To condense, or not to condense, that is the question

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Abstract. Recent key experimental results on the spectroscopy of $^{12}$C have been made, which, when taken together, provide much needed clarity on the structure of the ground- and Hoyle-state excitations and provided new avenues for future research. High excitation energies in light nuclei are being explored with evermore precision and recent examples of this in $^{12}$C, $^{16}$O and $^{20}$Ne are presented. These steps forward are examined and the outstanding issue of clustering versus condensation explored.

1. Introduction
The $\alpha$ particle is a very robust nucleus – possessing no bound excited states despite the first threshold lying at 19.81 MeV – and has a total spin of zero. These properties have two consequences: one is the geometric arrangement of pre-formed $\alpha$ particles in the nucleus, i.e. cluster structure, and the other is the possibility of the existence of gas-like states, obeying Bose statistics, namely $\alpha$ condensation.

Alpha clustering in light nuclei has been of great interest for many years. Theories of clustering date from the 1930’s and so far many varied theoretical models have been developed. Recently the idea of condensation of $\alpha$ particles has been intensely investigated and much theoretical progress has been made. Theoreticians and experimentalists from Japan have played key roles in this. One of the wonders of nature, the Hoyle state in $^{12}$C, has been proposed as the best condensate candidate and recently significant experimental and theoretical effort has been invested to understand, in detail, its unorthodox structure. In particular, the identification of excitations of the Hoyle state has been a key goal.

This paper addresses the current experimental progress in understanding excitations in $^{12}$C and also the search for analogues of the Hoyle-state in heavier $\alpha$-conjugate nuclei. The Birmingham group has recently undertaken an extensive programme investigating pertinent states in several nuclei. New studies at high-excitation energies in $^{12}$C, using inelastic scattering at the Birmingham cyclotron, and measurements of $^{20}$Ne made at Notre Dame, will be presented. Additionally, results for $^{16}$O relevant to the long-standing evidence for $^8$Be – $^8$Be cluster structures and four-$\alpha$ chain states will be discussed. Finally, the prospect of condensed bosonic-nuclear matter in heavier nuclei will be addressed.

2. Carbon-12 ground-state symmetries
Before understanding the crucial Hoyle state in $^{12}$C, it is revealing to first examine structures associated with the ground state band. In 2012 the Birmingham group performed an inelastic scattering experiment using a 40 MeV $^4$He beam impinging on a 100 $\mu g/cm^2$ nat-$^4$C target. An
array of four double-sided silicon strip detectors were placed symmetrically around the target position, to measure at least three of the four final-state particles; the scattered $^4\text{He}$ projectile and the $^{12}\text{C}^*$ break-up $\alpha$ particles. More details on the set-up can be found in [1, 2]. The data were reconstructed using energy and momentum conservation and events were selected by placing gates on both the total $Q$-value, defining the reaction of interest, and on those in which two of the four alpha-particles originated from the $^8\text{Be}$ ground state. The resulting energy spectrum of $^{12}\text{C}$ is shown in figure 1 with the additional requirement that the scattered beam particle and two of the three break-up particles were detected. A state at 22.4 MeV is clearly visible.

![Figure 1](image.png)

**Figure 1.** Carbon-12 excitation energy calculated from two detected $\alpha$ particles and the reconstructed third, undetected, break-up $\alpha$ particle. The newly observed state at 22.4(2) MeV can be clearly seen. The contamination by $^8\text{Be}$ states below 20 MeV is also observed. Based on figure 2 of [1].

