Budget Constrained Interactive Search for Multiple Targets

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ABSTRACT

Interactive graph search leverages human intelligence to categorize target labels in a hierarchy, which is useful for image classification, product categorization, and database search. However, many existing interactive graph search studies aim at identifying a single target optimally, and suffer from the limitations of asking too many questions and not being able to handle multiple targets.

To address these two limitations, in this paper, we study a new problem of budget constrained interactive graph search for multiple targets called kBM-IGS problem. Specifically, given a set of multiple targets $\mathcal{T}$ in a hierarchy and two parameters $k$ and $b$, the goal is to identify a $k$-sized set of selections $\mathcal{S}$, such that the closeness between selections $\mathcal{S}$ and targets $\mathcal{T}$ is as small as possible, by asking at most a budget of $b$ questions. We theoretically analyze the updating rules and design a penalty function to capture the closeness between selections and targets. To tackle the kBM-IGS problem, we develop a novel framework to ask questions using the best vertex with the largest expected gain, which provides a balanced trade-off between target probability and benefit gain. Based on the kBM-IGS framework, we first propose an efficient algorithm STBIS to handle the SingleTarget problem, which is a special case of kBM-IGS. Then, we propose a dynamic programming based method kBM-DM to tackle the MultiTargets problem. To further improve efficiency, we propose two heuristic but efficient algorithms, kBM-Topk and kBM-DM+. kBM-Topk develops a variant gain function and selects the top-$k$ vertices independently. kBM-DM+ uses an upper bound of gains and prunes disqualified vertices to save computations. Experiments on large real-world datasets with ground-truth targets verify both the effectiveness and efficiency of our proposed algorithms.

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1 INTRODUCTION

Crowdsourcing, such as Amazon’s Mechanical Turk [1] and CrowdFlower [2], allows organizations to design human-aided services in which humans can help solve tasks and get rewards. In real applications, many tasks such as object categorization [24], entity resolution [32, 33], filtering noisy data [15, 26], ranking [22], and labeling [5], are complex and difficult to solve algorithmically. With regard to human-aided object categorization, the graph search problem concerns leveraging human intelligence to categorize the target labels of a given object in a label hierarchy, which has a wide range of applications including image classification [10, 18], product categorization [23], and relational database search [30].

Recently, Tao et al. [30] investigated the problem of interactive graph search (IGS) to locate a unique target vertex in a hierarchy $\mathcal{H}$, with as few questions as possible. For example, Figure 1(a) shows a hierarchical tree with several labeled vertices. A directed edge from one vertex to another represents the concept-instance relationship, e.g., “pet” is a general concept of four instances “cat”, “dog”, “rabbit”, and “turtle”. Note that the target is unknown in advance. To identify the target, interaction is allowed to iteratively ask questions using the vertices in the hierarchy, e.g., “Is this a wild?”, “Is this a pet?”.

Assuming that the target is “pet” in Figure 1(a), we need to ask at least five questions of “Is this $x$?”, where $x \in \{\text{“pet”}, \text{“cat”}, \text{“dog”}, \text{“rabbit”}, \text{“turtle”}\}$, to get the answers (Yes, No, No, No, No) and then determine the exact target of “pet”. Effective algorithms with theoretical guarantee are proposed for finding the exact target using at most $\log_h n$ questions, where $n$ is the number of vertices, the maximum out-degree, and the hierarchy height in $\mathcal{H}$. However, two issues remain open:

- Finding nearly-optimal targets using a constrained budget. IGS [30] may incur a high cost to identify the exact target. It does not limit the number of questions that can be asked. In the worst case, the proposed algorithm asks $d \log_d n$ questions to optimally identify the target. In real hierarchy datasets, the out-degree $d$ could be large, e.g., ImageNet [10] has $d = 391$ and $n = 74,401$. Thus, users may need to answer 782 questions, which is not very practical. Even worse, asking questions is potentially costly [24], which motivates the problem of budget constrained IGS to bound the total cost.
- Identifying multiple targets. Existing studies on the IGS problem [18, 30] only consider a single target, where the answer has one and only one target. However, in real applications of
object categorization, an object may have multiple labels. Even worse, it is difficult to determine in advance how many labels the object may have. For example, Figure 1(b) shows an uncategorized image object. Both “cat” and “fish” are suitable to label the object, but either one alone is not good enough.

To address the above issues, in this paper, we propose a new kBM-IGS problem of interactive graph search for identifying multiple targets $T$ using a constrained budget to ask at most $b$ questions. Specifically, in each round, our kBM-IGS scheme asks a question in the form “Given a query vertex $q$ in tree $H$, can vertex $q$ reach one of targets in $T$?” and receives the answer from human-assisted interactions. On the basis of the previous answers, the kBM-IGS scheme determines the next question to ask. Finally, it selects a set of vertices to represent the targets after $b$ questions are answered.

However, it is significantly challenging to identify the most suitable selections in the kBM-IGS problem, for the following reasons. First, the number of targets is unknown in advance. Given an uncategorized object, it may have one or more ground-truth labels. Second, in the worst case, a total of $O(n)$ questions is needed to find the exact answers regarding targets, which makes the selection of $b$ questions difficult. Third, since the targets are unknown, another challenge is how to measure the goodness of a solution, i.e., the closeness between selections and targets.

In light of the above, the objective of our problem is formulated as finding a $k$-sized selection set of vertices to approach the targets as close as possible, w.r.t. an input number of $k$ and a budget of $b$ questions. For example, consider the hierarchy and the uncategorized image object in Figure 1. Assume that $k = 2$ and $b = 2$. We ask two questions of “Is this $x$?”1, where $x$ is “pet” and “fish”, respectively, and both answers are Yes. After that, we cannot ask any more questions to verify the other four specified pets, i.e., “cat”, “dog”, “rabbit”, and “turtle”. Thus, we select “pet” and “fish” as the solution. Assume that the targets are “cat” and “fish”. It can be seen that the selections of “pet” and “fish” are close to the targets, since “pet” is a generalization of “cat”. On the other hand, “animal” is also a good label, but it is far from “cat” and worse than our selection “pet”. More real-world applications of our problem are presented in Section 5. Moreover, a detailed comparison between the state-of-the-art methods [18, 24, 30] and our model is provided in Section 2.

To tackle the kBM-IGS problem, we propose a novel kBM-IGS framework, which uses a greedy strategy to ask the best question with the largest expected gain at each round. Specifically, vertices have different probabilities to be targets and may get Yes/No answers for questions asked. In general, a vertex at the top level of the hierarchy has a high probability of getting a Yes answer. However, the benefit of getting a Yes answer can be less than a No answer, which implies that none of descendants are targets. Therefore, we propose an expected gain to trade-off the target probability and benefit gain. Thus, the kBM-IGS framework can find the vertex with the largest expected gain to ask the next question. On the basis of the kBM-IGS framework, we first propose an efficient algorithm STIGS to solve the SingleTarget problem, which is a special case of kBM-IGS with $|T| = 1$. It can update the gains of all vertices in $O(n)$ for each question. Different from the SingleTarget problem, it is difficult to calculate the gains in the MultipleTargets problem. We then develop a $kBM$-DP method to calculate the optimal penalty between the $k$-sized selections and potential targets. The algorithm takes $O(bn^{2}d^{2}k^{2})$ time. To further improve efficiency, we propose two heuristic but efficient algorithms, kBM-Topk and kBM-DP++. The first method, kBM-Topk, uses an independent penalty function, which avoids costly enumerations to find the optimal result. The second method, kBM-DP++, improves kBM-DP by invoking an upper bound to quickly identify the best vertex to ask the next question before all expected gains are updated. To summarize, we make the following contributions:

- We propose a new kBM-IGS problem of budget constrained interactive graph search for identifying multiple targets in a hierarchical tree. We raise the problem of finding the $k$-sized selections close to multiple targets using a fixed number of $b$ questions, and formally design a penalty function to measure the closeness between selections and targets (Section 3).
- We give theoretical analysis of potential targets and Yes-candidate, which offers useful updating rules to prune disqualified candidates. Moreover, for a specific question, we discuss the probabilities of Yes and No answers, and analyze the expected gain of asking this question. On the basis of the updating rules and expected gains, we propose a novel kBM-IGS framework to tackle the kBM-IGS problem by asking $b$ good questions (Section 4).
- We investigate and tackle one instance of kBM-IGS problem, the SingleTarget problem, where the target involves a single answer. On the basis of the kBM-IGS framework, we derive new updating rules and propose a greedy algorithm STBIS. We further develop a DFS technique to accelerate STBIS (Section 5).
- We propose three efficient algorithms for identifying multiple targets based on the kBM-IGS framework. First, we propose a dynamic programming based technique kBM-DP, which computes the gain scores for all vertices optimally. To further improve efficiency, we propose two fast algorithms, kBM-Topk and kBM-DP+, based on an alternative function of independent gain score and a pruning upper bound for lazy gain computation, respectively (Section 6).
- We conduct extensive experiments on real-world datasets with ground-truth multiple targets to validate the efficiency and effectiveness of our proposed framework and algorithms (Section 7).

2 RELATED WORK

Our work is related to human-assisted data processing tasks [14, 17, 25, 26, 32, 34, 36] and object categorization problems [7, 11, 12, 16, 20, 29, 38]. Table 1 shows a detailed comparison of the three most relevant studies, IGS [30], Bing [18], HGS [24], and our kBM-IGS. Tao et al. [30] propose an interactive graph search (IGS) method for identifying a single target in a directed hierarchy. The general idea is to apply heavy-path decomposition to produce a balance representation of hierarchy and tackle the problem by binary searches. Li et al. [18] model the single target problem as a decision tree construction problem. They propose a greedy based method for interactive graph search (denoted as BinG), which improves the performance of IGS [30]. Both studies consider a single target and find the exact

1The question is equivalent to a search question in the form “Given a query vertex $q$ in tree $H$, can vertex $q$ reach one of targets in $T$?”
result using an unlimited budget of questions. Consider the example shown in Figure 2. Assume that the target is $r$. Both IGS [30] and BinG [18] would ask all its children to determine whether $r$ is the target, which takes $n - 1$ questions. Different from these two studies [18, 30], our proposed framework can tackle both SingleTarget and MultipleTargets problems and select the representative targets within a bounded budget.

