Building a novel multivariate nonlinear MGM(1,m,N|\gamma) model to forecast carbon emissions

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Abstract
With the proposal of the carbon neutrality target, China’s attention to carbon emissions has been further enhanced. Effective prediction of future carbon emissions is important for the formulation of carbon neutralization target and action plans in the region. Many factors affecting carbon emissions, cause their development trends may be nonlinear. To forecast the carbon emissions of coal and natural gas in the industrial sector more accurately, a new MGM(1,m,N|\gamma) model considering nonlinear characteristics is proposed in this paper. The new model introduces power function \gamma as nonlinear parameter, and the \gamma value is solved by nonlinear constraint function. We further deduce the simulation and prediction formula and then apply the improved model to the carbon emission forecast. The comparisons show that the nonlinear parameters can modify the trend of sequences and improve the prediction accuracy, which verifies the validity of the model. Finally, according to the influencing factors and forecast results, this paper analyzes the causes of high carbon emissions and puts forward reasonable suggestions for China’s carbon governance.

Keywords Grey system theory · Multivariate prediction model · Parameter optimization · Industrial sector

1 Introduction
Global warming, air pollution and other environmental problems have attracted more and more attention, but the essence of the problem is how to reduce carbon emissions. As a populous country, China’s total carbon emissions are increasing. In September 2020, China announced that it would strive to reach the peak of carbon emissions in 2030 and achieve carbon neutrality in 2060. Local governments actively respond to policy calls by analyzing
carbon emission paths (Chen & Lin, 2021; Wang, Huang, et al., 2022), exploring low-carbon energy utilization (Jiang, et al., 2017; Zou, et al., 2021), and studying carbon reduction optimization strategies (Cao, et al., 2021; Ma, et al., 2020). In this context, it is important to effectively predict the future carbon emissions, which will not only become the basis for local governments to formulate carbon emission strategies, but also become the constraints for enterprises to adjust the industrial structure and control carbon emissions in a planned way. More importantly, this is an indispensable part of China’s goal of carbon neutrality. Based on the social background of new development pattern (Zhu, et al., 2020), sharing economy (Yin, et al., 2021), and COVID-19 (Wang, Huang, et al., 2022; Wang, Li, et al., 2022), scholars have widely discussed China’s carbon emissions. The main prediction methods include STIRPAT model (Li & Du, 2019), scenario analysis method (Song, et al., 2021; Wang, et al., 2021) and LEAP model (Chen, et al., 2021; Wang, et al., 2020; Zhao, et al., 2021). These models have high requirements for the amount of data and are generally suitable for use when there is a large amount of data. While the grey model is suitable for small sample time series prediction, so it would have a good application effect in the prediction of carbon emissions with small samples (Chiu, et al., 2020; Han and Li, 2019; Ye, et al., 2021). Such as four univariate grey prediction models are used to fit the carbon emission data model of the top 20 countries with total carbon dioxide emissions since the twenty-first century (Hu, et al., 2021), the rolling discrete grey power model is built to forecast energy-related carbon emissions (Ding, et al., 2020), and grey multivariate Verhulst model predicts China’s carbon dioxide emissions by considering the characteristics of nonlinearity, independence and interaction (Jiang, et al., 2021). We will not list the application of grey model in carbon emission prediction one by one. Combined with the social background of China’s carbon neutralization target, this paper will build a new grey model and apply it to carbon emission prediction.

The GM(1,1) model is a common single-variable model in grey system models, it makes short-term trend prediction for only one variable (Deng, 2002). For grey multi-variable prediction models, such as MGM (1,m) model considers the correlation between m system behavior sequences (Zhai, et al., 1997); GM(1,N) model reflects the effect of N-1 related factor sequences on one system behavior sequence (Deng, 2002); MGM (1,m,N) Model is a combination of MGM(1,m) and GM(1,N), fully considers the correlation between m system behavior sequences and N-1 related factor sequences, it can more systematically describe the mutual restriction relationship between multiple variables (Xiong, et al., 2020).

At present, there are few studies about MGM(1,m,N) model. (Xiong, et al., 2020) proposed MGM(1,m,N) model for the first time and applied it to haze prediction, then popularized and applied it to new core and grey scale series, and effectively predicted PM$_{2.5}$ and PM$_{10}$ (Xiong, et al., 2022).

The existing MGM(1,m,N) model, like most grey models, is characterized by constant parameters of different sequences. Fixed parameters mean that sequences grow or fall at a fixed rate, showing a fixed linear relationship. However, under the influence of seasonal cycles, long-term trends and other factors, variables in complex systems tend to restrict each other (Anderson, 1995), so they do not always maintain a fixed trend. In other words, there may be nonlinear relationships between interacting variables. Therefore, the nonlinear characteristics between sequences deserve further study, which is the breakthrough of model optimization and the focus of this paper.

Nonlinear characteristics are studied more in the grey Bernoulli model, and are usually expressed by power functions. Flexible power exponent can determine data’s nonlinear change pattern (Pei, et al., 2018; Zeng and Li, 2021). Nonlinear model construction methods are different. (Zhou and Fang, 2010) constructed two nonlinear optimization models aiming
at the original value and the cumulative value, respectively. While (Wang & Song, 2019) built a nonlinear model aiming at background value. For parameter estimation of nonlinear models, ordinary least squares estimation is most commonly used. In addition, (Pei, et al., 2018) proposed nonlinear least squares estimation, and (Xie, et al., 2020) solved the convex optimization problem, both of which improved the ordinary least squares method.

Nonlinear characteristics can upgrade the original static model to dynamic model. Such optimization models have been well applied in resources and energy (Ma & Liu, 2018; Wu, et al., 2021; Xia, et al., 2019), disasters and environment (Keum, et al., 2020). Nonlinear parameters can change the trend of the sequence, relax the constraints and make the model more comprehensive. By constructing constraint function (Kong and Ma, 2018; Utkucan S¸ ahin., 2019; Zeng, et al., 2020), and then using optimization algorithms such as Levenberg–Marquardt (Shaikh, et al., 2017), self-memory (Guo, et al., 2019), particle swarm optimization (Li, et al., 2022; Wan, et al., 2021), and so on, we can continuously adjust the values of nonlinear parameters to obtain dynamic fitting accuracy. When the accuracy reaches the best, we can determine the optimal nonlinear parameter values.

