Multimode cat-state entanglement and network teleportation

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Schemes for generation and protocols for network teleportation of multimode entangled cat-states are proposed. Explicit expressions for probability of successful teleportation are derived for both symmetric and asymmetric networks.

Keywords: Multimode entanglement, cat-state, teleportation.

1. Introduction
Entanglement as the most characteristic trait of quantum mechanics is thought to be one of the few main keys to open the door to future cyber-information development including superdense coding, telecloning, quantum cryptography, quantum computation, etc. In particular, inspired by the more-than-surprising idea by Bennett et al. [1], who for the first time proposed a quantum way beautifully based on entanglement to teleport an unknown qubit between two space-like distant stations, various other teleportation protocols have been dealt with. Nowadays pure photonic [2], atomic [3] states as well as entangled states [4] and states with continuous variables [5] can be teleported successfully. Schrödinger cat-states [6] can also be teleported employing entangled coherent states [7] as a quantum channel. Recently, it turns out that single qubits and entangled qubits can be encoded using cat-states of bosonic fields [8]. As an example, Schrödinger cat-states have been proposed as well using pure [12–14] or mixed [15] quantum channels which are again types of (entangled) cat-states. The cat-state to be teleported is of the general multimode form.

$$|\Psi\rangle_{a_1a_2...a_L} = A_{a_1a_2...a_L} (x|\alpha\rangle_{a_1}|\alpha\rangle_{a_2}...|\alpha\rangle_{a_L} + y|\alpha\rangle_{a_1}|-\alpha\rangle_{a_2}...|-\alpha\rangle_{a_L}),$$

where

$$|\alpha\rangle_{a_j} = \exp[-|\alpha|^2/2] \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} (a_j^+)^n |0\rangle_{a_j},$$

with $\alpha \in \mathbb{C}$, $a_j^+$ ($a_j$) and $|0\rangle_{a_j}$ the photon creation (annihilation) operator and the vacuum of mode $j$, is a coherent state,

$$A_{a_1a_2...a_L} = (|x|^2 + |y|^2 + 2\exp(-2L|\alpha|^2)\Re(x^*y))^{-1/2}$$

is the normalization coefficient, $L = 1, 2, ...$ and $x, y$ are unknown complex amplitudes. We consider two types of networks: symmetric and asymmetric. By symmetric network we mean a network all parties of which are equivalent to each other. That is, any one can hold the state to be teleported and any another one is able to receive it. The quantum channel can be shared equally among the parties before an actual action of teleportation takes place and therefore teleportation can be performed between any pair of parties within the symmetric network. By asymmetric network we mean a network in which one party who is in possession with the state to be teleported is distinguished

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It can be proved by induction that the quantum channel cannot be shared by the parties until a decision is made of who is the teleposing party, i.e., who holds the state to be teleported. After this is decided the parties start to share the quantum channel but the sharing is unequal among the participants. Teleportation can go only from one certain party to another party in the asymmetric network. The structure of the cat-state to be teleported, state (1), suggests that we use as a quantum channel multimode entangled cat-states (also called multimode even/odd entangled coherent states) of the form [19]

\[ |\Psi_M^\pm\rangle = A_M^\pm |\Psi_M^\pm\rangle, \]  

with \( M = 2, 3, \ldots, \)

\[ |\Psi_M^\pm\rangle = |\Phi_M^+\rangle \pm |\Phi_M^-\rangle, \]  

\[ |\Phi_M^+\rangle = |\pm\alpha_1\rangle |\pm\alpha_2\rangle \ldots |\pm\alpha_M\rangle, \]  

and

\[ A_M^\pm = \left[ 2 \left( 1 \pm \exp(-2M|\alpha|^2) \right) \right]^{-1/2} \]  

the normalization coefficients.

We divide our paper into Introduction and three more sections. In section 2 we prepare the general multimode entangled cat-states by two schemes using only linear optical devices combined with simple parity measurements. These states are to be served as a quantum channel in section 3 to teleport an unknown state \( |\Psi_{a_1 a_2 \ldots a_L}\rangle \). Relevant discussions are provided in each section and the paper ends up with a conclusion, the final section.

2. Generation schemes

In this section we propose two schemes to generate the multimode entangled cat-states \( |\Psi_M^\pm\rangle \) with an arbitrary integer \( M \geq 2 \), given a single-mode odd/even coherent state (which is easily produced from a single-mode coherent state by means of a Kerr-medium [20]). In the first scheme only linear optics is needed while in the second scheme some measurements should be accompanied.

As is well-known, a lossless beam-splitter is described by the unitary operator

\[ \hat{b}_{k,l}(\theta) = \exp[i\theta(a_k^+ a_l + a_l^+ a_k)] \]  

and a phase-shifter by

\[ \hat{P}_j(\varphi) = \exp(-i\varphi a_j^+ a_j). \]  

We shall use two types of modified beam-splitters defined as follows

\[ \hat{B}_{k,l}(\theta) \equiv \hat{b}_{k,l}(\theta)\hat{P}_l(\pi/2), \]  

\[ \hat{B}_{k,l} \equiv \hat{P}_l(\pi/2)\hat{b}_{k,l}(\pi/4)\hat{P}_l(\pi/2). \]  

They act on coherent states as

\[ \hat{B}_{k,l}(\theta) |\alpha\rangle_k |\beta\rangle_l = |\alpha \cos\theta + i\beta \sin\theta\rangle_k |\alpha \sin\theta - i\beta \cos\theta\rangle_l, \]  

\[ \hat{B}_{k,l} |\alpha\rangle_k |\beta\rangle_l = \left| (\alpha + \beta)/\sqrt{2} \rightangle_k \left| (\alpha - \beta)/\sqrt{2} \right\rangle_l. \]  

While the first generation scheme uses operators \( \hat{B}_{k,l}(\theta) \), the second one is based on operators \( \hat{B}_{k,l} \). By reasons that will be made clear later, the first scheme is referred to as Ladder-scheme and the second one as Tree-scheme.

