Differential Evolution for Quantum Robust Control: Algorithm, Applications and Experiments

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Abstract—Robust control design for quantum systems has been recognized as a key task in quantum information technology, molecular chemistry and atomic physics. In this paper, an improved differential evolution algorithm of \textit{msMS\_DE} is proposed to search robust fields for various quantum control problems. In \textit{msMS\_DE}, multiple samples are used for fitness evaluation and a mixed strategy is employed for mutation operation. In particular, the \textit{msMS\_DE} algorithm is applied to the control problem of open inhomogeneous quantum ensembles and the consensus problem of a quantum network with uncertainties. Numerical results are presented to demonstrate the excellent performance of the improved DE algorithm for these two classes of quantum robust control problems. Furthermore, \textit{msMS\_DE} is experimentally implemented on femtosecond laser control systems to generate good signals of two-photon absorption and control fragmentation of halomethane molecules CH$_2$BrI. Experimental results demonstrate excellent performance of \textit{msMS\_DE} in searching effective femtosecond laser pulses for various tasks.

I. INTRODUCTION

Controlling quantum systems has become a fundamental task in promising quantum technology [1], and it is relevant to many emerging areas such as atomic physics, molecular chemistry, and quantum information science [2], [3], [4]. Some control methods, such as optimal control theory [1], learning control algorithms [2] and Lyapunov control approaches [5], have been developed for quantum systems. Among these methods, learning control is a powerful approach for many complex quantum control tasks [6] and has achieved great success in laser control of molecules since the method was presented in the seminal paper [7]. Many quantum learning control problems could be formulated as an optimization problem and a learning algorithm is employed to search for an optimal control field for achieving desired performance. Gradient algorithms have been demonstrated to be a good candidate for numerically finding an optimal field due to their high efficiency [8]. In many practical applications, the gradient information may not be easy to obtain and some complex quantum control problems may have local optimum. For these situations, it is necessary to employ stochastic searching algorithms to find a good control field. Genetic algorithm (GA) has been widely used in the area of quantum control and has achieved success in closed-loop learning control of molecular systems in the laboratory [2]. In this paper, we focus on robust control problems of quantum systems and explore the use of differential evolution (DE) [9] for searching robust control fields.

DE is a competitive form of evolutionary computation and has shown great performance for many complex optimization problems [10]. Recently, it has also been used for solving quantum control problems [11], [12]. For example, Zahedinejad et al. [11], [13] proposed a subspace-selective self-adaptive differential evolution (SUSSADE) algorithm to achieve a high-fidelity single-shot Toffoli gate and single-shot three-qubit gates. DE methods have been adopted to fulfill a desired state transition by designing optimal control fields for an open quantum ensemble [14]. Zahedinejad et al. [15] investigated several promising evolution algorithms and found DE outperformed GA and particle swarm optimization (PSO) for hard quantum control. In this paper, we employ DE algorithms to solve several classes of quantum robust control problems. The robustness of quantum systems has been recognized as an essential requirement for the development of practical quantum technology, and robust control methods could provide enhanced robustness performance for quantum systems [16], [17]. In particular, we propose an \textit{msMS\_DE} (multiple-samples and Mixed-Strategy DE) algorithm where a mixed strategy and an average performance with multiple samples are employed. The \textit{msMS\_DE} is used for three classes of quantum robust control problems: control of inhomogeneous open quantum ensembles, consensus of quantum networks with uncertainties and experimental fragmentation control using femtosecond laser pulses [18].

A quantum ensemble consists of a large number of single quantum systems (e.g., spin systems or molecules) and every individual system is referred to as a member of the ensemble [19]. In practical applications, the members of a quantum ensemble could have variations in the parameters characterizing the system dynamics. Such a quantum ensemble is called an inhomogeneous quantum ensemble [19], [20]. Inhomogeneous quantum ensembles have wide applications in fields ranging form long-distance quantum communication to magnetic-resonance imaging [21], [22]. Hence, it is highly desirable to design control laws that can
steer an inhomogeneous ensemble from an initial state to a desired state when variations exist in the system parameters. Some results have been presented for controllability analysis and control design of inhomogeneous quantum ensembles. For example, Li and co-workers [20], [21] presented a series of results to analyze controllability and design optimal control laws for inhomogeneous spin ensembles. A Lyapunov control design approach has been proposed to asymptotically stabilize a spin ensemble around a uniform state of spin $+1/2$ or $-1/2$ [23]. Chen et al. [19] presented a sampling-based learning control method to achieve high fidelity control of inhomogeneous quantum ensembles. In these results, decoherence and dissipation were usually not considered. The existence of decoherence and dissipation may irreversibly lead a quantum ensemble to becoming an open system [24], and the manipulation of inhomogeneous open quantum ensembles becomes more challenging than that without considering decoherence. For an inhomogeneous quantum ensemble, we cannot employ different control fields to control individual members. A practical solution is to find robust control fields that can drive all of the members in the ensemble into a given target state. In this paper, we employ an msMS\_DE algorithm to search such robust control fields for inhomogeneous open quantum ensembles aiming at achieving enhanced control performance.

Another problem under consideration is to drive a quantum network into a consensus state even in the presence of uncertainties. Achieving consensus is one of primary objectives in distributed control and coordination of classical (non-quantum) networked systems [25]. Consensus usually means that all of the nodes in a network hold the same state. Recent development in quantum technology has made it significant and feasible to analyze quantum networks where each node (agent) represents a quantum system such as a photon, an electron, a spin system or a superconducting quantum bit (qubit). Consensus of quantum networks may have potential applications in promising quantum communication networks, distributed quantum computation and one-way quantum computation [26], [27]. Since the nodes in a quantum network are described by quantum mechanics, some unique characteristics such as quantum entanglement and measurement backaction should be carefully considered, and the analysis and control of quantum networks raise new challenges. Some results have been presented for the consensus problem of quantum networks. For example, Sepulchre et al. [28] generalized consensus algorithms to noncommutative spaces and analyzed the asymptotic convergence to the consensus state of a fully mixed state. Ticozzi and co-workers [26], [29], [30] presented a series of results on consensus of quantum networks including several different definitions for quantum consensus, quantum gossip algorithms, and quantum consensus results within a group-theoretic framework. Shi et al. [27] presented a systematic investigation on consensus of quantum networks with continuous-time dynamics within the framework of graph theory. In this paper, we consider the basic problem of finding a robust control law to steer a quantum network to a reduced state consensus (as defined in [26]) and would not consider the distributed solutions to achieving quantum consensus. In particular, we employ the proposed msMS\_DE for driving a superconducting qubit network with uncertainties into a reduced state consensus.

