Impurity Effects on Quantum Depinning of Commensurate Charge Density Waves

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(recieved April 14, 2000)

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Abstract

We investigate quantum depinning of the one-dimensional (1D) commensurate charge-density wave (CDW) in the presence of one impurity theoretically. Quantum tunneling rate below but close to the threshold field is calculated at absolute zero temperature by use of the phase Hamiltonian within the WKB approximation. We show that the impurity can induce localized fluctuation and enhance the quantum depinning. The electric field dependence of the tunneling rate in the presence of the impurity is different from that in its absence.

KEYWORDS: charge-density wave (CDW), quantum depinning, commensurability, impurity, localized fluctuation, tunneling rate, WKB approximation

1 Introduction

The tunneling of macroscopic object, the so called Macroscopic Quantum Tunneling (MQT), is one of the most dramatic quantum effects. There has been many theoretical studies of MQT. [1]
Most of them are dealing with regular systems. In many actual cases, however, some kind of randomness exists and is expected to play an important role. Here we study the effects of randomness for the case of charge-density wave (CDW) as a typical example. MQT of the commensurate CDW below but close to the threshold field was investigated by Nakaya and Hida (NH). They employed the WKB approximation in order to obtain the tunneling rate as a function of the electric field. However the effects of randomness were not considered there. In the framework of classical mechanics it is shown that the randomness can make lower the threshold field for the depinning of the commensurate CDW with one impurity. In this paper we calculate the quantum tunneling rate of the commensurate CDW with one impurity. The 1D phase Hamiltonian is adopted, and the electric field dependence of the tunneling rate is obtained by use of Langer’s method at absolute zero. It is found that the impurity can induce localized fluctuation and enhance the tunneling rate near the threshold field. We have different electric field dependence of the tunneling rate in the presence of the impurity from that given by NH in its absence.

2 The Model and the Threshold Field

The CDW is the modulation of the charge density, \( \rho(X, T) = \rho_0 \cos(2k_F X + \phi(X, T)) \), where \( \rho_0 \) is the amplitude of the modulation, \( \phi \) is its phase and \( k_F \) is the Fermi wavenumber. We investigate the 1D commensurate CDW with one impurity located at \( X_i \) at absolute zero temperature. The action of the system is described by \( \phi \) as

\[
S = \int dT \int dX \left[ A \left( \frac{\partial \phi}{\partial T} \right)^2 - B \left( \frac{\partial \phi}{\partial X} \right)^2 + F_{\text{ele}} \phi - g (1 - \cos(M\phi)) + V \cos(2k_F X + \phi) \delta(X - X_i) \right].
\]

The first term in the integrand is the kinetic energy density with \( A = (\hbar v_F)/(4\pi v_F^2) \), where \( v_F \) is the Fermi velocity and \( v_\phi \) is the phason velocity. The second is the elastic one with \( B = (\hbar v_F)/(4\pi) \) and the third is the electric field one. Here \( F_{\text{ele}} = (eE_{\text{ext}})/(2\pi) \) and \( e < 0 \) is the charge of an electron and \( E_{\text{ext}} \) is the electric field. The fourth represents the energy resulting from the commensurability, \( M \equiv \pi/(k_F a) \), and \( a \) is a lattice constant. The last is the impurity potential energy with the amplitude \( V \). In this action there exist the characteristic length and time, \( \xi = \sqrt{(2B)/(M^2 g)} \) and \( \xi_t = \sqrt{(2A)/(M^2 g)} \), which are the phase coherence length and time due to the commensurability, respectively. We scale this action by \( \xi \) and \( \xi_t \), i.e. \( x = X/\xi \),

\[ \xi = \sqrt{(2B)/(M^2 g)} \] and \[ \xi_t = \sqrt{(2A)/(M^2 g)} \]
The Euclidean action is introduced as $S_E = iS$;

$$S_E = C \int d\tau \int dx \left[ \frac{1}{2} \left( \frac{\partial \phi}{\partial \tau} \right)^2 + \frac{1}{2} \left( \frac{\partial \phi}{\partial x} \right)^2 - \varepsilon \phi ight. + \left. \frac{1}{M^2} (1 - \cos(M\phi)) - v \cos(\chi(x_i) + \phi) \delta(x - x_i) \right],$$

where $\tau = -it$ is the imaginary time, $C = M^2 g\xi t = 2\sqrt{AB}$, $\varepsilon = F/(gM^2)$, and $v = V/(g\xi M^2)$. The typical magnitude of $C$ is given as

$$C/\hbar = \sqrt{m^*/m_b}/2\pi \simeq 2 \sim 2 \times 10.$$  

Here $m^*$ and $m_b$ are the effective mass of the CDW and the band mass, respectively. We define $x_i = X_i/\xi$ and $\chi(x_i) = (2\pi z)/M$, where $z = X_i/a$ characterizes the location of the impurity relative to the crest of the undeformed CDW with minimum commensurability energy. The range of $\chi$ can be set as $-\pi/M \leq \chi \leq \pi/M$, because the action except of the impurity term has the periodicity of $2\pi/M$ with respect to the phase variable. We consider the case of $v \geq 0$ hereafter.