On the basis of the above studies, this paper finds that the existing MGM (1, m, N) model ignores the nonlinear relationship between sequences, which cannot fully reflect the possible nonlinear situation in reality. To a certain extent, it will cause the deviation of the forecast result. Therefore, it is necessary to add parameters considering nonlinear characteristics. In this paper, the power exponent γ is introduced as a nonlinear parameter of the system behavior sequence, and a novel multivariate nonlinear MGM (1, m, N|γ) model is proposed to optimize the traditional MGM (1, m, N) model. Section 2 describes the modeling mechanism of the new model and the derivation of relevant formulas. Section 3 applies the new model to the prediction of carbon emissions from coal and natural gas in China’s industrial field to verify the effectiveness of the model, and then analyzes the causes of high emissions in detail according to relevant factors and puts forward some suggestions. Section 4 summarizes the research results of this paper.

The main contributions of this article are as follows:

1. Based on the complex multivariate MGM (1, m, N) model, the new model can consider the mutual interactions of multiple variables in a system, which is of great theoretical significance in multivariate simulation and prediction.
2. By introducing nonlinear parameters, the new model can upgrade the static model to the dynamic model. It is helpful to solve practical problems with nonlinear characteristics.
3. The optimal parameter value is confirmed by the iterative optimization algorithm of the computer, and the satisfactory effect is achieved.

2 Multivariate nonlinear MGM (1, m, N|γ) model

2.1 Modeling mechanism

Nonlinear relationships exist between system behavior sequences, related factor sequences, or system behavior sequences and related factor sequences. Due to the gradual deepening of the research, this paper only considers the nonlinear relationship between system behavior sequences, and introduces power exponent γ as a nonlinear parameter on their background
value $z_1^{(1)}, z_2^{(1)}, \ldots, z_m^{(1)}$. This section focuses on the modeling mechanism of nonlinear multivariate grey model—MGM $(1,m,N;\gamma)$ model.

**Definition 1** Assume that $Y^{(0)} = \left( Y_1^{(0)}, Y_2^{(0)}, \ldots, Y_m^{(0)} \right)^T$ is the system behavior matrix, $X^{(0)} = \left( X_1^{(0)}, X_2^{(0)}, \ldots, X_N^{(0)} \right)^T$ is the related factor matrix, $Y_i^{(0)} = \left( y_i^{(0)}(1), y_i^{(0)}(2), \ldots, y_i^{(0)}(n) \right)$, $i = 1, 2, \ldots, m$, and $X_j^{(0)} = \left( x_j^{(0)}(1), x_j^{(0)}(2), \ldots, x_j^{(0)}(n) \right)$, $j = 2, 3, \ldots, N$ are their original nonnegative data vector, respectively. $Y^{(1)} = \left( Y_1^{(1)}, Y_2^{(1)}, \ldots, Y_N^{(1)} \right)$ and $X^{(1)} = \left( X_1^{(1)}, X_2^{(1)}, \ldots, X_N^{(1)} \right)$ are their first-order accumulated generation matrices (1-AGO), respectively, where $Y_i^{(1)} = \left( y_i^{(1)}(1), y_i^{(1)}(2), \ldots, y_i^{(1)}(n) \right)$ and $X_j^{(1)} = \left( x_j^{(1)}(1), x_j^{(1)}(2), \ldots, x_j^{(1)}(n) \right)$, $j = 2, 3, \ldots, N$.

$$
\begin{align*}
Z_i^{(1)} &= \left( z_i^{(1)}(2), z_i^{(1)}(3), \ldots, z_i^{(1)}(n) \right) \\
&\text{is the background value sequence taken to be the mean generation of consecutive neighbors of } Y_i^{(1)}, \text{where} \\
z_i^{(1)}(k) &= 0.5 \left( y_i^{(1)}(k-1) + y_i^{(1)}(k) \right), i = 1, 2, \ldots, m, k = 2, 3, \ldots, n
\end{align*}
$$

The form of multivariate nonlinear MGM $(1,m,N;\gamma)$ model is defined as follows:

$$
\begin{align*}
y_1^{(0)}(k) &= a_{11}(z_1^{(1)}(k))^{\gamma_1} + \cdots + a_{1m}(z_m^{(1)}(k))^{\gamma_m} + b_{12}x_2^{(1)}(k) + \cdots + b_{1N}x_N^{(1)}(k) \\
y_2^{(0)}(k) &= a_{21}(z_1^{(1)}(k))^{\gamma_1} + \cdots + a_{2m}(z_m^{(1)}(k))^{\gamma_m} + b_{22}x_2^{(1)}(k) + \cdots + b_{2N}x_N^{(1)}(k) \\
&\quad \cdots \\
y_m^{(0)}(k) &= a_{m1}(z_1^{(1)}(k))^{\gamma_1} + \cdots + a_{mm}(z_m^{(1)}(k))^{\gamma_m} + b_{m2}x_2^{(1)}(k) + \cdots + b_{mN}x_N^{(1)}(k)
\end{align*}
$$

In Eq. (3), $\gamma_i$ are power exponents, reflecting the nonlinear action of the $i$-th sequence in the system behavior sequences. Apparently, when $\gamma_i = 1$, the nonlinear MGM $(1,m,N;\gamma)$ model degrades into MGM $(1,m,N)$ model, so it is a further extension of the traditional model.

For the convenience of readers, Eq. (3) can be written in matrix form, which is

$$
Y^{(0)} = AX^{(1)} + BX^{(1)}
$$
2.2 Parameter estimation

**Theorem** The parameter matrix is listed as $\hat{A}_i = (\hat{a}_{i1}, \hat{a}_{i2}, \ldots, \hat{a}_{im}, \hat{b}_{i2}, \ldots, \hat{b}_{iN})^T$ \(i = 1, 2, \ldots, m\) and its least-squares estimation should satisfy:

1) when \(n = m + N, \hat{a}_i = L^{-1}Y_i\);
2) when \(n > m + N, \hat{a}_i = (L^T L)^{-1}L^T Y_i\);
3) when \(n < m + N, \hat{a}_i = L^T (L^T L)^{-1} Y_i\);

where $L = \begin{pmatrix}
\left(z_1(1)\right)^{T_1} & \left(z_2(1)\right)^{T_2} & \ldots & \left(z_m(1)\right)^{T_m} \\
\left(z_1(2)\right)^{T_1} & \left(z_2(2)\right)^{T_2} & \ldots & \left(z_m(2)\right)^{T_m} \\
\left(z_1(3)\right)^{T_1} & \left(z_2(3)\right)^{T_2} & \ldots & \left(z_m(3)\right)^{T_m} \\
\vdots & \vdots & \ddots & \vdots \\
\left(z_1(n)\right)^{T_1} & \left(z_2(n)\right)^{T_2} & \ldots & \left(z_m(n)\right)^{T_m}
\end{pmatrix}$

