Spin Two-Body Problem of Classical Electrodynamics with Radiation Terms (I) – Derivation of Spin Equations

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Abstract The paper is devoted to extension of Synge equations describing motion of two mass charged particles in the frame of classical relativistic electrodynamics including radiation terms and spin equations. In previous papers we have derived equations of motion just with radiation terms. Our radiation terms correspond to the Dirac physical assumptions but they differ from Dirac’s terms on the mathematical derivation. They generate neutral type equations of motion with both retarded and advanced arguments. Here we consider two-body problem with spin following the Corben’s technique. As in the original Synge model the number of the spin equations is more than the number of the unknown functions. We prove that six spin equations are a consequence of the rest ones and in this manner we obtain six equations for the six unknown spin functions. This consequence is in weak sense obtained by scalar multiplication.

Keywords Two Body Problem of Classical Electrodynamics with Radiation Terms and Spin, Derivation of Equations of Motion

1. Introduction

The present paper is one of series devoted to study of the two-body problem of classical electrodynamics. In [1, 2] we have formulated 3-dimensional two-body problem based on the Synge relativistic model [3]. The system of equations of motion is of a neutral type with delays depending on unknown trajectories [4]. Later we have found a new form of the Dirac radiation term and formulated a two-body problem with radiation terms [5]. In this case the equations of motion become of neutral type but with both retarded and advanced arguments. In [6] we have proved existence-uniqueness of a periodic solution of the system of equation of motion utilizing fixed point method. An immediate consequence of this fact is that the spectrum of the hydrogen atom is discreet one.

In [7] we have estimated the radiation energy and thus we answer the question of why the moving electron around the nucleus is stable and not collided with a nucleus. In this manner we thus affirm the Bohr assumption for stationary states. It turns out that his assumption is a consequence of the laws of classical electrodynamics and does not violence it. Our intention to look for these results was caused by the famous results due to A. Sommerfeld [8, 9].

The main purpose of the present paper is to extend some our recent results concerning two-body problem of classical electrodynamics including spin equations. Without pretending to complete bibliography we mention some previous results with spin [10-18]. We note, however, that all considerations of spinning particles with radiation terms contain Dirac radiation terms with third derivative. We would like to emphasize once again that our new form of the Dirac radiation term is relativistic invariant. This enables us to use again Synge formalism for the two-body problem with radiation terms. We prove that the six spin equations are a consequence of the rest ones and in this manner we obtain six equations for six unknown spin functions. The last system jointly with the six equations of motion with radiation terms gives complete description of the two-body problem. We want to explicitly mention the researches due D.-A. Deckert et al. [19-22], and De Luca [23] based on Wheeler- Feynman approach.

Here we proceed from H. Corben considerations of classical spinning particles [12, 13].

The paper consists of eight sections. In Section 2 we repeat the derivation of Synge two-body equations. In Section 3 we derive two-body problem with radiation terms. In Section 4 the first group of spin equations is derived while in Section 5 the second group of spin equations is proposed. In Section 6 we write down spin equations in vector form. This enables us to show in Section 7 that some of them are consequences of the rest ones. In this way, we get as many equations as spin functions. An existence-uniqueness of the general system in a next paper will be proved.
2. Derivation of Synge Two-Body Equations of Motion

First we recall some denotations from [3], [1] and [2]. The considerations are in the Minkowski space. Roman suffixes run over 1, 2, 3, 4, while Greek – 1, 2, 3 with Einstein summation convention. By \( \langle \cdot, \cdot \rangle_4 \) we denote the scalar product in the Minkowski space, while by \( \langle \cdot, \cdot \rangle \) – the scalar product in three-dimensional Euclidean subspace.

We consider the equations of motion with radiation terms derived in [5] jointly with the spin equations (cf. [12], [13]):

\[
d\xi_{\gamma}^{(p)} / ds_1 = e_1 \left( F_{\alpha \nu}^{(p)} \lambda_{\nu}^{(p)} + F_{\alpha \nu}^{(p) rad} \lambda_{\nu}^{(p)} \right) / m_1 c^2,
\]

\[
d\sigma_{\gamma}^{(p)} / ds_1 = e_1 \left( F_{\alpha \nu}^{(p)} + F_{\alpha \nu}^{(p) rad} \right) \sigma_{\kappa}^{(p)} / m_1 c^2 \left( F_{\kappa \nu}^{(p)} + F_{\kappa \nu}^{(p) rad} \right) (pq) = (12), (21).
\]

Recall (cf. [1], [2]) that the quantities relating to the particles are:

\[
Q_p = -|e_1| |e_2| / m_p = -|e_p|^2 / m_p \quad (p = 1, 2); \quad e_{pq} = \left( e_{1pq}, e_{2pq}, e_{3pq}, e_{4pq} \right) = \left( e_{1pq}, e_{4pq} \right)
\]

\[
= \left( x_1^{(p)}(t) - x_1^{(q)}(t - \tau_{pq}), x_2^{(p)}(t) - x_2^{(q)}(t - \tau_{pq}), x_3^{(p)}(t) - x_3^{(q)}(t - \tau_{pq}), i c t \right) = (\vec{x}^{(p)}, i c t) - \text{space-time-coordinates of the moving particles;}
\]

\[
c \quad \text{– the speed of the light;}
\]

\[
L_p \quad \text{– the world lines;}
\]

\[
m_p \quad \text{– proper masses;}
\]

\[
e_p \quad \text{– charges} \quad (p = 1, 2).
\]

We suppose \( e_1 e_2 < 0 \) and since \( |e_1| = |e_2| \) then

\[
\left\{ \xi_{pq}^{(p)}, \xi_{pq}^{(q)} \right\}_4 = 0 \quad \text{or} \quad \tau_{pq}(t) = \frac{1}{c^2} \left( \sum_{\gamma=1}^{3} x_{\gamma}^{(p)}(t) - x_{\gamma}^{(q)}(t - \tau_{pq}(t)) \right)^2,
\]

\[
= \left( u_1^{(p)}(t), u_2^{(p)}(t), u_3^{(p)}(t) \right) = \vec{u}^{(p)}(t), \quad (p = 1, 2) - \text{velocities of the moving particles,}
\]

\[
\left( \vec{u}^{(p)}(t), \vec{u}^{(q)}(t) \right) = \sum_{\gamma=1}^{3} u_{\gamma}^{(p)}(t), u_{\gamma}^{(q)}(t), \quad \Delta_p = \sqrt{c^2 - \left( \vec{u}^{(p)}(t), \vec{u}^{(q)}(t) \right)} ,
\]

\[
\gamma_p = \sqrt{1 - \frac{\left( \vec{u}^{(p)}(t), \vec{u}^{(q)}(t) \right)}{c^2} = c / \Delta_p, \quad \left( \lambda_1^{(p)}, \lambda_2^{(p)}, \lambda_3^{(p)}, \lambda_4^{(p)} \right) = (\vec{\lambda}^{(p)}, ic / \Delta_p) = (\gamma_p u_{\alpha}^{(p)} / c, i \gamma_p) = (\vec{u}^{(p)} / \Delta_p, ic / \Delta_p),
\]

\[
(\alpha = 1, 2, 3 ; p = 1, 2) - \text{components of the unit tangent vectors to world lines,}
\]

\[
\lambda^{(q)} = \left( \frac{u_1^{(q)}(t - \tau_{pq}), u_2^{(q)}(t - \tau_{pq}), u_3^{(q)}(t - \tau_{pq}), ic}{\Delta_p, \Delta_p, \Delta_p} \right) = (\vec{u}^{(q)}(t - \tau_{pq}) / \Delta_p, ic / \Delta_p)
\]

where \( \Delta_p = \sqrt{c^2 - \left( \vec{u}^{(q)}(t - \tau_{pq}), \vec{u}^{(q)}(t - \tau_{pq}) \right)} ; ds^{(p)} = \Delta_p dt ;
\]

\[
\left\{ \lambda^{(p)}, \lambda^{(q)} \right\}_4 = \left( \vec{\lambda}^{(p)} / \Delta_p, \vec{\lambda}^{(q)} / \Delta_p \right) - c^2 / (\Delta_p \Delta_p) = \left( \vec{u}^{(p)}(t), \vec{u}^{(q)}(t - \tau_{pq}) \right) - c^2 / (\Delta_p \Delta_p);
\]

\[
\left\{ \xi^{(p)}, \xi^{(q)} \right\}_4 = \left( \vec{\xi}^{(p)} / \Delta_p, \vec{\xi}^{(q)} / \Delta_p \right) + ic \tau_{pq} / \Delta_p = \left( \vec{\xi}^{(p)}(t), \vec{u}^{(q)}(t) \right) - c^2 \tau_{pq} / \Delta_p;
\]

\[
\left\{ \xi^{(p)}, \xi^{(q)} \right\}_4 = \left( \vec{\xi}^{(p)}(t), \vec{u}^{(q)} / \Delta_p \right) + ic \tau_{pq} / \Delta_p = \left( \vec{\xi}^{(p)}(t), \vec{u}^{(q)} \right) - c^2 \tau_{pq} / \Delta_p ;
\]

\[
\frac{d \xi_{\alpha}^{(p)}}{ds_p} = \left\{ \frac{1}{\Delta_p} \frac{d u_{\alpha}^{(p)}}{dt} \right\} = \left\{ \frac{1}{\Delta_p} \frac{d u_{\alpha}^{(p)}}{dt} \right\} + \vec{u}_{\alpha}^{(p)}(t), \vec{u}_{\alpha}^{(q)}(t) / \Delta_4 ;
\]

\[
= \left( \vec{u}_{\alpha}^{(p)}(t) / \Delta_2 \right) + \vec{u}_{\alpha}^{(p)}(t), \vec{u}_{\alpha}^{(q)}(t) / \Delta_4 .
\]
\[
\frac{d\lambda_4^{(p)}}{ds_p} = \frac{ic}{\Delta_p} \frac{d}{dt}\left(\frac{1}{\Delta_p}\right) = \frac{ic}{\Delta_p^3} \left\{\hat{u}^{(p)}, \hat{u}^{(p)}\right\};
\]
\[
\frac{d}{ds_p} = \frac{1}{\Delta_p} \frac{d}{dt} \quad \frac{d}{ds_q} = \frac{1}{\Delta_p} \frac{d}{dt} \quad \frac{d}{dt} = \Delta_p \frac{d}{dt} \quad \frac{d}{dt} = D_{pq}
\]
and the derivative could be calculated from the equation
\[
t-t_{pq} = \frac{1}{c} \sqrt{\sum_{j=1}^{3} \left[x_j^{(p)}(t) - x_j^{(p)}(t_{pq})\right]^2}, \quad \text{i.e.}
\]
\[
\frac{dt}{dt_{pq}} = \frac{c}{\sqrt{\sum_{j=1}^{3} \left[x_j^{(p)}(t) - x_j^{(p)}(t_{pq})\right]^2}} = D_{pq}.
\]

Therefore
\[
d\lambda_4^{(q)} / ds_q = D_{pq} \hat{u}_{(q)}^{(q)}(t-t_{pq}) / \Delta_p^4 + \left(D_{pq} \left\{\hat{u}^{(p)}(t-t_{pq}), \hat{u}^{(p)}(t-t_{pq})\right\}\right) u_q^{(q)}(t-t_{pq}) / \Delta_p^4;
\]
\[
\frac{d\lambda_4^{(q)}}{ds_q} = icD_{pq} \left\{\hat{u}^{(q)}, \hat{u}^{(q)}\right\} / \Delta_p^4;
\]
\[
\left\{\xi^{(pq)}, d\lambda_4^{(q)} / ds_q\right\}_4 = D_{pq} \lambda_4^2 \left\{\hat{u}^{(pq)}, \hat{u}^{(pq)}\right\} + \left(D_{pq} \left\{\hat{u}^{(p)}, \hat{u}^{(p)}\right\}\right) \left\{\hat{u}^{(q)}, \hat{u}^{(q)}\right\} - c^2 \tau_{pq} \left\{\hat{u}^{(pq)}, \hat{u}^{(pq)}\right\};
\]
\[
1 + \left\{\xi^{(pq)}, d\lambda_4^{(q)} / ds_q\right\}_4 = D_{pq} / \Delta_p^4 \times \left[\left(\Delta_p^4 / D_{pq}\right) + \left(D_{pq} \left\{\hat{u}^{(pq)}, \hat{u}^{(pq)}\right\}\right) + \left(D_{pq} \left\{\hat{u}^{(p)}, \hat{u}^{(p)}\right\}\right) \left\{\hat{u}^{(q)}, \hat{u}^{(q)}\right\} - c^2 \tau_{pq}\right];
\]
\[
\left\{\xi^{(pq)}, d\lambda_4^{(q)} / ds_q\right\}_4 = D_{pq} / \Delta_p \lambda_4^4 \times \left(D_{pq} \left\{\hat{u}^{(pq)}, \hat{u}^{(pq)}\right\}\right) + \left(D_{pq} \left\{\hat{u}^{(p)}, \hat{u}^{(p)}\right\}\right) \left\{\hat{u}^{(q)}, \hat{u}^{(q)}\right\} - c^2 \left\{\hat{u}^{(pq)}, \hat{u}^{(pq)}\right\};
\]