Femtosecond (fs) (1 fs $= 10^{-15}$ second) laser [18] has found wide applications in controlling the molecular dynamics because of its short pulse duration, which is comparable to the time scales of electronic and nuclear motions of a molecule. The temporal structures of a femtosecond pulse could be manipulated by pulse shaping techniques [18], which is typically achieved by modulating the phase and/or amplitude of the laser frequency components, spatially separated in the Fourier plane, with a computer programmable spatial light modulator (SLM) before recombination into a “shaped pulse”. The advances in both fs lasers and pulse shaping techniques have spurred recent interest in laser-selective chemistry and quantum control using shaped fs laser pulses. In quantum control experiments, a practical approach is to use closed-loop learning control [2] to find an optimal field that can steer the quantum system towards the desired outcome, which has achieved a lot of successes. An evolutionary algorithm (e.g., GA) is often employed to assist the search of an optimal pulse. To the best of our knowledge, there have not been any experimental results reported where DE algorithms are used on femtosecond laser control systems. There are few quantum control experiments using femtosecond laser pulses that have investigated the robustness subject to variations in the control. In this work, we employ the msMS\_DE algorithm to an experimental quantum control problem, where the goal is to identify a robust solution (shaped fs laser pulse) that can maximize the CH$_2$Br$^+$/CH$_3$I$^+$ product ratio from the fragmentation of CH$_3$BrI molecule in a time-of-flight mass spectrometry (TOF-MS). The msMS\_DE algorithm is also used to identify the transform limited (TL) pulse via optimizing the two photon absorption (TPA) signal, which is carried out prior to the fragmentation experiment.

The main contributions of this paper are summarized as follows:

- Motivated by solving three classes of quantum robust control problems, an improved DE algorithm of $msM\_S\_DE$ is proposed where an average fitness function of multiple samples is utilized and a mixed strategy of mutation is employed.
- The control problem of inhomogeneous open quantum ensembles is investigated and the $msMS\_DE$ algorithm is used to learn robust control fields for inhomogeneous open quantum ensembles. Numerical results show that the control fields learned by $msMS\_DE$ usually have improved robustness performance compared with those learned by basic DE and GA.
- The task of driving a quantum network with uncertainties to a consensus state is investigated and robust control fields can be found by employing the $msMS\_DE$ algorithm. Numerical results demonstrate that $msMS\_DE$
has excellent performance in searching robust control fields for achieving consensus of quantum networks.

- Several experiments are implemented on femtosecond laser control systems where the msMS DE algorithm is employed to find effective femtosecond laser pulses for generating excellent TPA signal and achieving good fragmentation control of molecules. These experiments present the first result of testing DE for femtosecond laser quantum control systems as well as realize the sampling-based learning control method [19], [31], [32] in the area of quantum control.

The paper is organized as follows. Section II formulates three classes of quantum robust control problems under consideration. Section III briefly introduces the basic DE and then presents a systematic description of the msMS DE algorithm. Numerical results on quantum ensemble control and quantum network consensus are provided in Section IV. In Section V, we present experimental results on femtosecond laser control systems. Conclusions and discussions are given in Section VI.

II. THREE CLASSES OF QUANTUM CONTROL PROBLEMS

A. Control of inhomogeneous open quantum ensembles

We consider an inhomogeneous quantum ensemble where the members of the ensemble could have variations in the parameters that characterize the system dynamics. For example, the spins of an ensemble in nuclear magnetic-resonance (NMR) experiments may encounter large dispersion in the strength of the applied radio frequency field (rf inhomogeneity) and there also exist variations in the natural frequencies of these spins (Larmor dispersion) [20]. Several methods have been presented to design control laws for inhomogeneous quantum ensembles when dissipation and decoherence were not considered [19], [21].

For a practical quantum ensemble, it may be unavoidable to interact with its environment and each member in the quantum ensemble should be dealt as an open quantum system. For an open quantum system, its state can be described by a positive Hermitian density operator \( \rho \) satisfying \( \text{tr}(\rho) = 1 \). Under the assumption of a short environmental correlation time permitting the neglect of memory effects, a Markovian master equation for \( \rho(t) \) can be used to describe the dynamics of an open quantum system interacting with its environment [24]. Markovian master equations in the Lindblad form are described as [3], [33]

\[
\dot{\rho}(t) = -\frac{i}{\hbar}[H(t), \rho(t)] + \sum_k \gamma_k \mathcal{D}[L_k] \rho(t),
\]

where \( i = \sqrt{-1} \), \( H(t) \) is the system Hamiltonian, \( \hbar \) is reduced Planck constant (we set \( \hbar = 1 \) in this paper), the non-negative coefficient \( \gamma_k \) specify the relevant relaxation rates, \( L_k \) are appropriate Lindblad operators and

\[
\mathcal{D}[L_k] \rho = (L_k \rho L_k^\dagger - \frac{1}{2} L_k^\dagger L_k \rho - \frac{1}{2} \rho L_k^\dagger L_k).
\]

For an inhomogeneous open quantum ensemble, the Hamiltonian can be described in the form of

\[
H_\Theta(t) = g_0(\Theta_0)H_0 + \sum_{j=1}^{M} g_j(\Theta_j)u_j(t)H_j.
\]

Let \( \Theta = (\Theta_0, \Theta_1, \ldots, \Theta_M) \) and the functions \( g_j(\Theta_j) (j = 0, 1, \ldots, M) \) characterize possible inhomogeneities. For example, \( g_0(\Theta_0) \) corresponds to inhomogeneity in the free Hamiltonian (e.g., due to chemical shift in NMR). \( g_j(\Theta_j) (j = 1, \ldots, M) \) can characterize possible multiplicative noises in the control fields or imprecise parameters in the dipole approximation. We assume that \( g_j(\Theta_j) (j = 0, 1, \ldots, M) \) are continuous functions of \( \Theta_j \) and the parameters \( \Theta_j \) could be time-dependent and \( \Theta_j \in [1 - E_j, 1 + E_j] \). For simplicity, we assume that \( g_j(\Theta_j) = \Theta_j \), the nominal value of \( \Theta_j \) is 1 and \( E_0 = \ldots = E_j = \ldots = E_M = E \).

