On Repair with Probabilistic Attribute Grammars

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Program synthesis and repair have emerged as an exciting area of research, driven by the potential for revolutionary advances in programmer productivity. Among most promising ideas emerging for synthesis are syntax-driven search, probabilistic models of code, and the use of input-output examples. Our paper shows how to combine these techniques and use them for program repair, which is among the most relevant applications of synthesis to general-purpose code. Our approach combines semantic specifications, in the form of pre- and post-conditions and input-output examples with syntactic specifications in the form of term grammars and AST-level statistics extracted from code corpora. We show that synthesis in this framework can be viewed as an instance of graph search, permitting the use of well-understood families of techniques such as A*. We implement our algorithm in a framework for verification, synthesis and repair of functional programs, demonstrating that our approach can repair programs that are beyond the reach of previous tools.

1 Introduction

Program synthesis has attracted significant attention as a promising and ambitious approach to improve software development productivity [21, 33, 10, 30, 15, 12, 13, 23, 11, 26, 18, 20]. Among possibly deployment scenarios of synthesis is program repair, in which a tool localized an error repairs the program by synthesizing an alternative code fragment [24, 17].

Synthesis tools typically leverage both semantic and syntactic specification of the synthesis process. Typical forms of semantic information are specifications in the form of input-output examples, as well as reference implementations and predicates. These specifications are important to identify the specific intent of the developer. Among the syntactic forms of specification are simple types and probabilistic models of code. Syntactic models of code are appealing for two main reasons: they can improve the efficiency of search, and they can provide implicit, reusable baseline specifications that apply across multiple synthesis problems, reducing the need to provide near-complete semantic specifications of the desired functionality.

Our work aims to unify these two sources of specifications. To this extent, we describe a specification framework and an algorithms that efficiently searches for programs based on:

- a probabilistic and polymorphic attribute grammar that restricts and prioritizes the relevant space, and
- dynamically growing set of input examples along with test predicates, which narrows down the developer’s intent.

Our attribute grammars describe the generation of trees and can be thought of as a probabilistic extension of type systems used in previous work. The use of generics and attributes makes the grammars
compact and enables us to naturally describe constructs such as conditionals that are needed to generate more complex code fragments. Thanks to the fact that we use a language supporting generics and value classes (Scala), we show how to describe such derived grammars in the form of a wrapper library. The developers can therefore use mostly existing programming language constructs to describe the intended models of use of their libraries. The use of attributes enables the developers to incorporate domain knowledge, such as associativity of operators, which can break symmetries and reduce the search space. In addition to explicitly writing such models, we present an algorithm to automatically derive models from a corpus of code. Our results show that the derived corpus provides benefits in synthesis of meaningful code fragments.

Our algorithm efficiently supports specifications that include concrete inputs. In the case where the provided set of inputs is not sufficient to narrow down the search, our system can also make use of constraint solvers to generate additional inputs. To make search driven by probabilistic grammars efficient, we adopt a top-down approach for search. We introduce techniques to prune the search for expressions using partial evaluation of incomplete grammars on known input examples. Furthermore, we show how to incorporate equivalence modulo input-output examples in such a search algorithm, which is a technique that has so far been deployed primarily in bottom-up search techniques.

The practical focus of our techniques is on program repair. We use manually supplied and automatically derived examples to localize the expression likely to be responsible, then synthesize an alternative expression in its place. Examples are particularly relevant because they can be used to localize the error efficiently. The support of input examples in our algorithm makes it efficient for synthesizing candidate repairs. Furthermore, our support for probabilistic grammars enabled us to adopt the following approach to repair with probabilistic models: in addition to using grammars describing general Scala expressions, we bias the search for repairs towards the constructs specific for a program being repaired, making the synthesis more effective. We therefore show that our approach is able to produce program repairs whose size puts them beyond the reach of analogous existing non-probabilistic repair approaches.

2 Synthesis Algorithm

2.1 Synthesis problem

Let $\phi = \phi(\bar{a}, x)$ be the specification of a programming task, specifying a relation between a sequence of input parameters $\bar{a}$ and an output variable $x$. $\phi$ can be given as a logical predicate, a set of input-output examples, a reference implementation, or a combination thereof. Also, let $\Pi = \Pi(\bar{a})$ be the path condition for the problem, i.e. a relation satisfied by hypothesis by the input parameters. Let also $t = t(\bar{a})$ express a term in the target language.

The synthesis problem is then defined as asking for a constructive solution for the formula

$$\forall \bar{a}. \exists t. \Pi \implies \phi[x \mapsto t]$$

In other words, we want to find an expression $t$ such that replacing every occurrence of the output variable $x$ in the synthesis predicate $\phi$ causes the formula $\Pi \implies \phi$ to be valid. In this section, we will describe the probabilistic enumeration procedure used to solve such formulas.

2.2 Counter-example guided inductive synthesis (CEGIS)

The synthesis algorithm is an instantiation of CEGIS, a widely used framework to describe program synthesis algorithms [33][12]. The main technical difficulty with finding a $t$ which satisfies rule Terminal
is the universally quantified “∀a”, where the input variables a may range over a large, or even potentially infinite domain. We present the CEGIS routine in Algorithm 1. The procedure is parameterized by two sub-procedures, Search and Verify. At each step, we maintain a finite set A of concrete input points. In the search phase, the algorithm finds an expression t, such that

$$\bigwedge_{a \in A} \Pi \implies \phi[x \mapsto t].$$

Observe that for a specific choice of A, determining whether a given t satisfies requirement reduces to simply evaluating the resulting expression. In the verification phase, we confirm that the candidate expression t works for all inputs a. The procedure Verify(a, Π, φ, x, t) returns either: (a) valid, if t satisfies the synthesis predicate for all values of a, or (b) a counter-example input pex, such that Π[a ↦ pex] ∧ ¬φ[a ↦ pex, x ↦ t] i.e. pex satisfies the path condition Π, but t fails the synthesis predicate.

**Algorithm 1** CEGIS([a ↦ (Π ⊲ x)])

1. Initialize A := ∅. A is a finite set of concrete instantiations of a.
2. Repeat forever:
   (a) Let t = Search(a, A, Π, φ, x).
   (b) Let res = Verify(a, Π, φ, x, t).
   (c) If res = valid, then return t.
   (d) Otherwise, if res = pex, update A := A ∪ {pex}.

We discharge the calls to Verify by invoking Leon’s verification system [9]. The main technical contribution of this paper is in using probabilistic enumeration to instantiate the Search subroutine. We devote the rest of this section to describing this procedure.

