Fulfillment of expectations of precise measurements of the Casimir force

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We compare theoretical expectations for the Casimir force with the results of precise measurements. The force is calculated at finite temperature for multilayered covering of the bodies using the Lifshitz theory. We argue that the dielectric function of the metallization has to be directly measured to reach the necessary precision in the force calculation. Without knowledge of this function one can establish a well defined upper limit on the force using parameters of perfect single-crystal materials. The force measured in the torsion pendulum experiment does not contradict to the upper limit. Importance of a thin Au/Pd layer in the atomic force microscope experiments is stressed. The force measured with the microscope is larger than the upper limit at small separations between bodies. The discrepancy with the measurements is discussed. The simplest modification of the experiment is proposed allowing to make its results more reliable and answer the question if the discrepancy has any relation with the existence of a new force.

I. INTRODUCTION

The Casimir force [1] (see [2] for a review) between closely spaced macroscopic bodies is an effect of quantum electrodynamics (QED) and for this reason it could be predicted very accurately. The force acting between nonideal bodies can be found using the Lifshitz theory [3,4], where it depends on optical properties of used materials. Knowledge of these properties is the weakest element in the theory restricting the accuracy that can be achieved. Experiments measuring the Casimir force are of great importance because they are sensitive to the presence of new fundamental forces [5,6] predicted in many modern theories (see [7,8] and references therein). To distinguish a new force from the background, we should be able to calculate the Casimir force with a precision better than the experimental one. In the series of recent experiments this force has been measured with the torsion pendulum (TP) [9] in the range of distances $0.6 - 6 \mu m$ and with the atomic force microscope (AFM) [10–12] in the range $0.1 - 0.9 \mu m$. The corresponding precisions were 5% and 1%, respectively.

For two ideal plates the famous Casimir formula [1] for the force per unit area is

$$ F_{pl}^c(a) = \frac{\pi^2 \hbar c}{240a^4}, $$

where $a$ is the distance between plates. In the experiments the force is measured between metallized disc and sphere because for two plates it is difficult to keep them parallel. In this case (1) has to be modified with the proximity force theorem (PFT) [13]. This theorem allows to evaluate the force by adding the contributions of various distances as if they were independent and for plate and sphere it is reduced to

$$ F(a) = 2\pi R \int_a^{R+a} F_{pl}^c(x) \, dx, $$

where $R$ is the radius of curvature of the spherical surface. The PFT approximation is good for $R \gg a$ that holds true in all the experiments. If we use the Casimir expression (1) for the force in (2), then the force between plate and sphere will be

$$ F_{p0}^c(a) = \frac{\pi^3 \hbar c}{360} \frac{R}{a^3}. $$

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II. THEORY

Let us discuss first a reliable way to evaluate the Casimir force in the experimental configurations. The force per unit area between parallel plates arising as a result of electromagnetic fluctuations is generalized by the Lifshitz theory \( \text{[4]} \), where the real material is taken into account by its dielectric function at imaginary frequencies \( \varepsilon (i\omega) \):

\[
F_P(a) = \frac{kT}{\pi c^2} \sum_{n=0}^{\infty} \int_1^\infty dpp^2 \left\{ G_1^2 e^{2\nu_n a/c} - 1 \right\}^{-1} + \left\{ G_2^2 e^{2\nu_n a/c} - 1 \right\}^{-1}.
\]  

(7)
Here prime over the sum sign means that \( n = 0 \) term is taken with the coefficient 1/2 and

\[
G_1 = \frac{p + s}{p - s}, \quad G_2 = \frac{\varepsilon (i\zeta_n) p + s}{\varepsilon (i\zeta_n) p - s},
\]

\[
s = \sqrt{\varepsilon (i\zeta_n) - 1 + p^2}, \quad \zeta_n = \frac{2\pi nkT}{\hbar}. \tag{8}
\]

It is supposed that both bodies were made of identical materials. The function \( \varepsilon (i\zeta_n) \) cannot be measured directly but can be expressed via imaginary part of the dielectric function \( \varepsilon''(\omega) \) on the real axis with the dispersion relation

\[
\varepsilon (i\zeta) - 1 = \frac{2}{\pi} \int_0^\infty d\omega \frac{\omega \varepsilon''(\omega)}{\omega^2 + \zeta^2}. \tag{9}
\]

Information on \( \varepsilon''(\omega) \) can be extracted from the data on reflectivity and absorptivity of electromagnetic waves with the frequency \( \omega \) for a given material.

Applying PFT to Eq. (8) one can find the force between sphere and plate. The integration in (9) can be done analytically and we find

\[
F(a) = -\frac{kTR}{c^2} \sum_{n=0}^\infty \zeta_n^2 \int_1^\infty dp p \ln \left[ \left( G_1^{-2} e^{-2p\zeta_n a/c} - 1 \right) \left( G_2^{-2} e^{-2p\zeta_n a/c} - 1 \right) \right]. \tag{10}
\]

Special care needs to treat the first \( n = 0 \) term. The formal reason is that \( \zeta_n^2 \) becomes zero but the integral over \( p \) diverges. The physical reason is that this term corresponds to the static limit when for metallic bodies \( \varepsilon \to \infty \).

This means that any parameter characterizing the dielectric function of a metal cannot appear in the \( \varepsilon''(\omega) \) for a given material.

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\[ \varepsilon_p. \]
To take it into account, one has to generalize expression for the force (7) to the case of layered bodies. We have to add only the matching conditions for the Green functions on the layers interface. After some algebra the result will look exactly as (7) but with more complex $\varepsilon_p$ for the Drude dielectric function (see Eq. (13) below) with the damping frequency $\omega_c = 5 \cdot 10^{13}$ s\(^{-1}\). These values exceed the conservative estimate for the experimental errors $2 \cdot 10^{-6}$.

