On the $\rho^0$-meson production in the inclusive proton-proton collision

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Abstract

The production of the $\rho^0$-meson in the inclusive proton-proton scattering $p_A + p_B \rightarrow \rho^0 + X$ is studied using extension of parton model according to generalized vector meson dominance model (GVMD) in region of small $Q^2 = m_{\rho^0}^2$ and $\rho^0$-meson transverse momentum $\leq 1 - 2$ GeV/c. The realistic description of the experimental cross sections of $p_A + p_B \rightarrow \rho^0 + X$ for $4.9 \leq \sqrt{s} \leq 65$ GeV is achieved using an isotropic distribution of the $\rho^0$-meson. The resulting density matrix allows one to select values of the quark masses, which lead to the isotropic distribution of the emitted meson. It is demonstrated that the same cross sections of the reaction $p_A + p_B \rightarrow \rho^0 + X$ for the isotropic distribution of the $\rho^0$-meson can be obtained with different sets of the quark masses and corresponding coupling constants of the quark-meson vertex functions.
Introduction

Nowadays the transverse momentum dependent parton distribution function (TMD PDF) model is successfully applied for the description of the lepton pair production in numerous reactions with momentum transfer $\sqrt{Q^2} \geq 2.4 GeV$ [1, 2, 3, 4, 5]. Moreover, in [6, 7] detailed description of the production of the $\rho, \phi$ mesons in the exclusive electron-proton scattering is achieved in the framework of the generalized parton distributions (GPD) with the simple Gaussian functions for the distributions of the transverse quark momentum’s in the quark-meson wave functions. The fitting of the unintegrated pion multiplicities in the semi inclusive deep inelastic scattering (SIDIS) reactions $\ell N \rightarrow \ell' hX$ was fulfilled in the work of Torino group [8] using corresponding TMD PDF model. These successful applications of the TMD PDF model rely on existence of the factorization theorem [1, 9, 10, 11, 12], under which the cross sections of the considered reactions are a product of the cross sections of the parton annihilation process and the nucleon TMD PDFs. This condition was proved when $Q^2 >> k_T^2$ [1, 9], where $k_T$ denotes the transverse momentum of the final observed particle. For production of the $\rho$ meson with transverse momentum $k_T^2 \leq 1\,(GeV/c) : 2$ in the the inclusive reactions this condition is not fulfilled because $\sqrt{Q^2} = m_\rho = 0.775 GeV/c$ and $Q^2 \sim k_T^2$. The calculations [6, 7] and [8] of reactions $\ell p \rightarrow \ell' \rho(\phi)p'$ and $\ell p \rightarrow \ell' hX$ were performed within different TMD PDF models and different realization of the factorization theorem.

Other kind of successful description of the production of the light and vector mesons, $\Delta^{\pm, \pm}, \pi$ and $\Lambda$-resonances and proton and antiproton in the inclusive proton-proton collision at $\sqrt{s} = 27.5 GeV$ [15] with $p_T^2 \leq 4\,(GeV/c)^2$ was obtained within the FRITIOF model [16, 17] which based upon semiclassical approach and string dynamics. This description was obtained using parton model and corresponding standard PDF functions without restrictions coming from the spin of the particles and factorization theorem. Various applications of FRITIOF model for the pp, pA and AA collisions within parton model are given in refs. [18, 19, 20].

On the other hand existence of the transition $\gamma^* \leftrightarrow V$ allows one simply connect the cross sections of the reactions $p_A + p_B \rightarrow \gamma^*X$ and $p_A + p_B \rightarrow VX$ in the framework of the parton model (see e.g. ch. 14.3.3 [14]) based on the GVMD (generalized vector meson dominance) model [22, 23, 25, 26]. Moreover GVMD model [23, 24, 26] imply an extension of the PDF into region of very small $x$ and $Q^2 < 1 GeV^2$ (or $Q^2 = m^2_\gamma$) and large $p_T^2$. Within GVMD model was achieved explanation and good description of the various experimental data of deep inelastic scattering (DIS) in the region $Q^2 \leq 1 GeV^2$ and small $x$. An other way of extension of PDF in the region of the very small $x$ and $Q^2 < 1 GeV^2$ is given in [37, 38] for ultra cosmic ray and neutrino astronomy.

![Figure 1](image-url)

(a) Full amplitude of the reaction $p_A + p_B \rightarrow \rho^+X$ and its representation in the parton fusion model (b).
The purpose of this work is to examine validity of extension of the usual fusion parton model [13, 14] in the framework of the GVMD model for description of the \( \rho^0 \)-meson production in the inclusive proton-proton scattering \( p_A + p_B \rightarrow \rho^0 + X \rightarrow 1 + 2 + X \) \((1, 2 \equiv \pi^+\pi^- \text{ or } e^+e^-)\). In this approach the full amplitude of the reaction \( p_A + p_B \rightarrow V + X \) in Fig. 1a is replaced by the parton fusion diagram in Fig. 1b and in addition transverse momentum of partons is taken into account via the Gaussian multiplier in PDF. We will use the fusion model [15], where quarks and mesons are on the energy and mass shell.

