ON SOME SUBCLASSES OF STRONGLY STARLIKE ANALYTIC FUNCTIONS

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Abstract. The aim of the present article is to investigate a family of univalent analytic functions on the unit disc \( D \) defined for \( M \geq 1 \) by

\[ \Re\left(\frac{zf'(z)}{f(z)}\right) > 0, \quad \left| \left(\frac{zf'(z)}{f(z)}\right)^2 - M \right| < M, \quad z \in D. \]

Some proprieties, radius of convexity and coefficient bounds are obtained for classes in this family.

1. Introduction

Let \( \mathcal{A} \) be the set of analytic function on the unit disc \( D \) with the normalization \( f(0) = f'(0) - 1 = 0 \).

\( f \in \mathcal{A} \) if \( f \) is of the form

\[ f(z) = z + \sum_{n=2}^{+\infty} a_n z^n, \quad z \in D. \]

\( \mathcal{S} \) denotes the subclass of \( \mathcal{A} \) of univalent functions. A function \( f \in \mathcal{S} \) is said to be strongly starlike of order \( \alpha \), \( 0 < \alpha \leq 1 \), if it satisfies the condition

\[ \left| \text{Arg} \frac{zf'(z)}{f(z)} \right| < \frac{\alpha \pi}{2}, \quad \forall z \in D. \]
This class is denoted by $SS^*(\alpha)$ and was first introduced by D. A. Brannan and W. E. Kirwan [1] and independently by J. Stankiewicz [9].

$SS^*(1)$ is the well known class $S^*$ of starlike functions. Recall that a function $f \in S$ belongs to $S^*$ if the image of $D$ under $f$ is a starlike set with respect to the origin or, equivalently, if
\[
\Re\left(\frac{zf'(z)}{f(z)}\right) > 0, \quad z \in D.
\]
A function $f \in S$ belongs to $SS^*(\alpha)$ if the image of $D$ under $\frac{zf'(z)}{f(z)}$ lies in the angular sector
\[
\Omega_\alpha = \left\{ z \in \mathbb{C}, |\text{Arg}z| < \frac{\alpha\pi}{2} \right\}.
\]

W. Janowski [2] investigated the subclass $S^*(M) = \left\{ f \in S, \frac{zf'(z)}{f(z)} \in \mathcal{D}_M, \forall z \in D \right\}$, where
\[
\mathcal{D}_M = \left\{ w \in \mathbb{C}, |w-M| < M \right\}, \quad M \geq 1
\]

J. Sókol and J. Stankiewicz [8] introduced a subclass of $SS^*(\frac{1}{2})$, namely, the class $S_L^*$ defined by
\[
S_L^* = \left\{ f \in S, \frac{zf'(z)}{f(z)} \in \mathcal{L}_1, \forall z \in D \right\},
\]
where
\[
\mathcal{L}_1 = \left\{ w \in \mathbb{C}, \Re w > 0, |w^2 - 1| < 1 \right\}.
\]
$\mathcal{L}_1$ is the interior of the right half of the Bernoulli’s lemniscate $|w^2 - 1| = 1$.

In the present paper we are interested to the family of subclass of $S$
\[
S_L^*(M) = \left\{ f \in S, \frac{zf'(z)}{f(z)} \in \mathcal{L}_M, \forall z \in D \right\}, \quad M \geq 1,
\]
where

\[
\mathcal{L}_M = \left\{ w \in \mathbb{C}, \Re w > 0, |w^2 - M| < M \right\}.
\]

is the interior of the right half of the Cassini’s oval \(|w^2 - M| = M\). For the particular case \(M = 1\), \(S^*_L(1)\) stands for the class \(S^*_L\) introduced by J. Sókol and J. Stankiewicz [8]. Since \(\mathcal{L}_M \subset \Omega(\frac{1}{2})\), all functions in \(S^*_L(M)\) are strongly starlike of order \(\frac{1}{2}\).

Note that all classes above correspond to particular cases of the classes of \(S^*(\varphi)\) introduced by W. Ma and D. Minda [3],

\[
S^*(\varphi) = \left\{ f \in A, \frac{zf'(z)}{f(z)} \prec \varphi \right\}.
\]

where \(\varphi\) is Analytic univalent function with real positive part in the unit disc \(D\), \(\varphi(D)\) is symmetric with respect to the real axis and starlike with respect to \(\varphi(0) = 1\) and \(\varphi'(0) > 0\).

