Single-atom maser with engineered circuit for population inversion

A.A. Sokolova,1,2,3 G.P. Fedorov,1,2,3 E.V. Il’ichev,4,1 and O.V. Astafiev5,2,6,7

1Russian Quantum Center, Skolkovo village, Russia
2Moscow Institute of Physics and Technology, Dolgoprudnyi, Russia
3National University of Science and Technology MISIS, Moscow, Russia
4Leibniz Institute of Photonic Technology, 07745 Jena, Germany
5Skolkovo Institute of Science and Technology, Moscow, Russia
6Physics Department, Royal Holloway, University of London, Egham, Surrey TW20 0EX, United Kingdom
7National Physical Laboratory, Teddington, TW11 0LW, United Kingdom

(Dated: October 2020)

We present a blueprint for a maser with a single three-level artificial atom. The artificial atom is a superconducting quantum system of a transmon layout coupled to two resonators. The system is pumped via a two-photon process. To achieve a population inversion, we engineer the quantum system and optimize its parameters, particularly the relaxation via an auxiliary low-Q cavity coupled to a transition between two excited states. We show numerically that such a maser can operate both in the intermediate coupling regime with super-Poissonian photon statistics and in the strong coupling regime, where the statistics is sub-Poissonian. For the former, the maser exhibits thresholdless behavior and for the latter, there is a well-defined pumping threshold. An interesting side-effect of the auxiliary resonator is that it allows overcoming the photon blockage effect for the pump, which would otherwise prohibit reaching high photon population. Finally, we observe the bistability of the steady-state Wigner function and the self-quenching effect for some parameters.

I. INTRODUCTION

Conventional multi-atom lasers operate in the regime of weak coupling between the atoms and the light modes, possess a threshold and emit classical light. Single-atom lasers with strong coupling have been gaining increased attention since 1990s, as their behavior was predicted to be drastically different from usual lasers [1–12]. Among their non-standard features are presence or absence of the pumping threshold depending on the parameters [13], self-quenching, when the number of photons in the cavity reduces, even though the pumping rate is increased [1]; photon blockade of coherent pumping [13,15], which restricts laser from pumping much more than one photon in strong-coupling regime; bistability of Wigner function [16,17]; sub-Poissonian statistics of the emitted radiation leading to amplitude squeezing and pure quantum effects such as photon antibunching, or super-Poissonian statistics, and thus high intensity fluctuations and phase squeezing [1].

The non-classical light exhibits certain unconventional noise properties, providing a relatively wide range of applications [18,19]. In particular this includes quantum measurements [20] and quantum imaging [21,22]. Recently non-classical correlations of microwaves have been proposed to employ for quantum communications [23].

Experimentally, a single-atom maser has been first realized in 1985 on Rydberg atoms [24]. Then, several other platforms for single-atom lasing have been proposed, including trapped ions [13,25], trapped Cs atom [26,27], quantum dots [28,30], superconducting single-electron transistors [10,31] and flux qubits [32,34]. The single-atom masers based on superconducting cQED devices acquire particular attention since they can be used in tandem with superconducting artificial atoms [35].

In this work, we present a blueprint for a maser in this architecture using a transmon-type artificial atom [36]. The transmon can be regarded as a Ζ-system with the eigenstates |g⟩, |e⟩, |f⟩ referred to ground, first and second excited states correspondingly. A classical lasing scheme in Quantum Optics is based on a Lambda-configuration of a three-level atom [1]. However, transmon is a cascade system and cannot be excited directly to the second excited state. The lasing transition |e⟩ → |g⟩ is in resonance with the reservoir (lasing) cavity, and the pumping is done via two-photon excitation of the |g⟩ ↔ |f⟩ transition. The pumping processes for population inversion is described as |g⟩ 2ℏΩ∧ |f⟩ → |e⟩. To increase the effective pumping rate, we couple the transmon to another low-Q cavity resonant with the |f⟩ → |e⟩ transition, very similar to what was done before for a chain of transmons [37]. This cavity plays a role of artificial bath [38,41], which relaxes the system to |e⟩. Then, we solve the master equation numerically for the full system and study its transient and steady-state behavior for various combinations of parameters.

Our model is different from the ones studied before at least in two ways. First, we employ engineered |f⟩ → |e⟩ dissipation via an auxiliary resonator, which simultaneously significantly modifies the system energy levels providing convenient configuration. Second, we use a two-photon process as coherent pumping.