In the AFM experiments an additional Au/Pd layer of 20 nm [10] or 8 nm [11,12] thick was on the top of Al metallization of the bodies to prevent aluminum oxidation. It has to be included into consideration. This layer is transparent for the electromagnetic waves with high frequencies $\sim c/a$ since adsorption, proportional to $\varepsilon''(\omega)$, is small. For this reason the layer was ignored in [10,12]. However, the force depends on $\varepsilon(i\xi)$ for which the low frequencies dominate in the dispersion relation $\xi$ because of large $\varepsilon''(\omega)$ and that is why we cannot neglect the Au/Pd layer. To take it into account, one has to generalize expression for the force (7) to the case of layered bodies. Suppose that the top layer has the thickness $h$ and its dielectric function is $\varepsilon_1$. The bottom layer is thick enough to be considered as infinite and let its dielectric function be $\varepsilon_2$. The method described in [3] for deriving Eq. (7) can be easily generalized for layered plates. We have to add only the matching conditions for the Green functions on the layers interface. After some algebra the result will look exactly as (7) but with more complex $G_{1,2}$:

\[
G_1 = \left( \frac{s_1 + s_2}{p + s_1} + \frac{1}{p - s_1} \right) e^{i \omega_s h/c} + \left( \frac{s_1 - s_2}{p - s_1} + \frac{1}{p + s_1} \right) e^{-i \omega_s h/c},
\]

\[
G_2 = -\left( \frac{\varepsilon_2 s_1 + \varepsilon_1 s_2}{\varepsilon_2 p + s_1} + \frac{1}{\varepsilon_2 p - s_1} \right) e^{i \omega_s h/c} + \left( \frac{\varepsilon_2 s_1 + \varepsilon_1 s_2}{\varepsilon_2 p + s_1} + \frac{1}{\varepsilon_2 p - s_1} \right) e^{-i \omega_s h/c},
\]

where $s_{1,2}$ are defined similar to $s$ in (8). The force between plate and sphere is given by (10) with the above $G_{1,2}$.

To see qualitatively the effect of an additional layer, we found the finite conductivity correction up to the second order in this case. Than for the force one has

\[
F_c^0 (a, h) = F_c^0 (a) \left[ 1 - 4K(h) \frac{e}{a\omega_{1p} + \frac{72}{5} \left( K(h) \frac{c}{a\omega_{1p}} \right)^2} \right],
\]

where the function $K(h)$ depends on the plasma frequencies of the layers $\omega_{1p}, \omega_{2p}$ and the thickness of the top layer $h$

\[
K(h) = \frac{\omega_{1p} + \omega_{2p} \tanh (h \omega_{1p}/c)}{\omega_{2p} + \omega_{1p} \tanh (h \omega_{1p}/c)}.
\]

When $h = 0$ the force will depend only on $\omega_{2p}$ and in the case $h \to \infty$ on $\omega_{1p}$ as it should be. The effect of the top layer disappears if the plasma frequencies coincide. The top layer will be negligible if $h \omega_{1p}/c \ll 1$. For typical plasma frequencies $\sim 10^{16}$ s\(^{-1}\) it is definitely not the case even for $h = 8$ nm. The opposite conclusion made in [14] was based on the too small value of $\omega_p$ for gold as will be discussed below (see also [21]). Eq. (14) is not very good approximation and was discussed only for qualitative understanding of the effect. For actual calculations we will use the exact equations (11) and (13).

Importance of a thin metallic layer on the body surfaces has been stressed first in [26]. The general expression for the Casimir force between layered bodies has been presented in [21] but was not used for actual calculations. Significant role of the Au/Pd layer in the AFM experiments was indicated in our preprint [23], where it was demonstrated that the effect far exceeds the experimental errors. This conclusion was supported in [27] where the expressions (13) for $G_{1,2}$ were confirmed using a different method to deduce them. However, the authors were uncertain on applicability of (13) for thin films with $h < 25$ nm because the spatial dispersion of the dielectric function can be important for such films. We discuss this effect in Sec. V where we argue that the spatial dispersion can be neglected because of very short mean free path for the electrons in thin films.

### III. THE DIELECTRIC FUNCTION

Now we are able to evaluate the Casimir force in real geometry of the experiments if there is information on the dielectric functions of used materials: Au, Al, and Au$_{0.6}$Pd$_{0.4}$ alloy. Strictly speaking, one has to measure these
functions in wide range of wavelengths on the same samples which are used for the force measurement. It was not done in all of the experiments and to draw any conclusion from them we have to make some assumptions on the dielectric functions. At low frequencies Au and Al are well described by the Drude dielectric function

\[ \epsilon = \epsilon' + i \epsilon'', \]

\[ \epsilon'(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + \omega_0^2}, \quad \epsilon''(\omega) = \frac{\omega_p^2 \omega_0}{\omega (\omega^2 + \omega_p^2)}, \]  \hspace{1cm} (16)

where \( \omega_p \) is the free electron plasma frequency and \( \omega_0 \) is the Drude damping frequency. A simple test for validity of the Drude model is behavior of the material resistivity \[ \rho (\omega) = \text{Im} \left( \frac{1}{\epsilon_0 (1 - \epsilon(\omega)) \omega} \right) = \frac{\omega_0}{\epsilon_0 \omega_p^2}, \]  \hspace{1cm} (17)

where \( \epsilon_0 \) is the free space permittivity. The resistivity is frequency independent within the Drude approximation. For high-purity single-crystal samples of Au and Al (entries 2 in Table |) the frequency behavior of the resistivity in the infrared range of wavelengths \( 3 \mu m < \lambda < 32 \mu m \) is shown in Fig. 1. The data on the dielectric functions were taken from [23], where the results from many original works are collected. The data for \( \epsilon'(\omega) \) and \( \epsilon''(\omega) \) can be fitted with (16) to find the parameters \( \omega_p \) and \( \omega_0 \). The points and fitting curves for \( \epsilon''(\omega) \) is shown in the same Fig. 1. Palladium definitely cannot be described by (16) since its resistance significantly changes in the infrared range. However, it is known experimentally that amorphous metallic alloys can be described by the Drude approximation [29].

