Dark matter profiles of SPARC galaxies: a challenge to fuzzy dark matter

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Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT

Stellar and gas kinematics of galaxies are a sensitive probe of the dark matter distribution in the halo. The popular fuzzy dark matter models predict the peculiar shape of density distribution in galaxies: specific dense core with sharp transition to the halo. Moreover, fuzzy dark matter predicts scaling relations between the dark matter particle mass and density parameters. In this work, we use a Bayesian framework and several dark matter halo models to analyse the stellar kinematics of galaxies using the Spitzer Photometry & Accurate Rotation Curves database. We then employ a Bayesian model comparison to select the best halo density model. We find that more than half of the galaxies prefer the fuzzy dark model against standard dark matter profiles (NFW, Burkert, and cored NFW). While this seems like a success for fuzzy dark matter, we also find that there is no single value for the particle mass that provides a good fit for all galaxies.

Key words: cosmology: dark matter – galaxies: kinematics and dynamics – galaxies: haloes

1 INTRODUCTION

Dark matter (DM) was discovered more than 80 years ago, but its nature has remained a mystery. It is usually assumed to be composed of unknown particles which are not contained in the Standard Model of particle physics. However, despite considerable efforts, the origin and the parameters of those hypothetical particles are still unknown. The possible mass of proposed dark matter particles depends strongly on the specific model and the range spans dozens of orders of magnitude.

The most widely used model of dark matter is cold dark matter (CDM). It assumes a non-relativistic in the Early Universe, collisionless dark matter particles. This model has unquestionably been successful. Some of its achievements are predicting properties of the large-scale structure and some aspects of CMB anisotropies. CDM is typically assumed to be a type of weakly interacting massive particle (WIMP) with a mass of the order of GeV or larger.

The focus of this paper is an alternative to cold dark matter, commonly called fuzzy dark matter (FDM). These are axion-like particles which can appear for example in string theory (see, e.g., Arvanitaki et al. 2010).

Fuzzy dark matter is assumed to be composed of bosonic particles with masses $\sim 10^{-24}$–$10^{-19}$ eV and is the lightest dark matter candidate (see, e.g., Hu et al. 2000; Ferreira 2021; Hui 2021). FDM particles with masses $\lesssim 10^{-24}$ are ruled out by CMB observations (Hlozek et al. 2015), while the small-scale structures abundance in FDM with mass $\geq 10^{-19}$ eV are indistinguishable from CDM (Hu et al. 2000). The observed abundance of structures on small spatial scales is another way to constrain the FDM parameters. This approach uses the Lyman-$\alpha$ forest (Iršič et al. 2017; Nori et al. 2019; Rogers & Peiris 2021), the high-redshift galaxy luminosity functions (Bozek et al. 2015; Schive et al. 2016; Corasaniti et al. 2017; Ni et al. 2019; Schutz 2020), and the Milky Way subhalos (Nadler et al. 2021; Banik et al. 2021).

FDM possesses all the virtues of cold dark matter on cosmological scales but may have advantages on galactic scales. This is because a system composed of ultra-light bosonic particles has a non-negligible quantum pressure arising from the uncertainty principle. This keeps the central density of halos finite and avoids the core-cusp problem. It also decreases the amount of low-mass structures and thereby suppresses the power spectrum on small scales (see, e.g., Hu et al. 2000), solving the missing satellite problem. (see, e.g., extensive reviews Weinberg et al. 2015; Del Popolo & Le Delliou 2017; Bullock & Boylan-Kolchin 2017).

A prominent feature of a FDM halo is its peculiar density profile: a dense central soliton core with a sharp transition to the less dense outer region with granular structure (see, e.g., Schive et al. 2014a,b). This shape significantly differs from other cored density profiles, e.g., CDM halos modified by the baryonic feedback (see, e.g., Read et al. 2016a,b), profiles of fermionic DM (see, e.g., Shao et al. 2013; Macciò et al. 2013; Savchenko & Rudakovskiy 2019) or density profiles of self-interacting DM (see, e.g., Tulin & Yu 2018). Moreover, the model FDM implies a scaling relation which links the central density, the characteristic radius of the central soliton, and the FDM particle mass. Simulations also predict a relation between the virial halo mass and the soliton mass (see, e.g., Schive et al. 2014b; Hui et al. 2017). The predicted granular structure of the halo also allows one to constrain the ultra-light dark matter parameters (Amorisco & Loeb 2018; Dalal et al. 2021). All
this makes the observed kinematics of galaxies an appealing probe for the FDM model.

In this paper, we analyse the rotation curves of the SPARC galaxies (Lelli et al. 2016) within a Bayesian inference framework (see a review of Bayesian methods in cosmology in Trotta 2017). With the SPARC database we can test the predictions of various DM models for galaxies in a wide range of masses, luminosities, and morphological types. We use the Bayesian evidence to compare the FDM halo model with the Navarro–Frenk–White (NFW), cored NFW (coreNFW), and Burkert density profiles. This statistical approach naturally includes a penalty for models with additional parameters and allows one to reject models that are unnecessarily complex. We also obtain the credible intervals of the fuzzy dark matter mass for different galaxies in the SPARC sample and compare them with each other.

This paper is organised as follows: in Sec. 2 we describe the dark matter profiles under consideration, the rotation curve model and the data analysis framework. In Sec. 3 we report our findings and compare them to previously published results in Sec. 4. In Sec. 5 we discuss our results, and we conclude in Sec. 6. Throughout this paper we use the Hubble parameter $H_0 = 73 \text{km/s Mpc}$.  

