Self-Gravitational Corrections to the Cardy-Verlinde Formula and the FRW Brane Cosmology in $SdS_5$ Bulk

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Abstract

The semiclassical corrections to the Cardy-Verlinde entropy of a five-dimensional Schwarzschild de-Sitter black hole ($SdS_5$) are explicitly evaluated. These corrections are considered within the context of KKW analysis and arise as a result of the self-gravitation effect. In addition, a four-dimensional spacelike brane is considered as the boundary of the $SdS_5$ bulk background. It is already known that the induced geometry of the brane is exactly given by that of a radiation-dominated FRW universe. By exploiting the CFT/FRW-cosmology relation, we derive the self-gravitational corrections to the first Friedmann-like equation which is the equation of the brane motion. The additional term that arises due to the semiclassical analysis can be viewed as stiff matter where the self-gravitational corrections act as the source for it. This result is contrary to standard analysis that regards the charge of $SdS_5$ bulk black hole as the source for stiff matter. Furthermore, we rewrite the Friedmann-like equation in such a way that it represents the conservation equation of energy of a point particle moving in a one-dimensional effective potential. The self-gravitational corrections to the effective potential and, consequently, to the point particle’s motion are obtained. A short analysis on the asymptotic behavior of the 4-dimensional brane is presented.
1 Introduction

Concerning the quantum process called Hawking effect [1] much work has been done using a fixed background during the emission process. The idea of Keski-Vakkuri, Kraus and Wilczek (KKW) [2] is to view the black hole background as dynamical by treating the Hawking radiation as a tunnelling process. The energy conservation is the key to this description. The total (ADM) mass is kept fixed while the mass of the black hole under consideration decreases due to the emitted radiation. The effect of this modification gives rise to additional terms in the formulae concerning the known results for black holes [3]; a non-thermal partner to the thermal spectrum of the Hawking radiation shows up.

Recent astronomical observations of supernovae and cosmic microwave background [4] indicate that the universe is accelerating and can be well approximated by a world with a positive cosmological constant. If the universe accelerates indefinitely, the standard cosmology leads to an asymptotic dS universe. In addition, de Sitter spacetime plays an important role in the inflationary scenario where an exponentially expanding approximately dS spacetime is employed to solve a number of problems in standard cosmology. Furthermore, the quantum field theory on dS spacetime is also of considerable interest.

Much attention has currently been paid for the duality between de Sitter (dS) gravity and CFT in analogy to the AdS/CFT correspondence [5](for a very good review see also [6]). The reason is that the isometry of (n+1)-dimensional de Sitter space, $SO(n+1,1)$, exactly agrees with the conformal symmetry of n-dimensional Euclidean space. Thus, it might be natural to expect the correspondence between (n+1)-dimensional gravity in de Sitter space and n-dimensional Euclidean CFT (the dS/CFT correspondence). Moreover, the holographic principle between the radiation dominated Friedmann-Robertson-Walker (FRW) universe in n-dimensions and same dimensional CFT with a dual (n+1)-dimensional AdS description was studied in [7]. Particularly, one can see the correspondence between black hole entropy and the entropy of the CFT which is derived by making the appropriate identifications for the Friedmann equation with the Cardy-Verlinde formula.

There has been much recent interest in calculating the quantum corrections to $S_{BH}$ (the Bekenstein-Hawking entropy) [8]. The leading-order correction is proportional to $\ln S_{BH}$. In a recent work, Carlip [9] has deduced the leading order quantum correction to the classical Cardy formula. The Cardy formula follows from a saddle-point approximation of the partition function for a two-dimensional conformal field theory. This leads to the theory’s density of states, which is related to the partition function by way of a Fourier transform [10].

