Extracting CP violation and strong phase in $D$ decays

by using quantum correlations in

$\psi(3770) \rightarrow D^0 \bar{D}^0 \rightarrow (V_1 V_2)(V_3 V_4)$ and $\psi(3770) \rightarrow D^0 \bar{D}^0 \rightarrow (V_1 V_2)(K\pi)$

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Abstract

The charm quark offers interesting opportunities to cross-check the mechanism of CP violation precisely tested in the strange and beauty sectors. In this paper, we exploit the angular and quantum correlations in the $D\bar{D}$ pairs produced through the decay of the $\psi(3770)$ resonance in a charm factory to investigate CP-violation in two different ways. We build CP-violating observables in $\psi(3770) \rightarrow D\bar{D} \rightarrow (V_1 V_2)(V_3 V_4)$ to isolate specific New Physics effects in the charm sector. We also consider the case of $\psi(3770) \rightarrow D\bar{D} \rightarrow (V_1 V_2)(K\pi)$ decays, which provide a new way to measure the strong phase difference $\delta$ between Cabibbo-favoured and doubly-Cabibbo suppressed $D$ decays required in the determination of the CKM angle $\gamma$. Neglecting the systematics, we give a first rough estimate of the sensitivities of these measurements at BES-III with an integrated luminosity of 20 fb$^{-1}$ at $\psi(3770)$ peak and at a future Super $\tau$-charm factory with a luminosity of $10^{35}$ cm$^{-2}$s$^{-1}$.

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1 Introduction

Outstanding progress has been made over the last decade thanks to the data gathered at $B$-factories, confirming that the Cabibbo-Kobayashi-Maskawa (CKM) mechanism embedded in the Standard Model (SM) is the main source of CP violation in the quark sector. The impressive agreement between results from the $s$-quark and the $b$-quark sectors [1, 2] calls for further checks in less tested areas. The recent discussions concerning the leptonic decays of $D$ and $D_s$ mesons, about a possible disagreement between lattice results and experimental data [3–5], suggest that the charm sector has not been explored as extensively as other quarks [6]. Another illustration of this situation stems from $D$-meson mixing, which has only very recently provided interesting tests of the SM and its extensions [7–11].

Indeed, the D-meson sector is a remarkable place to improve our knowledge on CP violation in and beyond SM, for at least two different reasons. First, the SM predictions for CP violation in the charm sector are very small, due to the hierarchical structure of the CKM matrix and the difference of masses between the fermion generations. Any significant amount of CP violation would provide clear signals of New Physics, and a contrario, the absence of observation of CP violation already sets bounds on models beyond the SM [6]. Secondly, $D$ decays play a prominent role in determining $\gamma$, the least well known of the three angles from the $B$-meson unitarity triangle. A better understanding of the strong dynamics of related $D$ decays would help in reducing the current uncertainty on this angle [12–14].

On the experimental side, the final results from CLEO-c and the start of BES-III provide interesting opportunities. These charm factories are known to offer the possibility to exploit the quantum entanglement of $D\bar{D}$ pairs, as explained in several references [12, 14, 18]. In addition, it is also interesting to note that $D \rightarrow VV$ (vector-vector) modes exhibit rather large branching ratios, of similar size with respect to the pseudoscalar ($PP$) or vector-pseudoscalar ($VP$) modes, and provide further angular observables to study the above issues. In this paper, we investigate this question which had not been detailed so far.

In Section 2, we discuss production of coherent $D^0\bar{D}^0$ pairs from $\psi(3770)$ decay, and in particular the angular distribution when at least one of the $D$ meson decays into a pair of vector mesons. In Section 3, we apply these results to two different situations: the determination of CP-violating observables exploiting angular and quantum correlations in cases where both $D$ decay into vector pairs, and the extraction of $D \rightarrow K\pi$ hadronic parameters in relation with the measurement of the CKM angle $\gamma$. In Section 4, we briefly discuss the application of these results for BES-III and Super $\tau$-charm factory, before concluding.

2 Correlated $D$ decays

2.1 Basics

We want to describe the decay chain for $\psi(3770)$ as

$$\psi(3770) \rightarrow D^0\bar{D}^0 \rightarrow (M_1M_2)(M_3M_4).$$

where $M_1M_2$ and $M_3M_4$ are mesons from two-body decays of $D^0$ and $\bar{D}^0$, respectively (Hereafter, $\psi$ denotes $\psi(3770)$). Since we do not tag the $D$ mesons, and just observe their decay products, we can use two descriptions of $D$ mesons – either the flavour states $D^0$ and $\bar{D}^0$, or the CP eigenstates (neglecting, for the sake of simplicity, CP-violation in $D$ mixing):

$$|D_1\rangle = \frac{|D^0\rangle + |\bar{D}^0\rangle}{\sqrt{2}}, \quad |D_2\rangle = \frac{|D^0\rangle - |\bar{D}^0\rangle}{\sqrt{2}}$$

with respective CP-parity: $\eta_{CP}(D_1) = -1$ and $\eta_{CP}(D_2) = 1$ (we take the convention $CP|D^0\rangle = -|D^0\rangle$).
Due to the spin of $\psi$, the $D$-pair is emitted with an orbital momentum $L = 1$ corresponding to an antisymmetric coherent state:

$$|(D\bar{D})_{L=1}\rangle = \frac{|D_1\rangle|D_2\rangle + |D_2\rangle|D_1\rangle}{\sqrt{2}}.$$  \hfill (3)

One can in principle consider different situations as below, where $V$ stands for a vector and $P$ for a pseudoscalar meson:

- $(PP) + (PP), (PP) + (VP), (VP) + (VP)$: the only available observable is the branching ratio, since the partial waves and helicities are all fixed by angular momentum conservation.

- $(PP) + (VV), (VP) + (VP)$: $(VV)$ can have three helicity states, and thus there are new angular observables. This can be exploited for $(PP) = K\pi$ in connection with the measurement of the CKM angle $\gamma$.

