Compton dragged supercritical piles: The GRB prompt and afterglow scenario

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Abstract

We examine the prompt and afterglow emission within the context of the Supercritical Pile model for GRBs. For this we have performed self-consistent calculations by solving three time-dependent kinetic equations for protons, electrons and photons in addition to the usual mass and energy conservation equations. We follow the evolution of the RBW as it sweeps up circumstellar matter and assume that the swept-up electrons and protons have energies equal to the Lorentz factor of the flow. While the electrons radiate their energies through synchrotron and inverse Compton radiation on short timescales, the protons, at least initially, start accumulating without any dissipation. As the accumulated mass of relativistic protons increases, however, they can become supercritical to the ‘proton-photon pair-production - synchrotron radiation’ network, and, as a consequence, they transfer explosively their stored energy to secondary electron-positron pairs and radiation. This results in a burst which has many features similar to the ones observed in GRB prompt emission. We have included in our calculations the radiation drag force exerted on the flow from the scattered radiation of the prompt emission on the circumstellar material. We find that this can decelerate the flow on timescales which
are much faster than the ones related to the usual adiabatic/radiative ones. As a result the emission exhibits a steep drop just after the prompt phase, in agreement with the Swift afterglow observations.

1 Introduction

Despite the great progress in the GRB field made with CGRO and BeppoSAX ([1]), there are major issues concerning the dynamics and radiative processes of these events that still remain open. Some of these have been with us since the inception of the cosmological GRB models, while others are rather new, the outcome of the wealth of new observations made by Swift and HETE. Chief among them are the conversion of the RBW kinetic energy into radiation, the fact that the photon energy at which the GRB luminosity peaks is narrowly distributed around a value intriguingly close to the electron rest mass energy and the transition from the prompt to the afterglow emission. The purpose of the present note is to describe a process that provides a “natural” account of these generic, puzzling GRB features.

2 Blast Wave Dynamics and Radiation

We consider a Relativistic Blast Wave (RBW) moving with speed \( v_0 = \beta \Gamma c \), where \( \beta \Gamma = (1 - \Gamma^{-2})^{-1/2} \) and \( \Gamma \) the bulk Lorentz factor of the flow. It has a radius \( R(t) \) as measured from the origin of our coordinate system (assumed to be the center of the original explosion) and it is sweeping mass of density \( \rho_{\text{ext}} \). As the RBW sweeps up mass from the circumstellar matter (CSM), it starts slowing down. Following [2] we write two equations for the evolution of the RBW. One for the mass

\[
\frac{dM}{dR} = 4\pi R^2 \rho_{\text{ext}} \Gamma - \frac{1}{\epsilon \Gamma} \dot{E}
\]

(1)

and one for the Lorentz factor \( \Gamma \), which reflects the energy-momentum conservation

\[
\frac{d\Gamma}{dR} = -\frac{4\pi R^2 \rho_{\text{ext}} \Gamma^2}{M} - \frac{F_{\text{rad}}}{Mc^2}.
\]

(2)

Here \( \dot{E} \) is the radiation rate as measured in the comoving frame and \( F_{\text{rad}} \) is the radiation drag force which is exerted on the RBW from
any radiation field exterior to the flow. In the standard case (for a review see [3]) the above equations specify completely the dynamics of the RBW once the initial conditions of the flow have been specified. The profile of the external density $\rho_{\text{ext}}$ has also to be specified – in most cases either a constant density or a wind profile is assumed. The radiation rate $\dot{E}$ is usually set \textit{a-priori} varying between two extremes: When there is no radiation, the flow is considered as adiabatic, while when the hot mass is immediately radiated away the flow is considered as radiative. Either choice has a profound influence on the evolution of the RBW: In the adiabatic case the solution of Eqns (1) and (2) gives $\Gamma \propto R^{-3/2}$ while in the radiative case one gets $\Gamma \propto R^{-3}$. On the other hand, the role of $F_{\text{rad}}$ has not so far been investigated. Assuming that this force is exerted on the flow by the RBW photons scattered on the CSM, we can write (Mastichiadis & Kazanas, in preparation)

$$F_{\text{rad}} = \frac{64\pi}{9c} \tau_{\gamma} n_{e}^{CSM} \sigma_{T} R^{4} \dot{E}$$

(3)

where $\tau_{\gamma}$ is the Thomson optical depth of the RBW, $n_{e}^{CSM}$ is the electron density of the CSM and $\sigma_{T}$ is the Thomson cross section.

