Dark subhaloes and disturbances in extended H\textsc{i} discs

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ABSTRACT

We develop a perturbative approach to study the excitation of disturbances in the extended atomic hydrogen (H\textsc{i}) discs of galaxies produced by passing dark matter subhaloes. The shallow gravitational potential of the dark matter subhaloes (compared to the primary halo) allows us to use the epicyclic approximation, the equations of which we solve by modal analysis, that is, assuming a disc is composed of $N$ radial rings with $M$ modes. We show that properties of dark matter subhaloes can be inferred from the profile and amplitude of the modal energy of the disc. Namely, we find that the overall amplitude of the response gives the mass of the dark subhalo. Motivated by this modal analysis, we then show that the density response shows similar features. Finally, we show that our results agree with those from full hydrodynamic simulations. We find a simple scaling relation between the satellite mass and the low-order Fourier amplitudes of the resultant surface density of the gas disc where the effective Fourier amplitude (essentially a sum over the low-order modes) scales as $m_s^{1/2}$, where $m_s$ is the satellite mass. The utility of this relation is that it can be readily applied to an observed H\textsc{i} map to deduce the satellite mass without recourse to full numerical simulations. This will greatly aid us in analysing large samples of spiral galaxies to constrain the population of dwarf satellites in the Local Volume.

Key words: galaxies: general – galaxies: interactions – dark matter.

1 INTRODUCTION

Dark matter haloes grow by a sequence of mergers and accretion events in cold dark matter (CDM) cosmology. Due to the absence of a scale in this hierarchical structure formation scenario, DM haloes are expected to be merely scaled-up or down versions of each other with the total mass being the scaling parameter (Navarro, Frenk & White 1997). Observations of galaxies at different masses show that this is clearly not the case. At the high end, clusters have thousands of galaxies, that is, subhaloes, in line with theoretical predictions (Natarajan & Springel 2004). However, galactic-size haloes have a few bright satellites, for example, our Milky Way (MW), in contrast with theoretical models of the MW halo (e.g. Diemand et al. 2008), and dwarfs have no bright satellites. This qualitative difference at the faint end has come to be known as the substructure or ‘missing-satellites’ problem (Klypin et al. 1999; Kravtsov, Gnedin & Klypin 2004; Madau, Diemand & Kuhlen 2008; Kravtsov 2010).\textsuperscript{1}

Kravtsov (2010) recently reviewed the possible solutions to the ‘missing-satellites’ problem. To summarize, solving the ‘missing-satellites’ problem demands that a scale be introduced either in the structure formation of DM haloes or in the physics of galaxy formation to suppress the number of observed systems at small scales. Namely, the power spectrum of fluctuations must be suppressed at small scales via warm DM (Zentner & Bullock 2003; Hooper et al. 2007; Primack 2009) or star formation is suppressed in dwarfs due to reionization (Bullock, Kravtsov & Weinberg 2000; Okamoto & Frenk 2009; Busha et al. 2010; Ilić et al. 2011) or feedback from the first stars (Wise & Abel 2008).

Recent discoveries of dwarf galaxies in the MW and M31 have relieved some of this tension (for a review, see Willman 2010). These dwarfs are the most DM dominated (Strigari et al. 2008) and metal poor systems known (Kirby et al. 2008). While their unusual properties point to the difficulty of forming stars in these systems, the number density of these dwarfs may alleviate the ‘missing satellites’ problem once completeness corrections are accounted for (Tollerud et al. 2008).

The evidence from the MW observations and theoretical predictions points to the existence of many nearly-dark subhaloes hosting few stars. For instance, Bullock et al. (2010) recently pointed out that these stealth galaxies would be missed in current surveys, having surface brightness that is too diffuse. This motivates finding alternative means to constrain this dark subhalo population.

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\textsuperscript{1}To be precise, the ‘missing-satellites’ problem refers to the lack of observed satellites in the MW halo compared to what is expected theoretically.
In a recent series of papers (Chakrabarti & Blitz 2009, 2011, hereinafter CB09 and CB11, respectively), CB09 and CB11 made the assumption that the observed disturbances in the outer atomic hydrogen gas disc of the MW (Levine, Blitz & Heiles 2006) are due to the tidal interaction with a DM dominated dwarf galaxy. Proceeding under this hypothesis, CB09 and CB11 numerically simulated a suite of encounters with dwarf galaxies interacting with the MW, and developed an approach to infer the mass and current location of satellites from analysis of their tidal imprints on outer gas discs. The Fourier amplitudes (CB09) and phases (CB11) of the suite of simulations were compared with the observed amplitudes and phases to determine the pericentre approach, current position (in radius and azimuth) and mass of the subhalo that drove the observed disturbances. In doing so, CB09 and CB11 demonstrated how to characterize a dark subhalo from looking at its effects on the primary galaxy’s extended H\textsc{i} disc. Following this work, we demonstrated the validity of this method by performing the analysis with respect to galaxies with known optical companions (Chakrabarti et al. 2011, hereinafter CBCB). We showed that the method accurately recovers the mass of the satellite and its position purely from analysis of observed disturbances in the H\textsc{i} disc, without requiring any knowledge of the optical light from the satellites.

