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Minimal Supersymmetric Standard Model Parameter Space Exclusion by Analyzing Metastable Scalar Vacuum Configurations

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Parameter Space Exclusion by Analyzing
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The Standard Model is an accurate theory of the fundamental interactions and constituents of nature. It is not however a complete theory. There are still many questions left unanswered about its construction. In this paper, I will examine the construction of the Standard Model and the construction of its supersymmetric extension, the MSSM. I will present the parameters that make up both theories and show that the introduction of Supersymmetry to the standard Model introduces a tremendous number of parameters. I will use the stability of the vacuum (or rather the false vacuum) to find restrictions on this parameter space. I succeeded in examining three different false vacua and found significant parameter space exclusion in two of them.
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Introduction:
Particle Physics and the Standard Model

“If one attempts to count the number of parameters introduced into the SM we arrive at:
- $SU(3)_C \times SU(2)_L \times U(1)_Y$ : 3 couplings and the Weinberg angle
- Higgs Sector : 6 parameters and 9 couplings”

Figure 1: Standard Model Particle Spectrum. Gauge quantum numbers are given as $\langle SU(3)_C \times SU(2)_L \times U(1)_Y \rangle$
Particle physics is the study of the Standard Model (SM). The SM is the minimal renormalizable quantum gauge theory that accounts for all particles and particle interaction up to energy scales of a few hundred GeV. It is a theory with amazing predictive powers and accuracy. The electroweak sector includes Quantum Electrodynamics, which is the most accurate and well-tested physical theory to date. The strong sector has been thoroughly tested in the High Energy regime, though it’s Low Energy behavior is less well understood. We have therefore a seemingly complete view of the particle spectrum and interactions that nature has chosen for us. As complete as the picture may seem, there are several gaps within the predictions and phenomenology of the SM. It should be made clear that by “Standard Model” we mean the model that the scientific community agrees upon and in no way should convey that it represents a complete or closed theory. Therefore, the gaps in such a well-established and well-tested theory inevitably lead to new insights and/or new physics.

My thesis is organized as follows: First, an obligatory review of the SM is presented in Chapters 2 and 3. With it, I hope to develop all the necessary notation and formalism to make this a self-contained study. After the review, problems within the SM will be addressed. Chapter 4 will present the Minimally Supersymmetric Standard Model (MSSM) as a necessary cure for deficiencies in the Higgs sector. Our work on excluding the extensive MSSM parameter space is then presented in Chapter 5.
2

Electroweak and QCD Sectors

It is the purpose of these next two chapters to present the fundamental tenets of the SM. Here, the Electro-Weak (EW) sector and the Quantum Chromodynamic (QCD) sector are presented. The EW theory represents the fusion of two once separate phenomena, Electromagnetism (EM) and the Weak force. The quantized version of EM is Quantum Electrodynamics (QED) and it will be presented first, with special attention given to it’s $U(1)_{EM}$ group structure. The Weak force is presented and we then discuss the high energy unification of both these theories into the EW group, $SU(2)_L \times U(1)_Y$. The color group, $SU(3)_C$, and the QCD Lagrangian are then presented. The breakdown of the EW and QCD groups down to EM, Weak, and QCD is left for the next chapter where we present the Higgs sector.

2.1 QED: $U(1)_{EM}$

QED is the quantum extension of the very successful Maxwellian formulation of EM. To this day, it remains one of the most precisely tested theories, with some measurements accurate to one part in a billion. It is also the most accurate and well understood of gauge theories, due in no small part to the
simplicity of the particle spectra and their interactions as well as it's gauge group $U(1)_{EM}$. By the late 1800’s, EM was well understood in terms of the charged particles (at this stage only electrons) that interacted with other charged objects by means of the vector field $A_\mu$. The quantisation of this theory reinterpreted the vector field as a gauge field with a particle interpretation, the photon. The gauge group is $U(1)_{EM}$ and thus the photon mediates a U(1) phase over different points in space-time. This is the modern picture of EM as a gauge theory, QED. Because of its simplicity, the study of QED serves as a perfect foundation to understand the gauge concepts of systems with more complex symmetries.

In order to construct a global U(1) invariant Lagrangian, we postulate a free Dirac field, $\psi$, which transforms under a global U(1) gauge transformation as

$$\psi \stackrel{\gamma}{\rightarrow} e^{iQ_\alpha} \psi, \quad dy \psi \rightarrow e^{iQ_a} dy \psi, \quad \tag{2.1.1}$$

where $Q$ is the generator of the theory. Notice that both the field and its derivative transform the same way under U(1), i.e. they transform covariantly. The Lagrangian for these fields is

$$L_{\text{dirac}} = \bar{\psi} (i \gamma_m \partial_m - m_e) \psi. \quad \tag{2.1.2}$$

Minimization of this Lagrangian leads the usual Dirac Equation. When the symmetry is promoted from global to local ($a \rightarrow a(x)$), the fields above do not transform covariantly.
\[ \psi \mathcal{R} e^{iQ_a(x)} \psi, \quad d_m \psi \mathcal{R} e^{iQ_a(x)} \{ d_m \psi + iQd_m a(x) \}. \quad (2.1.3) \]

As we cannot construct a gauge invariant Lagrangian with non-covariant components, we set to fix the non-covariance of the derivative term in a minimal fashion. We start with the addition of a massless gauge field \( A_\mu \) that transforms under U(1) as

\[ A_m \mathcal{R} A_\mu = iQ^{-1} d_m a(x), \quad (2.1.4) \]

then modify the derivative term into what is called the covariant derivative

\[ d_m \mathcal{R} D_m = d_m - iQ^{-1} A_\mu. \quad (2.1.5) \]

Now both the field and its covariant derivative transform covariantly. Finally, we make the new gauge field dynamical by the addition of a kinetic term

\[ L_{gauge} = -\frac{i}{4} F_{mn} F^{mn}, \quad (2.1.6) \]

where \( F_{mn} \) is the field strength tensor

\[ F_{mn} = d_n A_m - d_m A_n. \quad (2.1.7) \]

We now have all the components needed to construct the U(1) invariant QED Lagrangian with a fermionic matter and photonic gauge spectrum

\[ L_{QED} = L_{dirac} + L_{gauge} \]

\[ = \overline{\psi} (iD_m - m_e) \psi - \frac{i}{4} F_{mn} F^{mn}. \quad (2.1.8) \]
2.2 Weak Theory: SU(2)$_L$

The weak theory is the next step in the evolution of the SM. The first theoretical formulation of beta decay

\[ n \rightarrow p + e^- + \bar{\nu}_n \quad (2.2.1) \]

was due to Fermi in 1933\textsuperscript{2}. His theory assumed a parity conserving vector current form for both the lepton (e and $\bar{n}$ )

\[ J^l_m = g \, e \, g_m \, \bar{n} \quad (2.2.2) \]

and hadrons (at the time, the neutrons and protons)

\[ J^h_m = g \, p \, g_m \, n. \quad (2.2.3) \]

These currents come about due to Noether’s theorem as applied to a U(1) gauge invariance. Twenty four years later, this formulation was altered by Yang and Lee\textsuperscript{3} who suggested non-conservation of parity as the reason for the observed kaon decay into two opposite parity states. Experiments by Wu et. al.\textsuperscript{4} actually provided the first phenomenological evidence of the structure of the parity violation. They observed exclusively left-handed electrons and right-handed antineutrinos as a result of angular momentum conservation in the decay of Co $\rightarrow$ Ni. The non-observance of right-handed electrons or left-handed antineutrinos was a sure sign of parity violation and in fact, is a sign of maximal parity violation. This prompted Gell-Mann and Feynman\textsuperscript{5} in 1958 to postulate
the Vector-Axial (V-A) theory of weak interactions whereby the current form is modified to

\[ (J_m^l)_{V-A} = g \bar{P}_m(1 - g_5)P, \]  

(2.2.4)

where P is any lepton or hadron. Therefore only left-handed leptons or hadrons,

\[ P_{\text{Left}} = \frac{1}{2}(1 - g_5)P, \]  

(2.2.5)

participate in weak interactions. However, the Fermi and Gell-Mann/Feynman theories are not acceptable beyond the Born approximation because they are non-renormalizable and violate unitarity at high energies. This can be attributed to the local nature of the four fermion interactions. The solution to this problem is reminiscent of the solution to the non-covariance of the electron’s kinetic term: we introduce a vector boson to mediate the force. Note that unlike the case of QED, the boson(s) must be massive so that the range of the weak force is small as experimentally observed. This means that we need to introduce spin one vector bosons. These bosons came to be known as Intermediate Vector Bosons (IVB) and the form for the weak charged current was given in 1961 by Rosenbluth and Yang:

\[ L_{\text{CC}} = g \sqrt{2} \left( J_m W_m^{+} + J_m^{\ast} W_m^{-} \right), \]  

(2.2.6)

with \( g^2 = (M_W)^2 \) and \( W_m^{\pm} \) the IVB’s. Hardly free of problems, we immediately run into problems with the massive formulation of the vector boson.
By making the boson massive, we give it a longitudinal polarization in addition to the two transverse polarizations of a massless boson. In realistic reactions such as $e^+ e^- \rightarrow W^+ W^-$, the transverse modes are well behaved, but the longitudinal modes still violate unitarity, albeit at a higher energy ($\sqrt{s} \sim 500$ vs. $\sqrt{s} \sim 300$). A temporary cure comes from the addition of a neutral IVB, which introduces a neutral current to our Lagrangian

$$L_{NC} = (g \cos^{-1} q_W) J^m_{NC} Z^m,$$  \hspace{1cm} (2.2.7)

$$J^m_{NC} = g_L \bar{f} g_m f_L + g_R \bar{f} g_m f_R,$$  \hspace{1cm} (2.2.8)

where $f$ is any fermion and $Z^\mu$ the neutral IVB. The real cure to the problems of the theory comes when, as in QED, we gauge the theory. We have 3 points to consider when choosing the gauge group: 1) The fundamental action of the weak interaction is to induce transitions between groups of different charge; 2) The V-A form of this interaction means that only left-handed fermions are sensitive to the weak force; 3) We must incorporate the IVB’s into the gauge theory. Motivated by these points, we group the left-handed fermions into SU(2) obeying doublets

$$E = \frac{\bar{\psi} e L}{\psi n L}, \quad M = \frac{\bar{\psi} m L}{\psi n L}, \quad T_t = \frac{\bar{\psi} t L}{\psi n L},$$  \hspace{1cm} (2.2.9)

and the right-handed fermions into SU(2) singlets

$$e_R, \quad m_R, \quad t_R.$$  \hspace{1cm} (2.2.10)
The three IVB’s of the theory ($W_{m}^{\pm}$ and $Z_{m}^{0}$) are replaced by the gauge propagators of a SU(2) group, $W_{m}^{a}$ ($W_{m}^{l}, W_{m}^{2}, W_{m}^{3}$). The covariant derivative is now defined as

$$d_{m} \otimes D_{m} = d_{m} - ig_{2} \frac{t_{m}}{2} W_{m}^{a}, \quad a = 1, 2, 3, \quad (2.2.11)$$

with $t_{m}$ the usual pauli matrices and $g_{2}$ the SU(2)$_{L}$ coupling constant. This left acting SU(2)$_{L}$ group is known as the isospin group. Left handed fermions are assigned isospin $t = \frac{1}{2}$ (e-like fermions have $t_{3} = -\frac{1}{2}$; neutrinos have $t_{3} = +\frac{1}{2}$) and right handed fermions have isospin $t = 0$.

### 2.3 Electroweak Unification: SU(2)$_{L}$ x U(1)$_{Y}$

Immediately we can identify the charged IVB

$$W_{m}^{\pm} = \frac{1}{\sqrt{2}} (W_{1} m i W_{2}) \quad (2.3.1)$$

However, the remaining gauge field $W_{3}$ can be neither the photon nor the neutral $Z_{m}$ as it has vector-axial coupling in disagreement with data. Glashow in his 1961 paper introduced the neutral current in an attempt to unify the weak and EM force at high energy as SU(2)$_{L}$ x U(1)$_{EM}$. The straight combination of the EM group into a high energy structure fails, but Weinberg and Salam postulated
that the U(1) group not be EM, but a new group U(1)_Y with hypercharge

\[ Y = Q - \frac{g_1}{g_2} t_3^Q \]

and gauge boson \( B_m \). The additional field will modify the
covariant derivative as

\[ D_m = d_m - \frac{ig_2}{2} t_m W_m^a + ig_1 Y B_m, \quad (2.3.2) \]

with \( g_1 \) the U(1)_Y hypercharge coupling constant. The neutral IVB and the photon
are now mixtures of the neutral bosons of SU(2)_L and U(1)_Y

\[ A_m = - \sin q_W W_3 + \cos q_W B_m, \quad (2.3.3) \]

\[ Z_m = \cos q_W W_3 + \sin q_W B_m, \quad (2.3.4) \]

where \( q_W \) is the Weinberg angle, defined as \( \tan q_W = \frac{g_1}{g_2} \) with \( \sin^2 q_W \approx 0.23 \).

This theory is known as the Glashow-Salam-Weinberg theory (GSW). The GSW
is a SU(2)_L x U(1)_{EM} invariant gauge theory with a Lagrangian given by

\[ L_{GSW} = L_{CC} + L_{NC} + L_{\text{free}} + L_{EM}, \quad (2.3.5) \]

\[ L_{\text{free}} = - \frac{1}{4} \sum_{a=1,2,3} \epsilon^{abc} W_a W_{a,m} + B_m B^m + 2(y D y )_L, \quad (2.3.6) \]

\[ L_{EM} = g J_{\text{EM}}^a A_m. \quad (2.3.7) \]

The spectrum consists of fermionic matter \( y \) and gauge fields \( W_m^a \) and \( B_m \) which
are rotated into the four physical fields \( W^\pm, A_m, \) and \( Z_m \).
2.4 Strong Sector

The strong sector is the realm of the hadrons such as protons and neutrons. The astounding number of different hadrons in the particle spectrum can be replaced by a more fundamental structure, that of the quark model, which when coupled with the non-abelian SU(3)$_C$ gauge theory, gives us our modern understanding of hadrons. In this section, we present the strong sector and its SU(3)$_C$ gauge structure.

