Decoherence of a qubit due to a quantum fluctuator or to a classical telegraph noise

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We investigate the decoherence of a qubit coupled to either a quantum two-level system (TLS) again coupled to an environment, or a classical fluctuator modeled by random telegraph noise. In order to do this we construct a model for the quantum TLS where we can adjust the temperature of its environment, and the decoherence rate independently. The model has a well-defined classical limit at any temperature and this corresponds to the appropriate random telegraph process, which is symmetric at high temperatures and becomes asymmetric at low temperatures. We find that the difference in the qubit decoherence rates predicted by the two models depends on the ratio between the qubit-TLS coupling and the decoherence rate in the pointer basis of the TLS. This is then the relevant parameter which determines whether the TLS has to be treated quantum mechanically or can be replaced by a classical telegraph process. We also compare the mutual information between the qubit and the TLS in the classical and quantum cases.

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I. INTRODUCTION

The interaction between a quantum system and its environment leads to loss of quantum coherence, or decoherence, in the system. Understanding decoherence is crucial for grasping the boundary between quantum and classical physics.1–4 It is also essential for testing theories describing quantum measurements.5–8

From an engineering point of view, the decay of coherence in quantum bit devices (qubits) is the most important obstacle for constructing a working quantum computer. Solid state qubits are leading candidates in the projects of designing quantum circuits, where the coherence times of the qubits are required to be sufficiently long to allow for manipulations and transfer of information by logical gates. The most important source of decoherence in many realizations of solid state qubits are believed to be bistable fluctuators – two level systems (TLSs), present as tunneling states in the amorphous substrate used to fabricate the qubit, or in the tunneling junction in superconductor-based devices.9–18

These TLSs are quantum mechanical systems that are, in turn, coupled to their own environments, which are conventionally considered as uncorrelated thermal baths. Usually one does not worry about the fine details of the environment of the TLSs, but rather uses simplified models. The most popular is the Bloch-Redfield approach,9,10 where the environment is taken into account by introduction of the relaxation and decoherence rates of the TLSs. If the TLSs couple more strongly to their own environment than to the qubit, they are usually treated classically. This means that the dynamical description of the TLSs is replaced by a classical dynamics of a fluctuator, which switches randomly between its two metastable states according to a random telegraph process (RTP).20,21 This approach is often referred to as the spin-fluctuator model.11,18,22 In many cases, however, the decoherence of the qubit is determined by only a few fluctuators that are more strongly coupled to the qubit than others.23–27 In such cases, one might question the validity of the classical model. From a practical point of view, it is therefore important to know when such a simplified classical description can replace the full quantum mechanical one. It is also of more fundamental interest in view of the decoherence approach to the quantum-classical transition.28,29

In this paper, we will develop a simple model allowing to show when a quantum system can in practice be replaced by a classical one, in the sense that interference effects can no longer be observed due to the entanglement with the environment. However, we believe that this is only a question of a system becoming in practice classical, i.e., when we can use a classical model to calculate a physical property of a quantum system. It does not shed any light on the real limitation of quantum mechanics such as the measurement problem, where one discuss deviations from linear quantum mechanics, see Ref.2 for a discussion.

Previously, the boundary between quantum and classical regime for the TLS has been explored in a model where the qubit is coupled to an impurity state, and an electron can tunnel between this state and an electron reservoir (metal).28,29 The qubit dephasing rate calculated in the quantum model was found to converge to the classical result in the high-temperature limit. In the study by Abel and Marquardt28 a threshold for strong coupling between the qubit and the TLS was defined by the onset of visibility oscillations in the qubit as a function of the ratio between the coupling to the qubit and the reservoir. The threshold for visibility oscillations was found for higher values of the qubit coupling in the quantum model compared to the classical model, the thresholds finally converge at high $T/\gamma$, where $\gamma$ is the TLS-reservoir coupling. Thus, both in the decoherence
rate and in the visibility oscillations the classical limit is recovered at high temperature. In this model, the temperature plays a dual role: It affects both the energy relaxation rate of the TLS, which maps to the switching rate of the RTP, and it affects the dephasing rate of the TLS. The usefulness of separation of the two effects is seen by the fact that it is perfectly possible to consider finite-temperature classical fluctuators by using an asymmetric RTP.\cite{20,21} This is never obtained in any limit of the model discussed in Refs.\cite{22} and\cite{23}.

