Hamiltonian Formalism in the Presence of Higher Derivatives

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ABSTRACT

A short review of basic formulas from Hamiltonian formalism in classical mechanics in the case when Lagrangian contains \( N \) time-derivatives of \( n \) coordinate variables. For non-local models \( N = \infty \).

1 Introduction

Theories with higher derivatives were long at the periphery of theoretical physics. The basic reason for this is that they do not appear in everyday physical problems. This in turn can have a reason: if, as many believe these days, observable world is described by a low-energy limit of some yet-unknown fundamental theory, then it is naturally governed by Lagrangian dynamics with lowest possible – this means a pair of first – time-derivatives and Newton law includes only acceleration. From this point of view there is no restriction on the number of derivatives in the \textit{fundamental} theory, and higher derivative terms are indeed present in most approaches, from QFT formulations of string and M-theory [1] to pure QFT models like asymptotically safe gravity [2] or (the quantum version of) the recent \( E_8 \) unification model [3]. It goes without saying that various non-local and innumerable non-commutative models all fit into category of higher-derivative theories. More than that, even the ordinary physical theories, like \textit{classical} electrodynamics, appear inconsistent without higher derivatives, if one includes radiation phenomena and allows space-time dimension to be greater than 4 – like one does, for example, in amusing TeV-gravity models [4]. In these circumstances the resolution of radiation friction and electromagnetic mass ”problems” requires inclusion into the bare (”fundamental”) action of counter-terms which not only renormalize mass (as in \( d = 4 \), but necessarily include higher derivatives [5]. Of course, higher derivatives are also used for purposes of UV regularization in more formal context, especially in gauge invariant and supersymmetric models [6], even if inclusion of such terms is not physically unavoidable. Last but not the least, higher-derivative terms are the common place in all effective theories, from solid state physics to quantum gravity.

For all these reasons the higher-derivative dynamics is slowly gaining new attention, see [7]-[18] for the relatively recent discussions from various viewpoints, as well as [19]-[34] for some classical papers and monographs. It should be emphasized that this almost-untouched ground is very attractive from the point of view of ”theoretical theory” and is intimately related to modern topological theory [35], \( L^{(N)} \) structures \textit{a la} [36], non-linear algebra [37] etc. Since [38] it is known that when such theories are required to be reparametrization invariant (what is the case in most of thinkable applications) new phenomena of outstanding beauty occur. Of special interest is symplectic geometry behind such theories [39, 40].

This short note is devoted to the 0-th chapter of higher-derivative theory. It contains a short list of elementary formulas – well-known to a narrow class of interested people ever since [19] – about \textit{classical} Lagrangian and Hamiltonian dynamics, which can be used for comparison with results of various more-advanced approaches.

2 Lagrangian formalism

Consider the classical mechanics with the action

\[
S\{q^\alpha(t)\} = \int L\,dt,
\]

where Lagrangian \( L\left(q^\alpha, d_1 q^\alpha, \ldots, (d_N q^\alpha)\right) \) depends on the first \( N \) time-derivatives \( q_i^\alpha = d_i^\alpha q^\alpha \) of \( n \) coordinate variables \( q^\alpha = q_0^\alpha, \alpha = 1, \ldots, n \). Obviously,

\[
q_i^\alpha = q_{i+1}^\alpha
\]
Introduce the variational derivatives w.r.t. $q^\alpha_i$ for all $i \geq 0$:

$$\delta_{i-1}^i \equiv \partial_i^i - d_i \partial_{i+1}^i + d_i^2 \partial_{i+2}^i - \ldots,$$

where $\partial_i^i = \partial/\partial q^\alpha_i$ and the momenta

$$\Pi^\alpha_i = \delta^\alpha_i L$$

These operators are related by time-derivatives:

$$\dot{\delta}_{i-1}^i \equiv d_t \delta_{i-1}^i = \partial_i^i - \delta_{i}^i$$

in a way, dual to (2).

