A parametric analysis of the “digitally-derived geometric design” of the façade of the Macau Holy house of Mercy

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\section*{ABSTRACT}

The Macau Holy House of Mercy was built in 1569 and rebuilt in the 18th century. Located in the historic center of Macao, the Macau Holy House of Mercy is included on the World Heritage List of UNESCO. With the Macau Holy House of Mercy, especially the façade of its neo-classical arcade added in 1905, as the research subject, this study applied the methods of historical document research, field measurement and digital analysis, especially parametric analyses with the assistance of the software of Rhino and Grasshopper. The following results were obtained: 1) its designers applied three-section partition in the horizontal and vertical directions and equal division in stratification in the overall dimension of the façade; 2) its designers also applied modularization (modular) design and combination in the columns and details of doorways; 3) this study found that the “digitally-derived geometric design” of the façade is closer to the “Golden Section” than “Root Rectangle Partition”.

This study applied the following research methods:

(a) A Cross-reference Research of Historical Documents and Literature: this study collected and conducted a cross-reference research on the related historical documents, literature and pictures about remaining Western Classic Buildings in Macao, while verifying the results of digital analyses.

(b) Field Measurements and Parametric Analyses: this study studied related drawings of the façade of the Macau Holy House of Mercy and digitalized them with the assistance of the software AutoCAD while revising them with reference to data acquired from field measurements. This study further conducted a parametric analysis of the design patterns of the Golden Ratio (nestings and combinations), modular, equal partition and successive subtraction in the façade with the assistance of the modeling software of Rhino and the software plug-in of Grasshopper (using...

\section{1. Introduction}

The Macau Holy House of Mercy was built in 1569 (Ljungstede 1997, 52) and rebuilt in the 18th century (Figure 1). In 1905, a neo-classical façade was added to the original building (Figure 2). Located in the historic center of Macao, the Macau Holy House of Mercy is included on the World Heritage List of UNESCO. It is of unparalleled protection and research value. Geometrical composition and rational order are beloved and common factors in Western Classic Buildings. Ratio plays a significant role in the “digitally-derived geometric design”. “Digitally-derived geometric design” is the “grammar” of Western Classic Buildings and the source of its rationalism. Digital analysis, especially parametric analysis, is an effective method to analyze the “digitally-derived geometric design.”

Professor Suzuki (1998, 43–46) has discussed the characteristics of the neo-classical architectures. There are limited (Hu and Yin 2017) digital studies (Tang 2018) on the Western Classic Buildings in the Far East (Ge 2005) and even less conducted from a parametric and detailed perspective.

\section{2. Research objectives and methods}

With the Macau Holy House of Mercy, especially the façade of its neo-classical arcade added in 1905 (Figure 2-3), as the research subject, this study synthetically applied the methods of historical document research, field measurement and digital analysis, especially parametric analyses and analyzed the modular, proportion relation and design pattern of the “digitally-derived geometric design” of the façade of the Macau Holy House of Mercy.
Rhinoceros 6.0 to place the façade and axes used in drawing analyses and using Grasshopper to write programs that composes data-mapping diagram).

3. Historical research

The Macau Holy House of Mercy, also named “Relief-providing Church” in A Brief History of Macao (Yin and Zhang 1992, 150), is located by the side of the Macau Senado Square. Built in 1569 (Ljungstede 1997, 52), the Macau Holy House of Mercy is one of the most important charity institution in the early times of Macao. It was founded by Bishop D. Belchior Carneiro (1156–1583) and has two hospitals and an orphanage (founding hospital). It was dedicated to Our Lady of Mercy as its patron, and modelled after the Roman charity associations, taking it as its own duty to provide daily necessities for those who poor and helpless. It also provided safety protection for the Portuguese “rich man and merchants”.

Bishop Carneiro allocated 5 percents of Macao’s tax revenue for the daily expenditure of the Macau Holy House of Mercy. It also set up an orphanage. “It was modelled after those inland, placing a basket by the side of the door and hanging a bell above. Those who abandoned their children put them in the basket and ringed the bell by pulling the rope attached to it. Clergymen inside on hearing the bell ring would come to pick up the abandoned children and raise them (Yin and Zhang 1992)“.

Bry’s City Map of Macao in the 1590s (1607; Bry 2000). recorded the shape of the architecture of the Macau Holy House of Mercy in the 16th century.

Located near the north-east side of the Leal Senado, the Holy House of Mercy was a two-storey building with a gable rooftop. According to Manuel Teixeira, this storehouse architecture style built with planks and bricks was common in Portugal at that time. It was obviously taller than the two-storey Leal Senado. The space between the Leal Senado and the Macau Holy House of Mercy came to make a square, which was later named the Largo do Senado (Xue 2012, 30).

The Macau Holy House of Mercy was rebuilt in the 18th century (Guo 1990, 72). Its image in Chinnery’s painting (Comissao Territorial de Macau para as Comemoracoes dos Descobrimentos Portugueses 1997, 73) in the middle of the 19th century (Figure 1) was a two-storey Western Classic architecture with pediments, white walls and sill-like stone railings. In 1905, a neo-classical façade was added to the original building (Figure 2).¹

4. Obtaining the horizontal and vertical controlling lines (axes) and hereby deducing the partition patterns in the overall dimension of the façade

4.1. Obtaining the Main controlling line in the horizontal direction (axis ①–②) and hereby deducing symmetry, three-segment in the horizontal direction and modular

(1) The overall width of the façade: the overall width of Axis ①–② (Figure 3, Table 1) is 22,000 mm and it is bilaterally symmetric along the Axis ①. The façade is also divided into three parts by Axis ① and Axis ②: the left part, the middle part and the

¹A Postcard of Macao’s World Heritage Site: the Macau Holy House of Mercy from June 22th, 1999.
right part. If the width of these three parts are respectively: a, b, a (a = 6,650 mm, b = 8,700 mm, a + b + a = 22,000 mm), their geometrical relationship would be (unit: mm, the same below):

22,000 (the overall width) = 11,000 × 2 (bilateral symmetry) = 6,650 + 8,700 + 6,650 (a + b + a, three-Segment in the horizontal direction)

(1) In the middle of the façade (Axis ①—⑫), the modular of the width of the double-column, c = 1,050 mm and the modular of the width of the arch hole, d = 1,500 mm. Dividing the middle of the façade lengthwise into parts of "c + d + c + d + c + d" wide brings about Axis ⑧, Axis ⑬, Axis ⑭, Axis ⑮, Axis ⑱ and Axis ⑳.

