The Gravitational Force Field of the Galaxy Measured From the Kinematics of RR Lyrae in Gaia

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ABSTRACT
From a sample of 15651 RR Lyrae with accurate proper motions in Gaia DR2, we measure the kinematics of the inner stellar halo between 1.5 kpc and 20 kpc from the Galactic centre. We find that their kinematics are strongly radially anisotropic, and their velocity ellipsoid nearly spherically aligned over this volume. Only in the inner regions \( < 5 \text{kpc} \) does the anisotropy significantly fall (but still with \( \beta > 0.25 \)) and the velocity ellipsoid tilt towards cylindrical alignment. In the inner regions, our sample of halo stars rotates at up to \( 50 \text{km s}^{-1} \), which may reflect the early history of the Milky Way, although there is also significant angular momentum exchange with the Galactic bar at these radii. We subsequently apply the Jeans equations to these kinematic measurements in order to non-parametrically infer the gravitational acceleration field over this volume, and by removing the contribution from baryonic matter, measure the contribution from dark matter. We find that the gravitational potential of the dark matter is nearly spherical with average flattening \( q_{p} = 1.03 \pm 0.08 \) between 5 kpc and 20 kpc, and by fitting parametric ellipsoidal density profiles to the acceleration field, we measure the flattening of the dark matter halo over these radii to be \( q_{p} = 1.00 \pm 0.09 \).

Key words: Galaxy: kinematics and dynamics – Galaxy: halo – dark matter

1 INTRODUCTION
Simulations in the LCDM paradigm have been extremely successful in producing many of the observational properties of galaxies across cosmic time. While dark matter only simulations produce dark matter halos with a characteristic profile (Navarro et al. 1996b) and highly flattened triaxial shapes with flattening \( q_{p} \equiv (c/a)_{p} \sim 0.5 \) (e.g. Dubinski & Carlberg 1991; Jing & Suto 2002; Allgood et al. 2006; Schneider et al. 2012), this is altered by the highly uncertain interplay between baryons and dark matter. In particular, their halos are expected to respond to baryonic infall by becoming more axiymmetric and slightly less flattened (Dubinski 1994; Abadi et al. 2010), although even for the massive, near maximal disk seen in the Milky Way (e.g. Boychuk & Rix 2013; Wegg et al. 2016) the increase in the flattening of the potential, \( q_{p} \), is typically only \( \sim 0.1 \) (Kazantzidis et al. 2010).

In external galaxies, we are typically only able to measure dark matter halo properties using samples of galaxies (e.g. van Uitert et al. 2012; Martinsson et al. 2013; Aniyan et al. 2015). In the Milky Way however, we can measure the detailed kinematics of individual stars. We can therefore measure the detailed kinematics of our dark matter halo, and use this as a prototype, a process referred to as near-field cosmology. However, despite the observational advantages of studying the Milky Way’s dark matter halo, there is still no consensus on either its shape or profile.

Probes of the shape of the Milky Way’s halo include tidal streams, halo kinematics, the flaring of the HI gas disk, and comparison of the local dark matter density with enclosed densities required by the rotation curve (see for example the review by Read 2014).

The tightest recent constraints on the shape of the halo have arisen from measurements of tidal streams. Initial work focused on the Sagittarius stream suggested the halo to be spherical (Ibata et al. 2001) while later models pointed to an oblate halo (Law & Majewski 2010). The stability of these models was questioned (Debattista et al. 2013), although this problem may be lessened by a halo whose shape changes with radius (Vera-Ciro & Helmi 2013).

However, the difficulty of using the complex Sagittarius stream to constrain the halo has led to a recent focus on other colder streams, particularly GD-1 (Grillmair & Dionatos 2006) and Pal-5 (Odenkirchen et al. 2001). These streams lie \( \approx 14 \text{kpc} \) and \( \approx 18 \text{kpc} \) from the Galactic centre, and while the modelling methods vary, the results generally point to a dark matter potential consistent with a spherical halo. For example, at the location of GD-1, the flattening of...
the overall potential has been measured to be $q\Phi = 0.87 +0.07 \pm 0.04$ by Kopelev et al. (2010), $q\Phi = 0.90 +0.05 \pm 0.10$ by Bowden et al. (2015) and $q\Phi = 0.95 \pm 0.04$ by Bovy et al. (2016). Similarly at the location of Pal-5 the overall potential was measured to be $q\Phi = 0.95 \pm 0.05$ by Küpper et al. (2015) and $q\Phi = 0.94 \pm 0.05$ by Bovy et al. (2016). Combining these constraints on the potential with baryonic models results in a dark matter halo with axes ratio $q = 1.05 \pm 0.14$ (Bovy et al. 2016), consistent with spherical, and therefore in tension with the expectations of cosmological \(\Lambda\)CDM simulations (Dai et al. 2018).

This tension is a tantalising prospect because halo shape can, in principle, be a probe of the nature of dark matter and its possible interactions (e.g. Peter et al. 2013).

The work here takes a different approach, instead applying Jeans modelling to the kinematics of halo stars. This approach has also been used several times recently to constrain the dark matter halo shape. However, unlike the stream modelling approach where different modelling techniques have produced similar results, the results using halo kinematics are more diverse. For example, Loebman et al. (2014) is the most conceptually similar work to ours. They apply the Jeans equations to SDSSSegue halo star kinematic measurements by Bond et al. (2010), finding the dark matter to have a flattened potential with $q = 0.8 \pm 0.1$ and density with $q = 0.4 \pm 0.1$ within 20kpc. However, in contrast, Bowden et al. (2016) favours a highly prolate dark matter potential with $q = 1.5 \pm 2.0$.

The present state of the art is therefore that stream modelling is providing consistent constraints that the halo is nearly spherical at radii $\approx 14$ kpc and $\approx 18$ kpc, while, inside this, the shape is highly uncertain. This situation is expected to rapidly change: the recent release of Gaia DR2 has provided measurements of the radial velocities of tens of millions of stars, and accurate astrometry of more than a billion, covering a large fraction of the Galaxy. Here, we take advantage of the accurate Gaia DR2 measurements of proper motions of RR Lyrae in the stellar halo and use them as kinematic tracers in order to measure the properties of the dark matter halo within 20 kpc of the Galactic center.

Our primary motivation in this work was to measure the shape of the dark matter halo and constrain its variation with radius. However, we also present results that impact two further important areas. (i) The dark matter density and mass profile inside 20 kpc. This is because the inner parts of our studied volume is a region which is particularly important in understanding whether the dark matter profile has a core as implied by the bulge measurements of Portail et al. (2017), or a cusp as seen in recent cosmological simulations of Milky Way mass haloes (Grand et al. 2017; Chan et al. 2015). (ii) The kinematics of the Galactic halo which is an extremely interesting topic in its own right.

The kinematics of the stellar halo are of particular interest because they provide a probe into the history of this fundamental population of stars (for a review of the stellar halo and its kinematics see section 6.1 of Bland-Hawthorn & Gerhard 2016, or Helmi 2008 for a dedicated but older introduction). Comparisons between samples must be made with care: the kinematics of the halo depends on metallicity (Kafle et al. 2013; Das & Binney 2016; Deason et al. 2017; Belokurov et al. 2018), and is therefore sensitive to the sample choice. Here, we study a sample of RR Lyrae without selection with respect to metallicity. The bulk of the halo does have [Fe/H] $> -2$, and we therefore largely sample from this, more metal rich part, of the halo. The reader most interested in the kinematics of the stellar halo should concentrate on section 3.

The paper proceeds as follows: in section 2 we construct a sample of RR Lyrae away from the Galactic plane with accurate transverse velocities, in section 3 we measure the kinematics of this sample, in section 4 we apply the Jeans equations to these kinematics to measure the Galactic acceleration field, and in section 5 we fit parametric dark matter profiles to these forces. We discuss and place our results in context in section 6, and conclude in section 7.

Throughout, we use a distance to the Galactic center of $R_0 = 8.2$ kpc (Bland-Hawthorn & Gerhard 2016), and an absolute value of the solar velocity $(U, V, W)_\odot = (11.1, 12.24 + 238, 7.25)$ km s$^{-1}$ (Schönrich 2012). We assess the impact of these assumptions when we assess our systematics.

2 A SAMPLE OF RR LYRAE WITH TRANSVERSE VELOCITIES

We construct a sample with which to trace the dynamics of the halo from the catalogue of RR Lyrae in PanSTARRS1 (PS1) provided by Sesar et al. (2017, hereafter S17). S17 classifies stars observed in the PS1 3sr survey as RR Lyrae using a machine learning approach. In their catalogue, each star has a score (score$_{ab}$), where high numbers indicate higher likelihood that the star is a type $ab$ RR Lyrae. We use a threshold of 0.6 which, for the relatively nearby RR Lyrae considered in this work, will provide a sample with greater than 95% purity and 95% completeness (table 3 of S17). The provided distances to these type $ab$ RR Lyrae are accurate to 3% (S17).

