Mass gap in the monopole-plus-sea-quarks system

Vladimir Dzhunushaliev, Vladimir Folomeev, and Albina Serikbolova

1 Department of Theoretical and Nuclear Physics, Al-Farabi Kazakh National University, Almaty 050040, Kazakhstan
2 Institute of Experimental and Theoretical Physics, Al-Farabi Kazakh National University, Almaty 050040, Kazakhstan
3 Academician J. Jeenbaev Institute of Physics of the NAS of the Kyrgyz Republic, 265 a, Chui Street, Bishkek 720071, Kyrgyzstan

(Dated: January 14, 2020)

Monopole solutions in SU(2) Yang-Mills theory which includes spinor fields described by the nonlinear Dirac equation are obtained. The physical interpretation of these solutions as describing a monopole-plus-sea-quarks system is suggested. It is demonstrated that the energy spectrum of such a system possesses a mass gap whose appearance is brought about solely by the nonlinear spinor fields. It is shown that the monopole solution obtained differs in principle from the ’t Hooft-Polyakov monopole in that it is topologically trivial. The statistical and thermodynamic properties of quark-gluon plasma filled with quasiparticles described by the solutions of the monopole-plus-sea-quarks type obtained here are studied. For such a plasma, we obtain an equation of state and calculate its internal energy.

PACS numbers: 12.38.Mh, 11.15.Tk, 12.38.Lg, 11.15.-q
Keywords: non-Abelian SU(2) theory, nonlinear Dirac equation, monopole, energy spectrum, mass gap

I. INTRODUCTION

The mass gap problem in quantum chromodynamics (QCD) is one of the central problems in the theory of strong interactions. This problem is closely related to the problem of confinement in QCD, and is believed to be resolved only using the methods of nonperturbative quantization, as applied to SU(3) Yang-Mills theory. Unfortunately, as of now, there is no universal method to go beyond the perturbative quantization, but there are only certain approaches that allow one to analyse some nonperturbative effects, including studies within a lattice gauge theory [1, 2].

Another possibility is to study simpler problems where quantum systems are replaced by approximate classical systems. As applied to a consideration of the mass gap problem, such an approach was used in our recent paper [3]. In that work we showed that, in non-Abelian Proca theory containing a Higgs scalar field and nonlinear spinor fields, there are regular particlelike solutions whose energy spectrum possesses a mass gap (with some limitations on the parameters determining these solutions). It was clarified that the reason for the appearance of the gap is the presence of the nonlinear spinor field. At the same time, the analysis of the corresponding equations indicates that, in the absence of the Higgs field, there are already no such solutions. A further analysis in this direction shows that, unlike non-Abelian Proca theory, in Yang-Mills theory, particlelike solutions can exist without a Higgs field. If only one could prove for this case that the energy spectrum of such particlelike objects has a mass gap, this would be of special interest.

Apparently, within nonlinear Dirac theory, the presence of a mass gap was first demonstrated in Refs. [4, 5] where it was shown that the energy spectrum of spherically symmetric solutions has a global minimum. The corresponding particle was called “the lightest stable particle,” since the term “mass gap” was not yet known at that time. Further investigations of the mass gap problem showed that even in the absence of quarks (i.e., in the case of purely gluon systems) there is still no precise understanding of how a gap can form in the energy spectrum [2]. In particular, glueball states predicted in QCD can be obtained within lattice-QCD calculations, but they, firstly, have not been unambiguously observed and, secondly, their predicted masses can be considerably modified by quark-antiquark mixing effects. This motivates some investigations on the subject. In Ref. [6], the influence of magnetic field on the behavior of the mass gap for quarks was studied. In Ref. [7], QCD-like gauge theories formulated on small $S^3 \times R^3$ spacetime were considered.

*Electronic address: v.dzhunushaliev@gmail.com
†Electronic address: vfolomeev@mail.ru
‡Electronic address: albeni2395@mail.ru
and a new mechanism of confinement was presented. The author of Ref. [3] showed that the vacuum condensate of dimensions 2 can provide the effective mass for gluons and ghosts; this can be the reason for the appearance of the mass gap and of confinement.

