NEUTRINO MASSES AND THE BARYON ASYMMETRY

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Abstract

Due to sphaleron processes in the high-temperature symmetric phase of the standard model the cosmological baryon asymmetry is related to neutrino properties. For hierarchical neutrino masses, with $B - L$ broken at the unification scale $\Lambda_{\text{GUT}} \sim 10^{16}$ GeV, the observed baryon asymmetry $n_B/s \sim 10^{-10}$ can be naturally explained by the decay of heavy Majorana neutrinos. We illustrate this mechanism with two models of neutrino masses, consistent with the solar and atmospheric neutrino anomalies, which are based on the two symmetry groups $SU(5) \times U(1)_F$ and $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_F$. We also review related cosmological bounds on Majorana neutrino masses and the use of Boltzmann equations.
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1 Introduction

The generation of a cosmological baryon asymmetry can be understood as a consequence of baryon number violation, C and CP violation, and a deviation from thermal equilibrium \[1\]. All these conditions can be naturally satisfied in the context of unified extensions of the standard model of strong and electroweak interactions. In particular the deviation from thermal equilibrium is realized in the out-of-equilibrium decay of heavy particles whose mass is related to the mass scale of unification \[2\]. The presently observed matter-antimatter asymmetry, the ratio of the baryon density to the entropy density of the universe,

\[ Y_B = \frac{n_B - n_B^\overline{\text{B}}}{s} = (0.6 - 1) \cdot 10^{-10}, \]

is then explained as a consequence of the spectrum and interactions of elementary particles, together with the cosmological evolution. A general overview of different scenarios for baryogenesis can be found in \[3\].

\[ \text{Sphaleron} \]

A crucial ingredient of baryogenesis is the connection between baryon number \((B)\) and lepton number \((L)\) in the high-temperature, symmetric phase of the standard model. Due to the chiral nature of the weak interactions \(B\) and \(L\) are not conserved \[4\]. At zero temperature this has no observable effect due to the smallness of the weak coupling. However, as the temperature approaches the critical temperature \(T_c\) of the electroweak phase transition \[5\], \(B\) and \(L\) violating processes come into thermal equilibrium \[6\].

Figure 1: *One of the 12-fermion processes which are in thermal equilibrium in the high-temperature phase of the standard model.*
The rate of these processes is related to the free energy of sphaleron-type field configurations which carry topological charge. In the standard model they lead to an effective interaction of all left-handed fermions [4] (cf. fig. 1),

\[ O_{B+L} = \prod_i (q_{Li}q_{Li}q_{Li}l_{Li}) , \]  

which violates baryon and lepton number by three units,

\[ \Delta B = \Delta L = 3 . \]  

The sphaleron transition rate in the symmetric high-temperature phase has been evaluated by combining an analytical resummation with numerical lattice techniques [7]. The result is, in accord with previous estimates, that \( B \) and \( L \) violating processes are in thermal equilibrium for temperatures in the range

\[ T_{EW} \sim 100 \text{ GeV} < T < T_{SPH} \sim 10^{12} \text{ GeV} . \]  

Sphaleron processes have a profound effect on the generation of the cosmological baryon asymmetry. Eq. 3 suggests that any \( B + L \) asymmetry generated before the electroweak phase transition, i.e., at temperatures \( T > T_{EW} \), will be washed out. However, since only left-handed fields couple to sphalerons, a non-zero value of \( B + L \) can persist in the high-temperature, symmetric phase if there exists a non-vanishing \( B - L \) asymmetry. An analysis of the chemical potentials of all particle species in the high-temperature phase yields the following relation between the baryon asymmetry \( Y_B \) and the corresponding \( L \) and \( B - L \) asymmetries \( Y_L \) and \( Y_{B-L} \), respectively [8],

\[ Y_B = C Y_{B-L} = \frac{C}{C - 1} Y_L . \]  

Here \( C \) is a number \( O(1) \). In the standard model with three generations and one Higgs doublet one has \( C = 28/79 \).

An important ingredient in the theory of baryogenesis is also the nature of the electroweak transition from the high-temperature symmetric phase to the low-temperature Higgs phase. A first-order phase transition yields a departure from thermal equilibrium. Since in the standard model baryon number, \( C \) and CP are not conserved, it is conceivable that the cosmological baryon asymmetry has been generated at the electroweak phase transition [8]. This possibility has stimulated a large theoretical activity during the past years to determine the phase diagram of the electroweak theory.

Electroweak baryogenesis requires that the baryon asymmetry, which is generated during the phase transition, is not erased by sphaleron processes afterwards. This leads
to a condition on the jump of the Higgs vacuum expectation value \( v = \sqrt{\phi^\dagger \phi} \) at the critical temperature [9],

\[
\frac{\Delta v(T_c)}{T_c} > 1.
\] (6)

The strength of the electroweak transition has been studied by numerical and analytical methods as function of the Higgs boson mass. The result for the \( SU(2) \) gauge-Higgs model is shown in fig. 2 [10], where the results of 4d lattice simulations [11], 3d lattice simulations [12] and perturbation theory [13] are compared. For Higgs masses above 50 GeV this is a good approximation for the full standard model. Since the present lower bound from LEP on the Higgs mass has reached almost 110 GeV, it is obvious that the electroweak transition in the standard model is too weak for baryogenesis. For supersymmetric extensions of the standard model a sufficiently strong first-order phase transition can still be achieved for special choices of parameters [14].

For large Higgs masses the nature of the electroweak transition is dominated by non-perturbative effects of the \( SU(2) \) gauge theory at high temperatures. At a critical Higgs mass \( m^c_H = \mathcal{O}(m_W) \) an intriguing phenomenon occurs: the first-order phase transition turns into a smooth crossover [15, 16, 17], as expected on general grounds [10]. This is shown in fig. 3 [18] where \( \lambda/g^2 \approx m^2_H/(8m^2_W) \) is plotted as function of \( \mu^2/g^4 \approx (3m^2_W+m^2_H)/(6\sqrt{2}G_Fm^4_W)(T^2-T^2_0)/T^2 \), with \( T^2_0 = \sqrt{2}m^2_H/(G_F(3m^2_W+m^2_H)) \). For small Higgs masses one has a first-order phase transition at a critical temperature \( T_c(m_H) \). At the endpoint of the line of first-order transitions, which is reached for \( m_H = m^c_H \), the phase transition is of second order [19].
The value of the critical Higgs mass can be estimated by comparing the W-boson mass $m_W$ in the Higgs phase with the magnetic mass $m_{SM}$ in the symmetric phase. For $m_{SM} = C g^2 T$ one obtains [18],

$$m_{cH} = \left( \frac{3}{4\pi C} \right)^{1/2} m_W \approx 74 \text{ GeV},$$

(7)

where we have used $C \simeq 0.35$ [20]. Numerical lattice simulations have determined the precise value $m_{cH} = 72.1 \pm 1.4 \text{ GeV}$ [21]. The estimate (7) can also be extended to the supersymmetric extensions of the standard model, where one obtains for the critical Higgs mass $m_{cH} < 130 \ldots 150 \text{ GeV}$ [22].

The detailed studies of the electroweak phase transition have shown, that for Higgs masses above the present LEP bound of about 110 GeV the cosmological baryon asymmetry did not change during this transition, except possibly for a small parameter range in the supersymmetric standard model. In particular, the electroweak transition may have been just a smooth crossover, without any deviation from thermal equilibrium. In this case it’s sole effect has been to switch off the $B - L$ changing sphaleron processes adiabatically.

Based on the relation (3) between baryon and lepton number we then conclude that $B - L$ violation is needed to explain the cosmological baryon asymmetry if baryogenesis took place before the electroweak transition, i.e. at temperatures $T > T_{EW} \sim 100 \text{ GeV}$. In the standard model, as well as its supersymmetric version and its unified extensions based on the gauge group $SU(5)$, $B - L$ is a conserved quantity. Hence, no baryon asymmetry can be generated dynamically in these models and one has to consider extensions with lepton number violation.
The remnant of lepton number violation at low energies is the appearance of an effective $\Delta L = 2$ interaction between lepton and Higgs fields (cf. fig. 4),

$$\mathcal{L}_{\Delta L=2} = \frac{1}{2} f_{ij} l_i^T \phi C l_j \phi + \text{h.c.}.$$  \hspace{1cm} (8)

Such an interaction arises in particular from the exchange of heavy Majorana neutrinos. In the Higgs phase of the standard model, where the Higgs field acquires a vacuum expectation value, it gives rise to Majorana masses of the light neutrinos $\nu_e$, $\nu_\mu$ and $\nu_\tau$.

Lepton number violation appears to be necessary to understand the cosmological baryon asymmetry. However, as we shall see in the following section, the required lepton number violation can only be weak, otherwise any baryon asymmetry would be washed out. These two conditions are the basis of the interesting constraints which the existence of the matter-antimatter asymmetry imposes on neutrino physics and on extensions of the standard model in general.

### 2 Cosmological bounds on neutrino masses

In the standard model (SM) neutrinos are massless. However, the exchange of heavy particles can give rise to an effective lepton Higgs interaction which, after electroweak symmetry breaking, generates neutrino masses. The lagrangian describing all fermion-Higgs couplings then reads

$$\mathcal{L}_Y = -h_{d_{ij}}^T \overline{d_{Ri}} q_{Lj} H_1 - h_{u_{ij}}^T \overline{u_{Ri}} q_{Lj} H_2 - h_{e_{ij}}^T \overline{e_{Ri}} l_{Lj} H_1 + \frac{1}{2} f_{ij} l_i^T H_2 C l_j H_2 + \text{h.c.}.$$ \hspace{1cm} (9)

Here $q_{Li}$, $u_{Ri}$, $d_{Ri}$, $l_{Li}$, $e_{Ri}$, $i = 1 \ldots N$, are N generations of quark and lepton fields, $H_1$ and $H_2$ are Higgs fields with vacuum expectation values $v_i = \langle H_i^0 \rangle \neq 0$. $h_d$, $h_u$, $h_e$ and $f$ are $N \times N$ complex matrices. For the further discussion it is convenient to choose a basis where $h_u$ and $h_e$ are diagonal and real. In the case $v_1 \sim v_2 \sim v = \sqrt{v_1^2 + v_2^2}$ the smallest Yukawa coupling is $h_{e11} = m_e/v_1$ followed by $h_{u11} = m_u/v_2$. 

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**Figure 4:** Effective lepton number violating interaction.
The mixing of the different quark generations is given by the Kobayashi-Maskawa matrix \( V_d \), which is defined by
\[
h_d^T V_d = \frac{m_d}{v_1}.
\] (10)
Here \( m_d \) is the diagonal real down quark mass matrix, and the weak eigenstates of the right-handed d-quarks have been chosen to be identical to the mass eigenstates. Correspondingly, the mixing matrix \( V_\nu \) in the leptonic charged current is determined by
\[
V_\nu^T f V_\nu = -\frac{m_\nu}{v_2^2},
\] (11)
where
\[
m_\nu = \begin{pmatrix}
m_1 & 0 & 0 \\
0 & m_2 & 0 \\
0 & 0 & m_3
\end{pmatrix}
\] (12)
is the diagonal and real mass matrix of the light Majorana neutrinos.

