Nonlinear dynamic analysis of a rotor-porous air journal bearing system with O-rings mounted

Kang Zhang · Kai Feng · Wenjun Li · Lijun Song

Received: 30 April 2021 / Accepted: 1 November 2021 / Published online: 18 November 2021
© The Author(s), under exclusive licence to Springer Nature B.V. 2021

Abstract Porous air journal bearings (PAJBs) with O-rings mounted are regarded as a potential choice to offer air levitation with a large load capacity and optimal start/stop characteristics, which are required in small-sized and high-efficiency turbomachinery. However, the coupling of the mechanical properties of the O-rings and lubrication induces strong nonlinear characteristics, thereby affecting the dynamic responses of the rotor. In this study, a numerical model, which is obtained by coupling the excitation frequency-influenced stiffness and damping characteristics of O-rings, Darcy’s laws, air motion equations, and rotor and bearing motion equations, is presented. The numerical model can efficiently reflect the influences of the actual mechanical property of O-rings and the coupling between aerodynamic and aerostatic effects on the performance of a rotor-PAJB system (RPS). The model is well verified by experimental results. The effects of different system parameters on the performance of the RPS are studied by analyzing bifurcation diagrams, orbit, Poincaré maps, and fast Fourier transform plots. Results show that the rotor motion is changed between periodic and quasi-periodic with the variation in rotational speed, external supply pressure, bearing clearance, and porous permeability. With O-rings mounted, the stability of the RPS can be increased to hinder the rotor motion from changing from periodic to quasi-periodic. The effectiveness in increasing the stability of the RPS for the O-rings with styrene butadiene rubber as the material is better than that with nitrile butadiene rubber but weaker than that with methyl vinyl silicone rubber. Moreover, only a limited number of O-rings have enhancing effects on the stability of the RPS.

Keywords Porous air journal bearings · O-rings mounted · Nonlinear dynamics · Theoretical and experimental analysis

1 Introduction

Air bearings have the advantages of simple structure, high precision, and low friction [1]. Porous air journal bearings (PAJBs), which are a special type of aerostatic bearings, restrict the external pressured air by using the porous material to realize a larger load capacity and better start/stop characteristics compared with those under aerodynamic bearings [2]. Hence, PAJBs are broadly used in small-sized and high-efficiency turbomachinery [3].

The first issue regarding the instability of PAJBs is air hammer because they belong to aerostatic bearings. Many studies [4–9] have been conducted to research
the air hammer instability of PAJBs. These studies were mainly concerned about the influences of permeability and porosity, which are special parameters of the porous material. However, Refs. [8, 9] found that whirl instability is still a problem for PAJBs used under high rotational speed. Air hammer and whirl instability interrelate, and the whirl of the rotor may cause air hammer. Jacob et al. [10] conducted a perturbation test to investigate the pneumatic stability of a rotor-PAJB system (RPS). The frequencies of whirl instability and air hammer vibration were determined using bode plots and full spectral cascade plots, respectively. The data demonstrated that air hammer is a nonsynchronous excitation of the system resonant frequency. If the rotor-bearing natural frequency exceeds this self-sustaining frequency, then air hammer instability occurs. Thus, in addition to air hammer, the stability of RPS must be examined, especially under high-rotational-speed conditions.

Liu et al. [11] measured the dynamic response of a high rotational rotor supported on PAJBs and discussed the stability characteristics. The results showed that the bearing supply pressure and imbalance should be carefully checked when coupling with high rotational speed because they significantly influence the rotor synchronous and sub-synchronous motions. Wang et al. [12] investigated the nonlinear characteristics of a flexible rotor supported by two PAJBs. They found through analyzing the bifurcation of the rotor system that varying the rotor speed and mass significantly influenced the system stability. Zhu and San Andrés [13] and Wu et al. [14] analyzed the stability characteristics of a system with the rotor supported by porous air bearings. They used tiling pad porous air bearings instead of PAJBs. This method can significantly increase the stability by changing the structural parameters of the bearings. Nevertheless, the consistency of the porous pads cannot be easily ensured. This design also adds to the complexity of the rotor-bearing system; accordingly, the system is difficult to adjust. These studies proved that the stability of RPS must be enhanced.

Adding a flexible support element to the system is an efficient method to improve the stability of the air bearing rotor system at a high rotational speed. Delgado and Ertas [15] adopted modular hermetically sealed squeeze film dampers to improve the system stability. Gu et al. [16] used high-static low-dynamic stiffness suspension to control the vibration of a rotating machine. Ertas [17] and Liu et al. [18] used metal mesh blocks to prevent system instability. Although these methods are novel, the new flexible support elements are either still in the concept stage or structurally complicated. In addition to these methods, using O-rings to support bearings is a common method of offering a flexible support element in the system with the advantages of low cost and high compaction [19].

Many researchers have studied the stability characteristics of a rotor-bearing system with O-rings mounted. Miyanaga and Tomioka [20] applied linear perturbation and nonlinear transient methods to analyze the effect of support stiffness and damping of O-rings on the stability characteristics of herringbone-grooved aerodynamic journal bearings. Shabaneh and Zu [21] analyzed the nonlinear dynamic characteristics of a rotor-bearing system with O-rings mounted. Dutt [22] presented the development of preliminary equations by using differential operator algebra to analyze the motion of a simple viscoelastic rotor-shaft-system. Dousti et al. [23] addressed the nonlinear dynamic behavior of O-ring seals as the centering spring in squeeze film dampers, and the model was incorporated in a linear and time transient rotor dynamic analyzer of a vertical turbocharger. Wau-mans et al. [24] used elastomeric O-rings in combination with a tunable squeeze film damper as the elastomer suspension in an experimental test to solve the dynamic stability problem of high-speed air bearings.

The above studies proved that O-rings have a significant influence on the stability of rotor-bearing systems. However, these studies traditionally considered the effect of O-rings by simply presetting several groups of stiffness and damping values. This method is unreasonable. The relationship of the stiffness and damping of O-rings depends on the material [25, 26] and frequency [27, 28], which means that the mechanical property of O-rings is nonlinear.

On the basis of the strength of O-rings, the application of PAJBs with O-rings mounted will be a simple and effective way to achieve a high-speed and stable operation of the rotor in turbomachinery, and related studies are necessary. To date, few studies on the stability of RPS with O-rings mounted have been performed. The nonlinear characteristics of RPS with O-rings mounted must be systematically studied.
because the mechanical properties of O-rings and air are nonlinear.

In this work, the stiffness and damping characteristics of O-rings are expressed on the basis of the Voigt model [29] with the consideration of the influence of excitation frequency. Then, Darcy’s laws and air motion equation are coupled to establish an air lubrication model for PAJBs. On the basis of these techniques, a numerical model for analyzing the nonlinear characteristics of O-rings mounted RPS is presented. The model can consider the coupling of aerostatic and aerodynamic effects and the influences of more realistic mechanical characteristics of O-rings. Two experimental test-rigs are designed and built to verify the numerical model. The dynamic characteristics of the RPS are tested on the basis of the rotor dynamic test-rig. The stiffness and damping characteristics of the O-rings used in the experiment are obtained in the excitation test-rig. The numerical results of the rotor dynamic characteristics of the RPS are first obtained by coupling the experimental tested stiffness and damping of the O-rings. Then, the numerical and experimental results of the rotor dynamic characteristics are compared to verify the numerical model. Lastly, the rotor orbit, fast Fourier transform (FFT) plots, Poincaré maps, and bifurcation diagrams are adopted to analyze the stability of the RPS in various parameters systematically [30–32]; the parameters include the materials and number of O-rings, the rotational speed of the rotor, air supply pressure, nominal clearance, and the permeability of the porous material.

