Reentrant Superconductivity in UTe₂

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Received May 3, 2020; revised May 12, 2020; accepted May 12, 2020

The reentrant superconductivity is the peculiar phenomenon observed in paramagnetic metal UTe₂ in magnetic field parallel to the hard magnetization axis. It is difficult to explain it in terms of field dependent intensity of magnetic fluctuations like it is done for explanation of the formally similar phenomena in the ferromagnetic uranium superconductors URhGe and UCoGe. Extremely large initial slope of the upper critical field temperature dependence suggests that the phenomenon has quasi-two-dimensional nature. Indeed, according to the recent band structure calculations by Y. Xu et al. (Phys. Rev. Lett. 123, 217002 (2019)) the Fermi surface of UTe₂ consists of two types slightly corrugated cylinders. The theory of reentrant superconductivity in UTe₂ based on its quasi-two-dimensional structure is presented here.

DOI: 10.1134/S0021364020120036

1. INTRODUCTION

The superconductivity in the metallic compound UTe₂ with orthorhombic structure has been discovered in December 2018 by Nicholas Butch and colleagues [1]. Almost immediately, the discovery has been confirmed by the joint French–Japanese team [2]. Since that time dozens of studies of different properties of this material have been published. The most impressive observation [3, 4] was that the superconducting state UTe₂ in the magnetic field aligned along the b axis, which is perpendicular to the easy magnetization direction parallel to a axis, persists up to 34.5 T where the superconductivity is destroyed by the metamagnetic transition. The magnetic field first reduces the superconducting transition temperature, then in the field interval (10 T, 27 T), T_c(H) is almost field independent, and finally near the metamagnetic transition, the transition temperature increases practically recreating its zero field magnitude [5], see Fig. 1. Even more astonishing was that sweeping magnetic fields above the metamagnetic transition through the angular range 20°–40° from the b axis towards the c axis reveals a superconducting phase inside the dome, with the maximum value exceeding 65 T, the maximum field possible in the measurements (see Figs. 1a, 1c in [3]). The critical temperature of transition in superconducting state in this material is 1.5 K. The extremely high upper critical field at such modest critical temperature indicates that the paramagnetic depairing mechanism does not work in this compound and the superconducting state is formed by the electron pairs with spin S = 1. At the same time, the usual orbital suppression of superconductivity also occurs ineffective.

The reentrant superconducting state under magnetic field perpendicular to the easy magnetization direction revealed in UTe₂ reminds phenomenon known in ferromagnetic uranium compounds URhGe and UCoGe [6]. For instance, in UCoGe, the magnetic field directed parallel to b axis, which is perpendicular to the direction of spontaneous magnetization along c axis, first suppresses the critical temperature but then at higher fields the critical temperature begins to increase. On the contrary, the field directed along b axis...
spontaneous magnetization effectively suppresses the superconducting state. The situation looks as a magnetic field in direction perpendicular to the easy magnetization direction stimulates pairing strength and a field directed along the easy magnetization direction suppresses the pairing interaction. The observations are not compatible with ordinary electron–phonon mechanism formation of superconducting state. This suggests that pairing interaction is produced by magnetic fluctuations that is the pairing amplitude is determined by the magnetic susceptibility. Magnetic moment saturates under magnetic field parallel to spontaneous magnetization. This leads to susceptibility suppression which yields the decrease in the electron effective mass. Both these effects produce the reduction of pairing interaction. As a result, in addition to the usual orbital depairing mechanism, another mechanism is added that accelerates the suppression of superconductivity, and the upper critical field \( H_c2(T) \) along the \( c \) axis exhibits an upward curvature [7]. The magnetic field perpendicular to the direction of spontaneous magnetization reduces the Curie temperature thereby increasing susceptibility in the direction of spontaneous magnetization [8]. The enhancement of susceptibility causes the electron effective mass increase. Thus, a magnetic field perpendicular to spontaneous magnetization strongly stimulates the superconductivity providing the high field reentrance of superconducting state.

\( \text{UTE}_2 \) is not a ferromagnet. Its low temperature magnetic susceptibility along \( b \) axis keeps a constant value until the metamagnetic transition [9]. The specific heat coefficient \( \gamma = C/T \) proportional to electron effective mass is also practically constant until the fields about 30 T and its increase appears only near the metamagnetic transition [9]. Thus, the mechanism responsible for the reentrance of superconducting state in uranium ferromagnets in huge magnetic fields perpendicular to spontaneous magnetization is not available in \( \text{UTE}_2 \) and cannot serve as an explanation of stability of superconductivity in this compound.

