Constraints on general primordial non-Gaussianity using wavelets for the Wilkinson Microwave Anisotropy Probe 7-year data

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ABSTRACT

We present constraints on the non-linear coupling parameter $f_{nl}$ with the Wilkinson Microwave Anisotropy Probe (WMAP) data. We use the method based on the spherical Mexican hat wavelet to measure the $f_{nl}$ parameter for three of the most interesting shapes of primordial non-Gaussianity: local, equilateral and orthogonal. Our results indicate that this parameter is compatible with a Gaussian distribution within the $2\sigma$ confidence level (CL) for the three shapes and the results are consistent with the values presented by the WMAP team. We have included in our analysis the impact on $f_{nl}$ due to contamination by unresolved point sources. The point sources add a positive contribution of $\Delta f_{nl}^{loc} = 2.5 \pm 3.0$, $\Delta f_{nl}^{eq} = 37 \pm 18$ and $\Delta f_{nl}^{ort} = 25 \pm 14$. As mentioned by the WMAP team, the contribution of the point sources to the orthogonal and equilateral forms is expected to be larger than the local one and thus it cannot be neglected in future constraints on these parameters. Taking into account this contamination, our best estimates for $f_{nl}$ are $-16.0 \leq f_{nl}^{loc} \leq 76.0$, $-382 \leq f_{nl}^{eq} \leq 202$ and $-394 \leq f_{nl}^{ort} \leq 34$ at 95 per cent CL. The three shapes are compatible with zero at 95 per cent CL ($2\sigma$). Our conclusion is that the WMAP 7-year data are consistent with Gaussian primordial fluctuations within $2\sigma$ CL. We stress however the importance of taking into account the unresolved point sources in the measurement of $f_{nl}$ in future works, especially when using more precise data sets such as the forthcoming Planck data.

Key words: methods: data analysis – cosmic background radiation – cosmology: observations.

1 INTRODUCTION

During the period of inflationary expansion in the very early stages of the universe, primordial perturbations were generated that are the seeds of the structures that we can observe today (Starobinskiı 1979; Guth 1981; Albrecht & Steinhardt 1982; Linde 1982, 1983; Mukhanov, Feldman & Brandenberger 1992). These primordial perturbations were linearly imprinted in the cosmic microwave background (CMB) anisotropies. Thus, the study of the CMB anisotropies is a powerful way to understand the physics of the early universe. Many observational CMB projects, for example the National Aeronautics and Space Administration (NASA) WMAP1 and European Space Agency (ESA) Planck2 missions, different ground-based 3D observational campaigns of large-scale structure and high-energy accelerators are enabling us to understand better the properties and the evolution of the universe. From the several observational approaches that are available, the search for departures from Gaussianity in the CMB anisotropies with a primordial origin has become a powerful way to discriminate among different inflationary scenarios. Inflationary models such as the widely accepted standard, single-field, slow roll inflation predict low levels of non-Gaussianity, whereas other models predict levels of non-Gaussianity that may be detected using the data from current experiments (Bartolo et al. 2004; Komatsu 2009, 2010; Yadav & Wandelt 2010). A detection of a deviation from Gaussianity with a primordial origin would rule out many inflationary models and would have far reaching implications in the physics of the early universe.

The level of primordial non-Gaussianity is usually parametrized by the non-linear coupling parameter $f_{nl}$ (Verde et al. 2000; Komatsu & Spergel 2001; Bartolo et al. 2004). This parameter measures departures from zero in the values of the third-order quantity known as the bispectrum, characterized through the shape function $F(k_1, k_2, k_3)$. The bispectrum is related to Bardeen’s curvature perturbations, $\Phi(k)$, through the three-point correlation function $\langle \Phi(k_1)\Phi(k_2)\Phi(k_3) \rangle = (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)F(k_1, k_2, k_3)$. Depending on the physical mechanisms of the different inflationary models, the shape function can take different forms.

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In this paper, we measure the levels of non-Gaussianity present in the WMAP data (corresponding to the three particular shapes (local, equilateral and orthogonal) that have been studied by the WMAP team (Komatsu et al. 2011). The shape function $F(k_1, k_2, k_3)$ of these types of non-Gaussianity, their CMB angular bispectra $b_{\ell_1 \ell_2 \ell_3}^{\text{loc}}$ and the inflationary scenarios that generate these non-Gaussianities are described below.