We first consider the threshold field at absolute zero temperature in the framework of classical static mechanics. The first variational equation of the action,

$$-\phi'' - \varepsilon + \frac{1}{M} \sin(M\phi) + v \sin(\chi + \phi) \delta(x - x_i) = 0,$$

determines the ground state configuration of the phase with the boundary conditions, $\phi(\pm \infty) = \phi_0$ and $\phi'(\pm \infty) = 0$. Here

$$\phi_0 \equiv \frac{1}{M} \arcsin(\varepsilon M)$$

is the ground state configuration in the absence of the impurity.

Depending on the range of $\chi$, there are two kinds of configurations. If $\chi$ is in the range of $-\pi/M \leq \chi \leq 0$, the solution is located in the positive side (Config.1) as shown in Fig. If $\chi$ is in the range of $0 \leq \chi \leq \pi/M$, the solution is located in the negative side (Config.2). We take electric field $\varepsilon \geq 0$. Under this condition, Config.1 has more tendency to depin than Config.2. Therefore, we consider only the case of $-\pi/M \leq \chi \leq 0$, because we are interested in the lowering of the threshold field.

The ground state solution, $\phi_\alpha(x)$, in the presence of the impurity which is determined by eq. (3) is obtained by the connection at the impurity site of two non-uniform solutions in the
absence of the impurity, $\phi_l(x)$ (Fig. 2). The non-uniform solution, $\phi_l(x)$, is given by

$$\phi'_l = - \left[ 2 \left( \frac{1}{M} \arcsin(\varepsilon M) - \phi_l \right) + \frac{1}{M^2} \left( \sqrt{1 - (\varepsilon M)^2} - \cos(M\phi_l) \right) \right]^{\frac{1}{2}},$$

(5)

which is derived by integration of

$$-\phi'' - \varepsilon + \frac{1}{M} \sin(M\phi) = 0,$$

(6)

with the boundary condition, $\phi(\pm \infty) = \phi_0$ and $\phi'(\pm \infty) = 0$. Integration of eq. (5) is obtained only by numerical treatment. When we carry out the integration practically, we need the information of the phase at the impurity site obtained analytically as follows. The ground state solution, $\phi_s(x)$, is given by

$$\phi_s(x) = \phi_l(|x - x_i| + c),$$

(7)

where $c$ represents the distance between the center of $\phi_l$ and $x_i$. Substituting eq. (7) into eq. (4), we obtain

$$-2\phi'_l(c) + v \sin(\chi + \phi_l(c)) = 0.$$  

(8)

By use of eq. (2), eq. (8) is rewritten as

$$\left[ 2 \left( \frac{1}{M} \arcsin(\varepsilon M) - \phi_l(c) \right) + \frac{1}{M^2} \left( \sqrt{1 - (\varepsilon M)^2} - \cos(M\phi_l(c)) \right) \right]^{\frac{1}{2}} + \frac{1}{2} v \sin(\chi + \phi_l(c)) = 0.$$  

(9)

This equation determines the phase, $\phi_l(c) = \phi_s(x_i)$, at the impurity site of the ground state solution, instead of $c$, when $\varepsilon$ is given. Substituting this $\phi_s(x_i)$ into eq. (3), we have the derivative at $x_i$, and then accomplish the numerical integration with the boundary conditions at $x_i$.

The threshold field is determined as the field where the ground state solution becomes unstable. The eigenvalue equation of the fluctuations, $\delta\phi(x)$, around the ground state solution, $\phi_s(x)$, is expressed as

$$\left[ -\frac{\partial^2}{\partial x^2} + V_s(x) \right] \delta\phi = \lambda \delta\phi,$$

(10)

where

$$V_s(x) \equiv \cos(M\phi_s(x)) + v \cos(\chi + \phi_s(x)) \delta(x - x_i),$$

(11)
and $\lambda$ is the eigenvalue. The threshold field is determined by the condition that the lowest $\lambda = 0$. In the absence of the impurity, $V_s(x)$ becomes
\[
\Delta_g = \sqrt{1 - (\varepsilon M)^2}.
\] (12)

