Analysis of students’ verbal and written mathematical communication error in solving word problem

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Abstract. Communication is an important ability for every human being to have. In this research, the focus of communication is on students' mathematical communication skills. Mathematical communication is divided into two parts, namely written communication and verbal communication. If students have good verbal and written mathematical communication skills, then they are easier to express ideas and strategies and are able to write the stages of problem solving well, especially word problem. This is because word problem requires special attention in their solution. Therefore, it is necessary to analyze mathematical communication errors experienced by students in solving mathematical word problem. This study aims to describe the students' written and oral mathematical communication skills in solving word problem. This type of research is qualitative-descriptive. The research subjects were three junior high school students. Based on the results of the study, there were still students who could not communicate their mathematical ideas well. Students' written communication error is in converting problem sentences into mathematical models. Student’s writing problem solving are still not in order or coherent. The research finding indicated that the students' verbal communication skills are better than their written communication skills.

1. Introduction
Mathematical communication is one of the most decisive factors in the process of solving mathematical problems. The urgency on aspects of mathematical communication in solving problems has been explained in the National Council of Teachers of Mathematics (NCTM) which states that mathematical communication is one of the basic abilities owned by children in solving mathematical problems [1]. NCTM states there are four mathematical communication standards in mathematics learning. The four mathematical communication standards are students can (1) organize and consolidate mathematical thinking through communication, (2) communicate mathematical thinking coherently and clearly to peers, teachers, and others, (3) analyze and evaluate mathematical ideas and strategies of people other; and (4) using mathematical language to present mathematical ideas correctly.

Good mathematical communication can provide opportunities for students to present ideas and strategies to solve mathematical problems clearly [2]. When students can convey their ideas in solving mathematical problems properly, it can facilitate the teacher in identifying the concept or procedure errors experienced by students [3]. Even though the mathematical ideas that students communicate are wrong, when the teacher can understand the purpose of the student's answers, the teacher will be able
to detect where the errors experienced by students. Conversely, if students have good ideas but students cannot communicate properly then the teacher will have difficulty in determining the right or wrong mathematical ideas that are communicated by students [4]. Students who can communicate their ideas well will indirectly affect the teacher's assessment of students' answers. The correct mathematical idea will be wrong if it is not well communicated because of the wrong interpretation of the person who reads or evaluates the student's answer [5]. Thus the mathematical communication skills are very important owned by students.

To be able to measure the ability of mathematical communication skills, the need for mathematical communication indicators. There are three indicators to measure the ability of mathematical communication, namely (1) interpreting the purpose of the problem by paying attention to the statement of purpose and completeness of the writing of the information provided, (2) displaying justification by paying attention to the type of justification as well as the truth of the completeness of the justification. The types of justification in question are vague / broad statement (explanation that cannot be understood), rule (use of special rules, algorithms, or definitions), procedural description (explanation of what has been done at a step, without an explanation of why the step is valid), relational justification (explanation of why a step is valid, suggesting an understanding link), (3) The use of representations by observing the types of representations and the accuracy and completeness of the representations used. The types of representation in question are verbal representations (students' own language and mathematical terminology), iconic representations (schematics or pictures), and symbolic representations (numeric symbols and / or algebra) [6].

Furthermore, there is the ability of mathematical communication can be improved by giving mathematical problems, especially word problems [6]. This is because in solving word problems, students need to make interpretations, justifications, and representations which are important components in mathematical communication [7]. Word problems are mathematical problems in the context of everyday life presented in the form of mathematical sentences to be solved [8]. Word problems can provide opportunities for students to develop skills in solving mathematical problems related to everyday life [1]. In solving word problems, students must translate the questions and change the mathematical sentences into mathematical models. The process of converting a problem sentence into a mathematical model is called modelling [9]. In this modelling process, students often make mistakes during problem solving. This modelling error is related to mathematical communication indicators namely the use of representations.

