Aspects of thermal strange quark production: 
the deconfinement and chiral phase transitions

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Abstract

We study the gluonic sector of the three-flavor PNJL model by obtaining the adjoint Polyakov loop and the gluon distribution function in the mean-field approximation. Besides, we explore the thermal strange quark pair-production processes, \( q \bar{q} \rightarrow s \bar{s} \) and \( gg \rightarrow s \bar{s} \), with the aid of the three-flavor PNJL model. The results help us identify the temperature where the gluonic contribution to the production rate becomes dominant, which is an innovative phenomenon compared with the result obtained in free perturbation theory.

1. Introduction

The enhanced production of the hadrons consisting of strange quarks, especially hyperons, was proposed to be one of the signatures of the formation quark-gluon plasma (QGP) in relativistic heavy-ion collisions \([1]\). In particular, the enhanced pair-production of thermal strange quarks was proposed to occur as an effect of quark and gluon deconfinement \([2, 3, 4, 5]\). The predicted enhancement in the production of hyperons has been observed in many experiments, e.g. in Ref. \([6, 7]\). In temperature domains around 200 MeV, deconfinement and and chiral symmetry restoration are the significant ingredients of the phase transformations of the QCD matter, and consequently they should play important roles in the thermal strange quark production. To elucidate their roles in the strange quark production, we study the probability of the production based on the thermodynamics. If the production rate is fast enough, the probability of thermal strange quark pair-production is

\[
\Gamma_s = \exp(-2m_s/T),
\]

where \( m_s \) is the constituent strange quark mass and \( T \) denotes the temperature. The probabilities of the strange quark pair-production in the hadronic gas (e.g. \( m_s = 528 \) MeV) and in the quark-gluon plasma (e.g. \( m_s = 135.7 \) MeV) are plotted respectively in Fig. 1(a). In Fig. 1(a), one can argue that the temperature dependence of \( \Gamma_s \) follows the dot-dashed (dashed) curve when \( T < T_c \) (\( T > T_c \)), where \( T_c \) is the critical temperature of the phase transformation. As the temperature increases, one would expect either a sudden or smooth jump of \( \Gamma_s \) in the vicinity of \( T_c \). Instead of the above argument, we focus on a more rigorous analysis on the temperature dependence of \( \Gamma_s \) in the following contexts.

The important physics aspects of \( \Gamma_s \) is sensitive not only to the deconfinement of quarks and gluons, but also to the chiral symmetry restoration, as can be seen from the dependence of \( \Gamma_s \) on \( m_s \). The recently developed Nambu-Jona-Lasinio model with the Polyakov loop (the PNJL model) incorporates both of the dynamics of deconfinement and chiral symmetry restoration in describing the phase transformations of the QCD matter \([8, 9]\). Therefore, in this paper, we study the temperature dependence of \( m_s \) by the three-flavor PNJL model in the mean-field approximation, thereby obtaining \( \Gamma_s \) valid for the whole temperature range. Besides, we investigate the tem-
perature dependence of the adjoint Polyakov loop and its implication for the gluon distribution function. Finally, we obtain the temperature dependence of the pair-production rates of thermal strange quarks using the Polyakov loop-suppressed quark and gluon distribution functions, and we identify the temperature where the gluonic contribution to the rate becomes dominant.

2. PNJL model

The three-flavor PNJL model [10, 11] incorporates the phase transformations of deconfinement and chiral symmetry restoration in one theoretical framework. The order parameters of deconfinement and chiral phase transitions are the Polyakov loops and the chiral quark condensates respectively. The Polyakov loop in color-SU(3) representation is defined as

$$L_r = P \exp \left( ig \int_{0}^{1/T} d\tau A_4 (x, \tau) \right),$$

where $P$ denotes that the exponential is path-ordered, $T$ denotes the temperature, and $A_4 (x, \tau)$ is the temporal component of the SU(3) gauge field in representation $r$. In particular, $L_3$ and $L_8$ denote the Polyakov loops in the fundamental and adjoint representations, respectively. The traces of the Polyakov loops are denoted as

