Time Walk Through the Quantum Cosmic String

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Abstract

This paper deals with the geometry of supermassive cosmic strings and its possible connection with the existence of closed timelike curves (CTCs), so as with inflationary expansion. The linear energy density, $G\mu$, of supermassive strings becomes so large that gravitational effects dominate, and therefore we have used an approach that enforces the spacetime of these strings to also satisfy the symmetry of a cylindric gravitational topological defect, that is a spacetime kink. In the simplest case of kink number unity the entire energy range of supermassive strings becomes then quantized so that only cylindrical defects with $G\mu = \frac{1}{4}$ (critical string) and $G\mu = \frac{1}{2}$ (extreme string) are allowed to occur in this range. It has been seen that the internal spherical coordinate $\theta$ of the string also evolves on imaginary values, leading to the creation of a covering shell of broken phase that protect the core with trapped energy, even for $G\mu = \frac{1}{2}$. Then, the conical singularity becomes a removable horizon singularity. We re-express the extreme string metric in the Finkelstein-McCollum standard form and remove the geodesic incompleteness by using the Kruskal technique. A $z=$const section of the resulting metric is the same as the $\theta = \frac{\pi}{2}$ section of a De Sitter kink, though it requires additional coordinate regions to completely describe it. It is shown that through such new regions the extreme string spacetime can accommodate CTCs. We discuss two of such curves and conclude that quantum effects allow them to exist, though it is also seen by using path integral formalism that quantum theory prevents these CTCs to be noticed by any observer. Also discussed is the idea that the extreme string is the unique local string which is able to spontaneously drive inflationary expansion in its core, without any fine tuning of the initial conditions.
1 Introduction

Topological defects consisting of confined regions of false vacuum, trapped inside bubbles of true vacuum, can occur in gauge theories with spontaneous symmetry breaking [1]. Among such defects, cosmic strings arising in the breaking of a local $U(1)$ gauge symmetry are of particular interest [1,2]. If local strings appeared in phase transitions taking place in the early universe, they could have served as seeds for the formation of galaxies and other larger-scale structures we now are able to observe [3]. Recently, the possibility that inflation can be driven in the core of topological defects has also been advanced [4,5]. In the case of local cosmic strings this extremely interesting possibility will critically depend [5] on the nature and strength of their gravity coupling $G\mu$, where $\mu$ is the string mass per unit length. It appears that only when the symmetry-breaking scale approaches the extreme supermassive limit $G\mu = \frac{1}{2}$, the size of the false vacuum region inside the string may exceed the horizon size that corresponds to the vacuum energy in the core and drives an inflationary process.

This can be seen for static defect solutions in simple models

$$V(\varphi) = \frac{1}{4} \lambda (\varphi_a \varphi_a - \eta^2)^2, a = 1, ..., N.$$  

Cosmic strings ($N = 2$) cannot exist for $G\mu \geq 1$ [6]. However, this fact is by no means meaning that inflation cannot be driven by local strings. Equalizing the core radius $\delta_0 = \eta(\pi V_0)^{-\frac{1}{2}}$, where $V_0 = \frac{1}{4} \lambda \eta^4$, and the horizon size corresponding to the vacuum energy $V_0$, $H_0^{-1} = (\frac{3}{8\pi G V_0})^{\frac{1}{2}}$, one obtains using $\mu = \eta^2$ a value for the string mass per unit length $\mu_I = \frac{3}{8\pi G}$, such that for $\mu > \mu_I$ inflation can be generated in the core.

Nevertheless, the concepts of string radius and mass per unit length for a source like the string core are not unambiguously defined. The above expression for $\delta_0$, which corresponds to the radius of the string cylinder, and the usual definition [7] of $\mu$, by which one simply integrates the uniform energy density of the string interior over the proper volume of the core, become only unambiguous for a perfectly spherical string interior i.e. when $2G\mu = 2G\eta^2 = 1$. It would then be expected that as one separates from a perfect spherical string for $G\mu < \frac{1}{2}$, the effective string radius would become somewhat smaller than that is predicted by $\delta_0 = \eta(\pi V_0)^{-\frac{1}{2}}$, so that inflation could be induced in the string core only when $\mu = \mu_e = \eta^2 = \frac{1}{2G}$, or for values of $\mu$ very close to $\mu_e$. Unfortunately, an extreme supermassive string with $2G\mu_e = 1$ does not seem to exist because it would correspond to a situation where all
the exterior broken phase is collapsed into the core, leaving a pure false-vacuum phase in which the picture of a cosmic string with a core region of trapped energy is lost [6-8]. This happens in all considered string metrics, i.e. for the Hiscock-Gott metric [10,11] and for the Laguna-Garfinkle [8] and Ortiz [9] metrics, in all the cases the spacetime possessing an unwanted singularity which cannot be smoothed out.

Strings for which $4G\mu < 1$ distort the underlying spacetime structure, but their own internal structure is not essentially modified by the structure of the underlying spacetime [7]. For such strings, spacetime is somehow adapted to the topological defect imposed by the matter gauge fields, resulting in a conical geometry, but this does not backreact over the string structure. This is no longer the case, however, for supermassive strings [8,9], $4G\mu \geq 1$, where the strength of gravity coupling is so great and its nature such that one should expect the string structure itself to be substantially modified by the characteristics it induces in the underlying spacetime as well; that is to say, one should expect supermassive cosmic strings to be those stable topological defects which smoothly follow the structural pattern dictated by a gravitational kink, i.e. the pattern of an allowed gravitational topological defect which can move about spacetime but cannot be removed without cutting [12].

By enforcing the interior metric of the string to follow the spacetime lightcone itinerary dictated by a cylindrically-symmetric gravitational kink, we show in this paper that the picture of a cosmic string with a core region of trapped energy is still retained even at the extreme value $2G\mu = 1$, and that the unwanted conical singularity becomes the apparent singularity (event horizon) of a De Sitter kink. This horizon singularity can now be removed by a suitable Kruskal extension. The resulting extreme supermassive string is then able to drive an essentially unique, gravitational inflationary process, without any fine tuning of the initial conditions. Regarding the extreme supermassive strings as initial baby universes, this inflationary expansion of the string core can then lead to a unique stationary picture of eternally self-reproducing inflating universes [13]. It is also shown that the extreme supermassive string model can accommodate finite spacetime closed timelike curves [14], without any imaginary mass states or causality violation [15].

The paper is outlined as follows. In section 1 we construct an explicit metric for the supermassive cosmic string kink, and discuss the constraints that the existence of the kink imposes on the internal geometry of the string. As a result of the surface identifications arising from the constraints, the energy of the supermassive cosmic string becomes quantized so that it can only take on values $4G\mu = 1$
and $2G\mu = 1$. We then re-express the metric of the cosmic string kink in standard form and obtain an analytical expression for the relevant time parameter entering that metric in section 3, where a discussion on surface identifications in kink space is moreover included. The geodesic incompleteness of the standard metric is removed in section 4 by maximally-extending this metric using the Kruskal technique. In section 4 we also show that the geodesically complete metric of a $2G\mu = 1$ cosmic-string kink describes a given section of a kinky De Sitter spacetime which can only be represented by using four different coordinate regions, while allowing the existence of an eternal inflationary process. Finally, we discuss the possibility for the existence of closed timelike curves in the above Kruskal spacetime first, in section 5, by explicitly determining the null-geodesic itinerary followed by two of such curves, and then, in section 6, by studying the quantum creation of particles induced by propagators along the null geodesics. It is concluded that although closed timelike curves are allowed to exist, no observer could ever notice their existence. Throughout the paper we use units so that $\hbar = c = 1$.

2 The Cosmic String Kink

The static, cylindrically symmetric internal metric of a straight cosmic string is [10,11]

$$ds^2 = -d\tau^2 + d\rho^2 + dz^2 + r_*^2 \sin \frac{\rho}{r_*} d\phi^2;$$

(1)

with $-\infty < \tau < \infty$, $-\infty < z < \infty$, $0 \leq \phi < 2\pi$, $0 \leq \rho \leq r_* \arccos(1 - 4G\mu)$, and

$$r_* = (8\pi G\epsilon)^{-\frac{1}{2}},$$

(2)

where $\epsilon$ is the uniform string density, out to some cylindrical radius $\rho_0$.

