Evidence for a low density Universe
from the relative velocities of galaxies

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The motions of galaxies can be used to constrain the cosmological density parameter, $\Omega$, and the clustering amplitude of matter on large scales. The mean relative velocity of galaxy pairs, estimated from the Mark III survey indicates $\Omega = 0.35^{+0.35}_{-0.25}$. If the clustering of galaxies is unbiased on large scales, $\Omega = 0.35 \pm 0.15$, so that an unbiased Einstein-de Sitter model ($\Omega = 1$) is inconsistent with the data.

The mean relative velocity for a pair of galaxies at positions $\vec{r}_1$ and $\vec{r}_2$ is $\vec{u}_{12} = H\vec{r}$, where $\vec{r} = \vec{r}_1 - \vec{r}_2$ and the constant of proportionality $H = 100 \, h \, \text{km} \, s^{-1} \, \text{Mpc}^{-1}$ is the Hubble parameter \((1,2)\). The quantity $0.6 < h < 1$ parametrizes uncertainties in $H$ measurements. This law is an
idealization, followed by real galaxies only on sufficiently large scales, corresponding to a uniform mass distribution. On smaller scales, the gravitational field induced by galaxy clusters and voids generates local deviations from the Hubble flow, called peculiar velocities. Correcting for this effect gives \( \vec{u}_{12} = H\vec{r} + v_{12}\vec{r}/r \). The quantity \( v_{12}(r) \) is called the mean pairwise streaming velocity. In the limit of large \( r \), \( v_{12} = 0 \). In the opposite limit of small separations, \( u_{12}(r) = 0 \) (virial equilibrium). Hence, at intermediate separations \( v_{12} < 0 \) and we can expect to observe gravitational infall, or the “mean tendency of well-separated galaxies to approach each other” (3). In a recent paper we derived an expression, relating \( v_{12} \) to cosmological parameters (4); in another, using Monte Carlo simulations we showed how \( v_{12} \) can be measured from velocity-distance surveys of galaxies (5). Our purpose here is to estimate \( v_{12}(r) \) from observations and constrain the cosmological density parameter, \( \Omega \).

The statistic we consider was first introduced in the context of the Bogolyubov-Born-Green-Kirkwood (BBGKY) kinetic theory describing the dynamical evolution of a self-gravitating collection of particles (3,6). One of the BBGKY equations is the so called pair conservation equation, relating the time evolution of \( v_{12} \) to \( \xi(r) \) — the two-point correlation function of spatial fluctuations in the fractional matter density contrast (3). Its solution is well approximated by (4)

\[
\begin{align*}
v_{12}(r) &= -\frac{2}{3} H r \Omega^{0.6} \xi(r) [1 + \alpha \xi(r)], \\
\bar{\xi}(r) &= \frac{3}{r^3} \int_0^r \xi(x) x^2 \, dx \left[ 1 + \xi(r) \right],
\end{align*}
\]

where \( \alpha = 1.2 - 0.65\gamma \), \( \gamma = -(d\ln\xi/d\ln r)_{\xi=1} \) and \( \Omega \) is the present density of nonrelativistic particles. The equations above have been obtained by interpolating between a second-order perturbative solution for \( v_{12}(r) \) and the nonlinear stable clustering solution. For a particle pair at
separation \( \vec{r} \), the streaming velocity is given by

\[
v_{12}(r) = \langle (\vec{v}_1 - \vec{v}_2) \cdot \hat{r} \rangle_{\rho} = \langle (\vec{v}_1 - \vec{v}_2) \cdot \hat{r} \, w_{12} \rangle ,
\]

(3)

where \( w_{12} = (1 + \delta_1)(1 + \delta_2)[1 + \xi(r)]^{-1} \) is the pair-density weighting, \( \vec{v}_A \) and \( \delta_A \) are the peculiar velocity and fractional density contrast of matter at position \( \vec{r}_A \), \( A = 1, 2, \ldots \), the separation \( r = |\vec{r}_1 - \vec{r}_2| \) is fixed for all pairs, the hats denote unit vectors, and \( \xi(r) = \langle \delta_1 \delta_2 \rangle \). The expression in square brackets in the definition of \( w_{12} \) ensures that \( \langle w_{12} \rangle = 1 \) and the pairwise velocity probability density integrates to unity. Note that the pair-weighted average, \( \langle \cdots \rangle_{\rho} \), differs from simple spatial averaging, \( \langle \cdots \rangle \), by the weighting factor \( w_{12} \). The pair-weighting makes the average different from zero, unlike the volume average \( \langle \vec{v}_1 - \vec{v}_2 \rangle \equiv 0 \), which vanishes because of isotropy.

