Research Article

Dimensionless Charts for Predicting the Range of Goaf Roof Caving

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In engineering, the method of charts can provide a convenient query for specific engineering problems. To provide the basis for the potential hazard evaluation and rational governance of the goaf, it is necessary to study the quantitative evaluation for the range of goaf roof caving. Undoubtedly, the charts used to visually query the caving range can simplify the workload of the quantitative evaluation. Therefore, the methods of dimensional analysis, numerical simulation, and linear interpolation are introduced to study the dimensionless charts for predicting the caving range. The dimensionless analysis is used to establish the fuzzy function relationship among the influence factors of the goaf roof caving, and the numerical simulation is used to calculate the dimensionless groups in the fuzzy function. Using the linear interpolation, the dimensionless charts in this work can predict the range of goaf roof caving under more working conditions. The results show that the characteristics of the goaf roof caving corresponding to the dimensionless curves are consistent with the actual situation. With the continuous increase of the goaf span, the dimensionless curves of the caving range experience zero growth, rapid growth, and steady growth. The growth degree varies with the fracture spacing. Especially in the zero growth phase, the duration of the relatively stable state in the overlying strata of the goaf increases with the increase of fracture spacing. Moreover, based on the case study of Shirengou Iron Mine, the dimensionless charts obtained in this work can predict the range of goaf roof caving under different working conditions, which indicates the findings of this study have certain guiding significance to the treatment of the goaf.

1. Introduction

The goaf is formed by manmade excavation or natural geological movement under the ground surface, which seriously threatens the safety production of the mine [1, 2]. The goaf roof caving, one of the main sources of disasters in the goaf, has been one of the research hotspots in the field of underground excavation engineering. Generally, the backfilling materials are used to fill the goaf to prevent the roof caving [3, 4]. Certainly, the measure of inducing roof caving can manage the mine safety hazards by optimizing mining technology, such as the induced caving method [5, 6], which includes the caving control engineering [7] and the certain safety measures [8]. Therefore, the research on the goaf roof caving has important practical significance for the underground excavation engineering.

Regardless of preventing or inducing the roof caving to ensure the safety production in the mine, the range of roof caving needs to be researched in advance. Most scholars [9–11] contributed to the study of the mechanism and law of the roof caving, which laid the foundation for predicting the range of goaf roof caving. Regarding the prediction of the range of goaf roof caving, the research methods can be divided into three categories: theoretical mechanics model, physical experiment, and numerical simulation. In the theoretical mechanics model, the caving arch mechanics model [12–14] was the most widely used. Based on that model, the relationship between critical caving span and caving height can be obtained, which can be used to evaluate the state of the goaf roof caving [15]. However, many physical and mechanical parameters affecting the goaf roof caving were neglected in the obtained
The dimensionless analysis in the present work is widely utilized to study the general law of the complex physical phenomena, which includes Rayleigh's method and the Buckingham pi theorem. Rayleigh's method is used to determine the functional relationships of physical phenomena with fewer variables. The Buckingham pi theorem can be used to research the physical phenomena containing multiple physical quantities. The functional relationships of physical phenomena with fewer variables can be expressed as

\[ f(x_1, x_2, \ldots, x_n) = 0. \]  

(1)

If the independent physical quantities in the functional relationship are \( m \) \((m < n)\), the independent physical quantities can be regarded as basic physical quantities, and the remaining physical quantities in the functional relationship are regarded as nonbasic physical quantities. Thus, the nonbasic physical quantities can be expressed by a compound quantity of the basic physical quantities, and a new function can be combined in the form of \((n-m)\) dimensionless numbers \( \pi \). Therefore, equation (1) can be expressed as

\[ f(\pi_1, \pi_2, \ldots, \pi_{n-m}) = 0. \]  

(2)

Obviously, a decrease in the number of physical quantities in equation (2) reduces the complexity of the physical phenomenon, which simplifies the theoretical analysis and experimental design alike.

The fractures in the natural rock mass are complex. Under the effect of joints and fractures, the goaf roof caving in the macroscopic is the tensile stress exceeds the tensile strength or the shear stress exceeds the shear strength. The microscopic performance is the development of rock mass fractures including new fracture development and the extension of original fracture. Therefore, the goaf roof caving is closely related to the fractures. To research the mechanical behavior of fractured rock mass, Warren and Root [38] introduced three groups of orthogonal fracture systems to represent the complex fracture systems. Then, the equivalent model for fractured rock mass can be established, as shown in Figure 2.

![Figure 1: Research strategy on the dimensionless charts for the range of goaf roof caving.](image-url)

### 2. Buckingham Pi Theorem

The goof roof caving is a relatively complex physical phenomenon, which involves many physical and mechanical parameters, such as the size of the excavation area and the properties of the surrounding rock. Therefore, it is difficult to visually predict the range of goaf roof caving. However, the method of charts can provide a convenient tool for specific engineering problems. The intuitive information can be obtained in the charts. For example, the support classification chart of the Q system can provide the support method of the excavation area according to the engineering information such as rock mass classification, excavation height, support spacing, and bolt length. Matthews stability graph can evaluate the slope stability through the stability number and hydraulic radius. The stability charts for rock slopes determine the safety factor of the slope according to the slope stability influencing factor. Based on the application of the above chart methods, this paper attempts to utilize the chart method to predict the range of goaf roof caving.

The research strategy on the dimensionless charts for the range of goaf roof caving is shown in Figure 1. In the following sections, considering that the dimensionless analysis is widely utilized to deal with complex physical phenomena containing multiple parameters, the Buckingham pi theorem in the dimensional analysis is first selected to establish the fuzzy function relationship between the range of roof caving and the influencing factors of the roof caving. Then, the discrete element software UDEC is applied to throw light on the fuzzy function relationship by numerical calculation. The large amounts data of the numerical calculation can obtain the dimensionless charts included the range of roof caving. Finally, based on a case study in the Shirengou Iron Mine, the range of goaf roof caving under the different working conditions can be predicted in the dimensionless charts using the method of linear interpolation.
The equivalent hydraulic conductivity $K$ of a set of parallel fractures in Figure 2 can be expressed as \[ K = \frac{g B^2}{12 \mu_k S}, \] \[ (3) \]
where $B$ is the fracture aperture and $S$ is the fracture spacing. The parameters $g$ and $\mu_k$ are the gravitational acceleration and the kinematic viscosity of water, respectively.

