Large-x power laws of parton distributions remain inconclusive

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Received 28 December 2021; accepted 15 January 2022

Focusing on hadron scattering at large partonic momentum fractions $x$, we compare nonperturbative QCD predictions for the asymptotic behavior of DIS structure functions and parton distribution functions (PDFs) to the $x$ and $Q$ dependence of phenomenological PDFs. In the CT18 NNLO global QCD analysis, higher-order radiative contributions and functional mimicry of PDF parametrizations result in about one unit of uncertainty in the effective powers of $(1-x)$ falloff of nucleon PDFs. Similar uncertainties are present in the case of the pion PDF, an object of growing interest in phenomenology.

Keywords: Parton distribution functions, hadron structure, quark counting rules.

DOI: https://doi.org/10.31349/SuplRevMexFis.3.0308085

1. Quark counting rules in the large-$x$ limit

Parton Distribution Functions (PDFs) are the key inputs to unveiling the hadron structure in high-energy scattering. At sufficiently large partonic momentum fractions, typically $x \gtrsim 0.1$, these nonperturbative QCD functions can be evaluated in theoretical models, effective theories, in lattice QCD or in the continuum, or alternatively determined by global analyses from experimental observations. While the factorization scale dependence of the PDFs is dictated by DGLAP evolution equations (starting from an initial scale of order 1 GeV), their dependence on the fraction of longitudinal momentum $x$ of the parent hadron is not predicted by perturbative QCD. Together with the first QCD principles, a few rules guide the shape of the PDFs in some specific regions: positivity constraints and quark counting rules (QCRs) are such examples.

In these proceedings, we summarize a recent study [1] of phenomenological implications of the QCRs in a broad range of scattering processes accessible in the global QCD analysis of PDFs. The QCRs have been first derived for DIS structure functions at large $x$ in early QCD models [2–5]. The QCRs reflect kinematic properties of the lowest-order scattering amplitudes for DIS cross sections in the regime dominated by semi-hard gluon exchanges. In this regime, the DIS structure functions decrease with Bjorken $x_B$ as $(1-x_B)^{2n_s-1+2|\lambda_q-\lambda_A|}$, where $n_s$ is the number of quark spectators, and $\lambda_q, A$ are the helicity of the active quark and parent hadron, respectively.

When the QCRs are extended from the large-$x_B$ structure functions to the large-$x$ universal parton distribution functions, various aspects of QCD must be taken into account. Those include QCD factorization, radiative contributions at the NLO and beyond, and, at the low scale values that are most relevant to accessing the large-$x$ regime, target-mass corrections as well as all-order resummation. Also, the QCRs are motivated by the kinematics of the lowest-order semi-hard QCD interactions in the low to mid-$Q$ regimes, supplemented by the DGLAP evolution of the PDFs to approximate the higher-order radiative contributions. In the comparisons to the nonperturbative predictions, on the other hand, one aims to bridge these phenomenological PDFs, expressed in terms of the perturbative degrees of freedom, to distribution functions evaluated in the low-energy approaches operating with non-perturbative representations of QCD. The task is gargantuan, yet future experiments and theoretical efforts should lead us in that direction.

2. Polynomial mimicry

When finding the PDFs $F(x, Q^2)$ from a phenomenological analysis, many assumptions are usually made. [Our notations omit the PDF flavor indices for brevity.] In particular, the $x$ dependence at the starting evolution scale $Q_0$ is parameterized by a functional form with fitted free parameters. A typical parametrization of a proton PDF modulates a baseline function that drives the behavior at $x \to 0$ and $x \to 1$ by a smooth function $\Phi$, as

$$F(x, Q_0^2) = x^{A_1} (1-x)^{A_2} \times \Phi(x; A_3, ..., A_n). \quad (1)$$

Similar parametrizations can and have been adopted for analyses of the pion PDF, see e.g. [7–9]. The best-fit values of the free parameters $A_{1,2}$ can be examined to learn about the PDF dynamics in the asymptotic limits [1, 6]. In particular, the parameter $A_2$ from a fit is often interpreted in literature as the primordial exponent of the large-$x$ falloff, but this can be misleading. It is important to note here that discrete data points in a finite $x$ range are compatible with more than one continuous functional form: that is, it is easy to show mathematically [1] that infinitely many functions $\Phi(x; A_3, ..., A_n)$ result in the same quality of a fit to the data at hand. The best-fit parameters $A_2$ determined this way are correlated with all parameters $A_n$ in $\Phi(x; A_3, ..., A_n)$. Therefore, it cannot be
proved that experimental data demand a \((1 - x)^{A_2}\) fall-off, where \(A_2\) would be larger than 3 for a proton and 2 for a pion. We call this feature a polynomial mimicry [1]. It can be demonstrated with exact interpolations based on Bézier curves, each rendering a unique polynomial solution for the chosen set of \((n + 1)\) sampled points.

