Inside-out star formation quenching and the need for a revision of bulge-disk decomposition concepts for spiral galaxies

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ABSTRACT

Our knowledge about the photometric and structural properties of bulges in late-type galaxies (LTGs) is founded upon image decomposition into a Sérsic model for the central luminosity excess of the bulge and an exponential model for the more extended underlying disk. We argue that the standard practice of adopting an exponential model for the disk all the way to its center is inadequate because it implicitly neglects the fact of star formation (SF) quenching in the centers of LTGs. Extrapolating the fit to the observable star-forming zone of the disk (outside the bulge) inwardly overestimates the true surface brightness of the disk in its SF-quenched central zone (beneath the bulge). We refer to this effect as $\delta_{\mu}$. Using predictions from evolutionary synthesis models and by applying to integral field spectroscopy data REMOVEYOUNG, a tool that allows the suppression of stellar populations younger than an adjustable age cutoff, we estimate the $\delta_{\mu}$ in the centers of massive SF-quenched LTGs to be up to $\sim$2.5 (0.7) $B(K)$ mag. The primary consequence of the neglect of $\delta_{\mu}$ in bulge-disk decomposition studies is the oversubtraction of the disk underneath the bulge, leading to a systematic underestimate of the true luminosity of the latter. Secondary biases impact the structural characterization (e.g., Sérsic exponent $\eta$ and effective radius) and color gradients of bulges, and might include the erroneous classification of LTGs with a moderately faint bulge as bulgeless disks. Framed in the picture of galaxy downsizing and inside-out SF quenching, $\delta_{\mu}$ is expected to differentially impact galaxies across redshift and stellar mass $M_*$, thus leading to systematic and complex biases in the scatter and slope of various galaxy scaling relations. We conjecture that correction for the $\delta_{\mu}$ effect will lead to a down-bending of the bulge versus supermassive black hole relation for galaxies below $\log(M_*/M_\odot)$$\sim$10.7. A decreasing $M_*/M_\odot$ ratio with decreasing $M_*$ would help to consistently explain the scarcity and weakness of accretion-powered nuclear activity in low-mass spiral galaxies. Finally, it is pointed out that a well-detectable $\delta_{\mu}$ (2 r mag) can emerge early on through inward migration of star-forming clumps from the disk in combination with a strong contrast of emission-line equivalent widths between the quenched protobulge and its star-forming periphery. Spatially resolved studies of $\delta_{\mu}$ with the James Webb Space Telescope, the Extremely Large Telescope, and Euclid could therefore offer key insights into the chronology and physical drivers of SF-quenching in the early phase of galaxy assembly.

Key words. galaxies: structure – galaxies: photometry – galaxies: spiral – galaxies: bulges – galaxies: evolution

1. Introduction

An explicit assumption in bulge-disk decomposition studies of late-type galaxies (LTGs) is that the exponential model for the visible part of the disk (i.e., outside the bulge) is valid all the way to the galaxy center (below the bulge). Subtraction of that model from the galaxy allows extracting the central luminosity excess of the bulge, the fitting of which with a Sérsic (1963) function yields the total magnitude, the Sérsic index $\eta$, effective radius $R_{\text{eff}}$, and the effective surface brightness $\mu_{\text{eff}}$ of the bulge. These quantities are fundamental to our understanding of the nature and structural characteristics of galaxy bulges, and to their empirical subdivision into classical bulges and pseudobulges (cf. Gadotti 2009; Fisher & Drory 2010, 2011; Fernández Lorenzo et al. 2014; Méndez-Abreu et al. 2017; Neumann et al. 2017, among others). They also constitute the observational foundation for various galaxy scaling relations (e.g., Kormendy 1977; Faber & Jackson 1976; Djorgovski & Davis 1987), including the correlation of the bulge versus supermassive black hole (SMBH) (Richstone et al. 1998; Ferrarese & Merritt 2000; Ho 2008; Simmons et al. 2013, see also, Kormendy & Ho 2013 for a review).

Regardless of whether the photometric analysis of the bulge is carried out in 1D or 2D and whether it is done sequentially, through subtraction of the disk first and subsequent fitting of the bulge, or simultaneously, through nonlinear fitting of both, the key assumption in all cases is that the intensity profile of the disk below the bulge (hereafter, inner disk, ID) is simply the inward extension of the exponential profile of the outer disk (OD).
However, it is worth contemplating what further assumptions are implicitly encapsulated in the postulate that the exponential model for the visible outer disk is valid all the way to the galaxy center. The first assumption is that the stellar surface density $\Sigma_\star$ (M$_\odot$ kpc$^{-2}$) of the disk follows a single exponential profile to $R^*=0''$, and the second assumption is that its stellar mass-to-light ratio ($M/L$) is spatially constant.

Here, we only focus on the second assumption. To a first approximation (leaving aside the age-metallicity degeneracy), a radially constant $M/L$ translates into a homologous spectral energy distribution (SED), a spatially uniform star formation history (SFH), and current specific star formation rate (sSFR), as previously pointed out in Breda et al. (2020b, hereafter B20b).

This, however, stands in diametric contrast to the well-established phenomenon of star formation quenching (SFQ) in the centers of massive ($L \geq L^*$) late-type galaxies. A strong depression or complete cessation of star formation (SF) activity within the bulge radius $R_B$ of these systems has been documented through photometry (e.g., Balcells & Peletier 1994; Peletier & Balcells 1996; de Jong & van der Kruit 1994) and, more recently, the centrally decreasing SF surface density, sSFR, and Hα equivalent width, and increasing age within $R_B$ using integral field spectroscopy (IFS) data (Pérez et al. 2013; Fang et al. 2013; González-Delgado et al. 2014; Catalán-Torrecilla et al. 2017; Belfiore et al. 2018; Zibetti et al. 2017; Breda & Papaderos 2018; Qua et al. 2019; Kalinova et al. 2021).

That SFQ commences early on in the dense centers of galaxies above typically log($M_\star$/M$_\odot$)$\sim$10.5 (Strateva et al. 2001) has observationally been established through studies of galaxies at higher redshift (Tacchella et al. 2015; Mosleh et al. 2017), in agreement with cosmological simulations (e.g., Tacchella et al. 2016). The most direct illustration of SFQ is through visual inspection of true-color images of local LTGs with their typically reddish bulge and surrounding blue star-forming disk (Fig. 1, left).

The physical origin and timescales of inside-out SFQ in galaxies, even though a fundamental subject that has motivated numerous previous investigations, is not of primary importance to our considerations next. Important is merely the simple argument that the mechanisms that inhibit SF in the bulge must also act toward inhibiting SF in the inner disk, given that these two stellar components are cospatial and their SF activity (if any at all) is fed by a common reservoir of gas (Breda et al. 2020a, hereafter B20a). Because partial or complete cessation of SF entails an increase in the $M/L$ ratio, a consequence of SFQ is that the $M/L$ ratio in the inner disk ($R^* \leq R_B$) must be higher than than in the outer disk ($R^* > R_B$). If this is so, the SF-quenched inner disk must be dimmer than the inwardly extrapolated model for the star-forming outer disk. Therefore, the immediate implication of SFQ for bulge-disk decomposition studies is the overestimation (and oversubtraction) of the disk inside $R_B$, consequently, the underestimation of the luminosity of the bulge.

A schematic illustration of this effect (dimmer inner disk than the inwardly extrapolated exponential fit to the outer disk) is given in the right panel of Fig. 1. For a thin face-on disk, the surface brightness profile (SBP; in $L_\odot$ pc$^{-2}$ or mag/arcsec$^2$, light blue) is the product of the stellar surface density $\Sigma_\star$ (dark blue) by the inverse $M/L$. For the sake of simplicity, $\Sigma_\star$ is here assumed to follow an exponential profile all the way to $R^*=0''$ (see, however, B20b and Sect. 4.4). If $M/L$ were to be constant throughout the disk (i.e., in the case of no SFQ), then the fit to the visible outer disk at $R^* > R_B$ (dashed line) would yield an exact match to the profile of the invisible inner disk ($R^* \leq R_B$), thus warranting a correct determination of the central luminosity excess of the bulge (shaded red area). In the case of SFQ, however, the central surface brightness $\mu_0$ of the disk is fainter by $\delta\mu_0$ mag than the value inferred from the exponential model for the visible outer disk. As a result, standard bulge-disk decomposition entails an overprediction of the luminosity of the disk underneath the bulge by a factor on the order of $\delta\mu_0/2.5$ $^{-1}$. In the following, we refer to the difference between the true integrated magnitude of the
The obvious next question is whether this $\delta_{\text{io}}$ effect is relevant at all. If it is small (e.g., $\lesssim 0.2$ mag), it might rightfully be argued that it is absorbed within the error budget of bulge-disk decomposition studies for spiral and lenticular galaxies and thus can be ignored. If it is significantly larger, however, then it should deserve a closer examination because it might systematically impact photometric and structural studies of bulges.

The aim of this pilot study is to draw attention to $\delta_{\text{io}}$ and motivate observational and theoretical work toward better understanding it. To this end, we provide empirical estimates of this effect and offer a concise discussion of the biases that neglecting it entails for the photometric characterization of galaxy bulges. Section 2 presents estimates based on simple evolutionary models and spectral population synthesis of IFS data for local LTGs and shows that $\delta_{\text{io}}$ can exceed 2 mag in the optical and 0.6 mag in the near-infrared (NIR). Section 3 addresses the effect of the neglect of $\delta_{\text{io}}$ on determinations of the luminosity and color of bulges. We also show that $\delta_{\text{io}}$ can prevent detection of a moderately faint bulge embedded within a centrally SF-quenched LTG and lead to its erroneous classification as bulgeless. In Sect. 4 we briefly comment on the mass-dependent evolution of $\delta_{\text{io}}$ from the perspective of galaxy downsizing and how neglecting it might be imprinted on the slope and scatter of galaxy scaling relations, including the relation between absolute magnitude $M_B$ of the bulge and SMBH mass $M_\bullet$. Finally, we discuss possible empirical approaches for estimating $\delta_{\text{io}}$ and accounting for it in galaxy decomposition schemes. A summary of our conclusions is given in Sect. 5.

### 2. Estimates of $\delta_{\text{io}}$

The exact value of $\delta_{\text{io}}$ depends on the integrated apparent magnitude of the disk inside the bulge radius $R_B$, denoted as $\delta_{\text{io}}(R_B)$, which can be calculated as

$$\delta_{\text{io}} = 2.5 \cdot \log(\psi)$$

where $\psi$ is the ratio of the mean $M/L$ of the disk inside and outside $R_B$ as

$$\delta_{\text{io}}(\text{mag}) \approx 2.5 \cdot \log(\psi) = 2.5 \cdot \log \left( \frac{M/L_{\text{disk}}}{M/L_{\text{io}}} \right).$$

The ratio $\psi$ in turn encapsulates the stellar mass assembly history and metallicity in the two disk zones, and obviously depends on the photometric filter considered. An estimate of $M/L$ can be obtained using standard semiempirical SFH parameterizations, according to which galactic disks are characterized by a nearly-constant star formation rate (SFR) (e.g., Gallagher et al. 1984) whereas stellar spheroids are better described by an exponentially declining SFR with an e-folding time $\tau$ that scales inversely with present-day stellar mass $M_\bullet$ (Sandage 1986; Guiderdoni & Rocca-Volmerange 1987, see also Poggianti et al. 1999, Gavazzi et al. 2002).