In this paper, we demonstrate the presence of a mass gap in SU(2) Yang-Mills theory containing also a spinor field described by the nonlinear Dirac equation. To do so, we seek monopole solutions with the source of magnetic field in the form of color charge created by the spinor field. In Ref. [3], we assumed that the nonlinear Dirac equation may approximately describe sea quarks interacting with sea gluons. Following this assumption, one can say that the monopole solutions obtained here describe a magnetic field created by a lump of sea quarks. Such a monopole-plus-sea-quarks object may act as a quasiparticle in a quark-gluon plasma introduced in Ref. [9] to understand the statistical physics of quark-gluon plasma consisting of quasiparticles of the monopole-plus-sea-quarks type. Finally, in Sec. VII we summarize the results obtained.

II. THEORY OF YANG-MILLS FIELDS COUPLED TO A NONLINEAR DIRAC FIELD

The Lagrangian describing a system consisting of a non-Abelian SU(2) field \( A^a \) interacting with nonlinear spinor field \( \psi \) can be taken in the form

\[
\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + i\hbar \bar{\psi} \gamma^\mu D_\mu \psi - m_f \hbar^2 \bar{\psi} \psi + \frac{\Lambda}{2} g \hbar \bar{\psi} (\bar{\psi} \psi)^2 .
\]  

(1)

Here \( m_f \) is the mass of the spinor field; \( D_\mu \) is the gauge-covariant derivative, where \( g \) is the coupling constant and \( \sigma^a \) are the SU(2) generators (the Pauli matrices); \( F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g \epsilon_{abc} A^b_\mu A^c_\nu \) is the field strength tensor for the SU(2) field, where \( \epsilon_{abc} \) (the completely antisymmetric Levi-Civita symbol) are the SU(2) structure constants; \( \Lambda \) is a constant; \( \gamma^\mu \) are the Dirac matrices in the standard representation; \( a, b, c = 1, 2, 3 \) are color indices and \( \mu, \nu = 0, 1, 2, 3 \) are spacetime indices.

Using Eq. (1), the corresponding field equations can be written in the form

\[
D_\mu F^{a\mu\nu} = \frac{g \hbar c}{2} \bar{\psi} \gamma^\mu \sigma^a \psi,
\]

(2)

\[
i\hbar \gamma^\mu D_\mu \psi - m_f c \psi + \Lambda g \hbar \bar{\psi} (\bar{\psi} \psi) = 0.
\]

(3)

Let us enumerate some distinctive features of the system under consideration: (i) The set of equations (2) and (3) has monopole-like solutions only for some special choices of the system parameters \( f_2 \) and \( u_1 \) [for their definition see Eq. (11)]; (ii) In the absence of the vector field \( A^a_\mu \), there exist particlelike solutions of the nonlinear Dirac equation (3) which describe a system with a mass gap \( \Lambda \neq 0 \); (iii) In the absence of the spinor field, the Yang-Mills equation (2) has no static globally regular solutions \( \Lambda = 0 \); (iv) To the best of our knowledge, in the case of linear spinor field (i.e., when \( \Lambda = 0 \)) the set of equations (2) and (3) has no static regular solutions as well.

To obtain particlelike solutions, Eqs. (2) and (3) will be solved numerically as an eigenvalue problem for the parameters \( f_2 \) and \( u_1 \), since apparently it is impossible to find their analytical solution.