$$\ell_3 = N^{-1} F_L L_3, \quad \bar{\ell}_3 = N^{-1} F_L L_3^\dagger, \quad \ell_8 = (N^2 - 1)^{-1} tr_A L_8,$$

where $tr_F$ and $tr_A$ denote the color traces in the fundamental and adjoint representation, respectively. The Lagrangian of the three-flavor PNJL model is [11, 12]

$$\mathcal{L} = \bar{\psi} (i \gamma \cdot D - \bar{m}_0) \psi - \mathcal{U}(\ell_3, \bar{\ell}_3; T) + \frac{g_S}{2} \sum_{m=1}^{8} \left[ (\bar{\psi} A^\mu \psi)^2 + (\bar{\psi} i \gamma_5 A^\mu \psi)^2 \right] + g_D [\det (1 - \gamma_5) \psi + h.c.],$$

where $D_\mu = \partial_\mu - g \delta_\mu A_4$ is the gauge-covariant derivative, $\mathcal{U}(\ell_3, \bar{\ell}_3; T)$ is the effective potential for the Polyakov loop, and $\bar{m}_0 = \text{diag}(m_{u,0}, m_{d,0}, m_{s,0})$. $g_S$ and $g_D$ are shown explicitly in Ref. [11, 12]. In the limit of isospin symmetry, $m_{u,0} = m_{d,0} = m_{q,0}$. From the above Lagrangian, we obtain the grand canonical thermodynamic potential per unit volume, $\Omega$, in the mean field approximation. We obtain the temperature dependence of the order parameters, $\langle \bar{q}q \rangle$, $\langle ss \rangle$, $\langle \ell_3 \rangle$ and $\langle \bar{\ell}_3 \rangle$, by minimizing $\Omega$ with respect to these order parameters and solving the equations simultaneously [12]. Besides, the constituent quark masses can be expressed as functions of the current masses and the chiral condensates

$$m_q = m_{q,0} - 2g_S \langle \bar{q}q \rangle - 2g_D \langle \bar{q}q \rangle \langle \bar{s}s \rangle, \quad (q = u, d)$$

$$m_s = m_{s,0} - 2g_S \langle \bar{s}s \rangle - 2g_D \langle \bar{q}q \rangle^2,$$

where $\langle uu \rangle = \langle \bar{d}d \rangle$, $m_{u,0} = m_{d,0}$, $m_q^0 = 0.0055$ GeV and $m_s^0 = 0.1357$ GeV [11]. The temperature dependence of $\langle \bar{q}q \rangle$, $\langle ss \rangle$, $\langle \ell_3 \rangle$, $\langle \bar{\ell}_3 \rangle$, $m_q$ and $m_s$ are depicted in Ref. [12]. In the PNJL model, we obtain $\Gamma_s$ as a function of temperature with $m_s$ input by Eq. (5), as shown in Fig. 1. We note that $\Gamma_s$ in the PNJL model exceeds $\Gamma_s$ of deconfined quarks for $T > 360$ MeV, which is due to an artifact of the PNJL model that $m_q < m_{q,0}$ and $m_s < m_{s,0}$ for $T > 360$ MeV. Moreover, in the PNJL model, $\Gamma_s \rightarrow 1$ at high temperature.
Figure 1: (a) The probabilities of thermal strange quark pair-production $\Gamma_s$ as functions of the temperature, with the constituent quark mass $m_s$ obtained for the PNJL model, the quark-gluon plasma and the hadronic gas respectively. (b) Thermal strange quark pair-production rates divided by $\alpha_s^2$ as functions of the temperature. The chemical potential for $u$ and $d$ quarks is $\mu = 0.1$ GeV.

