Design of multiple-deferred state sampling plans for exponentiated half logistic distribution

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Abstract: This paper aims to develop a multiple deferred state sampling plan for a time-truncated life test if the lifetime of the item follows exponentiated half logistic distribution. The optimal parameters of the proposed plan, such as the number of successive lots required for making the decision whether to accept or reject the current lot, sample size, the rejection and acceptance numbers are obtained using two points approach. The implementation of the proposed plan is illustrated with examples. Tables are constructed for various combinations of consumer’s and producer’s risks. Comparison is also made with existing sampling plans under exponentiated half logistic distribution.

Subjects: Applied Mathematics; Statistics & Probability; Optimization; SPC/Reliability/Quality Control; Statistical Computing; Statistics & Computing; Statistical Theory & Methods

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PUBLIC INTEREST STATEMENT

The exponentiated half logistic distribution (EHLD) is a commonly used distribution in many fields, including quality control and reliability. Due to its flexibility, the EHLD is widely used to various sampling plans. Motivated by this piece of information, for the present article develops a design of multiple deferred state sampling plans for EHLD. The optimal parameters of the sampling plan are obtained using two point approaches. Further, the results are compared with existing sampling plans and showed that the proposed sampling plan is more efficient than the existing sampling plans. The practitioners need to fit the data for the EHLD first and they can apply the proposed sampling plan for the inspection of the product. The proposed plan can be applied in any manufacturing industry for the testing of the product.
1. Introduction

Quality is a vital factor in any manufacturing process, and to conserve the reputation of the product in the market, each producer should produce a product with a superior quality. Using control charts technique the improvement of product quality can be examined, and these charts cane applied if the process is ongoing. The sampling inspection will be useful to check the quality of the product for finished goods. The consumer conducts the sampling inspection, which is based on random samples, to create a belief on a product, as well as the producer also conducts the sample inspection to expose the quality of the product.

The sampling methodology in which an acceptance or non-acceptance decision on the submitted lot is done based on the random samples drawn from the concerned lot is known as acceptance sampling. The acceptance criteria and the sample size are recommended by the acceptance sampling plan for disposing the submitted lot based on the results of sample items. In guaranteeing the quality of the manufactured products acceptance sampling plays a vital role in statistical quality control. When enforcing the acceptance sampling, the decision on the inclination of the lot is immediately made by the inspection of the sample items, so that the cost is reduced and the time will be saved for the inspection. There will be a chance of accepting a bad lot or rejecting a good lot because the decision is made using the results of random samples drawn from the concerned lot. When the bad lot is accepted, the consumer will be affected and when the good lot is accepted, the producer will be affected. Thus, the probability of rejecting a good lot is known as producer’s risk (a) and the probability of accepting a bad lot is known as consumer’s risk (β). The acceptance quality level (AQL) and the limiting quality level (LQL) are appropriate quality levels corresponding to producer’s and consumer’s risks, respectively.

The sampling plans based on time-truncated life tests have been used to study the product reliability. In acceptance sampling, the concept of time-truncated life test is gaining more popularity. Usually, a life test can be used to furnish lifetime assurance and it will be conducted to examine the lifetime of the product under a predetermined time. These sampling plans were proposed by various authors using life tests under various situations with different distributions. Kantam et al. (2001) developed acceptance sampling based on life tests for log-logistic model. Rao et al. (2016) established new acceptance sampling plans based on percentiles for exponentiated Fréchet distribution, Rao et al. (2019) studied new acceptance sampling plans based on percentiles for type-II generalized log logistic distribution. Al-Omari (2018) developed the transmuted generalized inverse Weibull distribution in acceptance sampling plans based on life tests. Yen et al. (2020) established a rectifying acceptance sampling plan based on the process capability index.

Various authors have developed generalization or exponentiation of the base distribution specifically with application of reliability [e.g., see Gupta et al. (1998), Mudholkar and Srivastava (1993), Mudholkar et al. (1995)]. Here, we stick to the terminology of Mudholkar and Srivastava (1993) as the exponentiated half logistic distribution with base distribution consider as half logistic distribution. Cordeiro et al. (2014) have developed an exponentiated half logistic distribution. More recently, Rao and Ramesh (2013, Rao & Ramesh Naidu, 2014) studied exponentiated half logistic distribution in the area of reliability and quality control for the product lifetime percentiles.