2 Theory

2.1 Operating principle of the RPS with O-rings mounted

Figure 1 shows the RPS with O-rings mounted. The PAJB is constructed using coaxially and tightly fitted porous and metal sleeves. The porous sleeve is obtained using the porous material. The porous sleeve is a special restrictor that has dense holes in the inner and surface of the material to restrict the external high-pressured air for ensuring that the bearing clearance can obtain a uniform distributed lubrication air film. This characteristic is called the aerostatic effect. Several grooves are made on the outer surface of the porous sleeve to ensure the air supply efficiency. The metal sleeve is obtained using a metal and intersected by a hole. Two bearings are inserted into a housing, and the housing is fixed on the base. The O-rings are mounted between the metal sleeve and housing to support PAJBs, and they are supported by the housing. The holes are also designed to intersect the housing and coaxial with the hole of the metal sleeve for allowing the external high-pressured air to enter the porous sleeve. The rotor structure of the turbomachinery is approximated as a rigid rotor with two disks at each end. One disk is designed with a mass equal to those of a turbine and a thrust runner. The other disk is designed at the opposite of the rotor to offer counterweight to balance the rotor mass. Accordingly, the weight of the rotor is shared on a 50–50 basis between two PAJBs.

The housing geometric center is defined as the origin of the coordinate. The position of the bearing geometric center relative to the housing center changes with various operating conditions because the PAJB is mounted by the O-rings, which are elastic. The geometric model of the PAJB with O-rings mounted is presented in Fig. 2. The inner radius of the bearing is \( r_{0} \), the thickness of the porous sleeve is \( r_{s} \), the radius of the rotor is \( r_{r} \), and the clearance of the bearing is defined as \( h_{0} = r_{0} - r_{r} \).

The O-ring is simplified as two pairs, which are in the \( x \) - and \( y \)-directions, of stiffness and damping to support the bearing. The rotor is rapidly rotated in the clockwise direction at the angular velocity \( \omega_{r} \), resulting in the aerodynamic effect of the air in the clearance. Consequently, the rotor is levitated by the air film pressure determined with the film thickness \( h \) by the coupling of the aerostatic and aerodynamic effects.
The eccentricity ratios of the rotor and bearing relative to the housing center are defined as \((e_{x}, e_{y})\) and \((e_{x}', e_{y}')\), respectively. Then, the film thickness in a dimensionless form can be obtained as

\[
H = 1 + (e_{x} \cos \theta + e_{y} \sin \theta) - (e_{x}' \cos \theta + e_{y}' \sin \theta).
\]

The dimensionless parameters are defined as follows:

\[h = h_0 H, \quad x_r = h_0 e_{x}, \quad y_r = h_0 e_{y}, \quad x_b = h_0 e_{x}', \quad y_b = h_0 e_{y}'\]

where \((x_r, y_r)\) and \((x_b, y_b)\) are the displacements of the rotor and bearing relative to the housing center, respectively, which are also called the absolute displacements.

The displacement of the rotor relative to the bearing center, which is also called relative displacement, in the \(x\)- and \(y\)-directions can be expressed as follows:

\[
\begin{pmatrix}
x_{rel} \\
y_{rel}
\end{pmatrix} = \begin{pmatrix}
x_r \\
y_r
\end{pmatrix} - \begin{pmatrix}
x_b \\
y_b
\end{pmatrix}.
\]

### 2.2 Lubrication model

Figure 3 shows the airflow model in the PAJBs. The airflow region is meshed by small control volumes. The small control volumes of the overall solution domain are classified into three types with the consideration of the characteristics of the structure and boundary. Different types of small control volumes have the same size in the bearing length direction and the same angle in the circular direction.

The size of the small control volume is different in the radial direction for the porous and air film regions. The air flowing in and out of the small control volume along three coordinate directions must satisfy the conservation of the mass flow rate, and it is expressed under the isothermal condition as follows:

\[
\dot{m}_o|_{\text{in}} - \dot{m}_o|_{\text{out}} + \dot{m}_r|_{\text{in}} - \dot{m}_r|_{\text{out}} + \dot{m}_z|_{\text{in}} - \dot{m}_z|_{\text{out}} - \Delta \dot{m}_i = 0
\]

(4)

where subscripts \(\text{in}\) and \(\text{out}\) represent the mass flow rate of the air flowing in and out of the small control volume, respectively; and \(\Delta \dot{m}_i\) represents the mass flow rate variation in a small control volume per unit time.

Type (a) shown in Fig. 3 represents the small control volume at the interior of the porous sleeve with a size in the radial direction defined as \(dr\). The air flows in and out of the small control volume from six surfaces. Assuming that the porous sleeve has the same permeability in different directions, the mass flow rate of the air in the porous sleeve is expressed on the basis of Darcy’s law as follows:

\[
\dot{m}_o = -\rho \frac{k \hat{c}_p}{\mu \sqrt{d \theta}} dr dz
\]

\[
\dot{m}_z = -\rho \frac{k \hat{c}_p}{\mu \sqrt{d \zeta}} rd dr
\]

\[
\dot{m}_r = -\rho \frac{k \hat{c}_p}{\mu \sqrt{d \rho}} rd dz.
\]

The dimensionless mass conservation for the airflow in the porous sleeve can be expressed by substituting Eqs. (5) into (4), as shown as follows:

\[
- \frac{d R}{R} \int_{r_{in}}^{r_{out}} \left[ \nabla \cdot \left( \frac{\partial \bar{p}}{\bar{R} \xi_0} \right) \right] dr + \frac{d R}{R} \int_{r_{in}}^{r_{out}} \left[ \nabla \cdot \left( \frac{\partial \bar{p}}{\bar{R} \xi_0} \right) \right] rd dr dR + \int_{r_{in}}^{r_{out}} \int_{\xi_{in}}^{\xi_{out}} \left[ \nabla \cdot \left( \frac{\partial \bar{p}}{\bar{R} \xi_0} \right) \right] rd dr dR d\xi
= \frac{\gamma}{\bar{c}} \frac{\bar{c} \bar{d}}{\bar{R}} dR d\bar{R} d\bar{R} d\xi.
\]

(6)

The dimensionless parameters are defined as follows:

\[
p = p_0 \bar{P}, \quad r = r_0 \bar{R}, \quad z = r_0 \bar{Z}, \quad \tau = \omega t, \quad \gamma = \frac{\mu_0 \omega \rho}{p_0 k}
\]

(7)
where $\rho$ is the air density, $\mu$ is the air viscosity, $p$ is the air pressure, $p_a$ is the ambient air pressure, $k$ is the permeability of the porous sleeve, $\eta$ is the porosity of the porous sleeve, $\tau$ is the dimensionless time, and $t$ is the dimensionless time.

Given that both end surfaces of the porous are sealed, the small control volume for meshing the end surfaces of the porous sleeve only has five surfaces for air flowing in and out, which is represented as type (b) in Fig. 3. The size in the radial direction is also defined as $dr$. Then, the dimensionless mass conservation of air on the sealed end surfaces of the porous layer is expressed as follows:

$$
\frac{\partial}{\partial t} \int_0^1 \rho \frac{\partial P}{\partial r} dr dz + \frac{\partial}{\partial r} \left[ \int_0^1 \frac{\partial P}{\partial r} r dz \right]_{in} - \frac{\partial}{\partial r} \left[ \int_0^1 \frac{\partial P}{\partial r} r dz \right]_{out} - \frac{dR}{2} \frac{\partial P}{\partial r} dr dz + \frac{dR}{2} \frac{\partial P}{\partial r} dr dz
$$

$$
= \left\{ \begin{array}{ll}
\int_0^1 \frac{\partial P}{\partial \theta} \frac{\partial \theta}{\partial r} dz & \text{for in along } z \text{- direction} \\
- \int_0^1 \frac{\partial P}{\partial \theta} \frac{\partial \theta}{\partial r} dz & \text{for out along } z \text{- direction}
\end{array} \right.
$$

Subscripts $in$ and $out$ for the $z$-direction are omitted. However, the air flowing in and out of the small control volume must be distinguished from the $z$-direction, which is indicated on the right side of Eq. (8).

