Improved sliding mode control of mine filling slurry concentration based on preview information

Weiqiang Tang\textsuperscript{a}, Haiyan Gao\textsuperscript{b} and Tianpeng Xu\textsuperscript{a}

\textsuperscript{a}College of Electrical and Information Engineering, Lanzhou University of Technology, Lanzhou, People's Republic of China; \textsuperscript{b}High-voltage Key Laboratory of Fujian Province, Xiamen University of Technology, Xiamen, People's Republic of China

ABSTRACT
Reducing the fluctuation of slurry concentration is the key to improve the quality of mine filling. This study aims to develop a novel sliding mode control strategy to improve the accuracy of slurry concentration. A mathematical model of a slurry preparation process is firstly established by using the system response method. Secondly, the preview information is integrated into the model by constructing an augmented system. Then, a sliding mode controller is designed by using an improved exponential reaching law. Besides, the uncertainty of the system is estimated and compensated. The results show that the designed control system has excellent dynamic performance, high accuracy and strong robustness. The problem of the large fluctuation range of the slurry concentration has been preliminarily solved. Finally, the effectiveness of the developed control strategy is verified by the numerical and experimental results.

1. Introduction

Mineral resources are not only an important material basis for the existence and development of human society, but also important means of production. Mining is the premise of the development and utilization of mineral resources, and the mining industry is the foundation of many industrial sectors, which provides raw materials for them. However, with the sharp increase in mineral demand, the environmental damage and waste discharge caused by the exploitation and utilization of mineral resources have become quite serious environmental and social problems (Hefni & Hassani, 2020; Rong et al., 2020). Filling mining method is an effective way to solve these problems according to the theory of industrial ecology. It takes filling as a part of mining, making use of mining waste, which can greatly reduce the output of mine waste and fundamentally solve the problems of mine environment and ecological destruction (Deng et al., 2017; Yin et al., 2020).

The purpose of the filling mining method is to use the filling body to manage the ground pressure, restrain the surface subsidence and surrounding rock caving, and protect the ecological environment balance of the mining site. From the point of view of materials, the filling method can be divided into the cemented one, the water-sand one and the dry one. Among them, the cemented filling method is the latest development of filling technology, which makes the materials into slurry or paste, and transports it to the filling areas by pumps. The application results show that it has the advantages of high filling strength, fast filling speed and large filling quantity (Wang et al., 2021a).

Due to the diversity of slurry components and their influence on each other, in addition, the stirring process takes a certain amount of time. Therefore, from the point of view of control, the slurry preparation process is a multi-input single-output, coupling and time-delay system. It is well known that it is difficult to directly design controllers for multivariable coupled systems. In practice, only one component needs to be adjusted and the other fixed according to the proportional relationship between components. Hence, a single-input system can be obtained, which brings convenience to the controller design and engineering applications.

Based on the simplified control system, designing a suitable controller is the key to slurry production. The control of deep cone thickener is a key problem of tailing paste filling, a novel control method based on data mining is proposed in the work (Xu et al., 2017), making the closed-loop system have good adaptability to different thickener structures and filling materials. In order to give full advantage of different methods, references
(Peng et al., 2013; Wang & Yan, 2013) combine the biological immunology, the fuzzy control, the expert system and the conventional proportional–integral–differential (PID) algorithm, and propose fuzzy immune PID and expert PID control strategies, respectively. These control strategies can achieve the better dynamic performance of slurry concentration than conventional PID control algorithms, reducing the overshoot and shortening the settling time. However, the poor robustness of these controlled systems makes the slurry concentration fluctuation large, which seriously affects the filling quality.

Sliding mode control is a robust control method, and one of its advantages is completely invariant to matched system uncertainties. Additionally, the design of sliding mode controller is relatively simple, which makes it widely used in engineering (Zhao et al., 2021; Derbeli et al., 2020; Wang et al., 2021b; Utkin et al., 2020). Obviously, applying sliding mode control to slurry preparation will enhance the system robustness, which is beneficial to reduce the fluctuation range of slurry concentration. However, the slurry preparation is a time-delay process, and there is a time mismatch between the control action and the system output, which makes the system output overshoot larger and the settling time longer. Therefore, for a time-delay system, its dynamic performance needs to be improved under the sliding mode control (Feng & Hao, 2021; Lee et al., 2017). And a large number of research results show that the preview information has a feedforward function, which can be used in control design to improve the dynamic behaviours and even the stability of a system (Alfadhl et al., 2018; Han et al., 2018; Liu et al., 2021).