According to equation (3), the parameters $B$ and $S$ determine the value of hydraulic conductivity $K$, which can be used to characterize the fracture development degree of rock mass. It indicates that the parameters $B$ and $S$ are closely related to the goaf roof caving. Taking the $x$-$z$ plane in Figure 2 as the analysis object, as shown in Figure 3, the original rock stress around the goaf is redistributed by the excavation of the initial goaf. The caving of the goaf roof occurs under the continuously increase of the goaf span $l$. Using the caving height $h$ to represent the range of goaf roof caving, as shown in Figure 3, the parameter $h$ is related to the goaf span $l$, the goaf initial height $h_b$, and the goaf depth $H$. The parameter $h_b$ is mainly used to control the compensation space of the goaf roof caving. In addition, the rock density $\rho$ and gravity acceleration $g$ determine the bulk weight of rock mass, which is related to the gravity stress of the goaf roof. The rock mass elastic modulus $E_m$ and Poisson’s ratio $\nu$ are also closely related to the excavation and subsequent caving of the goaf.

According to the analysis of the influence factors of the goaf roof caving in the fractured rock mass, the physical and mechanical parameters involved in the goaf roof caving include the parameters $h$, $l$, $h_b$, $H$, $\rho$, $g$, $E_m$, $\nu$, $S$, and $B$. The dimension of each parameter is shown in Table 1. The surrounding rock of the goaf in Table 1 can be any kind of rock or ore. The properties of the surrounding rock vary with the working conditions of the goaf. Based on the Buckingham pi theorem, a fuzzy function relationship among the influence factors of the goaf roof caving can be established, which can be expressed as

\[ f \left( \frac{h}{h_b}, \frac{l}{h_b}, H, \rho, g, E_m, \nu, S, B \right) = 0. \]
\[ (4) \]

The solutions to equation (5) is derived in Appendix and summarized as follows:

\[ \frac{h}{h_b} = f \left( \frac{l}{h_b}, \frac{\rho g H \nu}{E_m}, \frac{B}{S} \right). \]
\[ (5) \]

where the dimensionless group $h/h_b$ represents the extent of the goaf roof caving. The dimensionless group $l/h_b$ represents the ratio of width to height in the initial goaf. The dimensionless group $\rho g H \nu/E_m$ represents the basic properties of rock mass at a certain goaf depth. The dimensionless group $B/S$ represents the fracture development degree of rock mass.

3. Numerical Simulation

The numerical simulation method for the goaf roof caving can be conducted by the finite element software and the discrete element software. However, the finite element software is not applicable to study the large deformation and displacement of the block system. Therefore, the discrete element software is selected to study the goaf roof caving. The Universal Distinct Element Code (UDEC) is a two-dimensional discrete element program for processing discontinuities. It can be used to simulate the response of discontinuities (such as joints and fractures in the rock mass) under static or dynamic loads. In the discrete element software UDEC, the discontinuous surfaces are treated as boundary surfaces among blocks and the large displacement and rotation of the block along the discontinuity surface are allowed. Using this software, most scholars [41–45] have achieved abundant results in the excavation of fractured rock mass. Therefore, the discrete element software UDEC is selected in this work to study the range of goaf roof caving.
Table 1: Unit and dimension of physical quantities involved in the goaf roof caving.

| Number | Physical quantity            | Symbol | Unit | Dimension |
|--------|------------------------------|--------|------|-----------|
| 1      | Goaf roof caving height      | $h$    | m    | L         |
| 2      | Goaf span                    | $l$    | m    | L         |
| 3      | Goaf initial height          | $hb$   | m    | L         |
| 4      | Goaf depth                   | $H$    | m    | L         |
| 5      | Rock density                 | $\rho$ | kg/m$^3$ | ML$^{-3}$ |
| 6      | Gravity acceleration         | $g$    | m/s$^2$ | LT$^{-2}$ |
| 7      | Rock mass elastic modulus    | $Em$   | kg/(m$^2$.s$^{-2}$) | ML$^{-1}$T$^{-2}$ |
| 8      | Poisson’s ratio              | $\nu$  | —    | 1         |
| 9      | Fracture spacing             | $S$    | m    | L         |
| 10     | Fracture aperture            | $B$    | m    | L         |

3.1. Mechanical Parameters. The Mohr–Coulomb plastic model is selected as the constitutive model in the UDEC model. The mechanical parameters of rock mass in the discrete element calculation include the rock mass uniaxial tensile strength $\sigma_{mt}$, the rock mass cohesion $c_1$, the angle of internal friction of rock mass $\varphi_1$, the rock mass volume modulus $K_m$, and the rock mass shear modulus $G_m$, which can be obtained according to the rock mass strength criterion and continuity assumption.

3.1.1. Rock Mass Uniaxial Tensile Strength $\sigma_{mt}$. The rock mass uniaxial tensile strength $\sigma_{mt}$ can be obtained by the Hoek–Brown criterion. The generalized Hoek–Brown rock mass strength criterion can be expressed as [46]

$$\sigma_1 = \sigma_3 + \sigma_c \left( \frac{m}{\sigma_c} + s \right)^{a_1},$$

where $\sigma_1$ and $\sigma_3$ are the maximum and minimum principal stresses, respectively. The parameter $\sigma_c$ is the rock uniaxial compressive strength. The parameters $m$, $s$, and $a_1$ are constants related to rock mass materials, which can be expressed as

$$m = m_t \exp \left( \frac{GSI - 100}{28 - 14D} \right),$$

$$s = \exp \left( \frac{GSI - 100}{9 - 3D} \right),$$

$$a_1 = \frac{1}{2} + \frac{1}{6} \left( e^{-(GSI/15)} - e^{-(20/3)} \right),$$

where GSI is the geological strength index and $D$ is a factor, which depends upon the degree of disturbance to which the rock mass has been subjected to blast damage and stress relaxation [46]. The parameters GSI and $D$ can be obtained through field investigation. The parameter $m_t$ can be selected according to the Hoek–Brown constant charts of rock mass [47].

