Eigen-frequency analysis of spherical shell laminated composite plates with and without central cutouts using finite elements

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Abstract. This article presents the free vibration responses of spherical shell laminated composite panels (SSLCP) without and with central cutouts (square, circular and rectangular) using finite elements. The SSLCP midplane kinematics has been modeled using ABAQUS commercial finite element tool in the framework of first order shear deformation theory. Firstly, the validity of the present model is established by comparing the present natural frequencies values with the available benchmark results. Subsequently, the influence of both varying and constant aspect ratio for different central cutouts on the natural frequencies of SSLCP are investigated by solving appropriate numerical examples and findings are discussed in detail. The results are given in the form of tables and figures.

1. Introduction

Now a day's spherical shell laminated composite panels (SSLCP) are extensively used in various engineering applications such as ships, aircrafts, missiles, automobiles, mining equipments, railway wagon and civil structures etc. SSLCP are widely used due to weight sensitive, increase strength and stability, corrosion resistance, easy for fabrication and low cost.

Most of the researchers have used modal analysis for finding modal parameters by using different techniques. For dynamic analysis finite element analysis (FEA) is the best tool among the different numerical techniques. So many theories such as classical laminated plate/shell theory, first order deformation theory, higher order deformation theory etc. have been used for modal analysis of SSLCP.

Different computational models for laminated composite shell/plates were proposed by Reddy [1], Noor et al [2]. Reddy and Chandrashekh [3]. Chakravorty et al [4] solved many numerical problems of doubly curved laminated composite panels. Chakravorty et al [5] analyzed both free and force vibration on laminated shells with and without cutouts. Sahoo [6] solved numerical problems of laminated composite stiffened spherical panels with and without cutouts. Tornabene [7] formulated higher order layer wise theory for free vibration analysis of doubly-curved laminated composite panels. Singh [8] solved many numerical problems both static and free vibration behavior of laminated composite spherical shell panel. Fazzolari [9] formulated higher order shear deformation theory (HSDT) for free vibration analysis of cross-ply laminated composite both cylindrical and spherical shells. Chao et al [10] calculated ultimate strength of curved stiffened pates experimentally. Tornabene et al [11] formulated 2D higher order layer theory for free vibration of thin and thick
doubly curved laminated composite panels. Lee and Han [12] computed forced vibration of laminated composite plates subjected to arbitrary loading.

Thus, it is seen that free vibration of SSLCP with and without cutouts requires in depth for proper understanding of the mechanical behavior. The objective of this work is to model and simulate the free vibration behavior of spherical shell laminated composite panels (SSLCP) with and without cutouts having different boundary conditions. Firstly, the SSLCP is modeled using commercial FEA tool (ABAQUS) in the framework of first order shear deformation theory (FSDT). An eight-noded serendipity element having six degrees of freedom per node is utilized to discretize the panel. Subsequently, the validity of the present model is established along with the convergence. Finally, several numerical examples have been solved to study the influence of the variation of size of cutouts like square, rectangular and circular on the natural frequencies of the SSLCP and discussed in detail.

Figure 1. Simply supported (a) Spherical shell panel without cutout, (b) Spherical shell panel with central cutout

| Nomenclature |
|---------------|
| $a, b$ | length and width of SSLCP in plan |
| $a', b'$ | length and width of cutout in plan |
| $r$ | radius of circular cutout in plan |
| $h$ | thickness of panel |
| $E_{11}, E_{22}$ | elastic moduli |
| $G_{12}, G_{13}$ | shear moduli |
| $\nu_{12}$ | poisson’s ratio |
| $R_{xx}, R_{yy}$ | radii of curvature panel |
| $\rho$ | density of material |
| $\omega_n$ | natural frequency |
| $\bar{\omega}$ | non-dimensional natural frequency $\left[ \bar{\omega} = \omega_n \sqrt{\rho / E_{22}h^2} \right]$ |
2. Vibration analysis of SSLCP
The SSLCP made of orthotropic layers having equal thickness and placed symmetrically about its middle surface, which subjected to the simply supported boundary conditions as shown in Figure 1 (a) & (b) without and with central cutouts respectively. Neglect coupling effect due to bending-twisting and neglect shear extension. The middle plane dynamics for plate or shell structure is calculated based on the FSDT and given as:
\[
\begin{align*}
\mathbf{u}(x, y, z, t) &= u_0(x, y) + z\theta_x(x, y) \\
\mathbf{v}(x, y, z, t) &= v_0(x, y) + z\theta_y(x, y) \\
\mathbf{w}(x, y, z, t) &= w_0(x, y) + z\theta_z(x, y)
\end{align*}
\]
where, \(u\), \(v\) and \(w\) are the displacement vector of any point in the layer at time \(t\) along \(x\), \(y\) and \(z\) direction respectively. Modal analysis is used for this structure and its modal parameters are calculated by solving the eigen value equations:
\[
([K] - \omega^2[M])\varphi = 0
\]
where \([K]\), \([M]\), \(\omega\) and \{\(\varphi\)\} are the stiffness matrix, mass matrix, natural frequency of vibration and corresponding mode shape vector, respectively.

