Hadronic molecules in chiral dynamics

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Abstract. Hadrons are composed of quarks and gluons through the strong interaction. When the hadrons themselves are strongly correlated by the inter-hadron forces, a composite system of hadrons can be formed. This is called hadronic molecule state. We present recent developments in the study of the structure of such hadrons in chiral dynamics. It is shown that the novel structure of hadronic molecules can be realized by the strong attraction in chiral low energy interaction.

1. Introduction

Strong interaction of quarks and gluons is governed by quantum chromodynamics (QCD). At low energies, due to color confinement, asymptotic degrees of freedom become hadrons (mesons and baryons) which are regarded as elementary excitations of the QCD vacuum. Recently much attention is paid to the discussion of hadronic molecule states, in which two or more hadrons are loosely bound by the inter-hadron interactions, keeping their identities as hadrons.

Well known example of the hadronic molecule is the nucleus which is the composite system of protons and neutrons. For instance, deuteron is considered as the bound state of a proton and a neutron. As far as the quantum number is concerned, however, one could form the deuteron as a compact six-quark state, so that it is a single hadron as a whole. It is clear that the former picture is the correct one, but it should be verified using a theoretical argument. It has been indeed shown in Ref. [1] that the deuteron is not a single hadron but a composite system, by introducing a compositeness condition through the field renormalization constant.

At microscopic level, hadronic molecule structure can be interpreted as the clustering phenomena of quarks, given that the hadrons are composite by themselves. This quark-number picture is however not trivial when the antiquarks are taken into account. By writing the quark (antiquark) number as \( n_q \) (\( n_{\bar{q}} \)), the net quark number \( N = n_q - n_{\bar{q}} \) is conserved in QCD by vector U(1) symmetry. On the other hand, neither \( n_q \) nor \( n_{\bar{q}} \) is separately conserved, since the \( \bar{q}q \) pair creation/annihilation can take place. In other words, the number of valence quarks is not a well-defined classification scheme of the hadron structure.
Instead, the structure of hadrons can be classified by regarding them as the fundamental degrees of freedom. Even though the number of mesons are not conserved, one baryon state can be distinguished from the meson-baryon state. This is because the hadrons are the asymptotic states, and the two-body system forms continuous scattering states which are separated from the discrete levels of the one-body states. In contrast, multi-quark states in general do not form the continuous spectrum because of color confinement.

In this paper, we focus on the low energy chiral interaction as the driving force of hadronic molecules. We introduce a theoretical framework of chiral dynamics which describes the excited baryons as resonances in the meson-baryon scattering amplitude. We then review the recent attempts to pin down the structure of baryon resonances in chiral dynamics.

2. Chiral dynamics

QCD Lagrangian has approximate chiral symmetry for light flavor quarks. Chiral symmetry and its spontaneous breaking in the underlying theory dictate the low energy interaction of hadrons. In addition, the scattering amplitude should satisfy the unitarity condition which ensures the probability conservation. Refs. [2, 3, 4, 5] have established the dynamical framework in which the scattering amplitude is constructed consistently with the coupled-channel unitarity and obeys the chiral theorem at low energy. In the N/D method [4], we write down the general form of the amplitude for an s-wave single-channel meson-baryon scattering at total energy $\sqrt{s}$ as

$$ T(\sqrt{s}) = \frac{1}{V^{-1}(\sqrt{s}) - G(\sqrt{s}; a)}, \quad (1) $$

where $V(\sqrt{s})$ is the kernel interaction specified below. The once-subtracted dispersion integral $G(\sqrt{s}; a)$ can be identified as the meson-baryon loop function with dimensional regularization, and the subtraction constant $a$ plays the role of the cutoff of the loop integral. In the leading order of the low energy expansion, the interaction kernel is given by the Weinberg-Tomozawa term

$$ V(\sqrt{s}) = V_{WT}(\sqrt{s}) = -\frac{C}{2f^2}(\sqrt{s} - M_T), \quad (2) $$

where $C$, $M_T$, and $f$ are the group theoretical factor, the baryon mass, and the meson decay constant, respectively. Since Eq. (1) expresses nonperturbative resummation of the s-channel diagrams, resonances and bound states can be dynamically generated for sufficiently attractive interactions. The coupling strength $C$ is determined by group theoretical argument, and they are strong enough to generate resonances, especially in three flavor sector [6, 7]. Various hadron resonances are well described in this approach, together with the successfully reproduced hadron scattering amplitude, thanks to the universality of the low energy chiral interaction [8, 9].