During most of the development of the EW IVB theory, the hadrons were considered as elementary particles. They had their own quantum number, strangeness, and all hadrons were classified into integer spin mesons and half integer spin baryons. There are literally hundreds of hadrons and motivated by their increasingly complex structure, Gell-Mann and Ne’eman in 1961 proposed a larger symmetry group than SU(2) for the hadrons. They noted that all mesons and baryons could be grouped into irreducible representations of SU(3)$_f$ where the $f$ stands for flavor, as this is a symmetry in the different flavors of the hadronic spectrum. Then in 1964, Gell-Mann and Zweig noted that the lowest dimensional representation, the triplet of SU(3)$_f$, was not occupied by any of the known hadrons. The enormous hadronic spectrum motivated them to propose that new spin $\frac{1}{2}$ particles named quarks filled this fundamental representation and that all hadrons were actually composite quark states. Under this model,
the mesons are a quark-antiquark pair, and thus have integer spin, while baryons are a three quark composite state and thus have half integer spin as required. The quarks of the time were an up quark "u", down quark "d", and strange quark "s" all of which, though postulated to exist, have never seen observed free in nature. Using these three quarks, the entire known hadronic spectrum could be reconstructed and many more theoretically predicted states where indeed experimentally confirmed. The “3” in SU(3) seemed to be motivated by the 3 known quarks at the time, yet it was the resolution of the $\mathbf{V^{++}}$ paradox that would explain natures choice of 3.

The paradox is centered on the $\mathbf{V^{++}}$ wave function

$$\left| V^{++} \right\rangle \sim |uuu\rangle,$$

which is totally symmetric in space, flavor, and spin. However, being a state with 3 identical fermions, the total state should be antisymmetric. Gell-Mann's solution was to assign quarks a new quantum number, color, and thus by assigning each quark a different color in the $\mathbf{V^{++}}$ (i.e. completely antisymmetrized in color space)

$$\left| V^{++} \right\rangle \sim |u_r u_g u_b\rangle,$$

where $r$, $g$, and $b$ refer to the color quantum number of the particle, we no longer have identical fermions and hence the wave function must be completely symmetric in space, flavor and spin, as required by the natural classification.
The theory is now SU(3)$_C$ where we now understand the “3” as referring to the number of colors. Since particles with non-trivial color quantum number have not been observed in nature, all known hadrons must therefore be in color singlet states of SU(3)$_C$: the meson must therefore be a quark-antiquark pair and the baryon made of three quarks of different colors. Further experiments have added 3 more quark flavors to the theory, charm “c”, top “t”, and bottom “b”. We now have all the elements to construct the modern QCD theory.

QCD starts by postulating the existence of spin ½ particles called quarks. Quarks are only observed as the constituents of mesons or baryons. They have isospin and hypercharge, but also obey color symmetry SU(3)$_C$. Every hadron is in the color singlet representation of this group (i.e. colorless) while the quarks themselves are in the triplet representation (red, white, and blue or whatever colors your inner artist chooses). As color is an exact gauge symmetry, we need to include a massless dynamical gauge boson for each of the 8 generators of SU(3)$_C$. These vector bosons are known as gluons. Therefore, we have the following QCD spectrum:

\[
\begin{align*}
\text{Quarks and Antiquarks} & \quad q_i, \bar{q}_i : i = u, d, c, s, t, b \\
\text{Gluons} & \quad G^a_m : a = 1, \ldots, 8
\end{align*}
\]  

where each quark flavor is in a color triplet, \( q^i = (q^i_r, q^i_s, q^i_b) \), and thus the QCD Lagrangian becomes
with gluon field strength

\[ (G_{mn})^a = d_m G_n^a - d_n G_m^a + g_{s} f^{abc} G_m^b G_n^c. \]  

Under a SU(3)$_C$ transformation, the quark and it's covariant derivative transform as 

\[ e^{i \frac{q}{2} a} \lambda^a \], with $\lambda^a$ the Gell-Mann matrices, while the gluon transforms as 

\[ G_m^a \otimes G_m^a + g_3 \lambda^a \delta_{mn} + f^{abc} q^b G_m^c. \] 

Since hadrons, and therefore quarks, carry isospin and hypercharge, we expect them to obey the same gauge structure of the EW theory. Therefore, out of the 6 flavors of quarks we form 3 chiral SU(2) doublets and 6 chiral singlets

\[ U = \begin{pmatrix} u_R & d_R \\ \bar{c} & \bar{d}_L \end{pmatrix}, \quad S = \begin{pmatrix} s_R & c_R \\ \bar{s} & \bar{c}_L \end{pmatrix}, \quad T_t = \begin{pmatrix} t_R & b_R \\ \bar{t} & \bar{b}_L \end{pmatrix}. \] 

A wonderous structure emerges when we compare eqns. (2.2.9) and (2.2.10) with eqn. (2.4.7). The first set of leptons and quarks, E≡(e, n) and U≡(u, d), are known as the first generation and apart from getting more massive, the two remaining generations are identical! Thus the full matter content of the SM emerges as consisting of 3 generations multiplied by 2 families, the leptons and quarks. Experimental evidence suggests that 3 and only 3 generations exist.
with neutrinos lighter than \( \sim 40 \text{ GeV} \). It is singularly astounding that while there may be many more particles outside the reach of current experiments, all the particles we have predicted and observed with our theories and experiments are based on a simple template set at the low energy scale upon which we live.

The doublet and singlet structure imposed on the theory by the axial-vector nature of particles present us with an immediate problem: we cannot have explicit mass generations as mass terms of the form \( m_f f_L f_R \) {with \( f \) any fundamental fermion of our theory (leptons or quarks)} are expressly forbidden as they violate SU(2)_L \times U(1)_Y gauge invariance. We must find another mechanism by which to give our fermions their masses. We shall do this in the next chapter through the introduction of a complex scalar boson known as the Higgs field. In the process, the Higgs will break down our symmetry group from \( \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y \rightarrow \text{SU}(3)_C \times \text{U}(1)_{EM} \) and provide masses for our low energy IVBs as well.
At this stage in the theory’s construction, we have two unresolved questions:

1) If the high energy theory for EM and Weak interactions is indeed the EW theory, then what mechanism of symmetry breaking will allow us to recover the low energy theory of IVB’s and U(1)EM?

2) The chiral requirements of the weak interaction prevent us from explicit mass terms such as \( m^2 y_L \bar{y}_R \) since the left handed and right handed fermions transform differently under SU(2)_L.

Furthermore, since the SU(2)_L theory is exact, the gauge bosons are necessarily massless. How then can we obtain correct mass terms for fermions and bosons of the theory?

Both these questions were answered and the SM completed in its modern form by the introduction of the Higgs mechanism and with it the Higgs Sector. The Higgs mechanism allows not only the EW group to properly break down to its low energy limit, but provides the masses to all the fermions and gauge bosons as well. Let us look at global spontaneous symmetry breaking first and from it, develop the Higgs mechanism.
Symmetries

A fundamental principle in the SM is that of symmetries. In fact, the SM Lagrangian is constructed by observing the known particle spectrum, observing the symmetries they obey, assigning a group structure to the spectrum, and finally putting it all together in a renormalizable scheme. Symmetries serve as transformation groups under which the theory is invariant and as group representations by which particles are classified. In general, all the continuous symmetries in the SM will be finite dimensional compact semi-simple Lie groups. Continuous symmetries are classified as either space-time or internal. Space-time symmetries act directly on space-time. Examples include translations, rotations, and boosts. Internal symmetries act on the internal quantum numbers of a particle, in effect taking a particle into different particle state within the degenerate mass spectrum of the group. Internal symmetries are further classified based on their elements dependence on the space-time coordinates. If the elements do not depend on space-time, we say the symmetry is global or rigid. In this case, transformations occur instantaneously over all space and time. Much more relevant and interesting is the case when the elements of the symmetry do depend on the space-time coordinates. To mediate this dependence, we need to introduce bosonic fields known as gauge fields which act as generators of the symmetry group. For this reason, these are known as local or gauge symmetries.
Global symmetry breaking and the Goldstone theorem

It's important to consider how the symmetry groups behave at low energy, specifically how they act on the vacuum state of the theory. If we have a global continuous group denoted by G, it will have generators $g_i$ which obey the following commutation relation

$$[g_i, g_j] = i c_{ijk} g_k,$$  \hspace{1cm} (3.1.1)

with $c_{ijk}$ the group structure constants. If the generators of the theory annihilate the vacuum, ie

$$g_i |0\rangle = 0$$ \hspace{1cm} (3.1.2)

for all i, we have “i” number of degenerate set of ground states and the symmetry group is realized in a Wigner-Weyl manner. This will lead to a Fock space containing the usual particle states. If on the other hand, the generators do not annihilate the vacuum,

$$g_i |0\rangle \neq 0$$ \hspace{1cm} (3.1.3)

for some or all i, then the group is spontaneously broken down to a subgroup H and dim[G/H] massless scalars appear in the spectrum. This known as the Nambu-Goldstone realization of the symmetry and the massless scalars are known as the Nambu-Goldstone bosons. This process can be easily seen in the case of a complex scalar field

$$F = \frac{1}{\sqrt{2}}(f_1 + if_2)$$ \hspace{1cm} (3.1.4)
with the following Lagrangian

\[ L = \left( \frac{1}{m^2} F \right)^2 - m^2 F F^* - |F F^*|^2. \]  

(3.1.5)

When \( m^2 \geq 0 \), we can minimize the Hamiltonian such that the minimum of the theory is attained at \( F = 0 \). Note that both the Lagrangian and the vacuum of the theory are invariant under the global phase transformation \( F \xrightarrow{\varphi} e^{i\varphi} F \). However, if \( m^2 < 0 \), then the Hamiltonian is not minimal at \( F = 0 \), but rather at

\[ |F_0| = \frac{u}{\sqrt{2}}, \]  

(3.1.6)

where \( u \) is defined as the vacuum expectation value (vev) of the theory. The phase transformation transforms the vacuum into an inequivalent vacuum state and in order to construct a quantum theory, we have to fix the vacuum state unambiguously by, for example, choosing \( F_0 \) to be real. It is at this point that we have Spontaneous Symmetry Breaking (SSB) as the original Lagrangian remains symmetric, but the vacuum is itself not invariant under phase change. To study the SSB effects, we perturb the vacuum as

\[ F = \frac{1}{\sqrt{2}} (u + \phi_1 + i\phi_2), \]  

(3.1.7)

where \( \phi_1 \) and \( \phi_2 \) are small real fields fluctuating around the vev. The Lagrangian becomes
The SSB has changed a theory of a self-interacting complex scalar into a theory of two interacting real scalars. We see directly from the form of the Lagrangian that one of the scalars has mass $\sqrt{2}hu$, while the other scalar is massless. The appearance of this massless mode is a general and distinguishing feature of SSB models and is known as the Goldstone theorem: *When a global symmetry is broken, there will appear as many massless scalar modes (Goldstone bosons) as there are broken symmetries.* In our example above, we broke one symmetry by our choice of vacuum and hence obtain the one massless mode.

**(Local symmetry breaking and the Higgs mechanism)**

We mentioned above that promoting a global symmetry to a local symmetry requires the addition of a gauge boson to communicate (gauge) this symmetry to different points is space-time. If the symmetry is exact, then this gauge boson will necessarily be massless. In this local theory, we can still have SSB and Goldstone's theorem still applies. However, when we build up our theory form the vacuum state (by considering small perturbations to the vacuum), the goldstone bosons can be gauge transformed into longitudinal components of gauge bosons. Since gauge bosons with longitudinal components have three
polarizations and are thus massive, as many massless gauge bosons acquire mass as there would have been massless Goldstone Bosons; this is known as the Higgs mechanism. The minimal choice for the Higgs mechanism in a SU(2)XU(1) group is to introduce a charged scalar doublet

\[
F = \begin{pmatrix} f_1^+ + i f_2^0 \\ f_3^- - i f_4^0 \end{pmatrix}
\]

subject to the following potential

\[
V(F) = - m^2 F^\dagger F + |F^\dagger F|^2.
\]

When \( m^2 \geq 0 \), minimizing the Lagrangian to obtain the vacuum gives us a minimum at \( <F> = 0 \). This is the symmetric, non-broken theory. However, when \( m^2 < 0 \), the vacuum minima can be chosen as

\[
\langle F \rangle = \begin{pmatrix} \infty & 0 & 0 \\ 0 & u & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}
\]

where \( u \) is real and \( u = (2\sqrt{2}G_F)^{\frac{1}{2}} = 246 \text{ GeV} \). We now have one neutral scalar mode with a vev and three remaining massless modes. We will identify these three massless Higgs modes as the three massless Goldstone Bosons resulting from the breaking of SU(2)_L \times U(1)_Y (with generators \( t_1, t_2, t_3 \) and \( Y \)) down to U(1)_EM (with generator Q). When the symmetry is broken, these three massless modes will give masses to 3 gauge bosons. One boson, the photon,
will remain massless because of the remaining exact $U(1)_{\text{EM}}$ symmetry.

Counting up polarization states:

**Before SSB**
- 4 massless gauge bosons (x 2 polarizations): $W_m^a, B^m$
- 4 massless scalars: $f_1, f_2, f_3, f_4$

Total number of degrees of freedom: 12

**After SSB**
- 3 massive gauge bosons (3 pol.) : $W^\pm, Z$
- 1 massless gauge boson (2 pol) : $g$
- 1 massive scalar : $H$

Total number of degrees of freedom: 12

Note that we have preserved the total number of degrees of freedom by giving the low energy theory gauge bosons masses.

Let’s now show the Higg’s Mechanism explicitly at work in the SM.

Starting with the Higgs Sector Lagrangian,

$$L_H = |D_m F|^2 - V(F),$$  \hspace{1cm} (3.1.12)

the symmetry is broken by assigning $\Phi$ it’s non-symmetric vev, $F_0 = \langle F \rangle = \frac{u}{\sqrt{2}}$.

The $W$ and $Z$ bosons gain mass from the covariant derivative evaluated at $F_0$.
where $a = b (\cos^2 q_W - \sin^2 q_W )$ and $b = (- 2 \sin q_W \cos q_W )^{-1}$.

