All spin-2 cubic vertices
with two derivatives

Yu. M. Zinoviev *
Institute for High Energy Physics
Protvino, Moscow Region, 142280, Russia

Abstract

In this paper we provide a complete list of spin-2 cubic interaction vertices with two derivatives. We work in (anti) de Sitter space with dimension $d \geq 4$ and arbitrary value of cosmological constant and use simple metric formalism without any auxiliary or Stueckelberg fields. We separately consider cases with one, two and three different spin-2 fields entering the vertex where each field may be massive, massless or partially massless one. The connection of our results with massive (bi)gravity theories is also briefly discussed.

*E-mail address: Yurii.Zinoviev@ihep.ru
Introduction

One of the effective ways for investigation of possible higher spin interactions is the so called constructive approach. Here one assumes that the whole Lagrangian of the theory can be considered as a row

\[ \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \ldots \]

where \( \mathcal{L}_0 \) — free Lagrangian, \( \mathcal{L}_1 \) contains all cubic vertices and so on. Similarly, for gauge transformations one also assumes

\[ \delta = \delta_0 + \delta_1 + \delta_2 + \ldots \]

where \( \delta_0 \) — in-homogeneous terms, corresponding to gauge invariance of free Lagrangian, \( \delta_1 \) is linear in fields and so on. The first and very important step in all such investigations is the so called linear approximation, i.e. cubic vertices in the Lagrangian and linear in fields (hence the name) terms in gauge transformations. In this approximation gauge invariance requires that

\[ \delta_0 \mathcal{L}_1 + \delta_1 \mathcal{L}_0 = 0 \]

and it means that variation of the cubic vertex under the free gauge transformations must be proportional to free equations and thus can be compensated by appropriate corrections to gauge transformations. The importance of linear approximation comes from the fact that due to linearity of gauge transformations the properties of cubic vertex for any three given fields do not depend on the presence or absence of any other fields in the system. Thus to a large extent the results of linear approximation are model independent and this open the possibility for classification of all consistent cubic vertices.

For the last 10-15 years a lot of interesting and important results for the cubic vertices where obtained using very different approaches (light cone or Lorentz covariant, metric-like or frame-like) \[1\]-\[28\].

In spite of very simple formulation any real investigations of cubic vertices require very complicated and cumbersome calculations. To simplify this task the so called TT-approach was introduced \[29, 30, 31, 32\] where one try to construct the core part of cubic vertex that survives when all the fields entering the vertex are subject to transversality and tracelessness constraints. Such approach nicely work for the massless fields where gauge invariance allows one to reconstruct full vertex with the TT-constraints relaxed. But when massive or partially massless fields are present it is not at all evident if such TT-vertex (or some particular combination of them) can be uplifted to the full unconstrained one.

The aim of this paper — carefully consider may be the most simple case, namely all cubic vertices for spin-2 fields with two derivatives.\(^2\) Besides being an illustrative example for the construction of full cubic vertices, our results also have some relations with recent investigations of massive gravity \[34, 35, 36\], bigravity \[37, 38, 39\] and multigravity \[40, 41\] theories as well as with attempts to construct partially massless gravity or bigravity \[42, 43, 44, 45, 46, 47\]. We work in a simple metric formalism where spin-2 field is described by symmetric second rank tensor \( h_{\mu\nu} \) without any auxiliary or Stueckelberg fields. To be

---

\(^1\)Certainly, this list of References is not in any way complete, it serves just as an illustration of very different approaches used.

\(^2\)Even this simple case requires rather long calculations so that one must use some computer algebra system. In this work we used Reduce \[33\].
sure that we do not miss some particular cases we separately consider vertices with one, two and three different spin-2 fields, in this, any field can be massive, massless or partially massless one. We use "brute force" method, i.e. we construct the most general expression for cubic vertex and require that both transversality and tracelessness constraints follow from Lagrangean equations. Let us present here the main results of our work leaving technical details for the main text.

One field

- **Massless case.** There is a unique (up to possible field redefinitions) solution. Our choice corresponds to gauge invariance under

\[ \delta h_{\mu\nu} = D_{(\mu}\xi^{\nu)} + a_0(\xi^{\alpha}D_{\alpha}h_{\mu\nu} + D_{(\mu}\xi^{\alpha}h_{\nu)\alpha}) \]

- **Massive case.** General solution has the cubic potential of the form:

\[ \mathcal{L}_{10} = \frac{a_0m^2}{4}[(1 + 2b_3)h_{\mu\nu}h_{\nu\alpha}h_{\alpha\mu} - (1 + 3b_3)hh_{\mu\nu}^2 + b_3h^3] \]

In this scalar constraint has the structure:

\[ \mathcal{L} \sim h \oplus Dh Dh \oplus h^2 \]

Terms with derivatives are absent for \( b_3 = \frac{1}{2} \), in this

\[ \mathcal{L}_{10} = \frac{a_0m^2}{2}[h_{\mu\nu}h_{\nu\alpha}h_{\alpha\mu} - \frac{5}{4}hh_{\mu\nu}^2 + \frac{1}{4}h^3] \]

- **Partially massless case** \( m^2 = (d - 2)\kappa \). Only solution with \( b_3 = \frac{1}{2} \) admit partially massless limit in de Sitter space but for \( d = 4 \) only!

Two fields. Here we call ”first field” the one that enters the vertex linearly while ”second field” is the one entering it quadratically.

- **Both fields are massless.** There is a unique (up to possible field redefinitions) solution. Our choice corresponds to gauge invariance under

\[ \begin{align*}
\delta h_{1\mu\nu} & = D_{(\mu}\xi_{1
\nu)} + a_0(\xi_2^{\alpha}D_{\alpha}h_{2\mu\nu} + D_{(\mu}\xi_2^{\alpha}h_{2\nu)\alpha}) \\
\delta h_{2\mu\nu} & = D_{(\mu}\xi_{2\nu)} + a_0(\xi_1^{\alpha}h_{2\mu\nu} + D_{(\mu}\xi_1^{\alpha}h_{2\nu)\alpha} + \xi_2^{\alpha}D_{\alpha}h_{1\mu\nu} + D_{(\mu}\xi_2^{\alpha}h_{1\nu)\alpha})
\end{align*} \]

- **Both fields are massive.** General solution has cubic potential of the form:

\[ \mathcal{L}_{01} = \frac{a_0}{4}[(m_1^2 + 2m_2^2 + 2b_4)h_{1\mu\nu}h_{2\nu\alpha}h_{2\alpha\mu} - (m_1^2 + b_4)h_1h_{2\mu\nu}^2 - 2(m_2^2 + b_4)h_{1\mu\nu}h_2h_{2\mu\nu} + b_4h_1h_2^2)] \]

Scalar constraints for both fields are algebraic only for

\[ b_4 = \frac{m_1^2 + 2m_2^2}{2} \]
In this:

\[ \mathcal{L}_{01} = \frac{a_0}{2} \left[ (m_1^2 + 2m_2^2)h_{1\mu\nu}h_{2\alpha\beta} - \frac{3m_1^2 + 2m_2^2}{4}h_{1\mu\nu}^2 ight. \\
\left. - \frac{m_1^2 + 4m_2^2}{2}h_{1\mu\nu}h_{2\mu\nu} + \frac{m_1^2 + 2m_2^2}{4}h_{1\nu}^2 \right] \]

• **First field is massless** \( m_1 = 0 \). There is a unique solution:

\[ \mathcal{L}_{01} = a_0m_2^2 \left[ h_{1\mu\nu}h_{2\alpha\beta} - \frac{1}{4}h_{1\mu\nu}^2 - h_{1\mu\nu}h_{2\mu\nu} + \frac{1}{4}h_1h_2^2 \right] \]

Scalar constraint for second field is algebraic. In \( dS \) this solution admits partially massless limit \( m_2^2 \to (d - 2)\kappa \) without restrictions on \( d \).

