Side Lobe Reduction Using Non-Uniform Linear Phased Arrays

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Abstract. Non-uniform amplitude excitation arrays are nowadays very desirable in the modern wireless communication systems. They have ability to provide required radiation characteristics such as low side lobes and good directivity. The uniform amplitude excitation arrays are good in providing good directivity and narrow beam width. However, these desired radiation features are come at the cost of relatively high side lobes. Thus, these types of arrays are not widely used in the wireless communication systems especially when these arrays are operated in a noisy and crowded environment that contains many interfering signals. Non-uniform amplitude excitations such as Dolph and Taylor are considered in this paper and their performances are investigated under various array parameters. Simulation results show that as the amplitude excitations of the array elements decaying at the edge elements more reduction in the sidelobe patterns can be obtained.

1. Introduction

The performance in terms of the sidelobe level of the linear uniform amplitude excitations arrays is not acceptable in many practical applications. Many approaches can be used to improve the radiation characteristics of the antenna arrays. One of the simplest approaches is the modification in the amplitude excitations of the array elements such that it has highest value at the centre of the array and then decaying gradually toward the array ends [1-7]. By this way the sidelobe level in the corresponding array radiation pattern can be significantly reduced. However, the main beam is found to be broadening which is not desirable due to its negative effect on the directivity or gain of the array. Other methods used phase excitations instead of the amplitudes to control the sidelobe level in which the phase shifters are needed [8-9]. These approaches that depend on the element phase excitations are more complex because these phases are also responsible for main beam scanning. Thus, a good control on both sidelobe reduction and main beam scanning is not easy to be implemented in practice. Much more complicated systems may be encountered when combining the amplitude and phase excitations of the array elements in one design.

Other researchers suggested controlling the locations of the array elements to achieve the required sidelobe level [10-13]. In this case, lowest sidelobe levels can be obtained by modifying the separation distances of the array elements either in analytical or numerical methods. Many different optimization algorithms were also used to design such arrays. Nevertheless, the Dolph distribution has many advantages that can be fully exploited in the work [14].

In this paper, the authors are focusing on studying the simplest analytical method that can be used to modify the amplitude excitations of the array elements while the phase excitations are all assumed constant. The Dolph distributions [15-16] are analyzed and studied and their corresponding radiation...
patterns are shown. Some comparisons are also presented to show the effectiveness of the design. Many design parameters are investigated and different array sizes such as small, medium and large are considered.

The remainder of this paper is organized as follows. In section 2, the background theory of the linear uniform amplitude excitation arrays is presented. Then, in section 3, the non-uniform amplitude excitation arrays are explained in details, while the simulation results and discussion are elaborated in section 4. Finally, the conclusion remarks of the paper are drawn in section 5.

2. Background Theory

Phased and Linear antenna arrays can be classified according to their element amplitude distribution where in general there are equal and unequal element excitations. The equal element excitations are commonly referred to as uniform arrays [17]. Uniform arrays are simple and their practical designs are very cheap, thus, they are commonly and widely used in practice. Here the term uniform is in the sense that the separation distance between any two adjacent elements is constant and equal and all the array elements have the same amplitude currents. As an example, consider the end fire array in which the amplitude currents are all constant. Its radiation direction is along the line of the array elements. Whereas, the phase of each element in the end fire array must be fed with a value equal to the difference to the separation distance of the adjacent elements.

Another practical case is the broadside array, which is the most commonly used antenna array in practice. In broadside array, a number of identical parallel elements are arranged along a line perpendicular to the array axis line. The broadside pattern can be obtained in a linear array with identical elements by exciting them with equal magnitude and phase. The array factor of such arrays is given by [15]

\[
(AF)_n = \frac{\sin N (k \cos \theta + \beta/2)}{N (k \cos \theta + \beta/2)}
\]  

Where N represents number of elements, \(\beta\) progressive phase, k phase constant and d are the spacing between array elements. There are different array parameters such as Half Power Beam Width (HPBW), First Null Beam Width (FNBW), Side Lobe Level (SLL) and the Directivity (D) which their mathematical equations are given below [8].

\[
HPBW = 2 \left[ \pi/2 - \cos^{-1} \left( \frac{1.391 \lambda}{\pi N d} \right) \right] 
\]  

\[
FNBW = 2 \left[ \pi/2 - \cos^{-1} \left( \frac{A}{\pi N d} \right) \right]
\]  

\[
D = 2N \left( \frac{\lambda}{d} \right)
\]

3. Non-Uniform Amplitude Excitation Arrays

Non-uniform linear arrays are those in which the elements are fed with the currents of unequal magnitude. Some examples of the existing and well-known non-uniform arrays are Dolph-Tchebyscheff and Taylor arrays. Dolph arrays are considered as non-uniformly excited and equally spaced arrays in which the amplitudes of the antenna elements in the array are arranged according to the Dolph polynomial.

