Formation Control for Water-Jet USV Based on Bio-Inspired Method

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Abstract
The formation control problem for underactuated unmanned surface vehicles (USVs) is addressed by a distributed strategy based on virtual leader strategy. The control system takes account of disturbance induced by external environment. With the coordinate transformation, the advantage of the proposed scheme is that the control point can be any point of the ship instead of the center of gravity. By introducing bio-inspired model, the formation control problem is addressed with backstepping method. This avoids complicated computation, simplifies the control law, and smoothes the input signals. The system uniform ultimate boundness is proven by Lyapunov stability theory with Young inequality. Simulation results are presented to verify the effectiveness and robust of the proposed controller.

Key words: water-jet, USV, bio-inspired model, backstepping, formation control

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1 Introduction
USV control problem is focused by many researchers. Especially formation control (Cui et al., 2009; Do, 2012; Zhang and Zhang, 2014) achieve challenging and dangerous tasks such as mine clearance, patrol, investigation, transportation of strategic materials and marine science applications, which can not only decrease personal injury but also achieve tasks that single USV cannot complete. Most of the ship formation control strategies are belongs to complicated nonlinear system, which include leader-follower method (Fahimi, 2007; Meng et al., 2012), virtual structure strategy (Zhao et al., 2012), behavioral approach (Arrichello et al., 2006; Ma and Zeng, 2015), graph theory (Almeida et al., 2012), and artificial potential function (Li and Xiao, 2016). Ghomam and Mnif (2009) and Xie and Ma (2014) addressed the problem of maneuvering a group of underactuated ships along given path with constant disturbances by a path follower and coordinated controller. However, the way of centralized control depends too much on the overall information. Liu et al. (2016) Based on line-of-sight (LOS), extended state observer (ESO) and cascaded theory to realize the task of the multiple USV along one curve. Theoretical analysis has showed that the closed-loop system is ISS. Peng et al. (2013) developed an adaptive formation controller for USV by using the neural network and dynamic surface control, which avoided the calculations of the virtual signals by introducing the first-order filter. Shojaei (2016) adopted neural network combined with adaptive robust control strategy to deal with the formation problem and acquired ideal result. The limitations of the work have common styles with too many parameters to be adjusted and the complicated process. An adaptive control design method (Peng et al., 2017a) is presented for containment maneuvering of marine surface vehicles with the adaptation laws not limited to a Lyapunov-based design and modularity between the estimator and controller. An adaptive containment maneuvering controller (Peng et al., 2017b) is proposed according to neurodynamics-based output feedback control scheme and the observer-based containment maneuvering control laws is proposed at the kinematic level. The stability for closed loop network is verified based on ISS and cascade theory. Ding and Guo (2012) proposed a formation control law without external disturbances based on leader-follower. Børhaug et al. (2011) proposed a control law for underactuated surface vessels formation based on LOS path following control and a nonlinear synchronization controller without accounting the inherent limitations due to the communication and with the control law of straight line following. Liao et al. (2015) studied the path following problem of an USV by backstepping adaptive sliding mode control. The problem is transformed into a actuated problem based on Seret-Frenet frame. However, during the controller design large calculations of derivate are involved because of the virtual signals induced by backstepping.

In this work, the formation problem for USV with time-
varying disturbances is investigated. The leader-follower strategy is used as it can be easily implemented. During the control law design, a virtual is assumed to be a leader. All the USVs are needed to be coordinated to a controller to keep a desired distance and orientation, as produces a formation for the task. By changing the distance and orientation, the formation can be changed in time. With the coordination transformation, the output point position (the bow position) differs from the traditional point of the center of gravity (Peng et al., 2011). This guarantees the steady and avoids the chattering of course. Then being inspired by the bioinspired method (Yang and Zhu, 2011; Hodgkin and Huxley, 1952) three neural dynamic models are introduced. It can not only smooth the input signals and simplify the control law, but also can limit the signal to a certain range and adjust the attenuation rate by parameters. The formation control problem of underactuated USV is addressed to cooperate with the backstepping method.

2 Problem formulation

In this work, the frame definition, the motion model of a single USV and formation model are shown as follows.

