Neutrinos in the Family Replicated
Gauge Group Model

Colin D. Froggatt

Department of Physics and Astronomy
Glasgow University, Glasgow G12 8QQ, Scotland

Abstract

We present a discussion of the neutrino mass problem in the anti-grand unification theory based on three family replicated copies of the Standard Model gauge group (SMG). We consider two versions of the theory, with and without right-handed neutrinos, and present order of magnitude fits to the 17 measured quark-lepton mass and mixing angle variables. In the model without right-handed neutrinos, based on the gauge group $SMG^3 \times U(1)_f$ and an isotriplet Higgs, we obtain a good 4 parameter fit, but with a vacuum oscillation solution to the solar neutrino problem. In the second model, based on the gauge group $(SMG \times U(1)_{B-L})^3$, we obtain a good 5 parameter fit using the usual right-handed neutrino see-saw mechanism to generate an LMA-MSW solution to the solar neutrino problem. The CHOOZ mixing angle bound is easily satisfied in the first model and is close to the predicted value in the second model.

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1 Introduction

In this paper, I will update my report [1] on fermion masses in the Anti-Grand Unification Theory (AGUT) at the Bled 2001 workshop. There are two versions of the AGUT model based on three family replicated copies of the Standard Model (SM) gauge group, selected according to whether or not each family is supplemented by a right-handed neutrino. We note that neither model introduces supersymmetry. In the absence of right-handed neutrinos, the AGUT gauge group is $G_1 = SMG_3 \times U(1)_f$, where $SMG_3 \equiv SU(3) \times SU(2) \times U(1)$.

With the inclusion of three right-handed neutrinos, the AGUT gauge group is extended to $G_2 = (SMG_3 \times U(1)_{B-L})^3$, where the three copies of the SM gauge group are supplemented by an abelian $(B-L)$ (= baryon number minus lepton number) gauge group for each family. In each case, the AGUT gauge group $G_1 (G_2)$ is the largest anomaly free group [2] transforming the known 45 Weyl fermions (and the additional three right-handed neutrinos for $G_2$) into each other unitarily, which does NOT unify the irreducible representations under the SM gauge group.

Here we present good order of magnitude fits, with four and five adjustable parameters respectively, to the quark and lepton masses and mixing angles for the two versions of the AGUT model. In each case the fit to the charged fermion masses and quark mixings is arranged to essentially reproduce the original three parameter AGUT fit [1]. It is necessary to introduce a new mass scale into the theory, in order to obtain realistic neutrino masses. For the $G_1 = SMG_3 \times U(1)_f$ model we introduce a weak isotriplet Higgs field and obtain a vacuum oscillation solution to the solar neutrino problem, whereas for the $G_2 = (SMG_3 \times U(1)_{B-L})^3$ model we introduce the usual see-saw mass scale for the right-handed neutrinos and obtain a large mixing angle (LMA) MSW solution. During the last year, further data [3] from the Sudbury Neutrino Observatory have confirmed that the LMA-MSW solution is strongly favoured, with the Vacuum Oscillation and LOW solutions now allowed at the 3σ level while the SMA-MSW solution seems to be completely ruled out. We refer to [4] for a recent review of the phenomenology of neutrino physics.

2 The $SMG_3^3 \times U(1)_f$ Model

The usual SM gauge group is identified as the diagonal subgroup of $SMG_3$ and the AGUT gauge group $SMG_3 \times U(1)_f$ is broken down to this subgroup by four Higgs fields $S$, $W$, $T$ and $\xi$. Thus, for example, the SM weak hypercharge $y/2$ is given by the sum of the weak hypercharge quantum numbers $y_i/2$ for the three proto-families:

$$\frac{y}{2} = \frac{y_1}{2} + \frac{y_2}{2} + \frac{y_3}{2}$$

(1)

