High magnetic field ultrasound study of spin freezing in La1.88Sr0.12CuO4

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Abstract: High-Tc cuprate superconductors host spin, charge, and lattice instabilities. In particular, in the antiferromagnetic glass phase, over a large doping range, lanthanum-based cuprates display a glass-like spin freezing with antiferromagnetic correlations. Previously, sound velocity anomalies in La2−xSrxCuO4 (LSCO) for hole doping $p=x_{0.145}$ were reported and interpreted as arising from a coupling of the lattice to the magnetic glass [M. Frachet, I. Vinograd et al., Nat. Phys. 16, 1064 (2020)]. Here we report both sound velocity and attenuation in LSCO $p=0.12$, i.e., at a doping level for which the spin freezing temperature is the highest. Using high magnetic fields and comparing with nuclear magnetic resonance measurements, we confirm that the anomalies in the low temperature ultrasound properties of LSCO are produced by a coupling between the lattice and the spin glass. Moreover, we show that both sound velocity and attenuation can be simultaneously accounted for by a simple phenomenological model originally developed for canonical spin glasses. Our results point towards a strong competition between superconductivity and spin freezing, tuned by the magnetic field. A comparison of different acoustic modes suggests that the slow spin fluctuations have a nematic character.

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High magnetic field ultrasound study of spin freezing in La$_{1.88}$Sr$_{0.12}$CuO$_4$

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High-$T_c$ cuprate superconductors host spin, charge, and lattice instabilities. In particular, in the antiferromagnetic glass phase, over a large doping range, lanthanum-based cuprates display a glass-like spin freezing with antiferromagnetic correlations. Previously, sound velocity anomalies in La$_{2-x}$Sr$_x$CuO$_4$ (LSCO) for hole doping $p = x \geq 0.145$ were reported and interpreted as arising from a coupling of the lattice to the magnetic glass [M. Frachet, I. Vinograd et al., Nat. Phys. 16, 1064 (2020)]. Here we report both sound velocity and attenuation in LSCO $p = 0.12$, i.e., at a doping level for which the spin freezing temperature is the highest. Using high magnetic fields and comparing with nuclear magnetic resonance measurements, we confirm that the anomalies in the low temperature ultrasound properties of LSCO are produced by a coupling between the lattice and the spin glass. Moreover, we show that both sound velocity and attenuation can be simultaneously accounted for by a simple phenomenological model originally developed for canonical spin glasses. Our results point towards a strong competition between superconductivity and spin freezing, tuned by the magnetic field. A comparison of different acoustic modes suggests that the slow spin fluctuations have a nematic character.

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I. INTRODUCTION

The coupling of electronic instabilities to the crystal lattice plays a significant role in shaping the phase diagram of some high-$T_c$ cuprate superconductors. The case of La-based cuprates is emblematic. Upon cooling, La$_2$-Ba$_4$CuO$_4$ (LBCO) and rare-earth doped (Nd, Eu)$_{0.125}$La$_{2-x}$Sr$_x$CuO$_4$ [(Nd, Eu)-LSCO] evolve from a high-$T$ tetragonal (HTT) to a mid-$T$ orthorhombic (OMT) and finally to a low-$T$ tetragonal (LTT) crystal structure. The LTT order pins stripe order, a combination of mutually commensurate spin and charge modulations, initially found in Nd-LSCO [1]. Within this context sound velocity and attenuation are particularly relevant quantities. Ultrasound measurements directly probe the lattice properties and they are sensitive to any strain-dependent instability.

Among the La-based cuprate family La$_{2-x}$Sr$_x$CuO$_4$ (LSCO) appears peculiar. First, the OMT-LTT structural phase transition does not occur, although LTT-like distortions exist locally [2–4]. Moreover, scattering evidence for charge ordering inside the pseudogap phase has remained elusive until recently [5–8]. In LSCO around doping level $p = 0.12$, quasistatic charge modulation appears below $T_{CDW} = 70 \pm 15$ K with a maximal in-plane correlation length $\xi_{\parallel}(T_c) \simeq 30$ Å, a value practically one order of magnitude smaller than in LBCO at the same doping.

In the same compound incommensurate antiferromagnetic (AFM) correlations are also found at low field for $0.02 \leq p \lesssim 0.135$ [9]. The temperature at which these correlations appear static depends upon the probe frequency [9,10], revealing the glassy nature of the magnetic state. However, as in other La-based compounds close to $p \approx 0.12$, one observes that the incommensurabilities of charge and spin density waves (CDW and SDW, respectively) follow $2\delta_{\text{spin}} = \delta_{\text{charge}}$, a relation reminiscent of charge-spin stripe ordering [6].

