A Robust State Estimator for T-S Fuzzy System

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Abstract
In this paper, a robust filter is derived to estimate the state of a nonlinear system which is described by T-S fuzzy model. Firstly, this paper studies a robust state estimation algorithm based on the relationship between Kalman filter and regularized least square, and the algorithm considers the influence of model error caused by system parameter uncertainty of the system in any way. The form of the algorithm is similar to Kalman filter, and the computational complexity is similar to that of Kalman filter. Secondly, the robust filter is combined with the fuzzy model, and the fuzzy rule is used to approximate the nonlinear system. The new algorithm is used to estimate the state of the system. Finally, the new algorithm is compared with the fuzzy Kalman filter based on actual and nominal parameters by simulation experiment, and the simulation experimental results are given to illustrate the effectiveness of the proposed method, which proves that the proposed robust state estimator is better than the fuzzy Kalman filter based on nominal parameters.

Index Terms Kalman filter, model error, nonlinear system, robust filter, Takagi-Sugeno fuzzy model.

I. INTRODUCTION
State estimation plays an important role in the field of control theory and control engineering, it is a method to estimate the internal state of a dynamic system based on available measurement data. The input and output data onto the measurement system can only reflect the external characteristics of the system, while the dynamic law of the system usually needs to be described by the internal state variables which are usually not directly measured. Some approximate calculation methods can be used in engineering. The most commonly used one is the least square estimation [1], and there are some other common methods, such as the generalized Kalman filter based on local linearization, the Bayes [2], the maximum likelihood estimator [3], and the adaptive filter that can automatically modify parameters according to the historical knowledge of the filtering process [4]. Therefore, the state estimation is very important for understanding and controlling the system. In the real world, most dynamical systems are nonlinear, and the state estimation methods of nonlinear systems at present mainly include the unscented Kalman filter [5], the extended Kalman filter [6], the particle filter [7], etc. However, the effect of these filters on highly nonlinear systems is not ideal. If the fuzzy rule model is used to approximate the nonlinear system and the rule theory is used to seek the state estimation, then the estimation performance can be greatly improved [8].

T-S fuzzy model is to fit the same nonlinear system by multiple linear systems [9]. It uses fuzzy algorithm to deconstruct the input variables and uses fuzzy calculus reasoning to remove fuzziness, then generates several equations representing the relationship between input and output of each group [10]. T-S fuzzy model is a nonlinear system described by a set of “if … then…” fuzzy rules, and each rule represents a subsystem. Its original form of fuzzy implication condition sentence is if $x$ is $M$, then $y = f(x)$, where $f(x)$ is a linear function of $x$. In general, $f(x)$ is a polynomial function of $x$. When $f(x)$ is a first-order polynomial, the corresponding fuzzy inference system is called the first-order T-S fuzzy model. When $f(x)$ is a constant, it is called the zero order T-S fuzzy model [11].

In this paper, a new method for state estimation of nonlinear system is proposed, and the main idea of this new method includes the following parts: Firstly, a nonlinear system is approximated by the several linear subsystems by using fuzzy rules. Secondly, a robust state estimator in [12] is used to estimate the robust state of local subsystems. Then, the obtained local estimation is fuzzy fused, and the global state estimation is obtained after optimization. Finally, the effectiveness of the method is verified by a simulation example. The fuzzy inference theory is combined with robust state estimation, and the nonlinear model is approximated by linear method in this paper. By comparison, this robust state estimator can calculate some expectation matrix offline without optimizing the parameter selection. Its computational complexity is equivalent to Kalman filter, and it has better practicability.
in practical engineering. Some other state estimators for T-S fuzzy systems are also proposed in [13]–[17].