![Figure 2: A hierarchy has $n$ vertices with the target $r$.](image)

Paramswan et al. [24] also investigate both SingleTarget and MultipleTargets problems with bounded budgets and propose HGS to find multiple targets using $b$ questions. However, the novelty of our problem is the consideration of the interactive setting, which enables dynamic algorithm designs and brings significant performance benefits. First, the HGS scheme is a non-interactive algorithm that asks all $b$ questions in one go and then, based on the workers’ answers, does the best to figure out where the targets are. Its objective is to choose the $b$ questions wisely to minimize the size of the candidate set. As a result, it may not be able to find the candidates close to the targets. In contrast, our proposed approach leverages the answers of the previous $l$ questions ($1 \leq l \leq b - 1$) to dynamically determine the $(l + 1)$-th question. Such interaction allows our algorithm to quickly narrow down the search space and efficiently guide the search towards the targets. Second, the HGS algorithm divides the whole hierarchy into $b$ subtrees and asks a question on each root of the $b$ subtrees. It has an extremely time complexity of $O(bn^2)$ [24]. Different from HGS, our dynamic approach works by asking one question each time on a single vertex that achieves the largest expected gain based on the previous answers. In other words, our approach is a greedy algorithm that runs $b$ times to identify $b$ questions in $O(bn^2d^2k^2)$ time. It is more effective and efficient than HGS, as will be validated by the experiments in Section 7.

Furthermore, there exist a large number of studies on hierarchy construction using crowdsourcing techniques [6, 8, 28]. Sun et al. [28] build up a crowdsourcing system to construct a hierarchy by maintaining distributions over possible hierarchies. Chilton et al. [8] propose an automated workflow Cascade to create a taxonomy from the collective efforts of crowd workers. Bragg et al. [6] present an improved workflow DELUGE which uses less crowd labor to generate taxonomies. All these studies focus on how to build high quality category hierarchies. Orthogonal to these studies, our work utilizes the hierarchies and crowdsourcing ideas to categorize objects in an interactive way w.r.t. a given budget of questions.

### 3 PRELIMINARIES

In this section, we present definitions and formulate our problem.

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**Table 1:** Comparison with relevant studies IGS [30], BinG [18], and HGS [24]. Here, $b$ is the budget of questions, and $n, d, h$ are respectively the number of vertices, the maximum out-degree, and the height in the hierarchy.

| Method  | Interactive | Targets | Budget | Questions (Worst Case) | Time (Each Question) | Time (Total) |
|---------|-------------|---------|--------|------------------------|---------------------|--------------|
| IGS [30]| ✓           | Single  | ×      | $O((\log h)(1+\log n))+(d-1)\cdot\log n$ | $O(1)$             | $O(n\log n)$ |
| BinG [18]| ✓           | Single  | ×      | $n-1$ | $O(n)$ | $O(n^2)$ |
| HGS [24]| ×           | Single  | ✓      | $b$ | / | $O(n\log n)$ |
| ×       | Multiple    | ✓      | $b$ | $O(n\log n)$ |
| ✓       | Single ✓    | ✓      | $b$ | $O(n\log n)$ |
| ✓       | Multiple ✓  | ✓      | $b$ | $O(n\log n)$ |

**Figure 3:** The hierarchical tree used in the running example.

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### 3.1 Hierarchical Tree

Let $\mathcal{H} = (V, E)$ be a directed hierarchical tree rooted at $r$ with a set $V$ of vertices and a set $E$ of directed edges, where the root $r \in V$ and the edge set $E = \{(u, v) : v$ is the parent of $u\}$. Let the size of vertex set $n = |V|$ and the height of $\mathcal{H}$ be $h$. For a vertex $v \in V$, we denote its children of $v$ by $\text{child}(v) = \{u : (v, u) \in E\}$ and its unique parent by $\text{par}(v)$ where $\text{par}(v), v \in E$. A vertex $v$ with no children is called leaf, i.e., $\text{child}(v) = \emptyset$.

Given two vertices $u$ and $v$, we say that $v$ can reach $v$ (denoted as $u \rightarrow v$), if and only if there exists a directed path from $u$ to $v$ in $\mathcal{H}$. If $u$ cannot reach $v$, we use $u \not\rightarrow v$ to represent it. Note that $v \rightarrow v$ and $r \rightarrow v$ for any vertex $v \in V$. Moreover, the distance from $u$ to $v$ is denoted by $\text{dist}(u, v)$, as the length of the shortest path from $u \rightarrow v$ in $\mathcal{H}$. If $u \not\rightarrow v$, the distance $\text{dist}(u, v)$ is the height difference between two vertices $u$ and $v$ in $\mathcal{H}$. Note that $\text{dist}(u, v) = 0$ and $\text{dist}(u, v) = +\infty$ if $u \not\rightarrow v$. In addition, the ancestors of a vertex $v$, denoted by $\text{anc}(v)$, are defined to be the set of vertices that can reach $v$ in $\mathcal{H}$, i.e., $\text{anc}(v) = \{u \in V : u \rightarrow v\}$. Similarly, the descendants of a vertex $v$, denoted by $\text{desc}(v)$, are defined to be the set of vertices that are reachable from $v$ in $\mathcal{H}$, i.e., $\text{desc}(v) = \{v \in V : v \rightarrow u\}$. Table 2 lists the frequently used notations in the paper.

**Example 1.** Figure 3 shows an example of hierarchical tree rooted by $v_0$. The children of $v_3$ are $\{v_0, v_1, v_2\}$ and its parent is $v_2$. We have $\text{anc}(v_3) = \{v_0, v_1, v_2\}$ and $\text{desc}(v_3) = \{v_3, v_4, v_7, v_8\}$. The distance $\text{dist}(v_0, v_3) = 2$, $\text{dist}(v_3, v_0) = 0$ and $\text{dist}(v_2, v_3) = +\infty$.

### 3.2 kBm-IGS Interactive Scheme

In the following, we introduce the scheme of budget-based interactive graph search for identifying multiple targets. In contrast to IGS [30] and BinG [18], our kBm-IGS has new rules and features for asking a limited number of questions. The interactive scheme has four components: targets, questions, Yes-candidates, and selections.

**Targets.** The targets are a set of vertices in tree $\mathcal{H}$, denoted as $T \subseteq V$. The goal of our kBm-IGS is finding the target set $T$, which needs to be identified through a few rounds of question asking. The targets $T$ can be chosen arbitrarily from $V$. In other words, $T$ may be any hidden vertices distributed over the whole tree $\mathcal{H}$. 
The target set $T$ has two characteristics: \textit{variant cardinality} and \textit{target independence}. First, in terms of target cardinality, we categorize $T$ into two types, SingleTarget and MultipleTargets, following [24]. If the size of $T$ is known and $|T| = 1$, we call it SingleTarget. If the size of $T$ is unknown and variant with $|T| \geq 1$, we call it MultipleTargets, which does not constrain the size of $T$. On the other hand, the target set $T$ must satisfy the property of target independence, that is, any two vertices in $T$ are not related [24]:

$$\forall v, u \in T, \text{ if } v \neq u, \text{ then } v \not\rightarrow u.$$ 

Consider the example shown in Figure 1(a), where we assume the target set is $T = \{\text{"cat"}, \text{"fish"}\}$. Although “pet” is also a correct label to represent the object in Figure 1(b), it is not suitable to be added into $T$, as it would violate the property of target independence as “pet” reaches “cat” in Figure 1(a). Actually, “cat” is a more precise label to describe the object than “pet” in this example.

**Questions.** To identify targets, one can ask a search question in the form “Given a query vertex $q$ in tree $H$, can vertex $q$ reach one of targets in $T$?” Formally,

**Definition 1 (Questions).** Given a query vertex $q$ and targets $T$ in tree $H$, the search question is defined as reach($q$). The boolean answer of reach($q$) is either Yes or No. If reach($q$) $= \text{Yes}$, then $\exists t \in T$ such that $q \rightarrow t$; otherwise, reach($q$) $= \text{No}$, i.e., $\forall t \in T$, $q \not\rightarrow t$.

For example, in Figure 1(a), the question “Can the vertex labeled ‘bear’ reach one of targets in $T$?” will get the No answer and neither “bear”, “black bear”, nor “brown bear” will be the correct label. On the contrary, the question “Can the vertex labeled ‘pet’ reach one of targets in $T$?” will get the Yes answer. One-shot question asking is limited to figure out where the targets are in a large tree $H$. One can interactively ask more questions to identify the targets accurately. However, in our kBM-IGS setting, we have given a budget $b$ for the number of questions that can be asked. This is because asking questions is usually costly in real applications, e.g., on Mechanical Turk [1]. It is also not practical to ask users numerous questions as they may not be willing to answer too many questions. After $b$ rounds of question-asking and answer-checking, one finally makes a decision to choose the answers to represent the targets.

**Yes-candidates.** We say that a vertex $v$ is a Yes-candidate for targets $T$ if and only if $\exists t \in T$ such that $v \rightarrow t$. Obviously, the root $r$ is always a Yes-candidate for any targets $T$. We define the Yes-candidates as the set of Yes-candidate for targets $T$ as follows.

**Definition 2 (Yes-candidates).** Given the targets $T$ in tree $H$, and several rounds of asking questions $Q = \{q_0, q_1, \ldots, q_l\}$ where $l$ is a positive integer and $q_0 = r$, the Yes-candidates are defined as

$$Y = \bigcup_{q_i \in Q, \text{reach}(q_i) = \text{Yes}} \text{anc}(q_i).$$

**Example 2.** Assume that the targets $T = \{s_2, v_3\}$ in Figure 3 and the questions $Q = \{q_0, q_2, q_3, q_5\}$. The answers are $\text{reach}(v_2) = \text{Yes}$, $\text{reach}(v_3) = \text{Yes}$, and $\text{reach}(v_5) = \text{No}$, thus $Y = \{q_0, q_1, q_2, v_2, v_3\}$.

**Selections.** The selections, denoted as $S$, are a subset of Yes-candidates $Y$, which are selected by the algorithms to match the targets $T$ as closely as possible. For example, in Figure 1, assume that we have questioned “pet” and get the Yes answer. We can select “animal”, “domestication”, or “pet” because they must be the correct label.

Overall, the goal of kBM-IGS interactive scheme is to use a few questions to determine the selections $S \subseteq Y$ to approach the targets $T$ as closely as possible.

