Forward-Backward Asymmetries of Fourth Family Fermions
Through the $Z'$ Models at Linear Colliders

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Abstract

We investigate the forward-backward asymmetries in the pair production of fourth family fermions through the new $Z'$ interactions in the $e^+e^-$ collisions. The $Z'$ boson having family universal couplings can contribute to the pair production of fourth family fermions via the $s$-channel exchange. The linear colliders will provide a clean environment for the physics of $Z'$ boson to measure its couplings precisely. The effects of the $Z'$ boson to the asymmetries are shown to be important in some parameter regions for different $Z'$ models. Among these parameters, the invariant mass distribution $m_{\bar{F}F}$ will be an important measurement to constrain the $Z'$ models. Providing the fourth family fermion exist in an accessible mass range, a $\chi^2$ analysis can be used to probe the $Z'$ models at linear colliders.

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I. INTRODUCTION

Even though we observe three families of quarks and leptons of the Standard Model (SM), however there could be a fourth family if their masses and mixings are beyond our present experimental reach. A family extension to the SM fermion families contains the quarks \( t' \) and \( b' \), and charged lepton \( l' \) with its associated neutrino \( \nu' \). The allowed parameter space for a fourth family is restricted by the experimental searches, precision electroweak measurements, theoretical constraints from the requirements of unitarity and perturbativity.

Recent searches at Large Hadron Collider (LHC) experiments have considered pair production of the fourth family quarks. The data of 1 fb\(^{-1}\) from ATLAS experiment at the LHC (7 TeV) restricts the masses of \( t' \) and \( b' \) quarks: \( m_{t'} > 404 \text{ GeV} \) at 95\% CL. \(^1\) assuming \( t' \rightarrow W^+b \) and \( m_{b'} > 450 \text{ GeV} \) at 95\% CL. \(^2\) assuming \( b' \rightarrow W^-t \) and taking into account the subsequent decays into same-sign dilepton final state. A search for pair produced bottom-like quarks (\( b' \)) in the lepton+jets channel by the ATLAS Collaboration excludes a \( b' \) quark mass of less than 480 GeV \(^2\).

Using a data sample corresponding to an integrated luminosity of 4.9 fb\(^{-1}\) with the CMS detector at the LHC, the most stringent limits exclude the existence of a down-type (up-type) fourth family quark with masses below 611 GeV (565 GeV) \(^3\), \(^4\) assuming a branching fraction of 100\% for the decays \( b' \rightarrow W^-t \) and \( t' \rightarrow W^+b \).

From direct production searches at LEPII, there is a lower limit of the order of 100 GeV for the fourth family charged lepton and unstable neutrino. The precision measurements restrict the mass splitting between the fourth family leptons \( |M_{l'} - M_{\nu'}| \approx 30 - 60 \text{ GeV} \) and the fourth family quarks \( |M_{t'} - M_{b'}| \approx 50 - 70 \text{ GeV} \) \(^5\), \(^6\).

The fourth family quarks and leptons could also couple to an extra neutral gauge boson different from the three SM families. A new neutral gauge boson \( Z' \) can have family universal or non-universal couplings to fermions. The indirect searches of the \( Z' \) boson can also be performed at linear colliders where the discovery limits are related to the deviations from the SM predictions for the cross sections and asymmetries due to the interference effects between the propagators. The \( Z' \) boson having family universal couplings can contribute to the pair production of fourth family fermions via the \( s \)-channel exchange. The linear collider provides a clean environment for \( Z' \) physics, and can measure the couplings precisely.

In the extensions of the SM with \( U(1)_{\psi} \times U(1)_{\chi} \) gauge symmetry, the fields \( Z'_{\psi} \) and \( Z'_{\chi} \)
can be massive and their states can mix, therefore, a relatively lighter mass eigenstate can be written as \( Z'(\theta) = Z'_\psi \cos \theta + Z'_\chi \sin \theta \). A set of \( Z' \) models have some special names: the sequential \( Z'_S \) model has the same coupling to the fermions as that of the \( Z \) boson of the SM; the \( Z'_\psi \), \( Z'_\chi \) and \( Z'_\nu \) models corresponding to the specific values of the mixing angle \( \theta \) (0, \( \pi/2 \) and \( \arctan \sqrt{3/5} \), respectively) in the \( E_6 \) model have different couplings to the fermions; the \( Z'_{B-L} \) model has the couplings related to the minimal \( B - L \) (where \( B \) and \( L \) are baryon and lepton numbers, respectively) extension of the SM. The detailed descriptions of the \( Z' \) models, as well as the specific references can be found in Refs. [7–10].

