ROTATIONALLY SYMMETRIC HARMONIC DIFFEOMORPHISMS BETWEEN SURFACES

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Abstract. In this paper, we show that the nonexistence of rotationally symmetric harmonic diffeomorphism between the unit disk without the origin and a punctured disc with hyperbolic metric on the target.

1. Introduction

The existence of harmonic diffeomorphisms between complete Riemannian manifolds has been extensively studied, please see for example [1]-[34]. In particular, Heinz [17] proved that there is no harmonic diffeomorphism from the unit disc onto $\mathbb{C}$ with its flat metric. On the other hand, Schoen [25] mentioned a question about the existence, or nonexistence, of a harmonic diffeomorphism from the complex plane onto the hyperbolic 2-space. At the present time, many beautiful results about the asymptotic behavior of harmonic embedding from $\mathbb{C}$ into the hyperbolic plane have been obtained, please see for example [32, 14, 4, 5], or the review [33] by Wan and the references therein. In 2010, Collin and Rosenberg [10] constructed harmonic diffeomorphisms from $\mathbb{C}$ onto the hyperbolic plane. In [28, 29, 24, 7], the authors therein studied the rotational symmetry case. One of their results is the nonexistence of rotationally symmetric harmonic diffeomorphism from $\mathbb{C}$ onto the hyperbolic plane.

In this paper, we will study the existence, or nonexistence, of rotationally symmetric harmonic diffeomorphisms from the unit disk without the origin onto a punctured disc. For simplicity, let us denote

$D^* = D \setminus \{0\}$ and $P(a) = D \setminus \{|z| \leq e^{-a}\}$ for $a > 0$,

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here $\mathbb{D}$ is the unit disc, and $z$ is the complex coordinate of $\mathbb{C}$. We will prove the following results.

**Theorem 1.1.** For any $a > 0$, there is no rotationally symmetric harmonic diffeomorphism from $\mathbb{D}^*$ onto $P(a)$ with its hyperbolic metric.

And vice versa, that is:

**Theorem 1.2.** For any $a > 0$, there is no rotationally symmetric harmonic diffeomorphism from $P(a)$ onto $\mathbb{D}^*$ with its hyperbolic metric.

We will also consider the Euclidean case, and will prove the following theorem.

**Theorem 1.3.** For any $a > 0$, there is no rotationally symmetric harmonic diffeomorphism from $\mathbb{D}^*$ onto $P(a)$ with its Euclidean metric; but on the other hand, there are rotationally symmetric harmonic diffeomorphisms from $P(a)$ onto $\mathbb{D}^*$ with its Euclidean metric.

This paper is organized as follows. In section 2, we will prove Theorem 1.1 and Theorem 1.2. Theorem 1.3 will be proved in section 3. At the last section, we will give an alternative proof for the nonexistence of rotationally symmetric harmonic diffeomorphism from $\mathbb{C}$ onto the hyperbolic disc.

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2. **Harmonic maps from $\mathbb{D}^*$ to $P(a)$ with its hyperbolic metric and vice versa**

For convenience, let us recall the definition about the harmonic maps between surfaces. Let $M$ and $N$ be two oriented surfaces with metrics $\tau^2|dz|^2$ and $\sigma^2|du|^2$ respectively, where $z$ and $u$ are local complex coordinates of $M$ and $N$ respectively. A $C^2$ map $u$ from $M$ to $N$ is harmonic if and only if $u$ satisfies

\[(2.1) \quad u_{zz} + \frac{2\sigma_u}{\sigma} u_z u_{\bar{z}} = 0.\]

Now let us prove Theorem 1.1.

**Proof of Theorem 1.1.** First of all, let us denote $(r, \theta)$ as the polar coordinates of $\mathbb{D}^*$, and $u$ as the complex coordinates of $P(a)$ in $\mathbb{C}$, then the hyperbolic metric $\sigma_1|du|$ on $P(a)$ can be written as

\[(2.2) \quad -\frac{\pi|du|}{a|u| \sin \left( \frac{2}{a} \ln |u| \right)}\]
here \(|u|\) is the norm of \(u\) with respect to the Euclidean metric.

