DENSITY PROFILES OF ΛCDM CLUSTERS

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ABSTRACT

We analyze the mass accretion histories (MAHs) and density profiles of cluster-size halos with virial masses of \((0.6-2.5) \times 10^{14} \, h^{-1} M_\odot\) in a flat ΛCDM cosmology. We find that most MAHs have a similar shape: an early merger-dominated mass increase followed by a more gradual accretion-dominated growth. For some clusters the intense merger activity and rapid mass growth continue until the present-day epoch. In agreement with previous studies, we find that the concentration of the density distribution is tightly correlated with the halo’s MAH and with its formation redshift. During the period of fast mass growth, the concentration remains approximately constant and low, \(c_z \approx 3-4\), while during the slow accretion stages the concentration increases with decreasing redshift as \(c_z \propto (1+z)^{-1}\). We consider fits of three widely discussed analytic density profiles to the simulated clusters, focusing on the most relaxed inner regions. We find that there is no unique best-fit analytic profile for all the systems. At the same time, if a cluster is best fitted by a particular analytic profile at \(z = 0\), the same is usually true at earlier epochs out to \(z \sim 1-2\). The local logarithmic slope of the density profiles at 3\% of the virial radius ranges from \(-1.2\) to \(-2.0\), a remarkable diversity for the relatively narrow mass range of our cluster sample. Interestingly, for all the studied clusters the logarithmic slope down to the smallest resolved scale (≤1\% of the virial radius) becomes shallower with decreasing radius without reaching an asymptotic value. We do not find a clear correlation of the inner slope with the formation redshift or the shape of the halo’s MAH. We do find, however, that during the period of rapid mass growth the density profiles can be well described by a single power law \(\rho(r) \propto r^{-\gamma}\) with \(\gamma \sim 1.5-2\). The relatively shallow power-law slopes result in low concentrations at these stages of evolution, as the scale radius at which the density profiles reach the slope of \(-2\) is at large radii. This indicates that the inner power-law-like density distribution of halos is built up during the periods of rapid mass accretion and active merging, while the outer steeper profile is formed when the mass accretion slows down. To check the convergence and robustness of our conclusions, we resimulate one of our clusters by using 8 times as many particles and twice as good force resolution. We find good agreement between the two simulations in all the results discussed in our study.

Subject headings: cosmology: theory — dark matter — galaxies: clusters: general — methods: numerical

1 INTRODUCTION

During the last decade, there has been an increasingly growing interest in testing the predictions of variants of the cold dark matter (CDM) models on small scales. The interest was spurred by indications that the density distribution in the inner regions of dark matter halos predicted by CDM is at odds with the observed galactic rotation curves (Flores & Primack 1994; Moore 1994). This discrepancy is yet to be convincingly resolved and is still a subject of active debate (e.g., Côté, Carignan, & Freeman 2000; van den Bosch & Swaters 2001; Blais-Oullette, Amram, & Carignan 2001; de Blok, McGaugh, & Rubin 2001; de Blok, Bosma, & McGaugh 2003; Swaters et al. 2003). In addition, the CDM models face other apparent discrepancies with observations on galactic scales, such as the amount of substructure in galactic halos (Klypin et al. 1999; Moore et al. 1999a), the incorrect normalization of the Tully-Fisher relation, the angular momentum of disk galaxies (Navarro & Steinmetz 1997, 2000), the ellipticity of dark matter halos (Ibata et al. 2001), and others.

In the past several years, the density distribution in the cores of galaxy clusters has also become a subject of a related debate. CDM models predict cuspy density profiles without flat cores (Frenk et al. 1985; Quinn, Salmon, & Zurek 1986; Dubinski & Carlberg 1991). Navarro, Frenk, & White (1996, 1997, hereafter NFW) argued that the CDM halo profiles can be described by the following simple formula in all cosmologies and at all epochs:

\[
\rho(r) = \frac{\rho_0}{(r/r_s)(1 + r/r_s)^2}.
\]  

This analytic formula describes the density profile of a halo by using two parameters: a characteristic density, \(\rho_0\), and a scale radius, \(r_s\). These parameters are determined by the halo virial mass, \(M_v\), and concentration index, \(c \equiv r_s/r_v\), where \(r_v\) is the virial radius of the halo. In addition, NFW argued that there is a tight correlation between \(c\) and \(M_v\), which means that the halo profiles of different mass objects form a one-parameter family.

Moore et al. (1998; see also Ghigna et al. 2000) carried out a convergence study of the dark matter profiles and concluded that high mass resolution is required to resolve the inner density distribution robustly. They advocated the analytic density profile of the form \(\rho(r) \propto (r/r_s)^{-1.5}[1 + (r/r_s)^{1.5}]^{-1}\) as a better description of the density distribution of their simulated halos. This profile behaves similarly to the NFW profile at large radii (\(\propto r^{-3}\)) but is steeper at small radii (\(\propto r^{-1.5}\)).
Fukushige & Makino (1997, 2001, 2003) reached similar conclusions by using a set of independent simulations. Jing & Suto (2000) presented a systematic study of the density profiles of halos with masses in the range \((2 \times 10^{12}) - (5 \times 10^{14}) \, h^{-1} M_{\odot}\). They found that the inner slope at a radius of 1% of the virial radius is shallower (\(\approx -1.1\)) for cluster mass halos than for galactic halos (\(\approx -1.5\)). Recently, Hayashi et al. (2003) and Navarro et al. (2003) found that often the logarithmic slope of the density distribution at the convergence radius is steeper than \(-1\), as expected from the NFW profile, but significantly shallower than the \(-1.5\) inner slope found by Moore et al. (1998). Several other studies (Kravtsov, M"{u}cket, & Gottlöber 1997; Kravtsov et al. 1998; Avila-Reese et al. 1999; Jing 2000; Bullock et al. 2001; Klypin-Reese et al. 2001; Fukushige, Kawai, & Makino 2003) found a significant scatter in both the shape of the density profiles and halo concentrations, likely related to the details of the mass accretion histories (MAHs) of individual objects (Wechsler et al. 2002, hereafter W02; Zhao et al. 2003b). M"{u}cket & Hoeft (2003) and Hoeft, M"{u}cket, & Gottlöber (2004) study the radial dependence of the gravitational potential and the velocity dispersion and come to the conclusion that there does not exist a slope asymptote of the density profile over a wide range but that the slope increases with decreasing radius and reaches the value \(-0.58\) as \(r \rightarrow 0\).

Observational constraints on the inner slope of the dark matter density distribution in galactic halos are difficult because the distribution is affected by the cooling and dynamics of baryons. The dark matter profiles in clusters, on the other hand, should be less affected by cooling as a much smaller fraction of cluster baryons is observed to be in the cold condensed phase. Observational studies of the mass distribution in clusters by using weak lensing and hydrostatic analysis of the X-ray-emitting gas show that the overall mass distribution is in general agreement with CDM predictions (Allen 1998; Clowe et al. 2000; Willick & Padmanabhan 2000; Clowe & Schneider 2001; Sheldon et al. 2001; Arabadjis, Bautz, & Garmire 2002; Athreya et al. 2002; Bautz & Arabadjis 2004).

