CP violating angular asymmetries
of $b$ and $\bar{b}$ quarks in $e^+e^- \rightarrow t\bar{t}$

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Abstract

We obtain analytical formulae for the cross section and the angular distributions of the $b(\bar{b})$ quarks in the process $e^+e^- \rightarrow t\bar{t}$, with $t \rightarrow W^+b$ ($\bar{t} \rightarrow W^-\bar{b}$) assuming CP violation in the $\gamma t\bar{t}$ and $Zt\bar{t}$ vertices. We present CP violating asymmetries which measure separately the real and imaginary parts of the electroweak dipole moment form factors of the top, $d^\gamma(s)$ and $d^Z(s)$. We give a numerical analysis of these asymmetries within the Minimal Supersymmetric Standard Model with complex parameters. They turn out to be of order $\lesssim 10^{-3}$. 
1 Introduction

So far CP violation has been observed only in neutral kaon decays. An observation of CP violation in other reactions would be of crucial importance for a better understanding of the origin of this phenomenon. Top quark physics may offer new possibilities for CP violating observables at existing and future colliders. The reasons are both experimental and theoretical: Owing to its large mass the top quark decays before forming a hadronic bound state \([1]\). Therefore, its polarization can be determined by measuring the distributions of its decay products. The polarization of the top quark is sensitive to CP violation. For isolating the truly CP violating effects one has to compare the decays of the top quark with those of the anti–top quark. In future \(e^+e^-\) colliders, \(t\) and \(\bar{t}\) will be produced copiously, and the distributions of their decay products and in this way their polarizations can be measured in the same experiment.

Furthermore, because of the large top quark mass, the perturbative calculations are more reliable and free of uncertainties of hadronization models. This allows one to obtain clear theoretical predictions for CP violating observables.

In this paper we consider the process

\[
e^+e^- \rightarrow t\overline{t} \rightarrow W^+b
\]

and its CP–conjugate

\[
e^+e^- \rightarrow t\overline{t} \rightarrow W^-\overline{b}
\]

in the energy range of an \(e^+e^-\) Linear Collider. We discuss the possibility of examining CP violation in reactions (1) and (2) by analyzing the angular distributions of \(b\) and \(\overline{b}\) from the top quark decays.

Previously, the effects of the electroweak dipole moment form factors in (1) and (2) were considered in \([2]\). In this paper we give analytic formulae for the differential cross sections of (1) and (2) in the c.m.system. We work out formulae in terms of general CP violating couplings for the angular distributions of the \(b(\overline{b})\) quark and for suitable defined asymmetries. It is possible to integrate analytically over the four–particle phase space. These expressions are general and model independent. We also take into account longitudinal polarization of \(e^-\) and/or \(e^+\).
In the Standard Model (SM) the decay $t \to bW$ has 100% branching ratio. We assume CP violation to occur in the production process, induced by the electric $d^{\gamma}(s)$ and weak $d^{Z}(s)$ dipole moment form factors of the top quark in the $\gamma t\bar{t}$ and $Zt\bar{t}$ vertices. In the SM $d^{\gamma}(s)$ and $d^{Z}(s)$ get non-zero values through the complex phase of the CKM matrix. However, they are at least a second order loop effect and thus almost negligible. Therefore an observation of CP non-conservation in top quark physics would be related to physics beyond the SM. Supersymmetric models and models with more than one Higgs doublet are at present the most favoured candidates. These models can provide new sources of CP violation [3] so that $d^{\gamma}(s)$ and $d^{Z}(s)$ appear at one-loop level. A complete study of $d^{\gamma}(s)$ and $d^{Z}(s)$ in the Minimal Supersymmetric Standard Model (MSSM) with complex parameters has been performed in [4].

The previously proposed CP violating asymmetries involve measurements of triple product correlations [5, 6, 7, 8], lepton distributions, and other quantities [2, 9, 10, 11, 12]. In the present paper we show that measuring the angular distributions of the $b(\bar{b})$ quarks provides an alternative method to determine the CP violating parameters. For this purpose detection of $b$ and $\bar{b}$ jets is required. Looking at the angular distributions of the $b(\bar{b})$ jets coming from the $t$–decay instead of the angular distributions of the leptons coming from the $W$–decay [11, 12] has the advantage of a higher rate. CP violation in $t\bar{t}$ production at hadron colliders has been discussed in [13].

In section 2 we work out the general expressions for the differential cross sections of (1) and (2), in which the polarization four–vectors of $t$ and $\bar{t}$ enter explicitly. These are given in section 3. In section 4 we obtain the differential cross section in terms of these polarization vectors and in section 5 we give the analytic expressions in the c.m.system for the $\cos \theta_{b}$–distribution of the $b$ quarks of (1) and of $\cos \theta_{\bar{b}}$ of the $\bar{b}$ quarks of process (2), in terms of the real and imaginary parts of $d^{\gamma}(s)$ and $d^{Z}(s)$. Suitable angular asymmetries, sensitive to $\Re d^{\gamma,Z}(s)$ and $\Im d^{\gamma,Z}(s)$, are defined in section 6, where we also derive analytic formulae. In section 7 we present numerical results in MSSM with complex parameters, based on the calculations of $d^{\gamma}(s)$ and $d^{Z}(s)$ in [4]. A summary is given in section 8.
2 The formalism

In order to obtain analytic expressions for the cross sections of the sequential processes (1) and (2) we follow the formalism of [14]. According to it we write for (1) and (2):

\[ d\sigma^b_{\lambda\lambda'} = d\sigma^t_{\lambda\lambda'} \frac{d\Gamma_t}{\Gamma_t} \frac{E_t}{m_t}, \quad d\sigma^{\bar{b}}_{\lambda\lambda'} = d\sigma^{\bar{t}}_{\lambda\lambda'} \frac{d\Gamma_{\bar{t}}}{\Gamma_{\bar{t}}} \frac{E_{\bar{t}}}{m_{\bar{t}}}. \] (3)

Here \( d\sigma^t_{\lambda\lambda'} \) is the differential cross section for \( t (\bar{t}) \) production in \( e^+e^- \) annihilation, \( \lambda \) and \( \lambda' \) being the longitudinal polarization of \( e^- \) and \( e^+ \), respectively. \( d\Gamma_t (d\Gamma_{\bar{t}}) \) is the differential decay rate for \( t \to bW (\bar{t} \to \bar{b}W) \) when the top quark is polarized, its polarization vector \( \xi (\bar{\xi}) \) determined by the former production process, \( E_t (E_{\bar{t}}) \) is the energy of the \( t (\bar{t}) \) quark in the c.m. system, \( \Gamma_t \) is the total decay width of the top quark. For the differential cross section \( d\sigma^{b,\bar{b}}_{\lambda\lambda'} \) we obtain:

\[ d\sigma^b_{\lambda\lambda'} = \sigma^b_0 \left\{ 1 + \alpha_b m_t \frac{\xi_{p_b}}{(p_tp_b)} \right\} d\cos\theta_t d\Omega_b \] (4)
\[ d\sigma^{\bar{b}}_{\lambda\lambda'} = \sigma^{\bar{b}}_0 \left\{ 1 - \alpha_b m_t \frac{\bar{\xi}_{p_{\bar{b}}}}{(p_{\bar{t}}p_{\bar{b}})} \right\} d\cos\theta_{\bar{t}} d\Omega_{\bar{b}} \] (5)

We use a reference frame where the \( z^-\)axis points into the direction of \( \vec{q}_e; \vec{q}_e \) and \( \vec{p}_{t(\bar{t})} \) determine the \( xz^-\)plane; \( \cos\theta_{t(\bar{t})} \) is the scattering angle of \( t(\bar{t}) \), and \( d\Omega_{b(\bar{b})} = d\cos\theta_{b(\bar{b})} d\varphi_{b(\bar{b})} \).