### 3.3 Penalty between Selections and Targets

Given a budget of questions that can be asked and an unknown number of targets, it is challenging to determine the locations of targets in a large tree $H$. Instead of giving a simple boolean result, we develop an evaluation metric to quantify the goodness of our selections. In the following, we introduce another important feature of penalty in kBM-IGS. The penalty is an evaluation metric defined on the basis of distance, which measures the closeness between $S$ and $T$.

**Pair-wise Penalty.** Assume that we use a vertex $v \in V$ to cover a given target $t \in T$. If $v = t$, the choice $v$ exactly identifies the target $t$. If $v \neq t$, it needs to give a penalty score for using $v$ to cover the target $t$. On the basis of the distance, we give the definition of pair-wise penalty score as follows.

$$f(v, t) = \begin{cases} \text{dist}(v, t), & \text{if } v \in \text{anc}(t) \\ \text{dist}(r, t), & \text{if } v \notin \text{anc}(t) \end{cases}$$

(1)

By the above definitions, we consider two cases: 1) $v \in \text{anc}(t)$ and 2) $v \notin \text{anc}(t)$. First, for $v \in \text{anc}(t)$, indicating $v \rightarrow t$, the best selection $v$ should have $\text{dist}(v, t) = 0$. The further the distance $\text{dist}(v, t)$, the larger the penalty. Second, for $v \notin \text{anc}(t)$, indicating $v \not\rightarrow t$, we give a full penalty of the largest distance between $r$ and $t$, for using $v$ to cover $t$, i.e., $f(v, t) = \text{dist}(r, t)$. The deeper the location of target $t$, the larger the penalty. As a result, if a target can be reached by our selections, we use the shortest distance to indicate its closeness. Otherwise, if a target is not reachable from our selections, we give a distance-based penalty.

**Set-wise Penalty.** On the basis of the pair-wise penalty, we give the definitions of set-wise penalty distance below.

**Definition 3 (Penalty).** Given a set of targets $T$ and a set of selections $S$, the penalty of $S$ covering a target $t \in T$ is defined as the minimum penalty of using a vertex $v \in S$ to cover $t$, denoted as

$$f(S, t) = \min_{\forall v \in S} f(v, t) = \min_{\forall v \in \text{Suc}(t)} \text{dist}(v, t).$$

(2)

Moreover, the penalty of $S$ covering targets $T$ is defined as the total penalty sum of $S$ covering all targets $t \in T$, denoted by

$$f(S, T) = \sum_{t \in T} f(S, t) = \sum_{t \in T} \min_{\forall v \in \text{Suc}(t)} \text{dist}(v, t).$$

(3)
Obviously, if \( S = T \), the penalty is \( f(S, T) = 0 \). The smaller the penalty, the better the selections \( S \). In Figure 3, assume that \( S = \{v_2, v_3\} \) and \( T = \{v_2, v_5, v_6\} \), thus \( f(v_2, v_3) = 3 \) and \( f(v_3, v_6) = 1 \). The set-wise penalty \( f(S, v_3) = 1, f(S, v_5) = 2 \) and \( f(S, T) = 3 \).

3.4 Problem Formulation

On the basis of the above definitions, we formulate the problem of budget constrained interactive graph search for multiple targets (kBM-IGS).

**Problem 1 (kBM-IGS problem).** Given a hierarchical directed tree \( \mathcal{H} = (V, E) \) rooted at \( r \), a target set \( T \subseteq V \), a budget of \( b \geq 1 \) questions that can be asked, and a positive integer \( k \), the problem is asking \( b \) questions \( Q = \{q_0, q_1, ..., q_b\} \) one by one to determine a non-empty set of selections \( S^* \subseteq Y \) such that \( |S^*| \leq k \) and the distance \( f(S^*, T) \) is the smallest. Equivalently,

\[
S^* = \arg \min_{S \subseteq Y, |S| \leq k} f(S, T)
\]

s.t., \( Y = \bigcup_{q_i \in Q, \text{reach}(q_i) = \text{Yes}} \text{anc}(q_i) \).

Note that the maximum number of selections \( k \) where \( k \geq |S| \), could be either larger or smaller than \( |T| \) as we do not know the number of targets \( T \) in most applications of kBM-IGS. For the example in Figure 1, assume that we get Yes answer for questions “pet”, “fish” and No answer for questions “wild”, “shell”, and “whale”. The best selections \( S^* \) with \( k = 2 \) will be “pet” and “fish”.

3.5 Applications

We motivate the kBM-IGS problem with three applications.

**Image categorization.** New images (e.g., biomedical images, surveillance photos, and user-uploaded images in online social networks) are continuously being generated and need to be classified by humans to identify objects and labels [24, 30]. Our kBM-IGS scheme can leverage the crowd-aided intelligence to identify multiple objects in an image using a budget constrained interactive graph search. First, an image may have multiple labels, e.g., the image shown in Figure 1(b) has two labels “cat” and “fish”. Second, answering a question involves certain communication, latency, and monetary costs. Given a limited budget for rewards, it is necessary to constrain the total number of questions to be asked and select the most suitable labels to categorize the image.

**Manual curation.** Manual curation extends an existing taxonomy (e.g., Yago, Wikipedia, and web of concepts) by adding new entities to the taxonomy [30]. Given a new entity, interactive graph search can be used to find the nodes to parent the new entity via crowdsourcing. Different from the existing study [30], our kBM-IGS problem allows an entity to be a child of multiple parents, which is common in real-world applications, e.g., “whale” is a subclass of both “mammal” and “aquatic animal”. Moreover, on the basis of economic considerations, we can use a limited budget to constrain the total number of questions that can be asked.

**Cold-start recommendation.** Due to the lack of users’ preferences in cold-start recommendations, online platforms (e.g., Twitter, TikTok, and YouTube) can ask a few questions to identify users’ interests and then offer personalized recommendations in a more effective way. Users’ preferences may be diverse, which are usually not limited to a single interest, e.g., one may like “traveling”, “financial news”, “movies”, and so on. To avoid users becoming bored due to being asked too many questions, our kBM-IGS scheme can ask only a small number of questions using an interest hierarchy and adjusts its asking strategy dynamically based on the previous answers.

**Remark.** In practical crowdsourcing applications, while human mistakes are inevitable, they can be minimized or eliminated by adopting effective quality control measures such as expert review, majority voting, group consensus, and so on [9]. As validated in [30], the influence of such mistakes on the outcome of the graph search algorithms is negligible. Thus, as with the previous works [18, 24, 30], we assume in our algorithm design that the workers always give correct answers. For those cases where human mistakes are not eliminated and the workers give wrong answers, we will assess the quality of our methods in Section 7.

4 THE PROPOSED FRAMEWORK

In this section, we analyze the properties of kBM-IGS problem and briefly introduce our algorithmic framework.

4.1 Theoretical Analysis

We first give new definitions of potential targets and then analyze the relationships between questions and potential targets.

**Potential targets.** We first define the potential targets, denoted by \( P \), as a candidate set of vertices that could be exact targets of \( T \) where \( T \subseteq P \). Obviously, if no question has been asked, every vertex \( v \in V \) could be a potential target due to the limited prior information, i.e., \( P = V \). However, as more questions are asked, the potential targets could decrease as some vertices may be pruned from \( P \) for violating the target constraints, no matter whether the question answer is Yes or No. We have the following lemmas.

**Lemma 1.** Given a vertex \( q \in V \), if the question \( \text{reach}(q) = \text{No} \), all vertices \( u \in \text{des}(q) \) are not targets, which should be pruned from potential targets, i.e., \( \text{des}(q) \cap P = \emptyset \).

**Proof.** First, for \( \text{reach}(q) = \text{No} \), \( q \) cannot reach any target \( t \in T \). For each vertex \( u \in \text{des}(q) \), \( u \) also cannot reach any target \( t \in T \), \( u \notin T \). Thus, \( \text{des}(q) \cap T = \emptyset \), and all vertices \( \text{des}(q) \) can be pruned from potential targets, denoted as \( \text{des}(q) \cap P = \emptyset \).

**Lemma 2.** Given a vertex \( q \in V \), if the question \( \text{reach}(q) = \text{Yes} \), all vertices \( u \in \text{anc}(q) \setminus \{q\} \) are not targets, which should be pruned from potential targets, i.e., \( \text{anc}(q) \cap P = \{q\} \).

**Proof.** For \( \text{reach}(q) = \text{Yes} \), \( \exists t \in T \) satisfies \( q \rightarrow t \). On the basis of the independence property of targets, for other target \( t' \in
which achieves a considerable gain by reducing vertices by making a balanced trade-off between the probability of getting a Yes answer and low probabilities to get a No answer. On the contrary, the vertices at the bottom levels of $\mathcal{H}$ (e.g., leaf vertices in Figure 3) have high probabilities of getting a No answer and low probabilities to get a Yes answer. Therefore, for a vertex $v$, we denote the probabilities of reach($v$) = Yes and reach($v$) = No respectively as $p_{\text{yes}}(v)$ and $p_{\text{no}}(v)$, which satisfy $p_{\text{yes}}(v) + p_{\text{no}}(v) = 1$. The specific calculations of $p_{\text{yes}}(v)$ and $p_{\text{no}}(v)$ are based on the descendants des($v$), which will be introduced in Sections 5 and 6.

**Gain score.** First, we define the potential penalty. Instead of using the targets $T$ as in Problem 1, we define the potential penalty to measure the minimum distance between feasible selections $S$ and potential targets $P$ as we do not know the exact $T$, as follows.

**Definition 4 (Potential Penalty).** Given a set of potential targets $P$, a set of Yes-candidates $Y$, and a number $k$, the potential penalty is defined as $g(Y, P, k) = \min_{S \subseteq Y, |S| \leq k} f(S, P)$.

The potential penalty $g(Y, P, k)$ is to select the best $k$ vertices from the Yes-candidates $Y$ in order to identify the potential targets in $P$. The less $|P|$ and the closer $S$ to $P$ is, the lower score $g(Y, P, k)$ is, which is better. On the basis of potential penalties, we present the definition of gain score. For a given vertex $v \in V$ with existing $P$ and $Y$, we ask a new question reach($v$), and present two gain scores for the different answers reach($v$) = Yes and reach($v$) = No respectively, as follows.

