Control-Oriented Modeling of Pipe Flow in Gas Processing Facilities

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Abstract—Pipe flow models are developed with a focus on their eventual use for feedback control design at the process control level, as opposed to the unit level, in gas processing facilities. Accordingly, linearized facility-scale models are generated to describe pressures, mass flows, and temperatures based on sets of nonlinear partial differential equations from fluid dynamics and thermodynamics together with constraints associated with their interconnection. As part of the treatment, the divergence of these simplified models from physics is assessed since robustness to these errors will be an objective for the eventual control system. The approach commences with a thorough analysis of pipe flow models and then proceeds to study their automated interconnection into network models, which subsume the algebraic constraints of bond graph or standard fluid modeling. The models are validated and their errors are quantified by referring them to operational data from a commercial gas compressor test facility. For linear time-invariant models, the interconnection method to generate network models is shown to coincide with automation of Mason’s gain formula. These pipe network models based on engineering data are the first part of the development of general facility process control tools.

Index Terms—Control-oriented modeling, flow control, gas processing facility, Mason’s gain formula, model validation, thermodynamics.

NOMENCLATURE

\( A_c \) Cross-sectional area \([\text{m}^2]\).

\( c \) Speed of sound \([\text{m/s}]\).

\( c_p \) Specific heat \([\text{J/kg K}]\).

\( d \) Pipe inside diameter \([\text{m}]\).

\( d_o \) Pipe outside diameter \([\text{m}]\).

\( g \) Acceleration due to gravity \([\text{m/s}^2]\).

\( h(x) \) Pipe elevation \([\text{m}]\).

\( k_{\text{rad}} \) Lumped thermal conductivity pipe \([\text{W/m}^2 \text{K}]\).

\( L \) Pipe length \([\text{m}]\).

\( p(x,t) \) Pressure \([\text{kg/s}^2 \text{m}]\).

\( \tilde{p}(x,t) \) Pressure deviation from nominal point \([\text{kg/s}^2 \text{m}]\).

\( q(x,t) \) Mass flow \([\text{kg/s}]\).

\( \tilde{q}(x,t) \) Mass flow deviation from nominal point \([\text{kg/s}]\).

\( q \) Rate of heat flow per unit area \([\text{W/m}^2]\).

\( Re \) Reynolds number \([1]\).

\( R \) Specific gas constant \([\text{m}^2/\text{s}^2 \text{K}]\).

\( T(x,t) \) Temperature \([\text{K}]\).

\( \tilde{T}(x,t) \) Temperature deviation from nominal point \([\text{K}]\).

\( T_0 \) Nominal temperature \([\text{K}]\).

\( T_{\text{amb}} \) Ambient temperature \([\text{K}]\).

\( v(x,t) \) Velocity \([\text{m/s}]\).

\( \varepsilon \) Compressibility factor \([1]\).

\( \varepsilon_0 \) Constant compressibility factor \([1]\).

\( \rho \) Density \([\text{kg/m}^3]\).

\( \lambda \) Friction factor \([1]\).

I. INTRODUCTION

Gas processing facilities, where natural gas is received, treated, and compressed for onward transmission through a distribution pipeline network, provide motivation and embodiment for the development of systematic control-oriented modeling tools suited to the design of process control solutions based on plant schematics and layouts. The control of these plants involves the interconnection of a number of elements including pipes, compressors, heat exchangers, valves and valve manifolds, scrubbers, and other process units and volumes. The control splits into two distinct aspects: process control for system-wide operational efficiency and accuracy, and safety systems to ensure unit and plant protection. The two control aspects differ in their timescales and in their scope, with the safety system acting across a wide range of operating points (rather than around a single operating point), being both faster, more highly nonlinear, and more localized to specific unit operation, such as avoiding compressor surge. Our focus will be the process control side with an emphasis on unified plant-wide operational effectiveness. The aim of this article is to develop interconnectable and reconfigurable unit system models, which are amenable to control design, with an objective of bringing multi-input–multioutput (MIMO) control into the picture for gas processing facilities: first from engineering design specifications and then augmented by data-based tuning.

Control-oriented captures the modeling focus on eventual model-based feedback controller design reflecting: plant operational objectives, the presence and capabilities of selected actuators and sensors, and the possible reconfiguration of operations. More precisely, our models are designed to be used for the following conditions.

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1) **Plant**: Interconnected networks of pipes and processing elements located at one site on the order of tens of meters (rather than kilometers) in extent.

2) **Objective**: Bulk pressure regulation and disturbance flow rejection with flow as control input/manipulated variable.

3) **Sensing/Actuation**: Sampled at or below 1 Hz in line with the plant’s regulation objective. The focus is on widespread, reliable, and accurate pressure sensing in particular, and on actuation using flow control valves. Sensing of flow with orifice plates is there for corroboration more than for control. Temperature sensing is slow and of limited presence in the plant.

4) **Resonant and Acoustic Modes**: While ever-present in compression systems, they are at frequencies beyond the sensor and actuator bandwidth in plants of this size.

5) **Models**: They should facilitate control design for this regime and should be amenable to tuning by control-savvy plant engineers.

Although this is quite a specific scenario, it is fairly representative for gas processing facilities.

The models we seek will be linear(ized), time-invariant (LTI) state-space systems, optionally parameterized by nominal operating point, and capable of systematic interconnection of unit models into facility models using computer-based MIMO control design tools. Models with time delay do not fall into this category and are therefore approximated by control-compliant dynamics if necessary. The quid pro quo for this utility is that these models are necessarily simplistic and approximate, but that, by characterizing their nature, approximations might be addressed in control design. Inevitably, such modeling relies heavily on engineering knowledge of the specific application but admits fairly general applicability.

The subsystem models are based on simplified approximations of constituent equations from fluid dynamics, coupled partial differential equations (PDEs) plus algebraic equations, and are validated against plant data, including the assessment of model errors.

Fluid dynamics, particularly computational fluid dynamics, is a well-established subject centered on high-fidelity modeling of flows given design and boundary conditions; typically involving nonlinear PDEs and transport phenomena, which are not amenable to finite-dimensional control design but instead are targeted and tested for simulation. Other pragmatic modeling for pipeline distribution systems [1], [2], [3] yields ordinary differential algebraic equations (DAEs), which again are not well suited to control design. Although they can be used directly for controller synthesis in some circumstances [4] and, as noted in [1], if the DAE is of index 1; Benner et al. [1, Th. 4.1] established that the DAEs are indeed of index 1, and thus, it is possible to rewrite the DAE as an ordinary differential equation (ODE) without the algebraic constraints. Effectively, we complete this conversion here.

A generalization of such a differentiation index to partial DAEs (PDAEs) is developed in [5] and may also be applied to interconnected fluid systems such as we investigate here. In this way, reducing PDAEs to PDEs, linearizing, and only then discretizing the model may be advantageous for simulation accuracy [6]. Yet, our approach begins with discretization and thus avoids PDAEs since it meets our required utility for general engineers as described above and allows an explicit conversion scheme for arbitrary networks by leveraging linear systems theory, hence the control-oriented aspects.

For fluid or general mechanical systems, we take a lead from [1] and [7] as examples where graph-theoretic methods are applied to generate process models from component descriptions, with the latter paper specifically targeted at control design and the former at modeling for simulation. Williams et al. [7] is allied in its control objective with our work here and used energy as the *lingua franca* to map states between subsystems. The edges of their graphs are energy preserving connections with the dynamics occurring at the nodes. In contrast, Benner et al. [1] modeled the dynamics in the edges with the nodes applying the interaction constraints. For our target processes of gas processing plants, this latter structure accords better with the primary process control objective of pressure regulation and secondarily with flow estimation. Thermal energy is a byproduct and reflection of the inefficiency of the process. While of interest, the temperature is not the central manipulated variable. However, it is noteworthy that the energy formulation of Williams et al. [7] for composite aircraft systems allows conservation laws to be absorbed into the component models so that the aggregated state-space model can be directly applied for control design. A recent comprehensive survey of modeling and feedback control design for HVAC systems is provided by Goyal et al. [8], which cites Rasmussen and Alleyne [9], who concentrate explicitly on control-oriented modeling in vapor compression systems. However, because their pipes are short and well-insulated, the system structure again focuses on node dynamics rather than edge dynamics of our problem.

For large-scale domain-independent systems, works from [10], [11], and [12] follow a top-down approach, decomposing large-scale networks into smaller subsystems analyzing control-relevant notions, such as (structural) stability, reachability, and controllability. While their approach is generally applicable to the case of pipe flow, the logical direction differs: instead of decomposing, we compose interconnected systems from subsystems in a bottom-up approach under the assumption that structures are fixed (rendering structural stability [11] secondary). Furthermore, the control actuator and, to a lesser extent, sensor locations are few when compared with the number of subsystems or network elements.

