COLOR AND SPIN IN QUARKONIUM PRODUCTION

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I describe the NRQCD factorization approach to the inclusive production of heavy quarkonium, contrasting it with the color-singlet and color evaporation models. These approaches differ dramatically in their assumptions about the roles played by color and spin in the production process. They also differ dramatically in their predictions for the production of charmonium at large transverse momentum.

1 Introduction

The production of heavy quarkonium in high energy collisions is important, because there are several quarkonium states with clean experimental signatures through their decays into lepton pairs. Measurements of the production rate of these states can be used to test our understanding both of heavy-quark production in high energy collisions and of the formation of bound states from heavy quark pairs. The NRQCD factorization approach, which is based on the use of an effective field theory called nonrelativistic QCD to exploit the large mass of the heavy quark, has led to dramatic progress in our understanding of inclusive quarkonium production. Below, I describe the basic ideas underlying the NRQCD factorization approach, emphasizing its implications for the role played by color and spin in inclusive production.

2 Simple Production Models

Before 1995, most of the effort to understand the inclusive production of charmonium was carried out within either the color-singlet model or the color evaporation model. Both of these models have roots that go back to 1975 shortly after the discovery of charmonium. These two models make diametrically opposite assumptions about the roles played by color and spin in the production process.

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2.1 Color-singlet Model

In the color-singlet model, charmonium states are interpreted as nonrelativistic bound states of a $c$ and $\bar{c}$ interacting through a confining potential. The $J/\psi$, for example, is identified as a bound state of a $c\bar{c}$ pair in a color-singlet $^3S_1$ state. I denote such a state by $c\bar{c}_1(^3S_1)$, where the subscript specifies the color state (1 for singlet, 8 for octet) and the argument specifies the angular-momentum quantum numbers. The $\chi_{cJ}$ is identified as a $c\bar{c}_1(^3P_J)$ bound state.

The only role of gluons in the color-singlet model is to generate the potential that binds the $c$ and $\bar{c}$.

The color-singlet model provides a prescription for calculating not only the inclusive production rate for a quarkonium state, but also its inclusive decay rate into light hadrons and its decay rate into leptons or photons. For example, the formula for the decay rate of the $J/\psi$ into light hadrons in the color-singlet model is

$$\Gamma(J/\psi) = \hat{\Gamma}(c\bar{c}_1(^3S_1)) \frac{|R_{J/\psi}(0)|^2}{4\pi},$$

where $\hat{\Gamma}$ is proportional to the annihilation rate at threshold of a $c\bar{c}$ pair in the state $c\bar{c}_1(^3S_1)$. The last factor in (1) is the square of the wavefunction at the origin, which gives the probability for the $c\bar{c}$ pair in the $J/\psi$ to be close enough to annihilate.

In the color-singlet model, it is assumed that a $c\bar{c}$ pair that is produced in a high energy collision will bind to form a given charmonium state only if the $c\bar{c}$ pair is created in a color-singlet state with angular-momentum quantum numbers that match those of the bound state. For example, to form a $J/\psi$, the $c\bar{c}$ pair must be produced in a color-singlet $^3S_1$ state. The formula for the inclusive production cross section for the $J/\psi$ in the color-singlet model is

$$\sigma(J/\psi) = \hat{\sigma}(c\bar{c}_1(^3S_1)) \frac{|R_{J/\psi}(0)|^2}{4\pi},$$

where $\hat{\sigma}$ is proportional to the production rate for a $c\bar{c}$ pair at threshold in the state $c\bar{c}_1(^3S_1)$. The last factor in (2) is the probability that a pointlike $c\bar{c}$ pair in the state $c\bar{c}_1(^3S_1)$ will bind to form a $J/\psi$, which is again given by the square of the wavefunction at the origin.

The color-singlet model is an extremely economical phenomenological framework for calculating quarkonium production. The factors $\hat{\Gamma}$ and $\hat{\sigma}$ in (1) and (2) can be computed using QCD perturbation theory in terms of the charm quark mass $m_c$ and the running coupling constant $\alpha_s(m_c)$. The only other phenomenological parameters are a single wavefunction factor for each spin multiplet: $R_{J/\psi}(0)$ for the S-wave states $J/\psi$ and $\eta_c$, $R'_{\chi}(0)$ for the P-wave states $\chi_{c0}$, $\chi_{c1}$, $\chi_{c2}$, and $h_c$, etc. Moreover, these wavefunction factors

2
can be determined from experimental measurements of the decay rates. Thus, the color-singlet model predicts the inclusive production cross sections for all quarkonium states in terms of a single phenomenological parameter $m_c$.

The color-singlet model gives definite predictions for the dependence of the cross section on the polarization of the charmonium state. It also predicts that ratios $\sigma(H)/\sigma(J/\psi)$ of the cross sections of quarkonium states with different $J^{PC}$ quantum numbers should vary dramatically from process to process, because of angular momentum selection rules. Unfortunately, these predictions have proved to be wrong. Dramatic variations in the ratios $\sigma(H)/\sigma(J/\psi)$ have not been seen experimentally and there is little if any evidence for spin asymmetries in the cross sections.

