Supersymmetric Quantum Mechanics of Monopoles in N=4 Yang-Mills Theory

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Abstract

A supersymmetric collective coordinate expansion of the monopole solution of $N = 4$ Yang-Mills theory is performed resulting in an $N = 4$ supersymmetric quantum mechanics on the moduli space of monopole solutions.
1. Introduction

This article is an extension of the work of Harvey, Strominger [1], and Gauntlett [2] determining the low energy Lagrangians for solitons which are solutions of supersymmetric theories. Here we focus on monopoles in the $N = 4$ supersymmetric Yang-Mills theory. What we find is an $N = 4$ supersymmetric quantum mechanics of monopoles dancing on their moduli space. The $N = 0$ (bosonic) and $N = 2$ cases have been previously handled by Gibbons and Manton [3] and Gauntlett [4].

In this low energy approximation one considers only the dynamics of monopole zero modes, fields with classical trajectories that are geodesics on the moduli space. The trajectories are described by the collective coordinates of the moduli space corresponding to the parameters of a monopole solution that can be varied without altering the topological charge where gauge transformed solutions are identified. Equivalently, zero modes satisfy the equations of motion with the potential energy at its minimum, and their kinetic energy describes this geodesic motion. By limiting the dynamics to zero modes, the particles of the model (photons, massive gauge bosons, scalar fields, fermions) and internal massive excitations of the monopole are excluded from the picture.

The non-trivial monopole solution louses up two of the supersymmetries which are transformed into fermion zero modes. These two supersymmetries correspond to the collective coordinates of one monopole, and one can further construct two commuting fermion zero modes for each of the other collective coordinates. The fermion modes are constructed from bosonic zero modes and eight orthonormal spinors such that the number of independent bosonic modes is half that of the fermionic ones. The commuting fermion modes are paired with collective grassmann variables whose time dependence represents the motions of the fermion zero modes on the moduli space. Since two of the four supersymmetries leave the monopole solution invariant, the number of unblemished supersymmetries is that needed for the description of trajectories on the moduli space to be an $N = 4$ quantum mechanics.

2. The Monopole Solution of $N=4$ Yang-Mills Theory

In this section we discuss the $N=4$ Yang-Mills theory in three space plus one time dimensions and its monopole solution to the equations of motion. The notations used with
some minor changes are from Osborn [5]. The Lagrangian for this theory is

\[ \mathcal{L} = \left( \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} D_\mu A_i D^\mu A_i \right. \]

\[ \left. - \frac{1}{2} D_\mu B_j D^\mu B_j + \frac{1}{2} i \bar{\Psi} \gamma^\mu D_\mu \Psi \right) + \frac{1}{2} \bar{\Psi} [\alpha^i A_i + i \gamma_5 \beta^j B_j, \Psi] \] - V(A, B) \]

(2.1)

where the indices \( i, j = 1, 2, 3 \), and

\[ V(A, B) = -\frac{1}{4} \left( [A_i, A_j][A_i, A_j] + [B_i, B_j][B_i, B_j] + 2[A_i, B_j][A_i, B_j] \right) \]

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] \]

with \( D_\mu = \partial_\mu + [A_\mu, \cdot] \). All fields are in the adjoint representation of \( SU(2) \) (e.g. \( A_\mu = A_\mu^A T^A \)), and \( (T^A T^B) = \frac{1}{2} \text{tr}(T^A T^B) = -\delta^{AB} \). The metric used is \( g_{ij} = -\delta_{ij} \) for spatial indices while \( g_{00} = 1 \). The \( 4 \times 4 \) matrices \( \alpha^i, \beta^j \) satisfy the following relations:

\[ S^{ij} = [\alpha^i, \alpha^j] = -2 \epsilon^{ijk} \alpha^k \]

\[ V^{ij} = [\beta^i, \beta^j] = -2 \epsilon^{ijk} \beta^k \]

\[ U^{ij} = \{\alpha^i, \beta^j\} = -U^{ji} \]

\[ [\alpha^i, \beta^j] = 0 \]

\[ \{\alpha^i, \alpha^j\} = -2 \delta^{ij} \]

\[ \{\beta^i, \beta^j\} = -2 \delta^{ij} \].

