The Effect of 4D Effective Cosmological Constant On The Stability of Randall-Sundrum Scenario

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Abstract

We study the Rundall-Sundrum model with a small 4D effective cosmological constant on the brane, and drive a corrected radion potential following the Goldberger-Wise mechanism. We then discuss the effect of the 4D effective cosmological constant on the stability of the brane-system, and find that to quintessence determined by updated observation, the proper distance between the two branes required to solve the hierarchy problem can exist. However, during inflation, whether we can get an reasonable hierarchy scale is still uncertain.

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As motivated by the string theory [1] [2], there has been a lot of studies in the recent years on brane models. These models could provide a solution to problem of the hierarchy between the Plank scale and the electroweak scale [3] [4]. In the Randall-Sundrum (RS) [4] model, the fifth dimension has orbifold geometry $S_1/Z_2$, and two branes with opposite brane tension are located at the orbifold fixed points in a $AdS$ space with negative bulk cosmological constant. The exponential warp factor in the spacetime metric generates a hierarchy scale on the observational brane, which is determined by the distance between two branes. A mechanism of stabilizing this distance is suggested by Goldberger and Wise (GW) [5] using a bulk scalar field.

The existence of a small positive cosmological constant is strongly indicated by recent observational data [6], which is about $0.7$ of the critical density. This value is 120 orders of magnitude less than that from quantum field theory. Some attempts to get a small cosmological constant in variant of RS scenarios have been made [7]. In the meantime, the recent observations by BooMERANG and MAXIMA [8] on the location of the first peak in the Microwave Background Radiation anisotropy as an strong evidence of the flat universe strongly support the predictions of inflation. The corresponding inflation mechanism in the RS model have been proposed [9]. Therefore, the consideration of de Sitter phase on the brane is very important, which is studied in Ref. [10, 11].

In this letter, we study the effect of a small cosmological constant on the stability of the RS model. We drive the correction of a small four dimensional (4D) effective cosmological constant on the 3-brane to the GW radion potential, and then give some discussions.

We start with a 5D action

$$S_{\text{bulk}} = \int d^4x \sqrt{-g} \left( \frac{1}{2\kappa^2} R - \Lambda \right) - \int d^4x \left( \sqrt{-g_1\Lambda_1} + \sqrt{-g_2\Lambda_2} \right),$$

(1)

where $\kappa$ sets the 5D fundamental scale, $\Lambda$ is the cosmological constant of the bulk, $g_1$ and $g_2$ are the induced metrics on two 3-branes with the corresponding brane tensions are $\Lambda_1$ and $\Lambda_2$, located at $y = 0$ and $y = r$, respectively. A metric ansatz with maximal symmetry given by Ref. [10] is

$$ds^2 = \left( \cosh(ky) - \frac{\kappa\Lambda_1}{6(-\Lambda)} \sinh(ky) \right)^2 (-dt^2 + e^{2Ht}d\mathbf{x}^2) + dy^2,$$

(2)

The corresponding matching condition at $y = r$

$$k \left( 1 - \frac{\kappa^2\Lambda_1\Lambda_2}{6(-\Lambda)} \right) \sinh (kr) = \frac{\kappa^2}{6} (\Lambda_1 + \Lambda_2) \cosh (kr)$$

(3)

must be satisfied, where $k = \sqrt{-\frac{\kappa^2\Lambda}{6}}$ and

$$H^2 = \frac{\kappa^4}{36} \Lambda_1 - \frac{\kappa^2}{6} \Lambda.$$

(4)
is the Hubble parameter on two 3-branes. Assuming that the energy densities of two 3-
branes are expressed as $\Lambda_1 \rightarrow \sigma + \Lambda_{1\text{eff}}$ and $\Lambda_2 \rightarrow \sigma + \Lambda_{2\text{eff}}$, then fine-tuning $\Lambda = \frac{\kappa^2 \sigma^2}{6}$, when $\Lambda_{1\text{eff}}, \Lambda_{2\text{eff}} \ll \sigma$, which means a very small net cosmological constant on the 3-brane, we can rewrite Eqs. (3) and (4) as

