Dynamically Response and Synchronizing Characteristic for the Dual-Motor Driving System in Non-Inertial System

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Abstract: As one of the typical propulsion systems of the new-energy vehicles (NEVs), the dual-motor driving system (DMDS) is installed and fixed on the bodywork and moves together with the bodywork in space, that is, the DMDS works in non-inertial systems. The previous researches on the DMDS are based on the assumption that the bodywork is stationary. In fact, since the DMDS moves with the bodywork, besides its own excitation forces, it is inevitably affected by the additional inertial terms (AITs) caused by the change in the operating state of the NEVs. In order to investigate the dynamic response and synchronizing characteristic of the DMDS under different motion forms of NEVs, the dynamic model of the DMDS in a non-inertial system is established, considering the permanent magnet synchronous motors (PMSMs), time-varying meshing stiffness and transmission error of gears. Subsequently, the expressions of the AITs are deduced under different non-inertial conditions. The translational motion and circular motion of the vehicle are selected to analyze the dynamic response and synchronizing characteristics of the DMDS in a non-inertial system. The results show that the acceleration has a significant influence on the displacement response of gears, torque and speed of PMSMs, but the torque synchronization and speed synchronization between PMSMs are minimally influenced by both the acceleration and the AITs. Meanwhile, the AITs that affect the displacement response are analyzed and quantified.

Keywords: dual-motor driving system; non-inertial system; dynamic response; synchronizing characteristic; translational motion; circular motion

1. Introduction

It is generally believed that vehicles that use fossil fuels are one of the top sources of air pollution, and the depletion of oil is another serious problem that people have to face. As a consequence, new-energy vehicles (NEVs) are increasingly needed and have been listed as a development priority for all countries. Normally, the NEVs are driven by electric drive gear systems, such as the combination of a single motor or multiple motors with a gear transmission system. As one of the typical propulsion systems of NEVs, the dual-motor driving system (DMDS) has become the focus of more and more people, and has been reported in many literatures in the past. For example, Yang et al. designed a high power dual motor drive system to resolve the limitation of permanent magnet synchronous motor (PMSMs) application (such as temperature characteristics of permanent magnet materials and flux-weakening control problems) and utilize its good performances [1] and Tang provided a method and apparatus for optimizing the torque applied by each motor of a dual motor drive system of an all-electric vehicle [2]. In the traditional research on the dynamic response or synchronization property of DMDS, the DMDS has been considered to be stationary relative to the earth reference system. In fact, the DMDS is installed and fixed on the bodywork, and moves together with the bodywork in space, that is, the DMDS operates in a non-inertial system. Besides its own excitation forces, the DMDS is inevitably affected...
by additional inertial forces and gyroscopic moments, which affects the dynamic response and synchronization property of DMDS. Therefore, the effects of non-inertia should be considered when the NEVs perform a spatial motion.

In the past few decades, many scholars across the world have carried out meaningful research on the combination of a single motor or multiple motors with a gear transmission system attached to a moving body, including modeling, dynamic response and synchronization property, etc. In modeling and dynamic property terms, Inalpolat et al. established a transverse-torsional dynamic mode of a spur gear reducer, including periodically time-varying meshing stiffness and nonlinearities caused by tooth separations in resonance regions, and the influence of gear tooth indexing errors on the dynamic response was investigated [3]. Giagopulos et al. proposed a dynamic model of a gear-pair system considering the gear backlash and bearing stiffness non-linearities, and then the effect of non-linearities in the identification and fault detection of gear-pair system were investigated [4]. Setiawan et al. developed a mathematical model for multispeed transmissions in electric vehicles, and the transient dynamic response of the transmission was studied. The established model was subsequently validated by comparing its results with experiment results [5]. Sun et al. developed a coupling dynamic model of a multi-motor gear system by employing the proposed hierarchical modeling method and obtained the influence rules of system parameters on the natural frequency. Meanwhile, dynamic behaviors under different load conditions were also obtained [6]. Liu et al. used the proposed hybrid user-defined element method to establish the dynamic model of a gear transmission system in an electric vehicle, and investigated the effect of meshing impact force and meshing stiffness on the dynamic behavior of the system [7]. Tang et al. established a novel torsional vibration dynamic model of a hybrid power train, and the effect of a dual-mass flywheel on the torsional vibration characteristics of a power-split hybrid power train was studied [8]. Zhang et al. established a multi-degree-of freedom (M-D-O-F) dynamic model of a two-stage gear set that included the time-variant stiffness and gear backlash feature, and then the nonlinear response of the two-stage gear set under the combined periodic and stochastic excitations was investigated [9]. Yan et al. presented a new design by integrating a planetary gear train (PGT) within a DC motor to form a compact structure assembly, and then investigated the effects of the gear tooth on the dynamic responses of the DC motor associated with the kinematics of the PGT [10]. Mashimo et al. proposed a micro geared ultrasonic motor and its dynamic model was established to analyze the dynamic characteristics [11]. Kim et al. proposed a dual actuator unit (DAU) composed of two actuators and a planetary gear train; the mathematical model of the DAU was developed to investigate the performance of position tracking, stiffness variation, etc. [12]. In addition, Lee et al. also proposed a similar actuator system using a planetary gear and two motors; subsequently, a dynamic model of the actuator system was established to analyze the output velocity and torque, and obtain frequency response characteristics [13]. In addition, regarding synchronization characteristics, Wei et al. established a coupling dynamic model of a multi-source driving transmission system considering the flexible shafts, support bearing and meshing gear pair; the speed and torque synchronization properties under an impact load at different load change rates were studied [14]. Zhao et al. investigated the real-time speed synchronization of multiple induction motors during load variation and speed acceleration [15]. Xiong et al. addressed the system dynamic response synchronization and real-time problems by employing a dynamic simulation test system for electric vehicles with a dual drive circuit [16]. Shi et al. investigated the coupling characteristics and synchronization of multi-motors by using the presented synchronization control algorithm for a multi-motor speed synchronous driving system. The algorithm was employed to control the motion of the electronic circuit axis and virtual transmission shaft. The results showed that it improved the running response speed and synchronization accuracy between the motors [17]. Yang et al. established a translation-torsion coupling dynamic model of a multi-motor torque coupling system; the influence of system speed synchronization performance and torque synchronization performance on the load sharing behavior of the system were discussed [18].
At present, the related research on the combination of a single motor or multiple motors with a gear transmission system attached to a moving body mainly focuses on the dynamic response and synchronizing characteristics under inertial conditions, while related research under non-inertial conditions has rarely been reported. In fact, the gear transmission system is installed and fixed on the moving body and moves with the moving body. Obviously, under non-inertial conditions, the gear transmission system is not only affected by its own excitation forces, but is also affected by the additional inertial forces generated by the change in the operating state of the moving body. Therefore, it is necessary and meaningful to investigate the dynamic response and synchronizing characteristics of the combination of a single motor or multiple motors with a gear transmission system attached to a moving body. This paper will take the DMDS installed on the NEVs as an example to carry out the related research, and the organization of the paper is as follows. In Section 2, the dynamic model of DMDS in a non-inertial system is introduced. Then, the response and synchronizing characteristic of the DMDS under the translational motion and circular motion, two typical forms of NEV motion, are discussed in Section 3. Finally, some conclusions are drawn in Section 4.

