Neutrino oscillations in magnetized media and implications for the pulsar velocity puzzle

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After a brief presentation of the general techniques used to determine neutrino potentials in a magnetized medium I will discuss MSW resonant oscillations of active and sterile neutrinos in such environment. Using my results I will reconsider the viability of a solution of the pulsar velocity puzzle based on such a kind of neutrino oscillations.

1 Introduction

The effect of media on the propagation of neutrinos is nowadays a well-established research subject. Among the several relevant implications of this subject for particle physics and astrophysics, perhaps the most remarkable application was the discovery of the possibility to have neutrino resonant oscillations in the matter (MSW) and the study of the possible role that this effect may play to give a solution to the solar-neutrino problem.

Recently it has been proposed that MSW oscillation may help to solve another astrophysical puzzle, namely that of the high velocity of pulsars. Observations show that these velocities range from zero up to 900 km/s with a mean value of 450 ± 50 km/s. Among several models, the possibility that pulsar motion is a consequence of an asymmetric neutrino emission during the supernova (SN) explosion is particularly attractive. In fact, neutrinos carry more than 99% of the SN gravitational binding energy so that even a 1% asymmetry in the neutrino emission could generate the observed pulsar velocities. The strong magnetic field present during a SN explosion may induce such an asymmetry thanks to its interplay with the parity violating weak interaction in which neutrinos are involved. That this effect may actually take place in collapsed stars has been observed by several authors (see e.g.).

In this contribution I will discuss another realization of this idea based on neutrino resonant oscillations.

2 Neutrino potentials in a magnetized medium

The main tool to study neutrino propagation in a medium, like in the vacuum, is the Dirac equation. In the Fourier space this reads

\[(\partial_\mu \gamma^\mu - m_\nu - \Sigma(T, B, \mu_i)) \psi_\nu(k) = 0.\] (1)
Here $\Sigma(T, B, \mu_i)$, where $T$ is the heat-bath temperature, $\mu_i$ the chemical potential of the $i$-th particle species and $B$ the magnetic field strength, is the medium induced self-energy. The vacuum contribution to $\Sigma$ is already hidden in the neutrino physical vacuum mass $m_\nu$. Two kind of Feynman diagrams contribute to $\Sigma(T, B, \mu_i)$. These are the bubble and the tadpole diagrams discussed in ref.\[2]. The fermions running in the internal lines of these diagrams have to be intended as real (not virtual) particles belonging to the heat-bath. At the temperatures and densities generally present in SNs we can safely neglect any thermal population of gauge-bosons, so that $W$ and $Z$ in ours diagrams are the usual virtual ones. Thermal populations of muons and tauons are also negligible, so that only electrons, positrons, protons and neutrons give a relevant contribution to $\Sigma(T, B, \mu_i)$. If we neglect (see below) nucleon polarizations, the ambient magnetic field can affect $\Sigma$ only through its effect on the electron propagator. A very powerful tool to study this effect is given by the finite-temperature electron propagator in the presence of an external magnetic field. This propagator has been derived in refs.\[7] and applied for the first time to neutrino physics in\[6]. As the complete expression of this propagator is quite long I do not report it here. The interested reader can find it in refs.\[6,7]. With such a tool a complete one-loop expression for $\Sigma(T, B, \mu_i)$ can be determined. Inserting such a result into the neutrino dispersion relation

$$\det (\partial_\mu \gamma^\mu - m_\nu - \Sigma(T, B, \mu_i)) = 0 \quad (2)$$

we can then derive the matter induced potential of neutrinos propagating through an electrically neutral plasma

$$V(\nu_e) = \sqrt{2} G_F \left[ -\frac{N_n}{2} + N_e + 2 N_{\nu_e}^L + \sum_{i=e,\mu,\tau} N_{\nu_i}^L - \frac{1}{2} N_e^0 \cos \phi \right] \quad (3)$$

$$V(\nu_{\mu,\tau}) = \sqrt{2} G_F \left[ -\frac{N_n}{2} + \sum_{i=e,\mu,\tau} N_{\nu_i}^L + N_{\nu_{\mu,\tau}}^L + \frac{1}{2} N_e^0 \cos \phi \right]. \quad (4)$$

where $\phi$ is the angle between neutrino wave-vector and the magnetic field vector. In my notation $N_i$ represents the charge density of the $i$-th particle species. $N_e^0$ stands for the electron charge density in the lowest Landau level (LLL). Both quantities $N_e$ and $N_e^0$ are functions of $T, \mu_i$ and $\vec{B}$ and increase almost linearly with $B$ when $eB \gg \mu_e^2, T^2$. Since, due to the double degeneracy of the $n \geq 1$ Landau level, only the LLL contributes to the spin-polarization of the electron-positron gas, it is possible to rewrite the angular dependent terms in (3,4) in terms of a polarization parameter defined by $\lambda \equiv N_e^0/N_e$. 

