Predictable and unpredictable phenomena in optical fibers for space-division/mode-division multiplexing transmission: statistical analysis of coupling and mysterious behavior of modes

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Abstract The transmission capacity of single-mode single-core fibers is approaching the limit of about 100 Tbps owing to nonlinearity and the fiber fuse, which limit the input signal power. To avoid the so-called capacity crunch and dramatically increase the transmission capacity, a new approach utilizing space-division multiplexing (SDM) and mode-division multiplexing (MDM) has been proposed and demonstrated. These new multiplexing technologies, a problem that is unprecedented in conventional single-mode fibers, namely, the crosstalk between transmission channels, arises. Although the input signal channels are mixed through at the output end, the signal channel can be identified using multiple-input multiple-output (MIMO) signal processing technology at the output end. However, even using the MIMO digital signal processing technology, the problem that the computation time increases with the number of channels remains, resulting in increased latency, and so the number of channels is limited. To design a transmission system using the SDM and MDM technologies, the crosstalk should be analyzed precisely. However, the origin and behavior of crosstalk are different in SDM using multicore fibers (MCFs) and in MDM using few-mode fibers (FMFs). The behavior of crosstalk in single-mode MCFs is predictable to some extent by statistical analysis, and the system can be designed by considering the results of the analysis. On the other hand, the behavior of crosstalk of FMFs is less predictable. Since the mode launched at the input end is not the eigenmode, mode discrimination or accurate mode demultiplexing is difficult using a conventional mode demultiplexer. In addition, the eigenmode itself of FMFs is not always the hybrid mode predicted by the conventional theory but sometimes a linearly polarized (LP) mode, contradicting the conventional theory. In other words, the demultiplexed signal always involves crosstalk regardless of the transmission distance, and the quantity of crosstalk cannot be analyzed statistically. Therefore, the crosstalk and its behavior are unpredictable. In spite of these unpredictable phenomena, the signal channel can be identified using the MIMO signal processing as in single-mode MCFs. This means that the MDM technology using FMFs is established in the engineering or inductive logic sense, but still involves unexplained phenomena in the scientific or deductive logic sense. In this review, we discuss the predictable behavior of crosstalk in single-mode MCFs and also the unpredictable behavior of crosstalk in FMFs.

Keywords: space-division multiplexing, mode-division multiplexing, coupling, crosstalk, mode discrimination, eigenmode

Classification: Optical hardware (fiber optics, microwave photonics, optical interconnects, photonic signal processing, photonic integration and modules, optical sensing, etc.)

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1. Introduction

In spite of breakthroughs in optical fiber transmission, such as the Er-doped fiber amplifier (EDFA) [1, 2], wavelength division multiplexing (WDM) [3, 4, 5], and multilevel modulation including quadrature phase shift keying (QPSK) [6], communication traffic is expected to further increase in the future with the growth of advanced information devices, the Internet of Things (IoT), and the unexpected rapid increase in teleworking and remote school classes due to the spread of SARS-CoV-2 (COVID-19). For this reason, it is becoming increasingly difficult to support user demand continuously by transmission using single-mode single-core fibers (SCFs), and this concern is called the capacity crunch. According to Shannon’s information theory, the transmission capacity can be increased by increasing the input signal power when the noise level is constant. However, the signal power cannot be increased without limitations. The physical factor limiting the input signal power is the nonlinear optical effect induced by the high optical signal power [7] and fiber fuse, through which a destructive phenomenon propagates [8].

The capacity crunch of optical transmission systems using single-mode SCFs was first raised as an issue in 2008 [9], then mentioned in subsequent papers in 2009 [10, 11, 12] and in 2010 [13]. Space-division multiplexing (SDM) is an attempt to transmit a number of optical signals by using spatially separated communication channels, i.e., different multiple cores, in a single fiber called a multi-core fiber (MCF). On the other hand, mode-division multiplexing (MDM) utilizes spatial modes in few-mode fibers (FMFs) as transmission channels. There are many reports on SDM using uncoupled MCFs [9, 10], and MDM using coupled MCFs [11] and FMFs [14] formed by increasing the core diameter of SCFs.

In the early stage of research, multipath channel fibers, i.e., MCFs, and multimodal channel fibers, i.e., FMFs, were proposed. These concepts have recently been consolidated into few-mode MCFs as shown in Fig. 1 [15].

In these new multiplexing technologies, a problem that is unprecedented in conventional single-mode fibers, namely, the crosstalk between transmission channels, arises. In the very early stage of research on MCFs, it was assumed that the crosstalk is due to the coupling between cores with a constant mode coupling coefficient [10] and that the coupled
power oscillates periodically as a function of the propagation distance in accordance with the coupled-mode theory [16, 17, 18].

For an MCF whose cores are identical, which is called a homogeneous MCF [10], the core pitch (spacing) must be distant enough to suppress the coupling between cores. On the other hand, for an MCF whose cores are non-identical, which is called a heterogeneous MCF [10], the coupling is asymmetric and the maximum coupled power is reduced drastically to [16, 17, 18],

$$F = \frac{1}{1 + \frac{\beta_1 - \beta_2}{2K_{12}}}^2,$$

(1)

where $\beta_1$ and $\beta_2$ are the propagation constants of two single-mode cores, and $K_{12}$ is the mode-coupling coefficient between the two single-mode cores. In this case, the normalized coupled power $P_2(z)$, which is equivalent to the crosstalk, is predicted to be periodic function of the propagation distance $z$, as shown by the black dotted curve in Fig. 2, and the normalized coupled power takes its maximum value $F$ at the propagation distance $L_B (= \pi/2|K_{12}|)$ and its odd integer multiples.

It was revealed, however, that the coupling does not occur uniformly along the propagation distance but occurs unevenly owing to the structural parameter drift in the longitudinal direction resulting from the bending of the fiber [19]. As a result, the coupled power does not oscillate periodically but accumulates with the propagation distance similarly to the diffusion phenomenon. If the phase of the coupled mode is shifted by $\pi$ at the propagation distance $L_B$ and its integer multiples, the normalized cumulative coupled power will be maximal as shown by the blue dotted curve in Fig. 2. This maximized coupled power can be approximated using the following linear relation:

$$P(z) = \left(\frac{2|K_{12}|}{\pi}\right)z$$

(2)

Since an optical fiber in practical use suffers many types of perturbation, such as the fluctuations of core diameter and index, macro- and micro-bending, and twisting, coupling occurs unevenly along the propagation distance. Thus, the coupled power accumulates with increasing propagation distance and the actual curve is located lower than the blue dotted line as shown by the blue solid line in Fig. 2. This dependence of crosstalk on the propagation distance can be treated statistically [19, 20, 21]. If the propagation distance is so long enough that the powers of two cores are perfectly mixed, they approach 0.5 as shown in Fig. 2.

Let us consider a bent fiber as shown in Fig. 3(a) [20, 21]. The refractive index profile of such a bent waveguide can be expressed equivalently using a linearly slanted profile as shown in Fig. 4(a) [22].

In the case of a homogeneous MCF, the effect of bending on the index profile is equivalent to adding or subtracting $n_1\Lambda/R_b$ to the index, as shown in Fig. 4(b), where $\Lambda$ is the core pitch (spacing). Even if the fiber is straight, i.e., $R_b = \infty$, coupling occurs unevenly along the propagation distance owing to the fluctuations of structural parameters such as the refractive index and diameter of the cores along the propagation distance, and the coupled power accumulates with increasing propagation distance similar to the diffusion phenomenon. To analyze this phenomenon, a statistical approach is needed and coupled-power theory rather than coupled-mode theory is used [20, 21], because the sign of the perturbation term $n_1\Lambda/R_b$ can be positive or negative according to its position in the fiber cross section, as shown...
Fig. 3(b), and the magnitude of perturbation fluctuates with the twisting of the fiber.