The Tevatron experiments excluded the sequential \( Z'_S \) boson with a mass lower than 1 TeV at 95% CL. [11]. Recent measurements by the ATLAS and CMS Collaborations based on the data of 1 fb\(^{-1}\) excludes a \( Z'_S \) with mass lower than 1.83 TeV [12] and 1.94 TeV [13], respectively. With the data corresponding to an integrated luminosity of 5 fb\(^{-1}\) recorded by the ATLAS experiment, a lower limit of 2.21 TeV on the mass of sequential \( Z' \) boson has been set at 95% CL [14].

These limits on the \( Z' \) boson mass favors high energy (\( \geq 1 \) TeV) collisions for the observation of signal from most of the \( Z' \) models. It is also possible that the \( Z' \) bosons can be much heavy or weak enough to escape beyond the discovery reach expected at the LHC. In this case, only the indirect signatures of \( Z' \) exchanges may occur at the high energy colliders.

Recently, D0 and CDF Collaborations have measured the forward-backward (FB) asymmetries of top quark \( A_{FB}^t \) at Tevatron [15, 16] in the large \( t\bar{t} \) invariant mass region, while the \( A_{FB}^b \) was measured in the \( Z \) boson decays at LEP [17], which differ by about 3\( \sigma \) deviations from the SM expectations, without affecting significantly the well behaved total cross sections. Several models of new physics have been considered to explain these asymmetries (see Refs. [18–20] and references therein) at hadron colliders.

In this study, we investigate the forward-backward asymmetries \( A_{FB}^F \) for pair production of the fourth family fermions \( F_i (t', b', l', \nu') \) within the \( Z' \) models at linear collider energies of 1 TeV and 3 TeV. The linear colliders, namely the International Linear Collider (ILC) described in [21, 22] and the Compact Linear Collider (CLIC) described in [23, 24], have been designed to meet the baseline requirements for the planned physics programs. The effects of the \( Z' \) boson to the asymmetries of fourth family fermions at linear colliders are shown to be important in some parameter regions for the sequential model, some special \( E_6 \) models and the \( B - L \) model. We will typically consider the mass of the fourth family
II. INTERACTIONS WITH FOURTH FAMILY FERMIONS

The interactions of the fourth family quarks \( (Q_i) \) via neutral gauge bosons \( (g, \gamma, Z, Z') \) and fourth family leptons \( (L_i) \) via electroweak gauge bosons \( (\gamma, Z, Z') \) can be described by the following Lagrangian. We also include the interactions of fourth family fermions \( (F_i) \) with three known families of fermions \( (f_i) \) through the charged currents (via \( W^\pm \) bosons) to be read as

\[
L' = -g_s Q_i T^a \gamma^\mu Q_i G^a_\mu - g_e Q_F F_i \gamma^\mu F_i A_\mu - \frac{g}{2\sqrt{2}} V_{ij} F_i \gamma^\mu (1 - \gamma^5) f_j W_\mu - \frac{g Z}{2} F_i \gamma^\mu (C'_V - C'_A \gamma^5) F_i Z_\mu + \text{H.c.} \tag{1}
\]

where \( g_s, g_e, g_Z \) are the strong, electromagnetic and weak-neutral coupling constants, respectively. The \( G^a_\mu, A_\mu, W_\mu \) and \( Z_\mu \) are the fields for gluons, photon, \( W \) and \( Z \) bosons, respectively. The \( C'_V (C_V) \) and \( C'_A (C_A) \) are vector and axial-vector couplings with the \( Z' (Z) \) boson and they are given in Table I.

The decay widths into the heavy fermion pair \( FF \) and \( W^+W^- \) bosons are given as

\[
\Gamma (Z' \to FF) = \frac{g_{Z'}^2 N_c}{48\pi M_{Z'}} \sqrt{1 - \frac{4 M_F^2}{M_{Z'}^2}} \left[ (C'_A)^2 (-4 M_F^2 + M_{Z'}^2) + (C'_V)^2 (M_{Z'}^2 + 2 M_F^2) \right] \tag{2}
\]

\[
\Gamma (Z' \to W^+W^-) = \frac{g_W^2 \cos^2 \theta_W \kappa^2}{192\pi M_{Z'} M_W^4} \sqrt{1 - \frac{4 M_W^2}{M_{Z'}^2}} \left( \frac{M_Z}{M_{Z'}} \right)^4 \times \left[ M_{Z'}^6 + 16 M_W^2 M_{Z'}^2 - 68 M_W^4 M_{Z'}^2 - 48 M_W^6 \right] \tag{3}
\]

where \( N_c \) is the color factor (3 for quarks and 1 for leptons), and \( g_{Z'} \) is the coupling constant for \( Z' \) boson. The \( M_Z, M_{Z'}, \) and \( M_W \) are the masses for \( Z, Z' \) and \( W \) bosons, respectively. The \( M_F \) is the mass of heavy fermion. The mixing term between the \( Z' \) boson and \( Z \) boson is assumed to be of the order of \( M_Z^2/M_{Z'}^2 \), hence a mixing factor \( \kappa \) scales this extension depending on the specific \( Z' \) models. In the sequential model the factor \( \kappa \) is chosen to be
Table I: The family independent vector and axial-vector couplings to new Z' boson predicted by different models.