2.1. Ladder-scheme

It can be proved by induction that

\[ \prod_{q=M-1}^{1} \hat{B}_{q,q+1}(\theta_{M+1-q}) |\pm\alpha \sqrt{M}\rangle_1 |0\rangle_2 \ldots |0\rangle_M = |\Phi_M^\pm\rangle \]  

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where

\[ \theta_n = \cos^{-1} \frac{1}{\sqrt{n}}. \] (15)

Indeed, for \( M = 2 \), Eq. (14) reduces at once to Eq. (12) with the replacements \( \alpha \to \pm \alpha \sqrt{2}, \beta \to 0 \). We assume Eq. (14) being true for a given \( M > 2 \) and try to prove that

\[
\prod_{q=M}^{1} \hat{B}_{q,q+1}(\theta_{M+2-q}) \left| \pm \alpha \sqrt{M+1} \right\rangle_1 |0\rangle_2 \cdots |0\rangle_{M+1} = |\Phi^\pm_{M+1} \rangle
\] (16)

is also true. The l.h.s. of Eq. (16) can be rewritten as

\[
\prod_{q=M}^{2} \hat{B}_{q,q+1}(\theta_{M+2-q}) \hat{B}_{1,2}(\theta_{M+1}) \left| \pm \alpha \sqrt{M+1} \right\rangle_1 |0\rangle_2 \cdots |0\rangle_{M+1}
\] (17)

which by virtue of Eqs. (12) and (15) becomes

\[
|\pm \alpha \rangle_1 \prod_{q=M}^{2} \hat{B}_{q,q+1}(\theta_{M+2-q}) \left| \pm \alpha \sqrt{M} \right\rangle_2 |0\rangle_3 \cdots |0\rangle_{M+1}.
\] (18)

If in Eq. (14) a change of mode labelling \( q \to q + 1 \) is made this equation will look like

\[
\prod_{q=M}^{2} \hat{B}_{q,q+1}(\theta_{M+2-q}) \left| \pm \alpha \sqrt{M} \right\rangle_2 |0\rangle_3 \cdots |0\rangle_{M+1} = |\pm \alpha \rangle_2 |\pm \alpha \rangle_3 \cdots |\pm \alpha \rangle_{M+1}.
\] (19)

Taking Eq. (19) into account in Eq. (18) proves the correctness of Eq. (16). The desired states \( |\Psi^\pm_M \rangle \) are thus generated as

\[
|\Psi^\pm_M \rangle = A^+_M \hat{B}_M \left( |\alpha \sqrt{M} \rangle_1 \pm |\alpha \sqrt{M} \rangle_1 \right) |0\rangle_2 |0\rangle_3 \cdots |0\rangle_M
\] (20)

where the operator \( \hat{B}_M \) is defined by

\[
\hat{B}_M = \prod_{q=M}^{1} \hat{B}_{q,q+1}(\theta_{M+1-q}).
\] (21)

The schema of arrangement of beam-splitters \( \hat{B}_{q,q+1}(\theta_{M+1-q}) \), Fig. 1 a, looks like a ladder leading to the name Ladder-scheme.

2.2. Tree-scheme

Suppose that we have the state

\[
|\pm \alpha \sqrt{2} \rangle_1 |\pm \alpha \sqrt{2} \rangle_2 \cdots |\pm \alpha \sqrt{2} \rangle_{2^{Q-1}} |0\rangle_{1+2^{Q-1}} |0\rangle_{2+2^{Q-1}} \cdots |0\rangle_{2^Q} = \prod_{j=1}^{2^{Q-1}} |\pm \alpha \sqrt{2} \rangle_j \prod_{k=1}^{2^{Q-1}} |0\rangle_{k+2^{Q-1}}
\] (22)

with \( Q \) a positive integer. An action of the sequence of operators

\[
\hat{B}_{1,1+2^{Q-1}} \hat{B}_{2,2+2^{Q-1}} \cdots \hat{B}_{2^{Q-1},2^Q} \equiv \prod_{q=1}^{2^{Q-1}} \hat{B}_{q,q+2^{Q-1}}
\] (23)

on the state (22), on account of Eq. (13), yields the states \( |\Phi^\pm_{2^Q} \rangle \), i.e.

\[
\prod_{q=1}^{2^{Q-1}} \hat{B}_{q,q+2^{Q-1}} \prod_{j=1}^{2^{Q-1}} |\pm \alpha \sqrt{2} \rangle_j \prod_{k=1}^{2^{Q-1}} |0\rangle_{k+2^{Q-1}} = |\Phi^\pm_{2^Q} \rangle.
\] (24)
Repeated use of the above equation in its l.h.s. eventuates in
\begin{equation}
\widehat{B}_{2Q} \left| \pm \alpha \sqrt{2Q} \right\rangle \prod_{k=2}^{2Q} \left| 0 \right\rangle_k = \left| \Phi_{2Q}^{\pm} \right\rangle
\end{equation}

where the operator \( \widehat{B}_{2Q} \) is defined by
\begin{equation}
\widehat{B}_{2Q} = \prod_{l=Q}^{1} \left( \prod_{q=1}^{2^{l-1} - 1} \widehat{B}_{q,q+2^{l-1}} \right).
\end{equation}

The state \( \left| \Psi_{M}^{\pm} \right\rangle \) with \( M = 2^Q \) can thus be generated as
\begin{equation}
\left| \Psi_{M=2Q}^{\pm} \right\rangle = A_{2Q}^{\pm} \widehat{B}_{2Q} \left( \left| \alpha \sqrt{2Q} \right\rangle_1 \pm -\alpha \sqrt{2Q} \right\rangle_1 \prod_{k=2}^{2Q} \left| 0 \right\rangle_k .
\end{equation}

The schema of arrangement of beam-splitters \( \widehat{B}_{q,q+2^{l-1}} \), Fig. 1 b, resembles a tree suggesting the name Tree-scheme.

For \( M \neq 2^Q \) additional measurements are needed. To see what kinds of measurement are required let us suppose that we have the states \( \left| \tilde{\Psi}_K^{\pm} \right\rangle \) with \( K = 2, 3, ... \). It is a straightforward matter to verify the following equality
\begin{equation}
\left| \tilde{\Psi}_K^{\pm} \right\rangle = \frac{1}{2} \left( \left| \tilde{\Psi}^{-}_{K-1} \right\rangle \left| \mp \right\rangle_{K} + \left| \tilde{\Psi}^{+}_{K-1} \right\rangle \left| \pm \right\rangle_{K} \right) \end{equation}

where
\begin{equation}
\left| \pm \right\rangle_{K} = \left| \alpha \right\rangle_{K} \pm -\left| \alpha \right\rangle_{K} .
\end{equation}

Repeatedly applying Eq. (28) to its r.h.s. we have established the following general formula valid for an integer \( J \in [1,K] \)
\begin{equation}
\left| \tilde{\Psi}^{\pm}_{K} \right\rangle = \left| \tilde{\Psi}^{-}_{K-J} \right\rangle \left| F^{\mp}_{J} \right\rangle + \left| \tilde{\Psi}^{+}_{K-J} \right\rangle \left| F^{\mp}_{J} \right\rangle .
\end{equation}

In Eq. (30) the \( J \)-mode states \( \left| F_{J}^{\mp} \right\rangle \) are determined recurrently as
\begin{equation}
\left| F_{J}^{\mp} \right\rangle = \frac{1}{2} \left( \left| + \right\rangle_{K-J+1} \left| F^{\mp}_{J+1} \right\rangle + \left| - \right\rangle_{K-J+1} \left| F^{\mp}_{J-1} \right\rangle \right)
\end{equation}

with the conventions \( \left| F^{+}_{0} \right\rangle = 1 \) and \( \left| F^{-}_{0} \right\rangle = 0 \).

