Probing new physics in $B \to J/\Psi \pi^0$ decay

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Abstract

We calculate the branching ratio of $B \to J/\Psi \pi^0$ with a mixed formalism that combines the QCD-improved factorization and the perturbative QCD approaches. The result is consistent with experimental data. The quite small penguin contribution in $B \to J/\Psi \pi^0$ decay can be calculated with this method. We suggest two methods to extract the weak phase $\beta$. One is through the dependence of the mixing induced CP asymmetry $S_{J/\Psi\pi^0}$ on the weak phase $\beta$, the other is from the relation of the total asymmetry $A_{CP}$ with the weak phase $\beta$. Our result shows that the deviation $\Delta S_{J/\Psi\pi^0}$ of the mixing induced CP asymmetry from $\sin(-2\beta)$ is of $\mathcal{O}(10^{-3})$ and has much less uncertainty. The above $\mathcal{O}(10^{-3})$ deviation can provide a good reference for identifying new physics.

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B physics is entering the era of precision measurement. It is not far from revealing new physics beyond the Standard Model (SM). Many authors have studied the topics and suggest some windows for looking for new physics (NP) [1]-[9]. Because flavour-changing neutral current (FCNC) processes only occur at the loop-level in the SM, so they are particularly sensitive to NP interactions. It was pointed out that $B_0^0 - \bar{B}_0^0$ mixing and decays are good places for new physics to enter through the exchange of new particles in the box diagrams, or through new contributions at the tree level [10]-[12], so $B_0^0 - \bar{B}_0^0$ system has been studied in many papers for probing new physics [13, 17]. $B \rightarrow J/\Psi \pi^0$ decay is a good mode for looking for new physics and extracting the weak phase $\beta$. The direct CP asymmetry $C_{J/\Psi \pi^0}$ and the deviation $\Delta S_{J/\Psi \pi^0} \equiv S_{J/\Psi \pi^0} - \sin(-2\beta)$ of the mixing -induced CP asymmetry from $\sin(-2\beta)$ in this decay arise from quite small penguin contribution in the SM, so these quantities are sensitive to new physics effect. Comparing the prediction of CP asymmetry in the SM with the experimental data, one can find new physics signal. Thus it is essential to calculate the $\Delta S_{J/\Psi \pi^0}$ and $C_{J/\Psi \pi^0}$ in $B \rightarrow J/\Psi \pi^0$ in the SM accurately.

The deviation $\Delta S_{J/\Psi \pi^0} = S_{J/\Psi \pi^0} - \sin(-2\beta)$ or direct CP asymmetry $C_{J/\Psi \pi^0}$ in $B \rightarrow J/\Psi \pi^0$ decay have been studied in Ref. [18] by fitting to the current experimental data, the result is $C_{J/\Psi \pi^0} = 0.09 \pm 0.19$ which has very large uncertainty. In that case we can not say anything about new physics effects.

In order to reveal new physics effects, we need both better theoretical prediction and experimental measurement with less uncertainties. That is the aim of our present paper.

In what follows, we first evaluate the penguin pollution effect by a method which have been used to explain many B decays into charmonia successfully [19, 20]. We find the penguin pollution in the $B \rightarrow J/\Psi \pi^0$ decay is quite small, the deviation $\Delta S_{J/\Psi \pi^0} = S_{J/\Psi \pi^0} - \sin(-2\beta)$ in $B \rightarrow J/\Psi \pi^0$ decay is $\mathcal{O}(10^{-3})$, which means that the measured deviation $\Delta S_{J/\Psi \pi^0}$ at 1% will indicate the presence of new physics.

The latest experimental data of $\Delta S_{J/\Psi \pi^0}$ is $S_{J/\Psi \pi^0} = -0.4 \pm 0.4$ [21], which has large error, so we are expecting to have more precise measurement in the near future.

The decay rate of of $B \rightarrow J/\Psi \pi^0$ can be written as

$$\Gamma = \frac{1}{32\pi m_B} G_F^2 (1 - r_2^2 + \frac{1}{2} r_2^4 - r_3^2) |\mathcal{A}|^2. \quad (1)$$

with $r_2 = m_{J/\Psi}/m_B$, $r_3 = m_{\pi}/m_B$.

