Numerical simulation for homogeneous–heterogeneous reactions and Newtonian heating in the silver-water nanofluid flow past a nonlinear stretched cylinder

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Abstract

The present exploration aims to deliberate silver-water nanofluid flow with homogeneous–heterogeneous reactions and magnetic field impacts past a nonlinear stretched cylinder. The novelty of the presented work is enhanced with the addition of Newtonian heating, heat generation/absorption, viscous dissipation, nonlinear thermal radiation and joule heating effects. The numerical solution is established via Shooting technique for the system of ordinary differential equations with high nonlinearity. The influences of miscellaneous parameters including nanoparticles volume fraction ($0.0 \leq \phi \leq 0.3$), magnetic parameter ($1.0 \leq M \leq 4.0$), nonlinearity exponent ($1.0 \leq n \leq 5.0$), curvature parameter ($0.0 \leq \gamma \leq 0.4$), conjugate parameter ($0.4 \leq \lambda \leq 0.7$), heat generation/absorption parameter ($0.2 \leq E_c \leq 0.8$), radiation parameter ($0.7 \leq K^* \leq 1.0$), Eckert number ($0.1 \leq E_c \leq 0.7$), strength of homogeneous reaction ($0.1 \leq \kappa_1 \leq 1.8$), strength of heterogeneous reaction ($0.1 \leq \kappa_2 \leq 1.8$) and Schmidt number ($3.0 \leq S_c \leq 4.5$) on axial velocity, temperature profile, local Nusselt number, and skin friction coefficient are discussed via graphical illustrations and numerically erected tabulated values. It is examined that the velocity field diminishes while the temperature profile enhances for mounting values of the magnetic parameter. An excellent concurrence is achieved when our obtained numerical calculations are compared with an already published paper in limiting case; hence dependable results are being presented.

Keywords: homogeneous/heterogeneous reactions, nonlinear stretching cylinder, Newtonian heating, nonlinear thermal radiation, nanofluid

(Some figures may appear in colour only in the online journal)
1. Introduction

The feeble thermal conductivity of certain base fluids in numerous processes has been a big obstacle to shape a refined product. Certain techniques like pressure loss, abrasion and clogging were proposed by the researchers to overcome this deficiency but outcomes were not very encouraging. Nevertheless, the novel concept of nanofluid [1] (an amalgamation of suspended Nano metered sized metallic particles and some ordinary fluid such as oil, water or ethylene glycol) has revolutionized the modern industrial world. These nano-sized (<100 nm) metallic particles are comprised of metals, their oxides, and carbon nanotubes. Nanofluids possess certain unique properties that make them potentially worthwhile in numerous engineering and industrial heat transfer applications [2] like fuel cells, microelectronics, domestic refrigerators, machining, hybrid-powered engines, chillers, and grinding. Here, in all these applications enhanced thermal conductivity is seen when some metallic particles are injected into the base fluid [3]. The innovative idea of nanofluid was coined by Choi [4]. This was followed by numerous investigations highlighting varied aspects of nanofluids. To name some recent explorations in this regard includes the study by Nadeem and Muhammad [5] who examined three different ferrite nanoparticles in a ferromagnetic fluid. Rehman and Nadeem [6] analyzed the stagnation point water base nanofluid flow. Sheikholeslami [7] examined the flow of CuO and H₂O nanoliquid in a permeable channel numerically utilizing Lattice Boltzmann method. In another investigation Sheikholeslami [8] found numerical solution of electrodynamic nanofluid flow with impacts of free convection and thermal radiation numerically using control volume based finite element method. The flow of nanoliquid flow with suspended carbon nanotubes carrying the effects of activation energy and Cattaneo–Christov heat flux are examined numerically by engaging Runge–Kutta fifth-order Fehlberg technique was discussed by Lu et al [9] and many therein [10–20].

