Exact diagonalization study of domain structure in integer filling factor quantum Hall ferromagnets

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Opposite spin Landau levels in a quantum well can be brought into coincidence by tilting the magnetic field away from normal orientation. We demonstrate that the magnetotransport anomaly at integer filling factors that was recently discovered by Pan et al is due to such a coincidence. By performing exact diagonalization calculations using microscopically evaluated effective electron-electron interactions, we are able to establish that the electronic ground state at coincidence is an Ising quantum Hall ferromagnet and that the low energy excitations correspond to the formation of a domain wall.

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A magnetic filed component parallel to the plane of a quasi 2D electron gas can trigger transitions between correlated many-body states in the quantum Hall regime. For example, transport anisotropies induced \[ \nu = 5/2 \] and \[ \nu = 7/2 \] have been described theoretically \[ \nu = 9/2 \] and \[ \nu = 11/2 \], using exact diagonalization techniques, as arising from a transition between an incompressible paired quantum Hall state and a compressible stripe phase. In a separate tilted-field magnetotransport experiment Pan et al. \[ \nu = 9/2 \] and \[ \nu = 11/2 \] discovered, at \[ \nu = 9/2 \] and \[ \nu = 11/2 \], transitions, at critical tilt angles, between isotropic Fermi liquid states and anisotropic stripe states. An understanding of transitions like these that occur at large tilt angles can be achieved only by accounting for the large changes in Landau level wavefunctions caused by the tilts. The transitions studied by Pan et al. were explained \[ \nu = 9/2 \] and \[ \nu = 11/2 \] by combining many-body RPA/Hartree-Fock theory with self-consistent local-spin-density-functional (LSDA) descriptions of one-particle states in the experimental sample geometry \[ \nu = 9/2 \] and \[ \nu = 11/2 \]. Crudely described, the effect of the in-plane field in these experiments is to produce a crossing between a valence Landau level at small tilts, which is the lowest quantum well kinetic energy eigenstate of the second subband \( (N = 0, i = 2) \), and a Landau level that is a higher quantum kinetic energy eigenstate of the first subband \( (N = 2, i = 1) \). Stripe states occur only when the half-filled valence Landau level has dominant \( N = 2 \) character.

In this paper we address the magnetotransport anomaly \[ \nu = 9/2 \] observed in the same quantum well sample \[ \nu = 9/2 \] at integer filling factors for which the stripe states that are believed to be responsible for transport anisotropy in half-filled Landau levels have not yet been observed \[ \nu = 9/2 \]. By performing detailed quantum well electronic structure calculations we are able to establish that the experimental anomalies occur at the point where the \( (N = 2, i = 1) \) and the \( (N = 0, i = 2) \) Landau levels cross. We find that the ground state at the point of coincidence is always a uniform density Ising quantum Hall ferromagnet \[ \nu = 9/2 \], not a stripe state. We discuss possible sources of dissipation and anisotropy in the finite-temperature transport in these states, in particular the role of the domain wall excitations \[ \nu = 9/2 \].

The sample studied by Pan et al. \[ \nu = 9/2 \] contains a 35 nm wide GaAs quantum well with 2D electron density \( 4.2 \times 10^{11} \) cm\(^{-2} \). Due to the relatively large thickness of the quasi 2D system and the high electronic density, the orbital effects of the in-plane magnetic field, \( B_\parallel \), play an especially important role in determining many-body ground and excited state properties. In Fig.\[ \nu = 9/2 \] we plot single-particle Landau level spectra as a function of the magnetic field tilt \( \alpha \) \( (B_\parallel = B \sin \alpha) \), calculated using the self-consistent LSDA at filling factor \( \nu = 6 \). At \( \alpha = 0 \), the highest occupied energy level is the spin-down, \( N = 0 \) Landau level of the second subband \( (i = 2) \), and the lowest empty states are spin-up quasiparticles with indices \( N = 2 \) and \( i = 1 \). For non-zero in-plane fields, \( N \) and \( i \) are not good quantum numbers and the orbital part of the eigenfunction is described by a single index \( n \). The gap between the \( n = 3 \) level (emerging from the \( N = 0, i = 2 \) Landau level) and the \( n = 4 \) level (emerging from the \( N = 2, i = 1 \) Landau level) decreases with increasing \( B_\parallel \), eventually leading to coincidence at a high tilt angle. Note that the \( B_\parallel \)-enhancement of the Zeeman gap plays a minor role in bringing the two levels into the coincidence. In the LSDA, self-consistency pins the two levels close together over a finite range of \( \alpha \), hinting at the important role of correlations that are not captured by what is effectively a mean-field approximation for this system. As seen in Fig.\[ \nu = 9/2 \] the two levels remain nearly degenerate even at tilt angles as large as 84°. The ob-
ervation supports our contention of the close relation between the quantum Hall ferromagnet physics and the measured transport anomaly which, after an abrupt onset around tilt angle $82^\circ$, remains nearly unchanged up to the highest measured tilt angle $\alpha = 84.4^\circ$.