The amplitude $\mathcal{A}$ consists of factorizable part and nonfactorizable part. It can be written
FIG. 1: Nonfactorizable contribution to the $B^0 \to J/\psi \pi^0$ decay

as

$$A = A_{NF} + A_{VERT} + A_{HS}, \quad (2)$$

where $A_{NF}$ denote the factorizable contribution in Naive Factorization Assumption (NF), $A_{VERT}$ is the vertex corrections from Fig. 1 (a)-(d), $A_{HS}$ is the spectator correction from Fig. 1 (e)-(f).

The factorizable part $A_{NF}$ in Eq. (2) for $B \to J/\psi \pi^0$ decay can not be calculated reliably in the pQCD approach, because its characteristic scale is around 1 GeV. We parameterize the sum of the factorizable part $A_{NF}$ and the vertex corrections $A_{VERT}$ as,

$$A_{NF} + A_{VERT} = a_{eff} m_B^2 f_{J/\psi} F_{1}^{B-\eta}(m_{J/\psi}^2)(1-r_2^2), \quad (3)$$

where $f_{J/\psi}$ is decay constant of $J/\psi$ meson,

For the $B \to \pi$ transition form factors, we employ the models derived from the light-cone sum rules [22], which have been parameterized as

$$F_{1}^{B-\pi}(q^2) = \frac{r_1}{1 - q^2/m_{f1}^2} + \frac{r_2}{1 - q^2/m_{f2}^2}, \quad (4)$$
with \( r_1 = 0.744, r_2 = -0.486, m_1 = 5.32 GeV, m_{J/\psi}^2 = 40.73 GeV \) for \( B \to \pi \) transition.

The factorization and vertex correction from Fig. 11(a)-(d) can be calculated in the QCDF \[23\]. Summing up the factorizable part and vertex correction, we can get the Wilson coefficient \( a_{\text{eff}} \),

\[
a_{\text{eff}} = V_c^* \left[ C_1 + V_c^* C_2 \frac{C_F}{N_c} + \alpha_s \frac{C_F}{4\pi N_c} C_2 \left( -18 + 12 \ln \frac{m_b}{\mu} + f_I \right) \right] \\
- V_t^* \left[ C_3 + C_4 \frac{C_F}{N_c} + \alpha_s \frac{C_F}{4\pi N_c} C_4 \left( -18 + 12 \ln \frac{m_b}{\mu} + f_I \right) \right] \\
+ C_5 + \frac{C_6}{N_c} + \alpha_s \frac{C_F}{4\pi N_c} C_6 \left( 6 - 12 \ln \frac{m_b}{\mu} - f_I \right) + C_7 + \frac{C_8}{N_c} + C_9 + \frac{C_{10}}{N_c} \right] \tag{5}
\]

with the function,

\[
f_I = \frac{2\sqrt{2}N_c}{f_{J/\psi}} \int dx_2 \Psi^L(x_2) \left[ \frac{3(1 - 2x_2)}{1 - x_2} \ln x_2 - 3\pi i + 3 \ln(1 - r_2^2) - \frac{2r_2^2(1 - x_2)}{1 - r_2^2 x_2} \right] , \tag{6}
\]

The spectator corrections \( A_{HS} \) from Fig. 11(e)-(f), can be calculated reliably in the pQCD as in Ref. \[19, 20\],

\[
A_{HS} = V_c^* M_1^{(J/\psi)} - V_t^* M_4^{(J/\psi)} - V_t^* M_6^{(J/\psi)} , \tag{7}
\]

where the amplitudes \( M_1^{(J/\psi)} \) and \( M_6^{(J/\psi)} \) result from the \( (V - A)(V - A) \) and \( (V - A)(V + A) \) operators in the effective Hamiltonian, respectively. Their factorization formulas are given by the pQCD approach. In the calculation of \( M_1^{(J/\psi)} \) and \( M_6^{(J/\psi)} \), because \( J/\psi \) is heavy, we reserve the power terms of \( r_2 \) up to \( \mathcal{O}(r_2^2) \), the power terms of \( r_3 \) up to \( \mathcal{O}(r_3^2) \).

