Computationally efficient necking prediction using neural networks trained on virtual test data

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Abstract. The onset of localized necking under monotonic and non-monotonic loading can be well-predicted by the imperfection-based approach proposed by [1] (MK). However, a large number of virtual imperfections has to be investigated for an accurate necking prediction, making the MK approach computationally expensive and hence preventing the industrial application for full-scale vehicle models. To overcome these issues, a computationally efficient neural network (NN) model is proposed for replacing the MK model in the present work. An extended version of the MK model has been implemented into a User Material for an explicit crash solver. The model continuously computes the "distance to localized necking" as an important engineering quantity. Single shell element simulations are utilized for creating a comprehensive virtual test database for monotonic and non-monotonic loading for a 22MnB5 grade in an as-delivered state. A simple feed-forward NN model, featuring only one hidden layer, is trained and tested against the virtual data, where invariants of the stress and plastic strain tensors represent the input features of the NN and the "distance to necking" represents the output value of the NN. For comparison of the computational cost, the NN architecture has also been implemented in a User Material Routine for shell elements. The predictions of the NN and the MK model are in good agreement, where, due to the simple mathematical structure of the NN, the computational cost of the NN is significantly lower than for the MK implementation.

1. Introduction

Increasing environmental regulations continuously challenge the automotive industry for developing lighter and more fuel-efficient vehicles. At the same time, all safety standards have to be met. The occurrence of fracture during crash loading can have a large impact on the crashworthiness of a vehicle. Under plastic loading, thin-walled sheet metals often exhibit failure due to localized necking, which can be considered a conservative threshold for the occurrence of fracture. The numerical prediction of the onset of localized necking under the complex non-proportional plastic deformation occurring during a crash is very important, since it cannot be determined experimentally for arbitrary loading conditions. The term “localized necking” is abbreviated by “necking” throughout the remainder of this work.

A recent overview on modelling techniques for necking prediction of sheets can be found in [2], distinguishing classical necking models based on bifurcation theory, e.g. [3] and [4], imperfection-based models following the work by [1] (MK), or damage-driven models based on Gurson-Tvergaard-
Needleman (GTN) type models, in which the onset of necking is assumed to depend on the micro-void nucleation, growth and coalescence. Only few literature is available on using neural networks for the prediction of necking, e.g. [5], [6], [7]. In these publications, global forming process parameters are related to the obtained major and minor strain distribution of specific tests, based on a comprehensive real experimental test program.

In the present work, an extended imperfection-based MK model is used for creation of a comprehensive virtual data base for a quench-hardenable (22MnB5) steel grade in as-delivered condition. For the plasticity model, isotropic hardening and a non-quadratic isotropic yield criterion after [8] is applied. Further details regarding the underlying characterization and modelling is provided in [9].

A feed-forward neural network is trained on the virtual data base, completely replacing the computationally expensive MK model by a computationally efficient model. The NN is validated against numerical data obtained from the MK model, and also on real experimental data. A good agreement of the MK model and the NN model is achieved.

2. Extended MK model

[1] assumed a virtual imperfection line in a thin metal sheet, Fig. 1a, represented by parameter \( f_0 \), which is the ratio of initial sheet thickness \( t_0^A \) and slightly reduced initial thickness inside the imperfection \( t_0^B \).

\[
f_0 = \frac{t_0^B}{t_0^A}
\]

(1)

Considering arbitrary non-proportional loading, the orientation of the imperfection under which localized necking occurs, cannot be determined in advance. For an accurate analysis, different virtual imperfections oriented at different angles \( \psi \) have to be analysed simultaneously. In the present work, 90 imperfections, equally spaced within the range of 0° to 180°, have been analysed.

Plastic instability of a thin metal sheet is identified once the ratio of equivalent plastic strain increment inside the imperfection, \( d\varepsilon^B \), and equivalent plastic strain increment outside the imperfection, \( d\varepsilon \), exceeds a value \( N_f \):

\[
R_\varepsilon = \frac{d\varepsilon^B}{d\varepsilon} > N_f,
\]

