Full elimination of the gravity-gradient terms in atom interferometry

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Abstract

The A. Roura technique was modified to eliminate all terms in the phase of the atom interferometer that are linear in the gravity-gradient tensor. Full elimination occurs when the wave vectors of all Raman pulses change slightly. The full elimination technique would allow to relieve synchronization requirements when testing the Einstein equivalence principle. This technique also eliminates the systematic error of the absolute measurement of the gravitational field, which is due to the gravitational gradient. The error becomes three orders less and does not depend on the delay time between Raman pulses. In addition, a new differential scheme is proposed to observe the gravity-gradient term, independent on the atomic initial position and velocity.

1 Introduction

Atom interferometers [1, 2] are used as gravitational field sensors. In the gravitational field of the Earth, the dominant contribution to the phase of the atomic interferometer is the term [3]

$$\phi = k \cdot gT^2, \quad (1)$$

where $k$ is the effective wave vector of Raman pulses used as beam splitters, $T$ is the time separation between pulses. This phase is used to measure the local gravity field [4–6], the gravity gradient tensor [7], the Newtonian gravitational constant [8, 9], or when testing the Einstein's Equivalence Principle (EEP) [10–12]. For the local gravity sensor, one has to measure the absolute value of the phase, while for other applications one may use the differential schemes. If the field is slightly nonuniform, then the phase becomes sensitive to the atomic initial position $x$ [13] and velocity $v$ [14, 15]. By increasing the time of interrogation $T$, one can increase the accuracy of the measurement. But at the same time, the gravitational field slightly changes along the atomic trajectory which leads to another contribution to the phase, $\phi_{WT}$, which was first found by Wolf and Tourrence [16]. Piecing together all the contributions one finds

$$\phi = k \cdot gT^2 + \phi_R + \phi_{WT}, \quad (2a)$$

$$\phi_R = k \cdot \Gamma T^2(x + vT), \quad (2b)$$

$$\phi_{WT} = k \cdot \Gamma gT^2\left(\frac{7}{12}T^2 + TT_1 + \frac{1}{2}T_1^2\right), \quad (2c)$$

where $\Gamma$ is the gravity gradient tensor and $T_1$ is the delay time between the moment the atomic clouds are launched and the first Raman pulse. In this expression, only terms linear in $\Gamma$ are added to the main contribution (1), we neglected the rotation of the Earth and the quantum corrections. To eliminate the sensitivity to the initial position and speed, the term $\phi_R$, A. Roura proposed the use of an effective wave vector of the second Raman pulse slightly different from $k$, $k_2 = (1 + \Gamma T^2/2)k$. This effect has been recently observed [17, 18].

2 Non-eliminated term

I noted [19] that the WT-term $\phi_{WT}$ is not eliminated. After using A. Roura technique, one finds [19]

$$\phi_{WT} = \frac{1}{12} k \cdot \Gamma g T^4, \quad (4)$$

which means that the dependence on the time interval $T_1$ is completely eliminated, while the dependence on the interrogation time $T$ is only partially eliminated, it becomes seven times smaller. The analysis shows that at $T_1 = 0$, this partial elimination is due to the difference between the atomic acceleration extracted from the expression for the interferometer phase and the actual acceleration at the middle time $t = T$. Such a difference was first studied in the article [20].

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WT-term leads to errors in the absolute gravimetry and the EEP test. Let us consider these errors separately.

### 2.1 Absolute gravimeter systematic error

Measurement accuracy increases with longer time $T$. The interrogation time $T = 1.15 \, \text{s}$ was achieved in article [21]. For $\Gamma \approx 3 \times 10^{-10} \, \text{s}^{-2}$, $k = 1.6 \times 10^7 \, \text{m}^{-1}$, $T_1 = 0$. WT-term, $\phi_{WT} \approx 70 \, \text{rad}$, may affect the absolute gravity measurement on the level 0.3 ppm.

