On the symmetry of a Preisach map

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Abstract

At the very heart of the successful phenomenological model of magnetic hysteresis there is the so called Preisach distribution. In the existing literature it is implicitly assumed, that this distribution has a mirror symmetry. We show, by simple and convincing example, that this common assumption is plainly wrong. Dropping it, we gain the ability to model not only the usual hysteresis loops (major and minor) more accurately than ever before, but also those displaying the exchange bias effect, what is impossible within the framework of the symmetrical Preisach model. It is hoped, that our observation paves the way towards the unified description of all hysteretic systems, including, but not necessarily limited to, superconductors, (multi)layered structures, nanocrystalline materials, patterned media, and – perhaps – the other non-magnetic hysteretic phenomena.

Introduction

The major hysteresis loop, observed in the sizable samples of homogeneous ferromagnetic materials, exhibits well known symmetry:

\[ M_{lb}(H) = -M_{ub}(-H), \]

where \( M_{lb} \) (\( M_{ub} \)) denotes the lower (upper) branch of the sample’s magnetization, \( M \) vs. exciting field \( H \). The hysteresis curve, not necessarily the major loop, but also the response to the arbitrary sequence of exciting fields as well, can be described, or modelled, in many ways. One of them, the Classical Preisach Model (CPM), was first proposed by Ferenc Preisach [1] and then subsequently developed, generalized and tested by many researchers. In this model, the change of the sample’s magnetization is expressed by the double integral:

\[
\Delta M = M(H_f) - M(H_i) = 2M_s \int \int_{H_i \leq H \leq H_f} \varrho(H_1, H_2) \, dH_1 \, dH_2,
\]

for the monotonously increasing field (‘i’ – the initial state, ‘f’ – final) and by

\[
\Delta M = M(H_f) - M(H_i) = -2M_s \int \int_{H_i \leq H \leq H_f} \varrho(H_1, H_2) \, dH_1 \, dH_2,
\]

if the field is monotonously decreasing. \( M_s \) is the saturation magnetization, while the distribution \( \varrho(H_1, H_2) \geq 0 \), supported over the domain \( H_1 \geq H_2 \) is called the Preisach density. With the additional condition:

\[
\int \int dH_1 dH_2 \varrho(H_1, H_2) = 1,
\]

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the Preisach density is commonly regarded as a probability distribution. It describes the probability of encountering the so called hysteron, i.e. the hypothetical object characterized by elementary, rectangularly shaped, hysteresis loop with up- and down-switching fields equal to $H^\uparrow$ and $H^\downarrow$, respectively.

It is obvious, that the symmetry presented in Eq. (1) will be preserved, if the Preisach distribution, $\varrho$, is also symmetric, namely when

$$\varrho(H^\uparrow, H^\downarrow) = \varrho(-H^\downarrow, -H^\uparrow)$$

holds for any pair $(H^\uparrow, H^\downarrow)$ of its arguments, $H^\uparrow \geq H^\downarrow$. The mirror symmetry of $\varrho(H^\uparrow, H^\downarrow)$ with respect to the line $H^\downarrow = -H^\uparrow$ is therefore commonly assumed, before any reconstruction of the Preisach distribution is ever attempted. In this paper we are going to show, that symmetric hysteresis loops can be generated by Preisach distributions, which themselves have no mirror symmetry. In other words: while the mirror symmetry of $\varrho$ indeed implies

![Figure 1: An example of the non-symmetric Preisach map, which generates symmetric major hysteresis loop. Initially the only components of the map are two symmetrically located pairs of positive, delta-shaped peaks, with identical amplitudes, labeled as $(P, Q)$ and $(P', Q')$ respectively](image)

the experimentally observed symmetry properties of hysteresis loops, then the inverse need not to be true. The idea that the Preisach distribution has to be symmetric is present in the literature since almost 50 years [2]. It seems so obvious that nobody ever cared to prove it formally. For example, Mayergoyz (1986) wrote in [3]

... it can be easily proved that $\mu(\alpha, \beta) = \mu(-\beta, -\alpha)$

however he didn’t present the proof (Mayergoyz’s $\mu$ is our $\varrho$).