The flows of numerous fluids in attendance of magnetohydrodynamic (MHD) have extensive important applications in aerospace engineering, MHD generators, medicine, geothermal field, petroleum processes, nuclear reactors engineering and astrophysics. A reasonable number of explorations have been conducted featuring MHD fluid flows featuring an effort by Ramzan et al [21], they inspected the MHD flow of Jeffery nanofluid with radiation effects. Hayat et al [22] studied the MHD micropolar fluid flow with homogeneous–heterogeneous (h–h) reactions over a curved surface which is stretched in a linear manner. The study of MHD water-based nanofluid thin film using Homotopy analysis method past a stretched cylinder is considered by Khan et al [23]. Ramzan and Bilal [24] deliberated the 3D nanofluid flow in the attendance of MHD and chemical reaction. Ishak et al [25] utilized the stretching cylinder to examine the MHD flow. Qayyum et al [26] inspected the MHD stagnation point nanoliquid flow with Newtonian heat and mass conditions. Haq et al [27] scrutinized the MHD nanofluid flow with
thermal radiation via a stretching sheet near a stagnation point. Bhatti and Rashidi [28] studied Hall Effect on an MHD peristaltic flow. Nadeem and Hussain [29] examined the MHD Williamson flow of nanoliquid past the heated surface. Ramzan et al [30] numerically studied the MHD micropolar nanofluid past a rotating disk. Ibrahim [31] used the linearly stretched surface to discuss the MHD nanofluid flow in the occurrence of melting heat near a stagnation point.

A direct proportionate between heat transfer rate and the local temperature is called Newtonian heating. It is also named as conjugate convective flow. It is utilized in many processes like designing of heat exchangers, conjugate heat transfer around fins and convective flows in which heat is absorbed from solar radiators by surrounding bounded surfaces etc. Merkin [32] was the first to consider four distinct categories of heat transfer phenomenon from wall to ambient fluid namely (a) Newtonian heating (b) conjugate boundary conditions (c) constant or prescribed surface heat flux, and (d) constant or prescribed surface temperature. Lately, various researchers have used the impact of Newtonian heating because of its broad practical applications [33–40].

A literature survey indicates that abundant research articles are available pertaining to nanofluid flows with combined impacts of the h–h reactions and MHDs past linear/nonlinear stretching surfaces. Comparatively, less research work is done with nanoliquids past cylinders and this choice gets even narrower if we talk about nanoliquid flows over nonlinear stretching cylinders. As far as our knowledge is concerned no study so far is conducted for the nanoliquid flow (with silver nanoparticles and water) past a nonlinear stretched cylinder with impacts of both h–h reactions, Newtonian heating, and nonlinear thermal radiation. Thus, our prime objective is to examine the nanoliquid flow past a nonlinear stretching cylinder with Newtonian heating, nonlinear thermal radiation, and h–h reactions. This exploration is unique in its own way and will attract a good readership. Numerical solution of the system of equations is acquired with the Runge–Kutta method by shooting technique. A comparative study with an already established result is also made and an excellent concurrence of both results is obtained.

### 2. Flow analysis

Consider an incompressible Ag-water nanoliquid flow past a nonlinear stretching cylinder with h–h reactions. In addition, nonlinear thermal radiation and Newtonian heating effects are also considered. It is presumed that a magnetic field \( B = B_0 x^{(a-1)/2} \) is operated along the radial direction. The induced magnetic field is overlooked due to our supposition of small Reynolds number figure 1.

The homogeneous reaction for cubic autocatalysis can be communicated as given below:

\[
A_i + 2B_1 \rightarrow 3B_1, \quad \text{rate} = k_{2}ab^2,
\]
\[
A_i \rightarrow B_1, \quad \text{rate} = k_{1}a.
\]

These reaction equations guarantee that reaction rate vanishes in the outer tier of the boundary layer.

Usage of the boundary layer approximation, the continuity, momentum, temperature and concentration equations are appended below:

\[
\frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial r} = 0,
\]
\[
\frac{du}{dx} + \frac{v}{r} \frac{du}{dr} = u_{sf} \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) - \frac{\sigma B^2(x)}{\rho_{nf}} u,
\]
\[
\frac{\partial T}{\partial x} + \frac{v}{r} \frac{\partial T}{\partial r} = \alpha_{nf} \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) - \frac{1}{(\rho c_p)_{nf}} \frac{\partial p}{\partial r} + \frac{\mu_{nf}}{(\rho c_p)_{nf}} \left( \frac{\partial u}{\partial r} \right)^2 + \frac{\sigma B^2(x)}{(\rho c_p)_{nf}} u^2 + \frac{Q_{0}}{(\rho c_p)_{nf}} (T - T_{\infty}),
\]
\[
\frac{\partial a}{\partial x} + \frac{v}{r} \frac{\partial a}{\partial r} = D_a \left( \frac{\partial^2 a}{\partial r^2} + \frac{1}{r} \frac{\partial a}{\partial r} \right) - k_{1}ab^2,
\]
\[
\frac{\partial b}{\partial x} + \frac{v}{r} \frac{\partial b}{\partial r} = D_b \left( \frac{\partial^2 b}{\partial r^2} + \frac{1}{r} \frac{\partial b}{\partial r} \right) + k_{1}ab^2,
\]