In the present paper, we take into account corrections to the entropy of the five-dimensional Schwarzschild-de Sitter black hole (abbreviated to $SdS_5$ in the sequel) that arise due to the self-gravitational effect. Previous studies of the Cardy-Verlinde formula (or
corresponding Friedmann equation) in a dS/CFT context have thus far neglected this sub-dominant, but important, contribution.\(^1\)

# Schwarzchild-de Sitter black hole

The \(SdS_5\) black hole is a constant curvature solution of the Einstein equation which follows from the action

\[
S = \int d^5x \sqrt{-g} \left( \frac{R}{16\pi G_5} + \Lambda_5 \right),
\]

where \(R\) is the scalar curvature, \(\Lambda_5\) is the five-dimensional positive cosmological constant, and \(G_5\) denotes the five-dimensional Newton’s constant. The corresponding line element is given by

\[
ds^2 = -\left(1 - \frac{r^2}{l^2} - \frac{M\varepsilon_3}{r^2}\right)dt^2 + \left(1 - \frac{r^2}{l^2} - \frac{M\varepsilon_3}{r^2}\right)^{-1}dr^2 + r^2d\Omega^2_3,
\]

where \(\varepsilon_3 = \frac{16\pi G_5}{3V_3}\), \(M\) is the black hole mass \([13]\), \(V_3\) is the volume of the spherical hypersurface described by \(dS_3^2\), and \(l\) represents the curvature radius of the \(SdS_5\) bulk space. The cosmological constant and the curvature radius are related by \(\Lambda_5 = \frac{6}{l^2}\). The \(SdS_5\) cosmological horizon is described by the largest root of

\[
\frac{r^4}{l^2} - r^2 + M\varepsilon_3 = 0
\]

and thus it takes the form

\[
r_c^2 = \frac{l^2}{2} \left(1 + \sqrt{1 - 4M\varepsilon_3/l^2}\right),
\]

while the black hole horizon takes the form

\[
r_{bh}^2 = \frac{l^2}{2} \left(1 - \sqrt{1 - 4M\varepsilon_3/l^2}\right).
\]

It is evident that due to the presence of two horizons in \(SdS_5\) black hole spacetime, one can define two different Hawking temperatures. The temperature of the cosmological horizon \((r_c)\) is given as

\[
T_c = -\left(\frac{1}{2\pi r_c} - \frac{r_c}{\pi l^2}\right).
\]

\(^1\)On the contrary, self-gravitational corrections to thermodynamical quantities of AdS spaces have attracted a lot of attention. For instance, the authors have shown that the entropy of Achúcarro-Ortiz black hole (a locally AdS space) can be described by the Cardy-Verlinde formula \([11]\) and the self-gravitational corrections to this formula have been computed \([12]\).
while the temperature of black hole horizon \((r_{bh})\) reads

\[
T_{bh} = \frac{1}{2\pi r_{bh}} - \frac{r_{bh}}{\pi l^2}.
\] (7)

The entropy \(S\) for both horizons is

\[
S = \frac{V_3 r_H^3}{4G_5}
\] (8)

where \(r_H\) takes the values \(r_c\) and \(r_{bh}\) for the cosmological and the black hole horizons, respectively. The thermodynamical energy is given by

\[
E = \pm M
\] (9)

where + corresponds to the black hole horizon and − to the cosmological horizon.

3 Self-Gravitational Corrections to Cardy-Verlinde Formula

We are interested primarily in the corrections to the entropy \((S)\) that arise in the context of KKW analysis \([2]\) due to self-gravitational effect. Here we adopt the analysis as presented by Medved \([14]\) and thus we consider the incoming radiation from the cosmological horizon and, for the moment, ignore the outgoing radiation from the black hole horizon. Therefore, a pair of particles is spontaneously created just outside the cosmological horizon. The positive-energy particle is viewed as an incoming particle since it tunnels through the cosmological horizon and arrives to the bulk space. Thus, the black hole mass is increased while the background energy is lowered since the negative-energy particle remains behind the cosmological horizon. Let us remind that the key point to the KKW analysis is that the total energy of the spacetime under study is kept fixed while the black hole mass is allowed to vary. We therefore expect a black hole of initial mass \(M\) to have a final mass of \(M + \omega\) where \(\omega\) is the energy of the emitted particle. The point of particle creation, \(r_i\), is the radius of the cosmological horizon, i.e. \(r_c\),

