NEUTRINO MASSES AND MIXING FROM NEUTRINO OSCILLATION EXPERIMENTS

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Abstract

What information about the neutrino mass spectrum and mixing matrix can be inferred from the existing neutrino oscillation data? We discuss here the answer to this question in the case of mixing of three and four neutrinos. We present the constraints on the effective Majorana mass \(\langle m \rangle\) that can be obtained from the results of reactor neutrino oscillation experiments and from atmospheric neutrino data. We discuss the bounds on the oscillation probabilities in long-baseline neutrino oscillation experiments that follow from the results of short-baseline experiments. Some remarks on a model-independent approach to the solar neutrino problem are also made.

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1 Introduction

The problem of neutrino mass and mixing is the most important problem of today’s neutrino physics. The hypothesis of neutrino mixing was put forward by B. Pontecorvo in 1958 [1]. Many years later, after the appearance of GUT models, his idea became very popular. Today the investigation of neutrino masses and mixing is considered as one of the major ways of searching for new physics.

In accordance with the neutrino mixing hypothesis (see, for example, Refs. [2, 3, 4]), the fields $\nu_{\alpha L}$ of flavour neutrinos determined by the standard charged and neutral currents

$$
\begin{align*}
\hat{J}^{CC}_\rho &= 2 \sum_{\alpha=e,\mu,\tau} \bar{\nu}_{\alpha L} \gamma_\rho \ell_{\alpha L}, \\
\hat{J}^{NC}_\rho &= \sum_{\alpha=e,\mu,\tau} \bar{\nu}_{\alpha L} \gamma_\rho \nu_{\alpha L}.
\end{align*}
$$

are mixtures of the fields of neutrinos with definite mass:

$$
\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{k L}.
$$

Here $U$ is the unitary mixing matrix and $\nu_{k L}$ is the field of the neutrino with mass $m_k$.

Neutrino mixing can be quite different from the CKM quark mixing. Quarks are Dirac particles, whereas for neutrinos with definite masses there are two possibilities: neutrinos can be Dirac or truly neutral Majorana particles. Dirac masses and mixing of neutrinos can be generated by the standard Higgs mechanism. Majorana masses and mixing require a new mechanism of mass generation that does not conserve the total lepton charge. Let us notice also that the number of massive neutrinos in the general case of neutrino mixing can be more than the number of lepton flavours which, according to LEP data, is equal to three. In this case, in addition to Eq. (2) we have

$$
(\nu_{\alpha R})^c = \sum_k U_{\alpha k} \nu_{k L},
$$

where $\nu_{\alpha R}$ is a right-handed sterile field and $(\nu_{\alpha R})^c = C(\bar{\nu}_{\alpha R})^T$ ($C$ is the matrix of charge-conjugation).

From all existing data it follows that neutrino masses (if any) are much smaller than the masses of all the other fundamental fermions. There is the very attractive see-saw mechanism [5] of neutrino mass generation that connects the smallness of neutrino masses with the violation of lepton number at a very large scale $M$ that characterizes the right-handed Majorana mass term. In this case, for the neutrino masses we have the relations

$$
m_k \sim \frac{m_{F_k}^2}{M} \quad (k = 1, 2, 3),
$$

where $m_{F_k}$ is the mass of the up-quark or charged lepton in the $k^{th}$ generation and $M \gg m_{F_k}$. If the neutrino masses are generated with the see-saw mechanism, then 1) the number of massive neutrinos is equal to three, 2) massive neutrinos are Majorana particles, 3) there is a hierarchy of neutrino masses:

$$
m_1 \ll m_2 \ll m_3.
$$

At present there are three experimental indications in favour of neutrino mixing. The first indication was obtained in solar neutrino experiments: neutrino mixing is the most
natural explanation of the deficit of solar $\nu_e$'s observed in all solar neutrino experiments (Homestake, Kamiokande, GALLEX and SAGE \[6\]). The suppression of the solar $\nu_e$ flux can be due to resonant MSW transitions with a neutrino mass-squared difference $\sim 10^{-5}\text{eV}^2$.

The second indication in favour of neutrino oscillations was obtained in the Kamiokande, IMB and Soudan atmospheric neutrino experiments \[7\]. The observed deficit of muon neutrinos can be explained by neutrino oscillations with a mass-squared difference $\sim 10^{-2}\text{eV}^2$.

The third indication in favour of neutrino mixing was obtained in the LSND experiment \[8\]. The observed number of $\bar{\nu}_e$ events can be explained by $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations with a mass-squared difference $\sim 1\text{eV}^2$.

On the other hand, no indication in favour of neutrino masses and mixing was found in numerous reactor and accelerator oscillation experiments (see the reviews in Ref.\[9\]), in the experiments on the precise measurement of the high energy part of the beta-decay spectrum of tritium (see Ref.\[10\]) and in the experiments on the search for neutrinoless double-beta decay (see Ref.\[11\]).