From equation (9), the value of parameter $a_1$ is approximately equal to 0.5 when the parameter GSI $\geq 50$. Then, substituting $\sigma_1 = 0$ into equation (6), the rock mass uniaxial tensile strength $\sigma_{mt}$ can be expressed as

$$\sigma_{mt} = \frac{\sigma_c}{2} \left( m - \sqrt{m^2 + 4s} \right).$$

3.1.2. Rock Mass Cohesion $c_1$ and Angle of Internal Friction of Rock Mass $\varphi_1$. The parameters $c_1$ and $\varphi_1$ can be obtained by the Mohr–Coulomb criterion [48], which states that the relationship between $\sigma_1$ and $\sigma_3$ is linear.

$$\sigma_1 = \frac{1 + \sin \varphi_1}{1 - \sin \varphi_1} \sigma_3 + 2c_1 \cos \varphi_1,$$

Equation (11) can be expressed as

$$\sigma_1 = \sigma_{mc} + k\sigma_3,$$

where $\sigma_{mc}$ is the rock mass uniaxial compressive strength, $\sigma_{mc} = 2c_1\cos \varphi_1/(1 - \sin \varphi_1)$, and $k = 1 + \sin \varphi_1/(1 - \sin \varphi_1)$.

The Hoek–Brown criterion and Mohr–Coulomb criterion can predict rock strength well under the lower confining pressure. Hoek and Brown [49] proposed that the predicted results of both criteria are consistent under the condition of $0 < \sigma_3 < 0.25\sigma_c$. When $0 < \sigma_3 < 0.25\sigma_c$, the data of the parameters $\sigma_1$ and $\sigma_3$ can be calculated using equation (6). Then, using equation (12), the values of $\sigma_{mc}$ and $k$ can be obtained by fitting the data of principal stresses. Under the premise that the value of $k$ is known, the value of $\varphi_1$ is easy to find.

From $\sigma_{mc} = 2c_1\cos \varphi_1/(1 - \sin \varphi_1)$,

$$c_1 = \frac{\sigma_{mc}}{2\sqrt{k}}.$$  

According to equation (13), the rock mass cohesion $c_1$ can be obtained.

3.1.3. Elastic Modulus $E_m$, Volume Modulus $K_m$, and Shear Modulus $G_m$. The rock mass volume modulus $K_m$ and the rock mass shear modulus $G_m$ can be expressed as

$$K_m = \frac{E_m}{3(1 - 2\nu)}$$

$$G_m = \frac{E_m}{2(1 + \nu)}.$$  

The value of Poisson’s ratio $\nu$ is easy to obtain. When the elastic modulus of rock mass $E_m$ is obtained, the values of the parameters $K_m$ and $G_m$ can be calculated by equations (14) and (15). Therefore, the parameter $E_m$ is the key parameter to be determined.

The existence of joints and fractures directly affects the integrity of rock mass. Meanwhile, the joints and fractures determine the elastic modulus of rock mass, which the fracture spacing $S$ and fracture aperture $B$ have the most direct effect on the parameter $E_m$. A set of parallel fractures in Figure 3 is selected as the research object (as shown in Figure 4). It is convenient to regard the fractured rock mass as the continuous rock mass to study the mechanical behavior of fractured rock mass. Under the vertical stress $\Delta \sigma$, it can be known from Hooke’s law that
\[
\Delta \varepsilon_s = \frac{\Delta \sigma}{E}, \quad (16)
\]

where \(\Delta \varepsilon_s\) is the strain of fractured rock mass and \(E\) is the rock elastic modulus.

Then, the displacement \(\Delta u_b\) in the fracture spacing \(S\) can be expressed as
\[
\Delta u_b = S \Delta \varepsilon_s = \frac{S \Delta \sigma}{E}. \quad (17)
\]

Define the deformation of a single fracture is \(\Delta u_b\), then [39]
\[
\Delta u_b = \frac{\Delta \sigma}{k_n}, \quad (18)
\]

where \(k_n\) is the normal fracture stiffness.

As shown in Figure 4, the deformation of the continuous rock mass is \(\Delta u_{m}\), then,
\[
\Delta u_{m} = \Delta u_1 + \Delta u_b = \frac{(S + B) \Delta \sigma}{E_{m}}. \quad (19)
\]

Substituting equations (17) and (18) into equation (19) gives
\[
E_{m} = \frac{E k_n (S + B)}{S k_n + E} \quad (20)
\]

According to equations (6)–(20), the parameters \(\sigma_{m}, \varepsilon_{s}, \sigma_{m}, G_{m}\) can be calculated. The rock physical and mechanical parameters involved in the calculation can be obtained from the field investigation.

3.2. Numerical Calculation Flowchart. As shown in Figure 5, the fracture spacing \(S\), the goaf depth \(H\), and the goespan \(l\) are selected as the variables to obtain the dimensionless charts of the goaf roof caving height \(h\). The elastic modulus of rock mass \(E_{m}\) can be obtained by equation (20). When the rock physical and mechanical parameters in equations (6)–(20) are known, the parameter \(h\) can be obtained by the numerical simulation. Then, the dimensionless groups in equation (5) can be calculated. Subsequently, add the values of dimensionless groups into the dimensionless charts, then one calculation ends. In the next calculation, gradually increase the variables \(l, H\) and \(S\) for loop calculation. Finally, the whole loop calculation ends until the variable \(S\) takes the maximum value.

It is worth noting that the more times of the loop calculation, the higher the accuracy of the prediction of the goespan roof caving height in the dimensionless charts. Based on this point, the flowchart of numerical simulation in Figure 5 could increase the complexity of numerical simulation and the complexity of data processing. To overcome this difficult, the flowchart of dimensionless charts is established. As shown in Figure 6, the computer language Python is introduced to call the discrete element software UDEC using the os.system module. And then the numerical calculation results are directly provided to Python. Finally, using the NumPy module and Matplotlib module, the values of dimensionless groups can be calculated and the dimensionless charts can be plotted.

