Superelement simulation of dynamic characteristics of large-size combined systems «Foundation – reinforced concrete structures – metal structures»

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Abstract. The article presents the effectiveness description, verification, approbation and evaluation of the superelement technique for simulation the dynamic characteristics of large-sized combined systems “foundation - reinforced concrete structures - metal structures”.

Introduction
The key feature of the mathematical simulation of static and dynamic stress-strain state (SSS) of unique construction objects: the development and design optimization of connected large-sized basic subsystems "foundation", "reinforced concrete structures" and "metal cover structures" are independently held by different project design organizations. The combined building systems dimension can reach hundreds thousands of different types of structural elements and accordingly, tens of millions of freedom degrees in their finite element models. Such organizations are not able to build an adequate design model of the complete system, for example modern football stadiums. Various factors are obstacles to this, for example from the incompatibility of the file formats of computational models in various software packages and the large computational dimension of such models to commercial secrets.

To solve this problem it is necessary to develop the methodology for mathematical simulation, which makes it possible to substantiate the transition possibility to performing calculations within the framework of individual subsystem models (the “organizational” aspect) and to reduce the computational dimension of the problem.

The superelement method is a promising approach to solve this problem. Static [1] and dynamic superelement methods [2, 3] are widely used in the aerospace industry, mechanical engineering and the nuclear industry but are not adequately represented in civil engineering.

Superelement technique for simulation dynamic characteristics of large-size combined systems
Two approaches are possible to realize the research subsystems possibility within the separate models framework. In the first case comparing the dynamic characteristics of the complete system and
subsystems, evaluate their mutual influence and thereby justify the transition possibility to individual models. In the second universal case to use the super element technology. With the superelement approach, each organization develops the FE-model of its own subsystem forms a superelement (SE), which is a set of influence matrices. Further, the accountants’ teams exchange these super elements and join their “own” FE-model with the super element developed and formed by their colleagues. This ensures the transition from the complete system study to individual subsystems with the correct accounting for dynamic characteristics of the subsystems developed by subcontractors.

The component mode synthesis is used to calculate dynamic characteristics in this work. The essence of the component mode synthesis consists in transition from the full set of physical freedom degrees to the reduced set of generalized coordinates, i.e. to represent the substructure displacements, the Rayleigh-Ritz procedure is used where the displacement is represented as a superposition of basis vectors (coupled modes). The fixed-interface method and the free-interface method are considered as two variants of the component mode synthesis, differing in the limiting technique of the coupling freedom degrees. Connecting or interface freedom degrees are those that are used to dock the superelement with the FE model of the subsystem or other superelements.

Stiffness and mass matrices determine each substructure in the component mode synthesis.

\[
[M] [\ddot{u}] + [K] [u] = [F].
\] (1)

Partitioning the matrix equation into interface and interior freedom degrees (DOFs):

\[
\{u\} = \begin{bmatrix} u_m \\ u_s \end{bmatrix}, \quad [M] = \begin{bmatrix} M_{mm} & M_{ms} \\ M_{sm} & M_{ss} \end{bmatrix}, \quad [K] = \begin{bmatrix} K_{mm} & K_{ms} \\ K_{sm} & K_{ss} \end{bmatrix}, \quad [F] = \begin{bmatrix} F_m \\ F_s \end{bmatrix},
\] (2)

where subscripts \( m \) and \( s \) refer to: \( m \) – master DOFs defined only on interface nodes, \( s \) – all DOFs that are not master DOFs. The nodal displacement vector may be represented in terms of master DOFs completed by component-generalized coordinates:

\[
\{u\} = \begin{bmatrix} u_m \\ u_s \end{bmatrix} = [T]\begin{bmatrix} y_m \end{bmatrix},
\] (3)

where: \( y_m \) is a truncated set of generalized modal coordinates, \( [T] \) is a transformation matrix.

For the fixed-interface method, also commonly referred to as the Craig-Bampton method [4], the transformation matrix has the form:

\[
[T] = \begin{bmatrix} [I] & [0] \\ [G_{sm}] & [\Phi_s] \end{bmatrix},
\] (4)

where: \( G_{sm} = -[K_{ss}]^{-1}[K_{sm}] \), \( [\Phi_s] \) is the fixed-interface normal modes (eigenvectors obtained with interface nodes fixed).

