Andreev reflection through a quantum dot
coupled with two ferromagnets and a superconductor

Yu Zhu, Qing-feng Sun and Tsung-han Lin*

State Key Laboratory for Mesoscopic Physics and
Department of Physics, Peking University, Beijing
100871, China

Abstract

We study the Andreev reflection (AR) in a three terminal mesoscopic hybrid system, in which two ferromagnets (F₁ and F₂) are coupled to a superconductor (S) through a quantum dot (QD). By using non-equilibrium Green function, we derive a general current formula which allows arbitrary spin polarizations, magnetization orientations and bias voltages in F₁ and F₂. The formula is applied to study both zero bias conductance and finite bias current. The current conducted by crossed AR involving F₁, F₂ and S is particularly unusual, in which an electron with spin σ incident from one of the ferromagnets picks up another electron with spin σ̄ from the other one, both enter S and form a Cooper pair. Several special cases are investigated to reveal the properties of AR in this system.

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I. INTRODUCTION

Electron has spin as well as charge. The application of the electron spin property opens a fruitful field in the transport of ferromagnetic materials, such as the discovery of GMR and TMR effects [1]. On the other hand, there are growing interests on the mesoscopic normal-metal / superconductor (N/S) hybrid system [2], in which Andreev reflection (AR) at N/S interface plays an important role in low bias voltage regime [3]. In AR process, an electron incident with energy $E$ and spin $\sigma$ picks up another electron with energy $-E$ and spin $\bar{\sigma}$, both enter S and form a Cooper pair, leaving a Andreev reflected hole in N side. One may expect that the interplay of the spin property of AR process and the spin-dependent transport in ferromagnetic materials will add new physics to mesoscopic hybrid systems, and to the future applications of spintronics.

Several works have been devoted to this issue. In the pioneering work of de Jong et al. [4], the transport of a ferromagnet / superconductor (F/S) junction was studied by scattering matrix formalism. The conductance of AR is shown to be strongly affected by the spin polarization of F. The idea was verified by recent experiments in F/S thin film nanocontact [5] and F/S metallic point contact [6]. Especially, in Ref. [5], Soulen et al. successfully determined the spin polarization at the Fermi energy for several metals by measuring the differential conductance of F/S metallic point contact. Further calculations [7] implied that the Fermi velocity mismatch between F and S also affects AR conductance of F/S contact, and the conductance may even be enhanced in presence of spin polarization. In addition to simple F/S junction, F/S contact with S in d-wave symmetry [8,9], F/S nanostructure with giant proximity effect [10], and more complicated structures such as FSF double junctions [11,12], SFS double junctions [13,14], S/F superlattices [15], (NF)$_n$S multilayer structures [16,17] were also investigated.

In this paper, we propose an idea that two sources of spin polarized electrons with different orientations are injected into a superconductor, which can be achieved by a three terminal mesoscopic F/S hybrid structure shown in Fig.1. In this system, a central quantum dot (QD) is coupled via tunnel barriers to two ferromagnetic electrodes (F$_1$ and F$_2$) and a superconducting electrode (S) (hereafter, the system is simply referred as to (F$_1$,F$_2$)-QD-S). F$_1$ and F$_2$ are assumed to have arbitrary magnetization orientations, spin polarizations, and bias voltages. The bias voltage of S is set to zero as the ground. QD is designed to provide a link between F$_1$, F$_2$ and S, so that AR can take place through discrete energy states of QD.
Consider the special case that F_1 and F_2 are fully polarized, AR only involving F_1 and S or only involving F_2 and S are completely suppressed, while the crossed AR involving F_1, F_2, and S depends strongly on the magnetization orientations of F_1 and F_2, being suppressed if they are in ferromagnetic alignment, enhanced in anti-ferromagnetic alignment. In this paper, we will derive a current formula by using non-equilibrium Green function method, and investigate several special cases to illustrate the properties of ARs in this system.

During the preparation of this paper, we are aware that in the recent publication of Deutscher et al. [20], a device consisting of two point contacts between two ferromagnetic tips and a superconductor was proposed. For the two tips with fully but opposite spin polarizations, they suggested that “mixed” Cooper pair made of electrons coming one from each tip can be injected into the superconductor, leading to unusual properties of such device. Sec.IV of this paper is partially stimulated by their work.

The rest of this paper is organized as follows: In Sec.II, we present the model Hamiltonian and derive a general current formula for the hybrid system (F_1, F_2)-QD-S, by non-equilibrium Green function method. In Sec.III, we study the zero bias conductance, assuming \( V_1 = V_2 = 0^+ \). The explicit forms of the conductance are presented and numerically studied. In Sec.IV, we study the finite bias current with F_1 and F_2 in anti-ferromagnetic alignment, and the fully spin polarized case is discussed in detail. Finally, a brief summary is given in Sec.V.

