Planck Formula for the Gluon Parton Distribution in the Proton

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We describe the gluon parton distribution function (PDF) in the proton, deduced by data from the ATLAS and HERA experiments, in the framework of the parton statistical model. The best fit parameters involved in the Planck formula that describes the gluon distribution are consistent with the results obtained from analysis of deep inelastic scattering processes. Remarkably, the agreement between the statistical model and the experimental gluon distributions is found with the same value of the “temperature” parameter $\bar{x}$ found by fitting the valence parton distributions from deep inelastic scattering. This result corroborates the validity of the statistical approach in the gluon sector.

I. INTRODUCTION

The scale invariance in deep inelastic scattering of leptons (electrons, muons and neutrinos) on nucleons [1] had a crucial role in the proposal of quantum chromodynamics (QCD) [2] as the field theory of strong interactions. To describe the phenomenon, Feynman [3] proposed that the hadrons behave at large $Q^2$ as an incoherent set of point-like objects, called partons, characterized by a given probability of carrying a fraction $x \in [0, 1]$ of the hadron momentum in the rest frame of the final hadrons. While the charged partons have been identified with the quarks [4], a relevant fraction of the hadron momentum is carried by neutral partons, identified with the gluons, that play the role of gauge bosons of QCD. The $Q^2$ dependence of the parton distributions is implied by QCD, as described by the DGLAP equations [5–7], which have been experimentally confirmed [8]. Therefore, if one fixes an initial $Q_0^2$, sufficiently high that perturbative QCD is reliable for larger values, the parton distributions can be derived as a function of $Q^2$ by the DGLAP equations.

Polynomial functions are often considered for the boundary conditions, but theoretical ideas inspired by experimental facts suggest a different parametrization. The idea that Pauli principle implies $\bar{u}(x) > u(x)$ larger than $\bar{u}(x) > u(x)$ [9] has been confirmed by the defect [10] in the Gottfried sum rule [12] and by the experiments on Drell-Yan production of pairs in proton-proton and proton-deuteron scattering [13,14,15]. This fact has inspired to write parton distributions for the boundary low-$Q^2$ conditions [15] of DGLAP equations according to quantum statistical mechanics [16] in the variable $x$, which appears in the parton model sum rules. The “potentials” that appear in the Fermi-Dirac distributions of the valence partons depend on flavor ($q = u, d$) and helicity ($h = +, -$). This feature determines the intriguing possibility to describe both the unpolarized fermion distributions $q(x) = q^+(x) + q^-(x)$ and their polarized counterparts $\Delta q(x) = q^+(x) - q^-(x)$ [16,18].

An important constraint to the model is the hypothesis that, at the separation between the non-perturbative and the perturbative QCD regimes, there is equilibrium [19–21] for the elementary processes involved in the DGLAP equations. As a consequence, gluons must be described by a Planck formula, namely a Bose-Einstein distribution with vanishing chemical potential, while the isospin and spin asymmetries of the sea are related to the non-diffractive contribution to the valence parton distributions. This feature allows to predict, in agreement with the experiment (see Refs. [14,22]),

$$\Delta \bar{d}(x) < 0 < \Delta \bar{u}(x) < \bar{d}(x) - \bar{u}(x) < \Delta \bar{u}(x) - \Delta \bar{d}(x).$$

Deep inelastic processes are unable to probe the gluon distribution with precision, since gluons, that are singlets with respect to the electroweak group, appear in the logarithmic correction of the parton distributions of the fermions. Instead, they play an important role, as an octet of $SU(3)_c$, in the strong $p-p$ interactions measured at ATLAS. Purpose of this article is to describe the parton distribution deduced by the measurements at ATLAS [23] and HERA [24] in the light of the statistical model. More specifically, we will determine the parameters of the statistical gluon distribution by fitting experimental data, and compare the result with the outcomes of previous studies, which were obtained from constraints on QCD sum rules.