(2.3)

The fermions \( \Psi_{tu} \) are Majorana spinors with \( t \) a Lorentz spinor index acted on by the \( \gamma^\mu \) and \( u \) an \( SU(4) \) index transforming under \( S^{ij}, V^{ij} \), and \( iU^{ij} \) such that \( \bar{\Psi} = \Psi^T C = \Psi^\dagger \gamma^0 \) where \( C \) is the charge conjugate matrix. The above Lagrangian can be derived from the ten dimensional supersymmetric Yang-Mills action, and a global \( SO(6) \sim SU(4) \) symmetry exists as a consequence of the reduction of the Lorentz group [5]:

\[ \delta A_\mu = 0 \]

\[ \delta \Psi = \frac{-1}{8} [S^{ij} \epsilon^{S}_{ij} + V^{ij} \epsilon^{V}_{ij} + i \gamma_5 U^{ij} \epsilon^{U}_{ij}] \Psi \]

\[ \delta A_i = \epsilon^S_{ij} A_j + \epsilon^U_{ij} B_j \]

\[ \delta B_i = \epsilon^U_{ij} A_j + \epsilon^V_{ij} B_j \]

(2.4)
where \( \epsilon_{ij}^S, \epsilon_{ij}^V, \) and \( \epsilon_{ij}^U \) are constant and antisymmetric in their indices. The action is also invariant under the following \( N = 4 \) supersymmetries

\[
\delta \Psi = \frac{i}{2} \gamma^{\mu \nu} F_{\mu \nu} + \gamma^\mu D_\mu (\alpha^i A_i + i \gamma_5 \beta^j B_j)
- \frac{1}{2} i \epsilon^{ijk} \alpha^k [A_i, A_j] - \frac{1}{2} i \epsilon^{ijk} \beta^k [B_i, B_j]
- \alpha^i \beta^j [A_i, B_j] \gamma_5 \epsilon
\]

\delta A_\mu = \bar{\epsilon} \gamma_\mu \Psi
\delta A_i = -i \bar{\epsilon} \alpha^i \Psi
\delta B_j = \bar{\epsilon} \beta^j \gamma_5 \Psi

(2.5)

where \( \epsilon_{tu} \) is a constant, anticommuting Majorana spinor.

The equation of motion for the gauge field is

\[
D^\nu F_{\mu \nu} - [A_i, D_\mu A_i] - [B_j, D_\mu B_j] - \frac{1}{2} \bar{\Psi} [\gamma_\mu, \gamma_5 \Psi] = 0.
\]

(2.6)

Following the methods of Harvey, Strominger [1] and Gauntlett [2], we want to expand solutions of this equation in \( n = n_\partial + \frac{n_f}{2} \) where \( n_\partial \) is the number of time derivatives and \( n_f \) is the number of fermion fields. To zeroth order in \( n \) the solution as discussed by various authors [7] is a static monopole:

\[
B_i = D_i \Phi
A_0 = 0
V(A, B) = 0
(\Phi^2) = v^2 |\vec{x}| \to \infty
\]

(2.7)

where the scalar field \( \Phi \) is defined as a function of the \( A_i, B_j \)

\[
\Phi = a_i A_i + b_j B_j
a_i a_i + b_j b_j = 1,
\]

(2.8)

\( a_i, b_j \) constant, and the nonabelian magnetic field \( B_i \) is

\[
B_i = \frac{1}{2} \epsilon_{ijk} F_{jk}.
\]

(2.9)
The scalar field equations also are satisfied since $D_i B_i = 0$. Introducing Pauli matrices $\sigma_i$ and a projection matrix $P$,

$$\sigma_i = \frac{i}{2} \epsilon_{ijk} \gamma^j \gamma^k = -\gamma^0 \gamma_5$$

$$P = P^\dagger = \gamma_0 \gamma_5 \alpha^i a_i + i \gamma^0 \beta^j b_j$$

we see that

$$\delta \Psi = \sigma_i B_i (1 + P) \epsilon$$

and that this solution has broken half (those having $P \epsilon = \epsilon$) of the supersymmetries. The solution has also reduced the gauge symmetry from $SU(2)$ to $U(1)$ by specifying a direction in isospin space for the monopole on the two-sphere spatial boundary. The global symmetry is reduced to $SO(5)$ by this solution. To zeroth order in $n$ the Lagrangian is a topological charge invariant \[8\] under smooth deformations of $\Phi, A_i$ since

