Unexpected Scaling of the Performance of Carbon Nanotube Transistors

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(November 11, 2018)

We show that carbon nanotube transistors exhibit scaling that is qualitatively different than conventional transistors. The performance depends in an unexpected way on both the thickness and the dielectric constant of the gate oxide. Experimental measurements and theoretical calculations provide a consistent understanding of the scaling, which reflects the very different device physics of a Schottky barrier transistor with a quasi-one-dimensional channel contacting a sharp edge. A simple analytic model gives explicit scaling expressions for key device parameters such as subthreshold slope, turn-on voltage, and transconductance.

Recent decades have witnessed remarkable and continuing improvements in the performance of field-effect transistors (FETs). These improvements result largely from aggressive scaling of devices to smaller sizes. As further improvement of conventional FETs becomes increasingly difficult, attention has focused on new devices like carbon nanotube (CN) FETs. CNFETs have already shown very promising performance, despite the use of relatively thick gate oxides [1–3].

Here we examine the performance improvement of CNFETs upon scaling of the thickness and dielectric constant of the gate oxide. In both experimental measurements and numerical calculations, we find a very different scaling behavior than for conventional transistors, with important consequences for the design of CNFETs. Specifically, we find that key measures of device performance scale approximately as the square root of the gate-oxide thickness $t_{ox}$ or its inverse. These include the turn-on voltage, the transconductance, and the subthreshold slope.

We show that this surprising behavior can be captured in a simple analytic model, which gives a universal form for the saturation current versus gate voltage. Our model incorporates the recent recognition [1,2,4–6] that, in ambipolar CNFETs such as ours, transistor action is caused by modulation of the Schottky barriers (SBs) at the metal-nanotube contacts. The model also highlights the central role of the contact geometry in determining the scaling, with different geometries giving different power laws for the scaling. In contrast, the scaling of conventional FETs with $t_{ox}$ is independent of contact geometry in the long-channel limit.

In conventional transistors, there is great interest in oxides with high dielectric constants, because these increase the gate capacitance and thus the performance. Improved performance has also been obtained in this way for CNFETs [2,7]. However, for ballistic SB-CNFTs, the performance is not linked to the capacitance. We show that in this case, the improvement instead comes from changes in the electric field patterns due to the inhomogeneous dielectric used. For such geometries, the degree of improvement is tightly coupled to the gate-oxide thickness, with the most dramatic improvement occurring for thicker gate oxides.

![Graph showing representative transfer characteristics of CNFETs at different oxide thicknesses.](image)

**FIG. 1.** Representative transfer characteristics of CNFETs at different oxide thicknesses $t_{ox}$. The turn-on is characterized by the subthreshold slope (straight lines). The slope becomes steeper for thinner oxides: from 280 mV/decade for $t_{ox} = 20$ nm and $V_d = -0.5$ V, to 170 mV/decade for $t_{ox} = 5$ nm and $V_d = -0.5$ V, and 110 mV/decade for $t_{ox} = 2$ nm and $V_d = -0.2$ V. Comparable scaling is seen in n-type devices as well, e.g. 130 mV/decade for $t_{ox} = 2$ nm and $V_d = 0.2$ V.

Our CNFETs use a standard back-gated geometry [8], taking advantage of the precise thickness control and high quality of thermally grown SiO$_2$. Very thin gate oxides (2 and 5 nm) are grown in small, pre-patterned areas on a degenerately doped silicon wafer, which serves as the gate. Carbon nanotubes with diameter of about 1.4nm [9] are dispersed on the wafer. Source and drain electrodes, formed using electron-beam lithography and lift-off, contact those CNs lying on ultra-thin oxide. In order to suppress the leakage current, the rest of the source and drain contacts are separated from the back gate by a thicker (~ 120 nm) field oxide. Details of the
fabrication will be presented elsewhere.

Typical transport characteristics of bottom gate devices with thin oxides are shown in Fig. 1. A quantitative comparison can be made by measuring the subthreshold slope [10] \( S = (d \log_{10} I/dV_g)^{-1} \) where \( V_g \) is the gate voltage. The extracted values are given in Fig. 1. We see a steady improvement of the turn-on for thinner oxides. However, even the thinnest oxides give \( S \) significantly higher than the thermal limit of about \( kT \log 10 \sim 60 \) mV/dec at room temperature, and a key goal here is to understand the factors leading to further performance improvement.

(Lower \( S \) values have been reported in CNFETs using highly-doped, effectively ohmic contacts [7] or exotic designs [11]. However, these have their own advantages and disadvantages; and here we consider only straightforward scaling of existing designs that lend themselves to integration with Si-based devices.)

