Non-leptonic $B$-decays, CP violation & the UT

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We study the implication of the time-dependent CP asymmetry in $B \to \pi^+\pi^-$ decays on the extraction of weak phases taking into account the precise measurement of $\sin 2\beta$, obtained from the “gold-plated” mode $B \to J/\psi K_S$. Predictions and uncertainties for the hadronic parameters are investigated in QCD factorization. Furthermore, independent theoretical and experimental tests of the factorization framework are briefly discussed. Finally, a model-independent bound on the unitarity triangle from CP violation in $B \to \pi^+\pi^-$ and $B \to J/\psi K_S$ is derived.

1. Introduction

In the standard model (SM), the only source of CP violation is the Kobayashi-Maskawa phase $\alpha$, localized in the Unitarity Triangle (UT) of the Cabibbo-Kobayashi-Maskawa (CKM) matrix $M^U$. Thanks to the precise measurements at the current $B$-factories, CP violation could be established in $B_d \to J/\psi K_S$ [34], leading to a precise measurement of $\sin 2\beta$, where the current world average yields $\sin 2\beta = 0.739 \pm 0.048$. The extractions of the other two angles $\alpha$ and $\gamma$ are expected mainly through CP violation in the charmless $B$ decays, such as $B_d \to \pi\pi$ and similar modes [5]. The current $B$-factories measurements have been averaged to yield [5]:

$$S_{\pi\pi} = -0.74 \pm 0.16, \quad C_{\pi\pi} = -0.46 \pm 0.13.$$ 

On the theoretical side, the analysis is challenging due to the need to know the ratio of penguin-to-tree amplitude contributing to this process. In this talk, we present the result of [7,8], where a transparent method of exploring the UT through the Cabibbo-Kobayashi-Maskawa (CKM) parameters as functions of $S_{\pi\pi}$ and $\sin 2\beta$ has been proposed. A model independent lower bound on $\sin 2\beta$ is derived.

2. Basic Formulas

The time-dependent CP asymmetry in $B \to \pi^+\pi^-$ decays is defined by

$$A_{CP}^S(t) = -S_{\pi\pi} \sin(\Delta m_B t) + C_{\pi\pi} \cos(\Delta m_B t), \quad (1)$$

where

$$S_{\pi\pi} = \frac{2 \text{ Im } \xi}{1 + |\xi|^2}, \quad C_{\pi\pi} = \frac{1 - |\xi|^2}{1 + |\xi|^2}. \quad (2)$$

with $\xi = e^{-2i\beta} e^{-i\gamma}/\sqrt{r^2 + \eta^2}$, and $\beta$ and $\gamma$ are CKM angles which are related to the Wolfenstein parameters $\bar{\rho}$ and $\bar{\eta}$ in the usual way [9]. The penguin-to-tree ratio $P/T$ can be written as $P/T = r e^{i\phi}/\sqrt{\bar{\rho}^2 + \eta^2} = r e^{i\phi}/R_b$. The real parameters $r$ and $\phi$ defined in this way are pure strong interaction quantities without further dependence on CKM variables.

For any given values of $r$ and $\phi$ a measurement of $S_{\pi\pi}$ and $C_{\pi\pi}$ defines a curve in the $(\bar{\rho}, \bar{\eta})$-plane, expressed respectively through

$$S_{\pi\pi} = \frac{2\bar{\eta}R_b^2 - r^2 - \bar{\rho}(1 - r^2) + (R_b^2 - 1)r \cos \phi}{((1 - \bar{\rho})^2 + \bar{\eta}^2)(R_b^2 + r^2 + 2r\bar{\rho} \cos \phi)}, \quad (3)$$

and

$$C_{\pi\pi} = \frac{2r\bar{\eta} \sin \phi}{R_b^2 + r^2 + 2r\bar{\rho} \cos \phi}. \quad (4)$$

The penguin parameter $r e^{i\phi}$ has been computed in [10] in the framework of QCD. The result can be expressed in the form

$$r e^{i\phi} = -\frac{a_4^\pi + r_4^\pi a_6^\pi + r_4 [b_3 + 2b_4]}{a_1 + a_4^\pi + r_4 a_6^\pi + r_4 [b_1 + b_3 + 2b_4]}, \quad (5)$$

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where we neglected the very small effects from electroweak penguin operators. A recent analysis gives [7,8]

\[ r = 0.107 \pm 0.031, \quad \phi = 0.15 \pm 0.25, \quad (6) \]

where the error includes an estimate of potentially important power corrections. In order to obtain additional insight into the structure of hadronic $B$-decay amplitudes, it will be also interesting to extract these quantities from other $B$-channels, or using other methods. In this perspective, we have considered them in a simultaneous expansion in $1/m_b$ and $1/N_C$ ($N_C$ is the number of colours) in [6]. Expanding these coefficients to first order in $1/m_b$ and $1/N_C$ we find that the uncalculable power corrections $b_1$ and $H_{\pi\pi,3}$ do not appear in [6]. to which they only contribute at order $1/m_b$. Using our default input parameters, one obtains the central value $[7]$: $(r_{NC}, \phi_{NC}) = (0.084, 0.065)$, which seems to be in a good agreement with the standard QCDF framework at the next-to-leading order.

