STUDY OF CALIBRATION OF SOLAR RADIO SPECTROMETERS AND THE QUIET-SUN RADIO EMISSION

CHENGMING TAN1, YIHUA YAN1, BAOLIN TAN1, QIJUN FU1, YUYING LIU1, AND GUIRONG XU2

1 Key Laboratory of Solar Activity, National Astronomical Observatories of Chinese Academy of Sciences, Datun Road A20, Chaoyang District, Beijing 100012, China
2 Hubei Key Laboratory for Heavy Rain Monitoring and Warning Research, Institute of Heavy Rain, China Meteorological Administration, Wuhan 430205, China

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ABSTRACT

This work presents a systematic investigation of the influence of weather conditions on the calibration errors by using Gaussian fitness, least chi-square linear fitness, and wavelet transform to analyze the calibration coefficients from observations of the Chinese Solar Broadband Radio Spectrometers (at frequency bands of 1.0–2.0 GHz, 2.6–3.8 GHz, and 5.2–7.6 GHz) during 1997–2007. We found that calibration coefficients are influenced by the local air temperature. Considering the temperature correction, the calibration error will reduce by about 10%–20% at 2800 MHz. Based on the above investigation and the calibration corrections, we further study the radio emission of the quiet Sun by using an appropriate hybrid model of the quiet-Sun atmosphere. The results indicate that the numerical flux of the hybrid model is much closer to the observation flux than that of other ones.

Key words: methods: data analysis – methods: numerical – Sun: atmosphere – Sun: radio radiation

1. INTRODUCTION

Broadband radio spectrometers play an important role in observing and revealing the physical processes of the solar atmosphere, solar flares, coronal mass ejections, and other solar activities. When we utilize the data observed by spectrometers to study solar problems, calibration is a key procedure that dominates the reliability of the observational results. At present, there are several solar broadband radio spectrometers running in the world, such as Phoenix at ETH Zurich (100–4000 MHz; Benz et al. 1991), Ondrejov Radiospectrograph in the Czech Republic (800–5000 MHz; Jiricka et al. 1993), Brazil Broadband Spectrometer (200–2500 MHz; Sawant et al. 2001), and the Chinese Solar Broadband Radio Spectrometers (SBRS; 1.10–2.06 GHz, 2.60–3.80 GHz, and 5.20–7.60 GHz; Fu et al. 1995; Ji et al. 2005) at the Huairou Solar Observing Station. All the above spectrometers are single-dish telescopes that receive the radio emission of the full solar disk without spatial resolutions. So far, calibration techniques of single-dish radio telescopes can be classified as follows:

1. Absolute calibration. Skillful practices are needed to determine the absolute calibration values by using an approximate gain of the antenna based on radar and communication techniques, together with an approximate sensitivity derived from experiments with limited accuracy. It is still a difficult problem and usually only applied to polarimeters (Broten & Medd 1960; Findlay 1966; Tanaka et al. 1973).

2. Relative calibration. This method needs reference emission sources to calculate the emission flux (or brightness temperature). It is usually applied to spectrometers since the absolute calibration of spectrometers would be a huge and very complex mission at broadband frequencies (Messmer et al. 1999).

3. Nonlinear calibration. When the gain factor of the receiver exceeds its normal range during strong solar radio burst, it is necessary to consider nonlinear calibration (Yan et al. 2002).

Recently, we found that the calibration data have a strong correlation with the local air temperature at a frequency of 2.6–3.8 GHz of SBRS (Tan et al. 2009). The local air temperature is an important factor that needs to be taken into account in the calibration procedure. We also analyze the impact of the Sun elevation and other weather conditions on the calibration.

Moreover, the calibration of solar radio spectrometers is very important when we compare the observations with theoretical work in the study of the quiet-Sun radio emission. It supplies an accurate flux spectrum, which help us to study the basic nature of the quiet-Sun emission and provide a fundamental knowledge of the solar atmosphere. The radio quiet Sun has been studied for about 70 yr. Martyn (1946) considered the quiet Sun as a blackbody radiation and studied its radio spectrum. He showed the variation of radio brightness across the solar disk at various frequencies and indicated that limb brightening should be observed. Smerd (1950) studied the radio radiation from the quiet Sun with numerical analysis by applying radiative transfer equations to a typical ray trajectory (Jaeger & Westfold 1950). He presented a complete analysis of the quiet-Sun radio radiation, and the result is consistent with observations at wavelengths of 3 cm, 10 cm, 25 cm, 50 cm, 60 cm, 1.5 m, and 3.5 m (Pawsey & Yabes 1949; Christiansen & Hindman 1951). Since then, more and more observations and studies have been published (Allen 1957; Tanaka et al. 1973; Bastian et al. 1996). Kundu (1965) reviewed the basic theory of radiative transform and propagation in solar radio astrophysics. Tanaka et al. (1973) used the average value of the daily noon flux to obtain the radio flux spectrum of the quiet Sun. Other works (Nelson et al. 1985; Zirin et al. 1991) used the flux density of the quiet Sun around the minimum of the solar cycle. The numerical computation of the quiet-Sun radio emission has been discussed in those papers (Dulk 1985; Benz 1993; Selhorst et al. 2005). Benz (2009) pointed out that the radio emission of the quiet Sun is a well-defined minimum radiation level when the Sun has no sunspots for some weeks. The presence of sunspots enhances the radio emission and produces a slowly variable component. Shibasaki et al. (2011) reviewed radio emission of the quiet Sun in...
Radio Emission

\[ T_{\text{sun}} \text{(point the Sun)} \]
\[ T_{\text{sky}} \text{(point the sky)} \]

\[ A_{\nu}(\nu) \]
\[ T_{\nu} \]
\[ R_{\text{sun}} \]
\[ R_{\text{sky}} \]
\[ R_{n} \]
\[ R_{s} \]

Termination

Noise Source

Receiver

Computer

Intensity

2. OBSERVATIONS AND CALIBRATION

Three spectrometers of SBRS (Fu et al. 1995, 2004, Ji et al. 2005) are located at the Huairou Solar Observing Station in China. The first spectrometer (SBRS1) was upgraded three times. SBRS1 observed the Sun and only saved the burst data in the early years. It started routine and calibration observations in 1999 October. It was under construction after 2006 July and started to work again in 2013. The second spectrometer (SBRS2) began to observe the Sun in 1996 September. The third spectrometer (SBRS3) began observing the Sun in 1999 August. SBRS2 and SBRS3 shared the same antenna, with a diameter of 3.2 m. The main information and performance parameters of SBRS are listed in Table 1. During solar cycle 23, the SBRS observed the Sun and performed daily calibration at noon (avoiding the effect of radio bursts), except for the instruments under maintenance. They supply plenty of observations and calibration data to analyze the quiet-Sun radio emission. In this work, we do not consider the daily calibration data in less than 1 yr of observations because small data sets may result in larger error. We only select the daily calibration data from 1999 to 2004 October of SBRS1, from 1997 to 2007 of SBRS2, and from 2000 to 2007 of SBRS3, except for some big data gaps (>10 days) or without a noise source or termination (left panel of Figure 1). We go on to analyze the daily calibration data with the daily noon flux from the National Geophysical Data Center (NGDC), and we study some treatments to reduce the calibration errors. At last, the flux spectrum of the quiet Sun can be obtained from the observed data with calibration.