The most dramatic failure of the color-singlet model came in 1995, when the CDF collaboration measured the production cross sections for charmonium states at the Tevatron $p\bar{p}$ collider. They used a vertex detector to separate prompt production of charmonium from production through the decay of $b$ quarks. They found that the cross sections for the direct production of $J/\psi$ and $\psi'$ at large transverse momentum were larger than the predictions of the color-singlet model by about a factor of 30. This dramatic discrepancy marked the final demise of the color-singlet model. May it rest in peace.

### 2.2 Color Evaporation Model

In the color evaporation model, the probability that a $c\bar{c}$ pair produced in a high energy collision will bind to form a specific charmonium state is assumed to be almost completely independent of the color and spin state of the $c\bar{c}$ pair. The basic assumption is that the exchange and emission of soft gluons destroys any correlations between the color and spin state of the $c\bar{c}$ pair when it is created and the quantum numbers of the final $c\bar{c}$ bound state. In the color evaporation model, the formula for the inclusive cross section for producing a $J/\psi$ is conventionally written in the form

$$\sigma(J/\psi) = \tilde{\sigma}(c\bar{c} : 4m^2_c < s < 4m^2_D) f_{J/\psi},$$

where $\tilde{\sigma}$ is the cross section for producing a $c\bar{c}$ pair with invariant mass below the $D\bar{D}$ threshold. This cross section is summed over all the color and spin states of the $c\bar{c}$ pair. The factor $f_{J/\psi}$ in (3) is a phenomenological parameter that gives the fraction of $c\bar{c}$ pairs below the $D\bar{D}$ threshold that form a $J/\psi$.

The color evaporation model is much less ambitious than the color-singlet model. It makes no attempt to relate production cross sections to annihilation decays. However it is still a fairly economical phenomenological framework for calculating quarkonium production. The cross section $\tilde{\sigma}$ in (3) can be calculated using perturbative QCD as a function of $m_c, m_D$, and $\alpha_s(m_c)$. The
choice of $4m_D^2$ as the upper endpoint for the integration over $s$ is completely arbitrary, so we should regard $s_{\text{max}} = 4m_D^2$ as a phenomenological parameter. The remaining phenomenological parameters are a fraction $f_H$ for each quarkonium state $H$. Thus the color evaporation model gives predictions for inclusive production cross sections in all high energy processes in terms of $m_c$, $s_{\text{max}}$, and a fraction $f_H$ for each quarkonium state.

The color evaporation model gives simple predictions for the ratios of cross sections for different quarkonium states and for the dependence of the cross sections on the polarization of the quarkonium state. It predicts that ratios of cross sections are independent of the production process:

\[
\frac{\sigma(H)}{\sigma(J/\psi)} = \frac{f_H}{f_{J/\psi}}.
\]

The model also predicts that cross sections are independent of the polarization of the charmonium state. For example, the fraction of $J/\psi$'s that are transversely polarized is predicted to be 2/3 for any production process. Thus the color evaporation model can be ruled out by experimental measurements of a nonzero spin asymmetry for the $J/\psi$ in any process or of a variation of the $\chi_c$-to-$J/\psi$ ratio in different processes.

The color evaporation model is not yet dead. It is basically a phenomenological model, so it can only be killed by experimental data. I believe that its demise is only a matter of time until sufficiently accurate experimental data is available. The model is based on far too simplistic a picture of the effects of soft gluons on color and spin in quarkonium production.

3 The NRQCD Factorization Approach

The NRQCD factorization approach to quarkonium production was developed by Bodwin, Braaten, and Lepage in 1995. As in the color evaporation model, a $c\bar{c}$ pair in any color and spin state has a nonzero probability of binding to form a given charmonium state. As in the color-singlet model, that probability depends strongly on the color and angular-momentum state. Thus the roles of spin and color in this approach are somewhere intermediate between their roles in the color-singlet and color evaporation models. However, in contrast to these models, the NRQCD factorization approach is based firmly on QCD.

3.1 Quarkonium as a Nonrelativistic Bound State

The NRQCD factorization approach to annihilation decays and inclusive production is based on the fact that heavy quarkonium is a nonrelativistic bound
state. Its properties are determined by QCD in terms of essentially only 2 parameters: the QCD coupling constant $\alpha_s$ and the heavy quark mass $M$. However, because it is a nonrelativistic bound state, there are a number of different energy scales that play an important role in quarkonium physics. These scales include

- $M$, the mass of the heavy quark, which sets the scale for the mass of the bound state,
- $Mv$, the typical momentum of the heavy quark in quarkonium, which sets the scale for the size of a quarkonium state,
- $Mv^2$, the typical kinetic energy of the heavy quark, which is also the scale for splittings between radial excitations and between orbital angular momentum excitations,
- $\Lambda_{\text{QCD}}$, the scale of nonperturbative effects associated with light quarks and gluons.