The coefficient \( \alpha_b \) determines the sensitivity of the \( b \) quark to the polarization of the top quark:

\[ \alpha_b = \frac{m_t^2 - 2m_W^2}{m_t^2 + 2m_W^2}. \] (6)

The sensitivity to CP non–conservation in \( t \) quark production is determined both by the value of \( \alpha_b \) and the CP violating contribution to the \( t^-\)polarization. \( \sigma_0^{b,\bar{b}} \) determines the differential SM cross section of (1) and (2) for totally unpolarized decaying top quarks with longitudinally polarized initial electron–positron beams:

\[ \sigma_0^{b,\bar{b}} = \alpha_{em}^2 \frac{3\beta}{2s} \frac{\Gamma_{t,bW}}{\Gamma_t} \frac{m_t^2 E_{b(\bar{b})}^2}{(m_t^2 - m_W^2)^2} N_{\lambda\lambda'}^{t(\bar{t})} \] (7)

where \( E_{b(\bar{b})} \) is the energy of the \( b(\bar{b}) \) quark in the c.m. system:

\[ E_b = \frac{m_t^2 - m_W^2}{\sqrt{s} (1 - \beta \cos\theta_{tb})}, \quad E_{\bar{b}} = \frac{m_t^2 - m_W^2}{\sqrt{s} (1 - \beta \cos\theta_{\bar{t}b})}. \] (8)
$\sqrt{s}$ is the total c.m. energy, and
\[
\cos \theta_{tb} = \frac{\vec{p}_t \cdot \vec{p}_b}{|\vec{p}_t| |\vec{p}_b|} = \sin \theta_t \sin \theta_b \cos \phi_b + \cos \theta_t \cos \theta_b.
\] (9)

We take $m_b = 0$, $\beta = \sqrt{1 - 4m^2_b/s}$ is the velocity of the $t$ quark. $\Gamma_{t \to bW}$ is the partial decay width of the top quark for the decay $t \to bW$, and
\[
N^{\gamma(\ell)}_{\lambda \lambda'}(\cos \theta_i) = (1 + \beta^2 \cos^2 \theta_{t(\ell)}) F_1 + (1 - \beta^2) F_2 \pm 2 \beta \cos \theta_{t(\ell)} F_3. \tag{10}
\]
The dependence on the beam polarizations comes through the functions $F_i$, $i = 1, 2, 3$, given by
\[
F_i = (1 - \lambda \lambda') F_i^0 + (\lambda - \lambda') G_i^0 \tag{11}
\]
where
\[
F_1^0 = \frac{4}{9} - \frac{4}{3} c_V g_V h_Z + (c^2_V + c^2_A)(g^2_V + g^2_A) h_Z^2 \\
F_2^0 = \frac{4}{9} - \frac{4}{3} c_V g_V h_Z + (c^2_V + c^2_A)(g^2_V - g^2_A) h_Z^2 \\
F_3^0 = -\frac{4}{3} c_A g_A h_Z + 4 c_V c_A g_V g_A h_Z^2 \\
G_1^0 = -\frac{4}{7} c_A g_V h_Z + 2 c_V c_A (g^2_V + g^2_A) h_Z^2 \\
G_2^0 = -\frac{4}{7} c_A g_V h_Z + 2 c_V c_A (g^2_V - g^2_A) h_Z^2 \\
G_3^0 = -\frac{4}{7} c_V g_V h_Z + 2 (c^2_V + c^2_A) g_V g_A h_Z^2 \tag{12}
\]
The quantities $c_V = -(1/2) + 2 \sin^2 \Theta_w$, $c_A = (1/2)$ and $g_V = (1/2) - (4/3) \sin^2 \Theta_w$, $g_A = -(1/2)$, are the SM couplings of $Z$ to the electron and the top quark, respectively, and $h_Z = [s/(s - m^2_Z)]/\sin^2 2\Theta_w$.

In the SM at tree level the total cross section for the inclusive $b(\bar{b})$ production process (1) and (2) at tree level is
\[
\sigma_{tot}^{b(\bar{b})} = \frac{\pi \alpha^2_{em}}{s} \beta \frac{\Gamma_{t \to bW}}{\Gamma_t} [(3 + \beta^2) F_1 + 3(1 - \beta^2) F_2] = \frac{\pi \alpha^2_{em}}{s} \beta \frac{\Gamma_{t \to bW}}{\Gamma_t} N_{tot}, \tag{13}
\]
where we have introduced the convenient notation
\[
N_{tot} = (3 + \beta^2) F_1 + 3(1 - \beta^2) F_2, \tag{14}
\]
and the partial decay width is:
\[
\Gamma_{t \to bW} = \frac{\alpha_{em}}{16 \sin^2 \Theta_w (m_t^2 - m_W^2)^2 (m_t^2 + 2m_W^2)} \tag{15}
\]
3 The polarization vector of the top quark

The amplitude for $e^+e^- \rightarrow t\bar{t}$, assuming CP violation, is

$$\mathcal{M} = i\frac{\epsilon^2}{s} \bar{v}(q_e)\gamma_\mu u(q_e) (V_\gamma)^\mu - i\frac{g_2^2}{s - m_Z^2} \bar{v}(q_e)\gamma_\mu (c_V + c_A \gamma^5) u(q_e) (V_Z)^\mu$$ \hspace{1cm} (16)

where $g_Z = e / \sin 2\Theta_W$. The quantities $V_i$ define the $t\bar{t}\gamma$ and $t\bar{t}Z$ vertices:

$$(V_\gamma)^\mu = \frac{2}{3} \gamma^\mu - i(P^\mu/m_t)d'(s) \gamma^5$$ \hspace{1cm} (17)

$$(V_Z)^\mu = \gamma^\mu (g_V + g_A \gamma^5) - i(P^\mu/m_t)d^Z(s) \gamma^5$$ \hspace{1cm} (18)

Here $P^\mu = p_t^\mu - p_\bar{t}^\mu$, and $d'(s)$ and $d^Z(s)$ are functions of $s$, so that $d'(0)$ and $d^Z(m_\tilde{t}^2)$ determine the electric and weak dipole moments of the $t$ quark. These dipole moments can be induced only by CP violating interactions and have in general both real and imaginary parts.

Now we will give the expressions for the polarization four–vectors $\xi_\mu$ of the top quark and $\xi_\mu$ of the antitop quark, depending on the electric and weak dipole moment form factors.

