Ramon-Ramond (boundary) states

M. Billó
Instituut voor theoreetische fysica, K.U. Leuven, Celestijnenlaan 200D, B3001 Leuven, Belgium
Marco.Billo@fys.kuleuven.ac.be

R. Russo
Institut de Physique, Rue A.-L. Breguet 1 2000 Neuchâtel, Switzerland
Rodolfo.Russo@iph.unine.ch

Abstract

The description of D-branes as boundary states for type II string theories (in the covariant formulation) requires particular care in the R-R sector. Also the vertices for R-R potentials that can couple to D-branes need a careful handling. As an illustration of this, the example of the D0-D8 system is reviewed, where a “microscopic” description of the interaction via exchange of R-R potentials becomes possible.

1 Covariant superstring boundary states

D-branes can be incorporated in type II superstring theories by means of “boundary states” (b.s.) [1]. Such a state represents the insertion (at $\tau = 0$) of a boundary in the string world-sheet which lies within the D-brane world-volume from the target-space point of view. The b.s. formalism has been efficiently used to compute the interactions between moving D-branes [2] even in the presence of external fields [3]. Here, following [4], we will focus on some subtle aspects of the covariant b.s., which are relevant for the (anomalous) R-R couplings of D-branes and for “non-trivial” systems, like for instance the D0-D8 one [5, 6].

The boundary state $|B\rangle$ satisfies conditions that correspond, in the closed string channel, to the boundary conditions for open strings ending on the D-brane. In the simple case of a flat D-brane located at $y_i = 0$, the open string Neumann (Dirichlet) b.c.’s in the world-volume (transverse) coordinates $X^\alpha$ ($X^i$) translate into the conditions $\partial_\tau X^\alpha|_{\tau=0}|B_X\rangle = X^i|_{\tau=0}|B_X\rangle = 0$. Expanding the closed string fields into oscillators, these conditions become $(a_n^\mu + S_{\nu}^\mu \tilde{a}_n^\nu)|B_X\rangle = 0$ for $n \neq 0$ and $\hat{p}^a|B_X\rangle = \hat{q}^i|B_X\rangle = 0$ for the zero modes (we have introduced $S_{\mu \nu} = (\eta_{\alpha \beta}, -\delta_{ij})$). The solution to these conditions reads

$$|B_X\rangle = \delta^{d+}(\hat{q}) e^{-\sum_{n=1}^{\infty} a_n^\dagger S_{\mu \nu} \tilde{a}_n^\dagger |0\rangle}.$$ (1)

Analogous “overlap” conditions, i.e. $(\psi - i \eta S^A \tilde{\psi})|B_\psi, \eta\rangle = 0$ with $\eta = \pm 1$, arise for the fermionic fields $\psi^\mu, \tilde{\psi}^\mu$ fields whose mode expansion depends on which sector (Ramond or Neveu-Schwarz)

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2 See also the talks [7] of M. Bertolini and M. Serone at this conference, and references therein, for other applications of the b.s. formalism.
one considers. Actually, the GSO projection requires a combination of the two cases \( \eta = \pm 1 \), as we will see. Imposing the BRST invariance of the b.s., \((Q + \bar{Q})|B\rangle = 0\), conditions for its ghost and superghost parts are obtained. Thus finally one determines explicitly the expression

\[
|B, \eta\rangle_{\text{NS,R}} = \frac{T_p}{2} |B_X\rangle |B_{gh}\rangle |B_\psi, \eta\rangle_{\text{NS,R}} |B_{sgh}, \eta\rangle_{\text{NS,R}}.
\]

(2)

The D-brane tension, \( T_p = \sqrt{(2\pi\alpha')^{3-p} \cdot \text{fixed factorizing amplitudes involving boundary states} \}) Each term in Eq. (2) is expressed, like \(|B_X\rangle\) in Eq. (1), by a coherent-state-type exponential of non-zero-modes and a zero-mode part. In the NS sector, where there are no \( \psi \) zero-modes, one has \(|B_\psi, \eta\rangle_{\text{NS}} = \exp(i\eta \sum_{r=1/2}^{\infty} \bar{\psi}^I \cdot S \cdot \hat{\psi}^I)|0\rangle\). In the following we will focus on the fermion and superghost zero-mode part of the b.s. in the R-R sector. For complete expressions of the covariant b.s. see e.g. [3].

