The influence of confined acoustic phonon on the Quantum Ettingshausen effect in cylindrical quantum wire with an infinite potential in presence of strong electromagnetic wave

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Abstract. Based on the quantum kinetic equation method, the quantum Ettingshausen effect has been theoretically studied under the influence of confined acoustic phonon in a cylindrical quantum wire (CQW) with infinite potential in the presence of a strong electromagnetic wave. We considered a quantum wire in the presence of a constant electric field, a magnetic field, an electromagnetic wave (EMW) with an assumption that electron – confined acoustic phonon (CAP) scattering is essential. Analytical results obtained show that the EC depends on the amplitude and the frequency of the EMW in a non-linear way. Moreover, the impact of phonon confinement on the above effect characterized by \( m \)-quantum number in the expression of the EC. The theoretical results have been numerically calculated for the GaAS/AlGaAs cylindrical quantum wire model. The obtained results show that the phonon confinement contributes to the EC quantitatively and qualitatively. On the other hand, \( m \) is set to zero, the result obtained is similar to the case of unconfined phonon. Furthermore, by considering the quantum size effect, the values of the EC increases, the position of the magnetic-phonon resonance peak changes, and the number of peak resonant peak increases while the radius of quantum wire declines. These obtained results are different from bulk semiconductor and unconfined phonon case which donates to the theory of the Ettingshausen effect in low-dimensional semiconductor systems.

1. Introduction
In recently years, materials science has got many achievements in researching the properties of nanoscale structures. The published researches shown that due to the decrease in size, the semiconductor systems not only change remarkably in physical properties but also appear new effects namely the size effects. The quantum effects in the low-dimensional systems are very different from the previous work studied on the bulk semiconductor: the Shubnikov–de Haas oscillations observed in rectangular quantum wells of AlGaN/GaN does not depend on the temperature [1]; due to the confinement of electrons in quantum wires, the nonlinear
absorption coefficient of strong electromagnetic wave closely depends on the external field and the temperature of the systems [2]; the relative magneto-resistance decreases with increasing magnetic field in quantum well with parabolic potential [3].

The Ettingshausen effect is a thermo-magnetoelectric phenomenon that affects on the carrier current in materials. By using the Boltzmann classical kinetic equation, the Ettingshausen effect has been studied theoretically in bulk semiconductor [4]. In some low-dimensional semiconductor systems, the problem of the quantum Ettingshausen effect has solved successfully and the obtained results are different from the case of bulk semiconductors [5–9]. The quantum Ettingshausen coefficient oscillates according to the magnetic field in the case of electron-acoustic phonon scattering in two-dimensional compositional superlattice of GaAs/AlGaAs [5]. With increasing frequency of the laser radiation in quantum well, the EC fluctuates and reaches resonance point within phonon confinement [6] and increases in the case of unconfined phonon [7]. In quantum wire, the Ettingshausen coefficient decreases as the temperature increase [8] and the Shubnikov–de Haas oscillations in strong magnetic field domain are appeared [9]. In those aforementioned references, the electromagnetic wave is applied. In cylindrical quantum wire with an infinite potential, the Shubnikov-de Haas (SdH) oscillations are observed when investigating the dependence of EC on the magnetic field [9]. However, the results presented in Ref. [9] are obtained with the assumption that the electron-unconfined optical phonon is essential. On the other hand, the electron-phonon scattering mechanism and the confinement of phonon are well known as the key elements in the behavior of the kinetic effects in low-dimensional semiconductor systems. Therefore, how the quantum Ettingshausen effect occurs in the cylindrical quantum wire under the influence of CAP in the presence of a strong electromagnetic waves are an opened problem needed to be studied.

To study new effects in low-dimensional semiconductor systems, the Kubo-Mori method, the Green function method, the projection operator method, the Boltzmann classical kinetic equation and the quantum kinetics equations are proposed methods. Each method has its own advantages and disadvantages. In this work, the quantum kinetic equation method is used as an effective solution to study the effect of the confined phonons on the quantum Ettingshausen effect in the presence of electromagnetic waves under the low temperature condition because of its advantage of being true for all temperatures.

2. Calculation of the Quantum Ettingshausen coefficient in CQW by using quantum kinetic equation method

We consider a cylindrical quantum wire of radius R and length $L_z$ with the infinite confined potential $V(r) = 0$ inside the wire and $V(r) = \infty$ elsewhere. In this case, the wave function of an electron in cylindrical coordinates $(r, \phi, z)$ now becomes [10]

$$
\psi_{n,l,p_z}(r, \phi, z) = \begin{cases} 
0 & r > R \\
\frac{1}{\sqrt{V_0}} e^{in\phi} e^{ip_z z} \psi_{n,l}(r) & r < R
\end{cases}
$$

(1)

