A Light $Z'$ Heterotic–String Derived Model

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Abstract

The existence of an extra $Z'$ inspired from heterotic–string theory at accessible energy scales attracted considerable interest in the particle physics literature. Surprisingly, however, the construction of heterotic–string derived models that allow for an extra $Z'$ to remain unbroken down to low scales has proven to be very difficult. The main reason being that the $U(1)$ symmetries that are typically discussed in the literature are either anomalous or have to be broken at a high scale to generate light neutrino masses. In this paper we use for that purpose the self duality property under the spinor vector duality, which was discovered in free fermionic heterotic string models. The chiral massless states in the self–dual models fill complete 27 representations of $E_6$. The anomaly free gauge symmetry in the effective low energy field theory of our string model is $SU(4)_C \times SU(2)_L \times SU(2)_R \times U(1)_\zeta$, where $U(1)_\zeta$ is the family universal $U(1)$ symmetry that descends from $E_6$, and is typically anomalous in other free fermionic heterotic–string models. Our model therefore allows for the existence of a low scale $Z'$, which is a combination of $B - L$, $T_{3L}$ and $T_{3R}$. The string model is free of exotic fractionally charged states in the massless spectrum. It contains exotic $SO(10)$ singlet states that carry fractional, non–$E_6$ charge, with respect to $U(1)_\zeta$. These non–$E_6$ string states arise in the model due to the breaking of the $E_6$ symmetry by discrete Wilson lines. They represent a distinct signature of the string vacua. They may provide viable dark matter candidates.
1 Introduction

The consistency conditions of string theory necessitate the existence of additional gauge degrees of freedom beyond those that are observed in the Standard Model. Experimental observation of extra gauge degrees of freedom in contemporary experiment will lend evidence for the extra gauge degrees of freedom predicted in string theory. The Standard Model states may be neutral under some of these degrees of freedom and charged with respect to some others. The neutral sector is dubbed the hidden sector, and typically consists of a rank eight gauge group. The observable sector of the heterotic–string correspond to a rank eight group, whereas the Standard Model utilises four of these degrees of freedom. Naturally, the experimental signatures of extra vector bosons arising in the hidden and observable sectors will be markedly different. In this paper we consider the case with an extra vector boson arising in the observable sector.

Extra $U(1)$ gauge symmetries in string theory have been of interest since the mid–eighties and occupy a significant number of papers that use effective field theory methods to study their phenomenological implications [1, 2, 3, 4]. Surprisingly, however, the construction of viable heterotic–string models that admit an additional observable $U(1)$ vector boson that may remain unbroken down to low energies, has proven to be very difficult, for a variety of phenomenological restrictions. In fact, to date there does not exist a free fermionic heterotic–string derived model that allows an extra observable $U(1)$ symmetry to remain unbroken down to low scales.

One issue that must be addressed is that of simultaneously suppressing proton decay mediating operators, while allowing for a mechanism that suppresses the left–handed neutrino masses [3]. Embedding the Standard Model in $SO(10)$ extends the rank of the Standard Model gauge group by one. Hence giving rise to an extra $U(1)$, which is a combination of $B – L$, baryon minus lepton number, and $T_{3R}$, the diagonal generator of $SU(2)_R$. The existence of this extra $U(1)$ at low scales was already entertained in the late eighties [2]. The caveat is that since the lepton number is gauged the extra $U(1)$ symmetry forbids the formation of Majorana mass terms for the right–handed neutrinos. On the other hand, the underlying $SO(10)$ symmetry dictates the equality of the top–quark and tau–neutrino Yukawa couplings, and hence the equality of the tau–neutrino Dirac mass term and the top quark mass. Preserving $U(1)_{B–L}$ unbroken down to the TeV scale, entails a low seesaw scale and a tau neutrino mass scale of the order of $O(10\text{MeV})$ [5]. Ensuring neutrino masses below the eV scale necessitates that $U(1)_{B–L}$ is broken at a scale of order $O(10^{15}\text{GeV})$ [3].

Another problem arises from the fact that in many string models the additional family universal $U(1)$ symmetries, which are traditionally studied in string inspired constructions, are anomalous and are not viable at low scales. The reason is the particular symmetry breaking pattern that is realised in many of the quasi–realistic free fermionic heterotic–string models [6, 7, 8]. It can be seen to arise from the breaking of $E_6 \to SO(10) \times U(1)_{\xi}$, which results in $U(1)_{\xi}$ being anomalous, since the chiral
matter resides in incomplete $E_6$ representations [9]. The left–right symmetric heterotic string models [10] circumvent this symmetry breaking pattern and do produce anomaly free models. On the other hand, string inspired constructions that utilise the $U(1)_\zeta$ charge assignments of the left–right symmetric heterotic string models [4] disagree with the gauge coupling data [11]. The reason is that the charges of the Standard Model states under the extra $U(1)$ do not admit an $E_6$ embedding, which is a necessary ingredient for accommodating the gauge coupling data [11].

