Novel Detecting Methods of Shack-Hartmann Wavefront Sensor at Low Light Levels

A Zhang, C H Rao, Y D Zhang and W H Jiang

Lab of Adaptive Optics, Institute of Optics and Electronics, Chinese Academy of Sciences, Chengdu, Sichuan, China, 610209

E-mail: zhanganghm@yahoo.com

Abstract. A study of novel detecting methods of Shack-Hartmann wavefront sensor at low light levels has been made. Three methods of images processing before slope estimating are presented: Linear Enhancing (LE), Exponential Enhancing (EE) and Fourier Spectrum Filtering (FSF). The idea of LE method is to time the image intensity with a special coefficient before slope estimation. The image points are powered by a selected exponent in EE method. The FSF method is based on the spectrum difference between signal and noise. Most of noise spectrum is filtered and the noise is restrained. The simulated and experimental results show that the LE method does not work effectively, and the other two methods can improve the slope estimation when the Signal-to-noise ratio is higher than 3.0. When the Signal-to-noise ratio is less than 3.0, especially when it is less than 1.0, the FSF is the only method that can overcome the readout noise of the CCD detector.

1. Introduction

Shack-Hartmann wavefront sensor (SHWFS) has been widely applied on adaptive optics systems and optical testing systems [1-5]. The accuracy of Shack-Hartmann wavefront sensing depends on the subaperture slope measuring.

Readout noise is the main source of SHWFS based on CCD detector at low light levels [6]. Threshold value has been used to make down the influence of readout noise. J. M. Ruggiu etc. [7] proposed an alternative method to estimate the subaperture slope. Gram-Charlier expansion was used to model the point spread function of subaperture. Variable centroid estimating accuracy was obtained by changing the order of the Gram-Charlier fit.

Three new methods are proposed in this paper: Linear Enhancing (LE), Exponential Enhancing (EE) and Fourier Spectrum Filter (FSF). All the three methods are expected to improve the accuracy of centroid estimation. The LE method is to time all the points of the image with the same coefficient. Then the contrast of the image is increased and it may bring benefit for slope measuring. Pixels of image are powered by an exponential index in the EE method. The EE method is non-linear enhancement and it can exalt the Signal-to-Noise Ratio (SNR). The idea of FSF results from the different spectrum distributions between signal and noise. Image is transferred into spectrum space and the ingredients of noise spectrum are filtered by a selected filter and the noise is restrained.
2. Basic Principle of SHWFS
SHWFS, as shown in Figure 1, is composed of micro-lens array, matching lens and CCD camera. An array of identical positive lenslets is placed in the pupil of the optical beam to be measured. The wavefront of the source is brought to a separate focus by each lenslet, thus producing an array of spots in the focal plane. With a plane-wave input, each spot is located on the optical axis of its corresponding lenslet. Distortion of the input wavefront produces a local gradient \( g(x, y) \) over each lenslet, displacing each spot by a distance \( s(x, y) = g(x, y)Z \), where \( Z \) is the lenslet focal length. The lenslet array therefore converts the wavefront gradients into measurable spot displacements. The distorted wavefront can be obtained according to the measured wavefront gradients by use of the Zernike polynomials.

![Figure 1. Block diagram of Shack-Hartmann wavefront sensor.](image)

3. Analysis and Single Subaperture Simulated Results
The SNR of images is defined as:

\[
SNR = \frac{P_s}{\sigma_r}
\]

Where \( P_s \) is the peak value of signal; \( \sigma_r \) is the readout noise of CCD detector. It can be concluded from formula (1) that the SNR is proportional to the signal intensity at a special readout noise level.

The performance of Shack-Hartmann wavefront sensor depends on the centroid estimation of each subaperture. The three methods, LE, EE and FSF, are used to estimate the slope of a single subaperture in this section. The difference between detected centroid value and the referenced centroid is used as the criterion. In the simulating calculations, the reference slope is 3.052 pixels.