The article is organized as follows: in Section [11] we introduce the statistical model and briefly discuss previous findings concerning both the fermion and gluon sectors; in Section [11] we perform the fit of the gluon distribution function; in Section [14] we summarize the results and discuss their relevance.
II. PARTON STATISTICAL MODEL

The parameters of the statistical model found in the seminal work \[16\] at \(Q^2 = 4\,\text{GeV}^2/\text{c}^4\) were also successfully used to describe the polarized nucleon structure functions \[17\] \[18\]. Following studies determined the same parameters at \(Q^2 = 1\,\text{GeV}^2/\text{c}^4\) \[25\] and by comparison \[20\] with the parton distributions proposed in Ref. \[24\]. We report in Table \(I\) the values, found in the aforementioned studies, of the relevant parameters characterizing the quark and antiquark distributions \[16\]

\[
x_q^b(x) = \frac{AX_q^b x^b}{\exp(x - X_q^b/x) + 1} + \frac{\tilde{A}x^b}{\exp(x/x) + 1},
\]

(2)

\[
x_q^d(x) = \frac{AX_q^d x^d}{\exp(x + X_q^d/x) + 1} + \frac{\tilde{A}x^d}{\exp(x/x) + 1},
\]

(3)

(with \(q = u, d\) and \(h = +,-\)) and the gluon distribution \[10\]

\[
x_G(x) = \frac{AG x^G}{\exp(x/x) - 1}.
\]

(4)

The factors in the first terms of the fermion distributions may be explained by the extension to the transverse degrees of freedom \[28\] exactly for \(X_q^b\) and approximately for \((X_q^{-h})^{-1}\). The comparison among the parameters found in Refs. \[16\] \[25\] \[26\] shows stability for \(\bar{x}\), which we denote as the “temperature” of the model, and for the “potentials” \(X_q^b\) of the valence partons, depending on their flavor and helicity. Instead, the parameters \(A_G\) and \(b_G\), which appear in the Planck distribution \[4\] of the gluons, are characterized by a more striking variability. The same occurs for the parameters \(\tilde{A}\) and \(\tilde{b}\) that determine the diffractive term of the fermion distributions. The discrepancy of Ref. \[25\] with respect to Ref. \[16\] is due to the choice of a smaller \(Q^2\), lying in a region where the gluon and diffractive distributions are expected to become narrower as a consequence of scale dependence. In the case of Ref. \[26\], differences are due to the fact that the parameters of the statistical model were fixed to match the distributions proposed in Ref. \[24\]. In fact, the factor \((1 - x)^C\) of the standard parametrization and the Boltzmann factor \(\exp(-x/x)\) have a different behaviour, as stressed in Ref. \[15\], where the statistical description has been shown to be in a better agreement with the gluon distribution found in Ref. \[27\].

It is thus crucial to determine the gluon distribution measured at ATLAS \[23\]. In fact, while in deep inelastic scattering with incident leptons, gluons, that are singlets with respect to the electroweak gauge group, are fixed by their role in the DGLAP equations \[5\] \[7\], in proton-proton scattering they interact strongly as color octets. Therefore, one can hope to gain more information on the gluon distribution from LHC experiments. For this reason, we compare the prediction of the statistical approach with the experimental values, where they do not depend on the extrapolation following from the parametrization.

| Parameter | 16 | 25 | 26 |
|-----------|----|----|----|
| \(\bar{x}\) | 0.099 0.090 0.099 | | |
| \(X_u^+\) | 0.461 0.475 0.446 | | |
| \(X_u^-\) | 0.298 0.307 0.297 | | |
| \(X_d^+\) | 0.228 0.245 0.222 | | |
| \(X_d^-\) | 0.302 0.309 0.320 | | |
| \(A_G\) | 14.3 32.8 27.18 | | |
| \(b_G\) | 0.747 1.02 0.75 | | |
| \(\tilde{A}\) | 1.91 0.147 0.07 | | |
| \(\tilde{b}\) | −0.253 0.043 −0.25 | | |

TABLE I. Values of the statistical model parameters found in previous works. The temperature \(\bar{x}\) is involved in both the fermion and gluon distributions. The “potentials” \(X_q^b\), \(X_q^-\), \(X_d^+\) and \(X_d^-\) determine the non-diffractive parts of the fermion distributions, while \(\tilde{A}\) and \(\tilde{b}\) fix the diffractive ones. Finally, \(A_G\) and \(b_G\) appear in the gluon distribution.