4. Case Study

4.1. Background. The ore deposit in the Shirengou Iron Mine is an Anshan-type magnetite deposit with a surface elevation of ±145 m. The ore body is produced in layers with an inclination of 65°–75°. The surrounding rock is hornblende gneiss. Above the −60 m level, the shallow-hole mining method was adopted for the stable ore rock. The shallow-hole mining method had left many goafs of varying sizes, forming an intricate goaf group in space. The open stope subsequent filling method is used to mine the ore body below the −60 m level, and the section height is 15 m. At present, the Shirengou Iron Mine is mined to −210 m level. As shown in Figure 7, the roof of the goaf left by the illegal mining induced below the level of −210 m penetrated to the −210 m level and is intersected with the goaf of the operation area. Thus, a large irregular goaf (referred to as M2 main goaf in Figure 7) is formed.

It is measured that the height of the M2 main goaf is about 120 m, and the maximum width is about 102 m. Considering that the M2 main goaf is in an unstable state, it is necessary to carry out the potential hazard evaluation and rational governance to ensure the safety mining of the surrounding ore body. At this time, a quantitative evaluation for the range of the M2 main goaf caving is of important guiding significance to the safety measures of the M2 main goaf.

4.2. Calculation. To predict the range of goaf roof caving under different working conditions, this paper takes the Shirengou Iron Mine as the engineering background. As for the layered ore body, the goaf can be regarded as a 2-D model [50, 51]. The numerical model for the goaf roof caving is established (as shown in Figure 8). The field investigation shows that the fracture spacing \(S\) is mainly concentrated in 1~4 m, the joint occurrence is mostly 234°±0°, the goaf depth \(H\) is mainly concentrated in the range of 200~450 m, and the goaf span \(l\) is 10~102 m. Therefore, the range of the variables \(S, H, l\) in Figure 5 can be set to 1~4 m, 100~500 m, and 0~100 m, respectively. The excavation step distance of the goaf is set to 2 m, which can obtain massive data of the goaf roof caving.
The Mohr–Coulomb criterion is used as the elastic-plastic criterion in the discrete element calculation. The left and right boundaries of the numerical model are subjected to deformation constraints to limit the horizontal displacement. The bottom boundary of the numerical model is subjected to fixed constraints to limit the horizontal and vertical displacement, and the top is the free boundary. The numerical model just considers the gravity stress and neglects the tectonic stress and mining stress. The gravitational acceleration \( g \) is 9.8 m/s\(^2\). Physical and mechanical parameters of rock mass and joint are obtained by field investigation and relevant equations, which are shown in Table 2. The range of goaf roof caving corresponding to the excavation step distance can be obtained according to the flowchart of numerical simulation (as shown in Figure 5).

4.3. Results. When the goaf span is excavated to 100 m, the caving ranges of the goaf roof under different fracture spacing are shown in Figures 9–12. The results show that the caving height of goaf roof increases with the increase of the goaf depth \( H \) under the conditions of the same fracture spacing \( S \) and goaf span \( l \). When the goaf depth \( H \) and goaf span \( l \) are same, the caving height of goaf roof decreases with the increase of the fracture spacing \( S \). Using the goaf roof caving height corresponding to the excavation step distance to calculate the dimensionless group in equation (5), the dimensionless chart for the range of goaf roof caving (as shown in Figure 13) can be obtained.

As shown in Figure 13, the overlying strata of the goaf can remain relatively stable when the goaf span is small. However, the duration of the relatively stable state is closely related to the fracture spacing \( S \). As shown in Figures 13(a)–13(d), the points O1–O4 are the starting points of the goaf roof caving when the parameter \( H \) is 500 m, respectively. Obviously, the degree of difficulty of the goaf roof caving in the initial stage increases with the increase of fracture spacing \( S \), which indicates the duration of the relatively stable state increases with the increase of fracture spacing \( S \).

With the continuous increase of the goaf span, the exposed area of the goaf expands gradually. It results in the occurrence of the overlying strata instability under the action of the gravity stress and internal concentrated stress. As shown in Figures 13(a)–13(d), taking the dimensionless curve corresponding to the parameter \( H \) is 500 m as an
example, the caving height of the goaf roof \( h \) increases with the increase of the goaf span \( l \). The results show that there is a rapid increase process between the points \( O_j \) and \( R_j \) \( (j = 1, 2, 3, 4) \). The reason for this phenomenon is related to the accumulation of internal strain energy and the continuous development and expansion of fractures during the period when the overlying strata maintain a relatively stable state. The speed of caving can appear short acceleration when the caving of overlying strata is occurred.

Under the premise that the goaf span \( l \) is no longer expanded, a relatively stable natural balance arch can be formed when the roof caving reaches a certain height, and the roof caving could not occur for a long time. However, when the span of the goaf is increasing, the stress balance state of the roof rock is destroyed continuously, and the caving could continue to occur. As shown in Figures 13(a)–13(d), the overlying strata still caving with increase of the goaf span when the rapid growth period of the roof caving is over. At this time, the growth rate of the caving height is gradually smaller, but the scale of the caving is relatively large.

4.4. Application. The dimensionless charts obtained by the excavation step distance 2 m contain the goaf roof caving under lots of working conditions. Using the linear interpolation method, the range of goaf roof caving under other working conditions can be predicted. Taking the M2 main goaf of –210 m level in the Shirengou Iron Mine as an example, the goaf depth \( H \) is 355 m, the goaf span \( l \) is 72 m, and the fracture spacing \( S \) is 1.4 m. The dimensionless charts corresponding to \( S = 1 \) m and \( S = 2 \) m in Figure 13 are used to predict the range of goaf roof caving under this working condition.

Define \( \pi_1 = l/hb \), \( \pi_2 = h/hb \), and \( \pi_3 = \log_{10}(Em/pgH) \). The feature point of the goaf roof caving range predicted by the dimensionless chart is denoted as \( P(\pi_1, \pi_2, \pi_3) \). When the goaf depth \( H \) is 355 m and the goaf span \( l \) is 72 m, as shown in Figure 14, the coordinates of feature points predicted by the corresponding dimensionless charts corresponding to \( S = 1 \) m and \( S = 2 \) m are \( P1 = (4.8, 3.62, 3.52) \) and \( P2 = (4.8, 3.08, 3.77) \), respectively. Using the dimensionless group \( h/hb \), the ranges of goaf roof caving predicted by the two dimensionless charts are 54.3 m and 46.2 m, respectively. It can be known from the linear interpolation that the predicted range of the goaf roof caving is 51.06 m when the parameter \( S = 1.4 \) m.

