SPIN EFFECTS
IN HEAVY QUARK PROCESSES

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Abstract
In the infinite mass limit for a heavy quark its spin decouples from the QCD dynamics, which leads to the heavy-quark spin symmetry. After a short discussion of spin symmetry some applications are considered.

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1 Introduction

Over the last ten years the heavy mass limit has become a standard tool in heavy quark physics [1]. The main impact of the \(1/m_Q\) expansion is that the strong interaction connecting weak processes of heavy quarks with those of heavy hadrons can be handled in a more efficient way. This is mainly due to the presence of symmetries which appear in the heavy quark limit [2] and which restrict the nonperturbative quantities severely, at least in some cases. Furthermore, off the symmetry limit, i.e. the limit \(m_Q \to \infty\), one may compute or at least parametrize the corrections in a systematic fashion.

These corrections are characterized by two quantities, namely \(\alpha_s\) and \(\Lambda_{QCD}/m_Q\), both of which are small for a sufficiently heavy quark. The \(\alpha_s\) corrections are perturbative and may be calculated systematically using the Feynman rules of Heavy Quark Effective Theory (HQET) [3]. The other kind of corrections, the \(\Lambda_{QCD}/m_Q\) corrections, are nonperturbative and are usually parametrized in terms of certain matrix elements.

One of the two symmetries of the heavy mass limit is the so-called spin symmetry, which mainly tells us that in the heavy mass limit the spin of the heavy quark decouples from the dynamics. The second symmetry of the heavy mass limit is a heavy flavour symmetry which allows us to replace an infinitely heavy quark in some heavy hadron by another infinitely heavy hadron with the same four-velocity, but with a different flavour, without changing anything. This is true because in full QCD the dependence on the quark flavour enters only through the different masses.

We shall first give a brief account on the heavy flavour symmetries and then discuss a few applications of the heavy quark spin symmetry, such as the relations between \(B \to D\ell\bar{\nu}_\ell\) and \(B \to D^*\ell\bar{\nu}_\ell\), the polarization of \(b\) hadrons in \(Z_0\) decays, some polarization effects in \(\Lambda_c\) decays and finally the quark helicities in \(\Lambda_b \to \Lambda\gamma\).

2 Heavy Quark Limit and its Symmetries

Because of space and time limitations we shall not deduce the heavy quark limit from QCD, rather we refer the reader to one of the numerous reviews which are available on this subject [1]. The final result of some algebraic manipulations of the QCD Lagrangian and the field of the heavy quark is
their expansion in powers of $1/m_Q$, taking a limit in which the heavy quark velocity $p_Q/m_Q$ is kept fixed. More precisely, the heavy quark momentum is split into a “large” part scaling as $m_Q$ and a small residual part $k$, independent of $m_Q$, i.e. one writes $p_Q = m_Qv + k$. In this limit the relevant degree of freedom is the static heavy quark field $h_v(x)$ moving with the fixed velocity $v$ and having a residual momentum $kh_v(x)\equiv iDh_v(x)$.

If $Q(x)$ is the heavy quark field of full QCD, one obtains the $1/m_Q$ expansions

$$Q(x) = e^{-im_Qv \cdot x} \left[ 1 + \frac{1}{2m_Q}(i\not{D} \cdot \not{v}) + \frac{1}{4m_Q^2} \left( (v \cdot D)\not{D} \cdot \not{v} - \frac{1}{2} \not{D}^2 \right) + \cdots \right] h_v(x)$$

and

$$\mathcal{L} = \bar{h}_v(i\not{v} \cdot \not{D})h_v + \bar{K}_1 + \bar{M}_1 + \bar{E}_1 + \bar{K}_2 + \bar{M}_2 + \bar{E}_2 + \cdots \quad (2)$$

where we have defined the abbreviations

$$\bar{K}_1 = \bar{h}_v \frac{(iD)^2}{2m_Q^2}h_v, \quad \bar{M}_1 = \frac{(-i)}{2m_Q^2} \bar{h}_v \sigma_{\mu\nu}(iD^\mu)(iD^n)h_v, \quad \bar{E}_1 = \bar{h}_v \frac{(ivD)^2}{2m_Q^2}h_v$$

$$\bar{K}_2 = \frac{1}{8m_Q^2} \bar{h}_v [(iD^\mu), [(-ivD), (iD^\mu)]]h_v$$

$$\bar{M}_2 = \frac{(-i)}{8m_Q^2} \bar{h}_v \sigma_{\mu\nu} \{(iD^\mu), [(-ivD), (iD^\nu)\}]h_v, \quad \bar{E}_2 = \bar{h}_v \frac{(ivD)^3}{8m_Q^2}h_v \quad (3)$$

The leading term of these expansions together with the usual Lagrangian for the light degrees of freedom determines the dynamics of HQET. A remarkable feature of the leading term of the Lagrangian is that it has two additional symmetries which have not been present in full QCD.