Close to the hole doping level $p \approx 0.12$ elastic anomalies have been reported in both sound velocity and attenuation. Specifically, in single crystal studies and near the superconducting $T_c$, a broad sound velocity minimum has been observed in different acoustic modes [11,12]. In a similar range of temperature, an attenuation maximum of longitudinal waves has been found in polycrystals [13,14]. Different interpretations have been proposed to explain this peculiar behavior [11–13,15–17]. Recently, using nuclear magnetic resonance (NMR) and sound velocity measurements in high magnetic field in LSCO for $p \geq 0.145$, we showed that the anomalous sound velocity appears to be caused by a coupling of the AFM glass to the lattice [18].

In this study, we strengthen this interpretation with high magnetic field measurements of sound velocity and attenuation in LSCO $p = 0.12$. Comparing ultrasound attenuation...
with NMR measurements on crystals from the same batch, we reinforce the link between the slowing down of magnetic fluctuations and the ultrasound anomalies observed in the \((c_{11} - c_{12})/2\) and \(c_{11}\) elastic constants. Moreover, we show that the ultrasound properties of the \((c_{11} - c_{12})/2\) mode can be semiquantitatively reproduced by a phenomenological dynamical susceptibility model initially developed for canonical spin glasses. Finally, by comparing different acoustic modes, we find that the spin freezing produces an enhanced susceptibility in the \(B_{1g}\) channel, which is associated with nematicity in cuprates.

This paper is organized as follows. In Sec. II, we describe the sample studied and the experimental technique. Then, in Sec. III we report the experimental sound velocity and attenuation measurements. We present a phenomenological model of ultrasound in spin glasses and use it to analyze the ultrasound data in Sec. IV. Then, in Sec. V, we discuss the magnetic field effect on the ultrasound properties, the differences between the acoustic modes studied and the symmetry of the AFM fluctuations inferred from our measurements. We summarize our conclusions in Sec. VI.

II. METHODS

A high-quality LSCO single crystal was grown by the traveling solvent floating zone method. From this crystal, three samples were cut along different crystallographic directions to probe different elastic constants. Typical samples dimensions are \(2 \times 2 \times 2 \text{ mm}^3\). The hole doping \(p = 0.122 \pm 0.002\) has been determined by measuring \(T_a = 252\text{ K}\), the temperature of the HTT-OMT structural phase transition by sound velocity, as described in Ref. [18]. The different samples share a similar \(T_a\) and thus a similar doping. The superconducting transition temperature \(T_c = 29 \pm 3\text{ K}\) has been determined by sound velocity, in-plane resistivity, and magnetic susceptibility measurements.

A standard pulse-echo technique with phase comparison was used to measure variations of sound velocity \(\Delta v/v\) and sound attenuation \(\Delta \sigma\) [19,20]. Ultrasound was generated and detected using commercial LiNbO\(_3\) transducers glued onto parallel, clean, and polished surfaces of the samples. The excitation frequency \(\omega\) ranged from 50 to 300 MHz. For high symmetry propagation direction, the sound velocity variation of a given acoustic mode can be converted to the associated elastic constant change using \(\Delta c_{ii}/c_{ii} = 2\Delta v/v\).

Zero-field and static-field experiments were performed at the LNCMI Grenoble using 20 T superconducting and 28 T resistive magnets. Field-cooled conditions were used. Pulsed-field experiments up to 60 T were carried out at the LNCMI Toulouse. In all cases, the field was applied along the crystallographic \(c\)-axis.

III. RESULTS

A. Sound velocity in zero magnetic field

We begin with a zero magnetic-field study of different elastic constants in LSCO \(p = 0.12\) as shown in Fig. 1. The description of the different modes studied is reported in Table I. We use a tetragonal representation for the elastic constants even in the OMT phase since the sample is in a pseudotetragonal lattice [21]. The \(c_{44}\) acoustic mode follows the classical variation expected in solids: upon cooling the sound velocity increases continuously and eventually saturates at low temperature [22]. This behavior contrasts with the \(c_{33}\) elastic constant which shows a downward jump at \(T_c\). This mean-field anomaly at the superconducting transition is expected for a longitudinal mode and is related to the specific heat jump through the Ehrenfest relationship

\[
\Delta c_{ii}(T_c) = \frac{\Delta C_p(T_c)}{V_{mol}T_c} \left( \frac{dT_c}{dc_{ii}} \right)^2,
\]

with \(\Delta C_p(T_c)\) the specific heat jump at \(T_c\) and \(V_{mol}\) the molar volume. The amplitude of the anomaly \(\Delta v/v(T_c) \simeq 0.2 \times 10^{-3}\) is consistent with literature values on samples with similar doping levels [21,23].

