Superluminal warp drives are semiclassically unstable

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Abstract. Warp drives are very interesting configurations of General Relativity: they provide a way to travel at superluminal speeds, albeit at the cost of requiring exotic matter to build them. Even if one succeeded in providing the necessary exotic matter, it would still be necessary to check whether they would survive to the switching on of quantum effects. Semiclassical corrections to warp-drive geometries created out of an initially flat spacetime have been analyzed in a previous work by the present authors in special locations, close to the wall of the bubble and in its center. Here, we present an exact numerical analysis of the renormalized stress-energy tensor (RSET) in the whole bubble. We find that the RSET will exponentially grow in time close to the front wall of the superluminal bubble, after some transient terms have disappeared, hence strongly supporting our previous conclusion that the warp-drive geometries are unstable against semiclassical back-reaction. This result seems to implement the chronology protection conjecture, forbiddig the set up of a structure potentially dangerous for causality.

1. Introduction

Since they were introduced by Alcubierre\textsuperscript{[1]}, warp drives have been certainly one of the most studied spacetime geometries among those requiring exotic matter (\textit{i.e.} energy-conditions-violating matter) for their existence in the context of general relativity (see\textsuperscript{[2]} for a recent review). They are not only an exciting theoretical test-field for our comprehension of general relativity and quantum field theory in curved spacetimes, but they also provide, at least theoretically, a way to travel at superluminal speeds.

A warp-drive metric describes a bubble containing an almost flat region, moving at arbitrary speeds within an asymptotically flat spacetime:
\begin{equation}
 ds^2 = -c^2 dt^2 + (dx - v(r)dt)^2 + dy^2 + dz^2,
\end{equation}
where $r \equiv \sqrt{(x - v_0 t)^2 + y^2 + z^2}$ is the distance from the center of the bubble and $v_0$ is the warp-drive velocity. Here $v = v_0 f(r)$, with $f$ a suitable smooth function satisfying $f(0) = 1$ and $f(r) \to 0$ for $r \to \infty$.

After the warp-drive spacetimes were proposed, their most investigated aspect has been the amount of exotic matter that would be required to support them\textsuperscript{[3,4,5,6]}. It was found that, if the exoticity was provided by the quantum nature of a field, satisfying therefore the so-called Quantum Inequalities (QI)\textsuperscript{[7]}, then the violations of the energy conditions would have to be confined to Planck-size regions, making the bubble-wall thickness to be of Planck size. This bound turns into lower limits on the required amount of exotic matter ($\gtrsim 1 M_\odot$).
Less effort has been devoted to another important issue regarding the feasibility of these spacetimes: their stability under the switching on of quantum effects. It was noticed [2] that the backward and forward walls (along the direction of motion) look respectively as the future (black) and past (white) event horizon of a black hole, for an observer inside the warp-drive bubble. In the Heisenberg representation, by imposing over the spacetime a state which is vacuum at the null infinities, it was found that the renormalized stress-energy tensor (RSET) had to diverge on the horizons. However, from equivalent analysis in eternal-black-hole geometries we know that such divergences are related to an inappropriate vacuum choice (the Boulware vacuum) and, therefore, they are not physical.

We consider here the more realistic case of a warp drive created out of Minkowski spacetime and then accelerated up to some superluminal speed in a finite amount of time. Accordingly, the globally defined quantum state is chosen to reproduce the Minkowski vacuum at early times. Using this vacuum state, in a recent previous work [2], we approximately calculated the salient features of the energy density as seen by inertial observers inside the warp-drive bubble, in three specific locations: the center of the bubble, the black and the white horizons. In this work we show exact numerical calculations for the energy density in the entire interior of the bubble (using a specific model), which confirm our previous partial results. We find that some transient effects are generated when the horizons are created. However, these effects disappear in a short time and the behaviors found in our analytical work are recovered. We confirm the presence of a thermal flux of radiation with a Hawking temperature proportional to the surface gravity of the black horizon, which is inversely proportional to the wall thickness. As mention, if the supporting matter is subjected to the QI, then this wall thickness would be of the order of the Planck length, thus leading to a temperature of the order of the Planck temperature. More importantly, we find that the energy density measured by free-falling observers is negative on the white and black horizons (as expected for a vacuum polarization). While on the black one it is exponentially damped, on the white one it diverges exponentially to minus infinity. In [2], in addition to the black and white horizons already envisaged in [3], we discovered the presence of a Cauchy horizon. Rightgoing null rays approach this horizon at late times. As one should expect by analogy with other situation in which Cauchy horizons are present, as in Kerr-Newman black holes, we find that the energy carried by such rays diverges, this time contributing with a positive sign, when approaching this horizon, thus representing an additional source of instability.

