Universality in Transport Processes of Unconventional Superconductors

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We show that some of the low temperature transport coefficients (e.g., electrical and thermal conductivities, viscosity and sound attenuation) are universal, i.e., independent of the impurity concentration and phase shift for specific classes of unconventional superconductors. The existence of a universal limit depends on the symmetry of the order parameter and is achieved at low temperatures $k_B T \ll \gamma \ll \Delta_0$, where $\gamma$ is the bandwidth of the impurity induced Andreev bound states. The density of states is finite at zero energy and leads to the re-appearance of the Wiedemann-Franz law deep in the superconducting phase for $k_B T \ll \gamma$. Our findings also show that impurity concentration studies at low temperatures can distinguish between different order parameter symmetries.

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We investigate the behavior of the heat current, the electrical current, and the momentum current (transverse sound attenuation) for an unconventional superconductor, i.e., for an order parameter with reduced symmetry for which gapless excitations exist even at zero temperature. Such superconducting states have been argued to both exist in the cuprates and heavy fermion superconductors. A leading candidate in the cuprates is an odd-parity, spin triplet $E_N$ state, while in heavy fermion systems, a leading candidate is a singlet state with lines of nodes at the Fermi positions $p_{fx} = \pm p_{fy} \not= 0$. Similarly, the most promising candidates in the heavy fermion metal UPt$_3$ are the two-dimensional orbital representations coupled to a symmetry breaking field. For UPt$_3$, which has a hexagonal crystal structure ($D_{6h}$), phase diagram studies, and transport measurements lead to either an even-parity, spin singlet $E_{1g}$ or an odd-parity, spin triplet $E_{2u}$ pairing state. In both cases the order parameter vanishes at the Fermi surface on a line in the basal plane, $p_{fy} = 0$, and at points at the poles, $p_{fx} = p_{fy} = 0$.

Instead of examining the effects of the multi-sheeted Fermi surface on the heat current, charge current, and momentum current, we model the excitation spectrum by an excitation gap that opens at line and point nodes on the Fermi surface, and by the Fermi surface properties in the vicinity of the nodes (i.e., the Fermi velocities, $v_F$, and the density of states, $N_F$, near the nodes). Crystal symmetry determines the positions of the nodal regions of the excitation gap on the Fermi surface, but not the prefactors for the gap opening.

The low-temperature behavior of the transport coefficients probes lower-dimensional regions of the Fermi surface, $\epsilon \lesssim \gamma \ll \Delta_0 \ll E_f$, where the excitation gap vanishes, and is less sensitive to the overall geometry of the Fermi surface. The low-energy scale $\gamma$ is defined by the bandwidth of the impurity induced Andreev bound states, and reflects the formation of a novel metallic state deep in the superconducting phase; for strong scattering $\gamma \propto \sqrt{\Gamma_0 \Delta_0}$, where $\Gamma_0$ is the (elastic) normal-state scattering rate $h/2\tau(0)$. We parametrize the nodal regions of the gap with a minimal set of nodal parameters, and attempt to fit these parameters in order to achieve accurate low temperature limits for the different transport coefficients along the principal axes of the crystal. Thus, our order parameter model depends on angle ($p_f$) and the nodal parameters ($\mu_i$), $\Delta(p_f; \mu_i)$. The advantage of this approach is that we can quantitatively determine the phase space contributing to the low temperature transport coefficients and then examine in more detail the effects of impurity scattering and order parameter symmetry on the current response, without having to know the overall shape of the Fermi surface or basis functions.

The number of nodal parameters is fixed by the minimal number of symmetry unrelated point and line nodes. These parameters define the slope or curvature of the gap near a line or point node in a spherical coordinate system (uniaxial anisotropy is included by mapping an ellipsoidal Fermi surface onto a sphere). Specifically, for the E-rep models of UPt$_3$ we parametrize the gap in the vicinity of the equatorial line node by $|\Delta(\theta)| \approx \mu_0 |\Delta_0|^2 - \theta$, while near the poles $|\Delta(\theta)| \approx \mu_0 |\Delta_0|^2 \theta^n$, where the internal phase winding number is $n = 1$ for $E_{1g}$ and $n = 2$ for $E_{2u}$. This parametrization allows us to adjust independently the opening of the gap at the line and point nodes in order to describe the low-energy excitation spectrum. Note that the crucial difference between the $E_{1g}$ and $E_{2u}$ state
lies in the opening of the gap at the polar point nodes. A similar parametrization yields for the \textit{d}_{x^2-y^2}\textit{-}wave order parameter model in the cuprates with a cylindrical Fermi surface: \(\Delta(\phi) \approx \mu \Delta_0(\frac{\pi}{4} \phi), \phi \approx \pi/4\).