$$L^{(0)} = \int_{R^3} d(F \Phi) = -v \int_{S^2(\infty)} (\frac{1}{2} F \Phi - \frac{1}{2v^3} \Phi D \Phi \wedge D \Phi) = -4\pi k v$$

where $k$ is an integer, and the second term does not contribute because $D \Phi \sim O(\frac{1}{r^2})$ for smooth, finite energy configurations. The absolute value of this Lagrangian is the rest energy (mass) of $|k|$ monopoles or antimonopoles (for $k$ negative).

3. Bosonic Zero Modes and Geometry on the Monopole Moduli Space

The goal is to determine the low energy dynamics of monopoles. To accomplish it we need to consider bosonic zero modes and the geometry of the monopole moduli space \[8\]. For a monopole solution in $R^3$ with topological number $k$, there are $4k$ independent directions in which the solution can be deformed continuously, thus, preserving its topology or monopole number. These parameters correspond for $k = 1$ to the position of the monopole in $R^3$ and the parameter of the unbroken $U(1)$ gauge symmetry for gauge transformations with noncompact support. For general $k$ the monopole moduli manifold factorizes as $M^k = R^3 \times S^1 \times \frac{M^k}{Z_k}$ such that the center of mass sits in $R^3$, and the $S^1$ angle plays the same role as for $k = 1$. The $4k - 4$ dimensional manifold $M^k_0$ is coordinatized by parameters governing the relative motion of $k$ monopoles. Only the $M^2_0$ metric is known. The $Z_k$ identification reflects the fact that the $k$ monopoles are indistinguishable. For
large distances $M^k$ approximates $k$ copies of $M^1$ physically representing $k$ noninteracting monopoles. The manifolds $M^k$ and $M_0^k$ are hyperkahler implying that there exists a triplet of covariantly constant complex structures with the algebra

$$J_i J_j = -\delta_{ij} - \epsilon_{ijk} J_k.$$  \hfill (3.1)

We introduce time dependence by allowing the $4k$ parameters to become collective coordinates that depend on time. The time derivative of the $U(1)$ parameter is proportional to the total electric charge of the monopoles. Electrically charged monopoles are called dyons. The fields which depend on these collective coordinates also gain a time dependence. Consider the $k$-monopole sector with $4k$ collective coordinates $X^\alpha(t)$. The fields $A_i(x, X^\alpha)$ and $\Phi(x, X^\alpha)$ depend on these as well as on $R^3$. There are, thus, bosonic zero modes $\delta_\alpha A_i$, $\delta_\alpha \Phi$ tangent to the moduli manifold. They are defined as follows:

$$\delta_\alpha A_i = [s_\alpha, D_i]$$
$$\delta_\alpha \Phi = [s_\alpha, \Phi]$$ \hfill (3.2)

where $s_\alpha = \partial_\alpha + [\epsilon_\alpha, \quad]$. The gauge parameter $\epsilon_\alpha$ is fixed by requiring

$$D^i \delta_\alpha A_i - [\Phi, \delta_\alpha \Phi] = 0,$$ \hfill (3.3)

and equation (2.6) is still satisfied. To lie in the tangent space these zero modes must also satisfy

$$\epsilon_{ijk} D^j \delta_\alpha A^k = D_i \delta_\alpha \Phi + [\delta_\alpha A_i, \Phi].$$ \hfill (3.4)

The various geometrical structures on the moduli space are induced from these bosonic zero modes. The metric is a symmetric, nondegenerate tensor product of covectors. Since the zero mode equations (3.3) , (3.4) transform covariantly with respect to diffeomorphisms of $R^3$, global rotations of the gauge group, and global rotations of $(D_i, \Phi) \equiv (D_m)$, the metric should be invariant under these metamorphoses. The simplest Riemannian metric satisfying these conditions is

$$g_{\alpha\beta} = - \int d^3 x \left( \delta_\alpha A_i \delta_\beta A_i + \delta_\alpha \Phi \delta_\beta \Phi \right).$$ \hfill (3.5)