If \( \nu \) is a positive real number, the cumulative distribution function (cdf) of exponentiated half logistic distribution is given by

\[
F(t; \nu, \sigma) = [F(t; \sigma)]^{\nu} = \left( \frac{1 - e^{-t/\sigma}}{1 + e^{-t/\sigma}} \right)^{\nu}; \quad t \geq 0
\]

and the probability density function (pdf) of exponentiated half logistic distribution with \( \nu > 0 \) and \( \sigma > 0 \) is given by
\[ f(t; \nu, \sigma) = \frac{2\nu(1 - e^{-t/\sigma})^{\nu-1}e^{-t/\sigma}}{\sigma(1 - e^{-t/\sigma})^{\nu+1}} : t \geq 0 \]  

(2)

Where, \( \sigma \) is the scale parameter and \( \nu \) is shape parameter.

The 100\( q \)-th percentile of EHLD is given by:

\[ t_q = \sigma \eta_q \] where \[ \eta_q = \left[ \ln \left( \frac{1 + q^{1/\nu}}{1 - q^{1/\nu}} \right) \right] \]

(3)

Therefore, the quantile \( t_q \) specified in Equation (3) for the standard values of \( \nu = \nu_0 \), the function of scale parameter \( \sigma = \sigma_0 \) that is \( t_q \geq t^0_q \iff \sigma \geq \sigma_0 \), where

\[ \sigma_0 = t_q/\eta_q \]

(4)

Therefore, the median of EHLD becomes

\[ t_{0.5} = \sigma \left[ \ln \left( \frac{1 + 2^{1/\nu}}{1 - 2^{1/\nu}} \right) \right] \]

(5)

Let \( p \) is the probability of getting a failure within the life test schedule \( t_0 \). If the product lifetime follows an EHLD, then \( p = f(t_0) \). Usually, it would be appropriate to define the experiment termination time \( t_0 \) as \( t_0 = t^0_q \) for a constant \( \delta^0_q \) and the targeted 100q-th lifetime percentile, \( t^0_q \). Suppose \( t_q \) is the true 100q-th lifetime percentile. Then, \( p \) can be rewritten as:

\[ p = \frac{1 - \exp \left( -\delta^0_q t_q / t^0_q \right) }{1 + \exp \left( -\delta^0_q t_q / t^0_q \right)} , \text{where} \eta_q = \left[ \ln \left( \frac{1 + q^{1/\nu}}{1 - q^{1/\nu}} \right) \right] \]

(6)

It is important to note that the median percentile depends on both the shape and scale parameter but the failure probability \( p \) is independent of scale parameter.

2. MDS sampling plan based on EHLD

The multiple deferred (dependent) state (MDS) sampling plan is known as an attribute inspection procedure where the decision is made for each lot based on one of the three conditions: (1) accept the lot; (2) reject the lot; or (3) conditionally accept or reject the lot based on the disposition of future-related lots. Wortham and Baker (1976) introduced the MDS sampling concept. Since the decision is made using the results of the samples drawn from both the current and successive lots regarding the disposition of the current lot under the MDS plan and it is possible to reduce the sample size under this plan. Also, this plan is appropriate when the production is continuous and the lots are submitted serially for inspection in the order of production. Many authors have established MDS plans in different situations. For example, The MDS sampling plan using Bayesian methodology was studied by Balamurali et al. (2016). The selection of MDS sampling plans for given AQL and LQL was proposed by Govindaraju and Subramani (1993). For more details on MDS sampling plans, one may refer to Soundararajan and Vijayaraghavan (1990), Subramani and Haridoss (2012), and Aslam et al. (2014). The MDS sampling plan based on measurement data was derived by Balamurali and Jun (2007). Recently, the concept of MDS sampling has been used in control chart design. For example, Aslam et al. (2015) proposed an attribute control chart for monitoring the manufacturing process based on an MDS sampling approach. Khan et al. (2019) studied multiple dependent state repetitive sampling plans with or without auxiliary variable and Balamurali and Aslam (2019) developed determination of multiple dependent state repetitive group sampling plan based on the process capability index.
In this paper, when the lifetime of an item follows an EHLD we suggest an MDS sampling plan to assure the median life of the product based on a time-truncated life test. The conversion of EHLD will be given and the functioning process of MDS sampling plan under EHLD is also given. Some important facts of the MDS sampling plan under EHLD are provided in terms tables for analysis purpose. The performance of both existing sampling plans and the proposed sampling plan is given for comparison purpose.

