COMPARISON OF PROCESSES CONSTRUCT CONCEPT OF SOLO THEORY AND APOS THEORY IN MATHEMATICS LEARNING

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Article History: Received on 20th February 2019, Revised on 24th April 2019, Published on 15th May 2019

Abstract

Purpose of Study: To provide a deep understanding of learning of mathematics, it is necessary to make efforts to realize the conditions of students doing the construction of their understanding. There are many theories that describe thinking in constructing concepts in mathematics. By constructing their understanding, the students can be called doing meaningful learning as mentioned by Ausubel.

Methodology: This research used the comparative process of construct understanding of mathematics with SOLO and APOS theory. SOLO theory (Structure of the Observed Learning Outcome) has five parts namely pre-structural, uni-structural, multi-structural, relational, and extended abstract. While the theory of APOS (Action, Process, object, and Schema). The method used in this research is the study literature from various sources.

Results: This research found the differences construct of the concept in mathematics by using both theories.

Implications/ Applications: The SOLO Taxonomic Theory and APOS Theory about its use in constructing mathematical concepts. This means to achieve an individual process needs to do the action repeatedly.

Keywords: Mathematics Construction Concept, SOLO Theory, APOS Theory

INTRODUCTION

The mathematics lesson is very complex because of the various branches such as Logic, Arithmetic, Algebra, Geometry, Analysis, Numbers, Statistics, Opportunities, etc. When learning Logic, Geometry, and Algebra, students should understand and master them more easily than to study other branches of mathematics because they have been known and used in everyday life. However, the students’ practices in those fields still show that the students have difficulty in understanding. It has become common knowledge that mathematics is regarded as a difficult, uninteresting and frightening lesson.

So far there are two theories that are often used in learning. Both are behaviorism theories and constructivism, which are the two poles crossing. Behaviorism provides an understanding that students need external responses to build their knowledge, behaviorism that change due to stimulus and response relationships, they have no inherent ardo. While constructivism spirit builds awareness and construction knowledge from within oneself. Because during this time the spirit of building students' mathematical knowledge is more developed using behaviorism approach, they must be forced to learn, and more difficult to understand the concept of mathematics. Although the theory of constructivism has long been present, its application in mathematics is still taboo used.

Tall develops the theory of mathematical thinking to be grouped into three, namely (a) a world of being that begins with interaction with real-world objects and develops based on sensual experiences through verbal descriptions and definitions; (b) the symbolic world that evolves from action (such as counting) to symbolic calculations and manipulations that function in dual terms as processes and concepts (prosep), and (c) the formal world based on the axioms to build systems, based on the definitions to create new concepts, and based on formal evidence to construct coherent theory.

According to Meyer (2005), thinking includes three main components: (1) thinking is a cognitive activity, (2) thinking is a process involving some knowledge manipulation in the cognitive system, and (3) thinking directed to produce problem-solving action. Meanwhile, according to Sukowiyono (2013) said that in the process of thinking people arrange the relationship between the knowledge that has been recorded then it is considered as the meaning used to solve the problems faced.

The students will experience meaningful learning if they construct their own knowledge. Its opinion comes from constructivism theory initiated by Vygotsky. The constructivism theory provides the view that the students are able to build their own concept and understanding. "( Septiati, 2012) Through this approach, the students become more active,
creative, critical, and have a deeper understanding of their understanding of construction. Mathematical learning needs to be transformed to the students by constructivism so that they could have a deeper mathematical understanding.

To give an easier picture that determines the stages of the students construct and understand the concepts of mathematics, the SOLO and APOS theories can be used. SOLO (Structure Observed Learning Outcome) and APOS (Action, Process, Object, and Schema) are the theory that can be used to describe the thinking process in mathematics. According to Suryadi (2013), he APOS theory is the constructivism theory which describes the way how a person learns the mathematical concept. APOS is the cognitive theory. In this case, an object is an intelligence structure of people that explains how their mental process can construct mathematical concept defined in general concept. There is a relationship between the mental and mathematical concept that does not notice. (Maharaj, Aneshkumar, 2013).

Different from APOS, SOLO theory is divided into five levels, pre-structural, uni-structural, multi-structural, relational, and extended abstract. Biggs describes each level of SOLO theory taxonomy, those are: (1) uni-structural: people can capture information from one entity only, (2) multi-structural: people see some information but they cannot connect it, (3) relational: people can read some information that is demonstrated and connect it, (4) extended abstract: at this level, people can think inductively and generally. In addition to connecting some information, he is able to think in general and conceptual. It is limited guidance that explains or describes the steps to construct an understanding of mathematics using SOLO and APOS. The previous research in the form of solving math problems is measured only by using one of the theory. While there is no previous research that explains the process to construct understanding by using both theories. Therefore, this study will be described as the construct understanding of mathematics by using both theories. (Akbari, A., Abbasion, & Jansooz, 2013)

To facilitate the understanding and the different ways of constructing concepts in both theories, the subject matter taken in this research is Plane. The Plane has many various types that relate to each other, for example, a rectangular that relate to a square, or trapezoid, triangle, etc.