In which $V_0 = \pi R^2 L_z$ is the volume of quantum wire; $p_z = (0, 0, p_z)$ is the wave vector of the electron along the z-direction; $n=0, \pm 1, \pm 2, ...$ is azimuthal quantum number; $l=1,2,3,..$ is radial quantum number; $\psi_{n,l}$ is the Bessel function

$$
\psi_{n,l}(r) = \frac{1}{J_{n+1}(B_{n,l})} J_n(B_{n,l} \frac{r}{R})
$$

where $B_{n,l}$ is the $l^{th}$ root of the Bessel function of order $n$ $J_n(x) = 0$.
The cylindrical quantum wire is subjected to a crossed constant electric field $E_1 = (0, 0, E_1)$ and a magnetic field $B = (0, B, 0)$ in the presence of a strong EMW (laser radiation) $E = (E_0 \sin \Omega t, 0, 0)$ ($E_0$, $\Omega$ are amplitude and frequency of EMW). The energy spectrum of an electron is defined:

$$
\varepsilon_{n,l}(p_z) = \frac{\hbar^2 p_z^2}{2m} + \hbar \omega_H \left( N + \frac{n}{2} + \frac{l}{2} + \frac{1}{2} \right) - \frac{1}{2m} \left( eE_1 \right)^2
$$

(2)

Where $\hbar$ is the abridged Planck constant, $N=0,1,2...$ is the Landau level, $\omega_H = \frac{eB}{mc}$ is the cyclotron frequency.

The Hamiltonian of the confined electron-CAP system in the cylindrical quantum wire with infinite potential (CQWIP) in the second quantization presentation can be written:

$$
H_{e-p}^{(m_1,m_2)} = \sum_{n,l,p_z} \varepsilon_{n,l,p_z} \left( p_z - \frac{e}{\hbar c} A(t) a_{n,l,p_z}^+ a_{n,l,p_z} + \sum_{m_1,m_2,q_z} \hbar \omega_{m_1,m_2,q_z} b_{m_1,m_2,q_z}^+ b_{m_1,m_2,q_z} + \sum_{m_1,m_2,q_z} \varphi(q_z) a_{n',l',p_z+q_z}^+ a_{n,l,p_z} + \sum_{n,l,n',l'} \sum_{m_1,m_2,p_z,q_z} C_{m_1,m_2,q_z} f_{n,l,n',l'}^{m_1,m_2}(q_z) a_{n',l',p_z+q_z}^+ a_{n,l,p_z} \left( b_{m_1,m_2,q_z} + b_{m_1,m_2,-q_z}^+ \right) \right)
$$

(3)

In which $|C_{m_1,m_2,q_z}|^2 = \frac{\hbar E_0}{2 \rho v_s \sqrt{q_z^2 + \omega_{m_1,m_2}^2}}$ is the electron-CAP interaction constant; $\rho$ and $v_s$ are mass density and the velocity of acoustic wave, respectively; $E_0$ is the deformation potential constant; $I_{n,l,n',l'}^{m_1,m_2}(q_{m_1,m_2}) = \frac{2}{R^2} \int_0^R J_{|n-n'|}(q_{m_1,m_2}, R) \psi_{n',l'}^*(r) \varphi_{n',l'}(r) r dr$ is the form factor of electron [10]; $\omega_{m_1,m_2,q_z} = v_s \sqrt{q_z^2 + \omega_{m_1,m_2}^2} = v_s q_{m_1,m_2} [11]$ is the frequency of CAP; $q_{m_1,m_2} = \frac{x_{m_1,m_2}}{m_2} x_{m_1,m_2}$ is the $m_2^{th}$ zero of the $m_1^{th}$ order the Bessel function [12]; $m_1, m_2 = 1, 2, 3...$ being the quantum numbers characterizing phonon confinement; $m$ and $e$ are the effective mass and the effective charge of an electron, respectively; $A(t) = \frac{e}{\Omega} E_0 \sin \Omega t$ is the vector potential of laser field; $\varphi(q_z) = (2\pi i)^3 (eE_1 + \hbar \Omega [q_z, \hbar]) \frac{\partial}{\partial q_z} \delta(q_z)$ is the potential scalar; $a_{n,l,p_z}^+$ and $a_{n,l,p_z}$ ($b_{m_1,m_2,q_z}^+$ and $b_{m_1,m_2,q_z}$) is the creation and annihilation operators of electron (confined phonon), respectively. According to Eq.(3), the Hamiltonian is different form cases of bulk semiconductor and unconfined phonon [9] because of its dependencies on $m_1, m_2$ indices specific the confinement of acoustic phonon.

The quantum kinetic equation of the average number of electron $f_{n,l,p_z} = <a_{n,l,p_z}^+ a_{n,l,p_z} > t$ is:

$$
i\hbar \frac{\partial f_{n,l,p_z}(t)}{\partial t} = \left< \left[ a_{n,l,p_z}^+ a_{n,l,p_z}, H_{e-p}^{(m_1,m_2)} \right] \right>_t
$$

