Hartman effect and non-locality in quantum networks

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We study the phase time for various quantum mechanical networks having potential barriers in its arms to find the generic presence of Hartman effect. In such systems it is possible to control the ‘super arrival’ time in one of the arms by changing parameters on another, spatially separated from it. This is yet another quantum nonlocal effect. Negative time delays (time advancement) and ‘ultra Hartman effect’ with negative saturation times have been observed in some parameter regimes.

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I. INTRODUCTION

Quantum tunneling, where a particle has finite probability to penetrate a classically forbidden region is an important feature of wave mechanics. Invention of the tunnel diode [1], the scanning tunneling microscope [2] etc. make it useful from a technological point of view. In 1932 MacColl [3] pointed out that tunneling is not only characterized by a tunneling probability but also by a time the tunneling particle takes to traverse the barrier. There is considerable interest on the question of time spent by a particle in a given region of space [4, 5, 6]. The recent development of nanotechnology brought new urgency to study the tunneling time as it is directly related to the maximum attainable speed of nanoscale electronic devices. In a number of numerical [7], experimental [8, 9] and analytical study of quantum tunneling processes, various definitions of tunneling times have been investigated. These different time scales are based on various different operational definitions and physical interpretations. Till date there is no clear consensus about the existence of a simple expression for this time as there is no hermitian operator associated with it [10]. Among the various time scales, ‘dwell time’ [10] which gives the duration of a particle’s stay in the barrier region regardless of how it escapes can be calculated as the total probability of the particle inside the barrier divided by the transmission coefficient. In the case of ‘not too opaque’ barriers, the tunneling time evaluated either as a simple ‘phase time’ [5] or calculated through the analysis of the wave packet behaviour [11] becomes independent of the barrier width. This phenomenon is termed as the Hartman effect [12, 13, 14]. This implies that for sufficiently large barriers the effective velocity of the particle can become arbitrarily large, even larger than the light speed in the vacuum (superluminal effect). Though this interpretation is a little far fetched for non-relativistic Schrödinger equation as velocity of light plays no role in it, this effect has been established even in relativistic quantum mechanics.

Though experiments with electrons for verifying this prediction are yet to be done, the formal identity between the Schrödinger equation and the Helmholtz equation for electromagnetic wave enables one to correlate the results for electromagnetic and microwaves to that for electrons. Photonic experiments show that electromagnetic pulses travel with group velocities in excess of the speed of light in vacuum as they tunnel through a constriction in a waveguide [20]. Experiments with photonic band-gap structures clearly demonstrate that ‘tunneling photons’ indeed travel with superluminal group velocities [5]. Their measured tunneling time is practically obtained by comparing the two peaks of the incident and
transmitted wave packets. Thus all these experiments directly or indirectly confirmed the occurrence of Hartman effect without violating ‘Einstein causality’ i.e., the signal velocity or the information transfer velocity is always bounded by the velocity of light. This tunneling time could be interpreted as the ‘time of passage’ of the peak. Since the velocity of the ‘peak’ may exceed arbitrarily large numbers, this ‘fast tunneling’ has been frequently related to ‘superluminal propagation’ [20]. The ‘Hartman effect’ has been extensively studied both for nonrelativistic (Schrödinger equation) and relativistic (Dirac equation) [1, 5, 20] cases. Recently Winful [21] showed that the saturation of phase time is a direct consequence of saturation of integrated probability density under the barrier. Due to this saturation addition of a new particle in the incident side leads to an almost immediate release of another particle from the other side and these two particles are causally unrelated; i.e. even the superluminal tunneling does not violate causality. Equivalently, for electromagnetic waves the origin of the Hartman effect has been traced to stored energy. Since the stored energy in the evanescent field decreases exponentially within the barrier after a certain decay distances it becomes independent of the width of the barrier. The Hartman effect has been found in one dimensional barrier tunneling [18] as well as for cases beyond one dimension as in tunneling through mesoscopic rings in presence of Aharonov-Bohm flux [22]. In the current note we extend the study of phase times for branched networks of quantum wires. Such networks can readily be realized in optical wave propagation experiments.

In Section V we give the description of typical quantum network systems under consideration. The theoretical framework used to analyze these systems is provided in Section IV. All the main results showing Hartman effect and the effects of quantum non-locality are discussed in Section IV. Finally we summarize the results and draw conclusions in Section V.

