A method of paired zeroing of numbers in a residue system

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Abstract. The paper studies the problem of determining the lower boundary of number correction speed in computing systems operating in the basis of non-positional arithmetic of residual classes. The topicality of the problem is necessitated by the search of methods allowing reducing the time for digital processing of signals in non-positional neuroprocessors. The study considers several variants of error correction with a single and multiple control bases of the residue system. The calculations were performed that allowed determining the time necessary for zeroing operation. The method of paired zeroing of numbers in a system of residual classes was modified. It was demonstrated that the suggested solutions allow appreciably reducing the time consumption by digital processing of signals in neuroprocessors destined for operations of summation and multiplication.

1. Introduction
Large amount of calculations necessary for processing telemetry of oil and gas wells and modeling of bottom-hole zone treatment requires new solutions for reliable and fast implementation of basic arithmetic operations of summation and multiplication. The development of a non-positional neuroprocessor considerably accelerates the calculations. However, the provision of reliable calculations is still a challenge.

In [1], the authors consider the introduction of the term zeroing of numbers. Outstanding works allow developing the theory of error correction in a non-positional system of residual classes and suggesting a method of paired zeroing of numbers in a residue system. The peculiarity of the method is its focus on cutting time consumption for correction of single and multiple errors. The study solves the problem of combining operations of summing and sampling of the next zeroing constant in certain time cycles. It also solves the problem of sampling of the next constant and preparation of values of numbers that will be used at the next stage to select the next zeroing constant for the number.

2. Materials and Methods
Zeroing is one of the methods for determining the correctness of a number. It involves conversion of initial number \( A = (\alpha_1, \alpha_2, ..., \alpha_n, \alpha_{n+1}) \) into number \( A^{(n)} = (0, 0, ..., 0, \gamma_{n+1}) \) using the transformation
sequence that does not allow a single escape from working range $M = (m_1 \times m_2 \times \ldots \times m_n)$, where $m_i$ are the bases of the moduli of residual class system [1–3].

The process of number zeroing envisages consecutive subtraction of constants from this number. 

$$(m_{11}, m_{21}, \ldots, m_{n1}), m_{11} = 1, 2, \ldots, m_1 - 1;
(0, m_{22}, \ldots, m_{n12}), m_{22} = 1, 2, \ldots, m_2 - 1;
(0, 0, \ldots, m_{nn}, m_{n1n}), m_{nn} = 1, 2, \ldots, m_n - 1.\)$$

In this case, number $A = (\alpha_1, \alpha_2, \ldots, \alpha_{n+1})$ is consecutively transformed into $\bar{A} = (0, \alpha'_2, \ldots, \alpha'_{n+1})$, then into $A'' = (0,0, \ldots, \alpha''_{n+1})$, and so on.

After $n$ iterations, we get $A^{(n)} = (0,0, \ldots, 0, \gamma_{n+1})$.

If $\gamma_{n+1} = 0$, then the initial number is correct (lies within interval $[0, M)$; if $\gamma_{n+1} \neq 0$, then the number is incorrect and lies within interval $[jM, (j+1)M)$, $j=1, 2, \ldots, m_{n+1} - 1$. The value of interval number $(j+1)$, where gets operand $\bar{A}$ is determined from

$$j = \left\lfloor \frac{\Delta_i \times m_1 \times m_{n+1}}{m_i} \right\rfloor \mod m_{n+1} + \Delta_{0,1},$$

where $\Delta_{0,1}$ takes on value 0 or 1.

Error $\Delta A$ can lead a correct number $A$, lying in interval $[0, M)$, only into one of the two specified intervals.

Let $\bar{A} = A_{\Delta A}$, or $\bar{A} = (\alpha_1, \alpha_2, \ldots, \alpha_{n+1}) + (0,0, \ldots, \Delta \alpha_i, \ldots, 0)$.

Then, evidently, $\Delta A$ lies not in the first interval $[0, M)$, because number $A_0 = (0,0, \ldots, 0)$ lies in the first interval.

Let $\Delta A$ lie in the $k$-th interval: 

$$(k - 1)M \leq \Delta A < kM.$$

In the system of inequalities, let us

$$\begin{cases} 
0 \leq A < M \\
(k - 1)M \leq \Delta A < kM 
\end{cases}$$

sum up

$$(k - 1)M \leq A + \Delta A < (k + 1)M.$$

Let us assume $j = k - 1$, then we can write $jM \leq \bar{A} < (j + 2)M$, i.e. an error can turn correct operand into incorrect one lying only in one of the two intervals, $[jM, (j+1)M)$ or $[(j + 1)M, (j + 2)M)$.

The obtained result is applied for determining an alternative combination of numbers in the system of residual classes by the method of zeroing.

