CPT, STRINGS, AND THE $K\bar{K}$ SYSTEM

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ABSTRACT

This talk contains a summary of our work on dynamical CPT invariance and spontaneous CPT violation in string theories, including the possibility that stringy CPT violation could occur at levels detectable in the next generation of experiments. In particular, we present here an estimate for values of parameters for CPT violation in the kaon system.

1. Introduction

String theory is an ambitious proposal for a consistent quantum theory of gravity, simultaneously providing a framework for unification of the fundamental forces and interactions. Like any new physical model, string theory faces two issues. First, is it viable? Second, is it falsifiable? Viability includes the issues of internal self-consistency and physical realism. Falsifiability is satisfied if new effects are predicted that are experimentally observable.

Most of the work on string theory has addressed the question of viability, in particular the issue of realism. In contrast, relatively little has been done on the difficult issue of falsifiability, i.e., the existence and observability of stringy effects. A stringy effect can be taken as one violating some principle accepted as valid for a pure particle theory. Such effects are to be expected because strings are qualitatively different from particles, but there is a folklore that they are unobservable in current

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1 Invited talk given by V.A.K. at the Los Angeles meeting on Low Energy Signals from the Planck Scale, October 29-30 1992
experiments because a low-energy string theory looks like a point-particle theory. However, it is at least possible that a string-based low-energy effective theory is only approximately a standard, four-dimensional, renormalizable gauge field theory.

Proving or disproving the existence of such effects in general is evidently difficult in the absence of a completely realistic string theory. However, it is feasible to examine mechanisms by which the string might signal itself to us at low energies, without seeking to address the question of whether they must occur. Even this lesser task is daunting because any such effects should be highly suppressed. Present experiments can at best attain energies of the order of the electroweak scale $m_W$. This is only about $10^{-17}$ of the natural string scale, the Planck mass $m_P$. To minimize the impact of this large suppression factor, it is reasonable to focus on physical features that are believed to be exact and generic in particle theory and that can be tested to a high degree of precision.

This talk presents our work on one possible string signature, CPT violation. It outlines our published work [1] on dynamical CPT invariance and spontaneous CPT violation in string theories. In addition, we provide here more detail on the connection between stringy CPT violation and experimentally observable parameters in the $K\bar{K}$ system.

2. CPT and Strings

The CPT theorem [2] is a profound result holding for local relativistic point-particle field theories under a few mild restrictions. The theorem has substantial experimental support: at present, the tightest bound is found from observations of the $K\bar{K}$ system, where one figure of merit is

$$\frac{m_K - m_{\bar{K}}}{m_K} \leq 5 \times 10^{-18}. \quad (1)$$

The generality of the theoretical result for particles, together with the extended nature of strings and the existence of a high-precision test, make CPT a good candidate for a possible string signature.

Given a field theory, either for particles or for strings, several methods to investigate CPT properties are in principle available. The axiomatic method is perhaps the most general, but suitable extensions of the Wightman axioms for strings have not yet been formulated. An alternative is the constructive method, in which C, P, T are defined on anticommuting irreducible spinors and the CPT transformation properties of arbitrary irreducible tensor products are inferred. However, this would require more information on the general structure of covariant string theories. Instead, we use a practical method [4] that can be applied on a case-by-case basis. First, one seeks C, P, T transformations for the free theory that are consistent with usual notions and with free-field quantization. Assuming this is possible, one then examines the effect of the C, P, T transformations on the interacting fields by expanding them in free fields in the usual perturbation series. If the interactions change the canonical structure, which happens for strings, the consistency of the transformations under
the interacting-field quantization procedure must be verified. Additional constraints may arise if certain fields in the theory are to be identified with physical particles that have known transformation laws. In what follows, we take the massless modes of the string to transform like the particle fields in the standard model.

There are two issues to examine when considering CPT properties of a theory. One, dynamical CPT, is the behavior of the action under the transformation. The other is the possibility of spontaneous CPT violation, which occurs if the ground state is not invariant under CPT. Subsection 3 summarizes our results on dynamical CPT invariance, while subsection 4 contains a discussion of the possibility of spontaneous CPT violation.

