Analytical study of the nonisothermal flow of viscous fluid in an annular clearance at large pressure differentials

L S Prokhasko¹, R V Zalilov², N G Terenteva², K R Ovchinnikova³ and A V Shakhovskoy³

¹ South Ural State University (national research university), 76, Lenin Avenue, Chelyabinsk, 454080, Russia
² Nosov Magnitogorsk State Technical University, 38, Lenin Street, Magnitogorsk, 455000, Russia
³ K G Razumovsky Moscow State University of technologies and management (the First Cossack University), 73, Zemlyanoy Val, Moscow, 109004, Russia

E-mail: prokhaskols@mail.ru

Abstract. The questions of theory related to the study of viscous fluid flow in a small clearances at large pressure differentials are remarkably interesting for practical implementation. These are studies of operation of devices with an uncompressed piston, hydroextrusion with an uncompressed press-washer, hydroextrusion with an uncompressed shank, processes of continuous hydraulic pressing with viscous fluid, analysis of leaks in hydraulic drive elements and other technological spheres. The article presents the results of analytical study of the nonisothermal flow of a viscous fluid in a thin clearance with one movable boundary. The mathematical model takes into account the change of the fluid viscosity caused by pressure and temperature in accordance with the exponential law, as well as the elastic deformations of the walls which limit the clearance, directly proportional to the fluid pressure in a layer.

1. Introduction
The hydroextrusion method of solid materials processing has become known relatively not long ago, but it has already gained firm position in technologies of processing the materials that are difficult to process by other methods (hard-to-form materials, high-melting-point metals, brittle steels, etc.), or in technologies that require specifically high microstructural and operational properties [1-2]. Despite its popularity and high demand in various technologies around the world, this method still remains in the status of a developing area of knowledge both in theoretical terms and in the sphere of practical application. It can be explained by the complexity of production (technological) process: in order to apply this method it is necessary to create very high pressure (about 0.5–10 GPa), but in greater scale it is caused by the complexity of descriptive theory: when modeling the workflow and further developing the mathematical model it is necessary to take into consideration not only wide range of input parameters, technical properties etc. of the working fluid (which is used to implement the process), the extruded material and technological equipment (container, etc.), also it is necessary to account the changes in these properties during the hydroextrusion process. And most importantly, during study of the hydroextrusion model “fluid – extruded material – technological equipment” it is necessary to
simultaneously take into account as many parameters as possible, taking into account their interrelations with each other and their mutual influence.

Obviously the more correctly the mathematical model of hydroextrusion process is described and designed (it is referred to the completeness of parameters, boundary conditions, assumptions, etc.), the more reliable practical results can be obtained. One of those terms is to take into account the non-isothermicity of the fluid flowing through a thin annular clearance at large pressure differentials.

It is established that when a fluid flows in thin clearances under a large pressure differential, this fluid heats up [3, 4]. The similar heating the working fluid happens in a thin annular clearance during the processes of a solid body hydroextrusion. The authors of the article [5] point out that the nonisothermicity of a fluid is taken into consideration when studying temperature fields, hydrostatic pressure and rate of deformation of the extruded material.

However, this scientific topic is quite wide and versatile, it provides for its further development, so the material presented in this article is relevant both from the point of view of the development of the theory of nonisothermal flow of a working fluid in thin annular channels during hydroextrusion process, and in issues of accounting those moments in practical applications of this technology.

The aim of this articles is to develop a mathematical model for hydroextrusion processes to determine the parameters of annular clearance during the flow of a viscous fluid, taking into account its nonisothermicity at large pressure differentials.

Tasks:

- to propose a design scheme for fluid flow in a deformed clearance;
- for this calculation scheme to develop a mathematical model of the hydroextrusion process, taking the assumptions into account;
- to analyze the developed mathematical model.

2. Materials and methods

The design scheme of the fluid flow in the deformed clearance is shown in the figure 1.

The conventional notations shown in figure 1 are as follows: 1 – the surface of the plunger (shank); 2 – the surface of the container ; ℓ – the estimated length of the plunger (shank); P1 – the pressure in a working chamber (or in a radial cavity of the working chamber for schemes of hydraulic extrusion); P2 – Par for the uncompacted plunger (or pressure in the end cavity of the working chamber for schemes of hydraulic extrusion); V – the velocity of the plunger (shank); x, h – the axes of the fixed coordinate system; x1 – the direction of the tangent to the curved surface; ψ – the angle between the axes x and x1.