### 2.1 Chemical equilibrium

In a weakly coupled plasma with temperature \( T \) and volume \( V \) one can assign a chemical potential \( \mu \) to each of the quark, lepton and Higgs fields. In the SM with one Higgs doublet, i.e., \( H_2 = \tilde{H}_1 \equiv \varphi \), and \( N \) generations one has \( 5N+1 \) chemical potentials. The corresponding partition function is
\[
Z(\mu, T, V) = \text{Tr} e^{-\beta(H - \sum_i \mu_i Q_i)}.
\] (13)
Here \( \beta = 1/T \), \( H \) is the Hamilton operator and \( Q_i \) are the charge operators for the corresponding fields. The asymmetry in the particle and antiparticle number densities is then given by the derivative of the thermodynamic potential,
\[
n_i - \overline{n}_i = -\frac{\partial \Omega(\mu, T)}{\partial \mu_i}, \quad \Omega(\mu, T) = -\frac{T}{V} \ln Z(\mu, T, V).
\] (14)
For a non-interacting gas of massless particles one has
\[
n_i - \overline{n}_i = g T^3 \begin{cases}
\beta \mu_i + \mathcal{O}(\beta \mu_i^3), & \text{fermions}, \\
2\beta \mu_i + \mathcal{O}(\beta \mu_i^3), & \text{bosons}.
\end{cases}
\] (15)
The following analysis will be based on these relations for \( \beta \mu_i \ll 1 \). However, one should keep in mind that the plasma of the early universe is very different from a weakly coupled relativistic gas due to the presence of unscreened non-abelian gauge interactions. Hence, non-perturbative effects may be important in some cases.
In the high-temperature plasma quarks, leptons and Higgs bosons interact via Yukawa and gauge couplings and, in addition, via the non-perturbative sphaleron processes. In thermal equilibrium all these processes yield constraints between the various chemical potentials. The effective interaction (2) induced by the $SU(2)$ electroweak instantons yields the constraint \[ \sum_i (3\mu_{qi} + \mu_{li}) = 0 \] \[ (16) \]

One also has to take the $SU(3)$ QCD instanton processes into account [26] which generate the effective interaction

\[ O_A = \prod_i (q_L^i q_L^i u_R^c d_R^c) \] \[ (17) \]

between left-handed and right-handed quarks. The corresponding relation between the chemical potentials reads

\[ \sum_i (2\mu_{qi} - \mu_{ui} - \mu_{di}) = 0 \] \[ (18) \]

A third condition, which is valid at all temperatures, arises from the requirement that the total hypercharge of the plasma vanishes. From eq. (15) and the known hypercharges one obtains

\[ \sum_i \left( \mu_{qi} + 2\mu_{ui} - \mu_{di} - \mu_{li} - \mu_{ei} + \frac{2}{N}\mu_\varphi \right) = 0 \] \[ (19) \]

The Yukawa interactions, supplemented by gauge interactions, yield relations between the chemical potentials of left-handed and right-handed fermions,

\[ \mu_{qi} - \mu_\varphi - \mu_{dj} = 0 \], \[ \mu_{qi} + \mu_\varphi - \mu_{uj} = 0 \], \[ \mu_{li} - \mu_\varphi - \mu_{ej} = 0 \] \[ (20) \]

Furthermore, the $\Delta L = 2$ interaction in (3) implies

\[ \mu_{li} + \mu_\varphi = 0 \] \[ (21) \]

The above relations between chemical potentials hold if the corresponding interactions are in thermal equilibrium. In the temperature range $T_{EW} \sim 100$ GeV < $T$ < $T_{SPH} \sim 10^{12}$ GeV, which is of interest for baryogenesis, this is the case for all gauge interactions. It is not always true, however, for Yukawa interactions. The rate of a scattering process between left- and right-handed fermions, Higgs boson and W-boson,

\[ \psi_L \varphi \rightarrow \psi_R W \] \[ (22) \]

is $\Gamma \sim \alpha \lambda^2 T$, with $\alpha = g^2/(4\pi)$. This rate has to be compared with the Hubble rate,

\[ H \simeq 0.33 \frac{g_\star^{1/2} T^2}{M_{PL}} \simeq 0.1 \frac{g_\star^{1/2} T^2}{10^{18} \text{GeV}} \] \[ (23) \]
The equilibrium condition $\Gamma(T) > H(T)$ is satisfied for sufficiently small temperatures, 
\[ T < T_\lambda \sim \lambda^2 10^{16} \text{ GeV} \ . \] 
(24) 
Hence, one obtains for the decoupling temperatures of right-handed electrons, up-quarks,...,
\[ T_e \sim 10^4 \text{ GeV} , \quad T_u \sim 10^6 \text{ GeV} , \ldots \] 
(25) 
At a temperature $T \sim 10^{10} \text{ GeV}$, which is characteristic of leptogenesis, $e_R \equiv e_{R1}$, $\mu_R \equiv \epsilon_{R2}$, $d_R \equiv d_{R1}$, $s_R \equiv d_{R2}$ and $u_R \equiv u_{R1}$ are out of equilibrium.

Note that the QCD sphaleron constraint (18) is automatically satisfied if the quark Yukawa interactions are in equilibrium (cf. (20)). If the Yukawa interaction of one of the right-handed quarks is too weak, the sphaleron constraint still establishes full chemical equilibrium.

Using eq. (15) also the baryon number density $n_B \equiv BT^2/6$ and the lepton number densities $n_L \equiv LT^2/6$ can be expressed in terms of the chemical potentials. The baryon asymmetry $B$ and the lepton asymmetries $L_i$ read
\[ B = \sum_i (2\mu_{q_i} + \mu_{u_i} + \mu_{d_i}) \ , \] 
(26)
\[ L_i = 2\mu_{l_i} + \mu_{e_l} \ , \quad L = \sum_i L_i \ . \] 
(27)

### 2.2 Relations between $B$, $L$ and $B - L$

Knowing which particle species are in thermal equilibrium one can derive relations between different asymmetries. Consider first the most familiar case where all Yukawa interactions are in equilibrium and the $\Delta L = 2$ lepton-Higgs interaction is out of equilibrium. In this case the asymmetries $L_i - B/N$ are conserved. The Yukawa interactions establish equilibrium between the different generations,
\[ \mu_{l_i} \equiv \mu_l \ , \quad \mu_{q_i} \equiv \mu_q \ , \quad \text{etc.} \] 
(28)

Together with the sphaleron process and the hypercharge constraint they allow to express all chemical potentials, and therefore all asymmetries, in terms of a single chemical potential which may be chosen to be $\mu_l$. The result reads
\[ \mu_e = \frac{2N + 3}{6N + 3} \mu_l \ , \quad \mu_d = -\frac{6N + 1}{6N + 3} \mu_l \ , \quad \mu_u = \frac{2N - 1}{6N + 3} \mu_l \ , \] 
\[ \mu_q = -\frac{1}{3} \mu_l \ , \quad \mu_\varphi = \frac{4N}{6N + 3} \mu_l \ . \] 
(29)
The corresponding baryon and lepton asymmetries are
\[ B = -\frac{4N}{3} \mu_l \ , \quad L = \frac{14N^2 + 9N}{6N + 3} \mu_l \ , \] 
(30)
which yields the well-known connection between the $B$ and $B - L$ asymmetries \[27\]

$$B = \frac{8N + 4}{2N + 13}(B - L). \quad (31)$$

Note, that this relation only holds for temperatures $T \gg v$. In general, the ratio $B/(B - L)$ is a function of $v/T$ \[28, 29\].

Another instructive example is the case where the $\Delta L = 2$ interactions are in equilibrium but the right-handed electrons are not. Depending on the neutrino masses and mixings, this could be the case for temperatures above $T_e \sim 10^4$ GeV \[30\]. Right-handed electron number would then be conserved, and Yukawa and gauge interactions would relate all asymmetries to the asymmetry of right-handed electrons. The various chemical potentials are given by $(\mu_e = \mu_{e1}, \mu_\mu = \mu_{e2} = \ldots = \mu_{eN}),$

$$\begin{aligned}
\mu_e &= -\frac{3}{10N}\mu_e, \\
\mu_d &= -\frac{1}{10N}\mu_e, \\
\mu_u &= \frac{1}{5N}\mu_e, \\
\mu_l &= -\frac{3}{20N}\mu_e, \\
\mu_q &= \frac{1}{20N}\mu_e, \\
\mu_\phi &= \frac{3}{20N}\mu_e.
\end{aligned} \quad (32)$$

The corresponding baryon and lepton asymmetries are \[30\]

$$B = \frac{1}{5}\mu_e, \quad L = \frac{4N + 3}{10N}\mu_e, \quad (33)$$

which yields for the relation between $B$ and $B - L$,

$$B = -\frac{\frac{2N}{2N + 3}(B - L)}{1} \quad (34)$$

Note that although sphaleron processes and $\Delta L = 2$ processes are in equilibrium, the asymmetries in $B$, $L$ and $B - L$ do not vanish!

### 2.3 Constraint on Majorana neutrino masses

The two examples illustrate the connection between lepton number and baryon number induced by sphaleron processes. They also show how this connection depends on other processes in the high-temperature plasma. To have one quark-Higgs or lepton-Higgs interaction out of equilibrium is sufficient in order to have non-vanishing $B$, $L$ and $B - L$. If all interactions in \[9\] are in equilibrium, eqs. \[21\] and \[29\] together imply $\mu_l = 0$ and therefore

$$B = L = B - L = 0, \quad (35)$$

which is inconsistent with the existence of a matter-antimatter asymmetry. Since the equilibrium conditions of the various interactions are temperature dependent, and the $\Delta L = 2$ interaction is related to neutrino masses and mixings, one obtains important
constraints on neutrino properties from the existence of the cosmological baryon asymmetry.

The $\Delta L = 2$ processes described by (8) take place with the rate \[ \Gamma_{\Delta L=2}(T) = \frac{1}{\pi^3} \frac{T^3}{v^4} \sum_{i=e,\mu,\tau} m_{\nu_i}^2 . \] (36)

Requiring $\Gamma_{\Delta L=2}(T) < H(T)$ then yields an upper bound on Majorana neutrino masses,

$$\sum_i m_{\nu_i}^2 < \left( 0.2 \text{ eV} \left( \frac{T_{SPH}}{T} \right)^{1/2} \right)^2 .$$ (37)

For typical leptogenesis temperatures $T \sim 10^{10}$ GeV this bound is comparable to the upper bound on the electron neutrino mass obtained from neutrinoless double beta decay. Note, that the bound also applies to the $\tau$-neutrino mass. However, if one uses for $T$ the decoupling temperature of right-handed electrons, $T_e \sim 10^4$ GeV, the much weaker bound $m_{\nu} < 2$ keV is obtained \[30\].

Clearly, what temperature one has to use in eq. (37) depends on the thermal history of the early universe. Some information is needed on what kind of asymmetries may have been generated as the temperature decreased. This, together with the temperature dependence of the lepton-Higgs interactions then yields constraints on neutrino masses.

### 2.4 Primordial asymmetries

The possible generation of asymmetries can be systematically studied by listing all the higher-dimensional $SU(3) \times SU(2) \times U(1)$ operators which may be generated by the exchange of heavy particles. The dynamics of the heavy particles may then generate an asymmetry in the quantum numbers carried by the massless fields which appear in the operator.