The scales $Mv$ and $Mv^2$ owe their existence to the fact that a small parameter $v$ is generated dynamically in a nonrelativistic bound state by the balance between kinetic energy and potential energy. If the interactions between the quark and antiquark are described by a potential $V(R)$, the balance can be expressed in the form

$$Mv^2 \sim V(R) \quad \text{for} \ R \sim 1/(Mv).$$

(5)

If the mass $M$ is large enough, the potential is essentially Coulombic: $V(R) \sim \alpha_s(R)/R$. The condition (5) then reduces to $v \sim \alpha_s(Mv)$. If the mass $M$ is not so large, the potential may be dominated by the linear confinement region: $V(R) \sim \kappa^2 R$. In this case, the condition (5) gives $v \sim (\kappa/M)^{2/3}$. In either case, we get a dynamically-generated small parameter $v$.

In Table 1, I list the energy scales $M$, $Mv$, and $Mv^2$ for charmonium and bottomonium and also what they would be for toponium if the top quark did not decay too rapidly to form bound states. For the scale $M$, I have taken half the mass of the lowest quarkonium state. For the scale $Mv^2$, I have taken the average of the energy splittings for the first radial excitation and the first orbital angular momentum excitation. The scale listed for $Mv$ is just the geometric mean of $M$ and $Mv^2$. By dividing $Mv^2$ by $M$, we find that $v^2$ is about 1/3 for charmonium, 1/10 for bottomonium, and about 1/100 for toponium. While 1/3 is not tiny, it is small enough that we can hope to use it as an expansion parameter for relativistic corrections.
Table 1: Momentum scales for quarkonium systems

|       | $cc$  | $bb$  | $tt$  |
|-------|-------|-------|-------|
| $M$  | 1.5 GeV | 4.7 GeV | 180 GeV |
| $M_v$ | 0.9 GeV | 1.5 GeV | 16 GeV  |
| $M_{v^2}$ | 0.5 GeV | 0.5 GeV | 1.5 GeV |

In Table 2, I list the running coupling constant of QCD at each of the momentum scales for charmonium, bottomonium, and toponium. For charmonium and for bottomonium, the scale $M_{v^2}$ is definitely in the strong-coupling region. At the scale $M$, $\alpha_s$ is small enough that we should be able to use perturbation theory to calculate the effects of that scale.

3.2 Nonrelativistic QCD

The large mass of the charm and bottom quarks gives us two small numbers $\alpha_s(M)$ and $v^2$ that we can exploit to understand heavy quarkonium physics. In order to exploit the smallness of $\alpha_s(M)$, we need to solve the problem of separating the scale $M$ from the smaller momentum scales $M_v$, $M_{v^2}$, and $\Lambda_{QCD}$ that involve nonperturbative physics. In order to exploit the smallness of $v^2$, we need some way of organizing nonperturbative effects according to how they scale with $v$. These problems were solved by Peter Lepage and collaborators by introducing an effective field theory called nonrelativistic QCD (NRQCD).

NRQCD is a formulation of QCD in which heavy quarks are treated as nonrelativistic particles. Instead of describing the heavy quark and antiquark by a single 4-component Dirac field $\Psi$, they are described by separate 2-component Pauli fields $\psi$ and $\chi^\dagger$. The lagrangian for NRQCD has the form

$$L_{\text{NRQCD}} = L_{\text{light}} + L_0 + L_2 + \cdots,$$  \hspace{1cm} (6)

where $L_{\text{light}}$ is the usual QCD lagrangian for the gluons and light quarks and antiquarks, $L_0$ is the minimal NRQCD lagrangian,

$$L_0 = \psi^\dagger \left(iD_0 + \frac{D^2}{2M}\right)\psi + \chi^{\dagger}\left(iD_0 - \frac{D^2}{2M}\right)\chi,$$ \hspace{1cm} (7)

and $L_2, L_4, \ldots$ are a series of “improvement terms” that can be added to make NRQCD reproduce the physics of full QCD to higher and higher order in $v^2$. The minimal QCD lagrangian $L_0$ is invariant under heavy-quark spin symmetry, which mixes heavy quarks with spin up and spin down. The $v^2$-improvement term $L_2$ includes four terms: a $(D^2)^2$ term that includes the
Table 2: The QCD coupling constant at the momentum scales for quarkonium systems

|        | cc | bb | tt |
|--------|----|----|----|
| $\alpha_s(M)$ | 0.35 | 0.22 | 0.11 |
| $\alpha_s(Mv)$ | 0.52 | 0.35 | 0.16 |
| $\alpha_s(Mv^2)$ | $\sim 1$ | $\sim 1$ | 0.35 |

relativistic correction to the kinetic energy, a $\mathbf{D} \cdot \mathbf{E}$ term, and $\mathbf{D} \times \mathbf{E} \cdot \mathbf{\sigma}$ and $\mathbf{B} \cdot \mathbf{\sigma}$ terms that break the spin symmetry. The coefficients of these four terms depend only on the momentum scale $M$ and they can therefore be calculated as perturbation series in $\alpha_s(M)$.