Here, the functional procedure and designing methodology of MDS sampling plan under EHLD is given.

2.1. Operating procedure

Step-1. Draw a random sample of n items from the current lot. Then put them on life test for specified time $t_0$.

Step-2. Observe the number of items that failed before the test time $t_0$ and it will be denoted by d.

Step-3. If $d \leq c_1$, accept the lot otherwise reject the lot then the test will be terminated. If $c_1 < d \leq c_2$, accept the current lot provided that in m successive lots (preceding m lots in case of MDS sampling plan), the number of failures must be less than or equal to $c_1$ before the test time $t_0$.

The proposed MDS sampling plan is totally characterized by four parameters, namely, $n$, $c_1$, $c_2$ and $m$ where $n$ is the sample size, $c_1$ is the maximum number of allowable items that failed for unconditional acceptance $c_1 \geq 0$, $c_2$ is the maximum number of additional items that failed for conditional acceptance $c_2 > c_1$, and $m$ is the number of successive lots (previous) needed to make decision. The attributes MDS sampling plan converges to $m \rightarrow \infty$ and/or $c_2 = c_1 = c$ (say) and it is a general case of single sampling plan. The OC function can disclose the performance of a sampling plan. The OC function of the MDS sampling plan under EHLD for the time-truncated life test is defined using binomial probability law as given below:

$$P_0(p) = \sum_{d=0}^{c_1} \binom{n}{d} p^d (1-p)^{n-d} + \sum_{d=c_1+1}^{c_2} \binom{n}{d} p^d (1-p)^{n-d} \times \left[ \sum_{d=0}^{c_1} \binom{n}{d} p^d (1-p)^{n-d} \right]^m$$

(7)

The probability of lot acceptance at failure probability $p$ under binomial probability distribution can be attained using Equation (7).

2.2. Designing methodology

Usually, the available sampling plans have been designed to minimize the average sample number (ASN). Generally, the major objective of any sampling plan is to minimize the ASN because the analogous inspection time and inspection cost will be reduced. Similarly, we proposed the MDS sampling plan to minimize the ASN for EHLD under-truncated life tests. We use optimization technique to obtain optimal parameters and to minimize the ASN of the proposed MDS sampling plan as given below:

Minimize $ASN(p) = n$

Subject to $P_0(p_1) \geq 1 - \alpha$

$$P_0(p_2) \leq \beta$$

$n \geq 1$, $m \geq 1$, $c_2 > c_1 \geq 0$

(8)

Where, $p_1$ and $p_2$ are the quality levels corresponding to producer’s and consumer’s risks, respectively. These probabilities can be obtained using the following equations:
\[
P_d(p_1) = \sum_{d=0}^{\nu} \left( \frac{n}{d} \right) p_1^d (1 - p_1)^{n-d} + \sum_{d=1}^{\nu} \left( \frac{n}{d} \right) p_1^d (1 - p_1)^{n-d} \times \left[ \sum_{d=0}^{\nu} \left( \frac{n}{d} \right) p_1^d (1 - p_1)^{n-d} \right]^m
\]
and
\[
P_d(p_2) = \sum_{d=0}^{\nu} \left( \frac{n}{d} \right) p_2^d (1 - p_2)^{n-d} + \sum_{d=1}^{\nu} \left( \frac{n}{d} \right) p_2^d (1 - p_2)^{n-d} \times \left[ \sum_{d=0}^{\nu} \left( \frac{n}{d} \right) p_2^d (1 - p_2)^{n-d} \right]^m
\]