Using PÉGASE 2 (Fioc & Rocca-Volmerange 1997), we follow the photometric evolution of the outer disk ($R^*>R_B$) assuming...
continuous SF at a constant SFR and a fixed stellar metallic- 
ity Z_⊙/5 (hereafter SFH1). The bulge (R∗ ≤ R_B) is approximated 
by exponentially decreasing SFR models with a τ of 0.5 Gyr 
and 1 Gyr for solar metallicity (SFH2 and SFH3, respectively). 
These two models imply a currently low SFR and a predomi-
nantly old stellar population in the centers of LTGs, in agree-
ment with optical and NIR colors (de Jong 1996a,b; Gadotti & 
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dos Anjos 2001) and constraints from spectral synthesis (Gal-
azzi et al. 2005; Pérez et al. 2013; Zibetti et al. 2017; Breda 
& Papaderos 2018). Furthermore, the mass-weighted age im-
plied by the two latter SFHs (τM ≥ 12 Gyr) agrees well with 
determinations from spectral population synthesis for high-mass 
(M/L) and a leveling-off after ~8 Gyr (τ ~ 0.6) to a nearly constant δ_θ 
of ~2.5 mag in B, 1.6 mag in R, and 0.7 mag in K. 
The strongly simplified SFH parameterizations in Fig. 2 allow 
the insight that δ_θ becomes significant (>1 B mag) early on 
and is non-negligible in present-day LTGs. On the other hand, 
the bulge is often in mind that the underlying working hypothesis of 
stellar populations within R_B sharing a similar evolutionary 
history, although broadly consistent with observations and sup-
ported by plausibility arguments, does most certainly not capture 
due to the diversity of the complex and highly interlinked processes 
that shape the evolution of gas and stars at the centers of LTGs. 
The standard scenario (see, e.g., Kormendy & Kennicutt 2004, 
and references therein) distinguishes between classical bulges 
(CBs) and pseudobulges (PBs). The first are thought to emerge 
quasi-monolithically early on, prior to, and independently of the 
disk, and the second gradually build up over the billion-year-
long secular evolution of LTGs through in situ SF and inward 
migration of stars and star-forming clumps from the disk. The 
picture of two distinct routes to bulge formation is not undis-
pusted, however. For example, Breda & Papaderos (2018, here-
after BP18) found no evidence for the age bimodality implied by 
the standard scenario from a spectral synthesis study of a rep-
resentative sample of LTGs. Their analysis shows instead that 
bulges span a continuous sequence in age, with the latter follow-
ing a tight correlation with their present-day M_⋆ and Σ_*. 

Nevertheless, an elementary discussion of the relevance of δ_θ 
from the perspective of the standard bulge formation scenario 
may be attempted. Whereas a disk underneath the bulge is un-
likely to exist just after the dominant phase of CB formation, it is 
 conceivable that stellar diffusion and inward migration of ma-
terial from the outer disk subsequently lead to the gradual build-
up of an inner disk. However, it is not immediately apparent why 
such dynamical processes per se ensure the fine-tuning required 
to make the Σ_* profile of the inner disk an exact inward extension of the 
exponential Σ_* profile of the outer disk. Furthermore, dy-
namical heating of the inner disk through its billion-year-long in-
teraction with the kinematically hotter CB might inflate it into a 
triaxial entity that is hardly separable from the bulge itself. Such 
considerations suggest that CBs either lack an underlying inner 
disk, or that their inner disk, if present, substantially deviates 
from the exponential Σ_* slope of the outer disk (cf. the discus-
sion in Breda et al. 2020b). This would then call for a revision of 
structural determinations of CBs (e.g., absolute magnitude, Sė-
mic exponent, and effective radius) on the basis of an adequate 
image decomposition scheme that allows a down-bending of the 
disk inside R_B. Because a central depletion of the disk can from 
the photometric point of view be regarded as equivalent to an 
increase in the disk M/L to infinity, a high δ_θ should be a generic 
characteristic of CB-hosting LTGs and as such an indispensable 
element to consider in their study. 
The amplitude of δ_θ in PBs likely depends on the relative con-
tribution of migration and in situ SF to the stellar mass growth 
inside their R_B. The first process (inward migration of stars and SF 
clumps from the disk) leads to negative radial age gradients with 
a slope that inversely scales with the average migration velocity 
(B20a) and can amplify preexisting SFQ patterns at the centers 
of LTGs (cf. BP18 for a discussion). This is due to the stellar 
mass filtering effect discussed in Papaderos et al. (2002, see also 
Papaderos & Östlin 2012), namely the depopulation of the high-
mass end of the IMF with a main-sequence lifetime shorter than 
the migration timescale. As these authors remark, neglect of this 
effect can mimic a spatially varying IMF or lead to a system-
atic overestimation of age when color maps are used to age-date 
stellar populations. Quite importantly, because inward migration 
results in a higher M/L in the bulge than in the disk, it also nat-
urally acts toward enhancing δ_θ in PB-hosting LTGs. 
It follows from these considerations that the presence of δ_θ 
is consistent with the standard bulge formation scenario. On the 
other hand, without tight observational and theoretical con-
straints, it is difficult to establish whether an inner disk invariably 
exists in both CBs and PBs, and, if so, whether its SFH is similar 
to that of the bulge (and bar). 
Addressing these questions not only requires further observa-
tional work (e.g., correction of bulge color gradients for a possi-
ble underlying inner disk, or a combined spectrophotometric and 
kinematical bulge-disk decomposition), but also the understand-
ing of the influence of active galactic nuclei (AGN) on the SFH 
both within R_B and over the entire galaxy. This subject encapsu-
lates several poorly understood aspects, such as the directionality 
and chronology of negative AGN feedback, and its possible cou-
ping with galaxy mass M_⋆. For instance, BP18 (their Fig. 6f) 
concluded from diagnostic emission-line ratios after Baldwin et 
al. (1981, hereafter BPT) that gas excitation in bulges of lower-
mass LTGs (log(M_⋆/M⊙) < 10) is dominated by photoionization 
by OB stars, whereas more massive ones (log(M_⋆/M⊙) ≥ 10.5) 
generally fall on the locus of composites, LINERs, and Seyferts. 
This empirical insight, also supported by previous work (e.g., 
Strateva et al. 2001; Kormendy & Ho 2013), led these au-
torites to conjecture that the SMBH-to-bulge mass ratio M_⋆/M_0 
or, alternatively, the Eddington ratio or SMBH spin parameter) increase with M_⋆, with the transition between SF- and AGN-dominated 
gas excitation occurring at around 10^5 log(M_⋆/M⊙) < 10.5 and a 
Σ_* ≈ 10^7 M_0/kpc^2. Circumstantial support for this hypothesis 
comes from the observed inversion of radial stellar age gradients 
in bulges from zero and positive values in sub-L^* LTGs to pre-
dominantly negative values above log(M_⋆/M⊙) ~ 10.5 (B20a).
2.2. Observational estimate of $\delta_{io}$ from population spectral synthesis

A second estimate on $\delta_{io}$ can be obtained from a comparison of the luminosity contribution of young stellar populations inside and outside the bulge. This is possible through spectral modeling of spatially resolved IFS data in conjunction with postprocessing of population vectors (the best-fitting combination of spectral templates obtained from PSS) with the tool REMOVEYOUNG ($R_Y$; Gomes & Papaderos 2016). This stratigraphy (age slicing) technique consists of removing simple stellar population (SSP) spectra younger than an adjustable age cutoff $t_{cut}$ from the population vector, of the subsequent reconstruction of theSED of the residual older stellar component, and, finally, of the computation of its apparent magnitude in various bands through convolution of its SED with (currently 25) filter transmission curves. An additional output from $R_Y$ is the stellar mass $M_\star$ as a function of $t_{cut}$. The determination of the magnitude and mass of stellar populations within different age intervals is implicit to the concept of this tool and has been implemented in the IFS processing pipeline PORTO3D (Papaderos et al. 2013; Gomes et al. 2016a).

REMOVEYOUNG was for the first time systematically applied to IFS data for a representative sample of 135 LTGs spanning a stellar mass $8.9\leq\log(M_\star/M_\odot)\leq11.5$ by BP18. Their sample was compiled from the CALIFA survey (Sánchez et al. 2012; Sánchez et al. 2016), which was conducted with the Potsdam Multi-Aperture Spectrometer (PMAS; Roth et al. 2005) in its PPaK mode (Verheijen et al. 2004; Kelz et al. 2006) and the V300 grating, and reduced as described in García-Benito et al. (2015; and references therein). Spectral modeling was carried out in a spaxel-by-spaxel mode with STARLIGHT (Cid Fernandes et al. 2005) using SSPs from Bruzual & Charlot (2003) for 38 ages between 1 Myr and 13 Gyr (see BP18 for details). $R_Y$ was applied for nine $t_{cut}$ back to an age of 9 Gyr (an empirical estimate of the average lookback time where the age resolution of the centers of massive LTGs assemble and quench first (e.g., Pérez et al. 2013; Tacchella et al. 2015), it offers a novel route to the study of inside-out SFQ through the determination of the luminosity fraction of stellar populations of different age to the SBPs of galaxies. In particular, BP18 proposed to use the difference $\langle \delta_{io} \rangle$ (mag) between the mean $r$-band surface brightness within the bulge radius $r_{cut} = 0$ Gyr and 9 Gyr as a diagnostic for the evolutionary and physical properties of LTG bulges.

The rationale for this was that $\langle \delta_{io} \rangle$ tightly correlates with $M_\star$, $\Sigma_\star$, and the mass-weighted stellar age and metallicity of LTG bulges. Based on $\langle \delta_{io} \rangle$, BP18 tentatively subdivided their sample into three classes: galaxies hosting an bulges ($\langle \delta_{io} \rangle < -1.5$ mag) populate the low-mass end ($\log(M_\star/M_\odot) < 10.3$) of the LTG sequence and typically show the lowest bulge-to-total (B/T) mass ratio and bulge-to-disk age contrast. $\mathcal{B}$ bulges reside in LTGs with $\log(M_\star/M_\odot) > 10.5$ and show an intermediate $\langle \delta_{io} \rangle$, while high-mass $\mathcal{C}$ bulges in LTGs with $\log(M_\star/M_\odot) > 10.7$ are characterized by a $\langle \delta_{io} \rangle$ greater than $-0.5$ mag. This last class is characterized by the highest B/T ratio and bulge-to-disk age contrast (up to $\sim 3$ Gyr). The morphological and structural properties of these three LTG classes point to a loose association with bulgeless, pseudo-bulge, and classical-bulge LTGs, respectively. Consistently with the observed increase in the bulge-to-disk age contrast with increasing galaxy mass (BP18; their Fig. 7c), SBPs computed with $R_Y$ for different $t_{cut}$ witness a continued growth of the disk in all LTGs, regardless of their $M_\star$, in contrast to the evidence for a dependence of $SFQ$ on $M_\star$ within the bulge (cf. Fig. 5 in BP18): Suppression of stellar populations younger than 9 Gyr has virtually no effect on SBPs of high-mass $\mathcal{C}$ bulges, which implies that they have completed their assembly and entered the ensuing $SFQ$ early on.

In contrast, removal of young stellar populations leads to a strong dimming by 1–2 r mag in the disk of these most massive LTGs. At the antipodal end of low-mass LTGs (\textit{IA} class), suppression of stellar populations of increasingly high age leads to a roughly

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1 These authors defined $R_0$ as the extinction-corrected isophotal radius at which a Sérsic model to the central luminosity excess of the bulge has a surface brightness of $24 + r$ mag/arcsec$^2$. An alternative definition of $R_0$ involves the radius at which the surface brightness of the disk becomes equal to that of the bulge (e.g., Sánchez-Blázquez et al. 2014).

2 Determinations for a $t_{cut} = 0$ refer to the best-fitting stellar SED to an observed spectrum, that is, they only involve a correction for nebular line emission.
symmetric dimming both in the bulge and in the disk, which points to a nearly homologous radial growth of $\Sigma_*$. Therefore, the combined evidence from the analysis of LTGs with $R\mathcal{Y}$ suggests a trend for an increasing $\delta_\mu$ with increasing $M_*$. This is consistent with the fact that the mass fraction of stars older than 9 Gyr increases by a factor $\sim 4$ within the bulge (from 21% in iA bulges to 85% in iC bulges), but only by a factor $\sim 2$ in the outer disk (BP18).

