Linking $R_{K^{(*)}}$ anomalies to $H_0$ tension via Dirac neutrino

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The recently updated measurement from LHCb has strengthened the deviation of $R_K$, implying a stronger hint of new physics beyond the Standard Model. We show in this paper that, the long-standing $R_{K^{(*)}}$ anomalies can be explained by Dirac neutrinos embedded in a two-Higgs-doublet model. The explanation induces a thermalized right-handed Dirac neutrino in the early Universe, which contributes as extra radiation to the Hubble expansion and prompts a shift of the effective neutrino number, $N_{\text{eff}}$. In mimicking the favored scenarios for resolving the Hubble ($H_0$) tension, we show that the simultaneous explanation of $R_{K^{(*)}}$ anomalies and $H_0$ tension via one flavor right-handed Dirac neutrino can be readily tested or ruled out by the upcoming $H_0$ confirmation.

I. INTRODUCTION

In rare semi-leptonic $B$-meson decays, there exists a long-standing deviation with the Standard Model (SM) observed from BaBar [1], Belle [2, 3] and LHCb [4–6] collaborations for the ratios $R_{K^{(*)}}$, which are defined by

$$R_{K^{(*)}} = \frac{B(B \rightarrow K^{(*)}\mu^+\mu^-)}{B(B \rightarrow K^{(*)}\nu\bar{\nu})}.$$  

Compared to the SM prediction [7–9], $R_{K^{(*)}}^{\text{SM}} = 1.00 \pm 0.01$ in $1.1 \leq q^2 \leq 6 \text{ GeV}^2$, with $q^2$ the dilepton invariant mass squared, the discrepancy of $R_{K^{(*)}}$ is roughly at 2.5$\sigma$ level.

Strikingly, the recent update from LHCb [10],

$$R_K(1.1 \leq q^2 \leq 6 \text{ GeV}^2) = 0.846^{+0.042+0.013}_{-0.039-0.012},$$  

pushes the deviation to be even at the level of 3.1$\sigma$, and hence implies a stronger hint of new physics (NP) that violates the lepton-flavor universality (see also the recent reviews [11, 12]).

The $R_{K^{(*)}}$ anomalies have triggered a great deal of NP ideas that are being under extensive and intensive investigations (see Refs. [11, 12] and references therein). In particular, the two-Higgs-doublet model (2HDM) extended with right-handed neutrinos [13–17] is an interesting NP candidate, since it can connect the intriguing lepton-flavor universality violation to the neutrino masses – another big mystery in contemporary particle physics. Thus far, either Majorana [14–17] or Dirac [13] neutrinos have been considered to address the $R_{K^{(*)}}$ anomalies. In these scenarios, the dominant NP contributions to $b \rightarrow s\mu^+\mu^-$ come from the right-handed neutrinos running in the box diagrams, and the resulting NP Wilson coefficients reside in the direction, $C_{3\mu}^{\text{NP}} = -C_{10\mu}^{\text{NP}}$, which is persistently favored by the updated global fits following Eq. (2) [18–25].

Despite the intensive focus on Majorana neutrinos during the past decades, the topics with neutrinos treated as being of Dirac nature are attracting much attention, especially in connection with the big-bang nucleosynthesis (BBN) and cosmic microwave background (CMB) [26–30], as well as the baryon asymmetry of the Universe [31, 32]. The Dirac neutrino effects in the $b \rightarrow s\mu^+\mu^-$ process have been noticed in Ref. [13]. In fact, the loop function resulting from the box diagram for $b \rightarrow s\mu^+\mu^-$ transition is insensitive to the neutrino masses [14]. This implies that the difference between heavy Majorana and light Dirac neutrinos cannot be simply distinguished by the $R_{K^{(*)}}$ resolution. However, as discussed already in Ref. [13], since $O(1)$ Dirac neutrino Yukawa couplings are generically required to explain the $R_{K^{(*)}}$ anomalies, the thermalized right-handed Dirac neutrinos with such large couplings in the early Universe would make an undesired contribution as extra radiation, and hence generate a dangerous shift of the effective neutrino number, $N_{\text{eff}}$, at the BBN and CMB epochs.

