Model prototype utilization in the analysis of fault tolerant control and data processing systems

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Abstract. The procedure assessing the profit of control and data processing system implementation is presented in the paper. The reasonability of model prototype creation and analysis results from the implementing of the approach of fault tolerance provision through the inclusion of structural and software assessment redundancy. The developed procedure allows finding the best ratio between the development cost and the analysis of model prototype and earnings from the results of this utilization and information produced. The suggested approach has been illustrated by the model example of profit assessment and analysis of control and data processing system.

1. Introduction
Inclusion of structural and software redundancy is one of the primary approaches to the increasing of the fault tolerance of control and data processing systems [1, 2]. However, in this case the cost exposure increases, due to the fact that frequently it is not possible to get complete information during the systems formation of the class.

The problem is that there are two sources of uncertainty. The first one is expressed by the probability of the fact that research with using model prototype leads to the selection of system implementation variant on condition that this variant will be unsuccessful. The second one is expressed by the probability that model prototype utilization leads to the variant selection on condition that it will be successful [3-5].

One of the main methods of the profit expected value calculation with the use of model prototype during the choice of preferred data control and processing system implementation is utilization of Bayes’ theorem and its special cases [6, 7]. In addition, real costs under the different expenses for modeling and system debugging, which provide the various reliability levels for fault tolerance prediction of their individual elements, can be evaluated [8, 9].

Model prototype realization enables to obtain the information, which is sufficient for detection of the volume of input structural and software redundancy. Due to this fact, the alternative, which maximizes the profit of control and data processing system implementation, can be recommended.

2. Methodology
In the general case, obtaining of complete information of the real situation by prototype development method or other knowledge increasing methods is not possible.

There are two sources of uncertainty, which can be described through probability:
- $P(RM|UC)$ – the probability of that the research ($R$) leads to the choosing of radical ($M$) method providing that in real situation this method will be unsuccessful ($UC$ – unsuccessful case);

- $P(RM|SC)$ – the probability of that the research ($R$) leads to the choosing of radical method providing that in real situation this method will be successful ($SC$ – successful case).

As the prototype can not always adequately reproduce some of the technical details or can be just an approximate model of the real system, generally the probability $P(RM|UC)$ is different from zero.

Thus, the prototype can confirm choosing of radical alternative although in the reality the result can be unsuccessful.

The probability $P(RM|UC)$ is usually not equal to zero, as the prototype can contain errors which will be bug fixed in the real control and data processing system.

It indicates that there is a probability that the prototype will show redundancy invalidity and therefore provide support for choosing of conservative alternative whilst the radical one may become successful.

Based on the values of the variables according to probability theory we can get the following ratios:

$$P(RM) = P(RM|SC) \cdot P(SC) + P(RM|UC) \cdot P(UC);$$  \hspace{1cm} (1)

$$P(RC) = 1 - P(RM);$$  \hspace{1cm} (2)

$$P(SC|RM) = P(RM|SC) \cdot P(SC) / P(RM);$$  \hspace{1cm} (3)

$$P(UC|RM) = 1 - P(SC|RM)$$

Formula (1) corresponds to the case in which two variants of radical alternative can be chosen after prototype research. These variants are choosing of radical method when it is successful (the probability of this event is equals to $P(RM|SC) \cdot P(SC)$), and choosing of the radical method when in actual truth it leads to failure (the probability is equals to $P(RM|UC) \cdot P(UC)$).

Formula (1) equates probability of choosing radical method to the sum of probabilities of two mutually exclusive cases.

Formula (2) adds to (1): it corresponds to the prototype research, which leads to choosing of conservative alternative ($RC$).

Formula (3) shows the probability of a successful case under the condition of radical method and that in the result the prototype research leads to its choice and is determined in accordance with $P(SC|RM) = P$(choosing of $RM$ – radical method, in case of its successful completion) / $P$(choosing of $RM$).