In our analysis, we consider RR Lyrae with Galactocentric radius between 1.5 kpc and 20 kpc, which lie more than 20$^\circ$ from the Galactic plane in Galactocentric coordinates. However, we make several further cuts to ensure that the
The sample is clean and complete over a defined volume: (i) We only consider RR Lyrae with $|b| > 10$ deg because, closer to the Galactic plane, extinction causes the completeness of the sample to drop (S17). (ii) The nominal area of the PS1 survey is $\text{dec} > -30$ deg, however, to simplify selection near this boundary, we consider only stars with $\text{dec} > -29$ deg. (iii) We remove RR Lyrae that in projection lie within a conservative 10 half light radii of a galactic globular cluster. We use the catalogue of Harris (1996) and use 2 arcmin as the half light radius where none has been measured. (iv) To remove the Sagittarius dwarf galaxy and stream, we remove all RR Lyrae that lie more than 12 kpc from the Sun and lie within 10 degrees of the plane of the Sagittarius stream as defined by Majewski et al. (2003). Tests with the Sagittarius stream model of Law & Majewski (2010) indicate that this should remove more than 95% of the stream (see also Hernitschek et al. 2017, for an analysis of the Sagittarius stream in the S17 RR Lyrae).

The resulting 15813 RR Lyrae are cross matched with Gaia DR2 which provides astonishingly accurate absolute proper motions. We use a cross match radius of 0.5” and remove cross matches without a measured proper motion, or whose astrometric fit was poor (those with $\text{astrometric\_excess\_noise\_sig} > 10$).

We show in Figure 1 the difference in parallax between that measured in Gaia DR2 and the measured S17 distance modulus converted to parallax. The sample RR Lyrae are too distant and faint to have accurate parallax measurements in Gaia DR2. We therefore use the S17 RR Lyrae distances throughout our analysis, and their $\approx 3\%$ accuracy was the motivation for using this sample. We do however remove the 82 stars whose Gaia DR2 parallax lies more than $4\sigma$ from that predicted from their distances as measured by S17; these are likely to either not be genuine RR Lyrae, or have poor proper motion estimates. We note in passing that our sample has a slightly negative parallax zero point in Gaia DR2 of $-38\mu$as. This is similar to the $-29\mu$as found by Lindegren et al. (2018) with the difference likely resulting from the different distribution on the sky of our sources, which are more concentrated towards the Galactic centre than the quasar sample of Lindegren et al. (2018).

Finally, we remove two RR Lyrae which are clear outliers with respect to their transverse velocity. In Figure 2, we plot the transverse velocity distribution, and remove the two stars with apparent transverse velocity $> 1000\text{km}\text{s}^{-1}$.

Of the original 15813 RR Lyrae, 15651 remain after the Gaia cross matching. In Figure 3, we show the distribution of the sample, while in Figure 4 we show the distribution of the proper motion and proper motion errors. Note that...
although the GAIA DR2 parallaxes of our sample were not accurate (Figure 1), the proper motions are: the proper motion errors are generally less than 0.25 mas/yr and almost all smaller than 0.5 mas/yr. Even the larger value of 0.5 mas/yr corresponds to an error of 50 km s\(^{-1}\) at 20 kpc, allowing us to accurately measure the kinematics across the entire volume of our sample, provided this error is taken into account.

As we will see in section 3, our selected RR Lyrae in the inner halo are strongly radially anisotropic and have a nearly spherically aligned velocity ellipsoid. To preempt this, and motivate our choices of coordinates and binning, we illustrate the radial anisotropy directly from the data in Figure 5. In making this plot, we selected stars which, when projected onto the Galactic plane, lie within 25° of the tangent plane (see figure inset). For small \(|l|\) and \(|b|\), the transverse velocities of these stars trace the velocities in the meridional plane i.e. \((v_l,v_b) = (v_R,v_z)\). In the figure we extend to \(|l|\) and \(|b|\) values much larger than the strict applicability of this approximation, but we use this figure merely as a clear visual indication directly from the data that the inner halo, as traced by RR Lyrae, has a strongly radially anisotropic nature. On the basis of this figure, we choose to work in spherical coordinates throughout this work, these being more natural for our tracer population than cylindrical coordinates.

In Figure 6, we examine the density of the sample. Throughout this work, we work in spherical bins centred on the Galactic centre where \(\theta\) is 90° in the Galactic plane and 0° towards the North Galactic Pole. We use 9 logarithmically spaced bins between 1.5 kpc and 20 kpc in radius, and bins in \(\theta\) with edges at 0°, 25°, 40°, 50°, 60°, 70°. To convert from number counts in these bins to densities, we must account for selection effects. To do so, we simulate stars drawn from an ellipsoidal power-law profile:

\[
\rho \propto \left[ R^p + \frac{z^2}{q^2_r} \right]^{\alpha/2} \propto R^\alpha \left[ \sin^2 \theta + \frac{\cos^2 \theta}{q^2_r} \right]^{\alpha/2}.
\]

where \((R,z)\) are galactocentric cylindrical coordinates, and \((r,\theta)\) galactocentric spherical coordinates. In this estimate of selection fraction, we use gradient \(\alpha = \partial \log \rho / \partial \log r = -2.6\) and flattening \(q_{rr} = 0.72\) which we found best fit the sample overall. Hernitschek et al. (2018) investigated the structure of the S17 sample beyond 20 kpc and found a similar flattening of the RR Lyrae of \(q_{RR} = 0.8\) at the 20 kpc inner edge of their sample. We then compute the fraction of simulated stars in each bin which pass the selection cuts described above and use this fraction to correct the number of counts in each bin. Because almost all the selected stars pass our Gaia DR2 selection cuts, regardless of position, we do not simulate these. Instead we consider the much more important cuts described above (i)-(iv) i.e. the removal of RR Lyrae with \(\text{dec} < -29°\), \(|b| < 10°\), near globular clusters, or in the plane of the Sagittarius stream.

In what follows, we perform non-parametric modelling
of the tracer population of RR Lyrae. We use the parameterisation in Equation 1 only to compute the observed volume of each bin. We have found that the results are not sensitive to the details of the parameterisation used in the selection function because it is only important that its variation is approximately correct over a bin, and not globally. We also use this simulation of the selection function to remove poorly sampled bins: Any bin where less than 30% of the Monte Carlo simulated stars pass the selection cut is removed. This affects the bins which lie at low Galactic latitude due to the $|b| > 10$ deg selection, and one distant bin along the Galactic minor axis which is heavily contaminated by the Sagittarius stream.

We then compute the density using this selection fraction as a correction. In the upper left panel of Figure 6, we show the density as a function of galactocentric radius in each $\theta$ bin. It is noteworthy that, as predicted by Pérez-Villegas et al. (2016, Fig. 1), these densities appear extremely well, and both agree on their reconstruction in Figure 4 for details). The logarithmic gradient is generally between -3 and -2. In the lower left panel, we show the same density information, but plotted as a function of azimuthal angle $\phi$ within each of the radial bins. Notice that we have good coverage of azimuthal angle, and that variations with azimuth are relatively small, increasing to standard deviation 20% in our outermost radial bins.

### 3.1 The Intrinsic Kinematics Assuming Gaussian Velocities

If we assume that the velocities are Gaussian then, at each point in space, the distribution of velocities is

$$
\begin{align*}
  f(\mathbf{v}) &= \frac{1}{\sqrt{2\pi} |\Sigma|} \exp \left( -\frac{1}{2} (\mathbf{v} - \mathbf{v})^T \Sigma^{-1} (\mathbf{v} - \mathbf{v}) \right) \\
  &= \frac{1}{\sqrt{2\pi} |\Sigma|} \exp(-Q/2)
\end{align*}
$$

where $\mathbf{v}$ is the velocity in spherical coordinates $(\rho, \theta, \phi)$ i.e. $\mathbf{v} = (v_\rho, v_\theta, v_\phi)$, and $\Sigma$ is the velocity dispersion tensor in spherical coordinates:

$$
\Sigma = 
\begin{bmatrix}
  \sigma_{\rho\rho} & \sigma_{\rho\theta} & \sigma_{\rho\phi} \\
  \sigma_{\theta\rho} & \sigma_{\theta\theta} & \sigma_{\theta\phi} \\
  \sigma_{\phi\rho} & \sigma_{\phi\theta} & \sigma_{\phi\phi}
\end{bmatrix}.
$$

To find the resultant velocity distribution on the sky, we rotate this coordinate system into cartesian coordinates aligned with $d, l, b$ i.e. into $\mathbf{v}' = (v_d, v_l, v_b)$. Denoting this transformation as $\mathbf{R}$, then $\mathbf{v}' = \mathbf{R} \mathbf{v}$ and the quadratic form $Q$ becomes

$$
\begin{align*}
  Q &= (\mathbf{v}' - \mathbf{v})^T \mathbf{R} \Sigma^{-1} \mathbf{R}^T (\mathbf{v}' - \mathbf{v}) \\
  &= (\mathbf{v}' - \mathbf{v})^T \Lambda^{-1} (\mathbf{v}' - \mathbf{v})
\end{align*}
$$

where $\mathbf{v} = \mathbf{R} \mathbf{v}$ and $\Lambda = \mathbf{R} \Sigma \mathbf{R}^T$. The rotation $\mathbf{R}$, between spherical coordinates and $(d, l, b)$ is given explicitly in Ratnatunga et al. (1989), eqns. A1-A4) and we do not repeat it here.

