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Variation after projection with triaxially deformed nuclear mean field

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We implemented a variation after projection (VAP) algorithm based on a triaxially deformed Hartree-Fock-Bogoliubov vacuum state. This is the first projected mean field study that includes all the quantum numbers (except parity), i.e., spin (J), isospin (T) and mass number (A). Systematic VAP calculations with JTA-projection have been performed for the even-even sd-shell nuclei with the USDB Hamiltonian. All the VAP ground state energies are within 500 keV above the exact shell model values. Our VAP calculations show that the spin projection has two important effects: (1) the spin projection is crucial in achieving good approximation of the full shell model calculation. (2) the intrinsic shapes of the VAP wavefunctions with spin projection are always triaxial, while the Hartree-Fock-Bogoliubov methods likely provide axial intrinsic shapes. Finally, our analysis suggests that one may not be possible to associate an intrinsic shape to an exact shell model wave function.

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I. INTRODUCTION

Hartree-Fock-Bogoliubov (HFB) method has been very successful in describing the global properties of the ground states throughout the whole nuclear region. As a mean field method, HFB breaks the symmetries of the nuclear system, and can be used to study the intrinsic shapes. The HFB calculations with Gogny force show that almost all the calculated 1712 nuclei have axially symmetric HFB minima[1].

Projection can be done on a HFB vacuum to recover the symmetries that the Hamiltonian obeys. To test the quality of the projected wavefunctions, one can compare them with the exact shell model ones using a common Hamiltonian. HFB and variation after projected HFB calculations with shell model Hamiltonians have been reported by several authors [2–5]. For those calculations without projection, the HFB vacuum states are often assumed to be axially symmetric [4]. Indeed, we will see below that all the calculated HFB minima in sd-shell nuclei, except 24Mg, are exactly axial with the USDB Hamiltonian [6].

However, if one performs the variation of the projected HFB vacuum, usually called variation after projection (VAP) [7], it is likely that the intrinsic shape may change due to the inclusion of beyond mean field correlations. One typical example is the ground state (g.s.) of 32Mg, which is predicted to be spherical at the mean field level [8], but it turns out to have a quadrupole deformation when the correlations associated with the restoration of the broken rotational symmetry are considered [9]. Another example is 56Ni, whose ground state is spherical at the mean field level, but is slightly deformed when performing the projected energy surface calculation [10].

Moreover, the triaxial (γ) degree of freedom plays important roles on the low-lying collective dynamics in this mass region [12]. In 24Mg the possibility of the triaxial deformation in the ground state was discussed for decades [13–15], and it is still being used as the testing ground for modern theories involving angular momentum (spin) projection [16–18].

In this work, we perform VAP calculations of the even-even sd-shell nuclei using the USDB Hamiltonian. Here, we allow the γ degree of freedom in the HFB transformation. The shell model Hamiltonian conserves the spin (J), isospin (T), as well as the mass number (A). Hence a complete projection should recover all J, T, and A quantum numbers. This is generally very much time-consuming because of the 7-dimensional integration (3 for J, 3 for T, and 1 for A). Presently, we can only carry out such extensive studies in the sd shell. For efficiency, we use the new techniques of Refs. [19–21] to evaluate the kernels for projections.

II. THE VAP METHOD

From a randomly chosen HFB vacuum state |Ψ0⟩, one can construct a new HFB vacuum state |Ψ⟩ using the Thouless theorem [7]. Namely,

|Ψ⟩ = ℂN1/2 ∑ e µ ν δ µ ν a † µ a ν |Ψ0⟩,

where d is a skew symmetric matrix, and ℂ is the normalization factor. The triaxiality of the HFB vacuum can be treated similar to Ref. [22] so that the Q2±1 components of the quadrupole moment vanish.

Projecting |Ψ⟩ onto good quantum numbers J, T, and A, one gets the so called JTA-projection (similarly, TA-projection for T, A, etc.). The JTA-projected wavefunction can be written as

|ΨJT A,MM⟩ = ∑ P K K T P M K M T K T P A |Φ⟩
where $P^I_{MK}$, $P^T_{MK}$, and $P^A$ are the spin, isospin and mass number projection operators, respectively. The isospin projection operator is similar to the spin projection operator but in the isospin space. The corresponding $JTA$-projected energy is

$$ E_{JTA} = \langle \Phi | JTA, MM_T | H | JTA, MM_T \rangle = \sum_{K'K\gamma'_K} f_{K'K\gamma'_K \gamma_K} \langle \Phi | JTA | K'K\gamma'_K \gamma_K \rangle P^A | \Phi \rangle f_{K'K\gamma'_K} = 0, $$  