2.1 Single USV model

The 3-DOF dynamics and kinematics equations for a single water-jet USV are defined by Fossen (2002):

\[
\begin{align*}
\dot{x}_i &= u_i \cos \phi_i - v_i \sin \phi_i \\
\dot{y}_i &= u_i \sin \phi_i + v_i \cos \phi_i \\
\dot{\phi}_i &= r_i \\
\dot{u}_i &= \frac{m_2}{m_1} v_i r_i - \frac{d_{ii}}{m_1} + \frac{1}{m_1} \tau_{ui} + \frac{d_\tau}{m_1} \\
\dot{v}_i &= \frac{-m_1}{m_2} u_i r_i - \frac{d_{ii}}{m_2} + \frac{d_\tau}{m_2} \\
\dot{r}_i &= \frac{m_1 - m_2}{m_3} u_i v_i - \frac{d_{ii}}{m_3} r_i + \frac{1}{m_3} \tau_{ri} + \frac{d_\tau}{m_3}
\end{align*}
\]

where \( \eta_i = (x_i, y_i, \phi_i)^T \) denotes position (surge, sway displacements) and orientation (yaw angle) of the \( i \)-th USV in the north-east frame; \( \psi_i = (u_i, v_i, r_i)^T \) represents the \( i \)-th USV’s surge, sway and yaw angle velocities, respectively; \( m_{ii} \) is considered as the mass and inertia parameters of three axes. Hydrodynamic damping coefficients in the body coordinate is given by \( d_{ii} \). The signals \( \tau = (\tau_{ui}, \tau_{ri})^T \) are the force and torque inputs which are provided by water-jet thrusters. And \( W = [d_u, d_c, d_r]^T \) denote the surge, sway and yaw external disturbances, which are the time-varying disturbances in this work.

2.2 Leader-follower model

As start from simple, only two USVs are considered as a group. The formation mathematical model is shown in Fig. 1.

The distance between the leader USV and follower USV is given by \( l \); the angle of the follower relative to the leader is denotes by \( \theta \); \( l_s \) and \( l_t \) represent the components of \( l \) in 

![Fig. 1. Leader-follower formation configuration with the control point P.](image-url)

the north-east frame on the x and y axis, respectively. The bow position of USV is denoted by \( P \), with \( x_{FP}, y_{FP}, \) and \( \phi_{FP} \) of the position and orientation of \( P \); \( d \) is the distance between \( P \) and the mass center of follower. \( x_s, y_s, \phi_s, u_s, v_s, r_s, x_F, y_F, \phi_F, u_F, v_F, \) and \( r_F \) represent the position, orientation and surge, sway, and yaw velocities of the leader and follower, respectively. From Fig. 1 we can obtain

\[
x_{FP} = x_F + d \cos \phi_F; \quad y_{FP} = y_F + d \sin \phi_F.
\]  

In order to achieve the desired formation structure, following conditions should be satisfied:

\[
\lim_{t \to \infty} l - l_d \leq \zeta_1; \quad \lim_{t \to \infty} \theta - \phi_d \leq \zeta_2,
\]  

\( \zeta_1 \) and \( \zeta_2 \) are positive arbitrarily small constants.

If \( l \) and \( \phi \) can be confirmed, then \( l_s \) and \( l_t \) are unique. The control problem of \( l \) and \( \phi \) can be transformed to control \( l_s \) and \( l_t \). As can be seen from Fig. 1,

\[
l_s = l \cos \theta = x_L - x_F - d \cos \phi_F;
\]  

\[
l_t = l \sin \theta = y_L - y_F - d \sin \phi_F.
\]  

Then,

\[
l_s = u_L \cos \phi_L - v_L \sin \phi_L;
\]  

\[
\dot{l}_s = u_L \sin \phi_L + v_L \cos \phi_L;
\]  

\[
l_t = v_L \cos \phi_L - u_L \sin \phi_L;
\]  

\[
\dot{l}_t = v_L \sin \phi_L + u_L \cos \phi_L.
\]  

The desired distance between the two USVs is considered as \( l_d \); the projection weight in the north-east frame are \( l^d_s \) and \( l^d_t \); then

\[
l^d_s = -l_d \cos (\phi_d + \phi_L);
\]  

\[
l^d_t = -l_d \sin (\phi_d + \phi_L).
\]  

Differentiating Eq. (6) yields

\[
l^d_{s,t} = -l_d \dot{\phi}_d \cos (\phi_d + \phi_L) + l_d \phi_d \dot{\phi}_d \sin (\phi_d + \phi_L) + l_d \phi_d \dot{\phi}_L \sin (\phi_d + \phi_L);
\]  

\[
l^d_{s,t} = -l_d \phi_d \cos (\phi_d + \phi_L) + l_d \dot{\phi}_d \phi_d \cos (\phi_d + \phi_L) + l_d \dot{\phi}_L \cos (\phi_d + \phi_L).
\]  

