Cycling-Based Synthesis of Robust Output Estimators for Uncertain LPTV Systems

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Abstract: This paper studies a method of designing robust output estimators for discrete-time linear periodically time-varying (LPTV) systems with uncertainties. The key idea is to use the well-known lifting technique but that called cycling for dealing with LPTV systems in the estimator synthesis. Robustness for uncertainties in the estimation is evaluated with the separator-type robust stability theorem through such cycling-based treatment of systems. An advantage of our cycling-based approach, compared to the lifting-based approach, is that we can easily introduce restrictions on the coefficients of estimators in the synthesis for predetermining the estimator period regardless of the system period.

Key Words: periodic systems, robust estimation, cycling, scaling, LMI.

1. Introduction

The problem of estimating unmeasurable output signals only with available information such as measurable output signals is one of the important problems for practices in systems analysis and control. In particular, since the system models inevitably include modeling errors viewed as uncertainties, output estimation should be achieved even in the presence of uncertainties. In the earlier study [1], the synthesis of robust output estimators based on the idea of integral quadratic constraint (IQC) [2],[3] was discussed for discrete-time uncertain linear time-invariant systems. This paper is concerned with robust output estimator synthesis for discrete-time uncertain linear periodically time-varying (LPTV) systems. Hence, it can be regarded as studying an extension of the earlier study toward the case of periodic systems.

The IQC approach is known to be effective for ensuring robustness with respect to the uncertainties. In the case with linear time-invariant (LTI) uncertainties (and LTI nominal systems), the separator-type robust stability theorem [4] is further known to hold, which gives a necessary and sufficient condition for robust closed-loop stability; the theorem can also be exploited for robust performance analysis, as is the case with IQCs. Since this paper is focusing on linear uncertainties (and linear nominal systems), we make the terms consistent with those in the separator approach.

Although the separator-type robust stability theorem was originally derived for LTI systems as noticed above, it can be readily applied to LPTV systems if we use the lifting technique [5],[6]. Hence, one of the simplest ways of designing robust output estimators for uncertain LPTV systems is to exploit the result in [1] for using the separator-type robust stability theorem in the synthesis through the lifting-based treatment of systems. Nevertheless, this paper considers dealing with LPTV systems through the use of the technique called cycling [5],[6], instead of lifting; we call the cycling-based robustness analysis approach using the separator-type theorem the cycling-based LPTV scaling [7],[8]. The reason why we consider using not lifting but cycling in the synthesis is closely related to the following motivation of this study. Since the estimator with the period same as the target LPTV system generally requires more hardware costs than the time-invariant estimator from the viewpoint of implementation, the use of the latter would be a good option when it achieves sufficiently high performance even for the LPTV system. However, since the coefficient matrices in the lifted representation naturally involve products of the coefficient matrices in the state space representation of the original (i.e., lifting-free) system, the representation is actually incompatible with the time-invariant estimator synthesis; more specifically, the existence of the products makes it difficult to introduce meaningful restrictions on the decision variables in the associated inequality condition so that the lifting-free estimator can be reconstructed from the lifted estimator (which is first directly obtained) as a time-invariant estimator. In the cycling-based treatment, on the other hand, such products do not appear in the cycling-based LTI counterpart. Hence, we can easily introduce such restrictions in the inequality condition. By using the cycling technique, not only time-invariant estimators but also estimators with a predetermined period can be designed regardless of the system period.

Unlike the lifting-based approach, the ordinary use of the cycling technique leads us to the cycled systems with time-invariant coefficients and structured input/output signals, whose details will be reviewed later. Obviously, the systems with and without structural constraints on signals cannot simply be identified with each other even when they have common coefficients. Hence, it is not immediately clear whether we can deal with the above cycled systems as the usual time-invariant systems (i.e., without the constraints). This is an issue unique to our using the lifting technique for dealing with LPTV systems. Since we evaluate the performance of the estimators by using the $l_2$-induced norm, we first show in this paper the equivalence between the $l_2$-induced norm of the cycled system with the structured signals and that of the system obtained by viewing the cycled system as the usual time-invariant system (i.e., without the structural constraints on signals). Based on such
equivalence and the ideas in [1],[7],[8], we develop a method of designing robust output estimators, which allow us to pre-
determine the estimator period regardless of the system period.
Note that the same topic has been already dealt with in our con-
ference paper [9]. The major technical differences between the
present paper and the conference (i.e., brief) version are the ad-
dition of the proofs of key theorems (Theorems 1 and 3) and
the numerical example shown later.

The contents of this paper are as follows. In Section 2, the
problem of designing robust output estimators for LPTV sys-
tems is stated. In Section 3, our cycling-based treatment of
systems is described, and the equivalence associated with the
$l_2$-induced norm is shown. Then, a brief sketch about cycling-
based LPTV scaling is given in Section 4, and a linear matrix
inequality (LMI) condition for robust output estimator syn-
thesis is discussed in Section 5. An idea for designing estimators
with a predetermined period is also discussed, associated with
the introduction of the constraints on the LMI variables. Fi-
ally, a numerical example is provided in Section 6 to illustrate
our synthesis approach.

We use the following notation in this paper. The symbols $\mathbb{R}$
and $\mathbb{N}_0$ denote the set of real numbers and that of non-negative
integers, respectively, while $\mathbb{R}^n$ and $\mathbb{R}^{m \times n}$ denote the set of $n$-
dimensional real column vectors and that of $m \times n$ real matrices,
respectively. The symbol $\mathcal{D}$ denotes the unit circle. We use
diag(·) to denote the (block-)diagonal matrix, and $\|\cdot\|$ is used
to denote the Euclidean norm of the vector (·). We use
$l_2(\mathbb{N}_0,\mathbb{R}^{m_0})$ to denote the set of unilateral infinite sequences of
vectors $u_k \in \mathbb{R}^{m_0}$ such that $\sum_{k=0}^{\infty} \|u_k\|^2 < \infty$. For the matrix $V_1$
and the Hermitian matrix $V_2$ of the compatible size, we use the
simplified notation $[\cdot]^* V_2 V_1$ in referring to $V_1^* V_2 V_1$.

