Supersymmetry Then and Now

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ABSTRACT

A brief description of some salient aspects of four-dimensional supersymmetry: early history, supermanifolds, the MSSM, cold dark matter, the cosmological constant and the string landscape.
1 A brief history of the beginning of supersymmetry

Four dimensional supersymmetry (SUSY) has been discovered independently three times: first in Moscow, by Golfand and Likhtman, then in Kharkov, by Volkov and Akulov, and Volkov and Soroka, and finally by Julius Wess and me, who collaborated at CERN in Geneva and in Karlsruhe. It is remarkable that Volkov and his collaborators didn’t know about the work of Golfand and Likhtman, since all of them were writing papers in Russian in Soviet journals. Julius and I were totally unaware of the earlier work. For information on the life and work of Golfand and Likhtman, I refer to the Yuri Golfand Memorial Volume [1]. For information on Volkov’s life and work, I refer to the Proceedings of the 1997 Volkov Memorial Seminar in Kharkov [2].

Supersymmetry is a symmetry which relates the properties of integral-spin bosons to those of half-integral-spin fermions. The generators of the symmetry form what has come to be called a superalgebra, which is a super extension of the Poincaré Lie algebra of quantum field theory (Lorentz transformations and space-time translations) by fermionic generators. In a superalgebra both commutators and anticommutators occur. The study of superalgebras is relevant to the study of dynamical systems with both bosonic and fermionic quantities; very interesting work on such systems was done in Moscow by the mathematician F.A. Berezin and his collaborators. It is amusing that one of them, D.A. Leites, in a book he wrote on the subject, has attributed the origin of the prefix super to the exaggerated enthusiasm of physicists. The truth is that, like many other words in physics and mathematics the technical word ”super” never had any of the connotations it has in everyday language.

The work of Golfand and Likhtman and that of Volkov and collaborators went to a large extent unnoticed. Instead, the first three preprints Julius and I wrote aroused immediately the interest of numerous theoretical physicists, even before publication, and the subject took on a life of its own, to which we continued to contribute both together and separately, with other collaborators. Our early papers also gave rise to renewed interest by mathematicians in the theory of superalgebras. Eventually a complete classification of simple and semisimple superalgebras was obtained, analogous to Cartan’s classification of Lie algebras, and even the prefix super was adopted in mathematics. Unfortunately the Poincaré superalgebra is not semisimple, although it can be obtained by a suitable contraction; the situation is
similar to that for the Poincaré Lie algebra. The general classification of superalgebras does not appear to be very useful in physics, because, unlike Lie algebras, superalgebras cannot be used as internal symmetries, or so it seems.\footnote{This is true in Minkowski space. Recently the semisimple superalgebra of \textit{SO}(4/2) has been used in M-theory (a conjectured eleven dimensional superstring theory) in the background of \textit{pp} waves (a solution of Einstein’s gravitational equations which reduces to Minkowski space in a suitable limit; in the corresponding limit \textit{SO}(4/2) reduces to the Poincaré superalgebra.)}

The early work on supersymmetric field theories considered only one fermionic generator which is a Majorana spinor. The corresponding superalgebra is therefore called \( N = 1 \) SUSY. An important development was the study of extended (\( N > 1 \)) SUSY and the construction of quantum field theories admitting extended SUSY. It turns out that \( N = 1 \) SUSY in four space-time dimensions is still the best choice for a SUSY extension of the standard model of elementary particles, because of the chirality properties of physical fermions. I shall describe in a later section a popular version of such an extension, the minimal supersymmetric standard model (MSSM).

\section{Remarks on supermanifolds}

The influence of supersymmetry on mathematics can be seen by the great interest mathematicians have developed in the study of supermanifolds. From a physicist point of view this began with an important paper by A. Salam and J. Strathdee who introduced the concept of “superspace”, a space with both commuting and anticommuting coordinates and showed that \( N = 1 \) supersymmetry can be defined as a set of transformations in superspace. Wess, Ferrara and I then wrote some papers using the concept of “superfields” (fields in superspace). Eventually the technique of superpropagators was developed and shown to be a useful tool for supersymmetric perturbation theory.