Both the interior metric (2.1) and the exterior metric,

$$ds^2 = -d\tau^2 + d\rho^2 + dz^2 + (1 - 4G\mu)^2 \rho^2 d\phi^2,$$

(3)

define two surfaces at $z=$const, $\tau=$const, which can be simultaneously visualized by embedding the metrics in an Euclidean three-space [11]. Then, the geometries of such surfaces are, respectively, that of a spherical cap (interior region) and that of a cone with deficit angle $\Delta = 8\pi G\mu$ in the exterior vacuum region.

The change of coordinate

$$r = r_* \sin \frac{\rho}{r_*}$$

(4)
shows more transparently the invariance of the line element (2.1) under rotation about $\phi$ and translation along $z$ of the underlying cylindrical symmetry (i.e. under the two Killing vectors $\xi = \partial_\phi$ and $\zeta = \partial_z$ [16]), but gives rise to a singularity at $r = r_*$; that is

$$ds^2 = -d\tau^2 + \frac{dr^2}{1 - \frac{r^2}{r^2_*}} + dz^2 + r^2 d\phi^2. \quad (5)$$

Clearly, the divergence of (2.5) at $r = r_*$ must correspond to an apparent singularity and should therefore be removable by an appropriate coordinate transformation. A coordinate change which still transparently shows invariance under the Killing vectors and leads to no interior singularity is

$$u = \frac{\tau}{r_*} + \arcsin \frac{r}{r_*}, \quad v = \frac{\tau}{r_*} - \arcsin \frac{r}{r_*}. \quad (6)$$

In terms of coordinates $u$ and $v$ we obtain for the interior metric

$$ds^2 = -r_*^2 du dv + dz^2 + r^2 d\phi^2, \quad (7)$$

where

$$r = r_* \sin \left[ \frac{1}{2}(u - v) \right]. \quad (8)$$

In any case, the exterior metric still keeps a conical singularity.

The regular metric (2.7) would trivially describe the maximally extended interior region of a cosmic string when $4G\mu < 1$. The structure of the string core is then determined by the matter gauge theory (e.g. the Abelian Higgs model of Nielsen and Olesen [17]) rather than gravity, and should only lodge the real energy from the unbroken gauge phase. Nevertheless, according to that was discussed in the Introduction, at scales where the gravity coupling is very large, $4G\mu \geq 1$ [8-11], the cosmic string should also satisfy the conservation law for topological defects which are described by relativistic metric twists, and hence the coordinate $r$ entering metric (2.5) must vary along an extended interval in such a way that it allows metric (2.5) to also become a cylindrically-symmetric spacetime kink. In what follows we shall see that, at such scales, the maximally extended internal metric will also cover a given strip with the broken phase, surrounding the core of trapped real energy. These two internal phases are then separated by an event horizon at $r = r_*$.  

The relationship between a possible cosmic string kink and the apparent singularity in metric (2.5) can be appreciated by considering [18] a cylindrically-symmetric spacetime with manifest invariance under the action of the Killing vectors $\xi$ and $\zeta$ of the form

$$ds^2 = \cos 2\alpha (-dt^2 + dr^2) - 2 \sin 2\alpha dtdr + dz^2 + r^2 d\phi^2, \quad (9)$$
where $\alpha$ denotes the angle of tilt of the spacetime lightcones. We shall then ensure the presence of one kink (i.e. a gravitational topological defect with kink number unity) by requiring $\alpha$ to monotonously vary between 0 and $\pi$, in such a way that $\alpha$ will either increase from 0 to $\pi$ if this is the direction along which the time entering the relevant (explicitly showing the kink) metric increases, or decreases from $\pi$ to 0, otherwise.

We note that (2.9) can be transformed into (2.5) if we set
\[
\sin \alpha = \frac{r}{2\pi r_*},
\]
and change the time coordinate so that $\tau = t + G(\omega)$, where
\[
d\omega = d\tau + F(r)dr,
\]
with $F(r)$ a given function of $r$. Denoting $\frac{dG(\omega)}{d\omega}$ by $G'$, it follows
\[
dt = d\tau (1 - G' - G'F(r)dr).
\]
Metric (2.5) can then be obtained from (2.9) if
\[
G' = 1 - \frac{k_1}{\cos^2 \frac{\alpha}{2}}, \quad k_1 = \pm 1
\]
\[
G'F'(r) = \tan 2\alpha.
\]
Note that both $F(r)$ and $G'$ are singular at $r = r_*$. This simply reflects the singular character of metric (2.5).

Before discussing the above transformation, let us consider the limitations that the functional form (2.10) imposes on the very internal geometry of a supermassive cosmic string. From (2.4) and (2.10) it is obtained
\[
\cos^2 \theta = \cos 2\alpha,
\]
where we have denoted $\theta = \frac{\pi}{r_*}$, and $\alpha$ monotonously varies between 0 and $\pi$.

The question now is, how does $\theta$ vary under variation of $\alpha$ between 0 and $\pi$?. From (2.15) it follows that monotonous variation of $\alpha$ from 0 to $\frac{\pi}{4}$, and from $\frac{3\pi}{4}$ to $\pi$ induces monotonous variation of $\theta$, respectively, from 0 to $\frac{\pi}{2}$ and from $\frac{\pi}{2}$ to $\pi$ (or, likewise, from $\pi$ to $\frac{3\pi}{2}$ and from $\frac{3\pi}{2}$ to 0). Note that induced variations of $\theta$ in the interval ($\pi, 2\pi$) are also allowed but, since $\theta$ appears in the metric in the form $\sin^2 \theta$, these variations would lead to the same geometrical situations as from variations of $\theta$ in the interval (0, $\pi$). We obtain then that if $\alpha$ is allowed to run along its entire interval from 0 to $\pi$, and since the kink metric (2.9) depends on $\alpha$ only through the argument $2\alpha$ of the $\cos^2$ and $\sin$, the two opposite poles of the resulting interior two-sphere at $\theta = 0$ and $\theta = \pi$ should be identified.
Variation of $\alpha$ from $\frac{\pi}{4}$ to $\frac{3\pi}{4}$ induces variation of $\theta$ only along its imaginary axis, first from $\frac{\pi}{2}$ to $\frac{\pi}{2} + i \ln(2^{\frac{1}{2}} \pm 1)$ (at $\alpha = \frac{\pi}{2}$), and then to $\frac{\pi}{4}$ again. The choice of sign in the argument of the $\ln$ at the extremum value of $\theta$ corresponding to $\alpha = \frac{\pi}{2}$ should be made as follows. On the interval where $\theta$ is complex, $\theta = \frac{\pi}{2} + i \theta_i$, we have (see Refs. [10,11])

$$G\mu = \frac{1}{4} + \frac{i}{4} \sinh \theta_i$$ (16)

$$r = i r_* \coth \theta_i$$ (17)

Taking into account (2.17) it follows that the imaginary part of (2.16) corresponds to a real mass per unit imaginary length, or equivalently, a tachyonic mass per real unit length. Thus, once the critical mass $G\mu = \frac{1}{4}$ is reached, the interior of the string starts developing a new real region with maximum width $(2^{\frac{1}{2}} - 1)r_*$, covering the entire hemisphere, where the gauge symmetry is broken at the scale $\varphi = \pm \eta$. Then, the symmetry $\eta \rightarrow -\eta$ of the broken phase will imply an identification $\frac{\pi}{2} + i \ln(2^{\frac{1}{2}} + 1) \rightarrow \frac{\pi}{2} + i \ln(2^{\frac{1}{2}} - 1)$ on the extremum value of $\theta_i$ that corresponds to $\alpha = \frac{\pi}{2}$. We can therefore choose complex $\theta$ to vary first from $\frac{\pi}{2}$ to $\frac{\pi}{2} + i \kappa \ln(2^{\frac{1}{2}} + 1)$ as $\alpha$ goes from $\frac{\pi}{4}$ to $\frac{\pi}{2}$, and then from $\frac{\pi}{2} + i \kappa \ln(2^{\frac{1}{2}} - 1)$ to $\frac{\pi}{4}$ again as $\alpha$ goes from $\frac{\pi}{2}$ to $\frac{3\pi}{4}$, either for $\kappa = \pm 1$ (Fig. 1).

