Optimal sensor distance for damage detection considering wavelet sensitivity and uncertainties

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Abstract. Wavelet Transform (WT) is an efficient signal processing tool used extensively to detect damage in various types of structures. It is capable of describing signals in both time and frequency domains. However, improper sensor distance may result to false detection of damage, thus affecting the reliability of the method. A small sensor distance leads to large number of sensors, thus increases the financial cost and causes higher computational time, while large sensor distance may provide inadequate data for accurate damage detection. In this paper, the optimum sensor placement for damage detection is investigated by considering the effect of noise and sensitivity of WT. This involves using Continuous Wavelet Transform (CWT) to decompose the mode shape difference of a plate numerical model. Different measurement distances and different levels of uncertainties are added to the mode shape data to evaluate the optimum sensor distance that provide the best damage detection result. A numerical plate model with all four sides fixed is used as an example. The results indicate that increase of noise reduced the detectability of damage. It is also observed that excessive sensor distance increment significantly effects damage detectability.

1. Introduction

Vibration-based damage detection techniques that are applied to assess the structural condition are referred to as a global method of damage identification. These techniques are cheaper, faster and can be easily applied to large structures. The basic principle of these techniques in detecting damage in structures is based on the alteration of the physical properties (mass, stiffness and damping), dynamic properties (natural frequencies, mode shapes, damping ratios and frequency response function) [1] and dynamic responses (acceleration, velocity and displacement) [2]. Several methods have been developed to analyse these data for accurate detection, location and severity of damage; which includes modal data [3], frequency response function (FRF) [4] and time history [5].

In addition to the above mentioned techniques, Wavelet Transform (WT), an efficient signal processing tool has been extensively applied to detect damage in various types of structures. It is capable of describing signals in both time and frequency domains. Many studies have proven that WT is reliable in detecting damage. For example, Rucka and Wilde [6] analysed the estimated mode
shapes of a cantilever beam and steel plate with 1-D and 2-D CWT respectively for damage detection in these structures. Bagheri et al. [7] proposed an effective technique for detecting linear flaws in plate structures using 2-D Discrete Wavelet Transform (DWT) to decompose the mode shape signals. Ovanesova and Suarez [8] applied wavelet transform to detect cracks in plane frame structures using response signals from static and dynamic loading of the frames. Recently, Katunin and Przystalka [9] assessed damage in a composite structure using fractional wavelet transform. Also, Vafaei et al. [10] localised damage in a beam by decomposition of the mode shape using DWT. These authors have shown that with measurement data, damage can be identified in structures with WT.

In practice, there are two types of noise which may lead to false damage detection results. Modelling errors from inaccurate Finite Element (FE) parameters may result into the initial FE model not accurately representing the actual condition of the real structure. This type of noise is always the major issue in many methods that requires initial model to construct the relationship between the vibration parameters and damage detectability, such as artificial neural network (ANN) [11], Genetic Algorithm (GA) [12] and model updating [13]. On the other hand, the existence of measurement noise in the measured data due to environmental effects, electronic devices, measurement errors and etc., may also lead to false damage detection. Several studies have investigated the effect of noise in damage detection based on vibration parameters. For example, Chandrashekar and Ganguli [14] investigated the effect of noise in assessing structural damage with geometric and measurement uncertainties by mode shape curvatures. The authors employed a Fuzzy Logic System (FLS) to analyse a cantilever beam with single and multiple damage. The FLS identified multiple damages with 94% average accuracy when the measurement noise was 10%. Garcia-Gonzalez et al. [15] checked the capacity of ANN for noise filtering using natural frequencies as inputs in damage localisation in a beam structure. Bandara et al. [16] proposed a damage identification method to identify damage using noise polluted Frequency Response Function (FRF) which involved the calculation of FRF and damage index, development of an ANN and introduction of damage indices into the ANN method for damage detection. Aydin and Kisi [17] studied the effect of noise with ANN using frequencies, mode shape rotation and other beam properties as input to detect cracks in a beam. Most of the studies that investigated the effect of noise in damage detection have been using modal data as the main parameter to detect damage. However, with wide application of WT for damage detection, studies on noise effects are quite limited. Thus, it is very important to study the effect of noise in measured data of structures to wavelet ability to detect damage.