The challenge is therefore to construct three generation string models that allow for an extra family universal $U(1)$ symmetry with $E_6$ embedding of its chiral charges. The $E_6$ symmetry is broken directly at the string level and is not manifested at low scales. The fact that the chiral spectrum must be anomaly free entails that the chiral generations must come in complete $E_6$ multiplets. In this paper we use for that purpose the spinor vector duality that was observed in $Z_2 \times Z_2$ heterotic–string orbifolds with $SO(10)$ GUT symmetry [12, 13, 14, 15]. The spinor–vector duality entails that for every string vacuum with a number of $16 \oplus \overline{16}$, and a number $10$ representations of $SO(10)$, there exist another vacuum in which the two numbers are interchanged. The spinor vector duality was first noted in the classification of free fermion $SO(10)$ models [12, 13] in terms of the Generalised GSO projection coefficients. It was subsequently discussed in terms of discrete torsion in orbifold models [14, 15]. It was shown to arise generally from the breaking of the world–sheet supersymmetry from $(2,2) \rightarrow (2,0)$, and is induced by the spectral flow operator of the right–moving world–sheet supersymmetry [15]. A special class of models are the self–dual models under the spinor–vector duality, which contain an equal number of $16 \oplus \overline{16}$ and $10$, representations of $SO(10)$. In the self–dual models $U(1)_\zeta$ may be anomaly free without enhancement of the gauge symmetry to $E_6$. The reason being that the spinorial and vectorial states that form complete $E_6$ representations are obtained from different fixed points of the underlying $Z_2 \times Z_2$ orbifold. The next step in our construction is to add a basis vector that breaks the $SO(10)$ symmetry to the Pati–Salam subgroup [16, 8], while maintaining the spinor–vector self–duality. We present an exemplary three generation model with these characteristics, which is free of exotic fractionally charged states, and contains the Higgs states necessary for realistic phenomenology. An interesting property of the model is that while it is free of $SO(10)$ exotic states, it contains states that carry exotic charges with respect to $U(1)_\zeta$, i.e. states that are not descending from $E_6$ representations. Such states are therefore signature of the string models. Furthermore, these states fall into the general category of Wilsonian matter states, considered in ref. [17], and therefore may provide viable dark matter candidates. The reason being that breaking the $U(1)_{Z'}$ with $E_6$ states leaves a remnant discrete symmetry that forbids the decay of the exotic states to the Standard Model states, which carry standard $E_6$ charges.
2 A String derived extra $U(1)$ model

Our challenge is to construct three generation heterotic–string models that allow for an extra family universal $U(1)$ symmetry with $E_6$ embedding of the chiral charges. The $E_6$ symmetry is broken directly at the string level. The fact that the chiral spectrum must be anomaly free entails that the chiral generations must come in complete $E_6$ multiplets. In refs [11] and [18] the construction of $SO(6) \times SO(4) \times U(1)_\zeta$ and $SU(3)_C \times SU(2)_W \times U(1)_{B-L} \times U(1)_{T_R} \times U(1)_\zeta$ models was outlined. In both cases the symmetry is broken spontaneously to $SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_{Z'}$ by the vacuum expectation value of the Standard Model singlet in the spinorial $16$ representation of $SO(10)$. The outline of these constructions goes as follows. The spacetime vector bosons that produce the observable $E_8$ gauge symmetry in free fermion models are obtained from two sectors. The first is the untwisted sector and the second is the $x$–sector [19]. In the decomposition of $E_8 \rightarrow SO(16)$, the adjoint representation decomposes as $248 \rightarrow 120 + 128$, where the $120$ representation is obtained from the untwisted sector, whereas the $128$ is obtained from the $x$–sector. In many of the existing quasi–realistic free fermionic models the states from the $x$–sector are projected out. The consequence is that $U(1)_\zeta$ and consequently $U(1)_{Z'}$ is anomalous. The key to the proposals in [11, 18] is to construct models in which some of the vector bosons from the $x$–sector are retained in the spectrum and enhance the untwisted gauge symmetry.

An explicit realisation of such a construction is the $SU(6) \times SU(2)$ model of [20]. The caveat with this model is that the only scalar states available to break the gauge symmetry down to the Standard Model are obtained from the $27$ of $E_6$. It is therefore impossible to break the symmetry down to the Standard Model, while maintaining an unbroken extra $U(1)$ symmetry. The reason being that this model requires two stages of non–Abelian symmetry breaking. The first being the breaking of $SU(6) \times SU(2)$ to either the Pati–Salam or flipped $SU(5)$ subgroups, and the second being the breaking of these subgroups to the Standard Model. The strategy proposed in ref. [11, 18] is therefore to construct similar models, but in which the enhancement of the untwisted gauge group is to $SO(6) \times SO(4) \times U(1)_{\zeta}$ and $SU(3)_C \times SU(2)_W \times U(1)_{B-L} \times U(1)_{T_R} \times U(1)_\zeta$, respectively, rather than to $SU(6) \times SU(2)$. However, explicit string derived models that realise this construction were not presented in refs. [11, 18].

In this paper we adopt an alternative construction that exploits the spinor–vector duality observed in free fermionic models in ref. [12, 13]. The spinor–vector duality exchanges spinorial $16$ representations of $SO(10)$ with vectorial $10$ representations in the twisted sectors. For every vacuum with a total number of $16 \oplus 16$ multiplets and a number of $10$ multiplets, there exist a dual vacuum in which the two numbers are interchanged. The spinor–vector duality can be proved analytically in terms of the free fermion Generalised GSO (GGSO) phases of the one–loop partition function [13], or in terms of discrete torsions in an orbifold representation [14, 15]. It can be seen to arise due to the breaking of the $N = 2 \rightarrow N = 0$ world–sheet supersymmetry.
in the right–moving bosonic side of the heterotic–string. With \( N = 2 \) world–sheet supersymmetry the \( SO(10) \times U(1) \) GUT symmetry is enhanced to \( E_6 \). The chiral multiplets reside in the \( 27 \) and \( \overline{27} \) representations of \( E_6 \), which decompose as \( 27 = 16_2 + 10_{-1} + 1_{+2} \) and \( \overline{27} = 16_{-2} + 10_{+1} + 1_{-2} \), respectively, under \( SO(10) \times U(1) \). When the symmetry is enhanced to \( E_6 \) the total number of \( 16 + \overline{16} \) representations is equal to the total number of vectorial \( 10 \) representations. Hence, this case is self–dual under the spinor–vector duality. In this case the spectral flow generator on the bosonic side exchanges between the multiplets that are embedded in the \( E_6 \) representations. Breaking the \( N = 2 \) world–sheet supersymmetry to \( N = 0 \) induces the \( E_6 \rightarrow SO(10) \times U(1) \) breaking. In this case the spectral flow operator induces the spinor–vector duality map between the dual vacua [15]. Since, the \( E_6 \) symmetry is broken, the chiral spectrum resides in incomplete \( E_6 \) multiplets, and \( U(1)_\zeta \) is, in general, anomalous. A special class of models are the \( N = 0 \) self–dual models under the spinor–vector duality map. In these models the \( E_6 \) symmetry is broken to \( SO(10) \times U(1) \). However the total number of spinor plus anti–spinor representations is equal to the total number of vectorial representations. Hence, these models produce complete \( E_6 \) multiplets, but the gauge symmetry is not enhanced to \( E_6 \). This is possible if the different components of the \( E_6 \) multiplets are obtained from different fixed points of the underlying \( Z_2 \times Z_2 \) orbifold. Obtaining the spinorial and vectorial components at the same fixed point would necessarily imply that the \( SO(10) \times U(1) \) symmetry is enhanced to \( E_6 \). However, if the spinorial and vectorial components are obtained at different fixed points the symmetry is not enhanced. The chiral spectrum in the self–dual models may therefore arise in complete \( E_6 \) multiplets, with anomaly free \( U(1)_\zeta \), but without enhanced \( E_6 \) symmetry. The next stage in our construction is to break the \( SO(10) \times U(1)_\zeta \) symmetry to \( SO(6) \times SO(4) \times U(1)_\zeta \), while maintaining the spinor–vector self–duality.

