Fluid forces or impacts, what governs the entrainment of soil particles in sediment transport mediated by a Newtonian fluid?

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In steady sediment transport, sediment deposition is balanced by entrainment through fluid forces or particle-bed impacts. Both entrainment modes leave signatures in the vertical profile of the average horizontal particle velocity \(\langle v_z \rangle(z)\). Measurements of \(\langle v_z \rangle(z)\) for aeolian sediment transport (‘saltation’) have indicated an approximately constant value in natural units when extrapolated to the vertical location of the bed surface \(z_r\), which is widely recognized as a proxy evidencing that the transport is fully sustained through impact entrainment. In contrast, subaqueous sediment transport (‘bedload’) is thought to be fully sustained through fluid entrainment, and one thus expects a scaling of \(\langle v_z \rangle(z)\) with the local mean fluid velocity near \(z_r\). Here we use the numerical model of Durán et al. (Phys. Fluids \textbf{24}, 103306, 2012), which couples the Discrete Element Method for the particle motion with a continuum Reynolds-averaged description of hydrodynamics, to simulate steady sediment transport for a wide range of the particle-fluid density ratio \(s = \rho_p/\rho_f\), particle Reynolds number \(Re_p = \sqrt{(s-1)gd^3/\nu}\), and Shields number \(\Theta = \tau/(\rho_p - \rho_f)gd\), where \(\tau\) is the fluid shear stress, \(g\) the gravitational constant, \(d\) the mean particle diameter, and \(\nu\) the kinematic viscosity. These simulations show that the mode of entrainment is better characterized by the ‘bed velocity’ \(V_b\), which is obtained from averaging \(\langle v_z \rangle(z)\) over elevations near \(z_r\). We find transport is fully impact-sustained (i.e., \(V_b/\sqrt{gd} \approx \text{const.}\), where \(g = (s + 0.5)g/(s - 1)\) when either the ‘impact number’ \(Im = Re_p\sqrt{s + 0.5} \geq 20\) or \(\Theta \geq 5/\text{Im}\). These conditions are obeyed for the vast majority of transport regimes, including steady turbulent bedload, which has long been thought to be dominated by fluid entrainment. In fact, we find transport is fully fluid-sustained only for sufficiently viscous (at grain-scale) bedload (i.e., for \(Im \lesssim 20\) and \(\Theta \lesssim 1/\text{Im}\), where \(V_b/\sqrt{gd} \propto \text{Im}\)). Finally, we do not find a strong correlation between \(V_b\) or \(\langle v_z \rangle(z_r)\) and the transport-layer-averaged horizontal particle velocity \(\overline{v_x}\), which challenges the long-standing consensus that predominant impact entrainment is responsible for a linear scaling of the transport rate with \(\Theta\). For turbulent bedload in particular, \(\overline{v_x}\) increases with \(\Theta\) despite \(V_b\) remaining constant, which we propose is linked to the formation of a liquid-like bed on top of the static-bed surface.

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\section{I. INTRODUCTION}

Sediment transport in a Newtonian fluid, such as water and air, is one of the most important geological processes responsible for the alteration of sea and riverscapes, and dry planetary surfaces [1–12]. It can occur in a large variety of natural environments, e.g., viscous and turbulent transport of minerals and organics by Earth’s water streams [1–5], turbulent transport of dust and sand by Earth’s atmospheric winds [7–11], and wind-driven transport of minerals on planetary bodies with extremely thin atmospheres, such as Triton, Pluto, and comet 67P/Churyumov-Gerasimenko [12].

Different sediment transport regimes are documented. Very small particles, whose weight can be fully supported by the fluid turbulence, tend to be transported in turbulent suspensions [5, 10]. Medium and large particles, on the other hand, are transported close to the surface, in trajectories not much influenced by fluid turbulence [5, 10]. The latter case includes bedload and saltation transport. Bedload transport refers to particles rolling, sliding, and hopping in the vicinity of the sediment bed, which is typical for the transport of sand and gravel by water streams [13, 14]. Saltation transport refers to particles moving in ballistic trajectories along the bed, which is typical for the transport of sand by planetary winds [9–11].

It has become a widely-accepted hypothesis that the mechanisms sustaining bedload and saltation transport are fundamentally different: bedload transport being sustained through entrainment of soil bed particles directly by fluid forces [14–35] and saltation transport being sustained through particle-bed impacts ejecting bed particles [36–42], allowing transport even below the fluid entrainment threshold [9–12, 43–47]. However, recent studies have questioned this hypothesis by pointing out the role of particle inertia for sustaining bedload transport [48, 49], and by showing that an analytical model taking

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into account particle dynamics is able to reproduce measurements of bedload and saltation transport thresholds at the same time [12].