2. Problem of Robust Output Estimator Synthesis for
Uncertain LPTV Systems

In this paper, we deal with the system, shown in Fig. 1,
consisting of the generalized plant $P$, the uncertainty $\Delta$, and
the estimator $E$. The generalized plant $P$ is assumed to be
internally stable, finite-dimensional, linear, periodically time-
varying with period $N$ (i.e., $N$-periodic), and represented by

$$
\begin{bmatrix}
    x_{k+1} \\
    q_k \\
    v_k \\
    y_k
\end{bmatrix} =
\begin{bmatrix}
    A_k & B_{p,k} & B_{w,k} \\
    C_{q,k} & D_{q,k} & D_{q,w,k} \\
    C_{r,k} & D_{r,k} & D_{r,w,k} \\
    C_{y,k} & D_{y,k} & D_{y,w,k}
\end{bmatrix}
\begin{bmatrix}
    x_k \\
    p_k \\
    \bar{v}_k
\end{bmatrix},
\tag{1}
$$

where $k$ denotes the discrete time, $x_k \in \mathbb{R}^n$, $p_k \in \mathbb{R}^{m_0}$, $w_k \in \mathbb{R}^{m_0}$,
$q_k \in \mathbb{R}^{m_0}$, $v_k \in \mathbb{R}^{m_0}$, $y_k \in \mathbb{R}^{m_0}$, and the coefficient
matrices in (1) are $N$-periodic. The uncertainty $\Delta$ is assumed to
belong to some given set $\mathcal{A}$ satisfying the following assumption.

Assumption 1. The uncertainty set $\mathcal{A}$ satisfies the following conditions.

(i) Every $\Delta \in \mathcal{A}$ is a stable finite-dimensional linear $N$-
periodic system with $n_p$ inputs and outputs.

(ii) $\mathcal{A}$ is a connected set such that $0 \in \mathcal{A}$.

(iii) The closed-loop system consisting of $P$ and $\Delta$ is well-posed
and stable for each $\Delta \in \mathcal{A}$ (i.e., robustly with respect to $\Delta$).

The estimator $E$ is introduced to generate the estimate $\hat{v}$ of
the unmeasurable output $v$ of $P$ only with the measurable output $y$;
the estimation error is denoted by $z := v - \hat{v}$.

Let us denote by $G$ the system consisting of $P$ and $E$ (see
Fig. 1). Let us further denote by $G_\Delta$ the system (with input $w$
and output $z$) consisting of $G$ and $\Delta$. Then, for each $\Delta \in \mathcal{A}$, the
$l_2$-induced norm of $G_\Delta$ is defined by

$$
\|G_\Delta\|_2 := \sup_{w \in l_2(\mathbb{N}_0,\mathbb{R}^{m_0})} \frac{\|z\|}{\|w\|_2},
\tag{2}
$$

where $\|w\|_2$ and $\|z\|_2$ respectively denote the $l_2$ norms of $w$ and $z$.

This paper deals with the problem of designing an internally
stable linear periodically time-varying (LPTV) estimator
$E$ such that $\|G_\Delta\|_2 < \gamma$ ($\forall \Delta \in \mathcal{A}$) for given $\gamma > 0$,
where we also consider the case when the estimator is required to be
time-invariant. Note that $G_\Delta$ with internally stable $E$ becomes
robustly stable with respect to $\Delta$ by Assumption 1, and thus
$\|G_\Delta\|_2$ can be defined for every $\Delta \in \mathcal{A}$.

3. Equivalent Synthesis Problem with Cycling-Based
Treatment

In this section, we consider exploiting the cycling technique
[5],[6] for dealing with LPTV systems. In particular, we
show that the synthesis problem for LPTV systems stated in the
preceding section can be tackled as that for linear time-invariant
(LTI) systems through introducing what we call formal cycled
representations, whose details will be stated later.

3.1 Cycling of LPTV Systems

In this subsection, we first review the ordinary use of the
cycling technique [5],[6]. Under the notational restriction that $k$
is an integer multiple of $N$ (this restriction applies only to the
following equation to facilitate its description), the operation of
constructing the new signal

$$
\begin{bmatrix}
    w_k \\
    0 \\
    \vdots \\
    0
\end{bmatrix}
= \begin{bmatrix}
    \bar{w}_k \\
    \bar{w}_{k+1} \\
    \vdots \\
    \bar{w}_{k+N-1}
\end{bmatrix}
\tag{3}
$$

from $w$ (which is an input of the $N$-periodic $P$) is called the
cycling of the signal $w$. Through this operation, we can also
obtain the cycled representations $\bar{p}$, $\bar{q}$, $\bar{y}$, and $\bar{x}$ from $p$, $v$, $q$, and $x$, respectively. This operation induces the conversion of
the system $P$ with the signals $w$, $p$, $v$, $q$, and $x$ into the system
representation (denoted by $P_{eq}$) with the cycled signals $\bar{w}$, $\bar{p}$, $\bar{q},$
\( \hat{y}, \hat{q}, \text{and} \hat{x} \), and we call this conversion the cycling of the system \( P \). In addition, we call this \( \hat{P}_{eq} \) the equivalent cycled representation of \( P \). We can describe the equivalent cycled system \( \hat{P}_{eq} \) by

\[
\begin{bmatrix}
\hat{x}_{k+1} \\
\hat{q}_k \\
\hat{v}_k \\
\hat{y}_k
\end{bmatrix} =
\begin{bmatrix}
\hat{A} & \hat{B}_p & \hat{B}_w \\
\hat{C}_q & \hat{D}_{qp} & \hat{D}_{qw} \\
\hat{C}_v & \hat{D}_{vp} & \hat{D}_{vw} \\
\hat{C}_y & \hat{D}_{yp} & \hat{D}_{yw}
\end{bmatrix}
\begin{bmatrix}
\hat{x}_k \\
\hat{p}_k \\
\hat{w}_k
\end{bmatrix},
\]

(4)

where

\[
\hat{A} =
\begin{bmatrix}
0 & \cdots & 0 & \lambda_{N-1} \\
A_0 & 0 & 0 & 0 \\
0 & \ddots & 0 & \vdots \\
0 & 0 & \lambda_{N-2} & 0
\end{bmatrix},
\]

(5)

\[
\hat{C}_q = \text{diag}(C_{q0}, \ldots, C_{qN-1}),
\]

(6)

and \( \hat{B}_p \) and \( \hat{B}_w \) are constructed respectively with \( B_{pk} \) and \( B_{wk} \) \((k = 0, \ldots, N-1)\) as in (5) while \( \hat{C}_q, \hat{C}_v, \hat{D}_{qp}, \hat{D}_{qw}, \hat{D}_{vp}, \hat{D}_{vw}, \hat{D}_{yp}, \) and \( \hat{D}_{yw} \) are constructed respectively with \( C_{qk}, C_{vk}, D_{qp,k}, D_{qw,k}, D_{vp,k}, D_{vw,k}, D_{yp,k}, \) and \( D_{yw,k} \) \((k = 0, \ldots, N-1)\) as in (6); it is important that all of these coefficient matrices are time-invariant (i.e., the matrix on the right-hand side of (4) does not depend on \( k \); obtaining such a representation is exactly what we aimed at when we apply the cycling technique). We can also obtain the equivalent cycled representations \( \hat{A}_{eq} \) and \( \hat{E}_{eq} \) from \( A \) and \( E \), respectively.

