Unitarity of exclusive quark combination model: Exotic hadron production, entropy change and charmonium production for colour-singlet many-quark system

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Confinement indicates an asymptotic quark state not observable except its energy is zero. Unitarity indicates that the total probability of a definite state of quark system to transit to any final state is exactly one. This talk reviews some important conclusions/predictions from the basic properties like unitarity of the combination model, as addressed by the title.

1 Introduction: Unitarity of exclusive quark combination model

Quark Combination Model (QCM) was proposed in early seventies of 20th century (Anisovich, Bjorken) to describe the multi-production process in high energy collisions, based on the constituent quark model of hadrons. Various versions of QCM have succeeded in explaining many data. Recently in central gold-gold collisions at the Relativistic Heavy Ion Collider (RHIC), several ‘unexpected’ phenomena which lay difficulties for other hadronization mechanisms can be easily understood in quark combination mechanism. Common of all the hadronization models, QCM responds to describe the non-perturbative QCD phase. It includes two steps: 1) the ‘partons’ in various collisions turn into constituent quarks; 2) these quarks combined into hadrons according to certain rules. One can regard the combination model as a ‘reverse employment’ of the constituent quark model. In the following of this paper we concentrate on step 2, the combination process, which is the ‘realization’ of confinement for the constituent quarks. We will investigate the most general principles which a QCM has to respect, so that to see what can be reliably predicted by such a model, rather than seek how to employ a certain version of QCM to make a good postdiction and parameterization of the data. For this purpose, we deal with a colour-singlet (CS) system of many quarks prepared from step 1, but without addressing how the hard partons turned into constituent quarks, especially, ‘where is the gluon’? (Prof. Dixon asked after this presentation) in step 1. Charm and bottom quarks are produced from hard interactions. They in step 1 are ‘dressed’ to be a constituent but their momentum spectra are not largely modified. This special advantage will be discussed in the following.

Without digging into details of any special kind of QCM, one easily figures out two principles which it must respect: First of all, energy-momentum conservation is the principle law of physics, reflecting the basic symmetry space-time displacement invariance. The models must precisely (as precisely as possible, in practice) transfer the energy and momentum of the parton system into the constituent quark system and then the hadron system. Second, when applying the combination rules on a CS separated system of constituent quarks, it is necessary that all the quarks are combined into hadrons, or else there are free quarks with non-zero mass and energy, which obviously contradicts to any observations that suggest confinement. Moreover, these free quarks take away energy and momentum, hence make danger of the energy-momentum conservation. This second principle is referred as Unitarity of the relevant model. These two principles are closely connected, with the first one the natural result of the second one.

For a QCM which respects and can reflect unitarity, the combination process can be described by a unitary time-evolution operator $U$, with

$$\sum_{h} |< h|U|q >|^2 = < q| U^+ U |q > = 1.$$  \hspace{1cm} (1)

The quark state $|q>$, describes a CS quark system, and the corresponding hadron state $|h>$ describes the hadron system. The matrix element $U_{hq} = < h|U|q >$ gives the transition amplitude. For a separated system, the energy-momentum conservation is inherent, by the natural
commutation condition $[U, H] = [U, P] = 0$, with $H, P$ the energy and momentum operator of the systems. This is just the confinement which says that the total probability for the CS quark system to transit to all kinds of hadron is exactly 1, and agrees with the fact that all the constituent quark states and the hadron states are respectively two complete sets of bases of the same Hilbert space \(^1\), i.e., $\sum |q><q| = \sum |h><h| = 1$ for the colour-singlet system. So combination process never changes the degree of freedom of the system.

In the following sections we will address three relevant topics: Unitarity of the combination model naturally suppresses the production of exotic hadrons; Unitarity in exclusive QCM guarantees the non-decreasing of entropy in the combination process for a CS separated system; Unitarity does not introduce any new rules, when considering heavy quarks in the combination. Since lack of space, the Refs. are to be found from Ref. \(^1\).