In order to calculate self-consistently the $\dot{E}$ and $F_{\text{rad}}$ terms, we implement a numerical code for the radiation transfer in the RBW. Details have been given in [4] and [5] but for the sake of completeness we repeat here the basic points about it.

The equations to be solved can be written in the general form

$$\frac{\partial n_{i}}{\partial t} + L_{i} + Q_{i} = 0.$$  

(4)

The unknown functions $n_{i}$ are the differential number densities of protons, electrons and photons while the index $i$ can be any one of the subscripts ‘p’, ‘e’ or ‘$\gamma$’ referring to each species. The operators $L_{i}$ denote losses and escape from the system while $Q_{i}$ denote injection and source terms. The important physical processes to be included in the kinetic equations are proton-photon (Bethe-Heitler) pair production, photopion production, lepton synchrotron radiation, synchrotron self absorption, inverse Compton scattering (in both the Thomson and Klein-Nishina regimes) and photon-photon pair production. The processes involving photons can take place either with photons produced directly or with the photons which have been reflected by the CSM in front of the advancing shock. The above equations are solved in the fluid frame inside a spherical blob of radius $R_{b} = R/\Gamma$. This can
be justified from the fact that due to relativistic beaming an observer 
receives the radiation coming mainly from a small section of the RBW 
of lateral width $R/\Gamma$ and longitudinal width $R/\Gamma^2$ in the lab frame 
but $R/\Gamma$ on the comoving frame.

Despite the fact that the general frame adopted here is similar to 
[5], the present approach differs in two important aspects:

1. Hot protons accumulate continuously on the RBW. For this 
we introduce injection terms for protons and electrons, assuming that 
at each radius $R$ the RBW picks up an equal amount of electrons 
and protons from the CSM which have, upon injection, energies $E_p = \Gamma m_p c^2$ and $E_e = \Gamma m_e c^2$ respectively. Consequently, the proton energy 
injection rate is given by [6]

$$\left(\frac{dE}{dt}\right)_{inj} = 4\pi R^2 \rho_{ext} (\Gamma^2 - \Gamma)c^3$$  \hspace{1cm} (5)

while a fraction $m_e/m_p$ of it goes to electrons.

2. The scattering of the RBW photons takes place on the CSM 
matter in front of the advancing front, thus the column density is also 
uniquely determined from the initial conditions. In this way we relax 
the assumption of [5] about the optical depth of the mirror.

Eqns (1), (2) and (4), with the addition of Eqns (3) and (5) form 
a set of equations which can now be solved simultaneously to give us

Figure 1: Photon luminosity as a result of a proton supercriticality which is 
developed as the RBW sweeps up mass from the CSM. For the initial values 
of the run see text.
Figure 2: Evolution of the bulk Comptonized spectrum during the outburst shown in Fig.1.

the evolution of the RBW dynamics and radiated power. We should emphasize that this approach is self-consistent as the 'hot' mass injected through Eqn (5) corresponds to the RHS of Eqn (1) while the radiated luminosity $\dot{E}$ and the radiative force $F_{rad}$ are calculated from Eqns (4). Therefore, once the initial conditions which are (i) the energy of the explosion $E$, (ii) the initial radius at which the RBW starts sweeping up matter $R_0$, (iii) the value $\Gamma_0 = \Gamma(R_0)$ and (iv) the external density of the CSM are specified, one can solve the above set of equations forward in time. However, the radiation code needs one more parameter, which is the value of the magnetic field at position $R_0$ and an assumption on its profile with $R$. Without loss of generality we can assume that the magnetic field drops like $1/R$ as assumed in [7].
3 Proton Supercriticality

As emphasized in [8] and [9] relativistic protons can become supercritical in a network involving proton-photon (Bethe-Heitler) pair production and electron synchrotron radiation. This supercriticality is a radiative-type instability which can convert the free energy of the relativistic proton plasma into relativistic $e^+e^-$ pairs, once a kinematical and a dynamical criterion are simultaneously satisfied.