For our device, the additional coupling of auxiliary resonator reduces the effect of the photon blockade [14] caused by the vacuum Rabi-splittings [3], similar to the cascade laser proposed in 1995 [42], and allows to pump more than 40 photons in the strong-coupling regime. We find, that in our system a well-defined laser threshold should be observed, which we associate with the unusual energy level structure. Additionally, we find a phase transition with a sharp increase of the mean steady-state photon number and the intensity fluctu-
ations between regimes with and without the threshold, with sub-Poissonian statistics in the regime with the threshold. A similar transition between different lasing regimes was previously predicted for the strong coupling [16][43].

II. METHODS

A. Modeling the system

The conceptual scheme of the system is shown in Fig. 1(a). A tunable transmon is coupled to two cavities: to the reservoir (right) and the auxiliary cavity (left) with the coupling strengths \( g_r \) and \( g_a \), respectively. The first cavity is the lasing one, which has high internal and external Q-factors and should accumulate a considerable number of photons, and is in resonance with the \( |g⟩ \leftrightarrow |e⟩ \) transition of the transmon, at frequency \( \omega_{ge} \). The second low-Q cavity is resonant with the \( |e⟩ \leftrightarrow |f⟩ \) transition at frequency \( \omega_{ef} \), and thus provides an engineered metastability of the \( |f⟩ \) state. Factorized states are denoted as \( |\mu, M, N⟩ \) where \( \mu \) is transmon level (\( g, e \) or \( f \)), \( M \) and \( N \) are the number of photons in the auxiliary resonator and the reservoir, respectively. It is possible to create population inversion in a bare transmon by using the two-photon excitation of the level \( |f⟩ \) since the relaxation rate is normally \( \gamma_{fe} > \gamma_{eg} \), because in transmon the transition matrix element \( \langle e|v|f⟩ \) is a factor of \( \sqrt{2} \) larger than \( \langle g|v|e⟩ \). Simulations show that for significant inversion, and thus higher pumping rate, increased \( \gamma_{fe} \) is required.

The simplest description of the single-atom laser is based on the Jaynes-Cummings model [44]. Our implementation dictates obvious modification, and, in the frame of rotating wave approximation (RWA), the resulting Hamiltonian reads:

\[
\hat{H} = \hat{H}_t + \hat{H}_d + \sum_{\lambda=ge,ef} \hat{H}^{(\lambda)}_r + \hat{H}^{(\lambda)}_i,
\]

\[
\hat{H}_t = \hbar \Delta b^\dagger b + \frac{\hbar \alpha}{2} b^\dagger b (b^\dagger b - 1),
\]

\[
\hat{H}_d = \frac{\hbar \Omega}{2} (b + b^\dagger),
\]

\[
\hat{H}^{(\lambda)}_r = \frac{\hbar g_{\lambda}}{2} (a_\lambda^\dagger a_\lambda + 1/2),
\]

\[
\hat{H}^{(\lambda)}_i = \frac{\hbar g_{\lambda}}{2} (a_\lambda^\dagger a_\lambda + b_\lambda^\dagger b_\lambda),
\]

where \( \hat{H}_t \), \( \hat{H}_d \), \( \hat{H}^{(\lambda)}_r \) and \( \hat{H}^{(\lambda)}_i \) are transmon, drive, resonator and interaction Hamiltonians, respectively, and \( a_\lambda \), \( a_\lambda^\dagger \), \( b \), \( b^\dagger \) are the usual annihilation (creation) operators for the reservoir, auxiliary resonator and the transmon, correspondingly. The transmon parameters are \( \Delta = \omega_{ge} - \omega_d \), the detuning of drive frequency \( \omega_d \) from the \( |g⟩ \leftrightarrow |e⟩ \) transition; \( \alpha < 0 \), the transmon anharmonicity; \( \Omega \), which is the microwave drive amplitude. Next, \( \Delta^{(ge,ef)} = \omega^{(ge,ef)} - \omega_d \) are the detunings of the reservoir and auxiliary resonator, respectively. In the ideal configuration, when the reservoir is resonant with the \( |g⟩ \leftrightarrow |e⟩ \) transition, the auxiliary resonator is resonant with the \( |e⟩ \leftrightarrow |f⟩ \) transition, and the drive is at the two-photon frequency \( \omega_d = \omega_{gf}/2 = \omega_{ge} + \alpha/2 \), we can find \( \Delta^{fe} = \Delta = -\alpha/2 \) and \( \Delta^{ef} = \alpha/2 \). Finally, the interaction strengths between the systems is described by \( g_{ge,ef} \); the cavities do not interact directly with each other.