with allied boundary conditions

\[
u_{r=R} = U_{w}(x), \quad v_{r=R} = 0,
\]
\[
\frac{\partial T}{\partial r} \bigg|_{r=R} = h_{s}T, \quad D_{T} \frac{\partial a}{\partial r} \bigg|_{r=R} = k_{s}a,
\]
\[
D_{b} \frac{\partial b}{\partial r} \bigg|_{r=R} = -k_{s}a, \quad u_{r=\infty} \rightarrow 0,
\]
\[
T_{r=\infty} \rightarrow T_{\infty}, \quad a_{r=\infty} \rightarrow a_{0},
\]
\[
b_{r=\infty} \rightarrow 0.
\]
Here \( U_0(x) = U_0x^n \), and \( q_n = \frac{4\pi^2 \partial^2 \phi}{\partial T^2} = \frac{16\pi^2 T^2 \partial^2 T}{\partial y^2} \). The numerical value of specific heat, density, and thermal conductivity of nanoparticle (Ag) and conventional fluid water is given in Table 1.

The measured forms for the thermo-physical properties are given as:

\[
\begin{align*}
\alpha_{nf} &= \frac{k_{nf}}{(\rho c_p)_{nf}} = (1 - \phi) + \phi \frac{(\rho c_p)_h}{(\rho c_p)_f}, \\
\mu_{nf} &= \frac{\mu_f}{(1 - \phi)^{2.5}}, \quad \rho_{nf} = 1 - \phi \frac{\rho_f}{\rho_f}, \\
k_f &= \frac{(2k_f + k_s) + 2\phi(k_s - k_f)}{(2k_f + k_s) + \phi(k_f - k_s)}. \quad (9)
\end{align*}
\]

2.1. Similarity transformation

Utilizing the subsequent similarity transformations

\[
\eta = \frac{r^2 - R^2}{2R} \frac{U_w}{\sqrt{\nu_k x}}, \quad \psi = \sqrt{U_w/x} Rf(\eta), \quad \theta = \frac{T - T_\infty}{T_\infty} \frac{\nu}{a}
\]

\( a = a_0 h, \quad b = a_0 g. \)

(10)

The condition for equation (3) is agreed automatically and equations (4)–(7) take the form

\[
\begin{align*}
(1 + 2\gamma\eta)f'' + 2\gamma f'' + (1 - \phi)^2(1 - \phi + \omega_0) \\
\times \left( \frac{n + 1}{2} f'' - nf'^2 \right) - (1 - \phi)^2.5 MF' = 0, \quad (11)
\end{align*}
\]

\[
(1 + 2\gamma\eta) \left(\frac{k_{nf}}{k_f} + (1 + (N_r - 1)\theta) \right) \theta' = 0
\]

\[
+ \frac{Pr}{4K^{*}} \left( 1 - \phi + \phi \frac{(\rho c_p)_h}{(\rho c_p)_f} \right)
\]

\[
\times \left( \frac{n + 1}{2} f'' - nf'^2 + D_\theta \theta' \right)
\]

\[
= 2(1 + 2\gamma\eta)
\]

\[
\times (N_r - 1)(1 + (N_r - 1)\theta) \theta'^2 = 0.
\]

\[
\frac{1}{S_c} (1 + 2\gamma\eta) h'' + 2\gamma h' + \frac{2 K_i}{n + 1} h(1 - h)^2 = 0, \quad (13)
\]

\[
\frac{1}{S_c} (1 + 2\gamma\eta) g'' + 2\gamma g' + \frac{2 K_i}{n + 1} h(1 - h)^2 = 0.
\]

\[
\text{with boundary conditions}
\]

Table 2. Nusselt number \((Re_0^{-1/2}Nu_x)\) for several values of \(\gamma\) and \(Pr\) with \(M = 0, \phi = 0.0, \lambda = 0.0\).

