Exact Supersymmetric Amplitude for $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ Mixing

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Abstract

We present the most general supersymmetric amplitude for $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ mixing resulting from gluino box diagrams. We use this amplitude to place general constraints on the magnitude of flavor-changing squark mass mixings, and compare these constraints to theoretical predictions both in and beyond the Minimal Supersymmetric Standard Model.
Flavor changing neutral currents (FCNC’s) provide an important test of the radiative structure of the Standard Model and a sensitive probe of new physics beyond the Standard Model. Indeed, new particles proposed by alternative theoretical frameworks can generate observable FCNC effects, even for new particle masses well beyond the range of present and proposed accelerators. For many such frameworks, $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ mixing provide the most sensitive experimental probe of these FCNC’s.

During the past eight years, there have been a number of theoretical papers exploring FCNC’s in supersymmetry. These papers generally disagree on the magnitude of supersymmetric FCNC’s by as much as several orders of magnitude. These disagreements arise because there has been no systematic calculation of the relevant supersymmetric amplitudes for most FCNC processes comparable to those that have been performed in the Standard Model. This is because the supersymmetric contributions are more complicated, involving many new particles (e.g., six complex charge 2/3 squark fields) which mix in a complicated way.

In this paper, we present the exact Feynman amplitude for the dominant, gluino box contribution to $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ mixing. This calculation may be organized in either of two frameworks: first, using exact Feynman rules for mass-eigenstate squark fields with arbitrary off-generational squark-gluino couplings; second, as a perturbative expansion in small, off-generational squark mass insertions assuming diagonal squark-gluino couplings (and non-mass-eigenstate squark fields). The latter formulation affords the most convenient language both for expressing experimental constraints and for comparing these constraints against different theoretical predictions resulting from various supersymmetric models.

We have verified that the result in the mass insertion framework can be systematically derived from the result in the mass eigenstate basis by expanding around the universal squark mass. This provides an important consistency check on the results obtained from both computational frameworks. Since, for $M_{\tilde{q}} \lesssim 1$ TeV, the magnitude of the expansion parameters, $\delta \tilde{m}_{AB}^2 / M_{\tilde{q}}^2$, consistent with experiment are all much less than one, an expansion to first order in these small parameters provides an excellent approximation. Apart from algebraic corrections, the main improvements over earlier computations are the clarification of three distinct
terms involving left-right mass insertions \( \left[ \tilde{\delta} \tilde{m}_{d_{L_{SR}}}^2 \right]^2, \left[ \tilde{\delta} \tilde{m}_{d_{R_{SR}}}^2 \right]^2, \delta \tilde{m}_{d_{L_{SR}}}^2 \delta \tilde{m}_{d_{R_{SL}}}^2 \) and the identification of how the experimental constraints on, and theoretical predictions for, the mass insertions scale with the supersymmetry breaking scale.

The dominant supersymmetric contribution to \( \Delta S = 2 \) processes results from four topologically distinct gluino box diagrams (Figures 1a - d). Neglecting external quark momenta compared to heavy internal squark and gluino masses, these diagrams give rise to an effective interaction Lagrangian:

\[
\mathcal{L}_{\Delta S=2}^{\text{ef}} = \frac{\alpha_s^2}{216 M_q^2} \left\{ \left( \frac{\tilde{\delta} \tilde{m}_{d_{L_{SR}}}^2}{M_q^2} \right)^2 \left[ 66 \tilde{f}(x) + 24 x f(x) \right] (\bar{d}_i \gamma_{\mu} P_{L} s_i)(\bar{d}_j \gamma^{\mu} P_{L} s_j) \\
+ \left( \frac{\tilde{\delta} \tilde{m}_{d_{L_{SL}}}^2}{M_q^2} \right)^2 \left[ 66 \tilde{f}(x) + 24 x f(x) \right] (\bar{d}_i \gamma_{\mu} P_{R} s_i)(\bar{d}_j \gamma^{\mu} P_{R} s_j) \\
+ \left( \frac{\tilde{\delta} \tilde{m}_{d_{R_{SL}}}^2}{M_q^2} \right) \left( \frac{\tilde{\delta} \tilde{m}_{d_{R_{SR}}}^2}{M_q^2} \right) \left[ -72 \tilde{f}(x) + 504 x f(x) \right] (\bar{d}_i P_{L} s_i)(\bar{d}_j P_{R} s_j) \\
+ [120 \tilde{f}(x) + 24 x f(x)] (\bar{d}_i P_{L} s_j)(\bar{d}_j P_{R} s_i) \\
+ \left( \frac{\tilde{\delta} \tilde{m}_{d_{L_{SR}}}^2}{M_q^2} \right)^2 x f(x) [324 (\bar{d}_i P_{R} s_i)(\bar{d}_j P_{R} s_j) - 108 (\bar{d}_i P_{R} s_j)(\bar{d}_j P_{R} s_i)] \\
+ \left( \frac{\tilde{\delta} \tilde{m}_{d_{L_{SL}}}^2}{M_q^2} \right)^2 x f(x) [324 (\bar{d}_i P_{L} s_i)(\bar{d}_j P_{L} s_j) - 108 (\bar{d}_i P_{L} s_j)(\bar{d}_j P_{L} s_i)] \\
+ \left( \frac{\tilde{\delta} \tilde{m}_{d_{R_{SL}}}^2}{M_q^2} \right) \left( \frac{\tilde{\delta} \tilde{m}_{d_{R_{SR}}}^2}{M_q^2} \right) \tilde{f}(x) [108 (\bar{d}_i P_{L} s_i)(\bar{d}_j P_{R} s_j) \\
- 324 (\bar{d}_i P_{L} s_j)(\bar{d}_j P_{R} s_i)] \right\} \tag{1} \]

with

\[
f(x) = \frac{1}{6(1-x)^5} (-6 \ln x - 18 x \ln x - x^3 + 9 x^2 + 9 x - 17) \tag{2a}
\]

\[
\tilde{f}(x) = \frac{1}{3(1-x)^5} (-6 x^2 \ln x - 6 x \ln x + 1 x^3 + 9 x^2 - 9 x - 1) \tag{2b}
\]

\[
x \equiv M_{\tilde{g}}^2 / M_q^2 \tag{2c}
\]
where $M_{\tilde{q}}$ is the universal (or average) down-squark mass and the quantity $[66\tilde{f}(x) + 24xf(x)] \to -1$ for $x = 1$. The soft supersymmetry-breaking FCNC mass insertions $\delta \tilde{m}_{d_{AB}}^2$ ($A, B = L, R$) appearing in (1) are defined by

$$L \subset \tilde{d}_A \delta \tilde{m}_{d_{AB}}^2 \tilde{s}_B$$

where the squark fields $\tilde{d}_A, \tilde{s}_B$ reflect a “super KM” basis in which the $\tilde{g}, \tilde{\gamma}, \tilde{Z}$ gaugino couplings are flavor diagonal and the charged-current $\tilde{W}^\pm$ mixing angles linking quarks and squarks are equal to the standard KM angles.

One can easily verify that the interaction Lagrangian (1) reproduces the quark scattering amplitude in Figure 1, noting that the following operators give rise to associated amplitudes:

$$[\tilde{d}_i \gamma_{\mu} P_L s_\alpha] [\tilde{d}_j \gamma^{\mu} P_L s_\beta]$$

$$\to A = 2(\tilde{d}_\beta(k_2) \gamma_{\mu} P_L s_\alpha(k_1)) (\tilde{d}_\gamma(k_3) \gamma^{\mu} P_L s_\delta(k_4)) (\delta_{\alpha \beta} \delta_{\gamma \delta} + \delta_{\alpha \gamma} \delta_{\beta \delta})$$

$$[\tilde{d}_i P_A s_j] [\tilde{d}_k P_B s_l] \to A = (\tilde{d}_\beta(k_2) P_A s_\alpha(k_1)) (\tilde{d}_\gamma(k_3) P_B s_\delta(k_4)) \delta_{ij} \delta_{\beta \delta}$$

$$- (\tilde{d}_\gamma(k_3) P_A s_\alpha(k_1)) (\tilde{d}_\beta(k_2) P_B s_\delta(k_4)) \delta_{ij} \delta_{\beta \delta}$$

$$- (\tilde{d}_\beta(k_2) P_A s_\delta(k_4)) (\tilde{d}_\gamma(k_3) P_B s_\alpha(k_1)) \delta_{ij} \delta_{\gamma \delta}$$

$$+ (\tilde{d}_\gamma(k_3) P_A s_\delta(k_4)) (\tilde{d}_\beta(k_2) P_B s_\alpha(k_1)) \delta_{ij} \delta_{\gamma \delta}$$