For \(T \gg T_c\), the temperature dependence of \(c_{11}\) and \((c_{11} - c_{12})/2\) elastic constants is remarkable. In both of these modes, the normal state sound velocity decreases upon cooling, until the temperature hits \(T_c\) where it shows an upturn. Consequently, the sound velocity in these modes has a minimum at \(T_{min} \simeq T_c\) Figure 1 shows that the anomalous lattice softening appears only in acoustic modes having a \(B_{1g}\) strain compo-

-\(c_{11}\) & \([001]\) & \([001]\) & \(\epsilon_{xx}\) & \(A_{1g} + B_{1g}\) \\
-\(c_{33}\) & \([001]\) & \([001]\) & \(\epsilon_{zz}\) & \(A_{1g}\) \\
-\(c_{44}\) & \([001]\) & \([001]\) & \(\epsilon_{zz}, \epsilon_{xx}\) & \(E_{g}\) \\
-\((c_{11} - c_{12})/2\) & \([110]\) & \([110]\) & \(\epsilon_{xx} - \epsilon_{yy}\) & \(B_{1g}\) \\

This paper is organized as follows. In Sec. II, we describe the sample studied and the experimental technique. Then, in Sec. III we report the experimental sound velocity and attenuation measurements. We present a phenomenological model of ultrasound in spin glasses and use it to analyze the ultrasound data in Sec. IV. Then, in Sec. V, we discuss the magnetic field effect on the ultrasound properties, the differences between the acoustic modes studied and the symmetry of the AFM fluctuations inferred from our measurements. We summarize our conclusions in Sec. VI.
nent, namely $c_{11}$ and $(c_{11} - c_{12})/2$. Note that, so far, we have not been able to measure the $B_{2g}$ mode ($c_{66}$) for $T < T_{\alpha}$.

Finally, for $T \lesssim 15$ K or so, a rapid stiffening is observed in $c_{11}$ upon cooling. Indeed, the sound velocity in the $T = 0$ limit greatly exceeds what would be expected from an extrapolation of the high-temperature bare elastic constant (e.g., following the $c_{44}$ elastic constant). A similar upturn is found in $c_{33}$ and $(c_{11} - c_{12})/2$ upon cooling for $T \lesssim 15$ K, although much weaker than in $c_{11}$.

**B. Sound velocity and attenuation in applied magnetic field**

In Figs. 2 and 3 we investigate how the anomalous sound velocity, and the corresponding sound attenuation, evolve as a function of temperature at different magnetic fields, in the $c_{11}$ and $(c_{11} - c_{12})/2$ modes, respectively. In both these modes no signature of the vortex lattice is observed, as discussed in Appendix A.

The anomalous features of the zero field sound velocity in the $c_{11}$ and $(c_{11} - c_{12})/2$ acoustic modes are enhanced by a magnetic field: both the amplitudes of the lattice softening (for $T \geq T_{\text{min}}$) and stiffening ($T \leq T_{\text{min}}$) increase with increasing magnetic field. For both acoustic modes an attenuation peak is found at $T_{\alpha} \leq T_{\text{min}}$. The amplitude of this attenuation peak and $T_{\alpha}$ increase monotonically with increasing field.

The magnetic field dependencies of $T_{\alpha}$ and $T_{\text{min}}$ from $c_{11}$ measurements are shown in the phase diagram of Fig. 2(c). Within error bars, $(c_{11} - c_{12})/2$ and $c_{11}$ show at a given magnetic field similar $T_{\alpha}$ and $T_{\text{min}}$. In contrast with $T_{\alpha}$, $T_{\text{min}}$ has a nonmonotonic field dependence: it decreases for $0 \leq \mu_0 H \lesssim 2$ T and increases for higher fields. The initial decrease is

FIG. 2. Static field ultrasound measurements in the $c_{11}$ acoustic mode up to 28 T. (a) $\Delta v/v$ and (b) $\Delta \alpha$ as a function of temperature for different magnetic fields. The curves are shifted vertically for clarity. Reference value taken at $T \approx 30$ K. The arrows denote $T_{\text{min}}$ and $T_{\alpha}$ as indicated. (c) $H$–$T$ phase diagram: gray circles denote the vortex melting transition field $\mu_0 H_{\text{v}}$ inferred from in-plane resistivity $\rho(T)$ measurements [95% resistivity drop with respect to $\rho(T = 50$ K)], $T_{\alpha}$ (upward-pointing blue triangles) and $T_{\text{min}}$ (downward-pointing red triangles) refer to the maximum in attenuation and minimum in velocity, respectively. Error bars are smaller than the symbol size. Dashed lines are guides to the eye.