1.1 The Ramond-Ramond sector

In the R sector, the \(\psi\) field is integer moded and the anti-commutations \(\{\psi^\alpha, \bar{\psi}^\beta\} = \eta^{\mu\nu}\) imply that these zero-modes can be represented as \(\Gamma^\mu\) matrices, acting on a vacuum that carries spinor indices: \(|A\rangle|\bar{B}\rangle\). The zero-mode part of the b.s., satisfying the correct overlap conditions, turns out to be

\[
|B_\psi, \eta\rangle_{\text{R}}^{(0)} = \mathcal{M}(\eta)_{A|B]|A\rangle|\bar{B}\rangle, \quad \text{with} \quad \mathcal{M}(\eta) = C \prod_{\alpha} \Gamma^\alpha \left( \frac{1 + i\eta\Gamma_{11}}{1 + i\eta} \right),
\]

where \(C\) is the charge conjugation matrix. For the superghost systems it is necessary to choose a picture \((P, \bar{P})\); we consider thus a superghost vacuum \(|P, \bar{P}\rangle\) such that (e.g. in the l.n. sector) \(\gamma_m|P\rangle = 0\) for \(m > P + 1/2\) and \(\beta_m|P\rangle = 0\) for \(m > -P - 3/2\).

Insisting that the b.s. inserts a boundary in the world-sheet, thus turning e.g. a closed string sphere amplitude into an open string disk amplitude, implies that \(P + \bar{P} = -2\), due to the sphere and disk superghost anomaly [4]. Due to the moding of the superghost fields, \(P\) is integer in the NS and half-integer in the R sector. Thus, the boundary state \(|B\rangle_{\text{R}}\) has always \(P \neq \bar{P}\), and can couple only to R-R states in a left-right asymmetric picture. This point is crucial for our discussion. Let us choose for definiteness \(P = -1/2, \bar{P} = -3/2\); then the superghost zero-mode part of the b.s. is

\[
|B_{sgh}, \eta\rangle_{\text{R}}^{(0)} = \exp \left[ i\eta \tilde{\gamma}_0 \tilde{\beta}_0 \right] |P = -1/2, \bar{P} = -3/2\rangle.
\]

(4)

We need to perform on the b.s. the relevant type IIA or B GSO projection, obtaining

\[
|B\rangle_{\text{R}} = \frac{1 + (-1)^p(-1)^{F+G}}{2} |B, +\rangle_{\text{R}} = \frac{1}{2} \left( |B, +\rangle_{\text{R}} + |B, -\rangle_{\text{R}} \right),
\]

(5)

where \(p\) is even for Type IIA and odd for Type IIB. Notice that the fermion and superghost number operators reduce in the zero-mode sector as follows: \((-1)^F \to \Gamma_{11} \text{ and } G \to -\gamma_0 \tilde{\beta}_0\).

For later convenience we now rewrite the boundary state \(|B\rangle_{\text{R}}\) using 16-dimensional chiral and antichiral spinor indices \(\alpha\) and \(\dot{\alpha}\) for Majorana-Weyl fermions. Then, for the Type IIA theory we have

\[
|B\rangle_{\text{R}} = \frac{T_p}{2} |B_X\rangle |B_{gh}\rangle \left\{ (C\tilde{T}_0 \Gamma^I \ldots \Gamma^r)_{\alpha\beta} \cos \left[ \gamma_0 \tilde{\beta}_0 + \Theta \right] |\alpha\rangle_{-1/2} |\beta\rangle_{-3/2} \right. \\
+ \left. (C\tilde{T}_0 \Gamma^I \ldots \Gamma^r)_{\dot{\alpha}\dot{\beta}} \sin \left[ \gamma_0 \tilde{\beta}_0 + \Theta \right] |\dot{\alpha}\rangle_{-1/2} |\dot{\beta}\rangle_{-3/2} \right\},
\]

(6)

where \(\Theta\) contains the non-zero modes [3], and \(|\alpha\rangle_p = \lim_{z \to 0} S^\alpha(z)|P\rangle\).

