Broadband Fizeau Interferometers for Astrophysics

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ABSTRACT

Context. Measurements of the 2.7 K cosmic microwave background (CMB) radiation now provide the most stringent constraints on cosmological models. The power spectra of the temperature anisotropies and the E-mode polarization of the CMB are explained well by the inflationary paradigm. The next generation of CMB experiments aim at providing the most direct evidence for inflation through the detection of B-modes in the CMB polarization, presumed to have been caused by gravitational waves generated during the inflationary epoch around $10^{-34}$s. The B-mode polarization signals are very small ($\leq 10^{-8}$K) compared with the temperature anisotropies ($\sim 10^{-5}$K). Systematic effects in CMB telescopes can cause leakage from temperature anisotropy into polarization. Bolometric interferometry (BI) is a novel approach to measuring this small signal with lower leakage.

Aims. If BI can be made to work over wide bandwidth ($\sim 20$ – $30\%$) it can provide similar sensitivity to imagers. Subdividing the frequency passband of a Fizeau interferometer would mitigate the problem of ‘fringe smearing.’ Furthermore, the approach should allow simultaneous measurements in image space and visibility space.

Methods. For subdividing the frequency passband (‘sub-band splitting’ henceforth), we write an expression for the output from every baseline at every detector in the focal plane as a sum of visibilities in different frequency sub-bands. For operating the interferometer simultaneously as an imager, we write the output as two integrals over the sky and the focal plane, with all the phase differences accounted for.

Results. The sub-band splitting method described here is general and can be applied to broad-band Fizeau interferometers across the electromagnetic spectrum. Applications to CMB measurements and to long-baseline optical interferometry are promising.

Key words. cosmology: cosmic background radiation — techniques: interferometric — instrumentation: interferometers

1. Introduction

Interferometry has a long history for astronomical measurements at radio, millimeter, submillimeter, IR and optical wavelengths. In 1868 Hyppolyte Fizeau (Fizeau 1868) described how the diameters of stars could be measured by optical interferometry. He proposed ‘masking’ the aperture of a telescope to create interference fringes in the focal plane, similar to a Young’s double slit interference experiment. Today aperture masks are used to overcome atmospheric ‘seeing’ effects to reach the diffraction limit of single aperture telescopes (Tuthill et al. 2000). For even greater angular resolution, beam combination from widely separated apertures is used in long baseline optical interferometers. Michelson used this technique to measure, for the first time, the diameters of stars. He used flat mirrors to reflect the beams from separated apertures into a single telescope, which acted as the beam combiner.

This type of beam combination, in which beams are combined in the image plane of a telescope, is called ‘Fizeau interferometry,’ or ‘adding interferometry.’ An alternate approach involves combining the beams in the pupil plane of a telescope and is called ‘Michelson interferometry’ after the technique used in the Michelson-Morley experiment (Minni\textsuperscript{e}r 2003). Both types of combiners are used in long-baseline optical interferometers (Traub 2000). Conventional radio interferometers can also be thought of as Michelson interferometers. They typically mix the RF signals from each antenna in an array to lower frequencies (heterodyning) and interfere (multiply) them electronically one pair at a time with either an analog or digital correlator to measure visibilities. This approach is sometimes called ‘multiplying’ interferometry. These techniques are widely used for radio wavelengths to sub-mm wavelengths. As the number of antennas, $N$, increases the number of correlations to be performed grows as $N(N – 1)/2$. Although radio correlators are improving rapidly, they are currently limited to combining signals from about 100 antennas and bandwidths of a few GHz (Lawrence et al. 2008).

In this paper we describe a Fizeau ‘adding’ interferometer that overcomes the bandwidth and large-$N$ limitations of heterodyne interferometers. The instrument is optimized for precision measurements of the temperature and polarization anisotropy of the CMB and is sometimes called a ‘bolometric interferometer’ (Timbie et al. 2006; Tucker et al. 2008; Charlassier et al. 2008, Hamilton et al. 2008; Hamilton & Charlassier 2010). Interferometers are less sensitive to some kinds of systematic effects found in imaging instruments (Bunn 2007). The technique can be used at any wavelength.

