Application of nonlinear dynamic techniques to high pressure plasma jets

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1. Abstract

Arcs and arc plasmas have been known and used for welding, cutting, chemical synthesis and multitude of other industrial applications for more than hundred years. Though a copious source of heat, light and active species, plasma arc is inherently unstable, turbulent and difficult to control. During recent years, primarily driven by the need of new and energy efficient materials processing, various research groups around the world have been studying new and innovative ways of looking at the issues related to arc dynamics, arc stabilization, species non equilibrium, flow and heat transfer in a stabilized arc plasma device. In this context, experimental determination of nature of arc instabilities using tools of non-linear dynamics, theoretical model formulation, prediction of instability behavior under given operating conditions and possible control methods for the observed instabilities in arcs are reviewed.

Space selective probing of the zones inside arc plasma devices without disturbing the system is probably the best way to identify the originating zone of instabilities inside such devices. Existence of extremely high temperature and inaccessibility to direct experimentations due to mechanical obstructions make this task extremely difficult. Probing instabilities in otherwise inaccessible inner regions of the torches, using binary gas mixture as plasma gas is a novel technique that primarily rests on a process known as demixing in arcs. Once a binary gas mixture enters the constricted plasma column, the demixing process sets in causing spatial variations for each of the constituent gases depending on the diffusion coefficients and the gradient of the existing temperature field. By varying concentrations of the constituent gases in the feeding line, it is possible to obtain spatial variations of the plasma composition in a desired manner, enabling spatial probing of the associated zones. Detailed compositional description of different zones inside the torch may be obtained through appropriate numerical simulation. Potential of such experiments with judicious mix of numerical simulations is discussed.

2. Introduction

An arc plasma device is the hardware designed to convert the electrical energy of the arc to the thermal energy of the plasma jet. The physical and operational state of an arc plasma system is best described in terms of the static and the dynamic characteristics. The static characteristics normally indicate the steady state behaviour whereas the dynamic characteristics describe the evolution of the system behaviour with time. The instabilities arising out of interaction among the electromagnetic, fluid dynamic and the thermal fields give rise to erratic fluctuation of the plasma quantities inside the arc plasma devices. Such instabilities profoundly affect the system performance in most of the plasma processing applications. The challenge in development of the next generation high performance arc plasma devices is to understand the origin of such instabilities, predict the regime of stable/unstable operation and determine the control parameters for the excitation or elimination of these instabilities. This article offers a brief review of the problem and some of the recent developments.

Although there is no absolute definition of stability, a system in the state of equilibrium is called stable if any infinitesimal perturbations to the equilibrium state tend to decay in course of time. In all other cases the system is considered unstable and time evolution of the perturbation is termed as instability of the system [1]. Familiar concept of instability in hot magneto-plasmas basically refers to...
catastrophic instability of concerned plasma against deformation. Usual plasma instabilities like kink instability, sausage instability, two-stream instability, drift instability etc., once induced start growing until the system collapses. Investigation is concluded in such systems by studying the growth rate of such instabilities. Nonlinear dynamics in continuous media includes vast variety of dynamics and defines instability in a more general way. Lyapunov stability, asymptotic stability, Poincare or orbital stability and Lagrange or bounded stability are frequently used definitions [2]. A system, stable under one definition, may be unstable under the other. A solution $u(t)$ of a system of differential equations is said to be Lyapunov stable, if, given a small number $\varepsilon > 0$, there exists a number $\delta(\varepsilon) > 0$ such that for any other solution $v(t)$ for which $\|u - v\| < \delta$ at time $t = t_0$, satisfies $\|u - v\| < \varepsilon$ for all $t > t_0$. Stabilities of equilibrium solutions are usually described using this definition. However, if $\Gamma_1$ and $\Gamma_2$ are orbits corresponding to periodic solutions ‘u’ and ‘v’ of different periods, their motions evolve in different time scales and may not be stable under definition of ‘Lyapunov stability’. The solution is said to be stable under ‘Poincare stability’ if, given a small number $\varepsilon > 0$, there exists a number $\delta = \delta(\varepsilon) > 0$ such that if $\|u(t = 0) - v(t = \tau)\| < \delta$ for some $\tau$, then there exist $t_1$ and $t_2$ for which $\|u(t_1) - v(t_2)\| < \varepsilon$. A solution $u(t)$ of a system of differential equations is said to be ‘asymptotically stable’ if it is Lyapunov stable and $\lim_{t \to \infty} \|u - v\| \to 0$. The solution is said to be ‘Lagrange stable’ if, $\|u\| \leq L$ for all $t$, where $L$ is a finite positive quantity.