The subsequent considerations are based on the following qualitative picture: The dephasing of the qubit is caused by the generation of entanglement between the qubit and the environment. If the qubit and the TLS are strongly coupled, then they behave as a combined four-level quantum system and the quantum nature of the TLS will be important. In such a situation one cannot replace it by a classical RTP. On the other hand, if the TLS is sufficiently strongly coupled to the environment, it means that the information about its state is continuously transferred to the environment and this prevents any quantum interference to take place. From this we can guess that the relevant quantity determining whether the TLS can be considered either classical or quantum is the ratio of the qubit-TLS coupling (which determines the rate of entanglement generation between the qubit and TLS) and the TLS dephasing rate.

The goal of this paper is to study the applicability of the classical model for qubit decoherence due to a TLS. In order to achieve this we study a model where the dephasing rate of the TLS can be varied independently of the temperature, so that the classical limit can be taken at any temperature and correspond to the proper asymmetric RTP. We investigate the regime where the TLS is coupled weakly to the qubit. By use of a model borrowed from the study of TLSs in glasses, we compare the pure decoherence rate of the qubit subject to either a quantum TLS, in turn coupled to its environment, or a classical fluctuator, modeled by random telegraph noise. Our model allows us to separate the effects of temperature, coupling to the bath, and decoherence rate of the TLS. We find that the difference in the qubit decoherence rate predicted by the quantum model and the classical one depends on the ratio, $\xi/\bar{\gamma}_2$, where $\xi$ is the qubit-TLS coupling strength and $\bar{\gamma}_2$ is the decoherence rate of the TLS in the pointer basis.

II. MODEL

A. Quantum model for the TLS

We start by describing the quantum mechanical model for the TLS. The model we use for the TLS originates in the study of tunneling states in glasses, i.e., a particle, or a group of particles that can be approximated by a single configurational coordinate in a double-well potential.\cite{34,26} If the particle is charged, it may give rise to a potential on the qubit that depends on its position in the double-well.

Following Phillips\cite{29,32} the Hamiltonian for the coupled qubit-TLS is split into the Hamiltonians $H_q$ for the qubit, $H_f$ for the TLS, and $H_{fe}$ for the qubit-TLS interaction, $H_e$ for the environment and $H_{fe}$ for the TLS-environment interaction:

$$H = H_q + H_f + H_i + H_e + H_{fe},$$
$$H_q = E_q \sigma_z, \quad H_f = (1/2)(\Delta \sigma_z + \Delta_0 \sigma_x),$$
$$H_i = (1/2)\xi \tau_z \sigma_z$$

where the Pauli matrices $\tau_\alpha$, $\sigma_\alpha$ are operators in the Hilbert spaces of the qubit and the TLS, respectively.

The energy splitting, $\Delta$, and the tunnel amplitude, $\Delta_0$, can be calculated from the shape of the double-well potential.\cite{29} The energy of the qubit depends on the position of the particle in the double-well (we will in the following refer to the eigenstates of $\sigma_z$ as the position basis), and the coupling strength is given by $\xi$. In this work, we will assume the simplified case where the qubit does not directly interact with the environment and therefore has no intrinsic dynamics in the absence of the TLS. Furthermore, we consider a model where the qubit is subject to pure dephasing $[H_q, H_i] = 0$, there is no energy relaxation of the qubit in this model and the decoherence of the qubit is therefore insensitive to the qubit energy splitting $E_q$. When energy relaxation is present, coherent beatings between the qubit and resonant fluctuators are observed.\cite{29,32} In this strong coupling regime, the fluctuator has to be treated as a quantum system. Our present work concentrates solely on non-resonant fluctuators, which are typically modeled classically.