| \(\gamma\) | \(Pr\) | \(\text{Qusim et al [41]}\) | \(\text{Present result}\) |
|---|---|---|---|
| 0.0 | 0.72 | 1.236 64 | 1.236 651 |
| 1.0 | 1.000 00 | 1.000 000 |
| 6.7 | 0.333 30 | 0.333 310 |
| 10 | 0.268 76 | 0.268 770 |
| 1.0 | 0.72 | 0.870 18 | 0.870 190 |
| 1.0 | 0.72 | 0.870 18 | 0.870 190 |
| 6.7 | 0.296 61 | 0.296 620 |
| 10 | 0.242 17 | 0.242 180 |

\[
f(\eta) = 0, \quad f'(\eta) = 1, \quad \theta'(\eta) = -\lambda(1 + \theta(\eta)), \quad \eta = 0, \quad \theta(\eta) = 0, \quad f'(\eta) = 0, \quad h(\eta) = 1, \quad g(\eta) \to \infty, \quad (15)
\]

where \( \lambda = h_k(\nu_k x/U_w)^{1/2}, \quad M = \frac{a_0 B_0^2}{U_w}, \quad K^* = \frac{16\pi^2 T_\infty^3}{3 \pi^{1/2}}, \quad \gamma = \left( \frac{x_{y_*}}{R^* U_w} \right)^{1/2}, \quad N_r = T_r/T_\infty, \quad K = \frac{a_0 B_0^2}{U_w}, \quad K_s = \frac{k_s}{\rho_s c_{p_s}}, \quad S_c = \frac{y_c}{\rho_f c_{p_f}}.
\]

\[
E_r = \frac{\nu R^0 C_p}{C_p}, \quad D_f = \frac{\nu R^0 C_p}{C_p}.
\]

Here, it is anticipated that \( A_f \) and \( B_1 \) are analogous. This assumption implies that \( D_A \) and \( D_B \) (diffusion coefficients) are equivalent i.e. \( \delta = 1 \) and because of this assumption, we have

\[
g(\eta) + h(\eta) = 1.
\]

(16)

Using equations (16), (13) and (14) with corresponding boundary conditions take the form

\[
\frac{1}{S_c} (1 + 2\gamma\eta) h'' + 2\gamma h' + \frac{2 K_i}{n + 1} h(1 - h)^2 = 0, \quad (17)
\]

\[
h'(0) = k_2 h'(0), \quad h(\infty) \to 1.
\]

(18)

2.2. Local Nusselt number and Skin friction factor

The dimensional form of the skin friction factor \((C_f)\) and local Nusselt number \((Nu_x)\) are described as

\[
C_f = \frac{2w_{\tau w}}{\rho_f u_w^2}, \quad Nu_x = \frac{x_{q_w}}{k_s(T_f - T_\infty)}, \quad (19)
\]

where \( \tau_w \) and \( \rho_w \) are

\[
\tau_w = \mu_{nf} \frac{\partial u}{\partial y} |_{r=R}, \quad \rho_w = \frac{\partial T}{\partial y} |_{r=R}, \quad k_{nf} + (q_w)_{r=R}. \quad (20)
\]
Consuming equations (10) and (20), in equation (19), we get

\[
Re_{x}^{1/2} G_f = \left( \frac{1}{(1 - \phi)^{1.5}} \right) f''(0),
\]

\[
Re_{x}^{-1/2} Nu_x = \frac{k_{nf}}{k_f} \left( 1 + \frac{1}{\theta(0)} \right).
\]  

(21)

3. Numerical technique

Shooting technique is employed to find out the numerical solution of equations (11), (12) and (17) with associated boundary conditions (15) and (18). While finding the numerical solution, the third and second order differential equations are converted to first order by utilizing new parameters. In shooting technique, we select an initial guess that satisfies the boundary conditions and the equation asymptotically. For the present problem, tolerance is taken as $10^{-7}$. A comparison of the present analysis with already published paper Qasim et al [41] in limiting case is given in table 2 and all numerical calculations depict a good agreement.