We use the free fermionic formulation of the heterotic string in four dimensions [21] to construct our string derived model. In this formulation all the degrees of freedom required to cancel the world–sheet conformal anomaly are represented in terms of free fermions propagating on the string world–sheet. These fermions pick up a phase under parallel transport around the non–contractible loops of the worldsheet torus. The free fermion heterotic string models are fully described in terms of the boundary condition basis vectors \( v_i, i = 1, \ldots, N \)

\[ v_i = \{ v_i(f_1), v_i(f_2), v_i(f_3) \ldots \}, \]

for the 64 world–sheet real fermions \( f_j \) [21], and the associated one–loop GGSO coefficients \( c^{[N]}_{[v_j]} \). Taking all possible combinations of the basis vectors

\[ \eta = \sum N_i v_i, \quad N_i = 0, 1 \quad (2.1) \]

generates a finite additive group \( \Xi \). The physical states in each sector \( \eta \in \Xi \) are obtained by acting on the vacuum with fermionic and bosonic oscillators and by
imposing the GGSO projections
\[ e^{i\pi v_i F_S} |S> = \delta_S e^{\left[ S \right]} |S>, \tag{2.2} \]
where \( \delta_S = \pm 1 \) is the spacetime spin statistics index and \( F_S \) is a fermion number operator. In the usual notation the sixty-four worldsheet fermions in the light-cone gauge are: \( \psi^\mu, \chi^i, y^i, \omega^i, i = 1, \ldots, 6 \) (left-movers) and \( \bar{y}^i, \bar{\omega}^i, i = 1, \ldots, 6 \), \( \psi^A, A = 1, \ldots, 5 \), \( \eta^B, B = 1, 2, 3 \), \( \bar{\phi}^\alpha, \alpha = 1, \ldots, 8 \) (right-movers). Further details of the formalism and notation that we use in this paper are found in the literature [21, 6, 7, 8, 19, 22, 12, 23].

Our string model is constructed by using the methods developed in [25] for the classification of type IIB superstrings, and in [22] for the classification heterotic-string vacua with an unbroken \( SO(10) \) symmetry. It was adapted in [23] for the classification of Pati–Salam vacua, and in [24] for the classification of flipped \( SU(5) \) vacua. In this classification method the set of basis vectors is fixed and the enumeration of the models is achieved by varying GGSO phases. The set of basis vectors that we use here is identical to the one used in the classification of Pati–Salam vacua in ref [23] and is given by a set of thirteen basis vectors \( B = \{ v_1, v_2, \ldots, v_{13} \} \), where

\[ v_1 = 1 = \{ \psi^\mu, \chi^{1, \ldots, 6}, y^{1, \ldots, 6}, \omega^{1, \ldots, 6} \} \]
\[ v_2 = S = \{ \psi^\mu, \chi^{1, \ldots, 6} \} \]
\[ v_{2+i} = e_i = \{ y^i, \omega^i \} \}
\[ v_9 = b_1 = \{ \chi^{34}, y^{34}, \bar{y}^{34}, \bar{\omega}^{34}, \bar{\psi}^{1, \ldots, 5} \} \]
\[ v_{10} = b_2 = \{ \chi^{12}, y^{12}, \bar{y}^{12}, \bar{\psi}^{1, \ldots, 5} \} \]
\[ v_{11} = z_1 = \{ \bar{\phi}^{1, \ldots, 4} \} \]
\[ v_{12} = z_2 = \{ \bar{\phi}^{5, \ldots, 8} \} \]
\[ v_{13} = \alpha = \{ \bar{\psi}^{4, 5}, \bar{\phi}^{1, 2} \} \]

In the notation used in eq. (2.3) the fermions appearing in the curly brackets are periodic, whereas those that do not appear are antiperiodic. The untwisted gauge symmetry generated by this set is

observable : \( SO(6) \times SO(4) \times U(1)^3 \)

hidden : \( SO(4)^2 \times SO(8) \)

Additional spacetime vector bosons may arise from the sectors

\[ G = \left\{ \begin{array}{c}
  z_1, \\
  z_2, \\
  \alpha, \\
  \alpha + z_1, \\
  x, \\
  z_1 + z_2, \\
  \alpha + z_2, \\
  \alpha + z_1 + z_2, \\
  \alpha + x, \\
  \alpha + x + z_1
\end{array} \right\} \tag{2.4} \]

and enhance the four dimensional gauge group. In Eq. (2.4) we defined the combination

\[ x = 1 + S + \sum_{i=1}^{6} e_i + z_1 + z_2, \]
that may enhance the observable $SO(16)$ gauge group to $E_8$. For suitable choices of the GGSO phases the spacetime gauge bosons arising in the sectors of eq. (2.4) are projected out, and the gauge symmetry is generated solely by the vector bosons arising in the untwisted sector. A suitable choice of the GGSO projection coefficients guarantees the existence of $N = 1$ spacetime supersymmetry.

