Simulation of a RICH Detector for the CKM Experiment

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We will present here the simulations of RICH detectors which will be used in the CKM experiment. We will verify their performance, critical to the experiment.

\textit{Keywords:} RICH detector; phototubes; chromatic dispersion; rare kaon decay

En este trabajo, se presenta una simulación con el fin de verificar el desempeño de dos detectores RICH que se usarán en el experimento CKM.

\textit{Descriptores:} Detector RICH; fototubos; decaimiento raro de kaones; dispersión cromática

PACS: 14.40.Aq, 29.40.Ka, 78.20.Li, 85.60.Ha

\section{1. Introduction}

In this article we present results on simulating a RICH detector which will be used within the CKM experiment \cite{1}, a rare kaon decays experiment recently approved at Fermilab. The main objective is to verify if the resolution of the current design of the detector is sufficient for the CKM experiment.

After a short description of the CKM experiment, we will detail the requirements for the detector system, describe the current design, our method of simulation, and the results obtained so far \cite{2}.

\section{2. Description of the CKM Experiment}

CKM stands for “Charged Kaons at the Main Injector”. The main goal of this experiment is to measure the branching ratio of $K^+ \to \pi^+\nu\bar{\nu}$ to about 10\% statistical precision. This measurement can be used to extract in a theoretically clean way the magnitude of $V_{td}$, an element of the Cabibbo Kobayashi Maskawa matrix, with an overall precision of 10\% \cite{3}.

This measurement plays an important role in testing the description of $CP$ violation in the Standard Model, and is complementary to the current efforts of the B-factories.

The prediction of the Standard model \cite{3} for the branching ratio of $K^+ \to \pi^+\nu\bar{\nu}$ is $B(K^+ \to \pi^+\nu\bar{\nu}) = 0.75 \pm 0.29 \times 10^{-10}$ and was first calculated by Inami and Lim \cite{4}.

The E787 experiment at Brookhaven, with a stopped kaon beam, recently observed two clean events \cite{5}, leading to a measured branching ratio of $B = 1.57^{+1.75}_{-0.82} \times 10^{-10}$. A follow-up experiment at Brookhaven, E949, will exploit fully the stopped kaon beam technique, and expects to observe 5-10 events.

To reach the 100 event level, CKM will use kaon decay in flight. Due to kinematic backgrounds, the acceptance will only be in the 1 – 2\% range, and this, together with the spill structure of the Main Injector (1 s out of 3 s) and a running time of 2 years, requires at least 30,000,000 kaons passing the detector per second. This places high performance criteria on the detectors.

With an expected branching ratio of $10^{-10}$ and 100 signal events with less than 10 background events to observe, the experiment has the challenge of suppressing all sources of possible backgrounds to the $10^{-12}$ level.

The most obvious backgrounds are the main $K^+$ decays modes: $K^+ \to \mu^+\nu\bar{\nu}$ ($B = 64\%$) and $K^+ \to \pi^+\pi^0$ ($B = 21\%$). In the first, miss-identifying the $\mu^+$ as $\pi^+$, and in the second not registering the two photons from the $\pi^0$ decay lead to identical topologies like the signal $K^+ \to \pi^+\nu\bar{\nu}$.

We will describe here the experimental method on the $\pi^+\pi^0$ mode, other modes work the same. With a very efficient Photo-Veto-System \cite{9} CKM expects to reject this mode by $10^{-7}$. For the additional rejection down to the $10^{-12}$ level we use kinematic rejection: By measuring the invariant mass of the system the $\pi^+$ recoils (the missing mass), we obtain a peak at the $\pi^0$ mass in the $\pi^+\pi^0$ two-body decay, and a continuous distribution in the signal mode (a three-body decay). By excluding the mass region around the $\pi^0$, we obtain our signal events. The missing mass $M_{\text{miss}}$ is given (in a very good approximation) by

$$M_{\text{miss}}^2 = m_K^2(1 - p_\pi/p_K) + m_\pi^2(1 - p_K/p_\pi) - p_\pi p_K \theta^2$$

where $m_K$ is the mass of the $K^+$, $m_\pi$ is the mass of the $\pi^+$, $p_K$ and $p_\pi$ are their corresponding momenta, and $\theta$ is the decay angle. These are the only experimentally accessible quantities in the signal decay.