In particular, we focus on an approach to broadening the bandwidth of a Fizeau interferometer. Spectral resolution is not required for many applications where the source has a continuum spectrum (such as the CMB) and signal averaging over broad bands is required to uncover faint signals. As one increases the bandwidth the different wavelengths within the passband will smear out the interference fringes and reduce the sensitivity of the instrument. Furthermore, this ‘fringe washing’ reduces the resolution of the interferometer in the u-v plane. In this paper we present a simple and powerful technique, which we call ‘sub-band splitting’ to overcome the limitations caused by large bandwidths in Fizeau interferometers. We discuss how this approach can improve parameter estimation in the specific case of obser-
dictions of the CMB, where sensitivity and u-v space resolution are critical for constraining CMB power spectra.

We begin by motivating the need for adding interferometry in §2, followed by the reason sub-band splitting is required. §3 develops the formalism to describe how visibilities are measured by a Fizeau combiner (Appendix A) points out how this arrangement can be used as an imager. §4 describes in detail the technique for solving for visibilities in sub-bands.

2. Cosmological Context: The CMB and Adding Interferometry

We now have a `standard model’ of cosmology in which the inflationary paradigm describes many aspects of the universe accurately through well-quantified parameters. Precision measurements of the CMB are the most powerful probes for determining these parameters. In particular, measurements of the angular power spectrum of the CMB temperature and E-mode polarization anisotropy have yielded a wealth of information about the early universe (see Komatsu et al. 2009, for example). However, inflation is driven by physics that we do not currently understand (Baumann et al. 2009). The most direct way to probe inflation is through the so-called B-modes in CMB polarization. The amplitude of the B-mode signal is directly related to the energy scale of the particle interactions that occurred during inflation. However, the energy scale of inflation is not known. Recent measurements by Gupta et al. 2010, Chiang et al. 2010, are approaching the level at which B-modes are expected to appear in models in which inflation occurred at the GUT scale. However, inflation may have involved lower-energy interactions; there is no lower bound to the amplitude of the B-modes.

Current Takahashi et al. 2010, Hinderks et al. 2009 and planned CMB instruments Bock et al. 2009 all use some type of imaging technique. With focal-plane arrays of hundreds of background-limited detectors they are capable of detecting the B-mode signals predicted in the most optimistic models, at the level of $\sim 10^{-5}$ K. Systematic effects have been extensively studied for imaging polarimeters in the context of CMB measurements and appear to be controllable at this level as well (Hu et al. 2003, Bock et al. 2006). However, at some level all instruments can mix the relatively large temperature anisotropy and E-mode polarization signals into B-modes.

Different systematic effects are found in interferometers. It is for this reason that heterodyne interferometers have been used for many years to study the CMB temperature and polarization power spectra and the Sunyaev-Zel’dovich effect. These instruments multiply together the RF signals from all possible pairs of antennas (baselines) in an array to measure a set of visibilities, which are related to the sky image through a Fourier transform (Rohlfs & Wilson 2004). In fact, the first detection of CMB E-mode polarization was made by a multiplying interferometer: DASI (Kovac et al. 2002). DASI had 13 single-mode antennas and performed pairwise correlation of signals across the $K$ band, from 26 - 36 GHz.

Several groups have studied the possibility of building a new generation of mm-wave interferometers specifically to search for the small polarization signals in the CMB (Timbie et al. 2006). Compared to existing mm-wave interferometers, these new instruments would have to do the following: 1) collect more modes of radiation from the sky by including more antennas (> 100); 2) operate with broader bandwidth ($\sim 25\%$, corresponding to $> 10$ GHz), and 3) operate over a broader range of center frequencies, at least up to 90 GHz, to be able to detect and reject astrophysical foreground sources by their spectral signatures. Because pairwise correlation requires multiplying $N(N - 1)/2$ signals together, conventional heterodyne interferometers face a significant challenge to increasing the number of antennas.

On the other hand, adding interferometers have the ability to correlate large numbers of inputs over wide bands. Here we present an adding interferometer based on a Fizeau beam combiner. Combined with the optimal phase-shifting scheme described in Charlassier et al. 2003, this is a promising approach for measuring the B-mode polarization. The technique is compatible with either coherent receivers (amplifiers) or incoherent detectors (bolometers).

