The influence of polaron size on the conductivity of poly-DNA

Julia A. Berashevich, Adam D. Bookatz, and Tapash Chakraborty†
Department of Physics and Astronomy, University of Manitoba, Winnipeg, MB R3T 2N2, Canada

The velocity of polaron migration in the long poly-DNA chain (~40 base pairs) in an applied electric field has been studied within a polaron model. We found that the polaron velocity strongly depends on the polaron size. A small polaron shows a slow propagation and strong tolerance to the electric field, while a large polaron is much faster and less stable with increasing electric field. Moreover, the conductance of the DNA molecule within the polaron model is found to be sensitive to structural disorders in the DNA geometry, but that dependence diminishes with increasing temperature.

The application of the biological systems in developing nano电子ic devices has been recognized as one of the most intriguing and promising techniques in recent years. This is due to the molecular recognition and self-assembly properties that allows them to perform reparation of the damaged structures by the non-invasive technology and self-incorporation of molecular blocks into the well-structured systems. The DNA molecule is one such biological system. The interest on DNA, while originally derived as it is the source of genetic information, has been significantly elevated due to the discovery of DNA conductance [1, 2, 3, 4, 5, 6, 7, 8, 9].

Understanding the mechanism of charge migration in a complex molecule such as DNA is a difficult problem. Just as in the field of condensed matter, in simple DNA sequences, if the donor and the acceptor are separated by a single potential barrier, the charge transfer has been adequately explained by the competition of quantum tunneling and incoherent hopping [2, 3] that has also been experimentally confirmed [10, 11]. However, the observed conductance of long DNA molecule with both poly- and mixed sequences, varies from insulator to a metallic behavior [12, 13]. Further, the results for the decrease of conductance with decreasing temperature has been somewhat contradictory [4, 7]. For the description of transport phenomena in polymer chains, a polaron model has been known to be very successful [12, 13]. For DNA, this model has shown recently to provide promising results as well, especially for explaining the temperature dependence of the DNA conductance [4, 5, 6, 7]. Moreover, additional advantage of the polaron model, in comparison to the popular models such as the tight-binding approach which combines a quantum-mechanical treatment of the lattice distortion dynamics. Application of this model, which describes the charge tunneling from one DNA base site to another, is however limited to low temperatures. To avoid the temperature restriction we have investigated the poly-DNA molecule. In a poly-chain, each strand contains only one base type. Therefore, due to the large difference of ionization potential between the purine and pyrimidine bases (> 1.0 eV [14]) belonging to the opposite strands, the main mechanism of charge migration at low and at room temperature is longitudinal one-dimensional tunneling along a single strand containing purine bases.

An electric field is applied to the system using the method of Ref. [13], specifically, the charge transfer integral V is multiplied by a complex exponential phase factor that includes the electromagnetic vector potential A. For a uniform constant electric field E, we take A(t) = −cEt, thereby satisfying E = −dA/dt. Here, c is the speed of light, and we have taken A=0 at time t=0. The uniform electric field is included via its uniform vector potential, rather than with its non-uniform scalar potential [9], in order that periodic boundary conditions can be utilized.

With the modifications for the electric field, the coupled system of non-linear equations based on the PBH model is

\[ i\hbar \frac{d\Psi_n}{dt} = -V e^{-i(e\hbar/c)A(t)}\Psi_{n-1} - V e^{i(e\hbar/c)A(t)}\Psi_{n+1} + \chi \eta_n \Psi_n + \epsilon_n \Psi_n, \tag{1} \]