**Definition 5 (Yes& No Gains).** The gain of reach($v$) = Yes is denoted as $g_{\text{yes}}(v) = g(Y, P, k) - g(\hat{Y}_v, P_v, k)$ where $\hat{Y}_v, P_v$ are the updated potential targets and Yes-candidates after asking the question reach($v$) = Yes; Similarly, the gain of reach($v$) = No is denoted as $g_{\text{no}}(v) = g(Y, P, k) - g(\hat{Y}_v, P_v, k)$ where $\hat{Y}_v, P_v$ are the updated potential targets and Yes-candidates after asking the question reach($v$) = No.

On the basis of the target probabilities and gain scores, we define an integrated function of expected gain as follows.

**Definition 6 (Expected Gain).** Given a vertex $v \in \mathcal{H}$, potential targets $P$, Yes-candidates $Y$, and a number $k$, the expected gain of asking the question reach($v$) is denoted as

$$G(v) = g_{\text{yes}}(v) \cdot p_{\text{yes}}(v) + g_{\text{no}}(v) \cdot p_{\text{no}}(v).$$

The larger the expected gain of a vertex, the better the choice for questioning.

**Algorithm.** The algorithm of kBM-IGS framework is outlined in Algorithm 1. The general idea is to use a greedy strategy to select the vertex with the largest expected gain at each round of question-asking. The framework has an input of a hierarchy tree $\mathcal{H}$, a budget of $B$ questions that can be asked, and a number $k$. First, it initializes the Yes-candidates $Y$ as $\{r\}$ and the potential targets $P$ as the whole vertex set $V$ (line 1). Note that if the vertices have no probabilities, we can set all vertices to have the same probability as $\frac{1}{|P|}$ (line 2). The algorithm then iteratively selects one best vertex $q_i \in P \setminus Y$ and asks the question reach($q_i$) until the quota of $B$ questions is used up (lines 3-12). For each round, it calculates the target probabilities of $p_{\text{yes}}(v), p_{\text{no}}(v)$ and the Yes & No gains of $g_{\text{yes}}(v), g_{\text{no}}(v)$ for each vertex $v \in P \setminus Y$. Then, the expected gains of all vertices are
Algorithm 1: kBM-IGS Framework

Input: A hierarchy tree \( \mathcal{H} = (V, E) \), a budget \( b \), a number \( k \).
Output: Selections \( S \) with \( |S| \leq k \).

1. Let \( \mathcal{Y} \leftarrow \{r\} \), \( \mathcal{P} \leftarrow V \);
2. Initialize the probability \( \mathcal{P}(v) \) for every vertex \( v \in V \);
3. For \( i \leftarrow 1 \) to \( b \) do
   4. \( \quad \) for \( v \in \mathcal{P} \setminus \mathcal{Y} \) do
      5. \( \quad \quad \) \( \mathcal{P}(v) \leftarrow V \);
      6. \( \quad \quad \) \( \mathcal{P}(v) \leftarrow \mathcal{V} \cup \mathcal{E} \);
      7. \( \quad \quad \) \( \mathcal{P}(v) \leftarrow \mathcal{P}(v) \cup \mathcal{E} \);
      8. \( \quad \quad \) \( \mathcal{P}(v) \leftarrow \mathcal{P}(v) \);  
   9. End for
10. End for
11. Update the potential candidates \( \mathcal{P} \) and vertex probabilities \( \mathcal{P} \) accordingly if needed;
12. If \( \mathcal{P} \subseteq \mathcal{Y} \) then break by Theorem 1;
13. \( S' \leftarrow \arg \min_{S \subseteq \mathcal{Y}, |S| \leq k} f(S, \mathcal{P}) \);
14. return \( S' \).

Figure 5: An example of yes candidates and potential targets for single target.

5 SINGLE TARGET SEARCH

In this section, we investigate one special case of kBM-IGS problem, i.e., the SingleTarget problem, where \( |\mathcal{T}| = 1 \). [18, 30]. On the basis of the kBM-IGS framework, we develop a STBIS method to identify one vertex as \( S \) by asking \( b \) questions.

5.1 Single Target Problem Analysis

As an instance problem, the SingleTarget problem inherits all properties of kBM-IGS described in Section 4 and enjoys its own properties. Assume that the initial budget is \( \mathcal{P} = V \) and \( \mathcal{Y} = \{r\} \). We consider a question and interactively update \( \mathcal{P} \) and \( \mathcal{Y} \) as \( \mathcal{P}_\text{new} \) and \( \mathcal{Y}_\text{new} \) by obeying the two following rules.

First, as \( |\mathcal{T}| = 1 \). For each question, the best strategy is to ask a vertex \( v \) that is a potential target \( v \in \mathcal{P} \setminus \mathcal{Y} \). Otherwise, we consider two cases. First, if we ask a vertex \( v \in \mathcal{Y} \), the answer of reach(v) is always Yes; Second, if we ask a vertex \( v \in \mathcal{V} \setminus \mathcal{P} \cup \mathcal{Y} \), the answer of reach(v) is always No. Thus, it achieves no benefit gain but wastes one question from the budget. Moreover, in contrast to Lemma 2, in SingleTarget problem, more vertices can be pruned after Yes answer as follows.

Lemma 2. For a vertex \( q \in \mathcal{P} \setminus \mathcal{Y} \), if the question reach(q) = Yes, none of the vertices \( u \in \mathcal{P} \setminus \text{des}(q) \) are potential targets and we update \( \mathcal{P}_\text{new} = \text{des}(q) \cap \mathcal{P} \).

Proof. Because reach(q) = Yes, \( t \in S \) such that \( t \in \text{des}(q) \). Moreover, \( |\mathcal{T}| = 1 \), \( \mathcal{T} = \{t\} \subseteq \text{des}(q) \). Thus, \( \mathcal{T} \cap \mathcal{V} \setminus \text{des}(q) = 0 \) and all vertices \( u \in \mathcal{V} \setminus \text{des}(q) \) should be pruned from \( \mathcal{P} \).

Second, the Yes-candidates can be updated only when a question reach(q) = Yes by Def. 1, where the updated Yes-candidates \( \mathcal{Y}_{\text{new}} = \mathcal{Y} \cup \text{anc}(q) \). However, \( \mathcal{Y} \subseteq \mathcal{Y}_{\text{new}} \subseteq \text{anc}(t) \) always holds, i.e., all Yes-candidates lie along the path from root \( r \) to target \( t \). The penalty function \( f(S, \mathcal{T}) \) in Def 3 tells us that keeping one vertex \( s \in \mathcal{Y}_{\text{new}} \), closest to \( t \) is enough. In other words, it achieves the minimum penalty \( f(S, \mathcal{T}) = f(s) \). Thus, \( \mathcal{Y}_{\text{new}} \) can be updated as \( \mathcal{Y}_{\text{new}} = \{s\} \), where \( s \) has the largest depth dist(r, s) in \( \mathcal{H} \) and the question reach(s) = Yes.

Lemma 3. For \( \mathcal{Y} = \{s\} \), both \( \mathcal{Y} \cap \mathcal{P} = \{s\} \) and the penalty \( g(\mathcal{Y}, \mathcal{P}, 1) = f(\{s\}, \mathcal{P}) \) hold.

Proof. First, we prove \( s \in \mathcal{P} \). Since \( s \in \mathcal{Y} \), \( s \) will not be pruned from No questions according to Lemma 1. Furthermore, as \( s \) is the deepest vertex in \( \mathcal{Y} \), \( s \) will not be pruned from Yes questions according to Lemma 5. So, \( s \in \mathcal{P} \) and \( \mathcal{Y} \cap \mathcal{P} = \{s\} \). Moreover, since \( |\mathcal{Y}| = |\{s\}| = 1 \), \( g(\mathcal{Y}, \mathcal{P}, 1) = f(\{s\}, \mathcal{P}) \).

On the basis of the above properties, we have two useful updating rules.

Rule 1. For a vertex \( v \in \mathcal{P} \) with reach(v) = Yes, we update the potential targets \( \mathcal{P}_{\text{new}} = \text{des}(v) \cap \mathcal{P} \) and the Yes-candidates \( \mathcal{Y}_{\text{new}} = \{v\} \).

Rule 2. For a vertex \( v \in \mathcal{P} \) with reach(v) = No, we update the potential targets \( \mathcal{P}_{\text{new}} = \mathcal{P} \setminus \text{des}(v) \) and keep the Yes-candidates unchanged \( \mathcal{Y}_{\text{new}} = \mathcal{Y} \).

5.2 The STBIS Algorithm

In this section, we propose a greedy algorithm STBIS to find a single target. We begin with probability calculation.

Probability calculation. Before asking any questions, each vertex has an equal probability of being the target. Thus, we let each vertex \( u \) have a probability of \( \mathcal{P}(u) = \frac{1}{n} \) where \( n = |\mathcal{V}| \). Our proposed algorithm can be easily extended to other vertex probability distribution based on historical query logs as [18]. Therefore, for a vertex \( v \), the target probability for each vertex \( v \) follows \( \mathcal{P}(v) = \sum_{u \in \text{des}(v)} \mathcal{P}(u) \) and the no probability follows \( \mathcal{P}(v) = 1 - \mathcal{P}(v) \). As more question answers are discovered, the vertex probabilities need to be updated accordingly. The updated probability after each question is calculated as:

\[
\mathcal{P}(u) = \begin{cases} 
\frac{\mathcal{P}(u) \cdot |\mathcal{P}|}{\mathcal{P}_{\text{new}}}, & u \in \mathcal{P}_{\text{new}} \\
0, & u \notin \mathcal{P}_{\text{new}} 
\end{cases}
\]

where \( \mathcal{P}_{\text{new}} \) is the potential targets after asking a question reach(q) where \( 1 \leq i \leq b \). The general idea is to assign impossible targets
Algorithm 2 STBIS

Input: A hierarchy tree $H = (V, E)$, root $r$, budget $b$, and $k = 1$.
Output: One selection $s$.
1: Let $Y \rightarrow \{r\}$, $P \leftarrow V$;
2: Assign the probability $pr(v) = 1/n$ for $v \in V$;
3: for $i \leftarrow 1$ to $b$ do
4: for $v \in P \setminus Y$ do
5: $P_{\text{new}}(v) \leftarrow \text{dfsdes}(u) \setminus P_{\text{new}}(u)$ and $P_{\text{new}}(v) \leftarrow 1 - P_{\text{new}}(v)$;
6: Calculate $P_{\text{new}}$ as $P_{\text{new}} \leftarrow P \cap \text{des}(u)$ by Rule 1;
7: Update $g_{\text{yes}}(v) \leftarrow f(Y, P) - f(Y, P)$;
8: Calculate $g_{\text{new}}$ as $P_{\text{new}} \leftarrow P \setminus \text{des}(u)$ by Rule 2;
9: Update $g_{\text{no}}(v) \leftarrow f(Y, P) - f(Y, P)$;
10: $\text{Gain}(v) = g_{\text{yes}}(v) \cdot pr_{\text{yes}}(v) + g_{\text{no}}(v) \cdot pr_{\text{no}}(v)$;
11: $q_i \leftarrow \arg\max_{v \in P \setminus Y} \text{Gain}(v)$;
12: Ask the question reach($q_i$);
13: if reach($q_i$) = Yes then
14: $P_{\text{new}} \leftarrow P \cap \text{des}(q_i)$; $Y \leftarrow \{q_i\}$;
else
16: $P_{\text{new}} \leftarrow P \setminus \text{des}(q_i)$; $Y$ keeps unchanged;
17: Update $pr(u) = 0$ for $u \in P \setminus P_{\text{new}}$;
18: Update $pr(u) = pr(u) - ||P_{\text{new}}||$ for $u \in P_{\text{new}}$;
19: $P \leftarrow P_{\text{new}}$;
20: if $P = Y$ then return $s \in Y$;
21: return $s \in Y$.