• **Second field is massless** \( m_2 = 0 \). There is no solution except \( m_1 = 0 \).

• **First field is partially massless** \( m_1^2 = (d - 2)\kappa \). Solution exists only for

\[ m_2^2 = \frac{d(d - 2)\kappa}{4} \]

Scalar constraint for the second field is algebraic. Note that for \( d = 4 \) this corresponds to partially massless case.

• **Second field is partially massless** \( m_2^2 = (d - 2)\kappa \). Solution exists only for

\[ m_1^2 = 2(d - 3)\kappa \]

Scalar constraint for the first field is algebraic. Note that for \( d = 4 \) this again corresponds to partially massless case.

Three fields.

• **All three fields are massless**. There is a unique (up to possible fields redefinitions) solution. Our choice corresponds to gauge invariance under

\[ \delta h_{1\mu\nu} = D(\mu\xi_1h) + a_0(\xi_2^\alpha D_\alpha h_{3\mu\nu} + D(\mu\xi_2^\alpha h_{3\nu}) + \xi_3^\alpha h_{2\mu\nu} + D(\mu\xi_3^\alpha h_{2\nu})) \]
\[ \delta h_{2\mu\nu} = D(\mu\xi_2h) + a_0(\xi_1^\alpha D_\alpha h_{3\mu\nu} + D(\mu\xi_1^\alpha h_{3\nu}) + \xi_3^\alpha h_{1\mu\nu} + D(\mu\xi_3^\alpha h_{1\nu})) \]
\[ \delta h_{3\mu\nu} = D(\mu\xi_3h) + a_0(\xi_1^\alpha D_\alpha h_{2\mu\nu} + D(\mu\xi_1^\alpha h_{2\nu}) + \xi_2^\alpha h_{1\mu\nu} + D(\mu\xi_2^\alpha h_{1\nu})) \]

• **Two fields are massless** \( m_1 = m_2 = 0 \). No solution except \( m_3 = 0 \).

• **One field is massless** \( m_1 = 0 \). Solution exists for \( m_2 = m_3 \) only. Scalar constraints are algebraic. In \( dS \) this solution admits partially massless limit.
• All three fields are massive. General solution has potential

\[ \mathcal{L}_{10} = \frac{a_0}{2} \left[ (m_1^2 + m_2^2 + m_3^2 + 2b_5)h_{1\mu\nu}h_{2\nu\alpha}h_{3\alpha\mu} - (m_3^2 + b_5)h_{1\mu\nu}h_{2\mu\nu}h_3 \\ - (m_2^2 + b_5)h_{1\mu\nu}h_{2\mu\nu}h_3 - (m_3^2 + b_5)h_{1\mu\nu}h_{3\mu\nu} + b_5h_1h_2h_3 \right] \]

Terms with derivatives in scalar constraints are absent for

\[ b_5 = \frac{m_1^2 + m_2^2 + m_3^2}{2} \]

In this

\[ \mathcal{L}_{10} = a_0 [(m_1^2 + m_2^2 + m_3^2)h_{1\mu\nu}h_{2\nu\alpha}h_{3\alpha\mu} - \frac{m_1^2 + m_2^2 + 3m_3^2}{4}h_{1\mu\nu}h_{2\mu\nu}h_3 \\ - \frac{m_1^2 + 3m_2^2 + m_3^2}{4}h_{1\mu\nu}h_{2\mu\nu}h_{3\mu\nu} - \frac{3m_1^2 + m_2^2 + m_3^2}{4}h_{1\mu\nu}h_{2\mu\nu}h_{3\mu\nu} \\ + \frac{m_1^2 + m_2^2 + m_3^2}{4}h_1h_2h_3] \]

• One field is partially massless \( m_1^2 = (d - 2)\kappa \). There is a solution provided

\[ (m_2^2 - m_3^2)^2 + 2(m_2^2 + m_3^2)\kappa = d(d - 2)\kappa^2 \]

In this scalar constraints for both massive fields are algebraic. There are two particular solutions of the last relation that correspond to some two field cases given above.

- If masses of the second and third fields are equal \( m_2 = m_3 \) then this relation gives

\[ m_2^2 = m_3^2 = \frac{d(d - 2)\kappa}{4} \]

and this corresponds to the two field case where first field is partially massless.

- If the second field is also partially massless \( m_2^2 = (d - 2)\kappa \) then we obtain

\[ m_3^2 = 2(d - 3)\kappa \]

exactly as in the two field case where the second field is partially massless.

The layout of the paper is simple and straightforward. In Section 1 we provide all necessary kinematic formulas as well as our conventions. Section 2 devoted to the case of single spin-2 field. Surely, this case is rather well understood by now but it is instructive to reproduce these results by the same method that subsequently will be used for the case with two and three spin-2 fields. In Section 3 we consider two field case, while in Section 4 we consider relation of our results with formulation of bigravity [37, 38, 39]. At last, in Section 5 we consider cubic vertices for three fields with different masses.
1 Kinematics

We will work in (anti) de Sitter space-time with dimension \(d \geq 4\) and arbitrary value of cosmological constant \(\Lambda\). Indices are raised and lowered with non-dynamical background metric \(\eta_{\mu\nu}\), while \((A)dS\) covariant derivative \(\mathcal{D}_\mu\) is normalized so that

\[
[D_\mu, D_\nu] \xi_\alpha = -\kappa (\eta_{\mu\alpha} \xi_\nu - \eta_{\nu\alpha} \xi_\mu), \quad \kappa = \frac{2\Lambda}{(d-1)(d-2)} \tag{1}
\]

We use metric like formalism where spin-2 field is described by symmetric second rank tensor \(h_{\mu\nu}\) and choose free Lagrangian for massive field in the form:

\[
\mathcal{L}_0 = \frac{1}{2} \left[ D_\alpha h_{\mu\nu} D^\alpha h_{\mu\nu} - D_\mu h_{\nu\alpha} D_\nu h_{\mu\alpha} - (Dh)_\mu (Dh)_\mu + 2(Dh)_\mu D_\nu h - D_\mu h D_\nu h \right] - m^2 - \kappa (d-2) \left[ h_{\mu\nu}^2 - h^2 \right] \tag{2}
\]

where \((Dh)_\mu = D_\nu h_{\mu\nu}\) and \(h = h_{\mu\mu}\).

