Class dependency of fuzzy relational database using relational calculus and conditional probability

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Abstract. In this paper, we propose a design of fuzzy relational database to deal with a conditional probability relation using fuzzy relational calculus. In the previous, there are several researches about equivalence class in fuzzy database using similarity or approximate relation. It is an interesting topic to investigate the fuzzy dependency using equivalence classes. Our goal is to introduce a formulation of a fuzzy relational database model using the relational calculus on the category of fuzzy relations. We also introduce general formulas of the relational calculus for the notion of database operations such as 'projection', 'selection', 'injection' and 'natural join'. Using the fuzzy relational calculus and conditional probabilities, we introduce notions of equivalence class, redundant, and dependency in the theory fuzzy relational database.

1. Introduction
The relational database model was first introduced by Codd\cite{3, 6}. One of the advantage of relational model is soundness of consistency of data. A procedure of data processing is sometimes described by dynamic sequences of operations which may have ambiguities in its implementation. Since a procedure in the relational database model is defined by a static formula, we can avoid inconsistencies in their implementations.

The relational algebra for the possibility-distribution-fuzzy-relational model of fuzzy databases was introduced by Umano and Fukami\cite{12}. Further investigation of division operators are made by Nakata\cite{8}. Our formula of fuzzy relational database model is slightly different since we are using a notion of relational calculus. Okuma\cite{9} formulated the relational database model using a theory of relational calculus on the Dedekind Category. Since the Dedekind Category includes the category of fuzzy relations, we can formulate a fuzzy relational database model using the relational calculus on the category of fuzzy relations. Further, we introduce the notion of database operations using formulas of relational calculus. The advantage of our framework to approximate data reduction using fuzzy class clustering.

In traditional relational database, functional dependency (FD) is a set of constraints between two attributes in a relation. Functional dependency says that if two tuples have same values for attributes $A_1, A_2, \ldots, A_n$, then those two tuples must have to have same values for attributes $B_1, B_2, \ldots, B_n$. In fuzzy relational database, we can classification attribute value to class then we finding the dependency. Let see in Table 1 and 2. We classified the age 'young' from 19 to 22
Table 1. Traditional Relation Age and Salary

| Age | Salary |
|-----|--------|
| 19  | 125    |
| 19  | 100    |
| 22  | 100    |
| 22  | 125    |
| 34  | 150    |
| 38  | 150    |

Table 2. Equivalence Class Relation of Table 1

| Age       | Salary       |
|-----------|--------------|
| {Young}   | {Middle}     |
| {Old}     | {High}       |

and ‘old’ above 22. Also, the salary ‘middle’ from 100 to 125 and ‘High’ above 125. Using fuzzy dependency, we can reduce the data and also finding the rules that the salary depend with the age.

Fuzzy functional dependency is an important topic in the theory of fuzzy relational database. There are a lot of papers about data dependency analysis, based on such as similarity-based definition [11], proximity-based definition [2], possibility distribution based model[10], conformance-based definition [13] and semantic distance based definition [7]. But there are few investigations about an algorithm that would enable the identification of class of attribute relationships in fuzzy relations. In this paper, we introduce a definition of class dependency to define attribute dependency using a conditional probability method. We use the notions to reduce the data and finding the dependency then we can operate it using database operations.

2. Boolean relation theory

2.1. Basic notations

In this section, we introduce a notion of relational calculus. We follows the symbols and notations of relational calculus in [9, 5, 1].

Definition 2.1. A relation \( \alpha \) of a set \( A \) into another set \( B \) is a subset of the Cartesian product \( A \times B \) and denoted by \( \alpha : A \rightarrow B \).

Definition 2.2. The inverse relation \( \alpha^\# : B \rightarrow A \) of \( \alpha \) is a relation such that \( (b,a) \in \alpha^\# \) if and only if \( (a,b) \in \alpha \).

The composite \( \alpha \beta : A \rightarrow C \) of \( \alpha : A \rightarrow B \) followed by \( \beta : B \rightarrow C \) is a relation such that \( (a,c) \in \alpha \beta \) if and only if there exists \( b \in B \) with \( (a,b) \in \alpha \) and \( (b,c) \in \beta \).