2 Exotic hadron (multi-quark states) production

Two important points should be considered:

1. As a matter of fact from experiments,
   \[ \sum_{h=B,B,M} |<h|U|q>|^2 \sim 1 - \varepsilon, \varepsilon \to 0^+, \quad (2) \]
   here $B, \bar{B}, M$ denote baryon, antibaryon and meson respectively. Naïvely from the group theory, colour confinement seems not so restrict as Eq. (2). The CS state, i.e., the invariant, totally antisymmetric representation of the $SU_C(3)$ group, requires at least one quark and one antiquark, or three (anti)quarks, but more (anti)quarks can also construct this representation, hence possibly to form a CS “hadron”. They are to be called exotic hadrons (here not including glueball or hybrid). Until now, no experiment can definitely show $\varepsilon$ in Eq. (2) is exactly 0 or a small but non-vanishing number. If definitely $\varepsilon = 0$, there must be underlying properties of QCD which we still not very familiar. Even $\varepsilon$ is not vanishing, its smallness, definitely confirmed by experiments and shown in Eq. (2), also provides interesting challenges, especially on hadronization models. The small production rate of a special kind of exotic hadron seems easy to be adopted. However, taking into account so many possibilities to construct the CS representations by various numbers of (anti)quarks, that the total sum of them is still quite small, is very nontrivial as a property of QCD and even nontrivial for a hadronization model to reproduce.

2. Colour recombination destroys the distinction between multiquark state and molecule state. All kinds of Exotic hadrons have one common property: The (anti)quarks can be grouped into several clusters, with each cluster possibly in CS. However, the ways of grouping them into clusters are not unique, as it is simply known from group theory that the reduction ways for a direct product of several representations are not unique. Furthermore, these clusters need not necessarily be in CS respectively, since the only requirement is the whole set of these clusters in CS. For example, the system $q_1\bar{q}_2q_3\bar{q}_4$ (the constituents of a “tetraquark”) can be decomposed/clustered in the following ways: $(q_1q_3)\otimes (\bar{q}_2\bar{q}_4) \rightarrow 1$, $(q_1\bar{q}_2)\otimes (q_3\bar{q}_4) \rightarrow 1$, or $8 \otimes 8 \rightarrow 1$, ...

\(^a\)This is very natural, if one adopts that QCD is really the uniquely correct theory for the hadron physics, with its effective Hamiltonian $H_{QCD}$. Then all the hadron states with definite energy-momentum should be its eigenstates and expand the Hilbert space of states (though we do not know how to solve $H_{QCD}$ mathematically). While a model is proposed in language of constituent quarks which composite the hadrons, all of the quark states with definite energy-momentum should be eigenstates of the same $H_{QCD}$ (Here we consider constituent quark model, and ignore the rare probability of exotic hadrons like glueball, hybrid, hence need not consider gluon states). So these two sets of bases are of different representations, as is more easy to be recognized if one imagines that all the wave functions of hadrons are written in terms of quark states in some special framework of quark models and notices that the planer wave function as well as other special functions (bound state wave functions) are all possible to be complete bases for a definite functional space, mathematically.
In the above example, only the second case, when these two $q\bar{q}$ pairs are in CS respectively, it seems possible to be considered as a hadron molecule. But dynamically, the colour interactions in the system via exchanging gluons can change the colour state of each separate cluster, so each kind of grouping/reduction way seems no special physical meaning. Such an ambiguity, which has been considered in many hadronization and decay processes as “colour recombination/rearrangement” obstacles the possibility to consider the exotic hadron in a unique and uniform way, while leads to the possibility of introducing some phenomenological duality. Namely, even we consider the production of exotic hadron as “hadron molecule” formation, the subsequent colour interactions in the system can eventually transit this “molecule” into a “real” exotic hadron, at least by some probability.

From the above discussion, and in the calculation by Shandong QCM (SDQCM), one can introduce a model dependent definition of multiquark state, i.e., the number of quarks to be combined into the hadron is definite though quark pair could be created in the bound state. The fact $\epsilon \to 0^+$ is employed by introducing the parameter $x$. It is clear that to an extreme if we have infinite kinds of exotic hadrons, $x$ should be vanishing, expecting infinite number of vanishing variables (production rates corresponding to each certain exotic hadron) summing up to get a finite small result (the total production rate of all exotic hadrons). So it is predicted that if the Gell-mann Zweig quark model can be extrapolated to multiquark states, production of each of the species could be vanishing and not observable.

3 Entropy change

1. By the formula of entropy $S = -tr(\rho \ln \rho)$ for a separated system, we can conclude a unitary transition will not change the entropy.

$$\rho(t) = |t, i > P_i < i, t| = U(t, 0)|0, i > P_i < i, 0|U^\dagger(t, 0) = U(t, 0)\rho(0)U^\dagger(t, 0).$$

Here $U(t, 0)$ is the time evolution operator. $P_i$ is the probability of the state with index $i$. Taking $\rho(0)$ as the distribution of the constituent quark system just before combination, while $\rho(t)$ just after, of the hadron system, then $U(t, 0)$ is exactly the operator $U$ introduced in Eq. 1. This is a uniform unitary transformation on the Hermitian operator $\rho$, which does not change the trace of $\rho \ln \rho$. So the entropy holds as a constant in the combination process, same as energy and momentum.