Strong-lensing studies can probe the mass distribution in the inner region of clusters and thus test the “cuspsiness” of cluster halos. However, the results of strong-lensing analyses have so far been contradictory, even in the case in which the same system was studied. Tyson, Kochanski, & D"{e}ll’Antonio (1998), for example, argue that the density profile of cluster Cl 0024+1654 has a constant-density core, while Broadhurst et al. (2000) find that the mass distribution in this cluster is cuspy. Czoske et al. (2002) argue that Cl 0024+1654 is undergoing a major merger and that its density profile may not be representative. Sand, Treu, & Ellis (2002) find that the inner slope of the density profile in cluster MS 2137-23 is flatter than expected in CDM models, a conclusion they recently confirmed for six more clusters (Sand et al. 2004). Gavazzi et al. (2003), reanalyzing the same observations, argue that if the fifth demagnified image near the center of the lensing potential is not taken into account, then the inner slope may be consistent with CDM predictions.

Given the disagreement among the different analytical fits proposed for the density profiles of dark matter halos found in simulations and a possible discrepancy with strong-lensing observations, it is interesting to conduct a systematic study of the density profiles of clusters in the concordance \(\Lambda\)CDM model. The study of cluster mass halos is also interesting because the typical concentrations of their matter distribution are lower than those of galactic halos. Thus, if an asymptotic inner slope, suggested by the analytic profiles, does exist, it should be reached at a larger fraction of the virial radius in cluster halos and should be easier to detect.

The paper is organized as follows. In the following two sections, we describe the numerical simulations and halo-finding algorithm used in our analysis. In \S\ 4 we discuss the MAHs of the analyzed clusters. In \S\ 5 we present the convergence test and discuss the fitting procedure and our results on the shapes and inner slopes of the density profiles. We summarize our results and conclusions in \S\ 6.

2. NUMERICAL SIMULATIONS

We use the Adaptive Refinement Tree code (Kravtsov et al. 1997) to follow the evolution of cluster-size halos in the flat \(\Lambda\)CDM cosmology: \((\Omega_m, \Omega_{\Lambda}, h, \sigma_8) = (0.3, 0.7, 0.7, 0.9)\). We use the initial spectrum in the Holtzman approximation with \(\Omega_b = 0.03\) (see Klypin & Holtzman 1997). The code starts with a uniform 256\(^2\) grid covering the entire computational box. This grid defines the lowest (zeroth) level of resolution. Higher force resolution is achieved in the regions corresponding to collapsing structures by recursive adaptive refinement of all such regions. Each cell can be refined or de-refined individually. The cells are refined if the particle mass contained within them exceeds a certain specified threshold value. The code thus refines to follow the collapsing objects in a quasi-Lagrangian fashion.

The cluster halos were simulated in a box of \(80 \, h^{-1}\) Mpc. A low-resolution simulation was run first. A dozen cluster halos were identified, and multiple mass resolution technique was used to set up initial conditions (Klypin et al. 2001). Namely, a Lagrangian region corresponding to a sphere of radius equal to two virial radii around each halo was resampled with the highest resolution particles of mass \(m_p = 3.16 \times 10^8 \, h^{-1} M_{\odot}\), corresponding to an effective number of 512\(^3\) particles in the box, at the initial redshift of the simulation (\(z_i = 50\)). The high mass resolution region was surrounded by layers of particles of increasing mass with a total of three particle species. Only regions containing highest resolution particles were adaptively refined, and the threshold for refinement was set to correspond to a mass of four highest resolution particles per cell. Each cluster halo is resolved with \(\approx 10^6\) particles within its virial radius at \(z = 0\). The size of the highest refinement level cell was \(1.2 \, h^{-1}\) kpc. In addition, one of the clusters was resimulated with 8 times more particles \((m_p = 3.95 \times 10^7 \, h^{-1} M_{\odot})\) to study the convergence of the density profiles. In this simulation, the smallest cell size reached was 0.6 \(h^{-1}\) comoving kpc.

The time steps were chosen so that no particle moves by more than a fraction of the parent cell size in a single step. This criterion was motivated by the convergence studies presented by Klypin et al. (2001). For the analyzed simulations, the number of steps at the highest refinement level was \(\approx 250,000\), or \(\Delta t \approx (2-3) \times 10^4\) yr and a factor of 2 larger for each lower refinement level. For the high-resolution resimulation of one of the clusters used for the convergence check, the number of steps at the highest refinement level was \(\approx 500,000\). We analyze the cluster profiles and their MAHs by using 19 outputs from \(z = 10\) to 0, with a typical time interval between outputs of \(\approx 0.7\) Gyr.

3. HALO IDENTIFICATION

To identify cluster halos, we use a variant of the bound density maxima (BDM) halo-finding algorithm. The main
idea of the BDM algorithm is to find positions of local maxima in the density field smoothed at a certain scale and to apply physically motivated criteria to test whether the identified site corresponds to a gravitationally bound halo. The detailed description of the algorithm is given in Klypin & Holtzman (1997) and Klypin et al. (1999).

We start by calculating the local overdensity at each particle position by using the smoothed particle hydrodynamics smoothing kernel of 24 particles. We then sort particles within this sphere from further center search. After all the potential centers are identified, we analyze the density distribution and velocities of the surrounding particles to test whether the center corresponds to a gravitationally bound clump (Klypin et al. 1999). We then construct profiles by using only bound particles and use them to calculate the properties of halos, such as the maximum circular velocity, $V_{\text{max}}$, the mass, $M$, etc. In this study, we consider only isolated cluster-size halos. We should note that for isolated halos the BDM algorithm works very similarly to the commonly used spherical overdensity algorithm.

The virial radius is a convenient measure of the halo size. We define the virial radius as the radius within which the density is equal to 180 times the average density of the universe at a given epoch. The separation between halos is sometimes smaller than the sum of their virial radii. In such cases, the definition of the outer boundary of a halo and its mass are somewhat ambiguous. To this end, in addition to the virial radius, we estimate the truncation radius, $r_t$, at which the logarithmic slope of the density profile constructed from the bound particles becomes larger than $-0.5$, as we do not expect the density profile of the CDM halos to be flatter than this slope. In general, we consider the halo radius to be $r_h = \min(r_{\text{tr}}, r_t)$.

In our analysis we use only clusters with masses greater than $5 \times 10^{13} \, h^{-1} \, M_\odot$. In most cases two or more clusters were identified with this mass threshold in each run. To distinguish between the isolated cluster halos and massive sub-