2 METHODOLOGY

2.1 Dark matter halo models

NFW density profile

CDM halos are well described by the Navarro–Frenk–White profile inferred from the DM-only N-body simulations (Navarro et al. 1996, 1997):

$$\rho_{\text{NFW}}(r) = \frac{\rho_n}{\left(1 + \frac{r}{r_n}\right)^2}, \quad (1)$$

where $r_n$ is the half-mass radius, and $\rho_n = 4\rho_{\text{NFW}}(r_n)$ is a characteristic density.

coreNFW density profile

Stellar feedback can affect the inner slope of the DM density profile in galaxies. In this way, a cusped halo may be smoothed to one with a core in the inner region. Realistic high-resolution hydrodynamic simulations with baryonic feedback (Read et al. 2016a,b) show that the DM mass distribution in dwarf galaxies can be described by the following form:

$$M_{\text{coreNFW}}(r) = M_{\text{NFW}}(r) \left(\frac{r}{r_c}\right)^n, \quad (2)$$

in which the mass $M_{\text{NFW}}(r)$ is calculated according to the initial NFW density profile. The effect of stellar feedback is fully incorporated here through the last factor, in which the parameter $n$ ranges from 0 (corresponding to the NFW profile) to 1 (describing a completely cored profile). The parameter $r_c$ in this expression represents the characteristic radius of the stellar component, which is proportional to the stellar half-mass radius. Both $n$ and $r_c$ are considered as free parameters in our analysis.

Burkert density profile

We also consider the popular empirical Burkert profile Burkert (1995):

$$\rho_{\text{Burk}}(r) = \frac{\rho_b}{\left(1 + \frac{r}{r_b}\right)^2}, \quad (3)$$

where $r_b$ is the radius that contains 1/4 of the mass and $\rho_b = \rho_{\text{Burk}}(0)$ is the density at the halo centre.

Fuzzy dark matter density profile

A FDM halo is composed of a dense central soliton surrounded by an envelope of incoherent phase with granular structure, according to the hydrodynamic dark matter only simulations (Schive et al. 2014a,b; Veltmaat & Niemeyer 2016) and more sophisticated simulations such as finite-difference solving of the Schrödinger–Poisson system of equations for a self-gravitating free scalar field (Mina et al. 2020; Schwabe & Niemeyer 2021).

The density distribution in the central soliton is described by the ‘boson-star’ ground state solution of the Schrödinger–Poisson system of equations Schive et al. (2014a,b):

$$\rho_{\text{sol}}(r) = \frac{\rho_s}{\left(1 + 9.1 \cdot 10^{-2} \left(\frac{r}{r_s}\right)\right)^8}, \quad (4)$$

where $\rho_s$ is the density at the halo center, and $r_s$ is the characteristic scaling radius of the soliton, which contains approximately a quarter of the total soliton mass $M(r \leq r_s) = \frac{1}{3} M_{\text{sol}}$.

The FDM model predicts a relation between $r_s$ and $\rho_s$ which arises from the exact internal scaling symmetry of the Schrödinger–Poisson system (see, e.g. Schive et al. 2014a; Mina et al. 2020). It links the central density of the soliton to the characteristic radius and FDM particle mass $m = m_{22} \cdot 10^{-22} \text{eV}$ as follows:

$$\rho_s \approx \frac{1.9 \cdot 10^{-2} \cdot M_{\odot}}{m_{22}^2 \left(\frac{r_s}{\text{pc}}\right)^4}, \quad (5)$$

Note that according to our definition $m_{22}$ is dimensionless.

In the outer halo region, the structure is granular, and the density profile of the outer part is well described by the NFW profile on average. Therefore, we will here use the following form of FDM density profile:

$$\rho_{\text{FDM}}(r) = \begin{cases} \rho_{\text{sol}}(r), & r < r_a, \\ \rho_{\text{NFW}}(r), & r > r_a, \end{cases}, \quad (6)$$

where $r_a$ is the transition radius at which the density profile changes its behaviour. It was suggested in (Schive et al. 2014a; Mina et al. 2020) that $r_a = a r_s$ with $a \approx 3$. However, this relation is not exact and may vary from halo to halo.

We also consider the following relation between the soliton and halo masses (Schive et al. 2014b; Hui et al. 2017):

$$M_{\text{sol}} = 2.7 \cdot 10^8 \left(\frac{M_{\text{vir}}}{10^{10} \text{M}_\odot}\right)^{1/3} \text{M}_\odot, \quad (7)$$

It arises from the connection of the energy per unit mass of the central soliton and outer halo (see, e.g., Bar et al. 2018). The factor $\delta$ here describes the uncertainty up to $\pm 50\%$ in this relation (Schive et al. 2014b; Chan et al. 2022).
velocity is forces of luminous and dark matter; therefore, the observed rotation
We use the model of Li et al. (2020a) to calculate the rotational
2.2 Rotational velocity model
The parameters
where parameters
where
parametrization: logarithm of the mass of the FDM par-
characteristic radius of the initial NFW profile. The concentra-
where
while the density \( \rho_x \) and radius \( r_x \) is a natural parametrization of the NFW and Burkert density profiles, these parameters can vary in very wide ranges (orders of magnitude) from galaxy to galaxy. Because of this, following Li et al. (2020a), we use instead the rotation velocity \( v_{200} \) and concentration \( c_{200} \) as independent parameters. The velocity \( v_{200} \) is defined as
\[
v_{200}^2 = \frac{G \rho_N}{r_{200}} \int_0^{r_{200}} 4\pi \frac{r^2 \rho_{DM}}{r_2} \, dr,
\]
where \( r_{200} \) is the radius enclosing the region with average density 200 times greater than the critical density of the Universe.

The concentration is defined as
\[
c_{200} = \frac{r_{200}}{r_x},
\]
where \( r_x \) is the characteristic radius scale for the Burkert, NFW and coreNFW profiles.