Here we study the relevance of impact entrainment relative to direct fluid entrainment in a unified manner using direct numerical simulations of sediment transport in a Newtonian fluid with the model of Ref. [50], which belongs to a new generation of sophisticated grain-scale models of sediment transport [9, 12, 48, 50–61] and has been shown to reproduce many observations concerning viscous and turbulent sediment transport in air and water [9, 12, 50, 53] (e.g., see Fig. 6 of Ref. [53] and Extended Data Figs. 2, 3 of Ref. [12]), and bedform formation [54]. First, we present direct evidence from visual inspection of these simulations showing that impact entrainment events play a crucial role during both steady bedload and saltation transport. Then we quantitatively analyze the relative role of impact entrainment from our simulation data using a proxy that is similar but not identical to the classical ‘slip velocity’ (i.e., the average horizontal particle velocity at the bed surface [9, 44–47, 62–66]). This analysis reveals a crucial influence of a novel dimensionless number, henceforth called ‘impact number’, $\text{Im} = \text{Re}_p \sqrt{s + 0.5}$, where $s = \rho_p/\rho_f$ is the particle-fluid-density ratio and $\text{Re}_p = \sqrt{(s - 1)gd^3/\nu}$ the particle Reynolds number, with $g$ the gravitational constant, $d$ the mean particle diameter, and $\nu$ the kinematic viscosity. Finally, we shed light on possible links between impact entrainment and average transport characteristics, such as the scaling of the sediment transport rate $Q$ with the dimensionless fluid shear stress (the “Shields number” $\Theta = \tau_p/((\rho_p - \rho_f)gd)$). In fact, it is a widespread believe that impact entrainment inevitably causes a (nearly) linear scaling $Q \propto \Theta - \Theta_{ex}^s$, where $\Theta_{ex}^s$ is the extrapolated value of $\Theta$ at which $Q$ vanishes [38, 43–47, 62, 67]. Here we challenge this believe.

The reminder of the manuscript is organized as follows. Section II briefly summarizes important details of the numerical model. Section III presents and discusses the evidence for impact entrainment in both bedload and saltation transport obtained from visualizations of the numerical simulations. Section IV represents the core of the paper. It introduces the proxy we use to quantify the relative role of impact entrainment, explains why other proxies, such as the slip velocity, are inappropriate, and analyzes our proxy over the entire range of simulated conditions. Section V discusses possible links between impact entrainment and average transport characteristics. Finally, we discuss our results and draw conclusions in Section VI.

### II. NUMERICAL SIMULATIONS

The numerical model of sediment transport in Newtonian fluids of Ref. [50] couples a Discrete Element Method for the particle motion ($\approx 15000$ spheres, including $> 10$ layers of sediment bed particles) and a continuum Reynolds-averaged description of hydrodynamics. The Reynolds-Averaged Navier-Stokes equations are combined with an improved mixing length approximation, which can be used to calculate the turbulent mean fluid velocity at high particle concentrations. In contrast to the original model, which considers only gravity, buoyancy, and fluid drag forces acting on particles, we here also consider the added-mass force [9]. However, cohesive and higher-order fluid forces, such as the hindrance and lift force remain neglected. We also corrected two slight inaccuracies in the original model (with mostly negligible effect on the simulation outcome). We here take into account that the fluid shear stress is proportional to the fluid volume fraction and neglect the buoyancy contribution from the divergence of the fluid shear stress because the divergence of the Reynolds stress, previously considered, actually does not contribute to the buoyancy force.

We would also like to emphasize that all results presented in this study for the bedload transport regime usually do not significantly depend on contact parameters, such as the restitution coefficient $e$ and contact friction coefficient $\mu_c$. For instance, bedload transport simulations with $e = 0.9$ and $e = 0.01$ are nearly exactly the same on average, which is consistent with previous reports [60]. This finding implies that any dissipative interaction force that is proportional to the relative velocity of two approaching particles and acts at and/or close to particle contact, such as the lubrication force [69], does not significantly influence average bedload transport characteristics because the effect of such forces can be incorporated in $e$ and $\mu_c$ [57, 60, 70, 71].
We carried out simulations for $s$ and $Re_p$ within the range $s \in [2.65, 2000]$ and $Re_p \in [0.1, 100]$ and for $s = 10^7$ and $Re_p = 0.1$. For each pair of $s$ and $Re_p$, we varied $\Theta$ in regular intervals and obtained the cessation threshold ($\Theta^{cs}$) through extrapolation to vanishing sediment transport [50].

### III. VISUALIZATIONS OF IMPACT ENTRAINMENT EVENTS

Figure 1 shows snapshots of transport simulations near $\Theta^{cs}$ for turbulent bedload (Fig. 1a) and saltation transport (Fig. 1b) shortly before and after an impact entrainment event (see also Movies S1 and S2). In both cases, a transported particle collides with one or more bed particles, which subsequently become mobilized with a short time delay. In bedload transport, the transported particle impacts the bed from a very small height and entrains a bed particle by dragging it out of its trap, while in saltation transport, the transported particle impacts the bed from a very large height and entrains a bed particle by ejecting it. Close to the threshold, impact entrainment events are rare in both cases as impacts occur much less often, but are much more effective in saltation transport, the transported particle impacts the bed load from a very large height and entrains a bed particle by ejecting it. Close to the threshold, impact entrainment events are rare in both cases as impacts occur much less often, but are much more effective in saltation transport (Movies S1 and S2). As a consequence, bed particles remain in repose most of the time because fluid forces are too weak to entrain them directly. Sufficiently far from the threshold, impact entrainment events occur much more often in saltation transport (Movie S3), whereas it is impossible to determine single entrainment events in turbulent bedload transport as several layers of the bed are in continuous motion (Movie S4). Note that movie captions are provided in Appendix A.

