HARD EXCLUSIVE QCD PROCESSES AT THE LINEAR COLLIDER

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The next generation of $e^+e^-$-colliders will offer a possibility of clean testing of QCD dynamics. Recent progress in the theoretical description of exclusive processes permits for many of them a consistent use of the perturbative QCD methods. We find that already on the basis of Born approximation, the exclusive diffractive production of two $\rho$ mesons from virtual photons at very high energies should be measurable at the linear collider (LC).

1 Introduction

The high energy limit of strong interaction has a very long story, which started much before the development of QCD\textsuperscript{1}. The Regge limit corresponds to the kinematical regime of large scattering energy square $s$ and small momentum transfer square $t$, $s \gg -t$. The Regge model states that the amplitude $A_{el}(s, t)$ of elastic hadron-hadron collision can be expressed as a sum over amplitudes corresponding to the exchange of Regge trajectories in the $t$ channel. After partial wave expansion, they can be understood as states having continuous angular momentum $\alpha_i(t)$ ($i$ labels the trajectory). Such an hypothesis was supported experimentally by the famous Chew-Frautschi diagram, where one could see, when plotting spin as a function of mass square of known resonances, an impressive alignment of linear trajectories. It is a challenge for QCD to explain this experimental fact.

Through the optical theorem, it is possible to relate the total hadron-hadron cross-section with the imaginary part of the forward elastic amplitude. Thus, $\sigma_{tot} \simeq s^{\alpha_p(0)-1}$, where $\alpha_p(t)$ is the Pomeron trajectory named after Pomeranchuk, which is defined as the one carrying vacuum quantum numbers. $\alpha_p(0)$ is called the intercept of the Pomeron trajectory.

Using unitarity and analyticity of $S$ matrix, Froissart could prove the bound $\sigma_{tot} \leq const \ln^2 s$. 

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Since old studies of Donnachie and Lanshoff it is known that a satisfactory description of the elastic and inelastic hadronic data requires the soft Pomeron trajectory $\alpha_P(t) = 1 + 0.08 + 0.25t$, which explicitly violates Froissart bound. Thus, a need for unitarization was already present in the context of Regge models.

Soon after QCD was proposed as a theory for strong interactions, its Regge limit was studied by Balitsky, Fadin, Kuraev and Lipatov\(^2\). The evaluation of the elastic scattering amplitude of two infrared safe objects was performed, as an infinite series in $\alpha_s \ln s$. This so-called Leading Log Approximation (LLA), where small values of perturbative $\alpha_s$ are compensated by large values of $\ln s$, is expressed as an effective ladder with two reggeized gluons in $t$-channel (gluons dressed by interaction, resulting in appearence of Regge trajectories) interacting with $s$-channel gluonic rungs, through the effective Lipatov vertex which generalizes the usual triple Yang-Mills vertex. The net result for this hard Pomeron intercept is $\alpha_P(0) = 1 + c \alpha_s$, where $c$ is a strictly positive constant, which thus leads to a violation of the Froissart bound at perturbative level. Such an approach has intrinsic limitations. $\alpha_s$ is fixed in the LLA approximation. Along the effective ladder, typical transverse momenta of reggeons diffuse into the IR domain, with a typical gaussian shape, the so-called Bartels cigar, which broadness increases with $s$. Higher order correction in the N(ext)LLA approximation have been computed. They are large and highly dependent on the choice of scale of the running coupling constant. Thus, various resummation schemes have been proposed, in order to compute the effective Pomeron intercept from QCD. Because of the explicit violation of the Froissart bound by the BFKL Pomeron, an intense activity is now devoted to the problem of unitarization of QCD, and to the related problem of saturation, which avoids the unlimited growth of gluon density with increase of $s$.

2 Phenomenology of QCD Pomeron

In order to test the hard Pomeron, it is not enough to study large $s$ experiments. It is also compulsory to select processes where a hard scale enables one to use of perturbative QCD. Such an applicability is more intricate than for conventional QCD evolution\(^3\). Indeed, the usual Operator Product Expansion is not any more valid in the Regge limit of QCD, and one needs to use some generalized version of QCD factorization. In order to emphazise the effects of infrared singularities of QCD (responsible for soft Bremsstrahlung effects) with respect to collinear singularities (which are the source of the conventional DGLAP evolution), processes with comparable characteristic scales at both end of the effective Pomeron ladder have been objects of special interests. In hadron-
hadron colliders (Tevatron, LHC), processes with inclusive production of two high \( p_T \) jets with large relative rapidity \( Y \) (related to \( s \) by \( Y = \ln(s/s_0) \)), known as Mueller-Navelet jets, give access to the hard Pomeron at \( t = 0 \). Diffractive high energy jet production, with a large gap in rapidity between the two jets (no activity in the detector between them), at large \( t \) (which provides the hard scale), reveals the Pomeron structure at large \( t \).

In DIS, the virtuality of the photon naturally provides a hard scale. At the level of both total and diffractive cross-sections, it was possible to describe HERA data using models based on BFKL type of evolution, although the distinction with standard DGLAP evolution is not conclusive. Exclusive vector meson production was also proposed in order to see BFKL effects, selecting events with a large gap in rapidity between the vector meson and the proton remnants. These approaches needed however some ansatz for the coupling of the proton-Pomeron coupling.

3 \( \gamma^*\gamma^* \) processes: the gold plated experiment

Each of the phenomenological tests described above have various limitations, mainly related to the fact that non-perturbative inputs are always needed.