3.1. Linear Enhancing Method
Linear enhancing of image is to time all the points of the image with the same coefficient. The SNR of image is not boosted by linear enhancing, but the contrast of the image is improved after threshold value is used to it. Higher contrast of the image is expected to bring better estimation of the slope.

Three LE coefficients \( C_l \) of \( 2^0, 2^3, 2^5 \) are used, the simulated results are shown in Figure 2. The x-axis of the figure is SNR and the y-axis is the slope error. The threshold value of each curve is \( 3C_l\sigma_r \). It can be found out that LE method does not decrease the error compare with the original images.

![Figure 2. Results of LE method for single subaperture slope estimation. X-axis is the SNR and the y-axis is the slope error. The threshold value is \( 3C_l\sigma_r \).](image)

![Figure 3. Slope detecting error of EE method, the exponential index \( m \) of each curve is shown in the figure. The threshold value is \( (3\sigma_r)^m \). The reference coefficients \( c_1 \) equals \( 2^0, 2^3, 2^5 \) respectively. The slope is 3.052 pixels.](image)
3.2. Exponential Enhancing Method

Exponential Enhancing method is to make the value of image points be powered by \( m \). EE method can improve the SNR because of the different enhancement of the points in the image. Figure 3 shows the slope detecting error after EE method is used under different SNRs. Each curve in the figure corresponds to an exponential index. Curves show that when the SNR is closer to 1.0, the improvement of EE method decreases. When the SNR becomes as high as 9.0, 10.0 and the exponential index equals 2.5, the error of the slope detecting tends to 0.

3.3. Fourier Spectrum Filtering Method

The spectrum distributions of noise and signal have different properties, so the FSF method is prompted.

Given a subaperture of HSWFS with pixels of \( L \times L \), the photon-electronics distribution in the subaperture is

\[
I(x_i, y_j) = S(x_i, y_j) + n(x_i, y_j)
\]

\( (i, j = 1, 2,...L) \)  

Where the \( S \) and \( n \) is the signal and readout noise of CCD detector respectively. The intensity distribution of signal can be expressed by Gaussian function. Considering a signal with peak value \( S_0 \), Gaussian width \( \sigma_{s_x}, \sigma_{s_y} \) and centroid point \( (x_0, y_0) \):

\[
S(x, y) = S_0 \exp \left( -\frac{(x-x_0)^2}{2\sigma_{s_x}^2} - \frac{(y-y_0)^2}{2\sigma_{s_y}^2} \right)
\]  

The readout noise \( n \) is randomly distributed and its probability density function is:

\[
p(n) = \frac{2}{\sqrt{\pi} \sigma_r} e^{-\frac{n^2}{2\sigma_r^2}} \quad (n \geq 0)
\]

Where the \( \sigma_r \) is the readout noise deviation.

Transforming formula (2) into Fourier expressing:

\[
\mathcal{I}_I(jw_x, jw_y) = \mathcal{S}_I(jw_x, jw_y) + \mathcal{S}_n(jw_x, jw_y)
\]

Where the \( \mathcal{I}_I, \mathcal{S}_I \) and \( \mathcal{S}_n \) are the distributions of \( I, S \) and \( n \) in frequency space respectively. \( w_x, w_y \) are the frequency in \( x, y \) direction. \( \mathcal{S}_f(jw_x, jw_y) \) is the angle of \( \mathcal{S}_f(jw_x, jw_y), |\mathcal{S}_f(jw_x, jw_y)| \) is expressed as:

\[
|\mathcal{S}_f(jw_x, jw_y)| = S_p e^{\frac{x^2}{2\sigma_{s_x}^2} + \frac{y^2}{2\sigma_{s_y}^2}}
\]

Where the \( S_p \) is the peak value of \( |\mathcal{S}_f(jw_x, jw_y)| \). Formula (6) is the signal energy distribution on frequency axes. Integrating formula (6), we can find that

