Artificial Hawking black hole radiation using feedback-based mechanical circuits

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Black holes are considered among the most fascinating objects that exist in our universe, since in the classical formalism nothing, even no light, can escape from their vicinity. However, Hawking predicted that escape could still be possible for relativistic particles under certain conditions, known as Hawking radiation. Here we present a purely classical realization of this high energy phenomenon in a network of mechanical circuits, based on analogougs condensate matter formalism of tunneling through the event horizon. The underlying network couplings turn out incompatible with classical dynamics and are implemented by embedded active feedback interactions. We demonstrate the tunneling by propagating mechanical wavepackets through the network, achieving an exceptional correspondence to the quantum system both in momentum properties, and in the energy transmission rate exhibiting mass loss within the black hole. Our platform is table-top experimental-ready and reprogrammable, which opens up further possibilities for realizing inaccessible high energy physical phenomena.

In our universe, the existence of black holes [1] is theoretically predicted by Einstein’s theory of general relativity through the existence of space-time singularity in the well-known Schwarzschild metric [2]. This metric relates gravity with the curvature of space and time geometry. In classical gravity event horizon is thought as a boundary between the black hole and the visible universe. Any object that crosses this boundary is continuously dragged with the speed of light towards the center and ultimately becomes invisible. However, it was quite astonishing to discover that in the quantum realm this phenomenon of completely black is not entirely true. Rather, it was shown that a black hole radiates [3–5], and this radiation exists as fluctuation of quantum fields near the horizon with a temperature \( T_{\text{H}} \), famously known as the Hawking temperature. Equivalently, Hawking radiation can also be visualized as semi-classical tunneling of particles through the black hole event horizon [6,7].

Transmission of a particle with energy \( E \) through the horizon is compensated by an equivalent mass loss within the black hole for which the emission rate is given by

\[
\Gamma_{\text{H}} \approx e^{-\frac{\hbar c}{2\pi k_B T_{\text{H}}}}.
\]

Remarkably, Hawking temperature is proportional to the gravitational field strength \( g \) as \( K_B T_{\text{H}} = h g/2\pi c^3/8\pi G M \), where \( K_B \) is Boltzman constant, \( h \) is Planck constant, and \( G \) stands for the universal gravitational constant. As \( T_{\text{H}} \) is inversely proportional to the black hole mass \( M \), even for a small black hole currently present in our universe \( T_{\text{H}} \) will be of the order of \( \approx 10^{-6} \) Kelvin. As the value is six times smaller in magnitude than the cosmic microwave background, it is overshadowed by the background, thus remaining undetected till now. This motivated several studies to search for event horizon analogue, based on wave propagation in counterflowing fluids and associated ideas in photonics and Bose-Einstein condensates [8–13].

Recently, a condensed matter analogue of particle emission from the black hole has been obtained through wavepacket dynamics in Weyl semimetals (WSM), which are characterized by the Hamiltonian \( H_{\text{WSM}} = V_1 \cdot \mathbf{k} + v_f \sigma \cdot \mathbf{k} \) with a spatially-varying tilt profile \( V_1(r) \). Here, \( r \) and \( \mathbf{k} \) are respectively the spatial coordinate and the momentum in three dimensions, spanned by Pauli matrices \( \sigma \). The analogue was drawn pictorially by mapping Weyl cones into space-time geodesics (light cones), obtained from the Painlevé-Gullstrand-Lematre metric,

\[
ds^2 = c^2 dt^2 - (dr - V(r)dt)^2,
\]

in which \( V(r) \) incorporates the information of gravity, and the event horizon is denoted by the point \( r = r_s \), where \( V(r) \) is equal to the speed of light \( c \). In the WSM equivalence, the tilt \( V_1(r) \) and Fermi velocity \( v_f \) are mapped to the spatial profile \( V(r) \) and the speed of light \( c \) in (1), and the horizon is denoted by the point where the tilt strength equals the Fermi velocity \( v_f \). This condensed matter formalism of Hawking radiation provides a direct correspondence between the semi-classical tunneling rate in (1) and the attenuation rate of wavepackets transmitted through the analogous horizon [25,26].