$Y_i = \begin{pmatrix}
y_i^{(0)}(2) \\
y_i^{(0)}(3) \\
\vdots \\
y_i^{(0)}(n)
\end{pmatrix}$ \(i = 1, 2, \ldots, m\)

Then, the parameter matrices $\hat{A}$ of system behavior sequences and $\hat{B}$ of related factor sequences can be obtained:

$\hat{A} = \begin{pmatrix}
\hat{a}_{11} & \hat{a}_{12} & \cdots & \hat{a}_{1m} \\
\hat{a}_{21} & \hat{a}_{22} & \cdots & \hat{a}_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
\hat{a}_{m1} & \hat{a}_{m2} & \cdots & \hat{a}_{mm}
\end{pmatrix}_{m \times m}$, $\hat{B} = \begin{pmatrix}
\hat{b}_{12} & \hat{b}_{13} & \cdots & \hat{b}_{1N} \\
\hat{b}_{22} & \hat{b}_{23} & \cdots & \hat{b}_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
\hat{b}_{m2} & \hat{b}_{m3} & \cdots & \hat{b}_{mN}
\end{pmatrix}_{m \times N}$

**Proof** Substituting $k = 2, 3, \ldots, n$ into Eq. (3), we obtain:

\[
\begin{align*}
y_i^{(0)}(2) &= a_{i1}(z_i(1)_{a1})^{y_1} + \cdots + a_{im}(z_i(1)_{am})^{y_m} + b_{i2}x_{i2}^{(1)}(2) + \cdots + b_{iN}x_{iN}^{(1)}(2) \\
y_i^{(0)}(3) &= a_{i1}(z_i(1)_{a1})^{y_1} + \cdots + a_{im}(z_i(1)_{am})^{y_m} + b_{i2}x_{i2}^{(1)}(3) + \cdots + b_{iN}x_{iN}^{(1)}(3) \\
&\vdots \\
y_i^{(0)}(n) &= a_{i1}(z_i(1)_{a1})^{y_1} + \cdots + a_{im}(z_i(1)_{am})^{y_m} + b_{i2}x_{i2}^{(1)}(n) + \cdots + b_{iN}x_{iN}^{(1)}(n)
\end{align*}
\]

which can be written in matrix form as:

$Y_i = LH_i$

where $H_i = (a_{i1}, a_{i2}, \ldots, a_{im}, b_{i2}, \ldots, b_{iN})_{m+N-1}^T$, \(i = 1, 2, \ldots, m\)

1) When \(n = m + N, L\) is a reversible matrix, then $\hat{a}_i = H_i = L^{-1}Y_i$. 

\[\text{Springer}\]
2) When $n > m + N,L$ is a column full rank matrix, according to Moore–Penrose theorem, $L$ can be decomposed as $L = LI$, where $I$ is the identity matrix, then the generalized inverse matrix of $L$ is $L^+ = L^T (I^T L^T)^{-1} L^T$, therefore, $\hat{\alpha}_i = H_i L^+ Y_i = (L^T L)^{-1} L^T Y_i$.

3) When $n < m + N,L$ is a row full rank matrix, according to Moore–Penrose theorem, $L$ can be decomposed as $L = IL$, where $I$ is the identity matrix, then the generalized inverse matrix of $L$ is $L^+ = L^T (L L^T)^{-1} L^T$, therefore, $\hat{\alpha}_i = H_i L^+ Y_i = L^T (L L^T)^{-1} Y_i$.

2.3 Simulation and prediction of sequences

Theorem 2 On the basis of the above definition and theorem analysis, we can get the recursive equation of simulation and prediction for the system behavior sequence $Y^{(0)}(k)$, as follows:

$$
\begin{align*}
\hat{Y}^{(0)}(1) &= Y^{(0)}(1), \\ \hat{Y}^{(0)}(k) &= \hat{A} \begin{pmatrix} z_1^{(1)}(k) \\ z_2^{(1)}(k) \\ \vdots \\ z_m^{(1)}(k) \end{pmatrix}^{Y_i} + \hat{B} x^{(1)}(k), k \geq 2
\end{align*}
$$

Proof When $k = 1, \hat{Y}^{(0)}(1) = Y^{(0)}(1)$.

When $k \geq 2$, $x_j^{(1)}(k)$ and $z_i^{(1)}(k)$ have been explained in definition 1, they can be calculated from the actual values of system characteristic sequences (Eq. 2) and related factor sequences (Eq. 1). The estimated values of parameter matrices $\hat{A}$ and $\hat{B}$ can be calculated by theorem 1. The nonlinear parameters $(\gamma_1, \gamma_2, \ldots, \gamma_m)$ can be calculated by nonlinear optimization formula (6). So the predicted value $\hat{Y}^{(0)}(k)$ can be deduced by solving the linear equations, that is:

$$
\begin{align*}
\hat{y}^{(0)}_1(k) &= \hat{a}_{11}(z_1^{(1)}(k))^{Y_1} + \cdots + \hat{a}_{1m}(z_m^{(1)}(k))^{Y_m} + \hat{b}_{12} x^{(1)}_2(k) + \cdots + \hat{b}_{1N} x^{(1)}_N(k) \\
\hat{y}^{(0)}_2(k) &= \hat{a}_{21}(z_1^{(1)}(k))^{Y_1} + \cdots + \hat{a}_{2m}(z_m^{(1)}(k))^{Y_m} + \hat{b}_{22} x^{(1)}_2(k) + \cdots + \hat{b}_{2N} x^{(1)}_N(k) \\
& \quad \vdots \\
\hat{y}^{(0)}_m(k) &= \hat{a}_{m1}(z_1^{(1)}(k))^{Y_1} + \cdots + \hat{a}_{mm}(z_m^{(1)}(k))^{Y_m} + \hat{b}_{m2} x^{(1)}_2(k) + \cdots + \hat{b}_{mN} x^{(1)}_N(k)
\end{align*}
$$

It can be simplified to matrix form.

$$
\hat{Y}^{(0)}(k) = \hat{A} \begin{pmatrix} \left(z_1^{(1)}(k)\right)^{Y_1} \\ \left(z_2^{(1)}(k)\right)^{Y_2} \\ \vdots \\ \left(z_m^{(1)}(k)\right)^{Y_m} \end{pmatrix} + \hat{B} x^{(1)}(k)
$$
2.4 Nonlinear parameter optimization