Now, the maximum value of the internal spherical coordinate $\theta$, $\theta_M$, is related to the string mass per unit length $\mu$ by [10,11]

$$\theta_M = \arccos(1 - 4G\mu).$$

Then, in order to ensure the occurrence of one kink in the cosmic string interior, we must have $\theta_M = \pi$ and hence $G\mu = \frac{1}{2}$; or in other words, the gravitational coupling implies a quantization of supermassive cosmic strings so that only the critical hemispherical string at $G\mu = \frac{1}{4}$ (no kink present), and the extreme spherical string at $G\mu = \frac{1}{2}$ (one kink present) are allowed to exist along the entire interval, $\frac{\pi}{2} \leq \theta \leq \pi$, of possible classical supermassive cosmic strings. We regard this as a typical quantum-gravity effect.

If we ensure the presence of a kink, then no string exterior (except the portion which is incorporated into the interior metric by the complex evolution of $\theta$) is possible or needed [10,11], and the conical singularity becomes a removable horizon singularity on the core surface at $r = r_*$, separating the two gauge phases that make up the extended string interior. Thus, all of the possible geometry of the extreme string can be regarded as describable by a spacetime which is $\mathbb{R}^2 \times S^2$, showing still the picture of a cosmic string with a spherical core region of trapped energy surrounded by a shell of
true vacuum protecting the string from dissolving in the unbroken symmetry phase.

3 Lightcone Configurations

The results of section 2 allow us to deal with the interior metric of an extreme supermassive cosmic string (hereafter denoted as ”extreme string”) in a similar fashion to as it is done in the cases of the black hole kink [18] or the De Sitter kink [19].

The interior of an extreme string has still a geodesic incompleteness at \( r = r_* \), and can only be described by using a number of different coordinate patches. The identification and distinction of such patches can be achieved by transforming (2.9) into the standard metrical form proposed by Finkelstein and McCollum [18], adapted to cylindric symmetry. To accomplish such a transformation, it is convenient to introduce a new time coordinate, such that

\[
\bar{t} = t + G(\sigma) \quad (1)
\]

\[
\cos 2\alpha (1 - G^2 F_\sigma(r)) + 2 \sin 2\alpha G^i F_\sigma(r) = 0, \quad (2)
\]

where the new variable \( \sigma \) and the new functionals \( F_\sigma(r) \) and \( G \equiv G(\sigma) \) are defined as follows

\[
d\sigma = d\bar{t} + F_\sigma(r) dr
\]

\[
G^i \equiv G(\sigma)^i = \frac{dG}{d\sigma}.
\]

We obtain then

\[
G^i F_\sigma(r) = \tan 2\alpha - \frac{k_2}{\cos 2\alpha}, k_2 = \pm 1, \quad (3)
\]

so that metric (2.9) becomes

\[
ds^2 = -d\bar{t}^2 - \frac{2k_1k_2}{\cos \frac{\pi}{2} 2\alpha}d\bar{t}dr + dz^2 + r^2 d\phi^2, \quad (4)
\]

where \( k_1 \) is defined in (2.13). Metric (3.4) still is not the kink metric in standard form. This is obtained by making the re-definition

\[
k_1 d\bar{t} = k_1 \frac{d\bar{t}}{\cos \frac{\pi}{2} 2\alpha} = dt + (\tan 2\alpha - \frac{k_2}{\cos 2\alpha}) dr, \quad (5)
\]

so that we finally obtain

\[
ds^2 = -\cos 2\alpha d\bar{t}^2 - 2k_1k_2 d\bar{t}dr + dz^2 + r^2 d\phi^2. \quad (6)
\]

Since one can always write \( k_1k_2 = k = \pm 1 \), (3.6) becomes formally the same as the general line element in standard form given
by Finkelstein and McCollum [18], after exchanging spherical for cylindric symmetry. On the other hand, using (2.10) it can be readily seen that any \( z = \text{const} \) section of metric (3.6) is not but the \( \theta = \frac{\pi}{2} \) (hemispherical) section of the standard De Sitter kink metric [19] for a positive cosmological constant \( \Lambda = \frac{3}{r^2} \).

However, the facts that the sign definitions given by parameters \( k_1 \) and \( k_2 \) enter the formalism at different levels (Ref. Eqns (2.13) and (3.3)), and that the time parameter entering the standard metric for the De Sitter kink is defined in a similar fashion to as it is made in (3.1), rather than (3.5), leads to a sharp distinction between the geometries of De Sitter kink and extreme string kink. In order to clearerly appreciate the nature of that distinction, let us calculate now \( \bar{t} \) for the case \( t = 0 \) where we enforce the system to lie on the \( r \) axis. This involves calculating the following two integrals

\[
I_1 = \int_{0/A}^{r} \frac{dr}{\cos^2 2\alpha}, \quad I_2 = \int_{0/A}^{r} \frac{dr \sin 2\alpha}{\cos^2 2\alpha},
\]

where the lower limit \( 0/A \) refers to the choices \( r = 0 \) and \( r = A \equiv 21^2 r_\ast \), depending on whether the case \( k_2 = +1 \) or the case \( k_2 = -1 \) is being considered [19]. Taking

\[
\sin \alpha = \frac{r}{A}, \quad \cos \alpha = k_2(1 - \frac{r^2}{2r_\ast^2})^{\frac{1}{2}}, \quad \cos^2 2\alpha = k_1(1 - \frac{r^2}{r_\ast^2})^{\frac{1}{2}}, \quad \cos^{-\frac{1}{2}} 2\alpha = \frac{k_1}{A} \ln \left[ \frac{2k_1 A}{r_\ast}(1 - \frac{r^2}{2r_\ast^2})^{\frac{1}{2}} + 4(1 - \frac{r^2}{2r_\ast^2})^{\frac{1}{2}} \right] \Bigg|_{0/A}^{r}
\]

we obtain

\[
I_1 = -i r_\ast \ln \left[ \frac{r}{r_\ast} + k_1 \left(1 - \frac{r^2}{r_\ast^2}\right)^{\frac{1}{2}} \right] \bigg|_{0/A}^{r}
\]

and

\[
I_2 = \frac{k_2 r_\ast^2}{A} \left\{ k_1 [(1 - \frac{r^2}{2r_\ast^2})(1 - \frac{r^2}{r_\ast^2})^{\frac{1}{2}}]^{\frac{1}{2}} + \frac{r_\ast}{A} \ln \left[ \frac{2k_1 A}{r_\ast}(1 - \frac{r^2}{2r_\ast^2})^{\frac{1}{2}} + 4(1 - \frac{r^2}{2r_\ast^2})^{\frac{1}{2}} \right] \bigg|_{0/A}^{r} \right. \Bigg.
\]

From (3.8) and (3.9) it can be seen that it is not possible to obtain a unique, compact expression for the time parameter \( \bar{t} \). This fact distinguishes the extreme string kink from the De Sitter kink for which there is a well defined analytical definition of \( \bar{t} \) [19]. The geometrical reason for this is that time parameter \( \bar{t} \) actually corresponds to the case where a complete description of the geometry requires just the two patches associated with \( k_2 = \pm 1 \), which is the case for the De Sitter kink but not for the extreme string kink. Because of the presence of factor \( \cos^{-\frac{1}{2}} 2\alpha \) in the expression for
\( G' \) in (2.13), each of the two coordinate patches associated with \( k_2 = \pm 1 \) itself must unfold in the two different sets of coordinate regions that correspond to the new sign ambiguity \( k_1 = \pm 1 \). That unfolding can only be accounted for by the new time parameter \( \tilde{t} \) entering the standard metric (3.6) which, according to the sign definition introduced for \( k_2 \) in (3.5), will correspond to a physical time \( t \) running either forwards when \( k_1 = +1 \), and backwards when \( k_1 = -1 \). One should then expect a unique, compact analytical expression governing all coordinate domains only for time \( \tilde{t} \). By directly integrating (3.5) between the same limits as in (3.8) and (3.9) we in fact finally obtain [20]

\[
\tilde{t} \equiv \tilde{t}(k_1, k_2) = \int_{0/A}^{r} dt = k_1 t - r_\ast k_2 \left\{ \frac{A}{r_\ast} \left( 1 - \frac{r^2}{2r_\ast^2} \right)^{\frac{1}{2}} \right. \\
- \frac{1}{2} \ln \left[ \frac{(1 - \frac{r^2}{2r_\ast^2})^{\frac{1}{2}} + r_\ast (r_\ast - r)}{(1 - \frac{r^2}{2r_\ast^2})^{\frac{1}{2}} - r_\ast (r_\ast + r)} \right].
\]  

(10)

Note that all sign ambiguity arising from the square root in the argument of the \( \ln \) has been omitted in Eqn. (3.10). Such an ambiguity does not affect the discussion to follow as it only manifests as an additive constant term which leaves metric (3.6) unchanged. This ambiguity will be of decisive importance however for the consideration of the thermal properties of the extreme string which we shall deal with in section 6.

