Absorbers impact on the reliability of structures subjected to random vibrations

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Abstract. The paper considers the problem of optimizing the absorber parameters when random vibrations of the structure create a filtered Poisson process. A new parameter optimization criterion was proposed, consisting in minimizing the probability of the maximum vibrations of the structure exceeding a certain permissible value of displacements. This criterion means maximizing the reliability of the vibrating structure with regard to the maximum vibration amplitudes.

1. Introduction
Reliability analyses can help the designer to determine acceptable system’s tolerances and determine construction’s parameters for safe structures. The problem of reliability analysis of structures subjected to vibration is extremely important from the engineer's point of view. It is essential to develop vibration reduction for structures - an interesting option is using dynamic vibration absorbers. A Tuned Mass Damper (TMD) or a Dynamic Vibration Absorber (DVA) is a passive vibration device used to reduce resonant vibration. The different kinds of vibration absorbers are applied for reducing the vibration level in various engineering structures such as buildings, towers, bridges, etc.

The topic of a dynamic vibration absorber is well established in the literature. The first absorber was invented in the form of a spring supported mass in 1911 by Frahm [1]. On the other hand, the first damped vibration absorber was created in 1928 by Ormondroyd and Den Hartog [2]. Since Den Hartog [3] found an analytical optimal solution for an undamped primary system, many researches have tried to optimize Dynamic Vibration Absorber (DVA) or Tuned Mass Damper (TMD). The absorber optimization criteria proposed so far can be classified into three categories [4]: $H_\infty$ optimization, $H_2$ optimization, and stability maximization. The $H_\infty$ optimization is the earliest proposed criterion [2] and aims to minimize the maximum amplitude response. The $H_2$ optimization criterion was devised to minimize the kinetic energy over the entire frequency range of the primary system subjected to random excitation [5]. The stochastic vibrations were analysed in the field of correlation theory and spectral density analysis.

In the case of stochastic vibrations of structures, an important issue is the assessment of the impact of absorbers on the assessment of the reliability of this structure. As a criterion for assessing the effectiveness of the absorber in reducing vibrations, the study proposes the assessment of the reliability...
of the structure with regard to the established criterion. In stochastic structure dynamics, the problem of reliability is treated as the first-crossing problem \[6,7\].

In this paper, vibrations of the system (absorber and structure) caused by road traffic which is modelled by filtered Poisson process are considered. The very issue of structure reliability is analysed in the database of random variables, the infinite sequence (series) of which creates Poisson “white noise”. The probability of not exceeding a certain value was adopted as a measure of the reliability evaluation of the vibrating system. This value may indicate an acceptable level of vibrations due to people or may result from the reliability of the fixed structural element of the object. It can be deterministically defined or assume random values. In the paper it was assumed that all random variables have a log-normal distribution.

2. The problem

Let us consider vibration of a two degree freedom system caused by kinematic excitation (figure 1) where \( y_s(t) \) represent vibration of a primary system and \( y_a(t) \) of a dynamic vibration absorber.

![Figure 1. Two degree freedom system.](image)

Vibration of this system is described by system two differential equations:

\[
\frac{d^2 y_s(t)}{dt^2} + 2\xi_s\omega_s\frac{dy_s(t)}{dt} + \omega_s^2[y_s(t) - y_a(t)] = -\frac{d^2 z(t)}{dt^2} = f(t) \tag{1}
\]

\[
\frac{d^2 y_a(t)}{dt^2} + 2\xi_a\omega_a\frac{dy_a(t)}{dt} + \omega_a^2[y_a(t) + y_s(t)] + 2\xi_a\omega_a\mu\frac{dy_a(t)}{dt} + \omega_a^2\mu[y_s(t) - y_a(t)] = -\frac{d^2 z(t)}{dt^2} = f(t) \tag{2}
\]