3. Adjoint Polyakov loop and gluon distribution function

We obtain the thermal average of the adjoint Polyakov loop from that of the fundamental Polyakov loop by the following self-consistent procedure \[12\]. First, we define the thermal average with respect to the weight function,

$$W(\ell_3; T) = \exp(6 d \beta_3 \langle \ell_3 \rangle \text{Re}(\ell_3)),$$

where $d = 3$ and $\beta_3(T)$ is a fit parameter depending on temperature \[13, 14\]. Then, the thermal average of the fundamental Polyakov loop gives the constraint equation,

$$\langle \ell_3 \rangle = \langle \text{tr} F_L \rangle / 3 W,$$

where $\langle \ell_3 \rangle$ is, at each temperature, input by its mean-field value obtained in Sec. 2. We obtain $\beta_3(T)$ by solving this constraint equation, and then we evaluate $\langle \ell_8 \rangle$ as a function of the temperature. We verify that, with this averaging procedure, the adjoint and fundamental Polyakov loops satisfy the Casimir scaling,

$$\langle \ell_8 \rangle = \langle \ell_3 \rangle^{9/4},$$

which is observed in lattice QCD \[14, 15\]. Moreover, we obtain the color-averaged gluon distribution function \[12\]

$$f_g(k) = \frac{1}{8} \sum_{n=1}^{\infty} \langle \text{tr} A_L^n \rangle \exp(-n |k| / T).$$

We show that the color-averaged quark and gluon distribution functions are both suppressed by the Polyakov loops, and the degree of suppression for the gluon distribution is higher than that of the quark distribution \[12\].

4. Thermal strange quark pair-production

The pair-production of thermal strange quarks is contributed by two processes, $q\bar{q} \rightarrow s\bar{s}$ and $gg \rightarrow s\bar{s}$. The production rates was first obtained by QCD perturbation theory for free quarks \[2\]. In this paper, we take into account the effects of deconfinement and chiral symmetry restoration on the production rates. We obtain the strange quark production rate per unit volume by

$$A = \frac{dN}{dt d^3x} = A_q + A_g,$$

where $A_q$ and $A_g$ are the quark and gluon contributions respectively.
where the explicit expressions for $A_q$ and $A_g$ are shown in Ref. [12]. In Fig. 1(b), we plot the temperature dependence of the production rates divided by $\alpha_s^2$ for the PNJL model and those for free quarks respectively. The production rates for the PNJL model are smaller (larger) than those for free quarks when $T < 480$ MeV ($T > 480$ MeV). This phenomenon is due to either the addition or the competition of the following two effects. One effect is the suppression of the thermal quark and gluon excitations by the Polyakov loops, and the other effect is that, in the PNJL model, $m_s > m_{s,0}$ ($m_s < m_{s,0}$) when $T < 360$ MeV ($T > 360$ MeV). The later effect surpasses the former one when $T > 480$ MeV so that the production rates for the PNJL model become larger than those for free quarks. Besides, as proposed in [2, 4], the production rates for free quarks are enhanced in the deconfined phase, and the production rates for $gg \rightarrow s\bar{s}$ is dominant at all temperatures. In the PNJL model, the enhanced production is also obtained, but the production rates for $q\bar{q} \rightarrow s\bar{s}$ and $gg \rightarrow s\bar{s}$ cross over at $T_r \approx 240$ MeV. The production rate for $q\bar{q} \rightarrow s\bar{s}$ ($gg \rightarrow s\bar{s}$) is dominant when $T < T_r$ ($T > T_r$). Because the threshold $T_r$ is well beyond the temperature range in which the chiral phase transition occurs, and our neglect of the contribution from collective (hadronic) modes in the transition region near $T_c$ appears justified. Moreover, when $T < T_c$, the production rates for the PNJL model are very small because quark and gluon quasiparticles are strongly suppressed below $T_c$. In this temperature region, strangeness production is dominated by hadronic reactions, which were investigated by Rehberg et al. in the NJL model [16].

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