High transformer ratio resonant PWFA ideal working point design for EuPRAXIA@SPARC_LAB

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Abstract. In the context of plasma beam driven wakefield acceleration, we design and numerically test an ideal working point that exploits the resonant behavior of a train of driving bunches with ramped charge in order to accelerate a trailing bunch to high energy. The working point consists in a train of four bunches generated by an RF X-band photo-injector with the energy of 1.2 GeV. The bunch current profile is shaped by means of hybrid compression stage exploiting the combination of velocity bunching and magnetic chicane. The charges are properly calibrated in order to maximize the transformer ratio up to \( R_T = 8 \). The trailing bunch has a triangular shape and a peak current \( I = 3 \) kA. By means of a 2.4m long plasma channel we simulated the acceleration of the trailing bunch up to 5 GeV mainly preserving the quality of the accelerated beam. The simulations were performed in cylindrical symmetry with the hybrid kinetic-fluid code Architect.

1. Introduction
The Horizon 2020 Project EuPRAXIA ("European Plasma Research Accelerator with eXcellence In Applications") [1] is a European project with the target of building a highly compact and cost-effective facility with multi-GeV electron beams using plasma as the acceleration medium. The facility will exploit the electron beam accelerated in plasma in order to pilot a free electron laser (FEL) producing radiation in the range of the soft to hard X-rays. The EuPRAXIA collaboration investigated several schemes for the acceleration of electrons in plasma, identifying and studying in detail the most promising schemes that will be implemented in parallel in dedicated sites. The site for particle driven plasma wakefield acceleration (PWFA) scheme will be hosted in Frascati (EuPRAXIA@SPARC_LAB) [2]. A first version of the concept design report of the EuPRAXIA@SPARC_LAB facility has been recently addressed. In the CDR we proposed a new infrastructure that matches the requirements of the beam driven site of the EuPRAXIA project itself. This infrastructure will host the first-ever X-band 1 GeV linac, togheter with an FEL providing 3 nm radiation source and an upgraded version of the FLAME laser system.

Considerable effort has been spent in order to demonstrate numerically that the nowadays design of the linac is suitable to perform the first stage operations of EuPRAXIA exploiting the plasma medium. In this first stage, the linac produces a train of two bunches with an energy per particle \( \mathcal{E} \approx 0.5 \) GeV. The train is then transported at the injection of a plasma channel, where the trailing bunch (witness) is accelerated up to 1 GeV by the wake generated by the driving bunch (driver). The witness is then captured, transported downstream the plasma channel up
to the undulators and used to pilot the FEL. A start to end simulation presented in the CDR has provided a reasonable working point accomplishing this result, simulating with different codes every stage of the process \((T\text{-}step, \text{Astra, Architect, Genesis})\). In perspective of the draft of EuPRAXIA@SPARC_LAB technical design report, we explore through numerical simulations an ideal scenario suitable for the 5 GeV case. This scenario consists of a train of ultra-relativistic bunches \((\beta \approx 1)\) composed of 3 drivers and a witness with the initial energy of \(E = 1.2\) GeV. The train is injected inside a plasma channel in order to transfer the drivers energy to the witness [3]. The longitudinal shapes of the bunches are optimized in order to achieve a sufficient energy gain to reach the 5 GeV goal, at the same time preserving the accelerated witness quality.

2. Plasma and simulation parameters
The plasma channel used for the simulation is cylindrical with a radius \(R_p = 250\) µm. For the plasma channel shape we assume a 2.4 m long flat top plasma profile with a background density \(n_p = 2.5 \cdot 10^{16}\) cm\(^{-3}\), preceded by a 1 cm long injection ramp. The simulations are performed in 2D by means of the Hybrid Fluid Kinetic code \textit{Architect} [4]. The simulation box of any simulation is composed by a \(200\hat{u}_r \times 800\hat{u}_z\) grid cells, with \(\hat{u}_z\) and \(\hat{u}_r\) versors in the \(z\) and \(r\) dimension respectively. The mesh is square with a dimension 1 µm×1 µm. The drivers are discretized with \(2 \times 10^5\) particles. The witness is discretized with \(3 \times 10^4\) particles. The integration time step of the simulation is \(\Delta t = 0.59\) fs.

The choice of parameters of this working point is arbitrary since a real start-to-end simulation is still in progress, but it is based on the strong experience obtained in the design of the 1 GeV working point of the EuPRAXIA project [5]. Charges and currents have been chosen with the same order of magnitude of the driver and witness current of the 1 GeV working point, as well as the transverse emittance. Changing the configuration from 1 driver plus witness to 3 drivers plus witness could introduce minor variations in these parameters. This aspect, together with the analysis of the sensitivity of the working point and the comparison with the stability of a real photo-injector will be the treated in following works.