Our approximate solution of the pair conservation equation was successfully tested against N-body simulations in the dynamical range \( \xi \leq 10^3 \) \((4, 7)\). It is valid for universes filled with non-relativistic particles and it is insensitive to the value of the cosmological constant \((2, 4)\). Eqn.(1) was derived under the additional assumption that the probability distribution of the initial, small-amplitude density fluctuations was Gaussian.

Until now we have also implicitly assumed that (i) the spatial distribution of galaxies traces the mass distribution and that (ii) \( v_{12}(r) \) for the galaxies is the same as for the matter. If the galaxies are more clustered than mass, condition (i) is broken and we have “clustering bias”. The galaxy two-point correlation function is close to a power law, \( \xi_{\text{gal}}(r) \propto r^{-\gamma} \), over three orders of magnitude in separation \( r \) \((8)\). This is not true for the mass correlation function \( \xi(r) \) in structure formation models of the cold dark matter (CDM) family \((7)\). To reconcile theory with observation, one has to introduce a measure of bias that depends on separation and cosmological time, \( t \):

\[
b^2(r, t) = \xi_{\text{gal}}(r, t)/\xi(r, t) .
\]

Because of the pair-density weighting, clustering bias can in principle
induce “velocity bias” in a way similar to systematic error propagation. This is certainly true in
the most simplistic of all biasing prescriptions - the “linear biasing”, under which $b$ is a constant
and, moreover, $\delta_{\text{gal}} = b \delta$. The expression for $v_{12}^{\text{gal}}$ can be obtained from Eqn.(3) by formally
replacing the weighting function $w_{12}(\delta_1, \delta_2)$ with $w_{12}(\delta_1^{\text{gal}}, \delta_2^{\text{gal}})$. In the linear limit, $\xi \ll 1$, we get $v_{12}^{\text{gal}} = bv_{12}$ (9), in qualitative agreement with recent N-body simulations, which considered a whole
range biasing prescriptions, allowing nonlinear and/or non-local mapping of the mass density field
onto $\delta_{\text{gal}}$ (10). However, there are also simulations which show exactly the opposite: although the
galaxies do not trace the spatial distribution of mass, pairs of galaxies behave like pairs of test
particles moving in the gravitational field of the true mass distribution, and $v_{12}^{\text{gal}}(r) = v_{12}(r)$ (11).
Direct measurements of $v_{12}^{\text{gal}}(r)$ can help us decide which simulations and biasing schemes are more
believable than others. Indeed, one can measure $v_{12}^{\text{gal}}$ for different morphological classes of galaxies.
The linear bias model predicts $v_{12}^{(E)}/v_{12}^{(S)} = b^{(E)}/b^{(S)}$, where the superscripts refer to elliptical (E)
and spiral (S) galaxies. Observations suggest $b^{(E)}/b^{(S)} \approx 2$ and $b^{(S)} \approx 1$ (13). Hence, one expects
$v_{12}^{(E)}/v_{12}^{(S)} \approx 2$ if the linear bias model is correct and $v_{12}^{(E)}/v_{12}^{(S)} = 1$ in the absence of velocity
bias.