In a 2-element adding interferometer the electric field wavefronts from both antennas are added and then squared in a detector (Rohlfs & Wilson 2004). The result is a constant term proportional to the intensity plus an interference term. The constant term is an offset that is removed by phase-modulating one of the signals. Phase-sensitive detection at the modulation frequency recovers both the in-phase and quadrature-phase interference terms and reduces susceptibility to low-frequency drifts (1/f noise) in the detector and readout electronics. The adding interferometer recovers the same visibilities as a multiplying interferometer:

$$V(\mathbf{u}) = \int \int I(\hat{\mathbf{n}}, \nu)G(\hat{\mathbf{n}})e^{2\pi \mathbf{u} \cdot \hat{\mathbf{n}}} J(\nu) d\nu d\hat{\mathbf{n}}$$

where $I(\hat{\mathbf{n}}, \nu) \propto E_{0}(\hat{\mathbf{n}}, \nu)$ is the incident intensity and $E_{0}(\hat{\mathbf{n}}, \nu)$ is the incident electric field as a function of position of the source on the sky, $\hat{\mathbf{n}}$, and frequency, $\nu$. $G(\hat{\mathbf{n}})$ is the primary beam (power) pattern of the antennas (assumed identical and for simplicity assumed independent of frequency). $\mathbf{u}$ is the vector between the centers of the antenna apertures, measured in wavelengths, and has components $u$ and $v$. The frequency bandpass of the instrument is $J(\nu)$.

To combine signals from $N > 2$ antennas, we use a Fizeau beam combiner, a type of `image plane’ combiner (Traub 2000). This technique is analogous to the simplest interferometer in 1-d: the Young’s double (or multiple)-slit interferometer (Fig. 4). Schematics of Fizeau combination are shown in Fig. 5. The beams from the apertures can directly illuminate an array of detectors, or, more typically, they pass through a lens or telescope first to reduce the size of the instrument. While Fizeau combining is well-known, we stress here the fact that there are two types of path differences (and therefore phase differences) for rays traveling from a source to a detector: one path difference occurs outside the instrument, and the other inside the instrument. Compare this to a conventional interferometer (also shown in 1-d, though the extension to 2-d is straightforward) as shown in Fig. 4, where rays only undergo a phase difference before they enter the antennas. (Note that long-baseline optical interferometers usually include a `delay line’ between the apertures and the beam combiner. This element introduces an equal path length for all rays entering an aperture. In contrast, the internal path differences we are concerned with are different for each pixel in the focal plane.)

Let us explore what this combination of path differences achieves. We start by noting that the ‘external’ phase differences, which are present in any interferometer, are the reason that the visibility function is a Fourier transform of the image on the sky. The visibility measured by a single baseline essentially selects one Fourier mode from the image. In the Fizeau system, we have an additional set of phase differences. Without loss of generality, we may assign a negative sign to the phases introduced inside
the instrument. Now, if we sum over both the phases, we get a Fourier transform followed by an inverse Fourier transform - but this is the image itself! Thus, Fizeau combination enables imaging in an interferometer. The image formed in the focal plane of the Fizeau combiner is equivalent to the ‘dirty image’ measured by conventional radio interferometers. This topic is discussed later in this paper in Appendix A.

In addition, we show that, given enough detectors on the focal plane, we can extract some spectral information from the interference fringes and determine the visibilities in several sub-bands. The Fizeau system enables extraction of spectral information via geometry, without additional components like filters.

Without spectral information an interferometer with a large bandwidth suffers from a large radial width of each pixel in the u-v plane. This is shown in Fig. 3. Let ν be the center frequency and Δν the bandwidth. Then, a baseline of length B will measure the CMB power spectrum over a band centered on spherical harmonic number

$$\ell = \frac{\pi B}{A} = \frac{\pi \nu B}{c} = \frac{\pi}{c} \sqrt{\nu^2 + \nu^2}$$

where the width in ℓ-space is

$$\Delta \ell = \frac{\pi \Delta \nu B}{c}.$$  

As mentioned above, the additional spectral information that is available to us can be used to sub-divide the band in the u-v plane. We discuss this aspect in detail in Appendix [1].