As a relation of a set \( A \) into a set \( B \), the inclusion relation, union, intersection and difference of them are available as usual and denoted by \( \subseteq \), \( \sqcup \), \( \cap \) and \( \setminus \), respectively. The identity relation \( \text{id}_A : A \rightarrow A \) is a relation with \( \text{id}_A = \{(a,a) \in A \times A | a \in A \} \). The empty relation \( \phi \subseteq A \times B \) is denoted by \( 0_{AB} \). The entire set \( A \times B \) is called the universal relation and denoted by \( \nabla_{AB} \). The one point set \( \{\ast\} \) is denoted by \( I \). We note that \( \nabla_{II} = \text{id}_I \).

2.2. Relational model

In the relational database theory as originally defined by E.F. Codd[3], a relation \( R \) is a set of tuples \( (t_1,t_2,...,t_n) \), where each element \( t_i \) is a member of, a data domain \( D_i \). That is \( R \subseteq D_1 \times D_2 \times \ldots \times D_n \).

Lemma 2.3. Let \( A = \{a_1,a_2,...,a_n\} \) be a finite set of attributes. For any attribute \( a_i \in A \), there exists an attribute domain \( D_{a_i} \). Let \( X \) be a subset of \( A \). \( D_X \) is defined by \( D_X := \prod_{a \in X} D_a \). We denote the projection map from \( D_X \) to \( D_a \) by \( \rho_{X,a} : \prod_{a \in X} D_a \rightarrow D_a (a \in X) \).
For \( Y \subseteq X \) we define a relation \( \rho_{XY} : D_X \rightarrow D_Y \) as follows: 
\[
\rho_{XY} = \bigcap_{a \in Y} \rho_{X_a} \rho_{Y_a}^\#.
\]
If \( Y = \{a\} \) then it is trivial that \( \rho_{X(a)} = \rho_{X_a}(\forall a \in X) \), and \( \rho_{XX} = \text{id}_{D_X} \).

**Lemma 2.4.** A database scheme \( R = (A, \{D_a | a \in A\}) \) is a pair of an attribute set and a class of attribute domains. An element \( t \) of \( D_A \) is called a tuple.

**Lemma 2.5.** a database relation \( r \) is defined as \( r : D_A \rightarrow D_A \) with \( r \subseteq \text{id}_{D_A} \). That is \( r = \{(t, t) \in D_A \times D_A\} \).

**Example 2.6.** We have relation between name, education and salary that is shown by Intan [4].
Let \( A = \{\text{Name}, \text{Education}, \text{Salary}\} \), Domain of attribute ‘Name’ \( D_{\text{Name}} = \{N1, \ldots, N11\} \), 
\( D_{\text{Education}} = \{\text{Non}, \text{ES}, \text{JHS}, \text{SHS}, \text{BA}, \text{MS}, \text{PHD}\} \), \( D_{\text{Salary}} = \{0, \ldots, 500\} \).
where \( D_{\text{Name}} = D_{\text{Education}} = N \) and \( D_{\text{Salary}} = \Sigma \). \( D_A \subseteq D_{\text{Name}} \times D_{\text{Education}} \times D_{\text{Salary}} \).
Table 3 is a database relation \( r_{ES} \) is defined as \( r : D_A \rightarrow D_A \), in first tuple in table \((N1, \text{Master}, 400)\) means \(((N1, \text{Master}, 400), (N1, \text{Master}, 400))\) in \( r_{ES} \).

**Table 3. Education and Salary**

| Name  | Education            | Salary |
|-------|----------------------|--------|
| N1    | Master(ES)           | 400    |
| N2    | Senior High School(ES)| 150   |
| N3    | PHD                  | 470    |
| N4    | Junior High School(ES)| 200   |
| N5    | Elementary School(ES)| 125   |
| N6    | Senior High School(ES)| 250  |
| N7    | Master (MS)          | 420    |
| N8    | Senior High School(ES)| 175   |
| N9    | Master (MS)          | 415    |
| N10   | Senior High School(ES)| 275   |
| N11   | Non (N)              | 100    |