The first symmetry which arises is a heavy flavour symmetry. The interaction of the quarks with the gluons is determined by the color quantum numbers and the dependence on flavour enters in full QCD only through the different quark masses. For the light quarks the fact that the light quark current masses are small compared to the QCD scale $\Lambda_{QCD}$ yields the well known flavour symmetry for the light quarks; in the heavy mass limit a flavour symmetry arises in a similar manner: once the heavy quark is replaced by a static source of colour moving with a definite velocity the flavour does not matter anymore. In other words, for two heavy flavours $b$ and $c$ an $SU(2)$ symmetry emerges which relates $b$ and $c$ quarks moving with the same velocity.
For the case of two heavy flavours $b$ and $c$ one has to leading order the Lagrangian
\[
\mathcal{L}_{\text{heavy}} = \bar{b}_v (v \cdot D) b_v + \bar{c}_v (v \cdot D) c_v, \quad (4)
\]
where $b_v$ ($c_v$) is the field operator $h_v$ for the $b$ ($c$) quark moving with velocity $v$. This Lagrangian is obviously invariant under the $SU(2)_{HF}$ rotations
\[
\begin{pmatrix} b_v \\ c_v \end{pmatrix} \rightarrow U_v \begin{pmatrix} b_v \\ c_v \end{pmatrix} \quad U_v \in SU(2)_{HF}. \quad (5)
\]
We have put a subscript $v$ for the transformation matrix $U$, since this symmetry only relates heavy quarks moving with the same velocity.

The second symmetry emerging in the heavy mass limit is the so called spin symmetry. To leading order both spin degrees of freedom couple in the same way to the gauge field. We rewrite the leading-order Lagrangian as
\[
\mathcal{L} = \bar{h}_v^+ s_v (ivD) h_v^+ + \bar{h}_v^- s_v (ivD) h_v^-, \quad (6)
\]
where $h_v^\pm$ are the projections of the heavy quark field on a definite spin direction $s$
\[
h_v^\pm = \frac{1}{2} (1 \pm \gamma_5 \gamma^s) h_v, \quad s \cdot v = 0. \quad (7)
\]
This Lagrangian has a symmetry under the rotations of the heavy quark spin, which is formally again an $SU(2)_{SS}$ symmetry given by
\[
\begin{pmatrix} h_v^+ \\ h_v^- \end{pmatrix} \rightarrow W_v \begin{pmatrix} h_v^+ \\ h_v^- \end{pmatrix} \quad W_v \in SU(2)_{SS}. \quad (8)
\]
The spin rotations may explicitly be represented by
\[
W_v = \exp (-i \phi \gamma^s \gamma_5) \quad (9)
\]
where we have introduced the rotation axis $\epsilon$ satisfying $v \epsilon = 0$ and $\epsilon^2 = -1$ and the rotation angle $\phi$. In the rest frame $v = (1, 0, 0, 0)$ this reduces to the well known representation of rotations of spinors
\[
W_v = \exp (-i \phi \sigma^s \cdot \tilde{\sigma}) \quad (10)
\]
where $\tilde{\sigma}$ is the usual vector of the three Pauli matrices.
Thus in the heavy mass limit the heavy hadrons fall into spin symmetry doublets which may be characterized by the spin of the light degrees of freedom. Since the heavy quark spin decouples, the total angular momentum of the light degrees of freedom becomes a good quantum number. Hence the spin symmetry doublets of heavy hadrons are the ones with total angular momentum \( j + 1/2 \) and \( j - 1/2 \) (\( j = 1, 2, 3, \ldots \)), where \( j \) is the angular momentum of the light degrees of freedom.

