CHANDRA PHASE-RESOLVED X-RAY SPECTROSCOPY OF THE CRAB PULSAR

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1 INTRODUCTION

The Crab Nebula, the relic of a stellar explosion recorded by Chinese astronomers in 1054 C.E., has a special place in the history of astronomy. It is one of our closest laboratories, and possibly the most frequently observed, for high-energy astrophysics. Located at a distance of \( \approx 2 \) kpc, the system is energized by a pulsar of spin-down luminosity \( L_{\text{pulsr}} = 5 \times 10^{38} \text{ erg s}^{-1} \) and spin period \( P = 34 \text{ ms} \). The history and general properties of the system are nicely summarized in a recent review by Hester (2008). Optical and X-ray images (Hester et al. 1995, 2002; Weisskopf et al. 2000) of the inner nebula show features such as an inner ring, toroidal structure, knots, and two opposing jets originating from the pulsar. These latter are aligned with the proper motion vector (e.g., Kaplan et al. 2008) and presumably with the pulsar’s rotation axis. In X-rays the inner ring is commonly accepted as being the termination shock produced by the wind of particles emanating from the pulsar. Because of the nearby structure it has been impossible for any X-ray observatory, other than \( Chandra \), to isolate the pulsar from the surrounding nebulosity. The ability to accomplish this isolation is invaluable for determining an uncontaminated spectrum for the pulsar itself. Observations with instrumentation unable to resolve the pulsar from the nebula define the data in and near the pulse minimum to be “unpulsed,” subtracting it from the data at other pulse phases, and so deriving a “pulsed” spectrum. While such an approach is often appropriate, it is ultimately flawed as the Crab’s pulsar does not turn off in X-rays (Tennant et al. 2001) and therefore the “unpulsed” signal is contaminated with a pulsed component. Thus, as observations approach higher statistical accuracy, \( Chandra \) provides the only means to isolate, as fully as possible, the pulsed spectrum and to study variations with pulse phase at all pulse phases.

In Weisskopf et al. (2004)—hereafter Paper I—we utilized \( Chandra \) to obtain the first \( Chandra \) Low Energy Transmission Grating Spectrometer (LETGS) phase-averaged and phase-resolved X-ray spectra of the Crab Pulsar. Those observations isolated the pulsar from its surroundings to within \( Chandra \) spatial resolution and probed the spectral variation as a function of pulse phase. In Paper I we also set an upper limit to the thermal emission from the neutron star’s surface, once again essentially unblemished by any contaminating signal from the pulsar’s wind nebula. Here, we present the results of new \( Chandra \) observations that provide more data (by a factor of four) than previously and make use of the most recent \( Chandra \)-LETGS response functions.

ABSTRACT

We present a new study of the X-ray spectral properties of the Crab Pulsar. The superb angular resolution of the \( Chandra \) X-Ray Observatory enables distinguishing the pulsar from the surrounding nebulosity. Analysis of the spectrum as a function of pulse phase allows the least-biased measure of interstellar X-ray extinction due primarily to photoelectric absorption and secondarily to scattering by dust grains in the direction of the Crab Nebula. We modify previous findings that the line of sight to the Crab is underabundant in oxygen and provide measurements with improved accuracy and less bias. Using the abundances and cross sections from Wilms et al. we find \( [\text{O}/\text{H}] = (5.28 \pm 0.28) \times 10^{-4} \) (4.9 \( \times 10^{-4} \) is solar abundance). We also measure for the first time the impact of scattering of flux out of the image by interstellar grains. We find \( \tau_{\text{scat}} = 0.147 \pm 0.043 \). Analysis of the spectrum as a function of pulse phase also measures the X-ray spectral index even at pulse minimum—albeit with increasing statistical uncertainty. The spectral variations are, by and large, consistent with a sinusoidal variation. The only significant variation from the sinusoid occurs over the same phase range as some rather abrupt behavior in the optical polarization magnitude and position angle. We also compare these spectral variations to those observed in gamma-rays and conclude that our measurements are both a challenge and a guide to future modeling and will thus eventually help us understand pair cascade processes in pulsar magnetospheres. The data are also used to set new, and less biased, upper limits to the surface temperature of the neutron star for different models of the neutron star atmosphere. We discuss how such data are best connected to theoretical models of neutron star cooling and neutron star interiors. The data restrict the neutrino emission rate in the pulsar core and the amount of light elements in the heat-blanketing envelope. The observations allow the pulsar, irrespective of the composition of its envelope, to have a neutrino emission rate higher than \( \sim 0.2 \) of the standard rate of a non-superfluid star cooling via the modified Urca process. The observations also allow the rate to be lower but now with a limited amount of accreted matter in the envelope.

Key words: atomic processes – ISM: general – stars: individual (Crab Nebula) – techniques: spectroscopic – X-rays: stars
After briefly describing the observation and data reduction (Section 2), we discuss the analysis of the measured spectra (Section 3). We update the work of Paper I where we examined the interstellar abundances in the line of sight to the pulsar (Section 3.1). Here we only use one set of cross-sections and abundances for deriving spectral parameters. The reader is also urged to look at Paper I to see how the derived spectral parameters are influenced by the particular choice of the set of cross-sections and abundances. We present (Section 3.2) new, more precise, measurements of the variation of the non-thermal spectral parameters with pulse phase and the implications. We discuss constraints on the surface temperature of the underlying neutron star (Section 3.3) assuming two different models for the thermal emission. In Paper I we only considered a blackbody model; here we also examine the impact of the spectral parameters assuming a model with a neutron star atmosphere. Finally, in distinction from the approach in Paper I, when we analyze the data as a function of pulse phase we allow the power-law component to vary. This provides a less-biased approach to measuring or setting upper limits to the thermal spectral parameters. We discuss ramifications of the temperature measurements (Section 3.3.3) and summarize our findings (Section 4).