(3)

$E_{JTA}$ and the corresponding coefficients $f_{K'K\gamma'_K}$ are obtained by solving

$$ \sum_{K'K\gamma'_K} f_{K'K\gamma'_K \gamma_K} \langle \Phi | JTA | K'K\gamma'_K \gamma_K \rangle P^A | \Phi \rangle f_{K'K\gamma'_K} = 1. $$  

(5)

One can also perform the $TA$-projection by simply removing the spin projection from Eqs. (2-5),

$$ |\Psi_{TA, MT} \rangle = \sum_{KK_T} f_{KK_T} P^T_{MK_T} | \Phi \rangle, $$  

(6)

$$ E_{TA} = \langle \Psi_{TA, MT} | H | \Psi_{TA, MT} \rangle = \sum_{KK_T} f^*_{KK_T} f_{KK_T} \langle \Phi | JTA | K'K\gamma'_K \gamma_K \rangle P^A | \Phi \rangle. $$  

(7)

For the $A$-projection, the corresponding energy, $E_A$, is reduced to

$$ E_A = \frac{\langle \Phi | JTA | K'K\gamma'_K \gamma_K \rangle P^A | \Phi \rangle}{\langle \Phi | P^A | \Phi \rangle}. $$  

(8)

Without any projection, we define

$$ E_{HFB} = \langle \Phi | H | \Phi \rangle. $$  

(9)

It is natural that one may consider the neutron $(N)$ and proton $(Z)$ projection, as has been done in Refs. [2, 5]. However, this is essentially the same as the $MT_A$-projection ($MT = (N-Z)/2$). Here, we prefer to take $TA$-projection to recover the total isospin symmetry. In our case, the $MT$-projection is no longer necessary because the total isospin and the mass number are good quantum numbers. Thus all quantum numbers $J, T, N$ and $Z$ (parity is automatically good in the $sd$ valence space) have been recovered in the present work. The $sd$ valence space wave functions have the center-of-mass in its g.s., provided that harmonic oscillator single particle wave functions are considered.

VAP calculations can be performed by changing the $d$ matrix in Eq.(1). Here, we impose the following restrictions for the $d$ matrix: (1) $d$ is real, (2) keeping the time reversal symmetry, and (3) no mixing between neutron and proton in the HFB transformation. Therefore the total number of free VAP parameters for $sd$ shell is reduced to $N_{VAP} = 42$. In practice we start with $d = 0$ and and Nilsson+BCS vacuum states $|\Phi_0 \rangle$ obtained with randomly chosen quadrupole parameters [10]. The triaxial degree of freedom is also allowed in $|\Phi_0 \rangle$.

We follow the VAP algorithm whose details were introduced in Ref. [5]. Here are the main steps used in our VAP calculations. Given a certain $d$ matrix, one can get the corresponding HFB transformation for the vacuum $|\Phi \rangle$ [5]. Solving Eq. (4), one can obtain several $E_{JTA}$ eigen-energies for the single $|\Phi \rangle$. The lowest $E_{JTA}$ and the corresponding coefficients $f_{KK_T}$ are considered. Having fixed all $f_{KK_T}$, one can evaluate the partial derivatives $\frac{\partial E}{\partial \delta_{\mu\nu}}$ whose expression can be obtained from Eq.(3) [5]. If $E_{JTA}$ reaches a minimum, then $\frac{\partial E_{JTA}}{\partial \delta_{\mu\nu}} \approx 0$ for all selected $\delta_{\mu\nu}$ parameters and the VAP calculation terminates. Otherwise, we continue to search for a minimum using a gradient method [11] that updates the $d$ matrix and is going to the next iteration.