To analyse the laser behavior involving dissipation, we solve the Lindbladian master equation: for the transmon, the drive is at \( \omega_d = \omega_{gf}/2 \), for \( \Omega/2\pi = 25 \) MHz.

In all simulations, we keep \( \Omega/2\pi \) below 35 MHz to avoid direct pumping of the \( |g, 0, 0⟩ \) and \( |g, 0, 1⟩ \) levels, i.e., the direct off-resonant pumping of the cavity. The reasons why \( \omega_d \) has such a precise value despite energy levels splittings due to coupling are discussed in section III B. Since \( \kappa_a > > g_a \), the number of photons in auxiliary resonator is nearly zero in average, so we can truncate the Fock space of this resonator to one photon. During the optimization, we use fixed realistic values for \( \gamma = 0.1 \) MHz, \( \omega_{ge}/2\pi = 6 \) GHz, \( \alpha/2\pi = -200 \) MHz, \( \omega_{gf}/2\pi = 6 \) GHz, and \( \omega_p/2\pi = 5.8 \) GHz. For the optimal parameters we found \( n_{ss} = 49 \), which correspond to \( -132 \) dBm of emitted power.

Fabricating the resonators with megahertz accuracy in frequency should be experimentally attainable, and then the optimal configuration can be reached by tuning the transmon into the resonance with the reservoir via the external magnetic flux. Schematically, the electric scheme of the device is shown in Fig. 1(b).

The number of pumped photons is tolerant to small changes
of the parameters, which can occur during fabrication. \( n_{ss} \) becomes lower, but still enough to measure, due to mismatch between the auxiliary resonator frequency and \(|e\rangle \leftrightarrow |f\rangle \) transition up to 30 MHz, while \(|g\rangle \leftrightarrow |e\rangle \) frequency of the transmon can be always tuned directly to resonance with the reservoir cavity. The changes of \( \kappa_a \) do not affect \( n_{ss} \) significantly: higher \( \kappa_a \) does not change \( n_{ss} \) at all, while up to 40% lower \( \kappa_a \) allow for pumping enough photons to measure. Variation of \( g_f \) and \( g_e \) affects the number of pumped photons smoothly (see Section III C for detail), so their deviation from ideal values also should not be a problem. The only parameter, which significantly affects the pumping, is \( \kappa_r \): the lower \( \kappa_r \) allow for pumping much more photons, but higher \( \kappa_r \) may completely kill pumping. However, it is possible to achieve lasing with a higher \( \kappa_r \) by increasing \( g_r \), so one should consider it when designing the device.

**B. Dynamics and steady-state simulation**

To calculate numerically the dynamics of the system, we use a Python package named QuTiP [46]. To facilitate the interpretation of the simulation results, we split the task into several steps.

First of all, we simulate the transmon under a two-photon drive. The simulated evolution is shown in the upper panel of Fig. 2(a) (here and below we truncate the transmon subspace to four states). One can clearly see two-photon Rabi oscillations when the \(|f\rangle \) energy level is fully populated by the two-photon process. There are also small oscillations of the \(|e\rangle\)-level population from the virtual processes.

In the middle panel Fig. 2(a), we add natural dissipation to the transmon \(|e\rangle \rightarrow |g\rangle \). The dissipation \(|f\rangle \rightarrow |e\rangle \) is higher \( n_{12}/n_{01} \) times, where \( n_{ij} \) is matrix element of charge eigenstate in the basis of transmon eigenstates (from the definition of lowering operator). As one can see, in this case, some population inversion may be achieved without any additional efforts. However, when the auxiliary resonator is added to the model, the achievable population inversion becomes significantly larger (the low panel of Fig. 2(b)).