For the mesons the ground state spin symmetry doublet are the heavy pseudoscalar mesons (0\(^-\) states) and the corresponding vector meson states (1\(^-\) states). For the ground state baryons, the spin of the light degrees of freedom can either be \( j = 0 \) or \( j = 1 \). For \( j = 0 \) the corresponding baryon has spin 1/2 and is called \( \Lambda_Q \), and it is the simplest object from the point of view of heavy quark symmetry: The spin symmetry doublet are the two polarization directions of the \( \Lambda_Q \). For \( j = 1 \) the baryons can have either spin 1/2 (in which case they are called \( \Sigma_Q \)) or spin 2/3 (in which case they are called \( \Sigma^*_Q \)), and hence \( \Sigma_Q \) and \( \Sigma^*_Q \) form another spin symmetry doublet of heavy baryons.

Spin symmetry has some consequences for transition matrix elements, and we shall consider this here for mesons only. It is convenient to represent the mesons by representation matrices carrying a heavy quark spinor index \( A \) and a light quark index \( \alpha \). In fact, the matrix

\[
H(v) = H_{A\alpha}(v) = \frac{1}{2}\sqrt{m_H}\gamma_5(\not v - 1)
\]

represents the correct coupling of the heavy quark and the light degrees of freedom (also carrying spin 1/2) to a pseudoscalar meson of total spin 0. The heavy quark is on shell in the limit \( m_H \to \infty \), thus we must require

\[
(\not v - 1)H(v) = 0
\]

Likewise, the representation for a heavy vector meson is

\[
H^*(v, \epsilon) = H^*_{A\alpha}(v, \epsilon) = \frac{1}{2}\sqrt{m_H}\epsilon(\not v - 1)
\]

Rotations of the heavy quark spin rotate the heavy pseudoscalar mesons into the corresponding heavy vector mesons and vice versa. For a 90° rotation

\[
(\not v - 1)H(v) = 0
\]
of the heavy quark spin around the axis $\epsilon$ in a heavy pseudoscalar meson we obtain

$$W_v(\epsilon, 90^\circ)H(v) = \gamma_5\rlap/\!\!p\rlap/\!\!\epsilon H(v) = \frac{1}{2}\sqrt{m_H}\rlap/\!\!p (\rlap/\!\!p - 1)$$ (14)

which is the representation matrix of a heavy vector meson.

One may work out the group theory of heavy quark symmetries in more detail and study their consequences for transition matrix elements [2]. Without going into details, the final result is the analogue of the Wigner Eckhart theorem. If $H(v)$ denotes either $H(v)$ or $H^*(v, \epsilon)$ and if $|H(v)>$ denotes the corresponding state in the heavy mass limit, one finds

$$<H(v')|\bar{h}_v\Gamma h_v|H(v)> = \xi(v \cdot v') Tr \{\overline{H}(v')\Gamma H(v)\}$$ (15)

where $\Gamma$ is some arbitrary combination of Dirac matrices and $\xi(v \cdot v')$ is a nonperturbative form factor, the so-called Isgur Wise function.

Eq.(15) is the main result of heavy quark symmetry in the mesonic sector, since it relates every matrix element of bilinear heavy to heavy currents between two heavy mesons to the Isgur Wise function $\xi(v \cdot v')$. Furthermore, since the current

$$j_\mu = \bar{h}_v\gamma_\mu h_v$$

(16)

generates the heavy flavour symmetry, we have a normalization statement for the Isgur Wise function

$$\xi(v \cdot v' = 1) = 1$$ (17)

Note, finally, that the Isgur Wise function in a group theoretical language is just the reduced matrix element which is universal for the whole spin flavour symmetry multiplet. The trace in (15) in the language of the Wigner Eckart theorem is the Clebsch Gordan coefficient which is entirely determined by the current operator and the states of the multiplet.

Furthermore, since the spin symmetry violating terms are the ones proportional to the “strong Bohr magneton” $g/(2m_Q)$ one would expect that the splitting between the partners within a spin symmetry doublet scales as $1/m_Q$.

For mesons we may consider the quantity

$$\Delta = M^2(1^-) - M^2(0^-) = (M(1^-) + M(0^-))(M(1^-) - M(0^-))$$ (18)

$$\approx 2m_Q(M(1^-) - M(0^-))$$

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Table 1: Value of the splitting \( \Delta \) for the different systems, the data is from the Particle Data Group [4]

| System        | \( \Delta \) in GeV\(^2 \) |
|---------------|-----------------------------|
| \((B^*, B)\)  | 0.53                        |
| \((D^*, D)\)  | 0.54                        |
| \((K^*, K)\)  | 0.55                        |
| \((\rho, \pi)\)| 0.57                       |

Table 2: Value of the splitting \( \Delta' \) for the different systems

| System        | \( \Delta' \) in GeV\(^2 \) |
|---------------|-----------------------------|
| \((\Sigma_b^*, \Sigma_b)\) | 0.65                        |
| \((\Sigma_c^*, \Sigma_c)\) | 0.34                        |
| \((\Sigma^*, \Sigma)\)      | 0.50                        |
| \((\Delta, N)\)             | 0.64                        |

which we should expect to be a constant in the heavy quark systems. As can be seen in table 1, \( \Delta \) indeed turns out to be constant in the \( B \) and \( D \) meson systems, but to some surprise one obtains also the same constant looking into light quark systems.