As a second cross-check, one can extract $r$ and $\phi$ from $B^+ \rightarrow \pi^+\pi^0$ and $B^+ \rightarrow \pi^+K^0$, leading to the central value $[8]$: $(r_{SU3}, \phi_{SU3}) = (0.081, 0.17)$, in agreement with the above results$^2$, although their definitions differ slightly from $(r, \phi)$ (see [7] for further discussions).

3. UT through CP violation observables

It is possible to fix the UT by combining the information from $S_{\pi\pi}$ with the value of $\sin 2\beta$, well known from the “gold-plated” mode $B \rightarrow J/\Psi K_S$. The angle $\beta$ of the UT is given by

\[ \tau \equiv \cot \beta = \sin 2\beta \left( 1 - \sqrt{1 - \sin^2 2\beta} \right)^{-1}. \quad (7) \]

The current world average $[5]$: $\sin 2\beta = 0.739 \pm 0.048$, implies $\tau = 2.26 \pm 0.22$. Given a value of $\tau$, $\tilde{\rho}$ is related to $\bar{\eta}$ by $\rho = 1 - \tau \bar{\eta}$. The parameter $\rho$ may thus be eliminated from $S_{\pi\pi}$ in [3], which can be solved for $\bar{\eta}$ to yield

\[ \bar{\eta} = \frac{1}{(1 + \tau^2)S_{\pi\pi}} \sqrt{\tilde{S}(1 + r \cos \phi)} \quad (8) \]

$^2$One can compare also $r_{SU3}$ to its experimental value $r_{SU3}^{exp} = 0.099 \pm 0.014$.

\[ -\sqrt{(1 - S_{\pi\pi}^2)(1 + r^2 + 2r \cos \phi) - \tilde{S}^2 r^2 \sin^2 \phi}, \]

with $\tilde{S} = (1 + \tau S_{\pi\pi})$ The two observables $\tau$ (or $\sin 2\beta$) and $S_{\pi\pi}$ determine $\bar{\eta}$ and $\tilde{\rho}$ once the theoretical penguin parameters $r$ and $\phi$ are provided.

The determination of $\bar{\eta}$ as a function of $S_{\pi\pi}$ is shown in Fig. 1, which displays the theoretical uncertainty from the penguin parameters $r$ and $\phi$ in QCDF. Since the dependence on $\phi$ enters in $[3]$ only at second order, it turns out that its sensitivity is rather mild in contrast to $r$. In the determination of $\bar{\eta}$ and $\tilde{\rho}$ described here discrete ambiguities do in principle arise, however they are ruled out using the standard fit of the UT (see [7] for further discussions).

After considering the implications of $S_{\pi\pi}$ on the UT, let’s explore now $C_{\pi\pi}$. Since $C_{\pi\pi}$ is an odd function of $\phi$, it is therefore sufficient to restrict the discussion to positive values of $\phi$. A positive phase $\phi$ is obtained by the perturbative estimate in QCDF, neglecting soft phases with power suppression. For positive $\phi$ also $C_{\pi\pi}$ will be positive, assuming $\bar{\eta} > 0$, and a sign change in $\phi$ will simply flip the sign of $C_{\pi\pi}$.

In contrast to the case of $S_{\pi\pi}$, the hadronic quantities $r$ and $\phi$ play a prominent role for $C_{\pi\pi}$, as can be seen in [9]. This will in general complicate the interpretation of an experimental result for $C_{\pi\pi}$.

The analysis of $C_{\pi\pi}$ becomes more transparent if we fix the weak parameters and study the im-
impact of $r$ and $\phi$. An important application is a test of the SM, obtained by taking $\bar{\rho}$ and $\bar{\eta}$ from a SM fit and comparing the experimental result for $C_{\pi\pi}$ with the theoretical expression as a function of $r$ and $\phi$. In Fig. 2 a useful representation is obtained by plotting contours of constant $C_{\pi\pi}$ in the $(r, \phi)$-plane, for given values of $\bar{\rho}$ and $\bar{\eta}$.

Within the SM this illustrates the correlations between the parameters $(r, \phi)$ and observable $C_{\pi\pi}$.

As it has been shown in [7], a bound on the parameter $C_{\pi\pi}$ exists, given by

$$C_{\max} = \frac{2\kappa \sin \phi}{\sqrt{(1 + \kappa^2)^2 - 4\kappa^2 \cos^2 \phi}}, \quad (9)$$

with $\kappa \equiv r/R_b$ and where the maximum occurs at $\cos \gamma = -2\kappa \cos \phi/(1 + \kappa^2)$. If $\kappa = 1$, no useful upper bound is obtained. However, if $\kappa < 1$, then $C_{\max}$ is maximized for $\phi = \pi/2$, yielding the general bound $C < \frac{2\kappa}{1 + \kappa^2}$. For the conservative bound $r < 0.15$, $\kappa < 0.38$ this implies $C_{\pi\pi} < 0.66$. The bound on $C_{\pi\pi}$ can be strengthened by using information on $\phi$, as well as on $\kappa$, and employing [9]. Then $\kappa < 0.38$ and $\phi < 0.5$ gives $C_{\pi\pi} < 0.39$.