The matter states in the Pati–Salam heterotic–string models are embedded in $SU(4) \times SU(2)_L \times SU(2)_R$ representations as follows:

$$F_L (4, 2, 1) \rightarrow q \left( 3, 2, \frac{1}{6} \right) + \ell \left( 1, 2, \frac{1}{2} \right)$$

$$\bar{F}_R (\bar{4}, 1, 2) \rightarrow u^c \left( 3, 1, \frac{2}{3} \right) + d^c \left( 3, 1, -\frac{1}{3} \right) + e^c (1, 1, -1) + \nu^c (1, 1, 0)$$

$$h(1, 2, 2) \rightarrow h^d \left( 1, 2, \frac{1}{2} \right) + h^u \left( 1, 2, -\frac{1}{2} \right)$$

$$D (6, 1, 1) \rightarrow d_3 \left( 3, 1, \frac{1}{3} \right) + \bar{d}_3 \left( \bar{3}, 1, -\frac{1}{3} \right).$$

Here $\bar{F}_R$ and $F_L$ contain one Standard Model generation; $h^u$ and $h^d$ are electroweak Higgs doublets; and $D$ contains vector–like colour triplets. The Pati–Salam breaking Higgs fields, decomposed in terms of the Standard Model group factors, are given by:

$$\bar{H} (\bar{4}, 1, 2) \rightarrow u^c_H \left( \bar{3}, 1, \frac{2}{3} \right) + d^c_H \left( \bar{3}, 1, -\frac{1}{3} \right) + \nu^c_H (1, 1, 0) + e^c_H (1, 1, -1)$$

$$H (4, 1, 2) \rightarrow u^c_H \left( 3, 1, -\frac{2}{3} \right) + d^c_H \left( 3, 1, \frac{1}{3} \right) + \nu^c_H (1, 1, 0) + e^c_H (1, 1, 1)$$

The electric charge in the Pati–Salam models is given by:

$$Q_{em} = \frac{1}{\sqrt{6}} T_{15} + \frac{1}{2} T_{3L} + \frac{1}{2} T_{3R}$$

where $T_{15}$ is the diagonal generator of $SU(4)$ and $T_{3L}, T_{3R}$ are the diagonal generators of $SU(2)_L, SU(2)_R$, respectively.

The next ingredient required to define the string model are the GGSO projection coefficients that are obtained from the one–loop partition function $c_{[\nu_j]}$, spanning a $13 \times 13$ matrix. Modular invariance constraints imply that only the elements with $i > j$ are independent. There are therefore a priori 78 independent coefficients of which 11 are fixed by the requirement that the models possess $N = 1$ spacetime supersymmetry. The number of independent phases is reduced further to 51, by the requirement that only untwisted spacetime vector bosons are retained in the massless spectrum. Each distinct configuration of the GGSO phases corresponds to a distinct model, where some degeneracy in some of the phenomenological properties may still exist. A statistical analysis over the entire space of models was presented in [23]. Here
our interest is in the particular class of models that preserve the spinor–vector self-duality and that admit the additional anomaly free $U(1)_\zeta$ symmetry in the observable sector. The phases displayed in eq. (2.6),

\[
(v_i | v_j) = \begin{pmatrix}
1 & S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & b_1 & b_2 & z_1 & z_2 & \alpha \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
S & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
e_1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
e_2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
e_3 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
e_4 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
e_5 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
e_6 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
b_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
b_2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
z_1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\
z_2 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\
\alpha & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\
\end{pmatrix}
\]

(2.6)

where we introduced the notation $c_{[v_i]}^{[v_j]} = e^{i\pi (v_i | v_j)}$, represent a specific example of a heterotic–string model in this class. We estimate the existence of some $2 \times 10^5$ models with similar properties. The model was obtained using a fishing algorithm to extract a specific configuration with particular phenomenological properties [23]. An alternative method is to use the genetic algorithm proposed in ref. [26].

In tables 1, 2 and 3 we display the entire massless spectrum that arise in the model generated by the set of GGSO phases in eq. (2.6). The vector combination defined in the tables as $b_3 = b_1 + b_2 + x$ correspond to the third twisted plane of the $Z_2 \times Z_2$ orbifold.

In addition to the spacetime vector bosons that generate the four dimensional gauge group the untwisted sector gives rise to three pairs of $SU(4)$ sextets; six pairs of $SO(10) \times E_8$ singlets that are charged with respect to $U(1)$ symmetries; and six states that are neutral under the entire four dimensional gauge group. These states are displayed in table 1. The states arising from the untwisted sector are identical to all the Pati–Salam free fermionic models that use the basis set given in eq. (2.3) since the projections of the untwisted set only depend on the basis vectors and are independent of the choice of GGSO projection coefficients given in eq. (2.6).

The twisted sectors matter states obtained in the string model of eq. (2.6) generate the needed states for viable phenomenology. The massless spectrum contains three chiral generations; one pair of heavy Higgs states to break the Pati–Salam gauge symmetry; three light Higgs bi-doublets that can be used to break the electroweak symmetry and generate viable fermion mass spectrum; the twisted spectrum contains five sextet states of $SO(6)$, where at least one is required for the missing
Table 1: The untwisted matter states and \( SU(4) \times SU(2)_L \times SU(2)_R \times U(1)^3 \) charges.

partner mechanism. The massless spectrum is completely free of exotic fractionally charged states that are endemic in heterotic string vacua [27, 28, 29, 17]. Additionally, the spectrum contains a number of \( SO(10) \times U(1)_\xi \) singlet states, that can be used to produce a supersymmetric vacuum along \( F^- \) and \( D^- \) flat directions. Some of these states transform in non–trivial representations of the hidden sector gauge group. From table 2 it is seen that the spinor–vector self–duality at the \( SO(10) \) level is preserved in the Pati–Salam model.