Compared to the NFW profile, the coreNFW profile includes two additional parameters \( n \) and \( \beta_c = r_c / r_n \) (see Eq. 2), where \( r_n \) is the characteristic radius of the initial NFW profile. In general, the FDM profile can be described by four parameters: \( \rho_s \) and \( r_s \) for the central soliton, the transition radius \( r_{\alpha} \), and \( r_{\beta} \) for the NFW ‘tail’ (the NFW parameter \( \rho_n \) is fixed by the continuity condition). For convenience, we introduce the following baseline parametrization: logarithm of the mass of the FDM particle \( \log_{10} m_{22} \), the ratio between the transition and soliton radii \( \alpha = r_{\alpha} / r_s \), the parameter \( \delta \) (see Eq. 7) and the rotation velocity \( v_{200} \).

The parameters \( v_{200}, \log_{10} m_{22}, \alpha, \delta \) can be directly translated to \( \rho_s, r_s, \rho_n, r_n \) via the relations in Eq. 5, 7; see more in Appendix B.

We list the free parameters of each DM model in Tab. 1.

| Model      | Parameters |
|------------|------------|
| NFW        | \( v_{200}, \ c_{200} \) |
| Burkert    | \( v_{200}, \ c_{200} \) |
| coreNFW    | \( v_{200}, \ c_{200}, \ n, \ \beta_c \) |
| FDM        | \( v_{200}, \ \log_{10} m_{22}, \ \alpha, \ \delta \) |

**Table 1.** Free parameters of the DM models under consideration.

### 2.2 Rotational velocity model
We use the model of Li et al. (2020a) to calculate the rotational velocity. The total gravitational force is the sum of the attraction forces of luminous and dark matter; therefore, the observed rotation velocity is
\[
v = \sqrt{v_{\text{DM}}^2 + v_{\text{lum}}^2},
\]
where \( v_{\text{DM}}^2 \) and \( v_{\text{lum}}^2 \) are the contributions from dark and luminous matter, respectively.

The luminous matter contribution is defined by
\[
v_{\text{lum}}^2 = v_{\text{gas}}^2 \left| v_{\text{gas}} + v_{\text{disk}} \right| v_{\text{disk}} \left| v_{\text{disk}} + Y_{\text{bulge}} \right| v_{\text{bulge}} + v_{\text{bulge}}^2.
\]
where \( v_{\text{disk}} \) and \( Y_{\text{bulge}} \) are mass-to-light ratios of the disk and bulge, respectively.

1. Note that we consider \( n \) and \( \beta_c \) as free parameters, in contrast with Li et al. (2020a).
2. In expression Eq. 11, some baryonic components may contribute with negative sign to the total acceleration. This may be caused by the strong depression of the gas distribution in the innermost regions of some galaxies. In this case, the gravitation from the outer regions leads to the negative contribution to the total acceleration. To take this into account, the SPARC database sometimes provides negative velocity contribution from the baryonic component in the inner regions of galaxies.

### 2.3 Data analysis
We analyse the rotation curves of galaxies from the Spitzer Photometry & Accurate Rotation Curves (SPARC) database (Lelli et al. 2016). It provides detailed 3.6 μm photometry and rotational velocity data for 175 galaxies. The SPARC database includes galaxies of different Hubble types, spanning wide range of masses and luminosities. This allows one to robustly test the scaling relations predicted by the model of FDM halos.

Our model includes the DM density profile parameters (up to four parameters), the mass-to-light ratios \( Y_{\text{disk}} \) and \( Y_{\text{bulge}} \), the inclination angle \( i \), and the distance to the galaxy \( D \).

The observed total velocity and its uncertainty depends on the galaxy inclination angle \( i \) as (Li et al. 2018)
\[
v_{\text{obs}} \propto 1/\sin i, \quad \delta v_{\text{obs}} \propto 1/\sin i.
\]
The contribution of the luminous matter to the rotational velocity depends on the distance to the galaxy \( D \) (Li et al. 2018) as
\[
v_{\text{disk, bulge, gas}} \propto \sqrt{D}.
\]
The radius also scales with the distance:
\[
r \propto D.
\]
Since our velocity models have up to eight parameters, we select galaxies with at least eight data points. We also follow Lelli et al. (2016) and do not include galaxies with poor ‘quality’ of the data.

We specifically focus on low surface brightness (LSB) galaxies. They are useful for constraining the dark matter properties because they appear to be dark matter dominated, even at small radii. We take into account that LSB galaxies have surface brightness \( \log B_{\text{eff}} \leq 1.5L/\circ/p^2 \) (Pengfei Li, private communication).

We apply the Bayesian inference for constraining the model parameters and for model selection. According to the Bayes theorem, the posterior probability density of the model parameters \( \theta \) is given by
\[
P(\theta|d, M) = \frac{P(d|\theta, M) \pi(\theta, M)}{p(d|M)},
\]
where \( P(d|\theta, M) \) is the likelihood of the data \( d \), \( P(d|M) \) is the marginalized likelihood (evidence), and \( \pi(\theta, M) \) is the prior probability density.

The evidence is defined as
\[
Z = \int P(d|M) \pi(\theta) d\theta.
\]
We assume the likelihood probability density in the Gaussian form
\[
P(D|\theta, M) = \prod_i \frac{1}{\sqrt{2\pi} \sigma_i} \exp \left( -\frac{(v_{\text{obs}} - v_{\text{pred}}(\theta, M)(r_i))^2}{2\sigma_i^2} \right),
\]
where \( v_{\text{obs}} \) is the observed rotational velocity at radius \( r_i \), \( \sigma_i \) is the error of its determination, and \( v_{\text{pred}}(\theta, M)(r) \) is the rotational velocity predicted by model \( M \) with parameters \( \theta \).

