Giant quantized Goos-Hänchen effect on the surface of graphene in quantum Hall regime

Weijie Wu, Shizhen Chen, Chengquan Mi, Wenshuai Zhang, Hailu Luo and Shuangchun Wen
Laboratory for Spin Photonics, School of Physics and Electronics, Hunan University, Changsha 410082, China

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We theoretically predict a giant quantized Goos-Hänchen (GH) effect on the surface of graphene in quantum Hall regime. The giant quantized GH effect manifests itself as an angular shift whose quantized step reaches the order of mrad for light beams impinging on a graphene-on-substrate system. The quantized GH effect can be attributed to quantized Hall conductivity, which corresponds to the discrete Landau levels in quantum Hall regime. We find that the quantized step can be greatly enhanced for incident angles near the Brewster angle. Moreover, the Brewster angle is sensitive to the Hall conductivity, and therefore the quantized GH effect can be modulated by the Fermi energy and the external magnetic field. The giant quantized GH effect offers a convenient way to determine the quantized Hall conductivity and the discrete Landau levels by a direct optical measurement.

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I. INTRODUCTION

The well-known Snell’s law and Fresnel formulae provide a clear geometrical-optics picture to describe the interaction of a plane wave with an interface[1, 2]. In 1947, the spatial shift of a beam of light in total internal reflection from a dielectric surface was demonstrated which does not follow perfectly the geometric optics prediction. This spatial shift was firstly observed by F. Goos and H. Hänchen[3], and was therefore referred to as the Goos-Hänchen (GH) effect. In a simple explanation, such a spatial GH shift is attributed to the penetration of evanescent field. For the past few decades, the spatial GH shift has been studied in a variety of systems, such as plasmonics[4–6], metamaterials[7–18], and quantum systems[19, 20]. In addition, the angular GH shift has been predicted for the case of partial reflection, which can be explained as the Fresnel filtering[21, 22]. This remarkable deviation from geometric optics has been measured experimentally on the surface of bulk crystals[23, 24].

Recently, graphene, as a two-dimensional atomic crystal, has received considerable attention due to its extraordinary electronic and photonic properties[27–29]. It has been demonstrated that the Fresnel formulae based on the certain thickness and effective refractive index fails to perfectly explain the the light-matter interaction of graphene. However, the Fresnel formulae based on the zero-thickness interface can give a complete and convincing description of all the experimental observations[30, 31]. It would be interesting how the GH effect occurs on the zero-thickness interface of graphene. More recently, the quantized spatial shifts in GH effect have been theoretically predicted in the quantum Hall regime of graphene-substrate systems[32]. However, the quantized steps are just a fraction of a micrometer in terahertz regime. Therefore, how to enhance this tiny effect is still a challengeable problem.

In this paper, we theoretically predict a giant quantized GH effect on the surface of graphene in quantum Hall regime. A general propagation model is established to describe the GH shifts on the surface graphene-on-substrate system and freestanding graphene. Based on this model, both the spatial and the angular GH shifts are obtained when a light beam impinges on the surface of graphene. Most of previous works have demonstrated that the beam shifts can be significantly enhanced near Brewster angle on the surface of bulk crystals[33–35]. As excepted, a giant angular GH shifts are also obtained on the surface graphene. More importantly, we find that the quantized steps in angular GH shifts can be significantly enhanced and reaches the order of mrad. Furthermore, we examine the role of the Hall conductivity in quantized GH effect. We believe this work to be fundamental significance and maybe provide a possible scheme for the direct optical measurement of the quantized effect in graphene.

II. GENERAL PROPAGATION MODEL

We begin by analyzing optical reflection from a planar interface of graphene-substrate system. The Fig. 1 illustrates a monochromatic Gaussian beam of light with finite beam width and non-totally reflection impinging from air to a planar interface of graphene-substrate system. The z axis of the laboratory Cartesian frame (x, y, z) is normal to the air-graphene interface (z = 0), that separates empty space (typically air), where z < 0, from a substrate that is covered with a graphene sheet, and a static magnetic field B is applied along the z axis. We use the coordinate frames (x, y, z) and (x, y, z) to denote incident beam and reflected one, respectively. The electric field amplitude of such a beam can be written as[44–46]

$$E_i \propto \exp \left\{ ikz_i - \frac{k x_i^2 + y_i^2}{2 Z_R + i\delta} \right\} \times (\hat{x}_f f_p + \hat{y}_f f_z), \quad (1)$$

where $Z_R = \pi w^2 / \lambda$ is the Rayleigh range, the vectors $\hat{x}, \hat{y}$ represent the directions of parallel and perpendicular to the incidence plane, respectively. And the polarization of the beam is determined by the complex-valued unit vector $\vec{f} = (f_p\hat{x} + f_z\hat{y})/(|f_p|^2 + |f_z|^2)^{1/2}$.

