Search for $B^0 \to K^0_S K^0_S \gamma$ decays at Belle

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We report the first search for the penguin-dominated process $B^0 \to K_S^0 K_S^0 \gamma$ using the full data sample of $7.72 \times 10^6$ $B\bar{B}$ pairs collected with the Belle detector at the KEKB asymmetric-energy $e^+e^-$ collider. We do not observe any statistically significant signal yield in the $K_S^0$-pair invariant mass range $1$ GeV/$c^2 < M_{K_S^0K_S^0} < 3$ GeV/$c^2$, and set the following upper limits at $90\%$ confidence level: $B(B^0 \to K_S^0 K_S^0\gamma) < 5.8 \times 10^{-7}$, $B(B^0 \to f_2\gamma) \times B(f_2(1270) \to K_S^0 K_S^0) < 3.1 \times 10^{-7}$, and $B(B^0 \to f_2 \gamma) \times B(f_2(1525) \to K_S^0 K_S^0) < 2.1 \times 10^{-7}$. Further, $90\%$ confidence upper limits have also been set in the range of $[0.7-2.9] \times 10^{-7}$ on the $B^0 \to K_S^0 K_S^0 \gamma$ branching fraction in bins of $M_{K_S^0K_S^0}$.

Radiative $b \to s\gamma$ and $b \to d\gamma$ quark transitions are flavor-changing-neutral-current processes and not allowed at the tree level in the Standard Model (SM). Such decays proceed predominantly through the radiative loop diagrams and are sensitive to the contributions from non-SM particles which may enter the loop diagram. For example, the two-Higgs-doublet model (2HDM) introduces an additional Higgs doublet and the charged Higgs may appear in the loop instead of a $W$ boson. Wilson coefficients in the operator product expansion [1] are modified to include the effect of the 2HDM [2] and this new term depends on the mass of the charged Higgs [3]. Thus, any disparity in the branching fraction from the SM expectations can be interpreted as a new physics contribution.

Branching fractions of several exclusive $b \to s\gamma$ modes have been measured: $B \to K^*\gamma$ [4]; $B \to K_{1(1270)}\gamma$ [3]; $B \to \phi K\gamma$ [4]; $B \to K\eta'\gamma$ [4]; $B \to K\eta\gamma$ [3]. On the other hand, $B \to \rho\gamma$ and $B \to \omega\gamma$ are the only observed exclusive $b \to d\gamma$ modes [3], and a further study of an additional exclusive mode is important to constrain the ratio of Cabibbo-Kobayashi-Maskawa matrix elements $|V_{td}/V_{ts}|$ [10] and also to test the theoretical models.

The $B^0 \to K_S^0 K_S^0 \gamma$ decay shown in Fig. 1 is one such radiative electroweak penguin process that proceeds via $b \to d\gamma$ at the quark level. The angular momentum of an intermediate state decaying into two identical neutral pseudoscalar particles, $K_S^0$, is even due to Bose-Einstein statistics. The spin of the $K_S^0 K_S^0$ system must be at least two by the conservation of angular momentum in the $B^0 \to K_S^0 K_S^0 \gamma$ decay, since the photon is a massless vector particle. Therefore, spin-2 is the lowest spin state possible for the $K_S^0 K_S^0$ system. In this Letter, we present results from a search for the $B^0 \to K_S^0 K_S^0 \gamma$ decay.

![FIG. 1. $b \to d\gamma$ penguin diagram for $B^0 \to K_S^0 K_S^0 \gamma$ decay.](image-url)

The $\Upsilon(4S)$ meson is produced at the KEKB asymmetric-energy $e^+e^-$ collider [11] with electrons and positrons having the energies of 8 GeV and 3.5 GeV, respectively, and subsequently decays to $B\bar{B}$ pairs which are nearly at rest in the center-of-mass system (CMS). The $z$ axis is defined opposite to the $e^+$ beam direction. We search for the decay $B^0 \to K_S^0 K_S^0 \gamma$ using the full data sample of $(7.72 \pm 1.1) \times 10^6$ $B\bar{B}$ pairs collected at the $\Upsilon(4S)$ resonance with the Belle detector at the KEKB asymmetric-energy $e^+e^-$ collider. This is the first ever search for a $B^0$ decay to two pseudoscalars $K_S^0$ with a prompt photon in the final state. The inclusion of the charge conjugate modes is implied throughout this paper unless otherwise stated.