The spontaneously broken chiral AGUT gauge quantum numbers of the quarks and leptons protect the charged fermion masses and generate a mass hierarchy for them [5] in terms of the Higgs field vacuum expectation values (VEVs).
However the VEV of the Higgs field $S$ is taken to be unity in fundamental (Planck) mass units. Thus only the VEVs of the other three Higgs fields are used as free parameters, in the order of magnitude fit to the effective SM Yukawa coupling matrices $Y_U$, $Y_D$ and $Y_E$ for the quarks and charged leptons $[1]$. In this model, the large $\nu_\mu - \nu_\tau$ mixing required for atmospheric neutrino oscillations is generated by introducing a large off-diagonal element, $(Y_E)_{23} \sim (Y_E)_{33}$, in the charged lepton Yukawa coupling matrix. This is achieved by introducing a new Higgs field $\psi$ with a VEV equal to unity in Planck units and the following set of $U(1)$ gauge charges:

\[ \tilde{Q}_\psi = 3\tilde{Q}_\xi + 2\tilde{Q}_W + 4\tilde{Q}_T = \left( \frac{1}{2}, -\frac{13}{6}, \frac{5}{3}, -\frac{16}{3} \right), \]  

Here we express the abelian gauge charges in the model as a charge vector $\tilde{Q} = (y_1/2, y_2/2, y_3/2, Q_f)$.

We then have the Yukawa matrices for the quarks:

\[
Y_U = \begin{pmatrix}
W^2T^2\xi^2 & WT^2\xi & W^2T^\xi \\
WT^2\xi & WT^2 & W^2T \\
\xi & 1 & WT
\end{pmatrix}, \quad Y_D = \begin{pmatrix}
W^2T^2\xi^2 & WT^2\xi & T^\xi \\
WT^2 & WT^2 & T^3 \\
W^2T & W^3T^\xi & WT
\end{pmatrix}, \quad Y_E = \begin{pmatrix}
W^2T^2\xi^2 & W^3T^2 & W^2T^\xi \\
WT^2 & WT^2 & W^\xi \\
W^4\xi^2 & W^3T^\xi & WT
\end{pmatrix}
\]

and the charged lepton Yukawa matrix:

\[
Y_E = \begin{pmatrix}
W^2T^2\xi^2 & W^3T^2 & W^2T^\xi \\
WT^2 & WT^2 & W^\xi \\
W^4\xi^2 & W^3T^\xi & WT
\end{pmatrix}.
\]

We still obtain a good order of magnitude phenomenology for the charged fermion masses and quark mixing angles, similar to the original AGUT fit $[1]$, with the following VEVs in Planck units:

\[
< W >= 0.179, \quad < T >= 0.071, \quad < \xi >= 0.099. \quad (5)
\]

The unitary matrix $U_E$ which diagonalises $Y_EY_E^\dagger$ is then given by

\[
U_E \approx \begin{pmatrix}
1 & \frac{W\xi^3}{\xi} & \frac{W^2\xi^3}{\xi} \\
\frac{1}{T^3\sqrt{1+\frac{\xi}{T^2}}} & \frac{1}{\sqrt{1+\frac{\xi}{T^2}}} & \frac{T}{\sqrt{1+\frac{\xi}{T^2}}} \\
\frac{1}{T^4\sqrt{1+\frac{\xi}{T^2}}} & \frac{\xi}{\sqrt{1+\frac{\xi}{T^2}}} & \frac{1}{\sqrt{1+\frac{\xi}{T^2}}}
\end{pmatrix} \sim \begin{pmatrix}
1 & 0.05 & 1.7 \times 10^{-4} \\
-0.03 & 0.58 & 0.81 \\
0.04 & -0.81 & 0.58
\end{pmatrix}
\]

As we can see from the structure of $U_E$, we naturally obtain the large $\mu - \tau$ mixing required for the atmospheric neutrinos. We can now obtain suitable mixing and a suitable hierarchy of neutrino masses for a vacuum oscillation solution to the solar neutrino problem, by making the following choice of charges for a weak iso-triplet Higgs field, $\Delta$.

\[
\tilde{Q}_\Delta = \left( \frac{1}{2}, \frac{1}{3}, -\frac{5}{6}, 0 \right). \quad (7)
\]
We then have the neutrino mass matrix,

\[ M_\nu \sim <\Delta^0> \begin{pmatrix} W\xi^6 & W\xi^3 & WT\xi^2 \\ W\xi^3 & W & WT\xi \\ WT\xi^2 & WT\xi & WT^2\xi \end{pmatrix} \tag{8} \]

