Expanded calculation of weak-interaction-mediated neutrino cooling rates due to $^{56}\text{Ni}$ in stellar matter

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Abstract

An accurate estimate of the neutrino cooling rates is required in order to study the various stages of stellar evolution of massive stars. Neutrino losses from proto-neutron stars play a crucial role in deciding whether these stars would be crushed into black holes or explode as supernovae. Both pure leptonic and weak-interaction processes contribute to the neutrino energy losses in stellar matter. At low temperatures and densities, the characteristics of the early phase of presupernova evolution, cooling through neutrinos produced via the weak interaction, are important. Proton–neutron quasi-particle random phase approximation (pn-QRPA) theory has recently been used with success for the calculation of stellar weak-interaction rates of $^{fp}$-shell nuclide. The lepton-to-baryon ratio ($Y_e$) during early phases of stellar evolution of massive stars changes substantially, mainly due to electron captures on $^{56}\text{Ni}$. The stellar matter is transparent to the neutrinos produced during the presupernova evolution of massive stars. These neutrinos escape the site and assist the stellar core in maintaining a lower entropy. Here, an expanded calculation of weak-interaction-mediated neutrino and antineutrino cooling rates due to $^{56}\text{Ni}$ in stellar matter using the pn-QRPA theory is presented. This detailed scale is appropriate for interpolation purposes and is of greater utility for simulation codes. The calculated rates are compared with earlier calculations. During the relevant temperature and density regions of stellar matter the reported rates show few differences compared with the shell model rates and might contribute in fine-tuning of the lepton-to-baryon ratio during the presupernova phases of stellar evolution of massive stars.

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(Some figures in this article are in colour only in the electronic version.)

1. Introduction

It was the genius of Baade and Zwicky [1] that they were able to deduce the total energy released in a supernova explosion to be of the order of $(3 \times 10^{51}$ to $10^{55})$ erg on the basis of a few points of the light curve without any spectral information available at that time. Later, Colgate and White [2] and Arnett [3] presented their classical work on energy transport by neutrinos and antineutrinos in non-rotating massive stars. Since then, we have come a long way, and despite the immense technological advancements the explosion mechanism of core-collapse supernovae continues to pose challenges for collapse simulators throughout the globe. The prompt shock that follows the bounce stagnates and is incapable of producing on its own a supernova explosion. The stagnation is due to energy losses in disintegration of iron nuclei (so far cooked in the stellar pot) and through neutrino emissions (mainly non-thermal). The stellar matter was till then transparent to the neutrinos emitted. A few milliseconds after the bounce, the proto-neutron star accretes mass at a few tenths of solar mass per second. This accretion, if continued even for 1 s, can change the ultimate fate of the collapsing
core resulting in a black hole. Neutrinos have a crucial role to play in this scenario and radiate around 10% of the rest mass, converting the star to a neutron star. Initially, the nascent neutron star is a hot thermal bath of dense nuclear matter, \( e^−e^+ \) pairs, photons and neutrinos. Neutrinos, having the weak interaction, are most effective in cooling and diffuse outward within a few seconds, and eventually escape with about 99% of the released gravitational energy. Despite the small neutrino-nucleus cross sections, the neutrino flux generated by the cooling of a neutron star can produce a number of nuclear transmutations as it passes the onion-like structured envelope surrounding the neutron star. The microphysics involved in these extreme processes is indeed complex and one should be very cautious in interpolating and/or extrapolating values of stellar parameters during various phases of stellar evolution. A great many physical inputs are required at the beginning of each stage of the entire simulation process (e.g. collapse of the core, formation, stalling and revival of the shock wave and shock propagation). It is highly desirable to calculate these parameters with the most reliable physical data and inputs.

During the late phases of evolution of massive stars, an iron core develops (of mass around 1.5\( M_⊙ \)). Capture rates and photodisintegration processes contribute to lowering the degeneracy pressure required to counter the enormous self-gravity force of the star. Under such extreme thermodynamic conditions, neutrinos are produced in abundance. Eventually the collapse of the iron core begins. The mechanism of core-collapse supernovae is strongly believed to depend upon the transfer of energy from the inner core to the outer mantle of the iron core. Neutrinos seem to be the mediators of this energy transfer. As mentioned above, the shock wave produced as a result stalls due to photodisintegration and neutrino energy losses. Once again, the part played by neutrinos in this scenario is far from being completely understood. In the late-time neutrino heating mechanism, the stalled shock can be revived (about 1 s after the bounce) and may be driven as a delayed explosion [4]. However, to date there have been no successfully simulated spherically symmetric explosions. Even the two-dimensional (2D) simulations (addition of convection) performed with a Boltzmann solver for the neutrino transport fails to convert the collapse into an explosion [5]. (Recently, a few simulation groups (e.g. [6, 7, 8]) have reported successful explosions in 2D mode.) Additional energy sources (e.g. magnetic fields and rotations) were also sought that might transport energy to the mantle and lead to an explosion. Worldwide, core-collapse simulators are still working hard to come up with a convincing and decisive mode of producing explosions.

Neutrinos from core-collapse supernovae are unique messengers of the microphysics of supernovae and are crucial to the life and afterlife of supernovae. They provide information regarding the neutronization due to electron capture, the infall phase, the formation and propagation of the shock wave and the cooling phase. Cooling rate is one of the crucial parameters that strongly affect the stellar evolution. In stellar matter the neutrinos are produced from both weak-interaction reactions and pure leptonic processes. The latter includes pair annihilation, bremsstrahlung on nuclei, plasmon decay and \( \nu^- \)-photoproduction processes. White dwarfs and supernovae (which are the endpoints for stars of varying masses) both have cooling rates largely dominated by neutrino production. A cooling proto-neutron star emits about 3 \( \times 10^{53} \) erg in neutrinos, with the energy roughly equipartitioned among all species. The neutrino energy loss rates are important input parameters in multi-dimensional simulations of the contracting proto-neutron star. Parameter-free multi-dimensional models, with neutrino transport included consistently throughout the entire mass, yield conflicting results on the key issue of whether the star actually explodes. Reliable and microscopic calculations of neutrino loss rates and capture rates can contribute effectively in the final outcome of these simulations. Whereas neutrinos produced via pure leptonic processes dominate in the very high temperature–density domain during the very late phases of stellar evolution, the weak-interaction neutrinos also play an important role in cooling the core to a lower entropy especially during the early phases of stellar evolution. This work is primarily devoted to calculating the neutrino energy loss rates due to weak-interaction reactions (capture and \( \beta^- \) decays) on \( ^{56}\text{Ni} \).