2.1. Fundamental of Calibration

Since our data are obtained by broadband spectrometers, this work mainly focus on the relative calibration. The basic principle of the relative calibration was discussed in Messmer et al. (1999) and Yan et al. (2002) in detail. SBRS records a set of calibration data in daily routine observations. The left panel

![Figure 1](image-url)
of Figure 1 presents the schematic chart of the antenna and receiver system. The switch may turn to the antenna, noise source, and termination, respectively. Usually, the termination is a 50 ohm resistor. For each case there is an input to the receiver system. \( T_a \) is the equivalent antenna temperature \( (T_a = T_{\text{Sun}} \text{ or } T_a = T_{\text{sky}} \text{ when the antenna points to the Sun or the sky, respectively}) \). \( T_n \) and \( T_r \) are the equivalent temperature of the noise source and the terminal source, respectively. Therefore, there are four outputs at the computer at various frequencies: the Sun (marked as \( R_{\text{Sun}} \)), sky background (\( R_{\text{sky}} \)), noise source (\( R_n \)), and terminal source (\( R_t \)). They are shown in the right panel of Figure 1 by the data processing software. When the gain factor (\( G_r \)) of the receiver is in the linear range (Yan et al. 2002), they comply with the following relationships:

\[
\begin{align*}
R_{\text{Sun}}(\nu) &= (T_{\text{Sun}} + T_r(\nu)) \cdot G_r(\nu) \quad (1) \\
R_{\text{sky}}(\nu) &= (T_{\text{sky}} + T_r(\nu)) \cdot G_r(\nu) \quad (2) \\
R_n(\nu) &= (T_n(\nu) + T_r(\nu)) \cdot G_r(\nu) \quad (3) \\
R_t(\nu) &= (T_r(\nu) + T_r(\nu)) \cdot G_r(\nu). \quad (4)
\end{align*}
\]

Here \( \nu \) is the observed frequency, and \( T_r \) is equivalent temperature of the receiver system, which is very small. From Broten & Medd (1960) and Messmer et al. (1999), we have

\[
F_{\text{Sun}}(\nu) = \frac{2k_B \cdot T_{\text{Sun}}(\nu)}{A_c(\nu)}. \quad (5)
\]

\( F_{\text{Sun}} \) and \( F_{\text{sky}} \) are the radio fluxes of the quiet Sun and sky, respectively. The effective area of the antenna aperture \( A_c(\nu) \) is changeless and can be considered as a constant. \( k_B \) is Boltzmann’s constant. The real flux of the Sun \( F_0(\nu) \) should subtract the contribution of the sky \( F_{\text{sky}}(\nu) \), i.e., Equation (6). From Equations (1) and (2) we have

\[
F_0(\nu) = F_{\text{Sun}}(\nu) - F_{\text{sky}}(\nu) = \frac{2k_B}{A_c(\nu)} \cdot \frac{R_{\text{Sun}}(\nu) - R_{\text{sky}}(\nu)}{G_r(\nu)}. \quad (6)
\]

\( G_r(\nu) \) can be deduced with Equations (3) and (4). Then Equation (6) can be transformed into

\[
F_0(\nu) = \frac{2k_B \cdot (T_n(\nu) - T_r(\nu))}{A_c(\nu)} \cdot \frac{R_{\text{Sun}}(\nu) - R_{\text{sky}}(\nu)}{R_n(\nu) - R_t(\nu)}, \quad (7)
\]

where \( R_{\text{Sun}}(\nu), R_{\text{sky}}(\nu), R_n(\nu), \) and \( R_t(\nu) \) are daily calibration data. As written previously, \( k_B \) and \( A_c(\nu) \) are constant. \( T_n \) and \( T_r \) are equivalent temperatures of the noise source and the terminal source, respectively. The noise source and the terminal source are fixed electronic apparatuses and usually stable under the steady environment and no interference. Thus, in Equation (8), \( C(\nu) \) will be also stable under the steady environment and no interference. It is defined as calibration coefficient

\[
C(\nu) = 2k_B \cdot \frac{(T_n(\nu) - T_r(\nu))}{A_c(\nu)} = F_0(\nu) \cdot \frac{R_n(\nu) - R_t(\nu)}{R_{\text{Sun}}(\nu) - R_{\text{sky}}(\nu)}. \quad (8)
\]

For any observed data \( R_{\text{Sun}}(\nu) \), the corresponding real flux \( F_0(\nu) \) of the Sun is calibrated as follows:

\[
F_0(\nu) = F_{\text{Sun}}(\nu) - F_{\text{sky}}(\nu) = \frac{R_{\text{Sun}}(\nu) - R_{\text{sky}}(\nu)}{R_n(\nu) - R_t(\nu)} \cdot C(\nu). \quad (9)
\]

Equation (9) is very convenient to do calibration for it does not require that we know the values of \( T_n(\nu) - T_r(\nu) \) and \( A_c(\nu) \). The bandpass flatness of the spectrum is usually not good for two reasons: (1) the gain factor \( G_r(\nu) \) of the receiver varies along frequency; and (2) the antenna system \( A_c(\nu) \) will also vary more or less along frequency. The calibration with Equation (9) will eliminate the frequency property of the gain factor and flatten the frequency property of the antenna system with \( C(\nu) \) of each frequency. The most important calibration work is to analyze a suitable calibration coefficient \( C(\nu) \) with the right-hand side of expression (8) and reduce the calibration error. Usually we decide the daily noon flux from NGDC as \( F_0(\nu) \). It can be downloaded from the NGDC Web site,\(^8\) which provides standard flux of solar radio emission at nine fixed frequencies \((245, 410, 610, 1415, 2695, 2800, 4995, 8800, \text{and} 15400 \text{MHz})\). We can calculate flux at any frequency between 245 and 15,400 MHz when using linear interpolation.

2.2. Impact of the Weather in the Calibration

Generally, a constant coefficient is adopted in calibration. In practice, the calibration is influenced by the local air temperature more or less. Panels (b) and (c) of Figure 2 show the comparison between the local air temperature and the calibration result at 2800 MHz. Panels (d) and (e) of Figure 2 show that \( R_{\text{Sun}}(\nu), R_{\text{sky}}(\nu), R_n(\nu), \) and \( R_t(\nu) \) are all correlated with the local air temperature. From Equations (1) and (2) we can deduce that the gain factor \( G_r(\nu) \) is influenced by the local air temperature since \( T_{\text{Sun}}(\nu) \) and \( T_{\text{sky}}(\nu) \) have no relationship with air temperature and \( T_r(\nu) \) is very small. In fact, the gain factor \( G_r(\nu) \) of all bands of SBRS is influenced by the local air temperature more or less. Panels (d) and (e) also show two major jumps (arrows in the figure) at 2001 April 19 and 2002 November 09, respectively. We check the daybook of the observer and find that there is a change to the attenuation of the instrument. Panel (f) shows that the daily calibration coefficients \( C_d \) are correlated with the local air temperature. The bottom three panels of Figure 2 indicate that the correlation between \( C_d \) and the local air temperature varied because of adjustment of the instrument. We can deduce that \( T_n(\nu) - T_r(\nu) \) in Equation (8) is also influenced by the local air temperature. All these results imply that the electronic apparatuses of the instruments are influenced by the local air temperature. The relationship between the calibration result (panel (c) of Figure 2) and the humidity (panel (a) of Figure 2) or other weather conditions is not clear. The observation must be stopped when some extreme weather events occur (such as thunder, windstorms, rainstorms, and heavy snow). Until now we have found no distinct evidence that the calibration was influenced by normal weather conditions except for air temperature.

We will explain that the Sun elevation has a small effect on the calibration of this work. Tsuchiya & Nagame (1965) pointed out that solar flux can be measured without considering...

