Dynamical suppression of radiative decay via atomic deflection by a standing light wave

M.A. Efremov and M.V. Fedorov
General Physics Institute, RAS, 119991 Moscow, Russia

V.P. Yakovlev
Moscow State Engineering Physics Institute, 115409 Moscow, Russia

W.P. Schleich
Abteilung für Quantenphysik, Universität Ulm, 89069 Ulm, Germany

(Dated: 31st March 2022)

We consider the radiative decay of atoms scattered by a resonant standing light wave. Scattering is shown to suppress the Rabi oscillations and to slow down the atomic radiative decay giving rise to a power law behavior of the time-dependent level populations rather than the exponential one.

INTRODUCTION

Scattering of atoms by a resonant standing light wave is one of the basic phenomena of the atom optics. The number of publications on this subject is huge (see, e.g., the books [1, 2, 3] and the references therein). A very special place in this field belongs to a series of works of which is determined mainly by spontaneous radiative transitions to nonresonant atomic levels [4, 5, 6, 7, 8, 9].Realizability of such a scheme was demonstrated in the experiment [6, 9]. Ar* atoms were prepared initially in the metastable state \(| \tilde{m} = 2, J = 2 \rangle \) and, finally, to the ground level \( E_g \) (Fig. 1). In principle, in Ar* there is another transition, \(| 1 s_5, J = 2 \rangle \rightarrow | 2 p_1, J = 2 \rangle \), in which 98% of the width of the excited level is determined by spontaneous decay to levels different from \( E_m \) and, finally, to the ground level \( E_g \). In any case, for both transitions the model of a two-level system with a wide excited level decaying predominantly to levels different from the metastable one works reasonably well. This is the model to be considered in this work.

Both in the experiment [6, 9] and, most often, in the theory [10], the investigated regimes of scattering corresponded to the weak-scattering Bragg regime. This means that the atomic-beam incidence angle was close or equal to the Bragg angle and the resonance coupling was not too strong. In terms of the Rabi frequency \( \Omega \) and the width of the excited level \( \Gamma \), the last assumption implies that \(| \Omega | \ll | \Gamma | \). In this case, the decay dynamics of an atomic system was shown to obey the usual exponential law [10, 11]. Under some special conditions the effects like population trapping were predicted to take place [11]. But, again, the residual atomic population was shown to approach its asymptotic non-zero level exponentially [1].

In this work we will investigate the dynamics of spontaneous decay in the system under consideration at different conditions. First, we will consider the case of strong Rabi coupling, \(| \Omega | \gg | \Gamma | \) and \(| \Omega | t \gg 1 \). And, second, we will consider the case of normal (or almost normal) incidence of atoms upon the standing wave. This is the diffraction regime of scattering, in which many diffraction maxima can arise from the initially well collimated atomic beam.

By investigating the decay dynamics in such a regime we find that the total time-dependent populations of both metastable and excited levels fall non-exponentially. In contrast to standard predictions, the atomic populations are characterized by power-law dependencies on the interaction time \( t \). The effect is not connected with formation of any kind of grey or dark states (as in Ref. [12]) because asymptotically, at very long time, atomic populations tend to zero. But, owing to scattering, the radiative decay appears to be slowed down. In addition, we find that in atoms scattered by a standing light wave the Rabi oscillations of atomic populations appear to be strongly suppressed compared to a pure two-level system in a resonance field. The physical interpretation of these effects is given.

GENERAL EQUATIONS

The total wave function \( \Psi \) of an atom interacting with a light field depends on the atomic center-of-mass position vector \( \mathbf{r} \), intra-atomic variables, and time \( t \). The wave function obeys the Schrödinger equation

\[
\frac{i}{\hbar} \frac{\partial \Psi}{\partial t} = \left\{ -\frac{1}{2m} \nabla^2 + H_{at} - \mathbf{d} \cdot \mathbf{E}(\mathbf{r}, t) \right\} \Psi, \tag{1}
\]

where \( \hbar = 1 \), \( \nabla = \partial / \partial \mathbf{r} \), \( \mathbf{d} \) is the intra-atomic dipole moment, and \( \mathbf{E}(\mathbf{r}, t) \) is the electric field strength, for a
standing light wave given by
\[ \mathbf{E}(r, t) = 2\mathbf{E}_0 \cos(\omega t) \cos(kx). \]  
(2)

Here and below \( x \) is the center-of-mass coordinate along the axis parallel to \( k \); \( k \) and \( \mathbf{E}_0 \) are the wave vector and field-strength amplitude of one of the two identical counter-propagating travelling waves forming a standing light wave (Fig. 1), \( k = \omega/c \).