\[2\]
The reader should keep in mind that nucleon polarizations may also induce angular dependent terms in the neutrino potentials. Although nucleon anomalous magnetic moments are much smaller than the electrons', it is however possible that a strong degeneration of the electron gas suppress electron polarization. If, at same time, the nucleons are non-degenerate it may then happen that nucleon polarization is not negligible. A similar situation may actually be realized in the central regions of hot NSs. However, the effect of nucleons polarization on MSW oscillations is generally subdominant.

To conclude this section we summarise that the effect of the magnetic field on the neutrino propagation is two-folds. Strong magnetic fields may modify the bulk properties of the electron-positron gas inducing an enlargement of the potential to which the $\nu_e$ is submitted. Besides that, the magnetic field induces a polarization of particles having a non-vanishing magnetic moment. The latter effect gives rise to an angular dependence in the neutrino potential.

I finally observe that expressions $V(\nu_e)$ and $V(\nu_{\mu,\tau})$ disagree with those reported in [10] and [11] whereas they are consistent with those given in [8].

3 Resonant oscillations

I will now discuss some implications of the result reported in the previous section for MSW-type neutrino resonant oscillations. The general form of resonance condition for such a kind of oscillations is

$$V(\nu_i) + \frac{m_{\nu_i}}{2E} = V(\nu_j) + \frac{m_{\nu_j}}{2E}.$$  (5)

In the case of $\nu_e - \nu_{\mu,\tau}$ oscillations, substituting (3,4) in (5) we find (see also [8] for an independent derivation of the same result)

$$\frac{\Delta m^2}{2E} \cos 2\theta = \sqrt{2} G_F N_e (1 - \lambda \cos \phi)$$  (6)

where $\Delta m^2 \equiv m_{\nu_{\mu,\tau}}^2 - m_{\nu_e}^2$ and $\theta$ is the vacuum mixing angle. Since in a SN the charge asymmetry of neutrinos is usually negligible, I have ignored here the heath-bath neutrino contribution to the potential.

Resonant MSW oscillations between active and sterile neutrinos can also be studied. As $V(\nu_S) = 0$ we find the resonance conditions to be

$$\frac{\Delta m^2}{2E} \cos 2\theta = \sqrt{2} G_F \left[ N_e \left( 1 + \frac{1}{2} \lambda \cos \phi \right) - \frac{1}{2} N_n \right]$$  (7)

$$\frac{\Delta m^2}{2E} \cos 2\theta = \sqrt{2} G_F \left[ N_e \left( \frac{1}{2} \lambda \cos \phi \right) - \frac{1}{2} N_n \right]$$  (8)

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respectively for $\nu_e \leftrightarrow \nu_S$ and $\nu_{\mu,\tau} \leftrightarrow \nu_S$ oscillations. Here $\Delta m^2 \equiv m_{\nu_S}^2 - m_{\nu_e,\mu,\tau}^2$. Whenever $N_e < \frac{1}{2} N_n$ (typically below the $\tau$-neutrinosphere), we see from (8) that $\nu_e \leftrightarrow \nu_S$ resonant oscillations are only possible if $\Delta m^2 < 0$. On the contrary, if $\Delta m^2 > 0$ only $\bar{\nu}_e \leftrightarrow \bar{\nu}_S$ resonant oscillations are possible. Note that in such a case the asymmetry in the resonant surface would be rotated by $\pi$ with respect to the $\nu_e \leftrightarrow \nu_S$ case. Concerning $\nu_{\mu,\tau} \leftrightarrow \nu_S$ resonant oscillations, they are only possible for a negative value of $\Delta m^2$ or, in the opposite case, only between the respective anti-particles.

4 Consequences for pulsar velocities

The possibility that the angular dependence of the neutrino resonance surface in SNs could be the origin of the observed velocities of pulsars has been first proposed by Kusenko and Segre. To this purpose, Kusenko and Segre started considering $\nu_e - \nu_\tau$ oscillations. Their idea bears on the assumption that the resonance sphere lies between the $\tau$ and $e$-neutrinospheres. In fact, if this is the case the distortion of the resonance sphere would induce a temperature anisotropy of the escaping $\tau$-neutrinos produced by the oscillations, hence a recoil kick of the proto-neutron star. In order to account for the observed pulsar velocities a 1% asymmetry in the escaping neutrino total momentum is required.