As is well known the correct number of physical degrees of freedom requires that two constraints — vector and scalar ones — follow from the Lagrangean equations. It is easy to check that for the free Lagrangian given above we indeed have them:

\[
\mathcal{C}_\nu = D^\mu \frac{\delta \mathcal{L}_0}{\delta h_{\mu\nu}} = -m^2 ((Dh)_\nu - D_\nu h) = 0 \tag{3}
\]

\[
\mathcal{C} = (D^\mu D^\nu - \frac{m^2}{(d-2)} \eta^{\mu\nu}) \frac{\delta \mathcal{L}_0}{\delta h_{\mu\nu}} = -\frac{(d-1)}{(d-2)} m^2 [m^2 - \kappa (d-2)] h = 0 \tag{4}
\]

From these relations one can immediately note that there are two special cases. The first case is the massless one \(m = 0\) where we loose both constraints but the Lagrangian instead becomes invariant under the local gauge transformations with vector parameter:

\[
\delta_0 h_{\mu\nu} = D_\mu \xi_\nu + D_\nu \xi_\mu \tag{5}
\]

The second case is the so called partially massless one \([48, 49, 50, 51]\) \(m^2 = \kappa (d-2)\), in this we loose the second constraint but the Lagrangian becomes invariant under the local gauge transformations with scalar parameter:

\[
\delta_0 h_{\mu\nu} = (D_\mu D_\nu - \frac{m^2}{(d-2)} \eta_{\mu\nu}) \xi \tag{6}
\]

Note that to have correct number of physical degrees of freedom it is important that we still have vector constraint.

\(^3\text{Note that due to non-commutativity of } (A)dS \text{ covariant derivatives there is an ambiguity in the choice of combination of the second and third terms, in this the structure of the terms without derivatives depends on the choice made.}\)

\(^4\text{Here and in what follows dynamical equations will be of second order in derivatives, thus any relation containing field and its first derivative only will be considered as constraint.}\)
2 One field

In this Section we consider cubic vertex with one spin-2 field, i.e. its self-interaction. By now this case is rather well understood but it is instructive to reproduce known results by the same method that will subsequently be used for vertices with two and three different spin-2 fields.

We will look for the cubic vertex in the following form:

\[ \mathcal{L}_1 = \mathcal{L}_{12} + \mathcal{L}_{10} \]

where \( \mathcal{L}_{12} \) contains terms with two derivatives which without loss of generality can be chosen in the form \( ^3 \)

\[ \mathcal{L}_{12} \sim h D h D h \]

while \( \mathcal{L}_{10} \) contains terms without derivatives and looks like:

\[ \mathcal{L}_{10} = b_1 h_{\mu\nu} h_{\nu\alpha} h_{\alpha\mu} + b_2 h h_{\mu\nu}^2 + b_3 h^3 \] (7)

First of all we require that after switching on interaction we will still have vector constraint but in general with some corrections quadratic in field:

\[ \Delta C_\nu \approx D^{\mu} \delta \mathcal{L}_1 \frac{\delta \mathcal{L}_1}{\delta h_{\mu\nu}} \]

where here and in what follows \( \approx \) means "on the free mass shell", i.e. up to the terms proportional to free equations. Schematically this can be written as

\[ \Delta C_V = D^{\mu} \frac{\delta \mathcal{L}_1}{\delta h_{\mu\nu}} + (D h + h D) \frac{\delta \mathcal{L}_0}{\delta h} \]

where \( (D h + h D) \) denotes the most general operator linear in field \( h \) and of first order in derivatives. It turns out that the general solution for \( \mathcal{L}_{12} \) has five free parameters. Recall that in any case where interacting vertex has the same (or higher) number of derivatives as the free Lagrangian one always faces the ambiguities related with the possibility to make field redefinitions and hence obtain the families of physically equivalent theories. In the case at hands possible field redefinitions have the form:

\[ h_{\mu\nu} \Rightarrow h_{\mu\nu} + s_1 h_{\mu\alpha} h_{\alpha\nu} + s_2 h h_{\mu\nu} + s_3 \eta_{\mu\nu} h_{\alpha\beta}^2 + s_4 \eta_{\mu\nu} h^2 \] (8)

thus leaving us with only one non-trivial coupling constant. Using this freedom it is always possible to choose the form of \( \mathcal{L}_{12} \) such that in the massless case \( m = 0 \) the Lagrangian will be invariant under the following gauge transformations:

\[ \delta h_{\mu\nu} = D_\mu \xi_\nu + D_\nu \xi_\mu + a_0 [\xi_\alpha D_\alpha h_{\mu\nu} + D_\mu \xi_\alpha h_{\alpha\nu} + D_\nu \xi_\alpha h_{\mu\alpha}] \] (9)

Similarly to the free case due to non-commutativity of \( (A)dS \) covariant derivatives there are some ambiguities in the choice of expression for \( \mathcal{L}_{12} \) and this choice determines the explicit dependence for the coefficients in \( \mathcal{L}_{10} \) on the cosmological constant. In this, all physical results do not depend on the choice made.
Now let us turn to the scalar constraint. Here we also require that we will still have this constraint with possible corrections quadratic in field $h$:

$$\Delta C \approx (D^\mu D^\nu - \frac{m^2}{(d-2)\eta_{\mu\nu}}) \delta \mathcal{L}_1 \frac{\delta \mathcal{L}_1}{\delta h_{\mu\nu}}$$

or schematically

$$\Delta C = (D D - \eta) \frac{\delta \mathcal{L}_1}{\delta h} + (D^2 h + DhD + hD^2 + h) \frac{\delta \mathcal{L}_0}{\delta h}$$

where $(D^2 h + DhD + hD^2 + h)$ denotes the most general operator linear in field $h$ and of second order in derivatives. General solution has the form:

$$b_1 = \frac{m^2 a_0 + 8b_3}{4}, \quad b_2 = -\frac{m^2 a_0 + 12b_3}{4}$$

in agreement with the results of [34]. However, for general values of parameter $b_3$ the scalar constraint has the following structure:

$$\mathcal{C} \sim h + DhDh + h^2$$

so it contains terms with first derivatives of $h$. But such terms lead to the problems with causality [52, 46]. Happily, there is a special value of this parameter:

$$b_3 = \frac{m^2 a_0}{8}$$

when all these terms are absent so that scalar constraint remains to be purely algebraic, in this

$$\mathcal{L}_{10} = \frac{m^2 a_0}{2}[h_{\mu\nu}h_{\nu\alpha}h_{\alpha\mu} - \frac{5}{4} hh_{\mu\nu}^2 + \frac{1}{4} h^3]$$

Note that this is exactly the same structure that was obtained in [14] using gauge invariant description of massive spin-2 field. Moreover it is this solution that in de Sitter space admits partially massless limit in $d = 4$ [14, 53, 47]. In this the Lagrangian is invariant under the following gauge transformations (compare with Eq. (2.44) in [37]):

$$\delta h_{\mu\nu} = (D_{\mu}D_{\nu} - \frac{m^2}{2}\eta_{\mu\nu})\xi + \tilde{a}_0(D_{\mu}D_{\nu} - \frac{m^2}{2}\eta_{\mu\nu})(h\xi)$$

$$+ \frac{a_0}{4}[h_{\mu}^{\phantom{\mu}\alpha}D_{\nu}\xi - D_{(\mu}h_{\nu)}^{\phantom{\mu\nu}\alpha}\xi + 2D^{\alpha}h_{\mu\nu}D_{\alpha}\xi - m^2h_{\mu\nu}\xi]$$

where the terms with the factor $\tilde{a}_0$ correspond to redefinition of gauge parameter $\xi$ and could be discarded.