Figure 3 shows an example of the application of $R\mathcal{Y}$ to the iC LTG NGC 0776. From panel b, it is apparent that the Hα equivalent width $EW(\text{H}\alpha)$ increases from the center to the periphery, as also reflected in the radial distribution of values for individual spaxels (dots in panel c) for which the spectrum was fit with a mean percentual deviation $ADEV<6$ (cf. Cid Fernandes et al. 2005; Gomes et al. 2016a). Squares correspond to mean values within irregular isophotal annuli ($isan$) adapted to the morphology of the emission-line free continuum (cf. BP18 for details). Panel d shows the $g$-band SBPs after suppression of stellar populations younger than 0.03, 0.1, 0.3, 1, 3, 5, 7, and 9 Gyr, as obtained by postprocessing the spectral modeling output from STARLIGHT with $R\mathcal{Y}$. Removal of stellar populations for all $t_{\text{cut}}$ has little effect on the bulge, which, taken at face value, implies that SF at the center of the galaxy has been practically extinguished since at least 9 Gyr ago. In contrast, the outer disk ($R/\beta R>1$) has continued building up, as is apparent from the fact that suppression of stellar populations younger than 9 Gyr affects a dimming by $\sim 1.5 \text{ g mag}$.

The differential growth of bulge and disk is better illustrated in Fig. 4, where we show the difference between the observed $g$-band SBP relative to the SBPs computed for the eight aforementioned $t_{\text{cut}}$ values. The horizontal line corresponds to a dimming of 0 mag, and black symbols show $isan$ determinations for

Fig. 4. Dimming $\delta_\mu$ (mag) relative to the observed, emission-line corrected $g$-band SBP of NGC 0776 (black) as a function of normalized radius $R/\beta R$ after suppression of stellar populations younger than a $t_{\text{cut}}$ between 0.03 and 9 Gyr. The color-coding is same as in Fig. 3d. Gray lines show linear fits for $R/\beta R \geq 2$, i.e., in a radius interval in which the contribution of the bulge and bar become negligible.

Fig. 5. Visualization of the connection between present-day stellar surface density and stellar mass growth for the LTG NGC 0776. a) Difference $\delta M_*/\delta g$ between the $r$-band magnitude of the old ($\geq 9$ Gyr) and existing stellar component vs. logarithm of the stellar surface density $\Sigma_*$ ($M_*/\text{kpc}^{-2}$). Symbols have the same meaning as in Fig. 3c. b) Percentual mass fraction of stars formed over the past one Gyr (lower panel) vs. $\Sigma_*$.

Fig. 6. Comparison of the SFH inferred with STARLIGHT for the bulge and disk of NGC 0776 (lower and upper panel, respectively). Vertical bars, color-coded according to stellar metallicity (open blue, dark blue, green, and orange for $Z_\odot/10$, $Z_\odot/5$, $Z_\odot/2.5$, and $Z_\odot$, respectively) show the contribution (in percent) of individual SSPs to the monochromatic luminosity of the continuum at 5150 Å. The sum of all SSP contributions at a given age is shown in gray. Thin vertical lines mark the (38) ages available in the SSP library we used (see BP18 for details). The labels in the upper left part of each diagram inform about the luminosity fraction of stellar populations younger than 0.1, 1, and 5 Gyr, as well as the light-weighted stellar age and metallicity. The bulge luminosity is dominated by old stellar populations, whereas more than 50% of the emission in the disk comes from stars younger than 5 Gyr.
$t_{\text{cut}}=0$ Gyr, that is, after subtraction of nebular line emission only. Suppression of young (<30 Myr) ionizing stellar populations effects a dimming of $\delta \mu \sim 0.2$ mag, with the outer disk becoming fainter by ~1.5 mag at a $t_{\text{cut}}=9$ Gyr, that is, within the range of values previously estimated from evolutionary synthesis models in Sect. 2.1. The bar, originally traceable as a weak surface brightness enhancement at $R^*/R_B > 1$ in the observed SBP (see also Fig. 2 of BP18 for SBPs from higher-resolution SDSS data with an FWHM=1.3 as compared to an FWHM~2:7 for CALIFA) is now better visible out to $R^*/R_B \sim 2$. Moreover, a noticeable feature from Fig. 4 is a slight steepening of the disk $\delta \mu$ (cf. linear fits for $R^*/R_B > 2$) for an age >1 Gyr, which might indicate inside-out galaxy growth.

That the build-up timescale of stellar populations is inversely related to their present-day $\Sigma_*$(a trend referred to as local or subgalactic downsizing by Rosales-Ortega et al. 2012 and BP18 on the basis of spatially resolved IFS studies of stellar metallicity and age, respectively; see also Ganda et al. 2007) is illustrated in Fig. 5. Consistently with the evidence from Fig. 4, the upper panel shows that $<\delta \mu_{\text{io}}>$ increases by >1 mag from the low-$\Sigma_*$ disk periphery to the dense galactic center. As opposed to the stagnation of SF activity within $R_B$, the continued growth of the outer disk is also reflected in its mass fraction of stars younger than 1 Gyr (lower panel), which is higher by a factor $\gtrsim 4$.

From these considerations, it follows that only after removal of stellar populations of age $\lesssim 9$ Gyr does the excess emission of the outer disk (aka the $\delta \mu_i$ effect) in NGC 0776 become sufficiently suppressed and its residual older stellar component become roughly compatible with the bulge from the evolutionary point of view (i.e., with regard to its SED and light-weighted age). Only then would the total bulge-disk decomposition concept be logically sound and ensure an unbiased determination of the luminosity and color of the bulge (and bar). Clearly, the exploration of an optimal age threshold $t_{\text{cut}}$ that could permit such an age homogenization of stellar populations in the bulge and disk is a nontrivial task that requires further investigation.

Nevertheless, already the cursory discussion here shows that subtraction of an exponential model to the $\delta \mu_{\text{io}}$-corrected outer disk of an LTG can lead to a significant increase in the luminosity of the bulge, in addition to allowing a better recovery of a possibly present bar (Breda et al., in prep.). For instance, fitting an exponential to the SBP of NGC 0776 for $t_{\text{cut}}=9$ Gyr (dark red curve in Fig. 3d), that is, the $t_{\text{cut}}$ coming closest to the light-weighted age of the bulge (<9.7 Gyr; cf. lower panel of Fig. 6), yields a total magnitude for the disk that is fainter by 1.5 mag than that determined from the observed ($t_{\text{cut}}=0$ Gyr) SBP (14.47 mag and 12.95 mag, respectively). This in turn implies a luminosity for the excess emission above the disk (i.e., bulge + bar) that is higher by a factor ~3 and a rise in the B/T ratio of the galaxy from 0.15 to 0.43. Clearly, a positive trend between the B/T ratio and the minimum present-day age ($t_{\text{cut}}$) of stellar populations extracted from an IFS cube is to be expected in an inside-out galaxy growth (and SFq) scenario.

2.3. Effect of nebular emission on $\delta \mu_i$ in local LTGs

The effect of nebular emission on broadband magnitudes and colors has been examined in several previous studies (Huchra 1977; Krüger et al. 1995; Izotov et al. 1997; Papaderos et al. 1998; Zackrisson et al. 2001; Anders & Fritze-v. Alvensleben 2003; Schaerer & de Barros 2009; Atek et al. 2011; Papaderos & Östlin 2012, among others) showing that it becomes important when the emission-line EWs rise above a few percent of the effective width of broadband filters. Because the EW(Hα) in the disks of centrally SF-quenched local LTGs (typically, iB and iC systems with log$(M_*/M_\odot)>10.3$) is generally low (<40 Å; e.g., Kennicutt 1989; Catalán-Torrecilla et al. 2017; Belfiore et al. 2018; Breda & Papaderos 2018; Kalinova et al. 2021), nebular emission is practically negligible in the context of $\delta \mu_i$.

This can be illustrated again using NGC 0776 as an example. BPT line ratios place both the bulge and the disk of this galaxy in the locus of SF or composite sources (Fig. 7a). It may therefore be regarded as an example for a higher-mass LTG with an intermediate level of contamination (~40%) of the Hα+[NⅡ]6583,6548 blend by nitrogen lines. Panel b shows the observed EW(Hα) versus the enhancement $\delta \mu_i$(obs-fit) of the $r$ magnitude due to nebular line emission (i.e., the difference between the magnitude of the observed spectrum and the stellar fit to it, as obtained by applying $\text{Ry}$ for a $t_{\text{cut}}=0$). The low average EW(Hα) within individual $\text{is}$ ($<20$ Å) translates into a negligible $\delta \mu_i$ with the effect of nebular line contamination becoming appreciable (>0.1 mag) only locally for a few individual spaxels in HII regions in which the EW(Hα) rises above 10$^2$ Å. A linear fit yields the relation $\delta \mu_i = 0.74 r$ mag/kÅ, implying that nebular line emission does not notably enhance the surface brightness enhancement of the disk.

The situation is most certainly different in high-$z$ proto-LTGs, where rest-frame EWs in the disk likely exceed several hundred Ångström and nebular emission may even dominate optical broadband magnitudes. If the early phase of bulge growth is driven by inward migration and coalescence of massive (10$^8$–9 $M_\odot$) star-forming clumps emerging out of violent disk instabilities (Noguchi 1999; Bournaud et al. 2007; Elmegreen et al. 2008; Mandelker et al. 2014, 2017), then a strong $\delta \mu_i$ in these systems could develop early on, within the theoretically estimated clump migration timescale $t_m$ of a few 10$^7$ yr. In addition to negative age and M/L gradients (BP18), a plausible expectation from clump migration is a bulge-to-disk EW contrast that develops as early as within the first 1 Gyr of galaxy evolution.

For example, an SF clump with $Z_\star/20$ forming at a radius ~7 kpc and reaching the center of the galaxy after a $t_m$~700 Myr with a mean radial velocity of ~9.8 km/s will experience a decrease in its EW(Hα) from initially ~3 x 10$^2$ Å to 60 (2) Å for an exponentially declining SFR with an e-folding time of 300 (100) Myr. The EW(Hα) excess of the disk relative to the bulge translates by the empirical relation from Fig. 7b into a surface brightness enhancement $\delta \mu_{\text{io}}$ (~$\delta \mu_i$) of >2 r mag. The temporal evolution of the EW(Hα) profile will of course depend on various factors, such as the fraction of clumps being massive enough to survive SF-driven feedback and reach $R_B$ as dynamically bound entities (Tamburello et al. 2017), the level of in situ SF in the bulge that is fed by inflowing intraclump gas (Hopkins et al. 2012), the dilution of emission-line EWs by the inwardly increasing stellar continuum, and the stellar metallicity.

If, on the other hand, bulge formation starts with a phase of dissipative gas collapse (well compartment in the notation by Dekel & Burkert 2014), then a strong $\delta \mu_i$ should first emerge once a high-$\Sigma_*$ core surrounded by an outwardly propagating quenching wave (Tacchella et al. 2015) has developed. The radially
evolving high-sSFR ring around the SF-quenched proto-bulge that is inferred from postprocessing of cosmological simulations in Tacchella et al. (2016) probably implies an inversion of negative to positive EW(Hα) gradients as the galaxy completes its wet compaction phase and enters the inside-out SFQ.

Studies with the JWST, ELT, and Euclid could address whether positive EW gradients in high-\(z\) galaxies emerge prior to or after the appearance of a high-\(\delta_{\rm io}\) proto-bulge, thereby offering observational constraints on the relative role of clump migration and wet compaction during the early stage of bulge formation. Further valuable insights might be gained from high-resolution cosmological simulations incorporating a detailed treatment of nebular emission and its expected effect on radial color and EW profiles (e.g., Hirschmann et al. 2017).

