Cold Dark Matter from Dark Energy

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Dark energy/matter unification is first demonstrated within the framework of a simplified model. Geodetic evolution of a Λ-dominated bubble Universe, free of genuine matter, is translated into a specific FRW cosmology whose effectively induced dark component highly resembles the cold dark matter ansatz. The realistic extension constitutes a dark soliton which bridges past (radiation and/or matter dominated) and future (Λ-dominated) Einstein regimes; its experimental signature is a moderate redshift dependent cold dark matter deficiency function.

Dark energy/matter unification is the simplest most popular ansatz invoked by physicists, equipped with Einstein equations, when attempting to formulate the apparent clash between contemporary theoretical cosmology and the piling large scale observations. The freedom which characterizes the energy/momentum section of Einstein equations has opened the door for a vast army of non-conventional dark particles (MACHOs are dead, long live the WIMPs) and/or dark equations of state, none of which stands on solid theoretical/experimental grounds. Even the inflationary model, which successfully tackles some basic cosmological riddles and predicts the acoustic peaks in the CMB power spectrum, has not shed too much light on the dark corners. It elegantly explains why the curvature is almost negligible, but leaves us quite ignorant regarding the decomposition of the almost critical total energy density. The cosmological dark puzzle itself trifurcates into (i) The dark energy puzzle: What is the origin of the tiny positive cosmological vacuum energy Λ, and is it really a constant? (ii) The dark matter puzzle: What is the particle content of the Universe, and why is it really a constant? (iii) The dark coincidence puzzle: Is it a mere coin-cidence that a tenable dark companion effectively enters the game when deviating from the Einstein limit, we examine the idea that the apparently independent dark puzzles share a common origin. We refer to such an idea as dark unification. On pedagogical grounds, before diving into the realistic scheme, we first consider a simplified ‘dark matter from dark energy’ model of sufficient reality. To be more specific, we confront the standard matter infested Universe (ΛCDM), nicely fitted by

\[ \rho_{\text{standard}} = \Lambda + \rho_{\text{matter}}, \]  
\[ \rho_{\text{matter}} \sim a^{-3}(t), \]

with a unified bubble Universe, genuine matter free, whose evolution, when translated into the FRW language, is effectively governed by

\[ \rho_{\text{unified}} = \Lambda + \rho_{\text{dark}}, \]
\[ \rho_{\text{dark}}(\Lambda + \rho_{\text{dark}}) \sim a^{-8}(t). \]

Dark unification is manifest at this level by the fact that Λ is the only input parameter, with \( \rho_{\text{dark}}(\Lambda, a) \) being analytically derived. This dark component resembles the standard dark matter ansatz so closely, as demonstrated in Fig.5, that one may find it hard to practically distinguish between the two models. The corresponding Hubble plots, for instance, agree with each other at the 1% level for \( z \leq 10 \). The realistic extension of the simplified model, summarized in Fig.5, is the highlight of this paper. It is characterized by a dark soliton connecting past (radiation and/or matter dominated) and future (Λ-dominated) Einstein regimes, and offers an exclusive experimental signature.

Consider an embedded four dimensional brane, parameterized by \( x^\mu \), floating in some given (that is non-dynamical) \( N \)-dimensional background spanned by \( y^A (A = 0, \ldots, N - 1) \). Let the brane dynamics be described by the conventional Einstein-Hilbert Lagrangian on the brane, but non-conventionally, elevate the embedding vector \( y^A(x^\mu) \) to the level of the canonical gravitational field. This way, the brane metric tensor

\[ g_{\mu\nu}(x) = g_{AB}(y)y_A^\mu y_B^\nu \]

becomes an induced quantity. The field equations combined with the so-called fundamental embedding identity then guarantee automatic energy/momentum conservation, leaving us with

\[ (R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} - T^{\mu\nu}) \left( y^A_{\mu\nu} + \Gamma^A_{BC} y^{B}_{\mu\nu} y^{C}_{\rho\sigma} \right) = 0, \]

which, when using extrinsic curvatures, takes the geometrical oriented form

\[ (R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} - T^{\mu\nu}) K^{\mu\nu}_{BC} = 0. \]

These \( N \) \( - 4 \) equations describe a generalized geodetic motion of a bubble Universe. Clearly, every solution of Einstein equations is necessarily a solution of the geodetic brane equations.
Geodetic brane cosmology, formulated by virtue of 5-dim local isometric embedding in flat $M_5$ (or in an $AdS_5$ background, to be discussed soon), gives rise to a single independent equation of motion. Upon integration, using the energy/momentum conservation law $\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0$ as the integrability condition, we get

$$\rho a^3(\dot{a}^2 + k)^{1/2} - 3a(\dot{a}^2 + k)^{3/2} = \frac{1}{\sqrt{3}}\omega.$$ \hspace{1cm} (4)