We can study the remaining neutral Higgs mode by performing small oscillations about the vacuum

$$F \phi = \begin{pmatrix} 0 & 0 \\ \frac{1}{\sqrt{2}} (n + H(x)) & \phi \end{pmatrix}$$  \hspace{1cm} (3.1.14)

where $H(x)$ is the physical Higgs field. The potential in terms of these small oscillations is

$$V(F \phi) = 2l^2 n^2 H^2 = M_{H}^2 H^2$$  \hspace{1cm} (3.1.15)

and thus we obtain the mass of the leftover physical Higgs particle. All that remains is to assign masses to the fermions; the Higgs Mechanism will again do this for us through symmetry breaking. We introduce the Yukawa Lagrangian that couples the Higgs to leptons

$$L_{yukawa} = L_{YW} = \lambda \begin{pmatrix} e_L E_L F e_R + h.c. \end{pmatrix}$$  \hspace{1cm} (3.1.16)

and the quarks
\[ L_{YW\text{ QCD}} = \hat{\mathbf{a}}_{u,s,t} \bar{Q} d U_L F d_R + l \bar{u}_L (i t^a F^a) u_R + h.c \hat{\mathbf{U}} \quad (3.1.17) \]

After SSB, the Yukawa terms provide masses for the fermions \( f \). This is formally accomplished by the gauge transformation that transforms away the unphysical Goldstone modes. In the Kibble parametrization, this transformation is

\[ F = e^{i x_a t_a} \begin{pmatrix} 0 & \sigma^a \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 \end{pmatrix} \]

\[ F \psi = U(x) F, \quad U(x) = e^{-i x_a t_a} \quad (3.1.18) \]

with \( x_a \) the Goldstone modes, \( a=1,2,3 \). This gauge transformation has the effect of transforming the fermions to their physical state.

\[ L_{YW} = l \frac{n}{\sqrt{2}} f'_L f'_R = m_f f'_L f'_R, \quad (3.1.19) \]

where the prime indicates a left-handed doublet rotated by the action of the gauge transformation. After the transformation, the neutral quark and charged leptonic current remain flavor symmetric; for example

\[ L_{\text{neutral}} = \frac{u}{\sqrt{2}} \left( \bar{u}_L i D_L u_L + \bar{u}_R i D_R u_R + \text{d terms} \right) \]

\[ = \left( \bar{u} \sigma^a i D_L u \sigma^a + \bar{u} \sigma^a D_R u \sigma^a + \text{h.c.} \right) + d \psi \text{ term} \quad (3.1.20) \]

The charged quark current does not remain symmetric and thus we have transitions between up-type quarks and down-type quarks after SSB.
\[ L_{\text{charged}} = g u_L g_n d_L W_m^- + \text{h.c.} \]
\[ = g u_L g_n (S_u S_d) d_L W_m^- + \text{h.c.} \]
\[ = g u_L g_n (U_{ud}) d_L W_m^- + \text{h.c.} \]  (3.1.21)

\( U_{ud} \) is the Cabbibo-Kobayashi-Maskawa (CKM) matrix. It is a 3x3 unitary matrix and hence has \( 3^2 = 9 \) parameters. Five of these parameters can be absorbed as redefinitions of the phases of the quark fields. Under the Wolfenstein parametrization\(^1\), the CKM matrix becomes

\[
U_{ud} = \begin{pmatrix}
1 - \frac{l^2}{2} & l & A l^3 (r - i h) \\
- l & 1 - \frac{l^2}{2} & A l^2 \\
A l^3 (1 - r - i h) & A l^2 & 1
\end{pmatrix}
\]  (3.1.22)

This parametrization makes it very clear that the terms are strongest on the diagonal and weaker as we go off diagonal. Experimentally\(^1\): \( l \approx .22, A \approx .82, \sqrt{r^2 + h^2} \approx .4 \), and \( h \approx .3 \). The phase factor \( h \) is responsible for Charge Parity (CP) violation in the SM and its non-vanishing value means that CP violation in the SM is milliweak.
In conclusion, the addition of the Higgs Sector completes the SM. With it, we have a mechanism by which the proper high energy to low energy symmetry breaking are achieved and by which all masses in our model can be generated. All is not well however. As we shall see in the next section, while the Higgs sector effectively completes the theory, it too introduces a host of problems. When we start to apply loop corrections to the Higg’s mass, several disconcerting problems crop up. However, these problems ultimately open a window on new physics beyond our current experimental reach. In the next chapter, we examine the resolution to these problems and the new physics that come about as a result.
### Building the MSSM

“In a renormalizable supersymmetric field theory, the interaction and masses of all particles are determined just by the gauge transformation properties $T_{ab}$ and by the Superpotential $W_i$.”

| Chiral Supermultiplets | Quarks, Squarks | Leptons, Sleptons | Higgs, Higgsino |
|------------------------|-----------------|------------------|-----------------|
| $Q_L = \frac{2}{3}U_L + \frac{1}{3}D_L$ | $\langle \begin{array}{c} 3 \bar{A} 2 \bar{A} \\ 3 \bar{A} 1 \bar{A} - \frac{2}{3} \\ 3 \bar{A} 1 \bar{A} \frac{1}{3} \end{array} \rangle$ | $L_L = \frac{1}{2}E_L + \frac{1}{2}N_L$ | $\langle \begin{array}{c} 1 \bar{A} 2 \bar{A} - \frac{1}{2} \\ 1 \bar{A} 1 \bar{A} 1 \end{array} \rangle$ |
| $U_R$ | $\langle \begin{array}{c} 3 \bar{A} 2 \bar{A} \\ 3 \bar{A} 1 \bar{A} - \frac{2}{3} \\ 3 \bar{A} 1 \bar{A} \frac{1}{3} \end{array} \rangle$ | $E_R$ | $\langle \begin{array}{c} 1 \bar{A} 2 \bar{A} - \frac{1}{2} \\ 1 \bar{A} 1 \bar{A} 1 \end{array} \rangle$ |
| $D_R$ | $\langle \begin{array}{c} 3 \bar{A} 2 \bar{A} \\ 3 \bar{A} 1 \bar{A} - \frac{2}{3} \\ 3 \bar{A} 1 \bar{A} \frac{1}{3} \end{array} \rangle$ | |

### Gauge Supermultiplets

| | Gluon, Gluino | $G$ | $\langle \begin{array}{c} 8 \bar{A} 1 \bar{A} 0 \end{array} \rangle$ |
| | W boson, Wino | $W$ | $\langle \begin{array}{c} 1 \bar{A} 3 \bar{A} 0 \end{array} \rangle$ |
| | B boson, Bino | $B$ | $\langle \begin{array}{c} 1 \bar{A} 1 \bar{A} 0 \end{array} \rangle$ |

Figure 2: Chiral and Gauge MSSM Supermultiplet particle spectrum.
In constructing the SM, an interesting pattern of discovery emerges. Models are built, are successful for a time, and then fail due to problems. Fixing the new problems involved either adding a new particle or a new interaction. Hence, the model grows increasingly complex as more and more holes in the theory are plugged up. Ultimately we reach the stage of the modern SM where we seem to have completely accounted for all the low energy particles and interactions with only 19 free parameters. As we shall see in this chapter, we again encounter difficulties in the predictions of the theory, this time dealing with the quantum corrections of the Higgs mass. We shall start by reviewing how the mass renormalization of Higgs particles leads to problems. A solution to this problem, Supersymmetry (Susy), is then presented and predictably enough, it involves adding a host of new particles to the spectrum leading to an exact resolution of the Higgs problem to all orders of perturbation theory. The minimal addition of Susy to the SM is then presented and is known as the Minimally Supersymmetric Standard Model, the MSSM.
4.1 Problems with Higgs Mass Renormalization

The inclusion of quantum mechanics into the SM has the effect of introducing virtual particle production and these in turn lead to loop diagrams that modify the calculation of physical quantities. These loops can be treated as quantum perturbations upon a classical system. Certain loop integrals will be divergent and therefore the calculations of observables that depend on these loops lead to unphysical infinite result. As this is obviously a problem if we wish to have a model that makes sensible predictions, some way must be found to get rid of the infinities that plague the theory. Renormalization is the idea that we take these divergences as modifications to the free parameters in our model. The classical, tree level parameter is known as the bare parameter. Loop diagrams then modify this bare parameter in a calculable, albeit infinite, manner. The key is that we can then adjust the bare parameter (as a free parameter of our theory) to give the correct observed value and thus “absorb” the infinity. The heart of renormalization is that what we observe in nature is actually the renormalized parameter, given by the bare parameter plus the infinite contribution from loops, and not the bare parameter of the tree level theory. To see renormalization in action, we will first work out the fermion mass renormalization to illustrate a successful use of renormalization. Then we work out the Higgs mass renormalization by a fermion loop and run into a quadratically
divergent correction. We will then add the contributions from other scalar loops and see a way to resolve this (and other) problems.

**Fermion mass renormalization due to Higgs loop**

When we wish to calculate the fermionic self-energy in our present theory, virtual fermionic interactions with the Higgs have to be taken into account. That is to say, we have to consider the following one loop diagram:

![Figure 3: Fermion mass renormalization by a Higgs loop correction](image)

From the following Lagrangian

\[
L_y = i \gamma \cdot \overline{\psi} m \gamma + (\overline{\psi} m H)^2 - m_H^2 |H|^2 - \frac{l F}{2} (\overline{\psi} \gamma \cdot H + h.c.),
\]

we calculate the self-energy of the fermion arising from the scalar loop insertion

\[
- \mathcal{S}(p) = - (i)^2 \frac{g_y F}{\sqrt{2}} \frac{\gamma \cdot k}{\partial} \partial \frac{d^4k}{(2\pi)^d} \left[ k^2 - m_F^2 \right] \left[ (k - p)^2 - m_H^2 \right].
\]

The renormalized fermion mass will be 

\[(m_F^2)_{\text{renorm}} = m_F^2 + d m_F^2\]

with the delta correction coming form the fermion self energy calculated on-shell.
\[ dm_F = S ( p = m_F ) = \]
\[
\frac{i l_F^2}{32 p^2} \int_0^1 dx \hat{\partial} d^4 k \hat{\partial} \frac{m_F (1 + x)}{[k \hat{\partial} - (m_F x)^2 - m^2_F (1 - x)]^2} \]  
(4.1.3)

We shall use a momentum space cutoff to calculate this integral. First, we perform a wick rotation into Euclidian space

\[
k_0 \otimes i k_4,
\]
\[
d^4 k \hat{\partial} \otimes i d^4 k_{E},
\]
\[
k \hat{\partial} \otimes k^2_{E}.
\]

The integral now depends solely on \( k_E \). As it has a symmetric integrand, we can calculate the integral as

\[
\hat{\partial} \int_{-\infty}^{\infty} d^4 k_{E} f(k_{E}^2) = p^2 \int_{0}^{L^2} dy f(y),
\]
(4.1.5)

where \( \Lambda \) represents a high momentum cutoff at the scale of a more fundamental high energy theory (Unification: \( M_x \approx 10^{15} \text{ GeV} \), Planck: \( M_{pl} \approx 10^{18} \text{ GeV} \), etc).

Evaluating the self-energy integral, we obtain

\[
dm_F^2 = \frac{-3 l_F^2 m_F^2}{(8 p)^2} \log \frac{\Lambda^2 \hat{\partial} \otimes \hat{\partial}}{m_F^2 \hat{\partial} \otimes \hat{\partial}} + \ldots + O(L^{-2}).
\]
(4.1.6)

This is a well-defined expansion for the fermion mass as we can redefine our renormalized mass as

\[
m^2_F \text{renorm} = m^2_F \hat{\partial} \otimes \hat{\partial} - \frac{3 l_F^2}{(8 p)^2} \log \frac{\Lambda^2 \hat{\partial} \otimes \hat{\partial}}{m^2_F \hat{\partial} \otimes \hat{\partial}}
\]
(4.1.7)
Keep in mind that since $L \not\in Y$, we must adjust the bare mass such that $m_F^2 \not\in \log(...)^{-1}$ in order for the infinite divergence to cancel out. Fermion masses are thus labeled (by 't Hooft\textsuperscript{16}) as natural since $(m_F^2)_{\text{renorm}} \not\in 0$ as $m_F^2 \not\in 0$ (restoring chiral symmetry in the process) and because the leading order correction is $\sim \log \frac{L}{m_F \phi}$. Note that had we done this by dimensional regularization, we would still obtain an infinity term proportional to $(n-4)^{-1}$, where $n$ is the number of dimensions, which must go to 4 at the end of the calculations. As above, we still can renormalize the fermion mass by tweaking the bare mass to take this infinity into account.

**Higgs mass renormalization due to Fermion loop**

Let us now do the same with a fermion loop renormalizing the Higgs mass. The diagram of interest is

![Diagram of Higgs mass renormalization by a Fermionic loop](image)

Figure 4: Higgs mass renormalization by a Fermionic loop
The Higgs self energy for this process is given as

\[- S (p^2) = \frac{\alpha}{\sqrt{2}} \frac{\vec{F} \cdot \vec{\phi}}{\phi^2} (i)^2 (-1) \frac{d^4k}{(2p)^2} \frac{\text{Tr}\{k + m_F\}[(k - p) + m_F]}{(k^2 - m_F^2)[(k - p)^2 - m_F^2]} .\] (4.1.8)

Again integrating with a momentum space cutoff $\Lambda$, we find the renormalized mass $(m_{H}^2)_{\text{renorm}} = m_{H}^2 + dm_{H}^2$ with

\[dm_{H}^2 = - \frac{l_{F}^2}{8p^2} \frac{L}{\phi} (m_{H}^2 - 6m_F^2) \log \frac{\phi}{m_F} + \ldots + O(L^{-1}) \] (4.1.9)

Hence, to leading order, the Higgs mass is renormalized as

\[(m_{H}^2)_{\text{renorm}} = m_{H}^2 - \frac{l_{F}^2}{8p^2} \frac{L}{\phi} (m_{H}^2 - 6m_F^2) \log \frac{\phi}{m_F} .\] (4.1.10)

Like the fermionic renormalization, this is again divergent. However, the strongest divergence is quadratic. In order to obtain a proper Higgs mass, we could just add a counterterm of $O(\Lambda^{-2})$ to cancel this highly divergent term.