A fact deserving special attention in bulge-disk decomposition studies of higher-\(z\) LTGs is that strong emission lines lead to a selective surface brightness enhancement of the disk (consequently, an increased \(\delta_{\rm io}\)) in various redshift intervals, depending on photometric filter. For example, the \(r\)-band surface brightness of the disk is elevated at \(z \approx 0.17\) because of contamination by the H\(_{\alpha}\) line, with a second and third peak occurring at \(z \approx 0.54\) and \(\sim 1.18\) due to the [O\text{III}]\(\lambda 5007,\lambda 4959\) and [O\text{II}]\(\lambda 3727,\lambda 3729\) lines. Similarly, the \(H\)-band surface brightness of the disk is enhanced by hydrogen Paschen lines at \(z \approx 0.7\) and by the H\(_{\alpha}\) line at \(z \sim 1.4\). The essential aspects of this issue were discussed in Papaderos & Östlin (2012). These authors have examined the influence of nebular emission on photometric studies of high-sSFR galaxies that consist of a high-surface-brightness stellar core and a more extended nebular envelope. Whereas these authors used the local blue compact dwarf (BCD) galaxy I Zw 18 to exemplify a high-\(z\) protogalaxy, where SF-feedback leads to extended nebular emission and the spatial decoupling of ionized gas from ionizing stellar clusters, their considerations also apply to any other source of energy and momentum surrounded by a nebular envelope, for example, quasars with a Ly\(_{\alpha}\) halo (e.g., Villar-Martín et al. 2007; Humphrey et al. 2013; Borissova et al. 2016; Wisotzki et al. 2016; Leclercq et al. 2020). They showed that the shift of various strong emission lines to filter transmission curves across \(z\) leads to a wide range of combinations of colors and a core-to-envelope color contrast \(\delta_{\text{ce}}\). These, when interpreted in terms of purely stellar SEDs, or when ignoring the fact that the (stellar+nebular) SED of a galaxy varies with galactocentric radius, can prompt a variety of erroneous conclusions about the nature and evolutionary status of a higher-\(z\) galaxies. For example, as these authors remark, at 0.15 \(\leq z \leq 0.3\), the large \(\delta_{\text{ce}}\) (\(-0.8\) mag) in \(V-I\) and moderately blue colors of the core (0.5 mag) superficially suggest an old disk hosting nuclear SF activity. The opposite conclusion would be drawn from the \(\delta_{\text{ce}}\) in \(B-V\) (\(-0.5\) mag), which could be taken as evidence for a young stellar disk encompassing a slightly older core. Likewise, in other \(z\)-bands, observed-frame colors together with an overall low \(\delta_{\text{ce}}\) could lead to the classification of a starburst galaxy as quiescent.

From Fig. 15 by Papaderos & Östlin (2012) is also apparent that nebular line contamination does not boost \(\delta_{\text{io}}\) solely in narrow \(z\)-intervals that could easily be excluded from automated bulge-disk decomposition studies of large galaxy samples, but affects several broad windows in \(z\), a fact requiring a careful treatment of this issue. It is also important that the selective enhancement of \(\delta_{\text{io}}\) in these \(z\)-intervals, correspondingly an artificial dimming and reddening of the bulge relative to the disk (cf. Sect. 3), could affect the net (disk-subtracted) SED of the bulge such as to potentially mimic bulge growth in discrete major episodes at vari-

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**Fig. 7.** Diagnostic emission-line ratios and photometric impact of nebular emission in NGC 0776. **upper panel:** \(\log([\text{N\text{II}}]\lambda 6584/\text{H\alpha})\) vs. \(\log([\text{O\text{III}}]\lambda 5007/\text{H\beta})\) diagram. Single-spaxel determinations within \(R_d\) and in the outer disk (red and blue, respectively) are shown with dots, whereas squares depict average values within \(\delta_{\text{ce}}\). The grid of thin gray lines depicts the parameter space that can be accounted for by pure shock excitation, as predicted by Allen et al. (2008) for a magnetic field of 1 \(\mu G\), and a range of shock velocities between 100 and 1000 km/s for gas densities between 0.1 and 100 cm\(^{-3}\). It is apparent that the diagnostic line ratios both for the bulge and outer disk fall close to the empirical envelope characterizing SF regions and in the locus of composite sources (i.e., between the curves by Kauffmann et al. and Kewley et al.).

**lower panel:** Difference between the \(r\)-band magnitude of the observed spectrum and that of the stellar fit to it vs. EW(\text{H}\(_\alpha\)) (\(\AA\)) for individual spaxels (dots) and \(\delta_{\text{ce}}\) (squares). A linear fit (line) indicates that nebular line emission in this composite system enhances the \(r\)-band emission by \(-0.7\) mag per 10\(^3\) \(\AA\) in EW(\text{H}\(_\alpha\)).
ous $z$ or might lend superficial support to a duality in the origin of bulges.

3. Effect of $\delta_{\text{io}}$ on photometric properties of bulges

Before turning to potential implications of $\delta_{\text{io}}$ on galaxy scaling relations (Sect. 4), we briefly discuss here its expected principal effect on determinations of photometric properties of bulges. Additionally, we show that it is in practice impossible to photometrically rule out the presence of a centrally SF-quenched disk beneath the bulge, and that numerous combinations of a centrally downbending disk (cf. Fig. 1, right) with a Sérsic profile for the bulge can adequately reproduce typical SBPs of local LTGs over their entire morphological spectrum, from bulge-dominated all the way to prima facie bulgeless systems.

3.1. Effect of a centrally quenched disk on luminosity determinations for the bulge

The range of $\delta_{\text{io}}$ estimated in Sect. 2 (up to $\sim2.5$ mag in $B$ and $\sim0.7$ mag in $R$) translates into an overestimation of the disk below the bulge by a factor between $\sim1.7$ and $\sim9$. The underestimated bulge luminosity as result of the oversubtraction of the underlying disk depends on two competing factors, namely the $\delta_{\text{io}}$ itself and the luminosity ratio $B/ID$ of the bulge to the inner disk. For this reason, a faint bulge residing in a disk with a low $\delta_{\text{io}}$ might be more affected than a prominent bulge in a fully SF-quenched disk with a high $\delta_{\text{io}}$.

This can be illustrated by the example of a synthetic LTG (Fig. 8) that consists of an old bulge and a centrally SF-quenched disk. The $B$-band SBP of the bulge is approximated by a Sérsic profile,

$$\mu(R^*)_B = \mu_{0,B} + 1.086 \cdot (R^*/\beta)^{1/\nu},$$  \hspace{1cm} (3)

with $\mu_{0,B}$=18 mag/$\prime\prime$, $\beta$=0.4 and $\nu$=2.3, and the disk by the modified exponential (modexp) function proposed in Papaderos et al. (1996), which allows a depression of an exponential intensity profile inside a cutoff radius,

$$I(R^*) = I_{\exp} \cdot \left[1 - \epsilon_1 \exp(-P_3(R^*))\right],$$  \hspace{1cm} (4)

where $P_3(R^*)$ is defined as

$$P_3(R^*) = \left(\frac{R^*}{\epsilon_2 \alpha}\right)^3 + \left(\frac{R^*}{\alpha} - \frac{1}{\epsilon_1}\right).$$  \hspace{1cm} (5)

The first term in Eq. 4 stands for the standard exponential law $I_{\exp} = I_0 \cdot \exp(-R^*/\alpha)$, where $I_0$ and $\alpha$ are the central intensity and angular scale length of an exponential profile, respectively. The modexp function involves two further parameters, namely the central intensity depression $\epsilon_1 = \Delta I/I_0$ relative to the exponential model, and the core radius $R_c = \epsilon_2 \cdot \alpha$ within which the flattening occurs. The modexp was developed to approximate the underlying stellar host of local BCDs, for which indirect evidence for a central flattening exists (Papaderos et al. 1996; Noeske et al. 2003), similar to several normal and nucleated dwarf ellipticals (cf., e.g., Binggeli & Cameron 1993). This flexible functional form also permits simulating a centrally depleted disk profile (B20b).

The superposition of a Sérsic with a modexp profile yields a good match to the SBPs of typical LTGs (e.g., de Jong & van der Kruit 1994; de Jong 1996a; Andreddakis & Sanders 1994; Andreddakis et al. 1995; Gadotti & dos Anjos 2001; MacArthur et al. 2003; Domínguez-Palmero et al. 2008; Martinsson et al. 2013; Erwin 2015; Erwin et al. 2021; Barsanti et al. 2021; Costantin et al. 2021). Quite importantly, it is in practice impossible to visually infer the presence of even a strong ($\delta\mu_{0,D}$=2) central down-bending in the disk from such a composite profile because the bulge dominates the line-of-sight intensity out to $\sim R_{\text{h}}/2$ (see also B20b).

Figure 9 shows the net bulge emission that would be obtained by subtracting an exponential fit to the outer disk ($\gtrsim20''$) from the composite SBP in Fig. 8. This example, which essentially simulates the standard practice in 1D and 2D decomposition, illustrates that the deviation of the retrieved profile for the bulge...
from its true Sérsic form (red) increases with $\delta_{io}$, with a tendency for a steepening within the bulge effective radius ($R_{eff} \sim 11''$) and the appearance of a plateau at $R_{eff} \lesssim 2R_B$.

Depending on the specifics of the fitting procedure (in particular, the error assigned to the inner points and corrections for the point spread function, PSF; cf. Sect. 4.1), the Sérsic model parameters for the bulge can be biased in various ways (Breda et al. 2019, for further remarks). For instance, fitting only its inner part (1''$\lesssim R' \lesssim R_{eff}$) would in the case of $\delta_{io} = 2$ mag yield an $\eta \sim 1.75$, thus leading to the classification of the input classical bulge ($\eta=2.3$) as a pseudo-bulge. Evidently, the luminosity of the bulge is increasingly underestimated with increasing $\delta_{io}$ (aka $\delta_{io}$): the bulge apparent magnitude inside $R_B$ (14.64 mag) decreases to 14.75, 14.94, 15.09 and 15.21 for a $\delta_{io}$ of 0.5, 1.0, 1.5 and 2.0 mag, respectively, which for the synthetic model under study translates into an error of up to 40%. Additionally, the underestimation of the luminosity of the bulge scales inversely to the bulge luminosity itself, consequently, the described bias is aggravated for intrinsically faint bulges (i.e., a low B/I D ratio): if the simulated bulge in Fig. 8 were fainter by just one mag, then already a $\delta_{io}=0.5$ (1.0) mag would result in the underestimation of its luminosity by 47% (84%)$^4$.

Another insight from our foregoing remarks is that in the presence of SFQ, standard bulge-disk decomposition should lead to discrepant determinations of the bulge Sérsic $\eta$ in different filters, in the sense of a positive correlation between $\eta$ and central filter wavelength. This is because the effect of the oversubtraction of the star-forming outer disk on the net SBP of the bulge is stronger in the blue (because $\delta_{io} \propto \lambda^{-1}$; cf. Fig. 2). Together with the evidence from Fig. 9, this implies that the underestimation of Sérsic $\eta$ decreases with increasing $\lambda$ and becomes minimal in the NIR. Instead of attempting to eliminate this seeming discrepancy by forcing Sérsic model parameters to smoothly vary along $\lambda$ according to an empirical functional form (e.g., Haußer et al. 2013; Vika et al. 2014), we might take advantage of it in order to obtain indirect constraints on $\delta_{io}$ and the true radial intensity profile of the disk below the bulge.

$^4$ See Sect. A.2 for further quantitative estimates as a function of the true magnitude of the bulge and the intensity profile of the disk.

3.2. Bulgeless galaxies from the perspective of $\delta_{io}$

The existence of bulgeless galaxies, that is, LTGs with a minor, if any at all, central luminosity excess above the disk (e.g., Kormendy et al. 2010; Coelho et al. 2013; Bizzocchi et al. 2014; Grossi et al. 2018) has triggered an intense controversy (e.g., Governato et al. 2010; Pilkington et al. 2011), given that these galaxies are not expected in the $\Lambda$CDM cosmology. In the light of our previous considerations, it is worth contemplating whether the conclusion that these systems lack a bulge of appreciable mass remains compelling. Figure 10 simulates an LTG with a B/T ratio of $\sim 0.1$ through the superposition of the same $modexp$ model for the disk as in Fig. 8 (blue) with a shallower Sérsic profile for the bulge (red) that is given by $\mu_B(\delta_{io}=1-22$ mag/arcsec$^2$, $\beta=10''$ and $\eta=0.7$. The composite SBP (gray curve) is a nearly perfect exponential over 6 mag in surface brightness. A fit for the outer disk severely underestimates the luminosity of the bulge ($\sim 16.7$ mag, i.e., 1.2 mag fainter than its true apparent magnitude of 15.5 mag), which now appears as a low-surface brightness ($>24$ mag/arcsec$^2$) core enclosing just $<4\%$ of the total emission of the galaxy.