3.2 Robust Performance of Cycled Systems

It follows from the properties of cycling that the system consisting of cycling-free \( P, \Delta \), and \( E \) (i.e., \( G_d \)) is robustly stable with respect to \( A \) if and only if the system consisting of the equivalent cycled representations \( \hat{P}_{eq}, \hat{A}_{eq} \), and \( \hat{E}_{eq} \); a similar equivalence relation also holds for robust \( L_2 \) performance. This suggests that the synthesis problem stated in the preceding section could equivalently reduce to the problem of designing cycled \( \hat{E}_{eq} \) for \( \hat{P}_{eq} \) and \( \hat{A}_{eq} \). That is, \( \hat{E}_{eq} \) should satisfy an appropriate structural constraint so that it is the result of applying the cycling technique to some LPTV \( E \). However, when we are to design a robust estimator \( E \) through such an alternative problem, it would be natural that we are required to ensure all the signals somehow to have the structure shown in (3).

Viewing \( \hat{P}_{eq} \) and \( \hat{A}_{eq} \) simply as ordinary LTI systems so that we can apply the results for the synthesis of robust estimators for LTI systems (with the aforementioned structural constraint taken into account), however, does not ensure this requirement. This would be an obstacle when we consider using theory for LTI systems via the equivalent cycling-based treatment of \( G_d \).

To circumvent this issue, we consider the LTI system described by (4) (with coefficient matrices given as (5) and (6)) without assuming any restrictions on the structures of the signals \( \hat{w}, \hat{p}, \hat{v}, \hat{q}, \hat{y}, \text{and} \hat{x} \). In other words, we consider viewing the same system (4) just as an ordinary LTI system without such structural constraints on the associated signals. We denote such a system by \( \hat{P} \) and call it the formal cycled representation of \( P \). We also consider the formal cycled representations \( \hat{A} \) and \( \hat{E} \) for \( A \) and \( E \), respectively. Then, we can consider the system, shown in Fig. 2, consisting of \( \hat{P}, \hat{A}, \text{and} \hat{E} \). We denote \( G \) the system consisting of \( \hat{P} \) and \( \hat{E} \) and by \( G_d \) the system consisting of \( \hat{G} \) and \( \hat{A} \). We call this \( G_d \) the formal cycled representation of \( G_d \). This formal cycled representation is easier to deal with than the equivalent cycled representation. The issue here, however, is obviously whether dealing with this formal \( G_d \) is indeed helpful for the synthesis of an LPTV robust estimator \( E \).

A very closely related issue has been studied in [7], where robust stability of the formal cycled representation was shown to be equivalent to that of the original (cycling-free) system. In addition, we can show that a similar assertion also holds in our present situation about robust performance and obtain the following theorem, which constitutes a theoretical contribution in this paper.

**Theorem 1.** Suppose that \( A \) satisfies Assumption 1 and \( E \) is internally stable (i.e., \( G_d \) is stable robustly with respect to \( A \)). For each \( \Delta \in A \),

\[
\|G_d\|_2 < \gamma
\]

(7)

if and only if

\[
\|\hat{G}_d\|_2 < \gamma,
\]

(8)

where \( \|\hat{G}_d\|_2 \) is the \( L_2 \)-induced norm of the LTI system \( \hat{G}_d \) with input \( \hat{w} \) and output \( \hat{z} \).

**Proof.** (8) \( \Rightarrow \) (7): For the input \( w \in l_2(N_0, R^{N_0}) \cap \{0\} \) of \( G_d \), let us consider the corresponding cycled signal \( \hat{w} \) as the input of \( G_d \).

Then, the output \( \hat{z} \) of \( G_d \) becomes the cycled representation of the output \( z \) of \( G_d \) corresponding to the \( w \) above. Here, since \( \hat{w} \) has the structure shown in (3), its \( L_2 \) norm coincides with that of the original \( w \); a similar comment also applies to \( z \) and \( \hat{z} \). Since \( \|\hat{w}\|_2/\|\hat{z}\|_2 < \gamma \) by (8), this implies \( \|w\|_2/\|z\|_2 < \gamma \). Since these arguments are independent of \( w \in l_2(N_0, R^{N_0}) \cap \{0\} \), we have (7).

(7) \( \Rightarrow \) (8): Let us consider \( \hat{w} \in l_2(N_0, R^{N_0}) \cap \{0\} \) as the input of \( \hat{G}_d \), and introduce the partition \( \hat{w}_k := [w_{1k}^T, w_{2k}^T, \ldots, w_{Nk}^T]^T \) \((w_{jk} \in R^{N_0}; \ j = 1, \ldots, N)\) for the value of \( \hat{w} \) at \( k \). This \( \hat{w} \) does not have the structure shown in (3), in general. However, we can decompose it as

\[
\hat{w} = \hat{w}^{(0)} + \hat{w}^{(1)} + \cdots + \hat{w}^{(N-1)}
\]

(9)

in such a way that \( \hat{w}^{(i)} \) has the structure of the (standard) cycled representation (i.e., it has exactly the same structure as (3)) when \( i = 0 \), while for \( i = 1, \ldots, N-1 \), it has a modified cycled representation, in which the location of the nonzero subvector in \( \hat{w}_k^{(i)} \) for each \( k \in N_0 \) is a shifted one in the downward direction by \( i \) times (in a cyclic sense) with respect to that in the standard cycled representation for \( i = 0 \). Another equivalent explanation is that \( \hat{w}^{(i)} \) \((i = 1, \ldots, N-1)\) have the structures given by (3) with \( k \) replaced by \( k + i \) \((i = 1, \ldots, N-1)\), and a
more formal definition of this decomposition will be as follows. That is, the value of $\tilde{w}^{(k)}$ at $k$ denoted by

$$\tilde{w}^{(k)} := \begin{bmatrix} w_{1k}^{(0)} & w_{2k}^{(0)} & \cdots & w_{Nk}^{(0)} \end{bmatrix}^T (i = 0, \ldots, N - 1) \tag{10}$$
satisfies

$$w_{jk}^{(i)} = w_{ik}\delta_{ij} \quad (j = 1, \ldots, N), \tag{11}$$

$$l_k := (i + k \mod N) + 1, \tag{12}$$

where $\delta_{ij}$ denotes the Kronecker delta with respect to $l_k$ and $j$ and $(i + k \mod N)$ denotes the least positive remainder in the division of $i + k$ by $N$. In a similar fashion, we also decompose the output $\tilde{z}$ of $\tilde{G}_3$ as

$$\tilde{z} = \tilde{z}^{(0)} + \tilde{z}^{(1)} + \cdots + \tilde{z}^{(N-1)} \tag{13}$$

with the appropriately defined $\tilde{z}^{(i)} (i = 0, \ldots, N - 1)$.