**Experimental Details**

This research explained about the comparison of steps to construct understanding. The attempt to construct a mathematical understanding will be explained in detail by using both theories. This research uses a qualitative approach. The purpose is to explore the stages of constructing mathematical concepts with SOLO and APOS theories. Therefore, the method used in this research is a literature study. This study uses multiple information from various sources to explain the understanding of APOS and SOLO theory. The mathematical material taken in this research is the sub-subject. I collect the source as much as possible then analyzed it so that the valid data about the understanding of the theories can be obtained.

**RESULTS AND DISCUSSION**

**Stages of Constructing the Concept of SOLO Theory**

The SOLO theory was first acquainted by Biggs and Collin in 1989. The SOLO taxonomy emerged as the development of a theory based on dissatisfaction with Piaget's cognitive theory. The cognitive development stage introduced by Piaget is the structure of a child's understanding divided by age that applies to various intellectual domains. However, some experimental research results showed that Piaget’s assumption is not entirely correct. The SOLO theory which assumes a hidden hierarchy model and the accumulation of cognitive dimensions find that when students provide answers to the problem given can be grouped into five levels: pre-structural, uni-structural, multi-structural, relational, and extended abstract. (Maharaj, Aneshkumar, 2013; Retna, Milda, 2013; Syah, Muhibbin, 2004)

At the pre-structural stage, individual refuses to respond or provide illogical answers. The students respond to tasks with inconsistent approaches, repeating questions. The information obtained is irrelevant and not well organized. Although uni-structural can draw conclusions based on relationships, one data or information in concrete. Students respond using one concrete fact that is used consistently but with only one element. (Luo, Li, Peng, and Fan, (2018); Sunardi, Hartanto, 2013) At the Multi-structural stage, it is said that at this stage the individual may draw two or more relations, data or information but all are still in a separate state. Students respond to problems with two or more data or concepts that are suitable, stand-alone or separate. It is connected but not well integrated yet. The next stage is relational, at this level an individual is said to have been able to think inductively, drawing conclusions based on two or more relations, data or information in an integrated manner. Students respond to tasks with inductive thinking, can draw a conclusion based on matching data or concepts and see and establish relationships between the data or concepts. Finally, extended abstract, at this level the individual is able to construct all ideas and be able to give something new and different. Students respond to
tasks by thinking inductively and deductively, being able to establish or see relationships, hypothesize, draw conclusions and apply them to other situations. (Luo, Li, Peng, and Fan, (2018); Sunardi, Hartanto, 2013)

According to Biggs on the description of each level of SOLO taxonomy are (1) uni-structural: one can capture information from one entity only. (2) multi-structural: It sees some information but cannot connect it. (3) extended abstract: At this level, a person is able to think inductively and general, and in addition to connecting some information he is able to think in general and conceptually.

From the above description so that it can describe the stages of constructing mathematics on the Plane with SOLO theory as follows:

1. **Pre-structural**: At this stage, the teacher provides stimulation in the form of material in a demonstration or the students are asked to read the material wake up flat. Pre-structural here is a stage where there is no understanding of the material in the demonstration.

2. **Uni-structural**: At this stage, the individual can see and capture information from one entity only. Such as in this stage the teacher gives or demonstrates Plane. The teacher guides the students to see and understand Plane. At this stage, the students only see the shape of the wake.

3. **Multi-structural**: individual is able to find some information that is demonstrated, but from some information, he/she cannot connect one with others. At this stage, the teacher guides the students to understand the parts of the Plane. So, the students understand the part of the flat wake up separately.

4. **Relational**: an individual can capture some of the demonstrated information and be able to connect each of this information. At this stage, students are able to connect several entities of Plane through the guidance of the teacher. So, from the demonstration gained and based on the teacher's guidance, the students are able to understand the plane.

5. **Extended Abstract**: an individual can read all the information provided, the individual can link the information and can think conceptually. In this last stage, when the individual has understood the concept Plane through the guidance of the teacher, he/she has a pattern of understanding where the pattern of Plane, it can be used to understand other flat wakes.