These advantages are studied in the context of two novel millimeter-wave interferometers: MBI (the Millimeter-wave Bolometric Interferometer, Timbie et al. 2006) and QUBIC (Hamilton & Charlassier 2010). In these systems the apertures are replaced by an array of back-to-back single-mode horn antennas (see Fig. 2). The inward-facing horns illuminate a Cassegrain telescope that combines the beams at the focal (image) plane, which is tiled by an array of bolometers whose dimensions are small compared to the overall dimensions of the focal plane. Between the outward and inward facing horns are electronic phase modulators which can be operated independently. They are used to modulate the interference fringes that appear on the focal plane in such a way that fringes caused by different baselines can be distinguished from each other (Charlassier et al. 2003; Hyland et al. 2009). In principle this technique could work with multimode horns or antennas as well.

3. The Fizeau combiner output and its relation to visibilities

In this section, we study the output of the adding interferometer and its dependence on instrument parameters: number of detectors on the focal plane, number of antennas, etc. and relate this output to visibility from an interferometer, in order to describe bandwidth splitting in the following section. In what follows, we denote the output at the detectors as O. A simple adding interferometer with two antennas/apertures (single baseline) is shown in Fig. 4. A generalized Fizeau adding interferometer is shown in Fig. 5.

3.1. Simple Interferometer

In an adding interferometer, electric fields are added and then squared at the detector. If E₁(\hat{n}, ν) and E₂(\hat{n}, ν) are electric fields incident at antennas 1 and 2 respectively, then, denoting the primary beam of the outward-facing antennas by G(\hat{n}), an adding interferometer will detect

$$G(\hat{n})[E_1(\hat{n}, ν) + E_2(\hat{n}, ν)]^2$$

from a certain direction \hat{n}. Now E₁(\hat{n}, ν) and E₂(\hat{n}, ν) differ only by a phase factor \varphi = 2\pi B \cdot \hat{n}/λ, so that we can write the electric fields as E₀(\hat{n}, ν) and E₀(\hat{n}, ν) exp(\i\varphi), and the output at a single
frequency for the simple arrangement shown in Fig.(1) is
\[ G(\hat{u}) I(\hat{u}, \nu) \cos \varphi \equiv G(\hat{u}) I(\hat{u}, \nu) \Re \{ \exp(i\varphi) \} \]
where \( \varphi = 2\pi B \cdot \hat{u} / \lambda = 2\pi u \cdot \hat{u} \).

3.2. Fizeau Interferometer

The general arrangement of a Fizeau interferometer is shown in Fig.(5). Let \( B_k \) be the baseline formed by antennas at \( A_p \) and \( A_l \), such that \( B_k = A_p - A_l \) \( (k \in [1 \ldots N(N-1)/2] \) and \( p, l \in \{1 \ldots N\} \) \), and \( \hat{u} \) is a direction in the sky, as shown in Fig.(5). Then, \( u_k = B_k / \lambda \), so that the external path difference is \( (A_p - A_l) \cdot \hat{u} = B_k \cdot \hat{u} \), as shown in Fig.(5).

The difference between this arrangement and that of the simple adding interferometer in Fig.(1) is the set of internal phase differences introduced inside the instrument due to the fact that there is more than one detector, and geometry of the arrangement. Let \( x_{jk} \) denote the position of the \( j \)th detector on the focal plane. Then, the internal path difference between the rays from antennas \( p \) and \( l \) (which form baseline \( k \), is \( x_{jk}(x) = x_{jk} \cdot B_k \) as shown in Fig.(5). Thus, the intensity at the detector at \( x_{jk} \) contributed by baseline \( k \) is
\[ G(\hat{u}) I(\hat{u}, \nu) \Re \{ \exp(i\varphi_k + \frac{2\pi}{\lambda} x_{jk}) \} \]
where \( \varphi_k = 2\pi B \cdot \hat{u} / \lambda = 2\pi u_k \cdot \hat{u} \).