\[
r_i^2 = \frac{l^2}{2} \left( 1 + \sqrt{1 - \frac{4\varepsilon_3 M}{l^2}} \right)
\] (10)

while the classical turning point of motion is given by

\[
r_f^2 = \frac{l^2}{2} \left( 1 + \sqrt{1 - \frac{4\varepsilon_3 (M + \omega)}{l^2}} \right).\] (11)

\(^2\)Further developments in KKW analysis can be found in \([3]\).
To first order in the emitted energy ($\omega$), the aforementioned radii are related as follows

$$r_f^2 = r_i^2 \left(1 - \frac{\varepsilon_3}{r_i^2 \sqrt{1 - \frac{4\varepsilon_3 M}{l^2}}}\right)$$  \hspace{0.5cm} (12)

Consequently, the change in the entropy of the $SdS_5$ black hole during the process of emission takes the form

$$\Delta S = S_f - S_i = \frac{V_3}{4G_5} (r_f^3 - r_i^3) = -\frac{\omega}{T(\omega)}$$  \hspace{0.5cm} (13)

where $S_f$ is the modified entropy of the $SdS_5$ black hole due to the self-gravitational effect, $S_i$ is the standard formula for the entropy (Bekenstein-Hawking entropy) derived when the black hole mass is kept fixed, and $T(\omega)$ is the corrected temperature of the cosmological horizon due to the self-gravitational effect. The expression for the modified temperature of the cosmological horizon is given as

$$T(\omega) = \left(\frac{2r_i^2 - l^2}{2\pi l^2 r_i}\right) + \mathcal{O}(\omega)$$  \hspace{0.5cm} (14)

It is obvious that, to first order in $\omega$, the modified temperature is the Hawking temperature of the cosmological horizon, i.e. $T_c$. Substituting equation (14) into equation (13), we evaluate the corrected entropy of the $SdS_5$ black hole due to the self-gravitational effect

$$S_f = S_i - \frac{2\pi l^2 r_i}{(2r_i^2 - l^2)} \omega + \mathcal{O}(\omega^2)$$  \hspace{0.5cm} (15)

Since we are primarily interested to keep the leading-order correction (that is first order in $\omega$), we drop the last term in equation (15) and the corrected entropy of the $SdS_5$ black hole reads

$$S_f = S_i - \frac{2\pi l^2 r_i}{(2r_i^2 - l^2)} \omega$$  \hspace{0.5cm} (16)

It is interesting that the semiclassical correction to the black hole entropy is negative. This may be compared with the possibility for SdS black hole entropies to be negative in the context of higher derivative gravitational theory [15]. In this work, the negativity of black hole entropy was regarded as an indication of a new type of instability in asymptotically de Sitter black hole physics. Thus, the self-gravitational corrections presented here may also try to put the $SdS_5$ black hole under consideration to a less stable state.

Due to the dS/CFT correspondence, we are now ready to evaluate the corrections to the Cardy-Verlinde formula for the entropy of the $SdS_5$ black hole. An important physical quantity that gets involved in this calculation is the Casimir energy which is defined as the violation of the Euler identity and for the spacetime under study is given by

$$E_C = 4E_4 - 3T_c S_i$$  \hspace{0.5cm} (17)
where the four-dimensional energy $E_4$ which can be derived from the FRW equation of motion for a brane propagating in the $SdS_5$ bulk space, is given by \[7\]

\[ E_4 = \pm \frac{l}{r} M = \frac{l}{r} E \quad (18) \]

where ‘+’ corresponds to the black hole horizon and ‘−’ to the cosmological horizon. The temperature associated with the 4-dimensional brane, $T_{c}^{\text{brane}}$, should differ from expressions [10] and [17] by a similar factor [7]

\[ T_{c}^{\text{brane}} = \frac{l}{r} T_c \quad (19) \]