Based on a research result, mistakes that are often made by students in solving mathematical problems are errors in interpreting problem sentences and making mathematical models. This is in accordance with the results of previous studies that most of the research subjects had difficulty interpreting and making mathematical models when faced with a particular problem [10]. Another study showed the same results that most of the research subjects made mistakes in choosing and determining strategies to solve problems. One of the errors was in modeling problems in mathematical sentences [11]. This error causes incorrect written mathematical communication. Modelling is one of the most difficult steps for students to do. Errors in interpreting problem sentences will have an impact on students' mistakes in communicating their mathematical ideas [12]. Seeing this explanation, it is important to be able to see written mathematical communication errors when understanding story problems. Sometimes students' written communication is still not good will impact on the difficulty of the evaluation conducted by the teacher. Therefore, it is important to pay attention to students' verbal communication skills. As teachers, if we can find out the mistakes, we can take action according to the students' scaffolding needs. For students, this can train students in modelling the sentence problem so that the ideas communicated can be in accordance with the purpose of the given word problem. Previous studies that examined mathematical communication errors only focused on one form of communication that is verbal or written only (most of them are written because indicators can easily be made), in this study researchers used 2 forms of communication namely written and verbal in hopes the results shown are increasingly valid and can find out the subject's weaknesses accurately. This
implies that the main topic in this study is to find the location of mathematical communication errors both written and verbal subjects in solving word problems.

2. Literature Review
Communication is a fairly important part in school mathematics. Furthermore, NCTM explained that communication can help students understand mathematics well [1]. Not only that, students who do mathematical reasoning on a concept or solve mathematical problems well, then communicate it with other students can build the ability to convince their opinions to others. Students who listen to their peers' opinions on mathematics will also enrich their knowledge and sharpen their mathematical thinking. Students are expected to be able to solve problems systematically which are helped by good communication skills in the community later [13]. Mathematical communication is divided into two major parts, namely written and verbal mathematical communication [14].

Written mathematical communication is an intellectual activity that allows students to show their mathematical ideas through writing [14]. Written communication allows students to explain and illustrate deep mathematical understanding [15]. Mathematical communication of students can be used by teachers as a tool to measure how far students understand mathematical material. For example, by asking students to write coherent problem solving, the teacher can assess how far students understand mathematical concepts. Students who write up the process of coherence will experience an increase in the ability to solve problems [16]. Students who write down their thought processes in solving a problem will more easily solve more complex problems and can evaluate their work independently [17].

Oral mathematical communication in school mathematics is when students are able to say, express, or discuss their mathematical ideas or understanding in everyday language [1]. NCTM added that by using everyday language in communicating mathematical ideas, they will slowly understand the definition of a mathematical concept with more formal language. Discussion in study groups is very important to be applied in mathematics learning. Conveying thoughts into words that will be conveyed in discussion helps students in compiling their reasoning for a problem [16]. This is reinforced by the opinion [15] which states that one of the five competencies in communication skills is discussion. Oral mathematical communication skills in the form of discussion skills can be raised by students by continuously training them to share mathematical ideas and convey the work to a group of friends.

3. Method
This research is a qualitative-descriptive type because researchers will describe mathematical communication errors made by students in solving word problems. This research aims to describe a variable, symptom, or situation as it is. In addition, qualitative research is a research procedure that produces descriptive data in the form of written or verbal words of the behavior observed students [18]. The research was conducted at one of Junior High School in the province of East Java, Indonesia. Researchers gave written tests to 30 students. Based on 30 results of student work, researchers selected three results of student work that will be discussed further. The three subjects were then coded Y1, Y2, and Y3. The three results of the subjects' work would be analyzed in depth related to written and verbal mathematical communication in solving the given word problem. Indicators used to analyze students' written mathematical communication are obtained by combining mathematical communication standards presented in NCTM and written mathematical communication indicators and then adjusted to the step of solving word problems [1,6,19]. In addition, the subject will also be given an interview to find out how the subject's verbal communication skill. Following diagram is flow chart about research method in this study.
To measure the ability of communication subject, communication indicators are used shown in the following Table 1 and Table 2.

**Table 1. Written Mathematical Communication Indicators**

| Stages of Solving Word Problems | Written Mathematical Communication Indicators |
|---------------------------------|-----------------------------------------------|
| Reading and comprehension       | Write what is known and what is asked about the problem using words or mathematical symbols completely, clearly, and correctly |
| Translate the words into an equation | Use symbols, variables, or mathematical equations to model problems into mathematical sentences completely, clearly, and correctly |
| Answer the problem              | a. Write the procedure or steps to solve the problem completely, clearly and correctly |
|                                 | b. Use symbols, variables, or equations when writing problem solving |

![Diagram](image_url)
Stages of Solving Word Problems | Written Mathematical Communication Indicators
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Encoding | a. Change symbols, variables, or mathematical equations to the problem situation to write a conclusion  
b. Write conclusions and reasons when solving problems in a complete, clear, and correct way