Several observations can be noted from the twisted sectors states displayed in tables 2 and 3. First, it is seen that the chiral representations indeed form complete 27 representations and consequently \( U(1)_\xi \) is anomaly free. The string model contains two anomalous \( U(1) \) with

\[
\text{Tr}U(1)_1 = 36 \quad \text{and} \quad \text{Tr}U(1)_3 = -36,
\]

where the \( U(1)_{1,2,3} \) symmetries are generated by the right–moving complex worldsheet fermions \( \bar{\eta}^{1,2,3} \). Hence, the two combinations

\[
U(1)_\xi = U(1)_1 + U(1)_2 + U(1)_3 \quad (2.8)
\]

\[
U(1)_2 = U(1)_1 - 2U(1)_2 + U(1)_3 \quad (2.9)
\]

are anomaly free, whereas the combination

\[
U(1)_A = U(1)_1 - U(1)_3 \quad (2.10)
\]
is anomalous. The anomalous $U(1)$ symmetry generates a Fayet–Iliopoulos term that breaks supersymmetry near the Planck scale [30]. Supersymmetry can be restored along a flat directions by assigning a VEV to some $SO(10) \times U(1)_\zeta$ singlet fields in the string massless spectrum, for example by giving a VEV to $\Phi_{13}^-$. Assigning a VEV to the heavy Higgs field that breaks the Pati–Salam symmetry leaves unbroken the weak hypercharge combination

$$U(1)_Y = \frac{1}{3} U_C + \frac{1}{2} U_L,$$

and the $Z'$ combination given by

$$U(1)_{Z'} = \frac{1}{5} U_C - \frac{1}{5} U_L - U_\zeta, \quad (2.11)$$

where we used the $U(1)$ definitions traditionally used in free fermionic models $U_c = 3/2 U_{B-L}$ and $U_L = 2 T_{3R}$.

As noted from table 2 the $\chi_i^+$ states with $i = 1, \cdots, 5$ correspond to the $SO(10)$ singlet in the 27 representation of $E_6$. The corresponding $\chi_i^-$ states correspond to the twisted moduli [19, 31]. Thus, contrary to the cases [31] in which the twisted moduli are projected out, in the self–dual model of eq. (2.6) they are retained. The states $\zeta_a$ and $\bar{\zeta}_a$ with $a = 1, \cdots, 11$ are neutral under $SO(10) \times U(1)_\zeta$ and can therefore get non–trivial VEVs along supersymmetric $F$– and $D$–flat directions.

A particularly interesting class of states are the states $\phi_{1,2}$ and $\bar{\phi}_{1,2}$. These states are $SO(10)$ singlets and are charged with respect to $U(1)_\zeta$. These states therefore carry standard charges with respect to the Standard Model gauge group. However, they carry non standard charges with respect to $U(1)_\zeta$. That is, while these states are standard with respect to the Standard Model, they are exotic with respect to $E_6$.

The general characteristic of string vacua, due to the breaking of the non–Abelian GUT symmetries by Wilson lines with a left over unembedded $U(1)$ symmetry, is the existence of massless states that do not satisfy the $U(1)$ quantisation of the underlying GUT symmetry [27]. In many models the resulting exotic states carry fractional electric charge, which are severely constrained by experiments [32].

A theorem by Schellekens states that any string vacuum in which the non–Abelian GUT symmetry is broken by discrete Wilson lines, necessarily contains states with fractional electric charge, provided that the weak hypercharge possess the canonical GUT normalisation [28]. There exist, however, quasi–realistic string models in which the exotic fractionally charged states only appear in the massive spectrum and do not arise at the massless level [23]. Such exophobic three generation models were found when the $SO(10)$ GUT symmetry is broken to the Pati–Salam subgroup [23], whereas models in which the GUT symmetry is broken to the flipped $SU(5)$ gauge group with odd number of generations did not yield any exophobic models [24]. The model arising from the GGSO phases in eq. (2.6) is an exophobic Pati–Salam model. There are no fractionally charged states in the massless spectrum of this string vacuum.
The $\phi_{1,2}$ and $\bar{\phi}_{1,2}$ states in table 2 are similarly exotic states. Namely, they arise due to the breaking of $E_6$ by discrete Wilson lines in the string vacuum. However, as they carry standard charges with respect to the $SO(10)$ subgroup they are not exotic with respect to the Standard Model. Such states are therefore a particular signature of the string vacuum and may have interesting observational consequences [29]. Furthermore, if they remain sufficiently light they may be instrumental for generating an extended seesaw mechanism [33]. It should be remarked, though, that the $E_6$ exotic states are vector–like and are not chiral. Therefore, a priori there is no clear argument why they should remain light.

The $\phi_{1,2}$ and $\bar{\phi}_{1,2}$ states fall into the general category of Wilsonian matter states considered in ref. [17]. Namely, they arise as a general consequence of the breaking of the $E_6$ gauge symmetry by discrete Wilson lines [27]. However, they carry exotic non–$E_6$ charges only with respect to $U(1)_\zeta$, and consequently with respect to $U(1)_{Z'}$, whereas they carry the standard charges with respect to the $SO(10)$ subgroup of $E_6$. In fact, they are $SO(10)$ singlets. This is quite an intriguing situation as it renders them ideal dark matter candidates. The reason being that if only states with standard $E_6$ charges are used to break the $U(1)_{Z'}$ symmetry, say the $\chi^{\pm}$ states and their conjugates, then a local discrete symmetry [34] is left which forbids the decay of these states to the Standard Model states. The relic abundance of such states was considered in ref. [17] and it was shown that they may provide viable dark matter candidates. However, the singlet states considered in ref. [17] are Standard Model singlets, but not $SO(10)$ singlets. That is they carry exotic charges with respect to the $U(1)$ combination, which is a combination of $U(1)_{B-L}$ and $U(1)_{T_3^R}$, and must be broken at a high scale to suppress the left–handed neutrino masses. Hence, their relic abundance depends on an interplay between the reheating temperature following inflation and the extra $U(1)$ breaking scale. However, the states $\phi_{1,2}$ and $\bar{\phi}_{1,2}$ are $SO(10)$ singlets and are not constrained by the suppression of the left–handed neutrino masses. They may therefore remain light and stable down to the $Z'$ breaking scale, and may be within reach of forthcoming colliders.

