Towards Informative Statistical Flow Inversion

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ABSTRACT
A problem which has recently attracted research attention is that of estimating the distribution of flow sizes in internet traffic. On high traffic links it is sometimes impossible to record every packet. Researchers have approached the problem of estimating flow lengths from sampled packet data in two separate ways. Firstly, different sampling methodologies can be tried to more accurately measure the desired system parameters. One such method is the sample-and-hold method where, if a packet is sampled, all subsequent packets in that flow are sampled. Secondly, statistical methods can be used to “invert” the sampled data and produce an estimate of flow lengths from a sample.

In this paper we propose, implement and test two variants on the sample-and-hold method. In addition we show how the sample-and-hold method can be inverted to get an estimation of the genuine distribution of flow sizes. Experiments are carried out on real network traces to compare standard packet sampling with three variants of sample-and-hold. The methods are compared for their ability to reconstruct the genuine distribution of flow sizes in the traffic.

Categories and Subject Descriptors
G.3 [Probability and Statistics]: Distribution functions; C.2.3 [Computer-Communication Networks]: Network Operations—Network monitoring

General Terms
Statistical Inversion, Measurement

Keywords
Sampling, Inference, Inversion

1. INTRODUCTION
Routers at the core of the internet deal with millions of packets per second on multiple interfaces. From a network operations perspective, it is vital for the administrators to be aware of the volume and types of the packets that are traversing their networks. In order to achieve this objective, routers are required to collect management information but it is impossible to keep a record of all the packets. Thus, given the vast amount of information that needs to be collected, routers sample the traffic stream. This means that only a subset of the packets traversing any interface of the router are processed. Today, the most commonly implemented technique is packet sampling, where 1 packet out of every N is chosen on a random or periodic basis, and integrated into a flow record in the router memory.

In many practical cases, 1/N packet sampling is followed by multiplication of the recovered statistics by N (N-multiplication). This simple technique can be used to recover a number of packet level statistics of interest. For example, the number of SYN packets, TCP packets, ICMP packets or packets to or from given destinations in the original trace can be estimated by this process. However, the distribution of flow lengths cannot be recovered by this procedure (see Section 1.3).

The problem at the heart of this paper is that of recovering the distribution of flow lengths from sampled data. The flow inversion problem amounts to mathematically compensating for the effects of sampling in order to estimate the distribution of flow lengths which would have been observed in the original data. There has been great research activity around the flow distribution inversion [1, 2, 3] and this is discussed in Section 1.3. Based on the previous studies on analysis of the NetFlow performance [4], it is evident that N-multiplication upsets the flow level statistics of the original data stream [5].

1.1 Outline
This paper focuses on considering sampling methods and their use to estimate the flow length distribution in real traffic. Packet sampling has many useful statistical properties, but it is a hard problem to recover the flow distribution from packet-sampled data. This is sometimes called the flow inversion problem. In this paper we investigate three techniques based on the idea of the sample-and-hold method [6]. This method was originally conceived to track the largest flows in the traffic (with applications related to billing) [7].

Section 1.3 gives some basic information about packet
sampling as applied in real situations. Section 1.3 discusses other work on the problem of inferring flow distributions from sampled data. In Section 2 we describe the sampling methods we use and the inversion procedure used to recover the original flow distribution. There is also a brief overview on the router memory and resource requirements. In Section 3 we have applied our proposed algorithms on packet traces from a backbone network and have looked at the performance of our algorithm. In Section 4 we have summarized our results and discuss potential avenues for future work.

1.2 Definitions

The traffic on the Internet is carried in form of Internet Protocol (IP) packets and transmitted to the destination on a hop-by-hop basis by Internet routers. In order to keep account of the packets belonging to the same application, the concept of a flow is defined by router manufacturers. A flow is usually defined as a group of packets that have the same 5-tuple (IP protocol, source address, source port, destination address, destination port).

Usually, core Internet routers carry a large number of flows at any given time. This pressure on the router is controlled by using strict rules to remove from router memory (export) the statistics, and thus keep the router memory buffer and CPU resources available to deal with changes in traffic patterns by avoiding the handling of excessively large tables of flow records. Cisco NetFlow, the dominant standard on today’s routers, uses the following criteria for expiring flows in the cache entries:

1. Flows which have been idle for a specified time are expired and removed from the cache (15 seconds is default).
2. Long lived flows are expired and removed from the cache (30 minutes is default).
3. As the cache becomes full a number of heuristics are applied to aggressively age groups of flows simultaneously.
4. TCP connections which have reached the end of byte stream (FIN) or which have been reset (RST) will be expired.

As will be seen in Section 2.2, the selection of these parameters can greatly affect the nature of the sampled traffic.

After the flow records are terminated, they are grouped together and exported to an external aggregation point through a UDP (User datagram Protocol) stream. The collection of these NetFlow records enables system administrators to have a view of general trends in spatial traffic distribution, network host behavior, traffic matrix estimation, anomaly detection and other relevant measurements. However, the effectiveness of these applications is contingent upon the quality of the flow level statics recovered from the actual network measurements.