Of course, at higher frequencies when interband transitions are reached the Drude approximation fails. Nevertheless, it is very helpful since low frequencies dominate in the dispersion relation. Extrapolating (16) to all frequencies one finds

\[ \epsilon(i\zeta) = 1 + \frac{\omega_p^2}{\zeta (\zeta + \omega_0)}. \]  \hspace{1cm} (18)

Let us estimate the relative error inserted in (18) due to extrapolation. If \( \omega_0 \) is the frequency of the first resonance for a given metal, then the contribution in \( \epsilon(i\zeta) \) of the frequency range \( \omega_0 < \omega < \infty \), where the Drude model does not valid, will be \( (\omega_p/\zeta)^2 \cdot (\omega_0/\omega_0) \) for \( \zeta > \omega_0 \). This contribution one can take as an estimate for the absolute error and, therefore, for the relative error one has \( \sim \omega_0/\omega_0 \). For typical values \( \omega_0 \sim 10^{14} \text{ s}^{-1} \) and \( \omega_0 > 10^{15} \text{ s}^{-1} \) the error can be as large as 10% but error in the force is smaller. If we will use (18) for the force computation and change \( \omega_p \) by 5% (10% correction to \( \epsilon(i\zeta) \) at all frequencies) then the force is changed less than 2%. Moreover, since the interband transitions give a correction to (18) which is frequency dependent, it reduces the correction to the force further. Of course, we can take the interband transitions into consideration exactly using the handbook data in visible-ultraviolet range which are not very sensitive to the purity and defect density as it happens in the infrared range. However, we are intended to establish the upper limit on the force using \( \omega_p \) in (18) which is definitely larger than any real value and for this reason we can neglect the interband transitions.

Therefore, in all cases of interest we can use Eq. (18) to describe the dielectric function of a material on the imaginary axis. The question is how we should extract the parameters \( \omega_p \) and \( \omega_0 \) from the data. We proceeded as follows. The data for the complex refraction index \( n + i \kappa = \sqrt{\epsilon} \) have been taken from [30]. First, the validity of the Drude approximation was checked by calculating the frequency dependence of the material resistivity according to (17). In the investigated cases the resistivity is more or less constant in the wavelength range \( \lambda > 2 \mu m (\omega < 9.4 \cdot 10^{14} \text{ s}^{-1}) \). This range gives the most important contribution to the dispersion relation (9) and, therefore, it is the range where we have to extract the Drude parameters. Using \( \omega_p \) found from the high frequency region can happen to be wrong. For example, sometime the plasma frequency is estimated using the transition point in the reflectivity dependence on frequency. It works, not very good though, for Al but gives considerably smaller value for Au than that found from fitting \( \epsilon(\omega) \) in the infrared region. Probably such an estimate was taken in [30] for Au where very small value \( \omega_p = 3.6 \cdot 10^{15} \text{ s}^{-1} \) was used.

The optimal fitting procedure is described in [31]. The damping frequency is evaluated first from the ratio \( (1 - \epsilon')/\epsilon'' \) which depends linearly on frequency in the Drude model

\[ \frac{1 - \epsilon'}{\epsilon''} = \frac{\omega}{\omega_0}. \]  \hspace{1cm} (19)

After that \( \omega_0^2 \) can be extracted from \( 1 - \epsilon' \) by linear fit. The results together with the statistical errors are collected in Table II for those data in [30] which include the optical behavior of Au and Al in the infrared region. This table clearly
demonstrates that the Drude parameters depend significantly on the sample which is used to measure the optical data. These samples contained different densities of the defects (such as impurity atoms, vacancies, dislocations, etc.) that influence their optical properties. In this sense there are no universal material parameters. Reproducible parameters one can get only for high-purity single-crystals. In this connection all the attempts to use the handbook data for the Casimir force calculation can be considered only as estimates and cannot claim on high precision.

Actually in any of the experiment we do not know the Drude parameters even with 10% accuracy. That is because the optical properties of evaporated or spattered films which cover the bodies can be quite different from those of bulk materials and depend on technological details of film preparation. It is known, for example, that the film density is typically 0.7 from that of the bulk material if it was not annealed. For the resistivity of spattered and evaporated Au \([3,2]\) the value \(\rho_0 = 8.2 \, \mu\Omega \cdot \text{cm}\) has been reported in contrast with the bulk resistivity 2.25 \(\mu\Omega \cdot \text{cm}\). If a metal is evaporated or spattered on a substrate, it has a large number of defects. Relatively thick metallic films (> 100 nm) are usually exist in polycrystalline form. Defects will reduce the concentration of free electrons \(n\) which defines the plasma frequency of the material. They also will increase the damping frequency \(\omega_r\) and resistivity since the mean free path of electrons will shorten. To minimize these undesirable in practical applications effects the films are usually annealed at high temperature. In the experiments \([3,4]\) it was not reported were the bodies annealed or not but one can say definitely that it was not done in the AFM experiments because the polysterene ball cannot exist at the annealing temperature. Even more defects present in thin films (\(\leq 20 – 30 \, \text{nm}\)) which are usually amorphous. This explains why thin films have very large resistivity in comparison with the bulk material. Entries 1 and 4 for \(\text{Au}\) in Table I correspond to the data for thick film samples. They support our expectations that the plasma frequency for thin films should be smaller and the resistivity larger than those for the bulk material.

IV. THE UPPER LIMIT

Though we cannot use the handbook data to evaluate the force, one can constrain it for a given experiment. This statement is based on the observation that the force \((\Delta F)\) increases every time when \(\omega_p\) increases or \(\omega_r\) decreases. It has simple physical meaning: the force becomes larger when the metal reflectivity increases. For us it is important that any technological procedures will reduce \(\omega_p\) and increase \(\omega_r\) for a given material. A perfect single-crystal will have the largest plasma frequency and the smallest \(\omega_r\) and these parameters are well defined. One can use them to get the upper limit on the Casimir force. The plasma frequency \(\omega_p\) is defined by the concentration of free electrons in the metal \(n\) and their effective mass \(m_e^*\)

\[
\omega_p = \sqrt{\frac{e^2 n}{m_e^* \varepsilon_0}},
\]

where \(e\) is the electron charge. For good metals, which we are concerned, \(m_e^*\) is close but larger than the electron mass. It will be helpful for what follows to use Eq. (17) and instead of the damping frequency \(\omega_r\) take the static resistivity \(\rho(0) = \rho_0\) as a parameter. The later can be directly measured for any material.

A. Torsion pendulum experiment

In the TP experiment \([3]\) the quartz lens and plate were covered first with \(Cu\) of thickness 0.5 \(\mu\text{m}\) and then with \(Au\) of the same thickness. The \(Au\) layer is thick enough to be considered as infinite and \(Cu\) will not play any role. We will find the upper limit on the electron concentration if suppose that every \(Au\) atom produce a free electron with the mass \(m_e^* = m_e\). Then for the \(Au\) plasma frequency one finds \(\omega_{pAu} = 1.37 \cdot 10^{16} \, \text{s}^{-1}\). The resistivity for crystalline gold is \(\rho_0^{Au} = 2.25 \, \mu\Omega \cdot \text{cm}\). One can compare these parameters with that given in Table I to make sure that they correspond to the limit values. Substituting these parameters in (18) and calculating the force according to (10) one finds the upper limit on the Casimir force \(F_{\text{max}}(a)\) in the TP experiment.