\[
M_1^{(J/\psi)} = 16\pi m_B^2 C_F \sqrt{2N_c} \int_0^1 [dx] \int_0^\infty b_1 db_1 \Phi_B(x_1, b_1) \\
\times \left\{ [(1 - 2r_2^2 + r_2^4)(1 - x_2)\Phi_\pi(x_3)\Psi^L(x_2) + \frac{1}{2}(r_2^2 - r_2^4)\Phi_\pi(x_3)\Psi^t(x_2) \right. \\
- r_\pi(1 - r_2^2)x_3\Phi_\pi(x_3)\Psi^L(x_2) + r_\pi \left( 2r_2^2(1 - x_2) + (1 - r_2^2)x_3 \right) \Phi_\pi^t(x_3)\Psi^L(x_2) \right\} \\
\times E_1(t_3^{(1)}) h_d^{(1)}(x_1, x_2, x_3, b_1) \\
- \left[ (x_2 - x_2r_2^4 + x_3 - 2r_2^2x_3)\Phi_\pi(x_3)\Psi^L(x_2) \right. \\
+ r_\pi(2r_\pi^2x_3^2) - \frac{1}{2}(1 - r_2^2)\Phi_\pi(x_3)\Psi^t(x_2) \right. \\
- r_\pi(1 - r_2^2)x_3\Phi_\pi(x_3)\Psi^L(x_2) - r_\pi \left( 2r_\pi^2x_3^2 + (1 - r_2^2)x_3 \right) \Phi_\pi^t(x_3)\Psi^L(x_2) \right\} \\
\times E_1(t_3^{(2)}) h_d^{(2)}(x_1, x_2, x_3, b_1) \right) , \tag{8}
\]

\[
M_6^{(J/\psi)} = 16\pi m_B^2 C_F \sqrt{2N_c} \int_0^1 [dx] \int_0^\infty b_1 db_1 \Phi_B(x_1, b_1)
\]
relevant mesons, we list the wave functions in appendix.

We need to take the wave function of spectators in the pQCD, we need to take the wave function of

therefore the dependence of the mixing-induced CP asymmetry on weak phase $\beta$ we can determine weak phase $\beta$ through the dependence of $S_{J/\psi\pi^0}$ on $\beta$ as shown in Fig. 4 and Table 1.

For the $B^0$ decay, the CP asymmetry is time dependent,

$$A_{CP}(t) = \frac{\Gamma(B^0(t) \to J/\psi\pi^0) - \Gamma(B^0(t) \to J/\psi\pi^0)}{\Gamma(B^0(t) \to J/\psi\pi^0) + \Gamma(B^0(t) \to J/\psi\pi^0)} ,$$

$$= S_{J/\psi\pi^0} \sin(\Delta M t) - C_{J/\psi\pi^0} \cos(\Delta M t) ,$$

(10)

Where the mixing-induced asymmetry $S_{J/\psi\pi^0}$ and direct CP asymmetry is defined as

$$S_{J/\psi\pi^0} = \frac{2 \text{Im} \lambda_{J/\psi\pi^0}}{1 + |\lambda_{J/\psi\pi^0}|^2} ,$$

$$C_{J/\psi\pi^0} = \frac{1 - |\lambda_{J/\psi\pi^0}|^2}{1 + |\lambda_{J/\psi\pi^0}|^2} ,$$

(11)

where

$$\lambda_{CP} = \frac{V_{td}^* V_{ub}(J/\psi\pi^0)|H_{eff}|B^0}{{\bar V}_{td} V_{ub}^*(J/\psi\pi^0)|H_{eff}|B^0} .$$

(12)

