Finite-Time Average Consensus Control of Multi-Agent Systems Based on the Aperiodically Intermittent Control

YIPING LUO and JUNLING ZHU
Hunan Institute of Engineering, Xiangtan 411228, China
Corresponding author: Yiping Luo (lyp@hnie.edu.cn)

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ABSTRACT This study investigates the finite-time consensus problem of multi-agent systems under an aperiodically intermittent control. By designing an appropriate control protocol, the whole control process can be divided into two states, namely, the continuous communication time period and the interrupted communication time period. These two control states are aperiodic. Then, according to the designed control protocol, the appropriate Lyapunov function is constructed using the segmented state information. Subsequently, the corresponding state model of the multi-agent system is introduced, and the relevant conditions for the consensus of the multi-agent system under an aperiodically intermittent control are obtained using norm inequality and the corresponding properties of the Kronecker product. After determining the consensus of the general model, the nonlinear term and uncertainty term are added, and the consensus is realized using the control protocol. The control method presented in this research can avoid continuous communication, and the aperiodic control process is more in line with the relevant needs of the actual project than the periodic control process. As a result, the proposed method can effectively reduce energy consumption and fit actual projects. Finally, the effectiveness of the theory is proven via numerical simulation.

INDEX TERMS Aperiodically intermittent control, finite-time, consensus, multi-agent system.

I. INTRODUCTION

In the past ten years, the distributed coordinated control of multi-agent systems has been widely used in the fields of multi-robot cooperative control [1]–[5], UAV(Unmanned Aerial Vehicle) formation flight control [6]–[10], and communication network [11]–[13]; thus, it is also widely investigated. Scholars have conducted numerous research in this field and consequently achieved relevant results.

Consensus control is an important component of the coordinated control of multi-agent systems. Many fruitful results have been recently documented [14] and [15]. For example, the output stationary average consensus of heterogeneous linear multi-agent systems with unknown parameters under undirected or balanced directed networks was investigated in [14], and the average consensus of multi-agent systems with binary communication under directed topology was explored in [15]. In practical applications in which tasks need to be executed, a common expectation is for each agent to achieve a consensus within a limited time. For example, in multi-UAV formation flight, all UAVs need to reach a specified position as soon as possible, an objective requiring the design of a finite-time controller for controlling the whole multi-UAV system. Therefore, the multi-agent finite-time consensus has become a research hotspot in recent years [16]–[21]. Li et al. [16] studied the finite-time consensus tracking problem of uncertain nonlinear multi-agent systems with consensus error constraints. Wang et al. [17] studied the finite-time bipartite consensus problem of multi-agent systems on directed symbolic networks. Kaviarasan et al. [18] studied the finite-time consensus problem of stochastic nonlinear multi-agent systems with stochastic uncertainty and stochastic nonlinearity. Jin et al. [19] designed a distributed adaptive sliding mode controller for a class of uncertain multi-agent systems with interference factors, such as communication disturbance and
actuator failure, to eliminate the influence of these interferences and enable the multi-agent systems to be consistent within a limited time. Wang et al. [20] designed a new control protocol to enable multi-agent systems to achieve state consensus within limited time. Tong et al. [21] addressed the finite-time consensus problem of multi-agent systems by adopting a continuous time-varying interaction topology. However, in many practical applications, continuous communication is difficult to maintain because of the limitation of communication performance. For example, the instability of communication channels, the failure of physical equipment, and the limitation of the sensor range can interrupt communication. Aiming to solve this problem, intermittent control needs to be investigated.

The multi-agent consensus control problem based on intermittent control has also been studied [22]–[24]. In the research on the consensus of multi-agent intermittent control, the control strategy of periodically intermittent control is mainly considered. Wen et al. [22] studied the second-order consensus problem of multi-agent systems with inherent delay, nonlinear dynamics, and intermittent communication. Yang et al. [23] studied a new intermittent pulse scheme to achieve the consensus of time-varying delay of multi-agent systems. Liu et al. [24] studied the leader-following mean square consensus of stochastic multi-agent systems based on periodically intermittent event-trigger control. Findings showed that the development and change of a state need not be implemented in accordance with a fixed cycle. For example, during the formation flight of UAVs, the control is more likely to be aperiodic due to internal and external communication interference, and it manifests occasional electromagnetic wave disturbance, unstable voltage of the UAV internal communication module, or communication instability caused by the long distance from the control center. Aiming to effectively use the available resources and achieve a better control effect, the aperiodically intermittent control method was subsequently proposed. The adoption of aperiodically intermittent control theory has attracted the attention of many scholars. The methods attributable to aperiodically intermittent control theory have achieved rich results in many fields [25] and [26]. For example, Jing et al. [25] studied the finite-time synchronization problem of a class of time-delay dynamic networks with hybrid coupling. By designing a new control parameter, the aperiodically intermittent control scheme could ensure the finite-time synchronization of dynamic networks. Liu et al. [26] established a new differential inequality and studied the finite-time synchronization problem of time-delay dynamic networks by using the method of aperiodically intermittent control. However, many problems still need to be solved in this field, including how to achieve a consensus in multi-agent systems with uncertain parameters under aperiodically intermittent control.

In the present study, the finite-time consensus problem of a multi-agent system based on the aperiodically intermittent control is investigated. An aperiodically intermittent control protocol is designed, and then the Lyapunov function is constructed and the matrix analysis theory is adopted to obtain the sufficient conditions for the consensus of the multi-agent system. Moreover, the finite-time control problem of a class of uncertain nonlinear multi-agent systems is determined, and the sufficient conditions for the finite-time consensus of the system are obtained.

In summary, the main innovations of this research are as follows:

1. For the first-order linear-multi-agent system, we propose an aperiodically intermittent control method. This study has proven that the control method enables each agent to achieve a consensus in finite time under certain conditions.

2. For the class of first-order nonlinear uncertain multi-agent systems, we propose an aperiodically intermittent control method and design a class of aperiodically intermittent controller. This study has proven that the system can achieve a consensus in finite time under certain conditions.

3. Compared with the finite-time control in [16] and [20], this paper studies an aperiodically intermittent control method. Intermittent control can effectively reduce the communication cost. Compared with the intermittent control in [23] and [24], the aperiodically intermittent control method has been used in this paper. The advantage of this method is to prove that when the control signal of the system is interrupted for an indefinite time, the multi-agent system can still achieve the control purpose as long as the time length of signal interruption meets certain conditions.

II. PRELIMINARIES

We present in this section our knowledge of certain symbols, algebraic graph theory, and matrix theory.