We will prove this theorem by contradiction. Suppose \(u\) is a rotationally symmetric harmonic diffeomorphism from \(\mathbb{D}^*\) onto \(P(a)\) with the metric \(\sigma_1 d|u|\). Because \(\mathbb{D}^*\), \(P(a)\) and the metric \(\sigma_1 d|u|\) are rotationally symmetric, we can assume that such a map \(u\) has the form \(u = f(r)e^{i\theta}\). Substituting \(u\), \(\sigma_1\) to (2.1), we can get

\[
(2.3) \quad f'' + \frac{f'}{r} - \frac{f}{r^2} - \frac{\sin \left(\frac{\pi}{a} \ln f\right)}{f \sin \left(\frac{\pi}{a} \ln f\right)} \left(\left(f'\right)^2 - \frac{f^2}{r^2}\right) = 0
\]

for \(1 > r > 0\). Since \(u\) is a harmonic diffeomorphism from \(\mathbb{D}^*\) onto \(P(a)\), we have

\[
(2.4) \quad f(0) = e^{-a}, \quad f(1) = 1 \quad \text{and} \quad f'(r) > 0 \quad \text{for} \quad 1 > r > 0,
\]

or

\[
(2.4) \quad f(0) = 1, \quad f(1) = e^{-a} \quad \text{and} \quad f'(r) < 0 \quad \text{for} \quad 1 > r > 0.
\]

We will just deal with the case that (2.4) is satisfied, the rest case is similar. Let \(F = \ln f \in (-a, 0)\), then we have

\[
F' = \frac{f'}{f} > 0, \quad F'' = \frac{f''}{f} - \left(\frac{f'}{f}\right)^2.
\]

Using this fact, we can get from (2.3) the following equation.

\[
(2.5) \quad F'' + \frac{1}{r} F' - \frac{\pi}{a} \cotg \left(\frac{\pi}{a} F\right) \left(F'\right)^2 + \frac{1}{r^2} \frac{\pi}{a} \cotg \left(\frac{\pi}{a} F\right) = 0 \quad \text{for} \quad 1 > r > 0
\]

with \(F(0) = -a\), \(F(1) = 0\) and \(F'(r) > 0\) for \(1 > r > 0\).

Regarding \(r\) as a function of \(F\), we have the following relations.

\[
(2.6) \quad F_r = r^{-1} F^{-1}, \quad F_{rr} = -r^{-3} r_{FF}.
\]

Using these facts, we can get from (2.5) the following equation.

\[
(2.7) \quad \frac{r''}{r} - \left(\frac{r'}{r}\right)^2 + \frac{\pi}{a} \cotg \left(\frac{\pi}{a} F\right) \frac{r'}{r} - \left(\frac{r'}{r}\right)^3 \frac{\pi}{a} \cotg \left(\frac{\pi}{a} F\right) = 0
\]

for \(0 > F > -a\). Let \(x = (\ln r)'(F)\), from (2.7) we can get the following equation.

\[
x' + \frac{\pi}{a} \cotg \left(\frac{\pi}{a} F\right) \cdot x - \frac{\pi}{a} \cotg \left(\frac{\pi}{a} F\right) \cdot x^3 = 0.
\]

One can solve this Bernoulli equation to obtain

\[
x^{-2} = 1 + c_0 \left(\sin \left(\frac{\pi}{a} F\right)\right)^2
\]
here $c_0$ is a constant depending on the choice of the function $f$. So
\[ x = \frac{1}{\sqrt{1 + c_0 \left( \sin \left( \frac{\pi}{a} F \right) \right)^2}}. \]
Since $x = (\ln r)'(F)$, we can get
\[ (\ln r)(F) = \int_0^F x(t) \, dt = \int_0^F \frac{1}{\sqrt{1 + c_0 \left( \sin \left( \frac{\pi}{a} t \right) \right)^2}} \, dt. \]
Noting that $x(F)$ is continuous in $(-a, 0)$ and is equal to 1 as $F = -a$, or 0, one can get $x$ is uniformly bounded for $F \in [-a, 0]$. So the right hand side of (2.8) is uniformly bounded, but the left hand side will tend to $-\infty$ as $F \to -a$. Hence we get a contradiction. Therefore such $f$ does not exist, Theorem 1.1 has been proved. □

We are going to prove Theorem 1.2.

**Proof of Theorem 1.2.** First of all, let us denote $(r, \theta)$ as the polar coordinates of $P(a)$, and $u$ as the complex coordinates of $D^*$ in $C$, then the hyperbolic metric $\sigma_d |u|$ on $D^*$ can be written as
\[ (2.9) \quad \frac{|du|}{|u| \ln \frac{1}{|u|}}, \]
here $|u|$ is the norm with respect to the Euclidean metric.