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\(^8\) http://www.ngdc.noaa.gov/stp/SOLAR/ftpsolarradio.html
the weather conditions (atmospheric absorption) at frequency lower than 17 GHz. Considering the atmospheric absorption, the antenna temperature is given by

\[ T_T = T_{\text{source}} e^{-\tau} + T_{\text{sky}} \]  

and

\[ T_{\text{sky}} = \int_0^{\tau_0} T(\tau) e^{-\tau} d\tau. \]  

\[ \tau = \int_h^{\infty} \kappa \cdot dh \cdot \sec(z), \]  

where \( \kappa \) is the absorption coefficient of the atmosphere. It is small (<0.006 km\(^{-1} \)) at frequencies lower than 17 GHz (Tsuchiya & Nagame 1965). Moreover, the density of the water vapor and oxygen decreases rapidly along the height of atmosphere <10 km above the ground level. The absorption \( \kappa \) at the height of \( \sim 10 \) km will be about 1\% that of the ground level. Here \( h \) is height and \( z \) is the zenith distance. So the Sun elevation is equal to \( 90^\circ - z \). The daily calibration observation is done around noon. The Sun elevation at noon is larger than \( 26^\circ \), considering that the latitude of Beijing is \( 40^\circ \). The \( \sec(z) \) is smaller than 5.8 when the Sun elevation is greater than \( 10^\circ \). Thus, the optical depth in Equation (12) is also very small, \( \tau_0 \lesssim 0.02 \). It has only a small effect on the temperature of the emission source.

We also find no distinct evidence that the calibration was influenced by the Sun elevation. If the Sun elevation has a considerable effect on the observation, the following will result: (1) The variation profile of \( R_{\text{Sun}} \) (or \( R_{\text{sky}} \)) will show considerable difference from that of \( R_n \) (or \( R_t \)). The left panel of Figure 1 shows that \( R_n \) and \( R_t \) have no relationship with Sun elevation for no connection with antenna. (2) \( R_{\text{Sun}} \) (or \( R_{\text{sky}} \)) in summer (high elevation) will be larger than that in winter (low elevation). This is opposite to the observation shown in panel (d) of Figure 2. From all the above analyses,
the Sun elevation does not need to be considered in calibration.

We need to understand two problems: (1) the variety of daily calibration data and (2) the relationship between the daily calibration coefficients and the local air temperature. The first is related to the daily calibration data and daily standard flux of solar radio emission. The daily calibration data were recorded by SBRS. They consist of four recorded parameters: $R_{\text{Sun}}$, $R_{\text{sky}}$, $R_{\text{v}}$, and $R_t$ (right panel of Figure 1). The daily noon flux of radio emission at nine fixed frequencies can be downloaded from the NGDC Web site. Usually the observations from Learmonth Observatory are considered since its recording time (05:00 UTC) is very close to that of SBRS (04:00 UTC). The second problem is related to the local air temperature data of Beijing, which are collected from the China Meteorological Administration.

The daily calibration coefficients are calculated by the right-hand side of Equation (8) with the daily calibration data of SBRS during 1997–2007. There are several steps of pretreatment to exclude the abnormal data: (1) look up the notebook of the observer, check out the day when the instrument was not in good operation (testing, maintenance, no noise source, etc.), and rule out the data during these cases; (2) get rid of the calibration data with interference, ±NaN value or minus value. These data are wrong and will result in unpredictable errors in the mathematical processing. The main reason for the ±NaN, minus, and big value of the daily calibration coefficient comes from the electromagnetism interference of the environment. So $R_{\text{sky}}$ will be too big or even larger than $R_{\text{Sun}}$. The relationship between the abnormal data and bad weather conditions is still not clear. We will list all the possible reasons in Section 4.

2.3. Analysis of the Calibration

We take an example to analyze the calibration at 2800 MHz. At first, we compare the average value of the daily calibration coefficients ($\bar{C}$) with the Gaussian fitness value of the daily calibration coefficients ($\bar{C}$) at each frequency. They are $C(\nu)$ in Equation (9) when doing calibration. The Gaussian fitness value can exclude the contribution of big or very small values, which usually indicate radio bursts, sunspots, or abnormal observations. The left panel of Figure 3 shows the comparison between $\bar{C}$ and $\bar{C}$ at 2800 MHz. The vertical long-dashed line indicates values of $\bar{C}$. The black solid line is the histogram of the daily calibration coefficient. The dashed line is the Gaussian fitness of the black solid line. The Gaussian fitness is

$$f(x) = A_0 e^{-\frac{x^2}{2}} + A_3 + A_4 x + A_5 x^2, \quad z = \frac{x - A_1}{A_2} \quad (13)$$

The $x$ value of the maximum $f(x)$ is $\bar{C}$, marked as a vertical dashed line in Figure 3. When $|\bar{C} - \bar{C}| \leq 0.02 \bar{C}$, the average value is close to the Gaussian fitness value. This indicates that the observation of this frequency is reliable. In fact, $\bar{C}$ and $\bar{C}$ are close for most of the frequencies. The most probable value (MPV) is the maximum of the histogram (marked as a star in Figure 3). The rms value is the square root of $C_d$ (marked as a cross in Figure 3). Both the calibration error of the MPV and the rms are larger than those of $\bar{C}$ and $\bar{C}$. The right panel of Figure 3 shows the comparison between the standard deviation ($\sigma_v$) of calibration with $\bar{C}$ and the standard deviation ($\sigma_\bar{C}$) of calibration with $\bar{C}$. The standard deviation of calibration is calculated by

$$\sigma = \text{stddev}(F_H - F_N) \quad (14)$$

Here $F_H$ is the daily noon flux calculated by Equation (9) with the daily calibration data of SBRS and different $C$ ($\bar{C}$, $\bar{C}$, etc.). $F_N$ is the daily noon flux from NGDC. For 82% of frequencies, $\sigma_v$ are greater than $\sigma_\bar{C}$. For the remaining 18% of frequencies, $\sigma_v$ are less than or equal to $\sigma_\bar{C}$ because the Gaussian fitness deviates from the histogram of the daily calibration coefficients. Thus, $\bar{C}$ is used as the constant coefficient of calibration. The calibration with a constant coefficient ($\bar{C}$ or $\bar{C}$) is named the constant calibration.

The other two sets of calibration coefficients are related to the local air temperature, including calibration coefficients with temperature correction (TC) and temperature-wavelet correction (TWC). Figure 4 shows a strong correlation between the daily calibration coefficients (white cross in top panel) at 2800 MHz and local air temperature (black cross in bottom panel) during 1997–2007. Here we partition the data into three terms (1997–1999, 2000–2004, 2005–2007) according to the data gap during which the spectrometers are under maintenance. Those points that exceed the normal range during maintenance should be excluded. Then we decide in Equation (15) the linear relationship between the daily calibration coefficients $C_d(t)$ and the local air temperature $T_{\text{air}}(t)$ for each term. The variable $t$ is the time count in days. $C_1$ and $C_2$ can be obtained by least chi-square linear fitness between $C_d(t)$ and $T_{\text{air}}(t)$:

$$C_d(t) = C_1 + C_2 \times T_{\text{air}}(t) \quad (15)$$
Here $C_1$ is the first term of the linear fitness. In the second term of the linear fitness, $C_2$ is the correct factor that is related to the local air temperature. For each day, the calibration coefficient $C_{TC} = C_1 + C_2 \times T_{air}$ can be corrected by the local air temperature and replace $C(\nu)$ in Equation (9). This is the calibration with TC.

TWC is the calibration coefficient corrected by temperature-wavelet analysis. The top panel of Figure 4 plots the power spectrum (red color) of $C_d(t)$ after wavelet transform for three terms. The wavelet transform method was discussed by Sych & Yan (2002) in detail. Their work used a complex basis based on the Morlets wavelet, which is well localized in both the scale and frequency plane. This makes it possible to rapidly reconstruct the signal even in the presence of a singularity value or in the absence of data. We first transform $C_d(t)$ by wavelet to the wavelet data, which are complex numbers and give the power information along the time and period. The period of the maximum power is about 365 days, and the half-power beam widths (HPBWs) are in the range of 260–530 days. We go on to set the value of the wavelet data beyond the HPBW as zero (that is, we only keep the wavelet data within the HPBW), and we perform wavelet transform back to new coefficients $\tilde{C}(t)$. This numerical process is wavelet filtering, which filters out the information we do are not concerned with. The new coefficients $\tilde{C}(t)$ are plotted as a black smooth curve through $C_d(t)$ (white cross). In Equation (16), $T_{air}(t)$ is the local air temperature after wavelet filtering. So, $C_1$ and $C_2$ can be obtained by least chi-square linear fitness between $\tilde{C}(t)$ and $T_{air}(t)$:

$$\tilde{C}(t) = C_1 + C_2 \times T_{air}(t)$$

$$C_{TWC} = C_1 + C_2 \times T_{air}$$

when doing calibration. This is the calibration with TWC.