For an open quantum system in (1), we may define a coherent vector as \( \mathbf{y} := (\text{tr}(U_1^\dagger \rho), \text{tr}(U_2^\dagger \rho), \ldots, \text{tr}(U_M^\dagger \rho))^T \), where \( iU_1, iU_2, \ldots iU_m (m = n^2 - 1) \) are orthogonal generators of the special unitary group \( SU(n) \) with degree \( n \). Its density operator can be written as

\[
\rho = \frac{I}{n} + \frac{1}{2} \sum_{i=1}^M y_i U_i.
\]

Substituting (3) into (1), we can obtain the evolution of the coherent vector \( \mathbf{y} \) as:

\[
\dot{\mathbf{y}} = (\mathcal{L}_{H_0} + \mathcal{L}_D)\mathbf{y} + \sum_{j=1}^{M} y_j \mathcal{L}_{H_j} \mathbf{y} + l_0,
\]

where the superoperators \( \mathcal{L}_{H_0}, \mathcal{L}_D, \mathcal{L}_{H_j} (j = 1, 2, \ldots, M) \) and the term \( l_0 \) are explained in detail in [34], [35]. We choose the objective function to be maximized as follows [34]:

\[
J(u) = 1 - \frac{n}{8(n-1)} \| y_f - y(T) \|^2,
\]

where \( \| x \|^2 = x^T x \) is a vector norm and it is clear that \( J(u) \in [0, 1] \). And \( y_f \) and \( y(T) \) are the target state and the final state of the quantum system in terms of coherent vector, respectively.

The control problem of an inhomogeneous open quantum ensemble can be formulated as:

\[
\max_u J(u) := \max_u \mathbb{E}[J_\Theta(u)]
\]

s.t. \[
\begin{align*}
\begin{cases}
\dot{y}_\Theta(t) = (\Theta_0 \mathcal{L}_{H_0} + \mathcal{L}_D + \Theta_j \sum_{j=1}^{M} y_j(t) \mathcal{L}_{H_j}) y_\Theta(t) + l_0 \\
y_\Theta(0) = y_0, & t \in [0, T] \\
\Theta_j \in [1 - E_j, 1 + E_j], & j = 0, 1, \ldots, M
\end{cases}
\end{align*}
\]

where \( \mathbb{E}[J_\Theta(u)] \) denotes the average performance function regarding parameter inhomogeneities \( \Theta \).
B. Consensus in quantum networks

Achieving quantum consensus is a primary objective in the investigation of quantum networks. Existing results presented some distributed solutions to quantum consensus problems. For example, Mazzarella et al. [26] proposed a quantum gossip iteration algorithm where discrete-time quantum swapping operations between arbitrary two nodes are used to make a quantum network achieving consensus. A graphical method has been developed in [27] to build the connection between quantum consensus and its classical counterpart, and asymptotical convergence results on achieving a consensus state have been presented for a class of quantum networks with continuous-time Markovian dynamics. Here, we would not intend to develop a distributed algorithm for quantum consensus. In contrast, we consider how to design a robust control field to drive a quantum network from an initial state into a consensus state with high fidelity when uncertainties or inaccuracies may exist in the system dynamics. We consider the type of Reduced State Consensus that was defined in [26]. We denote \( \mathcal{H} \) a Hilbert space, \( A \otimes B \) returns the tensor product of \( A \) and \( B \), and \( |a \rangle \) with Dirac representation is a vector in \( \mathcal{H} \), i.e., \( |a \rangle \in \mathcal{H} \) [4]. In order to present the definition of reduced state consensus, we need to use the concept of partial trace defined as follows.

Definition 1 (Partial trace): [4] Let \( \mathcal{H}_A \) and \( \mathcal{H}_B \) be the state spaces of two quantum systems \( A \) and \( B \), respectively. Their composite system is described as a density operator \( \rho^{AB} \). The partial trace over system \( B \), denoted as \( \text{Tr}_{\mathcal{H}_B} \), is given in the following form

\[
\text{Tr}_{\mathcal{H}_B}(|a_1 \rangle \langle a_2| \otimes |b_1 \rangle \langle b_2|) = |a_1 \rangle \langle a_2| \text{Tr}(|b_1 \rangle \langle b_2|),
\]

where the vectors \( |a_1 \rangle, |a_2 \rangle \in \mathcal{H}_A \), and the vectors \( |b_1 \rangle, |b_2 \rangle \in \mathcal{H}_B \). When the composite system is in the state \( \rho^{AB} \), the reduced density operator for system \( A \) is defined as \( \rho_A = \text{Tr}_{\mathcal{H}_B} (\rho^{AB}) \) and the reduced density operator for system \( B \) is defined as \( \rho_B = \text{Tr}_{\mathcal{H}_A} (\rho^{AB}) \).

Reduced state consensus for a quantum network can be defined as follows.

Definition 2 (Reduced State Consensus): [26] A quantum network consisting of \( m \) nodes with the state \( \hat{\rho} \) is in Reduced State Consensus (RSC) if

\[
\hat{\rho}_1 = \hat{\rho}_2 = ... = \hat{\rho}_m,
\]

where \( \hat{\rho}_j = \text{Tr}_{\mathcal{H}_A^j} (\hat{\rho}) \) \( (j = 1, 2, ..., m) \) are defined as the reduced density operator for node \( j \) and can be calculated according to Definition 1.

We aim to steer a quantum network into a consensus state defined in Definition 2. In practical applications, the existence of noise (including extrinsic and intrinsic), inaccuracies (e.g., inaccurate operation in the coupling between nodes) and fluctuations (e.g., fluctuations in control fields) in quantum networks is unavoidable. We assume that the Hamiltonian with uncertainties can be written into

\[
H_{\Theta}(t) = \theta_0 H_0 + \sum_{j=1}^{M} \theta_j \mu_j(t) H_j.
\]

The problem can be formulated as follows:

\[
\max_u J(u) := \max_u \mathbb{E}[J_\Theta(u, \hat{\rho})]
\]

s.t.

\[
\begin{align*}
\rho(t) &= -i \left( \theta_0 H_0 + \sum_{j=1}^{M} \theta_j \mu_j(t) H_j, \rho(t) \right) \\
\rho(0) &= \rho^0, \quad t \in [0, T] \\
\theta_j &\in [1 - E, 1 + E], \quad j = 0, 1, 2, ..., M
\end{align*}
\]

where \( \mathbb{E}[J_\Theta(u, \hat{\rho})] \) denotes the average performance function with respect to the parameter fluctuations \( \theta \) and the target consensus state \( \hat{\rho} \), and \( E \in [0, 1] \) are the bounds of the parameter uncertainties.