Following [1], which deals with isothermal models of gas distribution networks, we commence by studying pipe flow in individual pipes before considering how these are connected into networks yielding automatable aggregation of subsystems. Benner et al. [1] proposed a network DAE with the algebraic part being the conservation of mass flows at the connection points. At this level of detail, this approach bears a strong resemblance to bond graph techniques [13] from which control design is problematic. However, since the resultant network DAE has index 1, the algebraic part can be solved locally to express some of the variables in terms of the others, thereby eliminating them. For our models,
algebraic equations arise when pipes join but not when they branch. For joints, a state variable is removed yielding a new network element subsuming the three joining pipes. These new elements preserve the linearity and other properties while also respecting the conservation laws. Furthermore, we show how these components might be aggregated into network equations to compute the larger state-space system, which we show subsumes Mason’s gain formula. That is, we are able to preserve the simplicity of the signal flow model of the pipe network as opposed to resorting to bond graphs or DAEs. Mass conservation appears at the network level through the presence of an integrator between net flow into the network and pressure (see [14] for a proof of this property and of its preservation in networks).

The pipe flow models developed are validated against 1-Hz operating data collected at the Gas Compressor Test Facility (GCTF), Solar Turbines Inc., San Diego, CA, USA. This is a well-instrumented site normally used to test compressor performance. We use engineering design values to derive the parameterized models, and then, experimental data from a number of recorded tests are used to compare the fit of the data and model outputs. The discrepancy between model and data is used to quantify and qualify the model performance. Specifically, we find that isothermal models, such as those used in [1], are subject to offsets and slow variations due to temperature gradients, which for this plant are measured but need not necessarily be. Accordingly, the control design needs to accommodate this known inaccuracy of the models. Indeed, the existing single-loop proportional-plus-integral (PI) controllers already give this clue and indicate that the principal plant objectives are the regulation of pressures and flows.

The design of network-ready models for pipe flow is the first stage of introducing model-based control design into these systems using engineering design information and data sheets. The project objective is to expand this to include other network elements, such as compressors, heat exchangers, vessels, and valves [15] validating their use for MIMO control design [14].

**PART I: CONTROL-ORIENTED PIPE MODELS**

We start with a deep dive into: modeling of individual pipe segments as nonlinear PDEs and boundary conditions, spatial discretization to nonlinear ODEs with input signals, and then linearized ODE models with inputs. These are then compared with experimental/operational data from the GCTF, yielding control-oriented finite-dimensional linear state-space models and an appreciation of their deviation from ideal behavior. We establish that these single pipe models inherently satisfy the conservation of mass flow.\(^1\) In Part 2, we explore how to move from pipe models to pipe network models.

**II. PDE MODELS**

We formulate the pipe dynamics as a 1-D flow with standing assumptions common in the literature (e.g. [1], [3], [16], [17]). We assume these throughout this article.

\(^1\)This central presence of mass conservation in flow models is more fully examined in our companion model-based control design paper [14], where conservation is shown to connect to integrators and inherent model structure at \(s = 0\), appreciation of which is critical for regulator design.

***Standing Assumption 1:*** For the 1-D pipe flow, the following conditions hold.

1. The cross-sectional area of each pipe segment is constant.
2. Average velocities across the cross section suffice for the computation of the mass flow.
3. There is no slip at the wall, i.e., the gas velocity at the inner pipe wall is zero.
4. Friction along the pipe can be approximated by the Darcy–Weisbach equation, see, e.g., [18].
5. The compressibility factor is constant along the pipe.
6. Capillary, magnetic, and electrical forces on the fluid are negligible.

Item 2) is a property of high Reynolds number turbulent flow. Under these assumptions, the constituent relations—continuity, momentum, energy, and gas equation—that serve as a basis for our model are

\[
\begin{align*}
\rho & = \rho R_T T_0 \\
\frac{d\rho}{dt} & = -\frac{\partial}{\partial x} (\rho v) \\
\frac{d}{dt} (\rho v) + \frac{\partial}{\partial x} (\rho v^2 + p) & = -\frac{k}{2d} \rho v |v| - g\rho \frac{dh}{dx} \\
\frac{\partial}{\partial x} (\rho v) & = \frac{\partial}{\partial t} \left[ \rho \left( c_v T + \frac{v^2}{2} + gh + \frac{p}{\rho} \right) \right] \\
& + \frac{\partial}{\partial t} \left[ \rho \left( c_v T + \frac{v^2}{2} + gh \right) \right] \\
p & = \rho R_T T_0 
\end{align*}
\]

which are derived, e.g., in [19] and whose parameters are defined in the Nomenclature. The boundary conditions

\[
p(0, t), \quad q(L, t), \quad T(0, t)
\]

are assumed to be known. The continuity equation (1a) captures the conservation of mass. The momentum equation (1b) is obtained by a Newtonian approach considering forces acting on a fluid. The total energy equation (1c) is the first law of thermodynamics in differential form (see, e.g., [20]). The gas equation (1d) closely describes the behavior of natural gas at the conditions pertaining to the handling facility.

We develop a dynamic model for \(p(L, t)\) and \(q(0, t)\) and, if required, also for \(T(L, t)\), and a related methodology that allows a systematic interconnection of pipe elements in a network. Toward this goal, in Section III, from the constituent relations above, we derive a nonisothermal, linear, 3-D state-space model with the pressure, mass flow, and temperature as state elements. Under the condition of a constant temperature, in Section IV, we revisit (1) and introduce a simplified isothermal 2-D model. In Section IV, we validate both models against operating data from the GCTF and compare them to the numerical solutions of the PDEs in (1). This analysis suggests using the isothermal model parameterized by spatially varying nominal temperature and managing small offsets and slow drifts with the controller design. Section VII treats the removal of algebraic constraints stemming from the DAEs and proposes a catalog of common network units in a state-space form, including a new pipe joint element. To interconnect these unit models to pipe networks, Section VIII contains a
matrix methodology, which we prove subsumes and automates Mason’s gain formula in the MIMO context. The properties of interconnected components are then illustrated by a numerical experiment in Section IX.\(^2\) We finish this article with a brief conclusion and directions for future research.

III. NONLINEAR AND LINEAR NONISOTHERMAL 3-D ODE MODELS

Toward a nonisothermal 3-D model with pressure, mass flow, and temperature as state elements, consider constituent relations (1). Notice that "3-D" refers to the number of states and not the spatial dimensionality. For the corresponding total energy equation [see (1c)], the heat flux, \(q\), is assumed to be limited to radial conduction through the pipe so that, similar to [21] and neglecting conduction through the gas,

\[
q \rho A_c dx = k_{rad} \pi \tilde{d} x (T_{amb} - T).
\]

(2)

This characterization enables the formulation of PDEs that isolate the time derivatives of the state variables.

**Proposition 1:** Let \(|v| \ll c = (\zeta R_c T_0)^{1/2}\). Then, constituent equations (1) and the heat flux described in (2) yield

\[
\begin{align*}
\frac{\partial p}{\partial t} &= \frac{R_s z_0}{A_c c_v} \left[ k_{rad} \pi \tilde{d} (T_{amb} - T) - \frac{\partial q}{\partial x} T (c_v + R_s z_0) + \frac{\partial p}{\partial x} R_s z_0 T q \rho}{p} \right. \\
&\left. - \frac{\partial T}{\partial x} (c_v + R_s z_0) + \frac{\lambda R_s^2 z_0^2 T^2 q^2 |q|}{2dA_c^2 p^2} \right] \quad (3a) \\
\frac{\partial q}{\partial t} &= -A_c \frac{\partial}{\partial x} \left( \frac{\partial p}{\partial x} - \frac{\lambda R_s T z_0 |q|}{2dA_c^2} \right) - A_c \frac{\partial q}{\partial x} \frac{\partial p}{\partial x} - A_c \frac{\partial q}{\partial x} \Delta g \frac{dh}{R_s z_0} \frac{dx}{p} \quad (3b) \\
\frac{\partial T}{\partial t} &= \frac{R_s z_0 T}{A_c c_v p} \left[ k_{rad} \pi \tilde{d} (T_{amb} - T) - \frac{\partial q}{\partial x} T R_s z_0 \right. \\
&\left. + \frac{\partial p}{\partial x} R_s z_0 T q \rho - \frac{\partial T}{\partial x} q (c_v + R_s z_0) \right. \\
&\left. + \frac{\lambda R_s^2 z_0^2 T^2 q^2 |q|}{2dA_c^2 p^2} \right]. \quad (3c)
\end{align*}
\]

The proof is provided in the Appendix. Proposition 1 enables us to obtain a linear 3D state-space realization, through spatial discretization and subsequent linearization of these PDEs.

