On Gauge Independence for Gauge Models with Soft Breaking of BRST Symmetry

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Abstract

A consistent quantum treatment of general gauge theories with an arbitrary gauge-fixing in the presence of soft breaking of the BRST symmetry in the field-antifield formalism is developed. It is based on a gauged (involving a field-dependent parameter) version of finite BRST transformations. The prescription allows one to restore the gauge-independence of the effective action at its extremals and therefore also that of the conventional S-matrix for a theory with BRST-breaking terms being additively introduced into a BRST-invariant action in order to achieve a consistency of the functional integral. We demonstrate the applicability of this prescription within the approach of functional renormalization group to the Yang–Mills and gravity theories. The Gribov–Zwanziger action and the refined Gribov–Zwanziger action for a many-parameter family of gauges, including the Coulomb, axial and covariant gauges, are derived perturbatively on the basis of finite gauged BRST transformations starting from Landau gauge. It is proved that gauge theories with soft breaking of BRST symmetry can be made consistent if the transformed BRST-breaking terms satisfy the same soft BRST symmetry breaking condition in the resulting gauge as the untransformed ones in the initial gauge, and also without this requirement.

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1 Introduction

The contemporary progress in high-energy physics and quantum field theory is strongly connected with the non-pertubative features of quantum theories. The electroweak and strong interactions are described by the Standard Model, in which Quantum Chromodynamics (QCD) is a constituent, and there are no experimental facts in conflict with QCD. While the Standard Model has been justified by the discovery of the Higgs boson, the problem of consistency in QCD is far from its solution, especially in view of the confinement phenomenon. The Lagrangian of QCD (and generally that of the Standard Model) belongs to the class of non-Abelian gauge theories \[1\], \[2\], \[3\]. It is well known that the BRST symmetry \[4\], being a special global fermionic descendant of gauge invariance, plays a fundamental role in quantum field theory, since the fundamental interactions of Nature, together with gravity and perhaps some yet unknown forces, can be described in terms of gauge theories. The covariant quantization of Yang–Mills theories by means of the Faddeev–Popov procedure cannot be realized correctly, even perturbatively, for the entire spectrum of the momenta distribution due to the well-known Gribov problem \[5\] in the deep infra-red region for gauge fields, once a gauge condition has been imposed differentially \[6\], since there remains an infinitely large number of discrete gauge copies even after gauge-fixing.

In order to fix the residual gauge freedom, Gribov has undertaken a detailed study of the Coulomb gauge and suggested a restriction of the domain of functional integration for gauge fields to the so-called first Gribov region, which has been effectively incorporated into the functional measure as the Heaviside $\Theta$-function, thus realizing the “no-pole” condition for the ghost propagator. Effectively, this restriction can be implemented, in the Landau gauge with a hermitian Faddeev–Popov operator, by a special addition introduced to the standard Faddeev–Popov action and known as the Gribov–Zwanziger functional \[9\], \[10\]. However, this addition is not gauge-invariant and is therefore non-invariant under the original BRST transformations.

The idea of using the Zwanziger action in order to take account of gauge field configurations has introduced to the path integral of the Yang–Mills theory the entire spectrum of frequencies, which has been examined in a number of papers \[11\] based on the breakdown of BRST symmetry in Yang–Mills theories. Notice that until now the considerations \[9\], \[10\], \[11\] of the Gribov horizon in Yang–Mills theories have been carried out basically in the Landau gauge. The analytical proof \[5\] of the presence of Gribov copies in the physical spectrum has been confirmed by lattice simulations in some QCD models, such as the $SU(2)$ gluodynamics (see, e.g., \[12\], \[13\] and references therein), which is an expected result due to the discovery of field configurations.

\[1\]There are some other recently suggested methods of solving the Gribov problem in a consistent way: first, the procedure of imposing an algebraic (instead of a differential one) gauge on auxiliary scalar fields in a theory which is non-perturbatively equivalent to the Yang-Mills theory with the same gauge group \[7\]; second, the procedure of averaging over the Gribov copies with a non-uniform weight in the functional integral and the replica trick \[8\].
within the same Landau gauge condition for the Faddeev–Popov action adopted to lattice calculations. For the sake of completeness, notice the study [14] of the Gribov problem (beyond the Landau gauge) in covariant $R_\xi$ gauges for a small value of the gauge parameter $\xi$ for an approximation of the quantum action being quadratic in the fields; let us also notice the proposal of a new form of the horizon functional in $R_\xi$ gauges [15] in the maximal Abelian gauges [16], [17], in the Coulomb gauge [18], and on a curved Riemannian background to study the influence of the curvature tensor on changing the size of the Gribov region [19].

There is a large freedom in the choice of admissible gauges used to obtain a correct path integral in Yang–Mills theories with account of the Gribov problem; it is also well known that the Green functions are gauge-dependent; however, this dependence has such a specific character that it should be cancelled in physical combinations such as the S-matrix. Contemporary proofs of the gauge-independence of the S-matrix in Yang–Mills theories are based on the BRST symmetry, see, e.g., [20], and also apply to more general gauge theories [21]. There arises an immediate problem of consistency for a gauge theory in case the BRST invariance of the resulting quantum action (such as the Gribov–Zwanziger action) turns out to be broken. The study of this problem has been initiated by [22], [23]. These studies investigated both Yang–Mills and general gauge theories, such as supergravity, superstrings with open algebras, and higher-spin fields as reducible gauge theories, see, e.g., [24], with an introduction of so-called soft breaking of the BRST symmetry (under the natural assumption of the existence of the Gribov horizon and Gribov–Zwanziger functional for any theory with a non-Abelian gauge algebra) and achieved their results on a basis of the field-antifield method [25], [26]. Namely, in [22], [23], it was shown (with some peculiar features studied in [27]) that the gauge-independence of the effective action for a gauge theory with soft BRST symmetry breaking on the mass shell requires the fulfillment of a quite strong condition for the BRST symmetry breaking term, and therefore we come to the conclusion [22]: “It is argued that gauge theories with a soft breaking of BRST symmetry are inconsistent.” The same statement has been shown to take place in the Gribov–Zwanziger theory with the $R_\xi$-gauge.

As a next step in solving the problem of determining the horizon functional for the Yang–Mills theory in gauges beyond the Landau gauge, there is the recent concept of so-called finite field-dependent BRST transformations [28], earlier used in the infinitesimal form [25], [26] in order to establish the gauge-independence of the vacuum functional, but now explicitly constructed to relate the Faddeev–Popov action in the Landau gauge and in the covariant $R_\xi$-gauge. The concept of field-dependent BRST transformations, first suggested [29] in a finite (however, different) form (see also [30], [31]) permits one to obtain perturbatively an explicit form of the Gribov horizon functional in the $R_\xi$-gauge [32], starting from its form in the Landau gauge. This provides a different perspective of the problem of gauge-dependence for the Gribov–Zwanziger theory and allows one to revisit this problem more generally in a gauge theory with soft BRST breaking symmetry.
In this work, we present a study of gauge-dependence in general gauge theories with soft breaking of the BRST breaking and develop, for this purpose, a concept of gauged (equivalently, field-antifield-dependent) BRST transformations. We present an explicit calculation of the Jacobian of the corresponding change of field-antifield variables in the partition function, to determine and solve a non-linear equation for an unknown field-antifield-dependent odd-valued parameter $\Lambda$. We establish a coincidence of the vacuum functional without a BRST broken term in a gauge determined by a gauge Fermion $\Psi$ with the vacuum functional in a different gauge determined by $\Psi + \Delta \Psi$. On this basis, we examine the properties of the average effective action within the approach of the functional renormalization group to the Yang–Mills and gravity theories. We also suggest the Gribov–Zwanziger horizon functional for a many-parameter family of linear gauges, including the Coulomb, the axial, and the $R_\xi$ gauges, used in non-Hermitian Faddeev–Popov operators.

The paper is organized as follows. In Section 2, we introduce the concept of finite gauged (field-dependent) BRST symmetry transformations and investigate the related change of variables in the functional integral for general gauge theories in the field-antifield formalism. In Section 3, we use the field-dependent BRST transformations to formulate the study of gauge-dependence for the generating functionals of Green’s functions for a general gauge theory with BRST-broken terms in arbitrary gauges (using a suitable regularization scheme); we also formulate the main result of this study. An application of the general results to the functional renormalization group approach to the Yang–Mills and gravity theories is considered in Section 4. In Section 5, we examine different choices for gauged BRST transformations in order to find the form of the Gribov–Zwanziger action and of the refined Gribov–Zwanziger action in a many-parameter family of gauges including the Coulomb, axial, Landau and covariant gauges, starting from the Landau gauge. Finally, in Section 6 we discuss some issues and perspectives related to the suggested procedure. In Appendix A we analyze the existence of a solution to a non-linear functional equation for an unknown field-antifield-dependent odd-valued parameter $\Lambda$, which establishes the coincidence of the vacuum functionals in different gauges.

We use the condensed notation of DeWitt [33] and our previous notation [22], [23], [15]. Derivatives with respect to sources and antifields are taken from the left, while those with respect to fields are taken from the right. Left derivatives with respect to fields are labelled by the subscript “$l$”. The Grassmann parity of a quantity $A$ is denoted by $\varepsilon(A)$.

2 Gauged BRST Symmetry Transformations

In this section, we recall the basic notions and properties of the field-antifield formalism for general gauge theories. We also introduce gauged (field-dependent) BRST transformations and calculate the Jacobian for the change of variables determined by these transformations.
2.1 Overview of Field-antifield Formalism

As the initial point of our study, we consider a theory of gauge fields, \( A^i, \ i = 1, 2, \ldots, n \), with \( \varepsilon(A^i) = \varepsilon_i \), determined by a classical action, \( S_0 = S_0(A) \), invariant under infinitesimal gauge transformations \( \delta A^i = R^i_{\alpha_0}(A)\xi^{\alpha_0} \) for \( \alpha_0 = 1, 2, \ldots, m_0 \), implying the Noether identities

\[
S_{0,i}(A)R^i_{\alpha_0}(A) = 0, \quad 0 < m_0 < n. \tag{2.1}
\]

Here, the gauge transformations are parameterized by \( m_0 \) arbitrary (usually supposed to be small) functions, \( \xi^{\alpha_0} \), of the space-time coordinates, with \( \varepsilon(\xi^{\alpha_0}) = \varepsilon_{\alpha_0} \), whereas \( S_{0,i} \equiv \delta S_0/\delta A^i \), while \( R^i_{\alpha_0}(A) \) are the generators of the gauge transformations, with \( \varepsilon(R^i_{\alpha_0}) = \varepsilon_i + \varepsilon_{\alpha_0} \).

The generators may be dependent in the case \( \text{rank} \| R^i_{\alpha_0} \|_{S_{0,i}=0} = m < m_0 \), implying the presence of zero eigenvectors, \( Z^\alpha_{\alpha_0}(A) \), \( \alpha = 1, \ldots, m_1 \), for the generators on the mass shell \( S_{0,i} = 0 \), thus determining a reducible gauge theory, so that in the case \( \text{rank} \| Z^\alpha_{\alpha_0} \|_{S_{0,i}=0} < m_1 \) the eigenvectors should be dependent as well. Thus, an \( L \)-th-stage reducible gauge theory of the fields \( A^i \) is determined by the relations

\[
Z^\alpha_s \equiv (A) Z^\alpha_{s+1} (A) = S_{0,i}(A)K^\alpha_{s+1}(A), \text{ for } \alpha_{s+1} = 1, \ldots, m_{s+1}, s = 0, \ldots, L - 1, \tag{2.2}
\]

and

\[
\text{rank} \| Z^\alpha_{s+1} \|_{S_{0,i}=0} = m_s, \quad \text{rank} \| Z^\alpha_{s+1} \|_{S_{0,i}=0} = m_{L}, \tag{2.3}
\]

where \( \varepsilon(K^\alpha_{s+1}) = \varepsilon_s + \varepsilon_{s+1}, \quad \varepsilon(R^i_{\alpha_0}) = \varepsilon_i + \varepsilon_{\alpha_0} \). \tag{2.4}

The total configuration space \( \mathcal{M} \) of all the fields \( \{ \Phi^A \} \) in the BV method depends on the irreducible \([25]\) or reducible \([26]\) nature of a given classical gauge theory. In the case of an \( L \)-th stage reducible theory, \( \mathcal{M} \) is parameterized by the fields

\[
\Phi \equiv \{ \Phi^A \} = \{ A^i, C^\alpha_s, C^\alpha_{s'}, B^\alpha_{s'} \}, \quad s = 0, \ldots, L, \ s' = 0, \ldots, s, \tag{2.5}
\]

with \( \varepsilon(C^\alpha_s, C^\alpha_{s'}, B^\alpha_{s'}) = (\varepsilon + s + 1, \varepsilon + s + 1, \varepsilon_s + s) \), \( \varepsilon(\Phi^A) = \varepsilon_A \) and the following ghost number distribution:

\[
gh(A^i, C^\alpha_s, C^\alpha_{s'}, B^\alpha_{s'}) = (0, s + 1, 2s' - s - 1, 2s' - s),
\]

which obeys an additive composition law when calculated on monomials. Here, the respective classical, minimal-ghost, antighost, extra-ghost and Nakanishi–Lautrup fields are explicitly indicated in the BV method. For \( L = 0 \), the gauge theory is irreducible, with \( C^\alpha_0, C^\alpha_0 \equiv C^\alpha_0, B^\alpha_0 \equiv B^\alpha_0 \) being the ghost, antighost and Nakanishi–Lautrup fields.

The BV method demands the introduction of an odd cotangent bundle \( \Pi T^* \mathcal{M} \equiv T^{*(0,1)} \mathcal{M} \), usually known as the field-antifield space (for a more involved geometry, based on the field-antifield formalism, see, e.g., Refs. \([34, 35, 36, 37, 38, 39, 40]\), where each field \( \Phi^A \) in \( \mathcal{M} \) has a corresponding antifield \( \Phi_A^* \equiv \Phi_A^* \),

\[
\{ \Phi_A^* \} = \{ A_A^*, C_{A_s}^*, C_{A_{s'}}^*, B_{A_{s'}}^* \}, \quad \text{with } (\varepsilon, gh)(\Phi_A^*) = (\varepsilon_A + 1, -1 - gh(\Phi^A)). \tag{2.6}
\]
In the total field-antifield space \( \{ \Phi_A, \Phi^*_A \} \), one defines a bosonic functional, \( \bar{S} = \bar{S}(\Phi, \Phi^*) \), being a special extension of the classical action to \( \Pi T^*M \) with the boundary condition of vanishing antifields \( \Phi^*_A \) and Planck constant, \( \bar{S}(\Phi, 0)|_{\hbar = 0} = S_0 \), with \( gh(S) = 0 \), encoding the gauge algebra functions and satisfying a quantum master equation (within the class of gauge-invariant regularizations, with \( \Delta \bar{S} \sim \delta(0) \neq 0 \) for a local \( \bar{S} \)) in two equivalent forms:

\[
\Delta \exp \left\{ \frac{i}{\hbar} \bar{S} \right\} = 0 \iff \frac{1}{2}(\bar{S}, \bar{S}) = i\hbar \Delta \bar{S}.
\] (2.7)

These equations are written in terms of a natural (in \( \Pi T^*M \)) odd Poisson bracket, \( (\cdot, \cdot) \), (known as the antibracket) and a nilpotent odd Laplacian, \( \Delta \),

\[
(\cdot, \cdot) = \frac{\delta \cdot}{\delta \Phi^*_A} \frac{\delta \cdot}{\delta \Phi^*_A} - \frac{\delta_r \cdot}{\delta \Phi^*_A} \frac{\delta_l \cdot}{\delta \Phi^*_A}, \quad \Delta = (-1)^{\varepsilon_A} \frac{\delta}{\delta \Phi^*_A} \frac{\delta}{\delta \Phi^*_A}.
\] (2.8)

We assume that formal manipulations with \( \Delta \) are supported by a suitable regularization scheme. This is a nontrivial requirement, since \( \Delta \) is not well-defined on local functionals because for any local functional \( F \) one finds that \( \Delta F \sim \delta(0) \). The standard way to solve this problem is to use a regularization similar to the dimensional one \([42]\), when \( \delta(0) = 0 \). In this paper, just as in \([23]\), we consider a more general class of regularizations.

The quantum action is constructed as a special representative from the set of solutions to the master equation (2.7) and is described by the transformation

\[
\exp \left\{ \frac{i}{\hbar} S_X \right\} = \exp \{ -[\Delta, X] \} \exp \left\{ \frac{i}{\hbar} \bar{S} \right\}, \quad \text{for } \varepsilon(X) = 1, \; gh(X) = -1,
\] (2.9)

with the supercommutator \([ \cdot , \cdot ]\) and some functional \( X = X(\Phi, \Phi^*) \), whose form controls the choice of a Lagrangian surface in \( \Pi T^*M \), on which the restriction of the Hessian for \( S_X \) should be non-degenerate. Choosing \( X = \Psi(\Phi) \) as the gauge fermion (e.g., \( \Psi(\Phi) = \bar{C}^a \chi_a(A, B) \) for irreducible theories with an admissible gauge \( \chi_a(A, B) = 0 \)), one makes the quantum action \( S_\Psi \) non-degenerate in the configuration space \( M \),

\[
\exp \left\{ \frac{i}{\hbar} S_\Psi \right\} = \exp \left\{ \frac{\delta \Psi}{\delta \Phi^*_A} \frac{\delta}{\delta \Phi^*_A} \right\} \exp \left\{ \frac{i}{\hbar} \bar{S} \right\} \iff S_\Psi(\Phi, \Phi^*) = \bar{S}(\Phi, \Phi^* + \frac{\delta \Psi}{\delta \Phi^*_A}).
\] (2.10)

By construction, the action \( S_\Psi \) satisfies the master equation (2.7) due to the supercommutativity of the operators \( \exp \{ -[\Delta, \Psi] \} \) and \( \Delta \), namely,

\[
[\Delta, \exp \{ -[\Delta, \Psi] \}] = 0 \implies \Delta \exp \left\{ \frac{i}{\hbar} S_\Psi \right\} = 0,
\] (2.11)

and is used to construct the path integral and the generating functionals of Green’s functions in the field-antifield formalism \([25], [26]\). The generating functionals of the usual, \( Z = Z(J, \Phi^*) \),
and connected, $W = W(J, \Phi^*)$, Green functions extended by external [those which do not enter the integration measure in (2.12)] antifields in the BV formalism \cite{25}, \cite{26} can be presented as

$$\exp \left\{ \frac{i}{\hbar} W \right\} = Z = \int D\Phi \exp \left\{ \frac{i}{\hbar} (S_\Psi(\Phi, \Phi^*) + J_A \Phi^A) \right\},$$

(2.12)

with sources $J_A$ ($\varepsilon(J_A) = \varepsilon_A$), whereas the effective action $\Gamma = \Gamma(\Phi, \Phi^*)$ is determined by the Legendre transformation of $W$ with respect to $J_A$,

$$\Gamma(\Phi, \Phi^*) = W(J, \Phi^*) - J_A \Phi^A, \quad \text{with} \quad \Phi^A = \frac{\delta W}{\delta J_A}, \quad \frac{\delta \Gamma}{\delta \Phi^A} = - J_A.$$

(2.13)

The standard properties of the above generating functionals are inherited from the gauge invariance of the classical action, transformed into the BRST invariance, being an invariance under global $N = 1$ supersymmetry transformations in the extended configuration space $\mathcal{M}$,

$$\delta_\mu \Phi^A = (\Phi^A, S_\Psi)\mu, \quad \delta_\mu \Phi^*_A = 0,$$

(2.14)

with a constant anticommuting parameter $\mu$.

