Neutralino Warm Dark Matter

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Abstract

In the supersymmetric (SUSY) standard model, the lightest neutralino may be the lightest SUSY particle (LSP), and it is a candidate of the dark matter in the universe. The LSP dark matter might be produced by the non-thermal process such as heavy particle decay after decoupling of the thermal relic LSP. If the produced LSP is relativistic, and does not scatter enough in the thermal bath, the neutralino LSP may contribute as the warm dark matter (WDM) to wash out the small scale structure of $O(0.1)$ Mpc. In this letter we calculate the energy reduction of the neutralino LSP in the thermal bath and study whether the LSP can be the WDM. If temperature of the production time $T_I$ is smaller than 5MeV, the bino-like LSP can be the WDM and may contribute to the small-scale structure of $O(0.1)$ Mpc. The Higgsino-like LSP might also work as the WDM if $T_I < 2$MeV. The wino-like LSP cannot be the WDM in the favoured parameter region.
Existence of the dark matter in the universe is one of the important observations for both cosmology and particle physics. The supersymmetric standard model (SUSY SM) provides good candidates for the dark matter in universe, since the R parity stabilizes the lightest SUSY particle (LSP) [1]. In the supergravity scenario the lightest neutralino with mass above O(50)GeV is the LSP. It is produced in the thermal processes of the early universe, and works as the cold dark matter (CDM) which explains the large-scale structure in the universe well [2].

On the other hand, it is not yet clear whether the neutralino is consistent with the small-scale structure formation of the universe. It is pointed out that the CDM tends to make cuspy structures in the halo density profiles [3]. Such cuspy structures may have drastic consequences to future observations in the dark matter search [4]. On the other hand, it is argued that the neutralino cuspy profile might be inconsistent with the radio emission from centre of the galaxy [5], although detailed studies are waited for to confirm it. The consistency between the observed structure of sub-galactic scale or cluster of galaxy [6] and the numerical simulation of N body system [7] has been discussed extensively at present. The inflation models with the small-scale perturbation suppressed [8] and some new candidates of the dark matter [9][10] have been proposed in order to explain it well.

Here we consider the neutralino dark matter produced by non-thermal processes. If the neutralino dark matter is produced after decoupling of the thermal relic neutralino, such neutralinos might remain without annihilating and contribute as the dark matter. The thermal relic of the LSP on the other hand may be washed away by the entropy production associated with the non-thermal production. Such a situation is realized in some models, those are, the heavier moduli decay with mass of the order of 10 ∼ 100 TeV at the late time [11][12], evaporation of cosmological defects [10], and so on. In those cases the produced LSP can be highly relativistic compared with the thermal background. If the LSP keeps most of its energy from the scattering processes in the thermal bath till the matter-radiation equality, the LSP behaves as the warm dark matter (WDM). The small-scale structure in the universe within the comoving free-streaming scale at the matter-radiation equality is washed out, and the cuspy profiles in the halo would not be formed [10].
If the reduction of the LSP energy by the scattering can be neglected, the comoving free-streaming scale at the matter-radiation equality $R_f$ is given as

$$R_f = \int_{t_I}^{t_{EQ}} \frac{v(t')}{a(t')} dt' \approx 2v_0 t_{EQ}(1 + z_{EQ})^2 \log \left( 1 + \frac{1}{v_0^2(1 + z_{EQ})^2} + \frac{1}{v_0(1 + z_{EQ})} \right)$$

where $z_{EQ}$ and $t_{EQ}$ are the red shift and cosmic time for the matter-radiation equality $[13]$. The $v_0$ is the current velocity of the LSP,

$$v_0 = \frac{T_0}{T_I m_{\chi^0_i}}$$

where $T_0$ and $T_I$ are temperatures for the current cosmic microwave background radiation and the production time of the LSP, $E_I$ and $m_{\chi^0_i}$ are the energy at the production time and the mass for the LSP. In order to explain the small-scale structure of $O(0.1)$ Mpc well, $v_0$ is preferred to be $10^{-7}$ from Eq. (1), and this means $[10]$\]

$$\frac{E_I}{T_I} = 2.1 \times 10^7 \times \left( \frac{m_{\chi^0_i}}{50 \text{GeV}} \right) \left( \frac{v_0}{10^{-7}} \right).$$

