To study the effect of inventory dependent consumption parameter for constant and time dependent holding cost

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Abstract. Economic production quantity models are effectively used to control inventory. In the present work EPQ model has been developed to study the effect of demand parameters on holding cost and total inventory. Demand used is inventory dependent during inventory buildup time and constant demand has been used during inventory depletion period. Rate of deterioration is considered to be constant. A numerical example has been included to validate the model. The sensitivity analysis shows that, higher inventory can attract customer to buy more. The result of models indicates that, the holding cost and total inventory is sensitive to change in inventory dependent demand parameter. Increase in demand parameter decreases the holding cost and total inventory.

1. Introduction
Every organization in many sectors of the economy has adopted some type of inventory system. Inventory is one of the most visible and tangible aspects of doing business. Unless inventories are controlled, they may be unreliable, inefficient and costly. The Economic Production quantity (EPQ model has been widely used for more than decades as an important tool to control the inventory. The EPQ is powerful to help practitioners and engineers to make a decision regarding inventory control. The complexity of a resulting EPQ model depends on assumptions one makes about various parameters of the inventory system. The effect of an imperfect production process, by assuming different rates of deterioration during the production process, on the optimal cycle time had been studied by (Rosenblatt and Lee, 1986). The Economic order quantity (EOQ model with power form of stock dependent demand for deteriorating items (Teng. et.al. 2005), investigated that the shape parameter of the demand and selling price is highly sensitive to optimal solution. The impact of random machine failure on the EPQ model (Teng. et.al. 2005) reveals the demand function and purchase cost is positive and fluctuating with time. The production process can shift at random from in control to out of control state. In such situation, the items produced out of control state deteriorate at higher rate than those at the normal rate, hence get consumed by demand under Last in First out (LIFO) policy (Garry and Dahl, 2006). Beerhouse and Babar, developed (EPQ) model by considering both the depreciation cost of stored items and process quality cost. Depreciation cost and process quality cost were assumed to be a continuous function of holding time and of production run length. Yuan et.al (2011) derived the optimal manufacturing batch size and number of shipment with scrap using mathematical modeling and algebraic approach. Ata (2011) discussed EPQ model for multi products single machine with discrete delivery. Naja and Naiki focused on EPQ model with a production capacity limitation and a random defective production rate. Gede and Hui(2010) analyzed an EPQ model for deteriorating items with stochastic machine unavailability and price-dependent demand. Jinn(2007), used time varying demand and cost to analyze EPQ model and characterize the influences of both demand and cost over the length of production run time and the economic production quantity. Hezari (2008) developed EPQ model by considering imperfect and defective items. Holding cost for defective items has not been considered in his study. David et. al (2009) considered partial backordering with constant demand to study inventory model. Jiang Tao (2009) considered a multi-item inventory system, where the vendor provides delay in payment to the retailer. Yao (2007) established a model for deteriorating items under delay in payment, in which the demand is a negative exponential function of price. Jia (2009) reported a supply chain system with trade credit. Liao established an EPQ model for deteriorating items.
with delay in payment. Ardak et al. (2017) developed EPQ model by considering mixed demand pattern. Ardak and Borade (2017) considered time dependent holding cost to develop EPQ model. Most of the researchers developed EPQ models by considering various demand patterns. The present study discusses the effect of inventory dependent parameter on holding cost and total inventory with the help of EPQ model. In the present study, it is assumed that the demand remains as inventory dependent during inventory buildup time and constant during inventory depletion period. This paper has five sections. Research motivation and literature is narrated in section 1. Section 2 contains notations and assumption. Methodology used to develop the model is discussed in section 3. Numerical and sensitivity analysis is discussed in section 4 and finally concluded in section 5.

2. Assumptions and Notation

2.1 Assumptions:-

The following assumptions are made in development of the model.

a) The production rate is known and treated as constant.
b) The production rate is greater than the demand.
c) The demand is inventory level dependent in up time and constant in down time.d) Deterioration of the items is varying.e) Inventory holding cost is known and termed as constant and time dependent.f) Deterioration of the items start as it enters into inventory.g) Shortages are not allowed.h) Every produced items needs inspection.

2.2 Notation:-

- D – Basic demand,
- H – Holding cost per unit,
- I₁ – Inventory level during production up time,
- I₂ – Inventory level during production down time,
- P – Rate of production,
- T – Production cycle time,
- T₁ – Production up time. Maximum inventory buildup in this time,
- T₂ – Production down time,
- Ci – Inspection cost per unit,
- I – Total Inventory,
- θ – Basic rate of Deterioration,
- α – Inventory dependent consumption rate parameter,

3. Mathematical Model.

This model deals with deteriorating items those consider the effect of different demand patterns for different time periods. Inventories are build up gradually during production up time, new production run will start after complete consumption of the buildup inventory. To keep the production process to its initial working condition, setup is essential. At the beginning, it is assumed that the inventory is zero.
During the time period \((0, T_1)\), as the production rate is greater than the demand rate, inventory is gradually buildup at a rate of \((P-D)\). Here the demand rate is inventory dependent. This rate is offset by a constant deterioration rate. At time \(T_1\) the inventory will be maximum. At this stage, production is terminated and on hand inventory used to meet the demand and to offset the loss due to deterioration. In this period, demand remains constant. The deterioration of the product will start soon it enters into the inventory. Production system can be described by the following differential equations.