Then the threshold is $\varepsilon_T = 1/M$. At $\varepsilon_T$ the barrier of the potential energy density in the absence of the impurity,
\[
U(\phi) = -\varepsilon \phi + \frac{1}{M^2}(1 - \cos(M\phi)),
\] (13)
disappears. The threshold field, $\varepsilon_c$, in the presence of the impurity [11] is lower than that in its absence under the condition of $-\pi/M \leq \chi < -\pi/(2M)$ and $v > 0$. When $v$ is fixed, $\varepsilon_c/\varepsilon_T$ goes to 1 as $\chi$ tends to $-\pi/(2M)$. We also note that the threshold field goes to a finite value, $\varepsilon_\infty$, even if $v \to \infty$. For $M = 4$ and $\chi = -\pi/M$, $\varepsilon_\infty/\varepsilon_T \simeq 0.725$, which is the lowest threshold of the present model.

First we consider the reason of the lowering of the threshold field in an aspect of the fluctuation. Due to the existence of the impurity, the potential term of the eigenvalue equation, $V_s(x)$, is non-uniform in space over the length scale of $\xi$ (Fig. 4(a)). We note that in the present dimensionless notation, $\xi$ is unity. We solve eq. (10) numerically by transforming a differential equation into a difference equation (the Numoev method) and the shooting method on $\lambda_n$. [12] [13] As the result two bound solutions, $\Phi_0(x)$ and $\Phi_1(x)$, (Fig. 4(b) and (c)), are obtained at most under the condition of $-\pi/M \leq \chi < -\pi/(2M)$ and $v > 0$ below but near the threshold field. Other solutions are non-bounded solutions whose eigenvalues form a continuous spectrum above $\Delta_g$. The eigenvalue of the bound state is smaller than the lowest eigenvalue, $\Delta_g$, of the continuous spectrum. Then applying the electric field, the lowest eigenvalue, $\lambda_0$, of the bound state becomes zero before $\Delta_g$ becomes zero. In this case the threshold field is lowered by the impurity than that in its absence. Here we note that in the absence of the impurity $V_s(x)$ is uniform in space and there exists the continuous spectrum only. In the region of $-\pi/(2M) \leq \chi$, however, there is no bound state solution of eq. (10), and the threshold is the same as that in the absence of the impurity.

The bound state corresponds to the localized fluctuation over the length scale of $\xi$ around the impurity (Fig. 4(b)). Owing to the localized fluctuation of the lowest eigenvalue, the CDW starts local sliding at the threshold field. The onset of a sliding in the absence of the impurity accompanies the spatially uniform and temporally continuous motion of the CDW. In the presence of the impurity, however, the sliding takes place around the impurity, i.e. the local sliding. Once the local sliding sets in, the CDW gains the electric field energy, which is transferred to the kinetic energy. Therefore, the sliding expands to the whole system and the whole CDW completely depins eventually, when dissipation is small enough. In the present
model dissipation is not taken into account. The issue of dissipation is discussed in the final section.

Next we clarify the meaning of $\varepsilon_c$ in an aspect of potential curves in the presence of the impurity. In the absence of the impurity the phase is uniform in the ground state, and hence the potential energy curve can be expressed with respect to the uniform phase variable as eq. (13). With the impurity, however, the phase is not uniform. Apparently we should consider the potential curve in a functional space in this case. We can reduce the problem, however, to a one-particle problem by adopting the phase variable at the impurity site, $\phi_i$, as a kind of generalized collective coordinate. For each value of $\phi_i$, we can determine the lowest energy configuration; we solve eq. (6) under the boundary condition, $\phi(x_i) = \phi_i$ and $\phi'(x_i) = \left[2 \varepsilon \left(\frac{1}{M} \arcsin(\varepsilon M) - \phi_i\right) + \frac{1}{M^2} \left(\sqrt{1 - (\varepsilon M)^2} - \cos(M\phi_i)\right)\right]^{\frac{1}{2}}$,

which is just the same as eq. (5) at the impurity site. Substituting this optimal configuration into the Euclidean Lagrangian, we obtain the potential energy,

$$U_{\text{imp}}(\phi_i) = -2\sqrt{2} \int_{\phi_i}^{\infty} d\phi \sqrt{E_0(\phi) - v \cos(\chi + \phi)},$$

where

$$E_0(\phi) = \varepsilon \left(\frac{1}{M} \arcsin(\varepsilon M) - \phi\right) + \frac{1}{M^2} \left(\sqrt{1 - (\varepsilon M)^2} - \cos(M\phi)\right).$$

The value of $\phi_i$ that minimizes $U_{\text{imp}}(\phi_i)$ is $\phi_s(x_i)$. Then we introduce a variable, $X \equiv \phi_i - \phi_s(x_i)$, and consider $U_{\text{imp}}$ as a function of $X$ hereafter. The curves of $U_{\text{imp}}(X)$ is shown in Fig. 3. For each electric field there is $\phi_{i-\text{max}}$, and for $\phi_i > \phi_{i-\text{max}}$ no such an optimal configuration exists. Therefore once the barrier disappears at $\varepsilon_c$, the sliding sets in because of the absence of any stable configuration, and then the CDW starts to depin. The value of $\phi_{i-\text{max}}$ is same as the maximum value of $\phi_l(x)$, $\phi^*$ (see Fig. 2).