First, the integrand in Eq. (2.12) for $Z_\Psi \equiv Z(0, \Phi^*)$ is invariant with respect to the transformations (2.14). Second, the vacuum functional $Z_\Psi$ is independent with respect to a variation of the gauge condition, $\Psi \rightarrow \Psi + \delta \Psi$, if one makes in $Z_{\Psi+\delta \Psi}$ the change of variables

$$\Phi^A \rightarrow \Phi'^A = \Phi^A + (\Phi^A, S_\Psi)\mu(\Phi), \quad \text{with} \quad \Phi'^*_A = \Phi^*_A,$$

(2.15)

referred to as field-dependent (i.e., gauged) BRST transformations, now with an arbitrary anticommuting $\mu(\Phi)$, $\mu^2(\Phi) = 0$, being, however, infinitesimal, $\mu(\Phi) = \frac{i}{\hbar} \delta \Psi$. Indeed, in this case we have $Z_{\Psi+\delta \Psi} = Z_\Psi + O(\delta \Psi)$.

The next consequence of the transformations (2.14), based on the equivalence theorem \cite{43}, is the presence of the Ward identities for $Z, W, \Gamma$, namely,

$$J_A \frac{\delta Z}{\delta \Phi^*_A} = 0, \quad J_A \frac{\delta W}{\delta \Phi^*_A} = 0, \quad (\Gamma, \Gamma) = 0.$$  

(2.16)

Finally, the study of gauge dependence for the generating functionals of Green’s functions $Z, W, \Gamma$ leads to the following variations \cite{21, 22, 23} under the change of the gauge condition $\Psi \rightarrow \Psi + \delta \Psi$:

$$\delta Z(J, \Phi^*) = \frac{i}{\hbar} J_A \frac{\delta}{\delta \Phi^*_A} \delta \Psi \left( \frac{\delta}{\delta \Phi^*_A} \right) Z(J, \Phi^*),$$

(2.17)

$$\delta W(J, \Phi^*) = J_A \frac{\delta}{\delta \Phi^*_A} \delta \Psi \left( \frac{\delta W}{\delta \Phi^*_A} + \frac{i}{\hbar} \frac{\delta}{\delta \Phi^*_A} \right),$$

(2.18)

$$\delta \Gamma(\Phi, \Phi^*) = - (\Gamma, \langle \delta \Psi \rangle) \quad \text{for} \quad \langle \delta \Psi \rangle = \delta \Psi(\widetilde{\Phi}) \cdot 1, \quad \Phi^A = \Phi^A + i\hbar (\Gamma''-1)^{AB} \frac{\delta \Psi}{\delta \Phi^B}.$$

(2.19)

\footnote{Despite the term “gauged”, the parameter $\mu(\Phi)$ should be considered as an odd-valued functional, i.e., not as an arbitrary space-time function, such as the gauge parameter $\xi^{\alpha}$.}
with the matrix \((\Gamma''^{-1})\) being reciprocal to the Hessian \((\Gamma'')\), with the elements

\[
(\Gamma'')_{AB} = \frac{\delta l}{\delta \Phi^A} \left( \frac{\delta \Gamma}{\delta \Phi^B} \right): (\Gamma'')^{-1 AC} (\Gamma'')_{CB} = \delta^A_B .
\]

The above local representation for \(\delta \Gamma\) can be rewritten with the use of differential consequences of the Legendre transformation (2.13) in a non-local form:

\[
\delta \Gamma = \frac{\delta \Gamma}{\delta \Phi^A} \left[ -\frac{\delta}{\delta \Phi^*_A} + (-1)^{\varepsilon_B (\varepsilon_A + 1)} (\Gamma''^{-1})_{BC} \left( \frac{\delta l}{\delta \Phi^C} \frac{\delta \Gamma}{\delta \Phi^*_A} \right) \frac{\delta l}{\delta \Phi^B} \right] \langle \delta \Psi \rangle .
\]

Indeed, in order to derive (2.17) we make the change of variables (2.15) with \(\mu(\Phi) = -\frac{i}{\hbar} \delta \Psi\) in the functional \(Z(J, \Phi^*) \equiv Z_\Psi(J, \Phi^*)\), constructed with respect to the action \(S_\Psi\), then, extracting the functional \(Z_{\Psi+\delta \Psi}(J, \Phi^*)\), we obtain, with accuracy up to the first order in \(\delta \Psi\),

\[
Z_\Psi(J, \Phi^*) = \int D\Phi' \exp \left\{ \frac{i}{\hbar} \left( S_\Psi(\Phi', \Phi'^*) + J_A \Phi'^A \right) \right\}
\]

\[
= \int D\Phi \exp \left\{ \frac{i}{\hbar} \left( S_{\Psi+\delta \Psi}(\Phi, \Phi^*) + J_A \Phi^A \right) \right\} \left( 1 - \left( \frac{i}{\hbar} \right)^2 J_A \frac{\delta S_\Psi}{\delta \Phi^*_A} \delta \Psi(\Phi) \right)
\]

\[
= Z_{\Psi+\delta \Psi}(J, \Phi^*) - \frac{i}{\hbar} J_A \frac{\delta}{\delta \Phi^*_A} \delta \Psi(\frac{\delta}{\delta \Phi^*_A}) Z_\Psi(J, \Phi^*) ,
\]

where the final line has been derived using the differentiation of the functional integral with respect to the sources \(J\) and external antifields \(\Phi^*\).

From the final variations of \(Z, W, \Gamma\), we can see, on the extremals, that for \(Z, W\) with \(J_A = 0\) and equivalently for \(\Gamma\) with \(\frac{\delta l}{\delta \Phi^*_A} = 0\), the corresponding variations given by Eqs. (2.17), (2.18) and (2.21) are vanishing. This result for \(Z, W\) is identical with that for the vacuum functionals \(Z_\Psi, W_\Psi = \frac{\hbar}{i} \ln Z_\Psi\). The same results are valid for renormalizable generating functionals \(Z_R, W_R, \Gamma_R\) with an appropriate gauge-invariant regularization respecting the Ward identities (2.16) and their differential consequences.

Due to Gribov’s [5] and, in general, Singer’s [6] results, we notice that the above-listed BV quantization rules correctly describe physics within the functional integral technique in the perturbative way only for Abelian gauge theories in any gauges and non-Abelian gauge theories in a connected domain of the configuration space where the Faddeev–Popov operator (for continuous gauges with space-time derivatives) for a theory in question has positive eigenvalues.

### 2.2 Gauged (Field-dependent) BRST Symmetry Transformations

Because of the crucial importance of gauged BRST transformations (2.15), we now consider them in detail, assuming that, in general, an infinitesimal value of the odd-valued parameter \(\mu\) can be changed to a finite nilpotent one, \(\Lambda(\Phi, \Phi^*)\), \(\Lambda^2 = 0\), being dependent, as a functional,
on the entire set of fields $\Phi^A$ and antifields $\Phi^*_A$ (however, not on the space-time coordinates in a manifest form) as follows:

$$\Phi'^A = \Phi^A + (\Phi^A, S_\Psi)\Lambda(\Phi, \Phi^*) \implies \delta \Phi^A \equiv S^A_\Psi \Lambda(\Phi, \Phi^*)$$

(2.23)

The corresponding extended (due to the antifields) Slavnov variation, $s_e F(\Phi, \Phi^*) = \frac{\delta F}{\delta \Phi^*} S^A_\Psi$, of an arbitrary functional $F(\Phi, \Phi^*)$ generally fails to be nilpotent,

$$s^2 e F(\Phi, \Phi^*) = \frac{\delta F}{\delta \Phi^*} S^A_\Psi \lambda_{\Psi, B} = \frac{\delta F}{\delta \Phi^*} (S^A_{\Psi, B} S^A_B - \frac{i}{\hbar} \Delta S^A_\Psi) (1)_{E_A} \neq 0,$$

(2.24)

even for a local action functional, when $\Delta S^A_\Psi \sim \delta(0)^4$. In spite of the result (2.24), i.e., that $(s_e)^2 \neq 0$, the observation that for any constant odd scalar parameters $\Lambda_1, \Lambda_2$ with $gh(\Lambda_1) = gh(\Lambda_2)$ there exists a real number $a$ such that $\Lambda_2 = a \Lambda_1$ implies that the right transformations $Gg(\Lambda) = (1 + s_e G\Lambda)$ acting on any functional $G = G(\Phi, \Phi^*)$ form an Abelian one-parametric supergroup, since $g(\Lambda_1)g(\Lambda_2) = g(\Lambda_1 + \Lambda_2)$ for any odd $\Lambda_i, i = 1, 2$, due to the fact that $\Lambda_1 \cdot \Lambda_2 = a \Lambda_1^2 = 0$.

The usual Slavnov variation $s F(\Phi)$ acting on a functional in the configuration space $F(\Phi) = F(\Phi, 0)$ and determined at the classical level (in the tree approximation) for $S_\Psi = \sum_{k \geq 0} S^{(k)}_\Psi$ is not nilpotent, compared to first-rank gauge theories, including the Yang–Mills theory [1],

$$s F(\Phi) = \frac{\delta F}{\delta \Phi^*} S^{(0)}_\Psi(\Phi, 0), \quad s F(\Phi) = s_e F(\Phi, \Phi^*)\big|_{\Phi^* = 0},$$

(2.25)

$$s^2 F(\Phi) = \frac{\delta F}{\delta \Phi^*} S^{(0)}_\Psi \lambda_{\Psi, B} S^{(0)}_\Psi^{B} \big|_{\Phi^* = 0} = \frac{\delta F}{\delta \Phi^*} S^{(0)}_\Psi \lambda_{\Psi, B} S^{(0)}_\Psi^{AB} (1)_{E_A} \big|_{\Phi^* = 0} \neq 0.$$  

(2.26)

This is explained by an open algebra, described here by the terms $S^{(0)}_\Psi^{AB} \big|_{\Phi^* = 0}$, which emerges in the general Lie bracket for the generators of gauge transformations, $R^i_{\alpha_0}(A),$

$$R^i_{\alpha_0, \beta_0}(A) R^j_{\alpha_1}(A) - R^i_{\alpha_0, \beta_0}(A) R^j_{\alpha_1}(A) = - R^i_{\alpha_0}(A) F^{\gamma_0}_{\alpha_0, \beta_0}(A) + S_{\alpha_1, \beta_1}(A) M^{ij}_{\alpha_0, \beta_0}(A),$$

(2.27)

in the form of the coefficients $M^{ij}_{\alpha_0, \beta_0}(A)$ at the extremals, which, together with the functions $F^{\gamma_0}_{\alpha_0, \beta_0}(A)$, satisfy the properties of generalized antisymmetry

$$[F^{\gamma_0}_{\alpha_0, \beta_0}, M^{ij}_{\alpha_0, \beta_0}] = - (-1)^{\varepsilon_{\alpha_0} \varepsilon_{\beta_0}} [F^{\gamma_0}_{\beta_0, \alpha_0}, M^{ij}_{\beta_0, \alpha_0}], \quad M^{ij}_{\alpha_0, \beta_0} = - (-1)^{\varepsilon_i \varepsilon_j} M^{ji}_{\alpha_0, \beta_0}. $$

Let us now calculate the Jacobian of the change of variables generated by the finite gauged BRST transformations (2.23), namely,

$$\text{Sdet} \left| \frac{\delta \Phi'^A}{\delta \Phi^B} \right| = \exp \left\{ \text{Str} \ln \left( \delta^A_B + \frac{\delta (S^A_\Psi \Lambda)}{\delta \Phi^B} \right) \right\} = \exp \left\{ - \text{Str} \sum_{n=1} \frac{(-1)^n}{n} \left( \frac{\delta (S^A_\Psi \Lambda)}{\delta \Phi^B} \right)^n \right\},$$

(2.28)

The fact that the odd operator $s_e$ is not nilpotent implies that one cannot restore a finite BRST flow (transformations) in $\Pi T^* M$ following the Frobenius theorem, because the odd-valued vector field $s_e (\Phi, \Phi^*) = \frac{\delta}{\delta \Phi^A} (s_e \Phi^A)$ does not have to be nilpotent.
where
\[ \text{Str}\left( \frac{\delta(S_A^B)}{\delta \Phi^B} \right)^n = \frac{\delta(S_A^B)}{\delta \Phi^B_1} \frac{\delta(S_B^A)}{\delta \Phi^B_2} \ldots \frac{\delta(S_{A}^{B_{n-1}})}{\delta \Phi^A} (-1)^{\varepsilon_A}. \] (2.29)

Explicitly, the supermatrix \( (S_A^B)_{B} \) in (2.29) can be presented as the sum of two terms:

\[ (S_A^B)_{B} = S_A^{B_1} \Lambda(-1)^{\varepsilon_B} + S_A^A \Lambda, B \equiv P_A^B + Q_B^A, \quad \text{for } \Lambda_B \equiv \Lambda, B, \] (2.30)

such that only the first supermatrix is nilpotent, \( S_A^{B_1} \Lambda S_{B_1}^A \Lambda = P_A^B P_B^C = 0 \), and furthermore

\[ P_A^B Q_B^C \ldots Q_{C_k}^{C_{k-1}} P_{D_k}^{C_k} Q_{D_{l-1}}^{D_{l-1}} = 0, \] (2.31)

for any natural numbers \( k, l \).

Using the property of supercommutativity for arbitrary even supermatrices \( F, G \) under the symbol “\( \text{Str} \)”, \( \text{Str}(FG) = \text{Str}(GF) \), we obtain from Eqs. (2.29), (2.30) the representation

\[ \text{Str}\left( P_A^B + Q_B^A \right)^n = \sum_{k=n-1}^{n} C_n^k \text{Str}\left( (P^{n-k})^A_B (Q^k)^B_A \right) = n \text{Str} (P_A^B (Q^{n-1})_B^C) + \text{Str} (Q^n)_B^A, \] (2.32)

with the number of combinations being \( C_n^k = \frac{n!}{k!(n-k)!} \).

Consequently, we have

\[
\begin{align*}
- \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \text{Str}\left( \frac{\delta(S_A^B)}{\delta \Phi^B} \right)^n & = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (\Lambda_C S_A^C)^n \\
- \sum_{n=2}^{\infty} (-1)^n (\Lambda_C S_A^C)^{n-2} \Lambda_A S_A^B S_B^A \Lambda + S_A^A \Lambda & = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (s_e \Lambda)^n - \sum_{n=2}^{\infty} (-1)^n (s_e \Lambda)^{n-2} \Lambda_A (s_e S_A^A) \Lambda + (\Delta S_\Lambda) \Lambda \\
& = - \ln (1 + s_e \Lambda) - \sum_{n=1}^{\infty} (-1)^{n-1} (s_e \Lambda)^{n-1} \Lambda_A (s_e S_A^A) \Lambda + (\Delta S_\Lambda) \Lambda \\
& = - \ln (1 + s_e \Lambda) - (1 + s_e \Lambda)^{-1} \Lambda_A (s_e S_A^A) \Lambda + (\Delta S_\Lambda) \Lambda. \quad (2.33)
\end{align*}
\]

As a result, we obtain the Jacobian for general gauged (field-dependent) BRST transformations:

\[ \text{Sdet} \left\| \frac{\delta \Phi^A}{\delta \Phi} \right\| = (1 + s_e \Lambda)^{-1} \exp \left\{ - (1 + s_e \Lambda)^{-1} \Lambda_A (s_e S_A^A) \Lambda + (\Delta S_\Lambda) \Lambda \right\} \]

\[ = (1 + s_e \Lambda)^{-1} \left\{ 1 - (1 + s_e \Lambda)^{-1} \Lambda_A (s_e S_A^A) \Lambda + (\Delta S_\Lambda) \Lambda \right\} \]

\[ = (1 + s_e \Lambda)^{-1} \left\{ 1 + \left[ s_e \Lambda \right] \right\} \left\{ 1 + (\Delta S_\Lambda) \Lambda \right\}. \] (2.34)

where \( G_{s_e} \equiv s_e G \), for any \( G = G(\Phi, \Phi^* \) and account has been taken of the identity \( \exp \{-a\Lambda\} = 1 - a\Lambda \), in view of \( \Lambda^2 = 0 \).

\textsuperscript{5}Let us emphasize that the superdeterminant (2.28) for vanishing antifields \( \Phi^* \) was calculated also in \[15\]:
Formula (2.34) is a natural extension of the result [28], obtained for Yang–Mills theories, for which \( s_e \) coincides with \( s \),

\[
(\Delta S_\Psi = 0, \quad S^A_\Psi S^B_\Psi = s^2 = 0) \implies \text{Sdet} \left[ \frac{\delta \Phi^A}{\delta \Phi^B} \right] = (1 + s\Lambda)^{-1},
\]

and which has also been taken into account as regards the independence of the generators \( S^A_\Psi(\Phi) \) of BRST transformations on the antifields \( \Phi^*_A \), whereas the odd-valued functional \( \Lambda \) should now be regarded as a field-dependent one, \( \Lambda = \Lambda(\Phi) \). The representation (2.35) is now valid for a gauge theory of rank 1 with a closed algebra, and with the additional requirement for the generators \( S^A_\Psi \) to be divergentless: \( \Delta S_\Psi = S^A_\Psi = 0 \).

For the functional integral

\[
G(\Phi^*) = \int D\Phi \exp \left\{ \frac{i}{\hbar} Q(\Phi, \Phi^*) \right\},
\]

the change of variables (2.23) leads to the representation

\[
G(\Phi^*) = \int D\Phi' \exp \left\{ \frac{1}{\hbar} Q(\Phi', \Phi^*) \right\} = \int D\Phi \exp \left\{ \frac{i}{\hbar} \left( Q(\Phi, \Phi^*) + s_e\Lambda(\Phi, \Phi^*) - i\hbar (\Delta S_\Psi)\Lambda - i\hbar \ln \left[ (1 + s_e\Lambda)^{-1}(1 + \frac{s_e}{s_e}\Lambda) \right] \right) \right\},
\]

which is different from the similar result [28] for the Yang–Mills theory due to the terms \( (1 + \frac{s_e}{s_e}\Lambda) \) in the final line and the presence of \( i\hbar (\Delta S_\Psi)\Lambda \).

A repeated application of gauged BRST transformations with the same gauged parameter \( \Lambda \) is not nilpotent due to the equality

\[
\delta_\Lambda (\delta_\Lambda F(\Phi, \Phi^*)) = \delta_\Lambda (s_e F(\Phi, \Phi^*)\Lambda) = -s_e^2 F(\Phi, \Phi^*)\Lambda^2 + s_e F(\Phi, \Phi^*)s_e(\Lambda)\Lambda,
\]

and due to the vanishing commutator \([\delta_{\mu_1}, \delta_{\mu_2}]F = 2(s_e^2 F)\mu_1\mu_2 = 0\) of global BRST transformations with constant parameters \( \mu_1, \mu_2 \), such that \( \mu_1 = a \cdot \mu_2 \), as shown by Eq. (2.24) for \( s_e^2 F \neq 0 \) and the subsequent relations. Of course, for a constant \( \Lambda \) (i.e., \( \Lambda = \mu \)) the nilpotency in however, any explicit calculation is absent. At the same time, the first version of the present work appeared as arXiv:1312.2092v1[hep-th] earlier than the above paper [44], which appeared as arXiv:1312.2802v1[hep-th]. Note that in the first version of [44] this superdeterminant was calculated in (3.10) incorrectly. A correct calculation algorithm, including a representation of the superdeterminant as a series in (2.33) in arXiv:1312.2092v1[hep-th]. A small error in the representation of the second series in the 3rd line, which appeared starting from the 4th line, was corrected in arXiv:1312.2092v3[hep-th]. Turning to [45], we have to pay attention to the fact that the introduction of “finite supersymmetric two-parametric transformations” linear in the anticommuting parameters \( \xi_a, a = 1, 2 \) in (2.3), (2.4) leads to the fact that the set of field-theoretic models which should be invariant under such transformations (2.3), (2.4) is empty because a realistic gauge field theory (e.g., Yang–Mills theories) should be invariant with respect to polynomial (in \( \xi_a \)) transformations, instead of (2.3); see [64] for details.
Eq. (2.37) is restored; however, as compared with the Yang–Mills theory we have the standard expression for the Jacobian:

\[ \text{Sdet} \left| \frac{\delta \Phi' A}{\delta \Phi B} \right| = \exp\{ \Delta S_{\Psi} \mu \}. \]

Let us now examine the generating functionals \( Z_\Psi(J, \Phi^*) \), \( Z_{\Psi+\Delta\Psi}(J, \Phi^*) \) in Eq. (2.22) for the same gauge theory, however, given by different (not necessarily related to each other by small variations) gauges described by the gauge fermions \( \Psi(\Phi) \), \( [\Psi + \Delta\Psi](\Phi) \), which differ by a Grassmann-odd functional, \( \Delta\Psi(\Phi) \), subject to the conditions \( (\varepsilon, gh)(\Delta\Psi(\Phi) = (1, -1) \).

