Nuclear effects in the Drell-Yan process

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Abstract

In the target rest frame and at high energies, Drell-Yan (DY) dilepton production looks like bremsstrahlung of massive photons, rather than parton annihilation. The projectile quark is decomposed into a series of Fock states. Configurations with fixed transverse separations in impact parameter space are interaction eigenstates for proton-proton (pp) scattering. The DY cross section can then be expressed in terms of the same color dipole cross section as DIS. We compare calculations in this dipole approach with E772 data and with next-to-leading order parton model calculations. This approach is especially suitable to describe nuclear effects, since it allows one to apply Glauber multiple scattering theory. We go beyond the Glauber eikonal approximation by taking into account transitions between states, which would be eigenstates for a proton target. We calculate nuclear shadowing at large Feynman-\(x_F\) for DY in proton-nucleus collisions and compare to E772 data. Nuclear effects on the transverse momentum distribution are also investigated.

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1 DY dilepton production in \textit{pp} scattering

Although cross sections are Lorentz invariant, the partonic interpretation of the microscopic process depends on the reference frame. As pointed out in \[1\], in the target rest frame, Drell-Yan (DY) dilepton production should be treated as bremsstrahlung, rather than parton annihilation (see also \[2\]). The space-time picture of the DY process in the target rest frame is illustrated in Fig. 1. A quark (or an antiquark) from the projectile hadron radiates a virtual photon on impact on the target.

![Figure 1](image.png)

Figure 1: A quark (or an antiquark) inside the projectile hadron scatters off the target color field and radiates a massive photon. The subsequent decay of the $\gamma^*$ into the lepton pair is not shown.

A salient feature of the rest frame picture of DY dilepton production is that at high energies and in impact parameter space the DY cross section can be formulated in terms of the same dipole cross section as low-$x_{Bj}$ DIS. The cross section for radiation of a virtual photon from a quark after scattering on a proton, can be written in factorized light-cone form \[1\], \[2\], \[3\],

\[
\frac{d\sigma(qp \to \gamma^*X)}{d\ln \alpha} = \sum_{T,L} \int d^2 \rho |\Psi_{T,L}^{q\gamma}(\alpha, \rho)|^2 \sigma_{qq}(\alpha \rho),
\]  

(1)

similar to the case of DIS. Here, $\sigma_{qq}$ is the cross section \[4\] for scattering a $q\bar{q}$-dipole off a proton which depends on the $q\bar{q}$ separation $\alpha \rho$, where $\rho$ is the photon-quark transverse separation and $\alpha$ is the fraction of the light-cone momentum of the initial quark taken away by the photon. For shortness, we do not explicitly write out the energy dependence of $\sigma_{qq}$. We use the standard notation for the kinematical variables, $x_1 - x_2 = x_F$, $\tau = M^2/s = x_1 x_2$, where $x_F$ is the Feynman variable, $s$ is the center of mass energy squared of the colliding protons and $M$ is the dilepton mass. In (1) $T$ stands for transverse and $L$ for longitudinal photons.

The physical interpretation of (1) is similar to the DIS case. The projectile quark is expanded in the interaction eigenstates. We keep here only the first eigenstate,

\[
|q\rangle = \sqrt{Z_2}|\text{bare}\rangle + \Psi_{\gamma^* q}^{T,L}(\gamma^*) + \ldots,
\]  

(2)

where $Z_2$ is the wavefunction renormalization constant for fermions. In order to produce a new state the interaction must distinguish between the two Fock states, \textit{i.e.} they have to interact differently. Since only the quarks interact in both Fock components the difference arises from their relative displacement in the transverse plane. If $\rho$ is the transverse separation between the quark and the photon, the $\gamma^* q$ fluctuation has a center of gravity in the
transverse plane which coincides with the impact parameter of the parent quark. The transverse separation between the photon and the center of gravity is \((1 - \alpha)\rho\) and the distance between the quark and the center of gravity is correspondingly \(\alpha\rho\). Therefore, the argument of \(\sigma_{qq}\) is \(\alpha\rho\). More discussion can be found in [3].