3Consider a closed string sphere amplitude \(|0\rangle \prod_{i=1}^{3} V_i(z) |B\rangle\). To cancel the sphere superghost number anomaly, the superghost charges of \(\prod_{i=1}^{3} V_i(z) |B\rangle\) must be \((-2 - P, -2 - \bar{P})\). If this amplitude has to coincide with the open string disk amplitude \((0) \sum_{r=1/2}^{\infty} \bar{\psi}^I \cdot S \cdot \hat{\psi}^I\rangle|0\rangle\), the (open) superghost charge of the vertices must cancel the disk anomaly: \(-2 - P - 2 - \bar{P} = -2\).
2 Ramond-Ramond gauge fields

Let us focus on the R-R massless states of the type IIA theory. The R-R states that are usually considered in the literature are in the symmetric (−1/2, −1/2) picture and are created by the following vertex operators:

\[ V_R(k; z, \bar{z}) = \frac{1}{2\sqrt{2}} (CF^{(m+1)})_{\alpha \beta} V_{-1/2}^\alpha(k/2; z) \tilde{V}_{-1/2}^\beta(k/2; \bar{z}) , \]  

where

\[ (CF^{(m+1)})_{\alpha \beta} = \frac{(CT_{\mu_1 \ldots \mu_{m+1}})_{\alpha \beta}}{(m+1)!} F_{\mu_1 \ldots \mu_{m+1}} \]  

with \( m \) odd, and \( V^\alpha_R(k; z) = c(z) S^\alpha(z) e^{ik \cdot X(z)} \). The form \( F^{(m+1)} \) has the right degree to be interpreted as a R-R field strength of the type IIA theory; indeed the vertex \( V_R \) is BRST invariant only if \( k^2 = 0 \), and \( dF^{(m+1)} = d * F^{(m+1)} = 0 \) which are precisely the Bianchi and Maxwell equations of a field strength. Thus, from the target space point of view, it is clear that the boundary state can not be saturated with the usual R-R states. In fact, \( |B\rangle_R \) is in the (−1/2, −3/2) picture of the R-R sector, and thus, to soak up the superghost number anomaly, it can only couple to states that are in the asymmetric (−3/2, −1/2) picture. These two observations are intimately related: in fact by considering the asymmetric picture \([3]\), it is possible to construct vertices that are equivalent to those of Eq. (7), but that contain gauge potentials instead of field strengths. They are expressed as the sum of infinite terms:

\[ W(k; z, \bar{z}) = \sum_{M=0}^{\infty} W^{(M)}(k; z, \bar{z}) . \]  

Each term contains the same polarisation tensor \( A^{(m)}_{\mu_1 \ldots \mu_m} \) with the right degree for a \( m \)-form potential. Explicitly the first two are

\[ W^{(0)}(k; z, \bar{z}) = (CA^{(m)})_{\alpha \beta} V^{\alpha}_{-1/2}(k/2; z) \tilde{V}^{\beta}_{-1/2}(k/2; \bar{z}) , \]  

\[ W^{(1)}(k; z, \bar{z}) = -(CA^{(m)})_{\alpha \beta} \eta(z) V^{\alpha}_{+1/2}(k/2; z) \partial \xi(\bar{z}) \tilde{V}^{\beta}_{-1/2}(k/2; \bar{z}) , \]  

where \( CA^{(m)} \) is given by an expression similar to Eq. (8). All subsequent term can be determined by requiring BRST invariance under the total charge \( Q + \tilde{Q} \).

