Set of Fuzzy Time Series Forecasting Models Based on the Difference Rate

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Abstract—Song & Chissom introduced the concept of fuzzy time series in 1993[1], and many fuzzy time series methods have been proposed, however, the prediction accuracy is not high, among which, Jilani, Burney and Ardil (2007) proposed prediction model has achieved a high accuracy. This paper improves their predicted model, and proposed the set of fuzzy time series forecasting models Based on the difference rate, simplified as SFBDR, it contains the predicted model SFBDR (0.000001, 0.0000003) and SFBDR (0.0000003, 0.0000001), in the historical enrollment of University of Alabama it can get the highest predicted accuracy of AFER=0% and MSE=0.

Keywords—fuzzy time series forecasting method; SFBDR fuzzy number function; SFBDR inverse fuzzy number function; SFBDR Predicted function

I. INTRODUCTION

Song and Chissom[1] (1993) put forward the fuzzy time series forecasting model by using the fuzzy set theory [2], and first of all studied the predicted problem of the enrollments of the University of Alabama. Although many fuzzy time series methods have been proposed, but the predicted accuracy is not high. Literature [5-9, 11, 12, 14-17] improved the predicted model of Jilani, Burney and Ardil [10, 13] (2007), the predicted accuracy of the predicted model rose obviously, but still cannot reach the ideal state. This paper further improve their forecasting model and dig out the set of fuzzy time series forecasting models Based on the difference rate, simplified as SFBDR, when the discussion of the forecasting process for the historical enrollments of University of Alabama, it contains the predicted model SFBDR (0.000001, 0.0000003) and SFBDR (0.0000003, 0.0000001), which can make the Average Forecasting Error Rate and the Mean Square Error to reach the ideal state: AFER=0% and MSE=0.

II. SET OF FUZZY TIME SERIES FORECASTING MODEL

SFBDR

This paper extends the concepts used in Jilani, Burney and Ardil[10,13], especially applying it's the basic idea. For the time series forecasting problem, Let $B$ be the historical universe of discourse, $B = \{B_1, B_2, ... B_n\}$, The Year-to-Year Percentage Change of historical data is calculated as $C_r = (B_r - B_{r-1}) / B_{r-1}$. Let $C$ be the differential rate discourse universe of the historical data $C_r = \{C_2, C_3, ..., C_n\}$.

The literature Jilani, Burney and Ardil[10,13] “prediction formula” is generalized to fuzzy number function, inverse fuzzy number function and predicted function are as follows:

Definition 1: for the time series prediction problem, Let $B$ be the universe of discourse of the historical data, $B = \{B_1, B_2, ... B_n\}$, Let $C$ be the difference rate for the universe of discoused of the historical data $C_r = \{C_2, C_3, ..., C_n\}$, let fuzzy number function $G_r(u_1, u_2)$ defined in the $C_r$.

$$
G_r(u_1, u_2) = \begin{cases}
\frac{1 + \mu_{u_1}}{C_2} + \mu_{u_2}, & \text{if } r = 2 \\
\frac{\mu_{u_1}}{C_{r-1}} + \frac{1}{C_r} + \frac{\mu_{u_2}}{C_{r+1}}, & \text{if } 3 \leq r \leq n-1, \\
\frac{\mu_{u_1}}{C_{n-1}} + \frac{1}{C_n}, & \text{if } r = n.
\end{cases}
$$

The independent variables $u_1 \in [0,1)$ and $u_2 \in [0,1)$, also called membership degree of the $G_r(u_1, u_2)$. When $u_1 = u_2 = u$, SFBDR’s fuzzy number function is denoted as $G_r(u, u)$ ($r=2,3,...,n$).

Definition 2: for the time series prediction problem, Let $B$ be the universe of discourse of the historical data, $B = \{B_1, B_2, ... B_n\}$, Let $C$ be the difference rate for the universe of discouraged of the historical data $C = \{C_2, C_3, ..., C_n\}$, let fuzzy number function $G_r(u_1, u_2)$ be defined in the $C$, the inverse fuzzy function $E_r(u_1, u_2)$ defined on $G_r(u_1, u_2)$ as follows:

$$
E_r(u_1, u_2) = \begin{cases}
\frac{1 + \mu_{u_1}}{C_2}, & \text{if } r = 2, \\
\frac{\mu_{u_1}}{C_{r-1}} + \frac{1}{C_r} + \frac{\mu_{u_2}}{C_{r+1}}, & \text{if } 3 \leq r \leq n-1, \\
\frac{\mu_{u_1}}{C_{n-1}} + \frac{1}{C_n}, & \text{if } r = n.
\end{cases}
$$