\(^{56}\text{Ni}\) is abundant in the presupernova conditions, and weak-interaction reactions on this nucleus are believed to contribute effectively to the dynamics of presupernova evolution. Aufderheide et al [9] ranked \(^{56}\text{Ni}\) as the third most important electron capture nucleus averaged throughout the stellar trajectory for 0.4 \( \leq Y_e \leq 0.5 \) during the presupernova evolution. Later Heger et al [10] also identified \(^{56}\text{Ni}\) as one of the most important nuclide for capture purposes for the presupernova evolution of massive stars (25\( M_⊙ \) and 40\( M_⊙ \)). Realizing the importance of \(^{56}\text{Ni}\) in astrophysical environments, Nabi and Rahman [11] reported the calculation of electron capture rates on \(^{56}\text{Ni}\) using the proton–neutron quasi-particle random phase approximation (pn-QRPA) theory (see also [12]) regarding the calculation of ground and excited state Gamow-Teller (GT) strength distributions of \(^{56}\text{Ni}\)). In a recent review of the theory of core-collapse supernovae, Janka et al [13] again discussed the importance of electron capture rates on \(^{56}\text{Ni}\) in presupernova evolution of massive stars. It can be seen from figure 1 ([13]) that during the onset of collapse (where \( t \approx 0 \) s) to the later stages (\( t \approx 0.11 \) s), iron and nickel isotopes are present in reasonable quantity. According to Aufderheide et al [9], for \( Y_e \) around 0.5, \(^{56}\text{Ni}\) is the most abundant nucleus having a mass fraction of around 0.99. Consequently, electron capture rates on \(^{56}\text{Ni}\) are an important process during these phases. During later stages after the bounce and shock propagation (\( t \approx 0.12 \) s) the photodisintegration of iron-group nuclei to alpha particles and protons results, which marks the beginning of the proto-neutron star.

In this paper, I analyze the weak-interaction neutrino energy loss rates (which I term as neutrino cooling rates throughout this text) due to this key isotope of nickel, which is so abundant during the silicon burning phases of the stellar core. Due to the extreme conditions prevailing in these scenarios, interpolation of calculated rates within large intervals of temperature–density points might pose some uncertainty in the values of weak rates for collapse simulators. In this paper, I describe the calculation of the neutrino and antineutrino cooling rates due to capture and decay rates on
$^{56}\text{Ni}$ on an expanded temperature–density grid suitable for collapse simulation codes. Section 2 briefly discusses the formalism of the pn-QRPA calculations and presents some of the calculated results. Comparison with earlier calculations during stellar evolution of massive stars is also included in this section. I summarize the main conclusions in section 3, and table 1 presents the expanded calculation of neutrino and antineutrino cooling rates due to $^{56}\text{Ni}$ in stellar matter.

**Table 1.** Weak-interaction-mediated neutrino and antineutrino cooling rates due to $^{56}\text{Ni}$ for selected densities and temperatures in stellar matter. log$\rho Y_e$ has units of g cm$^{-3}$, where $\rho$ is the baryon density and $Y_e$ is the ratio of the lepton number to the baryon number. Temperatures ($T_0$) are given in units of 10$^6$ K. The calculated Fermi energy is denoted by $E_f$ and is given in units of MeV. $\lambda_\nu$ ($\lambda_\bar{\nu}$) are the total neutrino (antineutrino) cooling rates as a result of $\beta^+$ decay and electron capture ($\beta^-$ decay and positron capture) in units of MeV s$^{-1}$. All calculated rates are tabulated in logarithmic (to base 10) scale. In the table, $-100$ means that the rate is smaller than $10^{-100}$ MeV s$^{-1}$.