For $d=5$, there is a unique operator, which has already been discussed above,

$$\left( l_{Li}\phi \right) \left( l_{Lj}\phi \right).$$ (38)

It is generated in particular by the exchange of heavy Majorana neutrinos whose coupling to the massless fields is

$$h_{\nu_{ij}} l_{\nu_{i}} l_{\nu_{j}} \phi .$$ (39)

The out-of-equilibrium decays of the heavy neutrinos can generate a lepton asymmetry, which is the well-known mechanism of leptogenesis. The decays yield asymmetries $L_i - B/N$ which are conserved in the subsequent evolution. The initial asymmetry in right-handed electrons is zero. In order to satisfy the out-of-equilibrium condition it is very
important that at least some Yukawa couplings are small and that the right-handed neutrinos carry no quantum numbers with respect to unbroken gauge symmetries.

In order to study possible asymmetries of right-handed electrons one has to consider operators containing $e_R$. A simple example, with $d=6$, reads

\[(q_L d_L)(u_R^c e_R^c).\]  

It can be generated by leptoquark exchange ($\chi \sim (3^*, 1, 1/3)$),

\[(\lambda^0_{ij} q^T_L c_L^j + \lambda^u_{ij} u^T_R C e_R^j)\chi.\]

Note, that $\tilde{\varphi}$ and $\chi$ form a $5^*$-plet of $SU(5)$. In principle, out-of-equilibrium decays of leptoquarks may generate an $e_R$ asymmetry. One may worry, however, whether the branching ratio into final states containing $e_R$ is sufficiently large. Furthermore, it appears very difficult to satisfy the out-of-equilibrium condition since leptoquarks carry colour. Maybe, all these problems can be overcome by making use of coherent oscillations of scalar fields [31] or by special particle production mechanisms after inflation. However, we are not aware of a consistent scenario for the generation of an $e_R$-asymmetry. Hence, it appears appropriate to take the bound eq. (37) on Majorana neutrino masses as a guideline and to examine its validity in each particular model.

### 2.5 Supersymmetry

At temperatures $T$ where some of the interactions are out of thermal equilibrium the effective theory acquires a larger symmetry. Examples discussed above are the $\Delta L = 2$ interactions and the Yukawa couplings of the right-handed electron. In the latter case the symmetry of the effective theory is generated by $Q_{eR}$, the number operator of right-handed electrons. As a consequence, the chemical potential $\mu_e$, i.e. the $e_R$-asymmetry, is a free parameter whose value influences $B$ and $L$.

In the supersymmetric standard model (SSM) one has Yukawa interactions given by the superpotential

\[W = h_{dij}D_i^c Q_j H_1 + h_{uij}U_i^c Q_j H_2 + h_{eij}E_i^c L_j H_1 + \frac{1}{2} f_{ij} L_i H_2 L_j H_2,\]

and mass parameters, the supersymmetric mass term $\mu H_1 H_2$ and soft supersymmetry breaking scalar masses and gaugino masses,

\[L_m = \frac{1}{2} m_\tilde{g}^2 \tilde{g}_L \tilde{g}_L + \frac{1}{2} m_\tilde{w}^2 \tilde{w}_L \tilde{w}_L + \frac{1}{2} m_\tilde{b}^2 \tilde{b}_L \tilde{b}_L + \text{h.c.},\]

for bino, wino and gluino, respectively. Naturally, the gaugino masses and the $\mu$-parameter are of order the gravitino mass, $m_i \sim \mu \sim m_G$. For unbroken supersymmetry,
the SSM lagrangian has two chiral U(1) symmetries in addition to those of the SM, an axial Peccei-Quinn symmetry and R-invariance.

Due to the additional fermions, the higgsinos $h_{L1}, h_{L2}$ and the gauginos, the effective instanton induced interactions are modified. For a Weyl fermion $\psi$ in the adjoint representation of a $SU(N)$ gauge theory the \textquoteleft t Hooft determinant reads,

$$O_{\text{adj}} = \psi_L^1 \cdots \psi_L^{2N} .$$  

For a theory with Weyl fermions in the fundamental and the adjoint representation the \textquoteleft t Hooft determinant is the product of the two single contributions. Hence, in the SSM eqs. (2) and (17) are replaced by

\begin{equation}
\hat{O}_B+L = \prod_i (q_{Li} q_{L1} l_{Li}) \tilde{h}_{L1} \tilde{h}_{L2} \tilde{w}_L \tilde{w}_L \tilde{w}_L \tilde{w}_L ,
\end{equation}

and

\begin{equation}
\hat{O}_A = \prod_i (q_{Li} q_{Li} u_{Ri} d_{Ri}) \tilde{g}_L \tilde{g}_L \tilde{g}_L \tilde{g}_L \tilde{g}_L \tilde{g}_L ,
\end{equation}

respectively. The corresponding relations for the chemical potentials read,

\begin{equation}
\sum_i (3\mu_{qi} + \mu_{li}) + \mu_{\tilde{h}1} + \mu_{\tilde{h}2} + 4\mu_{\tilde{w}} = 0 ,
\end{equation}

and

\begin{equation}
\sum_i (2\mu_{qi} - \mu_{ui} - \mu_{di}) + 6\mu_{\tilde{g}} = 0 .
\end{equation}

Due to the different Higgs content of the SSM also the zero hypercharge constraint is modified,

\begin{equation}
0 = \sum_i (\mu_{qi} + 2\mu_{ui} - \mu_{di} - \mu_{li} - \mu_{ei} + \frac{1}{N}(\mu_{h2} - \mu_{h1}) \\
+ 2\sum_i (\mu_{\tilde{Q}i} + 2\mu_{\tilde{w}} - \mu_{\tilde{d}i} - \mu_{\tilde{l}i} - \mu_{\tilde{e}i} + \frac{1}{2N}(\mu_{\tilde{h}2} - \mu_{\tilde{h}1}) .
\end{equation}

Note, that scalars contribute twice as much as fermions. Furthermore, one obtains from the gaugino interactions,

$$\mu_{\tilde{Q}i} = \mu_{\tilde{b}} + \mu_{\tilde{q}i} = \mu_{\tilde{w}} + \mu_{\tilde{q}i} = \mu_{\tilde{g}} + \mu_{\tilde{q}i} ,$$  

and therefore,

$$\mu_{\tilde{b}} = \mu_{\tilde{w}} = \mu_{\tilde{g}} .$$

The gaugino interactions also determine all chemical potentials of scalars in terms of the chemical potentials of the corresponding fermions.
If all mass terms, i.e., all effects of supersymmetry breaking, are in equilibrium, one has
\[ \mu_{\tilde{h}_1} + \mu_{\tilde{h}_2} = 0 \, , \quad \mu_{\tilde{b}} = \mu_{\tilde{w}} = \mu_{\tilde{g}} = 0 \, . \tag{52} \]
The number of parameters is then the same as in the SM and one obtains essentially the same relations between \( B \), \( L \) and \( B - L \).

However, at sufficiently high temperatures, \( T > T_{SB} \), supersymmetry breaking effects will not be in equilibrium. As an example, consider the case that all Yukawa interactions are in equilibrium, which implies \( \mu_{\tilde{t}_i} = \mu_t \), \( \mu_{\tilde{q}_i} = \mu_q \), etc. The above relations then determine all chemical potentials in terms of \( \mu_t \) and \( \mu_{\tilde{g}} \) \[32\]. The baryon and lepton asymmetries
\[ B = N \left( 2\mu_q + \mu_u + \mu_d \right) + 2N \left( 2\mu_{\tilde{Q}} + \mu_{\tilde{u}} + \mu_{\tilde{d}} \right) \, , \tag{53} \]
\[ L = N \left( 2\mu_L + \mu_e \right) + 2N \left( 2\mu_{\tilde{L}} + \mu_{\tilde{e}} \right) \, , \tag{54} \]
are then given by,
\[ B = -4N\mu_t + \frac{10N - 24}{N} \mu_{\tilde{g}} \, , \tag{55} \]
\[ L = \frac{14N^2 + 9N}{2N + 1} \mu_t + \frac{4N^2 + 18N + 3}{2N + 1} \mu_{\tilde{g}} \, . \tag{56} \]
Clearly, the simple proportionality between \( B \), \( L \) and \( B - L \) is lost. In particular, one obtains for \( B - L = 0 \),
\[ B = \frac{16N^3 + 212N^2 - 234N - 216}{22N^2 + 13N} \mu_{\tilde{g}} \, , \tag{57} \]
which yields \( B = 6\mu_{\tilde{g}} \) for \( N = 3 \). Hence, at temperatures \( T > T_{SB} \) also an asymmetry in gauginos affects the baryon asymmetry.

### 2.6 R-parity violating interactions

Like for Majorana neutrinos, cosmological bounds can also be derived on the strength of new baryon and lepton number changing interactions \[33, 34, 35\]. Of particular interest are R-parity violating interactions in supersymmetric theories which allow single production of supersymmetric particles at colliders.

Consider, as an example, the \( \Delta L = 1 \) Yukawa couplings
\[ W_{\Delta R} = \lambda_{ijk} D_i^c Q_j L_k \, , \tag{58} \]
which may appear natural since the Higgs multiplet \( H_1 \) and the lepton multiplets \( L_k \) have the same gauge quantum numbers. Requiring \( T_\lambda < T_{EW} \sim 100 \text{ GeV} \) and neglecting the masses of scalar quarks and leptons, one obtains from eq. \[24\]
\[ \lambda < 10^{-7} \, , \tag{59} \]
which would make the interaction (58) unobservable in collider experiments. A detailed study of $2 \rightarrow 1$ and $2 \rightarrow 2$ processes taking finite mass effects into account yields bounds slightly less stringent than (59).

It is important, however, that the bound (58) does not apply to all couplings $\lambda_{ijk}$. It is sufficient that $B - 3L_i$ is effectively conserved for a single lepton flavour. If the couplings $\lambda_{ijk}$ have such a flavour structure and if a $B - 3L_i$ asymmetry is indeed generated a primordial baryon asymmetry can be preserved. Since the corresponding couplings $\lambda$ are smaller than all other Yukawa interactions, the generation of a $B - 3L_i$ asymmetry is indeed conceivable.

### 2.7 Finite mass effects

The relation (15) between asymmetries and chemical potentials holds for an ideal relativistic gas. It is modified due to interactions. To leading order in the couplings their effect can be expressed in terms of thermal masses [35, 36, 37]. In the symmetric, high-temperature phase one finds [37],

\[
    n_i - \bar{n}_i = \frac{g T^2}{6} \mu_i \left( 1 - \frac{3 m_i^2(T)}{\pi^2 T^2} \right). \tag{60}
\]

For leptons, for instance, the thermal masses are given by

\[
    m_{Li}^2(T) = \left( \frac{3}{32} g_2^2 + \frac{1}{32} g_1^2 + \frac{1}{6} h_{ei}^2 \right),
\]

\[
    m_{Ri}^2(T) = \left( \frac{1}{8} g_1^2 + \frac{1}{8} h_{ei}^2 \right), \tag{61}
\]

where $g_1$ and $g_2$ are the $U(1)$ and $SU(2)$ gauge couplings, respectively.