Let \(m = 1 - \frac{1}{M}\) and \(\varphi_m\) be the function

\[
\varphi_m(z) = \sqrt{\frac{1 + z}{1 - mz}}, \quad z \in D
\]

where the branch of the square root is chosen so that \(\varphi_m(0) = 1\). We have

\[
S^*_L(M) = S^*(\varphi_m) = \left\{ f \in A, \frac{zf'(z)}{f(z)} \prec \varphi_m \right\}.
\]

Observe that \(S^*_L\) corresponds to \(m = 0\) so that \(S^*_L = S^*(\sqrt{1 + z})\).

2. SOME PROPERTIES OF THE CLASS \(S^*_L(M)\)

Let \(P\) the class of analytic functions \(p\) in \(D\) with \(p(0) = 1\) and \(\Re p(z) > 0\) in \(D\). For \(M \geq 1\), let

\[
P_L(M) = \left\{ p \in P, |p^2(z) - M| < M, \ z \in D \right\}.
\]

It is easy to see that \(P_L(M_1) \subset P_L(M_2)\) for \(M_1 \leq M_2\).

**Remark 2.1.** A function \(f \in A\) belongs to \(S^*_L(M)\) if and only if there exists \(p \in P_L(M)\) such that

\[
\frac{zf'(z)}{f(z)} = p(z), \quad z \in D.
\]

**Theorem 2.1.** A function \(f\) belongs to \(S^*_L(M)\) if and only if there exists \(p \in P_L(M)\) such that

\[
f(z) = z \exp \int_0^z \frac{p(\xi) - 1}{\xi} d\xi.
\]

**Proof.** (2.1) is an immediate consequence of the Remark 2.1
Let $f_m \in \mathcal{A}$ be the unique function such that
\begin{equation}
\frac{zf_m'(z)}{f_m(z)} = \varphi_m(z), \quad z \in \mathbb{D}
\end{equation}
with $m = 1 - \frac{1}{M}$. $f_m$ belongs to $S^*_L(M)$ and we have
\begin{equation}
f_m(z) = z \exp \int_0^z \frac{\varphi_m(\xi) - 1}{\xi} d\xi.
\end{equation}
Evaluating the integral in (2.3), we get
\begin{equation}
f_m(z) = \frac{4z \exp \int_1^{\varphi_m(z)} H_m(t) dt}{(\varphi_m(z) + 1)^2}, \quad z \in \mathbb{D},
\end{equation}
where
\[ H_m(t) = \frac{2mt + 2}{mt^2 + 1}, \quad m = 1 - \frac{1}{M} \]
For $M = 1$, $H_0$ is the constant function $H(t) = 2$ and we have
\[ f_0(z) = \frac{4z \exp \left(\frac{2\sqrt{1+z} - 2}{\sqrt{1+z} + 1}\right)}{(\sqrt{1+z} + 1)^2}\]
for $z \in \mathbb{D}$.
$f_0$ is extremal function for problems in the class $S^*_L$ (see [8]).
It is easy to see that
\begin{equation}
f_m(z) = z + \frac{m + 1}{2}z^2 + \frac{(m+1)(5m+1)}{16}z^3 + \frac{(m+1)(21m^2 + 6m + 1)}{96}z^4 + \ldots
\end{equation}
We need the following result by St. Ruscheweyh [5]

**Lemma 2.1.** [5, Theorem 1] Let $G$ be a convex conformal mapping of $\mathbb{D}$, $G(0) = 1$, and let
\[ F(z) = z \exp \int_0^z \frac{G(\xi) - 1}{\xi} d\xi.\]
Let $f \in \mathcal{A}$. Then we have
\[ \frac{zf'(z)}{f(z)} \prec G\]
if and only if for all $|s| \leq 1$, $|t| \leq 1$
\[ \frac{tf(sz)}{sf(tz)} \prec \frac{tF(sz)}{sF(tz)}\]

**Theorem 2.2.** If $f$ belongs to $S^*_L(M)$ then
\begin{equation}
\frac{f(z)}{z} \prec \frac{f_m(z)}{z}.
\end{equation}