(2) The modular of the width of the double-column (c = 1,050 mm) and the modular of the width of the arch hole (d = 1,500 mm) in the middle of the façade (Axis ①—⑫) also take effect in the left part of the façade (Axis ①—⑫) and the right part of the façade (Axis ①—⑫). Dividing these two parts of the façade into "the modular of the width of the double-column (c)-the modular of the width of the arch hole close to the inside (d)-the modular of the width of the broader double-column (e)-the modular of the width of the arch close to the outside (d)-the modular of the width of the double-column (c)-the modular of the width of the column at the break corner (f) (there exists a certain errors)" in sequence from within brings about Axis ②, Axis ③, Axis ④, Axis ⑤ and Axis ⑥, Axis ⑦, Axis ⑧, Axis ⑨, Axis ⑩, where e = 1,400 mm and c + f = 1,250 mm.

Accordingly, all the vertical Axes in the horizontal direction (Axis ①—⑫) are obtained. The combination of the modulars in the horizontal direction is: (f + c + d + e + d + c) + (c + d + c + d + c + d + c) + (c + d + e + d + c + f) = 6,650 + 8,700 + 6,650 (three-segment in the horizontal direction) = 22,000 (the overall width)

4.2. Obtaining the main controlling line in the vertical direction (axis ①—⑫) and hereby deducing equal stratification, three-segment in the vertical direction and modular

1) The top triangular pediment (Axis ①—⑫) and the golden rectangle (Rectangle A6-Q6-Q14-A14) compose the middle of the façade (Axis ①—⑫):

The peak of the triangle pediment is 16,000 mm high and drawing a horizontal line through this point brings about Axis ⑭. Shifting Axis ⑭ downward by 2,000 mm to the bottom edge of the triangle pediment brings about Axis ⑯. Shifting Axis ⑯ downward by 14,000 mm brings about Axis ⑱. Rectangle A6-Q6-Q14-A14 is 14,000 high and 8,700 mm wide. Its aspect ratio is 1.61, making it a golden rectangle.

(1) The bevel edge of the top triangular section and the triangular pediment is deduced by the following means:

The distance between the bottom edge of the triangle pediment (Axis ⑭) and the top edge of the parapet

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2 A Postcard of Macao’s World Heritage Site: the Macau Holy House of Mercy from June 22th, 1999.
Table 1. The corresponding relationship between axes and positions.

| Axis 1 | Axis 2 | Axis 3 | Axis 4 | Axis 5 | Axis 6 | Axis 7 | Axis 8 | Axis 9 | Axis 10 | Axis 11 | Axis 12 | Axis 13 | Axis 14 | Axis 15 | Axis 16 | Axis 17 | Axis 18 | Axis 19 | Axis 20 |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| The Horizontal Line of the Top Edges of the Parapets | The Right Side of the Right Arch Hole of the Middle of the Façade | The Right Side of the Middle Column in the Right Part of the Façade | The Left Side of the Middle Column in the Right Part of the Façade | The Right Side of the Left Arch Hole in the Middle of the Façade | The Left Side of the Right Arch Hole of the Middle of the Façade | The Right Side of the Middle Column in the Left Part of the Façade | The Left Side of the Left Arch Hole in the Middle of the Façade | The Right Side of the Right Arch Hole of the Middle of the Façade | The Left Side of the Right Arch Hole of the Middle of the Façade | The Right Side of the Middle Column in the Left Part of the Façade | The Left Side of the Left Arch Hole in the Middle of the Façade | The Right Side of the Right Arch Hole of the Middle of the Façade | The Left Side of the Left Arch Hole in the Middle of the Façade | The Right Side of the Right Arch Hole of the Middle of the Façade | The Left Side of the Left Arch Hole in the Middle of the Façade | The Right Side of the Right Arch Hole of the Middle of the Façade | The Left Side of the Left Arch Hole in the Middle of the Façade | The Right Side of the Right Arch Hole of the Middle of the Façade | The Left Side of the Left Arch Hole in the Middle of the Façade | The Right Side of the Right Arch Hole of the Middle of the Façade |
(Axis A) equals the modular of the width of the arch hole (d = 1,500 mm). Shifting Axis B downward by 900 mm, the same height of the parapet, brings about Axis C. The middle of the façade (Axis C) is 8,700 mm wide (b). Connecting M1 and R10 as well as M19 and R10, Line M1-R10 and Line M19-R10 intersect with Axis A and Axis C, forming the bevel edges of the triangle pediment.

3) The equal partition and stratification in the two stories (Axis C–A):

Drawing two squares with Line M1-M10 and Line M10-M19 as one side and the bottom sides of these two squares are on Axis C. The distance between Axis A and Axis C is 880 mm. These two squares is equally divided into two layers by the stratification controlling line of the first storey and the second storey, Axis C. Shifting Axis C upward by 900 mm brings about Axis A while shifting Axis C downward by 900 mm brings about Axis B. Axis C–A is the eaves of the first storey and Axis A–D is the column base and handrail of the second storey. Shifting the Axis C downward by 800 mm brings about Axis C and Axis A–C is the height of the column capitals in the first storey. Shifting Axis C downward by 800 mm brings about Axis A and Axis A–C is the height of the eaves in the second storey. Drawing a golden rectangle with Line A1-A6 as the long side, Line D1-A1 would be the short side and the highest points of arch holes in the first storey would be on Axis D. Shifting Axis C downward to the starting line of the circular arches brings about Axis C, the separating line of two-layer stacked column in the middle of the façade. Shifting Axis C upward by 2,000 mm brings about Axis C, the top edges of the circular arch casement. Shifting Axis C to the highest points of circular arches brings about Axis C.

4) On the overall scale of the façade, drawing a golden rectangle with Line C–A of 22,000 mm length on Axis A as the long side and Line A–C of 13,200 mm length on Axis C as the short side brings out Axis C, which lies the peaks of all sculpture embellishments of parapets.

Accordingly, all axes and the bevel edges of the top triangular section are brought about.

4.3. The partition patterns in the overall dimension of the façade

In the horizontal direction: Axis A and Axis C divide the façade into three parts. The middle part is the widest and highest and has one more arch hole and column than the right part and the left part, which is in accordance with the classic composition pattern of Three-segment in the horizontal direction. The number of circular arches in the left, middle and right parts of the façade are respectively 2-3-2. This pattern also takes effects in the number of columns, which is 3-4-3. This is in accordance with the pattern of modular and the rhythm of repetition. The central columns in the left part and the right part (A–C, C–D) and the side columns (A–C, D–C) are wider than the others. Such variation serves as adjustments.

In the vertical direction: Axis A and Axis C divide the façade into three parts. The bottom part (Axis A–C) is the column base. The middle part (Axis A–C) is stratified and equally divided by Axis A. The upper part (Axis A–C) are the parapets and the pediment. This is in accordance with the classic composition of Three-segment in the vertical direction.