By analysing the angular correlations of the initial break-up particles – $\alpha$ and $^8\text{Be}$ – for a subset of the data in figure 1, the periodicity of the oscillations for the 22.4 MeV resonance was found to be consistent only with $I = 5$. Given that the reaction exclusively involved spin-zero particles, the states populated are restricted to possessing natural parity leading to a spin-parity assignment of $I^\pi = 5^-$. This state has now also been observed in a second study exploiting $^3\text{He}$ inelastic scattering [3] (see later in this section). Calculations made using the algebraic cluster model, predict a characteristic sequence of states comprising, with increasing excitation energy, $0^+, 2^+, 3^-, 4^\pm$ and $5^-$. In this model, the three $\alpha$ particles lie at the corners of an equilateral triangle; $D_{3h}$ symmetry. The model allows for rotation about various axes as well as expansion and contraction. Other key features are softness (making nuclei more like liquids rather than rigid structures), a near equality of rotational and vibrational energies (such that it is difficult to distinguish between these two types of motion) and non-point-like identical constituents with a spatial extent comparable in size to the overall structure [4]. The energies predicted by the
algebraic model for $^{12}$C are shown in figure 2 together with the corresponding experimentally measured states. The $0^+$ and $2^+$ excitations correspond to rotations about an axis lying in the plane of the triangle with rotations about an axis perpendicular to the plane contributing to the higher lying spins and parities. The juxtaposed $4^−$ and $4^+$ states together with the newly observed $5^−$ state is crucial evidence in favour of $D_{3h}$ symmetry. See also [5].

![Figure 2](image_url)

**Figure 2.** Left: experimentally observed states currently assigned to the ground- and Hoyle bands. Right panel: corresponding predicted energies in $^{12}$C using the algebraic cluster model (see text for details) and performed by R. Bijker for [1].

In the process of obtaining evermore precise data on the higher-energy domain of $^{12}$C it is not just the $I^{π} = 5^−$ state that has been discovered in recent experimental work. The study that independently confirmed the existence of the 22.4 MeV resonance via the $^{12}$C($^3$He,$^3$He)$^{12}$C* break-up reaction at 46 MeV [3], additionally found evidence for a series of states at 16.3, 17.2, 18.4, 19.7 and 25.1 MeV. These can be seen in figure 3. These states all decay via the $^8$Be ground state and some of the higher-lying resonances may be related to excitations of the Hoyle state, beyond what is currently established (see section 3). Despite $^{12}$C being studied extensively, such $5^−$ and other newly observed resonances demonstrate the importance of understanding structures with precision to the excitation-energy limit.

3. Hoyle-state excitations

Returning to the calculations presented in figure 2, it can been seen that the algebraic model not only makes predictions for the ground-state band described in section 2, but also describes the Hoyle state and its excitations as a symmetric stretching vibration of the equilateral triangle. This is at odds with results from state-of-the-art chiral effective field theory (EFT) calculations on the lattice which predict a bend-arm or open triangle configuration [6]. While both the EFT lattice and algebraic models agree on the triangular structure of the ground-state band, the calculations diverge for excited structures. It is here where precision experimental results can play a pivotal discriminating role.
Carbox-12 high-excitation energy regime populated following break-up induced by $^3$He inelastic scattering. The states shown decay via the $^8$Be ground-state. The energies of the newly observed states are labelled in MeV in black and the $5^-$ in red. Intense, known excitations at lower energies are not labelled. Adapted from figure 5 of [3].

Recently several experimental advances have been made in identifying excited states built on the Hoyle state following a concerted international effort using a variety of techniques including break-up reactions and photo-disintegration via Compton backscattered photons. Beginning in 2004 by Itoh et al. [7] and later with Freer and coworkers [8] using inelastic proton scattering from carbon at the K600 spectrometer at IThemba Labs, South Africa, evidence for a possible $2^+$ excitation of the Hoyle state began to emerge. This culminated in the confirmation by several separate studies [9, 10, 11] and convergence on the properties of the newly discovered resonance: $I^\pi = 2^+$, $E_x = 9.8 - 10.0$ MeV and $\Gamma = 800$ keV [10, 11]. The proximity of the $2^+_2$ to the well known $3^-$ state at 9.641 MeV that typically dominates this excitation energy region has led to a more precise characterisation of the properties of the $3^-$ state in order to fully understand the various contributions at each point. Making use of the proton inelastic scattering data with an experimental resolution of 23(1) keV, an $R$-matrix fit to the data yields a consistent width of $48(2)$ keV [12] at laboratory scattering angles of 10°, 16° and 28°. This width is significantly larger than the literature value of $34(5)$ keV [13].