As $(p_\xi \xi) = 0$, in general the polarization vector $\xi_\mu$ can be decomposed along three independent four–vectors orthogonal to $p_\xi$: two of them, $Q_e^\mu$ and $Q_{\bar{e}}^\mu$ are in the production plane:

$$Q_e^\mu = q_e^\mu - \frac{(p_\xi q_e)}{m_t^2} p_\xi^\mu , \quad Q_{\bar{e}}^\mu = q_{\bar{e}}^\mu - \frac{(p_\xi q_{\bar{e}})}{m_t^2} p_\xi^\mu$$ \hspace{1cm} (19)

and the third one is normal to it: $\varepsilon_{\mu\alpha\beta\gamma} p_\xi^\alpha q_e^\beta q_{\bar{e}}^\gamma$. Most generally, we can write:

$$\xi_\mu = P_{e(\bar{e})}^\mu (Q_e)_{\mu} + P_{e(\bar{e})}^{CP} (Q_{\bar{e}})_{\mu} + D^\mu \varepsilon_{\mu\alpha\beta\gamma} p_\xi^\alpha q_e^\beta q_{\bar{e}}^\gamma .$$ \hspace{1cm} (20)

The components $P_{e(\bar{e})}^\mu$ get contributions from both SM and CP violating terms. The SM at tree level does not contribute to the normal component $\xi_{\mu}$. Thus we have:

$$P_{e(\bar{e})}^\mu = P_{e(\bar{e})}^{SM} + P_{e(\bar{e})}^{CP} , \quad D^\mu = D^{CP} .$$ \hspace{1cm} (21)

The polarization four–vector is determined by the expression $[14]$: \hspace{1cm} (22)

$$\xi_{\mu} = (g_{\mu\nu} - m_t^{-2} p_\mu p_\nu) \text{Tr}[\mathcal{M} \tilde{\Lambda}(p_t) \tilde{\Lambda}(p_t) \gamma^\nu \gamma^5] \cdot \text{Tr}[\mathcal{M} \tilde{\Lambda}(p_t) \tilde{\Lambda}(p_t)]^{-1}$$

Note the additional $i$ in front of $d'(s)$ and $d^Z(s)$, as compared to our previous paper $[8]$. 

1Note the additional $i$ in front of $d'(s)$ and $d^Z(s)$, as compared to our previous paper $[8]$. 

5
where $\mathcal{M}$ is the amplitude eq. (16). In the c.m. system the SM contribution to $P_{e(e)}^{SM}$ is at tree-level

\begin{align}
P_{e}^{SM}(\theta_{t}) &= \frac{2m_{t}}{s} \frac{1}{N_{\lambda'\lambda}} [(1 - \beta \cos \theta_{t})(G_{1} - G_{3}) + (1 + \beta \cos \theta_{t})G_{2}] \\
P_{\bar{e}}^{SM}(\theta_{t}) &= -\frac{2m_{t}}{s} \frac{1}{N_{\lambda'\lambda}} [(1 + \beta \cos \theta_{t})(G_{1} + G_{3}) + (1 - \beta \cos \theta_{t})G_{2}]
\end{align}

where $G_{i}$, $i = 1, 2, 3$ are given by:

\begin{equation}
G_{i} = (1 - \lambda \lambda')G_{i}^{0} + (\lambda - \lambda')F_{i}^{0}
\end{equation}

with $F_{i}^{0}$ and $G_{i}^{0}$ as defined in (13). The CP violating dipole moment form factors $d^{\gamma}(s)$ and $d^{Z}(s)$ induce two types of contributions: due to their real and imaginary parts. The absorptive parts $\Im d^{\gamma,Z}(s)$ contribute to $P_{e(e)}^{CP}$:

\begin{align}
P_{e}^{CP}(\theta_{t}) &= -\frac{2}{m_{t}} \frac{1}{N_{\lambda'\lambda}} [(1 + \beta \cos \theta_{t} - \beta^{2} \sin^{2} \theta_{t}) \Im H_{1} \\
&\quad - (\beta \cos \theta_{t} + \beta^{2}) \Im H_{2}] \\
P_{\bar{e}}^{CP}(\theta_{t}) &= -\frac{2}{m_{t}} \frac{1}{N_{\lambda'\lambda}} [(1 - \beta \cos \theta_{t} - \beta^{2} \sin^{2} \theta_{t}) \Im H_{1} \\
&\quad - (\beta \cos \theta_{t} - \beta^{2}) \Im H_{2}].
\end{align}

Here we have used the notation:

\begin{equation}
H_{i} = (1 - \lambda \lambda')H_{i}^{0} + (\lambda - \lambda')D_{i}^{0}
\end{equation}

where

\begin{align}
H_{1}^{0} &= \left(\frac{2}{3} - c_{V}g_{V}h_{Z}\right)d^{\gamma}(s) - \left(\frac{2}{3}c_{V}h_{Z} - (c_{V}^{2} + c_{A}^{2})g_{V}h_{Z}^{2}\right)d^{Z}(s) \\
H_{2}^{0} &= -c_{AG}h_{Z}d^{\gamma}(s) + 2c_{V}c_{AG}h_{Z}^{2}d^{Z}(s) \\
D_{1}^{0} &= -c_{A}h_{V}h_{Z}d^{\gamma}(s) - (\frac{2}{3}c_{A}h_{Z} - 2c_{V}c_{AG}h_{Z}^{2})d^{Z}(s) \\
D_{2}^{0} &= -c_{V}h_{Z}d^{\gamma}(s) + (c_{V}^{2} + c_{A}^{2})g_{A}h_{Z}^{2}d^{Z}(s).
\end{align}

The real parts of $d^{\gamma,Z}(s)$ determine the CP violating contribution $D^{CP}$ to the normal component of the polarization vector:

\begin{equation}
D^{CP}(\theta_{t}) = \frac{8}{m_{t}s} \frac{1}{N_{\lambda'\lambda}} [\Re D_{1} + \beta \cos \theta_{t} \Re D_{2}]
\end{equation}

Here

\begin{equation}
D_{i} = (1 - \lambda \lambda')D_{i}^{0} + (\lambda - \lambda')H_{i}^{0}
\end{equation}
Note that $H^0_i$ are C–odd and P–even, while $D^0_i$ are C–even and P–odd functions of the coupling constants in the production process $e^+e^−→t\bar{t}$. This implies that $H_i$ are C–odd and CP–odd, while $D^0_i$ are P–odd and CP–odd quantities.

The polarization four–vector $\xi$ for the anti–top is obtained through C–conjugation. This leads to the following replacements in the expressions for $\xi_\mu$, $F_i$, $G_i$, $H_i$, and $D_i$:

$$p_t → p_{\bar{t}}, \ (2/3)e → -(2/3)e, \ g_V → -g_V, \ d^{\gamma,Z}(s) → -d^{\gamma,Z}(s). \tag{32}$$

We have:

$$\xi_\mu = P^t_e(Q_e)_{\mu} + P^t_{\bar{e}}(\bar{Q}_e)_{\mu} + D^t\varepsilon_{\mu \alpha \beta \gamma} p_\ell^\alpha q_{\bar{e}}^\beta q_{\bar{e}}^\gamma. \tag{33}$$

where

$$Q^\mu_e = q_e^\mu - \frac{(p_t q_e)}{m^2_t} p^\mu_t, \quad \bar{Q}^\mu_{\bar{e}} = q_{\bar{e}}^\mu - \frac{(p_\ell q_{\bar{e}})}{m^2_\ell} p^\mu_\ell. \tag{34}$$