Finally, we include the reservoir cavity. The resulting level structure of the tripartite system is depicted in Fig. 2(b) for a low number of photons, and Fig. 2(c) for a high number of photons. One can see that there are two types of vacuum Rabi-splittings in the spectrum. The first, caused by the interaction between the reservoir and the \(|g\rangle \leftrightarrow |e\rangle \) transition of the transmon, leads to the splittings composed of

\[
|geN\rangle^\pm = \frac{1}{\sqrt{2}}( |g,0,N + 1\rangle \pm |e,0,N\rangle )
\]

Dressed levels. \( N \) denotes the number of photons in the reservoir. The splitting includes levels with a different number of photons \( (N \pm 1) \), and we designate the levels using the lower number (in this case, \( N \)). The size of the splitting is calculated as

\[
\Delta_{ge}(N) = 2g_r\sqrt{N + 1},
\]

where \( \Delta_{ge}(N) \) is the energy difference between \(|geN\rangle^+\) and \(|geN\rangle^-\). The second splitting is due to the coupling between three degenerate states \(|f,0,N\rangle, |e,1,N\rangle, \) and \(|g,1,N + 1\rangle\). When the degeneracy is lifted, triplet eigenstates appear:

\[
|gefN\rangle^0 = \sin \theta |g,1,N + 1\rangle - \cos \theta |e,1,N\rangle,
\]

\[
|gefN\rangle^\pm = \frac{1}{\sqrt{2}}( \cos \theta |g,1,N + 1\rangle \pm |e,1,N\rangle + \sin \theta |f,0,N\rangle ),
\]

where

\[
\tan \theta = \frac{g_a}{\sqrt{N + 1}g_r}
\]

is the mixing angle. The energy difference between \(|gefN\rangle^+\) and \(|gefN\rangle^-\) (Fig. 2(b)) is

\[
\Delta_{gef}(N) = 2\sqrt{(N + 1)g_a^2 + g_r^2}.
\]

As previously, we designate the states by the lower number of photons.
and dissipation operator

\[ C = a_a. \]  

III. RESULTS

A. A simplified analytical model

To calculate the approximate number of pumped photons, we reduce the system to two levels with effective pumping \( \Gamma \) and dissipation \( \kappa^\text{eff} \) (Fig. 3(d)). The pumping stops when \( \kappa^\text{eff} = \Gamma \), where \( \kappa^\text{eff} \) depend on \( N \). Such a simplification consists of several steps.

Firstly, one needs to take into account the effect of auxiliary resonator on the \( |f\rangle \rightarrow |e\rangle \) dissipation. For this purpose, we consider two coupled two-level systems (Fig. 3(a)). The first two-level system (on the left) represents \( |e\rangle \) and \( |f\rangle \) levels of transmon. The second one (on the right) – \( |0\rangle \) and \( |1\rangle \) levels of the auxiliary resonator. The coupling constant \( g_a \). The effective dissipation rate \( |f\rangle \rightarrow |e\rangle \) is \( \kappa^\text{eff} \), which need to be calculated.

We solve Master equation

\[ \dot{\rho} = -i[H, \rho] + \kappa_a(C\rho C^+ - \frac{1}{2}\{C^+C, \rho\}) \]  

with Hamiltonian

\[ H = g_a(b_a^+ + a_a b^+) \]  

and dissipation operator

\[ C = a_a. \]  

Assuming initial state \( |f\rangle \), the analytical solution in different regimes is the following. In strong coupling regime, when \( 4g_a > \kappa_a \), the population of \( |f\rangle \) is

\[ \rho_{ff} = e^{-\frac{4g'}{\cos^2 \theta}} \cos^2(g't + \theta), \]  

where \( 4g' = \sqrt{\kappa_a^2 - 16g_a^2} \), \( \tan \theta = \frac{\kappa_a}{g_a} \). The decay of oscillations is \( \frac{2g_a}{4} \), therefore, we can assume that

\[ \kappa^\text{eff} = \frac{\kappa_a}{2}. \]  

In the week coupling regime, when \( 4g_a < \kappa_a \),

\[ \rho_{ff} = e^{-\frac{2g'}{\cosh \theta}} \cosh^2(\gamma t + \theta), \]  

where \( \gamma = \sqrt{\kappa_a^2 - 16g_a^2} \), \( \tan \theta = \frac{\kappa_a}{g_a} \). The decay rate is determined by the slowest exponent, so

\[ \kappa^\text{eff} = \frac{\kappa_a - \sqrt{\kappa_a^2 - 16g_a^2}}{2}. \]  

One can see that, if \( g_a \) is fixed, increasing \( \kappa_a \) leads to increasing \( \kappa^\text{eff} \), while the condition for the strong-coupling regime is fulfilled, and then decreasing \( \kappa^\text{eff} \), when it turns to the weak-coupling regime. Physically, decreasing \( \kappa^\text{eff} \) can be explained by broadening of \( |1\rangle \) level, which prevents pumping energy from \( |f\rangle \). Thus, \( \kappa^\text{eff} \) reaches its maximum when \( \kappa_a = g_a \). It is consistent with our simulations (section II A).