It should be emphasized, nevertheless, that the implementation of BBN and CMB constraints depends on the number of thermalized right-handed Dirac neutrinos and the corresponding decoupling temperature. In addition, it further depends on what astrophysical and cosmological data are taken. This last proceeding of data analysis has also a close relation with the so-called Hubble tension, i.e., the locally measured values of the Hubble constant $H_0$ [33] differ from the standard $\Lambda$CDM prediction based on the Planck CMB measurements [34] at a level of 4-$\sigma$ (see the latest reviews [35–38]). In this respect, we are naturally led to an interesting situation, in which the resolution of $R_{K^{(*)}}$ anomalies via Dirac neutrinos can be further linked to the $H_0$ explanation if the current constraints from cosmology are somewhat relaxed to support the favored patterns of $H_0$ solutions.

Motivated by such a potential correlation, we consider here a 2HDM framework with flavor-specific right-handed Dirac neutrinos. Similar to our previous consideration [14], the $R_{K^{(*)}}$ anomalies will be addressed in the
direction: $C_{3u}^{NP} = -C_{10u}^{NP}$. However, the dramatic difference is that the flavor-specific Yukawa interaction only allows one flavor right-handed Dirac neutrino to provide a non-negligible contribution in the box diagram. Such a resolution indicates one (rather than three) thermalized right-handed neutrino in the early Universe, which can generate a shift of $\Delta N_{\text{eff}} = N_{\text{eff}} - N_{\text{eff}}^{\text{SM}} \simeq 1$ to mimic a possible explanation of the Hubble tension [39–41]. Although there is not yet a definite conclusion in favor of $\Delta N_{\text{eff}} \simeq 1$ partially due to some incompatibilities with the BBN constraints, the timely scenario presented in this paper can help to shed some light on the common cause of the BBN constraints, the timely scenario presented in this paper can help to shed some light on the common origin of the $R_{K^{(*)}}$ anomalies and the $H_0$ tension, and highlight the significant role of Dirac neutrinos in particle physics and cosmology. The model is also readily established for generating the fermion and gauge-boson anomalies and the $Z$ anomaly. Conclusions are finally made in Sec. V.

II. 2HDM WITH FLAVOR-SPECIFIC DIRAC NEUTRINOS

The 2HDM is an economic NP candidate with an additional Higgs doublet introduced to the SM particle content [42]. A specific 2HDM framework is characterized by its Yukawa interactions and scalar potential, both of which can be either specified by some symmetry background or by purely phenomenological considerations. For our purpose to address the $R_{K^{(*)}}$ anomalies, we embed three right-handed Dirac neutrinos into the 2HDM, while in constructing the Yukawa interactions and scalar potential, we follow here a mixed phenomenological and symmetric method. Explicitly, we consider the following Yukawa Lagrangian given in the mass basis:

$$\mathcal{L}_Y = \mathcal{L}_Y^{\text{SM}} - X_u \bar{Q}_L \tilde{H}_2 d_R - X_{\nu} \bar{E}_L \tilde{H}_2 \nu_R + \text{h.c.},$$

where $\tilde{H}_2 = i \sigma_2 H_2^{\ast}$, while $\bar{Q}_L$ and $\bar{E}_L$ are parameterized, respectively, as

$$\bar{Q}_L = (\bar{u}_L, \bar{d}_L, V^\dagger), \quad \bar{E}_L = (\bar{\nu}_L U^\dagger, \bar{e}_L),$$

with all the fermions $f_{L,R}$ $(f = u, d, e, \nu)$ defined in the mass basis, while $V$ and $U$ denoting the CKM and PMNS matrices, respectively. $\mathcal{L}_Y^{\text{SM}}$ is designed here to incorporate both the minimal SM Yukawa interactions and a neutrino Yukawa term $\bar{E}_L \tilde{H}_1 \nu_R$ for generating the Dirac neutrino masses via the Higgs mechanism.

