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Modelling the impact of health care providers in transmission dynamics of COVID-19  
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**ABSTRACT**

In this paper, a mathematical model is proposed and analysed to assess the impacts of health care providers in transmission dynamics of COVID-19. The stability theory of differential equations is used to examine a mathematical model. The results of both local and global stability of disease-free equilibrium points were determined by using Routh–Hurwitz criteria and Metzler matrix method which verified that was locally and globally asymptotically stable. Also, the endemic equilibrium point was determined by the Lyapunov function which showed that $E_*$ was globally asymptotically stable under strict conditions. The findings revealed that non-diagnosed and undetected health care providers seems to contribute to high spread of COVID-19 in a community. Also, it illustrates that an increase in the number of non-diagnostic testing rates of health care providers may result in high infection rates in the community and contaminations of hospitals’ equipment. Therefore, the particular study recommend that there is a necessity of applying early diagnostic testing to curtail the COVID-19 transmission in the health care providers’ community and reduce contaminations of hospital’s equipment.

**Introduction**

The ongoing Corona virus disease-2019 (COVID-19) was reported in the mid-December-2019 in Wuhan City at Hubei Province, China causing a severe acute respiratory syndrome corona virus- 2 (SARS-COV-2), where the first infected patient was identified after exposure into the seafood market [1,2]. The existence of this pandemic disease in the world affected many people without regarding their economic and development status.

The World Health Organisation in its 6th September, 2021 report showed that more than 220 563 227 were confirmed having COVID-19 infections worldwide and from which more than 4 565 483 confirmed deaths while more than 570 000 health care providers were infected and 2500 died due to the pandemic disease [3,4]. The corona virus is locally transmitted through either direct or indirect contact with infected individuals such as the droplets resulting after infected patients speaks, coughs and sneezes [5]. However, scientists and researchers proved that the transmission of the corona virus from an infected human to human spreads rapidly through eyes, nose, and mouth before washing contaminated hands with soaps or sanitizers [6]. Furthermore, the corona virus spreads through hospital’s equipment due to insufficient precautions taken by health care providers’ population resulted from the unavailability of equipment and poor facilities.

The infected individuals with COVID-19 expressing several signs of the disease. The foremost clinical symptoms are high fever, coughing, sneezing, headache, haemoptysis, diarrhoea, dysphonia, heart failure, and respiratory failure [6,7]. Since COVID-19 affecting many countries in the World, then East Africa as among the victims struggled to prevent further disease transmission by insisting people wearing face masks, using sanitizers, practising social distancing, introducing lock-down, and suspending all activities involving gathering such as closing learning institutions Mumbu & Hugo [1].

Various mathematical epidemiologists have been proposed different studies targeting the transmission dynamics and control the spread of COVID-19 in a population by introducing the best control measures such as personal protective equipments, face masks, disinfectants, sanitizers, and applying physical distancing but still the disease affects many people around the world. Some of the mathematical model includes:

The SEIRD model proposed by Viguerie et al. [8] applied partial differential equations to simulate the spread of COVID-19. The results inform the health authority in Italy of the best way of allocating medical resources and effective control measures regarding geographical location.

However, Mumbu & Hugo [1] developed the SMUEIHR model for the human population, by comparing the use and disuse of masks...
and observed that before and after introducing wearing face masks and hospitalization the basic reproduction number was \( R_0 = 3.8 \) and \( R_\infty = 0.698 \) respectively. Hence from the values of \( R_0 \), the researchers recommended that wearing face masks is mandatory whenever contacted with the crowded population as preventive measures for control spreads of COVID-19 transmission in Tanzania.

Besides, Baek et al. [9] developed SEIR containing compartments of Doctors, Nurses, Patients, and Caregivers model for assessing the effectiveness of early screening and quarantined in tertiary hospitals to identify infected patients. The results showed that early screening and quarantined in South Korea reduced the number of cases from 81.3% to 42% within 60 days after implementation. The study insists on implements early screening to all visitors and early stage of exposure to infections.

Furthermore, Mekonen et al. [5] studied SEIRDM model for self-protection behaviour changes from asymptotic individuals with COVID-19. The outcome showed that basic reproduction number depends on the contamination of environments and population. The study suggested that for preventing further COVID-19 transmission, the population should be aware with the environments.

Thus, the particular study needs to apply mathematical techniques to analyze the preventive measures required for minimizing the transmission dynamics of COVID-19 in healthcare providers’ population. Therefore, this study intends to analyze the impacts of health care providers’ population in transmission dynamics of COVID-19.

**Model formulation**

In this paper, a deterministic mathematical model on transmission dynamics of COVID-19 was formulated and analyzed. The biological model consists of seven sub-compartments namely: a susceptible health care providers’ population whose at high risk to be infected Alfred et al. [10] when contacted with infectious denoted by \( S_H \), exposed health care providers \( E_H \) are expected to be infectious with the disease but do not express clinical symptoms Zhang et al. [11]. The infected class is further subdivided into two classes that are diagnosed and detected \( I_D \) whose clinically identified having SARS-COV-2 infections while non-diagnosed and undetected \( I_U \) health care providers’ population are unidentified clinically and does not show disease symptoms due to either low virus concentrations or strong body immunity Kasali et al. [12], but treatment class denoted by \( T_H \) is a place where all infected individuals seek medications and supportive care services while recovery class \( R_H \) are all sick individuals who gained health either through medical treatments or natural body immunity. Finally, the virus concentrations attached on the hospital’s equipment is denoted by \( C_H \) and it influences susceptible individuals for getting COVID-19 infections indirectly from contaminated hospitals’ equipment.

It is also assumed that hospital’s equipment acquired infections through interaction of non-diagnosed and undetected health care providers’ population only since they are difficult to identify infected individuals. The total health care providers’ population at any time \( t \) is denoted by:

\[
N_H(t) = S_H(t) + E_H(t) + I_D(t) + I_U(t) + T_H(t) + R_H(t)
\]

Hence, the following parameters were constructed in formulating a mathematical model:

(i) Health care providers’ population get exposed either directly or indirect contacts to infectious through infection rate \( \lambda = \frac{\mu I_U + \beta C_H}{N_H} \) + \( \frac{\beta C_H}{G} \), where \( G \) represents the virus concentrations and capacity of hospital’s equipment to hold SARS-COV-2 [5], \( \beta \) is an effective probability of infection rate, \( \beta_C \) is probability of infection rate from \( C_H \) class, \( \zeta \) is a modification of parameter in reduction of disease transmission from non-diagnosed and undetected health care providers [5,12].