2. Dynamic Model of Dual-Motor Driving System in Non-Inertial System

The DMDS, as shown in Figure 1, is composed of PMSMs, pinions and a bull wheel, etc. The two pinions are driven by two PMSMs (the rated power of each one is 100 kW) separately and the wheel is driven by the two pinions together; the shaft is connected to the wheel via the spline. Based on the finite element theory, the DMDS can be divided into the following four basic coupling elements: meshing element, shafting element, connecting element and bearing element. The dynamic model and node diagram of DMDS are shown in Figure 2.

![Figure 1. Structure of DMDS.](image-url)
2.1. Dynamic Model of Basic Coupling Elements

**Meshing elements.** The meshing elements can be regarded as a bridge to couple two or more rotating shafts together and transmit motion and force, and can be described by a pair of rigid disks connected by a stiffness-damping spring system on the plane of action, as shown in Figure 3. \( \varphi, \beta \) and \( \alpha \) are the relative position angle, helix angle and pressure angle of gears, respectively. \( r_p \) and \( r_g \) are the base radius of gear \( p \) and gear \( g \), respectively. \( k_m \) is the meshing stiffness, which can be represented by rectangular wave and regarded as the time-varying function of gear rotational angular displacement, as shown in Figure 4. \( e_m \) and \( c_m \) are the transmission error and meshing damping, respectively, and they are the time-varying variables. Herein, \( e_m \) can be solved by referring to Ref. [18] and viewed as the time-varying function of gear rotational angular displacement. Therefore, the time-varying variables \( k_m, e_m \) and \( c_m \) can be expressed as Equation (2). \( x_i, y_i \) and \( z_i \) \( (i = p \) and \( g \) are the horizontal, vertical and axial direction displacements of gears, respectively. \( \theta_i \) \( (i = x_p, y_p \) and \( x_g, y_g \) is the swing angular displacement of gears around the \( x \)-axis or \( y \)-axis, \( \theta_i \) \( (i = z_p \) and \( z_g \) is the rotational angular displacement of gears around the \( z \)-axis. The dynamic model of the meshing elements can be expressed as

\[
\mathbf{M}_{pg} \ddot{\mathbf{x}}_{pg} + \mathbf{C}_{pg} \dot{\mathbf{x}}_{pg} + \mathbf{K}_{pg} \mathbf{x}_{pg} = \mathbf{F}_{pg}
\]  

where

\[
\mathbf{M}_{pg} = \text{diag}[m_{p} m_{p} m_{p} l_{xp} l_{yp} l_{zp} m_{g} m_{g} m_{g} l_{xg} l_{yg} l_{zg}], \quad \mathbf{x}_{pg} = [x_p y_p z_p \theta_{xp} \theta_{yp} \theta_{zp} x_g y_g z_g \theta_{xg} \theta_{yg} \theta_{zg}]^T
\]

\[
\mathbf{K}_{pg} = k_m \mathbf{V}_{pg} \mathbf{V}_{pg}^T, \quad \mathbf{F}_{pg} = (k_m e_m + c_m e_m) \mathbf{V}_{pg} \mathbf{x}_{pg}^T, \quad \mathbf{C}_{pg} = c_m \mathbf{V}_{pg} \mathbf{V}_{pg}^T
\]

\[
\mathbf{V}_{pg} = [\cos \beta \sin(\alpha - \varphi), \cos \beta \cos(\alpha - \varphi), \sin \beta, -r_p \sin \beta \sin(\alpha - \varphi), -r_p \sin \beta \cos(\alpha - \varphi), r_p \cos \beta, \cdots, -\cos \beta \sin(\alpha - \varphi), -\cos \beta \cos(\alpha - \varphi), -\sin \beta, -r_g \sin \beta \sin(\alpha - \varphi), -r_g \sin \beta \cos(\alpha - \varphi), -r_g \cos \beta]
\]
Figure 3. Equivalent model of meshing element.

Figure 4. Meshing stiffness.

\[
\begin{align*}
\mathbf{M}_{pg}, \mathbf{C}_{pg} \text{ and } \mathbf{K}_{pg} \text{ are the mass, damping and stiffness matrices of the gear pair, respectively. } \\
\mathbf{X}_{pg} \text{ and } \mathbf{V}_{pg} \text{ are the displacement vector and meshing matrix of the gear pair. } \\
\mathbf{F}_{pg} \text{ is the exciting force vector, which is a non-constant vector and is related to the time-varying variables } k_m, c_m \text{ and } \epsilon_m.
\end{align*}
\]

\[
\begin{cases}
k_m = \bar{k}_{pg} + \sum_{l=1}^{7} \tilde{k}_{pgl} \cdot \cos\left(\frac{l \cdot 2 \pi \gamma_p}{p_{bpg}} + \gamma_1\right) \\
c_m = 2 \xi \sqrt{k_m / (1/m_{eq,1} + 1/m_{eq,2})} \\
\epsilon_m = \bar{\epsilon}_{pg} + \sum_{l=1}^{7} \tilde{\epsilon}_{pgl} \cdot \cos\left(\frac{l \cdot 2 \pi \gamma_p}{p_{bpg}} + \gamma_1\right)
\end{cases}
\]

where \(\tilde{k}_{pgl}\) and \(\bar{k}_{pg}\) are the \(l\)th harmonic term amplitude and mean mesh stiffness, respectively. \(\tilde{\epsilon}_{pgl}\) and \(\bar{\epsilon}_{pg}\) are the \(l\)th harmonic term amplitude and mean transmission error, respectively. \(\gamma_1\) is the phase angle. \(\gamma_p\) and \(p_{bpg}\) denote the rotational angular displacement around the z-axis of gear \(p\), and the base pitch of gear \(p\), respectively. \(\xi\) is the damping ratio. \(m_{eq,1}\) and \(m_{eq,2}\) are the equivalent qualities of the gears.