The reflected angular spectrum of the electric field is associated with the boundary distribution by means of the rela-

*Electronic address: hailulu@hnu.edu.cn
In this section, we begin to reveal the giant angular GH shifts in graphene and discuss the relation between the shift and incident angle. Then we try to explore the relationship between the magnitude of Brewster angle and external conditions of magnetic field and Fermi energy. We now determine the centroid of the reflected beam. At any given plane $z_r = 0$, the reflected field is determined and can be written as

$$E_r \propto \exp\left(ik_{z_r} - \frac{k_x^2 + k_y^2}{2Z_r + i\lambda_r}\right)$$

where $f_p = a_p \in \mathbb{R}$, $f_s = a_s \exp(i\eta)$.

In addition, the Fresnel reflection coefficients of the graphene-substrate system with an external imposed magnetic field can be obtained as [48–50]

$$r_{pp} = \frac{\alpha_r^p \alpha_s^p + \beta}{\alpha_r^s \alpha_s^p + \beta},$$

$$r_{ss} = \frac{\alpha_r^s \alpha_s^s + \beta}{\alpha_r^s \alpha_s^s + \beta},$$

$$r_{ps} = -2 \left(\frac{\mu_0 k_0 k_s \sigma_H}{\varepsilon_0 \alpha_r^s \alpha_s^s + \beta}\right).$$

Here, $\alpha_r^p = (k_x \pm k_z \varepsilon k_0 + k_z \varepsilon k_s \sigma_L / \omega / \varepsilon_0, \alpha_r^s = k_x \pm k_z + \omega \mu_0 \sigma_T, \beta = \mu_0 k_0 k_s \sigma_H^2 / \varepsilon_0, \kappa_z = k_z \cos \theta_t, \kappa_x = k_x \cos \theta_t; \theta_t$ is the refraction angle; $\varepsilon_0, \mu_0$ are permittivity and permeability in vacuum, respectively; $\varepsilon$ is the permittivity of substrate; $\sigma_L, \sigma_T$ and $\sigma_H$ denote the longitudinal, transverse, and Hall conductivity, respectively.

When the external imposed magnetic field is strong enough, the Hall conductivity of the graphene is quantized in integer multiplies of the fine structure constant, and we have [32]

$$\sigma_H = 2(2n_e + 1) \text{Sgn}[\beta] \frac{\mu_e^2}{2\pi\hbar}.$$ 

Here, $n_e = \text{Int} \left( \frac{\mu_e^2}{2\hbar |\beta|} \right)$ is the number of occupied Landau levels, $\mu_e$ and $\nu_e$ are the Fermi energy and the Fermi velocity, respectively. Obviously, Landau levels play an important role in Hall conductivity. Note that in the linear optical response of an important 2D atomic crystal model of Fresnel coefficients in graphene has been developed by fixing both the surface susceptibility and the surface conductivity [51].

### III. THE GIANT ANGULAR GOOS-HÄNCHEN SHIFTS

In this section, we begin to reveal the giant angular GH shifts in graphene and discuss the relation between the shift and incident angle. Then we try to explore the relationship between the magnitude of Brewster angle and external conditions of magnetic field and Fermi energy. We now determine the centroid of the reflected beam. At any given plane $z_r = 0$, the reflected field is determined and can be written as

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Here, $\alpha_r^p = (k_x \pm k_z \varepsilon k_0 + k_z \varepsilon k_s \sigma_L / \omega / \varepsilon_0, \alpha_r^s = k_x \pm k_z + \omega \mu_0 \sigma_T, \beta = \mu_0 k_0 k_s \sigma_H^2 / \varepsilon_0, \kappa_z = k_z \cos \theta_t, \kappa_x = k_x \cos \theta_t; \theta_t$ is the refraction angle; $\varepsilon_0, \mu_0$ are permittivity and permeability in vacuum, respectively; $\varepsilon$ is the permittivity of substrate; $\sigma_L, \sigma_T$ and $\sigma_H$ denote the longitudinal, transverse, and Hall conductivity, respectively.