(ii) The infections of health care providers’ population increasing when interactions to the contaminated hospital equipment increases which induced by the rate of shedding off viruses \( \delta \) and progression rate \( \psi \) while the virus concentrations on hospital’s equipment decreases due to the natural death rate of SARS-COV-2 as time goes on which is denoted by \( q \) [5].

(iii) However, recruitment rate \( \Pi \) represents individuals visited at hospitals for medical treatment and consultations while \( \mu \) is a natural mortality rate occurred for each class of health care providers’ population.

(iv) The susceptible health care providers’ population increases at the rate of individual waned their body immunity \( \gamma \), recovered after using medications \( \sigma \) and diagnosed but not having infections rate \( \theta \). Furthermore, some of non-diagnosed and undetected having infection recovered at the rate \( \alpha \) and the rest who undergo severe clinical symptoms seeks medications at the rate \( \eta \) while all severe infected individuals are treated at the rate \( \rho \).

(v) The COVID-19 disease induced death rates \( \tau \) and \( \nu \) at diagnosed and detected, non-diagnosed and undetected classes respectively.

(vi) The proportion rates of diagnosed and non-diagnosed with respect to exposed health care providers’ population \( E_H \) is \( \epsilon \) and \( 1 - \epsilon \) respectively, while for those detected with COVID-19 infections seek treatments at the rate \( \omega \).

(vii) The virus concentrations in the hospital’s equipment increases at a rate of shedding off \( \delta \) from \( I_U \) while it decreases at decay rate \( \gamma \).

From the variables and parameters stated above, the transmission dynamics of the COVID-19 model were constructed as follows. Basing on the transmission dynamics of COVID-19 model in Fig. 1, we generate the following non-linear ordinary differential equations of the model system as:

\[
\begin{align*}
\frac{dS_H}{dt} &= \Pi - \theta I_D + (\gamma + \sigma) R_H - \lambda S_H - \mu S_H, \\
\frac{dE_H}{dt} &= \lambda S_H - (\mu + \phi) E_H, \\
\frac{dI_D}{dt} &= \epsilon \phi E_H - \theta S_H - (\mu + \tau + \omega) I_D, \\
\frac{dI_U}{dt} &= (1 - \epsilon) \phi E_H - (\mu + \eta + \alpha + \delta) I_U, \\
\frac{dR_H}{dt} &= \omega R_H + \eta I_U - (\mu + \nu + \tau) T_H, \\
\frac{dC_H}{dt} &= \alpha I_U + \mu T_H - (\gamma + \mu + \rho + v) R_H, \\
\frac{d\gamma}{dt} &= \delta I_U - \omega C_H.
\end{align*}
\]

For non-negative initial conditions we have:

\[
S_H(0) \geq 0; \quad E_H(0) \geq 0; \quad I_D(0) \geq 0; \quad I_U(0) \geq 0; \quad T_H(0) \geq 0; \quad R_H(0) \geq 0; \quad C_H(0) \geq 0;
\]

**Boundedness of solution**

In this subsection, the model system (2) is biologically well defined if all its solutions exists and is bounded within the positive invariant region such that:

\[
\Gamma = \left\{ (S_H(t), E_H(t), I_D(t), I_U(t), T_H(t), R_H(t), C_H(t)) \in R^7_+ : 0 \leq N_H(t) \leq \frac{\Pi}{\mu}, C_H(t) \in R^+_\varsigma : 0 \leq C_H(t) \leq \frac{\Pi \delta}{\mu q} \right\}
\]

**Theorem 1.** The solution of model system (2) lying within \( R^7_+ \) are uniformly bounded in the invariant region \( \Gamma \) for \( \forall t \geq 0 \).

**Proof.**

Let the bounded region be:

\[
\Gamma = \{ (S_H(t), E_H(t), I_D(t), I_U(t), T_H(t), R_H(t), C_H(t)) \in R^7_+ : 0 \leq N_H(t) \leq \frac{\Pi}{\mu}, \quad C_H(t) \in R^+_\varsigma : 0 \leq C_H(t) \leq \frac{\Pi \delta}{\mu q} \}
\]

for \( \forall t \geq 0 \)
Then, by differentiating equation (1) with respect to time \( t \) and substituting model system (2) of the first six equation, we obtain;

\[
\frac{d N_H}{dt} = \Pi + \theta I_D + (\gamma + \sigma) R_H - \mu S_H - (\mu + \phi) E_H + \epsilon \phi E_H - (\theta + \mu + \tau + \omega) I_D + (1 - \epsilon) \phi E_H
\]

\[
- (\mu + \eta + \gamma + \delta) I_U + \omega I_H + \eta I_U - (\mu + \rho + \nu) T_H + \rho T_H + \alpha I_U - (\gamma + \mu + \sigma) R_H
\]

Simplifying Eq. (3) and assuming that there is a well protections and control measures observed by health care providers’ population, then no infections induced by COVID-19 disease.