III. ANS"ATZE AND EQUATIONS

We seek monopole-like solutions to Eqs. (2) and (3) describing objects consisting of a radial magnetic field and a nonlinear spinor field. For this purpose, we employ the standard SU(2) monopole Ansatz

\[
A^a_\mu = \frac{1}{g} [1 - f(r)] \begin{pmatrix} 0 & \sin \varphi & \sin \theta \cos \theta \cos \varphi \\ 0 & -\cos \varphi & \sin \theta \cos \theta \sin \varphi \\ 0 & 0 & -\sin^2 \theta \end{pmatrix} , \quad i = r, \theta, \varphi \quad (\text{in polar coordinates}),
\]

(4)

\[
A^a_\nu = 0,
\]

(5)

and the Ansatz for the spinor field from Refs. [11, 12]

\[
\psi^T = \frac{e^{-i \Phi}}{gr \sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ -u \end{pmatrix} , \begin{pmatrix} u \\ 0 \\ -iv \cos \theta \end{pmatrix} , \begin{pmatrix} 0 \\ -iv \cos \theta \\ -iv \sin \theta \cos \varphi \end{pmatrix} , \begin{pmatrix} 0 \\ -iv \sin \theta \sin \varphi \end{pmatrix} ,
\]

(6)
where $E/\hbar$ is the spinor frequency and the functions $u$ and $v$ depend on the radial coordinate $r$ only. In Eq. (10), each row describes a spin-$1/2$ fermion, and these two fermions have the same mass $m_f$ and opposite spins and are located at one point. Aside from this, for each of such fermions, the energy-momentum tensors will not be spherically symmetric (due to the existence of nondiagonal components), but their sum will give a tensor compatible with spherical symmetry of the system under consideration.

Equations for the unknown functions $f, u,$ and $v$ can be obtained by substituting the expressions (1)-(4) into the field equations (2) and (4),

$$-f'' + f \left( f^2 - 1 \right) = 0,$$
$$f' f = u \left( -m_f + \tilde{\omega} - \tilde{\omega}^2 \right),$$
$$u' - f u = v \left( -m_f - \tilde{\omega} - \tilde{\omega}^2 \right).$$

Here, for convenience of making numerical calculations, we have introduced the following dimensionless variables: $x = r/r_0$, where $r_0$ is a constant corresponding to the characteristic size of the system under consideration; $\tilde{u} = \sqrt{r_0 u} g, \tilde{v} = \sqrt{r_0 v} g, \tilde{m}_f = r_0 m_f c / \hbar, \tilde{E} = r_0 E / (\hbar c), \tilde{\Lambda} = (g / c^2) \Lambda, \tilde{g}^2 = g^2 c$. The prime denotes differentiation with respect to $x$.

The parameter $r_0$ must depend only on constants of a theory; therefore one can take, say, $r_0 = \hbar / (m_f c)$.

The total energy density of the monopole-plus-spinor-fields system under consideration is

$$\tilde{\epsilon} = \tilde{\epsilon}_m + \tilde{\epsilon}_s = \frac{1}{g^2} \left[ f'^2 + \frac{(f^2 - 1)^2}{2 x^4} \right] + \frac{1}{2} \frac{(\tilde{u}^2 + \tilde{v}^2)}{x^2} + \frac{1}{2} \frac{(\tilde{u}^2 - \tilde{v}^2)}{x^4},$$

where the expressions in the square brackets correspond to the dimensionless energy densities of the monopole, $\tilde{\epsilon}_m \equiv (r_0^2 / \hbar c) \epsilon_m$, and of the spinor field, $\tilde{\epsilon}_s \equiv (r_0^2 / \hbar c) \epsilon_s$.

**IV. MONOPOLE-PLUS-SPINOR-FIELDS SOLUTIONS**

This section is devoted to the numerical study of monopole-plus-spinor-fields solutions of Eqs. (7)-(9). Because of the presence of the terms containing $x$ in the denominators of these equations, to perform numerical computations, we assign boundary conditions near the origin $x = 0$ where solutions are sought in the form of the Taylor series

$$f = 1 + f_2 x^2 + \ldots, \quad \tilde{u} = \tilde{u}_1 x + \frac{\tilde{u}_3}{3!} x^3 + \ldots, \quad \tilde{v} = \frac{\tilde{v}_2}{2} x^2 + \frac{\tilde{v}_4}{4!} x^4 + \ldots,$$

where $\tilde{v}_2 = 2 \tilde{u}_1 \left( \tilde{E} - \tilde{m}_f + \tilde{\Lambda} \tilde{u}_1 \right) / 3$ and the expansion coefficients $f_2$ and $\tilde{u}_1$ are free parameters whose values cannot be found from Eqs. (7)-(9).