Other energetic particles are likely associated with the LSP production. Therefore we mainly consider a case where $T_I$ at the production time is larger than about a few MeV so that the standard nucleosynthesis works. We will come back to this point later $[1]$

So far we assumed that the LSP does not lose its relativistic energy significantly in the scattering processes in the thermal bath. If the LSP is gravitino or axino, it does not lose its energy by the scattering because it couples with particles of the SUSY SM very weakly $[13]$. However, the neutralino LSP is weak interacting, and it may lose most of its energy by the scattering processes in thermal bath. In this letter we calculate the energy reduction in the successive scattering of the relativistic LSP in the thermal bath. We find the energy reduction is suppressed and the neutralino could be the WDM if some conditions are satisfied. For $v_0$ is $10^{-7}$, $T_I$ can not exceed over $\sim 5$ MeV for the bino-like LSP, $\sim 2$ MeV for the Higgsino-like LSP. The wino-like LSP cannot be the WDM.

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1 One might worry that the LSP is relativistic at the nucleosynthesis era and it may change the expansion rate significantly. Assuming that the LSP is the dark matter of the universe and Eq. (1)$[3]$, the energy density of the LSP at the nucleosynthesis era is $\sim 0.2\% (v_0/10^{-7})$ of that of three neutrinos, and it does not give any significant effect on the nucleosynthesis.
First, we review nature of the neutralino LSP. The neutralinos are composed of bino, wino, and two Higgsinos. The mass matrix is

$$M_N = \begin{pmatrix}
    M_\tilde{B} & 0 & -m_Z s_W c_\beta & m_Z s_W s_\beta \\
    0 & M_\tilde{W} & m_Z c_W c_\beta & -m_Z c_W s_\beta \\
    -m_Z s_W c_\beta & m_Z c_W c_\beta & 0 & -\mu \\
    m_Z s_W s_\beta & -m_Z c_W s_\beta & -\mu & 0
\end{pmatrix}. \quad (4)$$

Here $M_\tilde{B}$, $M_\tilde{W}$, and $\mu$ are the bino, wino, and supersymmetric Higgsino masses, respectively. $c_\beta(\equiv \cos \beta)$ and $s_\beta(\equiv \sin \beta)$ are for a mixing angle of the vacuum expectation values of the Higgs bosons, and $c_W(\equiv \cos \theta_W)$ and $s_W(\equiv \sin \theta_W)$ for the Weinberg angle. If $M_\tilde{B} \ll \mu, M_\tilde{W}$, the LSP is bino-like. On the other hand, if $M_\tilde{W}$ or $\mu$ is smaller than the others, the LSP is wino- or Higgsino-like. In the wino- and Higgsino-like cases, the LSP and the lighter chargino are degenerated in masses. The next lightest SUSY particle would be important for our discussion since the inelastic scattering of the LSP contributes to the energy reduction.

In the minimal supergravity model, the bino-component is dominant in the LSP. This is because $M_1 \sim 0.5M_2$ and $\mu$ tends to be larger than the gaugino masses due to the radiative breaking condition. However, if the universal gaugino mass condition at the gravitational scale is broken, LSP can be wino-like. Especially, in the anomaly mediation SUSY breaking model, the wino-component dominates over the others since the gaugino masses are proportional to the one-loop beta function of the gauge coupling constants in the SUSY SM [14]. For a very large universal scalar mass compared to the gaugino masses in the minimal supergravity model [13] or breakdown of universality of the scalar masses may lead to the Higgsino-like LSP. In this letter we do not assume any specific SUSY breaking models and discuss each the neutralino LSPs.

