Modification of the prediction model of the tensile strength of short fiber reinforced polymer composites

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Abstract: Based on the defect of COX model, the parameters related to the interface properties were introduced, and the fitting results were modified to be closer to the measured values.

1. Introduction

Fiber reinforced composite is a kind of multiphase material and its mechanical properties and failure laws depend on many factors. Researchers have proposed many models to predict the tensile properties of fiber reinforced polymer composites, and the most commonly used model is based on the linear superposition principle.

\[ F = F_1(x_1V_1) + F_2(x_2V_2) + F_3(x_3V_3) + \cdots \]  \hfill (1)

\( x_1, x_2, x_3 \) refers to the different components of composite materials. \( V_1, V_2, V_3 \) is corresponding to the volume fraction of each component. Based on the above principles, Weibull proposed a formula for calculating the tensile strength of single-fiber reinforced polymer composites [1]

\[ \sigma_c = \sigma_f V_f + \sigma_m (1 - V_f) \]  \hfill (2)

\( \sigma_c, \sigma_f \) and \( \sigma_m \) are the tensile strength of the composite fiber and polymer matrix respectively. \( V_f \) is the volume fraction of fiber.

Cox introduced fiber length factor and proposed Cox model for prediction of tensile strength of short fiber reinforced composites. [2]

\[ \sigma_c = \eta_{LE} \sigma_f V_f + \sigma_m (1 - V_f) \]  \hfill (3)

\( \eta_{LE} \) is the effective factor of fiber length and its calculation formula is given as follows.

\[ \eta_{LE} = 1 - \frac{\tanh(\beta L/2)}{\beta L/2} \]  \hfill (4)

\[ \beta = \frac{2G_m}{E_f \tau_f^2 \ln(R/\tau_f)} \]  \hfill (5)

\( L \) is the fiber length, \( \tau_f \) is the fiber radius, \( G_m \) is the shear modulus of matrix, \( R \) is half the distance of the adjacent fibers which can be obtained by equation 6:
If polymer matrix is isotropic material, the shear modulus $G_m$ of which can be calculated as follows.

$$G_m = \frac{E_m}{2(1+\nu_m)}$$ (7)

$E_m$, $\nu_m$ are the young's modulus and Poisson's ratio of the polymer matrix. On the basis of Cox model, Krenchel considered the effect of fiber orientation, introduced fiber orientation distribution factor $C_0$, and modified the prediction model of Equation 3-3 to Equation 8. [3]

$$\sigma_c = C_0\etaLE\sigma_f V_f + \sigma_m(1 - V_f)$$ (8)

Different composites molding process can get different value $C_0$. In general, the more oriented the fiber, the greater the value $C_0$. In this work, Sisal fibers were randomly distributed in PLA matrix and the value $C_0$ is 0.125 which was calculated by Fukuda when the orientation is randomly distributed in the three-dimensional direction. [4]

Through the single fiber pull-out experiment in our previous work, it was known that the interface strength of PLA/USFs and PLA/ASFs are different, and thus the interface strength of composites will certainly change due to the different proportion of USFs and ASFs.[1] Therefore, Untreated sisal fibers (USFs) and Alkali-treated sisal fibers (ASFs) are mixed into Hybrid SFs (HSFs) with different proportions, and then HSFs are mixed with PLA to prepare different PLA composites in this work. Our purpose is to consider introducing the interface strength into equation 3-8 to obtain a new modified model.

2. Experiment

2.1. Materials

PLA 3051D, Nature Works™, was purchased by Shenzhen Bright China Industrial Company (China). SF bundles (GB/T 15031-94) were provided by Guangxi Sisal Company of Guangxi Province, China. Other chemical reagents were commercial products.

2.2. Samples Preparation

SFs were submerged into 10% NaOH solution to prepare ASFs for 3 h. Then, ASFs were washed with distilled water and dried in the open air for 48 h. USFs were mixed with ASFs to fabricate HSFs in a mass ratio of 1:9, 2:8, 3:7, 4:6, 5:5, 6:4, 7:3, 8:2, 9:1. After that, PLA, USFs, ASFs and HSFs were dried at 100 degrees for 4 h. Then, PLA was compounded with USFs, ASFs and HSFs with a mass ratio 4:1 respectively using a two-roll plastic mill at 180 degrees. Eleven kinds of samples were made, as shown in Table 1. Finally, they were compressed into sheets by a hot press. The compression molding temperature and pressure were 190 degrees and 10 MPa, respectively.

| Composites | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 | C11 |
|------------|----|----|----|----|----|----|----|----|----|-----|-----|
| USFs:ASF   | 0:10 | 1:9 | 2:8 | 3:7 | 4:6 | 5:5 | 6:4 | 7:3 | 8:2 | 9:1  | 10:0 |
2.3 Mechanical test

An Instron Universal Testing Machine (model 5566, USA) was used to perform the tensile test, the crosshead speed of which was 5 mm min\(^{-1}\).