\[\text{In our conventions round brackets denote symmetrization without normalization factor and this may explain slightly different }\]
3 Two fields

In this Section we consider cubic vertex for two spin-2 fields with different masses \( m_1 \) and \( m_2 \), in this we will assume that first field \( h_{1\mu\nu} \) enters vertex linearly and the second one \( h_{2\mu\nu} \) — quadratically. Without loss of generality the part of the vertex with two derivatives can be chosen in the form (again with a lot of ambiguities due to non-commutativity of covariant derivatives):

\[
\mathcal{L}_{12} \sim h_1 h_2 D^2 h_2 + h_1 D h_2 D h_2
\]

while terms without derivatives will be written as follows:

\[
\mathcal{L}_{10} = b_1 h_{1\mu\alpha} h_{2\alpha\nu} + b_2 h_1 h_{2\mu\nu} + b_3 h_2 h_{1\mu\nu} + b_4 h_1 h_2^2
\]  \( (13) \)

First of all we require that we still have both vector constraints with optional quadratic corrections:

\[
\Delta C_{1\nu} \approx D^\mu \frac{\delta \mathcal{L}_1}{\delta h_{1\mu\nu}}, \quad \Delta C_{2\nu} \approx D^\mu \frac{\delta \mathcal{L}_1}{\delta h_{2\mu\nu}}
\]

or schematically:

\[
\Delta C_{1V} = D \frac{\delta \mathcal{L}_1}{\delta h_1} + (D h_2 + h_2 D) \frac{\delta \mathcal{L}_0}{\delta h_2}
\]

\[
\Delta C_{2V} = D \frac{\delta \mathcal{L}_1}{\delta h_2} + (D h_1 + h_1 D) \frac{\delta \mathcal{L}_0}{\delta h_2} + (D h_2 + h_2 D) \frac{\delta \mathcal{L}_0}{\delta h_1}
\]

It turns out that general solution for \( \mathcal{L}_{12} \) has ten free parameters, but taking into account possible field redefinitions that in this case look like

\[
\begin{align*}
 h_{1\mu\nu} &\Rightarrow h_{1\mu\nu} + s_1 h_{2\mu\alpha} h_{2\alpha\nu} + s_2 h_1 h_{2\mu\nu} + \eta_{\mu\nu}(s_3 h_{2\alpha\beta}^2 + s_4 h_2^2) \\
h_{2\mu\nu} &\Rightarrow h_{2\mu\nu} + s_5 h_{1\alpha}(h_{2\nu})\alpha + s_6 h_1 h_{2\mu\nu} + s_7 h_2 h_{1\mu\nu} + \eta_{\mu\nu}(s_8 h_{1\alpha\beta} h_{2\alpha\beta} + s_9 h_1 h_2)
\end{align*}
\]

we again see that there is only one non-trivial coupling constant here. Using this freedom one can always bring \( \mathcal{L}_{12} \) into the form such that in the massless case \( m_1 = m_2 = 0 \) the Lagrangian will be invariant under the following local gauge transformations:

\[
\begin{align*}
\delta h_{1\mu\nu} &= D_{\mu} \xi_{1\nu} + D_{\nu} \xi_{1\mu} + a_0 [\xi_2^a D_{\alpha} h_{2\mu\nu} + D_{(\mu} \xi_2^a h_{2\nu)\alpha}] \\
\delta h_{2\mu\nu} &= D_{\mu} \xi_{2\nu} + D_{\nu} \xi_{2\mu} + a_0 [\xi_2^a D_{\alpha} h_{1\mu\nu} + D_{(\mu} \xi_2^a h_{1\nu)\alpha} + \xi_1^a D_{\alpha} h_{2\mu\nu} + D_{(\mu} \xi_1^a h_{2\nu)\alpha}]
\end{align*}
\]

(15)

Now let us turn to the scalar constraints with possible quadratic corrections:

\[
\Delta C_1 \approx (D^\mu D^\nu - \frac{m_1^2}{(d-2)} \eta^\mu\nu) \frac{\delta \mathcal{L}_1}{\delta h_{1\mu\nu}}, \quad \Delta C_2 \approx (D^\mu D^\nu - \frac{m_2^2}{(d-2)} \eta^\mu\nu) \frac{\delta \mathcal{L}_1}{\delta h_{2\mu\nu}}
\]

or schematically:

\[
\Delta C_1 = (D D - \eta) \frac{\mathcal{L}_1}{\delta h_1} + (D^2 h_2 + D h_2 D + h_2 D^2 + h_2) \frac{\mathcal{L}_0}{\delta h_2}
\]

\[
\Delta C_2 = (D D - \eta) \frac{\mathcal{L}_1}{\delta h_2} + (D^2 h_1 + D h_1 D + h_1 D^2 + h_1) \frac{\mathcal{L}_0}{\delta h_2}
\]

\[
+ (D^2 h_2 + D h_2 D + h_2 D^2 + h_2) \frac{\mathcal{L}_0}{\delta h_1}
\]
In this case we have a number of possibilities because each spin-2 field can be massive, massless or partially massless one. In what follows we consider all of them one by one.

**Both fields are massive.** General solution has the form:

\[
\begin{align*}
    b_1 &= \frac{a_0(m_1^2 + 2m_2^2) + 8b_4}{4}, \quad b_2 = -\frac{a_0m_1^2 + 4b_4}{4}, \quad b_3 = -\frac{a_0m_2^2 + 4b_4}{2} \\
\end{align*}
\]  

(16)

so we have one free parameter \(b_4\). But as in the previous case for general values of this parameter both scalar constraints contain dangerous terms with derivatives:

\[
\begin{align*}
    C_1 \sim h_1 + Dh_2 Dh_2 + h_2^2 \\
    C_2 \sim h_2 + Dh_1 Dh_2 + h_1 h_2 \\
\end{align*}
\]

But there is a special value

\[
    b_4 = \frac{a_0(m_1^2 + 2m_2^2)}{8}
\]

(17)

when all these terms are absent, in this

\[
\begin{align*}
    b_1 &= \frac{a_0(m_1^2 + 2m_2^2)}{2}, \quad b_2 = -\frac{a_0(3m_1^2 + 2m_2^2)}{8}, \quad b_3 = -\frac{a_0(m_1^2 + 4m_2^2)}{4} \\
\end{align*}
\]

(18)

**First field is massless** \(m_1 = 0\). Here there exists unique solution:

\[
\begin{align*}
    b_1 &= -b_3 = a_0m_2^2, \quad b_2 = -b_4 = -\frac{a_0m_2^2}{4} \\
\end{align*}
\]

(19)

corresponding to usual gravitational interaction for massive spin-2 particle. Scalar constraint for massive field appears to be purely algebraic without any derivative terms. Moreover in de Sitter space this solution admits partially massless limit without any restrictions on the dimension \(d\).

**Second field is massless** \(m_2 = 0\). There is no solution here (except for the \(m_1 = 0\) in agreement with the results of [8] that such vertex requires as many as six derivatives.

