ENERGY-MOMENTUM (QUASI-)LOCALIZATION FOR GRAVITATING SYSTEMS

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Traditional approaches to energy-momentum localization led to reference frame dependent pseudotensors. The more modern idea is quasilocal energy-momentum. We take a Hamiltonian approach. The Hamiltonian boundary term gives not only the quasilocal values but also boundary conditions via the Hamiltonian variation boundary principle. Selecting a Hamiltonian boundary term involves several choices. We found that superpotentials can serve as Hamiltonian boundary terms, consequently pseudotensors are actually quasilocal and legitimate. Various Hamiltonian boundary term quasilocal expressions are considered including some famous pseudotensors, Møller’s tetrad-teleparallel “tensor”, Chen’s covariant expressions, the expressions of Katz & coworkers, the expression of Brown & York, and some spinor expressions. We emphasize the need for identifying criteria for a good choice.

1 Introduction

1.1 Energy-momentum

Energy-momentum is a fundamental conserved quantity associated with a symmetry of space-time geometry. Noether’s theorem and translation invariance leads to the canonical energy-momentum (EM) density tensor, $T^{\mu \nu}$, which is conserved: $\nabla_\nu T^{\mu \nu} = 0$. But this tensor is not unique: one can add an arbitrary “curl”, and it is still conserved, e.g., one can shift the “zero” of energy. Gravity, however, responds to any real EM, so it removes the ambiguity in $T^{\mu \nu}$. Thus gravity absolutely identifies EM.

The source of gravity is $T^{\mu \nu}$, the EM density for matter and all other interaction fields. But sources exchange EM with the gravitational field locally. This leads to the expectation that gravity should have its own local energy-momentum density (GEM).

1.2 Pseudotensors discredited

Attempts to identify a local “total EM density” for gravitating systems, via standard techniques, gave only various non-covariant, reference frame dependent pseudotensors, e.g., those of Einstein, Landau-Lifshitz, Møller, and Weinberg. Such objects cannot give a well-defined GEM localization. This is in accord with the equivalence principle (see, e.g., p 457) which implies that the gravitational field cannot be detected at a point. Thus the pseudotensor approach has been discredited.
2 Quasi-local energy-momentum

A new idea, quasi-local energy-momentum (QEM) (quasilocal ≡ associated with a closed 2-surface) is now regarded as the proper fundamental notion of energy.

There have been many quasilocal proposals and approaches including null ray geometry, twistors, background, symplectic reduction, spinors, Hamilton-Jacobi, and covariant-symplectic. Many criteria (see, e.g.,), such as good limits (in particular the ADM (spatial ∞), Bondi (null ∞), weak field and flat spacetime), have have been proposed. Positivity has also been advocated by many authors, but it not possible for a closed universe, and consequently, cannot be required in general. In any case, it has been noted that an infinite number of expressions satisfy all of these requirements! Clearly there is a need for additional principles and criteria.

3 The Hamiltonian Approach

Energy can be defined as the value of the Hamiltonian. The Hamiltonian for a gravitating system (for a finite region & any geometric gravity theory) is the Noether generator of translations along a displacement vector field, $N^\mu$:

$$H(N) = \int_\Sigma N^\mu H_\mu + \oint_{S=\partial\Sigma} B(N),$$

it includes a volume (i.e., spatial hypersurface) and a (2-dimensional) boundary term.

From Noether’s theorems it follows that $H_\mu \propto$ field equations, consequently $E := H(N)$ depends only on the boundary term $B$, which thus gives the quasi-local energy-momentum. However the boundary term $B$ can be modified. (This special case of the usual Noether current non-uniqueness is essentially just the aforementioned ambiguity in the canonical energy-momentum density.) Indeed $B$ must be adjusted to get the correct asymptotic GR values.

3.1 The Hamiltonian variation boundary term principle

Fortunately, $B$ is not arbitrary. Its form is controlled by another principle of the formalism, the Hamiltonian boundary variation principle: one should choose the Hamiltonian boundary term $B$ so that the boundary term in $\delta H$ vanishes, when the desired fields are fixed on S. There is thus a nice division: the Hamiltonian density $H_\mu$ determines the evolution and constraint equations, the boundary term $B$ determines the boundary conditions and the QEM.