It is easily seen that, to first order in $\omega$, the temperature of the brane is unchanged since the temperature of the cosmological horizon, i.e. $T_c$, is unchanged as stated before. The modified Casimir energy is given by

\[ \mathcal{E}_C = 4E_4 - 3T_{c}^{\text{brane}} S_i \quad (20) \]

where calligraphic letters denote that the corresponding quantity is modified due to the self-gravitational effect. The modified four-dimensional energy is

\[ E_4 = E_4 - \frac{l}{r} \omega \quad (21) \]

while the last term in \[20\] is given by

\[ T_{c}^{\text{brane}} S_i = T_{c}^{\text{brane}} S_f = T_{c}^{\text{brane}} S_i - \frac{T_{c}^{\text{brane}}}{T_c} \omega \]

\[ = T_{c}^{\text{brane}} S_i - \frac{l}{r} \omega \quad (22) \]

Substituting equations \[21\] and \[22\] in \[20\], the modified Casimir energy takes the form

\[ \mathcal{E}_C = E_C - \frac{l}{r} \omega \quad . \quad (23) \]

Due to the self-gravitational corrections, the modified Cardy-Verlinde formula for the entropy of the $SdS_5$ black hole is given as

\[ S_{CFT} = \frac{2\pi r}{3} \sqrt{|\mathcal{E}_C(2\mathcal{E}_4 - \mathcal{E}_C)|} \quad (24) \]

Using equations \[21\] and \[23\], the modified Cardy-Verlinde entropy formula becomes

\[ S_{CFT} = \frac{2\pi r}{3} \left| \left[ E_C - \frac{l}{r} \omega \right] \left[ (2E_4 - E_C) - \frac{l}{r} \omega \right] \right| \quad (25) \]

and keeping terms up to first order in the emitted energy $\omega$, it takes the form
where the small parameter $\varepsilon$ is given by

$$
\varepsilon = \frac{l}{r E_C (2E_4 - E_C)} .
$$

A welcomed but not unexpected result is that there is no entropy bound violation due to self-gravitational corrections to the Cardy-Verlinde entropy$^3$.

### 4 Self-Gravitational Corrections to FRW Brane Cosmology

We now consider a 4-dimensional brane in the $SdS_5$ black hole background. This 4-dimensional brane can be regarded as the boundary of the $SdS_5$ bulk background. Let us first replace the radial coordinate $r$ with $a$ and hence the line element described by equation (2) now takes the form

$$
\text{ds}^2 = -h(a) dt^2 + \frac{1}{h(a)} da^2 + a^2 d\Omega^2_{(3)} ,
$$

where

$$
h(a) = \left(1 - \frac{a^2}{l^2} - \frac{\mu}{a^2}\right)
$$

and

$$
\mu = M \varepsilon_3
$$

is the black hole mass parameter. It was shown that by reduction from the $SdS_5$ background and by imposing the condition

$$
- h(a) \left(\frac{\partial t}{\partial \tau}\right)^2 + \frac{1}{h(a)} \left(\frac{\partial a}{\partial \tau}\right)^2 = -1
$$

where $\tau$ is a new time parameter, one obtains an FRW metric for the 4-dimensional timelike brane

$$
ds^2_{(4)} = -d\tau^2 + a^2(\tau)d\Omega^2_{(3)} .
$$

Thus, the 4-dimensional FRW equation describes the motion of the brane universe in the $SdS_5$ background. It is easy to see that the matter on the brane can be regarded as radiation and consequently, the field theory on the brane should be a CFT.

