Practical Insight of Ferroconvection in Heterogeneous Brinkman Porous Medium

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Abstract: Ferromagnetic fluids are made up of magnetic particles, which are suspended in a carrier liquid such as water, hydrocarbon (mineral oil or kerosene) or fluorocarbon with a surfactant to avoid clumping [1, 2]. Ferrofluids possess with an extensive applications in several fields ranging from physics, electronics, electrical engineering, biomedical, micro and nanoelectromechanical systems, instrumentation in computer technology and various heterogeneous engineering applications [3-5]. The magnetization of ferrofluids will depend on the magnetic field, temperature and density. Since, the magnetic forces have propagated thermal state of fluid and it was derived from the ferromagnetic convection. Hence, the heat can transfer by ferromagnetic fluids and it will be emerged as one of the major areas to know the various applications of engineering sciences. The problem of ferromagnetic convective instability leads to magnetized ferrofluid layer, it was heated from below minimal temperature, this was investigated by Neuringer et al. Rosensweig et al. Finlayson et al. Recently, Afifah et al. studied various applications of ferroconvection in a magnetized ferrofluid with saturating porous medium. In Indian context a similar study was

1. Introduction

The ferromagnetic fluids are made up of magnetic nanoparticles, which are suspended in a carrier liquid such as water, hydrocarbon (mineral oil or kerosene) or fluorocarbon with a surfactant to avoid clumping [1, 2]. Ferrofluids possess with an extensive applications in several fields ranging from physics, electronics, electrical engineering, biomedical, micro and nanoelectromechanical systems, instrumentation in computer technology and various heterogeneous engineering applications [3-5]. The magnetization of ferrofluids will depend on the magnetic field, temperature and density. Since, the magnetic forces have propagated thermal state of fluid and it was derived from the ferromagnetic convection. Hence, the heat can transfer by ferromagnetic fluids and it will be emerged as one of the major areas to know the various applications of engineering sciences. The problem of ferromagnetic convective instability leads to magnetized ferrofluid layer, it was heated from below minimal temperature, this was investigated by Neuringer et al. Rosensweig et al. Finlayson et al. Recently, Afifah et al. studied various applications of ferroconvection in a magnetized ferrofluid with saturating porous medium. In Indian context a similar study was
reported by Sadrhosseini et al he observed that, the heat can transfer to ferrofluid by porous media inside canal with uniform heat flux on the wall and its effect was seen in magnetic field [6-9].

However, the ferromagnetic fluids analytical applications is very limited scope in worldwide due to paucity of literature and technological gap, there is a scope for further investigations on ferromagnetic convection in a heterogeneous Brinkman porous medium with internal heat source [10-12]. In this wide research gap, the present study is attempt to demonstrate the various applications of ferromagnetic convection with onset of penetrative ferromagnetic convection in a ferrofluid-saturated horizontal heterogeneous Brinkman porous layer (uniformly distributed internal heat source).

2. Mathematical Formulation

Consider an incompressible magnetized ferrofluid-saturated infinite horizontal Brinkman heterogeneous porous layer of thickness \( d \) with the presence of a uniform applied magnetic field \((0, 0, H_0)\) in the vertical direction. The lower surface was held at constant temperature \( T_L \), while the upper surface is at \( T_U \) (< \( T_L \)). A Cartesian co-ordinate systems \((x, y, z)\) used with the origin at the bottom of the porous layer and the z-axis directed vertically upward in the presence of gravitational field \( g \). In addition to that, the model is uniformly distributed with internal heat source in the ferrofluid saturated heterogeneous porous layer. The Boussinesq approximation density was estimated from the following equation.