2. OBSERVATIONS AND DATA REDUCTION

In this paper, we make use of data (ObsID 9765) we obtained in 2008 January. As with our previous observation (Paper I), these data were taken using Chandra’s LETG and High-Resolution Camera spectroscopy detector (HRC-S)—the LETGS. Due to a wiring error, the timetag associated with each event is really the time of the previous event.7 If the HRC does not saturate telemetry it is easy to shift each timetag back to recover the correct time of events. But if events occur faster than the telemetry can accommodate, then events will be left on the spacecraft and shifting will not always be correct. Tennant et al. (2001) showed that it was possible to limit the resulting error by only considering events close to each other in time thus any error from missing events must be smaller than this time separation. Applying this filter to the 50 ks ObsID 759 data set resulted in only 54,000 background subtracted counts in both orders of the dispersed pulsar spectrum.

For the 100 ks ObsID 9765 we reduced the HRC count rate by blocking some of the flux before it got to the detector. This approach used one of the two shutters that had been installed on the HRC to facilitate finding a good focus position. We were able to show that by turning off the outer plates (normally done for HRC timing mode) and inserting one of these blades into the beam, we would be able to block enough of the flux to reduce the Crab/HRC count rate to below telemetry saturation. For the highly absorbed Crab, turning off the outer plates had no impact since they are only useful for dispersed energies below 0.3 keV. By design, the blade would block most of the positive order, a significant fraction of the zero order, and some of the negative order flux. We then performed two brief on-orbit experiments (ObsIDs 8548 and 9764) to fine tune the precise blade position to maximize the count rate but still keep it below saturation. This method generated 230,000 background subtracted counts.

To correct the response, we first simulated the impact of the blade 257.861 mm above the HRC plate. We then restricted the energy range to 0.3–3.8 keV. Below the lower limit the first-order flux is effectively zero and the upper limit avoids contamination from the zeroth-order nebular image (see Figure 1 in Paper I). This simulation showed that the blockage was approximately a linear function over this spectral band. This can also be seen in Figure 1 which compares the background subtracted counts from the same extraction region for ObsID 9765 with blade) divided by the rate from ObsID 759 with no blade. The slight deviations at long wavelengths (long distances from the axis of symmetry) might well be hints of telescope-dependent effects which one will have to consider with longer integration times. We then corrected our response function by the blade correction factor for that location. Since the HRC has no energy response, we assume that the higher-order contribution also scales by the measured correction factor at the location on the detector where the counts were detected. This is reasonable for energies below the critical graze angle for the mirrors. In Section 3.1, we compare the phased-averaged-spectral parameters derived from the earlier observation without the blade, with the new data with the blade correction and find that the differences are minor. Given that the phase-averaged spectrum contains the most counts, we feel that our correction is more than adequate for the phase-resolved spectra.

We processed all data using Chandra X-ray Center (CXC) tools. Level 2 event files were created using the CIAO script hrc_process_events with pixel randomization off and CALDB 4.4.3.8 The program axbary was used to covert the time of arrival of events to the solar system’s barycenter. Using LEXTRACT (developed by one of us, A.F.T.) we extracted the pulsar’s dispersed spectrum from the images. The extraction used a ±12,348 pixel wide (1.63) region in the cross-dispersive direction and centered on the pulsar, in 2 pixel (dispersive) increments. This extraction width is the standard extraction region for which the CIAO-derived response functions apply for the central HRC-S detector. For the LETG’s 0.9912 µm grating period and 8.638 m Rowland-circle radius, the LETGS dispersion is 1.148 Å mm⁻¹. Consequently, the spectral resolution of the binned data (two 6.4294 µm HRC-S pixels) is 0.01476 Å.

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7 http://asc.harvard.edu/proposer/POG/

8 Earlier versions of CALDB tended to overestimate the higher-order flux.
Selecting the appropriate region of the data to be used as the background estimator requires care as one must deal with the dispersed flux from the nebula which includes such bright features as the inner ring (Weisskopf et al. 2000). To determine the background we studied the flux at the deepest portion of pulse minimum (phase range 0.65–0.70, in the spectral region corresponding to 0.3–3.8 keV in first order and projected onto the cross-dispersion axis (Figure 2). We chose pulse minimum for this study to avoid having the pulsar dominate the projected data. Based on the information shown in Figure 2, and to estimate the background, we extract data from two 15-HRC-pixel (1′98) wide bands starting 15 pixels to either side of the pulsar’s dispersed image. Note that just beyond (more negative) −15 pixels in Figure 2 there is a slight rise traceable to the dispersed spectrum of features near the pulsar, showing that extending the background region much wider would be a mistake. Of course the asymmetric nature of the background in the cross-dispersion direction due to features in the nebula makes the selection of regions to either side of the dispersed spectrum a necessity, so that the average provides an accurate background estimate.

3. ANALYSIS AND RESULTS

As in Paper I, whenever we fit data to a particular spectral model we allow for interstellar absorption using cross-sections from verner, Verner et al. (1993), and interstellar absorption from tbvarabs, Wilms et al. (2000, and references therein), allowing for absorption by interstellar grains. In addition, owing to the small effective aperture of our observation of the pulsar, we also allow for the effects of diffractive scattering by grains on the interstellar extinction using a formula based on the Rayleigh–Gans approximation (van der Hulst 1957; Overbeek 1965; Hayakawa 1970), valid when the phase shift through a grain diameter is small. Details may be found in Paper I. We analyzed the data using the XSPEC (v.11.3.2) spectral-fitting package (Arnaud 1996). To ensure applicability of the \( \chi^2 \) statistic, we merged spectral bins as needed to obtain at least 100 counts per fitting bin (before background subtraction). Response functions were first generated using the CIAO threads mkgarf and mkgarf and then corrected for the occultation of the shutter blade. The effective areas were generated using standard CIAO tools and CALDB 4.4.3.

3.1. Phase-averaged Spectrum

We present here the results of our analysis of the phase-averaged spectrum, although we will not use this spectrum to search for an additional thermal component or to measure phase-independent parameters. These tasks are more appropriately accomplished using the phase-resolved spectrum (Section 3.2), a point also emphasized by Jackson & Halpern (2005).

Fitting the phase-averaged data to a power-law spectrum and allowing for interstellar absorption, a variation in the abundance ratio of oxygen to hydrogen, and small-angle scattering by intervening dust give an excellent fit, \( \chi^2 \) of 1795 for 1810 degrees-of-freedom (v). The best-fit spectrum is shown in Figure 3.