The Belle detector is a hermetic magnetic spectrometre.
ter to detect the decay products of $B$ mesons that consists of a silicon vertex detector, a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters, a barrel-like arrangement of time-of-flight scintillation counters, and an electromagnetic calorimeter comprised of CsI(Tl) scintillation crystals (ECL). These detector components, providing high vertex resolution, good tracking, sophisticated particle identification capability, and excellent calorimetry, are located inside a superconducting solenoid coil providing a 1.5 T magnetic field. An iron flux-return is located outside of the magnetic coil which is instrumented to detect $K_L^0$ mesons and identify muons. The detector is described in detail elsewhere [12].

The event selections are optimized using simulated Monte Carlo (MC) samples. The MC samples for the signal and the backgrounds are generated with EvtGen [13] and the detector response is then simulated using GEANT3 [14]. Any environmental changes in the Belle detector and KEKB accelerator machine during the operations are reflected in the detector simulation. To generate the signal MC sample of $B^0$ decaying to a tensor meson (as an intermediate state) and a prompt photon, a two-body decay model is used with appropriate helicity amplitudes. Then, the intermediate state decays to two $K_L^0$. As the $K_L^0K_S^0$ structure in $B^0 \rightarrow K_L^0K_S^0\gamma$ is unknown, the mass of the intermediate state is evenly distributed between 1 GeV/$c^2$ and 3 GeV/$c^2$ in the signal MC sample.

Photons must have no associated tracks in the CDC, be in the ECL barrel region ($33^\circ < \theta_\gamma < 128^\circ$), and have a ratio of energy deposition in 3×3 ECL crystals to that in 5×5 ECL crystals centered on the crystal having the maximum energy above 95%. The CM energy of the prompt photon candidate, $E_\gamma$, must satisfy the requirement $1.6 \text{ GeV} < E_\gamma < 2.8 \text{ GeV}$. Most background photons originate from $\pi^0$ and $\eta$ to $\gamma$ decays. We combine the photon candidate with all other photons with momenta larger than 50 MeV/$c$ in the event and calculate the probabilities of the reconstructed particle to be $\pi^0$-like or $\eta$-like [15]. The background photons are suppressed by removing $\pi^0$-like and $\eta$-like photons by applying selection criteria on a likelihood based selector. About 86% of the photons from the signal $B$ are retained and about 62% from the accompanying $B$ are rejected. For more than one photon satisfying the selection criteria for the prompt photon candidate, the most energetic photon is chosen as the prompt photon candidate. The selection efficiency of the prompt photon is approximately 50% and 99.5% are truly matched photons for the signal MC.

$K_S^0$ candidates are reconstructed from two oppositely charged tracks. A displaced vertex consistent with $K_S^0 \rightarrow \pi^+\pi^-$ decay is required using a neural network (NN) discriminator with 20 inputs [16]; this selection also suppresses $\Lambda \rightarrow p\pi^-$ decays. The invariant mass of the pion pairs is then required to satisfy $|M_{\pi\pi} - m_{K_S^0}| < 4.7$ MeV/$c^2$, corresponding to a ±2.6σ interval in mass resolution, where $m_{K_S^0}$ is the nominal $K_S^0$ mass [17]. $B^0$ candidates are formed by combining two $K_S^0$ candidates and one prompt photon candidate. The energy difference $\Delta E \equiv E_B^{\text{cms}} - E_{\text{beam}}^{\text{beam}}$ and the beam-energy-constrained mass $M_{bc} \equiv \sqrt{(E_B^{\text{cms}})^2 - m_B^{2\text{ams}}^2 c^2 c^2}$, where $E_B^{\text{cms}}$ is the beam energy, and $E_\gamma^{\text{ams}}$ and $m_B^{\text{ams}}$ are the energy and momentum of the reconstructed $B^0$, respectively, are used to identify $B^0$ candidates. The candidates satisfying the requirements $5.20 \text{ GeV}/c^2 < M_{bc} < 5.29 \text{ GeV}/c^2$ and $|\Delta E| < 0.5 \text{ GeV}$ are retained for further analysis. We find that 6% of the events have more than one $B^0$ candidate. In case of multiple candidates, we choose the one with the smallest $\chi^2$ as defined by $\chi^2 = \sum_{i=1}^2 [(m_{K_S^0} - M_{\pi^+\pi^-})/\sigma_{\pi\pi}]^2$, where $\sigma_{\pi\pi}$ is the mass resolution for the reconstructed $K_S^0$.