II. DESCRIPTION OF THE SYSTEM

As a model system, we choose a network of thin wires. The width of these wires are so narrow that only the motion along the length of the wires is of interest (a single channel case). The motion in the perpendicular direction is frozen in the lowest transverse subband. In a three-port Y-branch circuit (Fig. 1) two side branches of quantum wire $S_1$ and $S_2$ are connected to a ‘base’ arm $S_0$ at the junction $J$. In general one can have $N(\geq 2)$ such side branches connected to a ‘base’ wire.

We study the scattering problem across a network geometry as presented schematically in Fig. 1.

Such geometries are important from the point of view of basic science due to their properties of tunneling and interference [23, 24] as well as in applications such as wiring in nano-structures. In particular, the Y-junction carbon nanotubes are in extensive studies and they show various interesting properties like asymmetric current voltage characteristics [23]. In our system of interest there are finite quantum mechanical potential barriers of strength $V_n$ and width $w_n$ in the $n$-th side branch. The number of side branches can vary from $n = 2, 3, \cdots N$. We focus on a situation wherein the incident electrons have an energy $E$ less than $V_n$ for all $n$. The impinging electrons in this subbarrier regime travels as an evanescent mode/wave and the transmission involve contributions from quantum tunneling and multiple reflections between each pair of barriers and the junction point. Here we are interested in a single channel case where the Fermi energy lie in the lowest subband. To excite the evanescent modes in the side branches one has to produce constrictions by making the width of the regions of wires containing barriers much thinner than that of other parts of the wires. The electrons occupying the lowest subband in the connecting wire on entering the constrictions experience a potential barrier (due to higher quantum zero point energy) and propagate as an evanescent mode [27]. In this work an analysis of the phase time or the group delay time in such a system is carried out.

III. THEORETICAL TREATMENT

We approach this scattering problem using the quantum wave guide theory [20, 28]. In the stationary case the incoming particles are represented by a plane wave $e^{ikz}$ of unit amplitude. The effective mass of the propagating particle is $m$ and the energy is $E = \hbar^2 k^2 / 2m$ where $k$ is the wave vector corresponding to the free particle. The wave functions, which are solutions of the Schrödinger equation, in different regions of the system considered (Fig. 1) can be written as,

\[ \phi_{\text{in}}(z) = e^{ikz} \]

\[ \phi_{\text{out}}(z) = \int \phi_{\text{in}}(z') \psi(z,z') \, dz' \]

\[ \psi(z,z') = \frac{1}{\sqrt{L_z}} e^{-iKz'} \]

\[ K = \frac{\hbar^2}{2m} \left( \frac{1}{L_z} \right)^2 \]

\[ S_{\text{in}}(z) = \int S_{\text{out}}(z') \psi(z,z') \, dz' \]

\[ S_{\text{out}}(z) = \int S_{\text{in}}(z') \psi(z,z') \, dz' \]

\[ \phi_{\text{in}}(z) = \phi_{\text{in}}(z') \psi(z,z') \]

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\[ S_{\text{out}}(z) = \int S_{\text{in}}(z') \psi(z,z') \, dz' \]

\[ \phi_{\text{in}}(z) = \phi_{\text{in}}(z') \psi(z,z') \]

\[ \phi_{\text{out}}(z) = \phi_{\text{out}}(z') \psi(z,z') \]
with \( \kappa_n = \sqrt{2m(V_n - E)/\hbar^2} \) being the imaginary wave vector in presence of rectangular barrier of strength \( V_n \).

\( \psi_{in}(x_0) = e^{ikx_0} + Re^{-ikx_0} \) \( (\text{in } S_0) \),
\[ \psi_{(n)}(x_n) = A_n e^{ikx_n} + B_n e^{-ikx_n} \] \( (\text{region I in } \text{S}_n) \),
\[ \psi_{(n)l}(x_n) = C_n e^{-\kappa_n(x_n - lb_n)} + D_n e^{\kappa_n(x_n - lb_n)} \] \( (\text{region II in } \text{S}_n) \),
\[ \psi_{(n)ll}(x_n) = t_n e^{ik(x_n - lb_n - w_n)} \] \( (\text{region III in } \text{S}_n) \),

In what follows, let us set \( \hbar = 1 \) and \( 2m = 1 \). We now proceed to analyze the behavior of \( \tau_n \) as a function of various physical parameters for different network topologies. We measure time at the far end of each barrier in the branched arms containing barriers and in the case of arms in absence of any barrier we measure the phase time at the junction points. We express all the physical quantities in dimensionless units i.e. all the barrier strengths \( V_n \) in units of incident energy \( E (V_n = V_n/E) \), all the barrier widths \( w_n \) in units of inverse wave vector \( k^{-1} (w_n = k w_n) \), where \( k = \sqrt{E} \) and all the extrapolated phase time \( \tau_n \) in units of inverse incident energy \( E (\tau_n = E \tau_n) \).