Inspired by the above analysis, a novel slurry concentration control method is proposed based on the combination of sliding mode control and preview information. This method not only maintains the robustness of sliding mode control, but also makes full use of the preview information to improve the system dynamic performance. The main contributions of this paper can be summarized as follows: (1) Based on the mechanism and actual operation, a control-oriented mathematical model of the slurry preparation process is established, which lays the foundation for subsequent analysis and design; (2) The typical exponential reaching law is modified to improve the control accuracy; (3) The system uncertainty is estimated and compensated in the control law, which enhances the robustness of the system.

The rest of the paper is organized as follows. Section 2 describes the slurry preparation process and its control model. The improved sliding mode control of the slurry preparation process is stated in Section 3. In Section 4, the numerical simulations and experimental verifications are carried out. Finally, a few remarks are made to conclude the paper in Section 5.

2. Preparation process and modelling of mine filling slurry

2.1. Preparation process of mine filling slurry

The filling station of a metal mine is shown in Figure 1, which adopts the high con-centration cemented filling method, that is, the high concentration slurry is prepared by mixing the cement, the tailings and the water according to a certain proportion and then transported to the goaf. It can be seen from Figure 1 that the tailings are filtered and transported by a belt to the stirred tank, and the cement used is provided by a powder storage container. Besides, the water needed for stirring is transported to the tank through a pipeline. The technical parameters of this station are shown in Table 1, from which some parameters, such as the pulping concentration range, the mixing mode and the production capacity, can be known.

The specific preparation process of the slurry is described in Figure 2. According to the different characteristics of the materials, the amount of water is controlled by an electric valve, the amount of cement is controlled by a screw feeder, and the amount of tailings is controlled by adjusting the speed of belt transmission. It should be noted that a nuclear concentration meter is installed at the outlet of the stirring tank to measure the slurry concentration. From the Figure 2, it can be observed that the slurry preparation process is a multi-input and single-output system. Moreover, the input variables restrict each other, which makes the direct controller design more challenging.

Fortunately, the ratio of the cement, the tailings and the water can be known according to the preparation process. If two input variables are fixed, only one input variable needs to be adjusted, which makes the controller design easier. Therefore, the multi-input system is simplified to a single-input system. Herein, the cement and the tailings are fixed, and the slurry with the desired concentration is produced by adjusting the opening of the electric valve. As a result, the control block diagram of the slurry preparation process can be given by the Figure 3.

2.2. Mathematical model of mine filling slurry preparation process

It can be seen from Figure 3, the slurry preparation process involves the electric valve, the stirring and the concentration measurement, which need to be modelled before the controller is designed. For the electric valve and the meter, their dynamic characteristics are usually...
Figure 1. A mine filling station.

Table 1. Technical parameters of the filling station.

| Parameter                      | Value                      | Preparation method of tailings | Disc filtration |
|--------------------------------|----------------------------|-------------------------------|-----------------|
| Total power                    | 72kW                       | Manual/Automatic              | Filling capacity (m³/h) 40–80 |
| Control mode                   | Manual/Automatic           | Filling capacity (m³/h) 100   | Stirring capacity (m³/h) 100 |
| Stirring particle size (mm)    | < 8                        | Stirring mode                 | Filling mode     |
| Stirring mode                  | Double horizontal axis     | Forced continuous type        | Pipeline Transportation |
| Slurry concentration range     | 55%-85%                    | Powder Tank volume (t) 100    | Voltage (V) 380  |
| Control algorithm              | PID                        |                               |                 |

Figure 2. Preparation process of the mine filling slurry.

Figure 3. Block diagram of slurry concentration control system.

simplified to first-order inertia elements, and their time constants can be obtained from the user manuals. For the stirring process, it includes physical and chemical reactions, and the reactions take a certain amount of time. In addition, some time is also spent from the entrance to the exit of the stirring tank. In short, the stirring process is a complex time-delay system, so it is difficult to apply the mechanism method to model it directly.

