Determination of parameters of a potential model for tetraquark study by studying all S-wave mesons

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Abstract The masses of low-lying S-wave mesons are evaluated in a constituent quark model (CQM) where the Cornell-like potential and one-gluon exchange spin-spin interaction are employed. To make the model applicable to both the light and heavy quark sectors, we introduce mass-dependent coupling coefficients. There are four free parameters in the model, which are determined by comparing the theoretical results with experimental data. The established model with one set of parameters may be applied to study higher excited meson states as well as multiquark systems in both the light and heavy quark sectors.

Keywords Cornell potential · meson · mass spectrum · quark model

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1 Introduction

Charged charmonium-like particles like \( Z_c^+(3900) \), \( Z_c^+(4010) \), \( Z_c^+(4050) \), \( Z_c^+(4055) \), \( Z_c^+(4100) \), \( Z_c^+(4200) \), \( Z_c^+(4250) \), and \( Z_c^+(4430) \) have been successively observed by experimental collaborations [1]. These charged charmonium-like states are beyond the conventional \( c\bar{c}\)-meson picture. Because of carrying one charge, these states are likely tetraquark systems with a quark content \( c\bar{c}ud \). Recently, a new resonance named \( Z_{c^+}(3985) \) has been observed by BESIII Collaboration [2], which is the first candidate of the charged charmonium-like tetraquark with strangeness, with a quark content \( c\bar{c}u\bar{s} \).

The significance of the resonance hypothesis is estimated to be 5.3\( \sigma \). Later, the LHCb Collaboration has reported four exotic states \( Z_{c^+}(4000)^+ \), \( Z_{c^+}(4220)^+ \), \( X(4685) \), and \( X(4630) \) with a quark content \( c\bar{c}u\bar{s} \) decaying to the \( J/\psi K^+ \) final state with high significance [3]. The first all-heavy multiquark exotic candidates \( X(6900) \) with a quark content \( c\bar{c}\bar{c}\bar{e} \) has been recently observed by LHCb Collaboration in the \( J/\psi \)-pair mass spectrum [4].

In addition to the large number of XYZ tetraquark candidates, the observations of four pentaquark-like states has been reported by LHCb Collaboration. \( P_c(4380)^+ \) and \( P_c(4450)^+ \) have been observed for the first time in the process \( \Lambda_b^0 \to J/\psi p K^- \), with more than 9\( \sigma \) significance [5][6]. In recent years, another narrow pentaquark state \( P_c(4312)^+ \) is observed in the process \( \Lambda_b^0 \to J/\psi p K^- \) with a statistical significance is 7.3\( \sigma \), and the \( P_c(4450)^+ \) pentaquark structure previously reported by LHCb has been confirmed and resolved at 5.4\( \sigma \) significance into two narrow states: the \( P_c(4440)^+ \) and the \( P_c(4457)^+ \) [7]. All four pentaquark-like states may have the quark content of \( uud\bar{c} \).

Various phenomenological research methods of hadron physics can be tested due to these observed multiquark states. A systematic spectrum study of the multiquark states would provide information for future experimental search for the missing higher excitations.

The mass spectrum of charmonium-like tetraquark has been studied in non-relativistic potential model [8][9], chromomagnetic interaction model [10][11], color flux-tube model [12], relativized diquark model [13], and QCD sum rules

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The paper is organized as follows. The constituent quark model applied in our previous work [35] is briefly reviewed in Sec. 2. In Sec. 3, meson mass spectra are evaluated in the constituent quark model, and all model parameters are determined by comparing the theoretical and experimental masses of light, strange, charmed, and bottom mesons. We study the multiquark systems in the nonrelativistic Hamiltonian, 

$$H = H_0 + H_{hyp}^{OGE},$$

$$H_0 = \sum_{k=1}^{N} \left( \frac{1}{2} M_k^{ave} + \frac{p_k^2}{2m_k} \right) + \sum_{i<j}^{N} \left( -\frac{3}{16} \lambda_i^C \cdot \lambda_j^C \right) (A_{ij} r_{ij} - B_{ij} \frac{r_{ij}}{r_{ij}}),$$

$$H_{hyp}^{OGE} = -C_{OGE} \sum_{i<j} \vec{\lambda}_i^C \cdot \vec{\lambda}_j^C \vec{\sigma}_i \cdot \vec{\sigma}_j,$$  