CKM will use two different types of spectrometer.
systems to measure the momentum vectors of the $K^+$ and the $\pi^+$, a magnetic spectrometer with high-rate MPWCs and straw-tubes, and a velocity spectrometer [6], a novel concept, consisting of two Ring Imaging Cherenkov counters [7]. The resolutions in momentum and angle of these detectors has to be good enough to allow for a narrow enough missing mass peak of the $\pi^0$ not to overlap too much with the signal region. It also requires a magnetic field homogeneity of better than a few per mil.

3. Requirements and Design Criteria for the Velocity Spectrometer

The velocity spectrometer has to measure independently of the conventional magnetic spectrometer the vector velocity of the incoming kaon and the outgoing pion. In a first approximation for designing the experiment [8] we assumed momentum resolutions of 0.5% and 1% respectively.

The RICH for measuring the outgoing $\pi^+$ was modeled after the very successful phototube based SELEX RICH detector [10], keeping the diameter of the vessel about the same, but doubling the vessel length to 20 m and correspondingly the radius of curvature of the spherical mirror to 40 m. As radiator gas we will use Neon, as in the SELEX RICH. The phototubes are sensitive down to 160 nm, requiring also sufficient reflectivity of the mirrors in the VUV.

The Kaon RICH, to measure the velocity vector of the incoming $K^+$, and also to discriminate against other beam particles (mostly $\pi^+$), had to be designed differently. We will still keep the main feature from the SELEX RICH, the use of photomultipliers for the detection of the Cherenkov photons.

The beam in CKM is very parallel, and has a diameter of only 10 cm. Since the detector is located just before the decay volume, we have to avoid interactions in detector material, asking for as less as possible amount of material in the beam. To reduce the effect of the phototube size on the overall resolution, we also decided to double the focal length, leading to a doubled ring radius. All this together lead us to a design shown in fig. 1.

The vessel has a length of 10 m, but we double-fold the light-path, with a (thin) plane mirror in the beam, a spherical mirror of 40 m radius (20 m focal length) and another plane mirror, both outside the beam.

A detailed design of the mirror arrangement is shown in fig. 2. The two plane mirrors have the same distance from the spherical mirror, as well horizontally as vertically, so that the tilt-angles of the two plane mirrors have the same absolute value. This assures normal incidence on the phototube plane without tilting it. With simple geometry it can be seen that the inclination angle $\alpha$ for the plane mirrors is given by

$$\alpha = \frac{1}{2} \theta = \frac{1}{2} \arctan \left( \frac{A}{L} \right)$$

where the focal length of the spherical mirror is given by $F = L + L / \cos \theta$.

The size of the mirrors can be determined from the Cherenkov angle, given by [11]

$$\cos \theta_C = \frac{1}{\beta n}$$  \hspace{1cm} (1)

where $\beta = v/c$ is the velocity of the particle as a fraction of the velocity of light in vacuum, and $n$ is the refractive index of the medium. The sizes of the 3 mirrors are determined assuming a $K^+$ with a momentum of 22 GeV/c, and the size of the photocathode (a matrix of 1/2 inch photomultipliers) is determined assuming a $\pi^+$ with the same momentum. The sizes obtained depend on the refractive index $n$ of the radiator gas. In table I we show typical values for the final parameters obtained.


| Mirror Type                | Diameter |
|---------------------------|----------|
| Thin plane mirror         | 43.5 cm  |
| Second plane mirror       | 90.5 cm  |
| Spherical mirror          | 79.5 cm  |
| Phototube array           | 90.5 cm  |

Table I: Sizes for the mirrors and the phototube array for CF$_4$ at 0.8 atm.

in this study.