| log$\rho Y_e$ | $T_0$ | $E_f$ | $\lambda_\nu$ | $\lambda_\bar{\nu}$ |
|--------------|-------|-------|---------------|---------------------|
| 0.5          | 0.50  | 0.065 | -6.917        | -100                |
| 0.5          | 1.00  | 0.000 | -6.826        | -81.247             |
| 0.5          | 1.50  | 0.000 | -6.152        | -54.126             |
| 0.5          | 2.00  | 0.000 | -5.389        | -40.456             |
| 0.5          | 2.50  | 0.000 | -4.782        | -32.190             |
| 0.5          | 3.00  | 0.000 | -4.290        | -26.636             |
| 0.5          | 3.50  | 0.000 | -3.873        | -22.363             |
| 0.5          | 4.00  | 0.000 | -3.506        | -19.609             |
| 0.5          | 4.50  | 0.000 | -3.167        | -17.232             |
| 0.5          | 5.00  | 0.000 | -2.843        | -15.312             |
| 0.5          | 5.50  | 0.000 | -2.529        | -13.724             |
| 0.5          | 6.00  | 0.000 | -2.223        | -12.386             |
| 0.5          | 6.50  | 0.000 | -1.928        | -11.241             |
| 0.5          | 7.00  | 0.000 | -1.645        | -10.249             |
| 0.5          | 7.50  | 0.000 | -1.376        | -9.378              |
| 0.5          | 8.00  | 0.000 | -1.121        | -8.606              |
| 0.5          | 8.50  | 0.000 | -0.879        | -7.917              |
| 0.5          | 9.00  | 0.000 | -0.650        | -7.297              |
| 0.5          | 9.50  | 0.000 | -0.433        | -6.735              |
| 0.5          | 10.00 | 0.000 | -0.227        | -6.222              |
| 0.5          | 20.00 | 0.000 | -0.821        | -0.000              |
| 0.5          | 30.00 | 0.000 | 3.787         | 1.457               |
| 1.0          | 0.50  | 0.113 | -6.916        | -100                |
| 1.0          | 1.00  | 0.000 | -6.826        | -81.249             |
| 1.0          | 1.50  | 0.000 | -6.151        | -54.126             |
| 1.0          | 2.00  | 0.000 | -5.388        | -40.456             |
| 1.0          | 2.50  | 0.000 | -4.781        | -32.190             |
| 1.0          | 3.00  | 0.000 | -4.289        | -26.636             |
| 1.0          | 3.50  | 0.000 | -3.872        | -22.363             |
| 1.0          | 4.00  | 0.000 | -3.505        | -19.609             |
| 1.0          | 4.50  | 0.000 | -3.166        | -17.231             |
| 1.0          | 5.00  | 0.000 | -2.842        | -15.310             |
| 1.0          | 5.50  | 0.000 | -2.528        | -13.723             |
| 1.0          | 6.00  | 0.000 | -2.222        | -12.385             |
| 1.0          | 6.50  | 0.000 | -1.927        | -11.240             |
| 1.0          | 7.00  | 0.000 | -1.644        | -10.246             |
| 1.0          | 7.50  | 0.000 | -1.375        | -9.375              |
| 1.0          | 8.00  | 0.000 | -1.119        | -8.604              |
| 1.0          | 8.50  | 0.000 | -0.877        | -7.915              |
| 1.0          | 9.00  | 0.000 | -0.648        | -7.294              |
| 1.0          | 9.50  | 0.000 | -0.431        | -6.732              |
| 1.0          | 10.00 | 0.000 | -0.225        | -6.220              |
| 1.0          | 20.00 | 0.000 | 2.436         | -0.817              |
| 1.0          | 30.00 | 0.000 | 3.790         | 1.461               |
| log$\rho Y_e$ | $T_9$ | $E_1$ | $\lambda_v$ | $\lambda_\nu$ |
|----------------|--------|--------|-------------|-------------|
| 3.0            | 0.50   | 0.311  | −6.828      | −100        |
| 3.0            | 1.00   | 0.046  | −6.773      | −81.476     |
| 3.0            | 1.50   | 0.005  | −6.138      | −54.141     |
| 3.0            | 2.00   | 0.001  | −5.384      | −40.459     |
| 3.0            | 2.50   | 0.001  | −4.780      | −32.191     |
| 3.0            | 3.00   | 0.000  | −4.288      | −26.636     |
| 3.0            | 3.50   | 0.000  | −3.872      | −22.635     |
| 3.0            | 4.00   | 0.000  | −3.504      | −19.608     |
| 3.0            | 4.50   | 0.000  | −3.165      | −17.231     |
| 3.0            | 5.00   | 0.000  | −2.842      | −15.310     |
| 3.0            | 5.50   | 0.000  | −2.527      | −13.722     |
| 3.0            | 6.00   | 0.000  | −2.222      | −12.384     |
| 3.0            | 6.50   | 0.000  | −1.927      | −11.239     |
| 3.0            | 7.00   | 0.000  | −1.644      | −10.246     |
| 3.0            | 7.50   | 0.000  | −1.375      | −9.375      |
| 3.5            | 8.00   | 0.000  | −1.119      | −8.604      |
| 3.5            | 8.50   | 0.000  | −0.877      | −7.914      |
| 3.5            | 9.00   | 0.000  | −0.648      | −7.294      |
| 3.5            | 9.50   | 0.000  | −0.431      | −6.732      |
| 3.5            | 10.00  | 0.000  | −0.225      | −6.219      |
| 3.5            | 20.00  | 0.000  | 2.436       | 0.816       |
| 3.5            | 30.00  | 0.000  | 3.791       | 1.463       |
| 3.5            | 50.00  | 0.000  | 6.682       | 100         |
| 3.5            | 100.00 | 0.000  | 13.747      | 25.289      |
| 4.5            | 0.50   | 0.464  | −6.009      | −100        |
| 4.5            | 1.00   | 0.309  | −5.953      | −82.804     |
| 4.5            | 1.50   | 0.130  | −5.768      | −54.561     |
| 4.5            | 2.00   | 0.044  | −5.281      | −40.565     |
| 4.5            | 2.50   | 0.020  | −4.741      | −32.230     |
| 4.5            | 3.00   | 0.011  | −4.270      | −26.654     |
| 4.5            | 3.50   | 0.007  | −3.862      | −22.645     |
| 4.5            | 4.00   | 0.005  | −3.498      | −19.614     |
| 4.5            | 4.50   | 0.004  | −3.162      | −17.235     |
| 4.5            | 5.00   | 0.003  | −2.839      | −15.313     |
| 4.5            | 5.50   | 0.002  | −2.526      | −13.724     |
| 4.5            | 6.00   | 0.000  | −2.221      | −12.386     |
| 4.5            | 6.50   | 0.002  | −1.926      | −11.240     |