We adopt the prior choice made in Li et al. (2020a) for \( c_{200} \) and
3 RESULTS

3.1 Model comparison

We first investigate which of the DM halo models fits the observed kinematics better. For this, we calculate the marginalised likelihoods for every galaxy and for each DM halo model under consideration. As mentioned above, this allows one to determine models that describe the data better in general. After that, we compute the Bayes factors for each pair of models. The distribution of the Bayesian coefficients for each pair of models is shown in Fig. 1.

We summarise the number of galaxies that substantially prefer or disfavor each DM halo model compared to other ones in Tab. 3. It clearly shows that half of the galaxies (72 out of 143) prefer the FDM profile compared to all other DM density profiles under consideration.

3.2 Astrophysical properties of the galaxies preferring FDM

Now we investigate correlations of the preference of the FDM profile with the properties of galaxies. As general characteristics of galaxies, we choose the morphological Hubble type, the total luminosity, the effective surface brightness, and the flat velocity (the last quantity is introduced only for galaxies that satisfy the velocity flatness criteria of Lelli et al. (2015) at the external point of the rotation curve). Their values are provided by the SPARC database and are model-independent.

Correlations are illustrated by histograms in Fig. 2. Red columns show the number of galaxies that substantially prefer the FDM model, while blue columns correspond to the total number of galaxies. We bin the continuous variables so that each bin has similar width while the morphology classification provides the natural binning.

Our analysis reveals that early Hubble spirals and lenticular galaxies prefer the FDM density profile among the profiles under investigations more often than late spirals or irregular galaxies. At the same time, among the continuous variables, the correlation is the strongest with the flat velocity, which correlates with the galaxy mass. Furthermore, we observe some correlation of the FDM preference with the total galactic luminosity, and a somewhat weaker correlation with the effective surface brightness. The latter correlation can easily be understood: massive galaxies often are more luminous and have higher surface brightness.\(^3\)

For a better quantitative estimate, we calculate the Pearson correlation coefficient between the FDM preference and each of the galactic parameters (flat velocity, total luminosity and effective surface brightness). We assign a discrete variable for the FDM preference, which is chosen to be equal to 1 if a galaxy strongly prefers FDM and 0 otherwise. The resulting correlation coefficients \(R\) with \(p\)-values (the probabilities that the data points are uncorrelated) are given in Tab. 4. The correlation coefficients and the corresponding \(p\)-values are calculated with PEARSON from scipy.stats python package.

These findings also confirm the presence of correlation between the astrophysical parameters of galaxies and their tendency to prefer FDM. More luminous and more massive galaxies as well as galaxies of early Hubble types and those with greater surface brightness prefer the FDM profile more often. The small \(p\)-values may be interpreted as a sign that the correlations are not accidental. However, the correlations are quite weak (with the correlation coefficients around 0.3).

3.3 Fuzzy dark matter particle mass

The nested sampling algorithm also gives us the posterior probability distribution of the model parameters. We provide an example of the posterior distribution of the FDM parameters for NGC 3741 in Appendix C. As expected, we find strong degeneracy between the astrophysical and DM halo parameters.

We focus on the constraints on the mass of fuzzy dark matter particle arising from individual galaxies. Since LSB galaxies are expected to be DM dominated, we use them for deriving the credible intervals for the mass parameter \(\log_{10} m_{22}\) for individual objects. We define the credible interval as a single highest-density interval, see Appendix D. The obtained credible intervals are shown in Fig. 3.

One can clearly see that 95% credible intervals for the mass parameter for individual galaxies are in tension with each other.

We find a similar tension for the \(\log_{10} m_{22}\) credible intervals from individual galaxies with intermediate and high surface brightness, \(\log B_{\text{eff}} > 1.5 L_\odot/pc^2\).

Different definitions of credible interval may give different values. We therefore check the reliability of our results with the Bayesian model comparison. For this, we calculate the Bayesian evidences for the FDM model with 21 fixed masses evenly spaced in the range \(-2 \leq \log_{10} m_{22} \leq 2\). For each galaxy, we select the FDM model with the maximal evidence and compute the Bayes factors for the FDM model with other masses against this ‘best’ model. The logarithm of the Bayesian factors for CamB, IC2574, UGC00128, UGC01281, UGC08837 are shown in Fig. 4. One can see that the FDM masses which are preferred for one particular galaxy are essentially disfavoured for some other galaxy in this collection (\(\log_{10} K \leq -1.0\)).

3.4 Scaling relations of FDM

If FDM comprises all of the DM in the universe, the soliton density profile parameters should follow the scaling relation Eq. 5. For an illustration of the comparison of our findings with the FDM prediction, we plot the best-fit core column densities \(\Sigma = \rho_s \cdot r_s\) vs the core characteristic radii in Fig. 5. The red dots correspond to the maximum a posteriori (MAP) parameters for each galaxy. The

\(^3\) However, the more massive galaxy does not always have to be more luminous or to has greater surface brightness.
Table 2. The parameters of the models and the corresponding priors. The free parameters of each DM model are listed in Tab. 1. We assume the same range of similar parameters in different models.

Figure 1. The number of galaxies that prefer the first model (red columns) or disfavour it (blue columns) as compared to the second model and the strength of preference.
Figure 2. The fraction of the galaxies that prefer FDM among the galaxies of each Hubble morphological type (a), flat velocity (b), total luminosity (c) and effective brightness (d). In histogram (b) galaxies that do not reach flat velocity on the observed rotation curve are filtered out.