Thus, Eq. (3) becomes;

\[
\frac{d N_H}{dt} \leq \Pi - \mu N_H \tag{4}
\]

It follows that solution of Eq. (6) is given as;

\[
N_H(t) \leq \frac{\Pi}{\mu} + \left( N_H(0) - \frac{\Pi}{\mu} \right) e^{-\mu t} \tag{5}
\]

Hence as \( t \to \infty \) in (5), we obtained;

\[
N_H(t) \leq \frac{\Pi}{\mu} \tag{6}
\]

From Eq. (6), the health care providers’ population implies that \( I \) is positively invariant bounded region of solutions in \( R_+^6 \) such that;

\[
\Gamma = \{ \{ S_H(t), E_H(t), I_D(t), I_U(t), T_H(t), R_H(t) \} \in R_+^6 : 0 \leq N_H(t) \leq \frac{\Pi}{\mu} \}
\]

However, for the virus concentrations on hospital’s equipment class, we have;

\[
\frac{dC_H}{dt} = \delta I_U - q C_H \tag{8}
\]

Since, \( I_U \leq \frac{\Pi}{\mu} \), then Eq. (8) becomes,

\[
\frac{dC_H}{dt} \leq \delta I_U - q C_H \tag{9}
\]

By using the concept of Gronwall inequality in (9) as applied by [5], for \( 0 \leq C_H(0) \leq \frac{\Pi \delta}{\mu q} \), we get;

\[
0 \leq C_H(t) \leq \frac{\Pi \delta}{\mu q} \tag{10}
\]

Hence, the virus concentrations in the hospital’s equipment are also bounded in the invariant region positively such as:

\[
\Gamma = \{ C_H(t) \in R_+ : 0 \leq C_H(t) \leq \frac{\Pi \delta}{\mu q} \}
\]

Therefore, combining the solutions of (7) and (11), the model system (2) is epidemiologically well defined in the bounded invariant region \( \Gamma \)

\[\square\]

**Model analysis**

In this section, we determine the equilibria points of the model equation, reproduction number, and analysis of the model.

The model (2) posses the following equilibria points:

1. **Free disease equilibrium point**;

   \[
   E_0 \left( S_H^0, E_H^0, I_D^0, I_U^0, T_H^0, R_H^0, C_H^0 \right) = \left( \frac{\Pi}{\mu}, 0, 0, 0, 0, 0 \right)
   \]

2. **Endemic equilibrium point** \( E_* \) of system (2) is expressed in terms of \( I_D^* \) at steady state, Then we have;

\[
S_H^* = \frac{\xi (\mu + \eta + \gamma + \delta)^{(\gamma + \mu + \nu)} (\mu + \rho + \nu + \epsilon) \bar{I}_D^*}{(\mu + \eta + \gamma + \delta)^{(\gamma + \mu + \nu)} (\mu + \rho + \nu + \epsilon) (\mu + \rho + \nu + \epsilon) \bar{I}_D^*},
\]

\[
I_D^* = \frac{(1 - \epsilon)(\mu + \rho + \nu + \mu)}{(\mu + \eta + \gamma + \delta) \bar{I}_D^*},
\]

\[
T_H^* = \frac{\mu (\mu + \rho + \nu + \mu)}{(\mu + \eta + \gamma + \delta) \bar{I}_D^*},
\]

\[
R_H^* = \frac{a (\mu + \rho + \nu) (1 - \epsilon)(\theta + \mu + \tau + \omega) + \xi \mu \nu (\mu + \eta + \gamma + \delta) (1 - \epsilon) (\mu + \rho + \nu + \epsilon) \bar{I}_D^*}{(\mu + \rho + \nu + \epsilon) (\mu + \eta + \gamma + \delta) (\mu + \rho + \nu + \epsilon) \bar{I}_D^*},
\]

\[
C_H^* = \frac{\delta (1 - \epsilon)(\theta + \mu + \tau + \omega)}{(\mu + \eta + \gamma + \delta) \bar{I}_D^*},
\]

where:

\[ A = (\mu + \eta + \gamma + \delta)(\gamma + \mu + \nu)(\mu + \rho + \nu + \epsilon) \theta \]
\[ (\gamma + \sigma) e^\rho_2 (\mu + \eta + \alpha + \delta) + (1 - \epsilon)(\theta + \mu + \tau + \omega) \times (a(\mu + \rho + \nu) + e^{\rho_1}), \]
\[ a_1 = \frac{\beta \epsilon (\mu + \eta + \alpha + \delta) + \beta_\psi (1 - \epsilon)(\theta + \mu + \tau + \omega)}{(\mu + \eta + \alpha + \delta) + \mu_0 N_H}, \]
\[ a_2 = \frac{\beta \epsilon (\theta + \mu + \tau + \omega) (1 - \epsilon)}{\delta (1 - \epsilon)(\theta + \mu + \tau + \omega) (1 - \epsilon)}. \]

Therefore, \( E_s = (S_H, E_H', I_H', T_H', R_H', C_H') \) is the endemic equilibrium point of the system (2).

**Basic reproduction number**

The basic reproduction number is used by the researchers to determine whether the disease exists or cleared out in the entirely community. The basic reproduction number is used by the researchers to determine whether the disease it will either continue existing or dying out in the community. In this, model system (2), to compute the basic reproduction number, we used the approach as applied by [2,13,14]. Hence, generation of new infections is denoted by matrix \( F \) and matrix \( V \) represents the disease transfer among compartments determined at disease free equilibrium that are given as follows:

\[
F = \begin{bmatrix}
0 & \beta & \beta \psi \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix},
\]

and;

\[
V = \begin{bmatrix}
\mu + \phi & 0 & 0 \\
-\epsilon \phi & \theta + \mu + \tau + \omega & 0 \\
-(1 - \epsilon) \phi & 0 & \mu + \eta + \alpha + \delta
\end{bmatrix}.
\]

Thus, basic reproduction number is the largest eigenvalue of the next generation matrix, in which after computation and further simplifications, we get:

\[
R_0 = \frac{\beta \epsilon (\mu + \eta + \alpha + \delta) + \beta_\psi (1 - \epsilon)(\theta + \mu + \tau + \omega)}{(\mu + \phi)(\theta + \mu + \tau + \omega)(\mu + \eta + \alpha + \delta)}. \tag{12}
\]

Biologically, the basic reproduction number \( R_0 \) shows that new number of infected health care providers’ population with COVID-19, it depends on \( \beta \) while keeping constants other parameter values:

If \( \beta < (\theta + \mu + \tau + \omega)(\mu + \eta + \alpha + \delta) \), then \( R_0 < 1 \) and health care providers’ population and hospital equipments will be free from COVID-19 infections.

If \( \beta > (\theta + \mu + \tau + \omega)(\mu + \eta + \alpha + \delta) \), then \( R_0 > 1 \) and the disease will invade and spreads in health care providers’ population which results to virus concentration on the hospital’s equipment.