3.1 Dolph-Tchebyscheff Arrays

This type of array was originally introduced by the Dolph and it has many important and practical applications. The Dolph method is based on the properties of the Chebyshev polynomials, which gives the possibility of obtaining the maximum gain for a fixed or required level of the side lobes. The Chebyshev polynomial is defined as having equal ripples in the region bounded by \( x = \pm 1 \).
Additionally, the amplitude is varied between +1 and −1. The polynomial value outside this region rises exponentially [15, 16]. The polynomial can be computed according to the following equations

\[ T_m(z) = \cos[m \cos^{-1}(z)] \quad -1 \leq z \leq +1 \]  
\[ T_m(z) = \cosh[m \cosh^{-1}(z)] \quad z < -1, \ z > +1 \]  

Additionally, the half-power beam width and the directivity can be found by introducing a beam broadening factor, \( f \), given approximately by [15]

\[ f = 1 + 0.636 \left( \frac{2}{R_0} \cosh \left( \sqrt{(\cosh^{-1} R_0)^2 - \pi^2} \right) \right)^2 \]  

While the directivity of this array type can be found as

\[ D_0 = \frac{2R_0^2}{1 + (R_0^2 - 1) f} \]  

The simplest method to compute the half-power beam width of the Tschebyscheff arrays is by multiplying the half-power beam width of the uniform array by the beam broadening factor.

### 3.2 Taylor Arrays

Taylor distributions are another popular method in designing the arrays. It was first introduced by Taylor in 1955 and the design makes a compromise between beamwidth and sidelobe level which are the most wanted and desirable radiation features. In the theory, the side lobes are maintained at an equal with certain level. Since the side lobes are of equal level in such distributions and extend to infinity, this implies an infinite power [15]. In other words, the Taylor technique leads to a pattern whose first few side lobes (which they are closest to the main beam) are maintained at an equal and certain level; the remaining other side lobes decay monotonically. In fact, even the level of the closest side lobes exhibits a slight monotonic decay. This decay is a function of the space \( u \) over which these side lobes are required to be maintained at certain equal level. As this space increases, the rate of decay of the closest side lobes decreases [16]. For a large space of \( u \) (over which the closest side lobes are required to have an equal ripple), the rate of decay is negligible. Theoretically the normalized space factor that yields a pattern with equal-ripple side lobes is given by [15]

\[ SF(\theta) = \frac{\cosh \sqrt{(\pi A)^2 - u^2}}{\cosh (\pi A)} \]  

The Taylor space factor is given by

\[ Sf(u, A, n) = \frac{\sin(u)}{u} \prod_{n=1}^{n-1} \frac{1 - \left( \frac{u}{\pi n} \right)^2}{1 - \left( \frac{u}{\pi n} \right)^2} \]  

While \( u \) and \( u_n \) are obtained using the following equations

\[ u_n = \pi v_n = \pi \frac{1}{\lambda} \cos \theta_n \]  
\[ u = \pi v = \pi \frac{1}{\lambda} \cos \theta \]  

where \( \theta_n \) represents the null locations. Additionally, the Taylor arrays have a parameter called scaling factor and it is represented by the symbol \( \sigma \). This factor controls the spaces of the inner nulls so that
they blend smoothly with the outer ones. This factor also controls the width of the beam width in the Taylor pattern in which its beam width is usually greater than that of the Dolph-Tschebyscheff, and it can be given by

$$\sigma = \frac{\bar{n}}{\sqrt{A^2 + (\bar{n} - \frac{1}{2})^2}}$$  \hspace{1cm} (13)

Where the peak sidelobe level in decibels can be approximately calculated

$$SLL \cong -20 \log(cosh(\pi A))$$  \hspace{1cm} (14)

Finally, the half power beam width is given by

$$HPBW \approx 2 \sin^{-1}\left\{\left[\frac{2\sigma}{\pi l}((cosh^{-1}R_0)^{-1} - (cosh^{-1}\frac{R_0}{\sqrt{2}})^{2/2}\right]^{1/2}\right\}$$  \hspace{1cm} (15)

It is worth to mention the main applications of the non-uniform amplitude excitations arrays which can be summarized as:

- Non-Uniform amplitude excitations such as Dolph and Taylor are generally used to generate low side lobe level in which they are extensively used in the wireless communication systems to suppress the interfering signals.
- They compromise between the required or wanted sidelobe level and the desired half power beam width.
- Rectangular planar arrays which are used in radar applications can be created by combining many linear arrays parallel to each other [5].
- The Dolph and Taylor arrays can be extended to generate other types of beam pattern such as fan beam which is useful for coverage a wide space in one plane and generate a narrow beam in the orthogonal plane.
- These arrays have proved their effectiveness in the shipboard applications [15].