Table 1. Continued.
| $\log Y_c$ | $T_0$ | $E_l$ | $\lambda_y$ | $\lambda_0$ |
|----------|-------|------|-----------|-----------|
| 7.0      | 30.00 | 0.021| 3.795     | 1.460     |
| 7.5      | 8.50  | 1.705| -2.203    | -100      |
| 7.5      | 10.00 | 1.693| -2.178    | -89.781   |
| 7.5      | 15.00 | 1.675| -2.139    | -59.752   |
| 7.5      | 2.00  | 1.648| -2.087    | -44.609   |
| 7.5      | 2.50  | 1.614| -2.026    | -35.444   |
| 7.5      | 3.00  | 1.573| -1.956    | -29.278   |
| 7.5      | 3.50  | 1.524| -1.881    | -24.830   |
| 7.5      | 4.00  | 1.468| -1.800    | -21.457   |
| 7.5      | 4.50  | 1.405| -1.710    | -18.804   |
| 7.5      | 5.00  | 1.336| -1.610    | -16.657   |
| 7.5      | 5.50  | 1.262| -1.493    | -14.878   |
| 7.5      | 6.00  | 1.183| -1.358    | -13.377   |
| 7.5      | 6.50  | 1.101| -1.204    | -12.093   |
| 7.5      | 7.00  | 1.018| -1.035    | -10.799   |
| 7.5      | 7.50  | 0.937| -0.857    | -10.004   |
| 7.5      | 8.00  | 0.858| -0.675    | -9.144    |
| 7.5      | 8.50  | 0.783| -0.494    | -8.378    |
| 7.5      | 9.00  | 0.714| -0.315    | -7.693    |
| 7.5      | 9.50  | 0.651| -0.140    | -7.077    |
| 7.5      | 10.00 | 0.593| 0.030     | -6.518    |
| 7.5      | 20.00 | 0.150| 2.473     | -0.854    |
| 7.5      | 30.00 | 0.066| 3.802     | 1.452     |
| 8.0      | 0.50  | 2.444| -1.253    | -100      |
| 8.0      | 1.00  | 2.437| -1.241    | -93.527   |
| 8.0      | 1.50  | 2.424| -1.221    | -62.269   |
| 8.0      | 2.00  | 2.406| -1.194    | -46.518   |
| 8.0      | 2.50  | 2.383| -1.160    | -36.994   |
| 8.0      | 3.00  | 2.355| -1.120    | -30.591   |
| 8.0      | 3.50  | 2.322| -1.074    | -25.978   |
| 8.0      | 4.00  | 2.283| -1.020    | -22.484   |
| 8.0      | 4.50  | 2.240| -0.959    | -19.739   |
| 8.0      | 5.00  | 2.192| -0.886    | -17.520   |
| 8.0      | 5.50  | 2.139| -0.799    | -15.682   |
| 8.0      | 6.00  | 2.081| -0.696    | -14.132   |
| 8.0      | 6.50  | 2.019| -0.577    | -12.804   |
| 8.0      | 7.00  | 1.952| -0.444    | -11.652   |
| 8.0      | 7.50  | 1.882| -0.301    | -10.640   |
| 8.0      | 8.00  | 1.808| -0.153    | -9.743    |
| 8.0      | 8.50  | 1.732| -0.003    | -8.941    |
| 8.0      | 9.00  | 1.653| 0.145     | -8.220    |
| 8.0      | 9.50  | 1.574| 0.290     | -7.566    |
| 8.0      | 10.00 | 1.523| 0.430     | -7.030    |
| 8.0      | 20.00 | 0.470| 2.551     | -0.935    |
| 8.0      | 30.00 | 0.209| 3.826     | 1.428     |
| 8.5      | 0.50  | 3.547| -0.270    | -100      |
| 8.5      | 1.00  | 3.542| -0.264    | -99.099   |
| 8.5      | 1.50  | 3.534| -0.253    | -65.998   |
| 8.5      | 2.00  | 3.521| -0.238    | -49.329   |
| 8.5      | 2.50  | 3.506| -0.218    | -39.257   |
| 8.5      | 3.00  | 3.487| -0.192    | -32.493   |
| 8.5      | 3.50  | 3.464| -0.161    | -27.623   |
| 8.5      | 4.00  | 3.438| -0.122    | -23.939   |
| 8.5      | 4.50  | 3.408| -0.076    | -21.048   |
| 8.5      | 5.00  | 3.375| -0.020    | -18.712   |
| 8.5      | 5.50  | 3.339| 0.047     | -16.781   |
| 8.5      | 6.00  | 3.299| 0.125     | -15.155   |
| 8.5      | 6.50  | 3.256| 0.215     | -13.763   |
| 8.5      | 7.00  | 3.209| -0.317    | -12.557   |
| 8.5      | 7.50  | 3.159| 0.428     | -11.498   |
| 8.5      | 8.00  | 3.106| 0.545     | -10.560   |
| 8.5      | 8.50  | 3.050| 0.666     | -9.722    |
| 8.5      | 9.00  | 2.990| 0.789     | -8.968    |
| 8.5      | 9.50  | 2.928| 0.910     | -8.285    |
| 8.5      | 10.00 | 2.863| 1.030     | -7.662    |
2. Calculations and results

The Hamiltonian of the pn-QRPA model, the model parameters and their selection criteria were discussed earlier in [12]. The neutrino (antineutrino) cooling rates can occur through four different weak-interaction-mediated channels: electron and positron emissions and continuum electron and positron captures. It is assumed that the stellar matter is transparent to the neutrinos and antineutrinos produced as a result of these reactions during the presupernova evolutionary phases and contributes effectively in cooling the system. The neutrino (antineutrino) cooling rates were calculated using the relation

$$\lambda_{ij}^{\nu} = \left[ \frac{\ln 2}{D} \right] \left[ r f_{ij}^{\nu}(T, \rho, E_i) + \left( \frac{g_i}{g_f} \right)^2 B(GT)_{ij} \right].$$

(1)
The value of $D$ was taken to be $6295$ s $^{14}$. $B_{ij}^\prime$ are the sum of reduced transition probabilities of the Fermi B(F) and GT transitions B(GT). The effective ratio of axial and vector coupling constants, $(g_A/g_V)$, was taken to be $-1.254$ $^{15}$. The $f_{ij}^\nu$ are the phase space integrals and are functions of stellar temperature ($T$), density ($\rho$) and Fermi energy ($E_F$) of the electrons. They are explicitly given by

$$f_{ij}^\nu = \int_{-\infty}^{\infty} w^{\nu - 1}(w_m - w)^3 F(\pm Z, w)(1 - G_\nu)dw,$$

and by

$$f_{ij}^{\nu} = \int_{w_1}^{\infty} w^{\nu - 1}(w_m + w)^3 F(\pm Z, w)G_\nu dw.$$