### IV. PROXY FOR RELEVANCE OF IMPACT ENTRAINMENT RELATIVE TO FLUID ENTRAINMENT

The finding from the previous section that impact entrainment events play crucial roles during both turbulent bedload and saltation transport highlights the need for a proxy conveying information about the relevance of impact entrainment relative to fluid entrainment. Here we discuss two potential proxies, which we obtain from the vertical profile of the average horizontal particle velocity $\langle v_x \rangle (z)$ (for the precise definition of the notation $\langle \cdot \rangle$, see Appendix B1). Section IV A discusses the standard proxy, the slip velocity $\langle v_x \rangle (z_r)$, where the location $z_r$ of the bed surface is here defined as the effective location of high-energy particle-bed rebounds (Appendix B3), and why it is inappropriate for our purposes. Therefore, Section IV B proposes and quantitatively analyzes a novel proxy, the ‘bed velocity’.

#### A. Slip velocity

Figure 2 shows the vertical profiles of (a) $\langle v_x \rangle / \sqrt{gd}$ and (b) $\sqrt{\langle v_x^2 \rangle / (gd)}$ relative to $z_r$ for the entire simulated range of $s$ and $Re_p$, and a value of $\Theta$ that is near the associated threshold $\Theta^{cs}(s, Re_p)$, where $\hat{g} = (s + 0.5)g/(s - 1)$.
is the value of $g$ reduced by the buoyancy and added-mass force. It can be seen that there is a very rough tendency of $\langle v_x \rangle / \sqrt{gd}$, but not of $\sqrt{\langle v_x^2 \rangle / (gd^2)}$, to collapse near $z_r$ when the impact number $Im \gtrsim 20$ (open symbols). In the Aeolian Research community, a constant value of $\langle v_x \rangle (z_r) / \sqrt{gd}$ is thought to be evidence that saltation transport is a fully impact-sustained transport regime because it is associated with a constant average outcome of particle-bed impacts [9, 10, 44-47, 62-66]. In fact, when impact entrainment dominates fluid entrainment, every particle trapped at the bed must be replaced by precisely one particle entrained through impacts on average. However, this line of reasoning is not entirely accurate because it indirectly assumes that all particle-bed impacts occur at the same vertical location $z_r$. However, particle-bed impacts actually occur at varying vertical locations and their range of influence often involves several layers of the sediment bed (e.g., see Movie S3), which makes this assumption problematic because $\langle v_x \rangle (z)$ increases exponentially with $z$ near $z_r$ (Fig. 2a), meaning small changes of $z$ have large effect. Indeed, Fig. 3a shows that the slip velocity exhibits significant fluctuations with $\Theta Im$ and thus with $\Theta$ even for typical saltation transport conditions in Earth’s atmosphere (e.g., $s = 2000$, $Re_p \geq 10$). Note that $\Theta Im = u_r^2 d / (\nu \sqrt{gd})$, where $u_r = \sqrt{\tau / \rho}$ is the fluid shear velocity, is the viscous, horizontal near-bed fluid velocity in natural units ($\sqrt{gd}$).

In the Aeolian Research community, it is the current consensus point of view that experiments (e.g., [68, 72-78]) show an approximately constant slip velocity for saltation transport [10], which would contradict our numerical finding if true. However, what the experiments truly show is instead that the extrapolation of $\langle v_x \rangle (z)$ from vertical locations $z \gtrsim z_r + 5\text{mm}$ to $z_r$ is approximately constant. In fact, reliable measurements of $\langle v_x \rangle (z) / \sqrt{gd}$ do not exist for vertical locations $z \lessim z_r + 5\text{mm}$ because large particle concentrations near $z_r$ strongly disturb the measurement apparatuses [68], which is supported by the fact that the few actual measurements (i.e., non-extrapolations) of $\langle v_x \rangle (z)/\sqrt{gd}$ reported for that region [72, 73, 76, 77] vary by more than an order of magnitude between about 0.5 [73] and 30 [76]. As shown in the inset of Fig. 2a, the data for $z \gtrsim z_r + 5\text{mm}$, where our simulations are consistent with measurements, suggest a linear trend of $\langle v_x \rangle (z)$ with $z$ even though the actual trend for $z \lessim z_r + 5\text{mm}$ is much closer to an exponential behavior. It explains why the extrapolation of $\langle v_x \rangle (z)$ to $z_r$ yields values very different from the actual slip velocity. Indeed, when we estimate the slip velocity from our transport simulations via linear extrapolation, we obtain values that are perfectly consistent with the likewise extrapolated measurements (inset of Fig. 3a).

Though the extrapolation of $\langle v_x \rangle (z)/\sqrt{gd}$ to $z_r$ might serve as a proxy for the relative relevance of impact entrainment for saltation transport in Earth’s atmosphere, which is characterized by a large transport layer, it is obviously meaningless for transport regimes with a small transport layer, such as bedload transport, and thus does not allow a unified treatment of all transport regimes. In
what follows, we therefore propose a novel proxy.