Eqs. (7)-(9) are solved numerically as a nonlinear problem for the eigenvalues $f_2$ and $\tilde{u}_1$ and the eigenfunctions $\tilde{u}, \tilde{v},$ and $f$, whose typical behavior is shown in Fig. 1. The corresponding computed values of the system parameters are given in Table 1.

Asymptotically (as $x \to \infty$), the behavior of the solutions is

$$f(x) \approx 1 - \frac{f_\infty}{x}, \quad \tilde{u}(x) \approx \tilde{u}_\infty e^{-x \sqrt{\tilde{m}_f^2 - \tilde{E}^2}}, \quad \tilde{v}(x) \approx \tilde{v}_\infty e^{-x \sqrt{\tilde{m}_f^2 - \tilde{E}^2}},$$

FIG. 1: The functions $\tilde{u}(x)/x, \tilde{v}(x)/x,$ and $f(x)$ for different values of the parameter $\tilde{E}$ with $\tilde{\Lambda} = 8, \tilde{m}_f = 1,$ and $\tilde{g} = 1.$
FIG. 2: The distributions of the color magnetic fields for different values of the parameter $\tilde{E}$: the radial component $\tilde{H}_r^a \equiv g r_0^2 H_{r}^a$ is given by Eq. (12) and the tangential components $H_{\theta,\varphi}^a \equiv g r_0 H_{\theta,\varphi}^a$ by Eq. (14).

### Table I: Eigenvalues $\tilde{u}_1$ and $f_2$ and the total energy $\tilde{W}_t$ from Eq. (15) for different values of the parameter $\tilde{E}$.

| $\tilde{E}$ | $f_2$  | $\tilde{u}_1$ | $\tilde{W}_t$ |
|-----------|-------|--------------|-------------|
| 0.555    | -0.21167 | 0.510757372 | 15.6339 |
| 0.655    | -0.1438 | 0.497238675 | 11.7187 |
| 0.755    | -0.092587 | 0.47145607 | 8.6202 |
| 0.855    | -0.0519 | 0.4184815 | 6.4621 |
| 0.955    | -0.016338 | 0.2834 | 5.8124 |
| 0.966    | -0.012473 | 0.2534 | 6.049896 |
| 0.977    | -0.0085451 | 0.2151 | 6.5242 |
| 0.988    | -0.0046615 | 0.163 | 7.3827 |

where $f_\infty$, $\tilde{u}_\infty$, and $\tilde{v}_\infty$ are integration constants.

It is of interest to follow the behavior of the magnetic Yang-Mills field. Its physical components can be defined as $H_i^a = -(1/2) \sqrt{\gamma} \epsilon_{ijk} F_{ajk}$, where $i, j, k$ are space indices. In our case this gives for the radial magnetic field

$$H_r^a \sim \frac{1 - f^2}{gr^2},$$

(12)

where $a = 1, 2, 3$ and we have dropped the dependence on the angular variables. The corresponding graphs for this component are shown in Fig. 2. In turn, its asymptotic behavior as $x \to \infty$ is

$$H_r^a \sim \frac{2 f_\infty}{gr^3}.$$  

(13)

It is seen from this expression that, by its asymptotic behavior, the system monopole-plus-nonlinear-spinor-fields differs in principle from the ’t Hooft-Polyakov monopole, whose magnetic field decreases as $r^{-2}$. Also, recall that in the present paper we assume that the nonlinear spinor field may approximately describe the quantum interaction between sea quarks and gluons (see Introduction). Thus the solution obtained here can be regarded as describing a monopole with a non-Abelian magnetic field and a lump of sea quarks.

