Absorption cross section of RN and SdS extremal black hole

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The nature of scalar wave functions near the horizon of Reissner Nordstrom (RN) extremal and Schwarzschild-de Sitter (SdS) extremal black holes are found using WKB approximation and the effect of reflection of waves from the horizon. The absorption cross section $\sigma_{abs}$ when RN extremal and SdS extremal black holes placed in a Klein-Gordon field is calculated.

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1. Introduction

Black holes are one of the most enigmatic constructs in the present day physics. On the one hand, they are the most simple predictions of the general relativity and on the other hand, they are the paradigmatic objects to test possible quantum theories of gravity. To obtain a deeper theoretical understanding of these extreme objects, many authors have considered the scattering of coherent waves. If the wave length of the incident wave is comparable with the size of the event horizon, then the wave will be diffracted by the black hole. Diffraction effects are responsible for many interesting phenomena in nature, so wave scattering from black holes is an interesting field of its own right, even if observations may not realizable in practice.

A considerable effort has taken place in studying the waves scattered off by black holes. Both numerical and analytical methods in solving the various wave equations in black hole scattering have been developed. Interest in the absorption of quantum waves by black hole was reignited in the 1970s, following Hawking’s
discovery that black holes can emit, as well as scatter and absorb, radiation. Hawking showed that the evaporation rate is proportional to the total absorption cross section. Unruh found the absorption cross section for massive scalar and Dirac particles scattered off by small non rotating black holes. In a series of papers Sanchez considered the scattering and absorption of massless scalar particles by an uncharged, spherically symmetric black hole.

Another quantum effect of interest is that event horizons need not be fully absorptive type but can reflect waves falling on it. It is also proposed that event horizons has a finite energy width. 't Hooft explained the horizon of the black hole as a brick wall so that the outer horizon \( r_+ \) spreads into a range of \( (r_+ - \Delta, r_+ + \Delta) \). Quantum horizon concepts were introduced by Mu-Lin Yan and Hua Bai. The relevant equation governing a scattering process in a black hole space time is analogous to Schrodinger type equations governing scattering phenomena in quantum mechanics. Hence the standard techniques used to study quantum scattering can be used to study scattering problems in black hole space time.

The work of Wang et al. shown that a non extremal RN black hole cannot turn into an extremal one by assimilating an in falling charged particle and shell. So in the present work we are interested in studying the scattering of scalar waves in the RN extremal and SdS extremal space-time. Earlier several authors have studied scattering of scalar and Fermi fields under different black hole space time and calculated absorption cross sections. In all these calculations the black hole is assumed to be capable of absorbing the radiation falling on it, but here we consider that both absorption and reflection could take place at the horizon of black holes.

The absorption cross section, of scalar waves in Schwarzschild-de Sitter space time, of charged scalar wave in RN space time and of dirac wave in Schwarzschild space time were found earlier. In the present work we use WKB approximation to get the solution of radial wave equation in the vicinity of the horizon of black holes. In section 2 we explain how to obtain the absorption cross section of RN extremal black hole in low energy limit. Section 3 contains a calculation of absorption cross section scalar wave scattered off by SdS extremal black hole. Here we take into consider both reflection and absorption properties of the black hole horizon. Section IV concludes the paper.

2. RN black hole - Extremal case

The RN black hole’s (event and inner) horizons in terms of the black hole parameters are given by, \( r_\pm = M \pm \sqrt{M^2 - Q^2} \), where \( M \) and \( Q \) are respectively mass and charge of black hole. In extreme case, these two horizons coincide, i.e, when, \( M = Q \), \( r_\pm = M = r_0 \). The metric then is given by,

\[
ds^2 = \left(1 - \frac{r_o}{r}\right)^2 dt^2 - \frac{1}{\left(1 - \frac{r_o}{r}\right)^2} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2).
\]
Klein-Gordon equation in a curved space time is,
\[ \left[ \frac{1}{\sqrt{-g}} \partial_\kappa \left( \sqrt{-g} g^{\kappa\nu} \partial_\nu \right) + \mu^2 \right] \Phi (r, \theta, \phi, t) = 0. \] (2)