In Sect. 2 we summarize the results of [2], using the specific model used for the numerical treatment. Our numerical results are presented in Sect. 3. Conclusions and final remarks follow in Sect. 4.

2. Dynamical warp-drives

2.1. Causal structure of a dynamical warp drive

We restrict our attention to the 1+1 dimensional case and write the metric (1) using coordinates $t$ and $r \equiv x - v_0 t$:

$$\begin{align*}
\begin{aligned}
\text{d}s^2 &= -c^2 \text{d}t^2 + [\text{d}r - \bar{v}(r) \text{d}t]^2 ,
\end{aligned}
\end{align*}
$$

where $\bar{v} = v - v_0$. As above, $v = v_0 f(r)$, with $f$ defined now on the whole real axis. We require $f(r)$ to be a smooth bell-shaped function with maximum $f(0) = 1$ and $f(r) \to 0$ for $r \to \pm \infty$. It is easy to check that $\bar{v}$ has a maximum in $r = 0$, $\bar{v}(0) = \bar{v}'(0) = 0$, and that $\bar{v}(r) \to -v_0 < -c$ for $r \to \pm \infty$. As a consequence, there are two positions $r_1$ and $r_2$ at which $c + \bar{v}(r) = 0$. As seen from an observer inside the bubble, these points will correspond to a black and a white horizons, $\mathcal{H}_C^+$ and $\mathcal{H}_R^-$. A dynamical warp-drive geometry is obtained by replacing $\bar{v}$ with a time dependent velocity $\hat{v}(t, r)$, such that the new metric interpolates between an initial Minkowski spacetime $[\hat{v}(t, r) \to 0$, for $t \to -\infty]$ and a final stationary superluminal ($v_0 > c$) bubble $[\hat{v}(t, r) \to \bar{v}(r)$, for $t \to +\infty]$. In Fig. 1 we plot the Penrose diagram of such a geometry (see Ref. [2] for its construction). One can note that a timelike observer can reach $\mathcal{H}_C^+$ and $\mathcal{H}_R^+$.
in a finite proper time. As a consequence the spacetime can be extended in the future, beyond these lines. However, $\mathcal{H}^+_C$ and $\mathcal{H}^+_R$ are on the boundary of the Cauchy development of $\mathcal{I}^-$. In this sense, they are Cauchy horizons given initial data assigned only on $\mathcal{I}^-$. 

2.2. Light-ray propagation

We now consider light-ray propagation in the above described geometry, specializing to the particular model used for numerical calculation. Adapting the method presented in [10, 11] for stellar collapses, in our framework, a dynamical superluminal warp-drive geometry is determined by a late time profile $\bar{v}(r)$ and a prescription for the transition between the early time Minkowski spacetime and the late time regime. We choose $\bar{v}$ as:

$$\bar{v}(r) = \alpha c \left[ f(r) - 1 \right] = \alpha c \left[ \frac{1}{\cosh(r/a)} - 1 \right], \quad (3)$$

with $\alpha > 0$ and we define a dynamical profile $\hat{v}$

$$\hat{v}(r, t) = \begin{cases} \bar{v}(\xi(t)) & \text{if } |r| \geq \xi(t), \\ \bar{v}(r) & \text{if } |r| < \xi(t). \end{cases} \quad (4)$$