The fact that for \( t = 0 \) both \( k_1 \) and \( k_2 \) enter (3.6) and (3.10) at exactly the same footing, so that all distinction between the coordinate domains that arise from different values of \( k_1 \) disappears to leave just two different coordinate patches corresponding to \( k = k_1 k_2 = \pm 1 \), is just an artifact coming from still keeping the removable geodesic incompleteness in metric (3.6). It will be seen in the next section that even for \( t = 0 \) the sign parameters \( k_1 \) and \( k_2 \) actually enter the coordinates defining the maximally-extended metric at different footings. Therefore, although \( \alpha \) may only vary within the interval \((0, \pi]\), there really are four distinguishable sets of coordinate regions that correspond to the distinct combinations of \( k_2 = \pm 1 \) with \( k_1 = \pm 1 \). In the general case \( t \neq 0 \) the coordinate regions can even be distinguished without removing the event singularity.

In the De Sitter space, where the geometry of a \( t=\text{const} \), \( r=\text{const} \) section is just a two-sphere, the spherical symmetry is manifested in the presence of a kink by just identifying the equators of the two hemispheres that, respectively, correspond to the two kink coordinate patches, but the poles of these hemispheres can never be identified [18,19]. In contrast, the real geometry of a \( t=\text{const} \),
$z=$-const section of extreme string kink is that of two real hemispheres whose mutual matching at their equators lies on the extremum imaginary values of the spherical coordinate $\theta$, $\ln(2^{\frac{k_1}{2}}+1) \to \ln(2^{\frac{k_1}{2}}-1)$. The imaginary values of $\theta$ are mapped onto real values of $\alpha$ in the kink, giving rise to an equator identification at $r = A$, provided the resulting values of $\alpha$ are either always increasing from 0 to $\pi$ (i.e. when $k_1 = +1$) as one goes along the two coordinate regions being joined from $r = 0$ to $r = 0$, or always decreasing from $\pi$ to 0 (i.e. when $k_1 = -1$) along the two entire joined coordinate patches. Thus, only the two equal-$k_1$ one-kink lightcone configurations (each involving two of such coordinate patches) given in Fig. 2 are in principle possible.

Nevertheless, the fact that we have obtained just a unique, compact expression for the time parameter $\tilde{t}$ should imply the existence of just a unique one-kink lightcone configuration involving all the four sets of coordinate regions $k_2 = k_1 = +1$, $k_2 = k_1 = -1$, $k_2 = -k_1 = +1$ and $k_2 = -k_1 = -1$. Therefore, at least one of the poles at $r = 0$ in one of the configurations of Fig. 2 ought to be identified to the pole having the same $\alpha$ and (in order to ensure the interval for $\alpha$ to be $(0, \pi)$) different $k_1$ at $r = 0$ in the other configuration. Inspection of Eqns. (2.13), (2.15) and (3.8) tells us that, in fact, the two poles in one of the two configurations should be identified in this way to the corresponding two poles in the other configuration. These identifications actually correspond by the mapping $\theta \Rightarrow \alpha$ to the identifications made in section 2 on the extreme values of real $\theta$. As it was expected, there is thus a unique one-kink closed configuration involving all four sets of coordinate regions described by the physical original regions $I_{k_1}^k$, $II_{k_1}^k$ in Fig. 2. A rather pictorial representation of such a unique lightcone configuration is given in Fig. 3. One can readily be convinced of the existence of this configuration by proving that if, along a null-geodesic itinerary, the above identifications are made then, by starting at e.g. $t = 0$ on a given surface $\phi =$-const, $z =$-const, $r = r_0$, $\tilde{t} = \tilde{t}_0$, we finally recover the same point $(r_0, \tilde{t}_0)$, also at $t = 0$, after completing the entire null-geodesic itinerary along the lightcone configuration of Fig. 3. We have checked by explicit calculation that this is actually the case. Insisting on the restriction of keeping $t = 0$ along a closed path requires equalizing points $(r, \tilde{t})$ on pole identifications, while changing the sign of $\tilde{t}$ on equator identifications.
4 Kruskal Extension of Extreme String Metric

The distinction between sign parameters $k_1$ and $k_2$ should manifest in the fact that $k_1$ must only be involved in the definition of time $\tilde{t}$, and $k_2$ should appear in the definition of both time $\tilde{t}$ and radial coordinate $r$ as well. As already noted before, that distinction could only be shown, even at $t = 0$, when the apparent singularity at $r = r_*$ is removed.

This geodesic incompleteness, which occurs in each of the four sets of coordinate regions described by metric (3.6), can be removed by the usual Kruskal technique [21]. Thus, we define the metric

$$ds^2 = -2F(U,V)dUdV + dz^2 + r^2d\phi^2,$$  \hspace{1cm}(1)$$

in this way straightening the null geodesics into lines parallel to the new $U$ and $V$ axis, and identify it with the standard metric (3.6), with $g_{\tilde{t}\tilde{t}} = -\cos 2\alpha$, $g_{\tilde{t}r} = -k_1k_2$ and $g_{UV} = -F$, in such a way that $k_1$ and $k_2$ be kept consistently distinguished and $F$ be finite, nonzero and depend on $r$ and $k_2$ alone. All of these requirements can be met by the choice

$$U = \mp e^{\beta k_1 \tilde{t}} \exp(2\beta k_2 \int_{0/A}^{r} \frac{dr}{\cos 2\alpha})$$ \hspace{1cm}(2)$$

$$V = \mp \frac{1}{r_*} e^{-\beta k_1 \tilde{t}}$$ \hspace{1cm}(3)$$

$$F = \frac{r_* \cos 2\alpha}{2\beta} \exp(-2\beta k_2 \int_{0/A}^{r} \frac{dr}{\cos 2\alpha}),$$ \hspace{1cm}(4)$$

with $\beta$ a constant which should be chosen so that $F$ has a finite limit as $r \to r_*$.

Using [19]

$$\int_{0/A}^{r} \frac{dr}{\cos 2\alpha} = \frac{1}{2} \ln \left(\frac{r_* + r}{r_* - r}\right),$$

$$\cos 2\alpha = 1 - \frac{r^2}{r_*^2},$$

we obtain from (4.4)

$$F = \frac{(r_*^2 - r^2)}{2\beta r_*} \left[\frac{(r_* + r)^2}{r_*^2 - r^2}\right]^{-\beta k_2 r_*}.$$

To avoid $F$ being either 0 or $\infty$ at $r = r_*$, we then choose $\beta = -\frac{1}{k_2 r_*}$, and arrive therefore at

$$F = -\frac{1}{2} k_2 (r_* + r)^2$$ \hspace{1cm}(5)$$
\[ U = \mp e^{-\frac{k_2 k_1 t}{r^*} \left( \frac{r^* - r}{r^* + r} \right)} \quad (6) \]

\[ V = \pm k_2 e^{\frac{k_1 k_2 t}{r^*}} \quad (7) \]

so that

\[ UV = -k_2 \frac{(r^* - r)}{(r^* + r)} \quad (8) \]

In terms of the coordinate product (4.8) we have finally

\[ F = \frac{2r^*_2 k_2}{(k_2 - UV)^2} \quad (9) \]

\[ r = r^* \frac{k_2 + UV}{k_2 - UV} \quad (10) \]

\[ \tilde{t} = k_1 t - k_1 k_2 r^* \left\{ (1 - \frac{4k_2 UV}{(k_2 - UV)^2})^{\frac{1}{2}} \right. \]

\[ + \left. \frac{1}{2} \ln \left[ \left( 1 + \left( 1 - \frac{4k_2 UV}{(k_2 - UV)^2} \right)^{\frac{1}{2}} \right) k_2 UV \right] \right\}. \quad (11) \]

The metric (4.1) becomes then

\[ ds^2 = -\frac{4k_2 r^2}{(k_2 - UV)^2} dU dV + dz^2 + \frac{r^2 (k_2 + UV)^2}{(k_2 - UV)^2} d\phi^2. \quad (12) \]

We notice that the \( z=\)const sections of this metric actually coincide with that is obtained for a hemispherical section (\( \theta = \frac{\pi}{2} \)) of the De Sitter kink [19], the sole though essential difference being that for each value of \( k_2 \) there are here two different values of time \( \tilde{t} \) and \( k_1 t \), corresponding to \( k_1 = \pm 1 \), at every value of \( r \) and \( t \). In Fig. 4 we give a representation in terms of coordinate \( U \), \( V \) of the four different sets of coordinate regions that occur in the one-kink geodesically-complete extreme string spacetime.