However there is no symmetry responsible for this cancellation, hence we would have to add a counter-term at each order of perturbation, each time adjusted to a precision of $\sim 10^{15}$ in order stabilize the divergences. This very unattractive prospect is known the Fine-Tuning problem. Furthermore, as the Higgs mass seems to be driven to the cutoff scale due to loop corrections, we question the lack of any new physics between the EW and cutoff scales. This is referred to as the Hierarchy problem. Before we get too carried away with our problems, let's proceed and renormalize the Higgs with a scalar loop.
**Higgs mass renormalization due to scalar loop**

The loop diagram that interests us in this case is Fig. 5 with Lagrangian

\[ L = L_{yH} + |\psi_{nf}|^2 + l_S f \bar{H} H. \]  

(4.1.11)

This leads to the following contribution to the renormalized mass

\[ dm_H^2 = \frac{l_S}{(4p)^2}(2L^2 + \ldots + O(L^{-1})). \]  

(4.1.12)

![Figure 5: Higgs mass renormalization by a Scalar loop](image)

Let’s now add up both the fermionic and scalar leading order contributions to the Higgs mass

\[ (m_H^2)_{\text{renorm}} = \frac{(l_S - l_{\tilde{F}})}{8p^2}L^2 + (m_{H_1}^2 - 6m_{\tilde{F}}^2)\log\frac{L}{m_{\tilde{F}}} + O(L^{-1}). \]  

(4.1.13)

Immediately we see that if

\[ l_S = l_{\tilde{F}}, \]  

(4.1.14)

then the quadratic divergences are automatically canceled leaving us with the more manageable logarithmic divergence. Furthermore, if we can actually...
attribute this equality to a symmetry, than the quadratic divergence will be canceled exactly to all orders of perturbation. This is exactly what Susy does!

4.2 Building a General SUSY Theory

Following the age-old precedent set before us in building the SM, we shall solve our problems by adding more particles to the spectrum. And taking our cue from before, we shall let a symmetry guide us. In fact, it will be no ordinary symmetry but a Supersymmetry! Susy will relate to every fermion (boson) in our present SM spectrum a bosonic (fermionic) partner. The operator responsible for this action will be an anti-commuting Dirac $1/2$ spinor $Q_i$

$$Q_i |boson\rangle = |fermion\rangle, \quad Q_i |fermion\rangle = |boson\rangle, \quad (4.2.1)$$

which obey the following commutation relations

$$\{Q_i, Q_j^\dagger\} = 2\delta^{ij} P_m, \quad (4.2.2)$$

$$\{Q_i, Q_j\} = \{Q_i^\dagger, Q_j^\dagger\} = 0, \quad (4.2.3)$$

$$[Q_i, P_m] = \delta^{ij} P_m Q^\dagger_j = 0, \quad (4.2.4)$$

with $P_\mu$ the space-time translation generator (Hamiltonian and momentum operator) and slash notation is with respect to the pauli matrices, $\tau_\mu$. This is a N=1 Susy model since we only have one set of Susy operators. One can in fact
try extended Susy with N>1, but these lead to theories that are phenomenologically unacceptable and hence won’t be considered further. It is obvious that $Q_i$ and $Q^\dagger_i$ commute with $-P^2_m = m^2$ and with the generators of the gauge transformation. Therefore, these partners will share all but their spin quantum numbers, including mass, isospin, hypercharge, etc. And since the partners share the same coupling constant, the scalar partner’s quadratic divergence will neatly cancel with the fermionic partner’s quadratic divergence in the Higgs renormalization mass in accordance to eqn. (4.1.14) hence solving our problems. In constructing a Susy theory, we start by noting that the one particle irreducible states naturally fall into what are known as Supermultiplets that contain both the bosonic and fermionic states. The Supermultiplet must contain exactly the same number of bosonic and fermionic degrees of freedom,

$$n_B = n_F,$$  \hspace{1cm} (4.2.5)

and all members of the Supermultiplet are related by supersymmetry transformations such that they share the same mass and quantum numbers, differing only by $\frac{1}{2}$ spin. The two Supermultiplets of interest are the chiral and gauge Supermultiplet, which I present below.
4.2.1 Susy Chiral and Gauge Free Supermultiplets

**Chiral multiplet**

The simplest chiral multiplet consists of a Weyl fermion, \( y \), of spin \( \frac{1}{2} \) and dimension 3/2 and a complex scalar field, \( A \), with spin 0 and dimension 1. These fields close the Susy algebra on-shell but not off-shell. In order to match \( n_B \) and \( n_F \) off-shell, we have to introduce an auxiliary complex scalar, \( F \), of spin 0 and dimension 2 which provides the missing degrees of freedom to close Susy off-shell. Hence, the chiral Supermultiplet contains

\[
C^0 (y, A, F),
\]

where \( C \) has four bosonic and fermions off-shell degrees of freedom as required by eqn. (4.2.5). A Susy transformation can be generated by an operator of the form

\[
S = yQ + y^\dagger Q^\dagger,
\]

where \( y \) and \( y^\dagger \) are constant anti-commuting Grassman numbers which parameterize the Susy transformation and spinor indices have been omitted. This operator acts on the fields as

\[
S y = \sqrt{2} (yF + iy^\dagger d_m A),
\]

\[
S A = \sqrt{2} yy^\dagger,
\]

\[
S F = i\sqrt{2} \partial_m y^i,
\]
with an anti-slash notation with respect to $\bar{t}_m \circ (t_0, -t_{1,2,3})$. A free, renormalizable chiral Susy Lagrangian (the Weiss-Zumino model) is given by

$$L(y, A, F) = -\bar{y}m_{\mu\nu}y - m A^{\mu\nu}A + F\bar{F} + m(AF - \frac{1}{2}yy + h.c.) + Y(A^2F - Ayy + h.c.).$$

(4.2.11)

with $m$ and $Y$ real parameters. Note that the F-term is non-dynamical which is expected as it was only introduced as an auxiliary field to make $n_b = n_f$ off-shell.

**Gauge multiplet**

The gauge multiplet consists of a gauge boson, $c$, of spin 1 and dimension 1, a Weyl fermion, $g$, of spin $\frac{1}{2}$ and dimension 3/2 and an auxiliary real scalar field, $D$, of spin 0 and dimension 2

$$V \circ (c, g, D).$$

(4.2.12)

D is introduced to close the algebra off shell as $F$ does for the chiral multiplet. Like the chiral multiplet, the Gauge multiplet has 4 fermionic and bosonic degrees of freedom. The supersymmetry operators transform the fields as

$$S c = -i\bar{g}g + i\bar{c}g,$$

(4.2.13)

$$S g = ixD + xt_{mn}F^{mn},$$

(4.2.14)

$$S D = -x\bar{F}_m\bar{g} - D_mg\bar{c},$$

(4.2.15)

with $4t_{mn} = t_m\bar{t}_n - t_n\bar{t}_m$. The free, renormalizable gauge Susy Lagrangian is
\[ L(c, g, D) = - \frac{1}{4} F_{mn}^a F^{a, mn} - i g^a \bar{D}_m g^a + \frac{1}{2} D^a D_a, \quad (4.2.16) \]

with covariant derivative and field strength tensor defined as

\[ D_m g^a = \partial_m g^a - g_{cc} f^{abc} c^b_m g^c, \quad (4.2.17) \]
\[ F_{mn}^a = \partial_m c^a_n - \partial_n c^a_m - g_{cc} f^{abc} c^b_m c^c_n, \quad (4.2.18) \]

with \( g_{cc} \) a gauge coupling constant. Again we see the presence of a non-dynamical auxiliary field, this time the \( D_a \) field.

### 4.2.2 Susy Interactions: The Superpotential

We have constructed two free theories for both types of Supermultiplets and we must now put them together. We must thus consider both gauge and non-gauge interactions. The gauge interactions are actually easy to incorporate. We merely change the normal derivatives to the following covariant derivative

\[ D_m A^i = \partial_m A^i + i g_{cc} c^a_m T^a_{ij} A^j, \quad (4.2.19) \]
\[ D_m y^i = \partial_m y^i + i g_{cc} c^a_m T^a_{ij} y^j, \quad (4.2.20) \]

with \( T^a_{ij} \) group operators. The gauge bosons are now coupled to the fermions and scalars of the theory. The non-gauge interactions are a bit more involved.

We start by considering chiral interactions. An interesting consequence of a Susy theory is that we can collect all these interactions in a single function called the Superpotential \( W \). The form of this function is severely restricted. Susy
forces it to be a holomorphic function in the scalar components of the superfields, $j$. Renormalizability restricts it to be at most a cubic polynomial in $j$, therefore

$$W(j) = m_{ij} i j^j + Y_{ijk} i j^j j^k.$$  \hspace{1cm} (4.2.21)

In terms of the Superpotential, the chiral interactions are given as

$$L_{\text{chiral}} = - \frac{1}{2} W_{ij} y^i y^j + F_i W^i + h.c.,$$ \hspace{1cm} (4.2.22)

with $W_i = \frac{\partial W}{\partial j^i}$ and $W_{ij} = \frac{\partial^2 W}{\partial j^i \partial j^j}$.

Explicitly inserting this into the free Lagrangian, we identify $m_{ij}$ with a mass term and $Y_{ijk}$ as a Yukawa coupling. Furthermore, by solving the equations of motions for the auxiliary field $F$ with the addition of the Superpotential, we obtain

$$\frac{\partial L}{\partial F} = F^i \dagger - W_i = 0,$$

$$\frac{\partial L}{\partial F^i} = F_i - W^{i \dagger} = 0.$$  \hspace{1cm} (4.2.23) (4.2.24)

We can use these equations to eliminate the $F$-fields in the Lagrangian in terms of the Superpotential. Finally, we consider the gauginos and D-field couplings. The only terms that will respect renormalizability are

$$g_{cc} \left[ (\bar{A} T^a c^a + h.c.) \right] \quad \text{and} \quad g_{cc} (\bar{A} T^a A) D^a.$$ \hspace{1cm} (4.2.25)

The addition of these terms to the Lagrangian has the effect of modifying the Susy transformation properties of $\psi$ and $F$, eqns. (4.2.8) and (4.2.10),
\[ S y = \sqrt{2}(yF + iy^\dagger \not{D}_m A) \quad (4.2.26) \]

\[ S F = ix^\dagger \not{D}_m y^i + \sqrt{2}g_{cc}x^\dagger \not{c}^a (T^a A). \quad (4.2.27) \]

The non-dynamical auxiliary fields \( D_a \) can now also be eliminated by using their equations of motion

\[ \frac{dL}{dD^a} = \overline{D}^a + g\left(\overline{A'}T^a_{ij} A^i\right) = 0. \quad (4.2.28) \]

Putting both the chiral and gauge free and interacting Lagrangians together, we obtain the most general interacting Susy Lagrangian involving a chiral and a gauge Supermultiplet

\[
L(y^i, A^i, F^i = - \overline{W_i}; \quad c_{m}^a, g^a, D^a = - g_{cc} \overline{A'}T^a_{ij} A^i) =
\]

\[
- \frac{1}{4} F^a_{mn} F^{a,mn} - D_{m} A^i D^m A^i - i g_{m}^a \not{D}_m g^a - i y^i \not{D}_m y^i
\]

\[
+ i\sqrt{2} g_{cc} \overline{A'}T^a_{ij} A^i g^a - \frac{1}{2} W_{ij} y^i y^j + h.c. \frac{\delta}{\delta} V(A, \overline{A}), \quad (4.2.29)
\]

where \( V(A, \overline{A}) \) is the scalar potential of the theory

\[
V(A, \overline{A}) = W_i W^i + \frac{1}{2} g_{cc}^2 (A^i T^a_{ij} \overline{A'})(\overline{A'} T^a_{ij} A^i)
\]

\[
= F_i F^i + \frac{1}{2} D_a D^a. \quad (4.2.30)
\]

The contributions to the scalar potential are referred to as the “F”-terms and “D”-terms respectively. Here we see a unique feature of Susy theories, namely that the scalar potential is completely determined by the other interactions of the
theory. The F-terms are fixed by the Yukawa terms and fermion masses (i.e. the Superpotential) and the D-terms are fixed by gauge interactions. In fact, once the superfields and their gauge structure are assigned, the only freedom left in constructing the Susy Lagrangian is in the choice of $W(j)$.

4.2.3 Soft Symmetry Breaking

We have built a seemingly complete Susy theory, yet one very important phenomenological fact is left out; experiments show that there are no superpartners in the low energy regime ($< M_Z$). After all, the SM contains no .511 MeV selectron nor do we observe low energy sHadrons built up of squarks! This means that Susy must be a broken theory at low energy. Since we have a symmetry that is not respected by the vacuum of the theory, it might be tempting to try to break Susy spontaneously as in EW breaking. Unfortunately, here we see how little we truly understand Susy for this idea runs into a host of problems. First, any model with Susy SSB does so by the introduction of more particles and interactions at high energies and there is no consensus on how to do this; we have very little concrete physics to guide us in our choice of high energy models. (Most of these attempts rely on some sort of GUT or Gravity mechanism to break Susy at high energies $\sim M_{PL}$.) A second phenomenological problem is that a SSB theory must obey a mass sum rule which for Susy takes the form

45
for gauge, chiral, and scalar particles. The problem is that we wish to put all our superpartners mass above present day experimental reach; a hard thing to do while obeying the above rule. For these reasons, most theorist have given up on SSB as the method to break Susy and we shall therefore resign ourselves to our ignorance by explicitly assigning soft mass terms to the superpartners at high energy. Soft here means that the breaking terms should have positive dimension ≤ 3 (for renormalizability) and should not introduce any further quadratic divergences into the theory (as this is the central reason to consider a Susy theory in the first place!). The most general soft Lagrangian terms are

\[ L_{soft} = - m^2 A_i A^i - \frac{1}{2} m_{ij} g_{i} g_{j} - (b_{ij} A_i A_j + a_{ijk} A_i A_j A_k + h.c.). \]  

Note that fermionic soft mass terms are not needed as they can be absorbed in a redefinition of the Superpotential and are therefore redundant. \( L_{soft} \) clearly breaks Susy as only the susy scalars and gauginos obtain an explicit mass, but not their SM partners. These soft mass terms renormalize the Higgs mass in a natural fashion

\[ dm^2_{soft} \sim m^2_{soft} \log \frac{L^2}{m^2_{soft}}. \]  

Setting \( L : m_{planck}, m_{soft} \), and hence the masses of at least the lightest superpartner, must be at most ~1TeV in order to give the correct \( W^\mu \) and \( Z^\mu \) mass
without any fine tuning. Hence, the superpartners must live in an energy regime between the GeV and TeV scale. Tantalizingly enough, Susy would conveniently solve the Hierarchy problem as well.