Figure 1. Penrose diagram of a superluminal warp-drive creation. Lines of constant $r$ (solid lines) and of constant $t$ (dashed lines). The lines of constant $r$ become null at the apparent horizon (heavy dashed lines). The warp-drive bubble is located in the central region ($r_1 < r < r_2$).
where $\xi(t)$ is a monotonically increasing function of $t$, such that $\xi(t) \to 0$, for $t \to -\infty$, and $\xi(t_H) = r_2 = -r_1$, which we called the kink function.\footnote{One may wonder whether defining a velocity profile with a kink, as in Eq. (4), may lead to unphysical phenomena. Indeed this computational trick induces some spurious effects, but these features are just transients and do not affect the results at late times.} The horizons are located at

$$r_{1,2} = \mp a \ln \left( \beta + \sqrt{\beta^2 - 1} \right), \quad \beta \equiv \frac{\alpha}{\alpha - 1} > 1.$$ 

(5)

We also define a signed surface gravity\footnote{In general, $\kappa_2$ could be different from $\kappa_1$, although it is expected to be comparable with $\kappa_1$.} on these horizons:

$$\kappa_{1,2} \equiv \frac{d\nu(r)}{dr} \bigg|_{r = r_{1,2}} = \frac{c}{a \beta} \left( \alpha - 1 \right) = \pm \kappa .$$ 

(6)

In [9] it was argued that the universal features of the RSET are determined only by right-going rays, just like outgoing modes do in the case of a black hole collapse [10, 12]. This result is confirmed by the numerical computation where also the effects of left-going rays are considered, but they turn out to be negligible at late times (see Sect. [3]). For the sake of conciseness, in this section we present only the features due to right-going rays, since treatment of left going rays is analogous.

First of all, we need the relation between the past null coordinate $U$ relevant on $\mathcal{I}^-$ and future null coordinate $u$ relevant on $\mathcal{I}^+$. Following [10], we write the differential equation describing right-going rays:

$$\frac{dr}{dt} = c + \bar{\nu}(r, t).$$ 

(7)

Integrating this equation at early times or close to $\mathcal{I}^+$ ($t \to +\infty$ and $r \to r_2$), one can define $U$ and $u$:

$$U = \lim_{t_i \to -\infty} \left( t_i - \frac{r_1}{c} \right),$$

$$u = \lim_{t_f \to +\infty} \left[ t_f + \frac{1}{\kappa} \ln (r_2 - r_f) \right].$$

(8)

(9)

We identify initial events $P \equiv (r_i, t_i)$, with $r_i \sim c t_i$, final events $Q \equiv (r_f, t_f)$ with $r_f \sim r_2 - e^{-\kappa t_f}$, and intermediate events $O \equiv (r_0, t_0)$. Integrating Eq. (7) between $P$ and $O$ in the in region of this spacetime (where the velocity profile depends only on $t$) and, respectively, between $O$ now in the out region (where the velocity profile depends only on $r$) and $Q$, one obtains for $r_1 < r_0 < r_2$:

$$U = t_0 - \frac{r_0}{c} + \frac{1}{c} \int_{-\infty}^{t_0} dt \bar{\nu}(\xi(t)),$$

$$u = t_0 + \frac{1}{\kappa} \ln [r_2 - r_0] - \frac{1}{\kappa} \ln [r_0 - r_1] + \frac{1}{\kappa} \ln [r_2 - r_1] + \int_{r_0}^{r_2} dr \left[ \frac{1}{c + \bar{\nu}(r)} - \frac{1}{\kappa (r_2 - r)} - \frac{1}{\kappa (r - r_1)} \right],$$

(10)

(11)

where the last integral is finite for each value of $r_0$. To find the relation between $U$ and $u$, one must choose a particular form of the kink function $\xi$. It is particularly useful to choose $\xi$ such that all light rays cross the kink $r = \mp \xi(t)$ only once. In this fashion, the relation $U = p(u)$ can be found by eliminating $t_0$ and $r_0$ between Eqs. (10) and (11), with $O$ on the kink, $r_0 = \mp \xi(t_0)$. We show below how the analytical results of [9] can be recovered for the kink model. To this
aim, one needs the limit for large and positive values of \( u \) (close to \( H^-_2 \), for \( r \to r_1, u \to +\infty \)) and for large and negative values of \( u \) (close to \( H^+_4 \), for \( r \to r_2, u \to -\infty \)). Let us focus to rays peeling off \( H^-_2 \), i.e., those rays which cross the left kink at \( r_0 = -\xi(t_0) \), when \( r_0 \) is very close to \( r_1 \).

