Symmetries and the cosmological constant puzzle

A.A. Andrianov\textsuperscript{a,b}, F. Cannata\textsuperscript{c,d}, P. Giacconi\textsuperscript{c}, A.Yu. Kamenshchik\textsuperscript{c,d,e}, R. Soldati\textsuperscript{c,d}

\textsuperscript{a} V.A. Fock Department of Theoretical Physics, Saint Petersburg State University, 198904, S.-Petersburg, Russia
\textsuperscript{b} Departament Estructura i Constituents de la Materia, Universitat de Barcelona, 08028, Barcelona, Spain
\textsuperscript{c} Istituto Nazionale di Fisica Nucleare, Sezione di Bologna, 40126 Bologna, Italia
\textsuperscript{d} Dipartimento di Fisica, Università di Bologna
\textsuperscript{e} L.D. Landau Institute for Theoretical Physics of the Russian Academy of Sciences, Kosygin str. 2, 119334 Moscow, Russia

Abstract

We outline the evaluation of the cosmological constant in the framework of the standard field-theoretical treatment of vacuum energy and discuss the relation between the vacuum energy problem and the gauge-group spontaneous symmetry breaking. We suggest possible extensions of the ’t Hooft-Nobbenhuis symmetry, in particular, its complexification till duality symmetry and discuss the compatible implementation on gravity. We propose to use the discrete time-reflection transform to formulate a framework in which one can eliminate the huge contributions of vacuum energy into the effective cosmological constant and suggest that the breaking of time–reflection symmetry could be responsible for a small observable value of this constant.

1 Introduction

The so-called cosmological constant problem has two quite different aspects, which are not always clearly distinguished in the literature. One of these aspects is genuinely classical or even geometrical in its origin. The corresponding question could be formulated as follows: the classical cosmological constant, which can be introduced into Einstein–Hilbert action and Einstein equations, should be equal to zero and if it should, why? In other words: does a symmetry exists which forces the vanishing of this constant? The second aspect is, instead, purely quantum-theoretical. Independently of the presence of the classical cosmological constant, the vacuum fluctuations of the quantum fields give the contribution to the energy-momentum tensor which behaves as a cosmological constant, i.e. has the equation of state \( p = -\rho \), where \( p \) is pressure and \( \rho \) is energy density. Naturally, this contribution is ultraviolet divergent. In the quantum field theory without gravity the problem is resolved by choosing the normal (Wick) ordering of creation and annihilation operators. This procedure is justified by fact that one measures the differences between energy levels, and not their absolute values. However, it is just the absolute value of terms in the energy-momentum tensor which stays in the right-hand side of the Einstein equations and in the presence of gravity the Wick’s normal-ordering loses its validity. A generally accepted procedure of the renormalization of vacuum energy does not exist, while the naive cutoff imposed on the integration in the four-momentum space at the Planck scale gives huge values. How can one cope with them? One of possible approaches consists in the search of symmetries, which prohibit the existence of the cosmological constant and eliminate it on both the levels: classical and quantum. It is well known that supersymmetry suppresses divergences due to the compensating role of fermions and bosons. However, there are some difficulties at the application of the supersymmetric models to both the cosmological constant problem
and to the correct description of the particle physics phenomenology, including the Higgs boson mass problem. Recently arguments were provided to force the vanishing of cosmological constant even limiting oneself to the bosonic sector [1]. The formulation of this symmetry requires to give meaning to the space–time coordinates complexification (see also [2], [3], where a similar transformation was proposed but in six dimensions). Here we would like to try to fold together the idea of complexification and the idea of compensation in a some new way.

The structure of the paper is as follows: in Sec. 2 we outline the evaluation of the cosmological constant in the framework of the standard field-theoretical treatment of vacuum energy and discuss the relation between the vacuum energy problem and the gauge-group spontaneous symmetry breaking. In Sec. 3 we study the proposal [1] and its possible extensions, starting from the formalism developed in Sec. 2, in particular, its complexification allows to introduce a suitable duality symmetry. Its effect on gravity is also discussed and possible ways to implement a correct gravity interaction are outlined. Finally, in Sec. 4 we propose to use the discrete time reflection to formulate a framework in which one can eliminate the huge contributions of vacuum energy into the effective cosmological constant and, moreover, we suggest that the breaking of the T symmetry could be responsible for a small observable value of this constant.

