Emergence of Superstring from Pure Spinor

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Abstract

Starting with a classical action where a pure spinor $\lambda^\alpha$ is only a fundamental and dynamical variable, the pure spinor formalism for superstring is derived by following the BRST formalism. In this formalism, not only the string variable $x^m$ but also the space-time spinor $\theta^\alpha$ are emerged as the Faddeev-Popov (FP) ghosts of a topological symmetry and its reducible symmetry. This study suggests that the fundamental theory behind the pure spinor formalism of the superstring might be a topological field theory.

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1 Introduction

It is interesting to inquire where superstring theory, which has been considered as a promising candidate of theory of everything, comes from and dream that it might be emerged from a quite trivial theory like a topological theory as in the scenario of creation of universes from nothing in cosmology. One of motivations in this article is to pursue such an idea and to suggest that the superstring in the pure spinor formalism [1] - [15] might be constructed out of a class of topological field theories [16].

Relevantly to our motivation, Berkovits has recently advocated a new interpretation of the BRST charge $Q_B$ in the pure spinor formalism of the superstring [17]. In this new interpretation, instead of regarding $(x^m, \theta^\alpha)$ and $\lambda^\alpha$ as matter and ghost variables, respectively, $(x^m, \lambda^\alpha)$ and $\theta^\alpha$ are regarded as matter and ghost variables. The matter variables then satisfy a twistor-like constraint instead of the Virasoro constraint. It turns out that quantizing this twistor-like constraint yields the fermionic Faddeev-Popov (FP) ghost $\theta^\alpha$ and the nilpotent BRST charge. After twisting the ghost number, it is shown that the BRST cohomology is related to the cohomology of the pure spinor formalism of the superstring. It is of interest to note that the fermionic coordinate $\theta^\alpha$ is emerged as the FP ghost via the standard BRST quantization procedure.

It is then natural to ask ourselves if in addition to the fermionic coordinate $\theta^\alpha$ the bosonic string coordinate $x^m$ could be emerged in a similar manner since the bosonic string coordinate $x^m$ is on the same footing as the fermionic coordinate $\theta^\alpha$ in a supersymmetric theory.

In this article, we would like to propose such a formalism where only the pure spinor $\lambda^\alpha$ is a dynamical variable while supersymmetric string coordinates $(x^m, \theta^\alpha)$ are emerged as the FP ghosts via the BRST formalism. In our approach, it is remarkable that starting with a trivial action of topological quantum field theory, the superstring coordinates $(x^m, \theta^\alpha)$ are appeared only at the quantum level through the gauge-fixing of a topological symmetry and its reducible symmetry. In this sense, the origin of the pure spinor formalism of the superstring might be a topological field theory.

The idea that the pure spinor formalism of the superstring stems from a topological field theory is supported by counting degrees of freedom in the both theories. Namely, it is well-known that topological field theories possess an equal number of bosonic and fermionic degrees of freedom. Thus, if the pure spinor formalism of the superstring is somehow derived from a topological field theory, the both theories should have the same number of bosonic and fermionic degrees of freedom. Indeed, the pure spinor formalism of the superstring has 32 bosonic and 32 fermionic degrees of freedom, therby giving us a $c = 0$ conformal field theory as in a topological field theory. Accordingly, there could be a possibility that the pure spinor formalism of the superstring has an origin of a topological field theory.
2 Superparticle

Before discussing a case of the superstring, it is worth investigating a case of the superparticle in ten dimensions even if the BRST-invariant action is slightly distinct from the usual superparticle action in the pure spinor formalism in that only 5 independent components of $x^m$ appear in the action.