Figure 3. 95% credible intervals for the parameter $\log_{10} m_{22}$ for LSB galaxies from the SPARC catalogue (red colour indicates the galaxies that strongly prefer FDM, blue colour indicates those that disfavour FDM, and grey colour indicates the galaxies without strong preference). Yellow lines are the prediction of the FDM scaling relation Eq. 5 for different FDM masses $m_{22}$. One observes a huge scatter in the values of the core column density relative to the theoretically predicted by Eq. 5. Moreover, the best-fit value of $\Sigma_{\text{sol}}$ significantly differs from the universal core column density $\Sigma = 75^{+55}_{-45} M_\odot/pc^2$ (blue lines in Fig. 5) reported in (Salucci & Burkert 2000; Donato et al. 2009; Kormendy & Freeman 2016), in contradiction with the findings of Burkert (2020).  

Our analysis also reveals a discrepancy between the theoretically predicted soliton–halo mass relation and the obtained MAP soliton

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4 Note that the universal core column density was derived for the usual (e.g., Burkert or isothermal) DM profiles, which are quite different from the FDM profile.
Table 3. The number of galaxies that strongly prefer or disfavour each dark matter model. The numbers in parentheses correspond to the subsample of LSB galaxies.

| Quantity          | R     | p-value          |
|-------------------|-------|------------------|
| Flat velocity     | 0.35  | 9 · 10⁻⁵         |
| Total luminosity  | 0.32  | 1 · 10⁻⁴         |
| Surface Brightness| 0.2   | 0.02             |

Table 4. Pearson correlation coefficient $R$ between the galaxy feature and the preference of the FDM density profile by this galaxy.

Figure 4. The logarithm of the Bayes factors for the FDM model with fixed values of the particle mass.

Figure 5. The core column density vs the characteristic soliton radius for galaxies from the SPARC database. The dots correspond to the MAP parameters found via Bayesian inference. The predictions from the FDM scaling relation (5) for particle masses in the range $10^{-23}–10^{-20}$ eV is shown by orange lines.

Figure 6. The mass of the FDM soliton against the virial halo mass for SPARC galaxies. The dots correspond to the MAP parameters found via Bayesian inference. The solid lines are the FDM scaling relation (7) for particle masses in the range $10^{-24}–10^{-19}$ eV.

Figure 5. The core column density vs the characteristic soliton radius for galaxies from the SPARC database. The dots correspond to the MAP parameters found via Bayesian inference. The predictions from the FDM scaling relation (5) for particle masses in the range $10^{-23}–10^{-20}$ eV is shown by orange lines.

3.5 The inner slope of the halo density profile

Are dark matter halo profiles cored or cusped? Our Bayesian model comparison does not reveal any galaxy with a strong preference for a cusped NFW profile. Moreover, the NFW density profile is disfavoured for most of the galaxies in the sample, 128 out of 138 (see Tab. 3). This result is in qualitative agreement with the findings of Li et al. (2020a). The coreNFW profiles naturally describe DM halos with different ‘degree of cuspiness’: the pure NFW profile corresponds to the parameter $n = 0$, while $n = 1$ corresponds to a completely cored density profile, see Eq. 2. We find that the credible intervals for the parameter $n$ are very large and often include values for both core-like and cusp-like halos. As an illustration, we provide 68% credible intervals for $n$ in Fig. 9. Nevertheless, galaxies with the maximal a posteriori value of $n$ close to 1 are much more frequent; see the histogram in Fig. 10.

4 COMPARISON TO PRIOR WORK

Our results are complementary to several constraints on FDM that were previously derived from the SPARC data. Bernal et al. (2017) obtained a confidence interval of $0.0212 \leq m_{22} \leq 0.270$ for the mass of FDM particles from the analysis of rotation curves of 18 LSB and 6 (4 from SPARC database) NGC galaxies. Similar to us, they found that the confidence intervals for the FDM mass don’t overlap for all galaxies. Also, scenarios with DM entirely composed of particles with mass in the range $10^{-22} < m_{22} < 10^2$ were disfavoured by the analysis of SPARC galaxies in (Bar et al. 2022). Chan & Fai Yeung (2021) ruled out values of $m_{22}$ in the range $0.14–3.14$ from the estimates of the soliton and halo mass of the ESO563-G021 galaxy, which is in agreement with our 95% credible.
Figure 7. Credible 2σ intervals for the transition parameter $\alpha$ of the FDM model for LSB galaxies from SPARC.

Figure 8. The transition radius parameter $\alpha$ vs the core radius for SPARC galaxies. The dots correspond to the MAP parameters found via Bayesian inference.

interval $\log_{10} m_{22} = -2.1^{+0.09}_{-0.10}$, Robles et al. (2019) found it difficult to simultaneously solve the Too-Big-To-Fail problem and explain the observed kinematics of SPARC galaxies via FDM halo fit based on FDM-only simulations (Schive et al. 2014a,b). Street et al. (2022) did not find any preferable mass for a single-flavour (composed of a single sort of particles) FDM from the combined analysis of data.

Also, Deng et al. (2018); Burkert (2020) concluded that it is impossible to explain the observed characteristic radii and core densities of the galaxies within relation (5) with fixed FDM mass. However, there is a concern about the reliability of the analysis in Deng et al. (2018); the authors implicitly used core sizes and central densities obtained for the Burkert profile and did not analyse the galaxy kinematics directly.

A big scatter between the values of the FDM mass preferred by individual objects was also found for dwarf spheroidal galaxies (dSphs), which are promising to probe fuzzy dark matter due to their largest mass-to-light ratios compared to other objects. González-Morales et al. (2017) asserted an upper 97.5% bound $m_{22} \leq 0.4$ from the analysis of the luminosity-averaged velocity dispersion of the stellar subpopulations of classical dSphs. Chen et al. (2017) found that FDM with $m_{22} \sim 1$–2 can fit different kinematic datasets of classical dwarfs. They also concluded that Fornax kinematics requires the ratio of the transition to soliton radii $\alpha \geq 2.5$.

