Saturnian mean motion resonances in meteoroid streams

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Accepted: 2013 May 16; Received: 2013 May 16; In Original Form: 2013 Apr 26; MNRAS Letters

ABSTRACT
Many previous works have shown the relevance and dynamics of Jovian mean motion resonances (MMR) in various meteoroid streams. These resonant swarms are known to have produced spectacular meteor displays in the past. In this work we investigate whether any MMR due to Saturn are feasible, and subsequently check whether such effects are strong enough to trap meteoroids so as to cause enhanced meteor phenomena on Earth. Extensive numerical simulations are done on two major meteoroid streams, which are known to exhibit exterior Jovian resonances. The roles of the 1:6 and 5:14 Jovian MMR have already been studied in the Orionids and Leonids respectively. Now we find strong evidence of 1:3 and 8:9 Saturnian MMR in Orionids and Leonids respectively. The presence of compact dust trails in real space due to these two Saturnian resonances is confirmed from our calculations.

Key words: 1P/Halley, 55P/Tempel-Tuttle, Orionids, Leonids, Saturn, Jupiter, Comet, Meteoroid, Resonance, Celestial mechanics

1 INTRODUCTION
Jovian mean motion resonances (MMR) are known to have played an important role in determining the long term evolution of many meteoroid streams (Asher & Emel’yanenko 2002; Ryabova 2003; Jenniskens 2006; Vaubaillon, Lamy & Jorda 2006; Soja et al. 2011). In many cases the compact dust trails due to Jovian resonances have led to observed outbursts and storms (Astartovich 1968; Yeomans 1981; Asher & Clube 1993; Rendtel & Betlem 1993; Brown et al. 2002; Trigo-Rodríguez et al. 2007; Rendtel 2008; Kero et al. 2011). Most of these observational records match with theoretical simulations to a very good degree. Generally it is seen that the orbital evolution of resonant meteoroids is dramatically different from that of the non-resonant ones (Sato & Watanabe 2007; Christou, Vaubaillon & Withers 2008). The remarkably different precession rates and particle concentrations of resonant swarms, in comparison to non-resonant meteoroids, lead to varying levels of meteor activity in various showers. Hence correlating enhanced meteor activity with known Jovian resonances (Asher, Bailey & Emel’yanenko 1999; Jenniskens et al. 2007; Rendtel 2007; Sekhar & Asher 2013) and subsequent predictions of future meteor outbursts have been a very active field for some decades. Such calculations have gained very high precision over the years (Jenniskens et al. 1998; McNaught & Asher 1999).

Nevertheless we realise that no detailed simulations or analysis were done regarding resonances in meteoroid streams due to Saturn’s gravitational effects except a brief mention of a possible 8:9 MMR in Leonids (Stoney & Downing 1898; Brown 2001). Most scientists seem to have presumed that Saturnian resonances are either too weak or practically non-existent when it comes to producing enhanced meteor phenomena on Earth. Our simulations here show that this assumption is not true. We find conclusive evidence that strong Saturnian resonances are feasible as well as effective in trapping large numbers of meteoroids which can lead to formation of compact dust trails in space. Even though Saturnian resonances are quite rare compared to Jovian resonances in the context of known meteoroid streams, the newly found Saturnian resonances in this work show significant strength and stability which can in turn relate to spectacular meteor outbursts in the past and future. This paper investigates such Saturnian resonances in two major streams which are known to exhibit exterior Jovian resonances (Emel’yanenko 2001) namely the Orionids and Leonids.

2 SEPARATING JOVIAN AND SATURNIAN RESONANCES
Since there is a well known 2:5 near commensurability, widely known as the great inequality (Kepler 1672; Halley 1676; Laplace 1785; Hill 1890; Lovett 1895; Brouwer & van Woerkom 1950; Milani & Knežević 1990), between the orbits
the interval until the next series of successive encounters. In
for the Earth to encounter a single Leonid/Orionid resonant zone.
Saturn. (b) Circulation of 2:15 (Jovian) resonant argument for
an Orionid test particle, confirming presence of 1:3 MMR with
axis confirms the presence of 1:3 Saturnian MMR (semi-major
of 1:3 MMR (Saturnian), multiplying this ratio by 2:5 gives
nearby resonances simultaneously. For example in the case
involving Saturn) of the meteoroid particle for these two
resonances (Peale 1976) of the form p:(p+q) where q is the
order of resonance and repeated conjunction occurs for every
p orbits of the particle.