For this reason, the stirring process modeling is carried out by the step response method. First of all, the reference slurry concentration is set to 60% and applied to the existing PID control system. And after the system output is stable, it is adjusted to 75% starting from 200s, that is, the concentration reference value is raised to a certain range. Finally, the recorded data is visualized, as shown in Figure 4. From the viewpoint of engineering, the slurry preparation process can be approximated as a high-order time-delay system. Combined with the previous analysis and field observation results, this process can be represented by the third-order inertia plus lag elements, namely

\[
G(s) = \frac{1}{(T_1s + 1)(T_2s + 1)(T_3s + 1)} e^{-\tau s},
\]

where \(\frac{1}{T_1s + 1}\) is the dynamic characteristics of the electric valve, \(\frac{1}{T_2s + 1}\) is the dynamic characteristics of the
concentration meter, and \( \frac{1}{10 s + 1} e^{-10 s} \) is the dynamic characteristics of the stirring process, \( \tau \) is the time lag. The mechanism analysis shows that the time constants \( T_1 \) and \( T_2 \) are relatively small, while the time constant \( T_3 \) is relatively large, usually having \( 0 < T_1 < 1.0 < T_2 < 1, T_3 > 1 \). The next thing to do is to determine the time constants \( T_1, T_2, T_3 \) and the lag time \( \tau \). According to the product manuals, the values of \( T_1, T_2 \) can be determined as 0.4 and 0.1, respectively. By using the results of Figure 4 and field observation, the values of \( T_3, \tau \) can be determined as 9 and 10. So far, the model of the slurry preparation process has been established, that is

\[
G(s) = \frac{1}{(0.4s + 1)(0.1s + 1)(9s + 1)(10s + 1)} e^{-10s}, \quad (2)
\]

The control task is to design a controller for the slurry preparation process based on the model (2), so that the fluctuation range of the concentration is within 5%.

3. Sliding mode control of mine filling slurry preparation process

It can be seen from the transfer function (2) that the time-delay term \( e^{-10s} \) brings difficulties to the controller design. It is necessary to deal with it without delay, here, the term \( e^{-10s} \) is replaced by a first-order inertia element \( \frac{1}{10 s + 1} \). For the slurry preparation, this delay-free treatment is feasible because the system is a slowly changing process. Additionally, this treatment is beneficial to transform the model, and makes the physical meaning of the transformed model clear (see the analysis below). Therefore, the transfer function (2) of the slurry production process can be given by

\[
G(s) \approx \frac{1}{(0.4s + 1)(0.1s + 1)(9s + 1)(10s + 1)} \quad (3)
\]

Under the condition of zero initial value, replacing the differential operator \( s \) with \( \frac{1}{T_s} \), the state space equation of the transfer function (3) can be written as

\[
\begin{align*}
\dot{x} &= A x + b u \\
y &= C x,
\end{align*}
\]

with \( x = [y \ y' \ y'']^T \), \( b_c = [0 \ 0 \ -0.2778]^T \), \( C_c = [1 \ 0 \ 0] \), and

\[
A_c = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
-0.2778 & -5.4167 & -27.65 & -12.7111
\end{bmatrix}, u \text{ is the electrical valve input voltage.} \ y \text{ denotes the slurry concentration, while } y', y'', y''' \text{ denote its first, second and third derivatives, respectively. It can be proved that the system (4) is controllable and observable. If the Padé approximation method (Basdevant, 1972) is used to deal with the time-delay term, such as second-order approximation, then giving}
\]

\[
e^{-10s} \approx \frac{s^2 - 0.6s + 0.12}{s^2 + 0.6s + 0.12} \quad (5)
\]

A transfer function (5) is transformed into the state space equation, the state variable no longer has the previous simple form of \( x \), which brings inconvenience to the system analysis. Discretize the system (4), yielding

\[
\begin{align*}
x(k + 1) &= A x(k) + b u(k) \\
y(k) &= C x(k)
\end{align*}
\]

where \( A = e^{A_c T_s}, b = \left( \int_0^{T_s} e^{A_c t} dt \right) b_c, C = C_c, T_s \) is the sampling period, which is chosen to be 1 s because the slurry production is a slow changing process. By this moment, a controller design-oriented model has been established. However, due to the influence of unmodeled dynamics, disturbances and discretization, a relatively complete model should be uncertain, namely

\[
\begin{align*}
x(k + 1) &= A x(k) + bu(k) + d(k) \\
y(k) &= C x(k)
\end{align*}
\]

where \( d(k) = [d_1(k) \ d_2(k) \ d_3(k) \ d_4(k)]^T \) is the lumped uncertainty.