3. Origin of resonances

In the scattering theory, single particle contribution to the scattering amplitude is called the Castillejo-Dalitz-Dyson (CDD) poles. The N/D method describes the amplitude by including the CDD pole contribution in the interaction kernel $V$. In the previous studies, CDD poles were introduced in $V$ as the explicit resonance propagator of the bare field, or as the contracted resonance contribution in the higher order chiral Lagrangian. In Ref. [10], it has been shown that the loop function $G$ can also contain the CDD pole contribution. This is understood from the viewpoint of the renormalization group; once the amplitude $T$ is determined by experiments, then the change of the interaction $V$ can be absorbed by modifying the renormalization parameter $a$ in the loop function $G$. In this case, the CDD pole contribution in the kernel $V$ may be effectively transferred to the loop function $G$. Thus, in order to study the origin of resonances in this approach, we should make the CDD pole contribution in the model under control.
For this purpose, “natural renormalization scheme” is developed [10]. In this scheme, we constrain the loop function $G$ to exclude the CDD pole contribution, by requiring $G$ to be negative below threshold. In addition, the amplitude $T$ is matched with the interaction kernel $V$ at certain low energy scale, based on the chiral expansion. These requirements uniquely lead to the renormalization condition

$$G(\sqrt{s}; a_{\text{natural}}) = 0 \quad \text{at} \quad \sqrt{s} = M_T,$$

from which we determine the subtraction constant $a_{\text{natural}}$.

Once the natural renormalization scheme is established, the origin of the resonances in the phenomenological approach can be studied. Suppose that the experimental data is fit by the leading order interaction $V_{\text{WT}}$ and the subtraction constant $a_{\text{pheno}}$. The scattering amplitude can also be expressed in the natural renormalization scheme with the subtraction constant $a_{\text{natural}}$ and the effective interaction $V_{\text{natural}}$. To obtain the same amplitude, it follows from Eq. (1) that

$$V_{\text{natural}}^{-1}(\sqrt{s}) - G(\sqrt{s}; a_{\text{natural}}) = V_{\text{WT}}^{-1}(\sqrt{s}) - G(\sqrt{s}; a_{\text{pheno}}).$$

By this equation we obtain $V_{\text{natural}}$ as

$$V_{\text{natural}}(\sqrt{s}) = -\frac{C}{2f^2}(\sqrt{s} - M_T) + \frac{C}{2f^2}(\frac{\sqrt{s} - M_T}{\sqrt{s} - M_{\text{eff}}}),$$

with an effective mass $M_{\text{eff}} \equiv M_T - (16\pi^2 f^2) / (CM_T \Delta a)$ and $\Delta a = a_{\text{pheno}} - a_{\text{natural}}$.

The effective interaction $V_{\text{natural}}$ has a pole term which is interpreted as the CDD pole contribution. The relevance of the pole term depends on the scale of the effective mass $M_{\text{eff}}$; for small $\Delta a$, the effective pole mass $M_{\text{eff}}$ is large and the second term of Eq. (5) can be neglected in the resonance energy region $\sqrt{s} \sim M_T + m \ll M_{\text{eff}}$. If the deviation $\Delta a$ is large, the effective mass $M_{\text{eff}}$ gets closer to the threshold and the pole contribution has substantial effect on the observables. Thus, the effect of the CDD pole contribution can be estimated from the values of the effective mass $M_{\text{eff}}$.

Applying this method to the physical baryon resonances, we find that the $\Lambda(1405)$ resonance is predominantly formed by the meson-baryon molecule component, while the $N(1535)$ resonance requires a substantial CDD pole contribution.

4. Compositeness of bound states

Next we introduce a quantitative measure of “compositeness” through the field renormalization constant [11, 12]. For simplicity, we consider the bound state with mass $M_B$. In the nonrelativistic quantum mechanics, the field renormalization constant $Z$ is given by the overlap of the bare state and the physical state [1]. Since $Z$ expresses the elementarity of the bound state, the quantity $1 - Z$ is interpreted as the compositeness.

In the relativistic field theory, the field renormalization constant is defined as the residue of the bound state propagator. For an $s$-wave bound state, it is given by

$$\Delta(\sqrt{s}) = \frac{Z}{\sqrt{s} - M_B}.$$

Utilizing a field theory with the Yukawa interaction, the constant $Z$ is expressed in terms of the renormalized coupling constant of the bound state to the scattering state $g$ and the derivative of the loop function $G$. Calculating the coupling constant $g$, we derive the compositeness of the bound state in chiral dynamics as [12]

$$1 - Z = \left[1 + \frac{G(M_B; a)}{(M_B - M_T)G'(M_B)}\right]^{-1}.$$
It is shown that the natural renormalization condition (3) gives a bound state which is dominated by the composite structure. This result reinforces the quantitative aspect of the discussion on the origin of resonances with the natural renormalization scheme. For instance, based on the result in Ref. [10], it is shown that more than 80% of \( \Lambda(1405) \) can be interpreted as a meson-baryon composite structure [12].

5. Summary

We have illustrated a description of hadron resonances in a dynamical approach with chiral symmetry, and have shown the recent progress in the study of hadron structure. With the help of the natural renormalization scheme, it is possible to extract the CDD pole contribution hidden in the loop function. This scheme, together with the phenomenological amplitude, enables us to clarify the origin of resonances in chiral dynamics. The compositeness of the bound state is then introduced to elaborate the quantitative aspect. These techniques reveal that the \( \Lambda(1405) \) resonance is a representative of the hadronic molecule state. It is important to clarify the property and the binding mechanism of the hadronic molecules for the understanding of the nonperturbative aspects of the strong interaction.

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