The difference for the gluon and the diffractive terms may be the consequence of a different value of \(Q^2\) chosen in Ref. \[25\], since the distributions are modified by the evolution, and by the comparison with the very different parametrization of HERA for the gluons: in fact the comparison with NNPDF \[27\] shows a better agreement for Eq. \[4\].

III. FIT OF THE GLUON DISTRIBUTION FUNCTION

Since we assume \(Q^2_0 = 4\,\text{GeV}^2/\text{c}^4\), we choose to limit the \(x\) range to \([0.05, 0.66]\), where the lower bound is fixed to avoid the QCD corrections proportional to \(\alpha_s(Q_0^2)\ln x\), while the upper bound ensures that the invariant mass:

\[
(M')^2 = M_p^2 + Q_0^2 \frac{1 - x}{x},
\]

(5)

with \(M_p\) the proton mass, is not too small. We choose to fit the central values of the gluon momentum distribution \(xg(x)\) obtained in the ATLAS experiment with free parameters \(A_G\) and \(b_G\), while we fix \(\bar{x} = 0.099\), since it was determined in Ref. \[16\] by the rapidity of the decrease of the fermionic parton distributions around their “potentials” and above them. The best fit of \(N = 74\) points in \([0.05, 0.66]\) provides

\[
A_G = 15.853 \pm 0.225, \quad b_G = 0.792 \pm 0.006.
\]

(6)

The agreement of this result with the experimental points is reported in Fig. \[1\]. The error bars therein are determined by three kinds of uncertainties, that are summed in quadrature: the first kind is obtained from variations of the experimental parameters; the second is related to uncertainties in the physical constants, such as the quark mass and the coupling constants, of the physical model; the third one is determined by variations in the
The good agreement of the Planck formula for the gluon parton distribution in the proton with experiment with the same value for the “temperature”, $\bar{x} = 0.099$, and the other parameters, $A_G$ and $b_G$, near to the ones found in the study of deep inelastic about twenty years ago, is a good point in favor of the parametrization inspired by quantum statistical mechanics, which allows a reliable extrapolation to the $x$ regions, where one has not a sufficient information from experiment. The proposal of boundary conditions for the DGLAP equations Ref. 17 fixed by statistical quantum mechanics Ref. 15 is inspired by the role of Pauli principle advocated in Ref. 9 and in Ref. 14 receives an important confirmation from the study of ATLAS data Ref. 23.

IV. CONCLUSIONS

We report in Table II the values of the integrated momentum distribution for low ($x < 0.2$) and high ($x > 0.2$) momentum values,

$$M_{g,\text{low}} = \int_{0}^{0.2} xg(x)dx, \quad M_{g,\text{low}} = \int_{0.2}^{1} xg(x)dx,$$

along with the total momentum

$$M_g = \int_{0}^{1} xg(x)dx = M_{g,\text{low}} + M_{g,\text{high}}$$

carried by the gluons, using the best fit values $0_{-0.084}^{+0.05}$ and $0_{-0.099}^{+0.13}$ for the low and high momentum distributions, respectively. The comparison with the values obtained in Refs. 15, 16, and 24 confirms that the prediction of the gluon distribution fit is in a very good agreement with the one obtained in Ref. 16 and the shape significantly differs from the one proposed by Ref. 24. More specifically there is good agreement at small $x$, but the difference becomes striking with increasing $x$. This effect is a consequence of the different behavior of the functions $\exp(-x/\bar{x})$ and $(1 - x)^C$, that predict the large $x$ gluon distributions in the statistical and in the standard approach. The good agreement with data of the Planck formula is a good point in favor of the statistical approach, considering furthermore that it is obtained considering the value of $\bar{x}$ found from the form of the Fermi-Dirac function for the non-diffractive term of the valence partons.

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