The constant of integration $\omega$, identified as the conserved bulk energy conjugate to the cyclic embedding time coordinate $y^0(t)$, serves to parameterize the deviation from the Einstein limit. A physicist equipped with the traditional Einstein formalism, presumably unaware of the underlying brane physics, would naturally re-organize the latter equation into

$$\dot{a}^2 + k = \frac{1}{3}(\rho + \rho_d)a^2,$$ \hspace{1cm} (5)

squeezing all ’anomalous’ pieces into dark $\rho_d$. Our physicist may rightly conclude that the Einstein evolution of the Universe is governed by a total $\rho_T \equiv \rho + \rho_d$, rather than by plain $\rho$. It is remarkable that the implicit formula for the dark component, namely

$$\rho_d^2(\rho + \rho_d) = \frac{\omega^2}{a^8},$$ \hspace{1cm} (6)

guarantees the definite positivity of the total energy density (which cannot vanish off the Einstein limit). We remark in passing that, in an $AdS_5$ background, the above master equation is elegantly generalized into $\rho_d^2(-\frac{1}{2}A_5 + \rho + \rho_d) = \omega^2a^{-8}$. This way, if $|A_5| \gg \rho_T$, the characteristic Randall-Sundrum $\rho_d^{RS} \sim a^{-4}$ piece emerges. Again, as far as this paper is concerned, $A_5 = 0$. The time is ripe now for the practical question: Can a simple input $\rho$, after taking into account the dark effect and setting $k = 0$, reasonably describe the observed Universe? A clue may come from the ’empty’ case

$$\rho = 0 \implies \rho_T = \rho_d(0,a) = \frac{\omega^{2/3}}{a^{8/3}},$$ \hspace{1cm} (7)

telling us that an empty bubble Universe evolves intriguingly similar to a matter dominated FRW Universe. However, a premature model, based on $\rho = 0$ or even on $\rho \sim a^{-3}$, falls short on realistic grounds; it would never generate enough negative pressure ($P_T < -\frac{\omega}{3}\rho_T$, whereas $P_T < -\frac{1}{3}\rho_T$ is needed) to support an accelerating Universe. In other words, we cannot get dark ’energy’ from dark matter. But can we still get dark ’matter’ from dark energy? To find out, let us first examine the simplest case

$$\rho = \Lambda \implies \rho_T = \Lambda + \rho_d(\Lambda, a),$$ \hspace{1cm} (8)

recognized as the geodetic brane distortion of the deSitter model, and focus on its single positive $\rho_d \geq 0$ branch. To present our results, it is crucial to fix a current value for $\Omega_d \equiv \rho_d/\rho_{crit}$. For the sake of definiteness, it is hereby taken to be around $\Omega_d \approx 0.3$. The first item on the list is the deceleration parameter $q = -\ddot{a}/\dot{a}^2$, comfortably given by

$$q = -\frac{3\Omega_d - 2}{2 + \Omega_d}.$$

(9)

For comparison, $\Lambda CDM$ comes with $q = \frac{1}{3}(3\Omega_m - 2)$. Evolving from $\frac{1}{3}$ towards $-1$, unified $q$ passes through 0 at $\Omega_d = \frac{4}{3}$, and is currently located around $\Omega_d \approx 0.5$. To extract the dark equation of state, first derive eq.(6) with respect to the cosmic time to obtain

$$\frac{\dot{\rho_d}}{\rho_d} + 8\frac{\dot{a}}{a} \left(\frac{\rho_d + \Lambda}{3\rho_d + 2\Lambda}\right) = 0,$$

(10)

which can now be interpreted as the dark conservation law $\dot{\rho_d} + 3\frac{\dot{a}}{a}(P_d + \rho_d) = 0$. This, in turn, fixes the dark equation of state, which takes the compact form

$$\gamma_d = \frac{P_d}{\rho_d} = -\frac{q}{3}.$$ \hspace{1cm} (11)

The total equation of state can be easily derived using a similar technique, and one finds

$$\gamma_T = \frac{P_T}{\rho_T} = \frac{-6 - 5\Omega_d}{3(2 + \Omega_d)}.$$ \hspace{1cm} (12)

The comparison with the standard equation of state $P_T = -(1 - \Omega_m)\rho_T$ is done in Fig.1. In a sense, the standard plot can be viewed as a linear approximation to the unified plot. A physicist ignorant of dark unification may naively interpret $\{\Lambda + \rho_d, \Omega_d\}$ as $\{\rho_X + \rho_m, \Omega_m\}$. In which case, the relation $(1 - \Omega_m)\gamma_X = \gamma_T$ opens the door for $\gamma_X$ to take values below or even above $-1$.