![Figure 1. A scheme of atom-light scattering and the internal structure of atomic levels.](image)

With respect to intra-atomic variable, the wave function \( \Psi \) can be expanded in a series of intra-atomic field-free wave functions \( |i⟩ \)
\[ \Psi = \sum_i \varphi_i(x, t)|i⟩ |i⟩ \exp \left(-i E_it + i\mathbf{p}_0 \cdot \mathbf{r} - i\frac{\mathbf{p}_0^2 t}{2m} \right), \]  
(3)

where \( \mathbf{p}_0 \) is the unperturbed-atom center-of-mass momentum.

In this work we will consider only the resonance case, when the light frequency equals the energy spacing between some two discrete nondegenerate atomic levels, \( E_e \) and \( E_m, \omega = E_e - E_m. \) We assume that \( E_m \) and \( E_e \) are, respectively, an infinitely narrow metastable and a wide excited atomic levels, and the width \( \Gamma \) of the excited level is determined predominantly by its spontaneous decay to the ground atomic level (inset of Fig. 1).

In the resonance case we keep only two terms in the expansion (3) with \( i = m \) and \( i = e. \) Moreover, in the rotating wave approximation we retain only one of the two terms in the Euler expansion for cosine \( \cos(\omega t) = \frac{1}{2}[\exp(i\omega t) + \exp(-i\omega t)] \) to drop the fast oscillating terms \( \propto \exp(\pm i\omega t) \). The arising equations for the metastable- and excited-state center-of-mass wave functions \( \varphi_m(x, t) \) and \( \varphi_e(x, t) \) can be written in the form of a matrix Schrödinger-like equation for the two-component function
\[ \Phi(x, t) = \left\{ \begin{array}{c} \varphi_m(x, t) \\ \varphi_e(x, t) \end{array} \right\} : \]  
(4)

\[ i\frac{\partial \Phi(x, t)}{\partial t} = \mathbf{H} \Phi(x, t) \]  
(5)

with the matrix Hamiltonian
\[ \mathbf{H} = \begin{pmatrix} -\frac{1}{2m}\nabla_x^2 - \frac{i}{m}p_0 x \\ \Omega' \cos(kx) - \frac{1}{2m}\nabla_x^2 - \frac{i}{m}p_0 x \end{pmatrix}, \]  
(6)

Here and below \( \nabla_x \equiv \partial/\partial x, \Omega = 2d_{me} \cdot \mathbf{E}_0 \) is the Rabi frequency, and \( d_{me} \equiv \langle m|d|e⟩ \) is the dipole matrix element.

**ADIABATIC APPROXIMATION**

Explicitly, equations for \( \varphi_m(x, t) \) and \( \varphi_e(x, t) \) equivalent to (3) have the form (in the case of normal incidence, \( p_{0x} = 0 \)):
\[ i\frac{\partial \varphi_m(x, t)}{\partial t} = -\frac{1}{2m}\nabla_x^2 \varphi_m(x, t) - \frac{\Omega}{2} \cos(kx) \varphi_e(x, t) \]  
(7)
and
\[ i\frac{\partial \varphi_e(x, t)}{\partial t} = -\frac{1}{2m}\nabla_x^2 \varphi_e(x, t) - \frac{\Omega'}{2} \cos(kx) \varphi_m(x, t). \]  
(8)

In this work we will use the adiabatic approximation in which we will drop the kinetic energy operator \( -\nabla_x^2/2m \) in the Hamiltonian \( \mathbf{H} \) (6) and, hence, the terms proportional to \( -\nabla_x^2/2m \) on the right-hand side of Eqs. (7) and (8) to get
\[ \mathbf{H} \approx \mathbf{H}_{ad} = \begin{pmatrix} 0 & -\frac{\Omega}{2} \cos(kx) \\ -\frac{\Omega'}{2} \cos(kx) & -\frac{1}{2}\nabla_x^2 \end{pmatrix}, \]  
(9)

\[ i\frac{\partial \varphi_m(x, t)}{\partial t} = -\frac{\Omega}{2} \cos(kx) \varphi_e(x, t), \]  
(10)
and
\[ i\frac{\partial \varphi_e(x, t)}{\partial t} = -\frac{\Omega'}{2} \cos(kx) \varphi_m(x, t). \]  
(11)

For the kinetic energy term \( -\nabla_x^2/2m \varphi_m(x, t) \) to be dropped from Eq. (10), it must be smaller than the other two terms retained in Eq. (10). Hence, qualitatively, the applicability criterion for Eq. (11) has the form
\[ \left\langle -\nabla_x^2/2m \right\rangle \ll \left\langle \frac{\partial}{\partial t} \right\rangle, \]  
(12)

where angular brackets denote averaging over the state \( \Psi. \) As for Eqs. (8) and (11), they contain additional terms \( \propto \Gamma, \) which can be large, and the adiabaticity criterion for Eq. (11) is given by
\[ \left\langle -\nabla_x^2/2m \right\rangle \ll \max \left\{ \Gamma, \left\langle \frac{\partial}{\partial t} \right\rangle \right\}. \]  
(13)
By comparing the conditions of Eqs. (12) and (13), we see that the first of them is sufficient for the kinetic energy operator to be dropped from the Hamiltonian $H$ and for the adiabatic approximation to be valid. An explicit form of the criterion (12) is discussed below.