In equations (2) and (3), $w$ is the total energy of the electron including its rest mass, $w_1$ is the total capture threshold energy (rest-kinetic) for positron (or electron) capture. $F(\pm Z, w)$ are the Fermi functions and were calculated according to the procedure adopted by Gove and Martin $^{16}$. $G_\nu$ is the Fermi–Dirac distribution function for positrons (electrons).

$$G_+ = \left[\exp\left(\frac{E + 2 + E_i}{kT}\right) + 1\right]^{-1},$$

$$G_- = \left[\exp\left(\frac{E - E_i}{kT}\right) + 1\right]^{-1},$$

where $E$ is the kinetic energy of the electrons and $k$ is the Boltzmann constant.

For the decay (capture) channel, equation (2) (equation (3)) was used for the calculation of phase space integrals. Upper signs were used for the case of electron emissions (captures) and lower signs for the case of positron emissions (captures). Details of the calculation of reduced transition probabilities can be found in $^{17}$.

The construction of parent and daughter excited states and calculation of GT transition amplitudes connecting these states within the pn-QRPA model is very important. The excited states in the pn-QRPA model can be constructed as phonon-correlated multi-quasi-particle states. The RPA is formulated for excitations from the $F^F = 0^+$ ground state of an even–even nucleus. The model extended to include the quasi-particle (q.p.) transition degrees of freedom yields decay half-lives of odd-mass and odd–odd parent nuclei with the same quality of agreement with experiment as for even–even nuclei (where only QRPA phonons contribute to the decays) $^{18}$. When the parent nucleus has an odd nucleon, the ground state can be expressed as a one-q.p. state, in which the odd q.p. occupies the single q.p. orbit of the smallest energy. Then there exist two different types of transitions: phonon transitions with the odd nucleon acting only as a spectator and transition of the odd nucleon itself. For the latter case, phonon correlations were introduced into one-q.p. states in first-order perturbation $^{19}$. The transition amplitudes between the multi-q.p. states can be reduced to those of single q.p. states as shown below.

Excited states of $^{56}$Ni can be constructed as two-proton q.p. states and two-neutron q.p. states. Transitions from these initial states to final proton–neutron q.p. pair states in the odd–odd daughter nucleus are possible. The transition amplitudes and their reduction to correlated (c) one-q.p. states are given by

\begin{align}
\langle p'f_1^n f_2^n \mid t_{u}\sigma_{-\mu} \mid p'_l p_2^n \rangle = & -\delta(p', p'_l)\delta(p'_2, p_2^n)\delta(n'_1, n'_2)\langle t_{u}\sigma_{-\mu} \mid p'_l p_2^n \rangle
\langle p'f_1^n f_2^n \mid t_{u}\sigma_{-\mu} \mid p'_l p_2^n \rangle = & +\delta(n'_1, n'_2)\delta(p'_2, p_2^n)\delta(n'_1, n'_2)\langle t_{u}\sigma_{-\mu} \mid p'_l p_2^n \rangle.
\end{align}

where $\mu = -1, 0$ and 1, are the spherical components of the spin operator and $t_{u}$ is the isospin raising and lowering operators.

For an odd–odd nucleus the ground state is assumed to be a proton–neutron q.p. pair state of smallest energy. States in $^{56}$Co, $^{56}$Cu are expressed in q.p. transformation by two-q.p. states (proton–neutron pair states) or by four-q.p. states (two-proton or two-neutron q.p. states). Reduction of two-q.p. states into correlated (c) one-q.p. states is given as

\begin{align}
\langle p'_1 p'_2 n'_1 f_2^n \mid t_{u}\sigma_{-\mu} \mid p'_l p_2^n \rangle = & -\delta(p'_1, p'_l)\delta(p'_2, p_2^n)\delta(n'_1, n'_2)\langle t_{u}\sigma_{-\mu} \mid p'_l p_2^n \rangle,
\langle n'_1 f_1^n f_2^n \mid t_{u}\sigma_{-\mu} \mid p'_l p_2^n \rangle = & +\delta(n'_1, n'_2)\delta(p'_1, p'_l)\delta(p'_2, p_2^n)\langle t_{u}\sigma_{-\mu} \mid p'_l p_2^n \rangle.
\end{align}

while the four-q.p. states are simplified as

\begin{align}
\langle p'_1 p'_2 n'_1 n'_2 f_2^n \mid t_{u}\sigma_{-\mu} \mid p'_1 p'_2 p_2^n p_1^n \rangle = & -\delta(p'_1, p'_l)\delta(p'_2, p_2^n)\delta(p'_2, p_2^n)\langle t_{u}\sigma_{-\mu} \mid p'_1 p'_2 p_2^n p_1^n \rangle
\langle p'_1 p'_2 n'_1 n'_2 f_2^n \mid t_{u}\sigma_{-\mu} \mid p'_1 p'_2 p_2^n p_1^n \rangle = & -\delta(p'_1, p'_l)\delta(p'_2, p_2^n)\delta(p'_2, p_2^n)\langle t_{u}\sigma_{-\mu} \mid p'_1 p'_2 p_2^n p_1^n \rangle.
\end{align}

\begin{align}
\langle p'_1 p'_2 n'_1 n'_2 f_2^n \mid t_{u}\sigma_{-\mu} \mid p'_1 p'_2 p_2^n p_1^n \rangle = & -\delta(p'_1, p'_l)\delta(p'_2, p_2^n)\delta(p'_2, p_2^n)\langle t_{u}\sigma_{-\mu} \mid p'_1 p'_2 p_2^n p_1^n \rangle.
\end{align}

