Casimir electromotive force in periodic configurations

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The possibility in principle of the existence of Casimir electromotive force (EMF) is shown for nonparallel nanosized metal plates arranged in the form of a periodic structure. It is found that EMF does not appear in strictly periodic structures with parallel plates. However, when the strict periodicity is disturbed in nonparallel plates, EMF is generated, and its value is equal to the number of pairs of plates in a configuration. Moreover, there are some effective parameters of the configuration (angles between plates, plate lengths and length to length ratios), at which the EMF generation per unit of the length of the periodic structure is maximal.

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INTRODUCTION

Recently in work [1], the possibility in principle of the existence of Casimir electromotive force has been shown for a single open perfectly conducting nanosized structure with nonparallel plates (wings). Naturally, in parallel metal plates in the classical configuration studied by Casimir [2–6], electromotive force must not generate. However, at the ends of the plates, there can be certain fluctuations of electric potentials due to Johnson-Nyquist thermal noise [7] and interference currents because of radio interference.

The possibility of Casimir EMF generation is associated with an effect similar to light-induced electron drag, which can appear in metals [8–10], graphite nano-films [11] and semiconductors [12]. In our case, the drag effect can arise in nonparallel and perfectly conducting wings due to the total uncompensated action of virtual photons upon electrons. Earlier it has been shown that in addition to the EMF generation in its wings, the system with nonparallel wings can also experience Casimir expulsion force [13, 14] and other interesting effects [15]. The force shows up as a time-constant expulsion of nonparallel plates in the direction of the smallest angle between them. It should be noted that this force significantly differs from Casimir pressure and expulsion which are capable of creating only effects of levitation above body-partners [16–20]. The uncompensated action of the forces upon nonparallel structures is due to the nonuniform action of Casimir forces upon the opposite ends of the configuration asymmetrical along one of the coordinates. Optimal parameters have been found for the opening of the angles between the plates and for the plate lengths, at which expulsion forces and EMF obtained should be maximal.

For obtaining large total uncompensated forces and EMF, it is important that the conditions for the existence of the discussed effects in periodic structures are investigated. In Ref. [3], for example, it is shown that there are no uncompensated forces in strictly periodic structures with parallel plates which are equally spaced. The expulsion effect arises when the wings are unparallel and the separation distance between the periods is larger than the minimal distance $a$ between the wings. There is an optimum of the $d/a$ relation at which the maximal effect is obtained.

It is natural to ask if the EMF excitation is possible in periodic structures constructed on the basis of configurations having the effect of Casimir EMF.

THEORY

Let us consider a periodic configuration with nonparallel nanosized metal plates for studying the possibility of the Casimir EMF existence. The inner and outer surfaces of the plates should have the properties of mirrors with the reflection coefficient $\rho$. The configuration can completely be merged into a material medium or be its part with the parameters of dielectric permeability different from those of physical vacuum. In a Cartesian coordi-
nate system, a single nonparallel configuration looks like two thin metal plates with the surface width $L = 1$ m (oriented along the $z$-axis) and length $R$ at the distance $a$ from one another; the angle $\varphi$ between them can be changed (at the same time and by the same value for both wings) as it is shown in Fig. 1.

In the Cartesian coordinates, the periodic configuration with pairs of nonparallel wings looks like it is shown in Fig. 2. Each figure in the configuration period is completely similar to the single figure depicted in Fig. 1. In the period, between the ends of the figures there is the distance $d$.

In the first approximation, Casimir EMF for one nonparallel wing $\Delta E_\parallel$ can be found in the form

$$\Delta E_\parallel = \frac{1}{2\pi e} \int_0^L dy \int_0^{r_{\max}} P(\Theta, r, \phi) dr. \quad (1)$$

Here $n_0$ is volume density, $e$ - electron charge, $\rho$ - reflection coefficient, $k$ - photon transmission, $P(\Theta, r, \phi)$ - local specific pressure at each point $r$ on the wing with the length $r_{\max}$ and width $L$

$$P(\Theta, r, \phi) = \frac{\hbar c^2}{2\pi \sin(2\Theta)} \int_0^{\Theta_2} d\Theta \sin(\Theta - 2\phi)^2 \sin \Theta \cos \Theta = -\frac{\hbar c^2}{2\pi \sin(2\Theta)} A(\varphi, \Theta_1, \Theta_2), \quad (2)$$

where

$$A(\varphi, \Theta_1, \Theta_2) = \frac{1}{50} [24\Theta_1 \sin 4\varphi - 24\Theta_2 \sin 4\varphi + 18 \cos 2\Theta_1 - 18 \cos 2\Theta_2 + 6 \cos(4\varphi - 4\Theta_2) - 6 \cos(4\varphi - 4\Theta_1) + 3 \cos(8\varphi - 2\Theta_2) - 3 \cos(8\varphi - 2\Theta_1) + \cos(8\varphi - 6\Theta_1) - \cos(8\varphi - 6\Theta_2)]. \quad (3)$$