3. Results and discussion

3.1. Tensile property of composites with different SFs

The tensile properties of pure PLA and elven PLA composites were shown in Figure 1, which could be observed that the tensile properties of PLA/USFs composite and PLA/ASFs composite were worse than those of pure PLA. It could also be seen that the addition of HSFs could improve the tensile properties of PLA effectively. Tensile strength and tensile modulus of PLA/HSFs are higher than those of PLA/USFs and PLA/ASFs except to C9 and C10 composites. C5 has the best tensile properties, whose tensile strength and tensile modulus were 53.9 and 2526.7 MPa, respectively, which improved by 41.5% and 48.1% compared with PLA/USFs, and 10.0% and 27.1% compared with PLA/ASFs. Those indicated that HSFs as reinforcement could improve the tensile properties of PLA composites more significantly, which mainly because that HSFs can give full play to the advantages of USFs in tensile properties and the advantages of ASFs in interfacial compatibility with PLA matrix. There is a synergistic effect between USFs and ASFs when they are mixed in polymer composites.

3.2 Discussion of the prediction model of the tensile strength of short fiber reinforced polymer composites

By substituting the data of USFs and ASFs into equation (8), it is surely found that the theoretical value is not equal to the measured data, and there is a considerable difference. Tensile failure of short fiber reinforced polymer composites involves fiber, matrix and interface which is a random process of cumulative microscopic damage, such as fiber fracture, matrix failure, interface debonding, etc. Therefore, a defect of Cox model lies in that it does not pay attention to the impact of interface properties on the failure process and strength, thus affecting the accurate judgment of tensile strength of composite.

The strength of the short fiber reinforced composite is mainly affected by three aspects, which are the fiber strength, the matrix strength and the interface strength. Based on the structure of composite, we can take a single fiber and its surrounding matrix as a unit, as shown in Figure 2.
The following stress analysis is performed for this model. When the composite is subjected to external forces, the matrix is the first to bear the force, and the force is transferred to the fiber as the reinforcement through the interface. When the adhesion between the fiber and the matrix is strong enough, external forces can be effectively transferred to the fiber through the interface, and the fiber thus plays a strengthening role. The mechanical analysis can be expressed as follows.

\[ F = F_m + F_f + F_i \]  \hspace{1cm} (9)

However, the interface between the matrix and the fiber was too weak due to too much USF after the mixing ratio was greater than 7:3, and the external force on the matrix could not be effectively transmitted through the interface. The fiber can not give full play to its strengthening function, but also causes the stress concentration in the composite due to the fact that the fiber itself is easy to aggregate. At this time, the prediction of the strength of composite is more complex, and there is no great significance and need to discuss because the fiber has no real strengthening effect.

The destruction of the composite is a random process, and there are various failure modes and thus \( F_i \) and \( \sigma_i \) exists nonlinear correlation. For short fiber reinforced polymer composites, Equation 3-8 can be corrected as Equation 10.

\[ \sigma_c = C_0 \eta_{LE} \sigma_i \nu_i + \sigma_m (1 - \nu_i) + f(\sigma_i) \]  \hspace{1cm} (10)

Then the tensile strength of hybrid fiber reinforced polymer composites can be calculated by Equation 13.

\[ \sigma_i = \lambda \sigma_{\text{ASF}} + (1 - \lambda) \sigma_{\text{USF}} \]  \hspace{1cm} (13)

\( \lambda \) is the mass ratio of the ASF to the hybrid fiber.

The relationship between \( f(\sigma_i) \) and \( \sigma_i \) is shown on Figure 3 from which it could be observed that \( f(\sigma_i) \) and \( \sigma_i \) have nonlinear negative correlation.

\[ f(\sigma_i) = \varepsilon - \xi \sigma_i^t \]  \hspace{1cm} (14)

\( \varepsilon, \xi \) and \( t \) are parameters related to interface properties of composites.
Fig. 3 Nonlinear curve fitting between $f(\sigma_i)$ and $\sigma_i$

By substituting equation (14) into equation (11), the model of short-fiber reinforced PLA composites can be obtained as follows.

$$\sigma_c = C_0\eta_L E_0 \sigma_f + \sigma_m (1 - \nu_f) + (\epsilon - \xi \sigma_f^t)$$ (15)

For hybrid fiber reinforced polymer composites

$$\sigma_c = C_0\eta_L E_0 (\sigma_{sf} + \sigma_{uf} + \nu_f) + \sigma_m (1 - \nu_f) + (\epsilon - \xi \sigma_f^t)$$ (16)

Through the fitting function of Origin software, values of $\epsilon$, $\xi$ and $t$ can be obtained.

$$\epsilon = 21.7469$$

$$\xi = 0.342$$

$$t = 1.5763$$

4. Conclusion

Tensile strength of short fiber reinforced polymer composite is mainly affected by three aspects which are fiber strength, the matrix strength and the interface strength. Based on the defect of COX model, the parameters related to the interface properties were introduced in this work. It can be seen that the fitting results were modified to be closer to the measured values.

Reference

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