There are two ways to extract weak phase $\beta$ through $B^0 \to J/\Psi \pi^0$ decay. The first way is through the dependence of the mixing-induced CP asymmetry on weak phase $\beta$. The $S_{J/\psi\pi^0}$ is not sensitive of input parameters, as shown in Fig. 4. That means that the theoretical uncertainties of $S_{J/\psi\pi^0}$ is quite small. If we measure the mixing-induced asymmetry $S_{J/\psi\pi^0}$, we can determine weak phase $\beta$ through the dependence of $S_{J/\psi\pi^0}$ on $\beta$ as shown in Fig. 4 and Table 1.
Another way is to use the relation of the total asymmetry $A_{CP}$ with the weak phase $\beta$. By integrating $A_{CP}(t)$ with respect to the time variable $t$, we can get the total asymmetry $A_{CP}$,

$$A_{CP} = \frac{x}{1 + x^2} S_{J/\psi\pi^0} - \frac{1}{1 + x^2} C_{J/\psi\pi^0},$$

with $x = \Delta m / \Gamma \simeq 0.723$ for the $B^0 - \bar{B}^0$ mixing in the SM [21].

Like the mixing-induced asymmetry, the total asymmetry is also not sensitive to the input parameters, so we can determine the weak phase through the relation of the total CP asymmetry with weak phase $\beta$ shown in Fig. 3.

The numerical calculation needs some parameters and meson distribution amplitudes as input, we list them in the appendix.

With the parameters and meson distribution amplitude in the appendix, we get the branching ratios of $B \to J/\psi \pi^0$ decays, $\Delta S_{J/\psi\pi^0}$ and $C_{J/\psi\pi^0}$,

$$Br(B^0 \to J/\psi\pi^0) = [1.89^{+0.182}_{-0.21}(\omega b)^{+1.0496}_{-0.02}(\mu)^{-0.103}(F_1)^{+0.015}_{-0.014}(f_{J/\psi})^{+0.04}_{-0.059}(\lambda)^{+0.04}_{-0.068}(A)] \times 10^{-5},$$

$$C_{J/\psi\pi^0} = [-9.936^{+0.866}_{-3.003}(\omega b)^{+1.173}_{-2.368}(\gamma)^{+6.914}_{-0.289}(\mu)^{+1.34}_{-1.18}(F_1)^{+0.54}_{-0.56}(\beta)] \times 10^{-3},$$

$$\Delta S_{J/\psi\pi^0} = [2.84^{+1.07}_{-1.00}(\omega b)^{+0.72}_{-0.35}(\gamma)^{+2.1}_{-0.17}(\mu)^{+0.29}_{-0.20}(F_1)^{+0.03}_{-0.05}(\beta)] \times 10^{-3}. \quad (14)$$

The main theoretical errors of the branching ratio are induced by the uncertainties below. The first error is from $\omega b = 0.4 \pm 0.04 GeV$, the second one is due to renormalization scale $\mu$ taken from $mb/2$ to $mb$, the third one is induced by 15% uncertainty of $B \to \pi$ form factor $F_1^{B \to \pi}$, the fourth one arise from decay constant $f_{J/\psi} = 0.405 \pm 0.05 GeV$, the fifth error is from CKM matrix parameter $\lambda = 0.2272 \pm 0.001$, the sixth one is from CKM matrix parameter $A = 0.818^{+0.007}_{-0.017}$.

Compared with the experimental data [21]

$$Br(B^0 \to J/\psi\pi^0) = (2.2 \pm 0.4) \times 10^{-5}, \quad (15)$$

| $\beta$(deg) | 18.0 | 18.3 | 18.6 | 18.9 | 19.2 | 19.5 | 19.8 | 20.1 |
|----------------|------|------|------|------|------|------|------|------|
| $S_{J/\psi\pi^0}$ | -0.58515 | -0.59357 | -0.60192 | -0.61021 | -0.61843 | -0.62658 | -0.63467 | -0.64269 |
| $\beta$ (deg) | 20.4 | 20.7 | 21 | 21.3 | 21.6 | 21.9 | 22.2 | 22.5 |
| $S_{J/\psi\pi^0}$ | -0.65063 | -0.65851 | -0.66631 | -0.67404 | -0.68170 | -0.68929 | -0.69680 | -0.70424 |
| $\beta$ (deg) | 22.8 | 23.1 | 23.4 | 23.7 | 24.0 | 24.3 | 24.6 | 24.9 |
| $S_{J/\psi\pi^0}$ | -0.71160 | -0.71888 | -0.72608 | -0.73321 | -0.74025 | -0.74722 | -0.75410 | -0.76090 |

