1. Introduction

The use of traction electric drives with AC electric motors in the rolling stock of main, city, and industrial transport is a current worldwide practice [1–3]. The most common are induction electric drives, which over the past two decades have been widely applied on trolleybuses produced in the European Union: “Solaris Bus & Coach” S. A. (Poland), Škoda Transportation (Czech Republic). As well on trolleybus manufacturers in other countries: “Busscar Ônibus S. A.” (Brazil), “Bogdan” (Ukraine), “Elektrorans” (Ukraine), “Elalon” (Ukraine). As well as trams manufactured by “Tatra-Yug” (Ukraine) and “Elektrorans” (Ukraine), and others [4]. On railroads, induction traction drive is also most...
common in rolling stock. For example, electric locomotives manufactured by “LORIC” (China), “Alstom”, “ADTranz”, “Bombardier”, “Siemens AG” (European Union), DS-3 manufactured by DEVZ (Ukraine), diesel trains DEL-02 manufactured by “Luganskteplovoz” (Ukraine), high-speed electric train EKR-1 manufactured by PAO “KVBZ” (Ukraine). They are also common for the rolling stock from foreign manufacturers, such as HRCS2 manufactured by Hyundai Rotem (South Korea) and EJ 675 by Skoda Transportation (Czech Republic), operated by Ukrzaliznytsia. The advantages of traction drive based on induction motors include the high energy parameters and mass-dimensional indicators, the simplicity of design and increased reliability, as well as a long maintenance interval [5]. Within global energy-saving trends, an alternative approach is the use of synchronous traction engines [8]. Installing permanent magnets in the engine rotor made it possible to completely refuse the contact apparatus but a large mass of highly-coercive permanent magnets significantly increases the cost of producing such electric motors [6]. To meet the requirements for compliance with a wide range of speed and torque adjustments, high energy indicators must be demonstrated not only under a rated mode but also at small and maximum rotational speeds. As indicated in work [7], such characteristics can be provided by permanent magnet-assisted synchronous reluctance motors (PMSynRM, PMA-SynRel). The analysis reported in [8] reveals that the mass-dimensional and cost indicators of the engine of this type are compared with those in the best induction ones. Thus, a drive based on permanent magnet-assisted synchronous reluctance motors is a direct competitor to the induction traction electric drive. Therefore, it is a relevant task to undertake a study aimed at improving the mass-size and energy indicators of permanent magnet-assisted synchronous reluctance motors.

2. Literature review and problem statement

Work [9] considers the design and principle of operation of permanent magnet-assisted synchronous reluctance motors with various engine rotors. In [10], it is noted that in a synchronous reluctance motor with permanent magnets, the stator is similar in design to the stator of an induction motor. Despite this, the structures of the rotor are very different. The rotor of the synchronous motor, in the radial direction, in order to increase the reactive torque, is divided into channels of flow barriers. A feature of the rotor is the presence of permanent magnets (rare earth elements such as NdFeB, ferrite Y30), inserted into its flow barriers [11], which line up in a line close to the armature’s reaction flow. The number of lines of barriers (holes in the core of the rotor) can be from one, perpendicular to the axis q of the motor, as indicated in [12], and along the d axis, described in [13], up to five, which are arranged at an angle to q axis, reported in [14]. A first type of rotor design belongs to the type with non-partitioned magnets, the rest are partitioned.

Magnets installed on the rotor are evenly magnetized in the direction of the q axis, actually forming it. Permanent magnets have approximately the same permeability as air. Therefore, the line of barriers (sections) creates a direction with high resistance and magnetic anisotropy in the direction of the q axis. Rotor’s iron has small segments that are necessary to build a solid rotor structure [7]. Rotor’s iron in segments has a large saturation with the rotor dispersion flows, so their magnetic permeability is significantly reduced. The size of the segments is chosen as a minimum. It is due to the strength indicators of the rotor [7]. For the d axis, the resistance to the magnetic flux is significantly reduced due to the fact that it passes along permanent magnets behind steel areas with low saturation. As noted in work [6], in terms of generating torque, the dispersion of the flow caused by the tangential ribs should be minimal.

According to the results of the analysis reported in [11], it is noted that determining the size and parameters of electric motors can be carried out through numerical field calculations in combination with multicriteria optimization, as shown in [15]. The design methodology, considered in work [7], provides an opportunity to determine only the main parameters of the engine stator, the thickness and width of permanent magnets. But their arrangement in the rotor, on which the operating properties of the engine depend, can be determined only based on a finite-element analysis of the magnetic field of the engines [16, 17]. All this suggests that it is advisable to conduct a study to define the optimal rotor geometry of the non-partitioned permanent magnet-assisted synchronous reluctance motor.

