A new tetra-quark interpretation of $X(3872)$

Kunihiko Terasaki

Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan

Institute for Theoretical Physics, Kanazawa University, Kanazawa 920-1192, Japan

A new tetra-quark interpretation of $X(3872)$ is presented. In this model, $X(3872)$ consists of two degenerate tetra-quark mesons $\sim \{[cn]([\bar{c}\bar{n}] \pm (cn)[\bar{c}\bar{n}])\}_{r=0}$, and, therefore, it is naturally understood that $X(3872)$ decays into two different eigenstates of $G$ parity.

A narrow charmonium-like resonance, which is called $X(3872)$, has been observed in the $\pi^+\pi^-J/\psi$ mass distribution from the $B^+ \to K^+\pi^+\pi^-J/\psi$ decay, and its existence has been confirmed by the CDF, D0 and Babar collaborations. Its mass and width have been compiled as $m = 3871.2 \pm 0.5$ MeV and $\Gamma < 2.3$ MeV, with $CL = 90\%$. Because the observed dipion mass spectrum is concentrated at high values, it was speculated that the decay might proceed through the $X(3872) \to \rho^0J/\psi$ reaction. However, this has not yet been established conclusively, because a search for its charged partners in $B \to K\pi^-\pi^0J/\psi$ decays has given a negative result and because the $X(3872) \to \pi^0\pi^0J/\psi$ decay has not yet been searched for. In addition, another resonance peak has been observed at the same mass in the $\pi^+\pi^-\pi^0J/\psi$ channel. By identifying these two resonances, the ratio of the measured rates has been determined.

This implies that conservation of $G$-parity and the above identification are incompatible. From the $\pi^+\pi^-\pi^0$ mass distribution in $X(3872) \to \pi^+\pi^-\pi^0J/\psi$, it has also been speculated that it proceeds through the sub-threshold decay $X(3872) \to \omega J/\psi$. However, if the parents of these decays were the same meson $X(3872)$, as implicitly assumed, such a process would imply that isospin conservation in strong interactions of $X(3872)$ is badly broken, because the measured rate for the decay $X(3872) \to D^0\bar{D}^0\pi^0$ is larger by an order of magnitude than that for the $X(3872) \to \pi^+\pi^-J/\psi$ decay.

To understand $X(3872)$, various theoretical models, for example, a loosely bound molecular state, a diquark-antidiquark $[cu][\bar{c}\bar{n}]$ (where $n = u, d$) state, a hybrid meson, a glueball with a $c\bar{c}$ admixture, etc., in addition to the conventional charmonium, have been proposed. However, it seems difficult to reconcile $X(3872)$ with the above charmonium with regard to mass. Among the abovementioned exotic models, the $D^0\bar{D}^{*0}$ molecule might easily explain the violation of isospin conservation speculated above, because the $D^+D^-$ molecule, the counterpart of $D^0\bar{D}^{*0}$, is not included. However, the absence of such a state leads to a problem in the production of $X(3872)$: specifically, the molecular model predicts $R_{molecule} < 0.1$, while experiments have found $0.13 < R_{exp} < 1.10$ at 90% CL.

In contrast, the diquark-antidiquark model, in which $X(3872)$ is assigned to a $[cn][\bar{c}\bar{n}]$ (where $n = u, d$) predicts that $R_{molecule} = 1$. In this sense, $R$ seems to favor the diquark-antidiquark model rather than the $D^0\bar{D}^{*0}$ molecule. However, the diquark-antidiquark model has predicted a large difference between the masses of $X_d \sim [cd][\bar{c}\bar{n}]$ and $X_u \sim [cu][\bar{c}\bar{n}]$, which are produced in the decays of $B^0$ and $B^+$, respectively, i.e., $m_{X_d} - m_{X_u} \simeq (7 \pm 2)$ MeV. This result is larger than the measured $(2.7 \pm 1.3 \pm 0.2)$ MeV. These experimental results suggest that isospin symmetry is compatible with the production of $X(3872)$. In addition, the diquark-antiquark model predicts the existence of its charged partners, in contrast with the negative result obtained from the experimental search, as mentioned above. In the models listed above, $X(3872)$ is assigned to a single meson state with a definite $G$-parity and thus they encounter the serious problem that the strong interactions of $X(3872)$ do not conserve $G$-parity, in contrast with the known ones. Thus, all the existing models seem to have some serious problem. Experimental data on $X(3872)$ and its theoretical interpretations are reviewed, for example, in Refs. 17 and 20.