### 3. Fuzzy Relational Database

**3.1. Basic notation**
We follow the basic notations in Okuma[9]. A fuzzy relation \( \alpha \) from a set \( A \) to another set \( B \) is a fuzzy subset of the Cartesian product \( A \times B \). i.e. \( \alpha : A \times B \rightarrow [0,1] \). It is denoted by \( \alpha : A \rightarrow B \). The inverse fuzzy relation \( \alpha^\# : B \rightarrow A \) of \( \alpha \) is a relation defined by \( \alpha^\# (b,a) := \alpha (a,b) \). The composite \( \alpha \beta : A \rightarrow C \) of \( \alpha : A \rightarrow B \) followed by \( \beta : B \rightarrow C \) is a fuzzy relation defined by \( \alpha \beta (a,c) := \bigvee_{b \in B} (\alpha(a,b) \land \beta(b,c)) \). As a relation \( \alpha : A \rightarrow B \) is a fuzzy subsets of \( A \times B \), the inclusion relation, union, intersection and difference of them are defined as fuzzy subset.

Let \( \alpha : A \rightarrow B \) and \( \beta : A \rightarrow B \) be fuzzy relations. We define the union as \( (\alpha \sqcup \beta)(a,b) := \alpha(a,b) \vee \beta(a,b) \), intersection as \( (\alpha \sqcap \beta)(a,b) := \alpha(a,b) \vee \beta(a,b) \), negation as \( \bar{\alpha}(a,b) := 1 - \alpha(a,b) \) and difference as \( (\alpha - \beta)(a,b) := \alpha(a,b) \land (1 - \beta(a,b)) \).

Let \( A = \{a_1, a_2, \ldots, a_n\} \) be a finite set of attributes. For any attribute \( a_i \in A \), we define a set \( D_{a_i} \) called an attribute domain. Let \( X \) be a finite subset of \( A \). \( D_X \) is the product set defined by \( D_X := \prod_{a \in X} D_a \). A database scheme \( R = (A, \{D_a\}_{a \in A}) \) is a pair of an attribute set and a class of attribute domains. We denote the projection function \( D_X \rightarrow D_a \) by \( \rho_a \) and \( D_X \rightarrow D_Y \) by \( \rho_{X,Y} \) where \( a \in A \) and \( Y \subseteq X \).
A database relation \( r \) is a fuzzy relation \( r : D_A \rightarrow D_A \) which satisfies \( r \subseteq \text{id}_A \) where \( \text{id}_A \) is an identity function on \( A \). We can consider a fuzzy relation \( r \) as a function \( r : D_A \times D_A \rightarrow [0, 1] \) and for a tuple \( t \in D_A \), we call \( r(t, t) \) the fuzzy value of \( t \).

4. Database Operations
The fuzzy database operations model was introduced by Umano and Fukami[12]. They have defined 'projection', 'join', and 'selection'. In their paper, the formulas were used to compute the value of fuzzy relation, not to compute how to get table of 'projection', 'join', and 'selection' from the complete table. In our work, we use relational calculus to compute database operations from the complete relation \( r : D_A \rightarrow D_A \). We use simpler and clearer formula to compute database operations.

(i) Let \( f \) and \( r \) be database relations \( f, r : D_A \rightarrow D_A \). The selection \( \sigma_f(r) \) of \( r \) with \( f \) is the database relation defined by \( \sigma_f(r) = r \cdot f : D_A \rightarrow D_A \).

(ii) Let \( X \subseteq A \) and \( r \) a database relation \( r : D_A \rightarrow D_A \). The projection \( \pi_{A,X}(r) \) of \( r \) to \( X \) is the database relation defined by \( \pi_{A,X}(r) = \rho_{A,X} \cdot r : D_A \rightarrow D_X \).

(iii) Let \( X \subseteq A \) and \( r_X \) a database relation \( r_X : D_X \rightarrow D_X \). The injection \( \eta_{X,A}(r_X) \) of \( r_X \) to \( A \) is the database relation defined by \( \eta_{X,A}(r_X) = (\rho_{A,X} \cdot r_X \cdot \rho_{A,X}^#) \cap \text{id}_A : D_A \rightarrow D_A \).

(iv) Let \( X, Y \subseteq A \) and \( Z = X \sqcup Y \). For database relations \( r_X : D_X \rightarrow D_X \) and \( r_Y : D_Y \rightarrow D_Y \), the natural join \( r_X \bowtie r_Y \) is the database relation defined by \( r_X \bowtie r_Y = \eta_{X,Z}(r_X) \cdot \eta_{Y,Z}(r_Y) : D_Z \rightarrow D_Z \).

