Using Windowed Interpolation Method to Suppress Power Frequency Noise of Vibration Signal

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Abstract. Power frequency noise interference is widespread. In order to suppress power frequency noise coupled into vibration signal, windowed interpolating method was studied. Firstly, three typical window functions were compared, including rectangular window, Hanning window and six-term cosine window. Secondly, the numerical simulation method was used to analyze the influence of window function on the interpolation results of power frequency noise, finding that different window function should be chose when the spectrum spacing between power frequency noise and useful components of vibration signal is different. Finally, the windowed interpolation method was applied to actual engineering vibration signal to suppress its power frequency noise, with the result that the interpolation result of vibration signal is the most accurate with Hanning window. Numerical simulation and engineering application show that the windowed interpolation method is effective for power frequency noise suppression.

Keywords: Power frequency noise; Windowed interpolating method; Vibration signal; Window function.

1. Introduction
Tilting pad bearing is widely used in large steam turbines, gas turbines, fans, pumps and other rotating machinery. Using eddy current displacement sensors to acquire the vibration signal of tilting pad bearing, when the power frequency noise is coupled to the vibration signal, the analysis of vibration characteristics will be badly affected, therefore it is necessary to study effective method to suppress power frequency noise.

Power frequency noise suppression is an important research topic. In order to suppress power frequency interference, scholars have proposed many methods, these methods can be mainly divided into six categories according to fundamental principles[1], methods based on filter theory[2], methods based on blind source separation[3], methods based on neural network[4], methods based on parameter fitting[5], methods based on signal decomposition theory[6] and methods based on singular value decomposition[7]. These methods are valid under certain conditions, but they also has some limitations. For example, the method based on filter theory requires constant power frequency 50Hz, obviously which is inconsistent with the facts, this method resulting not only suppressing power frequency noise, but also damaging the useful parts.

The windowed interpolating method[8] is widely used in power harmonic signal analysis domain, but it is seldom been found in power frequency noise suppression domain. Reference[6] uses spectrum interpolation method to eliminate power frequency components. However, the author doubts that it is inaccurate to simplify the power frequency spectrum as only one spectrum line. Besides the
characteristics of the window function spectrum itself, the characteristics of the processed signal itself should also be fully considered[9].

The structure of this paper is as follows. Firstly, analyse the characteristics of three types of window functions, including rectangular window, Hanning window and six-term cosine window. Secondly, compare the power frequency interpolation calculation results of vibration signal with these window functions by simulation. Finally, apply these window functions to actual vibration signal.

2. Window Functions and Interpolation Formulas

Rectangular window, Hanning window and six-term cosine window are cosine windows, but they have different terms and coefficients[10]. For rectangular window, the width of main lobe, peak value of side lobe and attenuation rate of side lobe are $4/N$, -13dB, -6dB/octave. For Hanning window, the main lobe width, side lobe peak value and side lobe attenuation rate are $8/N$, -32dB, -18dB/octave. For six-term cosine window, the main lobe width, side lobe peak value and side lobe attenuation rate are $24/N$, -88dB, -66dB/octave. These window functions have completely different characteristics and thus can reflect window functions comprehensively.

The parameters of periodic signal, including frequency, amplitude and phase, can be obtained by interpolating two adjacent spectral peaks on the main lobe after windowing and FFT. The interpolation formula[11] is used for rectangular window or Hanning window, while the interpolation formula[12] is applied for six-term cosine window. If the periodic signal is represented by $x_n(n = 0, 1, \ldots, N - 1)$, sampling frequency $f_s$, sampling interval $T$, $T = 1/f_s$. Use window function $w(n)$ to truncate $x_n(n)$, obtain discrete sequence $x(n)$:

$$x(n) = x_n(n) \cdot w(n)$$

Transform $x(n)$ with FFT. $h_m$, $f_m$ and phase($h_m$) respectively represent the amplitude, frequency and phase of the left spectrum line on the main lobe, $h_{m+1}$ and $f_{m+1}$ respectively represent the amplitude and frequency of the right spectrum line on the main lobe. The required frequency, amplitude and phase are represented by $f$, $A$ and $\theta$.