The distance between the sensors may also influence the degree of accuracy of damage identification. When the number of sensors is small, the distance between sensor are larger, thus lead to less accurate estimation of the structural damage, while a large number increases the financial cost of the monitoring system and causes higher computational time. Researchers have come up with several proposed methods for sensor placement using different damage detection methods. Li et al. [18] determined the optimal sensor location for structural vibration measurements using the uniform design method. This method appears to be simple, however, its application is limited to simple structures and not so accurate for complex structure. Xie and Xue [19] applied a hybrid algorithm for optimal number of sensors and placement by using improved reduced system and singular value decomposition. Yi et al. [20] proposed an optimal sensor location by reducing the maximum value of the off-diagonal elements contained in the modal assurance criterion matrix and presented a hybrid method for sensor placement based on multiple optimization methods respectively. In both studies, the methods presented are too complex and requires much computational efforts. Recently, Alem and Benazzouz [21] presented a structural adjacent matrix method of optimal sensor placement for fault detection by finding the vertex-disjoint paths. A detailed review on sensor placement can be found in Yi and Li [22], Li et al. [23] and Li et al. [24]. However, study on optimum sensor distance focusing on WT method is limited.

Therefore, this study investigates the optimum sensor placement for damage detection by considering the sensitivity of CWT and uncertainties (noise) in a plate-like structure. For the purpose of damage detection, the mode shape difference before and after damage served as the input rather
than conventional mode shape of the damaged structure [25]. The vibration data obtained from the numerical analysis of the plate structure is used in computing the mode shape difference. To detect and locate the damage, the mode shape differences are then decomposed using CWT. The effect of noise and sensors’ distances to CWT in detection of different degrees of damage severity in a plate structure is analysed in this study. To study the effect of noise to damage detectability, various levels of noise are added to the values of the mode shape differences. The effect of the sensor distance is investigated by varying the distance between the measurement nodes. The effects of noise and sensor distance are evaluated based on damage detectability by applying a damage index and pictorial representation of the decomposed signal.

2. Methodology

2.1. Continuous wavelet transform

A wavelet is an oscillation with amplitude which originates at zero, increases progressively, and then decreases back to zero. The function \( \psi(x) \), which is the mother wavelet, is localised in space and frequency domain, creating a wavelet family \( \psi_{u,s}(x) \).

\[
\psi_{u,s}(x) = \frac{1}{\sqrt{s}} \psi \left( \frac{x-u}{s} \right)
\]  

where \( s \) and \( u \) are scale and position respectively.

The continuous wavelet transform (CWT) of a 1-Dimensional signal \( f(x) \) is the integral of the product of the signal function with the complex conjugate \( \bar{\psi}(x) \) of the wavelet function. Taking the deflection of the structure as a 1-dimensional signal \( f(x) \), the CWT by Mallat [26] is

\[
Wf(u,s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} f(x) \psi \left( \frac{x-u}{s} \right) dx
\]  

\( Wf(u,s) \) is the wavelet coefficient of the wavelet \( \psi_{u,s}(x) \).

The \( n \) vanishing moment of a wavelet is vital in detecting singularity of signals, which is based on the equation

\[
\int_{-\infty}^{+\infty} x^k \psi(x) dx = 0, \ k = 0,1,2,\ldots,n-1.
\]

Rewriting equation (2) as,

\[
Wf(u,s) = \frac{1}{\sqrt{s}} f * \bar{\psi} \left( \frac{x-u}{s} \right) = f * \tilde{\psi}_s(u)
\]

\[\tilde{\psi}_s(u) = \frac{1}{\sqrt{s}} \psi \left( \frac{x-u}{s} \right)\]

The wavelet with \( n \) vanishing moment can be re-written as the \( n \)th order derivation of a smooth function \( \theta(x) \) [6], which can be expressed as a multiscale differential operator:

\[
Wf(u,s) = \frac{\partial^n}{\partial x^n} f * \bar{\theta} \left( \frac{x-u}{s} \right) = s^n \frac{\partial^n}{\partial u^n} f \bar{\theta}_s(u)
\]

\( f * \bar{\theta}_s \) denotes convolution of functions which is the average of \( f(x) \) over a domain proportional to the scale \( s \).

2.2. Noise

To analyse the effect of noise, noise levels of 1%, 2%, 5% and 10% (gaussian white noise) were added to the obtained mode shape difference data. The mode shape difference data were imported from the Finite Element Analysis (FEA) program SAP2000 to Matlab software. For a particular noise level \( n \%)\), the corresponding noise value \( e \) is obtained by:

\[
e = 20 \log_{10} \left( \frac{1}{n} \right)
\]

where \( n \) is noise level. This noise level \( n \) is added to the data \( m \) through the noise value \( e \). The noise value \( e \) for 10% noise level is 20. The values for 1%, 2% and 5% are 40, 33.9 and 26 respectively.
2.3. Sensor locations
The sensor locations are taken as the nodes in the models. The distance between these nodes are 80 mm, 40 mm, 20 mm, 10 mm and 5 mm, and this translates to distance ratio $r$ (total distance $l$ / measurement distance $l_e$) of 7, 14, 28, 56 and 112, and are represented by $i_{80}$, $i_{40}$, $i_{20}$, $i_{10}$ and $i_{5}$ respectively. This is tabulated in Table 1 and Figure 2 shows the distance between the nodes.

