Diffusion segregation of heavy elements in the Sun

A B Gorshkov and V A Baturin
Sternberg astronomical institute of Moscow State University, Russia
E-mail: gorshkov@sai.msu.ru

Abstract. Lowering of heavy element abundances $Z$ according to modern determinations presents a problem of the solar modeling. We estimate settling rates of several heavy elements such as C, N, O, Ne, Si and Fe from the solar convective zone during evolution according to the Bahcall-Thoul technique. Acceleration of the heavy elements settling due to partial ionization and radiation-driven slowing are taken into account. Diffusion profile in modern Sun reveals a narrow region under the convective zone where heavy elements abundance remains almost the same due to specific effect of thermal diffusion. A rate of the elements settling from the convective zone depends on the convective zone depth and on the overshooting mixing around the low convective boundary. According to our results, diffusion settling is fairly small to explain entirely a low $Z$ value in the photosphere.

1. Introduction
In this work we: 1) estimate the rate and its variation of different mechanisms of heavy elements diffusion inside the Sun, and 2) solve diffusion equation on the evolutionary set of solar models to get chemical composition profiles with the diffusion taken into account. Rotation, magnetic field and non-radial flows are ignored.

Solar models given initially by evolutionary track 721-0001 based on SAHA-S equation of state with hydrogen and helium diffusion included (S. Ayukov, private communication) as $T(r,t)$, $P(r,t)$ etc. and did not changed in simulations. We treat the solar convective zone as a region with ultrafast diffusion (taking into account that convection rapidly equalizes chemical composition over all convective zone).

2. Problem Formulation
For component $i$ of solar plasma (ion or electrons) we can write equations for momentum and energy conservation:

$$\nabla P_i - \rho_i \frac{F_i}{m_i} = \sum_k \left[ G_{ik}^{(1)} (w_i - w_k) + \mu_{ik} G_{ik}^{(2)} \left( \frac{h_i}{m_i} - \frac{h_k}{m_k} \right) \right]$$

(1)

$$\frac{5}{2} P_i \frac{\nabla T}{T} = \sum_k \left[ \frac{5}{2} \mu_{ik} G_{ik}^{(2)} (w_i - w_k) + G_{ik}^{(5)} h_i + G_{ik}^{(6)} h_k \right]$$

(2)

where $P_i$, $\rho_i$ – partial pressure and mass density, $T$ – temperature, external force $F_i = m_i g - q_i E$, $m_i$, $q_i$ – atomic mass and charge, reduced mass $\mu_{ik} = m_i m_k / (m_i + m_k)$, $G_{ik}^{(l)}$ – “friction” coefficients describing momentum and energy exchange in particles’ collisions, and, finally, $w_i$ and $h_i$ – diffusion velocities and partial residual heat flows correspondingly.
This hydrodynamic approach was proposed by [1] and later developed by [2], [3] and [4].
Following [3], we complement these equations with conditions of current neutrality and mass conservation ($\rho_{ei}$ – charge density of component $i$):

$$\sum_k \rho_{ek} w_k = 0, \quad \sum_k \rho_k w_k = 0.$$  \hspace{1cm} (3)

So, for $N$ species we have a linear system of $2N+2$ equations for $2N+2$ unknowns: $N$ diffusion velocities $w_i$, $N$ partial residual heat flows $h_i$, local electric field $E$ and gravitational acceleration $g$. Treatment gravity acceleration as unknown allows to not exclude explicitly one equation from the overdetermined system. Note that species $i$ can be different chemical elements as well as differently charged ions of the same element.

In every $(r, t)$ point of models we search a solution in a form ($K_0$ is numerical coefficient)

$$w_i = K_0 \frac{T^{5/2}}{\rho} \left( A_P \frac{d\ln P}{dr} + A_T \frac{d\ln T}{dr} + \sum_k A_{Xk} \frac{d\ln X_k}{dr} \right),$$  \hspace{1cm} (4)

and get coefficients $A$. Substituting $w_i(r, t)$ in the diffusion equation

$$\frac{\partial X_i}{\partial t} = -\frac{1}{\rho r^2} \frac{\partial (r^2 \rho w_i X_i)}{\partial r},$$  \hspace{1cm} (5)

and integrating (5) over $t$ with appropriate boundary ($dX/dr(0, t) = 0$, $dX/dr(R_{\text{Sun}}, t) = 0$) and initial ($X(r, 0) = \text{const}$) conditions, we finally calculate composition profiles $X_i(r, t)$ for every component $i$ of plasma.