where \( \omega_s^2 = k_s/m_s \), \( \omega_a^2 = k_a/m_a \), \( \alpha_s = c_s/2m_s = \xi_s\omega_s \), \( \alpha_a = c_a/2m_a = \xi_a\omega_a \), \( \mu = m_a/m_s \) and excitation process \( f(t) \) is a stochastic process, while \( m_s, c_s \) and \( k_s \) are the mass, damping coefficient and stiffness of the primary system, respectively, \( m_a, c_a \) and \( k_a \) are the mass, damping coefficient and stiffness of the absorber system, respectively. The function \( z(t) \) represents ground motion (kinematic excitation).
The stochastic process is assumed to be a series of pulses with random amplitudes occurring at random times. The excitation process has the form of filtered Poisson process:

$$f(t) = \sum_{n=1}^{N(t)} A_n \phi(t-t_n) = \sum_{n=1}^{N(t)} A_n \cdot (t-t_n) e^{-\alpha_j (t-t_n)} \sin \Omega_j (t-t_n)$$  \hspace{1cm} (3)

Figure 2. An example of the single excitation process.

whereas amplitudes are assumed to be random variables which are both mutually independent and independent of the instants $t_n$. It is assumed that the expected values $E[A_n] = E[A] = \text{const}$, $E[A_n^2] = E[A^2] = \text{const}$ are known and does not depend on $n$.

Figure 3. a) Response of the structure with overlapping effect. b) Random series of maximum response of the structure.

The excitation process, equation (3), is a sum of the pulses (figure 2) loading the system during observation. For this reason the response of the system is also a sum of the responses to pulses. Let introduce the dynamic influence functions $H_s(t-t_n)$ and $H_a(t-t_n)$ which are response of the system caused by a single impulse occurring in time $t_n$ ($A_n = 1$). Using dynamics response functions the displacement of system can be written in the form of a Stieltjes integral as

$$y_s(t_i) = A H_s(t_i) + \int_{t_0}^{t_i} A(\tau) H_s(t_i - \tau) dN(\tau) = y_s^{+}(t_i) + y_s^{-}(t_i)$$ \hspace{1cm} (4)

$$y_a(t_i) = A H_a(t_i) + \int_{t_0}^{t_i} A(\tau) H_a(t_i - \tau) dN(\tau)$$ \hspace{1cm} (5)

where $t_i$ is the moment when one of the impulses is located in the time-step in which the response of the system has the maximum value. [8] If $t_0 = 0$ then one considers transient vibration, and for $t_0 = -\infty$ one considers steady-state vibration case.
The symbol $dN(t)$ denotes increment of the process $N(t)$ in the time interval $(\tau, \tau + d\tau)$ and has the following properties

$$E[dN(t)] = \gamma dt, \quad E[dN^\alpha(t)] = \gamma dt$$

$$E[dN(t_1)dN(t_2)] = \gamma^2 dt_1 dt_2 \quad \text{for } t_1 \neq t_2,$$

where $\gamma$ is an intensity of the Poisson process $N(t)$.

The expected value and the variance of the system response can be obtained, after taking into account equations (4), (5) and (6). This yields

$$E[y_{x}(\infty)] = E[y_{u}] = E[A]H_x(t) + E[A]\gamma \int_0^\infty H_x(\tau)d\tau$$

(7)

$$E[y_{u}(\infty)] = E[y_{ul}] = E[A]H_u(t) + E[A]\gamma \int_0^\infty H_u(\tau)d\tau$$

(8)

$$\sigma_{y_{x}}^2(\infty) = \sigma_{y_{u}}^2(\infty) = \sigma_{A}^2H_{x}^2(t) + E[A^2]\gamma \int_0^\infty H_{x}^2(\tau)d\tau$$

(9)

$$\sigma_{y_{u}}^2(\infty) = \sigma_{A}^2H_{u}^2(t) + E[A^2]\gamma \int_0^\infty H_{u}^2(\tau)d\tau$$

(10)

where $\sigma_{A}^2 = E[A^2] \frac{1}{1 + v_A^2}$, while $v_A = 0.3$.

