Numerical investigations on entropy generation in cross-wavy channels

Xusheng Shi1,2, Yongwei Wang1,2* and Xiulan Huai1,2*

1Institute of Engineering Thermophysics, Chinese Academy of Sciences, Beijing, 100190, China
2University of the Chinese Academy of Sciences, Beijing, 100049, China
*Corresponding author: wangyongwei@iet.cn (Y. Wang), hxl@iet.cn (X. Huai)

Abstract. The numerical study to research the influence of ratio of channel amplitude to pitch of cross-wavy on entropy generation is conducted in this paper. In Reynolds number range of 200-1200, five types of cross-wavy channel are analyzed. The simulation results show the thermal entropy generation rising rate is fast when Reynolds number less than 700 and then flat. In addition, it is smaller with increasing channel amplitude to pitch value. The frictional entropy generation increases almost linearly with Reynolds number. However, it is much less than thermal entropy generation under the same conditions. In brief, the channels which have larger ratio of amplitude to pitch can reduce the total irreversibility to the least under the same flow conditions.

1. Introduction

In recent years, distributed power generation system develops rapidly and micro gas turbine is one of the core power equipment. For micro gas turbine, the thermal efficiency is only about 20% with simple cycle, while over 30% with recuperated cycle [1]. Therefore, recuperator which is the key part of gas turbine is often used in recuperated cycle to recover waste heat of exhaust gas and increase the inlet temperature of combustion. Cross-wavy (CW) primary surface heat exchangers have the advantages of high heat exchange efficiency, low pressure loss, compacted structure and light weight, and so on [2]. Therefore, it is often used as recuperator in gas turbine. Due to its compacted structure and small hydraulic diameter, the investigation of thermal-hydraulic performance in CW primary surface is useful.

Recently, many scholars have done lots of interesting study on CW primary surface by the method of experimental study and numerical simulation. Xiao et al. [3] reviewed recuperators used in micro gas turbines. The research provides a comprehensive understanding about optimization methods of recuperators. Stasiek and Ciofalo et al. [4, 5] researched thermal-hydraulic performance of corrugated passages. They obtained the local and average Nusselt number distribution data over channel surface. Utriainen and Sunden [6] studied thermal-hydraulic performance of cross-wavy duct by utilizing numerical simulation method. The results found amplitude of the duct have the great influence on heat transfer enhancement. Du et al. [7] implemented a three-dimensional numerical model of CW channel and investigated the influence of the value of the ratio L/A (wave length L, unit cell amplitude A) on comprehensive performance of channel.

Heat exchanger as an exchanging heat experiment is widely used to high energy-consuming sectors. The entropy generation analysis provides an insight that cannot be achieved by the energy analysis.
There is little research about the entropy generation in CW channels. In this paper, the influences of ratio of amplitude to pitch ($A/P$) on entropy generation are studied. It is meaningful to reduce energy consumption and improve the performance of recuperators.

2. Mathematical model

CW primary surface recuperator consists of many CW plates which are stacked and welded, as shown in figure 1. In the flow direction, the channels are corrugated. The hot channel and cold channel are alternating and this makes the structure more compact. The geometry parameters of the hot and cold channel, such as amplitude $A$, unit length $L$, pitch $P$ and channel inner height $H$, are shown in figure 1.

To reduce the computational complexity, three flow units of air channel are created as the numerical model, as shown in figure 2. Due to the flow and structure are periodic, the periodic boundary conditions can be used. It can be applied in the $S_1-1$ and $S_1-2$, $S_2-1$ and $S_2-2$, $S_3-1$ and $S_3-2$, $S_4-1$ and $S_4-2$, $S_5-1$ and $S_5-2$, $S_6-1$ and $S_6-2$. More details about multi-periodic boundary conditions can be found in literatures[8].

Given the influence of geometry parameters on distribution of the flow field of the channel, the dimensionless parameters $A/P$ is used as variable to research entropy generation performance of CW channel. The values of $A/P$ for different CW channel are 0.2333, 0.3333, 0.5000, 0.6667 and 0.8333, respectively.
2.1 Governing equations
Zhang and Che [9] researched the performance of cross-corrugated plates using various turbulence models, and the results of low Reynolds number turbulence (LBKE) model agree with experiments. Therefore, the LBKE turbulent model is used.

In this model, the continuum equation, momentum equation and energy equation are respectively expressed as [10]:

\[
\frac{\partial (\rho u_i)}{\partial x_i} = 0 \quad (1)
\]

\[
\frac{\partial (\rho u_i u_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \mu_{eff} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \mu_{eff} \frac{\partial u_i}{\partial x_j} \right) - \frac{\partial p_i}{\partial x_j} + \rho g_i \quad (2)
\]

\[
\frac{\partial (\rho u_i c_p T)}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \lambda_{eff} \frac{\partial T}{\partial x_i} \right) + \Phi \quad (3)
\]

where \( \Phi \) is the energy dissipation due to viscosity, \( \mu_{eff} \) is effective viscosity, which is equal to the sum of molecular viscosity \( \mu \) and turbulent viscosity \( \mu_t \).