**First field is partially massless** \(m_1^2 = \kappa(d - 2)\). There exists solution only for

\[
    m_2^2 = \frac{d(d - 2)\kappa}{4}
\]

(20)

Second scalar constraint is algebraic and the Lagrangian is invariant under the following gauge transformations:

\[
\begin{align*}
    \delta h_{1\mu
u} &= (D_\mu D_\nu - \frac{m_1^2}{2} \eta_{\mu\nu}) \xi \\
    \delta h_{2\mu\nu} &= \frac{a_0}{2} \left[ \frac{(d - 2)}{d} D_{(\mu} h_{\nu)\alpha} D_\alpha \xi - \frac{2}{d} D_{(\mu} h_{2\nu)\alpha} D_\alpha \xi + D^\alpha h_{2\mu\nu} D_\alpha \xi + \frac{(d - 6)\kappa}{2} h_{2\mu\nu} \xi \right]
\end{align*}
\]

(21)

Note that for \(d = 4\) this corresponds to partially massless case \(m_2^2 = 2\kappa\).

**Second field is partially massless** \(m_2^2 = \kappa(d - 2)\). There exists solution only for

\[
    m_1^2 = 2(d - 3)\kappa
\]

(22)
First scalar constraint is algebraic and the Lagrangian is invariant under the following gauge transformations:

\[
\delta h_{1\mu\nu} = \frac{a_0}{4}[h_2(\mu \alpha D_\nu)D_\alpha \xi - D(\mu h_{2\nu})^\alpha D_\alpha \xi + 2D^\alpha h_{2\mu\nu}D_\alpha \xi - 2\kappa h_{2\mu\nu} \xi]
\]
\[
\delta h_{2\mu\nu} = (D_\mu D_\nu - \kappa \eta_{\mu\nu})\xi + a_0(D_\mu D_\nu - \kappa \eta_{\mu\nu})(h_1 \xi)
+ \frac{a_0}{2}\left[\frac{(d-3)}{(d-2)}h_1(\mu \alpha D_\nu)D_\alpha \xi - \frac{1}{(d-2)}D(\mu h_{1\nu})^\alpha D_\alpha \xi + D^\alpha h_{1\mu\nu}D_\alpha \xi + \kappa(d-5)h_{1\mu\nu} \xi\right]
\] (23)

For \(d = 4\) this again corresponds to partially massless case \(m_1^2 = 2\kappa\).

## 4 Bigravity

As is well known (see e.g. [54]) any theory for two massless spin-2 fields with no more than two derivatives (and when both fields are physical one and not ghost) by field redefinitions can be brought into the form with two independent parts, each one being just usual gravity theory for some metric:

\[
\mathcal{L} = \mathcal{L}(g_{\mu\nu}) + \mathcal{L}(f_{\mu\nu})
\]

It is these separated metrics that where used in formulation of so called bigravity theory [37, 38, 39] where mixing appears in the potential terms only. The aim of this Section is to show how these two metrics can be related with massless and massive spin-2 fields.

Let us denote \(h_0\) the field that remains to be massless and \(h_m\) — the one that becomes massive. There are four possible cubic vertices for two massless spin-2 fields:

\[
\mathcal{L}_1 \sim a_{01}h_0^3 \oplus a_{02}h_0^2 h_m \oplus a_{03}h_0 h_m^2 \oplus a_{04}h_m^3
\]

which at this level are completely independent. But we already know that there is no solution for cubic vertex where massive field enters linearly, thus we put \(a_{02} = 0\). Then collecting the results from previous two sections for gauge transformations we will have:

\[
\delta h_{0\mu\nu} = D_\mu \xi_0 + D_\nu \xi_0 + a_{01}[\xi_0^\alpha D_\alpha h_{0\mu\nu} + D(\mu \xi_0^\alpha h_{0\nu}) \alpha] \\
+ a_{03}[\xi_m^\alpha D_\alpha h_{m\mu\nu} + D(\mu \xi_m^\alpha h_{m\nu}) \alpha]
\]
\[
\delta h_{m\mu\nu} = D_\mu \xi_m + D_\nu \xi_m + a_{03}[\xi_0^\alpha D_\alpha h_{m\mu\nu} + D(\mu \xi_0^\alpha h_{m\nu}) \alpha] \\
+ a_{04}[\xi_m^\alpha D_\alpha h_{m\mu\nu} + D(\mu \xi_m^\alpha h_{m\nu}) \alpha] 
\] (24)

A nice property of this form for gauge transformations is that their algebra can be closed without any corrections beyond linear approximation. For the case at hands this requires \(a_{01} = a_{03}\)\footnote{Note that this relation is nothing else but usual manifestation of universality of gravitational interactions.} while \(a_{04}\) can be arbitrary. But in this case if we make a change of variables:

\[
h_1 = h_0 \cos(\theta) + h_m \sin(\theta), \quad h_2 = -h_0 \sin(\theta) + h_m \cos(\theta)
\] (25)

and similarly

\[
\xi_1 = \xi_0 \cos(\theta) + \xi_m \sin(\theta), \quad \xi_2 = -\xi_0 \sin(\theta) + \xi_m \cos(\theta)
\] (26)
then by straightforward calculations we obtain
\[
\delta h_{1\mu\nu} = D_\mu \xi_{1\nu} + D_\nu \xi_{1\mu} + \frac{a_{01}}{\cos(\theta)} [\xi_{1}^\alpha D_\alpha h_{1\mu\nu} + D_{(\mu} \xi_{1\nu)} h_{1\nu}] \\
\delta h_{2\mu\nu} = D_\mu \xi_{2\nu} + D_\nu \xi_{2\mu} - \frac{a_{01}}{\sin(\theta)} [\xi_{2}^\alpha D_\alpha h_{2\mu\nu} + D_{(\mu} \xi_{2\nu)} h_{2\nu}] 
\]
(27)

provided
\[
\tan(2\theta) = -\frac{2a_{01}}{a_{04}} 
\]
(28)

Thus in terms of these new fields the parts of the Lagrangian with two derivatives are completely separated and the two metrics of bigravity can be written simply as:
\[
g_{\mu\nu} = \eta_{\mu\nu} + a_{01} [h_{0\mu\nu} + \tan(\theta) h_{m\mu\nu}] \\
f_{\mu\nu} = \eta_{\mu\nu} + a_{01} [h_{0\mu\nu} - \cot(\theta) h_{m\mu\nu}] 
\]
(29)

As for the potential terms, collecting the results of previous two sections we can write the most general form for the cubic part as:
\[
\mathcal{L}_{10} = a_{01} m^2 [h_{0\mu\nu}(h_{m\mu\alpha} h_{m\nu\alpha} - h_m h_{m\mu\nu}) - \frac{1}{4} h_0 (h_{m\mu\nu}^2 - h_m^2)] + \frac{a_{04} m^2}{4} [(1 + 8b_3) h_{m\mu\nu} h_{m\nu\alpha} h_{m\mu\alpha} - (1 + 12b_3) h_m h_{m\mu\nu}^2 + 4b_3 h_m^3] 
\]
(30)

Note that here we have changed the normalization of $b_3$, in particular, its special value now $b_3 = \frac{1}{8}$.