For rectangular window, let the interpolation coefficient $\delta$ be:

$$\delta = \frac{h_{m+1}}{h_m + h_{m+1}}$$

The interpolation formula is:

$$f = f_m + \left(\frac{\delta}{N \cdot T}\right), \quad A = \frac{\pi \cdot \delta \cdot h_m}{\sin(\pi \delta)}, \quad \theta = \text{phase}(h_m) - \pi \cdot \delta - 0.5\pi$$

For Hanning window, let the interpolation coefficient $\delta$ be:

$$\delta = \frac{2h_{m+1} - h_m}{h_{m+1} + h_m}$$

The interpolation formula is:

$$f = f_m - \left(\frac{1 - \delta}{N \cdot T}\right), \quad A = \frac{\pi \cdot \delta \cdot h_m \cdot (1 - \delta^2)}{\sin(\pi \delta)}, \quad \theta = \text{phase}(h_m) - \pi \cdot \delta - 0.5\pi$$

For six-term cosine window, let the interpolation coefficient $\delta$ be:

$$\delta = \frac{6h_{m+1} - 5h_m}{h_{m+1} + h_m}$$

The interpolation formula is:
3. Simulation Signal Analysis

Tilting pad bearing vibration experiment is carried out at different speeds. The frequency of the vibration signal collected by vibration sensor is also different, but the frequency of power frequency noise in the vibration signal is almost stable. Use simulation signal to simulate this working condition, find the appropriate window function to make the power frequency parameters calculated through the interpolation formula optimal. At each simulation test, only the frequency parameter of the vibration component varies, other parameters keep unchanged.

The simulation signal is represented by \( x(t) \) as shown in equation (8), \( x_1(t) \) simulating power frequency noise, \( x_2(t) \) simulating vibration, \( \xi(t) \) simulating Gaussian white noise, signal-to-noise ratio 28dB.

\[
x_1(t): \alpha_1=2, f_1=50.5Hz, \phi_1=\pi / 18; \ x_2(t): \alpha_2=1, \phi_2=7\pi / 9, \ f_2 \text{ varies at each test}, \ f_2: 30, 33, 35, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 65, 68, 70, \text{ unit Hz}, \\text{total 28 values}
\]

Using these parameters, total 28 simulation signals can be created. Rectangular window, Hanning window or six-term cosine window are plused to each simulation signal, then the corresponding interpolation formulas (3), (5) or (7) are used to calculate the parameters (amplitude, frequency, phase). The amplitude errors of these signals are plotted in Figure 1(a), the frequency errors are plotted in Figure 1(b), and the phase errors are plotted in Figure 1(c). From Figure 1, at the range of 48Hz~52Hz, the errors values are the smallest for rectangular window, the errors are the largest for six-term cosine window, the errors are between them for Hanning window.

\[
x(t) = x_1(t) + x_2(t) + \xi(t) = \alpha_1 \cos(2\pi f_1 t + \phi_1) + \alpha_2 \cos(2\pi f_2 t + \phi_2) + \xi(t)
\]

Figure 1. Power frequency parameter errors for different window functions.

In Figure 1, when the frequency exceeds the range of 48Hz~52Hz the errors of different window functions are very small, these three errors are overlapped and are difficult to distinguish from each other. In order to distinguish them, enlarge the lowest value part to obtain Figure 2. Compared with Figure 1, the error values of different window functions are easier to distinguish. In Figure 2(a), the amplitude parameter errors fluctuate greatly, without obvious difference between them. In Figure 2(b), the frequency parameter errors of rectangular window are the largest, the errors of Hanning window are the smallest, and the errors of six-term cosine window are between them. In Figure 2(c), the phase parameter errors of rectangular window and six-term cosine window fluctuates greatly, while the phase error of Hanning window varies little. Therefore, in terms of overall performance, the power frequency parameter error is optimal with Hanning window. Obviously, if Hanning window is applied to these signals, the caused damages are least.
Actual Vibration Signal Analysis

Tilting pad bearing has excellent performance and wide application. In order to develop a new type of tilting pad bearing for special application, relevant experimental research should be carried out to make the developed tilting pad bearing have the optimal vibration performance. The tilting pad bearing rotor experiment system was built at early project for this purpose, Kaman eddy current displacement sensors are installed for vibration monitoring. The sensor model is KD2306, accuracy 0.1μm. During experiment the power frequency noise was found coupled into the vibration signal, which influences the vibration signal analysis, thus this paper introduces the windowed interpolation method to suppress it. The frequency of power frequency noise is about 50Hz. The experiment for new tilting pad bearing development requires to be finished under several rotating speeds. According to the conclusion of above simulation signal analysis, the optimal window function should be selected to match the vibration signal characteristics with the windowed interpolation method.