### 3. The case of the pion PDFs

Can a set of discrete data points that sample a (pion) PDF demand the minimal suppression power for this (pion) PDF? One possible strategy to address this question is to examine the monomial expansions of the Bézier curves that fit the data [1],

\[
B^{(n)}(x) = \sum_{l=0}^{n} \tilde{c}_l \, (1 - x)^l. \tag{2}
\]

We can raise this question not only for a phenomenological PDF found from experimental measurements, but also when the PDF is predicted by a theoretical calculation, as long as it is presented in the form given by Eq. (1).

The monomial expansion shows that the parameters \(\tilde{c}_l\) of the reconstructed functional forms depend on the \(x\) range spanned by the data. More strikingly, we have observed a mixing of expansion coefficients, \(\tilde{c}_l\) with various \(l\), as well as the appearance of spurious coefficients. Mimicry therefore dilutes the connection between the data and theoretical truth. In other words, the available data are compatible, within a given uncertainty/confidence interval, with multiple polynomial solutions, including those suggested by the early QCD and nonperturbative models. This eliminates the concern about a possible contradiction between pion data and theory raised in Ref. [10].

For the pion asymptotics to be more meaningfully interpreted at large \(x\), data at \(x \gtrsim 0.9\) are necessary to minimize that mixing of the Bézier coefficients. Until then, only increased uncertainties at large \(x\) can appropriately represent the multiple choices for the functional forms in the region \(x \to 1\). This is best understood by considering the effective exponent at large-\(x\) values, defined as

\[
A_2^{\text{eff}}[\mathcal{F}(x, Q^2)] = \frac{\partial \ln (\mathcal{F}(x, Q^2))}{\partial \ln (1 - x)} = A_2[\mathcal{F}] + \text{correction}, \tag{3}
\]

\textit{i.e.,} the logarithmic derivative of the PDF \(\mathcal{F}(x, Q^2)\). In Fig. 1, we show \(A_2^{\text{eff}}\) for the pion PDF obtained through a Dyson-Schwinger formalism [11], given at a low hadronic scale of a few hundred MeV. Here the input PDF is of the form \((1 - x)^2\Phi(x)\). According to the figure, \(A_2^{\text{eff}}\) approaches the expected value of 2 in the limit \(x \to 1\) and quickly drops below 2 at smaller \(x\) values. In the phenomenological analyses, however, the \(x\) values above 0.9 are the most challenging, although not entirely hopeless in the pion case. These values, as already stated above, will determine the relevant corrections that must be accounted for.

### 4. The case of the proton PDFs

We will now illustrate the behavior of the effective asymptotic exponents for proton scattering, for which both precise theoretical and experimental inputs already exist.

We will use the CT18 ensemble of NNLO proton PDFs [14]. The CT18NNLO ensemble has been extended to include 363 functional forms as part of the study of its uncertainty bands. In Ref. [1], we have used those alternative functional forms to study the behavior of the falloff of the structure functions and PDFs at large \(x\). The DIS structure function \(F_2(x, Q^2)\) was found to be compatible with the quark counting rule expectation of \(A_2^{\text{eff}}(F_2) = 3\) as \(x \to 1\), within error bands, as shown in Fig. 2. The determination of PDF

\[\text{Figure 1. Effective } (1 - x) \text{ exponent } A_2^{\text{eff}}(x) \text{ for the pion PDF of Ref. [11].}\]
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In these proceedings, we have explored the phenomenology of PDFs at large $x$ in view of the constraints given by the quark counting rules. We have argued that the effective exponents predicted by the theoretical methods for valence quark PDFs are consistent with experimental observations within one-two units, namely: $0 \lesssim A_{2,\text{eff}}^V, \pi \lesssim 4$ and $1 \lesssim A_{2,\text{eff}}^P \lesssim 5$. Tests to a higher accuracy have been challenging because of two classes of uncertainties:

1. Higher-order and higher-power QCD contributions are often not assessed. To know the associated uncertainties, perturbatively stable factorization with universal
PDFs must be demonstrated in the relevant kinematic regions for the examined observables.

2. Determination of the asymptotic exponents relies on end-point extrapolations, analogously to measurements of a neutrino mass in weak decays or lattice extrapolations to the physical pion mass. In such determinations, the estimated derivatives are highly correlated with the end-point value of the extrapolated function as well as its higher-order derivatives. Theoretical or phenomenological estimates of $A_2$ involving inter/extrapolation are sensitive to user-chosen high-power terms of the extrapolating polynomial. Mathematical underpinnings of this functional mimicry are addressed in [1].

The published analyses most often neglect these systematic uncertainties, which are of theoretical rather than experimental nature. Control of these uncertainties is critical for testing the power laws incisively. Sections 3 and 4 elaborate on these issues in the contexts of the pion and proton analyses.

Acknowledgements

AC is supported by UNAM Grant No. DGAPA-PAPIIT IA101720 and CONACyT Ciencia de Frontera 2019 No. 51244 (FORDECYT-PRONACES). PN is partially supported by the U.S. Department of Energy under Grant No. DE-SC0010129.

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