**Shafting elements.** The rotating components (such as shafts and gears) of the DMDS can be equivalent to the shafting elements with different cross sections, including hollow circle cross-section, circle cross-section, etc. The force state and deformation state of the shafting elements can be analyzed by employing the Timoshenko beam theory \([19,20]\), as shown in Figure 5. The deformation of the shafting elements can be described by the displacement of the nodes and the dynamic model of the two-node shafting element can be express as

\[
\mathbf{M}_{ijkl+1} \ddot{\mathbf{X}}_{ijkl+1} + \mathbf{C}_{ijkl+1} \dot{\mathbf{X}}_{ijkl+1} + \mathbf{K}_{ijkl+1} \mathbf{X}_{ijkl+1} = \mathbf{0}
\]
where the damping matrix can be solved by Rayleigh damping, $C_{i} = \lambda_1 M_{i} + \lambda_2 K_{i}$, where $\lambda_1$ and $\lambda_2$ represent the proportionality coefficients.

**Connecting elements.** The connecting elements can be employed to describe the coupling relationship between two connecting components, such as couplings and splines. For example, as shown in Figure 6, the coupling relationship between nodes $i$ on the internal spline and node $j$ on the external spline can be described by means of a spring-damper system. Therefore, the dynamic model of the connecting elements can be expressed as

$$\begin{bmatrix} M_{i} & O \\ O & M_{j} \end{bmatrix} \begin{bmatrix} \dot{X}_{i} \\ \dot{X}_{j} \end{bmatrix} + \begin{bmatrix} -C_{co} & -C_{co} \\ -C_{co} & -C_{co} \end{bmatrix} \begin{bmatrix} X_{i} \\ X_{j} \end{bmatrix} + \begin{bmatrix} K_{co} & -K_{co} \\ -K_{co} & K_{co} \end{bmatrix} \begin{bmatrix} X_{i} \\ X_{j} \end{bmatrix} = 0$$  \hspace{1cm} (4)

**Bearing elements.** The bearings are used to support the rotating shafts and constrain its horizontal and vertical movement, which can be equivalent to a spring-damper system, as shown in Figure 7. $K_{bi}$ is the support stiffness matrix of the support node $i$ on the rotating shaft, and the Ref. [21] can be referred to in order to provide the specific form of $K_{bi}$. $C_{bi}$ represents the support damping matrix of the support node $i$ on the rotating shaft, and $C_{bi}$ has the same structure as $K_{bi}$. The dynamic model of the bearing element’s corresponding support node $i$ on the rotating shaft can be expressed as

$$M_{bi} \ddot{X}_{bi} + C_{bi} \dot{X}_{bi} + K_{bi} X_{bi} = 0 \hspace{1cm} (5)$$

$X_{bi}$ and $M_{bi}$ represent the displacement vector and mass matrix of the support node $i$ on the rotating shaft, respectively.

![Figure 5. Timoshenko beam.](image-url)  

$\mathbf{M}_{i,j}$ and $\mathbf{X}_{i,j}$ are the mass matrix and displacement vector, respectively. $\mathbf{K}_{i,j}$ is the stiffness matrix with a $12 \times 12$ order form and can be solved by referring to Ref. [20]. $\mathbf{C}_{i,j}$ is the damping matrix and can be solved by Rayleigh damping, $\mathbf{C}_{i,j} = \lambda_1 \mathbf{M}_{i,j} + \lambda_2 \mathbf{K}_{i,j}$, where $\lambda_1$ and $\lambda_2$ represent the proportionality coefficients.
2.2. Dynamic Model of Dual-Motor Driving System in Non-Inertial System

The rotating shafts of the DMDS are simulated by the shafting elements, and they can be assembled together by means of the meshing elements, connecting elements and bearing elements according to the node distribution diagram illustrated in Figure 2. The coupling matrix assemble method described in the Ref. [21] can be used to establish the overall system dynamic model of DMDS, which contains 44 nodes. The overall system dynamic model can be expressed as

\[
M\ddot{X}(t) + C(t)\dot{X}(t) + K(t)X(t) = F(t) + G(t) + M(t) + T(t) \tag{6}
\]

\(M\) and \(X(t)\) are the overall mass matrix and displacement vector, and their dimensions are \(264 \times 264\) and \(264 \times 1\), respectively. \(C(t)\) and \(K(t)\) represent the overall damping and stiffness matrices, and the dimensions of these matrices are \(264 \times 264\), respectively. \(F(t), G(t)\) and \(M(t)\) and \(T(t)\) are the additional inertial force vector, gravity and gyroscopic moment vector and external load excitation vector, and the dimensions of these vectors are \(264 \times 1\), respectively. \(T(t)\) can be expressed as

\[
T(t) = [0, 0, 0, 0, 0, 0, \cdots, 0, 0, 0, 0, T_{in}, \cdots, 0, 0, 0, 0, T_{out}, \cdots, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]^{T} \tag{7}
\]

\(T_{out}\) is the load torque that acts on the power output node. \(T_{in}\) is the output torque of PMSMs that acts on the power input node. \(T_{in}\) can be obtained by establishing the simulation model of PMSMs, and the modeling process is described in detail in Ref. [22], and the key parameters of PMSMs are listed in Table 1.