The invariance is obtained non-trivially. Recall that there is a natural splitting of the BRST charge \( Q \) into \( Q_0 \) (corresponding roughly to a current \( j_0(z) \propto c(z) T(z) \), \( T(z) \) being the stress-energy tensor), \( Q_1 \) (with current roughly \( j_1(z) \propto \gamma(z) G(z) \), \( G(z) \) being the supercurrent) and \( Q_2 \) (containing only ghosts and antighosts, needed for the closure). The commutation with \( Q_0 \) (and \( \tilde{Q}_0 \)) requires the conformal weight of the vertex to be zero, i.e. simply that \( k^2 = 0 \). The commutation with \( Q_2 \) can be arranged for all terms without imposing any condition.

The important part is the commutation with \( Q_1 + \tilde{Q}_1 \). We have \([\tilde{Q}_1, W^{(0)}] = 0\) and it turns out that for all the subsequent terms \([Q_1, W^{(M)}] + [\tilde{Q}_1, W^{(M+1)}] = 0\) provided the transversality condition for the polarisation is satisfied:

\[ \not{k} A^{(m)} + A^{(m)} \not{k} = 0 \Rightarrow d * A^{(m)} . \]  

This is the usual gauge condition for a potential. Notice that the constraint in Eq. (12) is independent of the on-shellness condition \( k^2 = 0 \). Thus the asymmetric R-R vertices share the property of the usual NS-NS massless vertices that they can be extended “softly” off-shell, namely spoiling
only the commutation with the \( Q_0 (\bar{Q}_0) \) part of the BRST charge\(^4\). It is argued in \([\text{1}]\) that from such an off-shell extension sensible result are obtained in the field theory limit when considering, as in \([\text{14}]\), the emission of closed strings from D-branes, where one is forced to go off-shell.

The state created by the vertex operator \([\text{3}]\) has a rather simple expression when written in the \((\beta, \gamma)\) system:

\[
|W\rangle = \left( C A^{(m)} \right)_{\alpha \beta} \cos \left( \gamma_0 \beta_0 \right) |\alpha; k/2\rangle_{-1/2} |\beta; k/2\rangle_{-3/2} + \left( C A^{(m)} \right)_{\alpha \beta} \sin \left( \gamma_0 \beta_0 \right) |\alpha; k/2\rangle_{-1/2} |\beta; k/2\rangle_{-3/2},
\]

(12)

where we have introduced the notation \(|\alpha; k\rangle \equiv \lim_{z \to 0} V_\alpha^\mu (k; z) \langle 0|\). The state \(|W\rangle\) is similar in form to the zero-mode part of the boundary state \(|B\rangle_R\), see Eq. \((\text{6})\).

2.1 Regulated scalar products and amplitudes

Having constructed the D-brane boundary states, and the closed string states that can couple to them, it is possible for instance to compute interactions between branes, \(\langle B_1 | D | B_2 \rangle\) (\(D\) being the closed string propagator), closed string fields emission from the D-brane, \(\langle B | D | W \rangle\), where \(|W\rangle\) represents the appropriate (off-shell) state, as well as to compute the scalar products in the Hilbert space of these states: \(\langle W^1 | W^2 \rangle\).

In all these cases an important subtlety is encountered in the R-R sector. Indeed the superghost zero-modes give rise to an infinite or ill-defined expression, which we need to properly regulate. The prescription we introduce (and that was devised for the purely Neumann boundary states in \([\text{12}]\)) consists in defining the “dangerous” expressions as the limit for \(x \to 1\) of the expressions modified by the introduction of the “regulator”

\[
\mathcal{R}(x) \equiv x^{2F_0 + G_0}.
\]

(13)

\(F_0\) and \(G_0\) are the zero-mode part of the fermion and superghost numbers respectively.

