Quantum network of superconducting qubits through opto-mechanical interface

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We propose a scheme to realize quantum networking of superconducting qubits based on the opto-mechanical interface. The superconducting qubits interact with the microwave photons, which then couple to the optical photons through the opto-mechanical interface. The interface generates a quantum link between superconducting qubits and optical flying qubits with tunable pulse shapes and carrier frequencies, enabling transmission of quantum information to other superconducting or atomic qubits. We show that the scheme works under realistic experimental conditions and it also provides a way for fast initialization of the superconducting qubits under 1 K instead of 20 mK operation temperature.

Superconducting qubits (SQs) constitute one of the leading candidate systems for realization of quantum computation\(^1\). Through the circuit resonators, SQs have strong coupling to the microwave photons\(^1\), which can be used for qubit interaction, state engineering of the photonic modes, and non-destructive readout of the qubits\(^2\,\(^3\)\). Universal quantum logic gates have been realized for SQs in circuit QED (cQED) systems with high fidelity and speed\(^4\). Through use of the noise insensitive qubits, the coherent time of the SQs has been increased by several orders of magnitude in recent years and pushed to the 100 μs region\(^5\,\(^6\)\). In a single circuit resonator, the number of SQs is still limited. Further scaling up the number of qubits requires linking distant cQED systems to form a quantum network. Microwave photons are sensitive to thermal noise and their quantum states only survive under cryogenic temperature. So it is hard to use them to link SQs in two different setups. Optical photons, on the other hand, are robust information carriers at room temperature and serve as ideal flying qubits for long-distance communication. They can carry quantum information to distant locations through an optical fiber.

In this paper, we propose a scheme to realize a quantum network of SQs through an opto-mechanical interface that couples optical photons in a cavity to microwave photons and SQs in a circuit resonator. The interface generates entangled states between SQs and photonic pulses with tunable pulse shape and carrier frequency. The photons then make a quantum link between distant SQs through either a measurement-based entangling protocol or a deterministic state mapping. Because of the tunability of shape and frequency of the emitted photon, the same scheme can also be used to realize a hybrid network between SQs and other matter qubits such as atomic ions\(^7\), quantum dots\(^8\), or defect spins in solids\(^9\). A hybrid network may allow combination of advantages of different kinds of qubits. For instance, SQs may be good for fast information processing while atomic qubits are ideal for quantum memory. Our scheme is based on the recent advance on the microwave-optical interface: there have been several proposals to realize this interface with ions\(^10\,\(^11\)\), cold atoms\(^12\), or a hybrid opto-mechanical system with superconducting resonators\(^13\,\(^18\)\). In particular, a recent experiment has demonstrated the transducer between microwave and optical photons using the opto-mechanical system at 4.5K temperature\(^19\). One hassle for an interface between SQs and optical photons is that thermal initialization of the SQs requires an operating temperature around 20 mK in a dilution fringe, while an interface to photons requires an optical window, which introduces heating due to black-body radiation and may significantly increase the system temperature. We circumvent this problem by showing that our proposed scheme can achieve fast initialization of the SQs at 1 K through optical sideband cooling by use of the same opto-mechanical interface.

![FIG. 1: (a) The schematic scheme of the opto-mechanical quantum interface. The dashed-orange box denotes the SQ coupled with the microwave mode \(a_2\) in a superconducting resonator (SR). The mechanical oscillator (MO) mode \(a_m\) for vibration of the interface couples simultaneously to the mode \(a_2\) of the SR and the mode \(a_1\) of the optical cavity (OC). Both \(a_2\) and \(a_1\) modes are driven by coherent classical fields on the red sideband. (b) The energy levels of the superconducting junction, where \(|g\rangle\) is the ground state, \(|e\rangle\) is the first excited state, and \(|s\rangle\) is the second excited state. The transition \(|g\rangle\) to \(|e\rangle\) couples to the mode \(a_2\) with coupling rate \(g_1\), while transition \(|e\rangle\) to \(|s\rangle\) is driven by a microwave field with Rabi frequency \(\Omega(t)\).](image)