The three complex structures are $J_{i\alpha\beta}$ where

$$J_{i\alpha\beta} = 2 \int d^3 x \left( \delta_{[\alpha} A_i \delta_{\beta]} \Phi - \frac{\epsilon_{ijk}}{2} \delta_{[\alpha} A_j \delta_{\beta]} A_k \right).$$ \hfill (3.6)
and $i = 1, 2, 3$. In addition to zero modes there are non-zero modes which together satisfy a completion condition

$$\delta^\alpha C^A_m(x)^p \delta_\alpha C^B_n(y)^p = \delta_{mn} \delta^{AB} \delta(x - y)$$  \hspace{1cm} (3.7)

where $C_m = A_i, \Phi$; $m = 1, 2, 3, 4$, and $p$ indexes the modes. The Christoffel symbols and Riemann curvature tensor can be calculated to take the form

$$\Gamma_{\alpha\beta\gamma} = -\frac{1}{2} \int d^3x \left( \delta_\alpha C_m [s_{\gamma}, \delta_\beta C_m] \right)$$  \hspace{1cm} (3.8)

$$R_{\alpha\beta\gamma\delta} = \frac{1}{2} \int d^3x \left( \delta_\alpha C_m [\Phi_{\gamma\delta}, \delta_\beta C_m] \right)$$  \hspace{1cm} (3.9)

where $\Phi_{\gamma\delta} = [s_{\gamma}, s_{\delta}]$. In these calculations we must remember that zero modes are orthogonal to nonzero modes and that only zero modes make up tensors that are tangent to the moduli manifold. Using (3.3), (3.4), and (3.7) we can show that the complex structures are covariantly constant and follow the correct algebra. We have started with $4k \times 4$ bosonic zero modes, but using the complex structures to relate the modes through (3.7) shows that only $4k$ of them are independent.

4. Fermion Zero Modes and the Low Energy Lagrangian of $N = 4$ Supersymmetric Monopoles

Let us now extend the picture to fermions and write down the supersymmetric quantum mechanics of $N = 4$ monopoles. Half of the supersymmetries are destroyed by the monopole solution, and these eight destroyed supersymmetries satisfy a zero mode equation

$$\gamma^i D_i \delta \Psi = i\gamma_5 \gamma^0 P[\Phi, \delta \Psi].$$  \hspace{1cm} (4.1)

The Majorana condition $B\Psi^* = \Psi$ where $B = \gamma^0 C^{-1}$, $B(\gamma^\mu)^* = -\gamma^\mu B$, and $PB = BP^*$ cuts the number of supersymmetries in half. As in the bosonic case there are fermionic fluctuations that satisfy the zero mode equation. These modes can be written explicitly in complex coordinates as

$$\Psi^a_\beta = (\sigma_i \delta_\beta A_i - i\delta_\beta \Phi)(\frac{1 + \sigma_2}{2}) \epsilon^a_\beta$$  \hspace{1cm} (4.2)

and

$$\Psi^{as*}_\beta = B(\Psi^{as}_\beta)^*$$  \hspace{1cm} (4.3)
where $\epsilon_{\alpha}^{as}$ is a commuting spinor. The broken supersymmetries are a linear combination of the eight $R^3 \times S^1$ modes. The precise form of these solutions has been chosen so that they are eigenfunctions of the complex structure $J_2$:

$$J_{2\alpha}{}^{\beta} \Psi_{\beta}^{as} = i \Psi_{\alpha}^{as}$$
$$J_{2\alpha}{}^{\beta} \Psi_{\beta}^{as*} = -i \Psi_{\alpha}^{as*}.$$  \hspace{1cm} (4.4)

We are using kahlerity to set $g_{\alpha\beta} = J_{2\alpha\beta} = 0$. The indices $a, s$ are both two dimensional. In fact, since $[P, \sigma_i] = 0$ we let

$$\sigma_i \epsilon_{as} = \epsilon_{as} (\sigma_i)_{s' s}$$  \hspace{1cm} (4.5)

with

$$\epsilon_{as}^\dagger \epsilon_{a' s'} = \delta_{aa'} \delta_{ss'}$$
$$\epsilon_{as}^\dagger B \epsilon_{a' s'} = 0$$
$$P \epsilon_{as} = \epsilon_{as}.$$  \hspace{1cm} (4.6)