II. MODEL DESCRIPTION

A. T-S FUZZY MODEL

A nonlinear function can be approximated by a group of linear functions. Based on the model in [8], for a discrete n-order nonlinear system, the fuzzy rules can be used to approximate the nonlinear system by several linear subsystems:

\[
R^i: \text{ if } Z_i \in F_1^i, \ldots, Z_n \in F_n^i, \text{ then:} \\
\begin{align*}
x_{k+1} &= A_k x_k + B_k U_k + C_k W_k, \\
y_k &= D_k x_k + V_k,
\end{align*}
\]

where \( F_j^i (j = 1, 2, \ldots, n) \) is the fuzzy set, \( x_k \) is the state variable, \( W_k \) is the process noise, \( y_k \) is the measurement output, \( V_k \) is the measurement noise. \( W_k \) and \( V_k \) are not unrelated, and \( E(W_k) = E(V_k) = 0 \). In addition, \( E[W_k^T(W_k)] = Q_k \delta_{ij} \), \( E[V_k^T(W_k)^T] = R_k \delta_{ij} \). Where \( Q_k \) and \( R_k \) are known positive definite matrices, \( \delta_{ij} \) is Kronecker delta function. Let \( \mu_i(z_k) \) denote the normalized membership function on the fuzzy set, where \( z_k \) is the variable. According to [18], \( \mu_i(z_k) \) describes the practicability of rule \( i \), then \( \sum_{i=1}^m \mu_i(z_k) = 1 \). Therefore, the local fuzzy subsystem can be fuzzy combined into a nonlinear global fuzzy system model:

\[
x_{k+1} = A_k x_k + B_k U_k + C_k W_k, \\
y_k = D_k x_k + V_k,
\]

where \( A_k = \sum_{i=1}^m \mu_i(z_k) A_i, B_k = \sum_{i=1}^m \mu_i(z_k) B_i, \)
\[
C_k = \sum_{i=1}^m \mu_i(z_k) C_i, D_k = \sum_{i=1}^m \mu_i(z_k) D_i.
\]

The above expression is a time-varying nonlinear system because \( z_k \) is a nonlinear function about \( x_k \) or \( U_k \). In order to estimate the state of the local subsystem represented by each fuzzy rule, it is necessary to construct the state and measurement output of the time linear invariant subsystem:

\[
x_i(k) = \mu_i(z_k) x_k, y_i(k) = \mu_i(z_k) y_k.
\]

Starting from the global system state model, we can get:

\[
x_{k+1} = A_k x_k + B_k U_k + C_k W_k \\
= \sum_{i=1}^m \mu_i(z_k) A_i x_k + \sum_{i=1}^m \mu_i(z_k) B_i U_k \\
+ \sum_{i=1}^m \mu_i(z_k) C_i W_k \\
= \sum_{i=1}^m A_i x_i(k) + \sum_{i=1}^m \mu_i(z_k) B_i U_k \\
+ \sum_{i=1}^m \mu_i(z_k) C_i W_k \\
= \sum_{i=1}^m x_i(k) + 1, \\
y_k = D_k x_k + V_k \\
= \sum_{i=1}^m \mu_i(z_k) D_i x_k + \sum_{i=1}^m \mu_i(z_k) V_k \\
= \sum_{i=1}^m D_i x_i(k) + \sum_{i=1}^m \mu_i(z_k) V_k \\
= \sum_{i=1}^m y_i(k).
\]

Then, the dynamic equation of local subsystem can be expressed as:

\[
\begin{align*}
x_i(k+1) &= A_i x_i(k) + \mu_i(z_k) B_i U_k + \mu_i(z_k) C_i W_k, \\
y_i(k) &= D_i x_i(k) + \mu_i(z_k) V_k.
\end{align*}
\]

B. IMPROVABLE MODEL

In the actual engineering systems, the interference is ubiquitous, some of which will directly affect the system parameters [19], so we need to consider the influence of model error when describing a system, which is reflected in the uncertainty of parameters that can affect the system model in any way.

In the T-S fuzzy system, it is necessary to consider the influence and disturbance of uncertain parameters. In [20], when studying the design method of robust observer, the existence of uncertain signals is taken into account, thus the subsystem expressed by the fuzzy rule \( i \) is obtained:

\[
R^i: \text{ if } Z_i \in F_1^i, \ldots, Z_n \in F_n^i, \text{ then:} \\
\begin{align*}
x_i(k+1) &= (A_i \pm \Delta A_i) x_i(k) + \mu_i(z_k) (B_i \pm \Delta B_i) U_k \\
&\quad + \mu_i(z_k) C_i \pm \Delta C_i) W_k, \\
y_i(k) &= (D_i \pm \Delta D_i) x_i(k) + \mu_i(z_k) V_k.
\end{align*}
\]

where \( \Delta A_i, \Delta B_i, \Delta C_i, \Delta D_i \) represents the uncertainty of parameters.