Table 3: The value of $g_{\text{yes}}, g_{\text{no}}, P_{\text{yes}}$, and $P_{\text{no}}$ of first question and the Gain of two questions. Here, $b = 2$ and $T = \{v_5\}$.

| Node | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | $v_6$ | $v_7$ | $v_8$ | $v_9$ |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $g_{\text{yes}}$ | 9.20 | 19.11 | 8.09 | 2.23 | 13.42 | 7.30 | 4.76 | 4.76 | 4.76 |
| $g_{\text{no}}$ | 19.12 | 19.12 | 19.12 | 19.12 | 19.12 | 19.12 | 19.12 | 19.12 | 19.12 |
| $P_{\text{yes}}$ | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 |
| $P_{\text{no}}$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $\text{Gain}^1$ | 6.23 | 3.17 | 6.23 | / | / | / | / | / | / |
| $\text{Gain}^2$ | 6.23 | 3.17 | 6.23 | / | / | / | / | / | / |

Algorithm 3 DFS-Gain($u$)

Input: A subtree $T_u$, the root of subtree $u$.
Output: Gain($u$).
1: $pr_{\text{yes}}(u) = pr(u)$, $P_{\text{yes}}(u) = 1$, $f(u), P_{\text{yes}}(u) = 0$;
2: for vertex $v \in \text{child}(u)$ do
3: if $v \notin P$ then continue;
4: $\text{DFS-Gain}(v)$;
5: $pr_{\text{yes}}(u) = pr_{\text{yes}}(u) + pr_{\text{yes}}(v)$;
6: $P_{\text{yes}}(u) = |P_{\text{yes}}(u)| + |P_{\text{yes}}(v)|$;
7: $f(u, P_{\text{yes}}(u)) = f(u, P_{\text{yes}}(u)) + f(u, P_{\text{yes}}(v))$;
8: $pr_{\text{no}}(u) = 1 - pr_{\text{yes}}(u)$;
9: Calculate $g_{\text{yes}}(v), g_{\text{no}}(v), f(u, P_{\text{yes}}(u))$ by Eq. 5, 6, 7;
10: Gain($u$) = $g_{\text{yes}}(u) \cdot pr_{\text{yes}}(u) + g_{\text{no}}(u) \cdot pr_{\text{no}}(u)$;
11: return Gain($u$);

Complexity analysis and optimizations. A straightforward implementation of STBIS in Algorithm 2 takes $O(n)$ time to compute $g_{\text{yes}}(v), g_{\text{no}}(v), P_{\text{yes}}(v)$ and $P_{\text{no}}(v)$ for each vertex $v \in V$. Thus, the calculation of Gain($v$) for all vertices $v \in P$ takes $O(n^2)$ time. However, these calculations have several repeated process, which can be avoided.

We consider the following deviations of Yes&No gains.

$$f((v), \hat{P}_v) = \sum_{u \in \text{child}(v)} f((u), \hat{P}_u) + |\hat{P}_v| - 1 \quad (5)$$

$$g_{\text{yes}}(v) = f((s), \hat{P}_v) - f((v), \hat{P}_v) \quad (6)$$

$$g_{\text{no}}(v) = f((v), \hat{P}_v) + \text{dist}(s, v) \cdot |\hat{P}_v| \quad (7)$$

We propose a fast approach in Algorithm 3 for computing expected gain Gain($u$) for all vertices $u$ based on Eqs. 5, 6 and 7. The algorithm can reduce the calculations to $O(n)$ time. Algorithm 3 shows the details of DFS algorithm. First, the algorithm initializes the yes probability $P_{\text{yes}}$, the size of potential targets $|P_{\text{yes}}|$ and $f((u), \hat{P}_u)$ (line 1). Next, it traverses the children of $u$ and updates $P_{\text{yes}}$, $P_{\text{no}}$ and $f((u), \hat{P}_u)$ (lines 2-7). Finally, the algorithm calculates $\text{Gain}(u)$ and returns them (lines 9-11). We can replace the procedure of lines 5-10 in Algorithm 2 by invoking DFS-Gain($u$) in Algorithm 3.

In summary, STBIS in Algorithm 2 takes $O(n)$ time to generate a question for users to answer. The overall time complexity of STBIS takes $O(bn)$ time in $O(n)$ space for generating $b$ questions.

6 MULTIPLE TARGETS SEARCH

In this section, we propose efficient algorithms for identifying $k$ selections for multiple targets. We first introduce the probability setting and updating rules for the identification of multiple targets. Then, we propose a kBM-DP method using dynamic programming techniques and improve its efficiency by leveraging the techniques of non-diverse selections and bounded pruning.

6.1 Multiple Targets Scheme

We start by presenting probability setting and updating rules for the Multiple Targets scheme.

Target probability. As there exist multiple targets with $|T| \geq 1$, we assume that each vertex has an independent probability of being a target. As our problem aims at finding $k$ selections and $|T|$ is
unknown, let \( pr(v) \) be the vertex probability of \( v \) and \( pr(v) = \frac{k}{n} \).
Thus, the target probability of a vertex \( v \) is computed as follows.
The probability of \( reach(v) = No \) is denoted as the No probability 
\( P_{No}(v) = \prod_{u \in \text{des}(v) \cap P} (1 - pr(u)) \), representing that none of vertices \( u \in \text{des}(v) \cap P \) is a target. Moreover, we update the target probabilities with more questions asked.

**Rules of updating \( P \) and \( Y \).** Following Def. 2 and Lemmas 1 and 2, we have the following rules for updating the potential targets \( P \) and Yes-candidates \( Y \).

**Rule 3.** For a vertex \( v \in P \) with \( \text{reach}(v) = Yes \), we update the potential targets \( P_{\text{new}} = P \setminus (\text{anc}(v) \setminus \{v\}) \) and update the Yes-candidates \( Y_{\text{new}} = Y \cup \text{anc}(v) \).

**Rule 4.** For a vertex \( v \in P \) with \( \text{reach}(v) = No \), we update the potential targets \( P_{\text{new}} = P \setminus \text{des}(v) \) and keep the Yes-candidates unchanged \( Y_{\text{new}} = Y \).

On the basis of Rules 3 and 4, we compute both \( g_{\text{Yes}}(v) \) and \( g_{\text{No}}(v) \), which equals \( g(Y, P, k) = g(Y_{\text{new}}, P_{\text{new}}, k) \) by Def. 5. However, as we know that \( g(Y, P, k) = \min_{S \subseteq Y, |S| \leq k} f(S, P) \), it is difficult to efficiently compute the penalty \( g(Y, P, k) \) with a straightforward enumeration of \( S \subseteq Y \) for \( k > 1 \). In the following sections, we mainly focus on developing efficient approaches to compute \( g(Y, P, k) \) and update Yes\&No gain scores \( g_{\text{Yes}}(v) \) and \( g_{\text{No}}(v) \).

### 6.2 kBM-DP Algorithm

In this section, we propose a kBM-DP algorithm for identifying multiple targets based on the kBM-IGS framework in Algorithm 1.

**The kBM-DP algorithm.** The algorithm of kBM-DP is presented in Algorithm 4, which implements the details of Algorithm 1. The algorithm first initializes the Yes-candidates \( Y \), the potential targets \( P \), and the independent probability \( pr(v) = \frac{k}{n} \) for each vertex \( v \in V \) (lines 1-2). Then, it iteratively selects one best vertex \( v \in P \setminus Y \) with

**Algorithm 5 kBM-DP: Calculated Expected Gains**

**Input:** \( H = (V, E), pr_{\text{Yes}}(.), pr_{\text{No}}(.), Y, P, k \).

**Output:** \( G(v) \) for all vertices in \( P \) and \( w \in \text{anc}(u) \) in Eq. 8.

1. Calculate \( DP(u, w, k) \) for all vertices in \( P \) and \( w \in \text{anc}(u) \) in Eq. 8.
2. for \( v \in P \setminus Y \)
3. \( \hat{P} \leftarrow P \setminus \{v\} \); // \( \text{reach}(v) = Yes \)
4. \( g_{\text{Yes}}(v) = g(Y, P, k) - \text{calg}_{\text{Yes}}(v) \);
5. \( P \leftarrow P \setminus \text{des}(v) \); // \( \text{reach}(v) = No \)
6. \( g_{\text{No}}(v) = g(Y, P, k) - \text{calg}_{\text{No}}(v) \);
7. \( G(v) = g_{\text{Yes}}(v) \cdot pr_{\text{Yes}}(v) + g_{\text{No}}(v) \cdot pr_{\text{No}}(v) \) by Def. 6;
8. procedure calg_{\text{Yes}}(v, k)
9. for \( v \in \text{anc}(u) \)
10. Recalculate \( DP_{Y}(u, k) \) by Eq. 9;
11. for \( w \in \text{anc}(v) \setminus \{v\} \)
12. Recalculate \( DP_{N}(u, w, k) \) by Eq. 10;
13. \( DP(u, w, k) \leftarrow \min \{DP_{Y}(u, w, k), DP_{Y}(u, k)\} \);
14. \( g(Y, \hat{P}, k) \leftarrow DP(r, r, k) \);
15. return \( g(Y, \hat{P}, k) \);
16. procedure calg_{\text{No}}(u, k)
17. for \( v \in \text{anc}(u) \)
18. for \( w \in \text{anc}(v) \cap Y \setminus \{v\} \)
19. Recalculate \( DP_{Y}(u, w, k) \) by Eq. 10;
20. if \( v \in Y \)
21. Recalculate \( DP_{Y}(u, k) \) by Eq. 9;
22. \( DP(u, w, k) \leftarrow \min \{DP_{Y}(u, w, k), DP_{Y}(u, k)\} \);
23. else
24. \( DP(u, w, k) \leftarrow DP_{N}(u, w, k) \);
25. \( g(Y, \hat{P}, k) \leftarrow DP(r, r, k) \);
26. return \( g(Y, \hat{P}, k) \);

