VARIABLE STARS: A NET OF COMPLEMENTARY METHODS FOR TIME SERIES ANALYSIS. APPLICATION TO RY UMA

I. L. Andronov\textsuperscript{1*}, L. L. Chinarova\textsuperscript{2,1†}

\textsuperscript{1} Odessa National Maritime University, Mechnikova st. 24, Odessa, 65029 Ukraine
\textsuperscript{2} Odessa I. I. Mechnikov National University, Dvoriaskaia st. 2, Odessa, 65000 Ukraine

\textbf{Abstract} – The expert system for time series analysis of irregularly spaced signals is reviewed. It consists of a number of complementary algorithms and programs, which may be effective for different types of variability. Obviously, for a pure sine signal, all the methods should produce the same results. However, for irregularly spaced signals with a complicated structure, e.g. a sum of different components, different methods may produce significantly different results.

The basic approach is based on classical method of the least squares (1994OAP....7...49A). However, contrary to common "step-by-step" methods of removal important components (e.g. mean, trend ("detrending"), sine wave ("prewhitening"), where covariations between different components are ignored, i.e. erroneously assumed to be zero, we use complete mathematical models.

Some of the methods are illustrated on the observations of the semi-regular pulsating variable RY UMa. The star shows a drastic
increase of an amplitude of pulsations and a wide minimum of the
mean brightness during the pulsation cycle.

Key words: Time Series Analysis; Data Analysis; pulsating stars;
stars: individual: RY UMa

1 Introduction

Variable stars represent very different types of signal shapes. The official classification is published in the “General Catalogue of Variable Stars” (GCVS) (Samus et al., 2017). In the current (September, 2019) version of the electronic catalogue of GCVS, there are 78 073 variable stars, which are classified into 569 combinations of 44 types. So many of these stars show multi-component variability.

The mathematical modelling of signals may be split into the “physical” and “phenomenological” methods. The first group tries to determine the physical parameters by fitting the theoretical curve to the observations. However, often the number of physical parameters is much larger than may be determined from the current observations. E.g., by determining visual brightness (one parameter), one may not determine the absolute brightness and the distance (two parameters). Other examples may be the size of the spot in the atmosphere and its relative brightness, etc.

So there may be “observational facts”, or “phenomenological parameters”, which may be then used in further models with an additional information.

In this short review paper, we list our main methods and show related references to the original papers. The illustration is presented for the semi-regular variable RY UMa.

2 Basic Methods

2.1 Test Function

The common method of the parameter determination is to minimize (sometimes, maximize) the test function $\Phi(x_k; t_k; C_\alpha)$, which is dependent on the observations $x_k$, $k = 1..n$ obtained at times $t_k$, and on a set of parameters $C_\alpha$, $\alpha = 1..m$. From the statistical point of view, the parameters should maximize the likelihood function (Anderson, 2003). Under a common assumption
that the statistical errors $\sigma_k$ of the observations $x_k$ are random numbers with a zero mathematical expectation (i.e. no systematic shifts) and normally distributed, the test function is typically defined as

$$\Phi_m(x_k; t_k; C_\alpha) = \sum_{k=1}^{n} w_k \cdot (x_k - x_C(t_k; C_\alpha))^2,$$

(1)

where $x_C(t_k; C_\alpha)$ is the "computed" value for a given argument $t_k$ and coefficients $C_\alpha$. The "weights" $w_k$ are to be defined as $w_k = \sigma_0^2 / \sigma_k^2$, where the "unit weight" error $\sigma_0$ may be, in principle, be any constant positive value. Often the programs (e.g. electronic tables) neglect the possible difference in weights, what is equal to set all of them $w_k = 1$.

In the "linear least squares" method, the approximation

$$x_C(t; C_\alpha) = \sum_{\alpha=1}^{m} C_\alpha \cdot f_\alpha(t),$$

(2)

where $f_\alpha(t)$ are called the "basic functions". For the "non-linear least squares", at least some of the basic functions are dependent on the coefficients. In this case, the test function is computed at a grid of values of these "non-linear coefficients", the position of the minimum is used as an initial "vector" (set of values) and then is corrected to more accurate values using the "differential corrections" (see Andronov, 1994a, 2003, Andronov and Marsakova, 2006 for more details). In this case, the basic functions may be extended to a definition

$$f_\alpha(t) = \frac{\partial x_C(t; C_\alpha)}{\partial C_\alpha},$$

(3)

The variance of the approximation is

$$\sigma^2[x_C(t; C_\alpha)] = \sigma_M^2 \cdot \sum_{\alpha\beta=1}^{m} A^{-1}_{\alpha\beta} f_\alpha(t) f_\beta(t),$$

(4)

where $\sigma_M^2 = \Phi_M/(n - M)$, and $M$ is a complete number of the parameters, including $m$ "linear" parameters. The matrix of normal equations

$$A_{\alpha\beta} \sum_{k=1}^{n} w_k \cdot f_\alpha(t_k) \cdot f_\beta(t_k).$$

(5)
2.2 Multi-Component Signals

Complicated models may be subdivided into "linear" (just a sum of larger summands in Eq. 2) or "non-linear" ones. To determine the parameters, it is natural, in both cases, to vary a complete set of parameters. However, often more simple models are used, which are applied consequently.