When a group of linear functions are used to approximate a nonlinear function, there will be errors naturally. For example, a nonlinear function \( F(x) = \sin x \) is expanded by Taylor formula, that is:

\[
\sin x = x - \frac{1}{3!} x^3 + \frac{1}{5!} x^5 + o(x^5)
\]

If the linear function \( f(x) = x \) is approximately equal to \( F(x) = \sin x \) in the interval \( (0, \pi/2) \), then there is \( F(x) = f(x) + \Delta x \), where the error is:

\[
\Delta x = -\frac{1}{3!} x^3 + \frac{1}{5!} x^5 + \ldots
\]

Similarly, if a nonlinear system is approximated by a linear system based on fuzzy rules, there will also be system errors. In this section, the robust state estimator of T-S fuzzy model is designed, that is, the system error is embodied in the parameter matrix in the form of model error, and the model error is substituted into the local subsystem model. At this time, the parameter matrix contains the uncertainty of the parameter, and then the subsystem dynamic equation expressed by the fuzzy rule \( i \) can be obtained:

\[
\begin{align*}
x_i(k+1) &= A_i(\epsilon_k) x_i(k) + \mu_i(z_k) B_i(\epsilon_k) U(k) \\
&\quad + \mu_i(z_k) C_i(\epsilon_k) W(k), \\
y_i(k) &= D_i(\epsilon_k) x_i(k) + \mu_i(z_k) V(k).
\end{align*}
\]

where \( \epsilon_k \) represents the system parameter model error of time \( k \), and several model errors are not related to each other. Compared with the subsystem model established in [19], it represents the “arbitrary form” of state parameter’s uncertainty, and the uncertainty range is broader. The problem of state
estimation for systems with parameter uncertainty is also studied in [21], [22]. However, in [21] and [22], the system uncertainty matrix is norm bounded. The simulation results show that the robust state estimator does not need optimization of design parameters, but only needs off-line calculation of a few expectation matrices, so it has a wider application compared with other methods.

### III. MAIN RESULTS

It can be seen from [21] that Kalman filter has a clear solution to a regularized least square problem, when the model error \( \epsilon_k = 0 \) in the above local subsystem model, it is regressed to Kalman filter as regularized least square solution:

\[
\hat{x}_i(k) = A_i(0)\hat{x}_i(k + 1) + \mu_i(z_k)B_i(0)U(k) \\
+ \mu_i(z_k)C_i(0)\hat{W}(k) + \mu_i(z_k)U(k). \\
\]

After considering the model error, we can get an improved cost function of the regularized least square problem:

\[
J_1(\alpha_k) = E\left[\left\|x_i(k) - \hat{x}_i(k)\right\|^2_{P_i^{-1}(k|k)} + \left\|W(k)\right\|^2_{Q_k} + \left\|y_i(k + 1) - D_i(\hat{x}(k + 1))\right\|^2_{R_k}\right].
\]

Let \( \partial J_1(\alpha_k) / \partial \alpha_k = 0 \), we get the unique minimum \( \alpha_k \)

\[
\Phi_k + E\left[\begin{bmatrix} A_{i}^T(\epsilon_k) \\
\mu_i(z_k)C_i^T(\epsilon_k) \end{bmatrix} D_i^T(\epsilon_k) + I_k \right] R^{-1}(k + 1) = 0,
\]

We define:

\[
H_{k1} = E\left[\begin{bmatrix} A_{i}^T(\epsilon_k) \\
\mu_i(z_k)C_i^T(\epsilon_k) \end{bmatrix} D_i^T(\epsilon_k) + I_k \right] R^{-1}(k + 1),
\]