Next, we calculate effective \( |g\rangle \rightarrow |e\rangle \) pumping rate \( \Gamma \). The simplified model of the system with two-photon pumping and \( |f\rangle \rightarrow |e\rangle \) dissipation is shown on Fig. 3(b). The two-photon process is replaced by equivalent one-photon process with the corresponding frequency \( \Omega_{2ph} \) (see Appendix A for detail). The dissipation rate \( \kappa^\text{eff} \) is calculated above. One need to take into account that such a model is approximate, since it does not consider that actually \( |g\rangle \rightarrow |f\rangle \) pumping is not resonant due to energy level splittings, discussed in section III B.

On Fig. 3(b) one can see that the tree-level system with pumping and dissipation is completely equivalent to the two coupled two-level systems on Fig. 3(a) with \( g_a \) replaced by \( \Omega_{2ph}/2 \) – matrix element of drive in RWA. One can check analytically that the solution for the such a system is the same: in the strong-coupling regime \( 4\Omega_{2ph}/2 > \kappa^\text{eff} \) the effective pumping rate is

\[ \Gamma = \frac{\kappa^\text{eff}}{2}, \]  

and for the weak-coupling regime \( 4\Omega_{2ph}/2 < \kappa^\text{eff} \)

\[ \Gamma = \frac{\kappa^\text{eff} - \sqrt{\kappa^\text{eff}^2 - 4\Omega_{2ph}^2}}{2}. \]

The strong-coupling regime, in this case, is not achievable due to restrictions on the pumping rate \( \Omega \): it should not be large enough to pump the reservoir directly. Consequently, we should use Eq. 21 to calculate effective pumping rate.
the optimal parameters from Section II A, it leads to \( \Gamma = 5.3 \) MHz.

The next step is calculation of reservoir effective dissipation rate \( \kappa_{eff} \) (Fig. 3(c)). Again, the system is equivalent to Fig. 3(a). Coupling strength \( g_a \) should not be much lower then \( g_a \), otherwise the Rabi splittings will disturb pumping (see next section for detail). Assuming this, for optimal \( g_a \) and \( \kappa_r \ll 1 \) MHz, the condition for the strong coupling regime \( 4\sqrt{N}g_r > N\kappa_r \) is met even for large number of photons (\( N \sim 100 \)). Since,

\[
\kappa_{eff} = \frac{n_{ss}\kappa_r}{2}.
\]

The condition, when the pumping stops, is \( \kappa_{eff} = \Gamma \) (Fig. 3(d)). Consequently, the maximum number of pumped photons can be expressed as following:

\[
n_{ss} = \frac{2\Gamma}{\kappa_r}.
\]

For the optimal parameters, this equation gives \( n_{ss} = 34 \). The exact simulated value is \( n_{ss} = 49 \). The 30% difference may be caused by energy level splittings due to coupling to both cavities, which leads to photon blockade and, in our system, to overcoming photon blockade (section III B), and which were not considered in this section.

B. Overcoming photon blockade

To illustrate the importance of the auxiliary cavity, one can consider a simple device that does not include an auxiliary resonator and operates in the strong-coupling regime. For such a device, the pumping at the \( |g,0,0\rangle \leftrightarrow |f,0,0\rangle \) frequency is inefficient for \( \langle N \rangle = \langle a_d^\dagger a_{ge} \rangle \geq 1 \) [3]. This is caused by the increasingly large \( |g\rangle \leftrightarrow |c\rangle \) vacuum Rabi splitting which detunes the transitions between \( |geN\rangle^\pm \) and \( |f,0,N\rangle \) from the pump frequency (see Fig. 2(b)). Due to the \( \sqrt{N} \) growth of the \( \Delta_{ge}(N) \), there is no single frequency that would satisfy the resonance condition for an arbitrary population of the reservoir. The same effect is observed for ordinary coherently pumped single-atom lasers: the detuning \( \delta = g_r\sqrt{N} \rightarrow \infty \) when \( N \rightarrow \infty \) leads to the photon blockade [13].