**Stages of Constructing the Concept of APOS Theory**

Unlike the SOLO theory that divides the stages into five levels, the APOS theory consists of Action, Process, Object, and Schema. This theory is used to describe how an individual thinking process to build understanding. APOS theory was first built by Dubinsky and Mcdonald. In this theory, it is explained that an individual has a cognitive stage to build an understanding of mathematical concepts. The cognitive structure such as Action, Process, Object and Schema used to learn a concept (Olesova, & Borisova, 2016). According to Mulyono theory is part of constructivist theory, thus describing how one can understand a mathematical concept and principle so that it can be used to elaborate the mental structure of action, process, object, and scheme.

The APOS theory postulates that the mathematical concept develops when one tries to transform the physical or mental objects that exist. According to Dubinsky stages in theory apos defined as follows: Action is object transformation that perceived by the individual basically external and needed, either explicitly or from memory, step by step instructions on how to operate. When an action is repeated and individuals reflect on it, he can create an internal mental construction called process. An object is built of a process when an individual becomes conscious of the process as a totality and realize that the transformation can be formed on it. Whereas the last right is the scheme, scheme for certain mathematical concepts is a collection of individual action, Processes, objects, and other schemes are linked by some general principles to form the Framework in the minds of individuals who may be brought in to deal with problem situations involving the Concept.

The process in each stage in this theory: Action is a transformation of the mental object to obtain other mental objects. The transformation of the matter was first forming action. The action formed as individual reactions to stimulate from the race will be outside. (Aydin, Sinan & Celal Mutlu, 2013) The transformation is done by taking action against an external guide, which provides details about what steps should be taken. The transformation was first understood as an action when it was the reaction Untuk individual stimulate perceived as an external thing. (DeVries, David 2001; Azizah, Mariani, Rochmad, 2012) Students process to perform a process if the student as an independent thinkers, such as receiving input and returns as output or value calculation imagine waking up flat without doing the actual calculations is
a mental construction process which occurs internally obtained when someone was able to perform the action level repeatedly. A process is a mental structure that performs the same operation as the action, but entirely in the mind of individuals. Object is the stage where the self has been able to prove the concept as a cognitive object that covers the ability to take action on the object and provide a reason or explanation of its properties. If one is aware of the process as a totality, realizing that transformation can act on that totality and can actually build such a transformation explicitly or in someone's imagination. (DeVries, David 2001; Dubinsky, McDonald, 2001; Makonye, Judah 2017) While the scheme is the overall knowledge that they connect consciously or unconsciously with certain mathematical concepts. Mathematical topics often involve many actions, objects, and processes that need to be arranged and linked into a coherent framework. López, Daniel Oscar Espinoza, and Silvia Sarzoa. 2018; Font, Vicenç, Trigueros, Maria., Badillo, Edelmira ., And Rubio, Norma. 2012.

Based on the definition and the above stages, it can be described in detail about the stages of the theory of APOS as the following table:

| Table 1: Table Stages of Constructing Reviewed from APOS Theory |
|---------------------------------------------------------------|
| **Action** | The action of is object transformation is perceived by the individual basically as external and what they needed, either is good in explicitly and memory. At this stage, the teacher gives a demonstration and guide students to understand some of the planes intensely until the students really understand about the plane. |
| **Process** | Students perform a process if the student is thinking itself as receive input and returns as output or value calculation imagine the plane without doing the actual calculations. a mental construction is going on internally obtained when someone was able to perform the action level for repeated. At this stage can reduce the involvement of teachers to guide students to understand a plane concept, because students have to take action repeatedly. |
| **Object** | The student has been able to prove the concept of the plane as a cognitive object that covers the ability to take action on the object and provide a reason or explanation of its properties from a plane. |
| **Schema** | Collection of individual actions, Processes, objects, and other schemes that are linked by some common principles to establish structure in the individual mind. So at this stage can be called a scheme. At this stage, the teacher only confirms the students’ knowledge of getting up the plane with either the task or the other. Because at this stage students have understood the scheme and formed a pattern of actions to understand a flat-build concept. |

**Intersection of APOS Theory and SOLO Theory**

Both SOLO and APOS are theories that can be used to explain the concept of one's thinking. If both are viewed from the aspect of philosophy, then it appears both are rooted in constructivism. Descartes explains that human beings from birth have innate ideas. Both of these theories see that students are not blank papers, but they have their own thinking structures. Since SOLO emerges as a response to dissatisfaction with Peaget's cognitive theory, SOLO can be referred to as neo-Peaget.