We will address here the following question: what information about neutrino mixing and the neutrino mass spectrum can be obtained from the existing data. Some predictions for the future experiments will also be discussed.

Let us start with neutrino oscillations in short-baseline (SBL) experiments. We will consider the general case of $n$ neutrinos with masses

$$m_1 < m_2 < \ldots < m_{r-1} \ll m_r < \ldots < m_n \quad (5)$$

and we will assume that only the largest mass square difference $\Delta m^2 \equiv m_n^2 - m_1^2$ is relevant for SBL oscillations \[12, 13, 14, 15\]:

$$\frac{\Delta m^2 L}{2p} \gtrsim 1, \quad \frac{\Delta m_{kj}^2 L}{2p} \ll 1 \text{ for } k < r \quad \text{ and } \quad \frac{\Delta m_{nk}^2 L}{2p} \ll 1 \text{ for } k \geq r, \quad (6)$$

where $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$, $L$ is the distance between the neutrino source and detector and $p$ is the neutrino momentum.

Using the unitarity of the mixing matrix, for the amplitude of $\nu_\alpha \to \nu_\beta$ transitions we have

$$|A_{\nu_\alpha \to \nu_\beta}| = \left| \delta_{\alpha\beta} + \left( \sum_{k=r}^{n} U_{\beta k} U_{\alpha k}^* \right) \left[ \exp \left( -i \frac{\Delta m^2 L}{2p} \right) - 1 \right] \right|. \quad (7)$$

The probability of $\nu_\alpha \to \nu_\beta$ transitions with $\alpha \neq \beta$ is given by

$$P_{\nu_\alpha \to \nu_\beta} = \frac{1}{2} A_{\alpha\beta} \left( 1 - \cos \frac{\Delta m^2 L}{2p} \right), \quad (8)$$

with the oscillation amplitude

$$A_{\alpha\beta} = 4 \left\{ \sum_{k=r}^{n} U_{\beta k} U_{\alpha k}^* \right\}^2 = 4 \left\{ \sum_{k=1}^{r-1} U_{\beta k} U_{\alpha k}^* \right\}^2. \quad (9)$$
For the survival probability of $\nu_\alpha$, from Eqs. (8) and (9) we find

$$P_{\nu_\alpha \rightarrow \nu_\alpha} = 1 - \sum_{\beta \neq \alpha} P_{\nu_\alpha \rightarrow \nu_\beta} = 1 - \frac{1}{2} B_{\alpha;\alpha} \left( 1 - \cos \frac{\Delta m^2 L}{2p} \right),$$

(10)

where

$$B_{\alpha;\alpha} = \sum_{\beta \neq \alpha} A_{\alpha;\beta} = 4 \left( \sum_{r=1}^{n} |U_{3r}|^2 \right) \left( 1 - \sum_{r=1}^{n} |U_{3r}|^2 \right).$$

(11)

It is obvious from Eqs. (8)–(11) that $0 \leq A_{\alpha;\beta} \leq 1$ and $0 \leq B_{\alpha;\alpha} \leq 1$.

The formulas (8) and (10) have the form of the standard expressions for the transition probabilities in the case of mixing of two neutrinos (see, for example, Refs. [2, 3, 4]). Therefore, if we identify $A_{\alpha;\beta}$ or $B_{\alpha;\alpha}$ with $\sin^2 2\theta$ (where $\theta$ is the mixing angle in the two-neutrino case), we can use the results of the standard analyses of the neutrino oscillation data.

2 Three massive neutrinos

Let us consider first the case of three massive neutrinos with the mass hierarchy [1], assuming that $\Delta m^2_{21}$ is relevant for the suppression of the flux of solar $\nu_e$’s [12]. In this case $n = r = 3$, SBL oscillations depend on $\Delta m^2$, $|U_{e3}|^2$, $|U_{\mu3}|^2$ (the unitarity of $U$ implies that $|U_{\tau3}|^2 = 1 - |U_{e3}|^2 - |U_{\mu3}|^2$), and the oscillation amplitudes are given by

$$A_{\alpha;\beta} = 4 |U_{\alpha3}|^2 |U_{\beta3}|^2, \quad B_{\alpha;\alpha} = 4 |U_{\alpha3}|^2 \left( 1 - |U_{\alpha3}|^2 \right).$$

(12)

In this section we do not consider the atmospheric neutrino anomaly, whose explanation, together with the explanations of the other indications in favour of neutrino oscillations, requires at least four massive neutrinos (see Section 4).