4. Result and discussion

4.1. Velocity profile for several parameters

Figures 2–11 are illustrated to exhibit the effects of sundry parameters on involved fields. The impacts of $\phi$, $M$, $n$ and $\gamma$ on axial velocity are shown in figures 2–5 respectively. The influence of solid volume fraction $\phi$ is inspected in figure 2. It is determined that the axial velocity boosts with enhancing the value of $\phi$. Figure 3 determines the impact of magnetic parameter $M$ on velocity distribution. The velocity profile diminishes with enhancement in values of $M$. The strong magnetic field augments the Lorentz force, thus the resistive force hinders the fluid’s flow and slows it down. That is why

\[
\gamma = 0.5, \quad M = 2, \quad n = 2,0, \quad \lambda = 0.1, \quad Sc = 1.0, \quad K' = 2.0
\]

\[
\phi = 0.0, 0.1, 0.2, 0.3
\]

Figure 2. Axial velocity for numerous estimates of $\phi$.

\[
\gamma = 0.5, \quad \phi = 0.1, \quad n = 2,0, \quad \lambda = 0.3, \quad Sc = 1.0, \quad K' = 3.0
\]

\[
M = 1.0, 2.0, 3.0, 4.0
\]

Figure 3. Axial velocity for numerous estimates of $M$.

\[
\gamma = 0.5, \quad \phi = 0.1, \quad M = 1.0, \quad \lambda = 0.3, \quad Sc = 1.0, \quad K' = 3.0
\]

\[
n = 2.0, 3.0, 4.0, 5.0
\]

Figure 4. Axial velocity for numerous estimates of $n$.

\[
\gamma = 0.5, \quad \phi = 0.1, \quad M = 1.0, \quad \lambda = 0.3, \quad Sc = 1.0, \quad K' = 3.0
\]

\[
n = 1.0, \quad \phi = 0.1, \quad M = 1.0, \quad \lambda = 0.1, \quad Sc = 1.0, \quad K' = 3.0
\]

\[
\gamma = 0.1, 0.2, 0.3, 0.4
\]

Figure 5. Axial velocity for numerous estimates of $\gamma$.  


Figure 6. Temperature field for numerous estimates of $\gamma$.

Figure 7. Temperature field for numerous estimates of $\lambda$.

Figure 8. Temperature field for numerous estimates of $K'$.

Figure 9. Temperature field for numerous estimates of $M$.

Figure 10. Temperature field for numerous estimates of $D_c$.

Figure 11. Temperature field for numerous estimates of $E_c$. 
reduction in the velocity of the fluid is noticed. The axial velocity diminishes with growing the value of nonlinear exponent \( n \), this effect is depicted in figure 4. This is due to the fact that fluid particles are disturbed for larger values of \( n \). Actually, more collision amongst fluid particles is witnessed that obstructs the movement of the fluid and ultimately a reduction in axial velocity is noticed. The impression of the curvature parameter \( \gamma \) on axial velocity is depicted in figure 5. It is seen that the axial velocity is a growing function of \( \gamma \). In fact, increased values of the \( \gamma \) result in squeezed radius and ultimately less contact area between the fluid and the cylinder is detected. This is the main reason behind the augmented axial velocity.

4.2. Temperature profile for several parameters

Figures 6–11 analyze the effect of curvature parameter, radiation parameter, conjugate parameter, and magnetic parameter on temperature field. Figure 6 exhibits the effects of curvature parameter \( \gamma \) on the temperature profile. The fluid’s temperature enhances for augmenting values of \( \gamma \). Actually, increase in heat transport is detected for augmented values of \( \gamma \), thus rise in temperature profile is witnessed. The impact of the conjugate parameter \( \lambda \) on the temperature field is characterized in figure 7. It is determined that the temperature profile upsurges with mounting values of \( \lambda \). Higher values of \( \lambda \) leads to stronger heat transfer coefficient and as a result more heat will transfer from the cylinder to the fluid. It is pertinent to mention that \( \lambda \rightarrow \infty \) exhibits the constant wall temperature and \( \lambda = 0 \) indicates the insulated wall. Figure 8 is drawn to analyze the effect of radiation parameter \( K^* \) on temperature profile. The temperature field enhances for augmented estimations of \( K^* \). In fact, for growing values of \( K^* \), the mean absorption coefficient decreases thus growth in radiative heat transfer rate is perceived. Figure 9 is portrayed to examine the impact of magnetic parameter \( M \) on temperature field. It is seen that for the growing values of magnetic parameter \( M \), the temperature profile enhances. This is because of the verity that the Lorentz force augments owing to increased estimates of \( M \) thus impedes the fluid’s movement. In this way, more collision between molecules of the fluid is observed and additional heat is produced thus increasing the temperature of the fluid. The impacts of heat generation/absorption parameter \( D_c \) and Eckert number \( E_c \) are illustrated in figures 10 and 11 respectively. The temperature of the fluid escalates for growing estimates of \( D_c \) and \( E_c \), which is an obvious veracity.