Thus, the above equation becomes
\[ \left[ \frac{1}{(1 - \frac{r_0}{r})^2} \frac{\partial^2}{\partial t^2} - \frac{1}{r^2} \frac{\partial}{\partial r} (r - r_0)^2 \frac{\partial}{\partial r} - \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial}{\partial \theta} - \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2} + \mu^2 \right] \Phi (r, \theta, \phi, t) = 0. \] (3)

Writing,
\[ \Phi (r, \theta, \phi, t) = \exp \left( -\iota \epsilon t \right) Y_{lm} (\theta, \phi) \Phi_l (r), \] (4)

where \( \epsilon, l, m \) are energy, momentum and its projection, while \( \Phi_l (r) \) is a radial function. Substituting Eq. (4) in Eq. (3) we get,
\[ \Phi''_l (r) + \frac{2}{r - r_0} \Phi'_l (r) + \left( \frac{\epsilon^2 r^4}{(r - r_0)^2} - \frac{\mu^2 r^2}{(r - r_0)^2} - \frac{l(l+1)}{(r - r_0)^2} \right) \Phi_l (r) = 0. \] (5)

To study the scattering problem, we divide the space time outside the black hole into 3 regions. We start the three different regions starting from the horizon as shown below.

2.1. Region 1: \( r \to r_0 \)

Now we solve the wave equation in the vicinity of horizon, i.e., as \( r \to r_0 \), using the WKB approximation \( \Phi = \exp \left( -\iota \int k(r) dr \right) \), in Eq. (5) and equating the real part we will get the radial wave number \( k(r, l, \epsilon) \) from the corresponding equation of motion: therefore,
\[ k (r) = \pm \left[ \epsilon^2 - \left( \frac{L^2}{r^2} - \mu^2 \right) \left( 1 - \frac{r_0}{r} \right)^2 \right]^{\frac{1}{2}} \frac{r^2}{(r - r_0)^2}. \] (6)

Thus near the horizon, i.e., as \( r \to r_0 \),
\[ k (r \to r_0) = \pm \frac{\epsilon r_0^2}{(r - r_0)^2}. \] (7)

Therefore, the wave function in the region \( r \to r_0 \) can be written as,
\[ \Phi_l (r) = \exp \left( \mp \iota \int \frac{\epsilon r_0^2 dr}{(r - r_0)^2} \right) = \exp \left( \mp \iota \frac{\epsilon r_0^2}{r_0 - r} \right), \] (8)

i.e.,
\[ \Phi_l (r) = \exp \left( \mp \iota \frac{\epsilon r_0^2}{r_0 - r} \right). \] (9)
Let us describe the radial motion with the help of the wave function \( \Phi_l(r) \). Using Eq.(8) the wave function in the vicinity of horizon can be written, assuming that the wave gets reflected at the horizon and having a reflection coefficient \(|R|\), as

\[
\Phi_l(r) = \exp \left( -i \frac{\epsilon r_0^2}{r_0 - r} \right) + \sqrt{|R|} \exp \left( +i \frac{\epsilon r_0^2}{r_0 - r} \right).
\]

(10)

### 2.2. Region 2: \( r > r_0 \)

This region is considered to be sufficiently away from the horizon, but not very far away from \( r = r_0 \). Assuming the terms in energy and momentum in Eq.(5) are very small compared to other terms, and considering the s wave case, we will get the equation as,

\[
\Phi_0''(r) + \frac{2}{r - r_0} \Phi_0'(r) = 0.
\]

(11)

Therefore,

\[
\Phi_0(r) = \frac{C}{(r - r_0)^2},
\]

(12)

where \( C \) is the constant of integration. Thus, the wave function will be of the form,

\[
\Phi_0(r) = -\frac{\alpha}{(r - r_0)} + \beta.
\]