Marginalising over the unobserved radial velocity $v_d$ provides the probability distribution of transverse velocities, $v_\perp = (v_l, v_b)$. This is a two-dimensional multivariate Gaussian:

$$
\begin{align*}
  f(v_\perp) &= \frac{1}{2\pi \sqrt{|\Lambda|}} \exp \left( -\frac{1}{2} (v_\perp - \bar{v}_\perp)^T \Lambda_\perp^{-1} (v_\perp - \bar{v}_\perp) \right) \\
  \Lambda_\perp &= 
\begin{bmatrix}
  \Lambda_{ll} & \Lambda_{lb} \\
  \Lambda_{bl} & \Lambda_{bb}
\end{bmatrix},
\end{align*}
$$

and $\bar{v}_\perp = \mathbf{R}_\perp \bar{v}$ where $\mathbf{R}_\perp$ is the rotation matrix $\mathbf{R}$ without the $d$ row i.e. it is a $(2 \times 3)$ matrix.

Because every star has a different position, the projection of the velocity ellipsoid is different for every star. If we then assume that the velocity ellipsoid is constant in each of our bins in $(r, \theta)$, then this allows us to recover the velocity ellipsoid without measurements of the radial velocities. For example, in bins near the Galactic plane, for stars that are in front or behind the Galactic centre, $v_l$ measures the velocity in the $\phi$ direction, while when tangent to the Galactic centre $v_l$ measures the velocity in the $\rho$ direction.

For each star, the likelihood of measuring $v_\perp$ is given by the convolution of Equation 5 with the error in transverse velocities. This error, when significant, is dominated by the uncertainty in the Gaia proper motions. Denoting the measurement covariance as $\mathbf{S}_\perp$ then the likelihood of measuring
\( \mathbf{v}_\perp \) is
\[
\mathcal{L}(\mathbf{v}_\perp) \propto \exp \left[ -\frac{1}{2} (\mathbf{v}_\perp - \mathbf{v}_\perp) \mathbf{A}_\perp^{-1} (\mathbf{v}_\perp - \mathbf{v}_\perp) \right] 
\times \exp \left[ -\frac{1}{2} (\mathbf{v}_\perp - \mathbf{v}_\perp) \mathbf{S}_\perp^{-1} (\mathbf{v}_\perp - \mathbf{v}_\perp) \right] d\mathbf{v}_\perp
\]
\[
= \frac{1}{2\sqrt{\det(\mathbf{C}_\perp)}} \exp \left[ -\frac{1}{2} (\mathbf{v}_\perp - \mathbf{v}_\perp) \mathbf{C}_\perp^{-1} (\mathbf{v}_\perp - \mathbf{v}_\perp) \right]
\]
where \( \mathbf{C}_\perp = \mathbf{S}_\perp + \mathbf{A}_\perp \).

In order to estimate the mean velocity, \( \mathbf{v} \), and dispersion tensor, \( \Sigma \), we consider the total log likelihood of all measurements:
\[
\log \mathcal{L} = \sum_i \log \mathcal{L}(\mathbf{v}_{\perp,i})
\]
\[
= -\frac{1}{2} \sum_i (\mathbf{v}_{\perp,i} - \hat{\mathbf{v}}_{\perp,i}) \mathbf{C}_\perp^{-1} (\mathbf{v}_{\perp,i} - \hat{\mathbf{v}}_{\perp,i}) - \frac{1}{2} \log(\det(\mathbf{C}_\perp)) - \log(2\pi)
\]
where \( \mathbf{v}_{\perp,i} \) is the measured transverse velocity of star \( i \), \( \mathbf{C}_{\perp,i} = \mathbf{S}_{\perp,i} + \mathbf{A}_{\perp,i} = \mathbf{S}_{\perp,i} + \mathbf{R}_{\perp,i} \Sigma R_{\perp,i}^\top \), and \( \hat{\mathbf{v}}_{\perp,i} = \mathbf{R}_{\perp,i} \hat{\mathbf{v}} \).

We wish to estimate the kinematic properties of the population i.e. \( \mathbf{v} \) and \( \Sigma \). To do so, we adopt a Bayesian approach and sample from this likelihood assuming flat priors on both \( \mathbf{v} \) and \( \Sigma \). In every bin, the number of stars is so large that the results are insensitive to prior choice.

### 3.2 The Intrinsic Kinematics For non-Gaussian Velocities

The reader may be concerned by the assumption of Gaussianity in subsection 3.1. To alleviate these fears, in this section we derive estimators for the mean velocity and second dispersion tensor which do not depend on the specific form of the velocity distribution. To do so, we generalise the method of DB98. The key assumption that allowed DB98 to recover the intrinsic kinematics from transverse velocities was that velocities and positions were uncorrelated i.e. that the velocity distribution did not depend on position. This was a good assumption for the solar neighbourhood sample of Hipparcos stars analysed in that work, but here we have Gaia data on stars across the inner Galaxy. We therefore make a different assumption: that positions and kinematics in spherical coordinates are uncorrelated in each of our bins.

We define the vector \( \mathbf{p} \) to be the transverse velocity measurement of our star in spherical coordinates with zero radial velocity:
\[
\mathbf{p} = \mathbf{R}^\top \begin{bmatrix} 0 \\ \mathbf{v}_l \\ \mathbf{v}_b \end{bmatrix}.
\]
This measurement results from measurements of a star with velocity \( \mathbf{v} \) through
\[
\mathbf{p} = \mathbf{A} \mathbf{v}
\]
where \( \mathbf{A} \) is the projection matrix which projects velocities onto \( \mathbf{v}_l = 0 \). Linear algebra instructs us how to construct the projection matrix \( \mathbf{A} \) (e.g. Bernstein 2011). Expressed in a form similar to DB98, this projection matrix is given by
\[
\mathbf{A} = \mathbf{I}_3 - \mathbf{R}_0^\top \mathbf{R}_0
\]
where \( \mathbf{I}_3 \) is the identity matrix, \( \mathbf{R}_0 \) is the matrix \( \mathbf{R} \), but with only the rows corresponding to \( \hat{d} \), and the \( l \) and \( \hat{b} \) rows zeroed, and \( \mathbf{R}_0^\top \) is the Moore-Penrose pseudo-inverse of \( \mathbf{R}_0 \).

The insight of DB98 was that, while Equation 9 obviously cannot be inverted, the mean velocity over each bin can be recovered from
\[
\langle \mathbf{p} \rangle = \langle \mathbf{A} \rangle \langle \mathbf{v} \rangle
\]
where we have used the key assumption that positions and velocities are independent. Then trivially
\[
\langle \mathbf{v} \rangle = \langle \mathbf{A} \rangle^{-1} \langle \mathbf{p} \rangle.
\]
Similarly the dispersion tensor \( \sigma \) can be obtained through the inversion of
\[
\langle p_i p_j \rangle = \sum_{ij} \langle A_{ij} A_{kl} \rangle \sigma_{ij}
\]
where \( \mathbf{p}' = \mathbf{p} - \mathbf{A} \langle \mathbf{v} \rangle \). In section 4, we will apply the Jeans equations to the second-moments of the velocity distribution, \( \langle v_i v_j \rangle \). We estimate these more directly from the inversion of
\[
\langle p_i p_j \rangle = \sum_{ij} \langle A_{ij} A_{kl} \rangle \langle v_i v_j \rangle.
\]

Observational errors in transverse velocity do not affect the mean velocity (Equation 12), but must be accounted for when estimating the dispersion (Equation 13) and moments (Equation 14). To do so, we subtract in quadrature the variance caused by these errors (Eqn 18 of DB98). We have tested the accuracy of this correction using mock data.

### 3.3 Testing the Kinematic Reconstruction

To test the kinematic measurements in subsections 3.1 and 3.2 (and later our reconstruction of the force field and dark matter halo properties) we have constructed a series of mock halos. These were constructed in the potential of the dynamical models of Portail et al. (2017, hereafter P17). These made-to-measure models were fitted to a range of data on the bar, bulge and inner Galaxy, while simultaneously matching the rotation curve and stellar surface density near the Sun. They however have no stellar halo, so, to construct our mock halos, we used dark matter halo particles as test particles from which to construct a stellar halo. These particles were then selected with a weighting by energy to have a ~ \( r^{-3} \) profile similar to our RR Lyrae sample, and by orbital radial extent to have a similar radial anisotropy. The details of this process are described in subsection A1, and the P17 models are described in more detail in section 5 where we use their baryonic part as our fiducial baryonic model.

We relegate the tests of our methods on these mock halos to the appendix. For each figure from Figure 6 onwards we have applied our code to the mock halos and show the equivalent plots in subsection A2.