**Figure 6:** A solution overview of dynamic programming.

The largest \( G(v) \) and asks question \( \text{reach}(v) \) until all \( b \) questions have been asked (lines 3-18). At the \( i \)-th round of asking question, it updates the target probabilities for all vertices \( P \setminus Y \) (lines 4-6). The algorithm invokes Algorithm 5 to calculate all expected gains (lines 7). Next, the algorithm asks question \( \text{reach}(q_{i}) \), and updates the vertex probabilities, \( P \) and \( Y \) accordingly by Rules 3 and 4 (lines 10-16). Finally, the algorithm returns the selections \( S^{*} = \arg \min_{S \subseteq Y, |S| \leq k} f(S, P) \) (line 19).

In the following, we introduce a dynamic programming algorithm for calculating the expected gains in Algorithm 5. We first consider the computation of \( g(Y, P, k) \) and then Yes\&No gain scores.

**Computing \( g(Y, P, k) \).** An intuitive approach enumerates all \( k \)-sized selections \( S \subseteq Y \) for finding the best selections \( S^{*} \), which is inefficient. Thus, we use a dynamic programming technique to calculate the minimum \( g(Y, P, k) \) efficiently. The general idea is to divide the global calculation into sub-problems of finding \( k' < k \) selection vertices optimally in a subtree \( T_{u} \) rooted by a vertex \( u \). The vertex \( w \in S \cap \text{anc}(u) \) is a selection closest to \( u \). Note that if \( S \cap \text{anc}(u) = \emptyset \), we consider \( w = r \). Obviously, let \( u = r, w = r \) and \( k' = k \), this subproblem is the same as the best selection of \( S \). Thus,
our objective is to calculate it from the sub-problems. We consider two cases of whether we select vertex u or not for each subtree Tu.

On one hand, if u ∈ Y and we select vertex u into the selections S, for each child node v1, v2,..., vx ∈ child(u) ∩ P, the sub-problem is how to find additional kx optimal vertices in the subtrees rooted by vx with the closest selected vertex u and ∑kx ≤ k′ − 1; On the other hand, if we do not select vertex u into the answer S, for each child node v1, v2,..., vx ∈ child(u) ∩ P, the sub-problem is how to find additional kx optimal vertices in Tu with the closest selected vertex w and ∑kx ≤ k′. The optimal answer is the best solution among the above two answers.

State and transfer equation. We first define the state of DP(Y, u, k) and DPN(u, w, k). In the subtree Tu, DP(Y, u, k) is the minimum value of f(S ∪ {u}, P ∩ des(u)) with selected k − 1-size set S ⊆ des(u). Similarly, DPN(u, w, k) is the minimum value of f(S ∪ {w}, P ∩ des(u)) with selected k-size set S ⊆ des(u) \ {u} and w ∈ anc(u) ∩ Y is the closest selected vertex to u. On the basis of DP(Y, u, k) and DPN(u, w, k), we define the state DP(u, w, k) as the optimal k-size selection in the subtree Tu with closest selected vertex w ∈ anc(u) ∩ P, which satisfies the equation as follows.

\[
DP(u, w, k) = \min \{DP(Y, u, k), DPN(u, w, k): u ∈ Y, k ≥ 1 \}
\]

Next, we propose the transfer equation of DP(Y, u, k) and DPN(u, w, k) as follows,

\[
DP_Y(u, k) = \min \left\{ \sum_{x ∈ \text{child}(u) ∩ P} \text{DP}(x, u, k_x) \right\}
\]

subject to \(k_x = k - 1\) \(\text{DP}_N(u, w, k) = \text{dist}(w, u) + \min \left\{ \sum_{x ∈ \text{child}(u) ∩ P} \text{DP}(x, w, k_x) \right\}
\]

subject to \(k_x = k\).

Furthermore, we can use the Knapsack dynamic programming technique [27] to tackle the transfer Eqs. 9 and 10. Assume that a number k represents the total capacity. Given a set of vertices child(u) ∩ P = \{x1, ..., xL\}, for each vertex x_i where 1 ≤ i ≤ L, \text{DP}(x_i, w, k_x) represents an item value and the item volume is k_x ≤ k. We assume that \(F(i, k')\) is the state that has the minimum value of the first i items with a total of k’ capacity. The equation of state transformation is shown as follows.

\[
F(i, k') = \min_{0 ≤ j ≤ k'} \{F(i - 1, k' - j) + \text{DP}(x_i, w, j)\}.
\]

For initialization, we set \(F(i, j) = +∞\) for \(1 ≤ i ≤ L, 0 ≤ j ≤ k\) and \(F(0, 0) = 0\). The return value is \(F(L, k) = \min(\sum_{x ∈ \text{child}(u) ∩ P} \text{DP}(x, w, k_x))\) with the constraint \(\sum_{x ∈ \text{child}(u) ∩ P} k_x = k\).

**Update gYes(u) and gNo(u).** For a vertex u that is questioned, only the state of arc(u) needs to be recalculated to update gYes(u) and gNo(u). Algorithm 5 presents the details of gYes(u) and gNo(u) calculation. First, the algorithm calculates DP(u, w, k) for all possible states (line 1). Then, for each vertex u ∈ P \ Y, it updates the corresponding states and calculates gYes(u) and gNo(u) (lines 2-7). For computing gYes (u), it needs to update the states DP(u, w, k) for all v ∈ anc(u), w ∈ anc(v) (lines 8-15). To computing gNo(u), it updates the states DP(u, w, k) for all v ∈ anc(u), w ∈ anc(v) ∩ Y (lines 16-26).

**Example 4.** Assume the targets are \(T = \{v_5, v_8\}\), the budget \(b = 2\), and \(k = 2\) in Figure 3. Table 4 shows the gains of each vertex. In the first question, the algorithm questions v3 and gets the Yes answer. Then, the vertices v0, v1 will be pruned from the potential targets. The vertex v5 gets the maximum gains in the second round and obtains the Yes answer. Note that some leaf vertices get gYes(v) = 0 because their parents are a better selection even if they get the Yes answer. The selections will be \(S = \{v_3, v_5\}\) and the penalty will be f(S, T) = 1.

**Complexity analysis.** The calculation of all states takes \(O(nhk^2)\) time. The update of each vertex takes \(O(hk^2)\) times using Algorithm 5. Overall, the kBM-DP in Algorithm 4 takes \(O(bnh^2k^2)\) time in \(O(nhk)\) space for generating b questions.

### 6.3 Fast algorithms: kBM-Topk and kBM-DP+

In this section, we propose two fast algorithms of kBM-Topk and kBM-DP+. The first method kBM-Topk uses an alternative penalty function to improve the calculation of expected gain. The second method kBM-DP+ develops an upper bound of Gain(\(x\)) to prune unnecessary vertices for updating the expected gains, which achieves the same results as kBM-DP in most cases.

#### 6.3.1 kBM-Topk

The penalty of \(g(Y, P, k)\) is complex to compute, due to the dependence relationship of selections in S. To deal with this issue, we propose an independent function to approximate \(g(Y, P, k)\), which can be efficiently computed. We begin with a new definition of selected gain as follows.

\[
\text{IG}(x) = \sum_{e ∈ P \cap \text{des}(x)} \text{dist}(r, x).
\]

The selected gain represents the reduced penalty after selecting x. The deeper the selected vertex and the larger the size of descendants, the higher the selected gain. Thus, the general idea of the approximate method is to select the top-k vertices that maximize the selected gain. On this basis, we propose the new potential penalty function as follows,

\[
g'(Y, P, k) = f(\{r\}, P) - \max_{S ⊆ Y, |S| ≤ k} \sum_{x ∈ S} \text{IG}(x).
\]

**kBM-Topk algorithm.** Similar to kBM-DP, kBM-Topk uses the same framework and the difference is the calculation of expected gains. Algorithm 6 shows the details of the calculation (replacing the line 7 of Algorithm 4). First, the algorithm calculates IG(\(x\)) for all the vertices \(v ∈ P \cup Y\) and uses max-heap H to maintain the top-k vertices in Yes-candidates Y (lines 1-2). Then, for each vertex u ∈ P \ Y, it updates the max-heap H and calculates gYes(u)
Algorithm 6 kBM-Topk: Calculate Expected Gains

