Free Surface Influence on Low Head Hydro Power Generation

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Abstract. The free surface influence on the power extraction of turbines in open-channel flow is analyzed under use of continuity, momentum and energy equations. The approach differs of previous models by avoiding two drawbacks: the exceeding of the available power with the Betz definition and the inaccurate assumption of an undisturbed approaching flow. The result is an energetic optimization focusing on the energy dissipation due to wake and shock losses downstream of the turbine.

1. Introduction

Figure 1 shows a turbine in a river or a man-made channel of width \( b \). We distinguish two types of installations: the run-of-the-river station (§2), with a dam that blocks the whole channel (blockage ratio \( \sigma = 1 \)) and the hydrokinetic turbine in a free flow environment (§3), that involves a bypass flow (\( \sigma < 1 \)). Both concepts are subjected to a similar kind of topology. As such turbines are placed in any shallow watercourse, the stream can significantly change, especially for high blockage ratio. We suppose the flow uniform at every considered section. The initial undisturbed flow of water depth \( h_0 \) and flow velocity \( u_0 \) with Froude number \( Fr_0 := u_0/\sqrt{gh_0} \) is altered as follows: the volume flow per unit width reduces \( q < q_0 \), the upstream depth (section [1] in figure 1) increases \( h_1 > h_0 \) and the Froude number decreases \( Fr_1 < Fr_0 \). Only the so-called effective head \( H_E := h_0 + u_0^2/2g + \Delta z \) remains unchanged. It is meaningful to forecast the energy that can be extracted by a turbine considering the above-mentioned changes and studying the free surface effects downstream. Pelz (2011), as did Betz (1920) for wind turbines, gave a new approach of this problem. Sections §2.1 and §2.2 are reminders of his results.

2. Optimal operation of a run-of-the-river station (blockage ratio \( \sigma = 1 \))

2.1. Available power

Albert Betz 1920 defined the available power by assuming an ideal machine without wake flow, completely absorbing the kinetic energy of the undisturbed flow: \( P_{\text{avail,BETZ}} := \rho u_0^3 A/2 \) (air density \( \rho \), turbine cross section \( A \)). By analogy for shallow water, Pelz (2011) defined the available power considering a hypothetical machine without tailwater flow. This is theoretically possible by a downstream moving blade. The converted power attributable to the hydraulic
stress vector for the moving panel is

\[ P_{T,h_2=0} = b \int_0^{h_3} \rho g y u_1 \, dy = \frac{1}{2} b \rho g h_1^2 u_1. \]  

Expressing this equation as a function of \( H_E \) and \( Fr_1 \) and optimizing it by the Froude number, we find the available power in a channel (figure 1):

\[ P_{\text{avail}} := 2 \rho b g^{3/2} \left( \frac{2}{5} H_E \right)^{5/2}. \]  

2.2. Optimal operation

The harvested power of a turbine blocking the whole channel (blockage ratio \( \sigma = 1 \)) writes, with the turbine efficiency \( \eta_T \) (Pelz 2011)

\[ P_T (q, h_2) = \eta_T \rho b g q \left( H_E - h_2 - \frac{q^2}{2gh_2^2} \right). \]  

Defining the dimensionless flow rate per unit width \( \bar{q} := q/(g^{1/2}H_E^{3/2}) \) and the dimensionless water depth in the tailwater \( \bar{h}_2 := h_2/H_E \), and dividing the expression by the available power (eq. 2), we obtain the following expression of the coefficient of performance:

\[ C_P (\bar{q}, \bar{h}_2) := \frac{P_T}{P_{\text{avail}}} = \eta_T \frac{1}{2} \left( \frac{5}{2} \right)^{5/2} \bar{q} \left( 1 - \bar{h}_2 - \frac{\bar{q}^2}{2\bar{h}_2^2} \right). \]  

It can also be expressed as a function of the dimensionless turbine head \( \bar{H}_T := H_T/(H_E\eta_T) \) and the Froude number \( Fr_2 \). Where the common definition of the turbine head is used \( H_T := P_T/(\rho gQ) \). The substitution is made with the energy balance between sections [1] and [2] \( 1 - \bar{H}_T = \bar{h}_2(1 + Fr_2^2/2) \) and the Froude number definition \( Fr_2 := \bar{q}/\bar{h}_2^{3/2} \). The final result reads

\[ C_P (\bar{H}_T, Fr_2) = \eta_T \frac{1}{2} \left( \frac{5}{2} \right)^{5/2} Fr_2 \bar{H}_T \left( \frac{1 - \bar{H}_T}{1 + Fr_2^2/2} \right)^{3/2} \leq \frac{1}{2} \eta_T. \]
The optimal operation point is found with the necessary condition: \( \partial C_P / \partial H_T = 0 \) and \( \partial C_P / \partial Fr_2 = 0 \). The solution of this system of equations yields \( H_{T, opt} = 2/5 \), and \( Fr_{2, opt} = 1 \).