We start with the following superparticle action in ten dimensions:

$$S_c = \int d\tau (\omega_\alpha \dot{\lambda}^\alpha + f_\alpha \lambda^\alpha),$$

(1)

where the spinor index $\alpha$ runs from 1 to 16 (and the vector index $m$, which appears below, runs from 0 to 9), and the dot denotes a derivative with respect to $\tau$. $\lambda^\alpha$ is a bosonic pure spinor variable of ghost number 1 satisfying the pure spinor condition

$$\lambda^\alpha \gamma^m \lambda^\beta = 0.$$ 

(2)

Now let us perform the canonical analysis of the action (1). The canonical conjugate momenta are defined as

$$\omega_\alpha = \frac{\partial S_c}{\partial \dot{\lambda}^\alpha},$$

$$\pi^\alpha = \frac{\partial S_c}{\partial \dot{f}_\alpha} \approx 0.$$ 

(3)

The last equality is a primary constraint. The Hamiltonian $H$ is then of form

$$H = -f_\alpha \lambda^\alpha.$$ 

(4)

Using this Hamiltonian, the time development of the primary constraint leads to a secondary constraint

$$\lambda^\alpha \approx 0.$$ 

(5)

It is easy to see that there is no ternary constraint and these constraints constitute the first-class constraints. Incidentally, the secondary constraint (5) renders the classical action (1) vanishing, thereby implying a topological nature of the classical theory at hand. Namely, the action (1) belongs to the Witten type of topological field theories [16].

The generator of a topological symmetry takes the form [18]

$$G = -\dot{\varepsilon}_\alpha \pi^\alpha + \varepsilon_\alpha \lambda^\alpha,$$

(6)

where $\varepsilon_\alpha$ is a bosonic local parameter. With this generator, the topological symmetry reads

$$\delta \omega_\alpha = \varepsilon_\alpha,$$

$$\delta f_\alpha = \dot{\varepsilon}_\alpha,$$

$$\delta \lambda^\alpha = 0.$$ 

(7)
Actually, the classical action (1) is invariant under this symmetry up to a surface term

$$\delta S_c = \int d\tau \frac{d}{d\tau}(\varepsilon_\alpha \lambda^\alpha).$$

(8)

By replacing the bosonic parameter $\varepsilon_\alpha$ with the fermionic ghost $p_\alpha$ of ghost number 0, one obtains the BRST transformation associated with the topological symmetry as follows:

$$\delta_B \omega_\alpha = p_\alpha,$$

$$\delta_B f_\alpha = \dot{p}_\alpha,$$

$$\delta_B \theta_\alpha = -b^\alpha,$$

$$\delta_B p_\alpha = \delta_B b^\alpha = 0,$$

(9)

where $\theta_\alpha$ is a fermionic antighost of ghost number 0 and $b^\alpha$ is a bosonic auxiliary field of ghost number 1.

We shall fix the topological symmetry by a gauge condition $f_\alpha = 0$, so that the gauge fermion is $\Psi_1 = -\theta^\alpha f_\alpha$. However, only 11 of 16 components of the secondary constraint (5) are independent, so we still have a reducible symmetry $\delta f_\alpha = \epsilon_\alpha$ of 5 components satisfying $\epsilon_\alpha \lambda^\alpha = \epsilon \gamma^{mn} \lambda = 0$. To treat this reducible symmetry in an appropriate manner, we introduce bosonic ghosts for ghosts $u_\alpha$, which have ghost number 1 and 5 independent components, such that $u_\alpha \lambda^\alpha = u \gamma^{mn} \lambda = 0$. Here the BRST transformation reads

$$\delta_B p_\alpha = u_\alpha,$$

$$\delta_B v^\alpha = B^\alpha,$$

$$\delta_B u_\alpha = \delta_B B^\alpha = 0,$$

(10)

where $v^\alpha$ is a bosonic antighost of ghost number $-1$ and $B^\alpha$ is a fermionic auxiliary field of ghost number 0. To fix this reducible symmetry, we take a gauge condition $\dot{p}_\alpha = 0$ so that the gauge fermion becomes $\Psi_2 = v^\alpha \dot{p}_\alpha$.