For the kinematical criterion to be fulfilled all one needs is that the synchrotron photons produced from the Bethe-Heitler pairs to be energetic enough as to produce more pairs on the relativistic protons (see [8]). As it was shown in [9], if the proton plasma itself is in relativistic bulk motion this criterion is satisfied if

$$b \Gamma^5 \geq 2$$

where $b = B/B_{\text{crit}}$ with $B_{\text{crit}} = \frac{m_e^2 c^3}{\hbar c} = 4.4 \times 10^{13}$ G, the critical magnetic field.

In our case, if the choice of the values of these parameters are such that the above criterion is not fulfilled initially, then it will not be fulfilled anywhere, provided that $\Gamma$ is in its coasting stage – as implied by eqn.(2). This happens because, as $B$ and $\Gamma$ drop outwards, $b \Gamma^5$ can only decrease as $R$ increases. Therefore this corresponds to the trivial case: The protons are accumulated at the rate given by eqn (1) and they remain inert no matter how much mass has been accumulated on the RBW. The electrons, on the other hand, can radiate their energies fast as their radiative lifetimes to synchrotron and/or ICS can be short compared to the light crossing timescale on the RBW frame. However, the produced $\dot{E}$ is rather low and cannot affect significantly the global eqns (1) and (2). Thus the RBW obeys, for all practical purposes, the non-radiative case, while its luminosity stays at a relatively low level.

Far more interesting is the case where the kinematical criterion is satisfied initially. Then whether the flow becomes radiatively unstable will depend on the second criterion. Qualitatively one can say that this criterion is satisfied if at least one of the synchrotron photons produced by the $e^+e^-$ pairs produces another pair before escaping the volume of the plasma in a reaction with a sufficiently energetic proton (i.e. one that fulfills the kinematic threshold). This results in a condition for the column density of the plasma which is identical to that of a critical nuclear chain reaction and can be examined numerically with the code described in the previous section. Running various cases we
can find those initial conditions which can make the flow unstable.

Figure 1 shows the photon evolution in the lab frame of a solution that becomes supercritical. The initial values of the run are $R_0 = 10^{15}$ cm, $n_{ext} = 10^5$ par/cm$^3$ (values consistent with those of the wind of a WR-star), $\mathcal{E} = 10^{54}$ erg, $\Gamma_0 = 200$, and $B_0 = 10$ G. Thus at the beginning of the swept-up phase the product $b_0 \Gamma_0^5 \simeq 7$ and the kinematic criterion (6) is satisfied. The swept-up protons become quickly supercritical and the photons grow exponentially. This eventually leads to fast proton cooling and photon saturation leading to the peak in the photon flux of Fig.1. Consequently an abrupt decrease in $\Gamma$ due to the $F_{rad}$ term in Eqn (2) leads to a steep decay of photons for the next hundreds of seconds. As the photon density decreases, lower energy protons continue cooling in a more controlled manner. This gives a characteristic flattening to the photon lightcurve for the next few thousands seconds. At even longer timescales (tens to hundred thousand of seconds) first the strength of the magnetic field $B$ and later $\Gamma$ start decreasing due to the RBW expansion. These effects cause a final drop in the observed photon luminosity. Thus, this model can, in principle, connect the prompt to the afterglow emission.

As it was stated in [9] and shown numerically in [5] the observed photon spectrum consists of two components: one that is observed directly and one that is produced from the bulk Comptonization of the aforementioned direct component scattered first on the CSM electrons and then on the cool pairs of the RBW. For loops operating close to threshold, this has a peak at $\simeq 1$ MeV independent of the initial value of $\Gamma$.

Figure 2 shows snapshots of the bulk Comptonized spectrum for the first thousand seconds after the photon burst for the run described above. This has a peak in the MeV region at maximum luminosity. Consequent proton cooling and $\Gamma$ reduction have as effects (i) the fast decrease of the luminosity, as already shown in Fig.1 and (ii) a softening of the spectrum from the MeV to sub-MeV / keV regime.

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