(i) **Local shape.** Significant non-Gaussianity of the local form can be generated for example in multifield inflation models (Komatsu, Spergel & Wandelt 2005; Komatsu 2010), the curvaton model (Lyth, Ungarelli & Wandls 2003), the inhomogeneous reheating scenario (Bartolo et al. 2004; Dvali, Gruzinov & Zaldarriaga 2004), models based on hybrid inflation (Lin 2009), etc. This shape is given by (see for example Creminelli et al. 2006; Fergusson, Liguori & Shellard 2010a; Yadav & Wandelt 2010; Komatsu et al. 2011)

$$F(k_1, k_2, k_3) = 2A^2 f_{\delta} \left[ \frac{1}{k_1^{3(n_s-1)}k_2^{3(n_s-1)}} + \frac{1}{k_1^{3(n_s-1)}k_3^{3(n_s-1)}} \right],$$

and its angular bispectrum is (see for example Fergusson et al. 2010a; Komatsu 2010; Yadav & Wandelt 2010)

$$b_{\ell_1 \ell_2 \ell_3}^{\text{loc}} = \int_0^\infty \! \! \! dx x^2 \left[ \alpha_\ell(x) \beta_{\ell_2}(x) \beta_{\ell_3}(x) + \beta_\ell(x) \alpha_\ell_2(x) \alpha_\ell_3(x) \right],$$

where $A$ is the amplitude of the power spectrum $P_{\delta}(k) = Ak^{n_s-4}$, $n_s$ is the spectral index and $\alpha_\ell(x)$ and $\beta_\ell(x)$ are the filter functions (see for example Komatsu & Spergel 2001; Komatsu et al. 2005; Fergusson et al. 2010a; Komatsu 2010).

(ii) **Equilateral shape.** Significant non-Gaussianity of the equilateral form can be generated for example by the Dirac–Born–Infeld inflation (Bartolo et al. 2004; Silverstein & Tong 2004; Linglois et al. 2008), ghost inflation (Arkani-Hamed et al. 2004), several single-field inflationary models in Einstein gravity (Chen et al. 2007) etc. This shape is given by (see for example Creminelli et al. 2006; Fergusson et al. 2010a; Yadav & Wandelt 2010; Komatsu et al. 2011)

$$F(k_1, k_2, k_3) = 6A^2 f_{\delta} \left[ \frac{1}{k_1^{3(n_s-1)}k_2^{3(n_s-1)}} - \frac{1}{k_1^{3(n_s-1)}k_3^{3(n_s-1)}} \right] - \frac{2}{(k_1k_2)^{3(4-n_s)/2}}$$

$$+ \left\{ \frac{1}{k_1^{3(4-n_s)/2}k_2^{3(4-n_s)/2}k_3^{3(4-n_s)/2}} + (\text{Sperm}) \right\},$$

and its angular bispectrum is (see for example Fergusson et al. 2010a; Yadav & Wandelt 2010; Komatsu 2010)

$$b_{\ell_1 \ell_2 \ell_3}^{\text{eq}} = \int_0^\infty \! \! \! dx x^2 \left[ -\alpha_\ell(x) \beta_{\ell_2}(x) \beta_{\ell_3}(x) + (2\text{perm}) \right.$$

$$\left. + \beta_\ell(x) \gamma_{\ell_2}(x) \delta_{\ell_3}(x) + (5\text{perm}) \right.$$

$$\left. - \frac{8}{3} \delta_{\ell_1}(x) \delta_{\ell_2}(x) \delta_{\ell_3}(x) \right],$$

where $\gamma_{\ell}(x)$ and $\delta_{\ell}(x)$ are the filter functions (see for example Fergusson et al. 2010a; Komatsu 2010).