FIG. 3. Temperature dependence of the $(c_{11} - c_{12})/2$ acoustic mode in pulsed-fields up to 60 T. (a) $\Delta v/v$ and (b) $\Delta \alpha$ both extracted from fixed magnetic field cuts of the pulsed-fields isotherms to which we add the zero field curve. Data are relative to value at $T \approx 40$ K. All lines are guides to the eye.
we compare the ultrasound attenuation $\Delta \alpha$ (blue line, right scale) and the $^{139}$La NMR spin-lattice relaxation rate $1/T_1$ (green circle, left scale), at $\mu_0 H = 28$ T. NMR and ultrasound samples are from the same batch. Both physical quantities show a peak when the excitation frequency becomes equal to the frequency of the spin fluctuations, i.e., $\omega \tau = 1$. A phenomenological linear background has been subtracted from the experimental $\Delta \alpha$ for clarity. The dashed green line is a guide to the eye. 

The good agreement between $1/T_1$ (blue line, right scale) and the La NMR spin-lattice relaxation rate (green circle, left scale), at $\mu_0 H = 28$ T. The comparison is striking, both systems show remarkably similar phenomenology. In the following, we focus on the transverse $(c_{11} - c_{12})/2$ acoustic mode shown in Fig. 3 and demonstrate that it can be semiquantitatively reproduced by a phenomenological model developed for spin glasses. The strong increase observed in $c_{11}(T)$ at low temperature is not explained by this model and will be discussed later.

We use the phenomenological dynamical susceptibility model developed by Dousineau et al. [32,36]. Sound velocity and attenuation are expressed in terms of a complex elastic constant, $c(\omega, T)$:

$$\chi_4(\omega, T) = c_0[1 - g^2 \chi_4(\omega, T)].$$

With $c_0$ the bare elastic constant, $g$ the spin-phonon coupling constant, and $\omega$ the ultrasound measurement frequency. Ultrasound quantities are deduced through

$$\Delta v/v = \frac{1}{2} \text{Re}(\Delta c/c),$$

$$\Delta \alpha(\text{dB/cm}) = \frac{\omega}{v} \log(10) \text{Im}(\Delta c/c).$$

Here $\chi_4(\omega, T)$ is a dynamical susceptibility defined as

$$\chi_4(\omega, T) = \int \frac{d \tau_4 \chi_4(\omega = 0, T)}{1 + i \omega \tau_4},$$

where $\chi_4(\omega = 0, T)$ is a static susceptibility and $\tau_4(T)$ is the correlation time of the spin fluctuations. In our case, since we are presumably dealing with spin-1/2 Cu$^{2+}$ moments, the magnetoacoustic coupling arises from the Waller mechanism (also called the exchange-striction mechanism), i.e., a modulation of the exchange interaction by the strain [37]. Consequently, the associated susceptibility is quadrupolar and the correlation time is involved in a four-spin correlation function. In contrast, the $1/T_1$ NMR relaxation rate is governed by a correlation time $\tau_2(T)$ which is involved in a two-spin correlation function. This can produce slight differences between $\Delta \alpha$ and $1/T_1$ in Fig. 4 [38]. We use the following expressions for $\tau_4(T)$ and $\chi_4(\omega = 0, T)$:

$$\tau_4(T) = \tau_\infty \exp(E_0/T),$$

$$\chi_4(\omega = 0, T) = \chi_0 + \frac{C_{\text{cure}}}{T}.$$
is the constant term of the susceptibility and finally $\tau_\infty$ is the correlation time of spin fluctuations for $T \gg E_0$. Note that Eqs. (6) and (7) are motivated by an analysis of $^{139}$La NMR $1/T_1$ [24,27] and ac-susceptibility measurements in the AFM glass of LSCO [39,40], respectively. As inferred from various experiments [41], the value of $\tau_\infty$ is fixed to $\exp(-30) \approx 10^{-13}$ s. Moreover, as usual in spin glasses [32,42], and especially in the AFM glass phase of LSCO [27,41], we consider that $\tau_4(T)$ is inhomogeneous using a Gaussian-distribution of $E_0$ with full width at half maximum $2\Delta E_0$. Within this framework it is possible to fit simultaneously $\Delta v/\nu$ and $\Delta \omega$, and to extract both $E_0$ and $\gamma^2 C_{\text{curie}}$ as a function of a magnetic field. A representative example is shown on Fig. 5(b): the model reproduces most of the salient features seen in the two ultrasound quantities.