The Fermi surface of \( \text{UTE}_2 \) found in the recently reported first-principles band structure calculations [10] consists of two separate electron and hole cylinders with axes parallel to \( c \)-direction. Thus, \( \text{UTE}_2 \) looks as quasi-two-dimensional metal with conducting layers parallel to \( (a, b) \) plane. It is known [11] that the magnetic field parallel to conducting layers of quasi-two-dimensional metal first suppresses the superconducting state and then at magnetic energy \( \hbar \omega _F \) comparable with the hopping amplitude between the conducting planes starts to recreate superconductivity. In high fields, the electrons move almost free along the open trajectories perpendicular to the conducting layers. Thus, the physical reason for the suppression of the orbital mechanism of the Cooper pair destruction is the suppression of the modulation of electron motion caused by the crystal field, in other words, the suppression of the trajectories curvature.

The observed experimentally [1, 2, 4, 5] abnormally large initial slope of the upper critical field temperature dependence (see Fig. 1) serves as another solid argument in support of quasi-two-dimensional nature of phenomenon of reentrant superconductivity in \( \text{UTE}_2 \).

Here, I apply the theory of superconductivity in quasi-two-dimensional metals developed in the papers [11, 12] to description of the upper critical field temperature dependence in \( \text{UTE}_2 \).

2. UPPER CRITICAL FIELD

In the magnetic field parallel to the \( b \) axis, it is reasonable to choose \((x, y, z)\) coordinate axes directed along \((c, a, b)\) crystallographic directions. The corresponding unit cell of the reciprocal space is limited by intervals \( \frac{\pi}{d} < p_x < \frac{\pi}{d}, \frac{\pi}{a} < p_y < \frac{\pi}{a}, \frac{\pi}{b} < p_z < \frac{\pi}{b} \). In this coordinate system according to [10] there are two different conducting bands with the Fermi surfaces consisting of two pairs corrugated cylinders with axes parallel to \( p_x \) direction located in the points \( (0, 0, \pm \frac{\pi}{b}) \) and \( (0, 0, \pm \frac{\pi}{b}) \). Thus, the conducting layers of quasi-two-dimensional metal are parallel to the plane \((a, b)\) and located at a distance \( d \) from each other. We will consider simplified single band model taking into account only the first band with following electron spectrum near the Fermi surface

\[
\xi(p) = \frac{1}{2m} \left( p_x + \left(p_z + \frac{\pi}{b}\right)^2 \right) - 2t \cos(p_zd) - \epsilon_F, \quad (1)
\]

such that \( \varepsilon \ll \epsilon_F \). In the magnetic field \( H = (0, 0, H) \), \( A = (-Hy, 0, 0) \) parallel to the \( b \) direction the electron wavefunction has the form

\[
\Psi(x, y, z) = \psi(p_x, y, p_z) \exp \left( ip_xx + i\left(p_z + \frac{\pi}{b}\right)z \right) \quad (2)
\]

and \( \psi(p_x, y, p_z) \) obeys the Schrödinger equation

\[
\left[ \frac{1}{2m} \left( \frac{d^2}{dy^2} + \left(p_z + \frac{\pi}{b}\right)^2 \right) - 2t \cos \left(p_zd - \alpha_yy / v_F \right) \right] \psi(p_x, y, p_z) = \epsilon \psi(p_x, y, p_z), \quad (3)
\]

where \( \alpha_y = ev_FdH/c \) and \( h = 1 \).

We do not take into account the band splitting due to the Zeeman interaction. In the case of equal spin triplet pairing, the latter does not produce paramagnetic suppression of superconducting state. However, yielding the opposite shifts in the Fermi momenta of spin-up and spin-down bands it produces the corresponding shifts in the density of states and changes the critical temperature of transition to the superconduct-
ing state (see, e.g., [7]). This effect is absent in quas\-2D case: the magnetic field parallel to the conducting layers changes the Fermi momenta but does not change the density of states near the Fermi surface.

