I. INTRODUCTION

It is usually considered that the Ginzburg-Landau parameter \( (\kappa = \lambda/\xi) \) of the London penetration depth \( \lambda \) and the coherence length \( \xi \) is temperature-independent. The effective tool for the experimental determination of the Ginzburg-Landau parameter near the critical temperature is given by the famous Abrikosov formula\(^1\) for the field derivative of magnetization near the upper critical field in the type-II superconductors

\[
\frac{dM}{dH} \bigg|_{H=H_{c2}} = \frac{1}{4\pi(2\kappa^2 - 1)}\beta_A. \tag{1}
\]

Here \( \beta_A = (|\Delta|^4/\langle|\Delta|^2\rangle^2) \) is the Abrikosov parameter. In practice, it is convenient to use the Ehrenfest formula which relates the slope of magnetization curve near \( H_{c2} \) to the specific heat jump at \( T = T_c(H_{c2}) \)

\[
\frac{\Delta C}{T} = \left(\frac{dH_{c2}}{dT}\right)^2 \left(\frac{dM}{dH}\right). \tag{2}
\]

From Eqs. \( \text{(1)} \) and \( \text{(2)} \) we obtain

\[
\kappa = \frac{1}{\sqrt{2}} \sqrt{1 + \frac{T}{4\pi\beta_A\Delta C} \left(\frac{dH_{c2}}{dT}\right)^2}. \tag{3}
\]

For strong type two superconductors with \( \kappa >> 1 \)

\[
\kappa \approx \left|\frac{dH_{c2}}{dT}\right| \sqrt{\frac{T}{8\pi\beta_A\Delta C}}. \tag{4}
\]

The slope of the upper critical field in the Ginzburg-Landau region is temperature-independent. The same is true for the ratio \( T/\Delta C \). However, experimentally in several heavy fermionic compounds there was revealed a fast drop of the GL parameter with decreasing temperature\(^2\). So, the GL parameter proved to be a function of temperature. Already in the earliest experimental study there was suggested\(^2\) that this temperature dependence is introduced by the Zeeman depairing effect. Then the temperature dependence of GL parameter has been discussed theoretically in the papers\(^3\) making use the numerical solution of the Eilenberger equations taking into account both the orbital and the paramagnetic effect. Later the analytical expression for the GL parameter below \( T_c(H) \) in the limit of strong paramagnetic effect has been found in the paper\(^2\). In spite of these results it is still of definite interest to give a simple analytic formula for GL parameter valid near the phase transition line taking into account both the paramagnetic and the orbital depairing. Here we find this dependence in a straightforward way. Then we compare this with result of paper\(^2\).

We note the temperature-dependent Ginzburg-Landau parameter \( \kappa(T) \) instead of \( \kappa_2 \) used in the experimental literature and reserve the term Maki parameter \( \alpha_M \) for the ratio of orbital critical field to the paramagnetic limiting field at zero temperature \( \sqrt{2}H_{c20}/H_p \). In a clean superconductor \( \alpha_M \approx T_c/mv_F^2 \), where \( v_F = k_F/m^* \) is the Fermi velocity, the orbital critical field is \( H_{c20} \approx 2\pi\xi_0^2 \) while \( H_p = \Delta_0/\sqrt{2}\mu \) is the Pauli-limiting field at zero temperature.

II. THE CRITICAL FIELD TEMPERATURE DERIVATIVE

The GL superconducting free energy density has the form

\[
F_s = F_{n0} + \frac{\hbar^2}{8\pi} + \alpha|\Delta|^2 + \beta|\Delta|^4 + \gamma|\Delta|^2. \tag{5}
\]

