Dependence of the oscillations amplitude on the thickness of magnetostrictive-piezoelectric bilayer structure in the theory of magnetoelastic effect

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Abstract. Dependence of the amplitude of mechanical oscillations on the thickness of magnetostrictive-piezoelectric bilayer structure in the form of rectangular plate is presented for the theory of linear magnetoelastic effect. The case of longitudinal orientation of the electric and magnetic fields was considered for structures in the form of a rectangular plate. The frequency dependence of the magnetoelastic effect is obtained using motion equation and electrostatic equations for both phases taking into account the boundary conditions on the interface. The theoretical dependencies of the displacement and stress distributions over the thickness of the sample in the magnetostrictive and piezoelectric phases are presented. These dependencies have a nonlinear character, and their accounting leads to a noticeable contribution to the magnitude of the effect.

Keywords: magnetoelastic effect, layered structure, magnetostriction, piezoelectricity.

1. Introduction
Magnetostrictive-piezoelectric structures have attracted much attention because in these structures there are effects which can be absent in magnetostrictive and piezoelectric components separately. They appear due to mechanical interaction between the magnetostrictive and piezoelectric subsystems. Magnetoelastic (ME) effect is one of such effects, which is caused by the elastic interaction of magnetostrictive and piezoelectric subsystems in bilayer structures. The action of the magnetic field in the magnetostrictive phase induces mechanical deformations which pass via mechanical coupling into the piezoelectric phase and leads to a change of the polarization of the sample.

The theoretical consideration of the ME effect in magnetostrictive-piezoelectric composites has two main methods at the present time. The method of effective parameters is one of these methods. Using this method, theory of the ME effect in bulk and multilayer composites was presented in works [1-6]. In these papers, an expression for ME voltage coefficient was defined by the method of effective parameters and the frequency dependence was analyzed. One of the disadvantages of the method of effective parameters is its usage limitation. The method is applicable when the characteristic size of the structural units of the composite is much smaller than the length of the acoustic oscillations, so that

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the composite can be considered as a homogeneous medium. Thus, the method of effective parameters is applicable for bulk composites. Bilayer structures attract much attention because of the value of ME effect, which is greater than in bulk composites [7]. But the method of effective parameters can not be used for these structures, because the thickness of the layers is comparable with the acoustic wavelengths. The second method is based on the simultaneous solution of the equations of motion and the constitutive relations for magnetostrictive and piezoelectric phases taking into account the boundary conditions on the interface. It was previously presented in [8-13]. The problem of this method is the consideration of the boundary condition between magnetostrictive and piezoelectric layers. These boundary conditions are presented formally by an interface coupling coefficient in [8-11] or by an assumption of ideal coupling [12,13]. The ME effect was investigated in laminated composite structure in work [14] where strain and ME voltage coefficient distributions were presented in piezoelectric, but in this work, the amplitude change of the oscillations over the thickness of the sample (in direction perpendicular to the interface) was not assumed. Recently the wave propagation and the ME effect in bilayer magnetostrictive-piezoelectric structure has been presented in case of a perfect bonding taking into account the changes of the amplitude in oscillations over the thickness of the sample in works [15-17]. But in these works, the spatial distribution of displacement and stress on the thickness of the sample was not analyzed and the influence of inhomogeneous distribution on the value of the effect was insufficiently analyzed.

In this paper, the dependencies of displacement and stress on the thickness of the sample are presented and their influence on the ME voltage coefficient is analyzed. The expressions for the ME voltage coefficient in the cases of longitudinal and transverse orientations of electric and magnetic fields were obtained using these dependencies and the open circuit condition. It is shown that the consideration of the change of oscillations amplitude over the thickness leads to a noticeable contribution to the magnitude of the effect.

2. Model and basic equations
As a model, we consider a bilayer structure in the form of a rectangular plate of length $L$ and width $W$, consisting of mechanically interacting magnetostrictive and piezoelectric layers of thickness $t_m$ and $t_p$, the values which are not assumed to be small (figure 1). Thin metal contacts are applied on the top and the bottom of the plate.