**Input:** \( \mathcal{H} = (V, E), p_{\text{res}}(\cdot), p_{\text{no}}(\cdot), P, \mathcal{Y}, k \).

**Output:** \( \text{Gain}(u) \) for all vertices \( u \in P \setminus \mathcal{Y} \).

1. Calculate \( \text{IG}(u) \) for all vertices \( u \in P \setminus \mathcal{Y} \).
2. Let \( H \) be a max-heap to maintain \( \text{IG}(u) \) for all vertices \( u \in \mathcal{Y} \).
3. for \( v \in P \setminus \mathcal{Y} \) do
   4. \( \hat{P} \leftarrow P \setminus (\text{anc}(v) \setminus \{v\}) \); // reach(\( v \)) = Yes
   5. \( g_{\text{Yes}}(v) = g(Y, P, k) - \text{calg}(v, k, H) \)
   6. \( \hat{P} \leftarrow P \setminus \text{des}(v); // reach(\( v \)) = No
   7. \( g_{\text{No}}(v) = g(Y, P, k) - \text{calg}(v, k, H) \)
   8. \( \text{Gain}(v) \leftarrow g_{\text{Yes}}(v) \cdot p_{\text{Yes}}(v) + g_{\text{No}}(v) \cdot p_{\text{No}}(v) \) by Def. 6;
   9. procedure calg(\( v, k, H \));
10. for \( v \in \text{anc}(u) \) do
11. \( \text{IG}_{\text{new}}(v) = \text{IG}(v) - |P \setminus \hat{P} \cap \text{des}(v)| \cdot \text{dist}(r, v); \)
12. if \( v \in \mathcal{Y} \) then
13. \( H(a, \text{IG}(u)) \leftarrow (a, \text{IG}_{\text{new}}(v)); \)
14. else
15. \( H, \text{push}(a, \text{IG}_{\text{new}}(v)); \)
16. \( f((r), \hat{P}) = f((r), P) - \sum_{a \in \hat{P}} \text{dist}(r, v); \)
17. return \( f((r), \hat{P}) - \max_{S \subseteq H|S| \leq k} \sum_{x \in S} \text{IG}_{\text{new}}(x); \)
18. procedure calg(\( v, k, H \));
19. for \( v \in \text{anc}(u) \) \( Y \) do
20. \( \text{IG}_{\text{new}}(v) = \text{IG}(v) - |P \setminus \hat{P}| \cdot \text{dist}(r, v); \)
21. \( H(a, \text{IG}(u), \hat{P}) \leftarrow (a, \text{IG}_{\text{new}}(v), \hat{P}); \)
22. \( f((r), \hat{P}) = f((r), P) - \sum_{a \in \hat{P}} \text{dist}(r, v); \)
23. return \( f((r), P) - \max_{S \subseteq H|S| \leq k} \sum_{x \in S} \text{IG}_{\text{new}}(x); \)

and \( g_{\text{No}}(u) \) (lines 3-8). For the \( g_{\text{Yes}}(u) \) calculation, \( \text{IG}(v) \) of vertex \( v \in \text{anc}(u) \) should be updated (lines 10-11). If the vertex \( v \) has been added into the heap \( H \), \( \text{IG}(v) \) will be replaced by \( \text{IG}_{\text{new}}(v) \) (lines 12-13). Otherwise, it will be added into the heap \( H \) because \( v \in \mathcal{Y}_{\text{new}} \) (lines 14-15). Finally, the algorithm updates the \( f((r), \hat{P}) \) and calculates \( \text{IG}_{\text{Yes}}(u) \) (lines 16-17). The calculation of \( g_{\text{No}}(u) \) is similar to that of \( g_{\text{Yes}}(u) \) (lines 18-23). The main difference is that the Yes-candidates \( \mathcal{Y} \) will not be changed and only the vertices \( v \in \text{anc}(u) \) \( Y \) should be updated (lines 19-21).

**Quality and complexity analysis.** We analyze the quality of our approximate penalty function \( g'(Y, P, k) \) as follows.

**Theorem 2.** It holds that \( LB_k \leq g'(Y, P, k) \leq UB_k \), where \( LB_k = g(Y, P, k) - (k-1) \cdot (f((r), P) - g(Y, P, k)) \) and \( UB_k = g(Y, P, k) \). If \( k = 1, g'(Y, P, 1) = g(Y, P, 1) \).

**Proof.** First, we prove \( g'(Y, P, k) \geq LB_k \). Assume that \( S' \) is the best selection in terms of potential penalty, i.e., \( g(Y, P, k) \leq f(S', P) \). Then \( g(Y, P, k) \leq f((r), P) - \sum_{x \in S'} \text{IG}(x) \geq g'(Y, P, k) \). Thus, \( g'(Y, P, k) \geq f((r), P) - k \cdot \max_{x \subseteq Y} \text{IG}(x) = f((r), P) - \text{calg}(v, k, \hat{P}) \). Second, we prove \( g'(Y, P, k) \geq LB_k \). We have \( g'(Y, P, k) \geq f((r), P) - k \cdot \max_{S \subseteq H|S| \leq k} \sum_{x \in S} \text{IG}_{\text{new}}(x) \leq LB_k \). Overall, \( LB_k \leq g'(Y, P, k) \leq UB_k \). For \( k = 1 \), we have \( UB_k = g(Y, P, k) = LB_k \). Thus, \( g'(Y, P, 1) = g(Y, P, 1) \) holds.

Next, we analyze the complexity of kBm-Topk. For each vertex \( u \in P \setminus \mathcal{Y} \), at most \( |\text{anc}(u)| \) vertices are calculated and each update in heap \( H \) takes \( O(\log |Y|) \) time. Thus, the score calculation of each vertex \( u \) takes \( O(h \log n) \). Overall, kBm-Topk takes \( O(bh \log n) \) time using \( O(n) \) space for generating \( b \) questions.

6.3.2 kBm-DP+. In this section, we propose a pruning optimization to accelerate the algorithm kBm-DP. The general idea is to design an upper bound of expected gain and skip the update of \( \text{Gain}(v) \) for those vertices that are disqualified for achieving the largest \( \text{Gain}(v) \) in \( P \setminus \mathcal{Y} \). In this way, we can prune lots of vertices in most cases at each round of question asking, and quickly identify a vertex \( q_i \) with the largest gain.

**An upper bound of \( \text{Gain}(v) \).** Consider a vertex \( v \in V \) at the \( i \)-th round of question asking where \( i \geq 1 \), the expected gain is denoted as \( \text{Gain}'(v) \). Then, we have an upper bound of \( \text{Gain}'(v) \), denoted as \( \text{Gain}^*(v) \), satisfying

\[
\text{Gain}^*(v) = UB_{\text{Yes}}^*(v) \cdot p_{\text{Yes}}(v) + UB_{\text{No}}^*(v) \cdot p_{\text{No}}(v),
\]

where \( UB_{\text{Yes}}^*(v) = g_{\text{Yes}}(i-1)(v) \) and \( UB_{\text{No}}^*(v) = g_{\text{No}}(i-1)(v) \). Note that if \( v \) is pruned in the previous round, we will set \( UB_{\text{Yes}}^*(v) = UB_{\text{Yes}}^i(v) \) and \( UB_{\text{No}}^*(v) = UB_{\text{No}}^i(v) \). We observe that both the Yes gain and No gain decrease with more questions asked in most cases. This is because that the potential targets decrease and the Yes-candidates increase with questions. Thus, we have \( UB_{\text{Yes}}^i(v) \geq g_{\text{Yes}}(v) \) and \( UB_{\text{No}}^i(v) \geq g_{\text{No}}(v) \). As a result, \( \text{Gain}^*(v) \geq \text{Gain}^i(v) \).

**Algorithm.** kBm-DP+ is a variant approach of kBm-DP in Algorithm 4 using the pruning optimization in Algorithm 7, which calculates expected gains and identifies the vertex \( q_i \) for question asking (replacing lines 7-8 of Algorithm 4). Specifically, the algorithm first computes all upper bounds for vertices \( v \in P \setminus \mathcal{Y} \) and then sorts the vertices in descending order of upper bounds (lines 2-3). Next, it calculates the expected gain \( \text{Gain}(v) \) and prunes disqualified vertices with an upper bound \( \text{Gain}^*(v) \leq \text{Gain}_{\text{max}} \) where \( \text{Gain}_{\text{max}} \) keeps updated with the largest value of all possible expected gains (lines 4-10). Finally, it returns a vertex \( q_i \) with the largest expected gain.

Note that we pre-compute the \( g_{\text{Yes}} \) and \( g_{\text{No}} \) of all vertices for the first question in \( H \).
Table 5: The averaged penalty of HGS, IGS, BinG and STBIS in SingleTarget problem on all datasets varied by budget $b$.

| Budget $b$ | Image-COCO | ImageNet | Yago3-I | Yago3-II |
|------------|-------------|----------|---------|----------|
|            | HGS | IGS | BinG | STBIS | HGS | IGS | BinG | STBIS | HGS | IGS | BinG | STBIS |
| 5          | 1.48 | 1.15 | 0.79 | 0.35 | 4.50 | 3.50 | 3.00 | 1.05 | 2.04 | 2.60 | 2.30 | 1.27 |
| 10         | 1.20 | 0.77 | 0.47 | 0.27 | 4.34 | 3.02 | 1.80 | 0.67 | 1.92 | 1.63 | 1.15 | 1.05 |
| 20         | 0.98 | 0.41 | 0.18 | 0.00 | 4.21 | 1.41 | 0.67 | 0.27 | 1.71 | 0.82 | 0.41 | 0.39 |
| 50         | 0.52 | 0.19 | 0.00 | 0.00 | 3.79 | 0.68 | 0.28 | 0.28 | 1.66 | 0.71 | 0.30 | 0.27 |
| 100        | N/A  | N/A  | N/A  | N/A  | N/A  | N/A  | N/A  | N/A  | 2.07 | 0.90 | 0.56 | 0.54 |

Table 6: The statistics of hierarchical tree datasets.

| Name       | $|V|$ | Depth | Avg Depth | Max Degree | # Queries |
|------------|-----|-------|-----------|------------|-----------|
| Image-COCO | 200 | 5     | 2.63      | 37         | 107,774   |
| ImageNet   | 74,401 | 19    | 8.78      | 391        | 16,188,196|
| Yago3      | 493,839 | 17    | 5.70      | 44,538     | 4,440,378 |

Datasets. We use three real datasets of hierarchical trees, whose detailed statistics are summarized in Table 6. First, ImageNet [3, 10] is a hierarchical image dataset based on WordNet. It has 74,401 taxonomy vertices and 16 million images with ground-truth labels. Second, we generate a small hierarchy with 200 taxonomy vertices from COCO [19] and ImageNet [10], denoted as Image-COCO, ensuring the successful and efficient running of all tested algorithms. For target search on Image-COCO and ImageNet, we randomly select a set of 1,000 images with a single label and another set of 1,000 images with multiple labels for the SingleTarget and MultipleTargets problems, respectively. Third, Yago3 [4, 21] is a knowledge base from multilingual Wikipedias. We use the ontology structure yagoTaxonomy as the hierarchy for testing. It contains 493,839 taxonomy vertices, where an edge $(v, u)$ means vertex $u$ is a “subClassOf” vertex $v$. Moreover, Yago3 contains 4,440,378 objects from yagoTypes, where each object may have a single label or multiple labels. For both the SingleTarget and MultipleTargets problems, we select two sets of objects with a single label and with multiple labels, respectively, using two methods. In the first method, we randomly select 1,000 labeled objects from Yago3, denoted as Yago3-I. In the second method, we first randomly select 1,000 categories from Yago3 and then pick a random labeled object under each selected category, denoted as Yago3-II.