and

\[ m \frac{d^2 y_n}{dt^2} = -\frac{dV_M(y_n)}{dy_n} - \frac{dW(y_n, y_{n-1})}{dy_n} - \frac{dW(y_{n+1}, y_n)}{dy_n} - \chi |\Psi_n|^2 - m\gamma \frac{dy_n}{dt}, \tag{2} \]
where $\Psi_n$ is the probability amplitude for the charge on the $n$-th base pair, $V$ is the nearest-neighbor transfer integral between the base pairs, $\chi$ is the charge-vibrational coupling constant, $\epsilon_n$ is the on-site energy for base site $n$, $y_n$ is the amount by which the $n$-th base site is displaced from its equilibrium position, $m$ is the polaron mass on the single site, $\gamma$ is the friction parameter, $V_M(y_n)$ is the Morse potential, and $W(y_n, y_{n-1})$ is the interaction of neighboring stacked base-pairs, $e$ is the electronic charge, and $d$ is the interbase spacing ($d=3.4\text{Å}$). The expressions and parameters for $V_M(y_n)$ and $W(y_n, y_{n-1})$ are taken from Ref. [9], and throughout this paper we take $\gamma=1$ ps$^{-1}$. In poly-DNA, $\chi$ and $V$ are constant with respect to the lattice site, and in the absence of disorder, we choose $\epsilon_n$ to be at the origin.

In our simulations, a polaron is initially created by solving the system of nonlinear equations (1) and (2) in a stationary situation (i.e., for $\frac{d^2 y_n}{dt^2} = \frac{dy_n}{dt} = 0$ and $i\hbar \frac{d\Psi_n}{dt} = \text{constant}$). Initial estimates for $y_n$ and $\Psi_n$ are chosen to be nonzero only for ten consecutive base pairs. Due to the uniformity and stability of the system, the solution is largely insensitive to the initial estimates. The resulting initial solution can be seen in Fig. 1 for $t=0$ ps. The lattice displacement $y_n$ and the charge density $|\Psi_n|^2$ coincide and have the same overall shape. Together they comprise a polaron that, for the parameters chosen here, is spread over approximately thirteen lattice sites but has a definite peak in the center. In the polaron’s vicinity, the displacement $y_n$, and therefore the distortion energy $\chi y_n$, are negative, creating a quantum well in the otherwise uniform energy profile of the poly-DNA. The wave function $\Psi_n$ is consequently localized within this well and the polaron is stable against small perturbations. Moreover, if no electric field is applied ($E=0$), the polaron remains completely stationary because the initial solution corresponds to a stable equilibrium.

In an electric field the positively charged polaron moves in the direction of the field (Fig. 1) for a constant electric field of $E=0.016 \text{mV/Å}$. Clearly, as the polaron migrates both the charge density and lattice displacement move in unison, i.e., the wave function and its formed quantum well travel together. Because of the stabilizing effect of the lattice distortion, the polaron retains the same overall shape as it migrates; however, our calculations indicate that in larger electric fields, the polaron becomes slightly more localized over time.

In the following, we study polaron shape and propagation in poly-DNA under the influence of an electric field. We focus specifically on the effect of three physical parameters, $\chi$, $V$, and $m$. The value of the charge-vibrational coupling constant $\chi$ determines the decrease in on-site energy in the charged-state geometry, and has been estimated theoretically to be in the range of 0.3 - 1.5 eV [15]. Its value predominantly depends on the nature of the state geometry and its extension, which can be influenced by the structural parameters of DNA and the solvent environment as well. The charge in DNA can be spread in two directions: in parallel to the propagation pathway – longitudinal direction and perpendicular – transverse direction. The spreading of the charge in the longitudinal direction significantly decreases $\chi$ [14]. For the transverse case, a charge can occupy a single purine base that suggests a larger value of $\chi$ than does a charge that is partially delocalized over a base pair [15]. The degree of charge delocalization also governs the value of the polaron mass $m$. If the charge is localized on a single base then $m \approx 150 \text{amu}$, and this value increases with extension of the geometry of the state in the transverse direction. The value of the charge transfer integral $V$ depends strongly on the configuration of the DNA geometry, and can be influenced by the solvent environment as well [17]. Theoretical estimations place the value of $V$ in the range of $0.05$ – $0.3 \text{eV}$ [14, 16], while experiments indicate the value of $V$ to be $\ll 0.01 \text{eV}$ [17, 18]. Throughout this paper, we use the values $\chi=0.6 \text{eV/Å}$, $V=0.1 \text{eV}$, and $m=300 \text{amu}$ as reference points, and consider the effects of deviating from these values as we study polaron shape and polaron response to applied electric fields.