Before introducing a new four-quark interpretation of $X(3872)$, we very briefly review four-quark mesons. They can be classified into four groups:

$$
\{q\bar{q}q\bar{q}\} = [qq][\bar{q}\bar{q}] \oplus [qq][\bar{q}\bar{q}] \oplus [qq][\bar{q}\bar{q}] \pm [qq][\bar{q}\bar{q}],
$$

(1)

where the parentheses and the square brackets denote symmetry and anti-symmetry, respectively, of the wavefunction under the exchange of flavors between them. Each term on the right-hand side (r.h.s.) of Eq. (1) is again classified into two classes, because there are two different ways to construct a color singlet $\{q\bar{q}q\bar{q}\}$ state, i.e., to take $3_c \times 3_c$ and $6_c \times 6_c$ of the color $SU_c(3)$. Although these two can mix with each other in general, here we ignore such mixing.
for simplicity. The allowed spins of low-lying four-quark mesons in the flavor symmetry limit are listed in Table I. As seen in the table, the \([qq][\bar{q}q]\) mesons with \(3_c \times 3_c\) have \(J^P = 0^+\). When \(q = u, d, s\), they accurately describe the observed low-lying scalar mesons \(\bar{3}_c\) \((a_0(980), f_0(980), \kappa\) and \(f_0(600)\), as suggested in Ref. [21] and supported recently by lattice QCD results [22]. However, the corresponding scalar, axial-vector and tensor mesons which arise from the \(6_c \times 6_c\) component, as seen in Table I, have not yet been observed near the scalar mesons. This implies that the \(6_c \times 6_c\) state cannot be bound, or it would be much heavier (even if it could be bound) than the \(3_c \times 3_c\), because the forces between \(qq\) (and \(\bar{q}q\)) are repulsive when \(qq\) (and \(\bar{q}q\)) are of \(6_c\) (and \(\bar{6}_c\)) [23]. As a result, the mixing between the \(3_c \times 3_c\) and \(6_c \times 6_c\) states might be small, and therefore, it is conjectured that ignoring the above mixing introduces only a small error.

When one of the light quarks in \([qq][\bar{q}q]\) is replaced by the charm quark, \(c\), open-charm scalar mesons can be obtained, and the well-known \(D_{s0}^+(2317)\) [3] has been successfully assigned to the iso-triplet \(F_0^+ \sim [cn][\bar{s}n]_{I=1}\) [24]. (Our notation for open-charm scalar mesons is defined in Ref. [24].) In fact, by adopting this assignment, its narrow width can be easily understood, because of the small overlap of the color and spin wavefunctions [25], and the experimental constraint [26],

\[
\frac{\Gamma(D_{s0}^+(2317) \to D_s^{++}\gamma)}{\Gamma(D_{s0}^+(2317) \to D_s^0\pi^0)} < 0.059,
\]

can be naturally satisfied [27, 28] in the approach in which the measured ratio of decay rates [5],

\[
\frac{\Gamma(D_s^{++} \to D_s^{+}\pi^0)}{\Gamma(D_s^{++} \to D_s^{+}\gamma)} = 0.062 \pm 0.008,
\]

is reproduced. The above ratio implies that isospin non-conserving interactions in the charm-strange system are much weaker than the electromagnetic ones, which are much weaker than the isospin conserving strong interactions; i.e., isospin conservation is approximately realized in the open-charm system, as in known strong interactions. In addition, it has been discussed [29] that the iso-singlet \(F_0^+\) might have already been observed in the radiative channel \(B \to \bar{D}D_s^{++}\) [20], and it has been conjectured that the neutral \(F_0^+\) and doubly charged \(F_1^{++}\) partners of \(D_{s0}^+(2317)\) will be found in hadronic weak decays of \(B\) mesons.