(1)

by solving the Schrödinger equation

$$H |\psi\rangle = E |\psi\rangle,$$  

(2)

where the meson wave function $|\psi\rangle$ is expanded in the complete bases defined in Eq. (2). Here $m_k$ are the constituent quark masses taken from the previous work [33].

$$m_u = m_d = 327 \text{ MeV}, \quad m_s = 498 \text{ MeV}, \quad m_c = 1642 \text{ MeV}, \quad m_b = 4960 \text{ MeV}. \quad (3)$$

$M_k^{ave}$ denotes the spin-averaged mass as $\frac{1}{2} M_{PS} + \frac{1}{2} M_V$ (except for $B_c, s\bar{s}, K^*$, and $q\bar{q}$), with $M_{PS}$ and $M_V$ being the mass of ground state pseudoscalar and vector mesons from experimental data [37]. The spin-averaged masses $M_k^{ave}$ for each kind of mesons are listed in Table 1. $A_{ij}$ and $B_{ij}$ are mass-dependent coupling parameters, taking the form

$$A_{ij} = A_0 \sqrt{m_{ij}}, \quad B_{ij} = B_0 \sqrt{\frac{1}{m_{ij}}},$$  

(4)

with $m_{ij}$ being the reduced mass of $i$th and $j$th quarks, defined as $m_{ij} = \frac{m_i m_j}{m_i + m_j}$. The hyperfine interaction, $H_{hyp}^{OGE}$ includes only one-gluon exchange contribution [38] [39]. We follow the CQM convention and adopt $C_{OGE} = C_0 \frac{2m_i m_j}{m_i + m_j}$ [10] with

$$C_{ij} = C_0 \left( \frac{m_i}{m_c} \right)^t \left( \frac{m_j}{m_c} \right)^{t},$$  

(5)

where $C_0$ and $t$ are constants. $m_c$ is the constituent quark mass of $c$ quark listed in Eq. (3). $\vec{\lambda}_i$ and $\vec{\sigma}_i$ in Eq. (1) are the quark color operator and the spin operator respectively. In a meson, the contribution of the color part $\vec{\lambda}_i^C \cdot \vec{\lambda}_j^C$ in Eq. (1) is $-16/3$, and the contribution of $\vec{\sigma}_i \cdot \vec{\sigma}_j$ in Eq. (1) is $-3$ for $S = 0$ and $+1$ for $S = 1$ mesons. According to the hyperfine interaction form, we have

$$\Delta M_{V-PS} = M_V - M_{PS} = C_0 \left( \frac{m_i}{m_c} \right)^t \left( \frac{m_j}{m_c} \right)^{t} \frac{m_i^2}{m_i m_j} (16 + \frac{16}{3}).$$  

(6)

Table 1: Spin-averaged masses $M_k^{ave}$ for various kinds of mesons. $M_{PS}$ and $M_V$ taken from PDG [37].

| Meson | $c\bar{c}$ | $b\bar{b}$ | $B_c$ | $B_s$ | $B$ | $D_s$ | $D$ | $s\bar{s}$ | $K^*$ | $q\bar{q}$ |
|-------|----------|----------|-------|-------|-----|-------|-----|----------|-------|----------|
| [MeV] |          |          |       |       |     |       |     |          |       |          |
| 3068  | 9444     | 6323     | 5404  | 5314  | 2076| 1972  | 955 | 819      | 683   |          |

[14] Fully-heavy tetraquark mass spectrum has also been studied in non-relativistic quark model [8] [15] [16] [17] [18] [19] [20], non-relativistic effective field theory [21], chromomagnetic interaction model [22], and QCD sum rules [23, 24]. The pentaquark structure has been studied in various suggestions in the last decades. For the new observed $P_c$ states, there are three possible explanations for the structure: compact multiquarks [25] [26] [27] [28] [29], hadronic molecules [30] [31] [32] [33], and their admixtures [34].

We are not to try to repeat the masses of the meson states by applying a large number of parameters but to develop a simple model with as less parameters as possible. We predetermined all model parameters by studying conventional $q\bar{q}$ mesons. The paper is organized as follows. The constituent quark model applied in our previous work [35] is briefly reviewed in Sec. 2. In Sec. 3, meson mass spectra are evaluated in the constituent quark model, and all model parameters are determined by comparing the theoretical and experimental masses of light, strange, charmed, and bottom mesons.