|           | down-type quarks | up-type quarks | charged leptons | neutrinos |
|-----------|------------------|----------------|-----------------|-----------|
|           | C'\_V            | C'\_A          | C'\_V           | C'\_A     |
| Z'\_S     | \(-\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W\) | -\frac{1}{2} | \(\frac{1}{2} - \frac{4}{3} \sin^2 \theta_W\) | \(-\frac{1}{2} + 2 \sin^2 \theta_W\) |
| \(Z'\_\psi\) | 0 | \(\sqrt{\frac{3}{6}} \sin \theta_W\) | 0 | \(\sqrt{\frac{3}{6}} \sin \theta_W\) |
| \(Z'\_\chi\) | \(\frac{\sqrt{3}}{3} \sin \theta_W\) | -\(\frac{\sqrt{3}}{3} \sin \theta_W\) | 0 | \(-\frac{\sqrt{3}}{6} \sin \theta_W\) |
| \(Z'\_\eta\) | \(\sin \theta_W\) | \(\frac{1}{3} \sin \theta_W\) | 0 | \(-\sin \theta_W\) |
| \(Z'\_B-L\) | \(\frac{2}{3}\) | 0 | \(\frac{2}{3}\) | -2 |

unity, which is a reference for the purpose of comparison. In Fig. 1 we present the decay width of Z' boson versus the mass M_{Z'} in case of four fermion families. As it can be seen from Fig. 1 that the total decay width \(\Gamma\) for \(Z'\_S\), \(Z'\_\psi\), \(Z'\_\chi\), \(Z'\_\eta\) and \(Z'\_B-L\) models are about 118, 23, 48, 129 and 10 GeV at a mass value of \(M_{Z'} = 3000\) GeV, respectively. For the sequential Z' model, the branching ratios of all fermionic modes are not much sensitive to \(M_{Z'}\), leading to the fractions of about 0.7, 0.2, 0.1 and 0.02 for \(Z' \rightarrow q\bar{q}, Z' \rightarrow \nu\bar{\nu}, Z' \rightarrow l^+l^-\) and \(Z' \rightarrow W^+W^-,\) respectively. Here, the \(q\bar{q}\) mode includes the quarks of four families, and the \(l^+l^- (\nu\bar{\nu})\) mode includes the charged leptons (neutrinos) of four families.

### III. CROSS SECTIONS

Using the interaction Lagrangian (1) we calculate the differential cross section for the pair production of fourth family quarks and leptons in the collisions of \(e^+\) and \(e^-\) beams. The analytical expressions for the differential cross section are given in the Appendix. We calculate the cross section for pair production of fourth family quarks (leptons) taking their masses in the interval 600-1000 (200-800) GeV. Table II shows the production cross sections.
Figure 1: The decay widths of $Z'$ boson predicted by different models in the case of four fermion families.

Figure 2: The branching ratios of sequential $Z'$ boson into different final states depending on the mass.

without $Z'$ contribution at ILC and CLIC energies. The ILC with $\sqrt{s} = 1\,\text{TeV}$ has the potential up to the kinematical range $(m_{l',\nu'} \leq 500\,\text{GeV})$ for the production cross section of fourth family lepton pairs. However, the CLIC with $\sqrt{s} = 3\,\text{TeV}$ extends the mass range for the fourth family fermions. In order to see the contributions from $Z'$ boson exchange and its interference we also calculate the cross sections assuming the reference mass values $M_{l'} = 650\,\text{GeV}$ and $M_{\nu'} = 100\,\text{GeV}$ with the constraints $M_{l'} - M_{l'} \simeq 50\,\text{GeV}$ and $M_{l'} - M_{\nu'} \simeq 100\,\text{GeV}$. The cross sections for $l'$ and $\nu'$ pair production through $Z'$ effects are shown in Table III for ILC with $\sqrt{s} = 1\,\text{TeV}$. In Table IV the cross sections for pair production of $t'$ and $b'$ quarks are presented for CLIC with $\sqrt{s} = 3\,\text{TeV}$ through different $Z'$ models. In Tables III and IV we assume the $Z'$ boson mass $m_{Z'} = 2500\,\text{GeV}$.