Here we draw attention to the comparison of the two kinematic reconstruction methods in subsections 3.1 and 3.2 on the mock halos. This is shown in Figure A1 without considering the selection function, and in Figure A2 with the selection function. When the survey is spatially complete, both perform equally well, but the method inspired by DB98 performs slightly better when the mocks are folded through the selection function. Both methods give very similar results on the real halo (see Figure 7). On the basis of the
3.4 The Measured Kinematics of RR Lyrae in the Inner Halo

In Figure 7, we show the resultant measured kinematics of the sample of halo RR Lyrae. In Figures 8 and 9, we show the same data in a more physically informative manner: Figure 8 shows the kinematics plotted in physical space, while Figure 9 shows the measured velocity ellipsoid in the meridional plane.

Several features are noteworthy in these measured kinematics:

(i) The dispersion tensor displays near spherical alignment, tilting towards cylindrical only in the innermost regions. This spherical alignment has been measured in local samples previously (Smith et al. 2009; Bond et al. 2010; Evans et al. 2016; Posti et al. 2018), but here we see that it is close to spherically aligned over the entire range from 4 kpc to 20 kpc. This near spherical alignment does not necessarily mean that the potential must be spherical (Evans et al. 2016), although in many cases it is likely to be (An & Evans 2016). As we shall see in subsection 4.2, the potential does appear nearly spherical in the Milky Way.

(ii) The RR Lyrae in the halo have a high radial anisotropy of $\beta \approx 0.8$. This decreases inside 5 kpc, but remains above $\beta \approx 0.25$ even in these inner regions. The $\beta \approx 0.8$ measured at solar galactocentric radii is slightly higher than the $\beta \approx 0.7$ measured in the overall halo locally (Smith et al. 2009; Bond et al. 2010), but as Belokurov et al. (2018) show, the local anisotropy of the stellar halo depends strongly on metallicity. Our result of $\beta \approx 0.8$ for RR Lyrae, which are likely to be drawn from the bulk of the halo metallicity distribution at $[\text{Fe/H}] \gtrsim -2$, agrees with the Belokurov et al. (2018) measurements at these metallicities. They argue that the extreme anisotropy of these higher metallicity halo stars, which have higher anisotropy than the $[\text{Fe/H}] < -1.7$ stars in their sample, can be most easily explained by a large fraction of the inner halo forming by the accretion of a massive satellite (see also Deason et al. 2018; Helmi et al. 2018). Here, we see the wider view that the entire inner halo is strongly radial anisotropic. Note that both features (i) and (ii) could be qualitatively anticipated directly from the data in Figure 5.

(iii) In the outer regions, beyond 10 kpc, the halo has mild counter rotation of $\sim 10 \text{ km s}^{-1}$. This measured outer rotation depends on the assumed velocity of the Sun, but this...
value is relatively well constrained by the proper motion of Sagittarius A*, assuming that the black hole is at rest with respect to the Galaxy. The mild counter rotation at 20 kpc galactocentric distance is at a similar level to that seen previously in diverse samples (Beers et al. 2012; Kafle et al. 2017; Helmi et al. 2018, although care must be taken: Fermani & Schönrich 2013). It likely results from the halo being built by a limited number of large mergers fragments at these radii (Koppelman et al. 2018), or the accretion of a single large SMC sized object (Helmi et al. 2018).

(iv) The halo at solar radii and inside has mild rotation. This rotation could reflect, in part, the accretion history of the inner halo. However, it is interesting that the shape of the rotation profile matches extremely well the rotation of the mock halo (see Fig. A1). The mock halos acquired their rotation by transfer of angular momentum from the bar (e.g. Athanassoula 2003). In the bulge, Pérez-Villegas et al. (2016) compared the kinematics of the RR Lyrae to barred models, finding that the rotation there could be matched by the spin up of an initially non-rotating population. Here however, the observed rotation is somewhat larger than the rotation in the mock. Whether this difference in level of rotation is a result of differing halo properties between our mock halo and the Milky Way, or whether this reflects the formation history is unclear. In addition if there are a significant fraction of in-situ, thick disk origin, stars in these bins, this would also increase the rotation profile (Haywood et al. 2018). It is worth noting that this rotation has been observed locally previously (e.g. Deason et al. 2017), and a similar result in LAMOST K giants was recently found using Gaia DR2 by Tian et al. (2018).

4 MEASURING THE GALACTIC FORCE FIELD

In this section, we apply the Jeans equations to the measured halo RR Lyrae kinematics in order to measure the acceleration field in the inner halo of the Galaxy. We first derive discretised versions of the Jeans equations in spherical coordinates (subsection 4.1). The novelty of this section is that we derive azimuthally averaged versions of the Jeans equations which do not assume axisymmetry either of the tracer or potential. From the subsequent application of discretised versions of these equations, we measure the azimuthal average of the gravitational force field of the Galaxy in subsection 4.2.

4.1 The Azimuthally Averaged Jeans Equations

We begin from the Boltzmann equation in spherical coordinates (r, θ, φ) where φ is the azimuthal angle and θ is the angle from the z-axis (BT87, P4-2):

\[
\frac{\partial f}{\partial t} + v_r \frac{\partial f}{\partial r} + \frac{v_\theta}{r} \frac{\partial f}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial f}{\partial \phi} + \frac{v_r^2 + v_\theta^2}{r} = \frac{\partial \Phi}{\partial r} \left( \frac{\partial f}{\partial v_r} + \frac{1}{r} \frac{\partial f}{\partial v_\theta} \cot \theta - v_r v_\theta - \frac{\partial \Phi}{\partial \theta} \right) \frac{\partial f}{\partial v_\theta} \]

\[
- \frac{1}{r} v_\phi (v_r + v_\phi \cot \theta) + \frac{1}{\sin \theta} \frac{\partial \Phi}{\partial \phi} \frac{\partial f}{\partial v_\phi} = 0 \quad (15)
\]

where f is the distribution function, Φ is the gravitational potential, and \(v_r, v_\theta, v_\phi\) are the velocities in the (r, θ, φ) directions respectively. Multiplying by \(v_r\) and integrating over
Figure 9. The velocity ellipsoid in the meridional plane. To maintain roughly equal stars per bin we use a logarithmic binning in radius and therefore split this figure to maintain visibility: the upper panel shows the kinematics between 5 kpc and 20 kpc, and the lower panel shows a zoom in on the region between 1.5 kpc and 5 kpc. The scale of each ellipsoid is shown in the inset.

The velocity ellipsoid gives

$$\frac{\partial (\rho \Sigma)}{\partial t} + \frac{\partial (\rho v_r \Sigma)}{\partial r} + \frac{1}{r} \frac{\partial (\rho v_r v_\theta \Sigma)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial (\rho v_r v_\phi \Sigma)}{\partial \phi} + \frac{\rho}{r} \left[ \frac{\Sigma}{v_r^2} v_\theta^2 - \Sigma v_r v_\phi \cot \theta + \Sigma v_\phi^2 \cos \theta \right] = -\frac{\rho \Phi}{r} \frac{\partial \Phi}{\partial r}$$

(16)

where $\Sigma$ denotes the density weighted azimuthal average of a quantity. Integrating over both velocity space and azimuth gives

$$\frac{\partial \langle v_r v_\theta \rangle}{\partial r} + \frac{1}{r} \frac{\partial \langle v_r v_\phi \rangle}{\partial \theta} + \frac{\rho}{r} \left[ 2\langle v_r^2 \rangle - \langle v_r^2 \rangle - \langle v_\phi^2 \rangle \right] \cot \theta = -\frac{\rho \Phi}{r} \frac{\partial \Phi}{\partial r}$$

(17)

This angular Jeans equation, termed the flattening equation by Bowden et al. (2016), is important because it allows the direction of the gravitational acceleration, and thereby the flattening of the potential and the dark matter, to be measured. Note that Equations 17 and 18 are the same as the axisymmetric Jeans equations in spherical coordinates (e.g. de Zeeuw et al. 1996), but do not assume axisymmetry, instead taking the azimuthal average of quantities.

We will use Equations 17 and 18 in a discretised form. Before discretising, we rewrite them as

$$(\langle v_r^2 \rangle) \frac{\partial \log \rho}{\partial r} + \frac{\partial \langle v_r v_\phi \rangle}{\partial \log r} + \frac{\partial \langle v_r v_\theta \rangle}{\partial \theta} + \frac{\partial \langle v_r v_\phi \rangle}{\partial \theta} \frac{\partial \log \rho}{\partial \theta} + \left[ 2\langle v_r^2 \rangle - \langle v_r^2 \rangle - \langle v_\phi^2 \rangle \right] \cot \theta = -\frac{\rho \Phi}{r} \frac{\partial \Phi}{\partial r}$$

(19a)

$$\langle v_r v_\phi \rangle \frac{\partial \log \rho}{\partial r} + \frac{\partial \langle v_r v_\phi \rangle}{\partial \log r} + \frac{\partial \langle v_r v_\theta \rangle}{\partial \theta} + \frac{\partial \langle v_r v_\phi \rangle}{\partial \theta} \frac{\partial \log \rho}{\partial \theta} + \left[ 3\langle v_r v_\theta \rangle + \langle v_\phi^2 \rangle \right] \cot \theta = -\frac{\rho \Phi}{r} \frac{\partial \Phi}{\partial \theta}$$

(19b)

These are more appropriate to use as the basis for the discretised equations because, while $\rho$ changes quickly with $r$ (approximately as $r^{-3}$), the logarithmic gradient, $\partial \log \rho/\partial \log r$, changes slowly.

