THE TRANSITION BETWEEN PERTURBATIVE AND NON-PERTURBATIVE QCD

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We study both polarized and unpolarized proton structure functions in the kinematical region of large Bjorken $x$ and four-momentum transfer of few GeV$^2$. In this region the phenomenon of parton-hadron duality takes place between the smooth continuation of the deep inelastic scattering curve and the average of the nucleon resonances. We present results on a perturbative-QCD analysis using all recent accurate data with the aim of extracting the infrared behavior of the nucleon structure functions.

1. Introduction

Parton-hadron duality is generally defined as the similarity between hadronic cross sections in the Deep Inelastic Scattering (DIS) region and in the resonance region. It encompasses therefore a range of phenomena where one expects to observe a transmogrification from partonic to hadronic degrees of freedom, a question, the latter, at the very heart of Quantum ChromoDynamics (QCD).

A number of experiments were conducted in the early days of QCD where the onset of parton-hadron duality was observed as the equivalence between the continuation of the smooth curve describing different observables from a wide variety of high energy and large momentum transfer reactions – structure functions, sum rules, $R(s)$ for $e^+e^- \to$ hadrons, heavy meson decays... – and the same observables in the low energy region characterized by low final state invariant mass values and resonance structure. A fully satisfactory theoretical description of this phenomenon, that became to be accepted as a “natural” feature of hadronic interactions, is still nowadays very difficult to obtain. Recent progress both on the theoretical
and experimental side \(^1\), has however both renovated and reinforced the hadronic physics community’s interest in this subject as also demonstrated by the very lively discussions among participants in the present Workshop.

In our contribution to the Workshop, we present evidence that standard Perturbative QCD (PQCD) approaches in the large Bjorken \(x\) region might not be adequate to describe parton-hadron duality. In particular, by conducting an analysis of the most recent polarized and unpolarized inclusive electron scattering data, we unravel a discrepancy in the behavior of the extracted power corrections from the DIS and resonance regions, respectively.

### 2. Overview of data and QCD-based Interpretation

Parton-hadron duality being the idea that the outcome of any hard scattering process is determined by the initial scattering process among elementary constituents – the quarks and gluons – independently from the hadronic phase of the reaction, is a well rooted concept in our current view of all high energy phenomena. Bloom and Gilman (BG) duality \(^3\) is the extension of this idea to a kinematical region characterized by lower center of mass energies of the hard scattering process. Recently, more accurate experimental data have been collected that allow us to explore in detail this phenomenon. Besides the already mentioned new data on both inclusive electron-proton scattering \(^1\), and polarized inclusive electron-proton scattering \(^2\), several additional data sets and hadronic reactions were measured in the resonance region: polarized inclusive electron-proton scattering at Jefferson Lab kinematics \(^4,5\), \(\tau \rightarrow \nu^+\) hadrons \(^6\), \(\gamma p \rightarrow \pi^+ n\) \(^7,8\), and, finally, inclusive electron-nucleus scattering \(^9\) (most of the recent data were presented and discussed at this Workshop, see also \(^10\) for a recent review of both theoretical and experimental results).

A particularly interesting result was found in studies of inclusive reactions with no hadrons in the initial state, such as \(e^+e^- \rightarrow \) hadrons, and hadronic \(\tau\) decays \(^6\). It was pointed out that, because of the truncation of the PQCD asymptotic series, terms including quark and gluon condensates play an increasing role as the center of mass energy of the process decreases. Oscillations in the physical observables were then found to appear if the condensates are calculated in an instanton background. Such oscillating structure, calculated in \(^6\) for values of the center of mass energy above the resonance region, is damped at high energy, hence warranting the onset of parton-hadron duality.
In what follows we examine a related question, namely whether it is possible to extend the picture of duality explored in the higher $Q^2$ region to the resonance region, or to the BG domain. A necessary condition is to determine whether the curve from the perturbative regime smoothly interpolates through the resonances, or whether, instead, violations of this correspondence occur. The latter would indicate that we are entering a semi-hard phase of QCD, where preconfinement effects might arise. Our analysis of “duality violations” requires a sufficiently large and accurate set of data. We have applied it therefore to both the unpolarized and polarized inclusive measurements of proton structure functions in the resonance region.