4. Simulation Results and Discussions

In this section, the results are obtained using the computer simulation and the theoretical approaches. In all considered arrays, the total number of the array elements is taken 10, 30, and 60 elements to illustrate the small, medium, and large array sizes. The array elements are distributed on the x-axis and on the linear form. The separation distance between any two successive elements is fixed and uniform at \(d=\lambda/2\). For simplicity, the mutual coupling between the array elements and other scattering effects are neglected. For uniform amplitude excitation arrays, the amplitude of each element is chosen to be one and its phase is set to zero.

In the first example, the number of array elements is chosen to be \(N=10\) elements. Figure 1 show the results of both Dolph and the Taylor array patterns. For comparison the uniform array pattern is also included in this figure. ‘Figure 2’ show the corresponding element amplitude excitations of these tested arrays. For Dolph and Taylor arrays, the required side lobe level is set in advanced during the design stage to be at -20 dB.
In the second example, the number of the array elements is chosen to be $N=30$ elements. Figure 3 show the results of the uniform, Dolph and the Taylor array patterns, while Figure 4 show the corresponding element amplitude excitations of these tested arrays. In this case, the wanted side lobe level of the Dolph and Taylor arrays is chosen at -30 dB.

Figure 1. Radiation patterns for uniform, Dolph, and Taylor arrays and $N=10$ elements

Figure 2. Amplitude excitations for the uniform, Dolph, and Taylor arrays and $N=10$ elements
In the third example, the number of the array elements is chosen to be N=60 elements. Figure 5 show the results of the uniform, Dolph and the Taylor array patterns, while Figure 6 show the corresponding element amplitude excitations of these tested arrays. In this case, the wanted side lobe level is chosen at -60 dB.

**Figure 3.** Radiation patterns for uniform, Dolph, and Taylor arrays and N=30 elements

**Figure 4.** Amplitude excitations for the uniform, Dolph, and Taylor arrays and N=30 elements
From these figures, it can be seen that as the number of array elements increases the amplitude excitations of the Dolph and Taylor arrays become more identical to each other. Further, in all cases, the required side lobe levels have been effectively satisfied at the cost of getting wider beam widths and relatively lower directivities. In order to highlight these important features, some numerical calculations were carried out and presented in Table 1 below for a number of array elements equal to N=10, 16, and 30 elements respectively. In this table, the Dolph array is considered and the wanted side lobe level is chosen to be at -20 dB. Furthermore, two new important measures that compute the area under the main beam and the side lobe regions are introduced. By this way, we may have an interesting idea about how these areas may change which they are related to the directivity and the sidelobe. The area under the main beam region (AUMB) is computed by taking only the areas

![Figure 5. Radiation patterns for uniform, Dolph, and Taylor arrays and N=60 elements](image1)

![Figure 6. Amplitude excitations for the uniform, Dolph, and Taylor arrays and N=60 elements](image2)
bounded between first null to null beam width, while the areas under the sidelobe regions (AUSL) are computed from areas that located outside the main beam region.

Table 1. Performance Measures of the uniform and Dolph Arrays

| Features       | Uniform Array | Non-Uniform Dolph Array |
|----------------|---------------|-------------------------|
|                | N=10          | N=16                    | N=30        | N=10          | N=16                    | N=30        |
| Directivity [dB]| 10.01         | 12.05                   | 14.78       | 9.84          | 11.87                   | 14.42       |
| HPBW[deg]      | 10.1          | 5.7520                  | 3.0         | 11.20         | 6.520                   | 3.52        |
| Taper Efficiency| 1             | 1                       | 1           | 0.9622        | 0.9609                  | 0.9207      |
| Peak SLL [dB]  | -13.23        | -13.23                  | -13.23      | -20           | -20                     | -20         |
| AUSL           | 0.038         | 0.041                   | 0.093       | 0.0442        | 0.0531                  | 0.068       |
| AUMB           | 0.256         | 0.16                    | 0.089       | 0.262         | 0.1796                  | 0.0917      |

From this table, it can be seen that the area under the sidelobe region is generally decreasing with lower side lobe level. More important, the area under the main beam is found to be increased with reduced side lobe level. Thus, the directivity of the Dolph array is little lower than that of the uniform array.

5. Conclusions

From this study, it is found that there is a great need for making a compromise between the wanted reduction in the side lobe level and the obtained broadening in the main beam. Further it is found that the side lobe level is inversely proportional to the width of the main beam or the directivity of the array. Different array sizes are considered and studied to generate certain side lobe levels. The simulation results show that, in general, the uniform amplitude excitation arrays are able to provide higher directivity and narrower half power beam width among all other tested non-uniform excitation arrays. Nevertheless, the side lobe level of all designed Dolph and Taylor arrays are found to be satisfactory and satisfying the wanted low side lobe levels. Further, the performances of both Dolph and Taylor arrays were found almost same for large array sizes. The practical implementation of the non-uniform amplitude excitation arrays is also a great challenging issue where they usually require attenuators and the power dividers at each single element to realize the computed amplitudes. This study can be further extended to include adaptive or optimization algorithms [18-20] that optimize between the side lobe level and the main beam width.