C. Femtosecond laser quantum control

We consider the experimental control of molecular fragmentation using shaped femtosecond laser pulses. Here, CH\(_2\)BrI is chosen as the target molecule. As a family member of Halomethane molecules, whose dissociative product plays a central role in ozone depletion, CH\(_2\)BrI has attracted wide attention because of its importance in environmental chemistry. In addition, it is one of the simplest prototype molecules containing different bonds, a stronger C-Br bond and a weaker C-I bond, which is ideal for the study of controlling the selective bond-breaking. Under strong fields with femtosecond laser pulses, CH\(_2\)BrI molecules will undergo ionization and dissociation, and their charged products can be separated and detected with a TOF-MS. In particular, we choose to optimize the photoproduction ratio of CH\(_2\)Br\(^+\)/CH\(_2\)I\(^+\) as our control objective, which corresponds to breaking the strong bond versus the weak bond. We apply closed-loop learning control, using our proposed msms\_DE algorithm, to search for a robust ultrafast laser pulse that maximizes this ratio.

In closed-loop learning control, the learning process can be conceptually demonstrated as follows. First, one applies trial input pulses to the molecules to be controlled and observes the results. Second, a learning algorithm suggests better control inputs based on the prior experiments. Third, one applies “better” control inputs to new molecules [2]. This approach has been employed to explore the quantum control landscape [36] to find the optimal control strategy where the control performance function \( J(u) \) reaches its maximum. In order to achieve good performance, we first need to identify a reference phase mask on the SLM that give shortest transform limited (TL) pulse, which can be obtained from optimizing the signal of two-photon absorption (TPA). We first use the proposed msms\_DE algorithm to search for a good control to obtain highest TPA signal. Then we apply the same algorithm to search for a good control for the fragment ratio of CH\(_2\)Br\(^+\)/CH\(_2\)I\(^+\). The consideration of robustness with multiple samples (MS) in DE would also ensure good transferability of the experimental results or photonic reagents [37]. That is, an optimal pulse identified from one laser system would also perform well (if not optimal) when transferred to another system despite the minor...
differences or uncertainties lying in the control parameters
(i.e., the spectral phases on the SLM).

III. DIFFERENTIAL EVOLUTION AND msMS DE ALGORITHM

DE was initially proposed in 1990s by Storn and Price [9], [39] and has derived many variants. DE algorithms have witnessed wide applications in diverse domains of science and engineering [10], [38]. In this section, we first briefly introduce the conventional DE algorithm, analyze DE with variant strategies and control parameters, and then propose an improved DE algorithm of msMS DE for these quantum robust control problems presented in Section II.

A. DE Algorithm

In DE, the individual trial solutions (which constitute a population) are termed as parameter vectors or genomes, usually represented in a vector \( X = [x_1, x_2, \ldots, x_D]^T \) where each parameter \( x_i \) is a real number. Solving an optimization problem using DE is basically a search for a parameter vector to minimize or maximize a fitness function (or objective function) \( f(X) \). The following operations are used to evolve a population of \( D \)-dimensional parameter vectors until a “best” individual is generated and found.

(a) Initialization. DE searches for a global optimum point in a \( D \)-dimensional real parameter space \( \mathbb{R}^D \). Here, we denote the population at the current generation as \( X_{i,G} = [x_{i1,D}^G, \ldots, x_{iD}^G], i = 1, \ldots, NP \) and let \( x_{iD}^G \in [x_{\text{min}}^D, x_{\text{max}}^D] \), \( j = 1, 2, \ldots, D \) since these parameters are usually related to physical variables with relevant bounds. We usually initialize the population (at \( G = 0 \)) as follows [43]:

\[
x_{i0} = x_{\text{min}}^j + \text{rand}(0,1) \cdot (x_{\text{max}}^j - x_{\text{min}}^j), \quad j = 1, 2, \ldots, D.
\]

where \( \text{rand}(0,1) \) is a uniformly distributed random number.

(b) Mutation. In DE, the key to “mutation” is to generate a difference vector by choosing three other distinct parameter vectors from the current generation (say, \( X_{r1,G}, X_{r2,G}, X_{r3,G} \)). The indices \( r_1, r_2, r_3 \in \{1, \ldots, NP\} \) are mutually exclusive integers randomly generated within the range \( [1, NP] \) and \( r_1, r_2, r_3 \neq i \). The donor vector \( V_{i,G} \) are generated by

\[
V_{i,G} = X_{r1,G} + F \cdot (X_{r2,G} - X_{r3,G}),
\]

where the scaling factor \( F \) is a positive control parameter typically in the interval \([0.4, 1] \).

(c) Crossover. To enhance the potential diversity of the population, a crossover operation comes into play after the mutation. The DE family has two types of crossover operations (i.e., exponential and binomial). The binomial (uniform) crossover is outlined as

\[
u_{i,j}^G = \begin{cases} 
v_{i,j}^G, & \text{if } \text{rand}(j) \leq CR \text{ or } j = \text{rand}(1,D), \\
x_{i,j}^G, & \text{if } \text{rand}(j) > CR \text{ and } j \neq \text{rand}(1,D), 
\end{cases}
\]

where \( j = 1, 2, \ldots, D \) and \( \text{rand}(j) \in [0,1] \) is a uniform random number. \( CR \) is a user-specified constant within the range \([0,1) \), \( \text{rand}(1,D) \in [1,2,\ldots,D] \) is a randomly chosen index.

(d) Selection. To keep the population size constant over subsequent generations, DE uses the following selection operation to determine whether the target vector or the trial vector survives to the next generation:

\[
X_{i,G+1} = \begin{cases} 
U_{i,G}, & \text{if } f(U_{i,G}) \geq f(X_{i,G}), \\
X_{i,G}, & \text{otherwise.}
\end{cases}
\]

If the new trial vector yields an equal or lower value of the objective function, it replaces the corresponding target vector in the next generation; otherwise the target vector survives.

Usually, it is the mutation operation that demarcates one DE scheme from another. The DE strategy with the mutation in (11) is referred to as “DE/rand/1” using the notation “DE/x/y”, where \( x \) represents a string denoting the base vector to be perturbed, \( y \) is the number of difference vector considered for perturbation of \( x \). Furthermore, when crossover is also considered, the notation “DE/x/y/z” is used, where \( z \) stands for the type of crossover (bin: binomial, exp: exponential). DE variants with different mutation strategies usually have different performance for solving optimization problems. “DE/rand/1/bin” is a commonly used strategy and it usually shows slow convergence speed and strong exploration capacity. The strategies relying on the best solution found so far such as “DE/rand-to-best/1/bin”, “DE/best/2/bin”, usually have a fast convergence speed and perform well when solving unimodal problems. However, they are more likely to get stuck at a local optimum and lead to premature convergence for multimodal problems.