**Local stability for disease free equilibrium point**

To determine the local stability of disease free equilibrium point of the model system (2), we applied the linearisation techniques to formulate a Jacobian matrix with corresponding to disease free equilibrium point, \( E_{0} \). However, in this study, we used the concept of Routh–Hurwitz criteria to prove local stability of the disease free equilibrium as applied in the work of [12,15].

**Theorem 2.** If \( R_0 < 1 \), then the disease free equilibrium point \( E_0 \) of the model system (2) is locally asymptotically stable, otherwise unstable.

**Proof.**

We need to attest that the real part of the eigenvalues of the Jacobian Matrix \( J \) at disease free equilibrium point \( E_0 \) has negative sings.

Hence, we have a Jacobian matrix \( J \) at disease free equilibrium point \( E_0 \) given as in Eq. (13) given in Box I. From matrix (11), we obtained the real parts of eigenvalues with negative sign as follows:

\[ \lambda_1 = -\mu, \lambda_2 = -(\mu + \lambda + \sigma), \lambda_3 = -(\mu + \rho + \nu). \]

Also, the remaining eigenvalues are determined by using matrix block reductions as applied in [10,16].

However, after cancellations of the first, fifth and sixth rows and columns respectively, the matrix (14), reduced to a 4 \( \times \) 4 matrix. Thus, the reduced Jacobian matrix \( J_{E_0} \) is given by:

\[
J_{E_0} = \begin{bmatrix}
-\theta - \mu - \tau - \omega & 0 & 0 & 0 \\
0 & -\mu - \eta - a - \delta & 0 & 0 \\
0 & 0 & \delta & -q - \lambda
\end{bmatrix}.
\tag{14}
\]

Now to obtain the eigenvalues of matrix (14), we determine its determinant as follows:

\[
\begin{vmatrix}
-(\phi + \mu) - \lambda & \beta & \beta \psi & \frac{\mu \epsilon}{\rho_1} \\
\phi & -\delta - \mu - \tau - \omega & 0 & 0 \\
(1 - \epsilon) \phi & 0 & -(\mu + \eta + \alpha + \delta) - \lambda & 0 \\
0 & 0 & \delta & -q - \lambda
\end{vmatrix} = 0.
\tag{15}
\]

Hence, characteristic equation \( J_{E_0} \) is a fourth order of degree polynomial which is written by;

\[
P(\lambda) = \lambda^4 + A \lambda^3 + B \lambda^2 + C \lambda + D,
\tag{16}
\]

where,

\[
A = a_1 + a_2 + a_3 + a_4,
B = \beta \epsilon \phi \psi - \beta \phi (\psi + \epsilon) + a_2 a_3 + a_4 a_1 + a_1 a_3 + a_1 a_2 + a_2 a_1 + a_4 a_3,
\]

Then \( B > 0 \) if \( \beta \epsilon \phi \psi + a_2 a_3 + a_1 a_4 + \beta \phi (\psi + \epsilon) > 0 \).

\[
C = a_1 a_2 a_3 - a_1 a_2 a_4 - \beta \phi a_4 + a_1 a_2 a_4 + a_1 a_3 a_4 + a_1 a_2 a_4 - a_1 a_2 a_4
\]

\[
= -\frac{\Pi \delta \phi \beta \psi a_1(1 - \epsilon)}{\mu G},
\]

Then \( C > 0 \) if \( a_1 a_2 a_3 + a_1 a_2 a_4 + \beta \phi a_4 + \Pi \delta \phi \beta \psi a_1(1 - \epsilon) > 0 \).

\[
D = a_1 a_2 a_3 a_4 - a_1 a_2 a_4 R_0 - \frac{\Pi \delta \phi \beta \psi a_1(1 - \epsilon)}{\mu G},
\]

Then \( D > 0 \) if \( a_1 a_2 a_3 a_4 > a_1 a_2 a_4 R_0 + \Pi \delta \phi \beta \psi a_1(1 - \epsilon) \).

Since \( A, B, C, D > 0 \), then we need to determine the necessary conditions for \( AB - C \) and \( ABC - C^2 - A^2 D \) to be greater than zero.

Consider \( AB - C \), then;

\[
AB - C = \beta \epsilon \phi \psi (a_1 + a_2 + a_4) + a_1 a_2 a_3 + a_1 a_2 a_4 + a_1 a_2 a_2
\]

\[
+ 2a_1 a_4 (a_2 + a_4)
\]

\[
+ 2a_2 a_1 (a_1 + a_2) + a_1 a_2 a_3 + a_1 a_2 a_3 + a_1 a_3 a_4
\]

\[
+ a_1 a_3 a_4 + a_2 a_2 a_4 + a_3 a_4
\]

\[
+ a_4 a_4 - \beta \phi (a_1 + a_2 + a_4) - \beta \phi a_1 (a_1 + a_4).
\]

Hence, \( AB - C > 0 \) if;

\[
\beta \epsilon \phi \psi (a_1 + a_2 + a_4) + a_1 a_2 a_3 + a_1 a_2 a_4 + a_1 a_2 a_2 + 2a_1 a_4 (a_2 + a_4)
\]

\[
+ 2a_2 a_1 (a_1 + a_2) + a_1 a_2 a_3 + a_1 a_2 a_3 + a_1 a_3 a_4 + a_2 a_2 a_4 + a_3 a_4
\]

\[
+ a_4 a_4 > \beta \phi (a_1 + a_2 + a_4) + \beta \phi a_1 (a_1 + a_4).
\]
Meanwhile;
\[ABC - C^2 - A^2 D = m(a_1 a_2 a_3 + a_1 a_2 a_4 + a_1 a_3 a_4 + a_2 a_3 a_4)
+ (\beta \phi)(a_1 + a_2 + a_3) + \beta \phi \zeta (a_1 + a_3)
+ \frac{\Pi \delta \phi \beta + \alpha (1 - \epsilon)}{2} a_3 R_0 + \beta \phi \zeta a_4
+ \frac{\Pi \delta \phi \beta + \alpha (1 - \epsilon)}{2} a_3 R_0 + \beta \phi \zeta a_4
+ (a_1 + a_2 + a_3 + a_4)^2 \left(a_1 a_2 a_3 a_4 \right)
\]}

