All tree level scattering amplitudes in Chern-Simons theories with fundamental matter

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We show that BCFW recursion relations can be used to compute all tree level scattering amplitudes in terms of $2 \rightarrow 2$ scattering amplitude in $U(N)$ $\mathcal{N} = 2$ Chern-Simons (CS) theory coupled to matter in fundamental representation. As a byproduct, we also obtain a recursion relation for the CS theory coupled to regular fermions, even though in this case standard BCFW deformations do not have a good asymptotic behavior. Moreover at large $N$, $2 \rightarrow 2$ scattering can be computed exactly to all orders in 't Hooft coupling as was done in earlier works by some of the authors. In particular, for $\mathcal{N} = 2$ theory, it was shown that $2 \rightarrow 2$ scattering is tree level exact to all orders except in the anyonic channel \cite{1}, where it gets renormalized by a simple function of 't Hooft coupling. This suggests that it may be possible to compute the all loop exact result for arbitrary higher point scattering amplitudes at large $N$.

I. INTRODUCTION

Chern Simons gauge theories coupled to matter fields have a wide variety of applications in areas as diverse as quantum hall physics, anyonic physics, topology of three manifolds, quantum gravity via the AdS/CFT correspondence, etc. These theories are conjectured to enjoy a strong-weak duality that has been tested in several intense computations at large $N, \kappa$, keeping the 't Hooft coupling $\lambda = \frac{2\pi}{N} \text{ fixed}$ \cite{2,30}. Recently, a finite $N, \kappa$ form of the duality was proposed \cite{31,39}. An example of the strong-weak duality is the duality between Chern-Simons gauge theory coupled to fundamental fermions and Chern-Simons gauge theory coupled to fundamental critical bosons. Other examples include self dual theories, such as $\mathcal{N} = 1$, $\mathcal{N} = 2$ supersymmetric CS matter theories. Very recently, at large $N$ it was demonstrated that the S matrix for the $2 \rightarrow 2$ scattering computed exactly to all orders in the 't Hooft coupling displays an unusual modified crossing relation \cite{4,15,25}. Moreover, for $\mathcal{N} = 2$ theory, the result is tree level exact \cite{1} except in the anyonic channel, where it gets renormalized by a simple function of the 't Hooft coupling.

A natural question to ask would be, is it possible to compute arbitrary $m \rightarrow n$ scattering amplitudes at all values of the 't Hooft coupling at large $N, \kappa$? Given the simplicity of the results at least in the supersymmetric case, it is also interesting to ask if the computability of scattering amplitudes extends to finite $N, \kappa$. As a first step towards these questions, we compute all tree level amplitudes for the $\mathcal{N} = 2$ theory and the regular fermionic theory. The self-dual $\mathcal{N} = 2$ supersymmetric theory is particularly interesting and important since via RG flow, we can obtain non supersymmetric dual pairs such as critical bosons coupled to CS and regular fermions coupled to CS \cite{14,21}.

The letter is organized as follows. In \textsection II we review the four point scattering amplitude in the fermionic and $\mathcal{N} = 2$ theory. In \textsection III we discuss general criteria for the BCFW recursions to hold for the $\mathcal{N} = 2$ theory. In \textsection IV we give a formal argument using background field method to show that BCFW works for the $\mathcal{N} = 2$ theory. In \textsection V we present a recursion relation for all tree level amplitudes for the $\mathcal{N} = 2$ theory. Furthermore in \textsection VI we discuss how to use the $\mathcal{N} = 2$ results to obtain recursion relations for all tree level amplitudes in fermionic theory. We end the letter with a discussion and possible future directions.

II. FOUR POINT SCATTERING AMPLITUDE

In this letter we compute scattering amplitudes in fermion coupled to $SU(N)$ Chern-Simons theory (FCS)

$$
\int d^3 x \left[ -\frac{\kappa}{4\pi} \epsilon^{\mu \nu \rho} \text{Tr} \left( A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho \right) + \bar{\psi} i \slashed{D} \psi \right],
$$

and in $\mathcal{N} = 2$ Chern-Simons matter theory coupled to a Chiral multiplet given by

$$
S_{N=2}^L = \int d^3 x \left[ -\frac{\kappa}{4\pi} \epsilon^{\mu \nu \rho} \text{Tr} \left( A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho \right) \right]
$$
\[ + \bar{\psi} i \mathcal{D} \psi - D^\mu \bar{\psi} D_\mu \phi + \frac{4\pi^2}{\kappa^2} (\bar{\phi} \phi)^3 + \frac{4\pi}{\kappa} (\bar{\phi} \phi)(\bar{\psi} \psi) \\
+ \frac{2\pi}{\kappa} (\bar{\phi} \phi)(\bar{\psi} \psi) \pmb.] \quad (2)

For our purposes, it is convenient to introduce the spinor helicity basis \[40\] defined by
\[ p_i^{a\beta} = p_i^\mu \sigma_\mu^{a\beta} = \lambda_i^a \lambda_i^\alpha, \quad (p_i + p_j)^2 = 2p_i \cdot p_j = (\lambda_i^a \lambda_i^{\alpha})^2. \] (3)