3. Monitoring the Transition between pQCD and npQCD

We outline two important procedures for the study of parton-hadron duality in structure functions: i) the continuation of DIS curve into the resonance region; ii) the averaging of the resonances. Although these concepts are equivalently found in a number of different reactions, and in different channels (see e.g. 10,6), in this contribution we concentrate on the proton structure functions $g_1$ and $F_2$, for polarized, and unpolarized electron scattering, respectively.

3.1. Continuation of DIS Curve

It is important to define exactly what one means by “continuation” of the DIS curve, in order to be able to define whether parton-duality can be considered to be fulfilled. The accuracy of current data allows us, in fact, to address the question of what extrapolation from the large $Q^2$, or asymptotic regime the cross sections in the resonance region should be compared to. In principle any extrapolation from high to low $Q^2$ is expected to be fraught with theoretical uncertainties ranging from the propagation of the uncertainty on $\alpha_s(M_Z^2)$ into the resonance region to the appearance of different types of both perturbative and power corrections in the low $Q^2$ regime. All of these aspects need therefore to be evaluated carefully.

Our approach applies to the large Bjorken $x$ behavior of inclusive data. We therefore consider:

(a) Non-Singlet (NS) Parton Distribution Functions (PDFs) evolved at Next to Leading Order (NLO);
(b) PQCD evolution using the scale $\approx Q^2(1-z)$ which properly takes into account integration over the parton’s transverse momentum $^{11,12}$. 
(c) Target Mass Corrections (TMCs).

We perform an extensive study of inclusive data in the resonance region by extrapolating to this region all available parameterizations of PDFs, which can be considered pure DIS, down to the measured ranges for $x$, $Q^2$, and final state invariant mass, $W^2 = Q^2(1 - x)/x + M^2$. Our general approach for both the unpolarized structure function $F_2(x, Q^2)$, and the polarized one, $g_1(x, Q^2)$ is described in detail in \textsuperscript{14}. An important point illustrated also in \textsuperscript{14} is that the uncertainty due to the use of different parameterizations can be taken into account by a band that is currently smaller than the experimental one in the region of interest. A potential theoretical error in the extrapolation of the ratios to low $Q^2$ could be generated by the propagation of the error in $\alpha_S(M_Z^2)$. However, because at large $x$ perturbative evolution involves only Non-Singlet (NS) distributions, we expect it to affect minimally the extrapolation of the initial pQCD distribution, even to the low values of $W^2$ considered.

The problem of resumming the large logarithm terms arising at large $x$ was first noticed in a pioneering paper \textsuperscript{11}. There it was shown how this type of resummation can be taken care of by considering the correct definition of the upper limit of integration for the transverse momentum in the ladder diagrams defining the leading log approximation. This implies replacing $Q^2$ with $\approx \tilde{W}^2 = Q^2(1 - z)$, i.e. an invariant mass, in the evolution equations. Such a procedure was used to obtain our results both in Refs.\textsuperscript{14,15}, and in the current contribution.

Finally, TMCs are expected to be important at $W^2 \rightarrow M^2$. Although TMCs are kinematic in nature and their contribution to the Operator Product Expansion (OPE) was calculated early on \textsuperscript{13}, their effect on structure functions evolution cannot be evaluated straightforwardly at leading twist since a truncation of the twist expansion brings inevitably to a mismatch between the Bjorken $x$ supports of the TM-corrected and the “asymptotic” results \textsuperscript{16}. The extent of this mismatch is, however, small in the DIS region thus rendering approximated treatments applicable \textsuperscript{13}. In the resonance region the discrepancies arising in the \textsuperscript{13} approach can be large. We adopt, therefore, the prescription of Ref. \textsuperscript{16}, according to which the twist expansion for the standard moments Mellin of the structure function including TMCs is truncated consistently at the same order in $1/Q^2$ for both the kinematic terms and the dynamical higher twists. As noticed already in \textsuperscript{15}, this method applies for sufficiently small values of the expansion parameter $\approx 4M^2x^2/Q^2$. In addition, we have control on the uncertainty which is a
3.2. Averaging Procedure