In the case of a heterogeneous MCF, the coupling is asymmetric between nonidentical cores and the coupled power can be reduced to a small value compared with the symmetric coupling case when the effect of bending, \( n_1 \Lambda/R_b \), is smaller than the difference in structural parameter between adjacent cores as shown in Fig. 4(c). If the effect of bending, \( n_1 \Lambda/R_b \), has the same order of magnitude as the difference in structural parameter between adjacent cores, the accumulation of crosstalk is large, and the crosstalk decreases when the bending radius decreases further. The critical bending radius \( R_c \), at which the effect of bending, \( n_1 \Lambda/R_b \), is equal to the difference in equivalent refractive index between two cores \( \nabla n_{eq} - \nabla n_{eq} \), is given by

\[
R_c \approx \Lambda \left( \frac{n_{eq}^{(1)} - n_{eq}^{(2)}}{n_{eq}^{(1)} - n_{eq}^{(2)}} \right). \tag{3}
\]

Here, the equivalent refractive index (also referred to as the effective refractive index) \( n_{eq} \) is defined by \( n_{eq} = \beta/k_0 \), where \( \beta \) is the propagation constant and \( k_0 \) is that in a vacuum.

In this manner, the crosstalk in single-mode MCFs can be analyzed statistically and structural parameters such as the core pitch, index difference, and the depth of the trench (the layer surrounding the core with a lower refractive index than the cladding) can be designed.

On the other hand, the crosstalk in MDM using FMFs is more complicated than that in SDM using single-mode MCFs. Since coupling occurs between modes in the core, it has been considered that multiple-input multiple-output (MIMO) digital signal processing is also inevitable in MDM transmission over a long distance. However, the necessity of the MIMO digital signal processing in MDM transmission is not due to the coupling between modes resulting from perturbations such as the imperfection of core–cladding boundary and index profile.

The polarization state of a laser source used for optical fiber transmission is linearly polarized, and so the excited electromagnetic field profile of the optical fiber is also linearly polarized. In MDM transmission, the basis of the mode utilized as the transmission channel is the so-called linearly polarized (LP) mode [23] owing to the above reason, and several mode multiplexers for LP modes have been developed [24]. Since LP modes are not the eigenmodes in a round optical fiber, the launched LP modes are expanded in terms of the eigenmodes (HE, EH, TE, and TM modes) [25] at the input end.

The propagation constants of eigenmodes constituting an LP mode are not identical to each other [26]; however, the electromagnetic field profile varies during the propagation. Therefore, mode discrimination or accurate mode demultiplexing is difficult using a conventional mode demultiplexer for LP modes, and the quantity of crosstalk cannot be analyzed statistically. Therefore, the mechanism of crosstalk in FMFs is different from that in single-mode MCFs and the behavior is unpredictable, because the quantity of crosstalk cannot be predicted analytically or statistically.

Although the crosstalk in MDM transmission is unpredictable, MIMO digital signal processing can identify the transmission channel by a process similar to cipher breaking by recursive numerical calculation. This means that although MDM technology using FMFs is established in the engineering or inductive logic sense, it still involves unexplained phenomena in the scientific or deductive logic sense.

In this review, we discuss the predictable behavior of crosstalk in single-mode MCFs and the unpredictable behavior of crosstalk in FMFs, as well as the eigenmodes.

2. Single-mode MCFs and crosstalk estimation using coupled-mode theory

Estimating the inter-core crosstalk (XT) in MCFs is very
important for determining their structural parameters. In this section, the coupled-mode theory (CMT) for XT analysis is briefly reviewed, mainly focusing on single-mode MCFs (SM-MCFs). Here, in order to derive a self-consistent CMT, a procedure for symmetrizing the mode-coupling coefficients for dissimilar cores that do not satisfy the reciprocity relation is specifically described.

For lossless MCFs, to satisfy the law of power conservation, the mode-coupling coefficients should satisfy the reciprocity relation expressed as

$$\kappa_{mn} = \kappa_{nm}, \quad (4)$$

where $\kappa_{mn}$ is the mode-coupling coefficient from core $m$ to core $n$, and $\kappa_{nm}$ is the mode-coupling coefficient from core $n$ to core $m$.

The reciprocity relation holds for homogeneous MCFs [10] with identical cores characterized as

$$\beta_m = \beta_n, \quad (5)$$

where $\beta_m$ and $\beta_n$ are the propagation constants of the modes in core $m$ and core $n$, respectively. However, the reciprocity relation does not hold for heterogeneous MCFs [10] ($\beta_m \neq \beta_n$), and in the case of the conventional, orthogonal CMT, the total power is not conserved, because the core modes that are not the eigenmodes of the entire MCF are not orthogonal to each other. To guarantee the self-consistency of the CMT for dissimilar cores, the cross power, which appears logically as a result of modal non-orthogonality, is necessary. For this reason, we start with a non-orthogonal CMT, which has been developed to describe the mode coupling between strongly coupled parallel waveguides [16, 27, 28, 29, 30], then we derive a self-consistent orthogonal CMT.

Coupled-mode equations with cross-power terms due to the mode non-orthogonality, $C_{mn}$ and $F_{nm}$, are written as [16, 27, 28, 29, 30]

$$\frac{dA_m}{dz} = -j \sum_{m \neq n} \tilde{\kappa}_{mn} A_n(z) \exp(j \tilde{\Delta} \tilde{\beta}_{mn} z) \quad (6)$$

with

$$\tilde{\kappa}_{mn} = \frac{\kappa_{mn} - C_{mn} \kappa_{nn}}{1 - C_{nn} C_{mn}}, \quad (7)$$

$$\tilde{\Delta} \tilde{\beta}_{mn} = \tilde{\beta}_m - \tilde{\beta}_n = -\Delta \beta_{mn}, \quad (8)$$

$$\tilde{\beta}_m = \beta_m + \kappa_{mn} - C_{mn} \kappa_{nn}, \quad (9)$$

where $A_m$ is the mode amplitude in core $m$, $z$ is the propagation direction, $\kappa_{mn}$ and $\kappa_{nn}$ are the self-coupling coefficients, and the cross-power terms are symmetric, $C_{mn} = C_{nm} \equiv C$.

In the case of the non-orthogonal CMT, the reciprocity relation is expressed as [16, 27, 28, 29, 30]

$$C \Delta \beta_{mn} = \kappa_{mn} - \kappa_{nn} \quad (10)$$

with $\Delta \beta_{mn} = \beta_m - \beta_n = -\Delta \beta_{nm}$ being the propagation constant difference between the modes in core $m$ and core $n$. This expression holds also for the quantities with a tilde as

$$C \Delta \tilde{\beta}_{mn} = \tilde{\kappa}_{mn} - \tilde{\kappa}_{nn}. \quad (11)$$

The maximum power-conversion efficiencies (also referred to as maximum power-transfer ratios) $F_{mn}$ from core $m$ to core $n$ and $F_{nm}$ from core $m$ to core $n$ are written as [16]

$$F_{mn} = \frac{(\tilde{\kappa}_{mn} - C \Delta \tilde{\beta}_{mn}/2)^2}{\tilde{\kappa}_{mn} \tilde{\kappa}_{nn} + (\Delta \tilde{\beta}_{mn}/2)^2} \quad (12a)$$

$$F_{nm} = \frac{(\tilde{\kappa}_{nm} - C \Delta \tilde{\beta}_{nm}/2)^2}{\tilde{\kappa}_{nm} \tilde{\kappa}_{nn} + (\Delta \tilde{\beta}_{nm}/2)^2}. \quad (12b)$$

It is confirmed from Eqs. (8) and (11) that the power is conserved, $F_{mn} = F_{nm}$. Using Eqs. (8) and (12), we redefine the mode-coupling coefficient as

$$K_{mn} = \frac{\tilde{\kappa}_{mn} - C \Delta \tilde{\beta}_{mn}/2}{\sqrt{1 - C^2}} = \frac{\kappa_{mn} + C \Delta \beta_{mn}/2}{\sqrt{1 - C^2}}. \quad (13)$$

From Eq. (13), we have

$$\tilde{\kappa}_{mn} = \sqrt{1 - C^2} K_{mn} + C \beta_{mn}/2, \quad (14a)$$

$$\tilde{\kappa}_{nm} = \sqrt{1 - C^2} K_{mn} - C \beta_{mn}/2. \quad (14b)$$

Substituting Eq. (14) into Eq. (12), we have

$$F_{mn} = F_{nm} = \frac{K_{mn}^2}{K_{mn}^2 + (\beta_{mn}/2)^2}. \quad (15)$$

This expression for the maximum power-transfer ratio is the same as that in the well-known self-consistent orthogonal CMT. From Eq. (14), the redefined mode-coupling coefficient is rewritten as

$$K_{mn} = \frac{\kappa_{mn} + \kappa_{nn} - C (\kappa_{mn} + \kappa_{nn})}{2 \sqrt{1 - C^2}}. \quad (16)$$