After the change of variables (2.23), due to the quantum master equation (2.11) for the action \( S_{\Psi} \), we obtain the generating functional \( Z_\Psi(J, \Phi^*) \), namely,

\[ Z_\Psi(J, \Phi^*) = \int D\Phi \exp \left\{ \frac{i}{\hbar} \left( S_{\Psi} + i\hbar \ln \left( 1 + s_e \Lambda \right) \right) + i\hbar \left( 1 + s_e \Lambda \right)^{-1} \Lambda_A (s_e S_{\Psi}^A) \Lambda + J_A (\Phi^A + \delta \Phi^A) \right\}. \]

In its turn, \( Z_{\Psi+\Delta\Psi}(J, \Phi^*) \) corresponding to a finite change of the gauge fermion takes the form

\[ Z_{\Psi+\Delta\Psi}(J, \Phi^*) = \int D\Phi \exp \left\{ \frac{i}{\hbar} \left( S_{\Psi}(\Phi, \Phi^*) + s_e (\overline{\Delta\Psi}(\Phi)) + \sum_{n \geq 2} \frac{1}{n!} \overline{\Delta\Psi}_{A_1} \cdots \overline{\Delta\Psi}_{A_n} S_{\Psi}^{A_n \cdots A_1}(\Phi, \Phi^*) + J_A \Phi^A \right) \right\}. \]

Consider a functional equation for an unknown odd-valued functional, \( \Lambda \), following the requirement of coincidence of the above representations (2.38) and (2.39) for \( J_A = 0 \): \( Z_{\Psi+\Delta\Psi}(0, \Phi^*) = Z_\Psi(0, \Phi^*) \)

\[ i\hbar \left\{ \ln \left( 1 + s_e \Lambda \right) + (1 + s_e \Lambda)^{-1} \Lambda_A (s_e S_{\Psi}^A) \Lambda \right\} = \sum_{n \geq 1} \frac{1}{n!} \overline{\Delta\Psi}_{A_1} \cdots \overline{\Delta\Psi}_{A_n} S_{\Psi}^{A_n \cdots A_1}(\Phi, \Phi^*) \]

\[ \iff -i\hbar \ln \left( (1 + s_e \Lambda)^{-1} (1 + \frac{\Lambda}{s_e} \Lambda) \right) = \left( \exp \left\{ - [\Delta, \overline{\Delta\Psi}] \right\} - 1 \right) S_{\Psi}. \]

Having in mind the fact that for an infinitesimal \( \overline{\Delta\Psi} = \delta\Psi \), with accuracy up to the first order in \( \delta\Psi \) from Eq. (2.40), we have a linearized (with respect to \( \Lambda \) and \( \Lambda_A \)) and easily solved equation,

\[ i\hbar s_e \Lambda = s_e \delta\Psi(\Phi) \implies \Lambda = -\frac{i}{\hbar} \delta\Psi \text{ and } \Lambda = \Lambda(\Phi). \]

This fact has been used to verify the gauge-independence property \( Z_\Psi = Z_{\Psi+\delta\Psi} \) for the vacuum functional \( Z_\Psi \) in Section 2.1.

Therefore, we hope that the highly non-linear equation (2.40), which provides a compensation for a finite change of the gauge Fermion in \( Z_\Psi \) by means of the Jacobian for the change
of variables generated by the gauged BRST transformations (2.23), also has a solution, which should be of the form

$$\Lambda(\Phi, \Phi^*|\Delta \Psi) = \Lambda(\Delta \Psi).$$

(2.42)

We analyze a justification of this representation in Appendix A.

Using the above result, we argue that for any finite change of the gauge $\Delta \Psi$ there exists a gauged (field-dependent) BRST transformation (2.15) with an odd-valued functional $\Lambda(\Delta \Psi)$ in (2.42) such that, due to the equivalence theorem [43, 44], there is a coincidence of the two representations (2.38) and (2.39), which is also valid for the vacuum functional:

$$Z_{\Psi+\Delta \Psi}(0, \Phi^*) = Z_{\Psi}(0, \Phi^*).$$

(2.43)

This is the main result of this section, which we use in the study of the gauge-(in)dependence problem for a theory with BRST symmetry breaking terms.

3 Gauge Dependence for Generating Functionals with Broken BRST Symmetry

Let us turn to the problem of gauge dependence for a gauge theory determined by Eqs. (2.1), (2.2), (2.4) with a quantum action $S_{\Psi}(\Phi, \Phi^*)$ additively extended along the lines of our previous study [22], [23] by a soft BRST breaking term $M(\Phi, \Phi^*)$ defined in a gauge $\Psi(\Phi)$ up to an action $S(\Phi, \Phi^*)$ determining the generating functional of Green’s functions, $Z_{M}(J, \Phi^*)$,

$$S = S_{\Psi} + M, \quad Z_{M}(J, \Phi^*) = \int D\Phi \ exp\left\{\frac{i}{\hbar} (S(\Phi, \Phi^*) + J_{A} \Phi^{A})\right\},$$

(3.1)

with the boundary condition

$$Z_{M}(J, \Phi^*) \big|_{M=0} = Z(J, \Phi^*).$$

We remind that, at the classical level, since we assume the bosonic functional $M(\Phi, \Phi^*)$ to have a regular decomposition in powers of $\hbar$, $M(\Phi, \Phi^*) = \sum_{n \geq 0} \hbar^{n} M_{n}(\Phi, \Phi^*)$, the condition of a soft breaking of BRST symmetry implies

$$(M_{0}, M_{0}) = 0 \text{ and } m_{0; i} R_{i0}^{\ell} \neq 0, \text{ for } m_{0}(\Phi) = M_{0}(\Phi, \Phi^*) \big|_{\Phi^*=0},$$

(3.2)

whereas in the case of a regularization more general than dimensional-like ones, the total generating equation for $M(\Phi, \Phi^*)$ reads [23]

$$\Delta \left\{ -\frac{i}{\hbar} M \right\} = 0 \iff \frac{1}{2} (M, M) = -i\hbar \Delta M.$$

(3.3)

6One may examine a more general BRST symmetry breaking functional, not satisfying Eq. (3.3) or Eq. (3.2), without changing the results for the dependence of the effective action (as will be seen later); however, we will follow the study of [22], [23], because the solution of these equations restricts the rank condition for the Hessian of $M$ to be no greater than $\dim M$, and to be such that the functional integral in (3.1) is well-defined.
As a consequence of Eqs. (2.11), (3.3), the total action now satisfies
\[ \frac{1}{2}(S, S) - i\hbar \Delta S = (S, M), \] (3.4)
so that, in the classical limit for \( S = S_0 + O(\hbar) \), Eq. (3.4) implies the equation
\[ \frac{1}{2}(S_0, S_0) = (S_0, M_0). \] (3.5)

The properties of the generating functionals of the usual, \( Z_M(J, \Phi^*) \), connected, \( W_M(J, \Phi^*) \), \((W_M = \frac{\hbar}{2} \ln Z_M)\) and vertex, \( \Gamma_M(\Phi, \Phi^*) \), Green functions, introduced via the Legendre transformation of \( W_M(J, \Phi^*) \) with respect to the sources \( J_A \),
\[ \Gamma_M(\Phi, \Phi^*) = W_M(J, \Phi^*) - J_A \Phi^A, \quad \Phi^A = \frac{\delta W_M(J, \Phi^*)}{\delta J_A}. \] (3.6)
have been studied in [22], [23]. These properties include the Ward identities and the calculation of variations of all the generating functionals under a variation of the gauge condition (Grassmann-odd functional), \( \Psi(\Phi) \rightarrow \Psi(\Phi) + \delta \Psi(\Phi) \). The properties were derived on the basis of functional averaging of the master equations (2.11) for \( S_\Psi \) in a dimensional-like regularization, as applied to the local functional \( S \) [22], and in more general regularizations [23].

These properties can only be obtained by means of global BRST and field-dependent (gauged) BRST transformations. There follow the Ward identities for \( W_M(J, \Phi^*) \), connected, and vertex, \( \Gamma_M(\Phi, \Phi^*) \), Green functions, introduced via the Legendre transformation of \( W_M(J, \Phi^*) \) with respect to the sources \( J_A \),
\[ \Gamma_M(\Phi, \Phi^*) = W_M(J, \Phi^*) - J_A \Phi^A, \quad \Phi^A = \frac{\delta W_M(J, \Phi^*)}{\delta J_A}. \] (3.6)
where the notation
\[ M_A(\frac{\hbar}{2} \frac{\delta}{\delta J}, \Phi^*) \equiv \frac{\delta M(\Phi, \Phi^*)}{\delta \Phi^A} \bigg|_{\Phi \rightarrow \frac{\Phi}{\delta J}} \quad \text{and} \quad M^{A*}(\frac{\hbar}{2} \frac{\delta}{\delta J}, \Phi^*) \equiv \frac{\delta M(\Phi, \Phi^*)}{\delta \Phi^{A*}} \bigg|_{\Phi \rightarrow \frac{\Phi}{\delta J}} \] (3.8)
has been used. In case \( M = 0 \), identity (3.7) is reduced to the usual Ward identity (2.16) for \( Z(J, \Phi^*) \), as well as to the Ward identities for \( W_M(J, \Phi^*) \), \( \Gamma_M(\Phi, \Phi^*) \), which follow from (3.7),
\[ \left( J_A + M_A(\frac{\hbar}{2} \frac{\delta}{\delta J}, \Phi^*) \right) \left( \frac{\delta W_M(\Phi, \Phi^*)}{\delta \Phi^A} - M^{A*}(\frac{\hbar}{2} \frac{\delta}{\delta J}, \Phi^*) \right) = 0, \] (3.9)
\[ \frac{1}{2}(\Gamma_M, \Gamma_M) = \frac{\delta \Gamma_M}{\delta \Phi^A} \tilde{M}^{A*} + \tilde{M}^A \frac{\delta \Gamma_M}{\delta \Phi^{A*}} - \tilde{M}^A \tilde{M}^{A*}. \] (3.10)
Here, we have used a notation introduced in [22]:
\[ \tilde{M}^A \equiv \frac{\delta M(\Phi, \Phi^*)}{\delta \Phi^A} \bigg|_{\Phi \rightarrow \tilde{\Phi}} \quad \text{and} \quad \tilde{M}^{A*} \equiv \frac{\delta M(\Phi, \Phi^*)}{\delta \Phi^{A*}} \bigg|_{\Phi \rightarrow \tilde{\Phi}}, \] (3.11)
with account taken for the conventions (2.19), (2.20), adapted to the case of broken BRST symmetry, i.e., according to the change $\Gamma \rightarrow \Gamma M$. For completeness, note that the functional $\Gamma M$ satisfies the functional integro-differential equation

$$\exp \left\{ \frac{i}{\hbar} \Gamma_M(\Phi, \Phi^*) \right\} = \int d\varphi \exp \left\{ \frac{i}{\hbar} \left[ S_\Psi(\Phi + \hbar^{\frac{1}{2}} \varphi, \Phi^*) + M(\Phi + \hbar^{\frac{1}{2}} \varphi, \Phi^*) \right] - \frac{\delta \Gamma_M(\Phi, \Phi^*)}{\delta \Phi} \hbar^{\frac{1}{2}} \varphi \right\},$$

(3.12)
determining the loop expansion $\Gamma_M = \sum_{n \geq 0} \hbar^n \Gamma_n M$. Thus, the tree-level (zero-loop) and one-loop approximations of (3.12) correspond to

$$\Gamma_0 M(\Phi, \Phi^*) = S_{\Psi 0}(\Phi, \Phi^*) + M_0(\Phi, \Phi^*),$$

(3.13)

$$\Gamma_1 M(\Phi, \Phi^*) = S_{\Psi 1}(\Phi, \Phi^*) + M_1(\Phi, \Phi^*) - \frac{i}{2} \ln \text{Sdet} \left\| (S''_0)_{AB}(\Phi, \Phi^*) \right\|,$$

(3.14)

so that the tree-level part of the Ward identity (3.10) for $\Gamma_0 M$

$$\frac{1}{2} (\Gamma_0 M, \Gamma_0 M) = \frac{\delta S_{\Psi 0}}{\delta \Phi^A} M_0 A^* + M_0 A \frac{\delta S_{\Psi 0}}{\delta \Phi^*_A} + M_0 A M_0 A^*$$

is fulfilled identically, due to the tree-level approximation to the generating equations (2.11) for $S_{\Psi 0}$ and (3.2) for $M_0$.

In order to study the gauge-dependence problem, we examine, first of all, the representation for $Z_M(J, \Phi^*)$ within the gauge determined by the gauge functional, $\Psi + \Delta \Psi$, similar to Eq. (2.39), but without the use of field-dependent BRST transformations:

$$Z_M(J, \Phi^*) = \int D\Phi \exp \left\{ \frac{i}{\hbar} \left( S_\Psi(\Phi, \Phi^*) + M(\Phi, \Phi^*) \right) + s_e(\Delta \Psi(\Phi)) 
+ \sum_{n \geq 2} \frac{1}{n!} \Delta \Psi_{A_1} \cdots \Delta \Psi_{A_n} S_{\Psi A_1 \cdots A_n}(\Phi, \Phi^*) + \Delta M(\Phi, \Phi^*) + J_A \Phi^A \right\},$$

(3.15)

where account has been taken for the fact that the functional $M = M_\Psi(\Phi, \Phi^*)$ should have the following representation in the above gauge, because of the relation of gauged BRST transformations with functional $\Lambda(\Phi, \Phi^* | \Delta \Psi)$ (2.42) which should compensate a finite change of the gauge $\Delta \Psi$ in $Z_\Psi(0, \Phi^*)$:

$$M_{\Psi + \Delta \Psi}(\Phi, \Phi^*) = M_\Psi(\Phi, \Phi^*) + \Delta M(\Phi, \Phi^*).$$

(3.16)

It should be noted that $M_{\Psi + \Delta \Psi}$ does not have the form of a gauge-invariant action, $S_{\Psi + \Delta \Psi}$, as regards the dependence on the variation $\Delta M$, despite the fact that an introduction of the additive term $\Delta \Psi$ by means of the transformation (2.9) applied to the action, $S_\Psi$,

$$\exp \left\{ \frac{i}{\hbar} S_{\Psi + \Delta \Psi} \right\} = \exp \left\{ -[\Delta, \Delta \Psi] \right\} \exp \left\{ \frac{i}{\hbar} S_\Psi \right\} = \exp \left\{ \frac{i}{\hbar} S_\Psi(\Phi, \Phi^* + \Delta \Psi_{\Phi^A}) \right\},$$

(3.17)
is a transformation which turns a solution of the soft BRST symmetry breaking equation \((3.3)\) for \(M_\Psi\) into another solution, however, not having the form \(M_{\Psi + \Delta \Psi}\). This takes place, since in the case of the functional \(M_\Psi\), being BRST-non-invariant, the gauge condition is not determined via a shift of the antifields:

\[
M_{\Psi + \Delta \Psi} \neq M_\Psi (\Phi, \Phi^* + \frac{\delta \Delta \Psi}{\delta \Phi}) .
\]

(3.18)

As we turn to Eq. \((3.15)\), let us present the finite change \(\Delta Z_M(J, \Phi^*) = Z_{M + \Delta M, \Psi + \Delta \Psi}(J, \Phi^*) - Z_{M, \Psi}(J, \Phi^*)\) in an equivalent form:

\[
\Delta Z_M(J, \Phi^*) = \int D\Phi \left[ \exp \left\{ i \frac{\hbar}{\lambda} (S(\Phi, \Phi^*) + J_\lambda \Phi^A) \right\} - 1 \right] \exp \left\{ i \frac{\hbar}{\lambda} (S(\Phi, \Phi^*) + J_\lambda \Phi^A) \right\}
\]

(3.19)

with allowance for the identity

\[
\left[ \exp \left\{ \frac{\delta \Delta \Psi}{\delta \Phi^A} \right\} - 1 \right] \exp \left\{ i \frac{\hbar}{\lambda} S \right\} = \left[ \exp \left\{ \frac{\delta \Delta \Psi}{\delta \Phi^A} \right\} \left( \frac{\delta^\ast}{\delta \Phi^A} - \frac{i}{\hbar} M^{A^*} \right) \right] \exp \left\{ i \frac{\hbar}{\lambda} S \right\} .
\]

(3.20)

Considering the general term \(\left\{ \frac{\delta \Delta \Psi}{\delta \Phi^A} \left( \frac{\delta^\ast}{\delta \Phi^A} - \frac{i}{\hbar} M^{A^*} \right) \right\}^n\), for \(n \geq 1\), inside the decomposition \((3.19)\) and integrating by parts in the path integral, we obtain

\[
\int D\Phi \exp \left\{ i \frac{\hbar}{\lambda} M \right\} \frac{\delta \Delta \Psi}{\delta \Phi^A} \left( \frac{\delta^\ast}{\delta \Phi^A} - \frac{i}{\hbar} M^{A^*} \right) \right\}^{n-1} \left( \frac{\delta^\ast}{\delta \Phi^A} - \frac{i}{\hbar} M^{A^*} \right) \exp \left\{ i \frac{\hbar}{\lambda} (S + J_\lambda \Phi^A) \right\}
\]

\[
= \frac{i}{\hbar} \int D\Phi \exp \left\{ i \frac{\hbar}{\lambda} M \right\} \left( \frac{\delta \Delta \Psi}{\delta \Phi^A} \right) \left( \frac{\delta^\ast}{\delta \Phi^A} - \frac{i}{\hbar} M^{A^*} \right) \right\}^{n-1} \left( \frac{\delta^\ast}{\delta \Phi^A} - \frac{i}{\hbar} M^{A^*} \right) \nonumber \]

\[
- \left\{ \sum_{k=1}^{n-1} \prod_{l=1}^{k-1} \frac{\delta \Delta \Psi}{\delta \Phi^A} \left( \frac{\delta^\ast}{\delta \Phi^A} - \frac{i}{\hbar} M^{A^*} \right) \right\} \left( \frac{\delta^\ast}{\delta \Phi^A} - \frac{i}{\hbar} M^{A^*} \right) \nonumber \]

\[
+ \left\{ \frac{\delta \Delta \Psi}{\delta \Phi^A} \left( \frac{\delta^\ast}{\delta \Phi^A} - \frac{i}{\hbar} M^{A^*} \right) \right\}^{n-1} \left( J_\lambda + M_\lambda \right) \left( \frac{\delta^\ast}{\delta \Phi^A} - \frac{i}{\hbar} M^{A^*} \right) \left( \frac{\delta^\ast}{\delta \Phi^A} - \frac{i}{\hbar} M^{A^*} \right) \nonumber \]