Recall that the Synge's equations of motion without radiation terms are
\[
d\lambda^{(p)} / ds_p = e_p F(p) \lambda^{(p)} / m_p c^2 \quad \text{(cf. [3])}.
\]
The elements of the electromagnetic tensor can be derived by Lienard-Wiechert retarded potentials
\[
\Phi_{n}^{(p)} = e_p F_{ik}^{(p)} \lambda^{(p)} / m_p c^2 \quad \text{(n = 1, 2, 3, 4)}
\]
\[
F_{ik}^{(p)} = \frac{\partial\Phi_{ik}^{(p)}}{\partial x_i^{(p)}} - \frac{\partial\Phi_{ik}^{(p)}}{\partial x_k^{(p)}} \quad \text{and then using denotations}
\]
\[ p^{(pq)}_{\alpha} = -\frac{1}{\Delta} \left( \frac{\delta^{(pq)} \cdot d\lambda^{(q)} / ds_q}{\left( \frac{\lambda^{(q)} \cdot \delta^{(pq)}}{\Delta^{(q)}} \right)^{3/4}} \right) + \frac{1}{\Delta} \frac{d\lambda^{(q)}}{ds_q} \]

\[ = \frac{D_{pq}}{\Delta^{(q)}} \left( \frac{\delta^{(pq)} \cdot d\lambda^{(q)} / ds_q}{\left( \frac{\lambda^{(q)} \cdot \delta^{(pq)}}{\Delta^{(q)}} \right)^{3/4}} \right) + \frac{1}{\Delta} \frac{d\lambda^{(q)}}{ds_q} \]

\[ = ic \left( \Delta^{2} + D_{pq} \left( \frac{\delta^{(pq)} \cdot d\lambda^{(q)} / ds_q}{\left( \frac{\lambda^{(q)} \cdot \delta^{(pq)}}{\Delta^{(q)}} \right)^{3/4}} \right) \right) \left( c^{2} \tau_{pq} - \left( \frac{\delta^{(pq)} \cdot d\lambda^{(q)} / ds_q}{\left( \frac{\lambda^{(q)} \cdot \delta^{(pq)}}{\Delta^{(q)}} \right)^{3/4}} \right) \right)^{3/2}; \]

\[ F^{(p)}_{ik} = \frac{\partial \Phi^{(p)}_{k}}{\partial \xi^{(p)}_{i}} - \frac{\partial \Phi^{(p)}_{i}}{\partial \xi^{(p)}_{k}} = e_{p} \left( p^{(p)}_{i} \delta^{(pq)}_{k} - p^{(p)}_{k} \delta^{(pq)}_{i} \right) \]

we obtain

\[ d\lambda^{(p)}_{i} / ds_{p} = e_{p} F^{(p)}_{ik}(\lambda^{(p)}_{k}) / m_{p}c^{2} = -\frac{e_{p}}{m_{p}c^{2}} \left[ \frac{1}{4} \left( \Delta^{(q)} + D_{pq} \left( \frac{\delta^{(pq)} \cdot d\lambda^{(q)} / ds_q}{\left( \frac{\lambda^{(q)} \cdot \delta^{(pq)}}{\Delta^{(q)}} \right)^{3/4}} \right) \right) + \frac{1}{4} \left( \frac{d\lambda^{(q)}}{ds_q} \right) \right] \]

\[ \left( \alpha = 1, 2, 3 \right), \left( pq \right) = \left( 21, 12 \right). \]

Let us set

\[ M_{pq} = \frac{\Delta^{2} + D_{pq} \left( \delta^{(pq)} \cdot d\lambda^{(q)} / ds_q \right)}{c^{2} \tau_{pq} - \left( \delta^{(pq)} \cdot d\lambda^{(q)} / ds_q \right)^{3/2}}, \quad N_{pq} = \frac{D_{pq}}{c^{2} \tau_{pq} - \left( \delta^{(pq)} \cdot d\lambda^{(q)} / ds_q \right)^{3/2}}. \]

Then \( P^{(pq)}_{\alpha} = M_{pq} u^{(pq)}_{\alpha} + N_{pq} \bar{u}^{(pq)}_{\alpha} \) and \( P_{4} = ic M_{pq} \).

Consequently

\[ P^{(pq)}_{\alpha} \left( \frac{\lambda^{(p)} \cdot \delta^{(pq)}_{i}}{\Delta^{(q)}} \right) = M_{pq} \left( \frac{\lambda^{(p)} \cdot \delta^{(pq)}_{i}}{\Delta^{(q)}} \right) + N_{pq} \left( \frac{\lambda^{(p)} \cdot \delta^{(pq)}_{i}}{\Delta^{(q)}} \right) \cdot \bar{u}^{(pq)}_{\alpha}. \]

For \( \left( \lambda^{(p)} \cdot P^{(pq)}_{\alpha} \right) / \Delta_{p} \) we have

\[ \left( \lambda^{(p)} \cdot P^{(pq)}_{\alpha} \right) / \Delta_{p} = \left( \lambda^{(p)} \cdot P^{(pq)}_{\alpha} \right) / \Delta_{p} + c^{2} M_{pq} / \Delta_{p}. \]

Then the 3-dimensional part of
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\[ \frac{d\lambda^{(p)}}{ds_p} = e_p F_{ik}^{(p)} \lambda_k^{(p)} / m_p c^2 = -e_p^2 \left[ p_i^{(pq)} \left\{ \lambda^{(p)}, \xi^{(pq)} \right\}_4 - \xi_i^{(pq)} \left\{ \lambda^{(p)}, P^{(pq)} \right\}_4 \right] / m_p c^2 \]

In view of \( Q_p = e_1 e_2 \), \( m_p = -e_p^2 / m_p \), becomes

\[ \frac{d\lambda^{(p)}}{ds_p} = Q_p \left[ p_i^{(pq)} \left( \xi_i^{(pq)}, \ddot{u}^{(p)} \right) - c^2 \tau_{pq} \right] / \Delta_p - \left( \dddot{\lambda}^{(p)}, \dddot{p}^{(pq)} \right) + c^2 M_{pq} / \Delta_p \]

**Remark 1.** We consider just the first three equations because the fourth equation is a consequence of the first three ones for fixed \( p \) (cf. [5], [6]).

### 3. Derivation of Two-Body Equations of Motion with Radiation Terms

The equations of motion for the two-body problem with radiation terms are derived in [5]. Using denotations from [5] we get

\[ \frac{d\lambda^{(p)}}{ds_p} = e_p \left( F_{ik}^{(p)} + F_{ik}^{(p), rad} \right) \lambda_k^{(p)} / m_p c^2 \]

or

\[ \frac{d\lambda^{(p)}}{ds_p} = \left( Q_p / c^2 \right) \left[ p_i^{(pq)} \left( \lambda^{(p)}, \xi^{(pq)} \right)_4 - \xi_i^{(pq)} \left( \lambda^{(p)}, P^{(pq)} \right)_4 \right] 

+ \left( Q_p / 2c \right) \left[ p_i^{(pq), ret} \left( \lambda^{(p), ret}, \xi^{(pq), ret} \right)_4 - \xi_i^{(pq), ret} \left( \lambda^{(p), ret}, P^{(pq), ret} \right)_4 \right] 

- p_i^{(pq), adv} \left( \lambda^{(p), adv}, \xi^{(pq), adv} \right)_4 + \xi_i^{(pq), adv} \left( \lambda^{(p), adv}, P^{(pq), adv} \right)_4 \right] \]

So we have

\[ \dddot{u}^{(p)} + \dddot{u}^{(p)} \left( \dddot{u}^{(p)}, \dddot{u}^{(p)} \right) / \Delta_p = Q_p \Delta_p^2 / 2c^2 \left[ 2 \left( \xi^{(pq)}, \ddot{u}^{(p)} \right) - c^2 \tau_{pq} \right] / \Delta_p 

- 2 M_{pq} \left( \Delta_p \Delta_{pq} - c^2 \right) + N_{pq} \left( \Delta_p \Delta_{pq} \right) \dddot{u}^{(p)} \right] / \Delta_p 

+ \left( \xi^{(pq), ret}, \ddot{u}^{(p)} \right) - c^2 \tau_{p, ret} \right] / \Delta_p \right] 

- \left( \xi^{(pq), adv}, \dddot{u}^{(p)} \right) - c^2 \tau_{p, adv} \right] / \Delta_p \right] 

\[ \dddot{u}^{(p)} + \xi^{(pq), adv} \right] / \Delta_p \right] 

G^{(p)}_d \]

In view of Remark 1 we consider just the following six equations for six unknown functions:
\[ u_\alpha^{(p)} + u_\alpha^{(p)} \left\langle \bar{u}^{(p)}, \check{u}^{(p)} \right\rangle / \Delta_p^2 = G_\alpha^{(p)}, (\alpha = 1, 2, 3; p = 1, 2) \]

or

\[
\left( 1 + \frac{u_1^{(p)} u_1^{(p)}}{\Delta_p^2} \right) u_1^{(p)} + \left( 1 + \frac{u_2^{(p)} u_2^{(p)}}{\Delta_p^2} \right) u_2^{(p)} + \left( 1 + \frac{u_3^{(p)} u_3^{(p)}}{\Delta_p^2} \right) u_3^{(p)} = G_1^{(p)};
\]

\[
\frac{u_2^{(p)} u_1^{(p)}}{\Delta_p^2} u_1^{(p)} + \frac{u_3^{(p)} u_1^{(p)}}{\Delta_p^2} u_1^{(p)} + \left( 1 + \frac{u_3^{(p)} u_3^{(p)}}{\Delta_p^2} \right) u_3^{(p)} = G_2^{(p)};
\]

\[
\frac{u_3^{(p)} u_1^{(p)}}{\Delta_p^2} u_1^{(p)} + \frac{u_3^{(p)} u_2^{(p)}}{\Delta_p^2} u_2^{(p)} + \left( 1 + \frac{u_3^{(p)} u_3^{(p)}}{\Delta_p^2} \right) u_3^{(p)} = G_3^{(p)}.
\]

Under the basic assumption

\[
\left\langle \bar{u}^{(p)}, \check{u}^{(p)} \right\rangle \leq \bar{c} < c \Rightarrow
\]

\[
\Delta = \begin{vmatrix}
1 + u_1^{(p)} u_1^{(p)} / \Delta_p^2 & u_1^{(p)} u_2^{(p)} / \Delta_p^2 & u_1^{(p)} u_3^{(p)} / \Delta_p^2 \\
1 + u_2^{(p)} u_2^{(p)} / \Delta_p^2 & u_2^{(p)} u_3^{(p)} / \Delta_p^2 \\
1 + u_3^{(p)} u_3^{(p)} / \Delta_p^2 & u_3^{(p)} u_3^{(p)} / \Delta_p^2
\end{vmatrix}
\]

\[
= c^2 / \Delta_p^2 \neq 0
\]

we obtain

\[
u_1^{(p)} = G_1^{(p)} - u_1^{(p)} \left\langle \bar{u}^{(p)}, \check{G}^{(p)} \right\rangle / c^2;
\]

\[
u_2^{(p)} = G_2^{(p)} - u_2^{(p)} \left\langle \bar{u}^{(p)}, \check{G}^{(p)} \right\rangle / c^2;
\]

\[
u_3^{(p)} = G_3^{(p)} - u_3^{(p)} \left\langle \bar{u}^{(p)}, \check{G}^{(p)} \right\rangle / c^2.
\]

Using vector denotations we have

\[
\dot{u}^{(p)} = \tilde{G}^{(p)} - \bar{u}^{(p)} \left\langle \bar{u}^{(p)}, \check{G}^{(p)} \right\rangle / c^2
\]

and in view of \( q_1 e_2 < 0 \) we obtain

\[
G_\alpha^{(p)} = \left( Q_p \Delta_p^2 / c^2 \right) \left( \left\langle \bar{\xi}^{(pq)}, \check{u}^{(p)} \right\rangle - c^2 \tau_{pq} \right) \left( M_{pq} \bar{u}^{(q)} + N_{pq} \check{u}^{(q)} \right) / \Delta_p
\]

\[
= - \left( M_{pq} \left( \frac{\bar{u}^{(p)} \check{u}^{(q)}}{\Delta_p^2 / \Delta_{pq}} - c^2 \right) + N_{pq} \left( \frac{\bar{u}^{(p)} \check{u}^{(q)}}{\Delta_p^2 / \Delta_{pq}} \right) \right) \bar{\xi}^{(pq)}
\]

\[
+ Q_p \Delta_p^2 \frac{1}{2 c^2} \left( \frac{\bar{\xi}^{(pq)} \check{u}^{(p)} - c^2 \tau_{p,ret}}{\Delta_p} \right) \left( M_{p,ret} \bar{u}^{(p)} + N_{p,ret} \check{u}^{(p)} \right)
\]

\[
- \left( M_{p,ret} \left( \frac{\bar{u}^{(p)} \check{u}^{(p)ret}}{\Delta_p^2 / \Delta_{p,ret}} - c^2 \right) + N_{p,ret} \left( \frac{\bar{u}^{(p)} \check{u}^{(p)ret}}{\Delta_p^2 / \Delta_{p,ret}} \right) \right) \bar{\xi}^{(pq)ret}
\]
- $\frac{Q_p \Delta_p^2}{2c^2} \left[ \frac{\tilde{\xi}(p)_{\text{adv}}, \tilde{u}^{(p)}(p)}{\Delta_p} - c^2 \tau_{p,\text{adv}} \left( M_{p,\text{adv}} \tilde{u}^{(p)}_{\text{adv}}(p) + N_{p,\text{adv}} \tilde{u}^{(p)}_{\text{adv}}(p) \right) \right]
\quad + \frac{Q_p \Delta_p^2}{2c^2} \left[ \frac{\tilde{\xi}(p)_{\text{adv}}, \tilde{u}^{(p)}(p)}{\Delta_p} - c^2 \tau_{p,\text{adv}} \left( M_{p,\text{adv}} \tilde{u}^{(p)}_{\text{adv}}(p) + N_{p,\text{adv}} \tilde{u}^{(p)}_{\text{adv}}(p) \right) \right] \frac{\tilde{\xi}(p)_{\text{adv}}}{\Delta_p}.
\quad - M_{p,\text{adv}} \left( \frac{\tilde{u}^{(p)}_{\text{ret}}, \tilde{u}^{(p)}(p)_{\text{ret}}}{\Delta_p} - c^2 \tau_{p,\text{ret}} \left( M_{p,\text{ret}} \tilde{u}^{(p)}_{\text{ret}}(p) + N_{p,\text{ret}} \tilde{u}^{(p)}_{\text{ret}}(p) \right) \right)
\quad \times \left. \frac{\tilde{\xi}(p)_{\text{ret}}, \tilde{u}^{(p)}(p)_{\text{ret}}}{\Delta_p} - c^2 \tau_{p,\text{ret}} \left( M_{p,\text{ret}} \tilde{u}^{(p)}_{\text{ret}}(p) + N_{p,\text{ret}} \tilde{u}^{(p)}_{\text{ret}}(p) \right) \right] - \frac{Q_p \Delta_p^2}{2c^2}
\quad + M_{p,\text{adv}} \left( \frac{\tilde{u}^{(p)}_{\text{ret}}, \tilde{u}^{(p)}(p)_{\text{adv}}}{\Delta_p} - c^2 \tau_{p,\text{adv}} \left( M_{p,\text{adv}} \tilde{u}^{(p)}_{\text{adv}}(p) + N_{p,\text{adv}} \tilde{u}^{(p)}_{\text{adv}}(p) \right) \right)
\quad \times \left. \frac{\tilde{\xi}(p)_{\text{adv}}, \tilde{u}^{(p)}(p)_{\text{adv}}}{\Delta_p} - c^2 \tau_{p,\text{adv}} \left( M_{p,\text{adv}} \tilde{u}^{(p)}_{\text{adv}}(p) + N_{p,\text{adv}} \tilde{u}^{(p)}_{\text{adv}}(p) \right) \right] \frac{\tilde{\xi}(p)_{\text{adv}}}{\Delta_p}.