The state \( \left| + \right\rangle \) \( \left| - \right\rangle \) has parity \( \pi_{j} = +1 \) \( -1 \) because it contains only an even (odd) number of photons. It is not difficult to realize from Eq. (31) that all the states \( \left| F^{+}_{k} \right\rangle \) \( \left| F^{-}_{k} \right\rangle \) with \( k > 0 \) have a positive (negative) parity. The cruciality of Eq. (30) rests in the fact that having \( \left| \tilde{\Psi}^{\pm}_{K} \right\rangle \) we are able to generate \( \left| \tilde{\Psi}^{\pm}_{K-J} \right\rangle \) or \( \left| \tilde{\Psi}^{-}_{K-J} \right\rangle \) by state-parity measurements to be described below. Let us have \( \left| \tilde{\Psi}^{\pm}_{K} \right\rangle \) and we wish to generate \( \left| \tilde{\Psi}^{\pm}_{K-J} \right\rangle \) \( \left| \tilde{\Psi}^{-}_{K-J} \right\rangle \). For that purpose, we measure state-parities of \( J \)-modes \( \{ K - J + 1, K - J + 2, ... , K \} \). If the outcome gives a total parity \( \prod_{k=1}^{J} \pi_{K-J+k} = +1 \) \( -1 \), then the collapsed state is the desired \( \left| \tilde{\Psi}^{\pm}_{K-J} \right\rangle \) \( \left| \tilde{\Psi}^{-}_{K-J} \right\rangle \). On the contrary, if we start from \( \left| \tilde{\Psi}^{\pm}_{K} \right\rangle \), then the outcome \( \prod_{k=1}^{J} \pi_{K-J+k} = +1 \) \( -1 \) means generation of the state \( \left| \tilde{\Psi}^{-}_{K-J} \right\rangle \) \( \left| \tilde{\Psi}^{\pm}_{K-J} \right\rangle \) instead.

We now exploit Eq. (30) to generate states \( \left| \Psi_{M}^{\pm} \right\rangle \) when \( M \) is not a power of 2. In this case, there always exist integers \( Q_{j} \) such that \( M < 2^{Q_{j}} \). Among \( Q_{j} \) let \( Q \) be the smallest one, i.e. \( Q = \min \{ Q_{j} \} \). To generate \( \left| \Psi_{M}^{\pm} \right\rangle \) we first act \( \widehat{B}_{2Q} \) (see the definition (26)) on \( A_{2Q}^{\pm} \left( \left| \alpha \sqrt{2Q} \right\rangle_1 \pm -\alpha \sqrt{2Q} \right\rangle_1 \prod_{k=2}^{2Q} \left| 0 \right\rangle_k \) to prepare the state \( \left| \Psi_{2Q}^{\pm} \right\rangle = A_{2Q}^{\pm} \left| \Phi_{2Q}^{\pm} \right\rangle \) which, on account of Eq. (30), reads
\begin{equation}
A_{2Q}^{\pm} \left( \left| \tilde{\Psi}^{-}_{M} \right\rangle \left| F_{2Q-M}^{-} \right\rangle + \left| \tilde{\Psi}^{+}_{M} \right\rangle \left| F_{2Q-M}^{+} \right\rangle \right).
\end{equation}
Then we measure the total state-parity of modes $M + 1, M + 2, \ldots, 2Q$. If the total parity $\prod_{k=1}^{Q-M} \pi_{M+k} = -1$ the state (32) collapses into the desired state $|\Psi_M^\pm\rangle$. The probability of successful generation is derived as

\[
P_{Q,M}^\pm = \left( \frac{A_{Q}^{-\pm}}{A_{M}^{\pm}} \right)^2 \left| 2^Q \langle n_{Q}^{2Q} | ... M + 2 \langle n_{M+2} | M + 1 \langle n_{M+1} | F_{2Q-M}^{\pm} \right|^2
\]

\[
= \left( \frac{A_{Q}^{-\pm}}{A_{M}^{\pm}} \right)^2 \sum_{\{n_j=0\}}^\infty \sum_{n_j=odd}^\infty |N_{n,M+1}(\alpha) N_{n,M+2}(\alpha) ... N_{n,Q}(\alpha)|^2
\]

\[
= \frac{(1 - e^{-2M|\alpha|^2})}{2(1 + e^{-2(Q+1)|\alpha|^2})(1 - e^{-2(Q-M)|\alpha|^2})}.
\]

In Eq. (33)

\[N_n(\beta) = k \langle n | \beta \rangle_k = \exp(-|\beta|^2/2) \beta^n / \sqrt{n!}
\]

is mode-independent. Alternatively, if we act $\hat{B}_{2Q}$ on $A_{2Q}^{-\pm} \left( |\alpha\sqrt{2Q}\rangle_1 - |\alpha\sqrt{2Q}\rangle_1 \right) \prod_{k=2}^{Q} |0\rangle_k$ to prepare the state $|\Psi_{2Q}^{-}\rangle = A_{2Q}^{-\pm} |\Psi_{2Q}^{-}\rangle$ and perform the same state-parity measurements, then the outcome $\prod_{k=1}^{Q-M} \pi_{M+k} = +1$ (not $-1$) means generation of the state $|\Psi_M^\pm\rangle$ with the probability of success equal to

\[
P_{Q,M}^- = \left( \frac{A_{Q}^{-\pm}}{A_{M}^{\pm}} \right)^2 \left| 2^Q \langle n_{Q}^{2Q} | ... M + 2 \langle n_{M+2} | M + 1 \langle n_{M+1} | F_{2Q-M}^{\mp} \right|^2
\]

\[
= \left( \frac{A_{Q}^{-\pm}}{A_{M}^{\pm}} \right)^2 \sum_{\{n_j=0\}}^\infty \sum_{n_j=even}^\infty |N_{n,M+1}(\alpha) N_{n,M+2}(\alpha) ... N_{n,Q}(\alpha)|^2
\]

\[
= \frac{(1 - e^{-2M|\alpha|^2})}{2(1 + e^{-2(Q+1)|\alpha|^2})(1 + e^{-2(Q-M)|\alpha|^2})}.
\]

Similarly, the state $|\Psi_M^+\rangle$ can be generated either from $|\Psi_{2Q}^+\rangle$ or from $|\Psi_{2Q}^-\rangle$ with the success probabilities given respectively by

\[
P_{Q,M}^{++} = \coth(M|\alpha|^2) \tanh(2Q|\alpha|^2) P_{Q,M}^-
\]

and

\[
P_{Q,M}^{+-} = \coth(M|\alpha|^2) \coth(2Q|\alpha|^2) P_{Q,M}^{-\pm}
\]

