Pre-main-sequence isochrones – I. The Pleiades benchmark

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ABSTRACT
We present a critical assessment of commonly used pre-main-sequence isochrones by comparing their predictions to a set of well-calibrated colour-magnitude diagrams of the Pleiades in the wavelength range 0.4 to 2.5 µm. Our analysis shows that for temperatures less than 4000 K the models systematically overestimate the flux by a factor two at 0.5 µm, though this decreases with wavelength, becoming negligible at 2.2 µm. In optical colours this will result in the ages for stars younger than 10 Myr being underestimated by factors between two and three.

We show that using observations of standard stars to transform the data into a standard system can introduce significant errors in the positioning of pre-main-sequences in colour-magnitude diagrams. Therefore we have compared the models to the data in the natural photometric system in which the observations were taken. Thus we have constructed and tested a model of the system responses for the Wide-Field Camera on the Isaac Newton Telescope.

As a benchmark test for the development of pre-main-sequence models we provide both our system responses and the Pleiades sequence.

Key words: stars: evolution – stars: formation – stars: pre-main-sequence – stars: fundamental parameters – techniques: photometric – open clusters and associations: general – Hertzsprung-Russell and colour-magnitude diagrams

1 INTRODUCTION
Our understanding of time-dependent physical processes in young pre-main-sequence (pre-MS) and main-sequence (MS) populations is fundamentally limited by poorly constrained timescales, the derivation of which requires the determination of ages for pre-MS stellar populations. These ages are generally derived via the comparison of photometric observations of pre-MS stars in young open clusters with theoretical models (or isochrones) using colour-magnitude diagrams (CMDs).

Many previous studies deriving stellar parameters from observations of young clusters in this way have shown large discrepancies between the photometric data and the model predictions. It has been well documented that pre-MS isochrones generally do not fit the entire dataset in CMDs; e.g. whilst tracing the sequence defined by the higher mass members they will not simultaneously follow the sequence in the lower mass regime (see for instance Hartmann 2003; Stauffer et al. 2007). In addition, pre-MS isochrones from different evolutionary models give ages that differ by a factor two for the same cluster (e.g. Dahm 2005; Mayne et al. 2007). Furthermore, the same dataset compared to isochrones in different colours can lead to different age determinations (e.g. Naylor et al. 2002).

The situation has become even more uncertain with the realisation that ages derived from the MS are systematically older by a factor 1.5 to 2 than ages derived from the pre-MS (e.g. Naylor 2009). To make further progress we require a precise test of the pre-MS models to determine whether the ages could be in error by such a large factor, and if any parts of the pre-MS or any colour combinations are reliable age indicators. Our longer term aim is that if we have a clear understanding of the limitations of pre-MS ages we may be able to create a reliable pre-MS age scale.

In this work, therefore, we set up a benchmark test for pre-MS isochrones. Most obviously that benchmark must be a cluster that contains a significant number of pre-MS objects, as well as a populated MS to show that the models at least fit these more evolved stars. In addition it should have distance and age determinations which are independent of the CMD, and preferably a low extinction (which might otherwise complicate transformations). Given that most pre-MS isochrones are only available for solar metallicity, the
cluster must be at least close to solar composition. Finally, it should be nearby so that the lowest possible masses can be accessed. The Pleiades is the only cluster which fits all these criteria, and Stauffer et al. (2007) have already undertaken a comparison in V and I_c, which shows that at least the V-band luminosity is not matched by two models available at that time.

In this paper we return to the Pleiades with the aim of assessing which of the most recent models best match the data. This then allows us to quantify the remaining mismatch so that we can make a realistic assessment of its impact on pre-MS ages. We trace the mismatch as a function of wavelength using observations in a set of filters which are contiguous in wavelength and cover 0.4 to 2.5 μm. The data reach cooler temperatures than the Stauffer et al. (2007) study, and are free of the uncertainties introduced by transforming heterogeneous datasets into a standard photometric system.

In Section 2 we describe the set of interior and atmospheric theoretical models we have studied and examine systematic differences between the different formalisms. In Section 3 we describe a set of observations, of both standard stars and the Pleiades, which we use to test the models. In Section 4 we compare the models to the Pleiades dataset using CMDs. Section 5 describes the comparison of the models to a sample of MS binaries with dynamically constrained masses. In Section 6 we quantify the mismatch between the models and the data as a function of wavelength. Our conclusions are detailed in Section 7.

2 THE MODELS

To derive fundamental parameters, such as age and distance, from fitting CMDs we require model isochrones, which must also be transformed into the requisite photometric system. Isochrones are constructed from stellar interior models that predict Lbol, Teff and surface gravity (log g). Colours are then calculated through a colour-Teff relation and magnitudes via bolometric corrections to Lbol. Both relations can be derived by folding atmospheric model flux distributions through the appropriate photometric filter responses. The colours and magnitudes must then be calibrated to a standard scale. Therefore, for this study we have adopted a set of pre-MS interior models and also a collection of atmospheric models with which to derive the required relations.

2.1 Pre-main-sequence interior models

We tested those pre-MS interior models that were publicly available, covered a wide range in stellar mass and had been designed specifically for the investigation of pre-MS evolution. The sets of interior models studied here are those of Baraffe et al. (1998), Siess, Dufour, & Forestini (2000), D’Antona & Mazzitelli (1997) and Dotter et al. (2008) (hereafter BCAH98, SDF00, DAM97 and DCJ08 respectively). Note that the recent Pisa models (Tognelli, Prada Moroni, & Degl’Innocenti 2011) do not extend to the age of the Pleiades (see Section 4.1) and therefore are not used in this study.

Fig. 1 shows the variation between the different pre-MS evolutionary tracks and the 4.6 Gyr isochrone for a range of masses. There are systematic differences in both the mass tracks, especially at young ages, and location of the MS, particularly for low-mass stars. These differences stem from variations in the treatment of various physical processes in addition to the values of adopted parameters. Most notable are the treatment of convection, the opacity sources and the treatment of the stellar interior/atmosphere boundary (see Hillenbrand & White 2004).

The BCAH98 models have been computed for two different mixing length parameters α = 1.0 (general mixing length parameter) and α = 1.9 (solar calibrated value), where for stellar masses below 0.6 M⊙ these models are identical as the dependence on the mixing length parameter is negligible in this regime (Baraffe et al. 2002). Further note that the publicly available SDF00 models have a slightly modified solar abundance and mixing length parameter from the solar calibrated model (cf. Z = 0.02 and α = 1.5 with Z = 0.0249 and α = 1.605 in Siess et al. 2000).