To overcome the influence of term (4), it was proposed [22] to define $g$ as

$$g = \phi \{ k T^2 \left[ 1 + \left( 1/12 \right) \Gamma zz T^2 \right] \}^{-1},$$

where $z$ is the vertical axis and $k \| \mathbf{g}$. However, even after such a redefinition, the precision of the gravitational measurement will be because of the error $\delta \Gamma zz$, of the gravity-gradient value,

$$\delta g \sim \frac{1}{12} \delta \Gamma zz T^2 g.$$  

(6)

Although starting from article [7], the methods of measuring gravity-gradient using atom interferometers are studied in many articles, I know only two in which the values of $\Gamma zz$ were published [23, 24]. Uncertainty $\delta \Gamma zz = 30 E$ was reached [24]. This uncertainty leads to an error $\delta g \sim 3 \, \text{ng}$. Since the WT-term grows as $T^4$, with an increase in precision, the relative weight of error (6) also increases as $T^2$.

### 2.2 EEP-test

Partial elimination, the existence of term (4), can also lead to important (to some extent) restrictions for the EEP test. Since one uses two atomic splices, $A$ and $B$, which may have different effective wave vectors, $\mathbf{k}_A$ and $\mathbf{k}_B$, one could use different Raman pulses that may be asynchronized, their delay times between pulses, $T_A$ and $T_B$, may slightly not match. Suppose both wave vectors are vertical and one measures the parameter

$$\eta' = \frac{2 \left( \phi_A - \phi_B \right)}{\phi_A + \phi_B}.$$  

(7)

Even neglecting the WT-term, one finds from Eq. (1) that

$$\eta' \approx \eta + \frac{T_A^2 - T_B^2}{T_A^2 + T_B^2}.$$  

(8)

where $\eta$ is the Eötvös parameter, and we neglected the second-order term proportional to $(g_A - g_B) \left( T_A^2 - T_B^2 \right)$. The state-of-art in atomic interferometry would allow one to measure the parameter $\eta'$ with an uncertainty of $\delta \eta = 1.5 \times 10^{-14}$ at $T = 1.04 \, \text{s}$ [25]. One sees that parameter (7) can be suitable for the EEP-test only if the Raman pulses are synchronized with a relative accuracy less than $\delta \eta$

$$\left| T_A - T_B \right| \lesssim \delta \eta T.$$  

(10)

To avoid this severe restriction, instead of the parameter (7), one can measure the parameter

$$\eta'' = \frac{2 \left( \phi_A - \phi_B \right)}{\phi_A k_A T_A^2 + \phi_B k_B T_B^2}.$$  

(11)

Then, owing to the WT-term (4), one gets

$$\eta'' \approx \eta \left[ 1 + \frac{1}{24} \Gamma zz \left( T_A^2 + T_B^2 \right) \right] + \frac{1}{12} \Gamma zz (T_A + T_B) (T_A - T_B).$$  

(12)

One sees that to avoid the systematic error caused by term (4), one has to synchronize Raman pulses better than

$$\left| T_A - T_B \right| \lesssim \frac{6 \delta \eta}{\Gamma zz T} \approx 30 \, \text{ns}.$$  

(13)

This time interval is 3 orders of magnitude shorter than the typical Raman pulse duration.

### 3 Full elimination

Neither error (6) in the absolute measurement of gravity, nor the constraint (13) will arise after the elimination of all terms (2b, 2c). For this purpose, I propose to extend the A.Roura technique and change the effective wave vectors of all three Raman pulses:

$$\mathbf{k}_i = \mathbf{k} + \Delta \mathbf{k}_i,$$  

(14)

where $\Delta \mathbf{k}_i$ are linear in the tensor $I$. In the limit $\hbar \to 0$, the phase of the atomic Mach–Zehnder interferometer, in which the effective wave vectors of the Raman pulses are different, is given by [26]

$$\phi = \mathbf{k}_1 \cdot \mathbf{x} (T_1) - 2 \mathbf{k}_2 \cdot \mathbf{x} (T_1 + T) + \mathbf{k}_3 \cdot \mathbf{x} (T_1 + 2T).$$  

(15)
\[ x(t) = x + vt + \frac{1}{2}g t^2 + \frac{1}{2} \mathbf{v} t^2 + \frac{1}{6} \mathbf{v}^2 t^3 + \cdots \quad (16) \]

is the trajectory of an atom in which only terms linear in \( \Gamma \) are kept. One requires that the terms proportional to \( x, v \) and \( 1 g \) be eliminated and gets a system of three equations for increments of the wave vectors \( \Delta \mathbf{k} \). The solution of those equations is

\[ \Delta \mathbf{k}_1 = \Delta \mathbf{k}_3 = -\frac{1}{12} \Gamma T^2 \mathbf{k}, \quad (17a) \]

\[ \Delta \mathbf{k}_2 = \frac{5}{12} \Gamma T^2 \mathbf{k}. \quad (17b) \]

We verified that this choice of wave vectors also eliminates the quantum correction to the phase of the order of \( \frac{\hbar}{M} \mathbf{k} \Gamma T^3 \) [14, 15], where \( M \) is the atom mass.