Table 1. Key parameters of PMSMs.

| Stator | Rotor | Unit |
|--------|-------|------|
| Resistance | 8.296 × 10^−3 | 7.11 × 10^−2 | Ω |
| Rated speed | — | 2400 | rpm |
| Rated voltage | 540 | — | VDC |
| Leakage inductance | 5.05 × 10^−5 | 5.05 × 10^−5 | H |
| Rotational inertia | 8.86 × 10^−1 | kg m^2 |
| Magnetizing inductance | 8.49 × 10^−5 | 8.49 × 10^−5 | H |
| Dimensions | ϕ380 × L399 | mm |
| Mass | 130 | kg |
| Rated power | 100 | kW |
| Rated torque | 400 | Nm |

**Additional force and moment.** The DMDS is installed and fixed to the bodywork and moves together with the bodywork in space. Therefore, the effects of non-inertia on the DMDS operating in moving bodywork should be considered, and the thermal changes between PMSM and the gear transmission system is ignored. As shown in Figure 8a, a non-inertial system is employed to describe the absolute motion of point \(Q\) in space, where \(OXYZ\) is a static coordinate system fixed on earth, and \(oxyz\) is a moving coordinate system that is relative to the \(OXYZ\) and fixed on moving bodywork. \(E_i\) and \(e_i\) (\(i = 1, 2\) and 3) are the base vectors of \(OXYZ\) and \(oxyz\), respectively. \(r_o\) and \(r_Q\) are the position vectors of origin \(o\) and moving point \(Q\), respectively and they are measured in \(OXYZ\). \(r_{o/Q}\) is the relative position vector of moving point \(Q\), which is measured in \(oxyz\). \(Ω\) is the angular velocity vector of \(oxyz\) and measured in \(OXYZ\). The position vector \(r_Q\) can be expressed as

\[
r_Q = r_o + r_{o/Q} = X_oE_1 + Y_oE_2 + Z_oE_3 + x_Qe_1 + y_Qe_2 + z_Qe_3 \tag{8}
\]

where \(X_o, Y_o, Z_o\) and \(x_Q, y_Q, z_Q\) are the variables. \(E_i\) and \(e_i\) (\(i = 1, 2\) and 3) are the constant vector and variable vector, respectively. The absolute velocity vector \(v_Q\) can be obtained
by taking the derivative of the position vector $r_Q$ with respect to time, and it can be expressed as

$$
v_Q = \frac{dr_Q}{dt} = X_Q E_1 + Y_Q E_2 + Z_Q E_3 + x_Q e_1 + y_Q e_2 + z_Q e_3 + x_Q \frac{de_1}{dt} + y_Q \frac{de_2}{dt} + z_Q \frac{de_3}{dt}
$$

$$
\frac{de_i}{dt} = \Omega \times e_i, \quad i = 1, 2, 3
$$

(9)

Then, the absolute acceleration vector $a_Q$ can be obtained by taking the derivative of the absolute velocity vector $v_Q$ with respect to time, and it can be expressed as

$$
a_Q = \frac{dv_Q}{dt} = a_0 + \frac{dv_0}{Q/Q} + \frac{d\Omega}{dt} \times r_0/Q + \Omega \times \frac{dr_0}{dt} + 2\Omega \times v_0/Q + \Omega \times (\Omega \times r_0/Q)
$$

(10)

(1) When the NEVs perform a variable speed translational motion, from Equation (10), it can be observed that the absolute acceleration term of the moving point $Q$ can be derived as $a_Q = a_0 + a_{\delta/Q}$, where $a_0$ is the convected acceleration term and $a_{\delta/Q}$ is the relative acceleration term. Since there is no relative movement between DMDS and the bodywork, this means that $a_{\delta/Q} = 0$. Therefore, when the NEVs perform a variable speed translational motion, the additional inertial force term $F(t)$ can be expressed as

$$
F(t) = -m_i \cdot a_{i/o} = -m_i \cdot [a_{1x/o}, a_{1y/o}, a_{1z/o}, 0, 0, 0, \ldots, a_{N_{i/x}/o}, a_{N_{i/y}/o}, a_{N_{i/z}/o}, 0, 0, 0, \ldots, 0, 0, 0]^T
$$

(11)

where $m_i$ is the mass of node $i$ on the rotating shaft, and $a_{ij/o}$ ($i = 1, 2, \ldots, N$) represents the convected acceleration vector of node $i$ on the rotating shaft and is measured in OXYZ. $a_{i/x/o}, a_{i/y/o}$ and $a_{i/z/o}$ are the components of $a_{i/o}$ in $x$, $y$ and $z$ directions, respectively. Besides $F(t)$, the node $i$ on the rotating shaft is also affected by gravity $G(t)$, and it can be expressed as

$$
G(t) = -m_i \cdot a_{ig} = -m_i \cdot [a_{1xg}, a_{1yg}, a_{1zg}, 0, 0, 0, \ldots, a_{N_{xg}}, a_{N_{yg}}, a_{N_{zg}}, 0, 0, 0, \ldots, 0, 0, 0]^T
$$

(12)

where $a_{ig}$ ($i = 1, 2, \ldots, N$) represents the gravitational acceleration vector of node $i$ on the rotating shaft. $a_{ixg}, a_{iyg}$ and $a_{izg}$ are the components of $a_{ig}$ in $x$, $y$ and $z$ directions, respectively.

(2) When the NEVs performs a variable speed circular motion, from Equation (10), the absolute acceleration term of moving point $Q$ can be derived as follows: $a_Q = a_{0/Q} + x r_0/Q + \Omega \times (\Omega \times r_0/Q) + 2\Omega \times v_0/Q$, where $\Omega \times r_0/Q$ and $\Omega \times (\Omega \times r_0/Q)$ are the first convected acceleration term and second convected acceleration term, respectively, and are denoted as $a_{r1}$ and $a_{r2}$, respectively. Herein, $a_r = a_{r1} + a_{r2}$. $2\Omega \times v_0/Q$ is the Coriolis acceleration term and is denoted as $a_c$. Similarly, since there is no relative movement between DMDS and the bodywork, the relative acceleration term $a_{\delta/Q} = 0$. Thus, when the

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Figure 8. (a) Kinematic analysis in non-inertial system; (b) analysis of additional gyroscopic moment.
NEVs performs a variable speed circular motion, the additional inertial force term \( F(t) \) can be expressed as

\[
F(t) = -m_i \cdot (a_{irx} + a_{irc}) = -m_i \cdot (a_{irx} + a_{icy} + a_{ircy} + a_{ircz} + a_{irz} + a_{ircz}, 0, 0, 0, \cdots, a_{iNrx} + a_{iNcy} + a_{iNcy} + a_{iNcz} + a_{iNrz} + a_{iNcz}, 0, 0, 0)^T
\]

where \( a_{ir} \) and \( a_{ic} \) (\( i = 1, 2, \ldots, N \)) represent the convected acceleration and Coriolis acceleration vectors of node \( i \) on the rotating shaft, respectively. \( a_{irx}, a_{icy}, a_{ircy}, a_{ircz}, a_{irz} \) and \( a_{ircz} \) are the components of \( a_{ir} \) and \( a_{ic} \) in the \( x, y, z \) directions, respectively. The gravity term \( \text{G}(t) \) can be calculated by Equation (12). Moreover, the rotating shaft will be subjected to an additional gyroscopic moment due to the rotating angular velocity \( \omega \) when the NEVs perform a circular motion. As shown in Figure 8b, the moment of momentum can be described as \( H \). The polar moment of inertia \( J \) is equal to 600 Nm. If one takes as an example the displacement responses of all the nodes on the shaft, respectively.

