MAGNIFICATION RATIO OF THE FLUCTUATING LIGHT IN GRAVITATIONAL LENS 0957 + 561

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ABSTRACT

Radio observations establish the B/A magnification ratio of gravitational lens 0957 + 561 at about 0.75. Yet, for more than 15 years, the optical magnification ratio has been between 0.9 and 1.12. The accepted explanation is microlensing of the optical source. However, this explanation is mildly discordant with (1) the relative constancy of the optical ratio and (2) recent data indicating possible non-achromaticity in the ratio. To study these issues, we develop a statistical formalism for separately measuring, in a unified manner, the magnification ratio of the fluctuating and constant parts of the light curve. Applying the formalism to the recently published data of Kundic and coworkers, we find that the magnification ratios of fluctuating parts in both the g and r colors agrees with the magnification ratio of the constant part in g band, and tends to disagree with the r-band value. One explanation could be about 0.1 mag of consistently unsubtracted r light from the lensing galaxy G1, which seems unlikely. Another could be that 0957 + 561 is approaching a caustic in the microlensing pattern.

Subject heading: gravitational lensing — quasars: individual (0957 + 561)

1. INTRODUCTION

The gravitational lens system 0957 + 561 has by now been observed at optical and radio wavelengths for nearly 20 yr (Walsh, Carswell, & Weymann 1979; Porcas et al. 1979). Radio studies have definitively established that the B/A magnification ratio of the lens, measured at the core of the radio images (which lies at the location of the optical point-source images), is close to 0.75; some recent measurements are 0.75 ± 0.02 (Garrett et al. 1994), and 0.752 ± 0.028 (Conner, Lehár, & Burke 1992). Note that while there is some controversy about the radio magnification ratio at the location of the radio jet, as opposed to the core (see, e.g., Garrett et al. 1994), only the core value interests us here.

Because the variability of the quasar in the optical is larger than in the radio, measurement of the B/A magnification ratio in the optical requires that the light curves be shifted by the correct time delay τ before the ratio is taken. Thus, the earliest determinations of B/A were incorrect. For example, Young et al. (1980) obtained a ratio of 0.76, comfortably—yet erroneously—close to the radio magnification.

However, at least since Vanderriest et al. (1989), who used a value for τ quite close to definitive recent determinations (Kundic et al. 1997), it has been clear that the B/A magnification ratio of the optical continuum in the B and A point sources is quite different from 0.75 and, moreover, has remained at least fairly constant for the full history of observation. Smoothing over observing seasons, Vanderriest et al. (1989) obtained a B/A ratio varying between about 0.9 and 1.05 over the observing seasons early 1980 through early 1986 (times referenced to the A component), with a single best-fit value of 0.97. It is debatable whether the variation around the best-fit value is actual time variation of the lens magnification ratio (note: as distinct from time variation in the quasar luminosity) or observational artifact. However, it does seem quite likely that the magnification ratio varied by no more than about ±8% during this time.

More recently, the value obtained by Kundic et al. (1995, 1997) for the 1995 season (A component) is 1.12 ± 0.01 in the g-band (with the error bar, a 95% confidence limit, depending somewhat on the method of reduction used). So it is quite plausible (and not contradicted by other measurements in the literature) that the optical B/A remained in the 0.9–1.12 range from 1980 through 1995, and possible that the variation has been considerably smaller than this range.

The discrepancy between the optical and radio magnification ratios has long been understood as due to microlensing (as predicted by Chang &Refsdal 1979 and Gott 1981). The proper radius of the Einstein ring from a 0.5 $M_{\odot}$ star at the lens galaxy redshift $z = 0.36$, illuminated by the quasar at redshift $z = 1.41$, is about $2 \times 10^{16} h^{-1/2}$ cm. Since the radio emission region is much larger than this scale, it averages spatially over the microlensing pattern and is magnified by the macrolens ratio of 0.75. If the optical magnification indeed differs by $\sim 30\%$ from the macrolens value, then the optically emitting region must be smaller, or at most a few times larger, than the Einstein ring scale. This accords nicely with, e.g., the size of an accretion disk smaller than 100 Schwarzschild radii around a $10^9 M_{\odot}$ black hole (a scale of $3 \times 10^{15}$ cm).