2 Renormalization of vacuum energy density and spontaneous symmetry breaking

Recall that summing the zero–point energies of all the normal modes of some field component of mass \( m \) up to a wave number cutoff \( K \gg m \) yields a vacuum energy density

\[
\langle \rho(m) \rangle = \frac{1}{2} \int_0^K \frac{4\pi k^2 dk}{(2\pi)^3} \sqrt{k^2 + m^2} = \frac{K^4}{16\pi^2} + \frac{K^2m^2}{16\pi^2} - \frac{m^4}{32\pi^2} \left[ \ln \frac{K}{m} - \frac{1}{4} + \ln 2 + O \left( \frac{m}{K} \right)^2 \right].
\] (1)

If we trust in general relativity up to the Planck scale \( M_p \), we might take \( K \simeq M_p = (8\pi G_N)^{-1/2} \), which would give

\[
\langle \rho \rangle \approx 2^{-10} \pi^{-4} G_N^{-2} = 2 \times 10^{71} \text{ GeV}^4.
\] (2)

But it is known that the observable value of the effective cosmological constant is less than about \( 10^{-47} \text{ GeV}^4 \), and, sometimes one writes that a huge fine-tuning seems to be at work. However, it is necessary to be careful with such statements [1] because naively cutoffed expression (1) does not correspond to the cosmological constant. Within the same regularization one can calculate the vacuum pressure,

\[
\langle p(m) \rangle = \langle \frac{1}{3} T_j^j \rangle = \frac{1}{3} (1 - m\partial_m) \langle \rho(m) \rangle.
\] (3)

When comparing (1) with (3) one finds that the quartic divergences behave like radiation \( p = \rho/3 \), the quadratic ones as a perfect fluid with the equation of state \( p = -\rho/3 \) and the logarithmic divergences reproduce the cosmological constant equation of state \( p = -\rho \). Evidently first two components are not entirely Lorentz-invariant but are determined in the rest frame of the Universe. On the other hand, as was pointed out by Zeldovich [4] the vacuum expectation of the energy-momentum tensor should be Lorentz-invariant and that means that it should be proportional to the metric tensor. That implies that the pressure is equal to the energy density taken with the opposite sign, or in other words, it means that that vacuum energy-momentum tensor must behave as a cosmological constant. The Lorentz-invariant part can unambiguously separated by averaging different components of the energy-momentum tensor over Lorentz transformations and it does not include any radiation background thereby starting from quadratic divergences only (see similar arguments in [5]). Still the vacuum energy remains huge as compared to the energy density related to observable cosmological constant.

\[\text{We are grateful to A.A. Starobinsky, who has attracted our attention to this problem.}\]
Meantime, in [4] it was shown that requiring the elimination of all the divergences due to some general renormalization procedure, equivalent to introducing a spectral function of some kind, one automatically deduces that the finite part of the energy-momentum tensor have a Lorentz-invariant form. Considering the spectral function not as a renormalization tool, but as giving a real particle spectrum, one can have a general restrictions on the particle physics models, providing the cancellation of the ultraviolet divergences in the energy-momentum tensor not only on the Minkowski, but also on the de Sitter background [6]. Here we discuss how various forms of regularization can be used to control the UV divergences.

We can try to regularize the zero–point energy density in terms of the dimensional regularization [7]: namely,

$$\langle \rho \rangle = \frac{1}{2} (2\pi)^{1-n} \int d^{n-1}k \left( k^2 + m^2 \right)^{\frac{1}{2}} = \frac{1}{2} (2\pi)^{1-n} \frac{2\pi^{(n-1)/2}}{\Gamma((n-1)/2)} \int_0^\infty dk \left( k^{n-2} (k^2 + m^2)^{\frac{1}{2}} \right).$$

(4)

Unfortunately there is no strip in the complex $n$–plane in which the above integral is well defined, so that dimensional regularization is not appropriate in order to give a meaning to the zero–point energy.

Alternatively we could also define the zero–point energy density in the path–integral formalism [7], which turns out to be quite convenient in view of its generalization to the curved space. Consider the classical action

$$S[\phi] = \frac{1}{2} \int d^4x \left[ \partial_\mu \phi \partial^\mu \phi - m^2 \phi^2 \right],$$

(5)

and define the kinetic invertible operator,

$$K_x := (\Box_x + m^2 - i\epsilon),$$

(6)

and its Feynman propagator ,

$$G_F(x-y) = - K_{xy}^{-1} = \int \frac{d^4k}{(2\pi)^4} \left( k^2 - m^2 + i\epsilon \right)^{-1} \exp\{ik \cdot (x-y)\}.$$  

(7)

Then we find the generating functional

$$Z[J] := \int \mathcal{D}[\phi] \exp\left\{ i S[\phi] + i \int d^4x J(x) \phi(x) \right\}$$

$$:= Z[0] \exp\left[ - \frac{i}{2} \int d^4x \int d^4y J(x) G_F(x-y) J(y) \right].$$

(8)

Now, in order to end up with a dimensionless generating functional, we can formally write

$$Z[0] = \mathcal{N} (\det ||\mu_0^{-2} K||)^{-1/2} = \mathcal{N} \exp \left[ \frac{1}{2} \text{Tr} \ln (\mu_0^2 K^{-1}) \right],$$

(9)

where $\mathcal{N}$ is an irrelevant numerical normalization constant that we shall omit in the sequel, whereas $\mu_0$ is an arbitrary wave number or momentum scale.