### II. MODEL AND FORMULATION

The system under consideration can be described by the following Hamiltonian:

\[
H = H_1 + H_2 + H_{dot} + H_s + H_T ,
\]

\[
H_1 = \sum_{k\sigma} (\epsilon_k - \sigma h_1 - \mu_1) a_{k\sigma}^\dagger a_{k\sigma} ,
\]

\[
H_2 = \sum_{k\sigma'} (\epsilon_k - \sigma' h_2 - \mu_2) b_{k\sigma'}^\dagger b_{k\sigma'} ,
\]

\[
H_{dot} = E_0 \sum_{\sigma} c_{\sigma}^\dagger c_{\sigma} ,
\]

\[
H_s = \sum_{p\sigma} \epsilon_p d_{p}^\dagger d_{p\sigma} + \sum_{p} \left[ \Delta d_{p\uparrow}^\dagger d_{-p\downarrow}^\dagger + h.c \right] ,
\]

\[
H_T = \sum_{k\sigma} \left[ t_{1\sigma} a_{k\sigma}^\dagger c_{\sigma} + h.c \right] + \sum_{k\sigma} \left[ t_{2\sigma} b_{k\sigma}^\dagger c_{\sigma} + h.c \right] + \sum_{p\sigma} \left[ t_{s} d_{p\sigma}^\dagger c_{\sigma} + h.c \right] .
\]
$H_1$ and $H_2$ are the Hamiltonians of $F_1$ and $F_2$ in the mean field approximation, with different magnetization orientations and chemical potentials. The spin bands of $F_1$ ($F_2$) are split by $2h_1$ ($2h_2$) due to the exchange energy. The magnetization orientation of $F_1$ is set as $z$ axis, while the orientation of $F_2$ as $z'$ axis which has an angle $\theta$ with respect to $z$ axis. The operators with the spin-quantization axis $z$ and the operators with the spin-quantization axis $z'$ are related by $D^{\frac{1}{2}}$ matrix as

$$
\begin{pmatrix}
(b^\dagger_{kz'}) \\
(b^\dagger_{kz})
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
(b^\dagger_{kz'}) \\
(b^\dagger_{kz})
\end{pmatrix}.
$$

$H_{dot}$ describes the quantum dot, in which only one spin degenerate level is considered and the intra-dot interaction is ignored for simplicity. $H_s$ is the Hamiltonian for a BCS superconductor with the chemical potential fixed to zero as the ground. $H_T$ depicts the tunneling between QD and $F_1$, $F_2$ and $S$, coupling different parts of the system together.

Since the current through QD can be expressed in terms of the Green functions of QD, we first derive the retarded and distribution Green functions by Dyson equation and Keldysh equation. To include the physics of Andreev reflections and the spin flip processes in a unified formulation, we introduce a $4 \times 4$ matrix representation, in which the Green function is defined as

$$
G \equiv \langle \langle 
\begin{pmatrix}
 c^\dagger_{z^\uparrow} \\
 c^\dagger_{z^\downarrow} \\
 c^\dagger_{z'^\uparrow} \\
 c^\dagger_{z'^\downarrow}
\end{pmatrix} | 
\begin{pmatrix}
 c_{z^\uparrow} \\
 c_{z^\downarrow} \\
 c_{z'^\uparrow} \\
 c_{z'^\downarrow}
\end{pmatrix}
\rangle \rangle. 
$$

Let $G^r$ denote the Fourier transformed retarded Green function of QD, and $G^r$ can be solved by Dyson equation:

$$
G^r = g^r + g^r \Sigma^r G^r,
$$

in which $g^r$ is the retarded Green function of an isolated QD and $\Sigma^r$ is the self-energy due to couplings between QD and leads. $g^r$ can be easily obtained as:

$$
g^r = 
\begin{pmatrix}
\frac{1}{\omega - E_0 + i0^+} & 0 & 0 & 0 \\
0 & \frac{1}{\omega + E_0 + i0^+} & 0 & 0 \\
0 & 0 & \frac{1}{\omega - E_0 + i0^+} & 0 \\
0 & 0 & 0 & \frac{1}{\omega + E_0 + i0^+}
\end{pmatrix},
$$
while $\Sigma^r$ consists of three parts,

$$
\Sigma^r = \Sigma_1^r + \Sigma_2^r + \Sigma_s^r.
$$

(6)

$\Sigma_1^r$ is the self-energy from the coupling between QD and F_1, given by

$$
\Sigma_1^r = -\frac{i}{2} \begin{pmatrix}
\Gamma_{1\uparrow} & 0 & 0 \\
0 & \Gamma_{1\downarrow} & 0 \\
0 & 0 & \Gamma_{1\downarrow}
\end{pmatrix},
$$