Although \(D_{s0}^+(2317)\) is a natural and feasible possibility for the iso-triplet scalar four-quark meson \(F_1^+\) with \(3_c \times 3_c\), as seen above, its straightforward extension to hidden-charm axial-vector mesons involves some problems, as discussed above. The \((cn)(\bar{c}n)\) system corresponding to the second term on the r.h.s. of Eq. (1) can have \(J^{PC} = 1^{++}\) for \(3_c \times 3_c\). However, it is not clear whether it can be bound. For example, if \((mn)(\bar{n}s)\) were bound, there would exist strange scalar mesons with \(I = 3/2\), and hence the \(\pi K\) phase shift with \(I = 3/2\) should pass through \(\pi/2\) at the resonant energy. However, no indication of such phenomena has been observed [31]. In addition, if \(X(3872)\) were assigned to \((cn)(\bar{c}n)_{I=0}\), its G-parity would be even, as long as isospin is a good quantum number. Therefore, it is difficult to avoid the problem of the non-conservation of G-parity, because \(X(3872)\) decays into the \(\pi\pi J/\psi\) and \(\pi\pi\pi J/\psi\) states with opposite G-parities.

Now we propose a new interpretation of \(X(3872)\). The last two components of Eq. (1), and hence the corresponding \([cq](\bar{c}q)\) and \((cq)[\bar{c}q]\), which belong to the ideally mixed \(60_f^I\) and \(60_f\) multiplets, respectively, of the flavor SU(4), have \(J^{PC} = 1^+\), as seen in Table I. Although the light-quark sector of \{\(qq\)[\(\bar{q}q]\)\} axial-vector mesons have been studied since long ago [21], they have not been identified with any observed axial-vector mesons since it is difficult to definitively assign observed axial-vector mesons to the members of this class of mesons. This is because their quantum numbers (flavors, isospin, G-parity, etc.) are included in the conventional \(qq\) and \((\bar{q}q)\) axial-vector mesons (and \([qq]\)[\(\bar{q}q]\) states with \(6_c \times 6_c\), if they can be bound), and because these mesons with the same quantum numbers might mix with each other through the common final states of their decays.

We now consider the hidden-charm sector. Because \(X(60_f) \sim [cn](\bar{c}n)_{I=0}\) and \(X(60_f) \sim (cn)(\bar{c}n)_{I=0}\) can be connected with each other under charge conjugation, they can have equal masses as long as charge-conjugation parity is conserved. However, neither of them can be an eigenstate of G-parity, and hence they mix with each other to form eigenstates of G-parity, i.e., \(X(\pm) \sim X(60_f) \pm X(60_f)\), under G-parity conserving strong interactions. In this case,
charm-strange mesons, we can have various exotic axial-vector mesons with or an iso-singlet four-quark meson), it would be difficult to naturally understand the above ratio, because the isospin of $X(\pm)$ will appear as two different states with different masses, and thus they could not be identified with $X(3872)$. However, if it is not sizable, then $X(\pm)$ will appear as a single meson state, $X(3872)$, with regard to mass, while they will behave as two different states in their decays, because of their different G-parities.

The mass difference $|\Delta m_X|$ caused by the mixing of hidden-charm tetra-quark mesons through common final states of their decays is expected to be small, because interactions causing their decays are weak, due to the small overlap of the color and spin wavefunctions, as discussed below, in contrast with the case of the light quark sector with smaller mass. This is similar to the result obtained in Refs. [27 and 28] that the rate for the $D_{s0}^+(2317) \rightarrow D_s^+ \pi^0$ decay at a higher energy scale is small, while the rate for $a_0(980) \rightarrow \eta \pi$ at a lower energy scale is large. With regard to this, it should be noted that a chiral model in the framework of broken $SU_f(4)$ symmetry [32] predicts two axial-vector mesons with opposite G-parities near $X(3872)$. Their mass difference is small (3 MeV), and it seems to be within energy resolutions of existing experiments on $X(3872)$. These axial-vector meson states could be realized by $X(\pm)$, as seen above. Therefore, by assuming that $X(3872)$ consists of $X(-)$ and $X(\pm)$, it could be understood that $X(3872)$ acts like a single meson state with regard to mass, while the former decays into $\pi\pi J/\psi$ and the latter into $\pi\pi\pi J/\psi$ without violating the usual G-parity conservation.

