The redefined SI and the electromagnetic quantities in detail – part I: current and voltage

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Abstract. This paper describes the electromagnetic units after the SI redefinition and the details regarding how they are realized. It is divided into two parts. In this first part, we review the state-of-the-art performance, the physical principles, diagrams, electrical circuits, equations and uncertainties of the ampere and volt realizations.

1. Introduction
The International System of Units (SI) formally began with the Metre Convention, signed in 1875 by 17 countries, including Brazil. Currently, there are 60 Member States to the Convention, plus 42 Associate States and Economies. This treaty created the International Bureau of Weights and Measures (BIPM, an intergovernmental organization whose mission is to assure the worldwide unification of the measurements), under the authority of the General Conference of Weights and Measures (CGPM, composed of delegates of the governments of the Member States and observers from the Associates of the CGPM) and under the supervision of the International Committee for Weights and Measures (CIPM). The CIPM is assisted by advisory committees from various areas related to the SI quantities; among them, we can mention the Consultative Committee for Electricity and Magnetism (CCEM) [1].

The seed of the SI (the metre, kilogram and second - MKS system) was created in 1889, with three base units: the metre (related to the circumference of the earth and realized through a metal bar) and the kilogram (related to the mass of a cubic decimeter of water and realized through a weight of iridium and platinum metal), both based on physical artifacts; plus the second (related to the rotation of the earth around its axis), based on astronomical realization. In 1946 the MKS system evolved to the MKSA system, with the introduction of the ampere base unit (and other electrical units) by the CIPM decision (ratified in 1948 by the 9th CGPM) [2], [3].

From then on, several changes on the SI were made. Such changes:
(a) were needed in order to keep the SI updated with the most advanced technologies; that means the measurements made within the SI cannot limit the research and development of new technologies and products;
(b) cannot be noticed by the public; that means they should have small numeric effect so they will not jeopardize world trade relations.

With the discovery of the Josephson effect in 1962, the voltage (through a Josephson junction) could be related to the irradiation frequency and the Josephson constant ($K_J = 2e/h$, where $h$ is the Planck constant and $e$ is the elementary electric charge). Similarly, the quantum Hall effect relates the electrical resistance (Hall resistance, $R_H$, which is equal to the Hall voltage divided by the applied current, measured on a two-dimensional electron gas) to the von Klitzing constant ($R_K = h / e^2$) [4].
Due to the unprecedented reproducibility of the results of the systems based on the Josephson and quantum Hall effects, the CIPM recommended the use of the Josephson effect to obtain the unit of electromotive force unit (volt) and the use of the quantum Hall effect to obtain the electrical resistance unit (ohm) representations [5], [6]. Thus, these units came to be represented by fundamental physical constants of nature (Planck constant and the elementary electric charge). Also, considering that the 1948 definition of the ampere proved difficult to be realized, the Josephson and quantum Hall effects were used as a practical realization of the ampere through Ohm's law [2].

Other definition of base units evolved following this path: from artifact standards through atomic standards to standards based on fundamental physical constants [7]. In this way, the natural evolution of the SI goes through the use of fundamental physical constants. Hence, in 2018 the CGPM decided to redefine the SI [8]. In this review paper (part I), we present the changes in the electric current and electric voltage units due to the redefined SI and the details regarding how they are realized. We discuss the physical principles, diagrams, electrical circuits, equations and uncertainties.

2. The Redefined SI and the Electrical Quantities

According to the resolution 1 of the 26th CGPM [8], the new SI (effective from 20 May 2019) is the system of units in which [9], [10]:

- "the unperturbed ground state hyperfine transition frequency of the caesium 133 atom \( \Delta \nu_{\text{Cs}} \) is 9 192 631 770 Hz;
- the speed of light in vacuum \( c \) is 299 792 458 m/s;
- the Planck constant \( h \) is 6.626 070 15 \times 10^{-34} \text{ Js};
- the elementary charge \( e \) is 1.602 176 634 \times 10^{-19} \text{ C};
- the Boltzmann constant \( k \) is 1.380 649 \times 10^{-23} \text{ J/K};
- the Avogadro constant \( N_A \) is 6.022 140 76 \times 10^{23} \text{ mol}^{-1};
- the luminous efficacy of monochromatic radiation of frequency 540 \times 10^{12} \text{ Hz}, \( K_{\text{cd}} \), is 683 \text{ lm/W},

where the hertz, joule, coulomb, lumen, and watt, with unit symbols Hz, J, C, lm, and W, respectively, are related to the units second, metre, kilogram, ampere, kelvin, mole, and candela, with unit symbols s, m, kg, A, K, mol, and cd, respectively, according to Hz = s\(^{-1}\), J = kg m\(^2\) s\(^{-2}\), C = A s, lm = cd m\(^{-2}\) = cd sr, and W = kg m\(^2\) s\(^{-3}\)."