$^3$A violation of the holographic entropy bound was observed when self-gravitational corrections to the Cardy-Verlinde entropy formula of the two-dimensional Achúcarro-Ortiz black hole were included [12] (see also [16]).
Within the context of AdS/CFT correspondence, Savonije and Verlinde studied the CFT/FRW-cosmology relation from the Randall-Sundrum type braneworld perspective [17]. They showed that the entropy formulas of the CFT coincide with the Friedmann equations when the brane crosses the black hole horizon. Analogously, one can assume holographic relations between the FRW universe and the boundary CFT which is dual to the $SdS_5$ bulk background in accordance with dS/CFT correspondence. In the case of a 4-dimensional spacelike\(^4\) brane\(^5\)

\[
ds_{(4)}^2 = d\tau^2 + a^2(\tau) d\Omega_{(3)}^2 ,
\]

one of the identifications that supports the CFT/FRW-cosmology relation is

\[
H^2 = \left( \frac{2G_4}{V} \right)^2 S^2
\]

where $H$ is the Hubble parameter defined by $H = \frac{1}{a} \frac{da}{d\tau}$ and $V$ is the volume of the 3-sphere ($V = a^3V_3$). The 4-dimensional Newton constant $G_4$ is related to the 5-dimensional one $G_5$ by

\[
G_4 = \frac{2}{l} G_5 .
\]

It was shown that at the moment that the 4-dimensional spacelike brane crosses the cosmological horizon, i.e. when $a = a_c$, the CFT entropy and the entropy of the $SdS_5$ black hole are identical. By substituting (26) into (34), we obtain the self-gravitational corrections to the motion of the CFT-dominated brane

\[
H^2 = \left( \frac{2G_4}{V} \right)^2 S^2_{CFT} (1 - \varepsilon \omega)^2 .
\]

It is obvious that from the first term on the right-hand side of (36) we get the standard Friedmann equation with the appropriate normalization

\[
H^2 = \frac{1}{a_c^2} - \frac{8\pi G_4}{3} \rho
\]

where $\rho$ is the energy density defined by $\rho = |E_4|/V$. Therefore, the correction to the FRW equation due to the self-gravitation effect is expressed by the second term in the right-hand side of equation (36). Keeping terms up to first order in the emitted energy ($\omega$), the modified Hubble equation due to the self-gravitation corrections is

\[
H^2 = \frac{1}{a_c^2} - \frac{8\pi G_4}{3} \rho - 2E_4 \frac{l}{a_c} \left( \frac{2G_4}{V} \right)^2 \left( \frac{2\pi a_c}{3} \right)^2 \omega .
\]

\(^4\)In this work we are interested in the radiation coming in the bulk space through the cosmological horizon. Timelike brane, i.e. a brane that has a Minkowskian metric, can only cross the black hole horizon. On the contrary, a spacelike brane, i.e. a brane with Euclidean metric, is able to cross both the black hole horizon and the cosmological horizon.

\(^5\)In order to derive the 4-dimensional spacelike brane, the imposed condition has to be slightly changed by replacing the ‘−’ with a ‘+’ on the right-hand side of it.
Taking into account that all quantities should be evaluated on the cosmological horizon, the modified Hubble equation, i.e. the first Friedmann equation, takes the form

\[ H^2 = \frac{1}{a^2} - \frac{8\pi G_4}{3} \rho + \frac{8\pi G_4}{3} \left[ \frac{4\pi G_4}{3} \frac{l}{a^2 V_3} \rho \right] \omega \] (39)

where the volume \( V \) is given by \( a^3 V_3 \). At this point it should be stresses that our analysis was up to now restricted to the spatially flat \( (k = +1) \) spacelike brane.

We will now extend the aforesaid analysis. We therefore consider an arbitrary scale factor \( a \) and include a general \( k \) taking values \( +1, 0, -1 \) in order to describe, respectively, the elliptic, flat, and hyperbolic horizon geometry of the \( SdS_5 \) bulk black hole. The modified Hubble equation is now given by

\[ H^2 = \frac{k}{a^2} - \frac{8\pi G_4}{3} \rho + \frac{8\pi G_4}{3} \left[ \frac{4\pi G_4}{3} \frac{l}{a^2 V_3} \rho \right] \omega \] (40)

where the volume \( V \) is now given by \( a^3 V_3 \) since all quantities that appear in equation (40) are defined for an arbitrary scale factor \( a \).