Type (c) shown in Fig. 3 represents that the small control volume is composed of two layers, which are porous and air film layers as the lower and upper layers, respectively. The surface between these two layers is also the inner surface of the porous sleeve. The thicknesses of the lower and upper layers are $dr/2$ and $h$, respectively. The air only flows in and out of the small control volume from five surfaces, excluding the upper surface, because the upper surface of the air film layer overlaps with the shaft surface. Meanwhile, the air in the lower and upper layers is also exchanged in the radial direction across the overlapped surface of these two layers. Accordingly, the air flowing in the porous and clearance can be coupled by small control volume of category (c). The air flow in the bearing clearance is assumed to be laminar. The mass flow rates in the upper layer are obtained on the basis of the air motion equation and expressed in the $\theta$, $r$, and $z$-directions as follows:

$$
\dot{m}_\theta = -\frac{h^3}{12\mu} \rho \frac{\partial P}{\partial \theta} \frac{\partial \theta}{\partial r} dz + \frac{\omega r_0}{2} \rho h dz
$$

$$
\dot{m}_z = -\frac{h^3}{12\mu} \rho \frac{\partial P}{\partial z} r_0 d\theta
$$

$$
\dot{m}_r = 0.
$$

The pressure gradient in the air film thickness direction is neglected for the ultrathin air film in the bearing clearance. Equation (9) shows that the mass flow rate in the $r$-direction in the clearance is zero. The dimensionless mass conservation for the air in the small control volume of category (c) is presented by substituting Eqs. (5) and (9) into Eq. (4), as shown below:
The dimensionless parameters are defined as follows:
\[
\Lambda = \frac{6K}{\rho a h_0^2}, \quad \Gamma = \frac{12r^2}{h_0^2}, \quad \sigma = \frac{12\mu \omega r^2}{p_a h_0^2}.
\]

Subscripts in and out for the \( r \)-direction are omitted. The air flowing in and out of the small control volume along the \( r \)-direction from the porous layer is distinguished by the right side of Eq. (10).

Equations (6) and (8) obtain the mass conservation based only on Darcy’s law. It reflects the throttling process of the high-pressured air in the porous, which directly influences the aerostatic effect. Equation (10) is obtained in accordance with Darcy’s law and the air motion equation simultaneously. It can reflect the coupling of the aerodynamic and aerostatic effects.

The external pressured air is supplied from the holes of the housing and metal sleeve to enter in the porous sleeve and gradually permeated into the clearance. The pressure value on the outer radial surface of the porous sleeve is equal to the external supply pressure, as shown as follows:
\[
\bar{P}_0 = \bar{P}\big|_{R=1}.
\]

The pressure in the air film and porous layers satisfies the continuity condition in the circular direction as follows:
\[
\bar{P}|_{R=1} + \frac{2\omega}{\omega_0} = \bar{P}|_{R=1}.
\]

The air pressure in the air film and porous layers satisfies the continuity condition in the circular direction as follows:
\[
\bar{P}|_{\theta=0} = \bar{P}|_{\theta=2\pi}.
\]

At the end of the bearings, the air pressure in the air film is equal to the atmosphere pressure, and it is expressed as follows:
\[
\bar{P}|_{Z=\pm\frac{L}{2}, \atop \bar{R}=1} = 1.
\]

The pressure in the air film is defined as \( \bar{P}_0 \) with the following dimensionless form for easy description:
\[
\bar{P}_0 = \bar{P}\big|_{R=1}.
\]

The pressure distribution in the air film and porous sleeve can be obtained by solving Eqs. (6), (8), and (10) with the given air film thickness by using the finite difference method and Newton’s iterative method. The bearing forces along the \( x \)- and \( y \)-directions are expressed by integrating the air film pressure over the inner surface of the porous sleeve as follows:
\[
\begin{bmatrix}
F_x \\
F_y
\end{bmatrix} = \int_{-\pi}^{\pi} \frac{F_x}{sin \theta} d\theta
\]

where \( L \) is the bearing length; and \( \bar{F}_x \) and \( \bar{F}_y \) are the dimensional film forces in the \( x \)- and \( y \)-directions, respectively.

2.3 Force and motion analysis of the RPS

Figure 4 shows the mechanical model of the RPS. The half rotor with mass \( m_s \) is supported by the air film. The bearing with mass \( m_b \) is mounted by the O-rings that are also supported by the housing. The O-rings offer elastic force, and the stiffness and damping, denoted as \( k_b \) and \( b_b \), respectively, can be obtained using the Voigt model [29] and expressed by empirical formulas as follows:
\[
k_b = k' \exp(k''f_v)
\]
\[
b_b = b' + \exp(-b''f_v).
\]

Equations (16) and (17) show that the stiffness and damping of the O-rings are frequency dependent. The value of coefficients \( k' [\text{Mn/m}], k'' [1/\text{kHz}], b' [\text{kN} \cdot \text{m/s}], \) and \( b'' [1/\text{kHz}] \) can be obtained from the experimental test data. \( f_v \) is the vibration frequency of the O-rings.

Equations (16) and (17) are exponential functions; that is, varying the vibration frequency will significantly change the stiffness and damping of the O-rings. Given that the rotational speed of the RPS is tens of thousands, it will induce a high vibration frequency to act on the O-rings. Therefore, the stiffness and damping characteristics of O-rings with
Various frequencies cannot be neglected in the analysis of the effect of O-rings on the performance of the RPS.

Ignoring the axial movement of the rotor, the transient motion of the rotor and bearing relative to the housing center in the x- and y-directions can be expressed as follows:

$$\begin{align*}
\ddot{x}_r &= (F_x + m_r b_{ab} \omega^2 \cos \omega t)/m_r + g, \\
\ddot{y}_r &= (F_y + m_r b_{ab} \omega^2 \sin \omega t)/m_r, \\
\ddot{x}_b &= (-F_x - k_b \dot{x}_b - b_b x_b)/m_b + g, \\
\ddot{y}_b &= (-F_y - k_b \dot{y}_b - b_b y_b)/m_b,
\end{align*}$$

(18)

(19)

where \((\ddot{x}_r, \ddot{y}_r)\) and \((\ddot{x}_b, \ddot{y}_b)\) are the absolute accelerations of the rotor and bearing, respectively, in the x- and y-directions; \((\dot{x}_b, \dot{y}_b)\) refers to the absolute velocities of the bearing in the x- and y-directions; \(m_r, m_b\), and \(e_{ab}\) are the masses of the rotor and bearing and the imbalanced mass of the rotor; and \(g\) is the gravitational acceleration.

The stiffness and damping of the O-rings are substituted into Eqs. (18) and (19). Thus, the transient motion of the RPS with coupling the frequency-influenced stiffness and damping characteristics of O-rings can be modeled. It can reflect more actual conditions.

2.4 Numerical solution method

The absolute velocity of the rotor and bearing varying with time can be obtained as follows by using Euler’s method to integrate Eqs. (18) and (19):

$$\begin{align*}
\dot{x}_r(\tau) &= \dot{x}_r(\tau - \Delta \tau) + \ddot{x}_r(\tau \Delta \tau), \\
\dot{y}_r(\tau) &= \dot{y}_r(\tau - \Delta \tau) + \ddot{y}_r(\tau \Delta \tau), \\
\dot{x}_b(\tau) &= \dot{x}_b(\tau - \Delta \tau) + \ddot{x}_b(\tau \Delta \tau), \\
\dot{y}_b(\tau) &= \dot{y}_b(\tau - \Delta \tau) + \ddot{y}_b(\tau \Delta \tau),
\end{align*}$$

(20)

(21)

where \((\dot{x}_r, \dot{y}_r)\) denotes the absolute velocities of the rotor in the x- and y-directions, and \(\Delta \tau\) is the dimensionless time interval.