Within Hartree-Fock theory, the ground state of a quantum Hall system at integer filling factors with two levels near the Fermi energy can be described by a Landau energy function $e_{HF}(n) = -bm_z - U_{zz}m_z^2$. In our case, pseudospin $m_z = -1$ corresponds to full occupation of quasiparticle states in the $n=3$ spin-down level and $m_z = +1$ corresponds to full occupation of the $n=4$ spin-up Landau level. The sign and magnitude of $U_{zz}$ can be evaluated from the microscopic wavefunctions of the crossing Landau levels. We find that $U_{zz}>0$ in the present case, implying that the quantum Hall ferromagnet has easy-axis (Ising) anisotropy in the Hartree-Fock approximation. Numerical exact diagonalization studies discussed below confirm that near the level crossing ($b \approx 0$) the ground state remains fully pseudospin polarized and uniform.

In an effort to achieve a more quantitative and confident understanding of the collective behavior of electrons in this quantum Hall system, we used the self-consistent LSDA one-particle states, obtained numerically for the specific and structural parameters of the sample to construct realistic effective interaction potentials. For the $\nu = 6$ quantum Hall ferromagnet we obtained two intra and one inter Landau level potentials for electronic states in the crossing levels. The influence of the five remote, fully occupied Landau levels was captured perturbatively using the RPA dielectric function. The pseudo-spin dependent effective interaction potentials can be approximated by $V^{s,s'}(q) = V_0^{s,s'}(q) + V_2^{s,s'}(q) \cos(2\phi)$ where $s,s'=\uparrow$ or $\downarrow$ are pseudospin labels, $q$ is the wave vector magnitude, and $\phi$ is the wave vector orientation relative to the in-plane field direction. At $B_\perp = 0$ the pseudospin dependent isotropic effective interactions $V_0^{s,s'}(q)$, shown in the left panels of Figs. (a)-(c), have a wave vector dependence similar to those of the effective interactions in infinitely narrow 2D layers. Due to in-plane field induced mixing between electric and magnetic levels in the quasi 2D system, the $q$-dependence of $V_0^{s,s'}(q)$ changes with increasing magnetic field tilt angle and the effective interactions becomes anisotropic, as seen from the non-zero $V_2^{s,s'}(q)$ coefficients plotted in the right panels of Figs. (a)-(c).

The many-body Hamiltonian with the LSDA/RPA effective interaction potentials was diagonalized numerically to obtain the exact spectrum in a finite size system of up to twelve electrons in the torus geometry. This geometry is well suited for detecting broken translational symmetry phases. The calculation of the low lying spectrum (for example as a function of the 2-D wavevector $K\parallel$) amounts to a “numerical crystallography” in these phases. In the case of stripes, the degeneracy $N_D$ of the ground state manifold corresponds to the number of electrons per stripe and their $K$-vectors form a 1-D array in reciprocal space with $\Delta K_{ij} = K_i - K_j = nQ$, where $n = 1,2,\ldots N_D-1$, $i,j = 1,2,\ldots N_D$, and $\lambda = 2\pi/|Q|$ is the wavelength of the stripes. Because of this property, stripe ground states are readily identified in finite-size exact diagonalization calculations. In a finite size system with $N_e$ electrons and periodic boundary conditions, the interaction potential $V^{s,s}(q_x,q_y)$ is only required at the $N_e^2$ values of $q$, where $N_e$ is the number of flux quanta. We obtain these by interpolating the LSDA/RPA potential which is calculated at a discrete set of $q$ values that do not in general coincide with the PBC values. We carried out our calculations for tilt angles $\alpha = 0,20,40,60$, and 80 degrees in a rectangular unit cell while we varied the aspect ratio. As shown previously, this helps to reveal any potential broken symmetry states. In all calculations we have restricted the Hilbert space to states within the two Landau levels of interest. (Processes which change the Landau level index of an electron can be neglected since they require reversal of an electron spin.) The eigenstate energies reported on below are in units of $e^2/4\pi\epsilon_0\ell_\perp$, where $\ell_\perp$ corresponds to the magnetic length associated with the out-of-plane component of the magnetic field $B_\perp$.