In this paper, we assume that the two extra Yukawa structures in Eq. (3) are given, respectively, as

$$X_{u,ij} = \kappa_i \delta_{i3} \delta_{j3}, \quad X_{\nu,ij} = \kappa_{\nu} \delta_{i2} \delta_{j3},$$

where $\kappa_{i,\nu}$ are the effective couplings, and the flavor index $s$ characterizes the one-flavor $\nu_R$ that couples to the muon in charged-scalar current. Note here that the explicit flavor is irrelevant and will be simply denoted as $\nu_{s,R}$. Such a setup comes from various data-driven considerations. In the quark sector, Eq. (5) would induce only neutral current in the third generation, while the generated charged currents, $d_{L,i} V_{L}^{\ast} X_{u,ij} \bar{u}_{R,j} \tilde{H}_2^\dagger$, have only significant interaction in the third generation due to the hierarchy of CKM matrix elements. This pattern also complies with the observation that only significant NP contributions are allowed in the third quark generation while the flavor-changing neutral scalar currents are severely constrained [43, 44]. For the neutrino part, on the other hand, Eq. (5) indicates that only one flavor right-handed Dirac neutrino has a significant coupling to the muon while couplings to the electron and tauon are strongly suppressed. Such a pattern follows closely the tight bounds from the charged lepton-flavor violating processes $\ell_i \rightarrow \ell_j \gamma$ mediated by $\nu_R$ at loop level [14] and the muon decay $\mu \rightarrow e \nu \nu$ mediated by the charged Higgs at tree level. Furthermore, the reason for allowing only one rather than three flavors of $\nu_R$ to interact with the muon is that, if all the three right-handed Dirac neutrinos have significant couplings to the SM, they would readily establish thermal equilibrium in the early Universe and generate, therefore, an unacceptable shift of $\Delta N_{\text{eff}}$ [13, 27]. Finally, it should be emphasized that, since we are here interested in the connection between the $R_{K^{(*)}}$ anomalies and the $H_0$ tension via a minimal setup, other couplings not presented here are unnecessary to be strictly zero in general, but rather signify the meaning of their phenomenological smallness. Besides, we will not concern here the symmetry underlying the flavor-specific Yukawa interactions given by Eq. (3), though interesting possibilities, such as the Branco-Grimus-Lavoura based scenarios [45] and the Minimal-Flavor-Violation (MFV) like mass-power textures [46], may deserve further exploitation.

In the Higgs sector, we consider an exact $Z_2$-symmetric scalar potential [42, 47]:

$$V_H = m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 + \frac{\lambda_1}{2} (H_1^\dagger H_1)^2$$

$$+ \frac{\lambda_2}{2} (H_2^\dagger H_2)^2 + \lambda_3 (H_1^\dagger H_1)(H_2^\dagger H_2)$$

$$+ \lambda_4 (H_1^\dagger H_2)(H_2^\dagger H_1) + \frac{\lambda_5}{2} (H_1^\dagger H_2)^2 + \text{h.c.},$$

where, by requiring the vacuum structure to be invariant under the $Z_2$ parities $Z_2(H_1) = 1$ and $Z_2(H_2) = -1$, the two Higgs doublets can be parametrized, respectively, as

$$H_1 = \left( \frac{G^+}{\sqrt{2}}, \nu + \mu + \tau \right), \quad H_2 = \left( \frac{H^+}{\sqrt{2}}, \nu^\dagger + \mu^\dagger + \tau^\dagger \right).$$