In analogy to eq.(21) we define:

$$P^{t(\bar{e})}_e = \bar{P}^{SM}_{e(\bar{e})} + \bar{P}^{CP}_{e(\bar{e})}, \quad D^t = \bar{D}^{CP}. \tag{35}$$

and obtain:

$$\bar{P}^{SM}_{e(\bar{e})}(\theta_t) = \frac{2m_t}{s} \frac{1}{N^{1}_{\lambda \lambda'}} [(1 - \beta \cos \theta_t)(G_1 + G_3) + (1 + \beta \cos \theta_t)G_2] \tag{36}$$

$$\bar{P}^{SM}_{\bar{e}}(\theta_t) = -\frac{2m_t}{s} \frac{1}{N^{1}_{\lambda \lambda'}} [(1 + \beta \cos \theta_t)(G_1 - G_3) + (1 - \beta \cos \theta_t)G_2] \tag{37}$$

$$\bar{P}^{CP}_{e(\bar{e})}(\theta_t) = -\frac{2}{m_t} \frac{1}{N^{1}_{\lambda \lambda'}} [(1 + \beta \cos \theta_t - \beta^2 \sin^2 \theta_t) \Im m H_1 + (\beta \cos \theta_t + \beta^2) \Im m H_2] \tag{38}$$

$$\bar{P}^{CP}_{\bar{e}}(\theta_t) = -\frac{2}{m_t} \frac{1}{N^{1}_{\lambda \lambda'}} [(1 - \beta \cos \theta_t - \beta^2 \sin^2 \theta_t) \Im m H_1 + (\beta \cos \theta_t - \beta^2) \Im m H_2] \tag{39}$$

$$\bar{D}^{CP}(\theta_t) = \frac{8}{m_t s} \frac{1}{N^{1}_{\lambda \lambda'}} [\Re \bar{D}_1 - \beta \cos \theta_t \Re D_2] \tag{40}$$

From the explicit expressions for $\xi_\mu$ and $\bar{\xi}_\mu$ together with

$$P^{SM}_{\pm} = P^{SM}_{e} \pm P^{SM}_{\bar{e}}, \quad P^{CP}_{\pm} = P^{CP}_{e} \pm P^{CP}_{\bar{e}} \tag{41}$$

we obviously obtain:

$$P^{SM}_{\pm}(\theta_t = \pi - \theta_t) = -P^{SM}_{\pm}(\theta_t), \quad P^{CP}_{+}(\theta_t = \pi - \theta_t) = P^{CP}_{+}(\theta_t) \tag{42}$$

$$P^{SM}_{\pm}(\theta_t = \pi - \theta_t) = P^{SM}_{\pm}(\theta_t), \quad P^{CP}_{-}(\theta_t = \pi - \theta_t) = -P^{CP}_{-}(\theta_t) \tag{43}$$

$$\bar{D}^{CP}(\theta_t = \pi - \theta_t) = \bar{D}^{CP}(\theta_t) \tag{44}$$
4 The differential cross section

Using the explicit expressions eqs.(21) and (33) for the top and the anti–top quark polarization four–vectors we obtain from (4) the analytic formula for the cross sections of (1) and (2) in the c.m.system:

\[ d\sigma^{b(\bar{b})}_{\lambda\lambda'} = \sigma_0^{b(\bar{b})}(\lambda, \lambda') \left\{ 1 \pm \alpha_b m_t \frac{\sqrt{2} E_{b(\bar{b})}}{m_t^2 - m_W^2} \left[ P^t_{\pm}(1 - \frac{1 - \beta \cos \theta_{b(\bar{b})}}{1 - \beta^2}) - P^-_{\pm}(\cos \theta_{b(\bar{b})} - \beta \cos \theta_{t(\bar{t})}) \right] + D_{\pm}(\frac{3\beta}{2} \langle q_e \hat{p}_{t(\bar{t})} \hat{p}_{b(\bar{b})} \rangle) \right\} \ d \cos \theta_{t(\bar{t})} \ d \Omega_{b(\bar{b})}, \]

where the triple product is defined as \( \langle q_e \hat{p}_s \hat{p}_b \rangle = \hat{q}_e \cdot (\hat{p}_s \times \hat{p}_b) \) with \( \hat{q}, \hat{p} \) being unit three–vectors in the direction of the particles. \( \sigma_0^{b(\bar{b})} \) is given in eq.(4). We use the notation

\[ P^t_{\pm} = P^t_e \pm P^t_{\bar{e}} = P^{SM}_{\pm} + P^{CP}_{\pm}, \quad P^\bar{t}_{\pm} = P^\bar{t}_e \pm P^\bar{t}_{\bar{e}} = \bar{P}^{SM}_{\pm} + \bar{P}^{CP}_{\pm}. \]

\( D^t \) and \( D^\bar{t} \) are given by eqs.(21) and (35).

5 The angular distributions of \( b \) and \( \bar{b} \) quarks

Integrating (4) and (3) over \( \cos \theta_t \) (\( \cos \theta_{\bar{t}} \)) and \( \phi_b \) (\( \phi_{\bar{b}} \)) we obtain the \( \cos \theta_b \) (\( \cos \theta_{\bar{b}} \))–distribution of the \( b(\bar{b}) \) quarks in the c.m.system:

\[ \frac{d\sigma^{b(\bar{b})}_{\lambda\lambda'}}{d \cos \theta_{b(\bar{b})}} = \frac{3\pi \alpha_e^2 \beta \Gamma_{t-\nu W}}{2s} \left( a_0^{b(\bar{b})} \pm a_1^{b(\bar{b})} \cos \theta_{b(\bar{b})} + a_2^{b(\bar{b})} \cos^2 \theta_{b(\bar{b})} \right) \]

where

\[ a_0^{b(\bar{b})} = a_0^{SM}(\pm), \quad a_1^{b(\bar{b})} = a_1^{SM}(\pm), \quad a_2^{b(\bar{b})} = a_2^{SM}(\pm), \quad a_0^{CP}, \quad a_1^{CP}, \quad a_2^{CP}, \]

\[ a_0^{SM} = (1 + \beta^2 - b) F_1 + (1 - \beta^2) F_2 - \alpha_b (b - \beta^2) G_3, \]

\[ a_1^{SM} = 2b F_3 - \alpha_b \left( (1 + \beta^2 - 2b) G_1 + (1 - \beta^2) G_2 \right), \]

\[ a_2^{SM} = (3b - 2\beta^2) F_1 + 3\alpha_b (b - \beta^2) G_3, \]

\[ a_0^{CP} = -2\alpha_b \Im m H_1, \quad a_1^{CP} = -4\alpha_b \Im m H_2, \quad a_2^{CP} = 6\alpha_b \Im m H_1, \]

\[ b = 1 - \frac{1 - \beta^2}{2\beta} \ln \left[ \frac{1 + \beta}{1 - \beta} \right]. \]
In “$(\pm)$” the “+” belongs to the $b$ and the “−” to the $\bar{b}$. These formulae coincide with the analogous SM expressions obtained in \cite{15} for the unpolarized $e^+e^−$ and with \cite{16} for polarized $e^+e^−$. The angular distribution of the leptons from the leptonic decay of $W$ is the same as that of the $b$ quarks because in the process $t \to bW \to b\ell\nu$ the $tbW$ and the $W\ell\nu$ vertices have the same Lorentz structure. Our expressions therefore coincide with the analogous formulae for the lepton angular distribution \cite{11}, if we replace $\alpha_b$ by $\alpha_\ell = 1$, $\Gamma_{t \to bW}$ by $\Gamma_{t \to b\bar{\ell}\nu}$, and $\theta_b$ by $\theta_\ell$.

