Some cosmological consequences of the five-dimensional Projective Unified Field Theory

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Abstract
The classical observational cosmological tests (Hubble diagram, count of sources, etc.) are considered for a homogeneous and isotropic model of the Universe in the framework of the five-dimensional Projective Unified Field Theory in which gravitation is described by both space-time curvature and some hypothetical scalar field ($\sigma$-field). It is shown that the presence of the $\sigma$-field can essentially affect conclusions obtained from the cosmological tests. The surface brightness-redshift relation can be used as a critical test for $\sigma$-field effects. It seems reasonable to say that the available experimental data testify that the $\sigma$-field decreases with time. It is concluded that the spatial curvature is positive or negative depending on whether the mass density is larger or smaller than some critical parameter which is smaller than the critical density and can even take negative values. It is shown that the increase in the number of the observational cosmological parameters as compared to the standard Friedmann model can essentially facilitate coordination of the existing observational data.

1 Introduction
It has been known that the cosmological tests [1,2] are a convenient method of studying cosmological gravitational fields. The most important of them are: magnitude-redshift relation (Hubble diagram), count of sources, angular size-redshift relation, etc. These tests allow one to find the Hubble constant $H_0$ and the deceleration parameter $q_0$. However recent estimates of these parameters, obtained from different tests in the framework of the standard Friedmann model, are in rather poor mutual agreement without special additional assumptions (see, e.g., [3,4] and references therein). The reasons for these difficulties can be both in unreliability of the observational data (which is mainly connected with evolution and selection effects) and in the restriction to the Friedmann model based on the equations of General Relativity (GR). In this context, a consideration of cosmological consequences of theories generalizing GR deserves attention. One of such theories is the 5-dimensional projective unified field theory (PUFT) developed by E. Schmutzer [5,6,7].

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As is well known, the idea of a 5-dimensional unified field theory goes back to the works of Kaluza and Klein [8,9]. The pioneers of the projective approach to this theory were Veblen and van Dantzig [10,11]. Later this approach was further developed by many authors (the corresponding references and a review of other higher-dimensional unified theories see in [12,13,14]).

In PUFT gravitation is described by both space-time curvature and some hypothetical scalar field (σ-field). To characterize the scalar field predicted in PUFT as a new fundamental phenomenon in Nature, E. Schmutzer introduced the notion “scalarism” (adjective: “scalaric”) by analogy with electromagnetism. The source of this “scalaric” field can be both the electromagnetic field and a new attribute of matter which Schmutzer has called “scalaric mass”. It should be noted that the presence of the σ-field can lead to essential additions to the general picture of the Universe evolution [15,16,17].

In this paper we shall consider a theory of classical cosmological tests within the framework of PUFT. Also, we shall investigate the observational parameters of a homogeneous and isotropic model on the basis of PUFT. It is obvious that the presence of the σ-field in the theory leads to an extension of the number of the observational cosmological parameters as compared to the standard Friedmann model. This circumstance, from our point of view, will allow us to make consistent the observational data existing now. That is primarily the data of cosmological tests, the problem of dark matter, etc. (see e.g. [18] and also [3,4]). All the results obtained will be compared with similar predictions of the standard Friedman cosmology.

2 Field equations of PUFT

The version of PUFT investigated here is based on the postulated 5-dimensional Einstein-like field equations. By projecting them into the 4-dimensional space-time one obtains the following 4-dimensional field equations (the cosmological term is omitted here) [6]:

\[ R_{mn} - \frac{1}{2} g_{mn} R = \kappa_0 (E_{mn} + \Sigma_{mn} + \Theta_{mn}) \] (1)

are the generalized gravitational field equations;

a) \[ H^{mn}_{\ ,n} = \frac{4\pi}{c} j^m \], b) \[ B_{mn,k} + B_{km,n} + B_{nk,m} = 0 \], c) \[ H_{mn} = \epsilon^{3\sigma} B_{mn} \] (2)

are the generalized electromagnetic field equations;

