Take-Off Trajectory Optimization of Tilt-rotor Aircraft Based on Direct Allocation Method

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Abstract. During the take-off stage, the tilt-rotor aircraft’s actuator is redundant and difficult to control. In order to take off more quickly and safety reach horizontal flight state with less power consumption, the take-off trajectory is necessary to be optimized. This paper introduces the direct allocation method to optimize the take-off trajectory of tilt-rotor aircraft. Firstly, the dynamic model of the take-off stage of the tilt-rotor aircraft is established. According to the actual flight conditions, the constraints and the optimization objective function of the take-off process are established. Then, the entire take-off process is divided into several segments according to time, and the continuous optimal control problem is discretized into a nonlinear programming problem by using the direct allocation method. The state and control variables at each segment point and midpoint are taken for optimization. Then the optimized variables are fitted by the third-order Simpson method. Finally, GPOPS toolbox in MATLAB was used to solve the nonlinear programming problem. The simulation results show that the optimized take-off trajectory can well meet the boundary constraints, and its solutions are stable.

1. Introduction

Tilt-rotor aircraft, with unique flight characteristics, integrates the advantages of traditional fixed wing aircraft and helicopters, which can not only take off and land vertically, but also cruise at a higher speed. So, it is widely concerned by the aviation industry [1-4]. The tilt-rotor aircraft has multiple flight phases. During the take-off phase, as the rotor tilt angle and airspeed increase, two types of actuators, thrust vector and rudder surface, simultaneously control the attitude and the movement of the aircraft. This makes the actuator redundant, which complicates the dynamic model at this phase [5]. In addition, during the entire vertical take-off to horizontal flight process, the tilt-rotor aircraft needs to ensure that the forward speed and altitude of the aircraft are within the safe range according to the rotor tilt angle and thrust limit, because the low forward speed will cause the aircraft to stall and dive into the ground [3, 6, 7]. When flying in fixed-wing mode, the tilt-rotor aircraft can maintain its flying height with the help of aerodynamic lift. The power consumption of the aircraft in level flight is far less than the power consumption in helicopter mode when taking off. Therefore, during the entire flight process of the tilt-rotor aircraft, the take-off and landing phrases are the main stages that affect the maximum endurance of the aircraft. And to some extent, the landing stage can be regarded as the reverse process of the take-off phrase. So, the optimal control of the take-off trajectory
of tilt-rotor aircraft, which enables the aircraft to quickly and efficiently switch from the take-off to the level flight, is the key to improving the endurance of the tilt-rotor [8-12].

The question of the optimal control of the take-off trajectory can be transformed into the question that searching for an optimal take-off trajectory within the feasible take-off trajectory range [10-13]. The maximum and minimum speed limit, the shortest time and the least energy consumption are usually used to evaluate the take-off trajectory [13-16]. The optimal control theory was widely used to solve the problem in order to get an analytic solution. However, it was difficult to obtain the analytical solution for the nonlinear system such as the takeoff process of the tilt-rotor aircraft. In recent years, the direct collocation method in numerical solution is often used for trajectory optimization [13, 14]. This method transforms the dynamic optimal control problem into a nonlinear programming problem with the idea of discretization, and then uses the nonlinear programming algorithm to solve it. Compared with the traditional indirect optimization method, the direct collocation method is insensitive to the initial parameters, and its solution is stable [17, 18]. In this paper, the optimal take-off trajectory of tilt-rotor aircraft is designed based on the direct collocation method, taking the shortest take-off time and minimum energy consumption as the optimization index.

2. Optimal take-off trajectory description of the tilt-rotor aircraft

The tilt-rotor mechanism studied in this paper is shown in Fig. 1. It is mainly powered by two main tilt rotors. The tail rotor is mainly used to maintain the attitude stability of the aircraft during take-off and landing. In order to take off as fast as possible and transfer to the level flight state with the minimum energy consumption, the tilt-rotor aircraft needs to tilt the rotors so that the aircraft can accelerate forward while taking off vertically. So, it can reach the minimum level flight speed as soon as possible, and make full use of the aerodynamic lift to reduce the energy consumption of the rotor. The take-off mode of tilt rotor aircraft is referred as slant take-off in this paper.