The leptonic decay mode of the $Z'$ boson has lower branching ratio than the hadronic one, but the cross section for the process $e^-e^+ \rightarrow l'^-l'^+$ is comparable with the $t\bar{t}$ pair production for some $Z'$ models. The intermediate goals after the discovery of the $Z'$ boson
Table II: The cross sections (fb) for the fourth family pair production processes (without $Z'$) at CLIC with $\sqrt{s} = 3$ TeV. The numbers in paranthesis shows the results for ILC with $\sqrt{s} = 1$ TeV.

| Mass (GeV) | $e^+e^- \rightarrow \nu'\bar{\nu}'$ | $e^+e^- \rightarrow l'^+l'^-$ | Mass (GeV) | $e^+e^- \rightarrow b'\bar{b}'$ | $e^+e^- \rightarrow t'\bar{t}'$ |
|------------|----------------------------------|--------------------------------|------------|----------------------------------|---------------------------------|
| 200        | 2.71(22.10)                      | 12.44(108.60)                  | 600        | 9.17                            | 18.80                           |
| 300        | 2.67(18.28)                      | 12.39(100.70)                  | 700        | 8.79                            | 18.30                           |
| 400        | 2.60(12.66)                      | 12.31(81.80)                   | 800        | 8.35                            | 17.70                           |
| 600        | 2.42                            | 12.05                          | 900        | 7.83                            | 17.00                           |
| 800        | 2.16                            | 11.56                          | 1000       | 7.22                            | 16.10                           |

Table III: The cross sections for the processes $e^-e^+ \rightarrow FF$ (where $F = l', \nu'$) at the collision center of mass energy $\sqrt{s} = 1$ TeV.

| Cross sections(fb) | $Z'_S$ | $Z'_{\psi}$ | $Z'_{\chi}$ | $Z'_{\eta}$ | $Z'_{B-L}$ |
|--------------------|--------|-------------|-------------|-------------|------------|
| $e^-e^+ \rightarrow l'^-l'^+$ | 105.17 | 107.65 | 99.92 | 105.50 | 104.78 |
| $e^-e^+ \rightarrow \nu'\bar{\nu}'$ | 16.04 | 25.55 | 27.03 | 24.18 | 24.48 |

and the fourth family fermions would be to understand their properties and couplings. The forward-backward asymmetry and the invariant mass spectrum of the heavy fermions could help to identify the nature of these new particles.

IV. FORWARD-BACKWARD ASYMMETRY

The forward-backward asymmetry $A_{FB}$ is defined as the relative difference between the cross sections with $\cos \theta > 0$ and $\cos \theta < 0$, being $\theta$ the angle between the heavy fermion $F$ and initial electron in the center of mass frame:

Table IV: The cross sections for the process $e^-e^+ \rightarrow FF$ (where $F = t', b', l', \nu'$) at $\sqrt{s} = 3$ TeV.

| Cross section (fb) | $Z'_S$ | $Z'_{\psi}$ | $Z'_{\chi}$ | $Z'_{\eta}$ | $Z'_{B-L}$ |
|--------------------|--------|-------------|-------------|-------------|------------|
| $e^-e^+ \rightarrow t'\bar{t}'$ | 84.75 | 15.72 | 26.46 | 45.74 | 24.14 |
| $e^-e^+ \rightarrow b'\bar{b}'$ | 95.13 | 16.16 | 14.91 | 7.48 | 6.84 |
| $e^-e^+ \rightarrow l'^-l'^+$ | 35.59 | 15.62 | 38.51 | 19.35 | 21.05 |
| $e^-e^+ \rightarrow \nu'\bar{\nu}'$ | 49.60 | 0.93 | 5.80 | 3.29 | 3.16 |
Figure 3: Forward-backward asymmetry for $t'$ (left) and $b'$ (right) production within different $Z'$ models at the center of mass energy of 3 TeV.

\[
A_{FB} = \frac{\sigma(\cos \theta > 0) - \sigma(\cos \theta < 0)}{\sigma(\cos \theta > 0) + \sigma(\cos \theta < 0)}
\]

Since the photon has only vector-like couplings to charged fermions the photon exchange can not generate an asymmetry, while the $Z$ boson and $Z'$ boson exchange and their interference can generate asymmetry for the fourth family fermions. If the heavy fermions are localized differently along a new dynamical symmetry breaking, one can then expect that the interactions of heavy fermions can be different from the ones of the light fermions. The presence of the $s$-channel resonance in $F \bar{F}$ production could be identified by an examination of the invariant mass distributions with sufficient statistics. It is seen from Fig. 3 that the asymmetry changes sign at a value of the $M_{Z'}$ near the center of mass energy. At relatively low $M_{Z'}$ the asymmetry value is around 0.6 (0.4) for $t'$ ($b'$) pair production with the contribution of sequential $Z'$ boson at the center of mass energy of 3 TeV. However, it has the value around 0.3 (0.45) for the large $M_{Z'}$ region. Figs. 4 and 5 show the asymmetries for the fourth family leptons depending on the $Z'$ mass. Concerning the other $Z'$ models different asymmetry behaviour of the fourth family fermions can be seen in Figs. 3-5 depending on the mass of $Z'$ boson.