Table 3 contains the vector–like matter states that transform non–trivially under hidden $E_8$ subgroup. All of these states are $SO(10)$ singlets, but carry nontrivial charges under $U(1)_{1,2,3}$. Furthermore, some of the hidden matter states in 3 carry exotic charges with respect to $U(1)_\zeta$. The hidden sector gauge group is broken to $SU(2)^4 \times SO(8)$. This model may therefore accommodate the self–interacting dark matter candidates proposed in ref [35].

2.1 The superpotential

Renormalisable and nonrenormalisable terms in the superpotential can be calculated by using the tools developed in [36]. The cubic level terms in the superpotential are shown in eq. (2.12–2.14). Eq. (2.12) displays the terms that contain fields that
transform nontrivially under the observable Pati–Salam group,

$$
\begin{align*}
\bar{F}_{1R} F_{1L} h_1 + \bar{F}_{1R} F_{3L} h_3 + \bar{F}_{1R} \bar{F}_{2R} D_4 + \bar{F}_{1R} \bar{F}_{4R} D_6 + \bar{F}_{1R} \bar{F}_{3R} \zeta_1 \\
+ \bar{F}_{1R} \bar{F}_{1R} D_1 + F_{1L} \bar{F}_{1L} D_2 + \bar{F}_{3R} \bar{F}_{3R} D_2 + \bar{F}_{2R} \bar{F}_{2R} D_2 + F_{2L} \bar{F}_{2L} D_2 \\
+ F_{1R} F_{1R} \bar{D}_1 + F_{3L} F_{3L} D_3 + \bar{F}_{4R} \bar{F}_{4R} D_3 + \bar{F}_{3R} \bar{F}_{1R} D_7 + F_{2L} F_{3L} D_7 \\
+ h_2 h_2 \Phi_{13} + h_3 h_3 \Phi_{13} + h_1 h_1 \Phi_{12} + h_1 h_2 \chi_5^+ \\
+ D_1 \bar{D}_2 \Phi_{12} + D_2 \bar{D}_1 \Phi_{12} + D_1 \bar{D}_2 \Phi_{12} + D_2 \bar{D}_1 \Phi_{12} \\
+ D_2 \bar{D}_3 \Phi_{23} + D_3 \bar{D}_2 \Phi_{23} + D_3 \bar{D}_2 \Phi_{23} + D_2 \bar{D}_3 \Phi_{23} \\
+ D_1 \bar{D}_3 \Phi_{13} + D_1 \bar{D}_3 \Phi_{13} + D_3 \bar{D}_1 \Phi_{13} + D_1 \bar{D}_3 \Phi_{13} \\
+ D_4 D_4 \Phi_{12} + D_5 D_5 \Phi_{13} + D_6 D_6 \Phi_{13} + D_6 \bar{D}_6 \Phi_{23} + D_7 D_7 \Phi_{23} \\
+ D_5 D_7 \chi_1^+ + D_3 D_4 \chi_1^+ + D_2 D_5 \chi_2^+ + D_2 D_6 \chi_3^+ + D_1 D_6 \chi_4^+ + D_1 D_7 \chi_5^+ \\
+ D_1 \bar{D}_6 \chi_1^+ + D_1 \bar{D}_7 \chi_5^+ + D_2 D_5 \chi_2^+ + D_2 D_6 \chi_3^+ + D_3 D_4 \chi_1^+ \\
+ D_6 D_6 \chi_1^+ + D_4 D_5 \chi_5^+ + D_4 D_7 \chi_2^+ .
\end{align*}
$$

(2.12)

A particular requirement that we impose on the selected string vacuum is the existence of a top quark mass term at the cubic level of the superpotential [37]. Such potential term may arise from the couplings to $h_1$ and $h_3$ in the first two terms in eq. (2.12).

Eq. (2.13) contains only states that are singlets of the observable and hidden non–Abelian group factors,