From the theoretical point of view, the best way for studying typical Regge behaviour in perturbative QCD is provided by the scattering of small transverse size objects. Such a reaction is naturally provided by a photons of high virtuality as produced in \( e^+e^- \) tagged collisions. This was investigated at the level of total \( \gamma^*\gamma^* \) cross section by various groups. Typical Pomeron enhancement can hardly be seen at LEP, but should be definitely measurable at LC. One of the key point in order to reveal this effect is that the detectors should be able to tag the outgoing particle with minimal tagging angle down to 20 mrad.

Another possibility is to select specific heavy bounds states (\( J/\Psi, \Upsilon, ... \)) in the final state. This has been studied in the case of double diffractive photo production of \( J/\Psi \). Several tens of thousand events are expected at LC, with an enhancement factor of the order of 50 with respect to the Born estimate.

We study the process of exclusive electroproduction of two \( \rho \)-mesons in the \( \gamma^*\gamma^* \) collisions. The virtualities \( Q_1^2 \) and \( Q_2^2 \) of the scattered photons play the role of the hard scales. This allows one to scan \( Q_1^2, Q_2^2 \), as well as \( t \) to test the structure of the hard Pomeron. It is also possible to study various polarizations of both photons and mesons. As a first step in this direction we shall consider this process with longitudinally polarized photons and \( \rho \)-mesons,

\[
\gamma_L^*(q_1) \gamma_L^*(q_2) \to \rho_L(k_1) \rho_L(k_2).
\]
The choice of longitudinal polarizations of both the scattered photons and produced vector mesons is dictated by the fact that this configuration of the lowest twist-2 gives the dominant contribution in the powers of the hard scales $Q^2_1, Q^2_2$. As a guiding line, one should remember the HERA data where the cross-section of diffractive photoproduction of $J/\Psi$ is comparable to the cross-section of diffractive electroproduction of $\rho$ when the virtuality of the photon is of the order of the mass squared of the $J/\Psi$, which can easily be understood heuristically by crossing symmetry arguments. So one may guess that $\gamma^*\gamma^* \rightarrow \rho\rho$ and $\gamma\gamma \rightarrow J/\Psi J/\Psi$ cross-sections to be comparable at $Q^2_1 = Q^2_2 \sim m_{J/\Psi}^2$.

We compute the Born order contribution to the process (1) using the impact representation, as illustrated in Fig.1.

![Figure 1: Amplitude for the process $\gamma^*\gamma^* \rightarrow \rho\rho$ at Born order. The dots denote the effective coupling of t-channel gluons to the impact factors. Virtualities are defined by $Q^2_1(2) = -q^2_{1(2)}$.](image)

The meson vertex is treated in the collinear approximation which neglects in the hard part of the amplitude the relative transverse momentum of the quarks. This results in appearance of the Distribution Amplitude (DA): the meson wave function integrated over the relative momentum of quarks.

The amplitude for the process reads

$$M = -i 4\pi \alpha_{em} s \alpha_s^2 f^2_\rho Q_1 Q_2 \frac{C_F}{N_c} \int_0^1 \int_0^1 dz_1 dz_2 z_1 \bar{z}_1 z_2 \bar{z}_2 \Phi(z_1) \Phi(z_2) M(z_1, z_2)$$

where $\Phi(z) = 6z\bar{z}$ is the asymptotic DA of the $\rho$ meson, $z$ ($\bar{z}$) being the light-cone fraction of the $\rho$ momentum carried by the quark (resp. antiquark). $M(z_1, z_2)$ is the transverse momentum convolution of the impact factors with 2 $t$-channel gluon propagators. It can be expressed through two integrals with respectively 3 propagators (1 massive, 2 massless) and 4 propagators (2 massive with different masses, 2 massless). These two integrals were computed.
exactly using a generalized version of a technique used in coordinate space when evaluating diagrams of massless two dimensional conformal field theories.

The result for $M(z_1, z_2)$ is regular in $z_1$ and $z_2$. After numerical integration over $z_1$ and $z_2$ and squaring, one obtains the differential cross-section $\frac{d\sigma_{\gamma^* \gamma^* \rightarrow \rho \rho}}{dt}$, shown in Fig.2a, for various values of $Q^2 (GeV^2)$. It is rapidly decreasing in $t$, and flat in $s$. Any BFKL type of resummation would give a rising shape in $s$. Integrating over $t$, one gets the $\sigma_{\gamma^* \gamma^* \rightarrow \rho \rho}$ cross-section. As can be seen from Fig.2b, it is a power like decreasing function of $Q$, as $1/Q^{10.5}$. This is due to the impact factors structure.

The expected number of events at LC, for a nominal luminosity of $100 fb^{-1}$, is of the order of 1000 events per year. This is only a lower bound since the contribution of the transverse photon case is to be added. Moreover, we expect a net and visible enhancement of this cross section, because of resummation effects à la BFKL.

4 Conclusions

Double diffractive $\rho$ production in $e^+ e^-$ collisions is a crucial test for QCD in Regge limit. The Born contribution for longitudinally polarized photon and meson gives a measurable cross-section. BFKL enhancement remains to be evaluated.

$e^+ e^-$ collisions would be also a very good place to observe and test the Odderon. Such an object is the partner of the Pomeron, with opposite charge conjugation. We propose to study double diffractive $\pi^0$ production from two highly virtual photons, which should be dominated by the $t-$channel exchange of an Odderon. In QCD, such a state is constructed from at least 3 gluons, and
resummation effects are expected in the Regge limit. To test the existence of Odderon at the amplitude level, one may study interference effects between Odderon and Pomeron exchange in $\gamma^*\gamma^* \rightarrow (\pi^+\pi^-)(\pi^+\pi^-)$ processes, using the fact that the C-parity is not fixed for such final states.

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