The form of $L_{\Delta S = 2}^{\text{eff}}$ in the “minimal susy limit” where $\delta \tilde{m}_{LL}^2$ dominates differs from previous analyses, which reported color octet operator structures in addition to the color singlet $(ii)(jj)$ operator shown in (1). (A color octet left-left operator can always be converted to a color singlet operator by a Fierz transformation.) One can use the effective Lagrangian (1) to assess the effects of off-generational mass mixings $\delta \tilde{m}_{d_{AB}}^2$ of arbitrary chirality which are radiatively induced in minimal supersymmetry or which may appear in non-minimal supersymmetric models. The $\Delta B = 2$ effective interaction Lagrangian can be read directly from (1) by the substitution $\delta \tilde{m}_{d_{AB}}^2 \to \delta \tilde{m}_{d_{AB}}^2$. The application of the low-energy effective $\Delta S = 2$ ($\Delta B = 2$) interaction Lagrangians derived above to $K^0 - \overline{K^0}$ ($B^0 - \overline{B^0}$) mixing requires estimating the
matrix elements of the various operators in $\mathcal{L}^{\text{eff}}$ between initial and final state mesons. The estimation of such hadronic matrix elements is notoriously difficult, and is generally accompanied by large uncertainties due to long-distance, non-perturbative strong-interaction physics. There are two factors which mitigate the effect of these hadronic uncertainties in the current phenomenological context:

1) The dominant supersymmetric contribution to $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ mixing in the Minimal Supersymmetric Standard Model (MSSM) results from a $\delta \tilde{m}_{LL}^2$ mass insertion, which gives rise to the same $(V - A)^2$ operator structure found in the Standard Model. This makes comparison of supersymmetric and Standard Model contributions relatively straightforward;

2) For the $B^0 - \bar{B}^0$ system, the valence quark approximation employed below is expected to be a good approximation. This belief is supported by lattice Monte-Carlo estimates of the $B^0 - \bar{B}^0$ matrix element “fudge factor” $B_B$ which give $B_B \simeq 1$.\(^{22}\)

The conventional result for the standard $(V - A)^2 \Delta S = 2$ operator is

$$\langle K^0 | [\bar{d}_i \gamma_\mu P_L s_i] [\bar{d}_j \gamma_\mu P_L s_j] | K^0 \rangle = \frac{2}{3} f_K^2 m_K^2 B_K^a$$

(5a)

where $f_K \simeq 165$ MeV is the K-meson decay constant and $B_K^a = 1$ corresponds to the “vacuum insertion” result. Various estimates of this matrix element place $B_K^a$ in the range of $0.3 - 1.0$,\(^{23}\) with a value $B_K^a \sim 0.7$ favored by lattice gauge results.\(^{22}\)

Matrix elements for the other hadronic operators appearing in $\mathcal{L}^{\text{eff}}_{\Delta S = 2}$ follow from current algebra:

$$\langle K^0 | [\bar{d}_\alpha P_L s_\alpha] [\bar{d}_\beta P_L s_\beta] | K^0 \rangle = \frac{5}{12} \left( \frac{m_K}{m_s + m_d} \right)^2 f_K^2 m_K^2$$

(5b)

$$\langle K^0 | [\bar{d}_\alpha P_L s_\beta] [\bar{d}_\beta P_L s_\alpha] | K^0 \rangle = -\frac{1}{12} \left( \frac{m_K}{m_s + m_d} \right)^2 f_K^2 m_K^2$$

(5c)

$$\langle K^0 | [\bar{d}_\alpha P_L s_\alpha] [\bar{d}_\beta P_R s_\beta] | K^0 \rangle = \left\{ \frac{1}{12} - \frac{1}{2} \left( \frac{m_K}{m_s + m_d} \right)^2 \right\} f_K^2 m_K^2$$