The normal state electron Green’s function derived in the same manner as in [11] is
\[
G_{\omega_n}(\phi, p_x, y, y_1) = -\frac{\text{sgn} \omega_n}{p_{0y}} \exp \left[ \mp \frac{m \omega_n}{p_{0y}} \right] \exp \left[ \pm i p_{0y} (y - y_1) \right] \times \exp \left\{ \pm i \frac{\omega_1 (y - y_1)}{2v_F} \cos \left[ p_{0y} (2 - \omega y + y_1) / 2v_F \right] \right\},
\]
\[
\pm \omega_n (y - y_1) > 0. \tag{4}
\]
Here, the Matsubara frequency \( \omega_n = \pi T (2n + 1) \) is shifted \( \tilde{\omega}_n = \omega_n + \frac{1}{2\pi} \text{sgn} \omega_n \) due to attenuation of electronic states produced by scattering on impurities, interaction with magnetic fluctuations, etc., \( \lambda = \frac{4A}{\omega_F} \), \( p_{0y} = p_0 \sin \phi \) and \( p_0 \) is the Fermi momentum.

The simplest equal spin pairing state has the order parameter
\[
\Delta(\phi, y) = \psi(\phi) \eta(y). \tag{5}
\]
Here,
\[
\psi(\phi) = A \left( \cos \phi + \frac{\pi}{b p_0}, \quad \frac{\pi}{2} < \phi < \frac{3\pi}{2}, \right. \tag{6}
\]
\[
\psi(\phi) = A \left( \cos \phi - \frac{\pi}{b p_0}, \quad -\frac{\pi}{2} < \phi < \frac{\pi}{2}. \right. \tag{7}
\]
where \( A \) is the normalization constant such that
\[
\int_{-\infty}^{\infty} \psi^2(\phi) d\phi = 1.
\]
This type of the order parameter belongs to either \( B_{2u} \) or \( B_{1u} \) irreducible representation of the orthorhombic point group \([10]\). The treatment of the equal spin pairing state belonging to the \( A_1 \) and \( B_{1u} \) representations is mathematically more cumbersome.

The function \( \psi(\phi) \) is an odd function in respect to the point \( \frac{\pi}{2} \): \( \psi(\frac{\pi}{2} + \phi) = -\psi(\frac{\pi}{2} - \phi) \), whereas the Green’s function is an even one. Hence, the self-energy corresponding to the order parameter is equal to zero \([12]\).

The linear equation for the function \( \eta(y) \) determining the upper critical field or the critical temperature \( T_c(H) \) of transition to the superconducting state has the form \([11, 12]\)
\[
\eta(y) = \tilde{g} \int_{-\infty}^{\infty} \psi^2(\phi) d\phi \int_{y-y_1}^{y+y_1} \frac{d\phi}{|\sin \phi|} \exp \left\{ \frac{-y - y_1}{v_F |\sin \phi|} \right\}
\]
\[
\times \left| \frac{2\pi T d\gamma}{v_F |\sin \phi|} \right| \exp \left\{ -\frac{y - y_1}{v_F |\sin \phi|} \right\} \sinh \left( \frac{2\pi T |y - y_1|}{v_F |\sin \phi|} \right)
\]
\[
\times \tilde{g}_0 \left[ 2\lambda \sin \left( \frac{\omega_1 (y - y_1)}{2v_F} \right) \sin \left( \frac{\omega_1 (y + y_1)}{2v_F} \right) \right] \eta(y_1), \tag{8}
\]
where \( \tilde{g}_0(x) \) is the Bessel function, \( \tilde{g} = \frac{mg}{4\pi l} \) is the product of the density of states and the pairing amplitude \( g \), \( a \) is a small distance cutoff, and \( l = v_F \tau \) is the mean free path.

A. Critical temperature. In the absence of a magnetic field, the equation
\[
1 = \tilde{g} \int_{-\infty}^{\infty} \frac{dz}{\sinh z} \exp \left( -\frac{z}{2\pi T \tau} \right)
\]
rewritten as
\[
\ln \frac{T_c}{T_{c0}} = \psi \left( \frac{1}{2} \right) - \psi \left( \frac{1}{2} + \frac{1}{4\pi T \tau} \right) \tag{10}
\]
determines the critical temperature. Here, \( \psi(x) \) is the digamma function,
\[
T_{c0} = \frac{v_F}{\pi a} \exp \left( -\frac{1}{\tilde{g}} \right) \tag{11}
\]
is the critical temperature in a perfect crystal without impurities \( l = \infty \). The superconducting state suppresses completely at \( l < \frac{v_F}{\pi T_{c0}} \), where \( \ln \gamma = C = 0.577... \) is the Euler constant.

