Dynamics of quantum collapse in coupled quantum dots

H. Cruz∗
Departamento de Física, Universidad de La Laguna, 38204 La Laguna, Tenerife, Spain
(Dated: November 20, 2021)

In this letter, we have considered an electron in a coupled quantum dot system interacting with a detector represented by a point contact. We present a dynamical model for wave function collapse in the strong coupling to the detector limit. In our model, the electron in the double quantum dot makes a fast transition minimizing the emission of electromagnetic radiation. In this way, a principle of least emitted radiation can provide a possible description of wave function collapse.

PACS numbers: 73.23.-b; 03.65.Ta; 73.50.-h

The problem of understanding whether a measurement process can be analyzed within the quantum mechanical formalism has long been a difficult unresolved issue in the foundations of quantum mechanics. On one hand, the quantum theory states that the vector corresponding to a physical system undergoes a continuous evolution governed by Schrödinger equation; on the other hand, the theory prescribes a sudden jump motion to the state of a physical system undergoing a measurement by an external device. Von Neumann’s projection rules [1] are indeed to be added to the quantum formalism in order to account for the transition from a pure to a mixed state (the so-called wave function collapse), and this makes quantum mechanics a non-self-contained theory. The renewed interest in the measurement problem is justified by the development of mesoscopic systems sensitive to the phase of the electronic wave function. Recent proposals suggested using mesoscopic devices, such as Josephson junctions or coupled quantum dots, as quantum bits (qubits), which are the basic elements of quantum computers [2]. Among various modern approaches to the measurement problem in mesoscopic structures let us mention the idea of replacing the collapse postulate by the gradual decoherence of the density matrix due to the interaction with the detector [3] and the approach of a stochastic evolution of the wave function [4].

Recently, Gurvitz et al. [5] have considered a qubit interacting with its environment and continuously monitored by a detector represented by a point contact. In such a case, the decoherence rate $\Gamma_d$ due to interaction with the detector is inversely proportional to the measurement time $\Delta t$. For strong coupling to the detector, i.e., $\Gamma_d \rightarrow \infty$ and $\Delta t \rightarrow 0$, the measurement is idealized to be instantaneous. Accordingly, the electron in the coupled quantum dot system instantaneously makes the transition $|\psi_i \rangle \rightarrow |\psi_f \rangle$ by measurement (the so-called quantum-jump [6]). However, we notice that one remaining key question in these theories is theoretical analysis of electromagnetic radiation emitted by a charged particle diverges. This implies an infinite value for the radiation reaction force. By contrast, an infinite value for the emitted radiation can be eliminated by considering a dynamical model for wave function collapse. This is the aim of the present work. In this letter, we present a dynamical model for wave function collapse in the strong coupling limit. In our model, the particle makes the transition $|\psi_i \rangle \rightarrow |\psi_f \rangle$ as soon as possible but not instantaneously. We assume that the electron in the coupled quantum dot makes a fast transition minimizing the emission of electromagnetic radiation. In this way, a principle of least emitted radiation can provide a possible description of wave function collapse:

$$\delta \int_{t_i}^{t_f} I(t)dt = 0,$$

being $I(t)$ the intensity of radiation per unit time and $t_i$, $t_f$ the initial and final time, respectively. A least emitted radiation principle will give us the minimum possible collapse times without radiation reaction divergences. Such a collapse time value will correspond to a minimum possible measurement time value in a double quantum dot system.

Let us now consider electrostatic quantum bit measurements in double quantum dots [8]. The qubit is a single electron and the detector is a point contact placed near one of the dots [9]. If the electron occupies the first dot, the transmission coefficient of the point contact decreases due to electrostatic repulsion generated by the electron. Thus, the electron position is monitored by the tunneling current. The qubit can be described by the Hamiltonian [5]

$$H = E_l a_l^\dagger a_l + E_r a_r^\dagger a_r + t (a_l^\dagger a_r + a_r^\dagger a_l)$$

where $a_l^\dagger (a_l)$ and $a_r^\dagger (a_r)$ are the creation (annihilation) operators in the left and right quantum wells and $t$ is the hopping amplitude between states $|l \rangle$ and $|r \rangle$ of the coupled quantum dot. The coupled quantum dot potential has alternating odd and even pure eigenstates ($|- \rangle$ and $|+ \rangle$). Superposition of these eigenstates can be constructed so as to give states well localized in the
left or right well. Then, the $(|l> > |r> >)$ states are given by