Here, the vacuum expectation value $v \simeq 246$ GeV is responsible for generating the fermion and gauge-boson...
masse, $h$ corresponds to the SM Higgs boson and $G_0^+$ are the Goldstone bosons, while $H^0, A$ are the two physical neutral scalars and $H^+$ the physical charged scalar. Note that the $Z_2$ symmetry is only broken by the Yukawa sector, which is in line with the MFV philosophy \[48, 49\]. Such a setup is also adopted to allow significant Yukawa contributions while evading the severe constraints from electroweak precision observables, $Z$-boson and $\tau$-lepton decays (see, e.g., Ref. \[50\] and references therein), because the NP contributions from scalars $H^*, H^0, A$ can be significantly canceled out by their quasi-degenerate masses in the small $\lambda_3$ region.

In the following sections, we will apply the structures given by Eq. (5) to address the $R_{K^{(*)}}$ anomalies and the subsequent connection to the $H_0$ tension, where the parameter $\kappa_\nu$ serves as the portal.

### III. EXPLAINING THE $R_{K^{(*)}}$ ANOMALIES WITH ONE-FLAVOR DIRAC NEUTRINO

Following Ref. \[14\], we can readily see that the dominant contribution to the $b \to s\mu^+\mu^-$ process comes from the box diagram, but now with only one flavor right-handed Dirac neutrino $\nu_{s,R}$ running in the loop, as shown in Fig. 1. It contributes to the effective Hamiltonian

$$\mathcal{H}_{	ext{eff}}^{\text{NP}} = -\frac{G_F a}{\sqrt{2}} V_{ts} V_{ts}^* (C_9 \mathcal{O}_9 + C_{10} \mathcal{O}_{10}),$$

with

$$\mathcal{O}_9 \equiv (\bar{s}_\mu P_L b)(\bar{\ell}_\gamma \ell), \quad \mathcal{O}_{10} \equiv (\bar{s}_\mu P_L b)(\bar{\ell}_\gamma \mu \gamma_5 \ell).$$

The resulting Wilson coefficients after turning the Dirac neutrino mass to zero are given by \[14\]

$$C_{9\mu}^{\text{NP}} = -C_{10\mu}^{\text{NP}},$$

$$= -v^4 |\kappa_{t4}|^2 |\kappa_{\nu}|^2 - \frac{1 - x_t + x_t \log x_t}{64s_W^2 m_W^2 m_{H^+}^2 (1 - x_t)^2},$$

where $s_W^2 \equiv \sin^2 \theta_W \approx 0.23$, $m_W$ is the $W$-boson mass, and $x_t \equiv m_t^2/m_{H^+}^2$, with $m_t$ and $m_{H^+}$ being the top-quark and the charged-Higgs mass, respectively. As the numerical dependence of the loop function on the neutrino mass is sufficiently weak, the $R_{K^{(*)}}$ resolution in the direction $C_9^{\text{NP}} = -C_{10\mu}^{\text{NP}}$ cannot uniquely determine the heavy Majorana or the light Dirac nature of the neutrino participating in the box diagram.

It should be mentioned that, under the phenomenological setup specified by Eq. (5), a sizable $\kappa_\nu$ can give significant contributions to the branching ratio $B(\bar{B} \to X_s \gamma)$ of the inclusive radiative decay $\bar{B} \to X_s \gamma$ and the mass differences $\Delta M_{b,s}$ of the $B_{d,s} - B_{d,s}$ mixings \[13-17\]. In addition, a large muon-philic coupling $\kappa_\nu$ could modify the precision observables involving the muon, such as the $Z \to \mu^+\mu^-$ and $W \to \mu\nu$ decays. Thus, we must take into account these constraints to find out the viable parameter space for the $R_{K^{(*)}}$ resolution. To this end, we show in Fig. 2 the contours in the $(\kappa_\nu, m_{H^+})$ plane under the considered B-physics constraints, which are independent of the coupling $\kappa_\nu$. It can be seen that the tightest constraint comes from the observable $\Delta M_d$ due to the updated SM prediction \[51-54\]. As a consequence, an $\mathcal{O}(1)$ $\kappa_\nu$ is only allowed by pushing the charged-Higgs mass to TeV scales \[50\].