### Table 1

| Halo  | $M_{\text{HI}}$ ($h^{-1} M_\odot$) | $r_{\text{HI}}$ (h$^{-1}$ Mpc) | $V_{\text{max}}$ (km s$^{-1}$) |
|-------|---------------------------------|---------------------------------|--------------------------------|
| Cl 1  | $2.5 \times 10^{14}$            | 1.58                            | 973                            |
| Cl 2  | $2.4 \times 10^{14}$            | 1.56                            | 1011                           |
| Cl 3  | $2.3 \times 10^{14}$            | 1.55                            | 904                            |
| Cl 4  | $1.3 \times 10^{14}$            | 1.59                            | 826                            |
| Cl 5  | $1.3 \times 10^{14}$            | 1.55                            | 798                            |
| Cl 6  | $1.2 \times 10^{14}$            | 1.25                            | 785                            |
| Cl 7  | $1.2 \times 10^{14}$            | 1.23                            | 587                            |
| Cl 8  | $1.2 \times 10^{14}$            | 1.23                            | 695                            |
| Cl 9  | $9.7 \times 10^{13}$            | 1.16                            | 630                            |
| Cl 10 | $8.6 \times 10^{13}$            | 1.11                            | 597                            |
| Cl 11 | $8.1 \times 10^{13}$            | 1.09                            | 670                            |
| Cl 12 | $8.1 \times 10^{13}$            | 1.09                            | 758                            |
| Cl 13 | $7.3 \times 10^{13}$            | 1.05                            | 607                            |
| Cl 14 | $5.8 \times 10^{13}$            | 0.98                            | 603                            |

halos, we use additional information, such as the virial-to-tidal radius ratio, the maximum circular velocity, and the number of gravitationally bound particles within $r_h$. We consider halos to be isolated if their separation is larger than $\frac{1}{2}$ of the sum of their virial radii. We list the present-day properties of the cluster halos included in our sample in Table 1. The masses and radii correspond to the cumulative overdensity of 180 times the mean density of the universe. In this list, clusters 4 and 12 and clusters 10 and 13 are close pairs, and clusters 6, 11, and 14 and clusters 7, 8, and 9 are triplets, while each of clusters 1, 2, 3, and 5 is a well-isolated system. The clusters thus sample a variety of environments.

We stress that the clusters in the analyzed sample were selected randomly; no specific criterion of relaxation or substructure was used. As Figure 1 shows, clusters in our sample span a wide range of MAHs and formation redshifts.

### 4. MASS ACCRETION HISTORIES

In the hierarchical structure formation scenario, halos are assembled via a continuous process of merging and accretion. Details of the MAH may affect the shape of the halo and its density distribution (Navarro et al. 1997; Bullock et al. 2001; W02; Zhao et al. 2003b). It is therefore interesting to study the accretion history of the halos in conjunction with the study of their density profiles. In this section we study the details of the assembly of the simulated cluster halos by following the most massive progenitor from $z = 10$ to the present. We discuss connections between the halo density profile and its MAH in §§ 5.3 and 6.

#### 4.1. Constructing MAHs

For each $z = 0$ cluster halo, we identify the most massive progenitor by using the halo catalogs of the previous time output. In what follows, if halo 1 is the most massive progenitor of halo 2, then halo 2 will be referred to as the offspring of halo 1. To identify the most massive progenitor of a halo, we first identify all its progenitors in the halo catalog. We then eliminate from the set of potential progenitors objects that are significantly tidally stripped (i.e., their virial-to-tidal radius ratio is greater than 3.5). We use the following criteria to identify the most massive progenitor among the remaining candidates: (1) we eliminate candidate progenitors with masses less than 20% of the offspring mass; (2) using the peculiar velocity of the offspring, we find the approximate location of the progenitor in the previous output and eliminate the candidates outside the sphere with radius $r = 10 v_p \Delta t$, where $v_p$ is the offspring peculiar velocity and $\Delta t$ the time elapsed between two successive outputs; (3) we require that candidate progenitor and offspring halos have a certain fraction of common particles. In what follows, $f_1$ ($f_2$) denotes the ratio of the number of particles that offspring and progenitor have in common to the number of particles in the offspring (progenitor).

The candidate with the largest number of common particles with the offspring and with $f_1 \geq 0.5$ is then chosen to be the most massive progenitor. At the same time the condition $f_2 \geq 0.5$ is also checked and found to be satisfied. If all the progenitors have $f_1 < 0.5$, as is often the case during major mergers, for the progenitor with the largest $f_1$ we also require that more than 95% of the particles within a comoving radius of $10 \, h^{-1}$ kpc from the most bound particle of the progenitor are also found in the offspring. Starting from $z = 0$, we repeat the identification of the most massive progenitor for all the 19

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4 To calculate the density we use the publicly available code SMOOTH: http://www-hpcc.astro.washington.edu/tools/tools.html.
simulation output epochs to \( z \simeq 10 \) or until the progenitor can no longer be identified. The MAHs constructed in this way for each of the analyzed clusters are shown in Figure 1 (solid lines).

Most of the MAHs have a qualitatively similar shape: a rapid increase in mass during the early epochs and a relatively slow increase at the later stages of evolution. Despite the similarities, the details of MAHs differ significantly from object to object. Cl 9, Cl 10, and Cl 13 have not yet reached the second, accretion-dominated stage of their evolution. Their mass is accumulated via intense merger activity up to the present epoch. Cl 7 and Cl 8 appear to have reached the slow accretion phase but experienced a late major merger. The masses of Cl 11 and Cl 14 at early epochs increase almost linearly with the expansion factor \( \log(M/M_0) \propto a \) and reach an \( M \approx \text{const} \) plateau at later epochs.

### 4.2. Major Mergers

In addition to the overall shape of the halo MAH, it is useful to have some more specific information on the major mergers experienced by the clusters. We use the term “major merger” to describe all events that result in more than a 30% increase in the mass of the main progenitor between the two output epochs. As we mentioned above, the average time elapsed

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**Fig. 1.—MAHs of the cluster halos (solid lines).** Also shown are the analytic fits of eq. (4), \( M(\tilde{a})/M_0 = \tilde{a}^p \exp(-\alpha(\tilde{a} - 1)) \) [where \( \tilde{a} \equiv a/a_0 \) and \( a = (1 + z)^{-1} \)]. The dashed lines show fits with both \( \alpha \) and \( p \) varied, while the dotted lines show the fits with the parameter \( p \) fixed to zero. The formation redshift \( z_f \), given by eq. (3) in terms of \( a_0 \), is labeled in each panel. The best-fit \( \alpha \) obtained from the two fits is nearly identical in all cases, except for Cl 9 and Cl 13. For Cl 9 and Cl 13, we also plot the fit obtained using eq. (5) (crosses). Finally, to show the effect of a recent major merger, for Cl 7 we plot the fit assuming an epoch of observation \( a_0 = 0.95 \) rather than \( a_0 = 1 \) as used for all the other fits (dash-dotted line). The \( z_f \) value obtained in this case is given in parentheses.
between two successive outputs of the simulation is \( \approx 0.7 \) Gyr, which is close to the crossing time of \( \approx 1 \) Gyr for a wide range of halo masses. The crossing time is a lower limit for the merger timescale, which means that the spacing of our outputs is appropriate for merger identification (Gottlöber, Klypin, & Kravtsov 2001; see also tests therein). We tabulate the redshift of the last major merger, \( z_{\text{LMM},i} \), as well as the corresponding fractional mass change, \( \Delta M/M_i \), for all the clusters in columns (2) and (3) of Table 2, respectively. One should keep in mind that these numbers are only indicative, since defining a major merger, e.g., as a 20\% mass increase, would render \( z_{\text{LMM}} \) for CI 4 equal to \( \approx 0.15 \).

### 4.3. Formation Redshift and MAH Shape

To characterize evolution of the halos, one can introduce the halo formation epoch (or redshift). Usually, the formation epoch is defined as the time when the mass in the most massive progenitor(s) is equal to some fraction of the halo’s final mass, \( M_0 \) (e.g., see Lacey & Cole 1993; Navarro et al. 1997). Taking this fraction to be equal to \( 1 \), we calculate the formation redshift, \( z_{1/2} \), which we tabulate in column (4) of Table 2. To find \( z_{1/2} \) we use linear interpolation between the successive outputs that bracket \( M/M_0 = 1/2 \). It is interesting to note that \( z_{1/2} \) is typically smaller than \( z_{\text{LMM}} \).