We measure the kinematics in bins across $(r, \theta)$ and insert these measurements into the discretised Jeans equations. Radially, we use $n_{\log r} = 9$ bins evenly spaced by $\delta \log r$ in $\log r$ between 1.5 kpc and 20 kpc. In elevation, we use 5 bins of $\theta$ with edges at 0°, 25°, 40°, 50°, 70°. These choices were made to have roughly the same number of stars in all bins while minimising the discretisation errors. It is necessary to choose a broad two-dimensional binning in order to obtain a large enough number of stars per bin, and therefore accurate force measurements. In order to reassure the reader that any systematic errors introduced by the discretisation are small, we analyse the mock halos, where the true potential and forces are known, with the same bins as in section A.

Denoting the differentials as $\Delta_{\log r}$ and $\Delta_{\theta}$, we use second order accurate differences apart from at the endpoints. For the evenly spaced grid in $\log r$ this is

$$\Delta_{\log r,i,j}(x) = \begin{cases} \frac{(x_{i+1,j} - x_{i,j})}{(\delta \log r)}, & i = 0 \\ \frac{(x_{i,j} - x_{i-1,j})}{(\delta \log r)}, & i = n_{\log r} - 1 \\ \frac{(x_{i+1,j} - x_{i-1,j})}{(2\delta \log r)}, & \text{otherwise} \end{cases}$$

(20)
where we have labeled the grid cells in $\log r$ by $i$ and $\theta$ by $j$. For the unevenly spaced grid in $\theta$, we use

$$\Delta_{\theta,j}(x) = \begin{cases} \frac{(x_{i+1} - x_i)}{(\theta_{j+1} - \theta_j)}, & \text{for } j = 0 \\ \frac{(x_{i} - x_{i-1})}{(\theta_{j-1} - \theta_j)}, & \text{for } j = n_\theta - 1 \\ \alpha x_{i,j+1} + \beta x_{i,j} + \gamma x_{i,j-1}, & \text{otherwise} \end{cases}$$

(21)

where $\theta_j$ is the mid-point of the $j$th bin in $\theta$ and $\alpha$, $\beta$ and $\gamma$ are chosen to give second order accuracy in the derivative (see e.g. `numpy.gradient` documentation).

Then, at each $(i,j)$ point on our grid, Equation 19 becomes

$$\begin{align*}
\langle v_{r,j}^2 \rangle_{\log r} \Delta_{\log r} \log \rho + \Delta_{\log r} \langle v_{\theta,j}^2 \rangle_{\log \theta} \\
+ \langle v_{r} v_{\phi,j} \rangle_{\log r} \Delta_{\log r} \log \rho + \Delta_{\log r} \langle v_{\phi,j} v_{\phi} \rangle_{\log \theta} \\
+ \frac{1}{2}(\langle v_{\phi}^2 \rangle_{\log \theta} - \langle v_{\phi}^2 \rangle) - \langle (v_{r})^2 \rangle_{\log r} \cot \theta = -r\langle F_r \rangle, \\
\langle v_{r} v_{\phi,j} \rangle_{\log r} \Delta_{\log r} \log \rho + \Delta_{\log r} \langle v_{\phi,j} v_{\phi} \rangle_{\log \theta} \\
+ \frac{1}{2}(\langle v_{\phi}^2 \rangle_{\log \theta} - \langle v_{\phi}^2 \rangle) + \langle (v_{r})^2 \rangle_{\log r} \cot \theta = -r\langle F_\phi \rangle,
\end{align*}$$

(22a)

(22b)

where we have omitted the $(i,j)$ subscripts for clarity, and substituted $\langle F_r \rangle$ and $\langle F_\phi \rangle$, the forces in the radial and azimuthal directions respectively, for the derivatives of the potential.

4.2 The Gravitational Force Field of the Inner 20 kpc of the Galaxy

With the discretized Jeans equations (Equation 22) and the required measurements of the velocity moments in hand, we proceed to measure $\langle F_r \rangle$ and $\langle F_\phi \rangle$. These forces fully specify the azimuthally averaged gravitational force of the Galaxy: $\langle F \rangle = \langle F_r \rangle \hat{r} + \langle F_\phi \rangle \hat{\phi}$. In the left panel of Figure 10, we plot these forces as vectors. It is immediately clear that the forces appear consistent with being nearly radial throughout the volume probed. In the next sections, we perform a more quantitative analysis of the force field and its implications.

5 THE PROPERTIES OF THE INNER DARK MATTER HALO

We now proceed to subtract models of the baryonic contribution to the forces in order to measure the properties of the dark matter halo.

As our fiducial baryonic model, we use a slightly modified version of the baryonic part of the model of Portail et al. (2017, hereafter P17). This model was constructed by using the made-to-measure method (Syer & Tremaine 1996; De Lorenzi et al. 2007) to adapt a barred N-body model to fit data on the inner Galaxy. Fitted data consisted of the 3D shape of the bulge measured by Wegg & Gerhard (2013), combined near-infrared star counts from the VVV, UKIDSS and 2MASS surveys (Wegg et al. 2015), and kinematics from the BRAVA (Kunder et al. 2012) and ARGOS (Ness et al. 2013) surveys. The result is a dynamical model that fits a range of data on the central 5 kpc of the Galaxy, which is where the majority of the stars lie, extremely well. However, it is also important that the model is accurate outside the central 5 kpc. The P17 model uses a local stellar surface density of $38 M_\odot$/pc$^2$ with an exponential scale length of 2.4 kpc, while for ISM it uses $13 M_\odot$/pc$^2$ with a scale length of 4.8 kpc. The scale heights of these components were set to 300 pc and 130 pc. The difference here to P17 is that, while in that work these disks were truncated at 10 kpc, here we do not truncate them. We test the effect of varying the baryonic model, and in particular the disk scale lengths and surface densities, in subsection 5.2.

The resultant forces from our fiducial baryonic model are shown as the middle panel of Figure 10. In the right panel, we subtract these from the measured forces to show the forces from the dark matter alone. Here the errors are
larger, particularly in the central regions where the force from the baryonic component dominates. It is already clear however that the forces are largely radial, meaning that the dark matter potential must be near spherical. To quantify this, we fit an ellipsoidal potential to the dark matter force in each radial bin using the ansatz that the dark matter potential is ellipsoidal:

$$\Phi_{\text{dm}}(m) = \Phi_{\text{dm}}\left(\frac{R^2 + z^2}{q^2}\right)^{1/2} \quad \text{and} \quad \frac{\partial \Phi_{\text{dm}}}{\partial \log m} = V_c^2$$

where $q_\Phi$ is the flattening of the potential, and $V_c$ is the in-plane circular velocity. Taking the derivative of Equation 23 with respect to $r$ and $\theta$ provides the forces. Because we fit these force measurements at a constant radius $r$, and not constant ellipsoidal radius $m$, in principle these forces involve a term of order $O(1-q_\Phi^2 \Phi''_{\text{dm}})$, which we neglect. In practice, we find near spherical potentials, making this term small. Furthermore, our tests on the mock halo which has $q_\Phi = 0.8$ (see Figure A6) show that we accurately recover the profiles of $q_\Phi$ and $V_c$ in this case.

We show the result of fitting for $q_\Phi$ and $V_c$ using the measured forces at each radius in Figure 11. From this figure, we see that, while the circular velocity is fairly flat outside the Sun, it drops inside. Meanwhile, from the lower panel we see that the potential is nearly spherical at all radii. Indeed the measurements between 5 and 20 kpc are consistent with a single value of potential flattening of $q_\Phi = 1.03 \pm 0.08$. However, in order to extract more quantitative overall measurements from the forces, we proceed to fit parametric dark matter density models.

### 5.1 Parametric Fits to the Dark Matter

In this section, we fit parametric dark matter halos to the gravitational force field to measure the properties of the central 20 kpc of the Milky Way’s dark matter halo.