Here we present a framework for constructing experimentally ready purely classical model, consisting of a network of active mechanical circuits, which realizes an artificial Hawking radiation based on the WSM condensed matter formalism. Classical circuit models mimicking quantum systems are common mainly for demonstrating topological phenomena [27–32], but rarely for high energy effects [33,34], as we propose here. Figure 1(a) illustrates a schematic of the WSM event horizon. It is represented by an interface of gradually tilted dispersion cones in one dimension, ranging from zero-tilted to over-tilted, which respectively stand for flat space and the black hole. The cones are formed by diagonalizing the underlying three dimensional quantum Hamiltonian \( H_q(k) \) along the principle path \( r = x \) in momentum space,

\[
H_q(k) = -\hat{\gamma}_t \sin k a \sigma_x + v_t (1 - \cos ka) \sigma_z - V_t(x) \sin ka \sigma_y,
\]

where \( v_t \) is the nearest neighbour hopping parameter, \( \hat{\gamma}_t \) represents the spin-orbit coupling strength, and \( a \) is the lattice constant. The tilt profile \( V_t(x) \) represents the interface and has smooth spatial dependence, e.g. \( V_t(x) = -(1 + \tanh \gamma_t x) \). Here, \( \gamma_t \) defines the rate of change of \( V_t(x) \) across the horizon,
FIG. 1. Artificial Hawking horizon tunneling model based on condensed matter formalism. (a) Schematic of the WSM along the principle path \( r = x \). The spatially varying tilt (gold cones) represents the interface between the black hole (over tilted cones) and flat space (zero tilted cones). The transition occurs at the critical tilt, which represents the event horizon. (b) The mechanical network model, consisting of masses A and B (black and gray cubes) that can move only in the out-of-plane direction. The blue, red, gray and yellow bars respectively represent the stable \( +t_1 \), unstable reciprocal \( -t_1 \), and unstable and non-reciprocal \( i\hat{t}_1 \) and \( iV_t(x) \) couplings. The inhomogeneity of \( iV_t(x) \) creates the dispersion tilt required for the artificial horizon. (c) Control loop details. The host structure features \( +t_1 \) couplings only, and control forces \( f_{mA} \) and \( f_{mB} \) (black arrows) applied to the masses. The forces are based on the measured velocities \( v_m \) and displacements \( u_m \) at adjacent sites, processed by corresponding controller gains, depicted by red, gray and yellow arrows, which respectively create the nonphysical couplings \( -t_1, i\hat{t}_1 \) and \( iV_t(x) \) in real time. (c) Schematic of the overall feedback control mechanism at the \( m^{th} \) unit cell.

which is directly mapped to the gravitational field strength \( g \) through \( \gamma_t = g/c \).

The corresponding mechanical network model is illustrated in Fig. 1(b). The underlying principle of this model is that its dynamical matrix needs to fully retrieve the quantum Hamiltonian \( (3) \). The network consists of an array of masses at sites \( A \) (black cubes) and \( B \) (gray cubes) with a single degree of freedom per site, e.g. vibrating in the out-of-plane direction. Following the quantum Hamiltonian that implies total four kind of electron hopping strength \( +t_1, -t_1, i\hat{t}_1 \) and \( iV_t(x) \), the masses A and B feature direct couplings of \( +t_1 \) (blue bars) for \( A \), \( -t_1 \) (red bars) for \( B \) and \( iV_t(x) \) (yellow bars) for both, as well as cross-couplings of \( i\hat{t}_1 \) (gray bars) for both. The quantum inhomogeneous potential \( V_t(x) \) therefore translates into space-dependent coupling, which changes its strength along the interface \(-L \leq x \leq L \). \( x = 0 \) is the critical tilt point indicating the artificial event horizon. The interface length \( L \) satisfies \( \tanh(L) \approx 1 \), which also ensures that outside the interface, i.e. at \( x < -L \) and \( x > L \), the function \( V_t(x) \) takes the constant end values of 0 and 2, respectively.
While $+t_1$ implies a conventional reciprocal linear spring, the $-t_1$, $i t_1$ and $i V_t(x)$ couplings are non-physical in the classical context. $-t_1$ is reciprocal but inherently unstable due to the negative sign, which is equivalent to a spring that expands when stretched. The $i t_1$ and $i V_t(x)$ couplings might seem nonphysical in classical mechanical systems due to being complex-valued. This, however, can be circumvented by invoking velocities in the frequency domain, as the Fourier transform of the out-of-plane masses velocity $v = \dot{u}$ equals $i\omega u$. The reason why these couplings are nonphysical is that they lack a restoring force, and are thus non-reciprocal. This also leads to overall system instability. We note, however, that in spite of the non-reciprocity the total system is Hermitian. Implementation of $-t_1$, $i t_1$ and $i V_t(x)$ thus requires the breaking of the originally Newtonian physics in the circuit. Here we achieve this by introducing an active feedback mechanism to be embedded in the network sites [33][45], as illustrated in Fig. [1][c,d].