The thermal masses also affect the relation between baryon and lepton numbers. In particular, one can have $B \neq 0$ with $B - L = 0$ if the lepton asymmetry is flavour dependent. In the symmetric phase one finds [37],

\[
    B = \frac{517}{2844 \pi^2} h_{r}^2 (L_\mu - L_\tau + L_\mu - L_\tau) \\
    \approx 9 \cdot 10^{-7} (L_\mu - L_\tau + L_\mu - L_\tau). \tag{62}
\]

Although this finite mass effect is very small, it can have interesting consequences if large flavour dependent lepton asymmetries are generated in a theory where $B - L$ is conserved.
3 Baryogenesis and neutrino masses

3.1 Baryogenesis through Majorana neutrino decays

As discussed in section 1, baryogenesis before the electroweak transition requires \(B - L\) violation, and therefore \(L\) violation. Lepton number violation is most simply realized by adding right-handed Majorana neutrinos to the standard model. Heavy right-handed Majorana neutrinos, whose existence is predicted by all extensions of the standard model containing \(B - L\) as a local symmetry, can also explain the smallness of the light neutrino masses via the see-saw mechanism \[38\].

The most general Lagrangian for couplings and masses of charged leptons and neutrinos reads

\[
L_Y = -h_{ej} \bar{e}_R^e l_j H_1 - h_{\nu ij} \bar{\nu}_R^i l_j H_2 - \frac{1}{2} h_{\nu ij} \bar{\nu}_R^i \nu_R^j R + \text{h.c.} .
\]

The vacuum expectation values of the Higgs fields, \(\langle H_1 \rangle = v_1\) and \(\langle H_2 \rangle = v_2 = \tan \beta \ v_1\), generate Dirac masses \(m_e\) and \(m_D\) for charged leptons and neutrinos, \(m_e = h_e v_1\) and \(m_D = h_\nu v_2\), respectively, which are assumed to be much smaller than the Majorana masses \(M = h_r \langle R \rangle\). This yields light and heavy neutrino mass eigenstates

\[
\nu \simeq V^T \nu_L + \nu^c L \nu^* , \quad N \simeq \nu_R + \nu^c R ,
\]

with masses

\[
m_\nu \simeq -V^T \mu \frac{1}{M} m_D V_\nu \quad \text{and} \quad m_N \simeq M .
\]

Here \(V_\nu\) is the mixing matrix in the leptonic charged current (cf. eqs. \[3\] and \[11\]).

The right-handed neutrinos, whose exchange may erase any lepton asymmetry, can also generate a lepton asymmetry by means of out-of-equilibrium decays. This lepton asymmetry is then partially transformed into a baryon asymmetry by sphaleron processes \[39\]. The decay width of the heavy neutrino \(N_i\) reads at tree level,

\[
\Gamma_{Di} = \Gamma (N_i \rightarrow H_2 + l) + \Gamma (N_i \rightarrow H_2^c + l^c) = \frac{1}{8\pi} (h_\nu h_\nu^c)_{ii} M_i .
\]

A necessary requirement for baryogenesis is the out-of-equilibrium condition \(\Gamma_{D1} < H|\_T=M_1\) \[40\], where \(H\) is the Hubble parameter at temperature \(T\). From the decay width \[63\] one then obtains an upper bound on an effective light neutrino mass \[34, 41\],

\[
\tilde{m}_1 = (h_\nu h_\nu^c)_{11} \frac{v_2^2}{M_1} \simeq 4 g_*^{1/2} \frac{v_2^2}{M_P} \frac{\Gamma_{D1}}{H} \bigg|_{T=M_1} < 10^{-3} \text{eV} .
\]

Here \(g_*\) is the number of relativistic degrees of freedom, \(M_P = (8\pi G_N)^{-1/2} \simeq 2.4 \cdot 10^{18}\) GeV is the Planck mass, and we have assumed \(g_* \simeq 100, v_2 \simeq 174\) GeV. More direct
bounds on the light neutrino masses depend on the structure of the Dirac neutrino mass matrix. The bound \((\ref{eq:bound})\) implies that the heavy neutrinos are not able to follow the rapid change of the equilibrium particle distribution once the temperature of the universe drops below the mass \(M_1\). Hence, the deviation from thermal equilibrium consists in a too large number densities of heavy neutrinos, as compared to the equilibrium density.

Eventually, however, the neutrinos will decay, and a lepton asymmetry is generated due to the CP asymmetry which comes about through interference between the tree-level amplitude and the one-loop diagrams shown in fig. 5. In a basis, where the right-handed neutrino mass matrix \(M = h_r \langle R \rangle\) is diagonal, one obtains

\[
\varepsilon_1 = \frac{\Gamma(N_1 \rightarrow l H_2) - \Gamma(N_1 \rightarrow l^c H_2^c)}{\Gamma(N_1 \rightarrow l H_2) + \Gamma(N_1 \rightarrow l^c H_2^c)} \approx \frac{1}{8\pi} \frac{1}{(h_{\nu} h_{\nu}^*)_{11}} \sum_{i=2,3} \text{Im} \left[ (h_{\nu} h_{\nu}^*)_{1i}^2 \right] \left[ f \left( \frac{M_i^2}{M_1^2} \right) + g \left( \frac{M_i^2}{M_1^2} \right) \right]; \tag{68}
\]

where \(f\) is the contribution from the one-loop vertex correction,

\[
f(x) = \sqrt{x} \left[ 1 - (1 + x) \ln \left( \frac{1 + x}{x} \right) \right], \tag{69}\]

and \(g\) denotes the contribution from the one-loop self energy \([42, 43, 44]\), which can be reliably calculated in perturbation theory for sufficiently large mass splittings, i.e., \(|M_i - M_1| \gg |\Gamma_i - \Gamma_1|\),

\[
g(x) = \frac{\sqrt{x}}{1 - x}. \tag{70}\]

For \(M_1 \ll M_2, M_3\) one obtains

\[
\varepsilon_1 \approx -\frac{3}{8\pi} \frac{1}{(h_{\nu} h_{\nu}^*)_{11}} \sum_{i=2,3} \text{Im} \left[ (h_{\nu} h_{\nu}^*)_{1i}^2 \right] \frac{M_1}{M_i}. \tag{71}\]

In the case of mass differences of order the decay widths one expects an enhancement from the self-energy contribution \([42]\), although the influence of the thermal bath on this effect is presently unclear.

The CP asymmetry \((\ref{eq:cp asymmetry})\) then leads to a lepton asymmetry \([40]\),

\[
Y_L = \frac{n_L - n_R}{s} = \kappa \frac{\varepsilon_1}{g^*}. \tag{72}\]
Here the factor $\kappa < 1$ represents the effect of washout processes. In order to determine $\kappa$ one has to solve the full Boltzmann equations \[47, 48\]. In the examples discussed in section 3 one obtains $\kappa \simeq 10^{-1} \ldots 10^{-3}$.

In a complete analysis one also has to consider washout processes. Particularly important are $\Delta L = 2$ lepton Higgs scatterings mediated by heavy neutrinos (cf. fig. 4) since cancellations between on-shell contributions to these scatterings and contributions from neutrino decays and inverse decays ensure that no asymmetry is generated in thermal equilibrium \[49, 50\].

Further, due to the large top-quark Yukawa coupling one has to take into account neutrino top-quark scatterings mediated by Higgs bosons \[17, 48\]. As we will see in section 5.1, these processes are of crucial importance for leptogenesis, since they can create a thermal population of heavy neutrinos at high temperatures $T > M_1$. As the CP asymmetry can be interpreted as a mean lepton asymmetry produced per neutrino decay, the requested baryon asymmetry can only be generated if the neutrinos are sufficiently numerous before decaying.

Various extensions of the standard model have been considered in early studies of leptogenesis \[51\]-\[63\]. In particular, it is intriguing that in the simple case of hierarchical heavy neutrino masses the observed value of the baryon asymmetry is obtained without any fine tuning of parameters if $B - L$ is broken at the unification scale, $\Lambda_{GUT} \sim 10^{16}$ GeV. The corresponding light neutrino masses are very small, i.e., $m_{\nu_2} \sim 3 \cdot 10^{-3}$ eV, as preferred by the MSW explanation of the solar neutrino deficit, and $m_{\nu_3} \sim 0.1$ eV \[58\]. Such small neutrino masses are also consistent with the atmospheric neutrino anomaly \[64\], which implies a small mass $m_{\nu_3}$ in the case of hierarchical neutrino masses. This fact gave rise to a renewed interest in leptogenesis in recent years, and a number of interesting models have been suggested \[65\]-\[81\].

### 3.2 Neutrino masses and mixings

Leptogenesis relates the cosmological baryon asymmetry and neutrino masses and mixings. The predicted value of the baryon asymmetry depends on the CP asymmetry \(68\) which is determined by the Dirac and the Majorana neutrino mass matrices. Depending on the neutrino mass hierarchy and the size of the mixing angles the CP asymmetry can vary over many orders of magnitude. It is therefore important to see whether patterns of neutrino masses \[82\] motivated by other considerations are consistent with leptogenesis. In the following we shall consider two examples.

An attractive framework to explain the observed mass hierarchies of quarks and charged leptons is the Froggatt-Nielsen mechanism \[83\] based on a spontaneously bro-
ken $U(1)_F$ generation symmetry. The Yukawa couplings arise from non-renormalizable interactions after a gauge singlet field $\Phi$ acquires a vacuum expectation value,

$$h_{ij} = g_{ij} \left( \frac{\langle \Phi \rangle}{\Lambda} \right)^{Q_i + Q_j}.$$  \hfill (73)

Here $g_{ij}$ are couplings $O(1)$ and $Q_i$ are the $U(1)$ charges of the various fermions, with $Q_\Phi = -1$. The interaction scale $\Lambda$ is usually chosen to be very large, $\Lambda > \Lambda_{GUT}$. In the following we shall discuss two different realizations of this idea which are motivated by the atmospheric neutrino anomaly \[64\]. Both scenarios have a large $\nu_\mu - \nu_\tau$ mixing angle. They differ, however, by the symmetry structure and by the size of the parameter $\epsilon$ which characterizes the flavour mixing.

### 3.2.1 $SU(5) \times U(1)_F$

This symmetry has been considered by a number of authors. Particularly interesting is the case with a non-parallel family structure where the chiral $U(1)_F$ charges are different for the $5^*$-plets and the $10$-plets of the same family \[84\]-\[88\]. An example of possible charges $Q_i$ is given in table 1.