In Table 1 we list the best-fit parameters, but now fix the dust scattering factoring as was done in Paper I to make comparisons. As a convenience for the reader, the first line in the table repeats the results from Paper I. The second line shows how these results change using the more recent response function, background extraction regions, etc. The reader will note that there are differences between line 1 and 2 of the table that are beginning to approach statistical significance. For example, the hydrogen abundance is now lower (3.68 as compared to 4.20). The differences are mostly a consequence of the evolution of the calibration of the LETGS effective area. The differences between the results for the same data (lines 1 and 2 of Table 1) emphasize that we have ignored (as many X-ray astronomers do) the possibility that there are any errors in the response functions. The final comparison is between our current results (line 3 of Table 1) and the updated results for ObsID 759. As before, there are differences, but none of them at the 3\( \sigma \) level. The largest differences, in the normalization and in the power-law index, may be a reflection of inaccuracies at the 5% level in our correction to the response function for the insertion of the blade.

3.2. Spectral Variation with Pulse Phase

Jodrell Bank (Lyne et al. 1993) routinely observes the Crab Pulsar (Wong et al. 2001) providing a period ephemeris.\(^9\)

\(^9\) http://www.jb.man.ac.uk/~pulsar/crab.html
for the pulsar’s phase-resolved power-law photon index, vary. This minimizes any bias produced by assuming a power-law index and normalization are allowed to vary. However, the spectral index and normalization are allowed to vary. The interstellar absorption and dust scattering parameters to vary, accounting for absorption by interstellar grains. M. Roberts & M. Kramer (2000, 2008, private communications) allowing for absorption by interstellar grains. Wilms et al. (2000, references therein) allowing for absorption by interstellar grains.

Table 1

| ObsID    | $\chi^2$/$\nu$ | $\Gamma_P$ | $N_H$ (10$^{21}$ cm$^{-2}$) | $[O/H]$ (10$^{-4}$) | Norm |
|----------|----------------|------------|----------------------------|---------------------|------|
| Paper Ib | 1539/1552      | 1.587 ± 0.019$^c$ | 4.20 ± 0.14           | 3.33 ± 0.44         | 0.506 ± 0.008 |
| 759$^b$  | 1447/1476      | 1.622 ± 0.023            | 3.68 ± 0.13            | 4.49 ± 0.42          | 0.479 ± 0.009 |
| 9765     | 1796/1811      | 1.547 ± 0.023            | 3.27 ± 0.09            | 5.04 ± 0.32          | 0.455 ± 0.008 |

Notes.

$^a$ Abundance models in XSPEC: wilm, Wilms et al. (2000); cross-section models in XSPEC: vern, Verner et al. (1993); tbvaraha, Wilms et al. (2000, references therein) allowing for absorption by interstellar grains.

$^b$ ObsID 759 (Paper I).

$^c$ Larger extraction width, hence more counts, hence larger number of degrees of freedom compared to line 2.

$^d$ Uncertainties are XSPEC’s estimate of the 1$\sigma$ error treating each variable as the one interesting parameter ($\chi^2$ at minimum +1.0).

$^e$ Paper I, but using the up-to-date response function and extraction widths.

Table 2

The Phase-independent Parameters Determined from a Phase-varying Power-law Fit$^a$ to the Chandra-LETGS Phase-resolved Spectrum of the Crab Pulsar

| ObsID | $\chi^2$/$\nu$ | $N_H$ (10$^{21}$ cm$^{-2}$) | $[O/H]$ (10$^{-4}$) | $\tau_{\text{scat}}$ at (1 keV) |
|-------|----------------|-----------------------------|---------------------|-------------------------------|
| 9765  | 3510/3546      | 3.22 ± 0.12$^d$             | 5.28 ± 0.28         | 0.147 ± 0.043                 |

Notes.

$^a$ Abundance models in XSPEC: wilm, Wilms et al. (2000); cross-section models in XSPEC: vern, Verner et al. (1993); tbvaraha, Wilms et al. (2000, references therein) allowing for absorption by interstellar grains.

$^b$ Uncertainties are XSPEC’s estimate of the 1$\sigma$ error treating each variable as the one interesting parameter ($\chi^2$ at minimum +1.0).

M. Roberts & M. Kramer (2000, 2008, private communications) kindly prepared ephemerides matched to our observation times. Phase zero in the following was set by the radio observations.

In performing the phase-resolved spectral analysis, we allow the interstellar absorption and dust scattering parameters to vary, but assume that they are identical for each pulse phase bin. However, the spectral index and normalization are allowed to vary. This minimizes any bias produced by assuming a power-law index that is independent of phase. The fit to the data was excellent: $\chi^2$ was 3156 on 3207 degrees of freedom. The best-fit values for the non-phase-varying parameters and their approximate (one-interesting-parameter) uncertainties are given in Table 2. Note $\tau_{\text{scat}}$, which we previously (Paper I) postulated must be present and accounted for, has now been detected at almost the 4$\sigma$ level. Accounting for $\tau_{\text{scat}}$ is important as it has an impact on the other spectral parameters. In this case, allowing for $\tau_{\text{scat}}$ increases the power-law index by approximately 3%, decreases the hydrogen column by about 8%, and decreases the oxygen-to-hydrogen ratio by about 12%. To the extent that the measures of such parameters are important at this level of precision, users of Chandra are cautioned to take account of dust scattering. Finally, Table 3 and Figure 4 summarize the results for the pulsar’s phase-resolved power-law photon index, $\Gamma_P$.