**Proof.** From (1.5), we obtain by applying Lemma 2.1 to the convex univalent function $G = \varphi_m$,
\[ \frac{tf(z)}{f(tz)} \prec \frac{tf_m(z)}{f_m(tz)}.\]
Letting $t \to 0$, we obtain the desired conclusion. \(\square\)
**Corollary 2.1.** Let \( f \) belongs to \( S^*_L(M) \) and \( |z| = r < 1 \), then

\[
-f_m(-r) \leq |f(z)| \leq f_m(r);
\]

\[
f'_m(-r) \leq |f'(z)| \leq f'_m(r).
\]

**Proof.** (2.7) follows from (2.6). Now if \( M \geq 1 \) we have \( 0 \leq m < 1 \). Thus for \( 0 \leq r < 1 \)

\[
\min_{|z|=r} |\varphi_m(z)| = \varphi_m(-r), \quad \max_{|z|=r} |\varphi_m(z)| = \varphi_m(r)
\]

From (2.6) and (2.9) we get (2.8) by applying Theorem 2 ([3], p. 162). \( \square \)

3. **Radius of convexity for the class \( S^*_L(M) \)**

In the sequel \( m = 1 - \frac{1}{M} \).

For \( M \geq 1 \), let \( P(M) \) be the family of analytic functions \( P \) in \( D \) satisfying

\[
P(0) = 1, \quad |P(z) - M| < M, \text{ for } z \in D.
\]

We have

\[
f \in S^*_L(M) \iff \exists P \in P(M) / \frac{zf''(z)}{f(z)} = \sqrt{P}.
\]

We need the two following lemmas by Janowski [2]:

**Lemma 3.1.** ([2] , Theorem 1) For every \( P(z) \in P(M) \) and \( |z| = r, 0 < r < 1 \), we have

\[
\inf_{P \in P(M)} \Re P(z) = \frac{1-r}{1+mr}.
\]

The infimum is attained by

\[
P(z) = \frac{1-\epsilon z}{1+\epsilon mz}, \quad |\epsilon| = 1.
\]

**Lemma 3.2.** (Theorem 2, [2]) For every \( P(z) \in P(M) \) and \( |z| = r, 0 < r < 1 \), we have

\[
\inf_{P \in P(M)} \Re zP'(z) = \frac{(1+m)r}{(1-r)(1+mr)}.
\]

The infimum is attained by

\[
P(z) = \frac{1-\epsilon z}{1+\epsilon mz}, \quad |\epsilon| = 1.
\]

**Theorem 3.1.** The radius of convexity of the class \( S^*_L(M) \) is the unique root in \((0, 1)\) of the equation

\[
4(1+mr)(1-r)^3 - (1+m)^2r^2 = 0.
\]
Proof. Let \( f \in S_L^*(M) \). From (3.2), there exists \( P \in \mathcal{P}(M) \) such that

\[
ZF'(z) = \sqrt{P(z)}, \quad z \in \mathbb{D}.
\]

(3.8) can be written

\[
z f'(z) = f(z) \sqrt{P(z)}
\]

which gives

\[
1 + z f''(z) + \frac{zf'(z)}{f(z)} = \sqrt{P(z)} + \frac{1}{2} \frac{zP'(z)}{P(z)}.
\]

This yields for \(|z| = r, \quad 0 < r < 1\),

\[
\mathbb{R}(1 + z f''(z) + \frac{zf'(z)}{f(z)}) \geq \inf_{P \in \mathcal{P}(M)} \mathbb{R} \left( \sqrt{P(z)} + \frac{1}{2} \frac{zP'(z)}{P(z)} \right).
\]

Replacing (3.3) and (3.5 in (3.9), we obtain

(3.10)

\[
\mathbb{R}(1 + z f''(z) + \frac{zf'(z)}{f(z)}) \geq \sqrt{1 - \frac{r}{1 + mr}} - \frac{1}{2} \frac{(1 + m)r}{(1 - r)(1 + mr)}.
\]

Let \( h_M \) be defined by

\[
h_M = \sqrt{1 + \frac{1}{1 + mr}} - \frac{1}{2} \frac{(1 + m)r}{(1 - r)(1 + mr)}.
\]

\( h_M \) is decreasing in the interval \((0, 1)\), \( h_M(0) = 1 \) and the limit of \( h_M \) in \( 1^− \) is \( -\infty \). Let \( r_{M-1} \) be the unique solution of \( h_M(r) = 0 \) in \((0, 1)\), then \( f \) is convex on the disc \(|z| < r_{M-1}\). On the other hand,

\[
1 + z f''_m(z) + \frac{zf'(z)}{f(z)} = \sqrt{1 + \frac{z}{1 - mz}} + \frac{1}{2} \frac{(1 + m)z}{(1 - mz)(1 + z)}
\]

vanishes in \( z = -r_{M-1} \). Thus \( r_{M-1} \) is the best value.