- $(VV) + (VV)$: this will be studied with an interest in new observables for CP-violation.

The relevant modes for our studies can be extracted from Ref. [19] for the branching ratios and Ref. [20] for the projected efficiency at BES-III.

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| $PP$ | $\eta_{CP}(P)\eta_{CP}(P)$ | Br (%) | Eff. ($\epsilon$) |
|------|-----------------------------|--------|------------------|
| $K^+K^-$ | +1 | 0.39 | 0.50 |
| $\pi^+\pi^-$ | +1 | 0.14 | 0.60 |
| $K_S\bar{K}_S$ | +1 | 0.038 | 0.30 |
| $\pi^0\pi^0$ | +1 | 0.08 | 0.24 |
| $K_S\pi^0$ | -1 | 1.22 | 0.33 |
| $K_S\eta$ | -1 | 0.40 | 0.26 |
| $K_{S0}(980) \rightarrow K_S(\eta\pi^0)$ | +1 | 0.67 | 0.18 |
| $K_{S0}(980) \rightarrow K_S(K^+K^-)$ | +1 | 0.31 | 0.10 |

Table 1: Branching ratios for $D$ decays into CP-eigenstates composed of two pseudoscalar mesons. In each case, the product of intrinsic CP parities and the estimated reconstruction efficiency at BES-III are indicated. Note that the efficiency is for both $D^0$ decaying into a $PP$ final state; for single $D$ decay, the efficiency is $\sqrt{\epsilon}$.

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| $PV$ | $\eta_{CP}(P)\eta_{CP}(V)$ | Br (%) | Eff. ($\epsilon$) |
|------|-----------------------------|--------|------------------|
| $\rho^0\pi^0$ | -1 | 0.37 | 0.29 |
| $\phi\pi^0 \rightarrow (K^+K^-)\pi^0$ | -1 | 0.06 | 0.10 |
| $K_S\rho^0$ | +1 | 0.77 | 0.27 |
| $K_S\phi \rightarrow K_S(K^+K^-)$ | +1 | 0.22 | 0.08 |
| $K_S\omega \rightarrow K_S(\pi^+\pi^-\pi^0)$ | +1 | 0.98 | 0.20 |
| $\bar{K}^*\eta \rightarrow (K_S\pi^0)(\pi^+\pi^-\pi^0)$ | +1 | 0.03 | 0.17 |
| $K^*0\eta \rightarrow (K_S\pi^0)(\gamma\gamma)$ | +1 | 0.06 | 0.17 |
| $K^*0\pi^0 \rightarrow (K_S\pi^0)\pi^0$ | +1 | 0.67 | 0.15 |

Table 2: Branching ratios for $D$ decays into CP-eigenstates composed of one pseudoscalar and one vector mesons. In each case, the product of intrinsic CP parities and the estimated reconstruction efficiency at BES-III are indicated. Note that the efficiency is for both $D^0$ decaying into a $PV$ final state; for single $D$ decay, the efficiency is $\sqrt{\epsilon}$.
The helicity formalism yields an amplitude of the form
\[ M_{12}^{\lambda_0} = \sum_{\lambda_V} A_{\lambda_0}^{\psi \rightarrow D_1 D_2} A_{\lambda_0}^{V_1 \rightarrow M_1 M'_1} A_{\lambda_0}^{V_2 \rightarrow M_2 M'_2} A_{\lambda_0}^{V_3 \rightarrow M_3 M'_3} A_{\lambda_0}^{V_4 \rightarrow M_4 M'_4} A_{\lambda_V}^{D_1 \rightarrow V_1 V_2} A_{\lambda_V}^{D_2 \rightarrow V_3 V_4} \]

or
\[ M_{12}^{\lambda_0} = \sqrt{\frac{3}{4\pi (4\pi)^2}} \sum_{\lambda_V} D_{\lambda_0,0}^{\lambda_V}(\phi, \psi, 0) H_{D_1 D_2} \]
The probability amplitude becomes

\[ M_{12}^m = \sqrt{\frac{9}{4\pi}} e^{im\phi_\psi} d_{m0}^l(\theta_\psi) H_{\psi V_1 V_2 V_4} \]

\[ \times \sum_\lambda e^{i\lambda\Phi_12} (-1)^\lambda d_{\lambda0}^l(\theta_\psi) d_{\lambda0}^l(\theta_\psi) H_{\lambda} D_{\lambda2}^l \sum_\kappa e^{i\kappa\Phi_{34}} (-1)^\kappa d_{\kappa0}^l(\theta_\psi) d_{\kappa0}^l(\theta_\psi) H_{\kappa2} D_{\kappa2}^l, \]

where we defined \( \Phi_{12} = \phi_{V_1} - \phi_{V_2} \) and \( \Phi_{34} = \phi_{V_3} - \phi_{V_4} \) (i.e. the angle between the two relevant vector mesons) and the combination of amplitudes

\[ H_{\psi V_1 V_2 V_4} = H_{D_1 D_2}^{\psi V_1 V_2 V_4} H_{V_1 V_2}^{V_1 V_2} H_{V_3 V_4}^{V_3 V_4} H_{M_1 M_2 M_3 M_4}^{V_1 V_2 V_3 V_4}, \]

\[ H_{\lambda1}^{D_1 V_1 V_2} = H_{\lambda}^{D_1 V_1 V_2}, \quad H_{\kappa2}^{D_2 V_3 V_4} = H_{\kappa2}^{D_2 V_3 V_4}. \]

We introduce the transversity amplitudes

\[ A_{||} = \frac{1}{\sqrt{2}} (H_{+1} - H_{-1}), \quad A_0 = H_0, \quad A_\perp = \frac{1}{\sqrt{2}} (H_{+1} - H_{-1}). \]