Table 3

| Phase Range x 1000 | $\Gamma_P$ | Norm |
|--------------------|------------|------|
| 001–017            | 1.627 ± 0.042$^a$ | 1.762 ± 0.053 |
| 017–031            | 1.583 ± 0.063          | 0.862 ± 0.031 |
| 031–051            | 1.463 ± 0.074          | 0.493 ± 0.020 |
| 051–075            | 1.462 ± 0.089          | 0.334 ± 0.016 |
| 075–120            | 1.450 ± 0.097          | 0.207 ± 0.010 |
| 120–330            | 1.342 ± 0.041          | 0.276 ± 0.008 |
| 330–370            | 1.472 ± 0.038          | 1.064 ± 0.031 |
| 370–390            | 1.500 ± 0.040          | 1.654 ± 0.049 |
| 390–400            | 1.536 ± 0.048          | 1.859 ± 0.059 |
| 400–410            | 1.622 ± 0.052          | 1.572 ± 0.051 |
| 410–430            | 1.604 ± 0.051          | 0.950 ± 0.031 |
| 430–470            | 1.656 ± 0.059          | 0.462 ± 0.016 |
| 470–650            | 1.699 ± 0.108          | 0.090 ± 0.005 |
| 650–830            | 1.886 ± 0.462          | 0.020 ± 0.004 |
| 830–950            | 1.503 ± 0.058          | 0.233 ± 0.008 |
| 950–960            | 1.733 ± 0.061          | 1.271 ± 0.044 |
| 960–970            | 1.611 ± 0.050          | 1.768 ± 0.057 |
| 970–980            | 1.649 ± 0.043          | 2.532 ± 0.076 |
| 980–984            | 1.651 ± 0.053          | 3.208 ± 0.105 |
| 984–992            | 1.627 ± 0.039          | 3.893 ± 0.114 |
| 992–001            | 1.594 ± 0.039          | 3.453 ± 0.101 |

Notes.

$^a$ Uncertainties are XSPEC’s estimate of the 1$\sigma$ error treating each variable as the one interesting parameter ($\chi^2$ at minimum +1.0).

et al. (1997) measurements, with the index increasing (the spectrum becoming softer) through the primary-pulse maximum and decreasing (the spectrum becoming harder) in the bridge between the primary and secondary pulses. It is difficult to be more quantitative in this comparison as the non-Chandra data were analyzed using different cross-sections, and different abundances. Also those data covered different spectral ranges. Moreover, Chandra provides the angular resolution needed to isolate the pulsar from the nebula, something that is essential to measure the spectral index for the pulse-phase range 0.5–0.9. Our analysis shows that the spectral index at pulse minimum is consistent with an apparent continuation of the increase (softening) of the spectral index until just before the onset of the primary pulse. The spectral-index uncertainty near pulse minimum is, of course, large because there are fewer counts. It is also interesting that the slope of the spectral variations appears to be the highest (the spectrum softest) during the peak of the two pulses.
In Figure 4, we also show the results of fitting a constant plus a sine wave to the variation of the spectral index. The fit to the sine wave is excellent. The phase of the variation is possibly an indication of the geometry. The point just before the rise to the pulse maximum, however, may not fit this simple picture. We discuss this point further in the following section.

3.2.1. Discussion

There are a number of interesting correlations shown also in Figure 4. In the middle panel we see the variation of the spectral index measured with the *Fermi Gamma-Ray Space Telescope* at energies above 100 MeV (Abdo et al. 2010a). The gamma-ray variation of spectral index with phase is strikingly similar to the X-ray spectral index variations with the hardest (smallest) index occurring midway between the two peaks and rising symmetrically through both peaks to reach maxima in the off-peak region. There is even a hint in the *Fermi* data of the small maximum preceding the first peak. The bottom two panels plot optical polarization data kindly provided by G. Kanbach and A. Slowikowska (Slowikowska et al. 2008). Here, we emphasize the aforementioned change in behavior in the X-ray power-law index just before the rise of the light curve to primary pulse maximum (phases 0.83–0.95), the apparent identical behavior in the gamma-ray measurements, and the clear abrupt change of optical polarization and position angle all take place in this same phase interval. These would appear to be correlated phenomena.

Indeed, the photon index variation is similar in other bright gamma-ray pulsars, including Geminga (Abdo et al. 2010b) and Vela (Abdo et al. 2010c), where the maximum preceding the first peak is even more pronounced. Yet, the Crab broadband spectrum is very different from that of Vela or Geminga. The Crab is one of the very few pulsars (including B1509−58) having equal or greater power in the X-ray band as in the hard gamma-ray band. The multiwavelength spectrum of the Crab Pulsar (Kuiper et al. 2001) seems to comprise two distinct components: one extending from UV to soft gamma-rays and one extending from soft to hard gamma-rays. In the phase-resolved spectra, the spectral indices of the two components tend to mirror each other, with the hardest spectra in soft X-rays and hard gamma-rays occurring in the bridge region and the softer spectra occurring in the peaks. The similarity of the *Chandra* and *Fermi* spectral index behavior is consistent with this trend.

The fact that the soft X-ray and hard gamma-ray spectra are part of two seemingly different radiation components, and most likely have different emission mechanisms, raises the question of why their spectral index variation with phase should be so similar. They must share a common property, such as the same radiating particles or the same locations in the magnetosphere.

It is now widely agreed that the high-energy emission from pulsars originates in their outer magnetospheres, since the measurement of the cutoffs in their gamma-ray spectra rules out attenuation due to magnetic pair production and therefore emission near the polar caps (Abdo et al. 2009). Several different outer-magnetosphere models, that advocate different emission mechanisms in the X-ray range, make predictions for phase-resolved spectral variations. In outer gap models (Cheng et al. 1986; Romani 1996), particles are accelerated in vacuum gaps that form along the last open magnetic field lines, from above the null-charge surface to near the light cylinder. In slot gap models (Muslimov & Harding 2004), particles are also accelerated along the last open field lines, but in a charge-depleted layer from the neutron star surface to near the light cylinder. In both models, the high-energy peaks in the light curve are caustics, caused by cancellation of phase differences along the trailing field lines (Morini 1983) or by overlapping field lines near the light cylinder.

Harding et al. (2008) presented a model for three-dimensional acceleration and high-altitude radiation from the slot gap, with application to the Crab Pulsar. In this model, emission in the optical to soft gamma-ray band is synchrotron radiation from pairs outside the slot gap undergoing cyclotron resonant absorption of radio photons. Hard gamma-rays come from primary electrons accelerating in the slot gap and radiating curvature and synchrotron emission. The common element for the X-ray and gamma-ray emission would then be the angles to the radio photons. Since Harding et al. (2008) assumed that the pair spectrum was constant throughout the open field volume, there was no spectral index variation with phase. However, polar cap pair cascade simulations (producing the X-ray emitting pairs in this model) show that there are large variations in the pair spectrum across the polar cap (Arendt & Eilek 2002). Thus, the detailed measurements of X-ray spectral index variation presented in this paper are mapping (and constraining) the variation in the pair spectrum across the open field lines.