(4)
By combining Eq.(3) and Eq.(4), we get the quantum kinetic equation:

\[
\frac{\partial f_{n,l,p_z}}{\partial t} + \left( \frac{eE_1}{\hbar} + \omega_H \left[ p_z, \hbar \right] \right) \frac{\partial f_{n,l,p_z}}{\partial p_z} = \frac{2\pi}{\hbar} \sum_{n,l,n',l'} \left| C_{m_1,m_2,q_z} f_{n,l,n',l'} \right|^2 \times \sum_{i=-\infty}^{\infty} J_i^2 \left( \frac{eE_0}{m\Omega} q_z \right)^2 \left[ f_{n',l',p_z+q_z} (N_{m_1,m_2,q_z} + 1) - f_{n,l,p_z} N_{m_1,m_2,q_z} \right] \times \delta \left( \varepsilon_{n',l',p_z+q_z} - \varepsilon_{n,l,p_z} - \hbar \omega_{m_1,m_2,q_z} - l\Omega \right) + \left[ f_{n',l',p_z-q_z} N_{m_1,m_2,q_z} - f_{n,l,p_z} (N_{m_1,m_2,q_z} + 1) \right] \times \delta \left( \varepsilon_{n',l',p_z-q_z} - \varepsilon_{n,l,p_z} + \hbar \omega_{m_1,m_2,q_z} - l\Omega \right) \right] \right) \right) \right)
\]

(5)

Where \( \mathbf{h} = \frac{\mathbf{H}}{H} \) is the unit vector in the direction of magnetic field, \( f_{n,l,p_z} \) is the unknown distribution function perturbed due to an external field; \( N_{m_1,m_2,q_z} = \frac{k_B T}{\hbar \nu_s \sqrt{q_z^2 + q_{m_1,m_2}^2}} \) is the equilibrium phonon distribution function, where \( k_B \) is the Boltzmann constant and \( T \) is temperature, respectively; \( J_l(x) \) is the \( l \)-th order the Bessel function of the argument \( x \); \( \delta(x) \) is the Dirac delta function.

Eq.(5) is the quantum kinetic equation for electrons and being used to derive analytical expressions of the kinetic tensors as well as EC in CQWIP. For simplicity, we limit the problem to case of \( l = 0, \pm 1 \). We multiply both sides of Eq.(5) by \( (e/m)p_z \delta(\varepsilon - \varepsilon_{n,l,p_z}) \) and if the temperature is low enough, the electron is considered degenerated and the distribution function takes in form of Heaviside function. So we get the following equation:

\[
\frac{\mathbf{R}(\varepsilon, m_1, m_2)}{\tau(\varepsilon)} + \omega_H \left[ \mathbf{h}, \mathbf{R}(\varepsilon, m_1, m_2) \right] = \mathbf{Q}(\varepsilon) + \mathbf{S}(\varepsilon, m_1, m_2)
\]

(6)

with

\[
\mathbf{Q}(\varepsilon) = -\frac{e}{m} \sum_{p_z} \left( \mathbf{F}, \frac{\partial f_{n,l,p_z}}{\partial p_z} \right) \delta(\varepsilon - \varepsilon_{n,l}(p_z))
\]

(7)

here \( \mathbf{F} = eE_1 - \frac{\varepsilon - \varepsilon_F}{T} \gamma T \) \( (\varepsilon_F \) is the Femi level of the electron)

\[
\mathbf{S}(\varepsilon, m_1, m_2) = \frac{2\pi e}{m} \frac{k_B T}{\hbar \nu_s \sqrt{q_z^2 + q_{m_1,m_2}^2}} \sum_{n,l,n',l'} \sum_{n,l,n',l'} \left| C_{m_1,m_2,q_z} f_{n,l,n',l'} \right|^2 \times \left( f_{n',l',p_z+q_z} - f_{n,l,p_z} \right) \left\{ \left( 1 - \frac{a^2}{2} \left( q_{m_1,m_2}^2 + q_z^2 \right) \right) \delta(\varepsilon_{n',l',p_z+q_z} - \varepsilon_{n,l,p_z} + \hbar \omega_{m_1,m_2,q_z} - l\Omega) + \delta(\varepsilon_{n',l',p_z+q_z} - \varepsilon_{n,l,p_z} + \hbar \omega_{m_1,m_2,q_z} + l\Omega) \right\}
\]

(8)

with \( a = \frac{eE_0}{m\Omega} \)

Solving the differential Eq.(6) we obtain the analytical expression of \( \mathbf{R}(\varepsilon, m_1, m_2) \) is a function
of $Q(\epsilon)$ and $S(\epsilon, m_1, m_2)$ (main variable is $\epsilon$ energy level)

$$R(\epsilon, m_1, m_2) = \frac{\tau(\epsilon)}{1 + \omega_H \tau^2(\epsilon)} \left\{ Q(\epsilon) + S(\epsilon, m_1, m_2) - \omega_c \tau(\epsilon) \left[ h, Q(\epsilon) + S(\epsilon, m_1, m_2) \right] \right\}$$

$+ S(\epsilon, m_1, m_2) + \omega^2 \tau^2(\epsilon) h \left( h, Q(\epsilon) + S(\epsilon, m_1, m_2) \right) \right\} \right\}$ \hfill (9)

In Eq.(9), the $R(\epsilon, m_1, m_2)$ function has the meaning of partial current density caused by electrons with $\epsilon$ energy level.