According to the assumptions, over time span \([0, T_1]\), deterioration rate is constant and demand rate is inventory dependent which makes variation of the inventory level with respect to time for the reference time governed by

\[
\frac{dI_1(t)}{dt} = P - D - \alpha I_1(t) - \theta I_1(t) \quad 0 \leq t \leq T_1
\]

In the time interval \((0,T_2)\), the system is affected by the combined effect of constant demand and deterioration. Hence, the change in inventory level is governed by the following differential equation

\[
\frac{dI_2(t)}{dt} = -D - \theta I_2(t) \quad 0 \leq t \leq T_2
\]

Above equations can be solved by using initial boundary conditions like at \(t = 0\), \(I_1(t) = 0\) and at \(t = T_2\), \(I_2(t) = 0\). And by using boundary condition \(I_1(T_1) = I_2(0)\), approximate value of production down time can be given as,

\[
T_2 \approx \frac{P-D}{D} \left[ T_1 \cdot \frac{(a+\theta)T_1^2}{2} \right]
\]

Total inventory is given by sum of inventory in up time and inventory in down time.

\[
TI = \int_0^{T_1} I_1(t) dt + \int_0^{T_2} I_2(t) dt
\]

By solving Equation no. 4, use Taylor’s series approximation and after some simplification, the approximate total inventory can be expressed as,

\[
TI \approx \frac{(P-D)T_1^2}{2} + \frac{DT_2^2}{2}
\]

Constant Holding cost is given by

\[
HC = H(TI)
\]

Time dependent Inventory holding cost is given by
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\[ I_H = \int_0^{T_1} H(t) I_1(t) dt + \int_0^{T_2} H(t) I_2(t) dt \]
\[ I_H = (P - D) T_1^2 \left[ \frac{a}{2} + \frac{b}{\alpha + \theta} \right] + D T_2^2 \left[ \frac{a}{2} + \frac{b}{\theta} \right] \]

Total cost = Set up cost + Holding cost + Inspection cost + Deteriorating cost.

\[ T_C = A + H(T_1) + I_C + C_d(T_D) \]

As total cost per unit time is a function of production up time, Optimum value of production up time can be derived by solving following equation.

\[ \frac{dT_C}{dT_1} = 0 \]

4. Numerical experiment and sensitivity analysis

Numerical example and sensitivity analysis has been carried out to validate the theoretical aspects. The numerical data is adopted from work published elsewhere (Ardak and Borade) except some additional data like deterioration cost, set up cost and inspection cost. Let \( A = \text{Rs}. 30 \) per production cycle, \( C_d = \text{Rs}. 5 \) per unit per unit time, \( C_i = \text{Rs} 2 \) per unit per unit time, \( H = \text{Rs}.2 \) per unit per unit time, \( P = 2500 \) units per unit time, \( D = 1200 \) units per unit time, \( \alpha = 0.5, \theta = 0.1 \)

The optimum value of \( T_1 \) can be found, as the total cost function is convex (fig. 2 and 3). The optimum value of \( T_1 \) when holding cost considered to be constant is 0.073 and 0.033 when holding cost is linear function of time.

Fig. 2 \( T_1 \) v/s TCT for constant holding cost

Fig. 3. \( T_1 \) v/s TCT for time dependent holding cost

Table 1 Values of constant HC, and TI for different values of \( \alpha \)

| \( \alpha \) | HC  | TI   |
|------------|-----|------|
| 0.3        | 106.3 | 53.15 |
| 0.4        | 102.11 | 51.055 |
Table 2. Values of time dependent HC and TI for different values of $\alpha$

| $\alpha$ | HC | TI |
|--------|-----|----|
| 0.3    | 29.100 | 1.97 |
| 0.4    | 28.520 | 1.62 |
| 0.5    | 28.470 | 1.46 |
| 0.6    | 27.960 | 1.29 |
| 0.7    | 27.590 | 1.14 |

Figure 4, 5, 6 and 7 shows the relation between inventory dependent demand parameter, total cost and holding cost. Demand used for this work is inventory dependent for production up time. Curve in Fig.6
and 7 indicates that total inventory and holding cost both are decreasing function of ‘α’. Inventory dependent consumption rate decreases the total inventory and holding cost, subsequently may lead in decreasing total cost. At higher inventory level, demand will be more and hence can reduce total cost. Decrease in holding cost implies that inventory dependent demand will be profitable. Thus selection of optimum value of ‘α’ attracts the attention of inventory managers.

5. Conclusion
The research findings of the present work will be useful to practitioners in realizing the importance of inventory dependent demand and time dependent holding cost on EPQ. During inventory buildup time, demand used was inventory dependent and during inventory depletion time it was constant. This approach has not been considered in previous research. The research indicates that the annual total cost function possesses convexities. It was found that holding cost and total inventory is sensitive to change in parameter ‘α’. Increase in demand parameter ‘α’ decreases the holding cost and total inventory. This attracts the attention of inventory managers to estimate the accurate value of the inventory dependent consumption rate parameter. This indicates that higher inventory stock can attract the customer to buy more.

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