4. Explicit Form of the First Three Spin Equations

In what follows we obtain the explicit form of the spin equations:

$$\frac{d\sigma^{(1)}_{\tilde{u}}}{ds^{(1)}} = \frac{e_1}{m_1 c^2} \left[ \left( F^{(1)}_{ik} + F^{(1),\text{rad}}_{ik} \right) \sigma^{(1)}_{ik} - \sigma^{(1)}_{ik} \left( F^{(1)}_{kj} + F^{(1),\text{rad}}_{kj} \right) \right]$$

$$\frac{d\sigma^{(2)}_{\tilde{u}}}{ds^{(2)}} = \frac{e_2}{m_2 c^2} \left[ \left( F^{(2)}_{ik} + F^{(2),\text{rad}}_{ik} \right) \sigma^{(2)}_{ik} - \sigma^{(2)}_{ik} \left( F^{(2)}_{kj} + F^{(2),\text{rad}}_{kj} \right) \right].$$

Introduce denotations analogous to the ones from [12], [13]:

$$\tilde{\vartheta}^{(p)} = \left( \vartheta_1^{(p)}(t), \vartheta_2^{(p)}(t), \vartheta_3^{(p)}(t) \right), \quad (p = 1, 2),$$

$$\vartheta^{(p)} = \left( \vartheta_1^{(p)}(t), \vartheta_2^{(p)}(t), \vartheta_3^{(p)}(t), u_4^{(p)}(t) = ic \right), \quad (p = 1, 2),$$

$$\vartheta^{(p)} = \left( \vartheta_1^{(p)}(t), \vartheta_2^{(p)}(t), \vartheta_3^{(p)}(t) \right), \quad (p = 1, 2),$$
\[
\left( \lambda_1^{(p)}, \lambda_2^{(p)}, \lambda_3^{(p)}, \lambda_4^{(p)} \right) = \left( \tilde{\lambda}^{(p)}, ic / \Delta_p \right) = \left( \vec{u}^{(p)} / \Delta_p, ic / \Delta_p \right), \quad (p = 1, 2)
\]
where
\[
\tilde{\theta}^{(p)} = \frac{1}{c} \left( \tilde{\lambda}^{(p)} \times \tilde{\sigma}^{(p)} \right) = \frac{1}{c} \begin{vmatrix}
\tilde{c}_1 & \tilde{c}_2 & \tilde{c}_3 \\
\lambda_1^{(p)} & \lambda_2^{(p)} & \lambda_3^{(p)} \\
\sigma_1^{(p)} & \sigma_2^{(p)} & \sigma_3^{(p)}
\end{vmatrix}
\]
\[
= \frac{\lambda_2^{(p)}}{c} \sigma_3^{(p)} - \frac{\lambda_3^{(p)}}{c} \sigma_2^{(p)} \tilde{c}_1 + \frac{\lambda_3^{(p)}}{c} \sigma_1^{(p)} - \frac{\lambda_1^{(p)}}{c} \sigma_3^{(p)} \tilde{c}_2 + \frac{\lambda_1^{(p)}}{c} \sigma_2^{(p)} - \frac{\lambda_2^{(p)}}{c} \sigma_1^{(p)} \tilde{c}_3
\]
we obtain the spin tensor
\[
\sigma_{\mu\nu}^{(p)} = \begin{pmatrix}
0 & -\sigma_2^{(p)} & i\theta_1^{(p)} \\
-\sigma_3^{(p)} & 0 & \sigma_1^{(p)} i\theta_2^{(p)} \\
\sigma_2^{(p)} & -\sigma_1^{(p)} & 0
\end{pmatrix}
\]
It is known (cf. [12], [13]) that \( \sigma_{ij}^{(p)} \lambda_j^{(p)} = 0 \). We verify the equalities for \( i = 1, 4 \):
\[
\begin{align*}
\sigma_1^{(p)} \lambda_2^{(p)} + \sigma_2^{(p)} \lambda_3^{(p)} + \sigma_3^{(p)} \lambda_4^{(p)} + \sigma_4^{(p)} \lambda_1^{(p)} &= \left( \frac{\lambda_2^{(p)}}{c} \sigma_3^{(p)} - \frac{\lambda_3^{(p)}}{c} \sigma_2^{(p)} \right) + \left( \frac{\lambda_3^{(p)}}{c} \sigma_1^{(p)} - \frac{\lambda_1^{(p)}}{c} \sigma_3^{(p)} \right) + \left( \frac{\lambda_1^{(p)}}{c} \sigma_2^{(p)} - \frac{\lambda_2^{(p)}}{c} \sigma_1^{(p)} \right) = 0;\\
\end{align*}
\]
\[
\begin{align*}
\sigma_4^{(p)} \lambda_1^{(p)} + \sigma_2^{(p)} \lambda_3^{(p)} + \sigma_3^{(p)} \lambda_4^{(p)} + \sigma_1^{(p)} \lambda_2^{(p)} &= \left[ -i \left( \lambda_2^{(p)} \sigma_3^{(p)} - \lambda_3^{(p)} \sigma_2^{(p)} \right) \lambda_1^{(p)} - i \left( \lambda_3^{(p)} \sigma_1^{(p)} - \lambda_1^{(p)} \sigma_3^{(p)} \right) \lambda_2^{(p)} - i \left( \lambda_1^{(p)} \sigma_2^{(p)} - \lambda_2^{(p)} \sigma_1^{(p)} \right) \lambda_3^{(p)} / c \right]
\end{align*}
\]
\[
= -i \begin{vmatrix}
\sigma_2^{(p)} & \sigma_3^{(p)} \\
\lambda_2^{(p)} & \lambda_3^{(p)}
\end{vmatrix} + \begin{vmatrix}
\sigma_1^{(p)} & \sigma_3^{(p)} \\
\lambda_1^{(p)} & \lambda_3^{(p)}
\end{vmatrix} + \begin{vmatrix}
\sigma_1^{(p)} & \sigma_2^{(p)} \\
\lambda_1^{(p)} & \lambda_2^{(p)}
\end{vmatrix} = 0.
\]

\textbf{Remark 2.} Let us consider the equations
\[
\frac{d\sigma_{ij}^{(p)}}{ds^{(p)}} = -\frac{e_p}{m_p c^2} \left( \left( F_{ik}^{(p)} + F_{ik}^{(p)rad} \right) \sigma_{kj}^{(p)} - \sigma_{ik}^{(p)} \left( F_{kj}^{(p)} + F_{kj}^{(p)rad} \right) \right) \equiv e_p \vec{F}_{i,j}^{(p)} / m_p c^2.
\]
We notice that
\[
F_{ik}^{(p)} = -F_{ki}^{(p)}; \quad F_{ik}^{(p)rad} = -F_{ki}^{(p)rad}; \quad \sigma_{ik}^{(p)} = -\sigma_{kj}^{(p)}.
\]
Therefore
\[
\vec{F}_{i,j}^{(p)} = \left( F_{ik}^{(p)} + F_{ik}^{(p)rad} \right) \sigma_{kj}^{(p)} - \sigma_{ik}^{(p)} \left( F_{kj}^{(p)} + F_{kj}^{(p)rad} \right) = -\left( \sigma_{jk}^{(p)} \left( F_{ki}^{(p)} + F_{ki}^{(p)rad} \right) - \left( F_{kj}^{(p)} + F_{kj}^{(p)rad} \right) \sigma_{ki}^{(p)} \right) = -\vec{F}_{ji}^{(p)}.
\]
It follows \( \vec{F}_{i,j}^{(p)} = 0 \) \( (i = 1, 2, 3, 4) \). Consequently we have derived the following spin equations for two particles \( (p = 1, 2) \):
\[
\frac{d\sigma^{(p)}_{12}}{ds^{(p)}} = \frac{e_p}{m_p c^2} \left[ \left( F^{(p)}_{1k} + F^{(p)\text{rad}}_{1k} \right) \sigma^{(p)}_{k2} - \sigma^{(p)}_{1k} \left( F^{(p)}_{k2} + F^{(p)\text{rad}}_{k2} \right) \right],
\]

\[
\frac{d\sigma^{(p)}_{13}}{ds^{(p)}} = \frac{e_p}{m_p c^2} \left[ \left( F^{(p)}_{1k} + F^{(p)\text{rad}}_{1k} \right) \sigma^{(p)}_{k3} - \sigma^{(p)}_{1k} \left( F^{(p)}_{k3} + F^{(p)\text{rad}}_{k3} \right) \right],
\]

\[
\frac{d\sigma^{(p)}_{23}}{ds^{(p)}} = \frac{e_p}{m_p c^2} \left[ \left( F^{(p)}_{2k} + F^{(p)\text{rad}}_{2k} \right) \sigma^{(p)}_{k3} - \sigma^{(p)}_{2k} \left( F^{(p)}_{k4} + F^{(p)\text{rad}}_{k4} \right) \right],
\]

\[
\frac{d\sigma^{(p)}_{14}}{ds^{(p)}} = \frac{e_p}{m_p c^2} \left[ \left( F^{(p)}_{1k} + F^{(p)\text{rad}}_{1k} \right) \sigma^{(p)}_{k4} - \sigma^{(p)}_{1k} \left( F^{(p)}_{k4} + F^{(p)\text{rad}}_{k4} \right) \right],
\]

\[
\frac{d\sigma^{(p)}_{24}}{ds^{(p)}} = \frac{e_p}{m_p c^2} \left[ \left( F^{(p)}_{2k} + F^{(p)\text{rad}}_{2k} \right) \sigma^{(p)}_{k4} - \sigma^{(p)}_{2k} \left( F^{(p)}_{k4} + F^{(p)\text{rad}}_{k4} \right) \right],
\]

\[
\frac{d\sigma^{(p)}_{34}}{ds^{(p)}} = \frac{e_p}{m_p c^2} \left[ \left( F^{(p)}_{3k} + F^{(p)\text{rad}}_{3k} \right) \sigma^{(p)}_{k4} - \sigma^{(p)}_{3k} \left( F^{(p)}_{k4} + F^{(p)\text{rad}}_{k4} \right) \right].
\]

First Spin Equation

\[
\frac{d\sigma^{(p)}_{12}}{ds^{(p)}} = \frac{e_p}{m_p c^2} \left[ \sigma^{(p)}_{k2} \left( F^{(p)}_{1k} + F^{(p)\text{rad}}_{1k} \right) - \sigma^{(p)}_{1k} \left( F^{(p)}_{k2} + F^{(p)\text{rad}}_{k2} \right) \right]
\]

\[
- \sigma^{(p)}_{1k} \left( \left( F^{(p)}_{1k} + F^{(p)\text{rad}}_{1k} \right) \xi_k^{(pq)} - \left( F^{(p)}_{k2} + F^{(p)\text{rad}}_{k2} \right) \xi_1^{(pq)} \right)
\]

\[
- \sigma^{(p)}_{2k} \left( \left( F^{(p)}_{1k} + F^{(p)\text{rad}}_{1k} \right) \xi_k^{(pq)} - \left( F^{(p)}_{k2} + F^{(p)\text{rad}}_{k2} \right) \xi_2^{(pq)} \right)
\]