Figure 1. Calculation scheme of fluid flow in a deformed clearance.
We present a mathematical model of the nonisothermal flow of a viscous fluid in an annular clearance at large pressure differentials. Let’s assume that the surface of the container is moving, and the plunger (shank) is slowed down. Due to fact that the thickness of a layer is small, we assume the following:

- the pressure and temperature do not change along the layer thickness;
- the fluid speed in this direction is equal to zero;
- in the analysis we neglect the compressibility of the fluid.

As a result of the assumptions obtained from the Navier-Stokes equations, we have the following:

$$\frac{dP}{dx} = \mu \frac{d^2 U}{dx^2},$$

(1)

where $\mu$ – is dynamic coefficient of fluid viscosity.

For calculations we accept the exponential dependence of the fluid viscosity on pressure and temperature:

$$\mu = \mu_0 \exp[\alpha(P - P_0) - \lambda(T - T_0)].$$

(2)

where $\alpha$ – is a pressure coefficient of viscosity; $\lambda$ – is thermical coefficient of viscosity; $T$ – is the temperature of the fluid. The index “0” refers to the parameters under normal conditions.

In accordance with Newton’s hypothesis for tangential stresses in a fluid, we have the following:

$$\tau_x = \mu \frac{dU}{dh},$$

(3)

where $\tau$ – is tangential (shearing) stresses in a fluid.

The energy balance equation for a fluid, moving in the clearance between the input $I$–$I$ and current cross-section $i$–$i$ (shown on the figure 1), is expressed in the following form:

$$Q(P_i - P) = \frac{1}{k}QF \rho c(T_i - T) + F_T V + F_0 V,$$

(4)

where $k$ – is the coefficient that defines the conditions of heat exchange between the fluid in the layer and the walls around the clearance. The range of variations of $k$ spans from 0 (isothermal process) to one (for the case of complete absence of heat transfer); $\rho$ – fluid density; $c$ – specific heat capacity of a fluid; $F_T$ – force of friction between a fluid and the moving surface; $F_0$ – the axial component of a force of fluid pressure on a curved moving surface; $Q$ – fluid flow rate through the cross-section of the annular channel; $V$ – velocity of the plunger (shank) movement.

By integrating equation (1) two times, taking into account the boundary conditions and the continuity equation in integral form for incompressible fluid, below we obtain the well-known Reynolds equation:

$$q = \frac{Vh}{2} \frac{h^3}{12\mu} \frac{dP}{dx},$$

(5)

where $q$ – is specific rate of flow through the clearance.

$$q = \frac{Q}{e},$$

(6)

where $Q$ – is full flow of fluid through the clearance; $e$ – is a width of clearance.

$$e \approx \pi d,$$

(7)

where $d$ – is diameter of a plunger (shank).
The clearance area is $S$, as $h/d << 1$, which is defined as the following ratio:

$$S = \pi h d.$$  \hfill (8)

The flow rate of a fluid is determined by the velocity of a plunger:

$$Q = \frac{\pi d^2}{4} V.$$  \hfill (9)

From which equation it follows:

$$q = \frac{Q}{\pi} = \frac{V d}{4},$$  \hfill (10)

a for hydroextrusion schemes with a non-compacted shank it is the following:

$$Q = \left(\frac{\pi d^2}{4} - \frac{\pi d_z^2}{4}\right) V,$$  \hfill (11)

where $d_z$ – is diameter of a blank:

$$q = \left(\frac{d - d_z^2}{d}\right) V.$$  \hfill (12)

Let’s introduce the notation

$$d_f = d - \frac{d_z^2}{d}.$$  \hfill (13)

Then equation (12) in the record format is similar to the equation (10), but instead of $d$ it is necessary to put into it $d_f$.

$$q = \frac{V d_f}{4}.$$  \hfill (14)

Substitute (10) into (5) and we obtain the following:

$$V = \frac{h^3}{3 \mu (d - 2h)} \frac{dP}{dx},$$  \hfill (15)

where $h$ – is a function of pressure.

The ratio for determination of the fluid viscosity in the clearance as a function of pressure can be obtained from equations (2) and (4). In equation (4) we neglect the last two terms at the first stage of the analysis, which is quite acceptable, as the friction force is quite small along with a small length of the plunger (shank). In this case the viscosity is determined by the following ratio:

$$\mu = \mu_1 \exp[\alpha \kappa (P - P_1)],$$  \hfill (16)

where

$$\mu_1 = \mu_0 \exp(\alpha P_f),$$  \hfill (17)

$$\alpha \kappa = \alpha + \frac{\lambda \kappa}{\rho c}.$$  \hfill (18)
The expression for the clearance, in the case when the container is not loaded in the axial direction, and the material of the plunger and the container have the same constants $E$ and $\mu$, has the following view:

$$h = h_0 - \frac{d}{2E} vP + \frac{d}{E (1 - k^2)} P. \tag{19}$$

Let’s introduce the following notation:

$$h_* = h_0 - \frac{d}{2E} vP_1,$$ \hspace{1cm} \tag{20}

$$a = \frac{d}{E (1 - k^2)},$$ \hspace{1cm} \tag{21}

$$d_* = d - 2h_*.$$ \hspace{1cm} \tag{22}

Taking into account the assumptions (20), (21), we write down equation (19) in the following form:

$$h = h_* + aP. \tag{23}$$

Substituting (17), (22) and (23) in equation (15), let’s write the result:

$$V = \frac{(h_* + aP)^3}{3\mu (d_* - 2aP) \exp[\alpha_k (P - P_1)]} \frac{dP}{dx}. \tag{24}$$

During the analyzing of the isothermal flow of a fluid the author of monography [6] neglected the Couette flow, that is the author discarded the term $Vh/2$ in equation (5). In our case this assumption requires discarding the term $2aP$ from the equation (24). The integral of equation (24) over the entire length of the clearance $\ell$ gives a ratio for the velocity of the plunger (shank) movement, similar to the author’s equation [6]:

$$V = \frac{\exp(\alpha_k P_1)}{3\mu_1 \ell c k} \left\{ h_*^3 + 3 \frac{a}{\alpha_k} h_*^2 + 6 \frac{a^2}{\alpha_k^2} h_* + 6 \frac{a^3}{\alpha_k^3} \right\} \exp\left(-\alpha_k P_1 \right) \left( h_* + aP1 \right)^3 + 3 \frac{a}{\alpha_k} \left( h_* + aP1 \right)^2$$

$$+ 6 \frac{a^2}{\alpha_k} \left( h_* + aP1 \right) + 6 \frac{a^3}{\alpha_k^3} \right\} \tag{25}$$

Below the result of integration of the non-truncated equation (24) is presented:

$$V = \frac{d^3 \exp(\alpha_k \left( -\frac{d_*}{2a} + P1 \right))}{48\mu_1 \ell \alpha_k} \left\{ E_i \left( \frac{\alpha_k}{2a} d_* \right) - E_i \left( \frac{\alpha_k}{2a} (d_* - 2aP1) \right) \right\} \left( \frac{\exp(\alpha_k P)}{a^3 \alpha_k^2} \right)$$

$$\left( \frac{h_*}{a} + 1 \right) \left( \alpha_k \frac{d}{2a} + 2 \right) + \frac{1}{6\mu_1 \ell \alpha_k} \left( \frac{h_*}{a} + P \right)^2 + \frac{\alpha_k^2 \ell^2}{4a^2} + \left( \alpha_k \frac{h_*}{a} + \alpha_k P1 + 1 \right)$$

$$\left( \frac{\alpha_k d}{2a} + 2 \right), \tag{26}$$
where $E_i$ – is the conventional notation for the Euler integral:

$$E_i(t) = \sum_{k=1}^{\infty} \frac{t^k}{k!}.$$  

(27)

3. Results and discussions

The developed mathematical model for calculation the parameters of hydroextrusion processes, taking into account the nonisothermicity of the working fluid, is of the fullest extent possible, since it allows taking into account a greater number of factors that affect the properties of movement – please refer to the formula (26). The effect of nonisothermicity is taken into account in equation (25) by the coefficient $k$. Coefficient $k$: in general case this coefficient is a function of the clearance, the velocity of the plunger (shank) movement, the thermophysical properties of the fluid and material of the container and the plunger (shank). For some cases it’s possible to use the results of the experiment [7]. It is necessary to note that as the clearance increases, the coefficient $k$ increases, and as the clearance decreases, it also decreases. The $k$ coefficient also decreases at the end of the clearance, where the difference between temperature of a fluid and the channel walls reaches its maximum value. The additional research is required to justify the choice of the coefficient $k$, which is included in the energy equation (4).

4. Conclusions

For the hydroextrusion process a design scheme was offered for the flow of a working fluid in a deformed clearance and a mathematical model was presented for determination of annular clearance parameters during a viscous fluid flow, taking into account its nonisothermicity at large pressure differentials, which can be used as a basis for calculating the various processes – fluid flow in devices with an uncompressed plunger, continuous hydraulic compression, hydroextrusion with an uncompressed shank, hydroextrusion with an elastically deformed bushing, etc. The presented results of the analytical research can be considered reliable, as they are based on the generally accepted laws of hydrodynamics, resistance of materials, theory of elasticity, etc.

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