Reply to Comment (cond-mat/0311174)

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We demonstrate that the experimental data on the temperature dependence of the resistivity $\rho(T)$ and conductivity $\sigma(T)$ for high-mobility Si MOSFETs, obtained for a wide range of densities ($\sim 1 : 7$) at intermediate temperatures, agree quantitatively with theory of interaction corrections in the ballistic regime, $T\tau > 1$. Our comparison does not involve any fitting parameters.

Recently [1], we compared the resistivity $\rho(T)$ data for high mobility Si MOSFETs with the interaction corrections calculated to the first order in $T$ [2], and found a quantitative agreement over a wide range of densities at intermediate temperatures (in the ballistic regime $T\tau > \hbar/k_B$) [1,2]. In the comparison, we used independently determined values of the renormalized $g$-factor and effective mass [3].

In the Comment by Shashkin, Dolgopolov, and Kravchenko (hereafter SDK) [4], several claims have been made; we respond to the claims below:

1. SDK claim that our paper [1] contradicts the earlier publication [5]. In fact, Ref. [1] focuses on electron transport in the ballistic regime ($T\tau > \hbar/k_B$), whereas Ref. [5] considers the single-particle interference in the diffusive regime ($\tau_e \gg \tau$); it has been shown that the weak localization and interaction corrections in the diffusive regime are irrelevant to the “metallic” $\rho(T)$. This conclusion does not contradict our analysis in Ref. [1].

2. SDK are concerned about our comparison with the theory of interaction corrections by Zala, Narozhny, and Aleiner (hereafter ZNA) [2] in terms of $\Delta\rho(T)$, and claim that “there is no linear-in $T$ interval” in our $\sigma(T)$ data.

Wherever the $T$-dependent corrections to the conductivity are small ($\Delta\sigma(T)/\sigma \ll 1$), one can treat $\Delta\rho(T)$ and $\Delta\sigma(T)$ on equal footing. At sufficiently large densities ($n > 3.5\times10^{11}$ cm$^{-2}$), this condition is fulfilled over a wide interval of temperatures ($\hbar/2\pi\tau < T_{\text{low}}$, $T_{\text{high}} \ll T_F$, $T_{\text{high}} - T_{\text{low}} \sim$ a few K); this enables to detect unambiguously the linear-in-$T$ regime of $\sigma(T)$. In Ref. [1] we analyzed resistivities, whereas here we compare the data for both samples from Ref. [1] with the theory in terms of $\sigma(T)$. Figures 1, 2 show that the $\sigma(T)$ data are in a quantitative agreement with the interaction corrections to the conductivity $\sigma = \sigma_D + \Delta\sigma(T)$ (solid lines). In the calculations, we have used the values for the $T$-dependent renormalized effective mass $m^*/m_b$ and $g$-factor $g^*/g_b = (\chi^*/\chi^0)(m_b/m^*)$, determined in independent experiments [3]; the value of $\tau$ for each curve is obtained from the experimental $\sigma_D \equiv (\rho(T \to 0))^{-1}$ data via the extrapolation procedure described in Refs. [1,2].

Once the $\tau$ value (and, hence, the Drude conductivity $\sigma_D$) is fixed, the slope $d\sigma/dT$ is obtained with no free parameters.

Recently [1], we compared the resistivity $\rho(T)$ data for high mobility Si MOSFETs with the interaction corrections calculated to the first order in $T$ [2], and found a quantitative agreement over a wide range of densities ($\sim 1 : 7$) at intermediate temperatures, agree quantitatively with theory of interaction corrections in the ballistic regime, $T\tau > 1$. Our comparison does not involve any fitting parameters.

In the comparison, in accord with Ref. [6], we considered only the temperature range $\hbar/2\pi\tau < T \ll T_F$, and ignored the lower-$T$ and higher-$T$ data. As was discussed in Ref. [1], the temperature range where the agreement holds, shrinks towards low densities. This might be expected because, as the density is decreased, the $\rho(T)$ dependence becomes so steep that the temperature changes of $\Delta\rho/\rho_D$ remain small only within a narrow range of $T$. The agreement of the data with the interaction corrections in Figs. 1, 2 holds over a reasonable range of tem-

![FIG. 1. Comparison of the $\sigma(T)$ data (symbols) for sample Si22 reproduced from Ref. [1] with calculated quantum corrections [2] (solid lines). Densities, from top to bottom, are (in units of $10^{11}$ cm$^{-2}$): 21.3, 18.9, 16.5, 14.1, 11.7, 10.5, 8.1, 7.5, 6.9, 6.3, 5.7.](image-url)
temperatures and in a wide range of densities (1:7), but not too close to the critical density value $n_c \sim 10^{11}\text{cm}^{-2}$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Comparison of the $\sigma(T)$ data (symbols) for sample Si15 [1] with the interaction corrections [2] (solid lines) and the “screening” theory [7] (dashed lines). Densities are indicated in units of $10^{11}\text{cm}^{-2}$.}
\end{figure}

Once again, no matter how good (or bad) the agreement at low densities is (this regime is beyond the applicability of the theory [2] anyway), the main conclusion of our paper [1] remains the same: the “metallic” behavior of $\rho(T)$ in high-mobility Si MOSFETs in the regime $\rho \ll h/e^2$ is in quantitative agreement with the predictions [2] based on considering the interaction effects in the ballistic regime.