B. Ginzburg–Landau region. Near the critical temperature \( T = T_c / \xi > \frac{\omega_F}{2\pi} \), the essential region of integration in Eq. (8) is limited by inequality \( \delta' < \frac{\omega_F}{2\pi T} \). Outside this region, the integrand is exponentially small. Hence, the product \( \frac{\omega_F}{2\pi T} \delta' < \frac{\omega_F}{2\pi T} \approx 1 \), and the argument of the Bessel function
\[
\frac{2\lambda}{\sin \phi} \left( \frac{\omega_1 (y - y_1)}{2v_F} \right) \sin \left( \frac{\omega_1 (y + y_1)}{2v_F} \right)
\]
\[
\left( \frac{2\pi T d\gamma}{v_F |\sin \phi|} \right) \eta(y_1)
\]
proves to be small. Here, \( \xi \) is the characteristic length on which the function \( \eta(y) \) changes. If \( \xi > \delta' \), then one can expand \( \eta(y_1) = \eta(y) + \eta'(y)(y - y_1) + \eta''(y)(y - y_1)^2/2 \) under integral in Eq. (8), and expand
the Bessel function $J_0(x) = 1 - x^2/4$. Thus, we come to the differential equation

$$\left[ \ln \frac{T_0}{T} + \psi\left(\frac{1}{2}\right) - \psi\left(\frac{1}{2} + \frac{1}{4\pi T \tau}\right) \right] \eta(y)$$

$$= -\frac{C^\nu J(\alpha)}{2}\left(\frac{v}{T}\right)^2 \eta''(y) + I(\alpha)\left(\frac{\omega \nu^2}{2\pi v T \tau}\right)^2 \eta(y),$$

(13)

where $\alpha = (2\pi T_c \tau)^{-1}$.

$$I(\alpha) = \int_{-\infty}^{\infty} \frac{1}{2\pi} \exp\left(-\frac{z}{2\pi T \tau}\right)$$

$$= 4\sum_{n=0}^{\infty} \frac{1}{(2n + 1 + \alpha)},$$

(14)

$$C_\psi = \int \psi^2(\phi) \sin^2 \phi \frac{d\phi}{\pi}.$$  

(15)

The lowest eigenvalue of Eq. (13) at $T = T_c$ is

$$\left[ \ln \frac{T_0}{T} + \psi\left(\frac{1}{2}\right) - \psi\left(\frac{1}{2} + \frac{1}{4\pi T \tau}\right) \right] = \frac{C^\nu J(\alpha) \omega}{2\sqrt{2\pi T_c^2}}.$$  

(16)

In the pure case $\alpha \approx \alpha_c = (2\pi T_c \tau)^{-1}$, we obtain

$$\omega_c^2(T) = \frac{\nu e d}{c} H_c^2(T)$$

$$= \frac{4\sqrt{2\pi^2}}{7\nu^2(3)\sqrt{C_\psi}} (T_0 - \omega^2/8\pi) (T_c - T),$$

(17)

where $\zeta(x)$ is the Riemann zeta function,

$$T_c = T_0 - \frac{\pi}{8\tau},$$

(18)

and

$$\beta = 2 - \frac{90\zeta(4)}{7\pi^2 \zeta(3)} = 0.83.$$  

(19)

C. High field region. The linear temperature dependence of $H_c^2(T)$ near $T_c$ changes to the more fast increase at lower temperatures. The formal continuation of the linear dependence Eq. (17) to $T = 0$ (see Fig. 1) yields

$$\omega_c^2(0) = \frac{\nu e d H_{c2}^2(0)}{c} = 10 \frac{T_c^2}{l},$$

(20)

which can be rewritten in dimensional units in the form

$$\frac{e\hbar}{m_0 c} H_{c2}^2(0) = 10 \frac{T_c^2}{h v_F d m_0},$$

(21)

convenient for numerical comparison with experiment. Here, $m_0$ is the electron mass in vacuum. According to the available experimental data [1, 2, 4, 5] the values $H_{c2}^2(T = 0)$ are from 25 to 30 T, $T_c = 1.5$ K. This gives us the possibility to estimate the magnitude of the interlayer hopping integral

$$t \leq \frac{1}{\hbar v_F d m_0} (Kelvin).$$

(22)

Using this estimate, we see that the combination $8t/\omega_c^2$ in the argument of the Bessel function in Eq. (8) begins to be smaller than unity at fields

$$H > H_0 = \frac{8}{(\hbar v_F d m_0)^2} (Tesla)$$

(23)

except the small interval of angles $\pi - \frac{8t}{\omega_c^2} \phi < \pi + \frac{8t}{\omega_c^2}$.