Then, the absolute displacements of the rotor and bearing can finally be obtained by integrating Eqs. (20) and (21) as follows:

$$\begin{align*}
x_r(\tau) &= x_r(\tau - \Delta \tau) + \int_0^\tau \dot{x}_r(\tau) \Delta \tau + \frac{1}{2} \int_0^\tau \ddot{x}_r(\tau) \Delta \tau^2, \\
y_r(\tau) &= y_r(\tau - \Delta \tau) + \int_0^\tau \dot{y}_r(\tau) \Delta \tau + \frac{1}{2} \int_0^\tau \ddot{y}_r(\tau) \Delta \tau^2, \\
x_b(\tau) &= x_b(\tau - \Delta \tau) + \int_0^\tau \dot{x}_b(\tau) \Delta \tau + \frac{1}{2} \int_0^\tau \ddot{x}_b(\tau) \Delta \tau^2, \\
y_b(\tau) &= y_b(\tau - \Delta \tau) + \int_0^\tau \dot{y}_b(\tau) \Delta \tau + \frac{1}{2} \int_0^\tau \ddot{y}_b(\tau) \Delta \tau^2.
\end{align*}$$

(22)

(23)

The final characteristics of the RPS are obtained via the iteration method, which is shown in Fig. 5. In the iteration at time \(\tau\), the absolute displacements of the rotor and bearing at time \(\tau - \Delta \tau\) are substituted into Eq. (1) to obtain the air film thickness. The corresponding film pressure can be obtained using Newton’s iterative method to solve Eqs. (1), (6), (8), and (10) with an appropriate condition of convergence being set. The film forces are obtained by integrating the film pressure over the inner surface of the porous sleeve on the basis of Eq. (15). Then, the film forces are substituted into Eqs. (18) and (19) to obtain the absolute acceleration of the rotor and bearing at time \(\tau\). In accordance with Eqs. (20) and (21), the absolute velocity at time \(\tau\) can be obtained using the absolute velocity at time \(\tau - \Delta \tau\) and absolute acceleration at time \(\tau\). Meanwhile, the absolute velocity and acceleration at time \(\tau\) are used as variates of Eqs. (22) and (23) coupled with the absolute displacement at time \(\tau - \Delta \tau\) to obtain the absolute displacement at time \(\tau\). The absolute position, speed, and acceleration of the rotor and bearing at time \(\tau\) are used as the initial variates of the iteration at \(\tau + \Delta \tau\). The characteristics of the RPS at time \(\tau + \Delta \tau\) can be obtained by repeating the solution process at time \(\tau\). The results are output until the iteration steps reach the preset.
3 Experimental verification

3.1 Experiment introduction

Two experiments are performed to verify the accuracy of the simulation program. The RPS experiment is conducted to obtain the dynamic motion characteristics of the rotor in different rotational speeds. The results are used to verify the validity of the numerical model and calculation process for assessing the rotor dynamic performance. The stiffness and damping characteristics of the O-rings, which are used in the RPS experiment, are obtained through excitation experiment. Meanwhile, the tested stiffness and damping of the O-rings are also used as the parameters in the numerical program to calculate the rotor dynamic performance.

3.2 Experiment of the RPS

Figure 6 shows the PAJB that is tested in the RPS experiment. The porous sleeve with graphite porosity as the material is coaxial with the metal sleeve that is made of 304 steel. The outer surface of the porous sleeve and the inner surface of the metal sleeve are locally glued with a superglue. Both end surfaces of the porous sleeve are sealed with a superglue. Several O-rings are mounted in the predesigned grooves, which are on the outer surface of the metal sleeve. The O-rings must be able to fasten the metal sleeve with slight elastic force, which can offer effective stiffness to support the bearing and damping to absorb vibration. The material of the O-rings is hydrogenated nitrile butadiene rubber (HNBR). An air supply hole is designed in the metal sleeve to allow the external high-pressured air to be supplied into the porous sleeve. Thus, the superglue must not drop into the supply hole of the metal sleeve and cover the outer surface of the porous sleeve to ensure that high-pressured air is successfully and efficiently supplied into the porous sleeve.

Figure 7a and b shows the schematic and physical map of the rotor-bearing test-rig, respectively. The rotor is supported by two PAJBs. A double-pulse turbine and a trust runner are designed at the same end of the rotor. Two foil thrust bearings are installed opposite to the two surfaces of the trust runner to balance the axial forces. A counterweight disk is installed at the other end of the rotor to balance the weight of the rotor. Two sets of eddy current sensors (ECSs. 1, 3 and 2, 4) are located at the two ends of the rotor. Each set of ECSs must be simultaneously orthogonally installed along the x- and y-directions to obtain the vertical and horizontal vibration of the rotor. Moreover, ECSs 5 and 6 are installed at the top of the housing and aimed at the center position of the...
PAJBs along the vertical direction to measure the movement of the bearings in the vertical direction. Meanwhile, a photoelectric tachometer is vertically aimed to the surface of the counterweight disk to measure the rotational speed of the rotor. All test data from the sensors are recorded and used to analyze the dynamic characteristics of the RPS.

To conduct the experiment, the high-pressured air is first supplied into the PAJBs to levitate the rotor. Accordingly, no friction exists between the PAJBs and rotor before the rotor rotates. Then, the high-pressured air from another air pipe, which is different from that in the PAJBs, is supplied along the radial direction of the turbine to drive the turbine. The turbine changes the energy of the high-pressured air to the kinetic energy of the rotor. Lastly, the rotor is rapidly initiated and rotated at a high speed as it is levitated by the air film force.

Table 1 lists the detailed parameters of the O-ring, PAJBs, rotor, and environment for the experimental test. The total mass of the rotor is 2.057 kg, and the load is equally divided between two PAJBs. Hence, the supporting load of each PAJB is 10.079 N. The external pressured air with pressure values of 0.5 and 0.55 MPa is supplied into the PAJBs. The driving air of the turbine is supplied with the pressure changing from 0 to 0.4 MPa to change the rotational speed of the rotor.

Figure 8 shows the tested waterfall plots (vertical direction) of the rotor and bearing centers relative to the housing center for the RPS with rotational speed changing from 12 to 30 krpm. With regarding the vibration characteristics of the rotor, the amplitude of the synchronous vibration is first increased and then gradually decreased to be nearly unchanged with the increase in rotational speed. The comparison of the results between the external pressure values of 0.5 and 0.55 MPa indicates that the vibration characteristics of the rotor are nearly not changed. In terms of the vibration results of the bearings, the amplitude of the synchronous vibration is small and approximately equal to 1.5 μm. This value is nearly not changed with the variation in rotational speed and external supply pressure.

3.3 Excitation testing of the O-rings

Figure 9 shows the schematic and physical map of the excitation test-rig. A new journal is designed with a special end. The structure and dimension parameters of the end are the same as those of the outer surface of the metal sleeve, which are presented in Fig. 6. The new journal is fixed using two clamping supports. A new sleeve is designed with its inner diameter being the same as that of the housing, which is presented in Fig. 7. The new sleeve and O-rings form a simple mechanical spring-mass damped system.

The new sleeve is excited by an electromagnetic exciter in the horizontal direction. Figure 8 shows that the vibration amplitude of the bearing is approximately 1.5 μm. The vibration amplitude of the exciter may be set at 1.5 μm when exciting the new sleeve. The influence of the gravity of the new sleeve in the vertical direction can be ignored because the vibration amplitude is small. The characteristics of the vibration in the horizontal direction are also tested with the sensors. The experimental data of the vibration characteristics can be processed in accordance with the research experiences of San Andrés et al. [33] to...
obtain the stiffness and damping of the O-rings at different frequencies.

The stiffness and damping of a HNBR O-ring in 200–500 Hz frequency are shown in Fig. 10. The stiffness is increased with the increase in excitation frequency, and the damping is decreased. The characteristics of the stiffness and damping varying with frequency are fitted in Eqs. (16) and (17). The values of the coefficients $k'_0$, $k''_0$, $b'_0$, and $b''_0$ are 0.8502, 1.46, 0.0121, and $-3.9257$, respectively.

### 3.4 Comparison of the results of the theoretical and experimental analyses

Through substituting the tested values of the stiffness and damping of the O-rings in the numerical calculation, the numerical results of the vibration characteristics of the rotor can be obtained with the consideration of the influence of the O-rings. The synchronous vibration of the rotor presented in Fig. 8 is used to compare the numerically predicted results. The comparison is presented in Fig. 11. The amplitude of the vibration first increases and then gradually decreases with the increase in rotational speed, regardless of the variation in the external supply pressure.