2.2 Calculation of entropy generation rate
The thermal and frictional volumetric entropy generation rate are respectively defined as [11]:

\[
S_{g,AT}^{*} = \frac{\lambda_{eff}}{T^2} \left[ \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 + \left( \frac{\partial T}{\partial z} \right)^2 \right] \quad (4)
\]

\[
S_{g,AP}^{*} = \frac{\mu_{eff}}{T} \left[ 2 \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial x} \right)^2 \right] + \left[ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right]^2 + \left[ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right]^2 + \left[ \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right]^2 \right] \quad (5)
\]

Where \( \mu_{eff} \) is effective viscosity, \( \lambda_{eff} \) is effective thermal conductivity which is equal to the sum of thermal conductivity \( \lambda \) and turbulent conductivity \( \lambda_{t} \).

The total volumetric entropy generation:

\[
S_g = S_{g,AT}^{*} + S_{g,AP}^{*} \quad (6)
\]

The corresponding dimensionless entropy generation rate [12]:

\[
N_{g,AT} = \iint_V \frac{S_{g,AT}^{*}}{\lambda / D_h^2} dV \quad (7)
\]

\[
N_{g,AP} = \iint_V \frac{S_{g,AP}^{*}}{\lambda / D_h^2} dV \quad (8)
\]

\[
N_g = \iint_V \frac{S_g^{*}}{\lambda / D_h^2} dV \quad (9)
\]

where \( V \) is fluid region.

3. Grid independence and validation
Different grid systems are used to test the grid independence for model of type 2 with \( Re = 451 \). The results shown average Nusselt number changes only 1.38% when grid number ranged from 7,262,830 to 9,230,730. Therefore, the grid number of 7,262,830 is adopted.
To demonstrate the accuracy of present numerical method, numerical results and experimental data studied by Ma et al. [13] are shown in Figure 3. From the figure, the maximum error of $Nu$ and $f$ are about 13.5% and 10.6%, respectively. The results agree well with the experimental data.

![Figure 3. Numerical method validation.](image)

4. Results and discussion

4.1 Volumetric entropy generation distribution

Figure 4 shows $S_{g,AT}$ distributions of type 2. At $z = 0.5L$, the maximum of $S_{g,AT}$ appeared at the channel center. However, at $z = L$, the value of $S_{g,AT}$ near the channel wall is higher than channel center. This is because the fluid mixing of adjacent passages is intense and the temperature gradient increases.

![Figure 4. Thermal entropy generation contours on several cross-sections of type 2.](image)

Figure 5 shows $S_{g,AP}$ distributions of type 2. At $z = 0.5L$, it can be seen that the maximum of $S_{g,AP}$ appeared at the center of the channel where the mixture of fluid is intense. However, at $z = L$, the value of $S_{g,AP}$ near the channel wall where the fluid mixing of adjacent passages is intense is higher than the center of the channel. In addition, the velocity gradient increasing results in $S_{g,AP}$ greater when Reynolds number increases.
4.2 Effects of channel amplitude to pitch value on entropy generation

Figure 6 shows the influence of A/P values on $N_{g,AT}$ and $N_{g,AP}$ for different Reynolds number. As shown in figure 6(a), $N_{g,AT}$ rising rate is fast when Reynolds number less than 700 and then flat. This is because the thermal boundary layer thickness is thinner and the temperature gradient increases with Reynolds number increases when Reynolds number less than 700. However, the thickness of thermal boundary layer has a little change with Reynolds number when it larger than 700. Besides, the greater A/P value is, the lower $N_{g,AT}$ becomes. For example, $N_{g,AT}$ has decreased by 32.9% at $Re = 1134$ with A/P value from 0.2333-0.8333. It is due to the greater channel amplitude results in the more drastic deflection of the fluid flow and fluid mixing which reduce the temperature gradient. Therefore, the thermal irreversibility of type 5 is the minimal. From figure 6(b), $N_{g,AP}$ increases almost linearly with Reynolds number. In addition, the $N_{g,AP}$ value of type 5 is greater when Reynolds number is larger than 700. This is because the greater amplitude of the channel results in more intense friction and impingement between fluid and channel wall. However, $N_{g,AP}$ is much smaller than $N_{g,AT}$ under same conditions.

As shown in figure 7, $N_g$ and $N_{g,AT}$ have the same variation trend. The $N_g$ value of type 5 is lowest under same conditions. Therefore, to minimize the irreversibility of heat transfer and fluid flow in cross-wavy channel, values of type 5 should be preferable considered.
Conclusion
In this paper, the influence of dimensionless parameter $A/P$ and Reynolds number on $N_{e,AT}$ and $N_{e,AP}$ distributions at typical cross-sections are presented. The simulation results show $N_{e,AT}$ rising rate is fast when Reynolds number less than 700 and then flat. In addition, it is smaller with increasing channel amplitude to pitch value. For example, thermal entropy generation has decreased by 32.9% at $Re = 1134$ with $A/P$ value from 0.2333-0.8333. However, $N_{e,AP}$ increases almost linearly with Reynolds number.

In a word, CW channels which have larger $A/P$ value can reduce the total irreversibility to the least under the same flow conditions. And the lower Reynolds number also can reduce the total irreversibility.

Acknowledgment
This study was supported by the Key Research Program of the Chinese Academy of Sciences (ZDRW-CN-2017-2).

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