The assignment of the same charge to the lepton doublets of the second and third generation leads to a neutrino mass matrix of the form \[84, 85\],

$$m_{\nu_{ij}} \sim \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \frac{\nu_2^2}{\langle R \rangle}. \hfill (74)$$

This structure immediately yields a large $\nu_\mu - \nu_\tau$ mixing angle. The phenomenology of neutrino oscillations depends on the unspecified coefficients $O(1)$. The parameter $\epsilon$ which gives the flavour mixing is chosen to be

$$\frac{\langle \Phi \rangle}{\Lambda} = \epsilon \sim \frac{1}{17}. \hfill (75)$$

The three Yukawa matrices for the leptons have the structure,

$$h_e, h_\nu \sim \begin{pmatrix} \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \end{pmatrix}, \quad h_\tau \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}. \hfill (76)$$

| $Q_i$ | $e_{R3}$ | $e_{R2}$ | $e_{R1}$ | $l_{L3}$ | $l_{L2}$ | $l_{L1}$ | $\nu_{R3}$ | $\nu_{R2}$ | $\nu_{R1}$ |
|-------|---------|---------|---------|---------|---------|---------|----------|----------|----------|
| 0     | 1       | 2       | 0       | 0       | 1       | 0       | 1        | 1        | 2        |

Table 1: Chiral charges of charged and neutral leptons with $SU(5) \times U(1)_F$ symmetry \[88\].
Table 2: Chiral charges of charged and neutral leptons with \( SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_F \) symmetry \[87\].

| \( \psi_i \) | \( e^c_{R3} \) | \( e^c_{R2} \) | \( e^c_{R1} \) | \( l_{L3} \) | \( l_{L2} \) | \( l_{L1} \) | \( \nu^c_{R3} \) | \( \nu^c_{R2} \) | \( \nu^c_{R1} \) |
|---|---|---|---|---|---|---|---|---|---|
| \( Q_i \) | 0 | \( \frac{1}{2} \) | \( \frac{5}{2} \) | 0 | \( \frac{1}{2} \) | \( \frac{5}{2} \) | 0 | \( \frac{1}{2} \) | \( \frac{5}{2} \) |

Note, that \( h_e \) and \( h_\nu \) have the same, non-symmetric structure. One easily verifies that the mass ratios for charged leptons, heavy and light Majorana neutrinos are given by

\[
m_e : m_\mu : m_\tau \sim \epsilon^3 : \epsilon^2 : 1, \quad M_1 : M_2 : M_3 \sim \epsilon^4 : \epsilon^2 : 1, \tag{77}
\]

\[
m_1 : m_2 : m_3 \sim \epsilon^2 : 1 : 1. \tag{78}
\]

The masses of the two eigenstates \( \nu_2 \) and \( \nu_3 \) depend on unspecified factors of order one, and may differ by an order of magnitude \[87, 88\]. They can therefore be consistent with the mass differences \( \Delta m^2_{\nu_1\nu_2} \approx 4 \cdot 10^{-6} - 1 \cdot 10^{-5} \) eV\(^2\) \[91\] inferred from the MSW solution of the solar neutrino problem \[92\] and \( \Delta m^2_{\nu_2\nu_3} \approx (5 \cdot 10^{-4} - 6 \cdot 10^{-3}) \) eV\(^2\) associated with the atmospheric neutrino deficit \[93\]. For numerical estimates we shall use the average of the neutrino masses of the second and third family, \( \bar{m}_\nu = (m_{\nu_2}m_{\nu_3})^{1/2} \sim 10^{-2} \) eV.

The choice of the charges in table 1 corresponds to large Yukawa couplings of the third generation. For the mass of the heaviest Majorana neutrino one finds

\[
M_3 \sim \frac{v_3^2}{m_\nu} \sim 10^{15} \text{ GeV}. \tag{79}
\]

Since \( h_{r33} \) and the gauge coupling of \( U(1)_{B-L} \) are \( \mathcal{O}(1) \), this implies that \( B-L \) is broken at the unification scale \( \Lambda_{GUT} \).

### 3.2.2 \( SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_F \)

This symmetry arises in unified theories based on the gauge group \( E_6 \). The leptons \( e^c_R, l_L \) and \( \nu^c_R \) are contained in a single \( (1, 3, \bar{3}) \) representation. Hence, all leptons of the same generation have the same \( U(1)_F \) charge and all leptonic Yukawa matrices are symmetric. Masses and mixings of quarks and charged leptons can be successfully described by using the charges given in table 2 \[87\]. Clearly, the three Yukawa matrices have the same structure\(^1\).

\[
h_e, h_\tau \sim \begin{pmatrix} \epsilon^5 & \epsilon^3 & \epsilon^{5/2} \\ \epsilon^3 & \epsilon & \epsilon^{1/2} \\ \epsilon^{5/2} & \epsilon^{1/2} & 1 \end{pmatrix}, \quad h_\nu \sim \begin{pmatrix} \bar{\epsilon}^5 & \bar{\epsilon}^3 & \bar{\epsilon}^{5/2} \\ \bar{\epsilon}^3 & \bar{\epsilon} & \bar{\epsilon}^{1/2} \\ \bar{\epsilon}^{5/2} & \bar{\epsilon}^{1/2} & 1 \end{pmatrix}, \tag{80}
\]

\(^1\)Note, that with respect to ref. \[87\], \( \epsilon \) and \( \tau \) have been interchanged.
but the expansion parameter in $h_\nu$ may be different from the one in $h_e$ and $h_r$. From the quark masses, which also contain $\epsilon$ and $\bar{\epsilon}$, one infers $\bar{\epsilon} \simeq \epsilon^2$.

From eq. (80) one obtains for the masses of charged leptons, light and heavy Majorana neutrinos,

$$m_e : m_\mu : m_\tau \sim M_1 : M_2 : M_3 \sim \epsilon^5 : \epsilon : 1,$$

$$m_1 : m_2 : m_3 \sim \epsilon^{15} : \epsilon^3 : 1.$$  

Like in the example with $SU(5) \times U(1)_F$ symmetry, the mass of the heaviest Majorana neutrino,

$$M_3 \sim \frac{v_2^2}{m_3} \sim 10^{15} \text{ GeV},$$

implies that $B - L$ is broken at the unification scale $\Lambda_{GUT}$.

The $\nu_\mu - \nu_\tau$ mixing angle is related to the mixing of the charged leptons of the second and third generation [87],

$$\sin \Theta_{\mu\tau} \sim \sqrt{\bar{\epsilon} + \epsilon}.$$  

This requires large flavour mixing,

$$\left(\frac{\langle \Phi \rangle}{\Lambda}\right)^{1/2} = \sqrt{\epsilon} \sim \frac{1}{2}.$$  

In view of the unknown coefficients $O(1)$ the corresponding mixing angle $\sin \Theta_{\mu\tau} \sim 0.7$ is consistent with the interpretation of the atmospheric neutrino anomaly as $\nu_\mu - \nu_\tau$ oscillation.

It is very instructive to compare the two scenarios of lepton masses and mixings described above. In the first case, the large $\nu_\mu - \nu_\tau$ mixing angle follows from a non-parallel flavour symmetry. The parameter $\epsilon$, which characterizes the flavour mixing, is small. In the second case, the large $\nu_\mu - \nu_\tau$ mixing angle is a consequence of the large flavour mixing $\epsilon$. The $U(1)_F$ charges of all leptons are the same, i.e., one has a parallel family structure. Also the mass hierarchies, given in terms of $\epsilon$, are rather different. This illustrates that the separation into a flavour mixing parameter $\epsilon$ and coefficients $O(1)$ is far from unique. It is therefore important to study other observables which depend on the lepton mass matrices. This includes lepton-flavour changing processes and, in particular, also the cosmological baryon asymmetry.
4 Boltzmann equations and scattering processes

4.1 Boltzmann equations

A full quantum mechanical description of baryogenesis has to take into account the interplay of all processes in the plasma, i.e. decays, inverse decays and scattering processes together with the time evolution of the system. Such a quantum mechanical treatment may be based either on the time evolution of the density matrix [93] or on the Kadanoff-Baym equations for the Green functions of the system. In the latter case a systematic perturbative expansion has recently been obtained which starts from a set of Boltzmann equations for distribution functions [94]. So far, however, all detailed studies of baryogenesis are based on the Boltzmann equations for number densities. In this section we therefore briefly review the basic ingredients of this approach. In particular it is assumed that between scatterings the particles move freely in the gravitational field of the expanding universe and that the interactions are described by quantum field theory at zero temperature.

Consider first a point-particle with mass $m \geq 0$ moving freely in a gravitational field. Its coordinates in phase space $x^\mu$, $p^\mu$ obey the geodesic equations of motion [95]

$$\frac{dp^\mu}{d\tau} + \Gamma^\mu_{\nu\alpha} p^\nu p^\alpha = 0,$$

$$\frac{dx^\mu}{d\tau} = p^\mu. \quad (86)$$

If $p^\mu$ is the 4-momentum of the particle, the affine parameter $\tau$ is uniquely determined, except for its origin. For a massive particle, $m > 0$, $s = m\tau$ is the proper time. With the Liouville operator

$$L = p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial}{\partial p^\alpha}, \quad (88)$$

the geodesic equations can be written as

$$\frac{dp^\mu}{d\tau} = L \left[ p^\mu \right], \quad (89)$$

$$\frac{dx^\mu}{d\tau} = L \left[ x^\mu \right]. \quad (90)$$

Consider now a gas of non-interacting particles, i.e. of particles whose motion is determined by the equations (89) and (90). Due to the absence of interactions, the phase space density $f_\psi(x, p)$ has to be constant, i.e. its total derivative with respect to the parameter $\tau$ has to vanish,

$$\frac{df_\psi(x, p)}{d\tau} = 0. \quad (91)$$
If we restrict the Liouville operator (88) to the mass-shell of these particles,

\[ L_m = p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma^i_{\beta\gamma} p^\beta p^\gamma \frac{\partial}{\partial p^i}, \]  

where \(i\) runs over spatial indices 1 to 3, the collisionless Boltzmann equation (91) reads

\[ L_m \left[ f_\psi(x, p) \right] = 0. \]  

In a spatially homogeneous and isotropic universe, described by the Robertson-Walker metric, the phase space density \( f_\psi \) can only be a function of \( t \) and \(|\vec{p_\psi}|\). The Boltzmann equation is then given by

\[ L_m [ f_\psi] = E_\psi \frac{\partial f_\psi}{\partial t} - H |\vec{p_\psi}|^2 \frac{\partial f_\psi}{\partial E_\psi} = 0, \]  

where \( E_\psi = \sqrt{\vec{p_\psi}^2 + m_\psi^2} \) and \( H \) is the Hubble parameter. The equilibrium phase space density \( f_{eq} = \exp \left( -E_\psi / T \right) \) solves this equation only in the limit \( m_\psi \to 0 \) or \( m_\psi \to \infty \), i.e., only extremely relativistic or non-relativistic particles can be in thermal equilibrium in a Robertson-Walker universe. This is due to the fact that the Robertson-Walker metric has no timelike spatially constant Killing vector [96]. Other solutions of (94) are the Bose-Einstein and the Fermi-Dirac distributions of massless particles.

However, one can come close to thermal equilibrium by including interactions, which are described by a collision term \( C \left[ f_\psi \right] \) in the Boltzmann equation:

\[ E_\psi \frac{\partial f_\psi}{\partial t} - H |\vec{p_\psi}|^2 \frac{\partial f_\psi}{\partial E_\psi} = C \left[ f_\psi \right]. \]  

Integrating over the phase space element

\[ \mathrm{d} \vec{p_\psi} = \frac{d^3 p_\psi}{(2\pi)^3 2 E_\psi}, \]  

yields

\[ \dot{n}_\psi + 3H n_\psi = \frac{g_\psi}{(2\pi)^3} \int \frac{d^3 p_\psi}{E_\psi} C \left[ f_\psi \right], \]  

where we have made use of the mass shell condition and the spatial isotropy, and \( n_\psi \) is the number density.