Kusenko and Segre computed the asymmetry in the $\tau$-neutrino flux in a weak field limit and assuming the electron to neutron number density ratio to be uniform close to the neutrino-spheres. However, such conditions are generally not fulfilled in a SN environment and a more general treatment is called for. Especially, electron relative abundance decreases steeply close to the neutrinospheres as a consequence of the strong deleptonization taking place in that region during the Kelvin-Helmholtz cooling phase. Furthermore, as a consequence of such an effect $eB$ may become larger or comparable to the electron Fermi momentum squared in this region, so that LL quantization cannot be ignored. Following (12) and by using (9) we determine the asymmetry in the escaping $\tau$-neutrino momentum to be

\[
\frac{\Delta k}{k} \approx \frac{1}{6} \int_0^\pi F_\nu \cos \phi \sin \phi d\phi \approx \frac{2 h_{N_e}}{9 h_T} \lambda
\]

(9)

where $F_\nu$ is the flux of the neutrino produced by the resonant oscillations and, $h_{N_e} \equiv |d \ln N_e/ dr|^{-1}$ and $h_T \equiv |d \ln T/ dr|^{-1}$ are, respectively, the variation scale heights of $T$ and $N_e$ at the resonance mean radius. The result reported in (9) is more general of that reported in (12) as it applies also to those case for which $eB \lesssim T^2 \mu^2$. Between the two neutrinospheres the value of the ratio
\( hN_e \) is typically \( \approx 1 \), the exact value depending on the adopted SN model. We then see from (9) that at least a \( \sim 10\% \) electron polarization is required to achieve the desired anisotropy. This requirement translates (see e.g. ref.\(^8\)) into a minimal value of the dipolar component of the magnetic field strength which lies in the range \( 10^{15} \div 10^{16} \) Gauss. This value is larger of at least one order of-magnitude than that claimed in\(^3\). In order for the resonance surface to lie between the two neutrino-spheres the \( \nu_\tau \) mass has to be of the order of 100 eV. This would be at odd with cosmological constraints unless \( \nu_\tau \) is unstable.

Active-sterile resonant neutrino oscillation may also play a role in accelerating pulsars. As the radius of the sterile-neutrinosphere is zero, in order to produce a recoil kick in this case we just need the resonance surface to be placed within the neutrinosphere of the active neutrino participating in the oscillations. The possible role that \( \bar{\nu}_{\mu,\tau} \leftrightarrow \bar{\nu}_S \) resonant oscillations may have in this context was considered, in\(^1\). As follows from (8), a value of the neutrino squared mass difference \( \Delta m^2 \approx 1^2 \) keV\(^2\) is, in this case, required. \( \nu_\mu \leftrightarrow \nu_S \) oscillations were instead ignored in\(^1\) as the \( V(\epsilon) \) angle dependence was erroneously disregarded in that work. A solution of the pulsar velocity puzzle based on this kind of oscillations is, however, still viable and appealing as it does not enter in conflict with cosmological bounds. In fact, using (7) we can write the equation determining the mean resonance position as

\[
\frac{\Delta m^2}{2E} = \frac{G_F \rho}{\sqrt{2}m_N} (3Y_\epsilon - 1) ,
\]

(10)

where, using electric charge neutrality, \( Y_\epsilon \equiv N_\epsilon/(N_p + N_n) = 1 - Y_n \) and \( m_N \) is the nucleon mass. We see from (9) that resonant oscillation may take place even for very small values of \( \Delta m^2 \) (eventually even compatible with a MSW inspired solution of the solar neutrino problem) provided that \( Y_\epsilon \approx Y_n/1 \approx 1/3 \). Indeed, this condition is expected to be fulfilled for \( \rho \approx 10^{12} \), that is in proximity of the \( e^- \) -neutrinosphere\(^1\). The required magnetic field strength is, also in this case, in the range \( 10^{15} \div 10^{16} \) Gauss.

We conclude that MSW oscillations may provide a very elegant solution to the problem of the origin of pulsar velocities. Although such a mechanism generally requires strong dipolar magnetic fields to be present in the inner core of SNs (even larger of what claimed in previous works), still, present observations does not exclude this intriguing possibility.

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