In recent studies of phase-resolved spectra of the Crab Pulsar in the outer gap model (Tang et al. 2008; Hirotani 2008), the optical to hard X-ray spectrum comes from synchrotron radiation of secondary pairs produced in situ in outer gap cascades while the gamma-rays come from inverse Compton radiation of pairs and curvature radiation of primary particles. This model does not match the observed X-ray spectral variations, although it does produce the observed gamma-ray spectral variations. Thus, this model lacks the essential physics that accounts for the X-ray spectral index variation and its similarity to that in gamma-rays.

3.3. Temperature of the Neutron Star and Superfluidity

Here, we investigate the hypothesis that there is a detectable underlying thermal component in addition to the non-thermal flux that we see from the pulsar. We fit the data as a function of pulse phase to spectral models that allow both components.
to establish the values of the phase independent parameters. In the latter case we use the data from the other 17 phase bins and in the second, we use the data from all 21 pulse phase bins; in the second, we are the hydrogen neutron star atmosphere. In XSPEC these models are the phase-independent thermal blackbody and then a phase-dependent power law together with a phase-independent blackbody component within statistics. We first consider a phase-dependent power law together with a phase-independent blackbody and then a phase-dependent power law together with a phase-independent thermal blackbody. The reader will note that the sensitivity to a thermal component was virtually the same in both approaches. However, the establishment of upper limits was computationally much faster using the data at pulse minimum. The latter is allowed to vary as a function of phase. Adding a phase-independent blackbody model to the spectral model. Right: using a powerlaw+NSA model. Multiple values of $kT_{\infty}$ arise as different combinations of $M$ and $R$ lead to the same (or similar) values of $\theta_{\infty}$. (See the text for details.)

**Table 4**

| Parameter               | Value       |
|-------------------------|-------------|
| $\chi^2/\nu$            | 3510/3544   |
| $N_{\text{H}}(10^{21} \text{ cm}^{-2})$ | 3.22 ± 0.13 |
| $[\text{O}/\text{H}]$  | (4.28 ± 0.30) × 10^{-4} |
| $\tau_{\text{scat}} \text{ at 1 keV}$ | 0.147 ± 0.045 |
| $kT_{\infty}(\text{keV})$ | 0.1 ± 7.2 |
| $\theta_{\infty}$      | 44 ± 31000 |

**Notes.** The latter is allowed to vary as a function of phase. Uncertainties are XSPEC estimates for the 1σ statistical errors based on one interesting parameter ($\chi^2$ at minimum +1.0).

We first consider a phase-dependent power law together with a phase-independent thermal blackbody and then a phase-independent model with a spectrum of radiation emergent from the hydrogen neutron star atmosphere. In XSPEC these models are the powerlaw, bbodyrad, and NSA (Pavlov et al. 1995).

We examined two approaches for the analysis. In the first, we use the data from all 21 pulse phase bins; in the second, we use the data from the 4 phase bins that are at pulse minimum. In the latter case we use the data from the other 17 phase bins to establish the values of the phase independent parameters $N_{\text{H}}$, $[\text{O}/\text{H}]$, and $\tau_{\text{scat}}$ for fitting the data at pulse minimum. We found that the sensitivity to a thermal component was virtually the same in both approaches. However, the establishment of upper limits was computationally much faster using the data at pulse minimum.

### 3.3.1. Blackbody Model

Adding a phase-independent blackbody model to the spectral fitting yields the results listed in Table 4. The large uncertainties in both the normalization, $\theta_{\infty}^2$, and the redshifted effective surface temperature, $kT_{\infty}$, clearly point to the absence of a blackbody component within statistics. $\theta_{\infty}$ is the angular size that would be determined by a distant observer, in XSPEC units—$\theta_{\infty} = (R_{\infty}/D_{10})$, with $R_{\infty}$ the apparent stellar radius in km units and $D_{10}$ the source distance in 10 kpc units. The left panel of Figure 5 shows the 2σ and 3σ upper limits to $kT_{\infty}$ for a range of values for $\theta_{\infty}^2$ that are relevant to neutron star models with realistic equations of state (EOSs). For example, the normalization of 6100 in the middle of the range corresponds to that of a neutron star of mass $1.4 M_\odot$ and a radius of 15.6 km, typical for neutron stars with moderately stiff EOSs (Haensel et al. 2007).

In Paper I, we found somewhat lower 3σ upper limits and higher 2σ upper limits to $kT_{\infty}$ than those shown in Figure 5. However, those previous results were erroneous and should have been higher. Unfortunately the error was only discovered while completing this paper. As an example of the changes due to the error, the 2σ and 3σ upper limits at $\theta_{\infty}^2 = 6100$ should have been 0.195 and 0.209 keV, respectively. Using the newer response function and signal and background extraction regions would have lowered these to 0.184 and 0.202 keV, respectively. Thus, the upper limits reported here, as expected, do represent a significant improvement over Paper I. (We note again that the difference in upper limits using the old and new response functions ignores the possibility that there are uncertainties associated with these responses.) Of course here we analyze the data as a function of pulse phase. As discussed above, this is a better approach as it reduces the possible bias produced by averaging a number of power laws, which, in turn will not be a power law and thus inadvertently create a spurious thermal component.

#### 3.3.2. Neutron Star Atmosphere (NSA) Model

The NSA model requires a number of inputs. The normalization was set assuming a distance to the Crab of 2 kpc. The surface magnetic field parameter was set at $1.0 \times 10^{13}$ G, although results are not terribly sensitive to this choice. The model also requires the gravitational mass $M$ and the circumferential radius $R$ of the neutron star. We examined a wide range of $M$ from 1.0 to 2.5 $M_\odot$ and $R$ from 8 to 15 km in creating the right panel of Figure 5 which shows the 2σ and 3σ upper limits to $kT_{\infty}$; hence the reason for the multiple values for a given $\theta_{\infty}$.

The reader will note that powerlaw+NSA fits yield a higher upper limit for given values of $\theta_{\infty}$ than powerlaw+bbodyrad.
Simulations with fake data have shown this to be correct. We believe that this happens because the NSA spectrum has a hard tail which makes it difficult to distinguish from the power law.