We compare the calibration results of three sets of calibration coefficients ($\tilde{C}$, $C_{TC}$, and $C_{TWC}$) at 2800 MHz in both the long term (more than 2 yr) and short term (1 yr). The standard deviations $\sigma_G$, $\sigma_{TC}$, and $\sigma_{TWC}$ are of calibration with $\tilde{C}$, $C_{TC}$, and $C_{TWC}$, respectively. They are calculated by Equation (14). Each panel of Figure 5 plots the daily noon flux observed by SBRS (calibration with $\tilde{C}$) versus the flux of NGDC. The upper three panels are the calibration results for three long observation terms. The bottom 11 panels show the yearly calibration results. The $\sigma_{TC}$ or $\sigma_{TWC}$ are about 10%–20% smaller than $\sigma_G$ at various observation terms. The difference between $\sigma_{TC}$ and $\sigma_{TWC}$ is small, ~2%. In short observation terms, $\sigma_{TC}$ is the smallest. In long observation terms, $\sigma_{TWC}$ is a little smaller than $\sigma_{TC}$ in most cases. We conclude that TC calibration is better for short observation terms, while TWC calibration is better for long observation terms. The relative standard deviations (RSDs) of calibration with $C_{TC}$ and $C_{TWC}$ are less than 10% in all cases, while RSDs of calibration with $\tilde{C}$ are greater than 10% sometimes because it cannot correct the influence of the air temperature. The RSD of the calibration will be larger when the influence of air temperature is stronger. The $C_{TC}$ of 2007 is used in the calibration during the years of 2008 and 2009. At 2800 MHz, $\sigma_{TC} = 6.9$ sfu is about the same as for the years of 2004–2007. In practice, the calibration coefficient $C(\nu)$ should be updated annually.

The daily calibration data of SBRS1 and SBRS3 also have a strong relationship with the local air temperature. The relationship between $C_d$ and the local air temperature is quite complex because SBRS1 and SBRS3 were examined and repaired many times. In short observing terms less than 1 yr, there is no significant difference (<5%) between constant calibration and TC (or TWC) calibration. The constant coefficients are used in calibration of them. At the 1.0–2.0 GHz band, the daily noon flux of SBRS1 at 1420 MHz is compared with the daily noon flux of NGDC data at 1415 MHz. The standard deviations are varied within 9–12 sfu in different observation terms. The same is done at the 1.10–2.06 GHz band. The standard deviations of calibration are varied within 8–12 sfu at 1420 MHz in different observation terms. At the 5.2–7.6 GHz band, there are no observation data from NGDC. The linear interpolated values between 4995 and 7270 MHz of NGDC are used for comparison. The standard deviations of calibration are varied within 5–18 sfu at 5900 MHz in different observation terms. The observation
term with the smallest standard deviation indicates a stable system and few occasional interferences, therefore signifying the best status of the instrument.

2.4. Observation Results

The daily noon flux observed by SBRS is calibrated by Equation (9) with the calibration coefficient $C(\nu)$. At the 2.6–3.8 GHz band, $C(\nu)$ is $C_{TC}$. At other frequency bands, $C(\nu)$ is the constant coefficient. The left panel of Figure 6 plots the daily noon flux at 1415, 2800, and 5900 MHz observed by SBRS during solar cycle 23. In each band of SBRS, we plot the same (or nearby) frequency as nine fixed frequencies of NGDC. The top right panel plots the correlation coefficients between the sunspot numbers and the daily noon flux at various frequencies of NGDC and SBRS. The bottom right panel plots the sunspot numbers (star) and the daily noon flux (cross) at 2800 MHz in 2008 August as an example. The gray cross is excluded because there are sunspots ±3 days.

Figure 5. Each panel plots the daily noon flux of SBRS after $C$ calibration vs. daily noon flux of NGDC. They are plotted as black crosses. The top three panels are the comparison of calibration results with $C$, $C_{TC}$, and $C_{TWC}$ at 2800 MHz. The bottom 11 panels are the yearly calibration results with $C$, $C_{TC}$, and $C_{TWC}$ at 2800 MHz.

Figure 6. Left: daily noon flux at 1415, 2800, and 5900 MHz observed by SBRS in solar cycle 23. The data gap is without daily calibration or during maintenance. Top right panel: correlation coefficients between the sunspot numbers and the daily noon flux at various frequencies of NGDC and SBRS. Bottom right panel: sunspot numbers (star) and the daily noon flux (cross) at 2800 MHz in 2008 August as an example. The gray cross is excluded because there are sunspots ±3 days.
minimum. In order to obtain the pure radio flux of the quiet Sun, we exclude the daily noon flux with sunspots ±3 days (gray cross in bottom right panel of Figure 6). During the solar minimum period of 2006–2009, 317 days are selected as the candidates. The Gaussian fitness of the daily noon flux of these 317 days is decided as radio flux of the quiet Sun statistically. They are 55, 67, and 68 sfu at 1415, 2695, and 2800 MHz of SBRS, respectively. The corresponding radio fluxes of the quiet Sun from NGDC are 11, 27, 35, 55, 67, 69, 118, 220, and 519 sfu at the nine fixed frequencies, respectively.

3. THEORETICAL ANALYSIS OF THE QUIET-SUN RADIO EMISSION

The quiet Sun radio emission is thermal radiation originated from the ambient plasma in absence of solar activity. The mechanism is bremsstrahlung from electrons interacting with ions in the presence of a relatively weak magnetic field (Shibasaki et al. 2011). In an irregular propagation medium, a wave cannot be represented by a single ray. Small fluctuations in density or magnetic field will distort the incident plane wave as the wave phase propagates at different speeds. Benz (1993) pointed out that the evidence of scattering of solar radio emission is ambiguous, while the scattering hypothesis has been successfully verified for pulsars and irregularities of the interstellar medium. Moreover, other papers (Aubier et al. 1971; Thejappa & MacDowall 2008) concluded that (1) the scattering effect decreases the intensity of the radio emission and enlarges the size of the radio source, and (2) the scattering effect increases with the radio wavelength (×λ4) and can be neglected at shorter meter wavelengths. We mainly studied the quiet-Sun radio emission at the frequency range of 245–15,400 MHz in this work; therefore, the scattering effect can be ignored. In numerical analysis of the quiet-Sun radio emission, most previous works are based on the theoretical treatments (Smerd 1950; Dulk 1985; Benz 1993). The quiet-Sun radio emission is calculated by using equations and treatments as follows.

The radiation transfer equation is

\[ \frac{d(1/\mu^2)}{ds} = \frac{\eta}{\mu^2} - \kappa \frac{I}{\mu^2}. \]  

(17)

Here I is the specific intensity of the radiation at frequency of ν, η is the volume emissivity, κ is the absorption coefficient, and \( \mu \) is the refractive index of the medium. The optical depth and the path element have the relationship \( d\tau = \kappa ds \). In this work, the electron temperature \( T_e \) can be treated as uniform in small segments because of the entirely numerical integration. Thus, under conditions of thermodynamic equilibrium in small segments, we have \( \eta = \mu^2\kappa B(T) \). In the radio frequency band, the Rayleigh–Jeans approximation is \( B(T) = \frac{2k_b}{c^2} T^3 \). Hence, the solution of Equation (17) can be obtained:

\[ I = \frac{2k_b}{c^2} \mu^2 T^3 \left( 1 - e^{-\mu}\right) + I_0(\mu/\mu_0) e^{-\mu}. \]  

(18)

Here the intensity \( I_0 \) and refractive index \( \mu_0 \) are at an optical depth of \( \tau_0 \), a0 is Boltzmann’s constant.

