Shortest Path Problem Under Interval Valued Neutrosophic Setting

Said Broumi, Assia Bakali, Mohamed Talea, Florentin Smarandache, K. P. Krishnan Kishore, Rıdvan Şahin

Laboratory of Information Processing, Faculty of Science Ben M' Sik, University Hassan II, B.P 7955, Sidi Othman, Casablanca, Morocco

Ecole Royale Navale, Boulevard Sour Jdid, B.P 16303 Casablanca, Morocco.

Department of Mathematics, University of New Mexico, 705 Garley Avenue, Gallup, NM 87301, USA

Bharathiar University, Coimbatore, India. Pincode – 641 046

Department of Mathematics, Faculty of Education, Bayburt University, Bayburt, Turkey

broumisaaid78@gmail.com
assiabakali@yahoo.fr
taleamohamed@yahoo.fr
fsmarandache@gmail.com; smarand@unm.edu
kishore2982@gmail.com
mat.ridone@gmail.com

Published online: 5 March 2018

Abstract—This paper presents a study of neutrosophic shortest path with interval valued neutrosophic number on a network. A proposed algorithm also gives the shortest path length using ranking function from source node to destination node. Here each arc length is assigned to interval valued neutrosophic number. Finally, a numerical example has been provided for illustrating the proposed approach.

Keywords—interval valued neutrosophic graph, score function, Shortest path problem

I. INTRODUCTION

Neutrosophy was pioneered by Smarandache in 1998. It is a branch of philosophy which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. Smarandache generalized the concepts of fuzzy sets [28] and intuitionistic fuzzy set [25] by adding an independent indeterminacy-membership. Neutrosophic set is a powerful tool to deal with incomplete, indeterminate and inconsistent information in real world, which have attracted the widespread concerns for researchers. The concept of neutrosophic set is characterized by three independent degrees namely truth-membership degree (T), indeterminacy-membership degree (I), and falsity-membership degree (F). Later on, Smarandache extended the neutrosophic set to neutrosophic overset, underset, and offset [46]. From scientific or engineering point of view, the neutrosophic set and set-theoretic operator will be difficult to apply in the real application. Afterwards, various kinds of extended neutrosophic sets such as single valued neutrosophic sets, interval valued neutrosophic sets, simplified neutrosophic sets, bipolar neutrosophic sets and so on. The subclass of the neutrosophic sets called single-valued neutrosophic sets [14] (SVNS for short) was studied deeply by many researchers. The concept of single valued neutrosophic theory has proven to be useful in many different field such as the decision making problem, medical diagnosis and so on. Later on, the concept of interval valued neutrosophic sets [15] (IVNS for short) appear as a generalization of fuzzy sets, intuitionistic fuzzy set, interval valued fuzzy sets [20], interval valued intuitionistic fuzzy sets [26] and single valued neutrosophic sets. Interval valued neutrosophic set is a model of a
neutrosophic set, which can be used to handle uncertainty in fields of scientific, environment and engineering. This concept is characterized by the truth-membership, the indeterminacy-membership and the falsity-membership independently, which is a powerful tool to deal with incomplete, indeterminate and inconsistent information. The concept of interval valued neutrosophic sets is more precise and flexible than single valued neutrosophic sets. Recently, the interval valued neutrosophic sets have become an interesting research topic. Additional literature on single valued neutrosophic and interval valued neutrosophic sets can be found in [1, 5, 6, 7, 8, 11, 13, 17, 18, 19, 20, 21, 22, 23, 28, 29, 30, 32, 35, 38, 42, 44]. As a generalization of single valued neutrosophic set, the concept of interval valued neutrosophic was proposed and studied. An increasing number of studies have dealt with indeterminate and inconsistent problems by applying interval neutrosophic sets. Recently, the concept of single valued neutrosophic set was combined with graph theory and new graph model was presented. This concept is called single valued neutrosophic graph [34, 37, 39]. The single valued neutrosophic graph model allows the attachment of truth-membership (T), indeterminacy-membership (I) and falsity-membership degrees (F) both to vertices and edges. The single valued neutrosophic graph is the generalization of fuzzy graph and intuitionistic fuzzy graph. In addition, the concept of interval valued neutrosophic set was combined with graph theory and new graph model was presented. This concept is called interval valued neutrosophic graph. The concept of interval valued neutrosophic graph [36, 43] generalized the concept of fuzzy graph, intuitionistic fuzzy graph, interval valued fuzzy graph and single valued neutrosophic graph. Up to the present, research on single valued neutrosophic graph and interval valued neutrosophic graph has been recently studied.