When \( t \to t_H \) (the time of the appearance of the trapping horizons) \( \xi \) can be expanded as:

\[
-\xi(t) = r_1 - \lambda (t - t_H) + \mathcal{O}[(t - t_H)^2],
\]

with \( \lambda \) a positive constant. From Eq. (10) one sees that

\[
U = U_{BH} - \frac{\lambda}{c} (t_H - t_0) + \mathcal{O}[(t_H - t_0)^2],
\]

where \( U_{BH} = U(t_H, -\xi(t_H)) \). Expanding Eq. (11) in the same limit, \( t_0 \to t_H \), one has

\[
u \simeq -\frac{1}{\kappa} \ln[\lambda(t_H - t_0)],
\]

Putting together the last two equations, one obtains

\[
U(u \to +\infty) \simeq U_{BH} + A_1 e^{-\kappa u} + \frac{A_2}{2} e^{-2\kappa u} + \ldots,
\]

where it can be shown that \( A_1 < 0 \). This relation is the standard result for the formation of a black hole through gravitational collapse. As a consequence, the quantum state which is vacuum on \( \mathcal{I}^- \) will show, inside the bubble, Hawking radiation with temperature \( T_H = \frac{\kappa_1}{2\pi} \).

By performing the same steps for light rays passing close to the white horizon one can find that

\[
U(u \to -\infty) \simeq U_{WH} + D_1 e^{\kappa u} + \frac{D_2}{2} e^{2\kappa u} + \ldots,
\]

where \( D_2 > 0 \).

Of course, this identification is not complete as this radiation does not travel towards an asymptotically flat region but towards the white horizon. However, for large-enough bubbles, the radiation at the center of the bubble will look approximately as Hawking free particles.

### 2.3. Renormalized stress-energy tensor

For the calculation of the RSET inside the warp-drive bubble we use the method proposed in [12]. In past null coordinates \( U \) and \( W \) the metric can be written as

\[
ds^2 = -C(U, W) dU dW.
\]

In the asymptotic out region, it is convenient to use a different set of coordinates: \( u \), Eq. (9), and \( \tilde{w} \), defined as

\[
\tilde{w}(t, r) = t + \int_0^r \frac{dr}{c - \bar{v}(r)}.
\]

In these coordinates the metric is expressed as

\[
ds^2 = -\tilde{C}(u, \tilde{w}) dud\tilde{w}, \quad \tilde{C}(U, W) = \frac{C(u, \tilde{w})}{p(u)q(\tilde{w})},
\]

where \( U = p(u) \) and \( W = q(\tilde{w}) \).
For concreteness, we refer to the RSET associated with a quantum massless scalar field living on the spacetime. The RSET components have the following form \[13\]:

\[
T_{UU} = -\frac{1}{12\pi} C^{1/2} \partial_u^2 C^{-1/2},
\]

\[
T_{WW} = -\frac{1}{12\pi} C^{1/2} \partial_w^2 C^{-1/2},
\]

\[
T_{UW} = T_{WU} = \frac{1}{96\pi} C R.
\]

If there were other fields present in the theory, the previous expressions would be multiplied by a specific numerical factor. Using the relationships

\[
\rho = T_{\mu\nu} u^\mu w^\nu = \rho_{\text{stat}} + \rho_{\text{dyn} - u} + \rho_{\text{dyn} - w},
\]

where we define a static term \(\rho_{\text{stat}}\), depending only on the \(r\) coordinate through \(\bar{v}(r)\),

\[
\rho_{\text{stat}} \equiv -\frac{1}{24\pi} \left[ \frac{(\bar{v}^4 - \bar{v}^2 + 2)}{(1 - \bar{v}^2)^2} \bar{v}^\prime 2 + \frac{2\bar{v}}{1 - \bar{v}^2} \bar{v}'' \right],
\]

and two dynamic terms \(\rho_{\text{dyn} - u}\) and \(\rho_{\text{dyn} - w}\)