Resonant data can be averaged over, according to different procedures. We considered the following complementary methods:

\[ I(Q^2) = \int_{x_{\text{min}}}^{x_{\text{max}}} F_2^{\text{res}}(x, Q^2) \, dx \]  \hspace{2cm} (1)

\[ M_n(Q^2) = \int_{0}^{1} dx \, \xi^{n-1} \frac{F_2^{\text{res}}(x, Q^2)}{x} \, p_n \]  \hspace{2cm} (2)

\[ F_2^{\text{ave}}(x, Q^2(x, W^2)) = F_2^{\text{Jlab}}(\xi, W^2) \]  \hspace{2cm} (3)

where \( F_2^{\text{res}} \) is evaluated using the experimental data in the resonance region. In Eq.(1), for each \( Q^2 \) value: \( x_{\text{min}} = Q^2/(Q^2 + W_{\text{max}}^2 - M^2) \), and \( x_{\text{max}} = Q^2/(Q^2 + W_{\text{min}}^2 - M^2) \). \( W_{\text{min}} \) and \( W_{\text{max}} \) delimit either the whole resonance region, i.e. \( W_{\text{min}} \approx 1.1 \text{ GeV}^2 \), and \( W_{\text{max}} \approx 4 \text{ GeV}^2 \), or smaller intervals within it. In Eq.(2), \( \xi \) is the Nachtmann variable, and \( M_n(Q^2) \) are Nachtmann moments. The r.h.s. of Eq.(3), \( F_2^{\text{Jlab}}(\xi, W^2) \), is a smooth fit to the resonant data, valid for \( 1 < W^2 < 4 \text{ GeV}^2 \); \( F_2^{\text{ave}} \) symbolizes the average taken at the \( Q^2 \equiv (x, W^2) \) of the data.

4. Results

After describing our program to address quantitatively all sources of theoretical errors started in Refs.\cite{15,14}, in Fig.1 we present our main results on the extraction of the dynamical Higher Twist (HT) terms from the resonance region. A clear discrepancy marking perhaps a breakdown of the twist expansion at low values of \( W^2 \) is seen for the unpolarized structure function, \( F_2 \) (upper panel). A comparison with other results obtained in the DIS region \cite{18} is also shown. For the polarized structure function, \( g_1 \), we added to our previous analysis data from Refs.\cite{4} at \( Q^2 = 0.65, 1, 1.2 \text{ GeV}^2 \). In addition, we used the experimental values of the ratio \( R = \sigma_L/\sigma_T \) from recent Jefferson Lab measurements in the resonance region \cite{21} which introduce an oscillation around the original result of about 2%, well within the error bars. A complete presentation and discussion of these results along with comparisons with other extractions \cite{19,20} will be given in a forthcoming paper \cite{22}. From

\textsuperscript{a}Similar formulae hold for the polarized structure function, \( g_1 \).
the figure one can see that although the trend seen in $Q^2 \approx 1 \text{ GeV}^2$ seems to be confirmed, more polarized data at large $x$ are needed in order to draw definite conclusions.

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Figure 1. Upper panel: Comparison of HT contributions for the structure function $F_2$ in the DIS and resonance regions, respectively. The full circles are the values obtained in the resonance region\(^{14}\). For $F_2$ these are compared with extractions using DIS data, from
\({\it \cite{18}}\). Lower panel: ratio of the experimental data on $g_1$\(^{2,4}\) and the PQCD extrapolation to the resonance region\(^{22}\). Notice that the new results obtained using data from\(^{4}\) agree with the trend of the Hermes data.
\[ R_{LT} = \frac{\tilde{\Gamma}_1 \text{res}}{\tilde{\Gamma}_1} \]

- \text{Data/NLO}
- \text{Data/NLO+TMC}
- \text{Data/NLO+TMC+LxR}

\[ Q^2 \text{ [GeV}^2\text{]} \]
$H(x)$

- BFL (2004) $W^2 < 4 \text{ GeV}^2$
- LSS (2003) DIS