![FIG. 1. Unified vs. Standard total equations of state. The standard plot can be viewed as a linear approximation to the curved unified plot.](image-url)
towards its future asymptotic value \( n \to 4 \), currently passing \( n_0 \approx 3.5 \), just above 3; an exact \( n = 3 \) matter-like behavior is locally detected at \( \Omega_d = \frac{2}{3} \). In other words, such a power dependence averagely resembles the standard \( n = 3 \) dark matter ansatz. To sharpen this \( \rho_d \leftrightarrow \rho_m \) similarity and appreciate the integrated effect, we now leave the pedagogical slow varying \( n \) approximation, and would like to compare the dark density ratio \( \rho_d(z)/\rho_{d0} \) with various powers of \((1 + z)\). This is carried out in Fig. 2 and comes with a clear message. It is amazing how fantastically close is the predicted dark ‘matter’ which accompanies our unified model to the standard dark matter ansatz. Note that whereas a small \( \Lambda_5 \) is tolerable (or even welcome), a large \( \Lambda_5 \), due to the enhanced RS dark piece \( \sim a^{-3} \), would spoil the matter-like behavior of \( \rho_d \) at small \( z \).

![FIG. 2. Unified dark ‘matter’ (solid line) can be nicely approximated by the standard \( n = 3 \) dark matter ansatz.](image)

The Hubble plot is perhaps the best tool for directly testing the \( \rho_d \leftrightarrow \rho_m \) similarity. Can it really tell the standard cold dark matter from the unified effectively induced dark imposter? To find out, we have calculated the luminosity distance

\[
d_L(z) = \frac{a_0^3}{a(z)} \int_{a(z)}^{a_0} \frac{\sqrt{3}da}{a^2 \sqrt{\Lambda + \rho_d(1 + z)}} = \frac{(1 + z)^{3/8}}{3H_0(1 - \Omega_d)^{1/4}} \left[ \frac{8x^{5/8}F(x)}{5(x - 1)^{1/4}} - \frac{(x - 1)^{1/4}x^{3/8}}{x^{3/8}} \right]_{x_0}^{x(z)},
\]

(14)

Here, \( F \) stands for the Gauss hypergeometric function \( F(x) \equiv 2F_1\left(1, \frac{11}{8}, \frac{13}{8} \right) \), and \( \xi(z) \geq 1 \) is nothing but a root of the cubic equation

\[
\xi(\xi - 1)^2 = \frac{\Omega^2_{d0}(1 + z)^8}{(1 - \Omega_{d0})^3},
\]

(15)

such that \( \xi_0 = (1 - \Omega_{d0})^{-1} \). The relevant parameter, to measure how close (numerically) are the unified and the standard Hubble plots, is obviously the relative luminosity distance \( d_L^{\text{unified}}/d_L^{\text{standard}} \). One may immediately verify, by plotting this quantity for \( \Omega_{d0} = \Omega_{m0} \approx 0.3 \) (we skip the plot due to length limitation), that the two Hubble plots agree with each other at the 1%-level for \( z \leq 10 \) (above and beyond the supernova data [11]). Another rewarding exercise is to calculate the age of the geodetically evolving \( \Lambda \)-bubble. The corresponding formula being

\[
\tau = \frac{1}{4} \sqrt{\frac{3}{\Lambda}} \left( \sqrt{\Omega_{d0} - \ln \frac{1 - \sqrt{\Omega_{d0}}}{1 + \sqrt{\Omega_{d0}}}} \right). 
\]

(16)

Plotting the unified and the standard ages in units of constant \( \sqrt{3/\Lambda} \) rather than in the conventional \( 1/H \) units, makes it easier to appreciate their numerical similarity for all \( \Omega_{d,m} \). For the sake of clarity, however, translating to conventional units and setting \( \Omega_{d0} \approx 0.7 \), we obtain

\[
\tau_{\text{unified}} \approx \frac{0.97}{H_0}, \quad \tau_{\text{standard}} \approx \frac{0.96}{H_0},
\]

(17)

in a remarkable \( O(1\%) \) agreement, and fully consistent with current data.