![Figure 1. Schematic view of the sample. 1-Magnetostrictive phase with $t_m$ thickness, 2-Piezoelectric phase with $t_p$ thickness, 3-Electrodes.](image)

Let us consider that the origin of the coordinates coincides with the boundary of the partition of magnetostrictive and piezoelectric layers. Preliminarily, the piezoelectric layer is polarized perpendicularly to the contact (Z axis). In the case of longitudinal orientation of the electric and magnetic fields (longitudinal effect) the magnetic fields (bias $H_0$ and alternating $H$ with the frequency $\omega$) coincide in direction with the polarization vector $P$. We restrict our consideration to the
planar vibrations propagating along the X axis. The alternating magnetic field causes elastic oscillations in the magnetostrictive component, transferring through the interface of the partition to the piezoelectric component through the shear stresses, which brings to interconnected oscillations of magnetostrictive and piezoelectric subsystems. As there is a nonuniformity along Z axis, under the first approximation we can assume that the displacements along plane Y are homogeneous and the nonzero components of stress are only $^\alpha T_{xx}$ and $^\alpha T_{xz}$. The value of the stresses will depend on the thickness of the sample, perpendicularly to the interface between magnetostrictive and piezoelectric phases. Therefore the equations of motion for the $^\alpha u_x$ displacement vector of the magnetostrictive and piezoelectric phases are of the form

$$\alpha \rho \frac{\partial^2 {^\alpha u}_x}{\partial t^2} = \frac{\partial \alpha T_{xx}}{\partial x} + \frac{\partial \alpha T_{xz}}{\partial z},$$

where upper-case index “$\alpha$” is correspondingly “$m$” for ferrite and “$p$” for piezoelectric, $\alpha \rho$ is the density of ferrite and piezoelectric layers, $^\alpha T_{xx}$ and $^\alpha T_{xz}$ are the components of the stress tensor.

The constitutive equations for the magnetostrictive and piezoelectric phases are

$$^p S_{xx} = \frac{1}{p Y} ^p T_{xx} + ^p d_{xx,z} ^p E_z,$$

$$^p S_{xz} = \frac{1}{p Y} ^p T_{xz},$$

$$^p D_z = ^p e_{zz} ^p E_z + ^p d_{xx,z} ^p T_{xx},$$

$$^m S_{xx} = \frac{1}{m Y} ^m T_{xx} + ^m q_{xx,z} ^m H_z,$$

$$^m S_{xz} = \frac{1}{m G} ^m T_{xz},$$

where $^\alpha S_{xx}$ and $^\alpha S_{xz}$ are the strain tensor components; $^p E_z$ and $^m H_z$ are the vector components of the electric and magnetic fields; $^p D_z$ is the vector component of the electric displacement; $^p Y$ and $^\alpha G$ are the Young's and shear moduli; $^p d_{xx,z}$ and $^m q_{xx,z}$ are the piezoelectric and piezomagnetic coefficients; $^p e_{zz}$ is the tensor component of permittivity.

Using the solution of equation for the displacement vector of the medium and the condition of mechanical equilibrium at the free surfaces of the plate, i.e. points $x = \pm L/2$, we obtain the expression for displacement of the magnetostrictive and piezoelectric media in final forms

$$m u_x = \left[ \exp(-2^m \kappa) \exp(\frac{m \kappa}{\chi_c}) + \exp(-\frac{m \kappa}{\chi_c}) \right] \sin(kx),$$

$$^p u_x = \frac{\exp(2^m \kappa) \exp(\frac{m \kappa}{\chi_c}) + \exp(-\frac{m \kappa}{\chi_c})}{\cos(\kappa) (1 + \exp(-2^m \kappa))} \sin(kx),$$