Since \(G_0 = -\gamma_0 \beta_0\), one can immediately compute the regulated superghost part. On the other side, one usually only considers \((-1)^{F_0} = \Gamma^{11}\). To give a meaning to \(F_0\) and thus to \(x^{2F_0}\), let us group the directions \(\mu\) into pairs. A two-state Hilbert space is associated to each pair. For instance, in the directions \((1, 2)\) the fermionic creator-annihilator couple is \(e^+_1 = (\Gamma^1 \pm i\Gamma^5)/2\), and the space is spanned by \(|\uparrow\rangle\) and \(|\downarrow\rangle\), with eigenvalues +1 and −1 resp. of the number operator \(N_1 = -i\Gamma^{12}\). Notwithstanding the explicit breaking of SO(10) to \(SO(2)^5\), if all directions were equivalent the final result would be still SO(10) invariant, see \([\text{12}]\). However, in presence of D-branes, care is needed in the pairing to obtain meaningful results, see \([\text{9}]\); see also Sec. \((\text{3.1})\).

In this framework we find (after Euclidean rotation) that

\[
(-1)^{F_0} = \Gamma^{11} = i \prod_{k=1}^{5} N_k = i \prod_{k=1}^{5} \exp(iN_k \pi/2) = (-i)^{\frac{5}{2}} \sum_{k=1}^{5} N_k.
\]

(14)

Thus \(F_0 = \frac{5}{2} \sum_{k=1}^{5} N_k\) and finally \(x^{2F_0} = \sum_{k=1}^{5} N_k\). We can now explicitly compute regulated expressions\(^5\). Notice that, because of Eq. \((\text{13})\) the fields \(\psi\) and the superghosts are not any more factorised; the regulator cancels the ill-defined contribution of the superghost fields with the one coming from two \(\psi\)'s, leaving a well-defined result that matches the one found in the light cone computations.

\(^4\)This off-shell extension is related to the scheme proposed in \([\text{14}]\) as the off-shell vertices still belong to the “restricted” cohomology \(Q' + Q'\) introduced there.

\(^5\)Typically we have to compute expressions like \(\text{tr} (x^{2F_0} \prod_{I} \Gamma^I)\) or \(\text{tr} (x^{2F_0} \prod_{I} \Gamma^I \Gamma^{11})\), that give rise in the various subspaces, depending on the set of indices \(\{ I \}\) and the chosen decomposition in pairs, to factors of \(\text{tr} (x^{N_k}) = (x + 1/x)\) or of \(\text{tr} (ix^{N_k} N_k) = i(x - 1/x)\).
2.2 Null states and “duality” relations among R-R potentials

A remarkable difference between the usual states and those of Eq. (12) is that the first ones have a definite chirality, while the second ones mix dotted and undotted indices. Thus one may wonder whether also for the asymmetric states it is possible to derive the Hodge conditions on the potentials that correspond to the usual constraints $F^{(p+2)} = *F^{(8-p)}$. As we will see, this feature arise in a non-trivial way from the Hilbert space structure of the new states.

The scalar product between a bra and a ket state for the R-R massless fields has to be defined as $\langle W'_1, W \rangle = \lim_{x \to 1} \langle W'_1 | R(x) | W \rangle$. It is easy to see that, with this prescription, the states of Eq. (12) have a definite norm. Moreover with this definition, the one-to-one mapping between asymmetric states and the usual ones becomes an isometry [6]. The most striking feature of the regulated scalar product is that forms of different order are, in general, not orthogonal to each other. Then there is a degenerate metric and therefore null-states.

For all states with transverse polarisations, the decoupling of the null states implements the dualities between R-R potentials. Consider a 1-form and a 7-form state with polarisations $A_{1}$ and $A_{2,8}$, transverse and on-shell: $k_0 = k_9$, other momenta zero. The Hodge duality $F_{01} = F_{2...9}$ implies $A_1 = A_{2,8}$. This equality is enforced in our context by setting to zero the null vector $|W_1⟩ - |W_{2,8}⟩$ that arises because of the non-orthogonality of the two corresponding asymmetric states $|W_1⟩$ and $|W_{2,8}⟩$, given by Eq. (12) with $CA^{(m)}$ equal to $CT^1$, resp. $CT^{1...8}$.