If $\Gamma \gg \langle \partial/\partial t \rangle$, the condition (12) can be invalid whereas, still, the condition (13) can be satisfied. In this case the kinetic energy operator in Eq. (8) has to be retained whereas in Eq. (9) it can be dropped, and the equations to be solved are (6) and (11). Such a generalization of the adiabatic approximation will be considered elsewhere.

By expressing $\varphi_e(x, t)$ from Eq. (11) via $\varphi_m(x, t)$, $\varphi_e(x, t) = -2i \dot{\varphi}_m/\Omega \cos (kx)$, and substituting $\varphi_e(x, t)$ into Eq. (11), we can reduce the two equations (10) and (11) to a single second-order equation for $\varphi_m(x, t)$

$$\ddot{\varphi}_m + \frac{\Gamma}{2} \dot{\varphi}_m + \left(\frac{\Omega^2 \cos^2(kx)}{4}\right) \varphi_m = 0.$$ (14)

QUASIENERGY SOLUTIONS

As the Hamiltonian (9) is stationary, Eq. (9) has solutions of the form

$$\Phi(x, t) = \exp(-i\gamma t) u_\gamma(x),$$ (15)

where $\gamma$ and $u_\gamma$ are the quasienergies and quasienergy wave functions to be found from the eigenvalue equation

$$H_{ad} u_\gamma(x) = \gamma u_\gamma(x).$$ (16)

In the adiabatic approximation solution of the quasienergy problem is very simple. As the operator $H_{ad}$ (9) does not contain any derivatives over $x$, the coordinate $x$ plays the role of a parameter or a quantum number, and the position-dependent eigenvalues of $H_{ad}$ are easily found to be given by

$$\gamma^{\pm}(x) = -i\frac{\Gamma}{4} \pm \frac{1}{2} \sqrt{-\frac{\Gamma^2}{4} + |\Omega|^2 \cos^2(kx)}.$$ (17)

The quasienergy $\gamma^+(x)$ is a direct generalization of the Chudesnikov-Yakovlev complex potential (9) (at zero detuning), which follows from Eq. (17) in the limit $|\Omega| \ll \Gamma$

$$\gamma^+(x) \approx -i\frac{|\Omega|^2 \cos^2(kx)}{2\Gamma} \equiv V_{CY}(x)|_{\omega=E_+ - E_-}.$$ (18)

In a general case, the complex quasienergies $\gamma^{\pm}(x)$ (17) determine both average 'center-of-mass' values $\text{Re}(\gamma^{\pm})$ and widths $\Gamma^{\pm} = -2\text{Im}(\gamma^{\pm})$ of the atomic quasienergy levels. Explicitly, the widths $\Gamma^{\pm}(x)$ are given by

$$\Gamma^{\pm}(x) = \frac{\Gamma}{2} \pm \text{Re}\left(\sqrt{-\frac{\Gamma^2}{4} - |\Omega|^2 \cos^2(kx)}\right)$$ (19)

The broadened quasienergy levels or zones are described in Fig. 2. The curves at these pictures correspond to boundaries of zones determined as $\text{Re}[\gamma^{\pm}(x)] + \frac{1}{2}\Gamma^{\pm}(x)$ and $\text{Re}[\gamma^{\pm}(x)] - \frac{1}{2}\Gamma^{\pm}(x)$. The spacings between the boundaries are equal to the widths of the zones $\Gamma^{\pm}(x)$. Two different zones are indicated by different shading. The pictures (a) and (b) correspond to weak $(2|\Omega| \ll \Gamma)$ and strong $(2|\Omega| > \Gamma)$ resonance or Rabi coupling of levels $E_m$ and $E_e$. In the case of weak Rabi coupling, one of the zones is much narrower than another, and the wide zone can be eliminated adiabatically to give rise to the description in terms of the potential $V_{CY}(x)$ (18). Mathematically such an elimination of a wide quasienergy zone is equivalent to dropping the second-order derivative term in Eq. (14), which gives

$$i \frac{\partial}{\partial t} \varphi_m(x, t) = V_{CY}(x) \varphi_m(x, t).$$ (20)

The condition under which the second-order derivative in Eq. (14) can be dropped is easily estimated with the help of Eq. (20): $\dot{\varphi}_m \sim V_{CY}(x) \varphi_m \sim V_{CY}^2 \varphi_m$. Hence, $\dot{\varphi}_m \ll \Gamma \varphi_m$ if $|V_{CY}| \ll |\Omega \cos (kx)| \ll \Gamma$.