3. The aim and objectives of the study

The purpose of this study is to devise a methodology for determining the optimal geometrical rotor parameters for the non-partitioned permanent magnet-assisted synchronous reluctance motor, which would make it possible to increase the electromagnetic momentum of the engine at the specified stator sizes.

To accomplish the aim, the following tasks have been set:
– to build a mathematical model of the magnetic field of a non-partitioned permanent magnet-assisted synchronous reluctance motor;
– to state the problem of optimizing the rotor parameters for a non-partitioned permanent magnet-assisted synchronous reluctance motor for the trolleybus wheels drive;
– to solve the problem of conditional optimization of the geometrical parameters for the motor rotor and analyze the results of rotor synthesis.

4. The study materials and methods

Based on the review of earlier research, it is proposed that the method of optimal design [18, 19], based on the solution to the problem of conditional optimization of engine rotor’s geometrical parameters, should underlie our procedure. To solve the analysis problem, it is proposed to apply a finite-element method for calculating the magnetic field, followed by determining the level of electromagnetic momentum of the engine. A similar approach was employed to optimize the parameters of induction motors in [20]. The adequacy of the results was confirmed by using the tested software packages for magnetic field simulation FEMM (USA) [16, 17], as well as optlab suites (Ukraine) [18, 19, 21].
5. Results of studying the geometrical parameters of the rotor for a non-partitioned permanent magnet-assisted synchronous reluctance motor

5.1. Mathematical model of the magnetic field in a non-partitioned permanent magnet-assisted synchronous reluctance motor

The main provisions of the design of permanent magnet-assisted synchronous reluctance motors are set out in work [7]. This procedure makes it possible to determine the thickness and approximate dimensions of permanent magnets. However, their mutual arrangement in the rotor can significantly affect the level of electromagnetic motor momentum. The area of research is to determine the most effective arrangement of permanent magnets in the engine rotor based on the criterion of the level of electromagnetic momentum. Therefore, the main criterion for the efficiency of the rotor is the level of electromagnetic momentum. This paper considers the statement of the analysis problem for a rotor with non-partitioned permanent magnets. The design of this type is one of the solutions acceptable for permanent magnet-assisted synchronous reluctance motors. The example considered here involves selecting an engine rotor geometry for the drive of trolleybus wheels. According to the preliminary calculations using the methodology from [7], we defined the estimation data for the engine, given in Table 1. The estimated data for the permanent magnet and its material are given in Table 2.

As the basic geometry of the engine, a structure with non-partitioned permanent magnets was chosen, which are in the cutouts of the rotor package. Such a structure is one of the competitive engine designs. Moreover, it has several advantages such as greater manufacturability and lower mechanical loads in the rotor design elements. The basic geometry is shown in Fig. 1.

To determine the electromagnetic momentum of the engine, it is proposed to use a finite-element method for the flat-parallel problem statement. This method makes it possible to take into consideration the distribution of magnetic flux in the rotor, the saturation of its parts depending on the location of magnets. The finite-element simulation package FEMM [6, 16, 21] was used to solve the problem. According to the method, the estimated region is divided into triangular finite elements. The density of the grid is adaptive, which makes it possible to determine in more detail the geometry of

### Table 1

| Specification                  | Value  |
|-------------------------------|--------|
| Engine power, kW              | 180    |
| Maximum electromagnetic moment, Nm | 970    |
| Armature current at maximum moment, A | 350    |
| Number of grooves             | 48     |
| The number of effective conductors in the groove | 6  |
| Number of parallel branches   | 1      |
| Step reduction factor         | 0.778  |
| The number of elementary conductors in the effective circuit | 3 (for height) |
| Estimated outer diameter of the stator, m | 0.46 |
| Boring diameter, m            | 0.3    |
| Axial length of the magnetic circuit, m | 0.25 |
| Unilateral air gap, m         | 0.002  |
| Dimensions of the stator winding conductor (without insulation) | 1.6–12.5 |
| Groove height, m              | 0.040  |
| Groove width, m               | 0.0145 |

### Table 2

| Specification                  | Value  |
|-------------------------------|--------|
| Number of pole magnets        | 2      |
| The thickness of the magnet, m | 0.03   |
| Magnet width, m               | 0.045  |
| Material of magnets           | Nd-Fe-B|
| Residual magnetic induction, Tl | 1.0    |
| Coercive force on magnetization, A/m | 850,000 |

Fig. 1. Basic geometry of the non-partitioned permanent magnet-assisted synchronous reluctance motor: 1 — cutouts in the core of the rotor for permanent magnets; 2 — permanent magnets; 3 — the core of the rotor

Fig. 2. A finite-element grid of the non-partitioned permanent magnet-assisted synchronous reluctance motor: a — general view; b — permanent magnet zone
the rotor and the air gap (Fig. 2). The value of the armature currents and the position of the rotor correspond to the rated mode of motor operation. The pattern of the engine magnetic field, based on the calculation results, is shown in Fig. 3. Based on the calculation of the magnetic field, the electromagnetic momentum of the engine is determined using the standard FEMM functions [16].