To finish, we observe that the equation \( h_M(r) = 0 \) is equivalent in the interval \((0, 1)\) to the equation

\[
4(1 + mr)(1 - r)^3 - (1 + m)^2 r^2 = 0.
\]

\( \square \)

**Remark 3.1.** As a consequence of Theorem 3.1 applying for \( M = 1 \), we find Theorem 4 [8] which gives \( r_0 \) the radius of convexity of the class \( S_L^* \): \( r_0 = \frac{1}{12} \left( 11 + \sqrt[4]{44928} - 181 - \sqrt[4]{44928 + 181} \right) \approx 0.5679591 \)

**Remark 3.2.** As observed above, \( S_L^*(M) \) increases with \( M \). Therefore \( r_{M-1} \) decreases when \( M \) increases.

Let

\[
r_\infty = \lim_{M \to +\infty} r_{M-1}.
\]

Substituting in (3.7), we obtain
\[(1 + r_\infty)(1 - r_\infty)^3 - r_\infty^2 = 0.\]

Solving this equation in \((0, 1)\), we get
\[r_\infty = \frac{1}{2} \left( 1 - \sqrt{2} + \sqrt{\sqrt{8} - 1} \right) \approx 0.46899.\]

We have
\[r_\infty \leq r_{\cdot - 1} \leq r_0.\]

**4. Coefficient bounds for \(S^*_{L}(M)\)**

**Theorem 4.1.** Let \(f(z) = \sum_{n=0}^{\infty} a_n z^n\) be a function in \(S^*_{L}(M)\). Then

for \(1 \leq M \leq 2\) we have

\[(4.1) \quad \sum_{n=2}^{\infty} ((1 - m)n^2 - 2)|a_n|^2 \leq 1 + m\]

and for \(M > 2\) we have

\[(4.2) \quad \sum_{n \geq \sqrt{\frac{1}{1 - m}}} ((1 - m)n^2 - 2)|a_n|^2 \leq 1 + m - \sum_{2 \leq k < \sqrt{\frac{1}{1 - m}}} ((1 - m)k^2 - 2)|a_k|^2.\]

with \(m = \frac{M - 1}{M} \).

**Proof.** If \(f \in S^*_{L}(M)\) there exists \(\omega \in B\) such that

\[(4.3) \quad (1 - m\omega(z))(zf'(z))^2 - f(z)^2 = \omega(z)f(z)^2, \quad z \in \mathbb{D}.\]

For \(0 < r < 1\) we have

\[(4.4) \quad 2\pi \sum_{n=1}^{\infty} |a_n|^2 r^2 = \int_0^{2\pi} |f(re^{i\theta})|^2 d\theta \geq \int_0^{2\pi} |\omega(re^{i\theta})||f(re^{i\theta})|^2 d\theta\]

Replacing (4.3) in the right side of (4.5) we obtain

\[2\pi \sum_{n=1}^{\infty} |a_n|^2 r^2 \geq \int_0^{2\pi} |(1 - m\omega(re^{i\theta}))(re^{i\theta}f'(re^{i\theta}))^2 - f(re^{i\theta})^2| d\theta \]

\[\geq \int_0^{2\pi} |(1 - m\omega(re^{i\theta}))(re^{i\theta}f'(re^{i\theta}))^2| d\theta - \int_0^{2\pi} |f(re^{i\theta})^2| d\theta \]

\[\geq (1 - m) \int_0^{2\pi} |(re^{i\theta}f'(re^{i\theta}))^2| d\theta - \int_0^{2\pi} |f(re^{i\theta})^2| d\theta \]

\[= 2\pi \sum_{n=1}^{\infty} (1 - m)n^2|a_n|^2 r^2 - 2\pi \sum_{n=1}^{\infty} |a_n|^2 r^2.\]
Thus
\[
2 \sum_{n=1}^{\infty} |a_n|^2 r^2 \geq \sum_{n=1}^{\infty} (1 - m)n^2 |a_n|^2 r^2.
\]
If we let \( r \to 1^- \), we obtain from the last inequality
\[
2 \sum_{n=1}^{\infty} |a_n|^2 \geq \sum_{n=1}^{\infty} (1 - m)n^2 |a_n|^2
\]
which gives,
\[
(4.5) \quad 1 + m \geq \sum_{n=2}^{\infty} \left( (1 - m)n^2 - 2 \right)|a_n|^2.
\]
Since \((1 - m)n^2 - 2 \geq 0\) for all \( n \geq 2 \) if and only if \( 1 \leq M \leq 2 \) then \((4.5)\) yields \((4.1)\) and \((4.2)\) according to the case \( 1 \leq M \leq 2 \) or \( M > 2 \).