\[
\rho_{\text{dyn} - u} \equiv \frac{1}{48\pi} \frac{f(u)}{(1 + \bar{v})^2},
\]

\[
\rho_{\text{dyn} - w} \equiv \frac{1}{48\pi} \frac{g(w)}{(1 - \bar{v})^2},
\]

where we have put \(c = 1\) and we have defined

\[
f(u) \equiv \frac{3\bar{p}^2(u) - 2\dot{p}(u) \dot{\bar{p}}(u)}{\bar{p}^2(u)},
\]

\[
g(w) \equiv \frac{3\bar{q}^2(\bar{w}) - 2\dot{q}(\bar{w}) \dot{\bar{q}}(\bar{w})}{\bar{q}^2(\bar{w})}.
\]

These dynamic terms, depending also on \(u\) (\(\bar{w}\)), correspond to energy travelling on right-going (left-going) rays, red/blue-shifted by a term depending on \(r\).

As said above, \(\dot{q}\) contains solely information associated with the dynamical details of the transition region. Moreover, for simple forms of this transition, as the one chosen in this work, \(\dot{q}(\bar{w})\) goes to a constant at late times, such that \(\dot{q}(\bar{w}) \to 0\). Thus, we briefly report the result of an analytical approximate analysis, by neglecting this term, even if in the numerics we retain the full expression of \(\rho\).

In the center of the bubble, at late times \(\rho_{\text{stat}} = 0\), because \(\bar{v}(r = 0) = \bar{v}'(r = 0) = 0\). Moreover \(u(t, 0) \to +\infty\) and one can evaluate \(\rho\) using a late-time expansion for \(f(u)\), which gives \(f(u) \approx \kappa_1^2\), so that \(\rho(r = 0) \approx \kappa_1^2/(48\pi) = \pi T_H^2 / 12\), where \(T_H \equiv \kappa_1/(2\pi)\) is the usual Hawking temperature.

This result confirms the presence of a thermal flux of radiation at temperature \(T_H\), given that the above expression is the energy density of a scalar field in \(1 + 1\) dimension at finite temperature \(T_H\).

On the horizons \(\mathscr{H}^+_1\) and \(\mathscr{H}^-_2\), both \(\rho_{\text{stat}}\) and \(\rho_{\text{dyn} - u}\) are divergent because of the \((1 + \bar{v})\) factors in the denominators. Using the late time expansion of \(f(u)\) in the proximity of the
horizons, one obtains that the diverging terms in \( \rho_{\text{st}} \) and \( \rho_{\text{dyn-u}} \) exactly cancel each other [3]. An analogous cancellation is found when studying the formation of a black hole through gravitational collapse [12]. It is now clear that the total \( \rho \) is \( O(1) \) on the horizon and does not diverge at any finite time (as expected from Fulling-Sweeny-Wald theorem [14]). By looking at the subleading terms for the black horizon, one sees that the contribution of the transient radiation (different from Hawking radiation) dies off with time exponentially, with a time scale \( \sim 1/\kappa_t \) while something distinctive occurs close to the white horizon. In fact, these subleading terms now becomes

\[
\rho = \frac{e^{2\kappa t}}{48\pi} \left[ 3 \left( \frac{D_2}{D_1} \right)^2 - 2 \frac{D_3}{D_1} \right] + D + O \left( r - r_1 \right). \tag{29}
\]

This expression shows an exponential increase of the energy density with time. This means that, moving along \( \mathcal{H}_2^- \), \( \rho \) grows exponentially, diverging at the crossing point between \( \mathcal{H}_2^- \) and \( \mathcal{H}_1^+ \).

In a completely analogous way, performing an expansion at late times (\( t \to +\infty \) for \( r \to r_2 \)), one finds that the RSET diverges on the whole \( \mathcal{H}_2^+ \). This result does not contradict the Fulling-Sweeny-Wald theorem [14] because \( \mathcal{H}_2^+ \) is a Cauchy horizon.