4.3. Concentration profile for different parameter

Figures 12 and 13 are illustrated to portray the strength of homogeneous and heterogeneous reactions’ impact on concentration profile respectively. It is observed that the concentration profile intensifies in both cases for growing values of \( \eta \), and after a certain estimate of \( \eta \), no impact on concentration distribution is seen for both cases of the strength of homogeneous and heterogeneous reactions.

Table 3 shows the numerical value of skin friction \( -Re^{1/2}C_f \) and local Nusselt number \( (Re_{\infty}^{1/2}Nu_e) \) for various estimates of parameters. It is detected that the numerical value of the Nusselt number and skin friction coefficient are enhanced for growing values of the nonlinearity parameter \( n \), curvature parameter \( \gamma \), and solid volume friction \( \phi \). While for the \( M \) (magnetic parameter) the skin friction coefficient enhances and the local Nusselt number diminishes. Further, for the value of temperature ratio parameter \( N_r \) and radiation parameter \( K^* \), the Nusselt number diminishes, while the skin friction coefficient is constant for temperature ratio parameter and slight change is observed for radiation parameter.

5. Concluding remarks

The problem of nanoliquid flow with silver nanoparticles and water (base fluid), is discussed with nonlinear thermal radiation with Newtonian heating past a nonlinear stretching
cylinder. The effects of heterogeneous/homogeneous reactions with MHDs are also examined. The shooting technique is engaged to solve the nonlinear ODEs. The key points of the current effort are appended as follows:

- For growing values of the solid volume fraction of nanoparticles, escalation in velocity field is observed.
- The velocity profile diminishes, and temperature profile enhances for augmented values of the magnetic parameter.
- For mounting values of radiation and curvature parameters, the temperature field enhances.
- Concentration field decreases versus increasing values of the strength of heterogeneous and homogeneous reactions.

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### Conflict of interest

Authors have no conflict of interest regarding this publication.

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### Table 3. Numerical values of \(-Re^{1/2}C_f\) and \(Re^{1/2}Nu_a\) for Ag-water with \(Pr = 6.2\).

| \(n\) | \(\gamma\) | \(\phi\) | \(M\) | \(N_x\) | \(K^*\) | \(-Re^{1/2}C_f\) | \(Re^{1/2}Nu_a\) |
|------|------|------|------|------|------|--------------|--------------|
| 1.0  | 1.0  | 0.3  | 1.0  | 0.1  | 2.0  | 1.062 40    | 0.828 64     |
| 2.0  |      |      |      |      |      | 2.100 20    | 0.931 09     |
| 3.0  |      |      |      |      |      | 2.692 80    | 1.093 50     |
| 2.0  | 1.0  |      |      |      |      | 2.100 20    | 0.931 09     |
| 2.0  |      |      |      |      |      | 2.552 00    | 0.992 47     |
| 3.0  |      |      |      |      |      | 2.967 20    | 1.037 70     |
| 1.0  | 0.1  |      |      |      |      | 1.113 80    | 0.684 40     |
| 0.2  |      |      |      |      |      | 1.937 40    | 0.797 11     |
| 0.3  |      |      |      |      |      | 2.100 20    | 0.931 09     |
| 0.3  | 1.0  |      |      |      |      | 2.100 20    | 0.931 09     |
| 2.0  |      |      |      |      |      | 2.974 70    | 0.926 88     |
| 3.0  |      |      |      |      |      | 3.232 50    | 0.923 04     |
| 1.0  | 0.1  |      |      |      |      | 2.100 20    | 0.931 09     |
| 0.7  |      |      |      |      |      | 2.100 21    | 0.877 70     |
| 1.0  |      |      |      |      |      | 2.100 21    | 0.420 70     |
| 0.1  | 1.0  |      |      |      |      | 2.100 20    | 0.978 27     |
| 2.0  |      |      |      |      |      | 2.100 20    | 0.931 09     |
| 3.0  |      |      |      |      |      | 2.100 20    | 0.883 91     |