Furthermore, in 2011, Freer et al. [14] obtained evidence using the two break-up reactions $^{12}$C($^4$He,$^4$He+$^4$He+$^4$He)$^4$He and $^9$Be($^4$He, $^4$He+$^4$He+$^4$He)n for a possible $4^+$ state – postulated to be the second excitation of the Hoyle state – lying at 13.3 MeV with a width of 1.7 MeV. This has been further elucidated by Marín-Lámbarri et al. [2], once again using angular correlations to confirm the $I^\pi = 4^+$ assignment.

With these new data, an early predictions for the underlying structure of the Hoyle state can be ruled out, specifically, a linear chain of three-$\alpha$ particles. This is by a simple calculation of the moment-of-inertia and relating this to the energy separation of the Hoyle state and the $2^+_2$ state, which is now known to be $\approx 2.2$ MeV. However, without further information, distinguishing between other predictions is challenging. For example, as mentioned earlier the chiral EFT lattice calculations predict a bent-arm structure and excitations of such a structure are not inconsistent with the present data. However, another theoretical approach treats the Hoyle state as a condensate of $\alpha$ particles all sharing a common $s$-orbit wavefunction [15]. In this model angular momentum is generated via exciting one of the $\alpha$-particle bosons to a $d$-wave orbital [16, 17]. Again, the current data do not rule out such a scenario and are also consistent
with the algebraic stretching mode discussed at the beginning of this section. What is missing to
differentiate between the model predictions are electromagnetic transition rates linking the band
members. Firstly, measuring enhanced $\gamma$ decay rates would verify the common structural lineage
and secondly, the transition rates constitute precision data to challenge the nuclear models. The
difficulties lie in reliably measuring the anticipated $10^{-5} - 10^{-7}$ branching ratios for such internal
transitions. Such investigations are not without precedent having been performed by Datar and
coworkers for the $4^+$ to $2^+$ transition in $^8\text{Be}$ [18] – a branching ratio of $1.5 \times 10^{-7}$.

With so much activity, both theoretical and experimental, the current status of our knowledge
of the Hoyle state has recently undergone a timely review by Freer and Fynbo [19].

4. Heavier systems
The discussion so far has concentrated on the $^{12}\text{C}$ system, and the Hoyle state in particular,
which although an important – and undoubtedly the most studied – multi-$\alpha$ state, it is expected
to be just one example with similar levels predicted in the heavier $\alpha$-conjugate nuclei $^{16}\text{O}$, $^{20}\text{Ne}$
etc. Indeed, there is long-standing evidence for a linear-chain of four $\alpha$-particles in $^{16}\text{O}$ as
evidenced by $^8\text{Be}$–$^8\text{Be}$ structures reported by Chevallier et al. [20]. Using the $^{12}\text{C}(^4\text{He},^8\text{Be})^8\text{Be}$
reaction, evidence for a rigid linear chain emerged from angular distributions for the observed
resonances indicating spin and parity assignments of $I^f = 2^+$, $2^+$, $4^+$ and $6^+$ at $E_x = 16.95$,
17.15, 18.05, and 19.35 MeV, respectively. To search for further evidence, e.g. an associated $8^+$
above 20 MeV, Curtis et al. [21] used the same reaction at Notre Dame, USA, measuring the
excitation function from 12.2 to 20.0 MeV in steps as small as 10 keV. By reconstructing the
kinematics of the break-up events, selections could be made for those originating from two $^8\text{Be}$
nuclei, or conversely, events that decayed via $^{12}\text{C}^* + \alpha$. The resulting two excitation functions
are shown in figure 4.

In order to extract spectroscopic information the $^8\text{Be}$ centre-of-mass angular distributions
were fitted with

\[
W(\theta) = \left| \sum_{L=0}^{L_{\text{max}}} \rho_L e^{i\phi_L} P_L(\cos(\theta)) \right|^2, \tag{1}
\]

where $\rho_L$ is the amplitude, $\phi_L$ the phase, $P_L$ Legendre polynomials and fitting was performed
for $L_{\text{max}} = 2, 4, 6$ and 8. However, the results of this phase-shift analysis not only found no
evidence of a $^8\text{Be}$–$^8\text{Be}$ $8^+$ structure, but also did not confirm the presence of the previously measured $6^+$ state, effectively ruling out the existence of a four-$\alpha$ linear chain band in this
region of the $^{16}\text{O}$ energy spectrum.