□

The following corollary is an immediate consequence of \((4.2)\).

**Corollary 4.1.** Let \( f(z) = \sum_{n=0}^{\infty} a_n z^n \) be a function in \( S^*_L(M) \). Then

for \( 1 \leq M \leq 2 \) we have
\[
(4.6) \quad |a_n| \leq \sqrt{\frac{1 + m}{(1 - m)n^2 - 2}}, \quad \text{for } n \geq 2
\]
and for \( M > 2 \) we have
\[
(4.7) \quad |a_n| \leq \sqrt{\frac{1 + m - \sum_{2 \leq k < \sqrt{\frac{2}{1 - m}}}(1 - m)k^2 |a_k|^2}{(1 - m)n^2 - 2}}; \quad \text{for } n \geq \sqrt{\frac{2}{1 - m}}.
\]
with \( m = \frac{M - 1}{M} \).

**Remark 4.1.** For \( M = 1 \), \((4.1)\) and \((4.6)\) give respectively Theorem 1 and Corollary 1 [6].

**Theorem 4.2.** Let \( f(z) = \sum_{n=0}^{\infty} a_n z^n \) be a function in \( S^*_L(M) \). Then

(i) \( |a_2| \leq \frac{m+1}{2} \), for \( 0 \leq m \leq 1 \);

(ii) \( |a_3| \leq \frac{m+1}{4} \), for \( 0 \leq m \leq \frac{3}{5} \);

(iii) \( |a_4| \leq \frac{m+1}{6} \), for \( 0 \leq m \leq \frac{\sqrt{5} - 1}{4} \).

This estimations are sharp.

**Proof.** If \( f \in S^*_L(M) \) there exists \( \omega(z) = \sum_{n=1}^{\infty} C_n z^n \in B \) such that
\[
(4.8) \quad (z f' (z))^2 - f(z)^2 = \omega(z)(m(z f' (z))^2 + f(z)^2), \quad z \in \mathbb{D}.
\]
Let \( f(z)^2 = \sum_{n=2}^{\infty} A_n z^n \), \((z f' (z))^2 = \sum_{n=2}^{\infty} B_n z^n \). \((4.8)\) becomes
\[
(4.9) \quad \sum_{n=2}^{\infty} (B_n - A_n) z^n = \left( \sum_{n=2}^{\infty} (mB_n + A_n) z^n \right) \left( \sum_{n=1}^{\infty} C_n z^n \right)
\]
Equating coefficients for \( n = 2, n = 3 \) in both sides of (4.9), we obtain

\[
(S_m) \begin{cases}
B_3 - A_3 = (mB_2 + A_2)C_1 \\
B_4 - A_4 = (mB_2 + A_2)C_2 + (mB_3 + A_3)C_1 \\
B_5 - A_5 = (mB_2 + A_2)C_3 + (mB_3 + A_3)C_2 + (mB_4 + A_4)C_1
\end{cases}
\]

A little calculation yields

\[
A_2 = a_1 = 1, \quad A_3 = 2a_2, \quad A_4 = 2a_3 + a_2^2, \quad A_5 = 2a_4 + 2a_2a_3
\]

and

\[
B_2 = a_1 = 1, \quad B_3 = 4a_2, \quad B_4 = 6a_3 + 4a_2^2, \quad B_5 = 8a_4 + 12a_2a_3.
\]

Replacing in \((S_m)\), we obtain

\[
\begin{cases}
(1) \quad 2a_2 = (m + 1)C_1 \\
(2) \quad 4a_3 + 3a_2^2 = (m + 1)C_2 + (4m + 2)a_2C_1 \\
(3) \quad 6a_4 + 10a_2a_3 = (m + 1)C_3 + (2m + 1)(m + 1)C_1C_2 + ((6m + 2)a_3 + (4m + 1)a_2^2)C_1
\end{cases}
\]