The dominant background is $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$) continuum events. We use another NN with four input variables calculated in the CMS to suppress this background [18]: the cosine of the polar angle ($\cos \theta_B$) of the $B^0$ candidate flight direction; the cosine of the angle ($\cos \theta_T$) between the thrust axis of the $B^0$ candidate and that of the rest of the event; a flavor-tagging quality parameter of the accompanying $B$ meson [19]; and a likelihood ratio obtained from the modified Fox-Wolfram moments [20]. The NN outputs for the signal and continuum MC events peak at +1 and −1, respectively. The figure-of-merit (FOM) is calculated as [21]:

$$\text{FOM} = \frac{\epsilon_S(t)}{a/2 + \sqrt{N_{\text{bkg}}(t)}},$$

where $t$ is the NN output; $\epsilon_S(t)$ is the signal efficiency as a function of $t$ determined by using the signal MC sample; $N_{\text{bkg}}$ is the remaining background events after NN selection and $a$ is taken to be 3 for a $3\sigma$ significance due to the low signal-to-background ratio, as suggested in Ref. [21]. The maximal FOM is obtained at 0.93 which rejects 99% of the continuum MC events and retains 37% of the signal MC events. Since we expect a few signal events and relatively large backgrounds, we further suppress the continuum background by using the helicity angle, $\theta_H$, which is the angle between the direction opposite to the $B^0$ candidate and that of the $K_S^0$ momentum in the rest frame of the $K_S^0K_S^0$ system. To maximize the FOM, we require $0.24 < |\cos \theta_H| < 0.86$ which removes 60% of the background while retaining 86% of the signal.

We use a Crystal Ball line shape [22] and a first-order polynomial for the signal and mis-reconstructed components, respectively. The signal region is defined as $-0.16 \text{ GeV} < \Delta E < 0.09 \text{ GeV}$ and $5.272 \text{ GeV}/c^2 < M_{bc} < 5.290 \text{ GeV}/c^2$, corresponding to ±3σ windows. About 99% of $B^0$ candidates in the signal region correctly match signal $B^0$ and all of them have the correct
prompt photon as the daughter particle of the signal $B^0$.

From continuum MC, we estimate 2.2 events in the signal region. In addition to the continuum, various $B\bar{B}$ background sources are also studied. Both neutral and charged $B\bar{B}$ MC samples corresponding to an integrated luminosity six times larger than that of the full data sample are used. We expect 0.3 events from generic $B\bar{B}$ decays in the signal region. The decay $B^0 \rightarrow D^0(\rightarrow K^0_S\pi^0)K^0$, with a branching fraction of $5.2 \times 10^{-5}$, is treated separately from the generic $B\bar{B}$ because its $\Delta E$ and $M_{bc}$ distributions are different from those of generic $B\bar{B}$ events. We estimate the background contribution from this decay to be 0.1.