B. Bed velocity

As explained above, the main issue with the slip velocity proxy is the fact that particle-bed impacts occur at varying vertical locations and their range of influence often involves several layers of the sediment bed. A straightforward way to mend this issue is to average \( \langle v_x \rangle(z) \) around \( z_c \) rather than evaluate it at \( z_c \). The averaging method we apply here is rather complex and explained in detail in Appendix B4. It results in what we call the ‘bed velocity’ \( v_b \).

Figure 3b shows that \( v_b/\sqrt{\hat{g} \hat{d}} \) is a much better proxy than \( \langle v_x \rangle(z_c)/\sqrt{\hat{g} \hat{d}} \) (Fig. 3a) as it reproduces the approximately constant behavior expected for saltation transport in Earth’s atmosphere. In detail, we observe two extreme regimes. When sediment transport is fully sustained through fluid entrainment, the bed velocity scales with the average near-bed fluid velocity (dashed line in Fig. 3b). In contrast, when sediment transport is fully sustained through impact entrainment, the dimensionless bed velocity is approximately constant, \( v_b/\sqrt{\hat{g} \hat{d}} \approx 1.0 \), and changes comparably little with \( \Theta \), \( s \), and \( \text{Re}_p \). A part of the variation of \( v_b/\sqrt{\hat{g} \hat{d}} \) in this regime can be attributed to small changes of the bed friction coefficient \( \mu_b = \mu(z_c) \), where \( \mu \) is particle-shear-pressure ratio (Appendix B2 and B3). In fact, we find that assuming \( v_b/\sqrt{\hat{g} \hat{d}} \propto \mu_b \) for fully impact-sustained conditions results in a significantly improved data collapse (Fig. 3c), which makes sense because once can expect that impact entrainment is the more difficult (larger \( v_b/\sqrt{\hat{g} \hat{d}} \)) the larger the granular resistance at the bed surface (larger \( \mu_b \)). Note that Fig. 3c corresponds to Fig. 3b when setting \( \mu_b = 0.8 = \text{const.} \).

The transition to a fully impact-sustained transport regime is determined by two independent conditions. First, the impact number has to exceed a critical value: \( \text{Im} \gtrsim 20 \) (Fig. 3, open symbols). This follows from the fact that the transport-layer average \( \overline{\tau_x} \) of the horizontal particle velocity (defined in Appendix B6) must be larger than the bed velocity since \( \langle v_x \rangle(z) \) increases with \( z \). For relatively viscous conditions at grain scale (\( \text{Re}_p < 5 \)) and close to the transport threshold (\( \Theta_{\text{ex}} \)), the scaling of the average particle velocity \( \overline{v_x} \) can be obtained from the proportionality of \( \overline{\tau_x} \) with the transport-layer average fluid velocity \( \overline{\tau_x} \) (Fig. 6) and a dynamic-friction condition (i.e., \( \overline{\tau_x} - \tau_{\text{tr}} \propto \text{Re}_p \sqrt{(s-1)\hat{g} \hat{d}} \) [45]). Thus, \( \overline{\tau_x} \propto \text{Re}_p \sqrt{(s-1)\hat{g} \hat{d}} = \text{Im}\sqrt{\hat{g} \hat{d}} \) and the condition \( \overline{\tau_x} > v_b \approx \sqrt{\hat{g} \hat{d}} \) implies \( \text{Im} \gtrsim \text{Im}_c \) with \( \text{Im}_c \approx 20 \).

The second condition for fully impact-sustained transport is \( \Theta \gtrsim 5/\text{Im} \) and follows from the proportionality of the dimensionless bed velocity with the dimensionless near-bed fluid velocity in the fully fluid-sustained regime (\( v_b/\sqrt{\hat{g} \hat{d}} \propto \Theta \text{Im} \)) and the fact that \( v_b/\sqrt{\hat{g} \hat{d}} \) cannot increase indefinitely (Figs. 3b,c). Therefore, for \( \text{Im} < 20 \), which exclusively characterizes viscous bedload transport conditions, increasing the Shields parameter leads to a transition from fully fluid-sustained when \( \Theta \lesssim 1/\text{Im} \) to fully impact-sustained transport when \( \Theta \gtrsim 5/\text{Im} \), the increasing bed velocity reaches the maximum value needed to replace every particle trapped at the bed by exactly one particle entrained through impacts (inset of Figs. 3b,c). Note that, near the threshold, the condition \( \Theta > 5/\text{Im} \) always implies \( \text{Im} \gtrsim 20 \) (i.e., \( \Theta_{\text{ex}} < 0.2 \) [79]), but not vice versa (e.g., \( s = 10^7 \), \( \text{Re}_p = 0.1 \) in Figs. 3b,c), and the condition \( \text{Im} \lesssim 4 \) always implies \( \Theta \lesssim 1/\text{Im} \).