For Einstein’s GR theory, (succinctly in terms of differential forms) with the connection one-form $\omega^\alpha_\gamma$, the curvature 2-form $\Omega^\alpha_\gamma := d\omega^\alpha_\gamma + \omega^\alpha_\mu \wedge \omega^\mu_\gamma$, and $\eta^\alpha_\gamma := *(\partial^\alpha \wedge \partial^\gamma \cdots)$, where $\partial^\alpha$ is the coframe one-form, we space-time split the Hilbert Lagrangian:

$$L = dt \wedge i_N L = dt \wedge i_N (\Omega^\alpha_\gamma \wedge \eta_\alpha_\gamma) = dt \wedge [L_N \omega^\alpha_\gamma \wedge \eta_\alpha_\gamma - H(N)],$$

(2)
to identify the Covariant Hamiltonian as
\[ H(N) = -N^\mu \Omega^\alpha_\gamma \eta^\gamma_\mu - i_N \omega^\alpha_\gamma D\eta^\gamma_\alpha + d(i_N \omega^\alpha_\gamma \eta^\gamma_\alpha). \] (3)

The total derivative term here integrates, via the generalized Stokes theorem, to a boundary term — which needs adjustment.

### 3.2 Covariant Boundary Terms

Although there are an infinite number of possible boundary terms, for each dynamical field C.M. Chen found\cite{18,23} using sympletic techniques\cite{24} that there are only two covariant choices for \( B \), depending upon what is held fixed on the boundary:

\[ B_\theta = \Delta \omega^\alpha_\gamma \wedge i_N \eta^\gamma_\alpha + i_N \omega^\gamma_\alpha \Delta \eta^\gamma_\alpha, \] (4)

\[ B_\omega = \Delta \omega^\alpha_\gamma \wedge i_N \eta^\gamma_\alpha + i_N \omega^\gamma_\alpha \Delta \eta^\gamma_\alpha, \] (5)

where for any quantity \( \Delta \alpha := \alpha - \alpha \), with \( \alpha \) determining a fixed reference configuration. For these choices of Hamiltonian boundary term, the boundary term in \( \delta H \) is a projection of a 4-covariant expression along the displacement vector field, respectively,

\[ di_N(-\Delta \omega^\alpha_\gamma \wedge \delta \eta^\gamma_\alpha), \quad di_N(\delta \omega^\alpha_\gamma \wedge \Delta \eta^\gamma_\alpha), \] (6)

representing a Dirichlet or Neumann type “control mode”.

In passing, we call attention to the important gauge term \( i_N \omega \). It plays a role in generating the correct dynamic evolution of the frame along with the proper variational boundary condition. However it also adds an unphysical, noncovariant “rotation of the reference frame” contribution to the quasilocal energy-momentum. This contribution can be isolated by inserting the identity

\[ (i_N \omega^\alpha_\gamma) \theta^\gamma \equiv DN^\alpha - L_N \theta^\alpha. \] (7)

It can then be removed simply by dropping the terms proportional to \( L_N \theta \).

Our “covariant” quasilocal expressions yield the correct total ADM and Bondi values\cite{25,26} Also, they have widely accepted limiting forms; in fact for many practical cases our quasilocal energy (likewise momentum) essentially reduces to the well known quasilocal expression of Brown & York\cite{16}

\[ E = \oint (K - K_0), \] (8)

although in general there are some important differences: in particular (i) our expressions are 4-covariant, (ii) they allow for general reference configurations, (iii) and more general displacements, (iv) they include a Møller-Komar like term, and (v) our boundary need not be orthogonal to \( \Sigma \).

### 3.3 Applications

As an example consider a static spherically symmetric star. In the exterior, where the geometry has the Schwarzschild form, for the quasilocal energy within a sphere
(using Dirichlet boundary conditions, a Minkowski reference configuration and the Minkowski timelike Killing vector for the displacement) we find

$$E = r(1 - \sqrt{1 - 2M/r}) = M(1 + M/2R) \tag{9}$$

in terms of the Schwarzschild $r$ (with necessarily $r > 2M$) and isotropic $R$, respectively. Note that this is a decreasing function which approaches $M$ asymptotically. Hence, for a region between two spherical boundaries in the exterior, the quasilocal energy is negative. Extending this to a black hole, using an inner boundary on the horizon and outer boundary at infinity, (according to this definition) the outer integral gives $M$ and the inner gives $2M$ so that the total value of the quasilocal energy within the intermediate region is $-M$.