A dedicated MC sample comprising of rare $B$ decays is prepared: various decays with branching fractions smaller than $O(10^{-4})$ are included and their total branching fraction is $O(10^{-3})$. Rare $B$ decays having one or two $K^0_S$ with $\gamma$ in the final state can peak in the $M_{bc}$ distribution. The backgrounds from the charged $B$ meson pairs do not show any peak in the $\Delta E$-$M_{bc}$ signal region. On the other hand, the background from the neutral $B$ meson peaks in the signal region and the largest contribution (34%) to the peak comes from $B^0 \rightarrow X_{d\bar{d}}(\rightarrow K^0_S\bar{K}^0_S)\gamma$. Herein, $X_{d\bar{d}}$ is a meson composed of a $d$-$\bar{d}$ quark pair. We regard this as a signal because the quark level transition and the final state are the same as for the signal. When we treat this decay mode as a signal by using MC information, the peaking background is removed. Neutral and charged rare $B$ backgrounds are estimated to be 1.0 and 0.9 events in the signal region, respectively.

Four more rare decay modes which are not included in the rare $B$ MC samples are considered: $B(B^0 \rightarrow K^0_S\bar{K}^0_S\pi^0) < 9 \times 10^{-7}$; $B(B^0 \rightarrow K^0_S\bar{K}^0_S\eta) < 1.0 \times 10^{-6}$; $B(B^0 \rightarrow K^0_S\pi^+\pi^-\gamma) = 1.99 \times 10^{-5}$; $B(B^0 \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0) < 9.1 \times 10^{-3}$. The first two decay modes occur via a $b \rightarrow s$ quark transition and become background when $\pi^0$ or $\eta$ are replaced by a photon. $B^0 \rightarrow K^0_S\pi^+\pi^-\gamma$ decays occur through a $b \rightarrow s\gamma$ quark transition and can be mis-identified as the signal. $B^0 \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0$ occurs in the tree level with $b \rightarrow u$ and can be mis-identified as the signal when $\pi^0$ is replaced by a photon. We estimate that the background contribution from these four decay modes is negligible.

We estimate the total number of background events in the signal region to be 4.5±0.7 via the counting method. To estimate the background events in the signal region using an extended unbinned maximum-likelihood fitting method, we fit the $M_{bc}$ distribution satisfying $-0.16 \text{ GeV} < \Delta E < 0.09 \text{ GeV}$ with an ARGUS function and a Crystal Ball line shape for the continuum and peaking background, respectively. The fitting parameters of the Crystal Ball line shape are fixed to those for the signal MC. We obtain 5.6±0.8 background events in the signal region. This result is consistent with that of the counting method.

The signal efficiency depends on the reconstructed $K^0_S$-pair mass $(M_{K^0_SK^0})$ as shown in Table I and is obtained by performing an extended unbinned maximum-likelihood fit to the $M_{bc}$ distribution satisfying $-0.16 \text{ GeV} < \Delta E < 0.09 \text{ GeV}$ and $5.2 \text{ GeV}/c^2 < M_{bc} < 5.9 \text{ GeV}/c^2$ in ten $M_{K^0_SK^0}$ bins of equal sizes between 1 GeV/$c^2$ and 3 GeV/$c^2$.

The systematic uncertainties from the number of produced $B\bar{B}$ pairs and the $\Upsilon(4S) \rightarrow B^0\bar{B}^0$ branching fraction are 1.4% and 1.2%, respectively. The systematic uncertainty in the peaking photon detection efficiency is studied using radiative Bhabha events and estimated to be 2.0%. Using a systematic uncertainty of 0.2% for $K^0_S$ reconstruction efficiency and per track uncertainty in efficiency of 0.4%, leads to our estimate of 1.4% for the reconstruction efficiency of two $K^0_S$. The systematic uncertainty due to the background suppression using the NN selection and $\pi^0/\eta$ veto is 0.6%. The signal efficiency depends on $M_{K^0_SK^0}$ and the MC statistical uncertainty in the efficiency varies between 0.5% and 0.7% depending on $M_{K^0_SK^0}$. The total systematic uncertainty is approximately 3.2% depending on $M_{K^0_SK^0}$ and is summarized in Table II.