We use $R$ to represent the set of real numbers, $R^n$ to represent the n-dimensional space, and $R^{n \times n}$ to represent the set of $n \times n$ real matrices. We define the two real integers of $N$ and $n$ to represent the number of agents in the multi-agent system, and the number of dimensions of each agent, respectively. $\text{sign}(\bullet)$ is a symbolic function, and $\otimes$ represents Kronecker product.

We use $G = (V, E, A)$ to represent an undirected topology, where $V = \{v_1, v_2, \cdots, v_n\}$ represents the set of nodes; $E \subseteq V \times V$ represents the set of edges for information exchange between agents; $A = [a_{ij}] \in R^{n \times n}$ represents the adjacency weight matrix, where $a_{ij}$ is a non-negative adjacency element. The edge of the topological graph is represented by $e_{ij}$. If the adjacent element $a_{ij}$ is positive, it means that there is a channel between agents $i$ and $j$ that can transfer information. In this paper, we assume that there is no self-circulation, that is $a_{ii} = 0$. For an undirected graph, the corresponding adjacency matrix $A$ is symmetric, that is $A^T = A$. In an undirected graph, if there are information transmission channels between any two different nodes $v_i$ and $v_j$, the undirected graph is connected. $d_i = \sum_{j=1}^{n} a_{ij}$ represents the penetration of undirected topological graph $G$, where $i \in V$, and Degree matrix of topological graph $G$ is $D = \text{diag} [d_1, d_2, \cdots, d_n]$, $L = D - A$ is the Laplacian matrix of topological graph $G$. 

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III. PROBLEM FORMULATION

Assuming that a multi-agent system is composed of \( N \) agents. The dynamic model of the \( i \)-th agent can be described as follows:

\[
\dot{x}_i(t) = \hat{A}x_i(t) + u_i(t),
\]

where \( x_i(t) \) represents the state of the \( i \)-th agent satisfying \( x_i(t) \in \mathbb{R}^n \) and \( i = 1, 2, \ldots, N \). The dimension of \( x_i(t) \) is \( n \), and its initial value is \( x_i(t_0) = x_i(0). \) \( \hat{A} \in \mathbb{R}^{n \times n} \) is a constant matrix, and \( u_i(t) \) represents the external input control received by the \( i \)-th agent.

The aperiodically intermittent control protocol in this study is defined as

\[
u_i(t) = \begin{cases} 
Bu_i^{(1)}(t) + u_i^{(2)}(t), & t_m \leq t < s_m \\
0, & s_m \leq t < t_{m+1},
\end{cases}
\]

where \( u_i^{(1)}(t) = \sum_{j=1}^{N} a_{ij}(x_j(t) - x_i(t)), \)

\( u_i^{(2)}(t) = \mu \sum_{j=1}^{N} a_{ij}\text{sign}(x_j(t) - x_i(t))[x_j(t) - x_i(t)]^\alpha, \)

The controller in this paper consists of two parts, and the control function of each part is not exactly the same. The main function of the first part \( u_i^{(1)}(t) \) is to make the multi-agent system achieve consensus, and the main function of the second part \( u_i^{(2)}(t) \) is to make the multi-agent system converge in a finite time. The function of the whole controller \( u_i(t) \) is to make the multi-agent achieve consensus in a finite time by controlling the agent.

Let

\[
sign(x_i) = \text{diag}(\text{sign}(x_{i1}), \text{sign}(x_{i2}), \ldots , \text{sign}(x_{in}));
\]

\( x_i = (x_{i1}, x_{i2}, \ldots , x_{in})^T ; \)

\( |x_i|^\alpha = (|x_{i1}|^\alpha, |x_{i2}|^\alpha, \ldots , |x_{in}|^\alpha)^T. \)

Then, we can change (2) into the following form:

\[
u_i(t) = \begin{cases} 
\mu \sum_{j=1}^{N} a_{ij}\text{sign}(x_j(t) - x_i(t))[x_j(t) - x_i(t)]^\alpha, & t_m \leq t < s_m \\
B \sum_{j=1}^{N} a_{ij}(x_j(t) - x_i(t)), & t_m \leq t < t_{m+1},
\end{cases}
\]

where \( \mu > 0 \) is an adjustable parameter and, \( \alpha \in [0, 1] \) is a real number. For any time interval \( [t_m, t_{m+1}) \), \( t_0 = 0 \) and \( m = 0, 1, 2, \ldots \) need to be satisfied. \( [t_m, s_m) \) and \( [s_m, t_{m+1}) \) are the \( m \)-th communication continuous time and the \( m \)-th communication interruption time, respectively. In these formulas, \( t_m \) and \( s_m \) represent the start time and end time of the \( m \)-th communication continuous time, respectively, and \( t_m + 1 \) represents the end time of the \( m \)-th communication interruption time.

We define the average state of the system as \( x^\ast(t) = \frac{1}{N} \sum_{i=1}^{N} x_i(t) \) and the state error of the \( i \)-th agent as \( e_i(t) = x_i(t) - x^\ast(t) \), where \( e_i(t) = (e_{i1}(t), e_{i2}(t), \ldots , e_{in}(t))^T. \)

The initial value of \( e_i(t) = e_i(t_0) = e_i(0). \) We use \( e(t) = (e_1^T(t), e_2^T(t), \ldots , e_N^T(t))^T \) to express the average error of the entire multi-agent system.

Assumption 1: The undirected communication topology \( G \) investigated in this study is connected.

Assumption 2: The aperiodic control strategy includes two positive scalars denoted by \( 0 < q < \sigma < +\infty. \) Let \( \inf(s_m - t_m) = \rho \) and \( \sup(t_m - t_{m+1}) = \sigma, \) where \( m = 0, 1, 2, \ldots \).

Assumption 3: With the negative constant \( q \geq 0 \), the nonlinear function \( f(\cdot) \) can satisfy the following inequality:

\[
(x - y)^T (f(x) - f(y)) \leq q(x - y)^T (x - y).
\]

In the formula, for any \( x, y \in \mathbb{R}^n. \)

Definition 1: For any initial state of agent \( i \), an appropriate control protocol \( u_i(t) \) can be designed such that there exists a constant \( T^* > 0 \). Furthermore, \( \lim_{t \to T^*} \| x_i(t) - x^\ast(t) \| = 0 \) is satisfied for \( i = 0, 1, 2, \ldots , N \), where \( x^\ast(t) = \frac{1}{N} \sum_{i=1}^{N} x_i(t). \)

In this manner, the multi-agent system can achieve an average consensus in finite time.