4.3 Introducing the MSSM

We are now in a position to build the Minimal Supersymmetric extension to the Standard Model, the MSSM. Since we are building up on an already complete theory, the SM, we have very little freedom in Susy model building. It is exactly this limitation that makes Susy such an attractive prospect to theorists! We begin by assigning superpartners to the SM particles and thus build up our Supermultiplets. The complete MSSM particle and gauge spectrum is given in Fig. 2 at the beginning of the chapter. The Superpotential and soft breaking terms are then given and complete the MSSM Lagrangian. Here we pay the price for our ignorance as the soft breaking terms introduce close to one hundred new parameters, most of them free and unconstrained.
4.3.1 MSSM Supermultiplets

We start by assigning superpartners to the fermions. These must be spin 0 and will naturally reside in a chiral Supermultiplet. We identify the scalar superpartner by prepending the corresponding SM fermion name by an "s" (hence a lepton’s superpartner: slepton, quark’s superpartner: squark, etc) and by placing a tilde over the SM fermion name ($\tilde{e}$, $\tilde{q}$, etc). The vector bosons of the SM reside naturally in a gauge Supermultiplet. Their spin $\frac{1}{2}$ superpartners are identified by appending “ino” to the corresponding SM particle name (Wino, Zino, etc) and again by placing a tilde over it’s SM partner’s symbol ($\tilde{W}$, $\tilde{Z}$,etc). After EW symmetry breaking, like their SM partners, the Wino and the Bino are mixed to produce the Zino and the Photino. Finally, the Higgs resides in a chiral multiplet with a Higgsino as its spin $\frac{1}{2}$ superpartner. However, it turns out that we need two Higgs chiral Supermultiplets in a Susy theory. There are two reasons for this:

1) The Higgsino has hypercharge and thus a single Higgsino would induce an uncompensated term in the U(1) triangle anomaly. The introduction of a second Higgsino (and hence a second full Higgs Supermultiplet) with opposite hypercharge will neatly cancel the anomaly once all fermionic contributions are taken into account.
2) The d-quark previously obtained it’s mass from the complex conjugate of the Higgs responsible for the mass of the u-quark. This strategy fails with Susy since the Superpotential must be analytic in the scalar fields and we again need two distinct Higgs.

Two Higgs’ are therefore necessary in a Susy theory: $H_u$ with $Y=+\frac{1}{2}$ to give mass to an u-quark and $H_d$ with $Y=-\frac{1}{2}$ for the d-quark. The choice of labels for both Higgs’ ($H_u$ and $H_d$) serves as a constant reminder of which Higgs couples with which quark. We will also label the Higgs’ as $H_1$ and $H_2$. This completes the presentation of the particle content of the MSSM.

**4.3.2 MSSM Superpotential**

The choice of the MSSM Superpotential is restricted by the form of eqn. (4.2.21) and by the necessity of having the Lagrangian of the SM contained within. To this effect, the most general SU(3)$_C \times$SU(2)$_L \times$U(1)$_Y$ Superpotential is given by

$$W = \sum_{3 \text{ Generations}} Y_u Q_L H_u U_R - Y_d Q_L H_d D_R - Y_e L_L H_d E_R + m H_u H_d . \quad (4.3.1)$$

Inserting this Superpotential into eqn. (4.2.22), the $Y$’s are dimensionless Yukawa couplings (3x3 matrices in family space) and $m$ is the Susy version of the Higgs mass. Here we explicitly see in action the two Higgs doublets as they give
mass to their respective quarks. The Superpotential as presented above is the most general Superpotential that respects baryon-lepton number conservation. However, nothing so far prevents the inclusion of the following terms into the Superpotential

\[ W_{\text{lepton/baryon violating}} = \frac{1}{3} L_i L_j E_k + \frac{1}{2} L_i Q_j D_k + \frac{1}{3} L_i D_j D_k, \quad (4.3.2) \]

which violate lepton (first two terms) or baryon (last term) number conservation and can easily lead to phenomenological disasters, such as proton decay at an unacceptable high level\(^2\). A solution, and it should be stressed that this is merely “a” solution as there are alternative\(^{11}\) ways to deal with B-L violating interactions, is to impose a new Susy symmetry called R-parity\(^{12}\) defined as

\[ R^{\circ} = (-1)^{B+L+s}, \quad (4.3.3) \]

where B and L are baryon and lepton number respectively for a spin s particle. Under R-parity, the SM particle have R=+1 and the Susy particles have R=-1. Imposing R-parity, or more specifically, that all terms in a Susy Lagrangian have a total R-parity of +1, explicitly forbids the terms of eqn. (4.3.2) and hence brings our Susy model in accordance with experimental observations. R-parity brings with it a host of predictions, including that Susy partners can only be pair produced from SM particles and the stability of the Lightest Susy Particle (LSP) to which all heavier Susy particle cascade decay into. R-parity is a large and important section of Susy model building and I refer the interested to the literature for more information.
4.3.3 MSSM Soft Terms

Setting all but the neutral Higgses equal to zero in anticipation of EW symmetry breaking, we derive the form of the Higgs potential

\[
V(h_u^0, h_d^0) = m^2 \left( |h_u^0|^2 + |h_d^0|^2 \right) + \frac{1}{8} (g_1^2 + g_2^2) \left( |h_u^0|^2 - |h_d^0|^2 \right)^2, \tag{4.3.4}
\]

where the quartic couplings are not independent parameters, but rather the gauge couplings \(g_1\) of \(U(1)_Y\) and \(g_2\) of \(SU(2)_L\). Here we see another reason for the MSSM soft terms. The neutral low energy Higgs potential of the MSSM is by necessity positive semi-definite and we see that \(V(h_u^0, h_d^0)\) is no exception.

Therefore, the minimum of this theory necessarily resides at \(\langle h_u^0 \rangle = \langle h_d^0 \rangle = 0\) and this corresponds to an unbroken theory; once again, Susy cannot be broken spontaneously!

The MSSM soft terms must now be provided to give the Susy partners an appropriate mass. We impose SM gauge invariance and R parity to obtain

\[
L_{\text{soft}} = \sum_{\text{generations}} \left( - A_u \left( \frac{\partial}{\partial L} H_u \frac{\partial}{\partial R} \right) + A_d \left( \frac{\partial}{\partial L} H_d \frac{\partial}{\partial R} \right) + A_e \left( \frac{\partial}{\partial L} H_e \frac{\partial}{\partial R} \right) \right)
\]

\[
- \sum_{\text{scalars}} m_{ij}^{2} \bar{u}_i \bar{d}_j - \frac{1}{2} \sum_{g, W, B} \left( m_{\tilde{g}} \bar{g} g + B H_u H_d \tilde{u} \tilde{d} \right) + \text{h.c.} \tag{4.3.5}
\]

where the “A” tri-linear scalar mixing terms are 3x3 matrices of mass 1 and the “B” bi-linear mixing term has mass 2.
The inclusion of soft terms into our theory expands the parameter space of the MSSM by

\[ (A_u, A_d, A_e, m_{ij}^2, m_g, B) . \] (4.3.6)

We therefore see that our ignorance about how Susy is broken has in fact introduced over 100 new parameters that cannot simply be rescaled or ruled out experimentally! It is this situation that we will address in the next chapter where our research in resucing this parameter space is presented.
Using Vacuum Stability to Reduce the Parameter Space of the MSSM

Let us summarize our results for the MSSM. Guided by the SM particle and gauge structure, we have been able to construct a free Susy theory with no new parameters aside from those already present in the SM. The addition of interactions introduced the Superpotential and with it, the introduction of Yukawa couplings and a Susy Higgs mass parameter. The fact that Susy is not observed below the $M_Z$ scale means that it must be a broken symmetry at low energy, though not spontaneously broken. This leads to the introduction of soft symmetry breaking terms to explicitly break Susy and this act alone skyrockets the number of parameters in our theory to over 100 (according to Haber, there are 124 free parameters in the MSSM, leading to his nomenclature MSSM-124). This tremendous parameter space stands in the way of realistic experimental searches as well model building and hence many strategies are used to reduce it, most taking advantage of mass degeneracy at the unification scale $M_X$ or, because it’s the heaviest matter particle, ignoring all but the top quark mass.

In the following sections I present our own attempts to cut down this parameter space. The essence of our approach lies in examining the lifetimes of the false vacuums obtained by an astute choice of fields. The decay of these
vacuums will lead to strong SM and cosmological signals which have not been observed and thus we are able to discard vast swaths of parameter space which lead to a false vacuum decay earlier than our present universes age. We will begin by exploring those directions in field space that lead to dangerous false vacuums, then continue to present the Bounce Instanton, and lastly put these two concepts together in an attempt to reduce the parameter space of the MSSM. The results of our investigations are presented at the end of the chapter.

5.1 MSSM Vacuum Structure

The MSSM vacuum should have the low energy SM pattern of symmetry breaking, $\text{SU}(2)_L \times \text{U}(1)_Y \rightarrow \text{U}(1)_{\text{EM}}$. The standard method of having the Higgs break the symmetry is complicated by the addition of a second Higgs as well as by the addition of several new scalar sparticles. We will now re-examine the Higgs mechanism with only the neutral Higgs' acquiring non-zero vev's that will generate new minima for the potential. The issue of dangerous directions in field space as other scalar fields are added is then discussed.
5.1.1 Realistic MSSM Minimum

**SU(2)xU(1) symmetry breaking**

In the SM, only the neutral components of the Higgs will acquire a vev under EW symmetry breaking. This mechanism is modified in the MSSM by having both of the neutral Higgs scalars acquire a vev

\[
\langle H_1 \rangle = \frac{\text{e}^0}{u_1} \frac{\delta}{\delta} \\
\langle H_2 \rangle = \frac{\text{e}^0}{u_2} \frac{\delta}{\delta} \tag{5.1.1}
\]

The tree level scalar sector of the MSSM is thus

\[
V_{\text{scalar}} = FF + DD - L_{\text{soft}} \\
= (m_1 |H_1|^2 + (m_2 |H_2|^2 + \frac{g^2}{8} (|H_2|^2 - |H_1|^2)^2 - 2m_3 |H_1||H_2|, \tag{5.1.2}
\]

where \( g^2 = g_1^2 + g_2^2 \) and the realistic minimum is given as

\[
V_{\text{min}} = -\frac{g^2}{8} (u_2^2 - u_1^2)^2, \tag{5.1.3}
\]

subject to

\[
m_1^2 + m_2^2 > 2m_3^2 > 2m_1m_2 \tag{5.1.4}
\]

in order for EW symmetry to be broken and \( V_{\text{min}} \) be Bounded From Below (BFB).

Before the SSB, we have two complex Higgs doublets for a total of eight polarizations. After SSB, three of these polarizations are gauge transformed to provide the gauge bosons with mass; the five remaining polarizations manifest themselves as a CP-odd neutral Higgs as well as two charged Higgs boson and two CP-even neutral Higgs.
The three free parameters of this sector

\begin{align*}
m_i^2 &= (m^2_{H_1})_{\text{soft}} + m^2,

m_2^2 &= (m^2_{H_2})_{\text{soft}} + m^2, \\
m_3^2 &= (Bm)^2,
\end{align*}

are constrained by the need for $M_W$ to have its proper value after SSB

\[ M_W^2 = \frac{1}{2}(g_2u)^2 = \frac{g_2^2}{2}(u_1^2 + u_2^2). \] (5.1.6)

This leaves us with two independent parameters:

\[ b \] such that

\begin{align*}
u_1 &= u \sin b, \\
u_2 &= u \cos b,
\end{align*}

and $M_A$

\[ M_A^2 = 2|m_3| \sin^{-1}(2b), \] (5.1.8)

the mass of the CP-odd neutral Higgs. From these two parameters and eqn. (5.1.6), the $m_3^2$ mass is fixed

\[ m_3^2 = \frac{|m_1 + m_2| \sqrt{(m_1^2 + m_{H_1}^2)(m_2^2 + m_{H_2}^2)}}{|m_1^2 + m_2^2 + 2m_W^2|}. \] (5.1.9)

$V_{\min}$ is our realistic low energy vacuum, since none of the sparticles are present. However, the addition of scalar sparticles will lead to a more complicated $V_{\text{scalar}}$ and can drive it into potentially disastrous directions.
**Dangerous directions in the MSSM vacuum**

In a non-broken Susy regime, a proper MSSM scalar potential has to include all the scalar particles of the model, not just the neutral Higgs. Since the early 80’s\(^2\), it’s been known that these scalar sparticles can acquire a vev and thus lead to Charge and/or Color Breaking (CCB) minima, which render the theory phenomenologically unacceptable. The vast complexity of the scalar sector of the MSSM meant that only two unstable field directions had been thoroughly studied through the 80’s.

The first bound is obtained when only trilinear scalar couplings are considered. Equal vev’s are assumed for all the scalar terms and the phases of the fields are taken to make the potential contribution from these terms negative. Fere et. al. and others\(^2\) showed that a very deep \((V_{CCB} > V_{min})\) CCB minima appears unless

\[
|A_\mu|^2 \leq 3(m_{\tilde{u}}^2 + m_{\tilde{u}}^2 + m_{\tilde{d}}^2)
\]

(5.1.10)

for the \(A_\mu Y_{\mu}(\bar{Q}H_2 u_h)\) trilinear term. This constraint is easily generalized to the other trilinear terms.

The second direction explored was by Komatsu\(^2\) who found that along the direction

\[
\tilde{L}^2 = H_2^2 + \bar{Q}^2,
\]

(5.1.11)

\[
\bar{Q}d = -\frac{\tilde{\nu}_d}{\tilde{L}_d}H_2,
\]

(5.1.12)
the MSSM potential is actually Unbounded From Below (UFB) unless the (Komatsu) constraint is satisfied

$$m_2^2 - m^2 + m_{t_c}^2 > 0.$$  \hspace{1cm} (5.1.14)

These minima are actually subsets of the CCB minima, as CCB→UFB for limiting cases of the CCB parameters. It should be stressed that these UFB directions are only a tree level approximation, as the potential eventually becomes BFB thanks to radiative corrections.