"Starting from the new definition of the SI described above in terms of fixed numerical values of the defining constants, definitions of each of the seven base units are deduced by taking, as appropriate, one or more of these defining constants to give the following set of definitions, effective from 20 May 2019:

- The second, symbol s, is the SI unit of time. It is defined by taking the fixed numerical value of the caesium frequency \( \Delta \nu_{\text{Cs}} \), the unperturbed ground-state hyperfine transition frequency of the caesium 133 atom, to be 9 192 631 770 when expressed in the unit Hz which is equal to s\(^{-1}\).
- The metre, symbol m, is the SI unit of length. It is defined by taking the fixed numerical value of the speed of light in vacuum \( c \) to be 299 792 458 when expressed in the unit m/s, where the second is defined in terms of \( \Delta \nu_{\text{Cs}} \).
- The kilogram, symbol kg, is the SI unit of mass. It is defined by taking the fixed numerical value of the Planck constant \( h \) to be 6.626 070 15 \times 10^{-34} \text{ when expressed in the unit J s, which is equal to kg m}^2\text{s}^{-1}, \text{where the metre and the second are defined in terms of } c \text{ and } \Delta \nu_{\text{Cs}}.
- The ampere, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge \( e \) to be 1.602 176 634 \times 10^{-19} \text{ when expressed in the unit C, which is equal to A s, where the second is defined in terms of } \Delta \nu_{\text{Cs}}.
- The kelvin, symbol K, is the SI unit of thermodynamic temperature. It is defined by taking the fixed numerical value of the Boltzmann constant \( k \) to be 1.380 649 \times 10^{-23} \text{ when expressed in
the unit J K⁻¹, which is equal to kg m² s⁻³ K⁻¹, where the kilogram, metre and second are defined in terms of h, c and ΔνCs.

- The mole, symbol mol, is the SI unit of amount of substance. One mole contains exactly 6.022 140 76 × 10²³ elementary entities. This number is the fixed numerical value of the Avogadro constant, Nₐ, when expressed in the unit mol⁻¹ and is called the Avogadro number. The amount of substance, symbol n, of a system is a measure of the number of specified elementary entities. An elementary entity may be an atom, a molecule, an ion, an electron, any other particle or specified group of particles.

- The candela, symbol cd, is the SI unit of luminous intensity in a given direction. It is defined by taking the fixed numerical value of the luminous efficacy of monochromatic radiation of frequency 540 × 10¹² Hz, Kcd, to be 683 when expressed in the unit lm W⁻¹, which is equal to cd sr W⁻¹, or cd sr kg⁻¹ m² s³, where the kilogram, metre and second are defined in terms of h, c and ΔνCs.

The complete set of the SI redefined base units is presented here because they are either directly (bold text above) or indirectly related to the electromagnetic quantities or for completeness.

2.1. Mise en pratique (MeP) for the definition of the ampere and other electric units in the SI

Some information from this subsection is a transcription from [10]. Two definitions are quite important and they are described in the BIPM MeP for each base unit [9]:

a) “In general, the term ’to realize a unit’ is interpreted to mean the establishment of the value and associated uncertainty of a quantity of the same kind as the unit that is consistent with the definition of the unit”.

b) “A primary method is a method having the highest metrological properties; whose operation can be completely described and understood; for which a complete uncertainty statement can be written down in terms of SI units; and which does not require a reference standard of the same quantity”.

Also, the definition of the base units of 2018 does not imply any particular experiment for its practical realization. Any method capable of deriving a quantity value traceable to the set of seven reference constants can, in principle, be used. Thus, the list of methods given is not meant to be an exhaustive list of all possibilities, but rather a list of those methods that are easiest to implement and/or that provide the smallest uncertainties and which are officially recognized as primary methods by the relevant Consultative Committee”.