![Figure 1. Structure of the tilt-rotor aircraft](image)

2.1. Kinematic equation of the tilt-rotor aircraft

The east-north-up geographic coordinate system is chosen as the navigation coordinate system ($O-X_nY_nZ_n$). The three axes of body coordinate system ($O-X_bY_bZ_b$) are pointing to transversal-forward-vertical direction of the tilt-rotor aircraft. The aircraft is symmetrical about the $YOZ$ plane. In the process of take-off, the transverse and heading motions of the tilt-rotor aircraft are kept stable by the attitude and position control. To simplify the problem, the transverse and heading motions are not considered in the optimization model. Therefore, this paper only analyses the forces in the $YOZ$ plane and establishes the particle motion model.

The slant take-off status of the tilt-rotor aircraft is shown in Fig. 2. In order to simplify the problem, the change of yaw, roll, sideslip angle and the movement in x-axis direction are not considered in the entire take-off process, and it is assumed that the longitudinal movement and the transversal movement of the tilt rotor aircraft are completely decoupled, and only the movement in the $YOZ$ plane will be considered in the take-off process. The take-off trajectory of the tilt-rotor aircraft is adjusted by changing the thrust and the tilt angle of rotors. In Fig. 2, $\phi$ is the angle between the tilt rotor and the
longitudinal axis of the aircraft, $V_a$ is the airspeed of the aircraft, $\alpha$ is the angle of attack, $A$ is the track angle, $D$ and $L$ are the air resistance and aerodynamic lift of the aircraft respectively. According to the Newton's second law, the dynamic equations of the tilt-rotor aircraft in the geographical coordinate system $O-X_nY_nZ_n$ can be expressed as:

\[
\begin{align*}
\dot{V}_U &= (T \sin(\varphi + \alpha + A) + L \cos(A) - D \sin(A) - mg) / m \\
\dot{V}_H &= (T \cos(\varphi + \alpha + A) - L \sin(A) - D \cos(A)) / m
\end{align*}
\]

(1)

Where, $V_U$ is vertical speed, $V_H$ is horizontal speed, track angle $A = \arctan \left( \frac{V_U}{V_H} \right)$, $V_a = \sqrt{V_U^2 + V_H^2}$, $T$ is the thrust provided by the rotors, and $m$ is the total weight of the aircraft.

![Figure 2. Longitudinal force analysis of tilt-rotor aircraft take-off process](image)

2.2. Constraint condition

In addition to the constraints of the above dynamic equations, three kinds of constraints are considered according to the practical application of the tilt-rotor aircraft.

2.2.1. Take-off start and end state constraints. The tilt-rotor aircraft takes off in a slant way. Initially, the aircraft is stationary on the ground, and its main rotors tilt forward at a certain angle, so that it can accelerate forward while taking off vertically. Finally, the aircraft reaches the set altitude and airspeed, and then it transfers to the horizontal flight phrase. According to the specification of the tilt-rotor aircraft, the following constraints on the start and end states of take-off phrase can be summarized as:

\[
\begin{align*}
V_U (t_0) &= 0 \text{ m/s}, V_U (t_f) = 0 \text{ m/s} \\
V_H (t_0) &= 0 \text{ m/s}, V_H (t_f) = V_{ac} \\
Z_a (t_0) &= 0 \text{ m}, Z_a (t_f) = Z_{ac} \\
A (t_0) &= 10^\circ, A (t_f) = 0^\circ \\
\alpha (t_0) &= 0^\circ, \alpha (t_f) = 2^\circ
\end{align*}
\]

(2)

Where, $t_0$ and $t_f$ respectively represent the starting and ending time of the take-off stage. $V_{ac}$ is the horizontal flight airspeed of the tilt-rotor aircraft, $Z_a$ is the vertical altitude of the aircraft, and $Z_{ac}$ is the horizontal flight altitude of the aircraft.