3 Quantum Depinning in the Presence of an Impurity

We derive the effective action based on the one-particle picture near the threshold field, $\varepsilon_c$, and calculate the tunneling rate in the presence of the impurity within the WKB approximation by Langer’s method. The tunneling rate is enlarged and its electric field dependence is changed by the impurity from that in its absence.
3.1 The Effective Action near the Threshold Field

To investigate quantum tunneling problems theoretically by the semiclassical approximation, we must choose a suitable tunneling process, or in other words, the bounce solution. If we can derive the effective potential as a function of a single variable, the bounce solution is easily obtained by solving the equation of motion of a single degree of freedom. We call such a variable as the tunneling variable. In the present case the tunneling variable is the phase variable at the impurity site, $\phi_i$ or $X$. The potential curve has been already obtained as a function of the tunneling variable in eq. (14).

In order to derive the kinetic term, we follow the method used to derive the effective action in the Tomonaga-Luttinger model with a single barrier. We introduce the partition function of the present model as

$$Z = \int \mathcal{D}\phi \exp \left( -S_E[\phi] / \hbar \right)$$

where $S_E[\phi]$ is the Euclidean action which is given by eq. (2). The phase, $\phi(\tau, x)$, is expressed by the sum of the static part, $\phi_s(x)$, which is the solution of eq. (4), and the fluctuation part, $\delta\phi(\tau, x)$;

$$\phi(\tau, x) = \phi_s(x) + \delta\phi(\tau, x).$$

Substituting eq. (16) into the action and expanding it in powers of $\delta\phi(\tau, x)$, we obtain the action up to the second order of $\delta\phi(\tau, x)$ as $S_E \simeq S_0 + S_2$, where

$$S_0 = C \int d\tau dx \left[ \frac{1}{2} \left( \frac{\partial \phi_s(x)}{\partial x} \right)^2 + V(\phi_s(x)) \right],$$

$$S_2 = C \int d\tau dx \left[ \delta\phi(\tau, x) \frac{1}{2} \left( -\frac{\partial^2}{\partial \tau^2} - \frac{\partial^2}{\partial x^2} + V''(\phi_s(x)) \right) \delta\phi(\tau, x) \right].$$

Here

$$V(\phi) = -\varepsilon\phi + \frac{1}{M^2} (1 - \cos(M\phi)) - v \cos(\chi + \phi) \delta(x - x_i).$$

In order to integrate the phase, $\delta\phi(\tau, x)$, under the condition, $\delta\phi(\tau, x_i) = \phi_i(\tau) - \phi_s(x_i) = X(\tau)$, auxiliary fields, $\Lambda(\tau)$, are introduced and the identity,

$$1 = \int \mathcal{D}X(\tau) \int \mathcal{D}\Lambda(\tau) \exp \left[ i \int d\tau \Lambda(\tau) (\delta\phi(\tau, x_i) - X(\tau)) \right],$$

is employed. Then the partition function is expressed as

$$Z \simeq \exp \left( -S_0 / \hbar \right) \int \mathcal{D}\delta\phi(\tau, x) \int \mathcal{D}X(\tau) \int \mathcal{D}\Lambda(\tau)$$

$$\times \exp \left[ -S_2 / \hbar + iC / \hbar \int d\tau \Lambda(\tau) (\delta\phi(\tau, x_i) - X(\tau)) \right].$$
We diagonalize $S_2$ by use of the eigenfunction;

$$\delta \phi (\tau, x) = \sum_{m,n} c_{m,n} \exp (i\omega_m \tau) \Phi_n(x),$$

where $\Phi_n(x)$ is the eigenfunction of eq. (11) (see eqs. (12) and (18), and note that $V''(\phi_s(x)) = V_\epsilon(x)$), and $c_{m,n}$ are the expansion coefficients. Then the partition function becomes

$$Z \simeq \exp (-S_0/\hbar) \int \prod_{m,n} dc_{m,n} \int D\tau \int D\Lambda(\tau)$$

