Fine structure of type III solar radio bursts from Langmuir wave motion in turbulent plasma

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The Sun frequently accelerates near-relativistic electron beams that travel out through the solar corona and interplanetary space. Interacting with their plasma environment, these beams produce type III radio bursts—the brightest astrophysical radio sources seen from Earth. The formation and motion of type III fine frequency structures is a puzzle, but is commonly believed to be related to plasma turbulence in the solar corona and solar wind. Combining a theoretical framework with kinetic simulations and high-resolution radio type III observations using the Low-Frequency Array, we quantitatively show that the fine structures are caused by the moving intense clumps of Langmuir waves in a turbulent medium. Our results show how type III fine structure can be used to remotely analyse the intensity and spectrum of compressive density fluctuations, and can infer ambient temperatures in astrophysical plasma, substantially expanding the current diagnostic potential of solar radio emission.

The Sun routinely accelerates electrons in its outer atmosphere that subsequently travel through interplanetary space. Electron beams generate Langmuir waves in the background plasma, producing bright solar radio emission1–3. Owing to our close proximity to the Sun, type III radio bursts are the brightest radio sources in the sky, providing a unique opportunity to understand particle acceleration and transport4,5. Type III bursts also provide us remote diagnostics of solar corona and solar wind properties, with stellar radio bursts having the diagnostic potential for stellar atmospheres6. Type III bursts present a remote indicator for energetic electrons escaping the Sun, which is important for our prediction of extreme space weather events7.

As electron beams propagate through plasma with decreasing density, and hence decreasing plasma frequency, type III bursts drift from high to low frequencies in dynamic spectra (frequency versus time plots). Type III frequency drift is often used to infer the bulk velocity of electron beams which drive type III emission. Fine frequency structures are often present in dynamic spectra of type III bursts, observed as nearly horizontal stria in the envelope of a type III burst8–11. The stria have short duration (for example, 1 s at 30 MHz) and have characteristic frequency fraction width Δff/ff ≈ 0.1, independent of frequency. However, the properties of these striae are puzzling and not understood, with a large number of competing theories8,12,13.

The drift rate of individual striae give derived speeds around 0.6 Mm s−1 (ref. 14). Intriguingly, these velocities are substantially smaller than electron beam velocities of 100 Mm s−1 (ref. 15), but much greater than coronal sound speeds of $\sqrt{k_b T/M}$ ≈ 0.2 Mm s−1, where $M$ is the ion mass, $k_b$ is the Boltzmann constant, and $T$ is the plasma temperature in kelvin15. It is therefore not clear what plasma process dictates striae drift rates and the origin and quantitative explanation of type III burst striae presents a major challenge. The power spectrum of type III striae has been shown for one event to have a roughly 5/3 power-law slope both for fundamental and harmonic emission16. It is likely that the striae are related to some compressive density perturbations8, hence can provide important insights into the magnetohydrodynamic waves at heights where extreme ultraviolet (EUV) observations are normally not available.

To establish the progenitor of striae in dynamic spectrum, one needs high-frequency-resolution observations combined with the numerical simulations of the solar radio bursts.

In this Article, we demonstrate that the observed properties of type III bursts fine structure are consistent with moving Langmuir waves through turbulent space plasma. Furthermore, the drift rate of the stria obtained in the observations is consistent with the group velocity of the Langmuir waves. The numerical simulation can reproduce high-resolution Low-Frequency Array (LOFAR) observations of type III fine structure in the solar corona. This more complete understanding of striae opens a new opportunity for diagnostics of density fluctuations parallel to the magnetic field within the solar corona, which are not available otherwise.

Type III observations

We present three examples of type III bursts showing obvious fine structure, observed using LOFAR17 on 16 April, 24 June and 16 September 2015. Figure 1 shows calibrated dynamic spectra of the fundamental emission from the different events observed between 30 MHz and 40 MHz. Analysis of observational properties from the event on the 16 April 2015 has been previously reported by refs. 10,13,14,16. Each event lasts a few seconds at these frequencies and all events had fundamental-harmonic pairs similar to the 16 April event that was shown by ref. 10. The large-scale frequency drift from high to low frequencies provides an estimate of electron beam bulk velocity (Methods) of $v_b = 88$ Mm s−1, 49 Mm s−1 and 46 Mm s−1, for the three events, in chronological order. Figure 1 also shows the LOFAR radio contours at different radio frequencies plotted over the EUV 171Å Sun, observed by the Atmospheric Imaging Assembly (AIA)18. The radio contour for each frequency channel is shown at varying intensities. We can quantify the characteristic intensity of the fine structure, the relative flux fluctuation amplitude $\Delta I/I$, between the frequencies 30 MHz and 40 MHz for all three events.

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(Methods). Taking the average variation from the mean intensity (Extended Data Fig. 1), we found typical values of $\Delta I/I = 1.41, 1.01$ and 1.35 for the three events, in chronological order. High values for $\Delta I/I$ are expected given that we selected these events for their abundant fine structure. However, if we want to use the intensity of radio fine structure as a plasma diagnostic tool, we must understand their origins.