Nonzero tangential components of the magnetic field are

$$H_{\theta}^a \sim \frac{1}{g} f', \quad H_{\varphi}^b \sim \frac{1}{g} f',$$

(14)

where $a = 1, 2, 3$ and $b = 1, 2$. Their behavior is shown in Fig. 2.

Thus in this section we have obtained the spherically symmetric solutions describing the self-consistent system consisting of the non-Abelian magnetic field and sea quarks. Let us emphasize the important feature of the monopole described by Eqs. (7)-(9): this monopole is topologically trivial, since for its existence the presence of a scalar field triplet, whose behavior at spatial infinity is topologically nontrivial, is not needed.

### V. ENERGY SPECTRUM

In this section we obtain the energy spectrum of the configuration under consideration as a function of the parameter $\tilde{E}$ and demonstrate the presence of a mass gap in such a system. For this purpose, we employ an expression for a dimensionless total energy of the system in question,

$$\tilde{W}_t = \frac{W_t}{\hbar c/r_0} = 4\pi \int_0^\infty x^2 \tilde{\epsilon} dx = \left(\tilde{W}_t\right)_m + \left(\tilde{W}_t\right)_s,$$

(15)
where the energy density $\tilde{\epsilon}$ is taken from Eq. (10). One can see from this formula that the total energy is split into a sum of energies of the monopole, $\tilde{W}_m$, and of the spinor fields, $\tilde{W}_s$, despite the presence of the direct interaction between the vector and spinor fields. The corresponding distributions of $\tilde{\epsilon}$ along the radius are shown in Fig. 3. In turn, using Eq. (15), we have calculated the magnitudes of the total energy given in Table I. Using them, we have plotted in Fig. 4 the corresponding energy spectrum of the system. According to the assumption of Ref. [3], such a system can be regarded as a SU(2) monopole with sea quarks that are approximately described by the nonlinear Dirac equation.

The calculated data for $\tilde{W}_t$ given in Table I can be interpolated by the fitting formula

$$[\tilde{W}_t]_{\text{fit}} = a\tilde{E}^\alpha + b \left(1 - \tilde{E}\right)^\beta$$

(16)

with $a = 4.80$, $\alpha = -2.02$, $b = 0.03$, and $\beta = -1.01$. Using this formula, we have plotted the curve in Fig. 4 whose minimum corresponds to a mass gap.

Fig. 4 also shows the magnitudes of the characteristic size of the system $\tilde{l}_0$ which is determined as the semi-width of the energy density graphs given in Fig. 3. These sizes can be interpolated by the expression

$$[\tilde{l}_0]_{\text{fit}} = c\tilde{E}^\gamma + d \left(1 - \tilde{E}\right)^\delta$$

(17)

with $c = 0.91$, $\gamma = -0.97$, $d = 0.06$, and $\delta = -0.75$. This formula will be used in calculations of Sec. VI B.

Thus in this section we have obtained the energy spectrum of the self-consistent monopole-plus-sea-quarks system modeled within SU(2) Yang-Mills theory containing the doublet of nonlinear spinor fields. The important result of the calculations is that the energy spectrum possesses the mass gap (see Fig. 4).

In this connection, let us note that here we have obtained the mass gap in the (3 + 1)-dimensional theory. For the sake of comparison, we may mention that in (2 + 1)-dimensional SU(2) Yang-Mills theory considered in Ref. [1] the presence of a mass gap was demonstrated from the analysis of the dependence of the mass on the coupling constant $g$, and the results obtained were compared to those found in lattice calculations (see Fig. 12.3 of Ref. [1]).