However, in presence of the auxiliary resonator, this detuning is calculated as

\[
\delta = (\Delta_{ge}(N + 1) - \Delta_{ge}(N))/2
\]

One can see that \( \delta \rightarrow 0 \) for \( N \rightarrow \infty \) (Fig. 2(c)), which means that it is possible to use a monochromatic pump at \( \omega_{gf}/2 \) for sufficiently large values of \( \langle \hat{N} \rangle \), even though it would be slightly off-resonant for low \( \langle \hat{N} \rangle \). In other words, if it is possible to pump several photons in the reservoir off-resonantly, then the subsequent pumping will become resonant and very efficient.

We demonstrate numerically that this is possible due to the large dissipative bandwidths of the states in the \( |gefN\rangle^{\pm,0} \) splittings. The low Q-factor of the auxiliary resonator allows pumping transitions to them even if the detuning is of the order of several MHz. In Fig. 4(a, b), we compare the steady-state solution with the transient solutions at 1 an 2 \( \mu s \). The steady-state picture shows that pumping at a frequency close to \( |g,0,0\rangle \leftrightarrow |f,0,0\rangle \) transition is indeed optimal in the long term, despite that at first the sideband \( |geN\rangle^\pm \leftrightarrow |gefN\rangle^\pm \) transitions have a higher pumping rate. However, these sidebands are subject to the Coulomb blockade and die off for large \( \langle \hat{N} \rangle \); for low \( \langle \hat{N} \rangle \), the upper sideband is the most efficient due to the highest two-photon transition rate (see Appendix A for details).

As a result, with optimal parameters, we can pump \( \approx 50 \) photons while maximizing output power (-132 dBm at the optimum). Maximizing the reservoir population allows pumping at least 90 photons in the steady state. Above this point, the simulation becomes intractable due to the size of the necessary Hilbert space.

C. Different lasing regimes and phase transitions

Despite difficulties in finding exact analytical expression for the number of pumped photons, it is possible to investigate it for different sets of parameters numerically. Fig. 4(c-f) shows the number of photons in the cavity and logarithm of the Fano factor \( F = \langle N^2 \rangle - \langle N \rangle^2 \) depending on the coupling strengths \( g_{r,a} \). \( F \) is a measure of intensity fluctuations of the laser field. \( F < 1 \) indicates sub-Poissonian statistics and photon antibunching, \( F = 1 \) – coherent Poissonian field, and \( F > 1 \) – super-Poissonian statistics with high intensity fluctuations.

For both driving strengths, one can see an area with a significant increase in the mean photon number in the cavity and the Fano-factor when the coupling strengths \( g_{r,a} \) are large enough. We find that this area marks the phase transition between two regimes of lasing: threshold-bearing and thresholdless. The lasing threshold is defined as the lowest pumping power value for which an increase of intensity fluctuations and usually of the steady-state photon number in the cavity is observed [10].

For the smaller drive amplitude, the transition is not very pronounced; however, for the larger drive, it is very sharp, so the system fully goes through the phase transition even for a small increase of the coupling strengths (orange dashed line). After the transition, the lasing is characterized by large \( \langle \hat{N} \rangle \) and \( F < 1 \). Finally, as one can see, the area of the threshold-bearing regime grows when the pumping rate is increased.

Qualitatively, the location of the threshold-bearing regime in the parameter space can be explained as follows. Firstly, the large coupling constant \( g_r \) allows pumping the reservoir quicker, which explains why the transition requires \( g_r/2\pi \) to exceed 12 or 8 MHz, for the weak and the strong drive, respectively. Secondly, when \( g_a/2\pi < 8 \) MHz, the coupling to the auxiliary resonator is too low to obtain a significant \( \gamma_{fe} \) and to produce sufficient population inversion. Finally, increasing \( g_r \) while keeping \( g_a \) fixed reduces the number of pumped photo-
Pumped photons and MHz, $\kappa g$

FIG. 4. (a, b) Average number of photons in the reservoir depending on the drive frequency for $g_r/2\pi = g_a/2\pi = 15$ MHz, $\kappa_r = 0.2$ MHz, $\kappa_a = 128$ MHz, $\Omega/2\pi = 20$ MHz (point A at (e, f)). (c, e) Pumped photons and (d, f) logarithm of Fano factor vs. coupling strengths for $\kappa_a = 138$ MHz, $\kappa_r = 0.2$ MHz. (c, d) $\Omega/2\pi = 15$ MHz, (e, f) $\Omega/2\pi = 20$ MHz. The letters are different sets of the parameters used in the other subsections (O depicts the optimal parameters). Orange dashed line marks the transition to the threshold-bearing regime and blue dashed line the thresholdless regime.

