Dynamical masses, time-scales, and evolution of star clusters

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Abstract. This review discusses (i) dynamical methods for determining the masses of Galactic and extragalactic star clusters, (ii) dynamical processes and their time-scales for the evolution of clusters, including evaporation, mass segregation, core collapse, tidal shocks, dynamical friction and merging. These processes lead to significant evolution of globular cluster systems after their formation.

1. Introduction

The Milky Way and probably all large galaxies contain old globular cluster populations (see the review by Harris 1991). These old star clusters have an approximately log-normal luminosity function, and the mean cluster luminosity is somewhat brighter than $M_V = -7$ with little dependence on the host galaxy luminosity. In the Milky Way their typical mass-to-light ratios are $M/L_V \approx 2$, and typical total masses are $\sim 2 \times 10^5 M_\odot$ (Pryor & Meylan 1993). It is widely assumed that the globular clusters we see today must be the part of an initially larger population that survived the internal and external dynamical processes leading to cluster destruction (e.g., Ostriker 1988).

One of the exciting results from HST has been the discovery of young star clusters in starburst and interacting galaxies. Whitmore & Schweizer (1995) found many hundreds of young clusters in the Antennae galaxies. Young cluster systems have now been discovered in other interacting and merging galaxies, in barred and starburst galaxies, and even dwarf starburst galaxies (e.g., ESO 338-IG04, Oestlin, Bergvall & Roennback 1998). The luminosity functions of the young clusters are not log-normal, but seem to be better described by power-laws, about $\propto L^{-2}$. Carlson et al. (1999) use population synthesis models to determine the ages of the blue clusters in the young merger remnant NGC 3597. Based on these models they argue that the difference in the observed luminosity function when compared to the Galactic globular clusters cannot simply be an age effect, even if the young clusters formed with an intrinsic age spread. Are these young cluster systems then a good model for what the Milky Way's globular cluster population could have looked like at birth?

This review gives a brief discussion of dynamical methods to determine masses of distant and nearby star clusters (Section 2). It then goes on to describe a number of dynamical processes and their time-scales which will lead to evolution and potentially destruction of star clusters over long time-scales.
Finally, the results of some evolution calculations for globular cluster systems are briefly summarized (Section 3).

2. Dynamical Mass Determination for Star Clusters

In this Section we discuss methods for estimating star cluster masses from structural and kinematic measurements. Mass estimates based on stellar population properties are discussed in U. Fritze von Alvensleben's article in these proceedings.

2.1. Virial Masses

A simple global mass estimate for a star cluster can be obtained from the virial theorem. This says that, in equilibrium, the radius of a stellar system is proportional to $GM/V^2$, where $M$ is the total mass and $V$ the rms three-dimensional velocity of the stars. The constant of proportionality generally depends on the stellar density profile, but Spitzer (1969) showed that if the relation is expressed in terms of the half–mass radius $r_h$, this dependence is weak and the constant is approximately 0.4 for realistic cluster profiles. If we furthermore assume that the cluster is spherical, $V^2 = 3\sigma^2_\parallel$, where $\sigma_\parallel$ is the one-dimensional rms velocity dispersion along the line-of-sight, and write $\sigma_{10} = \sigma_\parallel/10\text{ km s}^{-1}$ and $r_5 = r_h/5\text{ pc}$, then

$$M_V = 7.5\sigma_{10}^2 r_h/G = 8.7 \times 10^5 \sigma_{10}^2 r_5 M_\odot.$$  \hspace{1cm} (1)

When using this formula to estimate star cluster masses from observed velocity dispersions and radii, a few points should be noted:

(i) Because the virial mass (1) is a global estimate, it is independent of velocity anisotropy. For example, shifting some stars to radial orbits while keeping the (spherical) potential fixed, will result in a larger central velocity dispersion but also lead to reduced velocities in the cluster halo. To maintain virial equilibrium these changes must add in just such a way that the global $\sigma_\parallel$ remains the same.

(ii) The dynamical evolution of star clusters leads to mass segregation and the formation of a halo of low-mass stars on preferentially radial orbits (see §3.2 below). For evolved clusters the measured half-light radius will therefore in general underestimate the half-mass radius, the observed velocity dispersion will underestimate the rms velocity dispersion, and eq. (1) will underestimate the mass.

(iii) Sometimes only the velocity dispersion for stars in the core is known, or the velocity dispersion from integrated light within some aperture. In these cases, a dynamical model is needed to convert this to the rms $\sigma_\parallel$. This introduces some uncertainty in the mass estimate because the derived $\sigma_\parallel$ depends on anisotropy.