We commence with the spatial discretization using simple differences. Subscripts ‘\(t\)’ and ‘\(r\)’ connote variables at the left (entry) and right (exit) sides of the pipe. Input variables are identified with the pipe PDE boundary conditions, \(p_t, q_t,\) and \(T_t,\) and the state variables with the ODE solution, \(p_r, q_r,\) and \(T_r,\) where

\[
\begin{align*}
p_t &= p(0, t), \quad q_t = q(0, t), \quad T_t = T(0, t) \\
p_r &= p(L, t), \quad q_r = q(L, t), \quad T_r = T(L, t).
\end{align*}
\]

The subscripts are motivated by the definition of a positive \(x\)-direction from left to right but do not imply any specific flow direction, only that one is not free to prescribe both the pressure and flow at a single point. This yields the nonlinear nonisothermal 3-D model

\[
\begin{align*}
\dot{p}_r &= \frac{R_s z_0}{A_c c_v} \left[ k_{rad} \pi \tilde{d} (T_{amb} - T) \right. \\
&\left. - \frac{q_r - q_t}{L} T_r (c_v + R_s z_0) + \frac{p_r - p_t}{L} R_s z_0 T_r q_r \right. \\
&\left. - \frac{T_r - T_t}{L} q_r (c_v + R_s z_0) + \frac{\lambda R_s^2 z_0^2 T^2 q^2 |q_r|}{2dA_c^2 p_r^2} \right] \\
&\equiv f_p(p_t, p_r, q_t, q_r, T_t, T_r) \quad (4a) \\
\dot{q}_t &= -A_c \frac{p_r - p_t}{L} + \frac{\lambda R_s T z_0 q_t |q_t|}{2dA_c^2} - A_c \frac{\partial q}{\partial x} \frac{\partial p}{\partial x} - \frac{\partial q}{\partial x} \Delta g \frac{dh}{R_s z_0} \frac{dx}{p_t} \quad (4b) \\
\dot{T}_t &= \frac{R_s z_0 T}{A_c c_v p_r} \left[ k_{rad} \pi \tilde{d} (T_{amb} - T) - \frac{q_r - q_t}{L} T_r z_0 \right. \\
&\left. + \frac{p_r - p_t}{L} R_s z_0 T q_r \right. \\
&\left. - \frac{T_r - T_t}{L} q_r (c_v + R_s z_0) + \frac{\lambda R_s^2 z_0^2 T^2 q^2 |q_r|}{2dA_c^2 p_r^2} \right] \\
&\equiv f_T(p_t, p_r, q_t, q_r, T_t, T_r) \quad (4c)
\end{align*}
\]

We propose the discretization from (3) to (4) as it approximates reasonably well the original infinite-dimensional behavior at low frequencies relevant for our control problem, as discussed next.

Linearizing (4) results in the MIMO LTI 3-D state-space realization

\[
\begin{align*}
\dot{x}_t &= A x_t + B u_t \\
y_t &= x_t
\end{align*}
\]

where \(A \equiv (\partial f_t / \partial x_t)_{ss}\) and \(B \equiv (\partial f_t / \partial u_t)_{ss}\), with \(f_t \equiv \begin{bmatrix} f_p & f_q & f_T \end{bmatrix}^T\) and \((\partial f_t / \partial x_t)_{ss}\) indicating the Jacobian with respect to \(x\) evaluated at steady state (denoted by subscript \(ss\)) and \(B\) written accordingly. Furthermore, the state and input vectors are given by the following deviations from nominal/steady-state values:

\[
\begin{align*}
x_t &= [\tilde{p}_r \quad \tilde{q}_r \quad \tilde{T}_r]^T, \quad u_t = [\tilde{p}_t \quad \tilde{q}_t \quad \tilde{T}_t]^T
\end{align*}
\]

with

\[
\begin{align*}
\tilde{p}_t &= p_t - p_{t,ss}, \quad \tilde{p}_r = p_r - p_{r,ss} \\
\tilde{q}_t &= q_t - q_{ss}, \quad \tilde{q}_r = q_r - q_{ss} \\
\tilde{T}_t &= T_t - T_{t,ss}, \quad \tilde{T}_r = T_r - T_{r,ss}
\end{align*}
\]

We stress that for such a state-space realization, which is the basis for modern model-based control design, the preponderance of existing tools in linear systems theory is directly applicable, such as the determination of stability, dc gains, observability, and controllability. To assess sufficiency for control-oriented design, we will use this nonisothermal 3-D model as a benchmark for the reduced isothermal 2-D model introduced next. Where appropriate, we also compare the
solution of the linear system to both the nonlinear 3-D model (4) and the original PDEs (3).

IV. Isothermal 2-D Linear ODE Model

Assume that the temperature is constant, i.e., \( T(x, t) = T_0 \) for all \( x \in [0, L] \) and \( t \geq 0 \). The continuity, momentum, and gas equations in (1) suffice to obtain

\[
\begin{align*}
\frac{\partial p}{\partial t} & = -\frac{R_c T_0 z_0}{A_c} \frac{\partial q}{\partial x} \\
\frac{\partial q}{\partial t} & = -\frac{A_c}{\partial x} \frac{\partial p}{\partial x} - \frac{\lambda R_c T_0 z_0 q |q|}{2d A_c} + \frac{A_c g}{R_c T_0 z_0} \frac{dh}{dx} p
\end{align*}
\]

(6a)

(6b)

where for the mass flow, \( q \), we additionally used the relation \( q = \rho A_c v \). We also neglect the partial derivative of the inertia (or kinematic) term, \( \rho v^2 \), justified by the fact that the speed of sound, \( c \), usually greatly exceeds the velocity of the fluid [1, p. 174]. This is also consistent with Proposition 1.

Following [1], a spatial discretization of (6) yields

\[
\begin{align*}
\dot{p}_r & = -\frac{R_c T_0 z_0}{A_c L} (q_r - q_l) \\
\dot{q}_l & = -\frac{A_c}{L} (p_r - p_l) - \frac{\lambda R_c T_0 z_0 q_l |q_l|}{2d A_c} \frac{p_l}{p_l} - \frac{A_c g}{R_c T_0 z_0} \frac{h}{L} \frac{p_l}{p_l}
\end{align*}
\]

(7a)

(7b)

Linearizing around nominal points denoted by subscript ss and using tildes to denote perturbation variables, we obtain

\[
\begin{align*}
\dot{\tilde{p}}_r & = \alpha (q_r - \tilde{q}_l) \\
\dot{\tilde{q}}_l & = \beta \tilde{p}_r + \kappa \tilde{q}_r + \gamma \tilde{q}_l
\end{align*}
\]

(8a)

(8b)

with

\[
\begin{align*}
\alpha & = -\frac{R_c T_0 z_0}{A_c L} \\
\beta & = -\frac{A_c}{L} \\
\kappa & = \frac{A_c}{L} + \frac{\lambda R_c T_0 z_0 q_s |q_s|}{2d A_c} \frac{p_{l, ss}}{p_{l, ss}} - \frac{A_c g h}{R_c T_0 z_0 L} \\
\gamma & = -\frac{\lambda R_c T_0 z_0 |q_s|}{d A_c} \frac{p_{l, ss}}{p_{l, ss}}
\end{align*}
\]

The LTI ODEs (8) represent a system equivalently realized by

\[
\begin{align*}
\dot{x}_t & = \begin{bmatrix} 0 & -\alpha \\ \beta & \gamma \end{bmatrix} x_t + \begin{bmatrix} 0 \\ \kappa \end{bmatrix} u_t \\
y_t & = x_t
\end{align*}
\]

(9a)

(9b)

with \( x_t = [\tilde{p}_r \ \tilde{q}_l]^T \) as the state vector and \( u_t = [\tilde{p}_l \ \tilde{q}_r]^T \) as the input vector.

We note immediately several properties revealed by the linear model. The elements (\( \alpha, \beta, \gamma \)) of the system matrix are all negative and the matrix possesses two eigenvalues at \( (\gamma / 2) \pm (\gamma^2 / 4 - \alpha \gamma)^{1/2} \). The quantity \( \alpha \beta = R_c Z_0 T_0 / L^2 \) is the square of the resonant frequency of a pipe of length \( L \) since \( c = (Z_0 R_c T_0) \) is the speed of sound. The friction term \( \gamma \) is comparatively small. Thus, the linearized state-space model is that of a lightly damped resonant system.

In addition to stability, the control-oriented nature of the model allows us to deduce important properties, such as controllability. Input matrix \( B \) is full row rank, so \( (A, B) \) is reachable. If the pressure \( p_r \) is measured, then the system is also observable. Pressure is the simplest and most reliably measured process variable.

The dc gain from \( u_t \) to \( x_t \) can be readily extracted

\[
G_{DC} = -\frac{c A_c}{\beta} \begin{bmatrix} -\kappa & -\beta \gamma \\ 0 & -\beta \alpha \end{bmatrix}
\]

(10)

and reveals the following. In a steady state, the following conditions hold.

1) \( \tilde{p}_r \) is less than or equal to \( \tilde{p}_l \), with pressure loss due to nonzero flow and elevation.