(3.21)

where account has been taken of the generating equations \((2.11)\) for \(S_\Psi\) and \((3.3)\) for \(M_\), as well as the notation \(M^{A^*}_\lambda \equiv \frac{\delta}{\delta \Phi^A} (M^{B^*}_\lambda)\) and the following properties of the functional \(\Delta \Psi\):

\[
(\Delta \Psi)^2 = 0, \quad \text{and} \quad \Delta \Psi_{AB} \left( \frac{\delta}{\delta \Phi^A} - \frac{i}{\hbar} M^{B^*} \right) \left( \frac{\delta}{\delta \Phi^A} - \frac{i}{\hbar} M^{A^*} \right) = 0.
\]
The variation of the functional $Z_M(J, \Phi^*)$ can therefore be finally presented as

$$
\Delta Z_M = \int D\Phi \left[ \exp \left\{ \frac{i}{\hbar} \Delta M \right\} \sum_{n \geq 0} \frac{1}{n!} \left\{ \Delta \Psi_B \left( \frac{\delta}{\delta \Phi_B} - \frac{i}{\hbar} M^{B*} \right) \right\}^n - 1 \right] \exp \left\{ \frac{i}{\hbar} (S + J_A \Phi^A) \right\}
$$

$$
= \frac{i}{\hbar} \exp \left\{ \frac{i}{\hbar} \Delta M \left( \frac{\hbar}{i}, \frac{\delta}{\delta J}, \Phi^* \right) \right\} \left[ \Delta M_A \sum_{n \geq 1} \frac{1}{n!} \left\{ \Delta \Psi_B \left( \frac{\delta}{\delta \Phi_B} - \frac{i}{\hbar} M^{B*} \right) \right\}^{n-1} \right.
$$

$$
- \sum_{n \geq 1} \frac{1}{n!} \left\{ \sum_{k=1}^{n-1} \prod_{l=1}^{k-1} \Delta \Psi_{B_l} \left( \frac{\delta}{\delta \Phi_{B_l}} - \frac{i}{\hbar} M^{B_{l*}} \right) \Delta \Psi_{B_k} M^{B_{k*}} \prod_{l=k+1}^{n-1} \Delta \Psi_{B_l} \left( \frac{\delta}{\delta \Phi_{B_l}} - \frac{i}{\hbar} M^{B_{l*}} \right) \right\}
$$

$$
+ \sum_{n \geq 1} \frac{1}{n!} \left\{ \Delta \Psi_B \left( \frac{\delta}{\delta \Phi_B} - \frac{i}{\hbar} M^{B*} \right) \right\}^{n-1} \left( J_A + M_A \right) \left( \frac{\delta}{\delta \Phi_A} - \frac{i}{\hbar} M^{A*} \right) \Delta \Psi \left( \frac{\hbar}{i}, \frac{\delta}{\delta J} \right) Z_M
$$

$$
+ \left[ \exp \left\{ \frac{i}{\hbar} \Delta M \left( \frac{\hbar}{i}, \frac{\delta}{\delta J}, \Phi^* \right) \right\} - 1 \right] Z_M,
$$

where the arguments $\frac{\hbar}{i}, \frac{\delta}{\delta J}$ are implied to be substituted instead of the fields, $\Phi$, in $\Delta M_A, M^{B*}, M_A, M^{B_{k*}}, \Delta \Psi_B$ in the last equality, in accordance with the conventions \[(3.8)\].

The general result \[(3.22)\] for the variation of $Z_M$ in the approximation linear in powers of the variations $\Delta \Psi, \Delta M$ reads as follows:

$$
\Delta Z_M(J, \Phi^*) = \frac{i}{\hbar} \left[ (J_A + M_A \left( \frac{\hbar}{i}, \frac{\delta}{\delta J}, \Phi^* \right)) \left( \frac{\delta}{\delta \Phi_A} - \frac{i}{\hbar} M^{A*} \left( \frac{\hbar}{i}, \frac{\delta}{\delta J}, \Phi^* \right) \right) \Delta \Psi \left( \frac{\hbar}{i}, \frac{\delta}{\delta J} \right)
$$

$$
+ \Delta M \left( \frac{\hbar}{i}, \frac{\delta}{\delta J}, \Phi^* \right) \right] Z_M(J, \Phi^*), \tag{3.23}
$$

and coincides with the result for $\delta Z_M$ first obtained in \cite{23}.

In the particular case of the absence of BRST symmetry breaking terms, i.e., when $M = 0$, we derive from \[(3.22)\] a new representation for a finite variation of the functional $Z(J, \Phi^*)$ under a finite variation of the gauge condition,

$$
\Delta Z(J, \Phi^*) = \frac{i}{\hbar} \left[ \sum_{n \geq 0} \frac{1}{(n+1)!} \left( \Delta \Psi_B \left( \frac{\hbar}{i}, \frac{\delta}{\delta J} \right) \delta \right) \left( \frac{\delta}{\delta \Phi_A} \right)^n J_A \frac{\delta}{\delta \Phi_A} \right] \Delta \Psi \left( \frac{\hbar}{i}, \frac{\delta}{\delta J} \right) Z(J, \Phi^*), \tag{3.24}
$$

which is reduced, in the case of a small variation, $\Delta \Psi = \delta \Psi$, to the form \[(2.17)\], well-known in the BV formalism, with the notation $\Delta Z = \delta Z$.

To complete the research of gauge-dependence in the theory with broken BRST symmetry, let us calculate the variations of $W_M(J, \Phi^*)$ and $\Gamma_M(\Phi, \Phi^*)$ under a finite change of the gauge condition, taken into account for the relations $\Delta W_M = \frac{\hbar}{i} Z^{-1}_M \Delta Z_M$ and $\Delta W_M = \Delta \Gamma_M$. First of
all, $\overline{\Delta W}_M$ reads

$$
\overline{\Delta W}_M = \exp \left\{ \frac{i}{\hbar} \overline{\Delta M} \left( \frac{\delta}{\delta J} + \frac{\delta W_M}{\delta J}, \Phi^* \right) \right\} \left[ \overline{\Delta M}_A \sum_{n \geq 1} \frac{1}{n!} \left\{ \overline{\Delta \Psi}_B \left( \frac{\delta}{\delta \Phi^*_B} + i \frac{\delta W_M}{\delta \Phi^*_B} - i \frac{M^*_B}{\hbar} \right) \right\}^{n-1}
\times \left( \frac{\delta}{\delta \Phi^*_A} + i \frac{\delta W_M}{\delta \Phi^*_A} - i \frac{M^*_A}{\hbar} \right) \right] - \sum_{n \geq 1} \frac{1}{n!} \left\{ \sum_{k=1}^{n-1} \prod_{i=1}^{k-1} \overline{\Delta \Psi}_{B_i} \left( \frac{\delta}{\delta \Phi^*_{B_i}} + i \frac{\delta W_M}{\delta \Phi^*_{B_i}} - i \frac{M^*_B}{\hbar} \right) \right\}
\times \prod_{i=k+1}^{n-1} \overline{\Delta \Psi}_{B_i} \left( \frac{\delta}{\delta \Phi^*_{B_i}} + i \frac{\delta W_M}{\delta \Phi^*_{B_i}} - i \frac{M^*_B}{\hbar} \right) \right] \left( \overline{\Delta \Psi}_B \delta M^*_B \right) + \sum_{n \geq 1} \frac{1}{n!} \left( \frac{i}{\hbar} \right)^{n-1} \overline{\Delta M} \left( \frac{\delta}{\delta J} + \frac{\delta W_M}{\delta J}, \Phi^* \right),
$$

(3.25)

where use has been made of the Ward identity (3.9) and substitution of the arguments, $(\frac{\delta}{\delta J} + \frac{\delta W_M}{\delta J})$, instead of $\Phi$ in $\overline{\Delta M}_A, M^*_B, M_A, \overline{\Delta \Psi}_B$ should be made. Again, without BRST symmetry breaking terms for $M = 0$, we can obtain from Eq. (3.25) a new representation for a finite variation for the generating functional $W(J, \Phi^*)$, namely,

$$
\overline{\Delta W} = \left[ \sum_{n \geq 0} \frac{1}{(n+1)!} \left( \overline{\Delta \Psi}_B \left( \frac{\delta W}{\delta J} + \frac{\delta}{\delta J} \right) \right) n \frac{\delta}{\delta \Phi^*_A} \right] \overline{\Delta \Psi} \left( \frac{\delta W}{\delta J} + \frac{\delta}{\delta J} \right).
$$

(3.26)

For the first order in powers of $\overline{\Delta \Psi}$ and $\overline{\Delta M}$, the variation (3.25) for $\overline{\Delta W}_M(J, \Phi^*)$ has the form

$$
\overline{\Delta W}_M = \left( J_A + M_A (\frac{\delta W_M}{\delta J} + \frac{\delta}{\delta J}) \right) \frac{\delta}{\delta \Phi^*_A} \overline{\Delta \Psi} \left( \frac{\delta W_M}{\delta J} + \frac{\delta}{\delta J} \right) + \overline{\Delta M} \left( \frac{\delta W_M}{\delta J} + \frac{\delta}{\delta J}, \Phi^* \right),
$$

(3.27)

identical (after the change $\overline{\Delta} \rightarrow \delta$) with the one obtained in [22, 23].

Second, on order to derive a finite form of the gauge variation for the effective action, we can use the calculations of [23]. Namely, the change of variables $(J_A, \Phi^*_A) \rightarrow (\Phi^A, \Phi^*_A)$ from the Legendre transformation (3.6) implies

$$
\frac{\delta}{\delta \Phi^*_A} \bigg|_J = \frac{\delta}{\delta \Phi^*_A} \bigg|_\Phi + \frac{\delta \Phi}{\delta \Phi^*_A} \frac{\delta J}{\delta \Phi^*_A} \quad \text{and} \quad \frac{\delta W_M}{\delta \Phi^*_A} = \frac{\delta \Gamma_M}{\delta \Phi^*_A}.
$$

(3.28)

Then, the differential consequence of the Ward identities for $Z_M$ (3.7) and $W_M$ (3.9) implies

$$
- \left( \frac{\delta \Gamma_M}{\delta \Phi^*_A} - M_A \right) \frac{\delta \Phi^B}{\delta \Phi^*_A} = - \left( \frac{\delta \Gamma}{\delta \Phi^*_B} - \tilde{M}^B \right) (-1)^{\epsilon^B}
+ \frac{i}{\hbar} \left[ - M_A \frac{\delta \Gamma_M}{\delta \Phi^*_A} - \frac{\delta \Gamma_M}{\delta \Phi^*_A} \tilde{M}^A - \tilde{M}_A \tilde{M}^A, \Phi^B \right],
$$

(3.29)

with the same notation $[ , , ]$ for the supercommutator as in (3.17).
Using Eqs. (3.25), (3.28), (3.29) and the relation
\[
\frac{\delta \Phi^B}{\delta \Phi_A} = (-1)^{\varepsilon_B(\varepsilon_{A+1})} \frac{\delta W}{\delta J_B \delta \Phi_A} = -(1)^{\varepsilon_B(\varepsilon_{A+1})} (\Gamma'' \Gamma) \frac{\delta l}{\delta \Phi_A} \frac{\delta \Gamma}{\delta \Phi_A} ,
\]
we present the finite variation of the effective action in the form
\[
\Delta \Gamma_M = \exp \left\{ \frac{i}{\hbar} (\Delta M) \right\} \left\{ (\Delta M_A) \sum_{n \geq 1} \frac{1}{n!} \{ (\Delta \Psi_B) \left( - \hat{F}^B + \frac{i}{\hbar} \delta \Gamma_M \delta \Phi_B - \frac{i}{\hbar} \hat{M}^B \right) \right\}^{n-1}
\times \left\{ - \hat{F}^A + \frac{i}{\hbar} \delta \Gamma_M - \frac{i}{\hbar} \hat{M}^A \right\} - \sum_{n \geq 1} \frac{1}{n!} \left\{ \sum_{k=1}^{n-1} \prod_{l=1}^{k-1} (\Delta \Psi_{B_l}) \left( - \hat{F}^{B_l} + \frac{i}{\hbar} \delta \Gamma_M - \frac{i}{\hbar} \hat{M}^{B_l} \right) \right\}^{n-1}
\times \left\{ - \hat{F}^A + \frac{i}{\hbar} \delta \Gamma_M - \frac{i}{\hbar} \hat{M}^A \right\} + \sum_{n \geq 1} \frac{1}{n!} \left\{ (\Delta \Psi_B) \left( - \hat{F}^B + \frac{i}{\hbar} \delta \Gamma_M - \frac{i}{\hbar} \hat{M}^B \right) \right\}^{n-1}
\times \left\{ - \hat{F}^A + \frac{i}{\hbar} \delta \Gamma_M - \frac{i}{\hbar} \hat{M}^A \right\} + \sum_{n \geq 1} \frac{1}{n!} \left( \frac{i}{\hbar} \right)^{n-1} (\Delta M) .
\]

Here, we use the notation
\[
\langle \Delta \Psi \rangle = \Delta \Psi (\Phi) \cdot \mathbf{1} \quad \text{and} \quad \langle \Delta M \rangle = \Delta M (\Phi, \Phi^*) \cdot \mathbf{1} ,
\]
as well as the same notation for \( \langle \Delta \Psi_{B_k} \rangle, \langle \Delta M_A \rangle \), and introduce the operator \( \hat{F}^A \), derived from Eqs. (3.28), (3.29), (3.30), as follows
\[
\hat{F}^A = - \frac{\delta}{\delta \Phi_A} + (-1)^{\varepsilon_B(\varepsilon_{A+1})} (\Gamma'' \Gamma) \frac{\delta l}{\delta \Phi_A} \frac{\delta \Gamma}{\delta \Phi_A} .
\]

Now, we can deduce from Eq. (3.31) a new representation for a finite variation of the effective action \( \Gamma(J, \Phi^*) \) in a local form without BRST symmetry breaking terms \( (M = 0) \),
\[
\Delta \Gamma = - \sum_{n \geq 0} \frac{1}{(n+1)!} \left( \langle \Delta \Psi_B \rangle \left[ \frac{i}{\hbar} \delta \Gamma_M - \hat{F}^B \right] \right)_M \rangle^{n} (\Gamma, \langle \Delta \Psi \rangle),
\]
in the first order with respect to the variation \( \langle \Delta \Psi \rangle \), identical with the previously known representation (2.19).

For the first order in powers of \( \langle \Delta \Psi \rangle \) and \( \langle \Delta M \rangle \), the variation (3.31) of \( \Delta M_J(\Phi^*) \) takes the previously known [23] “local-like” form
\[
\Delta \Gamma_M = - (\Gamma_M, \langle \Delta \Psi \rangle) + \left( \hat{M}_A \frac{\delta}{\delta \Phi_A} + (-1)^{\varepsilon_A \hat{M}^A} \frac{\delta l}{\delta \Phi_A} \right) \langle \Delta \Psi \rangle
\]
\[
- \frac{i}{\hbar} \left[ \hat{M}_A \frac{\delta \Gamma_M}{\delta \Phi_A} + \hat{M}_A \hat{M}^A - \hat{M}_A \hat{M}^A \right] \langle \Delta \Psi \rangle + \langle \Delta M \rangle ,
\]
(3.35)
where coincidence with the final result of [23] is achieved by the change $\Delta \rightarrow \delta$.

To study gauged (field-dependent) BRST transformations in a theory with broken BRST symmetry, we follow the result of [22], [23] and present the variation, linear in $\langle \Delta \Psi \rangle$, $\langle \Delta M \rangle$, of the effective action (3.35) in an equivalent form, being the so-called non-local form, due to the explicit presence of $(\Gamma''_{M} - 1)_{BC}$ in $\hat{F}_{A}$ (3.33):

$$\Delta \Gamma_{M} = \frac{\delta \Gamma_{M}}{\delta \Phi^{A}} \hat{F}^{A}_{A} \langle \Delta \Psi \rangle - \hat{M}_{A} \hat{F}^{A} \langle \Delta \Psi \rangle + \langle \Delta M \rangle .$$

We now intend to revise our previous result [22], [23], which states that the variation (3.36) implies that the effective action with soft BRST symmetry breaking is generally gauge-dependent on the mass shell, since

$$\frac{\delta \Gamma_{M}}{\delta \Phi^{A}} = 0 \quad \Rightarrow \quad \Delta \Gamma_{M} \neq 0 .$$

Indeed, there is a hope that the introduction of broken BRST symmetry into the field-antifield formalism would be consistent only if the two final terms in (3.36) should cancel each other:

$$\langle \Delta M \rangle = \hat{M}_{A} \hat{F}^{A} \langle \Delta \Psi \rangle ,$$

which, at the classical level, imposes a condition on the gauge variation of $M$ under a change of the gauge-fixing functional $\Psi$,

$$\Delta M = \frac{\delta M}{\delta \Phi^{A}} \hat{F}^{A}_{0} \Delta \Psi \quad \text{where} \quad \hat{F}^{A}_{0} = (-1)^{\epsilon_{B}(\epsilon_{A}+1)}(S''_{-1})_{BC} \left( \frac{\delta_{l}}{\delta \Phi^{C}} \frac{\delta S}{\delta \Phi^{*}} \right) \frac{\delta S}{\delta \Phi^{B}} .$$

Of course, despite the fact that it seems to be a strong restriction that the BRST-breaking functional $M$ corresponding to the effective action should be gauge-independent on the mass shell (implying the gauge-independence of the physical S-matrix), the gauge-independence (but not invariance) can, in fact, be restored.