Similarly, for \( \Sigma_Q \)-like baryons we consider

\[
\Delta' = M^2(3/2) - M^2(1/2) \approx 2m_Q(M(3/2) - M(1/2))
\] (19)

which again should turn out to be a constant. In table 2 we list the corresponding quantities.

It is interesting to note that there seems to be a problem with the splitting for the heavy \( \Sigma_Q \)-type baryons, since the \( 1/m_Q \) scaling between the bottom and the charm systems does not seem to be satisfied. The data on \( \Sigma_b \) is from DELPHI [5] and is up to now only available from conference talks. One would expect that the uncertainties in this number are still large, at least much larger than the one on the data on the \( \Sigma_c \) and \( \Sigma_c^* \), which is dominated by CLEO data [6]. However, if one takes the numbers in the lighter systems
serious, hoping for a similar accident as for the mesons, the data in the charm system seem to be on the low side.

3 Applications of Spin Symmetry

3.1 Relation between exclusive semileptonic $b \rightarrow c$ transitions

It is well known that heavy quark symmetry allows to relate the decays $B \rightarrow D \ell \bar{\nu}_\ell$ and $B \rightarrow D^* \ell \bar{\nu}_\ell$. The relevant matrix elements are the ones involving the left handed current for a $b \rightarrow c$ transition, which are parametrized in general by six form factors

$$
\langle D(v')|\bar{c}\gamma_\mu b|B(v)\rangle = \sqrt{m_Bm_D}\left[\xi_+(y)(v_\mu + v'_\mu) + \xi_-(y)(v_\mu - v'_\mu)\right]
$$

(20)

$$
\langle D^*(v',\epsilon)|\bar{c}\gamma_\mu b|B(v)\rangle = i\sqrt{m_Bm_{D^*}}\xi_V(y)\varepsilon_{\mu\nu\rho}\epsilon^{*\nu}v'^\rho
$$

(21)

$$
\langle D^*(v',\epsilon)|\bar{c}\gamma_5 b|B(v)\rangle = \sqrt{m_Bm_{D^*}}\left[\xi_{A1}(y)(vv' + 1)\epsilon_\mu^* - \xi_{A2}(y)(\epsilon^* v)v_\mu
\right.
$$

$$
-\xi_{A2}(y)(\epsilon^* v)v'_\mu] ,
$$

(22)

where we have defined $y = vv'$. Of particular interest from the point of view of heavy quark symmetry is the edge of phase space where the final state $D$ meson is at rest, i.e. the point $y = 1$. At this kinematical point the decay rates take the following form

$$
\lim_{y \rightarrow 1} \frac{1}{\sqrt{y^2 - 1}} \frac{d\Gamma}{dy}(B \rightarrow D^* \ell \bar{\nu}_\ell) = \frac{G_F^2}{4\pi^3}(m_B - m_{D^*})^2m_{D^*}^3|V_{cb}|^2|\xi_{A1}(1)|^2
$$

(23)

and

$$
\lim_{y \rightarrow 1} \left(\frac{1}{\sqrt{y^2 - 1}}\right) \frac{d\Gamma}{dy}(B \rightarrow D \ell \bar{\nu}_\ell)
$$

$$
= \frac{G_F^2}{48\pi^3}(m_B + m_D)^2m_D^3|V_{cb}|^2\left|\xi_+(1) - \frac{m_B - m_D}{m_B + m_D}\xi_-(1)\right|^2
$$

(24)

Assuming that the $b$ and the $c$ quark are heavy one may relate all these form factors to a single one, the Isgur Wise function as introduced in (15)

$$
\xi_i(y) = \xi(y) \text{ for } i = +, V, A1, A3, \quad \xi_i(y) = 0 \text{ for } i = -, A2.
$$

(25)
Furthermore, again due to heavy quark symmetries, the Isgur Wise function is normalized at $y = 1$ as $\xi(y = 1) = 1$.