So \(ABC - C^2 - A^2 D\) is to be greater than zero if;
\[m(a_1 a_2 a_3 + a_1 a_2 a_4 + a_1 a_3 a_4 + a_2 a_3 a_4) + (\beta \phi)(a_1 + a_2 + a_3) + \beta \phi \zeta (a_1 + a_3)
+ \frac{\Pi \delta \phi \beta + \alpha (1 - \epsilon)}{2} a_3 R_0 + \beta \phi \zeta a_4
+ (a_1 + a_2 + a_3 + a_4)^2 \left(a_1 a_2 a_3 a_4 \right)
\]

where, \(a_1 = \phi + \mu, a_2 = \theta + \mu + \tau + \omega, a_3 = \rho + \eta + \alpha + \delta, a_4 = q,\)

Global stability of disease free equilibrium point

Theorem 3. The model system (2) is said to be globally asymptotically stable whenever \(R_0 < 1\) at disease free equilibrium point \(E_0\), otherwise it is unstable within invariant region \(\Gamma\).

To examine the global stability of disease free equilibrium point \(E_0\), we use the method done by [17,18]. Hence, we rewrite our model system (2) as:

\[\frac{dy_1}{dt} = -A_1(y_1 - y_1^*) + Ay_2\]
\[\frac{dy_2}{dt} = A_2y_2\]

where, \(y_1^*\) := uninfected classes of the model Eq. (2), \(y_2^*\) := infected classes of the model Eq. (2), \(y_1^*\) := disease free equilibrium point \(E_0\) of uninfected classes of model Eq. (2).

We define,
\[y_1^* = \left(\frac{\Pi \mu, \alpha, \beta, \gamma}{\mu, \alpha, \beta, \gamma}\right)^T\]

Thus, Theorem 2 is epidemiologically well posed if the following conditions holds the global stability of disease free equilibrium points \(E_0\):

i. \(A_1\) must be a matrix with real negative eigenvalues on the main diagonal.

ii. \(A_2\) must be a Metzler matrix (off diagonal matrix must have non-negative element).

Using Eq. (17) and (18), we obtained the following results:

\[A_1 = \begin{pmatrix}
-\mu & \gamma + \sigma & 0 \\
0 & -\mu - \rho - \nu & 0 \\
0 & \rho & -\gamma - \mu - \sigma
\end{pmatrix}\]
\[A_2 = \begin{pmatrix}
-\mu - \phi & \beta & \beta \zeta \\
(1 - \epsilon) \phi & -\theta - \mu - \tau - \omega & 0 \\
0 & 0 & -\gamma - \mu - \sigma
\end{pmatrix}\]

Since matrix \(A_1\) and \(A_2\) satisfies the necessary conditions (i) and (ii) respectively, we conclude that model system (2) at disease free equilibrium points \(E_0\) is globally asymptotically stable which makes epidemiologically meaningful in the invariant region \(\Gamma\).
Global stability analysis of endemic equilibrium point.

In this paper, the global stability of the endemic equilibrium point $E_*$ is determined by using the Lyapunov function as applied by Ngalya and Kuznetsov [19]. Therefore, to obtain global stability $E_*$, we need to prove that Lyapunov function $L$ is globally asymptotically stable contained in the invariant region $\mathcal{V}$. 

**Theorem 4.** If $R_0 \geq 1$, then endemic equilibrium point $E_*$ of the model system (2) is globally asymptotically stable defined in the invariant region $\mathcal{V}$ otherwise unstable.

**Proof.**

Using the Lyapunov function $L$, we prove that endemic equilibrium points $E_*$ is globally asymptotically stable if $R_0 \geq 1$.

Thus, we define the following Lyapunov function $L$ as:

$$L = \sum_{i=1}^{7} M_i (X_i - X_i^* \ln X_i)$$

(22)

where,

- $M_i$ = carefully chosen constants
- $X_i^*$ = represents HCPs at endemic equilibrium points
- $X_i$ = represents HCPs compartments ($S_H, E_H, I_H, I_U, T_H, R_H, C_H$)

![Fig. 2. Variations of diagnostic testing rates in health care providers’ population.](image1)

![Fig. 3. Variation of virus concentrations on hospitals’ equipment due to increasing the rate of shedding off viruses.](image2)
From (22), we generate the following equation:

\[
L = M_3(S_H - I_H^{\mu} \ln S_H) + M_2(E_H - E_H^0 \ln E_H) + M_3(I_D - I_D^0 \ln I_D)
+ M_4(I_U - I_U^0 \ln I_U)
+ M_5(T_H - T_H^0 \ln T_H) + M_6(S_H - S_H^0 \ln S_H) + M_7(C_H - C_H^0 \ln C_H)
\]

\[\text{(23)}\]

Differentiating Eq. (23) with respect to time t, we get:

\[
\frac{dL}{dt} = M_1 \left(1 - \frac{S_H^0}{S_H}\right) \frac{dS_H}{dt} + M_2 \left(1 - \frac{E_H^0}{E_H}\right) \frac{dE_H}{dt} + M_3 \left(1 - \frac{I_D^0}{I_D}\right) \frac{dI_D}{dt}
+ M_4 \left(1 - \frac{T_H^0}{T_H}\right) \frac{dT_H}{dt} + M_6 \left(1 - \frac{R_H^0}{R_H}\right) \frac{dR_H}{dt}
+ M_7 \left(1 - \frac{C_H^0}{C_H}\right) \frac{dC_H}{dt}.
\]

\[\text{(24)}\]