As pointed out by W02, defining the formation redshift as the redshift at which the ratio \( M/M_0 \) takes a specific value gives a formation redshift that depends on the time of observation of the halo. In addition, the definition uses the MAH of the halo at two epochs only (the formation and present epochs) and therefore makes it sensitive to the local jumps in the MAH and less sensitive to the overall MAH shape. CI 7 may serve as an illustration. This object had entered its quiescent stage of evolution relatively early. Nevertheless, the low value of its formation redshift, \( z_{1/2} \), is determined largely by the single late major merger. In view of these considerations, W02 argued that the MAHs can be better characterized by a formation redshift that is derived from a functional fit to the entire MAH. Namely, they propose to fit the MAHs of halos by a simple exponential,

\[
\tilde{M}(\tilde{a}) = \exp \left( \alpha (1 - 1/\tilde{a}) \right), \quad \tilde{a} \equiv a/a_0, \quad \alpha = (1+z)^{-1},
\]

where \( \tilde{M} \equiv M/M_0 \) and \( M_0 \) and \( a_0 \) are the virial mass of the halo and expansion factor at the epoch of observation, respectively. Using the fit, one can define the formation epoch independent of the epoch of observation as the redshift corresponding to a fixed value of \( d \log M/d \log a \equiv S \). The value of \( S \) is arbitrary, and we follow W02 and choose \( S = 2 \), since this is the value required to match the concentration index–collapse redshift relation found by Bullock et al. (2001). The formation redshift, \( z_f \), can then be defined by the relation

\[
z_f = \frac{2}{\alpha} \left( 1 + z_0 \right) - 1.
\]

The function in equation (2) is a special case of equation (4) for \( p = 0 \). The fits in which both \( \alpha \) and \( p \) were varied and the fits with fixed \( p = 0 \) are shown in Figure 1. These fits were obtained by \( \chi^2 \) minimization, even though the robustness of their relative quality with respect to the choice of merit function was tested. For CI 9 and CI 13 the fits with \( p = 0 \) are rather poor. For the two-parameter fits to the MAHs of these clusters, the value of \( \tilde{a} \) is close to zero, which means that the MAHs are better described by a power law in \( \tilde{a} \), rather than by an exponential in \( 1/\tilde{a} \). Detailed study of the MAH shapes clearly requires a larger sample of halos. Analyzing the MAHs in terms of the number of major mergers indicates that the power-law behavior may be related to the high frequency of major mergers up to the present epoch. We note that on average the galaxy-size halos studied by W02 formed earlier

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**Table 2**

| Halo | \( z_{\text{LMM}} \) | \( \Delta M/M_i \) | \( z_{1/2} \) | \( z_f \) | Best Fit | \( S \) | Slope |
|------|-----------------|-----------------|-------------|--------|----------|------|-------|
| Cl 1 | 1.24            | 0.65            | 1.09        | 0.99 ± 0.24 | M        | 9.7  | −1.23 ± 0.19 |
| Cl 2 | 0.85            | 0.56            | 0.83        | 1.37 ± 0.14 | JS       | 9.6  | −1.67 ± 0.15 |
| Cl 3 | 0.75            | 0.36            | 0.67        | 0.78 ± 0.19 | NFW      | 10.7 | −1.35 ± 0.20 |
| Cl 4 | 0.95            | 0.77            | 0.78        | 0.83 ± 0.21 | JS       | 5.4  | −1.89 ± 0.20 |
| Cl 5 | 0.85            | 0.34            | 0.69        | 1.32 ± 0.33 | M        | 12.2 | −1.30 ± 0.20 |
| Cl 6 | 0.95            | 0.45            | 0.91        | 1.33 ± 0.33 | JS       | 14.7 | −1.83 ± 0.19 |
| Cl 7 | 0.03            | 0.82            | 0.07        | 1.04 ± 0.25 | JS       | 13.6 | −2.01 ± 0.22 |
| Cl 8 | 1.74            | 1.83            | 0.41        | 0.80 ± 0.20 | M        | 11.5 | −1.25 ± 0.22 |
| Cl 9 | 0.15            | 0.95            | 0.19        | −0.04 ± 0.14 | M       | 3.5  | −1.68 ± 0.22 |
| Cl 10 | 0.25           | 0.78            | 0.22        | −0.11 ± 0.08 | M        | 2.3  | −1.78 ± 0.28 |
| Cl 11 | 0.85           | 2.11            | 0.80        | 0.21 ± 0.09 | NFW      | 8.4  | −1.38 ± 0.31 |
| Cl 12 | 1.24           | 1.67            | 1.23        | 1.64 ± 0.43 | M        | 12.6 | −1.50 ± 0.24 |
| Cl 13 | 0.03           | 0.33            | 0.25        | 0.41 ± 0.16 | JS       | 4.3  | −1.36 ± 0.42 |
| Cl 14 | 0.95           | 0.81            | 0.93        | 0.82 ± 0.20 | JS       | 10.8 | −1.42 ± 0.30 |

Note.—Col. (2): Redshift of last major merger; col. (3): fractional mass change during last major merger; col. (4): redshift at which half of the cluster’s current mass has been accreted; col. (5): formation redshift defined from MAHs; col. (6): best fit among the Navarro et al. 1996, 1997, Moore et al. 1998, and Jing & Suto 2000 profiles; col. (7): concentration index; col. (8): logarithmic slope as obtained by averaging the local logarithmic slope between the smallest resolved radius and 3\% of the virial radius.
than our halos, and thus only a small fraction (<5%) of their halos were similar to our Cl 9 and Cl 13.

Clearly, a smooth-fitting function for the MAHs cannot capture all the features of the actual evolution, such as minor and major merger events. These events, however, do influence the values of the best-fit parameters. In the case of Cl 7, one can see the effect of a very recent major merger. As can be seen in Figure 1, if we make the fit at $d_0 = 0.95$ instead of $d_0 = 1$ (i.e., with the observation epoch prior to the merger), we get a better overall fit.

Van den Bosch (2002) found that the average MAH of halos generated using the extended Press-Schechter formalism is described well by a different two-parameter function,

$$\log (\langle M \rangle) = -0.301(-1)^\nu \left( \frac{\log a}{\log (1 + z_f)} \right)^\nu,$$

(5)

with $z_f$ and $\nu$ the parameters to be determined. By definition, $z_f$ is the redshift that corresponds to $(M/M_0) = 1/2$. Typical best-fit values for $\nu$ are in the range 1.4–2.3. As before, clusters Cl 9 and Cl 13 are exceptions with $\nu \approx 1$ (i.e., their MAH is the power law $M \approx M_0 a^{d}$). We find that equation (5) gives fits equally good as those obtained with the two-parameter function of equation (4) in all cases. This function, however, is not convenient when used to calculate the formation redshift via the logarithmic derivative of the mass with respect to the scale factor. More specifically, in equation (5) (and its derivative) $z_f$ is by construction positive, and thus for a general $\nu$, $a$ has to be $\leq 1$. This is not the case for objects whose mass accretion rate reaches $S = 2$ in the future (negative formation redshift), and thus equation (5) cannot be used in those cases to obtain a formation redshift. We choose to use equation (4), as it is the simple extension of the function used by W02 and the definition of $z_f$ via the mass logarithmic derivative is guaranteed.