Two-difference-vectors-based strategies may result in better perturbations than one-difference-vector-based strategies. For control parameters of DE, Storn and Price [39] have proposed that a good initial choice of \( F \) was 0.5 and the range of \( F \) is usually set \([0.4, 1] \). The crossover rate \( CR \) may be \( CR \in [0, 1] \). Several results also proposed the techniques of self-adaptation to automatically find an optimal set of control parameters [10], [40] to provide improved performance.

B. msMS DE algorithm

When implementing DE, users need to determine the appropriate mutation-trial strategies and parameter settings to ensure the success of the algorithm [40]. It is a high-cost practice to perform a trial-and-error search for the most appropriate trial vector generation strategy and fine-tune its associated control parameter values, i.e., the values of \( CR, F \) for a given problem. Moreover, during different stages of evolution, different trial vector generation strategies coupled with specific control parameter values can be more effective and single strategy may result in premature convergence thus leading to a failure in complex problems such as nonseparable and multimodal functions [10], [41], [42]. Also, several variants of DE utilizing the idea of mixed strategies such as SaDE [40] and EPDE [43] have been proposed and exhibited good performance. Our numerical results show that DE with a single strategy might be enough for easy problems while DE variants with mixed strategies might emerge as a
promising candidate for quantum control problems with multimodal landscapes. Existing results of sampling-based learning control [19], [31], [44] have shown that the employment of an average objective function with multiple samples can provide improved performance for quantum robust control problems. Inspired by these observations, we adopt a mixed strategy and an average performance of multiple samples to present an improved DE algorithm (i.e., msMS\_DE) for these quantum robust control problems outlined in Section II.

We first choose one mutation scheme from a pool of strategy candidates where several mutation schemes with effective yet diverse characteristics are equally distributed. Subsequently, a binomial crossover operation is performed on the corresponding mutant vector to generate the trial vector. Note that we assign various values of $F$ and $CR$ for each individual during the current generation to increase the diversity of the population. To construct the candidate pool, we investigate several commonly used mutation strategies [42] and select four strategies with distinct capabilities at different stages of evolution as follows:

\begin{align*}
\text{DE/rand/1:} & \quad V_i = X_{r1} + F \cdot (X_{r2} - X_{r3}). \\
\text{DE/rand to best/2:} & \quad V_i = X_i + F \cdot (X_{best} - X_i) + F \cdot (X_{r1} - X_{r2}) + F \cdot (X_{r3} - X_{r4}). \\
\text{DE/rand/2:} & \quad V_i = X_{r1} + F \cdot (X_{r2} - X_{r3}) + F \cdot (X_{r4} - X_{r5}). \\
\text{DE/current-to-rand/1:} & \quad V_i = X_i + K \cdot (X_{r1} - X_i) + F \cdot (X_{r2} - X_{r3}).
\end{align*}

The indices $r_1, r_2, r_3, r_4$ and $r_5$ are mutually exclusive integers randomly chosen from the range $[1, NP]$ and all of them are different from the index $i$. $X_{best}$ is the best individual vector with the best fitness (i.e., the lowest objective function value for a minimization problem) in the population. The control parameter $K$ in the strategy DE/current-to-rand/1 is set as $K = 0.5$ to eliminate one additional parameter. As for the crossover operation, the first three mutation schemes are combined with a binomial crossover operation, while the fourth scheme directly generates trial vectors without crossover.

In the proposed msMS\_DE algorithm, the parameter $F$ is approximated by a normal distribution with mean value 0.5 and standard deviation 0.3, denoted by $N(0.5, 0.3)$. It is easy to verify that values of $F$ fall into the range $[-0.4, 1.4]$ with probability of 0.997 which helps maintain both exploitation (with small $F$ values) and exploration (with larger $F$ values). Similarly, we assume $CR$ obeys a normal distribution denoted by $N(0.5, 0.1)$ and the small standard deviation 0.1 is enough to guarantee that most values of $CR$ lies in $[0, 1]$ [40]. Consequently, a set of $F$ and $CR$ values are randomly sampled from normal distribution (denoted by Normrnd) and applied to each target vector in the current population. We may obtain some extraordinary values far from $[-0.4, 1.4]$ for scale factor $F$ and we usually accept them to increase diversity. While the crossover rate has probabilistic meaning for the chance of survival, we should abandon those fall outside $[0, 1]$, and generate another valid parameter by $CR = N(0.5, 0.1)$ to guarantee the practical meaning of crossover.

The msMS\_DE method is proposed for three classes of quantum control tasks. In order to design appropriate control laws to achieve good robustness performance, we integrate the idea of sampling-based learning control [19] into the msMS\_DE algorithm. To begin with, we prepare $N$ samples $\Theta_k = (\theta_1, \theta_2, \ldots, \theta_m)$ ($k = 1, 2, \ldots, N$) with different values of the uncertainty parameters. We compute the fitness values of these sample vectors. Then, we evaluate the average fitness value $\bar{f}$ for these samples, and $f$ is defined as follows

$$f(x_{iG}) = \frac{1}{N} \sum_{k=1}^{N} f(U_{iG}, \Theta_k).$$

The algorithmic description of the msMS\_DE is presented in Algorithm 1.

**Remark 1:** In simulation, after we obtain the nominal value of an individual, we may generate the other samples by perturbing the nominal value and then calculate the average fitness function. In experiment, we need to run an experiment for each sample to measure the fitness of each sample, and then calculate the average fitness. Usually, a larger number of samples $N$ may lead to a better robustness performance [19]. However, the computational or experimental time will significantly increase with the increase of $N$. In this paper, we use three samples for each uncertainty parameter for saving computational and experimental time.

**Remark 2:** After performing the mutation operation, we obtain new donor vectors, and some of them might survive into the next generation and serve as parents. Therefore, we add a procedure where each vector is evaluated in view of boundary constraints. If any parameter of the vector falls beyond the pre-defined lower or upper bounds, we will replace it with a random value within the allowed range.

**Remark 3:** In our proposed msMS\_DE, we preset a maximum generation $G_{\text{max}}$ as the termination criterion. During the implementation of the algorithm, the population evolves until the learning process reaches $G = G_{\text{max}}$. In numerical examples, we set $G_{\text{max}} = 50000$. In experimental examples, we choose $G_{\text{max}} = 150$.

