Spin-dependent Fragmentation Functions

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I will give an overview on fragmentation functions with particular emphasis on spin-dependence. A straightforward classification scheme permits to label all independent fragmentation functions for a given physical situation in an unambiguous way. In the context of light-cone quantisation the leading twist fragmentation functions have an intuitive probabilistic interpretation.

1. Motivation and Purpose of the Talk

A confusing variety of new and exotic fragmentation functions (FFs) is discussed recently, in particular in the context of the extraction of the third, leading twist – hitherto unknown – nucleon distribution function. This transversity distribution $h_1(x)$ (frequently also denoted $\delta q(x)$ or $\Delta_T q(x)$) involves a flip of quark chirality, and therefore, requires a second chiral-odd function to form a chiral-even observable. There is a number of FFs describing specific physical situations which offer themselves as possible chiral-odd partners for $h_1(x)$. These FFs are not only indispensable tools for the extraction of the transversity distribution, but of interest in themselves, since they bear witness to the process of hadronisation, or in other words, how confinement comes about. This holds true also for the chiral-even FFs.

The present contribution is intended to serve as a reminder that there is a systematic and unambiguous way to classify the FFs according to the physical situations they describe, and to exclude those not in accordance with general constraints from hermiticity of Dirac fields and their well-known behaviour under the parity transformation. In this contribution I will restrict myself to the consideration of jets initiated by quarks.

Disclaimer: Most of the content of this contribution is based on work done by others. I am indebted to all colleagues whose work guided my understanding of the topic. A complete list of references would exceed the allowed 6 pages easily. Therefore, the references listed are to be understood as proposed starting points for further reading; more references can be found in the cited ones. A more comprehensive source of information is the WWW database \[1\], a project of the TMR network ESOP dedicated to fragmentation functions.

2. Classification Scheme

The basic message first:

- the number of independent FFs describing a certain physical situation is limited
Let us take a look at the formal definition of FFs from soft hadronic matrix elements of quark field operators \[3\]. In a first step one defines a quark-quark correlation function for the fragmentation process of a quark to one (or two) hadron(s): 

\[ q \to h_1(h_2)X \]

\[
\Delta_{ij}(k, P_1, P_2) = \sum_{\chi} \int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi} \\
\times \langle 0 | U(0, \xi) \psi_i(\xi) | P_1, (P_2); X \rangle \langle P_1, (P_2); X | \overline{\psi}_j(0) | 0 \rangle \tag{1}
\]

Some properties of the correlation function can be derived easily, or are even evident from its definition. The quantity \( \Delta \) is a \( 4 \times 4 \) matrix in Dirac space, and depends on the momentum vectors of the quark \( k \) and of the observed hadron(s) \( P_1, (P_2) \) and possibly spin vector(s) \( S_1 (S_2) \) for spin-1/2 hadrons, or spin vector and tensor \( S_1, T_1 (S_2, T_2) \) for spin-1 hadrons, respectively. For instance, for the case a single spin-1/2 hadron with momentum \( P_h \) and spin vector \( S_h \) is observed the most general ansatz can be shown to have the form \[3\]

\[
\Delta(k, P_h) = B_1 M_h + B_2 \not{\!P}_h + B_3 \not{\!k} + (B_4/M_h) \sigma_{\mu\nu} P_\mu^h k^\nu \\
+ i B_5 (k \cdot S_h) \gamma_5 + B_6 M_h \not{\!S}_5 + (B_7/M_h) (k \cdot S_h) \not{\!P}_\gamma k^5 \\
+ (B_8/M_h) (k \cdot S_h) \not{\!k} \gamma_5 + i B_9 \sigma_{\mu\nu} \gamma_5 S_h^\mu P_\nu^h \\
+ i B_{10} \sigma_{\mu\nu} \gamma_5 S_h^\mu k^\nu + i (B_{11}/M_h) (k \cdot S_h) \sigma_{\mu\nu} \gamma_5 k^\mu P_\nu^h \\
+ (B_{12}/M_h) \epsilon_{\mu\nu\rho\sigma} \gamma_5 P_\mu^h k^\rho S_h^\sigma \tag{2}
\]

with real amplitudes \( B_i(\sigma, \tau) \) depending on the invariants \( \tau = k^2 \) and \( \sigma = P_h \cdot k \). Only terms are kept in the ansatz which are in accordance with general constraints derived from

1. the **hermiticity** of the Dirac fields

2. the known behaviour of Dirac fields under **parity transformation**.

We note in passing that the amplitudes \( B_4, B_5, B_{12} \) are **naive time-reversal odd** (T-odd), i.e. they would be forbidden by a constraint from the behaviour of Dirac fields under time-reversal, if this operation would not transform ‘in’ states and vice versa. The presence of final state interactions within a current jet, and the distortion of the ‘out’-states from plane waves, is sufficient to allow for non-vanishing T-odd amplitudes.