Plasma being a highly complex system having large number of influencing parameters, it is expected that such systems will exhibit vast variety of dynamical behavior. A look at the chronological development on the study of instabilities in plasmas reveals that the field is rich in both theoretical and experimental investigations, though mostly confined in the field of low-pressure gas discharges. Investigation on arc plasma devices basically followed two roots: numerical simulation of arc plasma column under steady state and experimentations to measure various plasma quantities [3-7]. Only few efforts have been devoted to understand the basic nature of the electrode phenomena in arc plasma devices [8-13]. Thermal, fluid dynamic and electromagnetic phenomena occurring inside the region generate inherent fluctuations in arc plasma devices through complex nonlinear interaction among themselves. Impact of such fluctuations on arc plasma devices and related applications has been understood in depth [8]. Study on dynamic and static behavior of dc vortex plasma torches, having button type cathodes are reported in Ref. [9]. Fluctuation in arc voltage, arc current (I), electrical power, optical radiation and acoustic signal generated from such devices are investigated in terms of various dimensionless numbers and frequency spectra of respective signals. Similar study is presented in Ref.[10] for dc vortex plasma torches having well type cathode. Fluctuation in arc voltages for various arc currents for spray plasma torches are reported in Ref.[11]. Fluctuating behavior of Sulzer Metco F4 DC plasma gun has been investigated in Ref.[12]. Temporal, spectral and statistical analyses are carried out on fluctuating arc voltage, arc current and optical output from the torch. Various features of arc root fluctuation like voltage jump and spot lifetime are studied in detail for upstream and down stream striking of dc spray plasma torch [13]. These fluctuations in plasma quantities were basically stamped as random and beyond control. Practically no investigation was carried out to investigate detail dynamics of such systems using tools of nonlinear analyses till then.

However, researchers in the field of mathematics have done intensive investigations on hydrodynamic and hydromagnetic instabilities. Excellent reviews on this are available in literature [14,15]. In the process standard recipe for extracting instability features in time dependent and time independent dynamical systems have come out. Concept of amplitude equation in study of one-dimensional dynamical systems was first introduced by L. Landau [16]. Extending his idea, a formal framework for obtaining amplitude equation in systems described by set of differential equations was devised by Arneodo, Coullet and Spiegel [ACS] [17]. ACS technique is verified to give true dynamics of the system through comparison of outcome of the technique with direct numerical simulation [18].
This technique is extremely useful to obtain qualitative nature of dynamics of the system without going into details of solution of each and every quantity involved. The problem of arc plasma instability is hydromagnetic in nature. Major contribution to such hydromagnetic instability comes from Prof. Chandrasekhar [14]. However, his related studies address mostly astrophysical objects and conducting non-plasma systems like mercury. Although, arc plasma devices encompass rich lode of hydrodynamic and hydromagnetic behavior, surprisingly nearly no investigation is reported by researchers on such systems along this line. Study of such systems attracts special attention due to the fact that they offer extremely easy means to detect nature of instabilities reigning inside the systems. Fluctuating arc voltage, emitted acoustic fluctuations and disturbance in the total optical output are the three easily measurable signals from the system that can provide almost every information regarding nature of the existing instabilities. The first definite experimental evidence of chaotic dynamics in arc plasma devices was reported at the author’s laboratory [19, 20] in the year 2000. It conclusively demonstrated that typical erratic fluctuations observed in arc plasma devices under usual operating conditions are macroscopic manifestation of underlying chaotic dynamics. Subsequent experimental and theoretical investigations by the same group substantiated this through development of a general theoretical model of the phenomenon [21-25].