The maximally-extended extreme string metric (4.12) does not possess any singularity and covers all the space of an extreme string, including both an unbroken-phase interior of radius \( r^* \) and a broken-phase covering shell of width \( \left( 2^{\frac{1}{2}} - 1 \right) r^* \), which has been converted into an unbroken phase containing a vanishing overall real energy by the mapping \( \theta \Rightarrow \alpha \) (see section 6). The first of these facts is in sharp contrast with the unavoidability of an unwelcome singularity found by other authors for supermassive cosmic strings [8,9]. The fact that (4.12) also describes an exterior shell that protects the spherical core from dissolving is, in turn, in contrast with the trivial maximal extension (2.7) of the interior metric of cosmic strings whose mass per unit length is smaller than the critical value \( G\mu_c = \)
In the latter case, the topological defect need not developing a broken-phase shell to protect itself against dissolving, as it is always immersed in the broken-phase with conical geometry.

On the other hand, the finding of an extreme string kink metric which on each $z=\text{const}$ section exactly possesses the symmetry of a hemispherical section of the De Sitter kink, may have two consequences of interest. First, since the size of the false vacuum region inside the extreme string appears to exceed the size of its corresponding horizon, the above symmetry makes it consistent to consider the emergence of a unique De Sitter inflationary process [22], without any fine tuning of the initial conditions. Secondly, it will allow continuous changes of topology from that of a two-sphere into that of two hemispheres joined at their poles, and vice versa [23]. This may ultimately lead to the possibility for closed time-like curves to occur throughout the maximally-extended spacetime of an extreme string. Moreover, if we interpret spherical extreme strings as incipient baby universes, then the first of these implications leads, in turn, to quite comfortably accommodate the concept of an eternal process of continually self-reproducing inflating universes [4,13] into the present picture. The resulting model would, furthermore, be implemented by the existence of a unique "genetic" hallmark which can be expressed by the condition $2G\mu = 2G\eta^2 = 1$. For any other value of $\mu$ the topological defect would not succeed in producing an observable universe.

5 Closed Timelike Curves in an Extreme String

Closed timelike curves (CTCs) in general relativity were already discussed by Gdel, back in 1949 [24]. Such curves appeared in solutions which were shown to require unphysical stress tensors [25]. The subject has recently been revived in two different contexts. The first involved tunnelling Lorentzian wormholes [26], and possesses two essential drawbacks: the used wormholes correspond to no vacuum solution of Einstein equations, and moreover, they require violation of the weak energy condition [26,27]. The second recent attempt involves solutions to Einstein equations for two rapidly moving, infinite parallel cosmic strings [28], and has also met serious difficulties. In this case, the CTCs can only appear at spatial infinity and, more importantly, correspond to spacetimes with imaginary total mass [29]. In what follows of the present section and in section 6, we shall discuss the possible existence of CTCs in the spacetime of a single extreme string and address the problem of their quantum characteristics.
It has already been mentioned that a single extreme string kink may accommodate CTCs. In fact, inspection of Fig. 3 might already suggest that null geodesics could be CTCs that somehow loop back through the new regions (i.e. regions $III_{k_1}$ and $IV_{k_2}$ on Fig.4), created by the extension process with coordinates $U$, $V$, in addition to the original regions (i.e. regions $I_{k_1}$ and $II_{k_1}$). However, before showing in more detail how the CTCs arise in the maximally-extended spacetime of a extreme string kink, one should extend the surface identifications discussed for metric (3.6) into the Kruskal language.

Clearly, to the conditions imposed for metric (3.6) (i.e. that the values of $r$ and $\tilde{t}$ have to be the same on each pair of identified points, and that whereas the value of parameter $k_1$ should be preserved on equator identifications and change on pole identifications, the value of parameter $k_2$ must change on all identifications) we must now add the two new conditions: (1) Surfaces on original regions can only be identified to surfaces on original regions, and (2) exactly the same conditions that are applied for identifying surfaces on original regions should also be applied to identify surfaces on the new regions which are created by the Kruskal extension (see Ref. [18]). The application of these conditions leaves us with the following identifications on the extreme-string Kruskal metric:

On the original regions of Fig. 4

- Upper hyperbola ($K_2 = k_1 = +1$) $\equiv$ Lower hyperbola ($k_2 = -k_1 = -1$)
- Upper hyperbola ($K_2 = -k_1 = +1$) $\equiv$ Lower hyperbola ($k_2 = k_1 = -1$)
- Right hyperbola ($K_2 = k_1 = +1$) $\equiv$ Right hyperbola ($k_2 = k_1 = -1$)
- Right hyperbola ($K_2 = -k_1 = +1$) $\equiv$ Right hyperbola ($k_2 = -k_1 = -1$)

On the new regions of Fig. 4

- Upper hyperbola ($K_2 = -k_1 = -1$) $\equiv$ Lower hyperbola ($k_2 = k_1 = +1$)
- Upper hyperbola ($K_2 = k_1 = -1$) $\equiv$ Lower hyperbola ($k_2 = -k_1 = +1$)
- Left hyperbola ($K_2 = k_1 = +1$) $\equiv$ Left hyperbola ($k_2 = k_1 = -1$)
- Left hyperbola ($K_2 = -k_1 = +1$) $\equiv$ Left hyperbola ($k_2 = -k_1 = -1$)

All other identifications being forbidden if we want to have one kink, with $\alpha$ continuously varying along the entire interval $0 \leq \alpha \leq \pi$, in both directions. In particular, the above identifications forbid the occurrence of any CTCs that follow curved itineraries which are
enforced to lie on the $r$ axis at $t = 0$, both in the original and new regions (see Fig. 4).

The addition of the new regions, created in the maximal, non-singular extension [21] of the one-kink extreme string, leads to the occurrence of two distinct one-kink lightcone configurations which respectively associate with two different CTCs for geodesics which start at time $t = 0$, evolve along nonzero values of $t$ through both, original and new regions, to finally loop unavoidably back to $t = 0$ again, at the same original value of $r$. These evolutions have been checked by explicit calculation of the physical time $t$ in all the situations where either $r = 0$ or $r = 2^{\frac{1}{z}} r_*$ by taking into account the fact that geodesic paths parallel to the $U$ axis preserve the value of the time parameter $\tilde{t}$ unchanged, but the value of $\tilde{t}$ will evolve according to

$$\tilde{t}(x_i) = \tilde{t}(x_j) + k_1 k_2 r_* \ln |(r_* + r(x_j))(r_* - r(x_i))|^{(r_* + r(x_i))(r_* - r(x_j))},$$

(1)

with $x_i$ and $x_j$ two points on the trajectory, along geodesic paths which are parallel to the $V$ axis. The value of the physical time $t$ at each chosen value of $r$ ($r = 0$ and $r = 2^{\frac{1}{z}} r_*$) was finally evaluated by the relation

$$t = k_1 (\tilde{t} - \tilde{t}_0),$$

(2)

where $\tilde{t}_0$ is the value of $\tilde{t}$ at $t = 0$ calculated by using (4.11) (or (3.10)).

The straight paths of the two null geodesics are represented on the coordinate regions given in Fig. 4 by a solid line for the CTC that starts at $r = 0$ on the original region $I^±_+$ (a point we denote as North$_+$), and by a broken line for the CTC that starts at $r = 0$ on the original region $I^±_-$ (a point we denote as North$_-$). These geodesic itineraries are also pictorially illustrated on Fig. 5, where we show how along the CTCs a spacetime geometry which initially is e.g. typically spherical evolves forth and back continuously into the geometry of two hemispheres joined at their poles, to finally loop back to the spherical geometry again. Also shown is Fig. 5 is the evolution of the directions of lightcones on the hypersurfaces. Finally, the itineraries and evolution of lightcone orientations are represented on a $r$-$t$ diagram for the two CTCs in Fig. 6.

