The String Landscape and the Swampland

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Abstract

Recent developments in string theory suggest that string theory landscape of vacua is vast. It is natural to ask if this landscape is as vast as allowed by consistent-looking effective field theories. We use universality ideas from string theory to suggest that this is not the case, and that the landscape is surrounded by an even more vast swampland of consistent-looking semiclassical effective field theories, which are actually inconsistent. Identification of the boundary of the landscape is a central question which is at the heart of the meaning of universality properties of consistent quantum gravitational theories. We propose certain finiteness criteria as one relevant factor in identifying this boundary (based on talks given at the Einstein Symposium in Alexandria, at the 2005 Simons Workshop in Mathematics and Physics, and the talk to have been presented at Strings 2005).

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1. Introduction

The central problem in string theory is how it will eventually connect with experiments. In this regard the notion of what kind of field theories can arise in string theory becomes a central question. The diversity of string vacuum constructions, which has been exploding in the past few years, leads one to the picture that basically any effective field theory which looks at least semiclassically consistent can arise in string theory. This is in sharp contrast, but not in contradiction, with the fact that the theory itself is believed to be unique.

If indeed any consistent looking effective field theory is actually consistent, one would naturally wonder about the wisdom of trying to construct string vacua using complicated geometries of internal manifolds etc. Instead one could just consider an effective field theory which looks consistent and which most closely resembles experimental data. In this context it is natural to postpone the question of its stringy construction until we probe high enough energies where the effective field theory description breaks down and a more stringy approach is called for. If consistency of a quantum theory of gravity does not pick out which effective field theories can arise, then string theory becomes of far less relevance for questions of IR physics.

In this note we argue that the pendulum has swung far enough: Not all consistent looking effective field theories are actually consistent and one should not be misled by a vast array of string vacuum constructions; we argue that the consistent looking effective field theories which are actually inconsistent are even more vast. This vast series of semiclassically consistent effective field theories which are actually inconsistent we shall call the ‘swampland’. Thus we view the vast stringy landscape of vacua as a relatively small island in an even more vast swampland of quantum inconsistent but semiclassically consistent effective field theories.

To argue the above picture we appeal to the fact that certain general features emerge in all string theory vacuum constructions, which have no apparent explanation as consistency conditions for effective field theories. Even though this by itself does not prove the above picture, it makes it look rather plausible. Moreover in some sense most of the consistent looking effective theories turn out to fall in the swampland.

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1 In this paper whenever we refer to string theory we have in mind all possible vacuum constructions including those which are more naturally phrased in terms of M-theory or F-theory.

2 In fact in the most naive sense of measure, the string landscape is of measure zero compared to the swampland.
The restrictions we focus on are finiteness properties including finiteness of volume of scalar fields, finiteness of the number of fields and finiteness/restrictions on the rank of the gauge groups. These finiteness properties can be relaxed if we turn off the gravity. In other words they are directly correlated with having a consistent quantum gravity coupled to matter. As it turns out these finiteness issues are highly non-trivial and are deeply related to the dualities in string theory and in fact imply the existence of certain S-dualities. We view these finiteness criteria as a universal feature of any potentially consistent quantum theory of gravity coupled to matter.

Some aspects of finiteness properties of string landscape have been independently noted by Michael Douglas [1]. The main motivation to consider the finiteness issues there was the requirement of being able to give a notion of a number distribution on the stringy landscape.

2. Finiteness of Scalar Field Moduli Space

Consider a quantum theory of gravity, coupled to massless scalar fields $\Phi^i$ with no potential

$$V(\Phi^i) = 0.$$ 

There could also be some other fields in the theory, but we will concentrate on the massless fields. We have the low energy effective action

$$S = \int M_P^{d-2} R + g_{ij}(\Phi) \partial \Phi^i \partial \Phi^j + ...$$

where ... represents higher derivative terms, as well as possibly other fields in the theory. Consider the volume of the scalar field space

$$V_\Phi = \int d\Phi \sqrt{g(\Phi)}$$

Let us be a bit more precise: As we vary the value of $\Phi$ the effective field theory may breakdown, due to the appearance of light states. Let us treat the effective field theory as being valid for energy scales

$$E \ll \Lambda$$
where \( \Lambda \) is some fixed scale. Let us consider regions in \( \Phi \) space where all the integrated out massive excitations have masses \( m \gg \Lambda \). Let us call this region \( B^\Lambda \). Consider the refined volume in the field space

\[
V^\Lambda_\Phi = \int_{B^\Lambda} d\Phi \sqrt{g(\Phi)}
\]

We define

\[
V_\Phi = V^\Lambda_\Phi \big|_{\Lambda \to 0}
\]

and view \( \Lambda \) as a regulator.