In order to justify the above proposition, let us subject the integrand in $Z_{M}$, with the gauge-fixing functional $\Psi(\Phi)$, to the change of variables (2.23), with a field-dependent odd-valued parameter $\Lambda (\Phi, \Phi^{*} | \Delta \Psi)$ in (2.42) being a solution of Eq. (2.40) [corresponding to the functional $\hat{\Lambda}(\Phi''_{0})$ from Eq. (A.9)], which provides the gauge-independence of the vacuum functional $Z_{\Psi}(0, \Phi^{*})$ (2.43):

$$Z_{M,\Psi}(J, \Phi^{*}) = \int D\Phi \exp \left\{ \frac{i}{\hbar} \left( S - i\hbar \ln \left[ (1 + s_{e} \Lambda (\Delta \Psi))^{-1} (1 + \frac{i\hbar}{s_{e}} \Lambda (\Delta \Psi)) \right] + s_{e} M(\Phi, \Phi^{*}) \Lambda (\Delta \Psi) + J_{A} [\Phi^{A} + (s_{e} \Phi^{A}) \Lambda (\Delta \Psi)] \right) \right\} .$$

Thus, taking into account the fact that for any variation of the gauge-fixing functional, $\Psi(\Phi) \rightarrow (\Psi + \Delta \Psi)(\Phi)$, in view of the result obtained in Section 2.2 there exists a parameter, $\Lambda (\Delta \Psi)$, of finite gauged BRST transformations, being a solution of Eq. (2.40) such that the action $S_{\Psi}$,
and therefore also the total quantum action $S$ in the gauge determined by $\Psi + \Delta \Psi$, takes the form

$$S_{\Psi + \Delta \Psi} + M_{\Psi + \Delta \Psi} = \left[ S_{\Psi} - i\hbar \ln \left\{ (1 + s_e \Lambda (\Delta \Psi))^\dagger (1 + \frac{s_e}{\hbar} \Lambda (\Delta \Psi)) \right\} \right]$$

$$+ \left[ M_{\Psi} + s_e M_{\Psi} (\Phi, \Phi^*) \Lambda (\Delta \Psi) \right],$$

(3.41)

where the first square brackets in the right-hand-side contain an expression for $S_{\Psi + \Delta \Psi}$, whereas the second brackets should contain an expression for $M_{\Psi + \Delta} = M_{\Psi} + \Delta M$. Therefore, based on the equivalence theorem [43], we have, first, a representation, being different form (3.22), for a finite change of the functional $Z_{M,\Psi}(J, \Phi^*)$ under a finite change of the gauge:

$$\Delta Z_{M,\Psi}(J, \Phi^*) = -\frac{i}{\hbar} J_A (s_e \Phi^A) \left( \frac{\delta}{\delta J} \right) \Lambda (\Delta \Psi) Z_{M,\Psi}(J, \Phi^*)$$

$$= (-1)^{\epsilon A} J_A \Lambda \left( \frac{\delta}{\delta J} \right) \left( \frac{\delta}{\delta \Phi^*} \right) M_{\Psi} (\Phi, \Phi^*) \Lambda (\Delta \Psi) Z_{M,\Psi}(J, \Phi^*)$$

(3.42)

which also leads to the on-shell coincidence (for $J_A = 0$) of the generating functionals $Z_{M}(J, \Phi^*)$ calculated in the gauges $\Psi$ and $\Psi + \Delta \Psi$, respectively; we also obtain the form of the soft BRST symmetry functional $M$ in the gauge determined by $\Psi + \Delta \Psi$, provided that in the gauge $\Psi$ the former is defined by the functional $M$,

$$\Delta M = (s_e M) \Lambda (\Delta \Psi).$$

(3.43)

In the approximation linear in $\Delta \Psi$, we have, making use of (3.33) and (2.41),

$$M_{\Psi + \Delta \Psi} = M - \frac{i}{\hbar} M_A S^A_{\Psi} \delta \Psi.$$

(3.44)

An important particular case, which covers practically all the known gauge models, corresponds to a gauge theory of first rank with a closed algebra, when Eqs. (A.6) for the quantum action are fulfilled. An explicit expression of the soft BRST-breaking functional similar to Eq. (3.44) in the gauge $(\Psi + \Delta \Psi)$ reads as follows:

$$M_{\Psi + \Delta \Psi} = M + M_A S^A_{\Psi} \Delta \Psi (s \Delta \Psi)^{\dagger} \left\{ \exp \left\{ \frac{-i}{\hbar} s \left( \Delta \Psi \right) \right\} - 1 \right\},$$

(3.45)

where account has been taken of Eq. (A.9).

Note, first of all, that the additional contribution to $M$ in (3.45) does not increase the maximal power in the antifields of the functional $M$. Second, the gauge variation of the BRST-symmetry-broken functional does not generally turn the solution of the soft BRST-breaking equation (3.3) into a solution. However, in the above case of a gauge theory with closed algebra (reducible or not) of first rank with $M = M(\Phi)$, Eq. (3.3) for the gauge-transformed functional $M_{\Psi + \Delta \Psi}$ is valid by construction, due to the representation (3.45).

Let us now check the validity of the representation (3.38) for a variation of the BRST symmetry breaking functional with accuracy up to the first order in the gauge variation $\Delta \Psi$. 
In fact, it is sufficient to compare two representations for a finite change of $Z_{M,\Psi}(J,\Phi^*)$, with (3.23) obtained from a change of the gauge condition and with (3.42) obtained via a change of variables generated by field-dependent BRST transformations with the parameter $\Lambda(\bar{\Delta} \Psi)$. Indeed, (3.23) can be presented as

$$
\bar{\Delta} Z_{M,\Psi} = \frac{i}{\hbar} J_A \left( \frac{\delta}{\delta \Phi_A} - \frac{i}{\hbar} M^{A*} \left( \frac{\delta}{\delta J}, \Phi^* \right) \right) \bar{\Delta} \Psi \left( \frac{\delta}{\delta J} \right) Z_{M,\Psi}
$$

(3.46)

and for the summand in the second line we have

$$
\frac{i}{\hbar} \left[ \frac{i}{\hbar} M_A \left( \frac{\delta}{\delta J}, \Phi^* \right) S^A \Phi \left( \frac{\delta}{\delta J}, \Phi^* \right) \bar{\Delta} \Psi \left( \frac{\delta}{\delta J} \right) + \bar{\Delta} M \left( \frac{\delta}{\delta J}, \Phi^* \right) \right] Z_{M,\Psi}
$$

(3.47)

due to the representation (3.43) for $\bar{\Delta} M$. The last expression in terms of the average fields (3.0) for the effective action $\Gamma_M$ is nothing else than the representation (3.38).

Thus, the coincidence of $\bar{\Delta} Z_{M,\Psi}(J,\Phi^*)$ in (3.23) with (3.42) is guaranteed due to (3.43) and (2.42).

As a consequence, the finite change of the functionals $W_M, \Gamma_M$ in the linear approximation in $\bar{\Delta} \Psi$ in the relations (3.27) and (3.36) should coincide (after change $\bar{\Delta} \rightarrow \delta$) with the variations of the functionals $W, \Gamma$, respectively, in (2.18) and (2.21) without any soft BRST symmetry breaking term $M$. Concerning the finite change of the $W_M, \Gamma_M$ in (3.25) and (3.31), as a result of the above-established correspondence between the finite change of the gauge $\bar{\Delta} \Psi$ and the parameter of gauged BRST transformation $\Lambda(\Phi, \Phi^* | \bar{\Delta} \Psi)$ in (2.42), the form of $\bar{\Delta} M$ should be chosen according to (3.43).

Thus, we have proved the following Statement: an addition to the quantum action $S_{\Psi}$, satisfying the master equation in the BV formalism (2.7), of a term, $M(\Phi, \Phi^*)$, breaking the BRST symmetry soft[7] (3.3), first, destroys the BRST invariance of the integrand in the generating functional of Green’s functions, $Z_M$, and therefore also the gauge-invariance of the total action ($S_{\Psi} + M$) in the tree approximation; second, this leads to an effective action, $\Gamma_M$, being gauge-independent upon a variation of the gauge condition within the class of admissible gauges on its extremals,

$$
\frac{\delta \Gamma_M}{\delta \Phi^A} = 0 \text{ and } \langle \bar{\Delta} M \rangle = \widehat{M_A} \widehat{F^A}(\bar{\Delta} \Psi) \rightarrow \bar{\Delta} \Gamma_M = 0 ,
$$

(3.48)

7Of course, any such term should be admissible, in order to have a well-determined path integral, at least in perturbation theory. Second, the requirement of soft breaking of the BRST symmetry may be weakened in order to consider only the breaking of BRST symmetry (see Footnote 6 for details).
providing the variation of the BRST symmetry breaking term $M$ in the form of gauged (field-dependent) BRST symmetry transformations (3.43).

In particular, this implies that if in the reference frame determined by the gauge fermion $\Psi$ the generating functional $Z_{M,\Psi}(J, \Phi^*)$ is described by (3.1) then in the reference frame $\Psi + \Delta \Psi$ it should have the form

$$Z_{M+\Delta \Psi+\Delta \Psi}(J, \Phi^*) = \int D\Phi \exp \left\{ \frac{i}{\hbar} \left( S_{\Psi+\Delta \Psi} + M_{\Psi} + (s_{e} M_{\Psi}) \Lambda (\Delta \Psi) + J_{A} \Phi^{A} \right) \right\}. \quad (3.49)$$

This fact makes the procedure of Lagrangian quantization of a gauge theory with soft BRST symmetry breaking consistent and leads, in particular, to a gauge-independent S-matrix within the conventional approach [1, 2, 3]. The problem of gauge dependence considered in a non-renormalized gauge theory with BRST broken terms should now be studied in a renormalized theory.

Let us turn to some field-theoretic examples and constructions where the concept of BRST symmetry breaking is realized.

4 Application to Functional Renormalization Group

In this section, we apply the above results to the study of the effective average action proposed in [48], [49], [50], which naturally arises within the functional renormalization group (FRG) approach to the Lagrangian quantization of Yang–Mills theories and was recently examined in [51].

The essence of FRG is to use, instead of $\Gamma$, the so-called effective average action $\Gamma^{k}$ with a momentum-shell parameter, $k$, coinciding with $\Gamma$ for vanishing $k$,

$$\lim_{k \to 0} \Gamma^{k} = \Gamma; \quad (4.1)$$

in such a way that the Faddeev–Popov action $S_{FP}(\Phi)$ for Yang–Mills theories should be extended by means of soft BRST symmetry breaking terms, $M$, having the form of the regulator action $S_{k}$ for $M = S_{k}$,

$$S_{k}(A, C, \bar{C}) = \frac{1}{2} A^{a}(R_{k,A})_{\mu \nu} A^{\mu}(x) A^{\nu}(x) + C^{a}(R_{k,gh})_{\alpha \beta} C^{\beta}(x), \quad (4.2)$$

In (4.2), we have specified the condensed notations, so that the total configuration space $\mathcal{M}$ of the Yang–Mills theory,

$$\{ \Phi^{A} \} = \{ A^{i}, C^{a}, \bar{C}^{a}, B^{a} \} = \{ A^{a}_{\mu}, C^{a}, \bar{C}^{a}, B^{a} \}(x) \quad \varepsilon(C^{a}) = \varepsilon(\bar{C}^{a}) = 1, \quad \varepsilon(A^{a}_{\mu}) = \varepsilon(B^{a}) = 0 ,$$

\[\text{However, another basic requirement for a quantum gauge field theory, i.e., the unitarity of the S-matrix, is destroyed when adding to the gauge theory any soft BRST symmetry breaking terms, and thus needs a special investigation.} \]
which determines this irreducible theory and includes the fields $A_\mu^a(x)$, the Grassmann-odd Faddeev–Popov (scalar) ghost and antighost fields $C^a$ and $\bar{C}^a$, as well as the Nakanishi–Lautrup auxiliary fields $B^a$, given in the $D$-dimensional Minkowski space-time $R^{1,D-1}$ and taking values in the adjoint representation of the Lie algebra $su(N)$. In turn, the regulator quantities $(R_{k,A})$, $(R_{k,gh})$, having no dependence on the fields, obey the property $(R_{k,A})_{ij} = (-1)^{\varepsilon_i\varepsilon_j} (R_{k,A})_{ji}$ and vanish as the parameter $k$ tends to zero.

The initial classical action $S_0$ of the Yang–Mills fields $A_\mu^a(x)$ and its gauge transformations have the standard form (with the coupling constant $g = 1$, for simplicity)

$$S_0(A) = -\frac{1}{4} \int d^Dx \, F_{\mu\nu}^a \, F^{\mu\nu a} \quad \text{for} \quad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc} A_\mu^b A_\nu^c , \quad (4.3)$$

$$\delta A_\mu^a = D_\mu^b \xi^b , \quad D_\mu^b = \delta^{ab} \partial_\mu + f^{abc} A_\mu^c , \quad \varepsilon(\xi^b) = 0 , \quad (4.4)$$

with the Lorentz indices $\mu, \nu = 0, 1, \ldots, D-1$, the metric tensor $\eta_{\mu\nu}$, $\eta_{\mu\nu} = \text{diag}(-, +, \ldots, +)$, the totally antisymmetric $su(N)$ structure constants $f^{abc}$, for $a, b, c = 1, \ldots, N^2 - 1$, the covariant derivative $D_\mu^b$, and arbitrary functions $\xi^b$ in $R^{1,D-1}$.

The corresponding set of odd momenta for the fields, i.e., antifields, reads

$$\{ \Phi_A^* \} = \{ A^{a\mu}, C^{*a}, \bar{C}\^{*a}, B^{*a} \}(x) \quad \text{with} \quad \varepsilon(A^{a\mu}) = \varepsilon(B^{*a}) = 1 , \quad \varepsilon(C^{*a}) = \varepsilon(\bar{C}\^{*a}) = 0 ,$$

which, in view of the identity $\Delta \tilde{S} = 0$, is also a solution to the classical master equation $(\tilde{S}, \tilde{S}) = 0$. The gauge-fixed action $S_\Psi(\Phi, \Phi^*) = \tilde{S}(\Phi, \Phi^* + \Phi_A^* + s \Psi(\Phi))$ obeys the same equations with a Grassmann-odd gauge-fixing functional $\Psi(\Phi)$, which can be chosen as

$$\Psi(\Phi) = \bar{C}^a \chi^a(A, B) \quad \text{with} \quad \chi^a = 0 \quad (4.5)$$

so that the non-renormalized Faddeev–Popov action $S_{FP}(\Phi)$ is obtained from $S_\Psi$ for vanishing antifields, $\Phi_A^*$,

$$S_{FP}(\Phi) = \left[ 1 - \Phi_A^* \frac{\delta}{\delta \Phi_A^*} \right] S_\Psi(\Phi, \Phi^*) = S_0(A) + \bar{C}^a K^{ab} C^b + \chi^a B^a$$

$$= S_0(A) + s \Psi(\Phi) , \quad (4.6)$$

where $K^{ab}$ and $s$ are the Faddeev–Popov operator and the Slavnov variation $(2.25)$, written for any functional $F(\Phi)$ as follows:

$$K^{ab} = \frac{\delta \chi^a}{\delta A^b_\mu} D_\mu^{bc} \quad \text{and} \quad s F(\Phi) = \frac{\delta F}{\delta \Phi_A} \delta S_\Psi . \quad (4.7)$$
Both actions $S_{\psi}(\Phi, \Phi^*)$, $S_{FP}(\Phi)$ are invariant with respect to the BRST transformation [compare with Eq. (2.14)]

$$\delta_\mu \Phi^A = S_{\psi}^A \mu \quad \text{with} \quad S_{\psi}^A = (D_\mu^a C^b, \frac{1}{2} f^{abc} C^c, B^a, 0),$$

and so does the integrand in the generating functional $Z_k(J, \Phi^*)$ of Green’s functions (introduced in [50], [51] with the obvious change of the notation $\Phi^*_A \equiv K_A$) for the vanishing sources, $J_A = (J^a, J^a_C, J^a_B)(x) = 0$, and the regulator action, $S_k = 0$,

$$Z_k(J, \Phi^*) = \int d\Phi \exp \left\{ \frac{i}{\hbar} [S_{FP}(\Phi) + \Phi^*_A S^A + S_k(\Phi) + J \Phi] \right\} = \exp \left\{ \frac{i}{\hbar} W_k(J, \Phi^*) \right\}. \quad (4.8)$$

Before taking the limit $k \to 0$, the integrand in the case $J = 0$ is not BRST-invariant, due to the easily verified inequality

$$\delta_\mu S_k(\Phi) \neq 0,$$

whereas in the limit $k \to 0$ the functionals $Z_k, W_k$ take correct values, identical with the usual generating functionals $Z, W$. The average effective action $\Gamma_k = \Gamma_k(\Phi, \Phi^*)$, being the generating functional of vertex functions in the presence of regulators, is introduced according to the rule described by Eq. (3.6) in Section 3

$$\Gamma_k(\Phi, \Phi^*) = W_k(J, \Phi^*) - J \Phi, \quad \Phi^A = \frac{\delta W_k}{\delta J_A}, \quad (4.9)$$

with the obvious consequences of the Legendre transformation (4.9), $J_A = (\delta \Gamma_k)/(\delta \Phi^A)$. Note, first of all, that the average effective action, by analogy with Eq. (3.12), satisfies an equation and possess tree-level and one-loop approximations which are similar to those for $\Gamma_M$ in Eqs. (3.13), (3.14), however, with $S_k$, instead of the functional $M$. Second, as to the regulator functions, we suppose that they model the non-perturbative contributions to the self-energy part of the Feynman diagrams, so that the dependence on the parameter $k$ enables one to extract some additional information about the scale dependence of the theory beyond the loop expansion [52]. Third, the Ward (Slavnov–Tailor) identities [53], [54] for the functionals $Z_k, W_k$ and $\Gamma_k$ are easily obtained from the general results (3.7), (3.9) and (3.10) for $Z_M, W_M$ (3.1) and $\Gamma_M$ (3.6), and, due to the property $M^{*A} = S^{*A} \equiv 0$, take the form:

For $Z_k$,

$$J_A \frac{\delta Z_k}{\delta \Phi^A} + \frac{\hbar}{i} \int d^D x \left[ (R_{k,A})^a_{\mu
u} \frac{\delta^2 Z_k}{\delta J^\mu_\nu \delta A^a_\mu} - (R_{k,gh})^a_{\mu
u} \frac{\delta^2 Z_k}{\delta J^\mu_\nu \delta C^a} + (R_{k,gh})^a_{\mu
u} \frac{\delta^2 Z_k}{\delta J^\mu_\nu \delta C^a} \right] = 0. \quad (4.10)$$

for $W_k$,

$$J_A \frac{\delta W_k}{\delta \Phi^A} + \frac{\hbar}{i} \int d^D x \left[ (R_{k,A})^a_{\mu
u} \frac{\delta^2 W_k}{\delta J^\mu_\nu \delta A^a_\mu} - (R_{k,gh})^a_{\mu
u} \frac{\delta^2 W_k}{\delta J^\mu_\nu \delta C^a} + (R_{k,gh})^a_{\mu
u} \frac{\delta^2 W_k}{\delta J^\mu_\nu \delta C^a} \right] = 0, \quad (4.11)$$
for $\Gamma_k$,  
\[
\frac{1}{2}(\Gamma_k, \Gamma_k) - \int d^4x \left[ (R_k, A)_{A^a} A^b - \frac{\delta \Gamma_k}{\delta A^a} - (R_k, e)_{A^a} C^b + (R_k, g)_{A^a} C^b \right] \delta \Gamma_k \delta C^{ab} \]
\[+ i\hbar \left\{ (R_k, A)_{A^a} \left( \Gamma_k^{(a)} \right)^{b \mu} \left( \delta A^a \right)^{b \mu} - (R_k, e)_{A^a} \left( \Gamma_k^{(a)} \right)^{b \mu} \left( \delta A^a \right)^{b \mu} \right\} = 0.
\] (4.12)

The supermatrix $(\Gamma_k^{(a)})$ is the inverse of $\Gamma_k^{(a)}$, with the elements determined by analogy with Eqs. (2.19), (2.20), with the obvious replacement $\Gamma \rightarrow \Gamma_k$. In the limit $k \rightarrow 0$, the identities (4.10), (4.11), (4.12) are reduced to the standard Ward identities (2.16).

The consistency of the FRG method, based on the introduction of Eq. (4.2), means that the values of the average effective actions $\Gamma_k$ calculated for two different gauges determined by $\chi^a + \Delta \chi^a$, corresponding, in view of Eq. (4.5), to the gauge functionals $\Psi$ and $\Psi + \Delta \Psi$, should coincide on the mass-shell for any value of the parameter $k$ (i.e., along the FRG trajectory, but not only in its boundary points). For completeness, let us recall that the FRG flow equation for $\Gamma_k$, which describes the FRG trajectory, reads [51] as follows, with account taken of the notation $\partial_t = \frac{d}{dk}$:
\[
\frac{\partial_t}{\partial_t} \Gamma_k = \frac{\partial_t}{\partial_t} S_k - \frac{i}{\hbar} \left\{ \frac{1}{2} \partial_t (R_k, A)^{ab} \left( \Gamma_k^{(a)} \right)^{b \mu} + \partial_t (R_k, g)^{ab} \left( \Gamma_k^{(a)} \right)^{b \mu} \right\},
\] (4.13)

which has the same form for the $\Phi^*_{A}$-independent part of $\Gamma_k$, due to the parametric dependence on $\Phi^*_{A}$ of all the terms in (4.13).