\[
\nabla \cdot \vec{q} = 0 \quad \rho_0 \left[ \frac{1}{\varepsilon} \frac{\partial \tilde{q}}{\partial t} + \frac{1}{\varepsilon^2} (\tilde{q} \nabla) \tilde{q} \right] = -\nabla p + \rho_f \vec{g} - \frac{\mu_f}{K(z)} \tilde{q} + \tilde{\mu}_f \nabla^2 \tilde{q} + \mu_0 (\tilde{M} \cdot \nabla) \tilde{H} \quad A \frac{\partial T}{\partial t} + (\tilde{q} \cdot \nabla) T = \kappa \nabla^2 T + Q
\]

where, \( \tilde{q} \) denotes velocity vector, \( P \) - pressure, \( \rho_f \) - fluid density, \( K(z) \) -variable permeability of the porous medium, \( \tilde{M} \) -magnetization of the ferrofluid, \( \tilde{B} \) - magnetic induction, \( \tilde{H} \) - magnetic field, \( \mu_f \) - fluid viscosity, \( \tilde{\mu}_f \) - effective viscosity, \( \mu_0 \) - magnetic permeability, \( \rho_0 \) - reference density, \( T \) - temperature, \( A \) - ratio of heat capacities, \( \varepsilon \) - porosity of the porous medium, \( \kappa \) - thermal diffusivity, \( Q \) - overall uniformly distributed effective volumetric internal heat generation, \( \alpha_i \) -thermal expansion coefficient, \( \chi = (\partial M / \partial H)_{H_0, T_0} \) -expressed magnetic susceptibility, \( K_p = -(\partial M / \partial T_f)_{H_0, T_0} \) derived the pyromagnetic coefficient,

\[
M_0 = M(H_0, T_u), \quad T_u = (T_L + T_U)/2, \quad H = |\tilde{H}|, \quad M = |\tilde{M}|, \quad \phi
\]

\[
\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}
\]

real values of Laplacian operator

In the basic conduction state, the following equation was formulated

\[
\tilde{q}_b = 0
\]

\[
\rho_b(z) = \rho_0 - \rho_0 \alpha_q \varepsilon \left[ \frac{Qz^2}{6k} - \frac{Qdz}{4k} \right] + \frac{\beta z}{2} \left[ \frac{z^2}{d} - \frac{zd}{2} \right] - \mu_0 M_0 K_p \left[ \frac{Qz^2}{2k} - \frac{Qdz}{2k} + \beta z \right]
\]

\[
T_b(z) = T_u - \beta \left( z - \frac{d}{2} \right) - \frac{Qz^2}{2k} + \frac{Qdz}{2k}
\]

\[
\tilde{H}_b(z) = \left[ H_0 - \frac{K_p}{1+\chi} \left( z - \frac{d}{2} + \beta \left( z - \frac{d}{2} \right) \right) \right] \hat{k}
\]
\[ \tilde{M}_b(z) = \left[ M_0 + \frac{K_p}{1 + \kappa} \left\{ \frac{Q}{2\kappa} z^2 - \frac{Qd}{2\kappa} z + \beta \left( \frac{z - d}{2} \right) \right\} \right] \hat{k} \]  

(4)

where, \( \beta = \Delta T / d = (T_b - T_i) / d \) is the temperature gradient, \( \hat{k} \) - unit vector in the z-direction and the subscript b clearly expressed the basic state. Later the equation was superimposed with perturbations, the basic solution becomes

\[ \tilde{q} = \tilde{q}^* \quad \tilde{p} = \tilde{p}^* \quad T = T_b + T' \quad \tilde{H} = \tilde{H}_b + \tilde{H}' \quad \text{and} \quad \tilde{M} = \tilde{M}_b + \tilde{M}' \]  

(5)

The linear stability analysis was performed with normal mode; non-dimensionalising of the real variables tends to be in the following form of equation

\[ \left( x^*, y^*, z^* \right) = \begin{bmatrix} \frac{x}{d} \\ \frac{y}{d} \\ \frac{z}{d} \end{bmatrix}, \quad t^* = \frac{K}{d^2} t, \quad W^* = \frac{d}{K} W, \quad \Theta^* = \frac{1}{\beta d} \Theta, \quad \Phi^* = \frac{1 + \chi}{K_p \beta d^2} \Phi \]  

(6)

Non-dimensional governing equations (asterisks for simplicity and noting that the principle of exchange of stability holds) are derived in the form of

\[ \begin{align*}
\left( D^2 - a^2 \right) F(z) - F(z) \left( D^2 - a^2 \right) W - DF(z)DW &= R_m a^2 \left[ N_s (1 - 2z) - 1 \right] \left( D\Phi - \Theta \right) + R_n a^2 \Theta \\
\left( D^2 - a^2 \right) \Theta &= \left[ N_s (1 - 2z) - 1 \right] W \left( D^2 - a^2 M_s \right) \Phi - D\Theta = 0 \end{align*} \]  