Similarly, the remaining redefined mode-coupling coefficient $K_{nm}$ is also given by Eq. (16). Substituting Eq. (7) into Eq. (16) and noting that $C_{mn} = C_{nm} \equiv C$, we have

$$K_{mn} = \frac{\kappa_{mn} + \kappa_{nn} - C (\kappa_{mn} + \kappa_{nn})}{2 \sqrt{1 - C^2}}. \quad (17)$$

It is interesting to note that by replacing the $(1 - C^2)$ term by 1, Eq. (17) is reduced to the earlier one in Ref. [30], where only the result is shown and the procedure for deriving the redefined mode-coupling coefficients is not specifically described. Since the cross power for uncoupled MCFs (also referred to as weakly coupled MCFs) is negligibly small, Eq. (17) is written as the average of the mode-coupling coefficients as [20, 30]

$$K_{mn} = \frac{\kappa_{mn} + \kappa_{nn}}{2}. \quad (18)$$

The redefined mode-coupling coefficients are symmetric, $K_{mn} = K_{nm}$, and therefore, we can use the conventional CMT and the law of power conservation is satisfied. The redefined mode-coupling coefficients have also been implemented into the coupled-power theory (CPT) [21] (see the next section).

Actual MCFs are longitudinally perturbed by bends, twists, and structure fluctuations, which are random processes [31, 32]. To cope with such perturbations, the
are respectively the statistical characteristics. To estimate the XT in MCFs more accurately, to obtain sufficiently accurate average XT values, when applying the CMT to MCFs with bend-induced perturbations, to consider the random part of the phase function, and to obtain the local power-coupling coefficients, we introduce a twisting model that has been incorporated into the CMT [34].

In [20, 31, 32], MCFs are assumed to be uniformly bent with a constant bending radius and twisted continuously at a constant rate [33]. In [20, 31, 32], MCFs are assumed to be uniformly bent with a constant bending radius and twisted continuously at a constant rate [33]. In [20, 31, 32], MCFs are assumed to be uniformly bent with a constant bending radius and twisted continuously at a constant rate [33]. In [20, 31, 32], MCFs are assumed to be uniformly bent with a constant bending radius and twisted continuously at a constant rate [33].


coupled-mode equations are modified as [20]

\[
\frac{d\Delta m}{dz} = -j \sum_{m n} K_{mn} A_n(\zeta) \exp(j \Delta \beta_{mn} \zeta) f(z),
\]

where \( \Delta \beta_{mn} \) in Eq. (6) is approximated as \( \Delta \beta_{mn} \) (for weakly coupled MCFs, \( \kappa_{mn} \equiv 0 \), \( \kappa_{nn} \equiv 0 \), and \( C \equiv 0 \)), \( K_{mn} \) is the redefined mode-coupling coefficient given by Eq. (18), and \( f \) is the phase function describing bending and twisting effects. Considering an MCF that is bent at a constant radius \( R_b \) and twisted continuously at a constant rate \( \gamma \), as shown in Fig. 3, the phase function is expressed as [20]

\[
f(z) = \exp\{j(\phi_m(z) - \phi_n(z))\} \delta f(z)
\]

with

\[
\phi_m(z) = \int_0^z \beta_m \frac{x_m \cos \gamma z' - y_m \sin \gamma z'}{R_0} dz',
\]

where \( \phi_m \) is the phase in core \( m \) that includes low-spatial-frequency perturbations induced by macro-bends and twists, \( \delta f \) is the random part of the phase function that includes high-spatial-frequency perturbations induced by longitudinal structure fluctuations and micro-bends, and \( x_m \) and \( y_m \) are respectively the \( x \) and \( y \) coordinates of the center of an arbitrarily located core \( m \) at \( z = 0 \) (initial state).

To consider the random part of the phase function, the total fiber length is divided into finite segments of arbitrary but equal length \( \Delta L \), as shown in Fig. 5, and then random phase offsets of \( \exp(j \phi_{rnd}) \) are applied to all cores at every segment. The segment length used in the CMT is a stochastic parameter corresponding to the correlation length of the phase function.

The statistical approach based on the CMT is extended to the XT analysis of few-mode MCFs (FM-MCFs) under a bent condition [33]. In [20, 31, 32], MCFs are assumed to be uniformly bent with a constant bending radius and uniformly twisted at a constant rate. Recently, the random twisting model has been incorporated into the CMT [34].

3. Single-mode MCFs and crosstalk estimation using coupled-power theory

When applying the CMT to MCFs with bend-induced perturbations, to obtain sufficiently accurate average XT values, a large number of simulations are required, since the XT obtained by the CMT with random phase offsets shows statistical characteristics. To estimate the XT in MCFs more easily, the CPT has been introduced [20, 21].

Coupled-power equations are written as [20]

\[
\frac{dP_m}{dz} = \sum_{m n} h_{mn}(z) [P_n(z) - P_m(z)]
\]

with \( P_m \) being the average power in core \( m \) and \( h_{mn} \) being the power-coupling coefficient. Since the power-coupling coefficients should be symmetric, \( h_{mn} = h_{nm} \), the starting point for deriving the power-coupling coefficients is the coupled-mode equations with the redefined mode-coupling coefficients.

Considering that the phase in core \( m \) caused by the bending and/or twisting is given by Eq. (21), the propagation constant of the mode in core \( m \) is perturbed as

\[
\beta_m'(z) = \beta_m \left( 1 + \frac{x_m \cos \gamma z - y_m \sin \gamma z}{R_b} \right),
\]

To obtain local power-coupling coefficients, we introduce the local propagation constant difference between the modes in core \( m \) and core \( n \) at a point \( z = z \), \( \Delta \beta_{mn}(z) = \beta_m'(z) - \beta_n'(z) \), expressed as [21]

\[
\Delta \beta_{mn}(z) = \Delta \beta_{mn} + B_{mn} \cos(\gamma z + \theta_{mn})
\]

with

\[
B_{mn} = \sqrt{(\beta_m x_m - \beta_n x_n)^2 + (\beta_m y_m - \beta_n y_n)^2},
\]

\[
\theta_{mn} = \tan^{-1} \frac{\beta_m y_m - \beta_n y_n}{\beta_m x_m - \beta_n x_n}.
\]

The coupled-mode equations in Eq. (19) are rewritten as

\[
\frac{d\Lambda_m}{dz} = -j \sum_{m n} K_{mn} A_n(\zeta) \exp(j \Delta \beta_{mn}' z) \delta f(z).
\]

If we assume that the random part of the phase function, \( \delta f \), is a stationary random process, it has an autocorrelation function that decreases above the correlation length. As a result, the average power in core \( m \) at a point \( z = z \) close to \( z = 0 \), \( P_m(z) = (|A_m(z)|^2) \) (ensemble average of \(|A_m(z)|^2 \)), is related to the average power in core \( n \) at a point \( z = 0 \), \( P_n(0) \) and given in terms of the solutions of Eq. (27) as [20, 21]

\[
P_m(z) = z P_n(0) K_{mn}^2 \int_{-\infty}^{\infty} R(\zeta) \exp(j \Delta \beta_{mn}' z) d\zeta,
\]

where \( R \) is the autocorrelation function, whose correlation length should be sufficiently smaller than the increment of the fiber length, \( z \). This is because when deriving Eq. (28), the upper and lower integration limits are replaced by \( +\infty \) and \( -\infty \), respectively.

Finally, we obtain the following longitudinally varying local power-coupling coefficient with the power spectrum density \( S(j \Delta \beta_{mn}') \), which is the Fourier transform of the autocorrelation function:

\[
h_{mn}(z) = \frac{P_m(z)}{z P_n(0)} = K_{mn}^2 S(j \Delta \beta_{mn}').
\]
Although the autocorrelation function is unknown, it has been reported that the experimental results of the XT in MCFs can be well fitted with the exponential autocorrelation function expressed as [20, 21]

\[ R(\xi) = \exp\left(-\frac{d|\xi|}{d}\right) \]  

(30)

with \( d \) being the correlation length.