Nevertheless, philosophically there is a meeting between cognitive and constructivism, that is in the aspect of thought. While APOS is also part of Peaget's theory. The distinguishing stages between the two are, on the cognitive and activity aspects. The solo theory labels the cognitive structure of the student in attempting to construct a stage of development and understanding, while the APOS labels activities to build an understanding. The path taken is different but the estuary is the same in terms of understanding.

The intersection of these two theories can be seen from each stage. Unstructural stages, there are efforts of students to be able to absorb information from one entity. This means that information enters the cognitive level of students as they construct a mathematical understanding so that students can read the information even though very little. While in APOS theory, Action stages indicate students giving response to the information from outside. This form of student response is part of an attempt to construct a mathematical understanding. Implied, Action on APOS is a response in the form of cognitive or action, while the marker in the cognitive review of the SOLO theory is included in the category of unstructural. So the un-structural stage can be used as a marker of students' cognitive processes in building mathematical concept constructs. When performing this action it means that a person undergoes a process of transformation from external to cognitive, although little information is obtained.
If repeated action occurs, a person has been able to perform calculations using cognitive then referred to as a process in APOS. At this stage can actually be seen that an individual is able to perform calculations without having to do it with actual calculations. Students do it in the mind without having to move their hands. This means there is a process of reflection in the minds of students for the steps to build the transformation process and can explain its steps although it is not done in real. But in understanding individual to a mathematical concept is not yet intact. This indicates that the process stages have similarities with the Multi-structural that can connect some information. Repeated student activity forms a process, indicating a behavioral habit of responding to external information. Because these activities occur repeatedly, and students are able to perform calculations in the cognitive without having to calculate the actual, has the meaning of students have been able to connect some information gained. It is capable of seeing a separate relationship of external entities.

When an individual has become accustomed to a process, then the activity becomes conscious action. He can prove a mathematical concept as an object of cognition and become aware that an understanding of a concept is built upon it then this is at the stage of the Object. ([Lian, dan Idris, 2006]) The awareness of the individual to build an understanding of a mathematical concept after taking action repeatedly forms a pattern. The individual does not require external stimuli, he can explain a concept, and explain the properties of a mathematical concept he has constructed in his cognitive. The students’ understanding at this stage has been completed in a mathematical concept, it can describe both from the beginning and from the back, from which the concept can be obtained. If we think deeply, this stage is the same as the Relational level in the SOLO taxonomy, wherein individual can relate some information gained to build a concept well. This means that at this stage of the object the individual has been able to think inductively, of course, it indicates the ability to connect all the information obtained so it is obtaining a complete understanding. (Borji, Vahid and Voskoglou, Michael Gr, 2016)

All of the individual understandings of action, process and object in APOS theory form a schema in the final stage. Schema gives a definite pattern to the individual to build a mathematical concept either consciously or unconsciously. He can relate these concepts to other mathematical concepts, so he can understand in various ways. (Mulyono, Apos 2011; Parraguez, Marcela, & Asuman Oktac, 2010 Putri, dan Manov, 2013) Because the process of understanding has become a pattern in student cognition. This, if further noted has similarities with abstract extended in SOLO taxonomy, where the schema is formed indicating that individuals are able to think conceptually. He is able to see the relationship of a concept with other concepts and able to develop against other problems. This last level in both theories becomes the culmination of students’ understanding of concept building, both of which indicate a fully developed understanding. Here we include tables to provide an explanation and simplify comparison of equations from APOS theory and SOLO theory:

| Teori Taksonomi SOLO | Teori APOS |
|----------------------|------------|
| Prestruktural Unistruktural | Aksi |
| Multistruktural | Proses |
| Relational | Objek |
| Extended Abstrak | Schme |

**CONCLUSION**

From the above description can be concluded about the differences of the SOLO Taxonomic Theory and APOS Theory about its use in constructing mathematical concepts. In SOLO theory there are five levels to explain how to construct concepts consisting of pre-structural, un-structural, multi-structural, relational and extended abstract. (Zare & Rajaeepr, 2013). The theory of APOS is composed of Action, Process, Object, and Scheme. The difference between the two lies in the attempt to construct, in which SOLO theory is more about seeing how an individual understands a concept and can be seen in the ability to construct from a level. Although the steps given to SOLO theory and APOS theories are almost identical, the APOS theory is more on attempting to construct a concept to achieve the scheme. But the equation of both theories is that an individual cannot reach the next stage if step by step is not understood. For example, in SOLO theory for pre-structural and un-structural level, an individual can understand a concept from one side only, and on multi-structural can understand a concept from some entity, whereas in APOS theory, an individual is unlikely to enter the process, unless the individual is trying to Understand and seek explanation of a concept until repeatedly. This means to achieve an individual process needs to do the action repeatedly.
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