In the case of strong Rabi coupling such adiabatic approximation and all the resulting equations can be invalid at $x$ close to the branching points of the root square in Eq. (17). Indeed, at these points the derivative $\nabla_x$ becomes infinitely large and the kinetic energy terms in Eqs. (3), (5), (8) cannot be dropped. But in the region of $x$ close to $\pi/2k$ (which is most important for the given below long-time analysis) even in the case $|\Omega| \gg \Gamma$ the width $\Gamma^+(x)$ (17) is very small and can be approximated
by a parabolic dependence on $x - \pi/2k$

$$\Gamma_+ \approx -2\text{Im}(V_{\text{CY}}) = \frac{\gamma^2 \cos^2(kx)}{\Gamma} \approx \frac{\gamma^2 k^2}{\Gamma} \left(x - \frac{\pi}{2k}\right)^2.$$  

(21)

As in this region $\Gamma_+ \approx \Gamma \gg \Gamma_+$, we get again a narrow quasienergy zone at the background of a wide one, and again adiabatic elimination appears to be applicable.

The role of "nonadiabatic" points, where $\Gamma \approx 2|\Omega \cos kx|$, will be discussed elsewhere. In principle, the arising peculiarities can be observed in experiments with scattering of narrow atomic wave packets aimed specifically at these points. Such a formulation of the problem will be discussed separately too.

**SOLUTION OF THE INITIAL-VALUE PROBLEM**

The found above quasienergies $\gamma_{\pm}(x)$ (17) are sufficient for solving the initial-value problem. By assuming that the interaction is turned on suddenly at $t = 0$ and that $\varphi_{m,0}(x) = 1$ and $\varphi_{e,0}(x) = 0$, we present the time-dependent functions $\varphi_{m,e}(x,t)$ in the form of superpositions

$$\varphi_{m,e}(x,t) = \sum_{\pm} A_{m,e}(x,t) \exp(-i\gamma_{\pm}(x)t),$$  

(22)

where the coefficients $A_{m,e}(x)$ are to be found from the initial conditions, which yield

$$A_{m}^{(+)} + A_{m}^{(-)} = 1, \quad A_{e}^{(+)} + A_{e}^{(-)} = 0,$$  

(23)

and the equations following from Eq. (14) (with the Hamiltonian (3))

$$A_{e}^{(\pm)} = -\frac{2\gamma_{\pm}}{\Omega \cos(kx)} A_{m}^{(\pm)}.$$  

(24)

Eqs. (23) and (24) are solved easily to give

$$A_{m}^{(\pm)} = \mp \sqrt{2} \frac{2\gamma_{\pm}}{\Delta^2 + 4|\Omega|^2 \cos^2(kx)}$$  

(25)

and

$$A_{e}^{(\pm)} = \mp \sqrt{\frac{\Omega^* \cos(kx)}{\Delta^2 + 4|\Omega|^2 \cos^2(kx)}}$$  

(26)

The corresponding time-dependent center-of-mass atomic wave functions are given by

$$\varphi_{m}(x,t) = \sum_{\pm} \frac{2\gamma_{\pm}(x) \exp(-i\gamma_{\pm}(x)t)}{\Delta^2 + 4|\Omega|^2 \cos^2(kx)}$$  

(27)

and

$$\varphi_{e}(x,t) = \sum_{\pm} \frac{\Omega^* \cos(kx) \exp(-i\gamma_{\pm}(x)t)}{\Delta^2 + 4|\Omega|^2 \cos^2(kx)}.$$  

(28)

The squared absolute values of the functions $\varphi_{m,e}(x,t)$ determine the probability densities to find an atom at a time $t$ in a vicinity of a point $x$ at the levels $E_m$ and $E_e$

$$\frac{dW_{m,e}(x,t)}{dx} = \frac{k}{\pi} |\varphi_{m,e}(x,t)|^2.$$  

(29)

Integrated over $x$ from zero to $\pi/k$, the probability densities $dW_{m,e}(x,t)/dx$ give the time-dependent total probabilities of scattering at a single period of a standing light wave

$$W_{tot}^{(m,e)}(t) = \int_{0}^{\pi/k} dx \frac{dW_{m,e}(x,t)}{dx}$$

$$= \frac{k}{\pi} \int_{0}^{\pi/k} dx |\varphi_{m,e}(x,t)|^2.$$  

(30)