(7)

in which $\Gamma_{1\uparrow}$ and $\Gamma_{1\downarrow}$ are the spin-dependent coupling strengths defined by $\Gamma_{1\sigma} \equiv 2\pi N_{1\sigma} |t_{1\sigma}|^2$, with $N_{1\sigma}$ being the density of states of spin $\sigma$ band of $F_1$. $\Sigma_2^r$ is the self-energy from the coupling between QD and $F_2$, given by

$$
\Sigma_2^r = -\frac{i}{2} \begin{pmatrix}
c^2\Gamma_{2\uparrow} + s^2\Gamma_{2\downarrow} & 0 & sc(\Gamma_{2\uparrow} - \Gamma_{2\downarrow}) & 0 \\
0 & c^2\Gamma_{2\uparrow} + s^2\Gamma_{2\downarrow} & 0 & sc(\Gamma_{2\uparrow} - \Gamma_{2\downarrow}) \\
sc(\Gamma_{2\uparrow} - \Gamma_{2\downarrow}) & 0 & c^2\Gamma_{2\downarrow} + s^2\Gamma_{2\uparrow} & 0 \\
sc(\Gamma_{2\uparrow} - \Gamma_{2\downarrow}) & 0 & c^2\Gamma_{2\downarrow} + s^2\Gamma_{2\uparrow}
\end{pmatrix},
$$

(8)

in which $s \equiv \sin\frac{\theta}{2}$, $c \equiv \cos\frac{\theta}{2}$, $\Gamma_{2\uparrow}$ and $\Gamma_{2\downarrow}$ are defined similarly to $\Gamma_{1\uparrow}$ and $\Gamma_{1\downarrow}$. $\Sigma_s^r$ is the self-energy from by the coupling between QD and $S$, given by

$$
\Sigma_s^r = -\frac{i}{2} \Gamma_s \rho(\omega) \begin{pmatrix}
1 & -\Delta & 0 & 0 \\
-\Delta & 1 & 0 & 0 \\
0 & 0 & 1 & \Delta \\
0 & 0 & \Delta & 1
\end{pmatrix},
$$

(9)

in which $\Gamma_s \equiv 2\pi N_s |t_s|^2$, with $N_s$ being the density of states when the superconductor is in normal state, $\rho(\omega)$ is the modified BCS density of states defined by

$$
\rho(\omega) \equiv \begin{cases} 
\frac{|\omega|}{\sqrt{\omega^2 - \Delta^2}} & |\omega| > \Delta \\
\frac{\omega}{i\sqrt{\Delta^2 - \omega^2}} & |\omega| < \Delta
\end{cases}.
$$

(10)

Thus, $G^r$ can be obtained by solving Dyson equation, Eq.(4).
Let $G^<$ denote the Fourier transformed distribution Green function of QD, and $G^<$ can be obtained by Keldysh equation:

$$G^<=G^r\Sigma^<G^a.$$  \hspace{1cm} (11)

Notice that the advanced Green function and self-energy are the Hermitian conjugations of the corresponding retarded Green function and self-energy. And $\Sigma^<$ can be obtained by applying the fluctuation-dissipation theorem to each of $\Sigma^<_1$, $\Sigma^<_2$ and $\Sigma^<_s$,

$$\Sigma^<=\Sigma^<_1+\Sigma^<_2+\Sigma^<_s,$$  \hspace{1cm} (12)

$$\Sigma^<_1=F_1(\Sigma^a_1-\Sigma^r_1),$$

$$\Sigma^<_2=F_2(\Sigma^a_2-\Sigma^r_2),$$

$$\Sigma^<_s=F_s(\Sigma^a_s-\Sigma^r_s),$$

in which

$$F_1=\begin{pmatrix} f_1 & 0 & 0 & 0 \\ 0 & \bar{f}_1 & 0 & 0 \\ 0 & 0 & f_1 & 0 \\ 0 & 0 & 0 & \bar{f}_1 \end{pmatrix},$$  \hspace{1cm} (13)

$$F_2=\begin{pmatrix} f_2 & 0 & 0 & 0 \\ 0 & \bar{f}_2 & 0 & 0 \\ 0 & 0 & f_2 & 0 \\ 0 & 0 & 0 & \bar{f}_2 \end{pmatrix},$$  \hspace{1cm} (14)

$$F_s=\begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & f & 0 \\ 0 & 0 & 0 & f \end{pmatrix},$$  \hspace{1cm} (15)

where $f_1$, $\bar{f}_1$, $f_2$, $\bar{f}_2$ and $f$ denote $f(\omega-V_1)$, $f(\omega+V_1)$, $f(\omega-V_2)$, $f(\omega+V_2)$ and $f(\omega)$, respectively, in which $f(\omega)$ is the Fermi distribution function.
Then, the current flowing from F_1 to the QD can be expressed in terms of G^r and G^< as

\[ I_1 = I_1^\uparrow + I_1^\downarrow = \frac{e}{\hbar} \int d\omega \left [ (G \Sigma_1)^< + h.c \right ]_{11+33}, \]  

