Surface wave propagation characteristics in atmospheric pressure plasma column

M Pencheva¹, E Benova² and I Zhelyazkov¹

¹ Faculty of Physics, Sofia University, 5 James Bourchier Blvd., BG-1164 Sofia, Bulgaria
² Department for Language Teaching and International Students, Sofia University, 27 Kosta Loulchev Street, BG-1111 Sofia, Bulgaria

E-mail: m_pencheva@deo.uni-sofia.bg

Abstract. In the typical experiments of surface wave sustained plasma columns at atmospheric pressure the ratio of collision to wave frequency \( \nu/\omega \) is much greater than unity. Therefore, one might expect that the usual analysis of the wave dispersion relation, performed under the assumption \( \nu/\omega = 0 \), cannot give adequate description of the wave propagation characteristics. In order to study these characteristics we have analyzed the wave dispersion relationship for arbitrary \( \nu/\omega \). Our analysis includes phase and wave dispersion curves, attenuation coefficient, and wave phase and group velocities. The numerical results show that a turning back point appears in the phase diagram, after which a region of backward wave propagation exists. The experimentally observed plasma column is only in a region where wave propagation coefficient is higher than the attenuation coefficient. At the plasma column end the electron density is much higher than that corresponding to the turning back point and the resonance.

1. Introduction

Surface-wave-sustained discharges (SWD) are intensively investigated because of an increasing number of applications in various fields, including technology and environmental protection [1–5]. The surface wave discharge plasmas offer several advantages compared to other types – positive column plasma of dc discharges or to other RF and microwave produced plasmas. They require no internal electrodes, and they can be applied over an extremely broad range of discharge conditions – wave frequency, gas pressure, discharge tube radius [6].

At low and intermediate gas pressures assumption that plasma is a weakly dissipative medium is applicable and the analysis of the wave dispersion relation is performed at the ratio of collision to wave frequency \( \nu/\omega \) equal to zero. This assumption simplifies considerably the numerical calculations and is the reason for significant progress in SWD modelling at these pressures [7]. In the typical experiments of SWD at atmospheric pressure the ratio \( \nu/\omega \) is much greater than unity. Therefore, one might expect that the usual analysis of the wave dispersion relation, performed under the assumption \( \nu/\omega = 0 \), cannot give adequate description of the wave propagation characteristics. The influence of collisions on the dispersion and attenuation characteristics has been investigated theoretically by Margot and Moisan [8] for values of the ratio \( \nu/\omega \) up to 5. It is shown that a turning back point appears in the phase diagram (the dependence of the wave number on the plasma frequency at fixed wave frequency) for all the values of \( \nu/\omega \neq 0 \). Similar result but for the dependence of the wave
number on the wave frequency at fixed plasma frequency (dispersion curves) is obtained by Cibin theoretically [9] and confirmed experimentally [10] for \( \nu/\omega_p \approx 0.04 \). After the turning back point a region of backward wave propagation is observed. The propagation coefficient is limited and does not tend to infinity as it is in the collisionless case \( \nu/\omega = 0 \).

At atmospheric pressure SWD the ratio \( \nu/\omega \) is typically greater than 10 and the effect of collisions on the wave propagation characteristics is even stronger than in the above mentioned cases. In order to study this effect we have analyzed the wave dispersion relationship for arbitrary \( \nu/\omega \). Our analysis includes phase and wave dispersion curves, attenuation coefficient, wave phase and group velocities. A subject of our study is stationary state of plasma column sustained by traveling electromagnetic wave at atmospheric pressure.

2. Basic equations
The plasma under consideration is produced by an azimuthally symmetric TM surface wave with angular frequency \( \omega \) propagating along a plasma cylinder with given radius \( R \) [11]. For simplicity we take up plasma surrounded by vacuum and we assume weak axial inhomogeneity of the plasma column using radially averaged plasma density \( n = (2/R^2) \int_0^R dr n(r) \) [12].