Dynamical systems, exhibiting chaotic solutions, fall in special category. Solutions of such systems are neither periodic nor stationary nor quasi-periodic but remain bounded in phase space. Motion is basically a superposition of very large number of unstable periodic motions. System constantly evolves from one periodic motion to another resulting in a long-term impression of randomness and short-term impression of order. Systems exhibit extreme sensitiveness to initial conditions. Exponential divergence of nearby trajectories is an important characteristic. They exhibit strange structures in phase space (called attractors), which are neither finite number of points nor closed curves nor torus nor even manifolds. Repeated folding of orbits and their exponential divergence result in fractal dimension of these attractors in phase space together with positive value of Lyapunov exponents. Obviously, such systems are unstable under definition of ‘Lyapunov stability’ and ‘Poincare stability’. However, in spite of exponential divergence, motion remains bounded in phase space within the attractor and hence maintains ‘Lagrange stability’. Catastrophic behavior [2] under certain operating condition, exhibited by many chaotic systems, results in solution that ultimately diverges to infinity for all initial conditions, leaving behind a trail of the initial chaotic dynamics called transient chaos. Period of transient chaos depends on the initial conditions. The existing attractor suddenly disappears from state space and post bifurcation response jumps to remote attractor that may or may not be bounded. In case of unbounded attractor, such systems are unstable under definition of ‘Lagrange stability’ too. The study of instabilities in arc plasma devices thus includes investigation of all these discussed behaviors of instability through experiment, development of theory and application of the developed theory to experimental systems. This articles reviews these developments and probable techniques to investigate such instabilities.

3. Experiments and analysis of arc plasma dynamics

Since arc exists inside every arc plasma devices, such experiments are rather easy and can be carried out in any of such arc devices. Whenever an arc device operates, three signals, namely, arc voltage, acoustic emission and optical emission from the device bear most of the information regarding ongoing dynamics inside the device. Analyses of these signals using tools of dynamical analysis give information regarding nature of the existing dynamics and possible control methods. Time series, frequency spectrum, phase portrait, dimension and Lyapunov exponents are the normally used tools for estimation of various dynamic properties associated with these signals. Signatures of five different types of dynamics as come out from different diagnostic tools are presented in Table-1.
Reaching a unique conclusion regarding existence of a particular nature of dynamics needs careful analyses of the results obtained through application of all theses tools together. Improper application of the tools may lead to false impression of dynamics [26-29].

Table 1. Signatures of five different types of dynamics on various diagnostic tools

| Sl. No | Type of dynamics | Time series | Phase portrait | Signature in FFT | Dimension | Lyapunov exponent |
|-------|------------------|-------------|----------------|------------------|-----------|------------------|
| 1     | Steady           | Shows constant value with time | Shows a point | Shows no peak except at zero frequency | Zero      | Zero             |
| 2     | Periodic         | Shows simple periodic repetition | Shows simple orbit | Shows sharp peak at particular frequency | Integer   | Zero             |
| 3     | Quasi-periodic   | Shows complex periodic repetition | Shows complex orbit | Shows sharp peaks at number of frequencies | Integer   | Zero             |
| 4     | Chaotic          | Shows no repetition | Shows specific strange looking pattern called ATTRACTOR | Shows continuous spectra | Fractal   | Positive         |
| 5     | Random           | Shows no repetition | Uniformly fills up the phase space | Shows continuous spectra | Infinite  | Zero             |

Typical fluctuating signals, corresponding power spectrum and reconstructed attractor in the phase space for arc dynamics are displayed in Fig.1. Schematic of the experimental set up and operating conditions used to obtain these signals is available in Ref. [19]. According to theory of generation of acoustic waves in plasma, the amplitude of acoustic signal should be proportional to the derivative of voltage signal under constant current operation. Genuineness of the received signals may be confirmed through verification of this fact [20]. In Fig.1, what is observed in each case is a seemingly erratic experimental signal associated with a strange structure in phase space and a continuous broadband power spectrum. As per Table-1, all these are characteristics of chaotic system. However, just looking just at the time series it is difficult to distinguish whether these signals belong to periodic, chaotic or random system. Part of the time series of a chaotic system may look like a periodic signal and a random looking signal may also be a chaotic signal [30-31]. The phase portrait of a system is one of the important tools for looking deep into the dynamics of the system. It is possible to distinguish periodic and aperiodic systems from these portraits but aperiodic and chaotic systems may look alike. In aperiodic systems, structures having fractal dimension (termed as strange attractors) may appear in state space. However, appearance of a strange structure in phase space does not necessarily prove existence of chaos. Chaotic attractors associate fractal dimensions together with a positive rate of expansion (Lyapunov exponent) of nearby trajectories in phase space. A number of strange attractors are reported [32–34] that do not satisfy positivity of Lyapunov exponent but show fractal dimensions [35-37].
A deterministic system can have as many Lyapunov exponents as dimension of the corresponding phase space. For a physical system whose phase portrait shows a complex and definite geometrical structure with orbits that are neither periodic nor quasi-periodic, it suffices to show at least one Lyapunov exponent positive to prove that the system as chaotic. The largest Lyapunov exponent ($\lambda_1$) is computed using an algorithm developed by Wolf et al. [38]. The idea is that, if at least a single exponent is positive it will be reflected in this case. Here it may be argued that other than chaotic systems, random noises also are reported to exhibit positive Lyapunov exponent and continuous broad band Fourier power spectra. Therefore, a comparison of results with similar analysis for a set of random data and regular signal is also essential. Fig.2. presents such an exercise including the results for theoretically generated signals discussed in the later part of this article. It is observed that for regular signal the value is zero, for random signal the exponent keeps on falling as dimension of state space increases and for the experimental and the theoretically generated signals the exponent value remains steady and positive with dimension of state space.