The energy loss of the relativistic LSP depends on the temperature at the LSP production time, $T_I$. If the LSP is produced below $T_C$,

$$T_C = 6.3\text{MeV} \left( \frac{m_{\chi_0}^0}{50\text{GeV}} \right)^{1/2} \left( \frac{v_0}{10^{-7}} \right)^{-1/2}, \quad (5)$$

it is typically non-relativistic in the CM frame of the scattering processes with particles in the thermal bath. In this case the energy reduction per one scattering $r(\equiv \Delta E/E)$ is

$$r = 4 \frac{qE}{m_{\chi_0}^2} \sin^2(\theta/2) \sin^2(\eta/2). \quad (6)$$
Here, $q$ is the energy of a particle in the thermal bath, which is $\sim 3T$, and $\theta$ is the relative angle between the LSP and the particle in the thermal bath, and $\eta$ the scattering angle of the LSP in the CM frame. Here we take a leading term of $O(q/E)$. Then, the energy reduction is suppressed by $O(TE/m_{\chi^0}^2)$ when $T \lesssim T_C$. On the other hand, if the LSPs are produced above $T_C$, they are relativistic in the CM frame of the scattering processes at the production time, and the energy reduction is unsuppressed as

$$r = \sin^2(\eta/2).$$

(7)

Provided that the event rate is faster than the Hubble expansion, the LSP loses the energy quickly so that LSP scattering becomes non-relativistic in the CM frame.

First, we consider the case where $T_I < \sim T_C$ and calculate the energy reduction of the LSP due to the two-body elastic scattering in the thermal bath. The evolution of the LSP energy is given as

$$\frac{dE}{dt} = -HE - \sum g_i \int \frac{d^3q}{(2\pi)^3} e^{-\frac{q}{T}} (rE) v_{rel} d\sigma_i dr dr.$$  

(8)

Here $H$ is the Hubble parameter. The index $i$ is for species of particle in the thermal bath with the degrees of freedom $g_i$, $v_{rel}$ and $\sigma_i$ are the relative velocity and the cross section of the elastic scattering between the LSP and the particle in the thermal bath. Since the LSP is neutral and $T_I$ is smaller than $T_C$ and larger than 1MeV, $i = e^-, \nu_e, \nu_\mu, \nu_\tau$, and the anti-particles. The contributing diagrams to the energy reduction come from the $Z$ boson and slepton exchanges. We will take a massless limit for the particles in the thermal bath for simplicity. The explicit calculation gives

$$\sum g_i \int \frac{d^3q}{(2\pi)^3} e^{-\frac{q}{T}} (rE) v_{rel} d\sigma_i dr = \frac{16}{\pi^3} \left(|A_L^{(e)}|^2 + |A_R^{(e)}|^2\right) \frac{E^4 T^6}{m_{\chi^0}^4}.$$ 

(9)
where \( C_{11} = ([O_N]^{2}_{13} - [O_N]^{2}_{14}) \) with \([O_N]\) the diagonalization matrix of \( M_N \), \( L_i = T_3 + Q s^2_W \), \( R_i = Q s^2_W \), and \( t_W \equiv \tan \theta_W \). The momentum transfers on the propagators of the exchanged particles are negligible compared with the masses, thus we replace the propagators of \( Z \) boson and sleptons to their mass squares \( m_Z^2 \) and \( m_{\tilde{l}}^2 \), respectively. By solving Eq. (8), the LSP energy at the radiation-matter equality is given as

\[
E_{EQ} = E_I (\frac{T_{EQ}}{T_I}) \left( 1 - \left( \frac{\Delta E}{E} \right)_{eff} \right),
\]

where

\[
\left( \frac{\Delta E}{E} \right)_{eff} = \frac{24\sqrt{5}}{7\pi^2} g_s^{-\frac{1}{2}} \sum_i \left( |A^{(i)}_L|^2 + |A^{(i)}_R|^2 \right) \frac{M_pl E^3 T^4_I}{m_{\tilde{\chi}^0_1}^4}.
\]