The shortest path problem is a fundamental algorithmic problem, in which a minimum weight path is computed between two nodes of a weighted, directed graph. This problem has been studied for a long time and has attracted researchers from various areas of interests such operation research, computer science, communication network and so on. There are many shortest path problems [2, 3, 4, 12, 31, 45] that have been studied with different types of input data, including fuzzy set, intuitionistic fuzzy sets, trapezoidal intuitionistic fuzzy sets vague set. Till now, few research papers deal with shortest path in neutrosophic environment. Broumi et al. [40] proposed an algorithm for solving neutrosophic shortest path problem based on score function. The same authors in [41] proposed a study of neutrosophic shortest path with single valued trapezoidal neutrosophic number on a network. Two key issues need to be addressed in a way to handle neutrosophic path problem. One is how to determine the sum of two edges. The other is how to compare the lengths of two different paths given that the length of each edge is represented by neutrosophic numbers. Therefore, in this study we extend the proposed method for solving neutrosophic shortest path proposed by Broumi et al. [40] for solving interval valued neutrosophic shortest path problems in which the arc lengths of a network are represented by interval valued neutrosophic numbers.

The remainder of this paper is organized as follows. In Section 2, we review some basic concepts about neutrosophic sets, interval valued valued neutrosophic graph. In Section 3, an algorithm is proposed for finding the shortest path and shortest distance in interval valued neutrosophic graph. In Section 4 an illustrative example is provided to find the shortest path and shortest distance between the source node and destination node. Finally, in Section 5 we provide conclusion and proposal for further research.

II. PRELIMINARIES

In this section, some basic concepts and definitions on neutrosophic sets, interval valued neutrosophic graphs are reviewed from the literature.

Definition 2.1 [9-10]. Let X be a space of points (objects) with generic elements in X denoted by x; then the neutrosophic set A (NS A) is an object having the form A = \{< x: T_A(x), I_A(x), F_A(x) >, x ∈ X \}, where the functions T, I, F: X→\[0,1]\[0,1] define respectively the truth-membership function, an indeterminacy-membership function, and a falsity-membership function of the element x ∈ X to the set A with the condition:

\[0 ≤ T_A(x) + I_A(x) + F_A(x) ≤ 3.\]  (1)

The functions T_A(x), I_A(x) and F_A(x) are real standard or nonstandard subsets of \[0,1\].

Smarandache in 1998 and later Wang et al. [15] proposed the concept of INS, which is an instance of a neutrosophic set, and introduced the definition of an INS.

Definition 2.2 [15]. Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set A (INS A) in X is characterized by truth-membership function T_A(x), an indeterminacy-membership function I_A(x), and a falsity-membership function F_A(x).

For each point x in X, there are T_A(x) = [T^{L}_{A}, T^{U}_{A}] ⊆ [0, 1], I_A(x) = [I^{L}_{A}, I^{U}_{A}] ⊆ [0, 1] and F_A(x) = [F^{L}_{A}, F^{U}_{A}] ⊆ [0, 1], and the sum T_A(x), I_A(x) and F_A(x) satisfies the condition 0 ≤ sup T_A(x) + sup I_A(x) + sup F_A(x) ≤ 3, then , an INS A can be expressed as

