Should the Pomeron and imaginary parts be modelled by two gluons and real quarks?

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We illustrate that solution of the Schwinger-Dyson equation for the gluon propagator in QCD does not support an infrared softened behaviour, but only an infrared enhancement. This has consequences for the modelling of the Pomeron in terms of dressed gluon exchange. It highlights that an understanding of the Pomeron within QCD must take account of the bound state nature of hadrons.
It has long been understood that at high energies total cross-sections for hadronic processes are controlled by cross-channel Pomeron exchange \cite{1,2}, where the Pomeron is believed to be a colour singlet with vacuum quantum numbers. Low and Nussinov \cite{2} proposed a QCD-inspired model for the Pomeron in terms of two gluon exchange and Landshoff and Nachtmann \cite{3} set up an explicit framework for phenomenological calculations of the resulting cross-sections. A key requirement of their model is that the dressed gluon propagator, $\Delta(k^2)$, should not have the singularity of the bare massless boson $\sim 1/k^2$ as $k^2 \to 0$, but should be softened so that the integral

$$\int_0^\infty dk^2 \Delta(k^2)^2$$

is finite, where $k$ is a Euclidean loop momentum. Here we discuss whether such infrared behaviour of the gluon propagator is possible in non-perturbative QCD.

The infrared behaviour of the gluon propagator is naturally studied in the continuum using the Schwinger-Dyson equations. It has been known since the work of Mandelstam \cite{4} and Bar-Gadda \cite{5} that an infrared enhanced gluon propagator, typically $\Delta(k^2) \sim 1/k^4$, is a possible solution of the truncated Schwinger-Dyson equations. Baker, Ball and Zachariasen (BBZ) \cite{6} also deduced such behaviour in axial gauges. However, their result depends crucially on setting one of the two axial gauge gluon renormalization functions to zero. West \cite{7} has proved that in axial gauges, in which only positive norm states occur, a behaviour more singular than $1/k^2$ is not possible and consequently the neglected axial gauge renormalization function must cancel any $1/k^4$ singularity in the infrared. More recently Cudell and Ross \cite{8} have shown that an alternative axial gauge solution with an infrared softened gluon propagator exists to Schoenmaker’s approximation \cite{9} to the BBZ equation. Unfortunately, this solution has now been recognised as only having been possible because of an incorrect sign in Schoenmaker’s approximate equation \cite{10}.

Because of the difficulty in justifying the neglect of one of the key gluon renormalization functions in axial gauges, we turn our attention to covariant gauges and the Landau gauge in particular. In such a gauge $\Delta(k^2) \sim 1/k^4$ has already been shown to be the behaviour of a self-consistent solution to the gluon Schwinger-Dyson equation \cite{4,5} — see \cite{11} for a full discussion of the approximations used. Such a $1/k^4$ solution, West \cite{12} has argued leads inexorably to a Wilson area law, which many would regard as a proof of quark confinement. However, such an infrared enhanced gluon is at variance with the Landshoff and Nachtmann picture of the Pomeron. Consequently we should search for an alternative softened solution to the Landau gauge gluon equation. We show, here, that no such behaviour is possible.
To do this we consider the Schwinger-Dyson equation for the gluon propagator in the Landau gauge. The gluon propagator is then represented by

$$\Delta^{\mu \nu}(k) = \Delta(k^2) \left( \delta^{\mu \nu} - \frac{k^{\mu}k^{\nu}}{k^2} \right)$$  \hspace{1cm} (1)

where $\Delta(k^2) = G(k^2)/k^2$ with $G(k^2)$ the gluon renormalization function. Since confinement must be a result of the non-Abelian nature of QCD, we consider a world without quarks. The gluon Schwinger-Dyson equation may be approximated by treating the ghosts perturbatively and neglecting 4-gluon interactions, as discussed by Mandelstam [4], and replacing the 3-gluon vertex by its longitudinal component determined by the Slavnov-Taylor identity [1]. The resulting truncated Schwinger-Dyson equation is displayed in Fig. 1.

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Figure 1: Approximate Schwinger-Dyson equation for the gluon propagator

Then $G(k^2)$ satisfies the following equation:

$$\frac{1}{G(k^2)} = 1 + \frac{g^2C_A}{96\pi^4} \frac{1}{k^2} \int d^4q \left[ G(q^2)A(q^2, k^2) + \frac{G(q^2)}{G(k^2)} G(q^2) B(q^2, k^2) \right.
\left. + \frac{G(q^2) - G(k^2)}{G(k^2)} C(q^2, k^2) + \frac{G(q^2) - G(k^2)}{q^2 - q^2} D(q^2, k^2) \right],$$  \hspace{1cm} (2)