In this paper, we continue this work and develop a simplified test-particle approach to studying the generation of these disturbances in extended H\textsc{i} discs by DM subhaloes. Using this approach, we develop relations between the satellite mass and Fourier amplitudes of the resultant surface density of the H\textsc{i} disc. These relations can be utilized by observers who wish to determine the satellite mass directly from the observed H\textsc{i} map, without having to take recourse to full numerical simulations. This simplified approach is motivated by the fact that DM subhaloes have a much lower velocity dispersion, that is, gravitational potential, than the host halo in which they sit. Thus, we calculate the motions of the gas (or stars) in the epicyclic approximation and show that this approximation matches the results of both test-body calculations and full numerical simulations.

This work is complementary to the work of CB09, CB11 and CBCB. Rather than comparing the results of numerical simulations to observations of specific extended H\textsc{i} discs, this work focuses on the physics of the subhalo excitation and exploration of its parameter space. In particular, we show that the low-order Fourier modes encode useful information on tidal encounters and show how some of this information can be extracted. In the future, we will apply these scaling relationships to a large sample of local spiral galaxies to determine the statistical viability of the tidal analysis method, that is, the incidence of false positives. Moreover, we will attempt to constrain the population of dwarf galaxies in the Local Volume.

This paper is organized as follows. In Section 2, we take the equations of motion for a test particle and develop its modal equivalent in the epicyclic approximation. We then show how the modal viewpoint can be related to the test-particle viewpoint. This modal analysis is shown to reproduce test-particle calculations accurately. In addition, the invariance of the modal energy response (in the absence of dissipation) provides us an unique vantage point to study the response of the disc. In Section 3, we study the modal energy response and the corresponding density response. We also compute global quantities from these responses as a measure of the global properties of the disturbances raised by the passing DM subhalo, in Section 4. We demonstrate a scaling relation between the disc response and the mass of the satellite where the amplitude of the modes scale like $M_s^{-1/2}$. To show that our analysis accurately models the relevant physics, we then compare our results with those computed from large-scale simulations, in Section 5. The good agreement between the smoothed particle hydrodynamic (SPH) simulations and our results is encouraging. In addition, the scaling which we derived from our simplified calculation continues to hold. Finally, we conclude in Section 6 with a discussion on how these methods can be used to constrain the properties of DM subhaloes in galaxy haloes.

2 BASIC EQUATIONS

We begin with the equations of motion of a test particle in 2D ($r, \phi$ coordinates):

$$\frac{dr}{dt} = v_r, \quad (1)$$

$$\frac{d\phi}{dt} = \frac{J}{r^2}, \quad (2)$$

$$\frac{\partial J}{\partial t} + v_r \frac{\partial v_r}{\partial r} + J \frac{\partial v_r}{\partial \phi} = \frac{J^2}{r^2} - \frac{\partial \Phi}{\partial r}, \quad (3)$$

$$\frac{\partial J}{\partial \phi} + v_r \frac{\partial J}{\partial r} + J \frac{\partial J}{\partial \phi} = -\frac{\partial \Phi}{\partial \phi}, \quad (4)$$

where $v_r$ is the radial velocity, $J = \dot{\phi} r^2$ is the angular momentum, $\phi(t)$ is the position of the particle and $\Phi$ is the gravitational potential. We assume a background state of circular orbits, that is, $v_r = 0$, about an axisymmetric potential, $\Phi$. Expanding equations (1)–(4) to linear order, for example, $r \rightarrow r + \delta r$, we find

$$\frac{\partial \delta r}{\partial t} = \delta v_r, \quad (5)$$

$$\frac{\partial \delta \phi}{\partial t} = \frac{\delta J}{r^2} - \frac{2J \delta r}{r^2}, \quad (6)$$

$$\frac{\partial \delta v_r}{\partial t} = \frac{J \delta v_r}{r^2} + \frac{2J \delta J}{r^2} - \frac{3J^2 \delta r}{r^3} - \frac{J \delta \Phi}{r^2} - \frac{\partial \delta \Phi}{\partial \phi}, \quad (7)$$

$$\delta J \frac{\partial \delta \phi}{\partial \phi} + \delta v_r \frac{\partial \delta \phi}{\partial r} + J \frac{\partial \delta J}{\partial \phi} = -\frac{\partial \delta \phi}{\partial \phi}. \quad (8)$$