Consequently, we have a gauge-fixed, BRST-invariant action

$$S \equiv S_c + \int d\tau \delta_B(\Psi_1 + \Psi_2)$$

$$= \int d\tau (\omega_\alpha \dot{\lambda}^\alpha + p_\alpha \dot{B}^\alpha + f_\alpha b^\alpha + v^\alpha \dot{u}_\alpha),$$

(11)

where we have defined $B^\alpha = B^\alpha + \theta^\alpha$ and $b^\alpha = b^\alpha + \lambda^\alpha$. Then, after integrating over $f_\alpha, b^\alpha$ and rewriting $B^\alpha$ as $\theta^\alpha$, we arrive at a quantum action

$$S = \int d\tau (\omega_\alpha \dot{\lambda}^\alpha + p_\alpha \dot{\theta}^\alpha + v^\alpha \dot{u}_\alpha).$$

(12)

Indeed, this action is invariant under the BRST transformation up to a surface term

$$\delta_B S = \int d\tau \frac{d}{d\tau}(u_\alpha \theta^\alpha).$$

(13)
Let us note that at this stage the BRST transformation is reduced to the form

\[
\begin{align*}
\delta_B \omega_\alpha &= p_\alpha, \\
\delta_B \theta^\alpha &= \lambda^\alpha, \\
\delta_B p_\alpha &= u_\alpha, \\
\delta_B v^\alpha &= \theta^\alpha, \\
\delta_B \lambda^\alpha &= \delta_B u_\alpha = 0.
\end{align*}
\]

(14)

The BRST charge then takes the form

\[
Q_B = \lambda^\alpha p_\alpha + u_\alpha \theta^\alpha.
\]

(15)

To verify that this BRST charge is related to that of the superparticle in the pure spinor formalism, it is enough to notice that \(u_\alpha\) can be described in terms of a space-time vector \(P_m\) as

\[
uard{u_\alpha = P_m (\gamma^m \lambda)_\alpha},
\]

(16)

since the both sides have 5 independent components. Then the BRST charge (15) can be rewritten as

\[
Q_B = \lambda^\alpha p_\alpha + P_m (\gamma^m \theta)_\alpha
= \lambda^\alpha [p_\alpha + P_m (\gamma^m \theta)_\alpha]
= \lambda^\alpha [\partial_\alpha - i (\gamma^m \theta)_\alpha \partial_m]
= \lambda^\alpha D_\alpha,
\]

(17)

where \(D_\alpha\) is the supersymmetric derivative, and we have set \(p_\alpha = \frac{\partial}{\partial \theta^\alpha} \equiv \partial_\alpha\) and \(P_m = -i \frac{\partial}{\partial v^m} \equiv -i \partial_m\).

Furthermore, with the definition of \(x^m \equiv - (v^m \gamma^m \lambda)\), which has ghost number 0, the action (12) is cast to the form

\[
S = \int d\tau (\omega'_\alpha \dot{\lambda}^\alpha + p_\alpha \dot{\theta}^\alpha + P_m \dot{x}^m),
\]

(18)

where we have defined \(\omega'_\alpha \equiv \omega_\alpha + P_m (\gamma^m v)_\alpha\). If we rewrite \(\omega'_\alpha\) as \(\omega_\alpha\), we finally have a BRST-invariant action for the superparticle

\[
S = \int d\tau (\omega_\alpha \dot{\lambda}^\alpha + p_\alpha \dot{\theta}^\alpha + P_m \dot{x}^m).
\]

(19)

Then, the BRST transformation is given by

\[
\begin{align*}
\delta_B \omega_\alpha &= p_\alpha + P_m (\gamma^m \theta)_\alpha, \\
\delta_B p_\alpha &= P_m (\gamma^m \lambda)_\alpha, \\
\delta_B x^m &= - (\theta \gamma^m \lambda), \\
\delta_B \theta^\alpha &= \lambda^\alpha, \\
\delta_B P_m &= \delta_B \lambda^\alpha = 0.
\end{align*}
\]

(20)
At first sight, it might appear that we have exactly obtained the BRST-invariant action for the superparticle in the pure spinor formalism, but this is an illusion, whose reason we will explain below in two different ways.