If we denote the output at the \( j \)th detector from the \( k \)th baseline as \( O_{jk} \), and integrate over all directions in the sky and the bandwidth \( \Delta \nu \), we get
\[ O_{jk} = \int \int G(\hat{u}) I(\hat{u}, \nu) \Re \{ \exp(i\varphi_k + \frac{2\pi}{\lambda} x_{jk}) \} \frac{J(\nu) d\nu}{\lambda} \]
where \( J(\nu) \) is a bandpass weighting. There is an integration over detector area as well in eq.(9), which will be implicit until 4.3 where we explore the effect of the detector area and make a suitable and practical approximation. Notice that \( x_{jk} \) does not depend on the direction \( \hat{u} \). For now, we make the (crude) approximation that \( \lambda \equiv \lambda_0 \) (the central wavelength), so that this ‘internal phase factor’ may be taken out of the integral over \( \nu \) as well. As discussed in 4.1 and 4.2 this crude approximation need not be made. We will instead choose to view the visibilities as ‘averages’ over a certain ‘sub-bandwidth’, in which case this ‘internal phase factor’ is an average over this ‘sub-bandwidth’. We will discuss a more efficient way to deal with this issue in 4.1 and 4.2. For now, eq.(9) reads
\[ O_{jk} = \Re \left\{ \exp \left( \frac{2\pi}{\lambda_0} x_{jk} \right) \right\} \int \int G(\hat{u}) I(\hat{u}, \nu) \exp \{i(\nu_k + \varphi_k)\} J(\nu) d\nu d\hat{u} \]

The quantity indicated by the underbrace in eq.(11) is the Visibility (defined in eq.(1)) from the \( k \)th baseline, \( V(u_k) \equiv V_k \). We can now denote the ‘internal phase differences’ \( (2\pi/\lambda)x_{jk} \) as \( \phi_k \), so that
\[ O_{jk} = \Re \left\{ \exp(i\phi_k)V_k \right\} \]

In Appendix A we show that by integrating over the entire focal plane, we can recover the image convolved with a ‘dirty beam’, as in radio interferometry.

In the next section, we consider the net signal from a single baseline and describe how it can be split into ‘sub–bands.’ For the sake of simplicity, the index \( k \) corresponding to baseline \( k \) is dropped, and all equations in 4.1 hold for every baseline.

4. Spectral information from an interferometer using a Fizeau approach

4.1. Preliminaries

The output measured at the detectors in a Fizeau interferometer contains the following phase information integrated over the entire bandwidth:

1. phase introduced because of the path difference between any two rays that arrive from the same part of the sky on the two outward-facing antennas that make up a baseline; and
2. phase introduced because of the path difference between any two rays that arrive from two different antennas on to the same point in the focal plane.

The phase in point 1 is due to the fact that we are considering a radio interferometer, and so the visibility that we measure must, by definition, include this phase. However, the phase in 2 introduced by the beam combiner needs to be factored out to recover visibility from each bolometer. If there were a way to calculate the ‘internal phase’ introduced by the beam combiner over the whole bandwidth, then all we would need to do is to divide the output at each point in the focal plane by this ‘internal phase’, and we would get visibility directly.

4.2. ‘Sub–band Splitting’

We can think of the effect of the instrument on the visibilities in the following way. Let us divide the entire bandwidth of the

1 A range of values of \( \nu \) will produce a range of \( \ell \)'s, or a band in \( \ell \)-space (as discussed in 4.2). A finite-width interferometer thus measures what is called a ‘bandpower’ instead of a single value of the power spectrum at one value of \( \ell \). But the power spectrum is just the variance of the visibilities for a circle (ring) in the \( u-v \) plane (see Malu 2007, Charlassier et al. 2010, Figs. 5.8, Figs. 1&2 respectively). And so we get different bandpowers for the same baseline and orientation but for different frequencies.
instrument into \( m \) sub-bands and let \( V_1, V_2, \ldots, V_m \) be the centre-frequencies of each one. Then for one baseline, one orientation, and one detector position, these will correspond to visibilities \( V_1, V_2, \ldots, V_m \) and to phase differences \( \phi_{j1}, \phi_{j2}, \ldots, \phi_{jm} \) (where \( j \) represents the detector). If we represent the output at the \( j \)th detector as \( O_j \) then we get (in the previous section, but dropping the index \( k \) for baseline, since we’re considering just one baseline):

\[
O_j = \sum_{a=1}^{m} R\{V_a \exp i\phi_{ja}\}.
\]  

(13)