However, each of the CTCs crosses twice horizons at $V = 0$ (i.e. the $U$ axes in Fig. 4). At the crossing points, both $\tilde{t}$ and the physical time $t$ become either $\pm\infty$. It could seem therefore that these sectors of the curves appear at time infinity, and hence our CTCs become suspect of meeting difficulties analogous to the spatial infinity of the Gott’s double-string device [28]. Nevertheless, the fact that the energy of extreme string becomes in the present
approach quantized to a value given by \((2G)^{-\frac{1}{2}}\) and, at the same time, the two-sphere radius for metric (4.12) is \(R = 2^\frac{1}{2}r_\ast\) and the volume of the cylinder circumscribed to the two-sphere is given by

\[ V_c = 4\pi R^3 G\mu = 2\pi R^3, \]

implies that \(r_\ast = G^\frac{1}{2}\). Then, since no string with energy larger than \((2G)^{-\frac{1}{2}}\) may exist and real energy has been assumed to be uniformly distributed throughout the string interior, it follows that no localized spacetime region with size smaller than \((2G)^{\frac{1}{2}}\) can be resolved inside the two-sphere. Therefore, only pairs of points such as \(r = 0\) and \(r = 2^\frac{1}{2}r_\ast\), separated a distance \((2G)^{\frac{1}{2}}\), can physically be considered simultaneously along the CTCs.

Once points at \(r = 0\) and \(r = 2^\frac{1}{2}r_\ast\) have been fixed on their respective hyperbolae, there will be no way to obtain any physical information about any particular trajectory joining such points. Thus, the ascending and descending straight solid lines of Fig. 6 do not represent real physical paths, but only directions of the quantum jumps between the fixed points on the extreme hyperbolae. These quantum transitions are only expressible as propagators given by integrals over all possible paths joining each pair of points on the hyperbolae. In a small neighbourhood of \(V = 0\), these paths would correspond to classical trajectories defined in the limit \(r_\ast \to 0\), where for real \(U\) and \(V\) we have

\[ |U| = \lim_{r_\ast \to 0} e^{-k_2 \frac{U}{r_\ast}}, \]

\[ |V| = \lim_{r_\ast \to 0} e^{k_2 \frac{V}{r_\ast}}, \]

so that the paths may cross the horizons \(V = 0\) at any finite values of the physical time \(t\).

The essential quantum nature of the spacetime of the extreme string makes thus possible the existence of quantum transitions between points of extreme hyperbolae on the original regions and points of extreme hyperbolae on the new Kruskal regions, and hence allow the occurrence of CTCs. These regions are always mutually separated by a \(V = 0\) horizon on which quantum theory allows the physical time to still keep any finite value.

According to the above results it appears that CTCs associated with extreme strings are possible and do not show any of the classical drawbacks that are present in the models mentioned at the beginning of this section. The viability and causality implications of these curves in a more complete quantum treatment will be discussed in the next section.
6 The Quantum Extreme String

The procedure followed in section 2 in order to make topological defects in gauge theories compatible with gravitational topological defects (kinks) actually led to quantization of the energy of supermassive cosmic strings. Basically, this quantization arose from the specific identifications of surfaces on the generally complex interior spherical coordinate $\theta$ that result from introducing one spacetime kink. Two such identifications were made. Identification of the surfaces corresponding to the two extreme values of real $\theta$ implied that only the energies associated with the critical and extreme strings were allowed along the entire range of supermassive strings. Identifications of the surfaces with maximum and minimum values of imaginary $\theta$ at real $\theta = \frac{\pi}{2}$ did not provide however the string with any observable, real energy, but only with a quantized exterior (true vacuum) shell with the tachyonic energy.

It would be expected that once mapped onto the kink variable $\alpha$, where even the evolution on imaginary $\theta$ becomes evolution on real $\alpha$ (Fig. 1), the true-vacuum tachyonic energy be converted into real energy. Because of the symmetry $\eta \to -\eta$ of the original broken phase, there will be equal contributions to this energy, $E$, from its positive and negative components. However, although the overall energy of the string, $(2G)^{-\frac{1}{2}}$, is thus always conserved, the energy emerging from the symmetry restoration in the mapping $\theta \to \alpha$ can still be stored in the different new "unphysical" (inobservable) regions $[30]$, created in the distinct coordinate patches of the Kruskal maximal extension, in such a way that if all of its positive component goes to one of such new regions for $k_2k_1 = \pm 1$, then all of its negative component goes to the same region for $k_2k_1 = \mp 1$, or to a different new region for $k_2k_1 = \pm 1$. On the coordinate patches of Fig 4, these would be, respectively, either regions $III_{\pm}^{\pm}$ and $IV_{\pm}^{\pm}$, or regions $IV_{\mp}^{\pm}$ and $III_{\mp}^{\pm}$. For our present purposes, it will suffice dealing with the storage of this energy in regions $III$ only. The treatment for regions $IV$ is completely parallel.

It appears that each of these energy components could only emerge as an observable quantity in the corresponding original exterior or interior region, $II_{\pm}^{\pm}$ or $II_{\mp}^{\pm}$, if some hypersurfaces in $III_{\pm}^{\pm}$ and $III_{\mp}^{\pm}$ are consistently identified with some hypersurface in $II_{\pm}^{\pm}$ and $II_{\mp}^{\pm}$, respectively. By a consistent identification we mean here that identification which is predicted by the geometry itself, not one which is introduced artificially (ad hoc), or by distorting the physical Lorentzian time $t$ to the Euclidean imaginary region $[31]$. On the other hand, since regions $III_{k_1}^{k_2}$ are essentially inobservable $[30]$, any manifestation of this energy distribution in regions $II_{k_1}^{k_2}$
should be in the form of a completely incoherent radiation which brought no information whatsoever about regions $III_{k_1}^{k_2}$. In what follows we shall show that such consistent identifications actually exist for the coordinate regions given in Fig. 4. We will also see that they give rise to a full protection against causality violation along the complete paths of the CTCs discussed in section 5.

The identification required between surfaces in $III_{k_1}^{k_2}$ and $II_{k_1}^{k_2}$ (or between surfaces in $IV_{k_1}^{k_2}$ and $I_{k_1}^{k_2}$) will simply come from explicitly displaying the sign ambiguity of the square root of the argument for the ln in Eqn. (3.10). Thus, if we want to express that argument as an absolute value then one has as the most general expression for metrical time

$$\tilde{t}(k_3) \equiv \tilde{t}(k_1, k_2, k_3) = \int_{0/A}^{r} d\tilde{t} = k_1 t - r_* k_1 k_2 \left( \frac{A}{r_*} \left(1 - \frac{r^2}{2r_*^2}\right)^{\frac{1}{2}} - \ln \left( \left| \frac{\left( A(1 - \frac{r^2}{2r_*^2})^{\frac{1}{2}} + r_* (r_* - r) \right)^{\frac{1}{2}}}{(A(1 - \frac{r^2}{2r_*^2})^{\frac{1}{2}} - r_*)(r_* + r)} \right| \right) \right) + \frac{i}{2} k_1 k_2 k_3 (1 - k_3) \pi r_*$$

$$= \tilde{t} + \frac{i}{2} k_1 k_2 k_3 (1 - k_3) \pi r_*,$$

where $\tilde{t} \equiv \tilde{t}(k_1, k_2)$ is the same as in (3.10), and $k_3 = \pm 1$ is a new sign parameter which unfolds the coordinate regions given in Fig. 4 into still two sets of four patches. Time (6.1) is the most general expression for the time $\tilde{t}$ entering the standard metric (3.6). One would again recover metric (3.6) from metric (4.1) with the same requirements as in section 4 using $\tilde{t}(k_3)$ instead of $\tilde{t}$ if we re-define the Kruskal coordinates $U, V$ such that

$$U = \pm k_3 e^{-\frac{k_1 k_2 k_3}{r_*} (r_* - r)} \frac{(r_* - r)}{(r_* + r)},$$

$$V = \mp k_2 k_3 e^{\frac{k_1 k_2 k_3}{r_*}},$$

where

$$\tilde{t}_c = \tilde{t} + i k_1 k_2 k_3 \pi r_*.$$
identification of the extreme imaginary values of $\theta$, and then stored in regions $II_{k_1}^{k_2}$ by mapping $\theta \Rightarrow \alpha$ and Kruskal extension) appear in the original regions $II_{k_1}^{k_2}$ as an observable real quantity. That this energy should appear in $II_{k_1}^{k_2}$ as a thermal bath of completely incoherent radiation is now easy to see if one invokes the general relation between metric periodicity and gravitational temperature \[31,32\].