The first term in the right-hand side of equation (40) represents the curvature contribution to the brane motion. The second term can be regarded as the contribution from the radiation and it redshifts as \( a^{-4} \). The last term in the right-hand side of equation (40) is the self-gravitational correction to the motion of 4-dimensional spacelike brane moving in the 5-dimensional Schwarzschild-de Sitter bulk background. Since this term goes like \( a^{-6} \), it is obvious that it is dominant at early times of the brane evolution while at late times the second term, i.e. the radiative matter term, dominates and thus the last term can be neglected. There are three different ways to interpret the last term in the right-hand side of equation (40).

The first choice is that the last term in right-hand side of equation (40) can be regarded as a \( \rho^2 \) term. Due to the opposite sign with respect to the \( \rho \) term, the FRW universe under consideration is led to an inevitable bounce [18]. However, in order this scenario to be materialized, one has to treat the emitted energy \( \omega \) as a 4-dimensional quantity. Thus, it has to be scaled according to

\[ \omega = \frac{l}{a} \omega_5 \] (41)

where \( \omega_5 \) has to be the shell of energy emitted by the 5-dimensional Schwarzschild-de Sitter bulk background as discussed in the preceding section within the context of KKW analysis.

The second choice is that the last term can be regarded as an anisotropy energy density. In this case, the modified Hubble equation (40) takes the form

\[ H^2 = \frac{k}{a^2} - \frac{8\pi G_4}{3} \rho + \frac{8\pi G_4}{3} \rho_{\text{shear}} \] (42)
where the anisotropy energy density is given by
\[ \rho_{\text{shear}} = \frac{\Sigma^2}{a^6} \] (43)
with
\[ \Sigma^2 = \frac{4\pi G_4}{3} \left( \frac{l}{V_3} \right)^2 M \omega . \] (44)

It should be mentioned that in standard cosmology where there are no corrections, the shear anisotropy term is a product of the anisotropic character of the brane geometry [19].

Finally, the third choice which looks more appealing, is to regard the last term in the right-hand side of equation (40) as stiff matter [20]. In particular, within the context of dS/CFT correspondence, Medved considered\(^6\) a Reissner-Nordstrom-de Sitter bulk background [21]
\[ ds^2 = -h(a)dt^2 + \frac{1}{h(a)}da^2 + a^2d\Omega^2_{(3)}; \] (45)

where
\[ h(a) = 1 - \frac{a^2}{l^2} - \frac{\varepsilon_3 M}{a^2} + \frac{3\varepsilon_3 Q^2}{16a^4}. \] (46)

and \( M, Q \) represent the black hole mass and charge, respectively. It was showed that the brane evolution is described by a Friedmann-like equation for radiative matter along with a stiff-matter contribution
\[ H^2 = \frac{1}{a^2} - \frac{\varepsilon_3 M}{a^4} + \frac{3\varepsilon_3 Q^2}{16a^6} = \frac{1}{a^2} - \frac{8\pi G_4}{3} \rho + \frac{3\varepsilon_3 Q^2}{16a^6}. \] (47)

If the following condition is satisfied
\[ Q^2 = \frac{8}{3} M \omega , \] (48)
then equations (40) and (47) are identical (for the case \( k = +1 \)) and the last term in the right-hand side of equation (40) is then regarded as stiff matter.

In passing, one should not worry about the opposite (wrong) sign of the stiff-matter term with respect to the radiative matter term. The reason is that the 4-dimensional spacelike brane has an Euclidean metric and typically such rotations (from the Minkowskian to the Euclidean sector of a theory) are followed by the transformation \( Q^2 \rightarrow -Q^2 \) [22].