In the modeling process, the parameters $\gamma_i$ that reflect the nonlinear relationship between the system behavior sequences are unknown, but in order to estimate the parameter matrices, we must determine their values in advance. Therefore, this paper establishes the following nonlinear constraint function, which aims at minimizing the prediction error, so as to obtain the optimal parameter values.

$$\arg\min_{\gamma} \frac{1}{mn} \sum_{i=1}^{m} \sum_{k=1}^{n} \left| \frac{\hat{y}_i^{(0)}(k) - y_i^{(0)}(k)}{y_i^{(0)}(k)} \right| \times 100\%$$

$$L = \left\{ \begin{array}{l}
\begin{pmatrix}
(z_1^{(1)}(2))^{y_1} \\
(z_2^{(1)}(2))^{y_2} \\
\vdots \\
(z_m^{(1)}(2))^{y_m}
\end{pmatrix} + \begin{pmatrix}
(z_1^{(1)}(3))^{y_1} \\
(z_2^{(1)}(3))^{y_2} \\
\vdots \\
(z_m^{(1)}(3))^{y_m}
\end{pmatrix} + \begin{pmatrix}
(1)\ldots(2) \\
(1)\ldots(3) \\
\vdots \\
(1)\ldots(N)
\end{pmatrix} \\
\vdots \\
\begin{pmatrix}
(z_1^{(n)}(1))^{y_1} \\
(z_2^{(n)}(1))^{y_2} \\
\vdots \\
(z_m^{(n)}(1))^{y_m}
\end{pmatrix} + \begin{pmatrix}
(z_1^{(n)}(2))^{y_1} \\
(z_2^{(n)}(2))^{y_2} \\
\vdots \\
(z_m^{(n)}(2))^{y_m}
\end{pmatrix} + \begin{pmatrix}
(1)\ldots(2) \\
(1)\ldots(3) \\
\vdots \\
(1)\ldots(N)
\end{pmatrix}
\end{array} \right\}$$

s.t. 

$$Y_i = \begin{pmatrix}
y_i^{(0)}(2) \\
y_i^{(0)}(3) \\
\vdots \\
y_i^{(0)}(n)
\end{pmatrix}$$

$$\hat{a}_i = L^T (L^T L)^{-1} Y_i$$

$$\hat{y}_i^{(0)}(k) = \hat{a}_{i1} (z_1^{(1)}(k)) + \cdots + \hat{a}_{im} (z_m^{(1)}(k)) + \hat{b}_{21} x_2^{(1)}(k) + \cdots + \hat{b}_{mN} x_n^{(1)}(k)$$

$$\hat{a}_i = (\hat{a}_{i1}, \hat{a}_{i2}, \ldots, \hat{a}_{im}, \hat{b}_{21}, \ldots, \hat{b}_{mN})^T$$

$$y'_1 y'_2 \cdots y'_m > 0$$

Equation (6) can be calculated by the optimization algorithm. Obviously, the nonlinear parameter values affect the background values of the system behavior sequences and determine the prediction error. The lowest error means the highest prediction accuracy. Only after the optimal values of $\gamma_i$ are determined can we use theorem 1 to estimate the parameter matrices A and B, and then use theorem 2 (Eq. 5) to simulate and predict sequences.

2.5 Test of the model accuracy

To ensure the high accuracy of the model, we need to test the error between the simulated and actual values point by point. The formulas of absolute percentage error (APE) and mean absolute percentage error (MAPE) are listed below:

$$\text{APE} = \left\| \frac{y_i^{(0)}(k) - \hat{y}_i^{(0)}(k)}{y_i^{(0)}(k)} \right\|$$

$$\text{MAPE} = \frac{1}{n} \sum_{k=1}^{n} \left\| \frac{y_i^{(0)}(k) - \hat{y}_i^{(0)}(k)}{\hat{y}_i^{(0)}(k)} \right\|$$

(7)
2.6 Modeling steps

The detailed modeling steps are described as follows:

Step 1: Select appropriate samples to determine which are system behavior sequences and which are related factor sequences, so as to determine the values of $m$ and $N$. Then use Eq. (3) to build MGM $(1, m, N | \gamma)$ model;

Step 2: Use Eq. (1) and Eq. (2) to calculate 1-AGO sequences $X^{(1)}, Y^{(1)}$ and the background value $Z^{(1)}$;

Step 3: Use Eq. (6) to calculate nonlinear parameters $\gamma_i$;

Step 4: Determine parameter matrices $\hat{A}$ and $\hat{B}$ according to theorem 1.

Step 5: Invoke theorem 2 (Eq. 5) to get the simulation and prediction values of system behavior sequences.

Step 6: Calculate APE and MAPE by Eq. (7).

The framework diagram is shown in Fig. 1

3 Empirical analysis

As an industrialized country, China’s use of coal and natural gas has penetrated into all fields and we cannot change the energy structure in the short term, resulting in severe carbon emissions, which put enormous pressure on environmental governance and carbon neutrality. This section takes the carbon emission of coal and natural gas in China’s industrial field and its influencing factors as the main research object. The following Fig. 2 shows the logic of case analysis.

3.1 Variable selection

In this context, we collects the data of carbon emissions generated by the use of coal and natural gas in China’s industrial sector during 2010–2018. There are many factors affecting carbon emissions. We use "Grey System Modeling Software" to calculate the grey comprehensive correlation between carbon emissions and each factor. Generally, when all the correlation degrees are greater than 0.5, we can preliminarily judge that this group of sample data meets the modeling conditions. Subsequently, we selected the following six factors that affect carbon emissions: GDP (hundred million yuan), Added value of the secondary industry (hundred million yuan), Total investment in environmental pollution control...
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Due to the lack of investment in environmental pollution control in 2018, we use discrete GM (1,1) model to forecast and fill in the data of this year.

The data are from the National Bureau of Statistics and the China Energy Statistics Yearbook, as shown in Table 2.