This has the hierarchy,

\[
\Delta m_{12}^2 \sim \Delta m_{23}^2, \tag{9}
\]
\[
\frac{\Delta m_{13}^2}{\Delta m_{12}^2} \sim 2T^3\xi^3 \sim 7 \times 10^{-7}, \tag{10}
\]

which is just suitable for the atmospheric neutrinos and the vacuum oscillation solution to the solar neutrino problem. The electron neutrino mixing is also large enough for the vacuum oscillation solution to the solar neutrino problem, as we can see from the matrix \( U_\nu \) which diagonalises \( M_\nu \):

\[
U_\nu \sim \begin{pmatrix} \frac{1}{\sqrt{2}}(1 + \frac{T\xi}{\sqrt{1 + T^2\xi^2}}) & \xi^3 & \frac{1}{\sqrt{2}}(1 - \frac{T\xi}{\sqrt{1 + T^2\xi^2}}) \\ \frac{1}{\sqrt{2}}(1 - \frac{T\xi}{\sqrt{1 + T^2\xi^2}}) & 1 & \frac{T\xi}{\sqrt{2}}(1 + \frac{T\xi}{\sqrt{1 + T^2\xi^2}}) \\ -\frac{1}{\sqrt{2}}(1 - \frac{T\xi}{\sqrt{1 + T^2\xi^2}}) & T\xi & \frac{1}{\sqrt{2}}(1 + \frac{T\xi}{\sqrt{1 + T^2\xi^2}}) \end{pmatrix}. \tag{11}
\]

Hence we have the lepton mixing matrix,

\[
U = U_E^\dagger U_\nu \sim \begin{pmatrix} \frac{1}{\sqrt{2}}(1 + \frac{T\xi}{\sqrt{1 + T^2\xi^2}}) & \frac{W\xi^4}{T^3\sqrt{1 + T^2\xi^2}} & \frac{1}{\sqrt{2}}(1 + \frac{T\xi}{\sqrt{1 + T^2\xi^2}}) \\ \frac{1}{\sqrt{2}}(1 - \frac{T\xi}{\sqrt{1 + T^2\xi^2}}) & \frac{T\xi}{\sqrt{2}}(1 + \frac{T\xi}{\sqrt{1 + T^2\xi^2}}) & \frac{T\xi}{\sqrt{2}}(1 + \frac{T\xi}{\sqrt{1 + T^2\xi^2}}) \\ -\frac{1}{\sqrt{2}}(1 - \frac{T\xi}{\sqrt{1 + T^2\xi^2}}) & \frac{1}{\sqrt{2}}(1 - \frac{T\xi}{\sqrt{1 + T^2\xi^2}}) & \frac{1}{\sqrt{2}}(1 - \frac{T\xi}{\sqrt{1 + T^2\xi^2}}) \end{pmatrix} \tag{12}
\]

\[
\sim \begin{pmatrix} 0.83 & 2.8 \times 10^{-2} & 0.58 \\ 0.47 & 0.58 & -0.68 \\ -0.34 & 0.813 & 0.49 \end{pmatrix}. \tag{13}
\]

If we take \( <\Delta^0> \sim 0.18 \text{ eV} \), then we have

\[
m_1 \sim <\Delta^0>(-WT\xi^2 + \frac{T^2\xi W}{2}) \sim -1.4 \times 10^{-5} \text{ eV}
\]
\[
m_2 \sim <\Delta^0>W \sim 3.2 \times 10^{-2} \text{ eV}
\]
\[
m_3 \sim <\Delta^0>(WT\xi^2 + \frac{T^2\xi W}{2}) \sim 3 \times 10^{-5} \text{ eV}. \tag{14}
\]