With the variables used in equation (4), \( h_{2, opt} = 2/5 \) and \( q_{opt} = (2/5)^{3/2} \). In these conditions the maximal value \( C_{P, opt} = \eta_T / 2 \) is reached. Hence, at this point, the effective head \( H_E \) is split in five equal parts of which two are the turbine head, two are the downstream water depth, and one is the downstream kinetic head.

2.3. Influence of the free surface downstream

This optimal operation is in fact reached when the energy downstream of the turbine is minimal \( (Fr_2 = 1) \), that is when there is no resistance in the tailwater. While in a man-made channel this situation can be approached, in natural stream adequate for energy harvesting, the Froude number is rarely higher than 0.5. Figure 2 reveals the energy reduction due to this resistance. The control of the downstream Froude number is then limited, an optimization is however possible changing the width of the draft tube in order to draw the downstream flow closer to the critical point (Metzler & Pelz 2015). The design of the draft tube is above all of primordial importance to minimize the dissipated energy. In figure 3, \( h_D \) is the height of the draft tube, respectively the height of the parallel stream flowing out of the turbine at the position \([\ast]\). For a non-optimized design of draft tube \( (h_D / h_2 < 1) \) two kinds of dissipative losses have to be distinguished. First, the stream at the position \([\ast]\) is a supercritical jet flow and a hydraulic jump separated from the structure occurs. Second, the draft tube is submerged and the shock is attached to the structure (figure 3). For both cases, the energy dissipation is calculated with momentum and mass conservation (the difference being the hydrostatic pressure on the wall). For the hydraulic jump (Chow 1959)

\[
\overline{h}_{\text{shock,H}} = \frac{(\overline{h}_2 - \overline{h}_D)^3}{4 \overline{h}_D \overline{h}_2},
\]

in the second case, considering \( \overline{h}_s \approx \overline{h}_2 \) (figure 3), the carnot shock loss writes (Becker 1986)

\[
\overline{h}_{\text{shock,C}} = \frac{\eta^2}{2 \overline{h}_D} \left( 1 - \frac{\overline{h}_D}{\overline{h}_2} \right)^2.
\]
The energy extractable by the turbine is reduced and we have the generalized efficiency of the system given by $\eta = \eta_T (1 - \varepsilon)$ with $\varepsilon$ the inefficiency of the draft tube defined as

$$\varepsilon(\bar{h}_D, \bar{q}, \bar{h}_2) := \frac{\bar{h}_{\text{shock}}}{1 - \bar{H}_2}.$$ 

Equation (4) writes thus

$$C_P \left( \bar{h}_D, \bar{q}, \bar{h}_2 \right) = \eta_T (1 - \varepsilon) \frac{1}{2} \left( \frac{5}{2} \right)^{5/2} \left( \frac{\bar{q}^2}{q} \right) \left( 1 - \bar{h}_2 - \frac{\bar{q}^2}{2\bar{h}_2^2} \right).$$

If the draft tube is optimized with $\bar{h}_D/\bar{h}_2 = 1$, the inefficiency reaches $\varepsilon = 0$; this is represented in figure 4 by the gray isolines. The black isolines show the power limitation of a submerged draft tube with dimensionless outlet height $\bar{h}_D = 1/4$ in downstream flow with Froude number $F_{r2} < 0.5$.

**Figure 3.** Carnot shock losses due to a submerged draft tube.

**Figure 4.** Isolines of the coefficient of performance $C_P/\eta_T$ as a function of $\bar{h}_2$ and $\bar{q}$ for a submerged draft tube with $F_{r2} < 0.5$ (black), for an optimized draft tube (gray).
3. Hydrokinetic turbine in a free flow environment (blockage ratio $\sigma < 1$)

3.1. Disc-actuator theory

The following is an extension of recent research results (Metzler & Pelz 2014, Pelz & Metzler 2016). The focus in this paper is the limiting character of the turbine wake losses on the power extraction. Figure 5 shows a hydrokinetic turbine (width $d = \sigma b$) blocking the whole height of the flow ($\sigma = d/b$). Neglecting $\Delta z$ (cf. figure 1), the flow around a turbine can be described using continuity, momentum and energy equations and modeling the turbine as an actuator disc. This has already been realized for open-channel flow. Whelan (2007) considered a bypass flow above and below a turbine that blocks the whole channel width. The model presented in this paper considers a bypass in the other direction. Houlsby (2008) proposed a model without any restriction in the turbine geometry but with strong assumptions on the free surface. The changes in area and velocity right before and behind the turbine were neglected (positions [+1] and [-1] in figure 5). The description of the flow that is made in figure 5 is justified by an experimental approach in an open-channel test rig where the turbine is modeled by a resistance plate. Velocity profiles were measured at each considered position.