Corrections to this symmetry limit may be systematically accessed using HQET. One important result concerning the $1/m_Q$ corrections is Luke’s theorem [7], which states that neither the normalization of $\xi_{A1}$ nor the one of $\xi_+$ receive corrections linear in $1/m_Q$. Hence the leading corrections to these two form factors are of the order $1/m_Q^2$. Furthermore, radiative corrections have been calculated up to next-to-leading order, and one obtains for $\xi_{A1}(1)$

$$\xi_{A1}(1) = \eta_A(1 + \delta_{1/m^2})$$  \hspace{1cm} (26)

where $\eta_A$ incorporates the (QCD and QED) radiative corrections to the axial-vector $b \to c$ current and $\delta_{1/m^2}$ parametrizes the corrections of order $1/m_Q^2$. Inserting numbers one finds

$$\xi_{A1}(1) = 0.92 \pm 0.03$$  \hspace{1cm} (27)

where the uncertainty is entirely due to the parametrization of the corrections of order $1/m_Q^2$.

Similarly, for the decay $B \to D\ell\bar{\nu}_\ell$ one may write for the relevant combination of form factors

$$\left| \xi_+(1) - \frac{m_B - m_D}{m_B + m_D} \xi_-(1) \right| = \eta_V(1 + \Delta_{1/m_Q})$$  \hspace{1cm} (28)

where $\eta_V$ incorporates the (QCD and QED) radiative corrections to the vector $b \to c$ current and $\Delta_{1/m_Q}$ are the $1/m_Q$ corrections induced by $\xi_-(1)$, which is not protected by Luke’s theorem. These corrections have been estimated recently [8]

$$\left| \xi_+(1) - \frac{m_B - m_D}{m_B + m_D} \xi_-(1) \right| = 0.98 \pm 0.07$$ \hspace{1cm} (29)

The absolute normalizations (27) and (29) have been used to determine the CKM matrix element $V_{cb}$ using the measured spectra of both $B \to D^*\ell\bar{\nu}_\ell$ and $B \to D\ell\bar{\nu}_\ell$ [8]. On the other hand, one may use the same data [10, 11] to test the helicity structure of the weak $b \to c$ transition current, since at $y = 1$ $B \to D^*\ell\bar{\nu}_\ell$ is sensitive to the axial current only and $B \to D\ell\bar{\nu}_\ell$ is sensitive to the vector current only. If we modify the weak $b \to c$ current by including coupling constants $g_V$ and $g_A$ according to

$$\bar{c}\gamma_\mu(1 - \gamma_5)b \longrightarrow \bar{c}\gamma_\mu(g_V - g_A\gamma_5)b$$ \hspace{1cm} (30)
the present data allow to constrain the possible values of the ratio of the coupling constants

\[ \left| \frac{g_A}{g_V} \right| = 1.02 \pm 0.28 \quad (31) \]

The large uncertainty in this number is due to the theoretical uncertainty in the \(1/m_Q\) corrections to \(B \to D\ell\bar{\nu}_\ell\) and to the experimental uncertainties also in \(B \to D\ell\bar{\nu}_\ell\).

### 3.2 Polarization of \(b\) hadrons from \(Z_0\) decay

There is a large data sample of \(b\) hadrons which originates from hadronization of \(b\) quarks produced from the decay \(Z_0 \to \bar{b}b\) at LEPI. The interesting feature of these bottom quarks is that their weak couplings are such that they are produced with a very high polarization

\[ P = \frac{g_A g_V}{g_A^2 + g_V^2} \approx 94\% \quad (32) \]

and thus the question arises how much of this polarization is retained in the polarization of the final state \(b\) hadrons.

Of course here the relevant symmetry is the spin symmetry. If one assumes that hadronization is a soft process, then we can describe it in the limit \(m_b \to \infty\), where the spin of the heavy quark decouples. This has already the obvious consequence that the \(\Lambda_b\) baryons from \(Z_0\) decay should be polarized to a similarly high degree as the \(b\) quark itself, and the corrections should be effects of order \(1/m_Q\). We shall give an estimate of this effect below.

However, only one out of ten \(b\) quarks hadronise into a \(\Lambda_b\) baryon and hence much more data is available on mesons, and we shall first analyze the situation for mesons along the lines of Falk and Peskin [12]. We start from a fully polarized \(b\) quark and represent the 100\% left handed state as \(| \downarrow \rangle\).