From the viewpoint of effective field theory there appears to be nothing inconsistent with

\[
V_\Phi = \infty.
\]

For example consider the case where we have one real field \( \Phi \) taking value in \( \mathbb{R} \) and where no masses depend on the value of \( \Phi \). So the region \( B \) is the entire real line and so \( V_\Phi = \infty \) in this case. However, as it turns out in all examples which come from string theory this is indeed finite as long as the Newton’s constant is not zero (i.e. \( M_p \neq \infty \) and gravity is not decoupled):

\[
V_\Phi \neq \infty.
\]

(more precisely the conjecture is that this could go to infinity \( \sim |\log \Lambda| \) as \( \Lambda \to 0 \) but no worse.) Let us give examples of this principle from string theory and also discuss how it can be evaded if we decouple gravity.

Consider type IIB in 10 dimensions. This theory has a complex scalar \( \tau \), which is the combination of string coupling constant combined with a scalar RR field. The conjecture applied to this case states that

\[
V_\tau = \text{finite}
\]

This is given by

\[
V_\tau = \int \frac{d^2 \tau}{\tau_2^2}
\]

If we did not use S-duality of type IIB this would indeed be infinite, violating the above conjecture. Thus we see that the finiteness of the scalar volume in this case is closely

\[3\] In the context of volume of field space for certain axion fields this finiteness was studied recently \[4\].
related to the existence of some non-trivial S-dualities. With S-duality taken into account this volume is indeed finite.

As another series of example consider compactifications of type II strings on $T^d$. The scalar fields are parameterized by the Narain moduli space

$$SO(d, d)/SO(d) \times SO(d) \times SO(d, d; \mathbb{Z})$$

which is known to be finite\(^3\). The existence of T-duality is crucial in making this volume finite.

Similarly if we consider compactifications of type II strings on $K3$, the scalar field space is given by

$$SO(20, 4)/SO(20) \times SO(4) \times SO(20, 4; \mathbb{Z})$$

which is finite, again thanks to the non-trivial T-dualities leading to quotient by $SO(20, 4; \mathbb{Z})$ in the above. Similarly one can consider compactifications of type II on Calabi-Yau manifolds. The vector multiplet have scalars which parameterize moduli space of complex structure (or quantum corrected Kähler structure) for type IIB (type IIA). These are believed to be finite in volume (see discussion in \(^4\)).

Note that this finiteness property correlates with having a finite Newton’s constant in the gravitational sector. In particular if we consider decompactifying the internal geometry, which leads to the lower dimensional gravitational sector to be non-dynamical, with vanishing Newton’s constant, the volume of scalar field space is no longer finite. For example consider type II strings on non-compact toric Calabi-Yau manifolds, which leads to vanishing Newton’s constant in the lower dimensional theory due to the fact that the internal volume is infinite. This leads to the volume of the scalar moduli space, i.e. moduli space of non-compact toric Calabi-Yau not being finite either (this is a well known fact in the context of geometric engineering, where the volume of the non-compact moduli space gets mapped to the volume of the space for scalar fields in the $\mathcal{N} = 2$ supersymmetric gauge theories, which is clearly infinite).

This finiteness can be rephrased in the context of AdS/CFT in the following way\(^5\). The moduli of scalar fields in the AdS context gets translated to the moduli of the CFT. Thus the finiteness of the moduli of scalar fields is the statement, in the CFT context, of

\(^{4}\) For $d = 1$ finiteness follows once we consider the cutoff dependent $V_\phi^A$ which behaves as $|\log \Lambda|$.  
\(^{5}\) This point was developed in discussions with E. Silverstein.
the finiteness of the moduli of the CFT. Finiteness of Newton's constant in the AdS context correlates with the existence of discrete spectrum of operators in the CFT (rather than a continuous spectrum). This is thus a purely field theoretic restatement of the gravitational finiteness type questions we are referring to:

*Is the volume of moduli space of CFT's, with a discrete spectrum of operators, finite?*

In all the examples we know of, for example superconformal field theories in 4 dimensions, this seems to be true. It also seems to be true for superconformal field theories in 2 dimensions. This it turns out gets related to the finiteness of volume of moduli space of scalar fields parameterizing internal geometry, in the context of string theory: In that case the worldsheet CFT moduli space is the same as the moduli space of the corresponding scalar fields parameterizing internal geometry. So this is at least a consistency check on this conjecture.