A first possibility is to understand the formal relationships [9] in terms of the $\zeta$–function regularization [9] that yields

$$\ln Z[0] = \frac{1}{2} L^4 \int \frac{d^4k}{(2\pi)^4} \frac{d}{ds} \left[ \mu_0^{-2} (-k^2 + m^2) \right]_{s=0}^{-s}.$$  

(10)

After changing the integration variable $k^0 = ik_4$ we find

$$\ln Z[0] = \frac{i m^4 L^4}{32\pi^2} \frac{d}{ds} \left[ \Gamma(s-2) \frac{(m/\mu_0)^{-2s}}{\Gamma(s)} \right]_{s=0}. $$

(11)
so that we can eventually write
\[ Z[0] = e^{iW} = \exp \left\{ -i L^4 \langle \rho \rangle_{\text{eff}} \right\} , \quad \langle \rho \rangle_{\text{eff}} = \frac{m^4}{64\pi^2} \left[ \ln (m^2/\mu_0^2) - 3/2 \right] . \] (12)

We see that the \( \zeta \)-function regularization drives to a result for the zero–point vacuum energy density which turns out to be IR logarithmically divergent and positive when the infrared cutoff \( \mu_0 \propto L^{-1} \) is removed. It seems that the \( \zeta \)-function regularization is not adequate in treating the cosmological constant problem as it re-directs the problem from UV to IR region.

Turning back to eq. (9), a second possibility is to use the ultraviolet cutoff regularization of the large wave number field modes: namely,
\[
\text{Tr} \left( \hat{\phi}(Q - \mathcal{K}) \ln(\mathcal{K}/\mu_0^2) \right) = \frac{i L^4}{16\pi^2} \int_0^{Q - m^2} q \ln \left[ (q + m^2)/\mu_0^2 \right] dq \tag{13}
\]
where \( k^0 = i k_4, \ k_E^2 = k^2 + k_4^2, \ Q \sim M_p^2 \), so that we eventually obtain
\[
\langle \rho \rangle_{\text{eff}} = \frac{1}{128\pi^2} \left[ Q^2(2 \ln [Q^2/\mu_0^2] - 1) - 4Q \ m^2(\ln [Q^2/\mu_0^2] - 1) + 2m^4 \ln (m^2/\mu_0^2) - 3m^4 \right] \tag{14}
\]
in a satisfactory agreement with eq. (11) up to a redefinition of the large wave number cutoff. Then one could fit the Lorentz invariant part of (1) to (13) choosing \( \ln Q/\mu_0^2 = 1/2 \) as for this choice the contribution \( \sim \Lambda^4 \) also vanishes in (13).

To be consistent with the standard model of particle physics we have to take care of spontaneous symmetry breaking in the field theory. In this framework the charged scalar field potential takes the form (with \( \mu^2 > 0, \ \lambda > 0 \))
\[
V(\phi) = V_0 - \mu^2 \phi^4/4 + \lambda(\phi^4/4). \tag{15}
\]
The classical minimum of this potential occurs at the constant field values \( \phi^4/4 = \mu^2/2\lambda \) so that it is convenient to parameterize the scalar field \( \phi \) by writing
\[
\phi(x) = U(x) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \sigma(x) \end{pmatrix} . \tag{16}
\]

We can now make a gauge transformation in order to eliminate \( U(x) \) from the lagrangian – unitary gauge – in such a way that
\[
V_{\text{min}} = V_0 - \frac{\mu^4}{4\lambda} . \tag{17}
\]

According to the review [8] it is apparently suggested that the classical potential should vanish at \( \phi = 0 \), which would give \( V_0 = 0 \), so that some classical negative contribution to the zero–point energy density would be there. In the electroweak theory, if we assume an Higgs boson mass \( m_H = \mu \sqrt{2} \simeq 150 \text{ GeV} \), this would give \( \rho_0 \simeq -(150 \text{ GeV})^2/16\lambda \), which even for \( \lambda \) as small as \( \alpha^2 \) would yield \( \rho_0 \simeq 10^{12} \text{ GeV}^4 \), larger than the observed value by a factor \( 10^{50} \). Of course we know of no reason why \( V_0 \) or \( \Lambda \) must vanish, and it is quite possible that \( V_0 \) or \( \Lambda \) cancels the term \( -\mu^4/\lambda \) (and higher order corrections), but this example neatly shows how un–natural is to get a reasonably small effective cosmological constant.