For the purposes of dimensional reduction we introduce matrices $\rho_i$ dependent on the choice of $a_i, b_j$ (2.8) so that the gamma matrices take the following form:

$$\gamma_i = \gamma_5 \gamma_0 \sigma_i$$
$$\gamma_0 = A \rho_2.$$  \hspace{1cm} (4.7)

where $\{A, \rho_i\} = 0$, $[\rho_i, \sigma_j] = 0$, $A^2 = -1$, and the $\rho_i$ act on the index $a$ analogously to (1.3). In these coordinates the fermion zero modes $\Psi_{\beta}^{as}$ and their charge conjugate modes are both zero. Also, the modes obtained by reversing the projection ($\sigma_2 \rightarrow -\sigma_2$) are either zero or can be obtained from the listed ones by multiplication of the matrix $J_3$. The number of zero modes agrees with the Callias index theorem [10] because taking into account the projection $\frac{1+\sigma_2}{2}$ and the Majorana condition (4.3), we are left with $2k \times 4$ zero modes. As mentioned above eight of the modes corresponding to the $R^3 \times S^1$ coordinates come from the broken supersymmetries.

The fermion zero modes do not alter the vacuum energy or mass of the monopoles, and we introduce time dependence analogously to the bosonic case by pairing the zero modes with anticommuting collective parameters $\lambda_{as}^\alpha(t)$. Then,

$$\Psi = \Psi_{\alpha}^{as} \lambda_{as}^\alpha + \Psi_{\alpha}^{as*} \lambda_{as}^{\alpha* T}$$  \hspace{1cm} (4.8)

and $J_{2\alpha}{}^{\beta} \lambda_{as}^\alpha = i \lambda_{as}^\beta$. 

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The next step is to solve (2.6) to first order in time \((n = 1)\). The following equation which can be derived using \(J_2\) is helpful:

\[
\begin{align*}
\{ 2i[\delta_{(\alpha^* A_2, \delta_\beta)}\Phi] & - i\epsilon_{2ij}[\delta_{(\alpha^* A_i, \delta_\beta) A_j}] \} \bar{\lambda}^{\alpha^*} \gamma_0 (1 + \sigma_2) \lambda^\beta =  \\
\{ [\delta_{\alpha^* A_i}, \delta_\beta A_i] + [\delta_{\alpha^* \Phi}, \delta_\beta \Phi] \} \bar{\lambda}^{\alpha^*} \gamma_0 (1 + \sigma_2) \lambda^\beta.
\end{align*}
\]

(4.9)

The solution after some algebra is

\[
A_0 = \epsilon_\alpha \partial_0 Z^\alpha + \epsilon_{\alpha^*} \partial_0 Z^{\alpha^*} + \frac{i}{2} \Phi_{\alpha^* \beta} \bar{\lambda}^{\alpha^*} \gamma_0 (1 + \sigma_2) \lambda^\beta
\]

\[
F_{0i} = \partial_0 Z^\alpha \delta_\alpha A_i + \partial_0 Z^{\alpha^*} \delta_{\alpha^*} A_i + i s_{(\alpha^*} \delta_{\beta)} A_i \bar{\lambda}^{\alpha^*} \gamma_0 (1 + \sigma_2) \lambda^\beta
\]

(4.10)

where \(\gamma^0 = \rho_2\) acting on the \(a\) index. The fermionic term of \(F_{0i}\) is not a zero mode and will not contribute to the low energy dynamics.