![Generic UFB potential for a single scalar field](image)

Figure 6: Generic UFB potential for a single scalar field

These studies lead to necessary, but not sufficient conditions to avoid these dangerous minima. In both cases, it was found that the heaviest particles of the spectrum would lead to the most dangerous directions, hence the practice to concentrate on the third generation of particles. Furthermore it is easy to see
in order for the potential to reach such deep minima, the F- and D- terms must be stabilized or set to zero as they are always positive. The situation remained unchanged until 1995, when Casas et. al. performed the first comprehensive study of these CCB minima. He found further constraints on both true CCB minima and UFB minima. At this point, I wish to exclusively concentrate on the UFB directions, as they are the relevant directions for our further study. In their paper, two general considerations for any UFB direction were given:

1) Trilinear scalar terms do not contribute to an UFB direction since for large enough values of the fields, the corresponding positive F-term contribution would be large enough to raise up the UFB potential and ultimately render it BFB.

2) At the $M_{\text{susy}}$ scale, all physical masses are positive, hence the only negative mass contributions in $V_{\text{scalar}}$ that will contribute to an UFB direction are $m_5^2 |H_2|^2$ and $-2 \sqrt{m_3^2} |H_1||H_2|$. Therefore, any UFB direction must involve $H_2$ and some additional fields to further stabilize the positive contributions arising from the F- and D-terms.

Using these two properties, Casas et. al. classified the UFB directions into the following three categories:
5.1.2 UFB-0

This is in a sense a trivial direction, as it is simply a model with $H_1$ and $H_2$ non-zero, all other scalars set to zero, and thus,

$$V_{UFB-0} = V_{\text{min}}.$$  \hfill (5.1.15)

The only UFB direction corresponds to the choice $|H_1| = |H_2|$ and we obtain a UFB potential unless eqn. (5.1.4) is satisfied. UFB-0 is nothing more than a statement on the boundedness of a realistic low energy minima.

5.1.3 UFB-1

To $H_1$ and $H_2$, we now add extra fields that will cancel or stabilize the D terms. The simplest choice is to include a selectron $L$ along the $\vec{n}_L$ direction leading to

$$W = mH_1H_2 - h_e \vec{\tilde{H}}_L \tilde{H}_2 \xi_R^L$$  \hfill (5.1.16)

and

$$V_{UFB-1} = (m_1 |H_1|)^2 + (m_2 |H_2|)^2 + (m_L |\tilde{L}|)^2 \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill \hfill (5.1.17)$$

$$+ \frac{g}{8} \left( |H_2|^2 - |H_1|^2 - |\tilde{L}|^2 \right)^2 - 2m_3^2 |H_1||H_2|.$$  \hfill (5.1.17)

We immediately identify the most dangerous directions as

$$|L|^2 = 4g^{-1}m_L^2 + |H_1|^2 - |H_2|^2.$$  \hfill (5.1.18)
\[ |H_I| = \frac{|H_2| m_3^2}{m_I^2 - m_L^2}, \]  

(5.1.19)  

provided that the following conditions are met  

\[ |m_3^2| < m_I^2 - m_L^2, \]  

(5.1.20)  

\[ |H_2|^2 > 4 g^{-2} m_L^2 \hat{q} l - m^I m_3^q \hat{u} \]  

(5.1.21)  

The tree level scalar potential along this direction is  

\[ V_{UFB-1} = \frac{\hat{q} m_2^2 + m_I^2 - \frac{|m_3|^4}{m_I^2 - m_L^2} H_2^2}{2 g^{-2} m_L^4}. \]  

(5.1.22)  

Hence, we obtain a UFB potential unless  

\[ m_2^2 + m_L^2 - \frac{|m_3|^2}{m_I^2 - m_L^2} > 0. \]  

(5.1.23)  

### 5.1.4 UFB-2

This last possibility involves exploring a direction where \( H_I = 0 \) and \( H_2 \neq 0 \). In this scenario, we need the help of the d-type squarks to stabilize the F-term contributions

\[ F_i F^i = \left| mH_2 + l_d \hat{Q}_L \hat{d}_R \right|^2 = 0. \]  

(5.1.24)
To stabilize the D-terms we make \( \langle d_e \rangle = \langle d_i \rangle = \langle \hat{a} \rangle \); this will eliminate the SU(3)_C D-term contribution. In order to stabilize the SU(2)_L and U(1)_Y D-terms, we need the addition of an extra field. Once again, we choose the selectron L along the \( \hat{n}_L \) direction, however from a different generation than our squarks. Our potential is now

\[
V_{UFB-2} = (m_2^2 - m^2) |H_2|^2 + m_{L_i}^2 |L_i|^2 + (m_{Q_i}^2 + m_{d_j}^2) |d_j|^2 \\
+ \frac{g^2}{8} \left( |H_2|^2 + |d_j|^2 - |L_i|^2 \right)^2,
\]

with i and j generational indices. This potential achieves its lowest value along the directions

\[
|L_i|^2 = 4g^{-2} m_{L_i}^2 + |H_2|^2 + |d_j|^2,
\]

\[
d_j = \frac{m}{l_d} H_2.
\]

If

\[
|H_2| > \sqrt{\frac{m^2}{(2l_d)^2} + \frac{3m_{L_i}^2}{g^2} - \frac{|m|}{2l_d}},
\]

then our scalar potential is

\[
V_{UFB-2} = (m_2^2 - m^2 + m_{L_i}^2) |H_2|^2 \\
+ \frac{|m|}{l_d} \left( m_{Q_j}^2 + m_{d_j}^2 + m_{L_i}^2 \right) |H_2| - 2g^{-4} m_{L_i}^4.
\]
Otherwise, if eqn. (5.1.27) is not met, the optimal value for \( L \) is \( L = 0 \) and the potential becomes

\[
V_{\text{UFB-2}} = \frac{g^2}{8} |H_2|^4 + (m_2^2 - m_r^2)|H_2|^2 + \frac{m_\phi}{l_d} m_Q^2 + m_d^2 + \frac{g_\phi^2}{8} |H_2|.
\tag{5.1.30}
\]

This is the direction originally studied by Komatsu and hence we obtain a UFB potential unless the Komatsu constraint, eqn. (5.1.14), is met.

From the above equations, it becomes clear that the larger the Yukawa coupling \( l_d \), the more restrictive our UFB bounds become. This justifies our earlier statement that we concentrate on the third generation of particles for the most dangerous direction.

We close this section by noting that the D-term could also have been stabilized by the \( \tilde{e}_L \) and \( \tilde{e}_R \) sleptons. In fact, all the results for the UFB potential above \( \text{(which I will now label UFB-2b)} \) remain the same under the substitution

\[
Q_L^l \otimes L_L^l, \quad d_R^l \otimes e_R^l, \quad l_d \otimes l_e, \quad \tag{5.1.31}
\]

with the most restrictive bounds again coming from considering the third generation. \textit{This substitutive direction will be labeled UFB-2a.}
5.2 The Bounce Instanton

In the previous section, all the possible UFB directions in the MSSM where identified. The theoretical and phenomenological danger of the existence of these directions should be obvious; the mere presence of directions in field space where the vacuum has no bottom may sound like the death of any theory. Early on, the choice of parameters that led to a CCB or UFB potential where sufficiently excluded. However, using metastable vacuum considerations as defined in the works of Sydney Coleman and others, a new possibility emerged. The SM can in fact reside in a MSSM false vacuum and our scalar potential have CCB/UFB directions as long as the decay into the true, deep vacuum is longer than the current age of the universe. Key to his method is the Semi-Classical Approximation (SCA), which we present first. Next, this formalism is applied to the vacuum transition probabilities in symmetric and skewed symmetric double well. A topologically conserved quantity, the Instanton, will emerge from the latter and a particular instanton, the Bounce, from the former. All these cases will first be developed using a spinless unit mass particle moving in a potential in one dimension

\[
L = \frac{i}{\hbar} \frac{\partial \phi}{\partial t} \left( \frac{\hbar}{2m} \frac{\partial^2 \phi}{\partial x^2} - V(x) \right). \tag{5.2.1}
\]

It will then be shown that the results derived can be trivially extended to a single scalar field in four dimensions.
Euclidean actions and the SCA

The fundamental tool in the current study is the Euclidean formulation of Feynman's sum over histories

$$\langle x_f | e^{\frac{HT}{\hbar}} | x_i \rangle = N \oint [dx] e^{-\frac{S}{\hbar}}.$$

(5.2.2)

The left hand side is comprised of the coordinate eigenstates $|x_i\rangle$ and $|x_f\rangle$ sandwiching the energy operator. Expanding into a complete set of energy eigenstates and considering large $T$, this will give us an expression for the energy and wave function of the lowest lying eigenstate. The right hand side is what will interest us most. Here, $N$ is a normalization factor, $S$ is the Euclidean action

$$S = \int_{-T}^{T} \frac{x^2}{2} + V(x) dt,$$

(5.2.3)

and $[dx]$ denotes integration over all functions $x(t)$ satisfying $x(-T)=x_i$ and $x(T)=x_f$.

In the semi-classical limit (that is the $\hbar \to 0$ limit), the right hand side can be readily evaluated, for in this limit, only the stationary points of the Lagrangian contribute to $S$. Denoting $\bar{x}$ as the stationary point

$$\frac{dS_E}{dx} + \frac{d^2\bar{x}}{dt^2} + V(\bar{x}) = 0,$$

(5.2.4)

where the prime denotes differentiation with respect to $\bar{x}$, and choosing $x_n$ to be eigenfunctions for the second variational derivative of $S$ at $\bar{x}$,
\[- \frac{d^2 x_n}{dt^2} + V(\bar{x}) \psi_n x_n = l_n x_n, \quad (5.2.5)\]

then in the small \(h\) limit, the integral becomes a product of gaussians

\[
\langle x_f \mid e^{iH T/h} \mid x_i \rangle = \sum_n N l_n \frac{1}{2 \pi} \int e^{-S(\bar{x})/h} \]

\[
= N \det \left( \frac{\partial}{\partial \bar{x}} \right) e^{\frac{1}{2} \int \frac{\partial^2}{\partial \bar{x}^2} \bar{x} - V(\bar{x})} \frac{1}{\sqrt{2\pi \hbar}} e^{-S(\bar{x})/h} \]

\[
= A e^{-S_E(\bar{x})/h} \left[ 1 + O(h) \right], \quad (5.2.6)\]

where the \([1+O(h)]\) is there to remind us that we are in the SCA and thus results are only valid up to order \(h\) in an expansion for \(x\)

\[
x = x_0 + x_1 h + x_2 h^2 + \ldots \quad (5.2.7)\]

Eqn. (5.2.4) is the equation of motion for a particle of unit mass moving in a potential \(-V\), therefore

\[
E = \frac{1}{2} \int \frac{\partial^2}{\partial \bar{x}^2} \bar{x} - V(\bar{x}) \]

\[
= \frac{1}{2} \int \frac{\partial^2}{\partial \bar{x}^2} \bar{x} - V(\bar{x}) \quad (5.2.8)\]

is a conserved quantity. From the above we see that to study a system in the SCA, it is enough to consider solving the Euclidean eqns of motion eqn. (5.2.4) and finding the Euclidean action, eqn. (5.2.3). This Euclidean extention formally amounts to analytically extending \( t \otimes t = it \).

Let us work out a trivial case of a single well

\[
V_0 = \frac{l x^2}{2}, \quad (5.2.9)\]

66
and consider $x_i = x_f = 0$ (i.e. vacuum to vacuum tunneling). Both the potential and the inverted potential are presented below.

![Figure 7: The potentials $V_0$ and $-V_0$](image)

Since we know that the functional integral is dominated by the stationary points of $S_E$, we look at the motion of the particle in the inverted potential and immediately note that the only stationary point with the given boundary conditions is $x = 0$. Hence,

$$\langle 0 | e^{iH_T/t} | 0 \rangle = \frac{1}{N} \text{det} \left[ \frac{1}{\hbar} \frac{\partial^2}{\partial t^2} + w^2 \frac{\partial^2}{\partial \phi^2} \right] U \left[ \frac{\partial}{\partial \phi} \right] e^{-w T/2},$$

(5.2.10)

where $w = V(0)$ (the calculation of $A$ used above follows Coleman's original work). In general, calculating the factor $A$ will be much more complicated. The ground state energy is $\frac{1}{2} \varphi \hbar$ and the probability for the particle to be in the ground state is $\frac{\varphi \hbar \varphi}{\hbar \varphi \hbar}$, both results as expected from semi-classical arguments.
Symmetric double well

Turning now to a less trivial example, consider a double well

\[ V_1 = \frac{1}{2} l (x^2 - a^2)^2. \] (5.2.11)

This potential and it’s inverted version are presented below.