The collision term counts the number of collisions a particle \( \psi \) undergoes in a time and volume element. For the process \( \psi + a + b + \ldots \to i + j + \ldots \) it is given by [97, 49, 47]:

\[ \gamma (\psi + a + b + \ldots \to i + j + \ldots) := -\frac{g_\psi}{(2\pi)^3} \int \frac{d^3 p_\psi}{E_\psi} C \left[ f_\psi \right] \]

\[ = \int \mathrm{d} \vec{p}_\psi \mathrm{d} \vec{p}_a \mathrm{d} \vec{p}_b \ldots \mathrm{d} \vec{p}_i \mathrm{d} \vec{p}_j \ldots (2\pi)^4 \delta^4 (p_\psi + p_a + p_b + \ldots - p_i - p_j - \ldots) \]

\[ \times |\mathcal{M} (\psi + a + b + \ldots \to i + j + \ldots)|^2 f_\psi f_a f_b \ldots (1 \pm f_i) (1 \pm f_j) \ldots, \]
where the squared matrix element $|\mathcal{M} (\psi + a + b + \ldots \to i + j + \ldots)|^2$ has to be summed over internal degrees of freedom of incoming and outgoing particles. A symmetry factor $1/n!$ has to be included if there are $n$ identical incoming or outgoing particles.

In a dilute gas the factors $(1 \pm f_i)$, where the upper (lower) sign refers to bosons (fermions), can be neglected, and we only have to consider decays $\psi \to i + j + \ldots$, two particle scatterings $\psi + a \to i + j + \ldots$, and the corresponding back reactions. Then the Boltzmann equation for $n_\psi$ reads:

$$\dot{n}_\psi + 3H n_\psi = - \sum_{i,j,\ldots} [\gamma (\psi \to i + j + \ldots) - \gamma (i + j + \ldots \to \psi)]$$

$$- \sum_{a,i,j,\ldots} [\gamma (\psi + a \to i + j + \ldots) - \gamma (i + j + \ldots \to \psi + a)].$$

(99)

Let us consider the decay term first. From the definition (98) it follows that

$$\sum_{i,j,\ldots} [\gamma (\psi \to i + j + \ldots) - \gamma (i + j + \ldots \to \psi)]$$

$$= \sum_{i,j,\ldots} \int d\tilde{p}_\psi d\tilde{p}_i d\tilde{p}_j \ldots (2\pi)^4 \delta^4 (p_\psi - p_i - p_j - \ldots) \times$$

$$\left[ |\mathcal{M} (\psi \to i + j + \ldots)|^2 f_\psi - |\mathcal{M} (i + j + \ldots \to \psi)|^2 f_i f_j \ldots \right].$$

(100)

In thermal equilibrium we have $f_i f_j \ldots = f_\psi$ because of energy conservation. Hence, due to the unitarity of the $S$-matrix, this decay term vanishes in thermal equilibrium. The same holds true for scattering processes, even if quantum corrections are included [49, 97, 94].

Further, one can distinguish elastic and inelastic scatterings. Elastic scatterings only affect the phase space densities of the particles but not the number densities, whereas inelastic scatterings do change the number densities. If elastic scatterings do occur at a higher rate than inelastic scatterings one can assume kinetic equilibrium, i.e., the phase space density is

$$f_\psi (E_\psi, T) = \frac{n_\psi}{n_{\psi eq}} e^{-E_\psi / T}.$$  

(101)

The Boltzmann equation (99) then takes the form

$$\dot{n}_\psi + 3H n_\psi =$$

$$- \sum_{i,j,\ldots} \left[ \frac{n_\psi}{n_{\psi eq}} \gamma^{eq} (\psi \to i + j + \ldots) - \frac{n_i n_j \ldots}{n_{\psi eq} n_i n_j} \gamma^{eq} (i + j + \ldots \to \psi) \right]$$

$$- \sum_{a,i,j,\ldots} \left[ \frac{n_\psi n_a}{n_{\psi eq} n_a eq} \gamma^{eq} (\psi + a \to i + j + \ldots) - \frac{n_i n_j \ldots}{n_{\psi eq} n_i n_j} \gamma^{eq} (i + j + \ldots \to \psi + a) \right],$$

where $\gamma^{eq}$ is the space time density of a given process in thermal equilibrium. Note, that elastic scatterings no longer contribute to the evolution of $n_\psi$.  

25
It is convenient to replace the particle density $n_\psi$ by a quantity which is not affected by the expansion of the universe. Consider the number of particles in a comoving volume element, i.e., the ratio of particle density and entropy density $s$,

$$Y_\psi \equiv \frac{n_\psi}{s}.$$  \hfill (103)

If the universe expands isentropically,

$$\dot{n}_\psi + 3Hn_\psi = s\dot{Y}_\psi.$$  \hfill (104)

Hence, $Y_\psi$ is not affected by the expansion, it can only be changed by interactions. Further, it is useful to transform to the dimensionless evolution variable $z = m_\psi / T$. In a radiation dominated universe the Boltzmann equation (102) then finally reads

$$\frac{sH(m_\psi)}{z} \frac{dY_\psi}{dz} = \sum_{i,j,...} \left[ \frac{Y_\psi Y_{eq}^i}{Y_{eq}^i Y_{eq}^j} \gamma^{eq}(\psi \rightarrow i + j + \ldots) - \frac{Y_i Y_j \ldots}{Y_{eq}^i Y_{eq}^j} \gamma^{eq}(i + j + \ldots \rightarrow \psi) \right] - \sum_{a,i,j,...} \left[ \frac{Y_\psi Y_a}{Y_{eq}^i Y_{eq}^j} \gamma^{eq}(\psi + a \rightarrow i + j + \ldots) - \frac{Y_i Y_j \ldots}{Y_{eq}^i Y_{eq}^j} \gamma^{eq}(i + j + \ldots \rightarrow \psi + a) \right],$$  \hfill (105)

where $H(m_\psi)$ is the Hubble parameter at $T = m_\psi$.

### 4.2 Reaction densities

The reaction densities in thermal equilibrium $\gamma^{eq}$ can be further simplified. The decay width of a particle with energy $E$ is

$$\tilde{\Gamma} = \frac{m}{E} \tilde{\Gamma}_{rs},$$  \hfill (106)

where $\tilde{\Gamma}_{rs}$ denotes the decay width in the rest system. The reaction density for the decay is then given by

$$\gamma^{eq}_D = n^{eq}_\psi \frac{K_1(z)}{K_2(z)} \tilde{\Gamma}_{rs};$$  \hfill (107)

here the ratio of Bessel functions is the thermal average of the time dilatation factor $m/E$ in eq. \hfill (106).

The decay rate $\Gamma_D$ for the particle $\psi$ is defined as the number of decays per time element, i.e., it is given by the ratio of $\gamma^{eq}_D$ and the number density $n^{eq}_\psi$:

$$\Gamma_D = \frac{K_1(z)}{K_2(z)} \tilde{\Gamma}_{rs}.$$  \hfill (108)
In the low and high temperature limits the decay rate is given by [9]:

\[
\Gamma_D \approx \begin{cases} 
\Gamma_{rs}, & T \ll m_\psi \\
\frac{m_\psi}{2T} \tilde{\Gamma}_{rs}, & T \gg m_\psi
\end{cases}
\] (109)

i.e., due to time dilatation, decays are suppressed at high temperatures.

In the Boltzmann equation (102) the expansion term 3Hn_\psi is responsible for deviations from thermal equilibrium, whereas interaction terms try to bring the system into equilibrium. Hence, the ratio of reaction rates \( \Gamma \) to the Hubble parameter \( H \) is a measure for the effectiveness of the interactions. If the reaction rates are too small the system will be driven out of equilibrium by the expansion of the universe.

For inverse decays we know from energy conservation that \( f_i^{eq} f_j^{eq} \ldots = f_\psi^{eq} \). If we further neglect possible CP violating effects we have

\[
M(\psi \rightarrow i + j + \ldots) = M(i + j + \ldots \rightarrow \psi),
\] (110)

i.e., in thermal equilibrium, the reaction densities for decays and inverse decays are equal. If the decay products are massless, the reaction rate for inverse decays is [10]:

\[
\Gamma_{ID} = \frac{n_\psi^{eq}}{n_\gamma^{eq}} \Gamma_D,
\] (111)

where \( n_\gamma^{eq} = (g/\pi^2)T^3 \) is the number density of a massless particle species with \( g \) internal degrees of freedom in thermal equilibrium. In the limit of low and high temperatures we now have

\[
\Gamma_{ID} \approx \begin{cases} 
\sqrt{\frac{\pi}{2}} \left(\frac{m_\psi}{2T}\right)^{3/2} e^{-m_\psi/T} \tilde{\Gamma}_{rs}, & T \ll m_\psi \\
\frac{m_\psi}{2T} \tilde{\Gamma}_{rs}, & T \gg m_\psi
\end{cases}
\] (112)

For a two particle scattering process one obtains [17]

\[
\gamma^{eq}(\psi + a \rightarrow i + j + \ldots) = \frac{T}{64\pi^4} \int_0^\infty ds \tilde{\sigma}(s) \sqrt{s} K_1 \left( \frac{\sqrt{s}}{T} \right),
\] (113)

where \( \tilde{\sigma}(s) \) is the reduced cross section

\[
\tilde{\sigma}(s) = \frac{2\lambda(s, m_\psi^2, m_a^2)}{s} \sigma(s).
\] (114)

Here, \( \sigma(s) \) is the usual cross section as a function of the squared centre of mass energy \( s \) and \( \lambda \) is a flux factor

\[
\lambda(x^2, y^2, z^2) = \sqrt{[x^2 - (y + z)^2][x^2 - (y - z)^2]}.
\] (115)
If $|\vec{v}|$ denotes the relative velocity of the two particles,

$$|\vec{v}| = \frac{\sqrt{(p_\psi \cdot p_a)^2 - m_\psi^2 m_a^2}}{E_\psi E_a},$$

(116)

the reaction density for the scattering can also be written as

$$\gamma_{eq}(\psi + a \rightarrow i + j + \ldots) = n_{eq}^\psi n_{eq}^a \langle \sigma |\vec{v}| \rangle.$$  

(117)

In the following we shall consider processes with up to three particles in the final state.

5 Baryon asymmetry

We are now in a position to evaluate the baryon asymmetry for the patterns of neutrino masses discussed in section 3.2. Before doing that, however, it is useful to consider some general aspects of leptogenesis, which are independent of any particular parametrization chosen for the neutrino mass matrices. This supplements the qualitative discussion of section 3.1.

5.1 Delayed decay

A characteristic feature of this baryogenesis scenario is that the generated asymmetry depends mostly on the mass parameter

$$\tilde{m}_1 = (h_\nu h^\dagger_\nu)_{11} \frac{v^2}{M_1}.$$  

(118)

This can be seen in fig. 3 where we have plotted the generated lepton asymmetry as a function of $\tilde{m}_1$ for three different heavy neutrino masses $M_1 = 10^8$ GeV, $10^{10}$ GeV and $10^{12}$ GeV. Further, we have assumed a fixed CP asymmetry $\varepsilon_1 = -10^{-6}$ and a fixed mass hierarchy for right-handed neutrinos, $M_2^2 = 10^3 M_1^2, M_3^2 = 10^6 M_1^2$.