Here, \( g_s \) is total number of the effective degrees of freedom for at the temperature \( T_I \). We assume that the universe is radiation dominant and use \( H = (4\pi^2/45)^{1/2} g_s^{1/2} T^2/M_{pl} \) for the Hubble parameter. The first bracket in the right-handed side in Eq. (10) comes from the red-shift due to the expansion of the universe, and the second one is the effect from the scattering of the LSP in the thermal bath. Here we expand \( E_{EQ} \) by \( E_I T_I/m_{\tilde{\chi}^0_1}^2 \) and keep the leading term in Eq. (11), assuming the energy reduction from the scattering is small. When \( (\Delta E/E)_{eff} \) is larger than one, \( T_I \) is replaced to the temperature at which the elastic scattering becomes ineffective to the LSP energy reduction, and \( E_I \) is given by the LSP energy at the \( T_I \). This means that our result is conservative.

In Eq. (11), \( (\Delta E/E)_{eff} \) is suppressed by \( T_I^4 \). This comes from the suppression in the amplitude and the phase space, in addition to the energy reduction in the non-relativistic limit of the LSP (Eq. (8)). The momentum transfers in the scattering processes \( \sim ET \) are smaller than the exchanged particle masses in the amplitude, and the phase space of the elastic scattering is also suppressed by \( ET/m_{\tilde{\chi}^0_1}^2 \). Thus, the event rate par a Hubble time is smaller in lower temperature by \( \propto T^3 \) as

\[
\frac{\Gamma}{H} = \frac{1}{H} \sum_i g_i \int \frac{d^3q}{(2\pi)^3} e^{-\frac{q^2}{4}} v_{rel} \sigma_i = \frac{45\sqrt{5}}{16\pi^2} g_s^{\frac{1}{2}} \sum_i \left( |A^{(i)}_L|^2 + |A^{(i)}_R|^2 \right) \frac{M_{pl} E^2 T^3_I}{m_{\tilde{\chi}^0_1}^2}.
\]

Note that the energy reduction \( (\Delta E/E)_{eff} \) is dominated by the contribution at \( T = T_I \) and is not sensitive to \( T_{EQ} \).
If the LSP is bino-like, the $Z$ boson exchange contribution is suppressed by $m_Z^2/\mu^2$ in the amplitude. Then, the slepton exchange contribution dominates if $\mu$ is larger than the slepton masses. Taking the pure bino limit,

$$\left(\frac{\Delta E}{E}\right)_{\text{eff}} = 3.9 \times 10^{-2} \left(\frac{m_{\tilde{\chi}_1^0}}{50 \text{GeV}}\right)^{-1} \left(\frac{m_{\tilde{\ell}}}{1 \text{TeV}}\right)^{-4} \left(\frac{v_0}{10^{-7}}\right)^3 \left(\frac{T_I}{1 \text{MeV}}\right)^7. \quad (13)$$

Here $m_{\tilde{\chi}_1^0} \simeq M_{\tilde{B}}$, and we take $m_{\tilde{\ell}_R} = m_{\tilde{\ell}_L} = m_{\tilde{\nu}_L}(\equiv m_{\tilde{l}})$. In order to suppress the energy reduction below the 10% so that the LSP can behave as the WDM, $T_I$ should be smaller than 1.1(3.1)MeV for $v_0 = 10^{-7}(10^{-8})$, $m_{\tilde{\chi}_1^0} = 50 \text{GeV}$ and $m_{\tilde{\ell}} < 1 \text{ TeV}$. This value corresponds to $E_I = 24(6.5)\text{TeV}$ from Eq. (3). If the LSP is heavier, the energy reduction is suppressed more, and a slightly larger $T_I$ is possible. For $m_{\tilde{\chi}_1^0} = 200 \text{GeV}$, $T_I$ should be smaller than 1.4(3.7)GeV for $v_0 = 10^{-7}(10^{-8})$. This means that $E_I < 118(32)\text{TeV}$.

