Lattice-QCD-based Schwinger-Dyson Approach for Chiral Symmetry Restoration at Finite Temperature

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We propose the Schwinger-Dyson (SD) formalism based on lattice QCD, i.e., LQCD-based SD formalism, for the study of dynamical chiral-symmetry breaking in QCD. We extract the kernel function $K(p^2)$ in the SD equation from the lattice data of the quark propagator in the Landau gauge. As remarkable features, we find infrared vanishing and intermediate enhancement of the kernel function $K(p^2)$ in the SD equation. We apply the LQCD-based SD equation to thermal QCD, and calculate the quark mass function at the finite temperature. We find chiral symmetry restoration at the critical temperature $T_c \sim 100$ MeV.

1. Introduction

Dynamical chiral-symmetry breaking (DCSB) is one of the most important nonperturbative features in QCD. For the study of DCSB, the Schwinger-Dyson (SD) formalism has been used as an interesting and powerful method. For QCD, the SD formalism consists of an infinite series of nonlinear integral equations which determine the n-point Green’s function of quarks and gluons, and therefore it includes the infinite-order effect of the QCD gauge coupling constant $g$.

For instance, the SD equation for the quark propagator $S(p)$ is described with the nonperturbative gluon propagator $D_{\mu\nu}(p)$ and the nonperturbative quark-gluon vertex $g\Gamma_{\nu}(p,q)$ as

$$S^{-1}(p) = S_0^{-1} + g^2 \int_q \gamma_{\mu} S(q) D_{\mu\nu}(p-q) \Gamma_{\nu}(p,q),$$

where $S_0(p)$ denotes the bare quark propagator and the simple notation $\int_q \equiv \int \frac{d^4q}{(2\pi)^4}$ has been used in the Euclidean metric.

In the practical calculation for QCD, however, the SD formalism is drastically truncated: the perturbative gluon propagator and the one-loop running coupling are used instead of the nonperturbative quantities in the original formalism. This simplification seems rather dangerous because some of nonperturbative-QCD effects are neglected.

In this paper, we formulate the SD equation based on the recent lattice QCD (LQCD) results, i.e., the LQCD-based SD equation, and aim to construct a useful and reliable analytic framework including the proper nonperturbative effect in QCD. Using the LQCD-based SD equation, we investigate also DCSB at finite temperatures.

2. The Quark Propagator in Lattice QCD

First, we briefly review the quark propagator $S(p)$ in lattice QCD. The inverse quark propagator in the Landau gauge is generally given by

$$S^{-1}(p) = Z(p^2)\{p + M(p^2)\}$$

in the Euclidean metric. Here, $M(p^2)$ is called as the quark mass function, and $Z(p^2)$ corresponds to the wave-function renormalization of the quark field. In the quark propagator, DCSB is characterized by the mass generation as $M(p^2) \neq 0$.

The quark mass function $M(p^2)$ in the Landau gauge is recently measured in lattice QCD at the quenched level [5], and the lattice data in the chiral limit is well reproduced by

$$M(p^2) = M_0/\{1 + (p/\bar{p})^\gamma\}$$

with $M_0=260$ MeV, $\bar{p}=870$ MeV and $\gamma=3.04$. The infrared quark mass $M(0) = M_0 \approx 260$ MeV seems consistent with the constituent quark mass in the quark model. Using this lattice result of $M(p^2)$, the pion decay constant is calculated.
as $f_\pi \simeq 87$ MeV with the Pagels-Stokar formula \[ \text{[83x-64]} \] and the quark condensate is obtained as $\langle \bar{q}q \rangle_{\lambda=1 \text{GeV}} \simeq -(220 \text{MeV})^3$. These quantities related to DCB seem consistent with the standard values.

3. The Schwinger-Dyson Equation

In this section, we formulate the SD equation for quarks in the chiral limit in the Landau gauge. By taking the trace Eq. (1), one finds

$$ M(p^2) = \frac{g^2}{4} \int_q \text{tr} \{ \gamma_{\mu} \frac{Z(q^2)}{\hat{A} + M(q^2)} \Gamma_{\nu}(p, q) \} D_{\mu\nu}(\tilde{q}) $$

(4)

with $\tilde{q} \equiv p - q$. For the quark-gluon vertex, we assume the chiral-preserving vector-type vertex,

$$ \Gamma_{\mu}(p, q) = \gamma_{\mu} \Gamma((p - q)^2), $$

(5)

which keeps the chiral symmetry properly. (In contrast, to be strict, the Higashijima-Miransky approximation \[ \text{[83x-231]} \] explicitly breaks the chiral symmetry in the formalism.) Then, one obtains

$$ M(p^2) = C_F g^2 \int_q \frac{Z(q^2)M(q^2)}{q^2 + M^2(q^2)} \Gamma((q^2)^2) D_{\mu\nu}(\tilde{q}) $$

(6)

with $C_F = 4/3$ being the color factor for quarks. In the Landau gauge, the Euclidean gluon propagator is generally expressed by

$$ D_{\mu\nu}(p^2) = \frac{d(p^2)}{p^2} \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right), $$