(7)

In the above equations, \( D = d / dz \) is the differential operator, \( a \) can explain horizontal wave number \( R_D = \alpha_d g \beta K_0 d^2 / \nu \kappa \) and Darcy-Rayleigh number was given by the equation \( Da = \mu J K_0 / \mu J d^2 \) and also the modified Darcy number was propagated in the form of \( M_i = \mu_0 K_0^2 \beta / (1 + \chi) \alpha_i \rho_0 g \) \( R_m = R_D M_i = \mu_0 K_0^2 \beta^2 d^2 / (1 + \chi) \mu \kappa \). Finally after algebraic solution, we have obtained the Darcy-Rayleigh number equation \( N_s = Q \beta d / 2 \kappa \beta^2 \) \( M_i = (1 + M_0 / H_0) / (1 + \chi) \) is the measure of non-linearity of magnetization and \( F(z) = K_0 / K(z) \) non-dimensional permeability heterogeneity function and \( K_0 \) is the mean value of \( K(z) \). The function \( F(z) \) was chosen in the following form of equation

\[ F(z) = 1 + \alpha_1 \left( z - \frac{1}{2} \right) + \alpha_2 \left( z^2 - \frac{1}{3} \right) \]  

(8)

where \( \alpha_1 \) and \( \alpha_2 \) is real valued constants and it was formulated in quadratic function with unit mean. For the homogeneous porous medium case \( \alpha_1 = 0 = \alpha_2 \).

Equations (7) - (8) we have solved that, the appropriate boundary conditions tend to be significantly homogeneous. The simulated boundaries are found to be rigid, ferromagnetic state, either in isothermal or insulated temperature perturbations. The boundary conditions was estimated in the following equation

\[ W = DW = \Theta \quad \text{or} \quad D\Theta = \Phi = 0 \quad \text{at} \quad z = 0, 1. \]  

(9)

The resulting eigenvalue problem was solved numerically by using Galerkin method. Accordingly, \( W(z) \) , \( \Theta(z) \) and \( \Phi(z) \) would be expanded in the series form

\[ W = \sum_{i=1}^{n} A_i W_i(z) , \quad \Theta(z) = \sum_{i=1}^{n} B_i \Theta_i(z) , \quad \Phi(z) = \sum_{i=1}^{n} C_i \Phi_i(z) \]  

(10)

where \( A_i , B_i \) and \( C_i \) are unknown coefficients. Multiplying Equation by \( W_j(z) \) , \( \Theta_j(z) \) and \( \Phi_j(z) \) respectively and integrate them across the layer. Using the boundary conditions, we obtain the following system of linear homogeneous algebraic equations

\[ C_{ji} A_i + D_{ji} B_i + E_{ji} C_i = 0 \]

\[ F_{ji} A_i + G_{ji} B_i = 0 \]

\[ H_{ji} B_i + I_{ji} C_i = 0 \]

The coefficients \( C_{ji} - I_{ji} \) can involve inner products of the basic functions and are given by

\[ \int_{0}^{1} W_i(z) W_j(z) dz = \int_{0}^{1} \Theta_i(z) \Theta_j(z) dz = \int_{0}^{1} \Phi_i(z) \Phi_j(z) dz \]
were obtained numerically in Table 1. The linear Fourier series is the modified Chebyshev polynomials, \( T_n \) are unknown, parameters were determined from the Darcy Rayleigh number \( R_D \) and \( a \). The characteristic equation was formed from (11)-(12) with the existence of non-trivial solution. (solved numerically with different values of \( a \)).