With the discovery of supergravity (SUGRA) the supersymmetric extension of Einstein’s gravity) it became natural to study the geometry of curved supermanifolds. Julius and I realized that the super Riemannian geometry proposed by R. Arnowitt and P. Nath had to be enlarged by the introduction of a supervielbein and a constrained, but nonvanishing, super-torsion. We also formulated the geometry in terms of exterior differential superforms, not unlike those introduced independently by F. Berezin.
3 The minimal supersymmetric standard model

Ordinary symmetries of elementary particle physics such as $SU(3) \otimes SU(2) \otimes U(1)$ arrange particles into multiplets of different internal quantum numbers but of the same total spin. Attempts to arrange particles of different spin in supermultiplets, analogous to Wigner’s supermultiplets in nuclear physics, failed to be consistent with the axioms of local relativistic quantum field theory. These attempts (such as “relativistic SU(6)”) involved operators which changed particles of integral spin into particles of a different integral spin and particles of half-integral into particles of a different half-integral spin. Their failure culminated in the proof of so called “no go theorems”. $N = 1$ SUSY overcomes these difficulties by using spin $\frac{1}{2}$ generators which change particle spins by $\frac{1}{2}$ and their statistics as well. SUSY quantum field theories are renormalizable theories consistent with the axioms of relativistic quantum field theory as is very clear already from the very first papers. Julius Wess and I wrote. Examples of particles belonging to a supermultiplet are:

\[
\begin{align*}
\{ & \text{GLUON, SPIN 1, BOSON} \\
& \text{GLUINO, SPIN } \frac{1}{2}, \text{ FERMION} \\
& \text{QUARK, SPIN } \frac{1}{2}, \text{ FERMION} \\
& \text{SQUARK, SPIN 0, BOSON}
\end{align*}
\]

It is customary to attach the ending “ino” to the fermionic superpartner of a boson and the initial “s” to the bosonic superpartner of a fermion, as indicated above. Thus the bosonic superpartner of the electron is called selection and denoted $\tilde{e}$, the fermionic superpartner of the W meson is called Wino and denoted $\tilde{W}$. It is customary to use a tilde for the superpartner of a known particle.

The Standard Model (SM) is very successful in describing particle physics, but some theorists are bothered by the so called “Hierarchy Problem”: due to quadratic radiative corrections the mass of the Higgs scalar would naturally be of order $M_p \sim 10^{18}$ GeV, the Planck mass. A SUSY version of the SM would not have this problem: in a SUSY Quantum Field Theory (QFT) the quadratic corrections cancel between boson and fermion loops, as Julius and I noticed in our second paper; other cancellations also occur. SUSY does solve the hierarchy problem, but it is historically incorrect to say that it was “invented” to solve the hierarchy problem, as some younger theorists claim. As explained before, it was invented to have spin supermultiplets;
then Julius Wess and I noticed the cancellation of divergences, as well as the fact that fewer renormalization constants are needed in SUSY quantum field theories.

The SM has both boson and fermion fields, but the obvious idea to arrange them in supermultiplets fails to agree with experiment. It turns out that one is forced to introduce a new superpartner field to every single field present in the SM. And in addition one must introduce a second Higgs doublet.

Let us remember the field content of the SM

Leptons: $L_i = \begin{pmatrix} v \\ e \end{pmatrix}_{L_i} = (1, 2, - \frac{1}{2})$

$e_{R_i} = (1, 1, -1)$

Quarks: $Q_i = \begin{pmatrix} u \\ d \end{pmatrix}_{L_i} = (3, 2, \frac{1}{6})$

$u_{R_i} = (3, 1, \frac{2}{3})$

$d_{R_i} = (3, 1, - \frac{1}{3})$

Higgs: $H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix} = (1, 2, \frac{1}{2})$

Here $i = 1, 2, 3$ is the “family” index, $L$ and $R$ refer to the left- and right-handed components of fermions and the numbers in parenthesis are the $SU(3) \otimes SU(2) \otimes U(1)$ quantum numbers.