5. Fuzzy Equivalence Class and Dependency
5.1. Fuzzy class partition
We begin this section by defining a fuzzy set using fuzzy relational database model. Let \( D_U \) be a classical set of objects \( D_X \), called universe of discourse. An element of \( D_U \) is denoted by \( u \).

**Definition 5.1.** Let a fuzzy relation set be \( f \), Domain of attribute \( X \) be \( D_X \) will be divided to \( n \)-class in domain of attribute class \( U \), \( D_U = \{ u_1, u_2, \ldots, u_n \} \). Let a collection attribute sets be \( A = \{ X, U \} \) with \( D_A \subseteq D_U \times D_X \). Hence, the fuzzy relation set \( f \) is denoted by the following \( f : D_A \times D_A \rightarrow [0, 1] \)

**Example 5.2.** From Example 3, we would like to divide domain of attribute education and salary, \( D_{\text{Education}}, D_{\text{Salary}} \). For \( D_{\text{Education}} \) be divided in 3 class, \( D_{\text{LE}} = \{ \text{High Education(HE)}, \text{Middle Education(ME)}, \text{High Education(HE)} \} \) and \( D_{\text{Salary}} \) be divided in 3 class, \( D_{\text{LS}} = \{ \text{High Salary(HS)}, \text{Middle Salary(MS)}, \text{High Salary(HS)} \} \).

| Table 4. \( f_{\text{ClassEducation}} \) |                |
|----------------------------------------|-----------------|
| Class Education | Education | Fuzzy Value |
| High Education | Non(N) | 1 |
| High Education | ES | 0.8 |
| High Education | JHS | 0.5 |
| Middle Education | ES | 0.2 |
| Middle Education | JHS | 0.5 |
| Middle Education | SHS | 0.9 |
| Middle Education | BA | 0.2 |
| Low Education | SHS | 0.1 |
| Low Education | BA | 0.8 |
| Low Education | MS | 1 |
| Low Education | PHS | 1 |
Table 5. \( f_{\text{ClassSalary}} \)

| Class Salary | Salary | Fuzzy Value |
|--------------|--------|-------------|
| Low Salary   | 100    | 1           |
| Low Salary   | 125    | 0.5         |
| Middle Salary| 150    | 1           |
| Middle Salary| 175    | 1           |
| Middle Salary| 200    | 1           |
| Middle Salary| 255    | 0.9         |
| Middle Salary| 275    | 0.2         |
| High Salary  | 255    | 0.1         |
| High Salary  | 400    | 1           |
| High Salary  | 415    | 1           |
| High Salary  | 420    | 1           |
| High Salary  | 470    | 1           |

Table 4 showed relation \( f_1 : D_{LE} \times D_{Education} \to [0, 1] \), in first tuple we have seen \((HE, Non(N)), 1)\) means that \( f_1((HE, Non(N)), (HE, Non(N))) = 1 \). Table 5 showed relation \( f_1 : D_{LS} \times D_{Salary} \to [0, 1] \), in first tuple we have seen \((LS, 100), 1)\) means that \( f_1((LS, 100)), (LS, 100)) = 1 \).

5.2. Degree of proximity relation

In this subsection, we extend the relational data model to handle fuzzy data, it is necessary to support integrity constraints expressing degree of proximity relation between member of class such that degree of 'approximately equal' or 'more or less equal'.

**Definition 5.3.** Let domain of attribute \( X \) be \( D_X \) will be divided to \( n \)-class in domain of attribute class \( U \), \( D_U = \{u_1, u_2, \ldots, u_n\} \). Let a collection attribute sets be \( A = \{X, U\} \), with \( D_A \subseteq D_U \times D_X \). Fuzzy relation be \( f : D_A \times D_A \to [0, 1] \). Degree of proximity relation between member of domain class \( u_i, u_j \), \( \mu_{PR}(u_i, u_j) \) follows:

Consider that \( u_i = t_i[U] \),

\[
\mu_{PR}(u_i, u_j) = 1 - \wedge \{ f(t_i, t_i) \vee f(t_j, t_j) \mid t_i[U] \neq t_j[U] \wedge t_i[X], t_j[X] \} \text{ and } \mu_{PR}(u_i, u_i) = 1
\]

**Example 5.4.** From table 4, we can compute \( \mu_{PR}(HE, ME) = 1 - \wedge \{(0.8 \vee 0.2), (0.5 \vee 0.5)\} = 0.5 \).