3.3.3. Implications

Current cooling theories (e.g., Page et al. 2006, 2009; Yakovlev & Pethick 2004; Yakovlev et al. 2008) state that any isolated neutron star of the Crab Pulsar age should be at the neutrino cooling stage with an isothermal interior. The preceding cooling stage of internal thermal relaxation lasts no longer than \( \sim 200 \) yr. Now the interior of the pulsar should be isothermal having the same internal temperature \( T_i(t) \) redshifted for a distant observer (Thorne 1977). The local (actual) temperature \( T_i \) in the isothermal interior is \( \sim 10\% \sim 30\% \) higher than \( T_i \). A noticeable temperature gradient survives only near the surface, in the outer heat-blanketing envelope (Gudmundsson et al. 1983) with thickness not higher than a few tens of meters. The temperature drop in this envelope depends on the matter composition and on the magnetic field (both factors affect heat conduction there; see Potekhin et al. 1997, 2003).

The cooling of the Crab Pulsar (as of all isolated neutron stars of ages \( t \lesssim 10^7 \) yr) is driven by neutrino emission from its interior, mainly from the superdense core. The internal cooling of a star with a given internal structure is the same for any heat-blanketing envelope (looks the same from inside) but the surface temperature is affected by thermal conduction in the envelope (looks different from outside).

Numerous cooling theories (cited above) are still poorly constrained by observations. The theories comprise different EOSs of neutron star cores, different neutrino emission properties, and models for superfluidity of baryons (which affect heat capacity and neutrino emission). The cooling of isolated thermally relaxed neutron stars is mostly regulated by the two factors,\(^{10}\) the neutrino cooling rate and the properties of the heat-blanketing envelope. Cooling of very compact stars is also affected by the stellar compactness \( x = r_s/R \) \( (r_s = 2GM/c^2 \) being the Schwarzschild radius) but we do not consider this effect which is weak for the majority of realistic models (Yakovlev et al. 2011).

The neutrino cooling rate \( \ell \) [K s\(^{-1}\)] is defined as

\[
\ell = \frac{L_\nu(\tilde{T})}{C(\tilde{T})}. \tag{1}
\]

Here, \( L_\nu(\tilde{T}) \) is the neutrino luminosity of the stellar core (redshifted for a distant observer), and \( C(\tilde{T}) \) is its heat capacity. It is convenient to introduce the normalized cooling rate (Yakovlev et al. 2011)

\[
f_\ell = \frac{\ell(\tilde{T})}{\ell_{SC}(\tilde{T})}. \tag{2}
\]

where \( \ell_{SC}(\tilde{T}) \) is the rate of the standard neutrino candle, a neutron star with the same \( M, R, \tilde{T} \), but with a non-superfluid nucleon core that is cooling via the ordinary modified Urca process of neutrino emission. For isolated neutron stars without extra internal heat sources, physically allowable values of \( f_\ell \) vary from \( \sim 10^{-2} \) to \( \sim 10^0 \). If \( f_\ell \approx 1 \) this implies standard neutrino cooling, \( f_\ell \approx 10^{-2} \) very slow cooling (e.g., when the modified Urca process is suppressed by superfluidity), and \( f_\ell \approx 10^2 - 10^6 \) fast cooling (accelerated by direct Urca processes, pion or kaon condensates, or by neutrino emission due to Cooper pairing of neutrons). The rate \( f_\ell \) is an important parameter of the neutron star core that can be inferred from observations. An inferred value of \( f_\ell \) can be realized by different physical models of neutron star cores (Yakovlev et al. 2011).

We employ the models of heat-blanketing envelopes of Potekhin et al. (1997) and Potekhin et al. (2003). The envelopes may contain some mass \( \Delta M \) of (light-element) accreted matter and have a dipole magnetic field \( B \) \((B = 3.8 \times 10^{15} \) G at the magnetic equator for the Crab Pulsar\). The effective temperature of a magnetized star varies over the surface with the magnetic poles being hotter than the equator. Cooling theory suggests using the effective temperature averaged over the surface (it defines the thermal luminosity of the star). The effect of the given magnetic field on the cooling is weak although included in our calculations. The effect of the envelope is regulated then by \( \Delta M \) which varies from \( \Delta M = 0 \) for a standard iron envelope to \( \Delta M_{\text{max}} \approx 10^{-7} M_\odot \) for a fully accreted envelope. For larger \( \Delta M \) light elements transform into heavier ones at the bottom of the envelope through electron capture and nuclear reactions.

Our new observational upper limits on \( T \) are high, comparable to the highest surface temperatures of a cooling neutron star. These limits allow the Crab Pulsar to have almost any neutrino cooling rate, from very fast to very slow. Figure 6 shows some observational cooling curves \( T_\infty(t) \) \((\infty \text{ refers to the redshifted temperature, with the data on the Crab and other cooling neutron stars. The data for other stars are taken from the same references as in Shtremin et al. (2011). The cooling curves are calculated for a neutron star model with nucleon core having the EOS of Akmal et al. (1998, hereafter APR). Specifically, we use a version of the APR EOS denoted as APR I in Gusakov et al. (2005). The maximum mass of a stable neutron star with this EOS is } M_{\text{max}} = 1.929 M_\odot \text{; the powerful direct Urca process}} \)

\( ^{10} \) This is true for all observed isolated neutron stars except for the star in the Cas A supernova remnant. For the Cas A star, one observes dynamical decrease of \( T(t) \) (Heinke & Ho 2010) which can give more information on neutron star structure (Page et al. 2011; Shtremin et al. 2011).
of neutrino emission is allowed in stars with $M > 1.829 M_{\odot}$. In Figure 6, we take a star with $M = 1.4 M_{\odot}$ ($R = 12.14$ km). The upper limit of $T_\infty$ for the Crab Pulsar ($\log T_{BB}^{\infty}(3\sigma)$[K] = 6.30; $T^{\infty}_{BB} \approx 2$ MK) is from the blackbody fits at the $3\sigma$ level for this choice of $M$ and $R$ (Section 3.3.1).