A related application of this formalism is to black hole thermodynamics. We considered the appropriate Hamiltonian for the region between an inner boundary on the horizon and a boundary at $\infty$ and obtained the first law. The outer integral gave the total energy and work terms, the integral over the horizon gives the expression for the entropy.

In an orthonormal Cartesian frame our Hamiltonian boundary term becomes

$$B_T := \omega^\alpha{}^\gamma \wedge i_N \eta_{\alpha\gamma}, \tag{10}$$

which yields Möller’s tetrad-teleparallel energy-momentum “tensor” $E_T$.

In a holonomic (coordinate) basis, our covariant boundary expressions correspond to controlling $\pi^{\alpha\sigma} := \sqrt{-g}g^{\alpha\sigma}$ (Dirichlet), or $\Gamma^\alpha\gamma\lambda \sim \partial g$ (Neumann), respectively, are

$$B_g = N^\mu\pi^{\gamma\sigma} \Delta \Gamma^\alpha_{\gamma\lambda} \delta_{\alpha\sigma\rho} + \mathcal{D}_\gamma N^{\alpha} \Delta (\pi^{\gamma\sigma}) \delta^\gamma_{\alpha\rho}, \tag{11}$$

$$B_T = N^\mu \pi^{\gamma\sigma} \Delta \Gamma^\alpha_{\gamma\lambda} \delta_{\alpha\sigma\rho} + \mathcal{D}_\gamma N^{\alpha} \Delta (\pi^{\gamma\sigma}) \delta^\gamma_{\alpha\rho}. \tag{12}$$

Our Dirichlet mode matches the recent expression of Katz, Bičák & Lynden-Bell, and Katz & Lerer which they derived via a Noether argument applied to gravity with a fixed global background (whereas we require our reference configuration only on the boundary). Their work includes a discussion of applications of this expression to cosmology and Mach’s principle. By modifying their construction we can likewise obtain our alternative Neumann control mode.

### 3.4 Choices

Selecting a Hamiltonian boundary term involves many choices:

1. the **representation** or choice of **dynamic variables**: metric, orthonormal frame, connection, spinors . . . ;

2. the **control mode** or **boundary conditions**: e.g., covariant Dirichlet/Neumann;

3. the **reference configuration**: e.g., Minkowski, (anti-)de Sitter, FRW cosmology, Schwarzschild;
• The physical meaning is that all of the quasilocal quantities vanish when the field has the reference values, so it fixes the zero of energy, etc.

• This could be determined either geometrically (by embedding and matching conditions) or dynamically (by an equation).

(4) the displacement vector field $N$: which timelike displacement gives the energy? which spatial displacement gives the momentum? A good choice is to use a Killing vector of the reference geometry.

As a consequence of such choices there are various kinds of energy, each associated with specific boundary conditions. The physical situation may be compared with thermodynamics (where we have enthalpy, Gibbs, Helmholtz, . . . ) or electrostatic energy in a region with fixed surface potential vs. fixed surface charge density.

4 Pseudotensors Rehabilitated

Consider the pseudotensor idea in detail. First select a superpotential $H_{\mu \nu} \equiv H_{\mu [\nu \lambda]}$, and then use it to split the Einstein tensor by defining the GEM pseudotensor:

$$\kappa \sqrt{-g} N_{\mu \nu} := -N_{\mu} \sqrt{-g} G_{\mu \nu} + \frac{1}{2} \partial_{\lambda} (N_{\mu} H_{\nu \lambda}^\mu).$$

(13)

Then Einstein’s equation, $G_{\mu \nu} = \kappa T_{\mu \nu}$, (with constant $N_{\mu}$) takes a form with the total effective EM pseudotensor as its source:

$$\partial_{\nu} (\kappa (-g)^{1/2} T_{\mu \nu}) := 2 \kappa (-g)^{1/2} (t_{\mu \nu} + T_{\mu \nu}).$$