Figure 1(a) shows the 1:3 Saturnian resonant argument
σ librating continuously for about 4 kyr, then briefly
becoming non-resonant (overall range of σ becomes 360° for
a short while) and subsequently falling into the same reso-
onance for a further 2 kyr, in the case of an Orionid test
particle. Figure 1(b) shows the 2:15 resonant argument (Jo-
van) clearly circulating during the same time frame. The
starting epoch is JD 1208900.18109 = 1404 B.C. October
15.68109, the oldest credible computed perihelion passage
time of 1P/Halley (Yeomans & Kiang 1981).

For the Leonids we confirm the presence of 8:9 Saturn-
nian MMR (aₙ = 10.32 au). Figure 2(a) shows σ for this
MMR librating for ~700 yr before starting to drift from the
libration centre (initial epoch = JD 2220280.1685 = com-
puted return time of 55P/Tempel-Tuttle in 1366). Figures
2(b) and (c) show the adjacent 16:45 (aₙ = 10.36 au) and
5:14 (aₙ = 10.33 au) resonant arguments (Jovian) clearly
circulating for the same test particle during the same time
frame. Multiplying 8:9 MMR (Saturnian) by 2:5 gives the
nearby 16:45 MMR (Jovian). In terms of aₙ, 5:14 MMR
(Jovian) is even nearer than 16:45 is to 8:9 MMR (Saturn-
nian). Hence both these Jovian cases were verified to get a
comprehensive conclusion.

In order to absolutely confirm the presence of Saturnian
MMR and rule out the presence of nearby Jovian MMR,
many of the different possible combinations (see Table 1)
of 1:3, 8:9 and 2:15, 16:45, 5:14 resonant arguments allowed
by d’Alembert rules (Murray & Dermott 1999, sections 6.7
and 8.2) were checked to confirm libration and circulation
respectively (cf. Sekhar & Asher 2013). The resonant
arguments plotted in Figures 1(a) and 2(a) are respectively

\[ \sigma = \lambda - 3\lambda_m + 2\pi_m \] and \[ \sigma = 8\lambda - 9\lambda_m + \pi_m \],
where λ and π denote mean longitude and longitude of pericentre,
subscripts s and m standing for Saturn and meteoroid par-
ticle. Because both Halley’s and Tempel-Tuttle’s orbits are
retrograde, here we use the modified definition of \( \pi = \Omega - \omega \)
(Saha & Tremaine 1993; Whipple & Shelus 1993).

These techniques clearly show that Saturnian reso-
nances are indeed real and not entwined with the near
commensurate Jovian resonances. In Section 3 we integrate
ranges of particles to show dense clusters of Saturnian reso-
nant meteoroids in space retaining their compact structure
for many kyr.

The numerical integrations in this work were done using
the MERCURY package (Chambers 1999) implementing the
RAOU algorithm (Everhart 1985) with accuracy parameter
set to 10⁻¹² and including the sun and eight planets, whose
orbital elements were retrieved from JPL Horizons (Giorgini
et al. 1996). Elements for the parent bodies 1P/Halley and
55P/Tempel-Tuttle were taken from Marsden & Williams
(2008). Radiation pressure and Poynting-Robertson effects
were not included in any integrations.

![Figure 1](image-url)

**Figure 1.** (a) Libration of 1:3 (Saturnian) resonant argument for
an Orionid test particle, confirming presence of 1:3 MMR with
Saturn. (b) Circulation of 2:15 (Jovian) resonant argument for
the same particle, confirming absence of 2:15 MMR with Jupiter.

**Table 1.** Order of resonances discussed in this work and applying
combinatorics to calculate resonant arguments permitted by
d’Alembert rules. **T** is the approximate number of successive years
for the Earth to encounter a single Leonid/Orionid resonant zone.
**P** is the interval until the next series of successive encounters.
In previous work (Sekhar & Asher 2013) on Orionids, **T ~ 5-6 yr &
P ~ 71 yr for 1:6 Jovian, and T ~ 1-2 yr & P ~ 77 yr for 2:13
Jovian.**

| MMR   | Order | Number of Possible Resonant Arguments | **P** (yr) | **T** (yr) |
|-------|-------|---------------------------------------|-----------|-----------|
| p:(p+q) | q     |                                       |           |           |
| 8:9 S | 1     | 2                                     | 33        | 2         |
| 1:3 S | 2     | 6                                     | 88        | 22        |
| 5:14 J | 9     | 110                                   | 33        | 1         |
| 2:15 J | 13    | 280                                   | -         | -         |
| 16:45 J | 29    | 2480                                  | -         | -         |