Given just one detector, it is impossible to extract every \( V_a \) for every sub-band, even though we know precisely what the \( \phi_{ja} \)’s are. However, if we have \( m \) detectors, then we can easily write the following system of equations for each baseline:

\[
O_1 = \mathbb{R}\{V_1 \exp i\phi_{11} + V_2 \exp i\phi_{12} + \cdots + V_m \exp i\phi_{1m}\}
\]

\[
O_2 = \mathbb{R}\{V_1 \exp i\phi_{21} + V_2 \exp i\phi_{22} + \cdots + V_m \exp i\phi_{2m}\}
\]

\[\cdots\]

\[
O_m = \mathbb{R}\{V_1 \exp i\phi_{m1} + V_2 \exp i\phi_{m2} + \cdots + V_m \exp i\phi_{mn}\}.
\]

(14)

Now, as discussed in the preceding section and in Fig. (1), we can apply a unique phase shift to each baseline. If we denote this phase shift by \( \Delta \phi \) (it is understood that integration is done over the area of one detector, so that we may write another set of \( m \) equations:

\[
O_1' = \mathbb{R}\{V_1 \exp i(\phi_{11} + \Delta\phi) + \cdots + V_m \exp i(\phi_{1m} + \Delta\phi)\}
\]

\[
O_2' = \mathbb{R}\{V_1 \exp i(\phi_{21} + \Delta\phi) + \cdots + V_m \exp i(\phi_{2m} + \Delta\phi)\}
\]

\[\cdots\]

\[
O_m' = \mathbb{R}\{V_1 \exp i(\phi_{m1} + \Delta\phi) + \cdots + V_m \exp i(\phi_{mn} + \Delta\phi)\}.
\]

(15)

This is a system of \( 2m \) equations with \( 2m \) unknowns - \( \mathbb{R}\{V_1\}, \mathbb{R}\{V_2\}, \ldots, \mathbb{R}\{V_m\} \), \( \mathbb{R}\{\phi_{11}\}, \mathbb{R}\{\phi_{12}\}, \ldots, \mathbb{R}\{\phi_{mn}\} \), and so we can solve for the values for each one of these ‘sub-band visibilities’. The beam combiner thus achieves far more than just separating the real and imaginary parts of visibilities.

### 4.3. Effect of finite detector size

We mentioned in \([4,3]\) that if the ‘internal phase’ is known, sub-band visibilities may be computed/estimated. However, calculating this internal phase is not easy, since integration over the bandwidth complicates the calculation, as seen in \([3.2] \) eq. \((11)\), where the ‘internal phase’ factor was taken out of the integral to define a visibility using a crude approximation (see Charlassier et al. 2010, §2.2). The internal phase differences in the Fizeau interferometer create fringe patterns on the focal plane. This is exactly the same as saying that the ‘visibilities’ in each sub-band are modulated by a fringe which depends on baseline length. To extract these visibilities, we need to separate the fringes. In order to do so, we need to realize that what we observe at every detector is the visibility on the sky times the fringe summed over the area of the detector as well as bandwidth.

The fringe pattern is different for every frequency in the bandwidth. Visibility is also different for different frequencies. Since we can compute the fringe corresponding to each frequency, it is also straightforward to sum up these fringes over a small ‘sub-band’ over the area of a single detector. The output at each detector from a baseline is known, and this can be written as a sum over the product of visibility for a ‘sub-band’ and the fringe for the respective ‘sub-band’. This system of equations can be solved for each baseline to yield the sub-band visibilities.

We have demonstrated this in \([4,2]\). Effectively, this amounts to not using the crude approximation in \([3.2] \) eq \((11)\) and instead writing the output as a product of a sub-band visibility and a fringe.

In the discussion so far, we have assumed that the collecting area of each detector is negligible, and we completely ignored the effect of the fringe pattern. Let us account for these effects in the following way. Let \( A \) be the effective collecting area of each detector. Let \( f(x, v_o) \) be the value of the fringe pattern (see discussion at the beginning of this section) at a point on the focal plane \( x \) and in a frequency sub-band marked by \( \alpha \). Then, equations \((14)) become

\[
O_j = \sum_{a=1}^{m} \mathbb{R}\{\int V_a \exp i\phi_{ja} (x) f(x, v_o) d^2 x\}
\]

\[
O_j' = \sum_{a=1}^{m} \mathbb{R}\{\int V_a \exp i(\phi_{ja} (x) + \Delta\phi) f(x, v_o) d^2 x\}
\]

(16)

where it is understood that integration is done over the area of the detector.