A careful study through computation of dimension is also necessary. The most widely used algorithm for computation of dimension is due to Grassberger and Procaccia [39]. This approach is based on the assumption that deterministic dynamics yield convergence of the slope of the correlation integral at a finite and small value, indicating a low dimensional attractor. Unfortunately, this assumption is not always true. An insufficient number of data points, linear correlation in the data, or other causes can lead to false convergence which creates illusion of proof of chaos where there is none [26-29]. Result of computation of correlation dimension for a typical experimental signal is shown in Fig.3. An algorithm developed and benchmarked in the laboratory [21] has been utilized. Similar
computation for a random signal having similar statistical properties is also shown in the same plot. Dimension of embedding state space in the computation is varied from 2 to 8. In each case $D_C$ is computed from a least square fit of the data in the linear portion of the plot of $H_2(\varepsilon)$ against $\ln(1/\varepsilon)$ [21]. It is seen from the figure that for the random signal, $D_C$ increases almost linearly with dimension of the state space, whereas for the experimental signal $D_C$ reaches almost a stationary level after state space dimension four and provides a dimension around 2.3. This fractal dimension together with positive Lyapunov exponent proves inherited dynamics in arc plasma to be definitely chaotic.

**Figure 2.** Computation of Lyapunov exponent for (a) regular sine wave (b) random noise signal (c) experimental acoustic signal and (d) theoretically generated acoustic signal.

**Figure 3.** Computation of fractal dimension of the experimental and a noise signal
For a theoretical understanding of the observed behaviour, let us consider the formation of arc inside an arc plasma device, illustrated in Fig.4. A gas passes through the space between the cathode and the nozzle. Electrical breakdown of the gas establishes a current path between the two electrodes forming an arc. The arc heats the gas leading to formation of plasma which finally comes out as a jet through the exit. Aerodynamic drag force tries to push the arc downstream while electromagnetic body force tries to keep the arc at minimum energy configuration. The arc should stay at a position where the two forces balances each other. Interestingly, although there is no other external force, such balance does not hold in reality and a certain kind of dynamics is always exhibited by the arc resulting in the observed apparently erratic signals. The theory aims at explaining this behaviour and starts from the postulate that the observed instability originates from the region near the arc root. Results of a novel experimental technique, described in the later part of this article, strongly supports this postulate.

The arc root region is modelled in terms of conservation of mass, momentum and heat coupled with the Maxwell’s equation and equation of state. Details of these modelling effort is available elsewhere [23,24]. Under Bousinesque approximation, the set of nonlinear differential equations for fluctuating plasma field quantities are first nondimensionalized and then simplified by removing pressure term and introducing stream function. Plasma field quantities are expressed in vector form, which transforms according to associated operators in the plasma field. The reduced set of governing equations depends on number of non-dimensional numbers like Rayleigh number, Prandtl number, Lewis number and Chandrasekhar number. Coupling of the electromagnetic effects into this problem comes through the Chandrasekhar number. In plasma parameter space, there is a critical hypersurface, where, the growth rate of linear theory vanishes. When plasma parameters are very near to this polycritical condition, the temporal behaviour of the system becomes complicated and may be chaotic. Components of the plasma field vector in this region are split into two parts: one in stable space and other in critical space. Temporal behaviour of amplitude of the plasma field vector is studied near the polycritical condition. Behaviour of the system under such condition is determined, first finding the linear amplitude equation and then computing the leading nonlinear terms. Critical polynomial and parameter values for the onset of lowest order instability are determined and system behaviour under different parametric conditions are generated from a resulting nonlinear amplitude equation [24-25]:

\[ \ddot{F} + \dot{F} + \Omega_0 \dot{F} + \Omega_1 F = \pm F^3 \]  

where, \( F \) is the amplitude of the instability and the coefficients \( (\Omega_0, \Omega_1) \) depend on properties of the plasma realized under that operating condition.