In general, if we turn to the shifted field \( \sigma(x) \) in the unitary gauge, we obtain the Lagrange density for the shifted field
\[
\mathcal{L}[\sigma] = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} (2\mu^2) \sigma^2 \mp \mu \sqrt{\lambda} \sigma^3 - \frac{\lambda}{4} \sigma^4 - V_0 + \frac{\mu^4}{4\lambda} . \tag{18}
\]

Accordingly, the zero–point energy density in the symmetry broken phase appears to be
\[
\langle \rho \rangle = \langle \rho \rangle_{\text{div}} + V_0 - \frac{\mu^4}{4\lambda} . \tag{19}
\]
It is clear that we can easily remove the divergent part of the zero-point energy density of the scalar field $\sigma(x)$ after the introduction of a real mirror free real scalar field $\phi(x)$ with a classical Lagrange density related to that one of eq. (18)

$$L[\phi] = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} (2\mu^2) \phi^2,$$

(20)

together with a ghost pair of scalar fields. These mirror and ghost fields are supposed not to interact directly to the SM fields. At the classical level the ghost fields are described by anticommuting, real Grassmann algebra valued, field functions $\eta(x) = \eta^\dagger(x), \tilde{\eta}(x) = \tilde{\eta}^\dagger$ with Lagrange density

$$L_{GP} = -i \partial_\mu \tilde{\eta} \partial^\mu \eta + 2i\mu^2 \tilde{\eta} \eta.$$

(21)

We have

$$\Pi_\eta = +i \partial_0 \tilde{\eta}, \quad \Pi_\eta = -i \partial_0 \eta,$$

(22)

so that correspondingly

$$H_{GP} = \int dx [\eta \Pi_\eta + \tilde{\eta} \Pi_\eta - L_{GP}] = -i \int dx [\partial_0 \tilde{\eta} \partial_0 \eta + \nabla \tilde{\eta} \cdot \nabla \eta + 2\mu^2 \tilde{\eta} \eta].$$

(23)

Now one can quantize the ghost pair with the help of canonical anti-commutation relations. As a consequence, after Fourier decomposition of ghost fields we eventually obtain

$$H_{GP} = i \int d\mathbf{p} p_0 [\tilde{\eta}^\dagger(\mathbf{p}) \eta(\mathbf{p}) - \eta^\dagger(\mathbf{p}) \tilde{\eta}(\mathbf{p}) + i (L/2\pi)^3],$$

(24)

so that we get the ghost pair negative contribution to the zero point energy density in the large wave number cutoff regularization with the Planck mass

$$\langle \rho \rangle_{GP} = -(2\pi)^{-3} \int_0^{M_P} 4\pi p^2 dp (p^2 + 2\mu^2)^{\frac{3}{2}}.$$

(25)

Thus the above mirror-symmetry, which does not mix the standard model multiplets, would admittedly resolve the cosmological constant problem in the scalar Higgs sector only. It can be extended onto the entire field content of the standard model, at the expense of introducing more ghost fields with wrong spin-statistics relation. Evidently, gravity will mix the standard model fields with their related mirror-replicas and ghost-pair, leading eventually to the breaking of the spin-statistics relation and even unitarity in the standard model world. A rather sophisticated proposal will be formulated in the next section to skip that nasty mixing and unitarity loss.

## 3 't Hooft-Nobbenhuis symmetry and cosmological constant

In this section we explore the symmetry against the change of the full metric sign by continuation of real space-time variables to complex values – in the original proposal [1] to imaginary ones – first in the flat Minkowski space-time,

$$\eta_{\mu\nu} = \text{diag } || +, -, - ||; \quad x^\mu \rightarrow -iy^\mu, \quad y^\mu = y^{\mu*}; \quad \partial_\mu \rightarrow i\partial_\mu; \quad k_\mu \rightarrow ik_\mu.$$

(26)

Moreover, for a real scalar field $\phi(x)$ we shall set :

$$\phi(x) = \phi(-iy) \rightarrow \tilde{\phi}(y) = \phi^*(y); \quad \int d^4x \rightarrow \int d^4 y;$$

(27)

We stress that $\tilde{\phi}(y) \neq \phi(-iy)$ since $\tilde{\phi}(y)$ is evidently real, whereas $\phi(-iy)$ is in general complex. Therefore, the 't Hooft-Nobbenhuis transformation is not merely an analytic continuation, as it involves an essential
change of the functional base-space. It is of course analogous to what we do when we make the transition
of the Euclidean formulation, e.g. \( \phi(t) \mapsto \phi(\tau = -it) \), in which we perform a simultaneous mapping of
one functional space – a subspace of the complex function space, to another one spanned by real functions
\( \phi(\tau) \) of the Euclidean–time coordinate \( \tau \).