(iii) **Orthogonal shape.** Significant non-Gaussianity of the orthogonal form can be generated in general by single-field models (Cheung et al. 2008; Senatore, Smith & Zaldarriaga 2010). This shape is given by (see for example Senatore et al. 2010; Yadav & Wandelt 2010; Komatsu et al. 2011)

$$F(k_1, k_2, k_3) = 6A^2 f_{\delta} \left[ \frac{3}{k_1^{3(n_s-1)}k_2^{3(n_s-1)}} - \frac{3}{k_2^{3(n_s-1)}k_3^{3(n_s-1)}} \right] - \frac{8}{(k_1k_2)^{3(4-n_s)/2}}$$

$$+ \left\{ \frac{3}{k_1^{3(4-n_s)/2}k_2^{3(4-n_s)/2}k_3^{3(4-n_s)/2}} + (5\text{perm}) \right\},$$

and its angular bispectrum is (see for example Yadav & Wandelt 2010; Komatsu 2010)

$$b_{\ell_1 \ell_2 \ell_3}^{\text{ort}} = \int_0^\infty \! \! \! dx x^2 \left[ -\alpha_\ell(x) \beta_{\ell_2}(x) \beta_{\ell_3}(x) + (2\text{perm}) \right.$$

$$\left. + \beta_\ell(x) \gamma_{\ell_2}(x) \delta_{\ell_3}(x) + (5\text{perm}) \right.$$

$$\left. - \frac{8}{3} \delta_{\ell_1}(x) \delta_{\ell_2}(x) \delta_{\ell_3}(x) \right].$$

Many studies have been performed to constrain $f_{\delta}$, especially for the local and the equilateral cases. The first constraints on $f_{\delta}$ were imposed using data sets with low resolution or small sky coverage, which led to large uncertainties in $f_{\delta}$. We can report analyses using the Cosmic Background Explorer (COBE) data (Komatsu et al. 2002; Cayón et al. 2003a), MAXIMA data (Cayón et al. 2003b; Santos et al. 2003), the Very Small Array (VSA) data (Smith et al. 2004), the Archeops data (Curto et al. 2007, 2008) and the BOOMERanG data (De Troia et al. 2007; Natoli et al. 2010).

Once the WMAP data were available, significant improvements were achieved in the precision of the estimation of $f_{\delta}$. Many studies have been developed to constrain $f_{\delta}$ using WMAP data and based on different estimators. We can mention the different bispectrum-based estimators (see for example Komatsu et al. 2003, 2009, 2011; Babich, Creminelli & Zaldarriaga 2004; Creminelli et al. 2006; Creminelli, Senatore & Zaldarriaga 2007; Fergusson & Shellard 2007, 2009, 2011; Spergel et al. 2007; Yadav & Wandelt 2008; Elsner & Wandelt 2009; Smith, Senatore & Zaldarriaga 2009; Bucher, van Tent & Carvalho 2010; Fergusson et al. 2010a; Fergusson, Liguori & Shellard 2010b; Liguori et al. 2010; Senatore et al. 2010; Smidt et al. 2010). The bispectrum is the most natural way to constrain $f_{\delta}$ given its linear dependence and the fact that in certain ideal conditions bispectrum-based estimators may be the optimal way to measure $f_{\delta}$. However, given that the data are contaminated by different non-Gaussian parasite signals and in most cases only a fraction of the sky can be used, it is convenient to use additional tools that can help to understand these effects better. We can mention the tests performed using the spherical Mexican hat wavelet (SMHW) (Mukherjee & Wang 2004; Curto et al. 2009a; Curto, Martínez-González & Barreiro 2009b, 2011), a healpix-based wavelet (Casaponsa et al. 2011a,b), a joint

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3 The improvement comes from a combination of large sky coverage, high angular resolution and good sensitivity. This combination improves the signal-to-noise ratio of $f_{\delta}$ which for the local case is proportional to $\log(\epsilon_{\text{max}})$ (Yadav & Wandelt 2010).
analysis with the SMHW and neural networks (Casaponsa et al. 2011b), needlets (Marinucci et al. 2008; Pietrobon et al. 2009, 2010a,b; Rudjord et al. 2009, 2010; Cabella et al. 2010; Pietrobon 2010), the Minkowski functionals (Hikage, Komatsu & Matsubara 2006; Gott et al. 2007; Hikage et al. 2008; Matsubara 2010; Takeuchi, Ichiki & Matsubara 2010), the $N$-point probability distribution function (N-PDF) distribution (Vielva & Sanz 2009, 2010) or a Bayesian approach (Elshner & Wandelt 2010; Elshner, Wandelt & Schneider 2010). Other works use the 3D distribution of matter on large scales (see for example Dalal et al. 2008; Matarrese & Verde 2008; Slosar et al. 2008; Seljak 2009; Desjacques & Seljak 2010; Xia et al. 2010; Baldauf, Seljak & Senatore 2011; Fergusson, Regan & Shellard 2010; Hamaus, Seljak & Desjacques 2011) to constrain the local $f_{\text{NL}}$.