The main insight from Fig. 10 is that even a modest $\delta_{io}$ ($\delta_{io} \sim 1$ mag) would comfortably allow a low-luminosity bulge to coexist with a centrally SF-quieted disk in a nearly perfect exponential composite SBP and evade detection with the standard bulge-disk decomposition technique. Although $\delta_{io}$ is mitigated in the I band and in the NIR, this alone does not warrant that a putative bulge can invariably be recovered in a bulgeless LTG because its detectability also depends on its luminosity (or the B/I D) and the core radius $R_c = e_\alpha \cdot \alpha$ of the SF-quieted central zone of the disk (Sect. 3.1 and A.2).

3.3. Effect of a centrally quenched disk on color profiles within the bulge

Color gradients within $R_B$ may hold valuable clues to bulge formation and evolution. It is thus worthwhile to examine how they are influenced by the line-of-sight intensity contribution of the underlying disk, both in the presence and absence of SFQ.
Regardless of whether the disk is centrally SF-quenched or not, the observed negative color gradient within $R_B$ (~ -0.5 mag/$R_B$) is purely a projection effect that is driven by the outwardly increasing line-of-sight contribution of the disk. Using this color gradient to place constraints on bulge evolution and demographics is obviously pointless. Without prior correction for the underlying disk, negative color gradients within $R_B$ cannot be interpreted in terms of a possible radial decrease in stellar age or metallicity within the bulge, and offer useful constraints on bulge formation and evolution. Unfortunately, a review of the rich literature about LTGs reveals only very few photometric studies acknowledging the influence of the disk on color gradients within the bulge (e.g., Head et al. 2014).

More generally, any superposition of evolutionary and spatially distinct stellar components results in (or alters existing) radial color gradients. One similar but reverse example is offered by blue compact dwarf galaxies: In their majority, these systems exhibit intense SF in the central part of an evolved (B-R ~ 1 mag) stellar host. The interplay between the radially decreasing SFR surface density and the increasing line-of-sight contribution of the old underlying stellar component manifests itself in positive color gradients out to approximately two exponential scale lengths of the host. Papaderos et al. (2002) have shown that correction for the host can reduce the integral color of the starburst component by ~0.5 B-R mag and change its radial color gradient by up to 1 mag/kpc. Thus, modeling and subtracting the underlying host is crucial for a proper age dating of the young stellar component in these systems. Evidently, similar considerations apply to any other radially resolvable luminosity-weighted observable in a 3D geometry (EW, light-weighted age and metallicity, stellar velocity dispersion, Lick indices, and diagnostic emission-line ratios).

Next, we comment on the color profile that would be obtained for the bulge if a correction for the underlying disk were attempted on the standard assumption that the intensity of the latter follows a purely exponential instead of a modexp profile. The color profile obtained in this way is overestimated throughout within the bulge, reaching at $R_{eff}$=0.74 mag and a maximum deviation of +0.5 mag from the true color. A correction of bulge color profile...
files (or color maps) for the underlying disk without taking SFQ into account (i.e., assuming a purely exponential model for the disk) leads to an increasingly overestimated color with increasing $\delta_0$. This is better illustrated in Fig. 12, where we plot the observed and disk-corrected color profile (solid and dashed curves, respectively) for a $\delta_0$ between 0 mag (no SFQ) and 2 mag. As $\delta_0$ increases, the color gradient at $R/B=2$ steepens from $-0.35$ mag$/R_B$ for $\delta_0=0$ mag (black) to $-0.76$ mag$/R_B$ for $\delta_0=2$ mag (red). The disk-corrected color of the bulge is overestimated for any $\delta_0>0$ mag, exhibiting around $R_{\text{eff}}$ a broad excess by 0.2 mag ($\delta_0=0.5$) and up to 0.9 mag ($\delta_0=2$).

Summarizing, negative color gradients within galaxy bulges can be entirely due to the outwardly increasing line-of-sight contribution of the underlying disk, both in the absence and presence of SFQ. In the case of a centrally SF-quenched LTG, correction for the disk using an inadequate model for its radial intensity (i.e., a purely exponential profile with constant color) leads to a local and global overestimation of the color of the bulge, in particular, to an artificially red circumnuclear zone that could be taken as evidence for enhanced dust obscuration. Additionally, the oversubtraction of the blue SED of the disk can lead to an abnormally red SED for the bulge and drive spectrophotometric bulge-disk decomposition studies toward an exceedingly high age and metallicity (B20b). This is particularly true for LTGs at redshift intervals where $\delta_0$ is enhanced by nebular line emission in the disk (cf. Sect. 2.3).

4. Discussion

The goal of this study is to draw attention to the implications of SFQ for structural studies of galaxies and motivate an exploration of this hitherto uncharted territory. Extending our discussion in Sect. 3, in the following we point out that the neglect of $\delta_0$ can introduce complex biases with considerable relevance to our understanding of the formation history and demographics of galaxy bulges (Sect. 4.1) and that these biases are not simple to overcome because they impact structural studies differentially, depending on galaxy mass and redshift (Sect. 4.2). In particular, the neglect of $\delta_0$ might appreciably affect the scatter and slope of galaxy scaling relations (Sect. 4.3), which calls for a closer observational and theoretical examination of this effect and the development of empirical approaches for its correction in a statistical sense. Possible routes toward this goal are discussed in Sect. 4.5.

4.1. Principle photometric biases due to the neglect of $\delta_0$

As discussed in Sect. 3, the neglect of $\delta_0$ in dedicated bulge-disk decomposition studies can result in biased determinations of the photometric properties of galaxy bulges. Expected effects include a) a systematic underestimated of the luminosity of the bulge in a manner that is inversely related to its prominence (specifically, the B/I0D ratio) and proportional to the degree ($\delta_0$), and radial extent ($R_e$) of SFQ, and b) a potential underestimate of the Sérsic exponent $\eta$. The latter entails an overestimation of the model-dependent effective radius of the bulge (because $R_{\text{eff}}$ is by definition inversely related to $\eta$; e.g., Trujillo et al. 2001), consequently also an underestimation of the mean stellar velocity dispersion $\sigma_\star$ therein. In practice, the adopted value for $\sigma_\star$ will depend on aperture corrections (e.g., Davies et al. 1987; Jørgensen et al. 1995; Ziegler & Bender 1997; Jablonska et al. 2007) which in turn require assumptions on the SBP of the bulge. The underestimation of $\eta$ likely results in a tendency for the classification of moderately luminous classical bulges as pseudo-bulges on the basis of the commonly adopted empirical cutoff at $\eta \sim 2$. Finally, as previously shown, the neglect of $\delta_0$ can result in the erroneous classification of low-B/D LTGs as bulgeless disks.

Whereas it is plausible that any galaxy decomposition scheme that lacks a relevant structural component or adopts an inadequate functional form for it can entail a bias, quantitative predictions on the concrete implications of this for a multicomponent bulge-(bar)-disk fit that involves multiple nonlinearly coupled free parameters is not straightforward. This is particularly true when photometric uncertainties, hence the statistical weights that largely guide the best-fitting solution, happen to correlate with evolutionary patterns in galaxies (colors, sSFR, and emission-line EWs). This is precisely the case in LTGs: because the S/N scales with the surface brightness (or $\Sigma_\star$), the highest weight is given to the SF-quenched core ($R^0<R_B$), that is, the radial zone showing the strongest negative color and age gradients simultaneously with the steepest positive sSFR and EW(H$\alpha$) gradients (cf., e.g., Balcells & Peletier 1994; Gadotti & dos Anjos 2001; Catalán-Torrecilla et al. 2017; Belfiore et al. 2018; Breda et al. 2020a). A further complication stems from the fact that this core is most affected by PSF smearing, which can propagate into further uncertainties in the Sérsic fit for the bulge (see the discussion in Breda et al. 2019).

How the interplay between these factors is imprinted on the best-fitting solution is not immediately clear. For instance, the scaling of photometric errors inversely to surface brightness can result in an artificial compaction of the disk (underestimation of its $\alpha$ and overestimation of its $\mu_0$), which in turn translates into an underestimation of B/I0D (the disk luminosity fraction inside $R_B$) and could bias the Sérsic model parameters for the bulge (Papaderos et al. 1996). The neglect of a possibly present bar poses a further complication (cf., e.g., Méndez-Abreu et al. 2008, BP18); this is especially true for higher-mass LTGs because the bar prominence increases with LTG luminosity (Bittner et al. 2017; Gadotti et al. 2020). Moreover, small-scale features such as nuclear rings (Buta & Combes 1996; Comerón et al. 2010) or nuclear disks (Bittner et al. 2020) or double bars (de Lorenzo-Cáceres et al. 2019) may have an influence given their high surface brightness (and weight in the fit).

It is thus conceivable that the neglect of $\delta_0$ is largely responsible for the fact that bulge-disk decomposition solutions in different bands are frequently mutually inconsistent or untenable from the evolutionary point of view. For instance, dos Reis et al. (2020) report several cases of formally (in terms of $\chi^2$) irreproducible fits with GALFIT (Peng et al. 2010) that translate into unphysical colors for the bulge and disk when the best-fitting solution is extrapolated beyond $R^0=R_{\text{eff}}$, however. As remarked in Sect. 3.1, forcing the Sérsic exponent $\eta$ to vary smoothly across $\lambda$ according to an ad hoc functional form might offer a remedy, but no solution to the problem.

An important step forward would be to empirically examine in synthetic multiband galaxy images how the neglect of SFQ is imprinted on determinations of structural properties of bulges ($\eta$, $R_{\text{eff}}$, $\mu_{\text{eff}}$, total magnitude $M_\epsilon$). To this end, standard bulge-disk decomposition into a Sérsic and a purely exponential model would need to be applied to synthetic galaxies whose disk is given by a modexp or other functional forms (e.g., a variant of the five-parameter core-Sérsic profile; Graham et al. 2003; Trujillo...
et al. 2004; Bonfini 2014), possibly with the addition of a Ferrers (1877) component approximating a bar. In addition to a grid of models that densely cover possible configurations of a central flattening or downbending of the disk inside the bulge (i.e., the $\epsilon_{1,2}$ parameters of Eq. 5), this task should ideally include the radially varying contribution of nebular emission to broadband SBPs in different evolutionary stages of an LTG (Sect. 2.3).

4.2. Galaxy downsizing and the differential nature of $\delta_{io}$

The factor $\delta_{LB}$ by which the luminosity of the bulge is underestimated due to the neglect of $\delta_{io}$ depends on two competing but interrelated factors, namely the $\delta_{io}$ itself and the luminosity fraction $B/ID$ of the bulge inside $R_{B}$. For the sake of simplicity, we assume that $B/ID$ roughly scales with $B/T$ (however, see Sect. 4.3).

Some qualitative predictions can be made from empirical insights and plausibility arguments. A reasonable expectation is that $\delta_{io}$ affects determinations of photometric and structural properties of bulges differentially, that is, in a manner that at a given age scales inversely with galaxy mass $M_{\star}$.

From the perspective of galaxy downsizing (Cowie et al. 1996), the formation timescale of galaxies is anticorrelated with $M_{\star}$, with the dominant phase of their build-up being delayed to a lower $z$ as $M_{\star}$ decreases (staged galaxy formation; Noeske et al. 2007). Manifestations of this phenomenon include the inverse relation between $M_{\star}$ and burst parameter or EW(\text{H}$\alpha$) in BCDs (Salzer et al. 1989; Krüger et al. 1995), and the anticorrelation between $M_{\star}$ (or $\Sigma_{\star}$) and various proxies to the ongoing and average past SF activity, such as EW(\text{H}$\alpha$) or SFR/$\langle$SFR$\rangle$ (Brinchmann et al. 2004), mass doubling time (Noeske et al. 2007) and luminosity-weighted age (Gallazzi et al. 2005). The advent of wide-field IFS helped to better recognize that an analogy to downsizing on galactic scales exists on subgalactic scales (subgalactic downsizing, in the notation by BP18 in the sense that the formation timescale of stellar populations is anticorrelated with their mean $\Sigma_{\star}$. As documented by several recent studies, the dense galaxy centers assemble first and is followed by a prolonged, still ongoing build up of their periphery (e.g., Pérez et al. 2013; Fang et al. 2013; González-Delgado et al. 2014; Gomes et al. 2016b), as also shown from UV observations in Muñoz-Mateos et al. (2007), Salim & Rich (2010), Salim et al. (2012), and Gil de Paz et al. (2007), for instance. In the specific context of LTGs, this trend is echoed by the increasing bulge-to-disk age contrast with increasing $M_{\star}$, for example, or by the radially increasing luminosity contribution of stellar populations of intermediate-to-young age (Sect. 2.2).

