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Gromov-Witten theory of complete intersections via nodal invariants. (English) Zbl 07738233
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Summary: We provide an inductive algorithm computing Gromov-Witten invariants in all genera with arbitrary insertions of all smooth complete intersections in projective space. We also prove that all Gromov-Witten classes of all smooth complete intersections in projective space belong to the tautological ring of the moduli space of stable curves. The main idea is to show that invariants with insertions of primitive cohomology classes are controlled by their monodromy and by invariants defined without primitive insertions but with imposed nodes in the domain curve. To compute these nodal Gromov-Witten invariants, we introduce the new notion of nodal relative Gromov-Witten invariants. We then prove a nodal degeneration formula and a relative splitting formula. These results for nodal relative Gromov-Witten theory are stated in complete generality and are of independent interest.

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