IV. NUMERICAL RESULTS FOR ENSEMBLE CONTROL AND QUANTUM NETWORK CONSENSUS

**A. Control of open inhomogeneous two-level quantum ensembles**

We consider a specific inhomogeneous open two-level ensemble with inhomogeneous parameter bound $E = 0.2$. Members of the ensemble are governed by the following Hamiltonian:

$$H(t) = \theta_0 \frac{1}{2} \sigma_z + \theta_1 u(t)(\cos \varphi \sigma_z + \sin \varphi \sigma_y),$$

(18)
Renew the best vector and $i \leftarrow i + 1$

\textbf{end} while

\textbf{end} if

\textbf{end} for

\textbf{end} for

\textbf{end} for

\textbf{end} for

\textbf{end} while

\textbf{for} $j = 1$ to $D$

\textbf{end for}

The dynamical equation for $\mathbf{r}$ can be written as

$$\dot{\mathbf{r}}(t) = \begin{pmatrix}
-0.045 & -\theta_0 & 0 \\
0 & 0 & -0.05 \\
0 & 2 \sin \varphi & 2 \cos \varphi \\
\end{pmatrix}
\mathbf{r}(t) + \begin{pmatrix}
0 \\
0 \\
0 \\
\end{pmatrix}
\begin{pmatrix} -2 \sin \varphi \\
-2 \cos \varphi \\
0 \\
\end{pmatrix} + \theta_1 u(t) \begin{pmatrix} 0 \\
0 \\
0 \\
\end{pmatrix}$$

The average fitness function is given as

$$E[f_0(u)] = \frac{1}{N} \sum_{\theta_0, \theta_1} \left[ 1 - \frac{1}{4} || \mathbf{r}_f - \mathbf{r}_0, \theta_0(T) ||^2 \right], \quad (22)$$

where $N$ is the total number of the chosen samples. An upper bound of the fitness function is 1 although we do not know the maximum that can be achieved. In the $msMS_{DE}$ algorithm, we choose three samples for each parameter, and here we have $N=9$. During learning control of the inhomogeneous quantum ensemble, we employ DE algorithms to seek for the optimal control $u^*(t)$. Then, we apply the optimal control field to additional samples with inhomogeneous parameters $(\theta_0, \theta_1)$ following uniform distributions within $[0.8, 1.2]$ to test its performances. We assume that the initial state and the target state are, respectively,

$$\rho_0 = \begin{pmatrix} 1 & 0 \\
0 & 0 \end{pmatrix}, \quad \rho_f = \begin{pmatrix} 0 & 0 \\
0 & 1 \end{pmatrix}. \quad (23)$$

The target time $T = 10$ and the time interval $[0, T]$ is equally divided into $D = 200$ time steps, and $\Delta t = 0.05$. The population size is set as $NP = 50$ for all the algorithms in this example. The simulation is implemented on MATLAB platform (version 8.3,0,532). The hardware environment for simulation is Intel(R)-Core(TM) i7-6700K CPU, dominant frequency @4.00GHz, and 16G(ARM).

To demonstrate the performance of the proposed $msMS_{DE}$ algorithm for the control problem of inhomogeneous quantum ensembles, we make performance comparisons between it and the $msDE$ (DE with multiple samples, i.e., using the average fitness function of multiple samples) with various parameters. To begin with, we present the results for the traditional DE (i.e., “DE/rand/1/bin”) using multiple samples with three typical sets of control parameters. Three cases with different control parameters are labeled as “msDE1”
More numerical results also show (F = 0.9, CR = 0.1), “ms\_DE2” (F = 0.9, CR = 0.9), and “ms\_DE3” (F = 0.5, CR = 0.3), and the training performance is presented in Fig. 1(a). It is clear that ms\_DE1 and ms\_DE3 have better performance than ms\_DE2 for the quantum control problem. ms\_DE1 can achieve the highest fitness 0.9566 among these three cases. We then compare the training performance of ms\_DE1, GA and ms\_MS\_DE, and the results are illustrated in Fig. 1(b). The ms\_MS\_DE algorithm achieve the highest fitness $J_{\text{max}} = 0.9798$, while ms\_DE1 and GA converge to a maximum value of 0.9566 and 0.9667, respectively. A comparison of testing performance for 2000 additional samples and training time between DE1 (using one sample), ms\_DE1, ms\_DE2, ms\_DE3, GA (with crossover probability $P_c = 0.8$ and mutation probability $P_m = 0.05$) and ms\_MS\_DE in Table 1 shows that ms\_MS\_DE is superior to ms\_DE and DE1. More numerical results also show that ms\_MS\_DE usually can find the control field with the best robustness among these algorithms because ms\_MS\_DE employs mixed mutation strategies as well as average performance using multiple samples. ms\_DE1, ms\_DE2, ms\_DE3, GA and ms\_MS\_DE also take similar time to find an optimal solution for the ensemble control problem. For example, ms\_MS\_DE takes 9 hours 20 minutes and GA takes 10 hours 18 minutes and 14 seconds.

![Fig. 1. (a) The training performance of the two-level open quantum ensemble via ms\_DE1 (F = 0.9, CR = 0.1), ms\_DE2 (F = 0.9, CR = 0.9) and ms\_DE3 (F = 0.5, CR = 0.3). (b) The training performance of ms\_DE1, GA and ms\_MS\_DE.](image)

### TABLE I

Performance comparison of different algorithms

| Algorithm | Parameters | Training time | $J(u)$ |
|-----------|------------|---------------|--------|
| DE1       | F = 0.9, CR = 0.1, N = 1 | 1h10m47s | 0.9408 |
| ms\_DE1   | F = 0.9, CR = 0.1, N = 9 | 9h27m47s | 0.9610 |
| ms\_DE2   | F = 0.9, CR = 0.9, N = 9 | 9h20m15s | 0.9537 |
| ms\_DE3   | F = 0.5, CR = 0.3, N = 9 | 9h40m5s  | 0.9601 |
| GA        | $P_c = 0.8$, $P_m = 0.05$, N = 9 | 10h18m14s | 0.9601 |
| ms\_MS\_DE| F = N(0.5, 0.3), CR = N(0.5, 0.1), N = 9 | 9h20m6s  | 0.9803 |

**B. Consensus in superconducting qubit networks**

The nodes in a quantum network could be photons, electrons, or other quantum systems. In this section, we consider a quantum network that consists of superconducting qubits as its nodes. Superconducting quantum circuits based on Josephson junctions are one promising candidate for building hardwares of quantum computers. These macroscopic circuits can behave quantum mechanically like artificial atoms that can also be used to test the laws of quantum mechanics on macroscopic systems [46]. Superconducting qubits have been widely investigated theoretically and implemented experimentally since they could be easily embedded in nanometer-scale electronic devices and scaled up to provide a large number of qubits for quantum computation [47]. Manipulation of superconducting qubits can be achieved by adjusting external parameters such as currents and voltages or by tuning the coupling between two superconducting qubits [48].