### 2.3 Probabilistic expression grammars

The goal of the Search(a, A, Π, φ, x) subroutine is to find a candidate expression t which satisfies the synthesis predicate for all input points a ∈ A. The expression t is drawn from the productions of a probabilistic context-free grammar (PCFG), such as that shown below:

```
IntExpr ::= 0 (Rule R0, p0 = 0.15)
  | 1 (Rule R1, p1 = 0.3)
  | x (Rule Rx, px = 0.3)
  | IntExpr + IntExpr (Rule R+, p+ = 0.15)
  | if (BoolExpr) { IntExpr } else { IntExpr } (Rule Rif, pif = 0.1)

BoolExpr ::= IntExpr ≤ IntExpr (Rule Rle, p≤ = 0.8)
  | BoolExpr & BoolExpr (Rule R& = 0.2)
```

The productions of IntExpr expand to expressions of type Int, and the productions of BoolExpr expand to expressions of type Bool. Each rule R of each non-terminal symbol N is annotated with a number pR, which indicates the probability of N expanding to R. Naturally, the probability pe of a production e is then defined as the product of the probabilities of all rules invoked in the derivation tree: for example, the production x + 1 has probability p+, px, p1 = 0.0135.

More generally, a PCFG is a pair $G = (\mathcal{N}, \mathcal{R})$, where:
1. \( \mathcal{N} \) is a finite, non-empty set of non-terminal symbols, where each non-terminal \( N \in \mathcal{N} \) is associated with a type \( T_N \).

2. \( \mathcal{R} \) maps each non-terminal \( N \) to a finite set of production rules \( \mathcal{R}(N) \). Each production rule \( R \in \mathcal{R}(N) \) is a well-typed construct of the form \( R = t(N_1, N_2, \ldots, N_k) \), where \( t \) is the top-level operator, \( N_1, N_2, \ldots, N_k \) are the child non-terminal symbols, and such that the output types of \( t \) and \( N \) coincide: \( T_t = T_N \), and

3. each rule \( R \) is associated with a probability \( p_R \in [0, 1] \) such that for all non-terminals \( N \), \( \sum_{R \in \mathcal{R}(N)} p_R = 1 \).

Notice that \( \mathcal{N} \) does not necessarily coincide with the set of types of the target language; we only require that the mapping from non-terminals to types is surjective.

We make two additional requirements of the PCFGs we consider in this paper: first, we require the probability of each rule, \( p_R < 1 \) (note the strict inequality), and second, we require that \( G \) be unambiguous, i.e. that every expression produced by the grammar have a unique derivation tree. If we relax the first assumption, then there are some technical difficulties in setting up the probability space, and results such as Theorem ?? will need additional constraints to be true. The second assumption simplifies the definition of \( P(e) \), which would otherwise be the sum of the probabilities over all derivation trees. Moreover, it reduces the search space to a tree (instead of a DAG), and thereby allows us to use stronger versions of the guarantees provided by search algorithms.

For simplicity of notation, and whenever the intent is clear, we will conflate productions \( e \) of the grammar with the expressions \( te \) that they encode. We write \( \mathcal{E}_N \) for the set of all productions of a non-terminal \( N \). Given a production \( e \in \mathcal{E}_N \), we define its probability, \( P(e) \), as the product of the probabilities of all rules used in \( e \). It can be shown that \( \sum_{e \in \mathcal{E}_N} P(e) = 1 \). The productions of \( G \) will constitute elements of the sample space under consideration, and the space of events is, as usual, the powerset \( 2^\mathcal{E}_N \) of productions. We will discuss the extraction of PCFGs from a code corpus in Section 3.

### 2.4 Top-down expression enumeration as graph search

In the search phase of the CEGIS loop, we want to find an expression \( t \) which satisfies the requirement in formula [1]. One approach to implementing \( \text{Search}(\bar{a}, A, \Pi, \phi, x) \) is to enumerate all candidate expressions, in some order, until a suitable answer is found. Recall that, because \( A \) is a finite set, for a given choice of \( A \) and \( t \), determining the satisfaction of formula [1] reduces to evaluating the conjunction, and does not involve expensive calls to an SMT solver. The enumerative SyGuS solver [34] uses a highly optimized form of expression enumeration in the search phase, and is currently among the most competitive SyGuS solvers. See Algorithm 2. In this section, we will show how we utilize a PCFG to enumerate expressions in order of decreasing probability in order to accelerate the search process. We will now describe this probabilistic enumeration algorithm.

**Partial productions.** Engineering the enumeration algorithm turns out to be much simpler if we enumerate expressions top-down rather than bottom-up. As a first step, we therefore extend the idea of a grammar production into the more general notion of a *partial* production:

\[
\tilde{e}_N \ ::= \ ?_N \ | \ t(\tilde{e}_{N_1}, \tilde{e}_{N_2}, \ldots, \tilde{e}_{N_k}),
\]

where \( R = t(N_1, N_2, \ldots, N_k) \) is a production rule of \( N \) in \( G \). In particular, notice that partial productions may contain incomplete sub-expressions, denoted by \(?_N\). Examples of partial productions include
Algorithm 2 Search($\bar{a}, A, \Pi, \phi, x$). Implements the search phase of the CEGIS loop by expression enumeration.

1. Let $G$ be the chosen PCFG, and $N$ be the starting non-terminal such that the types coincide, i.e. $T_N = T_x$.
2. For each $e$ emitted by Enumerate($G, N$):
   (a) Let $t_e$ be the expression encoded by $e$.
   (b) If $\bigwedge_{a \in A} \Pi \implies \phi[x \mapsto t_e] = \text{true}$, return $t_e$.
   (c) Otherwise, discard $t_e$ and continue enumeration.

\[ x + ? \] and \[ \text{if} \ (x \leq ?) \ \{ \ x \ \} \ \text{else} \ \{ \ ? \} \]. We write $\tilde{E}_N$ for the set of all partial productions of a non-terminal $N$. We can also speak of the probability of partial productions: $p_{\tilde{e}}$ of a partial production $\tilde{e}$ is the product of all production rules used to derive $\tilde{e}$. It will be mathematically more convenient to speak of negative log probabilities: the cost of a partial production $\tilde{e}$,

\[ \text{cost}(\tilde{e}) = -\log(p_{\tilde{e}}) = -\sum_{R \in \tilde{e}} \log(p_R). \] (2)

The expansion relation naturally induces a tree $\mathcal{G}$ on the partial productions of a grammar $G$. See Figure 1. The initial node is the starting non-terminal, $?$. There is an edge from the partial production $\tilde{e}_1$ to the partial production $\tilde{e}_2$ if they are related by the expansion relation, $\tilde{e}_1 \rightarrow \tilde{e}_2$, produced by an instantiation of the rule $R$, is annotated with the cost of the rule, $-\log(p_{\tilde{e}_1})$. Then, the resulting graph is a tree because of the unambiguity assumption on $G$, and the sum of the weights along the path to $\tilde{e}$ is equal to $\text{cost}(\tilde{e})$. The main insight of this paper is that Enumerate($G, N$) can be thought of as instantiations of various search algorithms on this tree.

Algorithm 3 Enumerate($G, N$). Emits a sequence of complete productions of $N$.