2.1.1. Practical realization of the ampere. In practice, the ampere can be realized:

a) “by using Ohm’s law, the unit relation A = V/Ω, and using practical realizations of the SI derived units the volt V and the ohm Ω, based on the Josephson and quantum Hall effects, respectively” [10] (the Josephson effect is discussed in Subsection 2.1.2). In this method of realization of the ampere, unknown current flows through a known standard resistor (traceable to the SI ohm through the quantum Hall effect), resulting in an electromotive force (emf) between its terminals. This emf is measured using a Josephson Voltage Standard (traceable to the SI volt through the Josephson effect). The current is estimated through the measured voltage and the known resistance.

It should be noted that standard resistors are highly sensitive to temperature variation. Even if some sort of environmental temperature control is employed (oil bath or air bath, for example), the higher the current, the higher the temperature variation. Thus, there exists a compromise between lower uncertainties and higher currents for currents measured using this technique. For instance, let us consider a standard reference resistor whose highest performance is obtained with a dissipation (=V²/Ω) of around 10 mW, with a minimum value of 1 Ω [11] and with an estimated standard uncertainty of around ±34 nΩ [12]). In this case, a maximum current of around 0.1 A and a maximum measured voltage of around 0.1 V are allowed (with an estimated standard uncertainty of around ±4 nV, considering the same uncertainty for 1 V measurements [12]). The estimated 0.1 A combined standard uncertainty is around ±3.5 × 10⁻⁷ A/A. For higher currents, the shunt resistances values are
lower than 1 Ω, hence commonly measured using impedance bridges, which increases the standard resistors uncertainties. Besides, the power dissipation can reach 10 W. Therefore, one can find 2 A, 10 A and 20 A combined standard uncertainties around ±1.5 × 10⁻⁴ A/A [12], ±2.3 × 10⁻⁶ A/A [12] and ±8 × 10⁻⁶ A/A [12], respectively.

b) “by using a single electron transport (SET) or similar device, the unit relation A = C/s, the value of e given in the definition of the ampere and a practical realization of the SI base unit the second s” [10].

Single electron transport or single electron tunneling (SET) is the process of transporting one electron at a time through a thin insulating barrier. Figure 1 shows the basic building blocks of SET devices [13]. According to figure 1(a), the proper instant increase of the voltage \( V \) allows a single electron to cross the insulating barrier double box (the top double box, for instance), what is known as “tunneling”. This single electron is then trapped in the island (the black dot). A single-electron tunneling transistor (SETT) is a switching device composed of two tunnel junctions sharing a common electrode (figure 1(b)). In this case, there are two control voltages, which allow a single electron to pass through both barriers, in a very controlled way. Hence, it is possible to control the amount of charge which flows during a known period of time, leading to a known electric current.

![Figure 1](image.png)

**Figure 1:** Basic building blocks of SET devices. (a) A single island (black dot) is defined by two tunnel junctions (double box symbols). Charge can flow on or off the island only in discrete units of \( e \). The voltage source \( V \) must supply the charging energy \( e^2/2C_g \) in order for the current to flow through the two junctions. (b) A single-electron tunneling transistor (SETT) is created by adding a gate capacitor and voltage source [13].

SET implementations still have technical limitations and often larger relative uncertainties than some other competitive techniques. There exists SET devices (chips with SET pumps [14], which deliver a very controlled electric current); however, current SET devices can deliver up to 100 pA with uncertainties between around ±1 × 10⁻⁴ A/A and ±1 × 10⁻⁷ A/A [13], [15], [16], [17]. Therefore, although these devices can be applied to realize the ampere, they cannot be used as electric current standards. Huge improvement is needed until this goal is achieved. Meanwhile, intermediary solutions are being provided, such as the Ultrastable Low-Noise Current Amplifier (ULCA), which is an electronic equipment composed mainly by operational amplifiers and resistor networks. It involves two stages, the first providing a 1000-fold current gain and the second performing a current-to-voltage conversion via a reference resistor. That means the ULCA can work as a highly accurate current-to-voltage converter with an effective transresistance \( A_{TR} \) of 1 GΩ, traceable to the SI ohm through the quantum Hall effect. In this case, a SET current source is applied to two ULCA inputs (ULCA A and ULCA B); the SET current \( I \) flows through ULCA A and it is converted into a voltage \( V_A = A_{TR}^A \times I \); the returning SET current \( I \) flows through ULCA B and it is converted into a voltage \( V_B = A_{TR}^B \times I \). A differential voltage measurement is taken using a Programmable Josephson Voltage Standard (PJVS, traceable to the SI volt through the Josephson effect) and an 8 ½ digit DVM, in order to estimate \( V_A \) and \( V_B \) voltages and therefore estimate the SET current value through ULCA A’s and ULCA B’s.
transresistances [17], [18], [19]. Figure 2 depicts the current measurement setup with two ULCA inputs.

