COMMUTATORS AND PRONILPOTENT SUBGROUPS IN PROFINITE GROUPS

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Abstract. Let $G$ be a profinite group in which all pronilpotent subgroups generated by commutators are periodic. We prove that $G'$ is locally finite.

1. Introduction

An element of a group $G$ that can be written in the form $x^{-1}y^{-1}xy$ for suitable $x, y \in G$ is called commutator. A number of outstanding results about commutators in finite and profinite groups have been obtained in recent years. In this context we mention the proof by Liebeck, O'Brien, Shalev and Tiep [6] of Ore’s conjecture: Every element of a finite simple group is a commutator. Another significant result is due to Nikolov and Segal, who proved that if $G$ is an $m$-generated finite group, then every element of $G'$ is a product of $m$-boundedly many commutators [8]. Here, as usual, $G'$ denotes the derived subgroup of $G$, that is, the subgroup generated by all commutators. One can ask a general question – given a group $G$ with certain specific restrictions on commutators, what kind of information on the structure of $G'$ one can deduce?

Let $G$ be a finite group and $P$ a Sylow $p$-subgroup of $G$. A corollary of the Focal Subgroup Theorem [3, Theorem 7.3.4] is that $P \cap G'$ is generated by commutators. From this one immediately deduces that if all nilpotent subgroups generated by commutators have exponent dividing $e$, then the exponent of the derived group $G'$ divides $e$, too. The routine inverse limit argument allows to easily obtain a related fact for profinite groups: if $G$ is a profinite group in which all pro-$p$ subgroups generated by commutators have finite exponent dividing $e$, then the exponent of the derived group $G'$ divides $e$. Recall that a group is said to be of finite exponent $e$ if $x^e = 1$ for each $x \in G$. A profinite group is a topological group that is isomorphic to an inverse limit of finite groups. The textbooks [2] and [14] provide a good introduction to the theory of profinite groups. In the context of profinite groups all the usual concepts of group theory are interpreted topologically. In particular, by a subgroup of a profinite group we mean a closed subgroup.

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A subgroup is said to be generated by a set $S$ if it is topologically generated by $S$.

The purpose of the present paper is to prove the following theorem.

**Theorem 1.** Let $G$ be a profinite group in which all pronilpotent subgroups generated by commutators are periodic. Then $G'$ is locally finite.

Recall that a group is periodic (torsion) if every element of the group has finite order. A group is locally finite if each of its finitely generated subgroups is finite. Periodic profinite groups have received a good deal of attention in the past. In particular, using Wilson’s reduction theorem [13], Zelmanov has been able to prove local finiteness of periodic profinite groups [15]. Earlier Herfort showed that there exist only finitely many primes dividing the orders of elements of a periodic profinite group [4]. It is a long-standing problem whether any periodic profinite group has finite exponent.

2. Preliminary lemmas

We will slightly abuse terminology and use the term “prosoluble group” to mean a topological group that is isomorphic to an inverse limit of finite soluble groups.

Let us denote by $\mathcal{X}$ the class of profinite groups with the property that every pronilpotent subgroup generated by commutators is periodic. Moreover, let us denote by $\mathcal{Y}$ the class of profinite groups in which every commutator has finite order and the derived subgroup of every Sylow subgroup is periodic. It is clear that the class $\mathcal{Y}$ is closed under taking subgroups and homomorphic images. We also remark that $\mathcal{X} \subseteq \mathcal{Y}$.

If $G$ is a finite soluble group, $h(G)$ denotes the Fitting height of $G$. Recall that this is the length of the shortest normal (or characteristic) series of $G$ all of whose quotients are nilpotent. If $G$ is a prosoluble group having a finite series of characteristic subgroups all of whose quotients are pronilpotent, we denote the shortest length of such a series by $ph(G)$. If $G$ has no such series, we write $ph(G) = \infty$. Of course $ph(G) < \infty$ if and only if $G$ is an inverse limit of finite soluble groups of bounded Fitting height.