Under normal operating conditions \( \Omega_1 \) exhibits nearly a constant value and \( \Omega_0 \) serves as the primary control parameter for the dynamics [24]. A numerical experiment through variation of \( \Omega_0 \)

![Figure 4. The formation of arc inside an arc device](image)
reveals a nice gradual transition from non-oscillatory to regular oscillatory to chaotic regime through period doubling route similar to the experiments but in a more magnificent manner. Fig.5 depicts this period doubling scenario as a route to chaos. Theory recognizes arc current and gas flow rate as the major externally controlling parameters in agreement with the experiment. The coefficient regimes, where an experimental system reaches near criticality are obtained from theory and found to be fairly within the experimentally favoured regime under normal operating conditions.

A comparison of the theoretically obtained signals with those obtained experimentally is interesting and presented in Fig.6. It is observed that the signals generated from the theory match very well in time series, phase portrait, Lyapunov exponent and dimension with the experimental data. Theory of acoustic wave generation in thermal plasma demands existence of an envelope function in the frequency domain of the acoustic signals [25]. Frequency spectra of Fig.6(b) include this envelope and offer an excellent agreement. Phase portraits of Fig. 6(c) and 6(d) show similar evolutions and structures. Slopes of the linear portion of the graphs in Fig. 6(g) and Fig.6(h) give dimensions of respective signals [21]. Both from theory and experiment we get a correlation dimension of 2.3.

To explore the features of simultaneous signals, theoretically obtained simultaneous voltage and acoustic signals are studied. Both are found to provide a correlation dimension of 2.3 in agreement with that observed experimentally. Invariability of the computed dimension is established by computing the dimension in various reconstruction state spaces having dimension ranging from 2 to 7. Identical fractal dimension of theoretical and experimental attractors essentially confirms that the phenomena modelled by the theory take place in the same state space, where the experimental phenomena happen. A detail study on Lyapunov exponent, carried out on regular oscillatory, random, experimental and corresponding theoretical signals, all having similar number of data points and same sampling interval, assures authenticity of the results obtained. These studies are carried out in reconstruction state space dimension ranging from 3 to 8 and the results confirmed both theoretical and experimental signals possess positive Lyapunov exponents very close to each other.

Possible excitations of higher order modes in arc plasma instability have also been investigated theoretically. It has been observed that for typical operations, excitation of the fundamental mode is the most probable and that few modes near to fundamental may also be excited.

![Figure 5. Period doubling route to chaos realized through variation of $\Omega_0$ in the theory. (a) period one orbit (b) period two orbit (c) period four orbit (d) reached in chaotic regime.](image-url)
However, when one mode is excited, possible other modes are damped. Higher order modes, if excited, offer similar dynamics but in different operating zone. It has been observed that under certain operating regimes current attractor suddenly disappears from state space and the post bifurcation response jumps to remote unbounded attractor [24]. In pre-bifurcation state, system exhibits usual oscillatory behaviour. Such catastrophic behaviour is similar to well known “Blue sky catastrophe” in nonlinear dynamics. Sudden extinction of arc, often encountered in certain operating regime of arc operation can be explained using this fact. Poor dependence on gravity related terms indicates that the phenomenon will be observed irrespective of alignment of the device (vertical/horizontal), a true fact observed in practice. In agreement with experimental signals, the theoretical voltage signal is found to produce saw-tooth time series and self-similar (saw-tooth inside saw-tooth) behaviour [24]. Both theoretical and experimental voltage signals exhibit similar nature of power spectra and phase portrait.