The transverse momentum distribution of DY pairs can also be expressed in terms of the dipole cross section [3]. The differential cross section is given by the Fourier integral

\[
\frac{d\sigma}{d\ln \alpha d^2 q_T} = \frac{1}{(2\pi)^2} \int d^2 \rho_1 d^2 \rho_2 \exp[iq_T \cdot (\vec{\rho}_1 - \vec{\rho}_2)]\Psi_{\gamma^* q}^*(\alpha, \vec{\rho}_1)\Psi_{\gamma^* q}^*(\alpha, \vec{\rho}_2)
\]

\[
\times \frac{1}{2} \left\{ \sigma_{qq}(\alpha \rho_1) + \sigma_{qq}(\alpha \rho_2) - \sigma_{qq}(\alpha (\vec{\rho}_1 - \vec{\rho}_2)) \right\}.
\]

(3)

after integrating this expression over the transverse momentum \(q_T\) of the photon, one obviously recovers (1).

The LC wavefunctions can be calculated in perturbation theory and are well known [2, 3]. The dipole cross section on the other hand is largely unknown. Only at small distances \(\rho\) it can be expressed in terms of the gluon density. However, several successful parameterizations exist in the literature, describing the entire function \(\sigma_{qq}(x, \rho)\), without explicitly taking into account the QCD evolution of the gluon density. We use the parameterization by Golec-Biernat and Wüsthoff [6] for our calculations, Fig. 2. This parameterization vanishes \(\propto \rho^2\) at small distances, as implied by color transparency [4] and levels off exponentially at large separations.

In Fig. 2 we compare to E772 data [7] on low \(x_2\) DY dilepton production [8]. Most of the data are quite well described without any \(K\)-factor, which does not appear in this approach since higher order corrections are supposed to be parameterized in \(\sigma_{qq}(\rho)\). Moreover, the calculation in the dipole approach agrees with the next-to-leading order (NLO) parton model calculation at low \(x_2\). Note that the dipole approach is valid only at low \(x_2\) [5]. At large \(x_2\), this approach is not applicable and differs strongly from the parton model calculation. The disagreement between the data and both of the calculations in some points is probably due to systematic errors in the measured cross section. Preliminary E866 data [9] agree well with the NLO parton model calculation.
Figure 2: The points represent the measured DY cross section in pD scattering from E772 [6]. The solid curve is calculated in the dipole approach, while the NLO parton model calculation (using CTEQ5M parton distributions [10]) is shown as dashed curve. The dipole approach is valid only at small $x_2$. 
2 Proton-nucleus \((pA)\) scattering

The main advantage of the dipole approach is its easy generalization to nuclear targets. Furthermore, it also includes some higher twist effects that are important in multiple scattering, and it provides insight into the physical mechanisms underlying nuclear effects, which are not easily accessible in the parton model [11].

Shadowing in DY is an interference phenomenon due to multiple scattering of the projectile quark inside the nucleus. In the target rest frame, where DY dilepton production is bremsstrahlung of massive photons, shadowing is the Landau-Pomeranchuk-Migdal (LPM) effect. These interferences occur (Fig. 1), because photons radiated at different longitudinal coordinates \(z_1\) and \(z_2\) are not independent of each other. Thus, the amplitudes have to be added coherently. Destructive interferences can occur only if the longitudinal distance \(z_2 - z_1\) is smaller than the so called coherence length \(l_c\), which is the time needed to distinguish between a quark and a quark with a \(\gamma^*\) nearby. It is given by the uncertainty relation,

\[
l_c = \frac{1}{\Delta P^-} = \frac{1}{m_N x_2 q_T^2 + (1 - \alpha) M^2 + \alpha^2 m_q^2}.
\]

Here, \(\Delta P^-\) is the light-cone energy denominator for the transition \(q \rightarrow q\gamma^*\) and \(q_T\) is the relative transverse momentum of the \(\gamma^*q\) Fock state. For \(z_1 - z_2 > l_c\), the radiations are independent of each other.