3.1. Design of sliding mode controller based on current information

Let the desired output of slurry concentration be \( r_d(k) \), then the desired state is \( x_{d}(k) = [r_d(k) \ 0 \ 0] \). Defined an error as \( x_e(k) = x(k) - x_d(k) \), so the state space equation about the error can be given by

\[
x_e(k + 1) = A x_e(k) + b u(k) + \tilde{x}_d(k),
\]

with \( \tilde{x}_d(k) = A x_d(k) - x_d(k + 1) + d(k) \). Based on the system (8), a sliding mode controller can be designed, including the design of a sliding mode surface and a control law.
Firstly, the following switching function is designed.

\[ s(k) = C_e x_{e}(k), \]  

(9)

where \( C_e \) is a sliding surface parameter which can be determined by the pole placement method (Behrouz et al., 2021), aiming to ensure that the state has good motion quality on the sliding surface. Secondly, if the bound of the uncertain term \( d(k) \) is known, then the controller can be designed based on the discrete-time exponential reaching law (Gao et al., 1995), i.e.

\[ s(k + 1) - s(k) = -qT_s s(k) - \varepsilon T_s \text{sgn}(s(k)), \]  

(10)

with \( \varepsilon > 0, q > 0, 1 - qT_s > 0 \). It has been proved that the (10) satisfies the reaching condition of sliding mode, that is \([s(k + 1) - s(k)]\text{sgn}(s(k)) < 0, [s(k + 1) + s(k)]\cdot \text{sgn}(s(k)) > 0\), hence the state can reach the sliding surface in finite time. The (10) reveals the dynamics of reaching process and takes the sampling period into account, which is the biggest feature that distinguishes it from other reaching conditions. The quasi-sliding mode control system will be established by using this reaching law, and the system finally enters an oscillation state rather than tends to the origin (Gao et al., 1995), and having

\[ |s(k)| = \frac{\varepsilon T_s}{2 - qT_s}. \]  

(11)

Due to the limitations of technology, equipment and other factors, the value of sampling period \( T_s \) can not be very small. Therefore, the value of the gain coefficient \( \varepsilon \) is the key to improve the control accuracy when the \( q \) is selected. Obviously, the smaller the \( \varepsilon \) is, the smaller the \( |s(k)| \) is, meaning that the state is closer to the origin. However, the value of \( \varepsilon \) is too small, it will affect the speed of the system state reaching the sliding surface. It follows that the value of \( \varepsilon \) should be time-varying. In the initial stage of system motion, i.e., the value of \( |s(k)| \) is larger, the \( \varepsilon \) also takes a larger value. With the increase of time, the value of \( |s(k)| \) is smaller, then the \( \varepsilon \) should also take a smaller value. Therefore, the value of \( \varepsilon \) had better change with the value of \( |s(k)| \). Consequently, the following improved reaching law is presented.

\[ s(k + 1) - s(k) = -qT_s s(k) - \alpha s^2(k) T_s \text{sgn}(s(k)), \]  

(12)

with \( \alpha > 0 \). As can be seen from (12), the gain coefficient \( \varepsilon' = \alpha s^2(k) \) in this case. The reaching law (12) not only has an adaptive function, but also can improve the control precision. Because the \( \varepsilon' \) can be arbitrarily small with the time going. Besides, the reaching law (12) has a special advantage due to the square operation. When \(|s(k)| > 1\), it can accelerate the reaching speed, while \(|s(k)| < 1\), it can quickly slow down the reaching speed, which can greatly reducing the chattering. Moreover, it has an adjustable scale factor \( \alpha \), which makes the application more flexible.