Contrary to common "step-by-step" methods of removal important components (e.g. mean, trend ("detrending"), sine wave ("prewhitening"), where covariations between different components are ignored, i.e. erroneously assumed to be zero, we use complete mathematical models.

Generally, the matrix $A_{\alpha\beta}$ is not diagonal (i.e. the fasic functions are not orthogonal), so not diagonal is the inverse matrix $A^{-1}_{\alpha\beta}$. This is often neglected, and the solutions and error estimates may significantly differ from the statistically optimal ones.

The oversimplification of the expressions was called the "matrix-phobia" by Prof. Z. Mikulášek (2007). It may change the estimates of the parameters by few dozen percent, and, in worst cases, by a factor of few times or even dozens times.

2.3 Periodogram analysis

For the periodogram analysis, we use a trigonometrical polynomial model of order $s$ (up to $(s - 1)$-th harmonic), which is added to an algebraic polynomial of order $q$:

$$x_C(t; C) = \sum_{\alpha=1}^{q+1} C_{\alpha} \cdot t^{\alpha-1} + \sum_{j=1}^{s} (C_{2j+q} \cdot \cos(j \omega t) + C_{2j+q+1} \cdot \sin(j \omega t)), \quad (6)$$

where $\omega = 2\pi f$, $f = 1/P$ is a trial frequency corresponding to a trial period $P$. As the test function for the periodogram, we use the ratio

$$S(f) = 1 - \frac{\Phi_{q+1+2s}}{\Phi_{q+1}}. \quad (7)$$

where $\Phi_{q+1+2s}$ corresponds to a complete model (Eq. 6) and $\Phi_{q+1}$ corresponds to the algebraic polynomial part. Because the basic functions are not orthogonal, the coefficients $C_{\alpha}$ are different for both models. The exact coincidence of the observations with the approximation corresponds to $S(f) = 1$, whereas the values at "bad frequencies" are typically much smaller.
Even if the preliminary values of the periods of a multi-periodic signal were estimated using one-period approximation, or “prewhitening”, the final values should be corrected using a complete model (e.g. Andronov and Kudashkina, 1988).

Our algorithms are pointed to the period search using trigonometric polynomials of different order with a possible trend, which is approximated by a polynomial of arbitrary order. Such approximations are effective for multi-periodic multi-harmonic signals superimposed on a slow trend. In the software MCV (Andronov and Baklanov, 2004), the approximations may be done for multi-harmonic models for (up to) 3 basic periods with a polynomial trend.

The second type of methods for periodogram analysis is called “non-parametric”. Andronov and Chinarova (1997) studied statistical properties of 9 modifications of the test-functions. They were implemented by software by various authors (e.g. Breus, 2007).

The optimal degree of the trigonometrical polynomial $s$ may be determined using the limit for the FAP (False Alarm Probability) (Andronov, 1994a). Kudashkina and Andronov (1996) made an atlas and catalogue of the Fourier characteristics of a group of LPVs (Long Period Variables). Kudashkina and Andronov (2017) have added “phase diagrams”, i.e. the dependence of the brightness on its derivative.

Recent reviews on LPVs are presented by Kudashkina (2019ab).

2.4 Scalegram and Wavelet Analysis

New effective characteristics of quasi-periodic signals based on the “$\sigma$–scalegram” analysis (Andronov, 1997) have been introduced, namely the effective amplitudes, periods (time scales) and slopes of the scalegram. The main idea is to compute the dependence of the r.m.s. deviation $\sigma(\Delta t)$ of the observations from the fit as a function of the filter half-width $\Delta t$. With an increasing $\Delta t$, the systematic differences of the approximation from the signal increase, thus one may estimate the effective ’period” (or cycle length and the amplitude). The scalegram was applied for additional classification of 173 semi-regular variables (Andronov and Chinarova, 2003).

Andronov, Kolesnikov and Shakhovskoy (1997) had found a fractal-type variability in AM Her at time scale from 3 sec to 30 years (7.5 orders of magnitude). Beyond, Andronov (2003) introduced the ”$\Lambda$–scalegram analysis, which is some kind of a periodogram analysis.
The wavelet analysis was improved for irregularly spaced data (Andronov, 1998) as a particular case of the scalegram analysis. E.g. the periodogram and wavelet analysis of the semi-regular variable supergiant Y CVn was presented by Andronov and Kudashkina (2010) with methodological details.