With high-resolution spectra and HST photometry virial masses can be determined for some young 'superclusters' seen in starburst galaxies. Masses for three clusters in M82 are compared by Smith & Gallagher in these proceedings, spanning a range from $3 \times 10^5 M_\odot$ to $2 \times 10^6 M_\odot$. 
2.2. Core Masses

Rood et al. (1972) gave a formula that is often used to determine core masses. This is based on the dynamics of King models (King 1966; Binney & Tremaine 1987) and assumes that the velocity distribution in the core is isotropic:

\[
\left( \frac{M}{L} \right) = \frac{9\sigma_0^2}{2\pi G I_0 r_c}.
\]

Here \(\sigma_0\) is the central velocity dispersion, \(I_0\) the central surface brightness and \(r_c\) the core radius. The product of the two last quantities is rather insensitive to errors caused by seeing. Eq. (2) is very accurate as long as the assumption of isotropy is met (Richstone & Tremaine 1986); but it can overestimate the mass by a factor \(\sim 2\) if the system is actually radially anisotropic (Merritt 1988).

Core mass-to-light ratios can only be determined for well-resolved Galactic star clusters for which the core parameters \(r_c\), \(I_0\) and \(\sigma_0\) can be estimated. Even with the resolving power of HST the cores of distant young clusters cannot be resolved.

2.3. Masses from Model Fitting

An alternative method of estimating cluster masses is fitting the photometric and kinematic data with dynamical models. A simple such scheme was used by Djorgovski et al. (1997) in their study of the M31 globular clusters mass-to-light ratios. They used structural and photometric parameters for these clusters obtained with HST and kinematic measurements in a rectangular aperture obtained with Keck and HIRES. They then estimated an aperture correction from King models to transform their measurements to central velocity dispersions, and used a formula analogous to eq. (1) to estimate masses with the constant again determined from models.

For some Galactic globular clusters large velocity samples are available and in such cases much more detailed model fitting is possible (Pryor et al. 1989). The masses of the Galactic globular clusters referred to in §1 (Pryor & Meylan 1993) have been determined by these techniques. A recent such study is Côté et al. (1995) who investigated the dynamics of the globular cluster NGC 3201, using a CCD surface brightness profile and a sample of 857 measured stellar radial velocities to trace the velocity dispersion profile to large radii.

In such work the data are fitted by single- or multi-mass King-Michie models. In the multi-mass models, a power-law mass function for the cluster stars is typically assumed, and for each mass bin \(m_i\), a distribution function of King-Michie type (Michie 1963, Binney & Tremaine 1987) is used:

\[
f_i(E, J) \propto e^{-\beta J^2} \left( e^{-A_i E} - 1 \right).
\]

Here \(E\), \(J\) are the specific energy and angular momentum of a star in the (spherical) star cluster potential, and \(\beta\) can be thought of as specifying an anisotropy radius. In the core of the cluster, stars of different masses are assumed to be in equipartition (§3.3), so that \(A_i \propto m_i\). In the fitting procedure the free parameters are the radius, velocity and luminosity scale, the cluster’s concentration parameter, the anisotropy radius, and the index of the mass function. These
parameters are determined from fitting to the measured surface brightness and velocity dispersion profile. This leads to a determination of the mass-to-light ratio profile and anisotropy profile, rather than just a single $M/L$ as for the previously described techniques. However, the fit is non-unique in the sense that adding even fairly large numbers of faint low-mass stars in the halo (expected there from mass-segregation and evaporation, see §3.2. below) have little effect on the observed profiles. In their study, Côté et al. (1995) find a steady rise in $M/L$ with distance from the cluster center, as expected from dynamical evolution theory, and a global $M/L_B \simeq M/L_V \simeq 2.0 \pm 0.2$.

2.4. Masses from Proper Motions

For nearby Galactic globular clusters, it is possible to measure stellar proper motions in addition to radial velocities. Proper motion measurements give information about the velocity dispersions in two directions on the sky (radial along projected $R$, and tangential), and for a spherically symmetric cluster they are therefore in principle sufficient to determine the velocity ellipsoid as a function of radius, and thus the mass profile free of assumptions about anisotropy. The projected proper motion dispersions $\sigma_R(R)$ and $\sigma_T(R)$ are related to the intrinsic velocity dispersions $\sigma_r(r)$ and $\sigma_t(r)$ by Abel integral equations and can thus be inverted (Leonard & Merritt 1989). Moreover, from the inferred $\sigma_r(r)$ and $\sigma_t(r)$ one can predict the line-of-sight velocity dispersions $\sigma_\parallel(R)$ and compare with independent radial velocity data. This provides a check on the modelling and also can be used to determine the cluster distance. In terms of global velocity dispersions, $\langle \sigma_\parallel^2 \rangle = (\langle \sigma_R^2 \rangle + \langle \sigma_T^2 \rangle)/2$ for a spherical cluster and the correct distance.