3. Numerical model
The mode shape of the plate is computed using the commercial Finite Element Analysis (FEA) program SAP2000 and the wavelet analysis was done with Wavelet Toolbox on Matlab software. The steel plate is square with dimensions: 560mm, 560mm and 2mm for length $l$, width $b$ and thickness $h$. Material properties are: Poisson’s ratio $\nu = 0.3$, Young Modulus $E = 200$Gpa and steel density $\rho = 7850$kg/m$^3$. Boundary condition at all four sides of the plate is fixed ends. Damage is assumed to be caused by rust effect and this is simulated by reducing the thickness of the plate at the specified damage area. Two damage locations are considered: middle and side damages of the plate. These points represent near and far from support damage conditions. Figure 1 shows a diagram of the depiction the positions of the damages. The damages are rectangular shaped with dimension of 80mm x 80mm which is approximately 2.041% of the plate’s area. The damages cases are simulated by reducing the thickness of the plate at the damage location, the reduced thickness ranged from 1.90mm to 0.25mm (see Figure 2). These cases are 190mm, 180mm, 175mm, 150mm, 125mm, 100mm, 75mm, 50mm and 25mm which are equivalent to 5%, 10%, 12.5%, 25%, 37.5%, 50%, 62.5%, 75% and 87% thickness reduction respectively. These damage cases are tabulated in Table 2. A total of 360 models were used for the accuracy and noise sensitivity analyses.

| Measurement distance (mm) $i$ | 80  | 40  | 20  | 10  | 5   |
|-------------------------------|-----|-----|-----|-----|-----|
| Distance ratio (MDR) $r$      | 7   | 14  | 28  | 56  | 112 |

Table 2. Thickness reduction of plate

| Case | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|------|----|----|----|----|----|----|----|----|----|
| Thickness reduction (%) | 5  | 10 | 12.5 | 25 | 37.5 | 50 | 62.5 | 75 | 87.5 |
| Plate thickness (mm)     | 1.90 | 1.80 | 1.75 | 1.50 | 1.25 | 1.00 | 0.75 | 0.50 | 0.25 |

4. Results
The results presented in this section are noise free. Damages are identified from the pictorial representation of the decomposed signal. This is based on the sensitivity of WT to sudden and abrupt change in the decomposed mode shape differences. Damages were not detectable at $i_{80}$, that is, measurements at 80mm. This is caused by the inability to capture adequate data for proper estimation of the state of the plate due to the large measurement distance. Figure 3 is a display of the decomposed signal of the plate with case 9 damage (middle and side) at $i_{80}$. Typical examples of detected damages are shown in Figure 4. The bar charts in figures 5 to 8 show the accuracy of the side and middle damages detected in the plate. The dark and lighter bars represent the accuracies of the $x$ and $y$ coordinates of the detected damage location, respectively.
In Figure 5, the results show damage detectability at $i_{80}$. Figure 5(a) shows the accuracy of middle detected damages. Damage of 5% thickness loss was not detected in this case. The damage was not captured in the decomposed signal to effect the detection of the damage because the severity was small (5%). Damages which were above 10% thickness loss were accurately detected, almost 100% accurate. Figure 5(b) shows the side damage accuracy. In this scenario, damages of 5% and above thickness loss were accurately detected. The detection of damage (side) of 5% thickness loss may be a result of its close proximity to the support. Considering the bar charts (Figures 5 to 8), damage is either detected with a high degree of accuracy or it is undetected. The detected middle damages are of higher accuracy. The detectability of middle damage is reduced as the measurement distance is increased i.e. for 10% thickness loss, middle damage is undetectable at $i_{40}$. This can be attributed to low damage severity and inability to capture the damage signal due to increase of the measurement distance. This is also noticed in the side damage as well, as the side damage accuracy decreases as $i$ increases. From the above results, by using this method, both side and middle damages can be accurately detected in a noise free signal.
5. Sensitivity to noise effect

The effects of noise in experimental and field experiments cannot be neglected. This is modelling error which is ascribed as the imperfection of numerical models to simulate real structures. In order to put these effects into consideration, various levels of noise were added to the numerical results i.e. 1%, 2%, 5% and 10% noise levels as described in the methodology section above. The Figures 9 – 12 represent the noise effects (sensitivity) for $i_{40}$, $i_{20}$, $i_{10}$ and $i_{5}$. The figures show the damage detection sensitivity to noise, $i$, and severity of damage.