Noting that the Fourier transform of the exponential function is the Lorentzian function and assuming that the change in \( \Delta \beta_{mn}(z) \) included in Eq. (28) is sufficiently gradual compared with the correlation length, the local power-coupling coefficient is given by

\[ h_{mn}(z) = \frac{2K^2d}{1+|\Delta \beta_{mn}(z)|d^2}. \]  

(31)

The above local power-coupling coefficient is averaged over the twist pitch \( 2\pi/y \) as

\[ \bar{h}_{mn} = \frac{\gamma}{2\pi} \int_0^{2\pi/y} h_{mn}(z)dz, \]  

(32)

then the analytical expression for the average power-coupling coefficient is derived as [21]

\[ \bar{h}_{mn} = \sqrt{2K^2d}\left[ \frac{1}{\sqrt{a(b+\sqrt{ac})}} + \frac{1}{\sqrt{c(b+\sqrt{ac})}} \right] \]  

(33)

with

\[ a = 1 + (\Delta \beta_{mn}d)^2 - \left( \frac{B_{mn}d}{R_b} \right)^2, \]  

(34a)

\[ b = 1 + (\Delta \beta_{mn}d)^2 - \left( \frac{B_{mn}d}{R_b} \right)^2, \]  

(34b)

\[ c = 1 + (\Delta \beta_{mn}d) + \left( \frac{B_{mn}d}{R_b} \right)^2. \]  

(34c)

Note that Eq. (33) can be understood as a closed-form solution of the convolution of the arc sine and Lorentzian distributions [35], and that, in this case, the average power-coupling coefficient is independent of the twist rate [21]. The arc sine and Lorentzian distributions represent the spectra of the perturbations induced by macro-bends and structure fluctuations, respectively [35].

From Eq. (33), the XT between cores with length \( L \) is easily estimated as

\[ XT = \tanh(\bar{h}_{mn}L). \]  

(35)

If the XT is very small, it can be approximated as \( XT \approx \bar{h}_{mn}L \). From Eqs. (33) and (34), we can see that the average power-coupling coefficient exhibits a peak XT value (maximum XT) at a particular bending radius. This critical bending radius is given by

\[ R_e = \frac{B_{mn}}{|\Delta \beta_{mn}|} = \frac{\beta_m\Lambda_m}{|\Delta \beta_{mn}|} = \frac{n_{eff,m}\Lambda_{mn}}{|\Delta n_{eff, mn}|}, \]  

(36)

where \( n_{eff,m} \) is the effective refractive index of the mode in core \( m \) and \( \Delta n_{eff, mn} \) is the effective refractive index difference between the modes in core \( m \) and core \( n \). If \( \beta_n/\beta_m \approx 1 \), \( B_{mn} \) given by Eq. (25) can be approximated as \( \beta_m\Lambda_{mn} \) with \( \Lambda_{mn} \) being the core-to-core distance (core pitch).

In the phase-matching region \( (R_b < R_e) \), the XT is dominated by the bending radius and, in the non-phase-matching region \( (R_b > R_e) \), on the other hand, the XT is dominated by the correlation length, in other words, statistical properties.

To roughly understand the behavior of the XT in MCFs on the basis of Eqs. (33) and (34), Tables I and II show the bending-radius dependence of the average power-coupling coefficients for large and small correlation lengths, respectively, where, for simplicity, the subscripts \( m \) and \( n \) are omitted.

From Table I for large correlation lengths, it is found that when the bending radius \( R_b \) is large (non-phase-matching region), the average power-coupling coefficient \( \bar{h} \) is inversely proportional to the correlation length \( d \) and also to the square of the propagation constant difference \( \Delta \beta \), suggesting that the XT can be suppressed in heterogeneous MCFs \( (\Delta \beta \neq 0) \), and that when \( R_b \) is small (phase-matching region), \( \bar{h} \) is proportional to \( R_b \) and independent of \( d \). On the other hand, from Table II for small correlation lengths, it is found that \( \bar{h} \) is proportional to \( d \) and independent of \( R_b \) over the entire bending-radius range.

In the case of homogeneous MCFs \( (\Delta \beta = 0) \) with large correlation lengths \( (d \gg R_b/(\beta \Lambda)) \), \( \bar{h} \) is reduced to

\[ \bar{h} = \frac{2K^2R_b}{\beta \Lambda}. \]  

(37)

It is worth noting that Eq. (37) coincides with the mean increase in XT per unit length derived from the CMT, which is proportional to \( R_b \) [32].

To investigate the performance of the analytical expression for the average power-coupling coefficients, we consider a heterogeneous seven-core fiber with a hexagonal close-packed structure [32], where the diameters of the center and
outer cores are 8.1 and 9.4 \textmu m, respectively, the relative refractive index difference between the core and the cladding is 0.38\% for all cores, the core pitch is 30 \textmu m, and the fiber length is 2 m.

Fig. 6 shows the bending-radius dependence of the average XT values from the center core to the outer core calculated using Eqs. (33) and (34) [21], where $R_c \equiv 65$ mm and $1/|\Delta \beta| \approx 5.3$ mm. From Fig. 6(a), where the correlation lengths are taken as 10, 50, 100, and 500 mm, we can see that the average XT values for a correlation length of 50 mm are in good agreement with the experimental results denoted by closed circles with error bars [32] and that the XT behavior is fairly consistent with that listed in Table I. From Fig. 6(b), where the correlation lengths are taken as 0.01, 0.05, 0.5, and 5 mm, we can see that the peak XT value decreases with decreasing correlation length and that the XT behavior is fairly consistent with that listed in Table II, except when $d = 5$ mm (in this case, $d \equiv 1/|\Delta \beta|$).

Recently, using the analytical expression for average power-coupling coefficients described here, the effect of twisting on XT reduction has been investigated [36]. By fitting the average power-coupling coefficients to the experimental results, the dependence of the XT on the bending radius and twist rate is evaluated. It is interesting to note that unknown parameters, such as effective refractive index difference, correlation length, and redefined mode-coupling coefficient that can describe the bending-radius dependence of the XT measured at each twist rate, can be theoretically determined. This fitting procedure is thought to be an approach to solving the inverse problem of starting with a set of observation data and then calculating the causal factors.

The analytical expression for average power-coupling coefficients has been widely used for estimating the XT not only in SM-MCFs but also in FM-MCFs [37, 38, 39, 40]. The XT in FM-MCFs is dominated by the XT between the highest-order modes. This is because the higher-order modes have relatively large effective areas compared with the lower-order modes and the field confinement of the higher-order modes is weaker than that of the lower-order modes. The XT between different propagating modes is small, because the effective refractive index difference between these modes is large. Therefore, FM-MCFs should be designed with the careful consideration of the XT related to the highest-order modes. Furthermore, when estimating the XT related to a mode with a higher azimuthal mode order, the azimuthal variation of the mode-coupling coefficient should be taken into account and usually the mode-coupling coefficient is averaged over the azimuth angle.

In FM-MCFs, not only the suppression of the XT for higher-order modes but also the control of the differential mode group delay (DMD) in each core is important. For long-distance transmission based on FM-MCFs, MIMO digital signal processing is deployed on the receiver side to separate coupled modes during propagation. The required number of taps depends on the DMD and each tap coefficient is appropriately determined with an adaptive algorithm. The signal-recovery computation becomes more complex as the total number of taps increases, and therefore, to reduce the signal-processing load in MIMO digital signal processing in a wide wavelength range, FM-MCFs with low DMD magnitudes and low DMD slopes are desired. The DMD in each core can be controlled with a multistep-index or graded-index profile.

Although the analytical expression for average power-coupling coefficients is very powerful for estimating the XT in MCFs, there are still a few unsolved problems in the XT analysis using the CPT. Here, we introduce two of them. One is the anomalously large correlation length and the other is the cladding-diameter dependence of the XT.