The base functions \( W_i(z) \), \( \Theta_i(z) \) and \( \Phi_i(z) \) is assumed in the following form:

\[
W_i = (z^4 - 2z^3 + z^2)T^*_{i-1}, \quad \Theta_i = (z^3 - 3z^2 + 2z)T^*_{i-1}, \quad \Phi_i = (z^2 - z)T^*_{i-1} \quad \text{for isothermal}
\]

\[
\Theta_i = (z^2 - z)T^*_{i-1} \quad \text{for insulated}.
\]

where, \( T^*_n \) is the modified Chebyshev polynomials, \( W_i(z), \Theta_i(z) \) and \( \Phi_i(z) \) will satisfies the corresponding boundary conditions. The characteristic equation was formed from (11)-(12) with the existence of non-trivial solution. (solved numerically with different values of \( M_3 \), \( N_s \), \( R_m \), \( Da \) and for different forms of \( F(z) \)). It was observed that, the numerical results were converged by using Galerkin method. The equation (8)-(9) was solved by considering initial value of \( z = 0 \)

For isothermal boundaries

\[
W(0) = 0, \quad DW(0) = 0, \quad D^2W(0) = 1, \quad D^3W(0) = \eta_1, \quad \Theta(0) = 0, \quad D\Theta(0) = \eta_2, \quad \Phi(0) = 0, \quad D\Phi(0) = \eta_3.
\]

For insulated boundaries

\[
W(0) = 0, \quad DW(0) = 0, \quad D^2W(0) = \eta_1, \quad D^3W(0) = \eta_2, \quad \Theta(0) = 1, \quad D\Theta(0) = 0, \quad \Phi(0) = 0, \quad D\Phi(0) = \eta_3.
\]

Here, the conditions \( D^2W(0) = 1 \) and \( \Theta(0) = 1 \) help us to break the scale invariance of the solution of isothermal and adiabatic boundary conditions. Further, the parameters of \( \eta_1, \eta_2 \) and \( \eta_3 \) are unknown, parameters were determined from the Darcy Rayleigh number \( R_D \) at isothermal and insulated temperature boundary condition. The shooting method was obtained by Runge-Kutta-Fehlberg method (RKF45), the formulation of the fitted equation will satisfy the regularity conditions of value \( z = 1 \),

\[
W(1) = 0, \quad DW(1) = 0, \quad \Theta(1) = 0 \quad \text{or} \quad D\Theta(1) = 0, \quad \Phi(1) = 0.
\]

Finally, the critical Darcy Rayleigh number \( R_{Dc} \) and the corresponding wave number \( a_c \) were obtained numerically in the different forms of \( F(z) \) as well as other values of physical parameters of rigid-rigid ferromagnetic (isothermal/insulated) boundary conditions. From (table 3), it was observed that, the numerical results were significantly obtained from the two methods, heterogeneous equilibrium was observed during the model fitting phase.

### 3. Results

The combined effect of internal heat source and the vertically stratified permeability at the onset of thermomagnetic convection was derived from the Brinkman porous method (heated below temperature). Simulation results have been investigated by driven application of Galerkin method to know the accuracy of the model, fitted model is very informative to know the provocative values of \( R_{Dc} \) and the corresponding \( a_c \) at the different levels of Galerkin approximation. The inspection of the results revealed that, the \( R_{Dc} \) turn out to be the same in vertically stratified permeability functions of type \( F_1 \) and \( F_2 \) as well...
as type $F3$ and $F4$, and also the model formulation was vertically stratified with permeability function of type $F4$. As per the findings, $F4$ was found to be more stable when compared to type $F1$ (ROC analysis was performed to know the accuracy of the model. As per the model, AUC was 0.91 and likelihood function was found to be significantly correlated for propagation of real expected values of permeability function). Further, it was observed that, the values of $R_{DC}$ will differ with different vertical heterogeneity of permeability functions at higher order Galerkin method.

![Figure 1](image)

**Figure 1.** Variation of (a) $R_{DC}$ and (b) $a_c$ with $N_5$ for different values of $Da$ when $R_m = 2$ and $M_3 = 1$ for different forms of $F(z)$.

| Models | $a_1$ | $a_2$ | Nature of $F(z)$ |
|--------|-------|-------|-----------------|
| $F1$   | 0     | 0     | $F(z) = 1$ (homogeneous) |
| $F2$   | 1     | 0     | $F(z) = 1 + \left( z - \frac{1}{2} \right)$ (linear variation in $z$) |
| $F3$   | 0     | 1     | $F(z) = 1 + \left( z^2 - \frac{1}{3} \right)$ (only quadratic variation in $z$) |
| $F4$   | 1     | 1     | $F(z) = 1 + \left( \frac{z}{2} \right) + \left( z^2 - \frac{1}{3} \right)$ (general quadratic variation in $z$) |