The double-well potential is in general perturbed by electromagnetic and strain fields modifying the asymmetry energy $\Delta$, while perturbations of the barrier height can usually be ignored. In our model we therefore assume that the environment couples to the TLS in the position basis, i.e., the eigenbasis of $\sigma_z$. Rather than formally specifying $H_e$ and $H_{fe}$ we consider two kinds of interaction between the TLS and the external environment, resonant and non-resonant. Resonant interaction, e.g., phonons with frequency close to the eigenfrequency of the TLS, are responsible for direct transitions between the eigenstates of the TLS, $|\psi_g\rangle$ and $|\psi_e\rangle$. We model this interaction by use of the generalized measurement operators defined for a small time step $\Delta t$ as:\cite{32}

$$M_1(\Delta t) = \sqrt{\gamma_{ab}(T)\Delta t} \mathbb{I} \otimes \sigma_x |\psi_g\rangle \langle \psi_g|,$$
$$M_2(\Delta t) = \sqrt{\gamma_{em}(T)\Delta t} \mathbb{I} \otimes \sigma_x |\psi_e\rangle \langle \psi_e|,$$
$$M_3(\Delta t) = \sqrt{1 - M_1^\dagger M_1 - M_2^\dagger M_2}. \quad (2)$$

Here $\mathbb{I}$ is the identity matrix in the Hilbert space of the qubit and the matrices $\sigma_x |\psi_g(e)\rangle \langle \psi_g(e)|$ ‘measures’ whether the TLS is in the ground (excited) state, projects the TLS onto this state and flips it. The rates for absorb-
Here \( T \) is the temperature, \( N(E) = \left( e^{E/T} - 1 \right)^{-1} \) is the Planck distribution, and \( E = \sqrt{\Delta^2 + \Delta_0^2} \) is the energy splitting of the TLS. The non-resonant interaction does not cause transitions between the eigenstates of the TLS. However, we might assume that in general the state of a phonon interacting with the TLS is perturbed by the interaction, and that the perturbation depends on the position of the system in the double-well. Schematically we can write

\[
|\psi_i\rangle|\phi_{0}^{\text{ph}}\rangle \rightarrow |\psi_i\rangle|\phi_{i}^{\text{ph}}\rangle,
\]

where \( i \in \{0, 1\} \) index the state of the TLS in the position basis, \(|\phi_{0}^{\text{ph}}\rangle\) is the initial state of the phonon and \(|\phi_{i}^{\text{ph}}\rangle\) is the state of the phonon after the interaction, conditioned upon that the TLS was initially in the state indexed by \( i \). The interaction, Eq. (4), results in entanglement between the phonon and the TLS, reducing the coherence of the latter. The rate of decoherence due to non-resonant phonons depends on the overlap element \( \alpha = \langle \phi_{0}^{\text{ph}}|\alpha\rangle \) and on the rate of phonons interacting with the system. We model this interaction by the single parameter \( \gamma_2 \), which is responsible for the decay rate of the off-diagonal density matrix elements of the TLS in the position basis.

In this model the equilibrium density matrix of the TLS will not necessarily lie along the \( z \)-axis of the Bloch sphere. The equilibrium density matrix is determined by the rate \( \gamma_2 \) of non-resonant phonons responsible for decay perpendicular to the \( z \)-axis on the Bloch-sphere and by relaxation along the \( z' \)-axis due to resonant phonons in the eigenbasis of the TLS, at the rate \( \gamma_1 \) towards a level determined by \( T \). The situation is illustrated in Fig. 1.

We model this interaction by the single parameter \( \gamma_2 \), the rate at which the off-diagonal density matrix elements decay in the basis where the density matrix is diagonal in equilibrium.

The time evolution in the quantum model is obtained by numerical integration of the von Neumann equation for the Hamiltonian given by Eq. (4), with two modifications. We add a damping term \( \gamma_2 \) to our differential equation

\[
\dot{\rho}_{aa'} = i \langle a| [\rho, H]|a'\rangle - \Lambda_{aa'}\rho_{aa'},
\]

where \( \rho \) is the density matrix of the system composed of the qubit and the TLS and \( \Lambda = \gamma_2 I \otimes \sigma_z \) which determines the decay of the off-diagonal density matrix elements of the TLS in the eigenbasis of \( \sigma_z \). In addition, the TLS absorb and emit phonons at the rates \( \gamma_{ab}(T) \) and \( \gamma_{em}(T) \). The absorption and emission of phonons is implemented as follows: for each timestep \( \Delta t \) we make a transform to the eigenbasis of the TLS

\[
\bar{\rho} = R(\theta)\rho R(\theta),
\]

using the rotation matrix

\[
R(\theta) = I \otimes \left( \begin{array}{cc} \cos \frac{\theta}{2} & \sin \frac{\theta}{2} \\ -\sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{array} \right), \quad \theta \equiv \arctan \left( \frac{\Delta_0}{\Delta} \right).
\]