(16)
in which we have used the compact notations [AB]^< ≡ A^r B^< + A^< B^s and [ ]_{11+33} ≡ [ ]_{11} + [ ]_{33}. After some algebra manipulations, the current can be divided into contributions from four conducting processes:

\[ I_1 = \frac{e}{\hbar} \int d\omega \left [ A_{11}(f_1 - \bar{f}_1) + A_{12}(f_1 - \bar{f}_2) + Q_{1s}(f_1 - f_s) + Q_{12}(f_1 - f_2) \right ], \]  

(17)
in which

\[ A_{11} = \Gamma_{1\uparrow}(\Gamma_{1\uparrow} | G_{12}^r |^2 + \Gamma_{1\downarrow} | G_{32}^r |^2) + \Gamma_{1\downarrow}(\Gamma_{1\uparrow} | G_{14}^r |^2 + \Gamma_{1\downarrow} | G_{34}^r |^2) \]

(18)
represents the Andreev reflection through F_1-QD-S,

\[ A_{12} = (c^2 \Gamma_{2\uparrow} + s^2 \Gamma_{2\downarrow})(\Gamma_{1\uparrow} | G_{12}^r |^2 + \Gamma_{1\downarrow} | G_{32}^r |^2) + \]

\[ (c^2 \Gamma_{2\uparrow} + s^2 \Gamma_{2\downarrow})(\Gamma_{1\uparrow} | G_{14}^r |^2 + \Gamma_{1\downarrow} | G_{34}^r |^2) + \]

\[ sc(\Gamma_{2\uparrow} - \Gamma_{2\downarrow})2 \text{Re}(\Gamma_{1\uparrow} G_{12}^r G_{12}^{rs} + \Gamma_{1\downarrow} G_{32}^r G_{32}^{rs}) \]

represents the crossed Andreev reflection through (F_1,F_2)-QD-S,

\[ Q_{1s} = \Gamma_{1\uparrow}\Gamma_{s}\hat{\rho} \left[ |G_{11}^r|^2 + |G_{12}^r|^2 + |G_{13}^r|^2 + |G_{14}^r|^2 + 2 \text{Re}(\frac{\Delta}{\omega} G_{11}^r G_{12}^{rs} + \frac{\Delta}{\omega} G_{13}^r G_{14}^{rs}) \right] \]

\[ \Gamma_{1\downarrow}\Gamma_{s}\hat{\rho} \left[ |G_{31}^r|^2 + |G_{32}^r|^2 + |G_{33}^r|^2 + |G_{34}^r|^2 + 2 \text{Re}(\frac{\Delta}{\omega} G_{31}^r G_{32}^{rs} + \frac{\Delta}{\omega} G_{33}^r G_{34}^{rs}) \right] \]

represents the single particle tunneling through F_1-QD-S, and \( \hat{\rho}(\omega) \equiv \frac{|\omega|}{\sqrt{\omega^2 - \Delta^2}} \theta(|\omega| - \Delta) \) is the ordinary BCS density of states,

\[ Q_{12} = (c^2 \Gamma_{2\uparrow} + s^2 \Gamma_{2\downarrow})(\Gamma_{1\uparrow} | G_{11}^r |^2 + \Gamma_{1\downarrow} | G_{31}^r |^2) + \]

\[ (c^2 \Gamma_{2\uparrow} + s^2 \Gamma_{2\downarrow})(\Gamma_{1\uparrow} | G_{13}^r |^2 + \Gamma_{1\downarrow} | G_{33}^r |^2) + \]

\[ sc(\Gamma_{2\uparrow} - \Gamma_{2\downarrow})2 \text{Re}(\Gamma_{1\uparrow} G_{11}^r G_{13}^{rs} + \Gamma_{1\downarrow} G_{33}^r G_{33}^{rs}) \]

represents the single-particle tunneling through F_1-QD-F_2. Similarly, one can obtain the current flowing from F_2 into QD simply by exchange the index 1 and 2.

The current formula Eq.(17) is the central result of this work, which can be applied to ferromagnetic electrodes F_1 and F_2 with arbitrary spin polarizations, magnetization orientations and bias voltages.
In the following numerical studies, we assume that $|eV_1|, |eV_2| < \Delta$ and $k_B T \ll \Delta$. $Q_{1s}$ process will vanish because of the factor $\tilde{\rho}$ and the Fermi function difference $(f_1 - f_s)$. $Q_{12}$ process will be ruled out in two special cases: $F_1$ and $F_2$ are either equally biased (Section III) or fully but oppositely polarized (Section IV). We will concentrate on AR processes $A_{11}$ (direct AR through $F_1$-QD-S) and $A_{12}$ (crossed AR through $(F_1,F_2)$-QD-S), and investigate several special cases to illustrate the properties of these two ARs.

