Bandgap-Assisted Quantum Control of Topological Edge States in a Cavity

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Quantum matter with exotic topological order has potential applications in quantum computation. However, in present experiments, the manipulations on topological states are still challenging. We here propose an architecture for quantum control of topological matter. We consider a topological superconducting qubit array with Su-Schrieffer-Heeger (SSH) Hamiltonian which couples to a microwave cavity. The light-matter interactions are analyzed by exploiting topological bandgap in the qubit array. With proper cavity-qubit couplings, edge states and topological phase transition can be spectroscopically probed by the cavity. And the reflection spectrum shows a signature of vacuum Rabi splitting for edge states. Moreover, with the protection of topological bandgap, cavity induces nonlocal interaction between edge states. Quantum interference of emissions from two edge states is discussed. Our work may pave a way for topological quantum state engineering.

Introduction — Characterization of topological matter is a crucial issue in condensed matter physics [1]. A hallmark of topological phases is the existence of topological invariants, e.g., Chern number and Zak phase, defined on energy bands of the systems [2–4]. According to edge-bulk correspondence, topological states emerge in the bandgaps and give rise to many novel transport phenomena [5, 6]. Due to their insensitivity to local decoherence, topological states have prospective applications in quantum information processing. In particular, zero-dimensional edge states, e.g., Majorana bound states are candidate to realize topological quantum computation [7–9], and have been observed experimentally in a range of materials, including semiconductor nanowires [10–13], ferromagnetic atomic chains [14] and iron-based superconductors [15]. However, the manipulations of edge states are rather challenging, for which reason topological materials with large bandgaps are explored [16–18].

Cavity quantum electrodynamics (QED), in which quantized electromagnetic fields are strongly coupled to an atomic system, was originally used for studying fundamentals of atomic physics and quantum optics [19]. With the superb control of quantum states, cavity QED is now applied to quantum information processing, in which the cavity field is proposed for manipulating, measuring, or transferring quantum states of atomic systems [20]. Circuit QED, in which a microwave transmission line resonator acting as a cavity is coupled to superconducting quantum circuit, is an extension of the cavity QED [21, 22]. The on-chip circuit QED system is not only a good platform for studying fundamental physics in microwave regime [23], but also a very promising candidate for realizing quantum computation and simulations [24–41]. The interacting qubits make it possible to explore many-body physics. For example, many-body localization [37, 38], Mott insulator of photons [40] and correlated quantum walk [41] are observed in 1D qubit arrays. With these experimental achievements, superconducting qubit systems are hopeful to simulate topological matter [42–46].

In this work, we study the interaction between a microwave cavity and the topological matter of a superconducting qubit array, described by the Su-Schrieffer-Heeger (SSH) Hamiltonian [47] which has been experimentally realized in a periodic driving way [48]. Different from the electronic transport detections of Majorana fermions [13, 49–51], the cavity spectroscopy method we study here unveils the edge states and topological phase transition with proper cavity-qubit couplings. In the superconducting qubit array, strong qubit-qubit interactions can give rise to large bandgaps. We pinpoint the role of topological bandgap in quantum manipulation of edge states, especially for small qubit arrays.

Spectroscopic characterization of a topological qubit array by a cavity — As schematically shown in Fig. 1(a), we study a theoretical model that a typical topological lattice in one-dimensional systems [52], with SSH interactions, is placed inside a cavity. Considering rapid progresses and flexible chip designs of superconducting quantum circuits, we here assume that the SSH array with N unit cells, formed by 2N superconducting qubits [43], is coupled to a microwave transmission line resonator, as schematically shown in Fig. 1(b). The Hamiltonian of the whole system is given as

\[
H/h = \omega_c \hat{a}^\dagger \hat{a} + \sum_{i=1,\mu=A,B}^{i=N} (\omega_0 \sigma_{i\mu}^+ \sigma_{i\mu}^- + g_{i\mu} \sigma_{i\mu}^+ \hat{a} + g_{i\mu}^\ast \hat{a}^\dagger \sigma_{i\mu}^-) \\
+ \sum_{i=1}^{i=N} (t_1 \sigma_{iA}^+ \sigma_{iB}^- + t_2 \sigma_{i+1A}^- \sigma_{iB}^+ + \text{H.c.}),
\]