This Einstein ring radius is, however, only marginally in accord with the apparent constancy of the microlensed magnification ratio: Since the Earth, the microlensing star (or stars, the effect being collective), and the quasar each have three-dimensional peculiar velocities of at least 300 km s$^{-1}$, the Earth should move through $\sim 100\%$ microlensing variations in $\sim 10$ yr (see Kochanek, Kolatt, & Bartelmann 1996 for related calculations). So the observed microlensing is about a factor of 10 too constant, and one is invited to speculate on whether something other than luck is the reason.

Another invitation to speculation is the fact that Kundic et al. obtain rather different magnification ratios in their r- and g-band data, with the r ratio being 1.22 ± 0.02: again, the error bar depends on the method of analysis used. By any interpretation of the error bars, however, the r and g results are strongly discrepant. (Again note that there is no assumption that the fluctuations themselves have the same amplitude in the two colors, but only that the magnification ratio should be the same.) Either a full 0.1 mag of r-band
galaxy light has escaped Kundić et al.’s careful subtraction in the B image, or something else is going on in the lens magnification ratio.

With these two hovering peculiarities (possible excess time constancy and possible nonachromaticity), it seems useful to try to get additional information on the magnification ratio. This paper therefore asks the question: Is the optical magnification ratio the same for the source region that produces the fluctuations in quasar light as it is for the source region that produces the constant light? And does the magnification ratio of the fluctuations (which we may call the AC magnification ratio or ACMR) agree more closely with the r- or with the g-band magnification ratio previously measured (here called the DC magnification ratio or DCMR)?

The answers to these questions can help diagnose the following situations: (1) If, as is true in many models, the size of the emitting region is much smaller than the microlensing scale, then all the magnification ratios should have the same value. (2) If there is a problem with r-band galaxy subtraction—or any other constant source of flux added to one lens component and not the other, then the ACMR should represent the “true” microlens magnification ratio, and we might further expect it to be close to the g-band DCMR (where galaxy subtraction is a much smaller effect). (3) If the optically emitting quasar accretion disk has a scale comparable to the microlensing scale and has (as seems almost inevitable) color gradients, then the r and g ACMRs, and r and g DCMRs, might all be distinct. Indeed, the two ACMRs and two DCMRs then provide four distinct windows on the convolution of the accretion disk source with the microlensing pattern. We note that Barkana (1997) has shown evidence for such microlensing gradients in the quadruple gravitational lens PG 1115+080.

Conceptually, one measures a DCMR and an ACMR as follows: Shift one of the light curves (A, B) in time by τ to undo the lens delay. Fit each light curve by a constant value plus a residual time-varying part. The ratio of the constant values is the DCMR. Now, for the two time-varying residuals, fit for a model that makes the B residual a constant times the A residual. The best-fitting constant is the ACMR.

This conceptual formulation, while simple, is actually not quite right. In the next section, we will give a statistical formulation of the problem that is more complete and also more directly applicable to unevenly sampled data. In § 3 we discuss some implementation details, and in § 4 we apply the formulation to the published data of Kundić et al. (1995, 1997). Section 5 is discussion and conclusions.

2. DERIVATION OF STATISTICAL METHOD

The spirit of this derivation is very much the same as that described in Rybicki & Press (1992, hereafter RP92), which the reader might wish to consult at this point. For the purposes of calculation, we assume that the underlying fluctuating light curve is generated by a Gaussian process f(t). This is of course only an approximation, and we will comment in § 3 below on its limits of validity; but the Gaussian assumption provides a clean analytic framework. Moreover, it is actually quite hard to find evidence for non-Gaussianity in the 0957+561 light curve (see Press & Rybicki 1997).

The process f(t) is completely characterized by its covariance,

\[ S(t_1, t_2) \equiv \langle f(t_1)f(t_2) \rangle, \tag{1} \]

so that the \( \chi^2 \) of a series of (for now, noiseless) observations \( f(t_1), f(t_2), \ldots, f(t_n) \) is

\[ \chi^2 = f^T S^{-1} f, \tag{2} \]

where \( f \) is the vector of f-values and \( S \) is the matrix of \( S \)-values relating all pairs of times occurring in \( f \). The relative probability of a given sequence \( f \) occurring is

\[ P(f) \propto (\det S)^{-1/2} \exp (-\chi^2/2). \tag{3} \]

We now model the observed A- and B-component light curves as

\[ a(t) = a_0 + f(t) + n_a(t), \tag{4} \]

\[ b(t) = b_0 + Rf(t) + n_b(t). \tag{4} \]

Here \( f(t) \) is a Gaussian process, as above, \( n_a \) and \( n_b \) are the (assumed Gaussian) noise processes in the A and B measurements, respectively, and \( R \) is the desired B/A magnification ratio, the ACMR. (The DCMR would be \( b_0/a_0 \).)