Here, since the structures of the coefficient matrices of $\tilde{F}$, $\tilde{A}$, and $\tilde{E}$ constituting $\tilde{G}_3$ are given as (5) and (6), we see that $\tilde{z}^{(i)}$ is determined only with $\tilde{w}^{(i)}$ for each $i = 0, \ldots, N - 1$ (provided that the initial state of $\tilde{G}_3$ is zero); in particular, if we construct the signals $\tilde{w}^{(i)}$ and $\tilde{z}^{(i)}$ by taking out the nonzero subvectors in the values of $w^{(i)}$ and $\tilde{z}^{(i)}$ for each $k$, respectively, then $\tilde{z}^{(i)}$ is given as the output for the input $w^{(i)} \in I_2(N_0, R^{n_w})$ (under the zero initial condition) for $G_3$ with time $i$ regarded as the associated initial time. Since the $L_2$-induced norm for LPTV systems does not depend on the way to determine the initial time, it follows from (7) that $\|\tilde{z}^{(i)}\|_2 < \gamma \|w^{(i)}\|_2$ (unless $w^{(i)} = 0$, in which case $\|\tilde{z}^{(i)}\| = 0$). By noting from the definition of the $L_2$ norm that

$$\|\tilde{w}\|_2^2 = \sum_{i=0}^{N-1} \|w^{(i)}\|_2^2 = \sum_{i=0}^{N-1} \|\tilde{w}^{(i)}\|_2^2, \tag{14}$$

$$\|\tilde{z}\|_2^2 = \sum_{i=0}^{N-1} \|\tilde{z}^{(i)}\|_2^2 = \sum_{i=0}^{N-1} \|	ilde{z}^{(i)}\|_2^2, \tag{15}$$

we readily see that $\|\tilde{w}\|_2 < \gamma \|\tilde{z}\|_2$ whenever $\tilde{w} \in I_2(N_0, R^{n_w} \setminus \{0\})$. This completes the proof.

This theorem, together with the equivalence relation about robust stability shown in [7], guarantees that the problem of designing a robust output estimator $E$ for the LPTV system shown in Fig. 1 can equivalently reduce to that of designing the formal cycled counterpart $\tilde{E}$ for the LTI system shown in Fig. 2. Hence, we could exploit the conventional results about LTI systems in the synthesis problem stated in Section 2 through this treatment, if we put aside the remaining issue that we have to restrict the structure of $\tilde{E}$ appropriately in the cycling-based synthesis (so that the implementable estimator $E$ can indeed be reconstructed from $\tilde{E}$). In the following section, as a step toward such synthesis, we discuss the robust $L_2$ performance condition with the matrix called a separator, which will be a basis for using the result in [1] about estimator synthesis for LTI systems.

4. Cycling-Based LPTV ($D, G$)-Scaling

The role of this section is to build a bridge between the present robust estimator synthesis problem and the result in [1], by giving a brief sketch of the robustness analysis approach called cycling-based LPTV scaling.

$\tilde{G}$ is an LTI system, we can define its transfer matrix and denote it by $\tilde{G}(\tilde{z})$, where $\tilde{z}$ denotes the variable for $z$-transforms. Corresponding to the input $[\tilde{p}^T, \tilde{w}^T]^T$ and the output $[\tilde{q}^T, \tilde{z}^T]^T$ of $\tilde{G}$, we partition $\tilde{G}(\tilde{z})$ as

$$\tilde{G}(\tilde{z}) = \begin{bmatrix} \tilde{G}_{pp}(\tilde{z}) & \tilde{G}_{pw}(\tilde{z}) \\ \tilde{G}_{zp}(\tilde{z}) & \tilde{G}_{zz}(\tilde{z}) \end{bmatrix}. \tag{16}$$

In addition, we also define the transfer matrix $\tilde{H}(\tilde{z})$ of $\tilde{A}$. Then, by exploiting the result in [4] about LTI systems through the cycling-based treatment with Theorem 1, we obtain the following cycling-based separator-type theorem² for robust performance analysis.

**Theorem 2.** Suppose that $A$ satisfies Assumption 1 and $E$ is internally stable. For $\gamma > 0$, if there exists $\tilde{\theta}(\tilde{z}) = \tilde{\theta}(\tilde{z})^* (\zeta \in \partial D)$ such that

$$[\star] \begin{bmatrix} \tilde{\theta}(\tilde{z})^* : 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I & 0 \\ \tilde{G}_{pp}(\tilde{z}) & \tilde{G}_{pw}(\tilde{z}) \\ \tilde{G}_{zp}(\tilde{z}) & \tilde{G}_{zz}(\tilde{z}) \end{bmatrix} < 0 \tag{17}$$

$$(\forall \zeta \in \partial D) \quad [\star] \begin{bmatrix} \tilde{\theta}(\tilde{z})^* \tilde{H}(\tilde{z})^* \end{bmatrix} > 0 \quad (\forall A \in A) \tag{18}$$

then $G_A$ is robustly stable with respect to $A$ and $\|G_A\|_2 < \gamma (\forall A \in A)$.

The matrix $\tilde{\theta}(\tilde{z})$ in the above theorem is called a cycling-based separator. By searching for $\tilde{\theta}(\tilde{z})$ satisfying (17) and (18), we can analyze robust $L_2$ performance of $G_A$; we call such a robustness analysis approach cycling-based LPTV scaling [7],[8]. Since it is numerically difficult to deal with a general class of separators $\tilde{\theta}(\tilde{z})$, we consider confining the class to a numerically tractable one in the following (in exchange for possible conservativeness in the corresponding robustness analysis); associated with this, we restrict our attention to the uncertainty set $A$ such that the $L_2$-induced norm condition $\|A\|_2 < \tilde{\delta}$ ($\forall A \in A$) holds for given $\tilde{\delta} > 0$.