By combining (8), (9) and (12), the sliding mode control law of the slurry preparation process is obtained as follows.

\[ u(k) = \frac{-1}{C_{eb}}[C_e(A-I)x_e(k) + C_e\tilde{x}_d(k) + qT_s s(k)] + \alpha s^2(k)T_s \text{sgn}(s(k)) \]  

(13)

where \( I \) is a unit matrix with corresponding dimensions. In order to facilitate the analysis of the control law (13), the expressions of \( x_e(k) \) and \( \tilde{x}_d(k) \) are substituted into it, giving

\[ u(k) = \frac{-1}{C_{eb}}[C_e(A-I)(x(k) - x_d(k)) + C_e(Ax_d(k) - x_d(k + 1)) + C_e d(k) + qT_s s(k)] + \alpha s^2(k)T_s \text{sgn}(s(k)) \]  

(14)

As shown in (14) that the \( u(k) \) is mainly composed of information at the time \( k \), i.e. the current information. Of course, the \( u(k) \) is also related to the desired output \( x_d(k + 1) \), that is \( r_d(k + 1) \), which indicates that the control law uses future reference information. However, the effect on improving the system performance is very limited using only one-step future information.

It should be pointed out that the control law (14) cannot be implemented because it contains the unknown term \( d(k) \). Thus, it is necessary to estimate it before the implementation of the control. Since \( d(k) \) is a vector and \( C_{ed}(k) \) is a scalar, it is easier to estimate \( C_{ed}(k) \). Define a new variable \( \hat{D}(k) = C_{ed}(k) \) and its estimated value is \( \hat{D}(k) = \hat{C}_{ed}(k) \). Due to the influence of uncertainty \( d(k) \), there is an error between the ideal switching function value and the actual switching function value. Then, the uncertainty can be estimated by using this error. The estimator in the form of predictor–corrector is used as follows.

\[ \hat{D}(k) = \hat{D}(k - 1) + L_D[s(k) - (1 - qT_s)s(k - 1)] + \alpha s^2(k - 1)T_s \text{sgn}(s(k - 1)) \]  

(15)

where \( L_D \) is the estimator gain coefficient. Substituting (15) into (14) leads to

\[ u(k) = \frac{-1}{C_{eb}}[C_e(A-I)(x(k) - x_d(k)) + C_e(Ax_d(k) - x_d(k + 1)) + \hat{D}(k) + qT_s s(k)] + \alpha s^2(k)T_s \text{sgn}(s(k)) \]  

(16)

In order to analyze the estimator (15), it is necessary to derive the dynamic equation of its estimation error.
selected as \( \hat{D}(k) = D(k) - \hat{D}(k) \). From (8), (9) and (16), the switching function at time \( k + 1 \) can be obtained.

\[
s(k + 1) = (1 - qT_s)s(k) - \alpha s^2(k)T_s \text{sgn}(s(k)) + \hat{D}(k)
\]

(17)

It can be found from (17) that \( \hat{D}(k + 1) = s(k) - (1 - qT_s)\hat{D}(k - 1) - \alpha s^2(k-1)T_s \cdot \text{sgn}(s(k-1)) \), hence (15) can be rewritten as

\[
\hat{D}(k) = \hat{D}(k + 1) - L_D \hat{D}(k - 1).
\]

(18)

Furthermore, the dynamic equation of the estimation error can be obtained as follows.

\[
\tilde{D}(k + 1) = D(k + 1) - D(k) - (L_D - 1) \hat{D}(k).
\]

(19)

As stated in the work (Eun et al., 1999), assume that \( \tilde{D}(k) = \tilde{D}_1(k) + \tilde{D}_2(k) \), then (19) can be given by

\[
\tilde{D}_1(k + 1) = -(L_D - 1) \tilde{D}_1(k),
\]

(20)

\[
\tilde{D}_2(k + 1) = D(k + 1) - D(k) - (L_D - 1) \tilde{D}_2(k).
\]

(21)

For (20), if \( (L_D - 1) \) is between 0 and 1, i.e. \( 1 < L_D < 2 \), then \( |\tilde{D}_1(k)| \to 0 (k \to \infty) \). For (21), if the rate of change of \( D(k) \) is bounded, i.e. \( |D(k + 1) - D(k)| < M_D(M_D > 0) \), then \( |\tilde{D}_2(k)| \leq \frac{M_D}{2 - L_D} \). This is because

\[
|\tilde{D}_2(k + 1)| \leq |D(k + 1) - D(k) - (L_D - 1)\tilde{D}_2(k)|
< |D(k + 1) - D(k)| + (L_D - 1)|\tilde{D}_2(k)|
< M_D + (L_D - 1)\frac{M_D}{2 - L_D} = \frac{M_D}{2 - L_D}.
\]