\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\mathcal{S}_f(jw_x, jw_y)| dw_x dw_y = 7.3 \times 10^{-6}
\]

It is indicated in formula (7) that most of signal is concentrated in the region of \( w_x \in [-3/\sigma_{s_x}, 3/\sigma_{s_x}], w_y \in [-3/\sigma_{s_y}, 3/\sigma_{s_y}] \), and then a filtering function can be selected as

\[
\Pi(jw_x, jw_y) = \begin{cases} 
1, w_x \in [-\frac{3}{\sigma_{s_x}}, \frac{3}{\sigma_{s_x}}], w_y \in [-\frac{3}{\sigma_{s_y}}, \frac{3}{\sigma_{s_y}}] \\
0, \text{Otherwise}
\end{cases}
\]

A filter expressed in formula (8) can restrain the noise of high frequency and does not bring loss of signal energy. So the photon-electronics after filtering is

\[
\hat{I}(x, y) = \text{IFFT}\left[ |\mathcal{S}_f(jw_x, jw_y)| \mathcal{S}_f(jw_x, jw_y) \Pi(jw_x, jw_y) + \mathcal{S}_n(jw_x, jw_y) \Pi(jw_x, jw_y) \right]
\]
Where the \textit{IFFT} is the inverse Fourier transforming process. Noting that the noise is statistically independent among pixels, so the Fourier spectrum of noise has the property:

\[ |n_F(j0,j0)| = |n_F(jw_x,jw_y)|, (w_x^2 + w_y^2 > 0) \] (10)

So the \( \hat{I}(x_i, y_j) \) should be amended as

\[
\hat{I}(x_i, y_j) = u\left[ IFFT \left[ S_F(jw_x, jw_y) \angle S_F(jw_x, jw_y) \prod (jw_x, jw_y) + n_F(jw_x, jw_y) \prod (jw_x, jw_y) \right] \right] - IFFT \left[ n_F(jw_x, jw_y) \right]_{x_i, y_j}
\]

\[ u(x) \text{ is the step function:} \]

\[ u(x) = \begin{cases} x, x > 0 \\ 0, \text{Otherwise} \end{cases} \] (12)

Figure 4 is the comparison of results without and with FSF method. When the SNR goes down to 0.5, FSF can decrease the centroid estimation error from more than 2.9 pixels to 1.8 pixels, which is less than the estimation error when SNR equals 7.0 without FSF. When SNR is larger than 3.0, estimation error is decreased to less than 0.2 pixels.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure4.png}
\caption{Detecting error of low SNRs. The line with circle marks is the result calculated without using FSF and the line with diamond marks is obtained with FSF method. The reference slope is 3.052 pixels.}
\end{figure}

4. Multi-subaperture Simulated Results with EE and FSF methods
The efficiency of LE, EE and FSF for single subaperture has been analyzed in section 3. Because LE has been proved invalid for low SNRs, it is not considered to be used in Shack-Hartmann image processing. The RMS value of the reconstructed error wavefront is used to examine the performance of EE and FSF methods. RMS value is defined as:

\[ RMS = \left( \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} (\omega_{i,j} - \bar{\omega})^2 \right)^{\frac{1}{2}} \] (13)

Where \( M, N \) is the resolution on 2 dimensions; \( \omega_{i,j} \) is the error wavefront at point \((i, j)\); \( \bar{\omega} \) is the average value of error wavefront.

The simulation conditions are as following: pixels in one subaperture 15×15, subapertures: 8×8.

4.1. Results with EE method
The simulated results of EE method is shown in Figure 5. Curves in the figure show that EE method can reduce the RMS value of error wavefront when SNR is larger than 3.0. When the exponential index increases from 1.5 to 2.5, the performance becomes better.
4.2. Results with FSF method
The LE and EE methods cannot improve the performance of Shack-Hartmann wavefront sensor when SNR is less than 3.0. In section 3 it has been proved that the FSF method is efficient to decrease the slope error for single subaperture even when SNR is less than 1.0. It can be concluded from Figure 6 that the FSF method can effectively improve the wavefront reconstruction accuracy. When the SNR is larger than 1.0, the wavefront measurement RMS-error can be less than $\lambda/10$.

