New Strategies to Extract Weak Phases from Neutral B Decays

Robert Fleischer
Theory Division, CERN, CH-1211 Geneva 23, Switzerland

Abstract

We discuss new, theoretically clean strategies to determine the angle $\gamma$ of the unitarity triangle from $B_d \to DK_{S(L)}$, $B_s \to D\eta^{(')}$, $D\phi$, ... decays, and point out that the $B_s \to DK_{S(L)}$ and $B_d \to D\pi^0, D\rho^0, ...$ modes allow very interesting determinations of the $B^0_q \to \overline{B}^0_q$ mixing phases $\phi_s$ and $\phi_d$, respectively. Their colour-allowed counterparts $B_s \to D^{(*)}_u \pm K^\mp$, ... and $B_d \to D^{(*)}_u \pm \pi^\mp$, ... also offer new methods to probe $\gamma$.

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1 Introduction

The time-dependent CP asymmetries of neutral $B_q$-meson decays ($q \in \{d,s\}$) into CP eigenstates, which satisfy $(\mathcal{CP})|f\rangle = \pm |f\rangle$, provide valuable information [1]:

$$\Gamma(B_q^0(t) \to f) - \Gamma(\overline{B}_q^0(t) \to f) \over\Gamma(B_q^0(t) \to f) + \Gamma(\overline{B}_q^0(t) \to f) = \frac{A_{\text{dir}}^{\text{CP}} \cos(\Delta M_q t) + A_{\text{mix}}^{\text{CP}} \sin(\Delta M_q t)}{\cosh(\Delta \Gamma_q t/2) - A_{\Delta \Gamma} \sinh(\Delta \Gamma_q t/2)}. \quad (1)$$

Here the CP-violating observables

$$A_{\text{dir}}^{\text{CP}} \equiv \frac{1 - |\xi_f^{(q)}|^2}{1 + |\xi_f^{(q)}|^2} \quad \text{and} \quad A_{\text{mix}}^{\text{CP}} \equiv \frac{2 \text{Im} \xi_f^{(q)}}{1 + |\xi_f^{(q)}|^2} \quad (2)$$

originate from “direct” and “mixing-induced” CP violation, respectively, and are governed by

$$\xi_f^{(q)} = -e^{-i\phi_q} \left[ A(B_q^0 \to f) \over A(B_q^0 \to f) \right], \quad (3)$$

where

$$\phi_q^{\text{SM}} \equiv 2 \arg(V_{tq}^* V_{tb}) = \begin{cases} +2\beta & (q = d) \\ -2\lambda^2 \eta & (q = s) \end{cases} \quad (4)$$

is the CP-violating weak $B_q^0 - \overline{B}_q^0$ mixing phase. The width difference $\Delta \Gamma_q$, which may be sizeable in the $q = s$ case, offers another observable $A_{\Delta \Gamma}$, which is, however, not independent from those in (2), and can be extracted from the following “untagged” rates:

$$\langle \Gamma(B_q(t) \to f) \rangle \equiv \Gamma(B_q^0(t) \to f) + \Gamma(\overline{B}_q^0(t) \to f) \propto [\cosh(\Delta \Gamma_q t/2) - A_{\Delta \Gamma} \sinh(\Delta \Gamma_q t/2)] e^{-\Gamma_q t}. \quad (5)$$

2 $B_d \rightarrow DK_{S(L)}$, $B_s \rightarrow D\eta^{(')}$, $D\phi$, ... and $B_s \rightarrow DK_{S(L)}$, $B_d \rightarrow D\pi^0$, $D\rho^0$, ...

Let us consider in this section $B_q^0 \rightarrow D^0 f_r$ transitions, where $r \in \{s, d\}$ distinguishes between $b \rightarrow Ds$ and $b \rightarrow Dd$ processes [2, 3]. If we require $(\mathcal{CP})|f_r\rangle = \eta^{f_r}_{\text{CP}}|f_r\rangle$, $B_q^0$ and $\overline{B}_q^0$ mesons may both decay into $D^0 f_r$, thereby leading to interference effects between $B_q^0 - \overline{B}_q^0$ mixing and decay processes, which involve the weak phase $\phi_q + \gamma$:

- For $r = s$, i.e. $B_d \rightarrow DK_{S(L)}$, $B_s \rightarrow D\eta^{(')}$, $D\phi$, ..., these effects are governed by a hadronic parameter $x_f e^{i\delta_f} \propto R_b \approx 0.4$, and are hence favourably large.