(14)

Consequently $\partial_{\nu} (\kappa (-g)^{1/2} T_{\mu \nu}) \equiv 0$, which thus integrates to a conserved energy-momentum:

$$N_{\mu} P_{\mu} := \int N_{\mu} T_{\mu \nu} (-g)^{1/2} (d^3 x)_{\nu},$$

(15)

whereas $\nabla_{\nu} T_{\mu \nu} = \partial_{\nu} T_{\mu \nu} - \Gamma_{\nu \mu \lambda} T_{\lambda \nu} + \Gamma_{\nu \lambda \nu} T_{\mu \lambda} = 0$ does not — unless $\Gamma = 0$ (flat space).

Minor variations of this procedure use $H^{\mu \nu \lambda} \equiv H_{\mu [\nu \lambda]}$, and even further $H^{\mu \nu \alpha} := \partial_{\nu} H^{\mu \alpha \gamma}$, with $H^{\mu \nu \gamma} \equiv H^{\nu \gamma \mu} \equiv H^{[\mu \nu \gamma]} \equiv H^{\mu \nu \gamma} \equiv 0$. The latter guarantees a symmetric pseudotensor and thus a simple definition of angular momentum. A special case:

$$H^{\mu \alpha \nu \gamma} (h) := h^{\mu \nu} h^{\alpha \gamma} - h^{\alpha \nu} h^{\mu \gamma},$$

(16)

generalizes to

$$H (h_1, h_2) := H (h_1 + h_2) - H (h_1) - H (h_2).$$

(17)

For suitable choices of $H$, $h$, $h_1$, $h_2$, these expressions include all of the famous pseudotensors. Formally, including the displacement vector field, and making adjustments like $N_{\mu} H_{\mu \nu} \rightarrow N_{\mu} H^{\mu \nu}$ allows us to cover all these particular forms.
We note that the superpotentials can serve as Hamiltonian boundary terms and consequently the associated pseudotensors are fundamentally \textit{quasilocal}:

\[- P(N) := - \int \Sigma N^\mu T_\mu^\nu \sqrt{-g}(d^3x)_\nu \equiv \int \Sigma N^\mu \sqrt{-g}(-t_\mu^\nu - T_\mu^\nu)(d^3x)_\nu \]

\[= \int \Sigma [N^\mu \sqrt{-g}(\frac{1}{\kappa} G_\mu^\nu - T_\mu^\nu) - \frac{1}{2\kappa} \partial_\lambda (N^\mu H_\mu^{\nu\lambda})](d^3x)_\nu \]

\[= \int \Sigma N^\mu \mathcal{H}_\mu + \oint_{S=\partial \Sigma} \mathcal{B}(N) \equiv H(N), \quad (18)\]

here \( \mathcal{H}_\mu \) is the covariant form of the ADM Hamiltonian and

\[\mathcal{B}(N) = -N^\mu (1/2\kappa)H_\mu^{\nu\lambda}(1/2)(d^2x)_{\nu\lambda}. \quad (19)\]

In all cases, to understand the \textit{physical meaning} of the associated quasilocalization we calculate the Hamiltonian variation:

\[\delta H(N) = \int \Sigma \text{field eqn. terms} + \oint_{\partial \Sigma} \text{ham. var. bound. term}. \quad (20)\]

The key is the \textit{Hamiltonian variation boundary term}:

\[- \frac{1}{4\kappa} [\delta \Gamma^\alpha_{\gamma\lambda} N^\mu \pi^\gamma_{\sigma\rho} \delta^\rho_{\alpha\sigma\mu} + \delta (N^\mu H_\mu^{\tau\rho})] dS_{\tau\rho}. \quad (21)\]

It shows what must be held fixed on the boundary.