(3) SDK claim that in the range of densities $n > 2 \times 10^{11}\text{cm}^{-2}$, the $\rho(T)$ changes can be described by the “traditional screening” theory by one of the authors [7]. The “screening” theory [7] predicts the dependence

$$
\delta \sigma(T)/\sigma(0) = 1 - 2C(\alpha)C(n)(T/E_F) + \ldots,
$$

with $C(\alpha) = 2 \ln 2$ (or $(8/3) \ln 2$) for charged impurities, $\alpha = -1$ (or for surface roughness, $\alpha = 0$) scattering, respectively, $C(n) = F(2k_F)(1 - G(2K_F))/(F(2k_F)(1 - G(2k_F)) + 2k_F/q_*)$, and $G(2k_F) = 0.2236$. By re-writing $T$ in units $x = T\tau/\hbar$ [2], and $\sigma$ in units of $e^2/(\pi \hbar)$, we obtain

$$
\frac{d\sigma}{dx} \approx -2C(\alpha, n) = -4C(\alpha)C(n).
$$

This expression differs from that derived in Ref. [2]:

$$
\frac{d\sigma}{dx} \approx 15 \frac{F_0^\alpha}{1 + F_0^\alpha} \left[ 1 - \frac{3}{8} f(x) \right] + \left[ 1 - \frac{3}{8} t(x) \right],
$$

where the functions $f(x)$ and $t(x)$ are defined in Ref. [2].

The difference between the predictions of the “screening” theory [7] and the theory of interaction corrections [2] is illustrated in Figs. 2,3. Note that the difference in the slopes $d\sigma/dT$ in Fig. 2 is a factor of $~2$ at the lowest density $n = 2.2 \times 10^{11}\text{cm}^{-2}$ (to calculate the interaction corrections to $\sigma$, we have used the experimental values of $F_0^\alpha(n)$ and $m^*(n)$ measured by us in independent experiments [3]). The theoretical predictions for the slopes $d\sigma/dx$ as functions of the density are shown in Fig. 3. The two density dependences cross each other at different $n$ depending on the specifics of impurity or interface scattering in the screening theory. Adjusting the latter parameters does not improve agreement between the theory [7] and our experimental data, if a sufficiently wide range of densities is considered. At the same time, the ZNA theory [2] is consistent with the data in the wide density range without any adjustable parameters. Theoretical aspects of the difference between approaches [7] and [2] have been clarified in Ref. [2].

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{Comparison of the slopes $d\sigma/dx$, calculated from Ref. [7] for two models of scatterers ($\alpha = -1$ and $\alpha = 0$), and from Ref. [2] with $F_0^\alpha$ and $m^*$ values determined in [3]. In the latter case, the slope slightly varies with the temperature (compare $x = 1$ and $x = 4$), due to the functions $f(x)$ and $t(x)$ in Eq. (2). Symbols depict the slope $d\sigma/dx$ of the data shown in Figs. 1, 2.}
\end{figure}
(4) SDK suggest that the low-temperature deviation of our $\rho(T)$ data from the theory, observed for several samples, was caused by electron overheating. Apparently, they overlooked the corresponding discussion in our paper (p.3, left column [1]), which analyzed and rejected this possibility. For the range of relatively high densities $n \gg n_c$, we explored in Ref. [1], the deviations from the theory occur at high temperatures, where overheating is highly unlikely. Moreover, the low temperature part of the data at $T \lesssim \hbar/2\pi\tau$ is irrelevant to the linear-in-$T$ dependence [6].

(5) SDK claim that the range of densities explored in [1] is “not related to the anomalous increase of $\rho(T)$ with temperature”. The term “metallic” refers to the sign of $d\rho/dT$, not to its magnitude; however, the “anomalous metallic” behavior is more commonly referred to the strong changes of $\rho(T)$ [6]. We emphasize that, though we have limited our quantitative analysis in [1] to the small changes $\Delta\rho(T)/\rho \ll 1$, where the theory [2] is applicable, the overall increase of the resistance with $T$ is large in the whole explored range of densities: $\rho$ increases with temperature by a factor of 2 even at $n$ as high as $15 \times 10^{11}$ cm$^{-2}$ [5].