TABLE I: Determination of weak phase $\beta$ through mixing-induced CP asymmetry $S_{J/\psi\pi^0}$
our prediction of the branching ratio for $B \to J/\Psi\pi^0$ is consistent with it.

Unlike the branching ratio, $\Delta S_{J/\Psi\pi^0}$ and $C_{J/\Psi\pi^0}$ is not sensitive to CKM matrix parameter $\lambda$ or $\Lambda$, because these parameter dependences cancel out. The independence of $\Delta S_{J/\Psi\pi^0}$ and $C_{J/\Psi\pi^0}$ on some CKM parameters is shown in Fig. 4(a),(b),and Fig. 5(a),(b).

To find new physics and to extract the weak phase $\beta$, we need reliable evaluation for the direct CP asymmetry $C_{J/\Psi\pi^0}$ and $\Delta S_{J/\Psi\pi^0}$, so we now consider the dependence of the direct CP asymmetry $C_{J/\Psi\pi^0}$ and $\Delta S_{J/\Psi\pi^0}$ with all parameters of input. The main uncertainties of $C_{J/\Psi\pi^0}$ and $\Delta S_{J/\Psi\pi^0}$ are induced by uncertainties of shape parameter $\omega b$, CKM matrix phase $\gamma$, renormalization scale $\mu$, $B \to \pi$ form factor $F_{B \to \pi}$ and the weak phase $\beta$. The uncertainties of $\Delta S_{J/\Psi\pi^0}$ and $C_{J/\Psi\pi^0}$ are shown in Fig. 4(c)-(f) and Fig. 5(c)-(f).

Comparing with the result in Ref. [18],

\[
C_{J/\Psi\pi^0} = 0.09 \pm 0.19 \quad \text{(16)}
\]
\[
S_{J/\Psi\pi^0} = -0.47 \pm 0.30 \quad \text{(17)}
\]

our results of $\Delta S_{J/\Psi\pi^0}$ and $C_{J/\Psi\pi^0}$ has much less theoretical uncertainties. So we conclude that if the measured deviation $\Delta S_{J/\Psi\pi^0}$ of the mixing-induced asymmetry is at 1% or the direct asymmetry $C_{J/\Psi\pi^0}$ is at the level of percentage then we can say that there should be new physics. We are expecting precise measurement to the CP asymmetry of $B^0 \to J/\Psi\pi^0$ in the near future.

**APPENDIX A: INPUT PARAMETERS AND WAVE FUNCTIONS**

We use the following input parameters in the numerical calculations

\[
\Lambda^{(f=4)}_{\overline{MS}} = 250\text{MeV}, \quad f_\pi = 130\text{MeV}, \quad f_B = 190\text{MeV},
\]
\[
m_0^2 = 1.4\text{GeV}, \quad M_B = 5.2792\text{GeV}, \quad \tau_{B^0} = 1.53 \times 10^{-12}\text{s},
\]  

(A1)

For the CKM matrix elements, we adopt the wolfenstein parametrization for the CKM matrix up to $\mathcal{O}(\lambda^3)$ [21],

\[
V_{CKM} = \begin{pmatrix}
1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \frac{3\lambda^2}{2} & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix},
\]  

(A2)
with the parameters $\lambda = 0.2272$, $A = 0.818$, $\rho = 0.221$ and $\eta = 0.340$.