2.2 Atmospheric models

We required atmospheric model flux distributions to calculate a theoretical colour-Teff relation and bolometric corrections. We produced a spectral library consisting of the PHOENIX BT-Settl model atmospheres for 400 ≤ Teff ≤ 7800 K and the ATLAS9 models with newly updated opacity distribution functions (ODFnew) for 8000 ≤ Teff ≤ 50000 K. Despite the differences in microphysics between the two sets of atmospheric models, we found that at the transitional Teff = 8000 K derived colours from both sets of models agree to within 0.02 mag in all colours.
2.2.1 The atlas9/ODFnew models

The atlas9/ODFnew atmospheric models are those of Castelli & Kurucz (2004)\(^2\). These models are based on the assumption of steady-state plane-parallel layers under the influence of local thermodynamic equilibrium and computed using the solar abundances of Grevesse & Sauval (1998). Line blanketing effects are computed statistically via the use of the opacity distribution functions which average the contribution from the various atomic and molecular species as described in Kurucz (1979). A pure mixing length theory (Böhm-Vitense 1958) is used in the computation of these models and we adopted those termed “no-overshoot” on the basis that these better recreate the observed spectra of stars with \(T_{\text{eff}}\) greater than the Sun (Castelli, Gratton, & Kurucz 1997). The mixing length parameter is set to \(\alpha = 1.25\) with a microturbulent velocity \(\xi = 2\) km\(\text{s}^{-1}\).

2.2.2 The BT-Settl models

The BT-Settl atmospheric models are those described in Allard, Homeier, & Freytag (2011)\(^3\). These models have been updated using an updated version of the phoenix atmospheric code (cf. Allard et al. 2001) to include the updated Barber et al. (2006) BT2 H\(_2\)O line list, the revised solar abundances of Asplund et al. (2009) and a sophisticated cloud model that accounts for the settling of dust grains (Allard et al. 2003). The effects of turbulent mixing in the atmosphere are calculated by interpolating from 2-D and 3-D radiation hydrodynamic models (Ludwig, Allard, & Hauschildt 2006), with the treatment of dust for cooler models (\(T_{\text{eff}} < 2600\) K) based on the dust formation cloud model that accounts for the settling of dust grains (Allard et al. 2003). The mixing length parameter is defined to be \(\alpha = 2.0\) with a microturbulent velocity of \(\xi = 2\) km\(\text{s}^{-1}\).

3 THE DATA

3.1 Observations

Our observations are a subset of a large survey of pre-MS clusters (the remainder of which will be published in an upcoming paper; Bell et al. in prep.) and were obtained using the 2.5-m Isaac Newton Telescope (INT) on La Palma. The survey was split over two runs, using the same instrumentation and filter set on both occasions, namely the four EEV 2k \(\times\) 4k CCD Wide-Field Camera (WFC) with a 34’ \(\times\) 34’ field-of-view and the \((Ugriz)_{\text{WFC}}\) filter set. A combination of long and short exposures were used to ensure that any bright stars saturated in the longer exposures could be measured in the shorter exposures. Whilst the \(( gri )_{\text{WFC}}\) filters are based on the Sloan Digital Sky Survey (SDSS) design there are no similar \(u\) and \(z\) filters for the WFC and so the Royal Greenwich Observatory (RGO) \(U_{\text{WFC}}\) and \(Z_{\text{WFC}}\) filters were used. The first set of observations were taken during the 21\textsuperscript{st} – 26\textsuperscript{th} Oct. 2007 with the second run during the 13\textsuperscript{th} – 19\textsuperscript{th} Sept. 2008.

Five overlapping fields within the Pleiades open cluster were observed over the course of the two runs (see Fig. 2 and Table 1). In addition, four fields in Stripe 82 of the SDSS were used as standard fields and routinely observed throughout both runs.

3.2 Data reduction

Each image was debiased and flat fielded using a median stacked bias and flat field frames. Both the \(i_{\text{WFC}}\)- and \(Z_{\text{WFC}}\)-band images were defringed using library fringe frames. The data were analysed using the cluster package as described in Naylor et al. (2002), Burningham et al. (2003) and Jeffries et al. (2004). The \(i_{\text{WFC}}\)-band images for a particular field were combined (after calculating spatial transformations between separate frames) with the resulting deep image used for object identification and detection. An object list was created and optimal photometry (Naylor 1998) was then performed providing a flux for each object at a given position. A spatially varying profile correction was calculated from the bright unsaturated stars in each frame so that the photometry could later be calibrated using standard star observations. Although the \(i_{\text{WFC}}\)-band images were combined, optimal photometry was carried out on each individual frame, so as to ensure that the good signal-to-noise in one frame was not outweighed by a poorer signal-to-noise in another frame upon combining the images. The individual profile corrected measurements of each frame were then combined by weighting each measurement in accordance with its signal-to-noise ratio and accounting for differences in air mass between separate frames. An additional statistical un-
Table 1. The central coordinates for each field-of-view and exposure times in the INT-WFC bandpasses for the observations of the Pleiades.

| Field | RA Dec. (J2000.0) | Filter | Exposure time (s) |
|-------|------------------|--------|------------------|
| Pleiades | 03h 46m 19.2s | $U_{WFC}$ | 1, 10 |
| Field A | +23° 38′ 02.4″ | $g_{WFC}$ | 1, 10, 100, 500($\times$ 2) |
| | | $r_{WFC}$ | 1, 7, 50, 250 |
| | | $i_{WFC}$ | 1, 10, 100($\times$ 2) |
| | | $Z_{WFC}$ | 1, 7, 50, 250 |
| Pleiades | 03h 45m 00.0s | $U_{WFC}$ | 1, 10 |
| Field B | +23° 44′ 45.6″ | $g_{WFC}$ | 1, 10, 100, 500($\times$ 2) |
| | | $r_{WFC}$ | 1, 7, 50, 250 |
| | | $i_{WFC}$ | 1, 10, 100($\times$ 2) |
| | | $Z_{WFC}$ | 1, 7, 50, 250 |
| Pleiades | 03h 43m 44.4s | $U_{WFC}$ | 1, 10 |
| Field C | +23° 58′ 08.4″ | $g_{WFC}$ | 1, 10, 100, 500($\times$ 2) |
| | | $r_{WFC}$ | 1, 7, 50, 250 |
| | | $i_{WFC}$ | 1, 10, 100($\times$ 2) |
| | | $Z_{WFC}$ | 1, 7, 50, 250 |
| Pleiades | 03h 42m 36.0s | $U_{WFC}$ | 1, 10 |
| Field D | +23° 58′ 08.4″ | $g_{WFC}$ | 1, 10, 100, 500($\times$ 2) |
| | | $r_{WFC}$ | 1, 7, 50, 250 |
| | | $i_{WFC}$ | 1, 10, 100($\times$ 2) |
| | | $Z_{WFC}$ | 1, 7, 50, 250 |
| Pleiades | 03h 43m 44.4s | $U_{WFC}$ | 1, 10 |
| Field E | +24° 24′ 28.8″ | $g_{WFC}$ | 1, 10, 100, 500($\times$ 2) |
| | | $r_{WFC}$ | 1, 7, 50, 250 |
| | | $i_{WFC}$ | 1, 10, 100($\times$ 2) |
| | | $Z_{WFC}$ | 1, 7, 50, 250 |