Comparison methods. We compare our algorithms with state-of-the-art methods HGS [24], IGS [30], and BinG [18]. Specifically,

- HGS: a dynamic programming Human-GS method for identifying multiple targets with a bounded number of questions, which generates $b$ questions offline in a non-interactive setting [24]. Following the algorithms in [24], we implement two methods, Single-Bounded and Multi-Bounded, for identifying a single target and multiple targets, respectively.
- IGS: an interactive graph search algorithm for identifying a single target [30]. The algorithm decomposes a hierarchy into connected paths and finds the target through a series of binary searches on individual paths.
- BinG: a greedy algorithm for identifying a single target, which asks questions using an optimal vertex that prunes the largest number of vertices [18]. It prunes the vertices $P \setminus \text{desc}(u)$ for reach($u$) = Yes. To identify multiple targets, we implement a variant BinG method, which only prunes $P \cap \text{anc}(u) \setminus \{u\}$ for reach($u$) = Yes.

Note that both IGS and BinG can ask unlimited questions to identify the targets. In our problem setting, we terminate the algorithms of IGS and BinG after asking $b$ questions. We also evaluate and compare our proposed algorithms as follows.

Figure 7: Quality evaluation of all methods for MultipleTargets problem, in terms of the averaged penalty.
Budget Constrained Interactive Search for Multiple Targets

- **STBIS**: identifies a single target in Algorithm 2.
- **kBM-DP**: a dynamic programming based method for identifying multiple targets in Algorithms 4 and 5.
- **kBM-Topk**: uses an independent penalty function to select top-\(k\) vertices in Algorithm 6.
- **kBM-DP+**: uses an upper bound pruning technique to accelerate kBM-DP in Algorithm 7.

After asking \(b\) questions, all algorithms return the selections from Yes-candidates in the same way following our kBM-IGS framework.

**Evaluation metrics and parameter settings.** For quality evaluation, we use the penalty \(f(S, T)\) to measure the closeness between selections \(S\) and targets \(T\) by Def. 3. For each experiment, we report the averaged penalty score of searching targets on 1,000 selected images/objects. By default, we set the budget \(b = 50\), and assign \(k = 1\) and \(k = 3\) respectively for the SingleTarget and MultipleTargets problems. The initial probability of each vertex \(v\) is set as \(pr(v) = \frac{1}{n}\) in a uniform way. We denote the running time as INF and the penalty result as N/A, if an algorithm cannot finish within 100 hours.

**EXP-1: Quality evaluation of the SingleTarget problem.** Table 5 shows the penalty results of four methods HGS, IGS, BinG, and STBIS for identifying a single target. For each dataset, we test five different budgets of \(b\), varying from 5 to 100. The smaller the penalty scores, the closer the selections to the hidden targets. All methods get lower penalty scores with increased budget \(b\) as more questions are asked to obtain better selections. Our method STBIS achieves the best performance in all tests, except for one case of \(b = 5\) on Yago3-I. In particular, it outperforms HGS by 23%-1,253%. While BinG has a competitive performance with STBIS, it is much worse than our methods in the more challenging MultipleTargets problem as will be shown in EXP-2.

**EXP-2: Quality evaluation of the MultipleTargets problem.** We evaluate four methods HGS, BinG, kBM-Topk, and kBM-DP+ for identifying multiple targets. Figures 7(a)-7(d) and Figures 7(e)-7(h) report the penalty results on all datasets by varying budget \(b\) and selection size \(k\), respectively. Several observations are made. First, HGS has the largest penalty scores on the small dataset Image-COCO as shown in Figures 7(a) and 7(e). On the three large datasets in Figures 7(b)-7(d) and 7(f)-7(h), HGS fails to finish within 100 hours, due to its high time complexity. Second, compared with BinG, our methods kBM-DP+ and kBM-Topk get smaller penalty scores by achieving an average of 2.1x better results. The main reason is that BinG tends to ask questions on the vertices at the bottom levels, which likely gets a No answer with little gain of reducing target penalties. In contrast, our methods kBM-Topk and kBM-DP+ aim at asking questions on the vertices with the largest expected gains based on the potential target distribution, thereby achieving a better performance. Moreover, with increased budget \(b\) and target number \(k\), kBM-Topk and kBM-DP+ get an even better performance with lower penalty scores. Finally, between kBM-DP+ and kBM-Topk, kBM-DP+ incurs less penalties because it has a better gain function for identifying diverse selections.

**EXP-3: Quality evaluation of kBM-IGS framework.** We evaluate the effectiveness of the kBM-IGS framework using different probability and gain score functions. We implement two variant methods of kBM-IGS as kBM-EP and kBM-Card. Both kBM-EP and kBM-Card follow the same framework in Algorithm 1. However, kBM-EP sets a uniform value of 0.5 for all vertices’ probability scores of \(p\text{Yes}\) and \(p\text{No}\). For two gain score functions \(g\text{Yes}\) and \(g\text{No}\), kBM-Card counts the number of potential targets in a vertex’s descendants, without considering distance. Figure 10(a) and Figure 10(b) report the results of kBM-IGS, kBM-EP, and kBM-Card varied by budget \(b\) and selection size \(k\) respectively. kBM-IGS wins over both kBM-EP and kBM-Card, demonstrating the effectiveness of the probability and gain score functions used in our kBM-IGS framework.

**EXP-4: Efficiency evaluation.** We evaluate the efficiency of generating questions for multiple targets on all datasets. Figure 8 shows the
running time results of five different algorithms HGS, BinG, kBM-DP, kBM-Topk, and kBM-DP+. Note that HGS fails to finish on ImageNet, Yago3-I, and Yago3-II, kBM-DP+ offline pre-calculates the UB\textsubscript{Yes} and UB\textsubscript{No} of the first question, which asks the same question for any targets. kBM-Topk is consistently faster (2.8x on average) than kBM-DP+ for different settings of parameter \( b \) in Figure 8(a)-(h). Because of the low efficiency of kBM-DP, we only run 10 cases for it. As shown in Figure 8(a)-(d), all methods take more time with increased budget \( b \), kBM-Topk and BinG generate questions fastest by adopting the top-\( k \) penalty function. Furthermore, all methods take less average running time for each question with increased budget \( b \) as shown in Figure 8(e)-(h).

EXP-5: Pruning ability evaluation. We conduct a pruning ability evaluation and report the results in Figures 11(a), 11(b). Figure 11(a) shows the number of potential targets by our methods kBM-DP, kBM-Topk, and kBM-DP+ and competitive method BinG. Our methods kBM-DP, kBM-Topk, and kBM-DP+ consistently outperform BinG with the increased number of budgets \( b \). However, in Figure 8, kBM-DP+ runs much faster than kBM-DP, due to its efficient expected gain calculation as shown in Figure 11(b). Compared with kBM-DP, kBM-DP+ takes three orders of magnitude less calculations of expected gain, which validates the pruning optimization strategies in Section 6.3.2.

EXP-6: Evaluation of the quality of kBM-IGS scheme incurred when wrong answers are received from the crowd. We conduct a quality evaluation of our methods where human mistakes are not eliminated and the workers give wrong answers. For each dataset, we randomly select \( X\% \) objects out of 1,000 objects and treat them as difficult objects. We vary \( X \in [0, 50] \) on the ImageNet and Yago3-I datasets. For each question that involves a difficult object, the workers have a probability of giving a wrong answer, denoted as \( p \). In the experiment, we set the wrong probability \( p = 10\% \) and budget \( b = 50 \). Figure 12 shows the penalty results when varying the percentage of difficult objects. As can be seen, the quality performances of kBM-Topk and kBM-DP+ are only slightly degraded with the increasing percentage of difficult objects, demonstrating their resilience to wrong answers. Moreover, our methods kBM-Topk and kBM-DP+ still win BinG by at least 40\%, even with wrong answers.

EXP-7: Case study of image categorization. We conduct a case study of interactive graph search to identify multiple targets on ImageNet. We extract an ‘animal’ sub-hierarchy of ImageNet, which contains nearly 4,000 labels. We use an image shown in Figure 1(b) with these targets \( T = \{ “fish”, “cat” \} \). We apply the kBM-DP+ method with a budget \( b = 12 \) and selection size \( k = 3 \). The left table in Figure 9 shows the detailed process and statistics of all interactive questions by kBM-DP+. For each question vertex \( q_i \), we report the label of \( q_i \), the answer \( \text{reach}(q_i) \), the depth of \( q_i \) in \( \mathcal{H} \), \(|\mathcal{P}|\), \(|\mathcal{Y}|\), and the penalty \( f(S^*, T) \). By default, \( q_0 = r \). We also show the questioned taxonomies in a simplified hierarchy on the right side of Figure 9. The red taxonomies get a Yes answer and the blue taxonomies get a No answer. The interactive process clearly shows that our questions approach the targets quickly in a top-down manner.
manner within 12 questions, which achieves a very small penalty of 1 between selections and targets. Finally, kBM-DP+ identifies the selections $S = \{\text{"fish"}, \text{"mammal"}, \text{"felid"}\}$. Note that "felid" means the cat family, which is the parent of the target "cat". The reason for selection "mammal" is because some subclass labels of "mammal", such as "primate" and "rodent", are potential targets.

8 CONCLUSION AND FUTURE WORK

In this paper, we study the problem of kBM-IGS to identify multiple targets in a hierarchy via a constrained budget of interactive questions. To effectively tackle the problem, we propose a novel kBM-IGS framework to select the vertex with the maximum expected gain to ask question. On the basis of the kBM-IGS framework, we develop STBS algorithm to identify a single target and a dynamic programming based method kBM-DP to identify multiple targets. To accelerate the efficiency, we propose two heuristic algorithms kBM-Topk and kBM-DP+ to ask question on the vertex with the best alternative gain. Extensive experiments validate the effectiveness and efficiency of our proposed algorithms. This paper also opens up interesting questions. One challenging direction is how to control the quality of target selections, given that crowd workers may give wrong answers. It takes non-trivial efforts to identify wrong answers in various possible forms and make a balanced budget cost between trusting the given answers by asking further questions and suspecting the given answers by repeating questions to double-check in a principled way.

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