Let us now compare the field content of the SM with that of the MSSM. The rules for building $N = 1$ SUSY gauge theories are to assign a vector superfield (VSF) to each gauge field and a chiral superfield ($\chi$SF) to each matter field. The field content of a VSF is one gauge boson and a Weyl fermion called gaugino, and of the $\chi$SF is one Weyl fermion and one complex scalar. The VSF’s transform under the adjoint of the gauge group, while the $\chi$SF’s can be in any representation. Since none of the matter fields of the SM transform under the adjoint of the gauge group, we cannot identify them with the gauginos. There are additional constraints dictated by the chirality and lepton number of the SM fields. The result is that the minimal choice is to attribute to the $\chi$SF’s of the MSSM the quantum numbers in the table. B and L are baryon and lepton number.
So, one is forced to introduce many new particles, which makes room for many new interactions not existing in physics, for instance baryon and lepton number violating interactions. To preserve B and L conservation (which is automatic in the SM) one introduces R parity conservation. R parity transforms

\[
\text{Particle} \rightarrow \text{Particle} \\
\text{Superpartner} \rightarrow -\text{Superpartner}
\]

It is a discrete invariance of the SUSY algebra.

It is well known that, in the SM, the coupling constants of the strong, electromagnetic and weak interactions run with energy according to the renormalization group equations to converge (almost) to a single value at \( \sim 10^{15} \text{GeV} \). If one uses the MSSM the running of the coupling constants is modified (especially because of the two Higgs supermultiplets, which together count as much as six ordinary Higgs fields) and they converge much better, now at \( \sim 10^{16} \text{GeV} \); is this a hint at unification with supergravity?

**SUSY breaking (the hidden sector)**

Exact SUSY implies that a particle and its superpartner have the same mass, which is clearly not true in the real world. So SUSY must be broken but not too violently in order not to lose the desirable features of SUSY quantum field theories, such as the cancellation of quadratic divergences, the unification of couplings etc. Like other symmetries SUSY can be broken “spontaneously”, which would satisfy that requirement; however spontaneous breaking still preserves relations among the masses (mass sum rules) which are not satisfied in reality. So one must find some other way to break SUSY “softly”.
A popular approach is to postulate the existence of a “hidden sector” where SUSY is broken spontaneously at much higher energy scales than the weak scale. The sector is hidden in the sense that its fields do not interact with the SM particles (“visible sector” except through “minimal” supergravity which will mediate the SUSY breaking to the visible sector. The idea of a hidden sector may seem far fetched, but it emerges naturally in some versions of superstring theory, e.g. heterotic string theory.

4 The cold dark matter problem

Many independent lines of cosmological evidence have led to the conclusion that the vast majority of matter in the universe is “dark” (it has evaded observation based on direct interaction with electromagnetic radiation). Nonbaryonic dark matter out-masses the ordinary matter by a factor of approximately 8. The dominant class of dark matter candidates are “Weakly Interacting Massive Particles” (WIMPS). There have been a number of suggestions for dark matter particles but it seems that the best candidate is provided by TeV-scale SUSY as the “neutralino”.

A particle dark matter candidate must satisfy the following criteria:

• It must be “stable” (long lifetime compared to the age of the universe) to contribute to structure formation.

• There must be an effective production mechanism to create the right amount in the early universe.

• It must be “nonrelativistic” during structure formation (“cold” dark matter).

• It must be “weakly interacting” to have escaped detection, electrically neutral and colorless.

These constraints are satisfied by the neutral Higgsinos ($\tilde{H}_u, \tilde{H}_d$), the neutral Wino ($\tilde{W}^0$) and the Bino ($\tilde{B}^0$), four Majorana fermions with the same quantum numbers, which can mix giving four mass eigenstates, the neutralinos $\chi^0_1, \chi^0_2, \chi^0_3, \chi^0_4$. The lightest one is a good candidate.