The errors of formation model can be defined as:

\[
\begin{bmatrix}
e_x \\
e_y \\
e_r \\
e_{\phi_L}
\end{bmatrix} =
\begin{bmatrix}
\cos \phi_F & \sin \phi_F & 0 & 0 \\
-\sin \phi_F & \cos \phi_F & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
l_s - l^d_s \\
l_t - l^d_t \\
\phi_d - \phi_d
\end{bmatrix}
\]  

(8)
The formation mathematics model can be written as follows:
\[
\begin{align*}
\dot{e}_x &= e_{x}\dot{r}_F - u_F + u_L \cos e_r - v_L \sin e_r - l_d \dot{r}_F \sin(\phi_d + e_r) + \mu_1; \\
\dot{e}_y &= -e_{y} \dot{r}_F - v_L + u_L \sin e_r - v_L \cos e_r - d_{r_F} \\
&\quad + l_d \dot{r}_F \cos(\phi_d + e_r) + \mu_2; \\
\dot{e}_r &= r_F - r; \\
\dot{u}_L &= m_{11} \frac{v_F}{m_{22}} u_L - \frac{d_{11}}{m_{22}} u_F + \frac{1}{m_{11}} \tau_{u_L}; \\
\dot{v}_L &= -\frac{d_{22}}{m_{22}} v_F - \frac{d_{22}}{m_{22}} u_F; \\
\dot{r}_F &= m_{11} - m_{22} u_F v_F - \frac{d_{33}}{m_{33}} \tau_{r_F} + \frac{1}{m_{33}} \tau_{r_F}; \\
\end{align*}
\]
where \( \mu_1 = l_d \dot{r}_L \sin(\phi_d + e_r) - l_d \dot{r}_F \sin(\phi_d + e_r); \)
\( \mu_2 = l_d \dot{r}_L \cos(\phi_d + e_r) + l_d \dot{r}_F \cos(\phi_d + e_r); \)
where \( \mu_1 \) and \( \mu_2 \) are bounded. Since \( |\sin(\phi_d + e_r)| \leq 1 \) and \( |\cos(\phi_d + e_r)| \leq 1 \), \( l_d \) and \( \phi_d \) are bounded.

**Assumption 1**: The control inputs \( \tau_{u_L}, \tau_{r_F} \) and velocities of USV are all bounded.

**Assumption 2**: Trajectory produced by the virtual leader is smooth enough.

**Assumption 3**: \( l_d, \phi_d \) and the inertial errors are bounded.

**Assumption 4**: The follower can contact with the leader in time to acquire the information of positions and velocities.

In a summary, the formation control problem of water-jet USV can be divided into two parts, kinematics and dynamics.

For kinematics, virtual surge and sway velocities are
\[
\lim_{t \to +\infty} l_x \to F \; \dot{r}_L, \quad \lim_{t \to +\infty} l_y \to F \; \dot{r}_L.
\]
(10)
For dynamics, the force and torque are designed to make actual velocities approximate to virtual velocities.

3 **Controller design**

To solve the formation control problem for USV, the backstepping strategy has been used. However, the backstepping method is too complicated to be employed. In order to address this problem, we combine it with the bio-inspired method by introducing three neural dynamic models.

**Step 1**: Lyapunov function is chosen as:
\[
V_1 = \frac{1}{2} e^2_x + \frac{1}{2} e^2_y.
\]
(11)
Differenitication Eq. (11), we can obtain
\[
\dot{V}_1 = e_x \left[ -u_y + u_x \cos e_r - v_L \sin e_r + e_y \right] \\
- l_d \dot{r}_F \sin(\phi_d + e_r) + \mu_1 \right] + e_y \left[ -v_y - e_x \right] e_F + u_L \sin e_r \\
- u_x \cos e_r - d_{r_F} + l_d \dot{r}_F \cos(\phi_d + e_r) + \mu_2.
\]
Virtual control variables are chosen
\[
u_d = k_1 e_x + u_x \cos e_r - v_L \sin e_r + e_x \dot{r}_F \\
- l_d \dot{r}_F \sin(\phi_d + e_r) + \mu_1; \\
u_o = k_2 e_y - e_x \dot{r}_F + u_L \sin e_r - v_L \cos e_r \\
- d_{r_F} + l_d \dot{r}_F \cos(\phi_d + e_r) + \mu_2,
\]
where \( k_1 \) and \( k_2 \) are positive constants. In order to avoid repeating derivative \( u_d \) and \( u_o \), we make \( u_d \) and \( u_o \) go through a neural dynamic model, and substitute \( u_d \) and \( u_o \) with \( u_t \) and \( v_t \) as the virtual variable in the process of backstepping design.