**LONG-TIME ASYMPTOTIC LIMIT**

The long-time asymptotic limit corresponds to the case $\Gamma \gg 1$. In this case the main contribution to the integral over $x$ in Eq. (30) is given by the most slowly decaying terms. In the case of a strong Rabi coupling $|\Omega| \gg \Gamma$ such slowly decaying terms correspond to the quasienergy $\gamma_{+}(x)$ and to the region of $x$ close to $\pi/2k$. By assuming that in this region the product $|\Omega|^2 \cos^2(kx)$ is small compared to $\Gamma^2/4$, we can reduce Eqs. (23) and (24) to the form

$$|\varphi_{m}(x,t)|^2 \approx \exp \left\{-\frac{|\Omega|^2 t}{\Gamma} \cos^2(kx) \right\}$$  

(31)

and

$$|\varphi_{e}(x,t)|^2 \approx \frac{|\Omega|^2}{\Gamma^2} \cos^2(kx) \exp \left\{-\frac{|\Omega|^2 t}{\Gamma} \cos^2(kx) \right\}.$$  

(32)

These equations follow also from Eq. (14), which is solved easily and which is valid under the condition $|\Omega \cos(kx)| < 2\Gamma$. The position-dependent metastable-state probability density was found earlier (Eq. (19) of Ref. [3] in which the spontaneous decay rate $\gamma$ should be substituted by $|\Omega|^2/2\Gamma$).

At $x$ close to $\pi/2k$ and with $\cos(kx)$ approximated by the parabolic function (21), Eqs. (31) and (32) take the form

$$|\varphi_{m}(x,t)|^2 \approx \exp \left\{-\frac{|\Omega|^2 t}{\Gamma} \left(kx - \frac{\pi}{2} \right)^2 \right\}$$  

(33)

and

$$|\varphi_{e}(x,t)|^2 \approx \frac{|\Omega|^2}{\Gamma^2} \left(kx - \frac{\pi}{2} \right)^2 \exp \left\{-\frac{|\Omega|^2 t}{\Gamma} \left(kx - \frac{\pi}{2} \right)^2 \right\}.$$  

(34)

The functions $|\varphi_{m}(x,t)|^2$ (33) and $|\varphi_{e}(x,t)|^2$ (34) are plotted in Fig. 3. The width $\Delta x$ of the interval where
regions close to nodes of a standing light wave give not
This is the condition under which only relatively narrow
E

Figure 3. Probability densities to find an atom at the levels
The ratio \( |\Omega| t \) is small if \( \Gamma t \gg 1 \) whereas in the opposite case, \( \Gamma t \ll 1 \), \( |\Omega_{eff}| > \Gamma \) (and, of course, \( \Omega_{eff} t \gg 1 \)). Hence, we expect that transitions between the metastable and excited levels have significantly different form at long and short times, \( \Gamma t \ll 1 \) and \( \Gamma t \geq 1 \).
In the first of these two cases (long-time asymptotic) the transitions \( E_{m} \rightarrow E_{e} \) have a form of irreversible transitions to the quasicontinuum of the wide excited level \( E_{e} \), whereas in the second case (short-time limit) they take a form of multiple Rabi oscillations. In the following section we will see how the dynamics of excitation is affected by a mixture of these two types of transitions.

The parameter \( \Delta x \) determines the relation between the heights of the curves \( |\varphi_{m}(x, t)|^2 \) and \( |\varphi_{e}(x, t)|^2 \): if \( |\varphi_{m}|^2_{max} = 1 \),
\[
|\varphi_{e}|^2_{max} = \frac{|\Omega|^2}{\Gamma^2} k^2 \Delta x^2 e^{-1} = \frac{e^{-1}}{\Gamma t} \ll 1,
\]
if \( \Gamma t \gg 1 \).
As \( \Delta x \) is the width of the "most important" region of \( x \), where \( \Gamma_{+} \) is small and the corresponding part of atoms decays slowly, we can estimate now a rigidity of the assumption about the normal incidence of atoms upon a standing wave. If \( p_{0x} \neq 0 \), the atoms move homogeneously along the \( x \)-axis. The arising displacement during the interaction time is \( p_{0x} t / m \), and it must be not larger than \( \Delta x \) to keep atoms decaying slowly, which gives
\[
k|v_{0x}|t \leq \frac{1}{|\Omega|} \sqrt{\frac{\Gamma}{t}} \ll 1,
\]
where \( v_{0x} = p_{0x} / m \), and the last inequality follows from Eq. (39).
At last, the definition of the characteristic "important" interval \( \Delta x \) can be used to evaluate the validity criterion of the adiabatic approximation. In this approximation the characteristic value of the kinetic energy \( -\nabla^2 _{x} / 2m \) is assumed to be small compared to the characteristic value of \( \partial / \partial t \). The latter is estimated as at \( \Gamma t \gg 1 \) and \( |x - \pi / 2k| \sim \Delta x \): \( \partial / \partial t \sim \Gamma_{+} \sim |\Omega|^2 k^2 \Delta x^2 / \Gamma \sim 1 / t \). In accordance with the uncertainty principle, we put \( \nabla \sim 1 / \Delta x \), which gives \( \nabla^2 _{x} / 2m \sim \omega_r |\Omega|^2 t / \Gamma \), where \( \omega_r = k^2 / 2m \) is the recoil frequency. With the help of these estimates the applicability criterion of the adiabatic approximation \( \nabla^2 _{x} / 2m \ll 1 / t \) can be reduced to the form
\[
|\Omega| t \ll \sqrt{\Gamma} / \omega_r.
\]
integrals over $x$ to integrals over the narrow-zone width $\Gamma_+$:
\[ W_{tot}^{(m)}(t) \approx \frac{\sqrt{\Gamma}}{\pi|\Omega|} \int_0^\infty \frac{d\Gamma_+}{\sqrt{\Gamma_+}} e^{-\Gamma_+ t} = \frac{\Gamma^{1/2}}{2|\Omega|\sqrt{\pi t}} \] (42)
and
\[ W_{tot}^{(e)}(t) \approx \frac{1}{\Gamma_+^{1/2}} \int_0^\infty \frac{d\Gamma_+}{\sqrt{\Gamma_+}} e^{-\Gamma_+ t} = \frac{1}{2|\Omega|\sqrt{\pi t^{3/2}}}. \] (43)
So, indeed, the long-time behavior of the total probabilities to find an atom after scattering at the metastable and excited levels is determined by power-law rather than exponential dependencies on the interaction time $t$.

