Numerical Analysis of a Lasing Output for the Quasi _Three _ Level Thin Disk Lasers

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Abstract. This paper reports theoretical treatment to drive the general equation for laser threshold power \( P_{th}^{pump} \), the efficiency \( \eta \) and the laser out power \( P_{out} \) in thin disk laser with a quasi-three–level pump plan by using the analytical solution of the rate equation for this laser system. In order to find a numerical solution, MATLAB program has been used. When a diode pumped high power, continuous wave to the Nd³⁺: YAG thin disc laser operated in a quasi-three-level. The pumping wavelength was (808 nm) and the laser wavelength was (946 nm) at Nd³⁺ doping concentration of 0.6 at %. While at (\( M^2 = 1.005 \)), this laser is operated in a single mode, all the laser output power and efficiency have increased with increasing the pumping power.

Keywords: laser, thin disc laser, diode pumped Neodymium, quasi three level lasers.

1. Introduction

The thickness of the crystal is small compared with the diameter of a thin disk and its coated with high reflectivity on one side of its faces for both lasing and pumping wavelength, fixing on a heat sink to remove the excess heat that appear on the disk [1]. If we assume that there is a large heat transfer coefficient over the whole region, then a temperature field inside the crystal is naturally equal to the temperature with the axis [1,2]. The disc can also be pumped by laser valves either longitudinally or radially. By using multiple passages for the pump radiation through the disk to increase the absorption of the pump's certification of a specific crystal thickness [2]. This disc can fit the resonator as false or like a folding mirror. This design can convert lasers in the range of several watts to kilowatts, as shown in Figure 1. The power volume can be increased by increasing the diameter of the pump to the disk at a constant pumping power density [2,3].
2. Analytical model

A quasi-3-level active medium, which is also termed a quasi-4-level active medium, it is characterized by being a large thermal count, low laser level. This means that when the temperature of the system with the four levels changes, it changes the properties of the laser, since at thermal equilibrium the laser does not contain a community, and thus the lower laser is filled in its entirety at thermal equilibrium [2]. The characteristics of the laser between the real 4-level system and the quasi-three-level system produce a major difference through re-absorption losses at the wavelength of the laser, and this leads to an increase in the three-dimensional losses in the resonator thus increasing the laser threshold [1,2]. The energy level system of Nd: YAG consists of stark split $I_{9/2}$ ground state and $F_{3/2}$ excited state. Because there is a small division of energy for each manifold, it is assumed that the relaxation times for the energy level within the manifold are also very small as shown in Figure 2. [2,5].

Figure 1. Multi-pass pumping in a Yb$^{3+}$: YAG thick-disk laser [14].

Figure 2. Energy level scheme of quasi three level Nd$^{3+}$: YAG [5].
Figure 2 illustrates the energy scheme for a Nd$^{3+}$: YAG crystal, where pumping wavelength ($\lambda_p=808$nm) and lasing wavelength ($\lambda_l=946$nm). $F_i (i=1,2)$ and $f_0 (j=1,2,3,4,5)$ act as a Boltzmann factors which refers to the occupation for the lower and upper manifolds, respectively.
\[ f_{1i} = \frac{\exp \left( -\frac{E_{1i}}{kT} \right)}{\sum_{p=1}^{n} \exp \left( -\frac{E_{1p}}{kT} \right)} \]  \hspace{1cm} \text{(1)}

\[ F_{0j} = \frac{\left( \exp \left( -\frac{E_{0j}}{kT} \right) \right)}{\sum_{q=1}^{n} \exp \left( -\frac{E_{0q}}{kT} \right)} \]  \hspace{1cm} \text{(2)}

where \((KB)\) Boltzmann occupation factor \((T)\) is the absolute temperature. \((E_{1i})\) \((i=1, 2)\) describe an energy to each stark level of excited divergent and \((E_{0j})\) \((j=1, 2, 3, 4, 5)\) describe an energy to each stark level of the ground divergent.