Note that calculation of the energy reduction rate is valid only when $\Gamma/H > 1$. In the bino dominant limit, the event rate of the elastic scattering process by the slepton exchange per a Hubble time is

$$\frac{\Gamma}{H} = 3.8 \left(\frac{m_{\tilde{\ell}}}{1 \text{TeV}}\right)^{-4} \left(\frac{v_0}{10^{-7}}\right)^2 \left(\frac{T_I}{1 \text{MeV}}\right)^5. \quad (14)$$

The event rate is not still sufficiently suppressed compared with the Hubble expansion.

In Eq. (13) we took the slepton masses 1TeV. However, some SUSY breaking models predict much heavier sleptons, which is not necessarily in conflict with the naturalness argument [16]. When sfermions are heavy, the thermal component of the bino-like LSP cannot annihilate sufficiently in the thermal processes so that the energy density might be too large beyond the critical density. However, if the huge entropy is supplied in the non-thermal process as mentioned before, it can be diluted and be harmless.

When the slepton exchange is sufficiently suppressed, the $Z$ boson exchange becomes dominant in the energy reduction of the LSP. The energy reduction by the $Z$ boson exchange is given as

$$\left(\frac{\Delta E}{E}\right)_{\text{eff}} = 6.9 \times 10^{-3} \left(\frac{m_{\tilde{\chi}_1^0}}{50 \text{GeV}}\right)^{-1} \left(\frac{\mu}{1 \text{TeV}}\right)^{-4} \left(\frac{v_0}{10^{-7}}\right)^3 \left(\frac{T_I}{1 \text{MeV}}\right)^7 \cos^2 2\beta. \quad (15)$$

Here we used the approximated solution $C_{11} = -m_Z^2 s_W^2 \cos 2\beta/\mu^2$ for $m_Z$, $M_{\tilde{B}} \ll \mu$. Recent LEP II searches of the light Higgs boson prefer $|\cos 2\beta| > 0.53$ [17]. Since the
Higgsino mass \( \mu \) is related with the Higgs boson mass, we cannot take too large a value for \( \mu \) compared to the weak scale from the naturalness argument. From Eq. (13), \( T_I \) should be smaller than 5MeV assuming \( \mu \) is smaller than 1TeV, \( m_{\tilde{\chi}_0^1} = 200\text{GeV} \), and \( v_0 = 10^{-8} \).

Next, let us consider the Higgsino-like LSP. In this case the slepton exchange contribution is suppressed by the small gaugino components and the Yukawa coupling constants, and the \( Z \) boson exchange contribution dominates in the elastic scattering processes. Then,

\[
(\frac{\Delta E}{E})_{\text{eff}} = 2.4 \left( \frac{m_{\tilde{\chi}_1^0}}{100\text{GeV}} \right)^{-3} \left( \frac{m_{\tilde{B}}}{500\text{GeV}} \right)^{-2} \left( \frac{v_0}{10^{-7}} \right)^3 \left( \frac{T_I}{1\text{MeV}} \right)^7 \cos^2 2\beta. \tag{16}
\]

Here, \( m_{\tilde{\chi}_1^0} \simeq \mu \) and it should be larger than about 100GeV from negative search for the Higgsino-like chargino. When the gaugino masses are heavier than \( \mu \) and \( m_Z \), \( C_{11} \) is given as

\[
C_{11} = \mp \frac{m_Z^2}{2\mu} \left( \frac{s_W^2}{m_B} + \frac{c_W^2}{m_W} \right) \cos 2\beta, \tag{17}
\]

for \( \mu \) positive (negative). It further reduces to

\[
C_{11} = \mp \frac{4 m_Z^2 s_W^2}{3 \mu m_B} \cos 2\beta \tag{18}
\]

by using the GUT relation \( M_B/M_W = 5/3 t_W^2 \simeq 1/2 \). In Eq. (16) we used this formula for simplicity. In order to suppress the energy reduction by the elastic scattering, \( T_I \) should be smaller than 0.85(2.3)MeV for \( v_0 = 10^{-7}(10^{-8}) \) and a relatively heavy LSP mass \( m_{\tilde{\chi}_1^0} = 200\text{GeV} \) as far as the gaugino masses are smaller than 1TeV.