3. Transformer ratio and resonant scheme
Forward operations of EuPRAXIA up to the energy of 5 GeV involve an energy increase of the incoming beam up to more than a factor 4. Such an energy increase is a challenging target for the state-of-the-art PWFA, since the energy increase of the witness is limited by the transformer ratio [6]. The transformer ratio is a quality factor that defines the amount of energy that can be transferred from a driving structure (that in general can be made up of one or more drivers) to a witness. The transformer ratio \(R_T\) is defined as the ratio between the module of the maximum accelerating field after the driver \(E_{max}^+\) and the module of the maximum decelerating field acting on the driver itself \(E_{max}^-\).

\[
R_T = \frac{|E_{max}^+|}{|E_{max}^-|}.
\] (1)

Assuming negligible energy spread, all the particles of the driver are injected with the same energy \(E_0\). After an accelerating length \(L_0 \approx \frac{E_0}{|E_{max}^-|}\), some particles of the driver become weakly relativistic, changing the longitudinal current profile of the driver that leads the shape of the wake. Since in most cases this condition is dramatic for the preservation of the witness quality, the accelerating length \(L_0\) is basically a limit in the accelerating length. Thus, the theoretical maximum energy per particle that can be transferred to a witness is \(\Delta E = R_T E_0\). Nevertheless, a more accurate way to evaluate the transformer ratio is using the average electric field acting on the witness including the beam loading effect (thus the rescaled average energy
gain of the witness) instead of $|E_{\text{max}}^n|$. We will refer to this number as effective transformer ratio.

Analytical evaluations of transformer ratio are possible if we assume to operate in the so called linear regime. If the bunches have a density that is much lower than the density of the background plasma channel $n_b/n_p \ll 1$ it is possible to find approximated analytical solutions for the equations of plasma. In this regime one can evaluate that the inequality $R_T \leq 2$ holds assuming a driver with an axially symmetric longitudinal current [7]. Several works showed that this limit can be overcome in blow-out regime for high density driver $n_b/n_p \gg 1$, i.e. [8]. Another way to overcome this limit is operating with a driving structure without longitudinal symmetry [6, 9]. A train of $N$ drivers properly shaped and with ramped charge could theoretically reach the transformer ratio $R_T = 2N$.

In the draft of the working point we combined these approaches, exploiting a resonant scheme in blow-out regime. The high density drivers maintain the required resonant behavior only operating in the so called quasi non linear regime [10]. Said $k_p^{-1}$ the plasma skin depth defined as $k_p = \sqrt{\epsilon^3 n_p / e m_e c^2}$, we can write the normalized charge as the ratio between the number of the electrons of the bunch $N_b$ and the number of the plasma electrons participating in the initial wave response, namely

$$Q = \frac{N_b k_p^3}{n_p}.$$  

The bunches in the train are supposed to be very short compared to the plasma wavelength. Such a scenario in a real case is very difficult to obtain because for a finite charge of the bunches, the currents are very high. In our design, we decided to assume driving bunches with a length that is comparable with the plasma wavelength. This causes an overlap of the drivers distribution, leading to a final design that is in-between a single bunch with triangular shape and a train of bunches.

Despite minor non-linearities start to occur for $\tilde{Q} > 0.5$, the resonant behavior is maintained up to $\tilde{Q} = 2$. The quasi non linear regime occurs when the limit $\tilde{Q} \leq 2$ is respected together with the condition for blow-out regime $n_b/n_p \gg 1$. As a preliminary approach we designed a working point exploiting a single driver. The driver is initialized at 1 mm before the beginning of the plasma ramp with an ideal bi-gaussian shape in all dimensions. Bunch parameters are reported in Table 1. The bunch charge has been set to $Q = 150$ pC in order to have a normalized charge at the plateau $\tilde{Q} = 1$. The emittance and the energy spread were chosen arbitrarily since at an ideal level of study they are not relevant. The energy $E = 1.2$ GeV is chosen as the maximum theoretical possibility of the linac. Conditions for emittance preservation of bunches injected inside plasma can be evaluated theoretically via the envelope equation. Imposing that at the beginning of plateau the Twiss $\beta$-function is $\beta_m = \sqrt{2\gamma} / k_p$ and the $\alpha$-function is 0, we evaluated as input parameters $\beta_{x,y} = 22$ mm and $\alpha_{x,y} = 1$. We chose the bunch length $\sigma_z = 33 \mu$m, compatible with the condition of maximum accelerating gradient in linear regime $k_p \sigma_z = 1$. With this single driver we were able to obtain a maximum accelerating gradient $|E_{\text{max}}^n| = 3.4$ GV/m with an ideal transformer ratio $R_T \approx 3.8$. We then tried to achieve the same maximum accelerating field using a train of three bunches with the same shape. Every bunch of the train is distant $\Delta z = 105.6 \mu$m from the previous one, corresponding to $\Delta z = 0.5\lambda_p$. The charges are $Q_1 = 40$ pC, $Q_2 = 140$ pC and $Q_3 = 270$ pC and were evaluated numerically in order to have the same maximum decelerating field acting on every bunch. The maximum accelerating gradient obtained with this scheme is $|E_{\text{max}}^n| = 3.5$ GV/m with an ideal transformer ratio $R_T \approx 8.75$. The current profile and the longitudinal field on axis for both schemes are reported in Fig. (1).
Figure 1. Longitudinal field on axis and longitudinal current profile for single bunch scheme (top) and 3 bunch train scheme (bottom) at $z = 0$.