5 Three fields

At last let us consider cubic vertex for three spin-2 fields with different masses. This time we choose the following general form for the terms with two derivatives:
\[
\mathcal{L}_{12} \sim h_1 h_2 D^2 h_3 \oplus h_1 D^2 h_2 h_3 \oplus h_1 D h_2 D h_3 
\]
while potential terms will be written as follows:
\[
\mathcal{L}_{10} = b_1 h_{1\mu\nu} h_{2\alpha\nu} h_{3\alpha\mu} + b_2 h_{1\mu\nu} h_{2\mu\nu} h_3 + b_3 h_{1\mu\nu} h_2 h_{3\mu\nu} + b_4 h_1 h_{2\mu\nu} h_{3\mu\nu} + b_5 h_1 h_2 h_3 
\]
(31)

Again we require that there are all three vector constraints with possible quadratic corrections:
\[
\Delta \mathcal{C}_{1\nu} \approx D^\mu \frac{\delta \mathcal{L}_1}{\delta h_{1\mu\nu}}, \quad \Delta \mathcal{C}_{2\nu} \approx D^\mu \frac{\delta \mathcal{L}_1}{\delta h_{2\mu\nu}}, \quad \Delta \mathcal{C}_{3\nu} \approx D^\mu \frac{\delta \mathcal{L}_1}{\delta h_{3\mu\nu}} 
\]
or schematically:
\[
\Delta \mathcal{C}_{1V} = D \frac{\delta \mathcal{L}_1}{\delta h_1} + (D h_2 + h_2 D) \frac{\delta \mathcal{L}_0}{\delta h_3} + (D h_3 + h_3 D) \frac{\delta \mathcal{L}_0}{\delta h_2} \\
\Delta \mathcal{C}_{2V} = D \frac{\delta \mathcal{L}_1}{\delta h_2} + (D h_1 + h_1 D) \frac{\delta \mathcal{L}_0}{\delta h_3} + (D h_3 + h_3 D) \frac{\delta \mathcal{L}_0}{\delta h_1} \\
\Delta \mathcal{C}_{3V} = D \frac{\delta \mathcal{L}_1}{\delta h_3} + (D h_1 + h_1 D) \frac{\delta \mathcal{L}_0}{\delta h_2} + (D h_2 + h_2 D) \frac{\delta \mathcal{L}_0}{\delta h_1} 
\]
In this case the general solution for $\mathcal{L}_{12}$ has sixteen free parameters, but we have fifteen possible fields redefinitions:

\begin{align*}
    h_{1\mu
u} & \Rightarrow h_{1\mu
u} + s_1 h_{2\alpha(\mu h_{3\nu})\alpha} + s_2 h_{2\mu\nu} + s_3 h_{3\mu\nu} + \eta_{\mu\nu}(s_4 h_{2\alpha\beta} h_{3\alpha\beta} + s_5 h_{2\nu}) \\
    h_{2\mu\nu} & \Rightarrow h_{2\mu\nu} + s_6 h_{1\alpha(\mu h_{3\nu})\alpha} + s_7 h_{1\mu\nu} + s_8 h_{1\mu\nu} + \eta_{\mu\nu}(s_9 h_{1\alpha\beta} h_{3\alpha\beta} + s_{10} h_{1\nu}) \\
    h_{3\mu\nu} & \Rightarrow h_{3\mu\nu} + s_{11} h_{1\alpha(\mu h_{2\nu})\alpha} + s_{12} h_{1\mu\nu} + s_{13} h_{2\mu\nu} + \eta_{\mu\nu}(s_{14} h_{1\alpha\beta} h_{2\alpha\beta} + s_{15} h_{1\nu})
\end{align*}

and hence only one non-trivial coupling constant. Using this freedom we choose explicit form for the $\mathcal{L}_{12}$ that in the massless case $m_1 = m_2 = m_3 = 0$ corresponds to invariance of the Lagrangian under the following gauge transformations:

\begin{align*}
    \delta h_{1\mu\nu} &= D_\mu \xi_{1\nu} + D_\nu \xi_{1\mu} + a_0 [\xi^2_2 D_\alpha h_{3\mu\nu} + D_\mu \xi^2_1 h_{3\nu\alpha}] + \xi_3^3 D_\alpha h_{2\mu\nu} + D_\mu \xi_3^3 h_{2\nu\alpha} \\
    \delta h_{2\mu\nu} &= D_\mu \xi_{2\nu} + D_\nu \xi_{2\mu} + a_0 [\xi^2_1 D_\alpha h_{3\mu\nu} + D_\mu \xi^2_1 h_{3\nu\alpha}] + \xi_3^3 D_\alpha h_{1\mu\nu} + D_\mu \xi_3^3 h_{1\nu\alpha} \\
    \delta h_{3\mu\nu} &= D_\mu \xi_{3\nu} + D_\nu \xi_{3\mu} + a_0 [\xi^2_1 D_\alpha h_{2\mu\nu} + D_\mu \xi^2_1 h_{2\nu\alpha}] + \xi_2^2 D_\alpha h_{1\mu\nu} + D_\mu \xi_2^2 h_{1\nu\alpha}
\end{align*}

Similarly, we require existence of all three scalar constraints with possible corrections:

\begin{align*}
    \Delta C_1 & \approx (D^\mu D^\nu - \frac{m_1^2}{(d-2)} \eta^\mu\nu) \frac{\delta \mathcal{L}_1}{\delta h_{1\mu\nu}} \\
    \Delta C_2 & \approx (D^\mu D^\nu - \frac{m_2^2}{(d-2)} \eta^\mu\nu) \frac{\delta \mathcal{L}_1}{\delta h_{2\mu\nu}} \\
    \Delta C_3 & \approx (D^\mu D^\nu - \frac{m_3^2}{(d-2)} \eta^\mu\nu) \frac{\delta \mathcal{L}_1}{\delta h_{3\mu\nu}}
\end{align*}

or schematically:

\begin{align*}
    \Delta C_1 &= (DD - \eta) \frac{\mathcal{L}_1}{\delta h_1} + (D^2 h_2 + D h_2 D + h_2 D^2 + h_2) \frac{\mathcal{L}_0}{\delta h_3} \\
    &\quad + (D^2 h_3 + D h_3 D + h_3 D^2 + h_3) \frac{\mathcal{L}_0}{\delta h_2} \\
    \Delta C_2 &= (DD - \eta) \frac{\mathcal{L}_1}{\delta h_2} + (D^2 h_1 + D h_1 D + h_1 D^2 + h_1) \frac{\mathcal{L}_0}{\delta h_3} \\
    &\quad + (D^2 h_3 + D h_3 D + h_3 D^2 + h_3) \frac{\mathcal{L}_0}{\delta h_1} \\
    \Delta C_3 &= (DD - \eta) \frac{\mathcal{L}_1}{\delta h_3} + (D^2 h_1 + D h_1 D + h_1 D^2 + h_1) \frac{\mathcal{L}_0}{\delta h_2} \\
    &\quad + (D^2 h_2 + D h_2 D + h_2 D^2 + h_2) \frac{\mathcal{L}_0}{\delta h_1}
\end{align*}

Here we again have a number of possibilities that we consider one by one. **Two fields are massless** $m_1 = m_2 = 0$. No solution except for $m_3 = 0$. 
One field is massless $m_1 = 0$. Solution exists for $m_2 = m_3$ only again in agreement with the results of [8] and (upon identification $h_2 = h_3$) corresponds to the case with two fields. Scalar constraints are algebraic, partially massless limit exists.