First one sees that leptogenesis is only possible in a rather narrow range of $\tilde{m}_1$, and that the washout processes, parametrized by the factor $\kappa$ in eq. (72), are important over the whole parameter range, since $\kappa < 0.1$. If $\tilde{m}_1$ is too low, the Yukawa interactions are too weak to produce a sufficient number of neutrinos at high temperatures, whereas for large $\tilde{m}_1$ the washout processes are too strong and destroy any generated asymmetry.

The asymmetry depends almost only on $\tilde{m}_1$ for small $\tilde{m}_1 \lesssim 10^{-4}$ eV, since then the generated asymmetry is determined by the number of neutrinos produced at high temperatures, i.e., on the strength of the processes in which a right-handed neutrino can be generated or annihilated. The dominant reactions are decays, inverse decays and scatterings with a top. In the Boltzmann equation (106) the reaction densities for heavy
Figure 6: *Generated B − L asymmetry as a function of \( \tilde{m}_1 \) for \( M_1 = 10^8 \, \text{GeV} \) (dotted line), \( M_1 = 10^{10} \, \text{GeV} \) (solid line) and \( M_1 = 10^{12} \, \text{GeV} \) (dashed line). The shaded area shows the measured value for the asymmetry.*

Neutrino production, \( \gamma_{\text{PROD}} \), all give contributions proportional to \( \tilde{m}_1 \) at high temperatures \( T > M_1 \),

\[
\frac{-z}{sH(M_1)} \gamma_{\text{PROD}} \propto \tilde{m}_1 .
\]

(119)

For large \( \tilde{m}_1 \gtrsim 10^{-4} \, \text{eV} \) on the other hand, the neutrinos reach thermal equilibrium at high temperatures, i.e., the generated asymmetry depends mostly on the influence of the lepton number violating scatterings \( \gamma_{\Delta L=2} \) at temperatures \( T \ll M_1 \). In contrast to eq. (119) the lepton number violating processes mediated by a heavy neutrino behave at low temperatures like

\[
\frac{-z}{sH(M_1)} \gamma_{\Delta L=2} \propto M_1 \sum_j \tilde{m}_j^2 ,
\]

(120)

with

\[
\tilde{m}_j = (h_{\nu} h_{\nu}^\dagger)_{jj} \frac{\nu^2}{M_j} ,
\]

(121)

where we have neglected interference terms. One therefore expects that the generated asymmetry becomes smaller for growing neutrino mass \( M_1 \) and this is exactly what one observes in fig. 6.
Eq. (120) can also explain the small dependence of the asymmetry on the heavy neutrino mass $M_1$ for $\tilde{m}_1 \lesssim 10^{-4}$ eV. The inverse decay processes which take part in producing the neutrinos at high temperatures are CP violating, i.e., they generate a lepton asymmetry at high temperatures. Due to the interplay of inverse decay processes and lepton number violating $2 \rightarrow 2$ scatterings this asymmetry has a different sign compared to the one generated in neutrino decays at low temperatures, i.e., the asymmetries will partially cancel each other. This cancellation can only be avoided if the asymmetry generated at high temperatures is washed out before the neutrinos decay. At temperatures $T > M_j$ the lepton number violating scatterings behave like

$$\frac{-z}{s H(M_1)} \gamma_{\Delta L=2} \propto M_1 \sum_j \frac{M_j^2}{M_1^2} \tilde{m}_j^2. \quad (122)$$

Hence, for fixed heavy neutrino mass hierarchy, the wash-out processes are more efficient for larger neutrino masses, i.e., the final asymmetry should grow with the neutrino mass $M_1$. The finally generated asymmetry is not affected by the stronger wash-out processes, since for small $\tilde{m}_1$ the neutrinos decay late, where one can neglect the lepton number violating scatterings.

5.2 Leptogenesis

We can now evaluate the baryon asymmetry for the two patterns of neutrino mass matrices discussed in sections 3.2.1 and 3.2.2. Since for the Yukawa couplings only the powers in $\epsilon$ are known, we will also obtain the CP asymmetries and the corresponding baryon asymmetries to leading order in $\epsilon$, i.e., up to unknown factors $O(1)$. Note, that for models with a $U(1)_F$ generation symmetry the baryon asymmetry is ‘quantized’, i.e., changing the $U(1)_F$ charges will change the baryon asymmetry by powers of $\epsilon$.

5.2.1 $SU(5) \times U(1)_F$

In this case one obtains from eqs. (68) and (76),

$$\epsilon_1 \sim \frac{3}{16\pi} \epsilon^4. \quad (123)$$

From eq. (72), $\epsilon^2 \sim 1/300$ and $g_* \sim 100$ one then obtains the baryon asymmetry,

$$Y_B \sim \kappa \ 10^{-8}. \quad (124)$$

For $\kappa \sim 0.1 \ldots 0.01$ this is indeed the correct order of magnitude! The baryogenesis temperature is given by the mass of the lightest of the heavy Majorana neutrinos,

$$T_B \sim M_1 \sim \epsilon^4 M_3 \sim 10^{10} \text{ GeV}. \quad (125)$$
Figure 7: Time evolution of the neutrino number density and the lepton asymmetry in the case of the SU(5) $\times$ U(1)$_F$ symmetry. The solid line shows the solution of the Boltzmann equation for the right-handed neutrinos, while the corresponding equilibrium distribution is represented by the dashed line. The absolute value of the lepton asymmetry $Y_L$ is given by the dotted line and the hatched area shows the lepton asymmetry corresponding to the observed baryon asymmetry.

This set of parameters, where the CP asymmetry is given in terms of the mass hierarchy of the heavy neutrinos, has been studied in detail \textsuperscript{58}. The generated baryon asymmetry does not depend on the flavour mixing of the light neutrinos, in particular the $\nu_\mu - \nu_\tau$ mixing angle. The solution of the full Boltzmann equations is shown in fig. 4 for the non-supersymmetric case. The initial condition at a temperature $T \sim 10M_1$ is chosen to be a state without heavy neutrinos. The Yukawa interactions are sufficient to bring the heavy neutrinos into thermal equilibrium. At temperatures $T \sim M_1$ this is followed by the usual out-of-equilibrium decays which lead to a non-vanishing baryon asymmetry. The final asymmetry agrees with the estimate \textsuperscript{124} for $\kappa \sim 0.1$. The dip in fig. 7 is due to a change of sign in the lepton asymmetry at $T \sim M_1$, as discussed in the previous section.

5.2.2 $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_F$

In this case the neutrino Yukawa couplings \textsuperscript{39} yield the CP asymmetry

$$\varepsilon_1 \sim \frac{3}{16\pi} \epsilon^5,$$

(126)
which correspond to the baryon asymmetry (cf. (72))

\[ Y_B \sim \kappa \ 10^{-6} . \]  

(127)

Due to the large value of \( \epsilon \) the CP asymmetry is two orders of magnitude larger than in the case with \( SU(5) \times U(1)_F \) symmetry. However, washout processes are now also stronger. The solution of the Boltzmann equations is shown in fig. 8. The final asymmetry is again \( Y_B \sim 10^{-9} \) which corresponds to \( \kappa \sim 10^{-3} \). The baryogenesis temperature is considerably larger than in the first case,

\[ T_B \sim M_1 \sim \epsilon^5 M_3 \sim 10^{12} \text{ GeV} . \]  

(128)

The baryon asymmetry is largely determined by the parameter \( \bar{m}_1 \) defined in eq. (67) [48]. In the first example, one has \( \bar{m}_1 \sim \bar{m}_\nu \). In the second case one finds \( \bar{m}_1 \sim m_3 \). Since \( \bar{m}_\nu \) and \( m_3 \) are rather similar it is not too surprising that the generated baryon asymmetry is about the same in both cases.

### 5.3 Supersymmetric leptogenesis

Without an intermediate scale of symmetry breaking, the unification of gauge couplings appears to require low-energy supersymmetry. Therefore, we are going to briefly review supersymmetric leptogenesis [53, 53, 60] in the following. As in the non-supersymmetric
Figure 9: Decay modes of the right-handed Majorana neutrinos and their scalar partners in the supersymmetric scenario.
case, a full analysis of the mechanism including all the relevant scattering processes is necessary in order to get a reliable relation between the input parameters and the final asymmetry.

The supersymmetric generalization of the lagrangian (63) is the superpotential

$$W = h_{eij}E^c_i L_j H_1 + h_{\nu ij}N^c_i L_j H_2 + \frac{1}{2} h_{r ij} N^c_i N^c_j R, \quad (129)$$

where, in the usual notation, $H_1$, $H_2$, $L$, $E^c$, $N^c$ and $R$ are chiral superfields describing spin-0 and spin-$\frac{1}{2}$ fields. The basis for the lepton fields can be chosen as in the non-supersymmetric case.

The heavy neutrinos and their scalar partners can decay into various final states (cf. fig. 9). At tree level, the decay widths read,

$$\Gamma_{rs}(N_1 \rightarrow \tilde{l} + \tilde{h}^c) = \Gamma_{rs}(N_1 \rightarrow l + H_2) = \frac{1}{16\pi} (h_{\nu} h_{\nu}^\dagger)_{11} M_1, \quad (130)$$

$$\Gamma_{rs}(\tilde{N}^c_1 \rightarrow \tilde{l} + H_2) = \Gamma_{rs}(\tilde{N}^c_1 \rightarrow l + \tilde{h}^c) = \frac{1}{8\pi} (h_{\nu} h_{\nu}^\dagger)_{11} M_1. \quad (131)$$

The CP asymmetry in each of the decay channels is given by

$$\varepsilon_1 = -\frac{1}{8\pi} \frac{1}{(h_{\nu} h_{\nu}^\dagger)_{11}} \sum_{i=2,3} \text{Im} \left[ (h_{\nu} h_{\nu}^\dagger)_{1i}^2 \right] f \left( \frac{M_i^2}{M_1^2} \right), \quad (132)$$

Figure 10: Same as fig. [8] for the supersymmetric version of leptogenesis.
Figure 11: Solution of the Boltzmann equations for supersymmetric leptogenesis for the parameters given by the $SU(5) \times U(1)_F$ symmetry. $Y_{N1}^{eq}$ stands for the equilibrium distribution of neutrinos $N_1$, and $Y_{N_1}$ and $Y_{1+}$ are the solutions for the neutrino number and the sum of scalar neutrino and scalar anti-neutrino number per comoving volume element, respectively. The generated lepton asymmetries in leptons and scalar leptons are denoted by $Y_{L_f}$ and $Y_{L_s}$, whereas $Y_{1-}$ stands for the difference of scalar neutrino and scalar anti-neutrino numbers.

where

$$f(x) = \sqrt{x} \left[ \ln \left( \frac{1+x}{x} \right) + \frac{2}{x-1} \right].$$

It arises through interference of tree level and one-loop diagrams shown in fig. 9. In the case of a mass hierarchy, $M_j \gg M_i$, the CP asymmetry is twice as large as in the non-supersymmetric case.

Like in the non-supersymmetric scenario lepton number violating scatterings mediated by heavy (s)neutrinos have to be included in a consistent analysis. In the supersymmetric case a large number of processes contribute which can easily reduce the generated asymmetry by two orders of magnitude. Similarly, the large number of (s)neutrino production processes makes leptogenesis possible for values of $\tilde{m}_1$ smaller than in the non-supersymmetric case [60].