For the followed estimate of $C_{9\mu}^{\text{NP}} = -C_{10\mu}^{\text{NP}}$, we will replace the parameter $\kappa_t$ in Eq. (10) with the charged-Higgs mass $m_{H^+}$ by saturating the $3\sigma$ upper bound from $\Delta M_5^\text{exp}/\Delta M^{\text{SM}}_d$, where the experimental data and the SM prediction are taken from the Particle Data Group \[55\] and the HQCD collaboration \[52\], respectively. As an illustration, we show in Fig. 3 the perturbative unitarity region $\kappa_\nu < \sqrt{4\pi}$ to reproduce the global-fit result, $C_{9\mu}^{\text{NP}} = -C_{10\mu}^{\text{NP}} = -0.35 \pm 0.08$ \[25\] (see also compatible results obtained in Refs. \[18-24\]). Note that the constraints from $Z \to \mu^+\mu^-$ and $W \to \mu\nu$ decays do not impose further significant bounds on the parameters and

![Figure 1](https://via.placeholder.com/150)

Figure 1. Significant NP box diagram contributing to the $b \to s\mu^+\mu^-$ transition, where only one flavor $\nu_{s,R}$ participates non-negligibly in the loop.

![Figure 2](https://via.placeholder.com/150)

Figure 2. Constraints from $B(\bar{B} \to X_s \gamma)$ and $\Delta M_{d,s}$. The regions above the curves are already excluded by $B(\bar{B} \to X_s \gamma)$ at $2\sigma$ and $\Delta M_{d,s}$ at $3\sigma$ level, respectively.

![Figure 3](https://via.placeholder.com/150)

Figure 3. Perturbative unitarity region $\kappa_\nu < \sqrt{4\pi}$ to reproduce the global-fit result, $C_{9\mu}^{\text{NP}} = -C_{10\mu}^{\text{NP}} = -0.35 \pm 0.08$ \[25\] (see also compatible results obtained in Refs. \[18-24\]). Note that the constraints from $Z \to \mu^+\mu^-$ and $W \to \mu\nu$ decays do not impose further significant bounds on the parameters and
IV. MIMICKING THE $H_0$ SOLUTION VIA AN EFFECTIVE NEUTRINO NUMBER SHIFT

It should be noted that a large $\mathcal{O}(1)$ $\kappa_\nu$ required to explain the $R_{K^{(*)}}$ anomalies can render the reaction $H^+ \rightarrow \mu^+ \mu^-$ and $W \rightarrow \mu \bar{\nu}$ decays [50]. From Fig. 3, it can be clearly seen that an $\mathcal{O}(1)$ $\kappa_\nu$ is required to explain the $R_{K^{(*)}}$ deficits, as found also in Refs. [13, 14].

$\Delta_N_{eff} = \left( \frac{10.75}{g^*_{eff}(T_{\nu_{\tau},dec})} \right)^{4/3},$ (11)

where $g^*_{eff}(T_{\nu_{\tau},dec})$ is the effective degree of freedom for the SM entropy density (see also Ref. [56] for details). The $\nu_{s,R}$ freezing-out temperature $T_{\nu_{s,R},dec}$ can be estimated from the instantaneous decoupling condition $\Gamma_{\nu_L=\nu_{s,R}} \lesssim H(T)$, where the Hubble expansion at the radiation-dominated epoch is given by

$$H = \sqrt{\frac{4\pi^3 g^*_{eff}(T)}{45 M_P^2}} T^2,$$ (12)