In order to see how the asymmetry changes depending on the heavy fermion mass $M_F$ we plot Fig. 6 without $Z'$ contribution. One may comment that the asymmetry $A_{FB}$ for $l'$ pair production is higher than that of the $\nu'$ at ILC and CLIC energies. As it can be seen from the differential cross section in the Appendix, fourth family fermions have different vector and axial couplings to the $Z$ boson. This can generate different asymmetries for the pair
Figure 4: Forward-backward asymmetry for $l'$ lepton within different $Z'$ models at the center of mass energies 1 TeV (left) and 3 TeV (right).

Figure 5: The same as Fig. 4 but for $\nu'$.

production of fourth family fermions at linear colliders. Pairs of heavy charged fermions can couple to photon and $Z$ boson and interfere with each other while the heavy neutrino pair can couple to $Z$ boson, therefore we expect a lower asymmetry for the neutrinos in

Figure 6: The forward-backward asymmetries for fourth family leptons ($l', \nu'$) at $\sqrt{s} = 1$ TeV (left) and for fourth family fermions ($t', b', l', \nu'$) at $\sqrt{s} = 3$ TeV (right) depending on their masses.
Table V: The collider beam parameters of the ILC and CLIC needed to calculate the ISR and BS.

|                                      | ILC (1 TeV) | CLIC (3 TeV) |
|--------------------------------------|-------------|---------------|
| Horizontal beam size (nm)            | 640         | 45            |
| Vertical beam size (nm)              | 5.7         | 1             |
| Bunch length (mm)                    | 0.3         | 0.044         |
| Number of particles in the bunch (N) | $2 \times 10^{10}$ | $3.72 \times 10^{9}$ |
| Design luminosity (cm$^{-2}$s$^{-1}$) | $2 \times 10^{34}$ | $5.9 \times 10^{34}$ |

the SM framework. Considering the current mass limits for $m_{\nu'}$ and $m_{\tilde{\nu}}$, the asymmetry is given for the CLIC energy. For the forward-backward asymmetry of the fourth family fermions depending on the invariant mass ($M_{F\bar{F}}$) and the initial state radiation (ISR) and beamstrahlung (BS) effects, we use CalcHEP \cite{25} with the beam parameters for the ILC \cite{21} and CLIC \cite{23} as presented in Table V.

The asymmetries ($A'_{FB}$) defined in terms of differential cross sections are presented in Fig. \ref{fig:asymmetry} depending on the invariant mass of heavy fermions for $M_{Z'} = 3500$ GeV at CLIC with $\sqrt{s} = 3$ TeV. For the sequential $Z'$ model, one obtains an asymmetry of 0.3 around $M_{\nu\bar{\nu}} = 2000$ GeV at CLIC. One may compare the distributions between the sequential $Z'$ model and $Z'_{\psi}$ model. It is seen that $Z'_{\psi}$ model generates more asymmetry for charged fermions depending on the invariant mass $M_{F\bar{F}}$. One should note that there is a threshold value for the invariant mass distributions of each type of fermions. These thresholds depend on the value of the heavy fermion mass. The asymmetry for the neutrino remains approximately at the same level for the interested mass region within these $Z'$ models.

V. ANALYSIS

In order to analyze the $Z'$ models we define a $\chi^2$ function given by

$$\chi^2 = \frac{(\sigma^{\text{with } Z'} - \sigma^{\text{no } Z'})^2}{\sigma^{\text{no } Z'}/(BR \epsilon L_{\text{int}})}$$

where $\sigma^{\text{with } Z'}$ and $\sigma^{\text{no } Z'}$ are the cross sections for pair production of the fourth family fermions with a $Z'$ boson and without $Z'$ boson, respectively. The integrated luminosity $L_{\text{int}}$ is taken as 200 fb$^{-1}$ at the center of mass energy $\sqrt{s} = 1$ TeV for ILC and 600 fb$^{-1}$ at
Figure 7: Asymmetry depending on the heavy fermion invariant mass for sequential $Z'_S$ model (left) and $Z'_{\psi}$ model (right) for $m_{Z'} = 3500$ GeV at CLIC with $\sqrt{s} = 3$ TeV.

Figure 8: The $\chi^2$ distribution for $t'$ (left) and $b'$ (right) pair production process depending on the $Z'$ mass at $\sqrt{s} = 3$ TeV.