**Radio fine structure driver**

The cause of type III fine structure has previously been thought to relate to modulations in the growth rate of beam-induced Langmuir waves, naturally leading to a modulation in the intensity of radio waves. Langmuir wave growth rates can be modulated via wave refraction, which is controlled by the level of density fluctuations in the background plasma (for example, refs. 19–22). Previous simulations\(^{20,21}\) were able to produce synthetic type III dynamic spectra with fine structure at frequencies corresponding to density variations in the background plasma. Some of the synthetic dynamic spectra showed discrete structures in frequency. However, the studies did not reproduce any frequency drift associated with these structures and the number of structures, particularly in fundamental emission, was much smaller than the number observed in type III striae bursts. The idea used by preceding studies and other related works that density fluctuations can modulate Langmuir waves and causes radio fine structure has been discussed for decades (for example, ref. 23, as a review). However, nobody has created a robust

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**Fig. 1 | Dynamic spectra and associated radio contours of solar type III radio bursts.** a, Dynamic spectra of the flux for fundamental type III bursts showing fine structure. Events from top to bottom occurred on the 16 April 2015 at 11:56:20 ut, 24 June 2015 at 12:18:20 ut and 16 September 2015 at 09:56:33 ut. The corresponding solar altitude assuming the Parker density model\(^{24}\) is also shown parallel to the frequency. b, The LOFAR contours at 75% of the peak flux of the type III bursts going from 40 MHz to 30 MHz in the colour sequence white-blue-green-yellow-red. The LOFAR beam contour at 75% for 30 MHz is shown in the top left corner in white. The background is the Sun in EUV at 171 Å observed by the AIA.
Theoretical model that allows one to reverse-engineer the level and spectrum of density fluctuations from the observed radio fine structure. Such a feat enables type III radio bursts to be used as a probe of space plasma density turbulence where in situ observations are not feasible.

We have shown (Methods) that the motion of Langmuir waves travelling with a group velocity $v_g = 3\nu_f/\nu$ has a shift $\delta v$ in Langmuir wave phase velocity, $v$, from refraction off density fluctuations with intensity $\Delta n/n$. The shift $\delta v$ causes the Langmuir waves to move out of resonance with the electron beam, suppressing wave growth. The intensity $\Delta n/n$ can directly be related to the intensity of radio fine structure $\Delta I/I$ via

$$\langle \Delta n^2 \rangle / n^2 = \left( \frac{v_{th}}{v_e} \right)^2 \left( \frac{\Delta F}{F} \right)^2$$  \hspace{1cm} (1)$$

where $v_{th} = \sqrt{k_b T_e/m_e}$ is the background thermal velocity, and $v_e$ is the bulk electron beam velocity (Methods). $T_e$ is the background electron temperature and $m_e$ is the electron mass. The above equation has broad implications, not only explaining how radio fluctuations are related to the turbulent intensity of the emitting astrophysical plasma but also showing that the radio fluctuations are the signatures of Langmuir wave motion. The velocities that dictate the intensity of radio fluctuations can be viewed as a ratio between electron beam and Langmuir wave spatial velocities $v_{th}^2/v_e^2 = v_g^2/(3\nu_f)$, given that the beam velocity $v_e$ is similar to the resonant Langmuir wave phase velocity $v$. Equation (1) quantifies how radio fluctuations can be a powerful remote probe of plasma density fluctuations in the solar (or stellar) atmosphere and wind.

### Langmuir wave modulation

We now show how key parameters of density fluctuation amplitude and wavelength affect the level of beam-driven Langmuir waves, and hence the type III radio bursts. We also show how the spatial motion of Langmuir waves at the group velocity also affects the level of beam-driven Langmuir waves.

We demonstrate this behaviour using one-dimensional kinetic simulations of an electron beam that is injected into the solar corona and then propagates out from the Sun. The simulations take into account the resonant interactions of electrons with Langmuir waves and other relevant physical mechanisms for electron propagation. The simulation model is described in Methods and is similar to previously used models (for example, ref. 2). The main difference for the simulations used in this section is that we have a constant background density of $n_e = 1.1 \times 10^5$ cm$^{-3}$, which corresponds to a plasma frequency of 30 MHz. This background density is then modulated by a single sinusoidal density fluctuation $n_0(r) = n_0(1 + A \sin(k_1 r + \phi))$, where $A$ is the amplitude of the density fluctuations, $k_1 = 2\pi/\lambda_1$ Mm$^{-1}$ and $\lambda_1 = 50$ Mm are the wave-number and wavelength of the density fluctuation, $r$ is distance from the Sun and $\phi$ is a random phase. We have used this unrealistic background density to remove the large-scale radial decrease in coronal background electron density so we can highlight the effects of the amplitude and wavelength of the sinusoidal density fluctuation. The real solar corona also has turbulent density fluctuations, which we simulate in the section ‘Type III simulations’.

The simulations include additional physical terms not considered for the derivation of equation (1), such as electron transport and the electron velocity diffusion due to Langmuir wave generation. The simulations highlight two important relations needed for understanding type III radio fine structure; how Langmuir wave growth mirrors the density fluctuations parameters, and how spatial Langmuir wave motion alters Langmuir wave growth.