2.1. Wave field components
From Maxwell equations one obtains the wave equation for the axial component \( E_z \) of the wave electric field in cylindrical coordinates as:

\[
\left[ \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} + \frac{\omega^2 \epsilon}{c^2} \right] E_z(r,z) = 0,
\]

where \( \epsilon \) is the dielectric permittivity being \( \epsilon = \epsilon_p \) for the plasma and \( \epsilon = \epsilon_v = 1 \) for the vacuum.

Similar equations are obtained for the radial electric field component \( E_r \) and for the azimuthal magnetic field component \( B_\phi \). Assuming weak axial density inhomogeneity we seek the solutions of the wave equations in the form:

\[
E_{r,z}(r,z) = \text{Re} \left[ F_{r,z}(r) E(z) \exp(-i \omega t + i \int_0^z dz' k'(z')) \right]
\]

\[
B_\phi(r,z) = \text{Re} \left[ G_\phi(r) B(z) \exp(-i \omega t + i \int_0^z dz' k'(z')) \right]
\]

In these expressions \( E(z) = B(z) = E(r = R) \) and \( F_z, F_r, G_\phi \) are cylindrical functions, solutions of the wave equations in dimensionless variables:

\[
F_z^p(\rho) = \frac{I_0(a_p \rho)}{I_0(a_p)}, \quad F_r^p(\rho) = -\frac{i}{a_p} \frac{I_1(a_p \rho)}{I_0(a_p)}, \quad G_\phi^p(\rho) = -\frac{\sigma \rho}{a_p} \frac{I_1(a_p \rho)}{I_0(a_p)}
\]

\[
F_z^v(\rho) = \frac{K_0(a_v \rho)}{K_0(a_v)}, \quad F_r^v(\rho) = -\frac{i}{a_v} \frac{K_1(a_v \rho)}{K_0(a_v)}, \quad G_\phi^v(\rho) = -\frac{\sigma}{a_v} \frac{K_1(a_v \rho)}{K_0(a_v)}
\]

Here \( \rho = r/R \) is the dimensionless radial coordinate, \( a_p^2 = k^2 - \sigma^2 \epsilon_p \) and \( a_v^2 = \tilde{k}^2 - \sigma^2 \), \( \tilde{k} = k'R + i k'R \) is the dimensionless complex wave number, \( \sigma = \omega R/c \) and \( I_0, I_1, K_0, K_1 \) are the modified Bessel functions of zeroth and first order. Te plasma permittivity is
where $\omega_p$ is the usual electron plasma angular frequency and $\nu_{\text{eff}}$ is the effective electron–neutral collision frequency for momentum transfer.

2.2. Local dispersion relation
The wave dispersion relation is obtained using the continuity of the azimuthal wave field components at the plasma–vacuum interface. It reads:

$$
\varepsilon_p^2 = 1 - \frac{\omega_p^2}{\omega(\omega + iv_{\text{eff}})} = 1 - \frac{\omega_p^2}{\omega^2} \left( 1 + \frac{v_{\text{eff}}^2}{\omega^2} \right)^{-1} + iv_{\text{eff}} \frac{\omega_p}{\omega} \left( 1 + \frac{v_{\text{eff}}^2}{\omega^2} \right)^{-1}
$$

(4)

where $\omega_p$ is the usual electron plasma angular frequency and $v_{\text{eff}}$ is the effective electron–neutral collision frequency for momentum transfer.

3. Results
3.1. Axially homogeneous plasma
Figure 1 shows the dispersion (a) and attenuation (b) diagrams for axially homogeneous plasma columns with different electron densities corresponding to plasma frequencies: (1) $\omega_p = 1.9 \times 10^{12}$ s$^{-1}$, (2) $\omega_p = 1.5 \times 10^{12}$ s$^{-1}$, (3) $\omega_p = 7.7 \times 10^{11}$ s$^{-1}$, (4) $\omega_p = 5.1 \times 10^{11}$ s$^{-1}$, (5) $\omega_p = 3.8 \times 10^{11}$ s$^{-1}$, (6) $\omega_p = 3 \times 10^{11}$ s$^{-1}$, (7) $\omega_p = 1.5 \times 10^{11}$ s$^{-1}$, (8) $\omega_p = 1 \times 10^{11}$ s$^{-1}$, (9) $\omega_p = 7.7 \times 10^{10}$ s$^{-1}$. The diagrams on figure 1a contain

