FEM Simulations of Bond Behaviour between Concrete and Seven-Wire Prestressing Strand

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Abstract. The article deals with the subject related to numerical tests of the concrete bond to seven-wire strands. Nonlinear contact concrete-steel calculations with using Finite Element Method (FEM) is applicable for concrete bond to reinforcing steel analyses. At the moment, the analyzes are the subject of independent research projects. The research using several scientific and commercial computing systems allows us to look deeper into the sample and obtain information about cracks development, stresses, and internal deformations. Such analyzes are impossible when tests only experimentally. The concrete-steel behaviour of materials can be modeled with a bond-slip mechanism where the relative slip of reinforcement and concrete is phenomenologically described. The behaviour of the slip zone is then mapped with a zero thickness interface (contact) element. The constitutive laws of bond-slip adhesion are based mainly on the theory of total deformation, which expresses the pulling force as a function of total relative displacement. The correct construction of the bond model requires consideration of physical phenomena occurring not only at the interface of the materials but also inside within each of them. When designing numerical research, it is necessary to keep in mind the limits in computing memory of computers. Performing static calculations for complex numerical models containing a number of geometric, material and contact nonlinearities may prove to be too time-consuming on a PC. For this reason, it is now reasonable to construct approximate, simplified models. FEM models with a strand modeled as a solid bar (with a sidewall accurate to the real one) made it possible to observe and research, destructive mechanisms identified in experimental pull-out tests.

1. Introduction
Nowadays, computational analyzes of the concrete bond to reinforcing bars are the subject of independent research projects. Conducted laboratory tests do not provide complete descriptions of phenomenons occurring in a concrete sample when pulling out a reinforcing bar. The application of the Finite Element Method (FEM) for nonlinear contact concrete-steel analysis comes with help. The research using several scientific and commercial computing systems allows us to look deeper into the sample and obtain information about cracks development, stresses, and internal deformations.

2. Capabilities of FEM concrete-steel bond modeling
Interaction between reinforcement and concrete in reinforced and prestressed concrete structures is defined mainly by longitudinal and secondary by transverse cracking of concrete in the vicinity of the reinforcement. This behaviour of materials can be modeled with a bond-slip mechanism where the
relative slip of reinforcement and concrete is phenomenologically described. The behaviour of the slip zone is then mapped with a zero thickness interface (contact) element [4, 15].

The constitutive laws of bond-slip adhesion are based mainly on the theory of total deformation, which expresses the pulling force as a function of total relative displacement. In advanced nonlinear FEA analysis programs (ie. Midas FEA, DIANA), the relationship between the normal traction ($t_n$) and normal relative displacement ($\Delta u_n$) is assumed to be linear elastic, whereas the relationship between shear traction ($t_s$) and the slip ($dt$) is assumed to be nonlinear function (eq. 1):

$$\begin{align*}
    t_n &= k_n \cdot \Delta u_n \\
    t_s &= f_s(dt)
\end{align*}$$

(1)

A number of models of bond-slip mechanism are available. Most of them are or can be easily implemented in the MES software. Some of them - selected by the author as the most popular - are presented in the article.

2.1. Cubic Function [7]

Dörr proposed a polynomial relation between shear traction and slip. His function has a limit if a slip is greater than a certain value $d_t^0$ (fig. 1).

$$t_s = f_s \left( \frac{5 \cdot f_t}{d_t^0} - 4.5 \cdot \left( \frac{dt}{d_t^0} \right)^2 + 1.4 \cdot \left( \frac{dt}{d_t^0} \right)^3 \right) \quad \text{if} \left( 0 \leq dt \leq d_t^0 \right)$$

(2)

2.2. Power Law [16]

The relation between shear traction and slip is shown in figure 2.
Figure 2. Power Law for bond-slip

The initial shear stiffness is kept linear to avoid an unrealistically high stiffness when the slip is smaller than an initial value \( d_t^0 \). \( a \) and \( b \) are coefficients, and \( b < 1 \). The formulation is given by a function (eq. 3):

\[
t_t = \begin{cases} 
    a \cdot (dt)^b & \text{if } (dt \geq dt^0) \\
    a \cdot (dt)^{b-1} \cdot dt & \text{if } (0 \leq dt \leq dt^0) 
\end{cases}
\]  

(3)

2.3. Shima bond-slip relation [18]

The relation between shear traction and slip proposed by Shima is shown in figure 3.