In superconducting circuit, the charging energy $E_C$ and the Josephson coupling energy $E_J$ have significant effect on the quantum mechanical behavior of a Josephson-junction circuit. Different kinds of superconducting qubits can be realized according to the regimes of $E_J/E_C$. For example, a charge qubit can form when $E_C \gg E_J$. In practical applications, the Josephson junction in the charge qubit is usually replaced by a dc superconducting quantum interference device (SQUID) with low inductance and a magnetic flux [49]. The equivalent Hamiltonian of a charge qubit can be described as [50], [51]

$$H = F_z(V_g)\sigma_z - F_x(\Phi)\sigma_x,$$  \hspace{1cm} (24)

where $F_z(V_g)$ can be adjusted through external voltage $V_g$, and $F_x(\Phi)$ corresponds to a tunable effective coupling with the external magnetic flux $\Phi$ in the SQUID. Hence, $F_z(V_g)$ and $F_x(\Phi)$ are related to external control fields.

Now, consider a quantum network consisting of three superconducting qubits with control fields acting on all qubits. We denote $\sigma_1^{(12)} = \sigma_1 \otimes \sigma_2 \otimes I$, $\sigma_1^{(23)} = I \otimes \sigma_2 \otimes \sigma_3$, $\sigma_1^{(13)} = \sigma_1 \otimes I \otimes \sigma_3$. Its free Hamiltonian can be described as

$$H_0 = \omega_{12} \sigma_1^{(12)} + \omega_{23} \sigma_1^{(23)} + \omega_{13} \sigma_1^{(13)}.$$  \hspace{1cm} (25)

Denote $\sigma_1^{(1)} = \sigma_1 \otimes I \otimes I$, $\sigma_1^{(2)} = I \otimes \sigma_2 \otimes I$, $\sigma_1^{(3)} = I \otimes I \otimes \sigma_3$, and $\sigma_1^{(4)} = \sigma_1 \otimes I \otimes I$, $\sigma_1^{(5)} = I \otimes \sigma_2 \otimes I$, $\sigma_1^{(6)} = I \otimes I \otimes \sigma_3$. We have the control Hamiltonian in the following form

$$H_c(t) = u_1^{(1)} \sigma_1^{(1)} + u_1^{(2)} \sigma_1^{(2)} + u_2^{(3)} \sigma_1^{(3)} + u_3^{(4)} \sigma_1^{(4)} + u_4^{(5)} \sigma_1^{(5)} + u_5^{(6)} \sigma_1^{(6)}.$$  \hspace{1cm} (26)

Our task is to drive the quantum network from an arbitrary initial state (usually three qubits having different reduced states) to a consensus state. Furthermore, if we withdraw the external control fields, the quantum network will remain in the consensus state under the free Hamiltonian. Denote $I_n$ as an $n$-dimensional matrix with all of its elements being 1. Let the target state be $\bar{\rho} = \frac{I_3}{8}$. We have the following result.

**Proposition 1:** The state $\bar{\rho} = \frac{I_3}{8}$ is a consensus state for the three qubit network. Also, $\bar{\rho}$ is invariant under
The action of free Hamiltonian $H_0 = \omega_{12}\sigma_2^{(12)} + \omega_{23}\sigma_2^{(23)} + \omega_{13}\sigma_2^{(13)}$.

Proof: For $\bar{\rho} = \frac{1}{2}I_n$, we can calculate the reduced states for three notes as follows:

$$\bar{\rho}_1 = \frac{1}{2}I_2, \quad \bar{\rho}_2 = \frac{1}{2}I_2, \quad \bar{\rho}_3 = \frac{1}{2}I_2.$$ 

It is clear that $\bar{\rho}_1 = \bar{\rho}_2 = \bar{\rho}_3$, that is, the state $\bar{\rho}$ is a consensus state for the three qubit network according to Definition 2.

A direct calculation shows that $[H_0, \bar{\rho}] = 0$. Hence,

$$\dot{\bar{\rho}} = [H_0, \bar{\rho}] = 0.$$

That is, $\bar{\rho}$ is invariant under the action of free Hamiltonian $H_0$.

The chosen target state $\check{\rho}$ is a symmetric state from which the superconducting qubit network will keep invariant with only free Hamiltonian $H_0$. The initial state is set as $\rho^0 = \rho^0_0 \otimes \rho^0_1 \otimes \rho^0_2$, where

$$\rho^0_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \rho^0_1 = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}, \quad \rho^0_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

The initial and target states of qubit network are illustrated in Fig. 2.

In practical applications, there may exist fluctuations in magnetic fields and electric fields in superconducting qubits. The practical control Hamiltonian is assumed to be

$$H_o(t) = \theta_u u_1^2 \sigma_1^{(1)} + \theta_u u_1^2 \sigma_1^{(2)} + \theta_u u_2^2 \sigma_2^{(2)} + \theta_u u_3^2 \sigma_2^{(2)} + \theta_u u_3^3 \sigma_3^{(3)} + \theta_u u_3^3 \sigma_3^{(3)}.$$

We apply the $msMS\_DE$ algorithm to search for the robust control fields to reach a consensus state in the above quantum network. Stimulation parameters are set as: the population size is set as $NP = 100$, the time interval $[0, 20]$ ns is equally divided into 100 smaller time intervals (i.e., $D = 100$), the control terms $u_1^2$, $u_1^2$, $u_2^2$, $u_2^2$, $u_3^2$, $u_3^2 \in [0, 1]$ GHz. Let $\omega_{12} = \omega_{23} = 0.1$ GHz. We assume that $\theta_u \in [0.98, 1.02]$ and $\theta_x \in [0.98, 1.02]$ (i.e., $E = 0.02$). For each uncertainty parameter, we choose three samples and we have $N = 9$ samples for training. We employ DE1 (one sample) for comparison. The training performance of driving qubit network is illustrated in Fig. 3. As we can see, $msMS\_DE$ achieves a rather high fitness $J_{max} = 0.9988$ (where 1 is an upper bound), while DE1 achieves the fitness of $J_{max} = 0.9561$. For the case $\theta_u = \theta_x = 1$, Fig. 4 shows the reduced states of three qubits from different trajectories asymptotically converging to the same trajectory using the control learned from $msMS\_DE$. Based on the control fields from DE1 and $msMS\_ED$, we test 2000 additional samples regarding the trace distance defined as (for $i = 1, 2, 3$)