2.2.2. Flight trajectory constraint. In order to avoid the large vibration of the tilt-rotor aircraft during the whole take-off process, and considering the critical stall angle of attack limit of the aircraft in low-speed flight, the trajectory constraints during the take-off process are summarized as follows:
2.2.3. Actuator constraints. In practical use, the tilt-rotor aircraft usually needs to reserve a certain amount of control margin. Therefore, when designing the optimal take-off trajectory of the tilt-rotor aircraft, the output of each actuator needs to be limited according to the actual working range of each actuator, which is limited as follows:

\[
\begin{align*}
0m/s & \leq V_v(t) \leq 5m/s \\
0m/s & \leq V_h(t) \leq V_{ac} \\
0m & \leq Z_a(t) \leq Z_{ac} \\
0^\circ & \leq A(t) \leq 15^\circ \\
-5^\circ & \leq \alpha(t) \leq 5^\circ
\end{align*}
\]  

(3)

Where, \(T_b\) and \(\phi_b\) are the thrust of tail rotor and the tilt angle in the \(OY\) direction respectively, and \(\delta_l\) and \(\delta_r\) are the deflection angles of the left and right ailerons respectively.

2.3. Optimization objective function

From the analysis above, it can be seen that the tilt-rotors need to provide thrust to overcome the gravity of the aircraft when it takes off and lands vertically, while when the rotors tilt to the horizontal direction in level flight, the thrust of rotors only needs to overcome the resistance. Therefore, it is necessary for the tilt-rotor aircraft to quickly transfer from the vertical take-off phase to the horizontal flight phase, and effectively use the aerodynamic lift to reduce energy consumption. In addition, in the entire slant take-off stage, the center of gravity and the moment of inertia of the aircraft will be changed by the tilt of rotors, and the control strategy changes accordingly. In order to ensure flight safety, it is necessary to reduce the time of slant take-off stage to the minimum within the allowable range of the control ability. To sum up, combined with the general form of the objective function of the optimal control problem given in [11], the optimal objective function in this paper is written as:

\[
J(t) = tf + \frac{1}{t_f-t_0} \int_{t_0}^{t_f} P(t) dt
\]  

(5)

Where \(P(t)\) is the total power of the tilt-rotor aircraft in the slant take-off stage. Generally, the power consumption of other actuators of the aircraft is negligibly compared with that of two rotors. Therefore, \(P(t)\) can approximately represent the total power of two rotors. According to [19], the power of the rotors can be described as:

\[
P(t) = T(t)V_v(t) + T(t)V_h(t)
\]  

(6)

Where \(V_v(t) = V_a \cos(\phi + \alpha)\) is the air velocity perpendicular to the propeller plane. \(V_h(t) = V_a \sin(\phi + \alpha)\) is the air velocity parallel to the propeller plane. \(v_i(t)\) is the induced velocity of the rotor, which is calculated by the following formula:

\[
v_i(t) = V_v(t) + V_h(t) + V_a^2(t)v_i^2(t) = V_a^4
\]  

(7)
Where \( v_g = \sqrt{mg / 2 \rho \pi R^2} \) is the induced velocity of the tilt-rotor aircraft in hover. \( \rho \) is the air density and \( R \) is the main propeller radius.

3. Direct allocation method

3.1. Method description

In the whole slant take-off process, the motion state and the optimization objective of trajectory of the tilt-rotor aircraft is a function of time, so the take-off trajectory optimization problem of the tilt-rotor aircraft can be transformed into a dynamic optimal control problem with constraints. Considering the problem is difficult to be solved by analytical method, the problem is transformed into a non-linear programming problem by using the direct allocation method described in this paper.