Measurements of $v_{12}(r)$ can be also used to determine $\Omega$. Indeed, if the mass correlation
function is well approximated by a power law, $\xi(r) \propto r^{-\gamma}$, $v_{12}$ at a fixed separation can be
expressed in terms of $\Omega$ and the standard normalization parameter $\sigma_8$. The latter quantity is the
root-mean-square contrast in the mass found within a randomly placed sphere of radius $8 \, h^{-1}\text{Mpc}$. Unlike the conventional linear perturbative expression for $v_{12}(r) \propto \Omega^{0.6} \sigma_8^2 r^{1-\gamma}$, our nonlinear
Ansatz provides the possibility of separating $\sigma_8$ from $\Omega$ by measuring $v_{12}$ at different values of $r$
[see the lowermost panel in Fig.1 below; see also ref. (12)].
We will now describe our measurements. The mean difference between radial velocities of a pair of galaxies is 
\[ \langle s_A - s_B \rangle_{\rho} = v_{12} \hat{r} \cdot (\hat{r}_A + \hat{r}_B)/2, \]
where \( s_A = \hat{r}_A \cdot \vec{v}_A \) and \( \vec{r} = \vec{r}_A - \vec{r}_B \). Here as before, the latin subscripts number the galaxies in the survey, \( A, B = 1, 2 \ldots \). To estimate \( v_{12} \), we minimize the quantity \( \chi^2(v_{12}) = \sum_{A,B} \left[ (s_A - s_B) - p_{AB} v_{12}/2 \right]^2 \), where \( p_{AB} \equiv \hat{r} \cdot (\hat{r}_A + \hat{r}_B) \) and the sum is over all pairs at fixed separation \( r = |\vec{r}_A - \vec{r}_B| \). The resulting statistic is (5)

\[ v_{12}(r) = \frac{2 \sum (s_A - s_B) p_{AB}}{\sum p_{AB}^2}. \] (4)

Monte-Carlo simulations show that this estimator is insensitive to biases in the way galaxies are selected from the sky and can be corrected for biases due to errors in the estimates of the radial distances to the galaxies (5). The survey used here is the Mark III standardized catalogue of galaxy peculiar velocities \((14,15,16)\). It contains 2437 spiral galaxies with Tully-Fisher (TF) distance estimates and 544 ellipticals with \( D_n - \sigma \) distances. The total survey depth is over 120 h\(^{-1}\)Mpc, with homogenous sky coverage up to 30 h\(^{-1}\)Mpc. The inverse TF and IRAS density field corrections for inhomogeneous Malmquist bias in the spiral sample agree with each other and give similar streaming velocities, with lognormal distance errors of order \( \sigma_{ln,d} \approx 23\% \). For the elliptical sample, \( \sigma_{ln,d} \approx 21\% \) and the distances assume a smooth Malmquist bias correction \((17)\).

The estimates from the spiral and elliptical are remarkably consistent with each other (Fig.1), unlike previous comparisons using the velocity correlation tensor \((18,19)\). For a velocity ratio \( R = v_{12}(E)/v_{12}(S) = 1 \), we obtain \( \chi^2 \simeq 1 \), while for \( R = 2 \) the \( \chi^2 = 2.1 \). The most straightforward interpretation of this result is that there is no velocity bias and the linear clustering bias model should be rejected. Its static character and the resulting failure to describe particle motion, induced by gravitational instability was pointed out earlier on theoretical grounds \((20)\). Our results can, however, be reconciled with linear bias model if it is generalized to allow scale-dependence, \( b = b(r) \).
Biasing factors for both galaxy types can be arbitrarily large at small separations, where $\xi(r) \gg 1$, if biasing is suppressed at large separations, where $|\xi(r)| < 1$. Indeed, in the nonlinear limit $w_{12}(b_1, b_2) \rightarrow b_1^2 \delta_1 \delta_2 / b_2 \xi = \delta_1 \delta_2 / \xi$, and hence $v^{\text{gal}}_{12}(r) \rightarrow v_{12}(r)$.

We obtained an estimate of $\sigma_8$ and $\Omega$ from the shape of the $v_{12}(r)$ profile as follows. We assumed that the shape of the mass correlation function $\xi(r)$ (but not necessarily the amplitude) is similar the shape of the galaxy correlation function estimated from the APM catalogue (8), consistent with a power law index $\gamma = 1.75 \pm 0.1$ (the errors we quote are conservative) for separations $r \leq 10 \, h^{-1}\text{Mpc}$. Given the depth of the Mark III catalogue we expect the covariance between estimates of $v_{12}(r)$ to be only weakly correlated at $r < 10 \, h^{-1}\text{Mpc}$; we use N-body simulations to determine the covariance of the estimates over this range of scales and use a $\chi^2$ minimization to obtain the 1-$\sigma$ constraints: $\sigma_8 \geq 0.7$ and $\Omega = 0.35^{+0.35}_{-0.25}$. Fixing $\sigma_8 = 1$ we obtain $\Omega = 0.35 \pm 0.15$ (Fig. 2).