\[ \sigma^{k\ ,k} = \kappa_0 \left( \frac{2}{3} \vartheta + \frac{1}{8\pi} B_{ik} H^{ik} \right) \] (3)

is the scalar field equation. Here \( R_{mn} \) is the Ricci tensor,

\[ E_{mn} = \frac{1}{4\pi} \left( B_{mk} H^{k\ n} + \frac{1}{4} g_{mn} B_{ik} H^{ik} \right) \] (4)

is the electromagnetic energy-momentum tensor,

\[ \Sigma_{mn} = -\frac{3}{2\kappa_0} \left( \sigma_{,m} \sigma_{,n} - \frac{1}{2} g_{mn} \sigma_{,k} \sigma^{,k} \right) \] (5)

is the scalaric energy-momentum tensor, \( \Theta_{mn} \) is the energy-momentum tensor of the nongeometrized matter (substrate), \( H_{mn} \) and \( B_{mn} \) are the electromagnetic induction and the field strength tensor, respectively, \( j^k \) is the electric current density, \( \vartheta \) is the scalaric substrate density,
\(\kappa_0 = 8\pi G/c^4\) is Einstein’s gravitational constant (\(G\) is Newton’s gravitational constant). Latin indices run from 1 to 4; the comma and semicolon denote partial and covariant derivatives, respectively; the signature of the space-time metric is +2.

These field equations lead to the following generalized energy-momentum conservation law and continuity equation for electric current density:

\[
\Theta^{mn;k} = \frac{1}{c} B^m_{\ ;k} j^k + \partial \sigma^m, \quad \text{b) } j^m;_m = 0. \quad (6)
\]

Using (2) and (6) it is possible to show [19] that in PUFT, as well as in GR, light rays propagate along null geodesics of space-time. However,

\[
(c^{3\sigma}T^{mn})_m = 0, \quad (7)
\]

where \(T^{mn}\) is the energy-momentum tensor of the photon beam. Thus the scalar \(\sigma\)-field can lead either to absorption of light or to its amplification.

Concluding this section, it should be mentioned that E. Schmutzer since 1995 has preferred new non-Einstein-like 5-dimensional field equations which he applied to cosmology and cosmogony in a series of papers [7, 17]. But this version of PUFT has slightly different 4-dimensional field equations as compared with the above-stated ones (one can find a detailed analysis of the geometric axiomatics of PUFT in [20]). It should be noted that both variants are physically acceptable and deserve a comprehensive study.

3 Basic equations for a homogeneous and isotropic cosmological model

Let us consider a homogeneous and isotropic cosmological model with the Robertson-Walker line element in the well-known form:

\[
ds^2 = R^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\varphi^2) \right] - c^2 dt^2, \quad (8)
\]

where \(R(t)\) is the scale factor and \(k\) takes the values 0 or \(\pm 1\). For an electrically neutral continuum which is described by the energy-momentum tensor of a perfect fluid the field equations (1) and (3) in the metric (8) lead to the following set of equations (the dot denotes a time derivative, \(\varrho\) is the mass density, \(p\) is the pressure):

\[
\frac{\ddot{R}}{R} = \frac{\kappa_0 c^2}{6} \left( \varrho c^2 + 3p \right) - \frac{1}{2} \dot{\sigma}^2, \quad (9)
\]

\[
\frac{\ddot{R}}{R} + 2\left( \ddot{R}^2 + kc^2 \right) = \frac{\kappa_0 c^2}{2} \left( \varrho c^2 - p \right), \quad (10)
\]

\[
\dot{\sigma} + 3 \frac{\dot{R}}{R} \dot{\sigma} = -\frac{2}{3} \kappa_0 c^2 \varrho, \quad (11)
\]

while the generalized energy conservation law (6) gives

\[
\dot{\varrho} + 3 \frac{\dot{R}}{R} \left( \varrho + \frac{p}{c^2} \right) = \frac{\dot{\varrho}}{c^2}, \quad (12)
\]
Eqs. (9) to (12) determine the dynamics of the cosmological model if the equations of state, i.e., \( p = p(\varrho) \) and \( \vartheta = \vartheta(\varrho) \), are known. The Friedmann model corresponds to the special case \( \vartheta = 0 \) and \( \dot{\vartheta} = 0 \) of our model. Unfortunately, the above set of differential equations leads to an Abel equation and till now was solved exactly only in some special cases [15, 16, 21, 22, 23].