To this end we consider a stable host structure consisting of the A and B unity masses connected by the $+t_1$ springs only, shown by the blue bars in the detailed schematic of Fig. [1][c). An active controller is embedded within each unit cell of the host structure, which at the $m^{th}$ cell generates commands for the external forces $f_{mA}$ and $f_{mB}$, operating in a real-time closed loop. The dynamical equations of motion at the unit cell in open loop thus read

\[
\begin{align*}
\ddot{u}_m^A & = \frac{1}{2} t_1 \left( -2 u_m^A + v_{m+1}^A + v_{m-1}^A \right) + f_{mA}, \quad (4a) \\
\ddot{u}_m^B & = \frac{1}{2} t_1 \left( -2 u_m^B + v_{m+1}^B + v_{m-1}^B \right) + f_{mB}, \quad (4b)
\end{align*}
\]

where $f_{mA} = f_{mA}^+ + f_{mA}^-$, and $f_{mB} = f_{mB}^+ + f_{mB}^-$. The components $f_{m(+/−)}^A$ and $f_{m(+/−)}^B$ are respectively responsible for generating the couplings $i t_1$ and $i V_t(x)$ with the correct sign in the equations of motion of the A and B masses, as implied by the Hamiltonian. $f_{mB}$ generates the excessive coupling $-t_1$ in the B chain. For example, in the $m^{th}$ site of the B chain $f_{mB}^+ f_{mB}^*$ receive measurements of the velocity signals $v_m^A, v_m^B$, shown by the gray and yellow arrows. $f_{mB}^+$ receives the measurement of the displacements $u_{m−1}, u_m, u_{m+1}$, shown by the red arrows. The measured signals are fed back into the respective controllers in real-time. The overall control operation for the $m^{th}$ unit cell is illustrated in Fig. [1][d]. The controller gains for the entire operation are embedded in the matrix C, which relates the control forces with the measured signals of the corresponding velocity $v_m = \begin{pmatrix} v_{m+1}^A & v_{m+1}^B & v_{m}^A & v_{m}^B & v_{m−1}^A & v_{m−1}^B \end{pmatrix}$ and displacement $u_m = \begin{pmatrix} u_{m+1}^A & u_{m+1}^B & u_{m}^A & u_{m}^B & u_{m−1}^A & u_{m−1}^B \end{pmatrix}$, as

\[
\begin{pmatrix} f_{mA} \\ f_{mB} \end{pmatrix} = -C \begin{pmatrix} v_m \\ u_m \end{pmatrix},
\]

where $C = \begin{pmatrix} -V_t^m & -V_t^m & -\beta \dot{t}_1 & -\dot{t}_1 & \dot{t}_1 & 2\beta t_1 & 0 & 0 & 0 \end{pmatrix}$.

The control matrix at the $m^{th}$ unit cell is given by

\[
C = \frac{1}{2} \begin{pmatrix} -V_t^m & -\beta \dot{t}_1 & -\beta \dot{t}_1 + 2\beta \dot{t}_1 & 2\beta t_1 & 2\beta \dot{t}_1 & 2(\beta - 2\dot{t}_1) & 2t_1 \end{pmatrix},
\]

where $V_t^m = V_t(x_m)$, and $\beta = 2\dot{t}_1$ is required for the overall system stability. With the nonphysical couplings obtained by the feedback operation, and setting the sign ($\pm$) for A and B, respectively, the closed loop dynamics becomes

\[
\begin{align*}
\ddot{u}_m^A & = \pm \frac{1}{2} t_1 \left( -2 u_m^{A/B} + v_{m+1}^{A/B} + v_{m−1}^{A/B} \right) + f_{m(A/B)}^+ f_{m(A/B)}^- + f_{m(A/B)}^+ f_{m(A/B)}^- \] \\
& \quad + \frac{1}{2} t_1 \left( -2 u_m^{A/B} + v_{m+1}^{A/B} + v_{m−1}^{A/B} \right) + f_{m(A/B)}^+ f_{m(A/B)}^- + f_{m(A/B)}^+ f_{m(A/B)}^-. \quad (7)
\end{align*}
\]