2) Regardless of the pressure, \( \tilde{q}_l \) is equal to \( \tilde{q}_r \), in steady state, as demanded by conservation of mass.

A more detailed analysis will be provided for the 3-D state model in Section IX.

Spatial Discretization: The spatial discretization of the PDEs using \( p_l \) and \( q_r \) as the input signals is neither capricious nor refractory but reflects two central matters; the boundary conditions required to specify the solution for pipe flow and the requirement for reachability of the resultant state-space model. The two properties are not disjoint. Assuming horizontal pipes, the two PDEs (6a) and (6b) may be combined to yield the damped wave equation

\[
\frac{q^2}{L^2} \frac{\partial^2 X}{\partial x^2} - \frac{\lambda c^2 q|q|}{2d A_c} \frac{\partial X}{\partial x} = \frac{q^2}{c^2} \frac{\partial^2 X}{\partial t^2} + \frac{\lambda \ q}{d A_c} \frac{\partial X}{\partial t}
\]

for either \( X(t, x) = p(t, x) \) or \( q(t, x) \) with distinct boundary conditions. This PDE is hyperbolic and requires Dirichlet, Neumann, or mixed boundary conditions at both ends to define the solutions [22]. Pressure \( p(t) \) provides the left Dirichlet boundary condition and, via (6b), \( q(t) \) provides the right mixed boundary condition.

An alternative view of this spatial discretization is that, drawing on the electrical transmission line analog of the pipe, the voltage/pressure and current/flow at one end of the line/pipe may not be independently prescribed since they are constrained by the driving-point impedance. From the control system perspective of this article, the selection of \( p_l \) and \( q_r \) as input signals would not yield the requisite system model reachability nor solubility mentioned above.

Cascaded Pipe Models: As discussed in Section I, it is our primary concern to provide sufficiently accurate models for frequencies below 1 Hz well-suited for process control for facilities with pipes of length of around tens of meters (rather than kilometers). Toward this goal, in Fig. 1, we compare the frequency responses of a single pipe of 30 m with those of two 15-m pipes and three 10-m pipes in series using the composite model for pipes from Section VII-C. This represents successively finer spatial discretizations of the PDE models. Thus, the ODE solutions are closer approximations of the PDE solutions. We observe that the behaviors for relevant low frequencies indeed coincide; changes for high frequencies are outside the relevant range and account for acoustical modes associated with the configurations and boundaries. Per the control objective of managing bulk pressures and mass flows, these low-frequency modes are preserved, while the zero-mean high-frequency resonances fall increasingly outside the sensor.
and actuator bandwidths. This direct analysis conforms to the control-orientated aspect of our model and serves as a guide to bound errors introduced through spatial discretization. We note that other fluid dynamic phenomena, such as turbulence and cross-pipe variation, are also neglected with this choice of PDEs.

A. Nonisothermal Modeling and Bernoulli

To ensure sufficient accuracy of linear models, it is important around which nominal point they are applied. Although one may use (7) to generate the corresponding values, we do so by solving the constituent equations in (1) directly for steady-state values. In this fashion, first, we are able to accommodate spatially varying temperatures, and second, we reveal the error inherent to the isothermal assumption and avoid its propagation.

Proposition 2: Suppose that at steady state, the change in density along the pipe is negligible. Then, the constituent equations in (1) yield

\[
q_{r, ss} = q_{L, ss} \tag{10a}
\]

\[
p_{r, ss} = p_\ell_{r, ss} \exp \left( \frac{\lambda L z_0 R_T r_{r, ss}}{2 d A c^2 p_{r, ss}} q_{r, ss} |q_{r, ss}| \right) - \frac{g h}{R_z z_0 T_{r, ss}} \tag{10b}
\]

If further \(|v|, |gh| \ll c, d \geq (\lambda / 2), \) and \(L |v| \ll c, \) then

\[
p_{r, ss} \approx p_\ell_{r, ss} \left( 1 - \frac{\lambda L z_0 R_T r_{r, ss}}{2 d A c^2 p_{r, ss}} q_{r, ss} |q_{r, ss}| \right)
\]

\[
- \frac{g h}{R_z z_0 T_{r, ss}} \right) \tag{10c}
\]

where \(T_{\ell, ss} = T(0) \) and \(T_{r, ss} = T(L) \) at steady state.

Proof: For brevity, we drop subscript ss in this proof. The nominal mass flow in (10a) follows directly from the continuity equation (1a) by setting the time derivative to zero. The nominal pressure in (1b), for the left-hand side, Lurie [19] showed that

\[
\frac{\partial}{\partial t} (\rho v) + \frac{\partial}{\partial x} (\rho v^2) = \rho \left( \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} \right).
\]

Now, at steady state, \((\partial v/\partial t) = 0 \) and (1b) holds

\[
v \frac{\partial v}{\partial x} = -\frac{1}{\rho} \rho \frac{\partial v}{\partial x} dx - \frac{\lambda}{2d} v^2 |dx - g dh
\]

\[
= -\frac{1}{g} d p - \frac{\lambda}{2d g} v |dx - d h
\]

with length \(dx\). We used the fact that the change in velocity, \(dv\), and pressure, \(dp\), along a control volume at steady state is exactly \(\partial(\cdot)/\partial x\rangle dx\). Without loss of generality, we now assume that the height at \(x = 0\) is zero. In addition, under the hypothesis and (10a), we can treat the velocity as a constant so that integrating (11) along the pipe using (1d) yields

\[
v^2 - v^2_s = -\frac{R_s z_0}{g} (T_r \ln p_r - T_\ell \ln p_\ell) - \frac{\lambda L}{2d g} |v| - h. \tag{12}
\]

As \(v_r = v_\ell \) and toward an expression for \(p_r \)

\[
0 = -\ln \left( \frac{p_r}{p_\ell} \right)^{T_r / T_\ell} - \frac{\lambda L}{2d R_s z_0} v_r |v_r| - \frac{g h}{R_s z_0} \tag{10b}
\]

which with (1d) gives (10b). By the additional hypothesis

\[
|gh| \ll c^2 = R_s T_r z_0
\]

\[
\frac{\lambda L}{2d v_r^2} \leq L v_r^2 \ll c^2 = R_s T_r z_0
\]

so that again using the gas equation (1d)

\[
p_r \approx p_\ell \exp \left( \frac{\lambda L}{2d R_s z_0 T_r} v_r |v_r| - \frac{g h}{R_s z_0 T_r} \right)
\]

\[
= p_\ell \left( 1 - \frac{\lambda L}{2d A c^2 p_r} q_r |q_r| - \frac{g h}{R_s z_0 T_r} \right). \tag{10b}
\]

1) On the Assumptions: For better understanding of conditions under which the assumptions hold and to underline the model’s suitability for control, consider methane with \(R_s = 518.28 \text{ J/mol K}\), a low temperature of \(T_r = 300 \text{ K}\), and a constant compressibility factor \(z_0 = 0.95\). The related speed of sound within the medium is \(c = 14.77 \times 10^4 \text{ m/s}\). Hence, the assumptions on the gas velocity, \(v\), height, \(h\), and
length, \( L \), conform to typical values in our control domain of gas processing facilities. Also, given a usual friction factor \( \lambda \ll 1 \), the lower bound on the diameter, \( d \), renders our formula applicable to many industrial scenarios.

2) Relation to Bernoulli’s Equation and Isothermal Model:
The proof of Proposition 2 is of interest in itself since it delineates the relation between the dynamic momentum equation (1b) and static Bernoulli’s equation (12) commonly used for computing static variables, including a term for head loss, \( H_l = (\lambda L/2d g)v|v| \), often referred to as the Darcy–Weissbach equation [18]. Furthermore, observe that the approximated nominal point (10c) coincides with that derived by the discretized model, (7), under the isothermal assumption and negligible change in density. In other words, Proposition 2 also quantifies the error induced through the isothermal assumption.

V. Model Validation
We now wish to assess both the isothermal and nonisothermal models in light of their suitability for control-oriented design, using operational industrial process data from the GCTF. The data fit the problem formulation: it is sampled at 1 Hz and describes pressure, mass flow, and temperature variations over a wide range for pipes on the order of tens of meters, at low frequencies. Accordingly, it is appropriate for model validation and tuning for this application, especially provided the linearization around one nominal point. Our conclusion is that, for pipe component modeling, the isothermal 2-D model is sufficient for model-based control because temperature variations in these elements are modest, temperature sensing devices can be both limited in number and variable in dynamic response, and variations with temperature can be accommodated by an appropriate controller since they are slowly varying and cause quantifiable gain fluctuations.