The time of zeroing is determined as $T = 2\pi \tau_{\Sigma}$, where $\tau_{\Sigma}$ is time of zeroed number summation with zeroing constant. The number of zeroing constants equals

$$k = \sum_{i=1}^{n} m_i - n,$$

while the number of digits of the storage of the constants in the non-positional neuroprocessor equals

$$c = (\sum_{i=1}^{n} m_i - n - i).$$

Obviously from the calculations, the most important of the strong points of the system of residual classes—high operation speed of modular operations—is lost during zeroing. The time of operation implementation is comparatively high, which lowers the efficiency of residue system application.

However, there is the method of paired zeroing of numbers in the system of residual classes [4–5]. According to the method, at each step, the zeroing is performed simultaneously in two bases. Zeroing time decreases twofold and equals $T = n \tau_{\Sigma} C_{\Sigma} L$.

The total number of zeroing constants equals

$$k = \sum_{i=1}^{n/2} m_i m_{n-i+1} - n/2,$$

while the number of digits of zeroing constants equals

$$c = \left(\sum_{i=1}^{n/2} m_i m_{n-i+1} - n/2\right)(n - 2i).$$
3. The study

Let us investigate the method of paired zeroing of numbers with preliminary sampling of digits. Here, the operations of summing and sampling of the next zeroing constant are combined in certain time cycles. In addition, the sampling of the next constant is performed with preparation of values of numbers that will be used at the next zeroing stage to select the next zeroing constant for number

\[ (0, ..., 0, \alpha_i, \alpha_{i+1}, \ldots, \alpha_{n-i}, \alpha_{n-i+1}, 0, \ldots, \beta_{n+1}). \]

Values \( \alpha_i \) and \( \alpha_{n-i+1} \) in an elementary member working in bases \( m_{i+1} \) and \( m_{n-i} \) can be used to prepare values \( \alpha'_{i+1}, \alpha'_{n-i} \) that will be exploited at the next zeroing stage to sample the constant. Indeed, values \( \Delta \alpha_{i+1}, \Delta \alpha_{n-i} \) that will be subtracted from \( \alpha_{i+1} \) and \( \alpha_{n-i} \), correspondingly, are determined only by values \( \alpha_i \) and \( \alpha_{n-i+1} \). During constant sampling using values \( \alpha_i \) and \( \alpha_{n-i+1} \) from corresponding tables in a single cycle values \( \alpha'_{i+1} \) and \( \alpha'_{n-i} \) can be used. In this case, it is unnecessary to have digits in bases \( m_{i+1} \) and \( m_{n-i} \), which will allow decreasing the capacity of zeroing constants

\[ C = \left( \sum_{i=1}^{\lfloor n/2 \rfloor} \Delta \alpha_i m_i m_{n-i+1} - n/2 \right) (n - 2l - 2). \]

The number of summations in suggested zeroing variant equals \( \left[ \frac{n+1}{2} \right] \), since the zeroing is simultaneously performed in all informational bases of the system of residual classes in pairs. After every two summations, one additional cycle is required for the formation of the next address and reference to the storage of the zeroing constants. In this connection every two summation cycles \( (\tau_S = T_0) \) correspond to one cycle without summation. If conventional paired zeroing at \( n = 5 \) (summation of two zeroing constants) requires four conditional time cycles \( 4T_0 \), then the considered method requires three cycles \( 3T_0 \).

Generally, the analytical dependence of the zeroing time on the number of information bases of the system of residual classes can be represented as

\[ T = \left[ \frac{n+1}{2} \right] \tau_S + \left[ \frac{n+1}{2} \right] / 2 \tau_{mem}. \]

where \([x]\) is the integral part of \( x \), that does not exceed \( x \), \( \tau_{mem} \) is the time of reference to the storage (table memory) of zeroing constants.

Considering that \( \tau_S = \tau_{mem} \), we get

\[ T = \left( \frac{n+1}{2} + \left[ \frac{n+1}{2} \right] / 2 \right) \tau_S. \]

At even \( n \):

\[ T' = \left( \frac{n}{2} + \left[ \frac{n+1}{2} \right] / 2 \right) \tau_S. \]

If \( \frac{n}{2} \) is even, then \( T'_{even} = \frac{3}{4} n \tau_S \).

If \( \frac{n}{2} \) is odd, then \( T'_{odd} = \left( \frac{3n+2}{4} \right) \tau_S \).

At odd \( n \), \( T'' = \left( \frac{n+1}{2} + \left[ \frac{n+1}{2} \right] / 2 \right) \tau_S. \)

If \( \frac{n+1}{2} \) is even, then \( T''_{even} = \frac{3}{4} (n + 1) \tau_S \).

If \( \frac{n+1}{2} \) is odd, then \( T''_{odd} = \left( \frac{3n+5}{4} \right) \tau_S \).