Evidently \( a(t) - a_0 \) and \( b(t) - b_0 \) are Gaussian processes. If \( a \) is a vector of A measurements and \( b \) is a vector of B measurements, then the \( \chi^2 \) value of a vector combining both measurements is immediately

\[ \chi^2 = \begin{pmatrix} a - a_0 & b - b_0 \end{pmatrix} M^{-1} \begin{pmatrix} a - a_0 \\ b - b_0 \end{pmatrix}, \tag{5} \]

where the matrix \( M \) has the block-partitioned form

\[ M = \begin{pmatrix} S_{aa} + N_a & R S_{ab} \\ R S_{ba} & R^2 S_{bb} + N_b \end{pmatrix}, \tag{6} \]

with \( N_a \) and \( N_b \) being the noise covariances (here and subsequently assumed to be diagonal) and the notation \( S_{ab} \), e.g., indicating a matrix of covariance values \( S \) relating the times of A observations and the times of B observations. The associated Gaussian probability, where we now emphasize the dependence on the unknown parameters, is

\[ P(a, b | a_0, b_0, R) \propto (\det M)^{-1/2} \exp (-\chi^2/2). \tag{7} \]

Noting that

\[ M = \begin{pmatrix} 1 & 0 \\ 0 & R \end{pmatrix} C \begin{pmatrix} 1 & 0 \\ 0 & R \end{pmatrix}, \tag{8} \]

where

\[ C = \begin{pmatrix} S_{aa} + N_a & S_{ab} \\ S_{ba} & S_{bb} + N_b \end{pmatrix}, \tag{9} \]

we also have

\[ \chi^2 = \begin{pmatrix} a - a_0 \\ R^{-1} b - R^{-1} b_0 \end{pmatrix} C^{-1} \begin{pmatrix} a - a_0 \\ R^{-1} b - R^{-1} b_0 \end{pmatrix}, \tag{10} \]

and

\[ \det M = R^{2M} \det C, \tag{11} \]

where \( M \) is the length of the vector \( b \) (the number of B-component data points).

We now arrive at a somewhat subtle, though important, issue: We do not want to assume that the fluctuating process \( f(t) \) has zero mean. Indeed, physically, \( f(t) \) might be everywhere positive, since a process can emit positive photons but not negative ones. Thus, in the context of equation (10), we want to use an unbiased \( \chi^2 \) that is independent of moving some constant flux from \( f(t) \) and into \( a_0 \)
and $b_0$ (in correct proportion). RP92 showed how such a “Gauss-Markov,” unbiased process is obtained by adding a term $\lambda E E^T$ to $C$ and then letting $\lambda \to \infty$ before calculating $C^{-1}$. (See RP92, especially the two paragraphs following eq. [23] of that paper, for an intuitive explanation of this limiting process.) Henceforth, we assume that this limit is always taken. In this case, $\chi^2$ becomes independent of any constant added to the vectors in the quadratic form, and equation (10) can be written as

$$\chi^2 = (a - R^{-1}b - c_0)^T C^{-1} (R^{-1}b - c_0),$$

(12)

where

$$c_0 \equiv R^{-1}b_0 - a_0 .$$

(13)

The point is that, for fluctuations not known to have zero mean, the three parameters $R$, $a_0$, and $b_0$ are not, even in principle, separately measurable. But the two parameters $R$ and $c_0$ are measurable. We refer to $c_0$ as the “contamination,” since in the ideal case that $B$ alone is contaminated by spurious constant light (e.g., an unsubtracted component of the lens galaxy’s light) then $Rc_0$ represents the spurious flux.

It is possible to separate, analytically, the contribution to $\chi^2$ of $c_0$ and $R$ as follows: Let

$$y \equiv \begin{pmatrix} a \\ R^{-1}b \end{pmatrix}, \quad q \equiv \begin{pmatrix} 0 \\ E_0 \end{pmatrix} \equiv (0, 0, \ldots, 0, 1, 1, \ldots, 1)^T ,$$

(14)

so that equation (12) becomes

$$\chi^2 = (y - c_0q)^T C^{-1} (y - c_0q) .$$

(15)