V. LINK BETWEEN BED VELOCITY AND AVERAGE TRANSPORT CHARACTERISTICS

It is commonly argued that the average horizontal particle velocity \( \overline{v_x} \) in the fully impact-sustained regime is constant [38, 46, 47] or nearly constant [43–45, 62, 67], and that the sediment transport rate therefore approximately scales as \( Q \propto \Theta - \Theta_{\text{ex}} \), because the slip velocity is constant. In light of our finding of a generally non-constant slip velocity, but constant bed velocity \( v_b \) in the fully impact-sustained regime (Fig. 3), one should actually rephrase this argument and say that \( \overline{v_x} \) is constant because the bed velocity \( v_b \) is constant in this regime. However, even when rephrased, we find this argument is not valid for fully impact-sustained bedload transport (Fig. 4). Although \( Q \) is, indeed, linear in \( \Theta \) when \( \Theta \lesssim 20\Theta_{\text{ex}} \), it transforms into a \( \Theta^{1.5} \)-dependency when \( \Theta \gtrsim 20\Theta_{\text{ex}} \). As a consequence, the scaling \( Q \propto \sqrt{\Theta(\Theta - \Theta_{\text{ex}}^2)} \), which is consistent with measurements of the bedload transport rate [80–82], provides a much better overall fit to the simulations.

The transition from a linear to a non-linear transport
law for turbulent bedload transport is consistent with a transition from a constant average particle velocity ($\overline{v_x}$) to one that increases with $\Theta$, which occurs even though $V_b$ remains nearly constant (inset of Fig. 4). A similar transition in $\overline{v_x}(\Theta)$ can be found for some other fully impact-sustained conditions (Fig. 7b) and a similar lack of correlation of $\overline{v_x}$ with $V_b$ for all fully impact-sustained conditions (Fig. 5a). Also the classical slip velocity $\langle v_x \rangle(z_r)$ usually does not correlate with $\overline{v_x}$ (Fig. 5b) with the exception of bedload transport conditions (Fig. 5c).

Rather than with the bed velocity, the horizontal particle velocity scales with the horizontal fluid velocity (Fig. 6). In detail, we find that the scaling of $\overline{v_x}$ depends on the relation between $\overline{v_x}$ and the size of the transport layer ($\tau$). When the transport layer is within the viscous sublayer of the turbulent boundary layer ($\tau/z_v \lesssim 5$), with the viscous length $z_v \equiv d/([\sqrt{\Theta}Re_p])$, $\overline{v_x}$ scales with the characteristic fluid velocity within the viscous sublayer: $\overline{v_x} \propto \overline{v_x}$, where $\overline{v_x}$ scales as $\overline{v_x} \approx \sqrt{\Theta/(s-1)}gd \tau/z_v$ when the particle-flow feedback (see below) can be neglected. On the other hand, when the transport layer extends beyond the viscous sublayer, the scale of the particle velocity is dominated by the characteristic fluid velocity in the logarithmic region of the velocity profile. That is, $\overline{v_x} \propto \sqrt{\Theta(s-1)}gd \equiv u_s$ when the particle-flow feedback can be neglected.

However, for saltation transport in Earth’s atmosphere ($s = 2000$, $Re_p \gtrsim 10$), it is well-known that there is a strong negative feedback on the flow generated by particle motion [83–85], which is a necessary condition to maintain a constant average impact velocity [9, 10]. As a consequence, the fluid shear velocity remains approximately constant with $\Theta$ in an extended region above the bed surface, and the particle velocity scales as $\overline{v_x} \propto \sqrt{\Theta/(s-1)}gd \equiv u_s$. This large fluid shear stresses ($\Theta \gtrsim 2\Theta u_s^2$ for turbulent bedload transport), a highly collisional layer of transported particles (a liquid-like or ‘soft’ bed [52]) develops because the bed surface becomes completely mobile (‘stage-3’ bedload transport [14], Movie S4). This liquid-like bed hinders particles moving over it from reaching the disturbed-flow region near the quasi-static-bed surface as they tend to rebound from the liquid-bed surface (Movie S4). Hence, these particles can remain extended periods of time in the nearly-undisturbed-flow region, leading to an increase of $\overline{v_x}$ with $\Theta$. This point is further supported by Fig. 7a, which shows that, for fully impact-sustained conditions, the beginning increase of $\overline{v_x}$ (Fig. 7b) approximately coincides with a beginning increase of the effective location of energetic particle rebounds relative to the quasi-static bed ($\Delta z_r$, Appendix B5).

VI. DISCUSSIONS AND CONCLUSIONS

Our study challenges the paradigm that sediment transport mediated by water [14, 22, 33] or heavy air
impact-sustained transport, the impact number scales as
\[
\frac{\Delta z}{\langle z_t \rangle_{\text{ex}} \sim 3}} \approx \Theta_{\text{ex}}
\]
Else: \( \nu_{t} \) or \( \nu_{u} \). Among the fully impact-sustained simulation cases, only those corresponding to the linear transport rate regime are shown in the inset (see text). The solid lines correspond to \( \nu_{t}/u_{\text{typ}} = 4.8 \sqrt{1 - \exp\left[-0.027 (\nu_{u}/u_{\text{typ}})^2\right]} \), where \( u_{\text{typ}} = u_{t} \) (main figure) or \( u_{\text{typ}} = u_{t}^{\text{ex}} \) (inset). For symbol legend, see Fig. 2.