As shown in Figure 7, the caving height of the M2 main goaf roof in the –210 m level is 46.66 m, which is less than the range of roof caving predicted by the dimensionless charts in this work. In fact, the M2 main goaf roof is always in an
Table 2: Physical and mechanical parameters of the Shirengou Iron Mine.

| Parameter                                           | Value                  | Method                          |
|-----------------------------------------------------|------------------------|---------------------------------|
| Geological strength index GSI                       | 76                     |                                 |
| Disturbance factor $D$                              | 0.7                    |                                 |
| Rock material constant $m_i$                        | 24                     |                                 |
| Gravitational acceleration $g$ (m/s$^2$)            | 9.8                    |                                 |
| Initial goaf height $h_b$ (m)                       | 15                     |                                 |
| Hydraulic conductivity $K$ (m/s)                    | $1.68 \times 10^{-8}$  | Field investigation             |
| Kinematic viscosity of water (20°C) $\mu_k$ (m$^2$/s) | $1.01 \times 10^{-6}$  |                                 |
| Rock density $\rho$ (kg/m$^3$)                      | 3.58                   |                                 |
| Rock uniaxial compressive strength $\sigma_c$ (MPa) | 99.44                  |                                 |
| Rock elastic modulus $E$ (GPa)                      | 80.3                   |                                 |
| Poisson’s ratio $\nu$                               | 0.21                   |                                 |
| Joint normal fracture stiffness $k_n$ (GPa)         | 9.67                   |                                 |
| Joint tangential fracture stiffness $k_t$ (GPa)     | 6.53                   |                                 |
| Joint cohesion $c_1$ (MPa)                          | 1.67                   | Field investigation and simulation experience |
| Joint internal friction angle $\varphi_1$ (°)       | 22.35                  |                                 |
| Joint tensile strength $\sigma_{t2}$ (MPa)          | 0.23                   |                                 |
| Fracture aperture $B$ (m)                           | $2.73 \times 10^{-4}$  | Equation (3)                    |
| Rock mass material constant $m$                     | 6.42                   | Equation (7)                    |
| Rock mass material constant $s$                     | 0.031                  | Equation (8)                    |
| Rock mass material constant $a$                     | 0.5                    | Equation (9)                    |
| Rock mass tensile strength $\sigma_{mt}$ (MPa)      | 0.48                   | Equation (10)                   |
| Rock mass cohesion $c_2$ (MPa)                      | 4.97                   | Using equation (11) to fit the principal stress data |
| Rock mass internal friction angle $\varphi_2$ (°)   | 40.98                  |                                 |
| Fracture spacing $S$ (m)                            | 1.0–4.0                | Field investigation             |
| Goaf depth $H$ (m)                                  | 100–500                |                                 |
| Goaf span $l$ (m)                                   | 0–100.0                |                                 |
| Rock mass elastic modulus $E_{mr}$ (GPa)            | Equation (20): the value of $E_{mr}$ varies with the parameter $S$ |
| Rock mass volume modulus $K_{mr}$ (GPa)             | Equation (14): the value of $K_{mr}$ varies with the parameter $E_{mr}$ |
| Rock mass shear modulus $G_{mr}$ (GPa)              | Equation (15): the value of $G_{mr}$ varies with the parameter $E_{mr}$ |

Figure 9: Caving range of the goaf roof (when $l = 100$ m, $S = 1$ m). (a) $H = 100$ m. (b) $H = 200$ m. (c) $H = 300$ m. (d) $H = 400$ m. (e) $H = 500$ m.
Figure 10: Caving range of the goaf roof (when $l = 100$ m, $S = 2$ m. (a) $H = 100$ m. (b) $H = 200$ m. (c) $H = 300$ m. (d) $H = 400$ m. (e) $H = 500$ m.

Figure 11: Caving range of the goaf roof (when $l = 100$ m, $S = 3$ m. (a) $H = 100$ m. (b) $H = 200$ m. (c) $H = 300$ m. (d) $H = 400$ m. (e) $H = 500$ m.
Figure 12: Caving range of the goaf roof (when \( l = 100 \text{ m}, S = 4 \text{ m} \)). (a) \( H = 100 \text{ m} \). (b) \( H = 200 \text{ m} \). (c) \( H = 300 \text{ m} \). (d) \( H = 400 \text{ m} \). (e) \( H = 500 \text{ m} \).

Figure 13: Continued.
unstable state. Occasionally, the M2 main goaf has the sound of caving rubble according to the field investigation. This phenomenon indicates that the M2 main goaf is in a state of caving activity, and the range of roof caving predicted by the dimensionless charts in this work is reasonable. Therefore, the dimensionless charts obtained by the dimensional analysis coupled with numerical simulation can preliminarily quantitatively evaluate the caving range of goaf roof under different working conditions, which has certain guiding significance to the treatment of the goaf.

5. Summary and Conclusions

The physical phenomenon of the goaf roof caving involves many physical quantities. Only considering the relationship between a single or a few physical quantities is not conducive to the study of the goaf roof caving. The method of the dimensional analysis coupled with numerical simulation in this work can be a useful technique to arrange the physical quantities and simplify the study of the goaf roof caving.

The dimensionless charts in this work are convenient for the direct acquisition of the range of goaf roof caving. Using the linear interpolation method, the range of goaf roof caving under different working conditions can be predicted, which can be used as a reference for the potential hazard evaluation and rational governance of the goaf. In addition, with the continuous increase of the goaf span \( l \), the dimensionless curves of the caving range in the dimensionless charts experience zero growth, rapid growth and steady growth. The growth degree varies with the fracture spacing \( S \).

In conclusion, the dimensionless charts in this work can comprehensively consider the influence factors of the goaf roof caving and can quantitatively evaluate the range of goaf roof caving under different working conditions, which have a certain referential value for similar engineering problems. However, the limitation of this study is that it is temporarily
applicable to the excavation of layered rock masses or ore bodies. The application of the excavation of other strata with different properties will be studied in the future work. More field applications contribute to the modification and supplementation of the dimensionless charts.