Straightforward matrix “completion of the squares” shows that equation (15) is the same as

$$\chi^2 = (q^T C^{-1} q - q^T C^{-1} c_0) + y^T H y ,$$

(16)

where

$$H = C^{-1} - \frac{q^T C^{-1} q}{q^T C^{-1} q} .$$

(17)

The probability associated with equation (16) is (using equations [7] and [11])

$$P(a, b | R, c_0) \propto (R^2 M \det C)^{-1/2} \exp \left( -\frac{\chi^2}{2} \right) .$$

(18)

Equations (16)–(18) could be used directly on data, to obtain, e.g., maximum likelihood estimates of $c_0$, which appears explicitly and only in one term, and $R$, which appears implicitly only through $y$. However, we want to take the more Bayesian viewpoint that $c_0$ is a nuisance variable that ought to be integrated over rather than maximized. (In practice, we find that $c_0$ is generally so well determined at fixed $R$ that it hardly matters whether we are Bayesians or not.) Bayes’s theorem now gives the simple result

$$P(R | a, b) \propto \int_{-\infty}^{\infty} dc_0 P(a, b | R, c_0) \times \exp \left[ -(1/2)y^T H y \right] \ (ACMR),$$

(19)

where, in the last equality, all constant factors that do not depend on $R$ have been dropped. The constant of proportionality is determined, in Bayesian manner, by requiring that the integral of equation (18) over $R$ be unity. Equation (18) is easily evaluated on a given data set $a, b$ for a range of values of $R$, giving either Bayesian probabilities or confidence intervals for $R$, which is the AC magnification ratio (ACMR, see the introduction).

What about the DC magnification ratio (DCMR)? From the above discussion, we now see that it is not directly measurable, without further assumption. The reason (to reiterate) is that some part of the light that is physically part of the fluctuating part, and which might fluctuate in the future, may happen to be constant, or nearly so, over a finite data set. The closest thing to what the observer means by DCMR is obtained by setting $c_0$ to zero in equation (15) and the following equations. That is, the observer corrects the data sets $a, b$ for all known constant sources of error (galaxy light subtraction, and so on), then fits for the best single ratio $R$ that directly relates the A and B data sets. The Bayesian probability corresponding to equation (19) is thus evidently, by equation (15),

$$P(R | a, b) \propto R^{-M} \exp \left[ -(1/2)y^T C^{-1} y \right] \ (DCMR) .$$

(20)

We will see that this DCMR is generally much better determined statistically than is the ACMR, but at the price of having unknown systematics (in the precorrection of the data). The ACMR is much less well determined, but is completely independent of such systematics. Thus, there is a synergy in computing both magnification ratios by the unified formalism given here.

3. HINTS AND LIMITATIONS

We need to discuss how limiting the original assumption of a Gaussian process is. The Gaussian assumption enters in two ways: First, it provides an “automatic” means of interpolating across the time intervals between measured A and B data points (which do not in general line up after shifting one by $\tau$). Experience has shown (see Press, Rybicki, & Hewitt 1992a, 1992b) that this use is quite robust—the interpolation is sensible and not very different from any other sensible method.

Second, the Gaussian assumption is used in associating $\chi^2$ values (and, more importantly, $\Delta \chi^2$ values) to probabilities. One should definitely be suspicious of this association in the tails of the distribution. If, however, one takes the resulting probability distributions as indicative of central value and uncertainty, rather than as correct in detail, then one is on relatively safer ground: the procedures described are essentially $\chi^2$ parameter estimations, and such estimations (in the limit of large numbers of data points, when the central limit theorem can apply) do not require any additional Gaussian assumptions. (Likewise, neither a Gaussian assumption, nor any interpolation procedure as such, needs to be assumed in estimating the time delay or magnification ratio of lens data; the method is essentially a minimum variance estimator.)

As in any $\chi^2$ parameter estimation, the method is valid only if the error model is close to correct. A simple test to be passed is whether the values of $\chi^2$ in the exponentials of equations (19) and (20) are consistent with the number of data points, i.e., whether the reduced $\chi^2$ is close to unity. When this is the case, we have found that additional fine tunings of the error model (fiddling the functional form of the covariance in the matrix $S$, or trade-offs between adjust-
ing the correlation model embodied in $S$ and the noise model embodied in $N$ have little effect on the output $P(R)$ probability functions. If, however, the original reduced $\chi^2$ is not close to unity (whether too low or too high), then any kind of rescaling procedure must be viewed as introducing unknown biases into the method given here.