The relation between density fluctuation parameters and Langmuir wave growth is best shown by setting the Langmuir wave group velocity to zero. Figure 2 shows a snapshot of the Langmuir wave energy density $U_{LW}$ at time $t = 7.5$ s for four simulations with density fluctuation amplitudes $A = 10^{-4}, 10^{-3}$ and $10^{-2}$. The wavelength in Langmuir wave fluctuations are the same as the wavelength of the gradient of the background density fluctuations $\partial n/\partial r$, indicated by the vertical dashed lines in Fig. 2. Increasing the amplitude of the density fluctuations increases the modulation of Langmuir waves. Langmuir wave energy density modulation is further highlighted by the background-subtracted power spectrum density in Fig. 2. The peak in each power spectrum occurs at $k_1 = 2\pi/\lambda_1$ Mm$^{-1}$ for $\lambda_1 = 50$ Mm, and the amplitude is governed by the amplitude of the sinusoidal density fluctuation. Modifying the wavelength $\lambda_1$ of the sinusoidal density fluctuation has a corresponding effect on the Langmuir wave energy density, shifting the peak in the Langmuir wave power spectral density to $k = 2\pi/\lambda_1$ Mm$^{-1}$, shown in Methods.

When we include the spatial motion of Langmuir waves motion at the group velocity, the connection between the density fluctuations and the Langmuir wave oscillations is modified. Langmuir waves move away from regions in space that have favourable growth conditions at the group velocity. This has the effect of smoothing the Langmuir wave energy at scales smaller than their lifetime multiplied by the group velocity. Figure 2 illustrates this behaviour for two simulations with and without group velocity. The power is reduced at higher wavenumbers where the Langmuir waves had time to disperse. However, at lower wavenumbers the power remains the same. Reducing the wavelength to, for example, $\lambda_1 = \lambda_1/5$ causes the power to be reduced for all the peaks in the spectrum, as the Langmuir waves exist for long enough that their motion smooths the wave energy beyond one wavelength of the density fluctuation (see Methods).

### Type III simulations

Solar-accelerated electron beams propagating out through the solar corona and solar wind can naturally produce type III radio emission with fine structure. Our kinetic simulations modelled the injection and propagation of an electron beam through the 1 MK solar corona that has a turbulent $-5/3$ spectrum (Kolmogorov) of coronal density fluctuations (Methods). The beam-driven Langmuir waves generate radio emission through the fundamental plasma emission mechanism. The resulting synthetic type III radio burst dynamic spectrum is shown in Fig. 3 between the frequencies 30 MHz and 40 MHz. The radio fine structure of the striae is evident. As demonstrated previously, the Langmuir wave level is affected by Langmuir wave refraction off density fluctuations in the background plasma. Intense populations of Langmuir waves then move through the solar corona, causing the individual striae.

Properties of the striae are strongly dictated by the temperature of the background plasma. Figure 3 shows a dynamic spectrum generated by the same electron beam propagating through a hotter 10 MK plasma. The higher temperature, and hence higher thermal velocity, relates to a faster Langmuir wave group velocity, $v_g = 3\nu_f/\nu$, than for a 1 MK plasma. The radio fine structure is noticeably different. Faster Langmuir wave motion blurs out striae with small frequency widths, increasing individual stria frequency width and drift rate. This increased striae frequency width and the stria drift magnitude makes the simulated striae incompatible with the LOFAR observations, consistent with 10 MK being an unrealistically high temperature for the upper solar corona. The duration of striae is also shortened as higher-temperature plasma has increased Landau damping of Langmuir waves generated at the back of the beam. This shortens the length of time radio is emitted at a given frequency (position).

Langmuir wave motion is absolutely essential to replicate the observed characteristic type III fine structure features. Figure 3 demonstrates that when the Langmuir wave motion was set to zero ($v_g = 0$), the dynamic spectrum is dramatically modified.
Wave growth is still modulated by local density gradients but the clumps of intense Langmuir waves do not propagate through space. Consequently, the fine structure has a much smaller width in frequency space and shows no motion in time, in stark contrast to type III observations.

Clumps of Langmuir waves moving at the group velocity consistently answers the outstanding puzzle of which physical process caused the derived velocity obtained from the frequency drift of individual stria. Langmuir waves propagate substantially slower than their resonant electrons, with relevant group velocities around 0.5 Mm s\(^{-1}\) for a 1 MK plasma, compared with electron speeds around 100 Mm s\(^{-1}\). Figure 4 shows the frequency drift from an individual stria observed by LOFAR. The linear fit estimates a group velocity of 0.69 Mm s\(^{-1}\) assuming the Parker density model\(^3\). Figure 4 also shows an individual stria from the simulations where the background temperature was 1 MK (Fig. 3, top). The linear fit of 0.69 Mm s\(^{-1}\) is very similar to the observed stria frequency drift rate, and is identical to the mean group velocity of the Langmuir waves responsible in the simulation. A quadratic fit indicates a change in the Langmuir wave group velocity with time; relating to the resonating electron beam velocities decreasing from the front to the back of the beam\(^1\). We cannot resolve any notable evolution in the observed stria frequency drift rate. However, the time duration for the observed stria is about 1 s smaller than the simulated stria. The stria time duration is probably related to the spread in electron velocities between the fastest and slowest electrons that excite the Langmuir waves. An increase in the Landau damping from a background plasma with a kappa-like distribution is stronger\(^2\), is more realistic of the solar wind and is likely to reduce stria duration. A decrease in the electron beam velocity spectral index would also cause a smaller spread in electron velocities and could reduce stria duration, in a similar way to how it reduces type III duration\(^3\).