To compare the upper limit on the force with the measured force \(F_{\text{exp}}\), it is more convenient to consider the residual force defined as \(\Delta F(a_i) = F_{\text{exp}}(a_i) - F_{\text{max}}(a_i)\),

\[
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\]

1Note that Lamoreaux \([3]\) used different definition of the residual force \(F_{\text{exp}}(a_i) - F_0^0(a_i)\).
where \( a_i \) are the separations for which the force has been measured. Theory and experiment will be in agreement if \( \Delta F \) will not be positive within the experimental errors. The original experimental data were presented for the lens curvature radius \( R = 11.3 \text{ cm} \) and the residual force in this case is shown in Fig. 2a. It clearly indicates the presence of some unexplained force at the smallest separations. However, later the author recognized [19] that he was working with aspheric lens which had the curvature radius \( R = 12.5 \pm 0.3 \text{ cm} \) in the place where the force was measured. The correction was published in erratum [33]. The points for \( \Delta F(a_i) \) with the corrected \( R \) are presented in Fig. 2b. This time the prediction obviously does not contradict to the experiment but dealing with the upper limit we cannot conclude that there is an agreement, either.

The question about surface distortions in TP experiment has been raised in [14]. Surfaces of the bodies have not been examined in [9] but roughness of the order of 30–40 nm is quite typical for a metallic film on a polished substrate and correlates with the substrate roughness. Quartz optics is used for near UV light and its surface has to be polished with a precision at least \( \lambda/10 \), where \( \lambda \sim 300 \text{ nm} \) is the UV wavelength. It supports the value above which is routinely observed with atomic force or tunnel microscope. According to [14] the short-scale stochastic distortions give only a few percent correction even for the smallest separation \( a = 0.6 \mu m \). Large-scale deviations seem potentially more dangerous [13] since the correction can be the first order in \( A/a \) especially if we take into account that the lens was not spherical. The radius \( r_{int} \) of the interaction one can estimate as \( r_{int} \sim \sqrt{R a} \sim 1000 \mu m \). Therefore, only small area on the lens takes part in the interaction. In this place the lens can be represented as part of the parabolic surface

\[
z = \frac{r^2}{2(R + \Delta R)} \approx \frac{r^2}{2R} \left( 1 - \frac{r^2}{2R^2} \Delta R \right),
\]

where \( \Delta R \) is the error in the curvature radius. Here the second term describes the error in the plate-lens separation and it can be taken as the distortion amplitude. This amplitude is maximal for \( r = r_{int} \) and for the relative amplitude one has an estimate

\[
\frac{A}{a} \sim \frac{1}{2} \frac{\Delta R}{R} \approx 1.2 \cdot 10^{-2}.
\]

This value is rather small and according to [14] the correction to the force will be less but comparable with the experimental errors. Moreover, negligible role of the large-scale distortions is actually an experimental fact. The region of the plate and sphere used for the force measurement in [9] was varied by tilting the lens with the adjustment screws and there was no evidence for any variation of the force depending on the region used for the measurement.

The first attempt to evaluate the Casimir force was undertaken by the author of TP experiment [9] who takes into account the first finite conductivity correction but used very small \( \omega_p \) for Au. Lamoreaux was the first who recognized the necessity of more rigorous approach to the force evaluation [21] and importance of thin films on the metallic surfaces [22]. His numerical results were not quite good due to the delicate problem with choice of \( \omega_p \) which we discussed above. The matter has been settled in [21,22] with the result which coincide with ours. However, our statement is that the calculated force represents the upper limit but not the force itself. The reason is that evaporated Au film will have smaller plasma frequency than 1.37 \( \times 10^{16} \text{ s}^{-1} \) due to large number of defects in the film. To know the exact value of the force, one has to measure the dielectric function of the bodies but not to take it from a handbook.

B. AFM experiment

Let us discuss now the upper limit on the Casimir force for the AFM experiment [10]. The plasma frequency for Al can be restricted using [22] if one supposes that every atom produces 3 free electrons. It gives \( \omega^p_{Al} = 2.40 \times 10^{16} \text{ s}^{-1} \) that coincide with the largest value in Table II. The resistivity of perfect crystals is \( \rho_0^{Al} = 2.65 \mu \Omega \cdot \text{cm} \). Since we successfully predicted the plasma frequencies for the best samples of Au and Al, the same way one can use to estimate \( \omega_p \) for Al/Pd. If each Au atom gives one and Pd atom gives not more than two free electrons, then \( \omega^p_{Au/Pd} = 1.69 \times 10^{16} \text{ s}^{-1} \). This alloy is used in microelectronics and resistivity of the bulk material is known to be \( \rho_0^{Au/Pd} \sim 30 \mu \Omega \cdot \text{cm} \) [14] in accordance with the statement that alloys have large Drude damping. These data allow to find the upper limit on the force using [11] with the functions \( G_{1,2} \) defined in [14].

Before comparing the upper limit with the measured force we have to discuss a few additional aspects concerning the experiment. Real surface of the bodies is always distorted and the distortions are especially important to treat the data in [10]. The distortion statistics were analysed with the atomic force microscope [19]. The force has to be averaged over the distorted surfaces and we use for this the procedure developed in [19]. The major distortions are the large separate crystals situated irregularly on the surfaces with a typical lateral size of 200 nm. The height of the
highest and intermediate distortions is about \( h_1 = 40 \text{ nm} \) and \( h_2 = 20 \text{ nm} \), respectively. The homogeneous stochastic background of the averaged height \( h_0/2 = 5 \text{ nm} \) fills the surface between the major distortions. The character of roughness on the plate and on the ball is quite similar. The part of the surface occupied by distortions with the height \( h_1, h_2, \) and \( h_0/2 \) was measured as \( v_1 = 0.11, v_2 = 0.25, \) and \( v_0 = 0.64, \) respectively. These values are the probabilities for the corresponding distortion to appear. The body surface is defined in such a way that averaging over distortions gives zero result. Then the averaged force is the sum of local forces for all possible kinds of distortions which face each other taken with the corresponding probabilities

\[
F^{\text{dist}}(a) = \sum_{i,j=0}^{2} v_i v_j F(a_{ij}),
\]

where \( a_{ij} \) are the local separations defined in [19]. For us it will be important that the minimal local separation is \( a_{11} = a - 54.8 \text{ nm} \). This procedure seems quite reliable but, of course, large distortions give the feeling of uncertainty. The upper limit on the Casimir force has to be averaged with the corresponding roughness parameters according to (24).