(5d)
\begin{equation}
\langle K^0 | [\bar{d}_\alpha P_L s_\beta] [\bar{d}_\beta P_R s_\alpha] | K^0 \rangle = \left\{ \frac{1}{4} - \frac{1}{6} \left( \frac{m_K}{m_s + m_d} \right)^2 \right\} f_K^2 m_K^2
d\end{equation}

(5e)

Similar fudge factors $B_K^{b-e}$ should be associated with each of the matrix elements (5b-e) above. Note that the matrix elements (5b-e) are enhanced relative to (5a) by a factor $(\frac{m_K}{m_s + m_d})^2 \sim 10$. This enhancement, which is significant for (5b and d), combined with large numerical coefficients for these non-standard chiral contributions to $L_{\Delta S=2}^{\text{eff}}$ (1), raise the prospect of interesting enhancements to $\Delta M_K$ outside of the MSSM, for which we will find that the $\delta \tilde{m}_{LL}^2$ mass insertion gives the dominant contribution. The corresponding results for the $\Delta B = 2$ matrix elements follow from (5) by setting $B_B \simeq (\frac{m_B}{m_b + m_d})^2 \simeq 1$.

| Phenomenological Upper Bounds on $\delta \tilde{m}_{\alpha'^{\prime} \alpha \beta}^{2}$ and $\delta \tilde{m}_{\alpha'b\beta}^{2}$ | $\left( \frac{\delta \tilde{m}_{\alpha'^{\prime} \alpha \beta}^{2}}{M_{\tilde{q}}^2} \right)$ | $\left( \frac{\delta \tilde{m}_{\alpha'^{\prime} \alpha b}^{2}}{M_{\tilde{q}}^2} \right)$ | $\left( \frac{\delta \tilde{m}_{\alpha'^{\prime} b \beta}^{2}}{M_{\tilde{q}}^2} \right)$ |
|------------------|------------------|------------------|------------------|
| $(\alpha \beta)(\alpha'^{\prime} \beta')$ | $\Delta M_K$ | $\epsilon_K$ | $\Delta M_B$ |
| $(LL)^2$ or $(RR)^2$ | $(0.10)^2$ | $(0.0080)^2$ | $(0.27)^2$ |
| $(LL)(RR)$ | $(0.0060)^2$ | $(0.00049)^2$ | $(0.073)^2$ |
| $(LR)^2$ or $(RL)^2$ | $(0.0082)^2$ | $(0.00066)^2$ | $(0.082)^2$ |
| $(LR)(RL)$ | $(0.044)^2$ | $(0.0035)^2$ | $(0.14)^2$ |

TABLE I. All numbers in the table must be multiplied by the factor $(M_{\tilde{q}}/\text{TeV})^2$. Numerical values assume $\sqrt{x} \sim M_{\tilde{q}}/M_{\tilde{q}} = 1$. Stricter (weaker) results generally apply to $x < 1$ ($x > 1$). Bounds derived from $\epsilon_K$ assume maximal CP violation. Bounds from $\Delta M_B$ must be scaled by (160 MeV/$f_B$).

Phenomenological constraints on the various FCNC mass mixings $\delta \tilde{m}_{\alpha'^{\prime} \alpha \beta}^{2}$ appearing in $L_{\Delta S=2}^{\text{eff}}$ can be derived by inserting the matrix elements (5) into $L_{\Delta S=2}^{\text{eff}}$ and requiring $\Delta M_K^{\text{supy}} \leq \Delta M_K^{\text{exp}}$ for each of the chiral contributions to the $K_L - K_S$ mass difference:

$$\Delta M_K \simeq 2 M_{12} = \frac{1}{m_K} \langle K^0 | - L_{\Delta S=2}^{\text{eff}} | K^0 \rangle = 3.52 \times 10^{-15} \text{GeV}$$

(6)

Barring accidental cancellations between the various chiral contributions, this constraint is conservative in that the short-distance, Standard Model charm-quark contribution is itself of order $\Delta M_K^{\text{exp}}$, and of the correct sign. Setting $B_K^{a} = 0.7$
and taking $x = 1$ yields the constraints on $\delta \tilde{m}_{AB}^2$ due to $\Delta M_K$ shown in Table I.