III. ZERO BIAS CONDUCTANCE

In this section, we study the zero bias conductance by taking $V_1 = V_2 = 0^+$. Since no bias voltage between $F_1$ and $F_2$, there is no net single-particle current flowing between them. For $k_B T \ll \Delta$, the single-particle current from $F_1$ or $F_2$ to $S$ is also negligible. Therefore, only ARs contribute to the conductance. For simplicity, we set $k_B T = 0$ and $E_0 = 0$, introduce the spin polarization $P_\beta \equiv \frac{\Gamma_\beta U - \Gamma_\beta D}{\Gamma_\beta U + \Gamma_\beta D}$, and the spin-averaged coupling strength $\Gamma_\beta \equiv \frac{1}{2} (\Gamma_\beta U + \Gamma_\beta D)$, with $\beta = 1, 2$ for $F_1$ and $F_2$ respectively.

First consider the simplest case in which $\Gamma_2 = 0$, $\Gamma_1 \equiv \Gamma_L$, $P_1 \equiv P$, $\Gamma_s \equiv \Gamma_R$, then the three-terminal system $(F_1,F_2)$-QD-S reduces to a two-terminal system $F$-QD-S, and the conductance is easily obtained from the current formula as

$$G_{F DS} = \frac{4e^2}{h} \frac{(1 - P^2)r^2}{(1 - P^2 + r^2)^2}, \quad (22)$$

where $r \equiv \Gamma_R/\Gamma_L$ is the ratio of the two coupling strengths. Analogous to the matching condition of the Fermi velocities in $F/S$ contact (i.e., $k_{F \uparrow}k_{F \downarrow} = k_S^2$), here $P^2 + r^2 = 1$ (i.e., $\Gamma_L/\Gamma_L = \Gamma_R^2$) plays the similar role. For $r > 1$, the matching condition can never be satisfied, so $G_{F DS}$ decreases monotonously with the increase of $P$ (Fig.2a). While for $r < 1$, it exists a certain value of $P$, say $P_0$, satisfying $P_0^2 + r^2 = 1$, so $G_{F DS}$ first increases with $P$, reaches its maximum value $\frac{4e^2}{h}$ at $P = P_0$, then drops to 0 when $P$ approaches to 1 (Fig.2b). This result warns us to be careful to deduce the spin polarization of $F$ from AR conductance of $F$-QD-S.

Next, consider the general case of the three terminal system $(F_1,F_2)$-QD-S. Similar to the composition of polarized light, the total current (or total conductance) of $F_1$ and $F_2$ are equivalent to that of an effective ferromagnet $\tilde{F}$. Introduce the spin polarization vectors $\tilde{q}_1$ and $\tilde{q}_2$, where $\tilde{q}_\beta$ has the magnitude of $\Gamma_\beta P_\beta$ and the direction of the magnetization direction of $F_\beta$, with $\beta = 1, 2$. It is easy to test that these vectors obey the vector composition rule, i.e.,
\[ \vec{q} = \vec{q}_1 + \vec{q}_2, \] in which \( \vec{q} \) is the spin polarization vector of \( \tilde{F} \). Therefore, the effective parameters of \( \tilde{F} \) are

\[
\tilde{\Gamma} = \Gamma_1 + \Gamma_2, \tag{23}
\]
\[
\tilde{P} = \frac{[(\Gamma_1 P_1)^2 + (\Gamma_2 P_2)^2 + 2\Gamma_1 P_1 \Gamma_2 P_2 \cos \theta]^\frac{1}{2}}{\Gamma_1 + \Gamma_2}.
\]

As a result, the total conductance of \( F_1 \) and \( F_2 \) can be obtained as

\[
G \equiv G_1 + G_2 = G_{FDS}(\tilde{P}, \tilde{r}), \tag{24}
\]

in which \( G_{FDS} \) has the same form as in Eq.(22), \( \tilde{P} \) is the effective polarization, and \( \tilde{r} \) is defined by \( \Gamma_s/\tilde{\Gamma} \). Then the conductance of \( F_1 \) and \( F_2 \) can be expressed by the total conductance multiplied by a sharing factor,

\[
G_1 = G \frac{\Gamma_1^2 + \Gamma_1 \Gamma_2 - (\Gamma_1^2 P_1^2 + \Gamma_1 P_1 \Gamma_2 P_2 \cos \theta)}{\Gamma_1^2 + \Gamma_2^2 + 2\Gamma_1 \Gamma_2 - (\Gamma_1^2 P_1^2 + \Gamma_2^2 P_2^2 + 2\Gamma_1 P_1 \Gamma_2 P_2 \cos \theta)}, \tag{25}
\]
\[
G_2 = G \frac{\Gamma_2^2 + \Gamma_1 \Gamma_2 - (\Gamma_2^2 P_2^2 + \Gamma_1 P_1 \Gamma_2 P_2 \cos \theta)}{\Gamma_1^2 + \Gamma_2^2 + 2\Gamma_1 \Gamma_2 - (\Gamma_1^2 P_1^2 + \Gamma_2^2 P_2^2 + 2\Gamma_1 P_1 \Gamma_2 P_2 \cos \theta)}. \tag{26}
\]