A total of 9 events are observed in the signal region. As shown in Fig. 2 we obtain $3.8\pm3.0$ signal and $5.6\pm0.8$ background events in the signal region with an extended unbinned maximum-likelihood fit to the $M_{bc}$ distribution with a Crystal Ball line shape including contributions from the peaking background for the signal and an ARGUS function for the background, respectively. The fitting parameters for the signal are fixed to those for the signal MC. The number of the background events in the signal region agrees well with that of the estimated background events in the signal region from MC samples.
TABLE I. Summary of the number of observed events ($N_{\text{obs}}$), number of estimated background events ($N_{\text{bkg}}$), efficiencies ($\epsilon_S$), upper limits on the signal yield ($S_{90}$), and branching fraction upper limits (U.L.) at the 90% C.L. in each $M_{K^0_S K^0_S}$ bin for the $B^0 \to K^0_S K^0_S \gamma$ decay.

| mass bin (GeV/c^2) | $\epsilon_S$ (%) | $N_{\text{bkg}}$ | $\sigma_{\text{sys}}$ (%) | $N_{\text{obs}}$ | $S_{90}$ | U.L. ($10^{-3}$) |
|-------------------|------------------|------------------|--------------------------|-----------------|---------|------------------|
| 1.0–1.2           | 3.3              | 0.8\pm0.3       | 3.2                      | 0               | 1.8     | 0.7              |
| 1.2–1.4           | 3.0              | 0.9\pm0.3       | 3.2                      | 3               | 6.5     | 2.8              |
| 1.4–1.6           | 2.7              | 0.8\pm0.3       | 3.2                      | 1               | 3.6     | 1.7              |
| 1.6–1.8           | 2.5              | 0.3\pm0.1       | 3.2                      | 0               | 2.1     | 1.1              |
| 1.8–2.0           | 2.3              | 0.8\pm0.3       | 3.2                      | 2               | 5.1     | 2.9              |
| 2.0–2.2           | 2.2              | 0.2\pm0.1       | 3.2                      | 1               | 4.2     | 2.5              |
| 2.2–2.4           | 2.2              | 0.4\pm0.2       | 3.2                      | 1               | 3.9     | 2.4              |
| 2.4–2.6           | 2.2              | 0.2\pm0.2       | 3.2                      | 0               | 2.2     | 1.3              |
| 2.6–2.8           | 2.3              | 0.0\pm0.0       | 3.2                      | 1               | 4.2     | 2.3              |
| 2.8–3.0           | 2.4              | 0.1\pm0.3       | 3.2                      | 0               | 2.3     | 1.2              |

The $|\cos \theta_H|$ distributions for events in the $\Delta E - M_{bc}$ signal region are shown in Fig. 3 for data and MC samples. The signal and background MC samples are normalized to have the same yields as obtained by fitting the data in the signal region. The results from data are consistent with MC simulation.

![FIG. 3. The helicity angle distribution of the observed events in the signal region.](image)

The observed number of events in each $M_{K^0_S K^0_S}$ bin is obtained by counting the events in the $\Delta E - M_{bc}$ signal region. Figure 4 shows the observed number of events ($N_{\text{obs}}$) in the full data sample and the estimated background events in each $M_{K^0_S K^0_S}$ bin. No significant excess over the estimated background is found in data and we derive an upper limit for the signal yield ($S_{90}$) at the 90% confidence level (C.L.) using the POLE program by taking into account the uncertainties associated with the signal selection efficiency, background expectation, and systematic uncertainty. The branching fractions are obtained from

$$B(B^0 \to K^0_S K^0_S \gamma) = \frac{S_{90}}{\epsilon_S \times N_{BB}}. \tag{2}$$

where $N_{BB}$ and $\epsilon_S$ are the number of $BB$ pairs and signal efficiency, respectively. We set 90% C.L. upper limits on the partial branching fractions for the decay $B^0 \to K^0_S K^0_S \gamma$ in ten bins of the $K^0_S$-pair for $1.0 \text{ GeV}/c^2 < M_{K^0_S K^0_S} < 3.0 \text{ GeV}/c^2$, which are listed in Table I.