Definition 2: For the aperiodically intermittent control, we define

\[
\Psi = \lim_{m \to +\infty} \sup_{t_m \leq t < s_m} \frac{s_m - t_m}{t_m - t_{m+1}},
\]

where \( 0 \leq \Psi < 1. \) When \( \Psi = 0 \), the aperiodically intermittent control becomes a continuous control.

Lemma 1 ([29]): For an undirected connected graph \( G \), the Laplacian matrix \( L \) has the following properties:

1) \( x^T L x = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij}(x_j - x_i)^2, x = (x_1, x_2, \ldots , x_N)^T. \)

2) \( 0 \) is a single eigenvalue of \( L \), \( 1_n \) is the corresponding eigenvector.

3) The second smallest eigenvalue of \( L \) is represented by \( \lambda_{\min}(L) \), and \( \lambda_{\min}(L) > 0 \). If \( 1_n^T x = 0 \), then \( x^T L x \geq \lambda_{\min}(L) x^T x. \)

Lemma 2 ([30]): For any given \( g_1, g_2, \ldots , g_n > 0 \) and \( 0 < p < 1 \), we have the following inequality:

\[
\left( \sum_{i=1}^{n} g_i \right)^p \leq \sum_{i=1}^{n} g_i^p.
\]

Lemma 3 ([31]): If Assumption 2 holds, then we have \( \Psi \leq 1 - \frac{\rho}{\sigma}. \)

Lemma 4 ([31]): For any given \( m = 0, 1, 2, \ldots \), we let

\[
\Psi(t) = \frac{t - s_m}{t_m - t} \in [s_m, t_{m+1}],
\]

Obviously, \( \Psi(t) \) is a strictly increasing function and \( \Psi(t) \leq \frac{t_{m+1} - t_m}{s_m - t_m}. \)

Lemma 5 ([32]): Suppose the function \( V(t) \) is continuous and nonnegative, then when \( t \in [0, +\infty) \) and \( V(t) \) meets the following conditions:

\[
\begin{cases} 
\dot{V}(t) \leq -\omega V^\ast(t) - \theta_1 V(t), & t_m \leq t < s_m \\
\dot{V}(t) \leq \theta_2 V(t), & s_m \leq t < t_{m+1}.
\end{cases}
\]
where \( m = 0, 1, 2, \ldots, \omega, \theta_1, \theta_2 > 0, 0 < \kappa < 1 \). If there exists a constant \( \Psi \in (0, 1) \), i.e.,
\[
\theta_1 - (\theta_1 + \theta_2)\Psi > 0,
\]
where \( \Psi \) is given in Definition 2, then we have
\[
V^{1+\kappa}(t) \exp \{(1 - \kappa)\theta_1 t \}
\leq \exp \{(1 - \kappa)(\theta_1 + \theta_2)\Psi t \} \left( V^{1+\kappa}(0) + \frac{\omega}{\kappa} \right) \exp \{-\beta(t) + \theta_2 t \}, 0 \leq t \leq T^*.
\]
The settling time \( T^* \) can be expressed as
\[
T^* \leq \frac{\ln(1 + \frac{\omega}{\kappa} V^{1+\kappa}(0))}{(1 - \kappa)(\theta_1 - (\theta_1 + \theta_2)\Psi)}.
\]

Lemma 6 ([33]): Suppose that \( Z, X, Q \) are some real matrices with appropriate dimensions and \( Q \) is positive define. Then for any vectors \( x, y \in \mathbb{R}^n \), we have
\[
x^T Z x \leq x^T Z Q x + y^T X^T Q T^* x.
\]

IV. MAIN RESULTS
A. CONSENSUS CONTROL OF GENERAL MULTI-AGENT SYSTEMS

Theorem 1: Suppose that Assumption 1-2 hold for multi-agent system (1). Thus, there exists a positive constant \( \theta_1, \theta_2 > 0 \) such that

(i) \( I_N \otimes A + (-L \otimes B) + \theta_1 \cdot I_N \times n \leq 0 \),

(ii) \( I_N \otimes A - \theta_2 \cdot I_N \times n \leq 0 \), and

(iii) \( \theta_1 - (\theta_1 + \theta_2)\Psi > 0 \),

where \( \Psi \in (0, 1) \) is given in Definition 2.

Then, for any initial condition \( x(0) \in \mathbb{R}^n \), multi-agent system (1) will achieve a consensus in finite time \( T_1^* \).

\[
T_1^* = \frac{\ln(1 + \frac{\omega}{\kappa} V^{1+\kappa}(0))}{(1 - \kappa)(\theta_1 - (\theta_1 + \theta_2)\Psi)}.
\]

where \( V(0) = e^T(0)e(0) \), and \( e(0) \) is the initial value of \( e(t) \),
\[
\omega = \frac{\lambda_{\min}}{\lambda_{\max}} (L(C)), \kappa = \frac{\alpha + 1}{2}.
\]

Proof: First, the Lyapunov equation is constructed as follows:
\[
V(t) = \frac{1}{2} e^T(t) e(t) = \frac{1}{2} \sum_{i=1}^{N} e_i^T(t) e_i(t),
\]

For \( t_m \leq t < s_m \), and \( m = 0, 1, 2, \ldots \), we can obtain the following state equation:
\[
\dot{x}_i(t) = \dot{A}x_i(t) + \eta \sum_{j=1}^{N} a_{ij}(x_j(t) - x_i(t))
+ \mu \sum_{j=1}^{N} a_{ij} \text{sign}(x_j(t) - x_i(t)) |x_j(t) - x_i(t)|^\alpha.
\]

For \( x^*(t) = \frac{1}{N} \sum_{i=1}^{N} x_i(t) \), we can further obtain the state model of average state as follows:
\[
\dot{x}^*(t) = \frac{1}{N} \sum_{i=1}^{N} \dot{x}_i(t)
\]

\[
= \frac{1}{N} \sum_{i=1}^{N} \dot{A}x_i(t) + \eta \sum_{j=1}^{N} a_{ij}(x_j(t) - x_i(t))
+ \mu \sum_{j=1}^{N} a_{ij} \text{sign}(x_j(t) - x_i(t)) |x_j(t) - x_i(t)|^\alpha.
\]