Knowing that the frequency drift of striae is driven by the Langmuir wave group velocity creates a powerful diagnostic tool for plasma temperature. Striae frequency drift contains information about the gradient of the background electron density, the velocity of the electron beam and the background thermal velocity such that

\[
\frac{d\nu}{dt} = \frac{f}{2n_e} \frac{\partial n_e \nu_b^2}{\partial \nu_b},
\]

where \(d\nu/dt\) is the striae drift rate, \(\partial n_e / \partial \nu\) is the local density gradient and \(\nu_b\), the electron beam is resonant with the Langmuir wave phase velocities. Velocity estimations\(^4\) using striae from the type III on the 16 April 2015 at 11:56, assuming the Newkirk coronal density model\(^3\), give an inferred velocity of \(\nu_b = 0.58\) Mm s\(^{-1}\) around 40 MHz. With the beam velocity estimated in the ‘Type III observations’ section as \(\nu_b = 88\) Mm s\(^{-1}\) using the Parker density model, we can obtain an estimate of the thermal velocity \(\nu_{Th} = \sqrt{\nu_b^2 c^2 / 2} = 4.1\) Mm s\(^{-1}\), corresponding to a background temperature of 1.1 MK. There is a small element of uncertainty in the derived temperature related to the background density model used. However, there are observational bounds for acceptable density models that limit these uncertainties. If instead we estimate the beam velocity using the Newkirk density model, we obtain a value of \(\nu_b = 118\) Mm s\(^{-1}\), which corresponds to a background temperature of 1.5 MK.

The characteristic levels of radio fine structure can now be used as a new remote sensor for density turbulence in the solar corona and solar wind. Using equation (1) and the derived values for \(\Delta I / I\) at \(\nu_b\) and \(\nu_{Th}\) from the LOFAR observations, we can estimate the...
amplitude of density fluctuations as $\Delta n/n = 0.3\%$. The simulations provide a robust check for equation (1) as they consider additional physical terms. Using multiple simulations with different values of $\Delta n/n$, Fig. 5 shows how $\Delta I/I$, found between 30 MHz and 40 MHz (Extended Data Fig. 2), corresponding to a length of nearly 100 Mm, increases when the intensity of the turbulence, $\Delta n/n$, is increased.

The simulations find a good agreement with equation (1), estimating the bulk electron beam velocity from the synthetic dynamic spectra. We also show in Fig. 5 the effect of varying the initial beam density. Electron beams travel at higher velocities for higher beam densities (with the same initial energy spectral index), on account of an increased electron energy density$^{25}$. Correspondingly, the value of $\Delta I/I$ is increased.

In contrast to previous studies$^{14,29}$, the value of $\Delta f/f$ measured from the striae bursts cannot be used to extrapolate the intensity of the background density turbulence $\Delta n/n$. As we demonstrated in Fig. 2, using a single characteristic wavelength to perturb a back-
ground plasma with constant mean density, modifying \(\Delta n/n\) does not change the length scale of the Langmuir wave enhancements. A similar result is demonstrated in the Extended Data Fig. 3 where the width \(\Delta f/\Delta f\) of the striae are not notably altered despite an order of magnitude change in \(\Delta n/n\). We note that the distribution of turbulence scales will affect the resultant value of \(\Delta f/\Delta f\) and indeed the value of \(\Delta f/\Delta f\) is not simple enough that one can be used to deduce the other.

Fourier analysis provides knowledge of the pertinent length scales of solar corona and solar wind density turbulence, relevant to the type III fine structure observed by LOFAR (Methods). This approach was previously done\(^{15}\), where spectral indices were found around \(-5/3\) for the event on 16 April in both fundamental and harmonic emission. Figure 6 shows all three type III fundamental radio flux power spectra as a function of wavenumber. At wavenumbers below \(2\pi\,\text{Mm}^{-1}\), the spectrum has a roughly \(-5/3\) spectral index. Around \(2\pi\,\text{Mm}^{-1}\), the power in the spectrum decreases, indicating a reduction in the amount of fine structure at the smallest scales. Figure 6 also shows the power spectrum from the simulated type III burst in the 2 MK corona. Similar behaviour is observed to the observations; a spectral index of \(-5/3\) at wavenumbers below \(2\pi\,\text{Mm}^{-1}\), and a drop in power around \(2\pi\,\text{Mm}^{-1}\). The drop in power is related to smoothing of Langmuir wave energy from the spatial motion at the group velocity. Figure 6 also shows the power spectrum of the type III bursts where no group velocity was simulated. The lack of spatial Langmuir wave motion leads to a heightened power at higher wavenumbers, not consistent with the LOFAR observations.

**Discussion**

Through a combined observational and simulation approach, we have quantitatively demonstrated how solar radio burst striae fine structure is developed. The individual radio striae are shown to be a combination of beam-driven Langmuir wave modulation by the turbulent solar wind, and the subsequent spatial motion of the Langmuir waves at their group velocity. The simulations indicate that the drift rate of stria is determined by the group velocity of Langmuir waves.