$$\times \exp \left[ -\frac{1}{2} \hbar \sum_{m,n} \left( |c_{m,n}|^2 (\omega_m^2 + \lambda_n) - i2c_{m,n}\Lambda(\omega_m)\Phi_n(x) + i2\Lambda(\omega_m)X(-\omega_m) \right) \right],$$

where

$$\Lambda(\omega_m) = \int d\tau \Lambda(\tau) \exp (i\omega_m \tau),$$

$$X(\omega_m) = \int d\tau X(\tau) \exp (i\omega_m \tau).$$

Twice Gaussian integrations bring

$$Z \simeq \exp (-S_0/\hbar) \int D\tau \int D\Lambda(\tau)$$

$$\times \exp \left[ -\frac{1}{2} \hbar \sum_{m,n} \frac{\Lambda(\omega_m)\Lambda(-\omega_m)}{\omega_m^2 + \lambda_n} |\Phi_n(x)|^2 + i2\Lambda(\omega_m)X(-\omega_m) \right]$$

$$\simeq \exp (-S_0/\hbar) \int D\tau \exp \left[ -\frac{1}{2} \hbar \sum_{m,n} \frac{X(\omega_m)X(-\omega_m)}{\sum_n \frac{|\Phi_n(x)|^2}{\omega_n^2 + \lambda_n}} \right].$$

Here unimportant prefactors are neglected. Then the harmonic terms of the effective action is given by

$$S_{\text{harm}} = \frac{1}{2} C \sum_m \frac{X(\omega_m)X(-\omega_m)}{\sum_n \frac{|\Phi_n(x)|^2}{\omega_n^2 + \lambda_n}}. \quad (19)$$

First we can now carry out the summation over $n$ in eq. (19). Equation (19) is expressed as a sum of the discrete eigenvalue part and the continuous one; i.e.

$$\sum_n \frac{|\Phi_n(x)|^2}{\omega_n^2 + \lambda_n} = \frac{|\Phi_0(x)|^2}{\omega_0^2 + \lambda_0} + \frac{L}{2\pi} \int_{-\infty}^{\infty} dk \frac{1}{\omega_m^2 + \lambda(k)}. \quad (20)$$
where \( L \) is the system size. Note that the first-excited state solution, \( \Phi_1(x) \), has odd parity and does not appear in the above equation. In eq. (20), \( k \) is a continuous parameter which characterizes the continuous eigenvalues instead of \( n \). The integration of the second term can be carried out analytically with the approximation, \( \Phi(k; x_i) \approx \sqrt{1/L} \exp(ikx_i) \). This is valid in the low temperature regime, which we are interested in, because the continuous states are higher energy states and give just a corrective contribution. Then

\[
\frac{L}{2\pi} \int_{-\infty}^{\infty} dk \frac{|\Phi(k; x_i)|^2}{\omega_m^2 + \lambda(k)} \approx \frac{1}{2\pi} \int_{-\infty}^{\infty} dk \frac{1}{\omega_m^2 + k^2 + \Delta_g} = \frac{1}{2} \frac{1}{\sqrt{\omega_m^2 + \Delta_g}}.
\]

Next we consider the summation over \( \omega_m \). In order to obtain the tunneling rate, we calculate the bounce solution and substitute it into the action. The bounce solution has a characteristic time scale, \( \omega_B^{-1} \), which is shown in Fig. 5. Then the main contribution to the summation over \( \omega_m \) comes from terms with \( \omega_m \approx \omega_B \). Now we expand \( S_{\text{harm}} \) into powers of \( \omega_m \) up to the second order;

\[
\sum_m \frac{X(\omega_m)X(-\omega_m)}{\omega_m^2 + \lambda_0} \approx \sum_m \left( M_0 \omega_m^2 + K_0 \right) X(\omega_m)X(-\omega_m),
\]

where

\[
M_0 = \frac{|\Phi_0(x_i)|^2 + \frac{\lambda_0^2}{4\Delta_g \sqrt{\Delta_g}}}{\left( |\Phi_0(x_i)|^2 + \frac{\lambda_0^2}{2\sqrt{\Delta_g}} \right)^2}
\]

and

\[
K_0 = \frac{\lambda_0}{|\Phi_0(x_i)|^2 + \frac{\lambda_0}{2\sqrt{\Delta_g}}}.
\]

This approximation is relevant under the condition of \( \omega_B^2/\lambda_0 \ll 1 \). Here \( \omega_B \) is the characteristic energy of the motion of the phase variable at the impurity site, \( X(\tau) \), and \( \lambda_0 \) is that of the spatial fluctuation, \( \Phi_0(x) \), around the ground state. Then condition, \( \omega_B^2/\lambda_0 \ll 1 \), means the spatial fluctuation around the ground state follows the motion of \( X(\tau) \) adiabatically. This condition is checked self-consistently later.