- For $r = d$, i.e. $B_s \rightarrow DK_{S(L)}$, $B_d \rightarrow D\pi^0$, $D\rho^0$, ..., these effects are tiny because of $x_f e^{i\delta_f} \propto -\lambda^2 R_b \approx 0.02$. 

1
2.1 $B_d \to DK_{S(L)}, B_s \to D\eta^{(
u)}, D\phi, ...$

Let us first focus on $r = s$. If we make use of the CP eigenstates $D_\pm$ of the neutral D-meson system satisfying $(CP)|D_\pm| = \pm |D_\pm|$, we obtain additional interference effects between $B_q^0 \to D^0 f_s$ and $B_q^0 \to \bar{D}^0 f_s$ at the decay-amplitude level, which involve $\gamma$. The most straightforward observable we may measure is the “untagged” rate

$$\langle \Gamma(B_q(t) \to D^\pm f_s) \rangle \equiv \Gamma(B_q^0(t) \to D^\pm f_s) + \Gamma(\bar{B}_q^0(t) \to \bar{D}^\pm f_s)$$

and

$$\Delta \Gamma_{\pm} \equiv [\Gamma(B_q^0 \to D^\pm f_s) + \Gamma(\bar{B}_q^0 \to \bar{D}^\pm f_s)] e^{-\Gamma_q t}$$

providing the following “untagged” rate asymmetry:

$$\Gamma_{\pm} \equiv \frac{\langle \Gamma(B_q \to D^\pm f_s) \rangle - \langle \Gamma(B_q \to D^\mp f_s) \rangle}{\langle \Gamma(B_q \to D^\pm f_s) \rangle + \langle \Gamma(B_q \to D^\mp f_s) \rangle}. \tag{6}$$

Interestingly, already this quantity offers valuable information on $\gamma$, since bounds on this angle are implied by

$$|\cos \gamma| \geq |\Gamma_{\pm}^f|. \tag{7}$$

Moreover, if we fix the sign of $\delta_{fs}$ with the help of the factorization approach, we obtain

$$\text{sgn}(\cos \gamma) = -\text{sgn}(\Gamma_{\pm}^f), \tag{8}$$

i.e. we may decide whether $\gamma$ is smaller or larger than $90^\circ$. If we employ, in addition, the mixing-induced observables $S_{\pm}^{fs} \equiv A_{\text{mix}}^{\text{CP}}(B_q \to D^\pm f_s)$, we may determine $\gamma$. To this end, it is convenient to introduce the quantities

$$\langle S_{fs} \rangle^\pm \equiv \frac{S_{fs}^+ \pm S_{fs}^-}{2}. \tag{10}$$

Expressing the $\langle S_{fs} \rangle^\pm$ in terms of the $B_q \to D^\pm f_s$ decay parameters gives rather complicated formulae. However, complementing the $\langle S_{fs} \rangle^\pm$ with $\Gamma_{\pm}^f$ yields

$$\tan \gamma \cos \phi_q = \left[ \frac{\eta_{fs} \langle S_{fs} \rangle^+}{\Gamma_{\pm}^f} \right] + [\eta_{fs} \langle S_{fs} \rangle^- - \sin \phi_q], \tag{11}$$

where $\eta_{fs} \equiv (-1)^L \eta_{\text{CP}}$, with $L$ denoting the $D_f$ angular momentum $[2, 3]$. If we use this simple but exact relation, we obtain the twofold solution $\gamma = \gamma_1 \lor \gamma_2$, with $\gamma_1 \in [0^\circ, 180^\circ]$ and $\gamma_2 = \gamma_1 + 180^\circ$. Since $\cos \gamma_1$ and $\cos \gamma_2$ have opposite signs, $[9]$ allows us to fix $\gamma$ unambiguously. Another advantage of $[10]$ is that $\langle S_{fs} \rangle^+$ and $\Gamma_{\pm}^f$ are both proportional to $x_{fs} \approx 0.4$, so that the first term in square brackets is of $O(1)$, whereas the second one is of $O(x_{fs}^2)$, hence playing a minor rôle. In order to extract $\gamma$, we may also employ $D$ decays into CP non-eigenstates $f_{\text{NE}}$, where we have to deal with complications originating from $D^0, \bar{D}^0 \to f_{\text{NE}}$ interference effects $[4]$. Also in this case, $\Gamma_{\pm}^f$ is a very powerful ingredient, offering an efficient, analytical strategy to include these interference effects in the extraction of $\gamma$ $[3]$.
2.2 $B_s \to D K_{S(L)}, B_d \to D \pi^0, D \rho^0, ...$