The mutual arrangement of the cutouts in the core of the rotor is imposed with restrictions on the mechanical strength of the engine structure. Using the procedure given in [7], we have established the dependence of the maximum angle of magnet inclination on size $a_1$. The dependence is shown in Fig. 4: it is approximated by the third-order spline function.

2) Restrictions imposed on the parameters of the optimization problem according to geometrical conditions. The geometrical dimensions of structural elements, which are parameters of the optimization problem, are subject to restrictions due to manufacturing technology:

\[0.01 \text{ m} < a_1 < 0.035 \text{ m}; \]
\[0 < \alpha < 80; \]  \hspace{1cm} (1)
\[0.001 \text{ m} < \delta < 0.0025 \text{ m}. \]
\[\theta - 45^\circ \text{ el.} < \theta < 45^\circ \text{ el.} \]

Optimization parameters are different in the order of determining. Therefore, to apply a single step of the optimization method in all parameters, it is advisable to switch to relative units when determining parameters. Thus, the parameters of the problem in relative units are determined from the following expressions:

\[x_1 = (a_1 - 0.01) / 0.025; \]
\[x_2 = \alpha / 80; \]  \hspace{1cm} (2)
\[x_3 = (\delta - 0.001) / 0.0015; \]
\[x_4 = (\theta + 45) / 90. \]

5.2. Statement of the problem of optimizing the rotor parameters of a synchronous reluctance motor for the drive of trolleybus wheels

The prerequisite for optimizing the rotor geometry of a non-partitioned permanent magnet-assisted synchronous reluctance motor is to define its parameters unambiguously. These parameters should accurately describe the geometrical position of the cutout in the rotor and the magnet in it.

As parameters for solving the optimization problem, it is proposed to choose the size $a_1$, which is the distance of the magnet from the interpole axis and the magnet’s angle of inclination $\alpha$ relative to it. In addition, the parameters should include the air gap of the engine $- \delta$ (Fig. 1), which determines the main magnetic resistance, as well as the change in the angle (\(\theta\)) of the load relative to the basic structure in electrical degrees. The basic load angle chosen for the structure is $135^\circ$ electric. The traction drive control system ensures maintaining the load angle under a rolling stock acceleration mode and reduces the load angle under the mode of maximum movement speed [5, 7, 9]. Therefore, determining the value for a load angle that corresponds to the maximum value of the electromagnetic momentum is necessary to define the parameters of the engine control system.

Optimization parameters are imposed with limitations that can be divided into two types:

1) Rotor strength limitations.

2) Restrictions imposed on the parameters of the optimization problem according to geometrical conditions. The geometrical dimensions of structural elements, which are parameters of the optimization problem, are subject to restrictions due to manufacturing technology:

\[0.01 \text{ m} < a_1 < 0.035 \text{ m}; \]
\[0 < \alpha < 80; \]  \hspace{1cm} (1)
\[0.001 \text{ m} < \delta < 0.0025 \text{ m}. \]
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\[x_3 = (\delta - 0.001) / 0.0015; \]
\[x_4 = (\theta + 45) / 90. \]
To improve the efficiency of the traction motor, it is necessary to increase the level of electromagnetic momentum. Since most optimization methods are aimed at minimizing the objective function, we select

$$F = -M_e \rightarrow \min,$$  \hspace{1cm} (3)

where $M_e$ is an electromagnetic momentum determined from the calculation of the magnetic field by a finite-element method.

5.3. Solving the problem of conditional optimization of the geometrical parameters of the motor rotor

To solve the problem of conditional optimization of the rotor’s geometrical parameters, it is proposed to use the optlab software package (Ukraine) [18, 19, 21]. This system makes it possible to choose a rational method for solving the optimization problem. The software package FEMM 4.2 (USA) [16, 17] is used to solve the analysis problem.

A number of test solutions to the optimization problem showed that the result of the solution does not depend on the starting point. This suggests finding one global minimum, that is, that the objective function is not multi-extremum. Therefore, according to [18, 19], it is rational to use the Nelder-Mead method.

The procedure of solving the problem is shown in Fig. 5 for coordinates $x_1$, $x_2$, $x_3$, and $x_4$. In Fig. 5, only the best points are marked. The starting point 1 of the search is indicated by a circle and the end point 2 is indicated by a diamond.