The results of Table I are significant and warrant comment. All the constraints from $\Delta M_K$ are useful constraints (i.e. $\delta \tilde{m}_{AB}^2/M_{\tilde{q}}^2 < 1$), even for heavy squark masses $M_{\tilde{q}} \gg 1$ TeV. Indeed the constraint on $\delta \tilde{m}_{LL}^2$, which is actually the weakest of the constraints from $\Delta M_K$, provides useful information on the down-squark mass matrix for $M_{\tilde{q}, \tilde{g}}$ as large as 10 TeV. Above 10 TeV, the upper bound on $\delta \tilde{m}_{\tilde{d}sL}^2/M_{\tilde{q}}^2$ rises above unity, at which point the perturbative expansion in powers of $\delta \tilde{m}^2/M_{\tilde{q}}^2$ breaks down, and no further useful information remains on the form of the squark mass matrix.

The constraint for $\delta \tilde{m}_{RR}^2$ is identical to that for $\delta \tilde{m}_{LL}^2$. However, Table I shows that in the presence of both types of mass insertions, even stronger constraints apply. This is because the numerical coefficients (e.g., 504) in $L_{\Delta S=2}^{\text{eff}}$ (1) and the operator matrix element (5) $\propto \left[m_K/(m_s+m_d)\right]^2$ are both enhanced relative to the $\delta \tilde{m}_{LL,RR}^2$ contributions. This results in useful constraints (i.e. $\delta \tilde{m}_{LL,RR}^2/M_{\tilde{q}}^2 < 1$) on the squark mass matrix for sparticle masses $M_{\tilde{q}, \tilde{g}}$ greater than 100 TeV. For the same reasons, the bounds on $\delta \tilde{m}_{\tilde{d}sL}^2$ and $\delta \tilde{m}_{\tilde{d}sL}^2$ are similarly stringent and furnish useful constraints for sparticle masses $M_{\tilde{q}, \tilde{g}}$ up to 100 TeV.

The most stringent bounds on $\delta \tilde{m}_{d_{AB}}^2$ follow from the CP impurity parameter $\epsilon_K$. Table I shows the constraints on the $\delta \tilde{m}_{d_{AB}}^2$ which result from setting $\epsilon^\text{susy}_K < \epsilon^\exp_K$ and assuming maximal CP-violating phases for each of the chiral contributions. Note that, in many cases, CP violation in $K^0 - \bar{K}^0$ is sensitive to supersymmetric contributions from sparticle masses $M_{\tilde{q}, \tilde{g}} > 1000$ TeV.

The analysis of $\Delta M_B$ is identical to $\Delta M_K$, with appropriate substitution of flavor indices. Although numerically less stringent than those from $\Delta M_K$, the constraints in Table I can be equally restrictive for many supersymmetric models (like the MSSM) which predict $\delta \tilde{m}_{d_{AB}}^2 \gg \delta \tilde{m}_{d_{ASB}}^2$.20

In the MSSM with universal soft supersymmetry breaking, FCNC mass insertions are generated by renormalization effects between the unification scale $M_U$ and $m_W$.2−5 The resulting low-energy mass insertions are predominantly left-left,
and exhibit the following approximate flavor dependence:

\[
\frac{[\delta \tilde{m}^2_{d_Ld_L}]_{ij}}{M^2_{\tilde{q}}} = c_{LL}(\xi_0, \xi_A)[V^\dagger \lambda^2_u V]_{ij}
\]  \hspace{1cm} (7)

where \(\lambda_u\) is the diagonalized charge 2/3 quark mass matrix and \(V\) is the KM matrix. The renormalization-group coefficient \(c_{LL}\) is flavor independent, and is plotted in Figure 2 as a function of \(\xi_0 \equiv m_0/m_{1/2}\) and \(\xi_A \equiv A/m_{1/2}\), where \(m_0, m_{1/2}, A\) are respectively the universal soft supersymmetry-breaking scalar mass, gaugino mass and trilinear coupling at \(\mu = M_U\). Using the experimental upper bound \(V_{td}^\ast V_{ts} < 7.3 \times 10^{-4}\) on the KM angles associated with the dominant \(\lambda_t^2\) contribution, we observe that the radiatively-generated \(\delta \tilde{m}^2_{d_Ls_L}\) is too small to contribute significantly to \(\Delta M_K\) or \(\epsilon_K\).