#### 5.1.1 Dark Matter Profiles

We explore four dark matter parameterisations: NFW, Einasto, pseudo-isothermal and generalised NFW. We treat all as ellipsoidal, writing them as a function of the ellipsoidal radius: $m^2 = R^2 + z^2/q^2$. For the NFW profile, we use (Navarro et al. 1996a)

$$\rho_{\text{dm}} \propto \frac{1}{m/m_s(1 + m/m_s)^2}.$$  \hspace{1cm} (24)

The use of this profile is inspired by dark matter only simulations. In these simulations, the halo mass inside the virial radius is correlated to the scale radius $m_s$. However, the dark matter profile, and therefore this relation, are altered by the uncertain interplay between dark and baryonic matter, and therefore we do not use this mass-concentration relation. We also fit a generalised version of this profile where the inner slope is free (Zhao 1996). This is referred to as the gNFW profile:

$$\rho_{\text{dm}} \propto \frac{1}{m/m_s(1 + m/m_s)^{3+\gamma}}.$$  \hspace{1cm} (25)

We also fit two other profiles whose central region is less cusped than the NFW profile: an Einasto profile (Einasto 1965)

$$\rho_{\text{dm}} \propto \exp\left[\frac{-2}{\alpha} \left(\frac{m}{m_{\alpha-2}}\right)^\alpha - 1\right],$$  \hspace{1cm} (26)

and a pseudo-isothermal profile (Sackett et al. 1994)

$$\rho_{\text{dm}} \propto \frac{m^2}{m_{\alpha}^2 + m^2}.$$  \hspace{1cm} (27)

When fitting these profiles, we assume uninformative flat priors in all parameters with the exception of $\gamma$ in the gNFW profile, which we assume to be flat between -5 and 5, $\alpha$ in the Einasto profile, which we assume to be flat between 0 and 8.

#### 5.1.2 Fitting Process

For each of the dark matter densities, we compute the forces ($F_r$) and ($F_\theta$) at the centre of each grid cell, add these to the baryonic model, and fit these model forces to the measured forces. The process is complicated by the measured forces in each grid cell being correlated. The correlations are introduced by the finite difference approximations used in the discretised Jeans equations (Equation 22) which connect measurements at neighbouring points. As a result, neighbouring force measurements are correlated and this must be taken account of during fitting.

To compute this correlation, we use bootstrap resampling of the data and compute the resultant forces from each resampling. We then estimate the covariance from the bootstrap resampled forces. The force measurements may be written as a vector $F$, of length $N = 90$, representing measurements from 5 angular bins, 9 radial bins, and 2 force...
directions. Using this notation, we estimate the covariance matrix of the forces $\mathbf{W}$ from

$$W_{mn} = \frac{1}{N} \sum_i(F_{mi} - F_m)(F_{ni} - F_n)$$

(28)

where $F_{mi}$ is the $i$-th resampling of force measurement $F_m$ and we have used $N = 10,000$ bootstrap resamplings.

We assume that the forces have normally distributed errors. This is expected from the central limit theorem because they arise from the kinematic measurements of more than 100 stars in each bin, and it appears to be a good approximation from the bootstrap resampling. Denoting the $N$ forces predicted from a dark matter profile with parameters $\mathbf{X}$ as $\mathbf{Y}(\mathbf{X})$ then the likelihood of measuring the forces $\mathbf{F}$ is

$$L(\mathbf{F}|\mathbf{X}) = \frac{1}{\sqrt{(2\pi)^N|\mathbf{W}|}} \exp\left[-\frac{1}{2} (\mathbf{F} - \mathbf{Y}(\mathbf{X}))^T \mathbf{W}^{-1} (\mathbf{F} - \mathbf{Y}(\mathbf{X})) \right].$$

(29)

We use an MCMC to sample from the posterior distribution of the parameters of our dark matter halos (Foreman-Mackey et al. 2013). We show the resultant maximum likelihood (or maximum a posteriori probability) parameters of the fits in Table 1. The parameters of the models are opaque and in some cases highly correlated, making their errors large and their values uninformative. We therefore plot in Figure 12 the parameters transformed into more physical quantities: the total circular velocity at the Sun including both dark and baryonic matter, $V_c(R_0)$, the dark matter density at the Sun, $\rho_{\text{dm}}(R_0)$ and the dark matter density flattening, $q_{\rho}$. Note that the Einasto and gNFW profiles have four parameters and so for these profiles there is one additional unplotted nuisance parameter to fully specify the dark matter profile. This extra parameter, which in both cases effectively describes the shape of the density profile, is poorly constrained.

All the parametric models fit the force field well, having $\chi^2$ values per degree of freedom of $\approx 0.95$. The AIC differs by less than 1 across all 4 models, meaning that we have insufficient information to distinguish between them in our sample. The errors are very similar with the exception of the Einasto profile, where the errors are larger. This is because the Einasto profile has more freedom to change its shape as can be seen from the range of shapes taken by the Einasto profile in Figure 11 (and later in Figure 13). Because of this we conservatively select the Einasto profile as our fiducial and conclude that $q_{\rho} = 1.00 \pm 0.09$, $V_c(R_0) = (222 \pm 6)$ km s$^{-1}$ and $\rho_{\text{dm}}(R_0) = (0.0109 \pm 0.0025) \, M_\odot / \text{kpc}^3$. From the MCMC samples of all of the profiles we find $q_{\rho} > 0.8$ at greater than 99% significance.

### 5.2 Systematics

In this section, we assess the errors induced by possible systematic errors in the data, and choices made in the analysis. In particular, we examine the effect of possible systematic errors in RR Lyrae distances, and in the the baryonic model. To estimate the uncertainties due to the baryonic model, we adjust each component in turn. When adjusting the stellar disk or ISM scale lengths, we fix the density at 4 kpc from the Galactic centre so that, by adjusting these scale lengths, we also adjust the surface density at the Sun. Our tests are summarised in Table 2.

Our parameters are relatively insensitive to any of the tested variations, changing within the formal statistical errors.
Our Jeans modelling assumes that the Galaxy, and our stellar sample, is in dynamical equilibrium. Because our halo is growing from the accretion of satellites, this assumption is broken in detail. Objects accreted into the inner halo are expected to phase mix relatively quickly, while retaining information in their integrals of motion or actions, making the distribution function less smooth. However, Jeans modelling does not require that the distribution function be featureless, only that the inner halo be well phase mixed i.e. that the distribution function can be taken as time independent. Features that have not had enough time to well phase mix could however present a problem: the Sagittarius stream is one such feature. There are likely to be others to be found in Gaia data (Malhan & Ibata 2018) and some are already known in the volume that we have studied (e.g. Ibata et al. 2018, and see also Mateu et al. 2017). However, because the fraction of RR Lyrae in these unmixed stellar streams in the halo inside 20kpc is small, the effect on our results is expected to be similarly small and we have not attempted to excise all streams. This is supported by subsection 5.2, in which we repeated our analysis without removing the Sagittarius stream. Despite being the most prominent non-equilibrium feature in our sample, the effects of not excising it are relatively small.

The Jeans analysis is attractive because it is non-parametric. However, as a result, its formal statistical power is lower than that of other methods. Parametric modelling could reduce the errors by effectively reducing the number of parameters considered in the modelling. Here, our halo is described by 45 density measurements, and 5×45 kinematic measurements, whereas parametric models of the halo distribution function reduce this to a handful of numbers. However, with complicated systems and high-dimensional data, it can be difficult to assess possible model degeneracies: while fitting a parametric model to data and assessing that the fitted model reproduces the data is straightforward, assessing whether a subtly different model could also reproduce the data, but with significantly different results requires a careful analysis. The non-parametric force measurements here circumvent that problem.

These non-parametric methods presented in this work has the further advantages that it is highly transparent: we can derive the force field in a clear manner from the kinematic measurements, and fit models directly to this. For example, during the initial analysis, the Sagittarius stream was not completely removed by our selection cuts. This, however, was immediately clear when the first force field was constructed because the bin from which the stream had not been excised was a clear outlier.

Our measurement of the dark matter flattening of $q_D = 1.00 \pm 0.09$ agrees with several recent measurements, but with somewhat smaller error. Measurements from streams also point towards a near spherical halo: in particular Bovy et al. (2016) found $q_D = 1.05 \pm 0.14$ towards the edge of our volume by combining measurements from the Pal-5 and GD-1 streams. In addition, very recently, Posti & Helmi (2018) performed action based modelling on 91 globular clusters

### Table 1. Parameters of the fitted dark matter profiles.

| Profile       | $q_D$    | $V_c(R_0)$ [km s$^{-1}$] | $\rho_{\text{halo}}(R_0)$ [M$_\odot$/kpc$^3$] | Best Fitting Parameters | Max log $\mathcal{L}$ | $\chi^2$/DOF | AIC  |
|---------------|----------|--------------------------|---------------------------------|--------------------------|-----------------------|--------------|------|
| NFW           | 0.99 ± 0.08 | 216.0 ± 3.5             | 0.0092 ± 0.0009                 | $m_s = 29$kpc            | −783.5                | 0.96         | 1573.0|
| Einasto       | 1.00 ± 0.09 | 222 ± 6                 | 0.0109 ± 0.0025                 | $m_{-2} = 8.7$kpc, $a = 1.4$ | −782.4                | 0.94         | 1572.7|
| gNFW          | 0.99 ± 0.09 | 218 ± 4                 | 0.0091 ± 0.0013                 | $m_s = 1.6$kpc, $\gamma = -3.9$ | −782.6                | 0.95         | 1573.2|
| Pseudosothermal | 0.99 ± 0.08 | 217 ± 4               | 0.0094 ± 0.0012                 | $m_c = 2.7$kpc          | −783.2                | 0.95         | 1572.4|