(13)

#### 2.2.1. Comparing Regions 1 and 2

Here we compare the wave functions in regions 1 and 2. Eq.(10) can be written for s wave as,

\[
\Phi_0(r) = 1 - i \frac{\epsilon r_0^2}{r_0 - r} + |R| \left( 1 + i \frac{\epsilon r_0^2}{r_0 - r} \right) = 1 + |R| + \frac{(1 - |R|) \epsilon r_0^2}{r - r_0}.
\]

(14)

Comparing Eq.(14) with the wave function in region 2 given by Eq.(13) we will get,

\[\alpha = - (1 - |R|) \epsilon r_0^2, \quad \beta = 1 + |R| .\]

(15)

### 2.3. Region 3: \( r \gg r_0 \)

This region is very far away from the horizon. Since, \( r \) is very large, we rewrite terms containing energy and momentum in Eq.(3) as,

\[
\frac{\epsilon^2 r^4}{(r - r_0)^4} = \epsilon^2 + \frac{4 \epsilon^2 r_0^2}{(r - r_0)^3} + \frac{4 \epsilon^2 r_0^2 (r + r_0)}{(r - r_0)^4} + \frac{\epsilon^2 r_0^2}{(r - r_0)^4} \epsilon^2 r^2 r_0^2 + 2 \frac{\mu^2 r_0}{(r - r_0)^2} + \frac{\mu^2 r_0^2}{(r - r_0)^4},
\]

(16)

and

\[
\frac{\mu^2 r^2}{(r - r_0)^2} = \mu^2 + \frac{2 \mu^2 r_0}{(r - r_0)} + \frac{\mu^2 r_0^2}{(r - r_0)^2}.
\]

(17)
Thus,

$$\frac{\epsilon^2 r^4}{(r-r_0)^4} - \frac{\mu^2 r^2}{(r-r_0)^2} = \epsilon^2 - \mu^2 + \frac{2 \epsilon^2 - \mu^2}{r-r_0} \frac{2r_0}{(r-r_0)^2} + \frac{4 \epsilon^2 - \mu^2}{r-r_0} \frac{r_0^2}{(r-r_0)^3} + \frac{\epsilon^2 r_0^2}{r-r_0} \frac{r^2}{(r-r_0)^3},$$

$$= p^2 + \frac{(p^2 + \epsilon^2) 2r_0}{r-r_0} + \frac{(p^2 + 3 \epsilon^2) r_0^2}{(r-r_0)^2} + \frac{\epsilon^2 r_0^2 (r + r_0)}{(r-r_0)^3} + \frac{\epsilon^2 r_0^2 r^2}{r-r_0}.$$

where $p^2 = \epsilon^2 - \mu^2$. Substituting Eq. (18) in Eq. (15) we get,

$$\Phi''_l(r) + \frac{2}{r-r_0} \Phi'_l(r) + \left( p^2 + \frac{(p^2 + \epsilon^2) 2r_0}{r-r_0} + \frac{(p^2 + 3 \epsilon^2) r_0^2}{(r-r_0)^2} + \frac{\epsilon^2 r_0^2 (r + r_0)}{(r-r_0)^3} + \frac{\epsilon^2 r_0^2 r^2}{r-r_0} \right) \Phi_l(r) = 0. \quad (19)$$

At $r \gg r_0$ we get,

$$\Phi''_l(r) + \frac{2}{r} \Phi'_l(r) + \left( p^2 + \frac{(p^2 + \epsilon^2) 2r_0}{r} + \frac{l(l+1)}{r^2} \right) \Phi_l(r) = 0. \quad (20)$$