### References

1. Buongiorno J 2006 Convective transport in nanofluids *J. Heat Transfer* **128** 240–50
2. Minkovyzcz W J, Sparrow E M and Abraham J P (ed) 2012 *Nanoparticle Heat Transfer and Fluid Flow*. vol 4 (Boca Raton, FL: CRC Press)
3. Kakac S and Pramuanjaroenkij A 2009 Review of convective heat transfer enhancement with nanofluids *Int. J. Heat Mass Transfer* **52** 3187–96
4. Choi S U S 1995 Enhancing conductivity of fluids with nanoparticles, ASME fluid Eng *Division* **231** 99–105
5. Muhammad N and Nadeem S 2017 Ferrite nanoparticles Ni-ZnFe2O4, Mn-ZnFe2O4 and Fe2O3 in the flow of ferromagnetic nanofluid *Eur. Phys. J. Plus* **132** 377
6. Rehman F U and Nadeem S 2017 Heat transfer analysis for three-dimensional stagnation-point flow of water-based nanofluid over an exponentially stretching surface *J. Heat Transfer* **140** 052401
7. Sheikholeslami M 2018 Numerical investigation for CuO-H2O nanofluid flow in a porous channel with magnetic field using mesoscopic method *J. Mol. Liq.* **249** 739–46
8. Sheikholeslami M 2018 Numerical investigation of nanofluid free convection under the influence of electric field in a porous enclosure *J. Mol. Liq.* **249** 1212–21
9. Lu D, Ramzan M, Ahmad S, Chung J D and Farooq U 2017 Upshot of binary chemical reaction and activation energy on carbon nanotubes with Cattaneo-Christov heat flux and buoyancy effects *Phys. Fluids* **29** 123103
10. Sheikholeslami M, Haq R U, Shafiee A and Li Z 2019 Heat transfer behavior of nanoparticle enhanced PCM solidification through an enclosure with V shaped fins *Int. J. Heat Mass Transfer* **130** 1322–42
11. Li Z, Ramzan M, Shafiee A, Saleem S, Al-Mdallal Q M and Chamkha A J 2018 Numerical approach for nanofluid transportation due to electric force in a porous enclosure *Microsyst. Technol.* 1–14
12. Lu D, Ramzan M, Ullah N, Chung J D and Farooq U 2017 A numerical treatment of radiative nanofluid 3D flow containing gyrotactic microorganism with anisotropic slip, binary chemical reaction and activation energy *Sci. Rep.* **7** 17008
13. Ramzan M, Ullah N, Chung J D, Lu D and Farooq U 2017 Buoyancy effects on the radiative magneto Micropolar nanofluid flow with double stratification, activation energy and binary chemical reaction *Sci. Rep.* **7** 12901
14. Ramzan M, Bilal M, Kanwal S and Chung J D 2017 Effects of variable thermal conductivity and non-linear thermal radiation past an Eyring Powell nanofluid flow with chemical reaction *Commun. Theor. Phys.* **67** 723
15. Sheikholeslami M 2019 New computational approach for exergy and entropy analysis of nanofluid under the impact of Lorentz force through a porous media *Comput. Methods Appl. Mech. Eng.* **344** 319–33
16. Sheikholeslami M 2019 Numerical approach for MHD Al2O3–water nanofluid transportation inside a permeable medium using innovative computer method *Comput. Methods Appl. Mech. Eng.* **344** 306–18
17. Sheikholeslami M, Mehrayan S A M, Shafiee A and Sheremet M A 2019 Variable magnetic forces impact on magnetizable hybrid nanofluid heat transfer through a circular cavity *J. Mol. Liq.* **277** 388–96
18. Sheikholeslami M, Shafiee A, Ramzan M and Li Z 2018 Investigation of Lorentz forces and radiation impacts on nanofluid treatment in a porous semi annulus via Darcy law *J. Mol. Liq.* **272** 8–14
19. Sheikholeslami M, Jafaryar M, Shafiee A and Li Z 2018 Investigation of second law and hydrothermal behavior of
nanofluid through a tube using passive methods *J. Mol. Liq.* **269** 407–16

[20] Sheikholeslami M 2018 Application of Darcy law for nanofluid flow in a porous cavity under the impact of Lorentz forces *J. Mol. Liq.* **266** 495–503

[21] Ramzan M, Bilal M, Chung J D and Mann A B 2017 On MHD radiative Jeffery nanofluid flow with convective heat and mass boundary conditions *Neural Comput. Appl.* **30** 2739–48