6 CP violating asymmetries

The following relation between the differential cross sections of processes (1) and (2) must hold in the case of CP invariance:

\[
\frac{d\sigma^b_{\lambda\lambda'}(\vec{p}_t, \vec{p}_b)}{d\cos\theta_t d\Omega_b} = \frac{d\sigma^{\bar{b}}_{\lambda\lambda'}(\vec{p}_{\bar{t}}, \vec{p}_{\bar{b}})}{d\cos\theta_{\bar{t}} d\Omega_{\bar{b}}}
\]

(54)

Note that in this equation (and in the following ones), the first lower index of $\sigma$ denotes the longitudinal polarization of the electron and the second one that of the positron.

The electroweak dipole moment form factors $d_{\gamma,Z}^{\gamma,Z}(s)$ have both real and imaginary parts. Therefore we consider two types of observables: sensitive to $\Re d_{\gamma,Z}^{\gamma,Z}(s)$ and to $\Im d_{\gamma,Z}^{\gamma,Z}(s)$.

First we consider observables sensitive to $\Im d_{\gamma,Z}^{\gamma,Z}(s)$. They are determined by the absorptive part of the loops at the $t\bar{t}\gamma$ and $t\bar{t}Z$ vertices.

1. We shall consider two CP violating forward–backward asymmetries. Let $\sigma^{b(\bar{b})}_{F}(\theta_0, \lambda, \lambda')$ and $\sigma^{b(\bar{b})}_{B}(\theta_0, \lambda, \lambda')$ denote the number of $b$ and $\bar{b}$ quarks produced in the forward and backward hemispheres, respectively:

\[
\sigma^{b(\bar{b})}_{F}(\theta_0, \lambda, \lambda') = \int_{\theta_0}^{\pi/2} \left( \frac{d\sigma^{b(\bar{b})}_{\lambda,\lambda'}}{d\cos\theta_{b(\bar{b})}} \right) \sin\theta_{b(\bar{b})} d\theta_{b(\bar{b})}, \quad (55)
\]

\[
\sigma^{b(\bar{b})}_{B}(\theta_0, \lambda, \lambda') = \int_{\pi/2}^{\pi-\theta_0} \left( \frac{d\sigma^{b(\bar{b})}_{\lambda,\lambda'}}{d\cos\theta_{b(\bar{b})}} \right) \sin\theta_{b(\bar{b})} d\theta_{b(\bar{b})}.
\]

(56)

The standard forward–backward asymmetries of $b$ and $\bar{b}$

\[
A^{b(\bar{b})}_{FB}(\theta_0, \lambda, \lambda') = \frac{\sigma^{b(\bar{b})}_{F}(\theta_0, \lambda, \lambda') - \sigma^{b(\bar{b})}_{B}(\theta_0, \lambda, \lambda')}{\sigma^{b(\bar{b})}_{F}(\theta_0, \lambda, \lambda') + \sigma^{b(\bar{b})}_{B}(\theta_0, \lambda, \lambda')},
\]

(57)
define the CP violating asymmetry $\mathcal{A}_{\lambda\lambda'}^{FB}(\theta_0)$

$$\mathcal{A}_{\lambda\lambda'}^{FB}(\theta_0) = A_{FB}^b(\theta_0,\lambda,\lambda') + A_{FB}^\bar{b}(\theta_0,-\lambda',-\lambda).$$

From (57) we obtain, keeping only the terms linear in $\Im d^{\gamma,Z}(s)$,

$$\mathcal{A}_{\lambda\lambda'}^{FB}(\theta_0) = -6\alpha_b b \cos \theta_0 \left( 2 \cdot \frac{\Im H_2}{N_{tot}(\theta_0)} - 3 \sin^2 \theta_0 \cdot \frac{a_1^{SM}}{N_{tot}(\theta_0)} \cdot \frac{\Im H_1}{N_{tot}(\theta_0)} \right),$$

where $N_{tot}(\theta_0)$ corresponds to the total SM cross section (54) with a $\theta_0$ cut:

$$N_{tot}(\theta_0) = 3a_0^{SM} + \cos^2 \theta_0 a_2^{SM} = N_{tot} - \sin^2 \theta_0 a_2^{SM}.$$

An analogous asymmetry to $\mathcal{A}_{\lambda\lambda'}^{FB}(\theta_0)$ was considered in [11, 12].

The difference of the number of $b$ quarks produced in the forward hemisphere and that of $\bar{b}$ quarks produced in the backward hemisphere defines our second CP violating asymmetry

$$\mathcal{A}_{\lambda\lambda'}^{A}(\theta_0) = \sigma_{b}^{\lambda}(\theta_0,\lambda,\lambda') - \sigma_{\bar{b}}^{\bar{b}}(\theta_0,\lambda,\lambda') - \sigma_{\bar{b}}^{\lambda}(\theta_0,\lambda',-\lambda) + \sigma_{b}^{\bar{b}}(\theta_0,\lambda',-\lambda).$$

This asymmetry is given by

$$\mathcal{A}_{\lambda\lambda'}^{A}(\theta_0) = -12\alpha_b b (\cos \theta_0 \, \Im H_2 + \sin^2 \theta_0 \, \Im H_1) \frac{2}{2N_{tot}(\theta_0) + 3 \cos \theta_0 a_1^{SM}}.$$

2. In order to obtain information about $\Im H_1$ we define a “central” asymmetry $\mathcal{A}_{\lambda\lambda'}^{C}(\eta)$ that measures the difference between the number of $b$ and $\bar{b}$ quarks in the central production region [11, 12]:

$$\mathcal{A}_{\lambda\lambda'}^{C}(\eta) = \frac{\sigma_{\bar{b}}^{b}(\eta,\lambda,\lambda') - \sigma_{b}^{b}(\eta,-\lambda',-\lambda)}{\sigma_{\bar{b}}^{b}(\eta,\lambda,\lambda') + \sigma_{b}^{b}(\eta,-\lambda',-\lambda)}.$$

where

$$\sigma_{\bar{b}}^{b}(\eta,\lambda,\lambda') = \int_{\eta}^{\pi-\eta} \left( \frac{d \sigma_{\lambda\lambda'}^{b}}{d \cos \theta_{\bar{b}}(\eta)} \right) \sin \theta_{\bar{b}(\eta)} d \theta_{\bar{b}(\eta)}.$$

From (57) we obtain:

$$\mathcal{A}_{\lambda\lambda'}^{C}(\eta) = \frac{-6\alpha_b b \sin^2 \eta \, \Im H_1}{N_{tot} - \sin^2 \eta \, [(3b - 2\beta^2)F_1 + 3\alpha_b(b - \beta^2)G_3]}.$$