Let assume $F_{y}(t)$ denote the log-normal distribution function of the random variables $y_i$, which expected value is given by equation (7) and variance by equation (9).

Let function $F_{y_{x}}(y, t)_{\text{max}}$ determine the distribution of structure response maxima in a given period $(0, t)$. This function is given by the formulae

$$F_{y_{x}}(y, t)_{\text{max}} = \sum_{n=1}^{\infty} P[N(t) = n] \prod_{i=1}^{n} (P(y_i \leq y)) = \sum_{n=1}^{\infty} p_n(t)F_{y}^{n}(y)$$

(11)

where $p_n(t)$ is probability of $n$ impulses in the time $(0, t)$.

Considering that the $N(t)$ process is a Poisson process, the above formula can be presented in the form

$$F_{y_{x}}(y, t)_{\text{max}} = \exp\{-\gamma t[1 - F_{y}(t)]\}$$

(12)

Above solutions can be used in optimization the parameters of the absorber. As the first criterion we assume minimum probability exceeding the permissible level $\Delta$ by a single pulse. This probability $p_f$ is given by formulae

$$p_f = 1 - p_r = 1 - \Phi\left[\frac{\ln(\Delta) - E[\ln y]}{\sigma_{\ln y}}\right]$$

(13)

where $\Phi(\cdot)$ - standard normal distribution.

The next criterion we assume the probability that the maximum vibrations in the period $(0, T)$ will not exceed $\Delta$ level. This probability is equal

$$p_r(T) = 1 - p_f(T) = \exp\{-\gamma T[1 - F_{y}(\Delta)]\}$$

(14)
where

$$F_{y} (\Delta) = \Phi \left[ \frac{\ln(\Delta) - E[\ln y]}{\sigma_{lny}^2} \right]$$  \hspace{1cm} (15)$$

Let’s consider the situation that the vibrations of the system caused by one shock are so dominant that the effect from the other shocks can be avoided, the question arises as to the distribution of structure response maxima in a given time interval (figure 4). Let $h$ be the maximum vibration amplitude caused by a single shock (pulse) with unit amplitude $A = 1$:

$$h = \max H_s(t), \quad 0 \leq t \leq \infty$$  \hspace{1cm} (16)$$

**Figure 4.** A simplified model of structure vibrations

If the system is loaded with an $A$ pulse, then the maximum displacement of the system is equal $Ah$. The excitation process (equation 3) is a pulse (figure 2) loading the system during observation. Equation (4) comes down to:

$$y_s(t) = y_s'(t) = AH_s(t)$$  \hspace{1cm} (17)$$

In above model the equations (11)-(15) need modification. Let function $F_{\max}(y,t)$ determine the distribution of structure response maxima in a given period $(0,t)$.

$$F_{\max}(x,t) = \sum_{n=1}^{\infty} P(N(t)=n) \prod_{i=1}^{n} P(A_h \leq x) = \sum_{n=1}^{\infty} p_n(t) F_{\delta}^n \left( \frac{x}{h} \right)$$  \hspace{1cm} (18)$$

Considering that the $N(t)$ process is a Poisson process, the above formula can be presented in the form

$$F_{\max}(x,t) = \exp \left\{ -\gamma t \left[ 1 - F_{\delta} \left( \frac{x}{h} \right) \right] \right\}$$  \hspace{1cm} (19)$$

where $F_{\delta}(x)$ is log-normal distribution of the amplitudes $A$.

In this case the probability $p_f$ is given by formulæ

$$p_f = 1 - \Phi \left[ \frac{\ln \left( \frac{\Delta}{h} \right) - E[\ln A]}{\sigma_{lnA}^2} \right]$$  \hspace{1cm} (20)$$
The probability the $\Delta$ level will be exceeded in the period $(0,T)$ is equal

$$
p_f(T) = \exp\left\{-\gamma T \left[1 - F_A\left(\frac{\Delta}{h}\right)\right]\right\}
$$

where

$$
F_A\left(\frac{\Delta}{h}\right) = \Phi\left[\frac{\ln\left(\frac{\Delta}{h}\right) - E[\ln A]}{\sigma_{\ln A}}\right]
$$

(21)

(22)

3. The numerical example

In the article, the calculations were made for two cases:

1) there is a lot of traffic, so vibrations overlap (figure 3),

2) the movement is so small that excitations can be considered separately (figure 4).