Finally we obtain the total effective action as

\[
S_{\text{total}}^{\text{eff}} = C \int d\tau \left[ \frac{1}{2} M_0 \left( \frac{\partial X}{\partial \tau} \right)^2 + U_{\text{imp}}(X) \right],
\]
where $U_{\text{imp}}(X)$ is the potential energy given by eq. (14) with $X = \phi_i - \phi_s(x_i)$. Here the harmonic term of the potential, $K_0X^2$ in eq. (21), is removed in order to avoid a double counting. We are interested in the tunneling near the threshold field, $\varepsilon_c$, where we can expand $U_{\text{imp}}$ into powers of $X$ and obtain

$$U_{\text{imp}}(X) \simeq a_{\text{imp}}X^2 - b_{\text{imp}}X^3,$$

where

$$a_{\text{imp}} = \frac{1}{2} \left[ -\frac{4}{M} \left( -\varepsilon + \frac{1}{M} \sin(M\phi_{si}) \right) \right]$$

and

$$b_{\text{imp}} = \frac{1}{6} \left[ -\left( 4 \left( -\varepsilon + \frac{1}{M} \sin(M\phi_{si}) \right) \right)^2 \right] + \frac{4}{(v \sin(\chi + \phi_{si}))^3}.$$  

Here $\phi_{si} \equiv \phi_s(x_i)$, which is determined by eq. (3) for each field. Then the effective action near $\varepsilon_c$ is

$$S_{\text{eff}} = C \int d\tau \left[ \frac{1}{2} M_0 \left( \frac{\partial X}{\partial \tau} \right)^2 + a_{\text{imp}}X^2 - b_{\text{imp}}X^3 \right].$$  

(22)

### 3.2 Quantum Tunneling Rate near the Threshold Field

We calculate the tunneling rate using the effective action near the threshold field. The tunneling rate within the WKB approximation is given by

$$\Gamma_{\text{imp}} \propto \exp \left[ -S_{\text{eff}}[X_B]/\hbar \right]$$

where $X_B(\tau)$ is the bounce solution of the equation of motion which is derived from $S_{\text{eff}}$ as

$$-M_0 \frac{\partial^2 X}{\partial \tau^2} + 2a_{\text{imp}}X - 3b_{\text{imp}}X^2 = 0.$$  

This equation of motion can be solved analytically and gives the solution as

$$X_B(\tau) = \frac{A_B}{\cosh^2(\omega_B \tau)},$$

where

$$A_B = \frac{a_{\text{imp}}}{b_{\text{imp}}},$$

$$\omega_B = \sqrt{\frac{a_{\text{imp}}}{2M_0}}.$$
Substituting $X_B(\tau)$ into $S_{\text{eff}}[X]$ and carrying out the $\tau$-integration, we obtain the tunneling rate as

$$\Gamma_{\text{imp}} \propto \exp \left[ -\frac{C}{\hbar} \frac{8}{15} \sqrt{2M_0 a_{\text{imp}}^2 b_{\text{imp}}^2} \right].$$  \hspace{1cm} (23)

The obtained tunneling rates are shown in Figs. 6 and 7. Here we take the ratio of the effective mass of the CDW to the mass of the band electron as $m^*/m_b = 1 \times 10^3$. This parameter determines a value of $C$ (see eq. (3)). The $v$-dependence of the tunneling rate is shown in Fig. 6 for $M = 4$ and $\chi = -\pi/M$, which gives the minimum value of the classical threshold field. On the other hand, the $\chi$-dependence of the tunneling rate is shown in Fig. 7 for $M = 4$ and $v = 1$.

The condition, $\omega_B^2/\lambda_0 \ll 1$, that means the present effective action is relevant, is satisfied in all the region in Figs. 6 and 7. We also note that the WKB approximation is relevant in the region of $-\ln \Gamma_{\text{imp}} = S_{\text{eff}}[X_B]/\hbar < 1$.

The results shown in Figs. 6 and 7 can be expressed as

$$\Gamma_{\text{imp}} \propto \exp \left[ -\gamma(v, \chi) \left( 1 - \frac{\varepsilon}{\varepsilon_c(v, \chi)} \right)^{\alpha(v, \chi)} \right].$$  \hspace{1cm} (24)

Here $\alpha$ is a kind of generalized critical exponent and $\gamma$ is some factor. The $v$ and $\chi$-dependencies of $\alpha$ are shown in Figs. 8 and 9, respectively. When $v$ exceeds 2 in the case of $\chi = -\pi/M$ ($M = 4$) and $\chi$ exceeds $-7\pi/(8M)$ ($M = 4$) in the case of $v = 1$, either of the two conditions corresponding to the relevances of the present effective action, $\omega_B^2/\lambda_0 \ll 1$, and of the WKB approximation gradually becomes invalid at any electric field. This means the present effective model is relevant in a limited region in the parameter space. Dependencies of $\gamma$ on $v$ and $\chi$ are shown in Fig. 10 and Fig. 11. In the relevant region, $\alpha$ depends on $v$ strongly, but does not on $\chi$.