The intensity of the radio fine structure is a powerful tool that can provide a diagnostic of the intensity of the background density turbulence via the relation \(\Delta I/I = (v_\parallel/v_e)\Delta n/n\). At certain scales, the observed spectrum of radio burst fine structure is very similar to the spectrum of density turbulence measured in the solar wind. Both the intensity and the spectrum of density turbulence is not well constrained in the solar corona. The LOFAR observations suggest that the level of parallel density fluctuations at heights 0.7–0.8\(\,\text{R}_\odot\) above the photosphere is around 0.1–0.3%. This value for \(\Delta n/n\) is similar to what was estimated using LOFAR observations of frequency width spread in coronal altitude\(^{46}\). The magnitude of parallel density fluctuations is probably smaller than perpendicular density fluctuations, found from anisotropic scattering ray-tracing estimates\(^{39,40}\), and similar to values of \(\delta B_\parallel/\delta B_\perp > 1\), for the magnetic

**Fig. 5** Relation between the level of background density fluctuations and type III radio burst intensity fluctuations. Simulated levels of radio emission fluctuations \(\Delta I/I\), for electron beams travelling through plasma with varying levels of density fluctuations \(\Delta n/n\). Values were taken between 30 MHz and 40 MHz from the simulated dynamic spectra. The case when an electron beam propagates through a homogeneous background plasma is indicated as a yellow dashed line, showing the minimum level of fluctuations obtained by the method. The initial beam density varied between \(10^{15}, 10^{16}\) and \(10^{17}\,\text{cm}^{-3}\) indicated in green, blue and red, respectively. The value of \(\Delta I/I\) is shown as a black triangle for the type III event observed by LOFAR on 16 April 2015, at the estimated value of \(\Delta n/n\). The black dashed line shows the fit found using equation (1) from the estimated values of \(v_\parallel\) and \(v_e\) obtained from the type III data.

**Fig. 6** Power density spectra from observed and simulated type III burst dynamic spectra. a. Power density spectra of the type III bursts as a function of wavenumber assuming the Parker density model. The 16 September 2015 (red) and the 16 April 2015 (blue) have been shifted up by \(10^4\) and \(10^5\) for clarity with the 24 June (green). b. Power spectral density for the simulated type III brightness temperature with a 1 MK (blue) and 10 MK (red) corona. Note the reduction in power for the 10 MK corona for higher wavenumbers. For both graphs, the black dashed lines are a power law with spectral index \(-5/3\), for reference. The power spectral density for the simulation with \(v_\parallel = 0\) is shown in green, multiplied by \(10^3\) to better fit into the plot. The lack of reduction in power at higher wavenumbers (shorter wavelengths) is evident.
field $\mathbf{B}$ measured in the solar wind\textsuperscript{32}. The levels of $\Delta n/n$ are smaller than was previously found\textsuperscript{31} using type III bursts at lower frequencies, which could be related to their isotropic scattering assumption and density turbulence being stronger in the solar wind.

The frequency drift of radio fine structure constrains the background thermal velocity, increasing the scope of solar radio bursts to be used as a remote plasma temperature diagnostic. The observation infers a corresponding coronal plasma temperature around 1.1 MK. The radio fine structure also provides an additional way to estimate the electron beam bulk velocity, which is mostly controlled by the beam energy density.

Our results have created a framework for exploiting the diagnostic potential of radio burst fine structure. This is especially relevant given the enhanced resolution of new-age ground-based radio telescopes that are resolving much more fine structure originating from the solar corona. Moreover, the closer proximity of Parker Solar Probe and Solar Orbiter to radio emission originating in the very high corona or solar wind, and hence higher sensitivity, allows fine structures to be detected in situ. Coupled with in situ plasma measurements from these spacecraft, the radial evolution of the turbulence can be studied throughout the inner heliosphere and should help to understand what drives solar wind turbulence.

**Methods**

**Observations.** LOFAR is a collection of interferometric antenna arrays distributed around Europe, with the core stations in the Netherlands. Our observations used the tied-array beam forming\textsuperscript{14} with 24 core stations, using the low-band antennas between 30 MHz and 80 MHz. Each beam pointed at a different part of the sky and recorded a simultaneous flux that is a convolution of the true source and the LOFAR point spread function. We calibrated the flux using an observation of Taurus A (Crab Nebula). We use a frequency channel resolution of 0.012 MHz and integrated the 0.01 s time resolution to 0.1 s to increase the signal to noise ratio. For imaging, the 124 (April observation) or 169 (June and September observations) individual beams performed a course mosaic of the solar disc and the solar corona, with a separation that is smaller than the full-width at half-maximum of the LOFAR point spread function. Images are then made using an interpolation grid.

The drift rate of the type III burst is related to the bulk electron beam velocity via

$$\frac{df}{dt} = \frac{f}{2n_e} \frac{dn_e}{dr} v_b.$$  

(3)

To determine the bulk electron beam velocities from the observations, we converted frequency $f$ to background electron density $n_e$ assuming fundamental plasma emission such that $2n_e^2 = \gamma k T_e/n_e^2$, where $\gamma$ is the electron charge. The altitude was then found using the Parker electron density model\textsuperscript{24}. The bulk electron beam velocity is obtained using a linear fit to the distances as a function of time, with a Savitsky–Golay filter to smooth the frequency fine structure.

The characteristic intensity of the frequency fine structure $\Delta I/I$ is obtained using

$$\frac{\Delta I}{I} = \left(\frac{(\delta f)^2}{(\delta I)^2}\right)^{1/4}.$$  

(4)

where $(\delta f)$ is the peak flux as a function of frequency and $(\delta I)$ is the difference between the peak flux and the smoothed peak flux, found using a Savitsky–Golay filter with a characteristic size of 3 MHz. Extended Data Fig. 1 shows the peak flux and the smoothed peak flux for each event as a function of frequency.