$\sqrt{s} = 3$ TeV for CLIC. The $BR$ and $\epsilon$ correspond to the branching ratio and efficiency for considered decay mode, respectively.

First, we take into account the pair production of fourth family quarks ($t'$ and $b'$) and their decays via $t'b' \rightarrow W^+bW^-\bar{b}$ and $b'b' \rightarrow W^-tW^+\bar{t} \rightarrow W^-W^+bW^+W^-\bar{b}$, respectively. For $t'$ pair production process, we consider the leptonic decay of one $W$ boson and hadronic decay of the other $W$ boson giving the signal $l^\pm + 2b_{jet} + 2j + \text{MET}$. In Fig. 8, we plot the $\chi^2$ distribution versus $M_{Z'}$ assuming the mass value $m_{t'} = 600$ GeV and CKM4 elements $|V_{t'b'}| = 0.993$, $|V_{tb}| = 0.115$, $|V_{t's}| = 0.034$, $|V_{t'd}| = 0.006$, $|V_{tb'}| = 0.115$, $|V_{cb'}| = 0.034$, $|V_{ub'}| = 0.014$ [26] at the linear collider center of mass energy $\sqrt{s} = 3$ TeV. For $b'$ pair production, we take into account the same sign $W$ bosons decay leptonically, while the others decay hadronically, leading to the signal $2l^\pm + 2b_{jet} + 4j + \text{MET}$.

For $t'$ ($b'$) pair production we can identify the $Z'_S$ model in the mass range $m_{Z'} = 3700 -$
5000 GeV at linear collider energy of $\sqrt{s} = 3$ TeV. However, the other models can be identified in the smaller mass range well above the experimental limits. Specifically, for the $b'$ pair production the $Z'_b$ model will give its signature up to a higher mass value.

Second, we consider the pair production of fourth family charged lepton and neutrino ($l'$ and $\nu'$) and their decays via $l'\bar{l}' \rightarrow W^- \nu'\bar{W}^+ \nu' \rightarrow W^- \mu^+\bar{W}^+ \mu^+ W^\pm$ and $\nu'\bar{\nu}' \rightarrow W^{\pm} \mu^+\bar{W}^\pm \mu^\mp$ assuming the Majorana nature of the neutrino, respectively. For $l'$ pair production, we assume three same sign $W$ bosons decay leptonically, while the other decays hadronically, giving the signal $3l^\pm + 2\mu^\mp + 2j + \text{MET}$. The results for this signal are given in Table VI and VII. However, we also consider the final state $l'\bar{l}' \rightarrow W^- \nu'\bar{W}^+ \nu' \rightarrow 8j + 2\mu^\pm$ which is more convenient to separate $Z'$ models at linear colliders for one year of operation.

In Fig. 9 we plot the $\chi^2$ distribution for the $2\mu^\pm + 8j$ signal versus $M_{Z'}$ assuming the mass value $m_{\nu'} = 200$ GeV and PMNS4 elements $U_{\nu'\nu'} > 0.996$ and $U_{\nu'b'} < 0.092$ [27] at the collider energies $\sqrt{s} = 1$ TeV and 3 TeV. For $\nu'$ pair production, we assume the $W$ bosons decay hadronically, leading to the signal $2\mu^\pm + 4j$. The $\chi^2$ distribution is given in Fig. 10 depending on the $M_{Z'}$ assuming the value $m_{\nu'} = 100$ GeV at the center of mass energies $\sqrt{s} = 1$ TeV and 3 TeV.

In the $l'$ pair and $\nu'$ pair search we can identify the $Z'_\chi$ model in the mass range $2000 < m_{Z'} < 2700$ GeV at $\sqrt{s} = 1$ TeV. A higher center of mass energy $\sqrt{s} = 3$ TeV expands the accessibility to the range $m_{Z'} = 3700 - 5000$ GeV.

The number of signal events for the fourth family pair production processes at the center of mass energies $\sqrt{s} = 1$ TeV and 3 TeV are given in Table VI and VII respectively. Here, we take the mass of fourth family $b'$ quark as $m_{b'} = 650$ GeV and fourth family lepton mass
Figure 10: The same as Fig. 9 but for the $\nu'$ pair production at $\sqrt{s} = 1$ TeV (left) and $\sqrt{s} = 3$ TeV (right).

