A Phenomenological Treatment of Chiral Symmetry Restoration and Deconfinement

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A phenomenological expression for the thermodynamic potential of gluons and quarks is constructed which incorporates the features of deconfinement and chiral symmetry restoration known from lattice simulations. The thermodynamic potential is a function of the Polyakov loop and chiral condensate expectation values. The gluonic sector uses a successful model for pure $SU(N_c)$ gauge theories in which the Polyakov loop eigenvalues are the fundamental order parameters for deconfinement. The quark sector is given by a Nambu-Jona-Lasinio model in which a constant background $A_0$ field couples the chiral condensate to the Polyakov loop. We consider the case of $N_f = 2$ in detail. For two massless quarks, we find a second order chiral phase transition. Confinement effects push the transition to higher temperatures, but the entropy associated with the gluonic sector acts in the opposite direction. For light mass quarks, only a rapid crossover occurs. For sufficiently heavy quarks, a first order deconfinement transition emerges. This simplest model has one adjustable parameter, which can be set from the chiral transition temperature for light quarks. It predicts all thermodynamic quantities as well as the behavior of the chiral condensate and the Polyakov loop over a wide range of temperatures.

Finite temperature QCD, in its variants which can be studied with lattice techniques, exhibits both deconfinement and chiral symmetry restoration. Depending on the model under study, there may be one or more phase transitions as the temperature is varied, or no phase transition at all. Finite density brings with it an additional rich set of possible behaviors at low temperatures. The transition from one phase to another may be marked by a first- or second-order phase transition, or two apparently different phases may be connected to one another, as in a liquid-gas transition. While we understand much about these complex behaviors, we lack a comprehensive, simple theoretical tool which gives us insight into possible behaviors. In short, we need a Landau-Ginzburg theory for finite temperature QCD.

The behavior of finite temperature QCD can be understood from the behavior of two order parameters: the Polyakov loop $P$, associated with deconfinement, and the chiral condensate, $\langle \bar{\psi} \psi \rangle$, associated with chiral symmetry restoration. Our aim is to construct simple phenomenological models incorporating both chiral symmetry restoration and deconfinement in a unified way, which can be applied to both quenched and unquenched behavior, for all values of $N_c$ and any number of flavors $N_f$. These models naturally give the behavior of $P$, $\langle \bar{\psi} \psi \rangle$, and thermodynamic variable such as pressure as a function of temperature and chemical potential. By incorporating both $P$ and $\langle \bar{\psi} \psi \rangle$ as order parameters, useful information about non-equilibrium behavior is also available.

Polyakov loop models have been used extensively to describe the deconfinement transition in pure gauge theories [1][2]. The two models of the deconfinement transition in $SU(N_c)$ gauge theories which we have developed [3], use the eigenvalues of the Polyakov loop as the fundamental order parameters. This accords with the results of high-temperature perturbation theory [4], and our models reproduce the leading $T^3$ behavior of perturbation theory at high temperature. Both of the models we have studied correctly predict a second order phase transition for $SU(2)$ and a first order phase transition for $SU(N_c)$ when

We gratefully acknowledge the support of the U.S. Dept. of Energy under DOE DE-FG02-91ER40628.
In the case of $SU(3)$, the center symmetry $Z(3)$ allows us to consider the vacuum expectation value of the trace of the Polyakov loop to lie along the real axis. The trace of the Polyakov loop in the fundamental representation can be parametrized along the real axis, as $P_F = 1 + \cos(2\pi/3 - \phi)$, where $\phi = 0$ corresponds to the confined phase and $\phi \neq 0$ corresponds to a non-zero value for $P_F$ signaling deconfinement. We will use our first model, model A, to describe deconfinement here. The free energy as a function of $\phi$ has the form

$$V_G = \frac{8\pi^2 T^4}{405} + \left(\frac{3T^2 M^2}{2\pi^2} - \frac{2T^4}{3}\right) \phi^2 - \frac{2T^4}{3\pi} \phi^3 - \frac{3T^4}{2\pi^2} \phi^4. \quad (1)$$

This model is obtained from a truncation of the free energy of massive gluons moving in a constant Polyakov loop background. Model A has a single free parameter, the mass scale $M$. The pressure, energy density and interaction measure derived from the above thermodynamic potential compare well with pure gauge simulations.