Fragmentation functions are obtained from the correlation function \( \Delta \) by projection with specific Dirac matrices \( \Gamma \), and integration over components of the quark momentum

\[
\Delta^{[\Gamma]}(z) \equiv \frac{1}{4z} \int dk^+ \int d^2k_T \ Tr[\Delta \Gamma] \bigg|_{k^- = P_h^- / z} \tag{3}
\]

or for \( k_T \)-unintegrated FFs

\[
\Delta^{[\Gamma]}(z, -zk_T) \equiv \frac{1}{4z} \int dk^+ \ Tr[\Delta \Gamma] \bigg|_{k^- = P_h^- / z} ; k_T \tag{4}
\]
where it turns out that the Dirac matrices $\Gamma \in \{\gamma^-, \gamma^-\gamma_5, i\sigma^\alpha\gamma_5\}$ project on leading twist FFs, and the remaining independent $4 \times 4$ matrices on higher twist FFs. Here, the notion of ‘twist’ is used in the sense of a ‘working redefinition’, or ‘effective twist’ as introduced by Jaffe [5]. An intuitive probabilistic interpretation arises in the context of light-cone quantisation. At leading twist the projected quark field operators can be rewritten as densities of “good” (or dynamically independent) components of the Dirac field operators. Chiral projectors $P_{R/L} = (1 \pm \gamma_5)/2$ allow a distinction of right- and left-handed components. Note that chirality and helicity are identical for the “good” components of a massless quark field. The matrix $\Gamma$ in the projection determines quark spin states as sketched in Fig. 1.

3. One Unpolarised Hadron [6,7]

The independent leading twist functions describing the fragmentation of a quark into one observed unpolarised hadron and the unobserved rest of the jet are listed in Fig. 2.

$$\Delta^{\gamma^-}(z) = D_1(z)$$
$$\Delta^{\gamma^-\gamma_5}(z) = 0$$
$$\Delta^{i\sigma^\alpha\gamma_5}(z) = 0$$

Additional $k_T$-unintegrated FFs

$$\Delta^{\gamma^-}(z, k_T) = \ldots$$
$$\Delta^{\gamma^-\gamma_5}(z, k_T) = 0$$
$$\Delta^{i\sigma^\alpha\gamma_5}(z, k_T) = \ldots + \frac{e_i^j k_{T_j}}{M_H} H_1^\perp(z, k_T)$$

There is only one FF $D_1(z)$, plus a second unintegrated, called $H_1^\perp(z, k_T)$ (Collins function [7]), occurring when transverse momentum dependent observables are considered. The latter describes the correlation between the spin orientation of a transversely polarised quark and the transverse momentum component $k_T$ of the observed hadron relative to the jet direction.
4. One Spin-1/2 Hadron

The independent leading twist functions describing the fragmentation of a quark into one observed spin-1/2 hadron and the unobserved rest of the jet are listed in Fig. 3. There are two distinct groups of additional unintegrated FFs occurring in transverse momentum dependent observables: With transverse momentum dependence the possibilities arise that longitudinal quark spin orientation is correlated to transverse hadron spin \( H_{1T}^T(z, k_T) \), or vice versa \( H_{1L}^T(z, k_T) \). Or two different transverse directions, say \( e_x \) and \( e_y \) for quark and hadron spin are correlated \( H_{1T}^T(z, k_T) \). This group is indicated by hatched hadrons in the pictograms of Fig. 3. The second group (indicated with crosshatched hadrons) comprises the functions \( H_{1T}^T(z, k_T) \) (Collins function) and \( D_{1T}^T(z, k_T) \); both are T-odd and can be regarded as twin partners, since they describe the correlation of the transverse momentum of the hadron with the transverse quark spin, or transverse hadron spin, respectively. Both are accessible measuring the azimuthal dependence of the hadron production.

5. One Spin-1 Hadron

By suitable adaptation of the ansatz Eq. (2) to the case of a spin-1 hadron the independent fragmentation functions can be derived following the method outlined in Sec. 2. Since the spin state of the hadron can be described by a spin vector \( S \) and an additional tensor \( T \), all FFs of the spin-1/2 case occur, plus additional ones associated to the spin tensor structure. The leading twist FFs are quoted in table 1 (adapted from [10], where also pictorial diagrams for a probabilistic interpretation can be found).