Formula (3) is a special case of Bayes’ formula and the main formula for determination of profit expected value with the utilization of imperfect prototype for the choosing of preferred development method of data control and processing system.
2.1. Formalization of procedure of full information profit expected value determination

Let us assume that $t$ is a set of alternatives (implementation variants of control and data processing system). $A_1, A_2, \ldots, A_m$ in the situation which has $n$ possible conditions $S_1, S_2, \ldots, S_n$, with probabilities values $P(S_1), P(S_2), \ldots, P(S_n)$, profit value of alternative $A_i$ choosing in condition $S_j$ are defined by gain matrix $V$.

The goal is to choose an alternative with the maximal expected value of gain. According to this definition, the profit can be expressed as dollars, quality indicator units, and utility function.

Based on expert based estimation of each condition probabilities profit expected value ($PEV$) when alternative $A_i$ is chosen:

$$PEV(A_i) = P(S_1)V_{i1} + P(S_2)V_{i2} + \ldots + P(S_n)V_{in}$$

Let us choose the alternative of maximum expected value:

$$PEV_{no\text{ info}} = \max [P(S_1)V_{i1} + \ldots + P(S_n)V_{in}], i = 1, \ldots, m.$$  

Let us choose the alternative with maximum profit when having full information on each case:

$$PEV_{full\text{ info}} = P(S_1)(\max V_{11}) + \ldots + P(S_n)(\max V_{in}), i = 1, \ldots, m.$$  

Then expected value of profit on full information ($FI$):

$$PEV = PEV_{full\text{ info}} - PEV_{no\text{ info}} = \sum_{j=1}^{n} P(S_j)(\max V_{ij}) - \max \sum_{j=1}^{n} P(S_j)V_{ij}, i = 1,\ldots,m.$$  

2.2. Formalization of procedure of incomplete information profit expected value determination

An alternative, which maximizes the profit, can be always recommended when the research provides the full information for task of choosing. $RA_i$ alternatives, recommended in the result of research, correlate with conditions $S_j$ by the following ratios:

- $P(RA_i| S_j) = 1$, if $A_i$ maximizes the profit for condition $S_j$,
- $P(RA_i| S_j) = 0$, in the contrary case.

Recommendation of $RA_i$ alternative is based on incomplete information on the conditions. Thus, for each alternative $A_i$ and condition $S_j$ the probability $P(RA_i| S_j)$ reflects the degree of possible departure from the ideal case (0 or 1) for the appropriate combination $A_i$ and $S_j$. The sum of probabilities $P(RA_i| S_j)$ on each conditions $S_j$ must equal to 1.
The general formula for profit expected value calculation in the task solution of alternatives choosing among \( RA_1, RA_2, \ldots, RA_m \) is:

\[
PEV(RA_1, RA_2, \ldots, RA_m) = \sum_{i=1}^{m} P(RA_i) \cdot PEV(\text{profit on acceptance of } RA_i) = \\
= \sum_{i=1}^{m} P(RA_i) \left( \sum_{j=1}^{n} P(S_j | RA_i)V_{ij} \right).
\] (4)

For calculation of \( PEV(RA_1, RA_2, \ldots, RA_m) \), where \( RA_i \) are RM and RC, \( S_j – SC \) and UC, the general formulas for the calculation of \( P(RA_i) \) and \( P(S_j | RA_i) \) according to the known values \( P(S_j) \) and \( P(RA_i | S_j) \) are needed. These formulas are the following:

\[
P(RA_i) = \sum_{j=1}^{n} P(RA_i | S_j)P(S_j),
\] (5)

\[
P(S_j) = P(RA_i | S_j)P(S_j) / P(RA_i).
\] (6)

The ratio (6) is a standard form of Bayes’ formula.

3. The procedure
The following procedure is suggested for the definition of the best ratio between the expenses on research making and profit on the application results of the information produced.

Step 1. Defining the set of alternatives of management information system development methods \( A_1, A_2, \ldots, A_m \).