(22)

From (22), it can be concluded that the (15) will achieve high estimation accuracy for uncertainty slowly varying systems. In particular, for a constant uncertainty, the unbiased estimation can be achieved. In addition, the gain coefficient \( L_D \) is also a factor that affects the estimation accuracy, and its small value is beneficial to the estimation accuracy. It should be noted that the use of such an uncertain estimation method can meet the control requirements due to the relatively stable environment of the slurry preparation process.

Remark 1: Both the proposed estimator and the extended state observer (Song et al., 2022) can estimate the system disturbance well, which provides a basis for subsequent compensation. However, the proposed estimator is discrete, has few design parameters, and can only estimate the disturbance. While the state observer is continuous, has many design parameters, and the system disturbance needs to meet the derivable condition. Nevertheless, it can also estimate the system state, which is convenient to design the output feedback control law.

3.2. Design of sliding mode control based on preview information

In order to improve the control performance of the system, more preview information needs to be used in the controller design. For the slurry preparation process, the preview information includes the future desired output and uncertainty. To begin with, define a tracking error as

\[
e(k) = r_d(k) - y(k).
\]

(23)

Besides, define a difference of the \( e(k) \) as \( \Delta e(k + 1) = e(k + 1) - e(k) \), so the following equation holds.

\[
e(k + 1) = \Delta e(k + 1) + e(k)
= \Delta r_d(k + 1) - C \Delta x(k + 1) + e(k)
\]

(24)

According to the system (7), it can be deduced that

\[
\Delta x(k + 1) = A \Delta x(k) + b \Delta u(k) + \Delta d(k).
\]

(25)

Substituting (25) into (24) yields

\[
e(k + 1) = \Delta r_d(k + 1) - CA \Delta x(k) - Cb \Delta u(k)
- C \Delta d(k) + e(k).
\]

(26)

Suppose that the number of previewed steps of \( r(k) \) is \( H_r \), then it remains unchanged, i.e. \( r_d(k + 1), \ldots, r_d(k + H_r) \) are known, while \( r_d(k + i) = r_d(k + H_r + i) \). Denote \( \Delta x_r(k) = [\Delta r_d(k) \Delta r_d(k+1) \ldots \Delta r_d(k+H_r)]^T \), then yield

\[
\Delta x_r(k + 1) = A_r \Delta x_r(k),
\]

(27)

with \( A_r = \begin{bmatrix} 0 & \cdots & 0 \\ 0 & 1 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 1 \end{bmatrix} \in \mathbb{R}^{(H_r+1) \times (H_r+1)} \) . For the future uncertainty, it usually can be known in practice. And hence, the uncertainty is much less than the desired output signal for the future information. Herein, the uncertainty at time \( k + 1 \), that is \( d(k + 1) \), is applied to the controller design. As the \( d(k) \) can be estimated by the (15), so the \( d(k + 1) \) can be estimated by the linear extrapolation method (Cho & Skidmore, 2006), described by

\[
d(k + 1) = 2d(k) - d(k - 1).
\]

(28)

Perform a difference operation on (28), giving

\[
\Delta d(k + 1) = 2\Delta d(k) - \Delta d(k - 1).
\]

(29)

Similarly, denote \( \Delta x_d(k) = [\Delta d(k-1)^T \Delta d(k)^T]^T \), then yield

\[
\Delta x_d(k + 1) = A_d \Delta x_d(k),
\]

(30)

with \( A_d = \begin{bmatrix} 0 & I_d \\ I_d & 2I_d \end{bmatrix} \in \mathbb{R}^{8 \times 8} \) . \( O \) are unit and zero matrices, respectively. In corporation with (25)-(27) and (30), the
following system can be obtained.