### Table 2. Systematic variations of the fiducial baryonic model and their affect on the fitted parameters. We show only the results of fits to an Einasto profile.

| Variation     | $V_c(R_0)$ [km s$^{-1}$] | $\rho_{\text{halo}}(R_0)$ [M$_\odot$/kpc$^3$] | $q_D$ |
|---------------|--------------------------|---------------------------------|------|
| Fiducial      | 222                      | 0.0109                           | 1.00 |
| $h_R, * = 2.15$kpc $^b$ | 220                      | 0.0111                           | 0.97 |
| $h_R, * = 2.68$kpc $^c$ | 221                      | 0.0104                           | 1.02 |
| $h_{R, \text{iso}} = 3 \times 2.4$kpc $^d$ | 220                      | 0.0110                           | 1.03 |
| $h_{R, \text{iso}} = 1.5 \times 2.4$kpc $^e$ | 221                      | 0.0113                           | 0.97 |
| P17 Boundary Model 1 $^f$ | 219                      | 0.0103                           | 0.99 |
| P17 Boundary Model 2 $^g$ | 221                      | 0.0107                           | 1.00 |
| RR Lyrae 0.03mag brighter $^h$ | 217                      | 0.0096                           | 1.00 |
| RR Lyrae 0.03mag fainter | 219                      | 0.0116                           | 0.98 |
| $R_0 = 8.0$kpc $^i$ | 220                      | 0.0098                           | 1.05 |
| $R_0 = 8.4$kpc $^j$ | 220                      | 0.0109                           | 0.98 |
| $\nu_0 = (11.1, 255, 7.25)$km/s $^k$ | 222                      | 0.0108                           | 1.00 |
| $\nu_0 = (11.1, 245, 7.25)$km/s $^l$ | 221                      | 0.0117                           | 1.01 |
| Fitting including Sgr Stream $^m$ | 222                      | 0.0083                           | 1.06 |

$a$ Uses stellar disk with scale length $h_R, * = 2.4$kpc, gas disk with scale length $h_{R, \text{iso}} = 2 \times 2.4$kpc, and best fitting model of P17. This model has bar pattern speed $\Omega = 40$kms$^{-1}$kpc$^{-1}$, mass-to-clump ratio 1000/M$_\odot$ and nuclear stellar mass 2×10$^7$M$_\odot$.

$b$ Dynamical disk scale length measured by Bovy & Rix (2013). Has $\Sigma_L(R_0) = 32$M$_\odot$/pc$^2$ to keep disk continuity at 5kpc. $^c$ Dynamical disk scale length measured by Piffl et al. (2014). Has $\Sigma_L(R_0) = 44$M$_\odot$/pc$^2$ to keep disk continuity at 5kpc.

$d$ Has $\Sigma_{\text{halo}}(R_0) = 16$M$_\odot$/pc$^2$ to keep disk continuity at 5kpc. $^e$ Has $\Sigma_{\text{halo}}(R_0) = 10$M$_\odot$/pc$^2$ to keep disk continuity at 5kpc.

$f$ Uses bar pattern speed $\Omega = 37.5$kms$^{-1}$kpc$^{-1}$, mass-to-clump ratio 900/M$_\odot$, and nuclear stellar mass 2.5×10$^7$M$_\odot$. $^g$ Uses bar pattern speed $\Omega = 42.5$kms$^{-1}$kpc$^{-1}$, mass-to-clump ratio 1100/M$_\odot$ and nuclear stellar mass 1.5×10$^7$M$_\odot$. $^h$ Estimated systematic uncertainty by S17

$i$ We remove the Sagittarius Dwarf, but leave the tail of the stream in the sample.

$\chi^2$/DOF = 2 per degree of freedom, and the Akaike information criterion.
with full 6D phase space information, finding $q_p = 1.30 \pm 0.25$. We expect these constraints to rapidly improve as the community begins to exploit Gaia DR2 in combination with other datasets.

If this emerging picture that the dark matter profile is nearly spherical holds into the Gaia era, then it appears in tension with the shapes expected from current cosmological simulations. Dissipation in baryonic simulations can make the highly triaxial halos seen in dark matter only simulations more spherical (Dubinski 1994; Kazantzidis et al. 2004; Debattista et al. 2008; Kazantzidis et al. 2010; Abadi et al. 2010). However, a completely spherical, or even mildly prolate halo, would be in tension with these simulations which typically predict increases in axis ratio of $q_p = 0.1 \sim 0.3$. It is possible that this tension could point to the physics of dark matter, one of the most studied examples being that halos are more spherical in self interacting dark matter models (Spergel & Steinhardt 2000; Yoshida et al. 2000; Peter et al. 2013). Careful assessment of these results is however needed: for example, Dai et al. (2018) reassessed the measurements of a near spherical halo by Bovy et al. (2016), and concluded that the data could also be reproduced in the mildly oblate potential of the Eris simulation.

We plot in Figure 13 the dark matter densities of our fitted models. The density that we find near the Sun is consistent with several recent measurements using the velocities of nearby disk stars. Piffl et al. (2014) finds a dark matter density of $0.0126q_p^{-0.89} M_\odot/pc^3$ with systematic errors estimated at 15\%, and we include this value in our plot. Likewise Bienaymé et al. (2014) finds $(0.0143 \pm 0.0011)M_\odot/pc^3$ which is slightly higher than our measurements. Interestingly, our value was not measured near the Sun but away from the Galactic plane. The closeness of these measurements points towards consistent picture of a near spherical dark matter halo at Solar galactocentric radii (Read 2014).

Our value of the circular velocity at the Sun of $V_c(R_0) = (222 \pm 6) \text{km s}^{-1}$ is lower than some other recent measurements. In particular, it is lower than the $(238 \pm 9) \text{km s}^{-1}$ measured by Schönrich (2012) and $(240 \pm 8)$ measured by Reid et al. (2014). This difference could be statistical, but non-axisymmetric motions could also play an role (Bland-Hawthorn & Gerhard 2016), despite the care taken by Schönrich (2012) and Reid et al. (2014). Many non-axisymmetric motions are obvious in the new data from Gaia (e.g. Katz et al. 2018; Antoja et al. 2018; Hunt et al. 2018), and we expect the value of $V_c(R_0)$ to soon be clarified with this data.

We also plot in Figure 13 the spherical cumulative mass profiles of the dark matter and the total mass including baryonic matter. We see that our mass enclosed inside 20 kpc is consistent with the measurements using Gaia DR2 data of the mass inside the same volume by Posti & Helmi (2018) and Watkins et al. (2018).

As expected, the profiles have quite similar dark matter densities at radii between 5 kpc and 20 kpc, where our method provides its most accurate measurements. Interestingly they are quite different inside 5 kpc. The dynamical models of P17 required fairly low dark matter fractions inside the bulge region of 17\%, corresponding to a mass of $(3.2 \pm 0.5) \times 10^9 M_\odot$. When combined with $V_c(R_0) = (238 \pm 9) \text{km s}^{-1}$ and the stellar surface density, this required that the dark matter have a core or shallow cusp (with $\gamma < 0.7$). However, the dark matter mass inside the bulge would be consistent with all our profiles at $\approx 1\sigma$. The reason that the NFW profile (which has central slope $\gamma = 1$) is still consistent with the P17 bulge mass measurement is most likely that the circular velocity found here is slightly smaller than the $V_c(R_0) = 238 \text{km s}^{-1}$ used in P17. This demonstrates the need for Gaia era dynamical modelling which connects data across the Galaxy to order to clarify whether the dark matter profile is shallow, as found by P17, or more steeply cusped as found in recent cosmological simulations (e.g. Chan et al. 2015; Grand et al. 2017).

7 SUMMARY AND CONCLUSIONS

Gaia has produced a truly transformational dataset with which, over the coming years, we will learn about how the
Milky Way and similar galaxies formed and evolved. The sample we have analysed is remarkable in having accurate transverse kinematic measurements across the entire inner halo, at least outside the Galactic plane. This is provided by combining the proper motions of Gaia with the accurate distances of the RR Lyrae sample of S17. This allowed us to investigate the kinematics of the stellar Halo between 1.5 kpc and 20 kpc.

By statistically reconstructing the kinematics in the absence of radial velocities, we found that, outside the central 5 kpc, halo RR Lyrae are highly radially anisotropic with $\beta \approx 0.8$ and have a nearly spherically aligned velocity ellipsoid. Between 1.5 kpc and 5 kpc, that anisotropy drops, but even there it remains above $\beta \approx 0.25$. Inside 10 kpc, our sample of Halo RR Lyrae rotates with a profile that rises to 50 km s$^{-1}$ in our innermost radial bin. This may reflect the early formation and accretion history of the halo, although there is significant transfer of angular momentum with the Galactic bar at these radii. Reaching firm conclusions regarding the origin of this rotation will require further modelling and data, including for example metallicity information.