The \( \Delta m^2 \) and mixing angle for the solar neutrinos are given by

\[
\Delta m_{13}^2 \sim 7 \times 10^{-10} \text{ eV}^2, \quad \sin^2 2\theta_\odot \sim 4U_{e1}^2 U_{e3}^2 \sim 0.93 \tag{15}
\]
which are compatible with vacuum oscillations for the solar neutrinos. Similarly
the $\Delta m^2$ and mixing angle for atmospheric neutrinos are given by,

$$\Delta m^2_{23} \sim 1 \times 10^{-3} \text{ eV}^2, \quad \sin^2 \theta_{\text{atm}} \sim 0.93. \quad (16)$$

Hence we can see that we have large (but not maximal) mixing for both the solar
and atmospheric neutrinos. We also note that the CHOOZ electron survival
probability bound is readily satisfied by $U_{e2} \sim 0.028 < 0.16$, which is the
relevant mixing matrix element since $\Delta m^2_{12} \sim \Delta m^2_{23} \gg \Delta m^2_{13}$. Thus we obtain
a good order of magnitude fit (agreeing with the data to within a factor of $2$)
to the 17 measured fermion mass and mixing angle variables with just 4 free
parameters ($W$, $T$, $\xi$ and $\Delta^0$), but assuming a vacuum oscillation solution to
the solar neutrino problem.

We note that we have really only used the abelian gauge quantum numbers to
generate a realistic spectrum of fermion masses; the non-abelian representations
are determined by imposing the usual SM charge quantisation rule for each of the
SMG factors in the gauge group. Furthermore two of the $U(1)$s (more precisely
two linear combinations of the $U(1)$s in the gauge group are spontaneously
broken by the Higgs fields $S$ and $\psi$, which have VEVs $< S > = < \psi >= 1$. Hence these $U(1)$s play essentially no part in obtaining the spectrum of fermion
masses and mixings. This means that we can construct a model based on the
gauge group $SMG \times U(1)^f$ with the same fermion spectrum as above. However it turns out $\dag$ that some of the quarks and leptons must have extremely large
(integer) $U(1)^f$ charges, making this reduced model rather unattractive. Also
three Higgs fields $W$, $T$ and $\xi$ are responsible for the spontaneous breakdown
to the SM gauge group, $SMG \times U(1)^f \rightarrow SMG$, and they have large relatively
prime $U(1)^f$ charges. So we prefer the better motivated $SMG^3 \times U(1)^f$ AGUT
model.

3 The $(SMG \times U(1)_{B-L})^3$ Model

In this extended AGUT model we introduce a right-handed neutrino and a
gauged $B - L$ charge for each family with the associated abelian gauge groups
$U(1)_{B-L,i}$ $(i = 1, 2, 3)$. The $U(1)_f$ abelian factor of the $SMG \times U(1)_f$ model
in the previous section gets absorbed as a linear combination of the $B - L$
charge and the weak hypercharge abelian gauge groups for the different families
(or generations). It is these 6 abelian gauge charges which are responsible for
generating the fermion mass hierarchy and we list their values in Table 1 for the
48 Weyl proto-fermions in the model. The see-saw scale for the right-handed
neutrinos is introduced via the VEV of a new Higgs field $\phi_{SS}$. However, in
order to get an LMA-MSW solution to the solar neutrino problem, we have to
replace $\dag$ the AGUT Higgs fields $S$ and $\xi$ by two new Higgs fields $\rho$ and $\omega$. The abelian gauge quantum numbers of the new system of Higgs fields for the
$(SMG \times U(1)_{B-L})^3$ model are given in Table 2.

As can be seen from Table 2, the fields $\omega$ and $\rho$ have only non-trivial quantum
numbers with respect to the first and second families. This choice of quantum

\[\text{Table 1: Abelian Gauge Quantum Numbers for Weyl Proto-Fermions}\]
\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\text{Family} & \text{U(1)_{B-L1}} & \text{U(1)_{B-L2}} & \text{U(1)_{B-L3}} & \text{U(1)_f1} & \text{U(1)_f2} & \text{U(1)_f3} \\
\hline
\text{Q1} & 1 & 0 & 0 & 1 & 0 & 0 \\
\text{Q2} & 0 & 1 & 0 & 0 & 1 & 0 \\
\text{Q3} & 0 & 0 & 1 & 0 & 0 & 1 \\
\text{U} & 1 & 0 & 0 & 0 & 1 & 0 \\
\text{D} & 0 & 1 & 0 & 0 & 0 & 1 \\
\text{L} & 0 & 0 & 1 & 0 & 0 & 0 \\
\hline
\end{array}
\]