$$\Lambda_{1\text{eff}} = -\Lambda_{2\text{eff}} \exp(-2kr)^2, \quad (5)$$

$$H^2 \approx \frac{\kappa^4}{18} \sigma \Lambda_{1\text{eff}} \equiv \frac{\kappa^4 \Lambda_{\text{eff}}}{3}, \quad (6)$$

where $\Lambda_{\text{eff}} \simeq \frac{\kappa^4}{6\kappa^4} \sigma \Lambda_{1\text{eff}}$ is the 4D effective cosmological constant on the 3-brane. For convenience, we define $\eta \equiv \sqrt{\frac{\kappa^4 \sigma^4}{6\Lambda}} - 1 \ll 1$, then see that $\eta \simeq \frac{\kappa^2}{\kappa^2 \Lambda_{\text{eff}}}$. When $\eta \rightarrow 0$, the 4D effective cosmological constant tend to 0. Therefore, we can regard $\eta$ as a measurement of 4D effective cosmological constant.

We can see from the metric (2) that the distance between two branes required to solve the hierarchy problem is less than that in the absence of cosmological constant, but when $\eta \exp(2ky) \simeq 2$, for $ky \gg 1$, the warp factor have a singularity in finite distance, thus the other brane ought to be placed in $\eta \exp(2ky) < 2$. Fig. 1 reflects the relation between the location of the other brane, which is required to generate the hierarchy between the Planck scale and the electroweak scale, and the 4D effective cosmological constant on the 3-brane. We see that with the increasing of the cosmological constant, the location of the other brane is gradually close to the singularity, but never arrive at it. Therefore, a model of RS type with the 4D effective cosmological and the proper distance without the singularity, which is required to generate the hierarchy, is reasonable.

To study the effect of a small effective cosmological constant to the stability of this model, following the GW mechanism [3], we introduce a bulk scalar field into the model,

$$S_{\text{bulk scalar}} = \frac{1}{2} \int d^4x \int dy \sqrt{-g} \left( g_{MN} \partial^M \phi \partial^N \phi - m^2 \phi^2 \right), \quad (7)$$

where $g_{MN}$ is the 5D metric given in (2), and the boundary potentials of the scalar field are

$$S_1 = -\lambda_1 \int d^4x \sqrt{-g_1} \left( \phi^2 - v_1^2 \right)^2 \quad \text{at} \quad y = 0 \quad (8)$$

$$S_2 = -\lambda_2 \int d^4x \sqrt{-g_2} \left( \phi^2 - v_2^2 \right)^2 \quad \text{at} \quad y = r \quad (9)$$

where $v_1$ and $v_2$ are the vacuum expectation values of bulk scalar field on two 3-branes, $\lambda_1$ and $\lambda_2$ are corresponding coupling constants.

\footnote{The appearance of this limitation is because we assume that the back-reaction of the bulk scalar field is negligible.}
We are interested in those configurations of the bulk scalar field where the boundary potentials are minimized. This essentially amounts to negligible dynamics of $\phi$ along the direction tangential to any of two 3-branes. This assumption is reasonable because we focus on the stability of two 3-branes system at the moment and do not study phenomenological consequence of possible coupling of the bulk scalar field $\phi$ to matter fields living on two 3-branes. Therefore, it suffices to concentrate on the equation of motion of $\phi$ only in $y$ direction, which is

$$\partial_y^2 \phi - 4f(y) \partial_y \phi - m^2 \phi = 0, \quad (10)$$

where

$$f(y) = \frac{k(e^{-ky} + \eta \cosh (ky))}{e^{-ky} - \eta \sinh (ky)}. \quad (11)$$

For $\eta \exp (2ky) \ll 1$, i.e. a very small cosmological constant, this equation is reduced to

$$\partial_y^2 \phi - 4k(1 + \eta e^{2ky}) \partial_y \phi - m^2 \phi = 0, \quad (12)$$