with $q' = k - q$ and where

$$A(q^2, k^2) = 48 \frac{(q \cdot q')^2}{q^2k^2q'^4} - 64 \frac{(q \cdot q')^3}{q^2q'^2q^4} + 16 \frac{(q \cdot q')^4}{q^2k^2q'^4} - 12 \frac{1}{q^2} + 22 \frac{k^2}{q^2q'^2} - 42 \frac{(q \cdot q')^2}{q^2q'^4} - 10 \frac{k^4}{q^2q'^4},$$

$$B(q^2, k^2) = -13 \frac{k^2}{q'^4} + 18 \frac{k^2(q \cdot q')}{q^2q'^4} - 2 \frac{(q \cdot q')^2}{q^2q'^4} - 4 \frac{k^4}{q^2q'^4} + \frac{k^2(q \cdot q')^2}{q'^4},$$

$$C(q^2, k^2) = 4 \frac{(q \cdot q')^2}{q^2q'^2} + 6 \frac{k^2(q \cdot q')}{q^2q'^2} + \frac{6(q \cdot q')}{q'^2} + \frac{8k^2}{q'^2},$$

$$D(q^2, k^2) = 12 \frac{q^2}{q'^2} - 48 \frac{(q \cdot q')^2}{k^2q'^2} + 48 \frac{(q \cdot q')^3}{q^2k^2q'^2} + 24 \frac{(q \cdot q')^4}{q^2k^2q'^2} - 5 \frac{k^2}{q'^2} - 40 \frac{(q \cdot q')^2}{q^2q'^2} + 9 \frac{k^2(q \cdot q')}{q'^2}.$$

Further approximating $G(q^2)$ by $G(k^2 + q^2)$, which should be exact in the infrared limit,
as first proposed by Schoenmaker [9], allows the angular integrals to be performed analytically, giving:

\[
\frac{1}{G(k^2)} = 1 + \frac{g^2 C_A}{48\pi^2 k^2} \left\{ \int_0^{k^2} dq^2 \left[ G_1 \left( -1 - 10 \frac{q^2}{k^2} + 6 \frac{q^4}{k^4} + \frac{q^2}{k^2 - q^2} \left( \frac{75}{4} - 39 \frac{q^2}{4 k^2} + 4 \frac{q^4}{k^4} - 5 \frac{k^2}{q^2} \right) \right) + G_2 \left( -\frac{21}{4} \frac{q^2}{k^2} + 7 \frac{q^4}{k^4} - 3 \frac{q^6}{k^6} \right) + G_3 \left( \frac{q^2}{k^2 - q^2} \left( -\frac{27}{8} - \frac{11 q^2}{4 k^2} - \frac{15 k^2}{8 q^2} \right) \right) \right] + \int_0^\infty dq^2 \left[ G_1 \left( \frac{k^2}{q^2} - 6 + \frac{k^2}{k^2 - q^2} \left( \frac{29}{4} + \frac{3 k^2}{4 q^2} \right) \right) + G_2 \left( \frac{3}{2} + \frac{1}{4} \frac{k^2}{q^2} \right) + G_3 \left( \frac{k^2}{k^2 - q^2} \left( \frac{3}{4} - \frac{67 k^2}{8 q^2} - \frac{3 k^4}{8 q^4} \right) \right) \right] \right\},
\]

where

\[
G_1 = G(k^2 + q^2),
G_2 = G(k^2 + q^2) - G(q^2),
G_3 = \frac{G(q^2)G(k^2 + q^2)}{G(k^2)}.
\]

In general, this equation has a quadratic ultraviolet divergence, which would give a mass to the gluon. Such terms have to be subtracted to ensure the masslessness condition

\[
\lim_{k^2 \to 0} \frac{1}{\Delta(k^2)} = 0,
\text{ i.e. } \frac{k^2}{G(k^2)} = 0 \text{ for } k^2 \to 0,
\]

is satisfied. This property can be derived generally from the Slavnov-Taylor identity and always has to hold. To determine possible self-consistent behaviour for the gluon renormalization function, \(G(k^2)\) is expanded in a series in powers of \(k^2/\mu^2\) for \(k^2 < \mu^2\) (including possible negative powers). Here \(\mu^2\) is the mass scale above which we assume perturbation theory applies and we demand that for \(k^2 > \mu^2\) the solution of the integral equation matches the perturbative result, i.e. we have \(G(k^2) = 1\) modulo logarithms.

To check whether Eq. (2) allows an infrared softened gluon propagator, i.e. the gluon renormalization function to vanish in the infrared, we take (cf. [8])

\[
G_{in}(k^2) = \begin{cases} (k^2/\mu^2)^{1-c} & \text{if } k^2 < \mu^2 \\ 1 & \text{if } k^2 > \mu^2 \end{cases}
\]

as a trial input function and substitute it into the right hand side of the integral equation, Eq. (2). \(c\) in Eq. (5) has to be positive to satisfy Eq. (4). Performing the \(q^2\)-integration,
we obtain, after mass renormalization:

$$\frac{1}{G_{\text{out}}(k^2)} = 1 + \frac{g^2 C_A}{48\pi^2} \left[ D_1 + D_2 \left( \frac{\mu^2}{k^2} \right)^{1-c} + D_3 \left( \frac{k^2}{\mu^2} \right)^{1-c} + D_4 \left( \frac{k^2}{\mu^2} \right)^c + \ldots \right],$$