An immediate consequence of this is that \(l_c\) has to be larger than the mean distance between two scattering centers in the nucleus (\(\sim 2\) fm in the nuclear rest frame). Otherwise, the projectile quark could not scatter twice within the coherence length and no shadowing would be observed.

We develop a Green function technique [3], which allows one to resum all multiple scattering terms, similar to Glauber theory, and in addition treats the coherence length exactly. The formalism is equivalent to the one proposed in [12] for the LPM effect in QED. Our general expression for the nuclear DY cross section reads

\[
\frac{d\sigma(qA \rightarrow \gamma^* X)}{d\ln \alpha} = A \frac{d\sigma(qp \rightarrow \gamma^* X)}{d\ln \alpha} - \frac{1}{2} \text{Re} \int d^2 b \int_{-\infty}^\infty dz_1 \int_{z_1}^\infty dz_2 \int d^2 \rho_1 \int d^2 \rho_2 \\
\times \left[ \Psi_{\gamma^*q}(\alpha, \rho_2) \right]^* \rho_A(b, z_2) \sigma_{q\bar{q}}(\alpha \rho_2) G(\vec{\rho}_2, z_2 \mid \vec{\rho}_1, z_1) \\
\times \rho_A(b, z_1) \sigma_{q\bar{q}}(\alpha \rho_1) \Psi_{\gamma^*q}(\alpha, \rho_1).
\]

The first term is just \(A\) times the single scattering cross section, where \(A\) is the nuclear mass number. The second term is the shadowing correction. The impact parameter is \(b\) and the nuclear density is \(\rho_A\). The Green function \(G\) describes, how the bremsstrahlung-amplitude at \(z_1\) interferes with the amplitude at \(z_2\).

To make the meaning of Eq. (4) more clear, let us first consider a limiting case for \(G\). In the simplest case, the coherence length, Eq. (4), is infinitely long and only the double scattering term is taken into account. Then \(G(\vec{\rho}_2, z_2 \mid \vec{\rho}_1, z_1) = \delta^{(2)}(\vec{\rho}_1 - \vec{\rho}_2)\) and one of the \(\rho\) integrations can be performed. The \(\delta\)-function means that at very high energy (infinite
coherence length) the transverse size of the $\gamma^*q$ Fock-state does not vary during propagation through the nucleus, it is frozen due to Lorentz time dilatation. Furthermore, partonic configurations with fixed transverse separations in impact parameter space were identified a long time ago [4] in QCD as interaction eigenstates. This is the reason, why we work in coordinate space. Namely, in coordinate space, all multiple scattering terms can be resummed and in the limit of infinite $l_c$ one obtains

$$G^{\text{frozen}}(\vec{p}_2, z_2 \mid \vec{p}_1, z_1) = \delta^{(2)}(\vec{p}_1 - \vec{p}_2) \exp \left( -\frac{\sigma_{qq}(\alpha \rho_1)}{2} \int_{z_1}^{z_2} dz \rho_A(b, z) \right).$$  

(6)

The frozen approximation is identical to eikonalization of the dipole cross section in Eq. (1). Thus, the impact parameter representation allows a very simple generalization from a proton to a nuclear target, provided the coherence length is infinitely long.

At Fermilab fixed-target energies ($\sqrt{s} = 38.8$ GeV for E772), this last condition is not fulfilled and one has to take a finite $l_c$ into account. The problem is however, that $l_c$, Eq. (4), depends on the relative transverse momentum $q_T$ of the $\gamma^*q$-fluctuation which is the conjugate variable to the size $\rho$ of this Fock-state and therefore completely undefined in $\rho$-representation. The quantum mechanically correct way to treat the $q_T^2$ in Eq. (4) is to represent it by a two-dimensional Laplacian $\Delta_T$ in $\rho$-space. The Green function which contains the correct, finite coherence length and resums all multiple scattering terms fulfills a two-dimensional Schrödinger equation with an imaginary potential,

$$\left[ i \frac{\partial}{\partial z_2} + \frac{\Delta_T(\rho_2) - \eta^2}{2E_q \alpha (1 - \alpha)} + i \frac{\rho_A(b, z_2)}{2} \sigma_{qq}(\alpha \rho_2) \right] G(\vec{p}_2, z_2 \mid \vec{p}_1, z_1) = i \delta(z_2 - z_1) \delta^{(2)}(\vec{p}_2 - \vec{p}_1),$$  