The numerically predicted and experimentally tested critical speeds are 18.53 and 20.00 krpm, respectively, with the external supply pressure being 0.5 MPa. They become 18.64 and 20.50 krpm when the external supply pressure is increased to 0.55 MPa. This condition proves that the critical speeds from the experimental and numerical results are well matched with a slight difference being less than 10%. The difference in the numerically calculated and experimentally tested amplitudes of the synchronous vibration is somewhat significant. The difference is caused by experimental errors. The errors come from three parts. First, the idealized installation of the excitation test-rig must be ensured with the centers of the new sleeve and journal being located on the excitation direction of the exciter. However, installation error is inevitable, and it will cause the error of the measured stiffness and damping of the O-rings. Second, the bearing length is 45 mm, and the diameter is 25 mm. Hence, it is a long bearing. The local error of the measured nominal clearance is inevitable. Third, the imbalance mass used in the numerical calculation is

### Table 1 Parameters of the O-rings, PAJB, rotor, and experimental environment

| Object      | Parameter                        | Value                  |
|-------------|----------------------------------|------------------------|
| O-ring      | Material                         | HNBR                   |
|             | Inside diameter                  | 34.5 mm                |
|             | Section diameter                 | 2.65 mm                |
|             | Compression ratio                | 15.1%                  |
|             | Distribution number              | 4 in uniform           |
| Bearing     | Bearing mass, $m_b$              | 0.183 kg               |
|             | Bearing diameter, $D$            | 25 mm                  |
|             | Bearing length, $L$              | 45 mm                  |
|             | Porous thickness, $r_s$          | 2.5 mm                 |
|             | Nominal clearance, $h_0$         | 15 μm                  |
|             | Porous porosity ratio, $\eta$    | 0.1                    |
|             | Porous permeability, $k$         | $5.979 \times 10^{-9}$ mm$^2$ |
|             | External supply pressure, $p_s$  | 0.5 MPa, 0.55 MPa      |
| Rotor       | Total mass, $2m_t$               | 2.057 kg               |
|             | Load                             | $2 \times 10.079$ N    |
|             | Mass eccentricity of the rotor, $e_{ab}$ | 0.674 μm             |
|             | Rotational speed                 | 0 – 30 krpm            |
| Environment | Ambient pressure, $p_a$          | 0.1 MPa                |
|             | Air viscosity, $\mu$             | $19.8 \times 10^{-12}$ MPa · s   |
|             | Temperature of the supplied air  | 294 K                  |
obtained through dynamic balance testing. It is a fixed value. However, the counterweight disk, thrust runner, and turbine must be dismantled and reassembled when assembling the rotor in the test-bed. The final imbalance mass presented when the test-rig is operating is easily changed and differs from the tested value. Consequently, the imbalance mass used in the numerical calculation is difficult to ensure to be equal to the actual value.

These three types of errors have various influences on the difference between the numerically calculated and experimentally tested results. The errors of the stiffness of O-rings and the nominal clearance influence the mechanical property of the rotor-bearing system to influence the critical speed. Given that the numerically predicted and experimentally tested critical speeds correspond well, the accuracies of the measured stiffness of O-rings and the nominal clearance of the bearing are credible. The damping of O-rings and the imbalance mass mainly influence the amplitude of the synchronous vibration. Considering that the damping and stiffness of O-rings are obtained on the basis of the same signal by data processing, the damping value of the O-rings used in the numerical calculation is also credible. Accordingly, the difference in the numerically calculated and experimentally tested amplitudes is mainly caused by the imbalance masses which used in the numerical calculation and presented when the test-rig is operating are not the same. Considering that the imbalance mass is easily changed while operating the test-rig, the gap between the numerically calculated and experimentally tested amplitudes is inevitable and acceptable.

4 Results and discussion

This section mainly discusses the characteristics of the RPS coupled with the influence of the O-rings. The design and operating parameters include the material and number of the O-rings, the rotational speed of the rotor, the external supply pressure, the bearing...
clearance, and the porous permeability. The necessary abbreviations are presented for simplicity. The O-rings with styrene butadiene rubber (SBR), methyl vinyl silicone rubber (VMQ), and nitrile butadiene rubber (NBR) as the materials are called SBRO, VMQO, and NBRO, respectively. The bearings with these three types of O-rings mounted are called BSBR, BVMQ, and BNBR. The bearing without an O-ring mounted is called BR for simplicity.

Figure 12 shows the stiffness and damping of these three types of O-rings varying with frequency and the corresponding fitted formulas, which were experimentally tested by Miyanaga and Tomioka [27]. The stiffness and damping of the O-rings are substituted into the numerical model of the RPS. The other parameters used in the numerical calculation are listed in Table 2. In the following text, the position of the rotor center relative to the housing center is represented by absolute vibration, absolute orbit, absolute position, or absolute response. The position of the rotor center relative to the bearing center is represented by relative vibration, relative orbit, relative position, or relative response.

4.1 Effects of O-rings with different materials

SBR, VMQ, and NBR are the three commonly used materials in obtaining O-rings. The effects of different O-rings, which are distinguished by materials, on the dynamic characteristics of the RPS are discussed in this part. The bifurcation diagram of the absolute vibration of the rotor versus rotational speed (12–60 krpm) with different O-rings mounted is presented in Fig. 13. The rotor absolute response in the vertical and horizontal directions with different O-rings shows 1 T-periodic motions at a low rotational speed. The quasi-periodic motions are gradually determined with the increase in rotational speed. The demarcation rotational speeds of the rotor absolute responses changed from periodic motions to quasi-periodic motions with the SBRO, VMQO, and NBRO mounted are 41, 38, and 40 krpm, respectively. When the
rotational speed is lower than the demarcation rotational speed, the coordinate value of the rotor center absolute position in the x-direction is larger than that in the y-direction. This phenomenon occurs because the gravity direction is along the x-direction. The absolute vibrations along the x- and y-directions simultaneously decrease with the increase in rotational speed but not beyond the demarcation value. Specifically, the rotor center is gradually close to the housing center. This phenomenon occurs because the frequency of the excitation force that acts on the O-rings increases with the increase in rotational speed, thereby resulting in the increase in the stiffness of the O-rings (Fig. 12). With the rotational speed being lower than the demarcation value, the absolute coordinates along the x-direction are arranged from small to large in the following order: that of the rotor supported by BNBR, BSBR, and BVMQ. The stiffness of the O-rings when arranged in descending order is that of NBRO, SBRO, and VMQO. The amplitude of the rotor absolute vibration for the bearings with SBRO mounted is smaller than that with NBRO mounted and larger than that with VMQO mounted when the rotational speed is increased to be larger than the demarcation rotational speed.

Figure 14 shows the bifurcation diagram of the relative vibration of the rotor versus rotational speed (12–60 krpm). The results for the O-rings mounted on the RPS with SBR, VMQ, and NBR materials are compared with those for the RPS in which the bearing is rigid supported. The rotor relative responses for the RPS with O-rings mounted are also changed from 1 T-periodic motions to quasi-periodic motions with the increase in rotational speed, and the demarcation rotational speeds of the change for the rotor supported by SBRO, VMQO, and NBRO are 41, 38, and 40 krpm, respectively. Without an O-ring mounted, the rotor motion changes from 1 T-periodic motion to 2 T-periodic motion with the rotational speed increased to 37 krpm. With the rotational speed continuing to increase and becoming larger than 52 krpm, the rotor motion changes from 2 T-periodic motion to quasi-periodic motion. When the rotor motion is not 1 T-periodic, the vibration amplitude of the rotor with BR supported is significantly higher than that of the rotor with O-rings mounted bearings.
supported. The relative vibration amplitude of the quasi-periodic motions for the RPS with SBRO mounted is higher than that with VMQO but lower than that with NBRO. Therefore, the effectiveness of O-rings in increasing the stability can be arranged from good to better in the following order: NBRO, SBRO, and VMQO. This characteristic is similar to the results presented in Ref. [25].

The relative orbit and Poincaré maps of the rotor center and the FFT of the rotor relative displacement in the horizontal direction are presented for a clear understanding of the characteristics and dynamic responses of the RPS. Figure 15 shows the results for the rotors supported on BR, BNBR, BSBR, and BVMQ.