The narrow width of $X(3872)$ can be understood as resulting from a small overlap of the color and spin wavefunctions, as in the case of $D_{s0}^+(2317)$ [27,28]. Its kinematically allowed two-body decays are $X(3872) \rightarrow \eta J/\psi$ and $\eta\omega$. Although they have not yet been observed, they could be dominant decays of $X(3872)$, because they can be $S$-wave decays. In fact, the measured branching fraction for the decay $X(3872) \rightarrow D^0\overline{D}^0 \pi^0$, which can proceed through the $S$-wave decay $X(3872) \rightarrow D^{*0}D^{*0} + " D^{*0}\pi^0 "$, is larger by about an order of magnitude than the measured $Br(X(3872) \rightarrow \pi^+\pi^- J/\psi)$, as mentioned above. Here $"D^{*0}"$ represents the virtual or kinematically allowed part of the $D^{*0}$ resonance. If the decay $X(3872) \rightarrow \pi^\pm\pi^- J/\psi$ proceeds through $X(3872) \rightarrow D^0 J/\psi$, as speculated by the Belle collaboration, it must be strongly suppressed, due to the isospin non-conservation. If it proceeds through $X(3872) \rightarrow f_0(600) J/\psi$, it must be a $P$-wave decay, so that it would be suppressed relatively to its $S$-wave decays.

With this in mind, we consider $X(3872) \rightarrow D^{*0}D^{*0} + " D^{*0}\pi^0 "$ as an example, and decompose $X_u(60_f) \sim [cu][\overline{c}u]$ into a sum of products of $\{\overline{c}u\}$ and $\{uc\}$ pairs. Then, we have

$$||\overline{c}u\rangle_{\overline{I}}(\overline{c}u)_{\overline{I}}\rangle_{\overline{I}} = \frac{1}{2\sqrt{3}}\{\overline{c}u\}_{\overline{I}}\{\overline{u}c\}_{\overline{I}}\langle_{\overline{I}} + \frac{1}{2}\{\overline{c}u\}_{\overline{I}}\{\overline{u}c\}_{\overline{I}}\langle_{\overline{I}} + \cdots $$

The coefficient of the first term on the r.h.s. in Eq. (4) provides the overlap of the color and spin wavefunctions between $X_u(60_f)$ and $D^{*0}D^{*0}$. It is small, as in the case of $D_{s0}^+(2317) \rightarrow D_s^+ \pi^0$. In the same way, a small overlap of the color and spin wavefunctions in the $\eta J/\psi$ and $\eta\omega$ decays of $X(3872)$ can be obtained. Here we recall that the rate for the $D_{s0}^+(2317) \rightarrow D_s^+ \pi^0$ decay is small, because of the small overlap of the color and spin wavefunctions, although the phase space volume is not necessarily small [27,28]. Similarly, the rates for the $X(3872) \rightarrow \eta J/\psi$ and $\eta\omega$ decays are expected to be small. The rate for the $X(3872) \rightarrow D^{*0}D^{*0} + " D^{*0}\pi^0 "$ decay should be smaller than the above decays, since its phase space volume is much smaller. In this way, we can understand the narrow width of $X(3872)$, although we cannot predict its value at the present stage, because we have no useful input data.

In addition, the present scheme is attractive, because it contains a rich spectrum of axial vector mesons. Among open-charm mesons, charm-strange axial-vector mesons, which are mixtures of $D_{s1}(60_f)_{\overline{I},0} \sim \{cn\} \{\overline{s}n\}_{\overline{I}=0,1}$ and $D_{s1}(60_f)_{\overline{I},0} \sim \{cn\} \{\overline{s}n\}_{\overline{I}=0,1}$, can exist, according to this model. There is reason to believe that the observed $D_{s1}^+(2460)$ [26] might be one of these $I = 1$ members, because the measured small ratio of decay rates [26], $\Gamma(D_{s1}^+(2460) \rightarrow \pi^0) / \Gamma(D_{s1}^+(2460) \rightarrow \pi^0) = 0.31 \pm 0.06$, can be easily understood in this assignment, as in the case of $D_{s0}^+(2317)$ [27,28]. By contrast, if $D_{s1}^+(2460)$ were assigned to an iso-singlet state (the conventional $\{c\}$ or an iso-singlet four-quark meson), it would be difficult to naturally understand the above ratio, because the isospin non-conserving interactions are much weaker than the electromagnetic ones, as seen above. In addition to the above charm-strange mesons, we can have various exotic axial-vector mesons with $C = \mp 2, S = \mp 1$, hidden-charm strange mesons, etc., as well as $X_I(\mp) \sim \{cn\} \{\overline{c}u\} \pm \{cs\} \{\overline{s}c\}$ and $X^\pm(\mp) \sim \{cs\} \{\overline{c}s\} \pm \{cs\} \{\overline{c}s\}$ as the partners of $X(\pm)$ in the present scenario, where $C$ and $S$ are the charm and strangeness quantum numbers, respectively. They will be studied elsewhere.