Fig. 2 shows the facility at Solar Turbines Inc. This is a well-instrumented site used for compressor testing, from which comprehensive datasets are available. The particular pipe section under consideration is sketched in Fig. 3. Notice that we simplify the stepped pipe geometry by neglecting the stub at the end of the vertical middle section, assuming instead a constant slope and an accordingly adjusted friction factor.\(^3\)

The relevant data are plotted in Fig. 4, with behavior in the relevant time scale for our goal of relatively slow process control. We observe that the output pressure, \( \tilde{p}_r \), closely follows the input pressure, \( \tilde{p}_i \), and is higher despite head losses through friction. This is due to the vertical middle section and heat flux causing changes in temperature. As shown in Fig. 4, the temperature is relatively constant but varies with changes in pressure and mass flow. We notice that \( \tilde{T}_L \) is measured by a more accurate sensor, given its lower quantization error, which can be observed in the middle zoom. We additionally note that \( \tilde{T}_L \) is lower than \( \tilde{T}_r \) for most of the time, as is also apparent in the zoom on the right, and it seems to be dynamically faster than \( \tilde{T}_r \), as shown in the left zoom, which may partly be caused by different thermal inertias and processing of the sensors. Given the speed, accuracy, and prevalence of pressure sensors, it is apparent that they will provide the primary signals used for feedback control and the quality of capturing the pressure state behavior should be the main model objective. We shall return to this shortly in Section V-C.

A. Linear Nonisothermal 3-D Model
We begin by validating the nonisothermal 3-D model from Section IV linearized around the nonisothermal nominal point developed in Proposition 2. In particular, let \( q_{r,ss} = \text{mean}(q_r(t)) \), \( p_{i,ss} = \text{mean}(p_i(t)) \), \( T_{L,ss} = \text{mean}(T_L(t)) \), and \( T_{r,ss} = \text{mean}(T_r(t)) \) so that Proposition 2 yields the corresponding nominal values \( p_{r,ss} \) and \( q_{r,ss} \). We then use \( \tilde{p}_i \)

\(^3\)We use Haaland’s formula [24] to estimate the friction factor for the straight pipes and empirical formulas in [18, Ch. 15] to approximate the friction losses induced by the bends and stub.
that the modeled mass flow is measured pressure but has a small offset. In addition, we observe the simulated output of the nonisothermal 3-D linear model with data as model inputs, compared against GCTF data and PDE model (1).

The lumped thermal conductivity, $k_{\text{lad}}$, is approximated at a quasi-steady state following [21, Sec. 3], using $T_{\ell,ss}$ and $T_{r,ss}$. The simulation results are shown in Fig. 5.

For the nonisothermal 3-D linear model, we see that the simulated output in a small neighborhood of the measured pressure but has a small offset. In addition, we observe that the modeled mass flow is close to the data input, $\tilde{q}_r$. This is expected for the short length of the pipe $L \approx 30$ m and the sampling rate of once per second. The zoom reveals that when the pressure increases at around 800 s, the model output first increases before the signal input, $\tilde{q}_r$, follows suit. This is consistent with a positive mass flow that increases first at the gas entry side of the pipe.

The temperature calculations from the successive models, while close (within 0.64 K), exhibit more variability than those of pressure and mass flow. The computed $\tilde{T}_r$ values also exceed the $\tilde{T}_r$ data at times, especially for the linear model. Furthermore, there are times, around 500 s for example, where the $\tilde{T}_r$ data also exceeds $\tilde{T}_r$ data. These discrepancies indicate three types of sources: the difference in predicted temperatures between the PDE model and linear model point to discretization and linearization errors; the entry and exit temperature sensors have differing response times and accuracies, as is common in application; and the heat flux model in (2) is too simplistic to capture the dependence of heat flux on velocity and geometry (see [25] for more detailed analysis of these phenomena). From a control-oriented perspective, this adds further weight to accommodating these slow variations—we quantify time constants shortly in Section V-C via eigenvalue analysis—through the design of the controller and to preserve the parsimony of the linear model, which captures the salient dynamics.

The linear model and the PDE model mostly coincide, especially for the predicted pressure and mass flow, which are our main concern for control purposes. This is also consistent with the frequency analysis from above.

**B. Linear Isothermal 2-D Model**

Consider now the isothermal 2-D model for which the model parameters and nominal point are equal to those of the nonisothermal 3-D model above, except the temperature, which we set to $T_0 = (T_{\ell,ss} + T_{r,ss})/2$. As before, $\tilde{p}_r$ and $\tilde{q}_r$ from the data are model inputs, and we compare the modeled pressure $\tilde{p}_r$ against $\tilde{p}_r$ from the data. The result is shown in Fig. 6. Notice that the modeled responses for pressure and mass flow seem congruent with those of the nonisothermal 3-D model, i.e., the modeled pressure is close to the measured pressure, but displays a small static offset. The mass flows at both ends of the pipe are close, consistent with conservation of mass at a steady state.

**C. Isothermal 2-D Versus Nonisothermal 3-D Model**

The results above are now evaluated in view of the control-oriented aspect of our approach. The similarity of both the isothermal and nonisothermal models and their accuracy characterize Fig. 7, which shows the relative error between the modeled and the measured pressure. The errors of the respective models are closely aligned, rather constant, and at most at a rate of $4 \times 10^{-3}$. Both the isothermal 2-D and the nonisothermal 3-D linear models exhibit almost identical small offsets in simulated pressure and both capture the pressure dynamics accurately. From a control design perspective, the controller can be constructed to accommodate this modeling error.

Computing the eigenvalues of the system matrices of the related isothermal 2-D and nonisothermal 3-D linear models,
respectively, $A_{\text{iso}}$ and $A_{\text{niso}}$, and of the truncation of $A_{\text{niso}}$ to its first two rows and columns, $[A_{\text{niso}}]_{1:2}$, we have

$$\text{eig}(A_{\text{iso}}) = (-3.90 \pm 12.47 i)$$
$$\text{eig}(A_{\text{niso}}) = (-3.90 \pm 14.31 i, -0.12)$$
$$\text{eig}([A_{\text{niso}}]_{1:2}) = (-3.88 \pm 14.31 i) \approx \text{eig}(A_{\text{iso}}).$$

From this, we conclude that the temperature state is both effectively decoupled from the pressure and mass flow states and, further, governed by a time constant approximately 30 times that of the reduced-order 2-D system, which preserves the dominant lightly damped oscillatory dynamics. Consequently, for moderate temperature gradients, it is reasonable to take the temperature as a constant and employ the isothermal 2-D model.

Pressing on with this control-oriented analysis, we note the respective dc gains

$$\lim_{t \to \infty} x_{2D} = A_{\text{iso}}^{-1} B_{\text{iso}} = \begin{bmatrix} 1.004 & -600.19 \\ 0 & 1 \end{bmatrix}$$
$$\lim_{t \to \infty} x_{3D} = A_{\text{niso}}^{-1} B_{\text{niso}} = \begin{bmatrix} 1.004 & -600.33 & -25.35 \\ 0 & 1 & 0 \\ 0 & 0.03 & 0.92 \end{bmatrix}.$$

Continuing the discussion in Section IV, steady-state conservation of mass flow follows for both models as the dc gains from $(\tilde{p}_r, \tilde{T}_l) \to \tilde{q}_l$ are zero and $\tilde{q}_r \to \tilde{q}_l$ is precisely one. For the steady-state pressure, there is (to two decimal places) a unity gain from $\tilde{p}_l \to \tilde{p}_r$, indicating that changes due to friction and height differences are marginal (cf. $\kappa$ in Section IV), and a drop of similar size for both models from $\tilde{q}_r \to \tilde{p}_r$ due to additional friction (cf. $\gamma$ in Section IV) for this example. The negative gain from $\tilde{T}_l \to \tilde{p}_r$ for the nonisothermal model may be due to larger heat losses to the environment, a characteristic not captured by the isothermal model. Yet, given the magnitude of the SI units used here and low-temperature variations in pipe elements, the consequential discrepancy is small, as corroborated by the simulations.

The isothermal 2-D model, which relies only on mass flow and pressure measurements, dovetails with the fact that in particular, pressure sensors (in contrast to temperature sensors) are usually well-distributed in gas processing facilities, fast and reliable. The 2-D isothermal model will be used for pipe segments and the control design will be expected to accommodate the small offsets and slow variation of dynamics with changing temperatures. The experimental results and customary practice of sparse temperature measurements (also due to slow temperature sensing responses) suggest that temperatures in typical pipes need not directly be modeled using the 3-D model; exceptions are heat exchangers, compressors, and other strongly temperature-affecting devices. Driven by this evaluation, we continue the exposition with a focus on this isothermal model.

### Part 2: Control-Oriented Models of Pipe Networks

#### VI. DAEs, Signal Flow Graphs, and Bond Graphs

Bond graphs [13] provide a systematic method for deriving dynamic equations for interconnected electromechanical–hydraulic systems. They combine effort variables and flow variables, with component properties linking the two types and conservation laws and continuity governing the flows at the interconnection. In the framework of fluid flow in pipe networks [1], [19], this leads to a set of PDEs for the dynamics combined with algebraic equations for the constraints. Discretizing the spatial derivative yields DAE system models, which are problematic for direct control design for these interconnected systems. In contrast, signal flow graphs (SFGs) correspond to systems described exclusively by ODEs, transfer functions in the linear case. Interconnected systems are directly managed by methods such as Mason’s gain formula for the linear case or by writing the composite state-variable ODEs without algebraic constraints. It is these latter model forms, which are amenable to control design tools.