![Diagram](image-url)

**Figure 5.** Flow pattern around a hydrokinetic turbine. [$i$] is the stream that flows through the turbine and [$o$] is the stream that bypasses the turbine. They mix between sections [*] and [2].
3.2. Physical description

Table 1. Continuity, momentum and energy equations applied to the control volumes of figure 5.

| Axiom | C. V. | Equation |
|-------|-------|----------|
| Continuity | [1] to [+| $\sigma \bar{h}_+^{3/2} F_{r+} = \beta_+ \bar{h}_1^{3/2} F_{r1}$ |
| Equation | [+| to [-| $\bar{h}_+^{3/2} F_{r+} = \bar{h}_-^{3/2} F_{r-}$ |
| [-| to [*| $\sigma \bar{h}_-^{3/2} F_{r-} = \beta_2 \bar{h}_2^{3/2} F_{r_i}$ |
| [1] to [*|, [0| $(1 - \beta_+) \bar{h}_1^{3/2} F_{r1} = (1 - \beta_-) \bar{h}_0^{3/2} F_{r0}$ |
| [*| to [2| $\beta_- \bar{h}_2^{3/2} F_{r2} + (1 - \beta_-) \bar{h}_2^{3/2} F_{r0} = \bar{h}_2^{3/2} F_{r2}$ |
| Momentum | [1] to [2| $\bar{h}_1^{2}(1/2 + F_{r1}^2) - \bar{h}_2^{2}(1/2 + F_{r2}^2) = \bar{D}$ |
| Equation | [*| to [2| $\beta_- \bar{h}_2^{2}(1/2 + F_{r2}^2) + (1 - \beta_-) \bar{h}_2^{2}(1/2 + F_{r2}^2) = \bar{h}_2^{2}(1/2 + F_{r2}^2)$ |
| [+] to [-| $\sigma \bar{h}_2^{2}(1/2 + F_{r2}^2) - \sigma \bar{h}_2^{2}(1/2 + F_{r2}^2) = \bar{D}$ |
| Energy | [1] to [+| $\bar{h}_1(1 + F_{r1}^2/2) = \bar{h}_+ (1 + F_{r2}^2/2)$ |
| Equation | [+| to [-| $\bar{h}_1(1 + F_{r1}^2/2) - \bar{h}_- (1 + F_{r2}^2/2) = \bar{H}_T$ |
| [-| to [*| $\bar{h}_- (1 + F_{r2}^2/2) = \bar{h}_2(1 + F_{r2}^2/2)$ |
| [1] to [*|, [0| $\bar{h}_2(1 + F_{r2}^2/2) = \bar{h}_1(1 + F_{r1}^2/2)$ |

The 12 equations extractable from the flow pattern represented in figure 5 are written in table 1. The system of equations is made dimensionless with the headwater specific energy, it comes then a thirteenth equation given by the definition of the specific head $1 = \bar{h}_1(1 + F_{r1}^2/2)$. The dimensionless force applied on the turbine is defined as $\bar{D} := D/(b g g H_0^2)$. We have then 13 non-linear equations for 16 variables. This system is solvable giving 3 independent variables ($\sigma$, and $\bar{H}_T$, $\bar{F}_{r2}$ or $\bar{h}_2$, $\bar{\eta} = F_{r2} \bar{h}_2^{3/2}$) as input and 13 proper conditions on variables to obtain a physically possible solution.

3.3. Influence of the free surface downstream

An energy dissipation (analog to §2.3) occurs in the turbine wake (between positions [*] and [2] in figure 5). Defining the inefficiency of the mixing as it was done with the shock losses in §2.3

$$\varepsilon(\sigma, \bar{q}, \bar{h}_2) := \frac{\bar{h}_{\text{wake}}}{1 - \bar{H}_2},$$

We obtain the generalized coefficient of performance:

$$C_p(\sigma, \bar{q}, \bar{h}_2) = \eta_T (1 - \varepsilon) \frac{1}{2} \left( \frac{5}{2} \right)^{5/2} \bar{q} \left( 1 - \bar{h}_2 - \frac{\bar{q}^2}{2 \bar{h}_2} \right).$$

The solving of the system of equations (presented in §3.2) gives the evolution of the energy losses in the system turbine-wake as a function of the turbine dimension and the stream situation. With a blockage ratio $\sigma = 0.75$ the limitation of the coefficient of performance due to the wake losses is represented in figure 6 with black isolines. By way of comparison the gray isolines represent the case $\sigma = 1$ treated in §2.
Figure 6. Isolines of the coefficient of performance $C_P/\eta_T$ as a function of $\bar{h}_2$ and $\bar{q}$ for a hydrokinetic turbine with $\sigma = 0.75$ and $Fr_2 < 0.5$ (black), with $\sigma = 1$ (gray).

4. Conclusion
This paper extends a new method to calculate and optimize the low head hydro power generation in a shallow water flow. The impact of the free surface is studied for a run-of-the-river station and a hydrokinetic turbine in a free flow environment, calculating the evolution of the coefficient of performance of each installation. An analogy is drawn between the blockage ratio $\sigma$ for the system hydrokinetic turbine - wake and the ratio $h_D/h_2$ (draft tube outlet height/downstream flow depth) for the system run-of-the-river turbine - draft tube. Besides its purely theoretical motivation, this basic analytical model gives first design hints and power extraction forecast avoiding the onerous numerical simulation approach.

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