First we take up a system similar to the Y-junction shown in Fig. 1 in presence of a barrier \( V_1 \) of width \( w_1 \) in arm \( S_1 \) but in absence of any barrier in arm \( S_2 \). For a tunneling particle having energy \( E < V_1 \) we find out the phase time \( \tau_1 \) in arm \( S_1 \) as well as \( \tau_2 \) in arm \( S_2 \) as a function of barrier width \( w_1 \) (Fig. 2). From Fig. 2(a) it is clear that \( \tau_1 \) evolves with \( w_1 \) and eventually saturates to \( \tau_{s1} \) for large \( w_1 \) to show the Hartman effect. Fig. 2(b)

\[
\psi_{in}(J) = \psi_{(n=1)l}(J) = \psi_{(n=2)l}(J) = \cdots = \psi_{(n=N)l}(J),
\]
and
\[
\frac{\partial \psi_{in}(x_0)}{\partial x_0} \bigg|_J = \sum_n \frac{\partial \psi_{(n)l}}{\partial x_n} \bigg|_J,
\]
at the junction \( J \). All the derivatives are taken either outward or inward from the junction [23]. In each side branch, at the starting and end points of the barrier, the boundary conditions can be written as

\[ \psi_{(n)}(lb_n) = \psi_{(n)ll}(lb_n), \]
\[ \psi_{(n)l}(lb_n + w_n) = \psi_{(n)ll}(lb_n + w_n), \]
\[ \frac{\partial \psi_{(n)l}}{\partial x_n} \bigg|_{(lb_n)} = \frac{\partial \psi_{(n)ll}}{\partial x_n} \bigg|_{(lb_n)}, \]
\[ \frac{\partial \psi_{(n)ll}}{\partial x_n} \bigg|_{(lb_n + w_n)} = \frac{\partial \psi_{(n)lll}}{\partial x_n} \bigg|_{(lb_n + w_n)}. \]

From the above mentioned boundary conditions one can obtain the complex transmission amplitudes \( t_n \) on each of the side branches.

**IV. RESULTS AND DISCUSSIONS**

Once \( t_n \) is known, the ‘phase time’ (phase time for transmission) can be calculated from the energy derivative of the phase of the transmission amplitude [7, 12] as

\[ \tau_n = \hbar \frac{\partial \text{Arg}[t_n]}{\partial E}, \]

where, \( v = \hbar k/m \) is the velocity of the free particle. The concept of ‘phase delay time’ was first introduced by Wigner [12] to estimate how long a quantum mechanical wave packet is delayed by the scattering obstacle.

![Fig. 2](image-url)
shows the phase time $\tau_2$ in arm $S_2$ which does not contain any barrier. This also evolves and saturates with $w_1$, the length of the barrier in the other arm $S_1$. This delay is due to the contribution from paths which undergo multiple reflection in the first branch before entering the second branch via junction point $J$. In absence of a barrier in the $n$-th arm the phase time $\tau_n$ measured close to the junction $J$ should go to zero i.e. $\tau_n \to 0$ in the absence of multiple scatterings in the first arm. Note that $\tau_{S1}$ and $\tau_{S2}$ change with energies of the incident particle (Fig. 2). From Fig. 2 it can be easily seen that $\tau_{S2}$ is always smaller than $\tau_{S1}$ for any particular $V_1$ i.e. the saturation time in the arm having no barrier is smaller. The phase time in both the arms show non-monotonic behavior as a function of $V_1$. As we decrease the strength of the barrier in arm $V_1$ the value of $\tau_{S1}$ ($\tau_{S2}$) decreases in the whole range of widths of the barrier and also the saturated value of $\tau_{S1}$ ($\tau_{S2}$) decreases until $V_1$ reaches 1.6 and with further decrease in $V_1$ the values of $\tau_{S1}$ ($\tau_{S2}$) as well as $\tau_{S1}$ ($\tau_{S2}$) starts increasing.