To extract the intrinsic shape, the quadrupole moment and the triaxial degree of freedom, $Q$ and $\gamma$, are defined such that

$$ Q \cos \gamma = \langle \Psi | \sqrt{\frac{16\pi}{5}} \frac{r^2}{b^2} Y_{20} | \Psi \rangle, $$  

(10)

$$ Q \sin \gamma = \langle \Psi | \sqrt{\frac{16\pi}{5}} \frac{1}{b^2} \frac{1}{\sqrt{2}} (Y_{22} + Y_{2-2}) | \Psi \rangle, $$  

(11)

where $b$ is the harmonic oscillator length. $|\Psi \rangle$ refers to an intrinsic state, which may have different forms. Explicitly, we define,

1. $Q_{HFB}$ and $\gamma_{HFB}$ for $|\Psi \rangle = |\Phi \rangle$,
2. $Q_A$ and $\gamma_A$ for $|\Psi \rangle = P^A |\Phi \rangle$, and
3. $Q_{TA}$ and $\gamma_{TA}$ for $|\Psi \rangle = |\Psi_{TA, MT} \rangle$.

### III. VAP CALCULATIONS FOR $^{24}$Mg

When performing the energy variation, one may find that there might be more than one energy minima. Therefore, the energy variation should be calculated several times with different starting $|\Phi_0 \rangle$ states which are randomly chosen. We then identify the lowest minimum, and denote it with $E^*$. Here and below, we only discuss the results corresponding to $E^*$.

In the present work, we adopt the USDB Hamiltonian [6]. The HFB energy for $^{24}$Mg is $E_{HFB} = -80.965$ MeV with the constraints $\langle \Phi | N | \Phi \rangle = N$ and $\langle \Phi | Z | \Phi \rangle = Z$. This is the only $sd$-shell nucleus for which the HFB calculation gives a non-axial shape with $Q_{HFB} = 18.659$ and $\gamma_{HFB} = 11.96^\circ$ (here and below the $Q^*$ and $\gamma^*$ are the shape parameters that can be associated with the absolute minimum for some VAP choice). Let’s first do the simplest VAP with only $A$-projection (called VAP-A). Since the particle number is already projected out, it
TABLE I: Results of the VAP-A calculations for $^{24}$Mg. We perform the VAP calculations for several times. Each time we start with different $|\Phi_0\rangle$ states. The numbers in the first column denote different $|\Phi_0\rangle$ states. The second column shows the converged energy $E_A$. Quantities in other columns are calculated with the converged $|\Phi\rangle$ vacua. Energies are in MeV.

| $|\Phi_0\rangle$ | $E_A^*$ | $Q_A$ | $\gamma_A$ | $E_{HF B}$ | $Q_{HF B}$ | $\gamma_{HF B}$ | $\langle A \rangle$ |
|-----------------|--------|-------|-------------|-------------|-------------|--------------|-------------|
| 1               | 81.358 | 18.284| 10.05       | -81.008     | 18.005      | 9.46         | 8.110        |
| 2               | 81.358 | 18.284| 130.05      | -90.178     | 18.371      | 128.94       | 9.013        |
| 3               | 81.358 | 18.284| -109.95     | -82.684     | 18.120      | -110.61      | 8.259        |
| 4               | 81.358 | 18.284| 10.05       | -79.720     | 17.905      | 9.05         | 8.000        |

might be unnecessary to impose a constraint to the average particle number of the HFB vacuum. To check this conjecture, we start from several different $|\Phi_0\rangle$ states and perform VAP-A. The results for few selected $|\Phi_0\rangle$ choices are shown in Table I. One can see that the VAP-A energies are identical ($E_A = -81.358$ MeV). However, the corresponding $E_{HF B}$, $Q_{HF B}$, $\gamma_{HF B}$ and $\langle A \rangle \equiv \langle \Phi | A | \Phi \rangle$ appear randomly, but after the A-projection, the $Q_A$ values are the same. Although the $\gamma_A$ values look different, the numbers indicate the same shape but with different orientations. All these results imply that although the converged vacua $|\Phi\rangle$ are not unique, they correspond to the same A-projected state. This can be further confirmed by calculating the overlaps between these projected states corresponding to different $|\Phi\rangle$. Our calculations show that all these overlaps among the converged HFB vacua in Table I are found to be 1 except for an arbitrary phase, i.e.

$$\langle \Phi | P^A | \Phi' \rangle = e^{i\delta},$$

where $\delta$ is a real number. $|\Phi\rangle$ and $|\Phi'\rangle$ are different converged HFB vacua, but $|\Phi'\rangle$ is the reoriented one whose $\gamma_A$ value should be the same as for $|\Phi\rangle$.

Therefore, one can adopt the values $Q_A = 18.284$ and $\gamma_A = 10.05$ to define the shape of the VAP-A minimum. If one imposes $\langle A \rangle = A = 8$, we still have $E_A^* = -81.358$ MeV, now the converged $|\Phi\rangle$ vacuum becomes unique, with $E_{HF B} = -79.720$, $Q_{HF B} = 17.905$, and $\gamma_{HF B} = 9.05$ (see the last line in Table I). However, for the VAP with T-projection, the situation becomes a little different.