The article consists of four Sections. The first Section deals with the relationship between cross sections of the reactions \( p_A + p_B \rightarrow \gamma^* + X \) and \( p_A + p_B \rightarrow V + X \) according to GVMD model. Unlike to the usual fusion model we take into account the vector and tensor coupling parts in the quark-meson vertex. An analytic expression for the density matrix of the reaction \( p_A + p_B \rightarrow \rho^0 + X \) and extraction of the isotropic distributions of the \( \rho^0 \)-meson are given in the Sect. 2. The numerical results for the cross sections of the reaction \( p_A + p_B \rightarrow \rho^0 + X \) are presented in Sect. 3. These calculations were performed for the isotropic distributions when the non-diagonal elements of the density matrix are equal to zero and diagonal elements are equal to 1/3. A brief conclusion is given in Sect. 4.

1. Vector meson dominance model (VMD) and cross section of the reaction

\[ p_A + p_B \rightarrow V + X \]

We shall describe \( \rho \)-meson production in the inclusive proton-proton scattering

\[ p_A(P_A S_A) + p_B(P_B S_B) \rightarrow \rho(kM) + X \]  

(1.1)

where \( \mathbf{k} \) and \( M \) denote a momentum and magnetic quantum number of the \( \rho \)-meson, \( P_A S_A \) and \( P_B S_B \) stand for the momentum and magnetic quantum number of the proton \( A \) and \( B \) correspondingly. In particular, in the c.m. frame of the protons \( A \) and \( B \) \( P = (P_A)_Z = -(P_B)_Z \) and \( \mathbf{k} \) is placed in the \( ZX \)-plane.

In this paper the differential cross section of the reaction \( p_A + p_B - \rho^0 X - \ell\ell^+ X \) (or \( p_A + p_B - \rho^0 X - \pi\pi X \)) is determined according to extension of the original Drell-Yan parton model for the dilepton photo production reaction \( p_A + p_B - \gamma^* X - \ell\ell^+ X \). This extension is performed within VMD. In particular, VMD model implies replacement of the quark-antiquark annihilation vertices \( q\overline{q} - \gamma^* \) with the vertices \( q\overline{q} - \rho^0 \). This procedure enables to replace the non-resonant Drell-Yan cross section

\[ \frac{d\sigma^{MM'}_{pA+pB\rightarrow\gamma^*X}}{dk_T^2dy} = \frac{m_N^2}{(4\pi)^2 P s^{1/2}} \sum_{n=u,d} \frac{2m_1m_2}{m_V^2} \int d^2q_{1T} \int d^2q_{2T} \delta(k_T - q_{1T} - q_{2T}) \left( f_{n/A}(x_1, q_{1T})f_{n/B}(x_2, q_{2T}) + (1 \leftrightarrow 2) \right) \]  

\[ \Gamma_{n\overline{q} - \gamma^*}^{MM'} \]  

(1.2a)

with the resonant cross section

\[ \frac{d\sigma^{MM'}_{pA+pB\rightarrow VX}}{dk_T^2dy} = \frac{m_N^2}{(4\pi)^2 P s^{1/2}} \sum_{n=u,d} \frac{2m_1m_2}{m_V^2} \int d^2q_{1T} \int d^2q_{2T} \delta(k_T - q_{1T} - q_{2T}) \left( f_{n/A}(x_1, q_{1T})f_{n/B}(x_2, q_{2T}) + (1 \leftrightarrow 2) \right) \]  

\[ \Gamma_{n\overline{q} - V}^{MM'} \]  

(1.2b)
where $k_T$ and $y = 1/2 \ln \left[ (k_v + k_Z)/(k_v - k_Z) \right]$ denote the transverse momentum and the longitudinal rapidity of the $\gamma^*$ or $\rho$-meson. $q$, $s$ and $n$ stand for a momentum, $Z$-projection of the spin and flavour $n = u, d$ of the quarks. The factor $\frac{m_N^2}{(4\pi)^2 Ps^{1/2}}$ arose in (1.2a,b) due to replacement of the variables $k$ with $k_T^2$ and $y$ in the standard differential cross sections [32], the multiplier 1/3 arose in (1.2a,b) from averaging over the color quantum numbers of the quarks [13, 14]; an additional 1/2 in (1.2b) is used according to the isotopic structure of the $\rho^0$-meson $|\rho^0 > = (|a\vec{a} > - |d\vec{d} >)/\sqrt{2}$. $\Gamma_{\pi\pi,\gamma}^{M,M'}$ and $\Gamma_{\pi\pi,\gamma}^{M,M'}$ are the product of the $\gamma^* - q\overline{q}$ or $V - q\overline{q}$ vertices in (1.2a,b). In the both cases $\Gamma_{\pi\pi,\gamma}^{M,M'} (V \equiv \gamma^*, V)$ have the same form