Fig.3 shows the curves of \( G \) vs \( \theta \) (also can be viewed as \( 2G_1 \) vs \( \theta \) or \( 2G_2 \) vs \( \theta \)) for the symmetric case, in which \( \Gamma_1 = \Gamma_2 \equiv \Gamma \) and \( P_1 = P_2 \equiv P \). For \( r = 1 \), \( G \) increases with the increase of \( \theta \) or decrease of \( P \). For \( r > 1 \), the curves of \( G \) vs \( \theta \) is qualitatively the same as those of \( r = 1 \), but the conductance is lowered and more sensitive to \( P \). For \( r < 1 \), the variation is more complicated: if \( P^2 < 1 - r^2 \), \( G \) decreases with the increase of \( \theta \) or decrease of \( P \); if \( P^2 > 1 - r^2 \), \( G \) has the maximum \( \frac{4e^2}{h} \) at \( \theta \) satisfying \( (P \cos \theta)^2 = 1 - r^2 \). These results are readily understood by the new matching condition \( \tilde{P}^2 + r^2 = 1 \) with the effective spin polarization \( \tilde{P} = P \cos \frac{\theta}{2} \).

Two points are noteworthy in the above result: (1) If \( F_1 \) and \( F_2 \) are regarded as a whole, the effective polarization can be tuned continuously by changing the angle of the mutual orientations, which is impossible for one chosen ferromagnet. (2) For \( r \geq 1 \), the total conductance for the two ferromagnets in anti-ferromagnetic alignment is larger than that in ferromagnetic alignment, which is completely different from the effect of GMR or TMR. To describe this new effect of magneto-resistance, define the ratio of Andreev reflected magnetic resistance (ARMR) in \((F_1,F_2)-QD-S\) by

\[
ARMR \equiv \frac{G_{AF} - G_F}{G_{AF} + G_F}, \tag{27}
\]

and the curves of \( ARMR \) vs \( P \) for various \( r \) are shown in Fig.4.
we only consider the antiparallel orientation of $F_1$ bias voltages (i.e., changing the spin orientation of $F_2$ block-diagonal due to $\theta$ polarization of $F_2$. We chose a half-metal material as $F_1$, the ferromagnetic material to be measured as $F_2$. In practice, one may chose a half-metal material as $F_1$, the ferromagnetic material to be measured as $F_2$, and changing the spin orientation of $F_1$ by applying an external magnetic field, then the spin polarization of $F_2$ can be deduced from the weak / strong dependence of $G_1$ on $\theta$.

**IV. FINITE BIAS CURRENT**

Now we turn to investigate the non-equilibrium transport of (F1,F2)-QD-S. For simplicity, we only consider the antiparallel orientation of $F_1$ and $F_2$ (i.e., $\theta = \pi$), with finite but small bias voltages (i.e., $|eV_1| < \Delta$ and $|eV_2| < \Delta$). Notice that the self-energy becomes to block-diagonal due to $\theta = \pi$, and the expression of current $I_1$ can be simplified as,

$$ I_1 = \frac{e}{h} \int d\omega \left[ A_{11}(f_1 - \bar{f}_1) + A_{12}(f_1 - \bar{f}_2) + Q_{1s}(f_1 - f_s) + Q_{12}(f_1 - f_2) \right], \quad (28) $$

$$ A_{11} = \Gamma_{1\uparrow} \Gamma_{1\uparrow} |G_{12}^r|^2 + \Gamma_{1\uparrow} \Gamma_{1\downarrow} |G_{34}^r|^2, $$

$$ A_{12} = \Gamma_{2\uparrow} \Gamma_{1\uparrow} |G_{12}^r|^2 + \Gamma_{2\uparrow} \Gamma_{1\downarrow} |G_{34}^r|^2, $$

$$ Q_{1s} = \Gamma_{1\uparrow} \Gamma_{s} \rho \left[ |G_{11}^r|^2 + |G_{12}^r|^2 + 2 \text{Re}(-\frac{\Delta}{\omega} G_{11}^r G_{12}^r) \right] + \Gamma_{1\downarrow} \Gamma_{s} \rho \left[ |G_{33}^r|^2 + |G_{34}^r|^2 + 2 \text{Re}(+\frac{\Delta}{\omega} G_{33}^r G_{34}^r) \right], $$

$$ Q_{12} = \Gamma_{2\uparrow} \Gamma_{1\uparrow} |G_{11}^r|^2 + \Gamma_{2\uparrow} \Gamma_{1\downarrow} |G_{33}^r|^2. $$