1. Let $\pi : \tilde{E}_N \rightarrow \mathbb{R}_{\geq 0}$ be a priority function mapping productions to non-negative real numbers.
2. Let $Q$ be a priority queue of partial productions $\tilde{e} \in \tilde{E}_N$ arranged in ascending order according to $\pi$. Initialize $Q := \{?_N\}$.
3. While $Q$ is not empty:
   (a) Let $\tilde{e}$ be the element at the front of $Q$. Dequeue $\tilde{e}$.
   (b) If $\tilde{e}$ is a complete production, emit $\tilde{e}$.
   (c) Otherwise, for every neighbor $\tilde{e}'$ such that $\tilde{e} \rightarrow \tilde{e}'$ in $\mathcal{G}$, insert $\tilde{e}'$ into $Q$.

See Algorithm 3. It is helpful to visualize the operation of the algorithm as shown in Figure 2. The algorithm maintains a priority queue $Q$ of still-unexpanded partial productions. At each step, the algorithm dequeues the element $\tilde{e}$ at the front of this priority queue and, if it is still incomplete, expands the leftmost $?$ with an instance of every applicable production rule $R$. This results in a set of partial
Figure 1: The tree $G$ of partial productions, connected by the expansion relation. The initial node is the empty partial production $\epsilon$ of the non-terminal IntExpr. Each edge indicates the replacement of the left-most incomplete sub-production by an instance of a production rule $R$. The edges are annotated with the negative log probabilities, $-\log(p_R)$: because $0 \leq p_R \leq 1$, $\log(p_R) \leq 0$, so the weights $-\log(p_R)$ are non-negative. A production is enclosed in a rectangle if it is complete.
sequence $\sigma$ of complete productions emitted by $\text{Enumerate}(G,N)$ is a sub-sequence of $\tau$, and therefore, the probability of productions in $\sigma$ monotonically decreases.

Now, arbitrarily choose a complete production $e \in \delta_N$. We will show that $\text{Enumerate}(G,N)$ will eventually emit $e$. Let $p_{hi}$ be the highest probability of any production rule in $G$. Recall our requirement that for each rule $R$, $P_R < 1$, and therefore, $p_{hi} < 1$ and $\log(p_{hi}) < 0$. Before processing $e$, $\text{Enumerate}(G,N)$ can only process those nodes which are at most $\log(P(e)) / \log(p_{hi})$ steps away from the root node $\tau$ of the search tree $\mathcal{T}$. If there are $N$ rules in the PCFG, then the out-degree of each node is at most $N$, and it follows that there are only finitely many nodes which can be processed before $\text{Enumerate}(G,N)$ processes $e$. It follows that $e$ is eventually emitted by the enumerator.

Unfortunately, when the priority function is instantiated to emulate Dijkstra’s algorithm, enumeration does not scale beyond a few hundred productions. The intuition may be found in the correctness proof above: before emitting $e$, the enumerator must explore all partial productions with probability bigger than $P(e)$. The number of these productions rapidly grows with decreasing $P(e)$, and furthermore, most of the processed partial productions are “very incomplete”, i.e. containing many instances of $\epsilon$, and therefore many edges away from turning into complete productions. This suggests other graph search algorithms, which we will now discuss.

**Non-terminal horizons and A* search.** When considering partial productions $\bar{e}$, we may speak of two quantities: (a) the cost already paid, which we have defined as $\text{cost}(\bar{e}) = -\log(P(\bar{e}))$, and (b) the minimum cost yet to be paid, before $\bar{e}$ turns into a complete production. We formalize this latter quantity as the horizon, defined as:

$$h(\bar{e}) = \min_{e_f \text{ s.t. } \bar{e} \rightarrow e_f} \text{cost}(e_f) - \text{cost}(\bar{e}),$$

where $e_f$ ranges over all complete productions reachable from $\bar{e}$. Recall the intuition for the failure of Dijkstra’s algorithm: before emitting $e$, we have to process every partial production with higher probability, and most such partial productions $\bar{e}'$ are themselves many steps away from complete productions. Since the horizon encodes the distance from the partial production to its nearest completion, it is natural to include it in the priority function $\pi(\bar{e})$. If we define $\pi(\bar{e}) = \text{cost}(\bar{e}) + h(\bar{e})$, then we obtain A* search.

The important property of $\pi(\bar{e})$ is that it is admissible, i.e. that $\pi(\bar{e})$ is always less than the cost of all complete productions descendant from $\bar{e})$. More formally, for all complete productions $e_c$ reachable from $\bar{e}$,

$$\pi(\bar{e}) = \min_{e_c \text{ s.t. } \bar{e} \rightarrow e_c} \text{cost}(e_c).$$

Figure 2: The operation of $\text{Enumerate}(G,N)$ in Algorithm 2. We describe the partial evaluation and indistinguishability-based optimizations, including production rewriting and queue deduplication, in Section 2.5.
We then have the following well-known result of the A* algorithm:

**Theorem 2.2.** If $\pi(\bar{e}) = \text{cost}(\bar{e}) + h(\bar{e})$, then:

1. The sequence of complete productions emitted by Enumerate$(G,N)$, $\sigma = e_1,e_2,e_3,\ldots$ contains all productions in $\mathcal{E}_N$ and has monotonically decreasing probability. (This is called the optimality property.)

2. For all complete productions $e$, every complete optimal algorithm $A'(G,N)$ will, before producing $e$, visit all strictly less expensive nodes, $\bar{e}$, such that $\pi(\bar{e}) < \pi(e)$. (This is the property of optimal efficiency.)

**Proof.** First, we show that Enumerate$(G,N)$ produces all complete productions $e \in \mathcal{E}_N$. Similar to Theorem 2.1, we bound the number of nodes that can be processed in line 3a before processing $e$. Observe that for all nodes, $\bar{e}$, $\pi(\bar{e}) \geq \text{cost}(\bar{e})$, and for complete productions $e$, $\pi(e) = \text{cost}(e)$. Therefore, all nodes processed by Enumerate$(G,N)$ before processing $e$ are no more than $\log(P(e))/\log(p_{hi})$ steps from the root of the search tree $\mathcal{G}$, where, as before, $p_{hi} < 1$ is the highest probability appearing in the PCFG.

We will now show that the productions emitted have monotonically decreasing probability. For this, first observe that for each complete production $e$, and for each of its (necessarily incomplete) ancestors $\bar{e}'$, $\pi(\bar{e}') \leq \pi(e)$. For the sake of contradiction, let there be some pair $e_1$, $e_2$ of complete productions, such that $P(e_1) < P(e_2)$, but Enumerate$(G,N)$ emits $e_1$ before $e_2$. Both $e_1$ and $e_2$ are therefore reachable from the initial node $\bar{e}$. Consider the state of $Q$ when $e_1$ is dequeued from it. It has to be the case that some ancestor $\bar{e}'_2$ is present in $Q$. However, it follows that $\pi(\bar{e}'_2) \leq \pi(e_2) < \pi(e_1)$, and therefore the priority queue must have made a mistake in its ordering. It follows that the sequence of productions emitted by Enumerate$(G,N)$ has monotonically increasing costs, or equivalently, monotonically decreasing probabilities.