Figure 3. Shima relation for bond-slip

The stiffness of Shima relation is infinite (the relation is increasing as long as slip is increasing). The function is highly dependent on concrete compressive strength \( f'_c \). \( a \) is a scaling factor. \( D \) is derived from the dimension of the interface element assuming a circular bar if not defined explicitly. The formulation is given by a function (eq. 4):

\[
t_t = a \cdot 0.9 \cdot f'_c^{2/3} \cdot \left[ 1 - e^{-40 \frac{d_t}{D}} \right]^{\frac{b}{0.6}}
\]

(4)

2.4. Multi-linear [4, 15]

The relationship between the shear traction \( t_t \) and the shear slip \( d_t \) is made with several sections. Every section is a line and is defined with two points (beginning and end). Each point can be defined with two values \((d_{ti}, t_{ti})\). The first point is the origin \((0, 0)\). The first point of the current section is also a second for the previous section. The diagram may also contain descending parts (softening). The Multi-linear curve is shown in figure 4.
2.5. fib Model Code bond-slip relation [9]

According to Chapter 6 of the fib Model Code for Concrete Structures 2010 [9], the bond-slip model back-bone curve consists of four sections. The curve is shown in figure 5.

In the first section (range $0 \div s_1$) the bond-slip stress $\tau$ develops from 0 to the maximum bond-slip stress $\tau_{\text{max}}$ at a relative slip displacement $s_1$. In the second section (range $s_1 \div s_2$) the bond-slip stress $\tau$ remains constant at $\tau_{\text{max}}$ until a relative slip displacement $s_2$. In the third section (range $s_2 \div s_3$) the bond-slip stress $\tau$ reduces linearly to the ultimate bond-slip stress $\tau_f$ at the relative slip displacement $s_3$. In the fourth section (range $s_3 \div \infty$) from relative slip displacement $s_3$ the bond-slip stress $\tau$ is kept constant at $\tau_f$.

2.6. General guidelines and comments

General guidelines about the bond-slip models are as follows:

- Shear relations are done in an absolute sense. Positive and negative values of slip are equal (refers to the positive part of the diagram).
- The incremental formulations make it possible to change the material parameters of the bond-slip curve in a phased analysis.

The choice of a bond-slip model cannot be optional. The bond model adopted for calculations should correspond to the results of experimental tests. The author's recommended model is the Multi-linear model, which allows to define the slip-force relation in any way (any curve course can be implemented).

3. Interface elements

The main motivation and the purpose of interface elements is to allow the displacement of nodes at the interface of two materials in the numerical model. The interface elements are created to model discrete
cracks in materials or relative movements at the boundaries of model parts. They are successfully used in the problems of modelling the bond of reinforcement in relation to slip. There are point, line and surface interface elements. They are defined by using the general finite element formulation, whereas the thickness of the elements is assumed to be zero. In order to define an interface element with zero-thickness from a numerical analysis point of view, a penalty stiffness is assigned for the interface element. If the stiffness is too high it might cause numerical problems and when is too low, the interface may yield with undesired relative displacements from both sides of the interface element. The recommended value is given by equation 5 [15].

\[ k = 1000 \cdot E \cdot d \]  

(5)

where:

- \( k \) - penalty stiffness [MPa],
- \( E \) - the highest Young’s modulus of used in the finite element model [MPa],
- \( d \) - representative element size [m].

The relationship between the relative displacement \( \Delta u \) and the traction \( t \) in the interface element is defined by equation 6. In a three dimensional case, it is described by the matrix equation 7. Explanation of symbols in the equations is shown in figure 6. \( t_n \) denotes the normal traction, \( t_s \) and \( t_t \) and denote the tangential tractions. \( \Delta u_n \) denotes the normal relative displacement \( \Delta u_s \) and \( \Delta u_t \) denote the tangential relative displacements [15].

\[ t = D \cdot \Delta u \]  

(6)

\[ t = \begin{bmatrix} t_n \\ t_s \\ t_t \end{bmatrix}, \quad D = \begin{bmatrix} k_n & 0 & 0 \\ 0 & k_s & 0 \\ 0 & 0 & k_t \end{bmatrix}, \quad \Delta u = \begin{bmatrix} \Delta u_n \\ \Delta u_s \\ \Delta u_t \end{bmatrix} \]

(7)

Figure 6. Tractions and relative displacements in 3D interface element [15]

The available types of surface interface finite elements are shown in figure 7. These elements can be used for the analysis of contact problems between solid, surface or solid and surface objects.
Figure 7. Surface interface elements: a) triangular - 3 nodes, b) quadrilateral - 4 nodes, c) triangular - 6 nodes, d) quadrilateral - 8 nodes [4]

4. A short review of literature
In the scope of modeling of regular reinforcement, the results of the analyzes are available for both plain and ribbed bars. Research attempts were made practically from the beginning of software development. In 2000, Gan carried out a review of the available FEM models of the concrete-steel bond to plain bars [10]. The results of analyzes for plain bars were presented by, among others: Lundgren and Gylltoft [14] in 2000 (with using DIANA software), Appa Rao and Kadhiran [2] in 2013 (with using FEMAP software). Results of analyzes of the bond of concrete to a ribbed bar in a full 3D model were published by, among others: Lettow [12] in 2004 (with using MASA software), Shafaie et al. [17] in 2009 (with using ANSYS system), rse ribbing in the ANSYS system. There are many similar studies available on this subject.