The evolution of the fitting parameters is shown in Figs. 5(c) and 5(d). Up to $\mu_0 H = 60$ T—i.e., well above our $T=0$ extrapolation of the vortex melting field $H_c$ on Fig. 2(c)—$E_0$ and $\gamma^2 C_{\text{curie}}$ increase continuously. The increase of $E_0$ is related to the nonsaturating values of the temperature scales $T_a$ and $T_{\text{min}}$. Regarding the raise of $\gamma^2 C_{\text{curie}}$, it is explained by the continuous increase of the amplitudes of the lattice softening and attenuation peak up to 60 T (see Fig. 3).

The NMR $1/T_1$ data at $\mu_0 H = 28$ T shown in Fig. 4 can be fitted with the BPP formula using Eq. (6) for $\tau_4(T)$ and a Gaussian distribution of activation energy $E_0$ [27,43]. This parametrization of $1/T_1$ data yields an activation energy in fair agreement with $E_0$ inferred from ultrasound data [see Fig. 5(c)]. It has been suggested previously that the activation energy is analogous to the spin-stiffness $2\pi \rho_s$ [25,43,44]. The value of $E_0 \approx 200$ K found here for $\mu_0 H = 20$ T is comparable to what is obtained in Nd-LSCO $x=0.12$ in zero magnetic field [25,44,45]. It is an order of magnitude smaller than the spin stiffness of the antiferromagnetic parent compound La$_2$CuO$_4$ where $2\pi \rho_s \approx 4$ [46].

Finally, in the paramagnetic state of a classical Néel AFM $C_{\text{curie}} \propto \mu^2$, where $\mu$ is the magnetic moment. Since the dynamical susceptibility model is purely phenomenological, we cannot extract microscopic information. As such, the increase of $\gamma^2 C_{\text{curie}}$ with magnetic field [see Fig. 5(d)] could originate from an enhanced $\mu$ [47,48] or from an increased magnetic volume [31].

V. DISCUSSION

Let us summarize our results so far. (i) The $(c_{11} - c_{12})/2$ and $c_{11}$ modes show a softening for $T \geq T_{\text{min}}$ and a hardening for $T \leq T_{\text{min}}$. Those features are enhanced by magnetic field and survive when superconductivity is strongly suppressed by the field. Consequently, neither feature is caused by superconductivity. We attribute this broad sound velocity minimum to the freezing of the AFM glass. (ii) The striking similarity of the ultrasound attenuation with the NMR relaxation rate $1/T_1$ shows that the AFM glass is also causing the anomalous
attenuation peak in high magnetic field. (iii) The behavior of the $(c_{11} - c_{12})/2$ elastic constant found in LSCO $p = 0.12$ in high magnetic field is remarkably similar to what is found in canonical spin glasses. A dynamical susceptibility model, developed in the context of spin glasses, reproduces all features of the anomalous ultrasound properties in the $(c_{11} - c_{12})/2$ mode.

The similar decrease of $T_{\text{min}}$ and $T_c$ with magnetic field $\mu_0 H \leq 14$ T in LSCO at $p \approx 0.14$ has previously motivated a scenario in which a competing lattice instability—that produces a lattice softening for $T > T_c$—is quenched by the onset of superconductivity that induces a hardening for $T < T_c$ [12]. While we observe the same behavior in LSCO $p = 0.12$ for $\mu_0 H \leq 2$ T (see Appendix B for more details), this scenario does not hold at higher field where we observe an increase of $T_{\text{min}}$. All measurements reported here in LSCO $p = 0.12$ support the interpretation that the ultrasound anomalies are caused by the AFM glass phase via spin-phonon coupling [18].

In the following we discuss some implications of the aforementioned results. In particular, we comment on the magnetic-field effect on the ultrasound properties, the relation of this study with previous elastic experiments and the symmetry of the AFM quasistatic fluctuations.

### A. Special coupling with $B_{1g}$ strain

In canonical spin glasses such as cobalt fluorophosphor- ate, the magnetic moments are frozen in a random manner. Consequently, longitudinal and transverse acoustic modes couple similarly to the spins in such systems (see Ref. [36]). The magnetic moments of LSCO have similar dynamical properties as canonical spin glasses: they gradually freeze as the system is cooled down, such that the onset temperature depends on the probe frequency [9,10]. However, the moments in LSCO arrange in a pattern displaying incommensurate AFM character, and Bragg peaks indicating correlation lengths as high as $\sim 200$ Å in LSCO $x = 0.12$ are observed in neutron diffraction experiments [47,48,51]. Consequently, in LSCO the coupling between the frozen spins and the lattice varies dramatically from one mode to another, as shown in Fig. 1. The anomalous softening for $T > T_{\text{min}}$ is observed only in modes transforming according to the $B_{1g}$ irreducible representation (see Table I and Fig. 1). Note, however, that we cannot exclude a similar coupling of the AFM glass to $B_{2g}$ mode. Nonetheless, this suggests a special role of the $B_{1g}$ mode.