To understand the scaling properties of carbon nanotube (NT) transistors, we calculate the current using the Landauer-Büttiker formula, and assuming ballistic transport:

\[
I = \frac{4e}{h} \int \left[ F(E) - F(E - eV_d) \right] T(E) dE .
\] (1)

Here \( V_d \) is the applied drain voltage and \( F \) is the Fermi function. The energy-dependent transmission \( T(E) \) through the SB is evaluated within the WKB approximation, using the idealized band structure [12]. This gives

\[
\ln T = -\frac{4}{3bV_x} \int_{z_1}^{z_2} \left( \Delta^2 - [E + eV(z)]^2 \right)^{1/2} dz ,
\] (2)

where \( b = 0.144 \) nm is the bond length, \( \Delta \) is half the NT band gap, \( V_x = 2.5 \) eV is the tight-binding parameter, and \( V(z) \) is the electrostatic potential along the NT. The integration is performed between the classical turning points \( z_1 \) and \( z_2 \). [E.g., for electrons in the conduction band, \( z_1 \) is determined from \( \Delta - eV(z_1) = E \); \( z_1 = 0 \) or, if \( E < \Delta \), is determined from \( -\Delta - eV(z_1) = E \).] For simplicity, we present numerical calculations only for the case of mid-gap Schottky barrier, though we have examined other cases. In this case, the current is symmetric with respect to the applied gate voltage; and without loss of generality, we limit the discussion to positive gate voltages, i.e. electron tunneling into the conduction band.

To obtain the electrostatic potential along the NT, we numerically solve the electrostatic boundary problem given by our device geometry, which is sketched in the upper inset of Fig. 2. We consider a device similar to the experiment with a bottom gate at \( \ell_{\text{ox}} \) from the NT, source and drain contacts that are 20 nm thick, and a top electrode which is far from the NT with respect to \( \ell_{\text{ox}} \). (The use of a top electrode in this geometry is for computational convenience; and whether it is grounded or kept at \( V_g \) has negligible impact on the results.) We further neglect charge on the NT as in [5]. This remains a good approximation throughout the regime studied here [13].

All numerical calculations here use a NT of 1.4 nm diameter and a band gap of 0.6 eV.

In Fig. 2 the inverse of the subthreshold slope, \( S^{-1} \), has been plotted versus the inverse oxide thickness \( \ell_{\text{ox}}^{-1} \). Experimentally, about 20 to 30 devices were characterized at each oxide thickness. Only devices with symmetric (or nearly symmetric) p- and n-conduction were included in the average values presented in Fig. 2. The \( S^{-1} \) values — even for very thin oxides — lie well below the thermal limit. The theoretical curve for the bottom gate geometry is in good qualitative agreement with experiment; the difference presumably arises from the specific contact geometry, which is not known precisely on the nm scale in the experiment.

To illustrate the importance of the contact geometry, we also consider an idealized double gate device (lower inset of Fig. 2). Its source and drain contacts are as thin as the NT diameter (1.4 nm), and both top and bottom gate oxides have thickness \( \ell_{\text{ox}} \). The general trend with \( \ell_{\text{ox}} \) is unchanged, but for any given \( \ell_{\text{ox}} \) the subthreshold slope is greatly improved relative to the bottom gate device with thicker contact.

![FIG. 2. Calculated and experimental values of the inverse subthreshold slope \( S \) as a function of the inverse oxide thickness \( \ell_{\text{ox}} \). Solid curves are fitted as described in text. Open red squares are experimental data from previous work, filled red squares are this work. Data at \( \ell_{\text{ox}} = 2 \) nm and 5 nm represent average values for many devices. For \( \ell_{\text{ox}} = 20 \) nm our result agrees with previous reports [3]. The upper left inset shows the bottom gate device geometry (open green circles) while the lower right inset shows the double gate device (filled orange circles).](image-url)
To better understand the behavior, we consider a simple model. For the double gate device, in the limit of vanishing contact thickness [inset of Fig. 3(b)], the electrostatic problem can be solved analytically [14]. The potential in the vicinity of the contact varies as $V(z) = 2V_g \pi^{-1/2} (z/t_{ox})^{1/2}$, where $V_g$ is the gate voltage and $z$ is the distance from the contact along the NT. Inserting this $V(z)$ into Eq. (2), and taking $V_d = V_g$ [as in inset of Fig. 3(a)], we calculate the saturation current. (We consider the limit of a thick gate oxide, where the position of the conduction band at the drain, $\Delta - eV_d$, can be replaced by $-\infty$). Then

$$I_{sat} = \frac{4e\Delta}{h} H \left( \frac{V_g}{V_{scale}^{dg}}, \frac{\Delta}{kT} \right),$$

(3)

where $H(x, y)$ is

$$H(x, y) = \int_{-\infty}^{\infty} \frac{\exp(-h(s)/x^2)}{1 + \exp(sy)} ds$$

(4)

and $h(s)$ is

$$h(s) = \int_{\max(0, -1-s)}^{1-s} t [1 - (s + t)^2]^{1/2} dt .$$

(5)

The “scaling voltage” for this idealized double-gate device is

$$V_{scale}^{dg} = \left( \frac{2\pi \Delta^3}{3be^2V_g} \right)^{1/2} t_{ox}^{1/2} .$$

(6)

Within this idealized geometry (and in the thick-oxide limit etc.), changing the oxide thickness is equivalent to simply rescaling the gate voltage by $V_{scale}^{dg}$. The turn-on voltage is proportional to $V_{scale}^{dg}$, and thus scales as $t_{ox}^{1/2}$. We can evaluate $S$ for $V_d$ at saturation, where $S = (d\log_{10} I_{sat}/dV_g)^{-1}$. Then $S$, like $I_{sat}$, is a function of $V_g/V_{scale}^{dg}$. Thus $S$ scales as $t_{ox}^{-1/2}$. Similarly, we find that the transconductance, $dI_{sat}/dV_g$, scales as $t_{ox}^{-1/2}$.