\begin{align}
\langle p'_1 p'_2 n'_1 n'_2 f_2^n \mid t_{u}\sigma_{-\mu} \mid p'_1 p'_2 p_2^n p_1^n \rangle = & -\delta(p'_1, p'_l)\delta(p'_2, p_2^n)\delta(p'_2, p_2^n)\langle t_{u}\sigma_{-\mu} \mid p'_1 p'_2 p_2^n p_1^n \rangle.
\end{align}

\begin{align}
\langle p'_1 p'_2 n'_1 n'_2 f_2^n \mid t_{u}\sigma_{-\mu} \mid p'_1 p'_2 p_2^n p_1^n \rangle = & -\delta(p'_1, p'_l)\delta(p'_2, p_2^n)\delta(p'_2, p_2^n)\langle t_{u}\sigma_{-\mu} \mid p'_1 p'_2 p_2^n p_1^n \rangle.
\end{align}

\begin{align}
\langle p'_1 p'_2 n'_1 n'_2 f_2^n \mid t_{u}\sigma_{-\mu} \mid p'_1 p'_2 p_2^n p_1^n \rangle = & -\delta(p'_1, p'_l)\delta(p'_2, p_2^n)\delta(p'_2, p_2^n)\langle t_{u}\sigma_{-\mu} \mid p'_1 p'_2 p_2^n p_1^n \rangle.
\end{align}

\begin{align}
\langle p'_1 p'_2 n'_1 n'_2 f_2^n \mid t_{u}\sigma_{-\mu} \mid p'_1 p'_2 p_2^n p_1^n \rangle = & -\delta(p'_1, p'_l)\delta(p'_2, p_2^n)\delta(p'_2, p_2^n)\langle t_{u}\sigma_{-\mu} \mid p'_1 p'_2 p_2^n p_1^n \rangle.
\end{align}

\begin{align}
\langle p'_1 p'_2 n'_1 n'_2 f_2^n \mid t_{u}\sigma_{-\mu} \mid p'_1 p'_2 p_2^n p_1^n \rangle = & -\delta(p'_1, p'_l)\delta(p'_2, p_2^n)\delta(p'_2, p_2^n)\langle t_{u}\sigma_{-\mu} \mid p'_1 p'_2 p_2^n p_1^n \rangle.
\end{align}
\begin{align*}
\langle n_1' n_2' n_3' n_4' | t_\pm | n_1 n_2 n_3 n_4 \rangle \\
= +\delta(n_1', n_1)\delta(n_2', n_2)\delta(n_3', n_3)\delta(n_4', n_4) | p_1 | + \sum_{i<j} \lambda_{ij} \frac{P_i\lambda_{ij}}{12} \sigma_{\pm \mu} | p_i | + \sum_{i<j} \lambda_{ij} \frac{P_i\lambda_{ij}}{12} | t_\pm | \sigma_{\pm \mu} | p_i | \\
- \delta(n_1', n_1)\delta(n_2', n_2)\delta(n_3', n_3)\delta(n_4', n_4) | n_1 n_2 n_3 n_4 | + \sum_{i<j} \lambda_{ij} \frac{P_i\lambda_{ij}}{12} \sigma_{\pm \mu} | n_i | + \sum_{i<j} \lambda_{ij} \frac{P_i\lambda_{ij}}{12} | t_\pm | \sigma_{\pm \mu} | n_i | \\
+ \delta(n_1', n_1)\delta(n_2', n_2)\delta(n_3', n_3)\delta(n_4', n_4) | n_1 n_2 n_3 n_4 | + \sum_{i<j} \lambda_{ij} \frac{P_i\lambda_{ij}}{12} \sigma_{\pm \mu} | p_i | + \sum_{i<j} \lambda_{ij} \frac{P_i\lambda_{ij}}{12} | t_\pm | \sigma_{\pm \mu} | p_i |. 
\end{align*}

For all the given q.p. transition amplitudes (equations (6)–(13)), the antisymmetrization of the single-q.p. states was taken into account:

\begin{align*}
p_1 | < p_2 | < p_3 | < p_4 | , \\
n_1 | < n_2 | < n_3 | < n_4 | , \\
p_1' | < p_2' | < p_3' | < p_4' | , \\
n_1' | < n_2' | < n_3' | < n_4'.
\end{align*}

GT transitions of phonon excitations for every excited state were also taken into account. Here I assumed that the q.p. in the parent nucleus remained in the same q.p. orbits. A detailed description of the formalism for the calculation of GT transition amplitudes can be found in [18].

The total neutrino cooling rate per unit time per nucleus is given by

\begin{equation}
\lambda^\nu = \sum_{ij} P_i \lambda_{ij}^\nu,
\end{equation}

where \( \lambda_{ij}^\nu \) is the sum of the electron capture and positron decay rates for the transition \( i \rightarrow j \) and \( P_i \) is the probability of occupation of parent excited states, which follows the normal Boltzmann distribution.

On the other hand, the total antineutrino cooling rate per unit time per nucleus is given by

\begin{equation}
\lambda^\bar{\nu} = \sum_{ij} P_i \lambda_{ij}^\bar{\nu},
\end{equation}

where \( \lambda_{ij}^\bar{\nu} \) is the sum of the positron capture and electron decay rates for the transition \( i \rightarrow j \).

The summation over all initial and final states was carried out until satisfactory convergence in the rate calculation was achieved. The pn-QRPA theory allows a microscopic state-by-state calculation of both sums present in equations (14) and (15). This feature of the pn-QRPA model greatly increases the reliability of the calculated rates over other models in stellar matter where there exists a finite probability of occupation of excited states.