A = \{< x: T_A(x), I_A(x), F_A(x) >, x ∈ X \}

= \{< x: [T^{L}_{A}(x), T^{U}_{A}(x)], [I^{L}_{A}(x), I^{U}_{A}(x)], [F^{L}_{A}(x), F^{U}_{A}(x)] >, x ∈ X \}  \]  (2)
Definition 2.3 [15]. An interval valued neutrosophic number 
\( \hat{A}_i = (T_i, I_i, F_i) \) is said to be zero interval valued neutrosophic number if and only if
\[
T_i^L = 0, T_i^U = 0, I_i^L = 1, I_i^U = 1, \text{ and } F_i^L = 1, F_i^U = 1 \tag{3}
\]
Definition 2.4 [33]. Let \( \hat{A}_1 = \left[ \left[ T_1^L, T_1^U \right], \left[ I_1^L, I_1^U \right], \left[ F_1^L, F_1^U \right] \right] \) and \( \hat{A}_2 = \left[ \left[ T_2^L, T_2^U \right], \left[ I_2^L, I_2^U \right], \left[ F_2^L, F_2^U \right] \right] \) be two interval valued neutrosophic numbers and \( \lambda > 0 \). Then, the operations rules are defined as follows;
(i) \( \hat{A}_1 \odot \hat{A}_2 = \left[ \left[ T_1^L + T_2^L, T_1^U + T_2^U \right], \left[ I_1^L + I_2^L, I_1^U + I_2^U \right], \left[ F_1^L + F_2^L, F_1^U + F_2^U \right] \right] \)
(ii) \( \hat{A}_1 \oplus \hat{A}_2 = \left[ \left[ T_1^L - T_2^L, T_1^U - T_2^U \right], \left[ I_1^L - I_2^L, I_1^U - I_2^U \right], \left[ F_1^L - F_2^L, F_1^U - F_2^U \right] \right] \)
(iii) \( \lambda \hat{A} = \left[ \left[ \lambda T_1^L, \lambda T_1^U \right], \left[ \lambda I_1^L, \lambda I_1^U \right], \left[ \lambda F_1^L, \lambda F_1^U \right] \right] \)

Definition 2.5 [33]. In order to make a comparisons between two IVNN, Rivan [33], introduced a concept of score function in 2014. The score function is applied to compare the grades of IVNS. This function shows that greater is the value, the greater is the interval-valued neutrosophic sets and by using this concept paths can be ranked. Let \( \hat{A}_i = (T_i, I_i, F_i) \) be an interval valued neutrosophic number, then, the score function \( s(\hat{A}_i) \), accuracy function \( a(\hat{A}_i) \) and certainty function \( c(\hat{A}_i) \) of an IVNN are defined as follows:

(i) \( s(\hat{A}_i) = \left( \frac{1}{4} \right) \left[ 1 + \left( T_i^L + T_i^U - 2I_i^L - 2I_i^U - F_i^L - F_i^U \right) \right] \tag{8} \)

(ii) Comparison of interval valued neutrosophic numbers
Let \( \hat{A}_1 = (T_1, I_1, F_1) \) and \( \hat{A}_2 = (T_2, I_2, F_2) \) be two interval valued neutrosophic numbers then
(i) \( \hat{A}_1 \preceq \hat{A}_2 \) if \( s(\hat{A}_1) \leq s(\hat{A}_2) \)
(ii) \( \hat{A}_1 \succeq \hat{A}_2 \) if \( s(\hat{A}_1) \geq s(\hat{A}_2) \)
(iii) \( \hat{A}_1 = \hat{A}_2 \) if \( s(\hat{A}_1) = s(\hat{A}_2) \)

Definition 2.6[36]. By an interval-valued neutrosophic graph of a graph \( (V, E) \) we mean a pair \( G = (A, B) \), where \( A = \left[ T_A^L, T_A^U \right], \left[ I_A^L, I_A^U \right], \left[ F_A^L, F_A^U \right] \) is an interval-valued neutrosophic set on \( V \), and \( B = \left[ T_B^L, T_B^U \right], \left[ I_B^L, I_B^U \right], \left[ F_B^L, F_B^U \right] \) is an interval-valued neutrosophic relation on \( E \) satisfying the following condition:

III. ALGORITHM OF INTERVAL VALUED NEUTROSOPHIC PATH PROBLEM
This algorithm illustrates the steps involved in finding the arc length in a network. This network considers the length of each arc as a neutrosophic number in intervals.

Step 1: Identify the first node length of the arc \( d_i = \min \{ d_i, d_j \} \) \( ; j = 2, 3, \ldots, \) and label the source node (say node 1) as \( d_1 = \min \{ 0, 0 \} \) and \( \{ 1, 1 \} \) respectively, and note that B is a symmetric interval valued neutrosophic relation on A.