We consider three fundamental interconnections of pipe elements: series connection, branching, and joining. Using the isothermal 2-D model above, we develop a catalog of composite models that describe common units in the form of interconnections of pipes. In this way, algebraic constraints and DAEs will be avoided, as exemplified through the component of joining pipes introduced first. For clarity, we limit this section to the 2-D model, but the methodology is equivalently applicable to the 3-D model and without loss of generality, we assume that the steady-state mass flow, $q_{ss}$, is positive, that is, $\gamma_l$ denotes the side where the steady-state mass flow enters the pipe and $\gamma_r$ the side with an outgoing mass flow, hence the denomination joint and branch to come.

The reduced state vector demonstrates that an interconnection of single pipes into more complex components, with corresponding algebraic constraints, cannot immediately be translated to an SFG using only single pipe models. We also point out that on the contrary, bond graphs [13] are able to represent more complex components, including algebraic constraints. However, constraints, such as those in (13), would lead to a causal conflict of type 1 and degree 1 [13, Definition 4.19], which in turn implies the existence of DAEs and therefore disaccords with our objective of control-oriented modeling.

#### VII. From DAEs of Index 1 to Composite Models

##### A. Joint

Consider the joint shown in Fig. 8(a) and let $p_{l,r}$ ($p_{r,l}$) be the pressure $p_{l,r}$ of pipe $P_l$. The mass flow is denoted...
may be rewritten as an unconstrained five-state system

\begin{align}
\tilde{p}_{1,r} &= \tilde{p}_{2,r} = \tilde{p}_{0,\ell} \\
\tilde{q}_{0,\ell} &= \tilde{q}_{1,\ell} + \tilde{q}_{2,\ell}.
\end{align}

The first equation is related to continuity and the second equation represents the conservation of mass at the junction. This composite joint model would have a state of dimension six: \([\tilde{p}_{0,\ell}, \tilde{p}_{1,r}, \tilde{p}_{2,r}, \tilde{q}_{0,\ell}, \tilde{q}_{1,\ell}, \tilde{q}_{2,\ell}]^\top\), in lexicographic ordering, plus the algebraic constraints [see (13)]. However, due to (13a), we can omit \(\tilde{p}_{2,r}\) as a state [which would naturally arise in three pipe models (9)].

Define \(\alpha_1\) and \(\alpha_2\) to be the parameters in (9) for pipes 1 and 2 and

\[\delta = \frac{\alpha_1}{\alpha_1 + \alpha_2}.\]

Then, the six-state composite joint system plus constraint (13) may be rewritten as an unconstrained five-state system

\[
\begin{align}
\dot{x}_i &= A_j x_i + B_j u_t \\
y_i &= C_j x_i + D_j u_t
\end{align}
\]

with

\[
A_j = \begin{bmatrix}
0 & 0 & -\alpha_0 & 0 & 0 \\
0 & 0 & \alpha_1 (1 - \delta) & -\alpha_1 (1 - \delta) & -\alpha_1 (1 - \delta)
\end{bmatrix}
\]

\[
B_j = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[D_j = 0_{3 \times 3}.\]

The state, input, and output vectors are now

\[
\begin{align}
x_i &= [\tilde{p}_{0,\ell}, \tilde{p}_{1,r}, \tilde{q}_{0,\ell}, \tilde{q}_{1,\ell}, \tilde{q}_{2,\ell}]^\top \\
u_t &= [\tilde{p}_{1,r}, \tilde{p}_{2,r}, \tilde{q}_{0,\ell}]^\top \\
y_i &= [\tilde{p}_{0,\ell}, \tilde{q}_{1,\ell}, \tilde{q}_{2,\ell}]^\top.
\end{align}
\]

The calculation of the steady-state gain from input three, \(q_{0,\ell}\), to outputs two, \(q_{1,\ell}\), and three, \(q_{2,\ell}\), shows that, in steady state

\[
\tilde{q}_{1,\ell} + \tilde{q}_{2,\ell} = \frac{\beta_1 \gamma_2}{\beta_1 \gamma_2 + \beta_2 \gamma_2} \tilde{q}_{0,\ell} + \frac{\beta_2 \gamma_1}{\beta_1 \gamma_2 + \beta_2 \gamma_2} \tilde{q}_{0,\ell} = \tilde{q}_{0,\ell}.
\]

That is, this five-state composite joint model satisfies the conservation of mass [see (13a)]. Constraint (13a) is redundant since the variables \(\tilde{p}_{2,r}\) and \(\tilde{q}_{0,\ell}\) have been removed; they can be computed from (13a). The new model parameter \(\delta\), defined in (14), describes the nominal proportion of flow \(\tilde{q}_{0,\ell}\) attributed to each of the feeding pipes. This is the formal process of removing the constraint from the DAE of index 1.

**B. Branch**

Differently from the joint, for the branch in Fig. 8(b), the equality constraint on the pressures relates the state variable, \(p_{0,\ell}\), to input signals of the single pipe model of the branching pipes, \(\tilde{p}_{1,\ell}\) and \(\tilde{p}_{2,\ell}\), i.e.,

\begin{align}
\tilde{p}_{0,\ell} &= \tilde{p}_{1,\ell} = \tilde{p}_{2,\ell} \\
\tilde{q}_{0,\ell} &= \tilde{q}_{1,\ell} + \tilde{q}_{2,\ell}
\end{align}

so that the dimension of the composite model does not reduce, but is equal to the direct sum of those of the single pipe models of the individual pipes. Similarly, constraint (16b) on the mass flows does not prescribe any interdependence of any input variables but rather sets the input signal of the single pipe model of pipe \(P_0\) as the sum of two other state variables. Hence, an additional parameter, such as \(\delta\) for the joint, is absent. The related matrices for a branch model are

\[
A_b = \begin{bmatrix}
0 & 0 & 0 & -\alpha_0 & \alpha_0 \\
0 & 0 & 0 & 0 & -\alpha_1 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
B_b = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

with state \(x_i = [\tilde{p}_{0,\ell}, \tilde{p}_{1,r}, \tilde{p}_{2,r}, \tilde{q}_{0,\ell}, \tilde{q}_{1,\ell}, \tilde{q}_{2,\ell}]^\top\), input \(u_t = [\tilde{p}_{0,\ell}, \tilde{q}_{1,\ell}, \tilde{q}_{2,\ell}]^\top\), and output \(y_i = [\tilde{p}_{1,r}, \tilde{p}_{2,r}, \tilde{q}_{0,\ell}]^\top\). The feedthrough matrix \(D_b\) is zero.

**Remark 1:** It is straightforward to expand these ideas to intersections comprising \(m\)-input pipes and \(n\)-output pipes. This construction is available at [26] and generalizes the systematic reduction of index-1 DAEs to systems of ODEs.

**C. Pipes in Series**

\(N\) pipes in series are shown in Fig. 9 and are of particular interest if pipe parameters (see Nomenclature) change along the dimension of \(x\) or the discretization error grows too large for a given length. For conciseness, we only state the relevant matrices here that result from the continuity conditions and conservation of mass, i.e.,

\[
\tilde{p}_{i,\ell} = \tilde{p}_{i+1,\ell}, \quad \tilde{q}_{i,\ell} = \tilde{q}_{i+1,\ell}
\]
with \( i \in \{0, 1, \ldots, N - 2\} \). The state, input, and output elements \( p_{i,r} \) and \( q_{i,\ell} \) in lexicographical order, i.e., \( x_i = \left[ \hat{p}_{0,r} \cdots \hat{p}_{N-1,r} \hat{q}_{0,\ell} \cdots \hat{q}_{N-1,\ell} \right]^\top \), \( u_i = \left[ \hat{p}_{0,t} \hat{q}_{N-1,t} \right]^\top \), and \( y_i = \left[ \hat{p}_{N-1,r} \hat{q}_{0,l} \right]^\top \), yield

\[
A_s = \begin{bmatrix} 0 & A_{s,12} \end{bmatrix}, \quad B_s = \begin{bmatrix} B_{s,1}^\top & B_{s,2}^\top \end{bmatrix}^\top
\]

\[
C_s = \begin{bmatrix} 0_{2,2(N-1)} & I_2 \end{bmatrix}, \quad D_s = 0,
\]

where the subscripts for 0 and 1 describe the dimension and \( \beta_0, \gamma_0 \) are the respective \( \beta \) and \( \gamma \) of (19) and (21) is given by

\[
\tilde{\dot{x}}_t = \bar{A} \tilde{x}_t + B \tilde{u}_t, \quad \tilde{y}_t = C \tilde{x}_t + D \tilde{u}_t
\]

the state-space models of the individual network components. With some abuse of notation

\[
\begin{align}
\dot{x}_i &= A x_i + B \bar{u}_t, \quad y_i(t) = C x_i + D \bar{u}_t \\
A &= \text{blkdiag}(A^{(1)}, A^{(2)}, \ldots, A^{(N)}) \\
B &= \text{blkdiag}(B^{(1)}, B^{(2)}, \ldots, B^{(N)}) \\
C &= \text{blkdiag}(C^{(1)}, C^{(2)}, \ldots, C^{(N)}) \\
D &= \text{blkdiag}(D^{(1)}, D^{(2)}, \ldots, D^{(N)})
\end{align}
\]
components. However, if only some variables constitute to the total output, modifying \( \tilde{y}_i \) is a simple exercise through the multiplication of \( \tilde{C} \) and \( \tilde{D} \) by an appropriate selection matrix. Given the fact that \( \tilde{D} \) maps input \( \tilde{w}_i \) to outputs \( \tilde{y}_i \) and \( \tilde{F} \) maps \( \tilde{y}_i \) to \( \tilde{u}_i \), if no algebraic loops exist, the matrix product \( \tilde{D} \tilde{F} \) is zero and \( (I - \tilde{D} \tilde{F}) \) is invertible.