$$
\Gamma_{\pi\pi,\gamma}^{M,M'} = \xi_{V}^\gamma(k, M) \xi_{V}^{\gamma*}(k, M')< q_1s_1, n; q_2s_2, \vec{p}|J_{\gamma\mu}(0)|0 > < 0|J_{\gamma\nu}(0)|q_1s_1, n; q_2s_2, \vec{p} >, \tag{1.3a}
$$

where $\xi_{V}^\gamma(k, M)$ is the polarization function of $\gamma^*$ or $V$-meson and $J_{\gamma\mu}(x) = (\Box + m_0^2)\Phi_{\gamma^*}(x)$ and $J_{\gamma\nu}(x) = (\Box + m_0^2)\Phi_{\gamma^*}(x)$ are the corresponding source operators.

The cross sections (1.2a,b) are a part of the cross section of the Drell-Yan reaction $p_a + p_B \rightarrow \ell^+\ell^- X$. Therefore $\Gamma_{\pi\pi,\gamma}^{M,M'}$ (1.3a) is ingredient of the cross section of the reaction $q\overline{q} - \gamma^* - \ell^+\ell^-$ or $q\overline{q} - V - \ell^+\ell^-$

$$
\sigma_{\pi\pi,\gamma}^{M,M',M,M'} = \frac{4\pi}{3} \frac{\Gamma_{\pi\pi,\gamma}^{M,M'}}{(p_\ell + p_{\ell^+})^2 - m_V^2} \Gamma_{\pi\pi,\gamma}^{M,M'}, \tag{1.3b}
$$

where $m_V^2 = 0$ and $m_V$ is the mass of the $V$-meson. $p_\ell$ and $p_{\ell^+}$ are four-momentum’s of the final electrons and $\Gamma_{\pi\pi,\gamma}^{M,M',M,M'}$ is defined through product of vertices $\ell\ell^- - V$ in analogy with (1.3a).

Unlike to the original Drell-Yan reaction [33], (1.2a) contains integration over the transverse momentum’s (see e.g. [21]). Validity of replacement of the intermediate photons with the vector meson follows from existence of the transition $\gamma^* \rightarrow V$ and it was tested many times in VMD models [22, 23, 26].

We take PDF $f_{n/A}(x, q_T)$ in (1.2a,b) same form as in TMD PDF model

$$
f_{n/A}(x, q_T) = f_{n/A}(x) \frac{e^{-q_T^2/(2b^2)}}{2b^2}; \quad f_{\pi/B}(x, q_T) = f_{\pi/B}(x) \frac{e^{-q_T^2/(2b^2)}}{2b^2}, \tag{1.4}\n$$

where $q_T$ denote the transverse momentum of the quark.

$$
x_1 = \frac{k_v + k_Z}{2P}; \quad x_2 = \frac{k_v - k_Z}{2P}, \tag{1.5}\n$$

where $k_v = \sqrt{m_V^2 + k_T^2 + k_Z^2}$.

The same Gaussian functions of transverse momentum distribution were used in [6, 8].

The factor $2m_1m_2/m_3^2$ in (1.2a,b) contain $m_1m_2$ which arose in the intermediate integration over the quarks with $m_1d^3q_1/q_1^2$ and $m_2d^3q_2/q_2^2$ according to [32]. This factor cancel with the corresponding expression in (2.6c). The factor $1/(2b^2)$ in (1.2a,b) insures the correct dimension of these cross sections.

In (1.3a) the vertex $q\overline{q} - V$ is

$$
< q_1s_1, n; q_2s_2, \vec{p}|J_{\mu}(0)|0 > = g_{\mu\nu}\overline{\gamma}(q_2)\gamma_{\mu}u(q_1) + g_{\mu\nu}\overline{\gamma}(q_2)\frac{i\sigma_{\mu\nu}(q_1 + q_2)}{m_1 + m_2}u(q_1), \tag{1.6}\n$$

where $u(q_2)$, $\overline{\gamma}(q_2)$, $\gamma_\mu$, $\sigma_{\mu\nu}$ are well-known spinors and Dirac matrices [32].
have energy shell. Then from the energy-momentum conservation in the meson rest frame we cross section in the rest frame of the $\rho$-meson through the masses of quarks and meson $v, \pi^\pm$ are determined by corresponding coupling constants $g^\rho_{\nu}$ and $g^\rho_{\mu}$ can be determined through the similar electromagnetic constants of the $n\pi - \gamma^*$-system in the same way as the coupling constants of the $\rho NN$-vertex are constructed via the coupling constants of the $\gamma^* NN$-vertex within the GVMD model [20, 23, 22], where

$$
g^{\rho NN}_\nu = G^2_\nu e^2; \quad g^{\rho NN}_\mu = G^2_\mu e^2. \quad (1.7)
$$

In analogy to (1.7), $g^{\mu}_{\nu}$ and $g^{\nu}_{\mu}$ are determined through the charges of quarks $e_n$ and antiquarks $\bar{c}_n$:

$$
g^{\mu}_{\nu} = g_{\nu} e_n; \quad g^{\nu}_{\mu} = g_{\mu} e_d; \quad q^{\nu}_{\mu} = g_{\nu} e_d, (1.9)
$$

where $e_u = 2/3$ and $e_d = -1/3$.