2. Energy conservation is kept for each combination step for the many quark CS separated system, by tuning the constituent quark masses in the programme. Then an ideal quasistatic process can be employed to calculate the entropy change. The result is again zero. The details are described in arXiv:1005.4664.

4 Charmonium production

In several combination models, including the SDQCM mentioned above, one has considered the open charm hadron production by introducing the charm quark into the bulk of the light quarks with its specific spectrum, to let all these four kinds of quarks to combine on equal footing. In this consideration, one has to deal with the case when a charm quark antiquark pair can be combined together under the combination rule to keep consistency. On the other hand, one can raise the question whether charmonium (or bottomium) can be produced under exactly the same mechanism as light $q\bar{q}$ hadron, i.e., via the common combination rules.

Charm/bottom is the kind of ‘constituent quarks’ which is more easy to be tracked than the light ones, and the ‘dressing process’ will not change the spectrum much. The light quark sector,
many of them come from gluon nonperturbative QCD transition, which is yet quite unclear, as described above. So it is more reliable talking about the charm distribution before combination. The above investigation of charmonium as well as the open charm in QCM can help the study of its energy loss in medium.

When according to the combination rules, a $c\bar{c}$ pair can be combined, we further restrict their invariant mass to lower than some definite value (say, that of $\Psi(3S)$) to be a charmonium. This will not change the unitarity mentioned above. This has a good analogy: the charmonium and open charm corresponding to the positronium and free electron (discrete and continuous spectrum), respectively. Such a restriction does not affect comparing with data, either, since charmonium resonances more massive than $\Psi(3S)$ almost all decay into open charms.

Our results indicate that at RHIC, the charmonium can be described exactly in the same way of the open charm particle by SDQCM without introducing any new rule. This check is to be done for LHC soon.

More details in the long write up will come soon. A preliminary figure can be seen from the presentation slides, P9; A table and formulae show the relative ratio of different kind of charmonium, see P8.

5 Postscript

In the above section, when comparing our result on $J/\Psi$ spectrum and concluding consistent with RHIC data, we neglect the contribution of bottom. In higher energies and larger transverse momentum $P_T$, e.g., in LHC, the contribution of B decay will increase and could be dominant for enough large $P_T$. In this case one need a separation of prompt in experiments, as done by CDF in Tevatron. To coincide with inclusive data, the theoretical calculation must include the bottom production. On the other hand, $J/\Psi$ is a good measure of B for large $P_T$ (Two body decay to $J/\Psi + h$ is an important way to measure B). Such a fact can be seen from the talks in this Rencontre (Z. Dolezal, K. Ulmer). Combined with the celebrating $J/\Psi$ suppression data in Pb-Pb of large $p_T$ reported by ATLAS in this Rencontre (B. Wosiek), it is easy to conclude that the bottom energy loss is almost the same as the light quark for large $P_T$, as expected by the author in discussions around the dinner table in La Thuile. The ‘non-photonic’ electron and forward muon data measured by ALICE presented on QM2011 (Annecy, May) is not contradicted with such a expectation, though the $R_{AA}$ a little larger comparing to e.g., pion. However, one sees $R_{AA}$ increasing with $P_T$ and the $P_T$ of electron/moun represents the behaviour of B around $2P_T$. Such a energy loss behaviour is well understood by considering the spacetime picture of the jet (heavy or light) medium interaction, as explained in the talk by the author in last year’s Rencontre. This kind of interaction has an analogy of hadron hadron interaction. The production process (pionization) is the main mechanism to lose energy. The produced particles composite the rapidity plateau, so that can be of large angle w.r.t the jet. Since the width and height of the plateau increase with the interaction energy, the energy loss also increases with jet energy. These have been confirmed by CMS measurements reported in La Thuile (F. Ma). $\frac{\Delta E}{E}$ is a constant for a large range of jet energy.

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1. The Refs. are to be found from arXiv:1005.4664, P.R.C80:035202,2009, SDU thesis of Shang and Yin (‘09,’10), and Li S.-Y., Yao, T. and Yin F., in preparation.