VI. THERMODYNAMICS AND STATISTICAL PHYSICS OF PLASMA CONTAINING QUASIPARTICLES OF THE MONOPOLE-PLUS-SEA-QUARKS TYPE

In this section we employ the solutions obtained above for describing quasiparticles in a quark-gluon plasma. In having the energy spectrum of such quasiparticles, we will calculate the statistical integral; this will permit us to obtain an equation of state for such a plasma and to determine some thermodynamic functions. This investigation continues the study of the thermodynamic properties of plasma consisting of quasiparticles that was begun by us in Ref. [12] where the solutions supported only by a nonlinear spinor field were obtained. Since the spinor fields under
investigation act as a source in the Yang-Mills equations, the next natural step in studying this type of plasma is to incorporate gauge fields into the consideration of statistical and thermodynamic properties of the plasma.

In the previous sections we have obtained the required solutions describing the doublet of nonlinear spinor fields which are the source of the SU(2) gauge field. For simplicity, in what follows we will examine a nonrelativistic plasma where quasiparticles do not interact with one another. Such a possibility follows from the fact that the quasiparticles under consideration are monopoles interacting through a color magnetic field. If the sizes of quasiparticles are neglected compared with the distances between them, then the force of their interaction will be defined by the generalized expression for the Lorentz force appearing in the right-hand side of Wong’s equations [14]

\[
m c \frac{d^2 x^\mu}{ds^2} = -g F^{a\mu}_\nu T^a \frac{dx^\nu}{ds},
\]

(18)

\[
dT^a = -g e_{abc} A^b_\mu T^c \frac{dx^\mu}{ds},
\]

(19)

where \( T^a \) are algebra generators (the color charge). The right-hand side of Eq. (18) is a non-Abelian generalization of the Lorentz force from Maxwell’s electrodynamics exerted on a charged particle of the mass \( m \). It is easy to show that, in a non-Abelian case, the force exerted on a particle by a non-Abelian magnetic field is perpendicular to the four-velocity \( dx^\mu/ds \); therefore the corresponding work done by the field on the particle is zero. This means that, in this approximation, the potential energy of interaction of monopoles is zero; therefore in calculating the statistical integral the total energy of a quasiparticle will contain only a kinetic energy term.

Now we recall some terms and definitions used in Ref. [13].

### A. Statistical and thermodynamic quantities

The total energy of a nonrelativistic quasiparticle \( W \) consists of two parts: the energy \( W_q \) (associated with the energy related to the presence of the internal field structure) and the kinetic energy \( W_k \). That is,

\[
W = W_q + W_k \equiv W_q + \frac{c^2 \rho^2}{2W_q},
\]

(20)

where we have taken into account that the mass of a quasiparticle is \( m_q = W_q/c^2 \).

The volume occupied by \( N \) quasiparticles is

\[
V_q = \sum_{i=1}^N v_i \approx NVq = Vn_q \overline{v}_q,
\]

(21)

where \( v_i \) is the characteristic volume occupied by the \( i \)-th quasiparticle; \( n_q \) is the concentration of quasiparticles; \( \overline{v}_q \) is the average value of the volume of one quasiparticle. After some simple algebra, the statistical integral can be written in the form (for details see Ref. [13])

\[
Z(T) \approx Z_{\text{quarks + gluons}} \left[ \int dVd\rho Z(\gamma) e^{-\frac{W_q(\gamma)}{T} + \frac{c^2 \rho^2}{2W_q} - \Delta \gamma} \right]^N = Z_{\text{quarks + gluons}} Z_{\text{quasiparticles}}.
\]

(22)

Here \( Z_{\text{quarks + gluons}} \) is the statistical integral for valence quarks and gluons; \( Z_{\text{quasiparticles}} \) is the statistical integral associated with the presence of the internal structure of quasiparticles; \( \gamma \) is the set of parameters on which the energy of quasiparticles depends; \( T \) is the temperature; \( \rho(\gamma) \) is the density of states. The constant \( \Delta \) corresponds to the minimum energy of a quasiparticle (the mass gap) from which the energy will be reckoned.