3.3 Zero-point stability

Standard field observations were taken on five separate evenings. On only one of the five nights were conditions photometric throughout, the other four nights were affected either at the beginning or the end by the presence of cloud. We therefore calculated a zero point for each $g_{WFC}$-band observation by comparing the instrumental magnitudes to those in the SDSS catalogue, and correcting for extinction using a mean $g_{WFC}$-band extinction coefficient ($k_{g_{WFC}} = 0.19$) for La Palma. We could then identify when the zero-point became unstable and define half nights of bona fide photometric conditions, which were then used for the photometric calibration procedure (the maximum deviation over the course of a single half night was 0.05 mag). Due to a paucity of blue stars in our standards catalogue, we opted to use the colour range which was most populated ($0.6 \leq (g - i)_{WFC} \leq 1.0$) and define our zero-point at a median colour of $(g - i)_{WFC} = 0.8$. This colour range cut in $(g - i)_{WFC}$ defines the necessary cuts in the other INT-WFC colours ($1.05 \leq (U - g)_{WFC} \leq 1.96$, $0.14 \leq (r - i)_{WFC} \leq 0.31$ and $0.02 \leq (i - Z)_{WFC} \leq 0.18$ with median values of $(U - g)_{WFC} = 1.505$, $(r - i)_{WFC} = 0.225$ and $(i - Z)_{WFC} = 0.1$). The seeing generally varied between 1 and 2 arcsec in the temporal regions defined above.

3.4 Transforming the data into the standard system

Traditional photometric calibration transforms instrumental magnitudes into magnitudes on a standard system. Typically colours and magnitudes are transformed using linear functions of the form

$$g_{\text{stand}} = \psi_g (g - i)_{\text{inst}} - k_g \chi + z_g,$$

$$\text{(1)}$$

$$\text{(g - i)}_{\text{stand}} = \psi_{g(i)} (g - i)_{\text{inst}} - k_{g(i)} \chi + z_{g(i)},$$

$$\text{(2)}$$

where $\chi$ is the airmass, $k$ the extinction coefficients, $z$ the zero-points and $\psi$ the colour terms ($k$, $z$ and $\psi$ are determined from observations of standard stars). Fig. 3 shows the $g$-band magnitude and $g - i$ colour differences between the standard SDSS and the natural INT-WFC photometric systems (see Appendices A and B for details). It is immediately obvious that there is no linear colour-dependent transformation that can be applied to INT-WFC observations of MS stars to calibrate them into the standard SDSS photometric system. Even if non-linear MS transformations were created, it is clear that they could not be used to transform observations of red ($(g - i)_{\text{SDSS}} \gtrsim 1.8$) pre-MS stars into the standard system. This is due to differences in the spectra between MS and pre-MS stars of the same colour. These differences can result in both $g$-band magnitude and $g - i$ colour errors of order 0.1 – 0.15 mag at a colour of $(g - i)_{\text{SDSS}} \approx 3$, culminating in stars that ultimately occupy the wrong position in CMD space. Even larger errors can be
Table 2. A sample of the full Pleiades photometric catalogue with colours and magnitudes in the natural INT-WFC photometric system. Due to space restrictions, we only show the $g_{\text{WFC}}$ and $(g-i)_{\text{WFC}}$ colours and magnitudes as a representation of its content. The full photometric catalogue (available online) also includes photometry in the $(U-g)_{\text{WFC}}$, $(r-i)_{\text{WFC}}$, and $(i-Z)_{\text{WFC}}$ colours and the $U_{\text{WFC}}$, $r_{\text{WFC}}$, $i_{\text{WFC}}$, and $Z_{\text{WFC}}$ magnitudes. Columns list unique identifiers for each star in the catalogue: field and CCD number (integer and decimal), ID, RA and Dec. (J2000.0), CCD pixel coordinates of the star, and for each of $g_{\text{WFC}}$ and $(g-i)_{\text{WFC}}$ there is a magnitude, an uncertainty on the magnitude and a flag (OO represents a "clean detection"; see Burningham et al. 2003 for a full description of the flags.)

| Field | ID | RA (J2000.0) | Dec. (J2000.0) | x   | y   | $g_{\text{WFC}}$ | $\sigma_{g_{\text{WFC}}}$ | Flag | $(g-i)_{\text{WFC}}$ | $\sigma_{(g-i)_{\text{WFC}}}$ | Flag |
|-------|----|--------------|----------------|-----|-----|-----------------|-------------------|------|------------------|---------------------|------|
| 21.04 | 8  | 03 46 34.136 | +23 37 26.25   | 1204.056 | 1439.980 | 8.236          | 0.010            | OO   | -0.412           | 0.014               | OL   |
| 22.02 | 16 | 03 43 41.521 | +23 38 56.30   | 998.324 | 2035.800 | 8.523          | 0.010            | OS   | -2.653           | 0.014               | SS   |

Figure 3. Calculated transformations between the SDSS and INT-WFC photometric systems. The transformations have been calculated for reddened (nominal $E(B-V) = 1$ mag; red) and unreddened (blue) ZAMS (crosses), 10 Myr (asterisks) and 1 Myr pre-MS stars (open circles). Upper panel: $\Delta g$ versus $(g-i)_{\text{SDSS}}$. Lower panel: $\Delta (g-i)$ versus $(g-i)_{\text{SDSS}}$. No single (even non-linear) transformation will calibrate observations of both red $(g-i)_{\text{SDSS}} \gtrsim 1.8$ MS and pre-MS stars into a standard system, and if either are reddened the situation becomes worse ($\gtrsim 0.15$ mag at a colour $(g-i)_{\text{SDSS}} \gtrsim 3$).

Figure 4. Magnitude difference in the $U_{\text{WFC}}$-band between the WFC$_{\text{calc}}$ and WFC$_{\text{obs}}$ photometric catalogues as a function of $(U-g)_{\text{WFC, calc}}$. Asterisks represent the median value of all points in a given bin of size 0.25 mag in colour. The error bars on these symbols are the standard error about the median value. Open circles represent individual data points and are plotted where the number of points is insufficient to calculate the median value in a given bin (defined as five). The error bars of these symbols are the individual uncertainties associated with that point. The dashed lines represent the $\pm 0.02$ mag level with respect to zero.

3.5 Calibrating the natural system

If the observations cannot be transformed into a standard photometric system, then the photometry must remain in the natural photometric system and the theoretical models transformed into this system. To achieve this we need a reliable model for the throughput for each bandpass as a function of wavelength. We emphasise that this is not simply a matter of using the filter responses, or even multiplying the CCD quantum efficiency by the filter response, but involves allowing for every element from the Earth’s atmosphere through to the CCD to create a system response.
We present our best estimates for the system responses in Appendix A.

