where \( \omega_k \) denotes the target frequency-domain point to be altered. Instead of directing towards the target frequency-domain point especially, multiple control points nearby the target frequency-domain point are utilized to achieve convergence of the response spectrum. 7 frequency-domain points including the target frequency-domain point are selected to be adjusted on their Fourier amplitudes, regarding to the side effect of the impact of nearby frequency-domain points on the target point. To make it easier to understand, the suggested modification is simplified in the following diagram.

Figure 2. Diagram of the different mechanisms between conventional and modified method

In case of Figure 2(b), it is verified that the iterative strategy of modified method is represented by green arrows while conventional method is represented by the black arrow. In the modified methodology, attenuation of the modifying scale of each frequency-domain point from target one to side one follows the pattern of Gaussian function.

4. Numerical simulation

A scenario response spectrum in Chinese Specification for Seismic Design of Highway Bridges (JTG/T 2231-01-2020)[7] is selected for case study of numerical simulation. The values of certain coefficients including characteristic period \( T_g \), seismic importance factor \( C_i \), field factor \( C_s \), damping impact factor \( C_d \) and basic peak ground acceleration \( A \) are shown in Table 1. The frequency range of control points for numerical simulation is prescribed at 0.125~34.94Hz (0.028~8s for periods). In total 82 control points are selected in the prescribed frequency range, evenly arranged on the logarithmic axis.

| Coefficients | Values |
|--------------|--------|
| \( T_g/s \)  | 0.45   |
| \( C_i \)    | 1.3    |
| \( C_s \)    | 1.0    |
| \( C_d \)    | 1.0    |
| \( A/g \)    | 0.2    |

10 artificial ground motions compatible with the design ground motion are generated by utilizing modified methodology and the other 10 ground motions are synthesized by conventional methodology as control group. Artificial ground motions in both test group and control group are simulated using the same sets of random Fourier phase spectra due to the principle of variable control. The duration for artificial signal is set at 40 seconds and the sampling frequency is set at 1024Hz. The fitting accuracy of both test group and control group is shown in Figure 3. Acceleration time history of one of artificial
ground motions in test group is shown in Figure 4.

![Figure 3](image)

**Figure 3. Fitting accuracy of response spectra in test group (a) and control group (b)**

It can be seen from Figure 3 that the fitting accuracy of test group is considerably better than control group, illustrating that the stubborn points are automatically recognized in test group and the modified methodology declines the deviations between artificial response spectra and design response spectrum.

5. Conclusion
The selection of input ground motions for structural seismic design is of huge importance. An innovative method modified from Stochastic Method is introduced in the paper, considering stubborn points in frequency domain automatically recognized and multiple control points exploited in iterative process. Numerical simulation is conducted with design response spectrum of highway bridge in China selected as the fitting target. The result shows that the artificial ground motions generated by the proposed method achieve a significantly better fitting accuracy to the target response spectrum than the conventional method. Therefore, the proposed method is regarded as a reliable means of synthesizing response spectrum-compatible ground motions.

For prospective study in the future, further applications of the methodology on other response spectra of different types require more practice and researches.

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