NUCLEAR PAIRING AT FINITE TEMPERATURE AND ANGULAR MOMENTUM *

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An approach is proposed to nuclear pairing at finite temperature and angular momentum, which includes the effects of the quasiparticle-number fluctuation and dynamic coupling to pair vibrations within the self-consistent quasiparticle random-phase approximation. The numerical calculations of pairing gaps, total energies, and heat capacities are carried out within a doubly folded multilevel model as well as several realistic nuclei. The results obtained show that, in the region of moderate and strong couplings, the sharp transition between the superconducting and normal phases is smoothed out, causing a thermal pairing gap, which does not collapse at a critical temperature predicted by the conventional Bardeen-Cooper-Schrieffer's (BCS) theory, but has a tail extended to high temperatures. The theory also predicts the appearance of a thermally assisted pairing in hot rotating nuclei.

1. Introduction

The effect of temperature and angular momentum on pairing properties is an interesting subject in the study of nuclear structure. Because of its simplicity, the BCS theory is often used, which offers a good description of pairing correlation in the macroscopic systems such as metallic superconductors. It predicts a collapse of the pairing gap at $T_c$, which signals the sharp superfluid-normal (SN) phase transition at finite temperature. The BCS theory, however, ignores quantal and thermal fluctuations, which are significant in finite small systems. Therefore, it needs to be corrected for the application to finite nuclei. Various theoretical approaches have

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been proposed to study the effects of fluctuations on nuclear pairing. Their results show that, at zero angular momentum, thermal fluctuations smear out the sharp SN phase transition, resulting in a pairing gap, which does not collapse at finite temperature. In rotating nuclei, a phenomenon of temperature induced pair correlations, which reflects the strong fluctuations of the order parameter in small systems, has also been predicted. The recent microscopic approach, called the modified BCS (MBCS) theory, has shown, for the first time, that the microscopic source causing the non-collapsing pairing gap is the quasiparticle-number fluctuation (QNF).

Recently, we proposed the self-consistent quasiparticle random-phase approximation (SCQRPA), which includes the QNF as well as the quantal fluctuations due to dynamic coupling to pair vibrations. The purpose of present work is to extend this approach to finite temperature and finite angular momentum.

2. Formalism

The pairing Hamiltonian is considered, which describes a system of N particles interacting via a pairing force with the parameter G and rotating with angular velocity \( \gamma \) and a fixed angular momentum projection \( M \) on the laboratory (or body) fixed \( z \)-axis:

\[
H = \sum_k \epsilon_k (N_k + N_{-k}) - G \sum_{k, k'} P_k^\dagger P_{k'} - \lambda N - \gamma M , \quad N_{\pm k} = a_{\pm k}^\dagger a_{\pm k}, \quad P_k = a_k^\dagger a_{-k}^\dagger ,
\]

where \( a_{\pm k}^\dagger (a_{\pm k}) \) is the operator that creates (annihilates) a particle with angular momentum \( k \), spin projection \( m_k \) or \(-m_k \), and energy \( \epsilon_k \). For simplicity, the subscripts \( k \) label the single-particle states \( |k, m_k\rangle \) with \( m_k > 0 \), whereas \(-k \) denote the time-reversal states \( |k, -m_k\rangle \). The particle number operator \( \hat{N} \) is defined as \( \hat{N} = \sum_k (a_{k}^\dagger a_k + a_{-k}^\dagger a_{-k}) \), whereas \( \hat{M} = \sum_k m_k (a_{k}^\dagger a_k - a_{-k}^\dagger a_{-k}) \) is the \( z \)-projection of total angular momentum. The variational procedure is applied to minimize the expectation value of this Hamiltonian in the grand canonical ensemble. The result yields the final equations for the pairing gap, particle number and total angular momentum, which include the effect of QNF in the form

\[
\Delta_k = \Delta + \delta \Delta_k = G \sum_{k, k'} u_{k'} v_k \langle D_{k'} \rangle + G \frac{\delta N^2_k}{\langle D_k \rangle} u_k v_k ,
\]

\[
N = 2 \sum_k \left[ v_k^2 \langle D_k \rangle + \frac{1}{2} (1 - \langle D_k \rangle) \right] , \quad M = \sum_k m_k (n_k^+ - n_k^-) ,
\]

where the quasiparticle energy \( E_k \) and renormalized single-particle energy \( \epsilon'_k \) are given as

\[
E_k = \sqrt{(\epsilon'_k - G v_k^2 - \lambda)^2 + \Delta_k^2}.
\]
\begin{equation}
\epsilon'_k = \epsilon_k + \frac{G}{\langle D_k \rangle} \sum_{k'} (u_{k'}^2 - v_{k'}^2) \left( \langle A_k^\dagger A_{k'} \rangle + \langle A_k^\dagger A_{k'}^\dagger \rangle_{k \neq k'} \right),
\end{equation}