Below we use the notation \[\langle \lambda_i^a \lambda_j^\alpha \rangle = \langle ij \rangle.\] For a supersymmetric amplitude, the standard procedure involves introduction of on-shell grassman variables \(\theta\) such that the super-creation and super-annihilation operators are given by
\[ A_i = a_i + \theta_i a_i, \quad A_i^\dagger = \theta_i a_i^\dagger + a_i^\dagger, \] (4)

where \((a_i^\dagger, a_i)\) \((a_i^{\dagger}, a_i)\) create and annihilate a boson/fermion with momenta \(p_i\) respectively. The two on-shell supercharges for \(n\) point scattering amplitudes are given by
\[ Q = \sum_{i=1}^n q_i = \sum_{i=1}^n \lambda_i \theta_i, \quad \bar{Q} = \sum_{i=1}^n \bar{q}_i = \sum_{i=1}^n \lambda_i \bar{\theta}_i. \] (5)

For FCS theory in \[1,2\], the tree level \(2 \rightarrow 2\) scattering amplitude is given by \[16\]
\[ A_i^F = \langle \bar{\psi}(p_1) \psi(p_2) \bar{\psi}(p_3) \psi(p_4) \rangle = \frac{\langle 12 \rangle}{\langle 23 \rangle} \delta^4 \sum_{i=1}^n p_i. \] (6)

For \(\mathcal{N} = 2\) theory in \[2\], the tree level \(2 \rightarrow 2\) super amplitude is given by
\[ A_i^S = \frac{\langle 12 \rangle}{\langle 23 \rangle} Q^2 = \frac{\langle 12 \rangle}{\langle 23 \rangle} \delta^4 \sum_{i=1}^n p_i \sum_{i=1}^n \langle ij \rangle \theta_i \bar{\theta}_j. \] (7)

Here \(A_i^S\) is the super-amplitude computed using the super-creation/annihilation operators defined in \[7\]. Any component amplitude can be obtained from \(\theta\) by picking up the coefficient of products of two \(\theta\)'s. For example, the four fermion amplitude is given by the coefficient of \(\theta_2 \theta_4\) that coincides precisely with \(\theta\).

III. HIGHER POINT SCATTERING AMPLITUDE

BCFW recursion relations are an efficient method to compute and express arbitrary higher point scattering amplitudes in terms of product of lower point amplitudes. Standard procedure for BCFW involves the deformation of two external momenta of the particles by a complex parameter \(z\). The deformation is such that the particles continue to remain 'on shell' and the total momentum conservation of the process continues to hold. In 3D, BCFW deformations are a little more involved than in 4D and were first discussed in \[11\] (We follow their notations closely). BCFW recursion relations are applicable in 3D provided that the higher point amplitudes are regular functions at \(z \rightarrow \infty\) and \(z \rightarrow 0\). In the following section we study the \(z \rightarrow \infty\) (and \(z \rightarrow 0\)) behavior of the amplitudes in the theories described earlier. We find it convenient to deform color contracted (we label them as '1' and '2') external legs. In 3-dimensions, momentum deformation of particles 1 and 2 can be written in terms of the spinor-helicity variables as
\[ \left( \frac{\lambda_1}{\lambda_2} \right) = R \left( \frac{\lambda_1}{\lambda_2} \right), \quad \text{where} \quad R = \left( \begin{array}{cc} z+z^{-1} & z^{-z^{-1}} \\ z^{-z^{-1}} & z+z^{-1} \end{array} \right). \] (8)

In the theories \[1,2\], all 3-point vertices involve gauge fields and since the Chern-Simons gauge field does not have an on shell propagating degree of freedom, it follows that only even-point functions are non-zero. This also implies that the 4-point functions are fundamental building blocks for higher point functions.

Under the deformation \[6\], any tree-level scattering amplitude for FCS in \[1\] is not well behaved at large \(z\) and hence doesn’t obey the requirements of BCFW. However this situation is cured for the \(\mathcal{N} = 2\) theory defined in \[2\]. Additionally, conservation of the super-charges in \(8\) require that the on-shell spinor variables \(\theta\) be deformed as
\[ \left( \frac{\bar{\theta}_1}{\bar{\theta}_2} \right) = R \left( \frac{\bar{\theta}_1}{\bar{\theta}_2} \right), \] (9)
where the \(R\) matrix is defined by \[8\].