We will now prove the last part of the theorem. Assume otherwise, so for some algorithm $A'(G,N)$ and some node $\bar{e}$ such that such that $\pi(\bar{e}) < \pi(e)$, $A'$ does not expand $\bar{e}$ before producing $e$. We know $\bar{e}$ has some descendant $e'$ such that $\pi(\bar{e}) = \text{cost}(e') < \pi(e)$. Because the search space $\mathcal{G}$ is a tree, if $A'$ does not expand $\bar{e}$, it follows that it did not enumerate $e'$ before $e$, violating the assumption that $A'$ is a complete optimal enumerator.

Note that, in a certain sense, the second part of Theorem 2.2 says that this is the best possible enumeration algorithm among all algorithms that use the same priority function, because Enumerate$(G,N)$ does not expand any nodes $\bar{e}'$ such that $\pi(\bar{e}') > \pi(e)$. This also captures our intuition for the superiority of this priority function over the simpler metric used by Dijkstra’s algorithm.

**Computing the horizon, $h(\bar{e})$.** It can be shown that the cost to be paid to complete $\bar{e}$ is the sum of the costs needed to expand each of its unexpanded nodes:

$$h(\bar{e}) = \sum_{\bar{e}' \in \bar{e}} h(\bar{e}') \quad (6)$$

Before starting the enumerator, we therefore compute the non-terminal horizons, $h(\bar{e}_N)$, for each non-terminal symbol $N$. Observe, that, by definition, this simply encodes the probability of the most likely expansion, $e \in \mathcal{E}_N$:

$$h(\bar{e}_N) = \min_{e \in \mathcal{E}_N} \text{cost}(e) = -\max_{e \in \mathcal{E}_N} \log(P(e)).$$

The values $h(\bar{e}_N)$ can be computed by the following fixpoint computation:
1. \[ h^0(N) = \min \{ -\log(p_R) \mid \text{Terminal rule } R \} \].

2. \[ h^{i+1}(N) = \min \{ h^i(R) \mid \text{Production rule } R \} \], where the horizon of a production rule \( R \) is defined as \[ h^i(R) = -\log(p_R) + \sum \{ h^i(N') \mid \text{child non-terminal } N' \text{ appearing in } R \} \].

### 2.5 Optimizations

**Eagerly discarding partial productions.** Given a single input variable \( a \), consider the synthesis predicate \( \phi \) given by:

```plaintext
if (a == 5) { x == 6 }
else if (a == 7) { x == 9 }
else { x == a }
```

During CEGIS, say the set of concrete input points, \( A = \{ 2, 5, 7 \} \). For this problem, \( \text{Enumerate}(G,N) \) will consider partial productions such as \( \tilde{e} = \text{if } (x < 6) \{ x \} \text{ else } \{ ? \} \).

Observe that the conjunction \( \bigwedge_{a \in A} \phi[x \mapsto e] \) used in the CEGIS loop can only be evaluated for complete productions \( e \). However the partial production \( \tilde{e} \) is already incorrect: for the input point \( a = 5 \), regardless of the completion of \( ? \), \( \tilde{e} \) will evaluate to 5, and this fails the requirement that \( x == 6 \).

This pattern of partial productions already failing requirements is particularly common with conditionals and match statements. Our first optimization is therefore the partial evaluation box shown in Figure 2. For each input point \( \tilde{a} \in A \), we partially evaluate the CEGIS predicate \( \Pi \Rightarrow \phi[x \mapsto \tilde{e}] \). If the result is \( \text{false} \), then we discard the partial production \( \tilde{e} \) without processing, as we know that all its descendants will fail the synthesis predicate. Otherwise, if it evaluates to \( \text{true} \) or unknown, then we process \( \tilde{e} \) as usual, and insert its neighbors back into the priority queue.

**Extending the priority function with scores.** We just observed that if a partial production fails the synthesis predicate by partial evaluation, then it can be discarded without affecting correctness. The dual heuristic optimization is to promote partial productions which definitely evaluate to \( \text{true} \) on some input points. We do this by modifying the priority function:

\[
\pi(\tilde{e}) = \text{cost}(\tilde{e}) + h(\tilde{e}) + c \times \log(\text{score}(\tilde{e})),
\]

where \( \text{score}(\tilde{e}) \) is the number of input points on which the partial production evaluates to \( \text{true} \), and \( c \) is a positive coefficient. Small positive values of \( c \) results in an enumerator which strictly follows probabilistic enumeration, while large positive values of \( c \) results in the enumerator favoring partial productions which already work on many points. While the use of this heuristic renders Theorem 2.2 inapplicable, we have observed improvements in performance as a result of this optimization.

**Indistinguishability.** Consider the partial production \( \tilde{e} = \text{if } (x < 5) \{ x - x \} \text{ else } \{ ? \} \), and focus on the sub-expression \( x - x \). This expression is identically equal to 0, and it is therefore possible to simplify \( \tilde{e} \) to \( \text{if } (x < 5) \{ 0 \} \text{ else } \{ ? \} \). If two expressions \( e \) and \( e' \) are equivalent, and \( P(e) > P(e') \), then a desirable optimization is to never enumerate \( e' \). Observe that, if properly implemented, this optimization produces exponential savings at each step of the search: if \( e \) and \( e' \) are equivalent, then, for example, it follows that both \( e + 3 \) and \( e' + 3 \) are equivalent, and only one of them needs to be enumerated to achieve completeness.
Indistinguishability [34] is a technique to mechanize this reasoning. Given a sequence of concrete input points $A = \{\overline{a_1}, \overline{a_2}, \ldots, \overline{a_n}\}$, a complete production $e$ can be evaluated to produce a set of output values $S_e = \{x_1, x_2, \ldots, x_n\}$. During enumeration, if an expression $e'$ is encountered such that for some previous expression $e$, $S_e = S_{e'}$, then $e'$ is discarded as a potential expansion. The original expression $e$ can thus be regarded as the representative of the equivalence class of all expressions whose signature is equal to $S_e$.

In this original formulation of indistinguishability-based pruning, the enumerator worked bottom-up, rather than in the top-down style we consider in this paper. As a result, the enumerator only needs to process complete productions, rather than the partial productions which populate $Q$ in our setting.

We have extended this optimization to work with partial expressions, and prune expressions in a top-down enumerator. The idea is to replace the priority queue $Q$ with a “deduplicating priority queue”, with the additional feature that it removes duplicate elements from being dequeued. Such a queue can be implemented as a combination of a traditional priority queue and a mutable set data structure commonly available in standard language libraries. Next, we maintain a dictionary mapping all previously seen signatures to the representatives of the respective equivalence classes. Every time we dequeue a partial production $\tilde{e}$ from $Q$, we evaluate every complete sub-production $e_{\text{sub}}$ on all input points $\overline{a} \in A$ to obtain its signature $S_{\text{sub}}$, and replace $e_{\text{sub}}$ with the canonical representative. This forms the box labelled “Indist. rewrite 1” in Figure 2.

