Optimization of Positioning of Interferometric Array Antennas Using Division Algorithm for Radio Astronomy Applications

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Abstract

The Square Kilometre Array (SKA) ushers in the new generation of large radio telescopes that will work at wavelengths between meters and centimeters. In order to competitively design interferometric antenna arrays such as SKA, it is crucial to focus on the optimization of system performance. In this paper, we contribute to the solution by introducing a new optimization algorithm called Division Algorithm (DA). This algorithm finds the optimal positions of antennas to simultaneously maximize \( u-v \) coverage and decrease sidelobe level (SLL). The DA is able to optimize the configuration of the interferometric array in both snapshot and Earth rotation synthesis observations. To demonstrate its efficiency, the DA is applied to configure an optimum 30-element array for the Giant Metrewave Radio Telescope. The proposed algorithm is able to improve the overlapped samples parameter by about 4% and the unsampled cells parameter by about 12%, at snapshot observation, compared to the Genetic Algorithm (GA). DA is able to improve these two parameters for a 6-hr tracking observation as well. Finally, the proposed algorithm is compared with the GA for different source declination. Results show that the DA is able to decrease the SLL better than the GA.

Key words: instrumentation: interferometers – telescopes

1. Introduction

Cross-correlation of antennas pairs in an interferometric array finds the visibilities in the \( u-v \) plane that are a measure of the sky brightness distribution. In order to achieve high resolution images with high signal-to-noise ratios (S/N), a uniform \( u-v \) distribution of antennas is favorable. At the same time, an optimized uniform array configuration would better fulfill the specific requirements of the observations (Su et al. 2004).

Typically, planning the locations of antennas in an interferometric array relies on optimizing the \( u-v \) plane coverage by decreasing Side Lobe Levels (SLL; see, e.g., Kogan 2000). Practically, to design an array configuration, various parameters such as source structures and positions, integration times, and application constrains (i.e., cost and geometrical locations) must be taken into account (Boone 2001).

There are several notable works done on optimization of antenna arrays, among them: Kogan 2000; Boone 2001, 2002; Cohanim et al. 2004; Su et al. 2004; Karastergiou et al. 2006; Jin & Rahmat-Samii 2008; Oliveri et al. 2010; Beardsley et al. 2012; Kiehbadroudinezhad et al. 2014. In particular, Karastergiou et al. (2006) obtained the most appropriate \( u-v \) plane sampling for low-density interferometers, based on the work of Keto (1997) and Boone (2001, 2002), but they did not consider SLL suppression, which is important in astronomical applications to capture a clear image of a radio point source. Oliveri et al. (2010) applied the Almost Difference Sets method based on binary sequences to improve the correlation properties in interferometric arrays. Particle swarm optimization was applied to an interferometric array for radio astronomy applications by Jin & Rahmat-Samii (2008). They applied the algorithm on closed-arm and open-arm configurations and showed that deploying antennas on three arms unequally can provide better \( u-v \) plane coverage with lower SLLs than placing antennas on three arms equally. The drawback of this algorithm is that it needs to be run separately for each observation to attain maximum \( u-v \) plane coverage. Beardsley et al. (2012) proposed a Bessel decomposition-based algorithm that is sensitive to large-scale over and under densities in the \( u-v \) plane without considering minimizing SLLs. In order to optimize the configuration of an interferometric array either in the snapshot or hour tracking observation, an algorithm named Sieving was developed by Su et al. (2004). The method did not consider SLL suppression and the algorithm was run separately for each observation to obtain maximum \( u-v \) plane coverage.

The more complicated problem of optimizing a correlator array that satisfies all possible observation conditions, such as suppressed SLL in the angular domain \((l-m)\) domain and uniform coverage in the spatial frequency domain \((u-v)\) domain, has been considered only recently by Kiehbadroudinezhad et al. (2014). Therefore, our contribution in the current paper focuses on a new algorithm that optimizes an interferometric array of antennas, in the two main aspects of the \( u-v \) sample distribution in the spatial frequency domain in both observations and SLLs in the angular domain, while considering the scientific application goals.