In an early study along these lines Leonard et al. (1992) investigated radial velocity and proper motion data for the globular cluster M13. They concluded that the mean anisotropy of this cluster $\beta = 3(\sigma_R^2 - \sigma_T^2)/(3\sigma_R^2 - \sigma_T^2) \simeq 0.3$ and that the effect of the anisotropy on the mass determination is $\sim 20\%$. Much more detailed modelling will be possible with the large proper motion surveys currently in progress.

2.5. Non-Parametric Cluster Mass Distributions

With large samples of stellar velocities at hand, radial velocities or proper motions, it is possible to infer the mass distribution of the cluster without making specific assumptions like King-Michie stellar distribution functions. This requires solving the Jeans and projection equations for the intrinsic density and velocity dispersions under some smoothness constraint, given the data. I do not give the equations here, but refer to the papers mentioned below.

When the data consist of several hundred radial velocities, some assumption about the anisotropy is still needed. Gebhardt & Fisher (1995) describe such a non-parametric analysis of radial velocity data for four Galactic globular clusters, assuming isotropy of the stellar orbits. With a few hundred stellar velocities in each case the results are still noisy, but indicate radially increasing $M/L$-profiles as expected. Merritt, Meylan & Mayor (1997) describe a similar analysis of the cluster $\omega$ Centauri, assuming that it is oblate and seen edge-on, and that it is described by a meridionally isotropic two-integral model. They find that the
mass distribution cannot be strongly constrained by their data, but appears to be slightly more extended than the luminosity distribution.

As discussed above, proper motion data result in two independent velocity dispersions in the plane of the sky, and thus, within a spherical model, they contain sufficient information to determine the anisotropy of the stellar orbits. With sufficiently large data sets it will therefore be possible to model the anisotropy profile and mass distribution of a spherical cluster non-parametrically.

3. Dynamical Evolution Processes and their Time-Scales

3.1. Relaxation

On dynamical time-scales large star clusters \((N >> 100)\) evolve collisionlessly. That is, for some time after birth they are described by a quasi-equilibrium phase-space distribution function \((df)\) which is a function of the integrals of motion (or of the stellar orbits) in the mean field potential. On longer time-scales, however, the graininess of the distribution of stars becomes important, and the dynamical evolution is no longer collisionless. Over a relaxation time two-particle interactions then deflect the cluster stars from the orbits they would otherwise have followed in the mean gravitational potential.

In the approximation of a homogeneous distribution of equal-mass stars, with density \(\rho_0\) and isotropic Maxwellian velocity distribution with dispersion \(\sigma_0\), the two-body relaxation time is (Spitzer & Hart 1971)

\[
t_{r0} = 0.34 \frac{\sigma_0^3}{G^2 m_* \rho_0 \ln \Lambda} = 1.8 \times 10^8 \sigma_{10}^3 n_4^{-1} m_{*\odot}^{-2} (\ln \Lambda)_{10}^{-1} \text{ yr.} \tag{4}
\]

Here \(n = 10^4 n_4 \text{ pc}^{-3}\) is the number density of stars, \(\sigma_0 = 10 \sigma_{10} \text{ km s}^{-1}\) the velocity dispersion, \(m_* = m_{*\odot} \text{ M}_\odot\) is the mean mass per star, \(\Lambda\) is of order the number of stars, and \((\ln \Lambda)_{10} = (\ln \Lambda)/10). Note that \(t_{r0}\) is inversely proportional to the stellar phase space density. Central relaxation times in globular cluster cores evaluated with eq. (4) are \(\sim 10^7 - 10^9\) yr.