From table 5, we can compute \( \mu_{PR}(LS, MS) = 1 - \wedge \{(0.5 \vee 0.5)\} = 0.5 \).

| \( \mu_{PR} \) | HE | ME | LE |
|---------------|----|----|----|
| HE            | 1  | 0.5| 0  |
| ME            | 0.5| 1  | 0.2|
| LE            | 0  | 0.2| 1  |

| \( \mu_{PR} \) | HS | MS | LS |
|---------------|----|----|----|
| HS            | 1  | 0.5| 0  |
| MS            | 0.5| 1  | 0.5|
| LS            | 0  | 0.5| 1  |

6. Dependency Using Conditional Probability Method

6.1. Class dependency

There are some theory to find dependency relation in every tuple(Table 8). Sozat and Yacizi [13] have defined conformance between tuple to compute dependency between tuple and attribute.
\[ \mu_{x_i|y_j} = \frac{S(P[x_i \cap y_j](h))}{S(P[y_j](h))} \]

\[ \mu_{y_j|x_i} = \frac{S(P[x_i \cap y_j](h))}{S(P[x_i](h))} \]

In this research, we use conditional probability method to compute dependency of class every attribute.

Let \( X, Y \) be collection attribute of a relation. Consider \( X, Y \subseteq A, Z = X \cup Y \). Let \( U_X \in X, U_Y \in Y \) be attribute classification. Domain of attribute \( U_X, D_{U_X} = \{x_1, x_2, ..., x_i\} \) and \( D_{U_Y} = \{y_1, y_2, ..., y_j\} \). For a database relation \( f : D_X \rightarrow D_X, g : D_Y \rightarrow D_Y \) and \( h = f \circ g, h : D_Z \rightarrow D_Z \). We define fuzzy value of \( x_i, y_j, x_i \land y_j \):

We abbreviate \( P[U_X] = x_i(h), P[U_Y] = y_j(h), P[U_X \times U_Y] = y_j(h) \) by \( P[x_i](h), P[y_j](h), P[x_i \cap y_j](h) \)

\[ P[x_i](h) = \sigma_{U_X=x_i}(h) : D_Z \rightarrow D_Z \]
\[ P[y_j](h) = \sigma_{U_Y=y_j}(h) : D_Z \rightarrow D_Z, \text{ and} \]
\[ P[x_i \cap y_j](h) = \sigma_{U_X=x_i \land U_Y=y_j}(h) : D_Z \rightarrow D_Z \]

Let \( D_a \subseteq R \), where \( R \) is a set of all real numbers. For database relation \( h : D_Z \times D_Z \rightarrow [0, 1] \) we define the fuzzy sum \( S(h) \) as follows:

\[ S(h) = \sum \{ h(t, t) \in [0, 1] | t \in D_a \} \]

So, we can define dependency between class \( x_i \rightsquigarrow_{H_a} y_j \),

**Example 6.1.** First, we transform table 3 to fuzzy relational model, \( r_{ES} : D_A \times D_A \rightarrow [1] \). After that we make natural between \( r_{ES}, f_1, \) and \( f_2, \) as we can see in table 9.

From table 10, 11, 12, we can get class dependency \( HS \rightsquigarrow HE, \mu_{HE|HS} = 0.881, \) and \( HE \rightsquigarrow HS, \mu_{HS|HE} = 0.911 \).

Table 13 and table 14 is complete result of dependency between class of attribute. For example, in first tuple we got in table 13 \( \mu_{LS|LE} = 0.6 \) and table 14 \( \mu_{LE|LS} = 0.882353 \), it means ‘Low Education’ more determine than ‘Low Salary’

6.2. Attribute dependency

In this subsection, we would like to redefine dependency between attributes, we change conformance in fuzzy relational database with “degree” of class dependency that we have defined in the previous section.