4. Model Independent bound on the UT

As has been shown in [8], the following inequality can be derived from [8] for $-\sin 2\beta \leq S_{\pi\pi} \leq 1$

$$\bar{\eta} \geq \frac{1 + r S_{\pi\pi} - \sqrt{1 - S_{\pi\pi}^2}}{(1 + \tau^2) S_{\pi\pi}}(1 + r \cos \phi). \quad (10)$$

This bound is still exact and requires no information on the phase $\phi$.

Assuming now $-90^\circ \leq \phi \leq 90^\circ$, we have $1 + r \cos \phi \geq 1$ and

$$\bar{\eta} \geq \frac{1 + \tau S_{\pi\pi} - \sqrt{1 - S_{\pi\pi}^2}}{(1 + \tau^2) S_{\pi\pi}}. \quad (11)$$

We emphasize that this lower bound on $\bar{\eta}$ depends only on the observables $\tau$ and $S_{\pi\pi}$ and is essentially free of hadronic uncertainties. Since both $r$ and $\phi$ are expected to be quite small, we anticipate that the lower limit (11) is a fairly strong bound, close to the actual value of $\bar{\eta}$ itself (see [7] for further details). We also note that the lower bound (11) represents the solution for the unitarity triangle in the limit of vanishing penguin amplitude, $r = 0$. In other words, the model-independent bounds for $\bar{\eta}$ and $\bar{\rho}$ are simply obtained by ignoring penguins and taking $S_{\pi\pi} \equiv \text{sin} 2\alpha$ when fixing the unitarity triangle from $S_{\pi\pi}$ and $\sin 2\beta$. Let us briefly comment on the second solution for $\bar{\eta}$, which has the minus sign in front of the square root in [8] replaced by a plus sign. For positive $S_{\pi\pi}$ this solution is always larger than [8] and the bound (11) is unaffected. For $-\sin 2\beta \leq S_{\pi\pi} \leq 0$ the second solution gives a negative $\bar{\eta}$, which is excluded by independent information on the UT (for instance from $\varepsilon_K$).

Because we have fixed the angle $\beta$, or $\tau$, the lower bound on $\bar{\eta}$ is equivalent to an upper bound on $\bar{\rho} = 1 - \tau\bar{\eta}$. The constraint (11) may also be expressed as a lower bound on the angle $\gamma$ or a lower bound on $R_t$ (see [7] for further details). In Figs. 3 we represent the lower bound on $\bar{\eta}$.
as a function of $S_{\pi\pi}$ for various values of $\sin 2\beta$. From Fig. 3 we observe that the lower bound on $\bar{\eta}$ becomes stronger as either $S_{\pi\pi}$ or $\sin 2\beta$ increase.

In Fig. 4 we illustrate the region in the $(\bar{\rho}, \bar{\eta})$ plane that can be constrained by the measurement of $\sin 2\beta$ and $S_{\pi\pi}$ using the bound in (11). We finally note that the condition $r \cos \phi > 0$, which is crucial for the bound, could be independently checked [12] by measuring the mixing-induced CP asymmetry in $B_s \to K^+ K^-$, the U-spin counterpart of the $B_d \to \pi^+ \pi^-$ mode [13].

5. Summary

In this talk, we have proposed strategies to extract information on weak phases from CP violation observables in $B \to \pi^+ \pi^-$ decays even in the presence of hadronic contributions related to penguin amplitudes. Assuming knowledge of the penguin pollution, an efficient use of mixing-induced CP violation in $B \to \pi^+ \pi^-$ decays, measured by $S_{\pi\pi}$, can be made by combining it with the corresponding observable from $B \to J/\psi K_S$, $\sin 2\beta$, to obtain the unitarity triangle parameters $\bar{\rho}$ and $\bar{\eta}$. The sensitivity on the hadronic quantities, which have typical values $r \approx 0.1$, $\phi \approx 0.2$, is very weak. In particular, there are no first-order corrections in $\phi$. For moderate values of $\phi$ its effect is negligible.

Concerning our penguin parameters, namely $r$ and $\phi$, they were investigated systematically within the QCDF framework. To validate our theoretical predictions, we have calculate these parameters in the $1/m_b$ and $1/N_C$ expansion, which exhibits a good framework to control the uncalculable power corrections, in the factorization formalism. As an alternative proposition, we have also considered to extract $r$ and $\phi$ from other $B$ decay channels, such as $B^+ \to \pi^+ \pi^0$ and $B^+ \to \pi^+ K^0$, relying on the SU(3) argument. Using these three different approaches, we found a compatible picture in estimating these hadronic parameters.

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