By applying discrentised versions of the Jeans equations to these kinematics, we subsequently measured the acceleration field inside 20 kpc. The acceleration field is largely radial, particularly in the outer regions. While the in-plane acceleration field of the Milky Way is relatively well constrained by the rotation curve, where our measurements are made, away from the Galactic plane, the situation is more uncertain. The most accurate previous measurements were limited to a small number of stellar streams in the halo.

By subtracting baryonic models, we inferred the force produced by the Milky Way’s dark matter. Because the forces from the dark matter are consistent with being directed in the radial direction, the resultant potential is consistent with spherical. We measured the profile of the shape of the dark matter potential between 5 kpc and 20 kpc and have found that these measurements are consistent with a single value of $q_\rho = 1.03 \pm 0.08$.

We have also fit parametric dark matter profiles to the forces. We found that the ellipsoidal flattening of these density profiles does not depend significantly on the profile and can be combined into a single constraint: $q_\rho = 1.00 \pm 0.09$. This is consistent with a spherical profile, as expected given the radial nature of forces, and the measured sphericity of the potential. Our fits indicate that density profiles as flattened as $q_\rho = 0.8$ are ruled out at higher than 99% significance.

The fitted dark profiles are also interesting, however, using the present data alone, we cannot determine the steepness of the inner profile and further dynamical modelling and data are required. If the kpc scale cored halo found in the inner Galaxy by P17 persists into the Gaia era, then this may also relate to the near spherical halo.

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APPENDIX A: MOCK HALO ANALYSIS

A1 Construction of Mock Samples

We have constructed observations of mock halos in order to test that our analysis reliably recovers both the intrinsic kinematics of the sample, and that we can reliably use these measurements to infer the Galactic potential. These mock halos were constructed from the made-to-measure models of the inner Galaxy constructed by Portail et al. (2017). The models do not include a stellar halo and so, instead, a sample of dark matter particles were selected. To do so, each dark matter particle was treated as a test particle with a statistical weighting factor determined so that the resultant halo had the desired properties.

To construct isotropic stellar halos, this statistical weighting was determined on the basis of each particles energy. We used a piecewise exponential of the energy:

$$\log f(E) = \alpha_i \frac{E - E_i}{E_{i+1} - E_i} + \beta_i$$

when $E_{i+1} > E > E_i$.  \(\alpha\) (A1)

We found that using 7 bins equally spaced in energy was sufficient to construct accurate radial profiles. Enforcing continuity leaves 8 free parameters. These were determined by fitting the density of particles within 20° of the minor axis between 1 kpc < |z| < 15 kpc to the target profile $\nu \sim -3$.

The energy is not an integral of motion in the non-axisymmetric inner Galaxy\textsuperscript{1}. Therefore, after choosing these statistical weights, we integrated the particles forwards in the models potential. After first integrating for 500 Myr (approximately the orbital timescale at 15 kpc) we selected particles every 100 Myr to be part of our model halo proportional to the statistical weight given by Equation A1.

The resultant mock halo had a minor axis profile near to the observed $\nu \sim r^{-3}$ but was nearly isotropic. This is because the dark halo particles from which the stellar halo was constructed was nearly isotropic. The Galaxy’s stellar halo is however non-isotropic and therefore we constructed a family of mock anisotropic halos. To do so, we used an approximate third integral inspired by axisymmetric models of Dhehnen & Gerhard (1993). We integrated each particle backwards in time for 10 Gyr and computed the normalised radial extent of each particles orbit in the equatorial plane: $D_r = (R_r - R_-)/R_+$ where $R_+$ and $R_-$ are the maximum and minimum radius reached at an equatorial crossing. We then adapted the statistical weight to be

$$\log f(E, D_r) = \alpha_i \frac{E - E_i}{E_{i+1} - E_i} + \beta_i + h(D_r, E)$$

when $E_{i+1} > E > E_i$. \(\alpha\) (A2)

where we chose $h(D_r, E)$ to provide a bias towards more radially extended orbits. Inspired by model 3 of Dhehnen & Gerhard (1993) we chose

$$h(D_r, E) = -\left(1 - D_r\right)^2 + \frac{1}{2} \nu_0^2$$

where $\nu_0$ is the parameter which determines the degree of radial anisotropy of our mock halo and $\nu(E)$ is a function

\textsuperscript{1} An alternative choice would be the Jacobi Energy, which is an integral of motion. However using this in place of the energy resulted in unrealistic halo shapes.
that ensures the halo is centrally isotropic:

\[ q(E) = \frac{R^2_{\text{c}}(E)}{R^2_{a} + R^2_{c}(E)} \]  

(A4)

where \( R_{\text{c}}(E) \) is the radius of an in-plane circular orbit with the energy \( E \) and \( R_{a} \) is an anisotropy radius for which we used 0.5 kpc.

We note in passing that a more elegant solution to constructing the mock halos might be to use the actions of the particles to select the statistical weights of each particle. However, both the energy and orbital turning points would be integrals of motion in an axisymmetric potential, and would therefore be functions of the actions, making both methods not fundamentally different. In addition, our method has the advantage of being straightforward and computationally fast, while still resulting in mock halos with a minor axis profile and anisotropy very similar to the Milky Way’s halo.

### A2 Mock Halo Kinematics

From the mock halos, we selected the halo with anisotropy parameter \( y_0 = 0.15 \) as being most similar to the Milky Way’s halo and show the results of its analysis here. Comparing our mock halo kinematics in Figure A1 to the measured Milky Way kinematics in Figure 7, we see that they are qualitatively very similar. We therefore proceed to test our analysis methods and code on this mock. We have also tested extensively with anisotropy \( y_0 = 0.10 \) and \( y_0 = 0.20 \) and obtained similar results. We use throughout exactly the code and methods developed for analysing the Milky Way halo, but here we have the advantage that we know both full 6D phase space information to test the kinematic reconstruction, and the potential in which the stars are moving in order to test the potential reconstruction.

We have constructed our mock halos to have a large number of stars in order to test that our methods asymptotically recover the kinematics, and the properties of the potential. In particular, we have 870,000 stars in our mock sample. Figures A1, A2, A3, A4 and A5 use this large sample to show that our analysis is asymptotically correct. In Figures A6 and A7, we compare the results obtained with the large sample and with a smaller subset of ~16,000 mock stars, similar to the number in the real sample.

In Figure A1, we compare the kinematics computed using the three dimensional galactocentric velocities of the particles (and therefore full 6D phase-space), to the kinematics reconstructed without radial velocity measurements (and therefore only 5D measurements). We use both methods described in section 3: the method assuming Gaussian velocities (subsection 3.1) and method inspired by DB98 (subsection 3.2). Both reconstruct the intrinsic kinematics accurately, being almost indistinguishable from the intrinsic kinematics.

Figure A1 was constructed assuming a complete sample over the entire inner Halo. In Figure A2, we instead apply the selection function described in section 2 to the mock halo before observing it. Again, both methods accurately reconstruct the velocity dispersion tensor. There are slight differences in the reconstruction of rotation, \( \langle v_\phi \rangle \), although...
Figure A2. As Figure A1 but instead showing the kinematics of our mock halo folded through the selection function described in section 2.

Figure A3. As Figure 8 but instead showing the reconstructed kinematics of our mock halo from proper motions, after folding the mock through the selection function described in section 2.

at a level $<10$ km s$^{-1}$. In general, the method inspired by DB98 performs slightly better. For this reason and because it explicitly does not depend on the assumption of a Gaussian velocity distribution, we use this reconstruction method as our fiducial, testing only that we find equivalent results with both in subsection 5.2.

In Figures A3 and A4, we show the kinematics of the same mock halo plotted in physical space so that the kine-
Figure A4. As Figure 9 but instead showing the reconstructed velocity ellipsoid in the meridional plane of the mock halo. Note that our mock halo also has a near spherically aligned velocity ellipsoid outside the central 5 kpc.

Figure A5. As Figure 10 but instead showing the reconstructed kinematics of our mock halo from proper motions, after folding the mock though the selection function described in section 2. As a result of the large numbers of stars in the mock the formal errors in the acceleration field are extremely small.
Figure A6. As Figure 11 but instead showing the results of fitting an ellipsoidal shaped dark matter potential to the forces in the mock halo. On the left we show the results of fitting the mock halo with 870,000 stars, and on the right the realistic sized mock sample of 16,000 stars. The true underlying potential is shown in blue. We also plot the results of fitting an Einasto dark matter density profile to the forces in red on the left for the mock halo with 870,000 stars, and in green on the right for the mock halo with 16,000 stars.

Figure A7. As Figure 12 but instead showing the parameters recovered after fitting an Einasto dark matter density profile to the forces. The results of fitting the large mock halo with 870,000 stars are shown in blue, and the results of fitting the mock halo with 16,000 stars are shown in red. The true parameters of the underlying potential are shown in dark gray.