**Abbreviations**

- $E_m$: Rock mass elastic modulus
- $K_m$: Rock mass volume modulus
- $G_m$: Rock mass shear modulus
- $\sigma_{mi}$: Rock mass uniaxial tensile strength
- $h_b$: Goaf initial height boundary
- $\sigma_c$: Rock uniaxial compressive strength
- $\sigma_1$: Minimum principal stress
- $\mu_c$: Kinematic viscosity of water
- $k_n$: Joint normal fracture stiffness
- $k_t$: Joint tangential fracture stiffness
- $m$: Rock material constant
- $c_1$: Rock mass cohesion difference from the joint
- $\varphi_1$: Rock mass internal friction angle difference from the joint
- $c_2$: Joint cohesion difference from the rock mass
- $\varphi_2$: Joint internal friction angle difference from the rock mass
- $\sigma_{t2}$: Joint tensile strength difference from the rock mass
- $\nu$: Poisson’s ratio
- $K$: Hydraulic conductivity
- $B$: Fracture aperture
- $S$: Fracture spacing
- $g$: Gravitational acceleration
- $\rho$: Rock density
- $h$: Goaf roof caving height
- $l$: Goaf span
- $H$: Goaf depth
- $GSI$: Geological strength index
- $D$: Disturbance factor
- $m$, $s$, and $a$: Rock mass material constants.

**Appendix**

The fuzzy function relationship among the influence factors of the goaf roof caving can be expressed as

$$f(h, l, h_b, H, \rho, g, E_m, \nu, S, B) = 0.$$ (A.1)

The dimension of Poisson’s ratio $\nu$ is 1, which can be regarded as a dimensionless group. Selecting $E_m$, $\rho$, and $h_b$ as the basic physical quantities, the remaining parameters $h$, $l$, $H$, $g$, $S$, and $B$ are the nonbasic physical quantities, then the dimensionless groups in the goaf roof caving can be expressed as

$$\pi_1 = \left( E_m^\alpha \rho^\beta h_b^\gamma \right) h_b^\eta_1,$$ (A.2)

$$\pi_2 = \left( E_m^\alpha \rho^\beta h_b^\gamma \right) f^\eta_2.$$ (A.3)

where $\alpha$, $\beta$, $\gamma$, and $\eta_i$ are the exponents of the influence factors and $i = 1, 2, 3, 4, 5, \text{and } 6$.

Analyzing the dimension of the parameters on both sides of equation (A.2), we can obtain

$$(MLT)^0 = (ML^{-1}T^{-1})^\eta_1 (ML^{-3})^\eta_2 (L)^{\eta_3}.$$ (A.4)

Based on the dimensional homogeneity, we can obtain $\alpha_1 = -\eta_1$, $\beta_1 = \gamma_1 = 0$. Then, equation (A.2) can be expressed as

$$\pi_1 = (h/h_b)^{\eta_1}. $$ (A.9)

Similarly, the remaining dimensionless groups can be expressed as

$$\pi_2 = (l/h_b)^{\eta_2}, $$ (A.10)

$$\pi_3 = (H/h_b)^{\eta_3}, $$ (A.11)

$$\pi_4 = (\rho gh_b/E_m)^{\eta_4}, $$ (A.12)

$$\pi_5 = (S/h_b)^{\eta_5}, $$ (A.13)

$$\pi_6 = (B/h_b)^{\eta_6}. $$ (A.14)

Substituting equations (A.9)–(A.14) and Poisson’s ratio $\nu$ into equation (2) gives

$$f\left[ \frac{h}{h_b}^{\eta_1}, \frac{l}{h_b}^{\eta_2}, \frac{H}{h_b}^{\eta_3}, \frac{\rho gh_b}{E_m}^{\eta_4}, \frac{S}{h_b}^{\eta_5}, \frac{B}{h_b}^{\eta_6}, \nu \right] = 0.$$ (A.15)

From equation (A.15), the parameter $\eta_i$ ($i = 1, 2, 3, 4, 5, 6$) is the only exponential of the influence factors. Generally, the value of the parameter $\eta_i$ is flexible, which has no influence on the evaluation results of the dimensionless charts [32, 36]. However, improper value could complicate the evaluation process of the dimensionless charts. To make the evaluation more convenient, we can take the $\eta_1 = \eta_2 = \eta_3 = \eta_4 = \eta_6 = 1$ and $\eta_5 = -1$. Then, equation (A.15) can be expressed as

$$f\left[ \frac{h}{h_b}, \frac{l}{h_b}, H, \rho gh_b, h_b, \frac{B}{h_b}, \frac{S}{h_b}, \frac{E_m}{h_b}, \nu \right] = 0.$$ (A.16)

The multiplication and division of dimensionless groups in equation (A.16) are allowed [32, 36]. To define a certain physical meaning to the dimensionless group, the dimensionless groups in equation (A.16) are arranged as

$$\pi_3 = \left( E_m^\alpha \rho^\beta h_b^\gamma \right) H^{\eta_5}, $$ (A.4)

$$\pi_4 = \left( E_m^\alpha \rho^\beta h_b^\gamma \right) g^{\eta_4}, $$ (A.5)

$$\pi_5 = \left( E_m^\alpha \rho^\beta h_b^\gamma \right) S^{\eta_5}, $$ (A.6)

$$\pi_6 = \left( E_m^\alpha \rho^\beta h_b^\gamma \right) B^{\eta_6}, $$ (A.7)
\[ f \left( \frac{h}{h_b}, \frac{l}{h_b}, \frac{\rho g H v B}{E_m}, \frac{B}{S} \right) = 0, \quad (A.17) \]

or

\[ \frac{h}{h_b} = f \left( \frac{l}{h_b}, \frac{\rho g H v B}{E_m}, \frac{B}{S} \right). \quad (A.18) \]

**Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

**Authors’ Contributions**

Jing Zhang and Rongxing He contributed to the formulation of the overarching research goals and aims. Jing Zhang and Zhihua Ouyang participated in dimensional analysis. Jing Zhang contributed to the numerical simulation. Jing Zhang wrote the manuscript, and Fengyu Ren checked the manuscript. Jing Zhang and Rongxing He contributed to the collection of the references. Rongxing He was involved in collection of the physical and mechanical parameters of rock mass.