To transform the above equations we notice that

\[
\sigma^{(p)}_{k2} \xi_k^{(pq)} = \sigma^{(p)}_{12} \xi_1^{(pq)} + \sigma^{(p)}_{32} \xi_3^{(pq)} + \sigma^{(p)}_{42} \xi_4^{(pq)}
\]

\[
= \sigma^{(p)}_{12} \xi_1^{(pq)} - \sigma^{(p)}_{1k} \xi_k^{(pq)} + (i / c) \left( \lambda_3^{(p)} \sigma_1^{(p)} - \lambda_1^{(p)} \sigma_3^{(p)} \right) \text{ic} \tau_{pq}
\]

\[
= \left[ \sigma^{(p)}_{12} \xi_1^{(pq)} - \sigma^{(p)}_{1k} \xi_k^{(pq)} \right] - \left( \lambda_1^{(p)} \sigma_3^{(p)} - \lambda_3^{(p)} \sigma_1^{(p)} \right) \text{ic} \tau_{pq}
\]
\[
\begin{align*}
\sigma_{k_2}^{(p)} F_k^{(pq)} &= \sigma_2^{(p)} F_1^{(pq)} + \sigma_3^{(p)} F_3^{(pq)} + \sigma_4^{(p)} F_4^{(pq)} \\
&= \sigma_3^{(p)} F_1^{(pq)} - \sigma_4^{(p)} F_3^{(pq)} - (i/c) \left( \lambda_3^{(p)} \sigma_1^{(p)} - \lambda_4^{(p)} \sigma_4^{(p)} \right) F_4^{(pq)} \\
&= -\left[ \sigma_1^{(p)} \sigma_3^{(p)} \right]_{p_1^{(pq)}} - \left( \lambda_4^{(p)} F_4^{(pq)} / c \right) \left[ \sigma_1^{(p)} \sigma_3^{(p)} \right]_{\rho_4^{(pq)}}; \\
\sigma_{k_1}^{(p)} F_k^{(pq)} &= \sigma_2^{(p)} F_2^{(pq)} + \sigma_3^{(p)} F_3^{(pq)} + \sigma_4^{(p)} F_4^{(pq)} \\
&= \sigma_3^{(p)} F_2^{(pq)} - \sigma_4^{(p)} F_3^{(pq)} + (i/c) \left( \lambda_3^{(p)} \sigma_2^{(p)} - \lambda_4^{(p)} \sigma_2^{(p)} \right) F_4^{(pq)} \\
&= -\left[ \sigma_2^{(p)} \sigma_3^{(p)} \right]_{p_2^{(pq)}} - \left( \lambda_4^{(p)} F_4^{(pq)} / c \right) \left[ \sigma_2^{(p)} \sigma_3^{(p)} \right]_{\rho_4^{(pq)}}; \\
\sigma_{k_1}^{(p)} F_k^{(pq)} &= \sigma_1^{(p)} F_1^{(pq)} + \sigma_2^{(p)} F_2^{(pq)} + \sigma_4^{(p)} F_4^{(pq)} \\
&= \sigma_1^{(p)} F_1^{(pq)} - \sigma_2^{(p)} F_2^{(pq)} + \left( \lambda_1^{(p)} \sigma_1^{(p)} - \lambda_2^{(p)} \sigma_2^{(p)} \right) F_4^{(pq)} \\
&= -\left[ \sigma_1^{(p)} \sigma_2^{(p)} \right]_{p_1^{(pq)}} - \left( \lambda_2^{(p)} F_4^{(pq)} / c \right) \left[ \sigma_1^{(p)} \sigma_2^{(p)} \right]_{\rho_4^{(pq)}}. \\
\end{align*}
\]

Therefore
\[
\begin{align*}
p_1^{(pq)} &= \sigma_{k_2}^{(p)} F_k^{(pq)} - \sigma_{k_2}^{(p)} F_k^{(pq)} - \sigma_{k_2}^{(p)} F_k^{(pq)} + \sigma_2^{(p)} \sigma_{k_2}^{(p)} F_k^{(pq)} \\
&= -p_1^{(pq)} \left[ \sigma_1^{(p)} \sigma_3^{(p)} \right]_{p_1^{(pq)}} - p_1^{(pq)} \left[ \sigma_1^{(p)} \sigma_3^{(p)} \right]_{p_1^{(pq)}} + p_1^{(pq)} \left[ \sigma_1^{(p)} \sigma_3^{(p)} \right]_{p_3^{(pq)}} + \xi_1^{(pq)} (i \rho_4^{(pq)} / c) \left[ \sigma_1^{(p)} \sigma_3^{(p)} \right]_{\rho_4^{(pq)}}. \\
\end{align*}
\]

For
\[
\begin{align*}
\sigma_{k_2}^{(p)} &= \sigma_{k_2}^{(p)} F_k^{(pq)} - \sigma_{k_2}^{(p)} F_k^{(pq)} - \sigma_{k_2}^{(p)} F_k^{(pq)} + \sigma_2^{(p)} \sigma_{k_2}^{(p)} F_k^{(pq)} \\
&= -p_1^{(pq)} \left[ \sigma_1^{(p)} \sigma_3^{(p)} \right]_{p_1^{(pq)}} - p_1^{(pq)} \left[ \sigma_1^{(p)} \sigma_3^{(p)} \right]_{p_1^{(pq)}} + p_1^{(pq)} \left[ \sigma_1^{(p)} \sigma_3^{(p)} \right]_{p_3^{(pq)}} + \xi_1^{(pq)} (i \rho_4^{(pq)} / c) \left[ \sigma_1^{(p)} \sigma_3^{(p)} \right]_{\rho_4^{(pq)}}. \\
\end{align*}
\]

We obtain analogous expressions. Consequently
\[
\frac{d \sigma_3^{(p)}}{ds_p} = \frac{Q_p}{e^2} \left[ -p_1^{(pq)} \left[ \sigma_1^{(p)} \sigma_3^{(p)} \right]_{p_1^{(pq)}} - p_1^{(pq)} \left[ \sigma_1^{(p)} \sigma_3^{(p)} \right]_{p_1^{(pq)}} \right] + \xi_1^{(pq)} (i \rho_4^{(pq)} / c) \left[ \sigma_1^{(p)} \sigma_3^{(p)} \right]_{\rho_4^{(pq)}}. \\
\]

\[
\begin{align*}
\frac{-Q_p}{2c^2} &\left[ -p_1^{(p)adv} \begin{vmatrix} \sigma_1^{(p)} & \sigma_3^{(p)} \\ \sigma_1^{(p)} & \sigma_3^{(p)} \end{vmatrix} - p_1^{(p)adv} \tau_{p,adv} \begin{vmatrix} \sigma_1^{(p)} & \sigma_3^{(p)} \\ \lambda_1^{(p)} & \lambda_3^{(p)} \end{vmatrix} + \frac{\varepsilon_1^{(p)adv}}{p_1^{(p)adv}} \right] \\
&\left[ + p_2^{(p)adv} \begin{vmatrix} \sigma_2^{(p)} & \sigma_3^{(p)} \\ \sigma_2^{(p)} & \sigma_3^{(p)} \end{vmatrix} + \frac{\varepsilon_2^{(p)adv}}{p_2^{(p)adv}} \right] \\
&\left[ + p_3^{(p)adv} \begin{vmatrix} \sigma_3^{(p)} \\ \lambda_3^{(p)} \end{vmatrix} \right].
\end{align*}
\]

Second Spin Equation

\[
\frac{d\sigma_1^{(p)} \sigma_3^{(p)}}{ds^{(p)}} = \frac{e_p}{m_p c^2} \left[ \left( F_0^{(p)adv} + F_1^{(p)rad} \right) \sigma_1^{(p)ad} - \sigma_3^{(p)ad} \right] \\
+ \frac{Q_p}{2c^2} \left[ \left( \sigma_1^{(p)ad} - \sigma_3^{(p)ad} \right) - \sigma_3^{(p)ad} \right] \\
+ \frac{Q_p}{2c^2} \left[ \left( \sigma_1^{(p)ad} - \sigma_3^{(p)ad} \right) - \sigma_3^{(p)ad} \right] \\
+ \frac{Q_p}{2c^2} \left[ \left( \sigma_1^{(p)ad} - \sigma_3^{(p)ad} \right) - \sigma_3^{(p)ad} \right].
\]

Then
\[- \frac{d\sigma_{2}^{(p)}}{ds_{p}} = \frac{Q_{p}}{c^{2}} \left[ \frac{1}{c} \left( p_{1}^{(p)} \right) \left| \begin{array}{cc} \sigma_{1}^{(p)} & \sigma_{2}^{(p)} \\ \xi_{1}^{(pq)} & \xi_{2}^{(pq)} \end{array} \right| - p_{1}^{(p)} \tau_{pq} \left| \begin{array}{cc} \sigma_{1}^{(p)} & \sigma_{2}^{(p)} \\ u_{1}^{(p)} & u_{2}^{(p)} \end{array} \right| - \frac{\xi_{1}^{(pq)}}{c} \left| \begin{array}{cc} \sigma_{1}^{(p)} & \sigma_{2}^{(p)} \\ p_{1}^{(pq)} & p_{2}^{(pq)} \end{array} \right| - \right. \]
\[+ \frac{P_{3}^{(pq)}}{c} \left( - \left| \begin{array}{cc} \sigma_{2}^{(p)} & \sigma_{3}^{(p)} \\ \xi_{2}^{(pq)} & \xi_{3}^{(pq)} \end{array} \right| + \tau_{pq} \right| \begin{array}{cc} \sigma_{2}^{(p)} & \sigma_{3}^{(p)} \\ u_{2}^{(p)} & u_{3}^{(p)} \end{array} \right| \left. \right] + \]
\[+ \frac{Q_{p}}{2c^{2}} \left[ \frac{1}{p_{1}^{(p)\text{ret}}} \left| \begin{array}{cc} \sigma_{1}^{(p)} & \sigma_{2}^{(p)} \\ \xi_{1}^{(pq)\text{ret}} & \xi_{2}^{(pq)\text{ret}} \end{array} \right| - P_{1}^{(p)\text{ret}} \tau_{p,\text{ret}} \left| \begin{array}{cc} \sigma_{1}^{(p)} & \sigma_{2}^{(p)} \\ u_{1}^{(p)} & u_{2}^{(p)} \end{array} \right| - \frac{\xi_{1}^{(pq)\text{ret}}}{c} \left| \begin{array}{cc} \sigma_{1}^{(p)} & \sigma_{2}^{(p)} \\ p_{1}^{(pq)\text{ret}} & p_{2}^{(pq)\text{ret}} \end{array} \right| - \right. \]
\[+ \frac{P_{3}^{(pq)\text{ret}}}{c} \left( - \left| \begin{array}{cc} \sigma_{2}^{(p)} & \sigma_{3}^{(p)} \\ \xi_{2}^{(pq)\text{ret}} & \xi_{3}^{(pq)\text{ret}} \end{array} \right| + P_{3}^{(pq)\text{ret}} \tau_{p,\text{ret}} \right| \begin{array}{cc} \sigma_{2}^{(p)} & \sigma_{3}^{(p)} \\ u_{2}^{(p)} & u_{3}^{(p)} \end{array} \right| \left. \right] + \]
\[+ \frac{Q_{p}}{c^{2}} \left[ \frac{1}{p_{1}^{(p)\text{adv}}} \left| \begin{array}{cc} \sigma_{1}^{(p)} & \sigma_{2}^{(p)} \\ \xi_{1}^{(pq)\text{adv}} & \xi_{2}^{(pq)\text{adv}} \end{array} \right| - P_{1}^{(p)\text{adv}} \tau_{p,\text{adv}} \left| \begin{array}{cc} \sigma_{1}^{(p)} & \sigma_{2}^{(p)} \\ u_{1}^{(p)} & u_{2}^{(p)} \end{array} \right| - \frac{\xi_{1}^{(pq)\text{adv}}}{c} \left| \begin{array}{cc} \sigma_{1}^{(p)} & \sigma_{2}^{(p)} \\ p_{1}^{(pq)\text{adv}} & p_{2}^{(pq)\text{adv}} \end{array} \right| - \right. \]
\[+ \frac{P_{3}^{(pq)\text{adv}}}{c} \left( - \left| \begin{array}{cc} \sigma_{2}^{(p)} & \sigma_{3}^{(p)} \\ \xi_{2}^{(pq)\text{adv}} & \xi_{3}^{(pq)\text{adv}} \end{array} \right| + \right. \]
\[+ P_{3}^{(pq)\text{adv}} \tau_{p,\text{adv}} \left| \begin{array}{cc} \sigma_{2}^{(p)} & \sigma_{3}^{(p)} \\ u_{2}^{(p)} & u_{3}^{(p)} \end{array} \right| \left. \right] - \right. \]
\[\frac{d\sigma_{2}^{(p)}}{ds_{p}} = \frac{Q_{p}}{c^{2}} \left[ \sigma_{k_{3}}^{(p)} \left( F_{k_{2}}^{(p)} + F_{k_{3}}^{(p)\text{rad}} \right) \right. \]
\[\left. - \sigma_{k_{2}}^{(p)} \left( F_{k_{2}}^{(p)} + F_{k_{3}}^{(p)\text{rad}} \right) \right] \]
\[= \frac{Q_{p}}{c^{2}} \left[ \frac{1}{c} \left( p_{2}^{(p)\text{pq}} \right) \sigma_{k_{3}}^{(p)} \xi_{k_{2}}^{(pq)} - \sigma_{k_{3}}^{(p)} p_{k_{2}}^{(p)\text{pq}} \xi_{k_{2}}^{(pq)} - \left( \sigma_{k_{2}}^{(p)} p_{k_{2}}^{(p)\text{pq}} \xi_{k_{2}}^{(pq)} - p_{k_{2}}^{(p)\text{pq}} \sigma_{k_{2}}^{(p)} \xi_{k_{2}}^{(pq)} \right) \right. \]
\[+ \frac{Q_{p}}{2c^{2}} \left[ \frac{1}{c} \left( p_{2}^{(p)\text{ret}} \right) \sigma_{k_{3}}^{(p)} \xi_{k_{2}}^{(pq)\text{ret}} - \sigma_{k_{3}}^{(p)} p_{k_{2}}^{(p)\text{ret}} \xi_{k_{2}}^{(pq)\text{ret}} - \left( \sigma_{k_{2}}^{(p)} p_{k_{2}}^{(p)\text{ret}} \xi_{k_{2}}^{(pq)\text{ret}} - p_{k_{2}}^{(p)\text{ret}} \sigma_{k_{2}}^{(p)} \xi_{k_{2}}^{(pq)\text{ret}} \right) \right. \]
\[+ \frac{Q_{p}}{2c^{2}} \left[ \frac{1}{c} \left( p_{2}^{(p)\text{adv}} \right) \sigma_{k_{3}}^{(p)} \xi_{k_{2}}^{(pq)\text{adv}} - \sigma_{k_{3}}^{(p)} p_{k_{2}}^{(p)\text{adv}} \xi_{k_{2}}^{(pq)\text{adv}} - \left( \sigma_{k_{2}}^{(p)} p_{k_{2}}^{(p)\text{adv}} \xi_{k_{2}}^{(pq)\text{adv}} - p_{k_{2}}^{(p)\text{adv}} \sigma_{k_{2}}^{(p)} \xi_{k_{2}}^{(pq)\text{adv}} \right) \right. \]
\[\left. \right]; \]
\[
\sigma_{k_{3}}^{(p)\text{pq}} = \xi_{k_{2}}^{(pq)\text{pq}} = \sigma_{k_{3}}^{(p)\text{pq}} + \sigma_{k_{3}}^{(p)\text{pq}} + \sigma_{k_{3}}^{(p)\text{pq}} + \sigma_{k_{3}}^{(p)\text{pq}} = - \xi_{k_{1}}^{(pq)\text{pq}} + \xi_{k_{2}}^{(pq)\text{pq}} + \xi_{k_{3}}^{(pq)\text{pq}} + \tau_{pq} \left( \lambda_{k_{1}}^{(p)\text{pq}} - \lambda_{k_{2}}^{(p)\text{pq}} \right) \]
\[= \left| \begin{array}{cc} \sigma_{1}^{(p)} & \sigma_{2}^{(p)} \\ \xi_{1}^{(pq)} & \xi_{2}^{(pq)} \end{array} \right| - \tau_{pq} \left| \begin{array}{cc} \sigma_{1}^{(p)} & \sigma_{2}^{(p)} \\ \lambda_{1}^{(p)} & \lambda_{2}^{(p)} \end{array} \right| + \]
\[
\sigma_{k_{3}}^{(p)\text{pq}} = \sigma_{k_{3}}^{(p)\text{pq}} + \sigma_{k_{3}}^{(p)\text{pq}} + \sigma_{k_{3}}^{(p)\text{pq}} + \sigma_{k_{3}}^{(p)\text{pq}} = - \sigma_{k_{2}}^{(p)\text{pq}} + \sigma_{k_{1}}^{(p)\text{pq}} + \sigma_{k_{2}}^{(p)\text{pq}} - \sigma_{k_{2}}^{(p)\text{pq}} \right| \frac{\alpha_{1}^{(p)\text{pq}} - \alpha_{2}^{(p)\text{pq}}}{c} \left| \right| \]
\[= \left| \begin{array}{cc} \sigma_{1}^{(p)} & \sigma_{2}^{(p)} \\ p_{1}^{(pq)} & p_{2}^{(pq)} \end{array} \right| + \frac{i}{c} \left| \begin{array}{cc} \sigma_{1}^{(p)} & \sigma_{2}^{(p)} \\ p_{1}^{(pq)} & p_{2}^{(pq)} \end{array} \right| + \left| \begin{array}{cc} \sigma_{1}^{(p)} & \sigma_{2}^{(p)} \\ \lambda_{1}^{(p)} & \lambda_{2}^{(p)} \end{array} \right| ; \]
The document discusses the derivation of spin equations in the context of classical electrodynamics with radiation terms. The equations are represented in a mathematical format, including various terms and operators. The text is complex and technical, focusing on the derivation of spin equations. The page contains mathematical expressions and equations, indicating a deep understanding of the subject matter. The document appears to be part of a larger work on classical electrodynamics and its implications for spin dynamics.
\( \sigma_{k4}(p) \xi_{k4}(p) = \xi_{k4}(p) \sigma_{14}(p) + \xi_{k2}(p) \sigma_{24}(p) + \xi_{k3}(p) \sigma_{34}(p) \)
\[
= \left( i/c \right) \left( \lambda_2(p) \sigma_{3}(p) - \lambda_3(p) \sigma_{2}(p) \right) \xi_{k1}(p) + \left( i/c \right) \left( \lambda_3(p) \sigma_{1}(p) - \lambda_4(p) \sigma_{3}(p) \right) \xi_{k2}(p)
\]
\[
+ \left( i/c \right) \left( \lambda_1(p) \sigma_{2}(p) - \lambda_2(p) \sigma_{1}(p) \right) \xi_{k3}(p) = \frac{i}{c} \begin{bmatrix} \xi_{k1}(p) \\ \xi_{k2}(p) \\ \xi_{k3}(p) \end{bmatrix} \begin{bmatrix} \lambda_1(p) \\ \lambda_2(p) \\ \lambda_3(p) \end{bmatrix};
\]
\[
p^{(pq)}_{p_k} \sigma_{k4}(p) = p^{(pq)}_{p_1} \sigma_{14}(p) + p^{(pq)}_{p_2} \sigma_{24}(p) + p^{(pq)}_{p_3} \sigma_{34}(p)
\]
\[
= p^{(pq)}_{p_1} \left( i/c \right) \left( \lambda_2(p) \sigma_{3}(p) - \lambda_3(p) \sigma_{2}(p) \right) + p^{(pq)}_{p_2} \left( i/c \right) \left( \lambda_3(p) \sigma_{1}(p) - \lambda_4(p) \sigma_{3}(p) \right) + p^{(pq)}_{p_3} \left( i/c \right) \left( \lambda_1(p) \sigma_{2}(p) - \lambda_2(p) \sigma_{1}(p) \right) = \frac{i}{c} \begin{bmatrix} p^{(pq)}_{p_1} \\ p^{(pq)}_{p_2} \\ p^{(pq)}_{p_3} \end{bmatrix} \begin{bmatrix} \lambda_1(p) \\ \lambda_2(p) \\ \lambda_3(p) \end{bmatrix};
\]
\[
\sigma_{1k}(p) \xi_{k4}(p) = \sigma_{1k}(p) \xi_{k4}(p) = p^{(pq)}_{p_1} \sigma_{12}(p) + p^{(pq)}_{p_3} \sigma_{14}(p) + p^{(pq)}_{p_4} \sigma_{14}(p)
\]
\[
= \sigma_{3}(p) p^{(pq)}_{p_2} - \sigma_{2}(p) p^{(pq)}_{p_3} (i/c) \left( \lambda_2(p) \sigma_{3}(p) - \lambda_3(p) \sigma_{2}(p) \right) p^{(pq)}_{p_4} = \frac{i}{c} \begin{bmatrix} p^{(pq)}_{p_2} \\ p^{(pq)}_{p_3} \\ p^{(pq)}_{p_4} \end{bmatrix} \begin{bmatrix} \lambda_2(p) \\ \lambda_3(p) \\ \lambda_4(p) \end{bmatrix};
\]
\[
\sigma_{1k}(p) \xi_{k4}(p) = \sigma_{1k}(p) \xi_{k4}(p) = p^{(pq)}_{p_1} \sigma_{12}(p) + p^{(pq)}_{p_3} \sigma_{14}(p) + p^{(pq)}_{p_4} \sigma_{14}(p)
\]
\[
= \sigma_{3}(p) p^{(pq)}_{p_2} - \sigma_{2}(p) p^{(pq)}_{p_3} - \sigma_{1}(p) p^{(pq)}_{p_4} (i/c) \left( \lambda_2(p) \sigma_{3}(p) - \lambda_3(p) \sigma_{2}(p) \right) p^{(pq)}_{p_4} = \frac{i}{c} \begin{bmatrix} p^{(pq)}_{p_2} \\ p^{(pq)}_{p_3} \\ p^{(pq)}_{p_4} \end{bmatrix} \begin{bmatrix} \lambda_2(p) \\ \lambda_3(p) \\ \lambda_4(p) \end{bmatrix};
\]