After some simplifications, we get the following expression for the statistical integral [13]:

\[
Z_{\text{quasiparticles}}(T) = Z_0 T^{3N/2} (V - V_q)^N \left[ \int W_q^{3/2}(\gamma) \rho(\gamma) e^{-\frac{W_q(\gamma) - \Delta}{T} + \Delta} \gamma \right]^N = Z_0 T^{3N/2} (V - V_q)^N (Z_q)^N = Z_0 T^{3N/2} V^N (1 - n_q \overline{v}_q)^N (Z_q)^N.
\]

(23)

Here \( Z_q \) is the statistical integral per one quasiparticle and all dimensional constants are collected in the normalization constant \( Z_0 \).
The internal energy of quasiparticles is defined as

$$U_{\text{quasiparticles}} = \frac{1}{Z_{\text{quasiparticles}}} \sum_{i=1}^{N} \left[ W_{q,i}(\gamma_i) + W_{k,i} - \Delta \right] e^{-\frac{\sum_{i=1}^{N} [W_{q,i}(\gamma_i) + W_{k,i} - \Delta]}{kT}} \prod_{i=1}^{N} dV_i dp_i \rho(\gamma_i) d\gamma_i,$$

(24)

where $W_{q,i}$ and $W_{k,i}$ correspond to energies of the $i$-th particle. Since

$$\int e^{-W_q(\gamma_i)+W_k \frac{x^2}{2}} dV dp d\gamma \sim T^{3/2} (V - V_q) \int W_q^{3/2}(\gamma_i) e^{-W_q(\gamma_i)} d\gamma,$$

(25)

the expression for the internal energy (24) takes the form

$$\frac{U_{\text{quasiparticles}}}{N} \equiv U_{\text{quasiparticle}} = \frac{\int W_q^{5/2}(\gamma_i) e^{-W_q(\gamma_i)} d\gamma}{\int W_q^{3/2}(\gamma_i) e^{-W_q(\gamma_i)} d\gamma} - \Delta + \frac{c}{2} \frac{\int W_q^{2}(\gamma_i) e^{-W_q(\gamma_i)} d\gamma}{\int e^{-W_q(\gamma_i)} d\gamma}.$$

(26)

The gas pressure of one quasiparticle is defined as

$$\frac{p_{\text{quasiparticles}}}{N} = -\frac{1}{N} \frac{\partial F_{\text{quasiparticles}}}{\partial V} = \frac{kT}{V - V_q} = \frac{kT}{V (1 - n_q \bar{\nu}_q)}.$$

(27)

**B. Equation of state and the internal energy of the plasma**

In order to describe the statistical and thermodynamic properties of the plasma consisting of quasiparticles of the monopole-plus-sea-quarks type, we use the expression for the total energy of a quasiparticle

$$W_q(\tilde{E}) = 4\pi \frac{\hbar c}{r_0} \int_0^\infty x^2 \tilde{w}_q(\tilde{E}) dx = 4\pi \frac{\hbar c}{r_0} \tilde{W}_q(\tilde{E}),$$

(28)

where the energy density of one quasiparticle $\tilde{w}_q \equiv \tilde{\epsilon}$ is taken from Eq. (10). Here, we have introduced the dimensionless energy $\tilde{W}_q(\tilde{E})$ whose value is actually determined only by one parameter $\tilde{E}$. Then, putting $\gamma = \tilde{E}$, the statistical integral $Z_q$ from (23) takes the form

$$Z_q \sim \Delta^{3/2} \int \tilde{W}_q^{3/2}(\tilde{E}) e^{-\frac{W_q(\tilde{E})}{kT}} d\tilde{E},$$

(29)

where, for convenience of making numerical calculations, we rescale the integrand variables $W_q$ and $T$ in Eq. (23) as follows: $\tilde{W}_q = W_q / \Delta$ and $\tilde{T} = T / \Delta$ with the dimensionless $\tilde{T} = T / T_0$ and $\Delta = \Delta / (kT_0)$ (here $T_0$ is some characteristic temperature).