Moreover, with respect to previous methods and algorithms (see, e.g., Kogan 2000; Boone 2001; Sodin & Kopilovich 2002; Cohanim et al. 2004; Su et al. 2004; Karastergiou et al. 2006; Oliveri et al. 2010; Gauci et al. 2013; Kiehbadroudinezhad et al. 2014), Kiehbadroudinezhad et al. (2014) applied the Genetic Algorithm (GA) to correlator arrays. The authors showed that the GA was able to cover the \( u-v \) plane more efficiently than the Giant Metrewave Radio Telescope (GMRT) under the same constraints. They showed that the GA could obtain an optimized solution that met almost all the desired requirements, simultaneously. Nevertheless, utilizing operators such as crossover and mutation decreases the speed of the
algorithm and increases the complexity. Therefore, the present study improves the previous work by proposing a new algorithm; the Division Algorithm (DA) without such operators. However, the DA has an easier, faster, and more flexible procedure than the algorithm was used in Kiehbadroudinezhad et al. (2014), as it will be demonstrated in the results. Even though the DA yields good results without using these operators, it could work more precisely if various operators of other algorithms such as mutation and crossover of GA are used. It is obvious that adding any operator would lead to lower speed of the algorithm.

The paper is organized as follows. In Section 2, the DA is presented. Section 3 describes the problem statement and how the optimization works on such a problem. In Section 4 preliminary results are shown to assess the validity of the proposed approach. A quantitative assessment of the array configuration’s effects on image efficiency for the GMRT is presented. Finally, conclusions are drawn in Section 5.
2. The Division Algorithm

In general, the application of the DA would be the optimization of problems that need gridding as well as uniform and smooth data coverage. Specifically, the proposed algorithm is based on the division of a given area by the number of possible candidate positions.

Consequently, this algorithm can be applied to find the optimal locations of antennas in a specific area of an interferometric array to enhance image quality of a distant radio source. Figure 1 illustrates the flowchart of the DA, and Figures 2–4 show an example of the DA to clarify the steps of the algorithm.

The DA has the following steps as shown in Figure 1:

1—Partitioning: the given area is partitioned.

1a—A main area (m.a.) is randomly created in the given area but the algorithm can create the m.a. with a known fit area.

1b—Two additional areas are selected, one smaller (s.a.) and one larger (l.a.) than the m.a.

To generate the l.a. and s.a., the m.a. is respectively multiplied and divided by 2 or by any other integer depending on the problem solving area. The main area is selected randomly, and it should be noticed that the area can get any geometric shape such as circular. It mainly depends on the problem (refer to Figure 2).

2—Division: Each of three areas from step 1 is further divided.

To accurately search for the optimum solutions, the algorithm divides each area into subareas, refer to Figure 2.

It is not necessary to divide the three areas by the same number m. Each single area can be divided by different numbers to generate different numbers of subareas.

3—Searching: N points/positions are selected in each defined subarea from step 2.

Depending on the problem solving, the algorithm selects N points in each subarea.

This selection can be random or based on a known fit evaluation function. More generally, it is not necessary for the algorithm to search for the same number (N) of candidate points in each subarea.

4—Evaluation: An evaluation value is assigned to each point from step 3.

The algorithm calculates evaluation values for all points/positions.

In the example shown in Figure 3, to simplify the algorithm, the DA looks for one solution in each subarea based on a defined fitness function, although the algorithm could seek for N number of points in each subarea.

5—Checking: The evaluation values from step 4 are ordered according to criteria.

Given the values from step 4 (the values of the fitness function), the algorithm checks that the criterion/criteria of the problem are met. Then it makes the decision to stop or continue based on the defined application requirements. If the criteria are met, it means that the current candidate points/positions (step 3) are our solutions and the algorithm ends. Otherwise, the DA goes to step 6 and from there it returns to step 2.

For the example illustrated in Figure 3, the criterion is to find two unique independent parameters that maximize the function \( f(x, y) = \frac{1}{1 + x^2 + y^2} \). The DA realizes that the optimum solution(s) are in the small area based on the calculated evaluation values, which show the closest values to unity are located in the small area.