Having obtained the system responses it is crucial that we obtain an estimate of how precise these are. In principle we could achieve this by folding the observed fluxes of spectrophotometric standards through our estimated system responses and seeing how closely the observed and predicted magnitudes match. In practice such standards have not been measured in sufficient numbers with the range of colours and magnitudes to prove useful for our purposes. Instead we took a sample of MS stars, used their observed SDSS colours to establish their effective temperatures, and then folded model atmospheres of the appropriate temperature through our estimated system responses to find their magnitudes in the INT-WFC system.

We defined our sample of MS stars using a series of colour-colour diagrams to isolate the observed MS in the Stripe 82 fields and trim any obvious outliers. To obtain colours and magnitudes in the INT-WFC system we then transformed the standard SDSS colours and magnitudes into the INT-WFC photometric system (hereafter WFC\textsubscript{calc}). These transforms are calculated by folding model atmospheres through both the SDSS and our estimated INT-WFC responses and are given in Table 3. The details of these calculations are given in Appendix B, and the transformed standards in the INT-WFC colours are given in Table 4.

With the INT-WFC standards established, we can evaluate how well constrained the calculated system responses of the INT-WFC are. Calculating zero-points in the colour ranges specified in Section 3.3 and applying these zero-points and extinction coefficients only (no colour terms) to all standard field observations we created a set of observed standard star colours and magnitudes in the INT-WFC system. To create a single catalogue for each of the four standard fields we performed the normalisation process as described in Section 3.2 and merged all four fields to create a catalogue of observed colours and magnitudes of Stripe 82 standards in the INT-WFC photometric system (hereafter WFC\textsubscript{obs}). After the normalisation procedure, we examined, for each star, the magnitude shift between each observation and the mean observed magnitude. This is an indicator of the inter-
Table 3. The calculated transformations between the SDSS and INT-WFC photometric systems for unreddened MS stars where Δ represents (INT-WFC – SDSS). The full table is available online, a sample is shown here as a representation of its content.

| Teff | log g | ∆g | (g − i)SDSS | ∆(u − g)SDSS | ∆(z − i)SDSS |
|------|-------|-----|--------------|--------------|--------------|
| 2775 | 5.25  | -0.29300 | -0.04765 | 3.83961 | 0.01264 | 4.12961 | 0.24688 | 2.20001 | -0.17476 | 1.22123 |
| 3290 | 5.11  | -0.23094 | -0.10391 | 2.68386 | -0.02563 | 3.06419 | 0.12898 | 1.20809 | -0.08704 | 0.66867 |
| 3528 | 4.98  | -0.21237 | -0.11299 | 2.35355 | -0.03558 | 2.87477 | 0.09764 | 0.94887 | -0.06707 | 0.53108 |
| 3683 | 4.92  | -0.20428 | -0.12031 | 2.19344 | -0.03595 | 2.83717 | 0.07940 | 0.81597 | -0.05550 | 0.45615 |
| 3829 | 4.87  | -0.19761 | -0.12806 | 2.05653 | -0.03751 | 2.81937 | 0.06188 | 0.70141 | -0.04429 | 0.38859 |

4 COMPARING THE MODELS AND THE DATA – THE PLEIADES

Having characterised the natural INT-WFC photometric system and calculated the corresponding system responses, we can now test the pre-MS interior and stellar atmospheric models by comparing them to our Pleiades data. We obtained a list of approximately 180 members, by cross-correlating our full catalogue with the combined membership catalogues of Stauffer et al. (2007) and Lodieu, Deacon, & Hambly (2012). This membership list is given in Table 5.

4.1 Model parameters

We used a distance modulus dm = 5.63 ± 0.02 which is based on the trigonometric parallax of Soderblom et al. (2005). We chose to rely on the lithium depletion age, since although it is based on the predictions of stellar interior models, Jeffries & Oliveira (2005) show there is a very high level of agreement between the various models. Furthermore we used the age based on the boundary in $K_s$ (130 Myr) of Barrado y Navascués, Stauffer, & Jayawardhana (2004), since as we shall show later, $K_s$ is the best predictor of luminosity available to us. We used a reddening of $E(B − V) = 0.04$ based on the mean extinction $A_V = 0.12$ (see Stauffer et al. 1998) and $R_V = 3.2$. Hence, to transform the theoretical models into the observable plane, we first reddened the atmospheric models by a nominal $E(B − V) = 0.04$ using the reddening law of Cardelli, Clayton, & Mathis (1989) and then calculated bolometric corrections using Eqn. B2.

4.2 Discussion

Fig. 9 shows four CMDs of the Pleiades with the pre-MS isochrones of BCAH98, SDF00, DAM97 and DCJ08 overlaid. The break between MS and pre-MS stars in the Pleiades occurs at $(g-i)_{WFC} ≃ 2$, and is marked by a paucity of stars in all CMDs. It is clear that the isochrones all follow the Pleiades MS reasonably well, though as we shall see in Section 6 this is in part due to compensating errors.
Table 5. A sample of the catalogue of colours and magnitudes in the natural INT-WFC photometric system for Pleiades members. The columns and content are in the same format as Table 2. The full photometric catalogue is available online.

| Field ID | RA (J2000.0) | Dec. (J2000.0) | x     | y     | g_{WFC} | σ_{g_{WFC}} | Flag | (g−i)_{WFC} | σ_{(g−i)_{WFC}} | Flag |
|----------|--------------|---------------|-------|-------|---------|-------------|------|-------------|----------------|------|
| 24.02    | 45           | 03 41 03.006  | +23   | 43    | 21.42   | 426.717     | 3696.221 | 0.006       | 2.553          | 0.006 |
| 24.02    | 118          | 03 41 05.230  | +23   | 50    | 14.82   | 526.182     | 2444.212 | 0.015       | 3.228          | 0.017 |

Figure 9. Optical/near-infrared CMDs of Pleiades members. The 130 Myr pre-MS isochrones of BCAH98 $\alpha = 1.0$ (red, continuous), BCAH98 $\alpha = 1.0$ (red, dashed), SDF00 (blue), DAM97 (green) and DCJ08 (cyan) are overlaid, adopting a distance modulus $dm = 5.63$ and a reddening $E(B-V) = 0.04$. The isochrones have been transformed into the observable plane using bolometric corrections derived by folding the atmospheric model flux distributions (see Section 2.2) through the calculated INT-WFC system responses. The black line in each panel represents the spline fit (by eye) to the Pleiades single-star sequence. The open circles represent the photometric data, with the associated uncertainties in colour and magnitude shown as the bars. Top left: $K_s,g_{WFC}$ CMD. Top right: $g_{WFC},(g-i)_{WFC}$ CMD. Bottom left: $r_{WFC},(i-Z)_{WFC}$ CMD. Bottom right: $K_s,J-K_s$ CMD.

in each individual band. None of the models, however, trace the observed pre-MS locus, with each predicting colours that are too blue for a given stellar mass. This issue has already been highlighted by Stauffer et al. (2007) (see their Fig. 13), though from their data one could only be certain there was a problem in the V-band, whereas our data show that all optical bands are affected.