In so doing, one finally comes to a theory of scalar tachyon – this is the reason why t’ Hooft and
Nobbenhuis actually neglect masses – namely,

\[
L_x = \frac{1}{2} \left( \partial_\mu \phi \partial^\mu \phi - m^2 \phi^2 \right) - \frac{\lambda}{4} \phi^4 \quad \rightarrow \quad L_y = \frac{1}{2} \left( \partial_\mu \phi \partial^\mu \phi + m^2 \phi^2 \right) + \frac{\lambda}{4} \phi^4 .
\]

(28)

with a repulsive quartic self-interaction, the issue of the vacuum stability against small time-dependent
perturbations keeping admittedly open.

We also remark that if the space-time is not flat, then the continuation towards complex coordinates
makes the metric also complex. However, since the background metric is the solution of Einstein equation,
this cannot be naively defined by analytic continuation. Rather, one has to perform the corresponding
mapping of the metric functional base-space and then solve the Einstein equations with a transformed
new energy–momentum tensor, because the matter fields are in turn suitably mapped. As a result,
one might expect the very same background metric, if at least the classical matter distribution remains
unchanged, what is far from being obvious for massive interacting matter fields.

Our complementary proposal: extended duality symmetry

Once that the ’t Hooft–Nobbenhuis proposal necessarily involves the mapping of the functional base–
space, one could naturally include into this mapping also the analytic continuation of the field variables.
To this concern, one could treat all the field amplitudes and coordinates, but the metric, on the same
footing: e.g. for real scalar fields,

\[
\phi(x) = \phi(-iy) \mapsto i\varphi(y) ; \quad \varphi(y) = \varphi^*(y) ; \quad L_y(\varphi) = \frac{1}{2} \left( \partial_\mu \varphi \partial^\mu \varphi - 2m^2 \varphi^2 \right) - \frac{\lambda}{4} \varphi^4 .
\]

(29)

This transformation does realize a link between two scalar field theories without and with spontaneous
symmetry breaking. The latter one has its classical minima at \( \langle \varphi \rangle = \pm m\lambda^{-1/2} \). After the field amplitude
shift \( \varphi \mapsto \langle \varphi \rangle + \varphi \), the effective lagrangian reads

\[
L_y(\varphi) = \frac{1}{2} \left( \partial_\mu \varphi \partial^\mu \varphi - 2m^2 \varphi^2 \right) - \frac{\lambda}{4} \varphi^4 = m\sqrt{\lambda} \varphi^3 .
\]

(30)

Moreover, for vector gauge fields we shall write

\[
A_\mu(x) \mapsto iV_\mu(y) ; \quad D_{\mu}^x = \partial_{\mu} + iA_{\mu}(x) \mapsto i \left[ \partial_{\mu} \right] + iV_\mu(y) ; \quad F_{\mu\nu}(x) \mapsto -G_{\mu\nu}(y)
\]

(31)

to preserve covariant derivatives. The above transformations just leave both the Maxwell’s lagrangian
and the action invariant. Finally, for massless fermions with Yukawa coupling \( g \)

\[
\psi(x) \mapsto \psi(-iy) \mapsto \Psi(y) ; \quad \bar{\psi}(x) \mapsto -i\bar{\psi}(-iy) \mapsto \bar{\Psi}(y) = \Psi^\dagger \gamma^0
\]

(32)

\[
L_x = \bar{\psi} \left( i \partial - A - g \phi \right) \psi \mapsto \quad L_y = \bar{\Psi} \left( i \partial - \gamma - g \varphi \right) \Psi
\]

(33)

so that the spontaneous symmetry breaking mechanism allows to generate the correct fermion masses.
We remark, had one included the bare fermion masses, then they would become imaginary with the above
rules. Therefore the transformed fermions would be unstable, having the imaginary bare part and the
real part generated by spontaneous symmetry breaking.
Vacuum energy under the extended duality symmetry

Suppose that the vacuum energy is compensated to zero in the original theory. This compensation can be described in an effective theory style by introducing a number of shadow fields – in analogy with the Pauli-Villars regularization scheme – with the same coupling constants but different masses. For instance, a real scalar field $\phi$ is supplemented by $N$ shadow fields $\varphi_j$, $j=1,\ldots,N$ with masses $M_j$, the same quartic coupling constant and the same effective lagrangian but with a positive or negative weight of their contribution into the effective action $\Gamma$

$$\Gamma = \Gamma(\phi,m,\lambda) - \sum_{j=1}^{N} (-1)^{p_j} \Gamma(\varphi_j,M_j,\lambda); \quad p_j = 1,2;$$

where we keep the same notation $\phi, \varphi_j$ to indicate the classical mean field variables in the Legendre functional transform. We stress that they are combined, at the level of the effective action, in order to exactly compensate the vacuum energy contributions. All of them are bosons, although some of them behave as ghosts in the leading quasi-classical approximation.

