Comment on “Measurement of quantum states of neutrons in the Earth’s gravitational field”

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In the paper by V.V. Nesvizhevsky et al., Phys. Rev. D 67, 102002 (2003), it is argued that the lowest quantum state of neutrons in the Earth’s gravitational field has been experimentally identified. While this is most likely correct, it is imperative to investigate all alternative explanations of the result in order to close all loopholes, as it is the first experiment ever claimed to have observed gravitational quantum states. Here we show that geometrical effects in the experimental setup can mimic the results attributed to gravity. Modifications of the experimental setup to close these possible loopholes are suggested.

1. INTRODUCTION

A well known property of quantum mechanics is the quantization of the energy levels of a confined particle, e.g., one trapped in a potential well. For instance, the electromagnetic and the strong nuclear forces create different kinds of quantized structure in atoms and nuclei. This suggests that a splitting of the energy levels should also be observed for particles in the Earth’s gravitational field, but since the gravitational interaction is much weaker, the effect is subtle and hard to detect.

Recently [1], Nesvizhevsky et al., described an experiment where such quantum effects of gravity acting on ultra-cold neutrons (UCN) were claimed to have been observed. The results of the experiment were also previously summarized in [2]. UCN were allowed to flow through a cavity with a reflecting surface below and an absorber above. By measuring the number of neutrons exiting the experimental setup, they claim to have observed discrete gravitational energy levels. They argue that the discrete data is related to the sudden increase of neutrons coming through at distinct widths between the reflecting surface and absorber. However, since the UCN are restricted by both the reflecting surface and the (non-ideal) absorber, also the geometric effect should be considered. The claimed result may even be explained by the use of geometrical arguments only.

2. THE EXPERIMENT

Here we give a brief review of the experiment reported in [1, 2]. A similar experiment was first suggested by V.I. Luschikov and A.I. Frank in 1978 [3].

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The absorber and the “mirror” create a slit through which the neutrons pass, eventually reaching a detector at the end of the experimental setup. UCN are essential to the experiment due to their crucial properties. First and foremost they are electrically neutral, making them insensitive to “stray” electric fields which could easily mask all gravitational effects. They also have an energy of about $10^{-7}$ eV, corresponding to a de Broglie wavelength of $\sim 500$ Å or a (horizontal) velocity of $\sim 10$ m/s, allowing them to undergo total reflection at all angles against a number of materials. The low energy also allows for high resolution, and since neutrons have a lifetime of the order of 900 s, it is possible to store them for periods of 100 s or more.

Nesvizhevsky et al., argue that when the neutrons are trapped in the potential formed by the mirror (an impenetrable “floor”) and the Earth’s effectively linear gravitational potential there will be a discrete set of possible energy levels, $E_n$, corresponding to the allowed eigenfunctions $\psi_n$. These are related through the time-independent Schrödinger equation, $H\psi_n = E_n\psi_n$, where $H = p^2/2m + V$, and $V = mgz$. For a theoretical treatment of this potential, see [4]. The four lowest theoretical energy eigenvalues are $E_1 = 1.4$ peV, $E_2 = 2.5$ peV, $E_3 = 3.3$ peV and $E_4 = 4.1$ peV.

The ground state energy $E_1$ corresponds to a classical height, $E_1 = mgz$, of about 15 $\mu$m. This leads the group to predict that when the slit-opening is less than this height no neutron transmission will occur. They argue that if the quantum mechanical wave function has a spatial extension larger than the opening, it will not “fit” without overlapping the absorber, and the neutrons have no chance of reaching the detector. In the experiment they observed a discrete increase in the number of detected neutrons as the slit opening was increased. In particular it was observed, as predicted, that when the slit-opening was less than $\sim 15$ $\mu$m no neutrons reached the detector, and that there occurred a sudden increase after 15 $\mu$m.

### 3. ALTERNATIVE EXPLANATIONS

A first thing to emphasize is that the energy eigenvalues themselves never were measured, i.e., all quoted energies are entirely theoretical. The only experimental data are the neutron counts $N$ at the detector, as a function of the mirror-absorber slit-width $\Delta h$. The experimental statistics for discrete steps corresponding to excited quantum levels is insufficient [1, 2]. Hence, the authors claim only to conclusively have identified the quantized ground state (first step). There is thus no absolute need to recreate the quoted energy eigenvalues, as one only needs to explain the first jump in the number of detected neutrons. The data also show a good fit to a “translated” classical curve $N \propto (\Delta h - h_1)^{1.5}$ (dotted curve in Fig. 5(c) of [1]) in which only the first discrete step is taken into account.