\[
\begin{align*}
\Delta^{[n]}(z) &= D_1(z) - (\cdot - \bigcirc) \\
\Delta^{[n-\gamma_5]}(z) &= \lambda_h G_1(z) - (\cdot - \bigcirc) \\
\Delta^{[i\sigma \gamma_5]}(z) &= S^i_{hT} H_1(z) - (\cdot - \bigcirc)
\end{align*}
\]

\[
\begin{align*}
\Delta^{[n]}(z, k_T) &= \ldots + \frac{e_T j i k_T S^j}{M_h} D_{1T}^1(z, k_T) \\
\Delta^{[n-\gamma_5]}(z, k_T) &= \ldots + \frac{k_T \cdot S_{hT}}{M_h} G_{1T}(z, k_T) \\
\Delta^{[i\sigma \gamma_5]}(z, k_T) &= \ldots + \frac{e_T j i k_T}{M_h} H_1^1(z, k_T) \\
& \quad + \frac{k_T}{M_h} \left( \lambda_h H_{1L}^1(z, k_T) + \frac{k_T \cdot S_{hT}}{M_h} H_{1T}^T(z, k_T) \right) \\
& \quad - (\cdot - \bigcirc) \\
& \quad - (\cdot - \bigcirc) \\
& \quad - (\cdot - \bigcirc)
\end{align*}
\]
Table 1
List of time-reversal even and odd, leading twist fragmentation functions $\Delta(k_T)$.

|       | $[\gamma^-]$ | $[\gamma^-\gamma_5]$ | $[i\sigma^+\gamma_5]$ |
|-------|---------------|------------------------|------------------------|
| T-even |               |                        |                        |
| T-odd  | $D_1$         | $G_{1L}$               | $H_{1L}^\perp$         |
| T      | $D_{1L}^\perp$| $G_{1T}$               | $H_{1T}$ $H_{1T}^\perp$|
| LL     | $D_{1LL}$    | (G_{1LT})              | (H_{1LT} $H_{1LT}^\perp$) |
| LT     | $D_{1LT}$    | (G_{1TT})              | (H_{1TT} $H_{1TT}^\perp$) |
| TT     | $D_{1TT}$    |                        |                        |

6. Two Unpolarised Hadrons [13–18]

A very promising option to access the transversity distribution $h_1(x)$ is to consider two hadrons produced in the same jet. There is one particular $k_T$-integrated function, in the present scheme named $H_1^q(z)$ (following [17]) which is chiral-odd and depends on the transverse component of the relative momentum $R = P_1 - P_2$, but not on the transverse momentum components of the hadron pair relative to the jet direction. Thus a measurement of an asymmetry involving a convolution of $h_1$ and $H_1^q$ combines several advantages: the asymmetry is a leading twist effect, collinear factorisation holds (no Sudakov suppression), and a spin measurement of final state hadrons is not required.

Consider a situation where a quark with momentum $k$ fragments to two unpolarised hadrons, say a pair of pions with momenta $P_1$ and $P_2$, and the rest of the jet. An adaption of the method outlined in Sec. 2 leads to the leading twist FFs depicted in Fig. 4.

$$\Delta[\gamma^-](z_1, z_2, k_T, R_T) = D_1(z_1, z_2, k_T, R_T) \begin{pmatrix} 0 \ 0 \end{pmatrix}$$

$$\Delta[\gamma^-\gamma_5](z_1, z_2, k_T, R_T) = \frac{e^{ij}_TR_Tk_T}{M_1M_2} G_{1L}^\perp \begin{pmatrix} -1 \ -1 \end{pmatrix} - \begin{pmatrix} -1 \ -1 \end{pmatrix}$$

$$\Delta[i\sigma^+\gamma_5](z_1, z_2, k_T, R_T) = \frac{e^{ij}_TR_T}{M_1 + M_2} H_1^q + \frac{e^{ij}_k k_T}{M_1 + M_2} H_{1L}^\perp \begin{pmatrix} 1 \ 1 \end{pmatrix} - \begin{pmatrix} 1 \ 1 \end{pmatrix}$$

Figure 4. Leading twist FFs for a quark fragmenting to two unpolarised hadrons (plus rest of the jet). The functions depend on the light-cone momentum fractions $z_1, z_2$ and the quantities $k_T^2, R_T^2$, and $k_T \cdot R_T$, where $R_T$ is the transverse component of $R \equiv (P_1 - P_2)/2$. 
7. Generalisations and Items Not Covered

The method outlined above lends itself to straightforward generalisations describing other physical situations like for instance the observation of hadrons with spin higher than 1/2, or the hadronisation in a gluon initiated jet [19].

A number of topics in the context of FFs could not be covered in the present short contribution, but some shall be mentioned at least (see [1] for more details):

higher twist The projections resulting in higher twist FFs involve a combination of “good” and “bad” (dependent) light-cone components of quark fields, and do not allow a simple interpretation. The “bad” components actually can be considered as quark gluon composites, which indicates the close relationship of higher twist projections of $\Delta$ to quark-gluon-quark correlation functions.

evolution FFs are subject to a logarithmic scale dependence similar to PDFs. Evolution of $k_T$-unintegrated FFs has been discussed up to now only in the large $N_c$ limit [20].

positivity bounds From the requirement of positivity of a helicity matrix a class of inequality relations between different FFs can be derived [21].

models FFs have been estimated in different model calculations, among them bag models, spectator models, and instanton models (references to be found in [1]).

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