The flow distribution is the distribution of flow lengths in a given traffic trace. The lengths are usually expressed in packets but sometimes in bytes. This can be thought of as the probability that a given flow has a particular length. That is, the distribution is \( \theta = P \text{ flow is of length } i \).

1.3 Packet sampling

In an analysis by Cisco [11] one NetFlow-enabled access router used up to 68% of its CPU on processing flow records when an average of 65,000 flows was kept in memory. When sampling was used, this utilization was decreased by more than 82%. There are three constraints on a core router which lead to the use packet sampling: the size of the record buffer, the CPU speed and the flow record look-up time. In packet sampling, in order to relax the pressure on the router while collecting measurements, 1 in \( N \) packets are chosen, and the rest are discarded. Sometimes this is done in a periodic way with every \( N \)th packet sampled. However, in the literature, independent and identically distributed (iid) sampling with a fixed probability \( p \) is often considered. The differences between periodic sampling and iid sampling can be important. Roughan [12] has shown iid sampling is useful in active probing and the concepts are also applicable to the case of passive measurement.

There are many advantages to iid packet sampling and it preserves many important characteristics of the traffic. However, this sampling does not preserve the flow length distribution. The reason for this should be clear but an example is illustrative. Imagine a situation where the flow distribution is such that half the flows in the original trace are of length two and half are of length one \( (\theta_1 = 0.5 \text{ and } \theta_2 = 0.5) \). Imagine these packets are sampled in an iid manner with \( p = 0.5 \). Half of the flows of length one will be sampled but only one quarter of the flows of length two will have both packets sampled. Another half of the flows of length two will have just one packet sampled and a final quarter will have no packets sampled. In the final sample the flow distribution will be \( (\theta_1 = 0.8 \text{ and } \theta_2 = 0.2) \). The problem of flow inversion is, therefore, defined as the problem of recovering the original distribution \( (\theta_i) \) from the sampled traffic.

The choice of sampling strategy will have a large impact on the quality of the data obtained from the network. This is why, to an extent, thereason why the usability of NetFlow sampled data has been questioned
by researchers [13]. The problems with packet sampling are twofold: stem from the following effects it has on sampled flows:

1. It is easy to miss short flows altogether. This is due to the fact that many flows are only a few packets long, and they may be temporally correlated. Thus, these constituent packets may cluster together and totally evade the sampling process.

2. It can be difficult to estimate the length of long flows. The major problem is that, for each flow, only a small subset of packets are seen with a given probability $p$ equal to the sampling rate. Thus, it is not clear how many packets actually were present, out of which a given number $X_i$ were been seen.

3. Flows may be mis-ranked. This means that, even though flow $A$ may seem to be larger than flow $B$ in the sampled statistics, this is not necessarily the case in reality [14].

4. Large flows may be split into smaller ones (creating sparse flows) [2]. This is due to the fact that some long flows have a bursty nature, and thus may include long periods of inactivity. During these periods, they might be mistakenly expired, and any new packets belonging to the same flow are mistakenly classified as part of a new flow.

For applications such as billing and monitoring, the naive inversion method of division of the final statistics by the sampling rate, or basically multiplying the final data by $(1/p)$ will simply lead to inaccurate results as pointed in [5].

On the other hand, the most important problem of the current NetFlow implementation of packet sampling is the fact that many flows are not sampled at all, as none of their packets are selected for sampling. In many cases, particularly in short-lived flows (like web and email applications, where a group of packets are sent together to reply a query), only one packet of the flow is captured. This results in NetFlow reports being dominated by single packet flows.

1.4 Practical implementation of sampling techniques

Even though sampling sampling techniques are used in order to simplify the processing of data collected at a router, in practice it is a complicated process in itself. In a simple implementation of packet sampling in a router, there are various points to be considered. A core or large access Internet router must constantly accommodate memory and processor resource constraints. Even though it is desirable to keep a large number of records in the flow cache, the fast growth of this number makes flow look-up and update a challenge. Thus, when sampling is implemented, the operator has to decide on a few parameters:

1. Sampling rate: The sampling rate has a direct impact on the quantity and the quality of the information formed from the data.

2. Flow time-out: The length of the time-out can have an impact on intermittent traffic flows, such as peer-to-peer file sharing or Instant Messaging, where the flows may not by transmitting packets at full rate the whole time.

3. Flow expiry: If many large flows are active, the flow cache of the router becomes progressively full, leaving no space for new flows. In order to avoid this, a value must be chosen for the expiration (timeout) of the flows.

4. Flow export frequency: If this is done too frequently, it increases the processing load on the router. However, if it is not done often enough, the loss of the UDP export packets in the path can effect the quality of the gathered statistics.

5. Flow cache size: The number of flows which are kept at the router plays a critical role on its performance. If this number is too large, flow look-up time becomes a difficult. On the other hand, if it is too small, many flows are bound to be dropped and frequent expiry and time-out of flows will be needed.

It can be observed that optimally setting all these values can be a challenging task for an operator. Changes in any of the above parameters can effect the length and number of the flows which are reported by a router. In Section 2.2 we look at some of the effects of the mentioned parameters.