We are ready to expand the Lagrangian to order \(n = 2\). Recalling that the moduli space is a Kahler manifold; using (3.5), (3.6), (3.8), (3.9), (4.9); and substituting (4.8), (4.10) in the kinetic energy; the result is

\[
L^{(2)} = \partial_0 Z^{\alpha^*} \partial_0 Z^\beta g_{\alpha^* \beta} - \frac{i}{2} (\bar{\lambda}^{\alpha^*} \gamma^0 (1 + \sigma_2) D_0 \lambda^\beta g_{\alpha^* \beta} + \text{c.c.})
\]

\[
- \frac{1}{2} \bar{\lambda}^{\alpha^*} \gamma^0 (1 + \sigma_2) \lambda^\beta \bar{\lambda}^{\gamma^*} \gamma^0 (1 + \sigma_2) \lambda^\delta R_{\alpha^* \beta \gamma^* \delta}
\]

(4.11)

where

\[
D_0 \lambda^\alpha = \partial_0 \lambda^\alpha + \Gamma^\alpha_{\beta \gamma} \partial_0 Z^\beta \lambda^\gamma.
\]

(4.12)

Substituting

\[
\lambda^\beta = \frac{1}{2} \left( \begin{array}{c}
-i \\
1
\end{array} \right) \lambda^{\beta_a} + \frac{1}{2} \left( \begin{array}{c}
i \\
1
\end{array} \right) \lambda^{\beta_\bar{a}}
\]

(4.13)

and writing \(L^{(2)}\) in terms of real coordinates yields

\[
L^{(2)} = \frac{1}{2} g_{\alpha \beta} \partial_0 X^\alpha \partial_0 X^\beta - \frac{i}{2} \bar{\lambda}^{\alpha^*} \gamma^0 D_0 \lambda^\beta g_{\alpha \beta} - \frac{1}{12} R_{\alpha \beta \gamma \delta} \bar{\lambda}^{\alpha} \lambda^\gamma \bar{\lambda}^\beta \lambda^\delta
\]

(4.14)

with a Fierz rearrangement of the curvature term. This Lagrangian represents a quantum mechanical system with \(N = 4\) supersymmetry. The supersymmetry transformations that leave the action invariant are:

\[
\delta X^\alpha = J^{\alpha}_{\mu \beta} \bar{\epsilon}^\mu \lambda^\alpha
\]

\[
\delta \lambda^\alpha = i (J^{\alpha}_{\mu \beta})^{-1} \gamma^0 \partial_0 X^\beta \epsilon^\mu - \Gamma^{\alpha}_{\beta \gamma} \delta X^\beta \lambda^\gamma
\]

(4.15)

where \(J^{\alpha}_{0 \beta} = \delta^{\alpha}_{\beta}\), \(\mu = 0, 1, 2, 3\) and \(\epsilon^\mu\) are real, two-dimensional anticommuting spinors.
5. Conclusions

After quantization the energy of a dyon can be written as

\[ E = 4\pi v + 2\pi vu^2 + \frac{\hbar^2 n^2}{8\pi}v \]  

(5.1)

where \( u \) is the velocity of the dyon, and \( q_e = \hbar n \) is the electric charge of the dyon. Since the charge \( q_e \) is conserved, dyons have a mass

\[ M_{dyon} = M_{monopole} + \frac{\hbar^2 n^2}{8\pi}v \]  

(5.2)

The energy is, of course, nonrelativistic, and one would expect relativistically \( E = \sqrt{p^2 + M_{monopole}^2} \) since the excess mass of the dyon is part of the kinetic energy. Taking \( u = 0 \) implies relativistically that

\[ M_{dyon}^{rel} = 4\pi v \sqrt{1 + \frac{\hbar^2 n^2}{(4\pi)^2}} = v \sqrt{q_m^2 + q_e^2} \]  

(5.3)

where the magnetic charge \( q_m = 4\pi \) in these units. This is indeed the formula derived from consideration of the central charges of the \( N = 4 \) supersymmetry [3].

One motivation for finding this low energy action of monopoles is that their quantum scattering can then be calculated. Gibbons and Manton [3] have performed these computations for the bosonic two-monopole. Some effort has been applied to extend their results to the supersymmetric case. By comparing these results to low energy scattering in the particle sector of the theory, one can possibly find evidence for the duality conjecture of Montonen and Olive [11]. This conjecture postulates an exchange of monopole and particle dynamics under the interchange of electric and magnetic charges and the inversion of the coupling constant \( g \to \frac{1}{g} \). Here, we have taken \( g = 4\pi \). The relativistic mass formula is valid for all particles and solitons. Osborn [3] has shown that the monopole supermultiplet contains the same spins as the particle one. Further endeavors to search for evidence of duality are being undertaken.

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