Step 2: Find the minimum of the length of node 1 with its neighbor node as \( d_j = \min \{ d_i \odot d_j \} \) \( ; j = 2, 3, \ldots, n \).

Step 3: If the minimum occurs in the node corresponding to unique value of \( i \) \( (i.e., i = r) \), then label node \( j \) as \( \{ d_j, r \} \).

Step 4: If the minimum occurs in the node corresponding to more than one values of \( i \) then it shows that there are more than one interval valued neutrosophic path between source.
node i and node j but interval valued neutrosophic distance along path is $d_{ij}$, choose any value of i.

**Step 5:** Let the destination node (say node n) be labeled as $\hat{d}_n$. Then the interval valued neutrosophic shortest distance between source node is $\hat{d}_n$.

**Step 6:** With respect to $[\hat{d}_n, l]$ find the interval valued neutrosophic shortest path between source node and destination node and check the label of node l. Let it be $[\hat{d}_j, p]$. Check the label of node p and so on. Repeat the same procedure until node 1 is obtained.

**Step 7:** The interval valued neutrosophic shortest path can be obtained by combining all the nodes obtained by repeating the process in step 4.

**Remark 5.1** Let $\hat{A}_i$; i = 1, 2, ..., n be a set of interval valued neutrosophic numbers. If $S(\hat{A}_i) < S(\hat{A}_j)$, for all i, the interval valued neutrosophic number is the minimum of $\hat{A}_k$.

**IV. ILLUSTRATIVE EXAMPLE**

This example illustrates the procedure of finding the shortest distance and shortest path between source node and destination node on the network of a interval valued neutrosophic graph.

![Fig.1.](image)

In this network each edge have been assigned to interval valued neutrosophic number as follows:

| Edges | Interval valued Neutrosophic distance |
|-------|--------------------------------------|
| 1-2   | $[0.1, 0.2], [0.2, 0.3], [0.4, 0.5]$ |
| 1-3   | $[0.2, 0.4], [0.3, 0.5], [0.1, 0.2]$ |
| 2-3   | $[0.3, 0.4], [0.1, 0.2], [0.3, 0.5]$ |
| 2-5   | $[0.1, 0.3], [0.3, 0.4], [0.2, 0.3]$ |
| 3-4   | $[0.2, 0.3], [0.2, 0.5], [0.4, 0.5]$ |
| 3-5   | $[0.3, 0.6], [0.1, 0.2], [0.1, 0.4]$ |
| 4-6   | $[0.4, 0.6], [0.2, 0.4], [0.1, 0.3]$ |
| 5-6   | $[0.2, 0.3], [0.3, 0.4], [0.1, 0.5]$ |

Table 1. weights of the interval valued neutrosophic graphs

The computation of the shortest path based on the algorithm of interval valued neutrosophic path problem is shown below. Since node 6 is the destination node, so $n=6$.

Assume $d_i = [0, 1, 1]$ and label the source node (say node 1) as $[0, 1, 1, 1]$, the value of $d_j$; j = 2, 3, 4, 5, 6 can be obtained as follows:

**Iteration 1** Since only node 1 is the predecessor node of node 2, so putting i = 1 and j = 2 in step 2 of the proposed algorithm, the value of $d_2$ is

$d_2 = \min \{ \hat{d}_i \oplus \hat{d}_{ij} \} = \min \{ \langle 0, 0, 0 \rangle, \langle 1, 1, 1 \rangle \} = \langle 0.1, 0.2, 0.3 \rangle, \langle 0.2, 0.3, 0.4, 0.5 \rangle$ = $\langle 0.37, 0.52 \rangle, \langle 0.1, 0.2 \rangle, \langle 0.4, 0.5 \rangle$.

Since minimum occurs corresponding to i = 1, so label node 2 as $[0, 1, 0.2, 0.2, 0.3, 0.5, 0.4, 0.5] > 1$.