The relaxation time often varies by large factors between the central and outer parts of a stellar system. It is then useful to define a half-mass relaxation time. For a virialized star cluster this is obtained from eq. (4) by replacing \(\rho_0\) with the mean density inside the system’s half-mass radius \(r_h = 5r_5\) pc and \(\sigma_0^2\) by one third of the rms \(V^2\), and then using the virial theorem to express \(V^2\) through \(r_h\) and the total cluster mass \(M = 10^5 M_5 \text{ M}_\odot\). The result is (Spitzer & Hart 1971)

\[
t_{rh} = \frac{0.14N}{\ln 0.4N} \left( \frac{r_h^3}{GM} \right)^{1/2} = \frac{N}{26 \log 0.4N} t_d = 7.2 \times 10^8 M_5^{1/2} r_5^{3/2} m_{*\odot}^{-1} (\ln \Lambda)_{10}^{-1} \text{ yr,} \tag{5}
\]

where \(N\) is the total number of stars in the system and

\[
t_d \equiv r_h/V = 1.58 \left( \frac{r_h^3}{GM} \right)^{1/2} = 8.3 \times 10^5 r_5^{3/2} M_5^{-1/2} \text{ yr} \tag{6}
\]

is the dynamical time. For comparison with the local formula eq. (4), the fiducial values used in eq. (5) correspond to a one-dimensional virial velocity dispersion \(\sigma \simeq 3.4 \text{ km s}^{-1}\) and a mean density \(n \simeq 96 \text{ pc}^{-3}\).
On short time-scales cluster evolution is still collisionless, so long as $t_d \ll t_{rh}$ (requiring $N \gg 100$); for example, during the violent relaxation at formation. The resulting quasi-equilibrium $df$ subsequently evolves slowly in response to collisions, which will tend to drive the system towards an isothermal energy distribution. One aspect of such slow evolution would be a decrease of ellipticity with dynamical age (Fall & Frenk 1985). This could be the reason for the significantly rounder globular clusters in M31 and the Milky Way as compared with the LMC and SMC clusters (Han & Ryden 1994).

### 3.2. Evaporation and Core Collapse

Collisions between single stars modify the stellar $df$ in two ways. The rarer process is ejection, in which a single close encounter leads one of the stars to acquire a velocity greater than the local escape velocity $v_e$ and to escape from the cluster. The time-scale for this is $t_{ej} \equiv -N/(dN/dt) \simeq 1.1 \times 10^3 \ln 0.4 N / t_{rh} \sim 10^3 t_{rh}$ (Hénon 1969). The more important process of evaporation is caused by the cumulative effect of many weak encounters, which gradually increase a star’s energy until $v \geq v_e$. It is easy to show that the rms escape velocity of the cluster is just twice its rms virial velocity. Thus, on average, a particle with $v \geq 2V = \sqrt{12} \sigma$ will escape. For a Maxwellian velocity distribution, a fraction $\epsilon \sim 0.74\%$ of stars have $v \geq 2V$; these stars will escape in one dynamical time, after which the high-velocity tail is repopulated only in $\sim t_{rh}$. Thus one expects the evaporation time scale of the cluster to be $\sim (\epsilon/t_{rh})^{-1}$; detailed calculations (Spitzer & Thuan 1972) show that

$$t_{ev} \equiv -N (dN/dt)^{-1} \simeq 300 t_{rh}. \quad (7)$$

Because the evaporation is dominated by weak encounters, escaping stars leave the cluster with only very small positive energy; thus the total energy of the remaining cluster is nearly constant, but must be shared among a shrinking number of stars. In virial equilibrium $N^2/r_h \simeq \text{const.}$ and thus $\rho \propto N/r_h^3 \propto 1/N^5$ and $t_{rh} \propto N r_h^3/M^{1/2} \propto N^{7/2}$. So as the cluster becomes denser, evaporation accelerates and the system contracts to negligible mass and radius in finite time.

This evaporation model, however, neglects the fact that the evolution of the stellar cluster is not homologous and that the rate of evaporation is much faster in the dense core than in the system’s outer parts. Stars gaining energy towards evaporation build up an extended halo where the time scale for further energy gain increases strongly, so that these stars may not in fact escape during the age of the cluster. On the other hand, the dense core loses stars to the halo on the much faster central relaxation time, and may collapse to very high densities before $M_{tot}$ and $r_h$ can change much.

This phenomenon of core collapse may be understood as a consequence of the fact that self-gravitating star clusters have negative specific heat (Lynden-Bell & Wood 1968): In virial equilibrium the total energy $E = -T$, where $T$ is the total kinetic energy, which is proportional to the virial temperature $T/M = V^2$. As energy is withdrawn from the cluster, its kinetic energy increases and so does the virial temperature. Since $r_h \simeq GM^2/(2|E|)$, the cluster thereby contracts. Vice-versa, an energy production mechanism (e.g., from binary stars) causes the cluster to cool and expand. Now the dense core of the cluster may be
approximately regarded as a virialized system in thermal contact with the rest of the cluster. It is normally hotter than its surroundings and therefore loses energy to them through stellar encounters. As a result it shrinks and becomes yet hotter, loses still more energy to the surrounding stars, and contracts to formally zero radius in finite time.