In terms of the modeling of concrete adhesion to prestressed reinforcements, the available literature is much poorer. Published scientific works in this area indicate the complexity of the problem [1, 3, 5, 11, 13, 19].

5. Purpose of the analyzes
The purpose of the FEM numerical analyzes was to create a computer model of a concrete bond to a seven-wire prestressing strand 7Ø5 mm. The 3D model will be used to obtain information on the concrete stress and strain distribution inside the concrete cube in the immediate vicinity of the strand at the moment of loss of adhesion and the image of the internal cracking of concrete during the tests. The assumption of the numerical FEM simulations is to supplement the results of the experimental tests with the description of internal destructive mechanisms [5, 6].

6. Numerical model implementation
The numerical model should represent the real sample both in terms of geometric as well as physical and mechanical. The preparation of the complex geometry of a strand consisting of 7 wires (1 central and 6 external ones spirally coiled around it) does not exceed the possibilities of the software. Embedding such a strand in a concrete cube is already very simple. Complications arise when the FEM mesh is generated. Correct solution requiring a very fine mesh. A proper mesh without any
incompatibilities must have dimensions below 1 mm. Such dense discretization leads to a time consuming FEM meshing process. Further difficulties arose in defining interface elements. The interface elements should appear not only at the interface between the material - concrete and the outer steel wires of the strand but also between the individual strand wires - allowing them to slip against each other. It should be emphasized that the material model of concrete, steel and contact elements should be based on the results of experimental tests - only then the FEM analysis will be reliable. After implementing the interface elements, the computational task becomes significantly complicated due to geometric, material and contact nonlinearities. This approach is practically beyond the ability to perform FEM analysis on a PC. The mesh generation itself may take several weeks.

**Figure 8. Model of a 705 mm strand: a) detailed, b) simplified solid**

Reliable performance of a task involves taking into account a greater number of subsequent factors, such as [1, 5, 8, 11]:

- creep and shrinkage of concrete,
- Hoyer effect (wedging),
- additional longitudinal forces acting on the concrete from the prestressing force,
- additional radial forces on concrete due to the prestressing force,
- changes in the value of the prestressing force in the strand along the transmission length,
- consideration of the impact of concrete cracking.

Simple models with a strand modeled as a solid bar with smooth sidewall (fig. 8a), make it possible to determine the averaged stress of adhesion, as well as to calculate the computational transmission length [1, 3, 13, 19]. Such analyzes from the engineering point of view are sufficient. The modeling of the accurate sidewall shape of the strand (fig. 8b), allows for more detailed analyzes, including identification of crack initiation in the concrete cover of the strand, determination of stress states in the sample, as well as determination of destructive mechanisms during pull-out tests [5].

**Figure 9. Detailed geometrical model**
Figure 10. Cracks initiation in the vicinity of strand - detailed analysis [5]

7. Conclusions

The implementation of an accurate model of the concrete-steel bond to a prestressed strand involves taking into account:

- the actual geometry of the strand consisting of 7 wires,
- mutual slippage of the wires in the strand during its pull-out attempt in relation to the strand pulling force,
- the relation between the slip of external wires and the concrete to the strand pulling force,
- creep and shrinkage of concrete,
- the Hoyer effect (wedging),
- additional longitudinal forces acting on the concrete from the prestressing force,
- additional radial forces on concrete due to the prestressing force,
- changes in the value of the prestressing force in the strand along the transmission length.

In order to perform reliable bond analyzes of concrete to seven-wire strand in the numerical model, the accurate geometry as well as the mechanical and physical phenomena occurring during the tests should be implemented. It should be indicated here that all mechanical phenomena occur at the moment when the prestress is transferred to the concrete. Consequently, the concrete itself does not experience any changes, apart from those imposed by the action of the strand. The correct construction of the bond model requires consideration of physical phenomena occurring not only at the interface of the materials but also inside within each of them. If we fulfill all of the above-mentioned assumptions, we encounter other ones related to the computing capabilities of computers. Performing static calculations for a complex numerical model containing a number of geometric, material and contact nonlinearities may prove to be too time-consuming on a PC. For this reason, it seems reasonable now to construct simplified, approximate models. FEM models with a strand modeled as a solid bar (with a sidewall accurate to the real one) made it possible to observe and research, destructive mechanisms identified in experimental pull-out tests.
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