![Figure 2](image2.png)
Figure 2. (a) Ply orientation (0°/90°/0°/90°) of laminated panel, (b) - (f) First mode shapes of SSLCP for different sizes of the central square cutouts

![Figure 3](image3.png)
Figure 3. (a)-(d) First mode shapes of SSLCP for different sizes of the central square cutouts, (e)-(h) First mode shapes of SSLCP for different sizes of the central rectangular cutouts

3. Results and discussions
Modal analysis of SSLCP is carried out using ABAQUS to calculate modal parameters and compared with Reddy [1], Chakravorty [4] [5] and Sahoo [6]. The material properties [4] are taken for modal analysis as given below:
\[
\frac{E_{11}}{E_{22}} = 25, \quad G_{12} = G_{11} = 0.5E_{22}, \quad \nu_{12} = 0.25.
\]
Element type S8R (from the ABAQUS element library) having eight nodes and six degrees of freedom per node has been used for discretization purpose. The first mode non-dimensional modal
frequencies (\( \sigma \)) are taken into account for analysis. Figure 2 (a) represent ply orientation of laminated composite panel and Figure 2 (b) shows SSLCP without cutouts.

In the present section, the non-dimensional modal frequencies (\( \sigma \)) are calculated for different mesh sizes to verify the convergence of present analysis. For convergence study, the simply supported spherical shells laminated composite panel (\( 0^\circ/90^\circ \)) is considered. The variation of non-dimensional modal frequencies (\( \sigma \)) with mesh sizes are shown in Table 1. It can be observed that, the values non-dimensional modal frequencies (\( \sigma \)) are nicely converging with refinement of mesh. Beyond 14 × 14 mesh size is not improved the results. The result of 14 × 14 mesh is good agreement with Reddy [1] and Chakravorty [4].

Table 1. Non-dimensional natural frequencies (\( \sigma = \omega_0 a^2 \sqrt{\rho / E_{22}h^2} \)) for simply supported SSLCP (\( 0^\circ/90^\circ \))

| Mesh Size | 4 × 4 | 6 × 6 | 8 × 8 | 10 × 10 | 12 × 12 | 14 × 14 | 16 × 16 | Reddy [1] | Chakravorty [2] |
|-----------|-------|-------|-------|---------|---------|---------|---------|-----------|----------------|
| \( \sigma \) | 47.862 | 46.841 | 45.959 | 46.358 | 45.940 | 45.799 | 46.002 | 45.801 |

\( \rho / R_{yy} = 1/300, a / b = 1, a / h = 100, E_{11} / E_{22} = 25, G_{12} = G_{13} = 0.5E_{22}, \nu_{12} = 0.25 \) [3].

3.1. Free vibration analysis of simply supported spherical shells laminated composite panel (SSLCP) without cutout

Non-dimensional natural frequency (\( \sigma \)) of SSLCP having different ply orientations (\( 0^\circ/90^\circ \) and \( 0^\circ/90^\circ/0^\circ \)) with different \( h / R_{xx} \) ratios are presented in Table 2 along with exact results obtained by Reddy [1] and Chakravorty [4]. In both cases the present results exhibit good agreement with others. With increase the \( R_{xx} \) or decrease the \( h / R_{xx} \), the non-dimensional natural frequency (\( \sigma \)) decreases.

Table 2. Non-dimensional natural frequencies (\( \sigma \)) for simply supported SSLCP without cutout

| \( h / R_{xx} \) | \( (0^\circ/90^\circ) \) | \( (0^\circ/90^\circ/0^\circ) \) |
|----------------|----------------|----------------|
| Reddy [1] | Chakravorty [4] | Present | Reddy [1] | Chakravorty [4] | Present |
| Plate | 9.6873 | 9.6893 | 9.7500 | 15.183 | 15.192 | 15.181 |
| \( 1/300 \) | 46.002 | 45.801 | 45.799 | 47.265 | 47.035 | 47.972 |
| \( 1/400 \) | 35.228 | 35.126 | 36.078 | 36.971 | 36.89 | 37.412 |
| \( 1/500 \) | 28.825 | 28.778 | 29.216 | 30.993 | 30.963 | 31.305 |

\( R_{xx} / R_{yy} = 1, a / b = 1, a / h = 100, E_{11} / E_{22} = 25, G_{12} = G_{13} = 0.5E_{22}, \nu_{12} = 0.25 \) [3].