Table VI: Number of signal ($l'$ and $\nu'$ pairs) and background events for relevant final states at $\sqrt{s} = 1$ TeV and $L_{\text{int}} = 200$ fb$^{-1}$. The numbers in the parenthesis denote corresponding signal significances.

| Signal | $Z'_S$ | $Z'_W$ | $Z'_X$ | $Z'_\eta$ | $Z'_{B-L}$ | Background |
|--------|--------|--------|--------|----------|------------|------------|
| $t^T \rightarrow 2\mu^- + 3\mu^+ + 2j + \text{MET}$ | 15.6(110.31) | 15.9(112.85) | 14.8(104.79) | 15.6(110.59) | 15.5(109.88) | $ZZW^+W^-$ 0.02 |
| $\nu'\nu' \rightarrow 2\mu^- + 4j$ | 151.9(159.29) | 241.9(253.66) | 255.9(268.29) | 229.0(240.09) | 231.8(243.00) | $W^+W^-W^+W^-$ 0.91 |

as $m_{\nu'} = 200$ GeV. The corresponding background events are also given in the last column of Table VI and VII for 200 fb$^{-1}$ and 600 fb$^{-1}$, respectively. When calculating the number of signal and background events we take into account the corresponding branching ratios and the efficiency factors for the given channel of signal. In the final state including $b$-quarks we take the $b$-tagging efficiency as $\epsilon = 0.5$.

In order to estimate signal significance for the production of fourth family fermions we use signal and background events at linear colliders with $\sqrt{s} = 1$ TeV and 3 TeV. For an illustration, considering $2\mu^- + 3\mu^+ + 2j + \text{MET}$ signal we have the signal significances

Table VII: The number of signal ($t'$, $b'$, $l'$ and $\nu'$ pairs) and background events for $\sqrt{s} = 3$ TeV and $L_{\text{int}} = 600$ fb$^{-1}$. The numbers in the parenthesis denote the significances.

| Signal | $Z'_g$ | $Z'_v$ | $Z'_\chi$ | $Z'_\eta$ | $Z'_{B-L}$ | Background |
|--------|--------|--------|----------|----------|------------|------------|
| $t^T \rightarrow l^+ + 2b_j + 2j + \text{MET}$ | 1582.8(62.15) | 293.6(11.53) | 194.2(19.41) | 854.3(33.55) | 450.8(17.70) | $W^+W^-W^-$ 648.5 |
| $b^B \rightarrow 2l^+ + 2b_j + 4j + \text{MET}$ | 206.0(193.82) | 34.9(32.92) | 32.3(30.39) | 16.2(15.23) | 14.8(13.93) | $W^+W^+W^-$ 1.13 |
| $t^T \rightarrow 2\nu^- + 3\nu^+ + 2j + \text{MET}$ | 15.6(44.95) | 6.8(19.72) | 16.9(48.64) | 8.5(24.45) | 9.2(26.59) | $ZZW^+W^-$ 0.12 |
| $\nu'\nu' \rightarrow 2\mu^- + 4j$ | 1385.5(624.02) | 26.0(11.72) | 162.0(72.96) | 91.7(41.32) | 88.3(39.76) | $W^+W^-W^+W^-$ 4.93 |
(S/\sqrt{B}) in the framework of \(Z'_S\) model as 110.3 (44.95) and 159.3 (624.02) for the \(l'\) and \(\nu'\) pair production at 1 (3) TeV, respectively. We will have more number of signal events if we choose the channel \(2\mu^\pm + 8j\) for the \(l'\) pair production. For this channel we expect quite low background due to at least six gauge bosons in the final state. At the ILC (CLIC), we obtain 460.5 (459.7), 471.4 (201.7), 437.5 (497.5), 461.9 (250.0) and 458.8 (271.8) events for the signal \(2\mu^\pm + 8j\) within the \(Z'_S\), \(Z'_\psi\), \(Z'_\chi\), \(Z'_\eta\) and \(Z'_{B-L}\) models, respectively. Concerning the fourth family quarks \(t'\) and \(b'\) we have the significances 62.15 and 193.82 for \(Z'_S\) model at \(\sqrt{s} = 3\) TeV. Providing the fourth family fermions exist within the considered mass range, the \(Z'\) models can be probed with a large significance at linear colliders.