The current density $J$ and the thermal flux density $q_e$ are given by

$$J(m_1, m_2) = \int_0^\infty R(\epsilon, m_1, m_2) d\epsilon$$

$\quad = \int_0^\infty (\epsilon - \epsilon_F) R(\epsilon, m_1, m_2) d\epsilon$ \hfill (10)

We also have:

$$J_{ik}(m_1, m_2) = \sigma_{ik}(m_1, m_2) E_k + \beta_{ik}(m_1, m_2) \nabla T_k$$

$$q_i(m_1, m_2) = \gamma_{ik}(m_1, m_2) E_k + \xi_{ik}(m_1, m_2) \nabla T_k$$

From formulas of the current density and the thermal flux density, we obtained the expressions for conductivity tensors and other kinetic tensors

$$\sigma_{xy}(m_1, m_2) = \frac{a \tau(\epsilon_F)}{1 + \omega_H \tau^2(\epsilon)} + b_T \sum_{n,m} \sum_{n',m} \left| \Gamma_{n,m,2}^{m_1} \right|^2 A \frac{\tau^2(\epsilon_F + h \omega_{m_1,m_2}, q_e)}{[1 + \omega_H \tau^2(\epsilon_F + h \omega_{m_1,m_2})]^2} \times \left[ 1 - \omega^2 \tau^2(\epsilon_F + h \omega_{m_1,m_2}, q_e) - h \Omega \right]$$

$$\times \left\{ B \frac{\tau^2(\epsilon_F + h \omega_{m_1,m_2}, q_e) - h \Omega}{[1 + \omega_H \tau^2(\epsilon_F + h \omega_{m_1,m_2})]^2} \left[ 1 - \omega^2 \tau^2(\epsilon_F + h \omega_{m_1,m_2}, q_e) - h \Omega \right] \right\}$$

$$\times \left\{ C \frac{\tau^2(\epsilon_F + h \omega_{m_1,m_2}, q_e) + h \Omega}{[1 + \omega_H \tau^2(\epsilon_F + h \omega_{m_1,m_2})]^2} \left[ 1 - \omega^2 \tau^2(\epsilon_F + h \omega_{m_1,m_2}, q_e) + h \Omega \right] \right\}$$

$$\sigma_{xy}(m_1, m_2) = \frac{a \tau(\epsilon_F)}{1 + \omega_H \tau^2(\epsilon)} + b_T \sum_{n,m} \sum_{n',m} \left| \Gamma_{n,m,2}^{m_1} \right|^2 A \frac{\tau^2(\epsilon_F + h \omega_{m_1,m_2}, q_e)}{[1 + \omega_H \tau^2(\epsilon_F + h \omega_{m_1,m_2})]^2} \times \left[ -2 \omega_H \tau(\epsilon_F + h \omega_{m_1,m_2}, q_e) \right]$$

$$\times \left\{ B \frac{\tau^2(\epsilon_F + h \omega_{m_1,m_2}, q_e) - h \Omega}{[1 + \omega_H \tau^2(\epsilon_F + h \omega_{m_1,m_2})]^2} \left[ -2 \omega_H \tau(\epsilon_F + h \omega_{m_1,m_2}, q_e) - h \Omega \right] \right\}$$

$$\times \left\{ C \frac{\tau^2(\epsilon_F + h \omega_{m_1,m_2}, q_e) + h \Omega}{[1 + \omega_H \tau^2(\epsilon_F + h \omega_{m_1,m_2})]^2} \left[ -2 \omega_H \tau(\epsilon_F + h \omega_{m_1,m_2}, q_e) + h \Omega \right] \right\}$$

$$\times \left\{ D \frac{\tau^2(\epsilon_F + h \omega_{m_1,m_2}, q_e) + h \Omega}{[1 + \omega_H \tau^2(\epsilon_F + h \omega_{m_1,m_2})]^2} \left[ -2 \omega_H \tau(\epsilon_F + h \omega_{m_1,m_2}, q_e) + h \Omega \right] \right\}$$

$$\times \left\{ E \frac{\tau^2(\epsilon_F + h \omega_{m_1,m_2}, q_e) - h \Omega}{[1 + \omega_H \tau^2(\epsilon_F + h \omega_{m_1,m_2})]^2} \left[ -2 \omega_H \tau(\epsilon_F + h \omega_{m_1,m_2}, q_e) - h \Omega \right] \right\}$$

$$\times \left\{ F \frac{\tau^2(\epsilon_F + h \omega_{m_1,m_2}, q_e) + h \Omega}{[1 + \omega_H \tau^2(\epsilon_F + h \omega_{m_1,m_2})]^2} \left[ -2 \omega_H \tau(\epsilon_F + h \omega_{m_1,m_2}, q_e) + h \Omega \right] \right\}$$
\[ \beta_{xx}(m_1, m_2) = - \frac{b_T}{m T} \sum_{n,l,n',l'} \sum_{m_1,m_2} |I_{n,l,n',l'}^{m_1,m_2}|^2 A(h \omega_{m_1,m_2}, q) \frac{\tau^2(\varepsilon_F + h \omega_{m_1,m_2}, q)}{[1 + \omega H \tau^2(\varepsilon_F + h \omega_{m_1,m_2}, q)]^2} \times \]