We also perform a second, fast indistinguishability rewrite after each partial production is expanded. This is done by maintaining a map from previously seen expressions to their canonical representatives. Consulting this map for every complete sub-production of the children, $\tilde{e}_1, \tilde{e}_2, \ldots$, of the original production $\tilde{e}$ is much faster than evaluating them on each concrete input point in $A$. There is some freedom in choosing the placement of various optimizations in Figure 2. Our motivation was that the priority queue can get very large, and many elements enqueued into the queue will never be dequeued, and that it is therefore wise to postpone as much processing as possible to when partial productions are extracted from $Q$.

3 Term Grammars

In the previous section, we explained our synthesis algorithm using a PCFG as a black box. In this section, we explain how the term grammars in our system are defined and extracted.

We designed our grammars with the following goals in mind:

- **Flexibility.** The system should allow the user to define and easily deploy a different grammar for each application or application domain.

- **Expressivity.** The grammars should be able to express complex relations between expressions, in a more elaborate way than simply mapping each type of the target language to a set of production rules. This allows the user to constrain the space of synthesized programs to exclude redundant or irrelevant ones, thus improving the scalability of the synthesizer.

- **Automation.** Apart from hand-coding the grammars, it should also be possible to automatically extract them from a corpus of programs, possibly taken from a specific application domain.

To this end, we introduce Probabilistic Attribute Grammars, which address all of the above points:

- A custom grammar is provided by the user along with the synthesis problem as a set of Scala source files (the grammar files), in which production rules are expressed as plain Scala functions.
Switching between grammars reduces to just including different files in compilation. It is also trivial to use any desired combination of such files. If the user does not want to manually provide a specific grammar for the task, they can always fall back to the system’s predefined grammar.

- The nonterminal symbols of the grammar are not plain types, but instead are types contained in a wrapper class that annotates them with extra information. The wrapped types can represent a subtype of the underlying type, or some other condition under which specific rules are accessible, which would not be expressible if we used plain nonterminals as types. While processing the input grammar, our system will desugar away those wrappers to produce values of the underlying plain type.

The approach of annotating types with attributes has already been introduced in [20], but there the attributes and their interpretation were predefined. Here we allow the user to provide their own wrapper classes.

It is of course also possible to omit the wrappers completely and use plain Scala types as nonterminals.

- We provide a system to automatically extract grammar files from a corpus of PureScala programs.

The grammar files undergo a series of preprocessing steps before generating a Probabilistic Attribute Grammar, the exact form of which is conditional to the synthesis problem. The synthesizer itself is oblivious to this procedure, which views the grammar just as a function from nonterminal symbols to production rules.

### 3.1 Grammar files, rules and frequencies

As mentioned above, the user can provide a series of grammar files for compilation along with the synthesis problem. The format of the files was designed with the aim to be readable and writable by a human user, but also straightforward to extract from a corpus of programs.

To illustrate the format though an example, let us look at a grammar file describing a set of simple arbitrary precision integer programs:

```scala
@production(10)
def plus(a: BigInt, b: BigInt): BigInt = a + b

@production(5)
def minus(a: BigInt, b: BigInt): BigInt = a - b

@production(5)
def o: BigInt = BigInt(1)

@production(10)
def z: BigInt = BigInt(0)

@production(20)
def vBigInt: BigInt = variable[BigInt]
```

A function annotated with `@production` is treated as a grammar production rule. A rule of the form `def pFoo(a: T1, b: T2): T3 = foo(a, b)` should be interpreted in CFG notation as...
T3 ::= foo(T1,T2)

The annotation’s argument indicates the absolute frequency with which this rule is encountered. The relative frequencies will be generated in the end of a processing procedure. In this example, though, they can be computed straightforwardly from the absolute frequencies.

An invocation of the built-in function variable[T], for some type T, indicates the absolute frequency of generating any variable of type T. This built-in compensates for the fact that we do not know the names of the variables accessible to the synthesizer a priori. When they become available, this production will be instantiated to a concrete production for each variable of the correct type, with the probability distributed equally among these variables. If, for example, we are synthesizing a function with two parameters (x: BigInt, y: BigInt), the above grammar file would generate the following grammar:

\[
\begin{align*}
\text{BigInt} & ::= \text{BigInt} + \text{BigInt} \quad (p_+ = 0.2) \\
& | \text{BigInt} - \text{BigInt} \quad (p_- = 0.1) \\
& | 0 \quad (p_0 = 0.1) \\
& | 1 \quad (p_1 = 0.2) \\
& | x \quad (p_x = 0.2) \\
& | y \quad (p_y = 0.2)
\end{align*}
\]

3.2 Generic productions

Generic grammar productions offer a concise way of representing operations on generic data-structures. For instance, we could describe common operations on a generic list data-structure with a few user-provided productions:

@production(..)
def single[A](a: A): List[A] = List(a, Nil)

@production(..)
def listInsert[A](a: A, l: List[A]): List[A] = List(a, l)

@production(..)
def listTail[A](l: List[A]): List[A] = l.tail

@production(..)
def listHead[A](l: List[A]): A = l.head

@production(..)
def listSize[A](l: List[A]): BigInt = l.size

When looking for productions of a specific ground type, we instantiate generic productions that can return that type, and introduce them as normal monomorphic productions. For example, if we are looking for productions of List[BigInt], we consider the first three rules above instantiated with \( T \rightarrow \text{BigInt} \), and the forth with \( T \rightarrow \text{List[BigInt]} \). The fifth rule is incompatible.

Because the set of types is typically infinite, generic productions with a type variable absent from the return type represent an infinite number of ground productions. This is for instance the case with listSize.
Our enumeration procedure described above requires that each non-terminal symbol has a finite number of ground productions. To solve this conflict, we abandon the theoretical completeness of the instantiation by only considering a set of reasonable types $\mathcal{T}$ during instantiation. This set of types is initialized with all types returned by ground productions and then iteratively expanded by discovering new return types of $\mathcal{T}$-instantiations of generic productions. For example, starting with $\mathcal{T} = \{\text{Int}\}$, running one iteration yields $\mathcal{T} = \{\text{Int}, \text{List[Int]}\}$ as $\text{single[Int]}(..)$ returns a value of type $\text{List[Int]}$.

The number of discovery iterations we perform has a big practical impact on the number of final productions, and thus on performance. On benchmarks with several generic data-structures, this discovery is typically exponential. By under-approximating the size of the smallest expression of any discovered type in $\mathcal{T}$, we bound the search to reasonable types. We leave to future work how to better integrate the instantiation of generic rules within the enumeration process.