This leaves us with an issue - that of deconvolving the \( V \)’s from the integrals. However, if the area of the detector is small compared to the width of fringes, then we can assume that the phase differences remain roughly constant over the collecting area of one detector, so that we may write

\[
O_j = A \sum_{a=1}^{m} \mathbb{R}\{V_a \exp i\phi_{ja} (x) F(x, v_o)\}
\]

\[
O_j' = A \sum_{a=1}^{m} \mathbb{R}\{V_a \exp i(\phi_{ja} (x) + \Delta\phi) F(x, v_o)\}
\]

(17)

where \( F(x, v_o) \) represents an ‘average’ value of the fringe pattern.

Equations \((17)) again have \( 2m \) variables and can be solved to get \( 2m \) quantities: the real and imaginary parts of \( m \) visibilities over the bandwidth.

**Application to CMB cosmology:** The QUBIC collaboration is implementing the technique described in this paper (Charlassier et al. 2010).

### 5. Conclusions

1. The Fizeau system makes it possible to recover spectral information without the need for filters.
2. The Fizeau system acts naturally as an imager.

In addition, by introducing phase modulators discussed in Hyland et al. 2009, Charlassier et al. 2009, we can measure visibilities for all baselines in a Fizeau system.

While it is possible to divide the bandwidth into many different sub-bandwidths, it isn’t possible to do this indefinitely. The beam for a single antenna determines the FOV of the instrument and limits the resolution in the u–v plane, as shown in Fig. 3.

It is also possible to operate the interferometer simultaneously as an imager. The additional modulation mentioned above opens up a range of possibilities, including the simultaneous measurement of visibilities and images. This is described in Appendix A.

In conclusion, the Fizeau system introduced here is potentially powerful tool for astrophysics: it could allow the recovery of more information than is possible with traditional interferometers or imagers and does not need significantly more re-
sources to build. Its application in CMB cosmology is straightforward and can be demonstrated in future version of QUBIC (Hamilton & Charlassier 2010).

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Fig. 3. The u–v plane coverage of one baseline of an interferometer for a single pointing in a single baseline orientation angle. Radial spread in a single pixel in the u–v plane due to bandwidth is shown. Resolution in the u–v plane is determined by the primary beam or size of field-of-view. The minimum size of each sub-band is also determined by this resolution, as shown.
Fig. 4. A Fizeau adding interferometer used as a beam combiner for long-baseline optical interferometry (figure reproduced from Traub (2000)). The baseline $B_{12}$ is the separation of the apertures, which can be widely spaced, and determines the angular resolution of the instrument. The Fizeau beam combiner has a different and much smaller baseline length (not labeled). The ‘external’ phase differences are marked $B_{12} \cdot \hat{n}$.

Fig. 5. A simple 1-d Fizeau system, similar to a figure in Traub (2000). $\mathbf{O}$ denotes the origin of the co-ordinate system, and is the center of the observation plane. $\mathbf{x}_i$’s denote positions of detectors on the focal plane, and $\mathbf{A}_i$’s denote positions of apertures, where $i \in [1 \ldots N]$. Notice that there are two sets of phase differences, marked $(A_p - A_i) \cdot \hat{n} \equiv B_{ij} \cdot \hat{n}$ and $\mathbf{x}_{jk}(\mathbf{x}) = \mathbf{x}_j \cdot (A_p - A_i)$ for external and internal phase differences respectively. The geometrical variation of $\mathbf{x}_{jk}(\mathbf{x}) = \mathbf{x}_j \cdot (A_p - A_i)$ allows “sub-band splitting”.

Appendix A: Imaging in a Fizeau system

In eq. (A.9), if we now integrate over the focal-plane area, omitting the integral over bandwidth, which is implicit (and remembering that \( \varphi = 2\pi B/\lambda \)):

\[
O = \int \int \Re \left( G(\hat{n},l,v) \right) \exp \left[ 2\pi \frac{B \cdot \hat{n} - \chi_{ij}}{\lambda} \right] d\hat{n}^2 d\chi \tag{A.1}
\]

where we have changed the sign on the ‘internal’ phase differences. This can be done without loss of generality, since the internal and external phase differences are independent of each other.