of Jupiter and Saturn, it is vital to cross check the evolution
of resonant arguments (for any particular resonance ratio
involving Saturn) of the meteoroid particle for these two
nearby resonances simultaneously. For example in the case
of 1:3 MMR (Saturnian), multiplying this ratio by 2:5 gives
the adjacent 2:15 Jovian resonant argument (aₙ = 19.93 au)
to avoid any misleading signature from this nearby Jovian
MMR. Here we use a = aₙ = the ‘nominal resonance location’
(Murray & Dermott 1999, section 8.4) for exterior
resonances (Peale 1976) of the form p:(p+q) where q is the
order of resonance and repeated conjunction occurs for every
p orbits of the particle.
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3 GEOMETRY OF RESONANT ZONES AND ECLIPTIC PLANE CROSSINGS

3.1 Orionids

Figure 2(a) shows the general picture of resonant zones for the 1:3 Saturnian MMR ($a_n = 19.84$ au): 7200 particles were integrated forward from 1404 B.C., varying the initial $a$ from 19.0 to 20.8 au in steps of 0.018 au, and initial $M$ from 0 to 360° in steps of 5°, keeping other elements (namely $q$, $i$, $\omega$ and $\Omega$) the same as 1P/Halley. Particles are plotted that librate continuously for 4 kyr. The distribution shows their initial semi-major axis $a$ and mean anomaly $M$. Resonant particles were identified (cf. Section 2) by a simple algorithm which looks at the overall range of resonant argument (for different combinations allowed by d’Alembert rules) for each particle every 10 yr during the whole 4 kyr. A snapshot of the same ($a, M$) phase space 4 kyr later shows a similar picture with three dense clouds of resonant particles. Our simulations show that 1:3 resonant meteoroids can retain compact structures for many kyr (4 kyr is typical). A resonant meteor outburst is a possibility if Earth passes through one of these three clumps in space. When the Earth misses these dense clouds, a normal meteor shower can still occur.

The $a$ range spanned by the resonant zone (Fig. 2h) is equivalent to perihelion tangential ejection speeds in the range of about +40 to +65 m s$^{-1}$ at the 1404 B.C. return of 1P/Halley. These ejection velocities are realistic in cometary activity (Whipple 1951). Moreover radiation pressure acts in the same way as positive (= forward) ejection velocities, i.e. increasing the orbital period, and for visual meteor sized meteoroids the effect of radiation pressure on the period is quantitatively comparable to these ejection speeds (cf. Kondrat’eva & Reznikov 1985; Williams 1997; Asher & Emel’yanenko 2002). Hence these positive ejection velocities (in the gravitational integration model) required to populate
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Figure 4. (a) Ecliptic plane crossings; and (b) distribution of heliocentric distances, for sets of 1:3 Saturnian MMR and non-resonant Orionid particles in 450 A.D. Both sets of particles evolved dynamically for 2 kyr but the dense clustering of resonant meteoroids contrasts with the large dispersion of non-resonant particles.

Figure 5. (a) Ecliptic plane crossings; and (b) distribution of heliocentric distances, for sets of 8:9 MMR (Saturnian) and non-resonant Leonid particles in 2000 A.D., both sets having evolved over the same time frame. The resonant case shows very compact dust trails whereas significant dispersion is indicated in the non-resonant case.

3.2 Leonids

To explore the 8:9 MMR ($a_n = 10.32$ au) in the Leonid stream, we integrated 7200 particles varying initial $a$ from 10.16 to 10.46 au in steps of 0.0030 au and $M$ in steps of $5^\circ$. Figure 4(b) shows the distribution of particles with various values of initial $(a,M)$ that have not circulated through $360^\circ$ for the entire period of 1000 years starting from 1366 A.D. Virtually all such particles are resonant for at least 700 yr (Fig. 2a, and similar plots for other particles not shown). The inherent mechanism of resonant zones is the same as described for Orionids (Section 3.1) except that there are nine dense clumps in this case. A snapshot of the same phase space after 700 years shows a similar picture with nine clumps of resonant particles. Our simulations show that 8:9 resonant meteoroids can retain compact structures for up to many centuries (typically ~700 yr; cf. Fig. 2a).

this 1:3 resonant zone at this epoch imply that significant numbers of real meteoroids were released by 1P/Halley into this resonance. Over centuries the comet will drift through the three 1:3 resonant zones and populate all of them.