**Definition 6.2.** Let \( U_X, U_Y \) be class of attribute \( X, Y, U_X = \bigcup_{i=1}^{n} x_i, U_Y = \bigcup_{i=1}^{m} y_j \). Fuzzy functional dependency \( X \rightsquigarrow^\theta Y \) (reads \( X \) determines \( Y \)) if for \( x_i \) and \( y_j \) in \( r \):

\[ \mu_{y_j|x_i} \geq \min(\theta, \mu_{x_i|y_j}) \]

where \( \theta \) is real number within the range [0,1] describing the linguistic strength. When \( \theta = 1 \), fuzzy functional dependency becomes functional dependency(Boolean), denoted as \( r : D_X \times D_Y \rightarrow 0 \) or 1.
Table 9. $r_{ES} \bowtie f_1 \bowtie f_2$

| Name | Class Education | Class Salary | Fuzzy Value |
|------|-----------------|--------------|-------------|
| N1   | High Education  | High Salary  | 1           |
| N2   | High Education  | Middle Salary| 0.1         |
| N3   | High Education  | High Salary  | 1           |
| N4   | Low Education   | Middle Salary| 0.5         |
| N4   | Middle Education| Middle Salary| 0.5         |
| N5   | Low Education   | Low Salary   | 0.5         |
| N5   | Middle Education| Middle Salary| 0.5         |
| N6   | High Education  | Low Salary   | 0.2         |
| N6   | Middle Education| Middle Salary| 0.2         |
| N7   | High Education  | High Salary  | 1           |
| N8   | High Education  | Middle Salary| 0.1         |
| N8   | Middle Education| Middle Salary| 0.9         |
| N9   | High Education  | High Salary  | 1           |
| N10  | High Education  | High Salary  | 0.1         |
| N10  | Middle Education| Middle Salary| 0.1         |
| N10  | Middle Education| High Salary  | 0.5         |
| N10  | Middle Education| Middle Salary| 0.5         |
| N11  | Low Education   | Low Salary   | 1           |

Table 10. $P([HE]$)

| Name | CE | CS | $P([HE]$) |
|------|----|----|-----------|
| N1   | HE | HS | 1         |
| N2   | HE | MS | 0.1       |
| N3   | HE | HS | 1         |
| N6   | HE | MS | 0.1       |
| N7   | HE | HS | 1         |
| N8   | HE | MS | 0.1       |
| N9   | HE | HS | 1         |
| N10  | HE | HS | 0.1       |
| N10  | HE | MS | 0.1       |

$S(P([HE])) = 4.5$

Table 11. $P([HS]$)

| Name | CE | CS | $P([HE]$) |
|------|----|----|-----------|
| N1   | HE | HS | 1         |
| N3   | HE | HS | 1         |
| N7   | HE | HS | 1         |
| N9   | HE | HS | 1         |
| N10  | HE | HS | 0.1       |
| N10  | ME | HS | 0.5       |

$S(P([HE])) = 4.6$

Table 12. $P([HE \cap HS])$

| Name | CE | CS | $P([HE]$) |
|------|----|----|-----------|
| N1   | HE | HS | 1         |
| N3   | HE | HS | 1         |
| N7   | HE | HS | 1         |
| N9   | HE | HS | 1         |
| N10  | HE | HS | 0.1       |

$S(P([HE])) = 4.1$

Example 6.3. From table 13, and table 14, we can compute Education $\rightarrow^\theta$ Salary:

(i) $(LE$ and $LS) \mu_{LS|LE} \geq \min(\theta, \mu_{LE|LS}) = 0.6 \geq \min(\theta, 0.82353)$

(ii) $(LE$ and $MS) \mu_{MS|LE} \geq \min(\theta, \mu_{LE|MS}) = 0.4 \geq \min(\theta, 0.188679)$

(iii) $(LE$ and $HS) \mu_{HS|LE} \geq \min(\theta, \mu_{LE|HS}) = 0. \geq \min(\theta, 0.)$
Table 13. $\mu_{\text{ClassSalary}|\text{ClassEducation}}$

| CS          | CE            | $\mu_{\text{CS}|\text{CE}}$ |
|-------------|---------------|-----------------------------|
| Low Salary  | Low Education | 0.6                         |
| Low Salary  | High Education| 0                           |
| Low Salary  | Middle Education| 0.0434783                |
| Middle Salary| Low Education | 0.4                         |
| Middle Salary| High Education| 0.0888889                |
| Middle Salary| Middle Education| 0.847826                |
| High Salary | Low Education | 0                           |
| High Salary | High Education| 0.911111                |
| High Salary | Middle Education| 0.108696                |