It should be noted, that, in principle, the non-exponential decay characterized by Eqs. (42), (43) can occur also in the case of weak Rabi coupling, $|\Omega| < \Gamma$, if only the conditions (36), (41) and (44) are fulfilled. The first of these conditions (36) at $|\Omega| < \Gamma$ can be fulfilled only if the interaction time $t$ is very large, $t \gg \Gamma/|\Omega|^2 \gg 1/|\Omega| \gg 1/\Gamma$. Such a long time can make the restriction of the transverse velocity $v_{tx}$ (40) too severe to be easily satisfied. For this reason, the case of strong Rabi coupling looks much more favorable than the case of weak coupling for observation of the effects described in this and the following sections. Compatibility of the conditions (36) and (41) requires the Rabi frequency to be not too small, $|\Omega| \gg \sqrt{\omega_0 \Gamma}$.

**PARTIAL PROBABILITIES OF SCATTERING INTO DIFFRACTION BEAMS**

To investigate in more details the time evolution of scattering, let us consider the Fourier transforms of the atomic center-of-mass wave functions $\varphi_{m,e}(x,t)$
\[ a_n^{(m,e)}(t) = \frac{k}{\pi} \int_0^\pi dx \varphi_{m,e}(x,t) \exp(-inx). \] (44)
The functions $a_n^{(m,e)}(t)$ and their squared absolute values
\[ W_n^{(m,e)}(t) = |a_n^{(m,e)}(t)|^2 \] (45)
are the probability amplitudes and partial probabilities to find an atom at the levels $E_m$ or $E_e$ in the $n$-th diffraction beam with the momentum $p_0 + nk$, which makes an angle $\theta_n \approx nk/p_0$ with $p_0$, $n = 0, \pm 1, \pm 2, \ldots$. Equations for $a_n^{(m,e)}(t)$, equivalent to Eqs. (36), (41), are given by
\[ i\dot{a}_n^{(m)}(t) = \left( n^2 \omega_r + n \delta \right) a_n^{(m)} - \frac{\Omega}{4} \left( a_{n-1}^{(c)} + a_{n+1}^{(c)} \right), \]
\[ i\dot{a}_n^{(e)}(t) = \left( n^2 \omega_r + n \delta - \frac{i\Gamma}{2} \right) a_n^{(e)} - \frac{\Omega^*}{4} \left( a_{n-1}^{(m)} + a_{n+1}^{(m)} \right). \] (46)
These equations look like equations for two coupled anharmonic oscillators in a resonance field with the resonance detuning $\delta = k p_0 \omega_r / m$ and the anharmonicity parameter coinciding with the recoil frequency $\omega_r$. The above-discussed adiabatic approximation corresponds to ignoring the anharmonicity terms $n^2 \omega_r$ in Eqs. (46). With the additional assumption about the normal incidence, $p_0 x = 0$, Eqs. (46) take the form
\[ i\dot{a}_n^{(m)}(t) = -\frac{\Omega}{4} \left( a_{n-1}^{(c)} + a_{n+1}^{(c)} \right), \]
\[ i\dot{a}_n^{(e)}(t) = -\frac{i\Gamma}{2} a_n^{(c)} - \frac{\Omega^*}{4} \left( a_{n-1}^{(m)} + a_{n+1}^{(m)} \right). \] (47)
The transition from Eqs. (46) to (47) can be considered as the Raman-Nath approximation for the two-dimensional (2D) system. It should be noted, however that both Eqs. (17) and their solutions presented below differ significantly from and are much more complicated than the standard 1D Raman-Nath equation and its solution (10).