Figure 15a shows the results with a rotational speed of 37 krpm. The synchronous frequency is 616.5 Hz. The rotor performs 2 T-periodic motion, as shown in a Poincaré map. With O-rings mounted, all the orbit shapes of the rotors are closed curves, only a spike with a frequency of 616.5 Hz exists in each FFT plot. The rotors perform 1 T-periodic motions, and the Poincaré map exhibits a point. The orbit sizes and synchronous amplitudes are evidently smaller than those without an O-ring mounted. The orbit size and synchronous amplitude of the rotor supported on BSBR are larger than those on BVMQ and smaller than those on BNBR.

Figure 15b shows the results with a rotational speed increased to 38 krpm. The synchronous frequency is 633.1 Hz. The rotor supported on BR still performs 2 T-periodic motion with a sub-synchronous frequency of 316.6 Hz. The orbit size and amplitude of the sub-synchronous and synchronous vibrations increase. The amplitude of the sub-synchronous vibration even becomes higher than that of the synchronous vibration. With the BVMQ supported, the rotor performs quasi-periodic motions, and the sub-synchronous frequency is 116.1 Hz. The amplitude of the sub-synchronous vibration is approximately 0.033 μm. The Poincaré map shows a dense curve. The results of the rotors supported on BSBR and BNBR indicate that the relative orbit shapes are still closed curves. The relationship of the synchronous vibration amplitude of the rotor supported on different O-ring mounted bearings is the same as that presented in Fig. 15a. The comparison results presented in Fig. 15a and b show that the synchronous vibration amplitude of the rotor decreases with the increase in rotational speed for the RPS with O-rings mounted.

Figure 15c shows the results of the bearing with a rotational speed of 40 krpm. The synchronous frequency is 666.4 Hz. The orbit size and vibration amplitude of the rotor with BR supported continuously increase. The amplitude of the sub-synchronous vibration with a frequency of 333.2 Hz becomes much higher than that of the synchronous vibration. The sub-synchronous vibration amplitude of the rotor supported on BVMQ also increases with sub-synchronous frequency of 111.1 Hz. The rotor supported on BNBR performs quasi-periodic motions with an exceptionally high sub-synchronous vibration amplitude. The sub-synchronous frequency is 222.1 Hz. The rotor still performs 1 T-periodic motions when it is supported on

| Table 2 Parameters for the numerical calculation |
|-----------------------------------------------|
| **PAJB parameters**                           |
| Bearing mass, \(m_b\)                        | 0.2 kg |
| Bearing diameter, \(D\)                      | 25 mm  |
| Bearing length, \(L\)                       | 45 mm  |
| Porous thickness, \(r_s\)                    | 2.5 mm |
| Nominal clearance, \(h_o\)                  | 10–30 μm |
| Porous carbon porosity ratio, \(η\)         | 0.1 |
| Porous permeability, \(k\)                  | \(0.1–2.0 \times 10^{-9} \text{ mm}^2\) |
| Supply pressure, \(p_s\)                    | \(0.2 – 0.4\) MPa |
| **Rotor parameters**                         |
| Total mass, \(2m_s\)                        | 2 kg |
| Mass eccentricity of the rotor, \(e_{ub}\)  | 2.83 μm |
| Rotational speed                            | 12–60 krpm |
| **Environmental parameters**                |
| Ambient pressure, \(p_a\)                  | 0.101 MPa |
| Air viscosity                               | \(1.98 \times 10^{-11} \text{ MPa} \cdot \text{s}\) |
| Temperature of the supplied air            | 294 K |
BSBR. The synchronous vibration amplitudes of the rotor supported on O-ring mounted bearings continuously decrease, and the relationship of the synchronous vibration amplitudes is also the same as that presented in Fig. 15a.

Figure 15d presents the results with a rotational speed of 41 krpm. The synchronous frequency is 683.1 Hz. The rotor supported on BSBR begins to perform quasi-periodic motions with a sub-synchronous frequency of 170.8 Hz. The amplitudes of the sub-synchronous vibration of the rotors supported on BR and BNBR increase (nearly double that of the synchronous one). The synchronous vibration amplitude of the rotor continually increases when it is supported on BR and decreases when supported on O-ring mounted bearings with the increase in rotational speed from 40 to 41 krpm. The synchronous vibration amplitudes of the rotor supported on different O-rings mounted bearings have the same relationship as that presented in Fig. 15a.

4.2 Effects of the number of O-rings

In this part, the effects of the number of O-rings on the performance of the RPS are analyzed. Figure 16 shows a comparison of the relative orbit of the rotor with bearings mounted by different numbers of SBRO when the rotational speeds are 35, 38, and 41 krpm.
The orbit shapes of the rotor are closed curves for all the conditions with a rotational speed of 35 krpm. The orbit size slightly decreases with the increase in the number of O-rings. When the rotational speed of the RPS with six O-rings mounted is increased to 38 krpm, the orbit size significantly increases, and the rotor
performs quasi-periodic motions. This phenomenon occurs because the total stiffness and damping of the six O-rings rapidly increase and decrease, respectively, with the increase in rotational speed. The orbit size of the RPS with four O-rings mounted is still smaller than that with two O-rings mounted. The total stiffness and damping of O-rings continually increase and decrease, respectively, with the increase in the rotational speed to 41 krpm. Accordingly, the rotor performs quasi-periodic motions with four O-rings mounted in the RPS.

Fig. 15 Predicted relative orbit and Poincaré maps of the rotor center and the FFT of the rotor center relative displacement in the horizontal direction with different rotational speeds: a 37 krpm; b 38 krpm; c 40 krpm; and d 41 krpm
4.3 Effects of external supply pressure with O-rings mounted

Figure 17 shows the bifurcation diagram of the relative vibration of the rotor in the vertical and horizontal directions versus external supply pressure when it is supported on (a) BR and (b) BSBR. The rotor in two directions of the RPS without O-rings mounted perform 2 T-periodic motion when the external supply pressure is maintained within the interval of $0.2 \text{ MPa} \leq P_s \leq 0.31 \text{ MPa}$. Then, it performs 1 T-periodic motion with the external supply pressure being increased to be larger than 0.31 MPa. For the rotor supported on BSBR, the performance...
changes from quasi-periodic to 1 T-periodic with increasing external supply pressure, and the demarcation pressure is 0.29 MPa. This phenomenon occurs because increasing the external supply pressure can increase the stiffness of the air film to prevent rotor orbit divergency. With increasing external supply pressure, the motion of the BSBR supported rotor first advances into 1 T-periodic. This situation is due to that the elastic suspension can increase the stability of the RPS.

Figure 18 shows the predicted relative orbit and Poincaré maps of the rotor center and the FFT of the rotor center relative displacement in the horizontal direction with supply pressure values of 0.2, 0.25, and...
0.3 MPa. The rotor is supported on BSBR. The orbit size gradually decreases with the increase in external supply pressure, and the orbit shape is changed from regular curves to a closed curve. The synchronous frequency is 633.1 Hz. The amplitude of the sub-synchronous vibration is extremely high with a supply pressure of 0.2 MPa. It rapidly decreases and finally disappears with the increase in external supply pressure. The sub-synchronous frequency is 168.8 Hz. Correspondingly, the rotor motion is changed from quasi-periodic to 1 T-periodic with the Poincaré map changing from a dot circle to a point. By contrast, the synchronous amplitude is less influenced by external supply pressure. The amplitude only slightly increases
at a high external supply pressure. The results are similar to those in Ref. [14].

4.4 Effects of bearing clearance with O-rings mounted

Figure 19 shows the bifurcation diagram of the relative vibration of the rotor in the vertical and horizontal directions versus the bearing clearance when it is supported on (a) BR and (b) BSBR. The rotor motion is 1 T-periodic motion at small clearance. With increasing clearance, the rotor motion is changed to 2 T-periodic and quasi-periodic for BR supported and directly changed to quasi-periodic for BSBR supported. This phenomenon occurs because increasing clearance decreases the film stiffness. The 1 T-periodic motions of the rotors with BR and BSBR supported disappear when the clearance is increased to be larger than 20 and 21 μm, respectively.

Figure 20 shows the predicted relative orbit and Poincaré maps and the FFT of the rotor center relative displacement in the horizontal direction with bearing...
clearances of 20, 25, and 30 \( \mu \text{m} \). The rotor is supported on BSBR.