\[ I = \frac{V_{PJVS} - V_{DVM}}{A_{TR} + A_{TR}^B} \]  

(1)

where \( V_{PJVS} \) is the quantum voltage generated by a Programmable JVS (PJVS) and \( V_{DVM} \) is the differential measurement made by the DVM. Using this technique, it was possible to measure a SET realization of the ampere generating 96 pA with combined standard uncertainty of \( \pm 1.6 \times 10^{-7} \) A/A [20].

c) “by using the relation \( I = C \frac{dV}{dt} \), the unit relation \( A = F V/s \), and practical realizations of the SI derived units the volt V and the farad F and of the SI base unit second s” [10]. This is done by applying a linear voltage ramp of a constant slope \( \frac{dV}{dt} \) to a capacitor of capacitance \( C \). Typical values for \( \frac{dV}{dt} \) and \( C \) are 10.0 mV/s and 1 nF, respectively [20]. That means the generated current is limited to around 10 pA, with combined standard uncertainty of \( \pm 4.7 \times 10^{-5} \) A/A [20]. Hence, this technique is suitable for the current range below 1 nA (usable for picoamperemeter calibrations, for instance).

2.1.2. Practical realization of the volt, V, SI derived unit of electric potential difference (voltage) and electromotive force. The volt V can be realized by using the Josephson effect and the following value of the Josephson constant \( K_J \):

\[ K_J = \frac{2e}{h} \approx 483 \, 597 \, 848 \, 416 \, 984 \, \text{GHz} \, V^{-1}. \]  

(2)

where \( e \) is the elementary charge and \( h \) the Planck constant.

Because \( e \) and \( h \) do have exact values, \( K_J \) also does have exact value (the exact rational number \( 2e/h \)). However, its numerical representation has infinite decimals digits. So, its numerical representation to practical application has been calculated to 15 significant digits, which is in error by less than 1 part in \( 10^{15} \) [10]. It is worth to mention that the best voltage measurements (direct Josephson comparisons) can reach a combined standard uncertainty of \( \pm 1.3 \times 10^{-10} \) V/V [21].

In a similar way to SET devices (see Subsection 2.1.1 (b)), electric current can flow through a thin insulating barrier (a Josephson junction) separating two superconductors [22]. When a Josephson...
junction is exposed to electromagnetic radiation of frequency $f$, its current-voltage characteristic exhibits precisely quantized voltage steps (see figure 3) described by the relation [22], [23], [24]:

$$V_J = \frac{n f}{K_J}$$  \hspace{1cm} (3)

where $V_J$ is the quantum voltage generated by the Josephson junction, $n$ is the step number and $K_J$ is given by equation (2).

Since $V_J$ is known and $V_{DVM}$ is measured, the voltage $V_Z$ of the Zener device under test ($Z_{DUT}$) can be estimated through equation (4), derived from the electrical diagram presented in figure 4:

$$V_Z = V_J - V_{DVM}$$  \hspace{1cm} (4)

The reader should bear in mind that there exist other effects which are not shown in the simplified circuit above, like thermal voltages, offsets, drift and noise. Please refer to [25] for further details regarding these effects, calibration algorithm and uncertainty budget.

The ampere was the only electrical quantity which was redefined. However, it does not mean that the other electrical quantities are not linked to the defining constants. The ampere works like an anchor to the defining constants. For instance, the voltage and the ohm are linked to the ampere through the Ohm’s law. Besides, the volt (and ohms) realization is directly linked to the elementary charge ($e$) and the Planck constant ($h$).
3. Conclusions
This review paper (part I) presented the changes in the electromagnetic units due to the redefined SI and the details regarding how the ampere and volt units are realized. The physical principles, diagrams, electrical circuits, equations and uncertainties were discussed as well. In the second part of this review paper [26], we present the details regarding how the electric resistance, electric conductance, electric charge, capacitance, inductance, electric power, magnetic flux density and magnetic flux units are realized in the redefined SI.

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