The raw force \( F_m \) measured in the experiments consists of a few components [10]

\[
F_m = F_c(a_1 + a_0) + F_e(a_1 + a_0) + C \cdot (a_1 + a_0).
\]

Here \( a_1 \) is the separation from the voltage applied to the piezo corrected to the cantilever deflection, \( a_0 \) is the parameter chosen in such a way that \( a = a_1 + a_0 \) is the absolute separation between bodies, the first term in (25) is the Casimir force, the second term is the electrostatic force corresponding to the measured contact potential \( 29 \text{ mV} \), the third term represents the linearly increasing coupling of the scattered light into the photodiodes (see [10] for details). The parameters \( a_0 \) and \( C \) were determined at large separations, where \( F_c \) is represented by the bare force (1). Then the Casimir force can be extracted from the raw data with the help of (25). Similar way to find the Casimir force from (25) but \( a \) was interpreted as the absolute separation between \( \text{Al} \) surfaces [10,19]. Effectively the \( \text{Au}/\text{Pd} \) layers were changed by \( \text{Al} \) which has larger \( \omega_p \) and, therefore, the force calculated theoretically was overestimated.

It was indicated [10] that the thickness of \( \text{Au}/\text{Pd} \) layer is less than \( 20 \text{ nm} \), that is why for calculations we use the conservative value \( h = 15 \text{ nm} \). This change makes the force only larger. The experimental points from [10] (triangles) and theoretical upper limit on the force including the roughness correction (solid line) are shown in Fig. 3 in the small separations range \( a < 250 \text{ nm} \). If the top layer is changed by \( \text{Al} \), it enlarges the force on \( 15 \text{ pN} \) at the smallest separation \( a = 120 \text{ nm} \). It is clear that the top layer definitely cannot be ignored in the force evaluation. Variation of \( \omega_p^{\text{Al}} \) on \( 10\% \) gives only \( 1 \text{ pN} \) change in the force because of screening effect of the top layer. The same variation in \( \omega_p^{\text{Au}/\text{Pd}} \) changes the force on \( 2 \text{ pN} \). The resistivity variation of the \( \text{Au}/\text{Pd} \) layer on \( 30\% \) gives \( 1 \text{ pN} \) effect. At larger separations all the effects become smaller.

We can see from Fig. 3 that the upper limit is smaller than the force measured at small separations and the difference is significant. This conclusion contradicts to that in [19], where good agreement between theory and experiment has been reported (dotted line) based on the detailed theoretical analysis. We have already stressed the importance of the \( \text{Au}/\text{Pd} \) layer but it is not the only reason of deviations. It comes also from poor behavior of the finite conductivity correction used in (11) at small separations \( a \). The correction was based on a simple interpolating formula (10) for the force between two plates

\[
F_c^{\text{plates}}(a) = F_c^0(\text{plates}) \left( 1 + \frac{11}{3} \frac{C}{\omega_p} \right)^{-\frac{11}{4}},
\]

which is applicable in wider range of separations than the expansion up to the second order (1). It was used to calculate the conductivity correction between sphere and plate. Applying the proximity force theorem to (24) one gets

\[
F(a) = 3F_c^0(a) \int_{1}^{\infty} \frac{dx}{x^4} \left( 1 + \frac{11}{3} \frac{C}{\omega_p} \right)^{-\frac{11}{4}},
\]

where the upper limit was moved to infinity since \( a \ll R \). This integral can be expressed via the Gauss hypergeometric function but in [19] it was expanded in the series up to the fourth order.
Fortunately, it is easy to restore the right data. In 
importance of the conclusion. It is stated in 
that the points have to be shifted to smaller separations. Even 
These detailed explanations are given not only to answer the criticism of our preprint [23] but also because of great 
To check the procedure, we successfully reproduced the force from the raw data in [10]. The force found from the 
presented in [10] and [12], where the procedure has been described in details. The data were available only for one scan 
that the right transformation was done we have tried to reproduce the measured force directly from the raw data 
and the equations (24) and (28) were used to get the theoretical prediction for the corrected Casimir force (dotted line 
in Fig. 3). We found that the interpolating curve which we got by numerical integation of (27) is very close to the 
exact force evaluated according to (10), (8) with the parameter 
\( \omega_p \) the force has to be smaller, but the actual relation is opposite and the difference is large. 

\[ F(a) = F^0_c(a) \left[ 1 - 4 \frac{c}{a\omega_p} + \frac{72}{5} \left( \frac{c}{a\omega_p} \right)^2 - \frac{152}{3} \left( \frac{c}{a\omega_p} \right)^3 + \frac{532}{3} \left( \frac{c}{a\omega_p} \right)^4 \right] \] (28)

and the equations (24) and (28) were used to get the theoretical prediction for the corrected Casimir force (dotted line 
in Fig. 3). We found that the interpolating curve which we got by numerical integration of (27) is very close to the 
extact force evaluated according to (10), (8) with the parameters \( \omega_p = 1.88 \cdot 10^{16} \text{ s}^{-1}, \omega_r = 0 \) from [19] and \( T = 300^\circ \text{K} \) in all range of distances. The small difference between the curves is the temperature effect which disappears when \( T \to 0 \). However, the expansion (28) works very bad at \( a < 100 \text{ nm} \). Although the separation in the experiment exceeds 100 nm, the local distance in (24) can be as small as 65 nm, where (28) is absolutely unacceptable. The same 
is true if one uses the expansion up to the fourth order found in [24] directly form (10). The dashed line in Fig. 3 
shows the force calculated according to (24), (27) with the parameters above. The divergence of the solid and dashed 
curves is the effect of the top layer and different parameters used for Al. The larger \( \omega_p \) which we are using for the 
upper limit is partly compensated by the top layer and that is why the dashed curve lies not too far from the solid one.