2 Theoretical model

We study the multiquark systems in the nonrelativistic Hamiltonian, 

$$H = H_0 + H_{hyp}^{OGE},$$

$$H_0 = \sum_{k=1}^{N} \left( \frac{1}{2} M_k^{ave} + \frac{p_k^2}{2m_k} \right) + \sum_{i<j}^{N} \left( -\frac{3}{16} \lambda_i^C \cdot \lambda_j^C \right) (A_{ij} r_{ij} - B_{ij} \frac{r_{ij}}{r_{ij}}),$$

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(6)
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Table 2: $\Delta M_{V-PS}$ for various kinds of mesons, $\Delta M_{V-PS}$ taken from PDG [37]. Units are MeV.

| meson | $b\bar{b}$ | $c\bar{c}$ | $B_s$ | $B$ | $D_s$ | $D$ |
|-------|-------------|-------------|-------|-----|-------|-----|
| $\Delta M_{V-PS}^{exp}$ | 61 | 113 | 48 | 46 | 144 | 137 |
| $\Delta M_{V-PS}^{cal}$ | 52 | 113 | 57 | 52 | 138 | 135 |

with $m_i$ and $m_j$ being the reduced masses of $i$th and $j$th quark respectively. The constants $C_0$ and $t$ in Eq. (6) are fixed by applying the least squares method to minimize the weighted squared distance $\delta^2$,

$$\delta^2 = \sum_{i=1}^{N} \omega_i \frac{(\Delta M_{V-PS}^{exp} - \Delta M_{V-PS}^{cal})^2}{\Delta M_{V-PS}^{exp}^2}$$

(7)

where $\omega_i$ are weights being 1 for all the states. The $\Delta M_{V-PS}^{exp}$ is the experimental data with the form $\Delta M_{V-PS}^{exp} = M_V - M_{PS}$, and the $\Delta M_{V-PS}^{cal}$ is the theoretical results with the form $\Delta M_{V-PS}^{cal} = C_0 \left( \frac{m_i}{m_c} \right)^\frac{3}{2} \frac{m_{m_j}^2}{m_{m_j}} (16 + \frac{i}{2})$. The fitting results of $\Delta M_{V-PS}$ are listed in Table 2.

The spin-averaged mass of $B_s$ meson $M_{B_s}^{ave}$ takes the form $M_{PS} + 16C_j \frac{m_c^2}{m_{m_j}}$ because of the lack of $M_V$ experimental data. The spin-averaged masses of $s\bar{s}$, $K^*$ and $q\bar{q}$ take the form $M_V - (16/3)C_j \frac{m_c^2}{m_{m_j}}$ to avoid the would-be Goldstone bosons of the chiral symmetry breaking.

The four model coupling parameters in Eq. (4) and Eq. (5) are fixed by comparing theoretical results with experimental data [37].

$$A_0 = 3219.51 \text{ MeV}^{3/2}, \quad B_0 = 31.728 \text{ MeV}^{1/2}, \quad C_0 = 133.558 \text{ MeV}, \quad t = 1.3$$

(8)

In this work, we concentrate on the S-wave meson states and do not consider the tensor and spin-orbital interactions. The fitting results of meson states are shown in Sec. 3.

3 Results and Summary

We construct the complete bases by using the harmonic oscillator wave function. In our calculations, the bases size is $N = 38$, and the length parameters of harmonic oscillator wave functions are adjusted to get the best eigenvalue of Eq. (2).

We calculate the mass spectra of light, strange, charmed, and bottom mesons which are believed mainly $q\bar{q}$ states in the Hamiltonian in Eq. (1). The theoretical results are collected, as shown in Table 3 with the experimental data taken from PDG [37] and some typical results of other works for comparison.

In Ref. [8], the S-wave tetraquark states with all quark configurations are systematically studied in a non-relativistic quark model. The parameters are fitted by reproducing all S-wave and P-wave ground state mesons. The charmonium-like tetraquark states $cu\bar{c}d$ are systematically studied in a color flux-tube model with a multi-body confinement potential [12]. The parameters are determined by fitting the masses of the ground states of mesons. The mass spectrum of the ground state mesons is obtained by solving the two-body Schrödinger equation. The fully-heavy tetraquark states $QQQQ$ are studied in several kinds of typical non-relativistic quark models in [17][18][19][20], where the Cornell-like potential are considered and model parameters are fixed by comparing the theoretical results of meson states with their experimental data.