Representation (4) of diffusion velocity makes possible to compare incomes to $w_i$ from barodiffusion, thermodiffusion etc.

Here we present diffusion simulation of mixture $H^+ + He^{2+} + Me^{Z+} + e^-$, with initial heavy element mass-fraction $X_{Me}(r, 0) = 0.02$, where $Me^{Z+}$ denotes heavy element under consideration with charge $Z$ (in general case $Z = Z(r, t)$).

3. Results

3.1. Effects of photodiffusion and radius-dependent ionization stage

Since diffusion rates depends on the plasma interaction cross section, highly ionized ion of the element has bigger cross section and lower diffusion velocity.

Hydrogen and helium are fully ionized everywhere in the solar interiors, but heavy ions are presented in solar plasma in several ionization stages, and a diffusion velocity is higher for ions with smaller charge. We calculate ionization stages according to SAHA-S EOS tables ([5]). $CNO$ and $Ne$ elements reveal to be fully ionized in the convective zone while Si and Fe do not reach full ionization even in the center of the Sun.

Since heavy elements are not fully ionized in solar interiors, the effect of interaction with radiation flux is noticeable (so called photodiffusion). We take into account radiative accelerations modifying gravitational acceleration $g$ in (1) by a factor $(1 - g_{\text{rad}} i/g)$, where radiative accelerations $g_{\text{rad}}$ are calculated according to [6].

We analyzed the settling of $C, N, O, Ne, Mg, Al, Si, S, Ar, Ca$ and $Fe$ in $Z(r, t) = Z_{\text{max}}$ approximation, and $C, N, O, Ne, Si, and Fe$ – with $Z = Z(r, t)$ and photodiffusion taken into account. We compare diffusion profiles for carbon and silicon (Fig. 1). Diffusional settling rates from the convective zone of six heavy elements are demonstrated on Fig. 2.
3.2. Elements settling as a function of mixing zone boundary position
To test the effect of shifting of the mixing zone boundary on the rate of hydrogen enhancement in solar envelope we performed several simulations with fixed evolutionary solar model sequence and different positions of mixing zone boundary. Resulting diffusion profiles are shown on the Fig. 3. From the experiment we conclude that diffusion enhancement (for hydrogen) and diffusion settling (for heavier elements) decrease in deeper convective zone.

3.3. Temperature profile effect
From equations (4) and (5) follows that $\frac{\partial X_i}{\partial t}$ is proportional to the second derivative of the temperature. Temperature gradient lowering in the narrow region under the mixing zone leads to the thermodiffusion acting against the gravity. As a result, a local maximum of heavy element profiles arises (Fig. 4). For additional check we performed simulations without taking thermodiffusion into account, and the local maximum of element profile has disappeared.
Figure 3. Hydrogen diffusion profiles for $t = 4.6$ Gyr in the case of the original boundary position ($0.73R$, solid line), and in cases of deeper mixing zones ($0.68R$ and $0.63R$, dashed and dot-dashed lines correspondingly). $X_H$ values in mixing zone are 0.749, 0.742 and 0.737 correspondingly. Initial abundance equals 0.716. The effect of nuclear burning on $X_H$ in solar core is not shown for clarity.

Figure 4. Effect of temperature gradient profile on element settling under the mixing zone boundary. Solid line – oxygen diffusion profile, dashed line – oxygen diffusion profile calculated without thermodiffusion, dotted line – temperature gradient profile for $t = 4.6$ Gyr.

4. Conclusions

- Taking into account effects of photodiffusion and radius-depended ionization stage provide more accurate treatment of diffusion. Superposition of these effects noticeably increase diffusion velocity for neon and heavier elements.

- According to [7] the heavy element abundance in solar atmosphere is lower by 0.006 in comparison to previous works. These figures are not directly related to our computations because we compute only differences with primordial abundances, which are generally unknown. Our results show that settling could not explain entirely the low-Z heavy element abundances in the atmosphere according to [7].

- Nevertheless, diffusional settling of elements from convective zone is sensitive to a location of the convective zone boundary – the most important value in low-Z problem consideration – and increases in shallower convective zone.

Acknowledgments

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