We will prove this theorem by contradiction. The idea is similar to the proof of Theorem 1.1. Suppose $\psi$ is a rotationally symmetric harmonic diffeomorphism from $P(a)$ onto $D^*$ with the metric $\sigma_d |u|$, with the form $\psi = g(r)e^{i\theta}$, then substituting $\psi$, $\sigma_d$ to $u$, $\sigma$ in (2.1) respectively, we can get
\[ (2.10) \quad g'' + \frac{g'}{r} - \frac{g}{r^2} - \frac{1 + \ln g}{g \ln g} \left( (g')^2 - \frac{g^2}{r^2} \right) = 0 \]
for $1 > r > e^{-a}$. Since $v$ is a harmonic diffeomorphism from $P(a)$ onto $D^*$, we have
\[ (2.11) \quad g(e^{-a}) = 0, \quad g(1) = 1 \quad \text{and} \quad g'(r) > 0 \quad \text{for} \quad 1 > r > e^{-a}, \]
or
\[ g(e^{-a}) = 1, \quad g(1) = 0 \quad \text{and} \quad g'(r) < 0 \quad \text{for} \quad 1 > r > e^{-a}. \]

We will only deal with the case that (2.11) is satisfied, the rest case is similar. Let $G = \ln g$, then the equation (2.10) can be rewritten as
\[ (2.12) \quad G'' + \frac{1}{r} G' - \frac{1}{G} (G')^2 + \frac{1}{r^2 G} = 0 \]
for $1 > r > e^{-a}$, with $G(1) = 0$ and $\lim_{r\to e^{-a}} G(r) = -\infty$. 
Regarding \( r \) as a function of \( G \), using a similar formula of (2.6), from (2.12) we can get
\[
\frac{r''}{r} - \left( \frac{r'}{r} \right)^2 + \frac{r'}{rG} - \frac{1}{G} \left( \frac{r'}{r} \right)^3 = 0, \quad G \in (-\infty, 0).
\]
Similar to solving (2.7), we can get the solution to (2.13) as follows.
\[
(\ln r)'(G) = \frac{1}{\sqrt{1 + c_1 G^2}}, \quad G \in (-\infty, 0)
\]
for some nonnegative constant \( c_1 \) depending on the choice of \( g \).
If \( c_1 \) is equal to 0, then \( g = r \), this is in contradiction to (2.11).
If \( c_1 \) is positive, then taking integration on both sides of (2.14), we can get
\[
\ln r(G) = \int_0^G \frac{1}{\sqrt{1 + c_1 t^2}} dt = \frac{1}{\sqrt{c_1}} \ln \left( \sqrt{c_1 G} + \sqrt{1 + c_1 G^2} \right).
\]
So
\[
r = \left( \sqrt{c_1 \ln g + \sqrt{1 + c_1 \ln^2 g}} \right)^\frac{1}{\sqrt{c_1}},
\]
with \( \lim_{g \to 0^+} r(g) = 0 \). On the other hand, from (2.11), we have \( r(0) = e^{-a} \). Hence we get a contradiction. Therefore such \( g \) does not exist, Theorem 1.2 has been proved.

3. Harmonic maps from \( \mathbb{D}^* \) to \( P(a) \) with its Euclidean metric and vice versa

Now let us consider the case of that the target has the Euclidean metric.

Proof of Theorem 1.3. Let us prove the first part of this theorem, that is, show the nonexistence of rotationally symmetric harmonic diffeomorphism from \( \mathbb{D}^* \) onto \( P(a) \) with its Euclidean metric. The idea is similar to the proof of Theorem 1.1 so we just sketch the proof here. Suppose there is such a harmonic diffeomorphism \( \varphi \) from \( \mathbb{D}^* \) onto \( P(a) \) with its Euclidean metric with the form \( \varphi = h(r)e^{ip} \), then we can get
\[
h'' + \frac{1}{r} h' - \frac{1}{r^2} h = 0 \quad \text{for} \quad 1 > r > 0
\]
with
\[
h(0) = e^{-a}, \quad h(1) = 1 \quad \text{and} \quad h'(r) > 0 \quad \text{for} \quad 1 > r > 0,
\]
or
\[ h(0) = 1, \ h(1) = e^{-a} \text{ and } h'(r) < 0 \text{ for } 1 > r > 0. \]