Other possibilities are: the lightest mixing of sneutrinos (apparently excluded by accelerator searches) or the gravitino (superpartner of the graviton, very hard to detect).

Depending on the model of SUSY SM, we are talking about the lightest superpartner (LSP).
5 The cosmological Constant, or Dark Energy, and the Vacuum Energy Problem.

It appears to be generally accepted, by astrophysicists and cosmologists, that the cosmological constant \( \Lambda \) (long believed to be zero) is actually positive but very small. As a consequence, the expansion of the universe accelerates. If we interpret \( \Lambda \) as the energy of the vacuum, dimensional arguments, as well as quantum field theory (QFT) calculations would give it a value of

\[ \Lambda_p = \frac{\text{Planck Mass}}{(\text{Planck Length})^3} \approx 10^{94} \text{ grams cm}^{-3} \]

The actual value is \( \Lambda \sim 10^{-120} \Lambda_p \).

In a SUSY QFT (without SUGRA), the vacuum energy vanishes to all orders in perturbation theory. In a generic QFT it diverges quadratically while if SUSY is broken only softly it diverges logarithmically. Still, for any reasonable cut-off, \( \Lambda \) comes out much larger than the above measured value. So, SUSY does not seem to explain the smallness of \( \Lambda \). Recently, within the framework of supersymmetric string theory an approach to this problem has emerged, which has been named the “String Theory Landscape” (Bousso, Polchinski, Susskind, Douglas and others) and which makes use of the so-called “Anthropic Principle”, in a form discussed some time ago by S. Weinberg.

Let us accept that the basic equations of superstring theory are given, in ten dimensions. These equations have many solutions and one is interested in those where six dimensions are compactified (most compactifications studied are in Calabi-Yau manifolds). the resulting theory in four dimensions depends on the topology of the manifold and on the values of various string theory fluxes of fields wrapped around handles of the manifold. Some string theorists count up to 500 handles and different numbers of flux lines (0 to 9). So, one could have \( 10^5 \text{ to } 10^9 \) parameters upon which physics, and the vacuum energy, in four dimension can depend (see figure). The vacuum energy for each valley corresponds to the local minimum, each valley corresponds to a stable (or metastable) set of physical laws. The figure assumes only one parameter: the size of the compact manifold. The true string theory landscape reflects all parameters and forms a topography with a vast number of dimensions. The entire visible universe exists within a region of space associated with a valley that happens to produce laws of physics suitable for the evolution of life. Weinberg considered a restricted form of the anthropic principle, in which one assumes that all constants of nature (e.g. the
time structure constant, mass ratios of elementary particles etc) have the observed values and only the constant $\Lambda$ is arbitrary. He showed that this requires a $\Lambda$ very close to the observed value, otherwise galaxies would not have formed.

6 Concluding remarks

In the present paper, which is based on a colloquium lecture, I necessarily had to limit myself to a very sketchy description of a few topics. For the reader who is interested in a deeper understanding, I can recommend some papers, books and review articles.

For Supersymmetry, Supergravity and Superstring theory I refer to \[3, 4, 5, 6, 7\]. A very clear description of the MSSM can be found in \[8\]. For the cold dark matter problem, there is a very comprehensive recent review \[9\]. For the cosmological constant problem and the anthropic principle see \[10\]; comprehensive reviews are \[11, 12\].

Before concluding, it should be mentioned that some theorists have argued that one should not worry too much about the hierarchy problem, which they consider a merely “philosophical” or “aesthetic” matter. This gives them the freedom to fine tune parameters arbitrarily and thus to invent new models (see H. Murayama’s “New SM”). If the SM is only an effective field theory, this point of view, which is at variance with arguments given above, is perhaps not totally unreasonable; however, it does not seem to have gained much acceptance.
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