The magnitude of this asymmetry depends strongly on the value we choose for the angle $\eta$. A direct consequence of the CPT theorem is $\mathcal{A}_{\lambda\lambda'}^{C}(\eta=0) = 0.$
\[ \Im m H_1 \text{ can be measured also by the asymmetry } A^Z_{\lambda \lambda'}(\eta): \]

\[ A^Z_{\lambda \lambda'}(\eta) = A^b_Z(\eta, \lambda, \lambda') - A^b_Z(\eta, -\lambda', -\lambda), \quad (66) \]

where

\[ A^b_Z(\eta, \lambda, \lambda') = \frac{\sigma_C^{b(\hat{\eta})}(\eta, \lambda, \lambda') - \sigma_P^{b(\hat{\eta})}(\eta, \lambda, \lambda')}{\sigma_C^{b(\hat{\eta})}(\eta, \lambda, \lambda') + \sigma_P^{b(\hat{\eta})}(\eta, \lambda, \lambda')}. \quad (67) \]

Here \( \sigma_P^{b(\hat{\eta})} \) is defined as complementary to \( \sigma_C^{b(\hat{\eta})} \):

\[ \sigma_P^{b(\hat{\eta})}(\eta, \lambda, \lambda') := \sigma_{\text{tot}}^{b(\hat{\eta})} - \sigma_C^{b(\hat{\eta})}(\eta, \lambda, \lambda') = (\int_0^\eta + \int_{\pi-\eta}^\pi) \left( d \frac{\sigma_{\lambda, \lambda'}^{b(\hat{\eta})}}{d \cos \theta_{b(\hat{\eta})}} \right) \sin \theta_{b(\hat{\eta})} d \theta_{b(\hat{\eta})}. \quad (68) \]

For \( A^Z_{\lambda \lambda'} \) we obtain:

\[ A^Z_{\lambda \lambda'} = -24 \alpha_b b \cos \eta \sin^2 \eta \Im m H_1/\Im m H_1. \quad (69) \]

For \( \cos \eta = 1/\sqrt{3} \) this asymmetry is largest.

The asymmetries \( A^E_{\lambda \lambda'}(\theta_0), A^A_{\lambda \lambda'}(\theta_0), A^C_{\lambda \lambda'}(\eta), \) and \( A^Z_{\lambda \lambda'}(\eta) \) are different from zero if CP invariance is violated in \( (4) \) and \( (4) \). Notice that the imaginary parts of \( d^{\gamma, Z}(s) \) enter in eqs. \( (\frac{3}{3}), (\frac{2}{2}), (\frac{5}{5}), \) and \( (\frac{6}{6}) \) as \( A^E_{\lambda \lambda'}(\theta_0), A^A_{\lambda \lambda'}(\theta_0), A^C_{\lambda \lambda'}(\eta) \) and \( A^Z_{\lambda \lambda'}(\eta) \) are even under time reversal transformation, in other words, the \( i \) in front of \( d^{\gamma, Z}(s) \) in the vertices \( V_\gamma \) and \( V_Z \) is compensated by the \( i \) in front of \( \Im m H_i \). If \( \lambda = \lambda' = 0 \) these asymmetries violate both C and CP invariance, and in accordance with this the analytic expressions are proportional to the C–odd and CP–odd functions \( H_i \). A measurement of these asymmetries allows one to determine the imaginary parts of the dipole moment form factors \( d^\gamma(s) \) and \( d^Z(s) \).

3. The real parts of \( d^{\gamma, Z}(s) \) can be singled out by measuring triple product correlations \( (\frac{3}{3}), (\frac{4}{4}) \). A suitable asymmetry is given by \( (\frac{8}{8}) \)

\[ A^T_{n(\lambda, \lambda')}(\eta) = \left( N[\langle \mathbf{q}_e \cdot \mathbf{p}_{t(\hat{\ell})}^T \mathbf{p}_{b(\hat{\eta})} > 0] - N[\langle \mathbf{q}_e \cdot \mathbf{p}_{t(\hat{\ell})}^T \mathbf{p}_{b(\hat{\eta})} < 0] \right)/\sigma_{\text{tot}}^{b(\hat{\eta})}, \quad (70) \]

where \( \langle \mathbf{q}_e \cdot \mathbf{p}_{t(\hat{\ell})}^T \mathbf{p}_{b(\hat{\eta})} \rangle = \mathbf{q}_e \cdot (\mathbf{p}_{t(\hat{\ell})}^T \times \mathbf{p}_{b(\hat{\eta})}) = \sin \theta_{t(\hat{\ell})} \sin \theta_{b(\hat{\eta})} \sin \phi_{b(\hat{\eta})} \). As \( \sin \theta_{t(\hat{\ell})}, \sin \theta_{b(\hat{\eta})} > 0 \), \( N[\langle \mathbf{q}_e \cdot \mathbf{p}_{t(\hat{\ell})}^T \mathbf{p}_{b(\hat{\eta})} > 0 | < 0] \) are the number of \( b(\hat{\eta}) \) quarks produced above/below (with \( \sin \phi_{b(\hat{\eta})} > 0 | < 0 \)) the production–plane \( \{ \mathbf{q}_e, \mathbf{p}_{t(\hat{\ell})} \} \) with given polarization \( \lambda, \lambda' \).
As in general $D$ gets also CP invariant contributions from absorptive parts in the SM amplitude, the truly CP violating contribution will be singled out through the difference:

$$A^T_{\lambda\lambda'} = A^T_b(\lambda,\lambda') - A^T_b(-\lambda',-\lambda)$$ (71)

where $A^T_b(-\lambda',-\lambda)$ refers to process $\bar{t}t\gamma$. A non–zero value of (71) would imply CP violation in the $t\bar{t}\gamma$ and/or $t\bar{t}Z$ vertices. From (45) we obtain:

$$A^T_{\lambda\lambda'} = -\alpha_b\frac{3\beta\pi\sqrt{s}}{2m_t} \Re D_1.$$ (72)

It is also possible to define a triple product asymmetry for determining $\Re D_2$. It is necessary to consider not only the space above and below the production plane as in eq.(71), but also in addition the forward and backward region with respect to the direction of the top quark.

The asymmetry $A^T_{\lambda\lambda'}$ is different from zero if CP invariance is violated in (1) and (2). We have obtained the analytic expression Eq. (71) for $A^T_{\lambda\lambda'}$ assuming CP violation in the production process only. However, the same expression will hold if CP violation occurs also in the decay vertex, i.e. $A^T_{\lambda\lambda'}$ is insensitive to CP violation in $t \to bW$. In order to measure CP violation in $t \to bW$ through triple product correlations one has to consider the three–body decay $t \to b\ell\nu$. As $A^T_{\lambda\lambda'}$ is odd under time reversal, the real parts of $d^{t\gamma,Z}(s)$ ($\Re D_i$) enter in (71). $A^T_{\lambda\lambda'}$ violates both P and CP invariance for $\lambda = \lambda' = 0$. Accordingly, the analytic expression obtained is proportional to $D_1$, which is a P–odd and CP–odd combination of the coupling constants.