The NRQCD collaboration has calculated the spectrum for bottomonium and charmonium using Monte Carlo simulations of lattice NRQCD. These calculations demonstrate convincingly that NRQCD provides an effective framework for describing heavy quarkonium physics. Using the lagrangian $\mathcal{L}_{\text{light}} + \mathcal{L}_0$, the splittings between spin-averaged energy levels are reproduced to an accuracy of about 30% for charmonium and 10% for bottomonium, which are consistent with errors of relative order $v^2$. Because this lagrangian has spin symmetry, it gives no spin splittings. After adding the $v^2$-improvement term $\mathcal{L}_2$, the accuracy of the spin-averaged splittings is improved to 10% for charmonium and 1% for bottomonium, consistent with errors of order $v^4$. This term also gives the correct spin splittings up to errors of order $v^2$. Higher accuracy could presumably be achieved by adding the $v^4$-improvement term $\mathcal{L}_4$.

NRQCD provides a simple solution to the problem of separating the scales $M$ from the lower momentum scales $Mv$, $Mv^2$, and $\Lambda_{\text{QCD}}$. The improvement terms in the NRQCD lagrangian have coefficients that must be tuned to reproduce the physics of heavy quarks in full QCD. These terms compensate for the fact that NRQCD does not treat correctly the physics of relativistic heavy quarks whose momenta are of order $M$. Their coefficients therefore depend only on the momentum scale $M$. Since they depend only on the physics at length scales of order $1/M$, we call them “short-distance coefficients.” In NRQCD, all effects of the scale $M$ are taken into account through the coefficients of the improvement terms in the NRQCD lagrangian. Thus the NRQCD lagrangian itself provides us with the desired separation of scales. The effects of the scale $M$ are all contained in the short-distance coefficients.

NRQCD also provides a solution to the problem of organizing nonperturbative effects in such a way as to exploit the smallness of $v^2$. It leads to a simple picture of the structure of a charmonium state, which is described most easily in terms of the Fock state expansion in Coulomb gauge. This gauge makes the dynamics of a nonrelativistic bound state particularly transparent, and it
allows for a sensible Fock state expansion since there are no negative norm states. For example, the Fock state decomposition for the $J/\psi$ in Coulomb gauge has the form

$$|J/\psi\rangle = \sum_{c\bar{c}} |c\bar{c}_1(3S_1)\rangle + \sum_{c\bar{c}g} |c\bar{c}_8(3P_J) + g\rangle + \cdots,$$

(8)

The probability of each Fock state scales in a definite way with $v$. Only the $c\bar{c}_1(3S_1)$ Fock state has a probability of order 1. All the higher Fock states have probabilities suppressed by powers of $v$. The higher Fock state with the greatest probability is $c\bar{c}_8(3P_J) + g$, whose probability is of order $v^2$. The next most important Fock state is $c\bar{c}_8(1S_0) + g$, whose probability is of order $v^3$. One of the next most important Fock states is $c\bar{c}_8(3S_1) + g$, whose probability is of order $v^4$. By exploiting this Fock space structure, one can derive velocity-scaling rules that determine how matrix elements of local operators in a quarkonium state scale with $v$. If nonperturbative effects can be organized into matrix elements, their relative magnitudes can be estimated using the velocity-scaling rules.

3.3 NRQCD Factorization Formula for Decays

The NRQCD factorization approach to annihilation decays involves separating the scale $M$ from the lower momentum scales $Mv$, $Mv^2$, and $\Lambda_{QCD}$, and then exploiting the fact that quarkonium physics involves two small numbers, $\alpha_s(M)$ and $v^2$. The smallness of $\alpha_s(M)$ is exploited by calculating the effects of the scale $M$ as perturbation series in $\alpha_s(M)$. The smallness of $v^2$ is exploited by using the velocity-scaling rules to estimate the magnitudes of nonperturbative matrix elements involving scales of order $Mv$ and smaller.

We proceed to outline the derivations of the factorization formula for annihilation decay rates. The inclusive rate for the decay of a quarkonium state $H$ into light hadrons can be expressed in the following schematic form:

$$\Gamma(H) = \frac{1}{2M_H} \sum_{QQ\text{ hadrons}} \sum_{QQ\text{ hadrons}} \left| \psi_{QQ}^H \otimes T_{QQ\to\text{hadrons}} \right|^2,$$

(9)

where $\psi_{QQ}^H$ is the wavefunction for $H$ to consist of a $Q$ and $\bar{Q}$ and $T_{QQ\to\text{hadrons}}$ is the T-matrix element for producing a particular final state consisting of light hadrons. The sum over all final-state hadrons in (9) can be replaced by a sum over all final-state partons. This simply amounts to a change of basis for color-singlet final states. The final-state partons can be separated into hard partons
with momenta of order $M$ and soft partons with much smaller momenta. The resulting expression for the decay rate has the form

$$\Gamma(H) = \frac{1}{2M_H} \sum_{Q\bar{Q}} \sum_{\text{hard}} \sum_{\text{soft}} |\psi^H_{Q\bar{Q}} \otimes T_{Q\bar{Q} \rightarrow \text{hard} + \text{soft}}|^2. \quad (10)$$