![Figure 8. Measurement at 40mm ($i_{40}$).](image)

![Figure 9. Measurement at 5mm ($i_{5}$).](image)

Figure 9 summarises the result (middle and side damages) for $i_{5}$ with noise level varying from 1% to 10%, and damage severity from 5% to 87.5%. When the noise level increases (and damage severity decreases), it is observed that the sensitivity to damage decreases. Take Figure 9(a) for example (middle damage), when noise is increased (to 2%, 5% or 10%), damage is not detectable until the severity is up to 25%. Also, when the damage severity is 12.5%, damage is detectable when the noise is reduced to 1%. The reason for this is that the sensitive damage features in the data have been submerged by noise. It is also observed that the accuracy of the detectability is high, close to 100%, and on the other hand, the non-detected damages are completely undetectable. Figure 9(b) shows the sensitivity of side damage for $i_{5}$. The side damage can be detected at 5% thickness loss, which was undetected in the middle damage. This could be as a result of the boundary effects. Figures 10 – 12 summarises the sensitivity $i_{10}$, $i_{20}$ and $i_{40}$, respectively. It is observed that for $i_{5}$ and $i_{20}$, the low intensity side damages are detectable even in the presence of 15% noise. Except for $i_{20}$, light damages imposed at the middle of plate are not detectable even when the noise level was minimal. Results also indicate that even severe damages may not be detected when the noise level is high and measurements are performed with larger distances. In short, it can be concluded that, measurement distances and noise levels have a great impact on the detectability of imposed damages. However, a direct relationship cannot be found between change in the noise level or measurement distance and detectability of damages. This may be related to the pattern at which the noise is being added to the data. Since the noise is added randomly to the mode shape differences data by considering the average, it is possible to have a distortion in the signal produced as a result of the addition of the noise. This distortion may aid or hinder damage detection.

![Figure 10. Measurement at 10mm ($i_{10}$).](image)

6. Parametric study

This section provides a more detailed sensitivity investigation of damage detection using WT. This analysis determines the sensitivity of this method to different values of $i$, damage severity (thickness loss) and noise (noise levels considered are 1%, 2%, 5% and 10%). Figures 13 to 16 summarize the damage detectability level of the plate in this study. The vertical lines correspond to the detectability.
limit for different levels of damage severities. The numbers in the rectangular boxes represent the $i_x$ value of each limit. The area on the left side of each line represents the area where damage is undetectable under those conditions. The area on the right side of each of these vertical lines represents the area where damage is detectable under those conditions. Figure 13(a) shows the middle damage detectability level at 1% noise. This figure shows that at $i_5$ and noise level at 1%, the minimum severity of damage (thickness loss) that can be detected is 10%. This implies that middle damage which is less than 10% thickness reduction (say 5%) cannot be detected with $i_5$ in the presence of 1% noise. On the other hand, when middle damage is 10% thickness loss (or more), an $i_x$ value of $i_5$ is capable of detecting this damage provided the noise level is not beyond 1%. This is because the damage signals captured overshadows the effect of the noise. Figure 13(b) represents the side (near support) damage case, and Figures 14 – 16 show the damage detectability at 2%, 5% and 10% respectively.

It is observed in the figures that in most cases, the detectability level decreases as the noise increases, this can be seen in both damage locations (far and near support). For example, at 1% noise level, the detectable damage for $i_{10}$ is 10% thickness reduction, whereas for 2% and 5% noise levels are 25% thickness reduction. A closer observation reveals that at low noise level (1%), middle damage can be detected with lower severity (10% thickness reduction) compared to damages near support (12.5%). This middle damage detectability decreases (25% thickness reduction) as the noise increases (25, 5%, and 10%) while the detectability of side damage remains at 12.5% even as the noise is increased to 10%. These results show that damage detectability depends on the damage severity, noise level and $i_x$ value.
Figure 15. Damage detectability at 5% noise.

Figure 16. Damage detectability at 10% noise.

7. Conclusion
The study presents an investigation on the Wavelet Transform (WT) sensitivity and attributed uncertainties for damage detection of plate-type structures using mode shape difference. The results have shown the sensitivity of WT to damage and its ability to accurately detect damage in the presence of noise. It is observed that damage is either detected accurately or it’s not detected at all. For all studied cases, damage was not detectable when the measurement distance was 80mm. In most cases, the presence of noise, increment of measurement distance and decrease of damage severity reduced the detectability of damage. Measurements at $i_5$, $i_{10}$, $i_{20}$ and $i_{40}$ are adequate for mode shape difference analysing with WT for damage detection.

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