4. Data Analysis and Results

After coding the full geometrical description of all relevant materials within the GEANT [12] package, and incorporating all the routines into the standard experiment Monte Carlo program, we studied as a first application the momentum resolution of the detector as a function of pressure for two different radiator gases, namely N$_2$ and CF$_4$. N$_2$ was selected because it’s refractive index at 1 atm of $n \approx 1 - 1/300 \cdot 10^{-6}$ gives the correct threshold for kaons of 22 GeV/c, but we were afraid of chromatic dispersion (change of $n$ as a function of wavelength [13]). For this reason we selected also CF$_4$, with lower dispersion [14], at the cost of running at a non-atmospheric pressure.

Using a 22 GeV $K^+$, where GEANT takes care of all possible interactions like multiple scattering etc., including the production and tracking of Cherenkov photons and simulation of the wavelength depending efficiency function of the phototubes [15], we obtained a list of hit phototubes for every event, we performed a simple ring-fit [16] to determine the radius of the Cherenkov ring, obtaining at the end an average radius $r$ and its standard deviation $\sigma_r$. The radius $r$ in a RICH as function of the momentum $p$ for a particle with mass $m$ is given by (with a small-angle approximation of eq. 1)

$$ r(p) = \frac{R}{\sqrt{2}} \left( 1 - \frac{1}{n(1 - m^2/p^2)} \right) \frac{1}{2} $$

and its derivative by

$$ \frac{dr}{dp} = \frac{R}{2\sqrt{2}} \left( 1 - \frac{(1 - m^2/p^2)^{-3/2}}{n} \right) \left( \frac{1}{n} \left( 1 - \frac{m^2}{p^2} \right)^{-3/2} \right) \left( m^2/p^3 \right) $$

(3)

where $R$ denotes the radius of curvature of the spherical mirror. With the simple relation $\sigma_p = \frac{dp}{dr} \sigma_r$, we can determine the momentum resolution $\sigma_p$ for any set of parameters we wish to simulate.

As an example we show in fig. 3 the radius $r$, the radius resolution $\sigma_r$, and the momentum resolution $\sigma_p$ as a function of the pressure for CF$_4$ as radiator gas. An increase in pressure corresponds to an increase in the refractive index $n$ of the radiator gas.

In fig. 4 we show the final result of this study, the momentum resolution $\sigma_p$ as a function of pressure for two possible radiator gases, CF$_4$ and N$_2$. The arrow indicates the required momentum resolution of 0.5% for a 22 GeV/c $K^+$. As seen, the required resolution can be obtained by using CF$_4$ as radiator gas. As shown in table II, we analysed the different contributions to the resolution. The main factor is the chromatic dispersion.

| Type                | $\sigma_p$ |
|---------------------|------------|
| Multiple Scattering | 0.09 cm    |
| Chromatic Dispersion | 0.49 cm   |
| Total               | 0.49 cm    |

Table II: Contributions to the ring radius resolution of the Kaon RICH with CF$_4$ at 0.65 atm as radiator gas.

5. Conclusions

In this work we performed a GEANT simulation of a RICH detector to determine its response and momen-
tum resolution as a function of pressure for two different radiator gases.

We obtained as a result that with CF$_4$ at 0.8 atm pressure as radiator gas we can achieve the required resolution of 0.5%.

The programs and routines written for this study were used extensively for other design studies for the second edition of the proposal for the CKM experiment [9]. The experiment was approved by the Fermilab Director in June 2001. We are now improving and testing the design of the RICH detectors in preparation for a Technical Design Report.

6. Acknowledgment

We would like to thank Peter Cooper and Erik Ramberg for fruitful discussions, and the other members of the CKM Collaboration for continuous and encouraging support. This work was supported by CONACyT-Mexico under Grant 28435-E and by FAI-UASLP.

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