At zero temperature and in the low bias regime, the current of $Q_{1s}$ process vanishes. Further assuming that both $F_1$ and $F_2$ are fully polarized, both $Q_{12}$ and $A_{11}$ process are also forbidden. Only the process of $A_{12}$, i.e., crossed AR involving $F_1,F_2$ and $S$ contributes to the current. $I_1$ and $I_2$ are derived as

$$ I \equiv I_1 = I_2 = \frac{e}{h} \int d\omega \Gamma_{2\uparrow} \Gamma_{1\uparrow} |G_{12}^r|^2 (f_1 - \bar{f}_2). \quad (29) $$

Notice that $I_1 = I_2$ holds even if $\Gamma_1 \neq \Gamma_2$ and $V_1 \neq V_2$, because $I_1$ is pure spin $\uparrow$ current and $I_2$ is pure spin $\downarrow$ current while $I_1 + I_2$ is required to be non-spin-polarized current by the superconductor. For simplicity, we further assume that $\Gamma_{1\uparrow} = \Gamma_{2\uparrow} \equiv \Gamma_L \ (\Gamma_{1\downarrow} = \Gamma_{2\downarrow} = 0$
due to \( P_1 = P_2 = 1 \) and \( \Gamma_s \equiv \Gamma_R \), then the system \((F_1,F_2)\)-QD-S is similar to a special N-QD-S one, in which the two spin bands of N have different chemical potentials controlled by \( V_1 \) and \( V_2 \). Define the transmission probability of crossed AR by \( T_{AR}(\omega) \equiv \Gamma_L |G_{12}|^2 \), the current can be expressed as

\[
I = \frac{e}{\hbar} \int_{-V_2}^{V_1} T_{AR}(\omega) d\omega .
\]

(30)

Notice that \( T_{AR}(\omega) \) is an even function of \( \omega \), the above current formula implies that the sign of \( I_1 \) or \( I_2 \) is not determined by \( V_1 \) or \( V_2 \) but by \( \frac{1}{2}(V_1 + V_2) \). This is quite unusual because it contains the case that \( V_1 > 0 \) and \( V_2 < 0 \) but \( I_1 = I_2 > 0 \) (This unusual property was first addressed in [20]). Generally, for a three-terminal system, one may expect that current flows out of the terminal with highest voltage and into the terminal with lowest voltage. But for the current conducted by crossed AR, the sign of current in each ferromagnetic terminal is linked to the averaged chemical potential of the two, because two ferromagnets cooperate with each other in this process, with total energy balanced. Fig.6 illustrates the conducting process corresponding to the case of \( I_1 = I_2 > 0 \) with \( \mu_1 > 0 \) and \( \mu_2 < 0 \) but \( \frac{1}{2}(\mu_1 + \mu_2) > 0 \).

We ignore the energy structure of QD in Fig.6 for simplicity, however, the current \( I \equiv I_1 = I_2 \) depends strongly on the transmission probability of QD. In fact, \( I \) is the integral of \( T_{AR}(\omega) \) over the range of \((-V_2, V_1)\). Fig7 shows the surfaces of \( I(V_1,V_2) \) and corresponding \( T_{AR}(\omega) \) spectrum for three typical cases of \( \Gamma_L \) and \( \Gamma_R \). In Fig.7a, \( \Gamma_L \ll \Gamma_R \), the spin degenerate level of QD is hybridized to two Andreev bound states due to coupling with S, while the coupling with \( F_1 \) and \( F_2 \) provides the small broadening to these bound states. \( T_{AR} \) has two peaks with the maximum of unity at each of the Andreev bound states. Correspondingly, the surface of \( I(V_1,V_2) \) has five steps: the highest step corresponds to \((-V_2, V_1)\) covering both of the peaks; the second step (including two patches) corresponds to \((-V_2, V_1)\) covering one of the peaks; the third step (including three patches) corresponds to \((-V_2, V_1)\) or \((V_1, -V_2)\) covering none of the peaks; the fourth step (including two patches) corresponds to \((V_1, -V_2)\) covering one of the peaks; and the lowest step corresponds to \((V_1, -V_2)\) covering both of the peaks. In Fig.7b, \( \Gamma_L = \Gamma_R \), the Andreev bound states are sufficiently broadened so that the two peaks in \( T_{AR} \) merge into one. The one peak structure of \( T_{AR} \) spectrum corresponds to three step pattern in \( I(V_1,V_2) \) surface. In Fig.7c, \( \Gamma_L \gg \Gamma_R \), the resonant level of QD is significantly broadened, as a result, \( T_{AR} \) is small and flat with tails at \( \omega = \pm \Delta \).