First let us note that our definition of \( x^m \equiv -(v \gamma^m \lambda) \) yields a null constraint \( x^m x_m = 0 \). The root of this problem is traced to the fact that our \( x^m \) satisfies an equation

\[
(\gamma^m \lambda)_\alpha = 0. \tag{21}
\]

Here recall that in the \( U(5) \) decomposition of \( SO(10) \), a space-time vector \( y^m \) and space-time spinor \( f^\alpha \) are described as \( y^m = y^a \oplus y_a \in 5 \oplus \bar{5} \) and \( f^\alpha = f^+ \oplus f_{ab} \oplus f^a \in 1 \oplus 10 \oplus 5 \) where the indices \( a, b \) take the values \( 1, \cdots, 5 \). Using the \( U(5) \) decomposition, Eq. (21) is divided into three equations

\[
\begin{aligned}
 x_b \lambda^{ba} + x^a \lambda^+ &= 0, \\
 \left( -\frac{1}{3!} \varepsilon^{abcde} \lambda_b x_a + \frac{1}{2} \lambda^d x^e \varepsilon_{defg} \right) &= 0, \\
 x^a \lambda_a &= 0. \tag{22}
\end{aligned}
\]

The general solution for (22) turns out to be

\[
x^a = \frac{1}{\lambda^+} \lambda^{ab} x_b, \tag{23}
\]

so our \( x^m \) has only 5 independent components, which is \( x_a \in \bar{5} \). Thus, the last term in the action (19) should be written as \( P^a \dot{x}_a \) instead of \( P_m \dot{x}^m \).

Next, let us show the same fact by counting the independent degrees of freedom of variables. We have started with a topological theory (1). It is known that topological field theories have an equal number of bosonic and fermionic degrees of freedom, so the action (1) should share such a feature. In fact, in the action (19) where \( P_m \dot{x}^m \) is replaced with \( P^a \dot{x}_a \), as bosonic degrees of freedom, we have \( 11 \omega_\alpha, 11 \lambda^\alpha, 5 P^a, 5 x_a \) so that we have in total 32 while as fermionic degrees of freedom, we have \( 16 p_\alpha, 16 \theta^\alpha \) so that we have totally 32. Thus it is certain that as required by topological field theories we have the same number of bosonic and fermionic degrees of freedom in the action (19) if we replace \( P_m \dot{x}^m \) with \( P^a \dot{x}_a \).

To close this section, it is valuable to point out that all the variables have proper ghost number assignment without twisting the ghost number. Of course, our ghost number assignment can be read out from a scale invariance of the action (19)

\[
\begin{align*}
P_m &\to P_m, \quad x^m \to x^m, \quad \omega_\alpha \to e^{-\rho} \omega_\alpha, \quad \lambda^\alpha \to e^\rho \lambda^\alpha, \\
p_\alpha &\to p_\alpha, \quad \theta^\alpha \to \theta^\alpha. \tag{24}
\end{align*}
\]

With this scale transformation, the ghost number can be defined to each variable as

\[
\begin{align*}
x^m(0), P_\alpha(0), \theta^\alpha(0), p_\alpha(0), \lambda^\alpha(1), \omega_\alpha(-1), 
\end{align*}
\]

where the values in the curly bracket after variables denote the ghost number. Note that as a result the BRST charge \( Q_B \) in (17) has ghost number 1 as desired.
3 Superstring

In this section, we move on to the superstring in ten dimensions, which is the main part of this article. A classical action for the superstring on the world-sheet is made out of the left and right-moving bosonic variables $\lambda^\alpha$ and $\hat{\lambda}^{\hat{\alpha}}$ satisfying the pure spinor conditions $\lambda\gamma^m\lambda = \hat{\lambda}\gamma^m\hat{\lambda} = 0$ as follows:

$$S_c = \int d^2z(\omega_\alpha \bar{\partial}\lambda^\alpha + \bar{\omega}_a \bar{\partial}\hat{\lambda}^{\hat{\alpha}} + f_a \lambda^\alpha + \hat{f}_{\hat{a}} \hat{\lambda}^{\hat{\alpha}}). \quad (26)$$

Here $\lambda^\alpha$ and $\hat{\lambda}^{\hat{\alpha}}$ have the same space-time chirality for the Type IIB superstring and the opposite space-time chirality for the Type IIA superstring. For simplicity, we shall confine ourselves to only the left-moving (holomorphic) sector of a closed superstring since the generalization to the right-moving (anti-holomorphic) sector is straightforward.