Experimental data were incorporated wherever available to strengthen the reliability of the calculation. The calculated excitation energies (along with their log \( ft \) values) were replaced with the measured ones when they were within 0.5 MeV of each other. Missing measured states were inserted and inverse and mirror transitions were also taken into consideration. If there appeared a level in experimental compilations without definite spin and/or parity assignment, theoretical levels were not replaced (inserted) with the experimental ones beyond this excitation energy. The detailed analysis of the pn-QRPA calculated ground and excited state GT\(_+\) strength distributions of \( ^{56}\text{Ni} \) was presented earlier in [12]. The pn-QRPA model calculated the centroid of the GT\(_+\) strength distribution to be around 5.7 MeV. This is to be compared with the FFN [20] value of 3.8 MeV and the large scale shell model range of 2.5–3.0 MeV [21]. Here I present the ground-state cumulative GT strength in both directions (\( \Sigma S_{p+} \)) for \( ^{56}\text{Ni} \) in figure 1. In the figure, the upper panel shows the calculated summed B(GT\(_+\)) strength distribution (\( \Sigma S_{p+} \)), whereas the bottom panel depicts the calculated summed B(GT\(_-\)) strength distribution (\( \Sigma S_{p-} \)). The abscissa shows the daughter excitation energy \( ^{56}\text{Co} \) in the upper panel and \( ^{56}\text{Cu} \) in the lower panel) in the units of MeV. It can be seen from figure 1 that the GT\(_+\) distributions are well fragmented and extend to high-lying daughter states. The model independent Ikeda sum rule is fulfilled in the calculation.

The pn-QRPA calculated neutrino cooling rates are depicted in figure 2. The figure shows the calculated rates as a function of stellar temperatures and densities. The upper, middle and lower panels depict the cooling rates at low (\( \rho Y_e [g \text{ cm}^{-3}] = 10^{0.5}, 10^1, 10^2 \) and \( 10^3 \)), medium (\( \rho Y_e [g \text{ cm}^{-3}] = 10^4, 10^5, 10^6 \) and \( 10^7 \)) and high (\( \rho Y_e [g \text{ cm}^{-3}] = 10^8, 10^9, 10^{10} \) and \( 10^{11} \)) stellar densities,
The neutrino energy loss rates are given in logarithmic scales (to base 10) in units of MeV s\(^{-1}\). In the figures and throughout the text, \(T_0\) gives the stellar temperature in units of 10\(^9\) K. It can be seen from figure 2 that at low stellar densities the cooling rates remain more or less the same as one increases the density by an order of magnitude. Considerable enhancement in neutrino cooling rates is witnessed as the stellar cores attain medium and high densities. This difference is more prominent in the low temperature domain \(T_0 < 5\). Especially in the high density region of the stellar core, the neutrino cooling rates increase by orders of magnitude as the core stiffens further. For a given temperature the neutrino energy loss rates increase monotonically with increasing densities.

The antineutrino energy loss rates are very small in magnitude as compared with the neutrino energy loss rates and as such these rates have a very small contribution to cooling the stellar cores. This is because the positron capture on \(^{56}\text{Ni}\) as well as the \(\beta\)-decay of \(^{56}\text{Ni}\) is relatively suppressed as compared to the electron capture rates on \(^{56}\text{Ni}\). The calculated antineutrino cooling rates are depicted in figure 3. Once again I show the result in a three-panel format as before. There is a sharp exponential increase in the antineutrino cooling rates as the stellar temperature increases up to \(T_0 = 5\). Beyond this temperature the slope of the rates reduces with increasing density. For a given temperature the antineutrino energy loss rates increase monotonically with increasing densities. The rates are almost superimposed on one another as a function of stellar densities in the low density domain. However, as the stellar matter moves from the medium high density region to high density region, these rates start to ‘peel off’ from one another. The neutrino and antineutrino energy loss rates are calculated on an extensive temperature–density grid point suitable for collapse simulations and interpolation purposes and presented in table 1. The calculated rates are tabulated in logarithmic (to base 10) scale. In the table, \(-100\) means that the rate is smaller than 10\(^{-100}\) MeV s\(^{-1}\). The first column gives \(\log(\rho Y_e)\) in units of g cm\(^{-3}\), where \(\rho\) is the baryon density and \(Y_e\) is the ratio of the electron number to the baryon number. Stellar temperatures \((T_0)\) are stated in 10\(^9\) K. Also stated are the values of the Fermi energy of electrons in units of MeV. \(\lambda_\nu\) (\(\lambda_{\bar{\nu}}\)) are the neutrino (antineutrino) cooling rates in units of MeV s\(^{-1}\). The electronic versions (ASCII files) of these rates may be requested from the author.

The calculation of neutrino cooling rates was also compared with previous calculations. For the sake of comparison, I took into consideration the pioneer calculations of FFN \([20]\) and those performed using the large-scale shell model (LSSM) \([22]\). The FFN rates were used in many simulation codes (e.g. KEPLER stellar evolution code \([23]\)), while LSSM rates were employed in recent simulation of presupernova evolution of massive stars in the mass range 11–40\(M_\odot\) \([10]\). The neutrino energy loss rates have contributions both from electron capture and positron decay rates (equation (1)). Both of these weak-interaction-mediated processes are governed by the ground- and excited-state GT\(_+\) strength distributions. As mentioned earlier, the pn-QRPA model places the centroid of the ground-state GT\(_+\) strength distribution around 3 MeV (2 MeV) higher than the LSSM (FFN) centroid. Accordingly, one expects a somewhat larger neutrino cooling rate due to previous calculations as compared with the reported rates. However, at low temperatures and densities the pn-QRPA calculated positron decay rates are orders of magnitude bigger, which causes an overall enhancement of the neutrino cooling rates (around a factor of 5). During intermediate stellar temperature and density domains the three calculations are in excellent agreement. At higher temperatures and densities the pn-QRPA neutrino cooling rates are smaller by up to an order of magnitude. The detailed comparison is presented below.