[The one-time partition of golden rectangles marks the diagonal lines; the two-times partition of golden rectangles marks the golden spiral lines (dotted lines); the three-times partition of golden rectangles marks the golden spiral lines (solid lines)]

After Analyses of the partitions (axes) of the façade in the horizontal and vertical direction (Table 1), the following will focus on the parametric analysis of diagonal lines (gradient of slope) and successive partitions (spiral lines).

5. Parametric analysis of the façade: the combination and successive partition of rectangles and spirals controlled by golden ratio

5.1. Assessment of “golden rectangles+squares” or “root rectangles” in the composition of the façade

There are mainly “Golden Rectangles+Squares” and “Root Rectangles” in the proportion partition in Western Classic Buildings.

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*Due to the limited space of this article, the analysis of the corresponding relations between the plane and the facade will be conducted in another research.*
A Golden Rectangle could be divided into a subordinate golden rectangle and a square in succession, that is to say “A main golden rectangle = a subordinate golden rectangle + a square”. Such divisions bring about a series of mutually-perpendicular /mutually-paralleled diagonals and golden spirals (Figure 4). The mathematical expression of Golden Ratio is \((\sqrt{5} + 1)/2 = 2/(\sqrt{5} - 1) \approx 1.618\). The Fibonacci Sequence, also known as the Golden Section Sequence, is: 1, 1, 2, 3, 5, 8, 13, 21, 34, ……

In actual construction, architects usually take round number in measurement so the Golden Ratio applied in actual construction is usually a range rather than an exact number like 1.618. In this study, the author take the range between 3.2 (1.5) and 5:3 (1.67) in the Fibonacci Sequence as the range of the Golden Ratio (Range Value: 0.17).\(^4\)

(a) Root Rectangle Composition:

Root Rectangle Section is one of partition methods. There are mainly \(\sqrt{2}\), \(\sqrt{3}\), \(\sqrt{4}\), \(\sqrt{5}\) rectangle composition (Figure 5).

In this study, the author import the facade into the Rhino 6.0 software and write parametric programs with the assistance of the software plug-in of Grasshopper. Based on the analysis of the slope \(k\) of the diagonals of rectangles (\(k\) of \(\sqrt{2}\) rectangles (Range: 1.32–1.49/Range Value: 0.17), \(k\) of golden rectangles (Range: 1.5–1.67/Range Value: 0.17), \(k\) of \(\sqrt{3}\) rectangles (Range: 1.68–1.85/Range Value: 0.17), \(k\) of \(\sqrt{4}\) rectangles (Range: 1.915–2.085/Range Value: 0.17) and \(k\) of \(\sqrt{5}\) rectangles (Range: 2.151–2.321/Range Value: 0.17) with 0.01 as the partition value of the range of \(k\), this study finds that the nodes and sides of golden rectangles have the largest numbers and coverage area and are usually the main controlling line of the division of the facade, and that these golden rectangles brings about golden spirals through successive partition.

Accordingly, this study concludes that Golden Ratio is an important foundation of the partition of the facade of the Macau Holy House of Mercy.

5.2. A parametric analysis of the facade: the complicated combination of rectangles and spirals controlled by golden ratio

(1) Golden Rectangles From the One-time Partition (Table 2)

1. The Main Rectangle 1 = Sub-rectangle 1 + Square 1

Main Rectangle 1: Line A1-A19 is 22,000 mm long and Line A1-P1 is 13,200 mm long. Their proportion is 1.67, approximate to the Golden Ratio, that is to say that the proportion of the overall width of the facade to the height of the top edge of the sculpture embellishments of parapets is in accordance with the Golden Ratio, bringing about Main Golden Rectangle 1: A1-A19-P19-P1.

Sub-rectangle1: Line A13-A19 is 7,700 mm long and Line A13-P13 is 13,200 mm long. Their proportion is 0.58, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 2: A13-A19-P19-P13.

Square 1: Line A1-A13 is 14,300 mm long and Line A1-P1 is 13,200 mm long. Their proportion is approximate to 1:1, which brings about Square 1: A1-A13-P13-P1

![Figure 4. The composition of golden section (the fibonacci sequence).](image-url)
2. The Main Rectangle 2 = Sub-rectangle 2 + Square 2

Sub-rectangle 2: Line A10-A16 is 6,800 mm long and Line A10-L10 is 10,800 mm long. Their proportion is 0.63, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 2: A10-A16-L16-L10.

Square 2: Line A1-A10 is 11,000 mm long and Line A1-L1 is 10,800 mm long. Their proportion is approximate to 1:1, which brings about Square 2: A1-A10-L10-L1.

1. The Main Rectangle 3 = Sub-rectangle 3 + Square 3

Main Rectangle 3: Line A1-A15 is 16,350 mm long and Line A1-K1 is 9,920 mm long. Their proportion is 1.65, approximate to the Golden Ratio, that is to say that the proportion of the distance between the left outside of whole façade and the right side of the left column in the right part of the façade to the height of the highest point of the arch holes in the second storey is in accordance with the Golden Ratio, bringing about Main Golden Rectangle 3: A1-A15-K15-K1.

Sub-rectangle 3: Line A9-A15 is 6,100 mm long and Line A9-K9 is 9,920 mm long. Their proportion is 0.61, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 3: A9-A15-K15-K9.

Square 3: Line A1-A9 is 10,250 mm long and Line A1-K1 is 9,920 mm long. Their proportion is approximate to 1:1, which brings about Square 3: A1-A9-K9-K1.

1. The Main Rectangle 4 = Sub-rectangle 4 + Square 4

Main Rectangle 4: Line A1-A6 is 6,650 mm long and Line A1-D1 is 4,000 mm long. Their proportion is 1.66, approximate to the Golden Ratio, that is to say that the proportion of the width of the left part of the façade to

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Table 2. Golden Rect from the one-time partition.