Here the Coulomb charge is $Z = (p^2 + \epsilon^2) r_0$. The solution to this equation will be,

$$\Phi_l(r) = \frac{1}{r} \left( A_l \exp (iz) + B_l \exp (-iz) \right), \quad (21)$$

where $z = pr - \frac{\mu l}{2} + \nu \ln 2pr + \delta_l^c$, $\delta_l^c = \arg \Gamma (l + 1 - \nu)$ and $\nu = \frac{Z}{2}$. Here the wave equation can be considered as governed by Coulomb problem. Introducing regular $F(r)$ and singular $G(r)$, solution of Coulomb problem, one can present the wave function as a linear combination:

$$\Phi_l(r) = \frac{1}{r} \left( aF_l (r) + bG_l (r) \right). \quad (22)$$

In the asymptotic limit, $r \to \infty$, the Coulomb functions takes the forms, $F_l (r) = \sin z$, $G_l (r) = \cos z$ where $z = pr - \frac{\mu l}{2} + \nu \ln 2pr + \delta_l^c$, thus Eq. (22) will be in the form:

$$\Phi_l (r) = \frac{1}{r} \left( a \sin z + b \cos z \right), \quad (23)$$

But we know that for l=0 [17],

$$F_0 (r) = cpr, \quad G_0 (r) = \frac{1}{c}, \quad (24)$$

where,

$$\epsilon^2 = \frac{2 \pi \nu}{1 - 2 \pi \nu}. \quad (25)$$

Thus Eq. (22) for s wave becomes,

$$\Phi_0 (r) = acp + \frac{b}{cr}. \quad (26)$$
2.3.1. Comparing Regions 2 and 3

By neglecting higher powers of \( \frac{1}{r} \), Eq. (13) can be written as,

\[
\Phi_0(r) = -\frac{\alpha}{r \left(1 - \frac{r_0}{r}\right)} + 2 \approx -\frac{\alpha}{r_0} + \beta, \tag{27}
\]

and using Eq. (26), Eq. (27) and Eq. (15) we get,

\[
a = \frac{1 + |R|}{pc}, \quad b = (1 - |R|) \omega r_0^2 c. \tag{28}
\]

2.4. Absorption cross section of RN extremal black hole

The two terms in Eq. (21) represents the incoming and outgoing waves. The \( S \) matrix can be written as the ratio of coefficient of the incoming and outgoing waves (\( A_l \) and \( B_l \)), Therefore,

\[
S_l = (-1)^{l+1} \frac{A_l}{B_l} \exp (2i\partial_l). \tag{29}
\]

We are here considering low energy absorption cross section for \( l = 0 \) (s wave). Thus from Eq. (21) and Eq. (22), we will find coefficients \( A_0 \) and \( B_0 \) as,

\[
A_0 = \frac{a + ib}{2i}, \quad B_0 = -\frac{a + ib}{2i}. \tag{30}
\]

Using Eq. (28) we get,

\[
A_0 = \frac{1 + |R| - \epsilon c^2 p (1 - |R|) r_0^2}{2pc}, \tag{31}
\]

and

\[
B_0 = -\frac{1 + |R| + \epsilon c^2 p (1 - |R|) r_0^2}{2pc}. \tag{32}
\]

Thus the \( S \)-matrix for the s-wave is given by,

\[
S_0 = -\frac{A_0}{B_0} \exp (2i\partial_0) = \frac{1 + |R| - \epsilon c^2 p (1 - |R|) r_0^2}{1 + |R| - \epsilon c^2 p (1 - |R|) r_0^2} \exp (2i\partial_0) \exp \left(\frac{1 - \epsilon c^2 \rho^2 \eta}{1 + \epsilon c^2 \rho^2 \eta}\right), \tag{33}
\]

where \( \eta = \frac{1 + |R|}{1 - |R|} \). The absorption cross section in the low energy limit is given by [17],

\[
\sigma_{abs} = \frac{\pi}{p^2} (1 - |S_0|^2) = \frac{\pi}{p^2} \frac{4\epsilon c^2 p r_0^2 \eta}{(1 + \epsilon c^2 p r_0^2 \eta)^2}, \tag{34}
\]

taking \( p = c\nu \), we get Eq. (33) as,

\[
\sigma_{abs} = \frac{4\pi c^2 r_0^2 \eta}{\nu (1 + \epsilon c^2 \nu r_0^2 \eta)^2}. \tag{35}
\]
3. Schwarzschild-de Sitter black hole - Extremal case

