Process for assessment of stress-strain state of a rock mass and its defects using data on shear measurements at outcrop surface

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Abstract. It is proposed to assess the stress-strain state of a rock mass and its defects by measuring displacements of mine working contour. Algorithms and software to solve the elastic and elastic-plastic problems are worked out for a case when a working contour shifts to failure domain. To compute of the working contour displacements, when all the shears except creep-strain occurred, the researchers set forth the process for continuous stress release in the contour of the preset geometry at continuous monitoring and recording of shear benchmark data at its boundary. The process is demonstrated on shear measurements on a circular hole in a rubber plate under tension load.

1. Introduction

It is the common knowledge that elastic and plastic deformations run almost instantly unlike creep deformations. Actually, when mine workings are driven in a rock mass, elastic and plastic deformations, including displacements corresponding to these deformations happen instantaneously at contours of mine workings. That is the reason why there is no sense to speak of displacements of benchmarks at working contours, provided that they do not relate to creep strain in a rock mass. Such statement of the “industrial” experiment with driving mine workings lead to loss of important information on shears at contours of mine workings. This information can be useful in search for stress values in “infinity” and defect imperfections of a rock mass with identification of hard inclusions and closed cavities [1–5]. The present paper proposes the process for determination of actually occurred displacements at mining contours using stress, applied in infinity.

2. Mathematical model and solution

The new-proposed approach is justified. For the sake of simplicity we consider a plane strain version. Assume, there is coordinate system \( xOy \) and deformation along axis \( z \) is plane. Let plane \( xOy \) is constrained with stress \( \sigma_x^{\infty} = -p \), \( \sigma_y^{\infty} = -p \), \( \sigma_z^{\infty} = -2\nu p \). The study case is presented in Figure 1a.

For plain strain the elasticity relations are:

\[
\sigma_r - \sigma_\theta = 2\mu(\varepsilon_r - \varepsilon_\theta), \quad \sigma_r + \sigma_\theta = 2k(\varepsilon_r + \varepsilon_\theta),
\]

\[
\varepsilon_r = \frac{du}{dr}, \quad \varepsilon_\theta = \frac{u}{r}, \quad 2\mu = E / (1+\nu), \quad 2k = 2\mu / (1-2\nu).
\]
Here, radial displacements \( u \) are counted at a certain preset point \( O \). Under stress compression of the plane \( \sigma_x^\infty = -p, \sigma_y^\infty = -p \) we obtain stresses in polar coordinates \( \sigma_r = -p \).

\[
\begin{align*}
\sigma_x^\infty &= -p \\
\sigma_y^\infty &= -p \\
\end{align*}
\]

Figure 1. Scheme of mine working driving in a rock mass: (a) in the initial hydrostatic stress field; (b) at the complete stress release at the contour concurrently to driving of a working.

From (1) it follows that \( \frac{du}{dr} + \frac{u}{r} = -\frac{p}{k} \) and \( \frac{du}{dr} - \frac{u}{r} = 0 \). Solving this system of equations we find:

\[
u = -\frac{p}{2k} r.
\]

Thus, under stress load \( \sigma_x^\infty = -p, \sigma_y^\infty = -p \) displacements of kind (2) take place.

Next, in the plane under consideration we drive a working of radius \( a \), this means that stress \( \sigma_r \) at its contour should be equal to zero. To gain this effect and to compensate compressive stress \( \sigma_r = -p \) we apply tension stress \( \sigma_r = +p \), vanishing in the infinity of the contour of the working of radius \( a \). We continue to keep to assumption that the material deforms under the elasticity law (1).