Therefore
\[
\left( i/c \right) d \left( \lambda_2(p) \sigma_{3}(p) - \lambda_3(p) \sigma_{2}(p) \right) / ds^{(p)} =
\]
\[
- \frac{Q_{p1}}{2c^2} \begin{bmatrix} p^{(pq)}_{p_1} \sigma_{12}(p) + i \frac{Q_{p1}}{c^2} \lambda_1(p) \sigma_{12}(p) + i \frac{Q_{p1}}{c^2} \lambda_2(p) \sigma_{12}(p) + i \frac{Q_{p1}}{c^2} \lambda_3(p) \sigma_{12}(p) \end{bmatrix}
\]
\[
- \frac{Q_{p2}}{2c^2} \begin{bmatrix} p^{(pq)}_{p_2} \sigma_{23}(p) + i \frac{Q_{p2}}{c^2} \lambda_2(p) \sigma_{23}(p) + i \frac{Q_{p2}}{c^2} \lambda_3(p) \sigma_{23}(p) \end{bmatrix}
\]
\[
- \frac{Q_{p3}}{2c^2} \begin{bmatrix} p^{(pq)}_{p_3} \sigma_{34}(p) + i \frac{Q_{p3}}{c^2} \lambda_3(p) \sigma_{34}(p) \end{bmatrix}
\]

Therefore
\[
\left( i/c \right) d \left( \lambda_2(p) \sigma_{3}(p) - \lambda_3(p) \sigma_{2}(p) \right) / ds^{(p)} =
\]
\[
\begin{bmatrix} p^{(pq)}_{p_1} \sigma_{12}(p) + i \frac{Q_{p1}}{c^2} \lambda_1(p) \sigma_{12}(p) + i \frac{Q_{p1}}{c^2} \lambda_2(p) \sigma_{12}(p) + i \frac{Q_{p1}}{c^2} \lambda_3(p) \sigma_{12}(p) \end{bmatrix}
\]
\[
- \frac{Q_{p2}}{2c^2} \begin{bmatrix} p^{(pq)}_{p_2} \sigma_{23}(p) + i \frac{Q_{p2}}{c^2} \lambda_2(p) \sigma_{23}(p) + i \frac{Q_{p2}}{c^2} \lambda_3(p) \sigma_{23}(p) \end{bmatrix}
\]
\[
- \frac{Q_{p3}}{2c^2} \begin{bmatrix} p^{(pq)}_{p_3} \sigma_{34}(p) + i \frac{Q_{p3}}{c^2} \lambda_3(p) \sigma_{34}(p) \end{bmatrix}
\]

Therefore
\[
\left( i/c \right) d \left( \lambda_2(p) \sigma_{3}(p) - \lambda_3(p) \sigma_{2}(p) \right) / ds^{(p)} =
\]
\[
\begin{bmatrix} p^{(pq)}_{p_1} \sigma_{12}(p) + i \frac{Q_{p1}}{c^2} \lambda_1(p) \sigma_{12}(p) + i \frac{Q_{p1}}{c^2} \lambda_2(p) \sigma_{12}(p) + i \frac{Q_{p1}}{c^2} \lambda_3(p) \sigma_{12}(p) \end{bmatrix}
\]
\[
- \frac{Q_{p2}}{2c^2} \begin{bmatrix} p^{(pq)}_{p_2} \sigma_{23}(p) + i \frac{Q_{p2}}{c^2} \lambda_2(p) \sigma_{23}(p) + i \frac{Q_{p2}}{c^2} \lambda_3(p) \sigma_{23}(p) \end{bmatrix}
\]
\[
- \frac{Q_{p3}}{2c^2} \begin{bmatrix} p^{(pq)}_{p_3} \sigma_{34}(p) + i \frac{Q_{p3}}{c^2} \lambda_3(p) \sigma_{34}(p) \end{bmatrix}
\]
Fifth Equation

\[
\frac{ds_{24}^{(p)}}{ds^{(p)}} = \frac{Q_p}{c^2} \left[ \left( F_{2k}^{(p)} + F_{2k}^{(p)\text{rad}} \right) \sigma_{k4}^{(p)} - \sigma_{k4}^{(p)} \left( F_{k4}^{(p)} + F_{k4}^{(p)\text{rad}} \right) \right];
\]

\[
(i/c) d\left( \lambda_3^{(p)} \sigma_1^{(p)} - \lambda_1^{(p)} \sigma_3^{(p)} \right) / ds^{(p)} =
\]

\[
\frac{Q_p}{c^2} \left[ \sigma_{k4}^{(p)} \left( p_{k4}^{(p)} z_4^{(p)} - p_{k4}^{(p)\text{rad}} z_4^{(p)} \right) - \sigma_{k4}^{(p)} \left( p_{k4}^{(p)} z_4^{(p)} + z_4^{(p)} p_{k4}^{(p)\text{rad}} \right) \right]
\]

\[
+ \frac{Q_p}{2c^2} \left[ \sigma_{k4}^{(p)} \left( p_{k4}^{(p)\text{ret}} z_4^{(p)\text{ret}} - p_{k4}^{(p)\text{ret}} z_4^{(p)\text{ret}} \right) - \sigma_{k4}^{(p)} \left( p_{k4}^{(p)\text{ret}} z_4^{(p)\text{ret}} + z_4^{(p)\text{ret}} p_{k4}^{(p)\text{ret}} \right) \right]
\]

\[
- \frac{Q_p}{2c^2} \left[ \sigma_{k4}^{(p)} \left( p_{k4}^{(p)\text{adv}} z_4^{(p)\text{adv}} - p_{k4}^{(p)\text{adv}} z_4^{(p)\text{adv}} \right) - \sigma_{k4}^{(p)} \left( p_{k4}^{(p)\text{adv}} z_4^{(p)\text{adv}} + z_4^{(p)\text{adv}} p_{k4}^{(p)\text{adv}} \right) \right];
\]

\[
\sigma_{k4}^{(p)} = \frac{z_1^{(p)} \sigma_{14}^{(p)} + z_2^{(p)} \sigma_{24}^{(p)} + z_3^{(p)} \sigma_{34}^{(p)} + z_4^{(p)} \sigma_{44}^{(p)}}{c}
\]

\[
\frac{i}{c} \left( \frac{1}{\lambda_1^{(p)}} \sigma_1^{(p)} - \frac{1}{\lambda_2^{(p)}} \sigma_2^{(p)} \right) \frac{z_3^{(p)}}{\lambda_3^{(p)}} = \frac{i}{c} \frac{z_1^{(p)}}{\lambda_1^{(p)}} \frac{z_2^{(p)}}{\lambda_2^{(p)}} \frac{z_3^{(p)}}{\lambda_3^{(p)}} \frac{z_4^{(p)}}{\lambda_4^{(p)}}
\]

\[
\sigma_{k4}^{(p)} = \frac{z_1^{(p)} \sigma_{14}^{(p)} + z_2^{(p)} \sigma_{24}^{(p)} + z_3^{(p)} \sigma_{34}^{(p)} + z_4^{(p)} \sigma_{44}^{(p)}}{c}
\]