6—New Division: The algorithm selects the best area/areas according to the highest evaluation(s) value(s) from the previous steps.

The best area is selected based on possible methods such as “Roulette-wheel” selection (see, e.g., Kiehbadrouinezhad et al. 2014), refer to Figure 3.

If the selected areas are in the:

5a—s.a.: the area is partitioned so that we have three new partitions. The largest one is assigned to the l.a., the medium one is assigned to m.a., and the smallest one becomes the new s.a.

5b—m.a.: the area is partitioned so that we have two new partitions. The largest one is assigned to the l.a., the medium one becomes the new m.a., and s.a. remains unaltered.

5c—l.a.: the algorithm finds an area larger than the current l.a. The largest one becomes the new l.a., the previous l.a. one becomes the new m.a., and the previous m.a. is the new s.a.

Then the algorithm will go back to the step 2.

If the algorithm continues, then it obtains new evaluation values for the new division as shown in Figure 4. Again, the optimum solution(s) are likely in the small area. The algorithm continues or stops whether the desired requirements are met or not.

Although the algorithm provides reasonable results, some parameters of other algorithms such as mutation, and crossover of GA could be used to make the algorithm more precise (see, e.g., Kiehbadrouinezhad et al. 2014). It should be noted that using these parameters might decrease the speed of the algorithm.

3. Applying DA to an Interferometric Array

To validate the algorithm, the DA is applied to the GMRT array. The GMRT is an approximately Y-shaped array of 30 parabolic dish antennas, 45 m in diameter each. The array is located at about 80 km north of Pune, India (19°5'47.46N 74°2'59.07E; Swarup et al. 1991).
Our objective is to optimize the position of \( n = 30 \) antennas to increase the distribution of \( u-v \) samples on \( u-v \) plane coverage while suppressing SLL, given the following parameters.

1. \( n \) = number of antennas;
2. \( f \) = operating frequency;
3. \( \text{maxlat} \) = maximum latitude of the area of interest;
4. maxlon = maximum longitude of the area of interest;
5. minlat = minimum latitude of the area of interest;
6. minlon = minimum longitude of the area of interest;
7. m = number of subareas in each area.

To apply the DA to localize the n antennas, some initial assumptions are made.
First of all, the algorithm takes the area as a known parameter and second, the number of subareas is based on the number of antennas.

The antennas location is typically given in spherical coordinates, but for simplicity in this paper, this system is converted to the rectangular coordinates (Kiehbadroudinezhad et al. 2013).

3.1. Partitioning, Division, and Searching

Two maximum and minimum values of latitudes and longitudes are given to identify the area of interest. In order to find the number of areas (nofarea), the number of antennas (n) is divided by the integer (m), according to the following equation:

$$\text{nofarea} = \text{fix}\left(\frac{n}{m}\right)$$  \hspace{1cm} (1)

where the function fix rounds \(\frac{n}{m}\) to the nearest lower integer.

Specifically, we choose \(m = 4\), therefore, the total number of areas (nofarea) is 7. It is worth noticing that the Equation (1) could be changed based on the application requirements.

Out of the 30 antennas, 4 are located at the boundary: 2 antennas in radial distance and another 2 in the middle lines of latitude and longitude, as shown in Figure 2. With this condition satisfied, the array has the desired resolution. The remaining 26 antennas are distributed one by one in the remaining subareas following the algorithm: each antenna is randomly located in one subarea (this distribution can also be changed according to the requirements).

Antennas in the GMRT were deployed based on several factors. Among all, the two primary contributing factors are: (1) obtaining the maximum coverage in the spatial frequency domain, and (2) the size of the sources to be studied (long baselines are adopted for small sources, whereas short baselines are used for extended sources). Because the antenna locations in the GMRT are fixed, both compact and extended arrays are employed to meet the desired requirements (Swarup et al. 1991). Hence, in order to take into account these factors in the application of the DA, we defined the distances \(D_{11}\) and \(D_{22}\), as follows (see Figure 5):

\[
\begin{align*}
D_{11} &= l \times \frac{(\text{max lat} - \text{meanlat})}{\text{nofarea}} \\
D_{22} &= l \times \frac{(\text{max lon} - \text{meanlon})}{\text{nofarea}} 
\end{align*}
\]

where \(l\) is an integer that varies according to the size of the sources one intends to study; extended or small source.