The present theory is developed for marginal instability only. Excellent agreement between experimental and theoretical behaviour suggests that the fluctuations observed in most of the arc plasma devices arise out of marginal instability in the systems. Such tendency may be a natural outcome of the non-equilibrium dynamics prevailing. However, issues regarding reason behind such tendency remains to be clarified. Because of non-dimensional nature of the formulation, the developed theory inherits general applicability and can be used to predict nature of instability in other experimental systems too. Literature is surveyed to identify important experiments worldwide on arc plasma instabilities and the developed arc instability theory has been applied to most of them. Results obtained are compared with the published experimental results and nice mach is observed in each case.

The theory is based on the postulate that the instability originates from the zone near the arc root. However, it may be argued that in Fig.4 the instability may arise from any part of the channel like the near cathode zone (A), the constriction zone after the cathode (B), the flow affected zone (C) or the near arc root zone (D). Recently, a novel experimental approach is devised to identify the originating zone of instabilities[40] which eventually supports the postulate of this theory. The method is based on a process known as demixing of component gases [41] in a binary gas mixture, illustrated in Fig. 7. In the illustration, a uniform mixture of argon and hydrogen enters through the left end of the channel. In the middle it encounters a radial distribution of temperature field due to an arc. When it finally comes out from the other end of the channel, the hydrogen atoms are mostly aligned along the periphery and the argon atoms are clubbed along the central axis. While Fig.4 is obviously an exaggeration, the actual measurement goes like one in Fig.8 [41]. In the measurement, a uniform mixture of argon and hydrogen (the proportion of hydrogen was 5% in one case and 1% in the other case) is used in a free burning arc with a 2% thoriated tungsten cathode of diameter 2.5 mm. At an axial distance of 1 mm below the tip of the cathode, argon and hydrogen concentrations are measured using laser scattering technique [41]. As observed (Fig.8), there is a significant increase in the relative concentration of hydrogen as one moves away from the central axis. Almost similar results were obtained 2.5 mm below the cathode. The overall observation is that for an argon-hydrogen mixture, the arc temperature field and the associated diffusion mechanism make the core region of the plasma channel dominated by argon and the boundary region dominated by hydrogen. Therefore, if one changes argon concentration in the uniform mixture, finally the core region will be affected. On the other hand, if hydrogen concentration is changed, the boundary region of the channel will be influenced. In other words, this phenomenon actually provides a means to probe the core and the boundary region behaviour by varying concentration of hydrogen and argon in the plasma gas at the entrance. Results of performing the described exercise are presented in the following section.
Figure 6. Comparison between experiment and theory for I=300A: (a) time series (b) Power spectra (c) Theoretically obtained phase portrait (d) Experimental phase portrait (e) Evolution of Lyapunov exponent for theoretical signal (f) Evolution of Lyapunov exponent for experimental signal (g) Dimension computation of the theoretical signal (h) Dimension computation of the experimental signal.

Figure 7. An illustrative sketch showing Ar-H demixing inside an arc plasma device.
In Fig.9 total hydrogen flow rate per minute ($F_{H}$) to produce an uniform mixture is kept constant (5 slm) but argon flow rate ($F_{Ar}$) for the same is varied over a range. Consequently, operating point D and E correspond to widely different Ar:H mass ratios for a uniform mixture and hence markedly different thermodynamic and transport properties of the working gas. If there is no demixing, uniformity of the mixture is maintained throughout the arc channel and distinctly different instability features are expected corresponding to operation at point D and point E (because of widely different properties of the gases). However, in actual experiment the instabilities recorded corresponding to point D and E do not show any sharp change (Fig.9(b)). Effect of increasing the argon flow rate in such cases has been investigated at different current levels [40] and it has been seen that other than highly linear increase in average arc voltage [Fig.10] other features do not show any distinct variation. So assumption of no demixing cannot explain the results. Now let us assume that there is demixing. In that case most of the hydrogen atoms are lined up along the periphery and concentration of that depends only on total flow rate of hydrogen (which is not changing in this case) whatever may be the flow rate of argon (within a range). Since composition of the layer over the anode surface (inner wall of the channel) is not changing, same instability behaviour is expected to be observed irrespective of the flow rate of argon. Thus demixing is able to explain the observed behaviour. However, together with this it gives us another information: the instability is originating from the layer over the anode surface. This is because with demixing the core region is dominated by argon and its concentration is changing each time the argon flow rate is changing. If the instability was originated from the core we would observe a change in the instability feature for every change in the argon flow.