In this paper, we focus on the measurement of non-Gaussianity for the previous mentioned shapes using the estimator based on wavelets that has been formerly used to constrain local $f_{\text{NL}}$ (Curto et al. 2009a, 2011; Curto, Martínez-González & Barreiro 2009b, 2010). We use the technique described by Fergusson et al. (2010a) to produce non-Gaussian maps with the local, equilateral and orthogonal bispectra for WMAP resolution in realistic conditions of partial sky coverage and anisotropic noise. These maps are later used to evaluate the expected values of the wavelet’s third-order moments $\alpha_{G}$ for each type of non-Gaussianity. We finally impose constrains on $f_{\text{NL}}$ for each shape using the wavelet estimator for the WMAP foreground reduced and raw data maps. As shown later, unresolved point sources produce a significant bias in $f_{\text{NL}}$ that should be considered in the analyses of WMAP data and in the forthcoming analyses of Planck data, especially for the equilateral and orthogonal shapes.

This paper is organized as follows. Section 2 presents the non-Gaussian maps that we have used to estimate the quantities needed for this analysis. In Section 3, we present the method and the estimator used in this analysis to constrain $f_{\text{NL}}$. The results of the analysis using WMAP data are presented in Section 4 and the conclusions are presented in Section 5.

2 NON-GAUSSIAN SIMULATIONS

Non-Gaussian Monte Carlo simulations are needed in order to calibrate the wavelet estimator. We have simulated our non-Gaussian maps following the algorithm described by Fergusson et al. (2010a). The non-Gaussian $a_{\text{NG}}^{G}$ coefficients can be written in terms of the bispectrum and the Gaussian $a_{\text{NG}}^{G}$ coefficients:

\[
a_{\text{NG}}^{G} = \frac{1}{6} \sum_{\ell, \ell_2, \ell_3} b_{\ell_2 \ell_3} C_{\ell_2} C_{\ell_3} \left[ a_{\text{NG}}^{G} \left( m_{\ell}, m_{\ell_2}, m_{\ell_3} \right) \delta_{\ell_2 \ell_3} \right]
\]

Using the fact that the shape functions of the local, equilateral and orthogonal bispectra are separable, we are able to reduce the number of sums in equation (7). This can be done in a straightforward way substituting equations (2), (4) and (6) in equation (7). However, as stated by Hansen et al. (2009) and Fergusson et al. (2010a), there are terms that may produce spurious divergences at low multipoles, large enough to affect the power spectrum of the final map. Fergusson et al. (2010a) located the divergent terms and provided equations for the local and equilateral shapes without these terms. A similar procedure can be performed with the orthogonal shape. In the following equations, we present the non-Gaussian $a_{\text{NG}}^{G}$ coefficients for each of the shapes without divergent terms.

(i) Local bispectrum:

\[
a_{\text{NG}}^{\text{L}} = \int_{0}^{\infty} dx d^{2}n Y_{\text{m}}^{*} (\vec{n}) M_{\text{m}} (x, \vec{n}) M_{\text{m}} (x, \vec{n}).
\]

(ii) Equilateral bispectrum:

\[
a_{\text{NG}}^{\text{E}} = \int_{0}^{\infty} dx d^{2}n Y_{\text{m}}^{*} (\vec{n}) M_{\text{m}} (x, \vec{n}) M_{\text{m}} (x, \vec{n})
\]

\[+ 2 \delta_{\ell} (x) \int d^{2}n Y_{\text{m}}^{*} (\vec{n}) M_{\text{m}} (x, \vec{n}) M_{\text{m}} (x, \vec{n})
\]

\[+ 6 \gamma_{\ell} (x) \int d^{2}n Y_{\text{m}}^{*} (\vec{n}) M_{\text{m}} (x, \vec{n}) M_{\text{m}} (x, \vec{n}),
\]