Within the framework of the dynamical susceptibility model, the lattice softening in the $B_{1g}$ mode is caused by the growth of a Curie-like susceptibility $\chi_1(\omega = 0, T)$. Equation (2) is reminiscent of the elastic constant $c = d^2F/\omega^2$ calculated using a Landau free energy $F$ containing a bilinear coupling $E = gQ$ [52], with $\epsilon$ a strain and $Q$ an order parameter. Indeed, within such a model, the softening is directly related to the increasing mean-field susceptibility of $Q$, $\Delta \nu/\nu \propto -g^2 \chi_0$. For this bilinear coupling to exist, both $\epsilon$ and $Q$ must transform according to the same irreducible representation. In this context, our result would suggest that the order parameter (and the fluctuations) associated with the AFM glass has a $B_{1g}$, i.e., nematic, character.

Although conjectural in the absence of a measurement of the $B_{2g}$ mode, this interpretation of the ultrasound data is evocative of the $B_{1g}$ susceptibility observed by symmetry-resolved Raman scattering in LSCO at $x = 0.10$ [53]. It is consistent with evidence of charge and spin stripe orders in this compound [6,51,54,55]. Nematicity can indeed result from fluctuating stripes [56]. We note that, at $p \sim 0.12$, the $B_{1g}$ susceptibility develops for $T \lesssim 70$ K, well below the pseudogap temperature $T^* \approx 130$ K [57]. The lack of $B_{1g}$ susceptibility at the pseudogap temperature is also reported in symmetry-resolved electronic Raman scattering experiments in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ [58]. The onset temperature of our detection of $B_{1g}$ susceptibility is actually comparable to the CDW onset temperature $T_{\text{CDW}} = 70 \pm 15$ K [5–7]. This suggests that, in LSCO $p = 0.12$, charge-stripe order triggers slow magnetic fluctuations [29,59,60] with nematic character.

### B. Effect of the magnetic field

Previous neutron scattering and $\mu$SR experiments have shown that the magnetism of LSCO at $p \approx 0.12$ is enhanced by a magnetic field [47,48,61–64], and this effect has been ascribed to a competition between superconducting and AFM order parameters. In line with this interpretation, we observe that the ultrasound signatures of the AFM glass are strengthened by a magnetic field (see Figs. 2, 3, and 5). The magnetic field dependence of the ultrasound properties does not saturate up to 60 T and the magnetic-field-induced softening appears at temperatures as high as $T \approx 50$ K (see Fig. 3). These observations are puzzling since at this doping $T_c \approx 29$ K and the extrapolation of the vortex melting line leads to $\mu_0 H_c(T \to 0) \approx 20$ T. This raises important questions on the effect of magnetic fields on the magnetic freezing and the possible resilience of superconducting fluctuations in high field.

We note that this behavior is reminiscent of the magnetoresistance producing an upturn in the resistivity of superconducting LSCO in high fields. This magnetoresistance is observed up to $T \approx 100$ K at the doping level $p = 0.12$ [65]. The spin freezing has been previously discussed as a cause of the resistivity upturn in La-based cuprates [66–70]. Consequently, it is possible that the large magnetoresistance observed in LSCO $p = 0.12$ above $T_c$ is related to the field-induced gradual slowing down of magnetic fluctuations observed here.

### C. Differences between $c_{11}$ and $(c_{11} - c_{12})/2$

Finally, we discuss the differences between the $c_{11}$ and $(c_{11} - c_{12})/2$ modes. As discussed above, the strength of the magnetoacoustic coupling is largest in the $(c_{11} - c_{12})/2$ mode, where the largest softening is observed (see Fig. 1). The second difference between the response in these two modes is the field-enhanced hardening that is seen in $c_{11}$ at low temperature. The situation is schematically depicted in Fig. 6. In the $(c_{11} - c_{12})/2$ mode, the difference between the measured sound velocity in the $T \to 0$ limit and the background velocity is negligibly small. On the other hand, this difference is significant in the $c_{11}$ mode, with the measured sound velocity being larger than the background velocity. This behavior echoes the results from previous studies performed...
In summary, we studied sound velocity and attenuation in La\textsubscript{1.88}Sr\textsubscript{0.12}CuO\textsubscript{4} in high magnetic field. The behavior of the \( c_{11} \) and (\( c_{11} - c_{12} \))/2 elastic constants is highly anomalous. By comparing the anomalies with \(^{139}\text{La}\) NMR \( 1/T_1 \) we confirm that they originate from the AFM glass phase via a magnetoacoustic coupling. A semiquantitative analysis of this contribution is made based on a phenomenological model of spin glass systems. Our ultrasound data points toward a strong competition between spin freezing and superconductivity in high magnetic field. A symmetry analysis reveals that the slowing down of spin fluctuations could be associated with a growing nematic susceptibility.