Next we demonstrate that the model in (6) fully satisfies the properties of Hawking radiation, both in momentum and
in real space. Substituting a plane wave solution of frequency \( \Omega \) for the displacement in \( \psi_m \),

\[
A/B \psi_m(t) = A/B \psi_m e^{i(\Omega t - kx)}.
\]

we obtain the quadratic eigenvalue problem

\[
\Omega^2 \sigma_0 + \left[t_1 \sin ka \sigma_x + V_t(x_0) \sin ka \sigma_0 \right] \Omega - t_1(1 - \cos ka) \sigma_x - 2\sigma_0 = 0.
\]

(8)

The solution of (8) gives the frequency spectrum in the momentum space of our classical-mechanical model, which we compare in Fig 2 with the corresponding energy spectrum of the quantum Hamiltonian (3). Both spectra are plotted for the entire range of the interface, from zero tilt \( V_t = 0 \) to the over tilt \( V_t = -2 \), through the critical tilt \( V_t = -1 \). In the classical spectrum, the stability correction \( \beta \) introduced by our control algorithm shifts the crossing point to a finite positive frequency. Remarkably, despite this inevitable correction, as well as the profound difference between the quantum first order and the classical second order dynamics, the shape of both dispersion curves evolves in a similar way for the entire span of the tilt strength across the horizon. In particular, the classical spectrum evolves from a linear and near-even dispersion at the zero tilt to the nonlinear and near-odd dispersion at the over tilt in accordance with the quantum spectrum.

Considering now the real space dynamics, we numerically simulate a finite chain embedded with the controller (4)- (6), which converges to (3) in closed loop. We excite the masses initial displacements and velocities with a Gaussian wavepacket of the form \( \Psi_m = \cos(k_i x_0) e^{(k_i - k_0)^2/4\delta^2} \phi_{k_0}^m \) in the over tilted region \( V_t = -2 \). The system hosts a two-band model with \( \alpha = 1, 2 \) shown by the red and black color respectively in Fig 2 and \( \phi_{k_0}^m \) represents the eigenvector associated with the frequency band \( \Omega_m \). The product of the eigenvector with the initial Gaussian envelope catches the required initial state at momentum \( k_i \). The velocity of the initial wavepacket along the y axis is determined by the time derivative of the displacement \( \dot{\Psi}_m \) to ensure unidirectional propagation through the interface.

Figures 3(a),(b) depict the numerical time domain evolution of a \( k_i = 2.35 \) \( (\Omega = 1.43) \) wavepacket, launched from \( x_m = 1600 (a = 1) \), as it crosses the interface (green vertical lines) and tunnels through the artificial horizon to the \( V_t = 0 \)
with $\omega$ as the difference between the initial wavepacket frequency and the classical spectrum crossing point. In parallel to that, we define the quantity

$$\chi_{q/e} = \frac{\sum_{n} N_{q}^n |\Psi_{m}^n|^2}{\sum_{m} N_{q}^n |\Psi_{m}^n|^2},$$

which represents the numerical decay rate for the quantum, $\chi_q$, and the classical, $\chi_c$, models. It is defined by the ratio of the squared amplitudes of the final wavepacket $\Psi_{f}^{in}$ and the initial wavepacket $\Psi_{m}^{in}$, obtained from time domain simulations. The dependence of $\Gamma_H$ (black), $\Gamma_s$ (gray), $\chi_q$ (green) and $\chi_c$ (orange) as a function of $\omega$ is depicted in Fig. 2. The length of the $V_t = 0$ and $V_t = -2$ regions, $N_L$ and $N_R$, as well as the starting point $\omega = 0.01$ frequency units, equal the values of the time domain simulation in Fig. 2. Remarkably, we observe that as a function of $\omega$, $\chi_c$ closely follows the profile of $\chi_q$, $\Gamma_s$ and $\Gamma_H$. This establishes our model as a complete classical analogue of Hawking phenomena.