The $r = d$ case also has interesting features. It corresponds to $B_s \to D K_{S(L)}, B_d \to D \pi^0, D \rho^0 \ldots$ decays, which can be described through the same formulae as their $r = s$ counterparts. Since the relevant interference effects are governed by $x_{fd} \approx -0.02$, these channels are not as attractive for the extraction of $\gamma$ as the $r = s$ modes. On the other hand, the relation

$$\eta_{fd}(S_{fd}) = \sin \phi_q + \mathcal{O}(x_{fd}^2) = \sin \phi_q + \mathcal{O}(4 \times 10^{-4})$$

(12)

offers very interesting determinations of $\sin \phi_q$ [2]. Following this avenue, there are no penguin uncertainties, and the theoretical accuracy is one order of magnitude better than in the “conventional” $B_d \to J/\psi K_S, B_s \to J/\psi \phi$ strategies. In particular, $\phi_s^{SM} = -2\lambda^2 \eta$ could, in principle, be determined with a theoretical uncertainty of only $\mathcal{O}(1\%)$, in contrast to the extraction from the $B_s \to J/\psi \phi$ angular distribution, which suffers from generic penguin uncertainties at the 10\% level.

3 $B_s \to D_s^{(*)\pm} K^{\mp}, \ldots$ and $B_d \to D^{(*)\pm} \pi^{\mp}, \ldots$

Let us now consider the colour-allowed counterparts of the $B_q \to D f_q$ modes discussed above, which we may write generically as $B_q \to D_q \pi_q$ [3]. The characteristic feature of these transitions is that both a $B_q^0$ and a $\bar{B}_q^0$ meson may decay into $D_q \pi_q$, thereby leading to interference between $B_q^0 - \bar{B}_q^0$ mixing and decay processes, which involve the weak phase $\phi_q + \gamma$:

- In the case of $q = s$, i.e. $D_s \in \{D_s^+, D_s^{*+}, \ldots\}$ and $u_s \in \{K^+, K^{*+}, \ldots\}$, these effects are favourably large as they are governed by $x_s e^{i\delta_s} \propto R_b \approx 0.4$.

- In the case of $q = d$, i.e. $D_d \in \{D^+, D^{*+}, \ldots\}$ and $u_d \in \{\pi^+, \rho^+, \ldots\}$, the interference effects are described by $x_d e^{i\delta_d} \propto -\lambda^2 R_b \approx -0.02$, and hence are tiny.

We shall only consider $B_q \to D_q \pi_q$ modes, where at least one of the $D_q, \pi_q$ states is a pseudoscalar meson; otherwise a complicated angular analysis has to be performed.

It is well known that such decays allow determinations of the weak phases $\phi_q + \gamma$, where the “conventional” approach works as follows [3, 7]: if we measure the observables $C(B_q \to D_q \pi_q) \equiv C_q$ and $C(B_q \to \bar{D}_q u_q) \equiv \bar{C}_q$ provided by the $\cos(\Delta M_q t)$ pieces of the time-dependent rate asymmetries, we may determine $x_q$ from terms entering at the $x_q^2$ level. In the case of $q = s$, we have $x_s = \mathcal{O}(R_b)$, implying $x_s^2 = \mathcal{O}(0.16)$, so that this may actually be possible, although challenging. On the other hand, $x_d = \mathcal{O}(-\lambda^2 R_b)$ is doubly Cabibbo-suppressed. Although it should be possible to resolve terms of $\mathcal{O}(x_d)$, this will be impossible for the vanishingly small $x_d^2 = \mathcal{O}(0.0004)$ terms, so that other approaches to fix $x_d$ are required [3]. In order to extract $\phi_q + \gamma$, the mixing-induced observables $S(B_q \to D_q \pi_q) \equiv S_q$ and $S(B_q \to \bar{D}_q u_q) \equiv \bar{S}_q$ associated with the $\sin(\Delta M_q t)$
terms of the time-dependent rate asymmetries must be measured, where it is convenient to introduce
\[ \langle S_q \rangle_\pm = \frac{S_q \pm S_q}{2}. \] (13)
If we assume that \( x_q \) is known, we may consider
\[ s_+ \equiv (-1)^L \left[ \frac{1 + x_q^2}{2x_q} \right] \langle S_q \rangle_+ = + \cos \delta_q \sin(\phi_q + \gamma) \] (14)
\[ s_- \equiv (-1)^L \left[ \frac{1 + x_q^2}{2x_q} \right] \langle S_q \rangle_- = - \sin \delta_q \cos(\phi_q + \gamma), \] (15)
yielding
\[ \sin^2(\phi_q + \gamma) = \frac{1 + s_+^2 - s_-^2}{2} \pm \sqrt{(1 + s_+^2 - s_-^2)^2 - 4s_+^2}, \] (16)
which implies an eightfold solution for \( \phi_q + \gamma \). If we fix the sign of \( \cos \delta_q \) with the help of factorization, a fourfold discrete ambiguity emerges. Note that this assumption allows us to extract also the sign of \( \sin(\phi_q + \gamma) \) from \( \langle S_q \rangle_+ \), which is of particular interest, as discussed in [5]. To this end, the factor \((-1)^L\), where \( L \) is the \( D_q \pi_q \) angular momentum, has to be properly taken into account.