$$|l> = \frac{1}{\sqrt{2}}(|+> +|->)$$

and by

$$|r> = \frac{1}{\sqrt{2}}(-|+> +|->),$$

respectively. The system under observation is in a pure quantum state $|\psi(t)> >$ at the beginning of the measurement. Then, it will be in a pure conditional state after the measurement, conditioned on the result. If the initial state is $|\psi_i> = |+>$, the unnormalized final state given the result $|\psi_f> = |r>$ at the end of the measurement becomes $|\psi_f> = (|r> < r|)|\psi_i>$, where $\{|l> < l|, |r> < r| \}$ represents a set of operators that define the measurements and satisfies

$$|l> < l| + |r> < r| = 1.$$  \hspace{1cm} (5)

When this occurs, the system undergoes an instantaneous evolution $|+> \rightarrow |r>$, called a quantum jump.

Let us now consider a principle of least emitted radiation, Eq. (1). From quantum electrodynamics we know that the intensity of dipole radiation is $[10]$

$$I(t) = \frac{1}{4\pi c} \frac{2}{3 \epsilon_0} |\ddot{\psi}(t)|^2,$$

being $c$ the speed of light, $p$ the dipole moment and $\epsilon$ the dielectric constant. This is directly analogous to the classical formula for the intensity of dipole radiation from a system of moving particles. We note the correspondence principle for the radiation intensity is valid not only in the quasi-classical but in the general quantum case \([10]\).

In the one-dimensional case, the dipole moment $p(t)$ of the electron in the coupled quantum dot is

$$p(t) = e x |\psi(x, t)|^2 dx = e < x(t) >$$

where $e|\psi(x, t)|^2$ is the electron charge density. In the pure dipolar case, Eq. (11) can be written as

$$\delta \int_{t_1}^{t_2} \left| \frac{\partial^2}{\partial t^2} < x > \right|^2 dt = 0.$$  \hspace{1cm} (8)

Now we solve Eq. (8) for a double quantum dot. We consider the transition $|+> \rightarrow |r>$ by measurement (Fig. 1). In such a case, and during the collapse, $< x >$ is given by

$$< x > = |1 - A(t)x_l + A(t)x_r|,$$

where $l|x|l > x_l$, $< r|x|r > x_r$ and $A(t)$ is a time-dependent coefficient ($0 \leq A(t) \leq 1$). For simplicity, let us assume that the coordinate origin is placed in the middle of the left quantum well, i.e., $x_l = 0$. We call $A(t)$ the true path and we take some trial path $A_1(t)$ that differs from the true path by a small amount which we will call $\eta(t)$. We calculate the emitted radiation $E_{rad}$ for the path $A(t)$ and $A(t) + \eta(t)$, respectively. The difference must be zero in the first-order approximation of small $\eta(t)$. We rearrange Eq. (5) by integrating by parts and considering $\eta(t_i) = \eta(t_f) = 0$ and $\eta(t_i) = \eta(t_f) = 0$. Leaving out second and higher order terms, we have for $\delta E_{rad}$

$$\delta E_{rad} = \int_{t_1}^{t_f} \eta(t)\ddot{A}(t) dt = 0.$$  \hspace{1cm} (9)

This means that the function $\ddot{A}(t)$ is zero. Equation $\ddot{A}(t) = 0$ can be easily solved considering $A(t) = 1/2$, $A(t_f - t_i) = 1$ and $A(t) = A(t_f - t_i) = 0$. Then, and during the measurement process, we have for $|\psi(t)>$

$$|\psi(t)> = \left(\begin{array}{c} \frac{t_3}{\tau_c^3} - \frac{3t_2}{2\tau_c^2} + \frac{1}{2} \\ \frac{-t_3}{\tau_c^3} + \frac{3t_2}{2\tau_c^2} + \frac{1}{2} \end{array}\right) |l> - e^{i\phi} \left(\begin{array}{c} \frac{-2t_3}{\tau_c^3} - \frac{3t_2}{2\tau_c^2} + 1 \\ 0 \end{array}\right) |r>$$

where $\phi$ is an arbitrary phase factor ($\phi = 0$ at $t = 0$) and $\tau_c = t_f - t_i$ is the collapse time. We define the operator $C(t)$ that verifies $|r> = C(t)|+>$. Taking into account Eq. (10), we have

$$C(t) = \left(\begin{array}{cc} \frac{2t_3}{\tau_c^3} - \frac{3t_2}{2\tau_c^2} + 1 \\ 0 \end{array}\right)$$

$$C(t) = \sqrt{2} e^{i\phi} \eta | > < r |,$$

being $| > < r |$ the standard Von Neumann projector. In our model, the collapse matrix is some sort of time-dependent projector.