The ray treatment of radiation (Jaeger & Westfold 1950) is based on the refraction of the ray path in the solar atmosphere. The equation of the path and the absorption of the rays can be deduced from Snell’s law. Figure 7 shows the ray trajectory in polar coordinates (ρ, θ). Here \( \rho = R/R_\odot \) is the distance from one point of the ray path to the solar center. These rays will emerge from the solar atmosphere parallel to the observer at a distance \( b \) from the center Sun–Earth line. Both \( \rho \) and \( b \) are in units of the Sun’s optical radius \( R_\odot \). (6.95 × 10^10 cm). The refractive index \( \mu \) of an ionized medium decreases with increasing electron density. It follows that a ray passing through the solar atmosphere experiences continuous bending by refraction. The point where the direction of propagation changes from that of decreasing \( \mu \) to that of increasing \( \mu \) is referred to as the “reflection point” or, better, as the turning point. All these rays are calculated with Equation (19) (Jaeger & Westfold 1950) and the solar atmosphere model in Fontenla et al. (1993, 2011). The path element \( ds \) of a trajectory \( b \) is given by

\[ \theta = \int_\rho^\infty \frac{bd\rho}{\rho \sqrt{\mu^2 - b^2}} \]  

(19)

\[ ds = R_\odot \sqrt{(d\rho)^2 + (\rho d\theta)^2} = \frac{R_\odot d\rho}{\sqrt{1 - b^2/\mu^2}}. \]  

(20)

The optical depth can be deduced from the equation of the path (20) and the absorption of the rays. The optical depth between two points \( \rho_1 \) and \( \rho_2 \) is as follows:

\[ \tau_{\rho_1, \rho_2}(b) = \int_{\rho_1}^{\rho_2} \frac{\kappa R_\odot}{\sqrt{1 - b^2/\mu^2}} \]  

(21)

The refractive index is \( \mu^2 \approx 1 - (\frac{\omega}{\omega_c})^2 \cdot (1 \pm \frac{\omega_c}{\omega} |\cos \theta|)^{-1} \approx 1 - (\frac{\omega_c}{\omega})^2 \) because the ray frequency is much greater than the local gyrofrequency \( \omega >> \omega_c \) along its propagation in the solar atmosphere of the quiet Sun (Benz 1993). The formulae of absorption coefficient \( \kappa \) differ in different papers. Some papers (Smerd 1950; Bracewell & Preston 1956; Thejappa & MacDowall 2010) used \( \kappa \) deduced by classical collision theory. The classical collision theory defined the absorption coefficient \( \kappa = \nu_e \kappa_{\text{elas}} \). Here \( \nu_e \) is the collision frequency of thermal electrons/ions. Some papers (Dulk 1985; Gary et al. 1990) used \( \kappa \) with the Gaunt factor deduced by quantum theory. The formula of \( \kappa \) in Dulk (1985) is the same as the approximate analytic formula of Novikov & Thorne (1973, p. 343N; Rybicki 1986) at radio wavelength (\( \nu < n_b T \)). We compare the Gaunt factor value of Dulk (1985) with that of van Hoof et al. (2014) at the solar atmosphere from 100 MHz to 30 GHz and find that the difference is of <2%, which has a very small impact on the numerical result. Thus in this work we still use the value calculated by the equation of Dulk (1985). Table 2 lists the equations and parameters from different papers. They are all approximately in the range of 0.15 0.25 μ^2/λ^2 under the condition of the solar atmosphere since the classical collision theory is valid and close to the quantum theory when \( \nu_e < n_b T \).

From Equation (21) and \( \kappa \) in Table 2, we have

\[ \tau_{\rho_1, \rho_2}(b) = \frac{9.78 \times 10^{-3} R_\odot}{\nu^2} \pi \int_{\rho_1}^{\rho_2} \frac{\mathcal{G}_{\text{dy}}(\nu, T) N^2}{T^{3/2} \sqrt{1 - b^2/\mu^2}} d\rho \]  

(22)
Figure 7. Ray trajectory of 245 MHz in polar coordinates. The bending curves of various colors indicate the ray trajectory of various distances $b$ from the center Sun–Earth line. The normal line of the refraction at the turning point is plotted as a dashed line. The arrow indicates the propagation direction of the ray to the observer.

Table 2  
Parameters of Collision Frequency $\nu_{e,i}$, Average Gain Factor $\bar{g}_b(\nu, T)$, and Absorption Coefficient $\kappa$  

| Paper                        | $\nu_{e,i}$ (cgs) | $\bar{g}_b(\nu, T)$ | $\kappa$ (cgs) |
|------------------------------|-------------------|----------------------|----------------|
| Smerd (1950)                 | $\frac{1.36N}{r^2}$ ln(1 + $\frac{4\nu T}{c^2}$) | ...                 | $3.65\times10^{-9}N$ ln(1 + $\frac{4\alpha T}{c^2N}$) |
| Bracewell & Preston (1956)   | $\frac{1.36N}{r^2}$ ln(1 + $\frac{4\nu T}{c^2}$) | ...                 | $0.2N^{0.5}$ |
| Dulk (1985)                  | $\frac{1.36N}{r^2}$ [18.2 + ln($\nu^2$)] | $9.78\times10^{-9}N^2$ [18.2 + ln($\nu^2$)] $T < 2 \times 10^4$ | $2.34\times10^{-9}N^2$ [24.5 + ln($\nu^2$)] $T > 2 \times 10^4$ |
| Dulk (1985)                  | $\frac{1.36N}{r^2}$ [24.5 + ln($\nu^2$)] | $9.78\times10^{-9}N^2$ [24.5 + ln($\nu^2$)] $T > 2 \times 10^4$ | $2.34\times10^{-9}N^2$ [10.81 + ln($\nu^2$)] (corona) |
| Thejappa & MacDowall (2010)  | $\frac{3.6N}{r^2}$ [10.81 + ln($\nu^2$)] | ...                 | ... |

Parameters of Collision Frequency $\nu_{e,i}$, Average Gain Factor $\bar{g}_b(\nu, T)$, and Absorption Coefficient $\kappa$.

$N$ and $T$ are the electron density and electron temperature in the solar atmosphere, respectively. They will be discussed in the next subsection. Equation (22) has a singularity point when $\mu = b/r$, $\rho = \rho_0$. This point was named the turning point or “reflection point” (Smerd 1950; Bracewell & Preston 1956), where the direction of propagation changes from that of decreasing $\mu$ to that of increasing $\mu$. The Appendix proves that the integration is convergent near the singularity point so long as the electron density $N$ increased limitedly and monotonously along the height $\rho$. Therefore, when $\rho > \rho_0$, the integration can be calculated with numerical integration.

3.1. The Electron Density and Temperature

There are many models of the electron density and electron temperature of the quiet-Sun atmosphere. The VAL III model (Vernazza et al. 1981) determined semi-empirical models for six components of the quiet solar chromosphere using EUV observations. The series of VAL models (Fontenla et al. 1993, 2009, 2011) built the semi-empirical models with the optical continuum and EUV/FUV observation. These two models are excellent for reconstructing the optical and ultraviolet observations but still have some discrepancies when describing the radio observations. The discrepancies will be illustrated in the next subsection. Selhorst et al. (2005) proposed a hybrid model that uses the FAL C model (Fontenla et al. 1993) from the photosphere to 1800 km, Zirin et al. (1991) model from 1800 to 3500 km in the chromosphere, and Gabriel & Sun (1992) model from 3500 to 40,000 km in the transition region and corona. These three models are selected in this work because the numerical results are not far apart from the observations. Figure 8 shows the electron density and electron temperature distribution of different models along the height above the photosphere. The Fontenla et al. (2011) model is plotted as a dashed line. It gave parameters from the chromosphere to the high corona until the height of $\sim 2 \times 10^5$ km. In the higher corona, the electron density is given by the Allen (1947) model as follows:

$$N = 10^8\left(1.55\rho^{-6} + 2.99\rho^{-16}\right) \text{ cm}^{-3}. \quad (23)$$

The Allen model (long-dashed line in Figure 8) will match the value of the Fontenla model at the height of $\sim 2 \times 10^8$ km. The Vernazza model is plotted as a cross in Figure 8. It gave parameters in the chromosphere and transit region until the height of 2439 km. We decide to use the same parameters as the Fontenla model in the corona and the Allen model in the higher corona as before. They are plotted as gray crosses in the figure. The Selhorst model is plotted as a solid line in Figure 8. It gives parameters from the chromosphere to the corona until the height of $3.93 \times 10^5$ km. We decide to use the Allen model after modification (gray long-dashed line in Figure 8) in the higher corona. Then three hybrid models that we used in this work are the FAL+Allen (F+A) model, VAL+FAL+Allen (V+F+A) model, and Selhorst+Allen (S+A) model. For all the models, the electron temperature in the higher corona is equal to the last value given by the corresponding model. We will compare the numerical results of three hybrid models with the observations in Section 3.3.