(iii) Orthogonal bispectrum:

\[
a_{\text{NG}}^{\text{O}} = \int_{0}^{\infty} dx d^{2}n Y_{\text{m}}^{*} (\vec{n}) M_{\text{m}} (x, \vec{n}) M_{\text{m}} (x, \vec{n})
\]

\[\int d^{2}n Y_{\text{m}}^{*} (\vec{n}) M_{\text{m}} (x, \vec{n}),
\]

\[+ 2 \delta_{\ell} (x) \int d^{2}n Y_{\text{m}}^{*} (\vec{n}) M_{\text{m}} (x, \vec{n}) M_{\text{m}} (x, \vec{n})
\]

\[+ 18 \gamma_{\ell} (x) \int d^{2}n Y_{\text{m}}^{*} (\vec{n}) M_{\text{m}} (x, \vec{n}) M_{\text{m}} (x, \vec{n}),
\]

where

\[
\alpha_{\ell} (x) = \frac{2}{\pi} \int_{0}^{\infty} k^{2} dk g_{\ell} (x, k),
\]

\[
\beta_{\ell} (x) = \frac{2}{\pi} \int_{0}^{\infty} k^{2} dk P_{\ell} (x) g_{\ell} (x, k),
\]

\[
\gamma_{\ell} (x) = \frac{2}{\pi} \int_{0}^{\infty} k^{2} dk P_{\ell} (x) g_{\ell} (x, k),
\]

and $g_{\ell} (x)$ is the radiation transfer function that can be evaluated using for example the CAMB\textsuperscript{4} or GTFA\textsuperscript{5} software. $P_{\ell} (x)$ is the linear power spectrum, $j_{\ell} (x, k)$ is the spherical Bessel function (of the first kind) and the $M (x, \vec{n})$ maps are defined as

\[
M_{a} (x, \vec{n}) = \sum_{\ell, m} a_{\ell, m}^{a} \frac{Y_{\ell, m} (\vec{n})}{C_{\ell}},
\]

\[
M_{b} (x, \vec{n}) = \sum_{\ell, m} b_{\ell, m}^{b} \frac{Y_{\ell, m} (\vec{n})}{C_{\ell}},
\]

\[
M_{c} (x, \vec{n}) = \sum_{\ell, m} c_{\ell, m}^{c} \frac{Y_{\ell, m} (\vec{n})}{C_{\ell}},
\]

\[
M_{d} (x, \vec{n}) = \sum_{\ell, m} d_{\ell, m}^{d} \frac{Y_{\ell, m} (\vec{n})}{C_{\ell}}.
\]

In Fig. 1, we plot the non-Gaussian spectrum of the non-Gaussian terms $a_{\ell, m}^{\text{NG}}$ defined by

\[
a_{\ell, m}^{\text{NG}} = \int_{0}^{\infty} dx d^{2}n Y_{\ell, m}^{*} (\vec{n}) M_{\ell, m} (x, \vec{n}),
\]

\[
a_{\ell, m}^{\text{NG}} = \int_{0}^{\infty} dx d^{2}n Y_{\ell, m}^{*} (\vec{n}) M_{\ell, m} (x, \vec{n}),
\]

\[
a_{\ell, m}^{\text{NG}} = \int_{0}^{\infty} dx d^{2}n Y_{\ell, m}^{*} (\vec{n}) M_{\ell, m} (x, \vec{n}),
\]

\[
a_{\ell, m}^{\text{NG}} = \int_{0}^{\infty} dx d^{2}n Y_{\ell, m}^{*} (\vec{n}) M_{\ell, m} (x, \vec{n}),
\]

\[
\text{The power spectrum of the Gaussian part is also plotted. We can see that these three terms add negligible extra power to the full Gaussian plus}
\]

\[\text{http://camb.info/}
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\[\text{http://gyudon.as.utexas.edu/~komatsu/CRL/index.html}
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Figure 1. From left to right, the power spectrum of the non-Gaussian $a_{nm}^{(NG)}$, $a_{nm}^{(DD)}$ and $a_{nm}^{(qRD)}$ coefficients (lower line) compared with the Gaussian part of the power spectrum (upper line).