In formula (2), $\hbar = \hbar/2\pi$ is reduced Planck constant, $c$ - speed of light. The functional expressions for limit angles between the wings $\Theta_1, \Theta_2$ and the $s$ parameter have the following form

$$\Theta_1 = \arccos \left( -\frac{r + a \sin \varphi - R \cos 2\varphi}{\sqrt{(a + R \sin \varphi + r \sin \varphi)^2 + (r \cos \varphi - R \cos \varphi)^2}} \right), \quad (4)$$

$$\Theta_2 = \arccos \left( \frac{r + a \sin \varphi}{\sqrt{a^2 + r^2 + 2ra \sin \varphi}} \right), \quad (5)$$

$$\sin(\Theta_2 - \Theta_1) = \frac{\sin(2\varphi + \Theta_1)(a + r \sin \varphi)}{\sin(\varphi - \Theta_2)}. \quad (6)$$

Thus, here the scheme is presented for calculating the EMF generation due to virtual photons in nanosized metal configurations in optical approximation. When such configurations are arranged in a periodic structure, it is necessary to take into consideration the following. The periodic arrangement of $s$ pairs of nonparallel wings with distance $d$ between them and against the $z$-axis with their wider sides leads to the formation of $n - 1$ pairs of wings with oppositely directed sides (see Fig. 2). In this case one wing of each configuration is at the same time a wing of the other configuration the wide opening of which is oppositely directed. Thus, for $n$ pairs of configurations periodically arranged along the $z$-axis it is possible to write the following expression for their total EMF [21] (if the wings as the sources of EMF are connected in a chain in series and not in parallel)

$$\Delta E_{\text{total}} = \sum_{n=1}^{n} \Delta E_n = n\Delta E(a) - (n - 1)\Delta E(d) \quad (7)$$

Here $\Delta E(a)$ is the EMF for the shortest distance $a$ between the pair of plates, and $\Delta E(d)$ is the EMF for the distance $d$ in formulae (1-6). Naturally, if all the wings in the periodic configuration of plates are connected in parallel and not in series, it is necessary to calculate the equivalent summed current in the system.

**CALCULATION RESULTS**

From formula (7) it follows that for $d = a$ in periodic configurations Casimir EMF is $\Delta E_{\text{total}} = \Delta E(a)$ at any $a$. It means that in the periodic structure, even for $d = a$, at $n \to \infty$ EMF is at the same level as that for one pair $(n = 1)$ of the configuration. However, it is clear that
at \( d \neq a \), the periodic configuration EMF will depend on the \( d/a \) ratio in accordance with formula (7) for different \( R/a \) ratios and angles \( \varphi \) between the wings as it is shown in Fig. 3a, c. When the number \( n \) of the pairs of wings grows, the behavior of the curve will be similar to those shown in Fig. 3, however, for any angles \( \varphi \) and parameters \( R/a \), however, naturally, the their EMF level will grow linearly depending on \( n \).

It is possible to determine the effectiveness \( Q \) of the EMF generation in \( n \) pairs of wings as the relation of \( \Delta E_{\text{total}} \) to the total configuration length along the \( z \)-axis

\[
Q = \frac{\Delta E_{\text{total}}}{n [a + d + 2R\tan(\varphi)]}. \tag{8}
\]

The dependence of \( Q \) on the relations \( R/a \) and \( d/a \) is shown in Figs. 3b, d. It can be seen, for example, that for any length \( R \) of the wings with different angles \( \varphi \), there is the maximal effectiveness \( Q \) of the generation of the total EMF \( \Delta E_{\text{total}} \).

As it is known \cite{1}, there is the maximum of the EMF generation for each pair of wings depending on the angle \( \varphi \) between the wings. The similar effect is also found for the periodic configuration shown in Fig. 4a. The combined dependences of EMF on two parameters, i.e. the angle \( \varphi \) and the relation \( d/a \), and also \( R/a \) and \( d/a \) are displayed in Fig. 4b, c. The corresponding effectivenesses \( Q \) are shown in Fig. 4b, d.

Let us note, that analytically and in the result of the numerical calculations, the value of the optimal relation \( d/a \to 1.62 \) at \( \varphi \to 0 \) and \( R/a \to \infty \) has been found for the periodic configurations. At the combined search with the use of angles and relation \( d/a \), the maximum of the EMF generation shifts to the values of the order \( d/a \to 1.8 \) at \( \varphi = 1 \) deg.

**CONCLUSIONS**

Thus, in the present paper, the possibility in principle is shown for the existence of Casimir EMF in nanosized nonparallel metal wings arranged in periodic structures. It is found that in the periodic structures with pairs of nonparallel wings, all Casimir currents are compensated, and EMF is not generated. However, when the periodicity of the pairs of nonparallel wings is disturbed, uncompensated currents are generated in them in the direction of the smallest angle between the wings, and consequently, EMF is generated. In this case, at any relations of the configuration parameters (angles between the wings and the wings lengths, the distance between the wings, etc.) and at any number of the pairs of wings at their series connection in the chain, there is the maximum of the effectiveness of the EMF generation in the period. The value of the total EMF in the nonparallel pairs of wings connected in series linearly grows depending on the number of elements in the chain.

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FIG. 5. The total EMF in the periodic structure at series connection in the chain of the wings depending on the angle $\varphi$ and the relation $R/a$ in the structure (a), and the corresponding structure effectiveness $Q$ (b). The total EMF (c) for different angles $\varphi$ depending on the relations $d/a$ and $R/a$, and the corresponding structure effectiveness $Q$.

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