The soft gluons in the final state can be emitted from the $Q$ or $\bar{Q}$, from the hard partons, or from the soft partons in the final state. The $T$-matrix element also involves virtual soft gluons that connect the initial $Q$ and $\bar{Q}$, the final hard partons, and the final soft partons. Standard factorization methods of perturbative QCD can be used to show that for every pair of hard partons whose 4-momenta are not collinear, there is a cancellation between the corrections from virtual soft gluons exchanged between the two partons and the interference terms between real soft gluons emitted by the two partons. A similar cancellation occurs between the corrections from virtual soft gluons exchanged between a hard parton and the $Q\bar{Q}$ pair and the interference terms from real soft gluons emitted by the hard parton and the $Q\bar{Q}$ pair. The only soft gluons that survive after these cancellations are the virtual gluons exchanged between the $Q$ and $\bar{Q}$ and the real gluons emitted from the $Q$ and $\bar{Q}$. But the effects of these gluons can be absorbed into the wavefunction $\psi^H_{Q\bar{Q} + \text{soft}}$ for $H$ to consist of a $Q\bar{Q}$ pair plus soft gluons. The resulting expression for the decay rate has the form

$$\Gamma(H) = \frac{1}{2M_H} \sum_{Q\bar{Q}} \sum_{\text{hard}} \sum_{\text{soft}} |\psi^H_{Q\bar{Q} + \text{soft}} \otimes T_{Q\bar{Q} \rightarrow \text{hard}}|^2. \quad (11)$$

The sum over $Q\bar{Q}$ states includes a sum over their color (1 or 8) and angular-momentum $(2S+1L_J)$ quantum numbers, which we denote collectively by $n$, and an integral over the magnitude of their relative momentum $q$. Because of the wavefunction factor, the integral over $q$ has support only for $q$ of order $Mv$. All dependence on the scale $Mv$ can be removed from the $T$-matrix element by expanding $T$ in powers of $q$. Denoting the expansion coefficients by $\hat{T}$ and associating the factors of $q$ and the integral over $q$ with the wavefunction factor, we obtain

$$\Gamma(H) = \frac{1}{2M_H} \sum_n \left( \sum_{\text{hard}} |\hat{T}_{Q\bar{Q}[n] \rightarrow \text{hard}}|^2 \right) \left( \int q^L \sum_{\text{soft}} |\psi^H_{Q\bar{Q}[n] + \text{soft}}|^2 \right). \quad (12)$$

The sum over $n$ includes all color and angular-momentum states of a $Q\bar{Q}$ pair. Each term in the sum is the product of a short-distance factor that involves only
the scale $M$ and a long-distance factor that involves only scales of order $Mv$ or smaller. The short-distance factor is proportional to the annihilation rate at threshold for a $Q\bar{Q}$ pair in the state $n$. The long-distance factor measures the probability of finding the $Q\bar{Q}$ pair at the same point in the state $n$ in the quarkonium state $H$.

The factorization formula (12) is simply a statement about the separation of scales in QCD, so it does not necessarily require NRQCD. However NRQCD provides a convenient prescription for carrying out this separation of scales. The long-distance factor in (12) can be expressed as the expectation value in the quarkonium state $H$ of a local gauge-invariant operator $O_n$ in NRQCD. The short-distance factor is proportional to the imaginary part of the coefficient of the operator $O_n$ in the NRQCD lagrangian. Thus (12) can be written

$$\Gamma(H) = \frac{1}{2M_H} \sum_n \hat{\Gamma}_{Q\bar{Q}[n]} \langle H|O_n|H \rangle. \quad (13)$$

This is the NRQCD factorization formula for the decay rate. Since the coefficient $\hat{\Gamma}$ involves only the scale $M$, it can be calculated as a perturbation series in $\alpha_s(M)$. The matrix elements $\langle H|O_n|H \rangle$ can in principle be calculated non-perturbatively using Monte Carlo simulations of lattice NRQCD. Thus the NRQCD factorization formula can be used to calculate the decay rate from first principles.

The NRQCD factorization formula (13) contains infinitely many terms. The utility of the formula relies on the fact that NRQCD also provides a way of exploiting the smallness of $v^2$. Each of the matrix elements scales with a definite power of $v$. In the case of $J/\psi$, the largest matrix element is $\langle J/\psi|O_1(3S_1)|J/\psi \rangle$, which is proportional to the square of the wavefunction at the origin and scales like $v^3$. The next most important matrix elements are the expectation values of $O_8(1S_0)$, which scales like $v^5$, and of $O_8(3S_1)$ and $O_8(3P_J)$, which scale like $v^7$. The relative magnitudes of the terms in the factorization formula (13) are determined by the order in $v$ of the matrix elements and by the order in $\alpha_s$ of the short-distance coefficients.

### 3.4 NRQCD Factorization Formula for Production

The NRQCD factorization approach to quarkonium production is based on the fact that a sufficiently inclusive cross section satisfies a factorization formula analogous to (12). All effects involving momentum scales of order $M$ and larger are contained in short-distance factors that involve the T-matrix element for producing a $Q\bar{Q}$ pair in the state $n$ with given 4-momentum $P$. All effects
involving momentum scales of order $Mv$ and smaller are contained in long-distance factors that involve the amplitude for a $Q\bar{Q}$ pair in the state $n$ to form the quarkonium state $H$ plus soft partons. The effective field theory NRQCD provides a convenient prescription for carrying out this separation of scales. It can also be used to exploit the small number $v^2$ by using velocity-scaling rules to determine how the long-distance factors scale with $v$.