The density matrix is then updated according to the rates of absorption and emission as

\[
\bar{\rho}' = M_1 \bar{\rho} M_1^\dagger + M_2 \bar{\rho} M_2^\dagger + M_3 \bar{\rho} M_3^\dagger,
\]

before we make the inverse transform \( \rho' = R^\dagger(\theta)\bar{\rho}' R(\theta) \), back to the position basis. Here \( \rho' \) is the density matrix after the (potential) interaction with the resonant phonons.

B. Classical telegraph noise

Pure dephasing of the qubit by a classical telegraph noise can be described by the interaction Hamiltonian

\[
H_i = (1/2)\xi(t)\tau_z,
\]

where \( \xi(t) = \pm \xi \) is the position of the fluctuator at time \( t \). For details on this model see, e.g., Ref. [33] and references therein. The probability for the fluctuator to switch from the state \( \xi_- \) to \( \xi_+ \) and from \( \xi_+ \) to \( \xi_- \) in the interval \( dt \) is given by \( \Gamma_{-+} dt \) and \( \Gamma_{++} dt \), respectively. To describe finite temperature we will consider the situation where the flipping rates \( \Gamma_{++} \) and \( \Gamma_{-+} \) of the fluctuator are in general not identical, but the states are symmetric \( \xi_- = -\xi_+ \). The situation with asymmetric switching rates was previously studied in Refs. [30] and [31]. The equilibrium average is given by

\[
\langle \xi \rangle = \xi (p_+^{eq} - p_-^{eq}) = \xi (\Gamma_{-+} - \Gamma_{++})/\Gamma,
\]
where 

$$\Gamma = \Gamma_{-+} + \Gamma_{+-},$$  

(10)

and $p_{\pm}(t)$ is the probability for the fluctuator to be found in the state $\xi_{\pm}$, respectively. The relaxation towards equilibrium is exponential with rate $\Gamma$. The decoherence of the qubit is obtained by averaging over the realizations and initial conditions of the noise process $\xi(t)$. For a given realization of $\xi(t)$, the Schrödinger equation yields a superposition of the eigenstates of the qubit with a contribution to the relative phase $\phi(t) = \int_0^t \xi(t')dt'$. Averaged over the realizations of the stochastic process $\xi(t)$ we obtain the qubit coherence $D(t) = \langle e^{i\phi(t)} \rangle$. Here we will use the transfer matrix method developed by Joynt et al., where we obtain directly the ensemble averaged Bloch-vector of the qubit.

The state of the qubit-fluctuator system can be stored in the six-dimensional vector

$$\vec{q}(t) = \vec{m}_+(t) \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} p_+(t) + \vec{m}_-(t) \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} p_-(t)$$  

(11)

where $\vec{m}_\pm$ is the Bloch vector of the qubit conditioned upon the state $\xi_\pm$ of the fluctuator. The propagator for $\vec{q}$ averaged over the individual realizations of the RTP can be expressed as $A(t) = e^{-Bt}$ where

$$B = I_3 \otimes V - i \frac{\xi}{2} L_z \otimes v_z, \quad V = \begin{pmatrix} \Gamma_{-+} & -\Gamma_{+-} \\ -\Gamma_{+-} & \Gamma_{-+} \end{pmatrix},$$

while $I_3$ and $L_z$ are generators of the SO$_3$ group and $v_z$ is the Pauli matrix. A direct advantage of this approach is that the qubit state conditioned upon whether the fluctuator is in the state $\xi_\pm$, $\rho_{\pm}^q$ follows directly from $\vec{q}$.