Table 2. Verbal Mathematical Communication Indicators

| Stages of Solving Word Problems | Verbal Mathematical Communication Indicators |
|---|---|
| Coherent | Communicate the results of his work verbally with coherent  
| Complete | Communicate the results of his work in verbal completely  
| Mention of mathematical notation | Mention mathematical notation that exists in the results of his work appropriately  

Data that obtained from this research were test result data, interviews, and data validation of research instruments. Research instruments are tools for collecting data [16]. The instrument used consisted of the main instrument and supporting instruments. The main instrument in this research was the researchers. Supporting instruments in this research were mathematics communication test sheets in the form of word problems for two-variable linear equation systems, interview guide sheets, validation sheets for research instruments, and cameras. Data source was the result of students' work in completing the test given along with information in the interview. Following are the mathematics communication test given to students.

Figure 2a. Mathematics Communication Test Sheets Part 1 (in Bahasa)
4. Results
The results of the work of 30 subjects given test sheet showed that there were still many students who solved the questions incorrectly. Subjects who failed to complete the test made mistakes in understanding the sentence so that the results of the written mathematical communication carried out were also inaccurate. In addition, there were also students who make mistakes in calculations when solving the test. Based on the analysis conducted by researchers, the results of student work tended to be divided into 3 categories presented in the following diagram.

The diagram in figure 3 above showed that 11 students experienced an error when modelling mathematical sentences to solve the problem. This error showed that students could not understand the sentences properly, so the written mathematical communication presented in the results of their work became inappropriate. There were 14 students who can made mathematical models well but made calculation errors that caused wrong final result. Finally, there were 5 students who could make a
mathematical model and the final result was also true. Based on the results of the whole subject which was divided into 3 categories, the researcher chooses 1 subject in each category for further discussion. Subject Y1 was a subject that made mistakes to make mathematical modelling. Subject Y2 was a subject that can make mathematical models well but made mistakes in calculations so the final result was wrong. Subject Y3 was a subject that could make mathematical models well and the final result of his work was also correct.

5. Discussion

5.1. Test Result of Y1
Subject Y1 was a representative of 11 students who made mistakes in modelling sentence problems in the form of mathematical equations (mathematical models) even though making mathematical models was the basis of solving mathematical problems. This is in accordance with the results of previous research which states that mathematical modelling is a very important process for students to solve story problems [20]. Errors in creating mathematical models are shown in Figure 3 below.

![Figure 4a. Y1’s Work Part 1](image)

![Figure 4b. Y1’s Work Part 2](image)

The Y1's work that were given a red coloured box in Figure 3a shows that Y1 made a mistake in understanding the sentence problem and making mathematical equations. Y1 made a mathematical equation by adding the number of sweaters and T-shirts in each store even though the information was given in the problem that the prices of each package offered at the two stores are different. Y1 made the mathematical equation with the aim of finding the price of one sweater and one shirt, assuming that the price of each item is the same. This can be seen from the results of the conclusions made by Y1 when finding the price of one sweater and one shirt. When writing the mathematical equation, Y1 also did not write the definition of the variables used. In addition, Y1 also did not write the questions asked in the answer sheet. Calculations made by Y1 are correct, Y1 understood that the solution of the two-variable linear equation system problem used the elimination-substitution method (mixed method). However, Y1 had an error in understanding the given word problem. The conclusion written by Y1 did not match what was asked in the problem, which is to determine which store sells the cheapest clothes, which mean the price of one sweater and one shirt at shop A and shop B were different. That was because Y1 experienced an error from the beginning of the process in solving the word problem, and had an impact on the wrong final result

Based on interviews, Y1 communicated their work mathematically and coherently. As written, Y1 coherently explained that the purpose of the problem was to find the value of x and y. When asked what were x and y, Y1 answered that the symbol was to express the sum of shirts and sweaters respectively. If examined more deeply the pronouncing of this symbol was slightly inaccurate because in fact the correct information was not the sum of shirt or sweater but the number of shirt or sweater. In addition, the problem solving steps could also be explained by Y1 correctly but the understanding of the problems given was still not right. The Y1 also stopped explaining when he had found the price of a sweater and a shirt.
Error in making the mathematical equation caused the solution made by Y1 to be wrong. Although the calculation process carried out by Y1 was correct but because of an error at the beginning, that is in understanding the sentence, the final result was wrong. This was consistent with the statement that making mathematical equations from word problem in order to solve word problem was a crucial factor in the success of problem-solving [20]. When the student made a wrong mathematical equation, the final result of students’ work also became wrong.