VAP calculations with T-projection (called VAP-TA) are listed in Table II. Unlike VAP-A, even if one imposes $\langle A \rangle = A = 8$ for $^{24}$Mg, the converged $|\Phi\rangle$ is still not unique as the $E_{HF B}$ energy appears randomly. Moreover, the $E_A$ energy is not unique either. Interestingly, after T-projection, those different $|\Phi\rangle$ vacuum states have exactly the same projected energy $E_A^* = -82.831$ (MeV) and the same $Q_{TA}^* = 17.295$. Similarly, we found (after rotation) $\gamma_{TA}^* = 0.09^\circ$, which describes an almost axial-shape. Again, our calculations show that the overlaps of the TA-projected states satisfy

$$\langle \Psi_{TA,M_T} | \Psi'_{TA,M_T} \rangle = \sum_{K_T K'_T} f^T_{K_T K'_T} | \langle \Phi | P^T_{K_T K'_T} | \Phi' \rangle | = e^{i\delta},$$

where $|\Phi'\rangle$ generating $|\Psi'_{TA,M_T}\rangle$ should be treated similar to that in Eq.(12). One can conclude that those VAP-TA projected states in Table II are essentially identical and the associated shape can only be described by $Q_{TA}$ and $\gamma_{TA}$.

A complete symmetry restoration is the T-projection. VAP results with T-projection (called as VAP-JTA) are shown in Table III. All the converged $E_{JTA}$ energies are $\approx 86.919$ MeV, significantly closer to the shell model result $E_{SM} = -87.105$ MeV. Overlap calculations clearly confirm that those JTA-projected states are identical, i.e.

$$\langle \Psi_{JTA,M_{MT}} | \Psi'_{JTA,M_{MT}} \rangle = \sum_{K K' T} f^K_{K K'} | \langle \Phi | P^K_{K K'} | \Phi' \rangle | = e^{i\delta}.$$

Here, $|\Phi'\rangle$ does not need to be reoriented due to the inclusion of the spin projection.

Again, both $E_A$ and $E_{HF B}$ in Table III can not be uniquely determined, even if one enforces the $\langle A \rangle = A$ constraint. Fortunately, with the additional spin projection, all $E_{TA}$ values are found to be $-79.879$ MeV, and similarly the corresponding shape is described by $Q_{TA} = 19.057$ and $\gamma_{TA} = 16.96^\circ$. Therefore, the quantities that can be associated with the shape of VAP-JTA wavefunction should be $Q_{TA}^* = 19.057$ and $\gamma_{TA}^* = 16.96^\circ$.

One can study the shape evolution of $^{24}$Mg from HFB to VAP-JTA. In VAP-JTA, $Q_{TA}^*$ looks smaller than $Q_{HF B}$ in HFB, and $\gamma_{TA}^*$ tends to be close to zero (axial shape). However in VAP-JTA, $Q_{TA}^*$ is larger than the $Q_{HF B}$ in HFB, and $\gamma_{TA}^*$ tends to describe a triaxial shape. This triaxiality in VAP-JTA, in comparison with VAP-TA, is likely caused by the spin projection. Nuclear triaxiality caused by the spin projection has been previously discussed by several authors [16–18, 23–25]. To determine if this phenomenon is more general, we performed systematic VAP calculations for a larger number of even-even $sd$-shell nuclei.