Let us now consider some specific examples, in each case taking our reference configuration as Minkowski space and using Cartesian coordinates. The \textit{Einstein pseudotensor} follows from the \textit{Freud} superpotential\[H_{\lambda}^{\mu\nu} = \pi^\gamma_{\sigma\tau} \Gamma^\alpha_{\gamma\rho} \delta^\rho_{\alpha\sigma\lambda} \equiv \frac{g_{\lambda\alpha}}{\sqrt{-g}} \partial_\gamma H^{\mu\alpha\gamma}, \quad (22)\]

where \( H^{\mu\nu} := \pi^\mu_{\nu\nu} - \pi^\mu_{\nu\nu} \). We find

\[\delta H = \text{field eqn. terms} + \oint \delta (N^\mu \pi^\gamma_{\tau\sigma}) \Gamma^\alpha_{\gamma\lambda} \delta^\rho_{\alpha\sigma\mu} (d^2x)_{\tau\rho}. \quad (23)\]

Hence we should use constant \( N^\alpha \) and control \( \pi^\alpha_{\gamma\tau} := \sqrt{-g}g^{\alpha\gamma} \) on the boundary. For the \textit{Landau-Lifshitz pseudotensor}: \( L^{\mu\nu} := \partial_\tau \partial_\alpha H^{\mu\nu} \equiv \partial_\alpha (\pi^{\mu\nu} H_\sigma^{\alpha\gamma}) \), by a similar calculation we find that we should take \( N^\alpha = \pi^\alpha_{\mu\nu} N^0_{\nu} \) for fixed \( N^0_{\nu} \), and control \( g^{\mu\nu} g^{\pi\gamma} \).

For the \textit{Møller pseudotensor} which has the superpotential

\[H_{\lambda}^{\alpha\sigma} = 2\pi^\rho_{[\alpha\sigma]} \Gamma^\tau_{\rho\lambda} = 2\pi^\rho_{[\alpha\sigma]} g^{\rho\tau} \partial_\tau g_{\nu\lambda}, \quad (24)\]

we must control the connection \( \Gamma \sim \partial g \).

A similar analysis can be applied to other expressions old and new — e.g., for the recently proposed “symmetric” expression of Petrov & Katz\[B_{KP} := \Delta \pi^{\sigma\tau} \tilde{D}_\mu N_{\nu\rho} + N_{\lambda} \tilde{D}_\gamma \left( \Delta \pi^{\alpha\lambda\sigma\gamma} \tilde{g}^{\delta\gamma} \right) \delta^\beta_{\alpha\sigma}, \quad (25)\]

we expect to get a mixed boundary condition.
5 Spinor Formulations

Using a spinor parameterization \( N^\mu \equiv \bar{\psi} \gamma^\mu \psi \), and adjusting the boundary term via a certain spinor-curvature identity \( 34 \) yields the Hamiltonian associated with the Witten positive energy proof \( 35 \),

\[
H_w(\psi) := 4D\bar{\psi} \gamma_5 \partial \wedge D\psi - i_N \omega^\alpha \gamma \eta_{\alpha \gamma} + i_N \omega^\alpha \gamma \eta_{\alpha \gamma} + dB_w ,
\]

where \( \partial \equiv \partial^\mu \gamma_\mu \) and

\[
B_w := 2(\bar{\psi} \gamma_5 \partial \wedge D\psi + D\bar{\psi} \wedge \gamma_5 \partial \psi) .
\]

The first term is the same as the main term in eq (4).

Another class of spinor quasilocal boundary expressions has been identified. Via other spinor-curvature identities \( 34 \) quadratic spinor Lagrangians (QSL) for GR have been found \( 37 \), \( 38 \). The Einstein-Hilbert scalar curvature Lagrangian equals (up to an exact differential) the QSL

\[
L_{qs} := 2D\bar{\Psi} \gamma_5 D\Psi \equiv R * 1 + d(D\bar{\Psi} \gamma_5 \Psi + \bar{\Psi} \gamma_5 D\Psi)
\]

where \( \Psi = \partial \psi \) is a spinor one-form field. The corresponding first order Lagrangian:

\[
L_\Psi := D\bar{\Psi} P + \bar{\Psi} D\Psi + \frac{1}{2} \bar{\Psi} \gamma_5 P ,
\]

yields the first order equations

\[
\bar{P} = 2D\bar{\Psi} \gamma_5 , \quad P = 2\gamma_5 D\Psi , \quad D\bar{P} = 0 = DP ,
\]

and can be used to construct the covariant Hamiltonian 3-form:

\[
H_\Psi := \bar{P} L_\Psi + L_\Psi \bar{P} - i_N L_\Psi \\
\equiv -i_N (\frac{1}{2} \bar{P} \gamma_5 P) - [i_N \bar{D} P + D\bar{\Psi} i_N P + \bar{\Psi} i_N \omega P - d(i_N \bar{\Psi} P) + c.c.] .
\]