The metric of a SdS space time is given by,
\[ ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2), \]
where \( f(r) = 1 - \frac{2m}{r} - \Lambda r^2 \) with \( \Lambda > 0 \) and \( m > 0 \). For \( 0 < 9\Lambda m^2 < 1 \) there exist two positive roots \( r_+ \) and \( r_{++} \) of \( f(r) \) such that \( 0 < 2m < r_+ < 3m < r_{++} \). The roots \( r_+ = \sqrt[3]{\Lambda} \cos \left( \frac{\alpha}{3} + \frac{\pi}{2} \right) \) with \( \cos \alpha = -3m\sqrt[3]{\Lambda} \), describes the black hole event horizon, and the root \( r_{++} = \sqrt[3]{\Lambda} \cos \left( \frac{\alpha}{3} \right) \) localizes the cosmological event horizon. As \( \Lambda \) approaches its extremal value, i.e, \( \Lambda \to \frac{1}{9m^2} \), the position of the black hole horizon \( r_+ \) monotonically increases and the cosmological horizon \( r_{++} \) decreases to a common value at \( r = 3m \). Here we analyze this extreme case of the SdS black hole which is characterized by the condition \( 9\Lambda m^2 = 1 \). In this case the \( f(r) \) becomes \[ f(r) = -\frac{1}{27m^2} (r - 3m)^2(r + 6m). \] Applying this in Klein Gordon equation, we will get,
\[
\left[ \frac{27m^2}{(r - 3m)^2(r + 6m)} \frac{\partial^2}{\partial t^2} + \frac{1}{27m^2} \frac{\partial}{\partial r} r(r - 3m)^2(r + 6m) \frac{\partial}{\partial r} \right.
\left. - \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial}{\partial \theta} - \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2} + \mu^2 \right] \psi(r, \theta, \phi, t) = 0.
\]
Real part is separated out by,
\[
\psi(r, \theta, \phi, t) = \exp \left( -i \epsilon t \right) Y_m(\theta, \phi) \Phi_l(r).
\]
where \( \epsilon, l, m \) are energy, momentum and its projection, while \( \Phi_l(r) \) is a radial function. Substituting Eq.(39) in Eq.(38) we get,
\[
\Phi_l''(r) + \left( \frac{1}{r} + \frac{2}{r - 3m} + \frac{1}{r + 6m} \right) \Phi_l'(r) + \left( \frac{\epsilon^2 27m^4}{(r - 3m)^4(r + 6m)^2} + \frac{\mu^2 27m^2}{(r - 3m)^2(r + 6m)} \right) \Phi_l(r) = 0.
\]
Let us describe the scattering of scalar waves by SdS extremal black hole with the help of Eq.(40). We will find solution of the wave equation in different regions outside the horizon.

3.1. Region 1: \( r \to 3m \)

This is the region very near to the horizon and is the last limit for a wave to reflect from the horizon. i.e., the wave function \( \Phi(r) \) in this region (i.e., as \( r \to 3m \)) will contain incident and reflected waves. To find \( \Phi(r) \) we use WKB approximation and write \( \Phi = \exp \left( -i \int k(r) dr \right) \), which will lead to,
\[
k(r \to 3m) = \pm \frac{\epsilon 27m^2 \times 3m}{(r - 3m)^2(3m + 6m)} = \pm \frac{9m^2 \epsilon}{(r - 3m)^2} = \pm \frac{\xi}{(r - 3m)^2}.
\]
Therefore the wave function in the region $r \rightarrow 3m$ can be written as,

$$\Phi_l(r) = \exp \left( \pm i \int \frac{\xi dr}{(r-3m)^2} \right) = \exp \left( \pm i \frac{\xi}{(r-3m)^2} \right), \quad (42)$$

where $\xi = \epsilon_9m^2$. Therefore, the wave function in the vicinity of horizon can be written, assuming that the wave gets reflected at the horizon, as

$$\Phi_l(r) = \exp \left( -i \frac{\xi}{3m-r} \right) + |R| \exp \left( +i \frac{\xi}{3m-r} \right). \quad (43)$$

where $|R|$ represents the reflection coefficient.