$$||\rho_i - \frac{1}{2}I_2||_T = \frac{1}{2}Tr(\rho_i - \frac{1}{2}I_2)^2(\rho_i - \frac{1}{2}I_2) = \frac{1}{2} \sum_{j=1}^{9} |\lambda_j|$$

where $\lambda_j$ are eigenvalues of $\rho_i - \frac{1}{2}I_3$. Since the maximum trace distance between two quantum states may be 1, we define the relative error between two quantum states $\rho_i$ and $\rho_j$ as $||\rho_i - \rho_j||_T \times 100\%$. Fig. 5 shows that the relative error between each qubit and its target state always keeps below 1.2% for the case using $msMS\_DE$ while the relative error between each qubit and its target state may exceed 10.0% for the case using DE1. We further show the trace distances between the quantum states of different qubits after the control Hamiltonian is withdrawn in Fig. 6. It is clear that the relative errors between the reduced states are always below 2.0% for the case of $msMS\_DE$ while the relative errors may exceed 12.0% for the case of DE1. The results demonstrate that the approximate consensus state achieved using $msMS\_DE$ has much better stability than that obtained using DE1.

V. EXPERIMENTAL RESULTS ON FEMTosecond LASER CONTROL SYSTEMS

The following quantum control experiments were carried out in Department of Chemistry at Princeton University.

A. Experimental setup

The experimental setup contains three major components: 1) a fs laser system, 2) a pulse shaper and 3) a time-of-flight mass spectrometry (TOF-MS). Briefly, the fs laser system (KMIlab, Dragon) consists of a Ti:sapphire oscillator and a amplifier, which produces 1 mJ, 25 fs pulses centered at 790 nm. The laser pulses are introduced into a pulse shaper with a programmable dual-mask liquid crystal spatial light modulator (SLM). The SLM has the capability of independent phase and amplitude modulation and has 640 pixels with 0.2 nm/pixel resolution [52], [53]. Typically, every 8 adjacent pixels are bundled together to form an array of 80 “grouped pixels”, which are the control variables. Each
control variable can have a phase value between 0 and 2π, and an amplitude value between 0 and 1. In this experiment, we do phase-only control, with all the amplitude values fixed at 1. The shaped laser pulses out of the shaper are focused into a vacuum chamber, where photoionization and photofragmentation occurs. The fragment ions are separated with a set of ion lens and passing through a TOF tube. The MS signals are recorded with a fast oscilloscope, which accumulates 1 second with 3000 laser shots each time before being collected with an MCP detector. The MS signals are recorded with a fast oscilloscope, which accumulates 1 second with 3000 laser shots each time before being collected with an MCP detector. The MS signals are recorded with a fast oscilloscope, which accumulates 1 second with 3000 laser shots each time before being collected with an MCP detector. 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The MS signals are recorded with a fast oscilloscope, which accumulates 1 second with 3000 laser shots each time before being collected with an MCP detector. The MS signals are recorded with a fast oscilloscope, which accumulates 1 second with 3000 laser shots each time before being collected with an MCP detector.
DE outperforms DE1 in terms of DE using DE. (a) Ratio \( \frac{CH}{I} \) using DE1 and these 100 testing samples can be written as optimal result is given in Fig. 9(b).

Femtosecond pulse optimized by signals. With 150 iterations, the optimized phases of 80 control variables for the final iteration. With 150 iterations, DE1 can find an optimized result corresponding to the maximum fitness.

The average TOF signal of 100 testing results for the same method as that in the experiments of optimizing TPA signals. With 150 iterations, msMS_DE can find an optimized pulse to make \( CHr/CHi \) achieve 2.41.

Figure 9 shows the results from the msMS_DE algorithm, in which three samples are measured in each experiment. Three samples for each individual were selected using the same method as that in the experiments of optimizing TPA signals. With 150 iterations, msMS_DE can find an optimized pulse for making the average \( CHr/CHi \) of three samples to achieve 2.67. The experimental results are shown in Fig. 9, where the average ratio \( CHr/CHi \) of three samples as the fitness function is presented in Fig. 9(a) and the optimized phases of 80 control variables for the final optimal result is given in Fig. 9(b).

After we obtained the optimal femtosecond control pulses using DE1 and msMS_DE, we can test the performance of the optimal pulses. The testing results are shown in Fig. 10, where Fig. 10(a) is the average TOF signal of 100 testing results with random noises between \(-7.5\%\) and \(+7.5\%\) with respect to the maximum phase \(2\pi\) for the femtosecond pulse optimized by DE1. In other words, if we denote the best individual as \( X_{best} = [x^b_1, x^b_2, \ldots, x^b_{80}]^T \), these 100 testing samples can be written as \( X^i = [x^i_1, 0.075(2\text{rand}(0,1) - 1) \times 2\pi, x^i_2, 0.075(2\text{rand}(0,1) - 1) \times 2\pi, \ldots, x^i_{80}, 0.075(2\text{rand}(0,1) - 1) \times 2\pi]^T \). Fig. 10(b) shows the average TOF signal of 100 testing results for the femtosecond pulse optimized by msMS_DE. The average \( CHr/CHi \) of the 100 testing samples can achieve 2.61 for the pulse from msMS_DE while the average \( CHr/CHi \) is only 2.12 for the pulse from DE1. It is clearly shown that msMS_DE outperforms DE1 in terms of reaching a better objective fitness value in this experiment.

VI. CONCLUSION

In order to solve three classes of quantum robust control problems, we have proposed improved msMS_DE using multiple samples for fitness evaluation and a mixed strategy for mutation. The msMS_DE algorithm shows excellent performance for the control problem of open inhomogeneous quantum ensembles and the consensus problem of quantum networks with uncertainties. We have experimentally implemented msMS_DE on femtosecond laser control systems in the laboratory to generate good TPA signal and control
fragmentation of chemical molecules. In future research, there is plenty of room for exploring the use of DE for emerging quantum control engineering. For example, it is worth adapting more efficient DE algorithms for high-dimensional quantum control problems [54]. In the laboratory, many quantum control problems involve multiple even many objectives that need to be optimized, and it is also worth adapting multi-objective and many-objective evolution optimization algorithms [55], [56] for these challenging quantum control problems.

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