Quantum-gravity phenomenology, Lorentz symmetry, and the SME

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Abstract. Violations of spacetime symmetries have recently been identified as promising signatures for physics underlying the Standard Model. The present talk gives an overview over various topics in this field: The motivations for spacetime-symmetry research, including some mechanisms for Lorentz breaking, are reviewed. An effective field theory called the Standard-Model Extension (SME) for the description of the resulting low-energy effects is introduced, and some experimental tests of Lorentz and CPT invariance are listed.

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INTRODUCTION

Perhaps the most challenging open question in present-day fundamental physics concerns a unified quantum theory of all interactions including gravity. To date, a tremendous amount of theoretical work has been devoted to various approaches addressing this question. However, experimental efforts in this line of research are hampered, primarily because of the expected Planck suppression of the corresponding effects at presently attainable energies. A possible avenue to circumvent this issue is to scan the predictions of a given candidate underlying theory for effects that could be present already at lower energy scales. For example, one can search for novel particles, such as those required by supersymmetry, or large extra dimension.

Another promising approach is to ask which type of effect can be measured with Planck precision, and then determine whether such effects are indeed allowed in theoretical approaches to a more fundamental theory. In this context, tests of invariance properties appear to be an excellent candidate: symmetries allow exact theoretical predictions, and they are typically amenable ultrahigh-precision experiments. From a quantum-gravity perspective, spacetime symmetries could be particularly promising: gravity governs the dynamics of space and time, so that a quantum theory of gravitation is likely to affect the structure of spacetime. For example, typical underlying models can involve more than four dimensions, operator-valued noncommuting coordinates, or a certain discreteness of space.

The present talk further explores the idea that spacetime symmetries—and in particular Lorentz invariance—may be violated by small amounts as a result of more fundamen-

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tal physics. We begin by studying the interplay between various spacetime symmetries. Then, two sample mechanisms for Lorentz and CPT breakdown in Lorentz-symmetric approaches to underlying physics are reviewed. The last section recounts the basic philosophy behind the construction of the Standard-Model Extension (SME), which provides the modern effective-field-theory framework for describing Lorentz and CPT violation. This section also lists some tests of Lorentz and CPT symmetry.

**THE INTERPLAY BETWEEN VARIOUS SPACETIME-SYMMETRY VIOLATIONS**

Spacetime transformations that, to the best of our knowledge, are associated with a symmetry of nature are closely intertwined in the Poincaré group. Consider the case in which symmetry is lost under one (or more) transformations. The question arises as to whether the remaining transformations still determine an invariance, or whether the violation of one set of spacetime symmetries typically leads to the breakdown of other invariances contained in the Poincaré group. The purpose of the present section is to explore this question.

Let us first consider the case of a discrete transformation—the combination of charge conjugation (C), parity inversion (P), and time reversal (T). The famous CPT theorem [1] states roughly that, under a few mild assumptions, CPT invariance is a consequence of conventional quantum mechanics and Lorentz symmetry. Thus, when CPT invariance is broken, one or more ingredients of the CPT theorem can no longer hold true. The question arises which one of these ingredients should be abandoned. Both the Lorentz and the CPT transformations involve spacetime, which suggests that CPT breaking implies Lorentz violation. This result has been proved in the framework of axiomatic field theory by Greenberg [2]. This “anti-CPT theorem” essentially asserts that in local, unitary, relativistic point-particle quantum field theories a breakdown of CPT symmetry always comes with Lorentz violation. Note, however, that the converse of this result, i.e., Lorentz violation implies CPT breakdown, is false in general. We see that as a corollary, CPT tests are at the same time Lorentz tests. We finally note that other types of CPT breaking would require further deviations from conventional physics, such as unconventional quantum mechanics [3].

