Oblique electroweak corrections and triple vector boson couplings

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Abstract

We relate all $C$– and $P$–invariant anomalous triple vector–boson couplings to the oblique electroweak parameters. LEP constraints on the latter then yield the strongest and most general simultaneous bounds to date on the former. Even if the oblique parameters assume their Standard Model values precisely, these bounds would not shrink to zero—thus underscoring the need for direct experimental probes at future colliders.

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The $SU(2)_L \otimes U(1)_Y$ theory of electroweak interactions, which is a part of the standard model (SM), has had dramatic confirmations in the last decade. Experiments at LEP are currently able to measure the mass of the $Z$ and its couplings to fermions at a less than 1% level and they agree with the theory considered upto one loop. This success of the SM has generated a feeling that even those of its parameters that have not been tested directly are likely to be in good agreement with observation. Such an impression has indeed been fostered in the literature during the past year [1] even with regard to forthcoming experiments such as those planned at LEP 200. While present precision measurements do constrain new physics vis-a-vis the parameters of the SM, such constraints need not be as restrictive in some sectors as in others.

In this Letter we examine how existing precision measurements constrain the general $C$– and $P$–conserving triple–electroweak–vector–boson (TEVB) vertices $WW\gamma$ and $WWZ$ which are predicted uniquely by the SM. Much effort has already gone into obtaining such constraints [2], but there have been two points of concern [3] involving some of these calculations. First, an $SU(2)_L \otimes U(1)_Y$ gauge non-invariant Lagrangian has been used in extending the SM gauge boson vertices. Second, the use of the cut–off procedure has not been made with due care needed in the case of new physics. In a recent publication [4], Burgess and London have clarified these issues somewhat. They show that any Lorentz and $U(1)_{em}$ gauge invariant Lagrangian, containing $W$’s and $Z$’s, automatically obeys $SU(2)_L \otimes U(1)_Y$ gauge invariance, realized nonlinearly in general. They have also pointed out misuses of the cutoff method in estimating sizes of loop diagrams and have recommended [4] the use of dimensional regularization instead.

In parameterizing new physics by an effective Lagrangian approach, one can characterize the issue of decoupling versus nondecoupling in the following way [5]. If a gauge invariant mass parameter ( e.g. the scale of new physics $\Lambda$) in the underlying theory is large, the corresponding particle decouples from the theory when the parameter tends to infinity. The decoupling theorem requires that the effective Lagrangian have a renormalizable form apart
from higher dimensional terms containing inverse powers of \( \Lambda \). If on the other hand, the mass parameter increases on account of a dimensionless coupling constant becoming large, or if the mass term is a gauge–variant one, then the particle does not decouple. This is the case, for example, with composite models of electroweak vector bosons, scenarios of Technicolor or with a heavy chiral fermion (such as the top–quark) in a spontaneously broken theory.

Restrictions on the TEVB couplings in the case of a decoupled Lagrangian are being studied by other authors [6]. These authors have chosen to use a Lagrangian with an explicit linearly realized \( SU(2)_L \otimes U(1)_Y \) gauge invariance. This can be done when the symmetry breaking is driven by elementary scalar fields. We consider the more general case [4] of a nonlinear realization of gauge invariance where the symmetry breaking sector need not be specified. Since a Lagrangian with a nonlinearly realized gauge invariance is equivalent [4] to one with \( W^\prime_s \) and \( Z^\prime_s \) and satisfying \( U(1)_{em} \) gauge invariance, we use the latter form to consider extensions to the TEVB interactions. This is tantamount to working with an \( SU(2)_L \otimes U(1)_Y \) invariant Lagrangian in the unitary gauge [4, 7].