![Potential and its inversion](image)

Figure 8: The potential \( V_1 \) and it’s inversion, \( -V_1 \)

Once again we look at the inverted potential to find the stationary points of the theory. Since we now have a system with two classically stable minimas, we find four stationary states

\[ \langle + a | e^{HT/h} | + a \rangle, \quad \langle - a | e^{HT/h} | - a \rangle, \] (5.2.12)

\[ \langle - a | e^{HT/h} | + a \rangle, \quad \langle - a | e^{HT/h} | + a \rangle. \] (5.2.13)

The first two correspond to the particle beginning at \( \pm a \) at \(-\infty\) and ending at the same place, \( \pm a \) at \(+\infty\). More interesting are the last two states corresponding to the particle starting at one hill \( \pm a \) at \(-\infty\) and arriving at the
other hill $ma$ at $+\gamma$. Just as before, we wish to find solutions to the Euclidean
equation of motion, this time for vanishing $E$

$$\frac{dx}{dt} = \left(2V\right)^{\frac{1}{2}} = \sqrt{l} \left(x^2 - a^2\right).$$  \hspace{1cm} (5.2.14)

The solution going from $-a$ to $+a$ is what is referred to as an Instanton. Their
names reflect their particle like nature and we see that for large $t$, $x \rightarrow +a$ and eqn.
(5.2.14) can be approximated as

$$\frac{dx}{dt} = (a - x)w,$$  \hspace{1cm} (5.2.15)

with $w = 2a\sqrt{l}$. Therefore

$$(a - x) \sim e^{-\omega t},$$  \hspace{1cm} (5.2.16)

and our Instantons are well localized objects having a size on the order of $w^{-1}$.
The solution from $+a$ to $-a$ is also possible and is referred to as the anti-
Instanton. Both are shown in Fig. 9. The form for the action of the Instanton is
very simple

$$S_0 = \int_{-a}^{a} \left\{ \frac{1}{2} \dot{x}\dot{x} + V(x) \right\} dt = \int_{-a}^{a} \left(2V\right)^{\frac{1}{2}},$$  \hspace{1cm} (5.2.17)

with $n$ Instantons (or anti-Instantons) contributing $nS_0$ to the total action. The
calculation of the actions corresponding to eqn. (5.2.13) is then trivial, as it
consists of the addition of an Instanton and an anti-Instanton as shown in Fig. 9 if
the graphs were connected.
**Skewed double well**

We will now modify the potential so that the minima are still at \( \pm a \), but their energy density modified such that \( E_- > E_+ \); this time the potential is

\[
V = V_{i} + \frac{e}{2a} (x - a).
\]  

(5.2.18)

As we see in Fig. 10, the additional epsilon contribution to the potential serves to skew the energy density between the two vacuua.

Figure 10: Non-symmetric double well potential and it’s inversion.

“\( e \)” is the classical escape point.
Note that once again we have two stable points of equilibrium. Classically, both minima are stable, however the introduction of barrier tunneling induces the probability that a particle in the local minimum, +a, will tunnel into the global minimum, -a, and a (much, much weaker) probability of the opposite process. Hence the local minimum becomes a false vacuum as particles are highly likely to tunnel into the lower energy vacuum, the true vacuum. The particle will start at the false vacuum, +a, and then tunnel to it’s classical escape point, e, the other zero of the theory. At this point, the particle emerges with zero velocity and propagates classically. Vacuum tunneling between these minima, as in eqns. (5.2.12) and (5.2.13), again leads to an instanton, though altered by the preference of the true vacuum. An interesting result obtains from considering tunneling where the particle is at the same minimum at $t = \pm \infty$. Specifically let’s consider the width of the process starting at the false minimum and tunneling to the true vacuum. To study the barrier penetration using the reasoning outlined in the previous section, we invert the potential and solve the Euclidean equations of motion. As this is a zero energy process, eqn. (5.2.8) becomes

$$E = \frac{1}{2} \dot{x}^2 - V(x) = 0.$$  \hspace{1cm} \text{(5.2.19)}
From the imaginary time version of the Euler-Lagrange equation, two important results ensue:

1) The velocity at the classical turning point is zero

$$\frac{dx}{dt} \bigg|_e = 0.$$  \hspace{1cm} (5.2.20)

As a consequence, the motion of the particle for $t^+$ is the time reversed version of its motion for $t^-$.

2) The false vacuum can only be reached asymptotically as $t \rightarrow -\infty$.

However, by 1) above, it is also reached as $t \rightarrow +\infty$; hence

$$\lim_{t \rightarrow \pm \infty} x = x_f.$$  \hspace{1cm} (5.2.21)

From the two points above, the Euclidean motion of the particle becomes clear. It will start on top of $+a$ at $t = -\infty$ (corresponding to starting at the false minimum), then roll to the classical escape point, where it must have zero velocity (corresponding to the particle tunneling through), and finally “bouncing” off the classical turning point as it rolls right back to $+a$ at $t = +\infty$. This is then our particular Instanton, the Bounce. It is specifically defined as the solution to the Euclidean eqns of motion, eqn. (5.2.4) subject to boundary conditions eqn. (5.2.20) and (5.2.21).

Our specific aim is to obtain the width for the process above. Using the arguments of the preceding section the width in the SCA will be of the form
\[ \frac{G}{V} = Ae^{B/h}, \quad (5.2.22) \]

where the “A” prefactor represents corrections from summing over small perturbations to the classical path and

\[ B = S_E = \int_{-\infty}^{\infty} L_E \, dt, \quad (5.2.23) \]

where the subscript E serves to remind us that these are the Euclidean actions and Lagrangian. This is of the same form as eqn. (5.2.3) and the action for one Bounce is

\[ S_I = \int_{0}^{0} L_E \, dt = \int_{x_f}^{x_f} (2V)^{i} dx. \quad (5.2.24) \]

Hence once A is determined, the problem of finding the decay probability for the particle depends on finding the action for the Bounce.

**Field theory Bounce**

The presentation above relies on a mechanical model to develop the Instanton and Bounce. I wish to do away with that model now and switch to the field theoretical description. The transcription is effortless in Euclidean space and we can develop the Bounce guided by the two previous sections. The Lagrangian is now of the form
with Euclidean equation of motion

\[
\left( \frac{d^2}{dt^2} + \tilde{N}^2 \right) f = U(\phi),
\]

(5.2.26)

and Euclidean action

\[
S_E = \int d^4 x d t \left[ \frac{1}{2} \frac{\delta}{\delta f} \left( \frac{\delta}{\delta f} \right) + \frac{1}{2} (\tilde{N} f) \right].
\]

(5.2.27)

Getting a field theoretical Bounce requires us to modify the boundary conditions, eqn. (5.2.20) and (5.2.21), into

\[
\left. \frac{df}{dt} \right|_e = 0,
\]

(5.2.28)

\[
\lim_{t \to \pm \infty} f = f_f.
\]

(5.2.29)

We need to add the following condition as well to insure that \( S_E \) remain finite

\[
\lim_{t \to \pm \infty} f = f_f.
\]

(5.2.30)

The last two limits naturally leads us to make the assumption that the solution is \( O(4) \) invariant and thus \( f \) purely a function of \( r \)

\[
r^2 = t^2 + |x|^2.
\]

(5.2.31)

Under this assumption, the action and equation of motion are
\[ S_E = 4p^2 \int_0 r^3 dr \left[ \frac{1}{2} \frac{d\bar{f}}{dr} \frac{d^2 \bar{f}}{dr^2} + U(\bar{f}) \right] \]  
\hspace{1cm} (5.2.32)

\[ \frac{d^2 \bar{f}}{dr^2} + \frac{3}{r} \frac{d\bar{f}}{dr} = U(\bar{f}), \]  
\hspace{1cm} (5.2.33)

where the prime denotes differentiation with respect to \( \bar{f} \). Eqns. (5.2.29) and (5.2.30) are combined into

\[ \lim_{r \to \infty} \bar{f} = f_f \]  
\hspace{1cm} (5.2.34)

and eqn. (5.2.28) becomes

\[ \left. \frac{df}{dr} \right|_b = 0, \]  
\hspace{1cm} (5.2.35)

with \( b \) denoting the field theoretical bounce escape point. If we now take the \( O(4) \) symmetric eqns of motion for the field theoretical Bounce and make a mechanical analogue (\( \bar{f} \) \text{ \rotatebox{90}{\( \approx \)}} x \text{ and } r \text{ \rotatebox{90}{\( \approx \)}} t \)), we have a particle moving in a \( -U \) potential \text{ and} subject to a viscous damping force. If we were to release the particle from it’s classical escape point, \( e \), it would in fact not reach the false vacuum. It would “undershoot”. On the other hand, if we were to start infinitesimally close to the true vacuum, the particle is in a metastable state (in the inverted potential); it will remain close to \( f_i \) for large \( r \). However, at large \( r \), the viscous damping term goes to zero and the particle will start to fall down the hill and, lacking the damping force, will “overshoot” the false vacuum.
Interpolating between these two results is the motion of the particle starting with zero velocity at the escape point “b" around $f_f$, and stopping exactly on top of $f_f$; in other words, a Bounce (more on this shooting procedure later). The O(4) field theoretical Bounce is formally defined as O(4) symmetric solutions to the Euclidean equation of motion eqn. (5.2.33) subject to boundary conditions eqns. (5.2.34) and (5.2.35).

**Cosmological bounds on metastable vacuua**

The field theoretical Bounce gives us is an easy way to measure the timescale on which a false vacuum will drop into the true vacuum. In the O(4) symmetric approximation, this drop will manifest itself as a bubble of true vacuum suddenly emerging in a vast false vacuum universe. Once the bubble is formed, it is stable and worse, immediately starts to expand at the speed of light. This would spell disaster to all systems built upon the false vacuum and should leave quite a strong cosmological signal in it’s wake. As we have no evidence for such a transition since the Big Bang, one of two possibilities emerges if we have a universe with false vacuua. The traditional mindset is that our Universe naturally rests at a global minimum. By definition, an infinitely old universe must reside in the lowest energy state available, hence our universe should reside in the true vacuum. The universe is not infinitely old, however. The second possibility is that the universe in fact resides in the false vacuum and will tunnel into the true
vacuum yet. If the tunneling timescale is large compared to the current age of
the universe, then the vacuum is stable and there are no problems with the
universe (i.e. the SM) to currently reside in a false vacuum. To obtain this
timescale, we calculate the product of the SCA tunneling width, eqn. (5.2.6), and
the past 4-volume light cone in terms of \( t_0 \), the age of the universe

\[
\frac{G}{V} x(V^4)_{\text{past l.c.}} = \frac{G}{V} (t_0 c)^3 t_0 .
\]  

(5.2.36)

If this product is \( < 1 \), then the false vacuum would have already nucleated out of
the universe and possibly left cosmological evidence of this event behind. If this
product is \( > 1 \), then the false vacuum is stable long enough for our universe to
have been in it since the Big Bang. If this product is of order unity, however, we
would have no warning as the false vacuum bubble wall hits us now!... and
dropped us to the true vacuum. The width is dominated by the exponential factor
\( \bar{S}_E \), which as we developed above, is nothing more than the Bounce action, \( B \).

For a \( t_0 \sim 10^9 \) yrs. old, the false vacuum will be unstable on cosmological
timescales to vacuum tunneling if

\[
B \leq 400 \hbar .
\]  

(5.2.37)

So if the Bounce action is greater than \( 400 \hbar \), the tunneling time is sufficiently
large and the false vacuum is stable for now. If the Bounce action is less than
\( 400 \hbar \), however, then the false vacuum would have already decayed into the true
vacuum by now, implying a transition we have yet to see. For these reasons,
models with Bounces less than $400\hbar$ can be sufficiently discarded as they represent unstable false vacuum on cosmological timescales.

So far, our discussion has centered around the Bounce action contribution to the width, but what of the prefactor "A" term? As we stated, this comes about as we sum up quantum corrections to the classical action. We will write

$$A = \hbar n^4$$  \hspace{1cm} (5.2.38)

where $\eta$ represents our ignorance of the full quantum corrections and $\nu$ is the characteristic mass scale. For the purposes of our project, we choose to set $\nu$ close to the EW breaking scale, $\nu=Q_0 \sim 200$ GeV, as this is the scale at which we expect strong Susy quantum corrections. This assumption is valid with $\eta \sim 1$ if the scalar masses and the inverse Bounce size are within a few orders of magnitude of $Q_0^{-30}$.

5.3 Bouncing to restrict MSSM parameters

Here I present our original work, as detailed in ref. [25]. As noted earlier, one of the biggest hurdles facing the MSSM right now is the overwhelming number of parameters introduced through soft Susy breaking. In the past, many groups have explored CCB or UFB directions and considered this a sufficient condition to discard the parameter space that lead to their existence. These
past explorations were always limited by the number of fields considered or by the assumptions that went into their models. Our contribution rests in the ability to parameterize multi-field configurations in such a way that we are left with a single scalar variable\(^3\). Using this method of reduction, we re-parameterize multi-field UFB scalar potentials down to a single field and the Bounce action can then be efficiently calculated for a vast set of parameters which make up the potential. The set of UFB parameters that lead to a violation of the cosmological bound \( B \leq 400h \) are thus sufficiently and necessarily excluded. I start by reviewing our shooting method and the field reparametrization that allows to use this shooting method on complicated scalar potentials. I end this section by presenting the computer methodology that allowed us to find the Bounce action for a varied set of parameters.

5.3.1 Definitions

*Shooting Method*

Our problem at this stage revolves around the calculation of the Bounce action. If greater than \( 400h \), our false vacuum is metastable on cosmological timescales. Lower than \( 400h \), and the vacuum should have already decayed into the true vacuum long ago. The shooting method is a simple numerical method of bisection that allows us to efficiently find the Bounce action of a single
scalar field. Since the Bounce is in the Euclidean formulation, we use the language of a particle rolling on the inverted potential, -U, Fig. 11.