Now we examine light propagation in a Robertson-Walker space-time. Consider light emitted from a point with the radial coordinate \( r_1 \) at the time \( t_1 \). The light, propagating along a null-geodesic line, will be received at the point \( r = 0 \) and at the time \( t_0 \) if

\[
\int_{t_1}^{t_0} \frac{c \, dt}{R(t)} = \int_0^{r_1} \frac{dr}{\sqrt{1 - kr^2}}. \tag{13}
\]

Then the redshift of the light source is given by the usual formula

\[
1 + z = \frac{R(t_0)}{R(t_1)}. \tag{14}
\]

On the other hand, the absolute luminosity \( L \) of the source and its apparent bolometric luminosity \( \ell \) are connected by the relation

\[
\ell(t_0) = \frac{L \, e^{-3\Delta\varrho}}{4\pi(1 + z)^2 R^2(t_0) r_1^2}, \tag{15}
\]

where \( \Delta\varrho \equiv \varrho(t_0) - \varrho(t_1) \). The presence of the multiplier \( e^{-3\Delta\varrho} \) in the last expression is a consequence of Eq. (7). Using (15), it is possible to show that the flux density of radiation \( S(\nu) \) (i.e. the power per unit area and per unit frequency interval of a receiver) is given by

\[
S(\nu) = \frac{P [\nu (1 + z)]}{(1 + z) R^2(t_0) r_1^4} e^{-3\Delta\varrho}, \tag{16}
\]

where \( P \) is the intrinsic source power per unit solid angle and per unit frequency interval.

With (15) the luminosity distance \( d_\ell \) to the source is determined by the following expression:

\[
d_\ell \equiv \sqrt{\frac{L}{4\pi\ell}} = (1 + z) \, r_1 \, R(t_0) \, e^{3\Delta\varrho/2}. \tag{17}
\]

If \( D \) is the linear size and \( \delta \) is the metric angular diameter of the source, then the angular diameter distance \( d_a \) has the form [2]:

\[
d_a \equiv \frac{D}{\delta} = \frac{R(t_0) r_1}{1 + z}. \tag{18}
\]

From Eqs. (17) and (18) we get

\[
d_\ell = d_a (1 + z)^2 e^{\frac{3}{2}\Delta\varrho}. \tag{19}
\]

Hence, taking into consideration the \( \sigma \)-field effects can cause changes in the construction of an extragalactic distance scale.

4 Observational cosmological tests

In deriving theoretical relations that describe the cosmological tests we refer to the small \( z \) (\( z \ll 1 \)) approximation. In this case we need not integrate rather complex cosmological equations of PUFT (9)–(12). It is only sufficient to require a relevant regularity of the functions \( R(t) \) and \( \sigma(t) \).
4.1 Hubble diagram

First we consider the $\sigma$-field influence on the relation $d_\ell(z)$. To this end, let us examine sources with the same intrinsic luminosity $L$. The quantities $d_\ell$ and $z$ of each source are bound to its unknown coordinates by the relations (13), (14) and (17). Assuming that $t - t_0$ and $r_1$ are small ($t_0$ corresponds to the present epoch), we can expand $R(t)$ and $e^{\sigma(t)}$ in the series

$$R(t) = R(t_0)[1 + H_0(t - t_0) - \frac{1}{2} q_0 H_0^2(t - t_0)^2 + \cdots],$$

$$e^{\sigma(t)} = e^{\sigma(t_0)}[1 + \lambda_0 H_0(t - t_0) + \cdots],$$

where $H_0$ and $q_0$ are the Hubble constant and deceleration parameter, respectively, defined in the usual way:

$$H_0 \equiv \frac{\dot{R}(t_0)}{R(t_0)}, \quad q_0 \equiv -\frac{\ddot{R}(t_0)R(t_0)}{\dot{R}(t_0)^2}.$$  \hspace{1cm} (22)