This approach has, however, always seemed rather mysterious as it gives no explanation to the use of the Euclidean version of spacetime \[33\]. In our model periodicity in the metric results, nevertheless, from a perfectly justifiable requirement for mathematical completeness, rather than a suggestive though not very convincing procedure, and entails no unjustified extension of the physical time $t$ into the imaginary axis. Our approach offers therefore a physically reasonable resolution of the paradox posed by the Euclidean gravity method, and provides the concept of gravitational temperature with a new physical background - the relation between event horizons and spontaneous symmetry breaking, discussed in this paper.

As it was concluded in section 5, the evolution of a field along any null geodesics in Fig. 4 should be described by a quantum propagator, rather than a classical path. If the field has mass $m$, such a propagator will be the same as the propagator $G(x', x)$ used by Gibbons and Hawking \[34\], and satisfy therefore the Klein-Gordon equation

$$ (\Box^2 - m^2)G(x', x) = -\delta(x, x'). $$

(5)

For metric (4.12), the propagator $G(x', x)$ becomes analytic \[34\] on precisely the strip of width $\pi r_*$ predicted by (6.4), here without any need to extend $t$ to the Euclidean regime. Then, the amplitude for detection of a detector \[34\] sensitive to particles of a certain energy $E$, in regions $II_{k_1}^{k_2}$, would be proportional to

$$ \Pi_E = \int_{-\infty}^{+\infty} d\tilde{\tau}_c e^{-iE\tilde{\tau}_c} G(0, \bar{R}'; \tilde{\tau}_c, \bar{R}), $$

(6)

where $\bar{R}'$ and $\bar{R}$ denote respectively $(r', z', \phi')$ and $(r, z, \phi)$. Note that the time parameter entering the amplitude for detection should be $\tilde{\tau}_c$, rather than $t$, as both the matter field and the detector must evolve in the spacetime described by metrics (3.6) and (4.12). Since time $\tilde{\tau}_c$ (but not $t$) already contains the imaginary term which is exactly required for the thermal effect to appear, we have not need to make the physical time $t$ complex. From (6.4) and (6.6), we can generally write

$$ \Pi_E = e^{k_1k_2k_3\pi r_*E} \int_{-\infty}^{+\infty} d\tilde{\tau} e^{-iE\tilde{\tau}} G(0, \bar{R}'; \tilde{\tau} + ik_1k_2k_3\pi r_*, \bar{R}). $$

(7)
Following Gibbons and Hawking [34], we now investigate the different particle creation processes that can take place on the extreme hyperbolae at \( r = 0 \) and \( r = 2^{1/2} r_* \), along the quantum itineraries of the CTCs discussed in section 5. Let us first consider the case \( k_3 = -1 \) for the original regions \( \Pi_{k_1}^{k_2} \) on the patches of Fig. 4. For \( k_1 = k_2 = +1 \), if \( x' \) is a fixed point on the hyperbola at \( r = 0 \) of region \( I_+^+ \), and \( x \) is a point on the hyperbola at \( r = 2^{1/2} r_* \) of region \( II_+^+ \), we obtain from (6.7)

\[
P_a^{II_+^+}(E) = e^{-2\pi r_* E} P_e^{II_+^+}(E),
\]

where \( P_a^{II_+^+}(E) \) generically denotes the probability for detector to absorb a particle with positive energy \( E \) from region \( II_{k_1}^{k_2} \), and \( P_e^{II_+^+}(E) \) accounts for the similar probability for detector to emit the same energy also to region \( II_{k_1}^{k_2} \).

An observer on the extreme hyperbola of the exterior original region of patch \( k_2 = k_1 = +1 \) will then measure an isotropic background of thermal positive-energy radiation at a temperature

\[
T_s = \frac{1}{2\pi r_*}.
\]

If, in turn, \( x' \) and \( x \) are fixed points on the extreme hyperbolae in regions \( I_+^- \) and \( II_+^- \), respectively, we obtain then for an observer on the hyperbola \( r = 0 \) in the interior original region

\[
P_a^{II_+^-}(-E) = e^{2\pi r_* E} P_e^{II_+^-}(-E).
\]

According to (6.10), there will appear an isotropic background of thermal radiation which is formed by exactly the antiparticles to the particles contained in the thermal bath detected in region \( II_+^+ \), at the same temperature \( T_s \) given by (6.9). The same probability relation as (6.10) is also obtained for the exterior physical region \( II_+^- \). Therefore, observers on the extreme hyperbola at \( r = 2^{1/2} r_* \) in that region will also detect a thermal bath of particles with energy \( E < 0 \), at temperature (6.9). Finally, for the point \( x \) on the extreme hyperbola of the interior original region \( II_-^- \), we derive an expression as (6.8), also for \( E > 0 \), and hence an interpretation for thermal properties as for region \( II_+^+ \). Had we similarly identified extreme hypersurfaces of regions \( I \) and \( IV \), and then carried out the parallel treatment for propagators \( G(x', x) \), the energy \( E \) from \( \theta \Rightarrow \alpha \) symmetry restoration stored at regions \( IV \) would have emerged as incoherent radiation with particles of negative energy in regions \( I_+^- \) and \( I_-^- \), and with particles with positive energy in regions \( I_+^+ \) and \( I_-^+ \). Thus, the overall balance of the energy being
created in the original regions would be exactly zero, with no correlation (information) whatsoever being exchanged between any of the different regions involved in the process. We note that, since \( r_\ast = G^{\frac{1}{2}} \), the temperature (6.9) becomes \((2\pi G^{\frac{1}{2}})^{-1}\), i.e. close to the Planck scale. Therefore, the emitted radiation is in all the cases formed by a single particle with nearly ± the Planck energy, which is uncorrelated to the region it comes from.

For \( k_3 = +1 \) we obtain similar hypersurface identifications as for \( k_3 = -1 \). In this case, the identification comes about in the situation resulting from simply exchanging the mutual positions of the original regions \( I^{k_2}_{k_1} \) and \( II^{k_2}_{k_1} \) for, respectively, the new regions \( III^{k_2}_{k_1} \) and \( IV^{k_2}_{k_1} \), on the coordinate patches given in Fig. 4, while keeping the sign of coordinates \( U, V \) unchanged with respect to those in (4.6) and (4.7); i.e. the points \((\tilde{t} + ik_1k_2\pi r_\ast, r, z, \phi)\) on the so-modified regions are the points on the original regions \( II^{k_2}_{k_1} \), on the same patches, again obtained by reflecting in the origins of the respective \( U, V \) planes, while keeping metric (4.12) and the physical time \( t \) real and unchanged.

In the so generated new set of coordinate patches one can now distinguish a new couple of CTCs, respectively starting at the poles \( South^\pm \) of the new regions. These additional CTCs would exactly follow the same itineraries as the CTCs considered in section 5 (see Fig. 4), but now looping back through original physical regions, to finally end at their respective starting points on the new regions.

Expressions for the relation between probabilities of absorption and emission for \( k_3 = +1 \) are obtained by simply replacing regions \( II^{k_2}_{k_1} \) for regions \( III^{k_2}_{k_1} \) (or regions \( I^{k_2}_{k_1} \) for regions \( IV^{k_2}_{k_1} \)), and energy \( E \) for \(-E\), in (6.8) and (6.10), keeping the same radiation temperature (6.9) in all the cases. Thus, we obtain that observers on the extreme hyperbolae will detect an isotropic thermal bath of particles with energy: (i) \( E < 0 \) in the exterior new region \( III^+_\pm \) and in the interior new region \( III^-_\pm \); and (ii) \( E > 0 \) in the interior new region \( III^-_\pm \) and in the exterior new region \( III^+_\pm \), with the particle energies being sign-reversed to these for radiation in the corresponding new regions \( IV^{k_2}_{k_1} \), in all the cases at an equilibrium temperature (6.9). Thus, the overall energy created in the new regions also vanishes.