Due to the presence of the internal structure of quasiparticles, the equation of state is modified as

$$p_q = -\frac{\partial F_q}{\partial V} = kT \frac{\partial \ln Z_q}{\partial V} = kN \frac{T}{V - V_q},$$

(30)

where we have used the expression (27). Remembering that the volume $V_q$ can be estimated as $V_q \approx N \bar{\nu}_q$, where $N$ is the quasiparticle number and $\bar{\nu}_q(T)$ is the average volume of one quasiparticle which depends on temperature, we can rewrite Eq. (30) in the form

$$\frac{p_q}{n_q} = \frac{kT}{1 - n_q \bar{\nu}_q(T)}.$$  

(31)

where we have separated out the term depending on temperature. Notice that from the physical point of view (in the spirit of the van der Waals equation) the expression $(n_q \bar{\nu}_q^3) \bar{\nu}_q^{-4}$ is nothing but the fraction of the volume $V$ occupied by the quasiparticles. It is evident that when this fraction tends to unity the pressure goes to infinity and a phase transition must occur. Fig. 5 shows the corresponding graph for the equation of state (31) where the average volume $\bar{\nu}_q(T)$ can be calculated using the expression for the characteristic size (17).
Finally, according to Eq. (26), a contribution to the internal energy from one quasiparticle, related to the internal structure of the quasiparticle, is defined by the formula

$$\tilde{U}_{\text{quasiparticle}}(\tilde{T}) \equiv \frac{\tilde{U}_{\text{quasiparticles}}(\tilde{T})}{N} = \int \frac{\tilde{W}_q^{5/2}(\gamma)}{\tilde{T}} e^{-\frac{\tilde{W}_q(\gamma)}{\tilde{T}}} d\gamma - \tilde{\Delta},$$

(32)

where the dimensionless \(\tilde{U}_{\text{quasiparticles}} = U_{\text{quasiparticles}}/(kT_0)\) and \(\gamma = \tilde{E}\). The behavior of this term is shown in Fig. 5.

Thus in this section we have obtained the equation of state and the internal energy for the plasma consisting of quasiparticles of the monopole-plus-sea-quarks type and shown that, with increasing temperature, in such a plasma a phase transition occurs. This phase transition is the result of the fact that the volume occupied by quasiparticles increases with rising temperature; this ultimately leads to the destruction of quasiparticles.

VII. SUMMARY

Within SU(2) Yang-Mills theory, we have studied a system consisting of monopole and doublet of nonlinear spinor fields, the inclusion of which enables us to describe sea quarks approximately, as was suggested in Ref. [3]. Summarizing the results obtained,

- We have found regular finite-energy solutions, the physical interpretation of which is that they describe a self-consistent monopole-plus-sea-quarks system.
- The key result is that we have shown the presence of a mass gap in the energy spectrum of such a system, whose appearance is caused by the nonlinear spinor fields.
- The solutions obtained have been used to describe quasiparticles (each of which is a monopole-plus-sea-quarks system) in a quark-gluon plasma introduced in Ref. [9].
- Within the simplified model of such a quark-gluon plasma, we have obtained the equation of state and the internal energy of the plasma and shown that, in the plasma, there will inevitably occur a phase transition caused by a reconstruction of the internal structure of quasiparticles.

In conclusion, we note that in some respect the nonlinear Dirac equation is similar to the Ginzburg-Landau equation: while the Ginzburg-Landau equation describes a Cooper pair of electrons connected by phonons, the nonlinear Dirac equation in our case describes a pair of quarks connected by a flux tube. In both cases (electron-electron and quark-quark pairs), the total spin of the pair is zero. However, in our approximation the quarks are located at one point; therefore the influence of the flux tube on the physical structure of the system under consideration is not taken into account.
Acknowledgements

This work was supported by Grant No. BR05236730 in Fundamental Research in Natural Sciences by the Ministry of Education and Science of the Republic of Kazakhstan. V.D. and V.F. also are grateful to the Research Group Linkage Programme of the Alexander von Humboldt Foundation for the support of this research.

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