We can obtain a more conservative constraint on $\sigma_8$ and $\Omega$ by examining a $v_{12}$ at a single separation, $r \equiv r_* = 10 \, h^{-1}\text{Mpc}$. Substituting $r = r_*$ and $\xi(r) \propto r^{-1.75}$ into Eqns. (1)-(2), we get

$$v_{12}(r_*) = -605 \sigma_8^2 \Omega^{0.6} (1 + 0.43 \sigma_8^2) / (1 + 0.38 \sigma_8^2)^2 \, \text{km/s}.$$  \hfill (5)

The above relation shows that at $r = r_*$, $v_{12}$ is almost entirely determined by the values of two parameters: $\sigma_8$ and $\Omega$. The uncertainties in the observed $\gamma$ lead to an error in eq. (5) of less than 10% for $\sigma_8 \leq 1$. In fact, at this level of accuracy and at this particular scale, our constraints depend only on the value of $\Omega$ and the overall normalization $\sigma_8$ but do not depend on other model parameters, such as the shape of $\xi(r)$. The streaming velocity, $v_{12}(r_*)$ depends on $\xi(r)$ only at $r < r_*$, so unlike bulk flows, it is unaffected by the behavior of $\xi(r)$ at $r > r_*$ [compare our Eqn. (1) with Eqn. (21.76) in ref. (2)]. Moreover, the dominant contribution to $v_{12}(r_*)$ comes from $\tilde{\xi}(r_*)$ -
an average of \( \xi(r) \) over a ball of radius \( r_\star \), so the details of the true shape of \( \xi(r) \) at \( r < r_\star \) have little effect on \( v_{12}(r_\star) \) as long as \( \sigma_8 \) (and hence, the volume-averaged \( \xi \)) is fixed. Hence, Eqn.\( (3) \) can provide robust limits on \( \sigma_8 \) and \( \Omega \) even if the assumption about the proportionality of \( \xi(r) \) to the APM correlation function is dropped. This statement can be directly tested by comparing predictions of Eqn.\( (3) \) with predictions of CDM-like models, all of which fail to reproduce the pure power-law behavior of the observed galaxy correlation function. When this test was applied to four models, recently simulated by the Virgo Consortium \( (7) \), we found that for fixed values of \( \sigma_8 \) and \( \Omega \), the predictions based on Eqn.\( (3) \) were within \( \leq 6\% \) of the \( v_{12}(r_\star) \), obtained from the simulations \( (21) \).

The measured value, \( -v_{12}(r_\star) = 280^{+68}_{-53} \) km/s (Fig.1), is inconsistent with \( \sigma_8 = 1 \) and \( \Omega = 1 \) at the 99% confidence level.

Our results are compatible with a number of earlier dynamical estimates of the parameter \( \beta \equiv \Omega^{0.6} \sigma_8 \) \( [\beta \text{ is sometimes defined as } \Omega^{0.6}/b, \text{ but } \sigma_8 \approx 1/b \text{ and the two definitions differ at most at the 10\% level, see, for example ref.\( (8) \)]. \) A technique, based on the action principle \( (22) \) gives \( \beta = 0.34 \pm 0.13 \); comparisons of peculiar velocity fields with redshift surveys based on the integral form of the continuity equation (called velocity-velocity comparisons) typically give \( \beta = 0.5 - 0.6 \) \( (23\text{-}26) \). Of the velocity-velocity comparisons, the one with the smallest error bars is the VELMOD estimate: \( \beta = 0.5 \pm 0.05 \) \( (24) \). This constraint has several advantages over others; in particular, it correctly takes into account cross calibration errors between different Mark III subcatalogues.