The factorization formula for the differential cross section for producing a quarkonium state $H$ with 4-momentum $P$ has the form

$$d\sigma(H(P)) = \sum_n d\widehat{\sigma}_{Q\bar{Q}[n,P]}(\mathcal{O}_n^H),$$

(14)

where the sum over $n$ includes all color and angular-momentum states of the $Q\bar{Q}$ pair. The short-distance factor $\widehat{\sigma}$ is proportional to the cross section for producing a $Q\bar{Q}$ pair at threshold in the state $n$ with total 4-momentum $P$. Since they involve only momentum scales of order $M$ and larger, the short-distance factors can be calculated as perturbation series in $\alpha_s(M)$. The long-distance factor $\langle \mathcal{O}_n^H \rangle$, which can be expressed as an NRQCD matrix element, is proportional to the probability that a $Q\bar{Q}$ pair produced at a point in the state $n$ will form the quarkonium state $H$ plus soft hadrons, whose energies in the $H$ rest frame are of order $Mv$ or smaller. The derivation of the factorization formula breaks down if the 4-momentum of the quarkonium is collinear with that of any hadrons in the initial state. It therefore does not apply in the diffractive region.

The NRQCD factorization formula (13) contains infinitely many terms. The relative importance of these terms depends on several factors. The relative magnitudes of the matrix elements depends on how they scale with $v$. The relative magnitudes of the short-distance coefficients depends on their order in $\alpha_s$, and also on how they scale with dimensionless ratios of kinematic variables, such as $m_c/p_T$ in the case of production at large transverse momentum. In practice, there are several matrix elements of phenomenological importance for any given quarkonium state. Unfortunately we have no effective nonperturbative methods for calculating these matrix elements, with a few exceptions. The exceptions are matrix elements like $\langle \mathcal{O}_{1/2}(3S_1) \rangle$ that can be related to the wavefunction of the dominant $c\bar{c}_1$ Fock state. The remaining matrix elements must be treated as phenomenological parameters. The predictive power of the factorization formula resides in the fact that the same matrix elements must describe inclusive quarkonium production in all high energy processes.

The factorization formula (13) is simply a statement about the separation of scales in QCD. One can ignore the prediction of NRQCD for the relative magnitudes of the matrix elements, and simply take the formula as a model-
independent framework for analyzing quarkonium production. Any model that is consistent with QCD at short distances must be expressible in this form for some choice of the NRQCD matrix elements. In particular, the color-singlet and color evaporation models can be expressed as specific assumptions about the NRQCD matrix elements. The color-singlet model reduces to the assumption that the only matrix element that is nonzero is the color-singlet matrix element whose angular-momentum quantum numbers correspond to those of the $c\bar{c}$ pair in the dominant Fock state. This matrix element is $\langle O_{J/\psi}^{3S_1} \rangle$ for the $J/\psi$ and $\langle O_{\chi_{cJ}}^{3P_J} \rangle$ for the $\chi_{cJ}$. The color evaporation model corresponds to the assumption that the S-wave matrix elements dominate and that they are equal up to simple color and spin factors:

$$\langle O_{H}^{3S_1} \rangle = 6 \langle O_{H}^{1S_0} \rangle = 6 \langle O_{H}^{3S_1} \rangle = 6 \langle O_{H}^{1S_0} \rangle. \quad (15)$$

The P-wave matrix elements are all suppressed by a factor of $(m_D^2 - m_c^2)/m_c^2$, which is of order $v^2$. NRQCD predicts a much more intricate pattern for the relative magnitudes of the matrix elements and this pattern depends on the $J^{PC}$ quantum numbers of the quarkonium state $H$. The matrix elements that scale with the leading power of $v$ include both the color-singlet model matrix element and one of the color evaporation model matrix elements in (15). For the $J/\psi$, the only leading matrix element is $\langle O_{J/\psi}^{3S_1} \rangle$, which scales like $v^3$. For the $\chi_{cJ}$, the leading matrix elements are $\langle O_{\chi_{cJ}}^{3P_J} \rangle$ and $\langle O_{\chi_{cJ}}^{3S_1} \rangle$, both of which scale like $v^5$.

As a phenomenological framework for calculating quarkonium production, the NRQCD factorization approach is much less economical than the color evaporation model. For each quarkonium state, there are several matrix elements of phenomenological importance. For $J/\psi$ production, the largest matrix element is $\langle O_{J/\psi}^{1S_0} \rangle$, which scales like $v^3$. Its value can be determined from decays of the $J/\psi$. The next most important matrix elements are $\langle O_{J/\psi}^{3S_1} \rangle$, $\langle O_{J/\psi}^{3P_J} \rangle$, and $\langle O_{J/\psi}^{3P_J} \rangle$, which scale like $v^5$, $v^7$, and $v^7$, respectively. These three color-octet matrix elements must be treated as independent phenomenological parameters, in contrast with the single phenomenological parameter $f_{J/\psi}$ in the color evaporation model.