**Iteration 2** The predecessor node of node 3 are node 1 and node 2, so putting i = 1, 2 and j = 3 in step 2 of the proposed algorithm, the value of $d_3$ is

$d_3 = \min \{ \hat{d}_i \oplus \hat{d}_{ij} \} = \min \{ \langle 0, 0, 0 \rangle, \langle 1, 1, 1 \rangle \} = \langle 0.2, 0.3 \rangle, \langle 0.3, 0.5 \rangle, \langle 0.1, 0.2 \rangle, \langle 0.4, 0.5 \rangle$ = $\langle 0.2, 0.4, 0.3, 0.5 \rangle, \langle 0.1, 0.2 \rangle, \langle 0.37, 0.52 \rangle, \langle 0.02, 0.06 \rangle, \langle 0.12, 0.25 \rangle$.

Since minimum occurs corresponding to i = 1, so putting i = 1, 2 and j = 3 in step 2 of the proposed algorithm, the value of $d_3$ is

$d_3 = \min \{ \hat{d}_i \oplus \hat{d}_{ij} \} = \min \{ \langle 0, 0, 0 \rangle, \langle 1, 1, 1 \rangle \} = \langle 0.37, 0.52 \rangle, \langle 0.02, 0.06 \rangle, \langle 0.12, 0.25 \rangle$.

Since minimum occurs corresponding to i = 1, 2 and j = 3 in step 2 of the proposed algorithm, the value of $d_3$ is

$d_3 = \min \{ \hat{d}_i \oplus \hat{d}_{ij} \} = \min \{ \langle 0, 0, 0 \rangle, \langle 1, 1, 1 \rangle \} = \langle 0.37, 0.52 \rangle, \langle 0.02, 0.06 \rangle, \langle 0.12, 0.25 \rangle > 1$.

**Iteration 3**. The predecessor node of node 4 is node 3, so putting i = 3 and j = 4 in step 2 of the proposed algorithm, the value of $d_4$ is

$d_4 = \min \{ \hat{d}_i \oplus \hat{d}_{ij} \} = \min \{ \langle 0.2, 0.4 \rangle, \langle 0.3, 0.5 \rangle, \langle 0.1, 0.2 \rangle \} = \langle 0.2, 0.3, 0.4, 0.5 \rangle, \langle 0.4, 0.5 \rangle$ = $\langle 0.36, 0.58 \rangle, \langle 0.06, 0.25 \rangle, \langle 0.04, 0.1 \rangle$.

Since minimum occurs corresponding to i = 3, so putting i = 1, 2 and j = 4 in step 2 of the proposed algorithm, the value of $d_4$ is

$d_4 = \min \{ \hat{d}_i \oplus \hat{d}_{ij} \} = \min \{ \langle 0.2, 0.4 \rangle, \langle 0.3, 0.5 \rangle, \langle 0.1, 0.2 \rangle \} = \langle 0.36, 0.58 \rangle, \langle 0.06, 0.25 \rangle, \langle 0.04, 0.1 \rangle > 3$.

**Iteration 4** The predecessor node of node 5 are node 2 and node 3, so putting i = 2, 3 and j = 5 in step 2 of the proposed algorithm, the value of $d_5$ is

$d_5 = \min \{ \hat{d}_i \oplus \hat{d}_{ij} \} = \min \{ \langle 0.1, 0.2 \rangle, \langle 0.2, 0.3 \rangle, \langle 0.1, 0.4 \rangle \} = \langle 0.1, 0.2 \rangle, \langle 0.2, 0.3 \rangle, \langle 0.1, 0.4 \rangle$ = $\langle 0.36, 0.58 \rangle, \langle 0.06, 0.25 \rangle, \langle 0.04, 0.1 \rangle > 3$.

Since minimum occurs corresponding to i = 3, so putting i = 1, 2 and j = 5 in step 2 of the proposed algorithm, the value of $d_5$ is

$d_5 = \min \{ \hat{d}_i \oplus \hat{d}_{ij} \} = \min \{ \langle 0.1, 0.2 \rangle, \langle 0.2, 0.3 \rangle, \langle 0.1, 0.4 \rangle \} = \langle 0.1, 0.2 \rangle, \langle 0.2, 0.3 \rangle, \langle 0.1, 0.4 \rangle$ = $\langle 0.36, 0.58 \rangle, \langle 0.06, 0.25 \rangle, \langle 0.04, 0.1 \rangle > 3$.
algorithm, the value of \( \tilde{d}_5 \) is:

\[
\min \{ \tilde{d}_2 \oplus \tilde{d}_{25}, \tilde{d}_3 \oplus \tilde{d}_{35} \} = \min \{<0.1, 0.2>, [0.2, 0.3], [0.4, 0.5], \oplus \langle 0.1, 0.3, [0.3, 0.4], [0.2, 0.3], \rangle \}
\]