Let us denote the \(2n\)-point super-amplitude as \(A_{2n}(\lambda_1, \lambda_2, \cdots, \lambda_{2n}, \theta_1, \theta_2, \cdots, \theta_{2n})\) and the deformed amplitude by \(A_{2n}(\hat{\lambda}_1, \hat{\lambda}_2, \cdots, \hat{\lambda}_{2n}, \hat{\theta}_1, \hat{\theta}_2, \cdots, \hat{\theta}_{2n})\). The deformed super-amplitude can be explicitly written as an expansion in the \(\theta\) variables as follows
\[ A_{2n}(z) = A^0(z) + A^1(z) \hat{\theta}_1(z) + A^2(z) \hat{\theta}_2(z) + A^{12}(z) \hat{\theta}_1(z) \hat{\theta}_2(z), \] (10)

where in the last line of \[10\] we have used \[8\] and the fact that \(\hat{\theta}_1(z) \hat{\theta}_2(z) = \theta_1 \theta_2\). We have also defined
\[ \left( \frac{A^1(z)}{A^2(z)} \right) = R^T \left( \frac{A^1(z)}{A^2(z)} \right), \] (11)
where \(R^T\) is the transpose of the \(R\) matrix defined in \[8\], with \(R R^T = 1\). The super-momentum conservation implies that the large \(z\) behavior of the super-amplitude \(A_{2n}(z)\) is identical to that of the components \(A^0, A^{12}\). Hence it is sufficient to show that either of \(A^0\) or \(A^{12}\) are well behaved since supersymmetric ward identity guarantees the required behavior for the rest of the amplitudes. It is convenient to write the fields in pair wise contractions since they transform in the fundamental representation of the gauge group. For instance we are interested in the large \(z\) behavior of amplitudes such as
we need minimum $\mathcal{N} = 2$ amount of supersymmetry for this to work.

V. RECURSION RELATIONS IN $\mathcal{N} = 2$ THEORY

\begin{equation}
A_{2n}(z = 1) = \sum_f \int \frac{d\theta}{p_f^2} \left( z_{a,f}^2 - 1 \right) A_L(z_{a,f}, \theta) A_R(z_{b,f}, i\theta) + (z_{a,f} \leftrightarrow z_{b,f}) ,
\end{equation}

where the integration is over the intermediate grassmann variable $\theta$ and $A_{2n}(z = 1)$ is the undeformed $2n$-point amplitude. In the above, $p_f$ is the undeformed momentum that runs in the factorization channel $f$ and the summation in (14) runs over all the factorization channels corresponding to different intermediate particles going on-shell. Here, $z_{a,f}$ and $z_{b,f}$ are given by

\begin{equation}
(z_{a,f}^2 , z_{b,f}^2) = -(p_f - p_2) \cdot (p_f + p_1) \pm \sqrt{(p_f - p_2)^2 (p_f + p_1)^2 - 4q \cdot (p_f - p_2)} ,
\end{equation}

where the null momenta $q$ are defined in terms of the spinor helicity variables as

\begin{equation}
q^{\alpha\beta} = \frac{1}{4}(\lambda_2 + i\lambda_1)^\alpha(\lambda_2 + i\lambda_1)^\beta , q^{\rho\sigma} = \frac{1}{4}(\lambda_2 - i\lambda_1)^\rho(\lambda_2 - i\lambda_1)^\sigma .
\end{equation}

For instance, for $\mathcal{N} = 1$ theory, the Lagrangian for which can be found in (Equation (21)), (13) is modified to $-2\pi/\kappa (41)w$ whereas (12) remains the same. Here $w$ is a free parameter in $\mathcal{N} = 1$ theory. This implies that only at $w = 1$, the $\mathcal{N} = 1$ theory has a good large $z$ behavior. This is exactly the point in the $w$ line where the supersymmetry of the theory gets enhanced to $\mathcal{N} = 2$. 

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IV. ASYMPTOTIC BEHAVIOR OF AMPLITUDES

To understand the large $z$ behavior of various scattering amplitudes, it is extremely useful to think from the background field method point of view introduced in [12]. Here $z$-deformed particles are considered as hard particles propagating in a background of soft particles. The amplitude is modified due to (a) modified propagator of intermediate hard particle; (b) the modified contribution of various vertices; and, (c) modified fermion wave function, in case an external deformed particle is a fermion. Detailed analysis shows (we follow closely [11, 42]) that the non-trivial $z \to \infty$ behavior of the amplitude is due to diagrams of the kind depicted in fig. 1. The values of these diagrams are:

\begin{align}
\text{Gauge-field exchange:} & \quad \frac{4\pi i}{\kappa} (k_4 | \gamma^\mu | 1) \frac{k_1 \cdot k_2^\prime}{(k_3 + p_2)^2} \epsilon_{\mu\nu\rho} \\
\text{Contact vertex:} & \quad -\frac{2\pi}{\kappa} (k_4 | 1) 
\end{align}

(12) (13)

Under the 1-2 $z$-deformations, [9], in the $z \to \infty$ limit the $\mathcal{O}(z)$ part of the amplitude cancels and the amplitude behaves as $\mathcal{O}(1/z)$. Hence this amplitude has a regular $z \to \infty$ behavior for $\mathcal{N} = 2$ theory. This cancellation works even for the 4-point function $\langle \bar{\psi}_1 \phi_2 \bar{\phi}_3 \psi_4 \rangle$, which receives contributions from the diagrams in fig. 1 with the blob removed and $k_3 \to p_3 , k_4 \to p_4$ are taken to be on-shell momenta. It is important to emphasize that we

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{The diagrams that have a non-regular $z \to \infty$ behavior. $\mathcal{O}(z)$ part of these two diagrams cancel against each other to give a regular $z \to \infty$ behavior of the total amplitude. In the above diagram, the solid lines correspond to fermions and the dashed lines correspond to bosons. This amplitude appears in $A^0$ in (10). The blue color lines correspond to deformed hard particle.
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Recursion formula for a $2n$ point amplitude: The black lines denote the undeformed legs, the external blue lines represent the deformed legs and $p_f$ represents the momentum in the factorization channel.
\end{figure}
Note that the formula (14) has a very similar form (but not quite the same as discussed below) to the one obtained in [41] for the ABJM theory that enjoys $\mathcal{N} = 6$ supersymmetry. It is remarkable that such recursion formulæ exist in a theory with much lesser supersymmetry such as the one in discussion.