Then, substituting the values of \(\frac{dS_H}{dt}, \frac{dE_H}{dt}, \frac{dI_D}{dt}, \frac{dT_H}{dt}, \frac{dR_H}{dt}, \frac{dC_H}{dt}\) into Eq. (24), gives the following results:

\[
\frac{dL}{dt} \leq M_1 \left(1 - \frac{S_H^0}{S_H}\right) \left[ \pi + \theta I_D + (\gamma + \sigma) R_H - \lambda S_H - \mu S_H \right] + M_2 \left(1 - \frac{E_H^0}{E_H}\right) \left[ \lambda S_H - (\mu + \phi) E_H \right]
+ M_3 \left(1 - \frac{I_D^0}{I_D}\right) \left[ \xi \phi E_H - (\theta + \mu + \tau + \omega) I_D \right]
+ M_4 \left(1 - \frac{T_H^0}{T_H}\right) \left[ (1 - \epsilon) \phi E_H - (\mu + \eta + \alpha + \delta) I_U \right]
+ M_6 \left(1 - \frac{R_H^0}{R_H}\right) \left[ \alpha I_U + \rho T_H - (\gamma + \mu + \sigma) R_H \right]
+ M_7 \left(1 - \frac{C_H^0}{C_H}\right) \left[ \delta I_U - q C_H \right].
\]

\[\text{(25)}\]

Moreover, using the endemic equilibrium point, \(E_*\) we get:

\[
\frac{dL}{dt} \leq M_1 \left(1 - \frac{S_H^0}{S_H}\right) \left[ \pi + \theta I_D + (\gamma + \sigma) R_H - \lambda S_H - \mu S_H \right] - \left[ \pi + \theta I_D + (\gamma + \sigma) R_H^* - \lambda S_H^* - \mu S_H^* \right]
+ M_2 \left(1 - \frac{E_H^0}{E_H}\right) \left[ \lambda S_H - (\mu + \phi) E_H^* \right] - \left[ \lambda S_H - (\mu + \phi) E_H^* \right]
+ M_3 \left(1 - \frac{I_D^0}{I_D}\right) \left[ (\xi \phi E_H - (\theta + \mu + \tau + \omega) I_D^* \right] - \left[ (\xi \phi E_H - (\theta + \mu + \tau + \omega) I_D^* \right]
+ M_4 \left(1 - \frac{T_H^0}{T_H}\right) \left[ (1 - \epsilon) \phi E_H - (\mu + \eta + \alpha + \delta) I_U^* \right] - \left[ (1 - \epsilon) \phi E_H - (\mu + \eta + \alpha + \delta) I_U^* \right]
+ M_6 \left(1 - \frac{R_H^0}{R_H}\right) \left[ \alpha I_U^* + \rho T_H^* - (\gamma + \mu + \sigma) R_H^* \right] - \left[ \alpha I_U^* + \rho T_H^* - (\gamma + \mu + \sigma) R_H^* \right]
+ M_7 \left(1 - \frac{C_H^0}{C_H}\right) \left[ \delta I_U^* - q C_H^* \right] - \left[ \delta I_U^* - q C_H^* \right].
\]

\[\text{(26)}\]

Again further simplifications, and collecting negative and positive terms together, we obtained the following results:

\[
\frac{dL}{dt} \leq M_1 \theta I_D \left(1 - \frac{I_D^0}{I_D}\right) \left(1 - \frac{S_H^0}{S_H}\right) + M_2 \lambda S_H \left(1 - \frac{S_H^*}{S_H}\right) + M_3 \frac{S_H}{S_H^*}
+ M_4 \phi E_H \left(1 - \frac{E_H^0}{E_H}\right) \left(1 - \frac{I_D^0}{I_D}\right)
+ M_5 (1 - \epsilon) \phi E_H \left(1 - \frac{I_D^0}{I_D}\right)
+ M_6 \alpha I_U \left(1 - \frac{T_H^0}{T_H}\right) \left(1 - \frac{R_H^0}{R_H}\right)
+ M_7 \beta I_U \left(1 - \frac{T_H^0}{T_H}\right) \left(1 - \frac{R_H^0}{R_H}\right)
+ M_8 I_U \left(1 - \frac{T_H^0}{T_H}\right) \left(1 - \frac{R_H^0}{R_H}\right)
+ \sigma R_H M_1 \left(1 - \frac{R_H^0}{R_H}\right) \left(1 - \frac{S_H^0}{S_H}\right)
\]

\[\text{(27)}\]

which can be written as:

\[
\frac{dL}{dt} = P - Q
\]

\[\text{(28)}\]

where,

\[
P = M_1 \theta I_D \left(1 - \frac{I_D^0}{I_D}\right) \left(1 - \frac{S_H^0}{S_H}\right) + M_2 \lambda S_H \left(1 - \frac{S_H^*}{S_H}\right) \left(1 - \frac{S_H^0}{S_H}\right)
+ M_3 \phi E_H \left(1 - \frac{E_H^0}{E_H}\right) \left(1 - \frac{I_D^0}{I_D}\right)
+ M_4 (1 - \epsilon) \phi E_H \left(1 - \frac{I_D^0}{I_D}\right)
+ M_5 \omega I_D \left(1 - \frac{T_H^0}{T_H}\right) \left(1 - \frac{R_H^0}{R_H}\right) + M_6 \alpha I_U \left(1 - \frac{T_H^0}{T_H}\right) \left(1 - \frac{R_H^0}{R_H}\right)
+ M_7 \beta I_U \left(1 - \frac{T_H^0}{T_H}\right) \left(1 - \frac{R_H^0}{R_H}\right) + M_8 I_U \left(1 - \frac{T_H^0}{T_H}\right) \left(1 - \frac{R_H^0}{R_H}\right)
\]

\[\text{and,}\]

\[
Q = M_1 \mu S_H \left(1 - \frac{S_H^0}{S_H}\right) + M_2 (\mu + \phi) E_H \left(1 - \frac{E_H^0}{E_H}\right)^2
+ M_3 \phi E_H \left(1 - \frac{E_H^0}{E_H}\right) \left(1 - \frac{I_D^0}{I_D}\right)
+ M_4 (1 - \epsilon) \phi E_H \left(1 - \frac{I_D^0}{I_D}\right)
+ M_5 \omega I_D \left(1 - \frac{T_H^0}{T_H}\right) \left(1 - \frac{R_H^0}{R_H}\right) + M_6 \alpha I_U \left(1 - \frac{T_H^0}{T_H}\right) \left(1 - \frac{R_H^0}{R_H}\right)
+ M_7 \beta I_U \left(1 - \frac{T_H^0}{T_H}\right) \left(1 - \frac{R_H^0}{R_H}\right) + M_8 I_U \left(1 - \frac{T_H^0}{T_H}\right) \left(1 - \frac{R_H^0}{R_H}\right)
+ \sigma R_H M_1 \left(1 - \frac{R_H^0}{R_H}\right) \left(1 - \frac{S_H^0}{S_H}\right)
\]
Fig. 4. Variations of diagnostic test rates in non-diagnosed and undetected health care providers.