The formation redshifts estimated using the best-fit parameters of equations (3) and (4) are given in column (5) of Table 2. Note that this definition of $z_f$ allows for future (negative) formation redshifts. The two definitions of the formation redshift, $z_{1/2}$ and $z_f$, are correlated at 98% probability level (a Spearman rank correlation of 0.58). In addition, to evaluate the effect of the early and late portions of the MAH on the formation redshift, we estimated the formation redshifts by using only the parts of the MAH for which $a < 0.65$ and $a > 0.65$; $z_f^{0.65}$ and $z_f^{0.65}$. The best-fit values of both $z_f^{0.65}$ and $z_f^{0.65}$ are consistent with the values of $z_f$ within errors. In addition, the values of $z_f^{0.65}$ are consistent with $z_f^{0.65}$ within (large) errors. Although our cluster sample is small, the significant spread in the values of $z_f$ for clusters of the same $M_{180}$ is apparent. This indicates that halos of the same mass exhibit a wide range of MAH shapes.

5. DENSITY PROFILES
5.1. Fitting Procedure

For each cluster halo, we fit the Navarro et al. (1997; the NFW profile), the Moore et al. (1998; the M profile), and the Jing & Suto (2000; the JS profile) analytic density profiles. For a general profile of the form

$$\rho(r) = \frac{\rho_s}{x^{\gamma}(1 + x)^{(\beta-\gamma)/\alpha}}, \quad x \equiv r/r_s,$$

(6)

the NFW, M, and JS profiles have values of $(\alpha, \beta, \gamma)$: (1, 3, 1), (1.5, 3, 1.5), and (1, 3, 1.5), respectively.

In what follows, we define the concentration of a halo as $c_2 \equiv r_{180}/r_2$ and $c_5 \equiv r_{63}/r_2$, where $r_2$ is the radius at which the logarithmic slope of the best-fit profile is equal to $-2$, $r_{180}$ is the radius within which the average density is equal to 180 times the mean matter density of the universe, and $r_{63}$ is the virial radius defined using the redshift-dependent virial overdensity ($\approx 180$ at $z > 1$ and $\approx 340$ at $z = 0$). To convert from $(M_{180}, r_{180})$ to $(M_{vir}, r_{vir})$ we use the fitting formulas of Hu & Kravtsov (2003). For the general profile of equation (6), the radius $r_2$ is given by

$$r_2 = \left( \frac{2-\beta}{2-2\beta} \right)^{1/\nu} r_s,$$

(7)

where $r_s$ is the scale radius of the corresponding analytic profile. Thus, $r_2 = r_s$ (NFW), $r_2 \approx 0.63 r_s$ (M), and $r_2 \approx 0.5 r_s$ (JS). In other words, $c_2 = c_{NFW}$, $c_2 \approx 1.5 c_{CM}$, and $c_2 = 2 c_{JS}$ if the best fit is found to be the NFW, the M, or the JS profile, respectively. We present the resulting concentration indices at $z = 0$ in column (7) of Table 2.

There are a number of factors that may affect the fits of analytic profiles to the profiles of the simulated clusters: the choice of binning, the merit function, the range of radii used in the fitting, the weights assigned to the data points, etc. For example, for the merit functions sensitive to the number of bins, such as $\chi^2$, the choices of binning and bin weights are extremely important.

In the following analysis, we use equal-size logarithmic bins to give more statistical weight to the inner regions of the halos. The number of bins is 30 for late epochs. For early epochs the number of bins is reduced to ensure that each bin contains a sufficiently large number (>100) of particles. We take as bin center the average radius of all particles in a bin. We checked that the fits are robust when varying the number of bins around the adopted value. We find, however, that the choice of binning affects the quality of the fit. For example, for a large number of bins the resulting profiles are quite noisy. Our choice of binning minimizes the noise. We weight the data points by the Poisson noise in the number of particles of each bin.

The presence of substructure may substantially bias fits of smooth analytic profiles. In particular, a substantial amount of substructure is present in the outer regions of halos, and the profiles in these regions are often nonmonotonic, exhibiting "bumps." To minimize the bias, we fit the profiles by using only the bins from a minimum resolved radius (see § 5.2) up to the radius within which the average density is equal to 500 times the critical density of the universe, $r_{500}$. This choice is motivated by the results of Evrard, Metzler, & Navarro (1996), who find that the material within this radius is generally relaxed and in hydrostatic equilibrium. We find that for the clusters in our sample $r_{500}/r_{180} \approx 0.36–0.37$ at $z = 0$. To the same end, the density profiles of Cl 9 and Cl 10 were obtained by averaging the $z = 0$ and $z \approx 0.05$ outputs. This renders the profiles less noisy and improves the quality of the obtained fits. The averaging does not change the best-fit parameters significantly.

It is important to understand that the formal quality of the fit may depend on the merit function, as well as the kind of binning and weighting used. In the following analysis, we fit the analytic profiles for the parameters $\rho_s$ and $r_s$ by minimizing the $\chi^2$. Klypin et al. (2001) show that the $\chi^2$ merit function applied to the profiles with logarithmic binning with
the Poisson error weights results in the fits of the NFW profile that are systematically below the simulated profiles in the inner regions. The logarithmic binning gives higher density of data points at small radii, thus creating bins with a smaller number of particles. The $\chi^2$ fits for the Cl 2 profile are shown in Figure 2. Also shown are the fits obtained when using the maximum fractional deviation (MFD) merit function, $\max_j |\bar{\rho}_j - \rho_{\text{data}}|/\rho_{\text{data}}$, which gives equal weight to all radial bins. Figure 2 shows that this merit function reduces the deviations in the inner region ($r \lesssim 0.02 r_{\text{vir}}$) at the expense of significant deviations at intermediate radii ($0.02 \lesssim r/r_{\text{vir}} \lesssim 0.2$). Although the MFD merit function is less sensitive to the choice of binning, it is more sensitive to the presence of substructure bumps in the profile than $\chi^2$. Both merit functions have their advantages and drawbacks.

Luckily, we find that regardless of the merit function and bin weighting used, the relative goodness of fits for different analytic profiles remains the same. If, for example, the $M$ profile is a better fit to the simulated profile than the NFW and JS profiles in the $\chi^2$ minimization, it is the resulting best fit in the maximum deviation minimization as well. The conclusion about which profile fits best is therefore robust. Note, however, that this is not true for the conclusions about the systematic ways by which a given fit fails. For example, the characteristic $S$ shape of the fractional deviation as a function of radius for the NFW fits found in various studies (e.g., Moore et al. 1999b; Ascasibar 2003) is not a robust result because it depends on the merit function, binning, and weighting.

In addition to the three widely used analytic profiles, we experimented with fits of more general analytic expressions of the form given by equation (6). Overall, the fitting procedure performed by varying all three parameters, $\alpha$, $\beta$, and $\gamma$, leads to strong degeneracies between parameters (see also Klypin et al. 2001). One can find several combinations of parameters that fit the data equally well. For example, good fits with inner asymptotic inner slopes as shallow as $\gamma = -0.3$ can be found.