We expressed the quality level as the percentile ratio \( t_q/t_q^0 \) of the median lifetime to the specified median lifetime. The percentile ratio \( t_q/t_q^0 \) is considered to be 1 for consumer’s risk and 0.25, 0.10, 0.05, and 0.01 for the producer’s risk. The optimal parameters of the proposed MDS sampling plan for EHLD with known shape parameter \( \nu = 2, 2.5, 3 \) under truncated life tests are given in Tables 1–3 by assuming that the consumer’s risk \( \beta = 0.25, 0.10, 0.05 \), and 0.01, producer’s risk \( \alpha = 0.05 \) and at 50th percentile. Also the optimal parameters of the proposed MDS sampling plan for EHLD with estimated shape parameter \( \nu = 1.728 \) for the real-time data are given in Table 4. The termination ratio is considered as \( \alpha = 0.5, \alpha = 0.7, \) and \( \alpha = 1.0 \). From Tables 1, 3, and 4, we can observe that when the termination ratio increases for specific values of \( \beta, t_q/t_q^0 \), and \( \nu \) the required sample size \( n \) decreases. From Table 2, we can observe that when the termination ratio increases for specific values of \( \beta, t_q/t_q^0 \), and \( \nu \) the required sample size \( n \) decreases, but the required sample size \( n \) also increases.

To illustrate the tables, the optimal parameters are selected from Table 1 are \( n = 32, c_1=2, c_2=4, \) and \( m = 1 \) with shape parameter value of \( \nu=2, \alpha=0.05, \beta=0.10, t_q/t_q^0=2 \) and \( \alpha = 1.0 \). The multiple deferred state sampling plans is established as follows: A sample of 32 observations will be selected at random for the submitted lot and put them on life test for 1000 hours. If the number of failed units before 1000 hours is less than or equal to 2 then the lot will be accepted and the lot will be rejected if the number of failed units is greater than 4. There will be disposition of the present lot is deferred until the next successive lot will be tested in case of the number of failed units are in between 2 and 4.

3. Application of the proposed MDS sampling plan in industry

Here, we describe the implementation of the proposed MDS sampling plan where the lifetime of the product follows EHLD with the shape parameter is unknown \( \nu \). If the shape parameter is unknown then we can estimate it using the past lifetime data. For example, we consider a real data set reported by Cordeiro et al. (2017) to estimate the unknown shape parameter. The data consists of 30 observations of March precipitation (in inches) in Minneapolis/St Paul. The observations are: 0.77, 1.74, 0.81, 1.20, 1.95, 1.20, 0.47, 1.43, 3.37, 2.40, 3.00, 3.09, 1.51, 2.10, 0.52, 1.62, 1.31, 0.32, 0.59, 0.81, 2.81, 1.87, 1.18, 1.35, 4.75, 2.48, 0.96, 1.89, 0.90 and 2.05.

The data set was used to demonstrate the goodness of fit for the given model and by plotting it shows that the exponentiated half logistic distribution is a best fit from the below graph and goodness of fit is underlined with Q-Q plot presented in (Figure 1) and also the multiple deferred state sampling plans for EHLD has not yet implemented. The MLE of the parameter of EHLD for March precipitation (in inches) in Minneapolis/St Paul are \( i=0.7280 \) and from the Kolmogorov–Smirnov test, we found that the maximum distance between the data and the fitted of the EHLD is 0.1192 with p-value is 0.7874.