The model. As show in Fig. 1, the system we consider contains an optical cavity (OC) and a microwave superconducting resonator (SR), which share an interface that can vibrate and forms a mechanical oscillator (MO)\(^20\,\(^21\)\). The shared vibrating interface between the OC and the SR has been proposed in several schemes\(^13\,\(^17\)\) and realized very recently in experiments\(^19\). For this system, the MO mode \(a_m\) of frequency \(\omega_m\) couples simultaneously to the optical mode \(a_1\) of...
frequency $\omega_1$ and the microwave mode $\alpha_2$ of frequency $\omega_2$. We have assumed that the coupling rate is much less than the mode spacing of either of these oscillators so that only one mode is relevant respectively for the OC, the MO, and the SR. The optical and the microwave modes $a_1$ and $a_2$ are driven at the red-sideband with optical frequency $\omega_{1\pm} = \omega_1 - \Delta_1$ and $\omega_{2\pm} = \omega_2 - \Delta_2$, respectively. Inside the SR, there are nonlinear Josephson junctions, with the lowest three anharmonic levels shown in Fig. 1b. The levels $|\pm\rangle$ and $|s\rangle$ make a SQ with coupling mediated by the middle level $|e\rangle$ with a coupling rate $g_e$ for the $|g\rangle \rightarrow |e\rangle$ transition and a Rabi frequency $\Omega(t)$ (driven by a microwave field with tunable shape) for the $|e\rangle \rightarrow |s\rangle$ transition.

The Hamiltonian of the system has the form $H = H_0 + H_I + H_{\text{drive}}$, where $H_0 = \sum_{i=1,2} \omega_i a_i^\dagger a_i + \omega_m a_m^\dagger a_m + \omega_e \sigma_{ee}$, $H_I = \sum_{i=1,2} g_i a_i^\dagger a_i (a_m + a_m^\dagger) + (g_e \sigma_{eg} a_2 + \text{h.c.})$, and $H_{\text{drive}} = \sum_{i=1,2} \frac{\nu}{\hbar} a_i e^{i\nu \Omega(t) + \text{h.c.}}$. We have set $\hbar = 1$ and taken the notation $\sigma_{ij} = |\mu\rangle \langle \nu|$ ($\mu, \nu = g, e, s$). The qubit is assumed to couple resonantly with the superconducting resonator with $\omega_2 = \omega_m$. The opto-mechanical coupling rates $g_i (i = 1, 2)$ are typically small, but their effect can be enhanced through the driving field $\Omega$. Under the driving, the steady state amplitude of the mode $a_i$ is given by $a_i = \Omega_i/2\Delta_i$. The opto-mechanical coupling terms can be expanded with $a_i - a_i^\dagger$ and the effective coupling Hamiltonian takes the form $[15, 16, 22]$

$$H_{\text{eff}} = \sum_{i=1,2} \left[ \Delta_i a_i^\dagger a_i + G_i (a_i^\dagger + a_i) (a_m + a_m^\dagger) + \omega_m a_m^\dagger a_m \right] + \omega_m a_m^\dagger a_m$$

where $G_i = a_i g_i$. We take the detuning $\Delta_i = \omega_m$ and assume $\omega_m \gg G_i$. Under the rotating wave approximation, the whole Hamiltonian in the interaction picture is given by

$$H_I = \left( G_1 a_1^\dagger + G_2 a_2^\dagger \right) a_m + G_e \sigma_{eg} a_2 + \text{h.c.}$$

The corresponding Langevin equations for the $a_j$ ($j = 1, 2, m$) modes and the SQ take the form

$$\dot{a}_j = -i[a_j, H_I] - \frac{k_j}{\sqrt{\kappa}} a_j^{\text{in}},$$

$$\dot{\sigma}_{ge} = -i[\sigma_{ge}, H_I] - \frac{\gamma}{\sqrt{\kappa}} \sigma_{ge} + \sqrt{\gamma} \sigma_{ee} a_m^{\text{in}},$$

where $\sigma_{ce} = \sigma_{ec} - \sigma_{gg}$, $\gamma$ is the decay rate of the level $|e\rangle$, and $k_j$ is the decay rate of the mode $a_j$.

**SQ-photon quantum interface.** In order to couple the SQ to an optical photon with controllable pulse shape, we prepare the SQ initially on the level $|s\rangle$ and drive the transition $|s\rangle \rightarrow |e\rangle$ by a microwave field with Rabi frequency $\Omega(t)$ and pulse duration $T$. The total Hamiltonian of the system is $H_{\text{tot}} = H_I + \Omega(t) \sigma_{ge} + \text{h.c.}$. Without loss of generality, we take $G_1 = G_2 = G$ for simplicity of notation. We may define the normal modes $b, b_\pm$ with $a_1 = (b_+ + b_- - \sqrt{2} b)/2$, $\alpha_2 = (b_+ + b_- + \sqrt{2} b)/2$, $a_m = (b_+ - b_-)/\sqrt{2}$, which diagonalize the opto-mechanical coupling Hamiltonian $[23, 24, 25]$. In the limit $\Delta_D \ll G, g, \kappa_1$, the modes $b, b_\pm$ are not populated and can be adiabatically eliminated. The effective Hamiltonian is simplified to $H_I = \Omega(t) \sigma_{ge} + \frac{\sqrt{2}}{\gamma} G \sigma_{eg} + \text{h.c.}$ The Hamiltonian $H_I$ has a dark state $|D\rangle = (|s\rangle(0) - r(t)|g\rangle)/\sqrt{1 + r(t)^2}$, where $r(t) = \sqrt{2} \Omega(t)/g_c$, and $|0\rangle, |1\rangle$ represent the Fock states of the mode $b$.