**Langmuir wave motion.** First, let us note that the decrease of Langmuir wave energy is due to locally increasing density, that is parallel density gradient $d n / dr > 0$. We can write the energy conservation equation\textsuperscript{16}

$$E_w + E_b = \text{constant}$$

where $E_w$ and $E_b$ are the Langmuir wave and the electron beam energy, respectively.

Since the electron beam is almost a plateau $f \approx n_b v_b$ up to velocity $v_b$

$$E_b = \int_0^{v_b} m_e v_b^2 n_b v_b\,dv$$

and

$$E_w = \int_0^{v_b} m_e /\omega_pe^2 (1 - v/v_b) n_0 v_b\,dv,$$

where $\omega_p$ is the plasma angular frequency and $n_0$ is the electron beam density. Due to the positive density gradient, Langmuir wave packet experiences refraction, so that the change of phase speed $\delta v$ (see equations (26) and (17) in ref. \textsuperscript{10}) is

$$\frac{\delta v}{v} \simeq \frac{1}{3} \frac{\delta \omega_p}{\omega_p} = \frac{1}{6} \frac{\delta n}{n}.$$  

(5)

where $v_e$ is the electron thermal speed.

Since the energy before the refraction and after should be the same, we can write (energy before = energy after), hence

$$\frac{m_e n_b}{6} \frac{v_b^2}{v^2} + \frac{m_e n_0}{12} \delta v^2 = \frac{m_e n_b}{6} (v + \delta v)^2 + (1 - \beta) \frac{m_e n_0}{12} (v + \delta v)^2$$

where $\beta = \delta E_w/E_w$. The energy from the waves goes to the beam that leads to an increase in the electron energy and must reduce the waves by a factor $1 - \beta = 1 - \delta E_w/E_w$ so that the energy is conserved.

In the limiting case, $1 - \beta = 0$ (all waves are absorbed), and the maximum beam velocity is $v_0 + \delta v = \sqrt{3/2}$ (for example, ref. \textsuperscript{32}). For weak absorption, $\beta < \ll 1$.

From the energy conservation, one finds

$$\frac{3}{2} v_0^2 = (3/2 - \beta/2) (v_0 + \delta v)^2$$

which gives us the connection between wave energy change $\delta E_w/E_w$ and the velocity increase $\delta v/v_0$ becomes

$$\beta = 3 \left(1 - (1 + \delta v/v_0)^{-2}\right)^2 = 3 \left(1 - (1 + 1/6 (v^2/v_0^2) \delta n/n)^{-2}\right)^2$$  

(6)

For small speed change $\delta v/v_0 \ll 1$, one finds

$$\delta v/v_0 \approx \delta \omega_p/\omega_p = \delta E_w/E_w.$$  

(7)

Combining the energy conservation and the phase velocity change, one finds for $\delta E_w/E_w$:

$$\delta E_w/E_w = \frac{\delta n}{n} = \frac{v_b^2}{v^2} \frac{\delta E_w}{E_w}.$$  

(8)

However, if the modulation is known from the observations we can determine the density fluctuations. Thus, taking 50% intensity modulation of radio flux (as observed by ref. \textsuperscript{10}), that is $\delta E_w/E_w = 1/2$, one finds from the equation (6):

$$\frac{\delta n}{n} \simeq 9 \times 10^{-4}$$

It is interesting to compare the spectrum of radio wave intensity modulation $(\delta I^2)$ and the spectrum of parallel density fluctuations $(\delta n^2)$. From equation (7), one obtains the relation

$$\frac{\langle \delta I^2 \rangle}{I^2} = \frac{v_b^2}{v^2} \left(\frac{\langle \delta n^2 \rangle}{n^2}\right).$$  

(8)

Observationally (from ref. \textsuperscript{10}), we have $(\delta I^2)/I^2 = 10^{-4}$ at 1 MHz. It is also interesting to note that the expression given by equation (6) saturates.

**Simulation description.** We model the self-consistent time evolution of an electron beam distribution function $f(n, r, t)$ (cm$^{-3}$s$^{-1}$) and their resonant interaction with the Langmuir waves spectral energy density $W(n, r, t)$ (erg cm$^{-3}$) with the following one-dimensional kinetic equations

$$\frac{df}{dt} + \frac{v}{M(r)} \frac{df}{dr} = 4 \pi e^2 c^3 \frac{d^2 W}{d n^2},$$

$$\frac{dW}{dr} + \frac{\partial W}{\partial n} \frac{dn}{dr} - \frac{\partial W}{\partial m} \frac{dm}{dr} = \frac{2}{\pi m_e} \frac{v_b^2}{v^2} \frac{dW}{dn} - (\gamma_e + \gamma_r) W + e^2 \omega_p v_b \gamma_e \frac{dW}{dn}.$$  

(9)

A complete description of equations (9) and (10) can be found in previous works (for example, ref. \textsuperscript{10}). Equation (9) simulates the electron propagation along...
a guiding magnetic flux rope, together with a decrease in density as the guiding magnetic flux rope expands (modelled through the cross-sectional area $A(r)$ of the expanding flux tube). Quasilinear terms\(^{19}\) in both equations describe the resonant wave growth ($\omega_0 = \omega_k$) and the subsequent diffusion of electrons in velocity space. The Langmuir wave angular frequency is $\omega_k$. The background plasma Landau damping rate is $\gamma_L$. Collisions of electron and waves ($\gamma_e$) modify $f(v, r, t)$ and $W(v, r, t)$ in the dense solar corona, where $\Lambda$ is the Coulomb logarithm.