![Figure 11: a) Undershoot b) Bounce c) Overshoot](image)

We wish to release the particle at an escape point, b, such that it will roll to the top of hill f and stay there for large $\rho$. This is the Bounce action Fig. 11b. As discussed previously, two other motions are possible, an undershoot, Fig. 11a, and an overshoot, Fig. 11c. Note that by virtue of the stokes-like damping term in eqn. (5.2.33), the classical escape point is an undershoot. The overshoot must correspond to an escape point to the right of the Bounce escape point; the undershoot escape point must be to the left. We place the particle at our initially chosen position and numerically evolve the solutions. At large $\rho$, we check to see where the particle is. If it is to the right of false vacuum, it has undershot and we must raise the initial release position. If at large $\rho$ we find the particle to the left of the false vacuum, then it has overshot and we must lower its release position. Therefore we iteratively adjust the release position based on whether there was an under- or overshoot. After several iterations, the release point “b” is
found that leads to the particle remaining on top of hill \( f \) for large \( \rho \) and this is our Bounce solution. Once we have found the Bounce after several iterations of the shooting method, it is easy to calculate its action from eqn. (5.2.32) since from our boundary conditions, at the Bounce escape point,

\[
\left. \frac{df}{dr} \right|_b = 0.
\]

**Methods of Reduction**

The above procedure works phenomenally well for a single scalar field. It is significantly complicated by additional scalar fields. Additional fields lead to scalar potential hyper-surfaces on which the simple bracketing algorithm is rendered useless. We might be tempted to attempt a brute force numerical minimization of the action in order to find the Bounce. However, as the Bounce is by definition a saddle point of the action, conventional minimization techniques are again rendered useless. We must therefore make modifications to our action to make it calculable. The first important modification we make is to include the effect of scale variations in our minimization. For an action

\[
S = T + V,
\]

a change of scale \( x \to \lambda x \) effects a change \( T \to \lambda^{d-2} T \) and \( V \to \lambda^d V \). Since the Bounce is an extremum of the action, we must have

\[
\left. \frac{d}{dl} S(f) \right|_{l=1} = 0,
\]

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which leads to the condition of stationarity with respect to scale

\[ K = [d - 2]T(f) + [d]V(f) = 0. \]  

(5.3.3)

We modify the action by adding terms that will stabilize the action against scale variations, such as for example

\[ S_{\text{improved}} = S + \hat{a}_n c_n |K|^{p_n}, \]  

(5.3.4)

where \( c_n \) and \( p_n \) are positive numbers, \( n=1,2,3,\ldots \), and by further imposing that these terms vanish at the Bounce. This modification has the remarkable effect of promoting the Bounce from a saddle point of the action to a local minimum. We are still left with the problem of a multi-dimensional hypersurface. The promotion to a local minimum of the action configurations helps, but in a multi-dimensional hypersurface, finding this local minimum is by no means guaranteed. Hence the next important modification is to reduce these hypersurfaces to a single scalar field by which the the Bounce can be found by the shooting method outlined above. To accomplish this, we choose a straight line passing through the point \( f_f = 0 \) and constrain all our fields to lie on this line. This simple parametric reduction in the number of fields is done by defining

\[ F = \hat{a} a_j, \quad \hat{a} a_j^2 = 1, \quad \text{for } j = 1, \ldots, n, \]  

(5.3.5)

Hence, \( n \) real scalar fields \( j \) are reduced to a single field \( F \). By imposing an \( O(d) \) symmetry
finding the Bounce is equivalent to solving the Euclidean equation of motion

\[ \frac{d^2 F}{dR^2} + \frac{3}{R} \frac{dF}{dR} = U(F), \]  

(5.3.7)

where the prime represents differentiation with respect to \( F \), and with boundary conditions

\[ \frac{dF}{dR} \bigg|_b = 0 \]  

(5.3.8)

These are exactly the equations for the field theoretical Bounce with the understanding that \( F \) is our reduced, O(4) symmetric field.

We now have all the elements necessary to start our investigation. Our plan of attack is as follows:

1) We choose a set of fields that will lead to a dangerous UFB potential.

2) The Bounce action is calculated for a range of parameters.

3) For the actions that violate the cosmological bound, eqn. (5.2.37), we discard the responsible sets of parameters.

4) We continue until we have sufficiently covered our parameter space.

We have already presented the dangerous field choices in Section 5.1 and the equations that define the Bounce action in Section 5.2. Before I present the
results of our investigation, I wish to take a slight aside to discuss the computer methodology that allow us to perform the necessary numerical computations.

**Computer Methodology**

The essence of the computation relies on numerically integrating O(4) symmetric Euclidean equation of motion for a reduced field, eqn. (5.3.7). This is accomplished by discretizing the fields and derivatives and then numerically advancing the solution from initial conditions using a fourth order Runge-Kutta routine. My program starts by calculating the potential for a set of parameters. A viable potential will have the realistic vacuum structure of eqns. (5.1.4) and (5.1.9) as well as satisfying the conditions for the existence of a UFB potential, which are unique to the choice of fields. The viable potential is then inverted and I set out to solve it using the shooting routine. To start, an initial escape point is chose far enough into the true vacuum (i.e. far to the right as in Fig. 11]) that when released (in the language of a particle rolling on the inverted potential), numerical integration causes the particle to overshoot. Once the overshoot is reached, the escape point is lowered (i.e. moved to the left) by a stepsize and released anew. If this new escape point also yields an overshoot, we return to the previous escape point, the stepsize is increased, the escape point is now lowered by this new step size, and released. If the new escape point is an undershoot, we know that we have bracketed the correct Bounce between and
over- and undershoot, so we are left with further bisecting this bracket to find it within tolerance. Formally, we return to the former escape point (always the overshoot point), the stepsize is reduced, the escape point lowered by the new stepsize, and released. The iterative change in stepsize speeds up the acquisition of a bracket for the Bounce as well as bracketing the correct escape point within tolerance. Once a Bounce is found, two important checks are made. The first check involves the rescaling factor

\[ \frac{(T - 2V)^2}{T^2} \]  

(5.3.9)

to make sure we have a global Bounce, as per eqn. (5.3.3). The second checks that the Bounce action be within a few orders of magnitude of \( Q_0 \) to justify \( \eta = 1 \) in the prefactor A term, eqn. (5.2.38). Once all these steps are taken, the parameters are classified by whether they lead to a Bounce greater than or less than \( 400h \) and saved to a file for analysis. All the results presented below have rescaling factors of less than \( 10^{-3} \) and Bounces within a few orders of magnitude of \( Q_0 \). The runtime varied from less than a minute to a few hours on a 533 MhZ alpha processor depending on the specific potential and on the parameter space scanned.
I now present the results of our investigations. Using the shooting method and the method of reduction presented above, three different UFB potentials were studied and extensive parameter space exclusion was found in two of them. I will be conservative with the tilde notation as we are dealing exclusively with the scalar components of any Supermultiplet.
5.3.2 UFB-1 Results

UFB-1 was presented in Section 5.1.3 and is defined by setting all scalars equal to zero except for $H_1^\circ$, $h_1^0$, $H_2^\circ$, $h_2^0$, and $N \equiv \frac{h_2}{H_1}$. The scalar potential for this case is

$$V_{UFB-1} = (m_1 |H_1|)^2 + (m_2 |H_2|)^2 + (m_n |N|)^2$$

$$+ \frac{g}{8} \left( |H_2|^2 - |H_1|^2 - |N|^2 \right)^2 - 2m_3^2 |H_1||H_2|. \quad (5.3.10)$$

Defining $y = H_1/H_2$, we have

$$U(H_2) = \left( m_2^2 + m_n^2 \right) - 2m_3^2 y + (m_1^2 - m_n^2) y^2 = H_2^2 f(y) \quad (5.3.11)$$

Thus the potential is UFB when $f(y) < 0$. Minimizing this potential to find the most dangerous direction, we obtain

$$y_m = \frac{m_3^2}{(m_1^2 - m_n^2)}. \quad (5.3.12)$$

provided that

$$m_1^2 - m_n^2 > 0 \quad (5.3.13)$$

and

$$0 < y_m < 1 \quad (5.3.14)$$

Therefore, to have a UFB direction along the fastest decreasing direction requires
\[ f(y_m) = (m_2^2 + m_n^2) - \frac{m_3^2}{(m_j^2 - m_n^2)} < 0. \] (5.3.15)

Having found the most dangerous direction, we reduce the scalar potential. We do this by taking a piecewise straight line in our 3 dimensional space \( \{H_1, H_2, N\} \) parametrized by the real number \( F \in [-\infty, +\infty] \).

For the interval \([-\infty, 0]\), we want only our false vacuum (corresponding to our realistic minimum), hence we set

\[ F^2 = H_1^2 + H_2^2, \] (5.3.16)

with endpoints \( \{u_1, u_2, 0\} \) and \( \{0, 0, 0\} \).

For the interval \((0, +\infty]\), we define

\[ f^2 = H_1^2 + H_2^2 + N^2, \] (5.3.17)

starting at \( \{0, 0, 0\} \) and defined by \( y_m = \frac{\partial H_2}{\partial H_1} \) and \( H_2^2 - H_1^2 - L^2 = 0 \). The former condition comes as a result of setting our line along a path where the D-term vanishes. Along this line, the reduced scalar potential becomes

\[ U(F) = aF^4 + bF^2 \quad \text{for } F \leq 0 \]
\[ = \frac{1}{2} f(y_m)^2 F^2 \quad \text{for } F > 0 \] (5.3.18)

with

\[ a = \frac{m_1 + m_2 x^2 - 2m_3 x}{1 + x^2}, \quad b = \frac{g \frac{\partial}{\partial y} \frac{x^2 y^2}{H_1 + x^2 \frac{\partial}{\partial y}}}{\partial H_1} \frac{\partial}{\partial H_1}, \quad x = \frac{m_1 + m_2 + \sqrt{(m_1 + m_2)^2 - 4m_3^2}}{2m_3}. \]
The reduced potential has the same form as shown in Fig. 6. The results of our numerical investigation are shown below.

Figure 12: UFB-1 Results. The masses are in units of 100 GeV and the iso-action lines in units of $h$.

The axis are given as

$$M_1^4 = (m_2^2 + m_n^2)(m_1^2 - m_n^2)$$

and

$$M_2^4 = m_3^4$$

and we see distinct areas emerge. The one marked “I” denotes the area in our parameter space where the conditions for vacuum stability, eqns. (5.1.4) and (5.1.9), could not be simultaneously met with the conditions for the existence of the UFB potential, eqns. (5.3.12), (5.3.13), and (5.3.14). For the narrow window
where we do have a proper vacuum, we find that all the Bounce actions are several orders of magnitude larger than $400\hbar$ (absolutely stable false vacuum) and thus we have no significant parameter exclusion in this direction.
5.3.3 UFB-2a Results

The UFB-2a direction is defined as having all scalars equal to zero except $H_2, E_L^i, e_R^i,$ and $N_j \not\in \{i,j\}$ with $i$ and $j$ generational indices and $i \neq j$. We will use the $e_R^i$ selectron (corresponding to setting $i=3$) and set $|e_R^i| = |E_L^i| = |E_i|$. The potential in this case is

$$V_{UFB	ext{-}2a} = (m_2^2 - m^2)|H_2|^2 + m_n^2|N_j|^2 + (m_R^2 + m_L^2)|E_i|^2 + \frac{g^2}{8} \left(|H_2|^2 + |E_i|^2 - |N_j|^2\right)^2. \tag{5.3.20}$$

In this potential, we require

$$|H_2|^2 + |E_i|^2 - |N_j|^2 = 0 \tag{5.3.21}$$

to stabilize the D-term contribution. Along this direction

$$U(H_2) = (m_2^2 - m^2 + m_E^2)H_2^2 \pm \frac{m}{l_E} \left(m_{Ei}^2 + m_{Ej}^2 + m_n^2\right)H_2, \tag{5.3.22}$$

where the sign ambiguity comes as a result of the phase choice for our fields.

We will choose the negative sign for a faster drop in the potential. The condition for vacuum stability is thus the aforementioned Komatsu constraint

$$m_2^2 - m^2 + m_n^2 \geq 0. \tag{5.3.23}$$

We will now reduce this three dimensional parameter space, $\{H_2, E_i, N_j\}$. As before, we choose a piecewise straight line parametrized by the real number $F \in [-\frac{1}{2}, +\frac{1}{2}]$ such that is follows a path starting at $\{0,0,0\}$ and obeying
\[ D = |H_2|^2 + |E_i|^2 - |N_j|^2 = 0 \text{ and } F=0. \] From \([-\infty, 0]\), we have \(H_1\) and \(H_2\) only and we obtain the same potential as in the \([-\infty, 0]\) case in UFB-1. Hence our potential in this case is

\[
U(F) = \begin{cases} 
0F^4 + bF^2 & \text{for } F \leq 0 \\
U(H_2) & \text{for } F > 0.
\end{cases}
\] 

(5.3.24)

For \((0, +\infty)\), \(F\) must be computed as a function of \(H_2\) by integrating along a differential curve

\[
dF^2 = \delta dH_2^2 + dE_{Li}^2 + dE_{Ri}^2 + dN_{ij}^2 \frac{\dot{U}}{U} \] 

(5.3.25)

The results for UFB-2a are shown below.

\[\text{Figure 13: Results for UFB-2a. Masses are in units of 100 GeV.}\]
The axes are

\[ M^4_3 = - \left( m_2^2 - |m|^2 + m_n^2 \right) \quad \text{and} \quad M^4_4 = \frac{3}{l_E} \left( |m| m_n^2 \right). \quad (5.3.26) \]

Our result is summarized as the broken line separating excluded from non-excluded parameter space. To the right of the line, a viable MSSM vacuum is ruled out because any or all of the following happens:

i) Eqns. (5.1.4) and (5.1.9) for vacuum stability are not satisfied

ii) slepton mass <45 GeV

iii) \( S < 400 \) h

Our boundary line is drawn so as to keep all the \( S < 400 \) h actions in the non-excluded area; it represents a modification of the Komatsu constraint

\[ M^2_3 x + .2 M^3_4 \cdot 9.5 x^3. \quad (5.3.27) \]

This condition is valid for \( M_3 > 0, x =100 \) GeV, and when all scalar masses are within two orders of magnitude of the EW breaking scale, \( Q_0 \sim 200 \) GeV.
5.3.4 UFB-2b Results

The UFB-2b direction is defined as the UFB-1a direction with the following substitutions

\[ e_R^i \otimes d_R^i, \quad L_L^i \otimes Q_L^i, \quad l_E \otimes l_D. \quad (5.3.28) \]

As with the selectrons \( E \), squark mass degeneracy has been taken advantage of and from the third generation we choose the bottom squark as it has the largest Yukawa coupling. The parameterization of UFB-2a is identical to UFB-1a allowing for the above substitutions. The results for UFB-2b are shown below.

Figure 14: Results for UFB-2b. Masses are in units of 100 GeV.
The axes are

\[ M_5^4 = - \left( m_2^2 - |m|^2 + m_6^2 \right) \quad \text{and} \quad M_4^4 = \frac{3}{f_D} \left( |m|m_3^2 \right) \quad (5.3.29) \]

The results for UFB-2b shows similar parameter space exclusion to the UFB-2a case. The resulting modified Komatsu constraint for this case becomes

\[ M_5^2 x + 2M_6^3 3^{\frac{3}{2}} 6x^3, \quad (5.3.30) \]

where again this is valid \( M_5 > 0, x = 100 \text{ GeV}, \) and when all scalar masses are within two orders of magnitude of the EW breaking scale, \( Q_0 \sim 200 \text{ GeV}. \)
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