In summary, we are able to evaluate all ground and first radial excited meson states in a general model with reliable accuracy. Other works are applicable for either the heavy mesons or for the ground state mesons. By introducing three mass-dependent model parameters $A_{ij}$, $B_{ij}$, and $C_{ij}$ in Eq. (4) and Eq. (5), the number of parameters of our calculations is less than others, but the fitting results are compatible with both experimental data and other works. In future work, the Hamiltonian in Eq. (1) with the predetermined model parameters will be applied to predict the masses of multiquark states, tetraquarks and pentaquarks.

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Table 3: Present the comparison of the theoretical results of low-lying meson states $M^{cal}$ with the experimental data $M^{exp}$ taken from PDG [37] and others. The D(%) column shows the deviation between the experimental and our theoretical mean values, $D = 100 \cdot (M^{exp} - M^{cal})/M^{exp}$. Units are MeV.

| Meson | $M^{exp}$ | $M^{cal}$ | D (%) | 8 | 12 | 17 | Model 1 | 17 | Model 2 | 18 | 19 | 20 |
|-------|-----------|-----------|-------|---|---|---|--------|---|--------|---|---|---|
| $\Upsilon' (1S)$ | 9460 | 9457 | 0.03 | 9433 | 9546 | 9503 | 9470 | 9460 | -- | -- | -- | -- |
| $\Upsilon' (2S)$ | 10023 | 10046 | -0.23 | -- | -- | 9949 | 10017 | 10017 | -- | -- | -- | 9981 |
| $\eta_c$ | 9399 | 9405 | -0.06 | 9415 | 9441 | 9498 | 9428 | 9390 | -- | -- | -- | -- |
| $\eta_c (2S)$ | 9999 | 9994 | 0.05 | -- | -- | -- | 9990 | -- | -- | -- | -- |
| $J/\psi (2S)$ | 3097 | 3096 | 0.03 | 3097 | 3102 | 3085 | 3102 | -- | -- | -- | -- | -- |
| $\psi(2S)$ | 3686 | 3686 | 0 | -- | -- | 3652 | 3658 | -- | -- | -- | -- | 3671 |
| $\eta_c$ | 2984 | 2983 | 0.03 | 3038 | 2912 | 3056 | 3006 | -- | -- | -- | -- | 2992 |
| $\eta_c (2S)$ | 3638 | 3573 | 1.79 | -- | -- | 3638 | 3621 | -- | -- | -- | -- | 3632 |
| $B_s$ | 6275 | 6275 | 0 | 6303 | 6261 | 6319 | 6293 | 6274 | -- | -- |
| $B_s (2S)$ | 6842 | 6865 | -0.34 | -- | -- | -- | -- | -- |
| $B_s^0$ | 5415 | 5418 | -0.06 | 5413 | 5430 | -- | -- | -- |
| $B_s^{*0}$ | 5367 | 5361 | 0.11 | 5372 | 5377 | -- | -- | -- |
| $B^*$ | 5325 | 5327 | -0.04 | 5350 | 5301 | -- | -- | -- |
| $B^{*0}$ | 5279 | 5275 | 0.08 | 5301 | 5259 | -- | -- | -- |
| $D_s^0 (2700)$ | 2112 | 2110 | 0.09 | 2101 | 2140 | -- | -- | -- |
| $D_s^*$ | 2708 | 2700 | 0.3 | -- | -- | -- | -- | -- |
| $D_s$ | 1968 | 1972 | -0.19 | 1996 | 1972 | -- | -- | -- |
| $D^*(2007)^0$ | 2107 | 2006 | 0.05 | 2020 | 2002 | -- | -- | -- |
| $D^0$ | 1870 | 1871 | -0.05 | 1886 | 1867 | -- | -- | -- |
| $\phi(1020)$ | 1020 | 1019 | 0.1 | 1017 | 1112 | -- | -- | -- |
| $\phi(1680)$ | 1680 | 1609 | 4.2 | -- | -- | -- | -- | -- |
| $K^*(892)$ | 892 | 892 | 0 | 909 | 974 | -- | -- | -- |
| $K^*(1410)$ | 1414 | 1482 | -4.8 | -- | -- | -- | -- | -- |
| $\rho(770)$ | 770 | 770 | 0 | 777 | 826 | -- | -- | -- |
| $\rho(1450)$ | 1450 | 1359 | 6.3 | -- | -- | -- | -- | -- |