3.2. The Optical Depth and Brightness Temperature across the Solar Disk

We do numerical integration entirely in this paper. In Equation (22), the optical depth $\tau_{n,n+1}(b)$ is calculated for
The Astrophysical Journal, 808:61 (14pp), 2015 July 20

**Figure 8.** Electron density (top panel) and electron temperature (bottom panel) distribution of different models along the height above the photosphere. The classical model of Allen (1947) is plotted as a long-dashed line only at a height of higher than $10^4$ km.

**Figure 9.** Left: absorption $\kappa_f$ (long dashed line) and optical depth $\tau_{f,\infty}$ (solid line) along the solar height, for each of the central lines at eight fixed frequencies. Right: brightness temperature across the solar disk at eight fixed frequencies. Black lines are the numerical result with the Selhorst et al. (2005) + Allen (1947) model. Dashed lines are the results of the approximation solution in Smerd (1950).

about 500 points from the turning point (inner limit) to the point after which the contribution is very small (outer limit). The Appendix proves that the integration of Equation (22) near the turning point is convergent. Beyond the outer limit, the absorption is very small ($<10^{-10}$ km$^{-1}$ in this work); thus, the optical depth can be ignored. The 500 points are decided as follows: (1) the zero point is the turning point $\rho_0$; (2) assume the first point $\rho_1 = \rho_0 + \Delta \rho_1$, $\Delta \rho_1 \leq 1R_{\odot}$, and calculate the optical depth $\tau_{f,0,1}$ between the two points; (3) set the midpoint between point 0 and point 1, $\rho_m = \rho_0 + (\rho_1 - \rho_0)/2$, and calculate the optical depth of $\tau_{f,0,m}$ and $\tau_{f,m,1}$, respectively; (4) if $\frac{|\tau_{f,0,m} + \tau_{f,m,1} - \tau_{f,0,1}|}{\tau_{f,0,1}} > 10^{-2}$, change the midpoint to point 1 and set a new midpoint. Do this circulotary calculation until $|\tau_{f,0,m} + \tau_{f,m,1} - \tau_{f,0,1}| \leq 10^{-2}$. The calculation error is very small. Then the first point $\rho_1 = \rho_0$ is decided. Usually, the first point is decided as $\Delta \rho_1 = 10^{-12} \sim 10^{-10} R_{\odot}$ by experience. (5) Decide the rest points with this method. It should be taken care that the electron density and electron temperature vary abruptly in the transition region. The step should be $10^{-12} R_{\odot} \leq \Delta \rho_n \leq 1.5 \times \Delta \rho_{n-1}$ to avoid the big error during calculation. For the points beyond the outer limit, the optical depths are approximated as zero. The left panel of Figure 9 plots the absorption $\kappa_f$ and optical depth $\tau_{f,\infty}$ of the central line calculated with the Selhorst+Allen model at eight fixed frequencies as an example. The right panel of Figure 9 shows the brightness temperature (or radiation intensity), which is calculated with Equations (18) and (22) for different $b$ at eight fixed frequencies. Black lines are the numerical result of the Selhorst+Allen model in this work. Dashed lines are the results of Smerd (1950) with the approximation solution. It shows differences between two results.

Figure 9 illustrates the main generating region of the quiet-Sun radio emission at varied frequencies. The result is consistent in the main but a little different from the classical results (Smerd 1950; Dela Luz et al. 2010). At high frequency (3–30 GHz), the absorption ($<10^{-5}$ km$^{-1}$) and optical depth ($<0.01$) are very small in the corona. The emissivity $\eta = \mu^2kB(T)$ is also very small in the corona. Thus, the thermal bremsstrahlung emission of the quiet Sun at high frequency (3 ~ 30 GHz) is mainly produced in the chromosphere. At low frequency (~300 MHz), the radio emission cannot propagate into the chromosphere. The thermal bremsstrahlung emission of the quiet Sun at low frequency (~300 MHz) is mainly produced in the corona.
bremstrahlung emission of the quiet Sun at intermediate frequency ($300 \text{ MHz} < \nu < 3 \text{ GHz}$) is produced in both the chromosphere and the corona. The limb brightening appears obviously at a frequency range of $200 \text{ MHz} < \nu \lesssim 30 \text{ GHz}$. The brightness temperature of the solar center radio emission is consistent with the electron temperature of the main generating region.

### 3.3. Brightness Temperature and Flux Density of the Quiet Sun

We calculate the brightness temperature spectrum and flux spectrum of the quiet-Sun radio emission for three hybrid models (F+A, V+F+A, S+A) at various frequencies. The total amount of radiation per unit time, unit frequency interval, and unit angle from the Sun to a distant observer is given by

$$E = 2\pi R_0^2 \int_0^\infty I(d) \, dd, \quad (24)$$

in units of erg cm$^{-2}$Hz$^{-1}$s$^{-1}$. In practice, the upper limit of this integral is replaced by a finite value $d_m$ out of which the radiation contribution is very small. The flux density of the quiet Sun can be transformed with the conversion 1 sfu = $10^{-22}$ Wm$^2$ Hz$^{-1}$ = $10^{-19}$ erg cm$^{-2}$ Hz$^{-1}$ s$^{-1}$. Then we compare the numerical results with observations and find the difference. The left panel of Figure 10 shows the comparison of brightness temperature spectrum between the observations and numerical results of different models. We choose the observations of Benz (2009) and Zirin et al. (1991) as the standard spectrum because the flux spectrum of Benz (2009) is well consistent with the observations of SBRBS/HSO and NGDC (middle panel of Figure 10). At frequency of 15400 MHz or nearby, the numerical results of all models and all the observations (Fuerst 1980; Zirin et al. 1991; Benz 2009) are consistent. This indicates that the parameters in the low chromosphere of all the models fit the observation well. At a frequency of 3–10 GHz, the numerical results of the F+A model (mainly Fontenla model) and V+F+A model (mainly Vernazza model) are close to the observations of Fuerst (1980), while the numerical results of the Selhorst+Allen model are close to the observations of Zirin et al. (1991) and Benz (2009). The radio emission at the frequency of 300 MHz–3 GHz comes from the transition region and corona. The numerical results of the Selhorst+Allen model (red solid line) are close to the observations of Benz (cyan solid line). Thus, we think that from the transition region to the low corona the parameters of the Selhorst model fit the observations well. It is complex to estimate whether the parameters in the higher corona are good or not owing to a lack of observations at low frequency (<300 MHz).