The structureless \( T_{AR} \) spectrum corresponds to a plain in \( I(V_1,V_2) \) surface, proportional to \( \frac{1}{2}(V_1 + V_2) \). In short, \( T_{AR} \) spectrum can be extracted from the measurement of \( I(V_1,V_2) \).
V. CONCLUSIONS

In this paper, we have investigated the Andreev reflection in a (F_1,F_2)-QD-S system. By using the non-equilibrium Green function, a general current formula is derived, allowing arbitrary spin polarizations, magnetization orientations and bias voltages in F_1 and F_2. The formula is applied to several special cases, revealing some interesting properties of this system: (1) Analogous to the Fermi velocity mismatch in F/S contact, the zero bias conductance in F-QD-S reaches its maximum \( \frac{4e^2}{h} \) if matching condition \( \Gamma_L \Gamma_L - \Gamma_R \Gamma_R = \Gamma_L^2 \) satisfied. (2) For total current (conductance) of (F_1,F_2)-QD-S with \( V_1 = V_2 \), the two ferromagnets F_1 and F_2 are equivalent to an effective ferromagnet \( \tilde{F} \), and the effective polarization \( \tilde{P} \) can be tuned by the angle between the spin orientations of F_1 and F_2. (3) There is a new effect of magneto-resistance in (F_1,F_2)-QD-S (named as ARMR), in which the conductance for F_1 and F_2 in anti-ferromagnetic alignment is larger than that in ferromagnetic alignment. Based on this effect, a possible way to measure the spin polarization of ferromagnetic material is proposed. (4) The non-equilibrium transport of this system is quite unusual. Especially, if F_1 and F_2 are fully but opposite polarized, the signs of current through F_1 and F_2 is determined by \( \frac{1}{2}(V_1 + V_2) \) rather than \( V_1 \) or \( V_2 \). Furthermore, the surface of \( I(V_1,V_2) \) depends strongly on the AR transmission probability, which can be applied to extract the latter. Finally, we believe that the suggested (F_1,F_2)-QD-S system is accessible of the up-date nano-technology, and we are eager to see relevant experiment on such appealing system.

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* To whom correspondence should be addressed.
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FIGURE CAPTIONS

Fig. 1 Schematic drawing of the three-terminal system under consideration. F$_1$ and F$_2$ represent two ferromagnetic electrodes with different magnetization orientations and bias voltages, QD is a quantum dot, and S is a superconductor with zero voltage as the ground.

Fig. 2 The zero bias conductance $G$ vs $P$ for F-QD-S, where $P$ is the spin polarization of F. $r \equiv \Gamma_R/\Gamma_L$ is the ratio of coupling strengths, with $r \geq 1$ for (a) and $r \leq 1$ for (b).

Fig. 3 The total conductance $G$ vs $\theta$ for (F$_1$,F$_2$)-QD-S, where $\theta$ is the angle between the orientations of F$_1$ and F$_2$, with $\Gamma_1 = \Gamma_2 \equiv \Gamma$, $P_1 = P_2 \equiv P$, and $r \equiv \Gamma_0/(\Gamma_1 + \Gamma_2)$.

Fig. 4 ARMR vs $P$ in (F$_1$,F$_2$)-QD-S, where $ARMR \equiv (G_{AF} - G_F)/(G_{AF} + G_F)$, $P$ and $r$ have the same meaning as in Fig.3.

Fig. 5 The conductance $G_1$ vs $\theta$ for different $P_2$, with $P_1 = 1$ and $r = 1$. $G_1$ has strong / weak dependence on $\theta$ for large / small $P_2$, which can be applied to measure the spin polarization of F$_2$.

Fig. 6 Schematic diagram of non-equilibrium transport in (F$_1$,F$_2$)-QD-S. F$_1$ and F$_2$ are in anti-ferromagnetic alignment, marked by left- and right- slanted shadows, respectively. S is marked by crossed shadow, with the energy gap region $\pm \Delta$ with respect to the chemical potential; QD is between the two barriers, and the energy structure is ignored for simplicity. The diagram illustrates an unusual property of the current conducted by crossed AR involving F$_1$, F$_2$ and S: the signs of $I_1$ or $I_2$ are determined by $\frac{1}{2}(\mu_1 + \mu_2)$ rather than $\mu_1$ or $\mu_2$.

Fig. 7 The current $I \equiv I_1 = I_2$ vs the bias voltages $(V_1, V_2)$ for three typical cases: (a) $\Gamma_L \ll \Gamma_R$; (b) $\Gamma_L = \Gamma_R$; and (c) $\Gamma_L \gg \Gamma_R$. The surface of $I(V_1, V_2)$ has close relationship to the spectrum $T_{AR}(\omega)$, which can be used to extract the latter.
Fig. 1
Fig. 3

(a) $r = 1$

(b) $r = 2$

(c) $r = 0.5$
Fig. 4
Fig. 5
Fig. 6
Fig. 7