The distances \(D_1\) and \(D_2\) used to complete the configuration are calculated using the Equation (3):

\[
\begin{align*}
D_1 &= \frac{(\text{max lat} - (\text{meanlat} + D_{11}))}{\text{nofarea}} \\
D_2 &= \frac{(\text{max lon} - (\text{meanlon} + D_{22}))}{\text{nofarea}} 
\end{align*}
\]


Note. 6 hr is used for hour tracking synthesis.

### 3.2. Evaluation and Checking

The next step is evaluation. To evaluate the position of each antenna in each subarea, the fitness function elaborated in Kiehbadroudinezhad et al. (2014) is used. Each new antenna/point in the current population is assigned with a fitness value based on Equations (4) and (5):

\[
\text{fitness}(k) = -\text{mean(first SLL + second SLL + third SLL)},
\]

where fitness(k) is the fitness value of kth baseline in nth generation, \( o_l(k) \) indicates the number of overlapped samples generated by kth baseline in nth generation, max(ol) provides the maximum value of overlapping that is in nth generation, D(k) is the distance from the grid center, A_tofo is the area generated by kth baseline’s samples in nth generation, and A_tofo is the total area generated by nth generation (Kiehbadroudinezhad et al. 2014). It follows that

\[
\text{fitness}(k) = -\text{mean(first SLL + second SLL + third SLL)},
\]

where first SLL, second SLL, and third SLL are the peak values of the first, second, and third SLL, respectively, generated from kth baseline in nth generation (Kiehbadroudinezhad et al. 2014).

The fitness value is assigned starting from the first random selected antenna/point in the first subarea. The assignment can be repeated r times until an optimum value is achieved. The evaluation procedure is applied to the second new antenna/point and continues up to the last antenna. In this implementation of DA, step 6 is not used because of the initial assumption on the fix area of the array, as explained at the beginning of Section 2.

### 4. Results

The algorithm is implemented in Matlab R2010a (MATLAB 7.10). The computer used is an Intel Core™ i5-2410 CPU @ 2.30 GHz Pentium 3.2 GHz with a 4 GB RAM. To optimize the \( u-v \) plane coverage and suppress the sidelobe levels in the synthesized beam, the DA algorithm is applied in this section. Results are shown in Figure 6 in terms of antennas configuration (6.a), \( u-v \) plane coverage for snapshot (6.b) and hour tracking (6.c). In Tables 1–7, results are given in terms of \( u-v \) plane parameters and SLL.

Table 1

| Configuration  | Overlapped Samples% (Snapshot) | Overlapped Samples% (Hour Tracking) | Unsampled Cells% (Snapshot) | Unsampled Cells% (Hour Tracking) |
|----------------|--------------------------------|------------------------------------|-----------------------------|----------------------------------|
| DA array       | 47.82                          | 50.11                              | 50.67                       | 50.78                            |
| 150th generation | 49.77                          | 53.36                              | 57.12                       | 54.65                            |
| GMRT           | 72.64                          | 74.12                              | 79.34                       | 75.13                            |

Table 2

| Configuration  | First SLL (dB) (Hour Tracking) | MeanSLL (dB) (Hour Tracking) | Peak SLL (dB) (Hour Tracking) |
|----------------|-------------------------------|-------------------------------|-------------------------------|
| DASLL array    | −31.55                        | −25.42                        | −22.14                        |
| 150th generation | −25.23                        | −23.07                        | −21.74                        |
| GMRT           | −11.71                        | −13.39                        | −10.68                        |

Note. 6 hr is used for hour tracking synthesis.