References

1. Brambilla, Nora and Eidelman, Simon and Hanhart, Christoph and Nefediev, Alexey and Shen, Cheng-Ping and Thomas, Christopher E. and Vairo, Antonio and Yuan, Chang-Zheng, The XYZ states: experimental and theoretical status and perspectives, Phys. Rept. 873, 1–154 (2020).
2. Ablikim, Medina, et al. (BESIII), Observation of a Near-Threshold Structure in the $K^+\pi^-\pi^+$ Decay, Phys. Rev. Lett. 126, 102001 (2021).
3. Aaij, Roel et al. (LHCb Collaboration), Observation of new resonances decaying to $J/\psi K^+$ and $J/\psi\phi$, arXiv: 2103.01803[hep-ex].
4. Aaij, Roel et al. (LHCb Collaboration), Observation of $J/\psi\phi$ Resonances Consistent with Pentaquark States in $\Lambda_c^0 \to J/\psi K^- p$ Decays, Phys. Rev. Lett. 115, 072001 (2015).
5. Aaij, Roel et al. (LHCb Collaboration), Model-independent evidence for $J/\psi\phi$ contributions to $\Lambda_c^0 \to J/\psi K^- p$ decays, Phys. Rev. Lett.117, 082002 (2016).
6. Aaij, Roel et al. (LHCb Collaboration), Observation of a narrow pentaquark state, $P_c (4312)^+$, and of two-peak structure of the $P_c (4450)^+$, Phys. Rev. Lett. 122, 222001 (2019).
7. Silvestre-Brac, B. and Semay, C., Systematics of $L=0$ q-2 anti-q-2 systems, Z. Phys. C, 57, 273–282 (1993).
8. Patel, Smruti and Shah, Manan and Vinodkumar, P, C, Mass spectra of four-quark states in the hidden charm sector, Eur. Phys. J. A, 50, 131 (2014).
9. Zhao, Lu and Deng, Wei-Zhen and Zhu, Shi-Lin, Hidden-Charm Tetraquarks and Charged Zs, States, Phys. Rev. D, 90, 094031 (2014).
10. Wu, Jing and Liu, Xiang and Liu, Yan-Rui and Zhu, Shi-Lin, Systematic studies of charmonium-, bottomonium-, and $B_{c}$-like tetraquark states, Phys. Rev. D, 99, 014037 (2019).
11. Deng, Chengrong and Ping, Jialun and Huang, Hongxia and Wang, Fan, Systematic study of $Z_{c}^+$ family from a multiquark color flux-tube model, Phys. Rev. D, 92, 034027 (2015).
12. Anwar, Muhammad Naeem and Ferretti, Jacopo and Santopinto, Elena, Spectroscopy of the hidden-charm $[qq][\bar{q}\bar{q}]$ and $[sc][\bar{s}\bar{c}]$ tetraquarks in the relativized diquark model, Phys. Rev. D, 98, 094015 (2018).
13. Wang, Zhi-Gang, Scalar or vector tetraquark state candidate: $Z_c(4100)$, Commun. Theor. Phys. 71, 1319-1327 (2019).
14. Brink, D. M. and Stancu, F., Tetraquarks with heavy flavors, Phys. Rev. D, 57, 6778-6787 (1998).
15. Berezhnoy, A. V. and Luchinsky, A. V. and Novoselov, A. A., Tetraquarks Composed of 4 Heavy Quarks, Phys. Rev. D, 86, 034004 (2012).
16. Wang, Guang-Juan and Meng, Lu and Zhu, Shi-Lin, Spectrum of the fully-heavy tetraquark state $QQQQ$, Phys. Rev. D, 100, 096013 (2019).
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18. Liu, Ming-Sheng and Li, Qi-Fang and Zhong, Xian-Hui and Zhao, Qiang, All-heavy tetraquarks, Phys. Rev. D, 100, 016006 (2019)
19. Debastiani, V. R. and Navarra, F. S., A non-relativistic model for the $cc\bar{c}\bar{c}$ tetraquark, Chin. Phys. C, 43, 013105 (2019)
20. Yang, Gang and Ping, Jialun and He, Lianyi and Wang, Qing, Potential model prediction of fully-heavy tetraquarks $Q Q\bar{Q}\bar{Q}$ ($Q = c, b$), arXiv: 2006.13756 [hep-ph]