**Table 1.** Various forms of vertical heterogeneity of permeability function $F(z)$.

| Approximations | $N_5$ | Model | $i=j=1$ | $i=j=2$ | $i=j=5$ | $i=j=6$ |
|----------------|-------|-------|---------|---------|---------|---------|
|                |       | $R_{NC}$ | $a_c$ | $R_{NC}$ | $a_c$ | $R_{NC}$ | $a_c$ | $R_{NC}$ | $a_c$ | $R_{NC}$ | $a_c$ | $R_{NC}$ | $a_c$ |
| 0              |       |         |       |         |       |         |       |         |       |         |       |         |       |
| $F1$           | 215.708 | 3.145 | 216.531 | 3.147 | 211.047 | 3.151 | 211.047 | 3.151 |           |       |         |       |         |       |
| $F2$           | 215.708 | 3.145 | 216.484 | 3.147 | 210.983 | 3.151 | 210.983 | 3.151 |           |       |         |       |         |       |
| $F3$           | 216.534 | 3.163 | 217.308 | 3.164 | 211.850 | 3.168 | 211.850 | 3.168 |           |       |         |       |         |       |
| $F4$           | 216.534 | 3.163 | 217.160 | 3.165 | 211.658 | 3.169 | 211.658 | 3.169 |           |       |         |       |         |       |
| 2              |       |         |       |         |       |         |       |         |       |         |       |         |       |         |       |
| $F1$           | 215.815 | 3.143 | 208.327 | 3.198 | 203.184 | 3.204 | 203.164 | 3.204 |           |       |         |       |         |       |
| $F2$           | 215.815 | 3.143 | 210.803 | 3.197 | 205.529 | 3.203 | 205.509 | 3.203 |           |       |         |       |         |       |
| $F3$           | 216.641 | 3.163 | 211.532 | 3.215 | 206.310 | 3.220 | 206.289 | 3.220 |           |       |         |       |         |       |
| $F4$           | 216.641 | 3.163 | 213.981 | 3.214 | 208.592 | 3.220 | 208.573 | 3.220 |           |       |         |       |         |       |
| 5              |       |         |       |         |       |         |       |         |       |         |       |         |       |         |       |
| $F1$           | 215.976 | 3.147 | 177.067 | 3.384 | 172.865 | 3.400 | 172.800 | 3.400 |           |       |         |       |         |       |
| $F2$           | 215.976 | 3.147 | 181.204 | 3.390 | 176.798 | 3.404 | 176.730 | 3.405 |           |       |         |       |         |       |
| $F3$           | 216.801 | 3.164 | 181.623 | 3.408 | 177.301 | 3.421 | 177.231 | 3.421 |           |       |         |       |         |       |
| $F4$           | 216.801 | 3.164 | 185.856 | 3.415 | 181.285 | 3.427 | 181.212 | 3.427 |           |       |         |       |         |       |
4. Model Application

4.1. Medical Science

Medical practitioner highly exhibits tremendous variation in decision making because of their normative approach, biological and clinical essence to deal with uncertainties round the clock. The test diagnostic decision also depends upon the physical experience, expertization and perception of the practitioners. As the complexity of the health care decision system, this model will be insight for taking clinical diagnosis at early stage without any bias for example, blood test and screening of HIV, biopsy test for cervical cancer and identification of rare diseases by using NGS method (Next generation gene sequencing), the \( F(z) \) will signify and provides sufficient analytical information to the practitioner at early stage, this system greatly converges large population level and deal with real concept of tremendous approximation of weak law of large numbers. The subtle of the approximation could provide signifying results for the practitioner at inception as well as last stage of decision, and also it can solidify precise what is imprecise in the world medicine. The fitted model shows an important role in medicine for symptomatic diagnostic cures in fuzzification techniques.