Let us consider just one term in the expression \( \Re (\ldots) \):

\[
O = \int \int I(\hat{n},v) \exp \left[ 2\pi \frac{B \cdot \hat{n}}{\lambda} \right] d\hat{n} \exp \left[ -i2\pi \frac{\chi_{ij}}{\lambda} \right] d^2 \chi \tag{A.2}
\]

If we include the effect of the primary beams of the inward-facing antennas \((G(x))\) and adopt \( I = I(\hat{n},v) \),

\[
O = \int G(x) \int G(\hat{n}) I \exp \left[ 2\pi \frac{B \cdot \hat{n}}{\lambda} \right] d\hat{n} \exp \left[ -i2\pi \frac{\chi_{ij}}{\lambda} \right] d^2 \chi \tag{A.3}
\]

The quantity in underbrace is clearly a fourier transform, and the expression can be written as

\[
O = \int G(x) \tilde{G}(GI) \exp \left[ -i2\pi \frac{\chi_{ij}}{\lambda} \right] d^2 \chi \tag{A.4}
\]

If the distance from the inward-facing antennas to the focal plane >> the collecting area for each bolometer,

\[
O = \tilde{\gamma}^{-1}(G\tilde{G})(GI) \tag{A.5}
\]

The beam needs to be deconvolved from the above expression in order to obtain an image from the instrument.

Now, eq. (A.4) can be split up over the focal plane:

\[
O = \sum_{i=1}^{N} \int G\tilde{G}(GI) \exp \left[ -i2\pi \frac{\chi_{ij}}{\lambda} \right] d^2 \chi \tag{A.6}
\]

where \( 1 \ldots N \) are labels for bolometers on the focal plane.

Each of the bolometer outputs then represents a pixel in image space. The total number of pixels depends on the resolution of the instrument, and not the number of bolometers on the focal plane. Therefore, if the number of bolometers on the focal plane are greater than the number of pixels in the image, we need to “repixelize” the image obtained, so that all pixels are independent of each other.

In general, this is how the beam is convolved with the image on the sky for the Fizeau beam combiner:

\[
O = \tilde{\gamma}^{-1}(G\tilde{G})(GI) \tag{A.7}
\]

\[
= \left[ (\tilde{\gamma}^{-1}G) \star (GI) \right] \tag{A.8}
\]

In traditional interferometry, eq. (A.7) would read

\[
O = \tilde{\gamma}^{-1}(\tilde{G})(GI) \tag{A.9}
\]

\[
= \tilde{\gamma}^{-1}(\tilde{G}G) \star (\tilde{G}I) \tag{A.10}
\]

\[
\equiv GI \tag{A.11}
\]

In eq. (A.10), the \( u-v \) space beam \( \tilde{G} \) needs to be multiplied by \( u-v \) coverage, which is a “mask”, say \( \mathcal{M}(u,v) \). Then, \( \tilde{\gamma}^{-1}(\tilde{G} \times \mathcal{M}(u,v)) \) is called the “dirty beam” in traditional interferometry. In eq. (A.7), the factor \( \mathcal{M}(u,v) \) is included. Eq. (A.8) thus tells us that the dirty beam for the image produced by the Fizeau combiner is more involved than the traditional interferometer dirty beam, but remains conceptually equivalent.

There are two assumptions inherent in the foregoing discussion:

1. The focal plane is large enough to receive most of the power from the inward-facing antennas

2. There are no “blank” areas on the focal plane for which the incident power is not absorbed by a bolometer

It is possible to detect the correlated signal from a pair of antennas as well. In order to separate every unique baseline, time-varying phase-shifts can be applied to the signal at the base of the skyward-facing antennas. The “correlated signal” from each pair of antennas is simply the visibility from a baseline, with one crucial difference: there is an “internal phase” added to every visibility due to the relative positions of antennas on the observation plane and detectors on the focal plane. These phase differences are geometrical. Since the output from all \( N \) antennas is incident on every detector, we can say that the output from a single detector contains information about \( N(N - 1)/2 \) visibilities.