To date, it has been believed that the correlation length ranges from several centimeters to several tens of centimeters. In some cases, however, the correlation length becomes anomalously large. As an example, Table III shows the cor-

![Fig. 6 Bending-radius dependence of XT in a heterogeneous seven-core fiber calculated with the analytical expression, Eqs. (33) and (34) [21]. (a) Dotted line: $d = 10$ mm, solid line: $d = 50$ mm, dashed line: $d = 100$ mm, dashed and dotted line: $d = 500$ mm, and closed circles with error bars: measurement data [32]. (b) Dotted line: $d = 0.01$ mm, solid line: $d = 0.05$ mm, dashed line: $d = 0.5$ mm, dashed and dotted line: $d = 5$ mm, and closed circles with error bars: measurement data [32].](image-url)
relation lengths for all the combinations of cores in a fabricated heterogeneous SM-MCF with 30 cores [41], where four types of cores (Cores 1, 2, 3, and 4) are arranged with a hexagonal close-packed structure. Cores 1, 2, and 3 have a trench-assisted-index profile and Core 4 has a step-index profile. Correlation lengths are estimated by comparing the results of the CPT with the measurement results. The estimated correlation lengths range from 1 m to 1000 m. A three-orders-of-magnitude variation of correlation lengths is not likely to occur and such large correlation lengths may not be acceptable, because the change in local propagation constant difference including the effects of bending and twisting in Eq. (24) is assumed to be sufficiently gradual compared with the correlation length. Further study to cope with this unresolved problem is desired.

Recently, the XT in two types of heterogeneous two-core fiber with different cladding diameters has been experimentally investigated [42]. The XT was found to be strongly dependent on the cladding diameter, and the XT in a two-core fiber with a cladding diameter of 229 μm was smaller than that in a two-core fiber with a cladding diameter of 134 μm under a winding-tension condition [42]. This phenomenon is clearly observed even under a free-coiling condition [43]. Noting that when micro-bends are applied to heterogeneous MCFs, the correlation length is reduced, and thus, the XT is increased [35] (see the rightmost column in Table I), it has been reported that a large cladding diameter can suppress the reduction in correlation length owing to the strong resistance against micro-bends [42, 43]. However, it has also been experimentally demonstrated that the cladding-diameter dependence of the XT is not observed for two-core fibers with cladding diameters of 125 and 149 μm [44]. This unsolved problem is also worth further study.

4. Few mode fibers and definition of modes

A rigorous mode theory of step-index cylindrical fibers was developed by Snitzer [25] in 1961. In 1971, the weakly guiding and LP mode approximations were introduced by Gloge [23], because a linear combination of quasi-degenerate pairs of HE and EH modes can form a linearly polarized field profile. However, since there is a small difference in propagation constant between true eigenmodes constituting an LP mode [26], the electromagnetic field profile of an LP mode evolves along with the propagation, and there was no exact analysis on this evolution of the field profile before Ref. [45].

4.1 Electromagnetic field of true eigenmodes and LP modes

Since the core and cladding of a round optical fiber are bounded by a cylinder, it is convenient to analyze the electric field of the optical fiber in terms of cylindrical coordinates \((r, \theta, z)\). The electric field component is, however, expressed in terms of Cartesian coordinates \((x, y, z)\), because the polarization state is usually analyzed and expressed in terms of Cartesian components.

Definitions of HE, EH, TE, and TM modes were given by Snitzer [25]. In the following analysis, we use his definitions and notations of true eigenmodes. Although Snitzer used the time- and \(z\)-dependent term \(\exp[i(\beta z - \omega t)]\), we use the notation \(\exp[j(\omega t - \beta z)]\).

Let us assume a step-index circular optical fiber whose index profile is given by

\[
n^2(r) = \begin{cases} n_1 & r \leq a \\ n_2 & a < r, \end{cases}
\]

where \(n_1\) and \(n_2\) are the refractive indexes of the core and cladding, respectively, and \(a\) is the core radius. The relative index difference \(\Delta\) is defined by

\[
\Delta = \frac{n_1^2 - n_2^2}{2n_1^2}.
\]

The solutions \(E_x\) and \(H_z\) of vector wave equations in the cylindrical coordinate system are expressed in terms of the Bessel function inside the core. The transverse electric field components \(E_x\) and \(E_y\) of the HE, EH, TE, and TM modes are derived from the \(E_x\) and \(H_z\) components as follows [25]:

a. HE\(_{\mu+1, m}\) mode (components of LP\(_{\mu, m}\) mode)

\[
E^{(\mu+1)}_x = -jB_{\mu+1}A_{\mu+1}e^{j(\omega t - \beta_{\mu+1}z)}J_{\mu}(kr)\cos(\mu\theta + \varphi_{\mu+1}),
\]

\[
E^{(\mu+1)}_y = jB_{\mu+1}A_{\mu+1}e^{j(\omega t - \beta_{\mu+1}z)}J_{\mu}(kr)\sin(\mu\theta + \varphi_{\mu+1}),
\]

where \(k\) is the transverse propagation constant defined by

\[
k = \sqrt{k_0^2n_1^2 - \beta^2},
\]

\(J_{\mu}(kr)\) is the Bessel function of first kind, and \(\varphi_{\mu+1} = 0\) and \(\pi/2\) correspond to even and odd modes, respectively. Since the field components in the cladding are obtained by replacing \(J_{\mu}(kr)\) by \((J_{\mu}(ka)/K_{\mu}(ya))K_{\mu}(yr)\), where \(y\) is the transverse decaying constant defined by

\[
y = \sqrt{\beta^2 - k_0^2n_1^2},
\]

and \(K_{\mu}(yr)\) is the modified Bessel function of second kind, we only discuss the field profile inside the core.

b. EH\(_{\mu-1, m}\) mode (components of LP\(_{\mu, m}\) mode)

\[
E^{(\mu-1)}_x = jB_{\mu-1}A_{\mu-1}e^{j(\omega t - \beta_{\mu-1}z)}J_{\mu}(kr)\cos(\mu\theta + \varphi_{\mu-1}),
\]

\[
E^{(\mu-1)}_y = jB_{\mu-1}A_{\mu-1}e^{j(\omega t - \beta_{\mu-1}z)}J_{\mu}(kr)\sin(\mu\theta + \varphi_{\mu-1}),
\]

c. TM\(_{0, m}\) mode (components of LP\(_{1, m}\) mode)

d. TE\(_{0, m}\) mode (components of LP\(_{1, m}\) mode)

In the case of the TE and TM modes, the electric fields are expressed by [25, 45]
\[ E_x^{(0)} = j \frac{B}{k} A_0 e^{i(\omega t - \beta_0 z)} J_0(kr) \cos(\theta + \varphi_0), \]
\[ E_y^{(0)} = j \frac{B}{k} A_0 e^{i(\omega t - \beta_0 z)} J_0(kr) \sin(\theta + \varphi_0), \]

where \( \varphi_0 = 0 \) and \( \varphi_0 = \pi/2 \) correspond to the TM\(_{0,m}\) and TE\(_{0,m}\) modes, respectively. Note that the propagation constant \( \beta_0 \) differs for the TM and TE modes, i.e., the TE\(_{0,m}\) and TM\(_{0,m}\) modes are not degenerate [26].

### 4.2 Synthesis of electric field of LP mode in terms of true eigenmodes

Let us consider the two-mode region of a step-index fiber corresponding to 2.405 < \( V < 3.81 \) that supports fundamental and first-order modes. Here, \( V \) is the normalized frequency defined by \( V = k_0 n_1 a \sqrt{2} \). In this region, in addition to the fundamental HE\(_{11}\) mode, two hybrid modes, i.e., HE\(_{21}\) and HE\(_{21}\) modes, and two transverse magnetic and electric modes, i.e., TM\(_{01}\) and TE\(_{01}\) modes, are supported as shown in Table IV. The fundamental HE\(_{11}\) mode has a linear polarization and corresponds to the LP\(_{01}\) mode.

On the other hand, since there are two orthogonal polarizations (\( x \) and \( y \) polarizations) and two orthogonal field profiles, i.e., even and odd modes for LP\(_{11}\) modes, six orthogonal LP modes are actually supported as shown in Table V.