21. Anwar, Muhammad Naeem and Ferretti, Jacopo and Guo, Feng-Kun and Santopinto, Elena and Zou, Bing-Song, Spectroscopy and decays of the fully-heavy tetraquarks, Eur. Phys. J. C, 78, 647 (2018)
22. Wu, Jing and Liu, Yan-Rui and Chen, Kan and Liu, Xiang and Zhu, Shi-Lin, Heavy-flavored tetraquark states with the $QQ\bar{Q}\bar{Q}$ configuration, Phys. Rev. D, 97, 094015 (2018)
23. Wang, Zhi-Gang and Di, Zun-Yan, Analysis of the vector and axialvector $QQ\bar{Q}\bar{Q}$ tetraquark states with QCD sum rules, Acta Phys. Polon. B, 50, 1335 (2019)
24. Wang, Zhi-Gang and Di, Zun-Yan, Analysis of the $QQ\bar{Q}\bar{Q}$ tetraquark states with QCD sum rules, Eur. Phys. J. C, 77, 432 (2017)
25. Maiani, L. and Polosa, A. D. and Riquer, V., The New Pentaquarks in the Diquark Model, Phys. Lett. B, 749, 289-291 (2015)
26. Lebed, Richard F., The Pentaquark Candidates in the Dynamical Diquark Picture, Phys. Lett. B, 749, 454-457 (2015)
27. Wang, Zhi-Gang, Analysis of $P_c(4380)$ and $P_c(4450)$ as pentaquark states in the diquark model with QCD sum rules, Eur. Phys. J. C, 76, 70 (2016)
28. Li, Guan-Nan and He, Xiao-Gang and He, Min, Some Predictions of Diquark Model for Hidden Charm Pentaquark Discovered at the LHCb, JHEP, 12, 128 (2015)
29. Takeuchi, Sachiko and Takizawa, Makoto, The hidden charm pentaquarks are the hidden color-octet $uud$ baryons?, Phys. Lett. B, 764, 254-259 (2017)
30. Roca, L. and Nieves, J. and Oset, E., LHCb pentaquark as a $D^*\Sigma_c^- - D^*\Sigma_c^-$ molecular state, Phys. Rev. D, 92, 094003 (2015)
31. He, Jun, $D^*\Sigma_c^-$ and $D^*\Sigma_c^+$ interactions and the LHCb hidden-charmed pentaquarks, Phys. Lett. B, 753, 547-551 (2016)
32. Chen, Rui and Liu, Xiang and Li, Xue-Qian and Zhu, Shi-Lin, Identifying exotic hidden-charm pentaquarks, Phys. Rev. Lett. 115, 132002 (2015)
33. Liu, Ming-Zhu and Pan, Ya-Wen and Peng, Fang-Zheng and Sánchez Sánchez, Mario and Geng, Li-Sheng and Hosaka, Atsushi and Pavon Valderrama, Manuel, Emergence of a complete heavy-quark spin symmetry multiplet: seven molecular pentaquarks in light of the latest LHCb analysis, Phys. Rev. Lett. 122, 242001 (2019)
34. Yamaguchi, Yasuhiro and Giachino, Alessandro and Hosaka, Atsushi and Santopinto, Elena and Takeuchi, Sachiko and Takizawa, Makoto, Hidden-charm and bottom meson-baryon molecules coupled with five-quark states, Phys. Rev. D, 96, 114031 (2017)
35. Xu, Kai and Kaewsnod, Attaphon and Liu, Xuyang and Srisuphaphon, Sorakrai and Limphirat, Ayut and Yan, Yupeng, Complete basis for the pentaquark wave function in a group theory approach, Phys. Rev. C, 100, 065207 (2019)
36. Xu, Kai and Kaewsnod, Attaphon and Zhao, Zheng and Liu, Xuyang and Srisuphaphon, Sorakrai and Limphirat, Ayut and Yan, Yupeng, Pentaquark components in low-lying baryon resonances, Phys. Rev. D, 101, 076025 (2020)
37. P. Zyla. et al. (Particle Data Group), 2020 Review of Particle Physics, Prog. Theor. Exp. Phys. 2020, 083C01 (2020)
38. Jaffe, Robert L., Multi-Quark Hadrons. 1. The Phenomenology of (2 Quark 2 anti-Quark) Mesons, Phys. Rev. D, 15, 267 (1977)
39. Jaffe, Robert L., Multi-Quark Hadrons. 2. Methods, Phys. Rev. D, 15, 281 (1977)