Suppose that the researcher would like to use the proposed multiple deferred state sampling plan to implement the median life percentile of the product where the product lifetime follows an exponentiated half logistic distribution with the shape parameter \( \nu=1.728 \). The producer suggests that given median life of his product is 800 hours but the researcher want to run the test for 600 hours. The
Table 1. Optimal parameters of the proposed MDS plan for EHLD with $\nu = 2$

| $\theta$ | $\frac{t}{s}$ | $\alpha = 0.5$ | $\alpha = 0.7$ | $\alpha = 1.0$ | $\alpha = 1.0$ |
|-----------|----------------|----------------|----------------|----------------|----------------|
|           | $n$ | $c_1$ | $c_2$ | $m$ | $P_d(p_1)$ | $n$ | $c_1$ | $c_2$ | $m$ | $P_d(p_1)$ | $n$ | $c_1$ | $c_2$ | $m$ | $P_d(p_1)$ |
| 0.25      | 2   | 18   | 3   | 1   | 0.9504   | 13  | 2   | 3   | 1   | 0.9680   | 7   | 2   | 3   | 2   | 0.9642   |
|           | 4   | 8    | 0   | 1   | 0.9805   | 0.9776   | 3   | 0   | 2   | 1   | 0.9631   |
|           | 6   | 0.9858 | 4   | 0   | 3   | 3   | 0.9951   | 3   | 0   | 2   | 1   | 0.9687   |
|           | 8   | 0.9986 | 4   | 0   | 3   | 3   | 0.9984   | 3   | 0   | 2   | 1   | 0.9995   |
|           | 10  | 1.0000 | 4   | 0   | 3   | 3   | 0.9993   | 3   | 0   | 2   | 1   | 0.9999   |
| 0.10      | 2   | 0.9512 | 17  | 2   | 4   | 1   | 0.9523   | 12  | 3   | 5   | 1   | 0.9736   |
|           | 4   | 0.9656 | 7   | 0   | 6   | 2   | 0.9568   | 4   | 0   | 1   | 1   | 0.9590   |
|           | 6   | 0.9924 | 7   | 0   | 6   | 2   | 0.9902   | 4   | 0   | 1   | 1   | 0.9908   |
|           | 8   | 0.9975 | 7   | 0   | 6   | 2   | 0.9967   | 4   | 0   | 1   | 1   | 0.9969   |
|           | 10  | 0.9990 | 7   | 0   | 6   | 2   | 0.9986   | 4   | 0   | 1   | 1   | 0.9985   |
| 0.05      | 2   | 0.9622 | 24  | 3   | 5   | 1   | 0.9596   | 14  | 3   | 7   | 1   | 0.9558   |
|           | 4   | 0.9511 | 10  | 0   | 3   | 1   | 0.9554   | 5   | 0   | 2   | 1   | 0.9534   |
|           | 6   | 0.9841 | 9   | 0   | 1   | 1   | 0.9884   | 5   | 0   | 2   | 1   | 0.9895   |
|           | 8   | 0.9946 | 9   | 0   | 1   | 1   | 0.9961   | 5   | 0   | 2   | 1   | 0.9965   |
|           | 10  | 0.9977 | 9   | 0   | 1   | 1   | 0.9984   | 5   | 0   | 2   | 1   | 0.9985   |
| 0.01      | 2   | 0.9548 | 35  | 4   | 7   | 1   | 0.9516   | 22  | 5   | 8   | 1   | 0.9673   |
|           | 4   | 0.9865 | 20  | 1   | 4   | 1   | 0.9937   | 11  | 1   | 5   | 1   | 0.9916   |
|           | 6   | 0.9771 | 13  | 0   | 1   | 1   | 0.9766   | 7   | 0   | 2   | 1   | 0.9802   |
|           | 8   | 0.9922 | 13  | 0   | 1   | 1   | 0.9920   | 7   | 0   | 2   | 1   | 0.9933   |
|           | 10  | 0.9967 | 13  | 0   | 1   | 1   | 0.9966   | 7   | 0   | 2   | 1   | 0.9972   |
| $\theta$ | $t_q/r_q$ | $\alpha = 0.5$ | $\alpha = 0.7$ | $\alpha = 1.0$ |
|---------|-----------|----------------|----------------|----------------|