3.2. Free vibration analysis of spherical shells laminated composite panel (SSLCP) with central square cutout

Non-dimensional natural frequency (\( \sigma \)) of SSLCP (\( 0^\circ/90^\circ/0^\circ/90^\circ \)) with central square cutouts in two boundary conditions, simply supported (SSSS) and clamped (CCCC), with different \( a' / a \) ratios are presented in Table 3 along with exact results obtained by Sahoo [6] and Chakravorty [5]. In Figure 2 (b)-(f) shows first mode non-dimensional natural frequency with varying aspect ratios (\( a' / a \)). In SSSS
boundary conditions, with increase the aspect ratios \((a'/a)\) the non-dimensional natural frequency \((\sigma)\) increases. But this type of results are not seen in CCCC boundary conditions.

Table 3. Non-dimensional natural frequencies \((\sigma)\) for SSLCP \((0^\circ/90^\circ/0^\circ/90^\circ)\) with central square cutouts as shown in Figure 2 (b)-(f) in different boundary conditions

| \(a'/a\) | Sahoo [6] | Chakravorty [5] | Present | Sahoo [6] | Chakravorty [5] | Present |
|---------|-----------|----------------|---------|-----------|----------------|---------|
| 0.0     | 47.100    | 47.109         | 47.866  | 117.621   | 118.197        | 118.205 |
| 0.1     | 47.114    | 47.524         | 48.522  | 104.251   | 104.274        | 104.578 |
| 0.2     | 48.801    | 48.823         | 49.763  | 97.488    | 98.299         | 99.015  |
| 0.3     | 50.920    | 50.925         | 51.789  | 113.226   | 113.766        | 114.236 |
| 0.4     | 53.788    | 53.789         | 54.503  | 110.094   | 110.601        | 110.986 |

\[ h/R_{yy} = h/R_{xx} = 1/300, a / b = a'/b' = 1, a / h = 100, E_{11}/E_{22} = 25, G_{12} = G_{13} = 0.5E_{22}, \nu_{12} = 0.25 \]

3.3. Free vibration analysis of simply supported spherical shells laminated composite panel (SSLCP) with central square, circular and rectangular cutouts

Non-dimensional natural frequency \((\sigma)\) of simply supported SSLCP \((0^\circ/90^\circ/0^\circ/90^\circ)\) with central square, circular and rectangular cutouts with different \(A'/A\) ratios are presented in Table 4. Here we take \(a'/b' = 2\) for rectangular cutout. Increase the \(A'/A\) ratio increase the non-dimensional natural frequency \((\sigma)\) for both square and circular central cutouts. But in case of rectangular cutouts, the non-dimensional natural frequencies \((\sigma)\) are more very at higher \(A'/A\) ratio. For constant \(A'/A\) ratio, the square cutouts give more natural frequencies followed by circular cutouts and then rectangular cutouts. But at higher \(A'/A\) ratio, the natural frequencies of circular cutouts give more value than square cutouts.

Table 4. Non-dimensional natural frequencies \((\sigma)\) for simply supported SSLCP \((0^\circ/90^\circ/0^\circ/90^\circ)\) with central square, circular and rectangular cutouts as shown respectively

| \(A'/A\) | Square cutout | Circular cutout | Rectangular cutout |
|---------|---------------|----------------|-------------------|
| 0.0     | 47.866        | 47.866         | 47.866            |
| 0.01    | 48.522        | 48.217         | 48.396            |
| 0.04    | 49.763        | 49.518         | 48.965            |
| 0.09    | 51.789        | 51.704         | 48.161            |
| 0.16    | 54.503        | 54.823         | 42.928            |

\[ h/R_{yy} = h/R_{xx} = 1/300, a / b = 1, A = ab, A' = a'b' = \pi r^2, \text{Square cutout (}a'/b' = 1), \text{Rectangular cutout (}a'/b' = 2), \text{Circular cutout (}r = \sqrt{a'b'}/\pi, a / h = 100, E_{11}/E_{22} = 25, G_{12} = G_{13} = 0.5E_{22}, \nu_{12} = 0.25\]
4. Conclusion
The following conclusions are drawn from the present study:
(a) Among simply supported and clamped boundary conditions, clamped boundary condition is the more desirable for SSLCP with or without cutouts.
(b) With increase the $R_{xx}$ or decrease the $h/R_{xx}$, the non-dimensional natural frequency ($\sigma$) decreases in case SSLCP without cutouts.
(c) In SSSS boundary conditions, with increase the aspect ($a'/a$) ratios the non-dimensional natural frequency ($\sigma$) increases. This type of results are not seen in CCCC boundary conditions. But its values are more desirable than former.
(d) Increase the $A'/A$ ratio increase the non-dimensional natural frequency ($\sigma$) in square, circular and rectangular central cutouts. But in case of rectangular cutouts, the non-dimensional natural frequencies ($\sigma$) are not so desirable as compared to other two cutouts. At higher $A'/A$ ratio, the rectangular cutouts give very less desirable outputs. For constant $A'/A$ ratio, the square cutouts give more natural frequencies followed by circular cutouts and then rectangular cutouts. But at higher $A'/A$ ratio, the natural frequencies of circular cutouts give more value than square cutouts.

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