Within the framework of quantum field theory, one can interpret part of this set with negative sign (even $p_j$) as originating from the evolution backward in time with anticausal prescription for propagators, i.e. with replacing $+i\varepsilon$ into $-i\varepsilon$ in [3],[7] – see an example in [11].

Concerning the zero-point energy, one normally has three types of leading divergences – see eq. (14)

$$\sim Q^2 \quad \sim Q m^2 \quad \sim m^4 \log (Q/m^2)$$

and with the help of a number of shadow fields one exactly cancels the divergences if

$$\sum_{j=1}^{N} (-1)^{p_j} = 1, \quad \sum_{j=0}^{N} (-1)^{p_j} M_j^2 = 0, \quad p_0 = 1 \quad M_0 \equiv m, \quad \sum_{j=0}^{N} (-1)^{p_j} M_j = 0 .$$

We assume of course the preliminary mass and coupling constant renormalization. One can also assume that the shadow world consists of sufficiently heavy particles in order to reduce their influence as much as possible on the physics accessible in the standard model real world. Then the minimal number of such fields is equal to five. The first sum rule can be interpreted as a “conservation law” of a number of matter sub-worlds evolving forward and backward in time in a certain accordance with no-time origin of our universe [13].

On the one hand, once the light shadow fields has been accepted, one can restrict himself to solely one species with negative sign of its effective action – compare with [11].

On the other hand, the cancellation of quartic and quadratic divergences has to be resorted to Planck’s scale physics, where the very notion of low energy fields with their Lagrangians of canonical dimension four is admittedly questionable. A self–consistent treatment at low energies must deal then with light mass scales and relatively light shadow fields (as compared to the Planck mass) and therefore only with the last relation, which involves the fourth powers of the shadow masses.

After the duality transformation $\varphi_j \leftrightarrow \varphi_j$ and the resolution of spontaneous symmetry breaking by shifting each field in $\langle \varphi_j \rangle = \pm M_j/\sqrt{\lambda}$, one finds the classical vacuum energy density

$$\langle \rho \rangle_{cl} = \sum_{j=0}^{N} (-1)^{p_j} \left[ -\frac{1}{2} M_j^2 \langle \varphi_j \rangle^2 + \frac{\lambda}{4} \langle \varphi_j \rangle^4 \right] = -\frac{1}{4\lambda} \sum_{j=0}^{N} (-1)^{p_j} M_j^4 = 0 .$$

Thus, quite remarkably, after the extended duality transformation the scalar field vacuum energy density remains equal to zero, whereas the masses are generated, both for fermions and for gauge bosons, thanks to the Higgs mechanism. Certainly all the standard model fields must be replicated in the shadow sectors, if one provides the zero cosmological constant. If those shadow fields do not interact with each other, it cannot be conceivably embedded into a minimal supersymmetry. On the contrary, if one starts from a minimal exact supersymmetry, this dressing by shadow fields and the subsequent extended duality transformation might lead to a spontaneous symmetry breaking for supersymmetry with zero cosmological constant in the outcome.
Hints for gravity

Suppose that shadow fields interact with our world only through gravity and therefore they belong to the dark side of the universe. Then we could exploit the shadow fields as a part of the matter in the universe and not merely just like regularizing fields. Since after the extended duality transformation we change the sign of derivatives but not of the metric, we have

$$R_{\mu\nu}(x) \quad \mapsto \quad -R_{\mu\nu}(y)$$  \hspace{1cm} (37)

$$S_g = -\frac{1}{16\pi G_N} \int d^4x \left[ R(x) - 2\Lambda \right] \quad \mapsto \quad \frac{1}{16\pi G_N} \int d^4y \left[ R(y) + 2\Lambda \right].$$  \hspace{1cm} (38)

There is no invariance, as the sign of cosmological constant is unchanged albeit gravity becomes anti–gravity. The possible solutions are:

1. anti–gravity in the symmetric phase $G_N < 0$ (repulsion supports this phase) is replaced by true gravity in the spontaneous symmetry broken phase, although one has to check classical solutions;