The Euler-Lagrange equations of motion are

$$\Pi_{i-1}^\alpha \equiv \delta_{i-1}^i L = 0.$$ 

These are equations of order $2N$ in time-derivatives. Initial conditions are imposed on $q^\alpha_i, d_t q^\alpha_i, \ldots, (d_{t}^{2N-1} q^\alpha).$

Note that superscripts are not powers. In what follows we suppress indices $\alpha$ in some formulas.

### 3 Hamiltonian formalism

The phase space is $2Nn$-dimensional with canonically conjugate coordinates $q^\alpha_i$ and $\Pi^\alpha_i$, where $i = 0, \ldots, N - 1$. The $q^\alpha_N$ are functions of these independent variables.

The closed symplectic 2-form

$$\Omega = d\theta = \sum_{i=0}^{N-1} d\Pi^\alpha_i \wedge dq^\alpha_i,$$ 

is conserved:

$$\dot{\Omega} = d_t \Omega = 0,$$ 

while the time-derivative of the associated pre-symplectic 1-form

$$\theta = \sum_{i=0}^{N-1} \Pi^\alpha_i dq^\alpha_i$$

is exact:

$$\dot{\theta} = dL.$$

Hamilton equations state that

$$\dot{q}_i^\alpha = \Pi_{i-1}^\alpha = \frac{\partial H}{\partial \Pi^\alpha_i},$$

$$\dot{\Pi}_{i+1}^\alpha = -\Pi_{i}^\alpha - \frac{\partial L}{\partial q_i^\alpha} = -\frac{\partial H}{\partial q_i^\alpha}$$

with the Hamiltonian

$$H(\Pi, q) = \sum_{i=0}^{N-1} \Pi^\alpha_i q_{i+1}^\alpha - L$$

and $q_N^\alpha$ expressed through canonical variables.

Consider $\tilde{S}(\tilde{q}, \tilde{\Pi})$ – the action (1), evaluated on the classical trajectory with the boundary conditions $\tilde{q}_i^\alpha = q_i^\alpha(i)$ and $\tilde{\Pi}_i^\alpha = \Pi_i^\alpha(i), i = 0, \ldots, N - 1$. Then

$$\frac{\partial \tilde{S}}{\partial \tilde{q}_i^\alpha} = -\tilde{\Pi}_i^\alpha,$$

$$\frac{\partial \tilde{S}}{\partial \tilde{\Pi}_i^\alpha} = \tilde{\Pi}_i^\alpha$$

All these relations are obvious generalizations of those in the simplest case of $N = 1$, see [41].
4 Proofs

The proofs of above relations are straightforward:

\[
d_t \left[ \sum_{i=0} \left( dq_i \wedge d\delta^i - dq_i \wedge d\delta^{i-1} + dq_i \wedge d\partial^i \right) \right] = -dq_0 \wedge d\delta - \sum_{i=0} dq_i \wedge d\delta^i
\]  

(12)

When acting on \( L \), the first term vanishes on equations of motion, \( \delta^{-1}L = 0 \), while the second term becomes

\[
\sum_{i=0} dq_i \wedge d\left( \partial_i L \right) = \sum_{i,j=0} \left( \partial^i L \right) dq_i \wedge dq_j = 0.
\]  

(13)

Similarly

\[
d_t \left[ \sum_{i=0} dq_i \delta^i \right] = \sum_{i=0} \left( dq_{i+1} \delta^i - dq_i \delta^{i-1} + dq_i \delta^i \right) = -dq_0 \delta^{-1} + d
\]  

(14)

Again, when acting on \( L \) the first term vanishes on equations on motion.