The dimensionless parameter $\lambda_0$, characterizing the scalar field, is given by

$$\lambda_0 \equiv \frac{1}{H_0} \frac{d\sigma(t_0)}{dt}.$$  \hspace{1cm} (23)

Taking into account Eqs. (13) and (14), the expansions (20) and (21) allow one to present $d_\ell$ (see (17)) as a power series in $z$:

$$d_\ell = \frac{c}{H_0} \left[ z + \frac{1}{2}(1 - q_0 + 3\lambda_0)z^2 + \cdots \right].$$  \hspace{1cm} (24)

This relation can be rewritten as a formula for the apparent luminosity:

$$\ell = \frac{LH_0^2}{4\pi c^2 z^2} \left[ 1 + (q_0 - 3\lambda_0 - 1)z + \cdots \right].$$  \hspace{1cm} (25)

Thus, if the equations of PUFT are valid, then from (24) and (25) it follows that, in the case $z \ll 1$, in astronomical observations, some effective deceleration parameter

$$q_{0,\text{eff}} = q_0 - 3\lambda_0,$$  \hspace{1cm} (26)

is measured, and the real deceleration parameter $q_0$ cannot be obtained from the Hubble diagram.

4.2 Counts of sources

Let us assume that the number of sources per unit physical volume with absolute luminosities within the bounds from $L$ up to $L + dL$ at the time $t_1$ is $n(t_1, L) \, dL$. Then the number of sources with radial coordinates from $r_1$ up to $r_1 + dr_1$ is given by

$$dN = \frac{4\pi R^3(t_1) r_1^2}{\sqrt{1 - kr_1^2}} \, n(t_1, L) \, dr_1 \, dL.$$  \hspace{1cm} (27)

From this relation, by taking account of (13) we obtain that the number of sources with redshifts smaller than $z$ and the apparent luminosity greater than $\ell$ is given by

$$N(<z, >\ell) = \int_0^\infty dL \int_{t_m}^{t_0} c \, dt_1 \frac{4\pi r_1^2 R^2(t_1)}{\sqrt{1 - kr_1^2}} \, n(t_1, L).$$  \hspace{1cm} (28)
Here \( t_m = \max \{ t_z, t_\ell(L) \} \), where \( t_z \) and \( t_\ell(L) \) are determined from Eqs. (14) and (15), respectively:

\[
a) \quad R(t_z) = \frac{R(t_0)}{1 + z}, \quad b) \quad \frac{r^2(t_\ell)}{R^2(t_\ell)} = \frac{L}{4\pi\ell R^4(t_0)} e^{-3[\sigma(t_0) - \sigma(t_\ell)]}. \tag{29}
\]

As in Friedmann’s cosmology [2], we shall assume that the spectrum of all sources has the form \( P \sim \nu^{-\alpha} \) with \( \alpha \approx 0.75 \). Then, using (13), we find that the number of sources with redshifts smaller than \( z \) and with the flux density at frequency \( \nu \) greater than \( S \) is

\[
N(<z, S; \nu) = \int_0^\infty dP \int_{t_0}^\infty c \, dt_1 \, 4\pi r_1^2 R^2(t_1) \, n(t_1, P; \nu) . \tag{30}
\]

Here \( t_m = \max \{ t_z, t_S(P) \} \), where \( t_S(P) \) is a solution of the equation

\[
r^2(t_S) \left[ \frac{R(t_S)}{R(t_0)} \right]^{-1-\alpha} = \frac{P(\nu) e^{-3[\sigma(t_0) - \sigma(t_S)]}}{S(\nu) R^2(t_0)} , \tag{31}
\]

and \( n(t_1, P; \nu) \, dP \) is the space density of sources with the intrinsic power at frequency \( \nu \) ranging from \( P \) up to \( P + dP \).