The middle panel of Figure 10 plots the flux spectrum of the quiet-Sun radio emission. The quiet-Sun fluxes of SBRBS observations (Gaussian fitness value) and the error bars ($3\sigma$) are plotted as blue crosses and short vertical lines. The quiet-Sun fluxes of NGDC (Gaussian fitness value) are plotted as black stars. The NGDC did not present the error of solar flux. Here we use three times the standard deviation ($3\sigma$) of the quiet-Sun data from NGDC as error bars (black short vertical line). The standard deviation is calculated separately at the low-value part and high-value part. We find that both the quiet-Sun flux spectra of SBRBS and NGDC fit the observation of Benz (cyan solid line) well. Thus, our work agrees with the brightness temperature spectrum and flux spectrum of Benz (2009) as the standard spectrum of the quiet-Sun radio emission. All the values of brightness temperature and flux density of the quiet Sun at various frequencies are listed in Table 3. The right panel of Figure 10 shows the observation flux and error bars of TC (solid blue line) and no TC (dashed blue line) in the range of 2500–4000 MHz as an example. It indicates that calibration of TC is more accurate than that of no TC. The red lines are the same numerical results of solar models as in the middle panel of Figure 10. The cyan line is the observation of Benz. As analyzed in Section 2.3, the difference between TC and TWC calibration is very small ($\sim 2\%$). The observation flux and error bars of TWC are not plotted together.

### 4. Conclusion and Discussion

SBRBS has been observing the Sun and obtaining plentiful data since 1994. This work adopted the observations of SBRBS to investigate the calibration procedure and study the quiet-Sun
radio emission. The calibration gives accurate observations, which is basic and important in the study of the quiet-Sun radio emission, while the study of the quiet-Sun radio emission is a scientific extension of the former part. We first study various impacts to the calibration results and conclude the following.

1. Generally, the calibration coefficient is constant and should be upgraded annually. Actually, the calibration result with constant coefficient is found to be related to the local air temperature at all frequency bands of SBRS. One possible reason is that the electronic apparatuses of the instruments are influenced by the local air temperature. Thus, the correlation between calibration and local air temperature will vary if there is an adjustment to the instrument.

2. The relationship between calibration results (panel c) of Figure 2 and the humidity (panel a) of Figure 2 or other weather conditions is not clear. There is no distinct evidence that the calibration was influenced by normal weather conditions except for air temperature.

3. The Sun elevation has a small effect on the calibration of this work. The absorption of the atmosphere should be considered only when the frequency is higher than 17 GHz (Tsuchiya & Nagame 1965) or the Sun elevation is lower than 10°.

The calibration accuracy is also influenced by the occasional abnormal signal. Some possible reasons for the abnormal signal are listed as follows.

1. Various kinds of interference: wireless communication, plane and airport, satellite, vehicle, lightning, etc. These will result in large values.
2. Unstability of the feed, cable, or noise source.
3. Sometimes, the gain factor may be out of normal range when there is a strong signal (Yan et al. 2002). The beam of the antenna will offset the Sun center if the tracking control is out of normal range. These will result in large or saturated values, or small values. The relationship between the abnormal signal and bad weather conditions is still not clear. The calibration with Equation (9) will eliminate the frequency property of gain factor $G_s(\nu)$ and flatten the frequency property of the antenna system with $C(\nu)$ of each frequency. The calibration with Equation (9) does not need the values of $A_x(\nu)$ and $T_x(\nu)$.

In order to improve the calibration accuracy, we investigate the influence and properties of the instrument from the data analysis and comparison between different calibration coefficients. All the investigations are under the fundamental principles of calibration. First, we exclude the abnormal data that are not well observed or are influenced by the interference. Then we compare the calibration results of four sets of calibration coefficients, including average value ($\bar{C}$), the Gaussian fitness value ($\bar{C}$), constant coefficients after TC ($C_{TC}$), and constant coefficients after TWC ($C_{TWC}$). The main analysis results are as follows: (1) In the 2.6–3.8 GHz band, the calibration errors $\sigma_{G}$ are smaller than $\sigma_{G}$ at 82% of frequencies. Comparing with other constant coefficients of calibration, $\bar{C}$ is the best. (2) At 2800 MHz, $\sigma_{TC}$ or $\sigma_{TWC}$ are about 10%–20% smaller than $\sigma_{G}$ at various observation terms. The RSDs of $\sigma_{TC}$ and $\sigma_{TWC}$ are less than 10%, while RSDs of $\sigma_{G}$ are greater than 10% sometimes because of the influence of the temperature. The $C_{TC}$ is used in the calibration in the years of 2008 and 2009. The calibration error $\sigma_{TC}$ = 6.9 sfu is about the same as that in the years of 2004–2007. (3) The calibrations of SBRS1 and SBRS3 also have a strong relationship with the local air temperature. But there is no significant difference between TC (or TWC) calibration and constant calibration of SBRS1 and SBRS3, which were examined and repaired many times. $\bar{C}$ is used in the calibration for several short observation terms. The standard deviations of calibration are varied within 8–12 sfu at 1415 MHz and within 5–18 sfu at 5900 MHz for different observation terms, respectively. The observation term with the smallest standard deviation indicates a stable system and few occasional interferences, therefore signifying the best status of the instrument.

The daily noon fluxes of SBRS are calibrated with improved $C(\nu)$. We select the daily noon flux without sunspots $\pm 3$ days as the flux of the quiet Sun during the solar minimum period of 2006–2009. The spectrum of daily noon flux observed by SBRS approaches the spectrum from NGDC well. The daily noon flux is correlated with sunspot numbers. The result is consistent with previous works (Christiansen & Hindman 1951).

Based on the above investigations, we further study the radio emission from the quiet Sun. The flux spectrum is strictly the observation without sunspots during the solar minimum period.

### Table 3

| $\nu$ (MHz) | $T_s$ (K) | $L_s$ (K) | $T_x$ (K) | $T_x$ (K) | $F_x$ | $F_x$ | $F_x$ | $F_x$ | $F_x$ | $F_x$ |
|-------------|-----------|-----------|-----------|-----------|------|------|------|------|------|------|
| F + A | V + F + A | S + A | Benz | F + A | V + F + A | S + A | Benz | F & | F & | F & |
| 245 | $1.00 \times 10^6$ | $1.00 \times 10^6$ | $9.69 \times 10^5$ | $7.88 \times 10^5$ | 17.3 | 17.3 | 15.7 | 11.2 | 11 | ... |
| 410 | $7.93 \times 10^4$ | $7.89 \times 10^5$ | $5.90 \times 10^5$ | $5.50 \times 10^5$ | 41.4 | 21.3 | 30.3 | 22.2 | 27 | ... |
| 610 | $5.56 \times 10^4$ | $5.27 \times 10^5$ | $2.84 \times 10^5$ | $3.58 \times 10^5$ | 69.8 | 68.9 | 41.9 | 32.3 | 35 | ... |
| 1415 | $1.68 \times 10^3$ | $1.48 \times 10^4$ | $7.43 \times 10^4$ | $9.70 \times 10^4$ | 139 | 127 | 60.8 | 46.9 | 55 | 55 |
| 2695 | $6.21 \times 10^4$ | $5.85 \times 10^4$ | $2.93 \times 10^4$ | $4.19 \times 10^4$ | 185 | 167 | 77.9 | 64.7 | 67 | 67 |
| 2800 | $5.76 \times 10^4$ | $5.49 \times 10^4$ | $2.76 \times 10^4$ | $3.68 \times 10^4$ | 189 | 172 | 80.3 | 66.8 | 69 | 68 |
| 4995 | $2.74 \times 10^4$ | $2.35 \times 10^4$ | $1.62 \times 10^4$ | $1.88 \times 10^4$ | 246 | 251 | 121 | 107 | 118 | ... |
| 8800 | $1.49 \times 10^4$ | $2.10 \times 10^4$ | $1.22 \times 10^4$ | $1.38 \times 10^4$ | 359 | 450 | 233 | 235 | 220 | ... |
| 15400 | $9.46 \times 10^4$ | $1.29 \times 10^4$ | $1.04 \times 10^4$ | $1.16 \times 10^4$ | 612 | 832 | 562 | 599 | 519 | ... |