In the thresholdless regime in the bottom left corner of Fig. 4 (c, e) (dashed blue line) it is still possible to pump a considerable number of photons (about 5-10) while $F \gtrsim 1$ in this area. This regime is standard for single-atom lasers in the intermediate and low-coupling regime. The coupling constants in the thresholdless regime are low, so the energy splittings $\Delta_{ge}(N)$ and $\Delta_{ge+f}(N)$ does not play any role since line broadenings are higher.

D. Threshold and bistability

Single-atom lasers previously discussed in literature were thresholdless in the strong coupling regime [10][13][26], but could have one or two thresholds in the regime of weak and intermediate coupling [13][25][49]. Therefore, below we discuss why in our case it is possible to observe a threshold even in the strong coupling regime.

Fig. 5 (a, b) shows the steady-state values for $\langle N \rangle$ and $F$ for five different combinations of coupling constants, corresponding to points B-F in Fig. 4 (e, f), depending on the drive amplitude $\Omega$. For the first four sets of parameters (B-E), there is no well-defined threshold. They show a thresholdless behavior (except for $g_r/2\pi = 5$ MHz, $g_a/2\pi = 8$ MHz (E), which has a small maximum of $F$ at $\Omega/2\pi = 12$ MHz). The photon distribution in the regimes B-E can be super-Poissonian as well as sub-Poissonian for different drive power. The observed behavior is consistent with previous works [9][49].

For the combination B, $g_r/2\pi = 1$ MHz and $g_a/2\pi = 2$ MHz, we also observe self-quenching, probably caused by the dressing of the energy levels by the strong drive. This effect is still not clear to us, because self-quenching remains even if we manually tune pumping frequency into resonance with the correct transition to one of the dressed states, even though the maximum of pumped photons shifts to the right in that case.

The most interesting feature in Fig. 5 (a, b) is the presence of a well-defined threshold for the combination G: $g_r/2\pi = 8$ MHz and $g_a/2\pi = 15$ MHz. Our simulations show that such a threshold is present for any point above the orange dashed line at Fig. 4 (e, f). Above the threshold, for some parameters there is also no manifestations of the photon blockade – the average number of photons increases steadily with growing $\Omega$. As mentioned above, this is different from the previous results for single-cavity systems with coherent pumping in the strong-coupling regime [25][27].

Fig. 5 (c, e, f) illustrate the cavity state in the vicinity of the threshold for the point G, and Fig. 5 (d, g, h) – for the point F. One can see bistability which manifests itself as a double-ring structure of the Wigner function. At a sufficiently high driving power for the bistability to vanish, the only outer ring of Wigner function remains. For such a state the Fano factor is significantly lower than one.

We explain the bistability in the studied system as follows. We consider the intermediate cavity states with $N$ approximately between 2 and 25 for Fig. 5 (c) and between 4 and 12 for Fig. 5 (d). These states can not be populated via the coherent pump due to the large detuning they have due to vacuum Rabi splittings (see Eq. 24), and the relaxation dominates their population dynamics. However, the upper states can be pumped resonantly, as discussed before, so the system can remain there indefinitely. Therefore, with increasing drive strength the higher levels gradually absorb more and more of the population from the lower ones as the population is leaking better and better through the “blocked” states. In the middle of the threshold transition, this results in the apparent bimodal distribution of photons and very high intensity fluctuations.

Additionally, in Appendix B we show that bistability would not be observed if the Rabi-splittings did not depend on the strength the higher levels gradually absorb more and more of the population from the lower ones as the population is leaking better and better through the “blocked” states. In the middle of the threshold transition, this results in the apparent bimodal distribution of photons and very high intensity fluctuations.

IV. CONCLUSION

We have proposed a single-atom maser based on two-photon coherent pumping of a transmon. We find that to attain high population inversion, one needs to artificially increase the relaxation rate of its $|e\rangle \rightarrow |f\rangle$ transition and show that this is feasible by coupling it to an auxiliary low-Q resonator.
Our master equation simulations predict two distinct regimes of operation for the device with the presence or absence of the lasing threshold. The boundary between them is determined by the coupling strengths between the transmon and both cavities.