\[
M(t) = H \times \Omega = J \cdot \omega \times \Omega = \left[ 0, 0, f_{1x} \omega_1 \Omega_{1z}, f_{1y} \omega_1 \Omega_{1y}, f_{1z} \omega_1 \Omega_{1z}, \cdots, 0, 0, f_{Nx} \omega_N \Omega_{Nz}, f_{Ny} \omega_N \Omega_{Ny}, f_{Nz} \omega_N \Omega_{Nz} \right]^T
\]

where \( f_{ij} \) and \( \omega_{ij} \) (\( j = x, y, z \)) are the moment of inertia and angular velocity of node \( i \) around the \( j \)-axis, respectively, and \( \Omega_{ij} \) (\( j = x, y, z \)) is the angular velocity of node \( i \) around the axes other than the \( j \)-axis.

### 3. Dynamic Response and Synchronizing Characteristic Analysis in Non-Inertial System

The software MATLAB/Simulink (license: 40927247) is used to build the simulation model of the DMDS, and the translational motion and circular motion of the NEVs are selected to investigate the response and synchronizing characteristics of the DMDS in a non-inertial system. The key parameters of the DMDS are listed in Table 2.

#### Table 2. Key parameters of DMDS.

| Parameters | Values | Parameters | Values |
|------------|--------|------------|--------|
| Tooth number (pinion/wheel) | 23/69 | Length of shaft 1 (or 3) \( L_1 \) (mm) | 32 |
| Module (mm) | 4 | Diameter of shaft 2 \( D/d \) (mm) | 65/45 |
| Width (mm) | 35 | Length of shaft 2 \( L_2 \) (mm) | 20 |
| Pressure angle \( \alpha \) (°) | 20 | Bearing translational stiffness \( (\text{Nm}^{-1}) \) | \( k_x = k_y = 1.2 \times 10^8; k_z = 9.5 \times 10^7 \) |
| Helix angle \( \beta \) (°) | 15 | Bearing torsional stiffness \( (\text{Nm}^{-1}) \) | \( k_{\theta x} = k_{\theta y} = 1.0 \times 10^7 \) |
| Diameter of shaft 1 (or 3) \( D/d \) (mm) | 60/45 | Transmission error amplitude (μm) | 20 |

#### 3.1. Case I: The NEVs Perform a Variable Speed Translational Motion

If one supposes that the NEVs travel in a straight line with variable speed in the \( x \)-direction, the force condition is shown as Figure 9, and the basic parameters of NEVs are listed in Table 3. For the gravity of node \( j \), \( \text{G}_j = -m_j a_{ic} (0 e_1 + e_2 + 0 e_3) \), as shown in Figure 9a. For the additional convected inertial force (ACIF) of node \( j \), \( \text{F}_j = -m_j a_{ir} (e_1 + 0 e_2 + 0 e_3) \), as shown in Figure 9b. The total gravity and ACIF of a shaft are equal to the sum of \( \text{G}_j \) and \( \text{F}_j \) of all the nodes on the shaft, respectively.

The curves of the translation acceleration \( a_t \) and translational speed \( v \) in the \( x \)-direction shown in Figure 10 are employed to investigate the response and synchronizing characteristics of the DMDS under different non-inertial conditions. The initial translational speed of the vehicle is 80 km/h and the rotating speeds of PMSMs are 1900 rpm, and the load imposed on the DMDS is 600 Nm. If one takes as an example the displacement responses
of node 14 (gear node) under different non-inertial conditions, as shown in Figure 11, it can be clearly observed that the displacement response curves in the x- and y-directions are significantly associated with the $a_x$-curve; the difference between both of them is that they exhibit a full reversal. The displacement responses increase with the increase in $a_x$, but the magnitude in the y-direction is markedly greater than that in the x-direction.

![Diagram](image-url)

**Figure 9.** Force analysis diagram: (a) force analysis when NEV travels in a straight line; (b) force analysis of node $j$.

**Table 3.** Basic parameters of NEVs.

| Parameters          | Values | Parameters          | Values | Parameters                  | Values |
|---------------------|--------|---------------------|--------|----------------------------|--------|
| Length (mm)         | 4675   | High (mm)           | 1500   | Front/rear track (mm)       | 1525/1520 |
| Width (mm)          | 1770   | Wheelbase (mm)      | 2670   | Weight (kg)                | 1531   |

![Graph](image-url)

**Figure 10.** The curves of translation acceleration $a_x$ and translational speed $v$.

![Graph](image-url)

**Figure 11.** Displacement responses of node 14 under different non-inertial conditions: (a) x-direction; (b) y-direction.

According to the theoretical analysis, when the NEVs perform a translational motion, the nodes are not only affected by the torque of PMSMs ($T_{in}$), but also by ACIF ($F$) in the x-direction and gravity ($G$) in the y-direction. Figure 12a shows the displacement responses with or without $F$ when $a_x$ changes from 10 to $-10$ m/s$^2$. As a consequence, the offset of the red line is significantly greater than that of blue line. Similarly, due to the effect of...
gravity $G$, the offset of the displacement response in the $y$-direction has increased, but is not obvious, as shown in Figure 12b.