The uniform system used in our simulations is ideal for studying the effect of model parameters on polaron shape since, as mentioned above, the polaron remains stationary in the absence of an electric field. Figure 2(a) shows the effect of changing the coupling constant $\chi$ under these circumstances. As is evident in the figure, increasing $\chi$ results in greater polaron localization: the polaron occupies fewer lattice sites, with a correspondingly larger charge density and lattice distortion at the center of the polaron. Conversely, decreasing $\chi$ has the opposite effect. Figure 2(b) shows the shape of the polaron at varying values of the charge transfer integral $V$. The effect is opposite to that observed when adjusting $\chi$: increasing (decreasing) $V$ causes a decrease (increase) in polaron localization. This is expected since the larger the charge transfer integral, the more the polaron will spread out to neighboring sites. In accordance with Eq. (2), the value
of $m$ has no effect on the polaron’s shape: in the stationary state the time derivative $m\gamma \frac{dy_n}{dt}$ in Eq. (2) equals zero.

After the polaron is initially created, a constant uniform electric field $E$ is applied and we study the resulting polaron motion in time. Simulations show that there exists a maximum electric field $E_{max}$ that the polaron can tolerate; for $E > E_{max}$, the lattice displacement and charge density become unsynchronized and irregular. The value of $E_{max}$ depends on the system’s parameters and in all cases studied, the more delocalized the polaron the smaller the value of $E_{max}$. As such, increasing $\chi$ or decreasing $V$ leads to a polaron capable of tolerating larger fields, whereas $E_{max}$ is mostly independent of $m$. A highly localized polaron causes a large localized lattice distortion, which effectively acts as a quantum well. By decreasing the energy in its vicinity, a more localized polaron is therefore more stable, and consequently, it is reasonable that the polaron would be able to remain well-formed in the presence of stronger electric fields.

The application of an electric field affects polaron migration (Fig. 1). Moreover, provided that $E < E_{max}$, the polaron remains well-formed and the motion is continuous. Further, in the presence of a field the polaron moves at a constant velocity. This result can be explained as follows: (i) our system represents poly-DNA under periodic boundary conditions, so the system parameters do not vary with location; (ii) the applied electric field is uniform and constant; (iii) the polaron maintains a constant shape as it travels; and (iv) the friction term $m\gamma \frac{dy_n}{dt}$ prevents the charge from continually accelerating. These factors ensure that after a brief period of acceleration from the initial stationary state, the polaron’s velocity does not vary.

The velocity does depend, however, on the system’s parameters and on the applied electric field strength. Figures 3(a), 3(b), and 3(c) show how the velocity of the polaron varies with $E$ for different values of $\chi$, $V$, and $m$ respectively. These graphs all display a similar behavior, but the magnitudes and slopes in the plots are very sensitive to the values of $\chi$, $V$, and $m$. In particular, an increase in the velocity – and therefore conductance – can be achieved by decreasing the coupling constant $\chi$, by increasing the charge transfer integral $V$, or by decreasing the polaron mass $m$. Analysis of both $\chi$ and $V$ suggest that an increase in polaron localization produces a decrease in velocity. It should be noted, however, that this correlation need not be a general mathematical rule; for example, by changing $m$ it is possible to change the polaron’s velocity without affecting its localization at all. Other model parameters, such as the friction constant $\gamma$, also affect the velocity, but do not change the qualitative behavior observed in Fig. 3.

Structural disorder in the poly-DNA can drastically alter the polaron propagation. An irregularity at a base

![Figure 2: Polaron shape in poly-DNA for varying values of (a) $\chi$ and (b) $V$. In (a), $V=0.05$ eV, $m=300$ amu, and $\chi$ is given in units of eV/Å. In (b), $\chi=0.6$ eV/Å, $m=300$ amu, and $V$ is given in units of eV.