Let us first consider the class of separators

$$\tilde{\theta}(\tilde{z}) = \begin{bmatrix} T_{N_0}(\tilde{z})^* \Gamma_{ij} T_{N_0}(\tilde{z}) \end{bmatrix}_{i,j=1,2} \tag{19}$$

with the fixed proper stable transfer matrix $T_{N_0}(\tilde{z})$ with $N_0$ inputs and the real symmetric matrix $\Gamma = \begin{bmatrix} \Gamma_{ij} \end{bmatrix}_{i,j=1,2} \begin{bmatrix} \Gamma_{i1} & \Gamma_{i2} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix}$ to be searched for. By defining $V_{1}(\tilde{z}) := \begin{bmatrix} T_{N_0}(\tilde{z})^* \end{bmatrix}_0^0$ and $V_{2}(\tilde{z}) := \begin{bmatrix} 0, T_{N_0}(\tilde{z})^* \end{bmatrix}$, we can rewrite the above separators as

$$\tilde{\theta}(\tilde{z}) = \begin{bmatrix} V_{1}(\tilde{z}) V_{2}(\tilde{z})^* \end{bmatrix}^* \begin{bmatrix} T_{N_0}(\tilde{z}) V_{2}(\tilde{z}) \end{bmatrix}. \tag{20}$$

Then, we further confine the above class by introducing the constraint that $\Gamma$ must satisfy

$$1 \text{ In Theorem 2, robust stability of the closed-loop system consisting of } P \text{ and } A \text{ is not necessarily required to be assumed (as in Assumption 1), since it can also be checked through (17) and (18). Neverthless, we employ the assumption because tackling robust performance analysis for } G_A \text{ becomes meaningless when the assumption is not satisfied. A similar comment also applies to Theorem 3 about estimator synthesis given later.}$$
\[ \Gamma_{21} = \Gamma_{12}^T, \quad \Gamma_{11} = -(1/\delta) \Gamma_{22}, \quad (21) \]
\[ V_2(\zeta)'TV_2(\zeta) > 0 \quad (\forall \zeta \in \partial D), \quad (22) \]

This restriction enables \( \tilde{\Theta}(\zeta) \) in (20) to have the \((D,G)\)-scaling [10] type structure
\[
\tilde{\Theta}(\zeta) = \begin{bmatrix} (1/\delta)^2 \tilde{\Theta}_d(\zeta) & \tilde{\Theta}_d(\zeta) \\ \tilde{\Theta}_d(\zeta)' & \tilde{\Theta}_d(\zeta) \end{bmatrix}, \quad (23)
\]
\[
\tilde{\Theta}_d(\zeta) > 0 \quad (\forall \zeta \in \partial D), \quad (24)
\]

where \( \tilde{\Theta}_d(\zeta) = T_{N_n}(\zeta)'T_{22}T_{N_n}(\zeta) \) and \( \tilde{\Theta}_d(\zeta) = T_{N_n}(\zeta)'T_{12}T_{N_n}(\zeta) \). The \((D,G)\)-scaling type separator is known to reduce (18) to the simple \( H_{\infty} \) norm condition \( \| \hat{M}(\zeta) \|_{\infty} < \delta \) (\( \forall A \in \Delta \)) if it further satisfies
\[
\tilde{\Theta}_d(\zeta)' \hat{\Theta}(\zeta) \tilde{\Theta}_d(\zeta) + \hat{\Theta}(\zeta)' \tilde{\Theta}_d(\zeta) = 0 \quad (25)
\]
\[
(\forall \zeta \in \partial D) \quad (26)
\]

for every \( \Delta \in \Delta \) and \( \zeta \in \partial D \); noting that \( \| \hat{\Theta}(\zeta) \|_{\infty} = \| \hat{\Theta} \|_b = \| \hat{A} \|_b \), this implies that (18) is automatically satisfied under the norm-bounded uncertainty set \( \Delta \). In this paper, we confine \( \Gamma \) to some appropriate class \( \Gamma_{\infty} \) such that the corresponding separators immediately satisfy all the above restrictions except (22) (i.e., (24)).

In this section, we gave a brief sketch of cycling-based LPTV \((D,G)\)-scaling. By searching for real symmetric \( \Gamma \in \Gamma_{\infty} \) satisfying (17) and (22), we can analyze robust \( l_2 \) performance of \( G_d \) with respect to the norm-bounded uncertainty set \( \Delta \) (because (18) is automatically satisfied). In the following section, we discuss robust estimator synthesis based on this scaling approach.

5. Robust Estimator Synthesis Based on Cycling-Based LPTV Scaling

In this section, we show a linear matrix inequality (LMI) condition for designing robust output estimators by exploiting the result in [1] about LTI systems. In particular, we consider restricting the classes of some of the decision variables in the LMI so that an implementable periodic estimator \( E \) can naturally be reconstructed from the \( \hat{\Theta}(\zeta) \). In addition, we provide an idea for designing estimators having any prescribed period (including time-invariant estimators as a special case) regardless of the period \( N \) of the systems \( P \) and \( \Delta \). Designing such estimators is closely related to our motivation of exploiting the cycling technique for dealing with LPTV systems.

5.1 LMI-Based Synthesis

In this subsection, we show an LMI condition for designing \( N \)-periodic \( E \) achieving minimal \( \gamma > 0 \) such that the \( l_2 \)-induced norm of the corresponding \( G_d \) is less than \( \gamma \) robustly with respect to \( \Delta \).

We represent the state space realizations of \( V_1(\zeta) \) and \( V_2(\zeta) \) in (20) by
\[
V_1(\zeta) = \begin{bmatrix} A_{V_1} & B_{V_1} \\ C_{V_1} & D_{V_1} \end{bmatrix}, \quad V_2(\zeta) = \begin{bmatrix} A_{V_2} & B_{V_2} \\ C_{V_2} & D_{V_2} \end{bmatrix}. \quad (27)
\]

Then, the realization of \([V_1(\zeta), V_2(\zeta)]\) can be written as
\[
[V_1(\zeta), V_2(\zeta)] = \begin{bmatrix} A_{V_1} & 0 & B_{V_1} & 0 \\ 0 & A_{V_2} & 0 & B_{V_2} \\ C_{V_1} & C_{V_2} & D_{V_1} & D_{V_2} \end{bmatrix}. \quad (28)
\]

We also represent the realization of \( \tilde{E} \) by
\[
\tilde{E} := \begin{bmatrix} \bar{A}_E & B_E \\ \bar{C}_E & D_E \end{bmatrix}. \quad (29)
\]

In addition, we define the constant matrices
\[
A_c := \begin{bmatrix} A_{c_1} \\ A_{c_2} \end{bmatrix}, \quad A_{c_3} := \begin{bmatrix} A_{c_1} & 0 \\ 0 & 0 \end{bmatrix}, \quad A_{c_4} := \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad (30)
\]

where this notation is introduced so that it becomes consistent in principle with that in [1]. Then, we can show the following theorem.