In view of these degeneracies, we choose not to use generalized fits but to simply complement the analytic fits with measurements of the logarithmic slope profiles $s(r) \equiv \partial \log \rho(r)/\partial \log r$. This analysis is complementary to the fits because the slope is sensitive to the local shape of the profile, while the fits may be sensitive to its global shape. The logarithmic slope is computed using the linear fit to $\log \rho - \log r$ locally. We use five neighboring profile bins centered on a given bin in the fit (i.e., two bins on either side) with a total of 100 bins for the whole range of radii from $r_{\text{min}}$ to $r_{\text{vir}}$. The choice of the number of bins is a tradeoff between the slope errors and spatial resolution. We experimented with fitting polynomials up to the fourth order but found no advantage over a simple linear fit. The local logarithmic slope is sensitive to the presence of transient massive substructures within the halo. For illustrative purposes only, to reduce the substructure-induced noise we also smooth the slope by using a top-hat filter.

5.2. Convergence Study

To study the effects of mass and force resolution, Cl 2 was resimulated with 8 times more particles ($m_p = 3.95 \times 10^7 h^{-1} M_\odot$) and with more refinements. The ART code performs mesh refinements when the number of particles in a mesh cell exceeds a specified threshold. Thus, the mass resolution is tightly linked to the peak spatial resolution achieved in simulation. The cell size of the highest refinement level, which we consider the formal resolution of the simulation, was 0.6 and 1.2 $h^{-1}$ kpc in the higher (HR) and lower resolution (LR) simulation, respectively. The HR simulation was initialized using the same set of modes as the LR. We therefore follow the formation of the same object with more particles. The comparison of density profiles allows us to check for the two-body relaxation effects, which may be important in cluster cores (Diemand et al. 2004), and numerical convergence.

We compare the density profiles of Cl 2 in the HR and LR simulations in Figure 3. To minimize the differences due to substructure, the profiles shown are obtained by averaging the $z = 0$, 0.02, and 0.1 outputs of the corresponding runs. The figure shows that the fractional difference between the profiles is $\lesssim 0.2$ down to $\sim 3$ formal resolutions of the LR run. This is in agreement with a previous convergence study for the ART code by using simulations with lower mass resolution (Klypin et al. 2001). A comparison of density profiles of clusters in ART simulations with the density profiles in simulations using the GADGET code (Springel, Yoshida, & White 2001) was recently performed by Ascasibar et al. (2003), who found excellent agreement between the two codes at the resolved scales.

Figure 3 (top) shows the local logarithmic slope of the density profiles as a function of radius (see § 5.1 for details). The error bars are computed by propagating the Poisson errors in the density profiles. At $r \gtrsim 200 h^{-1}$ kpc the strong non-monotonic variations of the slope are due to the presence of substructure. Despite the averaging, the small differences in the locations of substructures result in large differences in the slope value at a given $r$. At the same time, the slopes in the HR and LR runs agree well at scales $5 h^{-1}$ kpc $\lesssim r \lesssim 200 h^{-1}$ kpc. It is interesting to note that there is no evidence for a well-defined asymptotic inner slope. The local logarithmic slope in
Within which the average density equals 500 times the mean density for the universe, respectively. This is shown in Fig. 1. The cluster undergoes an intense merger event at $z \approx 1$. The radius within which the average density equals 340 and 180 times the mean density, and $r_{200}$, the radius within which the average density equals 340 and 180 times the mean density for the universe, respectively. Bottom: Fractional deviation between the LMM and HR profiles. The error bars are computed by propagating the shot noise in the density profiles. Top: Local logarithmic slope as a function of radius in the HR (squares) and the LR (triangles) runs.

Both runs increases monotonically with decreasing radius down to the smallest resolved scales.

On the basis of these results, in the subsequent analysis we conservatively consider only scales greater than $r_{\text{min}} = 10 h^{-1}$ kpc (or 4 times the formal resolution of the LR run) for both the fits and the plots. In addition, we require that more than 200 particles be contained within the minimum radius (Klypin et al. 2001). In our simulations this criterion is relevant only at early epochs ($z \gtrsim 2$), since at later epochs a $10 h^{-1}$ kpc radius always contains more than 200 particles for all clusters.

One of the main results of our analysis is an apparent diversity of the density profiles. At the same time, we find that if a profile is best fitted by a particular analytic profile at $z = 0$, it is generally best fitted by the same analytic profile at early epochs out to $z \sim 1$. Potentially, this is an interesting clue to the processes that determine the shape of the profile. It is therefore important to check that the conclusion does not change with resolution. Figure 4 shows the fits of the NFW and JS profiles to the profiles of Cl 2 in the LR and HR runs at different epochs. The fits were done using bins in the radial range $[r_{\text{min}}, r_{500}]$ (see § 5.1). The figure shows that at all shown redshifts the JS profile fits the simulated profiles at small radii better than the NFW profile in both runs. This is remarkable as the cluster experiences relatively rapid increase in mass and several violent mergers between $z = 1.5$ and 0. The mass changes by more than a factor of 5 during this period (see Fig. 1). The cluster undergoes an intense merger event at $z = 0.6$, and the last major merger for our definition occurred at $z = z_{\text{LMM}} \approx 0.85$.

We find that our fitting results are robust to changes in both the minimum and maximum radius used in the fits. For example, concentration changes by no more than $\sim 10\%–20\%$ if a different outer radius is used ($\sim 2/3 r_{180}$, and for some clusters an outer radius $\sim r_{180}$ did not change the results much). More importantly, the conclusion about the best-fit analytic profile remains the same, although in some cases the best fit changes from the M to the JS or vice versa. This can be expected because these analytic profiles are quite similar. We also repeated fits with a minimum radius ($\approx 20 h^{-1}$ kpc comoving) twice as large and find that the best-fit analytic profile remains the same and that the concentration changes by $\lesssim 10\%$.

5.3 Results

Our results on the density profiles at $z = 0$ are presented in Figure 5. In Figure 5 (middle of each plot) we plot the actual profiles, as well as the NFW, M, and JS fits. All profiles are plotted at $r < r_{180}$ but are fitted using only bins in the range $r_{\text{min}} < r < r_{500}$, as discussed in § 5.2. In Figure 5 (bottom of each plot) we present the fractional deviation as a function of distance for each of the analytic fits. The analytic profile that provides the best fit to the profile of each cluster is given in column (6) of Table 2. In Figure 5 (top of each plot) we plot the logarithmic slope as a function of radius for each of the analytic fits and the actual local logarithmic slope as calculated from the simulated profiles (see § 5.1).

