The light baryon spectrum and the exchange of pseudoscalar and vector mesons among constituent quarks

Fabio Cardarelli and Silvano Simula

Istituto Nazionale di Fisica Nucleare, Sezione Roma III, Via della Vasca Navale 84, I-00146 Roma, Italy

Abstract

The effects of the exchanges of both pseudoscalar and vector mesons among constituent quarks on the mass spectra of light baryons are investigated, paying particular attention to the contribution of tensor and spin-orbit terms. It is shown that the latter ones heavily affect the calculated spectra at variance with the empirical observation of the weakness of the baryon spin-orbit splittings. The relativistic suppression of the strength of the interaction among light quarks is argued to be a possible way to reproduce the light-baryon mass spectra.

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1 Introduction

The Constituent Quark Model is known to be a phenomenological model able to reproduce the hadron mass spectra. Within this model the constituent quarks ($CQ$’s) are the only relevant degrees of freedom in mesons and baryons, all the other degrees of freedom being frozen in the $CQ$ mass ($m_i$) and interaction. The baryon wave function $\Psi$ is therefore eigenfunction of a Schrödinger-type equation, viz. $\hat{H}\Psi = M\Psi$, where $\hat{H} = \hat{T} + \hat{V}$ with $\hat{T} = \sum_{i=1}^{3} \sqrt{[\vec{p}_i]^2 + m_i^2}$ being the kinetic term and $\hat{V} = \hat{V}_{conf} + \hat{V}_{s.r.}$ the interaction term, given by a long-ranged confining part $\hat{V}_{conf}$ and a short-ranged part $\hat{V}_{s.r.}$ responsible for the baryon mass hyperfine splittings.

The confining potential is usually derived from a Lorentz-scalar interaction and, as suggested by the spectroscopy and also by lattice QCD calculations, it can be taken linearly dependent on the quark-quark distance $r_{ij} \equiv |\vec{r}_i - \vec{r}_j|$, namely $\hat{V}_{conf} \rightarrow \hat{V}_s = \sum_{i<j} b \cdot r_{ij}$, where $b$ is the string tension. As for the short-ranged part of the quark-quark potential, two choices exist in the literature, based on two alternative mechanisms of boson exchange among $CQ$’s: the one-gluon-exchange ($OGE$) and the pseudoscalar Goldstone-boson exchange ($GBE$).

2 The $OGE$ + Confinement Potential

As it is well known, the non-relativistic reduction of the vector $OGE$ interaction leads to a Coulomb-like potential

$$\hat{V}_{s.r.} \rightarrow \hat{V}_v = \sum_{i<j} \alpha_s \frac{\vec{F}_i \cdot \vec{F}_j}{r_{ij}}$$

(1)

where $\alpha_s$ is the strong coupling constant and $\vec{F}_i$ is the color operator for quark $i$, being $\langle \vec{F}_i \cdot \vec{F}_j \rangle = -2/3 (-4/3)$ for baryons (mesons). The combination of the linear confinement and the Coulomb-type term gives the so-called Cornell-type potential, which was firstly proven to be successful in reproducing the mass spectra of heavy quarkonia. A similar success holds in the light-quark sector too, provided a spin-spin interaction term is added in order to take into account spin splittings (like, e.g., the $\pi - \rho$ and $N - \Delta$ splittings), namely

$$\hat{V}_{ss}^{(v)} = \frac{2}{3m_i m_j} \Delta \hat{V}_v (r_{ij}) \vec{s}_i \cdot \vec{s}_j$$

(2)

Since the spin-spin term (2) formally arises at first order in the $1/m^2$ expansion of the vector $OGE$ interaction, full consistency requires that spin-orbit and tensor terms at the same $1/m^2$ order have to be considered as well, viz.

$$\hat{V}_{ls}^{(v)} = \frac{1}{m_i m_j} \left[ \frac{d\hat{V}_v}{dr_{ij}} \left( \frac{\vec{r}_{ij} \times \vec{p}_i}{2m_i^2} \cdot \vec{s}_j + \frac{\vec{r}_{ij} \times \vec{p}_j}{2m_j^2} \cdot \vec{s}_i \right) - \frac{\vec{r}_{ij} \times \vec{p}_i}{m_i m_j} \cdot \vec{s}_j - \frac{\vec{r}_{ij} \times \vec{p}_j}{m_i m_j} \cdot \vec{s}_i \right]$$