with \( \langle D_k \rangle = 1 - n_k^+ - n_k^- \), and \( A_k^\dagger \equiv \alpha_k^\dagger \alpha_k^\dagger \). The expectation values \( \langle A_k^\dagger A_{k'} \rangle \) and \( \langle A_k^\dagger A_{k'}^\dagger \rangle \) are evaluated by solving a set of coupled equations, which contain the SCQRPA \( X \) and \( Y \) amplitudes. The QNF is given as

\begin{equation}
\delta N_k^2 = n_k^+ (1 - n_k^+) + n_k^- (1 - n_k^-),
\end{equation}

where the quasiparticle occupation numbers \( n_{\pm k} \) are found from the integral equations

\begin{equation}
n_{k}^\pm = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\gamma_{k}^\pm (\omega) (e^{\beta \omega} + 1)^{-1}}{\omega - E_k \pm \gamma m_k - M_k^\pm (\omega)^2 + [\gamma_{k}^\pm (\omega)]^2} d\omega,
\end{equation}

with the mass operators \( M_k^\pm (\omega) \) obtained by solving the set of equations for double-time quasiparticle Green’s functions and those of a quasiparticle coupled with SCQRPA pair vibrations. The quasiparticle dampings are given as \( \gamma_{k}^\pm (\omega) = \Im m [M_k^\pm (\omega \pm i\epsilon)] \).

The proposed approach is called the FTBCS1+SCQRPA theory. Neglecting the coupling to SCQRPA, i.e. the factors \( \langle A_k^\dagger A_{k'} \rangle \) and \( \langle A_k^\dagger A_{k'}^\dagger \rangle \), it becomes the FTBCS1 theory, which is different from the conventional FTBCS theory by the presence of the QNF. The violation of particle number at zero angular momentum is approximately removed by applying the Lipkin-Nogami (LN) method. The corresponding approaches are called the FTLN1+SCQRPA and FTLN1.

3. Results

The numerical calculations are carried out within the \( \Omega \) doubly degenerate equidistant model with the number \( \Omega \) of levels equal to that of particles, \( N \), as well as for \( ^{20}\text{O} \), \( ^{44}\text{Ca} \), \( ^{56}\text{Fe} \), and \( ^{120}\text{Sn} \). The results obtained show that, at zero angular momentum, under the effect of QNF within the FTBCS1 (FTLN1), the sharp SN phase transition predicted by the FTBCS theory is smoothed out. As the result, the pairing gap does not collapse at \( T = T_c \), but has a tail, which extends to high \( T \). The dynamic coupling to the SCQRPA vibrations significantly improves the agreement with the exact results for the total energies and heat capacities obtained for \( N = 10 \) as well as those obtained \( ^{56}\text{Fe} \) within the finite-temperature quantum Monte Carlo method \( \footnote{Figs. \( 1 \) (a) – \( 1 \) (c)} \). However, for heavy nuclei such as \( ^{120}\text{Sn} \), the SCQRPA corrections are found to be negligible in comparison with the FTBCS1 (FTLN1) results.

For \( ^{20}\text{O} \) and \( ^{44}\text{Ca} \), the FTBCS1 pairing gaps, obtained at different \( M \), decreases as \( T \) increases and do not collapses at high \( T \). At \( M \) higher than the critical value \( M_c \), where the FTBCS gap for \( T = 0 \) disappears, there appear thermally assisted pairing correlations, in which the FTBCS1 gap reappears at a given \( T_1 > 0 \), and remains finite at \( T > T_1 \) [Fig. \( 1 \) (d)]. This phenomenon is caused by the QNF within the FTBCS1 theory. At \( T = 0 \), the QNF is zero, so the FTBCS and FTBCS1 gaps are the same as functions of \( M \) (or \( \gamma \)), and both collapse at \( M = M_c \). However,
Fig. 1. Left panels: Pairing gaps (a), total energies (b), and heat capacities (c) obtained within the FTBCS (dotted lines), FTBCS1 (thin solid lines), FTLN1 (thin dashed lines), FTBCS1+SCQRPA (thick solid lines) and FTLN1+SCQRPA (thick dashed lines) for neutrons in $^{56}$Fe. Boxes and crosses with error bars connected by dash-dotted lines are results of Ref. 8. Right panels: pairing gaps as functions of $T$ at different $M$ (d), and as functions of $M$ (e) and $\gamma$ (f) at various $T$ obtained within the FTBCS1 theory for neutrons in $^{20}$O.

with increasing $T$, the FTBCS1 gaps, which are obtained at different $T$, collapse at $M > M_c$, and remain finite even at very high $T$, whereas those given by the conventional FTBCS theory vanish at $M \geq M_c$ and $T \geq T_c$ [Figs. 1(e) and 1(f)].

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