We study the radiation of thermal photons and dileptons likely to be produced in relativistic heavy ion collisions. We find that the thermal photon multiplicity scales with the charged pion multiplicity as \( (dN_{\text{ch}}/d\eta)^{\alpha} \) with \( \alpha \approx 1.2 \) for a transversely expanding system, contrary to the general belief in heavy ion collisions. It is expected that if the transverse expansion of the system which drastically alters the time scales in the system, at least with regard to the mixed and the hadronic phases is also affected by the equation of state \( \rho = P/T^4 \). What should be the equation of state of hot hadronic matter which is created in heavy ion collisions? It is expected that if the transition temperature is very close to the pion mass then the hadronic matter could possibly well be determined by pions and only a few low lying mesonic resonances. The hadronic matter is likely to be populated by heavier hadrons once the temperature is higher. Will these hadrons stay in chemical and thermal equilibrium, till the freeze-out? This will be decided by the competition between the collision frequency and the rate of expansion, and if valid, it will be reflected in uniquely determined particle ratios. It is not difficult to imagine that the equation of state or at least the composition of the hadronic matter may be somehow related to the manner in which the hadronization proceeds. This aspect has not yet received enough attention as it involves the non-perturbative realm of quantum chromodynamics.

The question of hadronization of the plasma gets very complex in view of the recent findings that the QGP formed at RHIC or at LHC will not reach chemical equilibrium during the life-time of the quark phase. We can understand this as follows. Recall that according to the Maxwell criterion, the phase transition temperature \( T_c \) is given by \( P_Q(T_c) = P_H(T_c) \), where \( P \) stands for the pressure in the corresponding phase. The pressure for non-equilibrated QGP can be approximated as

\[
P_Q(T) = \left[ 16\lambda_q + 21\lambda_g \right] T^4 \frac{\pi^2}{90} - B,
\]

where \( \lambda_k \) is the fugacity of the parton species \( k \), \( B \) is the bag-pressure, and we have considered a two flavor plasma with zero baryochemical potential. Approximating the hadronic pressure in terms of a temperature dependent effective number of degrees of freedom \( g_H(T) \) (\( g_H = 3 \), for a hadronic gas of mass-less pions, e.g.), we can write

\[
P_H(T) = g_H(T) \frac{\pi^2}{90} T^4,
\]

which leads to

\[
T_c^4 = \frac{90B/\pi^2}{16\lambda_q + 21\lambda_g} - g_H(T_c).
\]

We see that if we take the bag pressure to be a constant, then decreasing fugacities will, in general, increase the effective number of degrees of freedom of a hadronic gas.
the transition temperature. This is further accentuated by an increase in $g_T$ with temperature, which in fact reflects the change in the composition of the hadronic matter. The fugacities, measuring deviation from a chemical equilibrium, may also have a radial dependence for a transversely expanding plasma. These aspects encourage us to think of a scenario where even the composition of the hadronic matter may become $r$-dependent.

Till the development of such a complete description of the collision up to the point of freeze-out, hydrodynamics can provide a useful guideline. In as far as we can assume the validity of hydrodynamics, and additionally assume an adiabatic hadronization of the plasma, we can describe the collision in its entirety, provided that the equation of state of hadronic matter is known.

In a recent study we addressed the question of the equation of state in connection with the production of thermal photons and particles at CERN SPS in collisions involving lead nuclei, in a model calculation. Two approximations were used to describe the hadronic medium. In the first one, only a small number of mesons ($\pi$, $\rho$, $\omega$, and $\eta$) was used ("simplified equation of state") while in the second one we used all hadrons listed in the particle data table. Very different results were obtained for the transverse momentum distribution of photons and particles, depending on the assumptions made about the equation of state of hot hadronic matter as well as the nature of the initially created medium. In the present work, we show that a comparison of electromagnetic radiation for a number of final particle multiplicity densities may help us distinguish between the equations of state. We shall also see that the frozen-motion model, as a minimal extension of Bjorken hydrodynamics, provides us with a possibility to realize this distinction in an experiment done at a given energy.