Quarks break $Z(3)$ symmetry explicitly. This effect is small for large quark mass, and the deconfinement transition remains first order for sufficiently large quark masses. Within model A, we find that for a single heavy quark, deconfinement persists as a first-order phase transition until a second-order critical point is reached at $m_c \approx 2.57M$. For quark masses below this value, deconfinement occurs as a smooth crossover.

A complete discussion of QCD at finite temperature must also incorporate chiral symmetry restoration. We include these effects via a Nambu-Jona-Lasinio (NJL) model coupled to the background Polyakov loop. The free energy of $N_f = 2$ quarks at 1-loop order can be written as

$$V_F = \frac{\sigma^2}{4G} - N_f N_c tr\ln(i\gamma^\mu D_\mu + m_0 + \sigma), \quad (2)$$

where $G$ is the four-fermion coupling constant, the composite field $\sigma = \bar{\psi}\psi$, and $D_\mu$ is a covariant derivative containing the background $A_0$ field. The constituent mass $m$ of the quarks is given as $m = m_0 + \sigma$, where $m_0$ is the current mass. The coupling constant $G$ as well as the non-covariant cutoff $\Lambda$ required to regularize the theory are fixed by zero temperature phenomenology. The values used are $G = 5.02 \text{ GeV}^{-2}$ and $\Lambda = 653 \text{ MeV}$, which lead to $f_\pi = 93 \text{ MeV}$ and $\bar{\psi}\psi = -2(250 \text{ MeV})^3$. For sufficiently large temperature, the finite temperature contribution of the functional determinant can be evaluated using a high temperature expansion. The quark part of our model has no remaining free parameters once it is calibrated to give the correct zero temperature physics.

The only free parameter in model A coupled to
the NJL model is $M$. The chiral transition temperature $T_c$ is shown in figure 1 as a function of $M$. As $M$ is lowered, $T_c$ decreases until $M = 300$ MeV, below which $T_c$ is essentially frozen at 175 MeV, the value predicted by the NJL model with a trivial Polyakov loop. At very low values of $M$, the energy density divided by $T^4$ changes from a monotonically increasing function of temperature to one which approaches the blackbody behavior from above in stark contrast to lattice data. If one chooses $M = 596$ MeV, the value which yields a pure gauge deconfinement temperature of $T_d = 270$ MeV, then the chiral transition temperature is 201 MeV which is somewhat higher than lattice simulations indicate. Choosing the value $M = 300$ MeV yields a reasonable value of the transition temperature while avoiding the problem of poorly behaved thermodynamic functions. Once $M$ is fixed there are no free parameters in the model.

The chiral order parameter $\sigma$ and the Polyakov loop are plotted as a function of temperature in figure 2. Choosing a zero current quark mass yields a clear second order chiral transition while the deconfinement order parameter displays crossover behavior. The pressure divided by $T^4$ is plotted in figure 3. The interaction measure $\Delta = (\epsilon - 3p)/T^4$ falls to zero at high temperatures as $1/T^2$, a behavior also seen in the pure gauge theory model.

The effect of adding a small chemical potential can also be studied using a high temperature expansion of the quark determinant. In figure 4 the phase diagram is displayed for $M = 300$ MeV. The chiral transition appears to be second order out to at least $\mu = 130$ MeV which is roughly the limit of reliability of the high-temperature expansion. The dotted line shows an elliptical extrapolation to higher $\mu$. This line extrapolates to precisely the value predicted by the NJL model at zero temperature, displayed as a large point in the figure.

We plan to continue studying the phase diagram of this model. The dependence of the location of the tricritical point on the mass parameter $M$ remains to be worked out. The addition of a non-zero bare quark mass is expected to change the tricritical point to a second order critical point and shift its location slightly in the $T - \mu$ plane.

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