The \( WWV \) (\( V = \gamma/Z \)) vertex, assuming \( C \) and \( P \) invariance, can be parameterized by an effective Lagrangian [8]

\[
\mathcal{L}^V_{\text{eff}} = -i g_V \left[ g^V_1 \left( W^\dagger_{\alpha \beta} W^\alpha - W^\dagger W^\alpha \right) + \kappa_V W^\dagger_{\alpha \beta} W^\alpha \right] V^\beta + \frac{\lambda_V}{M_W^2} W^\dagger_{\alpha \beta} W^\beta \gamma^\sigma V^\sigma + \lambda^V \]

(1)

Here \( V_{\alpha \beta} = \partial_\alpha V_\beta - \partial_\beta V_\alpha, W_{\alpha \beta} = \partial_\alpha W_\beta - \partial_\beta W_\alpha \) and \( g_V \) is the \( WWV \) coupling strength in the SM with \( g_\gamma = e \) and \( g_Z = ec/s \), where \( c^2 \equiv 1 - s^2 \equiv M_W^2/M_Z^2 \). The SM values for the extra new couplings are \( g^V_1 = \kappa_V = \kappa_Z = 1, \lambda_Z = \lambda_Z = 0 \). Electromagnetic gauge invariance fixes \( g^V_1 \) to be unity. The other couplings \( g^Z_1, \kappa_V, \kappa_Z, \lambda_V, \lambda_Z \) have to be determined experimentally. The number of extra parameters is further reduced by taking \( \kappa_Z \), the weak neutral charge of the \( W \), equal to unity. The Lagrangian in (1) has the advantage that it could represent either decoupling or nondecoupling new physics at high energies and provides a practical way of calculating two point functions of physical vector bosons.

New physics beyond the SM can be usefully constrained in terms of the “oblique” electroweak [10, 11] parameters \( \tilde{S}, \tilde{T}, \) and \( \tilde{U} \) (or \( \Delta \epsilon_1, \Delta \epsilon_2, \) or \( \Delta \epsilon_3 \)). These are linearly related
to $\tilde{\Pi}_{WW}(q^2)$, $\tilde{\Pi}_{ab}(q^2)$ (where $a, b = \gamma, Z$), which are the new physics contributions to the electroweak vector boson self–energy functions. Several determinations \cite{13} have been made on these parameters using LEP data as well as lower energy information. If the anomalous TEVB couplings constitute the sole source of new physics, $\hat{T}$ and $\hat{U}$ will measure the weak isospin breaking induced by them, while $\hat{S}$ arises from their $SU(2)_L \otimes U(1)_Y$ breaking aspect through the involvement of the longitudinal vector boson modes.

One can work out the divergent contributions to the $\tilde{\Pi}$–functions from the general $WWV$ vertex by evaluating the 1–loop graphs with virtual vector bosons using (1) and dimensional regularization. (Anomalous four vector boson vertices, proportional to $\lambda_V$, make vanishing contributions to the seagull loops and hence to the $\tilde{\Pi}$–functions.) With $\epsilon = 4–$ dimension of spacetime and $\mu \equiv$ mass regulator, we can write

$$\tilde{\Pi}(q^2) = \hat{\Pi}(q^2) \left[ \frac{2}{\epsilon - \gamma_E} + \ln \left( \frac{4\pi \mu^2}{M_Z^2} \right) \right] + \cdots \quad (2)$$

where the dots denote finite terms which are small for $|q^2| \lesssim M_Z^2$. We compute

$$\tilde{\Pi}_{ab}(q^2) = -\frac{g_ag_bq^2}{192\pi^2} \left[ (\eta_a + \eta_b) (36 - 4r - r^2) + \eta_a \eta_b r (2 - r) \right. \\
\left. + (3\lambda_a + 3\lambda_b + \eta_a\lambda_b + \lambda_a\eta_b) (24 - 4r) + \lambda_a\lambda_b (36 + 8r - 2r^2) \right], \quad (3)$$

where $r \equiv q^2/M_W^2$ and $\eta_{a,b} \equiv 1 - \kappa_{a,b}$. For compactness, define

$$C_1 \equiv 36c^2 - 4 - c^{-2}, \quad C_2 \equiv 24c^2 - 4, \quad C_3 \equiv 2 - c^{-2}, \quad C_4 \equiv 36c^2 + 8 - 2c^{-2},$$

With $\hat{T}$, $\hat{S}$ and $\hat{U}$ defined analogously \cite{12} in terms of the $\hat{\Pi}$’s and denoting $\langle F \rangle \equiv s^2F_\gamma + c^2F_Z$ for any $F$, we obtain

$$\hat{S} = -\frac{1}{12\pi} \left[ (\eta_Z - \eta_\gamma) \{C_1 + \langle \eta \rangle C_3 + \langle \lambda \rangle C_2 \} + (\lambda_Z - \lambda_\gamma) \{C_2 (3 + \langle \eta \rangle) + \langle \lambda \rangle C_4 \} \right]. \quad (4)$$