We assume that the perturbation to the potential, $\delta \Phi$, arises from external perturbers, ignoring the self-gravity of the particles. This simplification is safe for typical H\textsc{i} disc parameters. Assuming solutions of the form $\exp(im(\phi - \Omega(r)t))$, where $\Omega(r) = \dot{\phi} r$, reduces equations (5)–(8) to $M$ separate wave equations, sourced by the Fourier components of the perturbing potential:

$$\frac{\partial \delta r}{\partial t} = \delta v_r, \quad (9)$$

$$\frac{\partial \delta \phi}{\partial t} = \frac{\delta J}{r^2} - \frac{2J \delta r}{r^2}, \quad (10)$$

$$\frac{\partial \delta v_r}{\partial t} = \frac{J \delta v_r}{r^2} + \frac{2J \delta J}{r^2} - \frac{3J^2 \delta r}{r^3} - \frac{J \delta \Phi}{r^2} \exp(im\Omega(r)t), \quad (11)$$

$$\frac{\partial \delta J}{\partial \phi} = -\delta v_r \frac{\partial \delta J}{\partial r} - im\delta \Phi \exp(im\Omega(r)t). \quad (12)$$

Solving these perturbed equations for a finite $M$ (say, $M = 10$) allows us to estimate the linear response of a circular ring of orbiting particles to external disturbances, that is, passing DM subhaloes. Thus, any individual particle’s perturbed orbit, whose guiding centre...
is at radius $r$, can be found by summing over this response. Namely, we reconstruct the particle positions from these modes using

$$r_i(t) = r_{i,0} + \delta r_i(t),$$  \hspace{1cm} (13)$$

$$\phi_i(t) = \phi_{i,0} + \delta \phi_i(t),$$  \hspace{1cm} (14)$$

where $r_{i,0}$ and $\phi_{i,0}$ are the initial radius and azimuth of particle $i$, and

$$\delta r_i(t) = \sum_{m=1}^{M} \delta r_m(r_{i,0}, t) \exp(i\phi_{i,0} - \Omega(r_{i,0})t),$$  \hspace{1cm} (15)$$

$$\delta \phi_i(t) = \sum_{m=1}^{M} \delta \phi_m(r_{i,0}, t) \exp(i\phi_{i,0} - \Omega(r_{i,0})t).$$  \hspace{1cm} (16)$$

This greatly simplifies this orbit into a sum of $M$ simple harmonic oscillators, whose natural frequency only depends on their radial position. This procedure of breaking the disc into rings has been followed by Professor Curt Struck and collaborators previously in the context of ring galaxies (Struck-Marcell 1990; Struck-Marcell & Lotan 1990; Struck-Marcell & Higdon 1993; Struck, 1999, 2010; Struck, Dobbs & Hwang 2011). For these ring galaxies, only the $m=0$ mode needs to be analysed which allows for an analytic solution for the propagation of these rings (Appleton & Struck-Marcell 1996; Struck 2010) in these galaxies due to interactions (à la Toomre & Toomre 1972). Here, we have extended this formalism to include $m \neq 0$, which allows us to study spiral patterns induced by interactions.

To illustrate the power of this formalism, we compare a direct numerical calculation of a collisionless particle disc with that of the modal calculation above and show the results of the direct test-particle calculation and our modal reconstruction in Fig. 1. Here, the broad features of the response is captured between the mode calculation and our modal reconstruction in Fig. 1. Here, it is evident that the modal calculation captures the details of the perturbation accurately.

Our formalism makes the analogy between the different modes of a radial ring and a collection of uncoupled simple harmonic oscillators, whose oscillator frequency just depends on their radius. Hence, it is natural to think of the modal energy,

$$E_m = \frac{1}{2} J_m^2 + \frac{1}{2} \kappa^2 \delta r_m^2,$$  \hspace{1cm} (17)$$

where $\kappa = \sqrt{R/dOmega/d\Omega + 4\Omega^2}$ is the epicyclic frequency and

$$\delta r_m = \frac{2\Omega J_m}{r \kappa^2}$$  \hspace{1cm} (18)$$

is the perturbed radial mode $m$ that accounts for the perturbation to the guiding centre. The oscillator energy has the major advantage that it remains constant in the absence of dissipation or excitation, whereas the density response of a disc varies with time. This is advantageous in elucidating the physics. In the next section, we show the benefit of this mode of analysis and relate its properties to the density response.