In order to select \( \sigma \)-field effects, we shall restrict our consideration to the case where there is no evolution of the sources. This means that the sources are not born and do not disappear, and also their luminosity does not depend on time. Then we have [2]:

\[
a) \quad n(t, L) = \left[ \frac{R(t_0)}{R(t)} \right]^3 n(t_0, L) , \quad b) \quad n(t, P; \nu) = \left[ \frac{R(t_0)}{R(t)} \right]^3 n(t_0, P; \nu) . \tag{32}
\]

At low \( z \) and large \( \ell \) or \( S \) we can use the expansions [20] and [21]. In this case, from Eqs. (28–32) we find

\[
N(<z) = \frac{4\pi}{3} \frac{c^3}{H_0^3} z^3 \int_0^\infty dL \, n(t_0, L) \left[ 1 - \frac{3}{2} (1 + q_0) z + \cdots \right] , \tag{33}
\]

\[
N(>\ell) = \frac{4\pi}{3} (4\pi\ell)^{-3/2} \int_0^\infty dL \, n(t_0, L) L^{3/2} \left[ 1 - 3 \left( 1 + \frac{3}{2} \lambda_0 \right) H_0 \frac{L}{c} \left( \frac{L}{4\pi\ell} \right)^{1/2} + \cdots \right] , \tag{34}
\]

\[
N(>S, \nu) = \frac{4\pi}{3} S^{-3/2} \int_0^\infty dP \, n(t_0, P; \nu) P^{3/2} \left[ 1 - \frac{3}{2} \left( 1 + \alpha + 3\lambda_0 \right) H_0 \frac{P}{c} \left( \frac{P}{S} \right)^{1/2} + \cdots \right] . \tag{35}
\]

Notice that Eq. (33) coincides with a similar result of GR. Thus at \( z \ll 1 \) the \( \sigma \)-field does not affect the magnitude \( N(<z) \). It is obvious that this follows from [24]. Hence, in principle, the experimental values \( N(<z) \) at low \( z \) could be used for determining the real deceleration parameter \( q_0 \). At the same time, measurements of \( N(>\ell) \) or \( N(>S, \nu) \) at large \( \ell \) or \( S \) do not give any information about \( q_0 \). But these measurements could be used to determine the parameter \( \lambda_0 \).

However, it should be noted that, as well as in the standard Friedmann cosmology, Eqs. (33), (34) and (35) are in conflict with observational data (see, e.g., [2] [13] and references therein). For example [2], the counts of radio sources testify that the function \( N(>S, \nu) \) decreases with
growing $S$ (at $S > 5 \times 10^{-26} \text{ W} \cdot \text{m}^{-2} \text{Hz}^{-1}$) approximately as $S^{-1.8}$ and definitely faster than $S^{-3/2}$, and only at low $S$ it begins to decrease slower than $S^{-3/2}$. Notice that according to the function $N(S, \nu)$ will decrease as $S^{-1}$, approximately as $S^{-1.8}$ and definitely faster than $S^{-3/2}$, and only at low $S$ it begins to decrease slower than $S^{-3/2}$. Notice that according to (35) the function $N(S, \nu)$ will decrease as $S^{-1}$ or faster provided that $\lambda_0 < -(1 + \alpha)/3 \approx -0.58$. However, it is difficult to explain such a complicated behaviour of the empirical function $N(S, \nu)$ only by means of the $\sigma$-field effects. Consequently, it is necessary to take into account the evolution of the sources. But in this case the reliability of the results obtained depend on the reliability of evolutionary suppositions. Under this circumstance the determination of cosmological parameters by means of the above test, including the parameter $\lambda_0$, becomes very complex.

4.3 Angular size-redshift relation

At low redshifts Eq. (18) for $d_\alpha$, taking account of (20) and (21), can be rewritten as

$$d_\alpha = \frac{D}{\delta} = \frac{cz}{H_0} \left[ 1 - \frac{3}{2} \left( 1 + \frac{q_0}{3} \right) z + \cdots \right]. \quad (36)$$

This outcome completely coincides with the similar result of standard cosmology. Hence, the $\sigma$-field does not influence the relation $\delta(z)$ at $z \ll 1$. Unfortunately, we cannot determine $q_0$ from this relation, because at low redshifts observational errors are much greater than the differences in $q_0$ expected for different cosmological models [3, 24].