| Golden Rect. From the One-time Partition | Width | Height | Ratio |
|-----------------------------------------|-------|--------|-------|
| Main Rect. 1                           | 22,000| 13,200 | 1.67  |
| Sub-Rect. 1                            | 7,700 | 13,200 | 0.58  |
| Square 1                               | 14,300| 13,200 | 1.08  |
| Main Rect. 2                           | 17,800| 10,800 | 1.65  |
| Sub-Rect. 2                            | 6,800 | 10,800 | 0.63  |
| Square 2                               | 11,000| 10,800 | 1.02  |
| Main Rect. 3                           | 16,350| 9,920  | 1.65  |
| Sub-Rect. 3                            | 6,100 | 9,920  | 0.61  |
| Square 3                               | 10,250| 9,920  | 1.03  |
| Main Rect. 4                           | 6,650 | 4,000  | 1.66  |
| Sub-Rect. 4                            | 2,450 | 4,000  | 0.61  |
| Square 4                               | 4,200 | 4,000  | 1.05  |
| Main Rect. 5                           | 13,600| 8,320  | 1.63  |
| Sub-Rect. 5                            | 5,000 | 8,320  | 0.60  |
| Square 6                               | 8,600 | 8,320  | 1.03  |
| Main Rect. 6                           | 13,600| 8,300  | 1.64  |
| Sub-Rect. 6                            | 5,000 | 8,300  | 0.60  |
| Square 7                               | 8,600 | 8,300  | 1.04  |
| Main Rect. 7                           | 6,600 | 4,000  | 1.65  |
| Sub-Rect. 8                            | 2,550 | 4,000  | 0.64  |
| Square 8                               | 4,050 | 4,000  | 1.01  |
| Main Rect. 9                           | 6,600 | 4,080  | 1.62  |
| Sub-Rect. 9                            | 2,550 | 4,080  | 0.63  |
| Square 9                               | 4,050 | 4,080  | 0.99  |

NOTE: "Rectangle" is abbreviated as "Rect." in this table.

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5 Please be noted that “Rectangle” is abbreviated as "Rect." in this table.
the height of the highest point of the arch holes in the first storey is in accordance with the Golden Ratio, bringing about Main Golden Rectangle 4: A1-A6-D6-D1.

Sub-rectangle 4: Line A4-A6 is 2,450 mm long and Line A4-D4 is 4,000 mm long. Their proportion is 0.61, which brings about Sub-golden-rectangle 4: A4-A6-D6-D4.

Square 4: Line A1-A4 is 4,200 mm long and Line A1-D1 is 4,000 mm long. Their proportion is approximate to 1:1, which brings about Square 4: A1-A4-D4-D1.

(1) The Main Rectangle 5 = Sub-rectangle 5 + Square 5

Main Rectangle 5: Line B1-B19 is 22,000 mm long and Line B1-Q1 is 13,120 mm long. Their proportion is 1.68, approximate to the Golden Ratio, that is to say that the proportion of the overall width of the façade to the distance between the bottom edge of the pediment and the top edges of the column bases in the first storey is in accordance with the Golden Ratio, bringing about Main Golden Rectangle 5: B1-B19-Q19-Q1.

Sub-rectangle 5: Line B13-B19 is 7,700 mm long and Line B13-Q13 is 13,120 mm long. Their proportion is 0.59, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 5: B13-B19-Q19-Q13.

Square 5: Line B1-B13 is 14,300 mm long and Line B1-Q1 is 13,120 mm. Their proportion is approximate to 1:1, which brings about Square 5: B1-B13-Q13-Q1.

(1) The Main Rectangle 6 = Sub-rectangle 6 + Square 6

Main Rectangle 6: Line B4-B16 is 13,600 mm long and Line B4-J4 is 8,320 mm long. Their proportion is 1.63, approximate to the Golden Ratio, that is to say that the proportion of the distance between the right side of the middle column in the left part of the façade and the left side of the middle column in the right part of the façade to the distance between the top edges of casements in the second storey and the top edges of the column bases in the first storey is in accordance with the Golden Ratio, bringing about Main Golden Rectangle 6: B4-B16-J16-J4.

Sub-rectangle 6: Line B12-B16 is 5,000 mm long and Line B12-J12 is 8,320 mm long. Their proportion is 0.60, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 6: B12-B16-J16-J12.

Square 6: Line B4-B12 is 8,600 mm long and Line B4-J4 is 8,320 mm. Their proportion is approximate to 1:1, which brings about Square 6: B4-B12-J12-J4.

(1) The Main Rectangle 7 = Sub-rectangle 7 + Square 7

Main Rectangle 7: Line C4-C16 is 13,600 mm long and Line C4-M4 is 8,300 mm long. Their proportion is 1.64, approximate to the Golden Ratio, that is to say that the proportion of the distance between the right side of the middle column in the left part of the façade and the left side of the middle column in the right part of the façade to the distance between the bottom edges of parapets and the separating line of two-layer stacked column in the middle of the façade is in accordance with the Golden Ratio, bringing about Main Golden Rectangle 7: C4-C16-M16-M4.

Sub-rectangle 7: Line C12-C16 is 5,000 mm long and Line C12-M12 is 8,300 mm long. Their proportion is 0.60, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 7: C12-C16-M16-M12.

Square 7: Line C4-C12 is 8,600 mm long and Line C4-M4 is 8,300 mm. Their proportion is approximate to 1:1, which brings about Square 7: C4-C12-M12-M4.

(1) The Main Rectangle 8 = Sub-rectangle 8 + Square 8

Main Rectangle 8: Line A7-A13 is 6,600 mm long and Line A7-D7 is 4,000 mm long. Their proportion is 1.65, approximate to the Golden Ratio, that is to say that the proportion of the distance between the left side of the left arch hole and the right side of the right arch hole in the middle of the façade and the height of the highest points of the arch holes in the first storey is in accordance with the Golden Ratio, bringing about Main Golden Rectangle 8: A7-A13-D13-D7.

Sub-rectangle 8: Line A11-A13 is 2,550 mm long and Line A11-D11 is 4,000 mm long. Their proportion is 0.64, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 8: A11-A13-D13-D11.

Square 8: Line A7-A11 is 4,050 mm long and Line A7-D7 is 4,000 mm. Their proportion is approximate to 1:1, which brings about Square 8: A7-A11-D11-D7.

(1) The Main Rectangle 9 = Sub-rectangle 9 + Square 9

Main Rectangle 9: Line K7-K13 is 6,600 mm long and Line K7-Q7 is 4,080 mm long. Their proportion is 1.62, approximate to the Golden Ratio, that is to say that the proportion of the left side of the left arch hole and the right side of the right arch hole in the middle of the façade to the distance between the bottom edges of the pediment and the highest points of the arch holes in the second storey is in accordance with the Golden Ratio, bringing about Main Golden Rectangle 9: K7-K13-Q13-Q7.

Sub-rectangle 9: Line K11-K13 is 2,550 mm long and Line K11-Q11 is 4,080 mm long. Their proportion is
0.63, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 9: K11-K13-Q13-Q11.

Square 9: Line K7-K11 is 4,050 mm long and Line K7-Q7 is 4,080 mm. Their proportion is approximate to 1:1, which brings about Square 9: K7-K11-Q11-Q7.