3 DISC RESPONSE

Our analysis on the disc response is divided into two different viewpoints. We first look at the modal energy response in Section 3.1 utilizing the advantage of our approach of separating the disc into $N$ rings with $M$ modes. As we stated earlier, this allows us to look at the response of the mode energy which is helpful in elucidating the physics. We then look at the density response, again from a Fourier perspective, in Section 3.2, which is the observed manifestation of the perturbed disc in order to compare it (ultimately) to observations. The low-order mode-by-mode comparison that we have undertaken in this second part is similar to the modal analysis first performed by Elmegreen & Elmegreen (1985), who use a similar low-order mode-by-mode comparison to elucidate the physics of galactic bars.

For the calculations presented in this work, we have constructed two separate codes. We first construct a collisionless test-particle code, which integrates the equations of motion using a fifth-order
Dark subhaloes and extended HI discs

Figure 2. Modal energies of the disc in response to the disturbances by a coplanar 1:100 perturber with $r_{\text{peri}} = 20 \text{kpc}$. In the upper left-hand corner of each panel is the radial distance of the perturber. Initially, when the perturber is far away (panel a), only the other disc responds to the perturber. However, as the perturber gets closer, more and more of the disc is perturbed. In panel (c), the perturber is at $r_{\text{peri}}$ and we note a kink in the $m = 0$ modal energy at $r = r_{\text{peri}}$. This kink in the modal energy shows up in the subsequent panels (d)–(f) in the other modes as well. This kink is the result of an increase in the modal energy response at $r_{\text{peri}}$ and serves as an indicator of $r_{\text{peri}}$ of perturbing subhaloes. We also note the advantage of studying the modal energy is that it is fixed after the interaction with the perturber in the absence of dissipation.

Runge–Kutta algorithm (Press et al. 1992) directly for the test particles. We also integrate the modal equations [equation (9)–(12)] and reconstruct the position of the particles based on these modes. In performing these calculations, we make a series of simplifying assumptions. First, we assume a static potential for both the primary galaxy and the subhalo, that is, we do not use a ‘live halo’. In particular, we assume the DM halo, which dominates the mass, follows a spherically symmetric Hernquist profile (Hernquist 1990). The parameters of the DM halo that we have used in our calculations are described in greater detail in CB09. Secondly, we assume a constant-density HI disc motivated by observations of the Galactic HI disc (Wong & Blitz 2002) and the extended HI discs of external galaxies (Bigiel et al. 2010). There also exists an exponential gas disc, but we ignore this for the sake of simplicity. Finally, we ignore the effect of self-gravity on the motions of the particles and modes which allows us to use the test-particle approximation. These assumptions allow us to develop a simplified approach to the modelling of disturbances in extended HI discs. We demonstrate the accuracy of the results of the simplified approach with respect to that of a full simulation later in Section 5.

3.1 Modal energy response

As we have mentioned earlier in Section 2, the modal equations reduce to a number of simple harmonic oscillators. A passing perturber excites these simple harmonic oscillators, each to a different amplitude and phase. In the absence of dissipation, the modal amplitude or energy remains constant. We illustrate this point in Fig. 2 where we show six snapshots of the orbit of a coplanar subhalo with $r_{\text{peri}} = 20 \text{kpc}$. Initially, the energy is zero, but rises dramatically after the disc suffers an encounter, before asymptoting to a constant. This property of the modal energy response makes them appealing for studying the response of the disc. We should point out that while the energy is already substantial when the perturber is at the pericentre, the density response of the disc does not happen until much later. This is because the energy imparted by the perturber at the pericentre serves mainly to increase the perturbed velocity. However, this perturbed velocity only translates into perturbed density after approximately a dynamical time.

How does the modal (energy) profile depend on the orbital parameter? To answer this question, let us first fix the mass and the orbit to be that of a parabolic ($E = 0$) coplanar orbit so that the modal profile depends only on the pericentre distance of the perturber. In Fig. 3, we plot modal energies as a function of radius for a 1:100 encounter for $r_{\text{peri}} = 10 \text{kpc}$ (left-hand panel) and 20 kpc (right-hand panel). Note the dramatic increase in modal energy for the oscillator at $r = r_{\text{peri}}$. Indeed this increase in modal energy is seen for 1:10 and 1:1000 coplanar encounters as shown in Fig. 4. We also note that the amplitude of the modal profile depends only on the mass and is only weakly dependent on $r_{\text{peri}}$ at least at large distances.