The normal mode $b$ decays through two channels, $a_1^{\text{out}}$ and $a_2^{\text{out}}$. The decay of $b$ mode is denoted as $\kappa = (\kappa_1 + \kappa_2)/2$. Typically, we have $\kappa_1 \gg \kappa_2$, so the photons go out dominantly through the $a_1^{\text{out}}$ channel. To solve the output pulse shape, we rewrite the dark state as $|D\rangle = \cos(\theta)|\langle 0| - \sin(\theta)|g\rangle|1\rangle$, with $\cos \theta = 1/\sqrt{1 + |r(t)|^2}$, and define an orthogonal bright state $|B\rangle = \sin(\theta)|\langle 0| + \cos(\theta)|g\rangle|1\rangle$. The wave-function of the whole system can be expanded as $|\Psi\rangle = (c_0|D\rangle + c_1|B\rangle + c_2|e\rangle)\otimes|\text{vac}\rangle + |g\rangle|\text{vac}\rangle$, where $|\text{vac}\rangle$ is the vacuum state of output field, and $|\varphi\rangle = \int d\omega_{\text{out}}d\omega_{\text{in}} a_{\text{out}}^\dagger a_{\text{in}}(\omega)|\text{vac}\rangle$ denotes the single-photon state of the output filed with frequency spectrum $c_{\omega_c}$. The dynamics of system is determined by the Schrödinger equation $i\hbar \partial_t |\Psi\rangle = H_I |\Psi\rangle$, where $H_I$ is the total Hamiltonian that includes the input-output coupling terms $[24]$. Using the method in Ref. [24], the output pulse shape $f(t)$, given by the Fourier transform of $c_{\omega_c}$, can be solved analytically in the adiabatic limit, with

$$f(t) = \sqrt{k} \sin \theta \exp(-\frac{k}{2} \int_0^t \sin^2 \theta(\tau) d\tau).$$

The pulse shape $f(t)$ is fully determined by $\theta(t)$.

To check whether the pulse shape of Eq. (4) derived under the adiabatic limit holds under typical experimental parameters, we compare in Fig. (2) the pulse shapes obtained from the analytic formula and from the exact numerical simulation. In numerical simulation, we solve the exact system dynamics by including the contribution of populations either in the bright state $|B\rangle$ or of all the three modes $b$ and $b_\pm$. As one can see from Fig. (2) if the pulse duration $T_D \gtrsim 20/\kappa$, the pulse shape from the analytic formula (4) overlaps very well with the exact result, with the mismatching error less than 1%. However, for a short pulse with $T_D \sim 5/\kappa$, there is a significant shape mismatching error and one should use the exact result instead of the approximate analytic formula. The exact result shows some oscillations in the pulse shape for a short driving field, resulting from the population oscillation in different modes $b, b_\pm$ when the condition of adiabatic elimination $T_D \ll G, g, \kappa_1$ is not well satisfied.

**Quantum networking of SQs.** In the above, we have shown how to couple a SQ to a single optical output photon with a controllable pulse shape. This ability is critical for building up a quantum network of SQs or a hybrid network between SQs and other matter qubits. Here, we mention two complementary schemes for quantum networking of SQs, requiring different kinds of pulse shape control.