For all tilt angles $\alpha$ and in every unit cell geometry, we find that the ground state is a fully pseudospin polarized uniform density state. For every $\alpha$ we fixed $\Delta_{\perp}$, the Zeeman gap, so that the two polarized ground states are degenerate, i.e. we have set the effective pseudospin field to zero. The entire spectrum is then $Z_2$ symmetric as required for an Ising ferromagnet. In the experiments of Pan et al the degeneracy occurs for only a single $\alpha$ (about 82 degrees). By fixing $\Delta_{\perp}$ in this way we have in effect studied the Ising ferromagnet, if it were to occur, at a general tilt angle $\alpha$. We find only quantitative differences between different tilt angles; the generic behavior is independent of any particular value of $\alpha$ and is present even at zero tilt. While the ground state remains the uniform density filled level, we can identify low lying states over a substantial range of aspect ratios that correspond to the formation of a domain wall. (These states have an excitation energy proportional to the linear size of the system and are therefore not low-lying elementary excitations in the thermodynamic limit.) We did not find any stripe order, either in the ground state or in the low-lying excitations. Fig. (b) shows how the low lying excitations for a tilt angle of 80 degrees change as a function of the aspect ratio for a 12 electron system with 6 reversed spins. Here, the long direction is along the $y$-axis and the domain wall is always along the $x$-axis (short direction). Fig. (c) corresponds to an in-plane field along the $x$-direction. The lowest lying manifold has degeneracy $N_D = 12 = N_e = N_\phi$ with wavevectors...
(0, 2mπ/L), where L is the length of the long side and m = 0, 1, 2, . . . , N − 1. This signifies a phase separated state with a domain wall along the short side of the unit cell [14].

The domain wall energy per unit length for N = 8, 10, and 12 electrons is shown in Fig. 4 which corresponds to B|| applied along the x-axis and the magnetic field tilt angle of 80 degrees. For other tilt angles we obtain similar results, except that the energy per unit length of the domain wall is consistently reduced as the tilt angle is increased. Our domain wall energy calculations also confirm that the in-plane component of the magnetic field induces an anisotropy in the domain structure of this Ising ferromagnet. Due to finite size effects, however, we were unable to conclusively establish whether the parallel orientation or perpendicular orientation of the domain wall is energetically more favorable for the studied quantum Hall system. This issue can be addressed separately, e.g. within a self-consistent Hartree-Fock theory, now that the Ising quantum Hall ferromagnet nature of the many-body system has been established by our exact diagonalization study. This calculation, however, is beyond the scope of this paper.

In concluding we discuss the relationship between our theoretical findings for the one-particle and many-body energy spectra and the measured transport anomalies. In a previous study [10], we proposed that enhanced dissipation would occur in Ising quantum Hall ferromagnets as a result of charge diffusion along domain walls. If this transport mechanism dominated in the sample studied by Pan et al. [6], a peak in the longitudinal resistivity would occur when the system breaks into domains with opposite pseudospin orientations, i.e., near the Landau level coincidence [10]. Our finding that the n = 3 spin-down and n = 4 spin-up Landau levels are nearly degenerate for tilt angles exceeding α > 81°, is therefore consistent with this interpretation of the experiments. The dissipation of the quasiparticle current in a system with an anisotropic domain structure could provide a mechanism for the measured transport anisotropy in this picture. In the measured sample, the in-plane field direction is the high resistivity direction so, intuitively, the domain walls are oriented perpendicular to the in-plane field direction. Measurements of the temperature dependence of the transport anomaly in samples with different in-plane magnetic field orientations relative to the crystal axes and corresponding microscopic calculations, mentioned in the previous paragraph, may provide useful information on how the anisotropy of tilted field quantum Hall ferromagnets is expressed in transport experiments [15] and on the combined effect of the in-plane magnetic field and disorder on the preferred orientation of the domain walls.

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FIG. 1: Self-consistent LSDA Landau level diagram plotted as a function of the magnetic field tilt angle. The numerical data are obtained for $\nu = 6$ and sample parameters of Ref. [6] for tilt angles indicated by filled circles. (Lines are plotted to guide the eye.) Solid (dashed) lines correspond to spin-up (spin-down) levels. The thick lines simultaneously approach the chemical potential at $\alpha = 81^\circ$.

FIG. 2: Isotropic term (left panels) and the anisotropy coefficient (right panels) of the effective 2D Coulomb interaction multiplied by the wave vector amplitude $q$ at different magnetic field tilt angles. a) Intra-level potentials for the $n = 3$ ($i = 2, N = 0$ for $\alpha = 0$) spin-down Landau level. b) Intra-level potentials for the $n = 4$ ($i = 1, N = 2$ for $\alpha = 0$) spin-up Landau level. c) Inter-level potentials.
FIG. 3: Low-lying spectrum for 12 particles with 6 reversed spins at total filling factor one as a function of aspect ratio at tilt angle of 80 degrees. The energy of the Ising ($Z_2$) ground state has been subtracted out. The lowest set of quasi-degenerate levels signify the formation of a domain wall between the two ground states. The domain wall begins to develop for aspect ratios greater than 1.2.

FIG. 4: The energy of the domain wall per unit length for the same system as in Fig. 3. Here $B_\parallel$ is along the x-axis. We have only included aspect ratios for which the wall is well formed (see Fig. 3).