The first-order LP modes consist of the superposition of eigenmodes as expressed by the following equation [45]:

\[
\begin{bmatrix}
E_{\text{LP even}}^{11-x} \\
E_{\text{LP odd}}^{11-x} \\
E_{\text{LP even}}^{11-y} \\
E_{\text{LP odd}}^{11-y}
\end{bmatrix} = \frac{1}{\sqrt{2}}
\begin{bmatrix}
1 & -1 & 0 & 0 \\
1 & 1 & 0 & 0 \\
0 & 0 & 1 & -1 \\
0 & 0 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
E_{\text{TM}_{01}} \\
E_{\text{HE}_{21}} \\
E_{\text{HE}_{21}} \\
E_{\text{TE}_{01}}
\end{bmatrix},
\]

where the LP\(_{\text{even}}^{11-x}\) mode means that the \( x \)-polarized light of the LP\(_{11}\) even mode and the parity (even or odd) is determined by the symmetry of the electric field profile in the horizontal (\( x \)) direction. The \( 4 \times 4 \) matrix on the right side of Eq. (48) can be separated into two \( 2 \times 2 \) matrices, and the LP modes of the first and second rows (LP\(_{\text{even}}^{11-x}\) and LP\(_{\text{odd}}^{11-y}\) modes) correspond to the TMH group (TM\(_{01}\) and HE\(_{21}\) modes), while the LP modes of the third and fourth rows (LP\(_{\text{odd}}^{11-y}\) and LP\(_{\text{even}}^{11-x}\) modes) correspond to the TEH group (HE\(_{21}\) and TE\(_{01}\), respectively [26].

Since the light source used in the transmission generally emits linearly polarized light, the excited modes are always LP modes. However, even if an LP mode having a specific polarization direction and mode parity (even or odd) is launched into a fiber, the electromagnetic field distribution changes during the propagation owing to the difference in propagation constant between the eigenmodes.

Let us suppose that the LP\(_{\text{even}}^{11-x}\) mode is launched into a fiber. The sum of Eq. (40) for the HE\(_{21}\) mode with \( \varphi_0 = 0 \) (corresponding to the HE\(_{21}\) mode) and Eq. (46) for the TM\(_{01}\) mode with \( \varphi_0 = 0 \) gives the \( x \) component of the electric field of the LP\(_{\text{even}}^{11-x}\) mode at the input end. In the same manner, the sum of Eqs. (41) and (47) gives the \( y \) component of the electric field of the LP\(_{\text{even}}^{11-x}\) mode at the input end. These relations can be seen from the vectorial summation of the electric field profiles in Tables IV and V.

When the input beam is x-polarized, \( -A_2 = A_0 = A_{LP1} \) should be satisfied to make the \( y \) component zero. In this case, the \( x \) and \( y \) components of the electric field at the propagation distance \( z \) are expressed by

\[
E_x^{LP_{11}} = j \frac{\beta_{aM}}{k} A_{LP1} e^{i(\omega t - \beta_{aM} z)} J_1(kr) \cos(\theta)[2 \cos(\delta_{BM} z)],
\]
\[
E_y^{LP_{11}} = j \frac{\beta_{aM}}{k} A_{LP1} e^{i(\omega t - \beta_{aM} z)} J_1(kr) \sin(\theta)[2 j \sin(\delta_{BM} z)].
\]

Here, \( \beta_{aM} \) is the average of the propagation constants of the HE\(_{21}\) and TM\(_{01}\) modes, and \( \delta_{BM} \) is half of the difference between these propagation constants,

\[
\beta_{aM} = \frac{\beta_{HE_{21}} + \beta_{TM_{01}}}{2},
\]
\[
\delta_{BM} = \frac{\beta_{HE_{21}} - \beta_{TM_{01}}}{2}.
\]

The intensity profile (the \( z \) component of the complex Poynting vector \( \vec{S}_z \)) is derived from Eqs. (49) and (50) and expressed by [45]

\[
\vec{S}_z = \frac{2 \beta_{aM}^3}{\omega \mu_0 k^2} (A_{LP1})^2 J_1^2(kr) \times [\cos^2(\delta_{BM} z) \cos^2(\theta) + \sin^2(\delta_{BM} z) \sin^2(\theta)].
\]

It can be seen from Eqs. (49) and (50) that the electric field profile is expressed by a locally elliptical polarization, whose ellipticity and direction of rotation depend on the position in the transverse cross section, because the phase of the \( y \) component in Eq. (50) differs from that of the \( x \) component of the electric field of the LP\(_{11}\) mode.
component in Eq. (49) by $\pm \pi/2$. This fact was not pointed out in Refs. [26, 46, 47, 48].

On the other hand, the intensity profile of the $\text{LP}_{11-x}$ mode evolves into that of the $\text{LP}_{11-\pm y}$ mode via a torus-shaped intensity profile, and this change is periodic with respect to the propagation distance. Therefore, the $\text{LP}_{11-x}^{\text{even}}$ and $\text{LP}_{11-\pm y}^{\text{odd}}$ modes belong to the same mode group called the TMH mode group [26], whose constituent true eigenmodes are the HE$_{21}^{x}$ and TM$_{01}^{y}$ modes.

For example, when the $\text{LP}_{11-x}^{\text{even}}$ mode is launched into a fiber, a periodic transition between the $\text{LP}_{11-x}^{\text{even}}$ and $\text{LP}_{11-\pm y}^{\text{odd}}$ modes of the TMH mode group occurs as shown in Fig. 7 [45]. The beat length $L_b$ of the interference between the true eigenmodes constituting an LP mode is expressed by

$$L_b = \frac{2\pi}{\beta_{\text{HE}_{21}} - \beta_{\text{TM}_{01}}} = \frac{\pi}{\delta \beta_{\text{M}}} \quad (54)$$

and ranges from several tens of centimeters (TEH group) to several meters (TMH group) [26, 45]. In the intermediate state of the periodic evolution between two LP modes that belong to the same mode group, the electric field has a donut-shaped intensity distribution with a locally different elliptical polarization in the cross section [45, 46, 47]. That is, it can no longer be expected that the input excited mode is reproduced at the output end of the FMF, and it is meaningless to demultiplex the output light using a mode demultiplexer for the LP modes.

5. Mode analysis using LP-mode-selective exciter and NFP observation system

It has been theoretically anticipated that the interference between true eigenmodes will occur when an LP mode is launched at the input end and that the electromagnetic field has a donut-shaped intensity distribution and locally different elliptical polarization states as shown in Fig. 7. To verify this interference experimentally, we need to construct an observation system that distinguishes the intermediate states of the transition between two LP modes consisting of two eigenmodes belonging to the same mode group (TMH or TEH group). For this purpose, as shown in Fig. 8 [49], it is appropriate to selectively excite an LP mode while changing the wavelength of the light source and observe the near-field pattern (NFP) of the light emitted from the FMF through a rotatable polarizer. This selective exciter can selectively excite any LP mode with an arbitrary polarization direction using a rotatable phase plate and a rotatable half-wave plate.

When the NFP is observed while rotating the polarizer at the exit and changing the wavelength, it can be determined whether the propagated light is formed by the interference between true eigenmodes or whether the propagated light can be considered as a mixture of eigenmodes. Therefore, even if the output polarizer is rotated, the shape of the observed pattern does not change. Theoretically, the intensity of the NFP observed here is sinusoidal with respect to the azimuth angle of the polarizer.

On the other hand, if a polarizer is inserted in the intermediate state (donut-shaped intensity distribution) in Fig. 7, the observed NFP changes with the azimuth angle of the polarizer, as shown in Fig. 9. Since the phase difference between the true eigenmodes constituting the output electromagnetic field depends on the wavelength, the NFP periodically transits between the $\text{LP}_{11-x}^{\text{even}}$ mode, the intermediate state, and the $\text{LP}_{11-x}^{\text{odd}}$ mode when the wavelength of the light source is changed, as shown in Fig. 7. On the other hand, we can de-
termine whether an LP\textsubscript{11} mode propagates as the eigenmode at a specific angle at which an LP\textsubscript{11} mode pattern appears by rotating the phase plate in the measurement system shown in Fig. 8 without the polarizer.

6. Experimental results

Using the measurement system shown in Fig. 8, we observed the electromagnetic field profile of four FMFs. Two of them were 2-LP-mode (LP\textsubscript{01} and LP\textsubscript{11} modes) single-core (SC) FMFs with a step-index (SI) profile and a graded-index (GI) profile in which the core only exists at the center. The remaining two were 4-LP-mode (LP\textsubscript{01}, LP\textsubscript{11}, LP\textsubscript{21}, and LP\textsubscript{02} modes) 12-core [50] and 19-core [51] FMFs. Both MCFs have a low-refractive-index trench in the region surrounding the core inside the cladding, and the core itself has a GI profile. The 12 and 19 cores are arranged in square as shown in Figs. 10(a) and triangular lattice patterns as shown in Figs. 10(b) and (c), respectively. The structural parameters of these FMFs are shown in Table VI.