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The Poynting vector related to the electromagnetic field can be obtained by $\hat{S} = \text{Re}(\partial \ln r_A / \partial \theta)$, where $r_A = R_A \exp(i\phi_A)$, $A \in \{pp, ss, ps\}$, $\rho_A = \text{Re}(\partial \ln r_A / \partial \theta)$, $\varphi_A = \text{Im}(\partial \ln r_A / \partial \theta)$, and $\chi_A = R_A^2 (\varphi_A^2 + \rho_A^2)$. If we consider vertical polarization (i.e. $a_p = 0$, $a_s = 1$, and $\eta = 0$), we can replace the $pp$ with $ss$ in above equation. Furthermore, since the vertical polarization is considered, the induced cross-polarization is $r_{ps}$ instead of $r_{pp}$.

Equation (11) gives the GH shift as a function of the beam propagation distance $z_r$. The first term is considered to represent the spatial GH shift, that is, the displacement will not change with $z_r$. If we use it in the conditions of total internal reflection and isotropy, we could get a result that is consistent with the Artmann formula [52]. Namely, when we make $r_{ps} = 0$ and $|R_{pp}| = 1$, we will get $D_{GH} = (\partial \phi_A / \partial \theta)/k$. Then the second term denote the angular GH shift. For more general cases, the derivative of the Fresnel reflection coefficients can be easily simplified, so an equation can be obtained by $\partial \ln r_A / \partial \theta = (\partial R_A / \partial \theta)/R_A + i\partial \phi_A / \partial \theta$. That is, the change of phase and amplitude reflectivity is responsible for spatial and angle shifts, respectively. We mainly discuss the angular shift, that is second term in above equation, so we get the important result

$$D_{GH} = \frac{2(R_{pp}^2 \rho_{pp} + R_{pp}^2 \rho_{ps})}{2k(R_{pp}^2 + R_{pp}^2)Z_R + \chi_{pp} + \chi_{ps}}. \quad (12)$$

As shown in the Fig. 2, the angular GH shifts are quantized functions of the Fermi energy and magnetic field. Plateaulike behavior can be observed by tuning Fermi energy, $\mu_F$, and magnetic field, $B$. The quantized Hall conductivity can be regarded as the physical origin of Plateaulike behavior. The quantized steps reach the order of $2\pi$ near the Brewster angle and can be determined by a direct optical measurement [53]. And, due to that the Hall conductivity is quantified, the quantized angular shift can be obtained. We now consider the difference of angular deviation for incidence angle near and far away from Brewster angle[Fig. 2(a) and Fig. 2(b)]. Remarkably, the angular shift for incidence angle near the Brewster angle will be greater, at the same Fermi energy and magnetic field.

But if we research it further, we will find that angular shift is not only influenced by incident angle, the shift is also related to the quantized Hall conductivity. In fact, the Landau levels are proportional to Fermi energy squared, but they are inversely proportional to magnetic field. And, Landau levels play an important role in Hall conductivity. From Eq. (2), it can be seen that the Hall conductivity is decreased as the Fermi energy decreased or the magnetic field increased. But it is worthy noting that the quantized steps width can be significantly enhanced. From Eq. (6), the quantized steps width of Hall conductivity have a close relationship with Brewster angle. In other words, from Fig. 3(b), the Brewster angle will dramatically change in the case of narrow quantized steps. This directly leads to the change of angular deviation. In the region of high magnetic field, this phenomenon is not obvious. This is consistent with our previous statement. Due to the wider quantized steps, the Brewster angle is insensitive to the changing magnetic field and Fermi energy. Compare Fig. 3(c) with Fig. 3(d), it can be seen that only when we maintain the magnetic field at a relatively small value ($B = 5T$), which leads a narrow quantized steps, the peak of angular shift will be observed by tuning Fermi energy.
FIG. 3: The role of quantized Hall conductivity in quantized angular GH shifts. (a) Hall conductivity as function of magnetic field in different Fermi energy, $\mu_F = 150, 250, 350, \text{ and } 450 \text{ meV}$. (b) The magnitudes of Brewster angle. (c) and (d) show the changes of the angular shift in different Fermi energy and the magnetic field.

become sensitive and move to the right with the increase of the Fermi energy. Of course, due to the magneto-optical effect, the behavior of angular deviation in the graphene-substrate system is different from that in the glass-air system [24]. In addition, the reflection coefficient, $r_{pp}$, will approach zero near the Brewster angle and change its sign across the angle, which means the electric field reverses its directions. So, when the incident angle is smaller than the Brewster angle, the angular shift is positive, and in the opposite case, the angular shift is negative. For vertical polarization, the angular deviation is very small. This rule of change seems to be an enlightenment for us, if we appropriately control the external conditions, the angular shift can be modulated.