3.2 Simulation experiments

In order to validate the application effect of multivariable nonlinear MGM(1, m, N|γ) model, simulation experiments and analysis are done. The detailed steps of experiments are described as follows:
Step 1: According to the analysis in Sect. 3.1, we set the carbon emissions of coal and natural gas as the system behavior sequences, and the other six sequences as the related factor sequences, so \( m=2 \), \( N=7 \). Then we use the data from 2010 to 2016 as simulations, and 2017 to 2018 as predictions. At this time, MGM(1,2,7|\( \gamma_1 \), \( \gamma_2 \)) model can be formed according to Eq. (3) as follows:

\[
\begin{align*}
\gamma_1^{(0)}(k) &= a_{11}(z_1^{(1)}(k))^{\gamma_1} + a_{12}(z_2^{(1)}(k))^{\gamma_1} + b_{12}x_2^{(1)}(k) + \cdots + b_{17}x_7^{(1)}(k) \\
\gamma_2^{(0)}(k) &= a_{21}(z_1^{(1)}(k))^{\gamma_2} + a_{22}(z_2^{(1)}(k))^{\gamma_2} + b_{22}x_2^{(1)}(k) + \cdots + b_{27}x_7^{(1)}(k)
\end{align*}
\]  

(8)

Step 2: We calculate the 1-AGO of each sequence \((X^{(1)}, Y^{(1)})\) and the background value \((Z^{(1)})\) according to Eq. (1) and Eq. (2).

Step 3: According to Eq. (6), we get that when \( \gamma_1 = 1.008, \gamma_2 = 1.01 \), the error is the smallest, so we determine that they are the optimal nonlinear parameters of two system behavior sequences respectively.

Step 4: According to Theorem 1, the parameter matrices of MGM(1,2,7|\( \gamma_1=1.008, \gamma_2=1.01 \)) model can be obtained by using least square estimation:

\[
\hat{A} = \begin{bmatrix}
-0.431 & -37.026 \\
0.051 & -4.980
\end{bmatrix}, \hat{B} = \begin{bmatrix}
-3.232 & -5.689 & 16.585 & 11.090 & 2.728 & -12.965 \\
-0.523 & -0.890 & 2.213 & 0.773 & 0.470 & -1.672
\end{bmatrix}.
\]

Step 5: According to theorem 2(Eq. 5), we substitute \( \gamma_1 = 1.008, \gamma_2 = 1.01, \hat{A} \) and \( \hat{B} \) into Eq. (8), so that the nonlinear MGM (1,2, 7|\( \gamma_1=1.008, \gamma_2=1.01 \)) model can be determined as follows:
Table 2  The detailed value of each sequence

| System behavior sequences | Related factor sequences |
|---------------------------|---------------------------|
| Carbon emissions of coal (ten thousand tons) | Carbon emissions of natural gas (ten thousand tons) | GDP (hundred million yuan) | Added value of the secondary industry (hundred million yuan) | Total investment in environmental pollution control (hundred million yuan) | Total population at year-end (ten thousand) | Household consumption level (yuan) | Urban area (km²) |
| Y₁ | Y₂ | X₂ | X₃ | X₄ | X₅ | X₆ | X₇ |
|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|
| 2010 | 582,616.19 | 13,131.07 | 412,119.3 | 191,626.5 | 7612.19 | 134,091 | 10,575 | 178,691.73 |
| 2011 | 652,457.35 | 16,673.88 | 487,940.2 | 227,035.1 | 7114.03 | 134,916 | 12,668 | 183,618.02 |
| 2012 | 692,238.95 | 18,682.08 | 538,580.0 | 244,639.1 | 8253.46 | 135,922 | 14,074 | 183,039.42 |
| 2013 | 695,002.98 | 22,167.51 | 592,963.2 | 261,951.6 | 9037.20 | 136,726 | 15,586 | 183,416.05 |
| 2014 | 670,190.71 | 24,018.11 | 643,563.1 | 277,282.8 | 9575.50 | 137,646 | 17,220 | 184,098.59 |
| 2015 | 639,730.22 | 24,844.31 | 688,858.2 | 281,338.9 | 8806.30 | 138,326 | 18,857 | 191,775.54 |
| 2016 | 618,575.90 | 27,214.80 | 746,395.1 | 295,427.8 | 9219.80 | 139,232 | 20,801 | 198,178.59 |
| 2017 | 624,132.34 | 32,604.28 | 832,035.9 | 331,580.5 | 9538.95 | 140,011 | 22,969 | 198,357.17 |
| 2018 | 642,840.29 | 40,698.29 | 919,281.1 | 364,835.2 | 10,083.90 | 140,541 | 25,245 | 200,896.50 |
Table 3  Actual and simulated carbon emissions of coal (ten thousand tons)—Y₁

| Year | Actual values | MGM(1,2,7|γ|) | MGM(1,2,7) | MGM(1,2|γ|) | MGM(1,2) | GM(1,7|γ|) | GM(1,7) | MGM(1,2|λ|) | GM(1,1) |
|------|---------------|----------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
|      |               | γ₁ = 1.008     | γ₁ = 1.001  | γ₁ = 1.021  | λ = 0.102   |             |             |             |             |             |
| 2010 | 582,616.19    | 582,616.19     | 582,616.19  | 582,616.19  | 582,616.19  | 582,616.19  | 582,616.19  | 582,616.19  | 686,101.07  |
| 2011 | 652,457.35    | 668,979.59     | 596,602.52  | 661,848.42  | 662,878.46  | 767,788.32  | 629,126.04  | 650,343.54  | 676,010.91  |
| 2012 | 692,238.95    | 920,272.78     | 814,713.58  | 680,621.16  | 682,082.72  | 1,105,550.06| 859,280.55  | 686,046.78  | 666,069.14  |
| 2013 | 695,002.98    | 882,048.80     | 762,560.91  | 686,285.42  | 687,786.77  | 1,175,130.13| 842,439.60  | 687,612.11  | 656,273.58  |
| 2014 | 670,190.71    | 805,001.40     | 691,606.65  | 677,594.30  | 678,675.71  | 1,124,317.77| 754,421.78  | 671,061.61  | 646,622.08  |
| 2015 | 639,730.22    | 741,075.56     | 639,046.79  | 653,504.43  | 653,648.88  | 1,026,396.17| 678,857.98  | 645,088.82  | 637,112.52  |
| 2016 | 618,575.90    | 689,102.31     | 597,724.03  | 613,215.88  | 611,863.21  | 965,002.84  | 637,223.23  | 614,406.33  | 680,357.67  |
| 2017 | 624,132.34    | 630,285.28     | 549,136.37  | 556,210.48  | 552,774.69  | 627,977.52  | 413,166.16  | 581,564.37  | 627,742.81  |
| 2018 | 642,840.29    | 574,112.41     | 501,487.24  | 482,287.07  | 476,176.76  | 380,294.95  | 254,700.13  | 547,938.72  | 618,510.90  |
Table 4  Actual and simulated carbon emissions of natural gas (ten thousand tons)—Y_2