Theorem 3. Suppose that \( \Delta \) satisfies Assumption 1 and every \( \Delta \in \Delta \) satisfies \( \| A \|_b < \delta \). If there exist \( \Gamma \in \Gamma_{\infty} \), \( X_1 = X_1^T > 0 \), \( Y = Y^T > 0 \), \( \bar{A}_E \), \( B_E \), \( \bar{C}_E \), and \( D_E \) such that

\[
X - Y > 0, \quad (31)
\]
\[
\begin{bmatrix} *' \end{bmatrix}^{'} = \begin{bmatrix} -X_1 & 0 & 0 \\ 0 & X_1 & 0 \\ 0 & 0 & \Gamma \end{bmatrix} > 0, \quad (32)
\]
\[
\begin{bmatrix} *[\star]' \end{bmatrix}^{'} = \begin{bmatrix} -X_1 & 0 & 0 \\ 0 & X_1 & 0 \\ 0 & 0 & \Gamma \end{bmatrix} > 0, \quad (33)
\]

\[
\begin{bmatrix} \bar{A}_E & B_E \\ \bar{C}_E & D_E \end{bmatrix} < 0, \quad (34)
\]

\[
Y^* := \begin{bmatrix} Y & Y \\ Y & X \end{bmatrix}, \quad (35)
\]

\[
X_{v}\bar{e} := \begin{bmatrix} X_{v}\bar{e} \\ X_{v}\bar{e} \end{bmatrix}, \quad (36)
\]

\[
\begin{bmatrix} A_1 & A_2 \\ B_1 & B_2 \end{bmatrix} := \begin{bmatrix} A_{c_1} & A_{c_2} \\ B_{c_1} & B_{c_2} \end{bmatrix}, \quad (37)
\]

\[
\begin{bmatrix} A_1 & A_2 \\ \bar{A}_E & B_E \end{bmatrix} := \begin{bmatrix} \bar{A}_E & \bar{B}_E \bar{C}_E - \bar{A}_E \bar{X}_c & \bar{B}_E \bar{C}_E \end{bmatrix}, \quad (38)
\]
\[
B_1^* := B_{23}, \quad B_2^* := \begin{bmatrix} \frac{Y}{x} \end{bmatrix}, \quad C_1^* := \begin{bmatrix} C_{11} & -D_{32} & C_{12} & -D_{31} \end{bmatrix}, \quad C_2^* := \begin{bmatrix} C_{21} \end{bmatrix}, \quad C_3^* := \begin{bmatrix} -D_{21} \end{bmatrix}, \quad C_4^* := \begin{bmatrix} C_{41} \end{bmatrix},
\]

then there exists a stable estimator \( \hat{E} \) such that the \( \ell_2 \)-induced norm of the corresponding \( \mathcal{G}_2 \) is less than \( \gamma \) robustly with respect to \( \Delta \), where the partition of \( X_{12} \) conforms with the partition of \( A_C \). In particular, \( \hat{E} \) with the coefficient matrices given by

\[
\begin{bmatrix}
\tilde{A}_E & \tilde{B}_E \\
\tilde{C}_E & \tilde{D}_E
\end{bmatrix} = \begin{bmatrix} X_2 & 0 \\ 0 & I \end{bmatrix}^{-1} \begin{bmatrix} \tilde{A}_E^* & \tilde{B}_E^* \\ \tilde{C}_E^* & \tilde{D}_E^* \end{bmatrix} \begin{bmatrix} X_3^{-1} X_2^* & 0 \\ 0 & I \end{bmatrix}^{-1}
\]

is one such estimator, where \( X_2 \) and \( X_3 \) are matrices satisfying \( X_2 X_3^* X_2^* = X - Y \).

**Proof.** In almost the same fashion as the proof of Theorem 3 in the earlier study [1], we can show that the \( \Gamma \) and \( \hat{E} \) obtained through the LMI also satisfy (17) and (22). Hence, we here summarize only the differences between the proof in the earlier study and that for the present theorem:

- Since we have not introduced the integral quadratic constraint for the disturbance \( w \) in the third term of the left-hand side of (3) in the earlier study is irrelevant and thus should be dropped (see the present (17)); this implies that the associated matrices (such as \( C_{12} \)) need not be introduced in the present (30).

- In this paper, we defined the input and output vectors of \( \hat{P} \) and \( \hat{G} \) as in (4) and (16). Since this definition, which we believe to be natural in terms of the system structure in Fig. 2, is not consistent with that in the earlier study, we have to consider the appropriate input/output permutation for the systems (and the associated matrices) before using the result in the study.

- In this paper, we took diag(1, \( \gamma I \)) as the \( \Theta \) appearing in the earlier study. Since this implies \( \Theta_{12} = 0 \), we can omit the corresponding trivial rows and columns associated with \( C_1^* \) and \( D_1^* \) (corresponding to \( C_1 \) and \( D_1 \) in the earlier study) in deriving the present (33); the remaining rows and columns associated with \( C_1^* \) and \( D_1^* \) in the present (33) correspond to those about \( \Theta_{11}/2 \) \( C_2 \) and \( \Theta_{11}/2 \) \( D_3 \) in (20) of the earlier study under \( \Theta_{11} = I \).

The above arguments, together with Theorem 2, complete the proof. \( \square \)

Inequality (33) is linear in the decision variables and \( \gamma^2 \). Hence, through the LMI optimization with respect to \( \gamma^2 \), we can obtain an estimator \( \hat{E} \) achieving minimal \( \gamma \). However, if we use the above theorem in a straightforward fashion, the resulting coefficient matrices (35) of \( \hat{E} \) are not ensured to have the structure described by (5) and (6). This implies that \( E \) cannot be reconstructed from \( \hat{E} \) as an implementable periodic estimator, in general. To circumvent this issue, we consider restricting the classes of \( X, Y, \tilde{A}_E^*, \tilde{B}_E^*, \tilde{C}_E^*, \) and \( \tilde{D}_E^* \) as follows:

\[
X - Y \in X^{\alpha \gamma}, \quad \tilde{A}_E^* \in X^{\alpha \gamma}_{\text{shifted}}, \quad \tilde{B}_E^* \in X^{\alpha \gamma}_{\text{shifted}}, \quad \tilde{C}_E^* \in X^{\alpha \gamma}, \quad \tilde{D}_E^* \in X^{\alpha \gamma}_{\text{shifted}}.
\]

with

\[
X^{\alpha \gamma} := \left\{ \begin{bmatrix} \text{diag}(X_0, \ldots, X_{N-1}) X_0, \ldots, X_{N-1} \in \mathbb{R}^{\alpha \gamma} \right\},
\]

\[
X^{\alpha \gamma}_{\text{shifted}} := \left\{ \begin{bmatrix} 0 \\ \text{diag}(X_1, \ldots, X_{N-1}) X_0, \ldots, X_{N-1} \in \mathbb{R}^{\alpha \gamma} \right\},
\]

This structure coincides with that of the cycled representation described by (5) and (6), and thus the above \( \hat{E} \) can be seen as the (formal) cycled representation of the \( N \)-periodic estimator \( E \), which is implementable.