In fig. 10 we have plotted the generated lepton asymmetry as function of $\tilde{m}_1$ for three different values of $M_1$, where we have again assumed the hierarchy $M_2^2 = 10^3 M_1^2$, $M_3^2 = 10^6 M_1^2$ and the CP asymmetry $\varepsilon_1 = -10^{-6}$. Fig. 10 demonstrates that in the whole parameter range the generated asymmetry is significantly smaller than the value
Figure 12: Solution of the supersymmetric Boltzmann equations in the case of the \( SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_F \) symmetry.

\[ 4 \cdot 10^{-9} \] which one obtains, if one neglects lepton number violating scattering processes. Baryogenesis is possible in the range

\[ 10^{-5} \text{ eV} \lesssim \bar{m}_1 \lesssim 5 \cdot 10^{-3} \text{ eV} . \]  

(134)

Comparing non-supersymmetric and supersymmetric leptogenesis one sees that the larger CP asymmetry and the additional contributions from the sneutrino decays in the supersymmetric scenario are compensated by the wash-out processes which are stronger than in the non-supersymmetric case. The final asymmetries are of the same order in the non-supersymmetric and in the supersymmetric case.

Like in the non-supersymmetric scenario it is interesting to see whether a parameter choice which gives light neutrino masses and mixing angles compatible with experimental results, can also explain the baryon asymmetry. To this end we again considered the two scenarios discussed in sections \([3.2.1]\) and \([3.2.2]\). The time evolution of the (s)neutrino number densities and the (s)lepton asymmetries in the case of the \( SU(5) \times U(1)_F \) symmetry is shown in fig. \([11]\). Although the CP-asymmetry is now larger than eq. \((123)\) by a factor of two, the wash-out processes are more effective than in the non-supersymmetric case, and the final asymmetry is again of order \(10^{-9}\), corresponding to a washout factor \(\kappa \sim 0.03\). Similarly, in the case of the \( SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_F \) symmetry (cf. fig. \([12]\)) the final asymmetry is again of order \(10^{-9}\). The baryogenesis temperature \(T_B \sim 10^{10} \text{ GeV}\) in the \( SU(5) \times U(1)_F \) model is consistent with the constraint from the allowed gravitino abundance \([98]\).
Leptogenesis can also be considered in extended models which contain heavy $SU(2)$-triplet Higgs fields in addition to right-handed neutrinos \cite{61,62}. Decays of the heavy scalar bosons can in principle also contribute to the baryon asymmetry. However, since these Higgs particles carry gauge quantum numbers they are strongly coupled to the plasma and it is difficult to satisfy the out-of-equilibrium condition. The resulting large baryogenesis temperature is in conflict with the ‘gravitino constraint’ \cite{99}.

5.4 Mass scales of leptogenesis and $B - L$ breaking

In the previous section we have considered baryogenesis in two models with hierarchical neutrino masses but rather different flavour structure. In both cases the Majorana masses of the right-handed neutrinos turned out to be rather large, i.e., $M_i \geq 10^{10}$ GeV, which implies large mass scales for the leptogenesis temperature and $B - L$ breaking.

In order to understand the model dependence of this result it is instructive to study the case of two generations for which bounds on light and heavy neutrino masses can be easily obtained (cf. \cite{41}).

Without loss of generality the heavy Majorana neutrino mass matrix can always be assumed to be diagonal with positive real eigenvalues,

$$ M = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}. \quad (135) $$

Similarly, in terms of the matrix $\hat{m}_D = i m_D V_\nu$ (cf. (65)) the light Majorana neutrino mass matrix reads

$$ m_\nu = \hat{m}_D^T \frac{1}{M} \hat{m}_D = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}, \quad (136) $$

where the eigenvalues $m_i$ are again real. With

$$ \hat{m}_D = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad b = \sqrt{\frac{M_1}{M_2}} \eta d, \quad c = -\sqrt{\frac{M_2}{M_1}} \eta a, \quad (137) $$

one obtains

$$ m_\nu = \begin{pmatrix} \frac{a^2}{M_1} & 0 \\ 0 & \frac{b^2}{M_2} \end{pmatrix} (1 + \eta^2). \quad (138) $$

Note that $a, c, d$ are complex. Since $m_1$ and $m_2$ are real there is only one independent phase. A complete set of parameters for the two neutrino mass matrices is given by $m_1, m_2, M_1, M_2$ and $\eta^2 = e^{i\alpha}$.

The out-of-equilibrium condition for the decay of the heavy neutrino implies an interesting upper bound on the mass of the lightest neutrino $m_- = \min\{m_1, m_2\}$. From
\( \Gamma_{\Delta_1} < H \mid_{T=M_1} \) one obtains for the effective neutrino mass \( \tilde{m}_1 \),

\[
\tilde{m}_1 = \frac{(\tilde{m}_D \tilde{m}_D^\dagger)_{11}}{M_1} < m_0 = 10^{-3} \text{ eV} .
\] (139)

With

\[
\tilde{m}_1 = \frac{m_1 + m_2 |\eta|^2}{|1 + \eta^2|},
\] (140)

one is led to

\[
m_- < m_- \frac{1 + |\eta|^2}{|1 + \eta^2|} < \tilde{m}_1 < 10^{-3} \text{ eV} .
\] (141)

Note, that this bound is only accurate up to a factor \( O(1) \), as the values for \( \tilde{m}_1 \) show which we have obtained for the two models discussed above. This difference illustrates the accuracy to which the out-of-equilibrium condition holds.

The upper bound (141) can always be satisfied by making the neutrino Yukawa couplings very small. However, leptogenesis also requires a sufficient amount of CP asymmetry. From (71) one obtains

\[
M_1 > \frac{16\pi v_2^2}{3|m_1^2 - m_2^2|} \varepsilon_0 \frac{(1 + \rho^2)^{1/2}(m_1 + m_2\rho)}{\rho} .
\] (143)

We now assume a maximal phase, i.e., \( \eta^2 = i\rho, \alpha = \pi/2 \), and \( |\varepsilon_1| > \varepsilon_0 \sim 10^{-8} \), which is the smallest value required by the observed baryon asymmetry \( Y_B \sim 10^{-10} \). This yields for the heavy neutrino mass \( M_1 \),

\[
M_1 > \frac{16\pi \varepsilon_0 v_2^2}{3|m_1^2 - m_2^2|} \varepsilon_0 \frac{(1 + \rho^2)^{1/2}(m_1 + m_2\rho)}{\rho} .
\] (143)

Since the right-hand side has a minimum at \( \rho = (m_1/m_2)^{1/3} \), one obtains a lower bound on \( M_1 \),

\[
M_1 > \frac{16\pi \varepsilon_0 v_2^2}{3} \frac{(m_1^{2/3} + m_2^{2/3})^{3/2}}{|m_1^2 - m_2^2|} .
\] (144)

For quasi-degenerate neutrinos the bound (141) implies \( m_1 \sim m_2 < m_0 \sim 10^{-3} \text{ eV} \). This yields

\[
M_1 > 10^{18} \text{ GeV} \varepsilon_0 \sim 10^{10} \text{ GeV} ,
\] (145)

independent of our knowledge about the solar and atmospheric neutrino anomalies. For hierarchical neutrinos all masses have to satisfy the upper bound on the electron neutrino
mass obtained from $\beta$-decays since the mass differences inferred from the atmospheric and solar neutrino anomalies are much smaller. Hence, one has $m_- \ll m_+ < 1\text{eV}$ which implies

$$M_1 > 10^{15} \text{ GeV} \varepsilon_0 \sim 10^7 \text{ GeV} .$$  \hfill(146)

The bounds (145) and (146) confirm the expectation, in accord with the explicit models discussed above, that the smallness of the light neutrino masses suggested by experimental data as well as the out-of-equilibrium condition requires heavy Majorana neutrinos far above the electroweak scale. The bounds (145) and (146) can be relaxed in the case $|M_1 - M_2| = \mathcal{O}(\Gamma_i)$. Then a resonant enhancement of the CP asymmetry can occur [45]. In this case the lower bound $M_1 > 10^5 \text{ GeV}$ has been obtained in a recent analysis [100].

In the context of a grand unified theory the large Majorana masses will be generated by a Higgs mechanism which breaks $B - L$ spontaneously. The corresponding vector bosons have to be sufficiently heavy such that the out-of-equilibrium condition for the decaying Majorana neutrino $N_1$ is not violated by processes like $N_1N_1 \rightarrow Z' \rightarrow e\bar{e}$ [47]. Assuming Higgs couplings $h_r < 1$ and a gauge coupling $\mathcal{O}(1)$ one obtains the lower bound on the scale of $B - L$ breaking

$$\Lambda_{B-L} \sim M_{Z'} > 10^{11} \left( \frac{M_1}{10^{10}\text{GeV}} \right)^{3/4} .$$  \hfill(147)

In the case of hierarchical heavy Majorana neutrinos the scale of $B - L$ breaking is naturally identified with the grand unification scale, as the models illustrate which we discussed in the previous section.

### 6 Outlook

Detailed studies of the thermodynamics of the electroweak interactions at high temperatures have shown that in the standard model and most of its extensions the electroweak transition is too weak to affect the cosmological baryon asymmetry. Hence, one has to search for baryogenesis mechanisms above the Fermi scale.

Due to sphaleron processes baryon number and lepton number are related in the high-temperature symmetric phase of the standard model. As a consequence, the cosmological baryon asymmetry is related to neutrino properties. Generically, baryogenesis requires lepton number violation, which occurs in many extensions of the standard model with right-handed neutrinos and Majorana neutrino masses. In detail the relations between $B$, $L$ and $B - L$ depend on all other processes taking place in the plasma, and therefore also on the temperature.

Although lepton number violation is needed in order to obtain a baryon asymmetry, it must not be too strong since otherwise any baryon and lepton asymmetry would be
washed out. Hence, leptogenesis leads to stringent upper and lower bounds on the masses of the light and heavy Majorana neutrinos, respectively.

The solar and atmospheric neutrino deficits can be interpreted as a result of neutrino oscillations. For hierarchical neutrinos the corresponding neutrino masses are very small. Assuming the see-saw mechanism, this suggests the existence of very heavy right-handed neutrinos and a large scale of $B - L$ breaking.

It is remarkable that these hints on the nature of lepton number violation fit very well together with the idea of leptogenesis. For hierarchical neutrino masses, with $B - L$ broken at the unification scale $\Lambda_{\text{GUT}} \sim 10^{16}$ GeV, the observed baryon asymmetry $n_B/s \sim 10^{-10}$ is naturally explained by the decay of heavy Majorana neutrinos. The corresponding baryogenesis temperature is $T_B \sim 10^{10}$ GeV.

The cosmological baryon asymmetry provides important constraints on neutrino masses and mixings. In addition the connection between lepton flavour and quark flavour changing processes can be studied in unified theories. In supersymmetric models implications for the mass spectrum of superparticles can be derived from the cosmological bound on the gravitino number density. It is intriguing that the baryogenesis temperature $T_B$ is of the same order as the supersymmetry breaking scale $\Lambda_S$. It will be interesting to see to what extent further theoretical work and experimental data will be able to identify or exclude leptogenesis as the dominant source of the cosmological matter-antimatter asymmetry. In order to obtain a theory of leptogenesis one also has to go beyond the classical Boltzmann equations and to treat the generation of the baryon asymmetry fully quantum mechanically, which presents a challenging problem of non-equilibrium physics.
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