All three fields are massive. General solution:

$$b_1 = \frac{a_0(m_1^2 + m_2^2 + m_3^2) + 4b_5}{2}, \quad b_2 = -\frac{a_0m_2^2 + 2b_5}{2}$$

$$b_3 = -\frac{a_0m_2^2 + 2b_5}{2}, \quad b_4 = -\frac{a_0m_1^2 + 2b_5}{2}$$

with $b_5$ as a free parameter. For general values of this parameter all three scalar constraints contain terms with derivatives which are absent only for

$$b_5 = \frac{a_0(m_1^2 + m_2^2 + m_3^2)}{4} \quad (34)$$

In this

$$b_1 = a_0(m_1^2 + m_2^2 + m_3^2), \quad b_2 = -\frac{a_0(m_1^2 + m_2^2 + 3m_3^2)}{4}$$

$$b_3 = -\frac{a_0(m_1^2 + 3m_2^2 + m_3^2)}{4}, \quad b_4 = -\frac{a_0(3m_1^2 + m_2^2 + m_3^2)}{4}$$

One field partially massless $m_1^2 = (d - 2)\kappa$. There is a solution provided the following relation holds:

$$(m_2^2 - m_3^2)^2 + 2(m_2^2 + m_3^2)\kappa = d(d - 2)\kappa^2 \quad (35)$$

In this, scalar constraints for both massive fields are algebraic and the Lagrangian is invariant under the following gauge transformations:

$$\delta h_{1\mu\nu} = (D_\mu D_\nu - \kappa \eta_{\mu\nu}) \xi$$

$$\delta h_{2\mu\nu} = \frac{a_0}{4}\left[\frac{m_2^2 + m_3^2 - \kappa(d - 2)}{m_2^2} h_{3(\mu} D_{\nu)} D_\alpha \xi + \frac{m_3^2 - m_2^2 - \kappa(d - 2)}{m_2^2} D_{(\mu} h_{3\nu)} D_\alpha \xi \right.
+ D^\alpha h_{3\mu\nu} D_\alpha \xi + (m_3^2 - m_2^2 + \kappa(d - 6)) h_{3\mu\nu} \xi \right] \quad (36)$$

$$\delta h_{3\mu\nu} = \frac{a_0}{4}\left[\frac{m_2^2 + m_3^2 - \kappa(d - 2)}{m_3^2} h_{2(\mu} D_{\nu)} D_\alpha \xi + \frac{m_2^2 - m_3^2 - \kappa(d - 2)}{m_3^2} D_{(\mu} h_{2\nu)} D_\alpha \xi \right.
+ D^\alpha h_{2\mu\nu} D_\alpha \xi + (m_2^2 - m_3^2 + \kappa(d - 6)) h_{2\mu\nu} \xi \right]$$

There are two particular solutions of the relation (35) that correspond to some two field cases given above.

- If masses of the second and third fields are equal $m_2 = m_3$ then this relation gives

$$m_2^2 = m_3^2 = \frac{d(d - 2)\kappa}{4}$$

and this corresponds to the two field case where first field is partially massless.

- If the second field is also partially massless $m_2^2 = (d - 2)\kappa$ then we obtain

$$m_3^2 = 2(d - 3)\kappa$$

exactly as in the two field case where the second field is partially massless.
Conclusion

In this paper we provide a complete (we hope) list of spin-2 cubic interaction vertices with two derivatives. To include not only massive and massless cases but also partially massless ones we work in (anti) de Sitter space with dimension $d \geq 4$ and arbitrary value of cosmological constant. To be as model independent as possible we work with a simple metric formalism without any auxiliary or Stueckelberg fields and use "brute force" method, i.e. we generate the most general form of cubic vertex and require that Lagrange's equations produce necessary constraints and/or lead to the invariance under some local gauge transformations.

For the massive cases we have seen that one obtains solutions with one free parameter but for the general values of this parameter the scalar constraints contain dangerous derivative terms that lead to the problem with causality. In all cases we considered there is a special value of this parameter when these terms are absent. Moreover, it is these special solutions that in de Sitter space admit partially massless limit. Thus the problem of causality and the existence of partially massless limit in massive (bi)gravity theories seem to be closely related.

For all cases where partially massless field were present we provide explicit form of the appropriate local gauge transformations with scalar parameter. Their rather complicated form suggests that they may arise from gauge invariant Stueckelberg formalism as a result of partial gauge fixing. Work in progress in this direction.

Acknowledgment Author is grateful to R. R. Metsaev and E. D. Skvortsov for discussions and correspondence. The work was supported in parts by RFBR grant No.11-02-00814.

References

[1] M. A. Vasiliev "Cubic Interactions of Bosonic Higher Spin Gauge Fields in $AdS_5$", Nucl.Phys. B616 (2001) 106-162; Erratum-ibid. B652 (2003) 407, arXiv:hep-th/0106200.

[2] K. B. Alkalaev, M. A. Vasiliev "$N=1$ Supersymmetric Theory of Higher Spin Gauge Fields in $AdS(5)$ at the Cubic Level", Nucl.Phys. B655 (2003) 57-92, arXiv:hep-th/0206068.

[3] K.B. Alkalaev "FV-type action for $AdS(5)$ mixed-symmetry fields”, JHEP 1103 (2011) 031, arXiv:1011.6109.

[4] M. Vasiliev "Cubic Vertices for Symmetric Higher-Spin Gauge Fields in $(A)dS_d”$, Nucl. Phys. B862 (2012) 341, arXiv:1108.5921.

[5] R. R. Metsaev "Cubic interaction vertices of massive and massless higher spin fields”, Nucl. Phys. B759 (2006) 147, arXiv:hep-th/0512342.

*Let us stress once again that it was important that besides these gauge symmetry we still have appropriate vector constraint.*
[6] R. R. Metsaev "Gravitational and higher-derivative interactions of massive spin 5/2 field in (A)dS space", Phys. Rev. D77 (2008) 025032, arXiv:hep-th/0612279.

[7] R.R. Metsaev "Cubic interaction vertices for fermionic and bosonic arbitrary spin fields", Nucl. Phys. B859 (2012) 13, arXiv:0712.3526.

[8] R. R. Metsaev "BRST-BV approach to cubic interaction vertices for massive and massless higher-spin fields", arXiv:1205.3131.

[9] N. Boulanger, S. Leclercq "Consistent couplings between spin-2 and spin-3 massless fields", JHEP 0611 (2006) 034, arXiv:hep-th/0609221.

[10] N. Boulanger, S. Leclercq, P. Sundell "On The Uniqueness of Minimal Coupling in Higher-Spin Gauge Theory", JHEP 0808 (2008) 056, arXiv:0805.2764.

[11] Nicolas Boulanger, E. D. Skvortsov, Yu. M. Zinoviev "Gravitational cubic interactions for a simple mixed-symmetry gauge field in AdS and flat backgrounds", J. Phys. A44 (2011) 415403, arXiv:1107.1872.

[12] Nicolas Boulanger, E. D. Skvortsov "Higher-spin algebras and cubic interactions for simple mixed-symmetry fields in AdS spacetime", JHEP 1109 (2011) 063, arXiv:1107.5028.