IV. VAP CALCULATIONS FOR EVEN-EVEN $sd$-SHELL NUCLEI

VAP calculations have been performed for the ground states of even-even $sd$-shell nuclei. The calculated energies relative to the shell model ones are shown in Figure 1a. The numerical results are given in Table IV. Here, we didn’t include the Oxygen isotopes and the $N = 20$
isotones because their VAP-JTA energies are exactly the same as the shell model results (\(E_{SM}\)). This special case is discussed below. The VAP-JTA energies are much lower than those of HFB and VAP-TA. Moreover, the VAP-JTA energies for \(^{20}\)Ne, \(^{28}\)Ne, and \(^{36}\)Ar nuclei are exactly the same as the shell model results (see also Figure 1b). This can be understood by comparing the number of VAP parameters, \(N_{VAP}\), with the shell model dimension, \(N_{JT}\) (the total number of the independent basis states with good \(J^\pi\)). Here, \(N_{VAP} = 42\). The \(N_{JT}\) values with \(J = 0\) and \(T = 0\) for both \(^{20}\)Ne and \(^{36}\)Ar are only \(21\). For \(^{28}\)Ne, \(N_{JT}\) for \(J = 0\) and \(T = 4\) is \(43\). It looks that when \(N_{JT}\) is less than, or close to \(N_{VAP}\), then the VAP-JTA energy is likely to be the same as the shell model one. Indeed, for all even-even oxygen isotopes and for the \(N = 20\) isotones, for which \(N_{JT} \leq N_{VAP}\), we have obtained \(E_{JTA}^* = E_{SM}\). In Figure 1b, one can also see that the energy difference \(E_{JTA}^* - E_{SM}\) increases with \(N_{JT}\). The largest \(E_{JTA}^* - E_{SM}\) is \(0.446\) MeV is obtained for \(^{26}\)Mg, corresponding to the largest \(N_{JT} = 1132\).

The quadrupole moment and the \(\gamma\) degree of freedom can be extracted using Eqs. (10) and (11). In Fig. 2a, the \(\gamma_{HFB}\) values in HFB are either 0° or 60°, except \(\gamma_{HFB} = 12°\) for \(^{24}\)Mg, thus supporting the conclusion that HFB likely presents axially deformed shapes. In Fig. 2b, the shapes in VAP-TA calculations still remain axially symmetric, except for \(^{26}\)Mg, which has \(\gamma_{JTA}^* = 25.7°\). Quite differently, the \(\gamma_{JTA}^*\) values in the VAP-JTA calculations (Fig. 2c) show that all these nuclei are non-axial without exception. Comparing these results with those of Fig. 2a, one can conclude that the triaxiality in VAP-JTA is definitely a beyond mean-field effect, which is likely to be a universal phenomenon. Fig. 2b, however, excludes the possibility that the isospin projection and the mass projection lead to triaxiality. Thus, the only possible cause of the triaxiality is the beyond mean-field spin projection.

To study directly the effect of spin projection, one can start from a Hartree-Fock (HF) Slater determinant (SD) and perform VAP calculations with only spin projection (called VAP-HF). The converged energies, \(E_{PHF}^*\), relative to \(E_{SM}\), are shown in Fig. 1a. The results show that VAP-HF is better than VAP-TA, and quite close to the VAP-JTA. The quadrupole moment \(Q_{PHF}^*\) and \(\gamma_{PHF}^*\) corresponding to \(E_{PHF}^*\) can be calculated using Eqs. (10) and (11) with \(|\Psi\rangle\) replaced by the converged SD. These quantities are uniquely determined, and are shown in Fig. 2d. Again, all the \(\gamma_{PHF}^*\) values are distributed in the interval (0°, 60°), which is very similar to Fig. 2c. Therefore, we could conclude that VAP results that include spin projection can always be associated with intrinsic states having triaxial deformation.

One more interesting phenomenon, however, is related to the VAP-JTA calculations for \(^{20}\)Ne, \(^{28}\)Ne, and \(^{36}\)Ar. We have shown above that the \(E_{JTA}^*\) energies of these nuclei are the same as the exact shell model results. Surprisingly, the corresponding \(Q_{TA}\) and \(\gamma_{TA}\) values are not unique, which is quite different from other nuclei with \(E_{JTA}^* > E_{SM}\). For example, the results for

| \(|\Psi\rangle\) | \(E_{JTA}^*(\text{MeV})\) | \(Q_{TA}\) | \(\gamma_{TA}\) | \(E_{PHF}^*(\text{MeV})\) | \(Q_{PHF}^*\) | \(\gamma_{PHF}^*\) | \(A\) |
|---|---|---|---|---|---|---|---|
| 1 | -86.919 | -79.879 | 19.057 | -16.963 | -75.644 | 17.520 | -20.068 | -73.845 | 16.281 | -23.506 | 8.000 |
| 2 | -86.919 | -79.879 | 19.057 | -16.963 | -75.644 | 17.510 | -20.119 | -73.830 | 16.264 | -23.604 | 8.000 |
| 3 | -86.919 | -79.879 | 19.057 | -16.963 | -75.644 | 17.482 | -20.119 | -73.830 | 16.264 | -23.604 | 8.000 |