Since minimum occurs corresponding to \( i = 2 \), so label node 5 as \(<[0.19, 0.44], [0.06, 0.12], [0.08, 0.15]>, 2 \>

\( \tilde{d}_6 = <[0.19, 0.44], [0.06, 0.12], [0.08, 0.15]> \)

Since minimum occurs corresponding to \( i = 3 \), so label node 5 as \(<[0.36, 0.58], [0.03, 0.1]>, 5 \>

\( \tilde{d}_6 = <[0.19, 0.44], [0.06, 0.12], [0.08, 0.15]> \)

Since minimum occurs corresponding to \( i = 4 \), so label node 5 as \(<[0.36, 0.58], [0.03, 0.1]>, 5 \>

\( \tilde{d}_6 = <[0.19, 0.44], [0.06, 0.12], [0.08, 0.15]> \)

\( \tilde{d}_6 = <[0.35, 0.60], [0.01, 0.04], [0.008, 0.075]> \)

Table 2. Tabular representation of different interval valued neutrosophic shortest path

| Node No.(j) | \( \tilde{d}_i \) | Interval valued Neutrosophic shortest path between jth and 1st node |
|-------------|-----------------|---------------------------------------------------------------|
| 2           | \(<[0.1, 0.2], [0.2, 0.3], [0.4, 0.5]> \) | \( 1 \rightarrow 2 \) |
| 3           | \(<[0.2, 0.4], [0.3, 0.5], [0.1, 0.2]> \) | \( 1 \rightarrow 3 \) |
| 4           | \(<[0.36, 0.58], [0.06, 0.25], [0.04, 0.1]> \) | \( 1 \rightarrow 3 \rightarrow 4 \) |
| 5           | \(<[0.19, 0.44], [0.06, 0.12], [0.08, 0.15]> \) | \( 1 \rightarrow 2 \rightarrow 5 \) |
| 6           | \(<[0.35, 0.60], [0.01, 0.04], [0.008, 0.075]> \) | \( 1 \rightarrow 2 \rightarrow 5 \rightarrow 6 \) |

V. CONCLUSION

In this paper we developed an algorithm for solving shortest path problem on a network with interval valued neutrosophic arc lengths. The process of ranking the path is very useful to make decisions in choosing the best of all possible path alternatives. We have explained the method by an example with the help of a hypothetical data. Further, we plan to extend the following algorithm of interval neutrosophic shortest path problem in an interval valued bipolar neutrosophic environment.

ACKNOWLEDGMENT

The authors are very grateful to the chief editor and reviewers for their comments and suggestions, which is helpful in improving the paper.

REFERENCES

[1] A. Thamaraiselvi and R. Santhi, A New Approach for
Optimization of Real Life Transportation Problems in Neurotropic Environment, Mathematical Problems in Engineering, 2016, 9 pages.

[2] A. Ngoo and M. M. Jaraburulla, Multiple labeling Approach For Finding shortest Path with Intuitionistic Fuzzy Arc Length, International Journal of Scientific and Engineering Research, V3, Issue 11, pp.102-1062012

[3] A. Kumar, and M. Kaur, Solution of fuzzy maximal flow problems using fuzzy linear programming, World Academy of Science and Technology, 87, 28-31, (2011).

[4] A. B. Ansari, M. Kaur and H. Pratap, A New Algorithm for Solving Shortest Path Problem on a Network with Imprecise Edge Weight, Applications and Applied Mathematics, Vol. 6, Issue 2, 2011, pp. 602 – 619.

[5] A. Q. Ansari, R. Biswas & S. Aggarwal, “Neutrosophication of Fuzzy Models,” IEEE Workshop On Computational Intelligence: Theories, Applications and Future Directions (hosted by IIT Kanpur), 14th July 2013.

[6] A. Q. Ansari, R. Biswas & S. Aggarwal, “Extension to fuzzy logic representation: Moving towards neutrosophic logic - A new laboratory rat,” Fuzzy Systems (FUZZ), 2013 IEEE International Conference, 2013, pp.1 – 8.