\( M_{12}^m \) is actually only one of the two “paths” that can be chosen. The total amplitude for a given projection \( m \) of the spin of \( \psi \) along an arbitrary \( z \)-axis is: \( M^m = (-M_{12}^m + M_{21}^m)/\sqrt{2} \). The differential decay width is obtained by averaging the squared modulus of the amplitude over the three possible values of \( m = +1, 0, -1 \). The three squared Wigner functions \( d_{m0}^l(\theta_\psi) \) add up to 1, so that the differential width is

\[ d\Gamma_{1V} = \frac{81}{32\pi^2} d(\cos \theta_\psi) d(\cos \theta_\psi) d(\cos \theta_\psi) d(\cos \theta_\psi) d(\cos \theta_\psi) d\Phi_{34} \times |A_{\psi V_1 V_2 V_4}|^2 \]

\[ \times \left[ \cos \theta_\psi \cos \theta_\psi A_0^{D_0 V_1 V_2} - \frac{1}{\sqrt{2}} \sin \theta_\psi \sin \theta_\psi \cos \Phi_{12} A_{||}^{D_0 V_1 V_2} - \frac{i}{\sqrt{2}} \sin \theta_\psi \sin \theta_\psi \sin \Phi_{12} A_\perp^{D_0 V_1 V_2} \right] \]

\[ \times \left[ \cos \theta_\psi \cos \theta_\psi A_0^{D_0 V_1 V_2} - \frac{1}{\sqrt{2}} \sin \theta_\psi \sin \theta_\psi \cos \Phi_{34} A_{||}^{D_0 V_1 V_2} - \frac{i}{\sqrt{2}} \sin \theta_\psi \sin \theta_\psi \sin \Phi_{34} A_\perp^{D_0 V_1 V_2} \right] \]

\[ \times \left[ \cos \theta_\psi \cos \theta_\psi A_0^{D_0 V_1 V_2} - \frac{1}{\sqrt{2}} \sin \theta_\psi \sin \theta_\psi \cos \Phi_{34} A_{||}^{D_0 V_1 V_2} - \frac{i}{\sqrt{2}} \sin \theta_\psi \sin \theta_\psi \sin \Phi_{34} A_\perp^{D_0 V_1 V_2} \right] \]
angular momentum between the two mesons. The favourite channels among the measured ones are normalized so that: \[ \Gamma = \frac{\Delta m}{\Delta \Gamma}, \]

CP-parity is an indication of violation in CP-parity. Therefore, we have indeed that the observation of \( (\psi \rightarrow D \rightarrow V V') \) for \( CP \)-violation is allowed: (0 vs 4 transversity amplitudes are allowed: (0, 4) or (0, 0). In terms of partial waves, 0

\[ \times \left[ \cos \theta_3 \cos \theta_4 A^{D \rightarrow V V'}_0 - \frac{1}{\sqrt{2}} \sin \theta_3 \sin \theta_4 \cos \Phi \right] \left[ \cos \Theta \cos \Phi \right] \]

\[ \left[ \sin \theta_3 \sin \theta_4 \sin \Phi \right] \left[ \sin \Theta \sin \Phi \right] \right] \right)^2. \]

We have an integration over [0, \pi] for \( \theta_3 \)'s and [0, 2\pi] for \( \Phi \). The amplitudes \( A \) are normalized so that: \( \Gamma = |A(X \rightarrow Y Z)|^2 \).

The above formalism can be adapted easily to describe the situation where one \( D \) meson decays into \( PP' \) rather than \( VV' \). Indeed, it amounts to considering only the longitudinal decay amplitude for \( D \rightarrow V V_1 \) and to remove the angular phase space related to the decay products of \( V_1 \) and \( V_2 \).

3 Observables from correlated \( D \) decays

3.1 Observables from \( \psi \rightarrow 2D \rightarrow 4V \) for \( CP \) violation

If we take the decay chain \([15][16]\)

\[ e^+ e^- \rightarrow \psi \rightarrow D^0 \bar{D}^0 \rightarrow f_a f_b \]

with \( f_a \) and \( f_b \) \( CP \) eigenstates of same \( CP \)-parity, we have

\[ CP|\psi \rangle = |\psi \rangle, \quad CP|f_a f_b \rangle = \eta_a \eta_b (-1)^{\ell} |f_a f_b \rangle = - |f_a f_b \rangle \]

since \( f_a \) and \( f_b \) are in a \( P \) wave. Therefore, the decay of \( \psi \) into states of identical \( CP \) parity is, by itself, a \( CP \)-violating observable \([15][16]\).

One obtains, neglecting \( CP \)-violation in \( D\bar{D} \) mixing, the following result for the combined branching ratio, which can be recovered from \([17]\)

\[ BR((D^0 \bar{D}^0)_{C=-1} \rightarrow f_a f_b) = 2BR(D^0 \rightarrow f_a)BR(D^0 \rightarrow f_b) |\rho_a - \rho_b|^2 + r_D |1 - \rho_a \rho_b|^2, \]

with the ratio of \( CP \)-conjugate amplitudes and the combination of \( D \)-mixing parameters

\[ \rho_f = \frac{A(D^0 \rightarrow f)}{A(D^0 \rightarrow \bar{f})}, \quad r_D = (x^2 + y^2)/2 < 10^{-4}. \]

where \( x = \Delta m/\Gamma \) and \( y = \Delta \Gamma/(2\Gamma) \) are the difference of masses and widths of the mass eigenstates in the \( D \bar{D} \) system, normalised by their average width \([3]\).