$$
\begin{align*}
\Phi_{12} \Phi_{13} \Phi_{23} + \Phi_{12} \Phi_{23} \Phi_{13} + \Phi_{12} \Phi_{23} \Phi_{13} + \Phi_{12} \Phi_{23} \Phi_{13} \\
+ \Phi_{12} \Phi_{13} \Phi_{23} + \Phi_{12} \Phi_{13} \Phi_{23} + \Phi_{12} \Phi_{13} \Phi_{23} + \Phi_{12} \Phi_{23} \Phi_{13} + \Phi_{12} \Phi_{23} \Phi_{13} + \Phi_{12} \Phi_{23} \Phi_{13} + \Phi_{12} \Phi_{23} \Phi_{13} + \Phi_{12} \Phi_{23} \Phi_{13} \\
+ \Phi_{12} \chi_1 \chi_1^+ + \Phi_{13} \chi_2 \chi_2^+ + \Phi_{13} \chi_3 \chi_3^+ + \Phi_{23} \chi_4 \chi_4^+ + \Phi_{23} \chi_5 \chi_5^+ \\
+ \Phi_{12} \xi_1 \xi_1 + \Phi_{12} \xi_2 \xi_2 + \Phi_{12} \xi_3 \xi_3 + \Phi_{12} \xi_4 \xi_4 + \Phi_{12} \xi_5 \xi_5 + \Phi_{12} \xi_6 \xi_6 \\
+ \Phi_{13} \xi_1 \xi_1 + \Phi_{13} \xi_2 \xi_2 + \Phi_{13} \xi_3 \xi_3 + \Phi_{13} \xi_4 \xi_4 + \Phi_{13} \xi_5 \xi_5 + \Phi_{13} \xi_6 \xi_6 \\
+ \Phi_{23} \xi_1 \xi_1 + \Phi_{23} \xi_2 \xi_2 + \Phi_{23} \xi_3 \xi_3 + \Phi_{23} \xi_4 \xi_4 + \Phi_{23} \xi_5 \xi_5 + \Phi_{23} \xi_6 \xi_6 \\
+ \Phi_{12} \xi_1 \xi_2 + \Phi_{12} \xi_2 \xi_1 + \Phi_{12} \xi_3 \xi_4 + \Phi_{12} \xi_4 \xi_3 + \Phi_{12} \xi_5 \xi_6 + \Phi_{12} \xi_6 \xi_5 \\
+ \Phi_{13} \xi_1 \xi_2 + \Phi_{13} \xi_2 \xi_1 + \Phi_{13} \xi_3 \xi_4 + \Phi_{13} \xi_4 \xi_3 + \Phi_{13} \xi_5 \xi_6 + \Phi_{13} \xi_6 \xi_5 \\
+ \Phi_{23} \xi_1 \xi_2 + \Phi_{23} \xi_2 \xi_1 + \Phi_{23} \xi_3 \xi_4 + \Phi_{23} \xi_4 \xi_3 + \Phi_{23} \xi_5 \xi_6 + \Phi_{23} \xi_6 \xi_5 \\
+ \Phi_{12} \xi_1 \xi_2 + \Phi_{12} \xi_2 \xi_1 + \Phi_{12} \xi_3 \xi_4 + \Phi_{12} \xi_4 \xi_3 + \Phi_{12} \xi_5 \xi_6 + \Phi_{12} \xi_6 \xi_5 \\
+ \Phi_{13} \xi_1 \xi_2 + \Phi_{13} \xi_2 \xi_1 + \Phi_{13} \xi_3 \xi_4 + \Phi_{13} \xi_4 \xi_3 + \Phi_{13} \xi_5 \xi_6 + \Phi_{13} \xi_6 \xi_5 \\
+ \Phi_{23} \xi_1 \xi_2 + \Phi_{23} \xi_2 \xi_1 + \Phi_{23} \xi_3 \xi_4 + \Phi_{23} \xi_4 \xi_3 + \Phi_{23} \xi_5 \xi_6 + \Phi_{23} \xi_6 \xi_5 .
\end{align*}
$$

(2.13)

Eq. (2.14) contains fields that transform nontrivially under the hidden sector group
factors,
\[
\begin{align*}
\Phi_{13} H_{34}^2 H_{34}^2 & + \Phi_{13} H_{12}^3 H_{12}^3 + \Phi_{13} H_{12}^2 H_{12}^2 + \Phi_{13} H_{34}^3 H_{34}^3 \\
& + \Phi_{23} H_{13}^3 H_{13}^3 + \Phi_{23} H_{34}^3 H_{34}^3 + \Phi_{23} H_{14}^3 H_{14}^3 \\
& + \Phi_{23} H_{12}^2 H_{12}^2 + \Phi_{23} H_{34}^2 H_{34}^2 + \Phi_{23} H_{14}^2 H_{14}^2 \\
& + \Phi_{12} H_{12}^1 H_{12}^1 + \Phi_{12} H_{14}^1 H_{14}^1 + \Phi_{12} H_{13}^1 H_{13}^1 \\
& + \Phi_{12} H_{34}^1 H_{34}^1 + \Phi_{12} H_{23}^1 H_{23}^1 + \Phi_{12} H_{24}^1 H_{24}^1 \\
& + \chi_2 H_{34}^5 H_{34}^5 + \chi_3 H_{14}^1 H_{14}^1 + \chi_3 H_{13}^1 H_{13}^1 + \chi_3 Z_5 Z_1 + \tilde{\phi}_2 H_{12}^3 H_{12}^3 + \tilde{\phi}_2 H_{34}^3 H_{34}^3 \\
& + \Phi_{13} Z_1 Z_1 + \Phi_{23} Z_2 Z_2 + \Phi_{23} Z_3 Z_3 + \Phi_{13} Z_4 Z_4 + \Phi_{12} Z_5 Z_5
\end{align*}
\]

(2.14)

As noted above a VEV that cancels the anomalous \(U(1)\) \(D\)-term, which is also \(F\)-flat to all orders in the superpotential is given by the VEV of \(\Phi_{13}\).

3 Conclusions

Extensions of the Standard Model by an Abelian gauge symmetry are among the most popular cases investigated in studies of physics beyond the Standard Model. Extra \(U(1)\) symmetries arise naturally in Grand Unified Theories with \(SO(10)\) and \(E_6\) gauge symmetry. Furthermore, the internal consistency conditions of string constructions mandate the existence of additional gauge symmetries, and may be viewed as a general prediction of string theory. Indeed, since the mid–eighties many authors explored the physics implication of a string inspired extra \(Z'\) vector boson in collider experiments and astroparticle observatories. Many of those studies are inspired by the heterotic–string, which also admit the appealing GUT structure, and gave rise to string inspired \(Z'\) models with \(E_6\) embedding. Surprisingly, however, the construction of explicit string derived models that allow the extra \(U(1)\) symmetry to remain unbroken down to low scales has proven to be a difficult challenge. The reason being that the extra \(U(1)\)s that arise in string models could not satisfy the phenomenological constraints that must be imposed on a viable \(U(1)\) symmetry down to low scales. Some of those constraints being: family universality; anomaly freedom; gauge coupling unification; suppressed left–handed neutrino masses. The main obstacle being the construction of an extra anomaly free \(U(1)\) with \(E_6\) embedding of its chiral charges.