To increase the accuracy for the studies of period (and other parameters) variations, the ”running sine” (Andronov and Chinarova, 2013) method was proposed, which is for the signals with high coherence (studied by global approximations) and low coherence (suitable for the wavelet analysis). The main idea is to use the ‘‘running approximation’’

\[ x_C(t; t_0; C_\alpha) = C_1 + C_2 \cdot \cos(\omega t) + C_3 \cdot \sin(\omega t) \]  

only in the ‘‘running’’ interval \( t_0 - \Delta t \leq t \leq t_0 + \Delta t \). Thus the parameters \( C_\alpha(t_0) \) are functions of \( t_0 \) and the filter half-width \( \Delta t \). Typically, we choose a ‘‘symmetrical’’ value \( \Delta t = 0.5P \), whereas, for large observational gaps, it may be enlarged to \( \Delta t = 1P \) or even more.

This method is effective for either ”nearly-periodic”, or ”modulated periodic” variations in intermediate polars, pulsating variables etc.

2.5 Special Shapes (Patterns)

For the signals with abrupt changes, a set of approximations using ”special shapes” was proposed. Particularly, the software NAV (“New Algol Variables”) is effective not only for the EA-type eclipsing variables (Andronov, 2012, Andronov et al., 2012), but also for EB and EW and allows distinguishing these types from non-eclipsing elliptic binaries (Tkachenko et al. 2016) while classifying. Using this phenomenological model for multi-color observations, Andronov et al. (2015) estimated physical parameters of the binary model.

For studies of ”near extremum” parts of the light curve, including the determination of ToM (Time of Minimum/Maximum), 19 functions (9 types of functions) were realized in the software MAVKA (Andrych and Andronov, 2019). Some of the functions were previously introduced by Andronov (2005), Andrych et al. (2015, 2017). These methods were applied to determine ToM of a group of pulsating variables (e.g. Tvardovskyi et al. 2018).

Non-polynomial spline-based functions are used for better approximations of the eclipses (Andronov et al., 2017a) and also for pulsating variables with asymmetric phase curves.
2.6 Other Methods

The statistically correct expressions for the auto-correlation functions of detrended signals were presented by Andronov (1994b). They improved previously known expressions for a removal only of a simple mean (Sutherland et al., 1978).

The Principal Component Analysis (PCA) was discussed by Andronov, Shakhovskoy and Kolesnikov (2003) and Andronov (2003).

3 Pulsations of the semi-regular variable RY UMa

The semi-regular pulsating variable RY UMa was analyzed on 6486 visual observations from the AFOEV database (http://cdsarc.u-strasbg.fr/afoev/). The time interval HJD 2451629 – 2458026 continues the previous interval studied in the “Catalogue of Main Characteristics of Pulsations of 173 Semi-Regular Stars” (Chinarova and Andronov, 2000), where the periodogram had shown 3 peaks at periods $P = 3926^d \pm 12^d, 303.74^d \pm 0.08^d$ and $285.29^d \pm 0.07^d$ days and corresponding semi-amplitudes $r = 0.197^m, 0.122^m$ and $0.122^m$, respectively.

Our new studies show a single peak with a period $287.00^d \pm 0.14^d$, initial epoch for the maximum brightness (minimum magnitude) $T_0 = 2454005.2 \pm 0.8$ and semi-amplitude $r = 0.211^m \pm 0.004^m$, superimposed onto a trend (which was approximated by a parabola).

These parameters were obtained by using a complete model, without any detrending or prewhitening, as realized in the software MCV (Andronov and Baklanov, 2004). The characteristics of the individual maxima and minima were determined using the new version of the software MAVKA (Andrych and Andronov, 2019).

The brightness at the individual maxima varies from $6.91^m$ to $7.29^m$, at the minima – from $7.52^m$ to $8.09^m$. For the analysis of the smooth variations of the mean brightness (over the cycle of pulsations), semi-amplitude and phase, the "running sines" method was applied (Andronov and Chinarova, 2013).

Results are shown in Fig. 1 and are explained in the captions. Except the "sine" approximation, all other show drastic variations in the shape of the individual cycles and the mean brightness. Similar "switchings" between the
states of 'nearly constant brightness' and oscillations are seen in some other stars, e.g. RU And (Chinarova, 2010).'

4 Conclusions

The methods have been applied (totally) to 2000+ variable stars of different types using own monitoring, as well as the photometric surveys from ground-based and space observatories. A wide range of types of variability initiated the elaboration of additional methods. They are briefly mentioned with extensive list of links to original papers. The methods are illustrated on the light curve of the semi-regular pulsating star RY UMa.

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Рис. 1: Approximations of the light curve of RY UMa from the AFOEV database. The points are the individual data. The “sine” corresponds to the simplest periodic (sine) approximation, i.e. the trigonometrical polynomial of order 1 without any trend (Eq. (6) with $q = 0$, $s = 1$). The best fit period is $P = 287.00^d \pm 0.14^d$. In this model, there is no variations of phase, amplitude or mean brightness (over the pulsation cycle). The “running parabola” shows a smooth approximation for all the data, whereas the “asymptotic parabola” corresponds to local approximations of separate intervals near extrema. For the “running sine” model, there are shown dependencies of 4 different parameters": extrapolated maximum (max) and minimum (min) brightness, mean brightness over the pulsation cycle (long-term variations) and the approximation of the current brightness (nearly periodic curve with changing amplitude and other parameters).
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