We now fix $r_{\text{peri}} = 20 \text{kpc}$ to study the effects of inclination in Fig. 5. In the coplanar case studied above, all the modes showed a similar feature at $r_{\text{peri}}$. Here, we instead see that only the $m = 0$ mode...
Figure 3. Modal energies of the disc for a 1:100 perturber with $r_{\text{peri}} = 20$ kpc (panel a) and 30 kpc (panel b). In both cases, the perturber sits at roughly $r_p \approx 80$ kpc after its interaction. As the modal energy is fixed at this point in the absence of dissipation, we can look in detail at the structure of the interaction. Note that in both cases, a kink in the modal energy appears at $r = r_{\text{peri}}$. This strengthens the point made in Fig. 2 that these kinks serve as a good proxy for $r_{\text{peri}}$.

Figure 4. Same as Fig. 3 but for a 1:1000 perturber with $r_{\text{peri}} = 20$ kpc (panel a) and 30 kpc (panel b).

consistently shows the same feature as the coplanar case. Specifically, for all inclinations, ranging from $+\pi/2$ (prograde coplanar) to 0 vertical to $-\pi/2$ (retrograde coplanar), the $m = 0$ modal energy increases dramatically as $r > r_{\text{peri}}$.

The effect of inclination on the $m \neq 0$ modal energies is more complicated. For the coplanar prograde interaction, we also see a large increase in the modal energy at $r = r_{\text{peri}}$ for all $m \neq 0$ modes. However, this is not true for the other inclinations. For instance, the $m = 1$ mode transitions from having its modal amplitude to rapidly increase at $r = r_{\text{peri}}$ for coplanar encounters to having its amplitude to peak at $r = r_{\text{peri}}$ and decline swiftly outwards. The $m = 2$ mode, on the other hand, transitions from having a rapid increase at $r = r_{\text{peri}}$ in mode amplitude for low-inclination encounters (either prograde or retrograde) to having its amplitude to peak at $r = r_{\text{peri}}$ for highly inclined encounters. The difference in the modal behaviour of these non-axisymmetric modes suggests that these low-order modes can encode additional information with regard to the tidal encounter beyond that of $r_{\text{peri}}$, which is most robustly encoded by the $m = 0$ mode. We will not pursue this encoding and how this encoding may be extracted in observations in this paper, but will leave this for future work.

In the examples above, we have studied the effect of varying the inclination for purely prograde encounters, that is, the case where the disc-crossing radius is equal to the pericentre distance. Though
we do not present them here, we have also studied the case of fixing \( r_{\text{peri}} \) and the inclination and varying the angle at which the perturber crosses the disc, that is, \( \phi \neq 0 \) for the disc-crossing point on the y-axis. In these examples, the modal energies also show a change at \( r_{\text{peri}} \), which suggests that looking for changes in the modal energy in the disc is a good way to deduce \( r_{\text{peri}} \). We briefly comment on this in the next subsection, but reserve a more extensive discussion on the operational details for future work.

### 3.2 Density response

The modal energies provide a good proxy for the response of the disc, but our analysis of modal energies here is new. We now link this modal energy response to the density response, which is the observed manifestation of these perturbations. To calculate the density response from the modal energies, we reconstruct the position of the particles using equations (13) and (14). From this, we obtain the density of particles as a function of \( r, \phi, \) and \( t \), from which the Fourier amplitudes can be calculated.

Our analysis of modal energies shows that they are localized to certain regions of the disc. This suggests that the density perturbations also follow a similar pattern. We show this in Fig. 6 where we show density perturbations from modal theory for the same six snapshots of the orbit of a coplanar subhalo with \( r_{\text{peri}} = 20 \) kpc as in Fig. 2. Note that we have not included the \( m = 0 \) mode here. This is because the axisymmetric perturbation is not well defined unless we know the initial unperturbed profile, which is something that is difficult to obtain for observed galaxies. Hence, we have ignored the perturbed \( m = 0 \) mode in the density response. In addition, we have normalized the \( m = 1 \) to \( 4 \) modes relative to the complete \( m = 0 \) mode, which is dominated by the unperturbed component to further facilitate comparisons to observations and simulations.

We also show the same six snapshots using a full test-particle calculation in Fig. 7. Note the similarity between Figs 6 and 7, which increases our confidence in our method of solution. Comparing Fig. 2 with Fig. 6 side by side, it is clear that the modal energies are already significant as the perturber crosses \( r_{\text{peri}} \), while the density perturbations occur later. As discussed above, this is because the modal energies are initially concentrated in the perturbed velocities and the density response only arises after approximately a dynamical time.