\[ [1 - \omega_H^2 \tau^2(\varepsilon_F + h \omega_{m_1,m_2}, q_s)] - \frac{c_T}{m T} \sum_{n,l,n',l'} \sum_{m_1,m_2} |I_{n,l,n',l'}^{m_1,m_2}|^2 \]

\[ \times \{ B(h \omega_{m_1,m_2}, q_s - h \Omega) \frac{\tau^2(\varepsilon_F + h \omega_{m_1,m_2}, q_s - h \Omega)}{[1 + \omega H \tau^2(\varepsilon_F + h \omega_{m_1,m_2}, q_s - h \Omega)]^2} \times [1 - \omega_H^2 \tau^2(\varepsilon_F + h \omega_{m_1,m_2}, q_s - h \Omega)] \}

\[ \gamma_{xx}(m_1, m_2) = - T \beta_{xx}(m_1, m_2) \]

\[ \gamma_{xy}(m_1, m_2) = \frac{1}{m} \left\{ b_T \sum_{n,l,n',l'} \sum_{m_1,m_2} |I_{n,l,n',l'}^{m_1,m_2}|^2 A(h \omega_{m_1,m_2}, q) \frac{\tau^2(\varepsilon_F + h \omega_{m_1,m_2}, q)}{[1 + \omega H \tau^2(\varepsilon_F + h \omega_{m_1,m_2}, q)]^2} \times \right. \]

\[ \times [1 - \omega_H^2 \tau^2(\varepsilon_F + h \omega_{m_1,m_2}, q_s - h \Omega)] \] \[ \left. \{ B(h \omega_{m_1,m_2}, q_s - h \Omega) \frac{\tau^2(\varepsilon_F + h \omega_{m_1,m_2}, q_s - h \Omega)}{[1 + \omega H \tau^2(\varepsilon_F + h \omega_{m_1,m_2}, q_s - h \Omega)]^2} \times \right. \]

\[ \left. [1 - \omega_H^2 \tau^2(\varepsilon_F + h \omega_{m_1,m_2}, q_s - h \Omega)] \right\} \]

\[ \xi_{xx}(m_1, m_2) = - \frac{1}{em T} \left\{ b_T \sum_{n,l,n',l'} \sum_{m_1,m_2} |I_{n,l,n',l'}^{m_1,m_2}|^2 A(h \omega_{m_1,m_2}, q) \frac{\tau^2(\varepsilon_F + h \omega_{m_1,m_2}, q)}{[1 + \omega H \tau^2(\varepsilon_F + h \omega_{m_1,m_2}, q)]^2} \times \right. \]

\[ \times [1 - \omega_H^2 \tau^2(\varepsilon_F + h \omega_{m_1,m_2}, q_s - h \Omega)] \] \[ \left. \{ B(h \omega_{m_1,m_2}, q_s - h \Omega) \frac{\tau^2(\varepsilon_F + h \omega_{m_1,m_2}, q_s - h \Omega)}{[1 + \omega H \tau^2(\varepsilon_F + h \omega_{m_1,m_2}, q_s - h \Omega)]^2} \times \right. \]

\[ \left. [1 - \omega_H^2 \tau^2(\varepsilon_F + h \omega_{m_1,m_2}, q_s - h \Omega)] \right\} \]

\[ a = \frac{e^2 \hbar LR^2}{8 m^3 \pi} \sum_{n,l} \Delta^{3/2}; \quad b_T = \frac{e k_B T \hbar^2 E_d^2 L}{192 \pi^2 m^3 v_f^2 \rho}; \quad c_T = \frac{\hbar^2 k_B T E_d^2 E_0^2 L}{768 m^3 v_f^2 \rho D^2} \]

\[ A = A_1(m_1, m_2) + A_2(m_1, m_2); \quad B = A_3(m_1, m_2) + A_4(m_1, m_2); \]