The solid line in Figure 6 is the basic cooling curve for a non-superfluid neutron star with the iron heat blanket ($\Delta M = 0$). This star cools via the modified Urca process, its $f_\ell = 1$; it is the standard neutrino candle. Its internal temperature at the Crab age would be $\tilde{T}_{SC} \approx 2.23 \times 10^8$ K, with the cooling rate $\epsilon_{SC} \approx 3.7 \times 10^4$ K yr$^{-1}$. The basic curve goes below the $T_\infty$ upper limit for the Crab Pulsar, i.e., the Crab can well be the standard candle.

Next consider the effects of the neutrino emission rate $f_\ell$ in the stellar core (to change the internal temperature $T_i$) and the amount $\Delta M$ of light elements in the envelope (to change $T_\infty$ for a given $T_i$). The long-dashed line in Figure 6 shows cooling of the same standard candle but with a fully accreted envelope. Light elements increase the thermal conductivity, make the envelope more heat transparent, and increase $T_\infty$ (for a given $T_i$). The increase is substantial but cannot raise the cooling curve above the $T_\infty$ upper limit for the Crab Pulsar. Thus, the Crab can be the standard candle inside and have a fully accreted envelope outside.

The pulsar can also be warmed by reducing its neutrino emission below the standard level. The dot-dashed line in Figure 6 shows the cooling of the same star with the iron heat-blanketing envelope, but with strong proton superfluidity in the core. This superfluidity greatly suppresses the modified Urca process and the bremsstrahlung emission of neutrino pairs in proton–proton and proton–neutron collisions (and also suppresses proton heat capacity in the core). Then the star cools via neutrino bremsstrahlung emission due to neutron–neutron collisions. In this scenario, the cooling rate is small, $f_\ell \approx 0.01$, and the core warmer. The critical temperature in the core for onset of proton superfluidity should be higher than a few times $10^8$ K to establish this very slow cooling regime. It is one of the slowest cooling regimes in a star (without extra heat sources). The dot-dashed line shows that this hottest star has about the same surface temperature as the standard neutrino candle with a fully accreted envelope; it is not forbidden by our observations.

The short-dashed curve in Figure 6 displays cooling of the neutron star with the same very slow cooling rate as previously but now with a fully accreted envelope. With respect to the basic solid cooling curve, its surface temperature is simultaneously increased by proton superfluidity in the core ($f_\ell \approx 0.01$) and accreted matter in the envelope ($\Delta M = \Delta M_{\text{max}}$). The large increase makes the surface exceptionally warm, with $T_\infty$ higher than the upper limit for the Crab Pulsar, refuting this particular model. Thus, if the Crab Pulsar does have strong proton superfluidity ($f_\ell \approx 0.01$) it can only have a partially accreted envelope, with $\Delta M \lesssim 10^{-9} M_{\odot}$. This is demonstrated by the dotted cooling curve calculated for $\Delta M = 10^{-9} M_{\odot}$; it hits the observational $T_\infty$ limit.

To visualize the effect of accretion envelopes, Figure 7 shows the dependence of $f_\ell$ on $\Delta M$ for the Crab Pulsar assuming several $T_\infty$, from 1.0 to 2.0 MK. We took the same APR 1.4 $M_{\odot}$ neutron star model as in Figure 6. At low mass of light elements, $\Delta M \lesssim 10^{-14} M_{\odot}$, cooling results are the same as for the iron heat blanket. At high mass $\Delta M \gtrsim 10^{-7} M_{\odot}$ we have a fully accreted envelope. Labels in the figure mark the neutrino cooling rate for the standard candle, for very slow neutrino cooling, and for the lowest enhanced cooling.

The lowest (thick) curve in Figure 7 corresponds to our $3\sigma$ upper limit on $T_{BB}^{\infty}$. All $f_\ell$ values above this curve are formally allowed for the Crab Pulsar but the values $f_\ell \lesssim 0.01$ are theoretically less realistic. With this in mind, the upper $T^{\infty}_{BB}(3\sigma)$ limit implies that the pulsar can have any $f_\ell \gtrsim 0.2$ for any $\Delta M$, and $f_\ell$ from $\sim 0.01$ to $\sim 0.2$ for $\Delta M \gtrsim 10^{-9} M_{\odot}$.

If $T_{\infty}$ were measured (below the current upper limit), Figure 7 would give a well-defined curve $f_\ell(\Delta M)$. Then we would be able to find $f_\ell$ within a factor of $\sim 50$ (depending on $\Delta M$). A more accurate determination of $f_\ell$ would require knowledge of $\Delta M$. Every decrease of $T_{\infty}$ by 0.2 MK would considerably increase $f_\ell$. A decrease of $T_{\infty}$ from 2 MK to 1 MK would drastically change the status of the Crab as a cooling neutron star.

Our conclusions are fairly independent of neutron star mass. We have considered a wide range of masses from 1 to 1.8 $M_{\odot}$ (for stars with the APR EOS). The cooling curves stay almost the same and the observational upper limits do not change much (Figure 5). For instance, $\log T_{BB}^{\infty}(3\sigma) = 6.31$ from the blackbody fits for an $M = 1.8 M_{\odot}$ ($R = 11.38$ km) APR star. Moreover, the cooling curves are nearly the same for a large variety of EOSs of dense nucleon matter. These include the nine original versions of the phenomenological PAL EOS (Prakash et al. 1988), as well as three other versions of this EOS with the symmetry energy of nuclear matter proposed by Page & Applegate (1992), and the SLy EOS (Douchin & Haensel 2001). The very weak dependence of the cooling curves on $M$, $R$, and the EOS for standard candles and for very slowly cooling neutron stars has been discussed in the literature starting from the paper by Page & Applegate (1992) (see, e.g., Yakovlev & Pethick 2004, for references).

Instead of the $3\sigma$ upper limits of $T_{\infty}$ we could also use less conservative $2\sigma$ limits. For instance, we have $\log T_{BB}^{\infty}(2\sigma) = 6.26$ ($T_{BB}^{\infty}(2\sigma) \approx 1.8$ MK) for the 1.4 $M_{\odot}$ neutron star with the APR EOS (the second thin curve above the thick one in Figure 7). Also, the upper limits of $T_{\infty}$, inferred from

![Figure 7](https://example.com/figure7.png)
the neutron star atmosphere fits (Section 3.3.2), are higher than for the blackbody fits (e.g., \( \log T^{\text{NSA}}(2\sigma) = 6.44 \) and \( \log T^{\text{NSA}}(3\sigma) = 6.45 \) for the 1.4 \( M_\odot \) APR star); they would be even less restrictive than for \( T_\infty = 2 \) MK.