As usual, if $G$ is a finite group, $\pi(G)$ is the set of primes dividing the order of $G$. For a profinite group $G$ we denote by $\pi(G)$ the (possibly infinite) set $\pi$ of primes such that $G$ is an inverse limit of finite groups of orders divisible by primes in $\pi$ only. In what follows we use the Wilson-Zelmanov Theorem that every periodic profinite group is locally finite without explicit references.

An important role in the present paper will be played by the following two theorems, due to Wilson [13, Theorems 2 and 3]:

**Theorem 2.** Let $p$ be a prime and $G$ a profinite group whose Sylow $p$-subgroups are periodic. Then $G$ has a finite series of closed characteristic subgroups in which each factor either is pro-$(p$-soluble) or is isomorphic to a Cartesian product of non-abelian finite simple groups of order divisible by $p$. 
Theorem 3. Let $p$ be an odd prime and $G$ a pro-$(p$-soluble) group whose Sylow $p$-subgroups are periodic. Then $G$ has a finite series of closed characteristic subgroups in which each factor is either a pro-$p$ group or a pro-$p'$ group.

One immediate corollary of Theorem 3 is that if $\pi$ is a finite set of odd primes and $G$ a prosoluble group whose Sylow $p$-subgroups are periodic for every $p \in \pi$, then $G$ has a finite series of closed characteristic subgroups in which each factor is either a pro-$\pi$ group or a pro-$\pi'$ group.

Another corollary is that if $G$ is a prosoluble group for which $\pi(G)$ is finite and all of whose Sylow subgroups are periodic, then $\text{ph}(G)$ is finite.

Lemma 4. Let $G \in X$. Then all Sylow $p$-subgroups of $G'$ are periodic.

Proof. If $p$ is a prime, a Sylow $p$-subgroup of $G'$ is of the form $P \cap G'$, for some Sylow $p$-subgroup $P$ of $G$. A profinite version of the Focal Subgroup Theorem (see Theorem 4.4 of [2]) implies that $P \cap G'$ is generated by commutators. Since $G \in X$, it follows that $P \cap G'$ is periodic. □

Lemma 5. Let $G$ be a profinite group all of whose Sylow subgroups are periodic. If $\pi(G)$ is finite, then $G$ is locally finite.

Proof. It is sufficient to show that $G$ is periodic. Choose $x \in G$ and consider the procyclic subgroup $\langle x \rangle$ generated by $x$. Write $\langle x \rangle = S_1S_2\ldots S_n$, where $S_i$ are the Sylow subgroups of $\langle x \rangle$. Since all the subgroups $S_i$ are finite cyclic and since there are only finitely many primes in $\pi(G)$, we conclude that $\langle x \rangle$ is finite. This completes the proof. □

Lemma 6. Let $G$ be a finite soluble group whose Hall $2'$-subgroups are abelian. Then $h(G) \leq 3$.

Proof. It is sufficient to show that if $G$ has no non-trivial normal 2-subgroups, then a Hall 2'-subgroup of $G$ is normal. So assume that $G$ has no non-trivial normal 2-subgroups and choose a Hall 2'-subgroup $H$ in $G$. Let $F$ be the Fitting subgroup of $G$. Then $F$ is a 2'-subgroup and therefore $F \leq H$. Since $C_G(F) \leq F$ (Theorem 6.1.3] and since $H$ is abelian, we conclude that $F = H$. The lemma follows. □

Let $G$ be a finite group acted on by a finite group $A$ such that $(|G|,|A|) = 1$. It is well-known that if $N$ is a normal $A$-invariant subgroup of $G$, then $C_{G/N}(A) = C_G(A)N/N$ (see for example [3] 6.2.2]). A profinite version of this result can be stated as follows.

Let $G$ be a profinite group on which a finite group $A$ acts by continuous automorphisms. Suppose that $\pi(G) \cap \pi(A) = \emptyset$. If $N$ is a closed normal $A$-invariant subgroup of $G$, then $C_{G/N}(A) = C_G(A)N/N$. A proof of this can be found in [10] (see also Lemma 2 in [4] for the case where $|A|$ is a prime).