The search for a chain state in $^{16}\text{O}$ was conducted in the <24 MeV region as the four-$\alpha$
threshold lies at 14.44 MeV. For the next $\alpha$ conjugate nucleus, $^{20}\text{Ne}$, the analogous five-$\alpha$
threshold lies at 19.17 MeV, delineating the region to look for Hoyle cognate states. An $\alpha$-
particle beam delivered by the Notre Dame tandem was once again used, but this time to explore
the resonant $^{16}\text{O}(^4\text{He},^{20}\text{Ne}^*)$ break-up reaction cross-section as a function of excitation energy.
The analysis probed both the $^8\text{Be}$–$^8\text{Be}$–$\alpha$ and $^{12}\text{C}^*$–$^8\text{Be}$ channels, the latter being a sub-set of
the former in this case. The excitation functions are shown in figure 5 and for the $^8\text{Be}$–$^8\text{Be}$–$\alpha$
channel (left panel) exhibit significant strength above 25 MeV. These results represent the first
data on these channels in high excitation energies in $^{20}\text{Ne}$ and offer a tantalising glimpse into
where to look for Hoyle counterparts in this nucleus.

5. Experimental challenges
These recent studies in $^{16}\text{O}$ and $^{20}\text{Ne}$ have focused on the emission of heavy clusters – i.e.
$^8\text{Be}$ and $^{12}\text{C}^*$ – as opposed to $\alpha$-particle emission. However, the question remains, as for the
$^{12}\text{C}$ system in section 3, about how to distinguish between a classically clustered state and a
condensed gas-like state of $\alpha$ particles?
Figure 4. Excitation functions for $^{16}$O [22]. (Top panel): for states decaying via the $^8$Be–$^8$Be channel; (bottom panel): for states breaking up into $^{12}$C* + $^{4}$He.

One such method, proposed in [24], is to search for a signature of these exotic gas-like states in reactions involving heavier nuclei [25] and compare the strengths of all the decay channel partitions. However, to do this requires a special experimental set-up the capabilities of which needs be: observation of the total kinetic energy distribution of the emitted particles (down to very low energies); comparison to known compact states and the ability to establish a shift due to Coulomb barrier differences with such a state; observation of high multiplicity $\alpha$ emission; extraction of decay probabilities for all particle partitions, i.e. a search for enhanced coherent multi-$\alpha$-particle emission e.g. $^8$Be, $^{12}$C* . . . . Finally, to confirm the population of a condensate rather than a clustered state would require the analysis to show that the decay probabilities are equal for all partitions.

The continued advances in particle detection mean that such experiments, though challenging, can now be designed.

6. Summary

Clustered states and their intriguing properties, still, after many decades of study, afford a unique insight into the manifestation of the nuclear force in many-body systems. The data on $^{12}$C has reached a level of precision where many of the outstanding questions are on the cusp of being resolved, both from an experimental and theoretical outlook. To resolve the issue of Hoyle analogue states the focus of study will need to shift to heavier $\alpha$ conjugate nuclei and the
initial steps have been taken in $^{20}$Ne, while activity on $^{16}$O is considerably advanced. One of the most stimulating areas for the future looks set to be the possibilities of more exotic states of matter such as $\alpha$-particle condensates or signatures for such states around heavy cores. The next SOTANCP workshop may well be the forum where matters related to condensation take centre stage.

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**Figure 5.** Excitation functions for $^{20}$Ne for events breaking up into (a) $^8$Be–$^8$Be–$\alpha$ and (b) $^{12}$C∗–$^8$Be. Adapted from figure 3 of [23]. The red dashed lines (corresponding to the right axes) demonstrate the change in detector efficiency for the two processes, obtained from Monte-Carlo simulations.