For a single-mass star cluster the late stages of core collapse are self-similar (Lynden-Bell & Eggleton 1980, Cohn 1980). As the core radius

\[ r_c \equiv 3 \sigma_c / \sqrt{4 \pi G \rho_c} \]  

shrinks, the central density \( \rho_c \) and velocity dispersion \( \sigma_c \) increase and the core mass \( M_c \) decreases according to

\[ \rho_c \propto r_c^{-2.23}, \quad \sigma_c \propto (\rho_c r_c^3)^{1/2} \propto r_c^{-0.11}, \quad M_c \propto \rho_c r_c^3, \propto t_c^{-0.77} \]  

until the core radius and mass formally shrink to zero at time \( t_{cc} \). Moreover, the density profile of the cluster outside the collapsing core has the same exponent:

\[ \rho \propto r^{-2.23} \]  

for \( r_c(t) \ll r \ll r_c(0) \).

The time-scale for core collapse is proportional to the central relaxation time; for a single mass cluster it is \( t_{cc} \approx 330 t_{rc} \) once the collapse is in the self-similar phase (Cohn 1980, Heggie & Stevenson 1988). The total time until core collapse in Cohn’s (1980) model is \( \sim 16 \) half-mass relaxation times or \( \sim 60 \) initial \( t_{rc} \). As Goodman (1993) has emphasized, the former number depends on the mass distribution of the cluster, and \( t_{cc}/t_{rh} \) will be less than 16 for clusters more centrally concentrated than Plummer models. By noting that

\[ r_c^{-1}dr_c/dt \propto t_{rc}^{-1} \propto \rho_c/\sigma_c^3 \propto r_c^{-1.89}, \]  

one can solve for the asymptotic time-dependence of the collapse:

\[ r_c \propto (t_{cc} - t)^{0.53}, \quad \rho_c \propto (t_{cc} - t)^{-1.18}, \quad \sigma_c \propto (t_{cc} - t)^{-0.06}, \quad M_c \propto (t_{cc} - t)^{0.41}. \]  

In summary, the collapse of a single mass cluster occurs in two stages (Cohn 1980). The longer part of the evolution is an evaporative phase, during which stellar collisions simultaneously populate a halo and make the core shrink and become denser. Only towards the end does the evolution accelerate and enter the gravothermal instability phase of self-similar collapse.

### 3.3. Equipartition, Mass Segregation, and Multi-Mass Core Collapse

When the cluster contains different stellar masses, energy can flow not only from the core to the halo, but also between stars of different masses. Stellar collisions drive the system towards equipartition of energy \( m_i \langle v_i^2 \rangle = \text{const} \). As eq. (4) shows, relaxation proceeds faster for more massive stars. The equipartition time-scale measures the rate at which a group of heavy stars with masses \( m_2 \) loses energy to lighter stars of mass \( m_1 \) (Spitzer 1969):

\[ t_{eq} = \frac{(\langle v_1^2 \rangle + \langle v_2^2 \rangle)^{3/2}}{8(6\pi)^{1/2}G^2m_1m_2n_1 \ln \Lambda} = 1.2 \frac{m_1}{m_2} t_{r0}(m_1) \]  

where we have assumed equal temperatures \( \langle v_1^2 \rangle = \langle v_2^2 \rangle \) and used eq. (4). Initially, \( \langle v_i^2 \rangle \) is independent of stellar mass; thus the massive stars lose kinetic
energy and sink to the center, while lighter stars gain kinetic energy in collisions and move outwards, a process called mass segregation. Moreover, eq. (11) shows that mass segregation of the massive stars occurs before relaxation of the cluster as a whole becomes significant.