Pozo et al. (2021) found signatures of transition from dense cores to halos in isolated dwarfs via the Jeans analysis in frames of the FDM model with $m_{22} = 1.5$. Safarzadeh & Spergel (2020) concluded that the FDM model cannot simultaneously explain the density profiles of ultra-faint and luminous dwarf satellites. Hayashi et al. (2021) obtained tight constraints $m_{22} = 1.1^{+8.3}_{-0.7} \cdot 10^3$ on the FDM mass from the kinematic data of ultra-faint dwarf spheroidal galaxy Segue I; however, this result is incompatible with the bounds based on luminous dwarfs, mentioned above. Dalal & Kravtsov (2022) suggested the lower bound $m_{22} \geq 4 \cdot 10^3$ from the stellar kinematics of Segue I and II. A very narrow range around $m_{22} \approx 10$ was claimed from the existence of a stellar cluster in Eridanus II, which would have to be heated and destroyed by the oscillations of soliton (Marsh & Niemeyer 2019). However, Chiang et al. (2021) argued that the core oscillation cannot significantly heat this stellar cluster.

We find a substantial preference for a soliton density profile from more than half of galaxies in our sample. A related finding was made in De Martino et al. (2020), who reported evidence for the presence of a solitonic core in the Milky Way (MW) from the kinematics of stars in the MW bulge. Moreover, such a central soliton may help to form the MW Central Molecular Cloud according to the recent hydrodynamic simulations by Li et al. (2020b). Toguz et al. 2022 ruled out mass values in the range $1.6 \leq \log_{10} m_{22} \leq 3.5$ and found a preference for $\log_{10} m_{22} \approx 1.5$ via the Jeans analysis of the MW nuclear star cluster.

It is worth noting that our results are in agreement with those by Zoutendijk et al. (2021b,a), who argued that the kinematics of ultra-faint dwarf satellites prefer the soliton halo model against NFW and Burkert profiles in terms of Bayesian evidences. On the other hand, Street et al. (2022) applied the Bayesian information criterion and found that most of the SPARC galaxies favour the Einasto profile against single-flavour FDM. However, these results were obtained for different classes of objects, different sets of considered DM profile models (Zoutendijk et al. 2021b,a) did not consider the Einasto profile and with using different FDM parametrizations and priors.

Many of the constraints listed above are in tension not only with each other, but also with the Lyman-α lower bounds $m_{22} \sim 10$–100 (Irišič et al. 2017; Nori et al. 2019; Rogers & Peiris 2021). Moreover, our analysis of many SPARC galaxies gives the credible intervals of the FDM mass that contradict the Lyman-α constraints. However, the Lyman-α constraints suffer from the degeneracy between the DM properties and the unknown astrophysics at high redshifts.

5 DISCUSSION

Throughout this paper, we use the popular FDM density profile proposed by Schive et al. (2014a,b). This profile was derived for an isolated relaxed spherically symmetric self-gravitating FDM halo without considering any impact from baryonic matter. This might
seem like a theoretical oversimplification, but this specific structure of the FDM halo, which is composed of a central soliton and an outer envelope, has been found as a common feature in simulations under various conditions (Schive et al. 2014a,b; Mocz et al. 2019; Schwabe et al. 2020; Niemeyer 2020; Nori & Baldi 2021; Davies & Mocz 2020; Veltmaat et al. 2020; Chan et al. 2022).

Moreover, it has been shown that solitons survive during FDM halo collisions (Veltmaat & Niemeyer 2016) and during mergers (Mina et al. 2020). The latter work also suggests that the scaling relation Eq. 4 holds for solitons created during mergers.

Ultra-light DM particles form a central soliton even in more complex systems. Schwabe et al. (2020) found that inner solitons also form in mixed FDM and CDM simulations, even if the fraction of FDM is only 10%.

In this work, we also report the deviation from the soliton-halo mass relation predicted in Schive et al. (2014b). However, different simulations demonstrate a variety in these relations. Mina et al. (2020) observed a power-law relation between the soliton and total halo mass, but with the power 5/9 instead of 1/3 in Eq. 7. A universal relation different from Eq. 7 was reported in Mocz et al. (2017). Nori & Baldi (2021) found that scaling relations are correct only for isolated spherically symmetric and relaxed halos, but are not valid in other cases. The simulations by Chan et al. (2022) established the power $0.515^{+0.111}_{-0.185}$ in the soliton–halo mass scaling relation; in this case, the lower bound is very close to 1/3 from Schive et al. (2014b) but the mean value is different.

The FDM density profile described in Sec. 2.1 was obtained without taking into account the effect of baryonic processes on the DM halo formation. Solitonic cores were also found in the simulations of baryons plus FDM with bayonic feedback (Mocz et al. 2019; Veltmaat et al. 2020). According to the simulation in Mocz et al. (2019), the DM density distribution was mostly unaffected by baryonic processes. Instead, the distribution of baryonic matter in galaxies follows the distribution of FDM halo. On the other hand, Veltmaat et al. (2020) showed that, due to the additional gravitational potential of baryons, FDM forms a denser soliton in the center, but the density profile of the soliton is different than in the case of DM-only simulations Eq. 4. They also found that the scaling relations Eq. 5, 7 change due to baryonic feedback. Davies & Mocz (2020) found that the FDM soliton is denser and smaller in the presence of a supermassive black hole. The difference between the results of the profile between FDM-only and FDM+baryons simulations may give a hint to the solution to the discrepancy between our FDM mass intervals preferred by individual galaxies.

