Generation of cluster states with Josephson charge qubits

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A scheme for the generation of the cluster states based on the Josephson charge qubit is proposed. The two-qubit generating case is first introduced, and then generalized to multi-qubit case. The scheme is simple and easily manipulated, because any two charge qubits can be selectively and effectively coupled by a common inductance. More manipulations can be realized before decoherence sets in. All the devices in the scheme are well within the current technology.

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Quantum entanglement plays one of the most important roles in the quantum information processing. Many ingenious applications of entanglement have been proposed, such as quantum dense coding [1], quantum teleportation [2], quantum cryptography [3], etc. In achieving the task of quantum communication, entangled states are used as a medium for transferring quantum information. Meanwhile, entangled states are also used for speeding up quantum computation. Therefore, the preparation of entangled states becomes a key step towards quantum computation. Though bipartite entanglement is well understood, the extensive researches of multipartite case are still difficultly proceeding. For a tripartite-entangled quantum system, there are two irreducible classes of entangled states [4]. Recently, Briegel and Raussendorf [5] introduced a new class of N-qubit entangled states, i.e., the cluster states, which have some special properties. In addition to the properties of both Greenberger-Horne-Zeilinger (GHZ) and W-class entangled states, they especially hold a large persistence of entanglement, that is, they (in the case of \( N > 4 \)) are harder to be destroyed by local operations than GHZ-class states. It has been shown that a new Bell inequality is maximally violated by the four-qubit cluster states, and isn’t violated by the four-qubit GHZ states [6]. More significantly, the cluster states are regarded as a resource of multiqubit entangled states, thus cluster states become an important resource in the physics, especially in quantum information.

Recently, much attention has been attracted to the quantum computer, which works on the fundamental quantum mechanical principle. The quantum computers can solve some problems exponentially faster than the classical computers. For realizing quantum computing, some physical systems, such as nuclear magnetic resonance [7], trapped ions [8], cavity quantum electrodynamics (QED) [9], and optical systems [10] have been proposed. These systems have the advantage of high quantum coherence, but can’t be integrated easily to form large-scale circuits. As is well known, the cluster states are mainly applied to quantum computing. In Ref. [11], Raussendorf and Briegel described the so-called one-way quantum computer, in which information is written onto the cluster and read out from the cluster by one-qubit measurements. A number of applications using cluster states in quantum computation have been proposed [12]. Thus the preparation of the cluster states has become the focus of research. Recently, Zou et al. proposed probabilistic schemes for generating the cluster states of four distant trapped atoms in leaky cavities [13] and linear optics systems [14]. Barrett and Kok proposed a protocol for the generation of the cluster states using spatially separated matter qubits and single-photon interference effects [15]. Yang et al. proposed an efficient scheme for the generation of the cluster states with trapped ions [16]. We also proposed two schemes for the generation of the cluster states via both cavity QED techniques and atomic ensembles [17].

As a solid-state qubit, Josephson charge qubit is one of the promising candidate for quantum computing. Accordingly, generation of the cluster states by Josephson charge qubit is of great importance. Josephson charge [18] and phase [21] [22] qubits, based on the macroscopic quantum effects in low-capacitance Josephson junction circuits [23] [24], have recently been used in quantum information processing because of large-scale integration and relatively high quantum coherence. Some striking experimental observations [25] [26] demonstrate that the Josephson charge and phase qubits are promising candidates of solid-state qubits in quantum information processing. In particular, recent experimental realizations of a single charge qubit demonstrate that it is hopeful to construct quantum computers by means of Josephson charge qubits [27]. In this paper, we propose a scheme for the generation of the cluster states using Josephson charge qubit. This scheme is simple and easily manipulated, because any two charge qubits can be selectively and effectively coupled by a common inductance. More manipulations can be realized before decoherence sets in. All of the devices in the scheme are well within the current technology. It is the efficient scheme for the generation of the cluster states based on the Josephson charge qubit.