Table 14. $\mu_{\text{ClassEducation}|\text{ClassSalary}}$

| CE            | CS          | $\mu_{\text{CE}|\text{CS}}$ |
|---------------|-------------|-----------------------------|
| Low Education | Low Salary  | 0.882353                   |
| Low Education | Middle Salary| 0.188679                |
| Low Education | High Salary | 0                           |
| High Education| Low Salary  | 0.117647                   |
| High Education| Middle Salary| 0.735849                |
| High Education| High Salary | 0.108696                   |

(iv) (HE and LS) $\mu_{\text{LS}|\text{HE}} \geq \min(\theta, \mu_{\text{HE}|\text{LS}}) = 0.043 \geq \min(\theta, 0)$

(v) (HE and MS) $\mu_{\text{MS}|\text{HE}} \geq \min(\theta, \mu_{\text{HE}|\text{MS}}) = 0.089 \geq \min(\theta, 0.075)$

(vi) (HE and HS) $\mu_{\text{HS}|\text{HE}} \geq \min(\theta, \mu_{\text{HE}|\text{HS}}) = 0.911 \geq \min(\theta, 0.89)$

(vii) (ME and LS) $\mu_{\text{LS}|\text{ME}} \geq \min(\theta, \mu_{\text{ME}|\text{LS}}) = 0.043 \geq \min(\theta, 0.117)$

(viii) (ME and MS) $\mu_{\text{MS}|\text{ME}} \geq \min(\theta, \mu_{\text{ME}|\text{MS}}) = 0.848 \geq \min(\theta, 0.7358)$

(ix) (ME and HS) $\mu_{\text{HS}|\text{ME}} \geq \min(\theta, \mu_{\text{ME}|\text{HS}}) = 0.11 \geq \min(\theta, 0.11)$

from that condition we can conclude that minimum value of $\theta$ is less than 0.043. It means that X and Y are weak relation. But, using class dependency we can get conclusion that in the some level X and Y has strong relation.

Compare with Kiss Model:
Let $A = \{A_1, A_2, \ldots, A_n\}$ be collection of attributes sets, $R$ be a fuzzy relation, $R : A_1 \times A_2 \times \ldots \times A_n \rightarrow [0,1]$, $t_1, t_2$ be tuples such that $t_1, t_2 \in R$.

"Degree" dependency of fuzzy database by Kiss is:

$$T_R(A_1, A_2) = 1 - \land \{R(t_1) \lor R(t_2)\}, t_1(A_1) = t_2(A_1) \land t_1(A_2) \neq t_2(A_2).$$

Using Kiss’s formula we can compute degree dependency between attributes.

For example, we can compute degree dependency between "Class of Education→ Class of Salary" $T_R(\text{LE}, \text{LS})$ from table 9. We got $\text{ClassEducation} \sim_{0.1} \text{Class Salary}$.

The Problem: Using Kiss’s formula, we can compute dependency between attribute but can not compute "degree" dependency between class of attribute. As we can see in table 13 and table 14, degree dependency of "High Education" to "High Salary" different with degree dependency of "High Education" to "Low Salary". It means using conditional probability, we can compute and analyze trend data more precisely. Using conditional probability, we introduce formula degree of class of functional dependency. In table 13 and table 14, we got
Low Education \Rightarrow_{0.88233} Low Salary and Low Salary \Rightarrow_{0.6} Low Education, from the result we can conclude that "Low Education" more give effect to "Low Salary".

7. Conclusion and Future Works
Generally, our research has shown how to extract intelligent information from fuzzy relational databases. In order to to find sub-relations where criteria for the existence of fuzzy dependencies. We have tried to identify possible dependency between class of attributes of fuzzy database, and also we try to find dependency between attributes using the previous theorem of fuzzy implications. The most important contribution of this research is characterization of the theory of partial dependency or class dependency in the context of developing tools and algorithms that facilitate identification of relations between class of attributes in the observed relational schemes. Using fuzzy relational calculus, we make new formula of fuzzy database operations, class of dependency, find proximity relation between class of attributes and dependency between attributes. Hopefully, the notion is very useful to make database analysis, knowledge discovery application in fuzzy relational database. Using fuzzy relational calculus, we can compute easily the functional dependency using conditional probability method. Using class dependency, we can making analysis deeper and accurate, because every class has different dependency. In the next research, we would like to continue in soundness and completeness theorem and making simplification for implication problem in fuzzy database using deduction category and others. Fuzzy relational database is a combination of relational database and fuzzy theory.

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