Uncontrollable or unobservable flow networks are easily constructed. For example, the connection of two identical branches in parallel would be uncontrollable and in series without a measurement at the junction would be unobservable. Thus, the nonminimality of the construction is apparent and mirrors that of electrical circuits, whose utility of analysis we seek to emulate. In [14], the composite network models presented here are used to develop digital controllers for process control applications. This includes the insertion of antialiasing filters around 0.4 Hz, matched to the 1-Hz sampling rate. These sharp-roll-off low-pass filters perform two functions: they render the collective acoustic resonant modes unobservable in the output signals and they limit closed-loop stability considerations to the passband of the controller since the systems are open-loop stable. This unobservability/uncontrollability is used to advantage in [14] to effect the dramatic reduction of model order prior to discrete control design. The relegation of acoustic modes to outside the bandwidth of the controller is part of the concept of control orientation in these models.

B. Subsuming Mason

Next, we show that the state-space realization above subsumes Mason’s gain formula [27]. The latter is a method to find transfer functions of SFGs with multiple inputs and multiple outputs and has also been established in a simple matrix form in, e.g., [28]. The interest in this equivalence result lies in its generality for linear systems and advantage over Mason’s gain formula via simple matrix manipulation without relying on symbolic matrix inversions with transfer functions as matrix elements. Furthermore, the calculation in Proposition 3 yields all the closed-loop transfer functions between each input and each output, versus Mason, which computes SISO transfer functions using Cramer’s rule.

Mason’s gain formula formulation in [28] starts by writing the vector of output signals, \( \tilde{y} \), as the interconnection of \( \tilde{y}_i^{(0)} \) and \( \tilde{u}_i \) with transfer function matrices

\[
\tilde{y}_t = \mathcal{Q} \tilde{y}_t + \mathcal{P} \tilde{u}_t.
\]

and Mason’s gain formula is then that the solution is given by

\[
\tilde{y}_t = (I - \mathcal{Q})^{-1} \mathcal{P} \tilde{u}_t.
\]

Proposition 4: A state-variable realization of Mason’s gain formula transfer function, \( (I - \mathcal{Q})^{-1} \mathcal{P} \), is given in (23). This is proven in the Appendix.

IX. NUMERICAL EXPERIMENT

We apply our modeling methodology to the loop shown in Fig. 10, which represents a hypothetical pipe loop at the GCTF. Such a feedback system creates problems for DAE methods, such as those in [1], because of the algebraic constraints. Here, we use it as a proof-of-concept test case and rely on, rather unrealistically but similarly to [1] for distribution networks, on isothermal models and treatment of the compressor and valve as static gains. Clearly, the thermal properties of compressors, heat exchangers, and valves play an important role on the spatial scales of gas processing facilities and these will form the focus for ongoing modeling. A comparison against a full PDE simulation as in Section V exceeds the scope and capabilities of this article, as dynamic models for components, such as compressors, valves, and intersections, require high-fidelity CFD simulations, which in turn would need to be validated.

The gas is methane and flows clockwise, entering through pipe \( P_1 \) and exiting through pipes \( P_6 \) and \( P_9 \). The aim is to regulate the pressures \( p_{3,9} \) and \( p_{7,10} \) in the face of leakage via \( P_6 \). The Haaland formula [18] and assumed parameters\(^4\) yield a friction factor for each pipe of \( \lambda = 0.0111 \).

A. Network Model

The compressor and valve, whose corresponding variables are, respectively, labeled by subscripts \( c \) and \( v \), are modeled as static gains

\[
D_c = \begin{bmatrix} k_c & 0 \\ 0 & 1 \end{bmatrix}, \quad D_v = \begin{bmatrix} k_v & 0 \\ 0 & 1 \end{bmatrix}
\]

where \( k_c = 4 \) and \( k_v = 0.8 \). Furthermore, pipes \( (P_1, P_2, P_3) \) are modeled as a joint, as in (15), and \( (P_5, P_6, P_7) \) and \( (P_8, P_9, P_{10}) \) are modeled as branches, as in (17). Composing the system according to (20) results in the component input vector

\[
\tilde{w}_t = \begin{bmatrix} \tilde{p}_{1,t} & \tilde{p}_{2,t} & \tilde{q}_{3,r} & \tilde{q}_{4,r} & \tilde{q}_{5,r} & \tilde{q}_{6,r} & \tilde{q}_{7,r} & \tilde{q}_{8,r} & \tilde{q}_{9,r} & \tilde{q}_{10,r} \end{bmatrix}^T
\]

and the total output vector

\[
\tilde{y}_t = \begin{bmatrix} \tilde{p}_{3,r} & \tilde{q}_{1,t} & \tilde{q}_{2,t} & \tilde{q}_{4,r} & \tilde{q}_{5,r} & \tilde{q}_{6,t} & \tilde{q}_{7,r} & \tilde{q}_{9,r} & \tilde{q}_{10,r} & \tilde{q}_{8,t} \end{bmatrix}^T.
\]

The inputs of the total system are

\[
\tilde{u}_t = \begin{bmatrix} \tilde{p}_{1,t} & \tilde{q}_{6,t} & \tilde{q}_{9,r} \end{bmatrix}^T.
\]

\(^4\) All pipes are assumed to have the same geometry, i.e., \( L = 10 \text{ m}, d = 0.7 \text{ m}, \) and roughness \( \approx 4.5 \times 10^{-3} \text{ m} \). Furthermore, we assume that \( Re \approx 1.168 \times 10^6, T_{0} = 300 \text{ K}, \gamma_0 = 0.95, \rho_g = 518.28 \text{ J/(mol K)}, \tilde{p}_{la,t} = 25 \times 10^5 \text{ Pa}, \) and \( \tilde{q}_{in} = 21 \text{ m/s}^2 \).

Fig. 10. Pipe network with compressor and valve \( = c \). In process control parlance, the fill pressure and vent flow are manipulated variables, the suction and distal pressures are controlled variables, and the flow from \( P_6 \) is a disturbance signal.

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With (22), the total input and output vector, $\tilde{u}$, and $\tilde{y}$, as well as the component input vector, $\tilde{u}_i$, are the basis for the construction of $F$ and $G$. For example, $[\tilde{u}]_l = \tilde{p}_{1,l}$ is an input of the total system and the first element of $\tilde{u}_l$. Hence, $[G]_{1,1} = 1$. Furthermore, $[\tilde{u}]_2 = \tilde{p}_{2,l}$ connects to $\tilde{p}_{10,r}$, and $\tilde{y}_{11,r}$ so that $[F]_{2,14} = 1$. Similarly, $[\tilde{u}]_11 = \tilde{q}_{6,r}$ is another total input, i.e., $\tilde{q}_{6,r} = [\tilde{u}]_2$, so that $[G]_{1,1,2} = 1$. In this way, by passing through $\tilde{u}_i$ and following (22), we can fill the matrices with ones at the appropriate location and zeros otherwise. The eigenvalues of the resulting interconnected system all have negative real part; hence, the stability is demonstrated. Some eigenvalues have large imaginary parts pointing to the highly oscillatory resonant modes, which we ignore in the control design due to the presence of antialiasing filters in the sensors [14].

### B. Steady State: Conservation of Mass

The isothermal LTI closed-loop system is stable with the overall pressure static gains from $\tilde{p}_{1,l}$ to all but $\tilde{p}_{2,r}$ greater than one. Increasing the compressor and/or valve gains can bring about instability, as might be expected. Furthermore, since the frequency response of each component is available, standard stability tests may be performed. Indeed, the control design is to construct a stabilizing two-input/two-output regulator to reject the effect of the disturbance flow.