2. The density matrix and isotropic distribution

In order to construct the density matrix from the cross sections (1.2b), we consider this cross section in the rest frame of the $\rho$-meson (Fig. 3), where $k^* = 0, \mathbf{q}_1 = -\mathbf{q}_2^*$

$$
\mathbf{q}_i^* = \mathbf{q}_i + \frac{k}{m_\nu} \left[ \frac{(k \mathbf{q}_i)}{m_\nu + k^0} - \mathbf{q}_i^* \right]; \quad i = 1, 2. \quad (2.1)
$$

In this paper meson and quarks in the $V$-meson-quark vertex (1.6) are on mass and energy shell. Then from the energy-momentum conservation in the meson rest frame we have $m_\nu = q_1^* + q_2^*$ and $\mathbf{q}_1 + \mathbf{q}_2 = 0$. These relations enables determine $|\mathbf{q}_1|^{2}$ and $|\mathbf{q}_1|^2$ through the masses of quarks and meson

$$
|\mathbf{q}_1|^2 = \frac{(m_\nu^2 - (m_1 + m_2)^2)(m_\nu^2 - (m_1 - m_2)^2)}{4m_\nu^2}; \quad q_{1,2} = \frac{m_2^2 + m_2^2}{2m_\nu}. \quad (2.2)
$$

Therefore, it is convenient to use the spherical coordinates

$$
\mathbf{q}^* = \mathbf{q}_1^* = |\mathbf{q}_1^*| \left( \sin \alpha \cos \beta^*, \sin \alpha \sin \beta^*, \cos \alpha^* \right), \quad (2.3)
$$

where

$$
\cos \alpha^* = \frac{q_{1z}}{|\mathbf{q}_1^*|} - \frac{k_z}{|\mathbf{q}_1^*|} \left( \frac{q_1^o + q_1^{*o}}{m_\nu + k^0} \right); \quad \tan \beta^* = \frac{q_{1y}}{q_{1x} \frac{q_1^o + q_1^{*o}}{m_\nu + k^0}}. \quad (2.4)
$$

A direct generalization of the Gordon identities for the particles with unequal masses $m_1$ and $m_2$

$$
\nabla(q_2^*) \frac{i \gamma_{\mu} \gamma_{\nu}(q_1^* + q_2^*)}{m_1 + m_2} u(\mathbf{q}_1^*) = -\nabla(q_2^*) \gamma_{\mu} u(\mathbf{q}_1^*) + \frac{m_2}{m_1 + m_2} u(\mathbf{q}_1^*), \quad (2.5)
$$

\n
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Figure 2: $\mathbf{q}_1$ and $\mathbf{q}_2$ (2.1) and the angles $\alpha^*$ and $\beta^*$ (2.3) in the rest frame of the $V$-meson.

allows one to represent (1.3) as

$$\sigma_{n,M}^{M'} = \epsilon_n^{2} \Sigma_{M'M}$$

(2.6a)

$$\Sigma_{M'M} = \xi^{\mu}(0, M) \xi^{\nu}(0, M') Tr \left[(g_V - g_T)^2 \gamma_\mu \frac{(q_1^* \gamma) + m_1}{2m_1} \gamma_\nu \frac{(q_2^* \gamma) - m_2}{2m_2} + g_T^2 \frac{(q_1^* - q_2^*)_\mu}{m_1 + m_2} \frac{(q_1^* \gamma) + m_1}{2m_1} \frac{(q_2^* \gamma) - m_2}{2m_2} + g_T (g_V - g_T) \left( \gamma_\mu \frac{(q_1^* - q_2^*)_\nu}{m_1 + m_2} \frac{(q_1^* \gamma) + m_1}{2m_1} \frac{(q_2^* \gamma) - m_2}{2m_2} \right) \right],$$

(2.6b)

where $(q^* \gamma) \equiv q^* \gamma$. Simple calculation leads to the following expression for $\Sigma_{M'M}$

$$\frac{2m_1 m_2}{m_V^2} \Sigma_{M'M} = (g_V - g_T)^2 \frac{m_V^2}{m_V^2} \left[ \delta_{M'M} + \delta Re \left( A_{M'M} \right) \right],$$