This is part of a broader picture where it is known that MS stars are similarly affected, at least in the V-band (see Section 5). It is well known that there are sources of opacity missing from the atmospheric models for cool stars ($T_{\text{eff}} \lesssim 3700$ K) in the optical regime of the spectrum (Leggett et al. 1996; Baraffe et al. 1998; Alvarez & Plez 1998; Baraffe et al. 2002). Although the BT-Settl models are computed using more complete line lists than previous generations of PHOENIX models, there is still a need for additional data to provide fully comprehensive line lists, especially for the molecular opacities ($\text{H}_2\text{O}$, TiO, CH$_4$, etc.; see Partridge & Schwenke 1997 and Schwenke 1998).

If we are to be quantitative about the missing opacity we need to measure the missing flux individually in each bandpass, rather than as a function of colour. The most straightforward way of achieving this is to identify a photometric bandpass where the missing sources of opacity...
are minimal, and create colours with respect to that band. This is equivalent to using the luminosity in that band to define the temperature. To test the temperature scale requires masses (from which the models predict temperatures) from binaries, data which are simply not available for the Pleiades. Instead, as the opacity problem affects MS stars as well as pre-MS stars (in the same $T_{\text{eff}}$ range), we will use MS binaries to identify a band we can use.

5 COMPARING THE MODELS AND THE DATA – MAIN-SEQUENCE BINARIES

Although there are many MS binaries with well determined masses and distances we could use to test the models, the outstanding problem is the small sub-sample of these which have individual magnitudes in many colours. Astrometric binaries are often separated in the infrared (IR), and so lack individual optical colours, and eclipsing binaries often have lightcurves in only a small number of colours. To overcome this problem we will model the system magnitude, which as we shall discuss later still gives significant insight as a function of stellar temperature.

5.1 The sample

To create our sample, data for low-mass binaries from Delfosse et al. (2000) were supplemented with data for higher-mass binaries from Andersen et al. (1987); Clausen et al. (2008); Clausen et al. (2009); Clement et al. (1997b); Clement et al. (1997a); Lacy et al. (1997); Lacy et al. (2005); López-Morales & Ribas (2005); Popper et al. (1986); Popper (1994); Popper (1997); Torres et al. (1997), and Torres et al. (2009). We imposed an upper limit to the mass uncertainty on both the primary and secondary components of 10 per cent, and in addition limited ourselves to non-contact binaries in which the effects of tidal forces and mass transfer may bias the results. We also limited the sample to binaries with associated uncertainties in the parallax of $\leq 11$ per cent; thus our conclusions are unaffected by the Lutz-Kelker bias (Lutz & Kelker 1973). The sample span the mass range $0.1 \leq M_\odot \leq 1.4$ for the individual components, with the higher mass limit chosen as the BCAH98 models only extend to such masses. For the low-mass systems in Delfosse et al. (2000), the apparent magnitudes were from the homogeneous dataset of Leggett (1992). A number of higher mass systems only had a Strömgren $b - y$ colour and so these were transformed into $V - I_c$ colours according to the relation of Bessell (1979) which is accurate to within 0.01 mag for MS stars. Near-IR measurements were taken from the 2MASS point source catalogue (Cutri et al. 2003). Parallax measurements were taken from the Hipparcos catalogue (Perryman et al. 1997), the Yale General Catalogue of Trigonometric Stellar Parallaxes (van Altena, Lee, & Hoffleit 1995) and Ségransan et al. (2000).

5.2 The models

We adopted 2 Gyr isochrones with which to compare observations against theory as we expect these binaries, which are located in the field, to be old systems. Note that at such ages, the mass scale is insensitive to changes in age. For each binary component, the bolometric luminosity ($L_{\text{bol}}$), defined by the stellar interior models, was converted into an absolute magnitude using bolometric corrections calculated using Eqn. B2 by folding the atmospheric models through the revised $UBVRI$ responses of Bessell & Murphy (2012) and the $JHK_s$ responses of Cohen et al. (2003) to create bolometric corrections which were used to derive the predicted absolute magnitudes. Upper panel: $\Delta V$. Upper middle panel: $\Delta I_c$. Middle panel: $\Delta J$. Lower middle panel: $\Delta H$. Lower panel: $\Delta K_s$.

The Pleiades benchmark

Figure 10. The difference between the observed and theoretically predicted system absolute magnitudes (where $\Delta$ implies calculated – observed) for the 28 binary systems as a function of the measured primary component stellar mass. Bolometric luminosities have been calculated using 2 Gyr interior models of BCAH98 $a = 1.9$ (red), SDF00 (blue), DAM97 (green) and DCJ08 (cyan). The atmospheric model flux distribution were folded through the revised $UBVRI$ responses of Bessell & Murphy (2012) and the $JHK_s$ responses of Cohen et al. (2003) to create bolometric corrections which were used to derive the predicted absolute magnitudes. Upper panel: $\Delta V$. Upper middle panel: $\Delta I_c$. Middle panel: $\Delta J$. Lower middle panel: $\Delta H$. Lower panel: $\Delta K_s$.
were reddened accordingly and the bolometric corrections calculated as discussed in Section 4.1.

5.3 Discussion

Fig. 10 shows the residuals (calculated – observed) in luminosity relation in different bandpasses. As the binary mass ratio $q \rightarrow 1$ both components have the same mass, whereas when $q \rightarrow 0$ the primary component is likely to dominate the flux output of the system. Thus we plot the difference in absolute magnitude as a function of the measured primary mass. The range of $q$ as defined by our sample is $q = m_2/m_1 = 0.33 – 1.0$, where for 65 per cent of these the ratio $q > 0.9$. Regardless of the choice of interior model, there are significant systematic deviations (especially in the optical regime) for masses below $\approx 0.4 – 0.5 M_\odot$. Fig. 10 shows that the differences between the observed and calculated system absolute magnitude decreases as one moves to redder bandpasses, from approximately 2.5 mag in the V-band to less than 1 mag in the $K_s$-band. In the near-IR, and in particular the $K_s$-band, there is far less spread for a given stellar mass (see also Martín et al. 2000 and Delfosse et al. 2000), with the interior models predicting similar luminosities. This further highlights the fact that there is a problem with the optical colours predicted by the atmospheric models for masses below $\approx 0.4 – 0.5 M_\odot$ ($T_{\text{eff}} \lesssim 3700$ K).

It has been noted in the literature that short-period, close-in binary system components have radii which are inflated of order 10 per cent with respect to evolutionary models of low-mass MS stars (e.g. Kraus et al. 2011). To test whether we observe this effect in the photometry of these systems, we plotted Fig. 10 as a function of physical separation. We found no evidence for the models to either under- or overpredict the absolute system magnitude for short-period binaries with smaller physical separations.