Figure 4 depicts the comparison of neutrino cooling rates due to \(^{56}\text{Ni}\) with earlier calculations at low densities. The upper panel displays the ratio of calculated rates to the LSSM rates, \(R_\nu\) (QRPA/LSSM), whereas the lower panel shows a similar comparison with the FFN calculation, \(R_\nu\) (QRPA/FFN). The graph is drawn for the low-density
regions \(\rho Y_e [\text{g cm}^{-3}] = 10^1, 10^3, 10^5\) as a function of stellar temperature. Both graphs follow a similar trend. At low densities and temperatures the pn-QRPA cooling rates are bigger by as much as a factor of 7 as compared with both FFN and LSSM rates. Otherwise the rates are in relatively good agreement especially as the core shifts to higher densities. As mentioned before, the neutrino energy loss rates have contributions from both electron capture and positron decay rates (equation (14)). At \(\rho Y_e [\text{g cm}^{-3}] = 10 \text{ and } T_9 = 1\), the pn-QRPA calculated electron capture rates are in reasonable agreement with the FFN and LSSM rates. On the other hand, the pn-QRPA calculated positron decay rates are bigger by roughly 8 orders of magnitude. The decay rates are a very sensitive function of available phase space \(\left(= Q_\beta + E_i - E_f \right)\). It is to be noted that Brink’s hypothesis is not assumed in the current calculation. Brink’s hypothesis states that GT strength distribution on excited states is identical to that from ground state, shifted only by the excitation energy of the state. In the current pn-QRPA calculation, all excited states are constructed in a microscopic fashion as discussed earlier. This greatly increases the reliability of calculated rates. Since the electron capture is the dominant process the overall neutrino cooling rate is bigger only by a factor of 7 at \(\rho Y_e [\text{g cm}^{-3}] = 10 \text{ and } T_9 = 1\). As the temperature increases the FFN and LSSM positron decay rates become in better agreement with the pn-QRPA rates, whereas the FFN and LSSM electron capture rates surpass the pn-QRPA rates (due to a lower placement of the centroid of the GT \(i \rightarrow f\) strength distribution). The reduced phase space at low temperatures is increased by finite occupation probabilities of parent excitation energies at high temperatures. At high temperatures the probability of occupation of the parent excited states \(E_i\) increases. FFN did not take into account the process of particle emission from excited states (this process is accounted for in the present pn-QRPA calculation). FFN’s parent excitation energies \(E_i\) are well above the particle decay channel and partly contribute to the enhancement of their weak rates at high temperatures.

The comparison with the previous calculations improves at higher stellar densities. The situation is depicted in figure 5 at stellar densities \(\rho Y_e [\text{g cm}^{-3}] = 10^6, 10^7, 10^8\). At low temperatures the comparison is excellent. This is roughly the region where weak interaction rates due to \(^{56}\text{Ni}\) are considered to be most effective during the presupernova evolution of massive stars. Core-collapse simulators might find it interesting to note that all three calculations agree very nicely for the above-mentioned range of stellar temperatures and densities. As \(T_9 \sim 10\), the LSSM and FFN cooling rates become stronger for reasons mentioned before. However, the abundance of \(^{56}\text{Ni}\) also decreases appreciably at high temperatures.

In high-density regions, the LSSM and FFN rates are bigger as expected. The situation is depicted in figure 6. Here, one notes that the LSSM rates are bigger by as much as a factor of 5 at \(\rho Y_e [\text{g cm}^{-3}] = 10^9, 1 \leq T_9 \leq 3\). At high stellar densities the weak rates are sensitive to the total GT strength rather than its distribution details. The LSSM calculated the total GT strength to be 10.1 [21] as compared to the pn-QRPA value of 8.9 [12]. The larger total GT strength of LSSM resulted in the enhancement of their rates at high densities. The corresponding FFN rates are enhanced at most by a factor of 3 at \(\rho Y_e [\text{g cm}^{-3}] = 10^{11}\) to an order of magnitude at \(\rho Y_e [\text{g cm}^{-3}] = 10^9\).

3. Conclusions

The pn-QRPA theory was used to calculate the weak-interaction-mediated neutrino and antineutrino cooling rates due to \(^{56}\text{Ni}\) on a detailed temperature–density grid point suitable for simulation purposes. At low temperatures and densities the pn-QRPA cooling rates are enhanced. Otherwise the rates are in reasonable agreement with previous calculations. For physical conditions considered to be most effective for electron capture rates on \(^{56}\text{Ni}\) \((\rho Y_e [\text{g cm}^{-3}] \sim 10^7, 1 \leq T_9 \leq 5)\) the three calculations are in very good agreement. Whereas for high stellar temperatures and densities the LSSM and FFN cooling rates are much bigger (up to an order of magnitude). However, the abundance of \(^{56}\text{Ni}\) decreases appreciably at high temperatures and densities.

According to Aufderheide and collaborators [9], for \(Y_e\) around 0.5, \(^{56}\text{Ni}\) is the most abundant nucleus having a mass fraction of around 0.99. The mass fraction of the most abundant nuclei decreases appreciably as the \(Y_e\) value...
decreases (e.g. it is of the order of $10^{-2}$ when $Y_e \sim 0.46$ and decreases by another two orders of magnitude for still lower values of $Y_e$). During the earlier phases of presupernova evolution, due to its high abundance, electron capture on $^{56}\text{Ni}$ is very important and the rate of change of $Y_e$ is roughly around 25% alone due to electron capture on $^{56}\text{Ni}$ [9]. It is expected that the neutrino cooling rates due to $^{56}\text{Ni}$ might also have an effect on the presupernova evolution of massive stars. It is expected that the reported rates (see also [11, 12]) might contribute to the fine-tuning of the $Y_e$ parameter during the various phases of stellar evolution of massive stars. Core-collapse simulators are suggested to be used to check for possible interesting outcomes using the reported neutrino cooling rates.

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