To illustrate the consistency of our results with velocity-velocity studies we will now compare our limits on \( \sigma_8 \) and \( \Omega \), derived from the shape of \( v_{12}(r) \) for a range of separations with constraints from our measurement of \( v_{12}(10\; h^{-1}\; \text{Mpc}) \) alone, combined with limits from VELMOD (Fig.2). Again we
find that a low-Ω universe is favoured: Ω < 0.65 and σ₈ > 0.7. The concordance region overlaps with the constraint derived from our measurements of v₁₂(r).

Our results disagree with the IRAS-POTENT estimate, β = 0.89 ± 0.12 (27). The IRAS-POTENT analysis is based on the continuity equation in its differential form; it uses a rather complicated reconstruction technique to recover the full velocity field from its radial component. The reason for the disagreement is not clear at present. One can think of at least two possible sources of systematic errors in the IRAS-POTENT analysis: (i) the reconstruction scheme itself (e.g., taking spatial derivatives of noisy data), and (ii) the nonlinear corrections adopted. The nonlinear corrections diverge like Ω⁻¹.₈ in the limit Ω → 0 (27). By contrast, the accuracy of the nonlinear corrections for the velocity-velocity is insensitive to Ω. The velocity-velocity approach is also simpler than IRAS-POTENT because it does not involve the reconstruction of the full velocity vector from its radial component measurements (although both approaches do require a reconstruction of galaxy positions from their redshifts). Note that our method is direct, not inverse: it does not require any reconstruction at all.

Finally, there is a potential caveat in the “no velocity bias” assumption in our own analysis. Although this assumption is based on empirical evidence from the two sets of galaxy types, the observational data is noisy and involves non-trivial corrections for Malmquist bias which could affect the two samples differently. Application of our approach to different data sets will clarify these issues. If contrary to our preliminary results, the streaming velocity turns out to be subjected to bias after all, such a finding may affect our estimates of σ₈ and Ω, based on the shape of the v₁₂(r) profile but not our rejection of the unbiased Einstein-de Sitter model. In this sense our differences with the IRAS-POTENT analysis do not depend on the presence or absence of velocity bias.
The advantages of the new statistic we have used here can be summarized as follows. First, $v_{12}$ can be estimated directly from velocity-distance surveys, without subjecting the observational data to multiple operations of spatial smoothing, integration and differentiation, used in various reconstruction schemes. Second, unlike cosmological parameter estimators based on the acoustic peaks, expected to appear in the cosmic microwave background power spectrum (28), the $\Omega$ estimate based on $v_{12}$ is model-independent. Finally, our approach offers the possibility to break the degeneracy between $\Omega$ and $\sigma_8$ by measuring the $v_{12}(r)$ at different separations.

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To linear bias enthusiasts, this result may appear puzzling. A textbook example of such a possibility is the spatial distribution of the gas and stars in a galaxy. Their distribution itself is biased, while their relative
velocities at separations comparable to the radius of the dark halo are not.

(12) N-body simulations show that the stable clustering solution \(u_{12} = 0, -v_{12}(r)/Hr = 1\) occurs for \(\xi > 200\) [ref. (7), see also B. Jain, Mon. Not. R. Astron. Soc. 287, 687 (1997)]. Note that in the limit \(r \to 0\) our Ansatz gives \(-v_{12}/Hr \to [2/(3 - \gamma)]\Omega_{0.6}^{1.0}(1 + \alpha)\), which for the range of parameters considered is generally different from unity (although of the right order of magnitude). It is possible to improve our approximation and satisfy the small separation boundary condition by replacing \((2/3)\Omega_{0.6}^{1.0}\) with the expression \([(1 - \gamma/3)F\xi(r) + (2/3)\Omega_{0.6}^{1.0}] / [F\xi(r) + 1]\), where \(F \approx 1/100\) is a fudge factor which ensures that \(-v_{12}/Hr \to 1\) when \(\xi \gg 100\) while the perturbative, large-scale solution remains unaffected. However, the stable clustering occurs at separations smaller than \(200h^{-1}\)kpc, which is a tenth of the smallest separation we consider here.

As a result, in our range of separations \(r\), Eqn. (1) is as close to fully nonlinear N-body simulations as the improvement, suggested above [see ref. (4), Fig. 2].