In his seminal work [11] Thompson showed that if $G$ is a finite soluble group acted coprimely by a finite group $A$, then $h(G)$ is bounded in terms of
h(C_G(A)) and the number of prime divisors of |A|, counting multiplicities. A survey of further results in this direction can be found in Turull [12]. We will require a profinite version of Thompson’s theorem. It can be deduced by the standard inverse limit argument using the above mentioned property that if N is a closed normal A-invariant subgroup of G, then C_G/N(A) = C_G(A)N/N. Therefore we omit the proof.

**Proposition 7.** Let G be a prosoluble group acted on by a finite group A and suppose that π(G) and π(A) are disjoint. If C_G(A) has a finite normal series with pronilpotent quotients, then also G has such a series.

**Lemma 8.** Let G be a non-abelian prosoluble group all of whose Sylow subgroups are periodic. Then there exists a subgroup of G which is non-abelian and finite.

**Proof.** Since G is non-abelian, we can choose two not necessarily distinct primes p and q such that a Hall \( \{p, q\} \)-subgroup M is non-abelian. Since by Lemma 5 M is locally finite, the result follows. □

3. **Proof of the Main Theorem**

**Proposition 9.** Let G be a prosoluble group all of whose Sylow subgroups are periodic and assume that every commutator in G has finite order. Then \( ph(G) \) is finite.

**Proof.** Assume that \( ph(G) \) is infinite. In what follows we will inductively construct a sequence of subgroups \( A_1, A_2, \ldots \), an increasing chain of normal closed subgroups

\[
1 = R_1 \leq R_2 \leq \ldots,
\]

and a decreasing chain of normal open subgroups

\[
G = H_1 \geq H_2 \geq \ldots \geq H = \cap H_i
\]

such that

1. there exist mutually disjoint sets of odd primes \( \pi_i \) with the property that the image of \( A_i \) in \( G/R_i \) is a finite non-abelian \( \pi_i \)-group for every \( i \);
2. \( [A_i, A_j] \leq R_j \) whenever \( i < j \);
3. \( ph(G/R_i) = \infty \) for every \( i \);
4. \( A_i \cap H_{i+1} = R_i \).

Later we will see that the above properties lead to a contradiction.

Since \( ph(G) \) is infinite, Lemma 6 implies that the Hall \( 2' \)-subgroups of G are non-abelian. By Lemma 8 we can find a non-abelian finite subgroup \( A_1 \) of odd order. Let \( \pi_1 = \pi(A_1) \) and set \( R_1 = 1 \). Next, choose a normal open subgroup \( H_2 \) in G such that \( A_1 \cap H_2 = 1 \). As \( \pi_1 \) is a set of odd primes it follows from Theorem 8 that \( H_2 \) has a finite series of closed characteristic subgroups \( \{N_i\} \) in which each quotient \( N_i/N_{i+1} \) is either a pro-\( \pi_1 \) or a pro-\( \pi_1' \) group. Since \( \pi_1 \) is finite, \( ph(N_i/N_{i+1}) \) is finite whenever the quotient
$N_i/N_{i+1}$ is pro-$\pi_1$. Since $ph(G) = \infty$, we have $ph(H_2) = \infty$. Therefore there exists a pro-$\pi'_1$ quotient $N_i/N_{i+1}$ such that $ph(N_i/N_{i+1}) = \infty$. Set $R_2 = N_{i+1}$. We will now pass to the quotient $G/R_2$.

The finite $\pi_1$-group $A_1$ acts in the natural way on the $\pi'_1$-group $N_i/R_2$. By Proposition 7 $ph(C_{N_i/R_2}(A_1)) = \infty$. Lemma 8 shows that Hall 2'-subgroups in $C_{N_i/R_2}(A_1)$ are non-abelian. Thus, by Lemma 8 there exists a subgroup $A_2$ such that $A_2/R_2$ is a finite non-abelian subgroup of odd order in $C_{N_i/R_2}(A_1)$. Set $\pi_2 = \pi(A_2/R_2)$. We note that $\pi_1 \cap \pi_2 = \emptyset$ and $[A_1, A_2] \leq R_2$.