3. Numerical results

In this section we present the result of the numerical calculation of \( \rho \) inside the bubble. The velocity profile is given in Eq. (3) with \( c = 1 \), \( \alpha = 2 \) and \( a = 1 \). With this choice, the surface gravity is \( \kappa = \sqrt{3}/2 \), the kink function is \( \xi(t) = \text{arccosh}(t+1) \) and the horizons appear at \( t_H = 1 \).

In Fig. 2 the energy density \( \rho \) (thick solid line) of Eq. (23) is plotted as a function of \( r \) at different times (\( t = 0.5, 1, 2, 3 \)) and \( r \) varies between \( r_1 \) and \( r_2 \) the locations of the \( \mathcal{H}_2^- \) and \( \mathcal{H}_1^+ \). The three terms in the right-hand-side of Eq. (23), \( \rho_{\text{st}}, \rho_{\text{dyn-u}} \) and \( \rho_{\text{dyn-w}} \) are plotted, respectively with thin-solid, dashed and dash-dotted lines. In the first plot, all the components of \( \rho \) vanish, for \(-0.96 \lesssim r \lesssim 0.96 \) because at time \( t = 0.5 \) the kink is located at \( r = \pm 0.96 \), and the space is flat for \( |r| > \xi(t) \).

The figure shows that the dynamic term \( \rho_{\text{dyn-w}} \) is transient, confirming the correctness of the approximation done in the analytical analysis, when this term had been neglected to study the behavior of \( \rho \) at late times. Note, however, that \( \rho_{\text{dyn-w}} \) gives an important contribution during the creation process of the warp drive.

In the center of the bubble (\( r = 0 \)) at late times, the energy density is \( \rho \approx \kappa^2/48\pi \approx 0.005 \) and it is due only to the dynamic term \( \rho_{\text{dyn-u}} \), which can be naturally interpreted as Hawking radiation. For \( t \lesssim 2 \) the transient term \( \rho_{\text{dyn-u}} \) cannot be neglected. For instance, at \( t = t_H = 1 \), it causes the energy density inside the bubble to be about twice its late-time value.

On the horizons \( r = r_1,2 \), both \( \rho_{\text{st}} \) and \( \rho_{\text{dyn-u}} \) are found to be divergent, but they cancel each other, leaving only a finite term, which vanishes with time on the black horizon (\( r = r_1 \)), while it diverges to \( -\infty \) on the white horizon (\( r = r_2 \)), as shown in Eq. (29). The claimed divergence on the Cauchy horizon is also found, even it is difficult to recognize it from such plots. Looking at the Penrose diagram in Fig. 1 one indeed realizes that the location (\( r = r_2, t = +\infty \)) does not represent a single point but a whole segment of the diagram. This means that one should disentangle the two effects (one at the Cauchy horizon, the other at the crossing point between \( \mathcal{H}_2^- \) and \( \mathcal{H}_1^+ \)) which both take place at \( r \) close to \( r_2 \). This can be done with a careful analysis: Exactly at \( r = r_2 \), as \( t \to +\infty \), the value of \( \rho \) goes towards \( -\infty \); this contribution is superposed with a positive energy pulse whose peak-value grows towards \( +\infty \) with time and whose center is located progressively closer and closer to \( r_2 \), approaching it as \( r - r_2 \approx e^{-\kappa t} \). This can be better appreciated in Fig. 3 where the same quantities of Fig. 2 are plotted at time \( t = 2, 3 \), with different scales, proportional to \( e^{2\kappa t} \) to compensate the exponential divergence of \( \rho \).

\[ \text{3} \] However, in analogy to the conclusions of [12], a slow approach to the black-horizon formation might lead to large values of the RSET and hence to a large back-reaction.
Figure 2. Energy density $\rho$ (thick solid line), $\rho_{\text{st}}$ (solid line), $\rho_{\text{dyn}-u}$ (dashed line) and $\rho_{\text{dyn}-w}$ (dash-dotted line) as functions of $r$, $r_1 < r < r_2$ at time $t = 0.5, 1, 2, 3$. Apparent horizons form at $t_H = 1$.

Figure 3. See caption of Fig. 2 for notation. Time slices at $t = 1, 3$. The scale of the $\rho$-axis is proportional to $e^{2\kappa t}$.