The direct allocation method has good robustness. Firstly, it discretizes the continuous dynamic optimal control problem. At the same time, it assumes that the parameters between two adjacent nodes change linearly with time, thereby the original problem will be transformed into a static optimization problem with constraints. Finally, the polynomial fitting method is used to obtain the optimization results over the entire interval \([20, 21]\). There are many ways to fit polynomials, such as two-step method, three-step Simpson method, four step Runge-Kutta method, etc. Considering the computational cost and calculation accuracy, the third-order Simpson method will be used in this paper.

3.2. Differential equations and constraints

According to the analysis in section 2, the differential equation of optimal control is expressed as follows:

\[
\begin{align*}
\dot{V}_U &= (T \sin(\varphi + \alpha + A) + L \cos(A) - D \sin(A) - mg) / m \\
\dot{V}_H &= (T \cos(\varphi + \alpha + A) - L \sin(A) - D \cos(A)) / m \\
\dot{Z}_n &= V_U \\
A &= (V_U V_H - V_U \dot{V}_H) / (V_H^2 + V_H^2)
\end{align*}
\]

(8)

Where the vertical velocity \( V_U \), horizontal velocity \( V_H \), altitude \( Z_n \), and track angle \( A \) are used as state quantities \( X = [V_U, V_H, Z_n, A] \). Tilt rotor thrust \( T \), rotor tilt angle \( \varphi \), and aircraft angle of attack \( \alpha \) as control variables \( U = [T, \varphi, \alpha] \). In addition, the constraint conditions are (2) – (4).

3.3. The third-order Simpson method

In the entire slant take-off phase of tilt-rotor aircraft, \( N-1 \) points are equidistant inserted in \([t_0, t_f]\), and the time is divided into \( N \) segments, each sub time interval is \([t_i, t_{i+1}]\) \((i = 0, 1, 2, \ldots, N-1, \ldots)\), where \( \Delta t = t_{i+1} - t_i = (t_f - t_0) / N \). The variables for the nonlinear programming are as follows:

\[
Z = [(X,U,U_m)_1, (X,U,U_m)_2, \ldots, (X,U,U_m)_i, \ldots, (X,U,U_m)_{N-1}, (X,U)_j, t_0, t_f]^T
\]

(9)

Where \( U_m \) is the control variable at the midpoint of the segmentation time period. The Simpson integral method is used in each segmentation time period to take the state variable, control variable and control variable at the midpoint of each segmentation point as optimization variables, and the state equation integration can be obtained:

\[
X_{i+1} = X_i + \Delta t / 6 \left[ f(X_i, U_i, t_i) + 4 f(X_m, U_m, t_m) + f(X_{i+1}, U_{i+1}, t_{i+1}) \right] (i = 0, 1, \ldots, N-1)
\]

(10)

Similarly, the optimization objective function is discretized by the third-order Simpson method, which is written as follows:
\[ J = t_N + \frac{1}{6} \sum_{i=1}^{N} \left[ P(t_i) + 4P(t_{i-1}) + P(t_{i+1}) \right] \]  

(11)

Finally, the constraints are applied to the segmentation point and the midpoint of the segmentation period.

4. Simulation and result analysis

4.1. Simulation parameters

The total mass of the tilt-rotor aircraft \( m = 67 \, \text{kg} \), the equivalent area of the wing is \( 4 \, \text{m}^2 \), and the maximum thrust of the rotors in take-off phrase is no more than \( 75 \, \text{kg} \).

| Table 1. The parameters of take-off phrase |
|------------------------------------------|
| Initial parameter of take-off | End parameter of take-off |
| \([V_U, V_H, Z_n, A, \alpha] = [0, 0, 0, 10^\circ, 0]\) | \([V_U, V_H, Z_n, A, \alpha] = [0, 33 \, \text{m/s}, 40 \, \text{m}, 0^\circ, 2^\circ]\) |