### Table 1
Parameter values used for numerical simulation.

| Symbol | Value | Source |
|--------|-------|--------|
| $H$   | $100,000$ | Assumed |
| $\beta$ | $0.5944$ | [1,21] |
| $\beta_c$ | $0.089$ | [5] |
| $\mu$ | $0.02857$ | [20] |
| $\rho$ | $0.1443, 1.428$ | [1,21] |
| $\omega$ | $0.0089$ | [5] |
| $\gamma$ | $0.125$ | Assumed |
| $\alpha$ | $0.0983$ | Assumed |
| $\theta$ | $0.00538$ | Assumed |
| $\theta$ | $0.0047876$ | [20] |
| $\tau$ | $0.00538$ | Assumed |
| $\eta$ | $0.0047876$ | [20] |
| $\epsilon$ | $0.0312$ | [20] |
| $\phi$ | $0.0142857$ | [22] |
| $\delta$ | $0.0323$ | [5] |
| $\psi$ | $0.5788$ | [5] |
| $\zeta$ | $0.025$ | Assumed |

Hence, if $P < Q$ then it implies that $\frac{dL}{dt} < 0$; for all values of $(S_H, E_H, I_D, I_U, T_H, R_H, C_H) > 0$.

However, $\frac{dL}{dt} = 0$, if and only if $S_H = S_H^*, E_H = E_H^*, I_D = I_D^*, I_U = I_U^*, T_H = T_H^*, R_H = R_H^*, C_H = C_H^*$. Therefore, by considering the La Salle’s invariant principle [23], Eq. (28) is strictly Lyapunov function if and only if $\frac{dL}{dt} < 0$ which signifies the endemic equilibrium point, $E_*$ is globally asymptotically stable whenever $R_0 \geq 1$ and otherwise unstable within the invariant region $\Gamma$. Epidemiologically, this informs that COVID-19 infections will be endemic in health care providers’ populations for a long time and this completes the proof. □

### Results and discussion

The implementation of mass diagnostic testing to susceptible health care providers’ population is an important approach that can help health authorities and stakeholders in general to control the transmission of COVID-19 disease in our community.

Fig. 2 shows as the rate of diagnostic testing increasing it results to identify the infected individuals from which can help to control the transmission of COVID-19 in health care providers’ population and reduce the possibility of contamination of hospital’s equipment. On the other hand, non-diagnosed and undetected increases slowly due to most of health care providers taking diagnostic testing and control measures. Also, the rate of shedding of virus into hospital’s equipment increases due to interactions of non-diagnosed and undetected with COVID-19 infections and contaminated equipment (Fig. 3) which results in disease existence, since an undetected individuals are difficult to identify them.

Fig. 4 shows that the non-diagnosed and undetected health care providers’ population decreasing due to most of them observing control measures which results to decrease the indirect contact and contamination of virus into hospital’s equipment.

Furthermore, the results from Figs. 5 and 6 shows that treatments of infected increasing as the number of detected and those with severe clinical symptoms from non-diagnosed and undetected health care providers increases.

In Figs. 7 and 8, we observed that most of the health care providers seek medical treatments and, non-diagnosed and undetected with low infections recovered from COVID-19 disease due to medications and their natural body immunity respectively.

Therefore, the findings from both analytical and numerical results showed that non-diagnosed and undetected health care providers with SARS-COV-2 infections were among the contributing the spreads of COVID-19 in the community, and contamination of virus into hospital equipment during the medical treatment and consultations.

On the other hand, results obtained by Mumbu & Hugo 2020 [1] showed that non-wearing face masks of infected individuals increase the human-to-human transmission of COVID-19 infections in a population. However, these findings concurred with the study done by [5,9,21,24,25] which showed that implementing of early screening helped
to detect all infected individuals and reducing further transmission of COVID-19.

Thus, from the results obtained we can recommend that all hospitals should introduce pre-diagnosis testing for all health care providers before and after medical consultation of any individuals.

**Conclusion**

Basing on the analytical and simulation results obtained from this study, it concluded that non-diagnosed and undetected health care providers’ population seems to contribute to high spread of COVID-19 pandemic in community and contamination of hospitals’ equipment; since, most health care providers have indirect interactions with contaminated hospitals’ equipment at the time of implementing medical consultations.

Hence, the study proved the necessity of implementing early diagnostic testing and taking medical treatments once detected having COVID-19 clinical symptoms to curtail the transmission of disease among health care providers’ population and keeping hospital’s equipment safe from infections.
Fig. 7. Variations of recovery HCPs population with natural recovery rate.

Fig. 8. Variations of recovery HCPs population with treatment rate.

**CRediT authorship contribution statement**

**Kulwa Maiga**: Data curation, Writing – original draft, Methodology, Writing – review & editing, Formal analysis, Software, Conceptualization. **Alfred Hugo**: Visualization, Investigation, Supervision, Validation, Editing, Project administration, Resources, Preparation.

**Declaration of competing interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

**Data availability**

The secondary data supporting this research article are obtained from the previously reported studies and included within the article.

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**References**

[1] Mumbu AJ, Hugo AE. Mathematical modelling on COVID-19 transmission impacts with preventive measures: a case study of Tanzania. J Biol Dyn 2020;14(1):748–66. http://dx.doi.org/10.1080/17513758.2020.1823494.
[2] Zhao S, Musa SS, Hebert JT, Cao P, Ran J, Meng J, He D, Qin J. Modelling the effective reproduction number of vector-borne diseases: The yellow fever outbreak in Luanda, Angola 2015–2016 as an example. PeerJ 2020;1–21. http://dx.doi.org/10.7717/peerj.8601.