The simplest way of finding $a_n^{(m,e)}(t)$ obeying Eqs. (47) is related to the calculation of the Fourier transforms (44) of the earlier found functions $\varphi_{m,e}(x,t)$ (27) and $\varphi_e(x,t)$ (24). Not dwelling upon the details of calculations, let us present here the arising results:
\[ W_{2n}^{(m)}(t) = e^{-\frac{\Gamma t}{2}} \left| J_{2n} \left( \frac{|\Omega t|}{2} \right) + \frac{\Gamma t}{4} \int_0^1 dz J_{2n} \left( \frac{|\Omega t|}{2} z \right) \right| \]
\[ \times \left[ \frac{I_1 \left( \frac{\Gamma t}{4} \sqrt{1 - z^2} \right)}{\sqrt{1 - z^2}} + I_0 \left( \frac{\Gamma t}{4} \sqrt{1 - z^2} \right) \right]^2 \] (48)
and
\[ W_{2n+1}^{(e)} = \left( \frac{|\Omega t|}{4} \right)^2 \exp \left( -\frac{i\Gamma t}{2} \right) \left[ \int_0^1 dz J_{2n+1} \left( \frac{\Gamma t}{4} \sqrt{1 - z^2} \right) \right]^2 \] (49)
whereas $W_{2n+1}^{(m)}(t) \equiv 0$ and $W_{2n}^{(e)}(t) \equiv 0$. In Eqs. (48), (49) $J_{2n}$ are the Bessel functions and $I_0$ and $I_1$ are the modified Bessel functions (11). It should be emphasized that in derivation of Eqs. (18) and (19) we did not make any assumptions about a value of the parameter $\Gamma t$ and, hence, these equations are valid both at short interaction times, $\Gamma t \leq 1$, and in the long-time asymptotic limit, $\Gamma t \gg 1$. The functions $W_{2n}^{(m)}(t)$ (18) and $W_{2n+1}^{(e)}(t)$ (49) are plotted in Fig. 4a and b. In contrast to the
well-known Rabi oscillations in a pure two-level system in a resonance field, oscillations of $W^{(m)}_n(t)$ are aperiodic and both positions of peaks and zeros of $W^{(m,c)}_{2n}(t)$ depend on $n$.

Alternatively to (30), the total probabilities of finding atoms after scattering at the levels $E_m$ and $E_e$ can be defined as sums over the diffraction beams

$$W^{(m,c)}_{\text{tot}}(t) = \sum_n |a^{(m,c)}_n(t)|^2.$$  \hspace{1cm} (50)

The dependencies $W^{(m)}_{\text{tot}}(t)$ and $W^{(c)}_{\text{tot}}(t)$ calculated numerically by summing $W^{(m)}_{2n}(t)$ and $W^{(c)}_{2n+1}(t)$ are shown in Figs. 5a and b. In contrast to $W^{(m)}_{2n}(t)$ and $W^{(c)}_{2n+1}(t)$, oscillations of $W^{(m,c)}_{\text{tot}}(t)$ are periodic, and their period coincides with that of the Rabi oscillations in a pure two-level system driven by a resonant field with the field-strength amplitude $2E_0$ (the dashed curves in Fig. 5). However, as seen well from Fig. 5, the quasi-Rabi oscillations of the functions $W^{(m,c)}_{\text{tot}}(t)$ and $W^{(c)}_{\text{tot}}(t)$ for scattered atoms are strongly suppressed compared to those of a two-level system. The effect of suppression of Rabi oscillation is explained mainly by a kind of inhomogeneous broadening. Oscillations of partial probabilities $W^{(m)}_{2n}(t)$ and $W^{(c)}_{2n+1}(t)$ are well pronounced and their amplitudes are large enough (Fig. 4). However, as "periods" of these oscillations are different for different $n$, summation over $n$ smoothes over the oscillations and decreases their amplitudes. In insets of Fig. 5 it’s seen clearly that the degree of suppression the Rabi oscillations increases with a growing value of $\Gamma t$ in the case $\Gamma t \gg 1$ oscillations are almost completely smoothed away. This effect is related to the discussed above change in the character of transitions between the levels $E_m$ and $E_e$: Rabi oscillations in the case $\Gamma t \lesssim 1$ and "discrete level - quasicontinuum" transitions in the case $\Gamma t \gg 1$.