The NRQCD factorization approach differs dramatically from the color evaporation model in its predictions for the dependence of the cross section on the polarization of the quarkonium state. The color evaporation model predicts no dependence on the polarization. The NRQCD factorization approach predicts a nontrivial dependence, because the NRQCD matrix elements $\langle O_n^H \rangle$ depend on the polarization of the quarkonium state $H$. 

12
4 Applications to Charmonium Production

The NRQCD factorization approach has been applied to charmonium production in almost all possible high energy processes. Some of these applications are described in two recent reviews. I will focus on one specific application for which the predictions of the NRQCD factorization approach differ significantly from both the color-singlet and the color evaporation models.

4.1 \(J/\psi\) at Large Transverse Momentum

The NRQCD factorization formula leads to a dramatic prediction for the production of prompt charmonium at large transverse momentum in \(p\bar{p}\) collisions. The prediction is that at sufficiently large \(p_T\), most of the \(J/\psi\)'s will be transversely polarized. This prediction follows from several simple steps, which I proceed to discuss in detail.

The first step in the argument involves the separation of the scale \(p_T\) from the lower momentum scales of order \(m_c\) and smaller. According to standard factorization theorems of perturbative QCD, the inclusive production cross section for any hadron at large \(p_T\) is dominated by fragmentation, which means that it is produced by the hadronization of a single high-\(p_T\) parton. As pointed out by Braaten and Yuan, this factorization theorem applies to charmonium states as well as to light hadrons provided that \(p_T \gg m_c\). In the case of the \(J/\psi\), the factorization formula is

\[
\frac{d\sigma}{dp}(p\bar{p} \to J/\psi(P) + X) = \sum_i \int_0^1 dz d\sigma(p\bar{p} \to i(P/z) + X) D_{i \to J/\psi}(z),
\]

(16)

where \(d\sigma(p\bar{p} \to i + X)\) is the inclusive cross section for producing a parton of type \(i\) with larger transverse momentum \(p_T/z\). This cross section can be calculated in terms of the parton distributions for \(p\) and \(\bar{p}\) and parton cross sections \(d\hat{\sigma}\) that involve only the scale \(p_T/z\) and can therefore be calculated as power series in \(\alpha_s(p_T/z)\). The fragmentation function \(D_{i \to J/\psi}\) gives the probability that the hadronization of the parton \(i\) produces a \(J/\psi\) carrying a fraction \(z\) of the parton momentum. All effects of momentum scales smaller than \(p_T\), including the effects of the charm quark mass and nonperturbative effects involved in the formation of the bound state \(J/\psi\), are contained within the fragmentation functions. The largest fragmentation probabilities are those for \(c, \bar{c}\), and the gluon. The sum over partons \(i\) in (16) is completely dominate by the gluon, because the cross section for producing gluons is so much larger than that for producing charm quarks. The corrections to the factorization formula (16) from nonfragmentation processes are suppressed asymptotically by
powers of $m_c^2/p_T^2$. Thus the factorization of the scale $p_T$ leads to the conclusion that $J/\psi$ production at large $p_T$ is dominated by gluon fragmentation.

The second step in the argument involves the separation of the scale $m_c$ from the lower momentum scales of order $m_c v$ and smaller. The NRQCD factorization formula (14) can be used to express the gluon fragmentation function in the form

$$ D_{g \rightarrow J/\psi}(z) = \sum_n \hat{d}_{g \rightarrow c\bar{c}[n]}(z) \langle O_{J/\psi}^{1}(3S_1) \rangle, \tag{17} $$

where $\hat{d}_{g \rightarrow c\bar{c}[n]}$ is the fragmentation function for producing a $c\bar{c}$ in the state $n$ carrying a fraction $z$ of the momentum of the gluon. This fragmentation function involves only momenta of order $m_c$, and it can therefore be calculated as a power series in $\alpha_s(m_c)$. All nonperturbative effects involving the binding of the $c\bar{c}$ pair to form a $J/\psi$ are factored into the NRQCD matrix elements.

According to the color-singlet model, the gluon fragmentation function (17) should be dominated by the $c\bar{c}1(3S_1)$ term. The leading contribution to the short-distance coefficient for this term is of order $\alpha_s^3$ and comes from the parton process $g^* \rightarrow c\bar{c}gg$. Keeping only this term in the fragmentation function (17), the cross section predicted by (16) is about a factor of 30 below recent CDF data on prompt $J/\psi$ production at the Tevatron. It is this result that spelled the final demise of the color-singlet model.