What is clear, however, is that of all the bands available, the $K_s$-band magnitude is closest to that predicted by the models for a given temperature, both in terms of mean magnitude and spread. This is unsurprising, since the $K_s$-band is less affected than either the J- or H-band by the missing sources of opacity that arise from incomplete H$_2$O line lists (see the discussion on the completeness of the BT2 H$_2$O line list as a function of wavelength by Barber et al. 2006). In what follows, therefore, we will use the $K_s$-band magnitude to determine the temperatures of Pleiades members.

6 QUANTIFYING THE DISCREPANCY

Our approach for quantifying the effects of the missing opacity is to determine the temperature of a given Pleiad by comparing its $K_s$-band magnitude with that predicted by a given model. We can then compare the predicted luminosity in any other band with that of the member in question to calculate the missing flux. In practice it is better to work with a sequence than individual members and so we defined the single-star Pleiades sequence in the INT-WFC and 2MASS colours by fitting a spline (by eye) to the observed sequence in various CMDs. We used only stars with uncertainties in both colour and magnitude of less than 0.1 mag. This spline lies slightly above the lower envelope of the sequence to account for photometric uncertainties and takes into consideration that the equal-mass binary sequence lies $\approx 0.75$ mag above the single-star sequence (with very few systems of higher multiplicity). Table 6 provides the single-star sequence in the (griZ)$_{\text{WFC}}, JHK_s$ bandpasses.

Starting, for instance, with the $K_s$ (griZ$-K_s$) Pleiades CMD, we compared the theoretical 130 Myr isochrone to the observed spline and thus calculated the difference between the colours at a given $T_{\text{eff}}$ (provided by the model isochrone). Assuming that the problem, in this case, lies in the $g$$_{\text{WFC}}$-band and not $K_s$, this then defines the required correction ($\Delta$BC; defined as BC$_{\text{obs}}$ – BC$_{\text{calc}}$) for the g$_{\text{WFC}}$-band at a specific $T_{\text{eff}}$. This process was then repeated over the $T_{\text{eff}}$ range of the theoretical isochrone or the colour range specified by the observed spline, whichever was more restrictive, thus providing $\Delta$BC as a function of $T_{\text{eff}}$ for the (griZ)$_{\text{WFC}}, JH$ bandpasses.

Our final results are shown in Fig. 11. In the Pleiades, stars cooler than $T_{\text{eff}} \approx 4000$ K are on the pre-MS, and all models fail to match the data in the optical in this regime. The discrepancy is large, 0.75 mag in g$_{\text{WFC}}$, but decreases with increasing wavelength. This is a clear improvement over the PHOENIX/GAIA models, for which we find a discrepancy of around 1 mag.

We can test what effect the overestimation in flux of the atmospheric models has on the derived ages for $T_{\text{eff}} < 4000$ K in the following way. If this discrepancy is independent of age and therefore log $g$, the roughly logarithmic age spacing between the isochrones in CMD space means that the resulting error in age is best represented as a fractional rather than an absolute difference. We begin by taking the BCAH98 $\alpha = 1.9$ isochrones and adding our $\Delta$BCs derived above to the theoretical bolometric corrections and colour-$T_{\text{eff}}$ relations at a given age. We then find the closest matching uncorrected BCAH98 $\alpha = 1.9$ isochrone (in CMD space). Comparing these, we find that for ages between 3 and 10 Myr, the difference in age is approximately a factor 3 in $g, g-i$ and 2 in $r, r-z$.

The message here is very clear, one should use the reddest waveband possible to determine an age. Unfortunately by the time one reaches the JHK$\alpha$ bandpasses, the pre-MS isochrones are vertical with a colour J – K$\alpha$ $\approx 0.85$ for $2500 < T_{\text{eff}} < 4000$ K. Although individual stars descend with age, the resulting sequences are almost degenerate with age, and for young clusters (< 10 Myr), observations are further complicated by the presence of discs. Thus the JHK$\alpha$ IR is unlikely to yield reliable young cluster ages. Masses do not suffer from such a degeneracy and the H-band may be a reliable mass indicator when $K_s$ is affected by circumstellar material.

Perhaps more surprising is the fact that in the MS regime, hotter than $T_{\text{eff}} \approx 4000$ K, only two models fit the data in the optical. The reason the BCAH98 $\alpha = 1.0$ and SDF00 models tend to overpredict the luminosity at a given $T_{\text{eff}}$ is that neither is tuned to the Sun, as explained in Section 2.1. Once the BCAH98 models have a tuned mixing length parameter ($\alpha = 1.9$) based on matching observed solar values they are a good match to the MS data. A possible reason why the DAM97 models fail to fit the MS may be because of their treatment of convection.
The corrections were calculated as the difference between the theoretically predicted colour and the observed colour of the Pleiades sequence at a given $T_{\text{eff}}$. Corrections have been calculated for the following models: BCAH98 $\alpha = 1.9$ (red, continuous), BCAH98 $\alpha = 1.0$ (red, dashed), SDF00 (blue), DAM97 (green) and DCJ08 (cyan). For $T_{\text{eff}} \lesssim 4000$ K all the models fail to match the data in the optical regime, with the magnitude of the mismatch decreasing with increasing wavelength.

Table 6. The Pleiades single-star sequence in the INT-WFC and 2MASS bandpasses. The full table is available online, a sample is shown here as a representation of its content.

| $g_{\text{WFC}}$ | $r_{\text{WFC}}$ | $i_{\text{WFC}}$ | $Z_{\text{WFC}}$ | $J$ | $H$ | $K_s$ |
|-----------------|-----------------|-----------------|-----------------|---|---|---|
| 10.807 | 10.377 | 10.270 | 10.251 | 9.370 | 9.082 | 8.987 |
| 10.972 | 10.505 | 10.389 | 10.364 | 9.484 | 9.181 | 9.078 |
| 11.133 | 10.645 | 10.516 | 10.481 | 9.594 | 9.275 | 9.166 |
| 11.292 | 10.783 | 10.640 | 10.596 | 9.699 | 9.364 | 9.251 |
| 11.445 | 10.914 | 10.758 | 10.704 | 9.796 | 9.445 | 9.330 |
| 11.593 | 11.040 | 10.871 | 10.809 | 9.887 | 9.520 | 9.405 |
| 11.735 | 11.161 | 10.979 | 10.908 | 9.970 | 9.588 | 9.473 |
| 11.870 | 11.273 | 11.079 | 11.000 | 10.043 | 9.646 | 9.534 |
| 11.998 | 11.378 | 11.172 | 11.086 | 10.110 | 9.699 | 9.589 |
| 12.120 | 11.478 | 11.261 | 11.168 | 10.169 | 9.745 | 9.637 |

7 CONCLUSIONS

We have carried out a precise test of a set of pre-MS models by examining how well they match the Pleiades sequence in optical and near-IR CMDs. The stages we have gone through to achieve this are as follows.