Hamiltonian derivatives in (9) are:

\[
\frac{\partial H}{\partial \Pi_i} = q_i + \left( \Pi_N - 1 - \frac{\partial L}{\partial q_N} \right) \frac{\partial q_N}{\partial \Pi_i},
\]  

\[
- \frac{\partial H}{\partial q_i} = - \left( \Pi_i - 1 - \delta_i, 0 \right) + \frac{\partial L}{\partial q_i} - \left( \Pi_N - 1 - \frac{\partial L}{\partial q_N} \right) \frac{\partial q_N}{\partial q_i}
\]  

(15)

The terms in brackets at the r.h.s. vanish because \( \Pi_N - 1 = \delta_N - 1 \).

Finally, the variation of action \( S\{q(t)\} \) under the variation \( \delta q(t) \) of its argument is equal to

\[
\delta S = \int \sum_{i=0}^N \delta x_i \delta^i L = \oint \delta x_i \delta^i L + \int \delta^{-1} L
\]  

(16)

On classical trajectory the second term vanishes, while the boundary contributions in the first term gives rise to (11).

5 Towards cohomological formulation

The key role in above calculations is played by the operator

\[
\hat{A} = \sum_{i=1} (-)^i \partial^{-1} \otimes \partial^{-i} = \frac{1}{1 \otimes 1 + \partial \otimes \partial^{-1}} 1 \otimes \partial^{-1} = \frac{1}{\partial^{-1} \otimes \partial + 1 \otimes 1} \partial^{-1} \otimes 1
\]  

(17)

It is a formal inverse of \( \partial_i \), which acts on the product by Leibnitz rule:

\[
\left( \partial \otimes 1 + 1 \otimes \partial \right) \hat{A} = \frac{1}{1 \otimes 1 + \partial \otimes \partial^{-1}} \left( \partial \otimes \partial^{-1} + 1 \otimes 1 \right) = 1 \otimes 1
\]

Conceptually, for \( \partial = d_t \)

\[
\Omega = \hat{A}_* dq \wedge \frac{\delta L}{\delta q}.
\]

Indeed, for

\[
\frac{\delta}{\delta q_i} = \frac{\partial}{\partial q_i} - \frac{\partial}{\partial q_{i+1}} + \ldots
\]

we have

\[
\partial \delta_{i+1} = \partial_i - \delta_i
\]

or

\[
\frac{\delta}{\delta q_{i+1}} = -\partial^{-1} \frac{\delta}{\delta q_i} = \ldots = (-)^i \partial^{-i} \frac{\delta}{\delta q}
\]

where \( \partial_i^{-1} \) properly takes care of the \( \partial / \partial q_i \) which lies in "cohomology" of \( \partial \).

A more careful treatment should take into account the difference between \( \hat{A}_* \) and \( \hat{A} \). It is this difference that makes the above \( \Omega \) non-vanishing, despite \( \delta L = 0 \). Only the application of time-derivative \( \partial \) eliminates \( \hat{A}_* \), but without \( \partial \) there is no vanishing.
6 Non-local examples

Hamiltonian formalism is immediately applicable to arbitrary functionals, including non-local. For \( q \)-quadratic examples one can take

\[
S\{q(t)\} = \int q(t) + \frac{1}{M-2} q^2 dt
\]

or

\[
S\{q(t)\} = \int q(t)q(t+\epsilon)dt = \int q(t)e^{\epsilon\partial_t}q(t)dt
\]

(solutions of Euler-Lagrange equations in this case are antiperiodic functions \( q(t+2\epsilon) = -q(t) \)), or, in general,

\[
S\{q(t)\} = \frac{1}{2} \int \left( \sum_n a_n \left( \partial^n q \right)^2 \right) dt
\]

with time-independent \( a_n \) (not a necessary restriction, of course).