### III. RESULTS

In order to compare the decoherence of the qubit subject to either the quantum TLS, or the classical telegraph noise, we calculate similar relaxation rates towards the equilibrium level in the two models. First we choose a set of parameters, $\Delta$, $\Delta_0$, $\gamma_1$, $\gamma_2$ and $T$ for the quantum model and prepare the TLS in the initial state $|\psi_1\rangle$. At this preliminary stage we are not interested in the qubit and consider the TLS and its environment decoupled from the qubit. We compute numerically the equilibrium occupation probabilities, $p_{\uparrow\downarrow}^q$ and $p_{\uparrow\downarrow}^c$, of the TLS in the position basis, as well as the relaxation rate $\Gamma$. Note that both the equilibrium occupations and the relaxation rate are in general complicated functions of all the parameters in our model. In this work, we always restrict ourselves to the regime where the TLS is overdamped $\Delta_0 \ll \gamma_2$, i.e., the decoherence rate is sufficiently large such that coherent oscillations are not observed in the TLS. Parameter is needed. Note also that since the states $|\psi_i\rangle$ are not eigenstates of the Hamiltonian, the occupation numbers $p_{\uparrow\downarrow}^c$ are not given by the Boltzmann weights at the bath temperature.

The decoherence rate is expressed through the rates $\Gamma_{\pm\mp}$ and the equilibrium occupancy $\langle \xi \rangle$ with the help of Eqs. (9) and (10). The qubit decoherence rate is in general a sum over multiple rates. For symmetric telegraph noise and pure dephasing the decay of coherence in the qubit $D(t)$ is given by $\langle \xi \rangle$. What is exact meaning of $D(t)$

$$D(t) = 2\mu \left[ (\mu + 1)e^{\Gamma t/2} + (\mu - 1)e^{-\Gamma t/2} \right],$$  

(12)

$\mu \equiv \sqrt{1 - (2\xi/\Gamma)^2}$, but in the regime where the coupling to the qubit is weak compared to the damping of the fluctuator, $\Gamma > \xi$, the long-time behavior of the decoherence is strongly dominated by a single rate,

$$\Gamma_q^c = \Gamma(1 - \mu)/2.$$  

We finally compute the decoherence rate, $\Gamma_q$, of the qubit when it is coupled to the same quantum fluctuator from which we calculated the relaxation rate and equilibrium occupations previously, but this time the initial state of the TLS is the thermal equilibrium state. The decoherence rate of the qubit is calculated by numerical simulation of the coupled qubit-fluctuator density matrix $\rho(t)$, from which we can find the qubit density matrix by tracing out the degrees of freedom of the quantum fluctuator. From the qubit density matrix, $\rho^q(t) = Tr_q[|\rho(t)|]$, we find the coherence $|\rho_{\uparrow\downarrow}^q(t)|$, where $\uparrow$ and $\downarrow$ denote the eigenstates of the qubit. Finally, $|\rho_{\uparrow\downarrow}^q(t)|$ is fitted to the exponential function $e^{-\Gamma_q t}$.

The relative difference in the decoherence rate of the qubit due to classical telegraph noise and the quantum fluctuator is defined as

$$\delta \Gamma_q = (\Gamma_q^c - \Gamma_q^c)/\Gamma_q^c,$$  

(13)

where $\Gamma_q^c$ and $\Gamma_q^c$ are the decoherence rate of the qubit subject to the quantum fluctuator and to the classical telegraph noise, respectively. This quantity is presented in Fig. 2 as a function of the dephasing rate of the fluctuator, $\gamma_2$, and temperature $T$. We have restricted ourselves to a parameter range where the TLS does not undergo coherent oscillations. It is evident that the relative difference in the qubit decoherence rate is small for strong decoherence of the TLS, and for high temperatures. In this case we can safely use the simple RTP model rather than the much more complicated quantum model. Superimposed on the contours for $\delta \Gamma_q$ we have plotted curves where the ratio $\xi/\gamma_2$ is constant. We find that the difference between the quantum and the classical fluctuator depends to a very good accuracy on the ratio $\xi/\gamma_2$.

When the qubit is put in contact with the quantum TLS, the qubit and the TLS will in general entangle due to their coupling. The mutual information, the information about the state of one of the systems that can be inferred by measuring the other, will for the quantum
The time evolution of the mutual information for a qubit coupled either to the quantum TLS or the classical fluctuator is shown in Fig. 3. The entanglement between the two systems builds up at a rate given by the coupling $\xi$, but is lost to the environment at a rate given by the decoherence rate of the TLS, $\gamma_2$. The increased information about the qubit encoded in the quantum TLS, compared to the classical fluctuator increases the transfer of entropy to the environment, thus increasing the decoherence rate of the qubit. This effect might explain the positive $\Delta T_q$ found for low values of $T$ and $\gamma_2$.