As an explicit demonstration, consider the six point amplitude $A_6(\lambda_1, \ldots, \lambda_6) \equiv \langle \phi_1\phi_2\phi_3\phi_4\phi_5\phi_6 \rangle$ in the $\mathcal{N} = 2$ SCS theory. This amplitude factorizes into two channels as shown in fig 3. The recursion formula can be explicitly written as

$$\langle \phi_1\phi_2\phi_3\phi_4\phi_5\phi_6 \rangle = \left( z_{a,f}^2 - \frac{1}{z_{a,f}^2} - \frac{\hat{z}_{a,f}^2}{\hat{z}_{a,f}^2} \right) \left( \frac{i}{p_f} \right)_{p_f = p_{234}}
+ \left( z_{a,f} \leftrightarrow z_{b,f} \right) \left( \frac{i}{p_f} \right)_{p_f = p_{256}},$$

(18)

where $z_{a,f}$ and $z_{b,f}$ are defined in (15). Fields with hats corresponds to deformed momenta. We have checked (18) explicitly by computing the relevant Feynman diagrams. It is a curious fact that, the total number of Feynman graphs that contribute to $A_6$ is 15. Of these, eleven are reproduced by the channel $p_f = p_{234}$ and the remaining four in the channel $p_f = p_{256}$.

VI. RECURSION RELATIONS IN THE FERMIONIC THEORY

In this section, we show that the BCFW recursion relations can be used to compute $2n$-point amplitude $A_{2n} = \langle \psi_1\psi_2 \cdots \psi_{2n-1}\psi_{2n} \rangle$ for the regular fermionic theory coupled to CS gauge field [1]. If we apply [5] to this amplitude, it is easy to show that, it does not have a good large $z$ (as well as $z \to 0$) behavior, hence we cannot readily apply the BCFW recursion relation to determine all higher point fermionic amplitudes. However, we show below that we can use the recursion relation of the $\mathcal{N} = 2$ to write a recursion relation for the fermionic theory.

As a first step towards this, let us note that the Feynman diagrams for any tree-level all-fermion scattering amplitude in the $\mathcal{N} = 2$ theory (2) is identical to that of the tree-level scattering amplitude in the fermionic theory [7]. In the previous section we proved for the $\mathcal{N} = 2$ theory that an arbitrary higher-point super-amplitude can be written only in terms of the 4-point super-amplitude. Same can be said for the component amplitudes including the purely fermionic component amplitude [2]. Let us note that for the four point super-amplitude, supersymmetry relates all the component 4-point amplitudes to one component amplitude, which for instance can be taken to be 4-fermion scattering amplitude (see [2]). Thus an arbitrary higher-point component amplitude can be written only in terms of 4-fermion amplitude. This can be recursively done for an arbitrary $2n$ point amplitude, however for simplicity we write the recursion relation for the six point amplitude below

$$\langle \bar{\psi}_1\bar{\psi}_2\bar{\psi}_3\bar{\psi}_4\bar{\psi}_5\bar{\psi}_6 \rangle =
\left( z_{a,f}^2 - \frac{1}{z_{a,f}^2} - \frac{2}{z_{a,f}^2} \right) \left( \frac{i}{p_f} \right)_{p_f = p_{234}}
+ \left( z_{a,f} \leftrightarrow z_{b,f} \right) \left( \frac{i}{p_f} \right)_{p_f = p_{256}}.$$
VII. DISCUSSION

In this letter we presented recursion relations for all tree level amplitudes in $\mathcal{N} = 2$ CS matter theory and CS theory coupled to regular fermions. Below we discuss some interesting open questions for future research.

It was shown in [1], that the $2 \rightarrow 2$ scattering amplitude in the $\mathcal{N} = 2$ theory does not get renormalized except in the anyonic channel, where it gets renormalized by a simple function of the ’t Hooft coupling. A natural question is, why in the $\mathcal{N} = 2$ theory the scattering amplitude has such a simple form, whereas the corresponding amplitudes in the fermionic [16] and other less susy $\mathcal{N} = 1$ theories are quite complicated. A possible explanation is that there exists some symmetry such as dual conformal invariance that appears in the $\mathcal{N} = 2$ theory and it protects the amplitude from loop corrections [13].