In addition, the relative influence proportion $\gamma_i$ is used to analyze the degree of influence of $F$ or $G$ on the displacement response, and it can be calculated by Equation (15). As shown in Figure 13a, the influence of $F$ on the displacement response in the $x$-direction is obviously greater than that of $G$ on the displacement response in the $y$-direction. When $a_x$ changes from $-15$ to $15$ m·s$^{-2}$, the relative influence proportion of $F$ has a numerically positive correlation with $a_x$, and then has a numerically positive correlation with $a_x$. When $a_x = -15$ or $15$ m·s$^{-2}$, the relative influence proportion reaches the maximum value, which is about 24%. However, when $a_x$ changes from $-15$ to $15$ m·s$^{-2}$, the relative influence proportion of $G$ has a numerically positive correlation with $a_x$, and then has a numerically negative correlation with $a_x$, and the maximum relative influence proportion only reaches 2.3% when $a_x = 0$ m·s$^{-2}$, as shown in Figure 15b. Therefore, the gravity $G$ is not the primary influence factor, and its influence can be ignored whether it is in inertial or non-inertial conditions.

$$
\gamma_i = \frac{(\delta_i - \bar{\delta}_i)}{\sum (\delta_i - \bar{\delta}_i)} \times 100\% 
$$

(15)

$\delta_i$ is the offset when the factor $i$ is not considered and $\bar{\delta}_i$ is the offset when all factors are considered.

The vibration of gears can be transmitted to the PMSMs and affect the torque and speed of PMSMs. Considering that the gravity $G$ has very little effect on the displacement responses of gears, we next focus on the effect of ACIF on the torque and speed of PMSMs. Herein, the synchronization error (SE) is applied to measure the synchronization of the output variables (torque and speed) between PMSMs. The greater the SE, the greater the difference in the output variables between PMSMs, which can be expressed as

$$
E_\lambda = \frac{(\lambda_I - \lambda_{mean})_{max}}{\lambda_{mean}} \times 100\%, \quad \lambda_{mean} = \frac{1}{N} \sum_{i=1}^{N} \lambda_I
$$

(16)
\( E_\lambda \) is the SE, \( \lambda = T_{in} \) and \( v, l = 1, 2, \ldots, N \). \( T_{in} \) and \( v \) represent the torque and speed of PMSMs, respectively. \( N \) is the number of PMSMs and \( N = 2 \) in this study.

The torque of PMSMs and its torque synchronization error (TSE) are shown in Figure 14. Except for the fact that there is some fluctuation at 4.0–4.5 s and 5.0–5.5 s, the curve of torque is very close to the \( a_x-t \) curve, as shown in Figure 14a. Obviously, the torque remains almost unchanged before and after considering the ACIF. Similarly, the TSE between PMSMs has hardly changed, and their amplitudes fluctuate between \(-2.1\%\) and \(2.3\%\) during acceleration and fluctuate between \(-14.2\%\) and \(18.8\%\) during deceleration. This phenomenon is firstly because the growth of the numerator in Equation (16) is lesser than that of the denominator, and second because the mean value of torque (namely, \( \lambda_{mean} \)) during acceleration is significantly greater than that during deceleration. Figure 14c shows the curve of the root mean square (RMS) value of TSE. When \( a_x \) changes in the range of \(-15 \) to \(15 \) m\( \cdot \)s\(^{-2}\), with the decrease in the numerical value of \( a_x \), the RMS value of TSE increases and then decreases and the maximum value appears when \( a_x = -2.5 \) m\( \cdot \)s\(^{-2}\) and is about \(2.8\%\). The reason for this is that the torque of PMSMs is at its minimum value when \( a_x = -2.5 \) m\( \cdot \)s\(^{-2}\), namely when \( \lambda_{mean} \) in Equation (16) is at its minimum value, so the calculated \( E_\lambda \) is at its maximum value. It indicates that the change in translation acceleration has a small effect on the synchronization of torque between PMSMs. Moreover, through the analysis and comparison, it is found that the change in the RMS value of TSE is minor before and after considering the ACIF, and the maximum difference between the two is only \(0.26\%\), indicating that the ACIF also has a small effect on the synchronization of torque between PMSMs.

![Figure 14](image_url)

*Figure 14.* Torque and TSE when \( a_x \) changes from 10 to \(-10\) m\( \cdot \)s\(^{-2}\): (a) torque of PMSMs; (b) TSE; (c) the changes in RMS of TSE with \( a_x \).

The speed of PMSM and its speed synchronization error (SSE) are shown in Figure 15. The curve of speed exhibits good correlation with the \( v-t \) curve, as shown in Figure 15a. There is no obvious change in speed at any of the times, regardless of whether the ACIF is considered or not. This can also be verified by Figure 15b and compared with the case without considering \( F \); the change in SSE is not noticeable, and its amplitude fluctuates between \(-2.2\%\) and \(2.3\%\). In addition, the curve of the RMS value of SSE under different translation acceleration conditions is plotted, as shown in Figure 15c. When \( a_x \) changes in the range of \(-15 \) to \(15 \) m\( \cdot \)s\(^{-2}\), with the decrease in the numerical value of \( a_x \), the RMS value of SSE increases and then decreases and the maximum value appears when \( a_x = -0 \) m\( \cdot \)s\(^{-2}\) and is about \(5.03\%\). This is because the speed of PMSMs is at its minimum value when \( a_x = 0 \) m\( \cdot \)s\(^{-2}\), namely when \( \lambda_{mean} \) in Equation (16) is at its minimum value, so the calculated \( E_\lambda \) is at its maximum value. This means that the changes in \( a_x \) have a small effect on speed synchronization between PMSMs. Meanwhile, it can also be clearly observed that the increase in the RMS value of SSE is not obvious after considering ACIF, and the maximum difference between the two is only \(0.27\%\), indicating that the ACIF has a very small influence on speed synchronization between PMSMs.
3.2. Case II: The NEVs Perform a Variable Speed Circular Motion

If one supposes that the NEVs perform a variable-speed circular motion with radius $R_{\Omega}$ around the $Y$-axis, $\theta_r$ is the angle displacement and $\Omega_Y$ is the rotating velocity of the vehicle around the $Y$-axis and they are measured in $OXYZ$, as shown in Figure 16. The gravity $G_j$ in case II is calculated in the same way as that in case I. For the additional inertial force of node $j$, $F_j = -m_j (a_{jr} + a_{o,j})$; $a_{jr}$ and $a_{o,j}$ are the convected acceleration vector and Coriolis acceleration vector of node $j$, respectively. For the additional gyroscopic moment of node $j$, $M_j = J_j \omega \times \Omega_Y$.