Let us now discuss the new strategies to explore the \( B_q \to D_q \pi_q \) modes proposed in [5]. If \( \Delta \Gamma_s \) is sizeable, the time-dependent “untagged” rates introduced in (5)
\[ \langle \Gamma(B_q(t) \to D_q \pi_q) \rangle = \langle \Gamma(B_q \to D_q \pi_q) \rangle e^{-\Gamma_q t} \] (17)
\[ \times [\cosh(\Delta \Gamma_q t/2) - A_{\Delta \Gamma}(B_q \to D_q \pi_q) \sinh(\Delta \Gamma_q t/2)] \]
and their CP conjugates provide \( A_{\Delta \Gamma}(B_s \to D_s \pi_s) \equiv A_{\Delta \Gamma s} \) and \( A_{\Delta \Gamma}(B_s \to \overline{D}_s u_s) \equiv \overline{A}_{\Delta \Gamma s} \), which yield
\[ \tan(\phi_s + \gamma) = - \frac{\langle S_s \rangle_+}{\langle A_{\Delta \Gamma s} \rangle_+} = + \frac{\langle A_{\Delta \Gamma s} \rangle_-}{\langle S_s \rangle_-}, \] (18)
where the \( \langle A_{\Delta \Gamma_s} \rangle_\pm \) are defined in analogy to (13). These relations allow an unambiguous extraction of \( \phi_s + \gamma \) if we fix again the sign of \( \cos \delta_q \) through factorization. Another important advantage of (13) is that we do not have to rely on \( O(x_q^2) \) terms, as \( \langle S_s \rangle_\pm \) and \( \langle A_{\Delta \Gamma_s} \rangle_\pm \) are proportional to \( x_s \). On the other hand, we need a sizeable value of \( \Delta \Gamma_s \). Measurements of untagged rates are also very useful in the case of vanishingly small \( \Delta \Gamma_q \), since the “unevolved” untagged rates in (17) offer various interesting strategies to determine \( x_q \) from the ratio of \( \langle \Gamma(B_q \to D_q \pi_q) \rangle + \langle \Gamma(B_q \to \overline{D}_q u_q) \rangle \) to CP-averaged rates of appropriate \( B^\pm \) or flavour-specific \( B_q \) decays.

If we keep the hadronic parameter \( x_q \) and the associated strong phase \( \delta_q \) as “unknown”, free parameters in the expressions for the \( \langle S_q \rangle_\pm \), we obtain
\[ |\sin(\phi_q + \gamma)| \geq |\langle S_q \rangle_+|, \quad |\cos(\phi_q + \gamma)| \geq |\langle S_q \rangle_-|, \] (19)
which can straightforwardly be converted into bounds on $\phi_q + \gamma$. If $x_q$ is known, stronger constraints are implied by

\[ |\sin(\phi_q + \gamma)| \geq |s_+|, \quad |\cos(\phi_q + \gamma)| \geq |s_-|. \tag{20} \]

Once $s_+$ and $s_-$ are known, we may of course determine $\phi_q + \gamma$ through the “conventional” approach, using (16). However, the bounds following from (20) provide essentially the same information and are much simpler to implement. Moreover, as discussed in detail in [5] for several examples, the bounds following from the $B_s$ and $B_d$ modes may be highly complementary, thereby providing particularly narrow, theoretically clean ranges for $\gamma$.