As a natural next step, it would also be interesting to explore an analog of the Aharonov-Bohm effect in the fermionic [16] and other less susy $\mathcal{N} = 1$ theories. It is also natural to ask, if the simplicity of the amplitudes continues to persist with higher point amplitudes. It is also interesting to explore an analog of the Aharonov-Bohm phase for higher point amplitudes. It may very well turn out that the Aharonov-Bohm phases of higher point amplitudes are products of the Aharonov-Bohm phases of the $2 \rightarrow 2$ amplitude. BCFW recursion relations provide a strong indication towards this result.

To answer the above questions, we need to compute higher scattering amplitudes to all orders in $\lambda$. A possible way is to investigate the Schwinger-Dyson equation. However, the Schwinger-Dyson equation approach is quite complicated even at the 6–point level. A refined approach might be to look for a larger class of symmetries such as Yangian symmetry [13] and use the powerful formulation of [14] to obtain results. Given the fact that, these theories are exactly solvable at large-$N$ as well as the fact that $\mathcal{N} = 2$ theory is self-dual, it could turn out that the $\mathcal{N} = 2$ theory may be one of the simplest play grounds to develop new techniques in computing S-matrices to all orders [14]. Furthermore exact solvability at large $N$ indicates that these models might even be integrable. One possible way to investigate integrability is to show the existence of an infinite dimensional Yangian symmetry. Since these theories relate to various physical situations, any of the above exercises may provide insight into finite $N, \kappa$ computations.

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[1] K. Inbasekar, S. Jain, S. Mazumdar, S. Minwalla, V. Umesh, and S. Yokoyama, JHEP 10, 176 (2015), arXiv:1505.06571 [hep-th]
[2] O. Aharony, G. Gur-Ari, and R. Yacoby, JHEP 1203, 037 (2012), arXiv:1110.4382 [hep-th]
[3] S. Giombi, S. Minwalla, S. Prakash, S. P. Trivedi, S. R. Wadia, et al., Eur.Phys.J. C72, 2112 (2012), arXiv:1110.4386 [hep-th]
[4] J. Maldacena and A. Zhiboedov, J.Phys. A46, 214011 (2013), arXiv:1112.1016 [hep-th]
[5] J. Maldacena and A. Zhiboedov, Class.Quant.Grav. 30, 104003 (2013), arXiv:1204.3882 [hep-th]
[6] C.-M. Chang, S. Minwalla, T. Sharma, and X. Yin, J.Phys. A46, 214009 (2013), arXiv:1207.4485 [hep-th]
[7] O. Aharony, G. Gur-Ari, and R. Yacoby, JHEP 1212, 028 (2012), arXiv:1207.4593 [hep-th]
[8] S. Jain, S. P. Trivedi, S. R. Wadia, and S. Yokoyama, JHEP 1210, 194 (2012), arXiv:1207.4750 [hep-th]
[9] S. Yokoyama, JHEP 1301, 052 (2013), arXiv:1210.4109 [hep-th]
[10] G. Gur-Ari and R. Yacoby, JHEP 1302, 150 (2013), arXiv:1211.1866 [hep-th]
[11] O. Aharony, S. Giombi, G. Gur-Ari, J. Maldacena, and R. Yacoby, JHEP 1303, 121 (2013), arXiv:1211.4843 [hep-th]
[12] S. Jain, S. Minwalla, T. Sharma, T. Takimi, S. R. Wadia, et al., JHEP 1309, 009 (2013), arXiv:1301.6169 [hep-th]
[13] T. Takimi, JHEP 1307, 177 (2013), arXiv:1304.3725 [hep-th]