1.5 Related work

Hohn and Veitch [3] considered in some depth the problem of producing an estimate of flow distribution from sampled packets. They first looked at methods for “inversion” to recreate the original flow distribution from the sampled packet data. They use two schemes to recreate the flow distribution from the packet sampled data, the first based upon a binomial sum and the second upon a Cauchy integral. These schemes can successfully recover the flow distribution for short length flows if the sampling rate $p$ is relatively high (for example, more than half the packets sampled is ideal). This is not a flaw in the methods described, but a fundamental limitation in the amount of information which can be retrieved from packets sampled in this manner.

Following this, their paper proposes a flow sampling model which can be used in an offline analysis of flow
records formed from an unsampled packet stream. In this method, all the packets are recorded and formed into flows. Then, a subset of these recreated flows are sampled using iid sampling with a given probability $p$. This sampling method proves extremely successful at recreating the flow distribution even when the sampling ratio $p$ is relatively small (say $p = 0.001$). However, the intensive computing and memory requirements makes the implementation of such a scheme on high speed routers a challenge.

Duffield et al [2] have looked at recovering the flow length distributions from a sampled packet trace. A scaling based, Maximum Likelihood Estimation method is proposed and, due to its complexity, an iterative Expectation Maximization algorithm is tested on the available trace files. The biggest issue encountered by the authors is the complexity of the process and the adjustment of the lowest order weights to reflect the underlying distributions. It is re-established by the authors that the estimation of flow level statistics from packet sampled data remains an open question.

In a subsequent paper, [15] the authors introduce threshold sampling as a sampling scheme that optimally controls the expected volume of samples and the variance of estimators over any classification of flows. The proposed scheme has packet capturing performed at routers, followed by flow formation and export and staging at a mediation station, and aggregation of records at a measurement collector.

Ribeiro et al [1] use several methods to estimate the flow distribution from sampled packets. They make use of several features of the TCP protocol, including the SYN flag, and the fact that sequence numbers can give information about the number of bytes between sampled packets. Their work uses maximum likelihood estimators to fit a the distribution of flow lengths up to some maximum flow length (maximum flow lengths of twenty and one hundred are used in the paper). The sequence numbers in particular prove helpful in extracting information about these short and mid-length flows. In addition they use a technique based on the Cramér–Rao bound to investigate the best possible (lowest variance) performance of unbiased flow distribution estimators given assumptions about the information available.

Estan and Varghese [6] propose two algorithms for identifying the large flows: sample and hold and multistage filters, which take a constant number of memory references per packet and use a small amount of memory. If $M$ is the available memory, the errors of the algorithms are proportional to $1/M$; by contrast, the error of an algorithm based on classical sampling is proportional to $1/\sqrt{M}$, thus providing much less accuracy for the same amount of memory. This scheme is intended for billing schemes where large flows are of higher interest to the operator.

Estan et al [10] have proposed an improvement to NetFlow by adapting the sampling rate, enabling the router to keep a pre-determined number of flows in the cache. As a result of change in sampling rate, at each stage a normalization step is performed which ignores the packets that would not have been sampled if the lower sampling rates had been chosen. This scheme produces more concise but less accurate reports, due to reduction in the collected information. The constant change in sampling rate and renormalization stage can be an exploitable threat to the router, allowing the degradation of its performance performance under some attack scenarios.

Barakat et al [14] study the possibility of detection and ranking of the largest flows on a link. A comparison is made between the blind ranking method and study how to detect and rank the largest flows on a link. The results indicate that at sampling rates of higher than 1 in a 100 it is difficult to identify the top flow with both methods.

Although not strictly relevant to the inversion problem it is worth noting that there is considerable research interest in the distribution of flow lengths in internet traffic. This is because the flow lengths are generally held to be heavy-tailed [17], that is they follow a distribution such that

$$P[\text{Length of flow} > x] \sim x^{-\alpha},$$

where $\alpha \in (0, 2)$ and $\sim$ means asymptotically proportional to as $x \rightarrow \infty$. This means that it is not sufficient simply to look at the flows under a given length, extremely long flows will also play an important part in the make-up of the traffic.

2. METHODOLOGY

Our results are based on a 30 minute long trace from an OC-48 link on the CAIDA [18] network on 24th April 2003. The trace contains 47,047,240 packets from which an average 83% are TCP, 7% are UDP. The rest are usually other network layer protocols, such as ICMP.

The sampling strategies used in this paper are referred to as

1. packet sampling,
2. sample-and-hold (by byte),
3. sample-and-hold (by packet) and
4. sample-and-hold (by SYN).

Sample-and-hold (by byte) is the original sample-and-hold technique developed in [6]. Packet sampling as has been previously described, is the commonly used technique of sampling each packet in an independent manner with a given probability $p$. This can be contrasted with techniques which are also commonly used whereby
for a given \( n \), every \( n \)th packet is sampled. The three sample-and-hold techniques are described in the next section.