To conclude, we proposed a purely classical realization for artificial Hawking black hole radiation, based on condensed matter formalism of semi-classical tunneling. Our model features a double-chain network of mass elements coupled in such a way that their collective dynamics is equivalent to WSM with inhomogeneous potential and the associated varying dispersion tilt, which is mapped to Hawking temperature. The resulting mechanical circuits required unstable and non-reciprocal connections, incompatible with Newtonian physics. We created these connections in real time using an embedded active feedback mechanism. We demonstrated, analytically and numerically, a complete correspondence of our model to the original high energy effect in terms of dispersion profile, and the launch and exit momenta of a wavepacket tunneled through the artificial horizon. Remarkably, we found that the wavepacket attenuation rate matches well the transmission probability of the quantum system, as well as the emission rate of the original black hole. The reprogrammable nature of our model enables realization of various quantum phenomena that require classically-incompatible dynamics.

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[1] V. P. Frolov and A. Zelnikov, Introduction to black hole physics (OUP Oxford, 2011).
[2] C. W. Misner, K. Thorne, and J. Wheeler, Gravitation WH Freeman and Co, San Francisco, 660 (1973).
[3] S. W. Hawking, Black hole explosions?, Nature 248, 30 (1974).
[4] S. W. Hawking, Particle creation by black holes, in Euclidean quantum gravity (World Scientific, 1975) pp. 167–188.
[5] J. D. Bekenstein, Black holes and entropy, in JACOB BEKENSTEIN: The Conservative Revolutionary (World Scientific, 2020) pp. 307–320.
[6] K. Srinivasan and T. Padmanabhan, Particle production and complex path analysis, Physical Review D 60, 024007 (1999).
[7] M. K. Parikh and F. Wilczek, Hawking radiation as tunneling, Physical Review Letters 85, 5042 (2000).
[8] W. G. Unruh, Experimental black-hole evaporation?, Physical Review Letters 46, 1351 (1981).
[9] L. J. Garay, J. Anglin, J. I. Cirac, and P. Zoller, Sonic analog of gravitational black holes in Bose-Einstein condensates, Physi...
[10] U. Leonhardt and P. Piwnicki, Relativistic effects of light in moving media with extremely low group velocity, Physical Review Letters 84, 822 (2000).

[11] U. Leonhardt, A laboratory analogue of the event horizon using slow light in an atomic medium, Nature 415, 406 (2002).

[12] S. Giovanazzi, Hawking radiation in sonic black holes, Physical Review Letters 94, 061302 (2005).

[13] R. Schützhold and W. G. Unruh, Hawking radiation in an electromagnetic waveguide?, Physical Review Letters 95, 031301 (2005).

[14] S. Weinertner, E. W. Redford, M. C. Penrice, W. G. Unruh, and G. A. Lawrence, Measurement of stimulated Hawking emission in an analogue system, Physical Review Letters 106, 021302 (2011).

[15] J. Hu, L. Feng, Z. Zhang, and C. Chin, Quantum simulation of Unruh radiation, Nature Physics 15, 785 (2019).

[16] G. E. Volovik, Black hole and Hawking radiation by type-II Weyl fermions, JETP Letters 104, 645 (2016).

[17] M. Zubkov, The black hole interior and the type II Weyl fermions, Modern Physics Letters A 33, 1850047 (2018).

[18] H. Huang, K.-H. Jin, and F. Liu, Black-hole horizon in the Dirac semimetal Zn 2 In 2 S 5, Physical Review B 98, 121110 (2018).

[19] L. Liang and T. Ojanen, Curved spacetime theory of inhomogeneous Weyl materials, Physical Review Research 1, 032006 (2019).

[20] Y. Kedem, E. J. Bergholtz, and F. Wilczek, Black and white holes at material junctions, Physical Review Research 2, 043285 (2020).

[21] V. Könye, C. Morice, D. Chernyavsky, A. G. Moghaddam, J. van Den Brink, and J. van Wezel, Horizon physics of quasi-one-dimensional tilted weyl cones on a lattice, Physical Review Research 4, 033237 (2022).

[22] P. Painlevé, La mécanique classique et la théorie de la relativité, Comptes Rendus Académie des Sciences (serie non specifique) 173, 677 (1921).

[23] A. Gullstrand, Allgemeine lösung des statischen einkörperproblems in der Einsteinschen gravitationstheorie (Almqvist & Wiksell, 1922).

[24] G. Lemaître, L’Univers en expansion, in Annales de la Société scientifique de Bruxelles, Vol. 53 (1933) p. 51.

[25] C. De Beule, S. Groenendijk, T. Meng, and T. Schmidt, Artificial event horizons in Weyl semimetal heterostructures and their non-equilibrium signatures, SciPost Physics 11, 095 (2021).