(3)

$$\hat{V}_{tens}^{(v)} = \frac{1}{m_i m_j} \left[ \frac{1}{r_{ij}} \frac{d\hat{V}_v}{dr_{ij}} - \frac{d^2\hat{V}_v}{d^2r_{ij}} \right] \left( \frac{\vec{s}_i \cdot \vec{r}_{ij}}{r_{ij}^2} \frac{\vec{s}_j \cdot \vec{r}_{ij}}{r_{ij}^2} - \frac{1}{3} \vec{s}_i \cdot \vec{s}_j \right)$$

(4)
However, once the above correction terms are considered, a flaw appears in the Confining + OGE model: the strength of the vector spin-orbit term is too large with respect to the one required by the light-baryon spectroscopy. This is known as the spin-orbit puzzle, which was solved by Isgur and coworkers [1] i) by partially compensating the vector spin-orbit term with the Thomas-Fermi precession spin-orbit term arising from the scalar confining interaction, i.e.

\[
\hat{V}_{ls}^{(s)} = -\frac{1}{2r_{ij}} \frac{d\hat{V}_s}{dr_{ij}} \left[ \frac{(\vec{r}_{ij} \times \vec{p}_i) \cdot \vec{s}_i}{m_i^2} - \frac{(\vec{r}_{ij} \times \vec{p}_j) \cdot \vec{s}_j}{m_j^2} \right] \tag{5}
\]

and ii) by using the relativistic factors $\sqrt{m_i m_j/E_i E_j}$ in the interquark potential, which yield a significant suppression of the strength in case of light quarks. The effective Hamiltonian model developed in ref. [1] was very successful: it reproduces a large amount of both meson and baryon experimental masses and solves the spin-orbit puzzle. Nevertheless, a residual problem still remains in the generally good picture given by the Isgur model: negative-parity states are below positive-parity ones in clear contrast to the observation.

3 The GBE + Confinement Potential

The GBE potential among CQ’s has been recently applied to the calculation of the light-baryon spectra in ref. [2]. The non-relativistic reduction of the GBE interaction leads to a potential of the form $\hat{V}_{ps} = \hat{V}^{\text{octet}} + \hat{V}^{\text{singlet}}$ with

\[
\hat{V}^{\text{octet}} = \sum_{a=1}^{3} V_\pi(r_{ij}) \lambda_i^a \cdot \lambda_j^a + \sum_{a=4}^{7} V_\kappa(r_{ij}) \lambda_i^a \cdot \lambda_j^a + V_\eta(r_{ij}) \lambda_i^8 \cdot \lambda_j^8 \] \tag{6}
\[
\hat{V}^{\text{singlet}} = \frac{2}{3} V_\eta'(r_{ij}) \vec{s}_i \cdot \vec{s}_j \tag{7}
\]

where $\lambda_i^a$ is a flavor operator for quark $i$ and $V_M (M = \pi, \kappa, \eta, \eta')$ are radial Yukawa-like functions

\[
V_M(r_{ij}) = \frac{g_{Mqq}^2}{4\pi} \frac{1}{12m_i m_j} \vec{s}_i \cdot \vec{s}_j \left[ \mu_M^2 e^{-\mu_M r_{ij}} - \Lambda_M^2 e^{-\Lambda_M r_{ij}} \right] \tag{8}
\]

The second term in the squared braket of eq. (8) is a smearing function, depending on a cutoff parameter $\Lambda_M$, and $g_{Mqq}$ is the quark-meson coupling costants.

The GBE model of ref.[2] predicts baryon masses in quite good agreement with the experimental data. In particular, due to the presence of the flavor-dependent factor $\vec{\lambda}_i^F \cdot \vec{\lambda}_j^F$, the GBE model is able to yield the correct ordering among positive and negative parity states. However, as in the OGE case, the full consistency requires to consider all the terms at the same $1/m^2$ order appearing in the non-relativistic reduction of the interaction. Thus, for the GBE interaction both the pseudoscalar tensor term

\[
\hat{V}_{\text{tens}}^{(ps)} = \frac{1}{m_i m_j} \left[ \frac{d^2\hat{V}_{ps}}{dr_{ij}^2} - \frac{1}{r_{ij}} \frac{d\hat{V}_{ps}}{dr_{ij}} \right] \left[ \frac{(\vec{s}_i \cdot \vec{r}_{ij})(\vec{s}_j \cdot \vec{r}_{ij})}{r_{ij}^2} - \frac{1}{3} \vec{s}_i \cdot \vec{s}_j \right] \tag{9}
\]
and the scalar spin-orbit term (5) have to be considered. These terms were not included in the calculations of ref. [2] and we will refer to the potential used in ref. [2] as the Graz version of the GBE interaction. We now briefly address the following question: if and to what extent the general goodness of the Graz model picture is preserved when the hyperfine terms (5) and (9) are included in the effective Hamiltonian.