If we assume that \( CP \) is conserved in decay, we have \( \rho_f = \eta_f \), and thus \( BR = 0 \) for \( a, b \) with same \( CP \)-parity. Therefore, we have indeed that the observation of \( (D^0 \bar{D}^0)_{C=-1} \rightarrow f_a f_b \) with \( a, b \) of same \( CP \)-parity is an indication of \( CP \)-violation. Let us notice that \( a \) and \( b \) must be different eigenstates (either different mesons, or for \( VV \), different partial waves), and that this branching ratio is sensitive to different aspects of \( CP \)-violation compared to uncorrelated decays of \( D \rightarrow f_a \) and \( D \rightarrow f_b \), since the latter would be sensitive to \( 1 - |\rho_a|^2 \) or \( 1 - |\rho_b|^2 \). We can thus construct observables for \( CP \) violation in \( VV \) decays by considering states with the same \( CP \) parity, which depends on the relative angular momentum between the two mesons. The favourite channels among the measured ones are \( K^+K^-, \pi^+\pi^-, K_S \pi^0, \rho^0\pi^0, K_S \rho^0, K^{*0}\rho^0 \rightarrow (K_S \pi^0)(\pi^+\pi^-) \) and \( \rho^0 \phi \).

The transversity amplitudes \( A \) for \( D_{1,2} \rightarrow VV' \) have simple transformation laws under \( CP \):

\[ A_{0}^{D \rightarrow V V'} \rightarrow +\eta_{CP}(V)\eta_{CP}(V')\eta_{CP}(D)A_{0}^{D \rightarrow V V'}, \]

\[ A_{\parallel}^{D \rightarrow V V'} \rightarrow +\eta_{CP}(V)\eta_{CP}(V')\eta_{CP}(D)A_{\parallel}^{D \rightarrow V V'}, \]

\[ A_{\perp}^{D \rightarrow V V'} \rightarrow -\eta_{CP}(V)\eta_{CP}(V')\eta_{CP}(D)A_{\perp}^{D \rightarrow V V'}. \]

Following eq. \([17]\), \( CP \) conservation at the level of the amplitude would require that only two combinations of transversity amplitudes are allowed: (0, \( \perp \)) or (\( \parallel \), \( \perp \)). In terms of partial waves, 0
and $||$ are combinations of $S$ and $D$ waves, whereas $\perp$ is $P$ wave, which means that $CP$ conservation at the level of the amplitude would impose the vector mesons to be emitted in $(S,D)$ waves on one side and $P$ wave on the other. Therefore, the following combinations of transversity amplitudes in the partial differential decay rate can be in principle $CP$ violating observables:

$$\{ (0,0), (0,||), (||,0), (||,||), (\perp,\perp) \}. \quad (22)$$

Let us notice that in the case of identical meson pairs in the final state, there is only one $CP$-violating configuration that is available: $(0,||)$, due to the Bose-Einstein statistics. It seems more interesting to consider two different meson pairs, both with longitudinal polarization, to get a larger $BR$. From the above table, the most interesting modes are $\bar{K}^0\rho^0 \rightarrow (K_S\pi^0)(\pi^+\pi^-)$ and $\rho^0\rho^0$. It is straightforward to construct the corresponding $CP$-violating observable:

$$\int d\Gamma_{4V} \frac{1}{128} \left( 5\cos^2\theta_{V_1} - 1 \right) \left( 5\cos^2\theta_{V_2} - 1 \right) \left( 5\cos^2\theta_{V_3} - 1 \right) \left( 5\cos^2\theta_{V_4} - 1 \right)$$

$$= |A^\psi_{V_1V_2V_3V_4}|^2 |A^D_{0\rightarrow V_1V_2}|^2 |A^D_{0\rightarrow V_3V_4}|^2 \times |\rho^0_{V_1V_2} - \rho^0_{V_3V_4}|^2. \quad (23)$$

Similar weights can be obtained for the other $CP$-violating combinations, exploiting orthogonality relationships for Legendre and Chebyshev polynomials to select specific angular dependences as in the previous case. For instance, we have:

$$\int d\Gamma_{4V} \frac{1}{32} \left( 5\cos^2\theta_{V_1} - 3 \right) \left( 5\cos^2\theta_{V_2} - 3 \right) \left( 5\cos^2\theta_{V_3} - 3 \right) \left( 5\cos^2\theta_{V_4} - 3 \right)$$

$$= |A^\psi_{V_1V_2V_3V_4}|^2 |A^D_{||\rightarrow V_1V_2}|^2 |A^D_{||\rightarrow V_3V_4}|^2 \times |\rho^\parallel_{V_1V_2} - \rho^\parallel_{V_3V_4}|^2. \quad (24)$$

and

$$\int d\Gamma_{4V} \frac{1}{32} \left( 5\cos^2\theta_{V_1} - 3 \right) \left( 5\cos^2\theta_{V_2} - 3 \right) \left( 5\cos^2\theta_{V_3} - 3 \right) \left( 5\cos^2\theta_{V_4} - 3 \right)$$

$$= |A^\psi_{V_1V_2V_3V_4}|^2 |A^D_{\perp\rightarrow V_1V_2}|^2 |A^D_{\perp\rightarrow V_3V_4}|^2 \times |\rho^\perp_{V_1V_2} - \rho^\perp_{V_3V_4}|^2. \quad (25)$$

In the case of the same $VV$ final state for both $D$ decays, one can obtain the appropriate observable corresponding to $(0,||)$ using for instance:

$$\int d\Gamma_{4V} [(5\cos^2\theta_{V_1} - 1)(5\cos^2\theta_{V_2} - 1)(5\cos^2\theta_{V_3} - 3)(5\cos^2\theta_{V_4} - 3)(4\cos^2\Phi_{12} - 1)$$

$$\cdot (5\cos^2\theta_{V_1} - 3)(5\cos^2\theta_{V_2} - 3)(5\cos^2\theta_{V_3} - 3)(5\cos^2\theta_{V_4} - 3)(4\cos^2\Phi_{34} - 1)]$$

$$= |A^\psi_{V_1V_2V_3V_4}|^2 |A^D_{0\rightarrow V_1V_2}|^2 |A^D_{0\rightarrow V_3V_4}|^2 \times |\rho^0_{V_1V_2} - \rho^0_{V_3V_4}|^2. \quad (26)$$