![Figure 1](image-url)
turning back points where the real part of the wave number attains its maximum value $k'_{\text{max}}$. After these points there is a region of backward wave propagation. In this region the wave phase and group velocities are with opposite signs (opposite directions, respectively). All of the curves end at point $k'R = 0$ and $(\omega / \omega_p)_{\text{end}}$ increasing with $\omega_p$. For diagrams 1–6 this last point $(\omega / \omega_p)_{\text{end}}$ is higher than the resonant value (the thin dotted line on figure 1a), $n_{\text{res}} = 2\pi n_{\text{cutoff}}$, $n_{\text{cutoff}} = m \omega^2 / 4\pi e^2$, e.g. the curves pass through a region of underdense plasma. The diagrams 7–9 end at point $(\omega / \omega_p)_{\text{end}}$ much lower than the resonant one, e.g. $n_{\text{end}} > n_{\text{res}}$.

The parts of the dispersion curves presented with solid lines satisfied the requirement the phase coefficient is higher or equal to the attenuation coefficient $k' \geq k''$ and the parts of them presented with dashed lines correspond to $k' < k''$. One can see that for $\omega_p \geq 5.1 \times 10^{11}$ s$^{-1}$ ($n_e \geq 8 \times 10^{13}$ cm$^{-3}$) there are regions in the curves where $k' \geq k''$. For high plasma densities these regions pass the turning back point and continue to the backward wave part of the dispersion diagrams (curves 1–3 on figure 1a). The existence of the backward wave propagation region in the dispersion curves is experimentally confirmed by Cibin [10]. For low plasma densities ($n_e < 4.6 \times 10^{13}$ cm$^{-3}$, $\omega_p < 3.8 \times 10^{11}$ s$^{-1}$) but still much higher than the resonant density $n_{\text{res}} = 1.5 \times 10^{14}$ cm$^{-3}$ ($n_{\text{cutoff}} = 7.466 \times 10^{10}$ cm$^{-3}$) there are not regions in the dispersion curves where $k' \geq k''$ e.g. the condition $k' < k''$ is valid for the entire curves (figure 1a, curves 5–9). The value of the wave number $k'' \equiv k' = k''$ corresponds to the point where the real part of the wave number becomes equal to the imaginary part.

In figure 1b are shown the attenuation diagrams for the same discharge conditions as in figure 1a. The attenuation coefficient increases with the decreasing of the plasma frequency and with the increasing of the wave angular frequency (at fixed plasma frequency). Exceptions are curves 1–3 where there are turning back points after which the imaginary part of the wave number decreases. The curves end at the same values of $\omega / \omega_p$ as the corresponding curves in the dispersion diagrams. One can see that for higher plasma densities the curves pass over the resonant line. Like the wave propagation coefficient the attenuation one also attains a maximum value $k''_{\text{max}}$. This maximum value in curves 1–3 is the turning back point while for the rest of the curves it is the point corresponding to $(\omega / \omega_p)_{\text{end}}$. In all diagrams $k''_{\text{max}} > k'_{\text{end}}$. It is necessary to point out that the regions of the dispersion and attenuation diagrams corresponding to $k'' > k'$ (figure 1, dashed lines) cannot be associated with wave propagation, since the wave disappears before making one turn in its phase.

Figure 2. Formation of the phase (a) and attenuation (b) diagrams (black solid and dashed lines) for atmospheric pressure plasma column obtained by using particular dispersion curves (thin solid and dashed lines, corresponding to weak and strong damping respectively).
3.2. Weakly inhomogeneous plasma column