4 An Investigation of the Full GBE Interaction

We have calculated light-baryon masses in case of both the Isgur and GBE potentials by expanding the baryon wave function in a truncated set of harmonic oscillators and using the Raleigh-Ritz principle to determine the corresponding variational coefficients. We have checked that the results of refs. [1]−[2] are correctly reproduced. Some of our results for the Graz potential are shown in the third column of Table 1. Then, we have introduced the pseudoscalar tensor term (9) of the GBE interaction with no addition of free parameters with respect to ref. [2], and we have solved the corresponding Schrödinger equation. The results obtained are shown in the fourth column of Table 1, labelled as Graz + tensor. It can be noticed that the $N - \Delta$ splitting is significantly lowered, while the $N - \text{Roper}$, $N - F_{15}$, $D_{13} - S_{11}$, and $S_{11}^* - S_{11}$ splittings are increased, so that the overall agreement between predicted and experimental masses is destroyed. The last step has been to add also the scalar spin-orbit term (6) and again no free parameters have been introduced. The results are shown in the last column of Table 1, labelled as full GBE. It can be seen that even more dramatic modifications are introduced in the predicted spectrum: the $N - \Delta$ and $N - F_{15}$ splittings appear to be largely underestimated, while the $S_{11}^* - S_{11}$ is strongly overestimated and, in addition, the $D_{13} - S_{11}$ splitting becomes too big, so that the spin-orbit puzzle reappears in the full GBE model.

Table 1. $N^* - N$ mass splittings (all values are in MeV).

| $N - N^*$ | exp. [3] | Graz | Graz + tensor | full GBE |
|----------|----------|------|---------------|----------|
| $N - \Delta$ | 294 ± 2 | 295 ± 6 | 217 ± 21 | 252 ± 19 |
| $N - \text{Roper}$ | 502_{-10}^{+10} | 528 ± 11 | 568 ± 20 | 557 ± 18 |
| $N - D_{13}$ | 582_{-5}^{+10} | 596 ± 6 | 651 ± 11 | 555 ± 11 |
| $N - S_{11}$ | 597_{-15}^{+20} | 596 ± 6 | 604 ± 12 | 665 ± 12 |
| $N - S_{11}^*$ | 712_{-10}^{+10} | 698 ± 6 | 850 ± 16 | 1044 ± 19 |
| $N - F_{15}$ | 792_{-5}^{+10} | 781 ± 9 | 854 ± 13 | 606 ± 16 |

5 Conclusions

In conclusion, we have shown that a constituent quark model based on the complete $1/m^2$ first-order expansion of the pseudoscalar Goldstone boson exchange interaction leaves the spin-orbit
problem unsolved in the light-baryon spectrum. Some ideas to overcome this problem may be considered, like: (a) to add the exchange of vector mesons; (b) to introduce the relativistic suppression factors $\sqrt{m_im_j/E_iE_j}$ in the $GBE$ interaction, as it has been already done for the $OGE$ case by Isgur and coworkers (both in the baryon and meson sectors).

Option (a) comes from the observation that the tensor interaction due to the vector meson exchange (eq. (4)) has opposite sign to the corresponding pseudoscalar interaction (eq. (9)). In fact we have checked that, by excluding all spin-orbit terms, the experimental mass splittings can be reproduced (i.e., vector mesons compensate pseudoscalar mesons in the tensor term). However, this is obtained at the price of having the quark - vector meson coupling of the same order of the quark - pseudoscalar meson one (which is not a welcome result at all). Moreover, the vector meson exchange exhibits a spin-orbit term with opposite sign with respect to the spin-orbit Thomas-Fermi precession term associated to the scalar confinement (see eqs. (3) and (5)). Nevertheless, we have found no compensation among them, mainly because the confining spin-orbit term is long-ranged, while vector mesons are massive. Thus, we can state that vector meson exchange cannot adequately help the $GBE$ model in reproducing the light-baryon spectrum.

Option (b) appears much more promising, because the relativistic suppression factors lower significantly the contribution of the spin-orbit and tensor terms in case of light quarks. Such momentum-dependent factors represent phenomenologically the effects of the non-locality in the effective quark-quark potential. A throughout investigation of this point is in progress and the results will be reported elsewhere.

References

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