Table 1. Driver(s) and witness parameters at the injection

|                | Driver(s) | Witness |
|----------------|-----------|---------|
| $Q$ [pC]      | 150/40-140-270 | 30      |
| $\gamma$      | 2348      | 2348    |
| $\epsilon_n$ [mm mrad] | 1         | 0.7     |
| $\sigma_E$ [%] | 0.1       | 0.1     |
| $\beta_{x,y}$ [mm]   | 22        | 22      |
| $\alpha_{x,y}$ [mm]  | 1         | 1       |
| $\sigma_z$ [$\mu$m] | 33        | 16 (3.8 rms) |

4. Witness acceleration

The witness has been designed assuming the integrated parameters listed in Table 1 that are compatible with the possibilities of the EuPRAXIA@SPARC_LAB linac. The charge has been set to 30 pC in accordance with the requirements of the EuPRAXIA facility. The energy is the same of drivers $E = 1.2$ GeV. As well both emittance and energy spread are chosen arbitrarily. The transverse distribution of the beam is bi-gaussian and the Twiss functions are set in order to have a waist corresponding to the matching condition at the beginning of the plateau. The current shape is triangular in order to minimize the energy spread growth [11]. The basis of the triangular shape has a length of $\sigma_z = 16$ $\mu$m corresponding to an rms bunch length $\sigma_z = 3.8$ $\mu$m. We optimized the bunch separation between the last driver and the witness up to $\Delta z = 0.46 \lambda_p \approx 97$ $\mu$m in order to minimize the energy spread growth. The integrated parameters evolution is reported in Fig.(2).
Figure 2. Integrated parameters evolution of Witness bunch. We report the evolution of the transverse spot size and emittance in a) (for the first 10 cm) and in b) (for the entire channel) along with the density of the plasma channel. We report in c) the evolution of energy and energy spread.

Figure 3. Witness phase space at the initialization (left) and at the end of the simulation (right). The transverse phase space is perfectly matched while the longitudinal phase space presents an energy spread growth mostly located on bunch tail.

witness are set with proper matching conditions on the $\beta$-function, the transverse evolution of the drivers doesn’t lead to major changes in the wakefield. The emittance of the witness is preserved along the entire plasma channel and the energy spread grows up to 0.4%. In Fig.(3) we report the witness phase space at the initialization and at the end of the simulation. The shape of longitudinal phase space at the exit of the channel can be explained by minor changes in the peak accelerating field during propagation of the drivers. The perfect beam loading compensation can be obtained only for rigid bunches, while for non-rigid bunches it is more convenient to find a setup that guarantees the minimum energy spread in the overall distribution. In our case this corresponds to a situation where the energy spread increase is concentrated on the tail where the bunch current is lower, preserving the head and the core, where the current is higher. The average accelerating gradient is $E_z = 1.65$ GV/m and the effective transformer ratio is $R_T = 3.65$.

5. Summary and future perspectives
By means of the resonant scheme, we designed and optimized an ideal working point for high energy gain plasma acceleration to be exploited at the EuPRAXIA@SPARC_LAB facility.
The final performances of the designed working point are an average accelerating field $E_z = 1.65$ GV/m and an effective transformer ratio of $R_T = 3.65$. We showed how this scheme can be theoretically exploited in order to accelerate a witness up to the final goal of EuPRAXIA@SPARC_LAB facility by means of a 2.4 meters long plasma channel. The final energy spread $\sigma_E = 0.4\%$ is far below the quality requirements of the facility ($< 1\%$). An ideal focusing of the bunch up to the theoretical values of the matching condition guarantees a good preservation of the brightness. These promising results guarantee a baseline for the development of a more realistic working point to be designed in the context of a start to end simulation.

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