(1)

where \(\omega_c = \omega_0\) and \(\omega_0\) are the frequencies of the cavity and qubits, respectively. The parameter \(g_{i\mu}\) denotes the coupling strength of the cavity to the qubit \(\mu\) in the \(i\)th unit cell. The operators \(|A_i\rangle\langle A_i|\) and \(|B_i\rangle\langle B_i|\) with the ground (excited) states \(|\alpha_i\rangle\) (\(|A_i\rangle\)) and \(|\beta_i\rangle\) (\(|B_i\rangle\)), respectively. The second line in Eq. (1) represents the SSH interaction Hamiltonian with tunable coupling strengths \(t_1\) and \(t_2\), which could be implemented in
superconducting qubit circuits [37, 38, 53–57]. Thus, we here assume that $t_1 = t_0(1 - \cos \varphi)$ and $t_2 = t_0(1 + \cos \varphi)$ with a tunable parameter $\varphi$. Note that the topological phase transition takes place at $\varphi = \pi/2$ ($t_1 = t_2$). The cases for $t_1 < t_2$ and $t_1 > t_2$ correspond to topological and non-topological phases, respectively.

To measure the topological qubit array, we assume that a probe field with the strength $\eta$ and the frequency $\omega_1 = \omega_c$ is applied to the qubit array via the cavity. Thus, the dynamics of the reduced density matrix $\rho$ of the whole system can be described by the master equation

$$\dot{\rho} = -\frac{i}{\hbar} [H, \rho] + \frac{\kappa}{2} [\hat{a}, [\hat{a}, \rho]] + \sum_{i=1}^{N} \gamma_{\text{dr}} \mathcal{D}[\sigma_{i\mu}^+] \rho.$$  

Here, $\kappa$ is the decay rate of the cavity, $\gamma_{\text{dr}} A$ and $\gamma_{\text{dr}} B$ are the decay rates of the qubits $A$ and $B$ at the $i$th unit cell, respectively. The dissipator superoperator is defined as $\mathcal{D}[O] \rho = \mathcal{L} \rho = \frac{1}{2} [O^\dagger O, \rho]$. The energy spectrum corresponding to both bulk and edge states of the SSH array can be measured by the reflection of the probe field, as shown in Fig. 1(c). The reflection spectrum is obtained by solving the master equation in Eq. (2).

Topological bandgap represents the energy separation between two bulk bands in topological phase. The cavity-qubit couplings we choose here allow the observation of topological phase transition. In superconducting qubit circuits, topological phases have recently been demonstrated experimentally [48, 58–65]. However, the quantum operations on topological states have not been implemented. Below, we study the manipulation of topological states in superconducting qubit array via microwave fields.

Vacuum Rabi splitting for resonant coupling between the cavity and edge modes. To show how to manipulate the qubit array by the quantized field in the cavity, we rewrite the states $|A_j\rangle$ and $|B_j\rangle$ of qubits $A$ and $B$ in the $i$th unit cell via eigenstates $|\Psi_j\rangle$ in the single-excitation subspace of the qubit array [46], i.e., $|A_j\rangle = \sum_{|j-1, j\rangle} \xi_{A,j} |\Psi_j\rangle$ and $|B_j\rangle = \sum_{|j+1, j\rangle} \xi_{B,j} |\Psi_j\rangle$. Here, $j = 1, \ldots, 2N$ is the label of the $j$th eigenstate from the lowest to highest energies, $|A_j\rangle = \sigma_j^+ |G\rangle$ and $|B_j\rangle = \sigma_j^+ |G\rangle$ with $|G\rangle$ being the ground state of the qubit array. Then, in the basis of these eigenstates, the Hamiltonian

![Figure 1](image1.png)