### Table 1. Quadrature in $x$ integration used to compute $a_{nm}^{(NG)}$.

| $0 \leq x < 9500$ | 64 points, Gauss–Legendre quadrature |
|---------------------|---------------------------------------|
| $9500 \leq x \leq 11000$ | 128 points, Gauss–Legendre quadrature |
| $11000 \leq x \leq 13800$ | 64 points, Gauss–Legendre quadrature |
| $13800 \leq x \leq 14600$ | 170 points, Gauss–Legendre quadrature |
| $14600 \leq x \leq 16000$ | 42 points, Gauss–Legendre quadrature |
| $16000 \leq x \leq 50000$ | 42 points, Gauss–Legendre quadrature |

non-Gaussian map. Once the $a_{nm}^{(NG)}$ terms are computed as a function of the bispectrum and the $a_{nm}^{(DD)}$, the $a_{nm}$ coefficients of a simulation with a given $f_{nl}$ can be written as $a_{nm} = a_{nm}^{(DD)} + f_{nl}a_{nm}^{(NG)}$.

In this paper, we have generated a set of 300 non-Gaussian maps for the local, equilateral and orthogonal $f_{nl}$. We have assumed a \Lambda \text{CDM} cold dark matter model using the parameters that best fit the WMAP 7-year data (Komatsu et al. 2011). We have computed a power spectrum $C$ and a transfer function $g_{x}(k)$ using these parameters as inputs for the CAMB software (Lewis, Challinor & Lasenby 2000) up to $\ell_{\text{max}} = 1535$. The integrals in equations (8), (9) and (10) have been performed using a Gauss–Legendre quadrature. We have used a large density of points near reionization and recombinations (see Table 1 for more details). A large number of points have been chosen in order to achieve convergence in the values of the Fisher matrix $\sigma_{ijk}^2(f_{nl})$ of the bispectrum for the three shapes. In Fig. 2, the Fisher matrix $\sigma_{ijk}^2(f_{nl})$ (Komatsu & Spergel 2001) obtained with the three bispectra is plotted for different $\ell_{\text{max}}$ values. Note that these values are comparable with the values presented, for example, by Yadav & Wandelt (2010). Once the $a_{nm}$ of the simulations with non-Gaussianity are generated, we transform them into WMAP maps for each radiometer by convolving with the appropriate window functions in the spherical harmonic space and by adding a Gaussian instrumental noise simulation in the real space (Bennett et al. 2003).

### 3 METHOD

We use an estimator that is based on third-order statistics generated by the different possible combinations of the wavelet coefficient maps of the SMHW evaluated at certain angular scales. For example Antoine & Vanderheystn (1998), Martínez-González et al. (2002), Vielva (2007) and Martínez-González (2008) for detailed information about this wavelet. This estimator is described and used to search for blind non-Gaussian deviations and constrain local $f_{nl}$ in Curto et al. (2009a,b, 2011).

We consider the same set of angular scales $R_\ell$ selected in Curto et al. (2011). After evaluating the wavelet coefficient map $w(R_\ell; b)$ for each angular scale $R_\ell$, we compute the third-order moments $q_{ijk}$ for each possible combination of three angular scales $\{i, j, k\}$. As mentioned in Curto et al. (2011), the expected values of the cubic statistics are linearly proportional to $f_{nl}$:

$$q_{ijk} = \alpha_{ijk} f_{nl},$$

(19)

where the $\alpha_{ijk}$ term is linearly related to the bispectrum. We evaluate these $\alpha_{ijk}$ quantities for the local, equilateral and orthogonal bispectra by averaging the values of the estimators obtained with the non-Gaussian simulations described in the previous section. We then compute a $\chi^2$ statistic in order to constrain each $f_{nl}$:

$$\chi^2(f_{nl}) = \sum_{ijk,rst} (q_{ijk}^{\text{obs}} - \alpha_{ijk} f_{nl}) C^{-1}_{ijk, rst} (q_{rst}^{\text{obs}} - \alpha_{rst} f_{nl}),$$

(20)

where $q_{ijk}^{\text{obs}}$ is the value of the statistics obtained for the actual data map and $C$ is the covariance matrix among the different statistics $q_{ijk}$. The covariance matrix is estimated using the set of 300 non-Gaussian simulations transformed into WMAP $V + W$ maps. Although we found analytical expressions for the covariance matrix $C_{ijk, rst}$ and the $\alpha_{ijk}$ quantities (see Curto et al. 2011), those expressions are only valid for the particular ideal case of full sky maps and white isotropic noise. For a realistic case, the analytical expressions become more complicated, and the best practical approach to compute those quantities is using simulations.