In most cases the best analytic fit provides by far the best fit compared with the other profiles. This is especially true for Cl 3 and Cl 11 (see Fig. 5), the two clusters best fitted by the NFW profile. The other two analytic profiles fail significantly compared with the NFW for these clusters. If we consider the similar M and JS as one family of profiles, the two families (NFW and M/JS) typically differ significantly in quality. As discussed above, the systematic way in which the NFW profile fails to fit the data can be attributed to the merit function used to obtain the fits. For our choice of merit function, the fits follow the actual profile well at intermediate distances. The largest deviations occur at the innermost regions. For large distances, the three fits are almost indistinguishable.
Figure 5 and Table 2 clearly show that there is significant dispersion in the shapes of the profiles, concentrations, and inner slopes. The dispersion of concentration parameter was studied in several analyses (Navarro et al. 1997; Jing 2000; Bullock et al. 2001; Eke, Navarro, & Steinmetz 2001; W02; Zhao et al. 2003b) and is thought to be related to the distribution of the halo formation epochs (W02). The typical values of scatter are $\sigma_{\log c} \approx 0.14$, with only a weak dependence on mass (Bullock et al. 2001; W02). Table 2 shows that the $c_{2}$ concentration indices of our clusters span a wide range of values, from 2.3 to 14.7. Making the appropriate conversion from $c_{2}$ to $c_{0}$, we find that the formal dispersion is $\sigma_{\log c} \approx 0.2$ at $z = 0$, larger than the dispersion for the smaller mass halos used to derive this scattering in other studies, which may reflect the more recent formation times of cluster halos, as well as their more diverse MAHs. For example, Klypin et al. (2003) and Colin et al. (2003) find a significantly smaller dispersion $\sigma_{\log c} \approx 0.1$ for a subsample of relaxed halos without significant substructure. Indeed, the clusters with the three lowest concentrations, Cl 9, Cl 10, and Cl 13, have all had a recent major merger (see $z_{\text{LMM}}$ in col. [2] of Table 2). The small concentration of these objects is due to the shapes of their density profiles, which are close to a power law over a wide range of radii. Careful examination of MAHs and merger histories indicates that the recent major merger activity results in a low concentration of density profiles. Cl 4, which also has a small concentration compared with the majority of the clusters, has a formal $z_{\text{LMM}} = 0.95$ for the definition of major merger adopted in our study. However, in agreement with the other low-concentration objects, it had a large merger ($\approx 22\%$.

![Figure 5](image-url)
fractional mass increase) at a very recent epoch \((z \approx 0.15)\). In addition, we find a strong correlation between \(z_F\) and \(c_{-2}\), in agreement with the correlation advocated by W02.

Figure 6 shows the redshift evolution of the median virial concentration, \(c_v\), of our sample. We also plot the predictions of the models by Bullock et al. (2001) and Eke et al. (2001). Our results seem to be in some general agreement with both model predictions. Overall, the Eke et al. model (2001) seems to be in better agreement than the Bullock et al. (2001) model. Recently, Dolag et al. (2004) found that the standard Bullock et al. (2001) recipe systematically underestimates concentrations of cluster-size halos at all redshifts. Figure 6 shows a similar trend in our simulations, although the difference we find is noticeably smaller. This may be due to the smaller mean mass of clusters in our sample. Zhao et al. (2003b), for example, show that discrepancy between simulations and the Bullock et al. (2001) model increases with increasing halo mass. In the mass range probed here, the difference from the analytic prescription of Bullock et al. (2001) is considerably smaller than the scatter in concentrations.

Figure 7 shows the evolution of the average concentration and the average MAH of our clusters. The figure shows that during the period of rapid mass growth the concentration is nearly constant at \(c_v \approx 3-4\), while during the period of gradual mass growth it increases with decreasing redshift as \(c_v \propto (1+z)^{-1}\). Therefore, the concentration of halos is approximately constant while they experience frequent major mergers, and there may exist a “floor” to the concentration values of \(c_{\text{min}} \approx 3\), while concentrations start to increase with time at \(z < z_F\). This behavior was pointed out by Zhao et al. (2003b, 2003a). However, unlike Zhao et al. (2003b), we find that the model of W02 does not underestimate the concentrations at the cluster-size halos of our sample. The evolution of average concentration in Figure 7 is similar to that found by Dolag et al. (2004). The mean concentration of cluster progenitors in their simulations (their Figs. 4 and 7) is approximately constant at \(z \gtrsim 1\).
We do not find a clear connection between the redshift of the last major merger and the best-fit profile (col. [6] in Table 2). We do find that none of the objects with low $z_{\text{LMM}}$ have the NFW profile as the best fit, but the statistics of our sample is too small to reach a firm conclusion. Nevertheless, for each individual system the shape of its density profile is remarkably stable during evolution. The best-fit analytic profile at $z = 0$ is typically also the best fit at earlier epochs, as was shown for Cl 2 in Figure 4. Figure 8 shows the evolution of the density profiles and the analytic fits at each epoch for Cl 1 and Cl 3. The NFW profile is a better fit than the JS at all epochs for $z < 1.5$ for Cl 1. This stability of the profile shape with time holds for most clusters with some exceptions. Cl 3 illustrates the case in which the best-fit analytic profile changes from epoch to epoch. We find this behavior for 4 of the 14 clusters in our sample.

Note that the NFW fits never reach their inner asymptotic slope of $-1$ at the radii we probe. The average logarithmic slopes estimated by averaging the local slope around $0.03r_{180}$ (see § 5.1) range from $-1.2$ to $-2$ and are given in column (8) of Table 2. In most cases, the local slope changes monotonically with radius with no sign of reaching the asymptotic inner slope. Note that for typical concentrations of cluster halos we expect the asymptotic slope to be reached at the resolved scales, at least for the M profile. This can be seen from the slope profiles for the best-fit analytic fits shown in Figure 5, top. The only cases in which the slope profiles are flat for relatively large radius ranges correspond to the halos with recent merger activity and rapid mass growth (i.e., Cl 4, Cl 9, Cl 10, and Cl 13).

5 From eq. (6), $r_s$ is the radius at which the logarithmic slope is equal to $-(3 + \gamma)/2$, with the asymptotic slope reached at $r \ll r_s$. The NFW and the JS profiles have typically smaller $r_s$ values than that of the M profile. As a result, they reach their asymptotic slopes at smaller distances. An analytic profile, of course, can be a good fit regardless of whether its inner asymptotic slope is resolved.
Interestingly, we find that the density profiles of systems that experience intense merger activity until the present epoch (clusters Cl 4, Cl 9, Cl 10, and Cl 13) can be well described by a single power law \( r^{-\gamma} \) with slope \( \gamma \) ranging from \(-1.5\) to \(-2\). Similar to other profile shapes, the power-law density profile for these systems is maintained for earlier epochs out to \( z \approx 1.5 \). In addition, there is evidence that the profiles of all clusters during their rapid mass growth stages are close to a power law. In particular, we find that the power law provides an increasingly better fit with increasing redshifts for all our clusters. At early epochs (\( z \gtrsim 1.5–2 \)), the power-law fit is always either comparable to or better than the NFW, M, and JS analytic profiles. The power law–like profiles relate to low concentrations. This is because they maintain a slope slightly shallower than \(-2\) out to large radii so that the scale radius is large. Indeed, the clusters with power-law profiles at \( z = 0 \) have the lowest concentrations in our sample. The decrease of concentrations at higher redshifts may thus reflect the power law–like density profiles of actively merging systems.

Fig. 5.—Continued

Fig. 6.—Median concentration vs. virial mass at different redshifts for the progenitors of clusters in our sample (points). The vertical error bars represent the 1 \( \sigma \) scatter in concentrations for the 14 clusters, while horizontal error bars show the mass range of the halos at each epoch. The predictions of the Bullock et al. (2001; thick lines) and the Eke, Navarro, & Steinmetz (2001; thin lines) models are plotted for comparison.