**FIG. 1.** (a) Schematic diagram for SSH qubit array with dimerized couplings $t_1$ and $t_2$ placed inside a cavity. Red and blue balls represent A and B qubits, respectively. (b) Design of (a) with superconducting qubit circuits where the couplings $t_1$ and $t_2$ are tunable. The microwave transmission line resonator acting as a cavity is coupled to qubits. (c) Reflection spectrum of the qubit array with 8 qubits. The frequencies of qubits and driving field are respective $\omega_1$ and $\omega_2 = \omega_1 \pm 2t_0$. The reflection at $\varphi = 0.25 \pi$ is shown in the right panel. Here we consider cavity-qubits couplings $g = g_0(-1, 1, 1, -1, 1, 1)$ with $g_0/2 \pi = 5$ MHz. Other parameters are: $\omega_1/2 \pi = 6$ GHz, $t_0/2 \pi = 100$ MHz, $\kappa/2 \pi = 20$ MHz, $\gamma_{\text{dr}} A = \gamma_{\text{dr}} B = 20 \times 2 \pi$ kHz. The white-dashed curves represent energy spectrum of the qubit array. (d) Cavity mediated couplings between qubits, denoted by the orange lines, in dispersive regime.

![Figure 2](image2.png)

**FIG. 2.** (a) The coupling strength between the cavity and eigenmodes of SSH qubit array with $N = 18$ unit cells (i.e., 36 qubits) for homogeneous cavity-qubit couplings. The number $j$ is the index of eigenmodes, and the middle two points (black dots) with $j = 18, 19$ are edge states. The bulk states with odd (blue dots) and even (red dots) numbers have zero and nonzero couplings to the cavity, respectively. The inset represents the coupling strengths for $j < 18$. (b) Vacuum Rabi splitting between edge states and the cavity for $N = 18$ unit cells. The lower anticrossing shows the coupling between the cavity and the edge states. The upper one is the coupling of the bulk states to the cavity. (c) The coupling strengths $\xi_N$ and $\xi_{N+1}$, denoted by $\xi_{N,N+1}$, between two edge modes and the cavity with different numbers $N$ of unit cells. Here we consider $\varphi = \pi/5$. Other parameters are the same to Figure 1(c).
in Eq. (1) can be rewritten as

$$H/h = 2N \sum_{j=1}^{2N} \omega_j \Psi_j^+ \Psi_j^- + \omega_c \hat{a}^\dagger \hat{a} + \sum_{j=1}^{2N} \left( \hat{\xi}_j \Psi_j^+ \hat{a} + \text{H.c.} \right),$$

with $\Psi_j^+ = |\Psi_j\rangle\langle G|$ and $\omega_j$ is the eigenenergy corresponding to the eigenstate $|\Psi_j\rangle$. The parameter $\hat{\xi}_j = \xi_j g_0$ is the effective coupling strength between the cavity and the $j$th eigenmode with $\bar{\xi}_j = \sum_i (\xi_{2i-1,j} + \xi_{2i,j})$ under the assumption that qubits have the homogeneous couplings to the cavity with the strength $g_0$, i.e., $g_{i\mu} = g_0$. Hereafter, we call $\Psi_j^+$ bulk or edge modes when $|\Psi_j\rangle$ are bulk or edge eigenstates. In Fig. 2(a), we show $|\xi_j|$ for the qubit array size $2N = 36$. The bulk modes have different couplings to the cavity because of their parities of wavefunctions. The odd-parity bulk states have zero coupling. However, the even-parity bulk states are coupled to the cavity. Two edge states have equal coupling strength to the cavity, i.e., $\xi_{18} = \xi_{19}$.

In Fig. 2(b), we show energy splitting produced by the qubits-cavity couplings. We assume that the qubit frequency is $\omega_0 = 2\pi \times 6$ GHz. The anticrossing near the driving frequency $\omega_1 = 2\pi \times 6$ GHz represents the Rabi splitting due to the resonant interaction between the cavity and edge modes. Because of the degeneracy of two edge states, the anticrossing here represents the couplings between the cavity and two edge states. If the frequency of the cavity is at resonance for the transitions from the ground to bulk states with high energies, a large anticrossing, as shown in upper part of Fig. 2(b), is produced around $\omega_1 = 2\pi \times 6.2$ GHz. The large energy gap of the SSH Hamiltonian protects the Rabi splitting of edge states. In Fig. 2(c), the coupling strengths $\xi_N$ and $\xi_{N+1}$ between the cavity and edge modes are plotted versus the unit cell number $N$. When the qubit array is small, e.g., $N \leq 14$, the edge states overlap with each other and form hybridized edge states with odd and even parities. The edge state with odd parity decouples from the cavity. With the increase of the unit cell number, two edge states are far separated from each other. The localized edge states lose parity, thus they have the same coupling strength to the cavity.