Indeed, the reason that we restrict ourselves, in the next section, to the data of Kundic et al. (1995, 1997) is that for other data sets (e.g., Vanderriest et al. 1989) we have not been able to formulate a satisfactory and self-consistent error model ($S$ and $N$ matrices).

A final technical note is to point out that the calculations implied by equations (19) and (20) can all be done using the “fast methods” for inverting $S$ and $N$ matrices described by Rybicki & Press (1995). These fast methods restrict the correlation model to a particular functional form, basically requiring the correlation structure function to be a linear function of lag time. However, this approximation appears to be adequate for the 0957+561 data, at least for these purposes, and the resulting reduction in computation time is enormous and would become essential for very large data sets.

4. APPLICATION TO KUNDIĆ ET AL. DATA

Figure 1 shows the results of applying the analysis described in the previous two sections to the data set published by Kundic et al. (1995, 1997). First we convert the data and error bars from magnitudes to fluxes $a(t)$ and $b(t)$. Next, we use the $A$-component light curves (separately in the $g$ and $r$ colors) to estimate a structure function $V(t) \equiv \langle [a(t) - a(t + \tau)]^2 \rangle$. Next, we use this structure function, along with the errors, to construct the $S$ and $N$ matrices appropriate for the “fast” method (Rybicki & Press 1995). It is at this stage that the Gauss-Markov (unbiased) limit is taken (see discussion before eq. [12] above).

The upper panel of the figure plots $P(R)$ as a function of $R$ for both the DCMR (eq. [20]; plotted as the “narrow” distributions) and the ACMR (eq. [19]; plotted as the “broad” distributions). In each case, the results for the $g$-band data are shown as solid curves, while the $r$-band data are shown as dotted curves.

Our ACMR values closely reproduce the $B/A$ flux ratios quoted by Kundic et al. in both the $r$ and $g$ bands, and our errors (widths of curves shown) are comparable to the latter’s quoted errors. Our ACMR values—novel to this work—are seen to be compatible, in both $g$ and $r$, with the $g$-band DCMR. Although there is no strict incompatibility among all four values (except the two DCMRs, as previously remarked) the distributions in Figure 1 tend to support the conclusion that the $r$-band DCMR is “odd man out.”

The lower panel in Figure 1 shows how many magnitudes of contamination (relative to the time-average flux) would need to be present in the $B$ image to move the DCMR peak from its plotted location to any other location. By definition, the curves go through zero at the DCMR peak centers. One sees that about 0.1 mag is required in the $r$ band to shift the DCMR to compatibility with $g$-band; however, even without shifting, the $r$-band DCMR is at least marginally compatible with the $r$-band ACMR.

5. DISCUSSION AND CONCLUSIONS

While these data, in this analysis, do not support any very definitive conclusions, we may make the following remarks:

Occam’s razor would seem to indicate that the $r$-band light curve of Kundic et al. has about 0.1 mag of residual, unsubtracted, constant light, as perhaps from unmodeled small-scale variations in the lens galaxy surface brightness. If this is the case, then all the data are compatible with a single magnification ratio for both colors and for both the fluctuating and constant pieces. This in turn suggests an accretion disk scale much smaller than the microlensing scale, in accord with theoretical prejudice. The utility of the ACMRs is that, taken together, they strongly favor the hypothesis that the $g$-band magnification ratio is the correct one and that nothing more exotic is going wrong. We note, however (E. Turner 1998, private communication), that the galaxy G1 is something like 2 magnitudes fainter than component B in the $r$ band; thus the amount of unsubtracted light would need to be comparable to the total brightness of G1, which seems quite unlikely.

It is up to the observers, not us, to decide whether Occam’s razor should rule in this case. If 0.1 mag of residual is not possibly present, then we must conclude that the accretion disk scale is comparable to the microlensing scale and that the constant $r$-band part of the disk is more strongly magnified than (at least some of) the other three regions. It seems likely on physical grounds that the fluctuating regions should be smaller than the constant regions and...
that the $g$-band regions should be smaller than the $r$-band regions (temperature decreasing outward in the disk). For the larger ($r$-band and constant) region to have a higher magnification ratio than an enclosed smaller region, the larger region must extend to a place where the magnification is a superlinear function of position in the sky. This might indicate at least a fair chance of the B image passing through a caustic in the near ($\sim 10$ yr) future. This possibility, as well as the reconciliation of the relative constancy of the magnification ratio over the last 15 years, will be explored by Monte Carlo simulations in another paper (Press & Kochanek 1999).

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