In this paper we constructed a string model that can satisfy these phenomenological constraints. The model that we constructed is a self–dual model under the spinor–vector duality map that was observed in free fermionic \(Z_2 \times Z_2\) orbifolds. As a consequence of the self–duality property the chiral states in the model form complete \(E_6 27\) representations. However, the gauge symmetry in the effective low energy field theory contains a subgroup of \(SO(10)\) and is not enhanced to \(E_6\). This is possible because the different components of the \(27\) are obtained from different fixed points of the \(Z_2 \times Z_2\) toroidal orbifold. Consequently, the family universal \(U(1)\) may remain unbroken down to low scales.
| sector     | field     | $SU(4) \times SU(2)_L \times SU(2)_R$ | $U(1)_Y$ | $U(1)_X$ | $U(1)_W$ | $U(1)_C$ |
|------------|-----------|--------------------------------------|----------|-----------|-----------|-----------|
| $S + b_1$  | $F_{1 L}$ | $(4,1,2)$                            | 1/2      | 0         | 0         | 1/2       |
| $S + b_1 + e_3 + e_5$ | $F_{1 R}$ | $(4,1,2)$                            | 1/2      | 0         | 0         | 1/2       |
| $S + b_2$  | $F_{2 L}$ | $(4,2,1)$                            | 0        | 1/2       | 0         | 1/2       |
| $S + b_2 + e_1 + e_2 + e_5$ | $F_{2 R}$ | $(4,2,1)$                            | 0        | 1/2       | 0         | 1/2       |
| $S + b_3 + e_1$ | $F_{3 L}$ | $(4,1,2)$                            | 0        | 1/2       | 0         | 1/2       |
| $S + b_3 + e_2$ | $F_{3 R}$ | $(4,1,2)$                            | 0        | 1/2       | 0         | 1/2       |
| $S + b_3 + x$ | $b_1$    | $(1,2,2)$                            | -1/2     | -1/2      | 0         | -1        |
| $S + b_2 + x + e_5$ | $b_2$    | $(1,2,2)$                            | -1/2     | 0         | -1/2      | -1        |
| $S + b_2 + x + e_1 + e_2$ | $b_3$    | $(1,2,2)$                            | -1/2     | 0         | -1/2      | -1        |
| $S + b_3 + x + e_1$ | $D_4$   | $(6,1,1)$                            | -1/2     | -1/2      | 0         | -1        |
| $S + b_2 + x + e_1 + e_5$ | $\chi_1^+$ | $(1,1,1)$                           | 1/2      | 1         | 1/2       | +2        |
| $S + b_2 + x + e_2$ | $\chi_2$ | $(1,1,1)$                            | 1/2      | -1        | 1/2       | 0         |
| $S + b_3 + x + e_1$ | $\chi_2$ | $(1,1,1)$                            | 1/2      | 0         | -1/2      | 0         |
| $S + b_2 + x + e_1 + e_5$ | $\chi_3$ | $(1,1,1)$                            | -1/2     | 0         | -1/2      | 0         |
| $S + b_2 + x + e_2$ | $\chi_3$ | $(1,1,1)$                            | -1/2     | 0         | -1/2      | 0         |
| $S + b_1 + x + e_3$ | $\chi_4^+$ | $(1,1,1)$                           | 0        | 1/2       | 1/2       | +1        |
| $S + b_1 + x + e_5$ | $\chi_4$ | $(1,1,1)$                            | -1       | -1/2      | -1/2      | -2        |
| $S + b_3 + x + e_1$ | $\chi_4$ | $(1,1,1)$                            | 1        | -1/2      | -1/2      | 0         |
| $S + b_3 + x + e_1 + e_5$ | $\chi_4$ | $(1,1,1)$                            | 0        | 1/2       | -1/2      | 0         |
| $S + b_3 + x + e_1 + e_5$ | $\chi_4$ | $(1,1,1)$                            | 0        | -1/2      | 1/2       | 0         |
| $S + b_3 + x + e_2 + e_3$ | $\chi_5$ | $(1,1,1)$                            | 0        | 1/2       | -1/2      | 0         |
| $S + b_3 + x + e_1 + e_5$ | $\chi_5$ | $(1,1,1)$                            | 0        | -1/2      | 1/2       | 0         |
| $S + b_4 + x + e_4 + e_6$ | $\phi_1$ | $(1,1,1)$                            | 0        | 1/2       | 1/2       | +1        |
| $S + b_4 + x + e_4 + e_5 + e_6$ | $\phi_2$ | $(1,1,1)$                            | 0        | -1/2      | -1/2      | -1        |

Table 2: Twisted matter spectrum (observable sector) and $SU(4) \times SU(2)_L \times SU(2)_R \times U(1)_Y$ quantum numbers.

Our three generation Pati–Salam model contains the Higgs fields required for generating a realistic mass spectrum. The model admits a mass term at the cubic level of the superpotential that may generate mass for the heavy fermion family at leading order. The massless spectrum of the model is free of exotic fractionally charged states.

Perhaps most tantalising is the appearance in the string model of vector–like states that carry standard charges with respect to Standard Model gauge group, but carry non–standard charges with respect to $U(1)_C$, which descends from $E_6$. Such states arise in the string models due to the breaking of the non–Abelian gauge symmetries.
by discrete Wilson lines. Combined observation of the extra $E_6 \times U(1)$ symmetry and of the $E_6$ exotic states will therefore provide strong evidence in favour of a string construction. Furthermore, as we discussed in section 2, they provide viable dark matter candidates. Existence of a light $Z'$ in these models may therefore not only be accompanied by the extra $E_6$ states, required for anomaly cancellation, but by the extra exotic states that serve as a distinct signature of the string vacua and provide viable dark matter candidates.

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