\[\text{Table 2: Abelian Gauge Quantum Numbers for Higgs Fields}\]
\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\text{Field} & \text{U(1)_{B-L1}} & \text{U(1)_{B-L2}} & \text{U(1)_{B-L3}} & \text{U(1)_f1} & \text{U(1)_f2} & \text{U(1)_f3} \\
\hline
\text{S} & 1 & 0 & 0 & 0 & 1 & 0 \\
\text{\xi} & 0 & 1 & 0 & 0 & 0 & 1 \\
\text{\rho} & 0 & 0 & 1 & 1 & 0 & 0 \\
\text{\omega} & 0 & 0 & 1 & 0 & 1 & 0 \\
\hline
\end{array}
\]
Table 1: All $U(1)$ quantum charges for the proto-fermions in the $(SMG \times U(1)_{B-L})^3$ model.

|                  | SMG1 | SMG2 | SMG3 | $U_{B-L,1}$ | $U_{B-L,2}$ | $U_{B-L,3}$ |
|------------------|------|------|------|-------------|-------------|-------------|
| $u_L, d_L$       | $\frac{1}{6}$ | 0 | 0 | $\frac{1}{6}$ | 0 | 0 |
| $u_R$            | $-\frac{1}{3}$ | 0 | 0 | $\frac{1}{3}$ | 0 | 0 |
| $d_R$            | $-\frac{1}{3}$ | 0 | 0 | $\frac{1}{3}$ | 0 | 0 |
| $e_L, \nu_{eL}$  | $-\frac{1}{3}$ | 0 | 0 | $-1$ | 0 | 0 |
| $e_R$            | $-1$ | 0 | 0 | $-1$ | 0 | 0 |
| $\nu_{\nu_R}$   | 0 | 0 | 0 | $-1$ | 0 | 0 |
| $c_L, s_L$       | 0 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ | 0 |
| $c_R$            | 0 | $-\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ | 0 |
| $s_R$            | 0 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ | 0 |
| $\mu_L, \nu_{\mu L}$ | 0 | $-\frac{1}{2}$ | 0 | 0 | $-1$ | 0 |
| $\mu_R$         | 0 | $-1$ | 0 | 0 | $-1$ | 0 |
| $\nu_{\nu_R}$   | 0 | 0 | 0 | $-1$ | 0 | 0 |
| $t_L, b_L$       | 0 | 0 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{2}$ |
| $t_R$            | 0 | 0 | $\frac{3}{2}$ | 0 | 0 | $\frac{3}{2}$ |
| $b_R$            | 0 | 0 | $-\frac{3}{2}$ | 0 | 0 | $\frac{3}{2}$ |
| $\tau_L, \nu_{\tau L}$ | 0 | 0 | $-\frac{1}{2}$ | 0 | 0 | $-1$ |
| $\tau_R$        | 0 | 0 | $-1$ | 0 | 0 | $-1$ |
| $\nu_{\nu_R}$   | 0 | 0 | 0 | 0 | $-1$ |

Table 2: All $U(1)$ quantum charges of the Higgs fields in the $(SMG \times U(1)_{B-L})^3$ model.

|                  | SMG1 | SMG2 | SMG3 | $U_{B-L,1}$ | $U_{B-L,2}$ | $U_{B-L,3}$ |
|------------------|------|------|------|-------------|-------------|-------------|
| $\omega$         | $\frac{1}{6}$ | $-\frac{1}{6}$ | 0 | 0 | 0 | 0 |
| $\rho$           | 0 | 0 | 0 | $\frac{1}{3}$ | $\frac{1}{3}$ | 0 |
| $W$              | 0 | $-\frac{1}{6}$ | $\frac{1}{6}$ | 0 | $-\frac{1}{3}$ | $\frac{1}{3}$ |
| $T$              | 0 | $-\frac{1}{6}$ | $\frac{1}{6}$ | 0 | 0 | 0 |
| $\phi_{W,S}$     | 0 | $\frac{1}{3}$ | $-\frac{1}{3}$ | 0 | $\frac{1}{3}$ | $-\frac{1}{3}$ |
| $\phi_{SS}$      | 0 | 1 | $-1$ | 0 | 2 | 0 |