[7] F. Smarandache, “Refined Limited Indeterminacy and the Multiplication Law of Sub-Indeterminacies,” Neutrosophic Sets and Systems, Vol. 9, 2015, pp.58.63.

[8] F. Smarandache, “Types of Neutrosophic Graphs and neutrosophic Algebraic Structures together with their Applications in Technology,” Universitatea Transilvania din Brasov, Facultatea de Design de Produs si Mediul, Brasov, Romania 06 June 2015.

[9] F. Smarandache, “Neutrosophic set - a generalization of the intuitionistic fuzzy set,” Granular Computing, 2006 IEEE International Conference, 2006, pp. 38 – 42.

[10] F. Smarandache, A geometric interpretation of the neutrosophic set — A generalization of the intuitionistic fuzzy set, Granular Computing (GrC), 2011 IEEE International Conference, 2011, pp.602 – 606.

[11] G. Garg, K. Bhutani, M. Kumar and S. Aggarwal, “Hybrid model for medical diagnosis using Neurotropic Cognitive Maps with Genetic Algorithms,” FUZZ-IEEE, 2015, 6page.

[12] G. Kumar, R. K. Bajaj and N. Gandomotra, “Algorithm for shortest path problem in a network with interval valued intuitionistic trapezoidal fuzzy number, Procedia Computer Science 70,2015, pp.123-129.

[13] H. Zhang, Y. Zhang, R. Sunderraman, “Truth-value based interval neutrosophic sets,” Granular Computing, 2005 IEEE International Conference, vol.1, 2005 pp. 274 – 277.

[14] H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, “Single valued Neutrosophic Sets,” Multispace and Multistructure, 2010, pp. 410-413.

[15] H. Wang, F. Smarandache, Zhang, Y.-Q. and R. Sunderraman, “Interval Neutrosophic Sets and Logic: Theory and Applications in Computing,” Hexit, Phoenix, AZ, 2005.

[16] H.J. Zimmermann, Fuzzy Set Theory and its Applications, Kluwer-Nijhoff, Boston, 1985.

[17] I. Deli, M. Ali, F. Smarandache, “Bipolar neutrosophic sets and their application based on multi-criteria decision making problems,” Advanced Mechatronic Systems (ICAMechs), 2015 International Conference, 2015, pp. 249 – 254.

[18] I. Deli, S. Yusuf, F. Smarandache and M. Ali, Interval valued bipolar neutrosophic sets and their application in pattern recognition, IEEE World Congress on Computational Intelligence 2016.

[19] I. Deli and Y. Subas, A Ranking methods of single valued neutrosophic numbers and its application to multi-attribute decision making problems, International Journal of Machine Learning and Cybernetics, 2016, 1-14.

[20] I. Turksen, Interval valued fuzzy sets based on normal forms, Fuzzy Sets and Systems, vol. 20, 1986, pp. 191-210.

[21] J. Ye, “Single-Valued Neutrosophic Minimum Spanning Tree and Its Clustering Method” Journal of Intelligent Systems 23(3), 2014, pp. 311–324.

[22] J. Ye, Interval Neutrosophic Multiple Attribute Decision-Making Method with Credibility Information, International Journal of Fuzzy Systems, pp 1-10, 2016.

[23] J. Ye, Trapezoidal fuzzy neutrosophic set and its application to multiple attribute decision making, Neural Computing and Applications, 2014. DOI 10.1007/s00521-014-1787-6.

[24] K. Atanassov, “Intuitionistic fuzzy sets,” Fuzzy Sets and Systems, vol. 20, 1986, pp. 87-96.

[25] K. Atanassov, “Intuitionistic fuzzy sets: theory and applications.” Physica, New York, 1999.

[26] A. Kumar and H. Pratap, “Interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems, vol.31, 1989, pp. 343-349.

[27] L. Zadeh, Fuzzy sets. Inform and Control, 8, 1965, pp.338-353

[28] M. Ali, and F. Smarandache, “Complex Neutrosophic Set,” Neural Computing and Applications, Vol. 25, 2016, pp.1-18.