The starting point of the search was chosen as a point with the relative coordinates $x_1=0.3$ rel. units, $x_2=0.3$ rel. units, and $x_3=0.2$ rel. units, and $x_4=0.5$ rel. units.

The results are given in Table 3.

| Parameter | Value          |
|-----------|----------------|
| $x_1$     | 0.3802 rel. units |
| $x_2$     | 0.449 rel. units  |
| $x_3$     | 0.2109 rel. units  |
| $x_4$     | 0.2625 rel. units  |
| $a_1$     | 0.0195 m         |
| $\alpha$  | 35.9°           |
| $\delta$  | 0.0013 m        |
| $\theta$  | −21.38° electric |
| $M_e$     | 1.120 Nm        |

According to the calculations, the value of the objective function decreased by 32.2 % (from −847 Nm to −1120 Nm), which makes it possible to significantly increase the electromagnetic momentum only with the help of the optimal arrangement of magnets on the motor rotor.

6. Discussion of results of synthesizing the rotor for a non-partitioned permanent magnet-assisted synchronous reluctance motor

Our optimal result (Table 3) proves that the arrangement of permanent magnets is close to [12]; however, it is impossible to completely turn the permanent magnets by an angle of $\alpha=45°$ due to the limitation for the mechanical strength of the rotor and the size of permanent magnets. Based on the calculations according to the proposed methodology, it has been determined that the optimal distance from the interpole axis and the angle of magnet rotation are subject to the limitation established for the strength of the rotor structure. In the resulting geometry, the flow of permanent magnets along the $q$ axis is the largest. According to the results from solving the optimization problem, the air gap underwent a slight increase of 5.5 %, while the load angle decreased by 21.38° electric, which is due to the significant saturation of engine rotor elements.

We have proposed the synthesis of rotor geometrical parameters for a non-partitioned permanent magnet-assisted synchronous reluctance motor.
synchronous reluctance motor, based on solving the problem of conditional optimization. The restrictions imposed on optimization parameters have been defined. The study results helped establish the dependence of limiting the angle of magnet rotation based on strength calculations, which is a feature of this problem.

The procedure devised could be used to design electric vehicle traction engines, such as trolleybuses, electric vehicles, electric trains, and trams. The range of engine capacities for which it is suitable is limited to capacities from 40 kW to 500 kW. With less power, it is rational to use partitioned permanent magnets; and with greater power – technological difficulties in assembling the rotor occur. The procedure holds for the use of motors with highly coercive permanent magnets. According to our calculations, the value of the objective function decreased by 32.2 % (from −847 Nm to −1,124 Nm), which makes it possible to significantly increase the electromagnetic momentum only with the help of the optimal arrangement of magnets on the motor rotor.

In several designs [12], an additional magnetic shunt (a hole in the rotor package) is installed in the rotor. This hole and its geometry make it possible to further increase the electromagnetic momentum of the engine. Not taking into consideration the shunt is likely a caveat of our work, which can be eliminated in further research.

In addition, the procedure that we devised for determining the optimal rotor geometry can be advanced through the optimization of rotor parameters with partitioned magnets and magnets that are arranged in an arc.

7. Conclusions

A mathematical model has been built for determining the electromagnetic momentum of a non-partitioned permanent magnet-assisted synchronous reluctance motor. It is based on the calculation of the motor’s magnetic field by the finite-element method in the flat-parallel statement of the problem. The use of an adaptive finite-element grid is proposed for solving the problem. For non-partitioned permanent magnet-assisted synchronous reluctance motors, a procedure has been devised for synthesizing optimal rotor geometric dimensions, which is based on solving the problem of conditional optimization. The following optimization criteria, which are defined in relative units, have been established: the magnet distance from the interpole axis, the angle of magnet inclination relative to the interpole axis, the air gap of the engine, as well as the change in the load angle relative to the base structure.

2. The problem of conditional optimization of the rotor for a synchronous reluctance motor has been stated on the basis of the rotor’s geometric criteria. As an analysis problem, it is proposed to use the mathematical model of the motor’s magnetic field. The limitation for the geometric and strength indicators has been defined in the form of dependence of the maximum angle of magnet inclination on size, which is approximated by the spline function of the third order. The Nelder-Mead method was chosen as the optimization method.

3. The results of solving the problem of synthesizing rotor parameters for a trolleybus traction motor have helped determine a 32.2 % increase in its momentum compared to the basic design, as well as define the optimal geometrical parameters for arranging permanent magnets (the distance from the boundaries of the pole axis, 0.0195 m; the angle of inclination, 35.9°), the air gap, which is 0.0013 m, and the engine load angle, which is 113.62° electric.
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