According to current theories, massive neutron stars cool faster than the standard neutrino candles due to enhanced neutrino emission in their cores. The mass range of stars which demonstrate faster cooling is very uncertain. Our \( T_\infty \) limits do not constrain the parameters of the Crab Pulsar if it is a rapidly cooling star.

Some conclusions above could be made from cooling calculations already available in the literature (particularly, Kaminker et al. 2006). We have repeated these calculations drawing special attention to the Crab Pulsar; our Figure 6 is similar to the right panel of Figure 2 of Kaminker et al. (2006). However, our short-dashed curve goes higher than the analogous curve in Kaminker et al. (2006). This is both because we assume a fully accreted heat-blanketing envelope \( \Delta M \sim 10^{-7} M_\odot \), while Kaminker et al. (2006) took \( \Delta M \sim 10^{-8} M_\odot \), and because we employ stronger proton superfluidity in the core. This difference of cooling curves does not affect our principal conclusions.

4. SUMMARY

We have obtained new Chandra data of the Crab Nebula and its pulsar. The new data were collected in such a manner to prevent telemetry saturation of the LETGS and thus enable efficient collection of high-time resolution data from the pulsar. We have analyzed these data and re-analyzed our previous observation of the phase-averaged spectrum to update spectral parameters. The updated phase-averaged spectral parameters no longer indicate that the Crab line of sight is underabundant in oxygen given the abundances and cross-sections employed in the spectral fitting. In all our analyses, we have accounted for the contribution of scattering by interstellar dust to the extinction of X rays in an aperture-limited measurement—a consideration (often ignored) in spectral analysis of point sources observed with Chandra’s exceptional angular resolution and, depending on the purpose and precision of the experiment, that may be important as we have discussed. Here we have measured, for the first time, the magnitude of that extinction in the direction of the Crab Pulsar and allowed for its impact on derived spectral parameters.

In addition, we have measured with a high precision the spectrum of the pulsar as a function of pulse phase and at all pulse phases. We find highly significant variation of the power-law spectral index as a function of phase and have discovered an unusual behavior of the spectral index as the pulse rises out of the pulse minimum on its approach toward the peak of the primary pulse. Interestingly, this behavior appears to be connected to a similar feature in the variation of the optical polarization as a function of phase as well as variations of the gamma-ray spectral index. In both slot- and outer-gap models for phase-resolved radiation from the Crab, the X-ray emission comes from synchrotron radiation of secondary pairs. The variations in X-ray spectral index are thus mapping the variations in pair spectrum with phase, although neither of these models currently includes the physical elements that produce the observed spectral variations. Therefore, the more accurate measurements presented in this paper will be a challenge to future modeling. It is hoped they will help us understand the pair cascade processes in pulsar magnetospheres.

We also use the spectral data to obtain new and more precise upper limits to the surface temperature of the neutron star for blackbody and neutron star atmosphere models of thermal surface radiation.

We have commented on the differences in measured parameters subsequent to analyzing the same data with different releases of the response functions. Our experience emphasizes the importance of accounting for the uncertainties in the response functions when analyzing data. One might estimate the level of those uncertainties by noting differences in spectral parameters using the old (Paper I) and new (this paper) response functions. However, this might be too extreme as the newer response functions are a product of several proven refinements. Perhaps these differences can serve as estimators of upper limits to the variations. We urge the various observatories to provide users with response functions with errors and the tools to use them.

Finally, we clarify the means by which the observational data as to the thermal emission may be connected to theories of neutron star cooling and neutron star structure. Our principal conclusions, only slightly dependent on the EOS of the pulsar core, pulsar mass, and radius, are the following.

1. Our upper limits to the surface temperature \( T_\infty \) of the Crab Pulsar weakly restrict the normalized neutrino emission rate \( f_\ell \) (in units of standard candles) in the pulsar core and the amount of light elements \( \Delta M \) in the heat-blanketing envelope.

2. Our observations allow the pulsar to have a neutrino emission rate \( f_\ell \gtrsim 0.2 \) (1/5 of the standard rate or higher) for any accreted mass \( \Delta M \). For lower rates from \( \sim 0.2 \) to 0.01 (the lowest rate on physical grounds, e.g., due to strong proton superfluidity in the core), our observations constrain the pulsar to have only a massive accreted envelope (with \( \Delta M \lesssim 10^{-9} M_\odot \) at \( f_\ell \sim 0.01 \)).

The absence of strong restrictions on the properties of the Crab pulsar follows from the current 3\( \sigma \) upper limit on \( T_\infty \). Nevertheless, this state of affairs has its own advantage. There is still a chance that the Crab Pulsar is warm, with the surface temperature \( T_\infty \) only slightly below the present upper limit. While our upper limit is not very restrictive, a real measurement of the surface temperature just below the present upper limit would be more restrictive, indicating that the Crab Pulsar is one of the warmest neutron stars. For instance, if the temperature \( T_\infty = 1.6 \times 10^8 \text{ K} ( \log T_\infty = 6.20 ) \) were measured, we would have (Figure 7) \( 0.06 \lesssim f_\ell \lesssim 10 \), depending on the mass of the accreted layer.

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REFERENCES

Abdo, A. A., Ackermann, M., Atwood, W. B., et al. 2009, ApJ, 696, 1084
Abdo, A. A., Ackermann, M., Ajello, M., et al. 2010a, ApJ, 708, 1254
Abdo, A. A., Ackermann, M., Ajello, M., et al. 2010b, ApJ, 720, 272
Abdo, A. A., Ackermann, M., Ajello, M., et al. 2010c, ApJ, 713, 154
Akmal, A., Pandharipande, V. R., & Ravenhall, D. G. 1998, Phys. Rev. C, 58, 1804
Arendt, P. N., & Eilek, J. A. 2002, ApJ, 581, 451