(1) Golden Rectangles and Spirals from the Two-times Partition (Table 3)

(2) Main Rectangle 10 = Sub-rectangle 10 + Square 10; Sub-rectangle 10 = Sub-rectangle 10.1 + Square 10.1

Main Rectangle 10: Line C6-C15 is 9,700 mm long and Line C6-J6 is 5,900 mm. Their proportion is 1.64, approximate to the Golden Ratio, that is to say that the proportion of the distance between the left side of the middle of the façade and the right side of the left column of the right part of the façade to the distance between the top edges of the casements in the second storey and the separating line of two-layer stacked column in the middle of the façade is in accordance with the Golden Ratio, bringing about Main Golden Rectangle 10: C6-C15-J15-J6.

Sub-rectangle 10: Line C12-C15 is 3,550 mm long and Line C12-J12 is 5,900 mm long. Their proportion is 0.60, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 10: C12-C15-J15-J12.

Square 10: Line C6-C12 is 6,150 mm long and Line C6-J6 is 5,900 mm long. Their proportion is approximate to 1:1, which brings about Square 10: C6-C12-J12 -J6.

Sub-rectangle 10.1: Line C12-C15 is 3,550 mm long and Line C12-F12 is 2,100 mm long. Their proportion is 1.69, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 10.1: C12-C15-F15-F12.

Square 10.1: Line F12-F15 is 3,550 mm long and Line F12-J12 is 3,800 mm long. Their proportion is approximate to 1:1, which brings about Square 10.1: F12-F15-J12-J15.

(1) Main Rectangle 11 = Sub-rectangle 11 + Square 11; Sub-rectangle 11 = Sub-rectangle 11.1 + Square 11.1

Main Rectangle 11: Line E13-E19 is 7,700 mm long and Line E13-J13 is 4,600 mm long. Their proportion is 1.67, approximate to the Golden Ratio, that is to say that the proportion of the distance between the right side of the right arch hole in the middle of the façade and the right side of the façade to the distance between the top edges of the casements in the second storey and the bottom edge of the capitals is in accordance with the Golden Ratio, bringing about Main Golden Rectangle 11: E13-E19-J19-J13.

Sub-rectangle 11: Line E17-E19 is 2,800 mm long and Line E17-J17 is 4,600 mm long. Their proportion is 0.61, which brings about Sub-golden-rectangle 11: E17-E19-J19-J17.

Square 11: Line E13-E17 is 4,900 mm long and Line E13-J13 is 4,600 mm long. Their proportion is approximate to 1:1, which brings about Square 11: E13-E17-J17-J13.

Sub-rectangle 11.1: Line E17-E19 is 2,800 mm long and Line E17-G17 is 1,700 mm long. Their proportion is 1.65, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 11.1: E17-E19-G19-G17.

Square 11.1: Line G17-G19 is 2,800 mm long and Line G17-J17 is 2,900 mm long. Their proportion is approximate to 1:1, which brings about Square 11.1: G17-G19-J19-J17.

| Table 3. Golden Rect from the two-times partition |
|-----------------------------------------------|
| **Main Rect. and Spirals from the Two-times Partition** | **Width** | **Height** | **Ratio** |
| Main Rect. 10 | C6-C15-J15-J6 | 9,700 | 5,900 | 1.64 |
| Sub- Rect. 10 | C12-C15-J15-J12 | 3,550 | 5,900 | 0.60 |
| Sub-Rect. 10.1 | C12-C15-F15-F12 | 3,550 | 2,100 | 1.69 |
| Square 10.1 | F12-F15-J15-J12 | 3,550 | 3,800 | 0.93 |
| Square 10.1 | C6-C12-J12-J6 | 6,150 | 5,900 | 1.04 |
| Main Rect. 11 | E13-E19-J19-J13 | 7,700 | 4,600 | 1.67 |
| Sub-Rect. 11 | E17-E19-J19-J17 | 2,800 | 4,600 | 0.61 |
| Sub-Rect. 11.1 | E17-E19-G19-G17 | 2,800 | 2,900 | 0.97 |
| Square 11.1 | G17-G19-J19-J17 | 4,000 | 4,600 | 1.07 |
| Main Rect. 12 | H5-H13-N13-N5 | 8,600 | 5,300 | 1.63 |
| Sub-Rect. 12 | H10-H13-N13-N10 | 3,300 | 5,300 | 0.62 |
| Sub-Rect. 12.1 | H10-J13-N13-N10 | 3,300 | 2,000 | 1.65 |
| Square 12.1 | J10-J13-N13-N10 | 3,300 | 3,300 | 1.00 |
| Main Rect. 13 | J6-J12-P12-P6 | 6,150 | 4,000 | 1.54 |
| Sub-Rect. 13 | J9-J12-P12-P9 | 2,550 | 4,000 | 0.64 |
| Sub-Rect. 13.1 | J9-J12-L12-L9 | 2,550 | 1,600 | 1.59 |
| Square 13.1 | L9-L12-P12-P9 | 2,550 | 2,400 | 1.06 |
| Main Rect. 14 | J8-J14-P14-P8 | 6,150 | 4,000 | 1.54 |
| Sub-Rect. 14 | J12-J14-P14-P12 | 2,550 | 4,000 | 0.64 |
| Sub-Rect. 14.1 | J12-J14-L14-L12 | 2,550 | 1,600 | 1.59 |
| Square 14.1 | L12-L14-P14-P12 | 2,550 | 2,400 | 1.06 |
| Square 14 | J8-J12-P12-P8 | 3,600 | 4,000 | 0.90 |
(1) Main Rectangle 12 = Sub-rectangle 12 + Square 12; Sub-rectangle 12 = Sub-rectangle 12.1 + Square 12.1

Main Rectangle 12: Line H5-H13 is 8,650 mm long and Line H5-N5 is 5,300 mm long. Their proportion is 1.63, approximate to the Golden Ratio, that is to say that the proportion of the distance between the left side of the right column in the left part of the façade and the right side of the right arch hole in the middle of the façade to the distance between the top edges of the parapets and the top edges of the handrails in the second storey is in accordance with the Golden Ratio, bringing about Main Golden Rectangle 12: H5-H13-N13-N5.

Sub-rectangle 12: Line H10-H13 is 3,300 mm long and Line H10-N10 is 5,300 mm long. Their proportion is 0.62, which brings about Sub-golden-rectangle 12: H10-H13-N13-N10.

Square 12: Line H5-H10 is 5,350 mm long and Line H5-N5 is 5,300 mm long. Their proportion is approximate to 1:1, which brings about Square 12: H5-H10-N10-N5.

Sub-rectangle 12.1: Line H10-H13 is 3,300 mm long and Line H10-J10 is 2,000 mm long. Their proportion is 1.65, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 12.1: H10-H13-J13-J10.

Square 12.1: Line J10-J13 is 3,300 mm long and Line J10-N10 is 3,300 mm. Their proportion is 1:1, which brings about Square 12.1: J10-J13-N13-N10.

