An improvement of a cryptanalysis algorithm

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Abstract In this paper we present experiments in order to show how some pseudo random number generators can improve the effectiveness of a statistical cryptanalysis algorithm. We deduce mainly that a better generator enhance the accuracy of the cryptanalysis algorithm.

Keywords Cryptanalysis · Markov Chain Monte Carlo · Pseudo Random Number Generators

1 Introduction

Cryptography refers to the science that concerns encrypting data so that, without a key, a third person other then the sender and the receiver can not recover the secret data [1]. At the same time, cryptanalyst try to break cryptosystems in order to prove that there is a security flaw. According to Kerckhoffs' Principle, The method must not need to be kept secret, and having it fall into the enemy’s hands should not cause problems [2]. In this paper we deal with classical cryptosystems and try to improve a cryptanalysis algorithm by pseudo random number generators. Classical cryptosystems operate at the byte level of data where as the modern one at the bit level. Pseudo random number generators have been deeply studied for their important applications in computer science. This paper is organized as follows, the next sections present Markov Chain Monte Carlo (mcmc) algorithm and a survey of the related
theory. Some Pseudo Random Number Generators (prng) with their output when applied to mcmc are given thereafter. We analyze those results and give our interpretation in the following section.

2 Markov Chain Monte Carlo

In this section, we give the definition of Markov Chain Monte Carlo. If a Markov chain $X_n$ on a finite or countable state space $X$ is irreducible and aperiodic, with stationary distribution $\pi$, then for every subset $A \subseteq X$,

$$\lim_{n \to \infty} P(X \in A) = \int_A \pi(x)dx$$

2.1 MCMC algorithm

In this section we rewrite the MCMC algorithm as described in [3].

- Choose an initial state $X_0 \in X$, where $X$ is the all possible states that the Markov Chain may takes. In probability theory we call $X$ the universe.
- For $n = 1, 2, 3, \ldots$
- Propose a new state $Y_n \in X$ from some symmetric proposal density $q(X_{n-1}, \ldots, X_0)$.
- Let $U_n \sim \text{Uniform}[0, 1]$, independently of $X_0, \ldots, X_{n-1}, Y_n$.
- If $U_n < (\pi(Y_n)/\pi(X_{n-1}))^p$, then "accept" the proposal by setting $X_n = Y_n$, otherwise "reject" the proposal by setting $X_n = X_{n-1}$

2.2 An MCMC algorithm to break a substitution-transposition cryptosystem

Here is the final version of the MCMC algorithm in [3]. The authors, after a deep study of MCMC, chose the best parameter that output the best decryption rate:

1. Choose an initial state (states here are all possible encryption keys), and a fixed scaling parameter $p > 0$.
2. Repeat the following steps for many iterations (e.g. 10 000 iterations).
   - Given the current state $x$, propose a new state $y$ from some symmetric density $q(x, y)$.
   - Sample $U_n \sim \text{Uniform}[0, 1]$, independently from all other variables.
   - If $u < (\pi(y)/\pi(x))^p$, then "accept" the proposal by replacing $x$ with $y$, otherwise reject $y$ by leaving $x$ unchanged.

\[
\pi(x) = \prod_{\beta_1, \beta_2} r(\beta_1, \beta_2)f(x, \beta_1, \beta_2)
\]

where $r$ is the frequencies of letters of the reference text and $f$ are those of the decrypted text using the key $x$. 
2.3 Testing methodology

We have tested the MCMC algorithm with different pseudo random number generators. The obtained results in terms of accuracy and number of successful decryption are given in the next paragraph.

### 3 drand48 pseudo random number generator

This generator generates a sequence of numbers according to this linear congruence

\[ X_{n+1} = (aX_n + c) \mod m \]

where \(a\), \(c\) and \(m\) are constants.

Note that in Table 1, the abbreviation are as follows:

- EN is the experience number, which refers to the five experiences done in this paper,
- AC is the average correctness computed as follows. For each experience, the text is encrypted with a different key and the cryptanalysis algorithm is run. The cryptanalysis algorithm outputs a decryption key. The AC measures the equality of the output of the cryptanalysis algorithm with the actual encryption key, in the overall.
- NSD is the number of successful decryptions, which refers to the number of successful decryptions out of the 100 performed for each experience.

### 4 xorshift pseudo random number generator

One of the properties of this generator is that it is a very fast algorithm with a great period \((2^{128} - 1)\) [4]. Also, as we see bellow, its design is simple. Also, it has been proved that this generator is successful in tests measuring the quality of a pseudo random number generator.

```c
#include <stdint.h>

uint32_t xor128(void) {
    static uint32_t x = 123456789;
    static uint32_t y = 362436069;
    static uint32_t z = 521288629;
    ...
```
Table 2 xorshift prng

| EN | AC | NSD |
|----|----|-----|
| 1  | 0.896 | 89 |
| 2  | 0.9161 | 89 |
| 3  | 0.8933 | 89 |
| 4  | 0.9156 | 89 |
| 5  | 0.8811 | 89 |

Table 3 CI prng

| EN | AC | NSD |
|----|----|-----|
| 1  | 0.8700 | 85 |
| 2  | 0.8744 | 82 |
| 3  | 0.88 | 87 |
| 4  | 0.8983 | 82 |
| 5  | 0.8478 | 87 |

static uint32_t w = 88675123;
uint32_t t;
t = x ^ (x << 11);
x = y; y = z; z = w;
return w = w ^ (w >> 19) ^ (t ^ (t >> 8));

Table 2 shows the obtained results.
Comparing with we can say that .......

5 chaotic iteration (CI) pseudo random number generator

This generator is obtained from reference [5]. As shown in this later, this generator bypass xorshift in some tests and a deep theoretical study proved that this generator has good randomness properties. The obtained results are tabulated in 3.

Here $x$ is a binary array of length $N$.

a := XORshift1();
m := a mod 2 + c
for i = 0, . . . , m do
b := XORshift2();
S := b mod N;
x[S] := 1 - x[S] mod 2;
end for
r := x;
return r;
Comparing 5 with 2 and 1 we can say that the xorshift has better results in all experiments in terms of AC and NSD.

6 Result discussion

In our experiments, we have shown that with different prng, the output of the cryptanalysis algorithm MCMC is different. More the statistical properties of the generator are better, more the accuracy and the number of successful decryption are great. This is due to the fact that the resulting Markov Chain has a better statistical behavior.

In the design of the MCMC algorithm, we see that prng plays a crucial role. prng will be used in different steps of the algorithm. We use it to pick a number in [0, 1] uniformly and to chose the initial state \(X_0\). The more those parameters are random, the more Markov Chain generated is accurate. Therefore, a good PRNG is of crucial importance in a MCMC.

Before picking any value \(X_n\) in the Markov Chain MA, we pick a random number uniformly in [0, 1] as stated in the algorithm. Naturally, a good prng will improve the convergence of the MC and a good prng will generate a MC close to the theoretical expected result. As the quality of a prng is measured with some standard tests, we may suppose that a prng generating good results in our experiment is of better statistical property.

7 Conclusion

In this paper we presented our experiments on decryption of classical cryptosystems. Our results show that xorshift prng are the most convenient to this kind of application since with this later we obtain the highest scores. Previous studies showed that CI prng have the best statistical proprieties, but surprisingly this does not implies that they are more suitable for all kind of applications like those we have done in this work.

References

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