where $B = \frac{m Y m q_{xx,z} (m H_z) + p Y \eta^p d_{xx,z} (p E_z)}{k \cos(\kappa) (1 + \exp(-2^m \kappa)) (m Y m \theta(\frac{m \kappa}{\chi_c}) + p Y \eta^p \frac{\theta(\frac{m \kappa}{\chi_c})}{p \kappa})}$, $\kappa = k L/2$ and $^\alpha \kappa = ^\alpha \chi t$ are non-dimensional parameters; $m \chi^2 = -2(1 + \nu) \left[ \frac{\alpha^2}{m V_L^2} - k^2 \right]$, $^p \chi^2 = 2(1 + \nu) \left[ \frac{\alpha^2}{p V_L^2} - k^2 \right]$, $\frac{1}{m V_L^2} = \frac{\alpha^2}{m Y}$, $\frac{1}{^p V_L^2} = \frac{\alpha^2}{^p Y}$.
Expressing the stress tensor components $\alpha T_{xx}$ and $\alpha T_{xz}$ by the strain tensor components in Eqs. (7), (8), (10) and (11), we obtain the final expressions

$$m T_{xx} = m Y \left[ k B \cos(kx) \left( \exp(-2m\kappa) \exp(m \chi_z) + \exp(-m \chi_z) \right) - m q_{xx,z} m H z \right],$$  
(9)

$$m T_{xz} = m G m \left[ \exp(-2m\kappa) \exp(m \chi_z) - \exp(-m \chi_z) \right] B \sin kx,$$  
(10)

$$p T_{xx} = p Y \left[ k B \cos(kx) \left( 1 + \exp(-2m\kappa) \cos(p \chi_z) - \tan(p \kappa) \sin(p \chi_z) \right) - p d_{xx,z} p E z \right],$$  
(11)

$$p T_{xz} = -p G p \chi \left[ 1 + \exp(-2m\kappa) \left( \sin(p \chi_z) - \tan(p \kappa) \cos(p \chi_z) \right) \right] B \sin kx.$$  
(12)

As can be seen in Eqs. (7) and (8), the solution is represented as plane waves whose amplitude is changed over the thickness of the sample. These dependencies have nonlinear character and depend on the frequency of oscillations in general. It is easy to show that in the case of low frequencies, when the dimensionless parameters $m \kappa$ and $p \kappa$ are less than one, the amplitude ceases to depend on the thickness of the sample. Thus, the results obtained earlier in [12], where the change of oscillations amplitude was not taken into account, take place only at low frequencies and thin layers. Using Eqs. (7) and (8), the distributions of relative displacements $\alpha u_x = \alpha u_x(z) / \alpha u_x(0)$ on the thickness of magnetostrictive and piezoelectric phases are presented in figure 2 and the distribution of relative displacement in the piezoelectric phase is presented for three different cases of frequency (figure 3) in bilayer nickel–lead zirconate titanate (Ni-PZT) structure.

![Figure 2](image1.png)

**Figure 2.** Relative displacement distributions on the thickness of magnetostrictive (line 1) and piezoelectric layers (line 2). Frequency of the ac magnetic field is $f = 300$ kHz.

![Figure 3](image2.png)

**Figure 3.** Relative displacement distribution on the thickness of piezoelectric layer in cases of different frequencies. Frequencies of the ac magnetic field are $f = 10$ kHz, $f = 150$ kHz, $f = 300$ kHz.

The following parameters of the structure were used in the calculations; for nickel $m Y = 204$ GPa , $m \rho = 8900$ kg/m$^3$, $m d_{xx,z} = 1156 \cdot 10^{-12}$ m/A ; and for the PZT: $p Y = 65$ GPa, $p \rho = 7600$ kg/m$^3$, $p d_{xx,z} = -175 \cdot 10^{-12}$ m/V, $p \varepsilon_{zz} / \varepsilon_0 = 1750$, ac magnetic field $H = 100$ Oe. Using Eqs. (9)-(12), the distributions of relative stresses $\alpha T_{xx} = \alpha T_{xx}(z) / \alpha T_{xx}(0)$ and $\alpha T_{xz} = \alpha T_{xz}(z) / \alpha T_{xz}(0)$ on the thickness of magnetostrictive and piezoelectric phases are presented for the same structure in figures 4 and 5.
Magnetoelectric voltage coefficient is defined as the ratio of the average value of the electric field to the average value of the external magnetic field, which induced it, i.e.:

$$\langle \alpha_E \rangle = \frac{\langle E \rangle}{\langle H \rangle},$$

(13)

where $\langle E \rangle = U/(m_t+p_t)$ is the average value of the electric field of the structure, $U$ is the potential difference between the electrodes.