(1) Main Rectangle 13 = Sub-rectangle 13 + Square 13; Sub-rectangle 13 = Sub-rectangle 13.1 + Square 13.1

Main Rectangle 13: Line J6-J12 is 6,150 mm long and Line J6-P6 is 4,000 mm long. Their proportion is 1.54, approximate to the Golden Ratio, that is to say that the proportion of the distance between the left side of the middle of the façade and the left side of the right arch hole of the middle of the façade to the distance between the peaks of sculpture embellishments of the parapets and the top edges of the casements in the second storey is in accordance with the Golden Ratio, bringing about Main Golden Rectangle 13: J6-J12-P12-P6.

Sub-rectangle 13: Line J9-J12 is 2,550 mm long and Line J9-P9 is 4,000 mm long. Their proportion is 0.64, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 13: J9-J12-P12-P9.

Square 13: Line J6-J9 is 3,600 mm long and Line J6-P6 is 4,000 mm long. Their proportion is approximate to 1:1, which brings about Square 13: J6-J9-P9-P6.

Sub-rectangle 13.1: Line J9-J12 is 2,550 mm long and Line J9-L9 is 1,600 mm long. Their proportion is 1.59, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 13.1: J9-J12-L12-L9.

Square 13.1: Line L9-L12 is 2,550 mm long and Line L9-P9 is 2,400 mm long. Their proportion is approximate to 1:1, which brings about Square 13.1: L9-L12-P12-P9.

(1) Main Rectangle 14 = Sub-rectangle 14 + Square 14; Sub-rectangle 14 = Sub-rectangle 14.1 + Square 14.1

Main Rectangle 14: Line J8-J14 is 6,150 mm long and Line J8-P8 is 4,000 mm long. Their proportion is 1.54, approximate to the Golden Ratio, that is to say that the proportion of the distance between the right side of the left arch hole in the middle of the façade and the right side of the middle of the façade to the distance between the peaks of sculptures embellishments of the parapets and the top edges of the casements in the second storey is in accordance with the Golden Ratio, bringing about Main Golden Rectangle 14: J8-J14-P14-P8.

Sub-rectangle 14: Line J12-J14 is 2,550 mm long and Line J12-P12 is 4,000 mm long. Their proportion is 0.64, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 14: J12-J14-P14-P12.

Square 14: Line J8-J12 is 3,600 mm long and Line J8-P8 is 4,000 mm long. Their proportion is approximate to 1:1, which brings about Square 14: J8-J12-P12-P8.

Sub-rectangle 14.1: Line J12-J14 is 2,550 mm long and Line J12-L12 is 1,600 mm long. Their proportion is 1.59, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 14.1: J12-J14-L14-L12.

Square 14.1: Line L12-L14 is 2,550 mm long and Line L12-P12 is 2,400 mm long. Their proportion is approximate to 1:1, which brings about Square 14.1: L12-L14-P14-P12.

(1) Golden Rectangles and Spirals from the Three-times Partition (Table 4)

1. Main Rectangle 15 = Sub-rectangle 15 + Square 15; Sub-rectangle 15 = Sub-rectangle 15.1 + Square 15.1; Sub-rectangle 15.1 = Sub-rectangle 15.1.1 + Square 15.1.1

Main Rectangle 15: Line B2-B15 is 15,100 mm long and Line B2-K2 is 9,040 mm long. Their proportion is 1.67, approximate to the Golden Ratio, that is to say that the proportion of the distance between the right side of the left column in the left part of the façade and the right side of the left column in the right part of the façade to the distance between the highest point of the arch hole in the second storey and the top edges of
the column bases in the first storey is in accordance with the Golden Ratio, bringing about Main Golden Rectangle 15: B2-B15-K15-K2.

Sub-rectangle 15: Line B10-B15 is 5,350 mm long and Line B10-K10 is 9,040 mm long. Their proportion is 0.59, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 15: B10-B15-K15-K10.

Square 15: Line B2-B10 is 9,750 mm long and Line B2-K2 is 9,040 mm long. Their proportion is approximate to 1:1, which brings about Square 15: B2-B10-K10-K2.

Sub-rectangle 15.1: Line B10-B15 is 5,350 mm long and Line B10-D10 is 3,120 mm long. Their proportion is 1.71, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 15.1: B10-B15-D15-D10.

Square 15.1: Line D10-D15 is 5,920 mm long and Line D10-K10 is 5,920 mm long. Their proportion is approximate to 1:1, which brings about Square 15.1: D10-D15-K15-K10.

Sub-rectangle 15.1.1: Line B10-B12 is 1,800 mm long and Line B10-D10 is 3,120 mm long. Their proportion is 0.58, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 15.1.1: B10-B12-D12-D10.

Square 15.1.1: Line B12-B15 is 3,550 mm long and Line B12-D12 is 3,120 mm long. Their proportion is approximate to 1:1, which brings about Square 15.1.1: B12-B15-D15-D12.

| Golden Rect. and Spirals from the Three-times Partition | Width   | Height   | Ratio   |
|--------------------------------------------------------|---------|----------|---------|
| Main Rect. 15  B2-B15-K15-K2                          | 15,100  | 9,040    | 1.67    |
| Sub-Rect. 15  B10-B15-K15-K10                          | 5,350   | 9,040    | 0.59    |
| Sub-Rect. 15.1 B10-B15-D15-D10                         | 5,350   | 3,120    | 1.71    |
| Sub-Rect. 15.1.1 B10-B12-D12-D10                       | 1,800   | 3,120    | 0.58    |
| Square 15.1   D10-D15-K15-K10                          | 3,550   | 3,120    | 1.14    |
| Main Rect. 16 A4-A19-L19-L4                           | 17,800  | 10,800   | 1.65    |
| Sub-Rect. 16  A14-A19-L19-L14                          | 6,650   | 10,800   | 0.62    |
| Sub-Rect. 16.1 A14-A16-D16-D14                         | 6,650   | 4,000    | 1.66    |
| Square 16.1   A16-A19-D16-D16                          | 4,200   | 4,000    | 1.05    |
| Square 16.1   D14-D19-L19-L14                          | 6,650   | 6,800    | 0.98    |
| Main Rect. 17 K7-K13-Q13-Q7                           | 11,150  | 10,800   | 1.03    |
| Sub-Rect. 17  K11-K13-Q13-Q11                          | 6,600   | 4,080    | 1.62    |
| Sub-Rect. 17.1 K11-K13-M13-M11                         | 2,550   | 1,680    | 1.52    |
| Square 17.1   K11-K12-M12-M11                          | 1,050   | 1,680    | 0.63    |
| Main Rect. 18 K3-K8-Q8-Q3                             | 6,400   | 4,080    | 1.57    |
| Sub-Rect. 18  K6-K8-Q8-Q6                              | 2,550   | 4,080    | 0.63    |
| Sub-Rect. 18.1 K6-K8-M8-M6                             | 2,550   | 1,680    | 1.52    |
| Square 18.1   K6-K8-M7-M7                              | 1,050   | 1,680    | 0.63    |
| Square 18.1   M6-M8-Q8-Q6                              | 2,550   | 2,400    | 1.06    |
| Main Rect. 19 J5-J11-P11-P5                           | 3,850   | 4,080    | 0.94    |
| Sub-Rect. 19  J8-J11-P11-P8                            | 6,100   | 4,000    | 1.53    |
| Sub-Rect. 19.1 J8-J11-L11-L8                          | 2,550   | 4,000    | 0.64    |
| Square 19.1   J8-J9-L9-L9                             | 1,050   | 1,600    | 0.66    |
| Main Rect. 20 J3-J8-P8-P3                             | 4,050   | 4,080    | 0.99    |
| Sub-Rect. 20  J6-J8-P8-P6                              | 6,400   | 4,080    | 1.57    |
| Sub-Rect. 20.1 J6-J8-L8-L6                             | 2,550   | 4,000    | 0.64    |
| Square 20.1   J6-J7-L7-L7                             | 2,550   | 2,400    | 1.06    |
| NOTE: “Rectangle” is abbreviated as “Rect.” in this table.
is 0.62, which brings about Sub-golden-rectangle 16: A14-A19-L19-L14.