The neural dynamic model is:
\[
\dot{u}_t = -A_1 u_t + (B_1 - u_t) f(u_d) - (D_1 + u_t) g(u_d), \\
\dot{v}_t = -A_2 v_t + (B_2 - v_t) f(u_o) - (D_2 + v_t) g(u_o),
\]
where \( u_t \) and \( v_t \) are the outputs of neural dynamic model, \( A_i (i = 1, 2, 3) \) are positive constants on behalf of the neurons of attenuation rate. The attenuation rate can be adjusted by parameter \( A_i, B_i \) and \( D_i (i = 1, 2, 3) \) are regarded as positive constants denoting the upper and lower bounds of neurons dynamic \( u_t \) and \( v_t \). It can limit the outputs to \([-D_i, B_i]\). The parameters are chosen based on the underactuate and actuator saturation characteristic, which approximate to the practice system. \( f(x) \) and \( g(x) \) are linear threshold functions of \( x \) as:
\[
f(x) = \begin{cases} 
1 & x \geq 0 \\
0 & x < 0 
\end{cases}
\]
(15)
**Step 2**: \( z_o, z_o, \) and \( e_o \) represent error variables, which are defined as:
\[
\begin{align*}
z_o &= u_t - u_x; \\
z_o &= v_t - v_y; \\
e_o &= u_o - u_t; \\
e_o &= v_o - v_t.
\end{align*}
\]
(16)

Then, \( e_o = u_t - u_x \).

By substituting Eq. (14) into Eq. (12), we obtain
\[
V_1 = -k_1 e_x^2 - k_2 e_y^2 - e_x e_u - e_y e_v - e_x \dot{e}_u - e_y \dot{e}_v.
\]
(17)

Considered the augmented Lyapunov function
\[
V_2 = V_1 + \frac{1}{2} e_o^2.
\]
(18)

The time derivative of Eq. (18) yields
\[
\dot{V}_2 = -k_1 e_x^2 - k_2 e_y^2 + e_o \left( \dot{u}_t - \dot{u}_x - e_x \right) - z_o e_x - e_o e_y - z_o e_y.
\]
(19)
where \( \dot{e}_x = -k_3 e_x - \dot{u}_x + e_x, \)
\( k_3 \) being a positive constant. Eq. (19) can be rewritten as:
\[
\dot{V}_2 = -k_1 e_x^2 - k_2 e_y^2 - k_3 e_x^2 - z_o e_x - e_o e_y - z_o e_y.
\]
(20)

Then the control input is selected as:
\[
\tau_{u_L} = m_{11}(k_3 e_x + e_x - u_x) - m_{22} u_y \dot{r}_F + d_{11} u_t.
\]
(21)

**Step 3**: As \( v \) is also a virtual control input, \( r \) is considered as a virtual input to control \( e_o \). Thus
\[
\dot{e}_v = v_t - v_o = -\frac{m_{11}}{m_{22}} u_y \dot{r}_F - \frac{d_{22}}{m_{22}} v_o - v_t.
\]
(22)
In this stage, the actual control input \( \tau_{u_L} \) is designed by considering Lyapunov candidate function which is given by
\[
V_3 = V_2 + \frac{1}{2} e_o^2.
\]
(23)

Time derivative of the Lyapunov candidate function
(23) results in
\[
\dot{V}_3 = -k_1 e_2^2 - k_2 e_2^2 - k_3 e_2^2 - z_0 e_2 - z_0 e_y + e_y - e_x a u_x r_e - \beta u_y - u_t) \tag{24} 
\]
where \( a = m_{11}/m_{22}, \beta = d_{12}/m_{22} \).

When the USV is moving on the sea, the surge velocity is a zero velocity, so we can choose a virtual variable \( r_d = \frac{1}{u_{ty}}(-e_y - \beta u_y - u_t + k_4 e_{x_0}) \), where \( k_4 \) is a positive constant.