It is well to bear in mind that, generally speaking, at high redshifts the function $\delta(z)$ will depend on the parameter $\lambda_0$, because the $\sigma$-field is present implicitly in $d_\alpha$ according to (18). But $R(t)$ and $r(t)$, contained in (18), depend on the $\sigma$-field. It is evident that this remark is correct for the test $N(< z)$ at high redshifts too.

4.4 Surface brightness-redshift relation

From Eqs. (15) and (18) it follows that the observed surface brightness of sources is given by

$$B \equiv \ell_\delta^2 = \frac{L}{4\pi D^2} \frac{e^{-3\Delta \sigma}}{(1+z)^4} = \frac{\text{const}}{(1+z)^4} e^{-3\Delta \sigma}, \quad (37)$$

where we assume that all these sources are identical, i.e. $L/D^2 = \text{const}$. Thus the presence of the $\sigma$-field can essentially change the simple surface brightness-redshift relation arising within the framework of GR:

$$B = \frac{\text{const}}{(1+z)^4} \quad \text{(GR).}$$

In the work [25] this equation was proposed to be used as a test for the redshift nature. Obviously in the framework of PUFT Eq. (37) can be used as a test for the presence of cosmological $\sigma$-field effects. From (37), taking account of (20) and (21), we find

$$B(z) = \frac{L}{4\pi D^2} \left[ 1 - (4 + 3\lambda_0)z + \cdots \right]. \quad (38)$$

Notice that the parameters $q_0$ and $H_0$ are absent in this expression. Consequently, at low redshifts the surface brightness-redshift relation allows one to estimate the $\sigma$-field effects in a pure form.

In the work [26], the relation $B(z)$ for a family of giant elliptical galaxies with small $z$ was investigated within the framework of the standard Friedmann model. In this paper the observed curve for the dependence of $B$ on $\log(1+z)$ is just a little more slanting than the straight line with a slope equal to $-4$. According to (38), it means that the parameter $\lambda_0 < 0$ if only we neglect the evolution effects. Hence, the $\sigma$-field has to decrease with time if we assume that $\sigma(t)$ is a monotonic function.
5 Cosmological parameters

5.1 Mass density and spatial curvature

It should be noted that for a correct interpretation of observational data in the framework of PUFT, obtained, in particular, from the cosmological tests, it is necessary to establish primarily a relationship between the observational cosmological parameters of PUFT and the mass density and spatial curvature of the present Universe. In order to solve this problem, we need the cosmological equations of PUFT (9)–(12). From Eqs. (9) and (10) with (22) and (23) one can find

\[ \rho_0 = \frac{3}{\kappa_0 c^4} \left[ \frac{k c^2}{R_0^2} + H_0^2 \left( 1 - \frac{\lambda_0^2}{4} \right) \right], \]

\[ p_0 = -\frac{1}{\kappa_0 c^2} \left[ \frac{k c^2}{R_0^2} + H_0^2 \left( 1 - 2q_0 + \frac{3}{4} \lambda_0^2 \right) \right]. \]

From the latter equation we obtain that the spatial curvature \( k/R_0^2 \) is positive or negative depending on whether the mass density is larger or smaller than some critical parameter \( \chi_c \):

\[ \chi_c = \rho_c (1 - \lambda_0^2/4), \]

where \( \rho_c = 3H_0^2/(\kappa_0 c^4) \) is the so-called critical density. Thus in PUFT the type of the Universe (open, spatially flat or closed) results from the comparison of \( \rho_0 \) with \( \chi_c \) instead of the comparison \( \rho_0 \) with \( \rho_c \). Notice that the parameter \( \chi_c \) takes negative values if \( |\lambda_0| > 2 \).