Square 16: Line A4-A14 is 11,150 mm long and Line A4-L14 is 10,800 mm long. Their proportion is approximately to 1:1, which brings about Square 16: A4-A14-L14-L4.

Sub-rectangle 16.1: Line A14-A19 is 6,650 mm long and Line A14-D14 is 4,000 mm long. Their proportion is 1.66, approximate to the Golden Ratio, which brings about sub-golden-rectangle 16.1: A14-A19-D19-D14.

Square 16.1: Line D14-D19 is 6,650 mm long and Line D14-L14 is 6,800 mm long. Their proportion is approximate to 1:1, which brings about Square 16.1: D14-D19-L19-L14.

Sub-rectangle 16.1.1: Line A14-A16 is 2,450 mm long and Line A14-D14 is 4,000 mm long. Their proportion is 0.61, which brings about Sub-golden-rectangle 16.1.1: A14-A16-D16-D14.

Square 16.1.1: Line A16-A19 is 4,200 mm long and Line A16-D16 is 4,000 mm long. Their proportion is approximate to 1:1, which brings about Square 16.1.1: A16-A19-D19-D16.

(1) Main Rectangle 17 = Sub-rectangle 17 + Square 17; Sub-rectangle 17 = Sub-rectangle 17.1 + Square 17.1; Sub-rectangle 17.1 = Sub-rectangle 17.1.1 + Square 17.1.1

Main Rectangle 17: Line K7-K13 is 6,600 mm long and Line K7-Q7 is 4,080 mm long. Their proportion is 1.62, that is to say that the proportion of the distance between the left side of the left arch hole and the right side of the right arch hole in the middle of the facade to the distance between the bottom edge of the pediment and the highest point of the arch hole in the second storey is in accordance with the Golden Ratio, which brings about Main Golden Rectangle 17: K7-K13-Q13-Q7.

Sub-rectangle 17: Line K11-K13 is 2,550 mm long and Line K11-Q11 is 4,080 mm long. Their proportion is 0.63, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 17: K11-K13-Q13-Q11.

Square 17: Line K7-K11 is 4,050 mm long and Line K7-Q7 is 4,080 mm long. Their proportion is approximate to 1:1, which brings about Square 17: K7-K11-Q11-Q7.

Sub-rectangle 17.1: Line K11-K13 is 2,550 mm long and Line K11-M11 is 1,680 mm long. Their proportion is 1.52, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 17.1: K11-K13-M13-M11.

Square 17.1: Line M11-M13 is 2,550 mm long and Line M11-Q11 is 2,400 mm long. Their proportion is approximate to 1:1, which brings about Square 17.1: M11-M13-Q13-Q11.

Sub-rectangle 17.1.1: Line K11-K12 is 1,050 mm long and Line K11-M11 is 1,680 mm long. Their proportion is 0.63, approximate to the Golden Ratio, that is to say that the proportion of the width of a column in the middle of the facade (Modular c) to the distance between the bottom edge of the parapet and the highest point of the arch hole in the second storey is in accordance with the Golden Ratio, bringing about Sub-golden-rectangle 17.1.1: K11-K12-M12-M11.

Square 17.1.1: Line K12-K13 is 1,500 mm long and K12-M12 is 1,680 mm long. Their proportion is approximate to 1:1, that is to say that the proportion of the width of a arch hole in the middle of the facade (Modular d) to the distance between the bottom edge of the parapet and the highest point of the arch hole in the second storey is in accordance with the one of a square, bringing about Square 17.1.1: K12-K13-M13-M12.

(1) Main Rectangle 18 = Sub-rectangle 18 + Square 18; Sub-rectangle 18 = Sub-rectangle 18.1 + Square 18.1; Sub-rectangle 18.1 = Sub-rectangle 18.1.1 + Square 18.1.1

Main Rectangle 18: Line K3-K8 is 6,400 mm long and Line K3-Q3 is 4,080 mm long. Their proportion is 1.57, approximate to the Golden Ratio, that is to say that the proportion of the distance between the left side of the middle column in the left part of the facade and the right side of the left arch hole in the middle of the facade to the distance between the bottom edge of the pediment and the highest point of the arch hole in the second storey is in accordance with the Golden Ratio, bringing about Main Golden Rectangle 18: K3-K8-Q8-Q3.

Sub-rectangle 18: Line K6-K8 is 2,550 mm long and Line K6-Q6 is 4,080 mm long. Their proportion is 0.63, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 18: K6-K8-Q8-Q6.

Square 18: Line K3-K6 is 3,850 mm long and Line K3-Q3 is 4,080 mm long. Their proportion is approximate to 1:1, which brings about Square 18: K3-K6-Q6-Q3.

Sub-rectangle 18.1: Line K6-K8 is 2,550 mm long and Line K6-M6 is 1,680 mm long. Their proportion is 1.52, which brings about Sub-golden-rectangle 18.1: K6-K8-M8-M6.

Square 18.1: Line M6-M8 is 2,550 mm long and Line M6-Q6 is 2,400 mm long. Their proportion is approximate to 1:1, which brings about Square 18.1: M6-M8-Q8-Q6.