We saw that the elastic scattering of the Higgsino-like LSP is suppressed for heavier gaugino masses. However, we have to check if the inelastic scattering of the LSP by the \( W \) boson exchange does not contribute to the energy reduction. As we noted before, the chargino is degenerate with LSP in masses. The Boltzmann suppression, \( \exp(-m_{\tilde{\chi}_1^0}\Delta m_{\tilde{\chi}}/2ET) \), may not be too small. Furthermore the coupling with \( W \) boson is not suppressed at all. Therefore the processes, such as \( \tilde{\chi}_1^0 e^- \rightarrow \tilde{\chi}_1^- \nu \), may be important over the Boltzmann suppression factor.

The mass difference between the chargino and the LSP is

\[
\Delta m_{\tilde{\chi}} \equiv (m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_1^0})
\]
= \left( 1 \pm \sin 2\beta \frac{m_Z^2 s_W^2}{M_B} + \frac{1 \mp \sin 2\beta m_Z^2 \epsilon_W^2}{2} \right) \tag{19}
= \frac{m_Z^2 s_W^2}{M_B} \left( \frac{4}{3} \mp \frac{1}{3} \sin 2\beta \right), \tag{20}

and this is about 5 GeV for $M_B = 500$ GeV. The energy reduction by one scattering of $\tilde{\chi}_1^0 e^- \rightarrow \tilde{\chi}_1^- \nu_e$ is

$$r = 4 \frac{q E}{m_{\tilde{\chi}_1^0}} \sin^2 (\theta/2) \sin^2 (\eta/2) - 2 \frac{\Delta m_{\tilde{\chi}}}{m_{\tilde{\chi}_1^0}}. \tag{21}$$

From the kinematics, $r$ is positive definite. Each chargino decay also reduces the energy of the order of $\Delta m_{\tilde{\chi}}/m_{\tilde{\chi}_1^0}$. The event rate of the inverse inelastic scattering processes of chargino, such as $\tilde{\chi}_1^- \nu_e \rightarrow \tilde{\chi}_1^0 e^-$, is suppressed by $120\pi (T/\Delta m_{\tilde{\chi}})^3$ compared with the decay rate, thus contribution to the energy reduction is negligible.

The event rate of the inelastic scattering of the Higgsino-like LSP per a Hubble time is

$$\frac{\Gamma}{H} = \frac{3\sqrt{5}}{4\pi^2} g_* \frac{\alpha}{\sqrt{2}} \frac{4}{3} \frac{g_1^4}{g_2^4} \frac{M_{Pl} T^2}{m_{W}^4} e^{-\frac{m_{W}^2 \Delta m_{\tilde{\chi}}}{2 T}} \left( \frac{\Delta m_{\tilde{\chi}}}{m_{\tilde{\chi}_1^0}} + 6 \frac{E T}{m_{\tilde{\chi}_1^0}^2} \right) N_F \tag{22}$$

where $N_F$ is the number of the inelastic processes. When the mass difference is larger than

$$\frac{2 T I E_I}{m_{\tilde{\chi}_1^0}^2} = 850 \text{MeV} \left( \frac{v_0}{10^{-7}} \right) \left( \frac{T_I}{1 \text{MeV}} \right)^2, \tag{23}$$

the inelastic processes are suppressed by the Boltzmann factor. Then, $\Gamma/H$ is sensitive to $T$, $v_0$, and $\Delta m_{\tilde{\chi}}$. If $v_0 = 10^{-7}$ and $\Delta m_{\tilde{\chi}} = 5$ GeV, $\Gamma/H$ is of the order of $10^5$ even for $T = 1$ MeV, and the energy reduction by the inelastic scattering cannot be suppressed. On the other hand, if $v_0 = 10^{-8}$ and $\Delta m_{\tilde{\chi}} = 5$ GeV, it is $10^{-20}$ (33) for $T = 1(2)$ MeV, and the energy reduction may be suppressed. The Higgsino-like WDM is marginally viable.