As the second case we take up another Y-junction which contain potential barriers in both its side branches as shown in Fig. 1. We fix the values of $V_1(=5)$ and vary $w_1$ for each values of $V_2$ to study the $w_1$-dependence of $\tau_1$ (Fig. 3). From Fig. 3 we see that $\tau_1$ decreases with increase in $w_1$ to saturate to a value $\tau_{S1}$ at each value of $V_2$ thereby showing ‘Hartman effect’ for arm ‘$S_1$’. But now, we can tune the saturation phase time at arm $S_1$ non-locally by tuning strength of the barrier potential $V_2$ sitting on another arm $S_2$. Thus ‘quantum nonlocality’ enables us to control the ‘super arrival’ time in one of the arms ($S_1$) by changing a parameter ($V_2$) on another, spatially separated from it. In the inset of Fig. 3 we plot $\tau_{S3}$ as a function of $V_2$. It clearly shows that when the barrier strengths $V_1$ and $V_2$ are very close the ‘phase time’ reaches its minimum value. In all other cases i.e. whenever $V_1 \neq V_2$, the value of $\tau_{S1}$ is larger.

We will show now another interesting result related to the Hartman effect. For this we keep $V_2(=5)$ unaltered and reduce $w_2$. For very small $w_2(=0.5)$ we see from Fig. 3 that $\tau_1$ is negative for almost the whole range of $w_1$-values showing ‘time-advancement’ and eventually after a sharp decrease saturates to a negative value of $\tau_{S1} = -4.514$ implying ‘Hartman effect’ with advanced time. It might be noted that, in principle, the ‘time-advancement’ (Fig. 3) can be measured experimentally as $|t_1|^2$ has a non-zero finite value for a small range of $w_1$ at lower $w_1$ regimes where $\tau_1$ is negative. In the inset we plot the corresponding $\tau_2$ and $|t_2|^2$ as function of $w_1$. Again the values of $\tau_2$ remains different from the one dimensional tunneling through a barrier of strength $V_2$ and width $w_2$ in the whole range of $w_1$ implying ‘quantum nonlocality’. In the cases discussed so far $\tau_2$ vary more sharply in small $w_1$ regime. Further the inset in Fig. 3 shows a dip in $\tau_2$ at parameter regimes where $|t_2|^2$ has a minimum. For a wave packet with large spread in real space it is possible that the leading edge of the wave packet reaches the barrier much earlier than the peak of the packet. This leading edge in turn can tunnel through to produce a peak in the other end of the barrier much before the incident wave packet reaches the barrier region, sometimes referred to as pulse reshaping effect. This, in general, causes ‘time advancement’.
V. CONCLUSIONS

We have studied Hartman effect and non-locality in quantum network consisting of a main one dimensional arm having \( N(\geq 2) \) side branches. These side branches may or may not have barriers. In presence of barrier the ‘phase time’ for transmission through a side branch shows the ‘Hartman effect’. In general, as the number of side branches \( N \) increases, the saturated ‘phase time’ decreases. Due to quantum nonlocality the ‘phase time’ and it’s saturated value at any side branch feels the presence of barriers in other branches. Thus one can tune the saturation value of ‘phase time’ and consequently the superluminal speed in one branch by changing barrier strength or width in any other branch, spatially separated from the former. Moreover Hartman effect with negative saturation times (time advancement) has been observed for some cases. In conclusion generalization of Hartman effect in branched networks exhibits several counter-intuitive results due to quantum non-locality. System parameters such as number of side branches \( N \) and barrier widths \( w_n \), strengths \( V_n \), distance \( lb_n \) (from \( J \)) and incident energy \( E \) etc. play very sensitive roles in determining delay times. The delay times are also sensitive to the junction S-matrix elements used for a given problem. In our present problem junction S-matrix is determined uniquely by the wave guide transport methods. Depending on \( lb_n \) there may be one or several bound states located between the barriers in different branched arms and as a consequence saturated delay time can be varied from the negative (ultra Hartman effect) to positive and vice-versa. We have verified this by looking at the transmission coefficient in the second arm \( S_2 \) which exhibits clear resonances as a function of \( lb_n \). Around these resonances, saturated delay time in the first arm \( \tau_{s1} \) changes the sign [the details of which will be published elsewhere \(^{31}\)]. Moreover the reported effects are amenable to experimental verifications in the electromagnetic wave-guide networks.

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[1] R. Tsu and L. Esaki, Appl. Phys. Lett. 22, 562 (1973).
[2] G. Binning and H. Rohrer, IBM J. Res. Dev. 30, 355 (1986).
[3] L. A. MacColl, Phys. Rev. 40, 621 (1932).
[4] R. Landauer and T. Martin, Rev. Mod. Phys. 66 (1994) 217.
[5] E. H. Hauge and J. A. Støvneng, Rev. Mod. Phys. 61 (1989) 917.