It is convenient to introduce the dimensionless density parameter by

\[ \Omega_0 \equiv \frac{\rho_0}{\rho_c}, \]

and the dimensionless critical parameter by

\[ \Omega_c \equiv \frac{\chi_c}{\rho_c} = 1 - \frac{\lambda_0^2}{4}. \]

This equality results in that \( \Omega_c < 1 \) is only valid if \( \lambda_0 \neq 0 \). Notice that the Universe is closed if \( \Omega_0 > \Omega_c \) and it is open if \( \Omega_0 \leq \Omega_c \).

Let us now find out how the spatial curvature and the mass density \( \rho_0 \) or \( \Omega_0 \) are connected with the observational cosmological parameters of PUFT \( q_0 \), \( H_0 \) and \( \lambda_0 \). In the case of the dust model \( (p = 0) \), from Eqs. (39), (40) and (22) we find

\[ \frac{k c^2}{R_0^2} = H_0^2 \left( 2q_0 - \frac{3}{4} \lambda_0^2 - 1 \right), \]

\[ \rho_0 = \rho_c (2q_0 - \lambda_0^2), \quad \Omega_0 = 2q_0 - \lambda_0^2. \]

Taking into account (44), one can obtain the conditions determining the type of the Universe:

\[ q_0 - \frac{3}{8} \lambda_0^2 > \frac{1}{2} \quad \Rightarrow \quad k = +1 \quad (\Omega_0 > \Omega_c), \]

\[ q_0 - \frac{3}{8} \lambda_0^2 = \frac{1}{2} \quad \Rightarrow \quad k = 0 \quad (\Omega_0 = \Omega_c), \]

\[ q_0 - \frac{3}{8} \lambda_0^2 < \frac{1}{2} \quad \Rightarrow \quad k = -1 \quad (\Omega_0 < \Omega_c). \]
In the case of a radiation-dominated Universe \((p = \frac{\varrho c^2}{3})\) Eqs. (44)–(46) have the form
\[
\frac{k c^2}{R_0^2} = H_0^2 \left( q_0 - \frac{1}{4} \lambda_0^2 - 1 \right),
\]
\[
q_0 = \varrho_c (q_0 - \frac{1}{2} \lambda_0^2), \quad \Omega_0 = q_0 - \frac{1}{2} \lambda_0^2, \tag{48}
\]
\[
q_0 - \frac{1}{4} \lambda_0^2 > 1 \Rightarrow k = +1 \quad (\Omega_0 > \Omega_c),
\]
\[
q_0 - \frac{1}{4} \lambda_0^2 = 1 \Rightarrow k = 0 \quad (\Omega_0 = \Omega_c),
\]
\[
q_0 - \frac{1}{4} \lambda_0^2 < 1 \Rightarrow k = -1 \quad (\Omega_0 < \Omega_c). \tag{49}
\]
Thus in PUFT, unlike to the Friedmann’s cosmology, by measuring only the deceleration parameter \(q_0\) it is impossible to determine whether the Universe is closed or open. For this purpose it is necessary to have the values of the two parameters, \(q_0\) and \(\lambda_0\) or \(\Omega_0\) and \(\lambda_0\).

\section*{5.2 Admitted regions for parameters}

It is interesting to note that in PUFT a spatially flat Universe can be realized for the whole range of values of the mass density \(\varrho_0\),
\[
0 \leq \varrho_0 \leq \varrho_c, \tag{50}
\]
because \(\varrho_0 = \varrho_c (1 - \lambda_0^2/4)\) if \(k = 0\). However, the condition (50) is necessary but not sufficient for the 3-dimensional space to be flat. Recall that in the Friedmann model the Universe is flat if and only if \(\varrho_0 = \varrho_c\). Taking into account this circumstance, it is useful to study in more detail the parameters of the theory.

First of all, let us find physically admitted regions for the parameters \(q_0\) and \(\lambda_0\). To this end we shall rewrite the natural inequality \(q_0 \geq 0\) taking into account (45) and (48):
\[
q_0 \geq \lambda_0^2/2. \tag{51}
\]
It is just the inequality which determines the admitted region of the parameter \(q_0\) depending on \(\lambda_0\) (this region is shaded in Fig. 1). Note that (51) is valid for both cases \(p = 0\) and \(p = \frac{\varrho c^2}{3}\).