Then

\[
\Pi_i = \delta^i L = \sum_{j=0} (-)^j a_{i+j+1} \partial^{i+2j+1} q
\]

and

\[
\Omega = \sum_{i=0} dq_i \wedge d\Pi_i = \sum_{i,j \geq 0} (-)^j a_{i+j+1} dq_i \wedge dq_{i+2j+1}
\]

Nota that terms \( dq_i \wedge dq_{i+2j} \) do not appear in this expansion. Since coefficients \( A_{ij} \) in the matrix

\[
\Omega = \sum_{i<j} A_{ij} dq_i \wedge dq_j
\]

are time independent, they are forced to be of the form \( A_{ij} = (-)^j A_{i+j} \) (familiar from the theory of Toda chain \( \tau \)-functions, see [42]) by the conservation condition:

\[
\dot{A}_{ij} + A_{i-1,j} + A_{i,j-2} = 0
\]

7 Reparametrization-invariant actions

Transformations \( t \to u(t) = t + \epsilon(t) \):

\[
q_1^0 \equiv q^0 \to uq_1^0 \to q_1^0 + \epsilon q_1^0, \\
q_2^0 \equiv q^0 \to u^2 q_2^0 + u\epsilon q_1^0 \to q_2^0 + 2\epsilon q_2^0 + \dot{\epsilon} q_1^0, \\
q_3^0 \to u^3 q_3^0 + 3u^2 \epsilon q_2^0 + u^2 q_1^0 + u^2 \epsilon q_1^0 \to q_3^0 + 3\epsilon q_3^0 + 3\dot{\epsilon} q_2^0 + \ddot{\epsilon} q_1^0,
\]

or

\[
q_k^0 \equiv \partial_k^k q^0 \to q_k^0 + \sum_{l=0}^{k-1} C_k^{k-l-1} q_{k-l} \partial_l^l \epsilon
\]

Invariance of the action means that for any \( \epsilon(t) \)

\[
\epsilon L = \epsilon \left\{ q^0 \frac{\partial}{\partial q^0} + 2q^0 \frac{\partial}{\partial q^0} + 3q_3^0 \frac{\partial}{\partial q_3^0} + 4q_4^0 \frac{\partial}{\partial q_4^0} + \ldots \right\} L + \\
+ \dot{\epsilon} \left\{ q^0 \frac{\partial}{\partial q^0} + 3q^0 \frac{\partial}{\partial q_3^0} + 6q_3^0 \frac{\partial}{\partial q_3^0} + \ldots \right\} L + \\
+ \ddot{\epsilon} \left\{ q^0 \frac{\partial}{\partial q_3^0} + 4q_3^0 \frac{\partial}{\partial q_3^0} + \ldots \right\} L + \\
+ (\partial^3 \epsilon) \left\{ q^0 \frac{\partial}{\partial q_4^0} + \ldots \right\} L + \ldots
\]
or

\[ \sum_{m=0}^{\infty} K_m \partial_m^m \epsilon = 0 \quad \text{i.e.} \quad K_m = \sum_{k \geq 1} C_{m+k}^{k-1} q_k^a \frac{\partial L}{\partial q_{k+m}} - L \delta_{m,0} = 0 \]  

(28)

where binomial coefficients \( C_{m+k}^{k-1} = \frac{(m+k)!}{(k-1)!(m+1)!} \).

As a corollary, the ordinary Hamiltonian (10) vanishes identically:

\[ H = \sum_{i=0}^{N-1} \Pi_i a_i^a + L = \sum_{k \geq 1} q_k^a \left( \frac{\partial L}{\partial q_k^a} \frac{d}{dt} \frac{\partial L}{\partial q_k^a} + \frac{d^2}{dt^2} \frac{\partial^2 L}{\partial q_k^a} \partial q_{k+2}\cdots \right) - L = \]

\[ = K_0 - \sum_{k \geq 1} \left( (k-1)q_k^a \frac{\partial L}{\partial q_k^a} + q_k^a \frac{\partial^2 L}{\partial q_{k+2}^a} q_{k+1}^a + q_k^a \frac{\partial^3 L}{\partial q_{k+2}^a \partial q_{k+1}^a} q_{k+2}\cdots \right) = \]

\[ = K_0 - \frac{dK_1}{dt} + \frac{d^2K_2}{dt^2} - \frac{d^3K_3}{dt^3} + \cdots = 0 \]  

(29)

As in every gauge invariant theory the Hamilton equations involve the constraint \( \Phi \) – generator of gauge transformation

\[ \dot{q}^a = \frac{\partial \Phi}{\partial \Pi_a}, \]

\[ \dot{\Pi}_a = -\frac{\partial \Phi}{\partial q^a} \]  

(30)

– instead of the naive Hamiltonian [25].