Finally, this estimator is also applied to a set of Gaussian maps in order to obtain an empirical estimate of the uncertainties of $f_{nl}$. Additionally, we also compute the value of the $f_{nl}$ Fisher matrix using the wavelet coefficients

$$\sigma_{ijkl}^2(f_{nl}) = \sum_{ij,kr} \alpha_{ijk} C^{-1}_{ijkl,kr} \alpha_{rst},$$

(21)

Note that for this estimator there is no need to subtract any linear term due to the anisotropic noise as in the case of the KSW estimator. The reason is that the non-ideal aspects of the analysis (as the mask, the anisotropic noise, etc.) are included in the covariance matrix and the $\alpha_{ijk}$ coefficients.
4 APPLICATION TO WMAP DATA

4.1 Constraints on \( f_{	ext{nl}} \) using WMAP data

We use the combined WMAP 7-year V- and W-band maps at the Healpix (Görski et al. 2005) resolution of \( N_{\text{side}} = 512 \). We consider both raw and foreground-reduced data maps as Komatsu et al. (2011). The maximum multipole chosen in this analysis is 3\( N_{\text{side}} \), although the noise contamination starts to be significant at \( \ell \sim 1000 \). For the three shapes, we find the best limits on \( f_{	ext{nl}} \) and provide the value of the Fisher and the simulated \( \sigma(f_{	ext{nl}}) \). During all the analysis, we use the WMAP KQ75 mask (Gold et al. 2011). In Table 2, we summarize our results.

We find that for the three cases, the parameters are compatible with zero at 95 per cent confidence level (CL). We would like to note the different effect that the foregrounds produce on different shapes: whereas it is negative for the local shape, it is positive for the equilateral and orthogonal shapes. For all the cases, \( \sigma(f_{	ext{nl}}) \) is lower than the value obtained with simulations (~95 per cent depending on the shape). We think that this small discrepancy is due to the limited number of simulations. We have checked that our estimator is unbiased. We have estimated the \( f_{	ext{nl}} \) values of 100 non-Gaussian simulations with an input \( f_{	ext{nl}} = 100 \) and used the remaining 200 non-Gaussian simulations to estimate the \( \sigma(f_{	ext{nl}}) \). The results are \( f_{	ext{nl}}^{\text{obs}} = 99.5 \pm 29.5 \), \( f_{	ext{nl}}^{\text{obs}} = 98 \pm 150 \) and \( f_{	ext{nl}}^{\text{obs}} = 97 \pm 118 \), which are clearly compatible with the input \( f_{	ext{nl}} \), taking into account the expected errors in the mean for the available number of realizations. Our best estimates for the clean maps are as follows.

(i) Local:

\[ f_{	ext{nl}} = 32.5 \pm 22.5 \] (68 per cent CL).

(ii) Equilateral:

\[ f_{	ext{nl}} = -53 \pm 145 \] (68 per cent CL).

(iii) Orthogonal:

\[ f_{	ext{nl}} = -155 \pm 106 \] (68 per cent CL).

The values match well with the results presented by Komatsu et al. (2011) within 1σ error bars. The differences can be explained by the different sensitivity of the bispectrum and wavelet estimators to the possible non-cosmological residuals present in the data.

4.2 Point-source contribution

We have also estimated the contribution of undetected point sources using the source number counts \( dN/d\ell \) derived from de Zotti et al. (2005). We have used point-source simulations based on this \( dN/d\ell \). We have chosen a maximum flux for the bright sources such that the power spectrum for the Q band is compatible with the value provided by the WMAP team. \( A_{\text{ps}} = 0.0090 \pm 0.0007 \mu \text{K}^2\text{sr} \) in antenna units (Larson et al. 2011). We have estimated the best-fitting \( f_{	ext{nl}} \) value for two sets of 1000 maps. The first set consists of 100 Gaussian CMB + noise maps and the second consists of the same Gaussian CMB + noise maps plus the point-source maps.

For each map with point sources, we estimate its best-fitting \( f_{	ext{nl}} \) parameter and compare it with the value obtained for the same map without point sources. The difference \( \Delta f_{	ext{nl}} \) provides an estimate of the impact on \( f_{	ext{nl}} \) due to the unresolved point sources. The point sources add a contribution of \( \Delta f_{	ext{nl}} = 2.5 \pm 3 \), \( \Delta f_{	ext{nl}} = 37 \pm 18 \) and \( \Delta f_{	ext{nl}} = 25 \pm 14 \) for the local, equilateral and orthogonal forms, respectively.