(2)

where typically \( 10 \leq N_f \leq 100 \). It is noted that \( R_\varepsilon \) remains at a roughly constant level close to unity until \( R_\varepsilon \) suddenly increases significantly, indicating the onset of necking. This instantaneous behaviour does not provide information about the "distance to necking" during the loading, which is a crucial information for the simulation engineer. Hence, an extension of the MK model is suggested, where at specific user-defined time intervals a complete MK analysis is performed, assuming a virtual scaling of the underlying plastic strain increment tensor \( d\varepsilon \) until the onset of necking is identified. In this process, the equivalent plastic strain in zone A at the beginning of each "distance to necking" computation is denoted \( \bar{\varepsilon}_\text{start} \). While scaling the plastic strain increment tensor, the necking condition, equation (2), is checked for all imperfections. The determined equivalent plastic strain \( \bar{\varepsilon}_\text{end} \) representing the onset of necking is used for computation of the distance to necking according to

\[
NECK = \frac{\bar{\varepsilon}_\text{start}}{\bar{\varepsilon}_\text{end}}
\]

(3)

where the necking state parameter lies in the range \( 0 \leq NECK \leq 1 \). The time evolution of \( NECK \) can be non-monotonic, e.g. for plane strain tension loading followed by biaxial tension loading.
3. Neural network (NN) representation of the MK model

The process steps for setting up the NN are explained below.

**Data creation:** A virtual test program for proportional (linear) and non-proportional (bi-linear) load cases is performed using a single shell element model (Fig. 1a). The extended MK model is implemented in a User Material routine of an explicit crash solver [10]. The process for performing the linear load cases is illustrated in Fig. 1b (blue curves), where different constant load states are applied up to the onset of necking. One bi-linear load case is shown in the figure (red curves), where in the illustration pre-straining is performed at a stress triaxiality of $\eta = 0.55$ up to almost 50% of the achievable necking strain under that loading condition. In secondary process steps, loading is continued at different load states up to necking. A total of 4378 load cases at different grades of pre-straining and pre-straining ratios have been performed.

**Data pre-processing:** The input data for the neural network is extracted from the simulation result files. The input data is represented by $R = 6$ quantities: the major and minor principal stresses $\sigma_1$ and $\sigma_2$, the accumulated major and minor principal plastic strains $\varepsilon_1$ and $\varepsilon_2$, the stress triaxiality $\eta$, defined as the ratio of hydrostatic stress an von Mises equivalent stress, and the equivalent plastic strain $\bar{\varepsilon}$. It is noted that all quantities are invariant with respect to the chosen coordinate system. Irrelevant data, such as data obtained during an elastic step and during the unloading/reloading phase for the bi-linear load cases, is removed from the data base.

**Neural network design and training:** The proposed fully-connected feed-forward neural network architecture is shown in Fig. 2. It is mathematically represented by

$$NECK = a = f_2[W_2 \ f_1[W_1p + b_1] + b_2],$$

(4)
where the input feature vector $\mathbf{p}$ contains the aforementioned quantities. For the activation functions $f_1$ and $f_2$, the rectified linear unit (ReLU) is chosen. The hidden layer contains $S_1=128$ neurons and the output of the network is the necking state $NECK$. In total, approximately 650k data points are available for the analysis. The data is randomly split into a training set (90%) and a testing set (10%), where only the training data set is used for training. Different network architectures have been investigated (using more hidden layers and more neurons). For the sake of brevity, only the finally chosen architecture is discussed here.

The Open Source software package Tensorflow [11] is used for training of the NN. Dropout regularization is applied, where the probability for keeping a neuron during an iteration is set to 0.75. The Adam algorithm is used for performing the optimization, where the initial learning rate is set to $5.0e^{-5}$. In each optimization iteration (epoch), the training data is randomly split into batches of 500 data points, being presented to the back-propagation algorithm for updating the weights ($W_1, W_2$) and biases ($b_1, b_2$). The loss (summed squared error) is minimized for the training data and the corresponding loss of the testing data is monitored independently. During the training, both losses continuously decreased to reach a steady state after approximately 1000 epochs, representing the optimized state.

For all investigated load cases of the virtual test program, the predicted equivalent plastic strain at the onset of necking ($NECK = 1$) of the trained NN is compared to the MK reference results, Fig. 3a. Some of the load cases do not exhibit necking at all. In this case the final strains are plotted. The majority of predictions lie within a corridor of ±15% with respect to the MK predictions.

**Comparison of computational costs**: Analogously to the extended MK model, the neural network is implemented in a User Material routine of an explicit crash solver [10], completely replacing the MK model. In every computation cycle, the required input features $\mathbf{p}$ are computed for the mid-plane integration points of the shell elements. The necking state $NECK$ is computed according to equation (4) using the determined weights/biases of the NN.