(2.6c)

where factor $\frac{2m_1 m_2}{m_V^2}$ is taken from (1.2a,b), $\delta_{M'M} = 1$ if $M = M'$ and $\delta_{M'M} = 0$ if $M \neq M'$,

$$A_{M'M} = \begin{bmatrix}
\frac{\sin^2(\alpha^*)}{2} & \frac{-\sin(2\alpha^*)}{2\sqrt{2}} e^{-i\beta^*} & \frac{-\sin^2(\alpha^*)}{2} e^{2i\beta^*} \\
\frac{\sin(2\alpha^*)}{2\sqrt{2}} e^{i\beta^*} & \frac{-\sin^2(\alpha^*)}{2} & \frac{-\sin(2\alpha^*)}{2\sqrt{2}} e^{-i\beta^*} \\
\frac{-\sin(2\alpha^*)}{2\sqrt{2}} e^{-2i\beta^*} & \frac{-\sin^2(\alpha^*)}{2} e^{2i\beta^*} & \frac{\sin^2(\alpha^*)}{2}
\end{bmatrix},$$

(2.6d)

$$\delta = \frac{m_V^2}{m_V^2} - \frac{(m_1 + m_2)^2}{m_V^2} \left[ \frac{g_T^2}{(g_V - g_T)^2} \frac{m_V^2}{(m_1 + m_2)^2} - \frac{2g_T}{(g_V - g_T)} - 1 \right],$$

(2.7)

Afterwards the cross section (1.2b) takes the form

$$\frac{d\sigma_{n,M}^{M'}_{p_A+p_B \rightarrow VX}}{d k_T^2 \ dy} = \frac{1}{6(4\pi)^2} \frac{m_N^2}{P^{s'1/2} 2\beta^2} \left< \Sigma_{M'M} \right>,$$

(2.8)

where

$$\left< \Sigma_{M'M} \right> = \frac{1}{d} \int d^2 \mathbf{q}_2 T \left( e_{d/A}^2 f_{d/A}(x_1, k_x - \mathbf{q}_2) \tilde{f}_{n/B}(x_2, \mathbf{q}_2) + e_{d/A}^2 f_{d/A}(x_1, k_x - \mathbf{q}_2) \tilde{f}_{n/B}(x_2, \mathbf{q}_2) + (1 \leftrightarrow 2) \right) \Sigma_{M'M}$$

(2.9a)
\[ d = \int d^2 q_{2T} \left( e_i^2 f_{u/A}(x_1, k_x - q_{2T}) f_{\gamma/B}(x_2, q_{2T}) + e_i^2 f_{d/A}(x_1, k_x - q_{2T}) f_{\gamma/B}(x_2, q_{2T}) + (1 \leftrightarrow 2) \right) \]

(2.9b)

and \( \langle F(\alpha^*, \beta^*) \rangle > \) denotes the average value of a function \( F(\alpha^*, \beta^*) \)

\[ \langle F(\alpha^*, \beta^*) \rangle = \frac{1}{d} \int d^2 q_{2T} \left( e_i^2 f_{u/A}(x_1, k_x - q_{2T}) f_{\gamma/B}(x_2, q_{2T}) + e_i^2 f_{d/A}(x_1, k_x - q_{2T}) f_{\gamma/B}(x_2, q_{2T}) + (1 \leftrightarrow 2) \right) F(\alpha^*, \beta^*). \]

(2.9c)

The elements of the density matrix \( \rho_{MM'} \) are defined as

\[ \rho_{MM'} = \frac{d\sigma_{\rho_{MM'}}}{d\sigma_{PA+PB\rightarrow VX}/d^{2}k_{X}/dy}; \quad \frac{d\sigma_{PA+PB\rightarrow VX}}{d^{2}k_{X}/dy} = \sum_{M} \frac{d\sigma_{MM'}}{d\sigma_{PA+PB\rightarrow VX}}. \]

(2.10)

Using formulas (2.8) and (2.9 a,b,c,d), we can obtain \( \rho_{MM'} \) (2.10) explicitly

\[ \rho_{oo} = \frac{\delta < \cos^2(\alpha^*) > + 1}{\delta + 3}; \quad \rho_{11} = \rho_{-1-1} = \frac{1}{2} \left( \frac{\delta < \sin^2(\alpha^*) > + 2}{\delta + 3} \right) \]

(2.11a)

\[ \text{Re}\rho_{1-1} = \frac{1}{2} \left( \frac{\delta < \sin^2(\alpha^*) \cos(2\beta^*) >}{\delta + 3} \right); \quad \text{Re}\rho_{00} = \frac{1}{2\sqrt{2}} \left( \frac{\delta < \sin(2\alpha^*) \cos(\beta^*) >}{\delta + 3} \right) \]

(2.11b)

\[ \text{Im}\rho_{1-1} = \frac{1}{2} \left( \frac{\delta < \sin^2(\alpha^*) \sin(2\beta^*) >}{\delta + 3} \right); \quad \text{Im}\rho_{10} = \frac{1}{2\sqrt{2}} \left( \frac{\delta < \sin(2\alpha^*) \sin(\beta^*) >}{\delta + 3} \right) \]

(2.11c)

From the density matrix \( \rho_{MM'} \) (2.11a,b,c), one can extract conditions for the isotropic distribution when

\[ \rho_{oo} = \rho_{11} = \rho_{-1-1} = 1/3; \quad \rho_{M\neq M'} = 0. \]

(2.12)

In particular, if

\[ \delta = 0, \]

(2.13)

then the conditions (2.12) are fulfilled.