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**References**

[1] M. Tutak and J. Brodny, “The impact of the strength of roof rocks on the extent of the zone with a high risk of spontaneous coal combustion for fully powered longwalls ventilated with the Y-type system—a case study,” *Applied Sciences*, vol. 9, no. 24, p. 5315, 2019.

[2] M. Tutak and J. Brodny, “Determination of particular endogenous fires hazard zones in goaf with caving of longwall,” *IOP Conference Series: Earth and Environmental Science*, vol. 95, Article ID 042026, 2017.

[3] C. X. Yan, X. Tao, P. L. P. Wasantha et al., “Dynamic disaster control of backfill mining under thick magmatic rock in one side goaf: a case study,” *Journal of Central South University (English Edition)*, vol. 27, pp. 3103–3117, 2020.

[4] W. Cai, Z. Chang, D. Zhang, X. Wang, W. Cao, and Y. Zhou, “Roof filling control technology and application to mine roadway damage in small pit goaf,” *International Journal of Mining Science and Technology*, vol. 29, no. 3, pp. 477–482, 2019.

[5] H. Chen, C. C. Wan, S. G. Zhang, and B. R. Zhou, “In-situ fracturing induced caving of a hard orebody and its Application,” *Geotechnical and Geological Engineering*, vol. 37, no. 4, pp. 2303–2313, 2018.

[6] X. M. Fan, F. Y. Ren, D. Xiao, and Y. C. Mao, “Opencast to underground iron ore mining method,” *Journal of Central South University*, vol. 25, no. 7, pp. 1813–1824, 2018.

[7] R. X. He, F. Y. Ren, and B. H. Tan, “Discussion on induced caving and block caving,” *Chinese Journal of Metal Mine*, vol. 3, pp. 9–14, 2017.

[8] J. L. Cao and F. Y. Ren, “Research of dispersions bedding’s safety thickness in processing deposits with induced caving,” *Chinese Journal of Metal Mine*, vol. 3, pp. 45–48, 2013.

[9] E. Mikhail, E. Gabriel, and S. Igor, “Numerical simulation of roof cavings in several kuzbass mines using finite-difference continuum damage mechanics approach,” *International Journal of Mining Science and Technology*, vol. 30, no. 2, pp. 157–166, 2020.

[10] S. X. Hu, L. Q. Ma, J. S. Guo et al., “Support-surrounding rock relationship and top-coal movement laws in large dip angle fully-mechanized caving face,” *International Journal of Mining Science and Technology*, vol. 28, no. 3, pp. 533–539, 2018.

[11] G. Zhou, Q. Zhang, R. Bai, T. Fan, and G. Wang, “The diffusion behavior law of respirable dust at fully mechanized caving face in coal mine. CFD numerical simulation and engineering application,” *Process Safety and Environmental Protection*, vol. 106, pp. 117–128, 2017.

[12] F. R. Haiying, Y. Zhao, and S. Ren, “Determination of the critical span for a large-caving above a mined-out area,” *Current Science*, vol. 116, no. 4, pp. 654–660, 2019.

[13] F. Y. Ren, J. Zhang, and Y. Liu, “Study on safe and efficient recovery technology of residual ore in lishugou iron mine,” *Chinese Journal of Metal Mine*, vol. 3, pp. 23–27, 2020.

[14] R. X. He, F. Y. Ren, D. L. Song et al., “Induced caving rule of inclined thick ore body in hemushan iron mine,” *Chinese Journal of Mining and Safety Engineering*, vol. 34, no. 5, pp. 899–904, 2017.

[15] D. J. Zhang, F. Y. Ren, and J. D. Wang, “Rock mass caving and movement mechanism of caving mining,” *Chinese Journal of Mining and Strata Control Engineering*, Article ID 033521, 2021.

[16] Y. Zhang, S. Tu, Q. Bai, and J. Li, “Overburden fracture evolution laws and water-controlling technologies in mining very thick coal seam under water-rich roof,” *International Journal of Mining Science and Technology*, vol. 23, no. 5, pp. 693–700, 2013.

[17] H. Kang, J. Lou, F. Gao, J. Yang, and J. Li, “A physical and numerical investigation of sudden massive roof collapse during longwall coal retreat mining,” *International Journal of Coal Geology*, vol. 188, pp. 25–36, 2018.

[18] D. F. Yang, Y. J. Zhang, W. X. Xu et al., “Analysis of the appearance characteristics of mineral pressure in the shallow Seam,” *Chinese Journal of Science Technology and Engineering*, vol. 20, no. 20, pp. 8099–8106, 2020.

[19] J. L. Cao, *Study on Falling Law and its Application of Magnesite Orebody and Rock with Multi-Mined-Out Areas*, Northeastern University, China, 2017.

[20] F. Y. Ren, D. J. Zhang, J. L. Cao, M. Yu, and S. Li, “Study on the rock mass caving and surface subsidence mechanism based on an in-situ geological investigation and numerical analysis,” *Mathematical Problems in Engineering*, vol. 2018, Article ID 6054145, 18 pages, 2018.

[21] Ü. Bahtiyar, A. H. Mehmet, T. Erhan et al., “Analysis of roof caving characteristics at a coal mine by using full scale 3D numerical modeling,” in *Proceedings of the 22nd Mine Planning and Equipment Selection*, pp. 501–509, Dresden, Germany, October 2014.
[22] C. C. Wei, C. G. Zhang, and I. Canbultal, “Numerical analysis of fault-slip behaviour in longwall mining using linear slip weakening law,” Tunnelling and Underground Space Technology, vol. 104, Article ID 103541, 2020.

[23] Y. M. Alshkane, A. M. Marshall, and L. R. Stace, “Prediction of strength and deformability of an interlocked blocky rock mass using UDEC,” Journal of Rock Mechanics and Geotechnical Engineering, vol. 9, no. 3, pp. 531–542, 2017.

[24] A. Lannuzze, A. D. Endice, T. V. Mele et al., “Numerical limit analysis-based modelling of masonry structures subjected to large displacements,” Computers and Structures, vol. 242, Article ID 106372, 2021.

[25] J. Hamdi, M. Souley, L. Scholtès, M. A. Heib, and Y. Gunzburger, “Assessment of the energy balance of rock masses through discrete element modelling,” Procedia Engineering, vol. 191, pp. 442–450, 2017.