Fragmentation means that the heavy quarks gets dressed with light degrees of freedom which have to have spin 1/2. Since there is no preferred spin direction for the light degrees of freedom, both \(| \uparrow \rangle\) and \(| \downarrow \rangle\) should have the same probability amplitude.

From these quark states we can form the following mesonic states

\[ | \downarrow \rangle | \downarrow \rangle = |B^*(\lambda = -1)\rangle \quad (33) \]
\[ | \downarrow \rangle \uparrow \rangle = \frac{1}{\sqrt{2}} [ | B \rangle - | B^*(\lambda = 0) \rangle ] \]

where \( \lambda \) is the helicity of the \( B^* \) meson. Since the \( B^* \) decays only electromagnetically, it has a very small width compared to the mass difference between the \( B^* \) itself and its spin symmetry partner \( B \), which has an even smaller width. Hence the two meson states involved in (33) do not overlap and hence they become incoherent before any decay can occur. Thus we may obtain from (33) the following table of probabilities

\[
\begin{align*}
P[B] &= \frac{1}{4} \\
P[B^*(\lambda = -1)] &= \frac{1}{2}, \\
P[B^*(\lambda = 0)] &= \frac{1}{4}, \\
P[B^*(\lambda = 1)] &= 0
\end{align*}
\]  

(34)

The \( B^* \) is identified by its decay \( B^* \to B\gamma \) which occurs after the time \( 1/\Gamma(B^*) \). Since the \( B \) mesons are pseudoscalar objects, no polarization information can be carried by them. One possibility would be the angular distribution of the photon emission, for which one obtains

\[
\begin{align*}
\frac{d\Gamma}{d\cos\theta}[B^*(\lambda = \pm 1) \to B\gamma] &\propto \frac{1}{2}(1 + \cos^2\theta) \\
\frac{d\Gamma}{d\cos\theta}[B^*(\lambda = 0) \to B\gamma] &\propto \sin^2\theta
\end{align*}
\]  

(35) (36)

where the constant of proportionality is the same in both cases and \( \theta \) is the angle between the boost direction of the \( B^* \) and the photon momentum. Using the table of probabilities (34), we end up with an isotropic distribution which again does not carry any polarization information.

Thus for the mesons the information on the polarization of the initial \( b \) quark is entirely transferred into the polarization of the emitted photon which is indeed left handed. The photon polarization can, however, not be measured with any of the LEP detectors, so not much can be done for the mesons.

As mentioned above the situation is more promising for baryons, in particular for the \( \Lambda_b \). Here one would naively expect a polarization of the order of 90\%, since the \( b \) quark polarization should be carried over to the \( \Lambda_b \) up to corrections of the order \( 1/m_Q \). This expectation is not supported by data \[13\], since much lower values are found experimentally. Thus one needs to analyze the \( 1/m_Q \) effects quantitatively.
This has been done by Falk and Peskin [12], who discuss the $\Lambda_b$ depolarization through $\Sigma_b$ and $\Sigma_b^*$ intermediate states, i.e. through the process $Z_0 \rightarrow \bar{b}b \rightarrow \Sigma_b^{(*)} \rightarrow \Lambda_b$.

In the case of baryons the hadronization process has to dress the $b$ quark with light degrees of freedom of either spin $S = 0$ or $S = 1$, if we restrict ourselves to the ground state baryons. Unlike for the simple case of mesons here two parameters enter the analysis. First there is the relative probability $A$ to have $S = 0$ for the light degrees of freedom compared to the $S = 1$ case, and the second parameter is the relative probability $\omega$ to have the light degrees transversely polarized $S_3 = \pm 1$ compared to $S_3 = 0$. In terms of these parameters one may again set up a table of probabilities for the various helicity states of the baryons, assuming again a fully left handed polarized $b$ quark in the initial state and incoherence of the various states. One finds

\[
\begin{bmatrix}
\begin{array}{llll}
\text{state} & \lambda = -\frac{3}{2} & \lambda = -\frac{1}{2} & \lambda = \frac{1}{2} & \lambda = -\frac{3}{2} \\
\Sigma_b^* & \frac{1}{2} \omega A & \frac{2}{3} (1 - \omega) A & \frac{1}{6} \omega A & 0 \\
\Sigma_b & -- & \frac{1}{3} (1 - \omega) A & \frac{1}{3} \omega A & -- \\
\Lambda_b & -- & 1 & 0 & --
\end{array}
\end{bmatrix} \cdot \frac{1}{1 + A}
\]

where the bracket means that all entries should be multiplied by the overall normalization $1/(1 + A)$.