To confirm this proposition another set of experiments were conducted in which the Ar:H$_2$ mass ratio was kept constant in the plasma gas. In Fig. 11(b), operating point A and C have the same Ar:H$_2$ mass ratio. If there were no demixing, identical uniform mixtures are present at both these two points and therefore, nearly similar instability features are expected at these two points (because gas composition is not changing). However, in actual experiment the results observed are presented in Fig.11(b). The instability features experienced a sharp change. Therefore, assumption of no demixing is not able to explain the features again. Now if demixing is present, operating point A associates 3 slm of hydrogen and operating point C associates 5 slm of hydrogen. Since now most of the hydrogen molecules are clubbed over the wall through demixing, each time there is a change in hydrogen flow, property of the layer over the wall experience a significant change and hence a change in the instability behaviour observed. Thus demixing again explains the observed fact nicely.
Figure 9. (a) Mass ratio of argon atom and hydrogen molecules per unit volume as a function of argon flow rate for a fixed hydrogen flow rate (5 slm) in the uniform mixture of gases. Operating point D: (Ar: 42 slm; H$_2$: 5 slm) and point E: (Ar: 63 slm; H$_2$: 5 slm) (b) Observed instability amplitude as a function of time corresponding to operating points D and E. Arc current 500A. For visibility, a DC shift of 30V is applied to the upper curve in Figure 9(b).

Figure 10. Effect of increase in argon flow rate at different hydrogen flow rates. Average arc voltage exhibits a highly linear increase with argon flow rate. For a fixed hydrogen flow rate instability behaviour remains almost unchanged over a range of argon flow rate [40]. Arc current 500A.

Figure 11. (a) Mass ratio of argon atom and hydrogen molecules per unit volume as a function of argon flow rate for different hydrogen flow rates in the uniform mixture of gases. Operating point A: (Ar: 42 slm; H$_2$: 3 slm) and point C: (Ar: 70 slm; H$_2$: 5 slm) (b) Observed instability amplitude as a function of time corresponding to operating points A and C. Arc current 700A. For visibility, a DC shift of 30V is applied to the upper curve in Fig. 11(b).
Results of the above two experiments provide strong evidences that the instability is originating over the boundary layer on the anode surface. It remains to be identified the particular location from which the instability is originating. A look at the schematic of Fig.4 indicates that the arc is attached over the anode surface at location like D (arc root) only. Current does not take a passage anywhere else over the anode surface other than this (D). Therefore, if the instability is originating from some location other than D (over the anode boundary layer), it will not associate any dependence on current. But experimentally, arc current has been found to be the most sensitive control parameter for the observed instability [40]. The experiment thus produces strong evidences that the arc instability originates from the region near the arc root.

4. Summary and discussion

Arc plasma technologies are now being widely used in numerous industrial processes. Plasma generating devices function as the key device in all of these applications. In spite of extensive research and development, basic problems in such devices lie with the inherent fluctuations in various plasma quantities associated with the generated plasma. So far it has been shelved as random and considered beyond control. A recent series of works on such instabilities indicates that the observed apparently erratic fluctuations in arc plasma devices are not random but chaotic under typical operating conditions. These works not only establish occurrence of chaos in arc plasma devices but also identifies chaotic and non-chaotic operating regime, chaos control parameters, and a theoretical basis for the observed fluctuations in arc plasma devices. Moreover, the works introduces a novel experimental technique based on the process natural demixing in binary gas mixtures to probe inner region of such devices. This is important because direct probing of inside of such plasma devices is not possible due to mechanical obstructions and existence of extremely high temperature. Application of the technique received a good success in exploring the instability features in arc plasma devices.

Identifying phenomenon of fluctuation in arc plasma devices as chaotic and not random is an important outcome of the work because unlike random systems, chaotic systems are controllable. The observed chaotic behavior in high pressure plasma beams may be linked to a number of possible physical effects within the torch like electrode losses due to erosion, puncturing of device, sudden extinction of arc, change of surface conditions of the electrodes, change in operating characteristics of torch, etc. The present series of works attempts at searching a path to control all these non-linear interactions employing knowledge of nonlinear dynamics. Once geometry, arc current, plasmagen gas and its flow rate are specified, static and dynamic behaviour of any arc plasma device is frozen. So far, standard fluid dynamic simulations were able to predict steady state behaviour of the system for these inputs but not dynamic behaviour. The series of works discussed in this article sets a new direction where dynamic behaviour of the arc plasma devices is also predictable based on the chosen operating conditions.

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