Considering energy conservation during transition $|+> \rightarrow |r>$, the total energy radiated $E_{rad}$ by an electron is $\Delta E/2$, ($\Delta E = E_+ - E_-$.). However, the electron may interchange energy $\Delta E_{xc}$ with the point contact during the measurement process. In such a case, we note that both quantities ($\Delta E_{xc}$ and $\Delta E$) should be of the same order of magnitude. If the interchange energy is higher than the level splitting between both quantum wells, the resonant condition is not obtained, and then, both symmetric $| > < r |$ pure states are destroyed. Accordingly, the particle may interchange energy with an efficient measurement device if $\Delta E_{xc} \sim \Delta E$. Taking into account this, in this work we assume that $E_{rad} \sim \Delta E$. The total energy radiated by an electron during measurement process $E_{rad}$ is

$$E_{rad} = \int_0^{\tau_c} I(t) dt = \frac{e^2 x_f^2}{2\pi \epsilon c^3 \tau_c^3},$$

where $t_i = 0$ and $t_f = \tau_c$. Now we consider the tunneling process between both quantum dots. We choose as our
initial state $|l> \rangle$ and monitor the time the particle takes to tunnel to the right-hand well when its state is $|r> \rangle$. This tunneling process occurs under free evolution, and the time taken is given by the inverse energy difference between the states $|+> \rangle$ and $|-> \rangle$, $\tau_t = \pi \hbar / \Delta E \ [11]$. Then, and from Eq. (13), the collapse time can be easily obtained

$$\tau_c = \left( \frac{e^2 x_r \tau_t}{2 \pi^2 \hbar^3 c^3} \right)^{1/3}. \ (14)$$

In Fig. 2, we have plotted the probability density in the right quantum well $|<r|\psi> |^2$ versus time during the measurement process. We have considered a GaAs/Ga$_{1-x}$Al$_x$As double quantum dot system which consists of two 150Å-wide GaAs quantum wells separated by a barrier of thickness $d = 15\AA$ and $d = 35\AA$. The barrier height and electron effective mass are taken to be 220meV and 0.067$m_0$, respectively. In Fig. 3 and 4 is shown both collapse and tunneling times versus barrier thickness. It is found that collapse times are three orders of magnitude lower than tunneling times. We should point out that the classical electron velocities associated with these collapse times are non-relativistic. In Fig. 4, it is clearly shown that collapse time is increased as we increase the barrier thickness. Such a result can be easily explained as follows. If the barrier thickness is increased, the level splitting between both quantum wells is decreased $[11]$ and both $\tau_t$ and $\tau_c$ are also increased.

Finally, we think that the emission of electromagnetic radiation during collapse process can be observed experimentally $[6]$. Each time a measurement is realized in a double quantum dot system, an emission of an electromagnetic radiation pulse takes place. The measured pulse width will correspond to the collapse time value.

In summary, in this work we have considered an electron in a double quantum dot system. We present a dynamical model for wave function collapse in the strong coupling to the detector limit. A leasts emitted radiation principle give us the minimum possible collapse times without radiation reaction divergences. Such collapse time values will correspond to minimum possible measurement time values in a double quantum dot system.

* Electronic address: hcruz@ull.es

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FIGURES

• **Fig. 1** A schematic illustration of the coupled quantum dot system. Conduction band potential and wave function amplitude of both $|r>$ and $|+>$ states.

• **Fig. 2** Probability density in the right quantum well $|<r|\psi>|^2$ versus time during the measurement process. Thin line: 15Å barrier thickness. Thick line: 35Å barrier thickness.

• **Fig. 3** Tunnelling time versus barrier thickness.

• **Fig. 4** Collapse time versus barrier thickness.