Such small FCNC results are a relatively unique prediction of the MSSM. Most nontrivial extensions of the MSSM contain extra Yukawa couplings, and these generically lead to large FCNC's. In supersymmetric Flipped SU(5), for example, we find potentially large FCNC's generated above the GUT scale:

\[
[\delta \tilde{m}^2_{d_LL,L}]_{ij} = -\frac{1}{8\pi^2} [\lambda^6_6 \lambda^7_6]_{ij} \ln \left(\frac{M_{Pl}}{M_{GUT}}\right)[3m^2_0 + A^2]
\]  \hspace{1cm} (8a)

\[
[\delta \tilde{m}^2_{d_Rd_R}]_{ij} = [\delta \tilde{m}^2_{d_Ld_L}]^*_{ij}
\]  \hspace{1cm} (8b)

\[
[\delta \tilde{m}^2_{d_Rd_L}]_{ij} = -\eta_{ij}^1 v = \frac{v A}{8\pi^2} [\lambda_6^6 \lambda^7_6]_{ij} \ln \left(\frac{M_{Pl}}{M_{GUT}}\right)
\]  \hspace{1cm} (8c)

\[
[\delta \tilde{m}^2_{d_Rd_L}]_{ij} = [\delta \tilde{m}^2_{d_Rd_R}]^*_{ij}
\]  \hspace{1cm} (8d)

where \(\lambda_6\) is an \textit{a-priori} unknown Yukawa coupling associated with a see-saw neutrino mass mechanism.

From the phenomenological constraints in Table I, we can derive useful bounds on the unknown GUT Yukawa \(\lambda_6\). The strongest bounds come from the \((\delta \tilde{m}^2_{LL})(\delta \tilde{m}^2_{RR})\) contributions to \(\Delta M_K, \Delta M_B\), from which we obtain:

\[
||[\lambda^6_6 \lambda^7_6]_{12}|| \ln \left(\frac{M_{Pl}}{M_{GUT}}\right) < 0.47 \frac{\xi_0^2 + 6}{3\xi_A^2 + \xi_0^2} \left(\frac{M_{\tilde{g}}}{1 \text{ TeV}}\right)
\]  \hspace{1cm} (9a)
\[ \left| [\lambda_6^T \lambda_6^T]_{13} \right| \ln \left( \frac{M_{Pl}}{M_{GUT}} \right) < 1.28 \frac{\xi_0^2 + 6}{3\xi_0^2 + \xi_A} \left( \frac{M_{\tilde{q}}}{1 \text{ TeV}} \right) \]  

(9b)

where we have used the approximate low-energy relation \( M_{\tilde{q}}^2 \approx (\xi_0^2 + 6)m_{1/2}^2 \). \(^{20}\)

Equation (9) provides important knowledge about the unknown matrix \( \lambda_6 \) and the pattern of soft supersymmetry breaking. For instance, if supersymmetry breaking takes the form of either \( m_0 \) or \( A \), then (9) requires certain elements of \( \lambda_6 \) to be quite small—smaller than expected from superstring theories, which relate Yukawa couplings to gauge couplings \( \lambda \approx g \approx 0.7 \) at \( \mu = M_{Pl} \). One may be forced to conclude that these couplings vanish at the tree level in such theories—or that soft supersymmetry breaking takes the form of \( m_{1/2} \).

Note that because of (8b), the Flipped contribution to CP violation \( \epsilon_K \) vanishes. Flipped SU(5) can, however, contribute significantly to “direct” CP violation \( \epsilon' \), \( K_L \to \pi^0 ee \) and \( K_L \to \pi^0 \nu \nu \), as shown in ref. 20.

We conclude that FCNC processes, and \( K^0 - \bar{K}^0 \) and \( B^0 - \bar{B}^0 \) mixing in particular, provide a sensitive probe of physics beyond the MSSM. In the presence of sparticles \( M_{\tilde{q}}, M_{\tilde{g}} \lesssim 1000 \text{ TeV} \), these FCNC processes provide an experimental window on the nature of physics at extremely high energies which should clearly be exploited to the fullest extent possible.

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