### 3.3 Annotated nonterminals

Simple grammars like the one displayed above, where the nonterminal symbols are plain types, are not expressive enough to handle all constraints that are obvious to a human programmer. For example, they cannot express that an operand of an addition cannot be 0. When used as-is, such grammars generate a large number of redundant programs, which prevent the synthesizer from scaling to larger program sizes.

To amend this, we provide the programmer with a general mechanism to more accurately express the restrictions of their grammar by enhancing their nonterminals with attributes in addition to the plain Scala type. Nonterminals with the same type but different attributes are considered distinct, and can have different production rules.

As an example, suppose the user wants to more accurately specify the grammar of Section 3.1 to exclude 0 from the operands of $+$, as well as the second operand of $-$. This can be captured with the following attribute grammar:

\[
\begin{align*}
\text{BigInt} & ::= \ 	ext{BigInt}\{\text{Bl}\} \\
\text{BigInt}\{\text{Bl}\} & | \ 	ext{BigInt}\{\text{NZ}\} \\
& | \ 0 \\
\text{BigInt}\{\text{NZ}\} & | \ 	ext{BigInt}\{\text{NZ}\} + \text{BigInt}\{\text{NZ}\} \\
& | \ 	ext{BigInt}\{\text{Bl}\} - \text{BigInt}\{\text{NZ}\} \\
& | \ x \\
& | \ 1
\end{align*}
\]

The above grammar contains two annotated nonterminals, $\text{BigInt}\{\text{NZ}\}$ and $\text{BigInt}\{\text{Bl}\}$. The former expresses nonzero integers and the second one the unrestricted integer type. Thus, generating $\text{BigInt}$ values reduces to generating values of the annotated type $\text{BigInt}\{\text{Bl}\}$, which is captured in the first rule of the grammar.

In Scala source code, this is expressed by defining implicit Scala classes containing a field of the base type, and annotating them with the @label annotation. The source file gets desugared by the system to the above grammar.

@label implicit class NZ(val v: BigInt)

@label implicit class BI(val v: BigInt)

@production(10) def plus(a: NZ, b: NZ): NZ = a.v + b.v
On Repair with Probabilistic Attribute Grammars

\@production(5) def minus(a: BI, b: NZ): NZ = a.v - b.v

\@production(5) def o: NZ = BigInt(1)

\@production(10) def z: BI = BigInt(0)

\@production(20) def vBigInt: NZ = variable[BigInt]

\@production(40) def nz2Bi(nz: NZ): BI = nz.v

\@production(1) def start(b: BI): BigInt = b.v

3.3.1 Built-in axiom system

In case the user does not want to manually implement a custom attribute grammar as above, they can reside to a set of default built-in axioms described in [20]. In case of the default grammars, those axioms are automatically applied to each combination of nonterminal and production rule. In case of a manually provided grammar, the user has to guide the system by tagging the production rules with a set of predefined tags. A couple of example tagged rules could be the following:

\@production(10)
\@tag("0")
def z: BigInt = BigInt(0)

\@production(10)
\@tag("commut")
def pFoo(b1: BigInt, b2: BigInt): BigInt = foo(b1, b2)

\@tag("0") indicates that this rule produces the constant 0 (e.g. it should be treated as neutral element of addition), whereas \@tag("commut") specifies foo as a commutative operation. These rules will undergo a set of predetermined optimizations described in [20].

3.4 Extracting an attribute grammar form a corpus

Whereas manually specifying grammars is very useful for specific tasks and small DSLs, it would be tedious to expect a user-provided grammar for generic programming tasks, where numerous types and functions are in scope.

To this end, we provide an automated system to extract a grammar from a corpus of Scala programs. We provide two different grammar extractors, described in the following paragraphs.

3.4.1 Unconditional Frequency Extractor

The simpler Unconditional Frequency Extractor (UFE) uses plain Scala types as nonterminals, but automatically tags them as in 3.3.1 so the built-in axioms can be instantiated on them. Each production rule generated by the UFE corresponds to a specific kind of expression encountered in the corpus, whereas the rule’s frequency reflects the number of occurrences of this kind. By kind of expression, we mean
the expression type along with all information required to reconstruct the expression from its arguments (except for variables, where the name is not preserved). More specifically,

- For variables, only the variable type (the name does not get preserved). All variables of a specific type will all be summarized by an invocation to the variable built-in.
- For literals, the literal itself (which takes 0 operands). E.g. 0, BigInt(42).
- For function invocations, the invoked function along with its actual type parameters (which may be generic). E.g. foo[BigInt](?, ?), foo[T](?, ?), where by ? we denote a hole to be filled during synthesis.
- For type constructors, the type constructor along with its actual type parameters. E.g. Cons[BigInt](?, ?), Nil[A]().
- For every other expression, only the type of the AST of that expression, and if needed its type instantiation. E.g. ? + ?, Set[BigInt]?.

Because the grammars generated by this extractor do not take into account the parent of the expression, we will call them depth-1 grammars.

For example, a corpus containing only the expression (x * 2) would result in the following file:

```scala
@production(1)
@tag("const")
def pBigIntInfiniteIntegerLiteral0(): BigInt = BigInt(2)

@production(1)
@tag("times")
def pBigIntTimes(v0 : BigInt, v1 : BigInt): BigInt = v0 * v1

@production(1)
@tag("top")
def pBigIntVariable(): BigInt = variable[BigInt]
```

### 3.4.2 Conditional Frequency Extractor

The second, more sophisticated extractor, extracts the conditional probabilities of the occurrence of each expression kind, given the expression’s relation with its parent. More specifically, along with each expression, it stores a pair (par, pos), where par is the type of AST of the parent of the expression, and pos the expression’s position in its parent’s operands. This pair is optional and omitted for a top-level expression. For example, if the extractor encountered the expression (x * 2), it would store the triples (? * ?, null, null), (<variable>, ? * ?, 0) and (2, ? * ?, 1). To express these conditional probabilities in a grammar file, it uses annotated nonterminals as described in 3.3.

Because these grammars express probabilities which are derived from an expression along with its parent, we will call them depth-2 grammars.

A corpus containing only the above expression would produce the following grammar file:

```scala
@label implicit class BigInt_TOLEVEL(val v : BigInt)
@label implicit class BigInt_0_Times(val v : BigInt)
@label implicit class BigInt_1_Times(val v : BigInt)
```
@production(1)
def pBigIntTimes(v0 : BigInt_0_Times, v1 : BigInt_1_Times): BigInt_TOPLEVEL =
v0.v * v1.v
@production(1)
def pBigIntVariable(): BigInt_0_Times = variable[BigInt]
@production(1)
def pBigIntInfiniteIntegerLiteral(): BigInt_1_Times = BigInt(2)
@production(1)
def pBigIntStart(v0 : BigInt_TOPLEVEL): BigInt = v0.v

3.5 Biasing grammars for repair

The flexibility and automation of the system give us a great opportunity to exploit the local structure of
the program. Especially for program repair, we can ease the task of generating a reasonable patch for
an erroneous program by collecting information about how the user has chosen to program in the same
module.