The independent variables $u_1 \in [0,1)$ and $u_2 \in [0,1)$, also called degree of membership of the $E_r(u_1, u_2)$. When $u_1 = u_2 = u$, SFBDR's inverse fuzzy number function is denoted as $E_r(u, u)$ ($r=2,3,...,n$). Comparison of different forecasting models is provided in Table I and Table II.
TABLE I. COMPARISON OF DIFFERENT FORECASTING MODELS

| Year | Enroll- | Wang, | Saxena, | Wang, | Feng, | Wang, | Wang, |
|------|---------|-------|---------|-------|-------|-------|-------|
|      | ments   | Guo,  | Sharma, | Guo,  | Wang, | Guo,  | Guo,  |
|      |         | Easo. | Easo.   | Wang, | Wang, | Feng, | Feng, |
|      |         |       |         |       |       |       |       |
| 1971 | 13055   | -     | -       | 13563 | -     | 13563 | -     |
| 1972 | 13563   | -     | -       | 13563 | -     | 13563 | -     |
| 1973 | 13867   | 13809 | 13896   | 13867 | 13867 | 13745 | 13845 |
| 1974 | 14696   | 14610 | 14698   | 14695 | 14696 | 14531 | 14729 |
| 1975 | 15460   | 15422 | 15454   | 15460 | 15461 | 15557 | 15412 |
| 1976 | 15311   | 15299 | 15595   | 15311 | 15312 | 15446 | 15317 |
| 1977 | 15603   | 15642 | 15600   | 15603 | 15604 | 15555 | 15620 |
| 1978 | 15861   | 15901 | 15844   | 15861 | 15860 | 15901 | 15895 |
| 1979 | 16807   | 16782 | 16681   | 16796 | 16804 | 16933 | 16786 |
| 1980 | 16919   | 16935 | 16916   | 16919 | 16920 | 16950 | 16961 |
| 1981 | 16888   | 16328 | 16425   | 16388 | 16387 | 16601 | 16334 |
| 1982 | 15433   | 15362 | 16567   | 15432 | 15430 | 15456 | 15461 |
| 1983 | 15497   | 15496 | 15480   | 15497 | 15496 | 15544 | 15497 |
| 1984 | 15145   | 15077 | 15214   | 15144 | 15143 | 15165 | 15094 |
| 1985 | 15163   | 15274 | 15184   | 15163 | 15163 | 15187 | 15133 |
| 1986 | 15984   | 15966 | 15959   | 15982 | 15976 | 15953 | 15972 |
| 1987 | 16859   | 16849 | 16861   | 16859 | 16858 | 16849 | 16805 |
| 1988 | 18150   | 18312 | 17965   | 18150 | 18150 | 18211 | 18183 |
| 1989 | 18970   | 18974 | 18967   | 18970 | 18974 | 19077 | 18990 |
| 1990 | 19328   | 19236 | 19329   | 19327 | 19326 | 19344 | 19338 |
| 1991 | 19337   | 19299 | 19378   | 19337 | 19338 | 19200 | 19346 |
| 1992 | 18876   | 18951 | 18984   | 18874 | 18872 | 18851 | 18822 |

In table I and table II, the mean square error MSE (Mean Square Error) and the average forecasting error rate AFER (Average Forecasting Error Rate) are calculated respectively as follows:

\[ \text{MSE} = \frac{1}{n} \sum_{i=1}^{n} (D_i - B_i)^2 \]

\[ \text{AFER} = \left( \frac{1}{n} \sum_{i=1}^{n} |D_i - B_i| / B_i \right) \times 100\% \]

Definition 3: For the time series prediction problem, Let \( B = \{B_1, B_2, \ldots, B_n\} \). Let \( C \) be the difference rate for the universe of discourse of the historical data, \( C = \{C_2, C_3, \ldots, C_n\} \). For each given \( r, r \in \{2, 3, \ldots, n\} \), the predicted function \( D_r(a_1, a_2) \) is defined as follows:
\[ D_r(\mu_1, \mu_2) = B_{r+1}(1 + E_r(\mu_1, \mu_2)) \]

The independent variables \(u_1 \in [0,1]\) and \(u_2 \in [0,1]\), also called membership degree of the \(D_r(u_1, u_2)\). \(B_r\) is \(r\) years of inverse fuzzy number function (2). When \(u_1 = u_2 = u\), \(D_r(u_1, u_2)\) \((r=2,3,...,n)\) is called SFBDR’s predicted formula, and also denoted as SFBDR\((u_1, u_2)\).