### 3.2 $\psi \rightarrow 2D \rightarrow (VV)(K\pi)$ for the extraction of $\gamma$

The measurement of $\gamma$ from the Atwood-Dunietz-Soni (ADS) method \[27\] requires the determination of the hadronic parameters $r$ and $\delta$. At BES-III, we can also take advantage of the coherence of the $D^0$ mesons produced at the $\psi(3770)$ peak to extract the strong phase difference $\delta$ between doubly-Cabibbo-suppressed and Cabibbo-favoured decay amplitudes that appears in the $\gamma$ measurements \[12\] \[14\]. Here we introduce, in the standard phase convention where $\delta$ vanishes in the SU(3) limit,

$$r \cdot e^{i\delta} = \frac{\langle K^-\pi^+|\bar{D}^0 \rangle}{\langle K^-\pi^+|D^0 \rangle}. \quad (27)$$
The process of one $D^0$ decaying to $K^−\pi^+$, while the other $D^0$ decaying to a $VV\ CP$ eigenstate can be described as (for our purposes, it will prove more convenient to express this decay rate in terms of $D^0$ and $\bar{D}^0$ amplitudes):

$$d\Gamma_{2V} = \frac{9}{4\pi}d(\cos \theta_{V_1})d(\cos \theta_{V_2})d\Phi \times |A^{V_1V_2}|^2 |A^{D^0\rightarrow K\pi}|^2$$

$$\times \left[ \cos \theta_{V_1} \cos \theta_{V_2} (A^{D_0\rightarrow V_1V_2}_0 - re^{i\delta} A^{\bar{D}_0\rightarrow V_1V_2}_0) - \frac{1}{\sqrt{2}} \sin \theta_{V_1} \sin \theta_{V_2} \cos \Phi (A^{D_0\rightarrow V_1V_2}_\parallel - r e^{i\delta} A^{\bar{D}_0\rightarrow V_1V_2}_\parallel) - \frac{i}{\sqrt{2}} \sin \theta_{V_1} \sin \theta_{V_2} \sin \Phi (A^{D_0\rightarrow V_1V_2}_\perp - r e^{i\delta} A^{\bar{D}_0\rightarrow V_1V_2}_\perp) \right]^2.$$  

We can introduce:

$$A_{0,\parallel,\perp}(\bar{D}^0 \rightarrow V_aV_b) = A_{0,\parallel,\perp}(D^0 \rightarrow V_aV_b) \rho_{V_aV_b}^{0,\parallel,\perp}.$$  

In the absence of $CP$ violation, which we will assume in this section, we have:

$$\rho_{V_aV_b}^{0,\parallel} = -\eta_{CP}(V_a)\eta_{CP}(V_b) = -\rho_{V_a,V_b}^{\perp}.$$  

Moreover, we notice that all the decays presented in Sec. 2.1 have CP parities such that $\rho^0 = -1$, which yields the further expression of the differential decay width in Eq. (28):

$$d\Gamma_{2V} = \frac{9}{4\pi}d(\cos \theta_{V_1})d(\cos \theta_{V_2})d\Phi \times |A^{V_1V_2}|^2 |A^{D^0\rightarrow K\pi}|^2$$

$$\times \left[ \cos^2 \theta_{V_1} \cos^2 \theta_{V_2} |A^{D_0\rightarrow V_1V_2}_0|^2 (1 + 2r \cos \delta + r^2) + \frac{1}{2} \sin^2 \theta_{V_1} \sin^2 \theta_{V_2} \cos^2 \Phi |A^{\bar{D}_0\rightarrow V_1V_2}_\parallel|^2 (1 + 2r \cos \delta + r^2) - \sqrt{2} \cos \theta_{V_1} \sin \theta_{V_2} \sin \theta_{V_2} \cos \Phi \Re[A^{D_0\rightarrow V_1V_2}_\parallel (A^{\bar{D}_0\rightarrow V_1V_2}_\parallel)^*] (1 + 2r \cos \delta + r^2) + \frac{1}{2} \sin^2 \theta_{V_1} \sin^2 \theta_{V_2} \sin^2 \Phi |A^{\bar{D}_0\rightarrow V_1V_2}_\perp|^2 (1 - 2r \cos \delta + r^2) + \sqrt{2} \cos \theta_{V_1} \sin \theta_{V_2} \cos \theta_{V_2} \sin \Phi \left\{ \Re[A^{D_0\rightarrow V_1V_2}_\parallel (A^{\bar{D}_0\rightarrow V_1V_2}_\parallel)^*] (2r \sin \delta) + \Im[A^{D_0\rightarrow V_1V_2}_\parallel (A^{\bar{D}_0\rightarrow V_1V_2}_\parallel)^*] (1 - r^2) \right\} - \sin^2 \theta_{V_1} \sin^2 \theta_{V_2} \cos \Phi \left\{ \Re[A^{D_0\rightarrow V_1V_2}_\parallel (A^{\bar{D}_0\rightarrow V_1V_2}_\parallel)^*] (2r \sin \delta) + \Im[A^{D_0\rightarrow V_1V_2}_\parallel (A^{\bar{D}_0\rightarrow V_1V_2}_\parallel)^*] (1 - r^2) \right\} \right].$$

We see that the differential decay width provides six different angular observables depending on the following (real) quantities:

- three products of moduli for $VV$ decays: $|A^{V_1V_2} A^{D_0\rightarrow K\pi} A^{\bar{D}_0\rightarrow V_1V_2}_0|^2$
- two relative phases between the three amplitudes $A^{D_0\rightarrow V_1V_2}_0$
- two strong parameters describing the $K\pi$ decay: $r$ and $\delta$

Whereas the full angular integration yields the sum of the three transversity amplitudes:

$$\int d\Gamma_{2V} = \frac{9}{4\pi} |A^{V_1V_2}|^2 |A^{D_0\rightarrow K\pi}|^2 \left[ \frac{8\pi}{9} |A^{D_0\rightarrow V_1V_2}_0|^2 (1 + 2r \cos \delta + r^2) + \right]$$

\(\text{(32)}\).
\[
\frac{8\pi}{9} |A_\|^{D^0\to V_1V_2}|^2 (1 + 2r \cos \delta + r^2) + \frac{8\pi}{9} |A_\perp^{D^0\to V_1V_2}|^2 (1 - 2r \cos \delta + r^2),
\]

one can easily separate the different contributions by choosing suitable weights for the angular integration (they can be obtained easily by exploiting orthogonality relations among Legendre polynomials).