Limiting ourselves to the regime where $\epsilon$ is small, $\epsilon \equiv \nu - 2 \approx \frac{m^2}{4k^2} \ll 1$, where $\nu \equiv \sqrt{2 + \frac{m^2}{k^2}}$, and only focusing on the effect of the 4D cosmological constant, we ignore the term of $\epsilon$ (for the effect of $\epsilon$, see Ref. [13]), and in the meantime reserve the first order term of $\eta \exp (2ky)$. Thus the solution of Eq. (12) is

$$\phi \simeq ae^{(2-\nu)ky} + be^{(2+\nu)ky} + \frac{4}{3}b(\eta e^{2ky})e^{(2+\nu)ky} + O((\eta e^{2ky})^2, \epsilon), \quad (13)$$

where $a$ and $b$ are two constant determined by appropriate boundary conditions on two 3-branes. Minimizing the boundary potential at $y = 0$ and $y = r$, we drive

$$a \simeq \frac{v_1 - v_2(1 + \frac{4}{3}\eta)e^{-(2+\nu)kr}}{1 - (1 + \frac{4}{3}\eta)e^{-2\nu kr}}, \quad (14)$$
 $$b \simeq \frac{-v_1 e^{-2\nu kr} + v_2 e^{-(2+\nu)kr}}{1 - (1 + \frac{4}{3}\eta)e^{-2\nu kr}}. \quad (15)$$

Substituting Eqs. (13), (14) and (15) into the action (7) and making integration of $y$, we get an effective potential for the radion from the bulk scalar field part

$$V_{\text{scalar}}(v) \simeq 4kv^4(v_2 - v_1v^\epsilon)^2 + \frac{8}{3}\eta kv^2(v_2 - v_1v^\epsilon)^2, \quad (16)$$

where $v = \exp (-kr)$ denotes the variability of the distance between two 3-branes. Compared with the potential in absence of the effective 4D cosmological constant [3], this has an additional correction from a 4D effective cosmological constant, but we see that this term do not change the minimum of the effective potential. However, there is still a contribution from the curvature of the universe in de Sitter brane [12],

$$V_{\text{curvature}} = \frac{1}{2k\kappa^2}v^2R_4, \quad (17)$$

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where $R_4 = 12H^2$ is the 4D scalar curvature. Consequently, combining Eq. (16) and (17), the effective potential is reduced to

$$V_{\text{eff}}(v) = 4kv^4 (v_2 - v_1 v^\epsilon)^2 + \frac{8}{3} \eta kv^2 (v_2 - v_1 v^\epsilon)^2 + \frac{12}{\kappa^2}k \eta v^2. \quad (18)$$

For $\eta \exp (2kr) \ll 1$, we plot Fig. 2, for $k \sim \frac{1}{\kappa} \sim m_p$ and $\exp (ky) \simeq 10^{16}$, which is required to generate the hierarchy between the Plank scale and electroweak scale. We see that a small cosmological constant can raise the value of the effective potential in the minimum and make this minimum unstable. When the cosmological constant increases to certain value, $\eta \sim 10^{-36}$, this minimum disappears, and the correspondent value of the cosmological constant is $\Lambda_{\text{eff}} \sim 10^{-36} m_p^4$ for $\Lambda \sim \frac{1}{\kappa^2}$.

In summary, we have studied the effect of a very small cosmological constant on the stability of the RS model and show that if there is a small cosmological constant on the observational 3-brane, the distance between two branes required to solve the hierarchy problem is less than that in the absence of cosmological constant and without singularity. We give an effective potential about this distance in some detail, and find that when the cosmological constant is very small, the corresponding minimum remain surviving, but is unstable. As the cosmological constant increases, this minimum disappear, and in this case we hardly determined the distance between two branes by this mechanism. For $k \sim \frac{1}{\kappa} \sim m_p$, $\Lambda \sim m_p^5$ and $\exp (ky) \sim 10^{16}$, while the minimum appears, the cosmological constant value is $\Lambda_{\text{eff}} \sim 10^{-36} m_p^4$. We see that to quintessence determined by updated observation, there exists a minimum, and we can get the proper distance between two 3-branes required to solve the hierarchy problem by the GW mechanism. However, to inflation in early universe, there requires a more high energy scale, in this period whether we can get an reasonable hierarchy scale is still uncertain.

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Figure 1: The $y$-axis is $\frac{\eta}{e^{2kr}}$ and the $x$-axis is $\log_{10}(\frac{\eta}{10^{-32}})$. The solid line and the dashing line denote the location of the other brane and the singularity, respectively.

Figure 2: The $y$-axis is $V(\phi)$ and the $x$-axis is $\phi$. The dashing line of the left figure is that without the 4D cosmological constant and The right figure reflects the variation of $V(\phi)$ with the increasing of the 4D cosmological constant.