\[
\frac{i}{c} \left( \frac{1}{\lambda_1^{(p)}} \sigma_1^{(p)} - \frac{1}{\lambda_2^{(p)}} \sigma_2^{(p)} \right) \frac{z_3^{(p)}}{\lambda_3^{(p)}} = \frac{i}{c} \frac{z_1^{(p)}}{\lambda_1^{(p)}} \frac{z_2^{(p)}}{\lambda_2^{(p)}} \frac{z_3^{(p)}}{\lambda_3^{(p)}} \frac{z_4^{(p)}}{\lambda_4^{(p)}}
\]

\[
\sigma_{k4}^{(p)} = \frac{z_1^{(p)} \sigma_{14}^{(p)} + z_2^{(p)} \sigma_{24}^{(p)} + z_3^{(p)} \sigma_{34}^{(p)} + z_4^{(p)} \sigma_{44}^{(p)}}{c}
\]

\[
\frac{d\sigma_{24}^{(p)}}{ds^{(p)}} = \frac{Q_p}{c^2} \left[ \left( F_{2k}^{(p)} + F_{2k}^{(p)\text{rad}} \right) \sigma_{k4}^{(p)} - \sigma_{k4}^{(p)} \left( F_{k4}^{(p)} + F_{k4}^{(p)\text{rad}} \right) \right];
\]

\[
(i/c) d\left( \lambda_3^{(p)} \sigma_1^{(p)} - \lambda_1^{(p)} \sigma_3^{(p)} \right) / ds^{(p)} =
\]

\[
\frac{Q_p}{c^2} \left[ \sigma_{k4}^{(p)} \left( p_{k4}^{(p)} z_4^{(p)} - p_{k4}^{(p)\text{rad}} z_4^{(p)} \right) - \sigma_{k4}^{(p)} \left( p_{k4}^{(p)} z_4^{(p)} + z_4^{(p)} p_{k4}^{(p)\text{rad}} \right) \right]
\]

\[
+ \frac{Q_p}{2c^2} \left[ \sigma_{k4}^{(p)} \left( p_{k4}^{(p)\text{ret}} z_4^{(p)\text{ret}} - p_{k4}^{(p)\text{ret}} z_4^{(p)\text{ret}} \right) - \sigma_{k4}^{(p)} \left( p_{k4}^{(p)\text{ret}} z_4^{(p)\text{ret}} + z_4^{(p)\text{ret}} p_{k4}^{(p)\text{ret}} \right) \right]
\]

\[
- \frac{Q_p}{2c^2} \left[ \sigma_{k4}^{(p)} \left( p_{k4}^{(p)\text{adv}} z_4^{(p)\text{adv}} - p_{k4}^{(p)\text{adv}} z_4^{(p)\text{adv}} \right) - \sigma_{k4}^{(p)} \left( p_{k4}^{(p)\text{adv}} z_4^{(p)\text{adv}} + z_4^{(p)\text{adv}} p_{k4}^{(p)\text{adv}} \right) \right];
\]

\[
\sigma_{k4}^{(p)} = \frac{z_1^{(p)} \sigma_{14}^{(p)} + z_2^{(p)} \sigma_{24}^{(p)} + z_3^{(p)} \sigma_{34}^{(p)} + z_4^{(p)} \sigma_{44}^{(p)}}{c}
\]

\[
\frac{i}{c} \left( \frac{1}{\lambda_1^{(p)}} \sigma_1^{(p)} - \frac{1}{\lambda_2^{(p)}} \sigma_2^{(p)} \right) \frac{z_3^{(p)}}{\lambda_3^{(p)}} = \frac{i}{c} \frac{z_1^{(p)}}{\lambda_1^{(p)}} \frac{z_2^{(p)}}{\lambda_2^{(p)}} \frac{z_3^{(p)}}{\lambda_3^{(p)}} \frac{z_4^{(p)}}{\lambda_4^{(p)}}
\]
Therefore

\[(i / c) \cdot d \left( \lambda_3^{(p)} \sigma_1^{(p)} - \lambda_4^{(p)} \sigma_3^{(p)} \right) / ds^{(p)} \]

\[= \frac{Q_p}{c^2} \left[ \begin{array}{c}
\lambda_1^{(p)} \\
\lambda_2^{(p)} \\
\lambda_3^{(p)}
\end{array} \right] \begin{bmatrix}
\sigma_1^{(p)} & \sigma_2^{(p)} & \sigma_3^{(p)} \\
\sigma_1^{(p)} & \sigma_2^{(p)} & \sigma_3^{(p)} \\
\sigma_1^{(p)} & \sigma_2^{(p)} & \sigma_3^{(p)}
\end{bmatrix} \left( i / c \right) - \frac{Q_p}{c^2} \left[ \begin{array}{c}
\lambda_1^{(p)} \\
\lambda_2^{(p)} \\
\lambda_3^{(p)}
\end{array} \right] \begin{bmatrix}
\sigma_1^{(p)} & \sigma_2^{(p)} & \sigma_3^{(p)} \\
\sigma_1^{(p)} & \sigma_2^{(p)} & \sigma_3^{(p)} \\
\sigma_1^{(p)} & \sigma_2^{(p)} & \sigma_3^{(p)}
\end{bmatrix} \left( i / c \right)
\]

\[- \left( \begin{array}{c}
\sigma_2^{(p)} \\
\sigma_3^{(p)}
\end{array} \right) \left( \begin{array}{c}
P_1^{(pq)} \\
P_2^{(pq)}
\end{array} \right) \left( \begin{array}{c}
P_3^{(pq)}
\end{array} \right) \left( i / c \right) \begin{bmatrix}
\lambda_1^{(p)} & \lambda_2^{(p)} & \lambda_3^{(p)} \\
\lambda_1^{(p)} & \lambda_2^{(p)} & \lambda_3^{(p)}
\end{bmatrix} \left( i / c \right) - \frac{Q_p}{c^2} \left[ \begin{array}{c}
\sigma_2^{(p)} \\
\sigma_3^{(p)}
\end{array} \right] \left( \begin{array}{c}
P_1^{(pq)} \\
P_2^{(pq)}
\end{array} \right) \left( \begin{array}{c}
P_3^{(pq)}
\end{array} \right) \left( i / c \right)
\]

\[- \left( \begin{array}{c}
\sigma_2^{(p)} \\
\sigma_3^{(p)}
\end{array} \right) \left( \begin{array}{c}
P_1^{(pq)} \\
P_2^{(pq)}
\end{array} \right) \left( \begin{array}{c}
P_3^{(pq)}
\end{array} \right) \left( i / c \right) \begin{bmatrix}
\lambda_1^{(p)} & \lambda_2^{(p)} & \lambda_3^{(p)} \\
\lambda_1^{(p)} & \lambda_2^{(p)} & \lambda_3^{(p)}
\end{bmatrix} \left( i / c \right) - \frac{Q_p}{c^2} \left[ \begin{array}{c}
\sigma_2^{(p)} \\
\sigma_3^{(p)}
\end{array} \right] \left( \begin{array}{c}
P_1^{(pq)} \\
P_2^{(pq)}
\end{array} \right) \left( \begin{array}{c}
P_3^{(pq)}
\end{array} \right) \left( i / c \right)
\]

\[- \frac{Q_p}{2c^2} \left( \begin{array}{c}
P_1^{(pq)} \\
P_2^{(pq)}
\end{array} \right) \left( \begin{array}{c}
P_3^{(pq)}
\end{array} \right) \left( i / c \right) \begin{bmatrix}
\lambda_1^{(p)} & \lambda_2^{(p)} & \lambda_3^{(p)} \\
\lambda_1^{(p)} & \lambda_2^{(p)} & \lambda_3^{(p)}
\end{bmatrix} \left( i / c \right) - \frac{Q_p}{2c^2} \left[ \begin{array}{c}
P_1^{(pq)} \\
P_2^{(pq)}
\end{array} \right] \left( \begin{array}{c}
P_3^{(pq)}
\end{array} \right) \left( i / c \right)
\]

Sixth Equation

\[
\frac{d x_4^{(p)}}{d s^{(p)}} = \frac{Q_p}{c^2} \left( \begin{array}{c}
\left( F_3^{(p)} + F_{3k}^{(p)\text{rad}} \right) \sigma_4^{(p)} - \sigma_3^{(p)} \left( F_3^{(p)} + F_{3k}^{(p)\text{rad}} \right) \\
\left( F_3^{(p)} + F_{3k}^{(p)\text{rad}} \right) \sigma_4^{(p)} - \sigma_3^{(p)} \left( F_3^{(p)} + F_{3k}^{(p)\text{rad}} \right)
\end{array} \right)
\]

\[
= \frac{Q_p}{c^2} \left[ \begin{array}{c}
\sigma_4^{(p)} \\
\sigma_4^{(p)}
\end{array} \right] \begin{bmatrix}
P_1^{(pq)} & P_2^{(pq)} & P_3^{(pq)} \\
P_1^{(pq)} & P_2^{(pq)} & P_3^{(pq)}
\end{bmatrix} \left( i / c \right) - \frac{Q_p}{c^2} \left[ \begin{array}{c}
\sigma_4^{(p)} \\
\sigma_4^{(p)}
\end{array} \right] \begin{bmatrix}
P_1^{(pq)} & P_2^{(pq)} & P_3^{(pq)} \\
P_1^{(pq)} & P_2^{(pq)} & P_3^{(pq)}
\end{bmatrix} \left( i / c \right)
\]

\[- \left( \begin{array}{c}
\sigma_4^{(p)} \\
\sigma_4^{(p)}
\end{array} \right) \left( \begin{array}{c}
P_1^{(pq)} \\
P_2^{(pq)}
\end{array} \right) \left( \begin{array}{c}
P_3^{(pq)}
\end{array} \right) \left( i / c \right) \begin{bmatrix}
P_1^{(pq)} & P_2^{(pq)} & P_3^{(pq)} \\
P_1^{(pq)} & P_2^{(pq)} & P_3^{(pq)}
\end{bmatrix} \left( i / c \right) - \frac{Q_p}{c^2} \left[ \begin{array}{c}
\sigma_4^{(p)} \\
\sigma_4^{(p)}
\end{array} \right] \left( \begin{array}{c}
P_1^{(pq)} \\
P_2^{(pq)}
\end{array} \right) \left( \begin{array}{c}
P_3^{(pq)}
\end{array} \right) \left( i / c \right)
\]

\[- \frac{Q_p}{2c^2} \left( \begin{array}{c}
P_1^{(pq)} \\
P_2^{(pq)}
\end{array} \right) \left( \begin{array}{c}
P_3^{(pq)}
\end{array} \right) \left( i / c \right) \begin{bmatrix}
P_1^{(pq)} & P_2^{(pq)} & P_3^{(pq)} \\
P_1^{(pq)} & P_2^{(pq)} & P_3^{(pq)}
\end{bmatrix} \left( i / c \right) - \frac{Q_p}{2c^2} \left[ \begin{array}{c}
P_1^{(pq)} \\
P_2^{(pq)}
\end{array} \right] \left( \begin{array}{c}
P_3^{(pq)}
\end{array} \right) \left( i / c \right)
\]

\[
\sigma_4^{(p)} = \sigma_3^{(p)} + \sigma_4^{(p)}, \quad \sigma_3^{(p)} + \sigma_4^{(p)} + \sigma_3^{(p)} + \sigma_3^{(p)}
\]

\[
= (i / c)(\lambda_2^{(p)} \sigma_3^{(p)} - \lambda_3^{(p)} \sigma_2^{(p)}), \quad (i / c)(\lambda_3^{(p)} \sigma_1^{(p)} - \lambda_1^{(p)} \sigma_3^{(p)}), \quad (i / c)(\lambda_1^{(p)} \sigma_2^{(p)} - \lambda_2^{(p)} \sigma_1^{(p)})
\]

\[
+ (i / c)(\lambda_1^{(p)} \sigma_2^{(p)} - \lambda_2^{(p)} \sigma_1^{(p)}) = - \frac{i}{c} \left[ \begin{array}{c}
\lambda_1^{(p)} \\
\lambda_2^{(p)} \\
\lambda_3^{(p)}
\end{array} \right] \begin{bmatrix}
\sigma_1^{(p)} & \sigma_2^{(p)} & \sigma_3^{(p)} \\
\sigma_1^{(p)} & \sigma_2^{(p)} & \sigma_3^{(p)} \\
\sigma_1^{(p)} & \sigma_2^{(p)} & \sigma_3^{(p)}
\end{bmatrix} \left( i / c \right)
\]