(6)

where $G_1, G_2$ and $G_3$ have been expanded for small $k^2$ and only the first few terms have been collected in this equation so that

$$D_1 = -\left(\frac{3}{2} + \frac{5 + 6c}{1-c} + \frac{25}{4} \ln \left( \frac{\Lambda^2}{\mu^2} \right) \right),$$

$$D_2 = -\left( \frac{3}{4(2-2c)} + \frac{3}{4} \ln \left( \frac{\Lambda^2}{\mu^2} \right) \right),$$

$$D_3 = -\left( \frac{1971}{60} + \frac{29c}{2} + \frac{37}{20c} + \frac{6 - 13c}{2(1-c)} + \frac{59 - 32c}{4(2-c)} + \frac{155 - 64c}{8(3-c)} \right)$$

$$+ \left( \frac{127 - 49c}{8(4-c)} + \frac{23 - 11c}{4(5-c)} - \frac{125 + 61c}{8(1-2c)} - \frac{55 + 6c}{8(2-2c)} + \frac{3}{4(3-2c)} \right) - 8(2-c)\Psi(-2c) - 8(2-c)\Psi(1),$$

$$D_4 = \frac{61 + 6c}{8(1-2c)},$$

where $\Psi$ is the logarithmic derivative of the Gamma function. Thus the dominant infrared behaviour is:

$$\frac{1}{G_{\text{out}}(k^2)} \to -\left( \frac{\mu^2}{k^2} \right)^{1-c},$$

(7)

and self-consistency is spoiled by a negative sign, just as in axial gauges [10]. Note that higher order terms in $k^2$ in the input form of Eq. (5) have no qualitative effect. We thus see that an infrared softened gluon is not possible. Even softer gluons resulting from the dynamical generation of a gluon mass, though often claimed, only arise if multi-gluon vertices have massless particle singularities that stop the zero momentum limit of the Slavnov-Taylor identity being smooth. Such singularities, though they occur in the vertices of Stingl et al. [13], should not be present in QCD. In contrast, similar arguments to the above show that an infrared enhanced behaviour $G(k^2) \sim 1/k^2$ for $k^2 \to 0$ is a consistent solution [10].

This gives the confining gluon behaviour of $\Delta(k^2) \sim 1/k^4$. Such a gluon has no Lehman spectral representation [14] and so is not a physical state, but is confined. How does this infrared behaviour of the gluon affect the Pomeron of Landshoff and Nachtmann? Their belief in an infrared softened, rather than enhanced, gluon rests on their model requirement that the integral

$$\int_0^\infty dk^2 \Delta(k^2)^2$$
should be finite. However, as we now explain we do not believe the issue of whether this integral is finite or not is relevant to the finiteness of total cross-sections. The Landshoff-Nachtmann picture is to imagine that the two dressed gluons that model their Pomeron couple to single quarks with other quarks in each initial state hadron being spectators (Fig. 2a).

![Diagram](image)

(a) Exchange of a gluon pair between two quarks (Landshoff-Nachtmann model),
(b) Exchange of a gluon pair between two hadrons.

Figure 2: Diagrammatic representation of the Pomeron in meson-meson scattering:
(The lines marked with an X are on-shell in the determination of the total cross-section.)
(a) Exchange of a gluon pair between two quarks (Landshoff-Nachtmann model),
(b) Exchange of a gluon pair between two hadrons.

In this way the forward hadronic scattering amplitude is viewed as essentially quark-quark scattering (Fig. 2a). The total cross-section is then just the imaginary part of this forward elastic quark scattering amplitude, by the optical theorem. However, an imaginary part is only generated if the quarks can be on mass-shell and have poles in their propagators, as an electron or pion does. This assumption is the key to the Landshoff-Nachtmann picture (Fig. 2a) and the subsequent phenomenology. However, quarks are confined particles; their propagators are likely entire functions and the elastic quark amplitude has no imaginary part. Only an infrared enhanced gluon propagator has been shown to produce a confined light quark propagator [14]. It is then the bound state properties of hadrons that are the essential ingredients of total cross-sections. It is the intermediate hadrons that have to be on-shell (Fig. 2b) and not the confined quarks. Confinement requires that hadronic amplitudes are not merely the result of free quark interactions. Only for hard short distance processes is such a perturbative treatment valid. In soft physics, the bound state nature of light hadrons has to be solved to compute observables. A programme of research to solve the appropriate Bethe-Salpeter equations is under way [15]. Indeed, no processes are softer than those that produce hadronic total
cross-sections. Consequently, an infrared enhanced gluon propagator is not at variance with the Pomeron, but is in fact in accord with quark confinement and with low energy properties of hadrons like dynamical chiral symmetry breaking.

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