**TABLE II: Similar to Table I but for the VAP-TA calculations.** \(A = 8\) is imposed.

| \(|\Psi\rangle\) | \(E_{JTA}^*(\text{MeV})\) | \(Q_{TA}\) | \(\gamma_{TA}\) | \(E_{PHF}^*(\text{MeV})\) | \(Q_{PHF}^*\) | \(\gamma_{PHF}^*\) | \(A\) |
|---|---|---|---|---|---|---|---|
| 1 | -86.919 | -79.879 | 19.057 | -16.963 | -75.644 | 17.520 | -20.068 | -73.845 | 16.281 | -23.506 | 8.000 |
| 2 | -86.919 | -79.879 | 19.057 | -16.963 | -75.644 | 17.510 | -20.119 | -73.830 | 16.264 | -23.604 | 8.000 |
| 3 | -86.919 | -79.879 | 19.057 | -16.963 | -75.644 | 17.482 | -20.119 | -73.830 | 16.264 | -23.604 | 8.000 |

**TABLE III: Similar to Table I but for the VAP-JTA calculations.** \(A = 8\) is imposed.
TABLE IV: Converged energies and associated shape parameters for even-even sd-shell nuclei calculated with the USDB Hamiltonian.

| Nucleus | \(N_{JT}\) | \(E_{SM}\) (MeV) | \(E_{JT\text{A}}\) (MeV) | \(Q_{JT\text{A}}\) (\(\text{sd}^0\)) | \(\gamma_{JT\text{A}}\) (\(\text{sd}^0\)) | \(E_{TA}\) (MeV) | \(Q_{TA}\) (\(\text{sd}^0\)) | \(\gamma_{TA}\) (\(\text{sd}^0\)) | \(E_{HF\text{B}}\) (MeV) | \(Q_{HF\text{B}}\) (\(\text{sd}^0\)) | \(\gamma_{HF\text{B}}\) (\(\text{sd}^0\)) | \(E_{PH\text{F}}\) (MeV) | \(Q_{PH\text{F}}\) (\(\text{sd}^0\)) | \(\gamma_{PH\text{F}}\) (\(\text{sd}^0\)) |
|---------|----------|----------------|----------------|----------------|----------------|----------|----------------|----------------|----------------|----------------|----------------|----------|----------------|----------------|
| \(^{20}\text{Ne}\) | 21 | -40.472 | -40.472 | – | – | -37.069 | 14.7 | 0.0 | -36.404 | 15.3 | 0.0 | -40.265 | 13.861 | 3.551 |
| \(^{24}\text{Ne}\) | 148 | -57.578 | -57.501 | 12.1 | 13.8 | -54.572 | 15.8 | 0.0 | -53.474 | 16.5 | 0.0 | -56.958 | 15.675 | 6.832 |
| \(^{28}\text{Ne}\) | 287 | -71.725 | -71.570 | 11.0 | 30.1 | -68.084 | 10.1 | 60.0 | -66.402 | 12.0 | 0.0 | -71.037 | 13.449 | 32.786 |
| \(^{26}\text{Ne}\) | 191 | -81.564 | -81.465 | 9.2 | 28.4 | -78.949 | 8.6 | 0.0 | -77.518 | 8.3 | 0.0 | -80.988 | 9.760 | 17.265 |

\(^{20}\text{Ne}\) is shown in Table V. With the same converged \(E_{JT\text{A}}\) = -40.472MeV, one can clearly see that starting with different initial states \(|\Phi_0\rangle\), the result for \(Q_{TA}\) and \(\gamma_{TA}\) could be different. These results indicate that it may not be possible to associate an unique intrinsic deformation with an exact eigenstate of the Hamiltonian.

V. SUMMARY

We implemented an algorithm that performs variation after projection (VAP) on spin, isospin, and mass number of a triaxially deformed Hartree-Fock-Bogoliubov vacuum state. This is the first projected mean field study that includes all these quantum numbers.