The reactor $\bar{\nu}_e$ and accelerator $\bar{\nu}_\mu$ disappearance experiments did not find any positive indication in favour of neutrino oscillations. From the exclusion plots obtained from the data of these experiments, at any fixed value of $\Delta m^2$ we have the following upper bound for the oscillation amplitudes:

$$B_{\alpha;\alpha} \leq B_{\alpha;\alpha}^0 \quad (\alpha = e, \mu).$$

(13)

The exclusion plots obtained by the Bugey [17] $\bar{\nu}_e$ disappearance experiment and by the CDHS and CCFR $\bar{\nu}_\mu$ disappearance experiments imply that the amplitudes $B_{\alpha;\alpha}^0$ are small for any value of $\Delta m^2$ in the wide interval

$$10^{-1} \lesssim \Delta m^2 \lesssim 10^3 \text{ eV}^2.$$

(14)
This means that the parameters $|U_{e3}|^2$ and $|U_{\mu3}|^2$ can be small or large (close to one, see Eq.(12)):

$$|U_{\alpha3}|^2 \leq a_\alpha^0 \quad \text{or} \quad |U_{\alpha3}|^2 \geq 1 - a_\alpha^0 \quad (\alpha = e, \mu),$$

(15)

where

$$a_\alpha^0 = \frac{1}{2} \left(1 - \sqrt{1 - B_{\alpha\alpha}^0}\right).$$

(16)

The quantity $a_\alpha^0$ is small ($a_\alpha^0 \lesssim 4 \times 10^{-2}$) for any value of $\Delta m^2$ in the range (14) and $a_\mu^0$ is small for $\Delta m^2 \gtrsim 0.3$ eV$^2$ ($a_\mu^0 \lesssim 10^{-1}$) (see Ref.[12]).

From the results of the solar neutrino experiments it follows that only small values of $|U_{e3}|^2$ are allowed. In fact, in the case of a neutrino mass hierarchy that we are considering, the probability of solar neutrinos to survive is given by (see Ref.[16])

$$P_{\nu e \rightarrow \nu e}(E) = (1 - |U_{e3}|^2)^2 P_{\nu e \rightarrow \nu e}(E) + |U_{e3}|^4,$$

(17)

where $P_{\nu e \rightarrow \nu e}(E)$ is the $\nu_e$ survival probability due to the mixing between the first and the second generations and $E$ is the neutrino energy. Eq.(17) implies that $P_{\nu e \rightarrow \nu e} \geq |U_{e3}|^4$. If $|U_{e3}|^2 \geq 1 - a_0^0$, we have $P_{\nu e \rightarrow \nu e} \geq 0.92$ at all neutrino energies, which is a bound that is not compatible with the solar neutrino data.

Thus, we come to the conclusion that only two schemes are possible:

(I) $|U_{e3}|^2 \leq a_e^0$, $|U_{\mu3}|^2 \leq a_\mu^0$,

(II) $|U_{e3}|^2 \leq a_e^0$, $|U_{\mu3}|^2 \geq 1 - a_\mu^0$.

(18)

(19)

The amplitudes of $^{(\nu)}_{\mu \rightarrow \nu e}$ transitions in the case of scheme I and $^{(\nu)}_{\nu e \rightarrow \nu \tau}$ transitions in case of scheme II have upper bounds bilinear in the small quantities $a_e^0$, $a_\mu^0$:

(I) $A_{\mu e} \leq 4 a_e^0 a_\mu^0$,

(II) $A_{e \tau} \leq 4 a_e^0 a_\mu^0$.

(20)

(21)

On the other hand, the upper bound for the amplitude of $^{(\nu)}_{\mu \rightarrow \nu e}$ transitions in both schemes is only linear in the small quantity $a_\mu^0$: $A_{\mu e} \leq 4 a_\mu^0$.

The inequality (20) implies that $^{(\nu)}_{\mu \rightarrow \nu e}$ transition are strongly suppressed. Is this inequality compatible with the results of the LSND experiment in which indications in favour of $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations were found? This question was considered in Ref.[12].

The upper bound obtained with the help of Eq.(20) from the 90% CL exclusion plots of the Bugey [7] $\bar{\nu}_e$ disappearance experiment and of the CDHS and CCFR [18] $^{(\nu)}_{\mu \rightarrow \nu e}$ disappearance experiments is represented in Fig.4 by the curve passing through the circles. The shadowed regions in Fig.4 are allowed by LSND at 90% CL. Also shown are are the 90% CL exclusion curves found in the BNL E734, BNL E776, KARMEN and CCFR [19] $^{(\nu)}_{\mu \rightarrow \nu e}$ appearance experiments and in the Bugey experiment. It is seen from Fig.4 that the bounds that were obtained from direct experiments on the search for $^{(\nu)}_{\mu \rightarrow \nu e}$ oscillations and the bound (20) obtained in the framework of scheme I are not compatible with the allowed regions of the LSND experiment [12].
Therefore, we come to the conclusion that the scheme with a hierarchy of neutrino masses and couplings between generations (scheme I) is not favoured by the existing experimental data. A confirmation of the LSND $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ signal would mean that neutrino mixing in the case of three massive neutrinos is quite different from quark mixing: there is no hierarchy of couplings and $\nu_\mu$ (not $\nu_\tau$) is the “heaviest” neutrino.