Solving differential equations of the problem we obtain that additional stress \( \sigma_r^\infty \) should be distributed under the law:

\[
\sigma_r^\infty = C_1 + \frac{C_2}{r^2}.
\]

At \( r \to \infty \), \( \sigma_r = 0 \), so \( C_1 = 0 \) in (3). Further, at \( r = a \) \( \sigma_r^\infty \) should be positive and equal to \( p \). As a result, formula for \( \sigma_r^\infty \) is:

\[
\sigma_r^\infty = \frac{pa^2}{r^2}.
\]

Thereto, \( \sigma_\theta = -\frac{pa^2}{r^2} \). Then \( \sigma_r^\infty + \sigma_\theta = 0 \), wherefrom we find \( \tilde{u} = \frac{C_2}{r} \). Substituting this term to the first relation (1) allows definition \( C_3 = -\frac{pa^2}{2\mu} \), then displacement additional to (2) is:

\[
\tilde{u} = -\frac{pa^2}{2\mu r}.
\]

It is shown in Figure 1b.

It is obvious that tension shear (5) directed inside of the mine working correspond to tension stresses (4). Overlapping of two stress states gives the resultant stress sum:
\[ \sigma_r = -p \left( 1 - \frac{a^2}{r^2} \right), \quad \sigma_\theta = -p \left( 1 + \frac{a^2}{r^2} \right). \] (6)

Displacements \( u \) can be also summed up:
\[ u = -\frac{p}{2k} r - \frac{pa^2}{2\mu r}. \] (7)

A question arises, provided that in a stressed rock mass a certain point at contour of a would-be working is marked by \( M \) (Figure 1a), what displacements: either (5) or (7) are obtained after the stress state is released. It is clear, at \( \nu = 1/2 \) difference between (5) and (7) vanishes, but you should mind that it happens only at \( \nu = 1/2 \).

Now let consider a more complicated case (Figure 2).

\[ \sigma_y = -q \]
\[ \sigma_x = -p \]

**Figure 2.** (a) Rock mass under two loads \( p \) and \( q \) and (b) formation of tangential stresses at contour of a would-be working.

Let a rock mass be under two loads \( \sigma_x = -p \), \( \sigma_y = -q \), \( (p, q > 0) \). Introducing polar coordinates \( r, \theta \), at plane \( xOy \) for stresses \( \sigma_r, \sigma_\theta, \tau_{r\theta} \), we obtain:
\[ \sigma_r = -p \cos^2 \theta - q \sin^2 \theta, \quad \sigma_\theta = -p \sin^2 \theta - q \cos^2 \theta, \]
\[ \tau_{r\theta} = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta = -\frac{p-q}{2} \sin 2\theta. \] (8)

In homogeneous stress and strain field \( \sigma_x = -p \), \( \sigma_y = -q \) correspond to displacements \( u_x \), \( u_y \) or displacements \( u_r \), \( u_\theta \) are:
\[ \begin{cases} u_r = -\frac{(1-2\nu)}{4\mu} (p + q) r + \frac{1}{2\mu} (q - p) r \cos 2\theta, \\ u_\theta = -\frac{p-q}{4\mu} r \sin 2\theta. \end{cases} \] (9)

It follows from (8) that compressive stresses \( \sigma_r \), positive and negative \( \tau_{r\theta} \) act in the contour of a would-be working with equation \( r = a \). To drive a working with boundary conditions \( \sigma_r = 0 \), \( \tau_{r\theta} = 0 \) requires to apply opposite-sign stresses to \( \sigma_r \), \( \tau_{r\theta} \) at the boundary \( r = a \):
\[ \sigma_r = p \cos^2 \theta + q \sin^2 \theta, \quad \tau_{r\theta} = \frac{p-q}{2} \sin 2\theta. \]
To define stresses, displacements in a rock mass with a working \( r = a \) Kolosov–Muskhelishvili formula is applicable [6]:

\[
\sigma_r - i\tau_{\theta \phi} = \Phi(z) + \Phi(z) - e^{2i\theta}(\Phi'(z) + \Psi(z)).
\] (10)