The associated Hamiltonian boundary term

\[
B(N) = i_N \bar{\Psi} P + \bar{P} i_N \Psi ,
\]

and the Hamiltonian boundary variation symplectic structure

\[
\delta H_\Psi (N) \simeq di_N (\bar{\Psi} \wedge P + \bar{P} \wedge \delta \Psi) ,
\]
are simple. Both are independent of any reference configuration. But we can introduce a reference configuration replacing the boundary term by one of

\[ B_\Psi := i_N \overline{\Psi} \Delta P + \Delta \overline{\Psi} i_N P + \text{c.c.} , \]  

\[ B_P := i_N \overline{\Psi} \Delta P + \Delta \overline{\Psi} i_N P + \text{c.c.} . \]  

(35) \hspace{1cm} (36)

Then the variation of the Hamiltonian will contain one of the boundary terms

\[ i_N (\Delta P \land \delta \Psi + \text{c.c.}) , \quad \text{or} \quad i_N (-\delta \overline{P} \land \Delta \Psi + \text{c.c.}) . \]  

(37)

Several remarks are in order.

(1) Note that there are no explicit connection terms in these spinor boundary terms. However they are present implicitly via \( P = 2 \gamma_5 D \Psi \).

(2) Only if we include an explicit reference configuration do we get \( i_N \omega \sim DN \) Møller-Komar type terms. Such terms have often been overlooked in earlier investigations. However we have found that they play a key role both in certain angular momentum calculations and in black hole thermodynamics.

(3) In addition to fixing the frame, the spinor field should be held fixed on the boundary. The physical meaning of fixing the spinor field is not yet so clear.

6 Concluding Discussion

We have noted that EM is both related to a symmetry under space-time displacements and is also the source of gravity. Sources interact with gravity and thus should exchange EM with the gravitational field yet there is no proper local density for gravitational or total EM; in the light of the equivalence principle it is not surprising that traditional methods lead only to reference frame dependent pseudotensors.

It is now widely believed that EM is really quasilocal. Identifying energy with the value of the Hamiltonian, we note that boundary terms in the Hamiltonian for a finite region (i) give the quasilocal EM, and (ii) via the Hamiltonian boundary variation principle, reflect the “control mode”, i.e., the boundary conditions.

There are many choices involved in selecting a Hamiltonian boundary term, e.g., variables, control mode, displacement, reference configuration. For GR we found that there are only two covariant control modes. Schwarzschild and black-hole thermodynamics applications were noted as well as a connection with the work of Brown & York and Katz and coworkers.

One class of Hamiltonian boundary term choices is to use a superpotential. In this way we show that the pseudotensors are actually quasilocal and legitimate expressions for EM. Each pseudotensor is then associated with the value of the Hamiltonian which evolves the variables subject to certain boundary conditions. Several examples of pseudotensors and quasilocal expressions were considered.

We also discusssed how our formalism deals with a couple of spinor quasilocal EM formulations.
In conclusion we stress the importance of identifying appropriate quasilocal energy-momentum criteria. In addition to good asymptotic and weak field/empty space limits (and realizing that quasilocal energy need not be positive) we see a need to relate the choice of boundary conditions to the physics. For electrodynamics (or thermodynamics) we have a good idea of the physical significance of our boundary conditions, but not for gravity. What does it mean to hold the metric (connection) fixed on the boundary (i.e., give it a prescribed time dependence)? What boundary condition corresponds to “thermal insulation”, i.e., no flow of energy-momentum through the boundary? Suppose that observers measured the metric and connection coefficients on the walls, floor and ceiling of this room, what component values in what reference frame would indicate that it was devoid of energy-momentum? Answering such questions should contribute much to our understanding of energy-momentum and its localization.

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