The diagrams presented in figure 1 depict the case of axially homogeneous plasma. In real experimental situations the plasma is axially inhomogeneous with plasma density linearly decreasing along the discharge from the launcher to the column end. The numerical calculations have been done for an argon plasma column investigated experimentally in [13] sustained by electromagnetic wave with frequency $\omega/2\pi = 2.45$ GHz, wave power 110 W, gas pressure 1 atm, tube radius 0.5 mm. The experimentally measured discharge length is 14 cm and the electron plasma densities at the launcher and at the column end, respectively are $7 \times 10^{14}$ and $1 \times 10^{14}$ cm$^{-3}$, which correspond to plasma frequencies $\omega_p^{\text{exc}} = 1.5 \times 10^{12}$ s$^{-1}$ and $\omega_p^{\text{end}} = 5.7 \times 10^{11}$ s$^{-1}$. The electron-neutral collision frequency obtained using a kinetic model for the processes in the plasma at these conditions is $\nu = 2 \times 10^{11}$ s$^{-1}$, which correspond to $\nu/\omega = 13$. The wave propagation characteristics are described by phase diagram ($\omega = \text{const}$). A series of dispersion curves is shown in figure 2a for different plasma densities, $\omega_p$ respectively, corresponding to the considered plasma column. In each of the curves there is one point for which $\omega/2\pi = 2.45$ GHz. Connecting these points with smooth line one obtains the phase diagram (the thick black line – solid and dashed – in figure 2a). It coincides with the phase diagram obtained by numerical solving the local dispersion relation. The thick black line in figure 2b is obtained in the same way. The phase diagram has got a turning back point as well which is corresponding to $k'^{\text{max}} R = 0.18$. It is not the only interpretation that the region of the phase diagram after the turning back point corresponds to backward wave propagation. Note that as wave group velocity can be determine from the dispersion diagrams only, the region after the turning back point in the phase diagram ($\omega = \text{const}$) has been built by using points belonging to the backward parts of curves of the dispersion diagrams ($\omega_p = \text{const}$). Therefore one may call this region a backward wave part of the phase diagram.

The experimentally determined plasma densities correspond to plasma frequencies $\omega_p^{\text{exc}}$ and $\omega_p^{\text{end}}$. The experimental column end is marked with red arrow. The corresponding value of $\omega_p^{\text{end}} (= 5.7 \times 10^{11}$ s$^{-1}$) is close to the calculated one ($\omega_p^{*} = 7.7 \times 10^{11}$ s$^{-1}$) for which the real part of the wave number becomes equal to the imaginary one (the red point). Hence: (i) the plasma column exists in an area of plasma frequencies (densities) where $k' \geq k''$, (ii) at the end of the discharge the electron concentration is almost three orders higher than the resonant one.

Similar behaviour of the phase diagrams is obtained and analyzed by Margot and Moisan for $\nu/\omega$ up to 5. The experimental study of the surface wave propagation in plasmas at atmospheric pressure as well as at reduced pressures is reported in [14]. They have noticed that the largest $k'$ value observed at a given pressure decreases as the pressure increased. At atmospheric pressure the experimentally observed $\omega_p$ domain corresponds to very small values (the beginning of the phase diagram). This is in good agreement with our result presented in figure 2a where one can see that the column ends at small values of $\omega_p$ (plasma density much higher than that corresponding to the turning back point and the resonance).

Having the dispersion curves (figure 1a) the phase and group velocities can be obtained. They are presented in figure 3. Up to the turning back point the values of the phase and group velocities are close. After this point the group velocity becomes negative. The region where the plasma column really exists is the one where $k' \geq k''$ and it is marked off by vertical dashed line. Note that after this line in the region where $k' \leq k''$ the WKB approximation is no longer valid.
4. Conclusions
For the purpose of investigation the propagation characteristics of surface wave, maintaining an atmospheric pressure plasma column we have analyze the wave dispersion relation for azimuthally symmetric wave at arbitrary $\nu/\omega$ ratio. The analysis includes phase and wave dispersion curves and attenuation diagrams. The formation of the phase diagram for fixed ratio of $\nu/\omega = 13$ (corresponding to a given atmospheric pressure plasma column) using particular dispersion curves is demonstrated. The numerical results show that a turning back point appears in the phase diagram, after which a region of backward wave propagation exists. The experimentally observed plasma column exists in a region where wave propagation coefficient is higher than the attenuation coefficient. At the plasma column end the electron density is much higher than that corresponding to the turning back point and the resonance. The results are in good agreement with the theoretical and experimental results obtained by other authors.

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