According to the color evaporation model, the gluon fragmentation function (17) should be dominated by the S-wave matrix element with the largest short-distance coefficient. We will see that the NRQCD factorization approach leads to the same conclusion. As first pointed out by Braaten and Fleming, the term that dominates is the $c\bar{c}8(3S_1)$ term, which corresponds to the formation of a $J/\psi$ from a $c\bar{c}$ pair that is created in a color-octet $3S_1$ state. At leading order in $\alpha_s$, this term in the fragmentation function is

$$ D_{g \rightarrow J/\psi}(z) \approx \frac{\pi \alpha_s(m_c)}{96m_c^4} \delta(1-z) \langle O_{8}^{J/\psi} (3S_1) \rangle. \tag{18} $$

The relative importance of the various terms in the fragmentation function (17) depends on several different factors. We will compare each of these factors for the $c\bar{c}1(3S_1)$ term and the $c\bar{c}8(3S_1)$ term. The magnitude of the NRQCD matrix element is determined by its order in $v$, which is $v^4$ for $\langle O_{1}^{J/\psi} (3S_1) \rangle$ and $v^7$ for $\langle O_{8}^{J/\psi} (3S_1) \rangle$. The magnitude of the short-distance coefficient $\hat{d}$ is determined by its order in $\alpha_s$. The coefficient of the $c\bar{c}1(3S_1)$ term comes from the parton process $g^* \rightarrow c\bar{c}gg$ and is of order $\alpha_s^3$, while the $c\bar{c}8(3S_1)$ term has a much larger coefficient of order $\alpha_s$ coming from the parton process $g^* \rightarrow c\bar{c}$. 

14
However the relative importance of the terms also depends on the shape of the $z$-distribution of $\hat{d}$. The reason is that in the expression (16) for the cross section, the fragmentation function is folded with the cross section for producing a gluon, which is a rapidly falling function of the transverse momentum $p_T/z$ of the gluon. Since $d\sigma/dp_T^2$ falls like the inverse fourth power of the gluon’s transverse momentum, the cross section is effectively weighted by $z^4$. This dramatically suppresses the $c\bar{c}_1(^3S_1)$ term, which has a rather soft $z$ distribution with $\langle z \rangle \approx 0.4$. In contrast, there is very little suppression of the $c\bar{c}_8(^3S_1)$ term in (18), since it is sharply peaked near $z = 1$. Putting all the factors together, the $cc_1(^3S_1)$ term scales like $\langle z^4 \rangle \alpha_s^2 v^3$ while the $c\bar{c}_8(^3S_1)$ term scales like $\alpha_s v^7$. The suppression of the color-singlet term by $\langle z^4 \rangle \alpha_s^2$ overwhelms the suppression of the color-octet term by $v^4$. Thus the factorization of the scale $m_c$ leads to the conclusion that the gluon fragmentation function is dominated by the $c\bar{c}_8(^3S_1)$ term. This term is singled out because its short-distance coefficient is the lowest order in $\alpha_s$ and the most sharply peaked near $z = 1$.

Unfortunately, we cannot use the formula (18) to predict the cross section at large $p_T$, because the matrix element $\langle O_{J/\psi}(^3S_1) \rangle$ has not yet been determined accurately from other production processes. However, the shape of the cross section $d\sigma/dp_T$ obtained by inserting (18) into (17) is in agreement with the Tevatron data at large $p_T$. The value of the matrix element can therefore be determined by fitting the data. The resulting value of $\langle O_{J/\psi}(^3S_1) \rangle$ is consistent with suppression by a factor of $v^4$ relative to the color-singlet model matrix element $\langle O_{J/\psi}(^3S_1) \rangle$.

The color evaporation model gives a prediction for the $J/\psi$ cross section at large $p_T$ that is similar to that of the NRQCD factorization approach, because the color evaporation model matrix elements in (18) include $\langle O_{J/\psi}(^3S_1) \rangle$. However the NRQCD factorization approach differs dramatically in its predictions for the dependence of the cross section on the polarization of the $J/\psi$. The color evaporation model predicts the absence of any spin asymmetry. As pointed out by Cho and Wise, the NRQCD factorization approach predicts that $J/\psi$’s should be predominantly transversely polarized at sufficiently large $p_T$. The leading term in the gluon fragmentation function contributes only to the production of $J/\psi$’s that are transversely polarized. The radiative corrections to the fragmentation function were examined by Beneke and Rothstein, and they concluded that the spin alignment at large $p_T$ remains greater than 90%. The largest corrections to the spin alignment at values of $p_T$ that have been measured at the Tevatron come from nonfragmentation contributions that are suppressed by $m_c^2/p_T^2$, relative to the terms in (14). These contributions have been calculated by Beneke and Kraemer and by Leibovich. Their prediction is that the spin alignment should be almost zero at small...
\( p_T \), that it should begin to turn on at a \( p_T \) of 3 to 5 GeV, and that it should reach 60\% to 80\% of its maximum possible value for \( p_T \) of 20 GeV. An experimental measurement of the spin alignment in agreement with these predictions would constitute a dramatic triumph for the NRQCD factorization approach. It would also spell the final demise of the color evaporation model.

5 Conclusions

The NRQCD factorization approach has led to dramatic progress in our understanding of inclusive quarkonium production. Within this approach, the roles played by color and spin in the production process are somewhere intermediate between the extreme assumptions of the color-singlet and color evaporation models. In some cases, the NRQCD factorization approach gives predictions that differ dramatically from those of both the color-singlet model and the color evaporation model. A particularly important example is the production of \( J/\psi \) at large \( p_T \) in \( p\bar{p} \) colliders, where a measurement of the spin asymmetry will provide a stringent test of our present understanding of inclusive quarkonium production.

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