Then, we can derive the differential equation of the state error as
\[
\dot{e}_i(t) = \dot{x}_i(t) - \dot{x}^*(t)
= \dot{A}x_i(t)
+ \mu \sum_{j=1}^{N} a_{ij} \text{sign}(x_j(t) - x_i(t)) |x_j(t) - x_i(t)|^\alpha
+ B \sum_{j=1}^{N} a_{ij}(x_j(t) - x_i(t)) - A x^*(t)
\]

\[
= \dot{A}x_i(t)
+ \mu \sum_{j=1}^{N} a_{ij} \text{sign}(x_j(t) - x_i(t)) |x_j(t) - x_i(t)|^\alpha
+ B \sum_{j=1}^{N} a_{ij}(x_j(t) - x_i(t))
\]

\[
= \dot{A} e_i(t)
+ \mu \sum_{j=1}^{N} a_{ij} \text{sign}(e_j(t) - e_i(t)) |e_j(t) - e_i(t)|^\alpha.
\]
According to (6), we can derive
\begin{align}
\dot{V}(t) &= \sum_{i=1}^{N} e_i^T(t) \dot{e}_i(t) \\
&= \sum_{i=1}^{N} e_i^T(t) \dot{A}_i e_i(t) + B \sum_{j=1}^{N} a_{ij}(e_j(t) - e_i(t)) \\
&\quad + \mu \sum_{i=1}^{N} a_{ij} \text{sign}(e_j(t) - e_i(t)) |e_j(t) - e_i(t)|^\alpha \\
&= \sum_{i=1}^{N} e_i^T(t) \dot{A}_i e_i(t) \\
&\quad + \mu \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} e_i^T \text{sign}(e_j(t) - e_i(t)) |e_j(t) - e_i(t)|^\alpha \\
&\quad + B \sum_{j=1}^{N} \sum_{i=1}^{N} a_{ij} e_i^T (e_j(t) - e_i(t)).
\end{align}

Then, let \( \dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) \), where
\begin{align}
\dot{V}_1(t) &= \sum_{i=1}^{N} e_i^T(t) \dot{A}_i e_i(t) + B \sum_{j=1}^{N} \sum_{i=1}^{N} a_{ij} e_i^T (e_j(t) - e_i(t)), \\
\dot{V}_2(t) &= \mu \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} e_i^T \text{sign}(e_j(t) - e_i(t)) |e_j(t) - e_i(t)|^\alpha,
\end{align}
We deal with \( \dot{V}_1(t) \) and \( \dot{V}_2(t) \) as follows.
For \( \dot{V}_1(t) \), we can obtain
\begin{align}
\dot{V}_1(t) &= e^T(t)(I_N \otimes \dot{A}) e(t) \\
&\quad + \frac{1}{2} B \sum_{i=1}^{N} \sum_{j=1}^{N} \left( a_{ij} e_i^T (e_j(t) - e_i(t)) \right) \\
&\quad - a_{ij} e_i^T (e_j(t) - e_i(t)) \\
&= e^T(t)(I_N \otimes \dot{A}) e(t) \\
&\quad - \frac{1}{2} B \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} (e_j(t) - e_i^T(t))(e_j(t) - e_i(t)) \\
&= e^T(t)(I_N \otimes \dot{A}) e(t) - e^T(t)(L \otimes B) e(t) \\
&= e^T(t)(I_N \otimes \dot{A}) + (-L \otimes B) e(t) \\
&= -\theta_1 V(t) \\
&\quad + e^T(t)(I_N \otimes \dot{A}) + (-L \otimes B) + \theta_1 \cdot I_{N \times n} e(t) \\
&\leq -\theta_1 V(t) , \tag{8}
\end{align}
where \((I_N \otimes \dot{A}) + (-L \otimes B) + \theta_1 \cdot I_{N \times n} \leq 0\).
For \( \dot{V}_2(t) \), we can obtain
\begin{align}
\dot{V}_2(t) &= \mu \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} e_i^T \text{sign}(e_j(t) - e_i(t)) |e_j(t) - e_i(t)|^\alpha \\
&= \frac{1}{2} \mu \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} e_i^T \text{sign}(e_j(t) - e_i(t)) |e_j(t) - e_i(t)|^\alpha \\
&\quad \times |e_j(t) - e_i(t)|^\alpha \\
&= \frac{1}{2} \mu \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} |e_j(t) - e_i(t)|^\alpha \\
&\quad \times \text{sign}(e_j(t) - e_i(t))|e_j(t) - e_i(t)|^\alpha \\
&= \frac{1}{2} \mu \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} |e_j(t) - e_i(t)|^\alpha \\
&\quad \times |e_j(t) - e_i(t)|^\alpha \\
&= \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} |e_j(t) - e_i(t)|^\alpha \\
&\quad \times |e_j(t) - e_i(t)|^\alpha \\
&= \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} |e_j(t) - e_i(t)|^{\alpha + 1} \\
&= \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \left( \left( \mu a_{ij} \right)^2 (e_j(t) - e_i(t))^2 \right)^{\frac{\alpha + 1}{2}} \tag{9}
\end{align}
Let \( L(C) = [c_{ij}] \), where \( c_{ij} = (\mu a_{ij})^2 \).
Then, according to Lemma 2, we have
\begin{align}
\dot{V}_2(t) &\leq - \left( \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{p=1}^{n} c_{ij} e_j(t) - e_i(t))^T \\
&\quad \times (e_j(t) - e_i(t))^T \right)^{\frac{\alpha + 1}{2}} \\
&= - \left( e^T(t)(L(C) \otimes I_n) e(t) \right)^{\frac{\alpha + 1}{2}} \\
&= - \left( e^T(t)(L(C) \otimes I_n) e(t) \right)^{\frac{\alpha + 1}{2}} \\
&= - \left( e^T(t)(L(C) \otimes I_n) e(t) \right)^{\frac{\alpha + 1}{2}} \\
&= - \left( e^T(t)(L(C) \otimes I_n) e(t) \right)^{\frac{\alpha + 1}{2}} , \tag{10}
\end{align}
where \( L(C) \otimes I_n \) is a symmetric positive definite matrix.
According to Lemma 1, we have
\begin{align}
e^T(t)(L(C) \otimes I_n) e(t) \geq \lambda_{\min} (L(C)) e^T(t) e(t).
\end{align}
Then, we can obtain
\begin{align}
\dot{V}_2(t) &\leq - \left( \lambda_{\min} (L(C) \otimes I_n) e^T(t) e(t) \right)^{\frac{\alpha + 1}{2}} \\
&= - \lambda_{\min}^{\frac{\alpha + 1}{2}} (L(C)) V^{\frac{\alpha + 1}{2}} (t) . \tag{11}
\end{align}
According to (8) and (11), we can obtain
\begin{align}
\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) \\
&\leq - \lambda^{\frac{\alpha + 1}{2}}_{\min} (L(C)) V^{\frac{\alpha + 1}{2}} (t) - \theta_1 V(t) . \tag{12}
\end{align}
For \( s_m \leq t < t_{m+1} \), and \( m = 1, 2, \ldots \), we can easily derive
\[
\dot{e}(t) = \hat{A}e(t).
\] (13)

We use the Kronecker product to expand the dimension
\[
\dot{e}(t) = (I_N \otimes \hat{A})e(t).
\] (14)

Then, we can further obtain
\[
\begin{align*}
\dot{V}(t) &= e^T(t)\dot{e}(t) \\
&= e^T(t)(I_N \otimes \hat{A})e(t) \\
&= \theta_2 V(t) + e^T(t)(I_N \otimes \hat{A} - \theta_2 \cdot I_N \times n)e(t) \\
&\leq \theta_2 V(t),
\end{align*}
\] (15)

where \( I_N \otimes \hat{A} - \theta_2 \cdot I_{N \times n} \leq 0 \).

