Finite Time Cascade Control Scheme for Tracking Control of an Underwater Exploring Robot

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Abstract: In this paper, we discuss the curve trajectory tracking problem of an underactuated underwater robot in the horizontal plane. We use global diffeomorphism transformation method to transform the sixth-order dynamic tracking error model into a nonlinear cascaded system, by using the cascaded systems’ finite time stability theory, along with the back stepping technique to design the controller. Two discontinuous finite time controllers are designed for the two subsystems respectively. In the existing literature, finite time control is used for the first order nonholonomic system, we extended this method to two order nonholonomic system in this paper. The finite time controller can tune not only the gain, but also the fractional power, it is more convenience for engineering implementation. The results show the control laws can achieve the control objective with good performance for the horizontal trajectory tracking control.

1. Introduction

This paper, we discuss the curve trajectory tracking problem of an under-actuated underwater vehicle (UUV) in the horizontal plane. The difficulty for controlling the under-actuated UUV is the limitations of the Brockett necessary condition [1]. To overcome the difficulty, the discontinuous feedback approach and time-varying feedback approach are often used [2]. The mathematical model of an under-actuated UUV [3] moving in a horizontal plane can be described as equation (1):

\[ \begin{align*}
  m_{11} \dot{u} - m_{22} v r + d_{11} u &= \tau_1, \\
  m_{22} \dot{v} - m_{11} u r + d_{22} v &= 0, \\
  m_{33} \ddot{r} + (m_{22} - m_{11}) uv + d_{33} r &= \tau_3,
\end{align*} \]

(1)

Where \( u, v \) and \( r \) represent the velocities of a UUV in surge, sway and yaw in the body-fixed frame respectively; \( x, y, \phi \) represent the position and orientation of a UUV in the earth-fixed frame. The parameters \( m_{ij} > 0 \) are the inertia and added mass effects of the UUV. The parameters \( d_{ii} > 0 \) represent the hydrodynamic damping of the UUV. The available input controls are the thruster force \( \tau_1 \), and the steering torque \( \tau_3 \). Due to the absence of actuators in sway direction, the discussed UUV...
is an under-actuated two order nonholonomic systems.

For under-actuated nonholonomic underwater vehicles, many control strategies have been proposed for AUV trajectory tracking controller, such as the local linearization, the back stepping method, adaptive neural network control, the Sliding Mode control method, intelligent control method, the Lyapunov design and redesign, etc. [4, 5].

Note that most of the aforementioned control schemes only consider the asymptotic stability, which means that the systems considered achieve convergence in infinite settling time, or have at best exponential convergence rate to the origin.

Finite time control aims to make the dynamical errors converging to the origin in finite time. Finite time stability means the controlled plant must be finite time convergence and satisfy the Lyapunov stability theorem. In [6], the sufficient condition for finite time stability of nonlinear dynamical systems was given. With the improvement of finite time control theory such as homogeneous theory and back stepping theory [7, 8], the design methods for non-Lipschitz continuous finite time controllers have developed rapidly and many new results for designing finite time controller have been obtained, see for [9-11] and the references therein. Compared with the classical asymptotic design scheme, the finite time controller possesses the property which can steer the dynamical errors converge to the origin in a finite time, furthermore, the finite time controller usually demonstrates higher tracking accuracy robustness against uncertainties and better disturbance-rejection performance [12].

The finite time control has attracted the interest of many scholars in recent years, but the research is mainly focused on the first order nonholonomic system, such as the wheeled mobile robots, the velocity along the plane perpendicular to the point of contact between the wheel and the ground is zero. The UUVs belong to two order nonholonomic system, and its control problem is more challenging. We extended the finite time method to two order nonholonomic systems in this paper.

Assume that a feasible reference trajectory \((u_d, v_d, r_d, x_d, y_d, \varphi_d)\) is given, i.e., a trajectory, \(u_d, v_d, r_d, x_d, y_d, \varphi_d\) satisfy the same dynamics equations with \(u, v, r, x, y, \varphi\).

We use the same coordinate and input transformations as in [13]. Let \(e=[e_1, e_2, e_3, e_4, e_5, e_6]^T = [z_1 - z_1d, z_2 - z_2d, z_3 - z_3d, z_4 - z_4d, z_5 - z_5d, z_6 - z_6d]^T\), we can obtain the tracing error dynamics equations

\[
\begin{align*}
\dot{e}_1 &= -ae_1 - ae_4 + (e_2 e_6 + z_2d e_6 + z_6d e_2) - b(e_5 e_6 + z_5d e_6 + z_6d e_5), \\
\dot{e}_2 &= e_4 e_6 + z_4d e_6 + z_6d e_4, \\
\dot{e}_3 &= e_6, \\
\dot{e}_4 &= F_1 - F_{1d}, \\
\dot{e}_5 &= c(e_1 e_6 + z_1d e_6 + z_6d e_1 + e_4 e_6 + z_4d e_6 + z_6d e_4) - c e_5, \\
\dot{e}_6 &= F_2 - F_{2d}
\end{align*}
\]

(2)

From the analysis of the above, the control objective moving the BYSQ-3 from the initial trajectory to a desired trajectory in the configuration space can be described as the stabilization of \([e_1, e_2, e_3, e_4, e_5, e_6]^T\).