\[ C = A_5(m_1, m_2) + A_6(m_1, m_2); \]
\[ \begin{align*}
A_1(m_1, m_2) &= -\frac{x_1^2}{\sqrt{\Delta_{n,l} \Delta_{11}}} \left( c_1 + d_1 - \frac{e^2 E_0^2}{m^2 \Omega t} q_{m_1,m_2}^2 - \frac{e^2 E_0^2}{2m^2 \Omega^4} (c_1^2 + d_1^2) \right) \\
&- \frac{x_2^2}{\sqrt{\Delta_{n,l} \Delta_{12}}} \left( c_2 + d_2 - \frac{e^2 E_0^2}{m^2 \Omega t} q_{m_1,m_2}^2 - \frac{e^2 E_0^2}{2m^2 \Omega^4} (c_2^2 + d_2^2) \right) \\
A_2(m_1, m_2) &= \frac{y_1}{\sqrt{\Delta_{n',l'} \Delta_{11}}} \left[ (y_1 - C_1) (C_1 - \frac{e^2 E_0^2}{m^2 \Omega t} q_{m_1,m_2}^2 + C_2^2) + (y_1 - D_1) \right] \\
&+ \frac{y_2}{\sqrt{\Delta_{n',l'} \Delta_{12}}} \left[ (y_2 - C_2) (C_2 - \frac{e^2 E_0^2}{m^2 \Omega t} q_{m_1,m_2}^2 + D_2^2) \right] \\
A_3(m_1, m_2) &= \frac{x_1^2}{\sqrt{\Delta_{n,l} \Delta_{21}}} \left[ g_1^3 + h_1^3 + q_{m_1,m_2}^2 (g_1 + h_1) \right] \\
&+ \frac{x_2^2}{\sqrt{\Delta_{n,l} \Delta_{22}}} \left[ g_2^3 + h_2^3 + q_{m_1,m_2}^2 (g_2 + h_2) \right] \\
A_4(m_1, m_2) &= \frac{x_1^2}{\sqrt{\Delta_{n',l'} \Delta_{11}}} \left[ (y_1 - G_1) (G_1^3 + q_{m_1,m_2}^2 G_1) + (y_1 - H_1) (H_1^3 + q_{m_1,m_2}^2 H_1) \right] \\
&+ \frac{y_2}{\sqrt{\Delta_{n',l'} \Delta_{12}}} \left[ (y_2 - G_2) (G_2^3 + q_{m_1,m_2}^2 G_2) + (y_1 - H_2) (H_2^3 + q_{m_1,m_2}^2 H_2) \right] \\
A_5(m_1, m_2) &= \frac{x_1^2}{\sqrt{\Delta_{n,l} \Delta_{31}}} \left[ v_1^3 + t_1^3 + q_{m_1,m_2}^2 (v_1 + t_1) \right] \\
&+ \frac{x_2^2}{\sqrt{\Delta_{n,l} \Delta_{32}}} \left[ v_2^3 + t_2^3 + q_{m_1,m_2}^2 (v_2 + t_2) \right] \\
A_6(m_1, m_2) &= \frac{x_1^2}{\sqrt{\Delta_{n',l'} \Delta_{11}}} \left[ (y_1 - V_1) (V_1^3 + q_{m_1,m_2}^2 V_1) + (y_1 - T_1) (T_1^3 + q_{m_1,m_2}^2 T_1) \right] \\
&+ \frac{y_2}{\sqrt{\Delta_{n',l'} \Delta_{12}}} \left[ (y_2 - V_2) (V_2^3 + q_{m_1,m_2}^2 V_2) + (y_1 - T_2) (T_2^3 + q_{m_1,m_2}^2 T_2) \right] \\
\Delta_{n,l} &= \frac{1}{h^2} \left( \frac{e E_1}{\omega_c} \right)^2 + \frac{2m}{h^2} \varepsilon_F - \frac{2 \omega_H m}{h} \left( N + \frac{n}{2} + \frac{l}{2} + \frac{1}{2} \right); x_1 = \sqrt{\Delta_{n,l}}; x_2 = -\sqrt{\Delta_{n,l}} \\
\Delta_{n',l'} &= \frac{2m}{h^2} \left( \varepsilon_F + \frac{1}{2m} \left( \frac{e E_1}{\omega_H} \right)^2 - \hbar \omega_H \left( N + \frac{n'}{2} + \frac{l'}{2} + \frac{1}{2} \right) \right); y_1 = \sqrt{\Delta_{n',l'}}; y_2 = -\sqrt{\Delta_{n',l'}} \\
\Delta_{11} &= x_1^2 - \frac{2m}{h} \left[ \left( \frac{n - n'}{2} + \frac{l - l'}{2} \right) \omega_H + v_s q_{m_1,m_2} \right]; \\
c_1 &= -x_1 + \sqrt{\Delta_{11}}; d_1 = -x_1 - \sqrt{\Delta_{11}} \\
\Delta_{12} &= x_2^2 - \frac{2m}{h} \left[ \left( \frac{n - n'}{2} + \frac{l - l'}{2} \right) \omega_H + v_s q_{m_1,m_2} \right]; \\
c_2 &= -x_2 + \sqrt{\Delta_{12}}; d_2 = -x_2 - \sqrt{\Delta_{12}} \\
\Delta_{11} &= y_1^2 + \frac{2m}{h} \left[ \left( \frac{n - n'}{2} + \frac{l - l'}{2} \right) \omega_H + v_s q_{m_1,m_2} \right]; \\
C_1 &= -y_1 + \sqrt{\Delta_{11}}; D_1 = -y_1 - \sqrt{\Delta_{11}} \\
\right. \]
\[ \Delta_{I2} = y_2^2 + \frac{2m}{\hbar} \left[ \left( \frac{n' - n}{2} + \frac{l' - l}{2} \right) \omega_H + \nu_{s, \xi(m_1, m_2)} \right] ; \]

\[ C_2 = -y_2 + \sqrt{\Delta_{I2}} ; D_2 = -y_2 - \sqrt{\Delta_{I2}} \]

\[ \Delta_{21} = x_1^2 - \frac{2m}{\hbar} \left[ \left( \frac{n' - n}{2} + \frac{l' - l}{2} \right) \omega_H + \nu_{s, \xi(m_1, m_2)} - \Omega \right] ; \]