Considering (8) and \( e^{2i\theta} = z / z = z^2 / a^2, \ z^2 = a^2 \), we obtain:

\[
P \frac{p+q}{2} - \frac{p-q}{2} z^2 = \Phi(z) + \Phi(z) - z^2 \left[ \frac{a^2}{z} \Phi'(z) + \Psi(z) \right].
\] (11)

Solution (11) is found as:

\[
\Phi(z) = \frac{B_2}{z^2}, \quad \Psi(z) = \frac{A_3}{z^2} + \frac{A_4}{z^4}.
\] (12)

Substituting (12) into (11) and comparing coefficients at equal powers \( z \) we find the system of three equations for determination of three unknowns \( B_2, A_2, A_4 \). Their values are equal to:

\[
A_2 = \frac{p+q}{2} a^2, \quad B_2 = \frac{p-q}{2} a^2, \quad A_4 = \frac{3}{2} (p-q) a^4.
\]

Further, using (10) we find stresses:

\[
\sigma_r = \frac{p+q}{2} \left( \frac{a}{r} \right)^2 + 2(p-q) \cos 2\theta \left( \frac{a}{r} \right)^2 - \frac{3}{2} (p-q) \cos 2\theta \left( \frac{a}{r} \right)^4,
\]

\[
\tau_{\phi \theta} = \left( p-q \right) \left( \frac{a}{r} \right)^2 - \frac{3}{2} (p-q) \left( \frac{a}{r} \right)^4 \sin 2\theta,
\]

\[
\sigma_r + \sigma_\theta = 4 \text{Re} \Phi(z).
\] (13)

We should note that stresses (13) at \( z \to \infty \) tend to zero.

To determine displacements \( u_r, u_\phi \) the second group of Kolosov–Muskhelishvili’s formula is available:

\[
2 \mu (u_r + i u_\phi) = e^{-i\theta}[\mathcal{N} \varphi(z) - z \varphi'(z) - \psi(z)],
\] (14)

where \( \varphi(z) = \int \Phi(z) dz = - \frac{p-q}{2} a^2, \quad \psi(z) = \frac{p+q}{2} a^2 - \frac{1}{2} (p-q) a^4, \quad \mathcal{N} = 3 - 4\nu. \) After identification of imaginary and real components in (14) we have:

\[
\frac{2 \mu u_r}{a} = - \frac{p+q}{2} \left( \frac{a}{r} \right) - \frac{p-q}{2} \left( \frac{a}{r} \right)^2 \mathcal{N} + 1 - \left( \frac{a}{r} \right)^2 \cos 2\theta,
\]

\[
\frac{2 \mu u_\phi}{a} = \frac{p-q}{2} \left( \frac{a}{r} \right) \sin 2\theta + \mathcal{N} - 1 + \left( \frac{a}{r} \right)^3.
\] (15)

At \( r = a \) from (15) it follows:

\[
\frac{2 \mu u_r}{a} = - \frac{p+q}{2} - \frac{p-q}{2} \mathcal{N} \cos 2\theta, \quad \frac{2 \mu u_\phi}{a} = \frac{p-q}{2} \mathcal{N} \sin 2\theta.
\] (16)

If in (16) values \( \theta \) are assumed equal to \( \theta = 0, \ \theta = \pi \) we obtain in total:

\[
2 \mu u_r \big|_{0,\pi} = - \frac{p+q}{2} - \frac{p-q}{2} \mathcal{N}, \quad u_\phi \big|_{0,\pi} = 0.
\] (17)

If in (16) values \( \theta \) are assumed equal to \( \theta = \frac{\pi}{2}, \ \theta = -\frac{\pi}{2} \) we have:
\[ 2\mu u_r \big|_{r=r_0} = -\frac{p+q}{2} + \frac{p-q}{2}N. \]  

(18)

From (17), (18) we derive formulas to find \( \sigma_x, \sigma_y \) values in “infinity”:

\[ p+q = -\frac{2\mu}{a}(u_r \big|_0 + u_r \big|_{r_0}), \quad p-q = -\frac{2\mu}{N a}(u_r \big|_{r=r_0} - u_r \big|_0). \]  

(19)

These formulas are analogous to formulas derived in the stress-release method [7].