We will just deal with the case that (3.2) is satisfied, the rest case is similar. Let \( H = (\ln h)' > 0 \text{ for } 1 > r > 0, \) then we can get

\[ H' + H^2 + \frac{1}{r} H = \frac{1}{r^2} \text{ for } 1 > r > 0. \]

Solving this equation, we can get

\[ H = \frac{1}{r} - \frac{2}{c_3 r^3 + r} = -\frac{1}{r} + \frac{2c_3 r}{1 + c_3 r^2} \]

here \( c_3 \) is a constant depending on the choice of \( h \) such that \( H > 0 \text{ for } 1 > r > 0 \) which implies

\[ 1 + c_3 r^2 > 2 \text{ or } 1 + c_3 r^2 < 0 \text{ for } 1 > r > 0. \]

So

\[ \ln h = -\ln r + \ln |1 + c_3 r^2| + c_4 \]

here \( c_4 \) is a constant depending on the choice of \( h \). Hence

\[ h = |1 + c_3 r^2| r^{-1} e^{c_4}. \]

From (3.5) and (3.7), we can get \( \lim_{r \to 0} h(r) = \infty \). On the other hand, from (3.2), \( h(0) = e^{-a}. \) We get a contradiction. Hence such a function \( h \) does not exist, the first part of Theorem 1.3 holds.

Now let us prove the second part of this theorem, that is, show the existence of rotationally symmetric harmonic diffeomorphisms from \( P(a) \) onto \( \mathbb{D}^* \) with its Euclidean metric. It suffices to find a map from \( P(a) \) onto \( \mathbb{D}^* \) with the form \( q(r)e^{i\theta} \) such that

\[ q'' + \frac{1}{r} q' - \frac{1}{r^2} q = 0 \text{ for } 1 > r > e^{-a} \]

with \( q(e^{-a}) = 0, \ q(1) = 1 \) and \( q' > 0 \text{ for } 1 > r > e^{-a}. \) Using the boundary condition and (3.7), we can get that

\[ q = (e^{2a} r^2 - 1) r^{-1} (e^{2a} - 1)^{-1} \]

is a solution to (3.8).

Therefore we finished the proof of Theorem 1.3. \[ \square \]
4. HARMONIC MAPS FROM $\mathbb{C}$ TO THE HYPERBOLIC DISC

In this section, we will give another proof of the following result which has been proved in [28, 29, 24, 7].

Proposition 4.1. There is no rotationally symmetric harmonic diffeomorphism from $\mathbb{C}$ onto the hyperbolic disc.

Proof. It well-known that the hyperbolic metric on the unit disc is 
\[ \frac{2}{1-|z|^2}|dz|. \]
We will also use the idea of the proof of Theorem 1.1. Suppose there is such a harmonic diffeomorphism $\phi$ from $\mathbb{C}$ onto $\mathbb{D}$ with its hyperbolic metric with the form $\phi = k(r)e^{i\theta}$, then we can get

\[
(4.1) \quad k'' + \frac{1}{r}k' - \frac{1}{r^2}k + \frac{2k}{1-k^2} \left[ (k')^2 - \frac{k^2}{r^2} \right] = 0 \quad \text{for } r > 0
\]
with

\[
(4.2) \quad k(0) = 0 \text{ and } k'(r) > 0 \quad \text{for } r > 0.
\]

Regarding $r$ as a function of $k$, setting $v = (\ln r)'(k)$, the equation (4.1) can be rewritten as

\[
(4.3) \quad (1-k^2)v' - 2kv + v^3(k + k^3) = 0.
\]

That is,

\[
(4.4) \quad (v^{-2})' + \frac{4k}{1-k^2}v^{-2} = \frac{2(k + k^3)}{1-k^2}.
\]

One can solve this equation to obtain

\[
v^{-2} = k^2 + c_5(1-k^2)^2
\]
for some nonnegative constant $c_5$ depending on the choice of the function $k$.

If $c_5 = 0$, then we can get $r = c_6k$ for some constant $c_6$. On the other hand, $\phi$ is a diffeomorphism, so $k \to 1$ as $r \to \infty$. This is a contradiction.

If $c_5 > 0$, then $b_1 \geq k^2 + c_5(1-k^2)^2 \geq b_2$ for some positive constants $b_1$ and $b_2$. So

\[
|(\ln r)'(k)| \leq \frac{1}{\sqrt{b_2}}.
\]

This is in contradiction to the assumption that $r \to \infty$ as $k \to 1$.

Therefore Proposition 4.1 holds. \qed
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