2. gravity is induced solely by matter and therefore the overall sign of the gravitational action remains the same under the extended symmetry transformation and the spontaneous symmetry breaking, albeit the compensation mechanism, if it is exact, does select out a vanishing coefficient in front of the scalar curvature;

3. there is a coupling to scalar fields: namely,

$$S_g = -\int d^4x \left( A + \sum_{j=0}^{N} B_j \phi_j^2(x) \right) R(x) \quad \mapsto \quad \int d^4y \left( -A + \sum_{j=0}^{N} B_j \varphi_j^2(y) \right) R(y)$$

$$= \left( A - \frac{1}{\lambda} \sum_{j=0}^{N} B_j M_j^2 \right) \int d^4y R(y) + \ldots = \frac{1}{16\pi G_N} \int d^4y R(y) + \ldots$$  \hspace{1cm} (39)

so that

$$A = \frac{1}{16\pi G_N} ; \quad \sum_{j=0}^{N} B_j M_j^2 = \frac{\lambda}{8\pi G_N}.$$  \hspace{1cm} (40)

If gravity is not principally induced by matter fields, then in the latter case there is no prescribed relation between the individual gravitational scalar couplings $B_j$. In such a circumstance, one could adjust them to support essential invariance under the extended duality transformation and a tiny cosmological constant might be generated via vacuum polarization.

## 4 Cosmological constant, time arrow and T violation

First of all, let us remark that according to recent observational results such as the discovery of the cosmic acceleration [14] it is reasonable to think that the real value of the cosmological constant is not strictly zero. Indeed, the so called ΛCDM cosmological model based on the presence of the cosmological constant has acquired the status of the standard cosmological model. Thus, the first “classical” aspect of the cosmological constant does not seem to be problematic anymore and the classical cosmological constant can have any value, being one of the fundamental constants.

The control of vacuum fluctuations is really important. The idea of the (almost) complete cancellation of the vacuum fluctuations seems very attractive because it permits to resolve both the cosmological and quantum field theoretical problems, connected with its treatment. Our idea is very simple. We are inspired by two facts.
the classical equations of motion are invariant in respect to the time inversion.

2. gravity being reparametrisation-invariant theory, does not have a time \([13]\). Indeed, at least for the closed cosmological models the Hamiltonian of the theory is equal to zero and the naïve notion of time loses sense. An effective time arises in the process of interaction with matter and due to the breakdown of the gauge (reparametrisation) invariance due to the gauge fixing choice – there is ample literature devoted to this topic \([15]\).

Hence, we suggest the following postulate: the vacuum state evolves time-symmetrically according to the evolution operator

\[
W(t) = \frac{1}{2} \left( T e^{-iHt} T^{-1} e^{iHt} + e^{-iHt} T e^{iHt} T^{-1} \right)
\]

(41)

where \(H\) is the Hamiltonian and \(T\) is the operator of time inversion. If the theory is invariant with respect to the time inversion operation, \(i.e.\)

\[
THT^{-1} = H,
\]

(42)

then \(W(t) = I\), which corresponds effectively to zero energy of the vacuum state.

In some respect this situation can be described in terms of negative (mirror) matter - some kind of shadow matter. The presence of two replicas of fields, having opposite signs of the vacuum energy results in the complete cancellation of vacuum energy in the same sense as the mirror energy reflection of ref. \([11]\) (see also \([12]\)). Much before a similar idea was elaborated by Linde \([16]\) – see also \([17, 18]\), where the idea of the second negative energy world was put forward. Certain hints from Superstring theory for shadow matter with negative vacuum energy were established in \([19]\). Thus, its contribution to the effective cosmological constant vanishes.

Therefore our prescription \([11]\) is equivalent to subtraction of the ground state energy only if time reversal holds \([42]\). In this it does not coincide with earlier proposals \([16, 11, 17, 18]\).

All written above assumes the exact time invariance of the fundamental physical theory. However, one could invoke a suitable small breaking of the time symmetry. Indeed, the violation of the CP invariance is an experimental fact, and the conservation of the CPT symmetry implies unavoidably the breakdown of the time symmetry. Such type of breakdown could occur even spontaneously as was suggested by Tsung Dao Lee in his seminal paper \([20]\). So, we are lead to suspect the existence of a connection between a small T (or CP) symmetry violation and a small observable value of the cosmological constant.

In a way our approach reminds that of the mirror world or mirror particles – see \([21]\) and references therein. The mirror symmetry is as well known as the symmetry with respect to spatial reflections or parity P symmetry. However, the absence of significant interactions between mirror particles and normal ones is imposed by the phenomenology and not by general principles like in the case of T reflection.