\[
+ (i / c)(\lambda_1^{(p)} \sigma_2^{(p)} - \lambda_2^{(p)} \sigma_1^{(p)}) = - \frac{i}{c} \left[ \begin{array}{c}
\lambda_1^{(p)} \\
\lambda_2^{(p)} \\
\lambda_3^{(p)}
\end{array} \right] \begin{bmatrix}
\sigma_1^{(p)} & \sigma_2^{(p)} & \sigma_3^{(p)} \\
\sigma_1^{(p)} & \sigma_2^{(p)} & \sigma_3^{(p)} \\
\sigma_1^{(p)} & \sigma_2^{(p)} & \sigma_3^{(p)}
\end{bmatrix} \left( i / c \right)
\]
\[
p_k^{(pq)} \sigma_{k4}^{(p)} = P_1^{(pq)} \sigma_{14}^{(p)} + P_2^{(pq)} \sigma_{24}^{(p)} + P_3^{(pq)} \sigma_{34}^{(p)}
\]
\[
= P_1^{(pq)} \frac{\lambda_2^{(p)} \sigma_3^{(p)} - \lambda_3^{(p)} \sigma_2^{(p)}}{c} + P_2^{(pq)} \frac{\lambda_3^{(p)} \sigma_1^{(p)} - \lambda_1^{(p)} \sigma_3^{(p)}}{c}
\]
\[
+ P_3^{(pq)} \frac{i}{c} \begin{vmatrix}
\sigma_1^{(p)} & \sigma_2^{(p)} & \sigma_3^{(p)} \\
\lambda_1^{(p)} & \lambda_2^{(p)} & \lambda_3^{(p)} \\
\end{vmatrix} = -i \frac{P_1^{(pq)} P_2^{(pq)} P_3^{(pq)}}{c} \begin{vmatrix}
\sigma_1^{(p)} & \sigma_2^{(p)} & \sigma_3^{(p)} \\
\lambda_1^{(p)} & \lambda_2^{(p)} & \lambda_3^{(p)} \\
\end{vmatrix} ;
\]
\[
p_k^{(pq)} \sigma_{3k}^{(p)} = P_1^{(pq)} \sigma_{31}^{(p)} + P_2^{(pq)} \sigma_{32}^{(p)} + P_4^{(pq)} \sigma_{34}^{(p)} = -\begin{vmatrix}
\sigma_1^{(p)} & \sigma_2^{(p)} & \sigma_3^{(p)} \\
\lambda_1^{(p)} & \lambda_2^{(p)} & \lambda_3^{(p)} \\
\end{vmatrix} \begin{vmatrix}
\sigma_1^{(p)} & \sigma_2^{(p)} \\
\lambda_1^{(p)} & \lambda_2^{(p)} \\
\end{vmatrix} \left( -i \frac{P_1^{(pq)} P_2^{(pq)} P_3^{(pq)}}{c} \begin{vmatrix}
\sigma_1^{(p)} & \sigma_2^{(p)} \\
\lambda_1^{(p)} & \lambda_2^{(p)} \\
\end{vmatrix} \right) ;
\]
\[
\sigma_{3k}^{(p)} \xi_{(pq)} = \sigma_{31}^{(p)} \xi_{(pq)} + \sigma_{32}^{(p)} \xi_{(pq)} + \sigma_{34}^{(p)} \xi_{(pq)} = -\sigma_{21}^{(p)} \xi_{(pq)} + \sigma_{21}^{(p)} \xi_{(pq)} - i \frac{\xi_{(pq)}}{c} \begin{vmatrix}
\sigma_1^{(p)} & \sigma_2^{(p)} \\
\lambda_1^{(p)} & \lambda_2^{(p)} \\
\end{vmatrix} \left( -i \frac{P_1^{(pq)} P_2^{(pq)} P_3^{(pq)}}{c} \begin{vmatrix}
\sigma_1^{(p)} & \sigma_2^{(p)} \\
\lambda_1^{(p)} & \lambda_2^{(p)} \\
\end{vmatrix} \right) ;
\]
\[
= - \begin{vmatrix}
\sigma_1^{(p)} & \sigma_2^{(p)} \\
\xi_{(pq)} & \xi_{(pq)} \\
\end{vmatrix} - \frac{i}{c} \frac{\xi_{(pq)}}{c} \begin{vmatrix}
\sigma_1^{(p)} & \sigma_2^{(p)} \\
\xi_{(pq)} & \xi_{(pq)} \\
\end{vmatrix}.
\]

Then
\[
(i / c) d \left( \lambda_1^{(p)} \sigma_1^{(p)} - \lambda_2^{(p)} \sigma_2^{(p)} \right) / ds^{(p)}
\]
\[
= \frac{Q_p}{c^2} \begin{vmatrix}
P_1^{(pq)} & P_2^{(pq)} & P_3^{(pq)} \\
\sigma_1^{(p)} & \sigma_2^{(p)} & \sigma_3^{(p)} \\
\lambda_1^{(p)} & \lambda_2^{(p)} & \lambda_3^{(p)} \\
\end{vmatrix} \left( -i \frac{P_1^{(pq)} P_2^{(pq)} P_3^{(pq)}}{c} \begin{vmatrix}
\sigma_1^{(p)} & \sigma_2^{(p)} \\
\lambda_1^{(p)} & \lambda_2^{(p)} \\
\end{vmatrix} \right) 
\]
\[
\left( - \begin{vmatrix}
\sigma_1^{(p)} & \sigma_2^{(p)} \\
P_1^{(pq)} & P_2^{(pq)} \\
\end{vmatrix} \right) \frac{i}{c} \frac{\xi_{(pq)}}{c} \begin{vmatrix}
\sigma_1^{(p)} & \sigma_2^{(p)} \\
\lambda_1^{(p)} & \lambda_2^{(p)} \\
\end{vmatrix} + \frac{Q_p}{2c^2} \begin{vmatrix}
P_1^{(pq)ret} & P_2^{(pq)ret} & P_3^{(pq)ret} \\
\sigma_1^{(p)ret} & \sigma_2^{(p)ret} & \sigma_3^{(p)ret} \\
\lambda_1^{(p)ret} & \lambda_2^{(p)ret} & \lambda_3^{(p)ret} \\
\end{vmatrix} \left( - \begin{vmatrix}
\sigma_1^{(p)} & \sigma_2^{(p)} \\
P_1^{(pq)ret} & P_2^{(pq)ret} \\
\end{vmatrix} \right) \frac{i}{c} \frac{\xi_{(pq)ret}}{c} \begin{vmatrix}
\sigma_1^{(p)} & \sigma_2^{(p)} \\
\lambda_1^{(p)} & \lambda_2^{(p)} \\
\end{vmatrix} 
\]
\[
\left( - \begin{vmatrix}
\sigma_1^{(p)} & \sigma_2^{(p)} \\
P_1^{(pq)ret} & P_2^{(pq)ret} \\
\end{vmatrix} \right) \frac{i}{c} \frac{\xi_{(pq)ret}}{c} \begin{vmatrix}
\sigma_1^{(p)} & \sigma_2^{(p)} \\
\lambda_1^{(p)} & \lambda_2^{(p)} \\
\end{vmatrix} + \frac{Q_p}{2c^2} \left( - \begin{vmatrix}
\sigma_1^{(p)} & \sigma_2^{(p)} \\
\xi_{(pq)ret} & \xi_{(pq)ret} \\
\end{vmatrix} \right) \frac{i}{c} \frac{\xi_{(pq)ret}}{c} \begin{vmatrix}
\sigma_1^{(p)} & \sigma_2^{(p)} \\
\xi_{(pq)ret} & \xi_{(pq)ret} \\
\end{vmatrix}.
\]
6. Vector Form of the Spin Equations

We present the above equations in vector form, which will facilitate us in further considerations.
If we introduce denotations
\[ \tilde{u}^\uparrow(p) = \left( u_1^\uparrow(p), u_2^\uparrow(p), u_3^\uparrow(p) \right); \]
\[ \tilde{u}^\downarrow(p) = \left( u_1^\downarrow(p), u_2^\downarrow(p), u_3^\downarrow(p) \right); \]
\[ \tilde{p}^\uparrow(q) = \left( p_1^\uparrow(q), p_2^\uparrow(q), p_3^\uparrow(q) \right); \]
\[ \tilde{p}^\downarrow(q) = \left( p_1^\downarrow(q), p_2^\downarrow(q), p_3^\downarrow(q) \right); \]
\[ \tilde{\sigma}(p) = \left( \sigma_1^p, \sigma_2^p, \sigma_3^p \right); \]
then the vector form of the above first three spin equations becomes
\[ d\tilde{\sigma}(p)/ds_p = (Q_p / (2c \ell^2)) \left[ \tilde{p}^\uparrow(p) \times \left( \tilde{\sigma}(p) \times \tilde{\sigma}^\uparrow(p) \right) + \tau_{pq} \left( \tilde{\sigma}(p) \times \tilde{u}^\uparrow(p) \right) \right] \]
\[ -\tilde{\sigma}^\downarrow(p) \times \left[ \left( \tilde{\sigma}(p) \times \tilde{p}^\downarrow(p) \right) + \left( i p_4^\downarrow / c \right) \left( \tilde{\sigma}(p) \times \tilde{u}^\downarrow(p) \right) \right] \]
\[ + (Q_p / 2c \ell^2) \left[ \tilde{p}^\uparrow(p) \times \left( \tilde{\sigma}(p) \times \tilde{\sigma}^\uparrow(p) \right) + \tau_{pq,ret} \left( \tilde{\sigma}(p) \times \tilde{u}^\uparrow(p) \right) \right] \]
\[ - \tilde{\sigma}(p,ret) \times \left[ \left( \tilde{\sigma}(p) \times \tilde{p}^\downarrow(p,ret) \right) + \left( i p_4^\downarrow / c \right) \left( \tilde{\sigma}(p) \times \tilde{u}^\downarrow(p,ret) \right) \right] \]
\[ - (Q_p / 2c \ell^2) \left[ \tilde{p}^\downarrow(p) \times \left( \tilde{\sigma}(p) \times \tilde{\sigma}^\downarrow(p) \right) + \tau_{pq,adv} \left( \tilde{\sigma}(p) \times \tilde{u}^\downarrow(p) \right) \right] \]
\[ - \tilde{\sigma}(p,adv) \times \left[ \left( \tilde{\sigma}(p) \times \tilde{p}^\downarrow(p,adv) \right) + \left( i p_4^\downarrow / c \right) \left( \tilde{\sigma}(p) \times \tilde{u}^\downarrow(p,adv) \right) \right] \]
while the second three become
\[ \frac{i}{c} d\tilde{\sigma}(p)/dv_\ell(p) = \frac{Q_p}{c^2} \left[ \left( \tilde{\sigma}(p) \times \tilde{p}^\uparrow(p) \right) \tilde{p}(p) - \left( i / c \right) \left( \tilde{p}^\uparrow(p) \times \tilde{u}^\uparrow(p,ret) \right) \tilde{\sigma}(p) \right] \]
\[ + \left( \tilde{\sigma}(p) \times \tilde{p}^\downarrow(p) \right) \tilde{p}(p,ret) + \left( i / c \right) \left( \tilde{p}^\downarrow(p) \times \tilde{u}^\downarrow(p,ret) \right) \tilde{\sigma}(p) \]
7. The Last Three Spin Equations are Consequences of the First Three Ones with Respect to Scalar Multiplication

Equations of motion in a vector form are (cf. [5], [6]):

\[
d\vec{\lambda}^{(p)}/ds_p = (Q_p/c^2)e^{\frac{1}{2}E_{pq}}\left[\frac{c^2\tau_{pq}}{\Delta_p}\left(\frac{\vec{\xi}(pq),\vec{\lambda}^{(p)}}{\Delta_p}\right) - \left(\frac{\vec{\lambda}^{(p)},\vec{p}(pq)}{\Delta_p}\right) + c^2M_{pq}/\Delta_p\right]\vec{\xi}(pq)
\]

\[
+(Q_p/c^2)e^{\frac{1}{2}E_{pq}}\left[\frac{c^2\tau_{pq}}{\Delta_p}\left(\frac{\vec{\xi}(pq),\vec{\lambda}^{(p)}}{\Delta_p}\right) - \left(\frac{\vec{\lambda}^{(p)},\vec{p}(pq)}{\Delta_p}\right) + c^2M_{pq}/\Delta_p\right]\vec{\xi}(pq)
\]

\[
-(Q_p/c^2)e^{\frac{1}{2}E_{pq}}\left[\frac{c^2\tau_{pq}}{\Delta_p}\left(\frac{\vec{\xi}(pq),\vec{\lambda}^{(p)}}{\Delta_p}\right) - \left(\frac{\vec{\lambda}^{(p)},\vec{p}(pq)}{\Delta_p}\right) + c^2M_{pq}/\Delta_p\right]\vec{\xi}(pq)
\]

and spin equations are

\[
\frac{d\vec{\sigma}^{(p)}}{ds_p} = \frac{Q_p}{c^2}e^{\frac{1}{2}E_{pq}}\left[\vec{p}(pq)\times\left(\vec{\sigma}(p)\times\vec{\xi}(pq)\right) + \tau_{pq}\left(\vec{\sigma}(p)\times\vec{\lambda}^{(p)}\right) - \vec{\xi}(pq)\times\left[\vec{\sigma}(p)\times\vec{p}(pq)\right] + ip_4\left(\vec{\sigma}(p)\times\vec{\lambda}^{(p)}\right)/c\right]
\]

or

\[
\frac{d\vec{\sigma}^{(p)}}{ds_p} = \frac{Q_p}{c^2}e^{\frac{1}{2}E_{pq}}\left[\vec{p}(pq)\times\left(\vec{\sigma}(p)\times\vec{\xi}(pq)\right) + \tau_{pq}\vec{p}(pq)\times\left(\vec{\sigma}(p)\times\vec{\lambda}^{(p)}\right) - \vec{\xi}(pq)\times\left(\vec{\sigma}(p)\times\vec{p}(pq)\right)/c\right]
\]

In what follows we show that if (1) and (2) possess solution then the last three (4-th, 5-th and 6-th) spin equations, namely...
\[
d\left(\tilde{\lambda}^{(p)}(\cdot) \times \tilde{\sigma}^{(p)}(\cdot)\right)/ds^{(p)} = (Q_p/c^2) \times \left[\left(\tilde{\xi}^{(pq)}(\cdot) \times \tilde{\lambda}^{(p)}(\cdot), \tilde{\sigma}^{(pq)}(\cdot)\right) \bar{\mathbf{p}}^{(pq)}(\cdot) - \left(\tilde{\mathbf{p}}^{(pq)}(\cdot) \times \tilde{\lambda}^{(p)}(\cdot), \tilde{\sigma}^{(p)}(\cdot)\right) \bar{\xi}^{(pq)}(\cdot)\right] \\
+ c^2 \tau_{pq} \left(\tilde{\sigma}^{(p)}(\cdot) \times \bar{\mathbf{p}}^{(pq)}(\cdot)\right) + c^2 M_{pq} \left(\tilde{\sigma}^{(p)}(\cdot) \times \bar{\xi}^{(pq)}(\cdot)\right) \\
+ (Q_p/2c^2) \left[\left(\tilde{\xi}^{(pq)\text{ret}}(\cdot) \times \tilde{\lambda}^{(p)}(\cdot), \tilde{\sigma}^{(pq)\text{ret}}(\cdot)\right) \bar{\mathbf{p}}^{(pq)\text{ret}}(\cdot) - \left(\tilde{\mathbf{p}}^{(pq)\text{ret}}(\cdot) \times \tilde{\lambda}^{(p)}(\cdot), \tilde{\sigma}^{(pq)\text{ret}}(\cdot)\right) \bar{\xi}^{(pq)\text{ret}}(\cdot)\right] \\
+ c^2 \tau^{\text{ret}}_{pq} \left(\tilde{\sigma}^{(p)}(\cdot) \times \bar{\mathbf{p}}^{(pq)\text{ret}}(\cdot)\right) + c^2 M_{\text{p,ret}} \left(\tilde{\sigma}^{(p)}(\cdot) \times \bar{\xi}^{(pq)\text{ret}}(\cdot)\right) \\
- (Q_p/2c^2) \left[\left(\tilde{\xi}^{(pq)\text{adv}}(\cdot) \times \tilde{\lambda}^{(p)}(\cdot), \tilde{\sigma}^{(pq)\text{adv}}(\cdot)\right) \bar{\mathbf{p}}^{(pq)\text{adv}}(\cdot) - \left(\tilde{\mathbf{p}}^{(pq)\text{adv}}(\cdot) \times \tilde{\lambda}^{(p)}(\cdot), \tilde{\sigma}^{(pq)\text{adv}}(\cdot)\right) \bar{\xi}^{(pq)\text{adv}}(\cdot)\right] \\
+ c^2 \tau^{\text{adv}}_{pq} \left(\tilde{\sigma}^{(p)}(\cdot) \times \bar{\mathbf{p}}^{(pq)\text{adv}}(\cdot)\right) + c^2 M_{\text{p,adv}} \left(\tilde{\sigma}^{(p)}(\cdot) \times \bar{\xi}^{(pq)\text{adv}}(\cdot)\right) \right]
\]

(3)

are satisfied.