### 3.2. Region 2: $r > 3m$

Now we consider the second region where $r > 3m$. As in section(2.3), here also we neglect the energy and momentum terms in Eq.(40) and for the s wave the resulting equation will be,

$$\Phi''_0(r) + \left( \frac{1}{r} + \frac{2}{r-3m} + \frac{1}{r+6m} \right) \Phi'_0(r) = 0, \quad (44)$$

i.e,

$$\ln \Phi'_0(r) = -2 \ln(r-3m) - \ln r - \ln(r+6m). \quad (45)$$

Therefore

$$\Phi_0(r) = -\frac{\alpha}{r-3m} + \beta. \quad (46)$$

#### 3.2.1. Comparing Regions 1 and 2

As in Section(2.3.1) here also we can compare the solution in the regions 1 and 2. Then Eq.(49) for s wave, can be written as,

$$\Phi_0(r) = 1 - i \frac{\xi}{3m-r} + |R| \left( 1 + i \frac{\xi}{3m-r} \right) = 1 + |R| + i \xi \left( 1 - |R| \right). \quad (47)$$

Comparing Eq.(47) with Eq.(46) we get,

$$\alpha = -(1- |R|) i \xi, \beta = 1+ |R| . \quad (48)$$

### 3.3. Region 3: $r >> 3m$

In this region, the terms containing energy and momentum in Eq.(40) can be simplified as,

$$\frac{\epsilon^2 \gamma^2 m^2 r^2}{(r-3m)^4 (r+6m)^2} + \frac{\mu^2 \gamma^2 m^2 r}{(r-3m)^2 (r+6m)} \simeq \epsilon^2 - \mu^2 + \frac{(\epsilon^2 - \mu^2)27 \times 2m^3}{(r-3m)^2 (r+6m)}$$

$$= p^2 + \frac{(\mu^2 + \epsilon^2)27 \times 2m^3}{(r-3m)^2 (r+6m)} \quad (49)$$
where $p$ is the momentum and is given by $p^2 = \epsilon^2 - \mu^2$. Thus Eq. (40) becomes,

$$\Phi''_l(r) + \left(\frac{1}{r} + \frac{2}{r - 3m} + \frac{1}{r + 6m}\right) \Phi'_l(r) + \left(p^2 + \frac{(p^2 + \epsilon^2)27 \times 2m^3}{(r - 3m)^2 (r + 6m)} - \frac{l (l + 1) 27m^2 r}{(r - 3m)^2 (r + 6m)}\right) \Phi_l(r) = 0.$$  

(50)

Since we are considering s wave we take $l = 0$. Now at region $r \gg 3m$ we can neglect higher powers of $\frac{1}{r}$ and we get the equation as,

$$\Phi''_0(r) + \left(\frac{2}{r - 3m} + \frac{1}{r + 6m}\right) \Phi'_0(r) + p^2 \Phi_0(r) = 0.$$  

(51)

Since $r + 6m > r > r - 3m$ we can approximate the coefficient of $\Phi'_0(r)$ as $\frac{2}{r}$. Therefore at large distance we can write the equation as,

$$\Phi''_0(r) + \frac{2}{r} \Phi'_0(r) + p^2 \Phi_0(r) = 0.$$  

(52)