\[
X(k + 1) = Ax(k) + Bu(k),
\]

where \( X(k) =
\begin{bmatrix}
  e(k) \\
  \Delta x(k) \\
  \Delta x_r(k)
\end{bmatrix}
\]

\[
A_X = \begin{bmatrix}
  1 - CA & G_r & G_d \\
  0 & A & 0 \\
  0 & 0 & A_d
\end{bmatrix}, \quad B_X = \begin{bmatrix}
  -Cb \\
  b \\
  O
\end{bmatrix}, \quad G_r = [0 \cdots 0], \quad G_d = [0 \cdots -c], \quad G_x = [\circ \circ].
\]

It is worth mentioning that the (31) is called an augmented error system commonly used in preview control. It has been proved that the system (31) is also controllable and observable if the system (7) is controllable and observable (Khalil & Fezans, 2021; Zhou et al., 2021). Next, the sliding mode control law of the system (31) will be designed. To do this, define its switching function as

\[
s_x(k) = C_X X(k),
\]

where \( C_X \) is the sliding mode surface parameter. According to the exponential reaching law (12), the control law can be obtained.

\[
\Delta u(k) = -\frac{1}{C_X B_X} [C_X(A_X - l_X)X(k) + q T_s s_X(k) + \alpha s_X^2(k) T_s \text{sgn}(s_X(k))],
\]

where \( l_X \) is a unit matrix with corresponding dimensions. Since the control increment is obtained by (33), it needs to be added to the last control when implementing, that is,

\[
u(k) = \Delta u(k) + u(k - 1).
\]

4. Results and analysis

According to the previous theoretical design and the control objective, this section will be verified from four aspects, namely, improved exponential reaching law, robustness, dynamic performance, and experiments.

4.1. Verification of the improved exponential reaching law

Consider a second-order discrete-time system

\[
x_s(k + 1) = (A_s + \Delta A_s)x_s(k) + (b_s + \Delta b_s)u_s(k) + f_s(k),
\]

where \( A_s = \begin{bmatrix}
  1 & 0.0001 \\
  0 & 0.9753
\end{bmatrix}, \quad b_s = \begin{bmatrix}
  0 \\
  -0.1314
\end{bmatrix}, \quad \Delta A_s, \Delta b_s, f_s(k)
\]

are the parameter perturbations and external disturbances, respectively. Besides, for the reaching laws (10) and (12), the parameters are selected as follows: \( T_s = 0.001s, \)

\[
x_s(k) = \begin{bmatrix}
  1 - 1 \\
  q = 10, \quad \alpha = 20, \quad \varepsilon = 1.5, \quad s_s(k) = 5x_{s1}(k) + x_{s2}(k).
\]

In the nominal case, the above two reaching laws are used to design controllers for the system (35), and the results are shown in Figure 5 and Figure 6.

As seen from Figure 5(a) and Figure 6(a), there is no big difference in the state convergence of the system, but the variation range of \( x_{s2} \) different, the range of design using the improved reaching law is larger than that of design using the conventional reaching law, reaching negative 3.5. Unlike the state, the switching function is very different. The switching function has chattering when the conventional reaching law is used for design, as shown in Figure 5(b). When the improved reaching law is used, the reaching speed of the system can be accelerated when \( s_s(k) > 1 \), as depicted in Figure 6(b). Due to the chattering of the switching function, the control input (see Figure 5(c)) will inevitably switch back and forth, which is usually disadvantageous to the actuator. Using the improved approach law, the control input is continuous, as shown in Figure 6(c). In summary, the proposed reaching law taking a variable gain mechanism has obvious advantages over the conventional reaching law in chattering suppression and reaching speed.

4.2. Verification of the robustness

In the uncertain case, assume that \( \Delta A_s = \begin{bmatrix}
  0.0001 & 0 \\
  0 & -0.01
\end{bmatrix}, \quad \Delta b_s = \begin{bmatrix}
  0.0001 \\
  0.01
\end{bmatrix}, \quad f_s(k) = \begin{bmatrix}
  0.00001 \\
  0.2 \cos(\pi k/1000)
\end{bmatrix}.

Based on the improved exponential reaching law, the influence of disturbance compensation on system robustness is discussed, and the results are shown in Figure 7 and Figure 8.