C. Improved AFM experiment

Very important progress has been achieved in [12] where controlled metal evaporation and smaller thickness of 
Au/Pd layer (\( h = 8 \text{ nm} \) instead of 20 nm) allowed to reduce the surface roughness to the level when the correction 
to the force becomes practically unimportant. Also the contact potential has been considerably reduced and the 
parameter \( a_0 \) defining the absolute separation of the surfaces has been found in independent electrostatic measure-
ments. Interaction between metallized ball and corrugated plate has been probed in [11]. It is not a subject of our 
consideration here. The data for flat plate and sphere in this work are actually in very good agreement with that given 
in [12] and we will not discuss them specially. The roughness parameters have been reduced to the following [12]: 
\( h_1 = 14 \text{ nm}, v_1 = 0.05; h_2 = 7 \text{ nm}, v_2 = 0.11; h_0/2 = 2 \text{ nm}, v_0 = 0.84 \). Unfortunately, the unjustified assumption 
that the force is insensitive to the presence of Au/Pd layer has been inserted in the procedure of the Casimir force 
extraction from the raw data and the following relation has been used instead of (23)

\[ F_m = F_c(a + 2h) + F_c(a) + Ca. \] (29)

For this reason we cannot directly use the data in [12] to compare with the theoretical prediction. It becomes obvious 
if we plot in the same figure the measured force in the experiments [10] and [12] (see Fig. 4). One would expect that 
for thicker Au/Pd layer the force has to be smaller, but the actual relation is opposite and the difference is large. 
Fortunately, it is easy to restore the right data. In [12] \( a_0 \) was found from an independent electrostatic measurement 
and the constant \( C \) was determined at large separations when the shift on \( 2h = 16 \text{ nm} \) in \( F_c \) argument was practically 
unimportant. The measured Casimir force \( F_{c-m} \) was expressed as

\[ F_{c-m}(a + 2h) = F_m(a) - F_c(a) - Ca, \] (30)

but the points in Fig. 4 taken from [12] represent the force as a function of true separation. Therefore, the force 
presented in the figure was calculated as

\[ F_{c-m}(a) = F_m(a - 2h) - F_c(a - 2h) - C(a - 2h). \] (31)

The Au/Pd layer certainly cannot be ignored and the right expression for the measured Casimir force must be

\[ F_{c-m}(a) = F_m(a) - F_c(a) - Ca. \] (32)

It is obvious that to restore the right data one has to shift the open squares in Fig. 4 on \( 2h = 16 \text{ nm} \) to larger 
separations. After this shift a good agreement between two different experiments is reached. To be absolutely sure 
that the right transformation was done we have tried to reproduce the measured force directly from the raw data 
presented in [10] and [12], where the procedure has been described in details. The data were available only for one scan 
and for this reason our calculations had restricted precision, but it was enough to make a conclusion on reproducibility. 
To check the procedure, we successfully reproduced the force from the raw data in [10]. The force found from the 
raw data in [12] according to (32) agrees much better with the shifted points than with the ones presented in [12]. 
These detailed explanations are given not only to answer the criticism of our preprint [23] but also because of great 
importance of the conclusion. It is stated in [33,27] that the points have to be shifted to smaller separations. Even
ignoring the arguments above it is obvious from Fig. 4 that such a shift would give drastic disagreement between two experiments made by the same method.

The upper limit on the Casimir force in conditions of the experiment [12] one can find exactly as was explained above. The only difference is the other set of roughness parameters, but in this case the roughness correction is on the level of experimental errors. The experimental points from [12] shifted on 16 nm as was discussed above and the corresponding upper limit (solid line) are presented in Fig. 8. Again we can see that the upper limit is smaller than the measured force and deviation increases at smaller separations. Moreover, even if we replace the 8 nm thick top layer by Al with the maximal plasma frequency \( \omega_p = 2.40 \times 10^{16} \text{ s}^{-1} \), this disagreement will not disappear as shows the dashed line. The residual force defined according (21) in the experiments [10] (triangles) and [12] (open squares) is shown in Fig. 6. It clearly demonstrates the presence of some unexplained attractive force which is decreasing rapidly when the distance between bodies increased. The points from two different experiments are in reasonable agreement with each other that means that the residual force is reproducible. The residual force becomes larger if we deviate the parameters from their limit values but the agreement between two experiments is not broken.

V. DISCUSSION

Let us discuss now possible reasons for disagreement between experiment and theoretical expectations. As was mentioned above the main problem is the values of the material parameters which can significantly deviate from their handbook values for evaporated or spattered metallic films. The idea of this paper was to find the upper limit on the force instead of the force itself. It allowed to use only well defined parameters of perfect single-crystal materials. We took the largest values for the plasma frequencies and the smallest ones for the resistivities. Any possible deviation from these values will make the force only smaller and disagreement between theory and experiment will be larger.

Some doubts were raised [26] about the possibility to describe the thin top layer by a dielectric function which depends only on frequency. It was stated that the spatial dispersion can be important for thin films because the distance traveled by electron during one period of the field can be larger than the film thickness

\[
\frac{v_F}{\omega} > h,
\]

where \( v_F \) is the velocity of the electron on the Fermi surface. This dimensional effect is really exist (see, for example, [30]) but it is difficult for observation at room temperature. The reason is that for thin metallic films the mean free path for electrons is very short (< 100 Å) because of large concentration of the defects. Typically the resistivity of very thing films is on the level of 100 \( \mu \Omega \cdot \text{cm} \). Then for \( \omega_p \sim 10^{16} \text{ s}^{-1} \) from (17) one finds \( \omega \tau \sim 10^{15} \text{ s}^{-1} \). The mean free path is estimated

\[
l = \frac{v_F}{\omega \tau} \sim 10 \text{ Å}
\]

that is smaller than the used Au/Pd film thickness and, therefore, the spatial dispersion can be neglected. That is why the standard dependence for the dielectric function \( \varepsilon (\omega) \) is widely used in optics of metals up to the film thickness in a few nanometers when quantum effects become involved. For the same reason one can neglect the anomalous skin effect for evaporated (spattered) films even for thick ones. Extremely high resistivity 2000 \( \mu \Omega \cdot \text{cm} \) for 60 nm thick Au/Pd film has been reported in [27]. However, the authors themselves stress that the resistance of the film is, to all appearance, dominated by grain boundaries but optical properties of the film are quite usual. This example shows once more that the details of the spattering technology cannot be ignored.