Our numerical finding that steady turbulent bedload \( \nu_{t} \) would be bounded between the horizontal velocities at take-off and impact and thus be approximately proportional to the slip velocity \( \langle v_{x} \rangle(z_{t}) \), which is the average particle velocity at the location of the bed surface \( z_{t} \). However, we find that \( \nu_{t} \) scales with the fluid velocity within the transport layer (Fig. 6) and generally not with \( \langle v_{x} \rangle(z_{t}) \) nor \( V_{b} \) (Fig. 5). We also find that the locally averaged vertical particle velocity \( \sqrt{\langle v_{z}^2 \rangle(z)} \) increases exponentially with elevation \( z \) near the bed surface (Fig. 2b), whereas an identical-trajectory model necessarily predicts a decrease. These discrepancies are evidence for a separation of particle velocity scales, which was already pointed out for saltation transport in a previous study (Fig. 21 of Ref. [9]). Near the bed surface, the average particle velocity is dominated by a comparably slow species of particle (‘reptons’ [62] or ‘leapers’/‘creepers’ [52]), whereas at larger elevations that cannot be reached by the slow species, a comparably fast species dominates (‘saltons’ [52, 62]).

Finally, for fully impact-sustained transport, our study predicts a relatively strong negative feedback of the particle motion on the flow for dimensionless fluid shear stress sufficiently close to \( \Theta_{\text{ex}} \). As a consequence, \( \nu_{t} \), which is controlled by the flow, remains approximately constant with \( \Theta \), leading to a linear scaling of the sediment transport rate \( Q \propto \Theta - \Theta_{\text{ex}} \). However, for turbulent bedload transport, this linear scaling becomes non-linear slightly above the threshold \( \Theta \approx 2\Theta_{\text{ex}} \), see Fig. 4) due to a sudden drop in the relative feedback strength, which is associated with the formation of a liquid-like bed of particles on top of the quasi-static bed surface (Fig. 7).

Our numerical finding that steady turbulent bedload

\[
\nu_{t} \approx \Theta_{\text{ex}}
\]

identical trajectories [7, 9, 38, 46, 47, 64, 89–96]. If this assumption was true, the transport-layer-averaged horizontal particle velocity \( \nu_{t} \) would be bounded between the horizontal velocities at take-off and impact and thus be approximately proportional to the slip velocity \( \langle v_{x} \rangle(z_{t}) \), which is the average particle velocity at the location of the bed surface \( z_{t} \). However, we find that \( \nu_{t} \) scales with the fluid velocity within the transport layer (Fig. 6) and generally not with \( \langle v_{x} \rangle(z_{t}) \) nor \( V_{b} \) (Fig. 5). We also find that the locally averaged vertical particle velocity \( \sqrt{\langle v_{z}^2 \rangle(z)} \) increases exponentially with elevation \( z \) near the bed surface (Fig. 2b), whereas an identical-trajectory model necessarily predicts a decrease. These discrepancies are evidence for a separation of particle velocity scales, which was already pointed out for saltation transport in a previous study (Fig. 21 of Ref. [9]). Near the bed surface, the average particle velocity is dominated by a comparably slow species of particle (‘reptons’ [62] or ‘leapers’/‘creepers’ [52]), whereas at larger elevations that cannot be reached by the slow species, a comparably fast species dominates (‘saltons’ [52, 62]).

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\]

identical trajectories [7, 9, 38, 46, 47, 64, 89–96]. If this assumption was true, the transport-layer-averaged horizontal particle velocity \( \nu_{t} \) would be bounded between the horizontal velocities at take-off and impact and thus be approximately proportional to the slip velocity \( \langle v_{x} \rangle(z_{t}) \), which is the average particle velocity at the location of the bed surface \( z_{t} \). However, we find that \( \nu_{t} \) scales with the fluid velocity within the transport layer (Fig. 6) and generally not with \( \langle v_{x} \rangle(z_{t}) \) nor \( V_{b} \) (Fig. 5). We also find that the locally averaged vertical particle velocity \( \sqrt{\langle v_{z}^2 \rangle(z)} \) increases exponentially with elevation \( z \) near the bed surface (Fig. 2b), whereas an identical-trajectory model necessarily predicts a decrease. These discrepancies are evidence for a separation of particle velocity scales, which was already pointed out for saltation transport in a previous study (Fig. 21 of Ref. [9]). Near the bed surface, the average particle velocity is dominated by a comparably slow species of particle (‘reptons’ [62] or ‘leapers’/‘creepers’ [52]), whereas at larger elevations that cannot be reached by the slow species, a comparably fast species dominates (‘saltons’ [52, 62]).

Finally, for fully impact-sustained transport, our study predicts a relatively strong negative feedback of the particle motion on the flow for dimensionless fluid shear stress sufficiently close to \( \Theta_{\text{ex}} \). As a consequence, \( \nu_{t} \), which is controlled by the flow, remains approximately constant with \( \Theta \), leading to a linear scaling of the sediment transport rate \( Q \propto \Theta - \Theta_{\text{ex}} \). However, for turbulent bedload transport, this linear scaling becomes non-linear slightly above the threshold \( \Theta \approx 2\Theta_{\text{ex}} \), see Fig. 4) due to a sudden drop in the relative feedback strength, which is associated with the formation of a liquid-like bed of particles on top of the quasi-static bed surface (Fig. 7).

