Multiple Scattering: Dispersion, Temperature Dependence, and Annular Pistons

Kimball A. Milton\textsuperscript{1}, Jef Wagner\textsuperscript{1}, Prachi Parashar\textsuperscript{1}, Inés Cavero-Peláez\textsuperscript{2}, Iver Brevik\textsuperscript{3}, and Simen Å. Ellingsen\textsuperscript{3}

Abstract We review various applications of the multiple scattering approach to the calculation of Casimir forces between separate bodies, including dispersion, wedge geometries, annular pistons, and temperature dependence. Exact results are obtained in many cases.

1 Quantum vacuum energy

Quantum vacuum energies, or Casimir energies, are important at all energy scales, from subnuclear to cosmological. Applications are starting to appear in nanotechnology. Furthermore it is most likely that the source of dark energy that makes up some 70\% of the energy budget of the universe is quantum vacuum fluctuations. In particular, the 7-year WMAP data is completely consistent with the existence of a cosmological constant \cite{1},

\[ w \equiv \frac{p}{\rho} = -1.10 \pm 0.14 \text{(68\% CL)}, \]  

which is precisely what would be expected if dark energy arose from this source \cite{2}. Finally, zero-point fluctuations may be the most fundamental aspect of quantum field theory.

\textsuperscript{1} H.L. Dodge Department of Physics and Astronomy, University of Oklahoma, Norman, OK 73019 USA e-mail: milton@nhn.ou.edu, wagner@nhn.ou.edu, prachi@nhn.ou.edu
\textsuperscript{2} Department of Theoretical Physics, Zaragoza University, 50009 Zaragoza, Spain e-mail: icaverop@gmail.com
\textsuperscript{3} Department of Energy and Process Engineering, Norwegian University of Science and Technology, N-7491 Trondheim, Norway e-mail: iver.h.brevik@ntnu.no, simen.a.ellingsen@ntnu.no
2 Multiple-scattering formulation

The multiple scattering formulation is easiest stated for a scalar field, which is rather ’easily’ generalized to electromagnetism. For example, see [3]. Vacuum energy is given by the famous trace-log formula,

\[ E = \frac{i}{2} \text{Tr} \ln G \rightarrow \frac{i}{2} \text{Tr} \ln GG_0^{-1}, \tag{2} \]

where in terms of the background potential \( V \),

\[ (-\partial^2 + V)G = 1, \quad -\partial^2 G_0 = 1. \tag{3} \]

Now we define the T-matrix,

\[ T = S - 1 = V(1 + G_0V)^{-1}, \tag{4} \]

and if the potential has two disjoint parts, \( V = V_1 + V_2 \), it is easy to derive the interaction between the two bodies (potentials):

\[ E_{12} = -\frac{i}{2} \text{Trln}(1 - G_0T_1G_0T_2) \tag{5a} \]

\[ = -\frac{i}{2} \text{Trln}(1 - V_1G_1V_2G_2), \tag{5b} \]

where \( G_i = (1 + G_0V_i)^{-1}G_0, \ i = 1, 2 \), and likewise \( T_i \) refers to \( V_i \).

3 Quantum vacuum energy—dispersion

Perhaps not surprisingly in retrospect, we find that the usual dispersive form of the electromagnetic energy [4]

\[ U = \frac{1}{2} \int (d\mathbf{r}) \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \left[ \frac{d(\omega\varepsilon)}{d\omega} E^2(\mathbf{r}) + H^2(\mathbf{r}) \right] \tag{6} \]

must be used, which, quantum mechanically, corresponds to the vacuum energy form

\[ \mathcal{E} = \frac{i}{2} \int (d\mathbf{r}) \int \frac{d\omega}{2\pi} \left[ 2\varepsilon \text{tr} \Gamma + \omega \frac{d\varepsilon}{d\omega} \text{tr} \Gamma \right], \tag{7} \]

in terms of the Green’s dyadic \( \Gamma \). This result follows directly from the trace-log formula for the vacuum energy

\[ \mathcal{E} = \frac{i}{2} \int \frac{d\omega}{2\pi} \text{Tr} \ln \Gamma, \tag{8} \]