According to a perfectly similar line of the argument to the superparticle, it is easy to obtain the following BRST-invariant action for the superstring

$$S = \int d^2z(\omega_\alpha \bar{\partial}\lambda^\alpha + p_a \bar{\partial}\theta^a + P^a \bar{\partial}x_a). \quad (27)$$

In order to have the superstring action in the pure spinor formalism, it is sufficient to choose

$$P^a = \partial x^a, \quad (28)$$

and then substitute it into the action (27) whose result reads

$$S = \int d^2z \left(\frac{1}{2} \partial x^m \bar{\partial}x_m + p_a \bar{\partial}\theta^a + \omega_\alpha \bar{\partial}\lambda^\alpha\right), \quad (29)$$

where we have used the relation

$$\partial x_a \bar{\partial}x^a + \partial x^a \bar{\partial}x_a = \partial x^m \bar{\partial}x_m. \quad (30)$$

This action is BRST-invariant under the BRST transformation generated by the BRST charge $Q_B$ for the superstring

$$Q_B = \oint dz\lambda^\alpha d_\alpha, \quad (31)$$

where $d_\alpha$ is the supersymmetric variable defined as

$$d_\alpha \equiv p_\alpha + [\partial x^m - \frac{1}{2}(\theta \gamma^m \partial \theta)(\gamma_m \theta)]_\alpha. \quad (32)$$

The term $-\frac{1}{2}(\theta \gamma^m \partial \theta)(\gamma_m \theta)_\alpha$ is added to $d_\alpha$ to compensate for the choice (28) in such a way that the BRST charge $Q_B$ becomes nilpotent using the pure spinor condition.
For instance, the BRST transformation is given by

\[ \delta_B \omega_\alpha = d_\alpha, \]
\[ \delta_B x^m = -(\lambda \gamma^m \theta), \]
\[ \delta_B \theta^\alpha = \lambda^\alpha. \]

(33)

Here it is valuable to mention that in contrast to the superparticle, we have precisely obtained the superstring in the pure spinor formalism. This fact is checked by counting the independent degrees of freedom as follows: Since we have started with a topological action (26), the BRST-invariant action (29) should have an equal number of bosonic and fermionic degrees of freedom. Actually, as bosonic degrees of freedom, we have 11\( \omega^\alpha \), 11\( \lambda^\alpha \), 10\( x^m \) so that we have in total 32 while as fermionic degrees of freedom, we have 16\( p_\alpha \), 16\( \theta^\alpha \) so that we have totally 32.

Finally, let us mention that the physical state condition for the Type II closed superstring is provided by

\[ Q_B \Phi = \hat{Q}_B \Phi = 0 \]

for physical states \( \Phi \) where \( \Phi \) is a functional of \( x^m, \theta^\alpha, \hat{\theta}^\alpha, \lambda^\alpha, \hat{\lambda}^\alpha \), that is, \( \Phi = \Phi(x^m, \theta^\alpha, \hat{\theta}^\alpha, \lambda^\alpha, \hat{\lambda}^\alpha) \). Given that a massless vertex operator \( \Phi = \lambda^\alpha \hat{\lambda}^\beta A_{\alpha \beta}(x, \theta, \hat{\theta}) \) with a superfield \( A_{\alpha \beta}(x, \theta, \hat{\theta}) \), the physical state condition yields the Type II supergravity equations

\[ \gamma_{\alpha \beta}^{m_1 ... m_5} D_\alpha A_{\beta \gamma} = \gamma_{\alpha \beta}^{m_1 ... m_5} \hat{D}_\alpha A_{\beta \gamma} = 0, \]

and \( \delta \Phi = Q_B \Omega + \hat{Q}_B \Omega \) gives us the gauge transformation of the Type II supergravity, \( \delta A_{\alpha \beta} = D_\alpha \hat{\Omega}_{\beta} + \hat{D}_\beta \Omega_{\alpha}, \gamma_{\alpha \beta}^{m_1 ... m_5} D_\alpha \Omega_{\beta} = \gamma_{\alpha \beta}^{m_1 ... m_5} \hat{D}_\alpha \hat{\Omega}_{\beta} = 0 \). Thus, our BRST cohomology for massless sector implies the Type II supergravity theory.