In the study of time series forecasting problems, when membership degree \(u_1\) and \(u_2\) are the specific values of the \([0,1]\), and the forecasting model SFBDR\((u_1, u_2)\) can be obtained by the predicted formula (3), its application steps are as follows:

Step1: Input time series prediction of the historical data table;
Step2: Input historical data in the field of \(B\) and the differential rate domain \(C\);
Step3: Input predicted formula SFBDR \((u_1, u_2)\);
Step4: SFBDR \((u_1, u_2)\) to calculate the historical data predicted value.

It can be seen that when a predicted formula SFBDR \((u_1, u_2)\) is determined, so predicted model SFBDR \((u_1, u_2)\) can be determined.

**Definition 4:** For the time series prediction problem, Let \(B\) be the universe of discourse of the historical data, \(B = \{B_1, B_2, ..., B_n\}\), the differential rate for the universe of discourse of the historical data is \(C = \{C_2, C_3, ..., C_n\}\), when the \(u_1\) and \(u_2\) are all values times on the half open interval \([0,1]\), the infinite time series predicted model SFBDR\((u_1, u_2)\) can obtained. All of the time series predicted model SFBDR\((u_1, u_2)\) consists of a collection of fuzzy time series forecasting model of differential rate the set SFTSFMBDR (The Set of Fuzzy based on Time Series FORECASTING Models Based on the Difference Rate), simplify for SFBDR. General element of SFBDR is SFBDR\((u_1, u_2)\), which represents a fuzzy time series predicted formula also said to SFBDR \((u_1, u_2)\) fuzzy time series predicted formula of the predicted model.

**Theorem:** For the time series prediction problem, the universe of discourse of the historical data \(B = \{B_1, B_2, ..., B_n\}\), the differential rate domain of the historical data is \(C = \{C_2, C_3, ..., C_n\}\). For each set of \(r \in \{2, 3, ..., n\}\), then

1). SFBDR’s fuzzy number function \(G_r(u_1, u_2)\), SFBDR’s the inverse fuzzy number function \(E_r(u_1, u_2)\) and SFBDR’s the predictive function \(D_r(u_1, u_2)\) are continuous functions;

2). When \(\mu_1 \rightarrow 0, \mu_2 \rightarrow 0\), the inverse fuzzy number function \(E_r(u_1, u_2)\) converges to the differential rate of the historical data \(C_r\):

\[ \lim_{\mu_1, \mu_2 \to 0} E_r(\mu_1, \mu_2) = C_r \]

3). When \(\mu_1 \rightarrow 0, \mu_2 \rightarrow 0\), the predicted function \(D_r(u_1, u_2)\) converges to the historical data \(B_r\):

\[ \lim_{\mu_1, \mu_2 \to 0} D_r(\mu_1, \mu_2) = B_r \]

4). When the membership degree \(u_1\) and \(u_2\) are small enough, the predicted function value \(E_r(\mu_1, \mu_2)\) of SFBDR equal to historical data \(B_r\) of the \(r\) years.

Table I and Table II show the predicted results obtained by using other applications of defuzzification technique proposed the fuzzy time series predicted model. Because the prediction accuracy of the prediction model using the defuzzification technique is higher, because the length limits, the forecast results proposed other types of fuzzy time series prediction models are not listed. Table II give the predicted results of SFBDR \((0.0001, 0.00004)\) and SFBDR \((0.00004, 0.00001)\) to predict the historical enrollments of the University of Alabama, obtained MSE=0 and AFER=0%, respectively the highest predicted accuracy. The predicted results in Table I and Table II show AFER and MSE of the SFBDR \((0.00001, 0.00004)\) and SFBDR \((0.00004, 0.00001)\) are the smallest

**III. Conclusions**

In the study of predicted problem historical enrollments of the University of Alabama, applying the prediction model proposed SFBDR\((0.00001, 0.00004)\) and SFBDR\((0.00004, 0.00001)\) in this paper, to get the Mean Square Error (MSE=0) and the Average Forecasting Error Rate (AFER=0%) for the predicted value of the enrollments (see Table II), to finish off the problem that the prediction accuracy of fuzzy time series prediction model is not high. The following work should be study the practical application using the prediction model.