We next look at a situation in which translational invariance is violated. (A mechanism for this effect is studied in a subsequent section of this talk.) In this case, the energy–momentum tensor $\theta^{\mu\nu}$, which generates translations, is usually no longer conserved. To see that this also affects Lorentz symmetry, we consider the angular-momentum tensor $J^{\mu\nu}$, given by

$$J^{\mu\nu} = \int d^3x \, (\theta^{0\mu} \chi^\nu - \theta^{0\nu} \chi^\mu).$$  \hspace{1cm} (1)

In the Poincaré-symmetric case, this tensor is the generator of Lorentz transformations. Note that energy–momentum tensor $\theta^{\mu\nu}$ appears in the definition of $J^{\mu\nu}$. Since $\theta^{\mu\nu}$ is taken as not conserved in the present case, $J^{\mu\nu}$ will usually depend on time. Then, the expression (1) no longer determines the conventional time-independent Lorentz-transformation generators. In fact, such generators will typically no longer exist, and
then Lorentz invariance ceases to be assured. In other words, translation-symmetry breaking is in such cases associated with Lorentz violation.

**EXAMPLES OF MECHANISMS FOR SPACETIME-SYMMETRY VIOLATION**

The preceding section has shown that the breakdown of certain subsets of spacetime symmetries can result in the violation of additional spacetime invariances. But the question remains how spacetime symmetries can be broken in a Poincaré-invariant underlying theory in the first place. Various mechanisms for symmetry violations have been devised in a number of approaches to fundamental physics, such as strings [4], spacetime foam [5], noncommutative geometry [6], nontrivial spacetime topology [7], and cosmologically varying scalars [8]. The present section gives a somewhat more detailed description of two of the above mechanisms—spontaneous Lorentz and CPT breaking in string theory and Lorentz and CPT violation through spacetime-dependent scalars.

**Spontaneous Lorentz and CPT violation.** Spontaneous breakdown of Lorentz and CPT symmetry can occur in the context of the field theory of the open bosonic string [4]. The mechanism of spontaneous symmetry violation is quite attractive from a theoretical viewpoint because the dynamics remains invariant under the symmetry in question. It is the ground-state solution of the system that fails to exhibit all invariances of the Hamiltonian, and is therefore said to break the corresponding symmetries. Instances of spontaneous symmetry violation can readily be identified in solid-state physics, in the physics of elastic media, and in elementary-particle theory.

In what follows, we will discuss three sample physical systems. The features of these examples will enable us to gain further intuition about spontaneous Lorentz and CPT breakdown in a step-by-step fashion. An illustration that supports these three examples is given in Fig. 1.

Our first system is classical electrodynamics. Within this context, the energy density $V(\vec{E},\vec{B})$ of any pattern of electric and magnetic fields $\vec{E}$ and $\vec{B}$, respectively, is determined by

$$V(\vec{E},\vec{B}) = \frac{1}{2} (\vec{E}^2 + \vec{B}^2),$$

where we have employed natural units. Given any solution of the classical Maxwell equations, Eq. (2) yields the energy stored in the electromagnetic fields. Any field configuration involving a nonzero field is associated with a strictly positive energy. Only when both $\vec{E}$ and $\vec{B}$ are zero everywhere, we have vanishing field energy. The lowest-energy configuration of a system is usually identified with its ground state or vacuum. It is thus apparent that in classical electrodynamics the vacuum contains no fields; the Maxwell vacuum is empty.

The second sample system we examine is a Higgs-type field $\phi$. Unlike the electromagnetic fields considered above, the Higgs field is a scalar. Scalar fields of this type are believed to occur in Nature; they are, in fact, part of the Standard Model of particle physics. As before, we consider the energy density of $\phi$. In the absence of spacetime
dependence, i.e., $\partial_\mu \phi = 0$, the energy density $V(\phi)$ obeys
\begin{equation}
V(\phi) = (\phi^2 - \lambda^2)^2 ,
\end{equation}
where $\lambda$ is a constant. A possible spacetime dependence of $\phi$ would add (positive definite) kinetic-type energy to this expression, which is uninteresting in the present context. We therefore see that the lowest possible energy state of $\phi$ possesses zero energy. As opposed to the electrodynamics case, this state is attained for finite field values $\phi = \pm \lambda$. It is thus apparent that in the presence of such Higgs-type fields the vacuum is filled with a condensate $\phi_{\text{vac}} \equiv \langle \phi \rangle = \pm \lambda$. In quantum theory, the condensate $\langle \phi \rangle$ is referred to as the vacuum expectation value (VEV) of $\phi$. One of the physical effects associated with the VEV of the Standard-Model Higgs is to cause mass terms for many elementary particles. It is important to note that $\langle \phi \rangle$ is a scalar, so it does not select a Lorentz-violating spacetime direction.