To estimate the field dependence of the critical temperature at $H > H_0$, let us divide the interval of integration over the angle $\phi$ in Eq. (8) as follows:

$$\frac{1}{\pi} \int_{\frac{3\pi}{2}}^{\frac{3\pi}{2}} (\ldots) d\phi = \frac{2}{\pi} \int_{\frac{3\pi}{2}}^{\frac{3\pi}{2}} (\ldots) d\phi$$

$$= 2 \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} (\ldots) d\phi + 2 \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} (\ldots) d\phi.$$

(24)

Thus, in the angular range $\pi/2 < \phi < \pi - \frac{8t}{\omega_c^2}$ at fields $H > H_0$, one can decompose the Bessel function $J_0(x) = 1 - x^2/4$ and substituting the rapidly oscillating trigonometric functions by their average values, we obtain

$$J_0 \left[ \frac{2\lambda}{\sin \phi} \sin \left( \frac{\omega_c(y - y_1)}{2v_F} \right) \sin \left( \frac{\omega_c(y + y_1)}{2v_F} \right) \right]$$

$$= 1 - \frac{t^2}{(\sin \phi)^2 \omega_c^2}.$$  

(25)

On the other hand, in the angular range $\pi - \frac{8t}{\omega_c^2} \phi < \pi$, it is enough to take in mind that the Bessel function is $J_0(x) < 1$.

Estimating the integrals over the angle $\phi$, we come to the equation for the critical temperature

$$1 = \frac{1}{\pi} \left[ 1 - \cos \left( \frac{8t}{\omega_c^2} \right) \right] \int_{\frac{\pi}{2}}^{\infty} \frac{dz}{\sinh z} \exp \left( -\frac{z}{2\pi T \tau} \right)$$

(26)

or

$$\ln \frac{T_c}{T_0^2} = \frac{1}{2} - \frac{1}{2} + \frac{1}{4\pi T \tau},$$

(27)

where

$$T_0^2(H) = T_0^2 \exp \left( -\frac{1}{\frac{\omega_c}{\omega_c^2}} \frac{t}{h v_F d m_0} \right) \quad \text{for} H > H_0.$$  

(28)
Hence, in the pure case
\[
T_c(H) = T_{c0}(H) \left(1 - \frac{\pi}{8\tau T_{c0}(H)}\right)_{H=H_{c1}} \rightarrow T_c. \quad (29)
\]
Thus, in high enough fields the critical temperature of transition to superconducting state tends to its zero field value.

3. CONCLUSIONS

We have demonstrated that the quasi-two-dimensional model allows describing the phenomenon of reentrant superconductivity in UTe$_2$ in magnetic fields parallel to the $b$ axis. The comparison of the linear upper critical field temperature dependence in the Ginzburg–Landau region with available experimental data gives the estimation of the hopping integral between the conducting layers. The smallness of its magnitude opens the possibility to the superconducting state recreation in reasonably high magnetic fields.

The treatment has several simplifications. We have considered single band model with parabolic spectrum, whereas the Fermi surface found in [10] consists of sheets corresponding to one electron and one hole band with more complex spectrum. Therefore, the given approach must be generalized taking into account the real band structure. This probably will allow explaining other peculiar observations made in UTe$_2$ such that half of conducting electrons seemingly do not participate in the superconductivity [1] and already mentioned the existence of the reentrant superconductivity in the extremely high magnetic fields in the angular range $20^\circ - 40^\circ$ from the $b$ axis towards the $c$ axis [3].

In our theory linear in respect of the order parameter we worked with the superconducting state belonging to $B_{2u}$ or $B_{3u}$ representations of the orthorhombic group $D_{2h}$. In the recent preprint [13] there was revealed the splitting of transition to the superconducting state on two subsequent transitions unnoticed in all earlier publications. For fields oriented along the $a$- and $b$-axes the splitting was found invariable in all fields. There was suggested the following sequence of transitions [13]. First, it is transition to the $B_{2u}$ state and then to the non-unitary state with the order parameter presenting the combination of the order parameters relating to $B_{2u}$ and $B_{3u}$ representations.

ACKNOWLEDGMENTS

I am grateful to Jean-Pascal Brison for the useful discussions of the results and to Sheng Ran for his interest in the work.

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