Then, we consider two special cases. Firstly, if we used a polarizer to eliminate cross-polarization component, namely $R_{ps}$, or used an isotropic reflected medium, which has also not cross-polarization component $R_{pp}$ and $R_{sp}$ in reflection matrix in the Eq. (2), a modified angular GH shifts can be obtained. Here, the horizontal polarization is only discussed. We now consider the effect of cross-polarization component $R_{ps}$ on the GH shifts at different magnetic field. So, we have modified expression

$$\Theta_{GH} = -2kR_{pp}^2\rho_{pp}/2kR_{pp}^2Z_R + \chi_{pp}.\quad (13)$$

Next, we compare the angular GH shift with the modified GH shift. From Fig. 4(a), there are giant quantized angular GH shifts with the change of magnetic field. And the peak of $\Theta_{GH}$ near the Brewster angle has a radian, which is more obvious in a area with narrow quantized steps. This proves our previous statement: If quantized steps were narrowed, the angular GH shifts would be sensitive to changing magnetic field and Fermi energy. So, the position of peak will be moved. From Fig. 4(b), the peak value of $\Theta'_{GH}$ near the area, where magnetic field is 5 T, is approximately 20 mrad larger than that of $\Theta_{GH}$ in the corresponding range, that is, beam propagation will produce a deviation of 2 centimeter per meter.

For further analysis of this difference, it is plotted that the magnitude of the cross-polarization reflection coefficients, $R_{ps}$. As shown in the Fig. 5(a), when magnetic field is decreased, the magnitude of $R_{ps}$ will increase. In fact, our previous statement still works. Widen quantized steps will cause $R_{ps}$ to be insensitive to the changing magnetic field and Fermi energy. From Fig. 5(b), it is clear that a significant difference

FIG. 4: Compare the original and the modified angular GH shifts. (a) The giant quantized GH shift in 450 meV Fermi energy (similarly hereinafter). (b) The modified angular GH shift.
FIG. 5: The role of cross-polarization in angular GH effect. (a) Relative amplitude of the cross-polarization, $R_{ps}$, as a function of angle of incidence, $\theta$, and magnetic field, $B$, for Fermi energy $\mu_F = 450$ meV. (b) The difference between the normal GH shift, $\Theta_{GH}$, and the modified GH shift, $\Theta_{GH}''$. Between the two cases near the Brewster angle. However, it is worthy to notice that, the difference will decrease in the range far away from the Brewster angle or in the interval of widen quantized steps. That is to say, there are no difference (or very small difference) of angular GH shifts in the above two range. At this point, we could get $\Theta_{GH}'' = -\rho_p / kZ_R$, which is in good agreement with the theoretical result of Aiello [44]. That is, the $R_{ps}$ is responsible for difference between two shifts only near the Brewster angle or in the interval of narrow quantized steps.

Finally, we consider the case of freestanding graphene [54], where we can make the relative refractive index of the substrate being tend to $n = 1$. As shown in Fig. 6, it can be seen that when the refractive index approaches 1, the magnitude of incidence angle, which is corresponding to peak, is increased, that is, the Brewster angle will approach grazing incidence on freestanding graphene. On the other hand, when the magnetic field is high or Fermi energy is low in quantum Hall regime, the magnitude of Brewster angle is decreased, the quantized steps are also wide at this time, which proves that the Brewster angle in this moment is insensitive to changing magnetic field or Fermi energy, namely, wide quantized steps lead to GH shift to be insensitive to changing magnetic field and Fermi energy.

IV. CONCLUSIONS

In conclusion, we have theoretically predicted a giant quantized Goos-Hänchen (GH) effect on the surface of graphene in quantum Hall regime. A strict model has been established and revealed a giant quantized angular GH shifts, which is dominated by a change of reflectance, for incidence angle near the Brewster angle on reflection. The quantized steps of angular deviation have been greatly enhanced for incident angles near the Brewster angle. We have found that when magnetic field is high or Fermi energy is low in quantum Hall regime, the quantized steps of Hall conductivity can be significantly widen. Meanwhile, quantized Hall conductivity is related to the discrete Landau levels in quantum Hall regime. In addition, we have demonstrated that cross-polarization component can not be ignored for incidence angle near Brewster angle or in the case of narrow quantized steps. And we have found that
Brewster angle would tend to grazing incidence in the case of freestanding graphene. We can determine the quantized Hall conductivity and the discrete Landau levels by a direct optical measurement. These findings provide a pathway for modulating the GH effect and thereby open the possibility of developing new nanophotonic devices.

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