When the rotating speed of tilting pad bearing rotor system is 3900 rpm (65Hz), the system operates at high speed, which situation needs special concern. If the vibration signal collected under this working condition is represented by \( x(t) \), its waveform and frequency spectrum are shown in Figure 3, and the power frequency noise is shown in Figure 3(b). The spectrum spacing is 15Hz between rotor working frequency (65Hz) and the frequency of the power frequency noise (50Hz). According to previous analysis, the Hanning window is more suitable for vibration signal analysis.

Apply rectangular window, Hanning window or six-term cosine window respectively to vibration signal \( x(t) \), then use interpolation formula (3), (5) or (7) to calculate power frequency parameters, fill the calculated results in Table 1. From the values in Table 1, it is obviously impossible to identify which window function is more proper.

![Figure 3. Vibration signal waveform and its spectrum.](image)

Table 1. Power frequency parameters.

| Window function          | Amplitude (mm) | Frequency(Hz) | Phase(°)  |
|--------------------------|----------------|---------------|-----------|
| Rectangular window       | 0.007251       | 49.925807     | 11.663451 |
| Hanning Window           | 0.007702       | 49.883141     | 18.018403 |
| Six-term cosine window   | 0.008950       | 50.896663     | -173.95131|

4. Actual Vibration Signal Analysis

Figure 2. Power frequency parameter errors with different window functions (Enlarged).
If the amplitude, frequency and phase in Table 1 are represented by A, F and P respectively, the power frequency noise can be expressed as formula (9)

\[ x_i(t) = A \cos(2\pi Ft + P) \]  

\( x(t) \) subtracts \( x_i(t) \), obtain the vibration signal \( x_0(t) \) without power frequency noise, then perform FFT on \( x_0(t) \), the spectrum diagram of \( x_0(t) \) is shown in Figure 4. Compare Figure 4(a) or Figure 4(b) with Figure 3(b) respectively, it can be seen the power frequency noise is effectively suppressed. Compare Figure 4(c) with Figure 3(b), it can be seen the power frequency noise is not suppressed at all. It can be concluded that rectangular window and Hanning window are more proper for power frequency noise suppression than six-term cosine window.

Figure 4. Vibration signal with power frequency noise suppressed.

For the power frequency noise is relatively stable, if the vibration signals are collected for several times for experiment, the calculated frequency parameters by the windowed interpolation method should also change little. Based on these facts another seven parts of signals are collected, the results of eight tests are summarized to get Figure 5. In Figure 5(a), the amplitude parameters fluctuate greatly, but their trends are basically same. In Figure 5(b), the frequency values with six-term cosine window fluctuate greatly, while the frequency value with Hanning window and rectangular window are relatively stable, and the frequency value with Hanning window fluctuates less. In Figure 5(c), the phase values for six-term cosine window fluctuate greatly, while the phase values for Hanning window and rectangular window have less fluctuation, but their trends are basically the same. For the signals are collected at different times, phase fluctuation is normal.

Figure 5. Power frequency parameters in vibration signal obtained by windowed interpolation method.

5. Conclusion
In order to apply windowed interpolation method to power frequency noise suppression of vibration signal, this paper selected three typical window functions and compared the power frequency interpolation calculation results for different window functions based on simulation signal and actual
tilting pad bearing vibration signal, finding when Hanning window is used the power frequency noise suppression effect is the best. The results show that the windowed interpolation method is effective for power frequency noise suppression. There are many window functions, how to select a window function more suitable for power frequency noise suppression is the content of follow-up research.

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