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#### Table 2

| Configuration  | First SLL (dB) (Hour Tracking) | MeanSLL (dB) (Hour Tracking) | Peak SLL (dB) (Hour Tracking) |
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| 150th generation | −25.23                        | −23.07                        | −21.74                        |
| GMRT           | −11.71                        | −13.39                        | −10.68                        |

Note. 6 hr is used for hour tracking synthesis.
Table 3
Different Parameters of u−v Coverage Using the Same SLL Population of Division Algorithm

| Configuration  | Overlapped Samples% (Snapshot) | Overlapped Samples% (Hour Tracking) | Unsampled Cells% (Snapshot) | Unsampled Cells% (Hour Tracking) |
|---------------|--------------------------------|------------------------------------|----------------------------|---------------------------------|
| DASLL array   | 52.18                          | 64.00                              | 56.45                      | 64.88                           |

Note. 6 hr is used for hour tracking synthesis.

Table 4
Different Parameters of u−v Coverage for Source Decl. = 60°

| Configuration  | Overlapped Samples% (Snapshot) | Overlapped Samples% (Hour Tracking) | Unsampled Cells% (Snapshot) | Unsampled Cells% (Hour Tracking) |
|---------------|--------------------------------|------------------------------------|----------------------------|---------------------------------|
| GMRT          | 51.37                          | 58.86                              | 65.14                      | 59.51                           |
| 150th generation | 50.34                          | 66.5                               | 59.44                      | 66.82                           |
| DA array      | 51.03                          | 69.98                              | 60.11                      | 70.26                           |

Note. 6 hr is used for hour tracking synthesis.

Table 6
Calculated SLL for Source Decl. = 60°

| Configuration  | First SLL (dB) (Hour Tracking) | MeanSLL (dB) (Hour Tracking) | Peak SLL (dB) (Hour Tracking) |
|---------------|-------------------------------|-----------------------------|------------------------------|
| GMRT          | −9.76                         | −11.26                      | −5.51                        |
| 150th generation | −11.55                        | −14.29                      | −11.5                        |
| DASLL array   | −16.49                        | −18.36                      | −13.51                       |

Note. 6 hr is used for hour tracking synthesis.

two additional source decl. The source decl. used in this part of the study are decl. = 60° and decl. = −30°.

Table 4 shows results of overlapped samples and unsampled cells for snapshot and hour tracking observations when the source decl. is 60°.

From the results, the DA and the GA have approximately the same performance in improving the overlapped samples and unsampled cells for both observations as compared to the configuration of the GMRT.

Table 5 indicates the improved ratios for 6 hr tracking and snapshot observations from the GA (150th generation) and DA algorithms with respect to the GMRT’s configuration for source decl. = −30°.

Finally, SLLs results are shown in Tables 6 and 7 for source decl. of 60° and −30°, respectively. It is then evident that for both decl. values, the DA (DASLL) is able to decrease the SLL better than the GA.

Therefore, the configuration using the DASLL array is the solution that provides the best results in almost all of the requirements in addition to reducing running time for all analyzed decl. (45°, 60°, and −30°).

5. Conclusions

The new algorithm (DA) presented in this study has been proven to be useful in achieving optimum solutions for interferometric arrays. Although the algorithm optimizes the solutions, it never converges to an optimum solution at each running time, as it is not as deterministic as GA. The version of the algorithm that has been tested is not specialized for astronomical applications constraints, but it could be specialized, as it is mentioned in the general description of the algorithm. Because the DA is able to be adapted to different constraints and geometries, it is a flexible optimization
algorithm. From the results of its application to interferometric arrays, one unique output configuration can be used for optimizing $u$–$v$ coverage for both snapshot and hour tracking observations as well as minimizing SLL in angular domain. Moreover, the optimum configuration is obtained after few repetitions. Although the DA in this work uses 30 telescopes with 45 m diameter each, the algorithm is designed to work on large populations, for different geometrical areas and for different sizes of telescopes. The proposed algorithm is able to distribute $u$–$v$ samples in the spatial frequency domain, and it decreases the SLL to enhance the quality of the simulated point source. The Earth rotation effect is included to simulate the hour tracking observations of the radio source. The DA is then compared with the GA under the same conditions of source decl. ($45^\circ$, $60^\circ$, and $-30^\circ$) and time duration. The DA used in this study shows better performance than the GA with respect to the uniform distributed samples, SLL, execution time, number of repetitions, and computational complexity.

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