There is a natural question whether some astrophysical objects can mimic the central soliton. Let us consider two such possibilities: giant molecular clouds and central black hole. Our analysis reveals that the MAP values of the central soliton mass are in the range $10^8$–$10^{11} M_\odot$, which is much larger than the typical mass of molecular clouds, $10^4$–$10^7 M_\odot$ (Fukui & Kawamura 2010). The obtained MAP soliton masses are comparable with the masses of supermassive black holes. However, the obtained maximal a posteriori soliton radii are usually greater than the radius of the innermost rotation curve bins. This can be regarded as an argument against the hypothesis that supermassive black holes mimic the central soliton inferred by our analysis. Also, we perform our fits with spherically symmetric model distributions of dark matter, whereas some SPARC galaxies may be explained better by a non-spherical DM distribution (Loizeau & Farrar 2022; Zatrimaylov 2021; Bariego Quintana et al. 2022). Non-spherical DM distributions were beyond the scope of the present paper.

Bayesian Inference is a powerful tool but must be used with due caution. For example, independent groups performed a Bayesian analysis on SPARC and reported both a non-detection (see, e.g., Rodrigues et al. 2018) and detection (McGaugh et al. 2018, see, e.g.,) of the fundamental acceleration scale (inspired by the Modified Newtonian Dynamics). Furthermore, Rodrigues et al. (2018) found an incompatibility between the confidence intervals of the fundamental acceleration scale for individual galaxies.

These controversial results caused vigorous discussion about the reliability of Bayesian inference in astrophysics. Cameron et al. (2020) suggested that Bayesian analysis of SPARC rotation curves may have pitfalls: i) the choice of an uninformative prior for the physical parameters; ii) the credible interval is not the same as the
confidence interval and can be defined in different ways. As a particularly illustrative example, Li et al. (2021) showed that a naïve application of Bayesian analysis to the SPARC data even allows one to rule out Newtonian gravity: different galaxies prefer incompatible values of the Newtonian gravitational constant $G_N$ if a flat prior for $G_N$ is used. However, this tension disappears if one chooses the log-normal prior. They attribute this ‘reductio ad absurdum’ to the uncertain nature of formal errors.

Our uninformative prior for $m_22$ is motivated by the fact that previous works reported different (and often contradictory) mass ranges. We apply the Bayesian evidences calculated for FDM with fixed mass on the grid to check the reliability of our $m_22$ results. This approach allows one to overcome the shortcoming of the naïve analysis that different definitions of credible intervals may give different results.

The difficulty of fitting the FDM profiles to galaxy data might be that our work (as many others) uses a simple halo model for relaxed spherically symmetric systems. Quite possibly, this is just insufficient to fit a large variety of galaxies. Progress might require a model of fuzzy dark matter that also takes baryonic matter into account. Naïve application of the Bayesian inference also may have hidden pitfalls. An analysis of ensembles of mock galaxy rotation curves in different dark matter models may be helpful for checking the reliability of the Bayesian analysis and justifying the DM constraints based on the observed kinematics.

6 CONCLUSION

In this work, we used Bayesian Inference to find the best dark matter profile for the rotation curves of galaxies in the Spitzer Photometry & Accurate Rotation Curves (SPARC) database. This approach naturally penalizes models with many parameters and provides us with a posterior parameter distribution.

We considered four dark matter profiles: cold dark matter (the Navarro–Frenk–White profile, NFW), cold dark matter modified by stellar feedback (cored NFW), empirical cored Burkert profile, and fuzzy dark matter profile. We have taken into account uncertainties in the relations between the soliton mass, the halo mass and the transition radius for the FDM model.

We found that although the FDM profile has the largest number of parameters, more than half of the galaxies in the sample substantially prefer this model. Moreover, only 13 out of 138 galaxies disfavor the FDM model. We have not found any galaxy that prefers the cusped NFW profile.

However, we also found it impossible to satisfactorily fit the rotation curves of all SPARC galaxies with a universal value of the FDM particle mass. The 95% credible intervals for the FDM particle mass are in tension for different individual galaxies.

The results of our analysis therefore speak both for and against fuzzy dark matter.

ACKNOWLEDGEMENTS

We thank Yuri Shtanov, Andrea Ferrara, Pengfei Li and Kirill Zatrymyailov for discussions and helpful comments. MKh and AR are grateful to the Armed Forces of Ukraine for protection and security. This work was supported by the National Research Foundation of Ukraine under Project No. 2020.02/0073. The work of AR was also partially supported by the ICTP through AF-06 and the priority project of National Academy of Sciences of Ukraine No. 0122U002259. MKh acknowledges support from Stiftung Politechnische Gesellschaft Förderung für ein kurzfristiges Projekt zur ‘Unterstützung geflüchteter junger Ukrainischer WissenschaftlerInnen’ and FIAS for the hospitality. The calculations presented here were performed on the BITP computer cluster.

DATA AVAILABILITY

The SPARC data underlying this paper are publicly available. The code that supports the findings of this study will be shared upon a reasonable request to the corresponding author.

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APPENDIX A: PRIOR PROBABILITY DENSITY FUNCTIONS

We assume the lognormal prior for $Y_{\text{disk}}$ and $Y_{\text{bulge}}$:

$$f(Y) = \frac{1}{\sqrt{2\pi} \sigma_Y} \exp \left\{ -\frac{(\ln Y - \mu_Y)^2}{2\sigma_Y^2} \right\},$$  \hspace{1cm} (A1)

where $\mu_{\text{disk}} = \ln 0.5$ and $\mu_{\text{bulge}} = \ln 0.7$. The standard deviation $\sigma = 0.1$ dex $= 10^{0.1} = 1.26$.

Gaussian priors are assumed in the following form:

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left\{ -\frac{(x - \mu_x)^2}{2\sigma^2} \right\},$$  \hspace{1cm} (A2)

where $\mu_x$ is the mean, and $\sigma$ is the variance. The flat prior is defined as follows:

$$f(\theta) = \begin{cases} 
\frac{1}{(\theta_{\text{max}} - \theta_{\text{min}})}, & \text{if } \theta_{\text{min}} \leq \theta \leq \theta_{\text{max}}, \\
0, & \text{otherwise},
\end{cases}$$  \hspace{1cm} (A3)

where $\theta_{\text{min}}$ and $\theta_{\text{max}}$ are the lower and upper bounds, respectively, of the allowed range for each parameter.