Due to the result (3.45) for a finite variation of the BRST symmetry breaking term, the variation of the regulator action $S_k$ under the variation of the gauge condition has the form
\[
\Delta S_k = S_k, A^s \Delta \Psi (s \Delta \Psi)^{-1} \left[ \exp \left\{ -\frac{i}{\hbar} s(\Delta \Psi) \right\} - 1 \right],
\] (4.14)

with the use of the Slavnov operator $s$. The corresponding gauged BRST transformation, leading to the variation of $S_k$, as applied to the generating functional $Z_k$ in the gauge $\Psi(\Phi)$ (4.5), must be characterized by the parameter $\Lambda(\Phi)$ given by
\[
\Lambda(\Phi) = \Delta \Psi(\Phi) (s \Delta \Psi(\Phi))^{-1} \left[ \exp \left\{ -\frac{i}{\hbar} s(\Delta \Psi(\Phi)) \right\} - 1 \right].
\] (4.15)

According to Eq. (3.31), in the case under consideration a finite variation of the average effective action $\Gamma_k$ with allowance for the explicit form of $\Delta S_k$ (4.14) takes the form
\[
\Delta \Gamma_k = \exp \left\{ \frac{i}{\hbar} \Delta S_k \right\} \left[ \left( \Delta S_{\Phi, A} \sum_{n \geq 1} \frac{1}{n!} \left\{ \Delta \Psi_B \right\} \left( -\hat{F} + \frac{i}{\hbar} \frac{\delta \Gamma}{\delta \Phi^o_B} \right) \right)^{n-1} \left( -\hat{F} + \frac{i}{\hbar} \frac{\delta \Gamma}{\delta \Phi^o_B} \right) \right] \left( \Delta \Psi \right) + \sum_{n \geq 1} \frac{1}{n!} \left( \frac{i}{\hbar} \right)^{n-1} \left( \Delta S_k \right),
\] (4.16)
where (3.32) has been taken into account, and the operator \( \hat{F}^A \) is now determined as

\[
\hat{F}^A = - \frac{\delta}{\delta \Phi_A^*} + (-1)^{e_B(e_A+1)}(\Gamma''_{k-1})^{BC} \left( \frac{\delta I}{\delta \Phi^*_C \delta \Phi_A^*} \right) \frac{\delta I}{\delta \Phi^*_B}.
\]  

(4.17)

Being linear in \( \langle \Delta \Psi \rangle \) and, due to (4.14), also in \( \langle \Delta S_k \rangle \), the variation \( \Delta \Gamma_k(J, \Phi^*) \) takes another “local-like” form; see Eq. (5.6) in [51]:

\[
\Delta \Gamma_k = - (\Gamma_k, \langle \Delta \Psi \rangle) + \hat{S}_{kA} \frac{\delta}{\delta \Phi_A^*} \langle \Delta \Psi \rangle - \frac{i}{\hbar} \left[ \hat{S}_{kA} \frac{\delta \Gamma_k}{\delta \Phi_A^*}, \Phi^B \right] \frac{\delta I}{\delta \Phi^*_B} \langle \Delta \Psi \rangle + \langle \Delta S_k \rangle.
\]  

(4.18)

There is a representation equivalent to Eq. (4.18) and similar to Eqs. (3.36), (3.33):

\[
\Delta \Gamma_k = \frac{\delta \Gamma_k}{\delta \Phi_A^*} \hat{F}^A \langle \Delta \Psi \rangle - \hat{S}_{kA} \hat{F}^A \langle \Delta \Psi \rangle + \langle \Delta S_k \rangle.
\]  

(4.19)

Due to the statement proved at the end of Section 3 [see Eq. (3.48)] regarding the presence in the gauge theory of a soft BRST breaking term, we can state that the average effective action \( \Gamma_k \), at least in the non-renormalized case, being evaluated at its extre ms, does not depend on the choice of the gauge condition:

\[
\frac{\delta \Gamma_k}{\delta \Phi_A^*} = 0 \text{ and } \langle \Delta S_k \rangle = \hat{S}_{kA} \hat{F}^A \langle \Delta \Psi \rangle \rightarrow \Delta \Gamma_k = 0,
\]  

(4.20)

provided that, in the approximation being linear with respect to \( \Delta \Psi \), the variation of the regulators \( S_k \) (4.14) takes the form

\[
\Delta S_k = - s(S_k) \frac{i}{\hbar} \Delta \Psi \text{ and } \Lambda(\Phi) = - \frac{i}{\hbar} \Delta \Psi(\Phi) + o(\Delta \Psi(\Phi)),
\]  

(4.21)

which, after averaging with respect to the mean fields \( \Phi^A \) by using \( \Gamma_k \), leads to

\[
\langle \Delta S_k \rangle + \langle s(S_k) \frac{i}{\hbar} \Delta \Psi \rangle = 0 \iff \langle \Delta S_k \rangle - \hat{S}_{kA} \hat{F}^A \langle \Delta \Psi \rangle = 0.
\]  

(4.22)

The result given by Eq. (4.20) allows one to revise (in comparison with [51]) the statement on the gauge-dependence of the average effective action, and therefore also on the consistency of its introduction within the Lagrangian quantization scheme for any value of the parameter \( k \). Indeed, the gauge dependence of the vacuum functional \( Z_{k,\chi} \) and of the average effective action \( \Gamma_k \) on its extre ms [51] was explicitly shown respectively in (4.12) and (5.9) therein. At the same time, the gauge independence of the average effective action in [51] was achieved on the mass-shell determined in a larger space of fields (see (6.31), (6.32) therein) with additional degrees of freedom, by means of considering the regulators \( S_k \) as composite fields, however, without taking into account the change of the regulators \( S_k \) under a change of the gauge condition in (6.22), (6.27), (6.30), (6.31). In this connection, note that the consideration of the regulators as the composite fields following to approach [69] – where their change under a variation of the gauge condition should be taken into account – allows one to provide the gauge independence of \( \Gamma_k \) on the mass shell determined by the usual average fields \( \Phi^A \) only.
Let us calculate the form of the regulator terms $S_k$ in different, but mutually related gauges, setting as $S_k^0$ the values in a fixed gauge; for the sake of definiteness, in the Landau gauge. To this end, let us consider a family of linear gauges given by the equation
\[
\chi^a(A, B) = \Lambda_\mu(\partial, \alpha, \beta, n)A^{\mu a} + \frac{\xi}{2}B^a = 0 \quad \text{with} \quad \Lambda_\mu(\partial, \alpha, \beta, n) = \alpha \partial_\mu + \beta \frac{K_{\mu\nu}}{n^2} n^\nu. \tag{4.23}
\]
Here, we have three numeric, $\alpha, \beta, \xi$, and one vector, $n^\mu$, gauge parameters. From $\alpha, \beta, \xi$, we can keep only two numbers, $\beta, \xi$, for instance, dividing $\chi^a(A, B)$ by $\alpha$.

Particular cases of these gauges can be obtained from the general many-parameter family under the choices
\[
\alpha = 1, \beta = 0 \rightarrow \text{family of } \xi\text{-gauges}, \tag{4.24}
\]
\[
\beta = -\alpha, \kappa_{\mu\nu} = n^\rho \partial_\rho \eta_{\mu\nu}, n^2 < 0, \xi = 0 \rightarrow \text{generalized Coulomb gauges}, \tag{4.25}
\]
\[
\alpha = 0, \kappa_{\mu\nu} = \eta_{\mu d-1} n^\nu, \xi = 0 \rightarrow \text{generalized axial gauges}. \tag{4.26}
\]

The Landau and Feynman gauges are obtained from the first family for the respective choices $\xi = 0$ and $\xi = 1$. The usual Coulomb, $\chi^a_C(A, B)$, and axial, $\chi^a_A(A, B)$, gauges are derived from the second and third families by setting, $n^\mu = (1, 0, ..., 0)$ and $n^\mu = (0, ..., 0, 1)$ for the respective parameters. For completeness, we have
\[
\chi^a_C(A, B) = \partial_\mu A^{\mu a} = 0, \quad \text{for} \quad \mu = (0, i), \tag{4.27}
\]
\[
\chi^a_A(A, B) = A^{d-1 a} = 0. \tag{4.28}
\]

Denoting the Landau gauge as $\chi^a(A, B)|_{\alpha=1, \beta=\xi=0} \equiv \chi^a(A)$, we can examine the form of the regulators which arises for arbitrary values of the parameters $\alpha, \beta, \xi, n^\mu$. Following Eq. (4.14), we immediately obtain the variation of the gauge fermion and its Slavnov variation, respectively,
\[
\overline{\Delta \Psi} = \bar{C}^a(\chi^a(A, B) - \chi^a(A)) = \int d^Dx \bar{C}^a \left( \{ (\alpha - 1) \partial_\mu + \beta \frac{K_{\mu\nu}}{n^2} n^\nu \} A^{\mu a} + \frac{\xi}{2} B^a \right), \tag{4.29}
\]
\[
s\overline{\Delta \Psi} = \int d^Dx \left( B^a \left( \{ (\alpha - 1) \partial_\mu + \beta \frac{K_{\mu\nu}}{n^2} n^\nu \} A^{\mu a} + \frac{\xi}{2} B^a \right) \right.
\]
\[
+ \bar{C}^a \left( ((\alpha - 1) \partial_\mu + \beta \frac{K_{\mu\nu}}{n^2} n^\nu) D^{\mu ab} C^b \right). \tag{4.30}
\]

so that the expression for $S_k = S_k^0 + \overline{\Delta S_k}$ reads
\[
S_k = S_k^0 + \int d^Dx \left\{ A^{a\mu}(x)(R_{k, A})_{\mu b}(x) D^{bc} C^c + (R_{k, gh})_{ab}(x) \left( \frac{1}{2} f^{bcd} C^a C^b C^d - C^b B^a \right) \right\}
\]
\[
\times \overline{\Delta \Psi} \left( s\overline{\Delta \Psi} \right)^{-1} \left[ \exp \left\{ -\frac{i}{\hbar} s\overline{\Delta \Psi} \right\} s\overline{\Delta \Psi} \right] - 1. \tag{4.31}
\]

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From Eqs. (4.29)–(4.31), we find an approximation linear in $\Delta \Psi$, 
\begin{align*}
S_k(\Phi) &= \int d^D x \left\{ \frac{1}{2} A^{\mu}(x)(R_{k,A})_{\mu\nu}(x) A^{\nu}(x) + \bar{C}^a(x)(R_{k,gh})^{ab}(x) C^b(x) \right\} \\
&\quad - \frac{i}{\hbar} \int d^D x \left\{ A^{\mu}(x)(R_{k,A})_{\mu\nu}(x) D^{bc\nu} C^c + (R_{k,gh})^{ab}(x) \left( \frac{1}{2} f^{bcd} C^a C^c C^d - C^b B^a \right) \right\} \\
&\quad \times \int d^D y \bar{C}^e(y) \left\{ \left( (\alpha - 1) \partial_\rho + \beta \frac{k_{\rho\nu}}{n^2} n^\nu \right) A^{\rho e}(y) + \frac{\xi}{2} B^e(y) \right\},
\end{align*}
(4.32)
depending now on all field variables and having the standard limit $S_k \to 0$ as the momentum-shell parameter tends to zero, $k \to 0$. For $\alpha = 1, \beta = \xi = 0$, the regulators $S_k(\Phi)$ are smoothly reduced to the initial ones $S^0_k(\Phi)$, given in the Landau gauge, whereas the expressions for $S_k(\Phi)$ in any gauges described by Eqs. (4.24)–(4.28) can now be explicitly obtained from Eq. (4.32).

In order to obtain the form of $S_k$ (4.32) without the terms $\frac{i}{\hbar}$, so that this functional should start from the tree-level term, we have to perform integration with respect to the Faddeev–Popov ghost fields in the functional integral $Z_k$ (4.8), and then extract the Faddeev-Popov operator (4.7), $K^{ab}$, in the resulting gauge and exponentiate it with help of the same Faddeev–Popov ghost fields.

It is interesting to investigate the consequences of the study of gauge-dependence in the case of the Pauli–Villars regularization [55], which does not preserve the gauge and therefore also the BRST invariance of the regularized quantum action in the regularization scheme without higher derivatives introduced in [3], but we leave this study outside this paper’s scope.

5 Standard and Refined Gribov–Zwanziger Actions in Many-parameter Family of Gauges

In this section, we apply the above general consideration developed in Section 3 and adopted to the case of the average effective action for Yang–Mills theories in Section 4 in the case of the so-called Gribov–Zwanziger [9], [10] and refined Gribov–Zwanziger theories, introduced in [56] and examined in [57], [58], [59], [60]. Let us remind that the Gribov–Zwanziger theory is determined by the Gribov–Zwanziger action $S_{GZ}(\Phi)$, given in the Landau gauge $\chi^a(A) = 0$,
\begin{align*}
S_{GZ}(\Phi) &= S_{FP}(\Phi) + M(A),
\end{align*}
(5.1)
which contains an additive non-local BRST-non-invariant summand, implying an inclusion of the Gribov horizon [5] and known as the Gribov horizon functional $M(A)$, with suppressed continuous space-time coordinates $x, y$,
\begin{align*}
M(A) &= \gamma^2 \left( f^{abc} A^b_{\mu}(K^{-1})^{ad} f^{dec} A^{\epsilon\mu} + D(N^2-1) \right), \text{ for } (K^{-1})^{ad}(K)^{db} = \delta^{ab},
\end{align*}
(5.2)
which is determined by means of the Faddeev-Popov operator $(K)^{ab} = \partial_\mu D^\mu_{ab}$ and the so-called thermodynamic (Gribov) parameter $\gamma$, introduced in a self-consistent way by the gap equation.
Here, the fields $\phi$ doublets \[\phi \] of the functional $S$. Both the non-local action $S$ equals the relation $G_{\xi}$ copies, and, second, a non-zero value for the Gribov parameter $\gamma$ is a manifestation of nontrivial properties of the vacuum of the theory as a consequence of restrictions on the Gribov horizon. The latter means that there exist additional reasons for non-perturbative effects, which can be encoded in a set of dimension-2 condensate, $\langle A^{\mu a} A_{\mu}^a \rangle$, in the case of a non-local Gribov–Zwanziger action with the Yang–Mills gauge fields $A^{\mu a}$ only, as well as in a similar set of dimension-2 condensates, $\langle A^{\mu a} A_{\mu}^a \rangle$, $\langle \varphi_{\mu}^{ab} \varphi_{\mu}^{ac} \rangle - \langle \varphi_{\mu}^{ab} \varphi_{\mu}^{ac} \rangle$, for a local Gribov–Zwanziger action, $S_{GZ}(\Phi, \phi)$, with an equivalent local representation for the horizon functional in terms of the functional $S_{\gamma}$, given in an extended configuration space with auxiliary variables, $\phi^{\hat{A}}$,

$$S_{GZ}(\Phi, \phi) = S_{FP}(\Phi) + S_{\gamma}(A, \phi)$$

$$S_{\gamma} = \bar{\varphi}_{\mu}^{ac} K^{ab} \varphi_{\mu}^{bc} - \bar{\omega}_{\mu}^{ac} K^{ab} \omega_{\mu}^{bc} + f^{amb}(\partial_{\nu} \varphi_{\mu}^{ac}) (D_{\nu}^{mp} e_{p}) \varphi_{\mu}^{bc} + \gamma f^{abc} A_{\mu}^{a} (\varphi_{\mu}^{bc} - \varphi_{\mu}^{bc}) - D(N^2 - 1) \gamma^2.$$ (5.5)

Here, the fields $\phi^{\hat{A}}$ contain tensors being antisymmetric with respect to the $su(N)$ indices,

$$\{ \phi^{\hat{A}} \} = \{ \varphi_{\mu}^{ac}, \varphi_{\mu}^{ac}, \omega_{\mu}^{ac}, \bar{\omega}_{\mu}^{ac} \},$$ (5.6)

even for $\varphi_{\mu}^{ac}, \varphi_{\mu}^{ac}$ (i.e., $\varepsilon(\varphi) = \varepsilon(\bar{\varphi}) = 0$) and odd for $\omega_{\mu}^{ac}, \bar{\omega}_{\mu}^{ac}$ ($\varepsilon(\omega) = \varepsilon(\bar{\omega}) = 1$), which form BRST doublets \[61\].

$$\delta_{\mu} \left( \varphi_{\nu}^{ac}, \bar{\varphi}_{\nu}^{ac} \right) = \left( \omega_{\nu}^{ac}, 0 \right)_{\mu} \quad \delta_{\mu} \left( \omega_{\nu}^{ac}, \bar{\omega}_{\nu}^{ac} \right) = \left( 0, \bar{\varphi}_{\nu}^{ac} \right)_{\mu}.\quad (5.7)$$

Both the non-local $M(A, \xi)$ and local $S_{\gamma}$ horizon functionals are not BRST-invariant:

$$sM = \gamma^2 f^{abc} f^{cde} \left[ 2D_{\mu}^{aq} C^{q}(K^{-1})^{ad} - f^{mpn} A_{\mu}^{b} (K^{-1})^{am} K^{pq} C^{q}(K^{-1})^{nd} \right] A^{\mu} \neq 0,$$ (5.8)

$$sS_{\gamma} = \gamma f^{ad} \left[ \left( D_{\mu}^{ae} C^{e}(\varphi_{\mu}^{ab} - \varphi_{\mu}^{ab}) + A_{\mu}^{d} \omega_{\mu}^{ab} \right) \right] \neq 0, (5.9)$$

where account has been taken of the relation $sK^{ab} = f^{abc} K^{cd} C^{d}$, with the latter Slavnov variation, together with the representation for $S_{\gamma}$, being different from those of \[32\]. The problem of finding the Gribov horizon functional in reference frames other than the Landau gauge has been considered in various papers. In \[14\], this problem was first solved in the approximation being quadratic in the fields for the linear covariant $R_{\xi}$-gauges given by Eqs. \[1.23, 1.24\] for a small value of the parameter $\xi$; another form of the functional $M(A, \xi)$ was suggested in \[15\], and also with the help of the gauged (field-dependent) BRST transformations in the recent paper \[32\]. Of course, the suggested result requires a verification of the fact that the functional derived actually satisfies the requirement that it should single out the first Gribov horizon region for the gauge fields $A^{\mu a}$ in the $R_{\xi}$-gauge, because an extraction of this region via
The functional $M(A)$ was determined non-perturbatively in the Landau gauge only, whereas a corresponding rigorous proof for $M(A, \xi)$, i.e., that it actually provides the restriction for the gauge fields $A^{\mu a}$ within the Gribov region $\Omega(\xi)$,

$$\Omega(\xi) = \left\{ A^{\mu a} | \chi^a(A, B) \big|_{\alpha=1, \beta=0} = 0, K^{ab}(\xi) \geq 0 \right\},$$

(5.10)
is absent in the literature in an explicit way.

As we turn to the refined Gribov–Zwanziger theory, let us propose the refined Gribov–Zwanziger action in a non-local form, and, along the lines of [56], [57], [58], [59], [60], also in a local form, as follows:

$$S_{GZ}(\Phi) \rightarrow S_{RGZ1}(\Phi) = S_{GZ} + \frac{m^2}{2} A^a_\mu A^{\mu a},$$

(5.11)

$$S_{GZ}(\Phi, \phi) \rightarrow S_{RGZ2}(\Phi, \phi) = S_{GZ}(\Phi, \phi) + \frac{m^2}{2} A^a_\mu A^{\mu a} - M^2 \left( \overline{\varphi}^a \varphi^b \mu ab - \overline{\omega}^a \omega^b \mu ab \right),$$

(5.12)

which can, of course, be considered as theories with composite operators.

The only non-vanishing Slavnov variations are those of the first composite fields:

$$S \left( \frac{m^2}{2} A^a_\mu A^{\mu a} \right) = m^2 A^a_\mu \partial^\mu C^a \neq 0, \quad \text{whereas} \quad S \left( M^2 \left( \overline{\varphi}^a \varphi^b \mu ab - \overline{\omega}^a \omega^b \mu ab \right) \right) = 0,$$

(5.13)

so that the only new BRST-non-invariant term is $\frac{1}{2} m^2 A^a_\mu A^{\mu a}$.