We start from a randomly chosen HFB vacuum state and carry out VAP calculations for \(^{24}\text{Mg}\) in sd-shell with various projections. In the VAP-A case the converged solution is independent of the Fermi level (chemical potential). Although the associated HFB vacuum does not have definite quadrupole moment \(Q_{HF\text{B}}\) and triaxial deformation parameter \(\gamma_{HF\text{B}}\), one can use the unique \(Q_A\) and \(\gamma_A\) to describe the intrinsic deformation of the VAP-A state. Similarly, in the VAP-TA calculations, \(Q_A\) and \(\gamma_A\) can not be uniquely determined, but \(Q_{TA}\) and \(\gamma_{TA}\) are unique and can be associated with the intrinsic deformation of the VAP-TA state. It is not possible to directly define deformation parameters \(Q\) and \(\gamma\) for the VAP-JTA wave function, which has the symmetries fully restored, but the \(Q_{TA}\) and \(\gamma_{TA}\) calculated with the VAP-JTA vacuum state \(|\Phi\rangle\) are also unique, and can be associated with the intrinsic deformation of the VAP-JTA state.

Systematical VAP calculations of even-even sd-shell nuclei have been performed using the USDB Hamiltonian. The VAP-JTA energies, \(E_{JT\text{A}}\), are very close to the shell model results, \(E_{SM}\). Moreover, the relative en-
ergy, $E_{TA} - E_{SM}$, increases with the shell model dimension $N_{JT}$. The shapes described by the HFB minima are always axial. However, with spin projection VAP calculations always produce triaxial shapes. We believe that such triaxiality is an universal phenomenon caused by the beyond mean-field dynamic correlations. Finally, we show that those VAP-JTA states reaching the exact shell model results do not have clearly defined intrinsic shapes.

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[1] J.-P. Delaroche, et al., Phys. Rev. C 81, 014303 (2010).
[2] I. Maqbool, J. A. Sheikh, P. A. Ganai and P. Ring, J. Phys. G38 045101 (2011).
[3] R. Rodríguez-Guzmán, Y. Alhassid, and G. F. Bertsch, Phys. Rev C 77, 064308 (2008).
[4] L. M. Robledo and G. F. Bertsch, Phys. Rev. C 84, 014312 (2011).
[5] K.W. Schmid, Prog. Part. Nucl. Phys. 52, 565 (2004).
[6] B. A. Brown and W. A. Richter, Phys. Rev. C 74, 034315 (2006).
[7] P. Ring and P. Schuck, The Nuclear Many-Body Problem (Springer Verlag, New York, Heidelberg, Berlin, 1980).
[8] P.-G. Reinhard, D. J. Dean, W. Nazarewicz, J. Dobaczewski, J. A. Maruhn, and M. R. Strayer, Phys. Rev. C 60, 014316 (1999).
[9] R. R. Rodríguez-Guzmán, J. L. Egido, and L. M. Robledo, Phys. Lett. B474, 15 (2000); Phys. Rev. C 62, 054319 (2000).
[10] Zao-Chun Gao, Mihai Horoi, and Y. S. Chen, Phys. Rev. C80, 034325 (2009).
[11] D. Liu and J. Nocedal, Mathematical Programming B 45, 503 (1989).
[12] D. Kurath, Phys. Rev. C 5, 768 (1972).
[13] W. Koepf and P. Ring, Phys. Lett. B 212, 397 (1988).
[14] P. Bonche, H. Flocard, and P. H. Heenen, Nucl. Phys. A 467, 115 (1987).
[15] R. K. Sheline, I. Ragnarsson, S. Åberg, and A. Watts, J. Phys. G 14, 1201 (1988).
[16] Michael Bender and Paul-Henri Heenen Phys. Rev. C78, 024309 (2008).
[17] J. M. Yao, J. Meng, P. Ring, and D. Vretenar Phys. Rev. C81, 044311 (2010).
[18] Tomás R. Rodríguez and J. L. Egido Phys. Rev. C81, 064323 (2010).
[19] Z.-C. Gao, Q.-L. Hu, Y.S.Chen Phys. Lett. B732, 360 (2014).
[20] Q.-L. Hu, Z.-C. Gao, Y.S.Chen Phys. Lett. B734, 162 (2014).
[21] L.-J. Wang, F.-Q. Chen, T. Mizusaki, M. Oi, Y. Sun, Phys. Rev. C 90 (2014) 011303(R)
[22] K. Hara, Y. Sun, Int. J. Mod. Phys. E 4, 637 (1995).
[23] A. Hayashi, K. Hara, and P. Ring, Phys. Rev. Lett. 53, 337(1984)
[24] K. Enami, K. Tanabe, N. Yoshinaga, and K. Higashiyama, Prog. Theor. Phys. 104, 757 (2000)
[25] J. M. Yao, J.Meng, P. Ring, and D. Pena Arteaga, Phys. Rev. C79, 044312 (2009)]