As shown in Figure 17a, the convected acceleration vector $a_{jr} = \Omega_Y \times r_{o,j} + \Omega_Y \times (\Omega_Y \times r_{o,j}) = \Omega_Y \times (r_{j/z} + r_{j/x} + r_{j/y}) + \Omega_Y \times (r_{j/z} + r_{j/x} + r_{j/y}) = \Omega_Y \times (r_{j/z} + r_{j/x} + r_{j/y}) + \Omega_Y \times (\Omega_Y \times (r_{j/z} + r_{j/x} + r_{j/y}))$. One must suppose that $a_{j\Omega1} = \Omega_Y \times (r_{j/z} + r_{j/x})$ is the additional first convected acceleration vector and $a_{j\Omega2} = \Omega_Y \times (\Omega_Y \times (r_{j/z} + r_{j/x}))$ is the additional second convected acceleration vector. Hence, the complex three-dimension spatial problem can be solved in a two-dimensional plane, i.e., the $xoz$ plane. As shown in Figure 17b, $(m, n)$ and $(m', n')$ represent the initial position coordinate and instantaneous position coordinate of node $j$ in the $xoz$ plane and $x_1o_1z_1$ plane, respectively, and the convected acceleration vector $a_{jr}$ can be derived as

$$\begin{align*}
a_{j\Omega1} &= |\Omega_Y \times r_{o,j}| \cdot \cos(\gamma_j) \cdot e_1 - |\Omega_Y \times r_{o,j}| \cdot \sin(\gamma_j) \cdot e_3; \\
a_{j\Omega2} &= -|\Omega_Y \times (\Omega_Y \times r_{o,j})| \cdot \sin(\gamma_j) \cdot e_1 - |\Omega_Y \times (\Omega_Y \times r_{o,j})| \cdot \cos(\gamma_j) \cdot e_3 \\
|\Omega_Y \times r_{o,j}| &= |\Omega_Y| \cdot \sqrt{(m + m')^2 + (n + n')^2}; \\
|\Omega_Y \times (\Omega_Y \times r_{o,j})| &= |\Omega_Y|^2 \cdot \sqrt{(m + m')^2 + (n + n')^2}; \\
\gamma_j &= \arctan((n + n')/(m + m'))
\end{align*}$$

Figure 15. Speed and SSE when $a_x$ changes from 10 to $-10$ m·s$^{-2}$: (a) speed of PMSMs; (b) SSE; (c) the changes in RMS of SSE with $a_x$.

Figure 16. Schematic diagram of circular motion.

Figure 17. Relative motion: (a) displacement vector of node $j$; (b) convected acceleration vector and Coriolis acceleration vector; (c) rotational angular velocity.
In Figure 17b, \( v_j \) is the velocity vector of node \( j \) in the \( xoz \) plane, and the Coriolis acceleration vector \( a_c \) can be derived as

\[
\{ \begin{align*}
  v_j &= \omega_y \times v_i + \omega_z \times j \times v_i = 2 \omega_y \times (v_j \times e_1 + v_j \times e_3) = 2 \omega_y \times v_j \times e_1 + 2 \omega_y \times v_j \times e_2 + e_3 = 2 \omega_y \times v_j \times e_1 - 2 \omega_y \times v_j \times e_3 \\
  a_c &= 2 \omega_y \times v_j = 2 \omega_y \times (v_j \times e_1 + v_j \times e_3) = 2 \omega_y \times v_j \times e_1 - 2 \omega_y \times v_j \times e_3 
\end{align*} \]

(18)

where \( \omega_y \) is an angular velocity vector in the \( y \)-axis, \( \omega_y \) is an angular velocity of node \( i \) around the \( y \)-axes and it is a number.

The initial positions and instantaneous positions of nodes \( i \) and \( j \) are shown in Figure 17c and \( L_{ij} \) represents the initial axial distance between \( i \) and \( j \). \((x'_i, y'_i, z'_i)\) and \((x'_j, y'_j, z'_j)\) are the instantaneous position coordinates of nodes \( i \) and \( j \). According to the Ref. [23], the additional gyroscopic moment \( M(t) \) of any node can be derived as Equation (19), and the terms with a value of zero are not given in Equation (19).

\[
M(t) = -\frac{J \cdot \omega \cdot \omega_y \cdot [L_{ij} + (z'_j - z'_i)]}{\sqrt{(x'_i - x'_j)^2 + (y'_i - y'_j)^2 + [L_{ij} + (z'_j - z'_i)]^2}} \cdot e_1 + \frac{J \cdot \omega \cdot \omega_y \cdot (x'_i - x'_j)}{\sqrt{(x'_i - x'_j)^2 + (y'_i - y'_j)^2 + [L_{ij} + (z'_j - z'_i)]^2}} \cdot e_3
\]

(19)

The \( \dot{\omega}_y - t \), \( \omega_y - t \) and \( \theta_y - t \) curves shown in Figure 18 are used to study the response and synchronizing characteristics of the DMDS when the NEVs perform a variable speed circular motion. Similarly, the initial speed of the vehicle, rotating speeds of PMSMs and load imposed on the DMDS are also 80 km/h, 1900 rpm and 600 Nm, respectively. The turning radius \( R \) is set to 5 m, and the rotating acceleration \( \omega_y \) is set to 5, 10 and 15 m·s\(^{-2}\), respectively. The displacement responses of node 14 (gear node) are shown in Figure 19. The displacement response curves in the \( y \)- and \( z \)-directions are significantly associated with the \( \dot{\omega}_y - t \) and \( \omega_y - t \) curves, respectively, and the difference is that the directions of the displacement response curves is completely opposite to that of the \( \dot{\omega}_y - t \) and \( \omega_y - t \) curves. However, due to the coupling effect of \( \omega_y \) and \( \omega_y \), the displacement response curves in the \( x \)-direction are neither completely related to \( \dot{\omega}_y - t \) nor \( \omega_y - t \). The displacement responses in three directions increase with the increase in \( \dot{\omega}_y \), but the increases are different.

Figure 18. The curves of rotating acceleration \( \dot{\omega}_y \), rotating speed \( \omega_y \) and angle displacement \( \theta_y \) in variable-speed circular motion.