For the free-interface method, also commonly referred to as the Herting method [5, 6], the transformation matrix has the form:
$$[T] = \begin{bmatrix} [I] & 0 & 0 \\ [G_{sm}] & [\Phi_{sr}] & [\Phi_{sr}] \end{bmatrix},$$  

(5)

where: $[\hat{\Phi}_s] = [[\Phi_s] - [G_{sm}][\Phi_m]]$, $[\Phi_m]$ is the master DOF partition matrix of free-interface normal modes (eigenvectors obtained with the interface DOFs free), $[\Phi_s]$ is a matrix of the slave DOF partition of the free-interface normal modes, $[\Phi_{sr}]$ is a matrix of inertia relief modes, $[\Phi_{sr}]$ is included only if rigid body modes are present. Any rigid body modes present are not included in $[\hat{\Phi}_s]$, $[\Phi_{sr}] = [K_{ss}]^{-1} [[M_{sm}] + [M_{ss}][G_{sm}]] [\Psi_{ms}]$, where $[\Psi_{ms}]$ is a matrix of the master DOF partition of the rigid body modes.

After applying the transformation in (refer with Eq.3) into the matrix motion equation (refer with Eq.1), the motion equation in the reduced space is obtained.

Verification and approbation studies were carried out on the basis of the universal software package ANSYS Mechanical, which implements superelement algorithms that are widely used in the developed technique.

**Verification**

Real object - the input block of the Volgamoll shopping center in the city of Volzhsky, which has a construction similar to the type of large-sized systems under investigation is considered as an example. This example analyzes the possibility of transition to calculate dynamic characteristics of subsystems within individual models; the possibilities and features of application of the substructures dynamic synthesis technique to combined systems of this kind. The influence choice of methods for record keeping of internal forms in substructure vibrations and number of internal natural frequencies taken into account and the substructure vibrations modes, the effectiveness evaluation are analyzed.

The following spatial shell-beam FE-models of supporting structures of the input block (refer with Figure 1) were built and verified: the complete system “reinforced concrete structures - metal structures”, subsystems and subsystems with superelements.

![FE-models of supporting structures of the input block.](image)

*Figure 1. FE-models of supporting structures of the input block.*

- a) «reinforced concrete frame structures - metal structures of the coatings»;
- b) «reinforced concrete frame structures»;
- c) «metal structures of the coating»

Finite elements SHELL181, implementing the theory of Mindlin-Reissner are used for modeling the foundation, walls, floor slabs, stair. BEAM188 are the finite elements based on Timoshenko beam theory used for modeling beams and columns. MATRIX50 (superelement / substructure) is a group of pre-assembled finite elements that is considered as a separate element and is represented by reduced matrices (stiffness, masses, loads).

Comparative analysis of calculated dynamic characteristics of the complete system and subsystems showed a significant effect of the supporting subsystem compliance on the behavior of the coating.
structures subsystem. This indicates the impossibility of carrying out subsystems independent calculations.

Comparison of the natural frequencies and modes of the complete system simulated head-on with finite elements and subsystems with superelements shows almost identical results. The difference in natural frequencies when accounting for a sufficient number of internal substructures modes for the fixed interface method was basically no more than 0.007\%, and for individual forms up to 0.295\%. For the free interface method - basically, no more than 0.005\%, and for individual vibration modes - up to 0.038\%.

For this type structures it is necessary to take into account the substructure internal modes in the frequency range 1.5–2 times the frequency range under study of the entire system.

For the “small” dimension problem considered the computational efficiency factor, as shown by the comparative analysis with full FE-models is not expectedly the strength of the developed superelement technique.

**Approbation**

Large-capacity stadium designed for the 2018 World Cup in Rostov-on-Don was the object for the technique approbation. Spatial shell-beam finite-element and superelement models of the stadium supporting structures were built and verified: the complete system model (1), subsystems models (2–4) and subsystems models of with superelements (5, 6) (refer with Table 1).

Direct block Lanczos method was used to calculate the significant part of the natural frequencies spectrum and mode shapes. The calculations were carried out taking into account the masses of only own weight of supporting structures. For each system and subsystem, the natural frequencies and mode shapes were calculated in the range from 0 to 6 Hz. In calculations with the superelements use, the substructure internal mode shapes were taken into account using both the fixed interface and free interface methods. The internal frequencies in the range up to 12 Hz were taken into account.

Comparative analysis of the natural frequencies and mode shapes of the complete system and subsystems of the stadium in Rostov-on-Don showed a significant mutual influence of the reference subsystem and the coating structures subsystem on dynamic characteristics. This indicates the impossibility of carrying out subsystems independent calculations without taking into account the stiffness and mass characteristics of all elements of the complete system.