6. References

[1] Mohammed, J. R. and Sayidmarie, K. H. 2012 A New Technique for Obtaining Wide-Angular Nulling in the Sum and Difference Patterns of Monopulse Antenna, IEEE Antennas and Wireless Propagation Letters, vol.11, pp. 1242-1245.

[2] Mohammed, J. R. 2013 Design of Printed Yagi Antenna with Additional Driven Element for WLAN Applications, Progress in Electromagnetics Research C, Vol. 37, 67-81, ISSN:1937-8718.
[3] Mohammed, J. R. 2013 Phased Array Antenna with Ultra-Low Sidelobes, Electronics Letters, vol. 49, issue 17, pp. 1055-1056.

[4] Sayidmarie, K. H. and Mohammed, J. R. 2013 Performance of a Wide Angle and Wideband Nulling Method for Phased Arrays, Progress in Electromagnetics Research M, vol. 33, pp. 239-249.

[5] Mohammed, J.R, and Sayidmarie, K. H. 2014 Sidelobe Cancellation for Uniformly Excited Planar Array Antennas by Controlling the Side Elements, IEEE Antennas and Wireless Propagation Letters, vol.13, pp. 987 - 990.

[6] Mohammed, J. R. and Sayidmarie, K. H. 2014 A Null Steering Method by Controlling Two Elements, IET Microwaves, Antennas & Propagation, vol. 8, issue 15, pp.1348-1355.

[7] A., R., and K., A. 2016 Design and Analysis of Broadside Arrays of Uniformly Spaced Linear Elements. International Journal of Computer Applications, 156(6), 19–24. https://doi.org/10.5120/ijca2016912477

[8] Mohammed, J.R. 2017 Optimal Null Steering Method in Uniformly Excited Equally Spaced Linear Array by Optimizing Two Edge Elements, Electronics Letters, vol. 53, issue 11.

[9] Mohammed, J.R. 2018 Element Selection For Optimized Multi-Wide Nulls in Almost Uniformly Excited Arrays IEEE Antennas and Wireless Communication Letters. vol. 17, issue 4, PP. 629-632.

[10] K.P.D. 2017 Study of Broadside Linear Array Antenna with Different Spacing and Number of Elements. International Journal of Advanced Engineering Research and Science, 4(5), 190–194. https://doi.org/10.22161/ijaers.4.5.30.

[11] Mohammed, J.R. 2018 Thinning a Subset of Selected Elements for Null Steering Using Binary Genetic Algorithm", Progress in Electromagnetics Research M, vol. 67, pp. 147-157.

[12] Mohammed, J.R. 2019 Obtaining Wide Steered Nulls in Linear Array Patterns by Controlling the Locations of Two Edge Elements AEÜ International Journal of Electronics and Communications, vol. 101, pp. 145–151.

[13] Panduro, M. A., Mendez, A. L., Dominguez, R., & Romero, G. 2006 Design of non-uniform circular antenna arrays for side lobe reduction using the method of genetic algorithms. AEU - International Journal of Electronics and Communications, 60(10), 713–717. https://doi.org/10.1016/j.aeue.2006.03.006.

[14] Mohammed, J.R Sayidmarie, K. H. 2017 Synthesizing Asymmetric SideLobe Pattern with Steered Nulling in Non-uniformly Excited Linear Arrays by Controlling Edge Elements International Journal of Antennas and Propagation, vol. 2017, Article ID 9293031, 8 pages.

[15] Balanis, C. A. 1996 Antenna Theory: Analysis and Design, 2nd Edition (2nd ed.). Wiley.

[16] Milligan, T. A. 2005 Modern Antenna Design. Wiley.

[17] Mohammed, J.R. 2017 Synthesizing Sum and Difference Patterns with Low Complexity Feeding Network By sharing Element Excitations International Journal of Antennas and Propagation, vol. 2017, Article ID 2563901, 7 pages.

[18] Mohammed, J.R and Sayidmarie, K. H. 2017 Performance Evaluation of the Adaptive Sidelobe Canceller with various Auxiliary Configurations AEÜ International Journal of Electronics and Communications, vol. 80, pp. 179–185.

[19] Mohammed, J.R. 2018 A New Antenna Array Pattern Synthesis Method with Sidelobe Control International Journal of Telecommunication, Electronics, and Computer Engineering, vol. 10. No, 3, pp. 31-36.

[20] Mohammed, J.R. 2019 An Optimum Side Lobe Reduction Method with Weight Perturbation Journal of Computational Electronics, US Springer, vol.18, no.2, pp.705-711.