In practice the best way to perform the experimental analysis is usually to do a maximum likelihood fit on Eq. (31).

In Table 4 we show that one is only sensitive to \( \cos \delta \) to \(-\delta\), while in the standard analysis with \( PP \) modes one is only sensitive to \( \cos \delta \) (neglecting the small mixing contributions which lift the ambiguity [12]).

Since the ratio \( r \) is already well known, \( r = 0.055 \pm 0.002 \) [28], our method may lead to a good measurement of \( \delta \).

The above constraint can be improved by exploiting our current or expected knowledge of the polarisation of \( D \to VV \). If we extract the relative size and phase of the three amplitudes from independent single \( D \to V_1V_2 \) decay (single-tag - ST) measurements, and if the three amplitudes are not too different in size (as seems to be the case for \( \rho^0 \rho^0 \)), the measurement of the \( M_i \) amplitudes in the correlated (double-tag - DT) \( DD \to (V_1V_2)(K\pi) \) decay leads to the determination of both \( r \) and \( \delta \) (more precisely, \( r, \cos \delta \) and \( |\sin \delta| \)). Since the ratio \( r \) is already well known, \( r = 0.055 \pm 0.002 \) [28], our method may lead to a good measurement of \( \delta \).

Note that for relatively low statistics, a simplified transversity analysis can be performed. Instead of considering the full angular distribution in both single and double \( D \) decays, one can perform a one-parameter fit to the distribution of the transversity angle \( \theta_{tr} \), which yields the perpendicular polarisation fraction in single-tag and double-tag decays:

\[
\begin{align*}
 f_{ST}^{\perp} &= \frac{|A_\perp|^2}{|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2}, \\
 f_{DT}^{\perp} &= \frac{|M_\perp|^2}{|M_0|^2 + |M_\parallel|^2 + |M_\perp|^2}.
\end{align*}
\]

Table 4: Weights used to select contributions from the transversity amplitudes for \( \psi \to (K\pi)(VV) \). M amplitudes are defined by Eq. (33).

| \( i \) | \( P_i(\theta_{V_1}, \theta_{V_2}, \Phi) \) | \( \int d\Gamma_{2VV}P_i/(|A_0^{V_1V_2}|^2|A_0^{D^0\to V_2K}|^2) \) |
|---|---|---|
| 1 | \( \frac{1}{9}(5 \cos \theta_{V_1}^2 - 1)(5 \cos \theta_{V_2}^2 - 1) \) | \( |M_0|^2 \) |
| 2 | \( \frac{1}{16}(5 \cos \theta_{V_1}^2 - 3)(5 \cos \theta_{V_2}^2 - 3)(4 \cos \Phi^2 - 1) \) | \( |M_\parallel|^2 \) |
| 3 | \( \frac{25}{4}(25 \cos \theta_{V_1} \cos \theta_{V_2} \sin \theta_{V_1} \sin \theta_{V_2} \cos \Phi \) | Re \( [M_0 M_\parallel^*] \) |
| 4 | \( \frac{25}{4}(5 \cos \theta_{V_1}^2 - 3)(5 \cos \theta_{V_2}^2 - 3)(4 \cos \Phi^2 - 3) \) | \( |M_\parallel|^2 \) |
| 5 | \( \frac{25}{4}(25 \cos \theta_{V_1} \sin \theta_{V_1} \cos \theta_{V_2} \sin \theta_{V_2} \sin \Phi \) | \( -\text{Re} [M_0 M_\parallel^*] \) |
| 6 | \( \frac{1}{9}(5 \cos \theta_{V_1}^2 - 3)(5 \cos \theta_{V_2}^2 - 3) \cos \Phi \sin \Phi \) | \( \text{Re} [M_1^* M_\parallel^*] \) |
The above observables leads to:
\[
\frac{|A_\perp|^2}{|A_0|^2 + |A_\parallel|^2} = \frac{f_{ST}^{f}}{1 - f_{ST}^{f}}, \quad \frac{|1 + re^{i\delta}|^2}{1 - re^{i\delta}} = \frac{f_{ST}^{f}}{1 - f_{ST}^{f}} \frac{1 - f_{VT}^{f}}{f_{VT}^{f}},
\]
which implies a one-dimensional parabolic constraint in the \((r, \cos \delta)\) plane. An independent constraint comes from the ratio of double-tag to single-tag widths proportional to \((|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2)/(|M_0|^2 + |M_\parallel|^2 + |M_\perp|^2)\), that can be expressed in terms of \(r, \cos \delta\) and \(f_{ST}^{f}\). This simplified transversity analysis allows one to determine \(r\) and \(\cos \delta\). However the main novelty of our proposal comes from the sensitivity of the complete correlated decay rate to \(|\sin \delta|\) terms, which needs the study of the full angular dependence.

### 3.3 CP-violation in \(D^0\bar{D}^0\) mixing

In the previous discussions, we have neglected the tiny CP-violation in \(D^0\bar{D}^0\) mixing in order to simplify the study of correlated \(D \to VV\) decays. The inclusion of this effect would impact our results in the following way:

- If CP violation is indeed measured through \(\psi(3770) \to D^0\bar{D}^0 \to (V_1V_2)(V_3V_4)\), we cannot \(a\) \(p\rior\) disentangle CP violation in mixing from CP violation in decay. Therefore, if we want to convert this result into a bound on fundamental parameters of a New Physics model, we will have to exploit external inputs on CP-violating parameters of the mixing (from other observables). On the other hand, such an input is not necessary if we only aim at setting a constraint on CP-violation itself.