In the absence of the impurity, an effective theory near the threshold field was given by Nakaya and Hida (NH) based on the scaling argument.  \cite{3} The effective action of this theory is obtained by expanding $U(\phi)$, eq. (13), at the inflection point, which yields

$$S_{\text{E-eff}} = C \left( \frac{a_1}{b_1} \right)^2 \int d\tilde{\tau} d\tilde{x} \left[ \frac{1}{2} \left( \frac{\partial \tilde{\phi}}{\partial \tilde{\tau}} \right)^2 + \frac{1}{2} \left( \frac{\partial \tilde{\phi}}{\partial \tilde{x}} \right)^2 + \tilde{\phi}^2 - \tilde{\phi}^3 \right]$$

where $\tilde{\phi} = (a_1/b_1)\phi_{\text{eff}}$, $\tilde{\tau} = \sqrt{a_1} \tau$, $\tilde{x} = \sqrt{a_1} x$, $\phi_{\text{eff}} = \phi - \pi/(2M) + (2/M)\sqrt{(1-\varepsilon/\varepsilon_T)/2}$, $\alpha_1 \equiv \sqrt{(1-\varepsilon/\varepsilon_T)/2}$, and $b_1 \equiv (1/3! M)$. Therefore we obtain the electric field dependence of
the tunneling rate, $\Gamma_0$, without any knowledge about the bounce solution as

$$\Gamma_0 \propto \exp \left[ -B_0 \left( 1 - \frac{\xi}{\xi_T} \right)^1 \right].$$

NH evaluated $B_0$ numerically as

$$B_0 \simeq \frac{C}{\hbar} \frac{6.2 \times 10^1}{M^2},$$

in the present notation. We compare the present result with NH’s. In the limit of vanishing impurity potential, $\alpha$ in eq. (24) tends to unity which was given by NH in the absence of the impurity. On the other hand $\gamma$ in the present model, around $6 \times 10^1$, is about three times larger than that given by NH ($B_0 \simeq 2.0 \times 10^1$ for the present parameter.).

We conclude that in the presence of the impurity $\alpha$ is larger than unity in the absence of the impurity. In the small limit of $v$, however, $\alpha$ becomes close to unity. Values of $\gamma$ given by the present calculation is about three times larger than that given by NH. Then present results in the small $v$ limit shows the consistency with those of NH qualitatively, however, does not show exact coincidence quantitatively. The present effective model is applicable when the localized fluctuation can follow the motion of the phase at the impurity site, $X(\tau)$, adiabatically. Then the origin of the $v$-dependence of $\alpha$ seems to come from the existence of the localized fluctuation over the length scale of $\xi$ around the impurity, as in the case of the lowering of the classical threshold field. We also conclude that the impurity can enhance the tunneling rate near the threshold field, because $\alpha > 1$ in that case. According to the scaling argument by NH, the lower dimensional system has larger $\alpha$: spatial dimension = 1, 2 and 3 have $\alpha = 1$, $3/4$ and $1/2$, respectively. Then $\alpha > 1$ means that the impurity brings the 1D system to the lower-dimensional system effectively.

4 Conclusion and Discussions

We have investigated the quantum depinning of the commensurate CDW with one impurity at absolute zero temperature theoretically. The impurity causes the different electric field dependence of the tunneling rate from that in its absence and enhances the quantum depinning. In this issue, the fluctuation around the ground state plays an important role. It is described by the eigenvalue equation (10). In the absence of the impurity the eigenvalue equation has no bound state. On the other hand, in the presence of the impurity under the condition of $-\pi/M \leq \chi < -\pi/(2M)$ the eigenvalue equation has a bound state, which means there is a spatially localized fluctuation around the impurity. This localized fluctuation which is induced by the impurity is the origin of the local sliding around the impurity and assists the depinning.
Finally we discuss future problems. The model which has been treated in the present paper offers some fundamental understandings of the effects of impurities on the depinning of the commensurate CDW. However, it is an idealized model. In actual cases, the following three points are important; dissipation, a finite density of impurities and three dimensionality. The dissipation is neglected in the present model. In the model once the local sliding occurs, the sliding expands to the whole system by the energy gain from the electric field. If a strong dissipation exists, however, the local sliding will be affected. Realistic systems contain not only one impurity but a finite density of impurities. However, the present result is expected to be applicable to the commensurate CDW with dilute but with macroscopic numbers of impurities where the inverse of the impurity density is larger than the phase coherence length, $\xi$. The depinning will be triggered by the local sliding at the optimal impurity site, $\chi = -\pi/M$. The effects of the high density of impurities is remained as a future problem. In one-dimensional systems, the impurity, a zero-dimensional object, has stronger effects than in higher dimensional systems. Generalization of the present model to three dimensional systems is left for future studies.