4. The CP violating angular asymmetries as defined above determine $H_i$ and $D_i$ and thus depend on the beam polarization. The beam polarization can strongly enhance (or decrease) the effects we are interested in. Measurements performed with opposite beam polarizations can be used to disentangle $H_i^0$ from $D_i^0$. We define the following polarization asymmetries analogous to the standard forward–backward asymmetry (77):

$$P_{FB}^{b(\bar{b})}(\theta_0) = \frac{(1 - \lambda \lambda')}{(\lambda - \lambda')} \cdot \left( \frac{\sigma^b_F(\theta_0,\lambda,\lambda') - \sigma^b_B(\theta_0,\lambda,\lambda')}{\sigma^b_F + \sigma^b_B(\theta_0,\lambda,\lambda') + \sigma^b_B(\theta_0,\lambda,\lambda')} \right) - \frac{(\sigma^b_F(\theta_0,\lambda',-\lambda) - \sigma^b_B(\theta_0,\lambda',-\lambda))}{\sigma^b_F + \sigma^b_B(\theta_0,\lambda',-\lambda') + \sigma^b_B(\theta_0,\lambda',-\lambda')}.$$ (73)

Then the CP violating asymmetry is

$$P_{FB}^{FB}(\theta_0) = P_{FB}^{b}(\theta_0) + P_{FB}^{\bar{b}}(\theta_0).$$ (74)
Again, keeping only the terms linear in $\Im d^\gamma Z(s)$, we obtain:

$$P^{FB}_{(\theta_0)} = -6\alpha_b \cos \theta_0 \left( 2 \cdot \frac{\Im D_2^0}{N_{tot}^0(\theta_0)} - 3 \sin^3 \theta_0 \cdot \frac{a_{1A}^{SM}}{N_{tot}^0(\theta_0)} - \frac{3 \Im H_1^0}{N_{tot}^0(\theta_0)} \right),$$

where

$$N_{tot}^0(\theta_0) = N_{tot}(\theta_0, \lambda = \lambda' = 0)$$

$$a_{1A}^{SM} = 2bG_3^0 - \alpha_b \left( (1 + \beta^2 - 2b)F_1^0 + (1 - \beta^2)F_2^0 \right).$$

We define a “central” polarization asymmetry $P^C(\eta)$

$$P^C(\eta) = \frac{(1 - \lambda\lambda') (\sigma_C^b - \sigma_C^\dagger)_{(\eta, -\lambda, \lambda')} - (\sigma_C^b - \sigma_C^\dagger)_{(\eta, -\lambda, -\lambda')}}{(\lambda - \lambda') (\sigma_C^b + \sigma_C^\dagger)_{(\eta, \lambda, \lambda')} + (\sigma_C^b + \sigma_C^\dagger)_{(\eta, -\lambda, -\lambda')}}$$

in order to measure $D_1^0$:

$$P^C(\eta) = \frac{-6\alpha_b \sin^2 \eta \cdot \Im D_1^0}{N_{tot}^0 - \sin^2 \eta \left( (3b - 2\beta^2)F_1^0 + 3\alpha_b (b - \beta^2)G_3^0 \right)},$$

where $N_{tot}^0 = N_{tot}(\lambda = \lambda' = 0)$.

### 7 Numerical results within MSSM

So far our formulae are general and model independent. Now we want to give numerical results for the CP violating observables eqs.\((72), (79), (84), (89), \text{ and } (71)\), defined in the previous section. We use the results for the electroweak dipole moment form factors $d^\gamma(s)$ and $d^Z(s)$ as obtained in \[4\]. There $d^\gamma(s)$ and $d^Z(s)$ were calculated within the MSSM \[17\] where we allowed for complex parameters. We included gluino, chargino, and neutralino exchange in the loop of the $\gamma t\bar{t}$ and $Z t\bar{t}$ vertex.

The observable quantities $A^{FB}_{AA'}(\theta_0)$, $A^{A}_{AA'}(\eta)$, $A^{Z}_{AA'}(\eta)$, $A^{C}_{AA'}(\eta)$, and $A^{T}_{AA'}$ depend on $d^\gamma(s)$ and $d^Z(s)$ and therefore on the parameters $M'$, $M$, and $m_{\tilde{g}}$, the mass parameters of the gauge groups $U(1)$, $SU(2)$, and $SU(3)$, respectively, $|\mu|$, the higgsino mass parameter, $\tan \beta = v_2/v_1$, with $v_i$ being the real vacuum expectation values of the Higgs fields, $m_{\tilde{t}_1}$, $m_{\tilde{b}_1}$, the masses of the two stops and the two sbottoms, $\theta_t$, $\theta_b$, their mixing angles, $\varphi_t$, $\varphi_b$, their mixing phases, and $\varphi_\mu$, the phase of the higgsino mass parameter. We use the GUT relations $m_{\tilde{g}} = (\alpha_s/\alpha_2)M \approx 3M$ and $M' = \frac{5}{3} \tan^2 \Theta_W M$, and take $m_{\tilde{w}} = 80$ GeV, $m_t = 175$ GeV, $m_b = 5$ GeV, $\sqrt{s} = 500$ GeV, $\alpha_s(\sqrt{s}) = 0.1$, and $\alpha_{em}(\sqrt{s}) = \frac{1}{123}$. For the parameters we choose:
\[
\begin{array}{|c|c|c|c|}
\hline
M & 230 \text{ GeV} & m_{\tilde{t}_1} & 150 \text{ GeV} \\
|\mu| & 250 \text{ GeV} & m_{\tilde{t}_2} & 400 \text{ GeV} \\
\tan \beta & 3 & \theta_{\tilde{t}} & \frac{\pi}{9} \\
\varphi_\mu & = \frac{4\pi}{3} & \varphi_{\tilde{t}} & = \frac{\pi}{6} \\
\hline
\end{array}
\]

Furthermore, the asymmetries depend on the polarizations of the electron and positron beams \((\lambda, \lambda')\), on the polar angle \(\theta_0\), (see eq.(62)) describing the forward–backward cut, and the polar angle \(\eta\) (see eqs.(69), (65)). For these quantities we take:

\[
\begin{array}{c}
\lambda = -\lambda' = -0.8, 0, 0.8 \\
\theta_0 = \frac{\pi}{12} \\
\cos \eta = \frac{1}{\sqrt{3}}
\end{array}
\]

For \(\lambda = -1\) the electron is purely left–handed. In all figures we show the asymmetries for \(\lambda = -\lambda' = \{0, -0.8, 0.8\}\) (full line, dashed line, dotted line).

In Fig. 1a we show \(A^{FB}_{\lambda\lambda'}(\theta_0)\) as a function of \(\sqrt{s}\) for our parameter set and different polarizations of the electron–beam. The curves exhibit spikes due to thresholds of intermediary particles in the \(t\bar{t}\)–production. The spikes are already present in the dipole moment form factors \(\Im m d^\gamma(s)\) and \(\Im m d^Z(s)\), as discussed in detail in [4].

Notice that the size of the asymmetry strongly depends on the polarization of the electrons. For \(\sqrt{s} \lesssim 700 \text{ GeV}\) it is much bigger (of order \(0.2 \times 10^{-3}\)) if the electrons are left polarized. Quite generally \(A^{FB}_{\lambda\lambda'}(\theta_0) \gtrsim 2A^{A}_{\lambda\lambda'}(\theta_0)\) for \(\sqrt{s} \gtrsim 500 \text{ GeV}\).

The dependence of \(A^{Z}_{\lambda\lambda'}(\eta)\) on \(\sqrt{s}\), shown in Fig. 2a, is very similar to that of \(A^{FB}_{\lambda\lambda'}(\theta_0)\) just discussed. It is, however, roughly 1.5 times bigger than \(A^{FB}_{\lambda\lambda'}(\theta_0)\) and two times as big as \(A^{C}_{\lambda\lambda'}(\eta)\). For the value \(\cos \eta = \frac{1}{\sqrt{3}}\) \(A^{Z}_{\lambda\lambda'}(\eta)\) is maximal.