When $|\alpha| \to \infty$ all the probabilities are approaching 0.5 but in the limit $|\alpha| \to 0$ their behaviors are different,

\[
\lim_{|\alpha| \to 0} P_{Q,M}^{+-} = 0, \quad \lim_{|\alpha| \to 0} P_{Q,M}^{-\pm} = M/2Q, \quad \lim_{|\alpha| \to 0} P_{Q,M}^{++} = 1, \quad \lim_{|\alpha| \to 0} P_{Q,M}^{+\pm} = 1 - M/2Q,
\]

as seen from Fig. 2 where the probabilities are plotted as a function of $|\alpha|^2$ for a fixed $Q$ and several values of $M$. The figure clearly indicates a preference of $|\Psi_{2Q}^+\rangle$ to $|\Psi_{2Q}^-\rangle$ in the process of generating $|\Psi_M^\pm\rangle$. Moreover, the probabilities $P_{Q,M}^{+-}$ and $P_{Q,M}^{+\pm}$ are always greater than or equal to 50% and both of them are larger for smaller difference between $2Q$ and $M$.

2.3. Discussion

The operator $\hat{B}_M$ defined in Eq. (21) has also been applied [18] to produce generalized multiparticle EPR states [21], i.e., eigenstates with zero total momentum and zero relative positions. The ladder-scheme looks quite simple through Eq. (20). Construction of the operator $\hat{B}_M$ defined by Eq. (21) is however difficult practically since all the involved beam-splitters differ one from another: each has to have its own carefully prepared parameter $\theta_n$ depending delicately on the transmissivity and reflectivity. The tree-scheme, on the other hand, requires only one type of beam-splitter, the 50:50 ones, which are commonly available. In case $M = 2Q$ the generation is as simple as via Eq. (27). Otherwise, additional measurements are necessary but these are of the state-parity kind which can easily performed, say, by
employing QED with dispersive atom-field interactions (see, e.g., Ref. [22]). Although the probability of success is not 100%, it is always above 50% no matter how “far” $M$ is from the smallest $2^Q$, if starting from the right state (i.e. $|\Psi^\pm_{2^Q}\rangle \rightarrow |\Psi^\pm_M\rangle$, but not $|\Psi^\pm_{2^Q}\rangle \rightarrow |\Psi^\mp_M\rangle$). Recently, similar but related to the center-of-mass vibrational modes of trapped ions multipartite entangled coherent states have been considered [23] (see also [24]). The generation scheme in [23] is based on entanglement swapping but the proposed generalized Bell-state measurement of electronic states seems so challenging even by means of controlled-NOT gates. Also, the motional entangled cat-states are not relevant for the teleportation task considered here. As for optical fields, multimode entangled cat-states may be obtained via the Raman atom-field interaction [25], but the entangled modes are confined in many separate cavities and not ready for distribution among distant users in free space.

Furthermore, it turns out that $|\Psi^\pm_M\rangle$ obey the eigen-equation

$$a^2_k a^2_j \cdots a^2_{j_K} |\Psi^\pm_M\rangle = \alpha^{2K} |\Psi^\pm_M\rangle$$

(39)

for $1 \leq K \leq M$. In particular, $K = M = 3$ corresponds to a special case of the so-called even/odd trio coherent states [26] which have been shown genuine nonclassical states. The states $|\Psi^\pm_M\rangle$ therefore intrinsically possess various types of nonclassical properties that may be exploited for practical purposes (see, e.g., Ref. [27] for weak-force detection). The total averaged number of photons in the states $|\Psi^\pm_M\rangle$ equals $M \pi^\pm$ with

$$\pi^\pm = |\alpha|^2 \times \left\{ \frac{\tanh(M|\alpha|^2)}{\coth(M|\alpha|^2)} \right\}$$

(40)

the averaged number in a mode which is mode-independent. At small values of $|\alpha|$ there are departures of $\pi^\pm$ from $|\alpha|^2$, the averaged number in a usual single-mode coherent state, but already for $|\alpha|^2 \geq 2$ the averaged number per mode, i.e. $\pi^\pm$, become coincident with $|\alpha|^2$ (see Fig. 3). An actual total photon number that can be found in $|\Psi^\pm_M\rangle$ and $|\Psi^\mp_M\rangle$, for any $M$, is respectively even and odd, an intuitive physical reason explaining why $\langle \Psi^+_M | \Psi^+_M \rangle = 0$ (mathematically such an orthogonality comes directly from the definition of $|\Psi^+_M\rangle$ in Eq. (4)).

A curiosity, as was noticed in [23], concerns the two limiting situations: $|\alpha| \rightarrow \infty$ and $|\alpha| \rightarrow 0$. When $|\alpha| \rightarrow \infty$ a qubit can be encoded as $|\alpha\rangle_j = |1L\rangle_j$ and $|\alpha\rangle_j = |0L\rangle_j$ where “$F$” stands for “Logical” (Note that in this limit $|\alpha\rangle_j$ and $|\alpha\rangle_j$ become orthogonal as $\langle \alpha | - \alpha \rangle_j = \exp(-2|\alpha|^2)$ vanishes. Then

$$|\Psi^+_M\rangle \rightarrow \frac{1}{\sqrt{2}} (|1L\rangle_1 |1L\rangle_2 \cdots |1L\rangle_M \pm |0L\rangle_1 |0L\rangle_2 \cdots |0L\rangle_M)$$

(41)

represent the “all or none” states, i.e. the GHZ-states [28]. Oppositely, $|\alpha| \rightarrow 0$ causes $|\Psi^+_M\rangle \rightarrow 0$ but

$$|\Psi^-_M\rangle \rightarrow \frac{1}{\sqrt{M}} (|F\rangle_1 |0F\rangle_2 \cdots |0F\rangle_M + |0F\rangle_1 |1F\rangle_2 \cdots |1F\rangle_M)$$

(42)

where “$F$” stands for “Fock state”. This is strictly due to entanglement between modes since in a factorizable case an individual mode $|\alpha\rangle_j$ tends to $|0F\rangle_j$, resulting in a multimode vacuum only. The state (42) corresponds to the “one for each” state, i.e. the W-state [29]. The above features seem to bridge between GHZ- and W-state. Yet, these do not at all mean that GHZ- and W-state are equivalent because they are being approached in two extreme limits, that cannot be realized by means of local operations and classical communication, in agreement with what discovered in Ref. [29].