(7)

where we refer to $d(p^2)$ as the gluon polarization factor. Therefore, Eq. (6) is rewritten as

$$ M(p^2) = 3C_F g^2 \int_q \frac{Z(q^2)M(q^2)}{q^2 + M^2(q^2)} \Gamma((q^2)^2) d(q^2) \cdot \frac{d(q^2)}{q^2}. $$

(8)

Here, we define the kernel function

$$ K(p^2) \equiv g^2 \Gamma(p^2) d(p^2) $$

(9)

as the product of the quark-gluon vertex $\Gamma(p^2)$ and the gluon polarization factor $d(p^2)$. Then, the SD equation is expressed as

$$ M(p^2) = 3C_F \int_q \frac{Z(q^2)M(q^2)}{q^2 + M^2(q^2)} K((p - q)^2), $$

(10)

In the Landau gauge, the quark wave-function renormalization is not so significant and seems to be approximated as $Z(p^2) = 1$, which reduces the SD equation to

$$ M(p^2) = 3C_F \int_q \frac{M(q^2)}{q^2 + M^2(q^2)} \frac{K((p - q)^2)}{(p - q)^2}. $$

(11)

4. Extraction of the Kernel Function in the SD Equation from Lattice QCD

In this section, we extract the kernel function $K(p^2) \equiv g^2 \Gamma(p^2) d(p^2)$ in the SD equation \[ \text{[83x-40]} \] using the quark mass function $M(p^2)$ obtained in lattice QCD. By shifting the integral variable from $q$ to $\tilde{q} \equiv q - p$, we rewrite Eq. (11) as

$$ M(p^2) = 3C_F \int_q \frac{M((p - q)^2)}{(p - q)^2 + M^2((p - q)^2)} \frac{K(q^2)}{q^2}. $$

(12)

Therefore, we obtain

$$ M(p^2) = \frac{3C_F}{8\pi^3} \int_0^\infty dq^2 \Theta(p, q) K(q^2), $$

(13)

where $\Theta(p, q)$ is defined with $M(p^2)$ as

$$ \Theta(p, q) \equiv \int_0^\pi d\theta \sin^2 \theta $$

$$ \frac{M(p^2 + q^2 - 2pq \cos \theta)}{p^2 + q^2 - 2pq \cos \theta + M^2(p^2 + q^2 - 2pq \cos \theta)}. $$

(14)

Since the quark mass function $M(p^2)$ is given by Eq. \[ \text{[83x-56]} \] in lattice QCD, we can calculate the kernel function $K(p^2)$ from Eq. \[ \text{[83x-175]} \].

As shown in Fig.1, we numerically obtained the kernel function $K(p^2) \equiv g^2 \Gamma(p^2) d(p^2)$ extracted from the lattice QCD result for the quark propagator in the Landau gauge.

As remarkable features, we find “infrared vanishing” and “intermediate enhancement” in the kernel function $K(p^2)$ in the SD equation. In fact, $K(p^2)$ seems consistent with zero in the very infrared region as

$$ K(p^2 \sim 0) \simeq 0, $$

(15)

while $K(p^2)$ exhibits a large enhancement in the intermediate-energy region around $p \sim 0.5 \text{GeV}$.

These tendencies of infrared vanishing and intermediate enhancement in the kernel function $K(p^2) \equiv g^2 \Gamma(p^2) d(p^2)$ are observed also in the
5. Chiral Symmetry at Finite Temperature

Finally, we demonstrate a simple application of the LQCD-based SD equation to chiral symmetry restoration in finite-temperature QCD.

At a finite temperature $T$, the field variables obey the (anti-)periodic boundary condition in the imaginary-time direction, which leads to the SD equation for the thermal quark mass $M_n(p^2)$ of the Matsubara frequency $\omega_n = (2n+1)\pi T$ as

$$M_n(p^2) = \int_{m,q} \frac{3C_F M_m(q^2) K(\omega_{nm}^2 + \tilde{q}^2)}{\omega_{nm}^2 + q^2 + M_m^2(q^2) \omega_{nm}^2 + \tilde{q}^2},$$

with $\int_{m,q} \equiv T \sum_{m=-\infty}^{\infty} \int \frac{d^3q}{(2\pi)^3}, \omega_{nm} = \omega_n - \omega_m$ and $\tilde{q} = p - q$.

Using the kernel function $K(p^2)$ obtained in the previous section, we solve Eq. (16) for the thermal quark mass $M_n(p^2)$. Figure 2 shows the preliminary result for the thermal infrared quark mass $M_0(p^2 = 0)$ plotted against the temperature $T$. We thus find chiral symmetry restoration at a critical temperature $T_c \sim 100\text{MeV}$.

Figure 2. The thermal infrared quark mass $M_0(p^2 = 0)$ plotted against the temperature $T$. The critical temperature $T_c$ is found to be about 100MeV.

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