For \(p = 0\), using (16) and (51), we obtain (see Fig. 1) that if \(q_0 > 2\) or \(|\lambda_0| > 2\), then a closed Universe is only possible \((k = +1)\), and if \(0 \leq q_0 < 1/2\), then an open Universe is only possible \((k = -1)\), while for \(1/2 \leq q_0 \leq 2\) all three types of the Universe are possible depending on the value of \(\lambda_0\).

Let us now take into consideration the available experimental data on the magnitude-redshift relation. These data, obtained in the framework of standard Friedmann cosmology (see, e.g., [3,4]), lead to the estimate \(0.5 \leq q_0 \leq 1\). In the case of PUFT, taking into account Eq. (26) and the remarks about this equation, we can suppose that the estimate \(0.5 \leq q_0^{\text{eff}} \leq 1\) is sufficiently reliable. On this basis we shall determine the possible values of the parameters \(q_0\) and \(\lambda_0\). Above all we note that the inequality (51) is consistent with Eq. (26) if \(q_0^{\text{eff}} \geq -4.5\). From (51) and (26) we find
\[
q_0^{\text{eff}} = 0.5 \quad \Rightarrow \quad \left\{ \begin{array}{l}
0.013 \leq q_0 \leq 19, \\
-0.16 \leq \lambda_0 \leq 6.2,
\end{array} \right.
\]
\[
q_0^{\text{eff}} = 1.0 \quad \Rightarrow \quad \left\{ \begin{array}{l}
0.05 \leq q_0 \leq 20, \\
-0.32 \leq \lambda_0 \leq 6.3.
\end{array} \right.
\]
From (52) we learn that, at large $q_0^{\text{eff}}$, models with the parameter $q_0$ tending to zero are possible. This circumstance can be used for coordination of experimental data of the different tests. It should be noted that in the framework of the Friedmann’s cosmology (see e.g. [3, 4]) the very low values for the deceleration parameter $q_0$, obtained from counts of sources, contradict the above-mentioned values of $q_0$ which follow from the magnitude-redshift relation.

6 Conclusions

We have considered the classical cosmological tests (Hubble diagram, count of sources, etc.) for a homogeneous and isotropic model of the Universe in the framework of the 5-dimensional Projective Unified Field Theory. The results show that the presence of the scalar $\sigma$-field predicted by PUFT can essentially affect the conclusions obtained from the cosmological tests.

We have shown, in particular, that in PUFT the deceleration parameter $q_0$ cannot be found from the Hubble diagram at low redshifts. We can only measure some effective deceleration parameter $q_0^{\text{eff}}$ given by (26). It should be noted that all the expressions describing cosmological tests in the small $z$ approximation do not depend on the choice of a specific model (the spatial curvature sign, choice of the equation of state, etc.). The surface brightness-redshift relation can be used as a critical test for $\sigma$-field effects, because the $\sigma$-field can essentially change the simple dependence of the surface brightness on the redshift in the form $B \sim (1 + z)^{-4}$ which results from Friedmann’s cosmology. It seems reasonable to say that the available experimental data testify that the $\sigma$-field decreases with time.

It is interesting to note that in cosmology, on the basis of the version of PUFT investigated here, the spatial curvature is positive or negative depending on whether the mass density is larger or smaller than some critical parameter $\chi_c$ determined by (41). Moreover, the parameter $\chi_c$ is smaller than the critical density and can even take negative values. It should be emphasized
that we did not take into account a cosmological constant in the field equations. On such a basis, in PUFT, a flat Universe with the current density parameter $\Omega_0 < 1$ is possible. These results can be used for solving the dark matter problem. Recall that in Friedmann’s cosmology the inflationary prediction of flat Universe is at odds with the current determinations of the matter density. Also, in PUFT the increase in the number of observational cosmological parameters in comparison with the standard Friedmann model can essentially facilitate the co-ordination of the observational data existing now. However, the comparison of cosmological theory with observations becomes technically more complicated.

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