In [21] uncertainty was expressed about applicability of the proximity force theorem. At the moment there is no any work where the force between sphere and plate was calculated from ”the first principles”. There are some heuristic approaches [3] allowing to calculate nonadditive Casimir force which agree well with the result found by using PFT (see additional discussion and references in [21]). The PFT states that the main contribution to the force can be found by adding the contributions of various distances as if they were independent and it is applicable to nonadditive forces. An example is the electrostatic force which is nonadditive because the surface charge density is nonuniform for curved surfaces. One can check that the PFT gives in this case the correct result with the accuracy \( \sim a/R \). For the discussed experiments the correction is very small \( (a/R \sim 0.001) \). Even if this term appears with a large coefficient, say \( \sim 10 \), the correction will be only on the level of the experimental errors.

In the AFM experiments the electrostatic attraction between bodies because of contact potential was carefully taken into account. Of course, the aluminum surfaces were partly oxidized and electrons could be trapped in the oxide. These charges can be potentially dangerous if the Au/Pd film is not continuous. In this case the trapped charges and their images in underlying aluminum will be the source of the dipole field. Then an additional force can arise as
a result of dipole-dipole interaction. However, it is difficult to make a reliable estimate for this effect because we do not know the concentration of trapped charges and the size of islands in $Au/Pd$ layer or even do discontinuities exist at all for the used layer thickness (it depends on details of the covering procedure). In this connection to make the experiment absolutely clear, it is preferable to use $Au$ instead of $Al$ metallization because its non-reactive surface has strong advantage over $Al$. It excludes also additional uncertainties connected with $Au/Pd$ layer. One can use as well silver or copper but they are not as inert as gold. It is difficult to measure the dielectric function at the wavelengths larger than 30 $\mu m$ but this range gives an important contribution to the dispersion relation. That is why the material behavior in this range has to be predictable. One can say definitely that the materials of platinum group cannot be used since they are not described by the Drude dielectric function at low frequencies.

One can speculate that the observed discrepancy is explained by a new Yukawa force mediated by a light scalar boson. Then interaction of two atoms is described by the Yukawa potential

$$V_Y (r) = -\alpha N_1 N_2 \frac{\hbar c}{r} \exp(-r/\lambda), \quad (35)$$

where $\alpha$ is a dimensionless interaction constant, $\lambda$ is the Compton wavelength of a particle responsible for the interaction, and $N_{1,2}$ is the number of nucleons in atoms of the interacting bodies. An additional advantage of $Au$ metallization is higher density of the bodies coating. In this case the Yukawa force will be enlarged roughly by the factor $(\rho_{Au}/\rho_{Al})^2 \approx 50$, where $\rho_{Au,Al}$ are the material densities. If the observed discrepancy has relation with the Yukawa interaction, the AFM experiment with $Au$ metallization of the bodies will definitely reveal this new force even without detailed knowledge of optical properties of the metallization.

VI. CONCLUSION

We have analysed the results of recent precise measurements of the Casimir force using the Lifshitz theory to evaluate the force. Layered structure of the bodies coating was taken into account in the frame of the Lifshitz approach. It was stressed that the force cannot be predicted with necessary accuracy if there is no detailed information on the dielectric function of the bodies coating. Fortunately, all the used materials ($Au$, $Al$ and $Al_{0.6}Pd_{0.4}$) are well described in terms of Drude parameters $\omega_p$ and $\omega_\tau$ in the infrared range which dominates in the dispersion relation for the dielectric function $\varepsilon (i\zeta)$. It was noted that one can find the upper limit on the Casimir force that realized for perfect single-crystal materials for which electrical and optical properties are well defined. The surface roughness and linear in temperature corrections were taken into consideration. It was shown that the upper limit on the Casimir force does not contradict to the result of the torsion pendulum experiment. The main conclusion of the paper is that the upper limit is smaller than the observed force in the AFM experiments and the difference far exceeds experimental errors and theoretical uncertainties for small separations between bodies. The simplest modification of the experiment is proposed allowing to reveal origin of the discrepancy.

[1] H.B.G. Casimir, Koninkl. Ned. Akad. Wetenschap. Proc. 51, 793 (1948).
[2] V.M. Mostepanenko and N.N. Trunov, The Casimir effect and its applications (Clarendon Press, Oxford, 1997).
[3] E.M. Lifshitz, Sov. Phys. JETP 2, 73 (1956).
[4] E.M. Lifshitz and L.P. Pitaevskii, Statistical Physics, Part 2, (Pergamon Press, Oxford, 1980).
[5] V.A. Kuz'min, I.I. Tkachev, and M.E. Shaposhnikov, JETP Letters (USA) 36, 59 (1982).
[6] V.M. Mostepanenko and I.Yu. Sokolov, Phys. Lett. A 125, 405 (1987).
[7] J.C. Long, H.W. Chan, and J.C. Price, Nucl. Phys. B 539, 23, (1999).
[8] E. Fischbach and C. Talmadge, The Search for Non-Newtonian Gravity (AIP Press/Springer-Verlag, New York, 1999).
[9] S.K. Lamoreaux, Phys. Rev. Lett. 78, 5 (1997); 81, 5475(E) (1998).
[10] U. Mohideen and A. Roy, Phys. Rev. Lett. 81, 4549 (1998); e-print physics/9805032.
[11] A. Roy and U. Mohideen, Phys. Rev. Lett. 82, 4380 (1999).
[12] A. Roy, C.-Y. Lin, and U. Mohideen, Phys. Rev. D 60, 111101 (1999); e-print quant-ph/9906062.
[13] J.Blocki, J. Randrup, W.J. Swiatecki, and C.F. Tsang, Ann. Phys. (N.Y.) 105,427 (1977).
[14] I. E. Dzyaloshinskii, E. M. Lifshitz, L. P. Pitaevskii, Sov. Phys. Uspekhi 4 (1961) 153.
[15] C. M. Hargreaves, Proc. Kon. Nederl. Acad. Wet. B68 (1965) 231; J. Schwinger, L.L. DeRaad, Jr., and K.A. Milton, Ann. Phys. (N.Y.) 115, 1 (1978).
V.M. Mostepanenko and N.N. Trunov, Sov. J. Nucl. Phys. (USA) 42, 812 (1985); V. B. Bezerra, G. L. Klimchitskaya, C. Romero, Mod. Phys. Lett. A 12 (1997) 2623.