Indeed, in view of \(\tilde{\lambda}^{(p)} = u^{(p)} / \Delta_p\) we obtain from (1)

\[
d\tilde{\lambda}^{(p)} / ds_p = (Q_p/c^2) \times \left[\left(\tilde{\xi}^{(pq)}(\cdot) \times \tilde{\lambda}^{(p)}(\cdot), \tilde{\sigma}^{(pq)}(\cdot)\right) \bar{\mathbf{p}}^{(pq)}(\cdot) - \left(\tilde{\mathbf{p}}^{(pq)}(\cdot) \times \tilde{\lambda}^{(p)}(\cdot), \tilde{\sigma}^{(pq)}(\cdot)\right) \bar{\xi}^{(pq)}(\cdot)\right] \\
- 2 \left(\tilde{\lambda}^{(p)}(\cdot), \bar{\mathbf{p}}^{(pq)}(\cdot)\right) + c^2 M_{pq} / \Delta_p \tilde{\xi}^{(pq)}(\cdot) + \tilde{\mathbf{p}}^{(pq)\text{ret}}(\cdot) - c^2 \tau^{\text{ret}} / \Delta_p \right]
\]

(4)

The cross product of the last equality by \(\tilde{\sigma}^{(p)}\) from the right is

\[
\left(d\tilde{\lambda}^{(p)} / ds_p\right) \times \tilde{\sigma}^{(p)} = (Q_p/c^2) \times \left[\left(\tilde{\xi}^{(pq)}(\cdot) \times \tilde{\lambda}^{(p)}(\cdot), \tilde{\sigma}^{(pq)}(\cdot)\right) \bar{\mathbf{p}}^{(pq)}(\cdot) \times \tilde{\sigma}^{(p)}(\cdot) - c^2 \tau_{pq} / \Delta_p \right] \bar{\mathbf{p}}^{(pq)}(\cdot) \times \tilde{\sigma}^{(p)}(\cdot)
\]

(5)

The cross product of (2) by \(\tilde{u}^{(p)} / \Delta_p\) from the left is

\[
\tilde{u}^{(p)}(\cdot) \times d\tilde{\sigma}^{(p)} / ds_p = Q_p / c^2 \left[\tilde{\lambda}^{(p)}(\cdot) \times \tilde{\sigma}^{(p)}(\cdot) \times \tilde{\xi}^{(pq)}(\cdot)\right] + \tau_{pq} \left(\tilde{\mathbf{p}}^{(pq)}(\cdot) \times \tilde{\lambda}^{(p)}(\cdot), \tilde{\sigma}^{(pq)}(\cdot)\right) - \tilde{\mathbf{p}}^{(pq)}(\cdot) \times \left(\tilde{\lambda}^{(p)}(\cdot), \tilde{\sigma}^{(pq)}(\cdot)\right) + c^2 M_{pq} \left(\tilde{\lambda}^{(p)}(\cdot) \times \tilde{\sigma}^{(pq)}(\cdot)\right) \\
+ \tau_{pq} \left(\tilde{\mathbf{p}}^{(pq)}(\cdot) \times \tilde{\lambda}^{(p)}(\cdot), \tilde{\sigma}^{(pq)}(\cdot)\right) - \tilde{\mathbf{p}}^{(pq)}(\cdot) \times \left(\tilde{\lambda}^{(p)}(\cdot), \tilde{\sigma}^{(pq)}(\cdot)\right) + c^2 M_{pq} \left(\tilde{\lambda}^{(p)}(\cdot) \times \tilde{\sigma}^{(pq)}(\cdot)\right) \\
- c^2 \tau_{pq} \left(\tilde{\lambda}^{(p)}(\cdot) \times \tilde{\sigma}^{(pq)}(\cdot)\right) - c^2 M_{pq} \left(\tilde{\lambda}^{(p)}(\cdot) \times \tilde{\sigma}^{(pq)}(\cdot)\right) \\
+ \tau_{pq} \left(\tilde{\mathbf{p}}^{(pq)}(\cdot) \times \tilde{\lambda}^{(p)}(\cdot), \tilde{\sigma}^{(pq)}(\cdot)\right) - \tilde{\mathbf{p}}^{(pq)}(\cdot) \times \left(\tilde{\lambda}^{(p)}(\cdot), \tilde{\sigma}^{(pq)}(\cdot)\right) + c^2 M_{pq} \left(\tilde{\lambda}^{(p)}(\cdot) \times \tilde{\sigma}^{(pq)}(\cdot)\right) \\
- \tau_{pq} \left(\tilde{\mathbf{p}}^{(pq)}(\cdot) \times \tilde{\lambda}^{(p)}(\cdot), \tilde{\sigma}^{(pq)}(\cdot)\right) - \tilde{\mathbf{p}}^{(pq)}(\cdot) \times \left(\tilde{\lambda}^{(p)}(\cdot), \tilde{\sigma}^{(pq)}(\cdot)\right) + c^2 M_{pq} \left(\tilde{\lambda}^{(p)}(\cdot) \times \tilde{\sigma}^{(pq)}(\cdot)\right) \\
- \tau_{pq} \left(\tilde{\mathbf{p}}^{(pq)}(\cdot) \times \tilde{\lambda}^{(p)}(\cdot), \tilde{\sigma}^{(pq)}(\cdot)\right) - \tilde{\mathbf{p}}^{(pq)}(\cdot) \times \left(\tilde{\lambda}^{(p)}(\cdot), \tilde{\sigma}^{(pq)}(\cdot)\right) + c^2 M_{pq} \left(\tilde{\lambda}^{(p)}(\cdot) \times \tilde{\sigma}^{(pq)}(\cdot)\right)
\]
We sum up (4) and (5) and obtain

\[
\frac{d\hat{J}(p)}{ds_p} \times \hat{\sigma}(p) + \hat{\alpha}(p) \times \frac{d\hat{\sigma}(p)}{ds_p} = \frac{Q_p}{c^2} \left[ \left( \frac{\hat{z}(pq)}{\hat{\rho}(pq)} - c^2 \tau_{pq}/\Delta_p \right) \hat{p}(pq) \times \hat{\sigma}(p) \right]
\]

\[
-\left( \left\langle \hat{z}(p), \hat{p}(pq) \right\rangle + c^2 \tau_{pq} / \Delta_p \right) \hat{\xi}(pq) \times \hat{\sigma}(p)
\]

\[
+ \frac{Q_p}{c^2} \left[ \left( \left\langle \hat{z}(pq), \hat{\alpha}(p) \right\rangle - c^2 \tau_{pq} / \Delta_p \right) \hat{p}(pq) \times \hat{\sigma}(p) \right] - \left( \left\langle \hat{z}(pq), \hat{\alpha}(p) \right\rangle - c^2 \tau_{pq} / \Delta_p \right) \hat{p}(pq) \times \hat{\sigma}(p)
\]

\[
+ \frac{Q_p}{c^2} \left( \hat{p}(pq) \times \left( \hat{\xi}(pq) \times \hat{\sigma}(p) \right) - \frac{Q_p}{c^2} \left( \hat{p}(pq) \times \left( \hat{\xi}(pq) \times \hat{\sigma}(p) \right) \right) \right) + \frac{Q_p}{c^2} \left( \hat{p}(pq) \times \left( \hat{\xi}(pq) \right) \right) + \frac{Q_p}{c^2} \left( \hat{p}(pq) \times \left( \hat{\xi}(pq) \right) \right)
\]

\[
(6)
\]

We have to check whether the right-hand sides of (3) and (6) coincide after a scalar multiplication by \( \hat{\sigma}(p) \). Indeed, we have

\[
\left\langle \hat{z}(pq) \times \hat{\sigma}(p) \right\rangle - \left\langle \hat{p}(pq) \times \hat{\sigma}(p) \right\rangle \hat{\xi}(pq) + c^2 \tau_{pq} \left( \hat{\xi}(pq) \times \hat{\sigma}(p) \right) + c^2 \tau_{pq} \left( \hat{\xi}(pq) \times \hat{\sigma}(p) \right)
\]

\[
+ \frac{Q_p}{c^2} \left( \hat{p}(pq) \times \left( \hat{\xi}(pq) \times \hat{\sigma}(p) \right) \right) = \left( \left\langle \hat{z}(pq) \times \hat{\sigma}(p) \right\rangle - c^2 \tau_{pq} / \Delta_p \right) \hat{p}(pq) \times \hat{\sigma}(p)
\]

\[
- \left( \left\langle \hat{z}(pq), \hat{\alpha}(p) \right\rangle + c^2 \tau_{pq} / \Delta_p \right) \hat{\xi}(pq) \times \hat{\sigma}(p)
\]

\[
+ \frac{Q_p}{c^2} \left( \left\langle \hat{z}(pq), \hat{\alpha}(p) \right\rangle - c^2 \tau_{pq} / \Delta_p \right) \hat{p}(pq) \times \hat{\sigma}(p)
\]

\[
+ \frac{Q_p}{c^2} \left( \hat{p}(pq) \times \left( \hat{\xi}(pq) \times \hat{\sigma}(p) \right) \right) = \left( \left\langle \hat{z}(pq), \hat{\alpha}(p) \right\rangle - c^2 \tau_{pq} / \Delta_p \right) \hat{p}(pq) \times \hat{\sigma}(p)
\]

By scalar multiplication with \( \hat{\sigma}(p) \) we obtain equality:
8. Conclusions

We have obtained a general system describing the movement of two charged spinning particles consisting of 12 equations for 12 unknown functions, six for the unknown trajectories and six for the spin functions: $(p=1,2)$

$$
\begin{align*}
\frac{d(\hat{p}(pq))}{ds_p} &= (Q_p / c^2) \left[ \hat{p}(pq) \left( \left( \hat{z}(pq), \hat{\lambda}(pq) \right) - c^2 \tau_{pq} / \Delta_p \right) \left( \left( \hat{\lambda}(pq), \hat{p}(pq) \right) + c^2 \mathbf{M}_{pq} / \Delta_p \right) \right] \\
&- (Q_p / 2c^2) \left[ \hat{p}^{p,adv} \left( \left( \hat{z}(pq), \hat{\lambda}(pq) \right) - c^2 \tau_p^{adv} / \Delta_p \right) \left( \left( \hat{\lambda}(pq), \hat{p}^{p,adv} \right) + c^2 \mathbf{M}_{p,adv} / \Delta_p \right) \right]
\end{align*}
$$

and

$$
\begin{align*}
\frac{d(\hat{\sigma}(pq))}{ds_p} &= \frac{Q_p}{c^2} \left[ \hat{p}(pq) \times (\hat{\sigma}(pq) \times \hat{z}(pq)) + \tau_{pq} \hat{p}(pq) \times (\hat{\sigma}(pq) \times \hat{\lambda}(pq)) \\
&- \hat{z}(pq) \times (\hat{\sigma}(pq) \times \hat{p}(pq)) - \left( \hat{p}(pq) / c \right) \hat{z}(pq) \times (\hat{\sigma}(pq) \times \hat{\lambda}(pq)) \right] \\
&+ \frac{Q_p}{2c^2} \left[ \hat{p}^{p,ret} \times (\hat{\sigma}(pq) \times \hat{z}(pq)) + \tau_{p^{ret}} \hat{p}^{p,ret} \times (\hat{\sigma}(pq) \times \hat{\lambda}(pq)) \\
&- \hat{z}(pq) \times (\hat{\sigma}(pq) \times \hat{p}^{p,ret}) - \left( \hat{p}^{p,ret} / c \right) \hat{z}(pq) \times (\hat{\sigma}(pq) \times \hat{\lambda}(pq)) \right] \\
&- \frac{Q_p}{2c^2} \left[ \hat{p}^{p,adv} \times (\hat{\sigma}(pq) \times \hat{z}(pq)) + \tau_{p^{adv}} \hat{p}^{p,adv} \times (\hat{\sigma}(pq) \times \hat{\lambda}(pq)) \\
&- \hat{z}(pq) \times (\hat{\sigma}(pq) \times \hat{p}^{p,adv}) - \left( \hat{p}^{p,adv} / c \right) \hat{z}(pq) \times (\hat{\sigma}(pq) \times \hat{\lambda}(pq)) \right]
\end{align*}
$$

In this way we have obtained a system with 12 equations for 12 unknown functions.

We will prove an existence-uniqueness of a solution for this system in a next paper.

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