### 2.1 Sample-and-hold techniques

Sample-and-hold (by byte) is a sampling technique developed in Estan–Varghese [6]. In this technique the router keeps track of certain flows and samples every packet on these flows until their expiry. The technique was developed with the aim of producing a sampling method in which flows that carry greater traffic volume (sometimes called “elephant” flows) are more likely to be sampled than smaller flows. Once a flow is expired, due to one of the reasons discussed in Section 1.2, it is marked for export and kept in the router cache, until a relatively large set of flows is ready for export to an external aggregation point. The process proceeds, in a packet by packet basis, as follows. When a packet is seen which is part of a flow being tracked, that packet is sampled. If the packet is part of a flow which is not being tracked, then there is a probability that this packet will be sampled and the flow will be added to the list of flows being tracked. Let \( b \) be the length of the packet being considered in bytes. Once \( p \) is a constant in \((0, 1)\). The probability of starting to sample this flow at the packet under consideration is \( p^b_\text{p} = 1 - (1 - p)^b \). This is equivalent to considering sampling every byte with probability \( p \).

Sample-and-hold (by packet) is an obvious variant of this technique where the probability of beginning to sample a flow at a given packet is a constant \( p \). This is equivalent to the technique from Estan–Varghese but with the probability fixed rather than depending on the length in bytes of the packet.

Sample-and-hold (by SYN) is another sample-and-hold variant based on the Transmission Control Protocol (TCP). A valid TCP flow is expected to begin with the SYN flag set exactly one packet with the SYN flag set. If a packet col (TCP). A valid TCP flow is expected to begin with the probability fixed rather than depending on the length in bytes of the packet.

The idea is that this SYN based sampling is as close as possible to a version of the flow-based sampling suggested by Holm–Veitch [3] which can be implemented without recording every packet and producing flows from them before sampling. Of course, in any given traffic trace, some TCP flows will have their SYN flag before the trace collection started. Other flows may have more than one SYN flag. This was observed previously by Duffield et al [2]. In their packet traces, they determined the proportion of those TCP flows containing at least one SYN packet that contained exactly one SYN packet. For one data set it was 98.8%; for another one it was 94.6%.

In the CAIDA data investigated here, 7% of flows which contained one SYN packet contained at least one other.

It should be noted that not all traffic in the traces analyzed is TCP traffic, and the Sample-and-hold (by SYN) method can only produce an estimate of the distribution of TCP flows. However, we have examined our algorithm on TCP since more than 90% of the traffic in our trace is TCP.

### 2.2 Flow termination dilemma

Memory constraints prevent routers from keeping flows active for long spans of time. The flow lifetime in the router cache is configurable by the user. If sampling is not used, it is impossible to keep the flows in the buffer for more than a few minutes on a heavily utilized router. For example, authors in [10] use a flow expiry timeout of 2 seconds, which they find to be the maximum before flow loss rates reach unacceptable levels. Figures 1 and 2 show the effects of the buffer size on the accuracy of the flow reports. It can be seen that the longer expiry times consistently help pick out more long flows. Additionally, it can be seen that the distribution obtained using a longer expiry time is more consistent with the straight line graph expected of a heavy-tailed flow distribution, as discussed in Section 1.5.

![Figure 1: Changes in the flow buffer can slice up or join flows.](image)

If the flow buffer memory in the router is chosen to be that of length \( t \), then each section of flow \( F_2 \) will be reported as a separate flow, and flow \( F_1 \) will also be sliced up into smaller flows, creating so-called sparse flows. However if the router can afford to have a longer time out for the flows (\( T \) in Figure 1), even if the smaller flows of \( F_2 \) are in reality individual, unrelated flows (though very unlikely due to the vast number of source ports available for TCP packets at least), they are reported as a single cumulative flow. Flow \( F_1 \) will be correctly reported as a complete flow.

Figure 3 displays the complimentary cumulative distribution function (CCDF) of flow lengths on the CAIDA data for two different flow expiry lengths. This shows clearly that, as would be expected, a shorter expiry time reduces the probability that the longer flows can be seen.
2.3 Flow construction from trace files

To build flow records out of trace files, we emulated the operation of NetFlow on a general purpose computer, relaxing the real-time memory requirements usually imposed on routers. Thus, we were able to greatly extend the amount of time that detailed flow records are kept in memory, and thus construct a baseline of unsampled measurements with which the results of our inversion procedures could be tested. However, the sheer amount of packets in some high-speed Internet core traces means that we cannot process all of it in one go. To address this, we divide time into analysis windows, over which flows are considered independently. For the 30 minute CAIDA datasets, however, we only used one analysis window per trace.

The algorithm we used for the sample-and-hold techniques of Section 1.4 is detailed in Algorithm 2.1 with the variables explained in Table 2.3. The algorithm for packet sampling is similar but simpler, since it does not need to track flows.

Basically, a trace file is explored and each of its packets considered in turn. If the packet belongs to a flow that had been previously selected for sampling, its information is aggregated into the current flow tables. If it is not, its flow is sampled with a probability \( p \). This probability is calculated on the basis of the sampling technique: in the case of sample-and-hold (by packet), the same \( p \) is used for all packets, while in sample-and-hold (by byte) the probability of sampling a packet is a function of the packet length. If a given packet is selected, its 5-tuple \( \phi \) is tracked, so that new packets with this same \( \phi \) and within the flow expiry timeout \( t_t \) are considered part of the same flow.