However, to show that it can be done, we choose as a first rough approximation a potential consisting of two infinite walls, i.e., a “neutron in an infinite box”. (The mirror can be seen as an “almost” infinite wall but the absorber is obviously poorly described by this.) The mirror introduces a non-gravitational “external” force to obtain the confining potential. This is very different from e.g., the Hydrogen atom where the “mirror” acting on the electron is internal, arising from electromagnetic interaction and quantum uncertainty only. The problem is trivial to solve analytically [4, 5], and the allowed energies are

$$E_n^{\text{Box}} = \frac{\hbar^2 \pi^2 n^2}{2ma^2},$$

where $a$ is the box-width. Thus, the first energy eigenvalue of a neutron trapped in a box
of width 15 µm is $E_1^{\text{Box}} = 0.9$ peV, of the same order of magnitude as the first energy eigenvalue of a neutron in the Earth’s gravitational field, $E_1 = 1.4$ peV. For more realistic potentials it is possible to reproduce the first energy level of $E_1 = 1.4$ peV at an opening of 15 µm, explaining the “gravitational quantum energy state” as merely a normal geometric cavity-effect\(^1\). A thorough investigation would necessitate a very exact and complicated modelling of the potential at the mirror and, especially, at the absorber.

However, as the final deciding factor in physics is experiment there is, at least in principle, a much simpler way to check this. Keeping everything else identical, turn the cavity from being horizontal to being vertical! If the effect is due to gravity it must then disappear as a potential well in the vertical direction no longer is present. If the effect is sustained in the vertical configuration it is solely due to the geometry of the cavity and the intrinsic properties of the neutron beam. (An even more ideal way would be to do the measurements in free-fall, but this seems virtually impossible to do in practice.) Nesvizhevsky et al., have controlled this by “reversing the geometry”, i.e., placing the absorber at the bottom instead of above. This control, however, is inconclusive. The absorber length is 13 cm, the mirror length is 10 cm. Outside the cavity formed between mirror and absorber the (unquantized) neutrons fall freely. Inside the cavity, even in the “reversed” case due to the fact that the absorber is non-ideal, there is a standing neutron-wave, meaning that the neutrons do not fall at all. When using the standard configuration the absorber is at the top, unable to absorb down-falling neutrons (outside the cavity). In the reversed configuration, however, there is a 3 cm “excess” of absorber outside, and below, the mirror, drastically reducing the neutron flux into the detector (as observed). Because of this, one can unfortunately not rule out a purely geometrical explanation of the measured effect, based on the performed tests.

4. CONCLUSIONS AND SUGGESTIONS FOR IMPROVEMENTS

Our conclusion is that a quantization of the gravitational ground state of a neutron has not been unambiguously identified. A “normal” cavity-effect can also explain the first discrete increase in the neutron count $N$, which is the only experimental result claimed to underlie the identification.

We therefore propose the following improvements of the experiment:

- Rotating the experimental setup by 90° keeping everything else, and especially the transverse neutron energies, constant. This gives a vertical instead of horizontal cavity. If the same result still occurs this would indicate that it is due only to the geometry of the experimental setup, as no gravitational quantum states can form in this case.

- Measuring where the neutrons strike the detector. This should be possible in principle, although not yet in practice. Since there is a standing neutron wave, the neutron is not falling in a classical sense, and the observed probability distribution should reflect $|\psi|^2$. This would directly discriminate between different theoretical explanations, as

\(^1\)If the transverse neutron temperature is 20 nK as stated in \(\text{[6]}\), corresponding to $\sim 1$ peV, even the simple infinite box potential can explain the first step. The smallest separation ($a \approx 15$ µm) then corresponds to the high energy “tail” of the transverse neutron energy. Any free-falling neutrons with higher transverse energies, which could traverse narrower slits, are “filtered out” by the absorber arrangement, just as in the original argument \(\text{[1, 2]}\).
the linear gravitational potential gives a very distinct probability distribution, different from those arising from “cavity-potentials”. However, it would require moving the (improved) detector right up to the end of the cavity, with no “free” mirror surface as in the present setup. As this test is not currently possible, we instead propose that measurements should be made of \( N = N(x) \) as a function of the cavity length, \( x \), accordingly varied. When the neutron wave function penetrates the absorber, the neutron count in the detector should be \( N \propto e^{-kx} \), where \( k = k(\Delta h) \) is proportional to the fraction of \( |\psi|^2 \) inside the absorber\(^2\). In this way it should be possible to differentiate between theoretical explanations (gravitation/cavity potentials) of the neutron counts.

The experimental group will also try to measure transitions between different quantum states. We close by noting that if this actually is accomplished it would be the first (indirect) measurement of a graviton spectrum, analogous to normal electromagnetic photon spectra from atoms. As one-graviton exchange is overwhelmingly more likely than multi-graviton exchange (suppressed by powers of the very small gravitational coupling constant), the transition energy difference, \( \Delta E \), will be carried away as a graviton with wavelength \( \lambda_{\text{grav}} = h c / \Delta E \sim 10^6 \) m.

\[1\] V. V. Nesvizhevsky et al., Phys. Rev. D 67, 102002 (2003).
[2] V. V. Nesvizhevsky et al., Nature 415, 297 (2002).
[3] V. I. Luschikov and A. I. Frank, JETP Letter 28, 559 (1978).
[4] S. Flügge, *Practical Quantum Mechanics* (Springer-Verlag, Berlin, 1999).
[5] S. Gasiorowicz, *Quantum Physics* (John Wiley & Sons, New York, 1996), 2nd ed.
[6] B. Schwarzschild, Physics Today 55, 20 (2002).

\[2\] A constant absorption probability per unit length, \( dN = -N k dx \), gives \( N = N_0 e^{-kx} \).