[14] S. Jain, S. Minwalla, and S. Yokoyama, JHEP 1311, 037 (2013), arXiv:1305.7235 [hep-th]
[15] S. Yokoyama, JHEP 1401, 148 (2014), arXiv:1310.0902
[16] S. Jain, M. Mandlik, S. Minwalla, T. Takimi, S. R. Wadia, et al., JHEP 1504, 129 (2015), arXiv:1404.6373 [hep-th].
[17] V. Gurucharan and S. Prakash, JHEP 1409, 009 (2014), arXiv:1401.7849 [hep-th].
[18] Y. Dandekar, M. Mandlik, and S. Minwalla, JHEP 1504, 102 (2015), arXiv:1407.1322 [hep-th].
[19] Y. Frishman and J. Sonnenschein, JHEP 1412, 165 (2014), arXiv:1409.6083 [hep-th].
[20] A. Bedhotiya and S. Prakash, JHEP 12, 032 (2015), arXiv:1506.05412 [hep-th].
[21] G. GurAri and R. Yacoby, JHEP 11, 013 (2015), arXiv:1507.04378 [hep-th].
[22] S. Minwalla and S. Yokoyama, JHEP 02, 103 (2016), arXiv:1507.04546 [hep-th].
[23] M. Geracie, M. Goykhman, and D. T. Son, JHEP 04, 103 (2016), arXiv:1511.04772 [hep-th].
[24] S. R. Wadia, Int. J. Mod. Phys. A31, 1630052 (2016).
[25] S. Yokoyama, JHEP 09, 105 (2016), arXiv:1604.01897 [hep-th].
[26] S. Giombi, in Proceedings, Theoretical Advanced Study Institute in Elementary Particle Physics: New Frontiers in Fields and Strings (TASI 2015): Boulder, CO, USA, June 1-26, 2015 (2017) pp. 137–214, arXiv:1607.02067 [hep-th].
[27] D. Radicevic, D. Tong, and C. Turner, JHEP 12, 067 (2016), arXiv:1608.04732 [hep-th].
[28] S. Giombi, V. Gurucharan, V. Kirilin, S. Prakash, and E. Skvortsov, JHEP 01, 058 (2017), arXiv:1610.08472 [hep-th].
[29] S. Giombi, (2017), arXiv:1707.06604 [hep-th].
[30] T. Nosaka and S. Yokoyama, (2017), arXiv:1706.07234 [hep-th].
[31] D. Radicevic, JHEP 03, 131 (2016), arXiv:1511.01902 [hep-th].
[32] O. Aharony, JHEP 02, 093 (2016), arXiv:1512.00161 [hep-th].
[33] N. Seiberg, T. Senthil, C. Wang, and E. Witten, Annals Phys. 374, 395 (2016), arXiv:1606.01989 [hep-th].
[34] A. Karch and D. Tong, Phys. Rev. X6, 031043 (2016), arXiv:1606.01893 [hep-th].
[35] A. Karch, B. Robinson, and D. Tong, JHEP 01, 017 (2017), arXiv:1609.04012 [hep-th].
[36] P.-S. Hsin and N. Seiberg, JHEP 09, 095 (2016), arXiv:1607.07457 [hep-th].
[37] O. Aharony, F. Benini, P.-S. Hsin, and N. Seiberg, JHEP 02, 072 (2017), arXiv:1611.07874 [cond-mat.str-el].
[38] F. Benini, P.-S. Hsin, and N. Seiberg, JHEP 04, 135 (2017), arXiv:1702.07035 [cond-mat.str-el].
[39] D. Galotto, Z. Komargodski, and N. Seiberg, (2017), arXiv:1708.06806 [hep-th].
[40] H. Elvang and Y.-t. Huang, Scattering Amplitudes in Gauge Theory and Gravity (Cambridge University Press, 2015).
[41] D. Gang, Y.-t. Huang, E. Koh, S. Lee, and A. E. Lipstein, JHEP 03, 116 (2011), arXiv:1012.5032 [hep-th].
[42] N. Arkani-Hamed and J. Kaplan, JHEP 04, 076 (2008), arXiv:0801.2385 [hep-th].
[43] K. Inbasekar, S. Jain, P. Nayak, T. Sharma, T. Niyogi, S. Majumdar, R. Sinha, and V. Unmesh, Work in Progress.
[44] N. Arkani-Hamed, J. L. Bourjaily, F. Cachazo, A. B. Goncharov, A. Postnikov, and J. Trnka, Grassmannian Geometry of Scattering Amplitudes (Cambridge University Press, 2016), arXiv:1212.5605 [hep-th].
VIII. APPENDIX