However, equipartition may never be reached. A simple case considered by Spitzer (1969) is one with two mass groups such that the heavy stars are much more massive than the light stars, \( m_2 \gg m_1 \), but the total mass in the cluster core is dominated by the light stars: \( M_2 \ll \rho_1 r_{c1}^3 \). In this case equipartition causes the formation of a small subsystem of heavy stars \((M_2, m_2)\) in the core of the distribution of lighter stars. Applying the virial theorem to the subsystem of heavy stars gives

\[
\langle v_2^2 \rangle \simeq \frac{0.4 \, G \, M_2}{r_{h2}} + \frac{4 \pi \, G \, \rho_{c1}}{3} \, k^2 \, r_{h2}^2,
\]

where the first term describes the self-energy of the subsystem \( M_2 \) and the second term its interaction with the system of light stars \((k \text{ is a constant of order unity})\). Spitzer (1969) noticed that the right-hand-side of eq. (12) has a minimum when regarded as a function of \( r_{h2} \). An equilibrium can therefore exist only if \( \langle v_2^2 \rangle \) is greater than this minimum, that is, assuming equipartition, if

\[
\frac{M_2}{\rho_1 r_{c1}^3} \leq 4.0 \, k^{-1} \left( \frac{m_1}{m_2} \right)^{3/2}.
\]

In other words, if its mass is too large, the subsystem of heavy stars remains a dynamically independent stellar system with mean square velocity greater than the equipartition value. It continues to lose energy to the lighter stars, becoming denser and hotter, and evolving away from equipartition all the time (mass stratification instability). Fokker-Planck calculations (Inagaki & Wiyanto 1984, Cohn 1985) show that in the end the subsystem of heavy stars core collapses independently from the cluster of light particles, just like a single mass system.

The evolution to core collapse with a spectrum of stellar masses has been considered by Inagaki & Saslaw (1985) and Chernoff & Weinberg (1990). The detailed evolution occurs in several phases: First, collisions trying to establish equipartition of energy lead to mass segregation and the formation of a heavy mass core. Then this core undergoes the gravothermal instability, i.e., contracts while remaining hotter than the mean temperature of the system and conducting energy outwards. This collapse accelerates towards core collapse, and finally goes over into a single-component collapse which reaches formally infinite central density. The time scale for this multi-mass core collapse evolution is faster than that for core collapse in any single component cluster, typically a factor of a few faster than for a cluster composed of the heaviest mass alone.

Deep in collapse, the density slopes of all mass groups \( m_k \) are characterized by separate power laws in the region where the heaviest component dominates the potential, such that approximately (Cohn 1985, Chernoff & Weinberg 1990)

\[
d \ln \rho_k / d \ln r \simeq -1.89 (m_k / m_u) - 0.35,
\]

where \( m_u \) is the mass of the heaviest species in the cluster. The overall mass profile is then not self-similar.
A multi-mass core collapse, however, may be strongly influenced by the stellar evolution of the more massive stars. This has two main effects: First, the mass loss from massive stars through winds may lead to an overall mass loss from the cluster, and thus cause an adiabatic expansion. Secondly, the finite stellar life-time $t_{MS}$ limits the time $t_{cc}$ during which they can core collapse, such that $t_{cc}(m_*) \lesssim t_{MS}(m_*)$. Both effects greatly increase the overall core collapse times; compared to a system of point masses within the range $(0.4 - 15) M_\odot$, Weinberg & Chernoff (1989) find an increase by about a factor of $30 - 60$ in their globular cluster models, including the expansion effects.

A reasonable approximation for the stellar lifetime of all but the most massive stars is $t_{MS} \simeq 9 \cdot 10^9 (m_*/M_\odot)^{-2.6} \text{yr}$. Thus if the most massive stars leave black hole remnants of $3 M_\odot$, these together with tight binaries will dominate the evolution after $5 \cdot 10^8 \text{yr}$, while if the most massive remnants are $1.4 M_\odot$ neutron stars, they and the binaries will dominate after $4 \cdot 10^9 \text{yr}$. The latter time scale approaches the time expected for core collapse in typical Milky Way globulars.

3.4. Reversing core collapse

A number of energy source mechanisms can stop core collapse (e.g., Goodman 1993): (i) Processes that generate kinetic energy in the core directly, such as binary stars transferring energy to the field stars in collisions. (ii) Mass loss processes that heat the core indirectly above its virial temperature, including: normal stellar evolution, accelerated stellar evolution by the formation of massive stars in mergers, and ejection of stars through binaries. In all cases the net result is adiabatic expansion and cooling of the core.