The third sample system we consider concerns a (hypothetical) vector field $\vec{C}$. Such a vector field clearly lacks observational evidence. However, such a field may be contained in candidate fundamental theories, such as strings. We focus here on the non-relativistic physics, which is sufficient to illustrate rotation violation; a relativistic generalization can easily be obtained. Paralleling the previous Higgs-field case, we take the potential-energy density for $\vec{C}$ to be given by
\begin{equation}
V(\vec{C}) = (\vec{C}^2 - \lambda^2)^2.
\end{equation}
Spacetime dependence of $\vec{C}$ would contribute positive-definite kinetic-energy contributions. Equation (4) shows that $V = 0$ is the lowest energy for the system. As for the Higgs field, this lowest-energy state requires $\vec{C}$ to be nonvanishing: $\vec{C}_{\text{vac}} \equiv \langle \vec{C} \rangle = \vec{\lambda}$. Here, $\vec{\lambda}$ is any spacetime-constant vector that obeys $\lambda^2 = \lambda^2$. As before, the vacuum fills with the VEV of the field $\vec{C}$ and is therefore not empty. Since $\langle \vec{C} \rangle = \vec{\lambda}$ is a constant vector, the vacuum contains an intrinsic direction, which violates rotational invariance and therefore also Lorentz symmetry.

**Cosmologically varying scalars.** As noted at the beginning of this section, a scalar varying on cosmological scales leads to Lorentz violation. This can be established with our result from the previous section that the breakdown of translational invariance (here via the spacetime dependence of the scalar) typically leads to the loss of Lorentz symmetry. This effect is independent of the mechanism driving the variation of the scalar. The remainder of this section gives a more detailed discussion of this result with the goal of providing further intuition.

We begin by establishing the effect at the Lagrangian level. To this end, consider two scalar fields $\phi$ and $\Phi$ and a varying coupling $\xi(x)$. It is the spacetime dependence of $\xi(x)$ that will lead to Lorentz violation; $\phi$ and $\Phi$ are sample dynamical variables that could in principle be replaced by vector or spinor fields. Let the Lagrangian $\mathcal{L}$ of the system contain a kinetic-type term of the form $\xi(x) \partial^\mu \phi \partial_\mu \Phi$. A suitable integration by parts at the level of the action will produce a boundary term that can be dropped while leaving unaffected the equations of motion. The resulting Lagrangian $\mathcal{L}'$ will then contain a term of the form
\begin{equation}
\mathcal{L}' \supset -K^\mu \phi \partial_\mu \Phi. 
\end{equation}
FIGURE 1. Spontaneous symmetry breaking. In Maxwell’s classical electromagnetism (1), the state with the lowest energy is attained for $\vec{E} = 0$ and $\vec{B} = 0$. The classical Maxwell vacuum is therefore empty. The Higgs-type field (2) possesses interactions with an energy density $V(\phi)$ that triggers a non-zero value of $\phi$ in the ground state. The vacuum contains a scalar condensate, which is shown in gray. Lorentz and CPT symmetry are still intact. (However, other, internal symmetries may be broken.) Vector fields occurring, for instance, in string field theory (3) can have energy densities analogous to those of the Higgs field. Such interactions would lead to a nonvanishing field value in the lowest-energy configuration. The resulting VEV of such a vector field causes a special direction in the vacuum, which breaks Lorentz and possibly CPT invariance.
FIGURE 2. Lorentz breakdown via spacetime-dependent scalars. The shade of gray displays the size of the scalar. Lighter areas correspond to smaller values and darker areas to larger values. The black arrows symbolize the gradient of the scalar. This gradient selects a preferred direction in spacetime, so that Lorentz symmetry is violated.

Here, the vector $K^\mu \equiv \partial^\mu \xi$ is an external nondynamical 4-vector. Such a quantity breaks Lorentz symmetry because it selects a preferred direction in spacetime. If $\xi$ varies on cosmological scales, $K^\mu$ is essentially a constant with respect to local physics, such as solar-system physics.