To evaluate the model in terms of conservation of mass, we also analyze the steady-state gains from the three loop inputs, $\tilde{p}_{1,l}, \tilde{q}_{6,r}$, and $\tilde{q}_{9,r}$, to each pipe’s mass flow. The corresponding dc-gain values are shown in Table I. Each column represents one model input and each row shows the corresponding steady-state change in mass flow from nominal due to a unit step change of the respective input and zero inputs otherwise.

1) **Step Response Fill Pressure Change:** Evaluating the first column with input $\tilde{p}_{1,l}$, a zero change in mass flows $\tilde{q}_{6,r}, \tilde{q}_{9,r}$ is consistent the other zero inputs, $\tilde{q}_{9,r} = \tilde{q}_{6,r} = 0$. As a result, the steady-state mass flow $\tilde{q}_{1,l} = 0$. A higher fill pressure leads to a larger mass flow around the loop, uniformly through all pipes, as evident by the numerical values of the other rows of the same column.

2) **Step Responses Vent and Disturbance Flow Changes:** Evaluating the second column with input $\tilde{q}_{9,r} = 1$ and zero disturbance flow, i.e., $\tilde{q}_{6,r} = 0$ (and hence $\tilde{q}_{6,r} = 0$), 1 kg/s enters the loop through $\tilde{q}_{9,r}$ so that $\tilde{q}_{9,r} = 1$. We further note that mass flow around the loop uniformly dropped by $-0.022$, excluding pipes $P_{10}$ and $P_2$. The flow through pipes $P_{10}$ and $P_2$ reduces by $-1.022$ as a result of the reduced overall flow and unit flow exiting through pipe $P_9$. Then, the additional flow $\tilde{q}_{1,l} = 1$ through pipe $P_1$ brings the flow back to $-0.022$. The same reasoning can be applied to the last column related to the disturbance input $\tilde{q}_{6,r}$.

Our analysis shows that the conservation of mass around the loop is captured through the use of composite models and the matrix methodology presented above, without imposing additional algebraic constraints. Furthermore, the linear LTI model model is amenable to direct feedback controller design and stability analysis.

### X. Conclusion and Further Directions

In this article, we present control-oriented models in the form of LTI state-space realizations that capture the dominant dynamics for the pressure, mass flow, and temperature in pipes at a scale appropriate for gas processing facilities. Validation against real-world data and simulation of the initial constituent equations illustrate their suitability for model-based controller design, which will incorporate requirements for robustness to minor static offsets and slow variations. Building on these models, we elaborate on the need for composite elements for interconnections to absorb DAEs and provide a corresponding catalog of composite models for common units. To increase the practical relevance of the proposed model, we also introduce a matrix methodology that enables a simple creation of pipe networks and illustrate its behavior with a numerical experiment. The analysis of costs and benefits of nonisothermal models indicates and quantifies inaccuracies of the models and distinguishes between models parameterized by nominal temperature versus those parameterized by measured temperatures. Here, we focus on process control; additional (nonlinear) control systems across multiple operating points may be employed for safety, start-up, and shutdown and these could be local to specific units and rapid in their action. Our methods are not targeted toward these latter controllers.

The control-oriented modeling developed here draws guidance at the formulation stage from the control objective specification in the introduction. Two companion works take these methods further. In [15], the modeling methods are applied to generate linear state-space models for a wider variety of network elements. This article provides a compendium of modeled elements together with their derivation and proof of internal satisfaction of conservation rules. The compendium also provides example MATLAB code illustrating the connection process for models. Thus, report [15] is a support document. The technical partner paper [14] on the other hand marries the control-oriented modeling with model-based control and provides strong evidence of the role played by model features here in subsequent controller development. In particular, we explore in detail the regulation control effect of the mass-conserving models [14].

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**Table I: DC (Steady-State) Gains From Inputs to Mass Flows**

| to | from | $\tilde{p}_{1,l}$ | $\tilde{q}_{9,r}$ | $\tilde{q}_{6,r}$ |
|----|------|------------------|------------------|------------------|
| $\tilde{q}_{1,l}$ | 0 | 1 | 1 | |
| $\tilde{q}_{2,l}$ | 0.184 | -1.022 | -0.8 | |
| $\tilde{q}_{3,l}$ | 0.184 | -0.022 | 0.2 | |
| $\tilde{q}_{4,l}$ | 0.184 | -0.022 | 0.2 | |
| $\tilde{q}_{5,l}$ | 0.184 | -0.022 | 0.2 | |
| $\tilde{q}_{6,l}$ | 0 | 0 | 1 | |
| $\tilde{q}_{7,l}$ | 0.184 | -0.022 | -0.8 | |
| $\tilde{q}_{8,l}$ | 0.184 | -0.022 | -0.8 | |
| $\tilde{q}_{9,l}$ | 0 | 1 | 0 | |
| $\tilde{q}_{10,l}$ | 0.184 | -1.022 | -0.8 | |
APPENDIX

A. Proofs

**Proposition 1:** By hypothesis and with \( q = \rho A_c v \), (3b) results directly from the momentum and gas equations, (1b) and (1d), see, e.g., [1].

For the pressure-related PDE, let \( a_1 = c_v / (R \rho z_0) - R \rho z_0 c_v^2 T / (2A_c^2 p^2) + gh / (R T z_0) \). Then, solving energy equation (1c) for \( \partial p / \partial t \) yields

\[
\frac{\partial p}{\partial t} = a_1^{-1} \left( q \rho + \frac{\partial T}{\partial t} \left( \frac{g h p}{R \rho T^2 z_0} - \frac{R \rho z_0 q^2}{2A_c^2 p} \right) - \frac{R \rho z_0 T q}{A_c^2 p} \right) \frac{\partial q}{\partial t} \]

- \( \frac{\partial}{\partial x} \left[ \frac{q}{A_c} (c_v T + gh) + \frac{q^3 R^2 T^2 z_0^2}{2p^2 A_c^2} + \frac{q R T z_0}{A_c} \right] \).

From continuity and gas equations, (1a) and (1d)

\[
\frac{\partial T}{\partial t} = \frac{z_0 R T^2}{\rho A_c} \frac{\partial q}{\partial x} + \frac{T}{p} \frac{\partial}{\partial x} \left( \frac{q}{A_c} + \frac{q^2}{2A_c^2} \right) \]

since \( (a_1 + (T/p)a_2) \). yields

\[
\frac{\partial p}{\partial t} = \frac{R \rho z_0}{c_v A_c} \left( q \rho + \frac{\partial q}{\partial x} \left( \frac{g h}{A_c} + \frac{3R^2 T^2 c_v^2}{2A_c^2 p^2} \right) + \frac{R \rho z_0 T q}{A_c^2 p} \right) \times \left[ \frac{\partial p}{\partial x} + \frac{\partial q}{\partial x} \left( \frac{z_0 R^2 T q^2}{A_c^2 p} - \frac{z_0 R T q}{A_c^2 p} \right) \right] \frac{\partial q}{\partial x} \times \frac{\lambda z_0 R T}{2DA_c^2 p} \frac{q |q| + g A_c}{z_0 R T} \frac{\partial}{\partial x} \left( c_v + gh \right) + \frac{q^3 R^2 T^2 z_0^2}{2p^2 A_c^3} \frac{q R T z_0}{A_c} \right) \]

From here and the definitions of \( w^1 \) and \( w^2 \), it is apparent that \( \gamma = \gamma_1 + \gamma_2 \) where

\[
\dot{x}_1 = [A + BF(I - DF)^{-1} C] \bar{x}_1 + BG \bar{u}^1 \]

\[
\gamma = (I - DF)^{-1} C \bar{x}_1 + \bar{u}^1 \]

and

\[
\dot{x}_2 = [A + BF(I - DF)^{-1} C] \bar{x}_2 + BF(I - DF)^{-1} DG \bar{u}.
\]

**System 1** [see (31) and (32)]: Denote the system transfer function \( \mathcal{H} = (I - DF)^{-1} C(sI - A)^{-1} B \) and rewrite (31) as

\[
\dot{x}_1 = A \bar{x}_1 + B(F \bar{y}_1 + G \bar{u}).
\]

In turn, writing this in terms of \( \mathcal{H} \) and using (30) for \( (I - \mathcal{H})^{-1} \), we have

\[
\gamma_1 = \mathcal{H} \bar{y}_1 + G \bar{u}.
\]

**System 2** [see (33) and (34)]: Directly comparing (30) to (33)–(34), we see that

\[
\bar{y}_2 = (I - \mathcal{H})^{-1} D \bar{G} \bar{u}.
\]

Combining (35) and (36), we have

\[
\bar{y} = \bar{y}_1 + \bar{y}_2.
\]
\[
(1 - \partial) = C(sI - A)^{-1}BG\bar{u} + (1 - \partial)^{-1}DG\bar{u}.
\]

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