\[ g_1 = -x_1 + \sqrt{\Delta_{21}} ; h_1 = -x_1 - \sqrt{\Delta_{21}} \]

\[ \Delta_{22} = x_2^2 - \frac{2m}{\hbar} \left[ \left( \frac{n' - n}{2} + \frac{l' - l}{2} \right) \omega_H + \nu_{s, \xi(m_1, m_2)} - \Omega \right] ; \]

\[ g_2 = -x_2 + \sqrt{\Delta_{22}} ; h_2 = -x_2 - \sqrt{\Delta_{22}} \]

\[ \Delta_{I11} = y_1^2 + \frac{2m}{\hbar} \left[ \left( \frac{n' - n}{2} + \frac{l' - l}{2} \right) \omega_H + \nu_{s, \xi(m_1, m_2)} - \Omega \right] ; \]

\[ G_1 = -y_1 + \sqrt{\Delta_{I11}} ; H_1 = -y_1 - \sqrt{\Delta_{I11}} \]

\[ \Delta_{I12} = y_2^2 + \frac{2m}{\hbar} \left[ \left( \frac{n' - n}{2} + \frac{l' - l}{2} \right) \omega_H + \nu_{s, \xi(m_1, m_2)} - \Omega \right] ; \]

\[ G_2 = -y_2 + \sqrt{\Delta_{I12}} ; H_2 = -y_2 - \sqrt{\Delta_{I12}} \]

\[ \Delta_{31} = x_1^2 - \frac{2m}{\hbar} \left[ \left( \frac{n' - n}{2} + \frac{l' - l}{2} \right) \omega_H + \nu_{s, \xi(m_1, m_2)} + \Omega \right] ; \]

\[ v_1 = -x_1 + \sqrt{\Delta_{31}} ; t_1 = -x_1 - \sqrt{\Delta_{31}} \]

\[ \Delta_{32} = x_2^2 - \frac{2m}{\hbar} \left[ \left( \frac{n' - n}{2} + \frac{l' - l}{2} \right) \omega_H + \nu_{s, \xi(m_1, m_2)} + \Omega \right] ; \]

\[ v_2 = -x_2 + \sqrt{\Delta_{32}} ; t_2 = -x_2 - \sqrt{\Delta_{32}} \]

\[ \Delta_{III1} = y_1^2 + \frac{2m}{\hbar} \left[ \left( \frac{n' - n}{2} + \frac{l' - l}{2} \right) \omega_H + \nu_{s, \xi(m_1, m_2)} + \Omega \right] ; \]

\[ V_1 = -y_1 + \sqrt{\Delta_{III1}} ; T_1 = -y_1 - \sqrt{\Delta_{III1}} \]

\[ \Delta_{III2} = y_2^2 + \frac{2m}{\hbar} \left[ \left( \frac{n' - n}{2} + \frac{l' - l}{2} \right) \omega_H + \nu_{s, \xi(m_1, m_2)} + \Omega \right] ; \]

\[ V_2 = -y_2 + \sqrt{\Delta_{III2}} ; T_2 = -y_2 - \sqrt{\Delta_{III2}} \]

with \( \tau = \tau(\varepsilon_F) \) being the momentum relaxation time.

The analytical expression of the EC is as follows:

\[
P(m_1, m_2) = \frac{1}{H} \frac{\sigma_{xx}(m_1, m_2)\gamma_{yy}(m_1, m_2) - \sigma_{yy}(m_1, m_2)\gamma_{xx}(m_1, m_2)}{\beta_{xx}(m_1, m_2)\gamma_{yy}(m_1, m_2) - \sigma_{xx}(m_1, m_2)(\xi_{xx}(m_1, m_2) - K_L)}
\]

(12)

Here \( K_L \) is lattice heat conductivity [13]. Similar to quantum wells [14], in quantum wire the value \( K_L \) is so small that it can be ignored for simplicity.

According to the Eq.(12), the ECs expression in CQWIP is much different from the bulk semiconductor [4], the unconfined phonon case in the same material [9] and the analytical results obtained in doped superlattice [15]. These differences are explained as a result of the change in the wave function and energy spectrum of the electron, the wave vector and frequency of the phonon and the confined potential in these systems. The EC depends heavily on quantities such as frequency and amplitude of the EMW, the temperature, the magnetic field, the quantities characteristic of the structure of the CQWIP including quantum wire length and radius. In addition, the EC depends on readings of the CAP in a complicated way. When the limit indices \( (m_1, m_2) \) are set to zero, the obtained results coincides with the unconfined phonon case. Next step, we study the quantum wire of GaAs/AlGaAs to clarify the above dependence clearly.
3. Numerical results and discussion
To clarify the obtained analytical results, in this section, we numerically calculate the quantum Ettingshausen coefficient in a cylindrical quantum wire subjected to the uniform crossed a magnetic field and a constant electric field in the presence of a strong EMW. For the numerical evaluation, we consider the cylindrical quantum wire of GaAs/AlGaAs with the parameters in table 1 [16,17].