The bipartite entanglement degree of the state $|\Psi^\pm_M\rangle$ can be assessed by an entanglement measure called concurrence [30]. The concurrence between two subsystems: subsystem $\{K\}$ consisting of modes $\{j_1, j_2, \ldots, j_K\}$ with $j_k \neq j_i \in \{1, M\}$, $1 \leq K < M$, and subsystem $\{M - K\}$ consisting of the remaining modes $\{j_{K+1}, j_{K+2}, \ldots, j_M\}$, can be calculated as follows. Each subsystem is assumed being linearly independent with respect to $\alpha$ and $-\alpha$ spanning a two-dimensional sub-space of the Hilbert space. According to the Gram-Schmidt theorem, we can always build in each sub-space an orthonormal basis $\{|0\rangle_i, |1\rangle_i\}, i = \{K\}, \{M - K\}$, which is determined as

$$|0\rangle_{\{K\}} = |\alpha\rangle_{j_1} \cdots |\alpha\rangle_{j_K},$$

(43)

$$|1\rangle_{\{K\}} = \sqrt{1 - \langle j_k (\alpha | - \alpha\rangle_{j_1} \cdots | - \alpha\rangle_{j_K} \rangle^2 $$

(44)

$$|0\rangle_{\{M - K\}} = |\alpha\rangle_{j_{K+1}} \cdots |\alpha\rangle_{j_M},$$

(45)
Given a network consisting of an arbitrary $N$ parties the task is to teleport from any one to any another party an unknown multimode cat-state of the form (1).

3.1. Single-mode cat-state

The simplest cat-state is with $L = 1$ in which case the state (1) reduces to

$$|\Psi\rangle_a = A_a (x |\alpha\rangle_a + y |\alpha\rangle_a).$$

with

$$A_a = (|x|^2 + |y|^2 + 2 \exp(-2|\alpha|^2) R(x^* y))^{-1/2}.$$
At party 1 station, the operator $\hat{B}_{a,1}$ is applied to mode $a$ and mode $1$ transforming state (56) into

$$
|\Theta\rangle_{a12\ldots N} = A_a A_N \left( x \left| \alpha \sqrt{2} \right. \right)_{a} |0\rangle_{1} |\alpha\rangle_{2} \ldots |\alpha\rangle_{N} \\
-x |0\rangle_{a} \left| \alpha \sqrt{2} \right. \rangle_{a} |\alpha\rangle_{2} \ldots |\alpha\rangle_{N} \\
+y |0\rangle_{a} \left| -\alpha \sqrt{2} \right. \rangle_{a} |\alpha\rangle_{2} \ldots |\alpha\rangle_{N} \\
-y \left| -\alpha \sqrt{2} \right. \rangle_{a} |0\rangle_{1} |\alpha\rangle_{2} \ldots |\alpha\rangle_{N}. \right)
$$

To fulfill the task mentioned above party 1 needs counting the photon numbers of mode $a$ and mode $1$, while parties 2, 3, ..., $N-1$ should respectively carry out the local number measurement of their modes. Let the measurement outcomes at party 1 station be $n_a$ and $n_1$, whereas $n_2$, $n_3$, ..., $n_{N-1}$ photons are detected at the stations of party 2, party 3, ..., party $N-1$, respectively. The structure of state $|\Theta\rangle_{a12\ldots N}$, Eq. (57), excludes the possibility of both $n_a \neq 0$ and $n_1 \neq 0$. The outcome $n_a = n_1 = 0$ may occur but does not help the task of teleportation (moreover, its probability depends on $x, y$ and is thus unknown). We are therefore left only with two possibilities: (i) $n_a = 0$, $n_1 > 0$ and (ii) $n_a > 0$, $n_1 = 0$. If Case (i) happens then the state at party $N$ station collapses into

$$
|\Psi^{(i)}\rangle_{N} = A_a \left[ (-1)^{n_2+n_3+\ldots+n_{N-1}} x |\alpha\rangle_{N} - (-1)^{n_1} y |\alpha\rangle_{N} \right].
$$

Clearly, if all the parties who performed the number measurement send their outcomes to party $N$ via a public (classical) channel and the outcomes are such that $n_1 + n_2 + \ldots + n_{N-1}$ is odd, then after obtaining the classical information party $N$ will apply the operator $\hat{P}_N(\pi)$ to Eq. (58) to get the state $|\Psi\rangle_{N} = \hat{P}_N(\pi) |\Psi^{(i)}\rangle_{N}$ which is nothing else but the desired state $|\Psi\rangle_{a}$ (up to a global unimportant phase constant sometimes). The probability of success is equal to

$$
\Pi_{N}^{(i)} = | \langle n_1 | 2 \langle n_2 | \ldots | n_{N-1} | \Theta \rangle_{a12\ldots N} |^2 \\
= (A_N^a)^2 \sum_{n_1=1, n_j>1=0, \sum n_k=odd}^{\infty} \left| N_{n_1} (\alpha \sqrt{2}) N_{n_2} (\alpha) \ldots N_{n_{N-1}} (\alpha) \right|^2.
$$

Alternatively, if Case (ii) occurs then the state at party $N$ station collapses into

$$
|\Psi^{(ii)}\rangle_{N} = A_a \left[ x |\alpha\rangle_{N} - (-1)^{n_2+n_3+\ldots+n_{N-1}} y |\alpha\rangle_{N} \right].
$$

Transparencyly, if $n_a + n_2 + n_3 + \ldots + n_{N-1}$ is odd, then, by doing nothing, party $N$ gets an exact replica of the desired state $|\Psi\rangle_{a}$. In this situation the success probability $\Pi_{N}^{(ii)} = | \langle n_1 | 2 \langle n_2 | \ldots | n_{N-1} | \Theta \rangle_{a12\ldots N} |^2$ is precisely equal to that in Case (i), i.e., $\Pi_{N}^{(ii)} = \Pi_{N}^{(i)}$. Hence, the total probability of successful teleportation can be calculated explicitly and, as a result,

$$
\Pi_{N}^{(1^{-})} = 2\Pi_{N}^{(i)} = \frac{1}{2} \left\{ 1 - \operatorname{csch}(N|\alpha|^2) \sinh [(N-2)|\alpha|^2] \right\}
$$

where the superscript “$(1^{-})$” indicates that the teleported cat-state is single-mode, and the state $|\Psi^-\rangle_{N}$ is used as a quantum channel. For $N = 2$ the formula (61) yields $\Pi_{2}^{(1^{-})} = 1/2$, independent of $\alpha$, recovering the result reported in Ref. [12]. For any $N > 2$, the probability of perfect teleportation tends to $1/N$ in the limit $|\alpha| \to 0$ and to $1/2$ in the limit $|\alpha| \to \infty$ (see Fig. 5).