6.1 Two-LP-mode single-core fibers

When the \(x\)-polarized LP\textsubscript{even}\textsubscript{11} mode was incident on the SC-FMF at a certain angle, the observed NFP that passed through the output polarizer rotated by a certain angle was quenched at a certain wavelength and it turned out to be linearly polarized light, as shown in Fig. 11 [49]. However, when the wavelength of the light source was slightly changed, an extinction angle did not appear, and a donut-shaped intensity distribution corresponding to the locally different elliptical polarization states was observed. For example, Fig. 11(a) is in good agreement with the theoretical interference between the TMH group of true eigenmodes (TM\textsubscript{01} and HE\textsubscript{even}\textsubscript{21}) [45].

Although the coordinate axes have been rotated in Fig. 11(b), if the direction of +45 degrees is defined as the \(x\)-axis, the pattern at a wavelength of 1540 nm corresponds to the LP\textsubscript{even}\textsubscript{11-x} mode and the pattern at a wavelength of 1547 nm corresponds to the LP\textsubscript{odd}\textsubscript{11-y} mode, whose polarization direction and parity are orthogonal to those of the LP\textsubscript{even}\textsubscript{11-x} mode. At a wavelength of 1549 nm, the intermediate state of the mode evolution corresponding to Fig. 9 can be observed. When the LP\textsubscript{11} mode of the TEH group was launched into the FMF, the interference between the true eigenmodes was also observed. In addition, the same result as described above was obtained for both SI- and GI-FMFs.

To measure the core ellipticity of these FMFs, we used an optical microscope and observed the dark-field image of the cleaved fiber etched with buffered hydrofluoric acid (BHF) at room temperature for 5 min. Here, the ellipticity \(e\) is defined by

\[
e = \frac{a - b}{a},
\]

where \(2a\) is the length of the major axis and \(2b\) is the length of the minor axis. The ellipticity of the SI-FMF was evaluated to be 1.9%. However, in the GI-FMF, a clear step at the core-cladding boundary was not formed by etching, and thus, the core shape could not be observed.

6.2 Four-LP-mode 12-core fiber

Next, the NFPs of light output from an outer core of the 4-LP-mode and 12-core GI-FMF were observed using the same measurement setup as in Fig. 8. When the output polarizer

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
Fiber & SC-SI-FMF & SC-GI-FMF & 12-core GI-FMF & 19-core GI-FMF \\
\hline
Number of LP modes & 2-LP & 2-LP & 4-LP & 4-LP \\
Core diameter [\mu m] & 13.3 & 22.2 & 10.1 & 10.1 \\
Index difference [\%] & 0.348 & 0.352 & 0.72 & 0.81 \\
Parameter \(\alpha\) & \infty (step) & \sim 2 & 2.0 & 1.81 \\
Core spacing [\mu m] & \text{(single core)} & 43.0-44.5 & 42.7-44.1 \\
Fiber length [m] & 100 & & & \\
Curvature radius [cm] & 10 & 15 & & \\
\hline
\end{tabular}
\caption{Structural parameters of FMFs [15].}
\end{table}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{image10}
\caption{Cross-sectional structures of (a) 4-LP-mode 12-core fiber and (b), (c) 4-LP-mode 19-core fiber [15]. Copyright (2018) The Japan Society of Applied Physics}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{image11}
\caption{NFPs observed from SC-FMF. (a) shows the observed results for the SI type and (b) shows those for the GI type. (a) and (b) correspond to the cases where the \(x\)-polarized LP\textsubscript{11} even mode is launched horizontally [49].}
\end{figure}
was not inserted, the mode pattern of LP_{11} appeared when the phase plate was rotated by a certain angle, as shown in the first row of Fig. 12(a) [49]. The mode pattern did not change even when the half-wave plate was rotated while the azimuth angle of the phase plate was fixed.

Next, we inserted the output polarizer and rotated the half-wave plate to launch the LP^{even}_{11-x} mode into the fiber. Then, even when the wavelength of the light source was changed, the mode pattern and the direction of polarization did not change as shown in Fig. 12(a). This pattern corresponds to the LP^{even}_{11-x} mode, and it was found that the incident LP^{even}_{11-x} mode propagated as the eigenmode or eigenstate of the core.

On the other hand, when the incident LP^{even}_{11-x} mode was rotated by 45 degrees in the azimuth direction, both the mode pattern and the polarization direction were rotated and varied with the wavelength as shown in Fig. 12(b) [49].

It is concluded from these results that the principal axis of the mode parity exists only in the horizontal and vertical directions in this core. When the parity of the incident light did not match the direction of the principal axis specific to the core, the interference between the orthogonal LP_{11} modes was observed.

Even when the LP_{11} mode of the TEH group was excited and launched into the core, the direction of the principal axis was the same as that of the TMH group. We measured another outer core and an adjacent inner core and observed almost the same behavior as that of the first core, except for the direction of the principal axis of the LP_{11} mode [49].

Next, we observed the cross section of the FMF using the dark field of an optical microscope, as shown in Fig. 10(a). Since we found the marker in the lower left of Fig. 10(a) after the measurement in Fig. 12, we could not identify which cores were the measured cores during the measurement in Fig. 12. However, we measured three cores of the multi-core FMF and found that these cores had their own principal axis of the LP_{11} modes that propagated in the FMF as the eigenmodes, as shown in Fig. 12(a). The ellipticity of the core was determined from the cross-sectional image in Fig. 10(a) with a larger magnification to have an average value of 2.8%. Since this ellipticity is not significantly different from that of an SC-SI-FMF, it is considered that the existence of the principal axis of the LP_{11} modes is not due to the elliptical deformation of the core. Furthermore, since the direction of the principal axis of the LP_{11} modes did not coincide with that of the symmetry axis of the core arrangement, it is difficult to directly attribute the origin of this phenomenon to stress.

Here, there are four differences in the structures from those of the SC-FMFs. (i) The outer cores of the 4-LP-mode 12-core GI-FMF are susceptible to stress since they are not located at the fiber center, (ii) the V-value is larger than that of 2-LP-mode FMFs, (iii) a trench structure exists in the refractive index profile, and (iv) multicore FMFs were fabricated by the rod-in-tube method. However, we could not conclude the cause of the LP_{11} mode becoming the eigenmode in this FMF. Anyway, although intentional LP mode transmission has been realized using extremely elliptical deformed core fibers [52, 53, 54, 55, 56], we discovered a new phenomenon that the LP modes propagate as the eigenmodes in a few-mode MCF with a small ellipticity [49]. This is an unpredictable phenomenon that contradicts the conventional theory [25, 26].

### 6.3 Four-LP-mode 19-core fiber

We also observed the NFP of the light output from a 4-LP-mode 19-core fiber using the measurement setup shown in Fig. 8. Before measuring this fiber, we cleaved the fiber end, etched the fragments with BHF, and confirmed the presence of the marker labeled in Fig. 10(b) by the microscopic observation of the cross section, as shown in Fig. 10(c) [15].

The input and output ends of the fiber were also observed using a microscope, and the numbering of the cores in the arrangement defined in Fig. 10(b) was identified in advance. Therefore, while measuring the direction of the principal axis of the LP_{11} modes, it was possible to specify into which core the light was launched. This fiber is identical to that reported in Ref. [51], and the cores are numbered in the same manner.

Since all the cores were identified, the NFPs of the light output from all the cores of the 4-LP-mode 19-core GI-FMF were observed using the same measurement setup shown in Fig. 8. As a result, it was found that the principal axis of the LP_{11} mode exists in all the cores observed except for four cores, and that the LP_{11} mode propagates as the eigenmode along this axis, as shown in Fig. 13. Here, LP_{11} mode could not be excited selectively in these four of the 19 cores, probably owing to the crimp on the surface of the input end.
which was formed by the cleavage of the fiber.

It can be seen from Fig. 13 that the direction of the principal axis of the LP$_{11}$ mode does not coincide with that of the symmetry axis of the hexagonal mesh structure in every core. In addition, it is interesting that the principal axis of the LP$_{11}$ mode exists in the center core #10. In this fiber, the ellipticity of each core and the major and minor axes of the ellipse were obtained from microscopic bright- and dark-field images. The average value of the measured ellipticity was as small as 2.0% and the maximum value was 3.9% [15].