J. Mehra, Physica 37, 145 (1967); L.S. Brown and G.J. Maclay, Phys. Rev. 184, 1272 (1969); J. Schwinger, L.L. DeRaad, Jr., and K.A. Milton, Ann. Phys. (N.Y.) 115, 1 (1978).

M. Bordag, G.L. Klimchitskaya, and V.M. Mostepanenko, Int. J. Mod. Phys A 10, 2661 (1995).

G. L. Klimchitskaya, A. Roy, U. Mohideen, and V. M. Mostepanenko, Phys. Rev. A 60, 3487 (1999); e-print quant-ph/9906033.

S.K. Lamoreaux, Phys. Rev. A59 R3149 (1999).

A. Lambrecht and S. Reynaud, Europ. Phys. J. D 8 309 (2000); e-print quant-ph/9907105.

A. Lambrecht and S. Reynaud, Phys. Rev. Lett., to appear; e-print quant-ph/9912087.

V.B. Bezerra, G.L. Klimchitskaya, V.M. Mostepanenko, e-print quant-ph/9912090.

V.B. Svetovoy and M.V. Lokhanin, e-print quant-ph/0001010.

G. L. Klimchitskaya, U. Mohideen, and V. M. Mostepanenko, e-print quant-ph/9907074.

S.K. Lamoreaux, e-print quant-ph/9907074.

P. Grosse, Free electrons in solids, (Springer-Verlag, Berlin,1979).

B.N.J. Persson, J.E. Demuth, Phys. Rev. B 31, 1856 (1985).

V.M. Zolotarev, V.N. Morozov, and E.V. Smirnov, Optical constants of natural and technical mediums (Chimiya, Leningrad,1984).

Handbook of Optical Constants of Solids, E.D. Palik ed. (Academic Press, New York 1995).

L. Lechevallier, G. Richon, J. Le Bas, and M. Bernole, Vacuum 41, 1218 (1990).

M. Bordag, B. Geyer, G. L. Klimchitskaya, and V. M. Mostepanenko, Phys. Rev. D 58, 075003 (1998).

Materials in mechanical engineering and automation. Handbook. Edited by Yu.M. Pyatin, 2nd ed. (Mashinostroenie, Moskva, 1982).

M. Bordag, B. Geyer, G. L. Klimchitskaya, and V. M. Mostepanenko, e-print hep-ph/0003011.

D.S. Campbell and A.K. Morley, Rep. Prog. Phys. 34, 283 (1971).

J.R. Kirtley, S. Washburn, and M.J. Brady, Phys. Rev. Lett. 60, 1546 (1988).

FIG. 1. This figure demonstrates validity of the Drude approximation for Al (triangles) and Au (circles) in the infrared range. The resistivity does not depend on frequency (left axis). Solid lines (right axis) show that $\varepsilon''(\omega)$ depends on $\omega$ according to (16) with the parameters given in Table I (entries 2).

FIG. 2. The residual force [21] as a function of sphere-plate separation $a$ for TP experiment [1]. For the original value of sphere radius $R = 11.3$ cm the points of closest approach in (a) demonstrate presence of some unexplained force. With the corrected value $R = 12.5$ cm [22] the residual force is shown in (b). In this case there is no contradiction between theory and experiment.

FIG. 3. The Casimir force measured in the AFM experiment [11] (triangles). The solid line represents the upper limit on the force. The dotted line is taken from [19] where the Au/Pd layer was ignored and expansion (28) used for the finite conductivity correction. This expansion fails at small separations and Eq. (27) has to be used instead. The result is represented by the dashed line. The difference between the solid and dashed lines is due to Au/Pd layer.

FIG. 4. Comparison of the results of two AFM experiments. The data from [11] are marked by solid triangles and the data from [12] presented as the open squares. The obvious contradiction of two experiments made with the same technique is connected with unjustified assumption of the transparency of Au/Pd layer in [12]. The right data can be restored by simple shift all the open squares on 16 nm to larger separations (see explanations in the text) and after that the experiments will agree with each other.

FIG. 5. The data from [12] shifted on 16 nm (open squares) and the upper limit on the Casimir force (solid line). The dashed line represents the force for the case when the 8 nm thick Au/Pd top layer is changed by Al with the maximal plasma frequency $\omega_p = 2.4 \cdot 10^{16}$ s$^{-1}$. 
FIG. 6. The residual force defined as (21) for the two AFM experiments. The data from [10] and [12] are presented as solid triangles and open squares, respectively. The figure demonstrates presence of some unexplained force which decreases rapidly with separation increase. The residual force is reproduced for both of the experiments.

|    | $\omega_p \cdot 10^{-15}$ | $\omega_\tau \cdot 10^{-13}$ | $\rho_0 \ \mu\Omega \cdot \text{cm}$ |
|----|----------------|----------------|------------------|
| Al |                |                |                  |
| 1  | 1.54 ± 0.01   | 15.5 ± 0.6    | 7.39             |
| 2  | 2.235 ± 0.001 | 12.49 ± 0.01  | 2.83             |
| 3  | 2.43 ± 0.05   | 14.4 ± 0.7    | 2.76             |
| 4  | 1.63 ± 0.03   | 18.2 ± 0.7    | 7.74             |
| Au |                |                |                  |
| 1  | 1.280 ± 0.001 | 3.29 ± 0.05   | 2.27             |
| 2  | 1.3720 ± 0.0006 | 4.060 ± 0.002 | 2.44             |
| 3  | 1.34 ± 0.02   | 7.08 ± 0.18   | 4.46             |
| 4  | 1.0513 ± 0.0007 | 6.24 ± 0.21  | 6.40             |

TABLE I. Parameters of the Drude dielectric function (16) for Al and Au found by fitting the data [30] of different measurements in the infrared range ($\lambda > 2 \mu m$). The statistical errors of fitting are indicated. The resistivity was calculated via $\omega_p$ and $\omega_\tau$ according to (17). The stars in the first column mark the data for film samples.