For the full range $1.0 \text{ GeV}/c^2 < M_{K^0_S K^0_S} < 3.0 \text{ GeV}/c^2$, we use the average efficiency of all bins, $(2.5\pm0.4\%)$. The standard deviation of efficiencies among $M_{K^0_S K^0_S}$ bins is assigned as a systematic uncertainty $(16.0\%)$. Adding to other systematic uncertainties listed in Table I in quadrature, the total systematic uncertainty is $16.2\%$. Using the POLE program with 9 observed events and expected background of $4.5\pm0.7$, we set the upper limit on the branching fraction for the $1.0 \text{ GeV}/c^2 < M_{K^0_S K^0_S} < 3.0 \text{ GeV}/c^2$ mass range to be $5.8 \times 10^{-7}$ at the 90% C.L.

We also set upper limits on branching fraction products for intermediate tensor $f_2$ states, $B(B^0 \to f_2 \gamma) \times B(f_2 \to K^0_S K^0_S)$. The signal mass regions are taken to be $1.00 \text{ GeV}/c^2 < M_{K^0_S K^0_S} < 1.44 \text{ GeV}/c^2$ and $1.44 \text{ GeV}/c^2 < M_{K^0_S K^0_S} < 1.63 \text{ GeV}/c^2$ for $f_2(1270)$ and $f_2'(1525)$, respectively. These mass regions contain 80% of signal events. The results are summarized in Table III.

In summary, we report on the search for radiative $B$ decays with the $K^0_S K^0_S \gamma$ final state using a data sample...
of $772 \times 10^6 \, B\bar{B}$ pairs. No significant signal is observed for the full data sample. The signal efficiency depends on $M_{K_S^0 K_S^0}$ and we set upper limits at the 90% C.L. on the partial branching fractions for the decay $B^0 \rightarrow K_S^0 K_S^0 \gamma$ in ten bins of the $K_S^0$-pair for $1.0 \, \text{GeV/c}^2 < M_{K_S^0 K_S^0} < 3.0 \, \text{GeV/c}^2$ to be $[0.7–2.9] \times 10^{-7}$. We also set an upper limit on its branching fraction as $5.8 \times 10^{-7}$ at the 90% C.L. for the $1.0 \, \text{GeV/c}^2 < M_{K_S^0 K_S^0} < 3.0 \, \text{GeV/c}^2$ mass range. The upper limits at the 90% C.L. on the products of the branching fractions $B(B^0 \rightarrow f_2 \gamma) \times B(f_2(1270) \rightarrow K_S^0 K_S^0)$ and $B(B^0 \rightarrow f_2' \gamma) \times B(f_2'(1525) \rightarrow K_S^0 K_S^0)$ are obtained to be $3.1 \times 10^{-7}$ and $2.1 \times 10^{-7}$, respectively. These results for the decay $B^0 \rightarrow K_S^0 K_S^0 \gamma$ are presented for the first time.

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\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
Mode & $\epsilon_S(\%)$ & $N_{\text{bkg}}$ & $\sigma_{\text{sys}}(\%)$ & $N_{\text{obs}}$ & $S_{\text{obs}}$ U.L. (10^{-7}) \\
\hline
$B^0 \rightarrow f_2(1270)(\rightarrow K_S^0 K_S^0)\gamma$ & 2.3 & 1.8±0.4 & 3.1 & 3 & 5.7 & 3.1 \\
$B^0 \rightarrow f_2'(1525)(\rightarrow K_S^0 K_S^0)\gamma$ & 2.2 & 0.8±0.3 & 3.1 & 1 & 3.6 & 2.1 \\
\hline
\end{tabular}
\caption{Summary of the number of observed events ($N_{\text{obs}}$), number of estimated background events ($N_{\text{bkg}}$), efficiencies ($\epsilon_S$), upper limits on the signal yield ($S_{\text{obs}}$), and branching fraction upper limits (U.L.) at the 90% C.L. for the $B^0 \rightarrow f_2\gamma$ and $f_2 \rightarrow K_S^0 K_S^0$ decays.}
\end{table}

FIG. 4. The $M_{K_S^0 K_S^0}$ distribution in the signal region. The dots represent data and the histograms are the estimated number of backgrounds from the MC background samples.
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