We now repeat the above arguments in the following way. Assume by induction that the subgroups $A_1, \ldots, A_{n-1}$ and the normal subgroups $R_1, \ldots, R_{n-1}$ and $H_1, \ldots, H_{n-1}$ with the prescribed properties have been found. For brevity we indicate with an overline the elements (or subgroups) modulo the subgroup $R_{n-1}$. Then the $\overline{A}_1, \ldots, \overline{A}_{n-1}$ are pairwise commuting finite subgroups of mutually coprime orders. In particular, $\overline{A} = \overline{A}_1 \cdots \overline{A}_{n-1}$ is a finite $\pi$-group, where $\pi = \cup_{i=1}^{n-1} \pi_i$. Let $J$ be a normal open subgroup of $G$ with the properties that $R_{n-1} \leq J$ and $J \cap \overline{A} = 1$. Let $H_n = H_{n-1} \cap J$. Note that $\pi$ consists of odd primes, so by Theorem 2 the subgroup $\overline{H}_n$ has a finite series of closed characteristic subgroups $\{N_i\}$ in which each quotient $\overline{N_i}/\overline{N}_{i+1}$ is either a pro-$\pi$ or a pro-$\pi'$ group. Since $\pi$ is finite, $ph(\overline{N}_i/\overline{N}_{i+1})$ is finite for each pro-$\pi$ quotient $\overline{N}_i/\overline{N}_{i+1}$. Taking into account that $ph(\overline{G}) = \infty$, we remark that also $ph(\overline{H}_n) = \infty$. Therefore there exists a pro-$\pi'$ quotient $T_n = \overline{N}_i/\overline{N}_{i+1}$ for some index $i$, such that $ph(T_n) = \infty$. Let $R_n/R_{n-1} = \overline{N}_{i+1}$.

The finite $\pi$-group $\overline{A}$ naturally acts on the $\pi'$-group $T_n$. By Proposition 7 $ph(C_{T_n}(\overline{A})) = \infty$. Lemma 6 shows that Hall 2'-subgroups in $C_{T_n}(\overline{A})$ are non-abelian. We therefore can use Lemma 8 and find a subgroup $A_n$ such that

(a) $A_n/R_n \leq C_{T_n}(\overline{A})$
(b) $A_n/R_n$ is a finite non-abelian $\pi'$-group of odd order.

It is now easy to see that the required properties are satisfied by $A_1, \ldots, A_n$, $R_1, \ldots, R_n$ and $H_1, \ldots, H_n$. This completes the inductive step. Hence we now assume that the non-abelian finite subgroups $A_i$, an increasing chain of normal closed subgroups $1 = R_1 \leq R_2 \leq \ldots$, and a decreasing chain of normal open subgroups

$$G = H_1 \geq H_2 \geq \ldots \geq H = \cap H_i$$

with the prescribed properties have been found.

Remark that $H$ contains $\cup_{n=1}^{\infty} R_n$. Since $H$ is closed, we can consider the quotient $G/H$ of $G$. The image in $G/H$ of the closed subgroup generated by all $A_i$ is isomorphic with the Cartesian product of finite non-abelian groups of mutually coprime orders. Obviously this group contains a commutator of infinite order. This yields a contradiction since all commutators in $G$ have finite order. The proof is now complete.

\[ \square \]
Lemma 10. Let $N$ be an abelian normal subgroup of a group $G \in \mathcal{Y}$. Then there exists an open normal subgroup $H$ of $G$ such that $[N,H]$ is locally finite.

Proof. Let $a$ be an element of $G$. As $N$ is abelian, the subgroup $[N,a]$ coincides with the set of commutators $\{[n,a] | n \in N\}$, and therefore it is a (closed) periodic group. In particular $[N,a]$ has finite exponent (see e.g. [9, Lemma 4.3.7]).