Figure 19. Displacement responses of node 14 under different rotating accelerations: (a) \( x \)-direction; (b) \( y \)-direction; (c) \( z \)-direction.
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Based on the above theoretical analysis, under the given non-inertial condition, except for the torque of PMSM \( T_{\text{m}} \), the nodes are also affected by the following five additional terms: first convected inertial force \( F_{\text{r1}} \), second convected inertial force \( F_{\text{r2}} \), Coriolis inertial force \( F_{\text{c}} \), gyroscopic moment \( M \) and gravity \( G \). The displacement responses of node 14 with or without additional inertial terms (AITs) are shown in Figure 20. In the x-direction, after considering AIFs, the displacement increases gradually during the II and III stages, and then decreases during the IV stage, and eventually reaches stabilization during the V stage. The displacement increases from about 0.62 µm to 1.18 µm, an increase of 90.3%, as shown in Figure 20a. In the y-direction, the displacement shows a small increase during the II-V stages, and increases from about 25 µm to 28 µm with an increase of 12% during the V stage. In the z-direction, the difference by not considering AITs is that the displacement increases gradually during the II-IV stages and then tends to become steady during the V stage. Finally, the displacement increases from approximately 3.6 µm to 7.5 µm, with an increase of 108.3%.

![Figure 20](image-url)

**Figure 20.** Displacement responses with or without AIF of node 14 when \( R = 5 \) m and \( \Omega_Y = 20 \) m/s²: (a) x-direction; (b) y-direction; (c) z-direction.

Herein, the Equation (14) is also used to calculate the relative influence proportion \( \gamma_i \) of the five additional terms and to analyze which terms are the major influence factors and which are the weakest ones, as shown in Figure 21. As demonstrated by the results in Figure 21a, during the I stage, the relative influence proportions of \( F_{\text{r1}} \), \( F_{\text{c}} \) and \( M \) are greater, reaching 2.9–76%, 9.1–48.6% and 10.6–48.6%, respectively. Then, their relative influence proportions decrease gradually during the II-V stages. However, during the II-V stages, the relative influence proportion of \( F_{\text{r2}} \) starts to increase quickly and becomes the main influence factor, and the maximum proportion can reach 93.9%. In Figure 21b, during the I stage, the relative proportions of \( G \), \( F_{\text{r1}} \), \( F_{\text{c}} \) and \( M \) are greater, and there is a little difference in their average proportions. Conversely, the relative influence proportion of \( M \) tends to become steady after the increase during the II-V stages, the maximum proportion reaches 99.3%, and the relative influence proportions of other additional terms are very small and their influences can be ignored. By comparing Figure 21a,b, it is found that the relative influence proportions of the five additional terms have a similar variation rule, but the shapes of the relative influence proportions of \( F_{\text{c}} \) and \( M \) in the two figures are exactly opposite. Likewise, the proportions of \( F_{\text{r1}} \), \( F_{\text{c}} \) and \( M \) are still greater during the I stage and then begin to decrease rapidly during the II-V stages. When it comes to stage II, the proportion of \( F_{\text{r2}} \) begins to increase quickly and then becomes the primary influence factor.

The torque and RMS value of TSE of the PMSMs are shown in Figure 22. Obviously, the curves of torque are very close to the \( \Omega_Y - t \) curve, and increase with the increase in rotating acceleration, as shown in Figure 22a. The results in Figure 22b show that the curves of RMS value of TSE are associated with the \( \Omega_Y - t \) curve, and exhibit a full reversal. Unlike torque, the RMS value of TSE decreases with the increase in rotating acceleration. When \( \Omega_Y \) changes from 5 to 15 m/s², the RMS value of TSE is only reduced from about 1.0% to 0.9%, and the reduction is no more than 10%, indicating that the rotating acceleration also has a small influence on the synchronization of torque between PMSMs. Moreover, the RMS values of TSE with or without AITs are shown in Figure 22c and it can be very clearly
observed that the red solid line almost coincides with the blue solid line at all times whether the AITs are considered or not, and the maximum difference does not exceed 0.05%, which means that the influence of AITs on the synchronization of torque between the PMSMs can almost be ignored when the NEVs perform a circular motion.

![Figure 21](image_url)

**Figure 21.** Relative influence proportion of each factor: (a) displacement response in x-direction; (b) displacement response in y-direction; (c) displacement response in z-direction.

![Figure 22](image_url)

**Figure 22.** Torque and RMS value of TSE: (a) torque of PMSMs under different rotating accelerations; (b) RMS values of TSE under different rotating accelerations; (c) RMS values of TSE with or without AITs.

The speed and RMS value of SSE of the PMSMs are shown in Figure 23. The curves of speed and RMS values of SSE are very close to the \( \Omega_y - t \) curve, but the curves of RMS values of SSE and \( \Omega_y - t \) have a diametrically opposite changing tendency. The speed increases as the rotating acceleration increases; conversely, the RMS value of SSE decreases as the rotating acceleration increases, with a maximum reduction of no more than 1.5‰, as shown in Figure 23a,b. Combining the results from Figure 23c, it is found that the influence of both AITs and rotating acceleration on the synchronization of speed between PMSMs can also be ignored when the vehicle performs a circular motion.

![Figure 23](image_url)

**Figure 23.** Speed and RMS value of SSE: (a) speed of PMSM under different rotating accelerations; (b) RMS values of SSE under different rotating accelerations; (c) RMS values of SSE with or without AITs.

4. Conclusions

The dynamic model of the DMDS installed on the NEVs was established considering the effects of the non-inertial system in the form of additional inertial terms. The response and synchronizing characteristics of the DMDS under different non-inertial conditions were investigated and compared, and the following conclusions were obtained:
(1) When the NEVs perform a variable speed translational motion, in addition to its own excitation forces and gravity, the components of the DMDS are subjected to an additional convected inertial force caused by the variable speed translational motion. There is a numerically positive correlation between the displacement response of the gears and the translation acceleration, and the contribution of the additional convected inertial force to displacement offset in the x-direction is greater than that of gravity in the y-direction.

(2) Due to the influence of multiple additional inertial terms’ superimposition, the variable speed circular motion of the NEVs results in greater displacement responses. For the displacement responses in the x and z directions, the additional second relative inertial term (namely, the $F_{r2}$ term) is the main influence factor among all the additional inertial terms that change the displacement offset. In the y-direction, the additional gyroscopic moment term (namely, the $M$ term) is the dominant influence factor, and its influence is more prominent under high-speed operating conditions.

(3) Regardless of whether the NEVs perform a variable speed translation motion or variable speed circular motion, the acceleration (translation acceleration or rotating acceleration) has a significant influence on the torques and speeds of PMSMs, and they increase with the increase in the acceleration. However, the torque synchronization and speed synchronization between PMSMs is minimally influenced by both the additional inertial terms and the acceleration.

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