In summary, we can derive
\[
\begin{align*}
\dot{V}(t) &\leq -\omega \max_{\min}(L(C)) V^{\frac{\alpha + 1}{\omega}}(t) - \theta_1 V(t), \quad t_m \leq t < s_m \\
\dot{V}(t) &\leq \theta_2 V(t), \quad s_m \leq t < t_{m+1}.
\end{align*}
\] (16)

According to Lemma 5, the state error of the multi-agent system converges to zero in finite time \( T_1^* \).
\[
\begin{align*}
T_1^* &= \frac{\ln(1 + \frac{\theta_1}{\omega} V^{1-\kappa}(0))}{(1 - \kappa)(\theta_1 - (\theta_1 + \theta_2)\Psi)}.
\end{align*}
\]

At this phase, the proof has been completed.

Remark 1: Under the condition of Theorem 1, the multi-agent system can achieve a finite-time average consensus under the action of the control protocol proposed in this study, in which \( T_1^* \) is the upper limit of the convergence time. The convergence time can be attuned by adjusting the control protocol parameters \( \omega \) and \( \alpha \). In contrast to the control protocol in the literature [20] and [22]–[24], the aperiodically intermittent control strategy is adopted in this study.

B. CONSENSUS CONTROL OF MULTI-AGENT SYSTEMS WITH NONLINEAR UNCERTAINTIES

The dynamic model of the nonlinear uncertain multi-agent system is defined as
\[
\begin{align*}
\dot{x}_i(t) &= \left(\hat{A} + \Delta A(t)\right)x_i(t) + f(t, x_i(t)) + u_i(t),
\end{align*}
\] (17)

where \( \Delta A(t) \) is a time-varying matrix representing the model error of the uncertain system that satisfies \( \Delta A(t) = MP(t)U \). \( M \) and \( U \) are constant matrices with appropriate dimension, and \( P(t) \) is an unknown time-varying matrix function that satisfies \( P(t)P(t) \leq 1 \). \( f(t, x_i(t)) = (f_1(t, x_i(t)), f_2(t, x_i(t)), \ldots, f_p(t, x_i(t)))^T \) is a continuously differentiable vector valued function representing the nonlinear dynamic behavior within the \( i \)-th agent, and it satisfies the Lipschitz condition.

Here, the control protocol we use is still protocol (2).

Then, the dynamic model of the system can be written in the following form:
\[
\begin{align*}
\dot{e}_i(t) &= \left(\hat{A} + \Delta A(t)\right)e_i(t) + f(t, x_i(t)) \\
&\quad + \mu \sum_{j=1}^{N} a_{ij}\text{sign}(e_j(t) - e_i(t))|e_j(t) - e_i(t)|^\alpha \\
&\quad + B \sum_{j=1}^{N} e_{ij} e_j(t) \\
&\quad + f(t, x_i(t)) - \frac{1}{N} \sum_{j=1}^{N} f(t, x_j(t)).
\end{align*}
\] (18)

Theorem 2: Suppose that Assumption 1-2 hold for multi-agent system (1). Thus, there exists the positive constant \( \theta_3, \theta_4, q > 0 \). Consequently,
\[
\begin{align*}
\dot{V}(t) &= e^T(t)e(t) \\
&\leq \frac{1}{2} \sum_{i=1}^{N} e_i^T(t)e_i(t).
\end{align*}
\] (21)

For \( t_m \leq t < s_m \), and \( m = 0, 1, 2, \ldots \),
\[
\dot{V}(t) = \sum_{i=1}^{N} e_i^T(t)\dot{e}_i(t)
\]
\[ = \sum_{i=1}^{N} e_i^T(t)(\hat{A} + \Delta A(t))e_i(t) \]
\[ + \sum_{i=1}^{N} e_i^T(t)\mu \sum_{j=1}^{N} a_{ij}\text{sign}(e_j(t) - e_i(t)) \times |e_j(t) - e_i(t)|^{q_i} \]
\[ + \sum_{i=1}^{N} e_i^T(t)(B \sum_{j=1}^{N} a_{ji}(e_j(t) - e_i(t))) \]
\[ + \sum_{i=1}^{N} e_i^T(t)f(t, x_i(t)) - \frac{1}{N} \sum_{j=1}^{N} f(t, x_j(t)). \]  

According to (8) and (9)-(11), we have
\[ \sum_{i=1}^{N} e_i^T(t)(B \sum_{j=1}^{N} a_{ji}(e_j(t) - e_i(t))) \]
\[ = -e^T(t)(L \otimes B)e(t), \]  
\[ \sum_{i=1}^{N} e_i^T(t)\mu \sum_{j=1}^{N} a_{ij}\text{sign}(e_j(t) - e_i(t))|e_j(t) - e_i(t)|^{q_i} \]
\[ \leq -\lambda_{\text{min}}^{\frac{q+1}{q}} (L(C)) V^{\frac{q+1}{q}}(t). \]  

Moreover, we have
\[ \sum_{i=1}^{N} e_i^T(t)(f(t, x_i(t)) - \frac{1}{N} \sum_{j=1}^{N} f(t, x_j(t))) \]
\[ = \sum_{i=1}^{N} e_i^T(t)(f(t, x_i(t)) - f(t, x^*(t))) \]
\[ + f(t, x^*(t)) - \frac{1}{N} \sum_{j=1}^{N} f(t, x_j(t))] \]
\[ = \sum_{i=1}^{N} e_i^T(t)\left(f(t, x_i(t)) - f(t, x^*(t))\right) \]
\[ + \sum_{i=1}^{N} e_i^T(t)\left(f(t, x^*(t)) - \frac{1}{N} \sum_{j=1}^{N} f(t, x_j(t))\right). \]  