| Year | Actual values | MGM(1,2,7|γ)| MGM(1,2,7) | MGM(1,2|γ)| MGM(1,2)| GM(1,7|γ)| GM(1,7)| MGM(1,2|λ)| GM(1,1) |
|------|---------------|----------------|-------------|------------|-----------|---------|---------|-----------|---------|---------|
|      |               | γ2 = 1.01      | γ2 = 1.001  | γ2 = 1.001 | λ = 0.102 |
| 2010 | 13,131.07     | 13,131.07      | 13,131.07   | 13,131.07  | 13,131.07 | 13,131.07| 13,131.07| 13,131.07  | 13,131.07| 13,131.07|
| 2011 | 16,673.88     | 15,352.45      | 13,131.07   | 16,532.95  | 16,594.67 | 14,277.07| 13,600.50| 16,384.84  | 17,477.85|        |
| 2012 | 18,682.08     | 22,213.29      | 19,893.40   | 19,107.25  | 19,215.20 | 20,034.51| 19,309.27| 19,277.74  | 19,160.25|        |
| 2013 | 22,167.51     | 24,951.10      | 22,042.65   | 21,489.16  | 21,638.38 | 22,682.08| 21,889.16| 21,606.27  | 21,004.59|        |
| 2014 | 24,018.11     | 26,936.60      | 23,661.85   | 23,610.68  | 23,792.88 | 24,689.48| 23,809.89| 23,610.00  | 23,026.46|        |
| 2015 | 24,844.31     | 27,896.27      | 24,477.20   | 25,403.57  | 25,607.09 | 25,683.64| 24,745.30| 25,419.50  | 25,242.96|        |
| 2016 | 27,214.80     | 29,799.05      | 26,167.86   | 26,800.89  | 27,010.67 | 27,957.65| 26,925.71| 27,106.11  | 27,672.81|        |
| 2017 | 32,604.28     | 32,965.44      | 28,751.40   | 27,738.49  | 27,936.21 | 32,857.02| 31,575.28| 28,709.04  | 30,336.56|        |
| 2018 | 40,698.29     | 36,230.30      | 31,471.97   | 28,156.65  | 28,321.06 | 38,092.29| 36,573.05| 30,249.96  | 33,256.72|        |
We then substitute $X^{(1)}$ and $Z^{(1)}$ into this equation, the corresponding simulation and prediction values can be obtained, the results are shown in Table 3 and Table 4.

Step 6: Finally, we calculate APEs between the simulated and the actual values, as well as MAPEs (Eq. 7). The results are shown in Tables 5, 6, 7.

### 3.3 Comparative analysis

In the traditional MGM(1,2,7) model without considering nonlinear characteristics, the parameter matrix $A'$ of system behavior sequences and $B'$ of related factor sequences are as follows:

$$
A' = \begin{pmatrix}
-0.513 & -39.343 \\
0.056 & -5.334
\end{pmatrix},
B' = \begin{pmatrix}
-3.185 & -4.727 & 19.693 & 10.360 & 2.537 & -15.124 \\
-0.550 & -0.808 & 2.655 & 0.724 & 0.462 & -1.989
\end{pmatrix}
$$

Affected by nonlinear parameters, the parameter matrices $A$ and $B$ of the two models are different, which leads to different simulation and prediction results.

Table 3 and Table 4 show the simulation results of two system behavior sequences under different models:

Where, MGM(1,2|γ) and GM(1,7|γ) are nonlinear models of MGM(1,2) and GM(1,7) respectively. MGM(1,2|λ) changes the value of $λ$ in traditional MGM(1,2). The parameters $γ$ and $λ$ are the optimized with the aim of minimizing error.

The comparison model we selected is relatively scientific, and each model has a certain progressive relationship. We show the gradual optimization of the model in Fig. 3:

It is worth affirming that the optimized model is only an extension and supplement of the original model and cannot be completely replaced. Only by properly selecting models and variables according to the actual situation, can the conclusions have more reference value. We can see that the optimal parameter value is basically close to 1. The main reason is that the original model itself is an optimized model and its effect is also good, while the new model has been optimized again. It is another round of optimization process that leads to a small gap in parameters.

Figure 4 and Fig. 5 show the curves of the modeling and original data of the eight models.

It can be seen from Fig. 4 and 5 that the fitting effect of system behavior sequence $Y_1$ and $Y_2$ is quite different. Particularly in the simulation region (data from 2010 to 2016), the simulation effect of $Y_1$ is worse than that of $Y_2$. Theoretically, this is due to the limitations of the grey model, which is mainly related to the development trend of the original sequence. The grey model is usually applicable to the sequence with regular overall trend. $Y_1$ shows a trend of rising first, then falling and then rising again, it’s development trend is extremely unstable. While $Y_2$ has a slow growth in the early stage and a fast growth in the late stage, but it is in an overall upward trend and relatively stable. Therefore, $Y_2$ is easier to capture new trends, while $Y_1$ has obvious errors at the fluctuation points such as those in 2012 and 2013.

From Fig. 4, we can see that the fitting degree of MGM(1,m) model in the simulation area is still very high (⊙⊙), but that of GM(1,N) model is low (⊙⊙⊙). The main reason...
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Table 5  APEs(%) of carbon emissions of coal—Y₁