Figure 1 gives information about the EC’s dependence on cyclotron energy in two cases of CAP (the red line) and unconfined acoustic phonon (the blue line) with $E_0 = 10^4(V/m)$. The graph is immediately obvious that the oscillation has appeared. The impact of confined acoustic phonon and electromagnetic waves on the EC is clearly observed. At the low temperature, in the domain of weak ($\leq 0.3.10^{-3}$) and strong ($\geq 0.5.10^{-3}$) cyclotron energy, the value of the EC both cases remained relatively unchanged. On the other hand, in the domain between these two-cyclotron energy values, the EC fluctuated wildly and reaches the resonance point while $\hbar\omega_H$ is just around 0.00036 (eV) with electromagnetic wave frequency $\Omega = 5.52.10^{11}(Hz)$. The peaks of the red line (with CAP) are not only higher but also experienced almost no changes from the peaks of the blue line (with unconfined AP). These results can be explained as follows: both CAP’s energy and interaction constant of electron-phonon systems depend on the quantum numbers $m_1,m_2$; the frequency of CAP is defined: $\omega_{m_1,m_2,q_z} = v\sqrt{q^2_{m_1,m_2} + q_z^2} = vq_{m_1,m_2}$. Therefore, the resonance condition in CQWIP changes with displacing values of the quantum numbers $m_1,m_2$ and makes the resonance peaks to climb.

Figure 2 illustrates the dependence of EC on EMW frequency in two cases with and without confined phonons with $E_0 = 10^5(V/m)$. Fig.2 indicates that two cases of acoustic phonon experienced a violent oscillation in the domain EMW frequency $\Omega = 5.2.10^{11} (+)$, it has many resonant peaks appearing in this electromagnetic wave frequency, the unconfined acoustic phonon (the blue line) experienced a stead oscillation whereas there was a wild fluctuation in CAP (the red line) and EC’s value tends to increase with large electromagnetic wave frequency. Furthermore, the presence of confined phonons and strong electromagnetic wave displaces peak resonance (red line), which was a characteristic that transforms EC in this work.

Figure 3 shows the dependence of the EC on temperature with $E_0 = 10^4(V/m)$. In the cases the unconfined phonon the EC depends on the temperature in a non-linear way like in the same in rectangular quantum wire [8] whereas the confined phonon fluctuates steadily and the EC approaches a negative constant. The obtained results consistent with the result of published study on the Ettingshausen effect in DSS [15]: the phonon confinement is the main cause of the

| Quantity | Symbol | Unit | Value |
|----------|--------|------|-------|
| Bolzmann constant | $k_B$ | J/K | $1.3807.10^{-23}$ |
| Planck constant | $h$ | Js | $1.05459.10^{-34}$ |
| Electron charge | $e$ | C | $1.60219.10^{-19}$ |
| Effect mass of electron | $m$ | kg | $0.067.9.1095.10^{-31}$ |
| Normalized radius | $R$ | nm | 15.6 |
| Femi energy | $\varepsilon_F$ | meV | 50 |
| Crystal density | $\rho$ | kg/m$^3$ | 5320 |
| Acoustic wave velocity | $v_a$ | m/s | 5370 |
| Momentum recovery time | $\tau$ | s | $10^{-12}$ |
| Potential deformation constant | $E_d$ | eV | 13.5 |
enhanced and stable EC in comparison with the unconfined phonon case. In contrast to the result presented in this work, the EC in DSS levelled off at a positive constant when the temperature is greater than 5 K [15]. This difference can be explained that the difference in structure, the wave function, the energy spectrum of the electron and the electron-phonon scattering between DSS and CQWIP lead to differences in analytical results.
4. Conclusion
We analytically investigated the quantum Ettingshausen effect in CQWIP under the influence of the confined phonon and the presence of strong electromagnetic waves. The electron - confined acoustic phonon scattering and the low temperatures and degenerated electron gas are taken into account. We noticed that the EC depends on quantities such as temperature, cyclotron energy, frequency and amplitude of the EMW, the radius of cylindrical quantum wire and $m_1, m_2$ quantum number $m$ - specific the confinement of phonon, respectively.

We estimated numerical values and graphs for a GaAs/AlGaAs quantum wire to show the dependencies clearly. Studying the dependencies of the EC on cyclotron energy, the frequency of the EMW under low-temperature conditions, we found that the phonon confinement displaces the resonance peaks and increases the value of the EC. In particular, the presence of electromagnetic waves causes the change of the amplitude of vibration considerably, the resonance peaks oscillate strongly around the domain of EMW frequency $\Omega = 5.2 \times 10^{11} \div 6.1 \times 10^{11}\text{ (Hz)}$. Investigating the dependence of the EC on temperature, we discovered that in the cases the confined phonon the EC depends on the temperature in a non-linear whereas the unconfined phonon fluctuates steadily. These results are very different from those of bulk semiconductors and 2-dimensional systems. We also get results consistent with the case of unconfined AP when quantum numbers $m_1, m_2$ disappear. Thus, the phonon confinement causes remarkable changes to the Ettingshausen effect in quantum wire with infinite potential and can not be ignored.

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