There are two conditions that trigger complete flow buffer exports in Algorithm 2.1. The first one, implemented by the boolean function \( \text{FlowBufferFull}() \), represents the flow buffer reaching its maximum capacity. The other one, corresponding to \( \text{FlowExportTimerExpired}() \), represents the expiry of a flow analysis window, that is, a periodic event over which the flow collection process is restarted. When a buffer export occurs (independently of its triggering condition) then the system stops tracking all flows and writes the flow statistics to disk. This means that, for every \( \psi \in \Psi \), the 3-tuple \( (\psi, T_p(\psi), T_b(\psi)) \) is written to a file and the relevant data structures in program memory are cleared.

It is informative to consider the difference between the 5-tuple \( \phi \) and the flow identifier \( \psi \) in Algorithm 2.1. If the buffer export timeout \( t_w \) is significantly longer than \( t_t \), it may be possible to encounter two (or more) different flows on the same 5-tuple \( \phi \) during the same analysis window (because the flow has been timed out

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1 As Algorithm 2.1 was implemented for emulation, the size of the flow buffer is not a hard parameter, but can be modified as appropriate.
and exported then seen again). Thus, \( \psi \) appends a discriminating string to \( \phi \), so that the identity of both flows is maintained. As a result of this procedure, after a flow expires its statistics are no longer updated, and it will be analyzed as a separate entity from other flows on the same \( \phi \). Of course, by choosing \( t_t > t_w \), individual flow expiration can be completely bypassed, and by setting \( t_w \) larger than the time spanned by the trace being analyzed, flow analysis window exports can be completely bypassed as well. This gives Algorithm 2.1 full flexibility to explore the influence of \( t_t \), \( t_w \) and \( N_f \) on the sampled flow statistics and our proposed inversion techniques.

**Algorithm 2.1: BuildFlows(trace)**

```plaintext
while packetsLeft(trace)

  \( P \leftarrow \text{ReadPacket}(trace) \)
  \( (\phi, t, N_b) \leftarrow \text{DecodePacket}(P) \)

  if flowsBeingTracked(\( \phi \))
    comment: Has the flow expired?
    if \( t_s(\phi) > t_i \)
      \( \psi \leftarrow \text{GetFlowID}(\phi) \)
      \( \text{TerminateFlow}(\psi) \)
      \( \psi \leftarrow \text{CreateFlow}(\phi) \)
    \( t_s(\phi) \leftarrow t \)
    \( T_p(\psi) \leftarrow T_p(\psi) + 1 \)
    \( T_b(\psi) \leftarrow T_b(\psi) + N_b \)
  do

  else
    comment: Is the flow going to be sampled?
    if flowSelectedForSampling(\( p, N_b \))
      \( t_s(\phi) \leftarrow t \)
      \( \psi \leftarrow \text{CreateFlow}(\phi) \)
      \( T_p(\psi) \leftarrow 1 \)
      \( T_b(\psi) \leftarrow N_b \)

    if flowBufferFull(|\( \Psi \)|, \( N_f \))
      or flowExportTimerExpired(\( t, t_w \))
        \{ exportFlowBuffer() \}
        \{ resetFlowBuffer() \}
```

For the rest of this paper we use a relatively large timeout \( t_t \) of five minutes for the flows. Even though this may be longer than the value usually applied in routers (of around fifteen to thirty seconds) it helps avoid unnecessary flow splitting. In this paper we use \( t_t = t_w = 5 \) minutes, so that all flow information is reset every 5 minutes. After this is done, a secon post-processing step is done where the output of this process is integrated as a single 30 minutes long file. This is done to reduce memory consumption while avoiding dropping flows due to lack of buffer space.

### 2.4 Inverting the sampled data

Two methods for inverting packet sampled data are given by Hohn and Veitch [3]. These techniques, while mathematically sound, are problematic in realistic cases. In particular, they are numerically unstable when estimating longer flows or rates of sampling with small values of \( p \) (where \( p \) is significantly less than 1/2). No results are presented here for inverting packet sampling but for an excellent discussion of the problem, the reader is referred to [3].

Inverting the sample-and-hold (by packet or by byte) is, on the other hand, a new problem.

For each flow which is not being sampled, there is a per-packet probability \( p \) that the flow will start being sampled at that point. Define \( q = 1 - p \). Let \( (\theta_1, \theta_2, \ldots) \) be the original flow length distribution before sampling, where \( \theta_i \) is the probability that a randomly chosen flow is exactly \( i \) packets long.