Hints for possible asymmetry Let us try to prove it now by reduction $\textit{ad absurdum}$. Assume that we have a nonsymmetric distribution and let’s see whether it necessarily has to generate the non-symmetric hysteresis loop. The distribution is non-symmetric, if there exists at least one point $(H^\uparrow_0, H^\downarrow_0)$ in its support, such that the condition $\varrho(H^\uparrow_0, H^\downarrow_0) = \varrho(-H^\downarrow_0, -H^\uparrow_0)$ is violated. But if this indeed is the case, then the integrals (2) and (3)
remain unchanged, since a single point is a zero-measure set on a Preisach plane! As we can see, it is impossible to prove the well known claim. One could argue that our finding has no physical meaning, since such kind of asymmetry cannot be detected in any real experiment and, consequently, it should be regarded only as a mathematical toy. Eventually we might modify the original claim to read: the Preisach distribution is symmetric almost everywhere on a Preisach plane. However, we are not going to stop at this point and shall present a well-founded, physical counterexample.

Figure 2: Stacked hysteresis loops generated by non-symmetric Preisach distribution presented in Fig. 1 for various amplitudes of the exciting field $[-H, +H]$. For small amplitudes there is a null response (hence the loop is symmetric), in the intermediate regime the loop becomes asymmetric and exhibits the so called exchange bias effect, while the major loop (lowest) is symmetric again.

**Derivation of the main result** Our counterexample uses four Dirac’s delta functions as the sole components of a Preisach distribution. This is not completely crazy idea, since the delta peaks are the only mathematical entities able to reproduce well known Barkhausen jumps. Those jumps appear sometimes very stable against repeated magnetization reversals. The same is true for the domain structures [4, 5]. The other advantage is that the single-point support of a delta function is no longer a zero-measure set for integrals (2) and (3). We are not original introducing delta functions into Preisach model (see Pescetti [6] 1989), so our derivation cannot be viewed only as a mathematical curiosity.

Consider Fig. 1. Initially the Preisach distribution consists of two delta peaks of equal amplitudes, located at points labeled as $P$ and $Q$, and their symmetrical images $P'$ and $Q'$, respectively. Such a distribution gives rise, of course, to symmetrical hysteresis loops. But now we destroy the initial symmetry by moving points $Q \rightarrow Q''$ and $P' \rightarrow P''$, as indicated by arrows. Having done so, we can observe (Fig. 2) that the major hysteresis loop, as well as some minor loops, remain symmetric, although their overall shape has changed. There are also some minor loops, which are asymmetric now, imitating the so called exchange bias effect.

It is also possible to rearrange the four initial delta peaks differently, in such a way as to preserve the symmetry of all minor loops with turning points of equal magnitudes. This can be achieved by rotating the pairs $(P, Q)$ and $(P', Q')$ around points $P$ and $Q'$ respectively by the same angle $\varphi$. Let initially $P = (H_0, -H_2)$, $Q = (H_0, -H_1)$, with $d = |H_1 - H_2|$. After rotation, the jumps on the lower branch of the hysteresis loop occur at fields: $H_0$, $H_0 + d \sin \varphi$, $H_1$ and $H_1 + d \cos \varphi$ (in increasing order). Upper branch jumps (in decreasing order) are located at: $-H_0$, $-H_0 - d \sin \varphi$, $-H_2 + d \cos \varphi$ and $-H_2$. One can easily see, that in all cases...
\[ H_{\text{jump}}^{\text{lb}} = -H_{\text{jump}}^{\text{ub}}, \]

what proves that all the relevant symmetry properties of the hysteresis loops are indeed preserved.

**Comments** Our construction strongly suggests that the true (asymmetric) Preisach distribution may be a conformal image of some ‘ideal’, i.e. symmetric map. It is worth to note, however, that conformal mapping of the Preisach triangle onto itself, if applicable at all, cannot be arbitrary. Ordinary Euclidean transformations, i.e. translations and rotations, are useless (shifted distribution fails to describe exchange biased loops, see [7]). It remains unclear, whether the asymmetry of the Preisach distribution alone would be able to describe correctly the recently reported [8] hysteresis loops exhibiting negative remanence.

**Conclusions** We have shown that, contrary to the common belief, the Preisach distribution need not to be symmetric. This seems to remove the apparent disagreement between the distributions recovered from experimental data on one of the classical ways [9, 10, 11] (which implicitly assume the existence of symmetry questioned here, sometimes even including the postulated particular shape of the map to be reconstructed) with those obtained by the recently introduced FORC (First Order Reversal Curves) [12, 13] diagrams technique, which reveals the asymmetry quite often; see [14, 15, 16] and especially impressive figures presented by Robb, Novotny and Rikvold [17].

The construction of FORC diagrams implicitly suppresses the reversible part of the Preisach map, i.e. the one located on the line \( H^\uparrow = H^\downarrow \). In our opinion, it is the combination of both techniques, what should constitute the complete tool for physical, as opposed to purely phenomenological, characterization of magnetic systems and their internal interactions.

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