Our numerical finding that steady turbulent bedload
transport is fully sustained through particle-bed impacts may be criticized because the simulations are quasi-two-dimensional and do only account for the turbulent mean flow, but not for turbulent fluctuations around the mean, which are known to be crucial for the initiation of bedload transport [28–30]. However, we believe that our finding is robust because our simulations quantitatively reproduce measurements of bedload transport cessation thresholds [12], which would not be expected if these neglected items played a crucial role for entrainment of bed sediment in steady bedload transport. Our reasoning is supported by Ref. [60], who compared three-dimensional bedload transport simulations with and without turbulent fluctuations. Their Fig. 6 indicates that, although the initiation threshold is strongly affected by turbulent fluctuations, the cessation threshold is nearly unaffected because the extrapolation of the simulated transport rates to vanishing transport remains nearly the same.

Appendix A: Movie captions

1. Movie S1

Time evolution of the simulated particle-fluid system for \( s = 2000, \text{Re}_p = 20, \) and \( \Theta \simeq 17.16\Theta_e^{in} \), considering weakly-damped binary collisions \( (e = 0.9) \). The flow velocity is shown as a background color with warm colors corresponding to high velocities and cold colors to small velocities. The horizontal and vertical axes are measured in mean particle diameters. Only 1/4 of the simulated horizontal domain is shown, which is why there are occasions at which no moving particle can be observed. This is an example for saltation transport, which is predominantly sustained through particle-bed impact entrainment. One can see that impacting particles tend to eject surface particles.

2. Movie S2

Time evolution of the simulated particle-fluid system for \( s = 2.65, \text{Re}_p = 20, \) and \( \Theta \simeq 13.89\Theta_e^{in} \), considering binary collisions that are nearly fully damped by the lubrication force \( (e = 0.01) \). The flow velocity is shown as a background color with warm colors corresponding to high velocities and cold colors to small velocities. The horizontal and vertical axes are measured in mean particle diameters. Only 1/4 of the simulated horizontal domain is shown, which is why there are occasions at which no moving particle can be observed. This is an example for turbulent bedload transport, which is predominantly sustained through particle-bed impact entrainment. One can see that impacting particles tend to drag surface particles out of they traps.

3. Movie S3

Time evolution of the simulated particle-fluid system for \( s = 2000, \text{Re}_p = 20, \) and \( \Theta \simeq 17.16\Theta_e^{in} \), considering weakly-damped binary collisions \( (e = 0.9) \). The flow velocity is shown as a background color with warm colors corresponding to high velocities and cold colors to small velocities. The horizontal and vertical axes are measured in mean particle diameters. Only 1/4 of the simulated horizontal domain is shown, which is why there are occasions at which no moving particle can be observed. This is an example for saltation transport, which is predominantly sustained through particle-bed impact entrainment. One can see that impacting particles tend to eject surface particles.

4. Movie S4

Time evolution of the simulated particle-fluid system for \( s = 2.65, \text{Re}_p = 20, \) and \( \Theta \simeq 13.89\Theta_e^{in} \), considering binary collisions that are nearly fully damped by the lubrication force \( (e = 0.01) \). The flow velocity is shown as a background color with warm colors corresponding to high velocities and cold colors to small velocities. The horizontal and vertical axes are measured in mean particle diameters. Only 1/4 of the simulated horizontal domain is shown. This is an example for turbulent bedload transport, which is predominantly sustained through particle-bed impact entrainment. However, it is impossible to determine single entrainment events as several layers of the bed are in continuous motion. These layers constitute the liquid-like bed, and it can be seen that energetic particles tend to rebound from its top.

Appendix B: Technical computation details

1. Local, mass-weighted ensemble average

We compute the local, mass-weighted ensemble average \( \langle A \rangle \) of a particle quantity \( A \) through \([58]\)

\[
\langle A \rangle = \frac{1}{\rho} \sum_n m^n A^n \delta(x - x^n)^E, \tag{B1}
\]

\[
\rho = \sum_n m^n \delta(x - x^n)^E, \tag{B2}
\]

where \( m \) is particle mass, \( x \) the location, and \( \delta \) the delta distribution. Furthermore, the sum iterates over all particles \( (n \in (1, N)) \), with \( N \) the total number of particles, and \( \tau^E \) denotes the ensemble average.

2. Particle stress tensor

Using the definition of the local, mass-weighted ensemble average, we obtain the particle stress tensor \( P_{ij} \)

\[
P_{ij} = \frac{1}{\rho} \sum_n \left( m^n \delta(x - x^n)^E \right) \left( \sum_k A_k^n \delta(x - x^n)^E \right) \left( \begin{array}{c} i \delta(x - x^n)^E \\ j \delta(x - x^n)^E \end{array} \right), \tag{B3}
\]
through [58]

\[ P_{ij} = \rho(v'_i v'_j) + \frac{1}{2} \sum_{m} F_{mn}^m (x_i^m - x_i^n) K(x, x^m, x^n), \]  
  \[ v' = v - \langle v \rangle, \]  
  \[ K = \int_0^1 \delta(x - ((x^m - x^n)s + x^n)) ds, \]

where \( v \) is the particle velocity and \( F_{mn}^m \) the contact force applied by particle \( n \) on particle \( m \) (\( F_{nn} = 0 \)).