In the threshold-less regime, our system demonstrates non-classical behavior similar to the previously known for single-atom lasers, including both sub-Poissonian and super-Poissonian statistic, and self-quenching.

In the other regime, for which the coupling strengths are higher, we observe a well-defined lasing threshold, marked by a large increase of intensity fluctuations. We associate its presence with modifications of the energy level structure caused by the auxiliary resonator, which leads to bistability of Wigner function. The light statistics in the cavity in this regime is sub-Poissonian, with $F < 0.5$, and the number of pumped photons of the order of 10. This modification of the energy structure leads lifts the photon blockade of the coherent pumping for high cavity population even in the strong coupling regime. This means that such a laser may have a wide range of applications due to its significantly sub-Poissonian statistics [18, 19, 21].

Finally, via numerical optimization, we find an optimal set of parameters for a device that allows pumping about 50 photons in the strong-coupling regime, which corresponds to $-132$ dBm of emitted power. The statistics in this regime is predicted to be sub-Poissonian with $F = 0.27$.

**Appendix A: Estimated pumping rate of the two-photon process**

The frequency of the two-photon Rabi oscillations for a three-level system is generally calculated as

$$\Omega_{2ph} = \frac{\Omega^2}{2\Delta},$$  \hspace{1cm} (A1)

where $\Delta$ is the detuning between the pump frequency and the intermediate level. In our case, this is detuning between $\omega_d$ and splitted energy levels $|geN\rangle$. In case of $|g,0,0\rangle \leftrightarrow |f,0,0\rangle$ pumping $\Delta = \alpha/2$. For $|ge0\rangle^\pm \leftrightarrow |gef1\rangle^\pm$-pumping

$$\Delta^\pm = \frac{\alpha}{2} + \frac{\Delta_{ge}(1) + \Delta_{ge}(0)}{4},$$

which we will call the “plus” and “minus” transitions. This leads to the fact that for the parameters in Fig. 4(a, b) the two-photon pumping rate of $|gef0\rangle^+$ level is 1.9 times higher then of $|gef0\rangle^-$ level. Since the pumping rate for the “plus” transition is higher then for the “minus” one for any $N$, in Fig. 4(a, b) the line corresponding to the “plus” transition is much brighter compared to the “minus” at the beginning of pumping. When $N \to \infty$, the frequencies of both types of transitions becomes equal, since $\delta \to 0$ (Eq. 24). So, it is not possible to distinguish them in the steady state picture in Fig. 4(a, b).

**Appendix B: Role of level splittings in bistability**

In Fig. 6(a), the population of the resonator-reservoir for various $\Omega$ is shown. One can see that the intermediate states...
Population a lowering operator with unity matrix elements so the detuning of the new Hamiltonian, the annihilation operator is replaced by intermediate transitions, we have constructed an artificial tuning of the intermediate pumping transition. To check if the steady states for such a system are shown in Fig. 6 (a). One can see that now there is only one peak and no bistability.

We have also simulated \( \langle N \rangle \) and \( F \) in the steady state depending on \( \Omega \) in Fig. 6 (c,d). One can see that a peak of \( F \), which follows the threshold and present for the system with normal \( H_{\text{int}} \) (\( F \approx 4 \)), is absent for the system with modified \( H_{\text{int}} \). Consequently, the threshold, discussed in the main text, is indeed connected with special energy level structure and bistability.

FIG. 6. Visualized states for the system with the following parameters: \( \kappa_{rf} = 138 \text{ MHz} \), \( \kappa_{pe} = 0.2 \text{ MHz} \), \( g_r/2\pi = 8 \text{ MHz} \), \( g_a = 11 \text{ MHz} \) (point F at Fig. 4 (c, f), for different \( \Omega \)). (a) With ordinary interaction Hamiltonian, (b) with modified interaction Hamiltonian

4-12 are never populated. We have checked that they are also never populated during the full-time evolution while the second peak just grows steadily similarly to what is shown in Fig. 6 (a).

In the main text, we explain the bistability by the large detuning of the intermediate pumping transition. To check if the unusual steady state is really caused by the detuning of these intermediate transitions, we have constructed an artificial interaction Hamiltonian of the reservoir and the transmon. In the new Hamiltonian, the annihilation operator is replaced by a lowering operator with unity matrix elements so the detuning does not depend on the number of photons in resonator any more. The steady states for such a system are shown in Fig. 6 (b).

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