Configuration mixing in the quark model

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Abstract. We discuss the flavor asymmetry and the strangeness content of the proton in an unquenched quark model. It is shown that the inclusion of hadron loops leads automatically to an excess of $\bar{d}$ over $\bar{u}$, but gives very small contributions to the strange magnetic moment and strangeness radius of the proton, both of which are in agreement with experimental data.

1. Introduction

There is strong evidence for the existence of exotic degrees of freedom (other than valence quarks) in hadrons, in particular for the need to include the effects of quark-antiquark pairs (hadron loops). The importance of these quark-antiquark configurations (or higher Fock components in baryon wave functions) is evident from measurements of the $\bar{d}/\bar{u}$ asymmetry in the nucleon sea [1], parity-violating electron scattering experiments [2, 3], the proton spin crisis [4], as well as from analysis of helicity amplitudes [5] and strong couplings of baryon resonances [6, 7].

The aim of this contribution is to review a recently proposed extension of the constituent quark model in which the quark-antiquark pairs are created in the $3P_0$ state with the quantum numbers of the vacuum [14, 15]. The present approach is based on a three-quark CQM to which the higher Fock components are added by configuration mixing, thereby employing a $3P_0$ model for the coupling between the $qqq$ and $qqq-\bar{q}\bar{q}$ configurations. The pair-creation mechanism is inserted at the quark level and the one-loop diagrams are calculated by summing over a complete set of intermediate baryon-meson states. Under these assumptions, the baryon wave function
consists of a zeroth order three-quark configuration $|A\rangle$ plus a sum over all possible higher Fock components due to the creation of $^3P_0$ quark-antiquark pairs

$$|\psi_A\rangle = \mathcal{N} \left[ |A\rangle + \sum_{BCl} \int d\vec{k} \ |BC\vec{k}lJ\rangle \frac{\langle BC\vec{k}lJ | T^\dagger | A\rangle}{M_A - E_B - E_C} \right].$$

Here $A$ denotes the initial baryon, $B$ and $C$ represent the intermediate baryon and meson, and $M_A$, $E_B$ and $E_C$ are their respective energies, $k$ and $l$ the relative radial momentum and orbital angular momentum of $B$ and $C$, and $J$ is the total angular momentum $J = J_B + J_C + \vec{l}$. The operator $T^\dagger$ creates a quark-antiquark pair in the $^3P_0$ state with the quantum numbers of the vacuum: $L = S = 1$ and $J = 0$ [8, 9, 16]. The coupling vertex $\langle BC\vec{k}lJ | T^\dagger | A\rangle$ can be derived in explicit form in the harmonic oscillator basis [16, 17].

In order to calculate the effects of quark-antiquark pairs on an observable, one has to evaluate the contribution of all possible intermediate states. By using a combination of group theoretical and computational techniques, the sum over intermediate states is carried out up to saturation and not only for the first few shells as in previous studies [14, 15]. Not only does this have a significant impact on the numerical results, but it is necessary for consistency with the OZI-rule and the success of CQMs in hadron spectroscopy [8, 18, 19].

The unquenching of the quark model is useful only if it preserves the phenomenological successes of the CQM. In applications to mesons it was shown that the inclusion of $q\bar{q}$ pairs does not destroy the good CQM results [18] and preserves the OZI hierarchy [19]. For baryons we studied the effect of the unquenching on the magnetic moments - historically one of the successes of the CQM - and found that, after renormalization, the good results for the magnetic moments are preserved [8].

The results presented in this contribution were obtained using harmonic oscillator wave functions up to four oscillator shells for the intermediate baryons and mesons with a single oscillator parameter for the baryons and another one for the mesons. All parameters were taken from the literature, without attempting to optimize their values in order to improve the agreement with experimental data [8, 9, 10].

3. Flavor asymmetry

The flavor asymmetry of the proton provides an important test for models of nucleon structure since it measures the difference between $\bar{d}$ and $\bar{u}$ quarks, and hence probes directly the existence of higher Fock components $qqq - \bar{q}\bar{q}$ in the proton wave function.

The flavor asymmetry of the proton $A(p)$ is related to the Gottfried integral $S_G$ for the difference of the proton and neutron electromagnetic structure functions as

$$S_G = \int_0^1 \frac{F_2^p(x) - F_2^n(x)}{x} \ dx = \frac{1}{3} - 2 \int_0^1 [\bar{d}_p(x) - \bar{u}_p(x)] \ dx = \frac{1}{3}[1 - 2A(p)].$$

Under the assumption of a flavor symmetric (or rather flavor independent) sea one obtains the Gottfried sum rule $S_G = 1/3$, whereas any deviation from this value is an indication of the $\bar{d}/\bar{u}$ asymmetry of the nucleon sea. All experiments show evidence that there are more $\bar{d}$ quarks in the proton than there are $\bar{u}$ quarks [1].