The key requirement of quantum networking is to generate entanglement between remote SQs. The first scheme for entanglement generation is based on a deterministic quantum state transfer between SQs in two remote cavities $[25]$. As absorption is the time reversal of the emission process, it has been shown in Ref. [25] that an emitted single-photon pulse can be completely absorbed by a matter qubit in a cavity if we
simultaneously reverse the temporal shape of the photon pulse and the driving filed \(\Omega(t)\). As shown in Fig. 2, with an appropriate control of the driving microwave field \(\Omega(t)\), we can transfer a quantum state from a SQ to a single-photon pulse with a symmetric temporal shape. This single-photon pulse, after propagation in an optical fiber, can then be absorbed by a SQ in another remote cavity, if the driving \(\Omega(t)\) of the second SQ is the time reversal of \(\Omega(t)\). The shape control of the driving microwave pulse \(\Phi(t)\) or \(\Omega(t)\) can be easily achieved through modulation by an arbitrary waveform generator. If we make a half transfer of the population from the first SQ to the photonic pulse, the generated state between the SQ and the output photon \(p\) has the form \((|s⟩|0⟩_p + |g⟩|1⟩_p) / \sqrt{2}\). Then, after absorption of the photon by the second SQ, we generate an entangled state \((|s⟩|g⟩_2 + |g⟩|s⟩_2) / \sqrt{2}\) between two remote SQs, as required for quantum networking.

The entanglement between remote SQs can also be generated in a probabilistic fashion through detection of interference of the emitted photon(s) \([24, 26]\). For instance, as shown in Fig. 4a, we have SQs in two remote cavities, each emitting a single-photon pulse with a small probability \(p_0 = 1 - \exp[-\kappa \int \sin^2 \theta(\tau) d\tau] \) through an incomplete adiabatic passage from the state \(|s⟩\) to \(|g⟩\). The emitted pulses, after propagation in optical channels, interfere at a 50 – 50% beam splitter, with outputs detected by single-photon counters. If we register only one photon from these detectors, the two SQs are projected to an entangled state \(|f\rangle = \sqrt{p_0} |\theta\rangle + (-1)^n |\phi\rangle\rangle / \sqrt{2}\) with a success probability proportional to \(p_0 \ll 1\). The unknown relative phase \(\phi\) can be canceled during the detection process \([27]\), or through the second round of entanglement generation by applying the same protocol again \([28]\). Compared with the deterministic scheme \([28]\), this probabilistic scheme has a lower efficiency as the protocol needs to be repeated until one successfully registers a photon count, however, it is more robust to noise as the photon loss in the optical channels does not influence the fidelity of this scheme.

A major challenge for quantum networking based on the photonic connection is to achieve the spectrum (shape) and frequency matching of the emitted photon pulses from different matter qubits. For solid-state qubits in particular, the coupling parameters usually vary for different systems and it is hard to get identical qubits or coupling rates. A remarkable advantage of the scheme based on the opto-mechanical interface is that all the mismatches in frequencies or pulse shapes can be easily compensated through the driving fields. For instance, the scheme works perfectly well if the coupling or decay rates are different for different systems. As the pulse shape only depends on \(\theta(t)\) from Eq. (4), we can always get identical shapes as difference in the coupling rates can be easily compensated by the microwave driving amplitude \(\Omega(t)\). Furthermore, the output optical frequency is purely determined by the eigenmode structure of the optical cavity and not limited by the qubit parameters. So, depending on the frequency and shape of the driving field, we can have a quantum interface between the SQ and the optical photon with widely tunable
carrier frequency and shape, which can then interfere with the photons emitted by other kinds of matter qubits, such as trapped ions [29], quantum dots [30], or diamond nitrogen vacancy centers [31]. The SQ-opto-mechanical interface can therefore work as a quantum transducer to generate entanglement links between different types of matter qubits. This leads to a hybrid quantum network, with an example illustrated in Fig. 3(b), which has the important advantage to combine the particular strength of each kind of matter qubits.

**Noises effects and opto-mechanical initialization of SQs.** In the above analysis, we assume the SQ couples dominantly to the output field of the optical cavity and neglect other dissipation channels. Now we take into account all the other dissipation processes and calculate their effects on the fidelity of quantum interface. Under the condition that the pulse duration $T_D^{-1} \ll G, g, \kappa_1$, we can adiabatically eliminate all the modes $a_j$ ($j = 1, 2, m$) in the Langevin equations (3) and arrive at the following decay equation for the SQ (see details in supplemental material):

$$\dot{\sigma}_{gc} = -\frac{\gamma_{eff}}{2} \sigma_{gc} + \sqrt{\gamma_{eff} \sigma_{gc}} d^{in}_{eff},$$