The direction of the principal axis obtained from the microscopic bright image is shown in Fig. 14. The principal axis of the LP$_{11}$ mode differs by more than 10 degrees from the elliptical major or minor axis in six cores, i.e., cores #4, #5, #7, #10 (center core), #16, and #18 [15].

7. Consideration on possible causes of principal axis of LP$_{11}$ mode

Possible causes of the LP$_{11}$ modes being the eigenstates of the FM-MCFs are the elliptical deformation of the core, internal stress, and the shift of the core position from the grid point of the hexagonal mesh structure.

7.1 Analysis of deformation of true eigenmodes of elliptical deformed core

To evaluate the deformation of true eigenmodes in a circular core due to the elliptical deformation of the core, we analyzed the eigenmodes of an elliptical deformed core using elliptical cylindrical coordinates and the Mathieu function [57, 58, 59].

Although Adams reported a method of analyzing true eigenmodes propagating in an elliptical core in Ref. [57], only the fundamental mode was calculated. In addition, since this calculation considered only the first-order terms in the trigonometric expansion of the Mathieu function, the accuracy of the approximated solution was insufficient for a large value of ellipticity.

Therefore, the solution of the eigenvalue equation was first found for the eigenmodes constituting the higher-order modes [15]. By substituting it into the equation of the electromagnetic field distribution expressed in the elliptical cylindrical coordinate system and converting the equation into Cartesian coordinates, a vector diagram and the intensity distribution of the mode were obtained. Furthermore, the terms up to the second-order terms in the trigonometric expansion of the Mathieu function and the Bessel function expansion of the modified Mathieu function were included to improve the accuracy of the calculation. Here, the TE$_{01}$ and TM$_{01}$ modes were analyzed, because the eigenvalue equations are simplified for these modes [15].

According to the calculated results, the eigenfunction is hardly deformed and does not become the LP$_{11}$ mode even when the core ellipticity is about 4% as shown in Figs. 15(a) and (b) [49]. The TE$_{01}$ mode becomes an LP$_{11}^{11}$ -like mode, and the TM$_{01}$ mode changes into an LP$_{11}^{11}$ -like mode owing...
to the elliptical deformation. Fig. 15(c) shows the calculated power ratios of the $x$ - and $y$-polarization components. Since the core ellipticity of the FMFs measured in this study is less than 4% (the average is 2.0%), it is considered that the cause of the LP$_{11}$ modes being the eigenstates of the FM-MCFs is not the elliptical deformation of the core.

Now, when an analysis of the electromagnetic fields of modes in an elliptic core is attempted using a numerical electromagnetic analysis tool with mesh-like area segmentation, the field profile changes abruptly from that of an eigenmode to that of an LP mode upon a very small elliptical deformation of less than 1%. This is a characteristic phenomenon of numerical tools with area segmentation when a symmetric structure is analyzed, and it appears to be difficult for a phenomenon such as switching to occur in a linear system. The field profiles of modes in cores that are slightly deformed from symmetric cores such as circular and rectangular cores can be numerically analyzed by a propagation simulation tool based on the finite-difference time-domain (FDTD) method or beam propagation method (BPM) by using a tapered structure that changes from a perfectly symmetric cross section to a deformed cross section with a very low tapering rate [60].

### 7.2 Internal stress

The second possible cause of the LP$_{11}$ modes being the eigenstates of the FM-MCFs is internal stress. To investigate this possibility, the birefringence distribution was measured using a two-dimensional birefringence measurement system (Photonics Lattice, PA-micro). The measured results are shown in Fig. 16 together with the principal axes of the LP$_{11}$ mode (white bars) and the differential core shift vectors (red bars), which will be described in the next section [15]. A polarization-maintaining fiber (PMF) was also measured at the same time and the maximum birefringence of the 4-LP-mode 19-core fiber, which was observed in the apex cores, was 15% of that of the PMF, and the birefringence of the inner cores was about 10% of that of the PMF. It can be seen by comparing Figs. 10 and 16 that the internal stress is mainly induced in the trench region surrounding the core. As can be seen from Fig. 16, the direction of the principal axis of the LP$_{11}$ mode does not coincide with the direction of birefringence (internal stress) in cores #1, #4, #7, #9, #10, #13, #16, #17, and #19. This fact implies that the principal axis of the LP$_{11}$ mode is not determined by only the birefringence (internal stress) and that this phenomenon has another origin.

### 7.3 Perturbation due to shift of core from grid point

The third possible cause of the LP$_{11}$ modes being the eigenstates of the FM-MCFs is the shift of the core position from the grid point of the hexagonal mesh structure. The center positions of the cores were measured from the image shown in Fig. 10(c) with a larger magnification, and the shift vectors of the core position of 19 cores are displayed on the image as shown in Fig. 17. In this figure, the grid of the hexagonal mesh structure is shown by white lines and the cores are displayed by blue circles.

The shift of the core position causes the unbalanced perturbation from adjacent cores as shown in Fig. 18 [61]. In the hexagonal mesh structure, there are three symmetrical axes, and so the directions of the perturbation from adjacent cores should be separately considered in these three directions. Since the shift of each core position linearly affects the spacing between adjacent cores, the perturbation can be regarded as a linear function of the shift of the core position. Thus, the differential shift vector of the core position should be considered as the direction of the perturbation. The differential shift vectors of individual core positions are also illustrated by red bars in Fig. 16. In cores #1, #4, #7, #10, #13, and #16, the direction of the principal axis of the LP$_{11}$ mode coincides with that of the differential shift vector as shown in Fig. 19. However, in the remaining cores #9, #17,
and #19, the direction of the principal axis of the LP$_{11}$ mode coincides with neither that of the differential shift vector nor that of the birefringence (internal stress).

Therefore, the direction of the principal axis of the LP$_{11}$ mode seems to be determined by a complex origin constituting a nonlinear combination or a vector sum of these causes, i.e., the internal stress and the shift of the core position.

8. Conclusion

In this review, we discussed the behavior of crosstalk in single-mode MCFs and FMFs. The crosstalk in single-mode MCFs is mainly caused by the core-to-core coupling due to the twist of the fiber and accumulates with increasing propagation distance. Therefore, the behavior of crosstalk in single-mode MCFs is predictable to some extent by statistical analysis, and the system can be designed by considering the results of the analysis. On the other hand, the behavior of crosstalk in FMFs is less predictable. The mode launched at the input end is not the eigenmode, because the light emitted from a laser diode is linearly polarized, but the eigenmodes of the optical fiber are generally not linearly polarized except for the fundamental mode. Therefore, mode discrimination or accurate mode demultiplexing is difficult using the conventional mode demultiplexer for LP modes. In addition, the eigenmode itself of FMFs is not always the hybrid mode predicted by the conventional theory, but sometimes a linearly polarized (LP) mode, which contradicts the conventional theory. This means that the demultiplexed signal always involves crosstalk independent of the transmission distance, and the quantity of crosstalk cannot be analyzed statistically.

In MDM, however, mode discrimination and accurate mode demultiplexing may be possible if the eigenmodes in an FMF are designed to be LP modes as described in Sections 6.2 and 6.3, and such LP modes are intentionally launched at the input end of an FMF. To make this possible, the reason why the LP$_{11}$ modes are the eigenstates of the FM-MCFs must be clarified, and a new design technology of FM-MCF having LP modes as the eigenstates should be established. Even if the eigenmodes are selectively excited at the input end, coupling may occur as in single-mode MCFs when the propagation distance is long. In the case of short-distance propagation, however, the crosstalk may be sufficiently reduced, and a MIMO-free MDM transmission may be possible without using largely deformed elliptical core fibers [52, 53, 54, 55].

In conclusion, some unsolved problems remain in the behavior of crosstalk in single-mode MCFs and FM-MCFs. In particular, the crosstalk in FM-MCFs cannot be predicted statistically. However, the signal channel can be identified using MIMO signal processing although the output modes cannot be accurately discriminated. This means that the MDM technology using FMFs is established in the engineering or inductive logic sense, but still involves unexplained phenomena in the scientific or deductive logic sense.

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