For the $B$ meson distribution amplitude, we adopt the model \[24\]

$$\phi_B(x,b) = N_B x^2 (1-x)^2 \exp \left[ -\frac{M_B^2 x^2}{2\omega_b^2} - \frac{1}{2} (\omega_b b)^2 \right],$$  \hspace{1cm} (A3)

where $\omega_b$ is a free parameter and we take $\omega_b = 0.4 \pm 0.05$ GeV in numerical calculations, and $N_B = 91.745$ is the normalization factor for $\omega_b = 0.4$.

The $J/\psi$ meson asymptotic distribution amplitudes are given by \[25\]

$$\Psi^L(x) = \Psi^T(x) = 9.58 \frac{f_{J/\psi}}{2\sqrt{2}N_c} x(1-x) \left[ \frac{x(1-x)}{1-2.8x(1-x)} \right]^{0.7},$$

$$\Psi^t(x) = 10.94 \frac{f_{J/\psi}}{2\sqrt{2}N_c} (1-2x)^2 \left[ \frac{x(1-x)}{1-2.8x(1-x)} \right]^{0.7},$$

$$\Psi^V(x) = 1.67 \frac{f_{J/\psi}}{2\sqrt{2}N_c} \left[ 1 + (2x-1)^2 \right] \left[ \frac{x(1-x)}{1-2.8x(1-x)} \right]^{0.7},$$  \hspace{1cm} (A4)

For the light meson wave function, we neglect the $b$ dependant part, which is not important in numerical analysis. We choose the wave function of $\pi$ meson \[26\]:

$$\Phi_\pi(x) = \frac{3}{\sqrt{6}} f_\pi x(1-x) \left[ 1 + 0.44C_2^{3/2}(2x-1) + 0.25C_4^{3/2}(2x-1) \right],$$  \hspace{1cm} (A5)

$$\Phi_\pi^P(x) = \frac{f_\pi}{2\sqrt{6}} \left[ 1 + 0.43C_2^{1/2}(2x-1) + 0.09C_4^{1/2}(2x-1) \right],$$  \hspace{1cm} (A6)

$$\Phi_\pi^t(x) = \frac{f_\pi}{2\sqrt{6}} (1-2x) \left[ 1 + 0.55(10t^2 - 10x + 1) \right].$$  \hspace{1cm} (A7)

The Gegenbauer polynomials are defined by

$$C_2^{1/2}(t) = \frac{1}{2}(3t^2 - 1), \quad C_4^{1/2}(t) = \frac{1}{8}(35t^4 - 30t^2 + 3),$$

$$C_2^{3/2}(t) = \frac{3}{2}(5t^2 - 1), \quad C_4^{3/2}(t) = \frac{15}{8}(21t^4 - 14t^2 + 1).$$  \hspace{1cm} (A8)

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FIG. 2: The dependence of the mixing-induced asymmetry $S_{J/\psi\pi^0}$ for $B^0 \rightarrow J/\Psi \pi^0$ on the weak phase $\beta$ in diagram (a). The dependence of the deviation $\Delta S_{J/\psi\pi^0}$ of the mixing-induced asymmetry from $\sin(-2\beta)$ on the weak phase $\beta$ in diagram (b)

FIG. 3: The dependence of the the mixing-induced asymmetry $S_{J/\psi\pi^0}$ for $B^0 \rightarrow J/\Psi \pi^0$ on the weak phase $\beta$ in diagram (a) can be used to extract the weak phase $\beta$. The dependence of total CP asymmetry $A_{CP}$ on the weak phase $\beta$ in diagram (b) can be used to extract the weak phase $\beta$ also.
FIG. 4: The uncertainties of $\Delta S_{J/\psi\pi^0}$ of the mixing-induced asymmetry from $\sin(-2\beta)$ are induced by that of renormalization scale $\mu$ in (c), that of $B \rightarrow \pi$ form factor in (d), that of the weak phase $\gamma$ in (e) and that of $\sin(2\beta)$ in (f).
FIG. 5: The uncertainties of the direct CP asymmetry $C_{J/\psi\pi^0}$ are induced by that of renormalization scale $\mu$ in (c), that of $B \to \pi$ form factor in (d), that of the weak phase $\gamma$ in (e) and that of $\sin(2\beta)$ in (f).