Afterwards, using a special choice of the masses of quarks \( m_1 \) and \( m_2 \) in \( \delta \) (2.7), one can satisfy the condition (2.13). There exist two possibilities:

1. \( m_1 \) and \( m_2 \) are at threshold \( |q^*|^2 = 0 \), where

\[ m_1 + m_2 = m_V \quad \text{or} \quad m_1 = m_2 = m_V/2. \]

(2.14a)

2. \( g_T/g_c \) satisfy the equation

\[ \frac{g_T^2}{(g_V - g_T)^2} \frac{m_V^2 - (m_1 + m_2)^2}{(m_1 + m_2)^2} - \frac{2g_T}{(g_V - g_T)} = 1 = 0. \]

(2.14b)
The solution of the equation (2.14b) determine \( g_T/g_V \) and \((g_V - g_T)^2\) through \(m_1\), \(m_2\) and \(m_V\).

\[
g_T = g_V \frac{m_1 + m_2}{m_V} \quad \text{which gives} \quad (g_V - g_T)^2 = g_V^2 \frac{(m_V - m_1 - m_2)^2}{m_V^2}. \quad (2.14c)
\]

The solution (2.14c) for the small (current) quark masses \(m_{1,2} \approx 5 - 10\,\text{MeV}\) gives the small values of \(g_T\). However, \(g_T/(m_1 + m_2) = g_V/m_V\) in the vertex (1.6) is not negligible. Thus the isotropic distribution with the current quark masses produces redefinition \(g_T/(m_1 + m_2) \rightarrow g_V/m_V\) in the quark-meson vertex (1.6).

Using the condition (2.13), one can rewrite the cross section (2.8) as

\[
\frac{d\sigma_{p_A+p_B \rightarrow VX}}{d^2k_X^2 dy} = \frac{(g_V - g_T)^2}{2(4\pi)^2} \frac{m_N^2}{P_s^{1/2} 2b^2} \int dq_T^2 \left( e^2 f_{u/A}(x_1, k_X - q_2T) f_{\bar{u}/B}(x_2, q_2T) + e^2 f_{d/A}(x_1, k_X - q_2T) f_{\bar{d}/B}(x_2, q_2T) + (1 \leftrightarrow 2) \right). \quad (2.15)
\]

This expression depends on \(m_1\) and \(m_2\) through \((g_V - g_T)^2\) only. It is easy to see that the different sets of \(g_V, g_T, m_1\) and \(m_2\) can give the same \((g_V - g_T)^2\) i.e. the same cross section (2.15).

In particular, for the current quark masses \(m_{1,2} \approx 5 - 10\,\text{MeV}\) the cross section (2.15) remains the same after replacement \((g_V - g_T)^2 \rightarrow g_V^2\) (current).

An other way to get the equal diagonal elements of the isotropic density matrix

\[
\rho^{00} = \rho^{-1-1} = \rho^{11} = 1/3 \quad (2.16a)
\]

turns out if

\[
< \cos^2(\alpha^*) > = \frac{1}{2} < \sin^2(\alpha^*) > = \frac{1}{3} \quad (2.17)
\]

In this case the nondiagonal elements of the density matrix are not equal to zero

\[
\rho^{M \neq M'} \neq 0 \quad (2.16b)
\]

and the distribution is anisotropic.

3. Numerical calculations of cross sections

We have numerically estimated the cross sections \(d\sigma_{p_A+p_B \rightarrow p^*X}/d^2k_X^2\), \(d\sigma_{p_A+p_B \rightarrow p^*X}/dy\) and \(\sigma_{p_A+p_B \rightarrow p^*X}\) based upon \(d\sigma_{p_A+p_B \rightarrow p^*X}/d^2k_X^2 dy\) (2.15) for the isotropic distribution (2.12).