The active role of 1:6 and 2:13 Jovian MMR in causing strong Orionid meteor outbursts has previously been demonstrated (Rendtel 2007; Sato & Watanabe 2007; Sekhar & Asher 2013). The 1:3 Saturnian MMR can produce similarly compact structures in the Orionid stream. Figure 4(a) shows the (ascending) nodal crossing distribution for particles from a single 1:3 resonant zone, contrasted with non-resonant particles (cf. librating and circulating model comparisons of Emel’yanenko & Bailey 1996, e.g. their fig. 3). In the resonant case, 2000 particles were integrated from Halley’s 1404 B.C. return ($M=0$), varying the initial $a$ from 20.0 to 20.1 in steps of $5 \times 10^{-5}$ au and keeping $q$, $i$, $\omega$ and $\Omega$ the same as the comet. All parameters were the same for the non-resonant particles except the starting epoch was adjusted by $\sim 2$ yr ($M=0$ at JD 1208171.10151 was chosen) so that the evolution of non-resonant Orionids can be studied for the same time frame. There were no significant close encounters with planets during this extra integration time. The very compact structures in Fig. 4(a) for the librating particles prove the physical presence of dense meteoroid concentrations in space. Moreover the particle distribution along the orbit shows the non-resonant ones to be dispersed over a very large range of heliocentric distances which in turn leads to very low meteoroid concentrations.
Meteoroid ejection velocities in the range of about $-20$ to $-4 \text{ m s}^{-1}$, perfectly feasible during outgassing activity, can populate the entire resonant zone (shown in Fig. 5b) at the 1366 return of 55P/Tempel-Tuttle. The requirement for negative ejection velocities (in a gravitational integration model) will have an opposite effect to that discussed in Section 3.1, i.e. the real population of resonant meteoroids will consist of particles less affected by radiation pressure. This would mean enhanced chances of narrow trails of larger meteoroids in turn leading to brighter meteor showers when they intersect Earth.

Asher et al. (1999) and Brown & Arlt (2000) have shown the relevance of 5:14 Jovian MMR in causing intense meteor outbursts in the recent past. Figure 5(a) shows the (descending) nodal crossing distribution in 2000 A.D. for meteoroids outbursts in the recent past. Figure 5(a) shows the (descending) nodal crossing distribution in 2000 A.D. for meteoroids. The resonance leading to the compact dust trails producing enhanced meteor activity, are comparable to those due to previously known Jovian resonances like 2:13 MMR in 8:9 MMR with Saturn for 4 kyr and Leonid meteoroids in 5:14 MMR with Saturn for $\sim 700 \text{ yr}$. It is verified that none of these resonant signatures are due to nearby Jovian resonances such as 2:15 and 5:14 in Orionids and Leonids respectively. The survival times (of the order of $10^3 \text{ yr}$) and density distributions of these Saturnian resonances, which can in turn lead to very compact dust trails producing enhanced meteor activity, are comparable to those due to previously known Jovian resonances like 2:13 MMR in the Orionids discussed in Sekhar & Asher (2013).

Generally the lower the order $q$ of a resonance, the higher its strength. In the resonances mentioned above, it is then fair to assume that the weaker gravitational effect (in comparison to Jupiter) of Saturn is compensated by the difference in order of resonance. In 8:9 and 1:3 MMR (Saturnian) $q$ is 1 and 2 respectively, very low compared to 9 and 13 in the case of 5:14 and 2:15 MMR (Jovian) respectively. Saturn’s effects can become very relevant and significant in such cases. Hence one cannot rule out the possibility of an interesting time frame (in past or future) when there could be spectacular Orionid or Leonid meteor displays due to Saturn’s effects comparable to those due to Jovian effects.

4 CONCLUSIONS

We have shown that Orionid meteoroids can stay continuously in 1:3 MMR with Saturn for $\sim 4 \text{ kyr}$ and Leonid meteoroids in 8:9 MMR with Saturn for $\sim 700 \text{ yr}$. It is verified that none of these resonant signatures are due to nearby Jovian resonances such as 2:15 and 5:14 in Orionids and Leonids respectively. The survival times (of the order of $10^3 \text{ yr}$) and density distributions of these Saturnian resonances, which can in turn lead to very compact dust trails producing enhanced meteor activity, are comparable to those due to previously known Jovian resonances like 2:13 MMR in the Orionids discussed in Sekhar & Asher (2013).

ACKNOWLEDGMENTS

The authors thank Jürgen Rendtel for helpful comments and express their gratitude to the Department of Culture, Arts and Leisure of Northern Ireland for the generous funding to pursue astronomical research at Armagh Observatory.

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