[22] Hayat T, Sajjad R, Ellahi R, Alsaedi A and Muhammad T 2017 Homogeneous-heterogeneous reactions in MHD flow of micropolar fluid by a curved stretching surface *J. Mol. Liq.* **240** 209–20

[23] Khan N S, Gul T, Islam S, Khan I, Alqahtani A M and Alshomrani A S 2017 Magnetohydrodynamic nanoliquid thin film sprayed on a stretching cylinder with heat transfer *Appl. Sci.* **7** 271

[24] Ramzan M and Bilal M 2016 Three-dimensional flow of an elasto-viscous nanofluid with chemical reaction and magnetic field effects *J. Mol. Liq.* **215** 212–20

[25] Ishak A, Nazar R and Pop I 2008 Magnetohydrodynamic (MHD) flow and heat transfer due to a stretching cylinder *Energy Convers. Manage.* **49** 3265–9

[26] Qayyum S, Hayat T, Shehzad S A and Alsaedi A 2017 Effect of a chemical reaction on magnetohydrodynamic (MHD) stagnation point flow of Walters-B nanofluid with Newtonian heat and mass conditions *Nucl. Eng. Technol.* **49** 1636–1644

[27] Haq R U, Nadeem S, Khan Z H and Akbar N S 2015 Thermal radiation and slip effects on MHD stagnation point flow of nanofluid over a stretching sheet *Physica E* **65** 17–23

[28] Bhatti M M and Rashidi M M 2017 Study of heat and mass transfer with Joule heating on magnetohydrodynamic (MHD) peristaltic blood flow under the influence of Hall effect *Propulsion Power Res.* **6** 177–85

[29] Nadeem S and Hussain S T 2016 Analysis of MHD Williamson nano fluid flow over a heated surface *J. Appl. Fluid Mech.* **9** 729–39

[30] Ramzan M, Chung J D and Ullah N 2017 Partial slip effect in the flow of MHD micropolar nanofluid flow due to a rotating disk—a numerical approach *Results Phys.* **7** 3557–66

[31] Ibrahim W 2017 Magnetohydrodynamic (MHD) boundary layer stagnation point flow and heat transfer of a nanofluid past a stretching sheet with melting *Propulsion Power Res.* **6** 214–22

[32] Merkin J H 1994 Natural-convection boundary-layer flow on a vertical surface with Newtonian heating *Int. J. Heat Fluid Flow* **15** 392–8

[33] Ramzan M 2015 Influence of Newtonian heating on three dimensional MHD flow of couple stress nanofluid with viscous dissipation and joule heating *PLoS One* **10** e0124699

[34] Qayyum S, Hayat T, Shehzad S A and Alsaedi A 2018 Mixed convection and heat generation/absorption aspects in MHD flow of tangent-hyperbolic nanoliquid with Newtonian heat/mass transfer *Radiat. Phys. Chem.* **144** 396–404

[35] Mohamed M K A, Salleh M Z, Noar N A Z and Ishak A 2017 Buoyancy effect on stagnation point flow past a stretching vertical surface with Newtonian heating *AIP Conf. Proc.* **1795** 020005

[36] Ramzan M 2015 Influence of Newtonian heating on three dimensional MHD flow of couple stress nanofluid with viscous dissipation and Joule heating *PLoS One* **10** e0124699

[37] Akbar N S and Nadeem S 2013 Mixed convective magnetohydrodynamic peristaltic flow of a Jeffrey nanofluid with Newtonian heating *Z. Naturforsch. A* **68** 433–41

[38] Makinde O D 2012 Computational modelling of MHD unsteady flow and heat transfer toward a flat plate with Navier slip and Newtonian heating *Braz. J. Chem. Eng.* **29** 159–66

[39] Rahman M U, Khan M and Manzur M 2017 Homogeneous–heterogeneous reactions in modified second grade fluid over a non-linear stretching sheet with Newtonian heating *Results Phys.* **7** 4364–70

[40] Khan I, Malik M Y, Salahuddin T, Khan M and Rehman K U 2017 Homogeneous–heterogeneous reactions in MHD flow of powell–eyring fluid over a stretching sheet with Newtonian heating *Neural Comput. Appl.* **30** 3581–88

[41] Qasim M, Khan Z H, Khan W A and Shah I A 2014 MHD boundary layer slip flow and heat transfer of ferrofluid along a stretching cylinder with prescribed heat flux *PLoS One* **9** e83930