(5)

where $\gamma_{eff} = \gamma + \tilde{\kappa}_1 + \tilde{\kappa}_2 + \tilde{\kappa}_m$, $d^{in}_{eff} = [-i \sqrt{\kappa_1} a^n_1 + i \sqrt{\kappa_2} d^{in}_2 + \sqrt{\gamma} d^{in}_m]/\sqrt{\gamma_{eff}}$, $\tilde{\kappa}_1 = \frac{\gamma g^2}{(\kappa_1 + \kappa_2 + \kappa_m) G^2}$, and $\tilde{\kappa}_m = \frac{\gamma g^2}{(\kappa_1 + \kappa_2 + \kappa_m) G^2}$. The physical meaning of Eq. (5) is clear: the SQ couples to four decay channels, the optical channel $a^n_1$ with decay rate $\tilde{\kappa}_1$, the microwave channel $d^{in}_2$ with decay rate $\tilde{\kappa}_2$, the mechanical channel $a^n_m$ with decay rate $\tilde{\kappa}_m$, and the intrinsic channel $d^{in}_m$ with decay rate $\gamma$. For each effective channel, the effective dissipation rate is given by $(\tilde{n}_j + 1) \tilde{\kappa}_j$, where $\tilde{n}_j = 1/(\exp(h\omega_j/k_B T) - 1)$ is the mean thermal photon (or phonon) number and $T$ denotes temperature of the system. For quantum networking of SQs through the optical decay channel, all the other dissipation channels contribute to noise, and the fidelity $F$ of the quantum interface can be estimated by the relative ratio of the optical decay rate to the total dissipation rate

$$F = \frac{\tilde{\kappa}_1}{\tilde{\kappa}_1 + (\tilde{n}_2 + 1) \tilde{\kappa}_2 + (\tilde{n}_m + 1) \tilde{\kappa}_m + (\tilde{n}_2 + 1) \gamma},$$

(6)

where we have taken $\tilde{n}_1 \approx 0$ at the optical frequency. The experimental parameters typically satisfy $G \sim 1 \gg \tilde{\kappa}_2, \tilde{\kappa}_m, \gamma$. In this case, $\tilde{\kappa}_1 \approx 4 g^2/\kappa_1$, $\tilde{\kappa}_2 \approx 4 g^2 k_2/\kappa_1^3$, and $\tilde{\kappa}_m \approx 4 g^2/\kappa_m G^2$. In Fig. 4, we show the fidelity as a function of the system temperature and the decay rate of the optical cavity. It is found that the fidelity is around 99% under typical values of the experimental parameters as listed in the figure caption (we assume temperature $T = 1$ K with $\tilde{n}_2 = 3.7$, and $\tilde{n}_m = 208$).

This opto-mechanical interface can also be used to initialize the SQ through the sideband cooling. If the SQ is initially in a mixture of $|g\rangle$ and $|e\rangle$ states [34], we can cool the system to the ground state $|g\rangle$ by driving the red sideband of the optical cavity [35-38]. This cooling is described by the Langevin equation (5) and the final probability $P_g$ for the SQ in the state $|g\rangle$ is determined by the stationary state under Eq. (5) (after a decay time of the order of $1/\tilde{\kappa}_1 \sim 10$ ns with $\tilde{\kappa}_1$)

$$P_g = \frac{\tilde{\kappa}_1 + (\tilde{n}_2 + 1) \tilde{\kappa}_2 + (\tilde{n}_m + 1) \tilde{\kappa}_m}{\tilde{\kappa}_1 + (2\tilde{n}_2 + 1)(\gamma + \tilde{\kappa}_2) + (2\tilde{n}_m + 1) \tilde{\kappa}_m}.$$  

(7)

Under experimental parameters list in the caption of Fig. (4) and 1 K system temperature, the fidelity $P_g$ for state initialization is larger than 99%.

Typically the SQ system is operated around 20 mK temperature, where the ground state cooling is achieved directly through thermal equilibrium. However, with an opto-mechanical interface, the system temperature may increase due to heating by the black-body radiation from the optical window. Here, we show that even under 1 K temperature, the state can still be initialized through the opto-mechanical sideband cooling. Another requirement for the system temperature is that the quasi-particle density in the superconducting circuit should be small, otherwise it will induce dissipation of the SQ. The quasi-particle density is proportional to $e^{-1.76 T_c/T}$, where $T_c$ is the critical temperature of the superconductor [39]. For niobium, the critical temperature $T_c$ is about 9.3 K, for which the quasi-particle density is negligibly small at 1 K temperature. For aluminum, the $T_c$ is about 1.2 K, where the quasi-particles can be neglected only at temperature in the order of 0.1 K.

In summary, we have proposed a scheme to realize a quantum network of SQs base on the opto-mechanical quantum interface. The interface can couple the SQs to optical photons with widely tunable carrier frequencies and pulse shapes. The same interface can also be used for fast initialization of the SQs through opto-mechanical sideband cooling.

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