5. Experimental results
A Shack-Hartmann wavefront sensor is used to measure known aberration. Signal intensity is switched to get different SNRs. The LE, EE and FSF methods are all used in the data processing. The known wavefront is shown in Figure 7. The RMS values are shown in Figure 8. It can be seen that LE and EE methods do not increase the performance obviously when then SNR is less than 3.0. While the FSF method decreases the RMS value from $1.0\lambda$ to $0.6\lambda$ when SNR equals 0.5 and the RMS is reduced from $0.8\lambda$ to about $0.1\lambda$ when SNR equals 3.2. The conclusion from the experiments is that the FSF method is the most efficient way to increase the detecting precision for low SNRs.

Figure 5. The RMS value when EE method is used. The exponential index of each curve is shown in the figure, the threshold value is $(3\sigma_p)^m$.

Figure 6. The RMS value when FSF method is used.

Figure 7. The reference wavefront with aberration of the experiments.
Figure 8. Experimental results when LE, EE and FSF methods are respectively used. The x-axis is SNR and the y-axis is RMS value of error wavefront. When the SNR equals 3.2, FSF method decrease the RMS value from 0.8λ to 0.12λ.

6. Conclusion
The LE, EE and FSF methods are proposed to increase the detecting performance of Shack-Hartmann wavefront sensor. When the three methods are used to single subaperture slope measuring, the EE method and FSF methods reduce the slope error though they have different efficiency. The LE method does not bring improvement even the SNR of the image is as high as 10.0. The EE and FSF methods are used to multi-subaperture wavefront detecting, the simulated results and experimental results both indicate that even when the SNR of the image is less than 1.0, FSF can improve the detecting accuracy, while EE works when SNR is larger than 3.0.

This paper is supported by National 863 High-tech Program.

References
[1] W.Jiang, H.Li, N.Ling and C.Guan 1993 A 37-element adaptive optics system with H-S wavefront sensor Proceedings of the ICO-16 Satellite Conference on Active and Adaptive Optics (International Commission for Optics 16 Secretariat, Garching, Germany) ed F. Merkle 127-135
[2] J.Liang, B.Grimm, S. Goelz and J.F. Bille 1994 Objective measurement of wave aberrations of the human eye with use of Hartmann-Shack sensors J. Opt. Soc. Am. A. 11 1949-57
[3] H. Li, H. Xian and W, Jiang Atmospheric turbulence parameter measurement using Hartmann-Shack wave-front sensor Proceedings of the ICO-16 Satellite Conference on Active and Adaptive Optics (International Commission for Optics 16 Secretariat, Garching, Germany, 1993) ed F. Merkle 21-25
[4] Changhui Rao, Wenhan Jiang and Ning Ling 1999 Measuring the power-law exponent of an atmospheric turbulence phase power spectrum with a Shack-Hartmann wave-front sensor Optics Letters 24 1008
[5] Jiang Wenhan, Xian hao and Shen feng 1998 Detecting error of Shack-Hartmann wavefront sensor Chinese Journal of Quantum Electronics 15 218-227
[6] Ang Zhang, Changhui Rao, Yudong Zhang and Wenhan Jiang 2004 Performance analysis of Shack-Hartmann wavefront sensor with variable subaperture pixels SPEI Proc. 5490
[7] J. M. Ruggiu, C. J. Solomon and G. Loos 1998 Gram-Charlier matched filter for Shack-Hartmann sensing at low light levels Optics Letters 23 235-237
[8] Ang Zhang Changhui Rao Yudong Zhang and Wenhan Jiang 2004 Sampling Error Analysis of Shack-Hartman Wavefront Sensor with Variable Subaperture Pixels J. of Modern Optics 51 2267-78