3. Dynamical CPT Invariance

To investigate the issue of dynamical CPT, we studied the field theory for the open bosonic string [5] and those proposed for the open superstring [6, 7]. As the string theory contains an infinite number of particle modes, the statement of dynamical CPT invariance implies the assignment of an infinite number of CPT transformation rules that are consistent with the free case, with the infinite number of interactions, and with the interacting-field quantization. Fortunately, the formalism of string field theory permits a direct treatment in terms of string fields. The CPT transformations of the modes are important as such, however, because experiments detect particles, not strings. Also, even though at present only the lowest string modes would be accessible, higher modes might nonetheless play a significant role because they are incorporated into the low-energy effective theory via functional integration. It is therefore useful to construct operators that implement C, P, and T at the mode level. This construction is also presented in [1].

Here is a summary of our results. The action of the open bosonic string is invariant under C, P, T and CPT. All the actions proposed for the open superstring violate C, P, and T but preserve CPT. While the P and T violations are expected for a ten-dimensional theory that contains massless chiral fermions, the C violation is not. However, we were able to find a modified action for the open superstring that preserves C and CPT.

In any event, it seems that the differences between particle and string quantum field theories are insufficient to cause dynamical CPT violation. The extended structure and corresponding smeared particle interactions are apparently smooth enough to preserve CPT, while the presence of an infinite number of fields does not cause trouble because each field enters as a finite representation of the Lorentz group. Our analysis makes use of some standard assumptions, such as the validity of the perturbation series, the completeness of the asymptotic Hilbert space, the reality of observables, and the connection between spin and statistics. As such, it is likely to
extend to field theories for other string models.

4. Spontaneous CPT Violation and the $K\bar{K}$ System

The remaining issue is whether the vacuum is CPT invariant. It is known [8] that there is a natural mechanism in string theory for the spontaneous breaking of Lorentz invariance. This mechanism can also break CPT.

To see how the mechanism might work, consider the open bosonic string. Among the interaction terms of this theory is a coupling $\phi A_\mu A^\mu$ between the tachyon $\phi$ and the massless vector $A_\mu$. The (static) effective potential for $\phi$ has an instability at the origin, suggesting the tachyon acquires a vacuum expectation value (vev). If this vev has the appropriate sign, the trilinear coupling above generates a negative contribution to the squared mass of the vector. This destabilizes in turn the effective potential for the vector, which could cause it to acquire a nonzero vev and hence to spontaneously break the (higher-dimensional) Lorentz symmetry. Note that this procedure cannot happen in a standard four-dimensional gauge field theory: particle gauge invariance excludes trilinear couplings of the necessary form, even though they are compatible with string gauge invariance. The mechanism can in principle involve any Lorentz tensor in a string model, because such tensors always have trilinear couplings to scalars. See ref. [8] for more details.

If the mechanism occurs, it can also break CPT. For instance, in the example above the vector $A_\mu$ changes sign under CPT, so, if it acquires a vev, terms breaking CPT appear in the lagrangian. More generally, if CPT induces a sign change in a field $f$ and if a vev is generated for $f$, then any interaction terms involving $f$ generate CPT violation when the shift to the new (stable) vacuum is made. This could in principle include violation of four-dimensional CPT.

Evidently, any such effects must be suppressed in the low-energy four-dimensional effective theory, perhaps because only the higher (Planck-mass) modes are involved. In any case, the natural suppression factor is the ratio of the low-energy scale $m_t$ to the Planck scale $m_P$, which is about $10^{-17}$ or less. For example, in the $K\bar{K}$ system one might expect [1]

$$\frac{m_K - m_{\bar{K}}}{m_K} \sim \frac{m_t}{m_P}, \quad (2)$$

which is in the range $10^{-20}$ to $10^{-17}$.

To make this more explicit, consider a class of four-dimensional low-energy effective interactions from string theory:

$$\mathcal{L}_I \supset \lambda m_P^{-k} T \cdot \bar{\psi} \Gamma (i\partial)^k \chi . \quad (3)$$

In this equation, $\lambda$ is a dimensionless coupling, the factor $m_P^{-k}$ is a dimensional factor correcting for derivative couplings and the compactification process, $T$ is a four-dimensional Lorentz tensor, $\psi$ and $\chi$ are four-dimensional fermions (possibly the same), $\Gamma$ represents a gamma-matrix structure, and $(i\partial)^k$ denotes the presence of
derivatives $\partial_\mu$. Since $\mathcal{L}_I$ is a Lorentz scalar, $T \cdot \Gamma(i\partial)^k$ is a spinor matrix with derivative entries.