In the simulation, the tilt-rotor aircraft takes off in a slant take-off way discussed in section 2 with the initial speed of \( 0 \, \text{m/s} \), the altitude of \( 0 \, \text{m} \), the track angle of \( 10^\circ \), and the angle of attack of \( 0^\circ \). After take-off, the rotors gradually tilt and finally, the aircraft reaches a stable horizontal flight state. At this time, the aircraft has the flight velocity of \( 33 \, \text{m/s} \), the altitude of \( 40 \, \text{m} \), the vertical velocity of \( 0 \, \text{m/s} \), and the track angle of \( 0^\circ \).

4.2. Results and analysis

In this paper, MATLAB software and GPOPS toolboxes are used for simulations. The direct allocation method is used to optimize the take-off trajectory of the tilt-rotor aircraft. The function curves of vertical altitude, horizontal velocity, rotor command, and rotor tilt angle are given as follows. The command value of the rotor represents the PWM signal duty cycle that controls the rotation of the rotor. The motor is running at the maximum speed when the duty ratio is 100%.

Figure 3. The vertical height over time

Figure 4. The horizontal velocity over time
It can be seen from Fig. 3 and Fig. 4 that the start and end values of vertical altitude and horizontal velocity well meet the boundary constraints. From Fig. 5 and Fig. 6, it can be seen that the rotor motor command changes smoothly, and the rotor tilt angle is turned to 0° in a short time. At the time, the tilt-rotor aircraft reaches the horizontal flight phrase. When the tilt-rotor aircraft just takes off, the rotor is maintained at a high power at the tilt angle of about 75°. At this time, the aerodynamic lift generated by the wings was small since the forward velocity of the aircraft is below 15 m/s. The aircraft still needs to overcome its own gravity and increases its altitude by the vertical component of the rotor thrust. When the forward velocity of the aircraft reaches 20 m/s, the aerodynamic lift generated by the wings increases significantly, which gradually increases in the role of overcoming the aircraft gravity, and the rotor gradually decreases in the role of overcoming the gravity. Then, the rotor can further tilt and accelerate forward. When the rotor is fully tilted to the horizon, the aircraft reaches the horizontal flight phrase. The aerodynamic lift generated by the wings is sufficient to overcome the weight of the aircraft and maintain the altitude. The rotor thrust is only used to overcome the air resistance to maintain airspeed. Therefore, the duty cycle of the rotor command in horizontal flight will be reduced to about 30% compared with that of the rotor command in take-off.

The constraints in the simulation parameters, such as the final altitude and final velocity of the aircraft, can be further adjusted according to the take-off location and take-off time of the tilt-rotor aircraft, so as to optimize the take-off trajectory applicable to different take-off locations. For example, the final altitudes are set to 20 m, 40 m, and 60 m respectively without changing other constraints, and the optimal flight trajectories are shown as follows:
Fig. 9 shows that setting different flight altitudes has little effect on the rotor command under the same take-off time, and three kinds of command curves are still smooth. It can be seen from Fig. 8 and Fig. 10 that the larger the target altitude is, the smaller the forward flight velocity is, and the smaller the rotor tilt angle is, so as to provide greater vertical thrust to reach the set flight altitude. The simulation results show that the direct allocation method can optimize the take-off trajectory of the tilt-rotor aircraft and achieve the set flight parameters.

5. Conclusion
In order to optimize the take-off trajectory of tilt-rotor aircraft, this paper analyzes the dynamic model of the aircraft in the take-off process, and sums up the initial and end state constraints of the aircraft, the flight trajectory constraint and the actuator constraint in this process. In addition, the optimization objective function is established with the total take-off time and total energy consumption. And then, the dynamic optimal control problem is transformed into a nonlinear programming problem by using the direct allocation method. Finally, MATLAB software and GPOPS toolbox are used to solve the nonlinear programming problem, and the optimal take-off trajectory and control parameters can be calculated with different constraint conditions.

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