3. Rebound height and bed friction coefficient

The bed friction coefficient \( \mu_b \) is the friction coefficient 
\( \mu = -P_{xz}/P_{zz} \) evaluated at the ‘bed surface’, which is, ideally, the vertical location at which particle-bed impacts occur. However, in a real system, particle-bed impacts are not localized at a fixed vertical location, but their range of influence rather involves several vertical layers of particles. We are thus looking for a definition of \( \mu_b \) that involves several particle layers around the bed surface. One way to do so is to exploit the fact that the average horizontal particle velocity \( \langle v_x \rangle \) decays exponentially within the sediment bed (Fig. 2), which implies that \( \langle v_x \rangle / dz \) exhibits a pronounced local maximum near the bed surface. This behavior suggests an approximate definition of \( \mu_b \) as

\[ \mu_b \approx \frac{\max \left(-P_{xz} \frac{d\langle v_x \rangle}{dz}\right)}{\max \left(P_{zz} \frac{d\langle v_x \rangle}{dz}\right)}. \]

Since the local maxima of \(-P_{xz} d\langle v_x \rangle/dz\) and \(P_{zz} d\langle v_x \rangle/dz\) in our simulations occur at almost the same location, we define \( \mu_b \) in the paper as

\[ \mu_b = \mu(z_r), \]

where the location \( z_r \) is defined through

\[ \max \left(P_{zz} \frac{d\langle v_x \rangle}{dz}\right) = \left[P_{zz} \frac{d\langle v_x \rangle}{dz}\right] (z_r) \]

The term \(-P_{xz} d\langle v_x \rangle/dz\) is the particle shear work, due to which fluctuation energy is produced [58]. Since, in a steady system, the location of maximal fluctuation energy production approximately corresponds to the location of maximal fluctuation energy dissipation, \( z_r \) should approximately correspond to the location where energetic rebounds of transported particles occur, which is usually near the bed surface. However, when sediment transport is sufficiently intense, a liquid-like bed can develop, and the most energetic rebounds occur near the liquid-bed surface rather than the quasi-static bed surface [52].

4. Bed velocity

Like for \( \mu_b \), we are looking for a definition of the bed velocity \( V_b \) that involves several particle layers around the bed surface because particle-bed impacts are not localized at a fixed vertical location. Fig. 2 shows that the exponential decay of \( \langle v_x \rangle \) within the sediment bed occurs within a characteristic decay height proportional to \( d \). This implies

\[ V_b \propto V'_{b} d, \]

where \( V'_{b} \) is a characteristic horizontal velocity gradient near the bed surface. We calculate \( V'_{b} \) as the ratio between \( \max(-P_{xz} d\langle v_x \rangle/dz) \), which is a suitable definition of an effective bed surface value of \(-P_{xz} d\langle v_x \rangle/dz\) (see Eqs. (B8)), and a suitable definition of the bed-surface averaged particle shear stress \(-P_{xz} \), namely,

\[ V'_{b} = \frac{\max \left(-P_{xz} \frac{d\langle v_x \rangle}{dz}\right)}{\max \left(P_{zz} \frac{d\langle v_x \rangle}{dz}\right)}. \]

Finally, we obtain the proportionality factor in Eq. (B9) by imposing that the bed velocity very roughly equals the classical slip velocity, \( V_b \approx \langle v_x \rangle(z_r) \), for the simulated turbulent saltation transport cases \((s = 2000, Re_p \geq 10)\). This constraint yields

\[ V_b = 0.33d \times \frac{\max \left(-P_{xz} \frac{d\langle v_x \rangle}{dz}\right)}{\tau - \frac{1}{\rho_0} \int_{-\infty}^{\infty} \rho^2 (a_z)dz}. \]

5. Relative rebound height

We define the location \( z_s \) of the interface between transport layer and quasi-static bed as the location at which the friction coefficient is equal to \( 0.7 \mu_b \), reading

\[ \mu(z_s) = 0.7 \mu_b. \]

This definition accounts for potential dependencies of the interface location \( z_s \) on the contact friction coefficient \( \mu_c \). For a typical value \( \mu_b = 0.6 \), it corresponds to \( \mu(z_s) = 0.42 \), consistent with previous studies of bedload transport [60]. We use Eq. (B13) to define the relative rebound height as

\[ \Delta z_r = z_r - z_s. \]
6. Transport layer average

We compute the transport layer average \( \mathcal{A} \) of a particle quantity \( A \) via [12]

\[
\mathcal{A} = \frac{\int_{z_r}^{\infty} \rho(A) dz}{\int_{z_r}^{\infty} \rho dz}.
\] (B15)

It describes a mass-weighted average of \( A \) over all particles within the transport layer defined through \( z > z_r \).

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