For longitudinal and scalar states (unphysical, but the one involved in Coulomb-like interactions!) a similar phenomenon occurs. Consider, for example, a 1-form state with polarisation $A_{0}$, off-shell in the kinematic region $k^i = 0$, $i = 0, 1, \ldots, 8$, relevant for the study of D-brane interactions as in Sec. (3.2), and a 9-form state $A_{0,8}$ in the same region. It is found that the corresponding states $|W_0⟩$ and $|W_{0,8}⟩$ have a degenerate metric admitting the null state

$$|\chi⟩ = |W_0⟩ + |W_{0,8}⟩.$$  \hfill (15)

While on-shell the scalar and longitudinal polarisations like always decouple from physical amplitudes, the presence of a D-brane forces an off-shell continuation and only linear combinations decouple. When we set $|\chi⟩ = 0$, we are led to identify $A_0$ with $-A_{0,8}$, recovering in this way an unusual “duality” also off-shell.

The Hilbert space structure is thus responsible both for the identification $A_1 = A_{2,8}$ for physical states, that is usually seen as an effect of the GSO projection of the symmetrical states, and for the “unusual” identification $A_0 = -A_{0,8}$, that is proper only of the longitudinal asymmetric states (12).

3 D0-D8 interaction: “microscopic” interpretation

We consider now the system of a D0 and a D8 brane at rest. This system has been widely investigated from different point of views [7, 8]. One of its important features is the creation of a fundamental string when the two branes cross each other maintaining the BPS condition. Although the description of this phenomenon, e.g. from the (M)atrix theory point of view [8], can be matched with the b.s. treatment, here we do not dwell on this aspect. We simply show how the peculiar properties of the asymmetric R-R states (12) shed light on the interpretation of the R-R part of the D0-D8 interaction.

3.1 D0-D8 interaction in the boundary states formalism

We take a D0-brane, with world-volume direction (0), and a D8-brane (012345678), at distance $b$ in the transverse direction. Since the number $\nu$ of “mixed” (ND + DN) directions is 8, such a configurations is BPS [8], and we expect the amplitude to vanish.
In the boundary state formalism, the amplitude $A = \langle B^8 | D | B^3 \rangle$ is straightforwardly computed in the NS-NS sector, with the result

$$A_{NS} = \frac{T}{2\pi} \frac{1}{\sqrt{8\pi^2\alpha'}} \int_0^\infty dt \left( \frac{\pi}{t} \right)^{\frac{3}{2}} e^{-b^2/(2\alpha't)} \left[ \left( \frac{f_1}{f_2} \right)^8 - \left( \frac{f_3}{f_2} \right)^8 \right] = -\frac{T}{4\pi\alpha'} |b| . \quad (16)$$

$T$ is the “volume” of the common world-volume dimension, the time, and the functions $f_i(q)$, with $q = e^{-t}$, are the usual infinite products given in $[4]$. In the first equality, the two terms within square brackets represent the oscillator contributions to the two part of the GSO projected amplitude, with or without the insertion of $(-)^{F+G}$. In the second step, one uses the “aequatio identica” $f_3^b - f_3^\bar{b} = f_3^b$ to replace 1 for their sum and carries out explicitly the integral. The final result represents a net repulsive Coulomb potential in the transverse direction, resulting from the exchange of graviton and dilaton, while the exchanges of massive fields cancel. Considering the amplitude in the dual channel, i.e. as the 1-loop free energy of the open strings suspended between the D-branes$[5]$ the two terms above correspond to the traces of $e^{-2i(\text{Loc}_a - a)}$ in the NS or R sector of the theory.