[29] M. Ali, I. Deli, F. Smarandache, “The Theory of Neutrosophic Cubic Sets and Their Applications in Pattern Recognition,” Journal of Intelligent and Fuzzy Systems, (In press), pp. 1-7.

[30] P. Biswas, S. Pramanik and B. C. Giri, Cosine Similarity Measure Based Multiple-attribute Decision-Making with Trapezoidal Fuzzy Neutrosophic numbers, Neutrosophic sets and systems, 8,2014,47-57.

[31] P. Jayagowri and G. Geetha Ramani, Using Trapezoidal Intuitionistic Fuzzy Number to Find Optimized Path in a Network, Volume 2014, Advances in Fuzzy Systems, 2014, 6 pages.

[32] R. Sahin, “Neutrosophic hierarchical clustering algorithms,” Neutrosophic Sets and Systems, vol. 2, 2014, 18-24.

[33] R Şahin and PD Liu (2015) Maximizing deviation method for neutrosophic multiple attribute decision making with incomplete weight information, Neural Computing and Applications, DOI: 10.1007/s00521-015-1995-8

[34] S. Broumi, M. Talea, A. Bakali, F. Smarandache, Single Valued Neutrosophic Graphs, Journal of New Theory, N 10, 2016, pp.86-101.

[35] S. Broumi, M. Talea, A. Bakali, F. Smarandache, “On Bipolar Single Valued Neutrosophic Graphs,” Journal of New Theory, N11, 2016, pp.84-102.

[36] S. Broumi, M. Talea, A. Bakali, F. Smarandache, Interval Valued Neutrosophic Graphs, Critical Review, XII, 2016, pp.5-33.

[37] S. Broumi, A. Bakali, M. Talea, and F. Smarandache, Isolated Single Valued Neutrosophic Graphs. Neutrosophic Sets and Systems, Vol. 11, 2016, pp.74-78

[38] S. Broumi, F. Smarandache, M. Talea and A. Bakali, An Introduction to Bipolar Single Valued Neutrosophic Graph Theory. Applied Mechanics and Materials, vol.841.2016, 184-191.

[39] S. Broumi, M. Talea, F. Smarandache and A. Bakali, “Single Valued Neutrosophic Graphs: Degree, Order and Size,” IEEE International Conference on Fuzzy Systems (FUZZ),2016,pp.2444-2451.

[40] S. Broumi, A. Bakali, M. Talea, F. Smarandache and L. Vladareanu, Computation of Shortest Path Problem in a Network with SV-Trapezoidal Neutrosophic Numbers, Proceedings of the 2016 International Conference on Advanced Mechatronic Systems, Melbourne, Australia, 2016, pp.417-422.

[41] S. Broumi, A. Bakali, M. Talea, F. Smarandache and L. Vladareanu, Applying Dijkstra Algorithm for Solving Neutrosophic Shortest Path Problem, Proceedings of the 2016 International Conference on Advanced Mechatronic Systems, Melbourne, Australia, 2016, pp.412-416.

[42] S. Broumi, F. Smarandache, “New distance and similarity measures of interval neutrosophic sets,” Information Fusion (FUSION), 2014 IEEE 17th International Conference, 2014, pp. 1 – 7.

[43] S. Broumi, F. Smarandache, M. Talea and A. Bakali, Decision-Making Method Based On the Interval Valued Neutrosophic Graph, Future Technology, 2016, IEEE, pp.44-50.

[44] Y. Subas, Neutrosophic numbers and their application to multi-attribute decision making problems (in Turkish), Master Thesis, 7 Arak university, Graduate School of Natural and Applied Science, Turkey, 2015.

[45] S. Majumdar and A. Pal, Shortest Path Problem on Intuitionistic Fuzzy Network, Annals of Pure and Applied Mathematics, Vol. 5, No. 1, November 2013, pp. 26-36.
[46] Florentin Smarandache, Neutrosophic Overset, Neutrosophic Underset, and Neutrosophic Offset. Similarly for Neutrosophic Over-/Under-/Off- Logic, Probability, and Statistics, 168 p., Pons Editions, Bruxelles, Belgique, 2016; https://arxiv.org/ftp/arxiv/papers/1607/1607.00234.pdf.