Since this degeneracy is due only to a bad choice of coordinates, it can be removed by using, for instance, the null coordinates $u$ and $\tilde{w}$. In Fig. 4 $\rho$ is plotted as a function of $-\tanh(\kappa u)$ (in order to put the horizon $u = \pm \infty$ to finite positions) and $\tilde{w}$. Here, $\rho \to -\infty$ for $u \to -\infty$ ($-\tanh(\kappa u) = 1$) and $\tilde{w} \to +\infty$, while $\rho \to +\infty$ for $u$ not too much negative and $\tilde{w} \to +\infty$. The former is the divergence on the white horizon, the latter is the divergence close to the Cauchy horizon: Whenever some energy travels on a $u$-ray, it is exponentially blue-shifted at late times $\tilde{w}$ (late times $t$), when approaching this horizon.

It is important to stress that these two divergences are of very different nature. In fact,
Figure 4. $\rho$ as a function of $-\tanh(\kappa u)$ and $\tilde{w}$.

divergence at the crossing point between $\mathcal{H}_2^-$ and $\mathcal{H}_C^+$ is intrinsically due to the inevitable transient disturbances produced by the formation of the white horizon. In this sense it is a new and very effective instability. On the contrary, the divergence on $\mathcal{H}_C^+$ is due to the well known infinite blue-shift suffered by light rays as they approach a Cauchy horizon. It is analogous to the often claimed instability of inner horizons in Kerr-Newman black holes [15, 16, 17]. In any case the backreaction of the RSET will doom the warp drive to be semiclassically unstable.

4. Conclusions

In this work a $1 + 1$ calculation was performed. Generally in spherically symmetric spacetimes this could be seen as a $s$-wave approximation to the exact results. However, this is not the case for the axisymmetric warp-drive configuration. Nonetheless, we do expect that the salient features of our results would be maintained in a full 3+1 calculation, given that they will still be valid in a suitable open set of the horizons centered around the axis aligned with the direction of motion.

Hence, we think that this work is convincingly ruling out the semiclassical stability of superluminal warp drives, confirming and extending the approximate analytic results found in our previous work [9], on the base of the following evidence.

(1) The central region of the warp drive behaves like the asymptotic region of a black hole: In both of these regions the static term $\rho_{st}$ vanishes and the whole energy density is due to the Hawking radiation generated at the black horizon. If one trusts the QI [3, 4], the wall thickness for a warp drive with $v_0 \approx c$ would be $\Delta \lesssim 10^2 L_P$, and its surface gravity $\kappa_1 \gtrsim 10^{-2} t_P^{-1}$, where $t_P$ is the Planck time. Hence, the Hawking temperature of this radiation would be unacceptably large: $T_H \sim \kappa_1 \gtrsim 10^{-2} T_P$, where $T_P$ is the Planck temperature, about $10^{32}$ K.
The formation of a white horizon produces a transient radiation which accumulates on the white horizon itself. This causes the energy density $\rho$ seeing by a free-falling observer to grow unboundedly with time on this horizon. The semiclassical backreaction of the RSET will make the superluminal warp drive to become rapidly unstable, in a time scale of the order of $1/\kappa^2$, the inverse of the surface gravity of the white horizon. In fact, in order to get even a time scale $\tau \sim 1$ s for the growing rate of the RSET, one would need a wall as large as $3 \times 10^8$ m. Thus, most probably, one would be able to maintain a superluminal speed for just a very short interval of time.

The formation of a Cauchy horizon gives rise to an instability, similar to inner horizon instability in black holes, due to the blue-shift of Hawking radiation produced by the black horizon.

A suggestive interpretation of these results can be argued in connection with the so called chronology protection conjecture [18]. In fact, a time machine [19] could be build through a couple of superluminal warp drives traveling in opposite directions. Thus, a protection mechanism seems to act at an early stage, forbidding the creation of a system which could be dangerous for causality.

Of course, all the aforementioned problems disappear when the bubble remains subluminal. In that case no horizon forms, no Hawking radiation is created, and neither strong temperature nor white horizon instability is found. Traveling at just 99% of the speed of light could be not that bad, after all.

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