Let \( \theta'_i \) be the probability that the algorithm would start sampling a randomly chosen stream \( i \) packets from the end of the stream. This is the probability that \( i \) packets are sampled. Note that \( \theta'_0 \neq 0 \).

\[
\theta'_i = \begin{cases} 
    \sum_{j=1}^{\infty} pq^{i-j} \theta_j & i > 0 \\
    \sum_{j=0}^{\infty} q^i \theta_j & i = 0
  \end{cases}
\]

Let \( X_i, i \in \mathbb{N} \) be the distribution of flow lengths which can actually be observed. This can be thought of as the distribution \( \theta'_i \) without the probability of zero length flows.

\[
X_i = \mathbb{P}[\text{Sample length} = i | \text{Sample length} > 0] = \frac{\theta'_i}{\sum_{k=1}^{\infty} \theta'_k} = \frac{\sum_{j=1}^{\infty} pq^{i-j} \theta_j}{\sum_{k=1}^{\infty} \sum_{j=k}^{\infty} pq^{j-k} \theta_j} = \frac{q^i \sum_{j=1}^{\infty} q^{i-j} \theta_j}{q^i \sum_{j=1}^{\infty} q^j \theta_j \sum_{k=1}^{j} q^{-k}}
\]

The sum \( \sum_{k=1}^{j} q^{-k} \) can be evaluated giving,

\[
X_i = \frac{(1 - q) \sum_{j=1}^{\infty} q^j \theta_j}{q^i \sum_{j=1}^{\infty} q^{j-i} \theta_j} = \frac{(1 - q) \sum_{j=i}^{\infty} q^j \theta_j}{q^i \sum_{j=1}^{\infty} (1 - q^j) \theta_j} = \frac{(1 - q) \sum_{j=i}^{\infty} q^j \theta_j}{q^i [1 - \sum_{j=1}^{\infty} q^j \theta_j]}
\]
Setting $i = 1$ and rearranging gives
\[
\sum_{j=1}^{\infty} q^j \theta_j = \frac{qX_1}{1 - q + qX_1}.
\]
Let $C = (1 - q)/(1 - \frac{qX_1}{1 - q + qX_1}) = (1 - q + qX_1)$ and therefore
\[
q^i X_i = C \sum_{j=1}^{\infty} q^j \theta_j.
\]
Giving the final estimate
\[
\theta_i = \frac{X_i - qX_{i+1}}{C} = \frac{X_i - qX_{i+1}}{1 - q + qX_1}.
\]

This method has certain obvious weaknesses. The factor $1 - q + qX_1$ is simply a normalization factor, the method wholly relies on the difference between $X_i$ and $X_{i+1}$. It is relatively insensitive to the particular value of $p$ when $p$ is small (which it would be for typical sampling rates) since the difference between $X_i - 0.99X_{i+1}$ and $X_i - 0.999X_{i+1}$ is usually not great. However, particularly at large flows this creates problems. In particular, if $X_{i+1} > X_i$ then the method will produce a negative estimate for the probability. This problem can be offset to some extent by pooling adjacent estimates so that, instead of estimating the probability that a flow has exactly length $i$, instead an estimate is given of the probability that the flow has a length in some range $i, i+1, \ldots, i+n$. This is discussed in the next section.

We did not find any obvious method of inverting the original Estan sample-and-hold (by byte). The method used in this paper is simply to assume that the data was obtained from sample-and-hold by packet with $p$ as the probability of sampling a packet of mean packet length using Estan’s method.

For SYN based sampling, the assumption that each TCP flow begins with exactly one SYN flag implies that no inversion should be needed. Unfortunately, the SYN based sampling will only sample TCP flows and can provide no information about the distribution of UDP flows. This is a weakness of the method.

### 2.5 Logarithmic binning

When examining the flow distribution, particularly for long flows, it is likely to be of more interest to know how many flows have a length in a given range, rather than the number of flows with a specific length. Therefore, we have used a pooling technique to average data using a logarithmic scale. The data relating to flow lengths is averaged over bins which contain data on a set of flow lengths (for example, one bin from all lengths from 1000 to 1100). The size of the bins are chosen so that they have a constant width (or as nearly as possible given they are integer valued) on a logarithmic scale. This technique is sometimes known as logarithmic binning.

Logarithmic binning is a simple way of smoothing sample data which is distributed on a logscale. Let $x_k$ be the number of observations of a flow with length $k$. Let $m$ be the largest flow observed. The values $x_1, x_2, \ldots, x_m$ will be combined into $n < m$ observations $X_1, X_2, \ldots, X_n$. Now, let $i_0, i_1, i_2, \ldots, i_n$ be some series of integers such that $i_0 < i_1 < i_2 \ldots$ with $i_0 = 1$, $i_n$ is larger than the largest flow length observed and $i_{k+1}/i_k$ is approximately constant for large $k$. Now we can derive a series $X_1, X_2, \ldots, X_n$ giving the average number of observations in the range $[i_{k-1}, i_k)$ – note that the integer $i_k$ is not in this range (but will be in the range of $X_{k+1}$).

\[
X_k = \frac{\sum_{j=i_{k-1}}^{i_k} x_j}{i_k - i_{k-1}},
\]

for $k = 1, 2, \ldots, n$. Note that, for display purposes, it makes sense to show the observation $X_k$ as occurring in the range $i_{k-1} - 0.5$ to $i_k - 0.5$.