[13] N. Boulanger, D. Ponomarev, E.D. Skvortsov "Non-abelian cubic vertices for higher-spin fields in anti-de Sitter space", arXiv:1211.6979.

[14] Yu. M. Zinoviev "On massive spin 2 interactions", Nucl. Phys. B770 (2007) 83-106, arXiv:hep-th/0609170.

[15] Yu. M. Zinoviev "On spin 3 interacting with gravity", Class. Quantum Grav. 26 (2009) 035022, arXiv:0805.2226.

[16] Yu. M. Zinoviev "On massive spin 2 electromagnetic interactions", Nucl. Phys. B821 (2009) 431-451, arXiv:0901.3462.

[17] Yu. M. Zinoviev "Spin 3 cubic vertices in a frame-like formalism", JHEP 08 (2010) 084, arXiv:1007.0158.

[18] Yu. M. Zinoviev "On electromagnetic interactions for massive mixed symmetry field", JHEP 03 (2011) 082, arXiv:1012.2706.

[19] Yu. M. Zinoviev "Gravitational cubic interactions for a massive mixed symmetry gauge field", Class. Quantum Grav. 29 (2012) 015013, arXiv:1107.3222.

[20] I. L. Buchbinder, T. V. Snegirev, Yu. M. Zinoviev "Cubic interaction vertex of higher-spin fields with external electromagnetic field", arXiv:1204.2341.

[21] Yu. M. Zinoviev "On massive gravity and bigravity in three dimensions", arXiv:1205.6892.
[22] I. L. Buchbinder, T. V. Snegirev, Yu. M. Zinoviev "On gravitational interactions for massive higher spins in AdS$_3$", arXiv:1208.0183.

[23] R. Manvelyan, K. Mkrtchyan, W. Ruehl "Off-shell construction of some trilinear higher spin gauge field interactions", Nucl. Phys. B826 (2010) 1, arXiv:0903.0243.

[24] R. Manvelyan, K. Mkrtchyan, W. Ruehl "General trilinear interaction for arbitrary even higher spin gauge fields", Nucl. Phys. B836 (2010) 204, arXiv:1003.2877.

[25] R. Manvelyan, K. Mkrtchyan, W. Ruehl "A generating function for the cubic interactions of higher spin fields", Phys. Lett. B696 (2011) 410, arXiv:1009.1054.

[26] R. Manvelyan, R. Mkrtchyan, W. Ruehl "Radial Reduction and Cubic Interaction for Higher Spins in (A)dS space", arXiv:1210.7227.

[27] G. L. Gomez, M. Henneaux, R. Rahman "Higher-Spin Fermionic Gauge Fields and Their Electromagnetic Coupling", JHEP 1208 (2012) 093, arXiv:1206.1048.

[28] A. Sagnotti, M. Taronna "String Lessons for Higher-Spin Interactions", Nucl. Phys. B842 (2011) 299, arXiv:1006.5242.

[29] E. Joung, M. Taronna "Cubic interactions of massless higher spins in (A)dS: metric-like approach", Nucl. Phys. B861 (2012) 145, arXiv:1110.5918.

[30] E. Joung, L. Lopez, M. Taronna "On the cubic interactions of massive and partially-massless higher spins in (A)dS", JHEP 07 (2012) 041, arXiv:1203.6578.

[31] E. Joung, L. Lopez, M. Taronna "Solving the Noether procedure for cubic interactions of higher spins in (A)dS", arXiv:1207.5520.

[32] E. Joung, L. Lopez, M. Taronna "Generating functions of (partially-)massless higher-spin cubic interactions", arXiv:1211.5912.

[33] REDUCE computer algebra system — www.reduce-algebra.com.

[34] Claudia de Rham, Gregory Gabadadze "Generalization of the Fierz-Pauli Action", Phys. Rev. D82 (2010) 044020, arXiv:1007.0443.

[35] Claudia de Rham, Gregory Gabadadze, Andrew J. Tolley "Resummation of Massive Gravity", Phys. Rev. Lett. 106 (2011) 231101, arXiv:1011.1232.

[36] S. F. Hassan, Rachel A. Rosen, Anghis Schmidt-May "Ghost-free Massive Gravity with a General Reference Metric", arXiv:1109.3230.

[37] S. F. Hassan, Rachel A. Rosen "Bimetric Gravity from Ghost-free Massive Gravity", arXiv:1109.3515.

[38] S. F. Hassan, Rachel A. Rosen "Confirmation of the Secondary Constraint and Absence of Ghost in Massive Gravity and Bimetric Gravity", arXiv:1111.2070.
[39] S. F. Hassan, A. Schmidt-May, M. von Strauss "On Consistent Theories of Massive Spin-2 Fields Coupled to Gravity", arXiv:1208.1515.

[40] Kurt Hinterbichler, Rachel A. Rosen "Interacting Spin-2 Fields", arXiv:1203.5783.

[41] S. F. Hassan, A. Schmidt-May, M. von Strauss "Metric Formulation of Ghost-Free Multivielbein Theory", arXiv:1204.5202.

[42] Claudia de Rham, Sebastien Renaux-Petel "Massive Gravity on de Sitter and Unique Candidate for Partially Massless Gravity", arXiv:1206.3482.

[43] S. F. Hassan, Angnis Schmidt-May, Mikael von Strauss "On Partially Massless Bimetric Gravity", arXiv:1208.1797.

[44] S. F. Hassan, A. Schmidt-May, M. von Strauss "Bimetric Theory and Partial Masslessness with Lanczos-Lovelock Terms in Arbitrary Dimensions", arXiv:1212.4525.

[45] S. Deser, E. Joung, A. Waldron "Partial Masslessness and Conformal Gravity", arXiv:1208.1307.

[46] S. Deser, M. Sandora, A. Waldron "Nonlinear Partially Massless from Massive Gravity?", arXiv:1301.5621.

[47] Claudia de Rham, Kurt Hinterbichler, Rachel A. Rosen, Andrew J. Tolley "Evidence for and Obstructions to Non-Linear Partially Massless Gravity", arXiv:1302.0025.

[48] S. Deser, A. Waldron "Gauge Invariance and Phases of Massive Higher Spins in (A)dS", Phys. Rev. Lett. 87 (2001) 031601, arXiv:hep-th/0102166.

[49] S. Deser, A. Waldron "Partial Masslessness of Higher Spins in (A)dS", Nucl. Phys. B607 (2001) 577, arXiv:hep-th/0103198.

[50] S. Deser, A. Waldron "Null Propagation of Partially Massless Higher Spins in (A)dS and Cosmological Constant Speculations", Phys. Lett. B513 (2001) 137, arXiv:hep-th/0105181.

[51] Yu. M. Zinoviev "On Massive High Spin Particles in (A)dS", arXiv:hep-th/0108192.

[52] S. Deser, A. Waldron "Acausality of Massive Gravity", arXiv:1212.5835.

[53] S. Deser, E. Joung, A. Waldron "Gravitational- and Self- Coupling of Partially Massless Spin 2", Phys. Rev. D86 (2012) 104004, arXiv:1301.4181.

[54] N. Boulanger, T. Damour, L. Gualtieri, M. Henneaux "Inconsistency of interacting, multi-graviton theories", Nucl. Phys. B597 (2001) 127, arXiv:hep-th/0007220.