This is achieved as follows: before repair, we generate an additional grammar file from the file under
repair, and include it for compilation along with the general-purpose grammar file extracted from the cor-
pus. This way, we bias the synthesis process towards local definitions, without completely disregarding
the general programming model.

4 Experimental Results

To experimentally evaluate the above ideas, we integrated them into the Leon synthesis and repair frame-
work [13] The Probabilistic Enumeration (PE) of Section 2 is deployed as a closing deductive synthesis
rule in the synthesis framework, in place of Symbolic Term Exploration (STE) [20].

For repair, we slightly modify Probabilistic Enumeration to generate terms similar to the erroneous
expression, in the spirit of [17]. This can be achieved by automatically modifying the grammar passed
to Enumeration, without any other changes to the algorithm. This rule is deployed along with plain PE
within the synthesis stage of the repair pipeline, after test generation and fault localization

In our evaluation, we aimed to measure how well our algorithms perform in isolation, rather than
how well they integrate with the synthesis framework. This is straightforward in repair, as the erroneous
implementation already defines the high-level structure of the function under repair, and the closing rules
can usually be deployed alone, without instantiating any decomposition rules first.

In the following tables, we compare the efficiency of synthesizing repairs for a set of erroneous
benchmarks taken from [20], along with a few new, more challenging ones.

In total, we test 4 tool configurations:

- Symbolic Term Exploration with built-in grammars and built-in axioms, i.e. the old version of
Leon (STE)
- Probabilistic Enumeration with built-in grammars and built-in axioms (PB)
- Probabilistic Enumeration with depth-1 extracted grammars and built-in axioms (PD1)
- Probabilistic Enumeration with depth-2 grammars, without built-in axioms (PD2)
In both the latter cases, the grammars come from a combination of grammar files extracted from a
corpus and the program under repair. We do not use built-in axioms with depth-2 grammars, as they
are redundant to some degree and they would overcomplicate the grammar, while yielding diminishing
returns.

The detailed results are given in 1. The first two columns of the table give an indication of the
complexity of the program by displaying the total size of the program and the size of the solution (which
might not be generated from scratch, but rather as a variation of the erroneous expression). The final
four columns display the repair times for the four deployments of the tool described above. Repair times
exclude compilation, test generation, and verification of the solution.

To summarize the results, the new enumerator is slightly faster for the built-in grammars, whereas
the depth-1 grammars are sometimes slightly slower, but manage to solve a couple more benchmarks.
Unfortunately, there is a relatively easy benchmark that depth-1 fails to solve. The depth-2 grammars,
though they show some promise (in the Compiler.semUntyped benchmark), they generally perform
badly. One reason for this behavior are the conflicts between annotated nonterminal symbols generated
from the general corpus with those from the file under repair. Both sources will generate annotated
non-terminals that express the same conditional probability, overloading the synthesizer with duplicated
work. This is a shortcoming we plan to address in the near future.

5 Related Work

5.1 Program synthesis

Program synthesis has been a challenge problem in computer science since its early days (see, for ex-
ample, [21]). With increasing computing power, and rapid improvements in the performance of SAT
and SMT solvers starting from the early 2000s, the problem of program synthesis has recently received
renewed attention. The seminal Sketch project [33] was the first to demonstrate the feasibility of using
modern constraint solving technology in synthesis tools, and used the popular counter-example guided
inductive synthesis (CEGIS) framework to solve synthesis problems [12].

Syntax-guided synthesis (SyGuS) has emerged as a common formulation and interchange format in
which to express many program synthesis problems [4]. Our problem formulation within Leon shares
two important similarities with SyGuS:

1. **Multi-modal specifications**, which can include any combination of input-output examples (such as
   “f(2, 3) = 8”), logical constraints (such as “∀x,y, f(x, y) > x + y + 2”), and even partial implemen-
tations (such as “∀x,y, x < y ⇒ f(x, y) = x + y + 3”).

2. **Expression grammars**, or equivalently, a library of components [15], are used to syntactically
   constrain the space of programs. This is important both to make synthesis feasible for complicated
   specifications, and for when the results are to be used in specialized domains, such as assembly
code for new computer architectures [25].

Of the various techniques to solve SyGuS problems, ESolver, which enumerates pairwise distinguishable
expressions [34], EUSolver, which combines indistinguishability-based enumeration with decision-tree
based condition inference [5], and the refutation-proof-based CVC4 solver [30] have broadly emerged as
the most efficient SyGuS solvers. Our present work may be viewed as an attempt to bias the enumerative
solver with statistical facts learned from a code corpus.

We would like to highlight that program synthesis and code repair within Leon is a much more
challenging problem than synthesis within SyGuS and related systems: the main difficulties include the
| Operation        | Sizes | STE | PB | PD1 | PD2 |
|------------------|-------|-----|----|-----|-----|
| Compiler.desugar | 717   | 3   | 1.9| 2.3 | 2.9 |
| Compiler.desugar | 715   | 2   | 5.2| 3.7 | 4.3 | 3.4 |
| Compiler.desugar | 719   | 7   | 3.0| 1.8 | 2.1 | 2.0 |
| Compiler.desugar | 719   | 7   | 3.8| 3.0 | 3.0 | 3.2 |
| Compiler.desugar | 719   | 14  | 5.2| 3.4 | 3.2 | 3.7 |
| Compiler.simplify| 765   | 4   | 2.8| 1.8 | 1.8 | 3.1 |
| Compiler.semUntyped| 738 | 5   |    |     | 16.8| 5.1 |
| Heap.merge       | 382   | 3   | 6.0| 4.9 | 5.1 | 5.2 |
| Heap.merge       | 382   | 1   | 2.8| 2.0 | 2.0 | 2.2 |
| Heap.merge       | 382   | 3   | 5.7| 4.7 | 5.1 | 5.3 |
| Heap.merge       | 382   | 9   | 4.4| 3.6 | 3.4 | 3.7 |
| Heap.merge       | 384   | 5   | 5.1| 6.4 | 5.2 | 15.6|
| Heap.merge       | 382   | 2   | 4.2| 2.6 | 2.7 | 2.9 |
| Heap.insert      | 345   | 8   | 3.0| 2.8 | 2.9 | 2.9 |
| Heap.makeN       | 384   | 7   | 5.6| 7.9 | 8.7 | 8.6 |
| List.pad         | 830   | 8   | 2.3| 1.5 | 1.6 | 2.7 |
| List.++          | 740   | 3   | 2.6| 1.5 | 1.2 | 8.1 |
| List.:+          | 772   | 1   | 3.0| 1.1 | 1.2 | 1.9 |
| List.replace     | 774   | 6   | 3.3| 1.7 | 1.7 | 1.7 |
| List.count       | 827   | 3   | 2.2| 1.2 | 1.4 | 1.3 |
| List.find        | 827   | 2   | 2.5| 1.7 | 1.6 | 1.7 |
| List.find        | 829   | 4   | 2.8| 1.6 | 1.8 | 1.7 |
| List.find        | 830   | 4   | 2.7| 1.7 | 1.9 | 1.7 |
| List.size        | 755   | 4   | 2.5| 1.1 |    |    |
| List.sum         | 774   | 4   | 2.3| 1.5 | 1.5 | 1.5 |
| List.-           | 774   | 1   | 1.5| 0.5 | 0.5 | 1.7 |
| List.drop        | 815   | 4   | 2.4| 1.4 | 1.6 | 1.5 |
| List.drop        | 815   | 3   | 4.0| 3.3 | 46.9| 2.4 |
| List.&           | 774   | 4   |    |     | 3.1 |    |
| List.count       | 774   | 1   |    | 170.7|    |    |
| Numerical.power  | 225   | 5   | 2.9| 1.1 | 1.2 |    |
| Numerical.moddiv | 174   | 3   | 1.9| 0.8 | 0.8 | 1.0 |
| MergeSort.split  | 284   | 5   | 3.6| 2.3 | 2.5 | 8.4 |
| MergeSort.merge  | 286   | 7   | 3.3| 2.3 | 2.5 | 2.2 |
| MergeSort.merge  | 286   | 3   | 5.9| 4.8 | 5.2 | 5.0 |
| MergeSort.merge  | 284   | 5   | 3.3| 2.4 | 2.3 | 13.7|
| MergeSort.merge  | 286   | 1   | 2.3| 1.6 | 1.7 | 1.7 |