[6] S. Anantha Ramakrishna and N. Kumar, Europhys. Lett., 60 (2002) 491; Colin Benjamin and A. M. Jayannavar, Solid State Commun., 121 (2002) 591.

[7] S. Collins, D. Lowe and J. R. Barker, J. Phys. C 20, 6213 (1987); R. S. Dumont and T. L. Marchioro II, Phys. Rev. A 47, 85 (1993).

[8] A. M. Steinberg, P. G. Kwiat, and R. Y. Chiao, Phys. Rev. Lett. 71 708 1993; R. Y. Chiao, P. G. Kwiat, and A. M. Steinberg, Scientific American, p. 38-46, August 1993.

[9] P. Guerent, E. Marclay, and H. Meier, Solid State Commun. 68, 977 (1988); Ch. Spielmann, R. Szipocs, A. Sting, and F. Krausz, Phys. Rev. Lett. 73, 2308 (1994); Th. Hills et al., Phys. Rev. A 58, 4784 (1998).

[10] F. T. Smith, Phys. Rev. 118, 349 (1960).

[11] M. Büttiker and R. Landauer, Phys. Rev. Lett. 49, 1739 (1982).

[12] E. P. Wigner, Phys. Rev. 98 (1955) 145.

[13] T. E. Hartman, J. Appl. Phys. 33 (1962) 3427.

[14] E. H. Hauge, J. P. Falck, and T. A. Fjeldly, Phys. Rev. B 36, 4203 (1987).

[15] M. Büttiker, J. Phys. Condensed Matter, 5 (1993) 9361; M. Büttiker, H. Thomas and A. Prêtre, Phys. Lett. A 180 (1993) 364

[16] M. Büttiker, Pramana-journal of physics, 58 (2002) 241.

[17] A. M. Jayannavar, G. V. Vijaygobindan and N. Kumar, Z. Phys. B 75 (1989) 77; Sandeep K. Joshi, A. Kar Gupta and A. M. Jayannavar, Phys. Rev. B58 (1998) 1092; Sandeep K. Joshi and A. M. Jayannavar, Solid State Commun. 106 (1998) 363; ibid 111 (1999) 547; For a review see Yan Fyodorov and H. Sommers, J. Math. Phys. 38 (1997) 1918.

[18] V. S. Olkhovsky, E. Recami and G. Salesi, Europhys. Lett. 57 (2002) 879; V. S. Olkhovsky and E. Recami, Physics Reports 214 (1992) 339.

[19] J. R. Fletcher, J. Phys. C 18, L55 (1985).

[20] A. Enders and G. Nimtz, J. Phys. I 2 (1992) 1693; 3 (1993) 1089; A. Enders and G. Nimtz, Phys. Rev. E, 48, 632 (1993); G. Nimtz, A. Enders and H. Spieker, ibid 4 (1994) 565.

[21] H. G. Winful, Phys. Rev. Lett. 91 (2003) 260401; H. G. Winful, Phys. Rev. E68 (2003) 016615; H. G. Winful, Opt. Express 10 (2002) 1491.

[22] Swarnali Bandopadhyay, Raishma Krishnan, and A. M. Jayannavar, Solid State Commun. 131, 447 (2004).

[23] S. Washburn and R. A. Webb, Adv. Phys., 35 (1986) 375.

[24] Y. Gefen, Y. Imry and M. Ya Azbel, Phys. Rev. Lett. 52 (1984) 129.

[25] B. C. Satishkumar, P. John Thomas, A. Govindaraj, and C. N. R. Rao, Applied Physics Letters 77(16) (2000) pp. 2530-2532.

[26] J. B. Xia, Phys. Rev. B45 (1992) 3593.

[27] A. M. Jayannavar and P. Singha Deo, Mod. Phys. Lett. B 8 (1994) 301-310; B. C. Gupta, P. Singha Deo and A. M. Jayannavar, Int. J. Mod. Phys. B 10 (1996) 3595; Colin Benjamin and A. M. Jayannavar, Phys. Rev. B68 (2003) 085325.

[28] P. S. Deo and A. M. Jayannavar, Phys. Rev. B50 (1994) 11629.

[29] S. Griffith, Trans. Faraday Soc. 49, 345 (1953) 650.

[30] J. G. Muga, I. L. Egusquiza, J. A. Damborenea, and F. Delgado, Phys. Rev. A, 66, 042115 (2002).

[31] Swarnali Bandopadhyay, A. M. Jayannavar, unpublished.