Similarly to \( u_d \) and \( v_d \), \( r_d \) is also going through a neural dynamic model,
\[
\dot{r}_d = -A_r r_t + (B_r - r_d) f(r_d) - (D_r + r_d) g(r_d) \tag{25} 
\]
Error variable are defined as \( z_r = r - r_d \) and \( e_r = r - r_t \).
Substituting them into Eq. (24) yields
\[
\dot{V}_3 = -k_1 e_2^2 - k_2 e_2^2 - k_3 e_2^2 - z_0 e_2 - z_0 e_y - e_x a u_x e_r + e_y (r_t - r - a u_y e_r) \tag{26} 
\]
Special 4: For the sake of controlling \( e_r \), an augmented Lyapunov function \( V_4 \) is considered.
\[
V_4 = V_3 + \frac{1}{2} z_r^2 \tag{27} 
\]
\[
\dot{V}_4 = -k_1 e_2^2 - k_2 e_2^2 - k_3 e_2^2 - k_4 e_2^2 - z_0 e_2 - z_0 e_y - e_x a u_x e_r + e_y (r_t - r - a u_y e_r) \tag{28} 
\]
Let
\[
\dot{r}_r = -k_5 e_r + a u_x e_r + r_t \tag{29} 
\]
where \( k_5 \) is a positive constant. We can obtain the control input of torque
\[
\tau_{ud} = m_{33} (-k_5 e_r + a u_x e_r + r_t) - (m_{11} - m_{22}) u_y e_r - d_{33} r_t \tag{30} 
\]
Substituting Eq. (29) into Eq. (28) yields
\[
\dot{V}_4 = -k_1 e_2^2 - k_2 e_2^2 - k_3 e_2^2 - k_4 e_2^2 - k_5 e_2^2 - z_0 e_2 - z_0 e_y - e_x a u_x e_r + e_y (r_t - r - a u_y e_r) \tag{31} 
\]
where \( \chi = -z_0 e_2 - z_0 e_y - e_x a u_x e_r \).

4 Stability analysis
For the formation system, we choose a Lyapunov function
\[
V_5 = V_4 + \frac{1}{2} z_\omega^2 + \frac{1}{2} z_{\omega 0}^2 + \frac{1}{2} z_r^2 \tag{32} 
\]
\[
\dot{V}_5 = -k_1 e_2^2 - k_2 e_2^2 - k_3 e_2^2 - k_4 e_2^2 - k_5 e_2^2 - z_0 e_2 - z_0 e_y - e_x a u_x e_r + e_y (r_t - r - a u_y e_r) \tag{33} 
\]
where \( z_\omega = \dot{u}_t - u_d \). According to Assumptions 3–4, we can obtain \( \dot{u}_t, v_d \), and \( r_d \) bounded, \( \Omega_1, \Omega_2 \), and \( \Omega_3 \) are assumed as the maximum value of \( u_d \), \( v_d \), and \( r_d \), respectively. Based on Eq. (15) with \( B_1 = D_1 \), it results in
\[
\dot{z}_\omega = -A_\omega u_t + B_1 r_d - u_d \tag{34} 
\]
where \( A_\omega = A_1 + f(u_d) + g(u_d) > 0 \). Similarly, let \( B_2 = D_2, B_3 = D_3 \), yields
\[
\dot{z}_\omega = -A_\omega u_t + B_2 v_d - u_d, \quad \dot{z}_r = -A_r r_t + B_3 r_d - r_d \tag{35} 
\]
where \( A_\omega = A_2 + f(r_d) + g(r_d) > 0, A_r = A_3 + f(r_d) + g(r_d) > 0 \). Let \( B_1 = A_\omega, B_2 = A_r, B_3 = A_r \). Eq. (33) can be rewritten as:
\[
\dot{V}_5 < -k_1 e_2^2 - k_2 e_2^2 - k_3 e_2^2 - k_4 e_2^2 - k_5 e_2^2 - z_0 e_2 - z_0 e_y - e_x a u_x e_r + e_y (r_t - r - a u_y e_r) \tag{36} 
\]
For the type of absolute value of the item, without the absolute value symbol all the items are assumed larger than 0. According to Young inequality (Ghommam and Saad, 2014), we can obtain
\[
|\omega| |\Omega_1| \leq \frac{\omega_1^2}{2} + \frac{\Omega_1^2}{2}; |\omega| |\Omega_2| \leq \frac{\omega_1^2}{2} + \frac{\Omega_1^2}{2}; \\
|\omega| |\Omega_3| \leq \frac{\omega_1^2}{2} + \frac{\Omega_1^2}{2}; \\
|\omega| \leq a e_r |u_y| |\omega| + |\omega| e |v| + |\omega| e |r| \\
< \left( a |u_e e_r| \right)^2 + \frac{e_1^2}{2} + \frac{e_2^2}{2} + \frac{e_3^2}{2} + \frac{e_4^2}{2}. \tag{37} 
\]
By substituting Eq. (37) into Eq. (36), inequality (37) is rewritten as:
\[
\dot{V}_5 < -k_1 e_2^2 - k_2 e_2^2 - k_3 e_2^2 - k_4 e_2^2 - k_5 e_2^2 - z_0 e_2 - z_0 e_y - e_x a u_x e_r + e_y (r_t - r - a u_y e_r) \tag{38} 
\]
For the reason of the system stability, \( k_1 > \frac{1}{2}, k_2 > \frac{1}{2}, k_3 > \frac{a |u_e e_r|}{2}, A_\omega > 1, A_r > 1, A_r > 1 \).