3.2. Two-mode cat-state

We now study the network teleportation problem for a two-mode cat-state ($L = 2$) of the form

$$
|\Psi\rangle_{ab} = A_{ab} (x |\alpha\rangle_{a} |\alpha\rangle_{b} + y |\alpha\rangle_{a} |-\alpha\rangle_{b})
$$

(62)

with

$$
A_{ab} = (|x|^2 + |y|^2 + 4 \exp(-4|\alpha|^2) \Re(x^*y))^{-1/2}.
$$

(63)

For a symmetric network of $N$ parties we use in this circumstance the state $|\Psi_{2N}^{-}\rangle$ as a quantum channel which is equally shared among all the parties by sending a pair of modes $\{2q - 1, 2q\}$ to a party $q$ where $q = 1, 2, \ldots, N$. The entire system is described by
\[ |\Phi\rangle_{ab12...2N} = |\Psi\rangle_{ab} |\Psi\rangle_{ab} \]

\[ = A_{ab} A_{2N} (x |\alpha\rangle_a |\alpha\rangle_b |\alpha\rangle_1 |\alpha\rangle_2 |\alpha\rangle_3 ... |\alpha\rangle_{2N} - x |\alpha\rangle_a |\alpha\rangle_b |\alpha\rangle_1 |\alpha\rangle_2 |\alpha\rangle_3 ... |\alpha\rangle_{2N} + y |\alpha\rangle_a |\alpha\rangle_b |\alpha\rangle_1 |\alpha\rangle_2 |\alpha\rangle_3 ... |\alpha\rangle_{2N} + y |\alpha\rangle_a |\alpha\rangle_b |\alpha\rangle_1 |\alpha\rangle_2 |\alpha\rangle_3 ... |\alpha\rangle_{2N} ) . \]  

(64)

Obviously, the symmetry allows teleportation from any party \( k \) to any other party \( l \) (\( l \neq k \)). Without loss of generality, we suppose that party 1 possesses the state \( |\Psi\rangle_{ab} \) and wishes to teleport it to party \( N \). Then, at party 1 location mode 1 is mixed with mode \( b \) and mode 2 with mode \( a \) (or mode 1 with mode \( a \) and mode 2 with mode \( b \), the result will be the same). That is, party 1 applies the operator \( \hat{B}_{a,2b,1} \) to his/her modes \( a, b, 1 \) and 2 transforming state (64) into

\[
|\Theta\rangle_{ab12...2N} = A_{ab} A_{2N} (x |\alpha\sqrt{2}\rangle_a |\alpha\sqrt{2}\rangle_b |0\rangle_1 |0\rangle_2 |\alpha\rangle_3 ... |\alpha\rangle_{2N} - x |0\rangle_a |\alpha\sqrt{2}\rangle_b |\alpha\rangle_1 |\alpha\rangle_2 |\alpha\rangle_3 ... |\alpha\rangle_{2N} + y |0\rangle_a |\alpha\sqrt{2}\rangle_b |\alpha\rangle_1 |\alpha\rangle_2 |\alpha\rangle_3 ... |\alpha\rangle_{2N} - y |\alpha\sqrt{2}\rangle_a |\alpha\sqrt{2}\rangle_b |0\rangle_1 |0\rangle_2 |\alpha\rangle_3 ... |\alpha\rangle_{2N} ) .
\]

(65)

After that party 1 counts the photon numbers of modes \( a, b, 1 \) and 2, while parties \( q \) (\( q = 2, 3, ..., N - 1 \)) carry out the local number measurement of modes \( 2q - 1 \) and \( 2q \) at their location. Let the outcome be \( n_a, n_b, n_{2q-1} \) and \( n_{2q} \) (\( q = 1, 2, ..., N - 1 \)). Similar arguments as in the previous subsection lead to only two situations: (i) \( n_a = n_b = 0, n_1 + n_2 > 0 \) and (ii) \( n_a + n_b > 0, n_1 = n_2 = 0 \). If Case (i) happens then the state at party \( N \) station collapses into

\[
|\Psi\rangle_N = A_{ab} \left[ (-1)^{n_3+...+n_{2(N-1)}-1} |\alpha\rangle_{2N-1} |\alpha\rangle_{2N} - (-1)^{n_1+n_2} y |\alpha\rangle_{2N-1} |\alpha\rangle_{2N} \right] .
\]

(66)

Clearly, if all the parties 1, 2, ..., \( N - 1 \) send their outcomes to party \( N \) via a public (classical) channel and the outcomes are such that \( n_1 + n_2 + ... + n_{2(N-1)} \) is odd, then after obtaining the classical information party \( N \) will apply the operator \( \hat{P}_{2N-1}() \hat{P}_{2N}() \) to Eq. (66) to get the state \( |\Psi\rangle_N = \hat{P}_{2N-1}() \hat{P}_{2N}() |\Psi\rangle_N \) which is nothing else but the desired state \( |\Psi\rangle_{ab} \) (up to a global unimportant phase constant sometimes). The probability of success is equal to

\[
\Pi_N^{(i)} = \left| \sum_{n_j=0}^{\infty} N_n \alpha \sqrt{2} N_n \alpha \sqrt{2} N_n \alpha \sqrt{2} N_n \alpha \sqrt{2} \right|^2
\]

(67)

with the prime to exclude the term with \( n_1 = n_2 = 0 \) and to sum over odd \( n_1 + n_2 + ... + n_{2(N-1)} \). Alternatively, if Case (ii) occurs then the state at party \( N \) station collapses into

\[
|\Psi\rangle_N = A_{ab} \left[ x |\alpha\rangle_{2N-1} |\alpha\rangle_{2N} - (-1)^{n_a+n_b+n_3+...+n_{2(N-1)}} y |\alpha\rangle_{2N-1} |\alpha\rangle_{2N} \right] .
\]

(68)

Transparencyly, if \( n_a + n_b + n_3 + ... + n_{2(N-1)} \) is odd, then, by doing nothing, party \( N \) gets an exact replica of the desired state \( |\Psi\rangle_{ab} \). In this situation the success probability \( \Pi_N^{(ii)} = \left| \sum_{n_j=0}^{\infty} N_n \alpha \sqrt{2} N_n \alpha \sqrt{2} N_n \alpha \sqrt{2} N_n \alpha \sqrt{2} \right|^2 \) is precisely equal to that in Case (i), i.e., \( \Pi_N^{(ii)} = \Pi_N^{(i)} \). The total probability of successful teleportation can be calculated explicitly as

\[
\Pi_N^{(2-)} = 2 \Pi_N^{(i)} = \frac{1}{2} \left\{ 1 - \cosh(2N|\alpha|) \sinh(2(N-1)|\alpha|^2) \right\}
\]

(69)

where the superscript \((“2-“)\) indicates that the teleported cat-state is two-mode, and the state \( |\Psi_{ab}^{2N-1}\rangle \) is used as a quantum channel. As in the case of \( L = 1 \) the success probability is 50% for \( N = 2 \) independent of \( \alpha \). For \( N > 2 \) it is less than but quickly tends to 50% with increasing \( |\alpha| \).