(i) We have demonstrated that traditional photometric calibration, using observations of standard stars to transform the data into a standard photometric system, should not be used for studies of pre-MS stars. Differences in log $g$ and $T_{\text{eff}}$ between a MS and pre-MS star of the same colour can, for instance, result in errors of order $0.1 - 0.15$ mag in both the $g$-band magnitude and $g - i$ colour for red stars. Hence it is crucial that precise photometric studies (especially of pre-MS objects) be carried out in the natural photometric system of the observations.

(ii) Therefore we have calculated system responses for the natural INT-WFC photometric system. Our observations show these are a good model of the photometric system.

(iii) We have demonstrated that for all optical colours no pre-MS model follows the observed Pleiades sequence for temperatures cooler than 4000 K. The models overestimate the flux by a factor two at 0.5 $\mu$m, with the difference decreasing with increasing wavelength. We believe there is little observable effect at 2.2 $\mu$m, as the $K_s$-band magnitude of MS binaries matches the models at this wavelength.
(iv) These differences in bolometric correction correspond to underestimating the ages of pre-MS stars younger than 10 Myr by factors of two to three, depending on the choice of colours used. Thus the errors in pre-MS isochrone models can explain the discrepancy between MS and pre-MS ages found by Naylor (2009), and implies the ages determined from the MS are more likely to be correct.

(v) We have provided both our system responses and our Pleiades sequence in such a way that it can be used as a benchmark for any new evolutionary models. These are given in Tables A1 and 6 respectively and will also be made available from the Cluster Collaboration homepage and the CDS archive.

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APPENDIX A: THE CALCULATED INT-WFC SYSTEM RESPONSES

Although the SDSS survey system responses are well constrained and available on the SDSS webpages\(^6\) (Doi et al. 2010), the responses on the Isaac Newton Group (ING) webpages for the WFC only combine the filter throughput and CCD quantum efficiency. To model the system responses of the INT-WFC we included the cumulative effects of the transmission of the Earth’s atmosphere, the reflectivity of the telescope mirror, the transmission of the prime focus corrector optics, the quantum efficiency of the detector, and the filter transmission. To calculate the atmospheric transmission we used the model for the La Palma atmosphere derived by King (1985), for an airmass typical for our observations of 1.4. This model varies smoothly as a function of wavelength and does not include the molecular absorption features. These atmospheric absorption bands (primarily due to H\(_2\)O, CO\(_2\) and O\(_3\)) were estimated using the spectrum of an F8 star observed using the Faint Object Spectrograph on the William Herschel Telescope. At low resolution the continuum of an F8 star is relatively smooth and thus the atmospheric bands can be identified by fitting a low order polynomial to the continuum and subtracting this from the spectrum (Shahbaz et al. 1996). The continuum is modelled between 4700 – 9800 Å. At prime focus the INT optical path involves a single reflection from the aluminium coated primary mirror and this was modelled using the aluminium reflectivity spectrum of Allen (1963). A three-element prime focus corrector is used which is coated to minimise reflection and thus improve efficiency and reduce ghosts. The first two elements are non-interchangeable and have a broadband single layer MgF\(_2\) coating. The third element, although not interchangeable, typically adds an additional single layer of MgF\(_2\) coating. These single layer coatings produce total reflectivities smaller than 2 per cent in the wavelength region 3500 – 7400 Å. Beyond this range, the reflectivity from the prime focus corrector increases to \(\zeta\) 3 per cent at 8000 Å. The filter and detector responses were from the ING webpages\(^7\). The detector is comprised of four EEV42-80 CCDs. The detector response on the ING webpages only extends to 3400 Å in the blue, however the blue edge of the RGO \(U\text{WFC}\) filter is at approximately 3000 Å and so the detector response was extended blueward using data for an almost identical EEV44-82 CCD to a wavelength of 3200 Å and further extrapolated to 3000 Å (Cavadore & Dorn 2000). The RGO \(Z\text{WFC}\) filter response data becomes noisy at wavelengths greater than 9000 Å and so this response was extended redward using calculated 4-mm Schott

\(^{6}\) http://www.sdss.org/dr7/instruments/imager/index.html\#filters

\(^{7}\) http://www.ing.iac.es/Astronomy/instruments/wfc/
RG850 data to \( \approx 10300 \, \text{Å} \) where the detector response approaches zero. Any negative values for the filter responses from the ING webpages were removed. Note also that the RGO \( U_{\text{WFC}} \) filter response used here does not include the recently discovered red leak near 7050 Å (Verbecue et al. 2012). The calculated INT-WFC system responses are shown in the top panel of Fig. A1, with the individual bandpass throughputs given in Table A1.

### APPENDIX B: CALCULATING THE THEORETICAL TRANSFORMATIONS

To investigate what effect variations in the system responses between the standard SDSS and natural INT-WFC photometric systems would have on the calibration of our photometric data, we calculated the magnitude and colour differences between the two photometric systems. We adopted the standard SDSS responses, the calculated INT-WFC responses and folded the atmospheric models through both. For this comparison we were interested only in the relative differences in the derived colours and magnitudes, and so our conclusions are not be affected by the use of atmospheric models.

Following the formalism of Girardi et al. (2002), a synthetic apparent magnitude \( m_{R_A} \) in a given filter with response function \( R_A \) is calculated as,

\[
m_{R_A} = -2.5 \log \left( \frac{\int_\lambda f_\lambda R_A d\lambda}{\int_\lambda f_\lambda^0 R_A d\lambda} \right) + m_{R_A}^0, \tag{B1}
\]

where \( f_\lambda \) is the stellar flux as observed at Earth and \( f_\lambda^0 \) denotes a reference spectrum with a known apparent magnitude \( m_{R_A}^0 \). The response function \( R_A \) is the product of the component responses described in Appendix A. Note that the integrands in Eqn. B1 imply photon counting across the filter which is appropriate for detectors such as CCDs.

Stellar atmospheric models only contain information about the flux at the stellar surface, \( F_\lambda \). The transformation from \( f_\lambda \) to \( F_\lambda \) requires knowledge of the stellar radius, its distance and an extinction curve \( A_\lambda \) to account for interstellar extinction. To incorporate these and remove the radial dependence, it is more convenient to deal in terms of bolometric corrections, which can be formulated as,

\[
BC_{R_A} = M_{\text{bol,⊙}} - 2.5 \log \left( \frac{4\pi(10\text{pc})^2F_{\text{bol}}}{L_\odot} \right) - 2.5 \log \left( \frac{\int_\lambda \lambda F_\lambda 10^{-0.4A_\lambda R_A d\lambda}}{\int_\lambda \lambda F_\lambda^0 R_A d\lambda} \right) - m_{R_A}^0,
\]

where \( F_{\text{bol}} = \sigma T_{\text{eff}}^4 \) is the total flux emergent at the stellar surface and all other symbols retain their original definitions. For the solar values we used \( M_{\text{bol,⊙}} = 4.74 \) and \( L_\odot = 3.855 \times 10^{33} \text{erg s}^{-1} \) (Bessell, Castelli, & Plez 1998).