In Fig. 1b we show the dependence of the asymmetry \(A^{FB}_{\lambda\lambda'}(\theta_0)\) on the mass parameter \(M\). Notice the interesting difference between left–handed and right–handed electrons in the lower mass region \(M \lesssim 360 \text{ GeV}\). \(A^{Z}_{\lambda\lambda'}(\eta)\) again shows the same behaviour being 1.5 times bigger. The decrease of \(A^{FB}_{\lambda\lambda'}(\theta_0)\) and \(A^{Z}_{\lambda\lambda'}(\eta)\) for bigger \(M\) can be explained by the fact that also the dipole moment form factors \(\Im m d^\gamma(s)\) and \(\Im m d^Z(s)\) decrease because the gaugino–higgsino mixing of the charginos and neutralinos becomes weaker.

The difference between left–handed and right–handed electrons is again apparent in the \(\varphi_\mu\) dependence of \(A^{FB}_{\lambda\lambda'}(\theta_0)\) as shown in Fig. 1c. In the case of left–handed electrons the asymmetry shows a \(\sin \varphi_\mu\) behaviour, whereas in the case of right–handed electrons practically no \(\varphi_\mu\) dependence is seen. When the CP violating phase \(\varphi_\mu\) vanishes, one has still an asymmetry due to the phase \(\varphi_{\tilde{t}}\) in the mass matrix of the \(\tilde{t}\) squarks.
In Fig. 2b one sees the different contributions to $A_{\lambda'\lambda}(\eta)$. The gluino is dominating only for small $M \lesssim 130$ GeV. For $M \gtrsim 130$ GeV the charginos give the main contribution, and for $M \gtrsim 230$ GeV the neutralinos are more important than the gluino. This analysis of the contributing diagrams holds for all asymmetries, $A_{\lambda'\lambda}(\theta_0)$, $A_{\lambda'\lambda}(\theta_0)$, $A_{\lambda'\lambda}(\eta)$, and $A_{\lambda'\lambda}(\eta)$, that depend on the imaginary parts of the dipole moment form factors. For the triple–product asymmetry $A_{\lambda'\lambda}$ the situation is a bit different. Although the gluino and the chargino contributions are quite big, they have opposite signs and therefore the neutralino contribution plays a more important role than in the other asymmetries.

The dependence of $A_{\lambda'\lambda}(\eta)$ on $\tan \beta$ is shown in Fig. 2c. An interesting fact is the very weak dependence on the polarization for large $\tan \beta$. There is a strong decrease of the asymmetry in $2 \lesssim \tan \beta \lesssim 10$. On the other hand the triple product asymmetry $A_{\lambda'\lambda}$ has nearly no dependence on $\tan \beta$ (not shown here).

In Fig. 3a we show the dependence of $A_{\lambda'\lambda}$ on $\sqrt{s}$. The similarity between this plot and the plots for $\text{Re} d(s)$ and $\text{Re} d'(s)$ in [3] can be clearly seen. The spikes at $\sqrt{s} = 400$ GeV and $\sqrt{s} = 590$ GeV are due to the thresholds of $\tilde{\chi}_1^+ \tilde{\chi}_1^-$ production with $m_{\tilde{\chi}_1^+} = 200$ GeV and $\tilde{\chi}_2^+ \tilde{\chi}_2^-$ production with $m_{\tilde{\chi}_2^+} = 295$ GeV respectively.

Fig. 3b shows the dependence of $A_{\lambda'\lambda}$ on $M$. Again the behaviour reflects that of the dipole moment form factors $\text{Re} d(s)$ and $\text{Re} d'(s)$. There is a big difference between left and right polarized electrons.

The dependence of the triple–product asymmetry on the phase $\varphi_\mu$, as shown in Fig. 3c, is weaker than for the other asymmetries.

The asymmetries $A_{\lambda'\lambda}(\theta_0)$, $A_{\lambda'\lambda}(\theta_0)$, $A_{\lambda'\lambda}(\eta)$, and $A_{\lambda'\lambda}(\eta)$ have all the same shape due to the similarity of the imaginary parts of the dipole moment form factors. The difference of $A_{\lambda'\lambda}(\eta)$ and $A_{\lambda'\lambda}(\eta)$ in the dependence of $\eta$ comes from the fact, that the denominator in $A_{\lambda'\lambda}(\eta)$ decreases proportionally to $\cos \eta$ and therefore this factor cancels. Although $A_{\lambda'\lambda}(\eta)$ reaches its maximum for $\cos \eta = 0$, there is no parameter space left for the measurement. Thus the best value for $\eta$ should be $\cos \eta = \frac{1}{\sqrt{3}}$, the maximum value for $A_{\lambda'\lambda}(\eta)$.

8 Summary and Conclusions

The process $e^+e^- \rightarrow t\bar{t}$, with $t(\bar{t})$ decaying into $W^+b(W^-\bar{b})$, is well suited to study CP violating effects beyond the Standard Model. We have derived analytic formulae
for the cross section and the angular distributions of the $b$ and $\bar{b}$ quarks assuming CP violation in the $\gamma t\bar{t}$ and $Zt\bar{t}$ vertices. Particular attention was paid to the $t(\bar{t})$ polarization. We then defined appropriate independent CP violating asymmetries, which quite generally allow one to determine both the real and imaginary parts of the electroweak dipole moment form factors of the top, $d\gamma(s)$ and $dZ(s)$. It was possible to integrate over the whole phase space and to obtain rather simple analytic expressions for these asymmetries. Thus far our study was quite general. We also performed a numerical analysis of these asymmetries within the Minimal Supersymmetric Standard Model with complex parameters. In this model the asymmetries turn out to be of order $\lesssim 10^{-3}$.

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Figure Captions

Figure 1: The asymmetry $A_{\lambda\lambda'}^{F\nu}(\theta_0)$ eq.(59) for the reference parameter set and a longitudinal electron beam polarization of -80% (dashed line), 0% (full line), and 80% (dotted line) depending on (a) $\sqrt{s}$ [GeV], (b) $M$ [GeV], and (c) $\varphi_\mu$.

Figure 2: The asymmetry $A_{\lambda\lambda'}^{Z\nu}(\eta)$ eq.(69) for the reference parameter set.
(a) Depending on $\sqrt{s}$ [GeV] for a longitudinal electron beam polarization of -80% (dashed line), 0% (full line), and 80% (dotted line).
(b) Depending on $M$ [GeV] for a longitudinal electron beam polarization of -80% (thick dashed line) together with the different contributions: chargino (thin dashed line), neutralino (thin dotted line), and gluino (thin dashed–dotted line).
(c) Depending on $\varphi_\mu$ for a longitudinal electron beam polarization of -80% (dashed line), 0% (full line), and 80% (dotted line).

Figure 3: The asymmetry $A_{\lambda\lambda'}^{T\nu}$ eq.(71) for the reference parameter set and a longitudinal electron beam polarization of -80% (dashed line), 0% (full line), and 80% (dotted line) depending on (a) $\sqrt{s}$ [GeV], (b) $M$ [GeV], and (c) $\varphi_\mu$. 
Figure 1: $A^{FB}_{\lambda\lambda'}$
Figure 2: $A_{\lambda\lambda'}^Z$
Figure 3: $A_{\lambda\lambda'}^T$