### 5.2 Synthesis of Robust Estimators Having a Predetermined Period

We next provide an idea for designing estimators having a period predetermined regardless of the period of \( P \) and \( \Delta \). Let us consider the situation where the period of \( E \) is required to be \( M \). Then, we can take positive integers \( M_0 \) and \( N_0 \) such that \( M = v M_0 \) and \( N = v N_0 \) with the greatest common factor \( v \) of \( M \) and \( N \). With such \( M_0, N_0, \) and \( v \), we can represent the least common multiple of \( M \) and \( N \) by \( v M_0 N_0 = M_0 N = MN_0 \). Noting that \( N \)-periodic systems can be seen as a special case of \( M \)-periodic systems, we can apply the cycling operation to \( P \) and \( \Delta \) under the period \( M_0 N \). Hence, by using Theorem 3 under this treatment of systems, it is immediate for us to design estimators if they are allowed to have period \( M N_0 \). Our basic strategy is to introduce some constraints in such an approach so that the resulting estimators become periodic with period \( M \), too.

To this end, let us introduce the matrix classes

\[
Y^{\alpha \gamma} := \begin{bmatrix} I_{N_0} \otimes \mathcal{F} \end{bmatrix}, \quad Y^{\alpha \gamma} := \begin{bmatrix} \text{diag}(Y_0, \ldots, Y_{M-1}) \end{bmatrix}, \quad Y^{\alpha \gamma} := \begin{bmatrix} \text{diag}(Y_0, \ldots, Y_{M-1}) \end{bmatrix}, \quad Y^{\alpha \gamma}_{\text{shifted}} := \begin{bmatrix} \text{diag}(Y_0, \ldots, Y_{M-1}) \end{bmatrix}, \quad Y^{\alpha \gamma}_{\text{shifted}} := \begin{bmatrix} \text{diag}(Y_0, \ldots, Y_{M-1}) \end{bmatrix},
\]

where \( \otimes \) denotes the Kronecker product. Then, through replacing \( X^{\alpha \gamma} \) and \( X^{\alpha \gamma}_{\text{shifted}} \) respectively by \( Y^{\alpha \gamma} \) and \( Y^{\alpha \gamma}_{\text{shifted}} \) in the arguments in the preceding subsection, we are led to \( \hat{E} \) with coefficient matrices satisfying

\[
\tilde{A}_E, \tilde{B}_E, \tilde{C}_E, \tilde{D}_E \in Y^{\alpha \gamma}_{\text{shifted}}, \tilde{B}_E, \tilde{C}_E, \tilde{D}_E \in Y^{\alpha \gamma}_{\text{shifted}}.
\]
right-hand side of (40) and (41). Because of this, we can see that the matrices $\tilde{A}_k$, $\tilde{B}_k$, $\tilde{C}_k$, and $\tilde{D}_k$ satisfying (42) have the structures corresponding to the cycled representation of an $M$-periodic system viewed as an $MN_0$ periodic system. Hence, an $M$-periodic estimator $\tilde{E}$ can be reconstructed from $\tilde{E}$ designed through the restricted classes (40) and (41).

Note that the above discussions are independent of the positive integer $M$. Hence, by taking $M = 1$, we can design time-invariant estimators for $N$-periodic $P$ and $A$. Since it is difficult to achieve this kind of synthesis in the case of using the lifting technique [5],[6] for dealing with periodic systems, this clearly shows the importance of our using cycling for the synthesis.

6. Numerical Example

In this section, we provide a numerical example for illustrating our estimator synthesis. Let us consider the 2-periodic internally stable generalized plant $P$ given by (1) with the coefficient matrices

$$A_0 = \begin{bmatrix} 0 & 1 \\ -0.9 & 0.2 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 1 \\ -0.4 & -0.2 \end{bmatrix},$$

$$B_{p,1} = [1, 1]^T, \quad B_{p,2} = [1, 1]^T,$$

$$C_{q,0} = [0.6, 0.8], \quad C_{q,1} = [1, 0.8], \quad C_{v,0} = [0.7, 0.6],$$

$$C_{v,1} = [0.4, 0.3], \quad C_{y,0} = [0.9, 0.4], \quad C_{y,1} = [0.6, 0.4],$$

$$D_{q,p,1} = 0, \quad D_{q,p,2} = 0, \quad D_{y,p,1} = 0, \quad D_{y,p,2} = 0,$$

$$D_{w,p,1} = 0, \quad D_{w,p,2} = 0 \quad (i = 1, 2)$$

and the uncertainty set $A = \{A : \|A\|_2 < \delta (\delta = 0.5), A_0 = A_1 = \delta \in \mathbb{R}\}$ (i.e., norm-bounded static LTI scalar uncertainties), which satisfies Assumption 1. For the system consisting of these $P$ and $A \in A$, we design 2-periodic and time-invariant estimators through our synthesis approach for robust output estimation.

Let us first consider the case of designing a 2-periodic estimator. Such synthesis can be tackled through Theorem 3 with constraint (36). As the separator class for this synthesis, we consider the finite impulse response (FIR) separators [8] with order $K$ given by (19) with

$$T_{N_0} = [\zeta^{-2}I_{N_0}, \zeta^{-1}I_{N_0}, I_{N_0}]^T,$$

$$\Gamma_{\delta} = \begin{bmatrix} \Gamma_1 & \Gamma_{11} \\ \Gamma_2 & \Gamma_{21} & \Gamma_{22} \end{bmatrix}, \quad \Gamma_{11} = \Gamma_{11}^T, \quad \Gamma_{21} = \Gamma_{21}^T, \quad \Gamma_{22} = \Gamma_{22}^T,$$

$$\Gamma_{12} + \Gamma_{21}^T = 0; \quad \Gamma_{12}, \Gamma_{22} \in \mathbb{R}^{2N_0 \times 2N_0},$$

where $N = 2$, $n_p = 1$, and $\bar{\delta} = 0.5$ in the present case; we can confirm that this class of separators satisfy the $(D, G)$-scaling type constraint stated in Section 4 (except (22)) to be satisfied through (32) in Theorem 3) since $A(x)$ in (25) and (26) becomes a real scalar matrix. Under this treatment of separators, we solved the LMI in Theorem 3 (with constraint (36)) with MATLAB, YALMIP [11] and MOSEK [12] on a laptop equipped with 8.00 GB RAM and Intel(R) Core(TM) i7-5600U CPU @ 2.60 GHz (this environment was also used for the time-invariant estimator synthesis discussed later). Then, we obtained 0.0726 as the minimal $\gamma$ satisfying the LMI, as well as the solution leading to a 2-periodic $E$, in 1.63 s. To confirm that such designed $E$ works properly as an output estimator, we computed the time responses of $\nu$ and $\nu$ with the disturbance input $w$ shown in Fig. 3 under the zero initial states. Then, for $A = \delta = -0.49$, we obtained the result shown in Fig. 4, where the solid line and the dotted line show the time responses of $\nu$ and $\nu$, respectively. We can see that $\nu$ gives an almost precise estimate of $\nu$ (similar results were obtained for other $A \in A$).