This (sub)galactic downsizing may be simulated with delayed SFH scenarios (Fig. 13) that yield a zero-order approximation to a $M_{\star}$-dependent formation history for bulges in the three LTG classes tentatively defined in BP18: Similar to Fig. 2, the disk outside the bulge is assumed to form continuously with a constant SFR, whereas the SFR of $iC$, $iB$, and $iA$ bulges is approximated by models involving an SFR that reaches its maximum at 0.47, 1.2, and 2.4 Gyr, respectively, and then exponentially declines with an e-folding time that scales inversely with $M_{\star}$. As in Fig. 2, nebular emission is taken into account and a constant metallicity of $Z_{0}$ ($Z_{0}$/5) is assumed for the bulge (disk).

Fig. 13. Mass-dependent SFH of bulges and resulting evolution of $\delta_{io}$ for a galaxy downsizing scenario. upper panel: SFR parameterizations simulating a downsizing trend for galaxy bulges by assuming that their age at the peak of their SFR and SF e-folding timescale inversely with $M_{\star}$. High-, intermediate-, and low-mass bulges ($iC$, $iB$, and $iA$, respectively, in the classification by BP18) reach their maximum SFR at 0.47, 1.2, and 2.4 Gyr. lower panel: Evolution of $\delta_{io}$ (approximately by 2.5 · log($\phi$), as in Fig. 2) in the $B$ and $R$ band (solid and dashed curves, respectively) for the three SFH scenarios for the bulge (upper panel) when a constant SFR for the outer disk is assumed. The shaded gray area depicts the redshift interval 0.76 ≤ $z$ ≤ 2.3 that will be covered both by the MOONS spectrograph at VLT (0.65-1.8 \text{\mu}m) and the $B$-band filter ($\sim$0.37-0.54 \text{\mu}m). The amplitude of $\delta_{io}$ at $z$ ~ 1 (vertical white line) depends on galaxy mass. It is moderate ($\sim$0.85 $B$ mag) for low-mass (<$L_{*}$) galaxies and reaches ~2 mag for massive LTGs in advanced stages of SF quenching.

The lower panel of Fig. 13 shows the evolution of $\delta_{io}$ in the $B$ and $R$ band for the three scenarios. $\delta_{io}$ becomes positive after 1-3 Gyr, as SF and nebular contamination gradually cease in the bulge and its $M_{\star}/L$ rises above that of the outer disk, and then smoothly increases to its present value of $\geq$2 $B$ mag and 1.5 $R$ mag.

An insight from this simplified parameterization of galaxy downsizing is that $\delta_{io}$ increases faster for massive LTGs and vice versa. Taking the evidence from Fig. 13 at face value, at $z$ ~ 1 (vertical line; roughly at the onset of the decline of the cosmic SFR density; cf. Madau & Dickinson 2014), massive galaxies in an advanced stage of SFQ have developed a $\delta_{io}$ that is higher by $\sim$1 mag than low-mass galaxies that still sustain a significant level of SF in their bulges. The gap between the two gradually shrinks to less than 0.7 $B$ mag since only ~4 Gyr ago ($z$ ~ 0.36).

The key implication is that (the neglect of) $\delta_{io}$ does not just add a scatter to any galaxy scaling relation that involves bulge luminosities, but affects its slope to a degree that depends both on galaxy mass and $z$. This calls for caution when (magnitude- or mass-selected) galaxy samples spanning a range in redshift are...
4.3. Effect of $\delta_{io}$ on the scatter and slope of fundamental scaling relations

A closer investigation of $\delta_{io}$ is of considerable interest also because correction for this effect in galaxy decomposition studies will likely influence the scatter and slope of some galaxy scaling relations, eventually opening new prospects for improving our understanding of bulge evolution and demographics. Because this correction will increase the bulge luminosity and probably also the Sérsic $n$, it will move some previously categorized pseudo-bulges into the locus of classical bulges, possibly leading to sharper observational photometric discriminators for bulge classification. For instance, it is imaginable that the failure to establish $n$ as a robust bulge classification means, despite several efforts over the past three decades (e.g., Neumann et al. 2017, for a recent review), partly stems from the neglect of $\delta_{io}$ (Sect. 3.1). Furthermore, the expected increase in the parametric (Sérsic model-dependent) effective surface brightness, effective radius, and $\sigma_e$ therein could be relevant to, for example, for the Kormendy (1977) relation (see also Kormendy & Djorgovski 1989 and Ziegler et al. 1999) and for the Faber & Jackson (1976) relation for bulges, as well as for any relation contrasting structural characteristics of bulges (e.g., $n$, $R_{e,eff}$, $\mu_{eff}$, $M_h$) with, for example, the $\alpha$ parameter (Fabricius et al. 2012), ellipticity, concentration index, velocity anisotropy, and nonthermal radio power. Furthermore, because disk-dominated LTGs are more affected by $\delta_{io}$ than bulge-dominated early-type galaxies, it might naturally be expected that bulges in these two galaxy classes describe slightly different slopes in various scaling relations.

The principle effect of accounting for $\delta_{io}$ might be illustrated with the example of the $M_{io}$-bulge relation (e.g., Magorrian et al. 1998; Ferrarese & Merritt 2000) in which the bulge mass is commonly approximated by its absolute $K$-band magnitude or mean $\sigma_e$ within $R_{e,eff}$. This relation is fundamental to our understanding of the regulatory role of AGN in galaxy evolution, as also reflected in the fact that deviations from the average ratio $M_{io}/M_{*}\approx0.002$ (Heckman & Best 2014) are regarded as signatures of distinct evolutionary routes in the coevolution of SMBHs and galaxy spheroids. For example, Kormendy et al. (2011) studied a sample of CBs (high-luminosity, pressure-supported bulges), PBs (intermediate-luminosity bulges with significant rotational support), and bulgeless disks, all having distinct kinematical determinations of their $M_{io}$. These authors find that similar to ellipticals, CBs obey the bulge versus $M_{io}$ relation, whereas PBs and bulgeless galaxies show a large scatter or systematic deviations from it (however, see Bennett et al. 2021). Five of the 11 PBs included in this study are indeed overluminous by 1-2 $K$ mag for their $M_{io}$.

A correction for $\delta_{io}$ could enhance the luminosity of bulges by a factor $\propto M_{io}^{-1}$ (cf. Sect. 4.2), which would have a greater effect for PBs than for CBs and would lead to a down-bending of the bulge-$M_{io}$ relation below log($M_{io}/M_\odot$)$\sim10.7$, as sketched in Fig. 14. From the perspective of the standard scenario (Kormendy & Kennicutt 2004, for a review), this down-bending could be taken as supportive evidence for the prevailing view that CBs and PBs are evolutionary distinct, with the former showing an affinity to massive stellar spheroids that form early on and experience a synchronous growth with their SMBH, whereas the latter primarily resulting from rearranged disk material that piles up into a central luminosity excess in the course of the secular evolution of disk-dominated galaxies.
Fig. 14. Schematic sketch of the relation between SMBH mass $M_\bullet$ and absolute magnitude of the bulge $M_B$, with the solid diagonal line depicting the commonly adopted mean ratio $M_\bullet/M_\star=0.002$ (cf., e.g., Heckman & Best 2014) and the dashed line its expected change after correction for the $\delta_\text{io}$ effect. Arrows in the upper part point to the amplitude of $\delta_\text{io}$ and the expected fractional under-estimation of the bulge luminosity $\delta L_B$ for the three LTG classes proposed in BP18. Bulgeless, pseudo-bulge, and classical-bulge galaxies may loosely be associated with type $iA$, $iB$, and $iC$ LTGs in the mass intervals log$(M_\bullet/M_\star)<10.3$, $10.3<\text{log}(M_\bullet/M_\star)<10.5$, and $>10.5$, respectively. Whereas $\delta_\text{io}$ increases with $M_\bullet$, its effect on $\delta L_B$ is likely comparatively small for high-mass LTGs because of the prominence of their bulges, while it is expected to increase for intermediate-mass LTGs that simultaneously show a significant $\delta_\text{io}$ and a modest B/D ratio. We expect $\delta L_B$ for low-mass LTGs to show a large scatter around a low median value because of their very low $\delta_\text{io}$ and B/D ratio. Alternatively, a down-bending of the bulge-$M_\bullet$ relation for sub-$L^*$ galaxies could strengthen the conclusion by Heckman & Best (2014, their Fig. 9b) that $M_\bullet$ is not a fixed fraction of $M_\star$ but decreases from a mean $M_\bullet/M_\star$ ratio of $10^{-2.5}$ at $M_\star \sim 10^{11.5} M_\odot$ to $10^{-4}$ at $M_\star \sim 10^{10} M_\odot$. Furthermore, a dependence of the $M_\bullet/M_\star$ ratio on $M_\star$ would be consistent with the scarcity or weakness of accretion-powered nuclear activity in low-mass LTGs (Kormendy & Ho 2013, see also BP18). Summarizing, in the light of our considerations above, a correction of bulge magnitudes for $\delta_\text{io}$ appears to be fundamental for sharpening observational constraints on the intrinsic scatter of the bulge-$M_\bullet$ relation (e.g., Gültekin et al. 2009) and the possible dependence of its slope on $M_\star$ (Fig. 14) or galaxy morphology (e.g., Sahu et al. 2020).

4.4. Combined effect of $\delta_\text{io}$ with a possible central depletion of the disk

Our foregoing considerations are based on the conservative assumption that the stellar surface density $\Sigma_\star(R^*)$ of the disk is exponential all the way to its center and the central flattening or down-bending of its SBP is solely due to a higher $M_\bullet$ inside the SF-quenched bulge. However, the exponentiality of $\Sigma_\star$ is not for granted. Theoretical work points to a central depletion of the disk (Kuijken & Dubinski 1995; Obreja et al. 2013; Widrow & Dubinski 2005; Du et al. 2020) although little is quantitatively known on its possible connection to $M_\bullet$ and other galaxy properties (e.g., angular momentum, presence of an AGN). As an example, Obreja et al. (2013) find from hydrodynamical simulations a steep decrease in the disk’s $\Sigma_\star$ by $\sim2.5$ dex within $R^*\sim1$ kpc ($\sim1/2 R_B$).

Circumstantial observational support for a central depletion in the disk comes from recent work by B20b. These authors showed that the stellar SED of the outer disk, if scaled to the light fraction implied within $R_B$ by inward extrapolation of a purely exponential model and then subtracted from the integrated spectrum therein, yields an unphysical (negative) or implausible net stellar SED for the bulge. The latter was deemed to be the case when the disk-corrected SED for the bulge is abnormally red, which would force spectral synthesis models to converge to the maximum possible age and metallicity allowed by the construction of the SSP library used, or yield for the net-bulge a $M_\bullet$ that exceeds the value obtained by fitting the integrated SED within $R_B$. These authors also tested the effect that a modexp model for the disk would have on the net SED of the bulge. They showed that models with a central flattening reduce the percentage of implausible solutions to 15%, whereas a down-bending of the disk to a central intensity of 0 (i.e., a modexp with $\epsilon_1=1$, cf. Eq. 4) in all cases yields an acceptable spectral fit. We recall that the first modexp model (centrally flat exponential profile) can be accounted for by a modest $\delta_\text{io}$, whereas the second model ($\epsilon_1=1$) cannot just be due to a variation in the $M_\bullet/L$ ratio, but requires a central evacuation of the disk. Because the pilot study by B20b has considered only these two modexp profiles, it was unable to pinpoint the form of the inner disk below the bulge or place constraints on the relative importance of $\delta_\text{io}$ and a possible central decrease in $\Sigma_\star$. For instance, it is conceivable that a $\delta_\text{io}$ at the upper range of estimated values ($\sim2.5$ B mag) could alleviate the need for a strong central reduction of $\Sigma_\star$. It is also important to bear in mind that modexp is a functional form developed in another context (dwarf galaxies) and may not be optimal for galactic disks.