We study the relation between the coupling strength $\xi_N (\xi_{N+1})$ and $\varphi$ in Fig. 3. For example, when the qubit array has $N = 6$ unit cells, the coupling strengths are described by the black-solid and blue-dashed curves. When $\varphi$ is small, the edge states have the same coupling to the cavity. However, the increase of $\varphi$ leads to hybridized edge states with even and odd parities. We find that the hybridized regime becomes smaller with the increase of the system size, e.g., $N = 18$ (green-solid and blue-dash-dotted curves) and $N = 78$ (red-solid and orange-dotted curves) as we show here. We also find that in topological phase (i.e., $\varphi < \pi/2$), the hybridized edge state with even parity has the coupling strength $\xi_< = \sqrt{2 \cos \varphi} g_0$. The couplings for separated edge states $\xi_L = \sqrt{\cos \varphi} g_0$.

**Cavity induced coupling between two edge modes**—. When the cavity is far detuned from qubits, i.e., $g_0 \ll \Delta_0$ (let $\Delta_0 = \omega_0 - \omega_c$), virtual-photons-mediated interactions among qubits $g_0^2 / \Delta_0$ can be obtained [38, 56], as shown in Fig. 1(d). In terms of the eigenmodes of the qubit array, the effective coupling strengths between ith and jth eigenmodes are

$$J_{jk} = \xi_j \xi_k \left( 1 \frac{1}{\Delta_j} + \frac{1}{\Delta_k} \right), \quad j, k \in [1, \cdots, 2N], \quad (4)$$

with $\Delta_{j/k} = \omega_{j/k} - \omega_c$. The eigenmodes with $j = 2N, 2N-2, 2N-4, \cdots$ have collective coupling strengths $\xi_j = \xi_j \sqrt{2N} g_0$. The coefficients $\xi_{2N}$ and $\xi_{2N-2}$ are shown in the inset of Fig. 3. Thus, these bulk modes have
dominant terms $\xi_{j} \xi_{k}$. Generally speaking, if the cavity-qubit coupling $g$ is given, effective couplings $J_{jk}$ are determined by the qubit-cavity detuning $\Delta_{0}$, the number $N$ of unit cells and qubit-qubit coupling strengths $t_{1}$ and $t_{2}$.

As schematically shown in Fig. 4(a), when the detunings of the bulk modes to the cavity are much larger than those of the edge modes to the cavity, and coupling strengths of the bulk modes to the cavity are comparable to those of the edge modes to the cavity, then the cavity induced couplings between bulk modes or between the bulk modes and the edge modes are negligibly small. When the energy splitting induced by hybridization of edge states is negligible (i.e., $\Delta_{N} \simeq \Delta_{N+1}$), the cavity mediated effective interaction Hamiltonian only contains the coupling between two edge modes with the strength

$$J = \cos \varphi \frac{g_{0}^{2}}{\Delta_{0}}.	ag{5}$$

In Figs. 4(b) and 4(c), we show the excitation dynamics of the left-edge qubit (qubit $A$ in the first unit cell is excited initially) in topological phase with $\varphi = 0.1 \pi$ and $0.3 \pi$, respectively. Figures 4(b) and 4(c) clearly show the population exchange between two edge states produced by the edge-mode coupling. In fact, finite topological bandgap makes the effective couplings between edge modes different from Eq. (5). In Figs. 4(d) and 4(e) with $\varphi = 0.5 \pi$ and $0.9 \pi$, the excitation propagates through the array and is bounded by the boundaries. In non-topological phase, excitation propagates along the qubit array with low velocity (see Fig. 4(e)), which is yielded by the smooth energy bands with large gap.