APPENDIX B: FDM DENSITY PROFILE PARAMETRIZATION

The soliton Eq. 4 and NFW Eq. 1 density profiles have the form:

$$\rho_{\text{sol}}(r) = \rho_s f_{\text{sol}}(r/r_s) , \quad f_{\text{sol}}(x) = \frac{1}{(1 + 0.091x^2)^8} ,$$ \hspace{1cm} (B1)

$$\rho_{\text{NFW}} = \rho_n f_{\text{NFW}}(r/r_n) , \quad f_{\text{NFW}}(x) = \frac{1}{x(1 + x)^2} .$$ \hspace{1cm} (B2)

We define the integrals expressing in dimensionless form the DM mass enclosed within some radius $x$:

$$I_{\text{sol}}(x) = \int_0^x dx' x'^2 f_{\text{sol}}(x') , \quad I_{\text{NFW}}(x_1:x_2) = \max \left[ 0, \int_{x_1}^{x_2} dx' x'^2 f_{\text{NFW}}(x') \right] .$$ \hspace{1cm} (B3) (B4)

In all equations below, we use the dimensionless radius $x = r/r_s$. Now we can write the expression for $v_{200}$ as:

$$v_{200}^2 = \frac{4\pi G}{r_{200}} \rho_s r_s^3 \int_{\alpha}^{c_{200}} I_{\text{sol}}(\min(\alpha, c_{200})) + K(\alpha, \beta, c_{200}) I_{\text{NFW}}(\alpha/\beta; c_{200}/\beta) \, d\alpha,$$ \hspace{1cm} (B5)

where $c_{200} = r_{200}/r_s$, $\beta = r_n/r_s$, $\alpha = r_s/r_s$, and $K(\alpha, \beta, c_{200})$ is a factor defined below. From the continuity condition for the soliton-halo transition, we have:

$$\rho_n = \rho_s f_{\text{sol}}(r_s) f_{\text{NFW}}(\alpha/\beta),$$\hspace{1cm} and the factor $K(\alpha, \beta, c_{200})$ is given by the relation:

$$K(\alpha, \beta, c_{200}) \rho_s r_s^3 = \rho_n r_n^3 f_{\text{sol}}(\alpha)f_{\text{NFW}}(\alpha/\beta).$$

Using the notation introduced above, we obtain an expression for the FDM velocity:

$$\frac{v_{200}^2}{\sqrt{2}} = \frac{c_{200}}{x} I_{\text{sol}}(\min(\alpha, x)) + K I_{\text{sol}}(\alpha/\beta; x/\beta) + K I_{\text{NFW}}(\alpha/\beta; c_{200}/\beta).$$ \hspace{1cm} (B6)

We also define an auxiliary function

$$\mu(x, \alpha, \beta) = I_{\text{sol}}(\min(\alpha, x)) + K I_{\text{sol}}(\alpha/\beta; x/\beta).$$ \hspace{1cm} (B7)

Then the expression for $v_{200}$ becomes

$$\frac{v_{200}^2}{\sqrt{2}} = \frac{c_{200}}{x} \mu(x, c_{200}).$$ \hspace{1cm} (B8)
Now, the FDM density profile is parameterized by the set of parameters \((v_{200}, c_{200}, \alpha, \beta)\). By taking into account also two scaling relations of the FDM model, we obtain parametrization in terms of \((v_{200}, m_{22}, \alpha, \delta)\), as stated in Tab. 1. For this purpose, we write the FDM scaling relations Eq. 5 and Eq. 7 in terms of new variables. Combining both scaling relations and the definitions of \(r_{200}, v_{200}\), and \(M_{200}\), we get the following equations:

\[
c_{200} = \frac{0.0537 m_{22} \delta v_{200}^{2/3}}{H_0^2\rho_{\text{crit}}}, \tag{B9}
\]

\[
\mu(c_{200}, \alpha, \beta) = \frac{0.350877 m_{22}^2 \rho_{\text{crit}} v_{200}^4}{c_{200} H_0^4}. \tag{B10}
\]

Equation B9 allows one to express the parameter \(c_{200}\) through the new FDM parameters. Equation B10 should be solved numerically to find the parameter \(\beta\). Equation B10 may not have a solution for some values of the parameters, even within the allowed range. This means that the FDM halo with such parameters does not exist. Therefore, we assign the log-likelihood value \(-\infty\) for such a set of parameters.

**APPENDIX C: POSTERIOR PROBABILITY DISTRIBUTION**

As an example, the posterior probability distribution of the FDM parameters for the irregular galaxy NGC 3741 is shown in Fig. C1. The 2D marginalised posterior distributions illustrate the degeneracy between the DM and astrophysical parameters.

**APPENDIX D: HIGHEST-POSTERIOR DENSITY INTERVAL**

We define the highest-posterior density interval \([\theta_1, \theta_2]\) for the parameter \(\theta\) via the following equations:

\[
P(\theta_1|D) = P(\theta_2|D), \quad \int_{\theta_1}^{\theta_2} P(\theta|D)\,d\theta = a, \tag{D1}
\]

where \(P(\theta|D)\) is a posterior probability density for the parameter \(\theta\), and \(a\) is the fraction of the posterior mass in the interval \([\theta_1, \theta_2]\).

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Figure C1. Corner plot showing the parameter constraints obtained from the Bayesian analysis of the rotation curve of NGC 3741: the 95% contours of the joint 2D marginalised posterior distributions with PDF along the diagonal.