Only hard binaries contribute to field star heating. Binaries are hard if their binding energy $E_b = -G m_1 m_2/a$ (with $a$ their semi-major axis) exceeds the mean kinetic energy, $E_b > 3 m_\ast \sigma^2$; those with $E_b < 3 m_\ast \sigma^2$ are called soft binaries. Heggie’s law (Heggie 1975, Hut 1983) states that, on average, hard binaries get harder by collisions with field stars, and soft binaries get softer. Essentially, the orbital velocity of a hard binary is on average greater than the velocity of an incoming field star, and the tendency towards energy equipartition therefore results in a net transfer of energy to the field star. The opposite is true for soft binaries, which gain net energy and eventually dissolve. The binary behaves like a mini-system with negative specific heat: as energy is withdrawn from it, the orbit shrinks, the orbital velocity increases, and the binary hardens. When the binary becomes sufficiently hard, the typical recoil from a collision with a single star becomes large, and the binary will eventually be kicked out of the cluster. Just like in the Sun, the binaries providing the nuclear energy source will eventually be ‘burned’.

Binaries can be formed by a close gravitational interaction of three stars (‘three-body binaries’), by dissipational tidal capture, or at the time of star formation (‘primordial binaries’). To be effective in reversing core collapse, binaries must have orbital semi-major axes

$$a < G m_\ast / 3\sigma^2 = 3 \sigma_{10}^{-2} m_\ast M_\odot \text{AU}. \quad (15)$$

The formation of a hard three-body binary requires a close encounter between two stars ($\delta v \simeq v$) with a third star in the immediate vicinity, such that
one of the three stars acquires additional energy, leaving the other two as a bound pair. Thus the time-scale is (Goodman 1984, Binney & Tremaine 1987)

\[ t_3 \simeq (np^2 v)^{-1} (np^3)^{-1} \simeq \frac{\sigma^9}{n^2 (Gm_*)^5} \simeq N_c^2 \ln N_c t_{r0}, \]

(16)

where \( p \simeq Gm_*/v^2 \), \( v = O(\sigma) \) because low relative velocities dominate, and we have used eq. (4) to express Goodman’s result in terms of the central relaxation time and the total number of stars in the core, \( N_c \). This implies that about \( 1/N_c \ln N_c \) three-body binaries form per central relaxation time. In other words, three-body binaries become important if the final core collapse is driven by fewer than 100 of the largest mass stars.

3.5. Tidal field and tidal shocks

A steady tidal field lowers the energy threshold beyond which stars are no longer bound to the cluster. It thus increases the mass loss rates from evaporation, both because the fraction of stars in the velocity distribution that escape in a dynamical time increases, and because the decreasing number of stars in the cluster leads to shorter relaxation times. The Quintuplet and Arches young clusters (Figer et al. 1999) in the inner Galactic bulge are two clusters for which these tidal effects are very important (Kim et al. 1999).

In reality, the tidal field is not stationary in the frame of the cluster stars. This complicates the escape process, but more importantly it leads to a new dynamical process in cluster evolution, referred to as gravitational shocking (Ostriker, Spitzer & Chevalier 1972). The tidal field acting on the cluster may suddenly increase in strength when the cluster passes through the disk of its host galaxy, or when it comes close to the high-density inner bulge near perigalacticon of its orbit. In both cases, the perturbations to the stellar orbits caused by the tidal shock lead to an effective energy input in the cluster which makes the cluster less bound and accelerates mass loss from internal processes.

A detailed recent discussion of this process is given by Kundic & Ostriker (1995) and Gnedin, Lee & Ostriker (1999). For stars in the outer parts of the cluster, the tidal perturbation can be approximated as impulsive because of the short time-scale of passage through the galactic disk. In the cluster’s central parts, on the other hand, the stellar orbital time-scales are short and adiabatic invariance reduces the effects of the perturbation. Traditionally these effects of the tidal shock were described by a first-order term \( \langle (\Delta E)_{ts} \rangle \), which denotes the net energy gain averaged over stellar orbits at a given position in the cluster. Kundic & Ostriker (1995) noticed that the second-order term \( \langle (\Delta E)^2_{ts} \rangle \) is typically even more important and competes with two-body relaxation near the half-mass radius in driving evolution of the cluster’s internal structure. This can speed up core collapse by a factor of three (Gnedin, Lee & Ostriker 1999). Cluster destruction is accelerated; recent modelling of the evolution of the Milky Way’s globular cluster system shows that the typical time to destruction becomes comparable to the typical age of the Galactic globulars (Gnedin & Ostriker 1997).
3.6. Dynamical friction and merging

As already noted by Tremaine, Ostriker & Spitzer (1975), massive star clusters experience dynamical friction against field stars as they move along their orbits through the host galaxy. Because of the frictional drag the cluster loses orbital energy and spirals into the galaxy center, where the tidal field becomes ever stronger and will eventually dissolve the cluster.