By virtue of the properties (5.8), (5.9) of the functionals $M(A)$ and $S_\gamma$, as well as due to Eq. (5.13) with the composite fields $M + \frac{1}{2} m^2 A^a_\mu A^{\mu a}$, and $S_\gamma + \frac{1}{2} m^2 A^a_\mu A^{\mu a} + M^2 (\overline{\varphi} \varphi - \overline{\omega} \omega)$, in Eq. (5.2), these functionals trivially satisfy both the quantum (3.3) and classical (3.2) conditions of soft BRST symmetry breaking, because of the independence on antifields.

To establish the gauge-independence of physical quantities in these theories, we have to examine the models in various gauges from the many-parameter family (4.23), thus explicitly extending the result of [32]. In this case, the Faddeev–Popov action is written as follows:

$$S_{FP}(\Phi, \alpha, \beta, n^\mu, \xi) = S_0(A) + C^a A^a_\mu (\partial, \alpha, \beta, n) D^{\mu b} C^b + \Lambda_\mu (\partial, \alpha, \beta, n) A^\mu a B^a + \frac{1}{2} B^a B^a.$$

(5.14)

The Faddeev–Popov operator $K^{ab} = N^a D^{ab}_\mu$ depends on $(\alpha, \beta, n)$, but not on $\xi$, and the functional $M$ should be removed from $(\alpha, \beta, n, \xi) = (1, 0, n, 0)$. However, since $K^{ab}$ cannot be Hermitian [14], [15] the application of the Zwanziger trick developed in the Landau gauge seems to be impossible. Now, we apply the result of the preceding Sections 3, 4 to gauged BRST transformations, and then, following Eqs. (3.45), (4.14), the variation of the gauge fermion $\overline{\Delta} \Psi$ and its Slavnov variation $s \overline{\Delta} \Psi$ are given by Eqs. (4.29), (4.30), so that the form of the Gribov horizon functional $M(\Phi, \alpha, \beta, n, \xi) \equiv \tilde{M}$ in the gauge under consideration reads, $\tilde{M} = M + \overline{\Delta} M$,

$$\tilde{M} = M(A) + \gamma^2 f^{abc} f^{de} \left[ 2 D^{pq}_A C^{q(K^-)^{nd}} - f^{mnp} A^b_m (K^-)^{am} K^{ps} C^q (K^-)^{nd} \right] A^{\mu a}$$

$$\times \overline{\Delta} \Psi \left( s \overline{\Delta} \Psi \right)^{-1} \left[ \exp \left\{ -\frac{i}{\hbar} s (\overline{\Delta} \Psi) \right\} - 1 \right],$$

(5.15)
In the linear approximation with respect to $\Delta \Psi$, we have

$$\tilde{M} = M(A) - \frac{i}{\hbar} \gamma^2 f^{abc} f^{cde} [2D_{\mu}^b C^q(K^{-1})_{ad} - f^{mpn} A^b_{\mu}(K^{-1})_{am} K_{pq} C^q(K^{-1})_{nd}] A^{\mu e}$$

$$\times \bar{C}_{\chi} \left\{ \left( (\alpha - 1) \partial_{\rho} + \beta \frac{K_{pq}}{n^2} n^\nu \right) A^{\rho h} + \frac{\xi}{2} B^h \right\}. \quad (5.16)$$

For $\alpha=1, \beta=\xi=0$, the Gribov horizon functional $M(\Phi, \alpha, \beta, n, \xi)$ reduces smoothly to $M(A)$ given in the Landau gauge, whereas the expressions for $M(\Phi, \alpha, \beta, n, \xi)$ in any linear gauges are now described by Eqs. (4.24)–(4.28). Thus, for $\alpha=1, \beta=0, \xi=1$ we deduce from Eq. (5.15) the Gribov horizon functional in the Feynman gauge as in [32], whereas in the Coulomb gauge $\chi^2_{\Phi}(A, B) = \partial_i A^{ia} = 0$, obtained by setting $n^{\mu} = (1, 0, 0, 0) \equiv n_0^{\mu}$, $\alpha = \beta = 1, \xi = 0$ in Eq. (4.23), in which the Gribov copies were first discovered [5], the functional $M(\Phi, 1, 1, n_0, 0) \equiv M_C$ has the form

$$M_C = M(A) + \gamma^2 f^{abc} f^{cde} [2D_{\mu}^b C^q(K^{-1})_{ad} - f^{mpn} A^b_{\mu}(K^{-1})_{am} K_{pq} C^q(K^{-1})_{nd}] A^{\mu e}$$

$$\times \bar{C}_{\chi} \left( s \bar{\Delta} \Psi_C \right)^{-1} \left\{ \exp \left\{ -\frac{i}{\hbar} s(\bar{\Delta} \Psi_C) \right\} - 1 \right\}. \quad (5.17)$$

For the linear $\gamma$-dependent part of the functional $S_\gamma$, which is now BRST-non-invariant, examined in the general gauge $\chi^a(A, B)$ from the family (4.23), we have an expression similar to Eq. (5.15),

$$\gamma \frac{\partial}{\partial \gamma} S_\gamma(\Phi, \phi, \alpha, \beta, n, \xi) = \gamma \frac{\partial}{\partial \gamma} S_\gamma(1, 0, n, 0) + \gamma f^{abc} \left[ (D_{\mu}^d C^e (\varphi^{abc} - \bar{\varphi}^{abc}) + A^d_{\mu} \omega^{abc}) \right]$$

$$\times \bar{\Delta} \Psi(s \bar{\Delta} \Psi)^{-1} \left\{ \exp \left\{ -\frac{i}{\hbar} s(\bar{\Delta} \Psi) \right\} - 1 \right\}. \quad (5.19)$$

On the other hand, in the Coulomb gauge we have the same expression for $\gamma \frac{\partial}{\partial \gamma} S_\gamma$, given by Eq. (5.19), however, with $\bar{\Delta} \Psi_C, s \bar{\Delta} \Psi_C$ given by Eq. (5.18). Finally, for the BRST-non-invariant term $m^2/2 A^{a}_{\mu} A^{a}_{\mu}$, we have a presentation in the gauge (4.23) with account taken of Eqs. (4.29), (4.30),

$$\frac{m^2}{2} A^{a}_{\mu} A^{a}_{\mu} \rightarrow \frac{m^2}{2} A^{a}_{\mu} A^{a}_{\mu} + m^2 A^{a}_{\mu} \partial^\mu \chi^{a} C^a \bar{\Delta} \Psi \left( s \bar{\Delta} \Psi \right)^{-1} \left\{ \exp \left\{ -\frac{i}{\hbar} s(\bar{\Delta} \Psi) \right\} - 1 \right\}, \quad (5.20)$$

and also in the Coulomb gauge,

$$\frac{m^2}{2} A^{a}_{\mu} A^{a}_{\mu} \rightarrow \frac{m^2}{2} A^{a}_{\mu} A^{a}_{\mu} + m^2 A^{a}_{\mu} \partial^\mu \bar{\Delta} \Psi_C \left( s \bar{\Delta} \Psi_C \right)^{-1} \left\{ \exp \left\{ -\frac{i}{\hbar} s(\bar{\Delta} \Psi_C) \right\} - 1 \right\}. \quad (5.21)$$

Summarizing, we state that the Gribov horizon functional and the local functional $S_\gamma$ are now obtained explicitly in an arbitrary gauge from the many-parameter family described by Eq.

$^9$It is formally possible to consider the Gribov horizon functional in the axial gauge $\chi^{a}_{\theta}$ following Eq. (5.15); however, it is an algebraic gauge without a space-time derivative, which ensures that there is no problem of Gribov copies due to Singer’s result [4].
as well as the total Gribov–Zwanziger action in its local and non-local forms. The same takes place for the refined Gribov–Zwanziger action, which is the principal result of this section. Note that the solution of this problem is based entirely on the concept of gauged (field-dependent) BRST transformations.

We can now revise our final statement of [22] and maintain that the soft breaking of BRST symmetry is not in conflict with the gauge-independence of physical quantities in Yang–Mills theories with the Gribov horizon both in the Gribov-Zwanziger and in refined Gribov-Zwanziger theories.

6 Conclusion

We have elaborated a treatment of general gauge theories with arbitrary gauge-fixing in the presence of soft breaking of the BRST symmetry in the field-antifield formalism. To this end, we have studied the concept of gauged (equivalently, field-dependent) BRST transformations for theories more general than the Yang–Mills theory, and calculated the exact Jacobian (2.34) of the corresponding change of variables in the path integral determining the generating functionals of Green’s functions, including the effective action. We have argued, on a basis of analyzing the non-linear functional equation (2.40) for an unknown field-dependent odd-valued parameter, which we call the “compensation equation” [32] that for any finite change of the gauge condition $\Psi \rightarrow \Psi + \Delta \Psi$ there exists a gauged BRST transformation with a field-dependent parameter $\Lambda(\Phi, \Phi^* | \Delta \Psi)$ in (2.42), depending on $\Delta \Psi$, which permits an entire compensation of the finite change of the vacuum functional, i.e., $Z_{\Psi} = Z_{\Psi + \Delta \Psi}$.

We have investigated the influence of BRST-non-invariant terms, $M$, added to the quantum action constructed within the BV formalism and satisfying the so-called soft BRST symmetry breaking condition, on the properties of gauge-dependence of the corresponding effective action $\Gamma_M$. To study this problem, we have, for the first time, calculated finite changes of the generating functionals $Z_M$, $W_M$ and the effective action $\Gamma_M$ under a finite change of the gauge condition (3.22), (3.25), (3.31) and found that, at least with accuracy up to the linear terms in the variation of the gauge-fixing functional $\Delta \Psi$, the effective action does not depend on its extremals on the choice of gauge, provided that the change of the BRST-broken term is subject to a corresponding gauged BRST transformation with the parameter $\Lambda(\Phi, \Phi^* | \Delta \Psi)$ determined by (3.43) and used in (3.48), which is our principal result. Thereby, the concept of soft BRST symmetry breaking does not violate the consistency of Lagrangian quantization within the perturbation theory, so that the suggested prescription allows one, first of all, to obtain perturbatively the form of the soft BRST symmetry broken term in a different gauge by

\[\text{Note that the term “compensation equation” has been recently suggested [62], [63] for BRST symmetry in the study of finite BRST–BFV and BRST–BV transformations, respectively, as well as for BRST-antiBRST symmetry in Yang–Mills [64] and general gauge theories in Lagrangian [65], [66] and generalized Hamiltonian [67], [68] formulations.}\]
means of Eq. (3.43) [for a gauge theory of rank 1 with help of (3.45)], at least for gauges being sufficiently close to each other, and, second, to restore the gauge-independence of the effective action at its extremals, and therefore also the gauge-independence of the conventional physical $S$-matrix. We believe that these results should also be valid for a renormalized theory with soft BRST symmetry breaking; however, this requires a detailed proof.

We have demonstrated the applicability of our statements in the case of the functional renormalization group approach to the Yang–Mills and gravity theories and found, within the many-parameter family of linear gauges (4.23), the form of the regulator functionals in arbitrary (4.31) and linear gauges (4.32) from the same family, starting from those given, e.g., in the Landau gauge. This construction allows one to restore the gauge-independence of the average effective action $\Gamma_k$ along the entire trajectory of a FRG flow (4.20) without having recourse to the composite fields technique. Finally, the general concept of the gauged BRST transformations related to the same gauge theory, however, given in different gauges, appears to be very useful in constructing the Gribov–Zwanziger and the refined Gribov–Zwanziger actions for a many-parameter family of gauges, including the Coulomb, axial and covariant gauges (5.16), (5.21). This result extends the Gribov–Zwanziger theory with $R_\xi$-gauges examined in [32]. At the same time, there arises a problem of comparing the form of the horizon functional in the Coulomb gauge obtained perturbatively by means of gauged BRST transformations (5.17), (5.18) with the horizon functional obtained following to the Zwanziger non-perturbative recipe [18], which is planned to consider as a separates study. Of course, our arguments are valid for gauge theories with soft breaking of the BRST symmetry in case the transformed BRST breaking terms satisfy the same conditions in the final gauge as the untransformed ones in the initial gauge, however, with a possible violation of the condition (3.3) of soft BRST symmetry breaking. For instance, this means that for the Gribov horizon functional in a different gauge amongst the examined family of gauges one needs to verify the validity of extracting the Gribov horizon precisely from the configuration space of Yang–Mills fields, perhaps with the examined dimension-2 condensate.

Finally, it may be hoped that, due to the appearance of the Higgs field in view of the spontaneous breaking of the initial gauge invariance related to the group $SU(2)$ for the electroweak Lagrangian, one can examine an addition (associated with the Higgs field) to the gauge-invariant (with respect to the $SU(2)$ group) action of a soft BRST-breaking term, so that the description of the resulting model will be made consistent in the conventional Lagrangian path integral approach developed in this paper. We consider this problem as the next one to be examined.

Concluding, let us mention, first, the treatment of the Gribov horizon functional as a composite field [69], second, the recently obtained BRST-antiBRST extension [64] of the Gribov–Zwanziger theory in different gauges in a way consistent with the gauge independence of the physical $S$-matrix, third, the concept of soft BRST-antiBRST symmetry breaking developed on a basis of finite field-dependent BRST-antiBRST transformations in [66].
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Appendix

A On Solution of Equation (2.40)

In this Appendix, we present arguments for the existence of a solution for Eq. (2.40) with respect to an unknown field-dependent odd functional, $\Lambda(\Phi, \Phi^*)$, in the form (2.42). In doing so, we follow a strategy partially based on some previously known facts. First, any gauge theory can be equivalently transformed to a gauge theory in the standard basis [46], with the generators and proper zero eigenvectors having the representation

\[
\{ R^{i}_\alpha, Z^{\alpha}_{0}, \ldots, Z^{\alpha}_{L-1}, Z^{\alpha}_{L}\} \rightarrow \left\{ \left( R^{i}_\alpha, 0 \right), \left( 0, \delta^\alpha_{B_0} \right), \ldots, \left( 0, \delta^\alpha_{B_{L-1}} \right), \left( 0, \delta^\alpha_{B_L} \right) \right\} \quad (A.1)
\]

for the division of indices $\alpha_s, s = 0, \ldots, L$ being related with the rank conditions (2.2), (2.4) as $\alpha_0 = (\alpha, B_0), \alpha_s = (\bar{\alpha}_s, B_{s+1})$, for $s = 1, \ldots, L - 1$ and $\alpha_L = B_L = m_L$. Note that the definition (2.2) of an $L$-stage reducible gauge theory in the standard basis (A.1) looks simple, $Z^{\alpha_s}_{\bar{\alpha}_s} Z^{\alpha_{s+1}}_{\bar{\alpha}_{s+1}} = 0$, for vanishing $K^{i\alpha_{s-1}}_{\bar{\alpha}_s}$, for $s = 0, \ldots, L - 1$. Second, a transition to the standard basis from the initial gauge theory can be realized as a non-degenerate (generally, non-local) change of variables, $\Phi^A \rightarrow \Phi'^A(\Phi)$, in $\mathcal{M}$, such that

\[
Z_{\Psi}(0, \Phi^*) = \int D\Phi \exp\left\{ \frac{i}{\hbar} S_{\Psi} \right\} = \int D\Phi' \exp\left\{ \frac{i}{\hbar} S_{\Psi}(\Phi') \right\},
\]

with $S_{\Psi}(\Phi') = S_{\Psi}(\Phi(\Phi')) - i\hbar \text{ Str ln} \left\| \frac{\delta\Phi^A}{\delta\Phi'^B} \right\|$. (A.2)

We then use the fact that any gauge theory with an open algebra of generators $R^i_\alpha$ (being already in standard basis) can be equivalently transformed to a theory with a closed algebra [17], so that in the new basis of the generators of gauge transformations, $R^i_\alpha$ [obtained by means
of additive extension of $R^i_\alpha$ by trivial gauge generators, $R^i_\alpha(A') = R^i_\alpha(A') + S_{0,j}(A') M^i_j(A')$, the Lie-type structure functions $F^\gamma_{\alpha\beta}(A')$ in relations such as (2.27),

$$R^{ij}_{\alpha \beta}(A') R^{ij}_{\beta \gamma}(A') - R^{ij}_{\beta \gamma}(A') R^{ij}_{\alpha \beta}(A') = - R^{ij}_{\alpha \beta}(A') F^\gamma_{\alpha \beta}(A'),$$  \hspace{1cm}  \text{(A.3)}

are the only ones to survive. A transition to the gauge theory subject to relations (A.3) may also be effectively realized as a non-degenerate change of variables, $\Phi'^A \rightarrow \Phi'^{A}(\Phi)$ in $\mathcal{M}$:

$$Z_\Psi(0, \Phi^*) = \int D\Phi' \exp \left\{ \frac{i}{\hbar} \hat{S}_\Psi(\Phi') \right\} = \int D\Phi'' \exp \left\{ \frac{i}{\hbar} \hat{S}_\Psi(\Phi'') \right\},$$

with $\hat{S}_\Psi(\Phi'') = \hat{S}_\Psi(\Phi'(\Phi'')) - i\hbar \text{Str} \ln \left\| \frac{\delta \Phi'^A}{\delta \Phi''^B} \right\|$. \hspace{1cm} \text{(A.4)}

Notice that the transformations $\Phi \rightarrow \Phi'$, $\Phi' \rightarrow \Phi''$ have a more general form than the gauged BRST transformations (2.15) and can be equivalently realized by a set of operations (2.9) with definite respective functionals, $X_i(\Phi, \Phi^*)$, for $i = 1, 2$ which convert a solution of the master equation (2.7) into another solution $\hat{S}_\Psi$,

$$\hat{S}_\Psi = \frac{\hbar}{i} \ln \left[ \exp \{-[\Delta, X_2]\} \cdot \exp \{-[\Delta, X_1]\} \exp \left\{ \frac{i}{\hbar} S_\Psi \right\} \right].$$ \hspace{1cm} \text{(A.5)}

Since the transformed action $\hat{S}_\Psi$ (A.4) has a form being linear in the antifields, $\hat{S}_\Psi(\Phi'', \Phi^*) = \Phi'^{A} \hat{S}_\Psi^{A}(\Phi'')$, we now obtain the relations (derivatives with respect to the fields in $\hat{S}^{A}_{\Psi,B}$ and $\hat{S}_{\Psi,B}$ are understood as taken for $\Phi'^{B}$, and we omit the Jacobi matrices of the above changes of variables for the sake of simplicity)

$$(\hat{S}^{AB}_{\Psi} = 0, \quad \Delta \hat{S}^{A}_{\Psi} = 0) \Rightarrow \hat{S}^{A}_{\Psi,B} \hat{S}^{B}_{\Psi} = 0, \quad \text{(A.6)}$$

which, first of all, imply the nilpotency of the Slavnov variation, $\hat{s}^2 = 0$, in the new basis of the gauge algebra and, second, allow one to present the equation (2.40) for a gauge theory with a closed algebra as an equation for the parameter $\hat{\Lambda}$\textsuperscript{11}

$$i\hbar \left\{ \ln \left( 1 + \hat{s} \hat{\Lambda} \right) \right\} = \hat{\Psi} (\bar{\Delta \Psi}(\Phi'')),$$ \hspace{1cm} \text{(A.7)}

where account has been taken of the fact that the generator $\hat{s}$ coincides with $\hat{s}_e$, being, however, expressed in terms of the action $\hat{S}_\Psi$ and fields $\Phi'^{A}$.