Finally a couple of comments are in order. Firstly, if one takes the \( \tau \) derivative of the modified Hubble equation (40) then the second modified Friedmann equation is obtained
\[ \dot{H} = -\frac{k}{a^2} + \frac{16\pi G_4}{3} \rho - 8\pi G_4 \left[ \frac{4\pi G_4}{3} \frac{l}{a^2 V_3} \rho \right] \omega . \] (49)

---

\(^6\)It should be noted that Medved considered a RN-dS background spacetime of arbitrary dimensionality. Here, for simplicity reasons, we reproduce his results for the case of \( n = 3 \), i.e. the RN-dS bulk background is five-dimensional.
Secondly, one can rewrite the modified Hubble equation \( \text{(40)} \) in such a way that it represents the conservation of energy of a point particle moving in a one-dimensional effective potential, \( V(a) \),

\[
\left( \frac{da}{d\tau} \right)^2 = k - V(a) \tag{50}
\]

where the variable \( a \) represents the position of the particle and the modified effective potential due to the self-gravitational effect reads

\[
V(a) = \frac{8\pi G_4}{3} a^2 \rho - \frac{8\pi G_4}{3} \left( \frac{4\pi l G_4}{3V_3} \rho \right) \omega
= \frac{\mu}{a^2} - \frac{\beta}{a^4} \tag{51}
\]

where

\[
\beta = 2 \left( \frac{4\pi G_4 l}{3V_3} \right)^2 M \omega \tag{52}
\]

The first term in the expression for the effective potential \( \text{(51)} \) is called “dark radiation” term.

Let us now recall the one-dimensional effective potential that appears in \( \text{(50)} \) for the case of standard FRW cosmology. In this framework, there are no corrections and hence the first term in \( \text{(51)} \) is actually the only term in the expression for the effective potential

\[
V(a) = \frac{\mu}{a^2} . \tag{53}
\]

The 4-dimensional spacelike FRW brane exists in the regions where \( V(a) \leq k \) so that \( H^2 \geq 0 \). It is clear that the only viable scenario is the case of \( k = +1 \) which is the spherical brane (spatially closed universe). The spherical brane starts at \( a = +\infty \) and reaches its minimum size at \( a_{\min} = \sqrt{\mu} \) and then re-expands.

On the contrary, when one includes quantum corrections, i.e. logarithmic corrections, then the effective potential takes the form [23]

\[
V(a) = \frac{\mu}{a^2} + 3\gamma \log \left( \frac{a}{\gamma l^2} \right) \tag{54}
\]

where \( \gamma \) is given by

\[
\gamma = \frac{2G_4}{V_3 l} . \tag{55}
\]

It is evident that in this case the behavior of the effective potential depends on the parameters \( \mu \) and \( \gamma \) (which in turn depends on \( G_4, V_3 \) and \( l \)). If one assumes that \( \gamma \leq \sqrt{\mu} \), then the behavior of the effective potential is almost the same as described above for the case of standard FRW cosmology. But if \( \gamma \) is large compared to \( \sqrt{\mu} \), then the behavior of the effective potential is modified\(^7\).

\(^7\)For a complete analysis on the behavior of the effective potential when logarithmic corrections are included see [23].
Comparing the expression for the effective potential when self-gravitational corrections (51) are taken into account with the one when the logarithmic corrections (54) are present, the important difference is that the self-gravitational corrections are subtractive while the logarithmic ones are additive (with respect to the “dark radiation” term). Furthermore, by substituting equation (51) into (50), we obtain

\[
\left( \frac{da}{d\tau} \right)^2 = k - \frac{\mu}{a^2} + \frac{\beta}{a^4}.
\]  

(56)

It is remarkable that the third term which is the contribution due to the self-gravitational effect is so small for large values of \(a\), i.e. at late times of the brane evolution, that it can be neglected. In this case the behavior of the effective potential is qualitative similar to that of the standard FRW cosmology presented before. On the contrary, for small values of \(a\), the contribution of the self-gravitational corrections is very important. Thus, the third term dominates with respect to the second term, i.e. the “dark radiation” term. The asymptotic behavior of the brane is as follows.