Acknowledgments

We would like to thank Hiroshi Kohno for fruitful discussions. The present work is a part of Doctor Thesis of one of the authors (M.Y). He is grateful for precious advice given by members of the committee for judgment: Hiroshi Fukuyama, Masao Ogata, Hajime Takayama, Seigo Tarucha and Miki Wadati (University of Tokyo).

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Figure 3: The potential curves in the presence of an impurity for a choice of $M = 4$, $\chi = -\pi/M$, $v = 1$ and $\varepsilon/\varepsilon_c = 0.90$. The origin of the potential is set at $\phi_i = \phi_s(x_i) = 0$ and $U_{\text{imp}}(\phi_s(x_i)) = 0$. The approximated $U_{\text{imp}}$ in terms of $X^2$ and $X^3$ ($X = \phi_i - \phi_s(x_i)$) is represented by the dotted line. For $\phi_i > \phi_{i-\text{max}}$, there is no optimal configuration.
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Figure 6: The tunneling rates in the presence of the impurity for $M = 4$ and $\chi = -\pi/M$. The electric field dependencies are different each other for various $v$ values. The dotted line shows the result of Nakaya and Hida.

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Figure 9: The $\chi$-dependence of $\alpha$ for $M = 4$ and $v = 1$. In this case, $\alpha$ does not depends on $\chi$ strongly. When $\chi$ exceeds $-7\pi/(8M)$ ($M = 4$) in the case of $v = 1$, the present calculation gradually becomes unreliable. Note that the left most data point (open square) should be considered as semiquantitative one because of irrelevancy of the approximation we took (see text).

Figure 10: The $v$-dependence of $\gamma$ for $M = 4$ and $\chi = -\pi/M$. Values of $\gamma$ distribute around the value, $6 \times 10^1$.

Figure 11: The $\chi$-dependence of $\gamma$ for $M = 4$ and $v = 1$. The value of $\gamma$ monotonically increases as $\chi$ decreases. Note that the left most data point (open square) should be considered as semiquantitative one because of irrelevancy of the approximation we took (see text).
\[ \frac{\phi}{(\pi/M)} \]

\[ \epsilon = 0.1 \quad \chi = -\pi/M \]

\[ M = 4 \quad v = 1 \]

\[ \phi^* \]

\[ \phi_1 \]

\[ \phi \]

\[ \chi \]

\[ s \]

\[ c \]

\[ x - x_1 \]

Fig. 2
\[ \frac{\varepsilon}{\varepsilon_c} = 0.90 \]

Approximated \( U_{\text{imp}} \)

\( U_{\text{imp}} \)

\( \phi_{i-\text{max}} \)

Fig. 3
\[ \frac{\varepsilon}{\varepsilon_c} = 0.95 \]
Fig. 4(b)

\[ \varepsilon / \varepsilon_c = 0.95 \]

\[ \lambda_0 = 0.3052 \]

\[ \Phi_0(0) = 0.3879 \]
$\Phi_1$ vs $x-x_1$

$\varepsilon/\varepsilon_c = 0.95$

$\lambda_1 = 0.5451$

$\Phi_1(0) = 0$

Fig. 4(c)
\[-\ln \Gamma_{\text{imp}} = S_{\text{eff}}[X_B]\]

Fig. 6

\[M = 4, \quad \chi = -\pi / M\]
\[-\ln \Gamma_{\text{imp}} = S_{\text{eff}} [X_B] \]

Fig. 7

\[ -\frac{\pi}{M} \]
\[ -\frac{31\pi}{32M} \]
\[ -\frac{15\pi}{16M} \]
\[ -\frac{7\pi}{8M} \]
\[ \alpha \]

Fig. 9

\[ M = 4 \\
\nu = 1 \]
Fig. 10

\[ M = 4 \]

\[ \chi = -\frac{\pi}{M} \]
$\gamma$ vs $\chi$

- $\chi = \frac{\pi}{M}$
- $\chi = \frac{7\pi}{8M}$
- $\chi = \frac{3\pi}{4M}$
- $\chi = \frac{\pi}{2M}$

$M = 4$

$\nu = 1$

Fig. 11