VI. CONCLUSIONS

We emphasize that exploring the \(F\,\overline{F}\) production cross sections and forward-backward asymmetry at linear colliders will allow further tests of the new models beyond the SM. Taking the masses of fourth family fermions as \(m_{t'} = 600\) GeV and \(m_{\nu'} = 200\) GeV with the constraints \(m_{b'} - m_{t'} = 50\) GeV and \(m_{\nu'} - m_{\nu} = 100\) GeV, the CLIC (ILC) can produce fourth family fermions \(t'\), \(b'\), \(l'\) and \(\nu'\) signal events 1582, 206, 15 (15), 1385 (151) per year, respectively. We study the dependence of \(A_{FB}^F\) on the heavy fermion invariant mass \(M_{F\overline{F}}\). At CLIC (ILC), the forward backward asymmetry for \(t'\), \(b'\), \(l'\) and \(\nu'\) pair production without \(Z'\) contribution can be calculated as 0.55, 0.60, 0.45 (0.40) and 0.10 (0.10) at their mass bounds, respectively. These asymmetries can be affected with the \(Z'\) masses in the framework of models. For an invariant mass of \(M_{F\overline{F}} = 2\) TeV, the \(t'\) and \(b'\) quarks can produce a forward backward asymmetry of \(A_{FB}^F \simeq 0.4\) if the \(Z'_\psi\) model is realized. However, heavy charged lepton FB asymmetry can also be measured at relatively low invariant mass range. Performing a \(\chi^2\) analysis using the cross sections we have the mass range for the \(Z'\) boson which can be accessible at the linear collider experiments. We found that the \(Z'\) models give different predictions for the observables and their correlations, and they may be distinguished by jointly studying these observables at linear colliders.

APPENDIX

The differential cross section for the process \(e^+e^- \rightarrow F\overline{F}\) is given by
\[
\frac{d\sigma (e^- e^+ \to F\bar{F})}{dt} = \frac{1}{16\pi s^2} \left\{ \frac{2g_e^4}{s^2} [A_1 Q_F^2] + \frac{g_Z^4}{8 \left[ (M_Z^2 - s)^2 + M_Z^2 \Gamma_Z^2 \right]} \right\} \\
\times \left[ (C_A^2 + C_V^2) \left( C_V^2 A_1 + C_A^2 A_2 \right) + 4C_A^2 C_V^2 C_V C_V^F s A_3 \right] \\
+ \left[ \frac{g_Z^4}{8 \left[ (M_Z^2 - s)^2 + M_Z^2 \Gamma_Z^2 \right]} \right] \left[ (C_A^2 + C_V^2) \left( C_V^2 A_1 + C_A^2 A_2 \right) \right] \\
+ 4C_A^2 C_V^F C_V C_V^F s A_3 \right] \\
- \frac{g_e^2 g_Z^2 (-Q_F) (M_Z^2 - s)}{2s \left[ (M_Z^2 - s)^2 + M_Z^2 \Gamma_Z^2 \right]} (C_A^2 C_A^F s A_3 + C_V^2 C_V^F A_1) \\
- \frac{g_e^2 g_Z^2 (Q_F) (M_Z^2 - s)}{2s \left[ (M_Z^2 - s)^2 + M_Z^2 \Gamma_Z^2 \right]} (C_A^2 C_A^F s A_3 + C_V^2 C_V^F A_1) \\
+ \frac{g_e^2 g_Z^2 (M_Z^2 - s) + M_Z^2 M_Z^2 \Gamma_Z^2 \Gamma_Z^2 + s (s - M_Z^2))}{(M_Z^2 - s)^2 + M_Z^2 \Gamma_Z^2} \left[ (M_Z^2 - s)^2 + M_Z^2 \Gamma_Z^2 \right] \\
\times \left\{ C_A^e \left[ C_A^e \left( C_A^e C_A^F A_2 + C_V^F C_V^F A_1 \right) + C_V^e s A_3 \left( C_A^F C_V^F + C_A^F C_V^F \right) \right] \right\} \\
+ C_V^e \left[ C_A^F \left( C_A^e C_V^F s A_3 + C_A^F C_V^F A_2 \right) + C_V^F \left( C_A^e C_V^F s A_3 + C_V^e C_V^F A_1 \right) \right]\}
\]

where \( A_1 = (s + t)^2 + t^2 - 4M_Z^2 t + 2M_F^4 \), \( A_2 = A_1 - 4M_F^2 s \) and \( A_3 = s + 2t - 2M_Z^2 \). The \( g_Z \)
and \( g_{Z'} \) are the coupling constants of the neutral current interactions with the gauge bosons \( Z \) and \( Z' \), respectively. The \( C_V^F \) \( (C_V^F) \) and \( C_A^F \) \( (C_A^F) \) are vector and axial-vector couplings with the \( Z' \) \( (Z) \) boson. The \( s \) and \( t \) are Mandelstam variables. \( M_Z \) and \( M_{Z'} \) are the masses of \( Z \) and \( Z' \) bosons; \( M_F \) is the heavy fermion mass. \( \Gamma_Z \) and \( \Gamma_{Z'} \) are decay widths for \( Z \) and \( Z' \) bosons, respectively. In order to obtain the differential cross section depending on the scattering angle, \( d\sigma/d\cos\theta \), the expression \( d\sigma/dt \) should be multiplied by the factor \( s\beta/2 \)
where \( \beta = \sqrt{1 - 4M_F^2/s} \), and here we use \( t = M_F^2 - s(1 - \beta \cos\theta)/2 \).

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