Equation (10) models the spontaneous emission of waves (for example, ref. 39), the propagation of waves, and importantly the refraction of waves on density fluctuations (for example, ref. 38).

We approximate a dynamic spectrum of fundamental emission from the Langmuir wave spectral energy density assuming a saturation level of plasma emission\(^{40–42}\). The brightness temperature $T_b(k, r, t)$ is found using

$$T_b(k, r, t) = \frac{(2\pi)^2}{k^2} \frac{2 \langle n \rangle}{\sqrt{\pi}} \hat{W}_l(k, r, t).$$

(11)

We use the peak value of $T_b(k, r, t)$ to obtain the brightness temperature at each position (frequency), $T_b(r, t)$, as the spread in $k$ is small.

The electron beam is injected as a source function

$$S(v, r, t) = A_s \exp\left(-\frac{r^2}{d^2}\right) A_s \exp\left(-\frac{(1 - t_{inj})^2}{c^2}\right).$$

(12)

The velocity distribution is a power law characterized by $\alpha$, the velocity spectral index. The constant $A$ scales the injected distribution such that the integral over velocity between $v_{min}$ and $v_{max}$ gives the number density $n_{be}$ of injected electrons. The spatial distribution is characterized by $d$ (cm), the spread of the electron beam in distance. The temporal profile is characterized by $t_{inj}$ (s), where $t_{inj}$ = 4$r$. It is normalized by $A_s$ such that the integral over time is 1.

The thermal level of Langmuir waves is set to

$$W^{th}(v, r, t) = \frac{h_k T_e}{4\pi^2} \frac{\alpha^2}{v^2} \ln \left(\frac{v}{v_{Th}}\right),$$

(13)

where $h_k$ is the Boltzmann constant and $T_e$ is the electron temperature. Equation (13) represents the thermal level of spontaneously emitted Langmuir waves from an uniform Maxwellian background plasma when Coulomb collisions are neglected.

We introduce a population of thermal electrons as a background plasma. This background Maxwellian population is characterized by a background temperature $T_b = 2$ MK that corresponds to a background thermal velocity of $v_{Th} = 5.5 \times 10^7$ cm s\(^{-1}\). The choice of 2 MK is related to the higher Langmuir damping that is predicted from the strahl present in the heliosphere (for example, ref. 38).

For the background electron density $n_b(r)$, we calculate the smooth background density profile using the Parker model that solves the equations for a stationary spherical symmetric solution with normalization factor found from satellites\(^{43}\).

$$r^2 n_b(r) = \text{constant}$$

(14)

where the critical velocity $v_c$ is defined such that $v_c \equiv (k_B T_b / \mu m_p)^{\frac{1}{2}}$ and the critical radius is defined by $v_c(r) = G M / 2 r^2$ (both independent on $r$). $T_b$ is the temperature of the solar wind, taken as 1 MK. $T_b$ used in the density model is different from the electron temperature $T_e$ that defines $v_{Th}$. $M$ is the mass of the Sun, $m_p$ is the proton mass and $\mu$ is the mean molecular weight. The constant appearing above is fixed by satellite measurements near the Earth’s orbit (at $r = 1$ au, $n = 6.69$ cm\(^{-3}\)) and equates to $6.3 \times 10^{10}$ cm\(^{-3}\). This model is static in time, set at the start of the simulations, justified through the electron beam moving at least two orders of magnitude faster than the solar wind velocity.

Static background fluctuations are added because the propagating electron beam is travelling much faster than any change in the background density. The spectrum of density fluctuations has a spectral index $\gamma = 5/3$, similar to observations near the Earth (for example, refs. 39–41) between wavelengths 1 Mm and 100 Mm, so that the perturbed density profile is given by

$$n(r) = n_b(r) \left[1 + C \sum_{i=1}^{N} \langle \Delta n_i^2 \rangle \sin(2\pi r / \lambda_i + \phi_i)\right].$$

(16)

where $N = 1,000$ is the number of perturbations, $n_b(r)$ is the initial unperturbed density as defined above, $\lambda_i$ is the wavelength of the $i$th fluctuation, $\mu = 5/3$ is the power-law spectral index in the power spectrum and $\phi_i$ is the random phase of the individual fluctuations. $C$ is the normalization constant that defines the root-mean-square deviation of the density $\sqrt{\langle \Delta n_i^2 \rangle}$ such that

$$C \sum_{i=1}^{N} \langle n_i^2 \rangle \sim 2 \Delta n_i^2.$$

(17)

Our one-dimensional approach means that we are only modelling fluctuations parallel to the magnetic field and not perpendicular.

We model an electron beam injection into the corona with a timescale $\tau = 0.001$ s that was near instantaneous to replicate energization via a magnetic reconnection. We used a reasonably dilute electron beam where $n_b/n_e = 10^{-3}$, accelerated in a region of longitudinal length 10 Mm, typical for coronal electron beams\(^{44}\). The electrons initially have a power-law distribution with a spectral index of 8 in velocity space, ranging from 4.4 to 38v_th (1.7–125 keV). Electron populations with smaller or larger velocities typically do not produce substantial levels of Langmuir waves above the thermal level.