A special case with no dependence on \( q_0^a \) and \( N = 1 \) was studied in [38].

8 Example of \( n = 2 \) and \( N = 2 \)

According to (27) in this case \( L \) is a function of \( q_1, q_2, v_1 = \dot{q}_1, v_2 = \dot{q}_2 \) and \( z = \dot{v}_1 \dot{a}_2 - \dot{v}_2 \dot{a}_1 = v_1 \dot{v}_2 - v_2 \dot{v}_1 \) of definite homogeneity degree:

\[ 3z \frac{\partial L}{\partial z} \bigg|_{v_1, v_2} + v_1 \frac{\partial L}{\partial v_1} \bigg|_{v_2, z} + v_2 \frac{\partial L}{\partial v_2} \bigg|_{v_1, z} = L \]  

(31)

The momenta are equal to:

\[ \Pi_0^1 = \frac{\partial L}{\partial v_1} \bigg|_{v_2, a_1, a_2} + v_2 \frac{d}{dt} \frac{\partial L}{\partial z} \bigg|_{v_1, z} + 2a_2 \frac{\partial L}{\partial z} \bigg|_{v_1, z} = \frac{\partial L}{\partial v_1} \bigg|_{v_2, z} + a_2 \frac{\partial L}{\partial z} \bigg|_{v_1, z} + \frac{d}{dt} \left( v_2 \frac{\partial L}{\partial z} \bigg|_{v_1, z} \right) = \]

\[ = \frac{\partial L}{\partial v_1} \bigg|_{v_2, z} + 2a_2 \frac{\partial L}{\partial z} \bigg|_{v_1, z} + v_2 \frac{d}{dt} \frac{\partial L}{\partial z} \bigg|_{v_1, z} = \frac{\partial L}{\partial v_1} \bigg|_{v_2, z} + 2a_2 \frac{\partial L}{\partial z} \bigg|_{v_1, z} + v_2 \left( \frac{\partial^2 L}{\partial z \partial q_1} + a_1 \frac{\partial^2 L}{\partial z \partial v_1} + (v_1 w_2 - v_2 w_1) \frac{\partial^2 L}{\partial z^2} \right) \],

\[ \Pi_0^2 = \frac{\partial L}{\partial v_2} \bigg|_{v_1, a_1, a_2} - \frac{d}{dt} \frac{\partial L}{\partial a_2} \bigg|_{v_1, a_1} \]

\[ \Pi_1^0 = \frac{\partial L}{\partial a_1} \bigg|_{v_1, a_1, a_2}, \quad \Pi_1^1 = \frac{\partial L}{\partial a_2} \bigg|_{v_1, a_1, a_2} \]  

(32)
Reparametrization invariance is always, not only in this example, represented as homogeneity condition for a function, which depends on peculiar combinations

\[ z_{ij} = \dot{q}_i \dot{q}_j - \ddot{q}_i \ddot{q}_j \sim \partial_t \left( \frac{\dot{q}_i}{\dot{q}_j} \right), \quad z_{ijkl} \sim \partial_t \left( \frac{\dot{z}_{ij}}{\dot{z}_{kl}} \right), \quad z_{ijkl';k'l'} \sim \partial_t \left( \frac{\dot{z}_{ij;kl'}}{\dot{z}_{i';j';k'l'}} \right), \ldots \]  

which are "elementary monomials", transforming homogeneously under the time-reparametrizations. Of course, they play essential role in the theory, see, for example, [40].

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