numbers makes it possible to express a fermion mass matrix element involving the first family in terms of the corresponding element involving the second family, by the inclusion of an appropriate product of powers of $\rho$ and $\omega$. With the system of quantum numbers in Table 2 one can easily evaluate, for a given mass matrix element, the numbers of Higgs field VEVs of the different types needed to perform the transition between the corresponding left- and right-
handed Weyl fields. The results of calculating the products of Higgs fields needed, and thereby the order of magnitudes of the mass matrix elements in our model, are presented in the following mass matrices (where, for clarity, we distinguish between Higgs fields and their hermitian conjugates):

the up-type quarks:

\[ M_U \simeq \frac{\langle \phi_{WS} \rangle}{\sqrt{2}} \begin{pmatrix} (\omega^1)^3 W^\dagger T^2 & \omega \rho^1 W^\dagger T^2 & \omega \rho^1 (W^\dagger)^2 T \\ (\omega^1)^4 \rho & W^\dagger T^2 & (W^\dagger)^2 T \\ 1 & 1 & 1 \end{pmatrix} \] (17)

the down-type quarks:

\[ M_D \simeq \frac{\langle \phi_{WS} \rangle}{\sqrt{2}} \begin{pmatrix} \omega^3 W(T^\dagger)^2 & \omega \rho^3 W(T^\dagger)^2 & \omega \rho^3 T^3 \\ \omega^2 \rho W(T^\dagger)^2 & W(T^\dagger)^2 & T^3 \\ \omega^2 \rho W^2(T^\dagger) & W^2(T^\dagger)^4 & W^2 \end{pmatrix} \] (18)

the charged leptons:

\[ M_E \simeq \frac{\langle \phi_{WS} \rangle}{\sqrt{2}} \begin{pmatrix} \omega^3 W(T^\dagger)^2 & \omega \rho^3 W(T^\dagger)^2 & \omega \rho^3 W^4(T^\dagger)^5 \\ \omega^6 \rho^3 W(T^\dagger)^2 & W(T^\dagger)^2 & W^4(T^\dagger)^5 \\ \omega^6 \rho^3 W^2(T^\dagger) & (W^\dagger)^2 T^4 & W^2 \end{pmatrix} \] (19)

the Dirac neutrinos:

\[ M^D_\nu \simeq \frac{\langle \phi_{WS} \rangle}{\sqrt{2}} \begin{pmatrix} (\omega^1)^3 W^\dagger T^2 & (\omega^1)^3 \rho^1 W^\dagger T^2 & (\omega^1)^3 \rho^1 W^2(T^\dagger)^7 \\ (\rho^1)^3 W^\dagger T^2 & W^\dagger T^2 & W^2(T^\dagger)^7 \\ (\rho^1)^3 W^4(T^\dagger)^3 & (W^\dagger)^4 T^8 & W^4 \end{pmatrix} \] (20)

and the Majorana (right-handed) neutrinos:

\[ M_R \simeq \langle \phi_{SS} \rangle \begin{pmatrix} (\rho^1)^3 T^6 & (\rho^1)^3 T^6 & (\rho^1)^3 T^6 \\ (\rho^1)^3 T^6 & T^6 & T^6 \\ (\rho^1)^3 W^3(T^\dagger)^3 & W^3(T^\dagger)^3 & W^3(T^\dagger)^3 \\ W^3(T^\dagger)^3 & W^6(T^\dagger)^3 & W^6(T^\dagger)^3 \\ W^3(T^\dagger)^3 & W^6(T^\dagger)^3 & W^6(T^\dagger)^3 \\ W^3(T^\dagger)^3 & W^6(T^\dagger)^3 & W^6(T^\dagger)^3 \end{pmatrix} \] (21)