For a comparison of the computational costs, a square (20x20 mm$^2$) patch of shell elements with an edge length of 1mm is investigated. The patch is fixed on one edge and the nodes of the edge on the opposite side are displaced, imposing a constant velocity of 1mm/ms over a simulation time of 5ms.

It is noted that the computation of the MK analysis using 90 imperfections is very costly, because the stress and plastic strain quantities in each virtual imperfection have to be solved at each simulation cycle and, additionally, the distance to necking computation has to be performed at a user-given frequency. Even for a reduced MK model, containing only 4 imperfections instead of 90, the computation time is roughly 10 times higher than for the proposed neural network.
4. Validation

The NN has been trained on linear and bi-linear load cases using single shell element simulations, Fig. 1a+b. For validation, the model shown in Fig. 1b is subjected to a continuously changing load state over the time. That is, the loading angle $\gamma$

$$\gamma = \arctan\left(\frac{\varepsilon_x}{\varepsilon_y}\right).$$

becomes a function of time:

$$\gamma = \max[\gamma_{\text{final}}, \gamma_{\text{init}} + \dot{\gamma}_0 t]$$

Figure. 2: Neural network architecture for necking prediction.
Figure 3: a) Equivalent plastic strain at the onset of necking (EPSN) for all investigated load cases of the virtual test program: NN versus MK; b) Evolution of the necking indicator $NECK$ for three selected load cases.

A sweep, starting at the load state of uniaxial tension, $\gamma_{init} = 116.565^\circ$, and approaching the load state of equi-biaxial tension, $\gamma_{final} = 45^\circ$, is considered for validation. Nine different rates, $\dot{\gamma}_0 \in [-0.4, -0.35, -0.3, -0.25, -0.2, -0.15, -0.1, -0.05, -0.025]$ rad/ms, are investigated.

The NN can represent the evolution of $NECK$ over time with good accuracy for 8 out of 9 of the investigated load cases. The evolutions of $NECK$ over time of representative load cases are shown in Fig. 3b. For load case #5, corresponding to $\dot{\gamma}_0 = -0.2$ rad/ms, the MK model already reaches $NECK = 1$ in the first ‘uniaxial tension to plane strain tension’ loading zone of the simulation. The NN predicts a slightly reduced progression of the $NECK$ state, reaching a value close to unity, until, at a time of approximately 2.3ms, the zone of ‘plane strain tension to equi-biaxial tension’ is entered, in which the $NECK$ state is temporarily reduced. In other words: simulation #5 represents a very sensitive load case, where very small differences of the $NECK$ response decide whether necking is triggered early or late.

An experimental validation is presented in Fig. 4. In the first loading phase, a sheet of the investigated material is pre-deformed under biaxial tension loading up to 7.5mm punch displacement, Fig. 4a. Then the specimen is removed and the cut-outs (Fig. 4b) are applied to the deformed sheet. Finally, the deformed sheet with cut-outs is again inserted into the test apparatus and the punch is further displaced until necking is observed under a load state close to plane strain tension in the specimen centre. The predicted load-displacement curves of the MK and NN models are in excellent agreement with the experimental curves. Also, the predictions for the punch displacement at the onset of necking are very close to the experimental observations of both models. The distribution of the necking state $NECK$ of MK and NN model is compared in Fig. 4c at the onset of necking. Again, the responses of the MK and NN model are very similar.
5. Conclusions

In the present work, the imperfection-based model of [1] has been extended by introducing a “distance to necking” computation. This computationally expensive model is then used for creating a virtual test data base under proportional and non-proportional loading. A computationally efficient feed-forward neural network featuring one hidden layer of 128 neurons is trained on the created data base, where the output of the necking state is a function of 6 inputs, represented by invariants of the stress and plastic strain tensors. The predictions of the neural network are in good agreement with the predictions of the imperfection-based reference model.

The developed neural network model has been validated using synthetic numerical tests, where the imperfection-based model serves as a reference. Additionally, both models have also been validated against real experiments. A good predictability was observed for the NN model for the investigated validation cases. Compared to the imperfection-based model, the computation time of the neural network model is massively reduced (factor >10), enabling industrial application within full-scale vehicle crash simulations.
6. References

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