If $T$ acquires a vev $\langle T \rangle$, $\mathcal{L}_I$ generates a contribution $\Delta K(p)$ to the fermion inverse propagator $K(p)$, given by

$$\Delta K(p) = \lambda m_p^{-k} \langle T \rangle \cdot \Gamma p^k, \quad k \geq 0 .$$

Suppose the vev $\langle T \rangle$ is written as

$$\langle T \rangle = t \left( \frac{m_l}{m_P} \right)^l m_P, \quad l \geq 0 ,$$

where $t$ is a numerical factor that may carry Lorentz indices and the factor $(m_l/m_P)^l$ allows for a suppression factor in line with the known hierarchy in nature. Then,

$$\frac{\Delta K}{K} \sim \left( \frac{p}{m_P} \right)^k \left( \frac{m_l}{m_P} \right)^{l-1} \left( \frac{m_l}{m_f} \right) ,$$

where $m_f$ is the fermion mass. If the fermions are taken to be light (observable) then $p \ll m_P$ and so $k + l \geq 1$. Moreover, if $T$ has nontrivial Lorentz structure and incorporates CPT breaking, a realistic model presumably has $\langle T \rangle \ll m_l$, i.e., $l \geq 2$. Then, the issue of whether any stringy spontaneous CPT violation might be accessible to experiment becomes the question of whether terms with $k = 0$ and $l = 2$ are observable.

Since such terms are suppressed by a factor of $10^{-17}$ or more, we need a sensitive (interferometric) experiment to observe their effects. A candidate is the $K\bar{K}$ system. For this case, take the fermions $\psi$ and $\chi$ to be $d$ and/or $s$ quarks. One can work in lowest-order perturbation theory where necessary, replacing $\Delta K(p)$ with its expectation in the quark wavefunction. For simplicity, suppose the expectation value $\langle T \rangle = t(m_l/m_P)^2 m_P$ and the gamma-matrix structure $\Gamma$ are such that the effect on the $d$ or $s$ inverse propagators is an energy shift. This happens, for instance, for interactions of the form $T_{\lambda\mu\nu} \bar{\psi} \gamma^\lambda \gamma^\mu \gamma^\nu \chi$ if $T$ acquires a vev $\langle T_{000} \rangle$. The energy shifts for the $d$ and $s$ quarks induce energy changes in the $K$ and $\bar{K}$, thereby affecting the kaon mass/decay matrix.

Before proceeding further, we summarize some notation and conventions. We take the states $|K^0\rangle$ and $|\bar{K}^0\rangle$ to satisfy

$$CP|K^0\rangle = -|\bar{K}^0\rangle \quad (7)$$
$$CP|\bar{K}^0\rangle = -|K^0\rangle \quad (8)$$

The effective hamiltonian governing the time evolution of a superposition of $|K^0\rangle$ and $|\bar{K}^0\rangle$ is a two-by-two matrix $H$:

$$H = M - i \Gamma \equiv \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} .$$

(9)
Under CPT, $H_{11} \rightarrow H_{22}$ and vice versa. A useful parametrization is

$$iH = \Gamma - iM \equiv \begin{pmatrix} D + iE_3 & iE_1 + E_2 \\ iE_1 - E_2 & D - iE_3 \end{pmatrix} .$$  \hspace{1cm} (10)

The eigenvalues $\lambda_L$ and $\lambda_S$ of this matrix and the corresponding eigenvectors $|K_L\rangle$ and $|K_S\rangle$ are given by

$$\lambda_L = D + iE , \quad |K_L\rangle = \frac{(1 + \epsilon - \delta)|K^0\rangle + (1 - \epsilon + \delta)|\bar{K}^0\rangle}{\sqrt{2(1 + |\epsilon - \delta|^2)}} ,$$  \hspace{1cm} (11)

$$\lambda_S = D - iE , \quad |K_L\rangle = \frac{(1 + \epsilon + \delta)|K^0\rangle - (1 - \epsilon - \delta)|\bar{K}^0\rangle}{\sqrt{2(1 + |\epsilon + \delta|^2)}} .$$  \hspace{1cm} (12)

In these equations,

$$E^2 = E_1^2 + E_2^2 + E_3^2 ,$$  \hspace{1cm} (13)

while

$$\epsilon = \frac{-iE_2}{E_1 + E}$$  \hspace{1cm} (14)

measures T violation and

$$\delta = \frac{-E_3}{E_1 + E}$$  \hspace{1cm} (15)

measures CPT violation.