Figure 4 shows the results of logarithmic binning on one of the data sets from Figure 2. The technique has two advantages for this study. Firstly, it produces

| $\phi$ | 5-tuple corresponding to packet $P$ |
| $t$ | Capture time of packet $P$ |
| $N_b$ | Number of bytes in packet $P$ |
| $t_s(\phi)$ | Trace time since the last packet on 5-tuple $\phi$ was seen |
| $T_0(\psi)$ | Set of all $\psi$ |
| $T_p(\psi)$ | Total number of packets in flow $\psi$ |
| $T_b(\psi)$ | Total number of bytes in flow $\psi$ |
| $p$ | Probability of starting to follow flow $\phi$ |
| $t_w$ | Flow buffer export timeout |
| $N_f$ | Flow buffer size in records |

Table 1: Variables for the flow construction algorithm
clearer information for large flows. In the large flows regime, we usually observe either one flow or no flows, and this can make the graph on that region harder to interpret. However, the logarithmic binning allows the graph to convey information about how many flows are in a given range, including those long-flow regimes where simple plots are usually uninformative. This also gives a clearer idea of the heavy-tailed nature of flow lengths. Secondly, it pools those estimators for which there is most uncertainty. Estimating the probability that there is a flow exactly (say) 10,005 packets long is a difficult task, requiring vast amounts of data and processing power. On the other hand, estimating the expected number of flows which are between 10,000 and 11,000 packets long allows the pooling of estimators to produce a more accurate estimate using a smaller data sets, and with lesser computational demands.

The results in this paper are all obtained on real network trace data. The traces are sampled using the techniques described in the previous section. The flow distribution is then produced on the sampled data after sampling inversion techniques are applied (where such techniques are available) in order to recreate the original flow distribution. This is compared with the correct flow distribution obtained from the unsampled data. In order to assist comparison, the sampling rates are chosen so that approximately one packet in every one hundred is sampled. That is, the methods used are all sampling approximately the same percentage of the data set and the storage requirements for each sampling method would not be dissimilar.

The logarithmic binning method is a simple but invaluable tool for the investigation of sampled flow length distribution. In addition to being a useful method for presenting the data it enables the pooling of otherwise unreliable estimates to get a reliable estimate over a range of values.

Packet sampling is an attractive sampling scheme for many purposes. It allows recovery of many important properties of the data, however, it is difficult to recover flow based information. Three sample-and-hold based schemes are used here, based upon the original sample-and-hold described by Estan and Varghese [6] (which is here referred to as sample-and-hold by byte). Like packet sampling, sample-and-hold can be implemented in a practical setting (for example in firmware) [19].

### 3.1 Packet sampling

As previously stated, inversion results are not given here for packet based sampling. This is due to the extreme difficulty of producing a flow distribution over the full range of possible flow lengths from packet sampled data (see the discussion in Section 1.5). The sampling was performed to get one packet in one hundred by setting $p = 0.01$. This gives 425,014 packets sampled in 207,126 flows, a mean of 2.1 packets per flow.

### 3.2 Sample-and-hold (by packet)

The value of $p$ was adjusted so that approximately one packet in every one hundred was sampled. The value of $p$ used was 0.000014 and this gave 413,702 packets and 614 flows (a mean flow length of 674 packets per flow).

At a higher sampling rate the inversion algorithm can be seen to be very good indeed. With $p = 0.001$ 10,333,134 of 47,047,240 packets were sampled on the CAIDA trace. This is a very high rate of sampling but suitable for an initial test of the sampling algorithm.

Figure 9 shows the density function for this experiment before and after inversion compared with the unsampled data. Figure 10 shows the distribution function for the same experiment.

![Figure 4: The effects of logarithmic binning.](image)

![Figure 5: Figure 2 replotted with logarithmic binning.](image)
Figure 6: The impact of packet sampling with $p = 0.01$ on the flow distribution.

Figure 7: Cumulative density function for packet based sample-and-hold with sampling of approximately one packet in every one hundred on the CAIDA data.

Figure 8: Density function for packet based sample-and-hold with sampling of approximately one packet in every one hundred on the CAIDA data.

Figure 9: Density function for packet based sample-and-hold with $p = 0.001$ on the CAIDA data.

Figure 10: Cumulative density function for packet based sample-and-hold with $p = 0.001$ on the CAIDA data.
The sample-and-hold by packet method focuses on large flows. At the high sampling rate in Figures 10 and 9 the sample-and-hold by packet method inverts almost precisely to the original distribution. However, nearly one in five packets were sampled and this is an unrealistically high sampling rate for a highly loaded router. At the more realistic sampling rates shown in Figures 7 and 8 the algorithm still performs relatively well, particularly at higher flow lengths. The distribution here was recovered from only 614 sampled flows. Another potentially useful property of the sample-and-hold by packet is that the packets sampled can be resampled to create a sample which would have been obtained from packet sampling with probability \( p \) (where this is the same \( p \) used for sample-and-hold by packet in the first place). This is done by sampling the first packet in each flow and then performing packet sampling with probability \( p \) on all subsequent packets.

### 3.3 Sample-and-hold (by byte)

Figure 11 shows the results from sampling the CAIDA data using the sample-and-hold (by byte) method as proposed by Estan and Varghese [6]. The \( p \) value has been tuned so that approximately one in one hundred packets are sampled. This gave 521,337 packets sampled in total over 527 flows, a mean flow length of 989 packets per flow.

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**Figure 11:** Cumulative density function for byte based sample-and-hold with sampling of approximately one packet in every one hundred on the CAIDA data.