Since $\hat{\Pi}_{ab}(0) = 0$, only $\hat{\Pi}_{WW}(0)$ contributes to $\hat{T}$ and we can write

$$\hat{\Pi}_{WW}(q^2) = \alpha M_W^2 \left[ \hat{T} - \frac{r}{48\pi s^2} \mathcal{R}(r) \right], \quad (5)$$

4
\[
\hat{T} = -\frac{3}{16\pi} \left\{ 4 + \eta_T \right\} \eta_T + \frac{\eta_Z}{c^2 s^2} \left\{ 2 \left( 2c^4 + 2c^2 - 1 \right) + \left( c^4 + c^2 - 1 \right) \eta_Z \right\},
\]
\[
\mathcal{R}(r) \equiv (4 - 2r) \left\{ 5\langle \eta \rangle + \langle \eta^2 \rangle \right\} + 28\eta_Z + 5\eta_Z^2 + 8 \left( 3 - r \right) \{ 3\langle \lambda \rangle + \langle \lambda \eta \rangle \}
\]
\[
+ 24\lambda \{ 3 + \eta_Z \} + 2 \left( 6 + 2r - r^2 \right) \langle \lambda^2 \rangle + 4 \left( 3 + 3c^{-2} + r \right) \lambda_Z^2.
\]

Consequently,
\[
\hat{U} = -\frac{1}{12\pi} \mathcal{R}(1) + \frac{1}{12\pi c^2} \left[ 2C_1 \langle \eta \rangle + C_3 \langle \eta \rangle^2 + 2C_2 \langle \lambda \rangle (3 + \langle \eta \rangle) + C_4 \langle \lambda \rangle^2 \right].
\]

Since \( \hat{T} \) depends only on \( \hat{\Pi}_W W(0) \) and \( \hat{\Pi}_Z Z(0) \) and not their \( q^2 \) variations, it is unaffected by the dimension 6 operators and is hence independent of \( \lambda_T \) and \( \lambda_Z \). Also, terms in \( \hat{S} \) are proportional to either \( \eta_Z - \eta_T \) or \( \lambda_Z - \lambda_T \) as they should, since \( \hat{S} \) originates from the mixing between weak hypercharge \( (Y) \) and the third component of weak isospin and the \( WWY \) vertex is linear in these differences.

The oblique parameters are not finite quantities here owing to a nonrenormalizable Lagrangian. In a cut–off dependent regularization scheme, this fact would manifest itself through a non–trivial functional dependence on the cut–off scale \[3\]. As a matching condition between two effective theories \[14\], we identify \( \mu = \Lambda \), the scale at which new physics becomes manifest (assumed to be \( \sim 1 \) TeV). Using the \( \overline{\text{MS}} \) scheme of renormalization, we can then write (see eqn.\[2\])
\[
\hat{S} \simeq \hat{S} \ln \frac{\Lambda^2}{M_Z^2}
\]
where we have retained only the largest logarithms. Similar relations hold for \( \hat{T} \) and \( \hat{U} \).

Observed bounds for \( \hat{S} \), \( \hat{T} \) and \( \hat{U} \) can now be translated onto \( \hat{S} \), \( \hat{T} \), \( \hat{U} \).

We use \[13\] \( \hat{S} = -0.31 \pm 0.49 \), \( \hat{T} = -0.12 \pm 0.34 \) and \( \hat{U} = -0.11 \pm 0.92 \), though our results are insensitive to the central values. \( \hat{T} \) allows only an elliptic band in the \( \kappa_T - \kappa_Z \) plane (Fig. 1a), where the width is given by the errors (95% C.L.) on \( M_W/M_Z \) and \( \hat{T} \). \( \hat{S} \) and \( \hat{U} \) then reduce the allowed region to only the small shaded part of the elliptic band. This is shown enlarged in Fig.(1b). Since \( \hat{S} \) is proportional to the differences of \( \gamma_- \) and \( Z^- \)–couplings, constraints on it generally (though not always) tend to make those converge.
Rather unexpectedly, $\bar{U}$ plays a significant role in constraining these anomalous couplings. In conjunction with $\tilde{S}$ and $\tilde{T}$, it serves to exclude a large part of the parameter space. We consider the $\kappa_\gamma - \lambda_\gamma$ plane (Fig. 2), since comparison with direct observations at $UA2$ and $CDF$ is then possible. Also shown is part of an ellipse, the interior of which represents (at 95% C.L.) the area allowed by the $UA2$ data [15]. Moreover, the two parentheses on the $\lambda_\gamma$–axis indicate the region in $\eta_\gamma$ allowed by the $CDF$ data [16] assuming $\lambda_\gamma = 0$. Of course the constraints are equally tight when expressed in any other form, say in the $\kappa_Z - \lambda_Z$ plane, as shown in Fig. 3.