In summary, we have studied the newly discovered resonance $X(3872)$ and discussed the fact that all the existing models of $X(3872)$ have some serious problems. Then, we presented the new tetra-quark interpretation that $X(3872)$ consists of two iso-singlet tetra-quark mesons, $X(\pm) \sim \{cn\} \{\overline{c}u\} \pm \{cs\} \{\overline{s}c\}$, with opposite G parities. In this scenario, we do not need to assume a large violation of isospin and G-parity conservation in the strong interactions of $X(3872)$.
Acknowledgments

The author would like to thank the members of the Yukawa Institute for Theoretical Physics at Kyoto University. This work was motivated by discussions during the workshop YKIS2006 on "New Frontiers on QCD". He also would like to thank Professor E. Oset and Mr. D. Gamermann, IFIC, Centro Mixto Universidad de Valencia-CSIC, for valuable discussions and the nuclear theory group of the RCNP, Osaka University, for their hospitality during his stay.

[1] S.-K. Choi et al., Belle Collaboration, Phys. Rev. Lett. 93 (2003), 26200.
[2] D. Acosta et al., CDF Collaboration, Phys. Rev. Lett. 91 (2004), 072001.
[3] V. M. Abazov et al., D0 Collaboration, Phys. Rev. Lett. 91 (2004), 162002.
[4] B. Aubert et al., Phys. Rev. D 71 (2005), 071103.
[5] W.-M. Yao et al., the Particle Data Group, J. of Phys. G 33 (2006), 1, and references quoted therein.
[6] B. Aubert et al., Phys. Rev. D 71 (2005), 031501.
[7] K. Abe et al., Belle Collaboration, hep-ex/0505037.
[8] M. Suzuki, Phys. Rev. D 72, 114013 (2005).
[9] K. Abe et al., Belle Collaboration. hep-ex/0505038.
[10] G. Gokhroo et al., Phys. Rev. Lett. 97 (2006), 162002.
[11] C. Hanhart, Yu. S. Kalashnikova, A. E. Kudryavtsev and A. V. Nefediev, hep-ph/0704.0605.
[12] N. A. Törnvist, hep-ph/0308277.
[13] L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. D 71 (2005), 014028.
[14] B. A. Li, Phys. Lett. B 605 (2005), 306.
[15] K. K. Seth, Phys. Lett. B 612 (2005), 1.
[16] T. Barnes and S. Godfrey, Phys. Rev. D 69 (2004), 054008.
[17] S.-L. Zhu, hep-ph/0703225, and references quoted therein.
[18] E. Braaten and M. Kusunoki, Phys. Rev. D 71 (2005), 074005.
[19] B. Aubert et al., Phys. Rev. D 73 (2006), 011101.
[20] T. Lesiak, hep-ex/0612042, and references quoted therein,
    B. D. Yabsley, hep-ex/0702012, and references quoted therein,
    E. S. Swanson, Phys. Rep. 429 (2006), 243, and references quoted therein.
[21] R. L. Jaffe, Phys. Rev. D 15 (1977), 267; ibid (1977), 281.
[22] K. F. Liu, hep-ph/0706.1262.
[23] S. Hori, Prog. Theor. Phys. 36 (1966), 131.
[24] K. Terasaki, Phys. Rev. D 68 (2003), 011501(R).
[25] K. Terasaki, hep-ph/0309270, (in HADRON SPECTROSCOPY, the proceedings of the 10th Int. Conf. on Hadron Spectroscopy, 31 Aug. – 6 Sept. 2003, Aschaffenbug, Germany, ed. E. Klempt, H. Koch and H. Orth, AIP Conf. Proc. 717 (2004), 556.
[26] D. Besson et al., CLEO Collaboration, Phys. Rev. D 68 (2003), 032002.
[27] K. Terasaki, hep-ph/0512285.
[28] A. Hayashigaki and K. Terasaki, Prog. Theor. Phys. 114 (2005), 1191.
[29] K. Terasaki, Prog. Theor. Phys. 116 (2006), 435; Eur. Phys. J. A 31 (2007), 676; hep-ph/0704.3299.
[30] P. Krokovny et al., Belle Collaboration, Phys. Rev. Lett. 91 (2003), 262002.
[31] P. Estabrooks et al., Nucl. Phys. B133 (1978), 490.
[32] D. Gamermann and E. Oset, hep-ph/0704.2314.