Quantum interference induced by topological state coupling—. As schematically shown in Fig. 5(a), we further consider that the left edge qubit $A_{1}$ is coupled to a waveguide, in which a probe field passes through. The left-edge qubit mainly contributes to the left edge state. Near resonance driving for the edge mode, the topological bandgap makes the bulk states to be negligible. Then the left edge state can be driven by fields passing through the waveguide. The single photons transmission amplitude can be given as

$$t = \frac{(i\Delta_{p} - 2\gamma_{L}) (i\Delta_{p} - 2\gamma_{R}) + J^{2}}{(i\Delta_{p} - 2\gamma_{L})(i\Delta_{p} - 2\gamma_{R}) + J^{2}},	ag{6}$$

and the susceptibility $\chi = -i (t - 1)/t$ is

$$\chi = \frac{\Gamma_{L}(\Delta_{p} + i\frac{2\pi}{\Delta_{p}}) + \Gamma_{R}(\Delta_{p} + i\frac{2\pi}{\Delta_{p}})}{2J^{2} - 2(\Delta_{p} + i\frac{2\pi}{\Delta_{p}})(\Delta_{p} + i\frac{2\pi}{\Delta_{p}})},	ag{7}$$

where $\Delta_{p}$ is the detuning between the probe field and the left edge state. As schematically shown in Fig. 5(b), the parameters $\gamma_{L}$ and $\gamma_{R}$ are the decay rates for left and right edge states, $\Gamma_{L}$ comes from the coupling between the left-edge qubit and the waveguide.

![FIG. 5. (a) Coupling between a waveguide and the left-edge qubit in the array. (b) Superatom model of (a). Here $\gamma_{L}$ and $\gamma_{R}$ are respective decays of left and right edge states, and $\Gamma_{L}$ is the waveguide induced decay rate of the left edge state. (c) Transmission of probe light for edge-state coupling $J = 0$ ($J = 0.035\Gamma_{L}$) is represented by the red-dashed (blue-solid) curve. (d) Real (solid) and imaginary (dashed) parts of the susceptibility. In both (c) and (d), the red and blue curves are for $J = 0$ and $J = 0.035\Gamma_{L}$, respectively. (e) The imaginary part of susceptibility can be decomposed into two Lorentzian peaks. In these three figures, we consider $\gamma_{L} = 0.15\Gamma_{L}$, $\gamma_{R} = 5 \times 10^{-4}\Gamma_{L}$.

The transmission of the probe field as a function of the detuning $\Delta_{p}$ is shown in Fig. 5 (c) with $J = 0$ and $0.035\Gamma_{L}$, respectively. When there is no coupling between edge states, the transmission vanishes at the resonance. However, when there is the coupling between two edge states, a transparency windows for the probe field appears. This can be further confirmed by the susceptibility, which is plotted as a function of the detuning $\Delta_{p}$ in Fig. 5(d) in the parameter regime $J \ll \Gamma_{L}$. This transparency window, in which the distance between two peaks is less than $2J$, is from the quantum interference as shown in Fig. 5(e), which is similar to electromagnetically induced transparency [66]. However, in the parameter regime $J > \Gamma_{L}$, the transparency window, in which the distance between two peaks equals to $2J$, is from the strong-coupling-induced energy splitting, which is similar to Autler-Townes splitting [67].

Conclusions and discussions—. In summary, we study cavity control and manipulations on topological degrees of freedom in one-dimensional systems with the SSH Hamiltonian. We show that the coupling between the cavity and edge modes are protected by topological bandgap, and topological phase transitions can be measured via the reflection spectrum of the probe field through the cavity. Due to topologically protected bandgap, the Rabi splitting, resulted from the resonant coupling of edge modes to the cavity, can be observed. When the cavity is largely detuned from
the edge modes, the long-range coupling between two edge states can be realized, this can further result in the quantum interference for emissions from two edge states when a qubit at the edge of the array is coupled to a waveguide. Meanwhile, we find that the topological properties of systems can also be detected by the cavity even for a small size, but the edge states are hybridized in small systems, and the hybridized states possesses the parity properties. We show that the parity engineering can also yield the coupling between edge states, as long as the splitting between hybridized edge states is small comparing to the detuning between cavity and qubits.

We also propose an experimental setup for implementing our approach by coupling superconducting qubit arrays to a transmission line resonator. This is because the tunable coupling between superconducting qubits can be experimentally realized via cavity or other superconducting elements [37, 38]. Moreover, the coupling strength between superconducting qubits can be sufficiently large such that the large topological bandgap of the system can be obtained, and thus the selective coupling of the edge states to the cavity is easier to be realized. We mention that our approach can also be applied to other systems. Our study on cavity QED for the topological matter might have potential applications in quantum information and quantum optics.

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