|        | MGM(1,2,7|γ) | MGM(1,2,7) | MGM(1,2|γ) | MGM(1,2) | GM(1,7|γ) | GM(1,7) | MGM(1,2|λ) | GM(1) |
|--------|-------|-----------|----------|----------|---------|---------|---------|----------|-------|
|        | γ₁ = 1.008 | γ₁ = 1.001 | γ₁ = 1.021 |         |         |         |         |
|        |        |           |           |           |         |         |         |         |
| simulations | 2010 | 0.00     | 0.00     | 0.00     | 0.00   | 0.00   | 0.00   | 0.00   | 0.00 |
|         | 2011  | 2.53     | 8.56     | 1.44     | 1.60   | 17.68  | 3.58   | 0.32   | 5.16 |
|         | 2012  | 32.94    | 17.69    | 1.68     | 1.47   | 59.71  | 24.13  | 0.89   | 2.34 |
|         | 2013  | 26.91    | 9.72     | 1.25     | 1.04   | 69.08  | 21.21  | 1.06   | 4.16 |
|         | 2014  | 20.12    | 3.20     | 1.10     | 1.27   | 65.99  | 12.57  | 0.13   | 2.08 |
|         | 2015  | 15.84    | 0.11     | 2.15     | 2.18   | 60.44  | 6.12   | 0.84   | 1.08 |
|         | 2016  | 11.40    | 3.37     | 0.87     | 1.09   | 56.00  | 3.01   | 0.67   | 3.00 |
| predictions | 2017 | 0.99     | 12.02    | 10.88    | 11.43  | 0.62   | 33.80  | 6.82   | 0.58 |
|         | 2018  | 10.69    | 21.99    | 24.98    | 25.93  | 40.84  | 60.38  | 14.76  | 3.78 |
is that when calculating the whitening time response function of GM(1, N), \( \sum b x^{(1)}(k) \) is regarded as a grey constant, which means that the sequence will increase or decrease with a fixed constant, so the model is suitable for the sequence with regular changes, such as \( Y_2 \). In case of fluctuation, it cannot be used for prediction, otherwise there will be large error, such as \( Y_1 \), because the fluctuating series does not increase or decrease with a fixed constant. Similarly, we know that MGM(1, m, N) model is the combination of MGM(1, m) and GM(1, N) model, although the new model optimizes GM(1, N) to a certain extent, it is more suitable for regular sequence, which has not been completely changed.
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Although the MGM model improves the prediction accuracy, it ignores the simulation accuracy, indicating that the degree of optimization is not enough. We deeply feel that we still need to optimize this problem, and we will continue to improve the model in the future.

To verify the validity of the new model, it is necessary to compare the accuracy of different models. The simulation and prediction accuracy can be defined as 1-APE. Figure 6 here gives the accuracy changes of these years in the form of a circle. From the center to the outside edge of this circle, the accuracy ranges from 50 to 100%. The closer to the center, the lower the accuracy. Different colors in Fig. 6 are arranged for eight models.

The APEs detail is listed in Table 5 and Table 6.

Table 7 compares the simulation and prediction MAPEs of two system behavior sequences under eight models, where we can clearly see the accuracy of each model.

Fig. 5 Comparison curves of carbon emissions of natural gas—Y2

Fig. 6 Simulation and prediction accuracy of carbon emissions
Table 6  APEs(%) of carbon emissions of natural gas—Y2

|          | MGM(1,2,7γ) | MGM(1,2γ) | MGM(1,2) | GM(1,7γ) | GM(1,7) | MGM(1,2λ) | GM(1,1) |
|----------|-------------|-----------|----------|----------|---------|-----------|---------|
|          | γ2 = 1.01   | γ2 = 1.001| γ2 = 1.001| λ = 0.102|         |           |         |
| Simulations |             |           |          |          |         |           |         |
| 2010     | 0.00        | 0.00      | 0.00     | 0.00     | 0.00    | 0.00      | 0.00    |
| 2011     | 7.93        | 18.95     | 0.85     | 0.48     | 14.37   | 18.43     | 1.73    | 4.82   |
| 2012     | 18.90       | 6.48      | 2.28     | 2.85     | 7.24    | 3.36      | 3.19    | 2.56   |
| 2013     | 12.56       | 0.56      | 3.06     | 2.39     | 2.32    | 1.26      | 2.53    | 5.25   |
| 2014     | 12.15       | 1.48      | 1.70     | 0.94     | 2.80    | 0.87      | 1.70    | 4.13   |
| 2015     | 12.28       | 1.48      | 2.25     | 3.07     | 3.38    | 0.40      | 2.32    | 1.60   |
| 2016     | 9.50        | 3.85      | 1.52     | 0.75     | 2.73    | 1.06      | 0.40    | 1.68   |
| Predictions |            |           |          |          |         |           |         |
| 2017     | 1.11        | 11.82     | 14.92    | 14.32    | 0.78    | 3.16      | 11.95   | 6.96   |
| 2018     | 10.98       | 22.67     | 30.82    | 30.41    | 6.40    | 10.14     | 25.67   | 18.28  |
Table 7 MAPEs(%) of two system behavior sequences

|                  | MGM(1,2,7|γ) | MGM(1,2,7) | MGM(1,2|γ) | MGM(1,2) | GM(1,7|γ) | GM(1,7) | MGM(1,2|λ) | GM(1,1) |
|------------------|-------|-----------|----------|--------|---------|--------|--------|--------|--------|
| **Simulations**  |       |           |          |        |         |        |        |        |        |
| coal-Y₁          | 15.68 | 6.09      | 1.21     | 1.23   | 46.99   | 10.09  | 0.56   | 2.55   |        |
| natural gas-Y₂   | 10.47 | 4.69      | 1.66     | 1.50   | 4.69    | 3.62   | 1.70   | 2.86   |        |
| average          | 13.08 | 5.39      | 1.44     | 1.37   | 25.84   | 6.86   | 1.13   | 2.71   |        |
| **Predictions**  |       |           |          |        |         |        |        |        |        |
| coal-Y₁          | 5.84  | 17.00     | 17.93    | 18.68  | 20.73   | 47.09  | 10.79  | 2.18   |        |
| natural gas-Y₂   | 6.04  | 17.24     | 22.87    | 22.36  | 3.59    | 6.65   | 18.81  | 12.62  |        |
| average          | 5.94  | 17.12     | 20.40    | 20.52  | 12.16   | 26.87  | 14.80  | 7.40   |        |
Table 7 and Fig. 7 show that in the prediction region, in terms of the mean absolute percentage error accuracy index of two system behavior sequences, the MAPE obtained by the new model is the lowest (5.94%), while the other models’ MAPEs are higher (17.12%, 20.40%, 20.52%, 12.16%, 26.87%, 14.80%, 7.40%). Therefore, the prediction effect of the new model is the best, but its simulation effect is not the best. It can be clearly seen from Fig. 7 that most of the comparison models have high simulation accuracy but poor prediction effect, which is divorced from our application requirements. Therefore, this paper aims to reduce the prediction error, so that the MAPE of the new model in the prediction area is much lower than that in the simulation area, while other models have the opposite situation. In fact, this is because the nonlinear MGM $(1,m,N|\gamma)$ model considers the nonlinear development characteristics between the two system behavior sequences, which can better reflect and utilize the information hidden in the latest data to forecast future trends. In other words, the nonlinear parameters can modify the development trend of the simulation sequence according to the trend of the prediction sequence. In the correction process, the errors of some simulated data are increased. Thereby achieving the purpose of reducing the prediction error and making the effect of the prediction region better. This nonlinear characteristic can also be seen in two other nonlinear comparison models (MGM$(1,2|\gamma)$ and GM$(1,7|\gamma)$).