Solution of Eq. (52) is obtained as a combination of $\sin z$ and $\cos z$ using Frobenius method. Thus solution can be written as,

$$\Phi_0(r) = \frac{1}{r} (A_0 \exp iz + B_0 \exp -iz),$$  

(53)

where $z = pr$. Here also, as in the previous case (section 2.3), the wave function can be written as a combination of two functions $F(r)$ and $G(r)$. Therefore

$$\Phi_0(r) = \frac{1}{r} (AF(r) + bG(r)),$$  

(54)

but by comparing with the solutions obtained using Frobenious method, we know that $F(r) = \sin pr$ and $G(r) = \cos pr$. From this we will deduce that,

$$A_0 = \frac{a + ib}{2i}, \quad B_0 = -\frac{a + ib}{2i}.$$  

(55)

On the other hand, since $\epsilon$ is low we can assume $pr \ll 1$ and thus,

$$F(r) \approx pr, \quad G(r) \approx 1.$$  

(56)

Thus Eq. (54) will become,

$$\Phi_0(r) = ap + \frac{b}{r}.$$  

(57)

3.3.1. Comparing Regions 2 and 3

By neglecting higher powers of $\frac{1}{r}$ Eq. (46) can also be written as,

$$\Phi_0(r) = -\frac{\alpha}{r \left(1 - \frac{3m}{r}\right)} + \beta \approx -\frac{\alpha}{r} + \beta,$$  

(58)

Thus Eq. (58) has the same form as Eq. (57). Therefore comparing the coefficients we will get,

$$a = \frac{1 + |R|}{p}, \quad b = i (1 - |R|) \xi.$$  

(59)
3.4. Absorption cross section of SdS extremal black hole

Now we will find an expression for absorption cross section of SdS extremal black hole. From Eq. (55) and Eq. (59) we will get,

$$A_0 = \frac{[1 + |R| - (1 - |R|) \xi p]}{2 ip c},$$

and

$$B_0 = -\frac{[1 + |R| + (1 - |R|) \xi p]}{2 ip c}.$$  

S-matrix for the s-wave is given by,

$$S_0 = -\frac{A_0}{B_0} \exp (2i \partial_0) = \frac{[1 + |R| - (1 - |R|) \xi p]}{[1 + |R| + (1 - |R|) \xi p]} \exp (2i \partial_0).$$  

It can also be written as,

$$S_0 = \frac{1 - \xi p \eta}{1 + \xi p \eta} \exp (2i \partial_0),$$

where \( \eta = \frac{1 - |R|}{1 + |R|} \). The absorption cross section in the low energy limit is given by,

$$\sigma_{abs} = \frac{\pi}{p^2} (1 - |S_0|^2) = \frac{\pi}{p} \frac{4 \xi \eta}{(1 + \xi p \eta)^2},$$

i.e,

$$\sigma_{abs} = \frac{36 \pi m^2 \eta}{v (1 + 9m^2 c^2 v \eta)^2}.$$  

4. Conclusion

We found the wave function \( \Phi_l(r) \) in the vicinity of outer horizon of RN extremal black hole i.e \( r \to r_0 \) for scalar field using WKB approximation. We have also studied the behavior of scattered scalar waves in the regions \( r > r_0 \) and \( r \gg r_0 \) in low energy limit. By comparing the solutions in the 3 regions viz., \( r \to r_0 \), \( r > r_0 \) and \( r \gg r_0 \), we found the S-matrix and the absorption cross section for RN extremal black hole by s-wave in the lower energy limit. Similarly we found the wave functions \( \Phi_l(r) \) in the vicinity of outer horizon \( r \to 3m \) and in the regions \( r > 3m \) and \( r \gg 3m \) of SdS extremal black hole for scalar field. We have also studied the behavior of scattered scalar waves in all these regions in low energy limit. By comparing the solutions in the 3 regions viz., \( r \to 3m \), \( r > 3m \) and \( r \gg 3m \), we found the absorption cross section for SdS extremal black hole by s-wave in the lower energy limit.

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