If network symmetry is not required a less expensive quantum channel can be used by a \((2N-1)\)-mode entangled state, i.e. by using the state \( |\Psi_{ab}^{2N-1}\rangle \). Since symmetry of the network is broken, the quantum channel cannot be shared by the parties before an announcement is made about who holds the state \( |\Psi_{ab}\rangle \). If party \( k \) holds it, one of
the $2N - 1$ modes of the state $|\Psi_{2N-1}\rangle$ should be sent to party $k$ and each of the remaining parties receives a pair of modes: an unequal distribution. For definiteness, we assume that party 1 holds the state $|\Psi\rangle_{ab}$. Then we send mode 1 to party 1 and a pair of modes $\{2(q-1), 2q-1\}$ to a party $q$ ($q = 2, 3, \ldots, N$). Party 1 applies the operator $\hat{B}_{b,1}$ to modes $b$ and 1 (or $\hat{B}_{a,1}$ to modes $a$ and 1, the result will be the same). All the parties except party $N$ who is supposed to get the teleported state should measure the photon number of their modes and send the outcome to party $N$. Similarly to the above described technique, depending on the measurement outcome, party $N$ is able to obtain the desired state $|\Psi\rangle_{ab}$ by doing nothing or by applying appropriate operations on his/her state. The total probability of successful teleportation now is

$$\bar{\Pi}^{(2-)}_{N} = \frac{1}{2} \{1 - \text{csch}[(2N - 1)|\alpha|^2] \sinh [(2N - 3)|\alpha|^2]\} \tag{70}$$

which is always $\alpha$-dependent, even for $N = 2$. Comparing Eq. (70) with Eq. (61) we establish the following relationship

$$\bar{\Pi}^{(2-)}_{N} = \bar{\Pi}^{(1-)}_{2N-1} \tag{71}$$

### 3.3. Multimode cat-state

It is straightforward to generalize the teleportation scenario to the case of an arbitrary $L$-mode cat-state, i.e. the state (1). The symmetric network of $N$ parties requires a quantum channel to be served by a state with $LN$ modes being entangled, i.e. the state $|\Psi_{LN}^{-}\rangle$ of which a collection of $L$ modes $\{(q-1)L + 1, (q-1)L + 2, \ldots, qL\}$ is sent to a party $q$ ($1 \leq q \leq N$). Note again that this equal sharing of the quantum channel can be done at any time before an actual teleportation takes place. For teleporting from a party, say, party 1, to another party, say, party $N$, party 1 mixes his/her modes pairwise: one from the group $\{a_1, a_2, \ldots, a_L\}$ and one from the group $\{1, 2, \ldots, L\}$. Then the parties 1, 2, $\ldots$, $N - 1$ carry out appropriate number measurement and communicate the outcome to party $N$ who will get the teleported state by the right action as mentioned above. The explicit expression of success probability in this general case can also be derived. It is

$$\bar{\Pi}^{(L-)}_{N} = \frac{1}{2} \{1 - \text{csch}(LN|\alpha|^2) \sinh [L(N - 2)|\alpha|^2]\} \tag{72}$$

where the superscript “$(L-)$” indicates that the teleported cat-state is $L$-mode, and the state $|\Psi_{LN}^{-}\rangle$ is used as a quantum channel. Generally $\bar{\Pi}^{(L-)}_{N}$ depends on $N$, $L$ and $\alpha$, but for $N = 2$ it is exactly 50% independent of both $L$ and $\alpha$.

In the case of asymmetric network we can use a less expensive quantum channel of only $(L(N - 1) + 1)$ modes, i.e. the state $|\Psi_{L(N-1)+1}^{-}\rangle$. Now only after knowing who holds the state to be teleported the parties start to share the quantum channel. Let the state $|\Psi\rangle_{a_1a_2\ldots a_L}$ be with party 1 and party $N$ receive it. Then, mode 1 should be sent to party 1 and a collection of $L$ modes $\{(q - 2)L + 2, (q - 2)L + 3, \ldots, qL + 1\}$ to a party $q = 2, 3, \ldots, N - 1$ : again an unequal sharing. After that party 1 applies the operator $\hat{B}_{a_j,1}$ to modes $a_j$ and 1 where $a_j$ is any one among the modes $\{a_1, a_2, \ldots, a_L\}$. All the parties except party $N$ measure the photon number of their modes and classically communicate their outcomes with party $N$. Depending on the communication content, party $N$ automatically gets the desired cat-state $|\Psi\rangle_{a_1a_2\ldots a_L}$ or is able by applying proper operations to convert the state in his/her location into $|\Psi\rangle_{a_1\ldots a_L}$. The total probability of success within such an asymmetric network is related to $\bar{\Pi}^{(1-)}_{M}$ as

$$\bar{\Pi}^{(L-)}_{N} = \bar{\Pi}^{(1-)}_{(N-1)L+1} \tag{73}$$

which always depends on $\alpha$, even for $N = 2$.

### 3.4. Discussion

As seen from the analysis, for a symmetric network the probability of successful teleportation reaches 50% when $N = 2$ independent of both $L$ and $\alpha$, i.e. $\bar{\Pi}^{(L-)}_{2} = 50\%$ no matter how many modes the state to be teleported is composed of and how intense the applied field is. For an asymmetric network an entangled state of lesser modes can be used as a quantum channel but the probability of success depends on $\alpha$ for all $N$ including $N = 2$. Generally for both symmetric and asymmetric networks the success probability quickly saturates to 50%. In fact, as Fig. 5 shows, the probability already reaches 50% starting from $|\alpha|^2 = 3$ for which $\bar{\Pi}$ well approaches $|\alpha|^2$ too (see Eq. (40) and Fig. 3). So, in a sense, for fields with an averaged photon number per mode equal to or greater than 3 the success of teleportation can be considered as being taken place with a probability 50%, independent of $\alpha$, $L$ and $N$ as well. However, the symmetric network has an advantage over the asymmetric one in the sense that the quantum channel can be shared equally among the participants at any time before the teleportation process and the network is ready for teleporting between any two locations at a later time.
In the proposed teleportation protocols state-parities rather than photon numbers themselves are needed. Thus the kind of required measurement remains the same as for preparation of the states \( |\Psi_M^+\rangle \), i.e. they are parity measurements. These can easily be performed, say, by coupling the Fock state appearing after a number measurement to a two-level atom via a dispersive interaction (see, e.g., Ref. [22]) governed by the interaction Hamiltonian \( H_{int} = f a_\alpha^+ a_\beta \sigma_x \) with \( f \) the coupling strength and \( \sigma_x = |g\rangle\langle e| + |e\rangle\langle g| \) the atomic ground (excited) state. If the atom is initially prepared either in the excited or in the ground state and is to be detected after an interaction time \( \tau \) such that \( \tau f = \pi/2 \), then a state-flipping (e.g., \( |g\rangle \rightarrow |e\rangle \) or \( |e\rangle \rightarrow |g\rangle \)) means an odd parity, whereas no-flipping indicates an even parity, no matter how many photons in the Fock state are.