Note that the energy of the chargino produced by the inelastic scattering is also reduced by the electromagnetic interaction. The life time of the Higgsino-like chargino is

$$\tau_{\tilde{\chi}_1^-}^{-1} = N_D \frac{g_1^4}{960 \pi^3} \frac{\alpha^5}{m_W^4} \Delta m_{\tilde{\chi}} \tag{24}$$

with $N_D$ the number of the decay modes. The energy reduction by the electromagnetic interaction is given as

$$\frac{dE}{dt} = -\frac{\pi^3 \alpha^2}{3} \Lambda T^2 \tag{25}$$

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where $\Lambda$ is of the order of 1 [18]. Then, the energy reduction rate of the Higgsino-like chargino in one life is

$$\left( \frac{\Delta E}{E} \right)_{\text{1-life}} \sim 1.3 \times 10^{-3} N_D \Lambda \left( \frac{T}{1 \text{MeV}} \right)^2 \left( \frac{m_{\tilde{\chi}_1^0}}{100 \text{GeV}} \right)^{-1} \left( \frac{\Delta m_{\tilde{\chi}}}[]{5 \text{GeV}} \right)^{-5}. $$

This effect may be harmless if $\Delta m_{\tilde{\chi}} = 5 \text{GeV}$.

When the LSP is wino-like, the elastic scattering can be suppressed if the slepton and the Higgsino masses are heavy, similar to the bino-like LSP. However, when the $Z$ boson contribution is suppressed by raising the Higgsino mass, the chargino and the LSP become more degenerate in masses than in the Higgsino-like case as

$$\Delta m_{\tilde{\chi}} = \frac{m_Z^4}{M_{\tilde{W}}^2 \mu^2} \frac{\gamma^2}{16 \pi^3} \frac{\sin^2 2\beta}{T_{C}} \left( \frac{L_1^2 + R_1^2}{\mu^4} \right) \frac{m_Z^4}{100 \text{GeV}} \left( \frac{\mu}{1 \text{TeV}} \right)^{-4} \left( \frac{v_0}{10^{-8}} \right) \left( \frac{T_{C}}{100 \text{MeV}} \right)^{-1} \cos^2 2\beta, $$

for $M_{\tilde{W}}$, $m_Z \ll M_{\tilde{B}}, \mu$. If $\mu$ is 1TeV and $M_{\tilde{B}}$ is 100GeV, $\Delta m_{\tilde{\chi}}$ is about 100MeV. The $\Gamma/H$ for the inelastic scattering by the $W$ boson exchange is of the order of $10^5$ for $\Delta m_{\tilde{\chi}} = 100\text{MeV}$ even if $T = 1\text{MeV}$ and $v_0 = 10^{-8}$. Since either the $Z$ or $W$ boson exchange contributions cannot be suppressed, the wino-like LSP cannot be the WDM.

Finally, we discuss the case for $T_I \gtrsim T_C$. In this case, the momentum on the exchanged particle is not negligible, and the event rate becomes larger than in the case of the lower temperature. Therefore the energy reduction becomes maximum at $T \simeq T_C$ and the LSP loses the relativistic energy till the temperature goes down to $T_C$.

As an example, we present the energy reduction of the bino-like LSP by the elastic scattering since the constraint on the $T_I$ is the weakest among the neutralino LSPs. Assuming the slepton exchange is suppressed by the heavy masses, the $Z$ boson contribution to the energy reduction when the typical momentum transfer is much larger than $m_Z^2$ ($ET \gg m_Z^2$) is expressed by

$$\sum_i g_i \int \frac{d^3q}{(2\pi)^3} e^{-\bar{T}(rE)} v_{rel} \frac{d\sigma_i}{dr} dr = \sum_i \frac{\gamma^4 h_W^4}{64 \pi^3} \left( \frac{L_1^2 + R_1^2}{\mu^4} \right) \frac{m_Z^4}{100 \text{GeV}} \left( \frac{\mu}{1 \text{TeV}} \right)^{-4} \left( \frac{v_0}{10^{-7}} \right) \left( \frac{T_{C}}{100 \text{MeV}} \right)^{-1} \cos^2 2\beta, $$