3. Finiteness of Matter Fields

Consider compactifying string theory on a manifold. The massless modes of the compactified theory, apart from the gravity multiplet, will have a number of other fields which get related to certain cohomology classes, or certain cohomology computations on the manifold. The number of such matter fields is thus directly bounded by the dimension of the relevant cohomology of the manifold. In all the examples we know in string theory, the dimension of cohomology of the manifold is finite. For example, we do not know of a sequence of Calabi-Yau threefolds for which the dimension of cohomologies can get unboundedly large. It is not known if this is actually a true mathematical fact, but experience with various examples in string theory gives one a strong feeling that this is going to be finite.

The interesting point is that again this finiteness correlates with the Newton’s constant being non-zero: If we consider local geometries for ‘compactification’ which do not have finite volume, then the dimension of the matter fields can grow arbitrarily large: Consider for example gauge theories in 6 dimensions. If we consider type IIA in the presence of $A_{N-1}$ singularities, we obtain a $U(N)$ gauge theory. There is no bound on $N$ and so we can have an arbitrarily large rank gauge group, in accordance with field theoretic semiclassical consistency conditions. But if we wish to get a non-trivial gravity in 6 dimensions by
having a non-zero Newton’s constant then we need to embed this in a compact geometry and it is known that this cannot be done for arbitrarily large \( N \) in \( K3 \) (indeed the rank of the matter field gauge group would be bounded by 20). Thus the consistency conditions for an effective gravitational theory should be more subtle than the corresponding condition for a consistent gauge theory which can be constructed in string theory for arbitrary rank.

3.1. A Lower Bound Conjecture

In the above discussion we have considered upper bounds on the number of fields. In this section we discuss the opposite restriction, namely that in some theories there should be some scalar fields, which is not demanded by naive consistency of the effective field theory. This again is going to be hard to prove. But let us present one potential candidate: Consider \( \mathcal{N} = 2 \) supersymmetric theories in 4d and ask if there are any consistent theories which have, as the low energy limit, only the supergravity multiplet and nothing else. Recall that the pure \( \mathcal{N} = 2 \) supergravity multiplet has no scalar. There is no known construction within string theory which accomplishes this. All known examples have extra scalars, such as the string coupling constant or the volume of the internal manifold etc. It is natural to conjecture that there is at least one such scalar in any such theory and that pure \( \mathcal{N} = 2 \) supergravity does not arise as the effective theory of a fully consistent quantum theory. It would be interesting to verify or disprove this.

4. Restrictions on Gauge Fields

The gauge fields that arise in string theory seem more restricted than would have been expected based on the requirement of semiclassical consistency of the effective field theory. There are many such examples. For example consider \( N = 1 \) supersymmetric theories in 10 dimensions with gauge groups:

\[
U(1)^{496}; U(1)^{248} \times E_8
\]

Both of these theories are consistent as far as anomalies are concerned. Thus they both appear to be consistent semiclassically. However, we have no way of constructing either of these theories in string theory. It is hard to believe that they actually exist, though we have no proof of this. It would be very interesting either to construct these or what
is more likely, to find what consistency condition goes wrong with these theories which renders them inconsistent.

There are many more such examples. For example if one considers consistent supersymmetric gauge theories coupled to gravity in 9 dimensions, there is a very restricted list known and it is difficult to imagine there are many other constructions within string theory. The importance of finding these restrictions for a small dimension of internal manifold is that it would be easier to be convinced that there are no other constructions missing. Were we to go further down in dimensions and in supersymmetry the available possibilities for compactifications, including the geometry and the flux/brane data would be so enormous that it would have been harder to make a definitive conclusion. Keeping the question at higher dimensions makes it essentially clear that there is no construction within string theory which gives other results.

5. Concluding Thoughts

We feel that finding suitable techniques to answer the types of questions raised in this short note are critical in the next level of our understanding of string theory and what it means to have a consistent quantum theory of gravity coupled to matter. We believe string theory can serve as a testing ground for any proposed consistency condition. The conjectures motivated in this note are motivated from the constructability or lack thereof within string theory. Examples of such potential questions include the interesting work which seeks to answer which minimal supergravities with at least one scalar field can be realized in string theory. It would be very interesting to find some general patterns of what are not possible to construct within string theory but that are naively allowed as consistent anomaly free effective theories (for some other possible constraints see ). There clearly should be many such criteria besides the ones discussed here. It would be

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6 One proposal along these lines, suggested by M. Douglas, is to assume the existence of the 1-brane electrically charged under the anti-symmetric tensor field and study the consistency conditions of the theory living on it. It would be interesting to see if this can be completed to an argument ruling out the existence of these theories.
exciting to uncover these conditions and come up with what it means to talk about the universality conditions for consistent quantum theories of gravity.

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