Sub-rectangle 18.1.1: Line K6-K7 is 1,050 mm long and Line K6-M6 is 1,680 mm long. Their proportion is 0.63, approximate to the Golden Ratio, that is to say that the proportion of the width of a column in the middle of the facade (Modular c) to the distance between the bottom edge of the parapet and the highest point of the arch hole in the second storey is in accordance with the Golden Ratio, bringing about Sub-golden-rectangle 18.1.1: K6-K7-M7-M6.
Square 18.1.1: Line K7-K8 is 1,500 mm long and Line K7-M7 is 1,680 mm long. Their proportion is approximate to 1:1, that is to say that the proportion of the width of an arch hole in the middle of the façade (Modular d) to the distance between the bottom edge of the parapet and the highest point of the arch hole in the second storey is in accordance with the one of a square, bringing about Square 18.1.1: K7-K8-M8-M7.

(1) Main Rectangle 19 = Sub-rectangle 19 + Square 19; Sub-rectangle 19 = Sub-rectangle 19.1 + Square 19.1; Sub-rectangle 19.1 = Sub-rectangle 19.1.1 + Square 19.1.1

Main Rectangle 19: Line J5-J11 is 6,100 mm long and Line J5-P5 is 4,000 mm long. Their proportion is 1.53, approximate to the Golden Ratio, that is to say that the proportion of the distance between the left side of the right column in the left part of the façade and the right side of the middle arch hole in the middle of the façade to the distance between the peaks of sculpture embellishments of the parapet and the top edge of the casement in the second storey is in accordance with the Golden Ratio, bringing about Main Golden Rectangle 19: J5-J11-P11-P5.

Sub-rectangle 19: Line J8-J11 is 2,550 mm long and Line J8-P8 is 4,000 mm long. Their proportion is 0.64, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 19: J8-J11-P11-P8.

Square 19: Line J5-J8 is 3,550 mm long and Line J5-P5 is 4,000 mm long. Their proportion is approximate to 1:1, which brings about Square 19: J5-J8-P8-P5.

Sub-rectangle 19.1: Line J8-J11 is 2,550 mm long and Line J8-L8 is 1,600 mm long. Their proportion is 1.59, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 19.1: J8-J11-L11-L8.

Square 19.1: Line L8-L11 is 2,550 mm long and Line L8-P8 is 2,400 mm long. Their proportion is approximate to 1:1, which brings about Square 19.1: L8-L11-P11-P8.

Sub-rectangle 19.1.1: Line J8-J9 is 1,050 mm long and Line J8-L8 is 1,600 mm long. Their proportion is 0.66, approximate to the Golden Ratio, that is to say that the proportion of the width of a column in the middle of the façade (Modular c) to the distance between the bottom edge of the eaves in the second storey and the top edge of the casement in the second storey is in accordance with the one of a square, bringing about Square 19.1.1: J8-J9-L9-L8.

Square 19.1.1: Line J9-J11 is 1,500 mm long and Line J9-L9 is 1,600 mm long. Their proportion is approximate to 1:1, that is to say that the proportion of the width of an arch hole in the middle of the façade (Modular d) to the distance between the bottom edge of the eaves in the second storey and the top edge of the casement in the second storey is in accordance with the one of a square, bringing about Square 19.1.1: J9-J11-L11-L9.

(1) Main Rectangle 20 = Sub-rectangle 20 + Square 20; Sub-rectangle 20 = Sub-rectangle 20.1 + Square 20.1; Sub-rectangle 20.1 = Sub-rectangle 20.1.1 + Square 20.1.1

Main Rectangle 20: Line J3-J8 is 6,400 mm long and Line J3-P3 is 4,000 mm long. Their proportion is 1.60, approximate to the Golden Ratio, that is to say that the proportion of the distance between the left side of the middle column in the left part of the façade and the right side of the left arch hole in the middle of the façade to the distance between the peaks of sculpture embellishments and the top edge of the casement in the second storey is in accordance with the Golden Ratio, bringing about Main Golden Rectangle 20: J3-J8-P8-P3.

Sub-rectangle 20: Line J6-J8 is 2,550 mm long and Line J6-P6 is 4,000 mm long. Their proportion is 0.64, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 20: J6-J8-P8-P6.

Square 20: Line J3-J6 is 3,850 mm and Line J3-P3 is 4,000 mm long. Their proportion is approximate to 1:1, which brings about Square 20: J3-J6-P6-P3.

Sub-rectangle 20.1: Line J6-J8 is 2,550 mm long and Line J6-L6 is 1,600 mm long. Their proportion is 1.59, approximate to the Golden Ratio, which brings about Sub-golden-rectangle 20.1: J6-J8-L8-L6.

Square 20.1: Line L6-L8 is 2,550 mm long and Line L6-P6 is 2,400 mm long. Their proportion is approximate to 1:1, which brings about Square 20.1: L6-L8-P8-P6.

Sub-rectangle 20.1.1: Line J6-J7 is 1,050 mm long and Line J6-L6 is 1,600 mm long. Their proportion is 0.66, approximate to the Golden Ratio, that is to say that the proportion of the width of a column in the middle of the façade (Modular c) to the distance between the bottom edge of the eave and the top edge of the casement in the second storey is in accordance with the Golden Ratio, bringing about Sub-golden-rectangle 20.1.1: J6-J7-L7-L6.

Square 20.1.1: Line J7-J8 is 1,500 mm long and Line J7-L7 is 1,600 mm long. Their proportion is approximate to 1:1, that is to say that the proportion of the width of an arch hole in the middle of the façade (Modular d) to the distance between the bottom edge of the eave and the top edge of the casement in the second storey is in accordance with the one of a square, bringing about Square 20.1.1: J7-J8-L8-L7.

The golden rectangles, spirals and their partitions is bilaterally symmetric along the Medial Axis.
6. Conclusions

The Macau Holy House of Mercy was built in 1569, rebuilt in the 18th century and accepted additional construction in 1905. Its façade is sophisticated in its design:

(1) Its designers applied three-section partition in the horizontal and vertical directions in the overall dimension of the façade and “equal stratification” in the first and second layers of the façade, which is different from the Upward Successive-subtraction in Stratification in the vertical direction in the Baroque-style St. Paul’s Church in Macao.
(2) Its designers applied modularization (modular) design and combination in details of the columns, windows and doorways.
(3) Parametric analyses with the assistance of the modeling software of Rhinoceros 6.0 and the software plug-in of Grasshopper found that the (successive) partition of the façade is closer to the “Golden Section” than “Root Rectangle Partition”. Further analyses found that the golden ratio has been applied in the partitions in all horizontal and vertical axes, and parts of these golden ratios brought about Golden Spirals (successive partitions) in the façade.

The underlying aesthetics-mathematics pattern of “digitally-derived geometric design” is of unparalleled historical value.

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