To check further these results, we have used an alternative method to estimate the point-source contamination to \( f_{	ext{nl}} \) given by the expression

\[
\Delta f_{	ext{nl}} = \frac{\sum_{ijk,\ell} (q_{ijk}) \alpha_{ijkl} C^{-1}_{ijkl} \sigma_{ij} \alpha_{ij}}{\sum_{ijkl} \alpha_{ijkl} C^{-1}_{ijkl} \sigma_{ij}},
\]

(22)

where \( (q_{ijk}) \) is the expected value of the third-order moments due to the point sources. The results are \( \Delta f_{	ext{nl}} = 2.5, 38 \) and 24 which agree with the values previously obtained with simulations. Taking into account the point-source contribution, our best estimates of \( f_{	ext{nl}} \) are as follows.

(i) Local:

\[ f_{	ext{nl}} = 30.0 \pm 22.5 \] (68 per cent CL).

(ii) Equilateral:

\[ f_{	ext{nl}} = -90 \pm 146 \] (68 per cent CL).

(iii) Orthogonal:

\[ f_{	ext{nl}} = -180 \pm 107 \] (68 per cent CL).

Fig. 3 contains the histograms of the best-fitting \( f_{	ext{nl}} \) values for each shape corresponding to 1000 CMB + noise Gaussian simulations and the values of the data after the point-source correction.
Note that the point sources add a significant contribution to the equilateral and orthogonal shapes. We agree with Komatsu et al. (2011) that the WMAP 7-year data are consistent with Gaussian primordial fluctuations for the three considered shapes. Planck will be able to address this issue with more detail due to its increased sensitivity and power to clean the signal.

5 CONCLUSIONS

We have imposed constraints on primordial non-Gaussianity with the WMAP 7-year data using the wavelet-based estimator. In this analysis, we have considered the combined $V+W$ maps and the KQ75 mask. In particular, we have focused on three shapes with particular interest for the physics of inflation in the early universe: the local, equilateral and orthogonal bispectra.

We have simulated the non-Gaussian maps for each of the considered shapes and estimated with these simulations the required quantities for our estimator. Our results are compatible with the values obtained by the WMAP team and our uncertainties are very similar to the error bars obtained with the optimal bispectrum estimator (Komatsu et al. 2011).

In addition, we have estimated the contribution of the point sources. In the particular case of the local $f_{\text{nl}}$, the contribution is $\Delta f_{\text{nl}}^{\text{clus}} = 2.5 \pm 3.0$. This is similar to the values obtained by the WMAP team (Komatsu et al. 2011) and its contribution to the parameter is not significant. However, we have detected a non-negligible contribution to the equilateral and orthogonal shapes due to the unresolved point sources. In particular, we have found $\Delta f_{\text{nl}}^{\text{clus}} = 37 \pm 18$ and $\Delta f_{\text{nl}}^{\text{clus}} = 25 \pm 14$ (68 per cent CL). These large values were already predicted by Komatsu et al. (2011), although they did not provide actual figures. This contribution should be taken carefully into account in future constraints on $f_{\text{nl}}$ with WMAP and Planck data. Considering the point sources, our best estimates of $f_{\text{nl}}$ are $f_{\text{nl}}^{\text{clus}} = 30.0 \pm 22.5$, $f_{\text{nl}}^{\text{clus}} = -90 \pm 146$ and $f_{\text{nl}}^{\text{clus}} = -180 \pm 107$. The three shapes are compatible with zero at 95 per cent CL. Our conclusion is that the $f_{\text{nl}}$ parameters are compatible with zero within the 2$\sigma$ CL and our results are in agreement with Komatsu et al. (2011).

The wavelet estimator has been tested and carefully checked with the available WMAP data in this and several previous works (Curto et al. 2009a,b, 2011). It is now ready and being upgraded to analyse the forthcoming Planck data. In future works we will also use the wavelet estimator jointly with neural networks to constrain these shapes along the lines of Casaponsa et al. (2011b), where this procedure has been already applied for the local $f_{\text{nl}}$ using WMAP data. This later process helps to speed up the calculations since it is not necessary to estimate the covariance matrix of the cubic statistics and it avoids all the possible complications in the computation of the inverse covariance matrix.

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