For every integer $m$ consider

$$S_m = \{a \in G | [N,a]^m = 1\}.$$ 

Note that the sets $S_m$ are closed and they cover $G$. By Baire’s Category Theorem [5, p. 200] there exist an integer $m$, an open normal subgroup $H \leq G$ and an element $a \in G$ such that $[N,aH]^m = 1$.

Since $[N,a]^G \leq N$ is abelian and generated by elements of orders dividing $m$, it has exponent dividing $m$. In particular $[N,a]^G$ is locally finite. We can now pass to the quotient $G/[N,a]^G$. Without loss of generality we assume that $[N,a] = 1$. For every $n \in N$ and $h \in H$ we have $[n,ah] = [n,h]$. Therefore $[N,H]$ is generated by elements of orders dividing $m$. Thus $[N,H]$, being abelian, has exponent dividing $m$, and so it is locally finite, as required. □

Lemma 11. Assume that $B$ is a profinite group in which every commutator has finite order, and let $A$ be an open normal abelian subgroup of $B$. Then $[A,B]$ is locally finite.

Proof. Let $b_1, \ldots, b_k$ be a set of representatives of the left cosets of $A$ in $B$. Then

$$[A,B] = \prod_{i=1}^k [A,b_i].$$

By the same argument as in Lemma 10 each $[A,b_i]$ is locally finite, thus $[A,B]$ is also locally finite. □

Proposition 12. Let $G \in \mathcal{Y}$ be prosoluble and assume that $\text{ph}(G)$ finite. Then $G'$ is locally finite.

Proof. Recall that each subgroup and each quotient of $G$ is in $\mathcal{Y}$. We prove the proposition by induction on $h = \text{ph}(G)$. If $h = 1$, then $G$ is pronilpotent and $G'$ is the Cartesian product of the derived subgroups $G'_p$ of the Sylow subgroups $G_p$ of $G$. As $G \in \mathcal{Y}$, each $G'_p$ is locally finite. Moreover $\pi(G')$ is finite, because otherwise $G$ would contain a commutator with infinite order, a contradiction. Therefore $G'$ is locally finite, as required.

Assume now that $h \geq 2$. The group $G$ has a characteristic series with pronilpotent quotients of length $h$. Let $N \leq K$ be the last two terms of the series. Thus, $K/N$ and $N$ are pronilpotent. Since $N$ is pronilpotent, our
argument on the case \( h = 1 \) yields that \( N' \) is locally finite. Factoring \( N' \) out, we can assume that \( N \) is abelian. By Lemma 10 there exists an open normal subgroup \( H \) of \( K \) such that \([N, H]\) is locally finite. Clearly we can assume that \( H \) is normal in \( G \) and pass to the quotient \( G/[N, H] \). Thus, we can assume that \([N, H] = 1\).

Then \( N \cap H \) is contained in the centre of \( H \). As \( H/N \cap H \cong HN/N \leq K/N \) is pronilpotent, we conclude that \( H \) is pronilpotent. Since in the case where \( h = 1 \) the lemma holds, we conclude that \( H' \) is locally finite. We now pass to the quotient \( G/H' \) and assume that \( H' = 1 \).

In this case \( H \) is an abelian subgroup of finite index in \( K \). By Lemma 11 \([H, K]\) is locally finite. Factoring \([H, K]\) out, we obtain that \( H \) is contained in the centre of \( K \). Hence \( K \) is nilpotent. By induction on \( h \) the proposition follows.

**Lemma 13.** Let \( H \in Y \) be a Cartesian product of non-abelian finite simple groups. Then \( H \) has finite exponent.