3 Neutrinoless double-beta decay

We will discuss now the limitations on the effective Majorana mass

$$\langle m \rangle = \sum_k U_{ek}^2 m_k$$

(22)

that can be obtained from the neutrino oscillation data in the framework of the 3-neutrino mass scheme (4) [21, 12, 22]. As it is well known, the effective mass $\langle m \rangle$ characterizes the contribution of Majorana neutrino masses and mixing to the matrix element of neutrino-less double-beta decay ($\beta\beta_0$) in the case of a left-handed interaction (see, for example, Refs. [3, 4]).

In the case of three massive neutrinos with the mass hierarchy (4) and $\Delta m^2_{21}$ relevant for the oscillations of solar neutrinos, from Eq. (22) and the constraint $|U_{e3}|^2 \leq a^0_e$ we have

$$|\langle m \rangle| \approx |U_{e3}|^2 \sqrt{\Delta m^2} \leq a^0_e \sqrt{\Delta m^2}.$$  

(23)

The solid line in Fig. 2 depicts this upper bound with $a^0_e$ obtained from the results of the Bugey [17] and Krasnoyarsk [23] reactor experiments. The straight line represents the unitarity bound $|\langle m \rangle| \leq \sqrt{\Delta m^2}$. The dashed lines where obtained from the sensitivity plots of the CHOOZ and Palo Verde [20] long-baseline (LBL) reactor experiments. The shadowed region enclosed by the dash-dotted line is allowed at 90% CL by the fit [22] of the Kamiokande atmospheric neutrino data (the black triangle corresponds to the best value of the parameters). In order to include in the figure this allowed region and the sensitivity curves of the CHOOZ and Palo Verde experiments, we considered $\Delta m^2$ in the wide range $10^{-4} \leq \Delta m^2 \leq 10^2$ eV$^2$.

As it is seen from Fig. 2, if $\Delta m^2 \lesssim 10^{-1}$ eV$^2$ we have $|\langle m \rangle| \lesssim 10^{-1}$ eV$^2$. If $\Delta m^2 \lesssim 10^2$ eV$^2$ we have $|\langle m \rangle| \lesssim 4 \times 10^{-2}$ eV$^2$. The results of the LSND experiment indicate that $0.3 \lesssim \Delta m^2 \lesssim 3$ eV$^2$; in this case $|\langle m \rangle| \lesssim 7 \times 10^{-2}$ eV$^2$. Finally, the Kamiokande atmospheric neutrino data imply that $|\langle m \rangle| \lesssim 7 \times 10^{-2}$ eV$^2$.

As it is well known, the upper bounds on $|\langle m \rangle|$ obtained from experimental data of ($\beta\beta_0$) decay experiments depend on the results of the calculation of nuclear matrix elements. The most stringent limit [24, 25] is given by the results of the $^{76}$Ge experiments: $|\langle m \rangle| < (0.6 \text{ -- } 1.6)$ eV. A big progress in searching for ($\beta\beta_0$) decay is expected in near future: several collaborations plan to reach the sensitivity $|\langle m \rangle| \sim (0.1 \text{ -- } 0.3)$ eV [24, 26].

Let us stress that the bound (23) is valid only in the case of the neutrino mass hierarchy (4). In the case of the inverted mass hierarchy ($n = 3$ and $r = 2$ in Eq. (5)) $m_1 \ll m_2 \lesssim m_3$, with $\Delta m^2_{32}$ relevant for solar neutrino oscillations, $|\langle m \rangle|$ is limited only by the unitarity bound $|\langle m \rangle| \leq m_3$. 

6
Therefore, we come to the conclusion that the observation of neutrinoless double-beta decay could allow to obtain important information about the spectrum of masses of Majorana neutrinos \[21, 12\].

4 Four massive neutrinos

We will consider now the schemes with mixing of four massive neutrinos which have three different scales of mass-squared differences that correspond to all existing indications in favour of neutrino mixing \[28\]. There are six possible schemes of such type. If the results of solar, atmospheric, LSND and all the other neutrino oscillation experiments are taken into account, only two schemes with the following mass spectra are preferable:

\[
\begin{align*}
\text{(A)} & \quad m_1 < m_2 \ll m_3 < m_4 \\
\text{(B)} & \quad m_1 < m_2 \ll m_3 < m_4 \\
\end{align*}
\]

In scheme A, $\Delta m^2_{21}$ is relevant for oscillations of atmospheric and LBL neutrinos and $\Delta m^2_{43}$ is relevant for solar neutrino oscillations. In scheme B, the roles of $\Delta m^2_{21}$ and $\Delta m^2_{43}$ are reversed.