3. Steady state Rate Equation.

Average equation for two divergent system is given by [2,4]:

\[ \frac{dN_{up}}{dt} = \frac{I_p}{\hbar\nu_p} \alpha_p - \frac{I_L}{\hbar\nu_L} g_L M_L \frac{N_{up}}{\tau} \]  \hspace{1cm} \text{(3)}

\[ \frac{dI_L}{dt} = I_L \left( g_L M_L d - \delta \right) \frac{c}{2L_{opt}} \]  \hspace{1cm} \text{(4)}

\[ g_L = \frac{(N_{up} - N) \sigma_L}{x_L} \]  \hspace{1cm} \text{(5)}

\[ X_L = \frac{f_{low}^{L}}{(f_{up}^{L} + f_{low}^{L})} \]  \hspace{1cm} \text{(6)}

\((N_{up})\) represent the ion density in upper divergent , \((I_p)\) is a medium pumping intensity through the crystal , \((I_L)\) is the energy intensity of the laser radiation scattered inside the cavity. \((M_L)\) is the number of laser passing in the crystal per one resonator round trip, \((\tau)\) is The duration of the excitation state. \((C)\) is the speed of light in a vacuum, \((L_{opt})\) Optical length on the lumen. \((d)\) Is the thickness of the crystal, \((g_L)\) is a laser gain coefficient. \((\sigma_p)\) and \((\sigma_L)\) Cross sections represent effective absorption at \((808\text{ nm})\) and \((946\text{ nm})\), respectively \((N)\) is the total ionic density , \((f_{low}^{L})\) and \((f_{up}^{L})\) represent the Boltzmann occupation factor in the lower and upper divergent for the laser level , respectively[2,4].

\[ \delta = -\ln \left( 1 - Tr \right) - \ln \left( 1 - \gamma \right) \]  \hspace{1cm} \text{(7)}

where \((Tr)\) the coupler is moved the output \((Tr = 1 - R^2)\) ....... (8)

\(\gamma)\) Calculate residual losses, such as those due to dispersion during one round-trip trip in the bore.

\(\delta\) It is the loss of the round trip and can be described by [4]; [4]:

\[ g_L = \frac{\delta}{M_L d} \]  \hspace{1cm} \text{(9)}

\[ N_{up} = X_L \left( N + \frac{g_L}{\alpha_L} \right) \]  \hspace{1cm} \text{(10)}

Furthermore, on the basis of a multi-pass pumping system, the absorbed pump intensity \((I_{abs})\) and the absorption efficiency \((\eta_{abs})\) are given by[2,4]:

\[ I_{abs} = I_p \alpha_p d = I_{pump} \eta_{abs} \]  \hspace{1cm} \text{(11)}

\[ \eta_{abs} = R_p (1 - \exp(-M_p \alpha d)) \]  \hspace{1cm} \text{(12)}

where \(I_{pump}\) is the input intensity \((R_p)\) is The total reflection of the multi-pass pumping system, \((M_p)\) the number of passes of the pump, and \((\alpha p)\) is the absorption coefficient. , and therefore, the laser intensity in the cavity \((I_L)\) , the pumping threshold intensity \((I_{pump}^{th})\)

output laser intensity \(I_{out}\) can be obtained by \(\frac{dN_{up}}{dt}\) The output

\[ I_L = \frac{v_L \eta_{abs} d_{pump}}{\delta} \left( I_{pump}^{th} - I_{pump}^{th} \right) \]  \hspace{1cm} \text{(13)}

\[ I_{pump}^{th} = \frac{h \nu_p N_{up} d}{\tau \eta_{abs}} \]  \hspace{1cm} \text{(14)}

And , the pump power , pump threshold power , and the laser power can be obtained by[2]:
\[ \text{PL} = \text{Ap} \cdot \text{IL} \quad \ldots (15) \]
\[ \text{Ppump} = \text{Ap} \cdot \text{Ipump} \quad \ldots (16) \]
\[ \text{P}^\text{L} = \text{Ap} \cdot \text{Ipump} \quad \ldots (17) \]

Where \((\text{Ap})\) is the given pumping area
\[ \text{Ap} = \pi \left( \text{rp} \right)^2 \quad \ldots (18) \]

\((\text{rp})\) is pump radius, the beam quality factor is obtained
\[ M^2 = \left( \frac{\alpha \text{rp}}{\sigma f} \right) \quad \ldots (19) \quad [6] \]

\((\text{rf})\) is the radius of a fundamental mode (TEM00), the output laser power
\[ \text{Pout} = \frac{V_L}{\lambda} \cdot \text{Tr} \cdot \frac{\delta \eta \text{abs}}{\lambda} (P_{\text{pump}} - P^\text{L}) \quad \ldots (20) \]