Introducing the masses and lifetimes of the $K_L$ and the $\bar{K}_S$ via

$$\lambda_L = \frac{1}{2} \gamma_L + im_L , \quad \lambda_S = \frac{1}{2} \gamma_S + im_S ,$$  \hspace{1cm} (16)

we get

$$E = \frac{1}{2} \Delta m + \frac{1}{4} i \Delta \gamma ,$$  \hspace{1cm} (17)

where $\Delta \gamma = \gamma_S - \gamma_L$ and $\Delta m = m_L - m_S$. Experimentally, $\Delta m \simeq \frac{1}{2} \Delta \gamma$, and one finds $E_1 \simeq E$, so defining the superweak phase

$$\phi_\epsilon = \tan^{-1} \frac{2 \Delta m}{\Delta \gamma}$$  \hspace{1cm} (18)

gives

$$\delta \simeq \frac{i \Delta m}{\sqrt{2} \Delta \gamma} e^{i\phi_\epsilon} .$$  \hspace{1cm} (19)

The quantities $\Delta m = (3.522 \pm 0.016) \times 10^{-15}\text{GeV}$ and $\phi_\epsilon = 43.68 \pm 0.14^\circ$ are experimentally determined [3].

This formalism can be used to obtain an expression for the CPT-violating parameter $\delta$ in terms of the quantities arising in the low-energy effective interactions. The energy shifts for the $d$ and $s$ quarks generate corresponding shifts in the kaon matrix $H$:

$$\Delta H_{11} = -\Delta H_{22} = h_d - h_s$$  \hspace{1cm} (20)
where
\[ h_{d,s} = r_{d,s} \lambda_{d,s} \frac{m^2}{m_P}. \tag{21} \]
The extra factors \( r_{d,s} \) are QCD corrections allowing for quark-binding effects. Setting \( \Delta H_{11} = E_3 \) gives
\[ \delta \simeq \frac{i}{\sqrt{2}} \frac{h_d - h_s}{\Delta m} e^{i \phi_e}. \tag{22} \]

The issue of observability of possible CPT-violating effects then reduces to estimating the net effect of contributions to \( \delta \) of the form (22) from different interactions. If the assumption is made that the energy shifts are real, we see that \( \text{Re} \delta \simeq -\text{Im} \delta \).

Then, for \( m_l \) lying in the region around \( m_K \) to \( m_W \), it is possible to get contributions to \( |\text{Re} \delta| \) lying below the current experimental limit of about \( 10^{-3} \) down to about \( 10^{-6} \). Part of this region may be explored by the \( \Phi \) factories under development at Frascati and Novosibirsk [10] or at the novel asymmetric \( \Phi \) factory being considered at UCLA [11].

In principle, CPT violating effects other than those included in \( \delta \) might arise in the decay amplitudes [12]. However, the energy shifts discussed above do not give a direct contribution to the matrix elements of the decays. There is an indirect effect that could arise through the energy-dependent normalizations of the eigenstates, but it is suppressed by at least one power of \( m_l/m_P \) and so is negligible. Another contribution comes from interactions of the form (3) when \( \psi \) is \( d \) and \( \chi \) is \( s \) (and vice versa). However, these either give negligible additional contributions to electroweak-type flavor-changing processes, or they generate standard-model-forbidden decays at unobservable levels because a suppression of at least two powers of \( m_l/m_P \) appears.

5. Summary

In summary, CPT is a good candidate for a string signature. We have demonstrated dynamical CPT invariance in field theories for the open bosonic string and the open superstring. A natural mechanism exists in string theory that could lead to spontaneous CPT breaking. It can induce violation in the \( K \bar{K} \) system at levels just beyond current limits. The signal would be a finite value for \( \delta \) and negligible other CPT-violating parameters.

6. Acknowledgments

We thank David Cline, Stuart Samuel, Julia Thompson, and Mike Zeller for useful conversations. V.A.K. thanks the Theory Division of CERN and the Aspen Center for Physics for hospitality while part of this work was done. This work was supported in part by the United States Department of Energy under contracts DE-AC02-84ER41025 and DE-FG02-91ER40661.

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