- In the determination of the strong phase in \(D \to K\pi\), the amplitudes exhibit in principle a small dependence on mixing effects. However, this dependence is very weak with respect to the dependence on the hadronic parameters \((r, \delta)\), and as a first approximation, it can be neglected. As in the previous case, we can use external information on the CP-violating parameters in mixing to include their impact when required by more accurate measurements of the partial decay rate.

### 4 Potential for BES-III and a super \(\tau\)-charm factory

In this section we give a first rough estimate of the expected sensitivity of the two different measurements discussed above, either at the BES-III experiment or at a Super \(\tau\)-charm factory.

#### 4.1 CP violation

As discussed in Section 3.3, the decay chain of \(e^+e^- \to \psi \to D^0\bar{D}^0 \to f_af_b\) can be described by Eq. (17), in which both CP conserving and violating processes can occur. We parameterize the ratio of amplitudes \(\rho_f\) in Eq. (18) as \(\rho_f = \eta_f (1 + \delta_f)e^{i\alpha_f}\), where the \(\delta_f\) is term from CP violation in decay, and \(\alpha_f\) is the phase difference between \(D^0\) and \(\bar{D}^0\) decay into the same final state \(f\).

The \(D^0\) decay channels in Table 1 can be directly used to search for CP violation by fully considering the correlation of \(D^0\bar{D}^0\) production at BES-III. The background is small, and the main dilution is due to the mis-identification of charged particles, which is suppressed by about \(10^{-4}\). The sensitivity of measurement of CP violation can reach about \(10^{-3}\) with a 20 fb\(^{-1}\) luminosity on the \(\psi(3770)\) peak at BES-III. As in the previous case, one must take care of the background due to the dilution from non-CP eigenstates that impact the quasi two-body decays of \(D^0\) meson listed in Table 2 (for \(D \to PV\)) and 3 (for \(D \to VV\)).

Final states consisting of two vector meson pairs are particularly interesting, since one can use information on transversity amplitudes to extract different combinations of CP-violating observables,
as discussed in Section 3.1 (0, 0), (0, |0⟩), (|0⟩, 0), (|0⟩, |0⟩), (|0⟩, |0⟩). For example, a back-of-the-envelope computation yields the most promising channel ρ₀ρ₀/̄K₀ρ₀:

\[ \mathcal{BR}(D^0D^0|_{C=-1} \rightarrow ρ₀ρ₀, ̄K₀ρ₀)_{\text{CPV}}^{|_{(0,|0⟩)} \simeq 8 \times \mathcal{BR}^0(D^0 \rightarrow ρ₀ρ₀) \cdot \mathcal{BR}^{|D^0 \rightarrow ̄K₀ρ₀|} \sin^2 \frac{α_a - α_b}{2}, \]

where \( \mathcal{BR}^0 \) means the branching fraction for longitudinal polarized \( D^0 \rightarrow ρ₀ρ₀ \) decay, \( \mathcal{BR}| \) means the parallel helicities fraction of \( D^0 \rightarrow ̄K₀ρ₀ \) decay, and where we have assumed that the CP-violating parameters \( δ_f \) vanish.

Assuming that no CP-violating signal events in \( D^0 ̄D^0 \) coherent decays are observed with 20 fb⁻¹ data at BES-III, we can provide an upper limit on the CP-violating branching fraction at 90% confidence level (C.L.), as indicated in Table 5. A Super τ-charm factory with 2 ab⁻¹ data yields naturally stronger constraints. If each polarized fraction is measured independently, an upper limit on the phase difference \( |α_a - α_b| \) can be set. For example, the current values for the polarized fractions in ρρ and ρK⁺ yields the upper limit \( |α_a - α_b| < 4.4^\circ \) at 90% confidence level from the channel \( (D^0 ̄D^0)_{C=-1} \rightarrow ρ₀ρ₀, ̄K₀ρ₀)_{(0,|0⟩) \). At a future Super τ-charm factory, with a data set of 2 ab⁻¹, the constraint would be more severe, \( |α_a - α_b| < 0.5^\circ \) at 90% confidence level.

A more realistic analysis requires a likelihood fit to the full angular dependence of the VV modes. Systematics will arise from the mis-reconstruction as VV CP-eigenstates of the events that actually come from other resonances or background contributions. In view of the sizable width of the vector resonances, we expect that these systematics will dominate the final result. Their precise estimate in the framework of each experiment is however beyond the scope of this paper.

| Reaction | Efficiency | Upper limits at BES-III (×10⁻⁷) |
|----------|------------|-------------------------------|
| \( D^0D^0 \rightarrow (ρ⁺ρ⁻)(K^{0+}ω) \) | 0.13 | 2.46 |
| \( D^0D^0 \rightarrow (ρ₀ρ₀)(K^{0+}ω) \) | 0.17 | 1.88 |
| \( ̄D^0D^0 \rightarrow (K^{0+}ρ₀)(K^{0+}ω) \) | 0.10 | 3.19 |
| \( D^0 ̄D^0 \rightarrow (K^{0+}ρ₀)(ρ₀φ) \) | 0.09 | 3.55 |
| \( D^0D^0 \rightarrow (K^{0+}ω)(ρ₀φ) \) | 0.08 | 3.99 |
| \( ̄D^0D^0 \rightarrow (ρ₀ρ₀)(̄K^{0+}ω) \) | 0.15 | 2.13 |
| \( D^0 ̄D^0 \rightarrow (ρ₀ρ₀)(ρ₀φ) \) | 0.13 | 2.46 |
| \( D^0 ̄D^0 \rightarrow (ρ⁺ρ⁻)(ρ₀φ) \) | 0.11 | 2.90 |
| \( D^0D^0 \rightarrow (ρ⁺ρ⁻)(K⁺⁺K⁻⁻) \) | 0.11 | 2.90 |

Table 5: The projected 90%-C.L. upper limits on CP violating branching fraction of some most interesting (VV)(VV) modes from correlated \( D^0 ̄D^0 \) pairs with 20 fb⁻¹ data taken at \( ψ(3770) \) peak at BES-III.