Or
\[ \text{Pout} = \frac{\lambda_p}{\lambda} \cdot \frac{V_L}{\lambda} \cdot \text{Tr} \cdot \frac{\delta \eta \text{abs}}{\lambda} (P_{\text{pump}} - P^\text{L}) \quad \ldots (21) \]

Where the laser extraction efficiency [4]
\[ \eta_L = \frac{1}{P^\text{L}_\text{pump}} \quad \ldots (22) \]

And [2]
\[ \eta = \frac{\lambda_p}{\lambda_L} \quad \ldots (23) \]

the output laser power can be
\[ \text{Pout} = \frac{\text{Tr}}{\delta} \eta \text{abs} \eta_L P_{\text{pump}} \quad \ldots (24) \]

4. Results and Discussion.
In this paper, the results of the numerical solution by using Mat lab for the output laser power and efficiency for a quasi-three level of Nd\(^{3+}\): YAG thin disk laser. The values of a coefficients for thin laser used in a Numerical solution illustrate in the Table 1.

| parameter | value       | unit     | Ref. |
|-----------|-------------|----------|------|
| \lambda_p | 808x10\(^{-9}\) | m        | [5, 7] |
| \sigma_p | 6.7 x 10\(^{-24}\) | m\(^2\) | [5, 7] |
| \sigma_p | 1.83 x 10\(^{2}\) | m\(^{-1}\) | [7] |
| \lambda_L | 946 x 10\(^{-9}\) | m | [5, 7] |
| \sigma_L | 3.7 x 10\(^{-4}\) | m\(^2\) | [5, 7] |
| \gamma   | 0.02 | --- | [7] |
| \alpha   | 0.85 | --- | [6] |
| Wood     | 195 x 10\(^{6}\) | m | [8] |
| L        | 5 x 10\(^{-3}\) | m | [5] |
| f\(_{\text{lower}}\) | 0.04 | --- | [7, 9] |
| f\(_{\text{upper}}\) | 0.51 | --- | [7, 9] |
| R\(_F\)  | 0.77 | --- | [4] |
| R\(_1\)  | 0.99 | --- | [11, 12] |
| R\(_2\)  | 0.85 | --- | [11, 12] |
| \tau     | 0.26 x 10\(^{-3}\) | sec | [5] |
| N        | 8.28 x 10\(^{25}\) | ion/m\(^3\) | [5] |
| M\(_L\)  | 4 | --- | [4] |
| M\(_P\)  | 54 | --- | [4] |
| \(d\)    | 2 x 10\(^{-3}\) | m | [4, 11, 12] |
| \(p\)    | 0.120 | w | [10, 12, 13] |
The first step in the numerical solution was determine the operation Nd³⁺: YAG thin disk laser of the type of the single-mode transverse or multi-mode transverse calculated (M⁰) by equation (20) and (P_pump typ) by equation (18) for two values of radius of the pumping beam falling on the thin disk (r_p) as shown in Table 2.

**Table 2.** The values of (M²) and (P_pump typ) to the of the radius of the pumping falling on the thin disk (r_p).

| Parameter       | Single-mode transverse | Multi-mode transverse | Unit |
|-----------------|------------------------|-----------------------|------|
| r_p             | 230*10⁶                | 727*10⁶               | M    |
| M²              | 1.005                  | 10.070                | -    |
| P_pump typ      | 2.549                  | 25.539                | W    |

Figure 3 illustrates the shape of the relationship between (p_out) and (p_pump), (p_out) is calculated by equation (26). In both modes, the laser output power increased with increasing of the pumping power, and the (p_out) is the highest values in a single mode operation.

Finally, the relation between the efficiency (η) and (pump) in both modes is shown in Figure 4. It is found that the efficiency of this laser increased with the increase of pumping power, and the values of (η) in single mode is highest.
6. Conclusions

In this paper, Nd\textsuperscript{3+}: YAG thin disk laser, were operated in a quasi-three-level system, at wavelength pump (808\texttimes10\textsuperscript{9} m) and wavelength laser (946\texttimes10\textsuperscript{9} m). The laser output power and efficiency are increased with pumping power, and the values of all (p\text{out}) and (\eta) in a single mode operation are highest than the value in a multi-mode operation for this design of laser.

6. References

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