Here, \( F_{n0} \) is the free energy density in normal state in absence of magnetic field, \( \mathbf{D} = -i\nabla + 2e\mathbf{A}, \mathbf{h} = \text{rot}\mathbf{A} \) is the local internal magnetic field, the induction \( B \) determined by the spatial average \( \mathbf{h} \equiv \mathbf{B} = B\hat{z} \) and the coefficient

\[
\alpha = \alpha_0 \frac{T - T_c}{T_c} + a \left(\frac{\mu B}{T_c}\right)^2 \tag{6}
\]

includes the paramagnetic depairing effect. The solution of the linearized GL equation as the linear combination of Landau wave functions with level \( n = 0 \) yields the equation for the upper critical field

\[
\alpha_0 \frac{T - T_c}{T_c} + a \left(\frac{\mu B}{T_c}\right)^2 + 2e\gamma B = 0, \tag{7}
\]

where \( T_c \) is the critical temperature at zero field. This formula is valid for a type of superconducting state with
singlet pairing and one component order parameter in a metal with a form of the Fermi surface. The value of coefficients can, of course, have different values in concrete materials with different purities for a concrete field orientation in respect to crystallographic axes. For a reader convenience we point out here their values valid near $T_c$ for a clean s-wave superconductor with spherical Fermi surface $\alpha_0 = N_0$, $a = \frac{\zeta(3)N_0}{4\pi^2}$, $b = \frac{\zeta(3)N_0}{16\pi^2}$, $\gamma = \frac{\zeta(3)N_0 e^2}{32\pi^2 T_c^2}$. Here, $N_0$ is the density of states at Fermi level and we put $k_B = h = c = 1$. For the more lower temperatures (say at $T \sim T_c/2$) one must use temperature and field dependent $\alpha, \beta$ and $\gamma$ coefficients.

Solving Eq. (7) we obtain

$$H_{c2} = \frac{e\gamma T_c^2}{a\mu^2} \left[ -1 + \sqrt{1 + \frac{\alpha_0 a \mu^2}{(e\gamma T_c)^2} \frac{T_c - T}{T_c}} \right]$$  \hspace{1cm} (8)

In the limiting case of pure orbital depairing that is at $\alpha_M << 1$ we obtain from Eq. (8) or directly from Eq. 7 the orbital critical field

$$H_{c2}^{orb} = \frac{\alpha_0}{2e\gamma} \frac{T_c - T_c}{T_c}.$$  \hspace{1cm} (9)

In the opposite case at $\alpha_M >> 1$, and $\frac{T_c - T_c}{T_c} > 1/\alpha_M^2$ the limited by paramagnetic effect critical field is

$$H_{c2}^p = \frac{\alpha_0}{\mu} \frac{T_c - T_c}{T_c}.$$  \hspace{1cm} (10)

By differentiation of Eq. (7) we have

$$- \frac{dH_{c2}}{dT} = - \frac{\alpha_0}{2e\gamma T_c} \frac{T_c - T_c}{T_c}.$$  \hspace{1cm} (11)

Substituting here the expression (5) we obtain

$$- \frac{dH_{c2}}{dT} = - \frac{\alpha_0}{2e\gamma T_c} \frac{T_c - T_c}{T_c}.$$  \hspace{1cm} (12)

This expression is valid for a type of superconducting state with singlet pairing and for the superconducting states with triplet but not equal spin pairing states in a metal with a form of the Fermi surface. For the clean superconductor one can rewrite this as follows

$$- \frac{dH_{c2}}{dT} = - \frac{\alpha_0}{2e\gamma T_c} \frac{T_c - T_c}{T_c}.$$  \hspace{1cm} (13)

$C$ in the denominator is a constant of the order of unity.

In the limit of small Maki parameters the critical field temperature derivative is determined only by the orbital effect. It is temperature independent and given by the numerator of Eq. (12). While in a superconductor with strong paramagnetic effect that is at large enough Maki parameters the value of $|dH_{c2}/dT|$ rapidly decreases with decreasing temperature, which leads in its turn to the fast decrease of the Ginzburg-Landau parameter (4).