5.2. Test Result of Y2
Y2 was a representative of 14 students who made mathematical model (or mathematical equation derived from problem) correctly but made calculation errors in finding the final results. In this regard, Y2 used a method of elimination-substitution to find the price of one sweater and one shirt at each store. This was consistent with research that found that elimination-substitution was a method that was often used by students to solve problems related to a system of two-variable linear equations [21]. This can be seen in Figure 4 below.

Figure 5. Y2’s work
Figure 5 showed that Y2 made a mistake in the calculation. Although Y2 did not write anything about what was known and asked, Y2 did mathematical modelling correctly. This could be seen in the grid lines in Figure 4 where subjects modeled the word problem into a system of two-variable linear equation using the symbols p and x. However, the circle in Figure 4, showed that Y2 made a calculation error. The blue circle showed that \( x - 2x = 2x \) operation was certainly wrong, the result of \( x - 2x \) should be \(-x\). In addition, the yellow circle showed that the Y2 made a mistake in calculating the value of the variable \( p \). The error of algebraic operations like this often occurred in students in accordance with the results of research which states that there are still many junior high school students who made mistakes in algebraic operations. Because the calculations made by Y2 were wrong, the final results obtained were also wrong. However, it needed to be appreciated when Y2 changed the form of answers from sentences problem to the mathematical equation to answer main question of the problem.

Based on interviews the Y2 communicated his work coherently, even though something unique happened when the subject explained his work. Y2 did the complete problem-solving process, he even revealed that \( p \) and \( x \) stated the number of shirts and sweaters. There was no problem when the subject mentioned the method name and symbol used. The unique thing happened when the subject communicated the results of his work verbally. Shortly after completing modelling the problem into mathematical equation, Y2 was silent for a moment, less than 1 minute the subject realized the mistakes he made and began to correct the answer. When asked whether double-check his work, Y2 claimed not to double-check at all, so errors like this often occurred. This is in accordance with the previous research which stated that most students are lazy to check back the answers obtained when finding only one answer [11].
5.3. Test Result of Y3
Y3 was a representative of 5 students who did modelling and calculation well so that the results of the Y3's work were also correct. Y3 made mathematical modelling well, it could be seen from the mathematical equation was in accordance with the sentence problem. This showed that the subject Y3 successfully interpreted the sentence problem into the mathematical equations [22]. These results were shown in Figure 5 below.

![Figure 5. Mathematical Equation]

Figure 6. Y3’s Work

Figure 6 showed that Y3 succeeded in making mathematical equation well. Calculations made by Y3 were also correct. So that Y3 could solve the problem given in accordance with what was asked in the problem. However mathematical communication errors are also seen where Y3 did not make modelling of the objects in the problem. Y3 immediately made a solving stage without writing the meaning of the symbols he wrote. Even though the final result of Y3's work was correct, Y3's written mathematical communication skills were poor.

Slightly different from written communication, verbal communication of Y3 was very good. Y3 explained the problem coherently and started by telling how the essence of the problem then proceed with modelling the problem very well to produce the right answer. When asked about the reason, Y3 did not write down what is known and asked, Y3 argued that he did not have a specific reason to do that and the most important thing was understanding the essence of the problem. This is in line with the statement that most students have a cognitive style that is “fast” which mean that they do not want to waste a lot of time to write down what was known and asked when working on mathematics problems [23]. From how the subject explained coherently, completely, and correctly in explaining the symbol or method used verbally, it could be categorized that Y3 had good verbal communication skills.

6. Conclusion
Based on the results of research and discussion, it can be concluded that: (1) there are still many students who cannot communicate the results of their work well. Students' written mathematical communication skills need to be improved. (2) Mathematical communication mistakes made by most students are errors in converting problem sentences into mathematical models or equations. This is related to the ability of students who are still weak in interpreting or understanding sentences in the problem. (3) There are still many students who communicate their answers unclearly (neither coherent nor complete). (4) Verbal mathematical communication of the subject is better when compared with written mathematical communication. The subject can tell a problem, explain what is known to convey the problem well verbally.
Suggestions for educators are as follows; (1) Students need to get used to write answers in a coherent and complete manner so that students are accustomed to communicate their mathematical ideas well. (2) The teacher needs to give more math word problem to train students' sensitivity in interpreting sentence problem into mathematical equation form. (3) The teacher periodically do the scaffolding to correct students' mathematical communication errors. (4) The teacher should provide opportunities for students to carry out verbal communication so independently so that they can encourage themselves to double-check the answers they have obtained.

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