With \( \sum_{i=1}^{N} e_i^T(t) = 0 \), we can obtain
\[ \sum_{i=1}^{N} e_i^T(t)\left(f(t, x^*(t)) - \frac{1}{N} \sum_{j=1}^{N} f(t, x_j(t))\right) = 0. \]  

According to Assumption 3 and (26) we have
\[ \sum_{i=1}^{N} e_i^T(t)(f(t, x_i(t)) - \frac{1}{N} \sum_{j=1}^{N} f(t, x_j(t))) \]
\[ = \sum_{i=1}^{N} e_i^T(t)\left(f(t, x_i(t)) - f(t, x^*(t))\right) \]
\[ \leq \sum_{i=1}^{N} e_i^T(t)\left(g(x_i(t)) - x^*(t)\right) \]
\[ = \sum_{i=1}^{N} q e_i^T(t)e_i(t). \]  

According to Lemma 6, we have
\[ \sum_{i=1}^{N} e_i^T(t)\left(\hat{A} + \Delta A(t)e_i(t)\right) \]
\[ = \sum_{i=1}^{N} e_i^T(t)\left(\hat{A} + MP(t)U)e_i(t)\right) \]
\[ = \sum_{i=1}^{N} e_i^T(t)\hat{A}e_i(t) + \sum_{i=1}^{N} e_i^T(t)(MP(t)U)e_i(t) \]
\[ \leq \sum_{i=1}^{N} e_i^T(t)(\hat{A}e_i(t) + \frac{1}{N} \sum_{j=1}^{N} [e_i^T(t)MM^T e_j(t)] \]
\[ + e_i^T(t)U^TP^T(t)P(t)e_i(t)) \]
\[ \leq \sum_{i=1}^{N} e_i^T(t)\left(\frac{1}{2}MM^T + \frac{1}{2}U^TU + \hat{A}\right)e_i(t) \]  

Thus, according to (23), (24), (27) and (28), we can obtain
\[ \dot{V}(t) \leq \sum_{i=1}^{N} e_i^T(t)\left(\frac{1}{2}MM^T + \frac{1}{2}U^TU + \hat{A}\right)e_i(t) \]
\[ - e^T(t)(L \otimes B)e(t) \]
\[ + \sum_{i=1}^{N} q e_i^T(t)e_i(t) - \lambda_{\text{min}}^{\frac{q+1}{q}} (L(C)) V^{\frac{q+1}{q}}(t) \]
\[ = e^T(t)\left(I_N \otimes \left(\frac{1}{2}MM^T + \frac{1}{2}U^TU + \hat{A}\right)\right)e(t) \]
\[ - e^T(t)(L \otimes B)e(t) \]
\[ + e^T(t)(q \cdot I_N \otimes e)(t) - \lambda_{\text{min}}^{\frac{q+1}{q}} (L(C)) V^{\frac{q+1}{q}}(t) \]
\[ = e^T(t)\left[I_N \otimes \left(\frac{1}{2}MM^T + \frac{1}{2}U^TU + \hat{A}\right) \right] \]
\[ + \left(-L \otimes B + q \cdot I_N \otimes e\right)(t) \]
\[ - \lambda_{\text{min}}^{\frac{q+1}{q}} (L(C)) V^{\frac{q+1}{q}}(t) \]
\[ \leq -\lambda_{\text{min}}^{\frac{q+1}{q}} (L(C)) V^{\frac{q+1}{q}}(t) \]
\[ + e^T(t)\left[I_N \otimes \left(\frac{1}{2}MM^T + \frac{1}{2}U^TU + \hat{A}\right) \right] \]
\[ + \left(-L \otimes B + q \cdot I_N \otimes e\right)(t) \]
\[ \leq -\lambda_{\text{min}}^{\frac{q+1}{q}} (L(C)) V^{\frac{q+1}{q}}(t) - \lambda_{\text{min}}^{\frac{q+1}{q}} (L(C)) V^{\frac{q+1}{q}}(t), \]  

where \( I_N \otimes \left(\frac{1}{2}MM^T + \frac{1}{2}U^TU + \hat{A}\right) \) + \( (-L \otimes B + (q + \theta_3) \cdot I_N \otimes e\right) \cdot I_{N \times N} \leq 0 \). For \( s_m \leq t < t_m + 1 \), and \( m = 0, 1, 2, \ldots \), we have
\[ \dot{e}_i(t) = \dot{x}_i(t) - \dot{x}^*(t) \]
\[ = (\hat{A} + \Delta A(t))e_i(t) + f(t, x_i(t)) \]
\[ - \frac{1}{N} \sum_{j=1}^{N} f(t, x_j(t)). \]
We can easily obtain

\[
\dot{V}(t) = \sum_{i=1}^{N} e_i^T(t) \dot{e}_i(t) \\
= \sum_{i=1}^{N} e_i^T(t)(\dot{A} + \Delta A(t))e_i(t) \\
+ \sum_{i=1}^{N} e_i^T(t)(f(t, x_i(t)) + \frac{1}{N} \sum_{j=1}^{N} f(t, x_j(t))) \\
\leq e^T(t) [I_N \otimes (\frac{1}{2}MM^T + \frac{1}{2}U^TU + \hat{\Delta})] \\
+ q \cdot I_{N \times n} e(t) \\
= \theta_4 V(t) \\
+ e^T(t) [I_N \otimes (\frac{1}{2}MM^T + \frac{1}{2}U^TU + \hat{\Delta})] \\
+(q - \theta_4) \cdot I_{N \times n} e(t) \\
\leq \theta_4 V(t),
\]

(31)

where \(I_N \otimes (\frac{1}{2}MM^T + \frac{1}{2}U^TU + \hat{\Delta})\) and \((q - \theta_4) \cdot I_{N \times n} \leq 0\).

In summary, we have

\[
\begin{cases}
\dot{V}(t) \leq -\lambda_{\text{min}}^{a+1}(L(C)) V^\frac{a+1}{2}(t) - \theta_3 V(t), t_m \leq t < s_m \\
\dot{V}(t) \leq \theta_4 V(t), s_m \leq t < t_{m+1}
\end{cases}
\]

(32)

Let \(\lambda_{\text{min}}^{a+1}(L(C)) = \omega\) and \(\frac{a+1}{2} = \kappa\). Then, we can obtain

\[
\begin{cases}
\dot{V}(t) \leq -\omega V^\kappa(t) - \theta_3 V(t), t_m \leq t < s_m \\
\dot{V}(t) \leq \theta_4 V(t), s_m \leq t < t_{m+1}
\end{cases}
\]

(33)

According to Lemma 5, the state error of multi-agent system converges to zero in finite time \(T_2^*\).

\[
T_2^* = \frac{\ln(1 + \frac{\theta_2}{\kappa})}{(1 - \kappa)(\theta_3 - (\theta_3 + \theta_4)\psi)}.
\]

At this phase, the proof has been completed.