Since the earliest Josephson charge qubit scheme [18] was proposed, a series of improved schemes [13] [28] have been explored. Here, we concern the architecture of Josephson charge qubit in Ref. [28], which is the first efficient scalable quantum computing (QC) architecture. The Josephson charge qubits structure is shown in Fig. (1). It consists of \( N \) cooper-pair boxes (CPBs) coupled by a common superconducting inductance \( L \). For the \( k \)th cooper-pair boxe,
conducting island with charge $Q_k = 2en_k$ is weakly coupled by two symmetric direct current superconducting quantum interference devices (dc SQUIDs) biased by an applied voltage through a gate capacitance $C_k$. Assume that the two symmetric dc SQUIDs are identical and all Josephson junctions in them have Josephson coupling energy $E_{Jk}$ and capacitance $C_{Jk}$. The self-inductance effects of each SQUID loop is usually neglected because of the very small size (1 μm) of the loop. Each SQUID pierced by a magnetic flux $\Phi_{k}$ provides an effective coupling energy $-E_{Jk}(\Phi_{Xk})\cos \phi_{k(A)}$, with $E_{Jk}(\Phi_{Xk}) = 2E_{J0}^2\cos(\pi\Phi_{Xk}/\Phi_0)$, and the flux quantum $\Phi_0 = h/2e$. The effective phase drop $\phi_{k(A)}$, with subscript $A(B)$ labeling the SQUID above (below) the island, equals the average value $[\phi_{k(A)}^e + \phi_{k(A)}^b]/2$, of the phase drops across the two Josephson junctions in the dc SQUID, with superscript $L(R)$ denoting the left (right) Josephson junction.

For any given cooper-pair box, say $i$, when $\Phi_{Xk} = \frac{1}{2}\Phi_0$ and $V_{Xk} = (2n_k + 1)e/c_k$ for all boxes except $k = i$, the inductance $L$ connects only the $i$th cooper-pair box to form a superconducting loop, as shown in Fig. 2(a). In the spin-$\frac{1}{2}$ representation, based on charge states $|0\rangle = |n_i\rangle$ and $|1\rangle = |n_i + 1\rangle$, the reduced Hamiltonian of the system becomes

$$H = \varepsilon_i(V_{Xi})\sigma_z^{(i)} - \mathcal{E}_{Ji}(\Phi_{Xi}, \Phi_e, L)\sigma_x^{(i)},$$  \hspace{1cm} (1)

where $\varepsilon_i(V_{Xi})$ is controlled by the gate voltage $V_{Xi}$, while the intrabit coupling $\mathcal{E}_{Ji}(\Phi_{Xi}, \Phi_e, L)$ depends on inductance $L$, the applied external flux $\Phi_e$ through the common inductance and the local flux $\Phi_{Xi}$ through the two SQUID loops of the $i$th cooper-pair box. By controlling $\Phi_{Xk}$ and $V_{Xk}$, the operations of Pauli matrix $\sigma_z^{(i)}$ and $\sigma_x^{(i)}$ are achieved. Thus, any single-qubit operations are realized by utilizing the Eq. (1).

To manipulate many-qubit, say $i$ and $j$, we configure $\Phi_{Xk} = \frac{1}{2}\Phi_0$ and $V_{Xk} = (2n_k + 1)e/c_k$ for all boxes except $k = i$ and $j$. In the case, the inductance $L$ is only shared by the cooper-pair boxes $i$ and $j$ to form superconducting loops, as shown in Fig. 2(b), the Hamiltonian of the system can be reduced to

$$H = \sum_{k=i,j}[\varepsilon_k(V_{Xk})\sigma_z^{(k)} - \mathcal{E}_{Jk}\sigma_z^{(k)}] + \Pi_{ij}\sigma_y^{(i)}\sigma_y^{(j)},$$  \hspace{1cm} (2)

where the interbit coupling $\Pi_{ij}$ depends on both the external flux $\Phi_e$ through the inductance $L$, the local fluxes $\Phi_{Xi}$ and $\Phi_{Xj}$ through the SQUID loops. In Eq. (2), if we choose $V_{Xk} = (2n_k + 1)e/c_k$, the Hamiltonian of system can be reduced to

$$H = -\mathcal{E}_{Ji}\sigma_z^{(i)} - \mathcal{E}_{Jj}\sigma_z^{(j)} + \Pi_{ij}\sigma_y^{(i)}\sigma_y^{(j)}. \hspace{1cm} (3)$$