The parameter \(B \equiv 1/(2b^2)\) of the transverse part of the PDF (1.4) was chosen as \(B_1 = 3.6(\text{GeV}/c)^{-2}\) (\(b = 0.4082\,\text{GeV}/c\)) and \(B_2 = 3.0(\text{GeV}/c)^{-2}\) (\(b = 0.3727\,\text{GeV}/c\)). The corresponding curves in Figs. 3, 4 and 5 are displayed with solid and dotted lines. The value of \(B = 3.6 \pm 0.4(\text{GeV}/c)^{-2}\) was used in [27] to describe experimental data at a momentum of the incoming proton 12\,\text{GeV}/c and 24\,\text{GeV}/c. \(B = 3.3 \pm 0.2(\text{GeV}/c)^{-2}\) was need in [30] to reproduce the experimental cross sections in the region \(23.6 \leq \sqrt{s} \leq 63.0\,\text{GeV}\) and \(B = 2.59 \pm 0.1(\text{GeV}/c)^{-2}\) was used in [15]. The magnitude of the parameter \(B = 2.6 - 3.6(\text{GeV}/c)^2\) in (1.4) differs largely from the same parameter \(B = 0.5 - 0.7(\text{GeV}/c)^2\) in the TMD PDF.
Unlike [27], in \( d\sigma/k \) the extension of \( f \) is described well the experimental data [27] excluding the region \( \rho^0 \) ferent PDF models are depicted in Fig. 3. These curves are rather close to each other and different quarks. In [37] PDF are constructed in the regions \( Q < 2 \) and for different reactions and different quarks. In [37] PDF are depicted in Figs. 3, 4 and 5 with red, green and blue curves, respectively. In [39 PDF are defined in regions \( Q^2 > 1.69(GeV)^2 \) and 0.2GeV/c < \( k_T < 0.9 GeV/c \).

Adjusted parameter \( (g_T - g_V)^2/(4\pi) \) is strongly correlated with \( B \). For \( B = 3.0(GeV/c)^{-2} \) we take \( (g_T - g_V)^2/(4\pi) \) = 32.1 and for \( B = 3.0(GeV/c)^{-2} \) we use \( (g_V - g_T)^2/(4\pi) \) = 32.1*1.6. These values are consistent with \( g^2_V/(4\pi) \) and \( g^2_T/(4\pi) \) within the VMD model [35] and the VMD model for the constituent quarks [36], where \( g^\rho_{NN}/g^V_{NN} = 5 \) or \( 6.6 \pm 0.6 \) and \( 2 \leq (g^\rho_{NN})^2/(4\pi) \leq 3 \) (\( g^V_{NN} \equiv g_V \)). By virtue of (2.14c), for the current quark masses \( \simeq 5 - 10 MeV \) one obtains the cross section (2.15) if \( g^2_V \). (current) \( \simeq (g_V - g_T)^2 \).

We have used three models of the standard PDF \( f_{n/A}(x) \) [39, 38, 37] which are depicted in Figs. 3, 4 and 5 with red, green and blue curves, respectively. In [39 PDF are defined in regions \( Q^2 > 1(GeV)^2 \) and \( 0.2 < x < 0.01 \), or \( x < 0.01 \), or \( x > 0.1 \) for the different reactions and different quarks. In [37] PDF are constructed in the regions \( Q^2 > 1(GeV)^2 \) and \( 0.2 < x < 0.01 \) and this region is extended for in \( x > 10^{-5} \) for the ultra cosmic ray and neutrino astronomy. The extension of \( f_{n/A}(x) \) in \( Q^2 < 1(GeV)^2 \) and in the ultra small \( x > 10^{-9} \) region is given [38].

\[
d\sigma_{\rho^0 A \to \rho^0 X}/d\kappa^2_X \text{ for the different parameters } B \text{ and corresponding } (g_V - g_T)^2, \text{ and different PDF models are depicted in Fig. 3. These curves are rather close to each other and describe well the experimental data [27] excluding the region } 0.5 \leq k^2_X \leq 0.75(GeV/c)^2. \]

Unlike [27], in \( d\sigma_{\rho^0 A \to \rho^0 X}/d\kappa^2_X \) (1.2) and (2.15) one can not extract exactly the dependence on \( k^2_X \) via factor \( e^{-Bk^2_X} \) because \( x_1 \) and \( x_2 \) (1.5) depend on \( k^2_X \) through \( k_o = \sqrt{m^2_V + k^2_X + k^2_Y} \). Similarly, unlike [13, 14], \( d\sigma_{\rho^0 A \to \rho^0 X}/dy \) contains integration over \( k^2_X \) of \( e^2 f_{u/A}(x_1) f_{\pi/B}(x_2) + e^2 f_{d/A}(x_1) f_{\pi/B}(x_2) \).
\( \frac{d\sigma}{dy} \) in Fig. 4 have a typical behavior for small \( y \) close to the constant, followed by a rapid drop. Similar behavior of these cross sections was also obtained in \cite{26} using an additional scaling condition between the cross sections of the light vector mesons and pions. In contrast to the distribution of \( k_X^2 \) in Fig. 3, the distribution of \( y \) in Fig. 4 significantly depend on the choice of the PDF and \( 2b^2 \) parameter in (1.4).