(7)

where $\eta^2 = (1 - \alpha)M^2 + \alpha^2m_q^2$. For details of the derivation, we refer to [3].

The imaginary potential accounts for all higher order scattering terms. The Laplacian implies that the Green function is no longer proportional to a $\delta$-function. This means the size of the $\gamma^*q$ fluctuation is no longer constant during propagation through the nucleus. One can say that an eigenstate of size $\rho_1$ evolves to an eigenstate of size $\rho_2 \neq \rho_1$, so transitions between eigenstates occur.

Calculations with Eqs. (3) and (4) are compared to E772 data [3] in Fig. 3. Note that the coherence length $l_c$ at E772 energy becomes smaller than the nuclear radius. Shadowing vanishes as $x_2$ approaches 0.1, because the coherence length becomes smaller than the mean internucleon separation. It is therefore important to have a correct description of a finite $l_c$ in this energy range. The eikonal (frozen) approximation, Eq. (6), does not reproduce the vanishing shadowing toward $x_2 \to 0.1$. The curves in Fig. 3 are somewhat different from the ones in [4], because we used a different parameterization of the dipole cross section. Note that for heavy nuclei, energy loss [15] leads to an additional suppression of the DY cross section.

Nuclear effects on the $q_T$-differential cross section calculated at RHIC energy are shown in Fig. 4. See [3] for details of the calculation. The differential cross section is suppressed at small transverse momentum $q_T$ of the dilepton, where large values of $\rho$ dominate. This
suppression vanishes at intermediate $q_T \sim 2$ GeV. The Cronin enhancement that one could expect in this intermediate $q_T$ region [14] is suppressed due to gluon shadowing [16].

A nuclear target provides a larger momentum transfer than a proton target and harder fluctuations are freed, which leads to nuclear broadening. Note, that not the entire suppression at low $q_T$ is due to shadowing. Some of the dileptons missing at low $q_T$ reappear at intermediate transverse momentum. At very large transverse momentum nuclear effects vanish.

### 3 Summary

We express the DY cross section in terms of the cross section for scattering a $q\bar{q}$ dipole off a proton. This is the same dipole cross section that appears in DIS. At low $x_2$ and for proton-proton scattering, calculations in the dipole approach agree with calculations in the NLO parton model. Some E772 data points are not well described by either of the approaches, which is probably due to a systematic error in the measured cross section.

At very high energy, the dipole approach is easily extended to nuclear targets by eikonalization. At lower fixed target energies (E772) the eikonal approximation is no longer valid, because the size of a Fock state varies during propagation through the nucleus. Therefore, transitions between interaction eigenstates (i.e. partonic configurations with fixed transverse separations) occur.

We develop a Green function technique, which takes variations of the transverse size into
Figure 4: Nuclear effects on the DY transverse momentum distribution at RHIC and LHC for dilepton mass \( M = 4.5 \text{ GeV} \) and Feynman \( x_F = 0.5 \).

account and resums all multiple scattering terms as well. For light nuclei, calculations with the Green function technique are in good agreement with DY shadowing data from E772. For heavier nuclei, also energy loss becomes important.

We have also calculated nuclear effects in the transverse momentum distribution of DY pairs at RHIC energy. The DY cross section is suppressed at low transverse momentum. The expected Cronin enhancement at intermediate \( q_T \sim 2 \text{ GeV} \) is reduced because of gluon shadowing. Nuclear effects vanish at very large \( q_T \).

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