with the effective degree of freedom for the energy density $g^*_{eff} \approx g^*_s$ and the Planck mass $M_P = 1.22 \times 10^{19}$ GeV. The effective four-neutrino interaction rate has the scaling $\Gamma \sim \kappa_\nu^4 T^5/m_S^4$, with $m_S \equiv m_{\mu^0} \approx m_A \approx m_{H^+}$ derived from Eq. (6) in the small $\lambda_3$ region [50]. To handle the twelve-dimensional phase-space integration in the four-neutrino interaction rate, we follow the numerical calculation presented in Ref. [28], which gives

$$\Gamma_{\nu_L=\nu_{s,R}} \approx 0.015 |U_{23}|^2 \left( \frac{\kappa_\nu}{m_S} \right)^4 T^5,$$ (13)

where the PMNS matrix element $U_{23}$ appears due to the Yukawa structure given by Eq. (5), with its best-fit value $|U_{23}| \approx 0.74$ [55].

Currently, the Hubble tension is still under intense debate [35–38]. The authors of Refs. [39–41, 57] pointed out that an effective neutrino number shift, with $\Delta N_{eff} \approx 1$, is able to solve the Hubble tension. However, such a large shift is at least naively in tension with the BBN measurements [58], such as the mass fraction of helium-4 ($Y_p$) and the deuterium abundance D/H. Nevertheless, as discussed in Ref. [37], there are possible ways to dilute or even evade the BBN constraints, allowing therefore a larger $\Delta N_{eff}$, which may be realized, e.g., by introducing secret SM neutrino interactions in the BBN regime [59] and/or by taking into account the systematic uncertainties of the helium abundance $Y_p$ [60]. Furthermore, it is also pointed out in Refs. [41, 61] that $\Delta N_{eff}$ is allowed to be larger if $Y_p$ is promoted as a free parameter in the BBN data analysis. On the other hand, large $\Delta N_{eff}$ is also severely constrained by the late-time CMB measurements. If one simply invokes the large $\Delta N_{eff}$ to solve the $H_0$ tension, the transparent discrepancy arises when confronted with the most severe bound from Planck combined with the baryon acoustic oscillation data, $\Delta N_{eff} < 0.45$ at $3\sigma$ [62]. As discussed also in Ref. [40], to comply with the late-time CMB measurements, additional NP interactions, such as the non-standard self-interactions of the SM neutrinos (i.e., the self-interacting $\nu_L$; see also the recent review [35]), are necessary and complementary to the $\Delta N_{eff}$ shift. In any case, there is currently no definite answer to the Hubble tension, and mimicking the Hubble solution via $\Delta N_{eff}$ would still remain viable, provided that additional NP effects can complementarily dilute the dangerous contributions caused by the $\Delta N_{eff}$ shift only.

In spite of such a controversial situation, our philosophy here is that, if the one-flavor $\nu_{s,R}$ is indeed responsible for the $R_{K^{(*)}}$ anomalies, the scenario presented here...
would then predict a significant $\Delta N_{\text{eff}}$ shift that serves as the NP origin of extra radiation, and hence can be further linked to the Hubble tension, while the complementary ingredients, such as additional self-$\nu_L$ interactions, go beyond the scope of the current work but deserve investigations in the future. Regardless of the details from the complementary ingredients, our scenario can be justified or ruled out once the final word about the Hubble tension is made. Bearing our philosophy and the open possibilities in mind, we can apply Eqs. (11)–(13) to find a connection between the $R_{K^{(*)}}$ anomalies and the $H_0$ tension. To this end, we should note that, once the decoupling temperature is given, a $\Delta N_{\text{eff}}$ shift and a restriction on $(\kappa_\nu, m_S)$ can be derived from Eqs. (11)–(13). The resulting restriction will further limit the parameter space allowed by the perturbative unitarity requirement $\kappa_\nu < \sqrt{4\pi}$ and hence affect the $R_{K^{(*)}}$ resolution. Thus, we can in general describe the NP Wilson coefficients $C^{\text{NP}}_{\nu\mu} = -C^{\text{NP}}_{10\mu}$ as a function of the $\Delta N_{\text{eff}}$ shift, which is in turn related to the $H_0$ explanation.