The qualitative description of the experimental cross sections \( \frac{d\sigma_{pA+pB\rightarrow hX}}{d\kappa_X^2} \) and \( \frac{d\sigma_{pA+pB\rightarrow hX}}{d\sigma_{pA+pB\rightarrow hX}} \) for production of the mesons \( \pi^{\pm,\circ}, \rho^{\pm,\circ}, \omega, \eta, f^{\circ}, K^{\pm}, K^{*\circ}, K^{*0}, \phi, \Delta^{++,+} \) and \( \Lambda \)-resonances and proton and antiproton in the inclusive proton-proton collision at \( \sqrt{s} = 27.5 GeV \) \cite{15} was obtained within the FRITIOF model \cite{16,17} within semiclassical approach and string dynamics. Relationship between Monte Carlo models for multiple jet production in pp, pA and AA collisions and parton model is given in refs. \cite{18,19,20}. 

Figure 4: The same as in Fig. 3, but for the distribution over the longitudinal rapidity \( y \).
Total cross sections $\sigma_{pA+pB\to\rho^0 X}$ in Fig. 5 also significantly depend on the choice of PDF and parameters $2b^2$ in (1.4) and $(g_T - g_N)^2$. However, these cross-sections have important features: 1) They increase dramatically in the energy region up to $\sqrt{s} \sim 20 GeV$. In particular,

$$\frac{\sigma_{pA+pB\to\rho^0 X}(P_{\text{beam}} = 24 Gev/c)}{\sigma_{pA+pB\to\rho^0 X}(P_{\text{beam}} = 12 Gev/c)} \sim 2$$

(3.1)

2) In the energy region $\sqrt{s} > 20 GeV$ these cross sections are growing much slower than before $\sqrt{s} < 20 GeV$. In the considered model, this behavior of $\sigma_{pA+pB\to\rho^0 X}$ is determined by the standard PDF. The dependence of $\sigma_{pA+pB\to\rho^0 X}$ on choice of PDF parameters $2b^2$ in (1.4) and related $(g_V - g_N)^2$ is much stronger in the energy region $\sqrt{s} > 20 GeV$. 

Figure 5: Same as in the Fig. 3 and 4, but for the total cross section as a function of $\sqrt{s}$. Experimental data at 4.93, 6.84 GeV from [27], at 1.2 GeV from [28], at 26.8 GeV from [29, 30], at 23.6, 30.6, 44.6, 52.8, 63.0 GeV from [31] at 52.5 GeV from [32], and at 27.5 GeV from [15].
4. Conclusion

We have shown that the parton model with an isotropic distribution of the \( \rho^0 \) meson (2.12) reproduces realistically the experimental cross sections of the inclusive proton-proton scattering \( p_A + p_B \rightarrow \rho^0 X \). This is consistent with the experimental results \[27\], according which the angular distributions of this reaction at \( P_{lab} = 12 \) and \( 24 GeV/c \) are roughly isotropic.

Present formulation of the reaction \( p_A + p_B \rightarrow \rho^0 + X \) based on extension of the original parton model for the Drell-Yan reaction \( p_A + p_B \rightarrow \gamma \ast + X \) within GVMD\[22, 23\]-\[25, 26\] which requires continuation of PDF in the region of the small \( x \) and \( Q^2 < 1 GeV^2 \). For this aim we have used three different PDF\[37, 38, 39\], where for \(|x| << 1 \) and \( Q^2 < 1 GeV^2 \) PDF are constructed exactly\[37, 38\] using the corresponding experimental data. Cross sections in Fig. 4,5 depend significantly on the choice of the PDF, but different PDF give qualitatively similar results. The isotropic cross section (2.15) strongly depends on the choice of the parameters \( 2b^2 \) of PDF (1.4) and the corresponding parameter \( (g_V - g_T)^2 \). Sensitivity on the correlated parameters \( 2b^2 \) and \( (g_V - g_T)^2 \) of the cross sections increases in the region \( 20 GeV < \sqrt{s} < 65 GeV \).

The cross section (2.15) for the isotropic distributions contains only two adjustable parameters: \( 2b^2 \) in the standard PDF (1.4) and the related constants \( (g_V - g_T)^2 \) from the \( \rho NN \) vertex (1.6). The considered cross section (2.15) depends on the quark masses through \( (g_V - g_T)^2 \) as it follows from (2.14a,b) and (2.14c) for the isotropic distribution (2.12). This allows one to get the several set of the different quark masses which yield the same cross section (2.15) with the same \( (g_V - g_T)^2 \). In particular, we have demonstrated that one can obtain the same isotropic distributions within the constituent and current quark models. The detailed theoretical and experimental investigation of the anisotropic contributions in the considered cross sections and the density matrix \( \rho_{MM'} \) (2.10)-(2.11a,b,c) will allow one to estimate the mechanisms of removing this degeneracy.

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