Similarly as for the mesons one now has to analyze the subsequent decays which are the decays $\Sigma_b^{(*)} \rightarrow \Lambda_b \pi$. We shall not discuss any of the details here and only quote the final result. For “reasonable” values (actually motivated by the Lund string model) of $A = 0.45$ and $\omega = 0$ one finds a significant depolarization of the $\Lambda_b$ baryons, namely

\[
\mathcal{P}(\Lambda_b@LEPI) \approx 68\%
\]

which is still not enough to explain the low experimental values. On the other hand, the analysis of Falk and Peskin has to be taken as an estimate depending on the two parameters $A$ and $\omega$, and if the experimental values remain as low as they are now, some more theoretical work is needed.
3.3 Polarization in $\Lambda_c$ decays

Heavy Quark Symmetries also restrict heavy to light transitions. While for mesons the number of form factors for e.g. $B \to \pi$ and $B \to \rho$ transitions is not reduced, some relations may be found for baryons. The $\Lambda_Q$ baryons are the simplest objects from the point of view of heavy quark symmetry and indeed spin symmetry imposes interesting constraints. Consider for example the matrix element of a current $\bar{q}\Gamma h_v$ between a heavy $\Lambda_Q$ and a light spin-1/2 baryon $B_\ell$, where $q$ is a light quark. This matrix element is described by only two form factors [14] according to

$$\langle B_\ell(p)|\bar{\ell}\Gamma h_v|\Lambda_Q(v)\rangle = \bar{u}_\ell(p)\{F_1(v \cdot p) + \gamma 2(v \cdot p)\}\Gamma u_{\Lambda_Q}(v). \quad (39)$$

Thus in this particular case spin symmetry drastically reduces the number of independent Lorentz-invariant amplitudes which describe the heavy to light transitions.

This has some interesting implications for exclusive semileptonic $\Lambda_c$ decays. For the case of a left handed current $\Gamma = \gamma_\mu(1 - \gamma_5)$, the semileptonic decay $\Lambda_c \to \Lambda\ell\bar{\nu}_\ell$ is in general parametrized in terms of six form factors

$$\langle \Lambda(p)|\bar{q}\gamma_\mu(1 - \gamma_5)c|\Lambda_c(v)\rangle = \bar{u}(p)\left[f_1\gamma_\mu + if_2\sigma_{\mu\nu}q^\nu + f_3g^\mu\right]u(p')$$
$$\quad + \bar{u}(p)\left[g_1\gamma_\mu + ig_2\sigma_{\mu\nu}q^\nu + g_3q^\mu\right]\gamma_5 u(p'), \quad (40)$$

where $p' = m_{\Lambda_c}v$ is the momentum of the $\Lambda_c$ whereas $q = m_{\Lambda_c}v - p$ is the momentum transfer. From this one defines the ratio $G_A/G_V$ by

$$\frac{G_A}{G_V} = \frac{g_1(q^2 = 0)}{f_1(q^2 = 0)}. \quad (41)$$

In the heavy $c$ quark limit one may relate the six form factors $f_i$ and $g_i$ ($i = 1, 2, 3$) to the two form factors $F_j$ ($j = 1, 2$)

$$f_1 = -g_1 = F_1 + \frac{m_\Lambda}{m_{\Lambda_c}}F_2 \quad (42)$$
$$f_2 = f_3 = -g_2 = -g_3 = \frac{1}{m_{\Lambda_c}}F_2 \quad (43)$$

from which one reads off $G_A/G_V = -1$. This ratio is accessible by measuring in semileptonic decays $\Lambda_c \to \lambda\ell\bar{\nu}_\ell$ the polarization variable $\alpha$

$$\alpha = \frac{2G_AG_V}{G_A^2 + G_V^2}. \quad (44)$$
which is predicted to be $\alpha = -1$ in the heavy $c$ quark limit. The subleading corrections to the heavy $c$ quark limit have been estimated and found to be small \[15\]

$$\alpha < -0.95,$$

and recent measurements yield

$$\alpha = -0.91 \pm 0.49 \quad \text{ARGUS} \ [16]$$

$$\alpha = -0.89^{+0.17+0.09}_{-0.11-0.05} \quad \text{CLEO} \ [17]$$

and are in satisfactory agreement with the theoretical predictions.