The problems arising in application of the spontaneous T symmetry breaking to cosmology and ways of their solution were considered in \([22]\).

It is important to emphasize, that if the connection between the T violation and the cosmological constant value does indeed exist, then it could be not connected with the presence of the standard CP-breaking terms in the CKM matrix, since we are interested in the vacuum expectation energy diagrams, to which those terms do not give a contribution. It would be rather connected with more subtle scheme, which could explain the small scale of the observable cosmological constant.

The idea of the presence of fields evolving backward in time and co-existing with “normal” fields evolving forward in time was used in many different contexts. First of all, one should cite the works by Wheeler and Feynman on time symmetric electrodynamics \([23]\) together with the so called transactional interpretation of quantum mechanics \([24]\). We should emphasize once again that there are no particles moving backward in time in our forward-in-time-world. The only influence which this time reversed world makes on us is just the presence of vacuum energy in the right-hand-side of the Einstein equations. Moreover, the appearances of such known observable quantum fluctuation effects like the Casimir effect could not be influenced by the energy reversed world as well because their observability is based on their interaction with normal particles, which provides boundary conditions responsible for these effects.
Such an interaction breaks the time symmetric evolution (41) which as we have suggested is valid only for vacuum state.

Notice that the idea that the direction of time can be connected with the existence of a cosmological term was first put forward by M.P. Bronstein in the context of Friedmann cosmology [25].

Acknowledgement

We are grateful to A.A. Starobinsky and G. Venturi for fruitful discussions. The work of A.A. was supported by Grants SAB2005-0140; RFBR 05-02-17477 and Programs RNP 2.1.1.1112; LSS-5538.2006.2. A.K. was partially supported by RFBR 05-02-17450 and LSS-1157.2006.2.

References

[1] G.'t Hooft and S. Nobbenhuis, Class. Quantum Grav. 23 (2006) 3819.
[2] G. Bonelli and A. Boyarsky, Phys. Lett. B 490 (2000) 147.
[3] R. Erdem, Phys. Lett B 621 11 (2005) 11; ibid. B 639 (2006) 348.
[4] Ya.B. Zeldovich, JETP Lett. 6 (1967) 316; Sov. Phys. - Uspekhi 11 (1968) 381.
[5] E.Kh. Akhmedov, hep-th/0204048.
[6] A.Y. Kamenshchik, A. Tronconi, G.P. Vacca and G. Venturi, Phys. Rev. D 75 (2007) 083514.
[7] N.D. Birrel, P.C.W. Davies, Quantum fields in curved space, Cambridge Univ. Press, Cambridge, U.K. (1982).
[8] S. Weinberg, Rev. Mod. Phys. 61, No. 1 (1989) 1.
[9] S.W. Hawking, Commun. Math. Phys. 55 (1977) 133.
[10] T. Kugo, I. Ojima, Suppl. Progr. Theor. Phys. 66 (1979) 1.
[11] D.E. Kaplan, R. Sundrum, JHEP 0607, 042 (2006) arXiv:hep-th/0505265.
[12] H. T. Elze, hep-th/0510267.
[13] B.S. DeWitt, Phys. Rev. 160 (1967) 1113.
[14] A. G. Riess et al., Astron. J. 116 (1998)1009; S. Perlmutter et al., Astroph. J. 517 (1999) 565.
[15] A.O. Barvinsky, Phys. Rept. 230 (1993) 237.
[16] A.D. Linde, Phys. Lett. B 200 (1988) 272.
[17] F. Henry-Couannier, Int. J. Mod. Phys. A 20 (2005) 2341; gr-qc/0410055.
[18] J.W. Moffat, Phys. Lett. B 627 (2005) 9; hep-th/0507020.
[19] A.A. Tseytlin, Phys. Rev. Lett. 66 (1991) 545.
[20] T.D. Lee, Phys. Rev. D 8 (1973) 1226.
[21] L.B. Okun, Talk at ITEP Meeting on the Future of Heavy Flavor Physics, Moscow, Russia, 24-25 Jul 2006, hep-ph/0606202.
[22] A.D. Dolgov, Phys. Rept. 222 (1992) 309; Lectures at 9th Int. Moscow School of Physics and 34th ITEP Winter School of Physics, Moscow, Russia, 21 Feb - 1 Mar 2006, hep-ph/0606230.

[23] J.A. Wheeler, R.P. Feynman, Rev. Mod. Phys. 17 (1945) 157; ibid., 21 (1949) 425.

[24] J.G. Cramer, Rev. Mod. Phys. 58 (1986) 647.

[25] Matvei P. Bronstein, Phys. Z. Sowjetunion, 3 (1933) 73.