### 3.4 Further discussion

Grey system model emphasizes the reference value of new information, but it is only suitable for short-term forecast. In this section, the nonlinear parameters $(\gamma_1 = 1.008, \gamma_2 = 1.01)$ have been obtained above. According to the characteristics of the model, we use the data of known related factors in 2019 and 2020 to predict the carbon emissions of coal and natural gas in the industrial field in 2019 and 2020. Fig. 8 shows the actual data and forecasted values of the two system behavior sequences, as well as the actual data of the six related factors studied in this paper.

The forecast results show that carbon emissions of coal and natural gas are on the rise in the short term. Affected by the existing data and the selection of relevant factors, we can only draw such a conclusion. However, it is worth noting that due to the influence of Covid-19, the shutdown of various industries is obvious, which will inevitably lead to the decline of industrial carbon emissions in 2020. We believe that there will be a big gap between the forecast results and the actual situation, but this does not affect the validity of
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the model, because Covid-19 belongs to a sudden event. We will define "carbon emission data in 2020" as a "abnormal point". The existence of abnormal points will cause large errors, but will not affect the long-term applicability of the model.

From the perspective of relevant factors, we analyze the causes of carbon emission as follows:

(1) GDP represents the level of China’s economic development, while vigorously developing the economy will damage the environment to a certain extent.
(2) At present, China’s industrial structure is still dominated by the secondary industry, which is increasing year by year. The industrial sector cannot leave the use of coal and natural gas in the short term, which will inevitably lead to an increase in carbon emissions.
(3) The total investment in environmental pollution control should have a positive effect on carbon emissions, but due to the lag in governance, it is often treated after pollution, which cannot inhibit carbon emissions in advance. The essence of increasing investment is that environmental pollution is becoming more and more serious, and we have to invest more funds for governance.
(4) China, as a populous country, will generate more demand in daily life, transportation and other aspects. Individual carbon emission behavior is increasingly becoming an issue that cannot be ignored.
(5) To meet people’s needs, industry, transportation, electricity and other aspects are bound to consume a lot of energy. Both direct and indirect energy consumption from population growth will also lead to an increase in carbon emissions.
(6) In the process of rapid urbanization, China needs to build a large number of urban infrastructure. Sustained urban construction and population gathering brings more economic activity, affecting energy consumption and carbon emissions.
In recent years, China’s overall economic focus is shifting from heavy and energy-intensive industries, economic growth is slowing down, and emphasis is placed on coal to gas and promotion of renewable energy, so the growth rate of carbon dioxide emissions is slowing down. With the implementation of carbon neutrality target and related policies, the energy structure will continue to adjust. It is expected that the carbon emissions of various industries will tend to be stable or even decline in the future, which means that the trend of the sequences will be changing. Therefore, the hidden information of new data should be emphasized when making predictions.

Nonlinear MGM(1,m,N|γ) model is a dynamic prediction model. Nonlinear parameters help to capture the latest trends of the sequence more accurately and modify the fitting degree in the simulation and prediction regions. Dynamic updating nonlinear parameters, which is the process of using new data information as much as possible, so as to improve prediction accuracy. This is the main reason why the forecast results of the new model are better, and it also makes the forecast process more flexible and adaptable.

In the context of China’s carbon neutrality target, reducing carbon emissions is receiving constant attention. It is necessary to refine climate policies, expand the scope of emission reductions and raise the corresponding targets for major economic sectors. Based on the above-mentioned forecast of carbon emission trend of energy consumption, the following reasonable suggestions are put forward for China’s carbon governance:

1. Accelerating the adjustment of industrial structure, vigorously developing high-tech and tertiary industries, reducing energy consumption per unit of GDP and achieving sustainable economic development are still the direction of efforts to achieve a low-carbon society.
2. We should promote pollution prevention and control in key industrial parks and enterprises, and encourage them to take the lead in achieving the peak of carbon emissions. Carrying out carbon emission intensity benchmarking activities in the industrial sectors and establishing their carbon emission peak targets should be taken seriously.
3. We can focus on the general requirements of "reducing pollution and carbon emissions", on the one hand, improve policies to increase support for green and low-carbon industries, on the other hand, introduce talents and technologies to encourage innovative actions for reducing emissions.
4. To guide the residents to live a greener life by strengthening the publicity and education. To force people to raise awareness of reducing carbon emissions by improving carbon emission laws and regulations.
5. Strengthening energy consumption control and actively carrying out public energy monitoring for low-carbon operation management is the direction of our future efforts. Adjusting policies based on the actual situation should also be done.
6. It is feasible to plan the urban area reasonably, control the urbanization properly, and combine urban infrastructure construction with environmental protection. The experience of building low-carbon communities in suitable areas can be replicated.

In the context of carbon neutrality, China is paying more attention to regional cooperation and building a nationwide carbon trading market. We believe that through the implementation of various measures, China will achieve carbon peak and carbon neutrality in advance, and make China’s contribution to the world’s environmental problems.
4 Conclusion

Based on the existing grey prediction model MGM (1, m, N), this paper introduces the power function γ as nonlinear parameter, and innovatively proposes nonlinear MGM (1, m, N|γ) model. The new model considers the nonlinear relationship between system behavior sequences, and is a successful optimization of the traditional grey model. The main conclusions of this paper are as follows:

(1) The forecast results of eight models in carbon emissions show that the new model has a higher prediction accuracy under the comparison of mean absolute percentage error index, which proves that its application effect is better than other comparative models.

(2) As dynamic nonlinear parameters, γ _i_ effectively capture the hidden latest information, make better use of the development trend of new data, so the accuracy of model prediction is greatly improved.

(3) The new model has a good effect on the prediction of carbon emissions of coal and natural gas in China’s industrial field, which can be extended to other countries with time series characteristics or other fields.

(4) Based on the current situations and development trend of carbon emissions from coal and natural gas studied in this paper, the causes of high carbon emissions are analyzed according to the related factor sequences, and several measures are proposed to promote the development of China’s carbon governance.

Multivariate nonlinear MGM (1, m, N|γ) model has great application and research value, but it still has shortcomings. First, it only considers the nonlinear relationship between system behavior sequences, and does not consider the situation between related factor sequences, or system behavior sequences and related factor sequences. Second, the optimization degree of simulation accuracy is not enough. Due to the gradual deepening and complexity of the research, we will further improve the research in the future to better apply it to the research and decision-making of carbon emissions.

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