In order to see that the results of neutrino oscillation experiments indicate the neutrino spectra \[(24)\], let us consider, for example, the neutrino spectra with one mass, $m_4$, separated from other three masses by a gap of about 1 eV ($n = r = 4$ in Eq.\(\text{(5)}\)):

\[
m_1 < m_2 < m_3 \ll m_4 .
\]

In this case SBL oscillations depend on four parameters, $\Delta m^2$, $|U_{e4}|^2$, $|U_{\mu4}|^2$, $|U_{\tau4}|^2$, and the oscillation probabilities are given by Eqs.\(\text{(8)}\) and \(\text{(10)}\) with the oscillation amplitudes

\[
A_{\alpha;\beta} = 4 |U_{\alpha4}|^2 |U_{\beta4}|^2 , \quad B_{\alpha;\alpha} = 4 |U_{\alpha4}|^2 \left(1 - |U_{\alpha4}|^2\right).
\]

Using the same reasoning as in Section \ref{sec:3-neutrinos} and taking into account the results of solar neutrino experiments, we come to the conclusion that $|U_{e4}|^2 \leq a^0_e$. Now we must also take into account the atmospheric neutrino anomaly. For the probability of atmospheric $\nu_\mu$’s to survive we have the lower bound $P_{\nu_{\mu} \to \nu_{\mu}} \geq |U_{\mu4}|^4 \left[14\right]$. From this inequality we conclude that, in order to explain the atmospheric neutrino anomaly from the two possibilities for $|U_{\mu4}|^2$ given by the results of SBL ($\nu_\mu$) disappearance experiments, $|U_{\mu4}|^2 \leq a^0_\mu$ and $|U_{\mu4}|^2 \geq 1 - a^0_\mu$, we must choose the first one. Therefore, the solution of the solar and atmospheric neutrino problems and the results of reactor and accelerator disappearance experiments lead to the constraints $|U_{e4}|^2 \leq a^0_e$, $|U_{\mu4}|^2 \leq a^0_\mu$ and for the amplitude of $\nu_\mu \to \nu_\mu$ transitions we have the same bound \[(20)\] as in the 3-neutrino scheme I. The experimental situation on $\nu_\mu \to \nu_e$ oscillations is presented in Fig.\[4\] and we conclude that the neutrino mass spectra under consideration are not favoured by the experimental data. By the same reasons, the mass spectra with the lightest mass $m_1$ separated from the other three masses by a gap of about 1 eV are also not favoured by the data.
We will consider now the two schemes (24) with the mass spectra A and B. For the transition amplitudes in SBL experiments, from the general expressions (9) and (11) in the heaviest neutrino mass $m_3$ the search for neutrinoless double beta decay have a good chance to reveal the effects of thus, if scheme A is realized in nature the tritium experiments and the experiments on the matrix element of neutrinoless double-beta decays. In fact, we have

$$A_{\alpha;\beta} = 4 \left| \sum_{k=1,2} U_{\beta k} U_{\alpha k}^* \right|^2 = 4 \left| \sum_{k=3,4} U_{\beta k} U_{\alpha k}^* \right|^2 ,$$

(27)

$$B_{\alpha;\alpha} = 4 \left( \sum_{k=1,2} |U_{\alpha k}|^2 \right) \left( 1 - \sum_{k=1,2} |U_{\alpha k}|^2 \right)$$

$$= 4 \left( \sum_{k=3,4} |U_{\alpha k}|^2 \right) \left( 1 - \sum_{k=3,4} |U_{\alpha k}|^2 \right).$$

(28)

Let us define the parameters

$$c_\alpha \equiv \sum_{k=1,2} |U_{\alpha k}|^2 \quad \text{and} \quad d_\alpha \equiv \sum_{k=3,4} |U_{\alpha k}|^2 .$$

(29)

It is obvious that the unitarity of the mixing matrix requires that

$$c_\alpha + d_\alpha = 1 .$$

(30)

Taking into account the results of the solar and atmospheric neutrino experiments and the constraints from the reactor and accelerator disappearance experiments, in the schemes A and B we have [14]

(A) \quad c_e \leq a^0_e , \quad d_\mu \leq a^0_\mu ,

(31)

(B) \quad d_e \leq a^0_e , \quad c_\mu \leq a^0_\mu .