Note that this claim by SDK is inconsistent with their own publications: indeed, in Ref. [8], they fitted the linear $\sigma(T)$ dependences at low densities and low temperatures ($T < 1$K) with the theory [2], whereas on pages 2,3 of Ref. [9], one of the authors concluded on the basis of analysis of $\sigma(T)$ over the same ranges of densities and temperatures that “the linear $\rho(T)$ dependence is different from the dependence seen at higher temperatures and higher densities”.

(6) SDK claim that the theory [2] “is inapplicable at high density $n > 5 \times 10^{11}$ cm$^{-2}$, where the surface roughness scattering becomes dominant”. In fact, the theory [2] assumes the short-range scattering (for more details, see Ref. [10]). This requirement is well justified for our samples over the entire range of $n = (2.2 - 30) \times 10^{11}$ cm$^{-2}$ as it has been shown in Ref. [1]: the ratio of the “large-angle” scattering time $\tau$ to the quantum life time (or “all-angle” scattering time) $\tau_q = \hbar/(2\pi T_D)$ is close to unity (here, $T_D$ is the Dingle temperature). Besides, it is expected that the difference between the effects of short-range and long-range disorder on $\rho(T)$ is negligible at $T < 0.1T_F$ [6].

(7) Criticizing our analysis of $\rho(T)$ at “high” densities $n \gg n_c$, SDK give preference to comparison with the theory at low densities $n \approx n_c$ and low temperatures. An attempt to analyze $\rho(T)$ in this regime on the basis of the theory [2], undertaken by SDK in Ref. [8], suffers from several drawbacks:

(i) At low densities $n \approx n_c$ and low temperatures, such as explored in [8], $T$ is of the order of $\hbar/2\pi\tau$, and the requirement for the ballistic regime is not satisfied (see, e.g. [6]). In this case, one cannot neglect the diffusive terms and crossover functions, as it was done in Ref. [8].

(ii) In the vicinity of $n_c$, the temperature dependence of $\sigma(T)$ is strongly affected by the procedure of sample cooling [11], this regime is beyond the scope of theory [2].

(iii) In the vicinity of $n = n_c$, the ratio $\tau_q/\tau = 3$, as observed by SDK [8,12] is clearly anomalous: this indicates that the standard models of disorder fail.

(iv) For low densities $n \sim 10^{11}$ cm$^{-2}$ (and, correspondingly, low values of $T_F \sim 2.5$K), the “metallic” behavior corresponds to the temperature range where the valley splitting $\Delta_v$ cannot be ignored. (Note that in our analysis [1], the valley splitting $\Delta_v \ll T$ and, therefore, is insignificant). The sample-dependent parameter $\Delta_v$ has not been measured for the SDK’s sample, and SDK made a doubtful assumption that the valley splitting $\gg 1$K [8], which contradicts their own Shubnikov-de Haas data.

Correspondingly, in fitting the above Eq. (3), the number of triplet components was incorrectly reduced from 15 to 7.

For the aforementioned reasons, the interaction corrections theory [2] can be hardly applied to the $\sigma(T)$-data analysis at the densities $n \approx n_c$. The “accurate measurements in the best samples” in Ref. [8] (though with mobility $2.9 \times 10^4$ cm$^2$/Vs [8] lower than $4 \times 10^4$ cm$^2$/Vs as in Fig. 2) have not been analyzed accurately enough to provide a reliable quantitative information on the renormalized effective mass. As the result of several unjustified simplifications, the authors of Ref. [8] overestimated the increase of the effective mass with decreasing density.

To summarize,

- Our $\rho(T)$ measurements performed over wide ranges of densities and temperatures, being combined with the independent measurements of renormalized $F_0^q(n)$ and $m^*(n)$ [3], enable a rigorous test of the theory [2]. The theoretical predictions are in a quantitative agreement with the experimental data, in terms of either $\rho(T)$ or $\sigma(T)$, over the density range where the theory can be applied ($\rho \ll \hbar/e^2$, $\Delta\rho(T)/\rho \ll 1$), and in the ballistic $T$-range.

- The alternative attempt of such comparison [8] in the regime of low densities $n \approx n_c$ and low temperatures suffers from several drawbacks, and does not provide a reliable quantitative information on renormalization of the effective mass.

- We point out that the predictions of the “screening” theory of Ref. [7] differ significantly from those of the theory of interaction corrections [2] over a wide range of densities, which is of primary importance for the problem of “metallic” resistivity of high-mobility Si MOSFETs.

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