Star formation quenching and a central depletion of the disk are presumably nonmutually exclusive phenomena, and both imply an upward revision of the luminosity of the bulge. A step forward toward their better understanding might be possible through spectrophotometric and kinematical decomposition of IFS data (e.g., by extending the method by B20b by a dense grid of modexp models for $0 \leq \epsilon_1 \leq 1$, or other functional forms that allow for a centrally depressed exponential disk). A combined analysis of observational constraints gained from this task with galaxy simulations that incorporate detailed prescriptions for SFG as well as Schwarzschild orbital superposition modeling (e.g., Zhu et al. 2018a,b; Du et al. 2020; Jagvaral et al. 2021) would be of considerable interest.

4.5. Approaches toward an empirical correction of $\delta_\text{io}$

Because the $\delta_\text{io}$ effect is a direct and causal consequence of star formation quenching, understanding it is almost tautologous to the understanding of spiral galaxy evolution, including the physical drivers, timescales, and spatial characteristics of inside-out cessation of SF. Elucidating this subject in its broad complexity is clearly a long-term endeavor. For this reason, it appears worthwhile to explore empirical recipes that could permit an approximate short-term a posteriori correction of the large existing body of photometric quantities for bulges from previous...
galaxy decomposition studies. Ideally, such corrective formulæ
should involve easily accessible observables (e.g., EWs, colors)
or quantities inferred from spectral modeling (bulge-to-disk age
contrast, $\delta\mu_{BC}$). This task can presumably only be achieved in
a statistical sense: The rich variety of substructures in the central
parts of LTGs (nuclear disks and rings, single- and double-bars,
barlenses; cf., e.g., Bittner et al. 2017; de Lorenzo-Cáceres et al.
2019; Laurikainen et al. 2018; Gadotti et al. 2020) together with
the significant scatter ($\sim 1.7\ Gyr/R_0$) of stellar age gradient deter-
minations therein (B20a) document a diversity in the biography of
individual systems.

The development of such a rectification concept will most likely
require determination and suppression of $\delta\mu_0$ in a large, repre-
sentative sample of local LTGs in conjunction with multiband
photometric analyses of simulated galaxies in various stages of
SFQ with the goal of elaborating empirical constraints on the im-
 pact of $\delta\mu_0$ on bulge-disk decomposition (Sect. 4.1). The first
choice could exploit image processing (e.g., unsharp masking), and
SED fitting of spatially resolved data in combination with an
age-slicing tool like $\mathcal{R}Y$.

a) One possibility is to mask or dimensionally subtract spi-
ral features prior to bulge-disk decomposition. This has been
done in detailed studies of small galaxy samples in order to
stabilize the initial guess on ellipticity and center (cf. the itera-
tive technique by Méndez-Abreu et al. 2008). While feasible,
such an approach requires careful interactive work and thus ap-
ppears impractical for automated studies of large galaxy sam-
ples. Machine-learning-supported suppression of nonaxissym-
metric features might be an alternative, but would require train-
ing on local galaxy samples, which could turn into a disadvan-
 tage for studies of irregular clumpy galaxies at a higher $z$. An ad-
 ditional concern is that high-contrast grand-design spiral features
are generally prominent in massive LTGs, whereas lower-mass
spirals tend to be flocculent and to show low-contrast spiral pat-
terns (e.g., Bittner et al. 2017). This could result in a bias against
lower-mass LTGs, especially when studying higher-$z$ samples
with PSF-degraded imaging data. Another limitation of this ap-
proach is that it is hardly suitable for suppression of diffuse ion-
ized gas (DIG) and intraspiral arm emission from moderately
evolved ($\sim 200$ Myr) stellar populations that experience signifi-
cant evolution in their $M/L$ over a galaxy rotation period. The
DIG, although faint (EW(Hα)<6 Å), can contribute up to ~50% of
the total Hα luminosity of star-forming galaxies (Ferguson et
al. 1996; Weilbacher et al. 2018; den Brok et al. 2019).

b) NIR photometry allows for the reduction of $\delta\mu_0$ by a factor ~3
(cf. Sect. 2.1), but it is prone to contamination by the nebular
continuum (e.g., Krüger et al. 1995). On the other hand, thanks
to the rapidly increasing amount of NIR data with an adequate
resolution (e.g., with VISTA and soon Euclid), it offers impor-
tant prospects for the suppression of $\delta\mu_0$ in low-$z$ galaxy samples.

c) Another possible approach could take advantage of $\mathcal{R}Y$
(Sect. 2.2) to suppress the excess emission from the star-forming
outer disk and obtain an estimate on the size and intensity pro-
file of the SF-quenched core (i.e., the $e_{1:2}$ parameters of Eq. 5).
Limitations of this approach stem from uncertainties in popula-
tion spectral synthesis (Gomes & Papaderos 2016; Breda & Pa-
paderos 2018, for a discussion), its high computational expense,
and the requirement for an objective definition of the optimal $t_{out}$
for a galaxy.

d) A complement to the aforementioned methods could in-
volve bulge-(bar)-disk decomposition of stellar surface density
(instead of surface brightness) maps (Lang et al. 2014, see
also Wuyts et al. 2012). In this regard, the availability of spa-
tially resolved, low-resolution ($R \sim 60$) SEDs from Javalambre
Physics of the Accelerating Universe Astrophysical Survey (J-
PAS; Molles et al. 2011; Bonoli et al. 2021) opens a promising
avenue.

Despite their advantages and disadvantages, a combination of
the approaches above could yield statistical estimates of $\delta\mu_0$ as
a function of $M_\star$ and B/D ratio and might support the refinement
of various spectrophotometric decomposition concepts proposed
recently (e.g., Johnston et al. 2012, 2017; Tabor et al. 2017,
2019; Méndez-Abreu et al. 2019; Breda et al. 2020b; Barsanti
et al. 2021). A correction for $\delta\mu_0$ could also help provide accu-
rate 2D $M/L$ and $\Sigma_*$ maps needed for advanced Schwarzschild
orbital decomposition and foster synergy between theory and ob-
servations toward a better understanding of the star formation
quenching phenomenon.

5. Summary and conclusions

The aim of this study was to draw attention to the implications
of star formation quenching (SFQ) for bulge-disk decomposition
studies of late-type galaxies (LTGs) and motivate further obser-
vational and theoretical work in this field. The standard practice
of bulge-disk decomposition is to fit and subtract an exponen-
tial model to the disk (more specifically, to its visible periphery
outside the bulge) in order to isolate the net luminosity of the
bulge. Regardless of whether the modeling of disk and bulge is
done sequentially or simultaneously or in 1D or 2D, the key as-
sumption in all cases is that the inwardly extrapolated fit to the
outer disk faithfully reproduces the radial intensity profile in its
observationally inaccessible inner zone, below the bulge.

However, this postulate would only be valid if the star formation
history (SFH) and specific SFR of the disk were spatially invari-
ant, which is irreconcilable with the well established fact of
SFQ inside the bulge radius $R_0$ of massive ($\geq L^*$) LTGs. A central
depression of star formation (SF) implies a higher $M/L$ ratio for
the inner SF-quenched disk ($R^< R_0$) than for the SF-enhanced
outer disk ($R^> R_0$). For this reason, the inwardly extrapolated
exponential model must lead to an overestimation (and oversub-
traction) of the disk inside $R_0$, consequently to a systematic un-
derestimation of the luminosity of the bulge. We refer to the dif-
ference between the true magnitude of the disk inside $R_0$ and
the $\delta\mu_0$ between the true and extrapolated central surface brightness of
the disk, or as $2.5 \log(\psi)$, where $\psi$ denotes the ratio of the mean
$M/L$ of the disk inside and outside $R_0$.

i) We showed that if the bulge and the inner disk share roughly
the same SFH, then $\delta\mu_0$ in present-day LTGs can reach 2.5 mag in
$B$ and 0.7 mag in $K$. This was demonstrated with evolutionary
synthesis models involving continuous SF in the outer disk and
an exponentially decreasing SF with an e-folding time of 0.5
and 1 Gyr within $R_0$. These estimates remain valid for SFH scenarios
that simulate a downsizing trend, with bulges in massive galax-
ies experiencing the dominant phase of their formation early on
and vice versa. Additionally, observational support for a $\delta\mu_0$ well
in excess of 1 $g$-band mag is provided through post-processing
of spatially resolved spectral synthesis models for local LTGs
with the age-slicing tool REMOVEYOUNG (Gomes & Papaderos
2016), extending previous work in Breda & Papaderos (2018).
ii) In the presence of SFQ, the exponential surface brightness profile (SBP) of the disk should show a central flattening or downbending inside the radius of the bulge. A suitable functional form for such an SBP is given by the modified exponential (modexp) distribution proposed in Papaderos et al. (1996).

iii) It is pointed out that the neglect of $\delta_{\text{io}}$ in dedicated bulge-disk decomposition studies can result in biased determinations of the photometric and structural properties of galaxy bulges. Expected effects include a) a systematic underestimation of the luminosity of the bulge in a manner that is inversely related to its prominence and proportional to the degree of SFQ (aka $\delta_{\text{io}}$), and b) a potential underestimation of the Sérsic exponent $n$. The latter entails an overestimation of the model-dependent effective radius of the bulge (because $R_{\text{e}}$ and $n$ are inversely related to each other), and consequently also an underestimation of the mean stellar velocity dispersion $\sigma_{*}$ therein. These biases likely lead to a tendency for the classification of moderately luminous classical bulges as pseudo-bulges on the basis of the commonly adopted empirical cutoff at $n \sim 2$. Additionally, we showed that the neglect of $\delta_{\text{io}}$ can readily result in the erroneous classification of low-B/D LTGs as bulgeless disks.

iv) Negative color gradients within the bulge can entirely be due to the outwardsly increasing line-of-sight intensity contribution of the blue star-forming disk. This applies both to centrally star-forming and SF-quenched disks. For this reason, correction for the underlying disk (which, in turn, requires an understanding of $\delta_{\text{io}}$) is crucial for a meaningful study of color gradients in LTG bulges and the evolutionary clues these may hold. If such a correction is attempted assuming a purely exponential SBP for the disk, then the net color of the bulge is invariably overestimated. This chromatic bias is aggravated by red rims that can be taken as evidence for dusty circumnuclear SF rings.

v) From a synopsis of insights from this study and existing observational and theoretical evidence, it is pointed out that massive LTGs develop a high $\delta_{\text{io}}$ early on because their centers quench first, whereas lower-mass LTGs experience a retarded evolution that leads to a slower rise of $\delta_{\text{io}}$ with time. This differential evolution of $\delta_{\text{io}}$ across galaxy mass ($M_\ast$) can lead to complex biases when galaxy samples spanning a range in redshift ($z$) or $M_\ast$ are structurally analyzed and compared with each other, and propagate into errors in the scatter and slope of any galaxy scaling relation that involves photometric quantities for bulges. As an example, we argued that correction for $\delta_{\text{io}}$ might lead to a downbending of the bulge versus supermassive black hole mass $M_\bullet$ relation below log($M_\bullet/M_\odot$)\lesssim10.7. A decrease of the $M_\bullet/M_\ast$ ratio with decreasing $M_\ast$ would offer an element toward understanding the virtual absence of accretion-powered nuclear activity in low-mass spiral galaxies.

vi) The above conclusions are drawn on the conservative assumption that the radial stellar surface density $\Sigma_{\ast}$ of the disk is purely exponential, and the central down-bending of its SBP results solely from an increase in its $M_\bullet/L_\ast$ ratio within the bulge. However, a possible central depletion of the disk (Breda et al. 2020b) would further enhance photometric biases due to $\delta_{\text{io}}$.

vii) A significant $\delta_{\text{io}}$ (22 mag in restframe r band) is expected in young high-z galaxies due to aging of SF clumps as they inwardly migrate from the disk. An important contribution to the early rise of $\delta_{\text{io}}$ likely comes from the outwardly increasing contamination of broadband fluxes by nebular emission, both due to the lower age of SF clumps and the lower dilution of their emission-line equivalent widths (EWs) by the local continuum. Spatially resolved studies of high-$z$ protogalaxies with the JWST, ELT, and Euclid could test whether positive EW gradients emerge prior to or after the appearance of a dense, SF-quenched bulge, and might in this way place observational constraints on the relative role of clump migration and dissipative gas collapse during the early phase of bulge formation.

viii) Possible approaches toward a statistical estimation of $\delta_{\text{io}}$ and its effect on the large existing body of photometric and structural determinations for bulges were discussed.