Let us next consider the case of designing a time-invariant estimator for the 2-periodic system. Such synthesis can be tackled through Theorem 3 with the idea provided in Section 5.2; in the present case, we take $v = M_0 = 1$ and $N_0 = 2$ for the classes shown in (40) and (41). For this synthesis, we consider the same class of FIR separators as in the above 2-periodic estimator case. Then, we obtained 2.1934 as the minimal $\gamma$ satisfying the LMI, as well as the solution leading to a time-invariant $E$, in 1.61 s. To confirm that such designed $E$ works properly as an output estimator, we computed the responses of $\nu$ and $\nu$ with the disturbance input $w$ shown in Fig. 3 under the zero initial states, as is the case with the 2-periodic estimator synthesis. Then, for $A = \delta = -0.49$, we obtained the result shown in Fig. 5. Compared to the result in Fig. 4, the performance achieved by the time-invariant estimator synthesis seems to be worse at least with the above $A$ in this example. However, this is not surprising because the class of time-invariant estimators is more restricted than that of 2-periodic estimators. Since the use of periodic estimators generally requires more hardware costs than that of time-invariant estimators, the latter would be a good option when we tackle problems such that the above deterioration of performance is acceptable.

Remark 1. In the above two cases of synthesis, we restrict some of the LMI variables as in (36) to circumvent the situation
where the resulting estimator does not become implementable. This treatment of LMI variables, however, might lead to inducing conservativeness in the evaluation of the robust $l_2$ performance in the synthesis, since $X$ and $Y$ in (36) correspond to the Lyapunov matrix and thus are related to the evaluation. Hence, if it is required to achieve more strict evaluation of robust $l_2$ performance that the designed $E$ can achieve, it is better to analyze the performance again without the above restrictions after the synthesis; such post-synthesis analysis will also enable us to achieve a fair comparison of the designed estimators (a different estimator period leads to a different restriction on the LMI variables and thus to a different evaluation of the performance).

For reference, we obtained 0.0566 in the 2-periodic estimator case and 2.0944 in the time-invariant estimator case as an upper bound of the minimal $\gamma$ satisfying $\|G_d\|_\infty < \gamma$ ($M \in \Delta$), through such post-synthesis analysis based on cycling-based LPTV (D,G)-scaling using FIR separators with order $K = 2$: if we increase this $K$, we can obtain further less conservative analysis results.

Remark 2. The estimator period, in practice, will be predetermined in accordance not only with the required performance but with other various external factors. In such general cases, the following fact would be helpful in predetermining the estimator period: it can be theoretically ensured that the $M_1$-periodic estimator designed with our cycling-based synthesis approach can achieve at least no worse performance than that with period $M_2$ (regardless of the plant period $N$) when $M_1$ is an integer multiple of $M_2$.

7. Conclusion

In this paper, we discussed an approach of robust estimator synthesis based on cycling-based LPTV scaling that can be applied to LPTV systems with uncertainties. A noteworthy property of our synthesis approach is that we can predetermine the estimator period regardless of the system period. The key idea for such synthesis is to appropriately restrict the LMI variables leading to the estimator in the synthesis under the cycling-based treatment of systems with the cycling period determined not only by the system period but also by the predetermined estimator period (recall the arguments in Section 5.2). Since this idea itself is independent of the present estimator synthesis problem, the idea is expected to have the application potentiality also for other synthesis problems with LPTV systems.

References

[1] C.-Y. Kao and M.-C. Chen: Robust estimation with dynamic integral quadratic constraints: The discrete-time case, IET Control Theory & Applications, Vol. 7, No. 12, pp. 1599–1608, 2013.
[2] A. Megretski and A. Rantzer: System analysis via integral quadratic constraints, IEEE Transactions on Automatic Control, Vol. 42, No. 6, pp. 819–830, 1997.
[3] C.W. Scherer and I.E. Köse: Robustness with dynamic IQCs: An exact state-space characterization of nominal stability with applications to robust estimation, Automatica, Vol. 44, No. 7, pp. 1666–1675, 2008.
[4] T. Iwasaki and S. Hara: Well-posedness of feedback systems: Insights into exact robustness analysis and approximate computations, IEEE Transactions on Automatic Control, Vol. 43, No. 5, pp. 619–630, 1998.
[5] S. Bittanti and P. Colaneri: Invariant representations of discrete-time periodic systems, Automatica, Vol. 36, No. 12, pp. 1777–1793, 2000.
[6] S. Bittanti and P. Colaneri: Periodic Systems: Filtering and Control, Springer-Verlag, 2009.
[7] M. Miyamoto, Y. Hosoe, and T. Hagiwara: Robust stability analysis with cycling-based LPTV scaling: Part I. Fundamental results on its relationship with lifting-based LPTV scaling, International Journal of Control, Vol. 90, No. 7, pp. 1345–1357, 2017.
[8] M. Miyamoto, Y. Hosoe, and T. Hagiwara: Robust output estimator synthesis based on cycling-based LPTV scaling, Proc. 20th IFAC World Congress, pp. 7839–7844, 2017.
[9] Y. Hosoe, M. Miyamoto, and T. Hagiwara: Robust output estimator synthesis based on cycling-based LPTV scaling, Part II. Its properties under the use of FIR separators, International Journal of Control, Vol. 90, No. 7, pp. 1358–1370, 2017.
[10] M.K.H. Fan, A.L. Tits, and J.C. Doyle: Robustness in the presence of mixed parametric uncertainty and unmodeled dynamics, IEEE Transactions on Automatic Control, Vol. 36, No. 1, pp. 25–38, 1991.
[11] J. Löfberg: YALMIP: A toolbox for modeling and optimization in MATLAB, Proc. 2004 IEEE International Symposium on Computer Aided Control Systems Design, pp. 284–289, 2004.
[12] MOSEK ApS: The MOSEK Optimization Toolbox for MATLAB Manual, Version 7.1 (Revision 28), http://docs.mosek.com/7.1/toolbox/index.html, 2015.

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