To gain further understanding of the Lorentz breakdown resulting from a varying scalar consider the following intuitive picture. The variation of the scalar implies that there is some region in which its 4-gradient is nonzero. Such a 4-gradient is associated with a preferred direction in spacetime. This is illustrated in Fig. 2. For instance, consider a particle that possesses interactions with this background scalar. When the motion of such a particle is along the gradient of the scalar, its propagation features may be different from the situation in which this particle moves perpendicular to the gradient. Since physically inequivalent directions correspond to anisotropies, rotational invariance, and thus Lorentz symmetry, must be violated.

**THE STANDARD-MODEL EXTENSION**

Once Lorentz and CPT violation is identified as a possible signature for physics underlying the Standard Model (and possibly arising at the Planck scale), it is desirable to have at ones disposal a test framework for the description of the resulting low-energy effects. Perhaps the most important use of such a test framework is the identification of suitable Lorentz tests. Another important advantage of a test model is provided by the fact that it can be used to compare different Lorentz and CPT tests if the model is broad enough. The consistency of the low-energy framework may also put some theoretical constraints on possible high-energy underlying models.

A test model for a certain high-energy underlying theory can usually be obtained by taking the low-energy limit of that theory. In the present case of Lorentz and CPT violation we do not follow this approach for two reasons. First, no complete and realistic fundamental theory is currently known. It is therefore desirable to have test framework for Lorentz and CPT symmetry that is relatively independent of the details of candidate underlying models. This permits a comprehensive search for Lorentz and CPT breakdown. Second, in some approaches to quantum gravity the physical vacuum state is cur-
rently unknown. For those models, the standard determination of a unique low-energy limit therefore fails. In light of these facts, we must rather proceed by constructing a test framework of sufficient generality to include the largest class of possible Lorentz and CPT violations consistent with certain more fundamental principles.

In what follows, we review the construction of the flat-spacetime limit of the Standard-Model Extension (SME), which is the modern framework for the description of the low-energy effects of Lorentz and CPT breakdown [9]. The basic idea is to start with the Standard-Model Lagrangian \( \mathcal{L}_{\text{SM}} \) and add terms that violate Lorentz and possibly also CPT symmetry:

\[
\mathcal{L}_{\text{SME}} = \mathcal{L}_{\text{SM}} + \delta \mathcal{L}.
\]  

(6)

Here, the SME Lagrangian is denoted by \( \mathcal{L}_{\text{SME}} \) and Lorentz- and CPT-breaking corrections are collected in \( \delta \mathcal{L} \). As discussed above, we need to construct \( \delta \mathcal{L} \) by hand. From the arguments in the previous sections we know that the symmetry-violating effects appear as background vectors or tensors in the vacuum. To be observable, they must couple to conventional fields. Because we insist on the fundamental principle of coordinate independence, this coupling must be a covariant contraction, so that \( \delta \mathcal{L} \) transforms as a scalar under changes of the (inertial) coordinate system. For example,

\[
\delta \mathcal{L} \supset b^\mu \bar{\psi} \gamma_5 \gamma_\mu \psi.
\]  

(7)

Here, \( b^\mu \) is a Lorentz- and CPT-violating background assumed to be generated in some underlying theory. It is a free coefficient, which can be searched for in suitable tests. Clearly, \( b^\mu \) must be extremely small on observational grounds. The quantity \( \bar{\psi} \gamma_5 \gamma_\mu \psi \) denotes the usual chiral current of a Standard-Model fermion. Note that all Lorentz indices are properly contracted, so that \( b^\mu \bar{\psi} \gamma_5 \gamma_\mu \psi \) is a coordinate scalar.

Clearly, this construction yields infinitely many contributions to \( \delta \mathcal{L} \), most of which would be expected to be subleading. For phenomenological purposes, it therefore seems practical and justified to consider only a subset of contributions to \( \delta \mathcal{L} \) that satisfies certain additional requirements. For example, power-counting renormalizability, translational invariance, and the usual gauge symmetries are commonly imposed. This “minimal SME” has provided the basis for a number of investigations of Lorentz and CPT breakdown involving mesons [11], baryons [12, 13, 14], electrons [15, 16, 17], photons [18], muons [19], and the Higgs sector [20]. An analysis involving the gravity sector has recently also been performed [21]. We note that neutrino-oscillation experiments offer discovery potential [9, 22].

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2 A similar methodology can also be applied in curved-spacetime situations involving gravity [21].
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