In a similar manner, we can define the massive physical states.

4 Conclusion

In this article, on the basis of the BRST quantization procedure, we have derived the superstring in the pure spinor formalism by starting with a classical action composed of only the pure spinor \( \lambda^\alpha \) as a dynamical variable. In our approach, the supersymmetric coordinates \( (x^m, \theta^\alpha) \) are emerged as the Faddeev-Popov (FP) ghosts stemming from the gauge-fixing of a topological symmetry and its reducible symmetry. Moreover, it turns out that the BRST cohomology describes the physical states of the superstring.

In order to understand the formalism at hand more deeply, it is useful to recall the worldline formalism of the Chern-Simons theory in three dimensions [19, 9] and the BF theory in arbitrary space-time dimensions [18]. It has been already shown that Chern-Simons theory can be described using the world-line action [19, 9]

\[ S = \int d\tau (P_\mu \dot{x}^\mu + l^\mu P_\mu) \]

\[ = \int d\tau (-x^\mu \dot{P}_\mu + l^\mu P_\mu), \]

(34)

\footnote{Going from the first equality to the second one, we have neglected a surface term for simplicity of the presentation. However, it should be remembered that such a surface term in general plays an important role in the Witten type of topological field theories.}
where the index $\mu$ takes the values 0, 1, 2 and $l^\mu$ is a Lagrange multiplier. In a similar way, we have also presented a world-line description of topological non-abelian BF theory in an arbitrary space-time dimension and shown that its BRST cohomology has a natural representation as the sum of the de Rham cohomology [18].

When we compare the action (34) with the superparticle action (1), we soon realize a similarity such that the superparticle action (1) is nothing but the world-line action (34) where the space-time vector $P_\mu$ is replaced with the corresponding spinor and the pure spinor condition is imposed on this spinor. Thus, roughly speaking, the world-line formalism with the pure spinor condition naturally leads to the superstring in the pure spinor formalism in ten dimensions.

From the viewpoint of $\beta - \gamma$ system [20, 21]

$$S = \int d^2z \dot{\beta}_i \bar{\partial} \gamma^i, \quad (35)$$

our classical action (26) of the superstring can be interpreted as that of the $\beta - \gamma$ system with a Lagrange multiplier enforcing a topological symmetry where $\gamma^i$ and $\beta_i$ is a pure spinor and its conjugate momentum, respectively. It would be also interesting to study the present formulation from this viewpoint further in future.

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References

[1] N. Berkovits, ”Super Poincare covariant quantization of the superstring”, JHEP 0004 (2000) 018, hep-th/0001035.

[2] N. Berkovits and B.C. Vallilo, ”Consistency of super Poincare covariant superstring tree amplitudes”, JHEP 0007 (2000) 015, hep-th/0004171.

[3] N. Berkovits, ”Cohomology in the pure spinor formalism for the superstring”, JHEP 0009 (2000) 046, hep-th/0006003; ”Relating the RNS and pure spinor formalisms for the superstring”, JHEP 0108 (2001) 026, hep-th/0104247; ”Covariant quantization of the superparticle using pure spinors”, JHEP 0109 (2001) 016, hep-th/0105050.