### 4.2 Strong phase in \( D^0 \rightarrow Kπ \)

The joint decay of \( D^0 \) into \( K^−π^+ \) and of \( D^0 \) into a CP eigenstate \( f_η \) can be described as

\[ Γ_{Kπ;f_η} = Γ[(K^−π^+)(f_η)] \approx A^2 A^2_f� |1 + iηre^{-iδ}|^2 \approx A^2 A^2_f |1 + 2ηr \cos δ|, \]

where \( A = |(K^−π^+|H|D^0)| \) and \( A_f = |(f_η|H|D^0)| \) are the real-valued decay amplitudes, \( η = ±1 \) is CP eigenvalue of the eigenstate \( f_η \), \( re^{-iδ} \) is defined in Eq. (27) and we have taken \( f_η \) to be a PP or VP CP-eigenstate, without any non trivial phase-space dependence. We also have neglected the subdominant \( r^2 \) term in Eq. (38). The following asymmetry can be used to determine \( δ \) [10]

\[ A \equiv \frac{Γ_{Kπ;f_+} - Γ_{Kπ;f_-}}{Γ_{Kπ;f_+} + Γ_{Kπ;f_-}}, \]

(39)
where $\Gamma_{K\pi,f_{\pm}}$ is defined in Eq. (38), which is the rate for the $\psi(3770) \rightarrow D^0\bar{D}^0$ configuration to decay into flavor eigenstates and a CP-eigenstates $f_{\pm}$. Eq. (38) implies a small asymmetry, $A = 2r \cos \delta$. In such a case, the error $\Delta A$ is approximately $1/\sqrt{N_{K-\pi^+}}$, where $N_{K-\pi^+}$ is the total number of events tagged with CP-even and CP-odd eigenstates, leading to:

$$\Delta(\cos \delta) \approx \frac{1}{2r \sqrt{N_{K-\pi^+}}}.$$  (40)

The expected number $N_{K-\pi^+}$ of CP-tagged events depends on the total number of $D^0\bar{D}^0$ pairs $N(D^0\bar{D}^0)$, the branching ratio to the CP-eigenstate $f_\eta$ and the tagging efficiency. Considering all decay modes listed in tables 1, 2 and 3 we find

$$\Delta(\cos \delta) \approx \frac{300}{\sqrt{N(D^0\bar{D}^0)}}.$$  (41)

At BES-III, about $72 \times 10^6$ $D^0\bar{D}^0$ pairs can be collected with four year running [20,29], which implies an accuracy of about 0.03 for $\cos \delta$, when considering both $K^-\pi^+$ and $K^+\pi^-$ final states.

As in the previous section a more realistic analysis requires a likelihood fit to the full angular dependence of the $VV$ modes, which in turn provides independent information on $|\sin \delta|$ as explained above. On the other hand the imperfect reconstruction of the $VV$ events as pure CP-eigenstates will presumably introduce sizable systematics in this discussion.

At a Super $\tau$-charm factory $^{30,31}$ with a 2 ab$^{-1}$ data set, we can expect a factor of ten improvement, but again the precise impact of the modeling of the vector resonances requires more studies.

5 Conclusion

The charm quark offers interesting opportunities to cross-check the mechanism of CP violation precisely tested in the strange and beauty sectors. The start of BES-III will allow for extensive measurements of charm properties. Among the various tests that can be considered, one may think of exploiting the quantum correlations in the $D\bar{D}$ pairs produced at $\psi(3770)$ resonance. In this paper, we exploit these correlations in $\psi(3770) \rightarrow D^0\bar{D}^0 \rightarrow (V_1V_2)(V_3V_4)$ in connection with CP violation, and $\psi(3770) \rightarrow D^0\bar{D}^0 \rightarrow (V_1V_2)(K\pi)$ for CKM angle $\gamma$ measurements, where all $VV$ pairs are reconstructed as CP-eigenstates.

In the case of $\psi(3770) \rightarrow D^0\bar{D}^0 \rightarrow (V_1V_2)(V_3V_4)$, the existence of correlations hinders some helicity configurations for the outgoing vector mesons in the absence of CP violation. This is mirrored by the angular distribution of the differential decay width, out of which CP-violating observables can be constructed. Such observables should be interesting to isolate significant New Physics effects in the charm sector. Assuming that there would be no CP-violating signal events observed in $D^0\bar{D}^0$ coherent decays with 20 fb$^{-1}$ data taken at $\psi(3770)$ peak at BES-III, we estimated an order of magnitude of the corresponding upper limit on CP-violating parameters, in particular for the channel $(D^0\bar{D}^0)_{C=-1} \rightarrow \rho^0\rho^0, K^{*0}\rho^0)$. Since the obtained bounds do not follow from a full angular fit and do not include the systematics corresponding to the separation of the wanted vector resonances from the background, further studies are needed.

CP-tagged $D \rightarrow K\pi$ decays give access to the strong phase difference $\delta$ between Cabibbo-favored and doubly-Cabibbo suppressed decays, and thus improve the uncertainty on the $\gamma$ measurement of the unitary triangle from $B^\pm \rightarrow D/\bar{D}K^{\pm}$ decays. At BES-III, with 20 fb$^{-1}$ data at $\psi(3770)$ peak, we estimate the error of $\cos \delta$ to be of a few percents, corresponding to an error on the $\delta$ of a few degrees.

We expect this estimate can be improved by taking into account the dependence of the full angular decay width to the sine of the strong phase. At the Super $\tau$-charm factory, the expected statistical error on $\delta$ could then fall below one degree. On the other hand a further study of experimental
systems related to the background identification is required since they will presumably dominate over the uncertainty quoted here.

Since our numerical estimates are quite promising we hope that the potential of such coherent $D$-decays into vector mesons at charm factories will be assessed more precisely in the future.

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