Since \(|C_1| \leq 1\) then (1) implies that \(|a_2| \leq \frac{1 + m}{2}\). This proves the assertion (i). On the other hand we have from (1) and (2)

\[
a_3 = \frac{1 + m}{4}C_2 + \frac{(5m + 1)(m + 1)}{16}C_1^2.
\]

Thus

\[
|a_3| \leq \frac{1 + m}{4}(|C_2| + \frac{5m + 1}{4}|C_1|).
\]

It is well known that \(|C_2| \leq 1 - |C_1|^2\). Therefore we obtain

\[
|a_3| \leq \frac{1 + m}{4}(1 - |C_1|^2 + \frac{5m + 1}{4}|C_1|)
\]

\[
= \frac{1 + m}{4}(1 + \frac{5m - 3}{4}|C_1|).
\]

Since \(5m - 3 \leq 0\) if and only if \(m \leq \frac{3}{5}\) then (4.10) yields the assertion (ii).

Replacing the values of \(a_2\) and \(a_3\) in the equation (3), we obtain

\[
a_4 = \frac{m + 1}{6}C_3 + \frac{(m + 1)(9m + 1)}{24}C_1C_2 + \frac{(m + 1)(21m^2 + 6m + 1)}{96}C_1^3
\]

\[
= \frac{m + 1}{6} \left(C_3 + \frac{9m + 1}{4}C_1C_2 + \frac{21m^2 + 6m + 1}{16}C_1^3\right).
\]

Let \(\mu = \frac{9m + 1}{4}\) and \(\nu = \frac{21m^2 + 6m + 1}{16}\). Under the assumption \(0 \leq m \leq \frac{\sqrt{7} - 1}{7}\), we have \((\mu, \nu) \in D_1\) (see [4], p. 127). Therefore by Lemma 2 [4] we obtain

\[
\left|C_3 + \frac{9m + 1}{4}C_1C_2 + \frac{21m^2 + 6m + 1}{16}C_1^3\right| \leq 1
\]
which yields from (4.11) the assertion (iii).

The sharpness of (i) is given by the function $f_{m}$. If we take in (4.8) $\omega(z) = z^2$ and $\omega(z) = z^3$ successively, we obtain two functions in $S^*_L(M)$:

$$f_{1,m}(z) = z + \frac{m + 1}{4}z^3 + \ldots$$

and

$$f_{2,m}(z) = z + \frac{m + 1}{6}z^4 + \ldots$$

which give respectively the sharpness of estimations (ii) and (iii).

Remark 4.2. The estimation (i) can be obtained directly from (2.6).

Remark 4.3. If we take $m = 0$ in Theorem 4.2, we obtain as particular case Theorem 2 [6].

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References

[1] D.A. Brannan, W.E. Kirwan, On Some Classes of Bounded Univalent Functions, Journal of the London Mathematical Society. s2-1 (1969), 431–443.

[2] W. Janowski, Extremal Problems for a Family of Functions with Positive Real Part and for Some related Families, Ann. Polon. Math. 23 (1970), 159-177.

[3] Wancang Ma, David Minda, A Unified Treatment of some Classes of Univalent Functions, Proceeding of the International Conference on Complex Analysis at the Nankai Institute of Mathematics, 1992, 157-169.

[4] Dimitri V. Prokhorov, Jan Szynal, Inverse Coefficients for $(\alpha,\beta)$-convex Functions, Ann. Univ. Mariae Curie-Skłodowska, XXXV (15) (1981), 125-143.

[5] S. Ruscheweyh, A Subordination Theorem for $\Phi$-Like Functions, J. Lond. Math. Soc. s2-13 (1976), 275–280.

[6] Janusz Sókol, Coefficient Estimates in a Class of Strongly Starlike Functions, Kyungpook Math. J. 49 (2009), 349-353.

[7] Janusz Sókol, On Some Subclass of Strongly Starlike Functions, Demonstr. Math. XXXI (1) (1988), 81-86.

[8] J. Sókol, J. Stankiewicz, Radius of Convexity of some Subclass of Strongly Starlike Functions, Folia Sci. Univ. Tech. Resoviensis, Math. 19 (1996), 101-105.

[9] J. Stankiewicz, Quelques problèmes extrémaux dans les Classes des Fonctions $\alpha$-angulairement étoilées, Ann. Univ. Mariae Curie-Skłodowska Sect. A, 20 (1966), 59-75.