The abrupt increase in the modal energies at \( r = r_{\text{peri}} \), which was previously identified by looking at modal energies, is also seen in the density response. The density response like the modal energies remains small in the regions \( r < r_{\text{peri}} \), but increases rapidly for \( r > r_{\text{peri}} \). What is also interesting is that the identification of \( r_{\text{peri}} \) from the density response in Figs 6 and 7 seems to be more clear than that of the modal energy response plotted in Fig. 2. In addition, the density response appears to be much more localized than the modal energy response (Fig. 2). Namely, it is strong between \( r_{\text{peri}} \) and \( r_{\text{peri}} + 10 \) kpc, whereas the modal energy response is strong all the way out to the end of the disc.

To help illustrate the effect of this density response, we define a synthetic measure of the power of these modes as

\[
\delta_{\text{tot}}(r) = \sqrt{\sum_{m=0}^{4} \frac{|\delta_{m}(r)|^2}{5}},
\]

and plot this in Figs 6 and 7.

We now use this synthetic measure (equation 19) to show that this localized response around \( r_{\text{peri}} \) continues to hold as we vary the orbit with different inclinations and different angles. We show the effects of different inclinations on the Fourier power of the resulting density structure, while holding the angle fixed. Comparing Fig. 8 with Fig. 5, we find again that the rapid increase

![Figure 5](https://example.com/figure5.png)

**Figure 5.** Modal energy profiles of a 1:100 perturber with \( r_{\text{peri}} = 20 \) kpc for different inclinations: \( i = 90^\circ \) (panel a), \( 60^\circ \) (panel b), \( 30^\circ \) (panel c), \( 0^\circ \) (panel d), \( -30^\circ \) (panel e), \( -60^\circ \) (panel f) and \( -90^\circ \) (panel g). The kink in the modal energy profile in the coplanar case (panel a) and Fig. 2 is only persistent for the \( m = 0 \) mode for all inclinations. However, the modal energy profile near \( r = r_{\text{peri}} \) shows features in all cases like peaks, for example, the \( m = 1 \) case for \( i = -30^\circ \) (panel e), \( -60^\circ \) (panel f) and \( -90^\circ \) (panel g).

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Fourier amplitudes of the density response of a disc suffering a 1:100 perturber encounter with $r_{peri} = 20$ kpc. The perturber has a coplanar orbit ($i = 90^\circ$). This corresponds to the modal energy calculation of Fig. 2, where we showed that the encounter generates a kink in the modal energy at $r = r_{peri}$. Here we see that the result of this modal energy kink is reflected in the density response of the disc in panels (d)–(f). Namely, the density response is peaked at $r \approx r_{peri}$. The response of the $m = 1$–4 modes is compared with that of the $m = 0$ mode. For the $m = 0$ line shown, we have only included the perturbation and compared it to the initial uniform background.

Same as Fig. 6 but using the test-particle calculation.
in the $m = 0$ power is a good discriminant for $r_{\text{peri}}$. The $m \neq 0$ modes also show good discriminating power though not as the $m = 0$ mode. However, what is really encouraging is that, while the $m = 1$ mode varies significantly with inclination in Fig. 5, its effect on the density response is much milder. By looking at these plots, it is clear that the $m = 0$ modes have the most discriminating power, but $a_{\text{tot}}$ appears to be adequate for determining $r_{\text{peri}}$.

4 SCALING RELATIONS: INFERRING THE SATELLITE MASS FROM FOURIER AMPLITUDES

Our discussion in Section 3 shows that, while the overall dynamical response of the disc depends on the orbital parameters, that is, $r_{\text{peri}}$, inclination and angle, the density response of the disc is generally localized around $r_{\text{peri}}$ and $r_{\text{peri}} + 10$ kpc. This localized response, especially for the $m = 0$ mode, allows us to constrain $r_{\text{peri}}$ of the orbit. We now discuss the effect of the subhalo’s mass on the disc response with an aim towards developing scaling relations between the response and the perturber’s mass.

We begin by defining an appropriate global measure of the response of the disc in much the same spirit as our earlier definition of $a_{\text{tot}}$ (equation 19). As we have already seen that the disc response is localized, we define the effective amplitude of the disc as

$$a_{\text{m,eff}} = (\Delta r)^{-1} \int_{r_{\text{peri}}}^{r_{\text{peri}} + \Delta r} |a_m(r)| \, dr,$$

where $\Delta r = 10$ kpc, motivated by the results of the previous section.

We then define the total effective amplitude as

$$a_{\text{eff}} = \sqrt{\frac{1}{4} \sum_{m=1}^{4} |a_{\text{m,eff}}|^2}.$$ 

A major advantage of defining such global quantities is that like the modal energies, their overall amplitudes settle into a relatively constant range after the initial excitation, in the absence of dissipation. We show this in Fig. 9, for DM perturbers with the mass ratios of 1:10, 1:100 and 1:1000. Note that $a_{\text{eff}}$ settles into a constant range that depends on the mass ratio $\approx 1$ Gyr after their interaction, which is roughly one rotation period of the outer disc.