2. Finite Time Controller Design

2.1. Designing the Finite Time Control Laws for \(e_3, e_6\)

Let \(\alpha(e_6)\) be the virtual input of \(e_6\) and \(\alpha(e_6) = -l_3\tanh^\rho(e_3)\), where \(l_3 > 0, 0 < \rho < 1\), define

\[
s_6 = e_6 - \alpha(e_6)
\]

substituting (3) into (2), we have:
\[
\begin{aligned}
\dot{e}_3 &= s_6 - l_3 \text{sig}^\theta(e_3) \\
\dot{s}_6 &= F_2 - F_{2d} - \dot{\alpha}(e_6)
\end{aligned}
\]  
(4)

Take the following Lyapunov function

\[V_1 = \frac{1}{2} (e_3^2 + s_6^2)\]  
(5)

Differentiating (5) along the solutions of the (4) satisfies

\[\dot{V}_1 = e_3 \dot{e}_3 + s_6 \dot{s}_6 = -l_3 e_3 \text{sig}^\theta(e_3) + e_3 s_6 + s_6 (F_2 - F_{2d} - \dot{\alpha}(e_6))\]

If we design

\[F_2 = F_{2d} + \dot{\alpha}(e_6) - e_3 - l_6 \text{sig}^\theta(e_6), \quad l_6 > 0,\]  
(6)

Then

\[\dot{V}_1 = -l_3 e_3 \text{sig}^\theta(e_3) + e_3 s_6 + s_6 (F_2 - F_{2d} - \dot{\alpha}(e_6))\]
\[\leq -h \left( \frac{1}{2} e_3^2 + \frac{1}{2} s_6^2 \right)^{1+\rho} \]
\[= -h V_1^\omega,\]

where \(h = 2^{\omega \min(l_3, l_6)}, 0 < \omega = \frac{1+\rho}{2} < 1.\)

From the theorem in [14], the control law moves the states \(e_3, e_6\) to zero in finite time \(t_1\). At the end of the first stage we have \(e_3 = e_6 = 0\)

Then the system (2) is converted into

\[
\begin{pmatrix}
\dot{e}_1 \\
\dot{e}_2 \\
\dot{e}_4 \\
\dot{e}_5
\end{pmatrix} =
\begin{pmatrix}
-ae_1 - ae_4 + e_2 z_{6d} - be_5 z_{6d} \\
e_4 z_{6d} \\
F_1 - F_{1d} \\
-ce_5 + c (e_4 z_{6d} + e_5 z_{6d})
\end{pmatrix}
\]  
(7)

We design the virtual control input of \(e_4\) to steer the state variables to origin in finite time. Considering the subsystem (7), let \(\alpha(e_4)\) be the virtual control input of \(e_4\) and define

\[s_4 = e_4 - \alpha(e_4),\]  
(8)

combining (7) and (8), we can obtain
Take the following Lyapunov function

$$V_2 = \frac{1}{2}(e_1^2 + e_2^2 + s_4^2 + e_5^2)$$  \hspace{1cm} (10)$$

Differentiating (10) along the solutions of the (9) satisfies

$$\dot{V}_2 = e_1 \dot{e}_1 + e_2 \dot{e}_2 + s_4 \dot{s}_4 + e_5 \dot{e}_5 =$$

$$-ae_1^2 - ce_5^2 + \alpha(e_4)(-ae_1 + e_2z_{6d} + ce_5z_{6d})^+$$

$$s_4(-ae_1 + e_2z_{6d} + ce_5z_{6d} + F_1 - F_{1d} - \ddot{a}(e_4)) + e_1e_2z_{6d} + (c-b)e_1e_5z_{6d}.$$

If we design $\alpha(e_4)$ as

$$\alpha(e_4) = l_1\text{sign}(e_4) - l_2z_{6d}\text{sign}(e_2) - l_3z_{6d}\text{sign}(e_5),$$

where $l_1, l_2, l_3$ are positive real numbers, $\sigma$ is a real number and $0 < \sigma < 1,$ and we design $F_1$ as

$$F_1 = F_{1d} + \ddot{a}(e_4) + ae_1 - e_2z_{6d} - ce_5z_{6d} - l_4\text{sign}(s_4),$$

where $l_4$ is a positive real number,

then, we can obtain

$$\dot{V}_2 = e_1 \dot{e}_1 + e_2 \dot{e}_2 + s_4 \dot{s}_4 + e_5 \dot{e}_5 =$$

$$-ae_1^2 - ce_5^2 + \alpha(e_4)(-ae_1 + e_2z_{6d} + ce_5z_{6d})^+$$

$$s_4(-ae_1 + e_2z_{6d} + ce_5z_{6d} + F_1 - F_{1d} - \ddot{a}(e_4)) + e_1e_2z_{6d} + (c-b)e_1e_5z_{6d}.$$

where

$$\Delta = e_1e_2z_{6d} + (c-b)e_1e_5z_{6d} - ae_1\left(-l_2z_{6d}\text{sign}(e_2) - l_3z_{6d}\text{sign}(e_5)\right)$$

$$+e_2z_{6d}\left(l_1\text{sign}(e_1) - l_3z_{6d}\text{sign}(e_5)\right) + ce_5z_{6d}\left(l_1\text{sign}(e_1) - l_2z_{6d}\text{sign}(e_2)\right).$$