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References

Allen, M.G., Groves, B.A., Dopita, M.A., Sutherland, R.S. & Kewley, L.J. 2008 ApJS, 178, 20
Anders, P. & Fritz von Alvensleben, U. 2003, A&A, 401, 1063
Andredakis, Y.C. & Sanders, R.H.L. 1994, MNRAS, 275, 874
Andredakis, Y.C., Peletier, R.F. & Balcells, M. 1995, MNRAS, 275, 874
Atek, H., Siana, B., Scarlata, C. et al. 2011, ApJ, 743, 121
Balcells, M. & Peletier, R.F. 1990, A&AS, 84, 107
Baldwin, J. A., Phillips, M. M. & Terlevich, R., 1981, PASP, 93 (5/BPT)
Bansanti, S., Owens, M.S., McDermid, R.M. et al. 2021, ApJ, 906, 100B
Belfiore, F., Maiolino, R., Bundy, K. et al. 2018, MNRAS, 477, 3014
Benno, V.N., Treu, T., Ding, X. et al. 2021, ApJ, 921, 36B
Binggeli, B., Cameron, L.M., 1993, A&AS, 98, 297
Bittner, A., Gadotti, D.A., Elmegreen, B.G. et al. 2017, MNRAS, 471, 1070
Bittner, A., Sánchez-Blázquez, P., Gadotti, D.A. et al. 2020, A&A, 643, 65
Bizzocchi, L., Filho, M.E., Leonardo, E. et al. 2014, ApJ, 782, 22
Bournaud, F., Elmegreen, B. G., Elmegreen, D. M., 2007, ApJ, 670, 237
Breda, I., Papaderos, P. 2018, A&A, 614, 48 (BP18)
Breda, I. 2019, PhD Thesis, University of Porto
Breda, I. & Papaderos, P. 2018, A&A, 614, 48 (BP18)
Breda, I., Papaderos, P., Gomes, J.M., Amarantidis, S. 2019, A&A, 632, A128
Breda, I., Papaderos, P., Gomes, J.M. et al. 2020a, A&A, 635, A177 (B20a)
Breda, I., Papaderos, P. & Gomes, J.M. 2020b, A&A, 640A, 20B (B20b)
Brinchmann, J., Charlot, S., White, S. D. M. et al. 2004, MNRAS, 351, 1151
Bonito, S. et al. 2021, A&A, 653A, 31B
Borissova, E., Cantalupo, S., Lilly, S.J. et al. 2016, ApJ, 831, 39
Bruzual, G. & Charlot, S., 2003, MNRAS, 344, 1000
Buta, R. & Combes, F. 1996, Fund. Cosmic Physics, 17, 95
Catalán-Torrecilla, C., Gil de Paz, A., Castillo-Morales, A. et al. 2017, ApJ, 848, 87
Cid Fernandes, R., Mateus, A., Sodré, L., Stasăska, G., Gomes, J. M., 2005, MNRAS, 358, 363
Cid Fernandes, R., Stasăska, G., Mateus, A. et al. 2011, MNRAS, 413, 1687

Article number. page 17 of 21
Zhu, L., van de Ven, G., Méndez-Abreu, J. & Obreja, A. 2018b, MNRAS, 479, 945
Zibetti, S., Gallazzi, A. R., Ascasibar, Y. et al. 2017, MNRAS, 468, 1902
Ziegler B.L. & Bender R., 1997, MNRAS, 291, 527
Ziegler, B.L., Saglia, R.P., Bender, R. et al. 1999, A&A, 346, 13
Appendix A: Supplementary notes to Sect. 3

A.1. \( \delta_{\mu_0} \) versus central intensity depression and cutoff radius of a modexp profile

In this section we supplement the discussion in Sect. 3.1 by a quantitative determination of \( \delta_{\mu_0} \) for different modexp profiles for the disk. We recall that modexp involves two further free parameters in addition to the central surface brightness \( \mu_0 \) (mag/arcsec\(^2\)) and scale length \( \alpha \) (denoted \( \theta \)) defining a purely exponential model. The first parameter \( \epsilon_1 = \Delta I/I_0 \) constrains the depression \( \Delta I \) of the disk relative to the central intensity \( I_0 \) of a purely exponential profile. Correspondingly, the dimming \( \delta \mu_0 \) (mag) of the disk at \( R^* \equiv 0'' \) relative to the central surface brightness of an exponential model is \(-2.5 \log(1 - \epsilon_1)\). In Sect. 3.1 we considered cases of a modest central dimming (0.5 \( \leq \delta \mu_0 \) (mag) \( \leq 2 \)), based on estimates with PEGASE on the \( M/L \) contrast between the star-forming disk and the SF-quenched bulge. This range of \( \delta \mu_0 \) appears adequate when we assume that the stellar surface density \( \Sigma \) profile of the disk is purely exponential all the way to \( R^* \equiv 0'' \), thus SFQ is the only cause for the surface brightness depression of the disk in its central part. If, additionally, the disk is centrally depleted (cf. Sect. 4.4), then a higher \( \delta \mu_0 \) will need be invoked (e.g., ~5 mag in the case of almost complete central evacuation with an \( \epsilon_1 \approx 0.99 \)).

The second free parameter \( \epsilon_2 \) quantifies the radius \( R_0 \) (in units of \( \alpha \)) inward of which modexp deviates from the exponential model. It can therefore be regarded as the galactocentric distance out to which the physical mechanisms behind SFQ appreciably influence the SFH (and radial \( M/L \) profile) of a galaxy. While to the best of our knowledge, theoretical or observational inferences do not exist for this radius, it is reasonable to assume that it depends on the nature and timescale of the dominant SF quenching mechanism. For example, in the context of morphological quenching (Martig et al. 2009), we may expect SFQ to be spatially confined to within \( R_0 \) (e.g., \( \epsilon_2 = 1 \)) for the synthetic galaxy in Fig. 8 where \( R_0 = \theta \) while extending well beyond \( R_0 \) in the case of AGN-driven SFQ (Silk 1997). As for other proposed mechanisms, such as inhibition of cold gas inflow from the cosmic web due to virial shocks in high-mass galaxies (Dekel et al. 2009), little can be quantitatively said about \( \epsilon_2 \) and the expected age and \( M/L \) gradients therein (see B20a for a discussion). Clearly, because \( \epsilon_2 \) might offer a discriminator between different models for SFQ, it would be interesting to explore techniques for its determination.

Figure A.1 (upper panel) illustrates the variation of modexp profiles for a disk model with the same \( \mu_0 \) and \( \alpha \) as in Fig. 8 (21.6 mag/arcsec\(^2\) and 20'', respectively) for three \( \epsilon_2 \) (0.67, 1.0, and 1.5; blue, orange, and red, respectively) and two values for \( \epsilon_1 \), the first one (0.84) characterizing the upper range of values that one might expect for a purely exponential \( \Sigma \) profile, and the second one (0.99) approximating a centrally depleted exponential profile. These \( \epsilon_1 \) values correspond to a \( \delta \mu_0 \) of 2 and 5 mag, respectively. The lower panel shows as a function of \( \epsilon_1 \) the variation of \( \delta \mu_0 \) and dimming \( \delta \mu_0 \) (mag) of the integrated magnitude of the disk within \( R^* \equiv \alpha \). The computed \( \delta \mu_0 \) for \( \epsilon_1 = 0.84 \) ranges between 0.9 and 1.25 mag for an \( \epsilon_2 \) of 1.0 and 1.5, respectively, reaching values of 1.4 and 2.5 mag for a depleted disk with \( \epsilon_1 = 0.99 \).

A.2. Retrieved versus true bulge magnitude when neglecting \( \delta \mu_0 \)

In this section we supplement our remarks on Fig. 9 with further quantitative inferences on the under-estimation of the luminosity of the bulge when bulge-disk decomposition of a centrally SF-quenched LTG is made assuming that the exponential model to the outer, SF-elevated zone of the disk is valid all the way to the center of the galaxy. In particular, we show that the fainter the
bulge is (i.e., the lower the B/D) the higher is the fractional underestimation of its luminosity (Sect. 3.1 & 4.2). This means that a low-luminosity bulge in a modestly SF-quenched disk can be more affected by $\delta_{\mu_0}$ than an intrinsically bright bulge in a fully SF-quenched disk. Clearly, the estimates below only address a narrow aspect of the influence of inside-out SFQ for galaxy decomposition. As pointed out in Sect. 4.1 a comprehensive investigation of this subject will require simulations of bulge-disk decomposition for synthetic multiband galaxy images. These should integrate realistic prescriptions for SFQ as a function of age and galaxy mass, and reproduce the observational imprints (e.g., radial age, $M/L$ and EW(H$\alpha$) gradients) of this process both for barless and barred galaxies. Additionally, observationally and theoretically motivated alternatives to modexp should be explored.

Similar to Fig. 9, the estimates next are obtained by decomposing synthetic SBPs that consist of a Sérsic model for the bulge and a modexp for the disk into, respectively, a Sérsic and a pure exponential profile. The net (disk-subtracted) profiles for the bulge were in turn used to infer how the neglect of $\delta_{\mu_0}$ affects the retrieved apparent magnitude of the bulge inside $R_B$. These simulations refer to a disk with an (extrapolated) central surface brightness of 21.6 mag/$^\prime\prime$ and an exponential scale length $\alpha = 20^\prime\prime$ (values identical to those in Fig. 8) and cover a range $0 \leq \epsilon_1 \leq 0.99$ (correspondingly, a $\delta_{\mu_0}$ between 0 and 5 mag) for three SFQ radii $\epsilon_2$ (0.67, 1 and 1.5 $\alpha$). Additionally, they address how at a given $\epsilon_1$ the fractional underestimation of the luminosity of the bulge increases as its intrinsic luminosity decreases; for this, we simulate a decrease of the magnitude of the bulge from its original value ($m_B = 14.27$ mag; cf. Fig. 8) to $\sim 18$ mag. The main insight from Fig. A.2 is that the underestimation of the bulge magnitude (vertical bar at the r.h.s. of each diagram) depends both on $m_B$ itself and the shape of the SF-quenched disk (i.e., on $\epsilon_1$). For example, a relatively bright bulge with the properties assumed in Fig. 8 is relatively immune to $\delta_{\mu_0}$, as its apparent magnitude within $R_B$ would be underestimated by merely $\approx 0.5$ (1) mag for a $\delta_{\mu_0} = 1$ (2) mag, when $\epsilon_2 = 1$. However, a bulge with the same compactness (i.e., $\beta$ and $\eta$) but lower luminosity will artificially dim by 1.4 mag if its $m_B$ were one mag fainter (15.27 mag), and by 3.7 mag if its $m_B$ were 16.27 mag. The detection bias against intrinsically faint bulges will be further aggravated for a larger cutoff radius. For instance, an $\epsilon_2 = 1.5$ will increase the underestimation of the bulge magnitude to 2.6 mag and 5.2 mag, respectively. The evidence from Fig. A.2 underscores our remarks in Sect. 4.3 that studies of intrinsically bright bulges in iC LTGs are only moderately affected by $\delta_{\mu_0}$, whereas a stronger effect is expected for lower-luminosity bulges in iB ($\sim L^*$) LTGs.

$\delta_{\mu_0}$ - $2.5 \log (1 - \epsilon_1)$ between 0 and 5 mag. The extrapolated central surface brightness $\mu_0$ (21.6 mag/$^\prime\prime$) and exponential scale length $\alpha$ of the disk are identical to those adopted in Fig. 8. The true apparent magnitude $m_B$ of the bulge (abscissa) varies between 14.27 mag (the value for the Sérsic profile in Fig. 8) and $\sim 18$ mag. Contours go from 0.5 mag to 9 mag in increments of 0.5 mag.