The orbit shape shows a closed curve when the clearance is 20 \( \mu \text{m} \). The orbit size significantly increases with the increase in clearance to 25 \( \mu \text{m} \), and its shape is changed to a regular curve. The synchronous frequency is 633.1 Hz. A comparison of the results with clearances of 20 and 25 \( \mu \text{m} \) shows that the amplitude of the synchronous vibration is nearly not changed. However, sub-synchronous vibration occurs with its frequency being 158.3 Hz, and its amplitude is extremely high. The rotor motion is 1 T-periodic with the Poincaré map showing a point when the clearance is 20 \( \mu \text{m} \). With the clearance being increased to 25 \( \mu \text{m} \), the rotor motion is changed to a quasi-periodic one when the Poincaré map is a smooth circle. The orbit shape, the amplitude of synchronous vibration, and the Poincaré map of the rotor are nearly not changed with the continued increase in clearance to 30 \( \mu \text{m} \) relative to the results with a clearance of 25 \( \mu \text{m} \). The sub-synchronous frequency is still 158.3 Hz. Nevertheless, the orbit size and the amplitude of the sub-synchronous vibration significantly increase.

4.5 Effects of material permeability with O-rings mounted

Figure 21 shows the bifurcation diagram of the relative vibration of the rotor in the vertical and horizontal directions versus the porous permeability when it is supported on (a) BR and (b) BSBR. The rotor center in two directions shows 2 T-periodic motion in small permeability when the rotor is

---

**Fig. 18** Predicted relative orbit and Poincaré maps of the rotor center and FFT of the rotor center relative displacement in the horizontal direction with different supply pressure values
supported on BR. Then, the motion is changed to 1 T-periodic with the permeability being increased to be larger than $1.1 \times 10^{-9}$ mm$^{-2}$. The rotor with BSBR supported is changed from quasi-periodic to 1 T-periodic with the increase in permeability, and the demarcation permeability is $0.9 \times 10^{-9}$ mm$^{-2}$. This phenomenon occurs because the film force can be increased with the increase in permeability, which increases the bearing stiffness, thereby resulting in the rotor center gradually showing a periodic motion. This finding is similar to the results with the influence of external supply pressure. Although a high permeability can increase the stability of the RPS, air hammer of the porous air bearings may occur with the increase in permeability. This result means that the whirl stability of the O-rings mounted RPS can be ensured at a low permeability to prevent air hammer efficiently.

Figure 22 shows the predicted relative orbit and Poincaré maps and the FFT of the rotor center relative displacement in the horizontal direction with porous permeabilities of $0.1 \times 10^{-9}$, $0.5 \times 10^{-9}$, and $1.0 \times 10^{-9}$ mm$^{-2}$. The rotor is supported on BSBR, and the synchronous frequency is 633.1 Hz. The orbit shape shows a regular curve with a large orbit size and an extremely high amplitude of the sub-synchronous vibration when the porous permeability is $0.1 \times 10^{-9}$ mm$^{-2}$. The sub-synchronous frequency is 158.3 Hz. The amplitude of the synchronous vibration is smaller than that of the sub-synchronous vibration. Moreover, the rotor shows a quasi-periodic motion with the Poincaré map showing a nearly closed curve. The orbit shape is still a regular curve when the porous permeability is increased to $0.5 \times 10^{-9}$ mm$^{-2}$, and the sub-synchronous frequency becomes 168.8 Hz. However, the orbit size and amplitude of the sub-synchronous vibration decrease. The amplitude of the synchronous vibration is still smaller than that of the sub-synchronous vibration, but it is slightly increased. The rotor shows quasi-periodic motion with the Poincaré map showing a dashed circle. The orbit shape is changed to a closed curve with a small size with the continued increase in permeability to $1.0 \times 10^{-9}$ mm$^{-2}$, and the sub-synchronous vibration disappears. The amplitude of the synchronous vibration continually slightly increases. Nonetheless, the rotor presents 1 T-periodic motion with the Poincaré map showing a point.
5 Conclusion

This paper presents a nonlinear analysis of an RPS with O-rings mounted. The lubrication film is modeled by combining Darcy’s law and the motion equation of air. The mechanical property of the O-rings is described using the Voigt model with the consideration of the influence of excitation frequency. The RPS performance is modeled by considering the interaction of rotor, bearings, and O-rings. The mechanical property of O-rings with HNBR as the material is first obtained in the excitation test-rig. Then, the tested stiffness and damping of the O-rings are substituted into the numerical model of RPS. Meanwhile, the tested O-rings are used in the rotor-bearing test-rig. The numerical and experimental results of the RPS performance are compared. Both results obtain good correspondence. On the basis of these steps and methods, a numerical model considering the coupling of aerostatic and aerodynamic effects and the influences of more realistic mechanical characteristics of O-rings is presented and verified. Subsequently, the characteristics of the RPS are analyzed using bifurcation diagrams, orbit, Poincaré maps, and FFT plots. The effects of different system parameters, including the material of O-rings, the number of O-rings, external supply pressure, bearing clearance, and porous permeability, on the performance of the RPS are studied. The numerical model and the analysis results can provide a theoretical basis and practical references for designing a high-rotational-speed RPS with O-rings mounted.

The influence of O-rings with SBR, VMQ, and NBR materials on the RPS performance is first studied.
by simultaneously analyzing the absolute and relative responses of the rotor. The results show that the motions are changed from 1 T-periodic to 2 T-periodic and then quasi-periodic when the rotor is supported by BR and changed from 1 T-periodic to quasi-periodic when the rotor is supported by O-rings mounted bearings. During this process, the orbit shape becomes a regular curve with the increase in orbit size, and sub-synchronous vibration occurs. Meanwhile, the Poincaré map is finally changed from a point to dotted curves. The demarcation rotational speeds for 1 T-periodic motion disappearance are 37, 41, 38, and 40 krpm with the rotor supported on BR, BSBR, BVMQ, and BNBR, respectively. After the 1 T-periodic motion disappearance, the orbit sizes and synchronous and sub-synchronous amplitudes of the rotor significantly decrease with O-rings mounted. Thus, O-rings can increase the stability of the RPS, and the effectiveness is arranged from good to better as follows: NBRO, SBRO, and VMQO.

Then, the effects of the other system parameters on the RPS performance are studied on the basis of the rotor supported on BSBR by analyzing the relative motion of the rotor. When the rotor performs periodic motions, the orbit size slightly decreases with the increase in the number of O-rings. However, increasing the number of O-rings causes the rotor to perform quasi-periodic motions easily relative to the conditions with fewer number of O-rings but not zero at higher rotational speed condition. Once the quasi-periodic motion is performed, the orbit size is significantly increased. Increasing the external supply pressure can make the quasi-periodic motioned rotor gradually perform 1 T-periodic motion. The orbit size and amplitude of sub-synchronous vibration significantly decrease with the increase in external supply pressure. The demarcation external supply pressure for motion changed to be 1 T-periodic is decreased with O-rings mounted. Increasing the bearing clearance significantly increases the orbit size and amplitude of sub-synchronous vibration to decrease the stability of the RPS, and the rotor motion is changed from periodic to quasi-periodic motions. With the rotor supported on the O-rings mounted bearings, the demarcation clearance, which changes the rotor motion from 1 T-periodic to 2 T-periodic or quasi-periodic, is increased. The rotor performance gradually demonstrates a 1 T-periodic motion with the increase in permeability. Moreover, increasing the permeability causes the orbit size and amplitude of
sub-synchronous vibration to decrease significantly. The demarcation permeability for 1 T-periodic motion disappearance is decreased with O-rings mounted.

Acknowledgements The authors acknowledge the financial support by National Key R&D Program of China (2019YFB1504600), National Natural Science Foundation of China (Grant No. 52005170), Natural Science Foundation of Hunan Province (2020J5065), the Foundation of China Academy of Space Technology (CAST), the Fundamental Research Funds for the Central Universities, China, The Presidential Foundation of China Academy of Engineering Physics (YZJLX2018010), Hunan Transformation and Industrialization Plan of Scientific and Technological Achievements (Grant No. 2020GK2069).