**Note.** The first column is frequency. The next three columns are numerical results of center-line brightness temperature with different models (VAL+FA; FAL+Allen; Selhorst+Allen). The fifth column is center-line brightness temperature in Benz (2009). From sixth to eighth columns are numerical results of flux density with different models. The last three columns are observation results of flux density of the quiet Sun (Benz 2009, NGDC online data, and SBRS).
of 2006–2009. The numerical simulation of the quiet-Sun radio emission in this work is improved with several semi-empirical solar models and entirely numerical integration instead of an approximate solution. The theoretical treatment has been testified well because it can deduce the same result as that of many papers (Jaeger & Westfold 1950; Thejappa & Kundu 1992; Thejappa & MacDowall 2010). Different works have different treatments, such as (1) different approximations of radiation transformation, (2) different absorption coefficients \( \kappa \), and (3) consideration of refractive index, scattering effect, or magnetic field. This work uses the radiation transfer equation and bremsstrahlung mechanism of emission without approximation and without scattering effect and magnetic field. The ray treatment of radiation is based on the refraction of the ray path in the solar atmosphere (Jaeger & Westfold 1950). The refractive index \( \mu \) and absorption coefficient \( \kappa \) in Dulk (1985) are considered since the Gaunt factor in Dulk (1985) is very close to the numerical value of van Hoof et al. (2014).

The VAL (Vernazza et al. 1981) and FAL (Fontenla et al. 2011) models are excellent for reconstructing the optical and ultraviolet observations but still have some discrepancies in describing the radio observations. Zhang et al. (2001) discuss the discrepancy and attribute the difference to an underestimation of Fe abundance used in the calculation of the UV line emission. Thus, Selhorst et al. (2005) proposed a hybrid model, of which the transition region is about 1000 km higher than in typical VAL and FAL models. We regulate solar models with a combination of different models and compare the numerical results with the observations. The numerical results of the models should be within the confines (error bars) of observations. Finally, we find an appropriate hybrid model that uses the Selhorst model from the chromosphere to the height of 3.93 \times 10^4 \text{ km} above the photosphere and the Allen model in the higher corona. The comparison (Figure 10) between the numerical results and the observations indicates the following: (1) The numerical results of all models and all observations are consistent at 15,400 MHz or nearby. The solar models in the low chromosphere are unanimous. The optical depth at 15,400 MHz is very small in the transition region or higher no matter what model is selected. (2) The argument happened at the transition region or nearby, which is the main emission region at 300 MHz–10 GHz. Only the flux spectrum of the Selhorst model is within the error bars of the observation. The theoretical results of the VAL and FAL models are about two times larger than observations. At the frequency of 300 MHz–10 GHz the optical depth varied mainly within 0.01 < \( \tau \) < 10 in the transition region or higher. It is difficult to modulate the solar models only with radio observations because each modulation in this region will impact the results of the whole band of 300 MHz–10 GHz. But the radio observations will help us to validate the models we know. (3) In the corona, the numerical results of the Allen model are close to the observations at the frequency of 100–300 MHz. There is a small argument on the solar models in the corona. But in this work we cannot assure exactly the parameters in the corona as a result of few observations and the scattering effect in low frequency (<300 MHz). (4) The observation with higher accurate calibration will help us further qualify the empirical/semi-empirical solar models. The solar model can be decided when the theoretical result is within the error.

The brightness temperature distribution across the solar disk in this work (right panel of Figure 9) shows that the limb brightening happens in the frequency range of >200 MHz–30 GHz. Kundu et al. (1977) showed that no limb brightening happens at the frequency of lower than \(~121\text{ MHz}\). Mercier & Chambe (2009, 2012) studied the radio images of the quiet Sun at 150–450 MHz but did not discuss limb brightening. However, their papers give us some clue that the limb brightening increased with the frequency in the range of 300–450 MHz. Some works also found limb brightening at a high frequency of 5 GHz (Kundu et al. 1979) and near the solar polar at 17 GHz (Selhorst et al. 2010). The result of higher frequency range (>30 GHz) was discussed in Dela Luz et al. (2011). Please note that the limb brightening of the numerical results is higher than in observations. This may be attributed to the spicules, which will decrease the limb brightening (Elzner 1976). Another reason is perhaps no consideration of fluctuation, scattering effect, and magnetic field. We need more work and more observations to understand this problem. The new construction of the Chinese solar radio heliograph (Yan et al. 2009) will provide more observations on the 2D image of the Sun at a wide frequency range of 400 MHz–15 GHz. In a word, this work gives an appropriate numerical method on the study of the quiet-Sun radio emission.

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**APPENDIX**

Here we discuss the convergence of the integration in Equation (22). At the turning point \( \rho_0 \), \( \mu_0 = d/\rho_0 \), \( \mu_0 \) is the refractive index at \( \rho = \rho_0 \). For \( \rho = \rho_0 + d\rho \), when \( d\rho \to 0 \), it can be proved that \( \frac{N^2 d\rho}{\sqrt{\mu^2 - b^2/\rho^2}} \to 0 \). The refractive index

\[ \mu^2 = 1 - \frac{e^2 N^2}{\pi m_f^2} \] (Benz 1993), where \( N = N_0 + dN \text{ cm}^{-3} \), and \( N_0 \) is the electron density at \( \rho = \rho_0 \). So,

\[ \mu^2 = 1 - \frac{e^2 (N_0 + dN)}{\pi m_f^2}. \]

Let the constant \( C_1 = \frac{e^2}{\pi m_f^2} \). Thus,

\[ \frac{N^2 d\rho}{\sqrt{\mu^2 - b^2/\rho^2}} = \frac{N^2 d\rho}{\sqrt{1 - C_1 (N_0 + dN) - b^2/\rho^2}}. \tag{25} \]

Assuming \( dN > 0 \) when \( d\rho > 0 \), we have \( \frac{1}{\rho_0 + d\rho} < \frac{1}{\rho_0} \)
and \( N_0 + dN < N_0 \). Thus, Equation (25) can be transformed as

\[ \frac{N^2 d\rho}{\sqrt{1 - C_1 (N_0 + dN) - b^2/\rho^2}} < \frac{N^2 d\rho}{\sqrt{1 - C_1 N_0 - b^2/\rho^2}}. \]

We put \( 1 - C_1 N_0 - b^2 \rho_0^2 = 0 \) and \( N^2 d\rho = \frac{1}{\rho^2} \), and

\[ \frac{1}{\rho^2} - \frac{2 \rho_0 d\rho + (d\rho)^2}{(\rho_0)^2 (\rho_0 + d\rho)^2} \]
into the right-hand side of
Equation (26). Then

$$\frac{N^2 d\rho}{\sqrt{1 - C_i N_0 - b^2 / \rho^2}} = \frac{N^2 d\rho}{\sqrt{d^2 \frac{2\rho_0 d\rho + (d\rho)^2}{(\rho_0)^2 (\rho_0 + d\rho)^2}}}$$

$$= \frac{\rho_0 (\rho_0 + d\rho) N^2 \sqrt{d\rho}}{d^2 (\rho_0 + d\rho)}.$$

(27)

When d\rho \to 0, \rho_0 + d\rho \to \rho_0, and N \to N_0. So

$$\frac{N^2 d\rho}{\sqrt{\mu^2 - b^2 / \rho^2}} < \frac{\rho_0 (\rho_0 + d\rho) N^2 \sqrt{d\rho}}{d^2 (\rho_0 + d\rho)}$$

$$< \frac{\rho_0 (2\rho_0 + d\rho) N^2 \sqrt{d\rho}}{d^2 (2\rho_0 + d\rho)}$$

$$< \frac{\rho_0 (N_0)^2 \sqrt{2\rho_0 d\rho}}{d} \to 0.$$

(28)

That is, the integration of Equation (22) near the turning point (singularity point) is convergent so long as the electron density N increased limitedly and monotonously along the height.

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