Another effect which is clearly seen in the insets of Figs. 5a and b is the scattering-induced slowing down of
the radiative decay. The long-time behavior of the probabilities \( W^{(m,e)}_{\text{tot}}(t) \) is described rather well by asymptotic formulas of Eqs. (42) and (43). The difference with exponential decay in a pure two-level system is rather well pronounced.

**CONCLUSION**

So, the main predictions of the carried out consideration are (i) suppression of the Rabi oscillations in the case of atom scattering (compared to a pure two-level system) and (ii) slowing down the radiative decay and formation of nonexponential (power-law) tails in the dependencies \( W^{(m,e)}_{\text{tot}}(t) \). Qualitatively, our interpretation of these effects consists of the following. The Rabi oscillations are suppressed owing to inhomogeneous-broadening-like effects when the partial probabilities of scattering are summed over the diffractions beams. The power-law dependencies and slowing down of the radiative decay arise because of the position-dependent modulation of the field-strength amplitude in a standing light wave and, hence, the position-dependent modulation of the decay rate of slowly decaying quasienergy atomic levels \( \Gamma_+(x) \). As the result, of this modulation the manifold of the arising quasienergy levels is characterized by continuously varying with \( x \) and approaching zero at \( x = \pi/2k \) width. Populations at these quasienergy levels decrease exponentially but with different, \( x \)-dependent rates, and their superposition gives rise to the non-exponential decay laws.

We assume that observation of these effects can be made in the framework of an experiment similar to [1, 3] though with some modifications, to provide the conditions for the strong Rabi coupling and the normal-incidence diffraction regime of scattering. To compare directly the time evolution of atomic populations in atoms scattered by a standing light wave and in a pure two-level system, one can make two series of similar measurements: in a standing light wave and in a single travelling wave of a doubled field strength amplitude. In the last case the standing-wave scattering effects disappear and the large-amplitude Rabi oscillations and the usual exponential decay have to be observed.

Finally, the above-described scattering-induced suppression of the radiative decay in the case of strong Rabi coupling reminds the effect of interference stabilization in Rydberg atoms in a strong light field [2, 3]. In both cases the effects of slowing down the decay processes are related to formation of narrow quasienergy levels and interference of transitions from different levels to the common continuum. We find this analogy important because it establishes links between different regions of physical phenomena and demonstrates a rather general character and fruitfulness of the idea of interference stimulated by sufficiently strong interactions.

**ACKNOWLEDGEMENT**

The work is supported partially by the Russian Foundation for Basic Research (grant # 02-02-16400). M. Fedorov gratefully acknowledges the support of the Humboldt foundation and the great hospitality enjoyed at the Abteilung für Quantenphysik, University of Ulm, Germany. The work of W. P. Schleich is partially supported by DFG.

[1] A.P. Kazantsev, G.I. Surdutovich, and V.P. Yakovlev, Mechanical Action of Light on Atoms, World Scientific, Singapore, 1990
[2] V.I. Balykin and V.S. Letokhov, Atom Optics with Laser Light, Harwood Academic, Chur, Switzerland, 1995
[3] W.P. Schleich, Quantum Optics in Phase Space, Wiley, New-York, 2001
[4] D.O. Chudesnikov and V.P. Yakovlev, Laser Phys., 1, 110 (1991)
[5] D.S. Krämer et al., In: Quantum Optics VI, Springer Proceedings in Physics, 77, 87; Editors: D.F. Walls & J.D. Harvey, Springer-Verlag, Berlin-Heidelberg, 1994
[6] M.K. Oberthaler, R. Abfalterer, S. Bernet, J. Schmiedmayer, and A. Zeilinger, Phys. Rev. Lett., 77, 4980 (1996)
[7] H. Batelaan, E.M. Rasel, M.K. Oberthaler, J. Schmiedmayer, and A. Zeilinger, Journ. of Modern Optics, 44, 2629 (1997)
[8] M.V. Berry and D.H.J. O’Dell, J. Phys A: Math. Gen., 31, 2093 (1998)
[9] M.K. Oberthaler, R. Abfalterer, S. Bernet, J. Schmiedmayer, and A. Zeilinger, Phys. Rev. A, 60, 456 (1999)
[10] M. Born and E. Wolf, Principles of Optics, Pergamon Press, Oxford, 1964
[11] M. Abramovitz and I.A. Stegun, Handbook of Mathematical Functions, Nath. Bur. Stand., Appl. Math. Ser., No 55, U.S. G.PO, Washington, DC, 1964
[12] M.V. Fedorov and A.M. Movsesian, J. Phys. B, 21, L155 (1988)
[13] M.V. Fedorov, Atomic and Free Electrons in a Strong Light Field, World Scientific, Singapore, 1991