[3] Erdem H, Lucery DR. Healthcare worker infections and deaths due to COVID-19: A survey from 37 nations and a call for WHO to post national data on their website. Int J Infect Diseases 2021;102:239–41. http://dx.doi.org/10.1016/j.ijid.2020.10.064.

[4] World Health Organisation (WHO). Corona virus disease-2019 (COVID-19): weekly operational update on COVID-19, 6 september 2021 issue no. 71. 2021, https://www.who.int/emergencies/diseases/novel-coronavirus-2019/situation-reports.

[5] Mekonen KG, Habtemicheal TG, Balcha SF. Modelling the effect of contaminated objects for the transmission dynamics of COVID-19 pandemic with self protection behavior changes. Results Appl Math 2021;9:100134. http://dx.doi.org/10.1016/j.rnam.2020.100134.

[6] Rothen HA, Byrareddy SN. The epidemiology and pathogenesis of coronavirus disease (COVID-19) outbreak. J Autoimmun 2020;2020:109. http://dx.doi.org/10.1016/j.jauto.2020.102433.

[7] Sohrabi C, Alsaﬁ Z, O’Neill N, Khan M, Kerwan A, AlJabir A, Iosifidis C, Agha R. World health organization declares global emergency: A review of the 2019 novel coronavirus (COVID-19). Int J Surg 2020;76(February):71–6. http://dx.doi.org/10.1016/j.ijsu.2020.02.034.

[8] Viguerie A, Lorenzo G, Auricchio F, Baroli D, Hughes TJR, Patton A, Reali A, Sohrabi C, Alsafi Z, O’Neill N, Khan M, Kerwan A, Al-Jabir A, Iosifidis C, Rothan HA, Byrareddy SN. The epidemiology and pathogenesis of coronavirus with infected immigrants in Tanzania. Afr J Pure Appl Sci 2021;2(2):118–26. http://dx.doi.org/10.33886/aajs.v2i2.215, June 2021 118126.

[9] Zhang Z, Zeb A, Alzahrani E, Iqbal S. Crowding effects on the dynamics of COVID-19 mathematical model. Adv Difference Equ 2020;2020(1). http://dx.doi.org/10.1186/s13662-020-03137-3.

[10] Kasali F, Ismail AIM. Modelling the dynamics of COVID-19 in Nigeria. Int J Appl Comput Math 2021;7(3):1–25. http://dx.doi.org/10.1007/s40819-021-01014-5.

[11] K. Maiga and A. Hugo

[12] Kasali F, Ismail AIM. Modelling the dynamics of COVID-19 in Nigeria. Int J Appl Comput Math 2021;7(3):1–25. http://dx.doi.org/10.1007/s40819-021-01014-5.

[13] Van den Driessche Pauline, Watmough J. Reproduction numbers and sub threshold endemic equilibria for compartmental models of disease transmission. Math Biosci 2002;180(1–2):29–48. http://dx.doi.org/10.1016/S0025-5564(02)00098-6.

[14] Van den Driessche Pauline. Reproduction numbers of infectious disease models. Infect Disease Model 2017;2(3):288–303. http://dx.doi.org/10.1016/j.idm.2017.06.002.

[15] Hugo, et al. An eco-epidemiological mathematical model with treatment and disease infection in both prey and predator population. J Ecol Nat Environ 2012;4(10):266–79. http://dx.doi.org/10.5897/jene12.013.

[16] Pedro SA, Abelman S, Ndjomatchoua FT, Sang R, Tomnang HEZ. Stability, bifurcation and chaos analysis of vector-borne disease model with application to rift valley fever. 2014;9(10). http://dx.doi.org/10.1371/journal.pone.0108172.

[17] Ega Tesfaye T, Luboosi Livingstone S, Kuznetsov Dmitry. Modelling the dynamics of rabies transmission with vaccination and stability analysis. Appl Comput Math 2015;4(6):409–19. http://dx.doi.org/10.11648/j.acm.20150406.13, 2015.

[18] Kamgang JC, Sallet G. Computation of threshold conditions for epidemiological models and global stability of the disease-free equilibrium (DFE). Math Biosci 2008;213(1):1–12. http://dx.doi.org/10.1016/j.mbs.2008.02.005.

[19] Ngalya C, Kuznetsov D. Modelling the impact of bemaßia tabaci in dynamics of tomato yellow leaf curl virus. Asian J Math Appl 2017;2017:1–15.

[20] Rafiq M, Macias-Díaz JE, Raza A, Ahmed N. Design of a non-linear model for the propagation of COVID-19 and its efficient non standard computational implementation. Appl Math Model 2021;89:1835–46. http://dx.doi.org/10.1016/j.apm.2020.08.082.

[21] Ivorra B, Ferrandez MR, Vela-Perez M, Ramos AM. Mathematical modeling of the spread of the corona virus disease 2019 (COVID-19) taking into account the undetected infections. The case of China. Commun Nonlinear Sci Numer Simul 2020;88:105303.

[22] Kim BN, Kim E, Lee S, Oh C. Mathematical model of COVID-19 transmission dynamics in South Korea: The impacts of travel restrictions, social distancing, and early detection. Processes 2020;8(10):1–18. http://dx.doi.org/10.3390/pr8101304.

[23] LoSalle JP. The stability of dynamical systems. CRMNS-NSF regional conference series in applied mathematics, vol. 25, Philadelphia: SIAM; 1976.

[24] Acuna-Zegarra MA, Santana-Cibrian M, Velasco-Hernandez JX. Modeling behavioral change and COVID-19 containment in Mexico: A tradeoff between lockdown and compliance. Math Biosci 2020;325(May):1088370. http://dx.doi.org/10.1016/j.mbs.2020.108570.

[25] Balsa C, Lopez I, Guarda T, Rufino J. Computational simulation of the COVID-19 epidemic with the SEIR stochastic model. Comput Math Organ Theory 2021;2021;45:6789. http://dx.doi.org/10.1007/s10588-021-09327-y.