Recently the CLEO collaboration also measured the ratio of the form factors $F_1$ and $F_2$, averaged over phase space. Heavy quark symmetries do not fix this form factor ratio, at least not for a heavy to light decay, while for a heavy to heavy decay the form factor $F_2$ vanishes in the heavy mass limit for the final state quark. CLEO measures \[18\]

$$\langle \frac{F_2}{F_1} \rangle_{\text{phase space}} = -0.25 \pm 0.14 \pm 0.08$$

which is in good agreement with various model estimates.

### 3.4 Quark Helicities in $\Lambda_b \to \Lambda \gamma$

Another interesting application of relation (39) is the rare decay $\Lambda_b \to \Lambda \gamma$ which is a flavour changing neutral current process of the type $b \to s \gamma$. The interesting part of this process is its short distance contribution due to the effective Hamiltonian

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{ts} V_{tb}^* C_7 \mathcal{O}_7$$

where $C_7$ is some short distance coefficient and

$$\mathcal{O}_7 = \frac{e}{32\pi^2} m_b \bar{s} \sigma_{\mu\nu} (G_V - G_A \gamma_5) b F^{\mu\nu}$$

where $F^{\mu\nu}$ is the usual electromagnetic field strength tensor.

The parameters $C_7$, $G_V$ and $G_A$ may be computed in the standard model where one finds $C_7 \approx 0.3$ and $G_V = 1 + m_s/m_b$ and $G_A = -1 + m_s/m_b$, i.e. the $b$ quark is practically right handed.
It has been speculated that $b \to s\gamma$ may open a window to physics beyond the standard model and the value of $C_7$ has already been tested to some extent in the decays $B \to X_s\gamma$ and also $B \to K^*\gamma$. However, the helicity structure of the effective Hamiltonian cannot be measured in the mesonic decays, they will only be accessible in the decays of $\Lambda_b$ baryons. In particular, the decay $\Lambda_b \to \Lambda\gamma$ is a good candidate, since the decay of the final state $\Lambda$ is self-analyzing.

The decay $\Lambda_b \to \Lambda\gamma$ has been analyzed in detail in [19]. Apart from the long distance effects, which have been estimated to be small, there is also a problem with the application of (39). This relation is expected to work best in the region of phase space where the final state light baryon moves slowly in the rest frame of the decaying $\Lambda_b$. Unfortunately, the relevant kinematic region of $\Lambda_b \to \Lambda\gamma$ is at the opposite side of phase space, since $q^2 = 0$ for the real photon implies for the energy $E$ of the light baryon $E \approx m_b/2$ which becomes large in the infinite mass limit. However, it has been argued in [19] that it is very likely that relation (39) still holds for the decay $\Lambda_b \to \Lambda\gamma$.

Assuming an unpolarized $\Lambda_b$ one may measure the polarization variable $\alpha'$

$$\Gamma = \Gamma_0 \left[1 + \alpha' \vec{n} \cdot \vec{S}_\Lambda\right]$$

where $\vec{n}$ is the direction of the outgoing $\Lambda$ in the rest frame of the $\Lambda_b$.

Computing the polarization variable using the CLEO measurement of the form factor ratio (48) as input one obtains [19]

$$\alpha' = 0.387 \frac{2G_V G_A}{G^2_V + G^2_A}$$

which will allow some test of the helicity structure of the effective short distance Hamiltonian, once enough data on $\Lambda_b \to \Lambda\gamma$ becomes available.

4 Conclusion

The fact that in the heavy mass limit the spin of the heavy quark decouples has many interesting consequences for processes involving heavy quarks. In the decays one may analyze the helicity structure of the transition operator for which the standard model makes definite predictions. Analogous to the Michel parameter analysis of the $\mu$ and $\tau$ decays one may check the left-handedness of the hadronic currents of heavy quarks in a model independent
way. However, for stringent tests one has to wait for the data coming from B-factories.

As far as production of heavy hadrons is concerned, LEPI provided the interesting possibility to observe the hadronization of highly polarized $b$ quarks. For the $b$ quarks hadronizing into mesons the polarization information is effectively lost, while some of the polarization is retained for hadronization into baryons. The amount of depolarization for the $\Lambda_b$ seems to be quite high, if the experimental values settle in the region as given in [13]. Unfortunately, for more data on polarized $b$ quark fragmentation one probably has to wait for the era of polarized hadron colliders.

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