We calculated in linear response (including vertex corrections), and in the long wavelength limit \((q \to 0)\), for sufficiently low temperatures and external frequencies \((k_BT, \hbar\omega \lesssim \gamma \ll \Delta_0)\) the transport coefficients in the limit of weak and strong impurity scattering. The asymptotic values, which we derived for the electrical and thermal conductivity, as well as for the transverse viscosity \(\eta\) and ultrasound attenuation \(\alpha\), are listed in Table I. Here, we restricted ourselves to the hydrodynamic limit and orientations of the wavevector \(q\) and polarization \(\varepsilon\), such that \(\alpha_{ij} = (q^2/\rho c_s)\eta_{ij}\varepsilon_i\varepsilon_j\), where \(\rho\) is the mass density and \(c_s\) the speed of sound.

Table I. Asymptotic low-\(T\) and low-\(\omega\) values of the transport coefficients for three different pairing states, where we have used \(\sigma_{00} = e^2 N_f \gamma^2 T_\text{FON}, \kappa_{00} = (\pi^2/3)k_B^2 N_f \gamma^2 T_\text{FON},\) and \(\alpha_{00} = (q^2/4\rho c_s)\eta_f N_f \gamma^2 T_\text{FON}\) with an effective \(\gamma_0 = \hbar/2\mu\Delta_0(0)\).

| Transport coeff. \(d_{x^2-y^2}\) | \(E_{1g}\) | \(E_{2u}\) |
|----------------------------------|-------|-------|
| \(\sigma_{xx}(T\to0)/\sigma_{00}\) | \(4/\pi\) | 1 |
| \(\kappa_{xx}(T\to0)/\kappa_{00}\) | \(2\mu\gamma/(\mu_2^2\Delta_0)\) | \(\mu/\mu_2\) |
| \(\alpha_{xy}(T\to0)/\alpha_{00}\) | \(2/\pi\) | 1 |
| \(\alpha_{xx}(T\to0)/\alpha_{00}\) | \(8\mu\Gamma_0/\Delta_0\) | \(2\mu\gamma/1+2\mu_2^2\Delta_0\) |

* In the strong scattering limit including vertex corrections.

The transport coefficients in the basal plane (or CuO\(_2\) planes) are universal, i.e., independent of the impurity concentration and scattering phase shift. Furthermore, the in-plane results do not distinguish between the two \(E\)-rep models in UPt\(_3\). Estimates of these coefficients are in good agreement with experiments [3]. However, transport measurements along the crystal \(c\) axis can distinguish between an \(E_{1g}\) or \(E_{2u}\) pairing state. For the electrical and thermal conductivity the \(E_{2u}\) state leads to a universal value at low temperatures, independent of impurity scattering, while the \(E_{1g}\) state has a non-universal value, strongly dependent on impurity scattering.

Independent of the universality of the transport coefficients we find that the Wiedemann-Franz law is re-established at low temperatures \(k_BT \lesssim \gamma\), due to the finite density of states (DOS) at zero energy. At finite temperatures the Lorenz ratio \(L(T)\) deviates significantly from Sommerfeld’s value \(L_S = \frac{\pi^2}{3}(k_B/e)^2\) even for elastic scattering, because of the different coherence factors in the electrical and thermal conductivity, and the energy dependence of the DOS. The Lorenz ratio depends sensitively on the scattering phase shift. For strong scattering the ratio \(L(T)/L_S\) is typically larger than one, whereas for weak scattering (Born) it is less than one [2]. This is a very robust feature, and remains true when we include inelastic scattering, modeled by a phenomenological temperature dependent relaxation time, as shown in Fig. 1.

![Fig. 1 Lorenz ratio \(L(T)\) for an \(E_{2u}\) state for weak (Born) and strong (unitarity) scattering and various phenomenological scattering rates \(\Gamma(T) = \Gamma_0(1+T^2/T_c^2)\).](image)

As a result, measurements of transport coefficients along different crystal axes at very low temperatures and for various impurity concentrations will distinguish between the \(E_{1g}\) and \(E_{2u}\) pairing models in UPt\(_3\), and also elucidate the pairing state in the cuprates.

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