Microscopic calculation of transition intensities for vibrational bands and high-K isomers

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Abstract

We investigate the effect of the Coriolis coupling and the residual interactions upon the inter-band transition rates for the vibrational bands and the decay of two-quasiparticle high-K isomers.

I. INTRODUCTION

The Coriolis coupling in rotating nuclei often leads to significant consequences. The back-bending phenomenon is a typical example in which a nucleon pair (Cooper pair) is broken by the Coriolis force. In order to understand the effect of Coriolis coupling, it is important to measure both the energy spectra and the inter-band electromagnetic transition rates. Theoretically, the cranking model provides a powerful tool to describe numbers of phenomena in rotating nuclei, such as the angular momentum alignment, the pairing phase transition, the superdeformation, etc. However, the cranking model has a apparent disadvantage of the semiclassical treatment of nuclear rotation, and does not produce an angular momentum eigenstate. Thus, for calculation of the inter-band transition rates, one may need the angular momentum projection which would be a heavy computational burden.

We have developed a new simple method to take account of quantum angular momentum algebra in the cranking approach II. The method allows us to calculate matrix elements of the intrinsic moments in the unified model and to see, from the microscopic point of view, how the Coriolis coupling affects the transition rates. The formula for M1, E1 and E2 transitions is recapitulated in section III.

The electromagnetic transitions between vibrational bands and the ground-state band are a good testing ground of the application, since the experimental data are available up to high spin. This will be discussed in section III A. The decay of high-K isomers by the Coriolis mixing mechanism would be another interesting example to see the higher-order Coriolis coupling effect, which will be discussed in section III B.
II. INTENSITY RELATION FORMULA

We show only the final formula for the transition amplitudes. See Ref. [1] for details.

Since we will discuss only even-even nuclei in this paper, we assume that the ground-state band has the \( K = 0 \) and \( \Delta K = K_f - K_i \leq 0 \) for transitions from the excited bands \((K_i, I_i)\) to the ground-state band \((K_f = 0, I_f)\). Within the lowest-order Coriolis coupling and the next order corrections, the \( \lambda \)-pole \((\lambda = 1, 2)\) transition amplitudes are written in a form

\[
\frac{\langle K_f I_f || M(\lambda) || K_i I_i \rangle}{\sqrt{2I_f + 1}} = \sqrt{\frac{(I_f - K_f)! (I_f + K_f + n)!}{(I_f - K_f - n)! (I_f + K_f)!}} \left\langle I_f K_i \lambda (\Delta K + n) | I_f (K_f + n) \right\rangle \\
\times Q_t (1 + q [I_t(I_t + 1) - I_i(I_i + 1)]) ,
\]

where the forbiddenness \( n = 0 \) for the \( K \)-allowed transitions and \( n = |\Delta K| - \lambda > 0 \) for the \( K \)-forbidden transitions.

The form of this formula is identical to the generalized intensity relation in the unified model [2], in which however there is no systematic method to calculate the intrinsic parameters \( Q_t \) and \( q \). We have shown that there is a correspondence between the \( I \)-expansion in the unified model and the \( \omega_{\text{rot}} \)-expansion in the cranking model, and have given a general method to calculate these parameters [1].

III. APPLICATIONS

A. \( \gamma \)- and octupole vibrational bands

The properties of vibrational motion in rotating nuclei may be investigated by means of a microscopic formalism of the random phase approximation (RPA) based on the cranked mean field (See Ref. [3] and references therein). The method mentioned in the previous section is easily implemented to the cranked RPA formalism. Then, one can calculate the
FIG. 2. Hindrance factors of $B(E2;6^+ \rightarrow 4^+_g)$ (left) and $B(M1+E2;6^+ \rightarrow 6^+_g)$ (right) for $K^\pi = 6^+$ isomers. The squares are the experimental data, while the circles indicate the calculated results. The triangles are the same as circles but with the proton chemical potential artificially increased by 70 keV. The effective charge $e_\pi = 2e$ and g-factor $g_{\text{eff}}^s = 0.7g^s_{\text{free}}$ have been used.

inter-band transition amplitudes in a straightforward manner. The calculation has been performed for quadrupole and octupole vibrations to investigate $M1$, $E2$ and $E3$ transitions [1]. In Fig. 1, we show $E2$ and $E1$ transitions for for $\gamma$ vibration in $^{166}$Er and the $K = 0$ octupole vibration in $^{232}$Th. For the $E2$ amplitudes of the $\gamma$ band, the agreement with the experiments is almost perfect. The $Q_t$ parameter for the $E1$ amplitude is difficult to reproduce (the absolute value is renormalized in Fig. 1), while the Coriolis coupling parameter $q$ shows a reasonable agreement.

B. Decay of High-$K$ Isomers

If the nucleus has an axially symmetric shape, the $K$ quantum number is supposed to be a good quantum number. However, one can expect that this symmetry is broken in a strict sense by the shape fluctuations and the Coriolis mixing. The shape fluctuation with respect to the $\gamma$-degrees of freedom has been studied by the $\gamma$-tunneling model [4]. We discuss the Coriolis mixing mechanism for the $K^\pi = 6^+$ two-quasiparticle (2qp) isomers in Hf isotopes.

The configuration of the isomer is assumed to be the lowest $K^\pi = 6^+$ 2qp state in each nucleus which is actually $\pi[402]5/2 \otimes [404]7/2$. The calculation involves the fourth (fifth) order of Coriolis coupling for $E2$ ($M1$) decays, and the hindrance factors are estimated microscopically (filled symbols in Fig. 2). The isotope dependence is qualitatively well reproduced, however, the calculation predicts the hindrance factors too small. Although the result is sensitive to the details of quasiparticle spectra (see difference between circles and triangles in the figure), the decay rates are overestimated for most cases, especially for the $M1$ transitions.

In order to investigate the effect of residual correlations, we introduce the spin-spin interaction, $V_0\hat{\delta}\hat{\delta}$, which may be responsible for the Gallagher-Moszkowski splitting. The interaction is diagonalized within a space of 2qp states ($E_{2qp} \leq 5$MeV), with $V_0 = 100$ keV being adopted in this calculation. The results are shown by open symbols in Fig. 2. The
hindrance factors are increased by 1∼2 orders of magnitude for $E2$ transitions and by 2∼3 orders for $M1$. The results still depend on the details of quasiparticle spectra, however, the dependence becomes weaker than the calculation without the residual correlation.

IV. CONCLUSION

In conclusion, we have developed a general method to calculate the intrinsic parameters in the intensity relations by using the microscopic cranking approach. The applications to the vibrational transitions and to decay of high-$K$ isomers are discussed. The calculated decay property of 2qp high-$K$ isomers seems to be improved by the inclusion of the residual correlations.

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