Figure 9. Plot of the synthetic Fourier amplitude of the density response as a function of time. Note that after the initial interaction, the amplitude stays relatively constant in the absence of gaseous dissipation. While this is expected for the modal energies, it is surprising that this is preserved in the density response as well. It points out that the synthetic amplitude is a good proxy for measuring the global properties of the response in a robust manner.
Motivated by this result, we average $a_{\text{t,eff}}$ after approximately one rotation period for different inclinations in Fig. 10. The error bars define the variation in $a_{\text{t,eff}}$ for different angles. The bands represent the variation in $a_{\text{t,eff}}$ for each mass ratio as we vary the inclination from $90^\circ$ to $-90^\circ$. However, the average amplitude over which $a_{\text{t,eff}}$ varies depends strongly on the mass ratio of the satellite. We show this mass ratio dependence more clearly in Fig. 11 where we collate the results of Fig. 10 and plot the overall disc response as a function of $m_s$. The error bars represent the variation in the inclination and angle (same as the banded structures in Fig. 10). In addition, we plot the following fitting formula (dotted line):

$$a_{\text{t,eff}} = 0.5 \left( \frac{m_s}{M_\star} \right)^{1/2}.$$  \hspace{1cm} (22)

Equation (22) provides a reasonable fit to the numerical results as shown by the dashed line in Fig. 11. It suggests that the energy of the modes $E \propto a^2 \propto M_s$.

This is different from the naive expectations of the impulse approximation which instead suggest that $E \propto M_s^2$ (Alladin & Narasimhan 1982). A similar deviation from the naive expectation of the impulse approximation has also been seen in the amount of mass-loss induced in flyby collisions of galaxies (Wallin & Stuart 1992; Struck 1999). Here, Struck (1999) has argued that the amount of mass-loss scales with the number of stars that have a significantly large impulse imparted to them to escape their parent galaxy. For an extended body, this scales with the area of the perturber rather than the mass. This gives a mass-loss rate that is linear with the perturber’s mass in agreement with the results of Wallin & Stuart (1992). For the purposes of this paper, if the waves are only significantly excited inside an area around the perturber as it impacts the disc, then this amount of wave excitation scales with the area of the perturber. Because the dark halo mass of the perturber scales as $1/r$ at small $r$, the mass $\alpha$ area and so we recover the above scaling.

The fact that our results differ from the impulse approximation means that the naive impulse approximation is not a good approximation for the interaction of subhaloes with extended H I discs, a point already discussed by CB09, CB11 and CBCB. However, the notion that the energy is given impulsively may still be valid, albeit, modified for extended bodies.

5 COMPARISON TO SPH SIMULATIONS

To confirm our analysis, we compare our results from the modal analysis above to full-scale numerical galactic simulations. We use the SPH code GADGET-2 (Springel 2005) using the methodology presented in CB09. GADGET-2 uses an $N$-body method to follow the evolution of the collisionless components, and SPH to follow the gaseous component. The simulations of disc galaxies tidally interacting with DM minihaloes reported here (unless otherwise noted) have gravitational softening lengths of 100 pc for the gas and stars, and 200 pc for the halo. The number of gas, stellar and halo particles in the primary galaxy is $4 \times 10^5$, $4 \times 10^6$ and $1.2 \times 10^8$, respectively, for our fiducial case. We refer the interested reader to CB09 for additional details on the simulation methodology.

We focus here on a few representative cases that span a reasonable range in parameter space. We list in Table 1 the parameters for the simulations that we use for this comparison. They vary in mass ratio between 1:10 and 1:300 and have coplanar and polar inclinations.

As an example, Fig. 12 shows the density response for a few time-snapshots of the interaction for the 1:100 coplanar example. The general response of the disc in the full simulation is visually similar to our modal calculations of Fig. 1, especially that of $t = 0.650$ Gyr. This is somewhat unsurprising as the dominant physics in both calculations are the gravity of the primary halo and the