To estimate the initial state, firstly define the initial state covariance: \( \Pi_0 = [x_0 - E(x_0)][x_0 - E(x_0)]^T \), then,

\[
J_2(\alpha_0) = E\left[\left\|x(0)\right\|^2_{\Pi_0^{-1}} + \left\|y(0) - D_0(\hat{x}_i(0))\right\|^2_{\Pi_0^{-1}}\right].
\]

Let \( \partial J_2(\alpha_0) / \partial \alpha_0 = 0 \), We can get the initial state:

\[
\hat{x}(0) = P_i(0)E\left[D_i^T(\epsilon_0) R^{-1}(0) y(0)\right].
\]
making:

\[
\begin{bmatrix}
\hat{P}_i^{-1}(k|k) \\
0
\end{bmatrix} = \begin{bmatrix}
0 \\
\hat{Q}_i^{-1}(k)
\end{bmatrix} + \hat{H}_k^T \psi_k \hat{H}_k \left( \tilde{x}_i(k+1) - \hat{x}_i(k|k) \right) \tilde{W}(k+1)
\]
\[
\begin{bmatrix}
\hat{P}_i(k|k) \\
0
\end{bmatrix} = -G_{k12}^T \hat{P}_i(k|k) I
\times \left( H_{k2} R^{-1}(k+1) y_i(k+1) - H_{k3} U(k) - H_{k4} \tilde{x}_i(k) \right).
\]

so we can get the following equation:

\[
\tilde{x}_i(k+1|k+1) = \left( I + P_i(k+1|k) D_i^T(0) R^{-1}(k+1) D_i(0) \right)^{-1} \times \left( A_i(0) \hat{P}_i(k|k) \left[ I 0 \right] + \hat{C}_i(0) \hat{Q}(k) \right)
\times \left( I - \left( I + P_i(k+1|k) D_i^T(0) \right)^{-1} \times P_i(k+1|k) D_i^T(0) \right)^{-1} \times \left( H_{k2} R^{-1}(k+1) y_i(k+1) + \left( I - H_{k2} R^{-1}(k+1) y_i(k+1) \right) D_i(0) \right) \times \left( \hat{A}_i(0) \hat{x}_i(k) + \hat{B}_i(0) U(k) \right)
\]

Let:

\[
P_i(k+1|k+1) = \left( I + P_i(k+1|k) D_i^T(0) \right)^{-1} \times P_i(k+1|k).
\]

According to the matrix lemma in [23]:

\[(A + BCD)^{-1} = A^{-1} - A^{-1} B (DA^{-1} B + C^{-1})^{-1} \times DA^{-1},\]

then:

\[
P_i(k+1|k+1) = P_i(k+1|k) - P_i(k+1|k) D_i^T(0) \times R^{-1}(k+1) D_i(0) P_i(k+1|k).
\]

where, \( \hat{R}(k+1) = R(k+1) + D_i(0) P_i(k+1|k) D_i^T(0) \).

From Equation (17), it can be specified that:

\[
\tilde{x}_i(k+1|k+1) \approx \tilde{x}_i(k+1|k+1),\]

then, the major results can be achieved:

\[
\tilde{x}_i(k+1|k+1) = \hat{A}_i(0) \hat{x}_i(k|k) + \hat{B}_i(0) U(k) + P_i(k+1|k+1)
\times \left( P_i^{-1}(k+1|k) \left[ A_i(0) \hat{P}_i(k|k) \left[ I 0 \right] + \hat{C}_i(0) \hat{Q}(k) \right) \times \hat{Q}(k) \left[ -G_{k12}^T \hat{P}_i(k|k) I \right] \times H_{k2} R^{-1}(k+1) y_i(k+1) + \left( I - H_{k2} R^{-1}(k+1) y_i(k+1) \right) D_i(0) \right) \times \left( \hat{A}_i(0) \hat{x}_i(k) + \hat{B}_i(0) U(k) \right)
\]