Remark 2: On the basis of Theorem 1, the multi-agent system takes into account the parameter uncertainty and the system nonlinear terms. Although this kind of model has been reported in the literature [28], the consensus of the system in finite time was not explored.

Remark 3: In [34]–[37], the finite-time convergence of several kinds of control systems is studied by using event-triggered control and sliding mode control. The systems they study are mainly semi-Markov stochastic systems. This paper studies the finite-time consensus of a class of deterministic systems and adopts the aperiodically intermittent control strategy. The purpose of this paper is to prove that when the control signal of multi-agent is interrupted within a certain time, multi-agent can still achieve the control purpose.

Remark 4: In [38], the leaderless and leader-following consensus of second-order nonlinear multi-agent under directed graph is studied. Reference [39] studies the problem of leaderless consensus control for high-order nonlinear multi-agent systems with completely unknown control direction. Although this paper studies a class of multi-agent finite-time consensus problem with bidirectional information exchange, it can be extended to one-way connected multi-agent consensus control under certain conditions.

V. NUMERICAL SIMULATION

We verify the effectiveness of the control algorithm via numerical simulation. Consider a multi-agent system composed of four agents, in which each agent is composed of three dimensions.

The state equation of each agent is

\[
\dot{x}_i(t) = \dot{A}x_i(t) + Bu_i^{(1)}(t) + u_i^{(2)}(t), \quad i = 1, 2, 3, 4,
\]

where

\[
\hat{\Delta} = \begin{bmatrix}
5 & -2 & -1 \\
-1 & 6 & -2 \\
-2 & -3 & 4
\end{bmatrix}, \quad B = \begin{bmatrix}
1.3 & 2 & 0.1 \\
0 & 1.7 & 0.1 \\
0 & 0 & 2
\end{bmatrix}.
\]

Let \(\mu = 1\) and \(\alpha = 0.2\). The error initial conditions of each agent are given by

\[
e_1 = \begin{bmatrix}
3 \\
-5 \\
0.2
\end{bmatrix}, \quad e_2 = \begin{bmatrix}
4 \\
-5 \\
0.3
\end{bmatrix},
\]

\[
e_3 = \begin{bmatrix}
5 \\
-5 \\
0.1
\end{bmatrix}, \quad e_4 = \begin{bmatrix}
4 \\
-6 \\
0.5
\end{bmatrix}.
\]

From the topology shown in Fig. 1, we can then determine the corresponding weight adjacency matrix \(A\) and degree matrix \(D\) as follows:

\[
A = \begin{bmatrix}
0 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 \\
1 & 1 & 0 & 1 \\
1 & 1 & 1 & 0
\end{bmatrix}, \quad D = \begin{bmatrix}
3 & 0 & 0 & 0 \\
0 & 3 & 0 & 0 \\
0 & 0 & 3 & 0 \\
0 & 0 & 0 & 3
\end{bmatrix}.
\]

Then, we obtain the Laplacian matrix as follows:

\[
L = D - A = \begin{bmatrix}
3 & -1 & -1 & -1 \\
-1 & 3 & -1 & -1 \\
-1 & -1 & 3 & -1 \\
-1 & -1 & -1 & 3
\end{bmatrix}.
\]

FIGURE 1. Topological graph.
step is 0.03 s. From the abovementioned initial values and parameter settings, we can then obtain the finite-time consensus time solution of the multi-agent system. Figs. 2 to 4 show the time-varying state error of each agent when \( \alpha \) takes different values, where \( e_{ij} \) in figures 2-4 represents the error value between the state of the \( j \)-th dimension of the \( i \)-th agent and the \( j \)-th dimension of the average state. Fig. 5 shows the time-varying control input of control protocol (2). We can change the convergence speed of the system by assigning the value of \( \alpha \) to the adjustable parameter \( a \) in the control protocol. As shown in the figure, the smaller the value of \( \alpha \) in a certain range, the faster the system converges. The simulation results indicate that the control protocol used in this study enables multi-agent system (1) to achieve a consensus, which proves the effectiveness of the abovementioned aperiodically intermittent control protocol.

VI. CONCLUSION

In this study, a new finite-time controller is designed for a class of first-order multi-agent systems. Under the action of the controller, the multi-agent system can achieve consensus in limited time. The innovation of this paper is to introduce aperiodically intermittent control into multi-agent consensus control, which is more in line with the actual needs than periodically intermittent control, and study the consensus of nonlinear uncertain multi-agent system model under aperiodically intermittent control. The application of aperiodically intermittent control in multi-agent systems still needs to be explored, such as system transmission time delay, excessive system transmission energy consumption, the performance of second-order multi-agent systems under aperiodically intermittent control, etc.

REFERENCES

[1] P. F. Huang, F. Zhang, J. Cai, D. Wang, Z. Meng, and J. Guo, “Dexterous tethered space robot: Design, measurement, control, and experiment,” IEEE Trans. Aerosp. Electron. Syst., vol. 53, no. 3, pp. 1452–1469, Jun. 2017, doi: 10.1109/TAES.2017.2671558.
[2] Y. Gao and S. Chien, “Review on space robotics: Toward top-level science through space exploration,” Sci. Robot., vol. 2, no. 7, pp. 5074–5085, Jun. 2017, doi: 10.1126/scirobotics.aan5074.
[3] L. Yan, W. Xu, Z. Hu, and B. Liang, “Multi-objective configuration optimization for coordinated capture of dual-arm space robot,” Acta Astronautica, vol. 167, pp. 189–200, Feb. 2020, doi: 10.1016/j.actaastro.2019.11.002.
[4] P. F. Huang, Y. S. Xu, and B. Liang, “Dynamic balance control of multi-arm free-floating space robots,” Int. J. Adv. Robot. Syst., vol. 2, no. 2, pp. 117–124, 2005, doi: 10.5772/5797.
[5] S. Pradhan, V. J. Modi, and A. K. Misra, “Order n formulation for flexible multibody systems in tree topology: Lagrangian approach,” J. Guid., Control, Dyn., vol. 20, no. 4, pp. 665–672, Jul. 1997, doi: 10.2514/2.4129.
[6] Y. Liu, H. Liu, Y. Tian, and C. Sun, “Reinforcement learning based two-level control framework of UAV swarm for cooperative persistent surveillance in an unknown urban area,” Aerosp. Sci. Technol., vol. 98, Mar. 2020, Art. no. 105671, doi: 10.1016/j.ast.2019.105671.
[7] J. Zhang, J. Yan, P. Zhang, and X. Kong, “Collision avoidance in fixed-wing UAV formation flight based on a consensus control algorithm,” IEEE Access, vol. 6, pp. 43672–43682, 2018, doi: 10.1109/ACCESS.2018.2864469.
[8] A. T. Hafez, A. J. Marasco, S. N. Govivi, M. Iskandaran, S. Yousefi, and C. A. Rabbath, “Solving multi-UAV dynamic encirclement via model predictive control,” IEEE Trans. Control Syst. Technol., vol. 23, no. 6, pp. 2251–2265, Nov. 2015, doi: 10.1109/TCST.2015.2411632.