As shown in Figure 7(a), the two states cannot converge to 0, especially the second state has a significant oscillation. The reason for this is that the gain coefficient of the improved exponential reaching law is very small near the sliding surface, which makes the system unable to suppress the uncertainty effectively, so that the system state cannot stay on the sliding surface, as shown in 7(b). Admittedly, it is necessary to compensate the uncertainty to improve the robustness of the system. As shown in Figure 8(a), the control performance of the system is greatly improved after compensating for the uncertainty. However, the convergence of the second state is not very ideal. This can be explained by two aspects. On one hand, the estimation is not unbiased, i.e. \( |D_2(k)| < \frac{M_0}{T\Delta t} \). On the other hand, only the overall role of uncertainty is compensated, that is \( D(k) = C_0 d(k) \), instead of every uncertainty. As a result, the switching of the system on the sliding surface is not timely, as described in Figure 8(b). Thereby, the designed control system can well compensate for the uncertainty.
Figure 5. System response using the conventional exponential reaching law. (a) The state, (b) The switching function, (c) The control input.

Figure 6. System response using the improved exponential reaching law. (a) The state, (b) The switching function, (c) The control input.
4.3. Verification of the dynamic performance

Some parameters are selected as $T_s = 0.01 s$, $H_r = 10$, $q = 14$, $\alpha = 25$. The desired slurry concentration is set to 75%. In order to highlight the advantages of preview information, a comparison between the control laws (16) and the (34) is made without the uncertainty. Clearly, the control law (34) uses the preview information, while control law (16) does not. The simulation results are shown in Figure 9.

As shown in Figure 9(a), the two control laws can make the slurry concentration track the desired signal, but the dynamic performance is obviously different. Concretely, no overshoot can be achieved under the control law (34), while a large overshoot occurs under the control law (16), up to 20%. Moreover, the settling time has become much longer due to the serious overshoot. The reason for these differences is that the different amount of preview information used by the controllers. The control law (34) uses multi-step pre-view information to make it have a feedforward function, while the control law (16) only uses one-step future information not enough to reflect the changing trend of the desired signal. Due to the great difference in the concentration response curve, the required control inputs are also greatly different, as shown in Figure 9(b). According to the above results, it can be concluded that the preview information is very helpful to improve the dynamic performance.

4.4. Experimental results

The experimental platform is a microcomputer control system and its core is a microcomputer system, as shown in Figure 10. The working process of the microcomputer system is as follows: firstly, collect the concentration, monitoring and control information, then process
the information, and finally carry on the output control according to the processing result. The more parameters and settings can be found in Table 2. Likewise, the desired value of slurry concentration is also set to 75%, and the control law (34) is applied to the practical slurry preparation process. Record and visualize the actual running data, and the results are shown in Figure 10. It can be seen from Figure 11(a) that the slurry concentration oscillates to a certain extent. However, the degree of oscillation is much weaker than that of PID control (see Figure 4). The range of oscillation decreases from about 8% to 3%, which
Table 2. Parameters of the microcomputer system.

| Component                        | Model/Manufacturer                  |
|----------------------------------|-------------------------------------|
| Programmable logic controller    | B&R Industrial Automation X20 series|
| Stirring driver                  | Siemens AG SINAMICS G120C(45kW)     |
| Transporting driver              | Siemens AG SINAMICS G120C(22kW)     |
| Cementing driver                 | Siemens AG SINAMICS G120C(2.2kW)    |
| Electric valve                   | Rosberg (Suzhou, China) Valve Co., Ltd. |
| Concentration meter              | Beijing Gongbiao Sensing Technology Co., Ltd. GB-CMR series |

achieves the control objective. Since the output slurry concentration oscillates, the required control input also oscillates slightly, as shown in Figure 11(b). The experimental results show that the proposed control strategy can greatly improve the slurry concentration accuracy.

5. Conclusions

Aiming at the problem of slurry concentration control in mine filling system, a novel sliding mode controller is presented, and it is analyzed and verified. The following conclusions can be made.

(1) The improved exponential reaching law can not only improve the control accuracy, but also help reduce the chattering.

(2) The designed control law can compensate the uncertainty well, thus improving the robustness of the closed-loop system.

(3) The use of preview information can effectively improve the dynamic performance of the system.

(4) The practical application results show that the designed control system can greatly reduce the fluctuation range of slurry concentration, and its advantage is obvious compared with the PID control system.

In the next step, more theoretical and verification work will be carried out to optimize the control design, so as to improve the engineering adaptability of the proposed control strategy.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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