The electron density is exponential in the solar corona (see ref. 34, for details), and has approximately $r^{-2}$ decrease in interplanetary space. In addition, turbulence over a spectrum of length scales typically below 100 Mm fluctuates the electron density with an intensity that increases with distance from the solar surface\(^{45}\), but remains much more constant over interplanetary distances\(^{46}\). We modelled the turbulence by increase the turbulent intensity $\Delta n/n = 10^{-5}$ near the Earth. The coronal plasma had a bulk temperature of 1 MK, giving a thermal velocity of 3.9 Mm s\(^{-1}\).

Density fluctuation wavelength. The simulations carried out in the ‘Langmuir wave modulation’ section were very similar to the main type III burst simulations but with some notable changes. The background density $n_b(r)$ was set constant to $1.1 \times 10^{10}$ cm\(^{-3}\), which corresponds to a plasma frequency of 30 MHz. The initial beam density was set as a broken power law in velocity space, varying as $r^{-4}$ when $v \geq v_{Th}$ with a break energy of 10 keV, and constant when $v < v_{Th}$. The reduction in electron flux at lower velocities was to account for the lack of Coulomb collisions in the lower corona. The second term in equation (9) did not include the radial expansion and such was $v_{Th}^2$ to enhance the level of Langmuir waves generated by the simulation.

The wavelength of background density fluctuations controls how regular the modulations occurs of beam-driven Langmuir waves. When the wavelength is reduced by a factor of five, from $\lambda = 50$ Mm to $\lambda = 10$ Mm, we can see modulation in Langmuir wave growth causes the energy density to oscillate with a correspondingly shorter length scale. This is shown in Extended Data Fig. 3. The oscillation causes a peak at higher wavenumbers, shown in Fourier space.

Radio fluctuations. We calculate the intensity of the fluctuations in the peak brightness temperature, obtaining $\Delta I / I$, between 30 MHz and 40 MHz, corresponding to a length of nearly 100 Mm. We subtract a smoothed function with a smoothing box of one-third the box size, 30 Mm to find the root-mean-square deviation. An example of the radio fluctuations and the smoothed function is given in Extended Data Fig. 2.

Comparing the simulated peak brightness temperature with the observed type III peak flux, the striae clumps display a similar pattern of steep rise and shallow decay. The clamp structures in the curves of Extended Data Fig. 1 show a similar pattern to the clamp structures in the simulated curves in Extended Data Fig. 2, increasing sharply and then decreasing. The rise and decay of the radio bursts is not as smooth as the simulated bursts, presumably related to smaller length scales that are present in the solar corona that were not simulated.

To calculate the power density spectra, we summed the square of the Fourier transform of the type III flux over the duration of the radio bursts between 30 MHz and 40 MHz. The corresponding altitude associated with each frequency was found using the Parker density model. For the simulated radio bursts, we found the Fourier transform between 30 MHz and 80 MHz to reduce the noise in the power spectrum.

Data availability

The simulation datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request. LOFAR data used during the current study are publicly available on the LOFAR Long Term Archive at https://lta.lofar.eu/ under the project codes LC3_012 and LC4_016. The data used to make the plots in the paper are available on the UCL Research Data Repository using https://doi.org/10.5525/04/14140077.

Code availability

The code used to make the plots in the paper is available on the UCL Research Data Repository using https://doi.org/10.5525/04/141400679. The code used to generate the datasets in our study is currently in preparation to be made publicly available.

In the interim period, the code can be made available from the corresponding author on reasonable request.

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Extended Data Fig. 1 | See next page for caption.
Extended Data Fig. 1 | Illustration of the observed type III radio intensity fluctuations. The peak flux for the fundamental emission of the three type III bursts observed by LOFAR, shown in Fig. 1, on a, 16th April, b, 24th June and c, 16th September. Errors calculated from the square root of the radio flux are indicated in red. The smoothed functions, used to calculate the values of $\Delta I/I$ are shown in green.
Extended Data Fig. 2 | Illustration of the simulated type III radio intensity fluctuations. a. Peak brightness temperature as a function of frequency for the simulated type III dynamic spectra propagating through plasma where $\Delta n/n = 2.5 \times 10^{-3}$. The smoothed function used to calculate $\Delta I/I$ is overplotted in green. b. Same as panel a but $\Delta n/n = 2.5 \times 10^{-4}$. 
Extended Data Fig. 3 | Simulated beam-drive Langmuir wave energy density and associated power density spectra. a. Beam-generated Langmuir wave energy density $U_{\text{lw}}$ as a function of distance. The background plasma has a sinusoidal perturbation with amplitude $A = 0.001$ and wavelength $\lambda = 50$ Mm in blue and $\lambda = 10$ Mm in red. The lower panel highlights the modulation in the background plasma gradient length scale $L = 0.5 n_e^{-1} d n_e/dx$. b. Power density spectrum of the Langmuir wave energy density minus the unperturbed case from the simulations shown in panel a. The peaks occur at the wavenumber $k = 2\pi/\lambda$ Mm$^{-1}$ and the harmonics. The simulation with a lower wavelength density fluctuations clearly modulates the Langmuir waves at higher wavenumbers.