For the simplicity of calculation, we assume $\mathcal{E}_{Ji} = \mathcal{E}_{Jj} = \Pi_{ij} = \frac{\mu_0}{4\pi}(\tau$ is a given period of time), which can be obtained by suitably choosing parameters. Thus Eq. (3) becomes

$$H = -\frac{\pi\hbar}{4\tau}(\sigma_z^{(i)} - \sigma_z^{(j)} + \sigma_y^{(i)}\sigma_y^{(j)}). \hspace{1cm} (4)$$

Below, we discuss problems on the basis $\{|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), |\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\}$. In order to generate the cluster states of two Josephson charge qubit, we prepare Josephson charge qubit $i$ in the state $|\phi\rangle = \frac{1}{\sqrt{2}}(|-\rangle_i + |+\rangle_i)$, and Josephson charge qubit $j$ in the state $|\phi\rangle = \frac{1}{\sqrt{2}}(|-\rangle_j + |+\rangle_j)$, so the initial state of the system is $|\phi\rangle_{ij} = \frac{1}{2}(|-\rangle_i + |+\rangle_i)\otimes|(-\rangle_j + |+\rangle_j)$. We assume $i = 1$ and $j = 2$, according to Hamiltonian $H$ of Eq. (4), we can obtain the following evolutions:

$$|+\rangle_{12} \rightarrow e^{-i\pi/4\tau} |+\rangle_{12}, \hspace{1cm} (5a)$$

$$|\rangle_{12} \rightarrow e^{-i\pi/4\tau} |\rangle_{12}; \hspace{1cm} (5b)$$

$$|+\rangle_{12} \rightarrow e^{-i\pi/4\tau} |+\rangle_{12}; \hspace{1cm} (5c)$$

$$|\rangle_{12} \rightarrow e^{i\pi\tau/4\tau} |\rangle_{12}; \hspace{1cm} (5d)$$
If we choose \( t = \tau \), which can be achieved by choosing switching time, and perform a single-qubit operation \( U = e^{i\pi/4} \), we can obtain

\[
|++\rangle_{12} \rightarrow |++\rangle_{12},
\]

(6a)

\[
|--\rangle_{12} \rightarrow |+\rangle_{12},
\]

(6b)

\[
|+\rangle_{12} \rightarrow |+\rangle_{12},
\]

(6c)

\[
|--\rangle_{12} \rightarrow |+\rangle_{12}.
\]

(6d)

The Eq. (6) have actually realized the operation of a controlled phase gate. These lead the state of Josephson junction charge qubits 1 and 2 to

\[
|\phi\rangle_{12} = \frac{1}{2} \left[ |\rangle_1 (|\rangle_2 + 1)(|--\rangle_2 + 1) + |\rangle_1 (|\rangle_2 + 1) \right] = \frac{1}{2} \left[ (|--\rangle_1|\rangle_1 + |\rangle_1 |\rangle_1 \right].
\]

(7)

where \( \sigma_z^2 = \langle ++ | - |-- \rangle \), and Eq. (7) is a standard cluster states of two-qubit. We generalize the above scheme for generating the cluster states of two-qubit to the multi-qubit case. We first prepare \( N (N \geq 2) \) Josephson junction charge qubits in the states

\[
|\phi\rangle_1 = 1\sqrt{|1\rangle_1 |1\rangle_1},
\]

(8a)

\[
|\phi\rangle_j = 1\sqrt{|1\rangle_j |1\rangle_j},
\]