Using the functional equation (A.7), we can express the variation $\bar{\Delta \Psi}(\Phi'')$ with accuracy up to BRST exact terms, $\hat{s} R(\Phi'')$,

$$\bar{\Delta \Psi}(\Phi'') = i\hbar \hat{\Lambda}(\Phi'') (\hat{s} \hat{\Lambda})^{-1} \left\{ \ln \left( 1 + \hat{s} \hat{\Lambda} \right) \right\},$$ \hspace{1cm} \text{(A.8)}

\textsuperscript{11}For simplicity, we use notation for the gauge fermion $\Psi$ and its variation $\bar{\Delta \Psi}$ in the case of a theory with a closed algebra which is the same as the notation used for a theory with an open algebra and the action $S_\Psi$.  

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which is identical with the variations for finite field-dependent BRST transformations in the Yang–Mills theory [28], now proved to be valid for a theory with a closed algebra. A solution of Eq. (A.7) with respect to an unknown \( \hat{\Lambda}(\Phi''') \) reads as follows:

\[
\hat{s}\hat{\Lambda}(\Phi''') = \exp\left\{-\frac{i}{\hbar}\hat{s}(\Delta\Psi(\Phi'''))\right\} - 1 \Rightarrow \hat{\Lambda}(\Phi''') = \Delta\Psi(\Phi''')\left[\exp\left\{-\frac{i}{\hbar}\hat{s}(\Delta\Psi(\Phi'''))\right\} - 1\right].
\]

(A.9)

Finally, in order to obtain a solution of the initial equation (2.40) which equivalently may be rewritten as

\[
(1 + s_e\Lambda)^{-1}(1 + \frac{\Delta}{s_e}\Lambda) = \exp\left[i\hbar\left(\exp\left\{-[\Delta, \Delta\Psi]\right\} - 1\right)S_\Psi\right]
\]

(A.10)

we have to make the inverse transformations \( \Phi'' \rightarrow \Phi' \rightarrow \Phi \) for \( \hat{\Lambda}(\Phi''') \) with respect to those used for the transition to the standard basis (A.1) and then to the gauge theory with a closed algebra (A.3), described in Eqs. (A.2), (A.4), and therefore a solution, \( \Lambda(\Phi, \Phi^*) \), of Eq. (2.40) does exist and is expressed by the variation \( \Delta\Psi(\Phi) \) in the form (2.42).

References

[1] M. Henneaux and C. Teitelboim, *Quantization of gauge systems*, Princeton University Press, 1992;

S. Weinberg, *The quantum theory of fields, Vol. II*, Cambridge University Press, 1996;

D.M. Gitman and I.V. Tyutin, *Quantization of fields with constraints*, Springer, 1990.

[2] N.N. Bogolyubov and D.V. Shirkov, *Introduction to theory of Quantized Fields*, John Wiley and Sons, New York, 1980.

[3] L.D. Faddeev and A.A. Slavnov, *Gauge Fields, Introduction to Quantum Theory*, second ed., Benjamin, Reading, 1990.

[4] C. Becchi, A. Rouet and R. Stora, *Renormalization of the abelian Higgs-Kibble model*, Commun. Math. Phys. 42 (1975) 127;

I.V. Tyutin, *Gauge invariance in field theory and statistical physics in operator formalism*, Lebedev Inst. preprint N 39 (1975), [arXiv:0812.0580[hep-th]].

[5] V.N. Gribov, *Quantization of nonabelian gauge theories*, Nucl.Phys. B139 (1978) 1.

[6] I.M.Singer, *Some remarks on the Gribov ambiguity*, Comm.Math.Phys. 60 (1978) 7-12.

\(^{12}\)In (A.10) the action of group-like element \( g(\Lambda(\Phi, \Phi^*)) = (1 + \frac{\Delta}{s_e}\Lambda) \), being trivial for nilpotent \( s_e \), measures difference of (2.40) with the equation (A.7) for the gauge theory with closed algebra
[7] A.A.Slavnov, Theor.Math.Phys. 170(2012),198-202;
A.Quadri,A.A.Slavnov, JHEP 07(2010) 087-109;
A.A. Slavnov, *Gauge fields beyond perturbation theory*, [arXiv:1310.8164[hep-th]].

[8] J. Serreau, M. Tissier and A. Tresmontant, *Covariant gauges without Gribov ambiguities in Yang-Mills theories*, [arXiv:1307.6019[hep-th]].

[9] D. Zwanziger, *Action from the Gribov horizon*, Nucl. Phys. B321 (1989) 591.

[10] D. Zwanziger, *Local and renormalizable action from the Gribov horizon*, Nucl. Phys. B323 (1989) 513.

[11] M.A.L. Capri, A.J. Gómes, M.S. Guimaraes, V.E.R. Lemes, S.P. Sorella and D.G. Tedesko, *A remark on the BRST symmetry in the Gribov-Zwanziger theory*, Phys. Rev. D82 (2010) 105019, arXiv:1009.4135 [hep-th];
L. Baulieu, M.A.L. Capri, A.J. Gomes, M.S. Guimaraes, V.E.R. Lemes, R.F. Sobreiro and S.P. Sorella, *Renormalizability of a quark-gluon model with soft BRST breaking in the infrared region*, Eur. Phys. J. C66 (2010) 451, arXiv:0901.3158 [hep-th];
D. Dudal, S.P. Sorella, N. Vandersickel and H. Verschelde, *Gribov no-pole condition, Zwanziger horizon function, Kugo-Ojima confinement criterion, boundary conditions, BRST breaking and all that*, Phys. Rev. D79 (2009) 121701, arXiv:0904.0641 [hep-th];
L. Baulieu and S.P. Sorella, *Soft breaking of BRST invariance for introducing non-perturbative infrared effects in a local and renormalizable way*, Phys. Lett. B671 (2009) 481, arXiv:0808.1356 [hep-th];
M.A.L. Capri, A.J. Gómes, M.S. Guimaraes, V.E.R. Lemes, S.P. Sorella and D.G. Tedesko, *Renormalizability of the linearly broken formulation of the BRST symmetry in presence of the Gribov horizon in Landau gauge Euclidean Yang-Mills theories*, arXiv:1102.5695 [hep-th];
D. Dudal, S.P. Sorella and N. Vandersickel, *The dynamical origin of the refinement of the Gribov-Zwanziger theory*, arXiv:1105.3371 [hep-th].

[12] I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker, and A. Sternbeck, *Lattice gluodynamics computation of Landau gauge Green’s functions in the deep infrared*, Phys. Lett. B676 (2009) 69, arXiv:0901.0736[hep-lat];
V. Bornyakov, V. Mitrjushkin, and M. Muller-Preussker, *SU(2) lattice gluon propagator: Continuum limit, finite-volume effects and infrared mass scale m(IR)*, Phys. Rev. D81 (2010) 054503, arXiv:0912.4475[hep-lat].

[13] V.G. Bornyakov, V.K. Mitrushkin and R.N. Rogalyov, *Gluon propagators in 3D SU(2) theory and effects of Gribov copies*, arXiv:1112.4975[hep-lat].
[14] R.F. Sobreiro and S.P. Sorella, *A study of the Gribov copies in linear covariant gauges in Euclidean Yang-Mills theories*, JHEP 0506 (2005) 054, [arXiv:hep-th/0506165].

[15] P. Lavrov and A. Reshetnyak, *Gauge dependence of vacuum expectation values of gauge invariant operators from soft breaking of BRST symmetry. Example of Gribov-Zwanziger action*, to appear in Proc. of QUARKS'2012, arXiv:1210.5651[hep-th].

[16] D. Dudal, M.A.L. Capri, J.A. Gracey et al., *Gribov Ambiguities in the Maximal Abelian Gauge*, Braz. J. Phys. 37 (2007) 320-324, [arXiv:hep-th/0609160].

[17] Sh. Gongyo and H. Iida, *Gribov-Zwanziger action in SU(2) Maximally Abelian Gauge with U(1) Landau Gauge* Phys.Rev. D 89 (2014) 025022, arXiv:1310.4877[hep-th].

[18] D. Zwanziger, *Equation of State of Gluon Plasma from Local Action*, Phys.Rev. D 76 (2007) 125014, [arXiv:hep-th/0610021].

[19] M. de Cesare, G. Esposito and H. Ghorbani, *Size of the Gribov region in curved spacetime*, Phys. Rev. D 88 (2013) 087701, arXiv:1308.5857[hep-th].

[20] P.M. Lavrov and I.V. Tyutin. *On the structure of renormalization in gauge theories*, Sov. J. Nucl. Phys. 34 (1981) 156;

P.M. Lavrov and I.V. Tyutin. *On the generating functional for the vertex functions in Yang-Mills theories*, Sov. J. Nucl. Phys. 34 (1981) 474.

[21] B.L. Voronov, P.M. Lavrov and I.V. Tyutin, *Canonical transformations and gauge dependence in general gauge theories*, Sov. J. Nucl. Phys. 36 (1982) 292.

[22] P. Lavrov, O. Lechtenfeld and A. Reshetnyak, *Is soft breaking of BRST symmetry consistent?*, JHEP 1110 (2011) 043, arXiv:1108.4820 [hep-th].

[23] P. Lavrov, O. Radchenko and A. Reshetnyak, *Soft breaking of BRST symmetry and gauge dependence*, MPLA A27 (2012) 1250067, arXiv:1201.4720 [hep-th].

[24] M. Vasiliev, *Higher spin gauge theories in various dimensions*, Fortsch. Phys. 52 (2004) 702–717, [arXiv:hep-th/0401177];

D. Sorokin, *Introduction to the classical theory of higher spins*, AIP Conf. Proc. 767 (2005) 172–202, [arXiv:hep-th/0405069];

N. Bouatta, G. Compère, A. Sagnotti, *An introduction to free higher-spin fields*, [arXiv:hep-th/0409068];

X. Bekaert, S. Cnockaert, C. Iazeolla, M.A. Vasiliev, *Nonlinear higher spin theories in various dimensions*, [arXiv:hep-th/0503128];

A. Fotopoulos, M. Tsulaia, *Gauge Invariant Lagrangians for Free and Interacting Higher Spin Fields. A review of BRST formulation*, Int.J.Mod.Phys. A24 (2008) 1–60, [arXiv:0805.1346[hep-th]];
I.L. Buchbinder and A. Reshetnyak, General Lagrangian Formulation for Higher Spin Fields with Arbitrary Index Symmetry. I. Bosonic fields, Nucl. Phys. B 862 (2012) 270-323, [arXiv:1110.5044[hep-th]].

A. Reshetnyak, General Lagrangian Formulation for Higher Spin Fields with Arbitrary Index Symmetry. 2. Fermionic fields Nucl. Phys. B 869 (2013) 523-597, [arXiv:1211.1273[hep-th]].

[25] I.A. Batalin and G.A. Vilkovisky, Gauge algebra and quantization, Phys. Lett. 102B (1981) 27;

[26] I.A. Batalin and G.A. Vilkovisky, Quantization of gauge theories with linearly dependent generators, Phys. Rev. D28 (1983) 2567.

[27] O. Radchenko and A. Reshetnyak, Notes on soft breaking of BRST symmetry in the Batalin-Vilkovisky formalism, Russ.Phys.J. 55 (2013) 1005-1010, arXiv:1210.6140 [hep-th].

[28] P. Lavrov and O. Lechtenfeld, Field-dependent BRST transformations in Yang-Mills theory, Phys.Lett. B725 (2013) 382-385, arXiv:1305.0712[hep-th].

[29] S.D. Joglekar and B.P. Mandal, Finite field dependent BRS transformations, Phys. Rev. D51 (1995) 1919.

[30] S.D. Joglekar, Connecting Green’s functions in an arbitrary pair of gauges and an application to planar gauges, IJMPA 16 (2001) 5043.

[31] S. Upadhyay, S.K. Rai and B.P. Mandal, Off-Shell Nilpotent Finite BRST/Anti-BRST Transformations, J. Math. Phys. 52 (2011) 022301, arXiv:1002.1373hep-th].

[32] P. Lavrov and O. Lechtenfeld, Gribov horizon beyond the Landau gauge, Phys.Lett. B725 (2013) 386-388, arXiv:1305.2931[hep-th].

[33] B.S. DeWitt, Dynamical theory of groups and fields, Gordon and Breach, 1965.

[34] O.M. Khudaverdian and A.P. Nersessian, On the geometry of the Batalin-Vilkovisky formalism Mod.Phys.Lett. A8 (1993) 2377-2386, [arXiv:hep-th/9303136].

[35] I.A. Batalin and I.V. Tyutin, On possible generalizations of field - antifield formalism, Int.J.Mod.Phys. A8 (1993) 2333-2350, [arXiv:hep-th/9211096];

On the multilevel generalization of the field - antifield formalism Mod.Phys.Lett. A8 (1993) 3673-3682, [arXiv:hep-th/9309011];

On the multilevel field - antifield formalism with the most general Lagrangian hypergauges Mod.Phys.Lett. A9 (1994) 1707-1716, [arXiv:hep-th/9403180].

[36] A.S. Schwarz, Geometry of Batalin-Vilkovisky quantization, Commun.Math.Phys. 155 (1993) 249-260, [arXiv:hep-th/9210588];

M. Alexandrov, M. Kontsevich, A. Schwarz, O. Zaboronsky, The geometry of the master equation and topological quantum field theory, Int. J. Modern Phys. A 12 (1997) 1405-1429.
[37] P.M. Lavrov, P.Yu. Moshin and A.A. Reshetnyak, Superfield formulation of the Lagrangian BRST quantization method, Mod.Phys.Lett. A10 (1995) 2687-2694, [arXiv:hep-th/9507104].

[38] D.M. Gitman, P.Yu. Moshin and A.A. Reshetnyak, Local superfield Lagrangian BRST quantization, J.Math.Phys. 46 (2005) 072302, [arXiv:hep-th/0507160]; An Embedding of the BV quantization into an N=1 local superfield formalism, Phys.Lett. B621 (2005) 295-308, [arXiv:hep-th/0507046].

[39] A.A. Reshetnyak, The Effective action for superfield Lagrangian quantization in reducible hypergauges, Russ.Phys.J. 47 (2004) 1026-1036, [arXiv:hep-th/0512327].

[40] I.A. Batalin and K. Bering, On generalized gauge-fixing in the field-antifield formalism, Nucl.Phys. B739 (2006) 389-440, [arxiv:hep-th/0512131].

[41] A. Kiselev, The geometry of variations in Batalin-Vilkovisky formalism, Journal of Physics: Conference Series 474 (2013) 012024, 1-51 [arXiv:1312.1262 [math-ph]].

[42] G. Leibbrandt, Introduction to the technique of the dimensional regularization, Rev. Mod. Phys. 47 (1975) 849.

[43] K.E. Kallosh and I.V. Tyutin, The equivalence theorem and gauge invariance in renormalizable theories, Sov. J. Nucl. Phys. 17 (1973) 98.

[44] I.V. Tyutin, Once again on the equivalence theorem, Phys. Atom. Nucl. 65 (2002) 194-202, [arxiv:hep-th/0001050].

[45] S.R. Esipova, P.M. Lavrov and O.V. Radchenko, Int. J. Mod. Phys. A 29 (2014) 1450065, arXiv:1312.2802[hep-th].

[46] I.A. Batalin, P.L. Lavrov and I.V. Tyutin, An Sp(2)covariant quantization of gauge theories with linearly dependent generators, J. Math. Phys. 32, (1991) 532.

[47] B.L. Voronov, I.V. Tyutin, Formulation of gauge theories of general form. I, Theor. Math. Phys. 50 (1982) 218-225.

[48] C. Wetterich, Average Action And The Renormalization Group Equations. Nucl. Phys. B352 (1991) 529.

[49] M. Reuter and C. Wetterich, Average action for the Higgs model with abelian gauge symmetry, Nucl. Phys. B391 (1993) 147.

[50] M. Reuter and C. Wetterich, Effective average action for gauge theories and exact evolution equations, Nucl. Phys. B417 (1994) 181.

[51] P. Lavrov and I. Shapiro, On the Functional Renormalization Group approach for Yang-Mills fields, JHEP, 1306 (2013) 086, [arXiv:1212.2577[hep-th]].
[52] J. Polchinski, *Renormalization and effective lagrangians*, Nucl. Phys. B231, 269 (1984).

[53] A.A. Slavnov, *Ward identities in gauge theories*, Theor. Math. Phys. 10 (1972) 99.

[54] J.C. Taylor, *Ward identities and charge renormalization of the Yang-Mills field*, Nucl. Phys. B33 (1971) 436.

[55] W. Pauli, F. Villars, *On the Invariant Regularization in Relativistic Quantum Theory*, Rev. Mod. Phys. 21 (1949) 434-444.

[56] D. Dudal, J. A. Gracey, S.P. Sorella et all, *A refinement of the Gribov-Zwanziger approach in the Landau gauge: infrared propagators in harmony with the lattice results*, Phys.Rev. D78 (2008) 065047, arXiv:0806.0348[hep-th].

[57] D. Dudal, J.A. Gracey, S.P. Sorella et al , *The Landau gauge gluon and ghost propagator in the refined Gribov-Zwanziger framework in 3 dimensions* Phys.Rev. D78 (2008) 125012, arXiv:0808.0893[hep-th].

[58] D. Dudal, S. Sorella, N. Vandersickel, H. Verschelde, *A Renormalization group invariant scalar glueball operator in the (Refined) Gribov-Zwanziger framework*, JHEP 0908 (2009) 110, [arXiv:0906.4257[hep-th]].

[59] S. Sorella, D. Dudal, S. Guimaraes, N. Vandersickel, *Features of the Refined Gribov-Zwanziger theory: Propagators, BRST soft symmetry breaking and glueball masses*, PoS FACESQCD (2010) 022, [arXiv:1102.0574[hep-th]].

[60] D. Dudal, S. Sorella, N. Vandersickel, *The dynamical origin of the refinement of the Gribov-Zwanziger theory*, Phys.Rev. D84 (2011) 065039, [arXiv:1105.3371[hep-th]].

[61] D. Dudal, S.P. Sorella and N. Vandersickel, *More on the renormalization of the horizon function of the Gribov-Zwanziger action and the Kugo-Ojima Green function(s)*, Eur. Phys. J. C 68 (2010) 283, [arXiv:1001.3103 [hep-th]].

[62] I.A. Batalin, P.M. Lavrov and I.V. Tyutin, *A systematic study of finite BRST-BFV transformations in generalized Hamiltonian formalism*, arXiv:1404.4154[hep-th].

[63] I.A. Batalin, P.M. Lavrov, I.V. Tyutin, *A systematic study of finite BRST-BV transformations in field-antifield formalism*, arXiv:1405.2621[hep-th].

[64] P.Yu. Moshin and A.A. Reshetnyak, *Field-dependent BRST-antiBRST Transformations in Yang-Mills and Gribov-Zwanziger Theories*, Nucl. Phys. B 888C (2014) 92-128, arXiv:1405.0790 [hep-th].

[65] P.Yu. Moshin and A.A. Reshetnyak, *Finite BRST-antiBRST Transformations in Lagrangian Formalism*, arXiv:1406.0179[hep-th].

[66] P.Yu. Moshin and A.A. Reshetnyak, *Field-Dependent BRST-antiBRST Lagrangian Transformations*, arXiv:1406.5086[hep-th].
[67] P.Yu. Moshin and A.A. Reshetnyak, *Finite BRST-antiBRST Transformations in Generalized Hamiltonian Formalism*, Int. J. Mod. Phys. A (2014), arXiv:1405.7549 [hep-th].

[68] I.A. Batalin, P.M. Lavrov and I.V. Tyutin, *A systematic study of finite BRST-BFV Transformations in Sp(2)-extended generalized Hamiltonian formalism*, arXiv:1405.7218[hep-th].

[69] A. Reshetnyak, *On composite fields approach to Gribov copies elimination in Yang-Mills theories*, Phys.Part.Nucl. 11 (2014) 1-4, arXiv:1402.3060[hep-th].