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**APPENDIX A: VORTEX LATTICE CONTRIBUTION TO THE ULTRASOUND PROPERTIES**

In our experiments with \( H \parallel [001] \), the \( c_{11} \) acoustic mode probes the compression modulus of the vortex lattice (VL), whereas (\( c_{11} - c_{12} \))/2 probes its shear modulus. This last is notorious for its small value, beyond our resolution. On the other hand, the compression modulus of the VL should make a detectable contribution. This contribution should give a step-like increase of the sound velocity and an attenuation peak when decreasing temperature through the depinning transition of the vortex lattice. Both these features should move to lower temperatures with increasing field and disappear for \( H \geq H_c \).

In Figs. 2(a) and 2(b) there is no evidence of the vortex lattice depinning transition. It has a negligible contribution to the longitudinal sound velocity in LSCO \( p = 0.12 \). This is due to the low value of the irreversibility field, and presumably also
because of the two-dimensional and disordered character of the VL at this doping level [73]. Consequently no signature of the vortex lattice depinning transition is observed in Figs. 2(a) and 2(b) and Fig. 3.

APPENDIX B: IMPACT OF SUPERCONDUCTIVITY ON THE ULTRASOUND PROPERTIES

We discuss the effect of superconductivity on the ultrasound properties which is best observed at low fields. Below 5 T or so, $T_{\text{min}}$ coincides with $T_c$ in both $c_{11}$ and $(c_{11} - c_{12})/2$ in our LSCO $p = 0.12$ sample (see Figs. 1 and 2). This behavior has also been observed in LSCO $p \approx 0.14$ up to 14 T by Nohara and coworkers [12]. In this field range $T_{\text{min}}$ appears to be primarily set by superconductivity.

In LSCO, superconductivity can induce an increase of the sound velocity for $T < T_c$ via two mechanisms. First, the superconducting order parameter has a direct coupling with the lattice for $T < T_c$. In cuprates, this coupling produces a hardening in the superconducting state, for both longitudinal and transverse modes [21]. Consequently, an upturn can occur at $T_c$ in both $(c_{11} - c_{12})/2$ and $c_{11}$. The second possible mechanism is indirect, and involves the competition between superconductivity and magnetism. In zero and low fields, the growth $\chi_2(\omega = 0, T)$, signaled by the softening of the sound velocity, can be tempered by the onset of superconductivity. If $\chi_2(\omega = 0, T)$ is sufficiently modified through $T_c$, it can result in an upturn at $T_c$. As a result of these two possible mechanisms, at zero and low magnetic field, we observe $T_{\text{min}} = T_c$, and $T_{\text{min}}$ decreases as field increases.

However, for $\mu_0 H \geq 5$ T, $T_{\text{min}}$ increases with magnetic field, meaning that the mechanism causing the softening for $T > T_{\text{min}}$ and the hardening for $T < T_{\text{min}}$ observed for $\mu_0 H \geq 5$ T in LSCO $p = 0.12$ does not involve the coupling of the superconducting order parameter to the lattice. Increasing the magnetic field above $\mu_0 H = 14$ T at $p \approx 0.14$ leads to the same observation [18]. As field increases the superconducting contribution to the sound velocity becomes weaker and the spin freezing contribution larger. For $\mu_0 H \geq 2$ T the superconducting contribution is dwarfed by the contribution from the magnetic slowing down. This explains why the difference between $T_{\text{min}}$ and $T_u = T_c$ is large and strongly field dependent for $\mu_0 H < 5$ T, while smaller and constant for higher fields [see Fig. 2(c)].

