PETER AND ANTI-PETER PRINCIPLE AS THE DISCRETE LOGISTIC EQUATION

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Abstract

In this work Peter principle (in the hierarchical structure any competent member tends to rise to his level of incompetence) is consistently interpreted as the discrete form of the well-known logistic (Verhulst or Maltusian) equation of the population dynamics. According to such interpretation anti-Peter principle (in the hierarchical structure any incompetent member tends to rise to his level of competence) is formulated too.

As it is well-known remarkable Peter principle [1], [2] states that in the hierarchical structure any competent member tends to rise to his level of incompetence. Even if Peter principle is seemingly paradoxical it is in a satisfactory agreement with situations existing in real social hierarchical structures. There are different attempts of the interpretation or mathematical foundation of Peter principle. In this work an original interpretation will be suggested. Namely, in this work Peter principle will be consistently interpreted as the discrete logistic (Verhulst or Maltusian) equation of the population dynamics. According to such interpretation anti-Peter principle (in the hierarchical structure any incompetent member tends to rise to his level of competence) is formulated too.

Thus, as it is well-known logistic (Verhulst or Maltusian) equation in the population dynamics has form

\[
\frac{dx}{dt} = ax(1 - \frac{x}{r}) \quad \text{for} \quad a, r > 0 \quad \text{and} \quad x \leq r \quad (1)
\]

where \( t \) represents the time moment, \( x \) - (human or some other species) population, \( a \) - growth parameter and \( r \) carrying capacity. Simple solution of this equation, representing a sigmoid function, is

\[
x = x_0 r \exp[ar] \frac{1}{r - x_0 - x_0 \exp[ar]} \quad (2)
\]

where \( x_0 \) represents the initial population smaller than \( r \). Obviously, when \( t \) tends toward infinity \( x \) tends toward \( r \) and \( \frac{dx}{dt} \) toward zero. Given logistic dynamics describes population growth limited by negative species self-interaction.

It is well known too that there is anti-logistic equation corresponding to (1)

\[
\frac{dx}{dt} = -ax\left(\frac{x}{r} - 1\right) \quad \text{for} \quad a, r > 0 \quad \text{and} \quad x \geq r \quad (3)
\]
where \(-a\) represents the decrease parameter. It holds simple solution

\[
x = x_0 r \frac{1}{x_0 - (x_0 - r) \exp[\alpha t]}
\]

where \(x_0\) represents the initial population smaller than \(r\). Obviously, when \(t\) tends toward infinity \(x\) tends toward \(r\) and \(\frac{dx}{dt}\) toward zero. Given logistic dynamics describes population decrease limited by positive species self-interaction.

Finally, it is well-known that both, logistic and anti-logistic, equations have significant application not only in population dynamics, i.e. in the biology and demography, but also in the chemistry, mathematical psychology, economics and sociology.

We observe that sigmoid form of the solution of logistic equation satisfactory corresponds to predicted and observed form of the successful member competence time evolution. For this reason we suppose that, in the first approximation, successful member competence time evolution is described by (1). But now \(x\) represents competence in the time moment \(t\), \(x_0\) - initial competence, \(a\) - competence growth parameter and \(r\) - level of incompetence, all of which are characteristic for given member.

But in distinction of a biological species where individuals number can be very large so that population can be effectively treated as a continuous variable satisfying logistic differential equation (1), number of the members in a hierarchical sociological structure (e.g. factory, university, etc.) can be relatively small. For this reason member competence can be a discrete function that satisfies a discrete dynamics. Nevertheless, it is not hard to see that such discrete dynamics must correspond to the discretized form of the logistic equation (1) (which will be not discussed detailedly). According to such discretization any competent member of the hierarchical structure can rise to its incompetence level in a finite time interval.

Moreover, mentioned interpretation of the Peter principle (by discretized form of the logistic equation) admits formulation of the anti-Peter principle by discretized anti-logistic equation (3). But now \(x\) represents competence in the time moment \(t\), \(x_0\) - initial competence, \(-a\) - competence decrease parameter and \(r\) - level of competence, all of which are characteristic for given member. This principle states that in the hierarchical structure any incompetent member tends to rise to his level of competence. It is, of course, in full agreement with discretized form of (4) (which will be not discussed detailedly). Also, according to such discretization, any incompetent member of the hierarchical structure can rise to its competence level in a finite time interval.

In conclusion, it can be shortly repeated and pointed out the following. In this work Peter principle (in the hierarchical structure any successful member tends to rise to his level of incompetence) is consistently interpreted as the discrete form of the well-known logistic (Verhulst or Maltusian) equation of the population dynamics. According to such interpretation anti-Peter principle (in the hierarchical structure any non-successful member tends to rise to his level of competence) is formulated too.

1 References

[1 ] L. J. Peter, R. Hul, The Peter Principle: why things always go wrong (William Morrow and Company, New York, 1969)

[2 ] A. Pulchino, A. Rapisarda, C. Garofalo, The Peter Principle Revisited: A Computational Study, soc-ph/0907.0455 and references therein