Table 1 shows the natural frequency and modes of the studied models. Green background indicates the natural frequencies of the complete model and the submodels where the modes match. Orange background indicates those natural frequencies of “non-superelement” models 2, 3 and 4, where the modes do not correspond to the modes of the full model or are absent. In Table 2 data are arranged in such a way that natural frequencies in a single line are close in value and have similar or coinciding mode shapes.

**Table 1.** Comparison of natural frequencies and vibration modes of the complete FE-model «basis - reinforced concrete constructions of foundations and stands - metal structures of the roof» (model 1) of stadium in Rostov-on-Don, subsystem FE-models (models 2-4), subsystem SE-models (models 5-6).
Similar comparison of the natural frequencies and mode shapes of complete systems and subsystems with superelements shows that calculations of dynamic characteristics using the component mode synthesis give results similar to those obtained in the calculation of the complete system. Discrepancies in the values of the calculated natural frequencies for most mode shapes do not exceed 0.050%, and for individual modes in the studied range up to 0.900%.

Comparison of calculations results of dynamic characteristics that are calculated by various variants of the component mode synthesis (refer with Table 2) allows formulating the following recommendations: it is preferable to use the free-interface method for the subsystem "foundation - reinforced concrete foundation structures and stands"; fixed-interface method. It is with this approach that the minimum divergence of the nature frequencies and mode shapes of the complete system and subsystems is achieved using superelements. The recommendation was also confirmed to retain for the substructure all of its own mode shapes, which frequencies are 1.5–2 times higher than the frequency range under investigation for the entire system.

Table 2. Comparison of natural frequencies and mode shapes of the complete system (refer with model 1) and subsystems with superelements (refer with models 5 and 6) of the stadium in Rostov-on-Don.
The total “machine” time for the superelement formation and the dynamic characteristics calculation of the subsystem with its application, taking into account a sufficient number of internal vibration shapes of the substructure is comparable to the time spent on the FE calculation of the complete system. The time for calculating the “metal structures of the coating” subsystem, taking into account the “foundation – reinforced concrete structures” superelement (without taking into account the machine time spent on the superelement formation) is 15-30 times less than the total system spent on the calculation (refer with Figure 2). This gives a significant gain in computational efficiency when conducting multivariate computational studies of the subsystem “metal structures of the coating” in order to optimize it, since there is no need to regenerate the superelement.

**Figure 2.** Comparison of the computation time of the dynamic characteristics in the complete system and subsystems with superelements of the stadium in Rostov-on-Don.

| Model 1 | Model 5 | Model 6 |
|---------|---------|---------|
| Ne 350  | mode shape/frequency | Ne 350  | mode shape/frequency | Ne 350  | mode shape/frequency |
| $f_{350} = 5.9914$ Hz | | $f_{350} = 5.9915$ Hz | | $f_{350} = 5.9915$ Hz | | $\Delta_1, \%$ 0.002 |
| $\Delta_2, \%$ 0.002 | | | | | |

Frequency range up to 6 Hz (350 natural frequencies and modes)
Summary

- Efficient superelement technique for mathematical simulation of the dynamic characteristics of large-size systems “foundation - reinforced concrete structures - metal structures of the coating” was developed and programmed, which allows proceeding to the study of the subsystems “foundation”, “reinforced concrete structures” and “metal structures of the coating” within individual models.
- Verification of the technique was carried out, the effectiveness and peculiarities of using the developed superelement methods for studying the dynamic characteristics of the combined systems of a similar type were demonstrated.
- Approbation of the technique was carried out on examples of real large-size combined systems “foundation - reinforced concrete structures - metal structures” - the large-capacity football stadium in Rostov-on-Don.
  - The most valuable, “organizational” efficiency of the developed technique was demonstrated for a real combined large-sized system “foundation – reinforced concrete foundation structures and stands – metal coating structures” using superelement approaches. The computational competitiveness of the developed models with superelements was confirmed (as compared with the full FE models).
- The presented research results allow us to recommend the developed superelement technique for use for a wide class of computational studies of combined large-sized systems of unique buildings and structures.
- The prospect of the further development of this topic is the development and application of the proposed superelement methodology for mathematical modeling of combined large-sized systems for solving problems in physically, geometrically, structurally and genetically non-linear modes.

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