[4] P.A. Grassi, G. Policastro, M. Porrati and P. van Nieuwenhuizen, ”Covariant quantization of superstrings without pure spinor constraints”, JHEP 0210 (2002) 054, hep-th/0112162; ”The Massless spectrum of covariant superstrings”, P.A. Grassi, G. Policastro and P. van Nieuwenhuizen, JHEP 0211 (2002) 001, hep-th/0202123.
[5] N. Berkovits and O. Chandia, "Lorentz invariance of the pure spinor BRST cohomology for the superstring", Phys. Lett. B514 (2001) 394, hep-th/0105149; "Superstring vertex operators in an AdS(5) x S**5 background", Nucl. Phys. B596 (2001) 185, hep-th/0009168.

[6] I. Oda and M. Tonin, "On the Berkovits covariant quantization of GS superstring", Phys. Lett. B520 (2001) 398, hep-th/0109051.

[7] N. Berkovits and P. Howe, "Ten-dimensional supergravity constraints from the pure spinor formalism for the superstring", Nucl. Phys. B635 (2002) 75, hep-th/0112160.

[8] M. Matone, L. Mazzucato, I. Oda, D. Sorokin and M. Tonin, "The Superembedding origin of the Berkovits pure spinor covariant quantization of superstrings", Nucl. Phys. B639 (2002) 182, hep-th/0206104.

[9] N. Berkovits, "ICTP lectures on covariant quantization of the superstring", hep-th/0209059.

[10] N. Berkovits, "Towards covariant quantization of the supermembrane", JHEP 0209 (2002) 051, hep-th/0201151; "Multiloop amplitudes and vanishing theorems using the pure spinor formalism for the superstring", JHEP 0409 (2004) 047, hep-th/0406055; "Pure spinor formalism as an N=2 topological string", JHEP 0510 (2005) 089, hep-th/0509120.

[11] Y. Aisaka and Y. Kazama, "A New first class algebra, homological perturbation and extension of pure spinor formalism for superstring", JHEP 0302 (2003) 017, hep-th/0212316; "Operator mapping between RNS and extended pure spinor formalisms for superstring", JHEP 0308 (2003) 047, hep-th/0305221; "Relating Green-Schwarz and extended pure spinor formalisms by similarity transformation", JHEP 0404 (2004) 089, hep-th/0404141; "Origin of pure spinor superstring", JHEP 0505 (2005) 046, hep-th/0502208.

[12] I. Oda and M. Tonin, "On the b-antighost in the pure spinor quantization of superstrings", Phys. Lett. B606 (2005) 218, hep-th/0409052; "Y-formalism in pure spinor quantization of superstrings", Nucl. Phys. B727 (2005) 176, hep-th/0505277; "Y-formalism and b ghost in the non-minimal pure spinor formalism of superstrings", Nucl. Phys. B779 (2007) 63, arXiv:0704.1219 [hep-th].

[13] N. Berkovits and N.A. Nekrasov, "The Character of pure spinors", Lett. Math. Phys. 74 (2005) 75, hep-th/0503075; "Multiloop superstring amplitudes from non-minimal pure spinor formalism", JHEP 0612 (2006) 029, hep-th/0609012.

[14] C.R. Mafra, "Superstring scattering amplitudes with the pure spinor formalism", arXiv:0902.1552 [hep-th] and references therein.
[15] L. Mazzucato, "Superstrings in AdS", arXiv:1104.2604 [hep-th] and references therein.

[16] E. Witten, "Topological quantum field theory", Comm. Math. Phys. 117 (1988) 353.

[17] N. Berkovits, "Pure spinors, twistors, and emergent supersymmetry", arXiv:1105.1147 [hep-th].

[18] I. Oda and M. Tonin, "Worldline approach of topological BF theory", Phys. Lett. B625 (2005) 155, arXiv:hep-th/0506054.

[19] E. Witten, "Chern-Simons gauge theory as a string theory", Prog. Math. 113 (1995) 637, hep-th/9207094.

[20] N.A. Nekrasov, "Lectures on curved beta-gamma systems, pure spinors, and anomalies", hep-th/0511008.

[21] P.A. Grassi, I. Oda and M. Tonin, "Y-formalism and curved beta-gamma systems", Nucl. Phys. B806 (2009) 1, arXiv:0803.0236 [hep-th].