The elegance of the geodetically evolving \( \Lambda \)-dominated bubble Universe need not fool us. In many respects, although being good news for the forthcoming realistic model, which must exhibit exactly such a behavior at the small-\( z \) region where baryonic matter and radiation are negligible, it cannot constitute the full picture. Adding the missing ingredients to the game, that is starting from

\[
\rho = \Lambda + Ba^{-3} + Ra^{-4},
\]

the pretentiously realistic dark component is then the positive root of

\[
\rho_d^2 \left( \Lambda + \frac{B}{a^3} + \frac{R}{a^4} + \rho_d \right) = \frac{\omega^2}{a^8}.
\]

(18)

The Einstein limit is approached as \( E(\rho, a) = \frac{\rho^{2/3}}{\rho a^{8/3}} \to 0 \). Near the Einstein limit

\[
\rho_T = \rho \left( 1 + E^{3/2} + \ldots \right).
\]

(19)

The fact that associated with the two pieces which constitute \( \rho \), namely \( \Lambda \) and \( Ba^{-3} + Ra^{-4} \), are powers of \( a(t) \) above and below \( \frac{1}{3} \), respectively, is crucial for our analysis. For \( a \to \infty \), we have \( \rho \simeq \Lambda a^0 \) such that \( E \sim a^{-8/3} \to 0 \), signaling the future Einstein limit. For \( a \to 0 \), on the other hand, we have \( \rho \simeq Ba^{-4} \) leading to \( E \sim a^{4/3} \to 0 \), marking the past Einstein limit (\( E \sim a^{1/3} \to 0 \) for \( R = 0 \)), with \( \rho_d \) serendipitously mimicking a curvature term. The complete cosmological evolution, subject to the present data [11], is depicted in Fig. 3. It describes a bubble Universe in geodetic transition from radiation to \( \Lambda \) domination, and is characterized by a dark soliton which connects these two distinct (past and future) Einstein regimes. The dark effect peaks at

\[
1 + \epsilon_{\text{max}} = \left( 8\Omega_{d0}/\Omega_{B0} \right)^{1/3}.
\]

(20)
from early future (Λ-dominated) Einstein regimes. The dark era extends a considerable amount of ‘matter’ in comparison with standard ΛCDM.

In other words, the Λ relative amount of ‘matter’ in comparison with standard wisdom. In particular, (i) The amount of such dark Universe evolution by means of the traditional Einstein effective entrance when attempting to formulate bubble are added to the game. Artifact dark ‘matter’ makes its alistic level, some genuine baryonic matter and radiation the choice of the canonical gravitational fields. At the rational action, but conceptually differ from each other by models are associated with the one and the same gravitational field. At the reduced level, the sole input of our geodetic gravity is spelled out as dark unification. At a fundamental underlying physical principle. In this re-

Special attention should be devoted to the combination ρB + ρd, the unified analog of the standard ρm. At any given a(t), the ratio η ≡ (ρB + ρd)/ρm measures the relative amount of ‘matter’ in comparison with standard ΛCDM-like FRW evolution is governed by Λ + η(a)a^{-3}. Normalized to unity today, the moderate η-function (plotted in Fig.4) tends to ρB0/ρm0 at very early (and very late) times. Such a detailed deficiency function of matter density appears to be the main testable prediction of dark unification.

FIG. 3. Dark soliton connecting past (R-dominated) and future (Λ-dominated) Einstein regimes. The dark era extends from early z ≃ 600 to very recent z ≃ 0.3, peaking at z ≃ 4.2

To summarize, the name of our game is simplicity. Our idea is to trade the standard cold dark matter ansatz by a fundamental underlying physical principle. In this respect, simplicity is spelled out as dark unification. At the simplified level, the sole input of our geodetic gravity model is the cosmological constant Λ, and nothing but Λ. The corresponding Λ-bubble Universe model is then the exact analog of deSitter model of Einstein gravity; both models are associated with the one and the same gravitational action, but conceptually differ from each other by the choice of the canonical gravitational fields. At the realistic level, some genuine baryonic matter and radiation are added to the game. Artifact dark ‘matter’ makes its effective entrance when attempting to formulate bubble Universe evolution by means of the traditional Einstein equations. In particular, (i) The amount of such dark ‘matter’ is controlled by the conserved bulk energy which parametrizes the deviation from the Einstein limit, (ii) The functional behavior of the emerging dark ‘matter’ soliton is fully dictated by the theory, (iii) The numerics involved highly resemble the standard cold dark matter ansatz, and (iv) The dark era extends from z ≃ 600 until today. Dark unification does leave, however, an exclusive experimental fingerprint in the shape of a moderate z-dependent matter deficiency function; its consequences are currently under investigation.

FIG. 4. The matter deficiency function measures the relative amount of ‘matter’ in comparison with standard ΛCDM.

Clearly, reflecting its underlying first principles, the universality of dark unification must be absolute. Thus, our major challenge is a galactic scale realization of the cosmological dark ‘matter’ idea, presumably but not necessarily in the form of some dark soliton bridging two Einstein regimes. The main theoretical obstacle at the moment is the exact radially symmetric (and time dependent) geodetic brane analog of the Schwarzschild or Schwarzschild-deSitter solution, which is still unknown. An important progress in this direction, namely the recovery of the Newtonian limit in an ‘empty’ (in the sense of eq.7) dark cosmological background, can already be reported.

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