[26] D. Sabsovich, P. Wunderlich, V. Fleurov, D. I. Pikulin, R. Ilan, and T. Meng, Hawking fragmentation and Hawking attenuation in Weyl semimetals, Physical Review Research 4, 013055 (2022).

[27] R. Süssstrunk and S. D. Huber, Observation of phononic helical edge states in a mechanical topological insulator, Science 349, 47 (2015).

[28] L. M. Nash, D. Kleckner, A. Read, V. Vitelli, A. M. Turner, and W. T. Irvine, Topological mechanics of gyrosopic metamaterials, Proceedings of the National Academy of Sciences of the USA 112, 14495 (2015).

[29] S. H. Mousavi, A. B. Khanikaev, and Z. Wang, Topologically protected elastic waves in phononic metamaterials, Nature Communications 6, 8682 (2015).

[30] R. K. Pal and M. Ruzzene, Edge waves in plates with resonators: an elastic analogue of the quantum valley Hall effect, New Journal of Physics 19, 025001 (2017).

[31] R. Chauvasi, C.-W. Chen, and J. Yang, Subwavelength and directional control of flexural waves in zone-folding induced topological plates, Physical Review B 97, 054307 (2018).

[32] Y. Zhou, P. R. Bandaru, and D. F. Sievenpiper, Quantum-spin-Hall topological insulator in a spring-mass system, New Journal of Physics 20, 123011 (2018).

[33] X. Jiang, C. Shi, Z. Li, S. Wang, Y. Wang, S. Yang, S. G. Louie, and X. Zhang, Direct observation of Klein tunneling in phononic crystals, Science 370, 1447 (2020).

[34] L. Sirota, Klein-like tunneling of sound via negative index metamaterials, Physical Review Applied 18, 014057 (2022).

[35] T. Hofmann, T. Helbig, C. H. Lee, M. Greiter, and R. Thomale, Chiral voltage propagation and calibration in a topological Chern circuit, Physical Review Letters 122, 247702 (2019).

[36] M. Brandenbourger, X. Loscin, E. Lerner, and C. Coulais, Non-reciprocal robotic metamaterials, Nature Communications 10, 18 (2019).

[37] L. Sirota, F. Semperlotti, and A. M. Annaswamy, Tunable and reconfigurable mechanical transmission-line metamaterials via direct active feedback control, Mechanical Systems and Signal Processing 123, 117 (2019).

[38] A. Darabi, M. CollEt, and M. J. Leamy, Experimental realization of a reconfigurable electroacoustic topological insulator, Proceedings of the National Academy of Sciences 117, 16138 (2020).

[39] C. Scheibner, W. T. Irvine, and V. Vitelli, Non-Hermitian band topology and skin modes in active elastic media, Physical Review Letters 125, 118001 (2020).

[40] M. I. Rosa and M. Ruzzene, Dynamics and topology of non-Hermitian elastic lattices with non-local feedback control interactions, New Journal of Physics 22, 053004 (2020).

[41] T. Helbig, T. Hofmann, S. Imhof, M. Abdelghany, T. Kiessling, L. Molenkamp, C. Lee, A. Szameit, M. Greiter, and R. Thomale, Generalized bulk–boundary correspondence in non-Hermitian topological circuits, Nature Physics 16, 747 (2020).

[42] L. Sirota, R. Ilan, Y. Shokef, and Y. Lahini, Non-Newtonian topological mechanical metamaterials using feedback control, Physical Review Letters 125, 256802 (2020).

[43] L. Sirota, D. Sabsovich, Y. Lahini, R. Ilan, and Y. Shokef, Real-time steering of curved sound beams in a feedback-based topological acoustic metamaterial, Mechanical Systems and Signal Processing 153, 107479 (2021).

[44] L. Sirota, Quantum tunneling analogue in real-time-controlled mechanical metamaterials, in 2021 Fifteenth International Congress on Artificial Materials for Novel Wave Phenomena (Metamaterials) (IEEE, 2021) pp. 041–043.

[45] L. Zhang, Y. Yang, Y. Ge, Y.-J. Guan, Q. Chen, Q. Yan, F. Chen, R. Xi, Y. Li, D. Jia, et al., Acoustic non-Hermitian skin effect from twisted winding topology, Nature Communications 12, 1 (2021).