Moreover, two excitation variants were considered: one was close to the resonance frequency $\lambda_1 = \Omega_{j_1} / \omega_z$, and the second variant of the excitation was far from the resonance frequency - $\lambda_2 = \Omega_{j_2} / \omega_z$. All the graphs below present the results for the data $\kappa = \omega_a / \omega_z = \{0,3\}$, $\mu = m_a / m = \{0,0.8\}$ and the excitation $\lambda_i = \Omega_{j_i} / \omega_z = 1.1$ or $\lambda_2 = \Omega_{j_2} / \omega_z = 2.2$. One can see results with a single impulse $y_1(t)$ (equation 17) and with series of impulses $y_2(t)$ or the sum $y_3(t)$ (equation 4). The graphs on the left below show the results when a single excitation with frequency $\lambda_1 = 1.1$, while on the right the graph shows results when $\lambda_2 = 2.2$

Figure 5. The variance of the system response for $\lambda_1 = 1.1$ and $\lambda_2 = 2.2$.

Figure 6. The expected value of the system response for $\lambda_1 = 1.1$ and $\lambda_2 = 2.2$. 
Figures 5-6 illustrates results with a single impulse $y_1^*(t)$ - blue plane - and with series of impulses $y_2^*(t)$ - red plane - or the sum $y_s(t) = y_1^*(t) + y_2^*(t)$ – pale yellow surface. Comparing those figures, the following conclusions were obtained:

1. The level of the function of the variance of the system response $\sigma^2_{y_s}$ is mostly stabled (figure 5). In the case of $\lambda_1 = 1.1$ (when the excitation is close to a resonance) results are more various - one can see a very significant decrease in the level of the function $\sigma^2_{y_s}$ for $\kappa = \omega_s / \omega_1 \approx 1.1$. In the second case, $\lambda_2 = 2.2$ (when the excitation is far from a resonance) is similar situation for $\kappa = \omega_s / \omega_1 \approx 2.3$ but not so significant as in the first case.

2. The level of the function of the expected value of the system response $E[y_s]$ is mostly stabled (figure 6). Moreover results with a single impulse $y_1^*(t)$ are almost the same as the sum $y_s(t)$ - the planes are coincided. Similarly to the function of the variance of the system response, in the case of $\lambda_1 = 1.1$ - there is a significant decrease in the level of the function $E[y_s]$ for $\kappa = \omega_s / \omega_1 \approx 1.1$ but only for the sum $y_s(t)$. In the second case, $\lambda_2 = 2.2$ - the planes are almost flat.

4. Conclusions

The article considers the problem of optimizing the parameters of an absorber in a situation where random vibrations of the structure create a filtered Poisson process, e.g. vehicles passing by a building. A parameter optimization criterion was proposed, consisting in minimizing the probability of the maximum vibrations of the structure, i.e. the maximum reliability of the structure in relation to the maximum vibration amplitudes.

Two load models were analysed – a frequent vehicle traffic and a single vehicle passing. Numerical simulations indicate that both the variance and the mean value depend on the ratio of the frequency of the absorber to the structure $\kappa = \omega_s / \omega_1$ in both the first and second load models. However, it seems that they hardly depend on the ratio of the absorber mass to the structure mass $\mu = m_s / m_1$. There is significant decrease in the level of the variance function of the excitation when a frequency of the absorber is close to a frequency of the structure ($\kappa = \omega_s / \omega_1 \approx 1.1$).

Probability criterion allows objectify criterion limits the vibration of the structure.

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