Table 1: Benchmarks for repair
rich type system and algebraic data types of Scala, and the synthesis of recursive functions. In this light, our problem is most similar to Escher [1] and $\lambda^2$ [10]: the main differences are that Escher enumerates programs bottom-up rather than top-down, and our use of PCFGs to improve the performance of the enumeration algorithm.

5.2 Type-driven synthesis

One appealing approach to program synthesis is to phrase the problem as the type-inhabitation problem in a sufficiently expressive type system. Examples of such systems include InSynth [13], and Synquid [26]. The Myth project [23, 11] extends the type system of the host language with refinement types corresponding to input-output examples. It can therefore be viewed as an attempt to provide a type-theoretic interpretation to Escher. The Synquid system [26] is an elegant extension of this idea, by encoding the specification as part of the type of the required expression using the liquid type framework.

Previous work on synthesis within Leon [18, 20] may also be viewed as being implicitly type-driven. The path conditions and postcondition obtained after each invocation of the deduction rules can be rephrased as additional typing constraints on the expression being synthesized. As discussed in Section 2, the present PCFG-based enumeration is intended as a drop-in substitute for the symbolic term exploration rule previously used by Leon.

5.3 Statistical analysis of code corpora

With increasing availability of large open-source code repositories such as GitHub and Bitbucket, the statistical analysis of code corpora has become an exciting research problem. Code repositories have been used to learn coding idioms [3], to automatically suggest names for program elements [2], and to deobfuscate JavaScript code [28]. Our paper is an attempt to apply similar techniques to accelerate program synthesis.

The program completion tool Slang [29] uses n-grams and recurrent neural networks to predict missing API calls in code snippets. There are two main aspects which distinguish our work from Slang: (a) the presence of hard correctness requirements in Leon in the form of pre-/post-conditions, and (b) program synthesis in Leon is fundamentally about synthesizing expressions rather than API call sequences, and prediction systems such as n-grams are insufficient to create the nested recursive structure inherent in the output we produce. The DeepCoder tool [6] uses a recurrent neural network (RNN) to predict the presence of elements in the program being synthesized. The output of this neural network is then used to guide a more exhaustive search over the space of possible programs. In this sense, DeepCoder is similar to the PCFG-guided enumeration of Leon, and the use of neural networks to accelerate the program synthesis problem is an intriguing direction for future research.

Probabilistic context-free grammars (PCFGs) are a classical extension of context-free grammars [16]. They may be used both to model ambiguity (for applications in natural language processing), and to model probability distributions over the generated language, which motivates our application in accelerating code synthesis. The more recent model of probabilistic higher-order grammars (PHOGs) [7] [27] extends PCFGs by allowing the expansion probabilities of a non-terminal node to depend on attributes such as node siblings and DFS-predecessors. Experiments indicate that the PHOG model is significantly better at predicting elements of JavaScript programs than PCFGs. Extending the probabilistic model of our paper to use PHOGs instead of PCFGs is an immediate area of future work.
5.4 Search algorithms

One of the main technical insights of this paper is to phrase syntax-guided program synthesis as an instance of graph search. In this space, *A* is a classical algorithm used for path finding and graph traversal. As described in Section 2, *A* orders the queue of still-unexplored nodes by their priority values, \( f(n) = g(n) + h(n) \), where \( g(n) \) is the cost required to reach the node \( n \), and the heuristic function \( h(n) \) is the estimated cost from \( n \) to the target node. The historically older Dijkstra’s algorithm for shortest paths through graphs is a special case, where the heuristic function \( h(n) \) is identically equal to 0.

Recall that the graph of partial expansions forms a tree: in this setting, if the cost function \( h(n) \) is admissible, i.e. that \( h(n) \leq \tilde{h}(n) \), where \( \tilde{h}(n) \) is the actual cost of the shortest path from \( n \) to the target node, then *A* discovers the shortest path to the target node (optimality). The second important property of *A* is that of optimal efficiency: of all algorithms using the same heuristic function, *A* explores the fewest nodes to reach the target. In the setting of this paper, optimality implies discovering the expansion with highest probability. By extending \( h(n) \) to include the number of satisfied examples, we have relaxed optimality in favour of empirical performance improvements.

*A* is simply the best known of a large family of search algorithms (see chapters 3 and 4 of [32]), including iterative deepening *A* (IDA*, [19]), simplified memory-bounded *A* (SMA*, [31]), fringe search [8], beam search, swarm search, and various instantiations of genetic algorithms [22]. Extending the techniques of this paper to these algorithms is an exciting direction of future work.

6 Conclusion + Future Work

In this paper, we present a technique to use statistical information extracted from code corpora, encoded as probabilistic attribute grammars, to guide program synthesis techniques. The method is based on rephrasing expression enumeration as graph search, and applying classical techniques such as *A* to solve these problems. We implemented the algorithm in the Leon synthesis and verification system, and obtained a much more robust and scalable synthesizer in comparison to the previous implementation. Our end goal is a predictable one-touch program synthesizer in a rich programming environment such as Scala: there are several exciting directions of future work, such as investigating other search methods including swarm search and genetic algorithms, incorporating richer models of statistics such as probabilistic higher-order grammars (PHOGS) [7], and combining type- and syntax-driven and statistical techniques into a highly expressive and performant program synthesizer.

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