Experimentally, since the composite density matrix $\rho_{qf}$ is required, the mutual information can only be extracted in the case where one has access to measurement on both the qubit and the fluctuator simultaneously. Since the fluctuator by definition is a system of the environment outside our control, this cannot be achieved. However, the mutual information could potentially be studied in two coupled qubits, where one of the qubits are subject to controlled noise and takes the role of the fluctuator. Qubits subject to engineered noise under the control of the experimentalist has been realized in optically trapped $^{8}$Be$^+$ ions, where also the required quantum gates has already been implemented in a similar systems.

IV. DISCUSSION

In general, the dynamics of a quantum TLS in an environment depends on three parameters: the relaxation rate $\gamma_1$, the dephasing rate $\gamma_2$ and the temperature $T$ determining the equilibrium occupations. In this paper, we use a model where the processes responsible for pure dephasing couple to the position basis, while the relaxation processes take place in the eigenbasis of the TLS. This model was used in order to study the relevance of the classical RTP model for description of decoherence of a qubit. If the interaction responsible for pure dephasing processes (characterized by $\gamma_2$) is diagonal in the eigenbasis of the TLS, i.e., $\Delta_0 = 0$, it will not have any effect on the decoherence rate of the qubit, as long as the qubit couples weakly to the TLS $\xi/\gamma_1 \ll 1$, and the TLS is prepared in the thermal equilibrium state. The TLS will in this case always behave as a classical fluctuator, and can therefore straightforwardly be modeled by the classical telegraph noise.

In general, the difference in decoherence rate $\delta \Gamma$ depends on the ratio $\Delta_0/\Delta$ as well as $\xi/\gamma_2$, where the contours of constant $\xi/\gamma_2$ in the $\ln T$ versus $\gamma_2$ plot, match those of constant $\delta \Gamma$ for all values of $\Delta_0/\Delta$.

Furthermore, we notice that our results do not tell us that it is, in principle, not possible to construct a classical telegraph model providing the same decoherence rate for the qubit as the quantum TLS, even in the regime where the deviation $\delta \Gamma_q$ between the two models are large according to Fig. 2. We show that the decoherence rate of the qubit differ in the two models in the case where the relaxation rates of the classical and quantum fluctu-
ator are identical. To the best of our knowledge, there exist no general relationship between the quantum TLS model and the classical spin-fluctuator model. Therefore, one should be careful in applying the classical telegraph model unless one expects the decoherence rates of the fluctuators to be much larger than the qubit-fluctuator coupling. $\xi/\bar{\gamma}_2 \ll 1$. However, in systems such as glasses this inequality is usually expected to hold, and the TLS can be treated effectively as a classical fluctuator with an exception if the system is subject to an external AC field.

The pointer states of a quantum system are defined as the pure states that are the least affected by environmental decoherence. It is generally believed that when the dynamics of the system is dominated by the interaction with the environment, the pointer states are the eigenstates of the interaction Hamiltonian. On the other hand, when the system is weakly coupled to the environment, the pointer states are assumed to be the eigenstates of the isolated system. Our model can be considered to interpolate between the two extremes. If we define the pointer basis as the basis where the Bloch-vector of the system lies along the $z$-axis in equilibrium, the decoherence rate $\bar{\gamma}_2$ of the system is the rate of decay of the off-diagonal elements of the density matrix in this basis.

In conclusion, we have constructed a model for the quantum TLS where we can study its effect on the qubit as a function of both temperature and the decoherence rate of the TLS due to its interaction with the environment. We have compared the decoherence rate of the qubit found in this model, and in the widely used classical telegraph noise model. We find that the difference in the qubit decoherence rates depends on the ratio $\xi/\bar{\gamma}_2$ between the strength of the qubit-TLS coupling and decoherence rate of the TLS in the pointer basis. In the limit $\xi/\bar{\gamma}_2 \ll 1$, the TLS behaves essentially classically and the qubit decoherence rate can accurately be predicted by the telegraph noise model.

This work is part of the master project of one of the authors (H. J. W.) and more details can be found in his thesis.

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