Combining all the above results it follows that there exists a close connection between the CTCs and the thermal processes induced by the presence of the event horizon of an extreme string. Along the entire CTC itinerary starting at \( North^+_\pm \), all the possible observers on the extreme hyperbolae of exterior and interior (original and new) regions would always detect the CTC by the presence of a totally incoherent radiation with positive energy at the Planck temperature. No information whatsoever could therefore be trans-
ferred from the CTC to such observers, or *vice versa*, so spacetime causality could never be violated. The situation for the second of the CTCs considered in section 5 is essentially the same, the only difference being that the CTC starting at *North* is always manifested by a thermal radiation with negative Planck energy. For CTCs starting at new regions one basically achieves identical conclusions, with the CTC starting at *South* giving always rise to a thermal bath with negative Planck energy, and the CTC starting at *South* giving always rise to a thermal bath with positive Planck energy, both at equilibrium temperature (6.9) as well.

If no inflationary expansion occurred, temperature (6.9) and radiated particle energy would always reach a value close to the Planck scale. One would however expect the inflationary process discussed in section 4 to take unavoidably place as soon as an extreme string is formed. In this case, CTCs would be associated to thermal processes quite less energetic.

In summary, the treatment carried out in the present section seems to indicate that there is no quantum obstruction preventing the existence of CTCs in possible cosmological models based on minimally inflating extreme strings, in this sense contradicting the Hawking’s chronology protection conjecture [35]. What the present model really prevents is the possibility that any physically realistic observer may get any experimental direct proof about the existence of such curves. Or, re-paraphrasing Stephen Hawking [35], there could perfectly be hords of tourists visiting us from the future, but neither they nor we could know anything about their trip. For them it would be a touring which costs a lot and rewards nothing.

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References

1 T.W.B. Kibble, J. Phys. A9, 1387 (1976); A. Vilenkin, Phys. Rep. 121, 236 (1985).

2 A. Vilenkin, in 300 Years of Gravitation, eds S.W. Hawking and W. Israel (Cambridge Univ. Press, Cambridge, 1987).

3 A. Vilenkin, Phys. Rev. Lett. 46, 1169 (1981); Phys. Rev. D24, 2082 (1981); T.W.B. Kibble and N. Turok, Phys. Lett. 116B, 141 (1982).

4 A.D. Linde and D.A. Linde, Phys. Rev. D50, 2456 (1994).

5 A. Vilenkin, Topological Inflation, gr-qc/940240, preprint (1994).

6 A. Vilenkin and E.P.S. Shellard, Cosmic Strings and Other Topological Defects (Cambridge Univ. Press, Cambridge, 1994).

7 A. Vilenkin, Phys. Rev. D23, 852 (1981).

8 P. Laguna and D. Garfinkle, Phys. Rev. D40, 1011 (1989).

9 M.E. Ortiz, Phys. Rev. D43, 2521 (1991).

10 W.A. Hiscock, Phys. Rev. D31, 3288 (1985).

11 J.R. Gott, Astrophys. J. 288, 422 (1985).

12 D. Finkelstein and C.W. Misner, Ann. Phys. (N.Y.) 6, 230 (1959).

13 A.D. Linde, Inflation and Quantum Cosmology (Academic Press, Boston, 1990).

14 C.W. Misner and A. Taub, Soviet Phys. JETP 28, 122 (1969).

15 S. Deser and A.R. Steif, in Directions in General Relativity, eds. B.L. Hu, M.P. Ryan Jr., and C.V. Vishveshwara (Cambridge Univ. Press, Cambridge, 1993), Vol. 1.

16 D. Kramer, H. Stephani, M. MacCallum and E. Herlt, Exact Solutions of Einstein’s Field Equations (Cambridge Univ. Press, Cambridge, 1980).

17 H.B. Nielsen and P. Olesen, Nucl. Phys. B61, 45 (1973).

18 D. Finkelstein and G. McCollum, J. Math. Phys. 16, 2250 (1975).

19 K.A. Dunn, T.A. Harriott and J.G. Williams, J. Math. Phys. 35, 4145 (1994).
Note that the first term of the rhs in Eqn. (7) of Ref. [19] contains the sign ambiguity here denoted as $k_2$. This is incorrect and possibly due to a typing error.

M.D. Kruskal, Phys. Rev. 119, 1743 (1981).

A.H. Guth, Phys. Rev. D23, 347 (1981).

The connection between kinks and topology change has already been suggested by G.W. Gibbons and S.W. Hawking, Phys. Rev. Lett. 69, 1719 (1992). Criticisms to this idea were however raised by A. Chamblin and R. Penrose, Twistor Newsletter 34, 13 (1992).

K. Gdel, Rev. Mod. Phys. 21, 447 (1949).

V.P. Frolov and I.D. Novikov, Phys. Rev. D42, 1057 (1990).

M.S. Morris, K.S. Thorne and U. Yurtsever, Phys. Rev. Lett. 61, 1446 (1988); M.S. Morris and K.S. Thorne, Am. J. Phys. 56, 395 (1988).

S.W. Hawking and G.F.R. Ellis, *The Large Scale Structure of Space-Time* (Cambridge Univ. Press, Cambridge, 1973).

J.R. Gott, Phys. Rev. Lett. 66, 1126 (1991).

S. Deser, R. Jackiw and G. ’t Hooft, Phys. Rev. Lett. 68, 267 (1992).

R.M. Wald, *General Relativity* (The University of Chicago Press, Chicago, 1984).

J.B. Hartle and S.W. Hawking, Phys. Rev. D31, 2188 (1976).

S.W. Hawking, in *General Relativity. An Einstein Centenary Survey* (Cambridge Univ. Press, Cambridge, 1979).

N. Pauchapakesan, in *Highlights in Gravitation and Cosmology*, eds B.R. Iyer, A. Kembhavi, J.V. Narlikar and C.V. Vishveshwara (Cambridge Univ. Press, Cambridge, 1988).

G.W. Gibbons and S.W. Hawking, Phys. Rev. D15, 2738 (1977).

S.W. Hawking, Phys. Rev. D46, 603 (1992).
Legends for Figures.

• Fig. 1. Identifications \(\circ\) on the orientable, complex spherical structure of an extreme string, and mapping from that structure into that of the corresponding one-kink. These identifications and mapping lead to quantization of supermassive cosmic strings.

• Fig. 2. Separate one-kink lightcone configurations for the extreme string. Also represented are some geodesics passing through the different regions \(I_{k1}^{k2}\) and \(II_{k1}^{k2}\).

• Fig. 3. Pictorial representation of the one-kink lightcone configuration connecting all the four possible spacetime regions \(I_{k1}^{k2}\) and \(II_{k1}^{k2}\) of the extreme string.

• Fig. 4. The different coordinate regions of the one-kink extended extreme string metric. In the figure, \(A = 2^{\frac{1}{2}} r_*\), \(\tilde{t}_\alpha = -r_*(2^{\frac{1}{2}} + \ln(2^{\frac{1}{2}} - 1))\), and \(\tilde{t}_\beta = r_* \ln(2^{\frac{1}{2}} - 1)\). Each point on the diagrams represents an infinite cylinder. The null geodesics starting at \(North_+\) (solid line) and at \(North_-\) (broken line) are two CTCs. The solid curved lines at \(t = 0\) do not correspond to CTCs.

• Fig. 5. Directions of the lightcones along the CTC itineraries starting at (a) \(North_+\) and (b) \(North_-\), showing the continuous change of spherical symmetry into the symmetry of two hemispheres joined at their poles.

• Fig. 6. \(r - t\) diagrams for the two CTCs, starting at (a) \(North_+\) and (b) \(North_-\). The directions of the lightcones given in Fig. 4 are also represented. The transitions between hyperbolae on \(r = 0\) and \(r = 2^{\frac{1}{2}} r_*\), diagrammatically shown in the figure as straight lines, actually represent the set of all admissible quantum paths between \(x\) and \(x'\), at either of such hyperbolae, for the path integral that gives the propagator \(G(x, x')\) [31,34] (see section 6).
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