$l = 2\tau l', l' = \min\{a_l, l_2z_{6d}, l_4, cl_5z_{6d}\}$

$0 < \tau = (1 + \sigma)/2 < 1.$

If we choose proper real numbers $l_i (i = 1, 2, \ldots, 5)$ such that the following inequality can hold:

$$l > p + \Delta/V_2^p, p > 0,$$

then, we can obtain

$$\dot{V}_2 \leq -pV_2^p$$

Then from the theorem in [14], we know the control law moves the states $e_1, e_2, e_4, e_5$ to zero in finite time $t_2.$

3. Simulations

The hydrodynamic coefficients are as follows:

$m_{11} = 45$ kg, $m_{22} = 46$kg, $m_{33} = 45.5$ kg,

$d_{11} = 0.4kg/s, d_{22} = 0.2kg/s, d_{33} = 0.001kgm^2/s$,

The desired trajectory is chosen as a circle given by

$x_d = 5\cos t, y_d = 5\sin t, \varphi_d = t + \pi/2, u_d = 3, v_d = 0, r_d = 1.$

The initial conditions are

$x(0) = 5m, y(0) = 5m, \varphi(0) = pt/2, u(0) = 0m/s, v(0) = 0m/s, r(0) = 0 rad/s.$

Thus, we get a group of control parameters as
The other parameters are selected as
\[ k_1 = 2, k_2 = 8, k_3 = 3, k_4 = 0.8, k_5 = 1.2, k_6 = 0.8, k_7 = 1 \]
\[ \alpha = 0.8, \rho = 0.6. \]

The simulation results are shown in figures 1-6.

The AUV moves from stationary state. Its initial tracking errors are \(|x_e| = 0 \text{ m}, |y_e| = 5 \text{ m}| \), figures 1, 2 show the time responses of the position and velocity of the AUV. From these simulation results, the error \( x_e \) is less than 0.10 m and \( y_e \) is less than 0.07 m.

In figure 3, the simulation trajectory, under the action of the finite time tracking controller is demonstrated. It can be seen obviously that the robot can accomplish the tracking mission accurately in finite time.

From figure 3, it can be seen that the sway displacement, surge displacement converges to desired trajectory infinite time. From figure 3, it can be seen that the angle increased uniformly, which conform to the situation of tracking circular trajectories.

**References**

[1] Wadoo S A and Kachroo P 2010 Autonomous underwater vehicles: modeling, control design, and simulation. CRC Press.

[2] Do K D and Pan J 2009 Control of Ships and Underwater Vehicles, Advances in Industrial Control.

[3] Aguiar A P and Hespanha J P 2007 *IEEE Transac. Automatic Control* Logic-based switching control for trajectory-tracking and path-following of underactuated autonomous vehicles with parametric modeling uncertainty, 52(8) 1362-79.

[4] Du J and Guo C 2004 2004 *American control conf.* Nonlinear adaptive ship course tracking control based on backstepping and nussbaum gain, 3845-50.
[5] Holzhü T 1997 *IEE Proceedings* LQG approach for the high-precision track control of ships. Control Theory and Applications, 144(2) 121-7.

[6] Haimo V T 1986 *SIAM J. Control Optim.* Finite time controllers. 24(4) 760-70.

[7] Bhat S P and Bernstein D S 1998 *IEEE Transac. Automatic Control* Continuous finite-time stabilization of the translational and rotational double integrators, 43(5) 678-82.

[8] Bhat S P and Bernstein D S 2005 *Mathemat. Control Signals Syst.* Geometric homogeneity with applications to finite-time stability, 17(2) 101-27.

[9] Huang X, Lin W and Yang B 2005 *Automatica* Global finite-time stabilization of a class of uncertain nonlinear systems, 41(5) 881-8.

[10] Zhai J 2014 *Circuits Syst. Signal Process* Finite-time output feedback stabilization for stochastic high-order nonlinear systems, 33(12) 3809-37.

[11] Zhang X, Feng G and Sun Y 2012 *Automatica* Finite-time stabilization by state feedback control for a class of time-varying nonlinear systems, 48(3) 499-504.

[12] Du H B, Li S H and Qian C J 2011 *IEEE Transac. Automatic Control* Finite-time attitude tracking control of space craft with application to attitude synchronization, 56(11) 2711-17.

[13] Ma B L 2009 *Syst. Control Lett.* Global K-exponential asymptotic stabilization of underactuated surface vessels, 58(3) 194-201.

[14] Li S and Tian Y P 2007 *Int. J. Control* Finite-time stability of cascaded time-varying systems, 80(4) 646-57.