### III. COMPARISON WITH THE PAPER(3)

We have found the temperature dependence of the Ginzburg-Landau parameter basing on the Ehrenfest relation (2). Meanwhile as we already pointed out there was derived an expression for $\kappa$ valid in the limit of strong paramagnetic depairing. To compare these results it is convenient begin with the general formula for the spacial average of superconducting energy density

$$\overline{\mathcal{F}_s} = \mathcal{F}_{n\kappa} + \frac{B^2}{8\pi} - \frac{\left(\mathcal{F}_2(\Delta, A_\alpha)\right)^2}{4 \left(\mathcal{F}_4(\Delta, A_\alpha) - \frac{B^2}{8\pi}\right)},$$  \hspace{1cm} (14)

where $\mathcal{F}_2$ and $\mathcal{F}_4$ collect together quadratic and quartic terms with respect to $\Delta$, respectively. Just below the upper critical line defined by $H_{c2}(T)$, the magnetic field is partially screened by supercurrents and we decompose $\mathbf{h} = \mathbf{H} + \mathbf{h}_1$, such that $H_1 = 0$, and, correspondingly, $\mathbf{A} = A_\alpha + A_1$.

Starting this formula one can derive general expression for $\kappa$ at arbitrary Maki parameter value. However, to escape the cumbersome formulation we consider only the situations with $\alpha_M << 1$ and $\alpha_M >> 1$. In the first case

$$\mathcal{F}_2(\Delta, A_\alpha) = 2e\gamma \left(\mathcal{F}_2(\Delta, A_\alpha)\right)^2,$$  \hspace{1cm} (15)

in the second one

$$\mathcal{F}_2(\Delta, A_\alpha) = \mathcal{F}_2(\Delta, A_\alpha) - \kappa \left(\mathcal{F}_2(\Delta, A_\alpha)\right)^2.$$  \hspace{1cm} (16)

Here,

$$\kappa = \left(\frac{\partial \alpha}{\partial B}\right)_{B=H_{c2}^p} = \frac{2\alpha_0 H_{c2}^p}{T_c^2}$$  \hspace{1cm} (17)

In any case

$$\mathcal{F}_4(\Delta, A_\alpha) = \beta \beta_\alpha \left(\mathcal{F}_4(\Delta, A_\alpha)\right)^2.$$  \hspace{1cm} (18)

Then, taking into account the screening currents term in denominator of Eq. (14) we come to equation

$$\overline{\mathcal{F}_s} = \mathcal{F}_{n\kappa} + \frac{B^2}{8\pi} - \frac{(B - H_{c2}(T))^2}{8\pi |1 + \beta_\alpha (\mathcal{F}_2(\Delta, A_\alpha) - \frac{B^2}{8\pi})|},$$  \hspace{1cm} (19)

valid at any the Maki parameter value. But at $\alpha_M << 1$ one must put here the upper critical field as determined by Eq. (9) and the Ginzburg-Landau parameter is

$$\kappa = \kappa_{GL} = \frac{\sqrt{\beta}}{4\sqrt{\pi e}\gamma}.$$  \hspace{1cm} (20)

Whereas at $\alpha_M >> 1$ and $\frac{T_c - T_c}{T_c} > 1/\alpha_M^2$ one must use the upper critical field as determined by Eq. (10) and the Ginzburg-Landau parameter is

$$\kappa = \sqrt{\frac{\beta}{2\sqrt{\pi \varepsilon}}}.$$  \hspace{1cm} (21)
The latter for a clean superconductor can be rewritten as

\[ \kappa \approx \frac{\kappa_{GL}}{\alpha_M \sqrt{\frac{T_c - T}{T_c}}} \]  \quad (22)

This expression is in obvious correspondence with Eqs. (13) and (4).

IV. CONCLUSION

The derived temperature dependence of the Ginzburg-Landau parameter is consistent with experimental observations in several heavy fermionic superconductors CeCoIn$_5$, URu$_2$Si$_2$, NpPd$_5$Al$_2$. In all these compounds the phase transition to the superconducting state becomes of the first order at low temperature - high field region, that directly demonstrates the dominant role of paramagnetic depairing mechanism.

Similar observations have been done recently in heavy fermionic compound UBe$_{13}$. This case demands further investigation because it seems that this material having extremely high upper critical field and $T^3$ behavior of specific heat at low temperatures belongs to triplet superconductors with point nodes in the quasiparticle spectrum.

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