[9] J. Hu, X. Sun, S. Liu, and L. He, “Adaptive finite-time formation tracking control for multiple nonholonomic UAV system with uncertainties and quantized input: Adaptive finite-time formation tracking control for multiple nonholonomic UAV system with uncertainties and quantized input,” Int. J. Adapt. Control Signal Process., vol. 33, no. 1, pp. 114–129, Jan. 2019, doi: 10.1002/acs.2954.

[10] Z. Yu, Y. Qu, and Y. Zhang, “Distributed fault-tolerant cooperative control for multi-UAVs under actuator fault and input saturation,” IEEE Trans. Control Syst. Technol., vol. 27, no. 6, pp. 2229–2417, Nov. 2019, doi: 10.1109/TCST.2018.2868038.

[11] R. Heidari, A. Khajehzadeh, and M. Eslami, “A consensus event-triggered control of networked multi-agent systems with non-ideal communication network,” Ain Shams Eng. J., vol. 12, no. 4, pp. 3783–3790, Dec. 2021, doi: 10.1016/j.asej.2021.04.010.

[12] S. Xu and B. Wang, “Noisy information based control of multi-agent systems in time-varying communication networks,” IEEE Access, vol. 9, pp. 70313–70321, 2021, doi: 10.1109/ACCESS.2021.3078173.

[13] M. Radenkovic and V. S. H. Huynh, “Cognitive caching at the edges for mobile social community networks: A multi-agent deep reinforcement learning approach,” IEEE Access, vol. 8, pp. 179561–179574, 2020, doi: 10.1109/ACCESS.2020.3027707.

[14] M. H. Rezaei and M. B. Menhaj, “Adaptive output stationary average consensus for heterogeneous unknown linear multi-agent systems,” IET Control Theory Appl., vol. 12, no. 7, pp. 847–856, May 2018, doi: 10.1049/ietcta.2017.0877.

[15] T. Wang, H. Zhang, and Y. Zhao, “Average consensus of multi-agent systems under directed topologies and binary-valued communications,” IEEE Access, vol. 6, pp. 55995–56006, 2018, doi: 10.1109/ACCESS.2018.2872755.

[16] X. Li, X. Luo, J. Wang, and X. Guan, “Finite-time consensus of nonlinear multi-agent system with prescribed performance,” Nonlinear Dyn., vol. 91, no. 4, pp. 2397–2409, Mar. 2018, doi: 10.1007/s11071-017-4020-1.

[17] H. Wang, W. Yu, G. Wen, and G. Chen, “Finite-time bipartite consensus for multi-agent systems on directed signed networks,” IEEE Trans. Circuits Syst. I, Reg. Papers, vol. 65, no. 12, pp. 4336–4348, Dec. 2018, doi: 10.1109/tcisl.2018.2838087.

[18] B. Kaviarasan, R. Sakthivel, Y. Li, D. Zhao, and Y. Ren, “Nonfragile adaptive finite-time consensus control for stochastic switching systems with semi-Markovian switching and application to boost converter circuit model,” IEEE Trans. Circuits Syst. I, Reg. Papers, vol. 68, no. 2, pp. 786–796, Feb. 2021, doi: 10.1109/TCSI.2020.3038647.

[19] W. Q. Y. Hou, G. Zong, and C. K. Ahn, “Finite-time event-triggered control for semi-Markovian switching cyber-physical systems with FDI attacks and applications,” IEEE Trans. Circuits Syst. I, Reg. Papers, vol. 66, no. 6, pp. 2665–2674, Jun. 2021, doi: 10.1109/TCSI.2021.3071341.

[20] W. Q. G. Zong, and X. Zheng, “Adaptive-event triggered SMC for stochastic switching systems with semi-Markov process and application to boost converter circuit model,” IEEE Trans. Circuits Syst. I, Reg. Papers, vol. 68, no. 2, pp. 786–796, Feb. 2021, doi: 10.1109/TCSI.2020.3038647.

[21] W. Q. Y. Hou, Z. L. Zhang, J. Cao, and J. Cheng, “Non-fragile H∞ SMC for Markovian jump systems in a finite-time,” J. Franklin Inst., vol. 358, no. 9, pp. 4721–4740, Jun. 2021, doi: 10.1016/j.jfranklin.2021.04.010.

[22] X. Jin, W. Qi, and G. Zong, “Finite-time synchronization of delayed semi-Markov neural networks with dynamic event-triggered scheme,” Int. J. Control, Autom. Syst., vol. 19, no. 6, pp. 1–12, 2021, doi: 10.1109/TCSII.2021.3071341.

[23] G. Wang, C. Wang, L. Li, and Z. Zhang, “Designing distributed consensus protocols for second-order nonlinear multi-agents with unknown control directions under directed graphs,” J. Franklin Inst., vol. 354, no. 1, pp. 571–592, 2017, doi: 10.1016/j.jfranklin.2016.10.034.

[24] G. Wang, “Distributed control of higher-order nonlinear multi-agent systems with unknown non-identical control directions under general directed graphs,” Automatica, vol. 110, pp. 108559–108565, Dec. 2019, doi: 10.1016/j.automatica.2019.108559.

YIPING LUO received the Ph.D. degree in control theory and control engineering from the South China University of Technology, Guangzhou, China, in 2006. He is a Professor with the Hunan Institute of Engineering, Hunan, China, and he is also the Academic Leader of Hunan Province. He was working with the New Century 121 Talent Program of Hunan Province. His current research interests include complex networks and multi-agent systems.

JUNLING ZHU was born in Nantong, Jiangsu, China, in 1998. He received the bachelor’s degree from the College of Electrical and Information Engineering, Jiangsu University of Science and Technology, in 2018. He is currently pursuing the master’s degree in electrical engineering with the Hunan Institute of Engineering. His main research interests include multi-agent systems and aperiodically intermittent control.