[26] N. Barton, R. Lien, and J. Lunde, “Engineering classification of rock masses for the design of tunnel support,” Rock Mechanics and Rock Engineering, vol. 6, no. 4, pp. 189–236, 1974.

[27] Q. S. Liu, X. Y. Wang, and X. Huang, “Prediction model of rock mass class using classification and regression tree integrated adaboost algorithm based on TBM driving data,” Tunnelling and Underground Space Technology, vol. 106, pp. 1–13, 2020.

[28] K. E. Mathews, E. Hoek, D. C. Wyllie et al., Prediction of Stable Excavation Spans at Depths below 1000 Metres in Hard Rock Mines, Canada Centre for Mining and Energy Technology, Ottowa, Canada, 1981.

[29] X. D. Zhao and J. A. Niu, “Stability evaluation and parameter optimization of stope based on extended mathews stability graph method,” Chinese Journal of Metal Mine, vol. 2, pp. 141–147, 2020.

[30] C. W. Sun, J. R. Chai, Z. G. Xu et al., “3D stability charts for convex and concave slopes in plan view with homogeneous soil based on the strength-reduction method,” International Journal of Geomechanics, vol. 17, no. 5, pp. 215–225, 2017.

[31] C. W. Sun, J. R. Chai, Z. G. Xu et al., “Stability charts for rock slopes based on the method of reduction of hoek-brown strength,” Chinese Journal of Rock Mechanics and Engineering, vol. 214, no. 4, pp. 838–851, 2018.

[32] N. C. Alberto, Fundamentals of Dimensional Analysis, Springer Nature Singapore Pte Ltd, Singapore, 2021.

[33] H. H. Olsen, “Buckingham’s pi-theorem,” Mathematical Modelling, pp. 1–7, 2004.

[34] S. Du, "Potential laws on the changes of shale in acid erosion process based on the fast matching method of dimensional analysis," International Journal of Hydrogen Energy, vol. 46, no. 11, pp. 7836–7847, 2021.

[35] S. Khandoozi, M. R. Malayeri, and M. Riazi, “Inspectional and dimensional analyses for scaling of low salinity waterflooding (LSWF): from core to field scale,” Journal of Petroleum Science and Engineering, vol. 189, pp. 1–23, 2020.

[36] F. Y. Ren, J. Zhang, Z. H. Ouyang, and H. Hu, “Calculation of elastic modulus for fractured rock mass using dimensional analysis coupled with numerical simulation,” Mathematical Problems in Engineering, vol. 2021, Article ID 2803837, 14 pages, 2021.

[37] J. I. Onue, J. O. Ademiluyi, and J. Agunwamba, “Buckingham pi dimensional analysis of cake yield from sludge filtration process,” American Journal of Sciences and Engineering Research, vol. 3, no. 2, pp. 55–69, 2020.

[38] J. E. Warren and P. J. Root, “The behavior of naturally fractured reservoirs,” Society of Petroleum Engineers Journal, vol. 3, no. 3, pp. 245–255, 1963.

[39] Z. Ouyang and D. Elsworth, “Evaluation of groundwater flow into mined panels,” International Journal of Rock Mechanics and Mining Sciences and Geomechanics Abstracts, vol. 30, no. 2, pp. 71–79, 1993.

[40] J. Liu and D. Elsworth, “Three dimensional effects of hydraulic conductivity enhancement and desaturation around mined panels,” International Journal of Rock Mechanics and Mining Sciences, vol. 34, no. 8, pp. 1139–1152, 1997.

[41] Z. P. Yang, Y. W. Jiang, B. Li et al., “Study on the mechanism of deep and large fracture propagation and transfixion in karst slope under the action of mining,” Chinese Journal of Geomechanics, vol. 26, no. 4, pp. 459–470, 2020.

[42] X. L. Li, C. Y. Hu, Q. F. Sun et al., “Research on stress distribution in collapse process of mine goaf by UDEC software,” Chinese Journal of Gezhouba Geology, vol. 36, no. 3, pp. 254–260, 2019.

[43] B. Wu, X. Wang, J. Bai, W. Wu, X. Zhu, and G. Li, “Study on crack evolution mechanism of roadside backfill body in gobi side entry retaining based on UDEC trigon model,” Rock Mechanics and Rock Engineering, vol. 52, no. 9, pp. 3385–3399, 2019.

[44] M. Kalilillo and Y. Y. Xia, “UDEC-based stability analysis of jointed bedding slope and slope parameter optimization suggestions: a case study,” SN Applied Sciences, vol. 2, Article ID 1943, 2020.

[45] A. Bindlish, M. Singh, and N. K. Samadihya, “Modeling of ultimate bearing capacity of shallow foundations resting on jointed rock mass,” Indian Geotechnical Journal, vol. 43, no. 3, pp. 25–266, 2013.

[46] E. Hoek and E. T. Brown, “The hoek-brown failure criterion and GSI — 2018 edition,” Journal of Rock Mechanics and Geotechnical Engineering, vol. 11, no. 3, pp. 445–463, 2019.

[47] E. Hoek and M. S. Diedrichs, “Empirical estimation of rock mass modulus,” International Journal of Rock Mechanics and Mining Sciences, vol. 43, no. 2, pp. 203–215, 2006.

[48] O. Mohr, “Welche umstande bedingen die elastizitatsgrenze und den bruch eines materials,” Zeitschrift des Vereins Deutscher Ingenieure, vol. 44, pp. 1524–1530, 1900.

[49] E. Hoek and E. T. Brown, “Practical estimates of rock mass strength,” International Journal of Rock Mechanics and Mining Sciences, vol. 34, no. 8, pp. 1165–1186, 1997.

[50] Y. Sun, J. Zuo, M. Karakus, and J. Wang, “Investigation of movement and damage of integral overburden during shallow coal seam mining,” International Journal of Rock Mechanics and Mining Sciences, vol. 117, pp. 63–75, 2019.

[51] S. Cao, W. Song, D. Deng, Y. Lei, and J. Lan, “Numerical simulation of land subsidence and verification of its character for an iron mine using sublevel caving,” International Journal of Mining Science and Technology, vol. 26, no. 2, pp. 327–332, 2016.