In [8], through the design of Fuzzy Kalman filter, the conclusion is drawn that based on m fuzzy rules, m local states \( x_i(k) \) can be obtained. After the Kalman filter formula is used to derive the local target state estimation, an unbiased estimation can be obtained:

\[
\hat{x}_{k|k} = P_{k|k} X_k, X_k = \left( \begin{array}{c}
\hat{x}_1(k) \\
\vdots \\
\hat{x}_m(k)
\end{array} \right), P_{k|k} = \left( \begin{array}{ccc}
P_{11} & & \\
& \ddots & \vdots \\
& & P_{nn}
\end{array} \right)
\]

where \( P_k \) is the renewable state covariance derived from the formula in the derivation process of Kalman filter for each local system. It can be seen that for a nonlinear system, the global robust state estimation \( \hat{x}_{k|k} = P_{k|k} X_k \) obtained by T-S fuzzy rule is unbiased for \( x \), because each local estimation is linear and asymptotically unbiased, and the global estimation is a weighted form for them. According to the theory of matrix theory, the state estimation constructed
in this way is still asymptotically unbiased [8]. Therefore, an unbiased global estimation can also be obtained by robust state estimation:

\[
\hat{x}_{k|k} = \begin{pmatrix} \tilde{x}_1(k|k) \\ \vdots \\ \tilde{x}_m(k|k) \end{pmatrix}.
\] (25)

IV. NUMERICAL SIMULATION

In this numerical simulation, the performance of robust state estimator is compared with Kalman filter based on nominal parameters and actual parameters for each local linear subsystem, and each simulation experiment is conducted 500 times. Taking the first-order nonlinear system as an example, for each local system, the statistical mean value of the estimation error at each time is approximately calculated by the mean value of its actual’s square state to its estimated Euclidean distance, the statistical mean value is as follows:

\[
e_i \|x_i(k) - \hat{x}_i(k|k)\|^2 \approx \frac{1}{500} \sum_{j=1}^{500} \|x_i(k) - \hat{x}^{(j)}_i(k)\|^2.
\] (26)

Then, the global system estimation error can be obtained:

\[
E_k = \begin{pmatrix} e_1 \|x_1(k) - \hat{x}_1(k|k)\|^2 \\ \vdots \\ e_m \|x_m(k) - \hat{x}_m(k|k)\|^2 \end{pmatrix}.
\] (27)

In the form of Euclidean norm, the estimation error of global fuzzy system can be displayed directly:

\[
\|E_k\| = \sqrt{E_k^TE_k}.
\]

The system parameters of the numerical simulation are quoted from the tractor-car system. From the angle information of the vehicle in the system, it can be seen that the system is highly nonlinear. \(x^1(k)\) is the tractor steering angle, \(x^2(k)\) is the direction angle of the car, the state vector is \(x_k = [x^1(k) x^2(k)]\), the previous variable is \(z_k = x^2(k) - \frac{vt}{2L} x^1(k)\). If the fuzzy set is set up with \([-\frac{\pi}{2}, 0], [0, \frac{\pi}{2}]\) as the interval, the membership function can be selected as:

\[
\mu_1(z_k) = 1 - \frac{1}{1 + \exp(-\frac{z_k - 0}{\frac{\pi}{2}})}, \mu_2(z_k) = 1 - \mu_1(z_k).
\]

Using the T-S fuzzy rule theory, the system can be expressed as:

\[\begin{align*}
R^1: & \text{ if } Z_k \in F^1, \text{ then:} \\
& \begin{cases} \\
x^1(k+1) = (1 - \frac{vt}{L})x^1(k) + \frac{vt}{L}x^2(k) + \frac{vt}{L}U(k), \\
x^2(k+1) = (1 - \frac{vt}{L})x^2(k).
\end{cases}
\]

\[\begin{align*}
R^2: & \text{ if } Z_k \in F^2, \text{ then:} \\
& \begin{cases} \\
x^1(k+1) = (1 - \frac{vt}{2L})x^1(k) + \frac{vt}{L}x^2(k) + \frac{vt}{L}U(k), \\
x^2(k+1) = (1 - \frac{vt}{2L})x^2(k).
\end{cases}
\]

where, \(U(k)\) is the steering angle, \(L\) is the length of the tractor, \(v\) is the sampling period, \(v\) is the constant speed. Taking into account the system error when the linear system approximates the nonlinear system, it is substituted into the model in the form of model error, then the matrix parameters are:

\[
A_1(\varepsilon_k) = \begin{bmatrix} 1 - \frac{vt}{L} & \frac{vt}{L} + \Delta \varepsilon_k \\ -\frac{vt}{L} & 1 - \frac{vt}{L} \end{bmatrix},
\]

\[
A_2(\varepsilon_k) = \begin{bmatrix} 1 - \frac{vt}{2L} & \frac{vt}{2L} + \Delta \varepsilon_k \\ -\frac{vt}{2L} & 1 - \frac{vt}{2L} \end{bmatrix},
\]

\[
B_1(\varepsilon_k) = B_2(\varepsilon_k) = \begin{bmatrix} \frac{vt}{L} \\ 0 \end{bmatrix}.
\]

the numerical values in the simulation experiment are \(v = 98 \text{ cm/s}, t = 0.1 \text{ s}, k = (0, 0.1, 0.2, \ldots n), L = 500 \text{ cm}\). Then, the matrix parameters are as follows, where the parameter before model error \(\varepsilon_k\) is adjustable, which represents the ‘size’ of uncertainty:

\[
A_1(\varepsilon_k) = \begin{bmatrix} 0.9804 & 0.0196 + 0.099 \varepsilon_k \\ 0 & 0.9804 \end{bmatrix},
\]

\[
A_2(\varepsilon_k) = \begin{bmatrix} 0.9902 & 0.0196 + 0.099 \varepsilon_k \\ 0 & 0.9902 \end{bmatrix},
\]

\[
B_1(\varepsilon_k) = B_2(\varepsilon_k) = \begin{bmatrix} 0.0196 \\ 0 \end{bmatrix},
\]

\[
Q = \begin{bmatrix} 1.9608 & 0.0195 \\ 0.0195 & 1.9605 \end{bmatrix},
\]

\[
R = 1.0000, \Pi_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},
\]

\[
C_1(\varepsilon_k) = C_2(\varepsilon_k) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},
\]

\[
D_1(\varepsilon_k) = D_2(\varepsilon_k) = \begin{bmatrix} 1 & -1 \end{bmatrix}.
\]
FIGURE 2. Estimation error variance - when the value of model error is time-varying.

FIGURE 3. Estimation error variance - when the value of model error is fixed and the uncertainty is "increased".

FIGURE 4. Estimation error variance - when the value of model error is time-varying and the uncertainty is "increased".

When the model error $\varepsilon_k$ is generated by the uniform distribution of $[-1 1]$, and each experiment keeps the same value, the experimental results are as follows (see Figure 1). In all of the following drawings, each line is marked with that ♦: Kalman filter based on nominal parameters, ∗: Robust state estimator, □: Kalman filter based on actual parameters.

When the model error $\varepsilon_k$ at each time is generated by the uniform distribution of $[-1 1]$ (see Figure 2).

The uncertainty is "increased", that is, the element of one row and two columns of matrix $A_i(\varepsilon_k)$ is modified to $0.0196 + 0.5\varepsilon_k$, and the model error $\varepsilon_k$ is generated by the uniform distribution of $[-1 1]$, and each experiment keeps the same value (see Figure 3).

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V. CONCLUSION
In this paper, a robust state estimation method is introduced for T-S fuzzy system, which takes the influence of model error into account while modeling. This robust state estimator can be obtained by off-line calculation of some expectation matrices without optimizing the parameter selection, while the computational complexity is equivalent to that of Kalman filter, so it has better practicability in practical engineering.

It can be seen from the simulation results that the Kalman filter based on nominal system parameters is very sensitive to the variation of model error amplitude when the uncertainty of each experiment is fixed. More concretely, when the uncertainty "increases", the performance of Kalman filter deteriorates seriously. When the uncertainty of each experiment is fixed or time-varying, the robust state estimator quoted in this paper is stable for the variation of model error amplitude. Therefore, in terms of the stability and accuracy, the robust state estimator introduced in this paper is better than the fuzzy Kalman filter based on nominal parameters.

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