(32)

The schemes A and B give different predictions for the effective neutrino mass $m_\nu(3\text{H})$ measured in tritium experiments and for the effective Majorana mass $\langle m \rangle$ that determines the matrix element of neutrinoless double-beta decays. In fact, we have

$$m_\nu(3\text{H}) \simeq d_e m_4 \simeq m_4 , \quad \langle m \rangle \simeq \left| \sum_{k=3,4} U_{\alpha k}^2 \right| m_4 \leq d_e m_4 \simeq m_4 ,$$

(33)

$$m_\nu(3\text{H}) \simeq d_e m_4 \leq a^0_e m_4 \ll m_4 , \quad \langle m \rangle \leq d_e m_4 \leq a^0_e m_4 \ll m_4 .$$

Thus, if scheme A is realized in nature the tritium experiments and the experiments on the search for neutrinoless double beta decay have a good chance to reveal the effects of the heaviest neutrino mass $m_4$.

Let us consider now neutrino oscillations in long-baseline experiments. The probabilities of $\nu_\alpha \to \nu_\beta$ transitions in LBL experiments in the schemes A and B are given by

$$P_{\nu_\alpha \to \nu_\beta}^{(\text{LBL},A)} = \left| \sum_{k=1,2} U_{\beta k} U_{\alpha k}^* \exp \left( -i \frac{\Delta m^2_{k1} L}{2 p} \right) \right|^2 + \left| \sum_{k=3,4} U_{\beta k} U_{\alpha k}^* \right|^2 ,$$

(34)

$$P_{\nu_\alpha \to \nu_\beta}^{(\text{LBL},B)} = \left| \sum_{k=1,2} U_{\beta k} U_{\alpha k}^* \right|^2 + \left| \sum_{k=3,4} U_{\beta k} U_{\alpha k}^* \exp \left( i \frac{\Delta m^2_{4k} L}{2 p} \right) \right|^2 .$$

(35)
These formulas have been obtained taking into account the fact that in LBL experiments $\Delta m_{32}^2 L / 2p \ll 1$ in scheme A and $\Delta m_{21}^2 L / 2p \ll 1$ in scheme B and dropping the terms proportional to the cosines of phases much larger than $2\pi$ ($\Delta m_{kj}^2 L / 2p \gg 2\pi$ for $k = 3, 4$ and $j = 1, 2$), which do not contribute to the oscillation probabilities averaged over the neutrino energy spectrum. The oscillation probabilities for antineutrinos are given by the same expressions with the changes $U_{\beta k} \rightarrow U_{\beta k}^*$ and $U_{\alpha k}^* \rightarrow U_{\alpha k}$. With the help of the Cauchy–Schwarz inequality, from Eqs.(34) for the survival probability of $(\nu_\alpha \rightarrow \nu_\alpha)$ and $(\nu_\alpha \rightarrow \nu_\beta)$ transitions in LBL experiments in the scheme A we have the following bounds:

$$d_\alpha^2 \leq P_{(\nu_\alpha \rightarrow \nu_\alpha)}^{(\text{LBL})} \leq c_\alpha^2 + d_\alpha^2 , \quad (36)$$

$$\frac{1}{4} A_{\alpha;\beta} \leq P_{(\nu_\alpha \rightarrow \nu_\beta)}^{(\text{LBL})} \leq c_\alpha c_\beta + \frac{1}{4} A_{\alpha;\beta} , \quad (37)$$

where $A_{\alpha;\beta}$ is the amplitude of $(\nu_\alpha \rightarrow \nu_\beta)$ oscillations in SBL experiments (see Eq.(9)). The corresponding bounds in the scheme B can be obtained with the change $c_\alpha \leftrightarrow d_\alpha$. It is clear from Eqs.(31) and (32) that the bounds for the oscillation probabilities in LBL experiments are equal in the schemes A and B.

From Eqs.(36) and (37), using the limits on the parameters $c_\varepsilon$ and $A_{\mu;\varepsilon}$ obtained from the results of reactor and accelerator SBL oscillation experiments, it is possible to obtain rather strong constraints on the probabilities of $(\nu_\varepsilon \rightarrow \nu_\varepsilon)$ and $(\nu_\mu \rightarrow \nu_\varepsilon)$ transitions in LBL experiments [13].

For the transition probability of $(\nu_\varepsilon)$ into all possible states, from Eqs.(31), (32) and (36) we have the following bound

$$1 - P_{(\nu_\varepsilon \rightarrow \nu_\varepsilon)}^{(\text{LBL})} \leq a_\varepsilon^0 (2 - a_\varepsilon^0) . \quad (38)$$

The curve corresponding to this limit obtained from the 90% CL exclusion plot of the Bugey [17] experiment is shown in Fig.3 (solid line). The range of the SBL parameter $\Delta m^2$ considered is $10^{-1} \leq \Delta m^2 \leq 10^3 \text{eV}^2$. The dash-dotted and dash-dot-dotted vertical lines depict the minimal probability (sensitivities) that will be reached by CHOOZ and Palo Verde long-baseline reactor neutrino experiments. The shadowed region in Fig.3 is allowed by the results of LSND experiment. Thus, in the framework of the schemes A and B, the CHOOZ experiment could reveal LBL neutrino oscillations if $\Delta m^2 \gtrsim 4\text{eV}^2$.