The temperature scale $T_u$ is insensitive to a direct contribution from superconductivity. While in conventional superconductors, the opening of the superconducting gap causes an attenuation drop, there is no corresponding behavior in LSCO $p \approx 0.12$. However, $\Delta \alpha(T)$ can be indirectly impacted by the onset of superconductivity at low field because the latter modifies the spin dynamics that controls $\Delta \alpha(T)$. This is best illustrated by the zero field $\Delta \alpha(T)$ that shows a remarkable kink at $T_c$ and then a maximum at $T_{\text{min}} \approx 9.5$ K [see Fig. 2(b)]. Within the dynamical susceptibility model, the ultrasound attenuation is mostly governed by the energy scale $E_0$ entering $\tau_4$ as indicated by Eq. (4). The kink anomaly at $T_c$ in $\Delta \alpha(T)$ in zero field can be interpreted as a decrease of $E_0$ for $T < T_c$ caused by the onset of superconductivity.

Because the dynamical susceptibility model does not take into account the impact of superconductivity on spin dynamics, we use an alternative scheme in order to extract $E_0$ for $\mu_0 H < 5$ T. At $T = T_a$, the condition $\omega \tau_4(T) = 1$ is met. Solving Eq. (6) for $E_0$ at $T = T_a$ hence yields $E_0 = -T_a / \ln(\omega \tau_4)$ [43]. In Fig. 8 we compare this $T_a$ derived $E_0$ with $E_0$ of Fig. 5(c) obtained with the parametrization of the ultrasound data. Good agreement is found between the two estimations of $E_0$. As seen in Fig. 8, $-T_a / \ln(\omega \tau_4)$ decreases rapidly at low fields, dropping from $E_0 \sim 150$ K for $\mu_0 H = 5$ T to $E_0 = 92$ K for $H = 0$. This rapid drop reflects the competition between spin freezing and superconductivity.

APPENDIX C: PREVIOUS EXPERIMENTS ON POLYCRYSTALS

In Sec. VC we discuss a scenario that would explain the lattice hardening seen mainly in the $c_{11}$ elastic constant at low temperature. Here, we detail previous putative explanations that have been proposed to explain a similar phenomenology from ultrasound and anelastic studies of polycrystalline La-based compounds.

We first consider previous ultrasound experiments. Around $p \approx 0.12$, in several La-based cuprates and, in particular, LSCO [13,14,71,74], the sound velocity of longitudinal waves increases markedly at low temperature while an attenuation peak occurs. This behavior is most likely related to the anomalous hardening of the $c_{11}$ elastic constant, as suggested by the similar temperature scales and field-enhancement. Interestingly, those previous ultrasound studies have shown that the temperature scale of the lattice hardening evolves smoothly from LBCO $p = 0.12$ to LSCO $p = 0.12$, and correspond to the coincident OMT-LTT and charge-stripe transition in the former [13,71]. Based on these experiments it has been proposed that in LSCO $p \approx 0.12$ the low temperature lattice hardening arises from the parallel development of local and/or fluctuating LTT distortions and charge-stripenes. However, in this scenario it is unclear why the lattice hardening is observed only at low temperature—comparable to $T_u$—

![FIG. 8. Comparison between the $E_0$ energy scale (circles, left scale) extracted from the fitting procedure of Fig. 5 and $-T_a \times \ln(\omega \tau_4)$ (diamonds, right scale) determined directly from the data shown in Fig. 2(b), $\omega = 2\pi \times 110$ MHz and $\tau_\infty \approx 1 \times 10^{-15}$ s. The energy scale $-T_a \times \ln(\omega \tau_4)$ corresponds to an experimental determination of $E_0$ solely based on $T_a$, using the condition $\omega \tau_4(T) = 1$ in Eq. (6) (see text). The rapid drop of $-T_a \times \ln(\omega \tau_4)$ at low $H$ is most likely due to the impact of superconductivity on the spin dynamics. Error bars on this quantity are smaller than the size of the symbols.](image-url)
whereas, in LSCO $p \approx 0.12$, LTT-type tilts are found at temperatures as high as $T = 100 \text{ K}$ in electron diffraction experiments [75,76] and LTT-type reflections are observed up to the OMT-HTT transition temperature $T_\text{O}$ in x-ray diffraction [77].

Now let us consider anelastic experiments in LSCO and LBCO polycrystals. These have shown that the Young modulus increases markedly at low temperature [77]. The elastic energy loss coefficient shows a step-like increase for $T < T_\text{O}$ and a plateau down to the lowest $T$ [72]. Based on a comparison with nuclear quadrupolar resonance (NQR) experiment, as well as the effect of oxygen vacancy, it has been inferred that the anelastic anomalies arise from a strain induced motion of the antiferromagnetic domain walls of the AFM glass phase [72]. In particular, this naturally explains why these anomalies are observed at temperatures of the order of $T_\text{O}$. However, these experiments are performed at much lower frequency than the ultrasound ones (within the kHz range) and thus do not necessarily probe the same relaxation process as ultrasound experiments [41].

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