Inequality Eq. (38) can be rewritten as:
\[
\dot{V}_5 < -2 k_\phi V_5 + \Delta \tag{39} 
\]
To solve the inequality, we can obtain
\[
V_5 \leq \left( V_5(0) - \frac{\Delta}{2k_\phi} \right) e^{-2 \phi t} + \frac{\Delta}{2k_\phi} \leq V_5(0) e^{-2 \phi t} + \frac{\Delta}{2k_\phi} \forall t > 0. \tag{40} 
\]
It shows that the Lyapunov function \( V_5 \) is smaller than \( \frac{\Delta}{2k_\phi} \). This means that the control error is bounded. The radius of convergence \( \frac{\Delta}{2k_\phi} \) can be small enough by adjusting the value of \( k_1 \) and \( A_\omega, A_r \).

5 Numerical simulation
To verify the effectiveness of the proposed formation controller of USVs, the mathematical results of three USVs from Do et al. (2002) are simulated.
\[
m_{11} = 120 \times 10^3 \text{kg}; m_{22} = 217.9 \times 10^3 \text{kg}; m_{33} = 636 \times 10^3 \text{kg} \;
\]
\[
d_{11} = 215 \times 10^3 \text{kg/s}; d_{22} = 117 \times 10^3 \text{kg/s}; d_{33} = 802 \times 10^3 \text{kg/s}. \tag{41} 
\]
In the simulation, USV1 is a virtual leader, USV2 and USV3 are followers. The system initial states are given as \( u_t = [0.5, 0, 0]^T \), \( u_\omega = [0, -5, 0]^T \). The desired position and orientation respect to USV1 are as follows: in the first 20 s, \( l_{d12} = 5 \text{m} \), \( l_{d13} = 5 \text{m} \), \( \phi_{d12} = \pi/2, \phi_{d13} = -\pi/2 \), during the 20 s to 150 s, \( l_{d12} = 5 \text{m} \), \( l_{d13} = 5 \text{m} \), \( \phi_{d12} = 5\pi/6, \phi_{d13} = -5\pi/6 \). We
select \( k_1 = k_2 = k_3 = 1, k_4 = 5, A_1 = A_2 = A_3 = 4, B_1 = D_1 = 10, B_2 = D_2 = 8, B_3 = D_3 = 8 \). The disturbances are defined as time-varying disturbances \( W = 4 \times 10^3 [\sin(0.2t), 1 + 2\sin(0.3t), 2]^T \).

The moving trajectories of three USVs under the proposed control law are shown in Fig. 2. It can be seen that in the first 20 s, the distances of USV2 and USV3 respect to USV1 are 5 m with the orientations of \( \pi/2 \) and \(-\pi/2\), respectively. After that, the distances of USV2 and USV3 relative to USV1 are still 5 m, the orientation are \( 5\pi/6 \) and \(-5\pi/6\). It is verified the effectiveness of the proposed control law. Fig. 3 shows the positions and orientation of three USV. We can see from it that in the first 20 s, the positions of \( x \) (north) are same and the positions of \( y \) (east) vary in a range of \( \pm 5 \). Thus the distances are \( \sqrt{x^2 + y^2} = 5 \). Fig. 4 shows the velocities of three USV. As can be seen from Fig. 5, the forces and torques of USV2 and USV3 are smooth except to the moment of formation changed.

### 6 Concluding remarks

The development of a bio-inspired formation controller for USV has been presented in the presence of disturbances caused by external environment. Compared with the backstepping method, the bio-inspired based scheme shows some advantages to cope with the derivation repeating problem and smooth the input signal. Stability analysis verifies that the control law can guarantee the system globally asymptotically stable. The simulation results reveal that the formation can be changed at any time with satisfying performance and robustness. The effectiveness of the control law has been validated by the simulation results and theoretical analysis.

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