A clear illustration of the effect of the equation of state on the flow pattern at, say, LHC energies is seen in Fig. 1. We have plotted the space-time boundaries (at $z = 0$) of the QGP, the mixed, and the hadronic phases, using the approach described in Ref. [4]. We assume the system to be created in a state of QGP and use a boost invariant cylindrically symmetric transverse expansion. We see that the richer equation of state leads to a shorter-lived mixed phase and consequently the system edges past the freeze-out much more quickly. In fact this picture holds the key to the entire discussion that is to follow. One can, namely, immediately guess that the number of thermal photons or thermal dileptons produced for the richer equation of state would be smaller. One may also witness a unique sensitivity on the extent of the strangeness equilibration from such varied flow profiles of the system. These large differences could also be observed from pion interferometry.

In order to proceed with our discussions we consider central collisions involving lead nuclei at a number of particle rapidity densities. Assuming an isentropic expansion of the plasma we relate the particle rapidity density ($dN/dy$) to the likely initial temperature ($T_i$) as

$$T_i^3 \tau_i = \frac{2\pi^4}{45\zeta(3)} \frac{1}{4a_Q \pi R_T^2} \frac{dN}{dy} , \quad (4)$$

where $a_Q = 37\pi^2/90$ for a QGP consisting of $u$ and $d$ quarks and gluons, $R_T$ is the transverse dimension of the system, and we use the canonical value of 1 fm/c for the initial time $\tau_i$. We shall fix the transition temperature at 160 MeV and the freeze-out temperature at 100 MeV.

The photon $p_T$-spectrum is obtained by convoluting the rates for their production with the space-time history of the system as

$$\frac{dN}{d^2p_T dy} = \int \tau d\tau \int d\phi \int d\eta \left[ f_Q g_0 \frac{dN_Q}{d^2xd\eta} + (1 - f_Q) g_0 \frac{dN^H}{d^2xd\eta} \right] \quad (5)$$

where the function $f_Q(r, \tau)$ denotes the fraction of the quark gluon plasma in the system. The rates in the quark and the hadronic matter are evaluated according to methods developed earlier. Thus, we have included the Compton and the annihilation contributions from the quark matter, and the reactions involving $\pi$, $\rho$, $\omega$ and $\eta$ mesons, along with the $\pi\rho \rightarrow a_1 \rightarrow \pi\gamma$ reaction in the hadronic matter.

We have shown the photon multiplicities as a function of the multiplicity of the charged particles in Fig. 2. The slopes of the transverse momentum distributions for the two cases were found to be quite similar and are not shown here. We see that the “simplified equation of state” for the hadronic matter leads to about twice as many photons. This difference is likely to increase if the transition temperature is higher than 160 MeV, which we have used, as then the difference between the equations of state will become even more magnified.

We have also marked the likely locations of the expected values for collisions at SPS, RHIC and LHC energies using estimates for the particle multiplicities from Ref. [14]. It is quite clear that the availability of these results will go a long way in fixing the equation of state for the hadronic matter. From these results we can easily see that the thermal photon multiplicity is given by

$$\frac{dN_\gamma}{dy} \simeq K \left( \frac{dN_{ch}}{dy} \right)^\alpha , \quad (6)$$

where $\alpha \simeq 1.2$ and $K$ depends on the equation of state.

In Fig. 3 we have repeated this analysis for the production of thermal dileptons, using the procedures discussed in Ref. [1], for dileptons having invariant mass $M$ between 0.5 and 1 GeV. We have included the most dominant contribution of quark annihilation in the QGP and the pion annihilation proceeding via the formation of a $\rho$ meson in the hadronic matter. We again find that twice as many thermal dileptons are produced for the simplified equation of state (see also Ref. [1]) and a scaling similar to Eq. (3) with $\alpha \simeq 1.1$ is satisfied. Apparently the slightly lower value of $\alpha$ in this case could be related to
the fact that virtual photons carrying an invariant mass, are differently affected by the transverse flow.