Then the light neutrino mass matrix – effective left-left transition Majorana mass matrix – can be obtained via the see-saw mechanism:

\[ M_{\nu}^{\text{eff}} \simeq M^D_\nu \ M_R^{-1} \ (M^D_\nu)^T \] (22)

with an appropriate renormalisation group running from the Planck scale to the see-saw scale and then to the electroweak scale. The experimental quark and lepton masses and mixing angles in Table 3 can now be fitted, by varying just 5 Higgs field VEVs and averaging over a set of complex order unity random numbers, which multiply all the independent mass matrix elements. The best fit is obtained with the following values for the VEVs:

\[ \langle \phi_{SS} \rangle = 5.25 \times 10^{15} \text{ GeV} , \ \langle \omega \rangle = 0.244 \ , \ \langle \rho \rangle = 0.265 \ , \ \langle W \rangle = 0.157 \ , \ \langle T \rangle = 0.0766 \ , \] (23)
Table 3: Best fit to conventional experimental data. All masses are running masses at 1 GeV except the top quark mass which is the pole mass.

|               | Fitted         | Experimental |
|---------------|----------------|--------------|
| $m_u$         | 4.4 MeV        | 4 MeV        |
| $m_d$         | 4.3 MeV        | 9 MeV        |
| $m_s$         | 1.6 MeV        | 0.5 MeV      |
| $m_c$         | 0.64 GeV       | 1.4 GeV      |
| $m_s$         | 295 MeV        | 200 MeV      |
| $m_t$         | 111 MeV        | 105 MeV      |
| $M_t$         | 202 GeV        | 180 GeV      |
| $m_b$         | 5.7 GeV        | 6.3 GeV      |
| $m_{\tau}$   | 1.46 GeV       | 1.78 GeV     |
| $V_{us}$      | 0.11           | 0.22         |
| $V_{cb}$      | 0.026          | 0.041        |
| $V_{ub}$      | 0.0027         | 0.0035       |
| $\Delta m^2_{\odot}$ | $9.0 \times 10^{-5}$ eV$^2$ | $5.0 \times 10^{-5}$ eV$^2$ |
| $\Delta m^2_{\text{atm}}$ | $1.7 \times 10^{-3}$ eV$^2$ | $2.5 \times 10^{-3}$ eV$^2$ |
| $\tan^2 \theta_{\odot}$ | 0.26           | 0.34         |
| $\tan^2 \theta_{\text{atm}}$ | 0.65           | 1.0          |
| $\tan^2 \theta_{\text{chooz}}$ | $2.9 \times 10^{-2}$ | $\lesssim 2.6 \times 10^{-2}$ |

where, except for the Higgs field $\langle \phi_{SS} \rangle$, the VEVs are expressed in Planck units. The resulting 5 parameter order of magnitude fit, with an LMA-MSW solution to the solar neutrino problem, is presented in Table 3.

Transforming from $\tan^2 \theta$ variables to $\sin^2 2\theta$ variables, our predictions for the neutrino mixing angles become:

$$\sin^2 2\theta_{\odot} = 0.66 \ , \ \sin^2 2\theta_{\text{atm}} = 0.96 \ , \ \sin^2 2\theta_{\text{chooz}} = 0.11 \ . \quad (24)$$

Note that our fit to the CHOOZ mixing angle lies close to the 2$\sigma$ Confidence Level experimental bound. We also give here our predicted hierarchical left-handed neutrino masses ($m_i$) and the right-handed neutrino masses ($M_i$) with mass eigenstate indices ($i = 1, 2, 3$):

$$m_1 = 1.4 \times 10^{-3} \ \text{eV} \ , \ M_1 = 1.0 \times 10^6 \ \text{GeV} \ , \quad (25)$$
$$m_2 = 9.6 \times 10^{-3} \ \text{eV} \ , \ M_2 = 6.1 \times 10^9 \ \text{GeV} \ , \quad (26)$$
$$m_3 = 4.2 \times 10^{-2} \ \text{eV} \ , \ M_3 = 7.8 \times 10^9 \ \text{GeV} \ . \quad (27)$$

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