In Refs. [14,15] the nonorthogonal cat-states have been reconstructed in orthogonal bases and Bell-like state measurements are to be performed for a teleportation task. However, unlike the original Bell states discriminating Bell-like states is not trivial, and an experimental setup was suggested in [15]. The setup is probabilistic and does not allow a measurement to be performed for a teleportation task. However, unlike the original Bell states discriminating Bell-like states the parity of the measured modes turns out to be even (odd) in which case the state at party \( L \) of the symmetric network, and hence no-flipping, means an even parity, no matter how many photons in the Fock state are.

For the same teleportation task we could alternatively use the state \( |\Psi_M^+\rangle \) as a quantum channel. Going along similar line we have derived formulae for the probability of successful teleportation as

\[
P_N^{(L+)} = \frac{1}{2} \left\{ 1 - \frac{1}{\text{sech}(LN|\alpha|^2)} \cosh[L(N-2)|\alpha|^2] \right\}
\]

for the symmetric network, and

\[
\overline{P}_N^{(L+)} = P_N^{(1+)}
\]

for the asymmetric network with \( P_M^{(1+)} \) for any \( M \geq 2 \) given by

\[
P_M^{(1+)} = \frac{1}{2} \left\{ 1 - \frac{1}{\text{sech}(M|\alpha|^2)} \cosh[(M-2)|\alpha|^2] \right\}.
\]

Although both \( P_N^{(L+)} \) and \( \overline{P}_N^{(L+)} \) saturate to 1/2 for large \( |\alpha| \), they tend to vanish in the limit \( |\alpha| \rightarrow 0 \) (see the dashed curves in Fig. 4). This gives an advantage of the state \( |\Psi_M^+\rangle \) over the state \( |\Psi_M^+\rangle \) in teleportation utilizing low intensity fields.

It should also be kept in mind that in the proposed protocols the teleportation cannot be reliable (i.e. with 100% probability of success). There may happen that while using \( |\Psi_M^+\rangle \) as a quantum channel the total state-parity of the measured modes turns out to be even (odd) in which case the state at party \( N \) station differs from the original state \( |\Psi\rangle_{a_1 a_2 ... a_L} \) by a relative phase factor. Since no unitary transformation that casts \( x |\alpha\rangle_{a_1} |\alpha\rangle_{a_2} ... |\alpha\rangle_{a_L} - y |\alpha\rangle_{a_1} |\alpha\rangle_{a_2} ... |\alpha\rangle_{a_L} \) into \( x |\alpha\rangle_{a_1} |\alpha\rangle_{a_2} ... |\alpha\rangle_{a_L} + y |\alpha\rangle_{a_1} |\alpha\rangle_{a_2} ... |\alpha\rangle_{a_L} \) for any \( \alpha \) has been available, the teleportation fails.

4. Conclusion

In conclusion, we have proposed two schemes to generate general multimode entangled cat-states and have then used them to teleport within a network of \( N \) parties an unknown state which is also a kind of multimode entangled cat-state. If the state to be teleported is \( L \)-mode the quantum channel state needs to be \( NL \)-mode for a symmetric network and \( (L(N-1)+1) \)-mode for an asymmetric network. Both the proposed generation schemes and teleportation protocols require only linear optical devices (beam-splitters) and simple parity measurements. The analytical results show that for medium and high intensity fields (\( |\alpha|^2 \geq 3 \)) perfect teleportation can be achieved with a probability 50% for any \( L \) and \( N \). The teleportation protocols meet all the requirements imposed by no-cloning theorem and special relativity. In fact, no more than one party can get an exact replica of the teleported state because except party \( N \) all the other parties counted the photon number of their modes and thus destroyed the original state at their locations. Also, no party stands outside the game. Without participation of any of the parties the teleportation can never be completed since the measurement outcomes of all the \( N-1 \) parties should be collected at party \( N \) before that latter party is able to infer the teleported state. Since the collection of measurement outcome takes a finite time via public communication superluminal signaling does not occur.

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Figure captions

Fig. 1: Arrangement of beam-splitters in different schemes. a) Ladder-scheme for generating state $|\Psi^\pm_M\rangle$ from the input in the form of a single-mode cat-state $A^\pm_M(|\alpha\sqrt{M}\rangle \pm -\alpha\sqrt{M}\rangle)$ with $M = 6$. Each 45°-inclined bar represents a beam-splitter $\hat{B}_{k,l}(\theta)$ defined by Eq. (10). b) Tree-scheme for generating state $|\Psi^\pm_{2Q}\rangle$ from the input in the form of a single-mode cat-state $A^\pm_{2Q}(|\alpha\sqrt{2Q}\rangle \pm -\alpha\sqrt{2Q}\rangle)$ with $Q = 3$. Each vertical bar represents a beam-splitter $\hat{B}_{k,l}$ defined by Eq. (11).

Fig. 2: The probabilities a) $P^{+\pm}_{Q,M}$ (downwards), b) $P^{-\pm}_{Q,M}$ (upwards), c) $P^{\pm\pm}_{Q,M}$ (upwards) and d) $P^{-\pm\pm}_{Q,M}$ (downwards) as a function of $|\alpha|^2$ for $Q = 3$ and $M = 5, 6, 7$.

Fig. 3: The averaged photon number per mode $n^-$ (upper solid curve) and $n^+$ (lower solid curve) in dependence on $|\alpha|^2$ for $M = 3$. The dashed line represents $|\alpha|^2$ which is drawn for comparison.

Fig. 4: The concurrences $C_{\{K\},\{M-K\}}^-$ (solid curves, upwards) and $C_{\{K\},\{M-K\}}^+$ (dashed curves, upwards) versus $|\alpha|^2$ for a) $M = 6$, $K = 1(5), 2(4)$ and 3, and b) $M = 7$, $K = 1(6), 2(5)$ and 3(4).

Fig. 5: The probability of successful teleportation $\Pi^{(1-)}_N$ (solid curves, downwards) and $\Pi^{(1+)}_N$ (dashed curves, upwards) as a function of $|\alpha|^2$ for $N = 2, 4$ and 6.
Fig. 1, Nguyen
Fig. 4, Nguyen