These constraints on TEVB vertices are stronger than those achieved so far by direct experiments. Whereas we obtain the (95% C.L.) bounds $-6.1 \lesssim \kappa_\gamma - 1 \lesssim 4.1$, $-6.0 \lesssim \lambda_\gamma \lesssim 4.5$, $-2.0 \lesssim \kappa_Z - 1 \lesssim 0.3$ and $-4.5 \lesssim \lambda_Z \lesssim 1.9$, earlier $UA2$ analysis [15] had yielded $-8.4 \lesssim \kappa_\gamma - 1 \lesssim 12.1$ (for arbitrary $\lambda_\gamma$) and $-8.5 \lesssim \lambda_\gamma \lesssim 6.5$ (for arbitrary $\kappa_\gamma$). The allowed regions represent solutions to polynomial equations, which are curves that thicken into bands on account of experimental error bars on the coefficients. As the errors shrink to zero, the allowed parameter space collapses into the curves still permitting wide ranges of values for the anomalous couplings, e.g. the solid curves in Figs. (1b, 2, 3) correspond to the SM point $\tilde{S} = \tilde{T} = \tilde{U} = 0$. (This is a consequence of cancellations between various contributions to the $\tilde{\Pi}$-functions.) Direct studies of the TEVB vertices at Fermilab, LEP 200 or the NLC will be necessary to probe regions much closer to the origin [17].

To conclude, our use of precision measurements at LEP and at lower energies, in terms of $\tilde{S}, \tilde{T}$ and $\tilde{U}$, constrains the anomalous $WW\gamma$ and $WWZ$ vertices quite stringently. Unlike previous efforts, which could constrain only $\kappa_\gamma$ and $\lambda_\gamma$, we are able to restrict $\kappa_Z$ and $\lambda_Z$ as well. These bounds are much stronger than the all existing limits, though comparable limits may be directly achieved at Fermilab in the near future. The three oblique parameters do constrain the $WW\gamma$ and $WWZ$ anomalous couplings, but cannot exclude all regions in the space of the four coupling constants — there is always a set of limiting curves. Stronger restrictions cannot be imposed from these measurements alone; direct experimental study of
TEVB vertices at future colliders would be necessary to go beyond. This point of contention has been resolved by the present work.

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Figure Captions

1. (a) The dashed elliptical band gives the constraints from $\tilde{T}$ alone on the $\kappa_\gamma-\kappa_Z$ plane for $\Lambda = 1 \, TeV$. In this and the following figures, the shaded part indicates the region allowed by $\tilde{S}$, $\tilde{T}$ and $\tilde{U}$ constraints put together. (b) Enlarged section of the shaded area in (a). In this and subsequent figures, the solid curve represents solutions for $\tilde{S} = \tilde{T} = \tilde{U} = 0$.

2. Constraints (at 95% C.L.) from $\tilde{S}, \tilde{T}$ and $\tilde{U}$ on the $\kappa_\gamma-\lambda_\gamma$ plane for $\Lambda = 1 \, TeV$. UA2 data constrain the parameters to be bound by the dashed ellipse. The parentheses on the $\lambda_\gamma$ axis give the bounds of Ref. [16].

3. Constraints (at 95% C.L.) from $\tilde{S}, \tilde{T}$ and $\tilde{U}$ on the $\kappa_Z-\lambda_Z$ plane for $\Lambda = 1 \, TeV$. 

8
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Figure 1

Figure 2

Figure 3