Let us consider now $(\nu_\mu \rightarrow \nu_\varepsilon)$ transitions in LBL experiments. From Eqs.(31), (32) and (37), we have the upper bound

$$P_{(\nu_\mu \rightarrow \nu_\varepsilon)}^{(\text{LBL})} \leq a_\varepsilon^0 + \frac{1}{4} A_{\mu;\varepsilon}^0 . \quad (39)$$

where $A_{\mu;\varepsilon}^0$ is the upper bound for the amplitude of $(\nu_\mu \rightarrow \nu_\varepsilon)$ transitions found in SBL experiments. Another inequality can be obtained from Eq.(38) using the unitarity of the mixing matrix:

$$P_{(\nu_\mu \rightarrow \nu_\varepsilon)}^{(\text{LBL})} \leq a_\varepsilon^0 (2 - a_\varepsilon^0) . \quad (40)$$
The curves corresponding to the limits (10) (solid lines) and (39) (long-dashed line) obtained from the 90% CL exclusion plots of the Bugey [17] experiment for $a^0_{\nu_e}$ and of the BNL E734, BNL E776 and CCFR [19] experiments for $A^0_{\mu e}$ are shown in Fig.4. Sensitivities of the KEK–SK, MINOS and ICARUS [29] LBL accelerator experiments are represented in Fig.4 by the dotted, dash-dotted and dash-dot-dotted vertical lines. The shadowed region is the region allowed by the results of the LSND experiment. As it is seen from Fig.4, the sensitivities of the MINOS and ICARUS experiments are much higher than the bounds that we have obtained. The solid line in Fig.4 represents also an upper bound for the probability of $^{(-)}\nu_e\rightarrow^{(-)}\nu_{\tau}$ transitions.

In the framework of schemes A and B the results of SBL neutrino oscillation experiments do not put any constraints on the probabilities of $^{(-)}\nu_\mu\rightarrow^{(-)}\nu_\mu$ and $^{(-)}\nu_\mu\rightarrow^{(-)}\nu_{\tau}$ transitions in LBL experiments. From our analysis it follows that $^{(-)}\nu_\mu\rightarrow^{(-)}\nu_\mu$ and $^{(-)}\nu_\mu\rightarrow^{(-)}\nu_{\tau}$ are the preferable channels for future LBL accelerator neutrino experiments.

5 Solar neutrinos

In this Section we will present a few remarks about solar neutrinos. Most analyses of the data of solar neutrino experiments are based on the Standard Solar Model [31]. In spite of the great success of the model, it is very important to check its predictions and to obtain model-independent information about neutrino mixing from solar neutrino experiments.

It was shown in Ref.[30] that when the data of the Super-Kamiokande (S-K) [32] and SNO [33] experiments will be available it will become possible: 1) to check whether there are transitions of solar $\nu_e$’s into other states; 2) to measure the initial flux of solar $^8\text{B}$ $\nu_e$’s; 3) to determine the probability of solar $\nu_e$’s to survive directly from experimental data.

Important features of future solar neutrino experiments will be the detection of solar neutrinos via CC and NC reactions and the relatively large statistics of events. In the S-K experiment solar neutrinos are detected by the observation of the elastic scattering (ES) process $\nu e^-\rightarrow\nu e^-$. The spectrum of the recoil electrons in this process can be written in the form

\[ n^{\text{ES}}(T) = \int_{E_m(T)} dE \left[ \frac{d\sigma^{\nu_e e}}{dT}(E, T) - \frac{d\sigma^{\nu_\mu e}}{dT}(E, T) \right] \phi_{\nu_e}(E) \]

\[ + \int_{E_m(T)} dE \frac{d\sigma^{\nu_\mu e}}{dT}(E, T) \phi^0_{\nu_e}(E) \sum_{\beta=e,\mu,\tau} P^{\text{sun}}_{\nu_e\rightarrow\nu_\beta}(E). \]

Here $T$ is the kinetic energy of the recoil electron, $E_m(T) = \frac{T}{2} \left( 1 + \sqrt{1 + \frac{2m_e}{T}} \right)$, $\frac{d\sigma^{\nu_e e}}{dT}(E, T)$ is the differential cross section of the process $\nu_\alpha e \rightarrow \nu_\alpha e$ ($\alpha = e, \mu$), $\phi_{\nu_e}(E)$ is the spectrum of solar $\nu_e$’s on the Earth and $\phi^0_{\nu_e}(E)$ is the spectrum of initial $^8\text{B}$ neutrinos, which can be written as

\[ \phi^0_{\nu_e}(E) = \Phi_B X(E) \]
were $X(E)$ is a known normalized function and $\Phi_B$ is the total flux. We can rewrite the relation (42) in the form

$$\frac{\Sigma^{\text{ES}}(T)}{X_{\nu_{\mu e}}(T)} = \left\langle \sum_{\beta=e,\mu,\tau} P_{\nu_{\mu e} \to \nu_{\beta}}^{\text{sun}} \right\rangle_T \Phi_B,$$  