Sample-and-hold by byte was engineered to focus on the largest flows (sometimes referred to in the literature as “elephant” flows). It is no surprise that the flow distribution in Figure 11 shows this clearly. The largest flows are tracked. The inversion algorithm in this paper was designed to work with the packet based sample-and-hold and it is no surprise that it does not perform particularly well in this case. As discussed, negative values occur in the predicted “probabilities” and the “distribution” does not total to one as it should. This is a result of applying an algorithm which is not quite appropriate for the data set. Of course these issues could be fixed by forcing a minimum of zero and normalizing the distribution artificially. Nonetheless, the inverted distribution is an improvement over the original at longer flow lengths although performs poorly over short flows. The distribution in Figure 11 is reconstructed from only 527 flows so it is, perhaps, impressive that it is as close to the original as it is.

The advantages of sample-and-hold by byte are that it focuses clearly on those “elephant” flows which can dominate traffic. It has been previously studied in the literature and implemented in software for real sampling applications. On the other hand, the disadvantages are that no good inversion algorithm exists as yet. In addition there is no obvious way to recover a packet sampled data set from the sample-and-hold by byte data.

### 3.4 Sample-and-hold (by SYN)

Sample and hold by SYN was run with \( p \) tuned to get approximately one packet in every one hundred. If the assumption of one SYN packet per flow was correct this would simply mean setting \( p = 0.01 \). But, as previously discussed, this assumption is not met in the real data. In this sample, 520,116 packets were sampled in 68,618 flows, a mean of 7.6 packets per flow.

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**Figure 12:** Cumulative density function for SYN based sample-and-hold with sampling of approximately one packet in every one hundred on the CAIDA data.

The sample-and-hold by SYN is actually surprisingly good at recovering the sampled distribution as can be seen in Figures 12 and 13. However, this is somewhat misleading. As can be seen if Figure 13 the distribution from all SYN flows (effectively SYN sampling at a rate of 1) is, in fact, very different from the distribution of all the flows. This is not unexpected. It seems that in
this trace the non SYN flows (mostly UDP and some ICMP) are shorter and, in particular there are more flows which are just one or two packets long.

It has already been noted that SYN sampling does not provide an unbiased estimate of SYN flows because, in reality, flows can have more than one SYN packet. Many of these flows with more than one SYN packet are short flows (perhaps because a flow with multiple SYNs packets can result from trying to initiate a connection to a machine which is not responding). When SYN sample-and-hold is used then it will be more likely to sample those flows with multiple SYN packets (unless the sample rate is one, of course). The presence of such a protocol behaviour has fortuitously cancelled out the error in the other direction and the good recovery of the flow distribution is a product of two errors in opposite directions cancelling rather than a true measure of the success of the algorithm. This gives a large element of uncertainty to the use of SYN sampling as a method for recovering flow distributions since the basic assumption (that TCP flows begin with a single SYN packet) is not met in real data.

A major disadvantage of sample-and-hold by SYN packet is that it cannot provide information about non TCP packets. A major advantage is that it gives an approximate flow distribution with no need for inversion because it is an approximation to flow sampling.

4. CONCLUSIONS AND FUTURE WORK

Producing flow distributions for sampled packet traces is a difficult problem. Several authors have approached the problem of producing flow distributions from traces sampled using standard packet sampling. However, different sampling methods can be used to provide a sample which makes the recovery of the flow distribution easier while at the same time not putting an undue requirement on memory and storage for the hardware performing the sampling.

Of the sample-and-hold based methods studied here all have advantages and disadvantages. The byte based sample-and-hold originally proposed in [6] was intended to focus on the longest flows and does this better than any other method. An inversion to recover the original flow distribution has been attempted in this paper and is partially successful. Further work in this area might improve this algorithm.

Packet based sample-and-hold has two advantages. Firstly, it can be inducted well to produce a reasonable estimate of the flow distribution even for relatively low sampling rates (approximately one in one hundred packets sampled). Secondly, it can be resampled to get a packet sample and recover those quantities which can be measured at the packet, not flow, level. A disadvantage is that the estimated probabilities are not guaranteed positive.

SYN based sample-and-hold is near to the original flow-based sampling proposed by Hohn and Veitch [3] but it has problems due to packets with more than one SYN flag. It is possible that further work could correct for this problem. However, the problem that this technique will only ever be useful for TCP traffic remains a major issue.

Our future work on this topic will focus on two main issues. Firstly, there is more information to be gained from other parts of the TCP header, notably the authors know of no results which use the FIN or RST flag for inversion. While these are problematic since not every flow terminates correctly, still it would seem that valuable information is contained in these flags. Secondly, using multiple sample sources could be a rich topic for research. Some work has already been done in this area: [1, Section 6] provides a start on this topic focusing on packet sampling and [20] provides another approach. Investigating different sampling techniques which might take advantage of network topology (for example, if samples are available from two directions on the same link) could provide more information which might be used to develop better sampling techniques and also to provide more information for the inversion problem.

Acknowledgments

The authors would like to acknowledge CAIDA [18] for providing the trace files. This work is conducted under the MASTS project (EPSRC grant GR/T10503).

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