The received correlations refine the lower limit of error correction speed for specified values \( n \) and \( \tau_S \).

The reduction of error correction time by the method of paired zeroing of numbers with preliminary sampling of digits in comparison with simple zeroing is

\[ \Delta_t^{even} = 2 n \tau_S - \frac{3}{4} n \tau_S = \frac{5}{4} n \tau_S. \]

\[ \frac{\Delta_t^{even}}{2n \tau_S} \cdot 100\% = 62.5\%; \]
In comparison with paired zeroing:

\[
\Delta_1^{even} = n\tau_S - \left(\frac{3n+2}{4}\right)\tau_S = \left(\frac{5n-2}{4}\right)\tau_S; \\
\frac{\Delta_1^{even}}{n\tau_S} \cdot 100\% = \left(\frac{62.5 - 25}{n}\right)\%;
\]

\[
\Delta_1^{odd} = n\tau_S - \left(\frac{3n}{4}\right)(n+1)\tau_S = \left(\frac{5n-3}{4}\right)\tau_S; \\
\frac{\Delta_1^{odd}}{n\tau_S} \cdot 100\% = \left(62.5 - \frac{37.5}{n}\right)\%;
\]

\[
\Delta_2^{even} = n\tau_S - \left(\frac{3n}{4}\right)\tau_S = \left(\frac{n-3}{4}\right)\tau_S; \\
\frac{\Delta_2^{even}}{n\tau_S} \cdot 100\% = \left(25 - \frac{75}{n}\right)\%;
\]

\[
\Delta_2^{odd} = n\tau_S - \left(\frac{3n+5}{4}\right)\tau_S = \left(\frac{n-5}{4}\right)\tau_S; \\
\frac{\Delta_2^{odd}}{n\tau_S} \cdot 100\% = \left(25 - \frac{125}{n}\right)\%.
\]

Let the determination of distorted residual of operand \( \bar{A} \) requires \( k = 5 \) operations of determination of alternative combination, and let \( \Delta t_{AC} = \Delta t_n, \Delta t_i = \text{const} \) and \( \Delta t_i = 0 \). In this case \( \Delta t_{error} = 5 \Delta t_{AC} \). For the system of residual classes \( n = 8 \) \( \Delta t_{AC} = \frac{3\Delta t_{AC}}{4}, \) then \( \Delta t_{error} = \frac{2K}{3} \cdot \Delta t_{AC} = \frac{K\Delta t_{AC}}{2} \) (see Table).

4. Conclusion

Above we have studied the method of paired zeroing of numbers in the system of residual classes with preliminary sampling of digits. The method allows determining the lower limit of error correction speed in a residue system. The decrease of the time for convergence of an alternative combination to the error basis by 50% allows sharply increasing the efficiency of application of the system of residual classes through the possibility of correction of multiple errors.

The developed method can serve as the basis for synthesizing the error correction unit similar to that described in [6]. The unit should include operation and storage registers, deciphers, selectors, unit of zeroing of constants, memory unit for errors of constants, summation unit, valves, keys and commutators. The practical application of the studies is aimed at the solution of problems described in [7–10].
Table 1. The number of information bases of the system of residual classes sufficient to provide the number representation range.

| Number of information bases $n$ | Correction time $T/c_S$ | Benefit [%] |
|---------------------------------|-------------------------|-------------|
|                                 | Conventional zeroing    | Paired zeroing | Zeroing with PSD | Conventional zeroing | Paired zeroing |
| 1                               | 2                       | 1            | 2               | 0                   | –             |
| 2                               | 4                       | 2            | 2               | 50                  | 0             |
| 3                               | 6                       | 3            | 3               | 50                  | 0             |
| 4                               | 8                       | 4            | 3               | 62.5                | 25            |
| 5                               | 10                      | 5            | 5               | 50                  | 0             |
| 6                               | 12                      | 6            | 5               | 58.5                | 16.7          |
| 7                               | 14                      | 7            | 6               | 57.5                | 14.3          |
| 8                               | 16                      | 8            | 6               | 62.5                | 25            |
| 9                               | 18                      | 9            | 8               | 55.6                | 11.2          |
| 10                              | 20                      | 10           | 8               | 60                  | 20            |
| 11                              | 22                      | 11           | 9               | 59                  | 18            |
| 12                              | 24                      | 12           | 9               | 62                  | 25            |

5. Acknowledgments

The study was supported by Scientific and Production Enterprise “Oilwell Innovations of Ufa state petroleum technological university” within agreement No. 07-18n/10-2 as of 09.11.2018 on Increase of oil recovery from producing reservoirs with application of an implosion drill and gasdynamic reservoir fracturing charges (within agreement with Lukoil Perm Company as of 20.10.2018 No. 18z1773).

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