![Figure 3: Propagation velocity of a polaron through poly-DNA as a function of applied electric field for varying values of (a) $\chi$, (b) $V$, and (c) $m$. Points corresponding to $E > E_{max}$ are absent as they do not represent accurate values. Unless specified otherwise in the figure, $\chi=0.6$ eV/Å, $V=0.1$ eV, and $m=300$ amu.](image-url)
site \( k \) can produce a quantum barrier or well, which we represent with a non-zero value of \( \epsilon_k \). If \( \epsilon_k > 0 \) (a barrier), the polaron is usually unable to traverse site \( k \) for any \( E < E_{max} \), instead coming to a stop just before the irregularity. Only for very low barriers (\( \epsilon_k \approx 0.01 \) eV) in conjunction with the polaron size and a large electric field (\( E \approx 0.1 \) mV/Å) can the large polaron migrate through the barrier. The situation for quantum wells (\( \epsilon_k < 0 \)) is very different. For very shallow wells (\( \epsilon_k \gtrsim -0.05 \) eV), the polaron completely enters the well for any \( E \), although a large \( E \) can dislodge the polaron if the well is extremely shallow. For deeper wells, the polaron sometimes splits into two, with one polaron halting prior to the well and second entering the well (Fig. 4). In these cases, the initial polaron stops before the irregularity, but then part of it tunnels from the polaron-induced well (\( \chi y_n \)) into the disorder-induced well (\( \epsilon_k \)). It should be emphasized that the above analysis applies only to poly-DNA. In less uniform situations, the system is generally unstable, and the polaron often tunnels through barriers to reach distant wells [15].

In conclusion, the velocity of polaron propagation and hence the DNA conductance is mostly determined by the polaron size in transverse and longitudinal directions. A decrease of \( \chi \) induced by partial delocalization of the charge from the purine base to the whole base pair [14] provides the extension of polaron size in the longitudinal direction and significantly increases the DNA conductance. For example, a decrease of \( \chi \) by 0.2 eV can increase the conductance by \( \sim 6 \) – 10 times depending on the electric field value. The polaron can be destroyed at high electric fields, which then changes the mechanism of charge transfer in the DNA molecule and therefore, will cause a discontinuity at the conductance characteristics. A large polaron has higher velocity but is less tolerant to the electric field. The structural disorders, which form a barrier \( \lesssim 0.05 \) eV (well) on the polaron pathway, depending on energetic conditions and electric field magnitude, can stop (trap) the polaron and cause a fast decrease of DNA conductance resulting in an insulating behavior. The influence of disorder on the DNA conductance decreases with increasing temperature.

Acknowledgments

The work has been supported by the Canada Research Chair Program and the NSERC Discovery Grant.

[1] Electronic mail: tapash@physics.umanitoba.ca
[2] Charge Migration in DNA: Perspectives from Physics, Chemistry, and Biology, edited by T. Chakraborty (Springer, New York, 2007); Long-range charge transfer in DNA, edited by G.B. Schuster (Springer-Verlag, Heidelberg, New York, 2004).
[3] J. Jortner, M. Bixon, T. Langenbacher, and M.E. Michel-Beyerle, Proc. Natl. Acad. Sci. (USA) 95, 12759, (1998).
[4] T.C. Berlin, A.L. Burin, and M.A. Ranter, J. Phys. Chem. A 104, 443 (2000).
[5] K.-H. Yoo, et al., Phys. Rev. Lett. 87 198102 (2001).
[6] K. Taniguchi, and T. Kawai, Physica E 33, 1 (2006).
[7] D. Corah, A. Besryadin, S. deVries, C. Dekker, Nature 403, 635 (2000).
[8] E. Conwell, Top. Curr. Chem. 237, 73 (2004).
[9] P. Maniadis, et al. Phys. Rev. E 72, 021912 (2005).
[10] F.D. Lewis, et al. Angew. Chem. Int. Ed. 45, 7982 (2006).
[11] B. Giese, et al. Nature 412, 318 (2001).
[12] W.P. Su, J.R. Schrieffer, A.J. Heeger, Phys. Rev. B. 22, 2099 (1980).
[13] Y. Ono, and A. Terai, J. Phys. Soc. Jap. 59, 2893 (1990).
[14] J. Berashevich, and T. Chakraborty, Chem. Phys. Lett. (2007) in press.
[15] J. Berashevich, and T. Chakraborty, cond-mat/0709.0954.
[16] K. Senthilkumar et al., J. Am. Chem. Soc. 127, 14894 (2005).
[17] F.D. Lewis, et al. J. Am. Chem. Soc. 122, 12346 (2000).
[18] M. A. O’Neill, et al. Angew. Chem. Int. Ed. 42, 5896 (2003).