The R-R contribution to the amplitude is more delicate. We need to regulate the expression; to this effect we may split the directions into the pairs (1,2), (3,4), . . . , (9,0). It is essential (and rather intuitive) that one must not pair NN or DD directions with “mixed” ones, see $[6]$. Considering the amplitudes before GSO projection, using the definitions of Sec. (1) one finds for the superghost zero-modes

$$\langle B^8 \langle B^8_{sgh}, \eta | x^{2G_0} B^0_{sgh}, \eta \rangle \rangle_R^{(0)} = \left( 1 - \eta_1 \eta_2 x^2 \right)^{-1} . \quad (17)$$

For the fermionic zero-modes one has to compute

$$\langle B^8 \langle B^8_{\psi}, \eta | x^{2F_0} B^0_{\psi}, \eta \rangle \rangle_R^{(0)} = -\text{tr} (x^{2F_0} \Gamma^0) \delta_{\eta\eta'}, -1 - \text{tr} (x^{2F_0} \Gamma^0 \Gamma^{11}) \delta_{\eta\eta'}, 1 . \quad (18)$$

Employing the techniques devised in Sec. (2) to treat Eq. (18) one finally obtains

$$\lim_{x \to 1} \langle B^1 \langle B^1, \eta | R(x) | B^2, \eta \rangle \rangle_R^{(0)} = 16 \delta_{\eta\eta'}, 1 . \quad (19)$$

Comparing with the GSO projected expression $[3]$ one sees that it is only the term without insertion of $(-)^{F+G}$ (the “odd spin structure”) that contributes, contrarily to what happens for all cases with $\nu \neq 8$.

It is easy to see that for the odd spin structure the contribution of all non-zero modes vanishes; taking into account the contribution of the bosonic zero-mode, which does not depend from the sector, we finally get

$$A_R = \frac{T}{2\pi} \frac{1}{\sqrt{8\pi^2\alpha'}} \int_0^\infty dt \left( \frac{\pi}{t} \right)^{\frac{3}{2}} e^{-b^2/(2\alpha't)} = \frac{T}{4\pi\alpha'} |b| = -A_{NS} . \quad (20)$$

The computation of the R-R amplitude is not parity-invariant, due to the $\Gamma^{11}$ in Eq. (18). If we switch position between the two branes, $x^9 \to -x^9$, we get $A_{NS} - A_R = -T |b|/(2\pi\alpha')$. The BPS zero-force condition is maintained if a fundamental string with tension $1/(2\pi\alpha')$ is created.

### 3.2 Interpretation in terms of field theory exchange diagrams

Thus, the repulsive NS contribution to the D0-D8 amplitude is cancelled by an attractive Coulomb force in the “odd spin structure” (R-R sector): this is different from what happens in the D0-D6
brane system where the odd-spin structure encodes the Lorentz force [4]. What kind of exchange can be responsible for this Coulomb interaction? The D8 brane naturally couples to the R-R 9-form potential, and the D0 brane to the 1-form potential, and the two seem not to communicate. However the forms emitted by the static D-branes have to be off-shell (they carry only transverse momentum). Moreover, whatever is exchanged between the two can only have momentum along the 9th direction (as appropriate for the interpretation of the Coulomb interaction as due to an exchange diagram). Therefore the R-R states that may appear in the factorisation of the D0-D8 amplitude are precisely the “unphysical” states $|W_{01...8}\rangle$ and $|W_0\rangle$ discussed in Sec. (2.2) above. The peculiar “duality” relation between them, i.e. the existence of the null vector $|\chi\rangle$ of Eq. (15) allows an interpretation of the R-R part of the D0-D8 interaction as the exchange of a state that can be described equivalently by the 1-form $A_0$ or by the 9-form $A_{0...8}$. Namely, the Fourier transform of the R-R amplitude (20) factorises as follows:

$$\hat{A}_R(k_9) \propto_R \langle B^8 | (|W_{01...8}\rangle - |W_0\rangle) \frac{1}{k_9^2} (\langle W_{01...8} | - \langle W_0 |) B^0 \rangle_R ,$$

which explains also the attractive character of this interaction, as opposed to the usual R-R repulsive force in the other static D-brane interactions.

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