In Fig. 4, we plot the induced $C^{\text{NP}}_{\nu\mu} = -C^{\text{NP}}_{10\mu}$ for given $\Delta N_{\text{eff}}$ values, with the scalar mass $m_S$ varied within the ranges [500, 2000] GeV (dark green) and [500, 3000] GeV (light green), respectively. Here the $\Delta N_{\text{eff}}$ shifts result from the different decoupling temperatures chosen in the interval $T_{\nu,\text{dec}} = [1, 150]$ MeV, and the corresponding values of $C^{\text{NP}}_{\nu\mu} = -C^{\text{NP}}_{10\mu}$ are subject to both the B-physics constraints and the out-of-equilibrium condition $\Gamma_{\nu_L=\nu_L,R} \lesssim H(T_{\nu,\text{dec}})$. The horizontal bands correspond to the $1\sigma$ and $2\sigma$ global-fit results of $C^{\text{NP}}_{\nu\mu} = -C^{\text{NP}}_{10\mu}$ [25], respectively. Besides, we also show the $3\sigma$ upper bound of $\Delta N_{\text{eff}}$ from Planck TT+lowE [62], as well as the $1\sigma$ intervals of $\Delta N_{\text{eff}}$ required by the $H_0$ explanation induced by dark interacting radiation (DIR) [41] and moderately interacting neutrino (MI$\nu$) [40] patterns.

From Fig. 4, it can be seen that the $R_{K^{(*)}}$ explanation via the one-flavor $\nu_{s,R}$ can indeed mimic the favored $\Delta N_{\text{eff}}$ shift for addressing the $H_0$ tension, i.e., the generated shifts reside in the interval suggested by the DIR and MI$\nu$ patterns. As far as the $1\sigma$ explanation of the $R_{K^{(*)}}$, deficits is concerned, however, the desired values of $\Delta N_{\text{eff}}$, while being in some tension with the Planck TT+lowE result [62], $\Delta N_{\text{eff}} < 0.81$ at the $3\sigma$ level, are still compatible with the WMAP measurement [63], $\Delta N_{\text{eff}} < 1.19$ at the $1\sigma$ level. In fact, it is the dependence of the final conclusion on the specific data taken that comprises an important part of the $H_0$ debate [35–38]. Finally, it can also be inferred from Fig. 4 that the proposal with three thermalized right-handed Dirac neutrinos to address the $R_{K^{(*)}}$ anomalies will exceed any bounds considered here.

V. CONCLUSION

We have shown in this paper that the $R_{K^{(*)}}$ anomalies can be explained by one flavor right-handed Dirac neutrino, which couples exclusively to the muon with a sizable Yukawa parameter. Being dramatically different from the three-flavor right-handed Dirac neutrino scenarios, the consequence of such a flavor-specific explanation is a moderate but significant shift of the effective neutrino number at the early BBN and late CMB epochs, which can be linked to the favored resolution for the Hubble tension. Nevertheless, solving the Hubble tension via a $\Delta N_{\text{eff}}$ shift will prompt additional impact on other observables at the early BBN and late CMB epochs, which requires additional NP effects beyond the right-handed Dirac neutrino considered in our minimal setup.

Despite the currently intense debate on the $H_0$ tension, as well as on the details of complementary ingredients, such as additional self-$\nu_L$ interactions, the striking point presented in this paper is that the simultaneous explanation of the $R_{K^{(*)}}$ anomalies and the $H_0$ tension via the one-flavor, rather than three-flavor, right-handed Dirac neutrino can be readily tested or ruled out by the upcoming $H_0$ confirmation. Our scenario can also help to decipher the light Dirac rather than the heavy Majorana nature of the neutrinos.

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