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**References**

1. Q Song, B S Chissom. Fuzzy series and its models. Fuzzy Sets and Systems, Vol. 54, pp.269-277, 1993.
2. L A Zadeh. Fuzzy set Fuzzy sets and Systems, Vol. 8, pp. 338-353, 1965.
3. Q Song, B S Chissom. Forecasting enrollments with fuzzy time series—Part I. Fuzzy Sets and Systems, Vol.54, pp. 1-9,1993.
4. Q Song, B S Chissom. Forecasting enrollments with fuzzy time series—Part II. Fuzzy Sets and Systems, Vol.62, pp. 1-8,1994.
5. Fang Hongxu, Guo Jianchun, Feng Hao, Jin Hailong. A new forecasting model of fuzzy time series. 2014 3rdInternational Conference on Mechantronics and Control Engineering (ICMCE 2014), Applied Mechanics and Materials, Vol. 678(2014), PP: 59-63, 2014.
6. Preetika Saxena, Kalyani Sharma, Santhosh Easo. Forecasting enrollments based on fuzzy time series with higher forecast accuracy rate. Int. J. Computer Technology& Applications, Vol.3 (3), pp. 957-961, 2012.
7. Wang Hongxu, Wu Zhenxing. Preliminary Theory of Set SDR of Fuzzy Time Series Forecasting Model. 2014 3rdInternational Conference on Mechantronics and Control Engineering (ICMCE 2014), Applied Mechanics and Materials, Vol. 678(2014), PP: 261-265, 2016.
8. Hongxu Wang, JianchunGuo, Hui Wang, HaoFeng. A fuzzy time series forecasting model based on yearly difference of the student enrollment number \([C]\). 2014 2nd International Conference on Soft Computing in
Information Communication Technology (SCICT2014), The Authors- Published by Atlantis Press, pp. 172-175, 2014.

[9] Hao Feng, Jianchun Guo, Hongxu Wang, Fujin Zhang. A modified method of forecasting enrollments based on fuzzy time series [C]. 2014 2nd International Conference on Soft Computing in Information Communication Technology (SCICT2014), The Authors- Published by Atlantis Press, pp. 176-179, 2014.

[10] Tahseen A Jilani, S M Aqil Burney, and C Ardil. Multivariate high order fuzzy time series forecasting for car road accidents. World Academy of Science, Engineering and Technology, Vol. 1, pp: 288-293, 2007.

[11] Hong Xu Wang, JianChun Gum, HaoFeng, HaiLong Jin. A fuzzy time series forecasting model based on percentages. 2nd International Conference on Frontiers in Computer Education (ICFCE2014), December 24-25, 2014, Wuhan, China. ICT IN EDUCATION. Frontiers in Computer Education. pp: 11-14, 2014.

[12] Wang Hongxu, GuoJianchun, FengHao, Jin Hailong. An improved forecasting model of fuzzy time series. 2014 3rd International Conference on Mechatronics and Control Engineering (ICMCE 2014), Applied Mechanics and Materials, Vol. 678(2014), PP: 64-69, 2014.

[13] T A Jilani, S M A Burney, C Ardil. Fuzzy metric approach for fuzzy time series forecasting based on frequency density based partitioning. Proceedings of World Academy of Science, Engineering and Technology, Vol. 34, pp, 333-338, 2007.

[14] Hong Xu Wang, JianChun Guo, Hao Feng, HaiLong Jin. A fuzzy time series forecasting model based on data differences. 2nd International Conference on Frontiers in Computer Education (ICFCE2014), December 24-25, 2014, Wuhan, China. ICT IN EDUCATION. Frontiers in Computer Education. pp: 15-18, 2014.

[15] Meredith Stevenson and John E. Porter. Fuzzy time series forecasting using percentage change as the universe of discourse. Proceedings of World Academy of Science, Engineering and Technology, Vol. 55, pp, 154-157, 2009.

[16] Wang Hongxu, Wu Zhenxing. Preliminary Theory of Set DR of Fuzzy Time Series Forecasting Model. 2016 International Conference on Mathematical, Computational and Statistical Sciences and Engineering (MCSSE2016), pp:256-260, 2016.

[17] H X Wang, J C Guo, H Feng, F J Zhang. A new model of forecast enrollment using fuzzy time series. Education Management and Management Science, 2014 International Conference on Education Management and Management Science (ICEMMS 2014), 7-8 August, 2014, Tianjin, China, pp: 95-98, 2014.