The SDSS photometric system is based on a monochromatic AB magnitude system (see Oke & Gunn 1983; Fukugita et al. 1996) which by definition means that a reference spectrum of constant flux density per unit frequency \( f_\nu^0 \) will have AB magnitudes \( m_\nu^0 = 0 \) at all frequencies \( \nu \). By converting this reference spectrum in Eqn. B2 to energy per unit wavelength according to \( f_\lambda = c/\lambda^2 f_\nu \), this definition can then be extended to any photometric system with the result that \( m_\lambda^0 = 0 \) are the reference magnitudes. Thus the magnitude and colour differences for a given flux distribution between the INT-WFC and SDSS photometric systems were then simply calculated as the difference in the derived bolometric corrections.

We required bolometric corrections for the MS and pre-MS regimes. To model the changing \( T_{\text{eff}} \) and \( \log g \) values along the MS (from \( \log g \approx 4 \) for early B-type stars to \( \approx 5.5 \) for late M-type) we used the calculated ZAMS of SDF00 for stellar masses in the range \( 0.1 - 7 M_\odot \). For the pre-MS values we adopted the SDF00 interior models for ages of 1 and 10 Myr, calculating \( T_{\text{eff}} \) and \( \log g \) over the same mass range. The reason for choosing the SDF00 interior models over other pre-MS stellar interior models was simply because they span a greater range of stellar masses and so more thoroughly map the MS. Due to the degree of model dependency in pre-MS evolutionary models, we repeated the calculation of the theoretical transformations using the ZAMS \( T_{\text{eff}} \), \( \log g \) values from the BCAH98, DAM97 and DCJ08 models and found that all four sets of evolutionary models agree to within 0.02 mag over the range of stellar masses.

Fig. 3 shows the calculated magnitude and colour differences for reddened and unreddened objects (for illustrative purposes we adopted a nominal reddening of \( E(B-V) = 1 \)). The simple formalism adopted in the derivation of the bolometric corrections allowed us to include the effects of interstellar reddening by applying a given extinction curve \( A_\lambda \) and calculating the bolometric correction as shown in Eqn. B2. We adopted the Cardelli et al. (1989) extinction curve of \( R_V = A_V/E(B-V) \) and thus derived the extinction in a given bandpass at a specific \( E(B-V) \). We adopted \( R_V = 3.2 \) as this value best reproduced the colour dependent reddening vector of Bessell et al. (1998) upon folding the ATLAS9 “no-overshoot” atmospheric models (Kurucz 1992) through the \( UBVRI \) responses of Bessell (1990). Magnitude and colour differences were calculated for each \( T_{\text{eff}}, \log g \)
Table A1. The normalised calculated INT-WFC system responses.

| $\lambda$ | $U_{WFC}$ | $g_{WFC}$ | $r_{WFC}$ | $i_{WFC}$ | $Z_{WFC}$ |
|-----------|-----------|-----------|-----------|-----------|-----------|
| 3050      | 0.000     | 0.000     | 0.000     | 0.000     | 0.000     |
| 3100      | 0.002     | 0.015     | 0.013     | 0.010     | 0.000     |
| 3150      | 0.011     | 0.099     | 0.142     | 0.050     | 0.002     |
| 3200      | 0.042     | 0.203     | 0.384     | 0.142     | 0.005     |
| 3250      | 0.097     | 0.409     | 0.586     | 0.285     | 0.015     |
| 3300      | 0.216     | 0.463     | 0.684     | 0.462     | 0.041     |
| 3350      | 0.356     | 0.618     | 0.812     | 0.605     | 0.104     |
| 3400      | 0.528     | 0.676     | 0.904     | 0.745     | 0.223     |
| 3450      | 0.614     | 0.736     | 0.957     | 0.863     | 0.391     |
| 3500      | 0.694     | 0.781     | 1.000     | 0.915     | 0.570     |
| 3550      | 0.767     | 0.813     | 0.980     | 0.873     | 0.736     |
| 3600      | 0.832     | 0.855     | 0.998     | 0.906     | 0.861     |
| 3650      | 0.902     | 0.864     | 0.986     | 0.956     | 0.906     |
| 3700      | 0.954     | 0.905     | 0.976     | 1.000     | 0.973     |
| 3750      | 1.000     | 0.918     | 0.966     | 0.978     | 0.974     |
| 3800      | 0.932     | 0.930     | 0.923     | 0.945     | 0.971     |
| 3850      | 0.885     | 0.959     | 0.946     | 0.968     | 0.956     |
| 3900      | 0.684     | 0.976     | 0.909     | 0.980     | 0.966     |
| 3950      | 0.425     | 0.986     | 0.848     | 0.931     | 0.829     |
| 4000      | 0.206     | 0.979     | 0.856     | 0.736     | 0.702     |
| 4050      | 0.074     | 0.996     | 0.894     | 0.923     | 0.751     |
| 4100      | 0.021     | 0.998     | 0.817     | 0.919     | 0.687     |
| 4150      | 0.005     | 0.972     | 0.758     | 0.889     | 0.645     |
| 4200      | 0.001     | 0.997     | 0.684     | 0.848     | 0.691     |
| 4250      | 0.000     | 1.000     | 0.845     | 0.819     | 0.640     |
|           | 5000      | 0.970     | 0.809     | 0.828     | 0.493     |
|           | 5300      | 0.985     | 0.784     | 0.800     | 0.390     |
|           | 5350      | 0.972     | 0.789     | 0.789     | 0.502     |
|           | 5400      | 0.939     | 0.721     | 0.759     | 0.458     |
|           | 5450      | 0.851     | 0.526     | 0.686     | 0.473     |
|           | 5500      | 0.285     | 0.455     | 0.769     | 0.426     |
|           | 5550      | 0.011     | 0.329     | 0.669     | 0.433     |
|           | 5600      | 0.000     | 0.061     | 0.686     | 0.400     |
|           | 7050      | 0.008     | 0.660     | 0.366     |
|           | 7100      | 0.003     | 0.652     | 0.333     |
|           | 7150      | 0.001     | 0.641     | 0.299     |
|           | 7200      | 0.000     | 0.341     | 0.272     |
|           | 8550      | 0.068     | 0.245     |
|           | 8600      | 0.013     | 0.219     |
|           | 8650      | 0.003     | 0.192     |
|           | 8700      | 0.000     | 0.167     |

combination in both the MS and pre-MS regimes via linear interpolation.