3. Laboratory test results

It should be outlined that the considered scheme to form a mine working is extremely primitive because of disregard for rock mass failure inside the contour \( r = a \). Moreover, the unloading and secondary loading can happen because of both tension loads and shears within/out of the contour. In view of this fact after a mine working is formed, \( r = a \) can appear at its contour concurrently as displacements of both kinds (2) and (5); along with (9) and (15).

Two problems are stated in the present paper. The first problem deals with experimental research aimed at development of a process for computing displacements at working contour. The second concerns the formula to determine a load in “infinity” based on values of displacements at working boundary and to establish defects of a rock mass.

To solve the first problem we implement the following experiments. We take a sheet of an elastic material, let it be rubber. We make an attempt to load it by two ways, shown in Figure 3. The first scheme in Figure 3a implies that a circle of 20 cm in diameter with center coinciding with the center of the rubber sheet is plotted on the rubber sheet of 1 x 1 m dimensions and 1.5 mm in thickness. 12 points are evenly marked at circle boundary like in a clock.

Next, interior of the circle is cut out; the rubber sheet with a circular hole is placed vertically with fixation of its upper edge and is subjected to a tension test, viz., different loads from 10 to 20 kGs are applied to the bottom edge of the sheet. A digital camera is used to identify displacements \( u_r \) and \( u_\theta \) at 12 points of the circle.

![Figure 3. Two loading schemes for a rubber-like material.](image)

The second loading scheme in Figure 3b implies the following. The initial rubber sheet is positioned vertically, the preset tension load is applied at the lower edge (the top edge of the sheet is fixed). The test sheet is in a tension state; cliché of the circle of the same radius as in Figure 3a is placed on the sheet surface and delineated. 12 points are marked at the boundary of the new-made circle. The circle is cut under tensile state of the test sheet. A digital camera is used to record displacements \( u_r, u_\theta \) at the circle boundary.
Figure 4. Tension of material sheets with a hole by applying two loading schemes.

Next, displacements obtained by two methods are compared. Loading results shown in Figure 4 demonstrate that tension of a material sheet with a hole leads to greater displacements $u_1$ in direction of tension load action as compared to tension of a material, free of an initial hole. Difference is not significant, but it is available.

The proposed process to determine displacements is deposition of an initial contour of a mine working on an even surface in a rock mass, its unloading follows; and displacements of hole contour are recorded under unloading.

4. Conclusions
Displacements of a rock mass are determined for cases without mine workings and considering formation of mine workings on the basis of elasticity relationships.

The experiments are undertaken on rubber-like materials in view to estimate displacements in a rock mass loaded by two methods.

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References
[1] Schwab AA 1989 Ill-posed static problems in the theory of elasticity Izv. AN SSSR. Mekh. Tverd. Tela No 6 pp 24–27
[2] Chanyshhev AI 2001 To the problem of deformable medium failure Part I Basic equations Journal of Mining Science Vol 37 No 3 pp 273–288
[3] Chanyshhev AI and Abdulin IM 2008 Characteristics and the relations on them at the stage of post-limit deformation in rocks Journal of Mining Science Vol 44 No 5 pp 451–463
[4] Kabanikhin S 2009 Inverse and Ill-Posed Problems Novosibirsk: SO RAN (in Russian)
[5] Alekseev AS 1967 Inverse dynamic seismic problems Some Methods and Algorithms for Interpreting Geophysical Data Moscow (in Russian)
[6] Muskheilishvili NI 1949 Some Basic Problems of the Mathematical Theory of Elasticity Moscow (in Russian)
[7] Baklashov IV 2004 Geomechanics: Textbook for High Schools. Fundamentals of Geomechanics Moscow: MGGU Vol 1 pp 208 (in Russian)