Fig. 7.—Average MAH (top) and average concentration of cluster progenitors (bottom) as a function of scale factor measured in units of the formation scale factor, \( a_f \). The error bars in both panels represent the 1 \( \sigma \) spread around the mean. The figure shows that the concentration is approximately constant at \( c_v \approx 3–4 \) during the period of rapid mass accretion (\( a/a_f < 1 \)) and increases with decreasing redshift during the period of slow mass growth (\( a/a_f > 1 \)). For comparison the dashed line shows \( (1+z)^{-1} \) evolution.
6. DISCUSSION AND CONCLUSIONS

We have studied the MAHs and density profiles of 14 cluster-size halos simulated using the ART code in a flat ΛCDM cosmology. In agreement with previous studies, we find that most MAHs have a similar shape: an early, merger-dominated mass increase followed by a more gradual, accretion-dominated growth. To obtain a formation redshift that characterizes the overall shape of the MAH, we perform analytic functional fits. The typical MAHs are well described by the one-parameter exponential function proposed by W02 (eq. [2]). Two of the clusters in our sample experience intense merger activity and rapid mass growth until the present-day epoch. The MAHs of these systems are better described by a one-parameter power-law function in the scale factor. We therefore generalize the form proposed by W02 into a two-parameter form (eq. [4]) to encompass both exponential and power-law MAHs. For each class, however, the fit reduces to a one-parameter fit.

We check the convergence of halo density profiles by using a resimulation of one of the clusters with 8 times more particles and better force resolution. We show that both the halo profiles and their local logarithmic slopes converge at scales larger than about 4 times the formal resolution of the low-resolution run, in agreement with a previous convergence study of the ART code by Klypin et al. (2001). We fit the density distribution of the clusters with the NFW, M, and JS analytic profiles. Experiments show that the choice of merit function, weighting, and binning affects the absolute quality of a fit and may bias conclusions about how well a particular analytic profile fits simulation results. We find, however, that the relative goodness of fit for the three analytic profiles and our conclusions about the best-fit profile are robust to the changes in binning and merit function.

The main result of our study is a remarkable diversity of the MAHs, profile shapes, concentrations, and inner slopes for cluster-size halos in a relatively narrow mass range. The concentrations of cluster-size halos at the present-day epoch exhibit a scatter of σₗoconc ≈ 0.20. This scatter is related to the diversity of halo MAHs and formation redshifts. We find a statistically significant correlation between the formation redshift and the concentration of a halo, in agreement with results of W02. There is a more detailed connection between the MAH and concentration. The concentration of a halo is approximately constant at cₜ ≈ 3–4 during the period of rapid mass growth and frequent major mergers (z > zf) and increases with decreasing redshift when the mass accretion rates slow down at z < zf. This behavior was recently pointed out by Zhao et al. (2003b, 2003a). The implied floor in the concentration is not accounted for in the currently used models for cₜ(M) (Bullock et al. 2001; Eke et al. 2001), which predict a monotonic decrease of concentration with increasing mass and may thus underestimate concentrations of the most massive (∼5 × 10¹⁴ h⁻¹ M☉) halos. This may have important implications for estimates of the expected number of wide-separation quasar lenses (e.g., Kuhlen, Keeton, & Maddox 2004) and other results sensitive to the concentrations of very massive clusters.

The inner logarithmic slope of cluster profiles at 3% of the virial radius (or 10–50 kpc) ranges from −1.2 to −2. In the best-resolved clusters, the logarithmic slope does not seem to reach a specific asymptotic value down to the smallest resolved scales in our simulations (r/r₁₈₀ ≈ 0.007). A similar conclusion was reached by Klypin et al. (2001) for galaxy-size halos and several recent studies (Power et al. 2003; Ascasibar et al. 2003; Fukushige et al. 2003; Hoefl et al. 2004; Hayashi et al. 2003). It is still not clear whether the density profiles in our simulations are consistent with the density distribution of observed clusters. We note, however, that at the scales probed in observations the slope is not expected to be shallower than −1. The asymptotic value of the slope has been a subject of much numerical effort in the last several years. Our results indicate that a universal asymptotic slope may not exist. We should note that the resolution of current dissipationless simulations is sufficiently high to converge on the density profile at scales smaller than the size of a typical central galaxy in clusters and groups (∼30–50 kpc). Further improvement in profile modeling should therefore include realistic dynamics and cooling of the baryonic component as contraction of gas is expected to significantly modify dark matter distribution at these scales.

One of the most interesting results of our study is the existence of systems with density profiles that can be well described by a power law ρ ∝ r⁻γ, with γ ranging from ≈−1.5 to ≈−2. All these systems are still in their rapid mass growth stage and have experienced a recent major or minor merger. Remarkably, these halos maintain the power-law density profiles at earlier epochs out to at least z ≈ 1.5. The relatively shallow γ > −2 power-law slopes result in low concentrations as the scale radius at which the density profile reaches the slope of −2 is at large radii. There are also indications that the profiles of all clusters are power law—like during their rapid mass growth stages. We find, for example, that the power law provides an increasingly better fit with increasing redshifts for all our clusters. At early epochs (z ≈ 1.5–2), the power-law fit is always either comparable to or better than the NFW, M, and JS analytic profiles. We did not find any correlation of the power slope with the details of the cluster MAH. It would be interesting to look for such correlations by using a larger sample of objects.

When the mass growth slows down at z > zf, an outer steeper density profile is built up. As pointed out by Zhao et al. (2003b), the difference in density profiles during the two mass accretion regimes may be due to more violent and thorough relaxation during the period of rapid mass growth. Although Zhao et al. (2003b) focused on halo concentrations and did not
consider density profile shapes, their results are consistent with our conclusions. In particular, they find that during the rapid mass growth stage the circular velocity is nearly constant from the scale radius to the virial radius. This behavior is consistent with a power-law density distribution with a slope close to $-2$.

In a recent study, Ascasibar et al. (2003) find that objects that experienced a recent merger event$^6$ have lower concentrations and steeper inner profiles than more relaxed systems. This is consistent with our findings described above. At the same time, Ascasibar et al. (2003) find that relaxed systems are better fitted by the NFW, while systems with a recent major or minor merger are better fitted by the M profile (see also Ascasibar 2003). They thus associate a particular shape of the profile with a recent merger history. In contrast, our results show that the shape of density profiles is set early in the halo evolution and is usually stable over the past 10 billion years. Clusters with density profiles best described by the NFW rather than a JS at $z = 0$ tend to have NFW-like profiles at earlier epochs as well. The reverse is also true. We tested this conclusion by using the high-resolution resimulation of one of the clusters in our sample. Also, we do not find any correlation between the redshift of the last major merger (in our definition) or the formation redshift and the best-fit analytic profile.

The origin of the distinctive density profile shape of the CDM halos remains poorly understood. Our results and results of other recent studies indicate that the shape is tightly linked to the halo MAH. During the period of rapid mass accretion the violent relaxation is significant and results in a power law—like density distribution. This stage of evolution usually occurs early, when the universe is dense and builds up the inner dense regions of halo. At this point, the logarithmic slope of the density distribution is shallower than $-2$ over a large fraction of the halo volume and its concentration is small. At later epochs, as the mass accretion rate slows down, the outer regions of the halo are built, while its central regions remains nearly intact. This can be seen in Figure 8 and Figures 10–13 of Fukushige & Makino (2001). This picture can explain why the best-fit analytic profiles tend to be the same at various redshifts. The fits are sensitive to the density distribution at small and intermediate ($\sim r$) radii, which are set early. However, it is still unclear which process(es) determine a particular shape of the profiles. The key to understanding these processes appears to be in the details of early evolutionary stages of CDM halos, which will be the subject of a future study.

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$^6$ Note that Ascasibar et al. (2003) identify a recent merger by the presence of massive substructures within virial radii of their systems. This is different from our definition, which identifies major mergers directly from mass accretion tracks.
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