A. Large $z$ behavior of the six point scattering amplitude $(\bar{\phi}(p_1)\psi(p_2))(\bar{\psi}(p_3)\phi(p_4))(\bar{\phi}(p_5)\phi(p_6))$

In this section, we compute the six point scattering amplitude $(\bar{\phi}(p_1)\psi(p_2))(\bar{\psi}(p_3)\phi(p_4))(\bar{\phi}(p_5)\phi(p_6))$ and demonstrate that it is well behaved under the BCFW deformations. The Feynman diagrams that contribute to the six point function under consideration are displayed in fig 4.
We give below explicit expression for each diagram appearing in Fig.4

\[ A_1 = -\frac{16\pi^2i}{\kappa^2} \frac{p_1.(p_2-p_3)}{p_{23}^2 p_{45}^2 p_{123}^2} \langle 23 \rangle \langle 45 \rangle \langle 56 \rangle \langle 46 \rangle \]  
\[ A_2 = \frac{16\pi^2i}{\kappa^2} \frac{p_4.(p_2-p_3)}{p_{23}^2 p_{16}^2 p_{234}^2} \langle 23 \rangle \langle 16 \rangle \langle 65 \rangle \langle 15 \rangle \]  
\[ A_3 = \frac{4i\pi^2}{\kappa^2} \frac{(16)\langle 45 \rangle}{p_{10}^2 p_{45}^2 p_{126}^2} \langle 21 \rangle \langle (34) \langle 5 \rangle p_{12} \langle 6 \rangle + \langle 35 \rangle \langle 6 \rangle p_{12} \langle 4 \rangle \rangle + \langle 26 \rangle \langle (34) \langle 1 \rangle p_{26} \langle 5 \rangle + \langle 35 \rangle \langle 1 \rangle p_{26} \langle 4 \rangle \rangle \]  
\[ B_1 = \frac{8\pi^2i}{\kappa^2} \frac{1}{(14)} \frac{\langle 1 \rangle p_{56} \langle 3 \rangle \langle 24 \rangle + \langle 3 \rangle p_{56} \langle 4 \rangle \langle 21 \rangle}{p_{14}^2 p_{356}^2} \]  
\[ B_2 = \frac{8\pi^2i}{\kappa^2} \frac{1}{(14)} \frac{\langle 1 \rangle p_{56} \langle 2 \rangle \langle 34 \rangle + \langle 2 \rangle p_{56} \langle 4 \rangle \langle 31 \rangle}{p_{14}^2 p_{256}^2} \]  
\[ C_1 = -\frac{8\pi^2i}{\kappa^2} \frac{1}{(14)} \frac{\langle 2 \rangle p_{1} \langle 3 \rangle (p_{23} p_{6} - p_{23} p_{6})}{p_{23}^2 p_{16}^2} \]  
\[ C_2 = -\frac{8\pi^2i}{\kappa^2} \frac{1}{(14)} \frac{\langle 2 \rangle p_{5} \langle 3 \rangle (p_{23} p_{4} - p_{23} p_{5})}{p_{23}^2 p_{45}^2} \]  
\[ D_1 = \frac{8\pi^2i}{\kappa^2} \frac{\langle 2 \rangle p_{14} \langle 3 \rangle}{p_{124}^2} \]  
\[ D_2 = \frac{8\pi^2i}{\kappa^2} \frac{\langle 2 \rangle p_{56} \langle 3 \rangle}{p_{256}^2} \]  
\[ E = -\frac{4\pi^2i}{\kappa^2} \frac{(45)\langle 23 \rangle}{(12)^2 - (13)^2} \frac{\langle 46 \rangle \langle 56 \rangle - \langle 26 \rangle^2 - \langle 36 \rangle^2 \langle 14 \rangle \langle 15 \rangle}{p_{23}^2 p_{45}^2 p_{16}^2} \]
\[ F_1 = \frac{8\pi^2 i \langle 23 \rangle(16)\langle 65 \rangle(15)}{\kappa^2 p_{16}^2 p_{234}^2} \]
\[ F_2 = \frac{8\pi^2 i \langle 23 \rangle(45)\langle 56 \rangle(46)}{\kappa^2 p_{15}^2 p_{231}^2} \]
\[ G = \frac{4\pi^2 i \langle 2|p_{16}\rangle 3}{\kappa^2 p_{126}^2} \]
\[ H_1 = \frac{4\pi^2 i \langle 16 \rangle}{\kappa^2} \left( \frac{\langle 12 \rangle(6)p_{12} + \langle 26 \rangle(1)p_{26}}{p_{16}^2 p_{126}^2} \right) \]
\[ H_2 = -\frac{4\pi^2 i \langle 45 \rangle}{\kappa^2} \left( \frac{\langle 35 \rangle(2)p_{16} + \langle 34 \rangle(2)p_{16}}{p_{15}^2 p_{126}^2} \right) \]

It is easy to verify that the asymptotic behavior of the full set of diagram is well behaved by deforming the momentum \( p_1 \) and \( p_2 \), as discussed in sections III and IV. We apply the BCFW deformations in the large \( z \) limit using
\[ p_2 \rightarrow q z^2 \]
\[ p_1 \rightarrow -q z^2 \]
and obtain the asymptotic behavior of the amplitudes to leading order in \( z \) as follows. The diagrams \( A_2, C_1, D_1, F_1 \) in fig 4 go as \( O(1/z) \) to the leading order in the large \( z \) limit.
\[ A_2 \sim \frac{2\pi^2 i (q.p_4)(q.3)(q.6)(65)(5) + O \left( \frac{1}{z^3} \right)}{\kappa^2 z^2 (q.p_3)(q.p_4)(q.p_6)} \]
\[ B_2 \sim \frac{2\pi^2 i (q.4)(q|p_{56}|3) + O \left( \frac{1}{z^3} \right)}{\kappa^2 z^2 (q.p_4)(q.p_{56})} \]
\[ C_1 \sim -\frac{2\pi^2 i (q|p_{56}|3) + O \left( \frac{1}{z^3} \right)}{\kappa^2 z^2 (q.p_4)(q.p_{56})} \]
\[ D_2 \sim -\frac{2\pi^2 i (q|p_{56}|3) + O \left( \frac{1}{z^3} \right)}{\kappa^2 z^2 (q.p_{56})(q.p_{55})} \]
\[ F_1 \sim \frac{2\pi^2 i (q.3)(q.6)(65)(5) + O \left( \frac{1}{z^3} \right)}{\kappa^2 z^2 (q.p_6)(q.p_{14})} \]

For the remaining diagrams we just display the leading large \( z \) behavior. They are given by
\[ A_3 \sim -\frac{8\pi^2 i z (q.3)(45)(56)(46)}{\kappa^2 p_{16}^2 p_{231}^2} + O \left( \frac{1}{z} \right) \]
\[ B_1 \sim \frac{8\pi^2 i z (q|p_{56}|3) + O \left( \frac{1}{z} \right)}{\kappa^2 p_{356}^2} \]
\[ A_3 \sim -\frac{4\pi^2 i z (34)(q|p_{34}|5) + (35)(q|p_{35}|4)}{\kappa^2 p_{15}^2 p_{126}^2} + O \left( \frac{1}{z} \right) \]
\[ C_2 \sim \frac{2\pi^2 i z (q.5)(3q)(45)}{\kappa^2 p_{15}^2 (q.p_3)} + O \left( \frac{1}{z} \right) \]
\[ G \sim -\frac{4\pi^2 i z (q|p_{45}|3) + O \left( \frac{1}{z} \right)}{\kappa^2 p_{25}^2 p_{126}^2} \]