Declarations

Conflict of interest The authors declare that they have no conflict of interest. All authors have approved the manuscript and agreed with submission to Nonlinear Dynamics.

Data availability The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

References

1. Chen, G., Ju, B., Fang, H., Chen, Y., Yu, N., Wan, Y.: Air bearing: academic insights and trend analysis. Int. J. Adv. Manuf. Technol. 106(3), 1191–1202 (2020)
2. Gao, Q., Chen, W., Lu, L., Huo, D., Cheng, K.: Aerostatic bearings design and analysis with the application to precision engineering: state-of-the-art and future perspectives. Tribol. Int. 135, 1–17 (2019)
3. San Andrés, L., Yang, J., Devitt, A.: On tilting pad carbon-graphite porous journal bearings: measurements of imbalance response and comparison to predictions of bearing performance and system dynamic response. Tribol. Trans. (2021). https://doi.org/10.1080/10402004.2021.1875091

4. Gururajan, K., Prakash, J.: Roughness effects in a narrow porous journal bearing with arbitrary porous wall thickness. Int. J. Mech. Sci. 44(5), 1003–1016 (2002)

5. Yamada, K., Onishi, N., Sekiya, K., Yamane, Y.: Permeability control method with laser for porous bushings of aero static bearings. Int. J. Precis. Eng. Manuf. 14(5), 779–784 (2013)

6. Panzera, T.H., Christoforo, A.L., Campos Rubio, J.C., Bowen, C.R., Borges, P.H.R., Silva, L.J.: Evaluation of compacted cementitious composites for porous bearings. Int. J. Appl. Ceram. Technol. 10(3), 474–483 (2013)

7. da Silva, L.J., Panzera, T.H., Vieira, L.M., Duduch, J.G., Bowen, C.R., Campos Rubio, J.C.: Carbon nanotubes and superplasticizer reinforcing cementitious composite for aero static porous bearing. Proc. Inst. Mech. Eng. Part J J. Eng. Tribol. 231(11), 1397–1407 (2017)

8. Otsu, Y., Miyatake, M., Yoshimoto, S.: Dynamic characteristics of aerostatic porous journal bearings with a surface-restricted layer. J. Tribol. 133(1), 011701 (2011)

9. Miyatake, M., Yoshimoto, S., Sato, J.: Whirling instability of a rotor supported by aerostatic porous journal bearings with a surface-restricted layer. Proc. Inst. Mech. Eng. Part J J. Eng. Tribol. 220(2), 95–103 (2006)

10. Jacob, J.S., Yu, J.J., Bently, D.E., Goldman, P.: Air-hammer instability of externally pressurized compressible-fluid bearings. In: Proc. ISICORMA-1, pp. 20–24 (2001)

11. Liu, W., Feng, K., Huo, Y., Guo, Z.: Measurements of the rotordynamic response of a rotor supported on porous type gas bearing. J. Eng. Gas Turbines Power 140(10), 102501 (2018)

12. Wang, C.C., Lo, C.Y., Chen, C.O.K.: Nonlinear dynamic analysis of a flexible rotor supported by externally pressurized porous gas journal bearings. J. Tribol. 124(3), 553–561 (2002)

13. Zhu, X., San Andrés, L.: Rotordynamic performance of flexure pivot hydrostatic gas bearings for oil-free turbomach inery. J. Eng. Gas Turbines Power 129(4), 1020–1027 (2007)

14. Wu, Y., Feng, K., Zhang, Y., Liu, W., Li, W.: Nonlinear dynamic analysis of a rotor-bearing system with porous tilting pad bearing support. Nonlinear Dyn. 94(2), 1391–1408 (2018)

15. Delgado, A., Ertas, B.: Dynamic characterization of a novel externally pressurized compliantly damped gas-lubricated bearing with hermetically sealed squeeze film damper modules. J. Eng. Gas Turbines Power 141(2), 021028 (2019)

16. Gu, Y., Ma, Y., Ren, G.: Stability and vibration characteristics of a rotor-gas foil bearings system with high-static-low-dynamic-stiffness supports. J. Sound Vib. 397(9), 152–170 (2017)

17. Ertas, B.H.: Compliant hybrid journal bearings using integral wire mesh dampers. J. Eng. Gas Turbines and Power 131(2), 022503 (2009)

18. Liu, W., Feng, K., Lyu, P.: Bifurcation and nonlinear dynamic behaviors of a metal mesh damped flexible pivot tilting pad gas bearing system. Nonlinear Dyn. 91(1), 655–677 (2018)

19. Bättig, P., Schiffermann, J.: Flexible support for gas-lubricated bearing bushings. Tribol. Trans. 63(3), 494–508 (2020)

20. Miyanaga, N., Tomioka, J.: Effect of support stiffness and damping on stability characteristics of herringbone-grooved aerodynamic journal bearings mounted on viscoelastic supports. Tribol. Int. 100, 195–203 (2016)

21. Shabaneh, N., Zu, J.W.: Nonlinear dynamic analysis of a rotor shaft system with viscoelastically supported bearings. J. Vib. Acoust. 125(3), 290–298 (2003)

22. Dutt, J.K.: A simple viscoelastic model of rotor-shaft systems. In: IUTAM Symposium on emerging trends in rotor dynamics, pp. 143–151 (2011)

23. Dousti, S., Kaplan, J.A., He, F., Allaire, P.E.: Elastomer O-rings as centering spring in squeeze film dampers: application to turbochargers. In: Turbo Expo: Power for Land, Sea, and Air, pp. V07BT30A018 (2013)

24. Waumans, T., Peirs, J., Al-Bender, F., Reynaerts, D.: Aerodynamic journal bearing with a flexible, damped support operating at 7.2 million DN. J Micromech Microeng 21(10), 104014 (2011)

25. Belforte, G., Colombo, F., Raparelli, T., Viktorov, V.: High-speed rotor with air bearings mounted on flexible supports: test bench and experimental results. J. Tribol. 132(2), 021103 (2008)

26. Belforte, G., Raparelli, T., Viktorov, V., Colombo, F.: An experimental study on a high-speed rotor supported by air bearings mounted on O-rings. Eng. Syst. Des. Anal. 42509, 995–1000 (2006)

27. Miyanaga, N., Tomioka, J.: Stability threshold of herringbone-grooved aerodynamic journal bearings with considering frequency dependence of external stiffness and damping elements. J. Jpn. Soc. Des. Eng. 51(6), 2015–2659 (2016)

28. Tomioka, J., Miyanaga, N.: Measurement of dynamic properties of O-rings and stability threshold of flexibly supported herringbone grooved aerodynamic journal bearings. Tribol. Online 3(7), 366–369 (2008)

29. Aktürk, N., Gohar, R.: Damping the vibrations of a rigid shaft supported by ball bearings by means of external elastomeric O-ring dampers. Proc. Inst. Mech. Eng. Part J J. Eng. Tribol. 208(3), 183–190 (1994)

30. Avancó, R., Tusset, A., Suetake, M., Navarro, H., Balthazar, J., Nabarrete, A.: The pendulum dynamic analysis with DC motors and generators for sea waves energy harvest.
In: DSTA2017 Vibration, Control and Stability of Dynamical Systems, pp 1–12 (2013)
31. Avanço, R.H., Tusset, A.M., Balthazar, J.M., Nabarrete, A., Navarro, H.A.: On nonlinear dynamics behavior of an electro-mechanical pendulum excited by a nonideal motor and a chaos control taking into account parametric errors. J. Braz. Soc. Mech. Sci. Eng. 40(23), 1–17 (2018)
32. Guan, H., Feng, K., Yu, K., Cao, Y., Wu, Y.: Nonlinear dynamic responses of a rigid rotor supported by active bump-type foil bearings. Nonlinear Dyn. 100, 2241–2264 (2020)
33. San Andrés, L., Ryu, K., Kim, T.H.: Identification of structural stiffness and energy dissipation parameters in a second generation foil bearing: effect of shaft temperature. J. Eng. Gas Turbines Power 133(3), 032501 (2011)

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.