At first this slower increase of the number of thermal photons with the charged particle rapidity density looks very surprising, as it is generally believed that this dependence should be nearly quadratic for thermal sources [15]. It is not clear as to how this belief got currency, as it was pointed out by Feinberg [16] two decades ago that the number of thermal photons or dileptons should scale as $N_{ch}^{3/2}$; from very general arguments. In view of the importance attached to this scaling behavior for identifying new sources of photons and dileptons it is necessary to understand it more clearly. Consider a system consisting of $N_{ch}$ charged particles. The number of thermal photons $N_\gamma$ will then be given by

$$N_\gamma \sim e^2 N_{ch} \nu$$

(7)

where $\nu$ is the number of collisions that each particle suffers. If the system lives long enough, as when it is confined to a box, every particle will have a chance to collide with every other particle, and then $\nu \sim N_{ch}$. This will lead to the quadratic dependence suggested earlier [3]. However, the number of collisions suffered by the particles will be given by $R/\lambda$, where $R$ is the size and $\lambda$ is the mean free path of the particles, for a system created in heavy ion collisions [7]. Realizing that the number of particles will scale as $R^3$, we immediately get the scaling envisaged by Feinberg (see Ref. [10] for arguments when the system may be undergoing expansion, which might bring in an additional factor of $\sqrt{\ln(N_{ch})}$). We also realize that the transverse expansion of the system drastically reduces the time scales in the system [3]. In the absence of the transverse expansion the life-time of the system can become extremely large; for example it will be several thousand fm/$c$, for the case discussed in Fig. 1 (see table I in Ref. [3]). This would then mimic the case of particles contained in a box, and lead to the quadratic scaling reported by authors of Ref. [3].

Of course the transverse flow will considerably enhance the production of photons having large transverse momenta, which however contribute only marginally to the total number. This is quite a sobering thought, as it suggests that it should be more difficult to disentangle thermal production from the background as the total yields in both cases show a very similar dependence on the final state multiplicity. The $p_T$ distribution of the background and the thermal photons will thus have to play a decisive role in identifying the latter.

If one could assume that the minimal extension of the Bjorken hydrodynamics invoked by a number of authors [3] to evaluate the production of thermal photons and dileptons at non-central rapidities is reliable, then one could also easily distinguish between different equations of state by comparing the results at different rapidities. It will be interesting to see whether a more detailed hydrodynamic model of the collision providing a fuller description at all rapidities [3] can continue to support this distinction between the two equations of state.

It is interesting to note that the excess dileptons observed at SPS energies for sulphur induced collisions by the CERES and HELIOS experiments [10-11], which are measured at different rapidities, scale as $(dN_{ch}/dy)^\beta$, with $\beta > 1$. The statistics, however, is still not sufficient to distinguish the hadronic equation of state.

In summary, we have evaluated the radiation of thermal photons and dileptons from relativistic heavy ion collisions for a range of multiplicities. Assuming the formation of a quark gluon plasma in the initial state, and properly accounting for the transverse expansion, we find that thermal production exhibits a much weaker than quadratic dependence on the final state multiplicity normally assumed. It is also shown that the results are quite sensitive to the equation of state used to describe the hadronic matter. In order to distinguish and fix the equation of state we suggest to utilize the scaling rule of Eq.(3), reasonably valid for both real and virtual photons, as the proportionality constant is determined by the equation of state. We are investigating whether this constant of proportionality is also a measure of the time-scale in the system Ref. [10].

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FIG. 1. The space-time boundaries for the QGP phase, the mixed phase and the hadronic phase at LHC energies. The equation of state for the hadronic matter consists of only $\pi$, $\rho$, $\omega$, and $\eta$ mesons (solid curves) and of full list of hadrons in the Particle Data Book (dashed curves).

FIG. 2. Variation of rapidity density of thermal photons with the charge particle rapidity density for the two equations of state for the hadronic matter.

FIG. 3. Variation of rapidity density of thermal dileptons with the charge particle rapidity density for the two equations of state for the hadronic matter, having $0.5 < M < 1$ GeV.