4.2. Extension of Artificial Intelligence (AI) in Engineering Science

An artificial neural network (ANN) is an information processing model that is able to capture and represent complex inputs-output relationship. The motivation of \( F(z) \) critical Darcy-Rayleigh and the corresponding wave numbers for different orders of approximations in the Galerkin expansion would be extended for AI system that could process information in the same way the human brain. ANNs resemble human brain in two respects learning process and storing experimental knowledge. An AI network learns and classifies problem through repeated adjustments of the connecting weights between the elements.

4.3. Galerkin Expansion (GE) in Bio Informatics

The GE logic can be easily used to implement systems ranging from simple, small or even embedded up to large networked ones. The GE logic is that it accepts the uncertainties that are inherited in the realistic inputs and it deals with these uncertainties in their affect is negligible and thus resulting in a precise outputs. The GE reduces the design steps and simplifies complexity that might arise since the first step is to understand and characterize the system behavior by using knowledge experience. It is successfully applied to several areas in practice like for building knowledge based system of following areas. Increasing flexibility of protein, studying differences between various nucleotides, analyzing experimental data sets using GE adoptive resonance theory, aligning sequencing by separate algorithms, complex trait analysis, NGS and Mendelian experimental data analysis etc. [12, 13].

5. Discussion

Asper the resulted findings we have observed that, the linear stability criterion expressed in terms of the critical Rayleigh number (< the number the system is very stable and > the number system tends unstable) [14, 15]. For each of the forms of \( F(z) \), the effect of increasing \( N_s \); \( R_m \) and \( M_s \) indicates that, the decreased trend of the Darcy Rayleigh number, while in opposite condition, the trend was significantly increased values of \( Da \). From the (Figure 1.) depicted that, the vertical permeability of heterogeneous function of type \( F_4 \) is more stable followed by type \( F_3 \) and \( F_2 \), least effect was seen in \( F_1 \) with presence of internal heat source ( \( N_s \neq 0 \)). The effect of increased internal heat source strength shall express large amount of incremental deviation was propagated simultaneously [16, 17, 18, 19]. In this Juxtapose, the distribution system was induced instability. We have noted that, similar findings were reported by Shivakumara et al and it was found to be significantly associated with permeability heterogeneous function of type \( F_4 \). The least stable was observed in the absence of internal heat source strength ( \( N_s = 0 \) ) his findings were completely matched with present intervention. The variation of critical Rayleigh number and the corresponding wave number of \( N_s \) in different parameter values would be generated heterogeneous simulation.
On the onset of convection will trigger the function of constants in different forms of $F(z)$, a study revealed that, the effect of $N_s$ destabilizing the system of homogeneous porous medium considering brief summary of results found, the increased values of $N_s$ and $R_m$ was found to be positively associated with critical wave numbers. Thus, the wave number effect leads to reduction of convection cells. Further, the inspection of the illustration findings depicted that, the values of $a_c$ was found to be higher in $F4$ followed by $F3$ and $F2$. The insignificant effect was seen in $F1$ in the presence of magnetic Rayleigh number [22, 23].

The non-linearity of magnetization $M_3$ has not been influence on the onset convection and also the critical wave numbers. Resulted model findings found that, $F1$ is greatly converges with real values of Rayleigh number and propagates very small values [24, 25].

Variation of critical Rayleigh number of different forms of $F(z)$ was presented in Figure 1 which are demonstrated by two values of $Da = 0.05$ and 0.06 and function of $N_s$. From the figure 1 we have seen that, the system was more stable in the form of $F(z)$ with type $F4$ followed by $F3$ and $F2$ and least stable equilibrium was observed in $F1$ till observation is turned out to be the same in isothermal boundaries [26, 27]. Besides, an insulated boundary was found to be more destabilizing when compared to isothermal boundaries [28-30].

6. Conclusions

The present study concludes that, the fitted model is found to be more unstable for insulating boundary as compared with isothermal boundary. However, an increased study state in the value of internal heat source strength $N_s$, magnetic Rayleigh number $R_m$ and the measure of non-linearity of magnetization $M_3$ is to be hasten for the onset of ferromagnetic convection, while increasing the Darcy number $Da$ shows stabilizing equilibrium effect on the system due to increased rate of viscous diffusion state.

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