The time-scale for dynamical friction for a cluster on a circular orbit at initial radius \( r_i = 2r_{i,2} \) kpc in a singular isothermal sphere with circular velocity \( v_c = 250 \) \( v_{250} \) km s\(^{-1} \) is (Chandrasekhar 1943, Binney & Tremaine 1987)

\[
t_{df} = \frac{1.17r_i^2v_c}{\ln \Lambda GM} = 2.64 \times 10^{11} r_{i,2}^2 v_{250} M_5^{-1} (\ln \Lambda)_{10}^{-1} \text{ yr}
\]

where \( M_5 \) is again the cluster mass in units of \( 10^5 M_\odot \). The friction time-scale thus scales with the square of the cluster’s initial radius in the potential, and is inversely proportional to its mass. It is the inner, most massive clusters which are affected first.

If we continue to model the inner parts of the host galaxy as an isothermal sphere with \( M_G(r) = v_c^2 r/G \) and use the virial theorem \( \text{[eq. (1)]} \) for the cluster, we can determine the radius at which the incoming cluster will dissolve as

\[
r_{\text{dis}} \equiv r_h \left( \frac{M_G(r_{\text{dis}})}{M} \right)^{1/4} = \frac{r_h v_c}{\sqrt{1.5} \sigma_{\parallel}} = 46 r_5 \sigma_{10}^{-1} v_{250} \text{ pc.}
\]

Young clusters formed in the high-density regions of starburst galaxies would thus contribute to the build-up of the nuclear bulge after being dragged inwards by dynamical friction and tidally shredded by the tidal field.

In some circumstances it may be possible that several young clusters are born close enough to each other to tidally interact and even merge. To quantify this we use an approximate merging criterion fitted by Aarseth & Fall (1980) to the results of N-body merger simulations. For the escape velocity of the clusters at pericenter \( p \) of their relative orbit we take an approximate expression assuming two overlapping Plummer spheres, \( v_e^2(p) = 27.6 \sigma_{\parallel}^2 / (1 + p^2/1.2r_h^2)^{1/2} \) (see also the discussion in Gerhard & Fall 1983). Then the criterion for merging becomes

\[
\left( \frac{p}{4r_h} \right)^2 + \left( \frac{v_p}{6\sigma_{\parallel}} \right)^2 \left( 1 + \frac{p^2}{1.2r_h^2} \right)^{1/2} \lesssim 1
\]

where \( v_p \) is the relative velocity at pericenter. Here we have used the virial equation \( \text{[eq. (2)]} \), and \( \sigma_{\parallel} \) is again the one-dimensional rms velocity dispersion of the cluster. For head-on collisions, this formula predicts merging for \( v_p \lesssim 6\sigma_{\parallel} = 60 \sigma_{10} \text{ km s}^{-1} \) (slightly more than \( \sqrt{2} \) times the rms escape velocity from each cluster), or \( \Delta v = \sqrt{36 - 27.6 \sigma_{\parallel}} \simeq 3\sigma_{\parallel} \simeq 30 \sigma_{10} \text{ km s}^{-1} \) for their relative velocity at large separations. It also shows that merging requires the two clusters to come within several half-mass radii of each other for merging to occur, at correspondingly smaller approach velocities. The most likely situation for this to happen would be when two clusters are born from the same giant molecular cloud complex.
3.7. Evolution of globular cluster systems

The evolution of globular cluster systems has recently been modelled in a number of studies, among others by Gnedin & Ostriker (1997), Murali & Weinberg (1997), Baumgardt (1998) and Vesperini (1998). These models combine assumptions about the initial cluster mass function and cluster locations with evolutionary models for individual clusters. In the models the various processes described above are considered, and treated in some studies by parametrized mass loss rates or analytic approximations to the results of N-body simulations, in others as diffusion terms in Fokker-Planck models. Some of the main results of these studies are:

(i) Globular cluster systems in galaxies evolve significantly. In the Milky Way the typical cluster destruction time is of order the age of the system, and about half of the present globulars will be destroyed in the next Hubble time.

(ii) Clusters in the inner regions of their host galaxy are disrupted most rapidly. Similarly, clusters on eccentric orbits are preferentially destroyed over clusters on tangential orbits.

(iii) Low-mass and high-concentration clusters are disrupted by evaporation, loosely bound clusters and those on central or eccentric orbits by tides, and massive inner clusters by dynamical friction and tides.

(iv) Low-mass clusters are destroyed most efficiently and initial power-law mass distributions tend to become transformed towards approximately log-normal mass distributions.

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