(44)

where

$$\Sigma^{\text{ES}}(T) \equiv n^{\text{ES}}(T) - \int_{E_m(T)} dE \left[ \frac{d\sigma_{\nu_{e e}}}{dT}(E,T) - \frac{d\sigma_{\nu_{\mu e}}}{dT}(E,T) \right] \phi_{\nu_e}(E)$$  

(45)

and

$$X_{\nu_{\mu e}}(T) \equiv \int_{E_m(T)} dE \frac{d\sigma_{\nu_{\mu e}}}{dT}(E,T) X(E)$$  

(46)

is a known function. The quantity $\left\langle \sum_{\beta=e,\mu,\tau} P_{\nu_{\mu e} \to \nu_{\beta}}^{\text{sun}} \right\rangle_T$ is the average over $d\sigma_{\nu_{\mu e}}/dT(E,T) X(E)$ of the total transition probability of solar $\nu_e$’s into all possible active states. If there are no transitions of solar neutrinos into sterile states, we have $\left\langle \sum_{\beta=e,\mu,\tau} P_{\nu_{\mu e} \to \nu_{\beta}}^{\text{sun}} \right\rangle_T = 1$.

The function $\Sigma^{\text{ES}}(T)$ can be determined by combining the S-K measurement of the spectrum of recoil electrons with the measurement of the spectrum $\phi_{\nu_e}(E)$ of $\nu_e$’s on the Earth. Such measurement will be done in the SNO experiment by the investigation of the CC process $\nu_e + d \to e^- + p + p$.

If it is found that the function $\Sigma^{\text{ES}}(T)/X_{\nu_{\mu e}}(T)$ depends on energy, we will have a model-independent proof that solar $\nu_e$’s transfer into sterile states (if the function $\Sigma^{\text{ES}}(T)/X_{\nu_{\mu e}}(T)$ does not depend on energy, it could mean either that solar neutrinos do not transfer into sterile states or the probability of this transition does not depend on energy). The function $\Sigma^{\text{ES}}(T)$ was calculated in Ref.[34] in a model with $\nu_e-\nu_s$ mixing. The values of the mixing parameters were taken from the fit of solar neutrino data. In Fig.3 we present the function

$$R^{\text{ES}}(T) \equiv \frac{\Sigma^{\text{ES}}(T)}{X_{\nu_{\mu e}}(T)} / \left( \frac{\Sigma^{\text{ES}}(T)}{X_{\nu_{\mu e}}(T)} \right)_{\max},$$  

(47)

where the subscript max indicates the maximum value in the allowed range of $T$. Figure 3 illustrates the rather strong dependence of the ratio $R^{\text{ES}}(T)$ on $T$ in the model.

We have described one possible test that could reveal the presence of sterile neutrinos in the flux of solar neutrinos on the Earth. Other model-independent tests are discussed in Refs.[31, 34]. If there are no transitions of solar neutrinos into sterile states, the initial flux of $^8$B neutrinos can be determined directly from experimental data. From Eq. (44) we have

$$\Phi_B = \frac{\Sigma^{\text{ES}}(T)}{X_{\nu_{\mu e}}(T)}.$$  

(48)

If the total flux $\Phi_B$ is known, the survival probability of solar neutrinos can be determined from the CC measurement of the flux of $\nu_e$’s on the Earth:

$$P_{\nu_{e} \to \nu_{e}}^{\text{sun}}(E) = \frac{\phi_{\nu_e}(E)}{X(E) \Phi_B}.$$  

(49)
6 Conclusions

The present and future neutrino oscillation experiments will check the existing indications in favour of neutrino mixing. We have shown here with different arguments that these experiments have a good potential to obtain model-independent information about the neutrino mass spectrum and the elements of the neutrino mixing matrix. It is clear that this information will be extremely important for the future theory of neutrino masses and mixing.

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\[ \Delta m^2 (\text{eV}^2) \]

Figure 15
\[ \Delta m^2 (\text{eV}^2) \]

\[ |\langle m \rangle| = \sqrt{\Delta m^2} \]

Figure 2
\[ \nu_e \rightarrow \nu_e \]

- CHOOZ
- Palo Verde
- LSND

\( \Delta m^2 (\text{eV}^2) \)

\( 1 - P_{\nu_e \rightarrow \nu_e}^{(\text{LB})} \)

Figure 3
$\nu_\mu \rightarrow \nu_e$

$\Delta m^2 (eV^2)$

Figure 4
Figure 5