### Table 1. Parameters of SPH simulations and $a_{\text{f, tot}}$

| Mass ratio | $r_{\text{peri}}$ (kpc) | Orientation | $a_{\text{f, tot}}$ |
|------------|--------------------------|-------------|---------------------|
| 1:10       | 30                       | Coplanar    | 0.22                |
|            |                           | Polar       | 0.11                |
| 1:100      | 32                       | Coplanar    | 0.061               |
|            |                           | Polar       | 0.055               |
| 1:300      | 34                       | Coplanar    | 0.039               |
|            |                           | Polar       | 0.036               |
subhalo. However, it does highlight that much of the physics that have been ignored in the simplified calculation, that is, a live DM halo, tidal stripping of the subhalo, gas dissipation, star formation and self-gravity, do lead to some difference in the appearance of the disturbances in the H\textsubscript{I} disc. A main physical effect that has not been taken into account in these calculations is gas dissipation. Neglecting this effect allows for the simplicity of calculations as the modal energy (and the Fourier amplitudes once generated) is constant in the absence of dissipation. In the presence of gas dissipation, the Fourier amplitudes decrease as a function of time as disturbances in the gas disc damp out in approximately a dynamical time. We do not model this effect here, which would be needed to identify the current location of a satellite from observed disturbances in the H\textsubscript{I} disc as done earlier in the SPH calculations by CBCB.

Motivated by the visual agreement between the SPH and modal calculations, we focus on comparing $a_{\text{eff}}$ for the SPH simulation and the modal calculation. Here the agreement is striking. In the final column of Table 1, we list $a_{\text{eff}}$ for the various SPH simulations that we have performed. We also list $a_{\text{eff}}$ for the modal calculation, denoted by the symbol 'x', in Figs 10 and 11. The SPH simulation and the modal calculation show striking agreement especially for the 1:100 case. We have also computed the 1:300 case and it too shows excellent agreement between the modal calculation and the SPH calculation. The 1:10 case shows some mild disagreement between the modal calculation and the simplified modal calculation. This is not unexpected as linear theory should start to break down for sufficiently massive satellites (mass ratios $\gtrsim 1:10$). The overlaid points from the SPH simulation in Fig. 11 give additional support to the $a_{\text{eff}} \propto m_1^{1/2}$ scaling that we initially found from the modal calculation.

6 CONCLUSIONS AND DISCUSSION

Subhalo interactions with the H\textsubscript{I} discs of galaxies should leave measurable effects on the gas long after the subhalo has passed by as demonstrated by CB09, CB11 and CBCB. In this paper, we look carefully at the dynamics of these interactions in the context of a simplified epicyclic approximation.

(i) By performing a modal analysis on the equations of motion, we show that we can reproduce the orbital dynamics and distribution of particles of a test-particle calculation for identical conditions. While this may not seem surprising, given the mass range of the perturbers we are interested in ($\lesssim 0.1$), we show that performing such an analysis, we can understand crucial aspects of the nature of such interactions. In particular, we show that the modal energy shows sharp variations at $\sim r_{\text{pen}}$.

(ii) By developing a sense of what to look for from the modal analysis, we then showed that such variations also show up in the Fourier modes of the perturbed density. Namely, we find sudden increases in the low-order Fourier modes of the density response at $\sim r_{\text{pen}}$. Such sharp variations at large radii are a distinct signature of tidal interactions and would argue against a secular origin. The signatures of tidal encounters are thus imprinted in low-order Fourier modes of the galactic H\textsubscript{I} discs, allowing for observations of these low-order modes to provide useful information on these encounters.

(iii) We also showed that this simplified analysis agrees with test-particle calculations as well as more extensive SPH simulations. This demonstrates that the physics ignored in the simplified approach (a live DM halo, tidal stripping of the subhalo, gas dissipation effects, self-gravity and star formation, and dynamical friction) do not significantly alter the production of the disturbances in the H\textsubscript{I} discs.
We have also calculated the caustic pattern and showed that it matches \( \propto m_{\text{pert}}^{416} \), maps and determine the mass of the perturbing satellite without having to take recourse to full numerical simulations.

Finally, after this paper was initially submitted, Struck et al. (2011) demonstrated similar results in stellar discs and grand design spirals. Namely, they found that long-live grand-design spiral patterns can emerge as a result of flyby encounters where non-linear interactions between waves generate caustics to produce these patterns. Their results qualitatively match ours, namely, that these patterns emerge only after one dynamical time after the impulsive encounter and that the pattern of the spiral mode matches that from a caustic calculation.

In future work, we will apply these scaling relations to analyse results from cosmological simulations to determine how the dark subhaloes will impact the disc. To do this, we will generalize this method to incorporate multiple perturbers. The effect of multiple perturbers is not yet known, but it may be the case that multiple perturbers do not significantly alter our results. Namely, if the timescale between encounters is significantly longer than a dynamical time, then the disturbances in the gas disc from the first encounter will be damped out. This short-term memory of the gaseous component allows us to more cleanly disentangle the effect of the last perturber from the ones that impacted the disc previously. Simulations predict that impacts with \( \sim 1:100 \) mass ratio perturbers occur only once every \( \sim \) Gyr (Kazantzidis et al. 2008), which bodes well for this possibility.

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