For very thin gate oxides, some of these approximations break down — in particular, the simple form of $V(z)$ near the contact, and the infinite limits of integration in Eq. (4). But in any case, $S$ must eventually saturate at the thermal limit of $kT \ln 10 \sim \sim 60$ mV/dec. This suggests the interpolation formula $S = (\alpha t_{ox} + (kT \ln 10)^2)^{1/2}$, with $\alpha$ a fitting parameter. Fig. 2 shows that this fitting works well for the experiment and the calculation for both device geometries. For thick oxides, $S$ scales as $t_{ox}^{1/2}$, while for very thin oxides it gradually approaches the thermal limit.

For a more complete description of the device behavior, we examine the saturation current vs. gate voltage. Figure 3(a) shows the results of the full numerical calculation for the bottom-gate geometry. These are well described by the analytic model (3), over a large range of $t_{ox}$, if we replace the analytic $V_{scale}^{bg}$ with $V_{scale}^{bg} = 2.2V_{scale}^{dg}$, where the single empirical factor of 2.2 is sufficient to account for the difference in geometry. Figure 3(b) shows that all curves are nearly identical to the analytic form (3), with the same $t_{ox}^{1/2}$ scaling as in the idealized model (aside from the one empirical factor of 2.2). As discussed above, the analytic model becomes inaccurate for very thin oxides, and this is observed in Fig. 3(b) for $t_{ox} = 2$ nm.

### FIG. 3. Calculated saturation current versus gate voltage. (a) shows a set of curves for bottom gate devices with $t_{ox} = 35, 20, 10, 5, \text{and } 2$ nm, from right to left, and the thermal limit given by the dotted line. Solid lines are calculated with Eq. (3) and $V_{scale}^{bg} = 2.2V_{scale}^{dg}$, while dashed lines show the full calculation. (The inset shows the band diagram for saturating current, i.e. $V_d = V_g$.) (b) displays the same curves versus $V_g/V_{scale}^{bg}$. All solid lines become one black line, i.e. Eq. (3). (The inset is a sketch of the double gate geometry with a semi-infinite sheet contact.)

In contrast to conventional FETs, the scaling of the performance here with $t_{ox}$ depends on the specific contact geometry. If the contact were infinitely thick, then the electrostatic potential near the right-angle corner would depend on distance as $z^{2/3}$ [14]. For such a geometry, the turn-on voltage and $S$ would scale as $t_{ox}^{2/3}$, while the
transconductance would scale as $t_{\text{ox}}^{-2/3}$. Thus there is not really a universal power law; rather, the behavior depends on the contact geometry. Nevertheless, the two geometries considered here are rather well described by a single simple power-law behavior.

In addition to reducing the oxide thickness, the performance can be improved by using an oxide (or other gate dielectric) having higher dielectric constant [2,7,11]. In conventional transistors, this improves performance by increasing the gate capacitance, and hence the charge in the channel. However, in ballistic SB-CNFTs, the charge in the channel is unimportant for the turn-on regime, so the improvement observed in such devices [2] must have a different origin.

In fact, within the approximations made here, simply increasing the dielectric constant everywhere would have no effect on the performance. However, most actual devices studied use a thin gate oxide below the NT, with air above it.

Our numerical calculations indicate that with high ratios $\varepsilon_b/\varepsilon_t$, large improvements can be achieved for thick oxides. However, the changes become rather small for very thin oxides. For example, for $t_{\text{ox}} = 20$ nm, replacing the homogeneous oxide ($\varepsilon_b/\varepsilon_t = 1$) by HfO$_2$ below and air above the NT ($\varepsilon_b/\varepsilon_t = 11$) changes the slope of the transconductance $S$ from 240 mV/dec to 120 mV/dec. This improvement agrees well with experimental data [2]. For $t_{\text{ox}} = 2$ nm, though, $S$ changes only from 95 mV/dec to 75 mV/dec.

We have focused here on the case of a mid-gap Schottky barrier. However, we have also examined the behavior for other values of the Schottky barrier. As long as the current is limited by tunneling through a SB, as for barrier heights of 0.2 eV or more, the scaling behavior derived here continues to hold.

We thank Bruce Ek for expert technical assistance and Jörg Appenzeller for helpful discussion. S. H. thanks the Deutsche Forschungsgemeinschaft for financial support under the Grant number HE3292/2-1.

\[ T(E) \approx 1. \]

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**FIG. 4.** Contour lines of the electrostatic potential at the source contact for an interface between a high dielectric and vacuum (solid red lines, $\varepsilon_b/\varepsilon_t = 11$) and a homogeneous dielectric (dotted blue lines). The value of adjacent contour lines differs by 0.1 V. ($V_d = V_g = 0.5$ V.)

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