Let us now further exploit the complementarity between the $B_s^0 \to D_s^{(*)+}K^-$ and $B_d^0 \to D^{(*)+}\pi^-$ modes. If we look at their decay topologies, we observe that these channels are related to each other through an interchange of all down and strange quarks. Consequently, the $U$-spin flavour symmetry of strong interactions implies $a_s = a_d$ and $\delta_s = \delta_d$, where $a_s = x_s/R_b$ and $a_d = -x_d/(\lambda^2 R_b)$ are the ratios of hadronic matrix elements entering $x_s$ and $x_d$, respectively. There are various possibilities to implement these relations. A particularly simple picture emerges if we assume that $a_s = a_d$ and $\delta_s = \delta_d$, which yields

\[
\tan \gamma = -\left[ \frac{\sin \phi_d - S \sin \phi_s}{\cos \phi_d - S \cos \phi_s} \right]_{\phi_s = 0^\circ} = -\left[ \frac{\sin \phi_d}{\cos \phi_d - S} \right]. \tag{21}
\]

Here we have introduced

\[ S = -R \left[ \frac{\langle S_d \rangle^+}{\langle S_s \rangle^+} \right] \tag{22} \]

with

\[ R = \left( \frac{1 - \lambda^2}{\lambda^2} \right) \left[ \frac{1}{1 + x_s^2} \right], \tag{23} \]

which can be fixed from untagged $B_s$ rates through

\[ R = \left( \frac{f_K}{f_\pi} \right)^2 \left[ \frac{\Gamma(B_s^0 \to D_s^{(*)+}K^-) + \Gamma(B_s^0 \to D_s^{(*)+}\pi^-)}{\Gamma(B_s \to D_s^{(*)+}K^-) + \Gamma(B_s \to D_s^{(*)+}\pi^-)} \right]. \tag{24} \]

Alternatively, we may only assume that $\delta_s = \delta_d$ or that $a_s = a_d$. Apart from features related to multiple discrete ambiguities, the most important advantage with respect to the “conventional” approach is that the experimental resolution of the $x_q^2$ terms is not required. In particular, $x_d$ does not have to be fixed, and $x_s$ may only enter through a $1 + x_s^2$ correction, which can straightforwardly be determined through untagged $B_s$ rate measurements. In the most refined implementation of this strategy, the measurement of $x_d/x_s$ would only be interesting for the inclusion of $U$-spin-breaking effects in $a_d/a_s$. Moreover, we may obtain interesting insights into hadron dynamics and $U$-spin-breaking effects.
4 Conclusions

We have discussed new strategies to explore CP violation through neutral $B_q$ decays. In the first part, we have shown that $B_d \to DK_{S(L)}$, $B_s \to D\eta(\prime)$, $D\phi$, ... modes provide theoretically clean, efficient and unambiguous extractions of $\tan \gamma$ if we combine an “untagged” rate asymmetry with mixing-induced observables. On the other hand, their $B_s \to D_{\pm}K_{S(L)}$, $B_d \to D_{\pm}\pi^0$, $D_{\pm}\rho^0$, ... counterparts are not as attractive for the determination of $\gamma$, but allow extremely clean extractions of the mixing phases $\phi_s$ and $\phi_d$, which may be particularly interesting for the $\phi_s$ case. In the second part, we have discussed interesting new aspects of $B_s \to D_{s}^{(*)}\pm K^{\mp}$, ... and $B_d \to D_{s}^{(*)}\pm\pi^{\mp}$, ... decays. The observables of these modes provide clean bounds on $\phi_q + \gamma$, where the resulting ranges for $\gamma$ may be highly complementary in the $B_s$ and $B_d$ cases, thereby yielding stringent constraints on $\gamma$. Moreover, it is of great advantage to combine the $B_d \to D_{s}^{(*)}\pm\pi^{\mp}$ modes with their $U$-spin counterparts $B_s \to D_{s}^{(*)}\pm K^{\mp}$, allowing us to overcome the main problems of the “conventional” strategies to deal with these modes. We strongly encourage detailed feasibility studies of these new strategies.

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