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(20) Under realistic circumstances one expects that gravitational growth of clustering pulls the mass with the galaxies. As a result, any clustering bias, introduced at the epoch of galaxy formation is likely to evolve towards unity at late times, at variance with the linear bias model, where \(b\) is time-independent [J. N. Fry, Astrophys. J. 461, L65 (1996); P. J. E. Peebles, astro-ph/9910234]. A similar behavior was seen in N-body simulations (11). Note also that the linear bias relation \(\delta^{\text{gal}} = b\delta\) is a conjecture which generally does not follow from the relationship between the correlation functions \(\xi^{\text{gal}} = b^2\xi\) unless we restrict our model to a narrow subclass of random fields.
Imagine that one of the CDM-like models is a valid description of our Universe. Let us choose the so-called ΛCDM model, recently simulated by the Virgo Consortium (7). It is defined by parameters $h = 0.7$, $\Omega = 0.3$, $\sigma_8 = 0.9$ and $\Omega_\Lambda = 0.7$, which is the cosmological constant's contribution to the density parameter. This model requires scale-dependent biasing because the predicted shape of $\xi(r)$ differs widely from the observed galaxy correlation function: at separations $hr/\text{Mpc} = 10$, 4, 2 and 0.1 the logarithmic slope of the mass correlation function reaches the values, given by $\gamma = 1.7$, 1.5, 2.5 and 1, respectively. As we expected, however, these wild oscillations do not affect the resulting $v_{12}(r_*)$. Indeed, the N-body simulations (7) give $v_{12}(r_*) = -220$ km/s, while substituting $\sigma_8 = 0.9$ and $\Omega = 0.3$ in Eqn.(5) gives $v_{12}(r_*) = -225$ km/s.

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Figure 1: The streaming velocities of 2437 spiral galaxies (top panel) and 544 elliptical galaxies (center panel) estimated from the Mark III catalogue. The error bars are the estimated 1σ uncertainties in the measurement due to lognormal distance errors, sparse sampling (shot noise) and finite volume of the sample (sample variance). The error bars were estimated from mock catalogues described in ref. (5). The small sample volume also introduces correlations between measurements of $v_{12}(r)$ at different values of $r$. To guide the eye, and show that although the two samples have different noise levels (because of much smaller number of galaxies in the elliptical sample), the $v_{12}(r)$ signal in both cases is similar, we also plot $v_{12}(r)$ calculated from equation (1) for a $\xi \propto r^{-1.75}$ power-law model with $\sigma_8 = 1.25$ and $\Omega = 0.3$. Three theoretical $v_{12}(r)$ curves are plotted (bottom panel) with $\xi \propto r^{-1.75}$, $\sigma_8 \Omega^{0.6} = 0.7$ and $\sigma_8 = 0.5$ (solid line), 1 (dotted line) and 1.5 (dashed line). These curves show how measurements of $v_{12}(r)$ can break the degeneracy between $\Omega$ and $\sigma_8$.

Figure 2: The blue region constrains the viable values of $\Omega$, the fractional mass density of the universe, and $\sigma_8$, the variance of mass fluctuations at $r = 8h^{-1}$Mpc, from the combination of the constraints on the streaming velocities (red region) and $\beta = \sigma_8 \Omega^{0.6}$ (green region). The streaming velocities are constrained at $r = 10h^{-1}$Mpc from the Mark III catalogue of peculiar velocities and $\beta$ is measured using the VELMOD comparison between the Mark III catalogue and the velocity field inferred from the IRAS redshift survey. The dashed line defines the 1-σ obtained from comparing expressions 1 and 2 with the Mark III catalogue from $2h^{-1}$Mpc to $10h^{-1}$ Mpc.
\[ \Omega \]

\[ \sigma_8 \]

- **Concordance**
- **0.45<\beta<0.55**
- **227\text{km/s} < \text{\(v_{12}\)} < 348\text{km/s}**
  (at 10 h\(^{-1}\)Mpc)

68\% C.L.

(1h\(^{-1}\)Mpc < r < 10h\(^{-1}\)Mpc)