In the unquenched quark model, the flavor asymmetry of the proton can be calculated directly from the difference of the number of $\bar{d}$ and $\bar{u}$ sea quarks in the proton, even in the absence of explicit information on the (anti)quark distribution functions. The inclusion of the $qq\bar{q}$ pairs leads automatically to an excess of $\bar{d}$ over $\bar{u}$ quarks [9]. In the UCQM, the flavor asymmetry for the proton $A(p)$ is calculated to be 0.151, which corresponds to a value of the Gottfried integral of 0.232, in remarkable agreement with the experimental data (see Fig. 1).
The main contribution to the flavor asymmetry of the proton is due to pion loops, especially the $n\pi^+$ intermediate state, thus confirming in an explicit calculation the explanation given in Refs. [24] in the context of the meson-cloud model. In addition, we find that there are important contributions from the $\Delta\pi$ channel and, especially, from the off-diagonal terms $p\pi^0-\eta_8$ and $p\pi^0-\eta_1$ which together are of the order of 15-20% of that of the $N\pi$ channel. The contribution of the intermediate vector mesons is very small due to a cancellation between the $n\rho^+$ and the $\Delta\rho$ channels and the cross terms $p\rho^0-\omega_8$ and $p\rho^0-\omega_1$ [25].

Since the unquenched quark model is valid not only for the proton, but for all baryons (ground state or resonance), it is straightforward to calculate the flavor asymmetries of the other octet baryons. In Table 1, we show a comparison of some predictions for the flavor asymmetry of the $\Sigma^+$ and $\Xi^0$ hyperons relative to that of the proton. In the unquenched quark model, the flavor asymmetry of the proton is predicted to be of the same order as that of the $\Sigma^+$ hyperon and much larger than that of the cascade particle

$$A(p) \sim A(\Sigma^+) \gg |A(\Xi^0)|.$$  \hspace{1cm} (3)

This behavior is very different from that obtained in the chiral quark model $A(\Sigma^+) = 2A(p) = 2A(\Xi^0)$ [26], the balance model $A(\Sigma^+) > A(\Xi^0) > A(p)$ [27], and the octet model.
Table 2. Strange magnetic moment of the proton

| Reference            | \( \mu_s [\mu_N] \) |
|----------------------|-----------------------|
| SAMPLE [43]          | 0.01 ± 0.29 ± 0.31 ± 0.07 |
| Global Analysis [38] | 0.12 ± 0.55 ± 0.07    |
| Global Analysis [39] | 0.29 ± 0.21           |
| LQCD [40]            | −0.046 ± 0.019        |
| LQCD [41]            | −0.017 ± 0.025 ± 0.007 |
| Two-component [37]   | 0.315                 |
| Unquenched QM [10]   | 0.0018                |

\( A(p) > |A(\Xi^0)| > A(\Sigma^+) \) [28]. The values for the chiral quark model and the balance model were taken from [29].

In order to distinguish between the predictions of the different models and to obtain a better understanding of the non-perturbative structure of QCD, new experiments are needed to measure the flavor asymmetry of hyperons. In particular, the flavor asymmetry of charged \( \Sigma \) hyperons can obtained from Drell-Yan experiments using charged hyperon beams on the proton [28] or by means of backward \( K^\pm \) electroproduction [30].

4. Strangeness of the proton

Other interesting observables that directly probe the sea quarks of the nucleon are provided by the strange form factors of the proton. Even though the net strangeness of the nucleon is zero, the nucleon may have a nonzero distribution of strangeness, in the same way as the neutron has a nonvanishing charge distribution while its electric charge is zero.

Experimentally, the strange form factors can be determined by combining the measurement of parity-violating asymmetries of the proton with information on the electromagnetic form factors of the proton and neutron [31, 32].

\[
G_{E,M}^s(Q^2) = \left( 1 - 4 \sin^2 \theta_W \right) G_{E,M}^p(Q^2) - G_{E,M}^n(Q^2) - G_{E,M}^Z(Q^2),
\]

(4)

where \( G_E^s \) and \( G_M^s \) denote the strange electric and magnetic form factors, respectively. The first measurements of the parity-violating asymmetries showed surprisingly large and positive values of \( G_M^s \), e.g. the SAMPLE collaboration found \( G_M^s = 0.61 \pm 0.17 \pm 0.21 \pm 0.19 \) at \( Q^2 = 0.1 \) (GeV/c)^2 [33]. However, more recent experiments by the HAPPEX [2, 34, 35], PVA4 [3] and G0 [36] collaborations showed that the contribution of strange quarks to the electric and magnetic form factors, \( G_E^s \) and \( G_M^s \), is much smaller and compatible with zero within the experimental errors.