(8b)

where \( j = 2, 3, \cdots, N \). So the total state of \( N \) Josephson charge qubits is

\[
|\phi\rangle_{1j} = \frac{1}{2N/2(|1\rangle_1 + 1)} \left( \bigotimes_{j=2}^N (|--\rangle_j + |\rangle_j \right),
\]

(9)

By choosing the suitable parameters (e.g. \( \varepsilon_k(V_X k), \phi_k, \), et al.), the interaction only occurs between Josephson charge qubit \( i \) and Josephson charge qubit \( j \), while other qubits’ interaction don’t involved.

Firstly, let Josephson charge qubit 1 only act with Josephson charge qubit 2 without other qubits’ interactions, and make both qubits undergo the same evolutions as Eq. (6). This leads Eq. (9) to

\[
|\phi\rangle_{1j} = \frac{1}{2N/2}\left( |\rangle_1 |\rangle_1 + 1 \right) \left( \bigotimes_{j=2}^N (|--\rangle_j + |\rangle_j \right).
\]

(10)

Next, let Josephson charge qubit 2 only act with Josephson charge qubit 3 without other qubits’ interactions. After the same interaction as Josephson charge qubit 2 with Josephson charge qubit 1, Eq. (10) becomes

\[
|\phi\rangle_{1j} = \frac{1}{2N/2}\left( |\rangle_1 |\rangle_1 + 1 \right) \left( \bigotimes_{j=2}^N (|--\rangle_j + |\rangle_j \right).
\]

(11)

From the form of the above states, we can conclude if we let two Josephson charge qubits interact without other qubits’ interactions every time, step by step, we can obtain the cluster states of Josephson charge multi-qubit easily. In other words, let Josephson charge qubit \( j \) only act with Josephson charge qubit \( j+1 \) without other qubits’ interactions. Thus the cluster states of Josephson charge \( N \) qubits can be obtained

\[
|\phi\rangle_N = \frac{1}{2N/2}\left( \bigotimes_{j=2}^N (|--\rangle_j + |\rangle_j \right),
\]

(12)

where \( \sigma_z^N_{j+1} \equiv 1 \).

Below, we briefly discuss the experimental feasibility of the current scheme. For the used charge qubit in our scheme, the typical experimental switching time \( \tau^{(1)} \) during a single-bit operation is about \( 0.1ns \) [28]. The inductance \( L \) in our used proposal is about \( 30nH \), which is experimentally accessible. In the earlier design [19], the inductance \( L \) is about \( 3.6\mu H \), which is difficult to make at nanometer scales. Another improved design [23] greatly reduces the inductance \( L \) to \( \sim 120nH \), which is about 4 times larger than the one used in our scheme. The fluctuations of voltage source and fluxes result in decoherence for all charge qubits. The gate voltage fluctuation plays the dominant role in producing decoherence. The estimated dephasing time is \( \tau_d \sim 10^{-5}s \) [23], which allow in principle \( 10^6 \) coherent single-bit manipulations. Owing to using the probe junction, the phase coherence time is only about \( 2ns \) [27, 30]. In this setup, background charge fluctuations and the probe-junction measurement may be two of the major factors in producing decoherences [23]. The charge fluctuations are principal only in the low-frequency region and can be reduced by the echo technique [30] and by controlling the gate voltage to the degeneracy point, but an effective technique for suppressing charge fluctuations still needs to be explored.

In summary, we have investigated a simple scheme for generating the cluster states based on the Josephson charge qubit. We first introduce the two-qubit case, and then generalize it to multi-qubit case. Our scheme is simple and easily manipulated, because any two charge qubits can be selectively and effectively coupled by a common inductance. The architecture of our proposal is made by present scalable microfabrication technique. More manipulations can be realized before decoherence sets in. All the devices in the scheme are well within the current technology. It is the efficient scheme for the generation of the cluster states based on the Josephson charge qubit.
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