In order to define a theoretical expression for the ME voltage coefficient, we use the method developed earlier [15,16]. Substituting the expression in Eq. (11) into Eq. (4), using the condition of open circuit and the fact that the potential difference between the electrodes is defined as $U = (p E_{zz}^p) t$, we obtain the expression for the ME voltage coefficient in the case of longitudinal orientation of the fields in the final form

$$\alpha_{E,L} = \frac{p Y \rho d_{xx,zz}^m q_{xx,zz}^m}{p E_{zz}^p \Delta L} \frac{m Y^m}{m^2} \frac{\tan(m \kappa)}{\kappa} \frac{\tan(p \kappa)}{p \kappa} \frac{\tan(p \kappa)}{p \kappa} \frac{p_t}{m_t + p_t},$$

(14)

where $\Delta L = 1 - K_p^2 \left( 1 - \frac{p Y \rho f}{m Y^m \tan(m \kappa) + p Y \rho f \tan(p \kappa)} \frac{\tan(p \kappa)}{p \kappa} \right)$, with $K_p^2 = \frac{p Y (p d_{xx,zz})^2}{p E_{zz}^p}$ the squared coefficient of electromechanical coupling.

3. Results and discussion

The theoretical investigation was conducted for bilayer nickel–lead zirconate titanate (Ni-PZT) structure in the form of a rectangular plate. As can be seen from figure 2, the amplitude of oscillations in magnetostrictive phase is practically unchanged over the thickness of its layer. This is due to the alternating magnetic field which excites oscillations of the magnetostrictive phase simultaneously throughout the thickness of the magnetostrictive phase. Oscillations in the piezoelectric phase are excited due to the shear deformations which are transferred via the ferrite-piezoelectric interface.
As can be seen from figure 3, the distributions strongly depend on the frequency of oscillations. The dependencies have a nonlinear character at frequencies $f = 150$ kHz and $f = 300$ kHz. It is shown that in the case of low frequency ($f = 10$ kHz) the amplitude ceases to depend on the thickness of the layer and it has a linear character. Thus, the results obtained in [12] are incomplete and they can take place only at low frequencies and thin layers. Figure 4 shows that the shear stresses have the maximum values on the interface ($Z = 0$) and are equal to zero at the top and the bottom surfaces of the sample in accordance with the theory.

Calculation of the ME voltage coefficient, with the use of Eq. (14) obtained in this work, leads to a value not equal to the one obtained earlier in [12], where the inhomogeneity of the oscillations amplitude over the thickness was not taken into account. Thus, the value of ME voltage coefficient is $\alpha_{E,L} = 513$ mV/cm-Oe, whereas the use of the expression obtained in [12] leads to overestimated value $\alpha_{E,L} = 860$ mV/cm-Oe. Thereby, the consideration of inhomogeneity of oscillations amplitude at frequency of 300 kHz causes a change of the ME voltage coefficient. This value differs more than 40% from the result calculated with an assumption that the oscillations amplitude is the same over the thickness of the sample. Both models yield the same values for the ME voltage coefficient at low frequencies.

4. Conclusion
The inhomogeneity of the structure, associated with the presence of the ferrite-piezoelectric interface, leads to the change of displacement and stress amplitudes over the thickness of the sample. This inhomogeneity can be neglected, and we can assume that the amplitude is not changed over the thickness of the sample only at low frequencies. However, this change makes a significant contribution in the value of the ME voltage coefficient at the region of frequencies near 100 kHz.

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