**Proof.** Let \( H \) be a Cartesian product of the non-abelian finite simple groups \( S_i, i \in I \). If \( H \) has infinite exponent, then for each \( n \in \mathbb{N} \) there exists an index \( i_n \) and an element \( s_{i_n} \in S_{i_n} \) such that \( |s_{i_n}| > n \). By the positive solution of Ore’s Conjecture [6] each \( s_{i_n} \) is a commutator, say \( s_{i_n} = [x_{i_n}, y_{i_n}] \). Then the element

\[
\left[ \prod_{n \in \mathbb{N}} x_{i_n}, \prod_{n \in \mathbb{N}} y_{i_n} \right] = \prod_{n \in \mathbb{N}} [x_{i_n}, y_{i_n}]
\]

has infinite order, a contradiction. \( \square \)

We can now complete the proof of Theorem 4.

**Proof of Theorem 4.** Recall that we wish to prove that \( G' \) is locally finite whenever \( G \in X \). By Lemma 4 for every group \( G \in X \) all Sylow subgroups of \( G' \) are periodic. The theorem will follow once it is shown that

\((\ast)\) If \( G \in Y \) and all Sylow subgroups of \( G' \) are periodic, then \( G' \) is locally finite.

If \( 2 \notin \pi(G') \), then \( G' \) is prosoluble [1] and so is \( G \). Thus, by Proposition 3 \( ph(G') \), and hence also \( ph(G) \), are finite. In this case by Proposition 12 \( G' \) is locally finite, as required.

So, assume that \( 2 \in \pi(G') \). By Theorem 2 \( G' \) has a finite series of closed characteristic subgroups in which each factor either is pro-\((2\text{-soluble})\) or is isomorphic to a Cartesian product of non-abelian finite simple groups. Since finite \( 2' \)-groups are soluble, \( G \) has a finite series of closed characteristic subgroups

\[ 1 = G_{s+1} < G_s < \ldots < G_1 < G_0 = G \]

in which each factor either is prosoluble or is isomorphic to a Cartesian product of non-abelian finite simple groups; moreover we can assume \( G_1 \leq G' \).
We will prove (*) by induction on the minimal number $n$ of non prosoluble factors in this series. If $n = 0$, then $G$ is prosoluble, and, as above, the theorem follows from Propositions 5 and 12.

So let $n \geq 1$. If the last term $G_s$ is isomorphic to a Cartesian product of non-abelian finite simple groups, then by Lemma 13 it is locally finite. Since the corresponding series of $G/G_s$ has $n - 1$ non-prosoluble quotients, the result follows by induction.

Therefore we assume that $G_s$ is prosoluble. Since $G_s \leq G'$, all Sylow subgroups of $G_s$ are periodic. Hence, by Propositions 9 and 12 the derived subgroup $G_s'$ is locally finite. Passing to the quotient $G/G_s'$, we can assume that $G_s$ abelian. Then Lemma 14 states that there exists an open normal subgroup $H$ of $G$ such that $[G_s, H]$ is locally finite. Passing to the quotient $G/[G_s, H]$, we can assume that $G_s \leq Z(H)$.

Since $G_{s-1}/G_s$ is a Cartesian product of non-abelian finite simple groups, by Lemma 13 $G_{s-1}/G_s$ has finite exponent. Set $K = H \cap G_{s-1}$. Then $Z(K) \geq Z(H) \cap G_{s-1} \geq G_s$. It follows that $K/Z(K)$ has finite exponent. A theorem of Mann states that if $B$ is a finite group such that $B/Z(B)$ has exponent $e$, then the exponent of $B'$ is bounded by a function of $e$ [7]. Applying a profinite version of this theorem we deduce that the exponent of $K'$ is finite. Thus $K'$ is locally finite. Passing to the quotient $G/K'$, we can assume that $K$ is abelian.

Since $|G : H|$ is finite, $K$ has finite index in $G_{s-1}$. By Lemma 11 the subgroup $[K, G_{s-1}]$ is locally finite. Passing to the quotient $G/[K, G_{s-1}]$ we assume that $K \leq Z(G_{s-1})$. Again, by the profinite version of Mann’s theorem, we conclude that $G_{s-1}'$ is locally finite. Then we apply the inductive hypothesis to $G/G_{s-1}'$ and conclude that $G'$ is locally finite. The proof is complete. □

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