where $\zeta = (2 \log \frac{4ET}{m_Z^2} - 5 - 2\gamma)$. The energy reduction rate is given as

$$\left( \frac{\Delta E}{E} \right)_{\text{eff}} = 1.0 \times 10^3 \zeta \left( \frac{m_{\chi_1^0}}{50 \text{GeV}} \right)^{-1} \left( \frac{\mu}{1 \text{TeV}} \right)^{-4} \left( \frac{v_0}{10^{-7}} \right) \left( \frac{T_{C}}{100 \text{MeV}} \right)^{-1} \cos^2 2\beta, $$

and it is difficult for the LSP to keep the relativistic energy.
In this letter we calculate the energy reduction of the LSP which is produced by the non-thermal process and study whether the LSP can be the warm dark matter or not. If the temperature of the production time $T_I$ is smaller than 5MeV, the bino-like LSP can be the WDM and may contribute to the small-scale structure of $O(0.1)$ Mpc. The Higgsino-like LSP might also work as the WDM if $T_I < 2$MeV. The wino-like LSP cannot be the WDM.

We now discuss the some of the aspects on the mechanism to produce relativistic neutralino. Here we discuss the LSP produced from heavy moduli decay. Such a moduli might dominate the energy density of universe before its decay. Therefore the moduli decay associates with the large entropy production and reheating. For this case, the LSP energy density over the entropy density at present might be written as

$$m_{\tilde{\chi}_1^0} Y_{\tilde{\chi}_1^0} \simeq 0.75 \times 10^{-6} \text{GeV} \bar{N}_{\tilde{\chi}_1^0} \left( \frac{m_{\tilde{\chi}_1^0}}{100 \text{GeV}} \right) \left( \frac{T_I}{1 \text{MeV}} \right) \left( \frac{m_\phi}{100 \text{TeV}} \right)^{-1}. \quad (30)$$

Here we identify $T_I \equiv T_R$, $\bar{N}_{\tilde{\chi}_1^0}$ is average number of the LSP from a moduli decay. On the other hand, $m_{\tilde{\chi}_1^0} Y_{\tilde{\chi}_1^0} \sim 10^{-9}$ GeV is preferred as the dark matter density. This leads $\bar{N}_{\tilde{\chi}_1^0}$ must be a order of $10^{-3}$. Such a small branching ratio is expected for the case where the moduli decay into gravitino is suppressed [12]. The small branching ratio also means that many energetic particles are produced associated with the moduli decay, thus the effect on the nucleosynthesis must be considered. For $T_R \gtrsim 2.5$–4 MeV such an effect would be small enough [19]. On the other hand, if the energy density of the heavy moduli does not exceed over that of one neutrino spices at the time of nucleosynthesis and it decays before $10^4$ sec, the associated high energetic particles are thermalized before they hits to nuclei, provided that the hadrons are not produced in the decay. In such case, the decay can be harmless to the nucleosynthesis.

For the neutralino LSP to stay warm and produced above 1 MeV, the LSP must be either nearly pure bino or Higgsino, in order to suppress the scattering in the thermal bath. This means counting rate at the conventional dark matter detectors would be very small. Discovery of the dark matter signal in any forthcoming experiments [20] will suggest the LSP is not the warm dark matter. For the bino-like LSP the slepton masses also need to be very heavy. If deviation of the muon anomalous magnetic moment from the standard model prediction is observed, the warm bino-like LSP is disfavoured [21].
Note Added: After completion of this work, there appears a paper where the energy reduction of the WIMP by scattering in the thermal bath is also discussed\cite{22}. They assume the WIMP is produced by the non-thermal process, but the WIMP is not the LSP. Also, they impose $\Gamma/H < 1$ for the scattering processes, and do not calculate the energy reduction rate of the WIMP by scattering.

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