### 4 Error analysis

Let us now estimate the error of the full elimination technique when measuring the absolute value of the gravitational field. Ideally, changes in wave vectors (17) lead to the fact that all systematic errors associated with terms linear in the tensor \( \Gamma \) disappear. But since \( \Gamma \) is known only with limited accuracy, the elimination (17) also leads to some error in the interferometer's phase.

If one changes the wave vectors in the ratio \((1:5:1)\), that is, if \( \Delta \mathbf{k}_1 = \Delta \mathbf{k}_3 = -\alpha, \Delta \mathbf{k}_2 = 5\alpha \), where \( \alpha \) is the vector fitting parameter, which is varied around the desired value

\[ \alpha = \frac{1}{12} \Gamma T^2 \mathbf{k} + \delta \alpha, \quad (18) \]

then one finds for the phase (15)

\[ \phi = \mathbf{k} \cdot \mathbf{g} T^2 - 12 \delta \alpha \cdot x - 12 (T_1 + T) \delta \alpha \cdot \mathbf{p} \]

\[ - (7T^2 + 12T_1 T + 6T_1^2) \delta \alpha \cdot \mathbf{g}. \quad (19) \]

To determine \( \delta \alpha \), one can initially use the differential technique [17]. In two identical interferometers, atoms are triggered from two points \( x_1 \) and \( x_2 \). If the distance between the atom clouds \( L = x_1 - x_2 \), the vector \( \delta \alpha \) and the initial pulse are directed along the vertical \( z \)-axis, then one gets

\[ \delta \alpha_z = -\frac{\delta \phi}{12 L}. \quad (20) \]

where \( \delta \phi \) is the phase difference, which can be reduced to the level of phase noise. With phase noise \( \delta \phi \sim 10^{-3} \), distance \( L \sim 10 \text{ m} \), the uncertainty of the fitting parameter \( \delta \alpha_z \sim 10^{-5} \text{ m}^{-1} \). Then, from Eqs. (19, 20), the error in the magnitude of gravity \( g = \phi/kT^2 \) is given by

\[ \delta g \approx \frac{5 \delta \phi}{12 k L} \approx 3 \text{ pg}, \quad (21) \]

where we assumed that \( x \ll g T^2, T \gg T_1 \) and one exploits the fountain technique [28], i.e. \( \mathbf{p} = -M \mathbf{g} T \). Comparing this error with the error (6) of the method proposed in [22], one sees that implementing our proposal to completely eliminate gravity-gradient terms could allow us to reduce the error by 3 orders of magnitude. Moreover, since the error (21) is \( T \)-independent, it does not increase with a higher precision of gravity measurement.

### 5 New differential scheme to observe the WT-term

Let us consider the WT-term again. Since this term does not depend on the atomic position and velocity, it cannot be observed using existing differential schemes. To make this term visible, we propose the following modification of the differential scheme. If atomic clouds are launching at different times, namely the second cloud at a time \( \delta T_1 \) before the first, and if the interval \( \delta T_1 \) is small enough to neglect the vibration noise, then the WT-term leads to a phase difference

\[ \delta \phi = \mathbf{k} \cdot \mathbf{g} T^2 (T + T_1 + \delta T_1/2) \delta T_1. \quad (22) \]

It is also interesting to note that this phase difference disappears when using any of the elimination techniques.

### 6 Conclusion

In conclusion, we propose to modify the technique of Roura [13], i.e. change in a certain proportion the effective wave vectors of all three Raman pulses to eliminate all terms linear in the gravity-gradient tensor. This complete elimination may allow one to increase the accuracy of absolute gravitational measurements and mitigate the problem of synchronization of Raman pulses when testing the EEP.

In addition, we propose a new differential scheme in which the WT-term can be observed.

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