Step 2. Determining of all possible situations \( S_1, S_2, \ldots, S_n \), which may influence the result of methods application.

Step 3. Defining the \( V \) gain matrix elements, where \( V_{ij} – \text{gain of method } A_i \text{ utilization in the case } S_j \).

Step 4. Defining the probabilities \( P(S_j) \) of each situation \( S_j \).

Step 5. \( PEV_{\text{no info}}, PEV_{\text{full info}}, \text{and } PEV_{FII} \).

Step 6. If the value \( PEV_{FII} \) can be neglected, then the additional research may not be pursued. In this case, the method providing the maximum \( PEV_{\text{no info}} \) should be chosen, then its implementation should be started.

Step 7. Definition of \( P(RA_i | S_j) \) for each research type, i.e. the probability that it will lead to the recommendation of alternative \( A_i \) in the situation \( S_j \).

Step 8. Calculation of \( PEV \) on utilization of information produced during the research \( k \), \( PEV(R_k) \) according to the formulas (4)-(6).

Step 9. Calculation of the real costs for each research.

4. Results and discussion
Let us consider an example of the real costs assessment of control and data processing system under the different expenses on the prototype development.
The prototype cost and the correspondent values of profit and system real costs expected values are given in Table 1. Here $EV_1$ means the expected value of profit, $EV_2$ means the expected value of profit on information, and $EV_3$ means the expected value of real costs under prototype development.

**Table 1. Expected value of real costs and prototype cost**

| Prototype cost, thousand c. u. | $P(RM|UC)$ | $P(RM|SC)$ | $EV_1$, thousand c. u. | $EV_2$, thousand c. u. | $EV_3$, thousand c. u. |
|--------------------------------|-----------|-----------|----------------|----------------|----------------|
| 0                              | 0.4       | 0.6       | 57             | 0              | 0              |
| 5                              | 0.3       | 0.7       | 66.5           | 9.5            | 4.5            |
| 10                             | 0.25      | 0.75      | 71.25          | 14.25          | 4.25           |
| 20                             | 0.1       | 0.9       | 85.5           | 28.5           | 8.5            |
| 30                             | 0         | 1         | 95             | 38             | 8              |

There is a possibility to increase research results reliability, take off some sources of uncertainty, decrease $P(RM|UC)$ up to 0.1 and increase $P(RM|SC)$ up to 0.9, when spending 20 thousand c.u. on prototype development. In this case, the expected profit equals to 85.5 thousand c.u. and expected value of profit on information produced during the prototype research equals to 28.5 thousand c.u. Real costs amounts 8.5 thousand c.u.

The dependency between prototype cost and real cost of control and data processing system is shown in Figure 1.

**Figure 1.** Dependency between real costs and prototype cost.

It may be concluded that the expenses on prototype implementation equal to 10 thousand c.u. is the best decision. Low costs do not provide reasonable profit. High costs provide more information, but at the high price, which decreases profit.
5. Conclusions
The suggested procedure of information profit determines the whole series of steps for the
detection of necessary analysis of costs and income during fault tolerant control and data
processing system implementation.
This procedure serves to the decision of many key problems during the fault tolerant
control and data processing system composition. In particular, it allows determining the
necessary tools for the following steps:

1. Research of project feasibility (users questioning, conceptual analysis, modeling,
speculation, job assignment), before making the final choice of particular system
variant.
2. Analysis of the alternative variants of control and data processing system realization
implementation (job and tasks assignment, walkthroughs, performance analysis),
before making the final choice of particular system variant.
3. Risk analysis (imitation, prototype research, user interaction study, task assignment,
modeling, sensitivity test), before the concretization of control and data processing
system requirements and project implementation.
4. Verification and assertion (requirement development, design, critical testing, real
tests), before exploitation of the developed control and data processing system.

Model prototype development as a tool of assessment and analysis of control and data
processing systems enables calculation of profit on its implementation, and the necessary
detail level and required investment for system model prototype development and research.

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