- **k=-1**
  
  The 4-dimensional brane expands from an initial state of vanishing spatial volume, \(a = 0\), and reaches its maximal size \(a_{\text{max}}\) when the constraint equation \(V(a) = -1\) is satisfied. Then the negatively curved brane undergoes a recollapse to zero volume.

- **k=0**
  
  As in the case of negatively curved brane, the 4-dimensional flat brane starts form \(a = 0\) and reaches its maximal size \(a_{\text{max}}\) when the constraint equation \(V(a) = 0\) is satisfied. A recollapse to zero volume follows.

- **k=+1**
  
  In this case there are a number of different outcomes for the positively curved brane. If the constraint equation \(V(a) < +1\) is always satisfied, then the 4-dimensional positively curved brane expands to infinity. But if the potential exceeds the critical value of unity then the brane reverse its direction of motion. Solving the equation \(V(a) = +1\), two solutions are obtained \(a_1\) and \(a_2\) (say \(a_1 < a_2\)). Therefore, if the constraint equation \(V(a) < +1\) is satisfied, then the 4-dimensional positively curved brane either starts from \(a = 0\), reaches its maximal size \(a_1\) and finally undergoes a recollapse to zero volume, or starts from infinite spatial volume, undergoes a collapse to a minimal size \(a_2\) and reexpands to infinity. This is the behavior of a “bouncing” universe.
5 Conclusions

One of the striking results for the dynamic dS/CFT correspondence is that the Cardy-Verlinde formula on the CFT-side coincides with the first Friedmann equation (Hubble equation) in cosmology when the brane crosses the horizon, i.e. when $a = a_c$, of the $SdS_5$ black hole. This means that the Hubble equation knows the thermodynamics of the CFT.

In this paper we have considered the dynamics of a 4-dimensional spacelike FRW brane propagating in an 5-dimensional dS bulk space containing a Schwarzschild black hole. Taking into account the semiclassical corrections to the black hole entropy that arise as a result of the self-gravitational effect, and employing the dS/CFT correspondence, we obtained the self-gravitational corrections to the Cardy-Verlinde formula. A welcomed but not unexpected result was that the modified entropy doesn’t violate any entropy bound since the additional term due to the self-gravitational effect is subtractive. These self-gravitational corrections to the Cardy-Verlinde entropy formula express the existence of a deep connection between semiclassical thermodynamics and de Sitter holography.

Furthermore, the self-gravitational corrections to the associated Friedmann-like brane equations are obtained. The additional term in the Hubble equation due to the self-gravitation effect goes as $a^{-6}$. We presented several ways of interpreting this term. It seems that the most appealing choice since it doesn’t require a scaling of the emitted energy, or a generalization of the FRW brane geometry to an anisotropic one, is the stiff matter. Thus, the self-gravitational corrections act as a source of stiff matter contrary to standard FRW cosmology where the charge of the black hole plays this role.

Finally, we rewrite the Hubble equation in a way that represents the conservation equation of energy of a point particle moving in a one-dimensional effective potential. The self-gravitational corrections to the effective potential and, consequently, to the conservation equation of energy of the point particle are presented. The term that describes the semiclassical corrections becomes dominant at early times of the brane evolution. On the contrary, at late times the “dark radiation” term is the dominant one. A short analysis of the asymptotic behavior of the brane follows. It is remarkable that in the case of the positively curved brane, there is the possibility of getting a bouncing universe. A feature that is not possible in the framework of standard FRW cosmology.

At this point a couple of questions are raised. First, is it possible for an observer that lives on the brane and observes the stiff matter to discriminate the kind of source that produces the stiff matter? The second question arises when one includes both semiclassical and quantum corrections. Which is the dominant correction and when does this dominance take place during the brane evolution? We plan to address these interesting issues in a future work.
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