Even though some of the individual diagrams are divergent linearly in \( z \), the divergences in the total amplitude cancel pair wise in the large \( z \) limit as is evident from the way we have written the results. For example linear in \( z \) behavior cancelling pair wise in \( A_1, F_2 \), \( B_1, D_1 \) etc. Thus the total amplitude is well behaved as \( z \rightarrow \infty \). A straightforward computation yields the analogous result for the \( z \rightarrow 0 \) limit. Thus the amplitude \( A_6(\hat{\phi}(p_1)\hat{\psi}(p_2))(\hat{\psi}(p_3)\hat{\varphi}(p_4))\langle \hat{\varphi}(p_3)\hat{\varphi}(p_4)\rangle \) is well behaved under the BCFW deformations both at \( z \rightarrow \infty \) and \( z \rightarrow 0 \).

Towards the end of IV we had mentioned that four of the diagrams are reproduced in the factorization channel \( p_f = p_{256} \), these diagrams are \( B_1, B_2, D_1, D_2 \) in fig 4. The remaining eleven diagrams in fig 4 are reproduced in the factorization channel \( p_f = p_{234} \).

\(^7\) \( q \) is defined in [16].
B. A Dyson-Schwinger equation for all loop six point correlator

As we saw earlier in \( \S II \), the basic building block of higher point amplitudes in the Chern-Simons matter theories at the tree level is the four point amplitude. In this section we describe the Dyson-Schwinger construction of the all loop six point correlator

\[
\langle \bar{\Phi}_i(p, q, \theta_1) \Phi_i(2p, 2q, \theta_2) \Phi_j(3p, 3q, \theta_3) \Phi_k(4p, 4q, \theta_4) \rangle
\]

using the superspace Schwinger-Dyson construction developed in \( [1] \). In the above \( \Phi^i \) is a complex scalar superfield in \( \mathcal{N} = 1 \) superspace defined by

\[
\Phi^i = \phi^i + \theta \psi^i - \theta^2 F^i
\]

where \( \phi^i \) is a complex scalar, \( \psi^i \) is a complex fermion and \( F^i \) is a complex auxiliary field. The \( \mathcal{N} = 2 \) theory can be written in \( \mathcal{N} = 1 \) superspace in terms of \( \Phi^i \). For more details see \( [1] \). Before presenting the central idea it is informative to understand the color structure of the tree level and one loop amplitudes in the theory. In the supersymmetric Light cone gauge these are described succinctly in fig 5 and in fig 6. It turns out that, there are six different diagrams for

\[ a) \quad b) \quad c) \]

\[ a) \quad b) \quad c) \]

FIG. 5. Six point correlator: We display tree diagrams in supersymmetric light cone gauge. For simplicity we have only displayed the ladder diagrams. The tree diagrams are of order \( \mathcal{O}(\frac{1}{\kappa^2}) \) since the gauge field propagator contributes a factor of \( \mathcal{O}(\frac{1}{\kappa^3}) \).

\[ a) \quad b) \quad c) \]

\[ a) \quad b) \quad c) \]

FIG. 6. Six point correlator: We have listed the various contributions to the one loop correlator in supersymmetric light cone gauge. For simplicity we have displayed only the ladder diagrams. In fig a) and c) the three gauge field propagators contribute a factor of \( \mathcal{O}(\frac{1}{\kappa^2}) \) and the single color loop gives a factor of \( \mathcal{N}_c \) leading to a contribution of the order \( \mathcal{O}(\frac{1}{\kappa^4}) \). Note that this is of the same order in \( \kappa \) as the tree level diagram displayed in fig 5. On the other hand fig b) has three gauge fields and no color loops, rendering it to be \( \mathcal{O}(\frac{1}{\kappa^3}) \).

\footnote{A similar discussion can be carried out for higher point function.}
a given color contracted correlator. We have displayed only one in Fig. 5 for brevity. The situation is a little bit more complicated at one loop as three different type of diagrams can appear as displayed in Fig 6. Note that diagrams like fig 6 b) are suppressed in the large $N, \kappa$ limit (keeping $\lambda = \frac{N}{\kappa}$ fixed). So they don’t contribute to the Schwinger-Dyson equation at this order. It can be checked that these type of diagrams continue to remain suppressed at higher loops.

This paves way for the construction of all loop higher point correlators entirely in terms of all-loop four point correlators at least in the planar approximation. The case for the six point correlator is displayed in (see fig 7). It

FIG. 7. Six point correlator in superspace: The grey boxes represent the $\mathcal{N} = 2$ all-loop four point correlator computed in [1].

is straightforward to write down the correlator for the first diagram in 7, the second contribution however requires a loop integration over both intermediate grassmann and momentum variables and is quite complicated, we defer a detailed treatment to future works.