In the majority of theoretical analyses, the strangeness content to the nucleon is discussed in terms of its static properties, the strange magnetic moment and the strangeness charge radius

\[
\mu_s = \left. G_M^s(Q^2) \right|_{Q^2=0},
\]

\[
r_s^2 = -6 \left. \frac{dG_E^s(Q^2)}{dQ^2} \right|_{Q^2=0}.
\]

(5)

A compilation of theoretical values of \( \mu_s \) and \( r_s^2 \) shows that, even though calculations in a variety of hadronic models and effective hadronic theories vary widely both in absolute value and sign, most studies agree on a small negative strangeness radius and a moderate negative strange magnetic moment [32, 37]. On the contrary, the results of a combined fit of the strange electric
Table 3. Strangeness radius of the proton

| Reference       | $r_s^2$ [fm$^2$]           |
|-----------------|---------------------------|
| HAPPEX [34]     | 0.098 ± 0.108 ± 0.026     |
| HAPPEX [2]      | -0.006 ± 0.042 ± 0.021    |
| HAPPEX [2]      | 0.012 ± 0.044             |
| PVA4 [3]        | -0.053 ± 0.040 ± 0.020    |
| G0 [36]         | 0.015 ± 0.038 ± 0.019 ± 0.018 |
| Global Analysis [38] | 0.014 ± 0.096         |
| Global Analysis [39] | 0.019 ± 0.038         |
| LQCD [42]       | -0.007 ± 0.004 ± 0.002 ± 0.021 |
| LQCD [41]       | -0.0024 ± 0.0015 ± 0.0007 |
| Two-component [37] | 0.005                 |
| Unquenched QM [10] | 0.012                |

and magnetic form factors measured by SAMPLE, PVA4 and HAPPEX at $Q^2 \sim 0.1$ (GeV/c)$^2$ yielded the opposite sign for $\mu_s$ [38, 39]. Recent lattice calculations give small negative values for both $\mu_s$ and $r_s^2$ [40, 41, 42] (see Tables 2 and 3).

In the unquenched quark model, the strange magnetic moment of the proton is given by the expectation value of the operator

$$\bar{\mu}_s = \frac{1}{\epsilon_s} \sum_i \mu_{i,s} \left[ 2\bar{s}(q_{i,s}) + \bar{\ell}(q_{i,s}) - 2\bar{s}(\bar{q}_{i,s}) - \bar{\ell}(\bar{q}_{i,s}) \right].$$

(6)

Here, $\mu_{i,s}$ is the magnetic moment of the strange quark $i$ (not be confused with the strange magnetic moment of the proton $\mu_s$). The strange magnetic moment of the proton is entirely due to the sea quarks. There is a contribution from the quark spins of the $s\bar{s}$ pair 0.0012 $\mu_N$, as well as from its orbital motion 0.0006 $\mu_N$. Both contributions are small and give a total strange magnetic moment $\mu_s = 0.0018 \mu_N$ [10].

Similarly, the strange radius of the proton is calculated as the expectation value of the operator

$$r_s^2 = \sum_{i=1}^{5} \left( \bar{r}_{i,s} - \bar{R}_{CM} \right)^2,$$

(7)

where $\bar{r}_{i,s}$ and $\bar{R}_{CM}$ are the coordinates of the strange quark $i$ and the center of mass, respectively. The strangeness radius of the proton is calculated to be $r_s^2 = 0.012$ fm$^2$ [10].

In conclusion, the effects of the higher Fock components on the strange magnetic moment and the strange radius of the proton are found to be very small. Tables 2 and 3 show that the results for the unquenched quark model are compatible with the latest experimental data and recent lattice calculations.

5. Summary and conclusions

In this contribution, we presented some recent work on unquenching the quark model by configuration mixing between the valence quark and $qqq$ – $q\bar{q}$ higher Fock configurations. As application, we studied the flavor asymmetry and the strangeness of the proton.

The inclusion of the $q\bar{q}$ pairs leads automatically to an excess of $d$ over $\bar{u}$ quarks, in agreement with the observed flavor asymmetry of the proton. The results for the flavor asymmetry of the proton are dominated by the $N\pi$ channel, but with important contributions from the $\Delta\pi$ channel and the off-diagonal $N\pi-N\eta$ terms.
On the other hand, the effects of the higher Fock components on the strange magnetic moment and the strangeness radius of the proton are found to be very small, in agreement with the latest experimental data and recent lattice calculations.

Even though different models of hadron structure may show similar results for the properties of the proton, often their predictions for the other octet baryons exhibit large variations. Therefore, in order to be able to distinguish between the predictions of different models of hadron structure and to obtain a better understanding of the non-perturbative structure of QCD new experiments are needed to measure the flavor asymmetry and spin content of other octet baryons.

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