|         | $n$       | $c_1$ | $c_2$ | $m$ | $P_a(p_1)$ | $n$ | $c_1$ | $c_2$ | $m$ | $P_a(p_1)$ | $n$ | $c_1$ | $c_2$ | $m$ | $P_a(p_1)$ |
| 0.25    | 2         | 19    | 3    | 2   | 0.9778 | 9   | 1    | 8    | 3   | 0.9675 | 5   | 1    | 2    | 1   | 0.9576 |
|         | 4         | 10    | 0    | 3   | 2    | 0.9947 | 5   | 0    | 4    | 2   | 0.9931 | 3   | 0    | 2    | 1   | 0.9928 |
|         | 6         | 10    | 0    | 3   | 2   | 0.9993 | 5   | 0    | 4    | 2   | 0.9990 | 3   | 0    | 2    | 1   | 0.9990 |
|         | 8         | 10    | 0    | 3   | 2   | 0.9998 | 5   | 0    | 4    | 2   | 0.9998 | 3   | 0    | 2    | 1   | 0.9997 |
|         | 10        | 10    | 0    | 3   | 2   | 0.9999 | 5   | 0    | 4    | 2   | 0.9999 | 3   | 0    | 2    | 1   | 0.9999 |
| 0.10    | 2         | 43    | 2    | 3   | 1   | 0.9522 | 18  | 2    | 3    | 1   | 0.9663 | 9   | 2    | 8    | 2   | 0.9707 |
|         | 4         | 21    | 0    | 1   | 1   | 0.9837 | 8   | 0    | 1    | 1   | 0.9876 | 4   | 0    | 1    | 1   | 0.9833 |
|         | 6         | 21    | 0    | 1   | 1   | 0.9976 | 8   | 0    | 1    | 1   | 0.9982 | 4   | 0    | 1    | 1   | 0.9975 |
|         | 8         | 21    | 0    | 1   | 1   | 0.9994 | 8   | 0    | 1    | 1   | 0.9996 | 4   | 0    | 1    | 1   | 0.9994 |
|         | 10        | 21    | 0    | 1   | 1   | 0.9998 | 8   | 0    | 1    | 1   | 0.9999 | 4   | 0    | 1    | 1   | 0.9998 |
| 0.05    | 2         | 68    | 3    | 6   | 1   | 0.9784 | 21  | 2    | 12   | 2   | 0.9588 | 11  | 2    | 5    | 1   | 0.9589 |
|         | 4         | 30    | 0    | 3   | 2   | 0.9593 | 10  | 0    | 1    | 1   | 0.9810 | 5   | 0    | 2    | 1   | 0.9810 |
|         | 6         | 30    | 0    | 3   | 2   | 0.9937 | 10  | 0    | 1    | 1   | 0.9972 | 5   | 0    | 2    | 1   | 0.9972 |
|         | 8         | 30    | 0    | 3   | 2   | 0.9984 | 10  | 0    | 1    | 1   | 0.9993 | 5   | 0    | 2    | 1   | 0.9993 |
|         | 10        | 30    | 0    | 3   | 2   | 0.9995 | 10  | 0    | 1    | 1   | 0.9998 | 5   | 0    | 2    | 1   | 0.9998 |
| 0.01    | 2         | 9     | 1    | 8   | 3   | 0.9675 | 33  | 3    | 6    | 1   | 0.9708 | 17  | 3    | 8    | 1   | 0.9534 |
|         | 4         | 5     | 0    | 4   | 2   | 0.9931 | 15  | 0    | 3    | 1   | 0.9716 | 7   | 0    | 2    | 1   | 0.9646 |
|         | 6         | 5     | 0    | 4   | 2   | 0.9990 | 15  | 0    | 3    | 1   | 0.9957 | 7   | 0    | 2    | 1   | 0.9946 |
|         | 8         | 5     | 0    | 4   | 2   | 0.9998 | 15  | 0    | 3    | 1   | 0.9989 | 7   | 0    | 2    | 1   | 0.9986 |
|         | 10        | 5     | 0    | 4   | 2   | 0.9999 | 15  | 0    | 3    | 1   | 0.9996 | 7   | 0    | 2    | 1   | 0.9995 |
Table 3. Optimal parameters of the proposed MDS plan for EHLD with \( \nu = 3 \)

| \( \theta \) | \( t_x/\xi \) | \( \alpha = 0.5 \) | \( \alpha = 0.7 \) | \( \alpha = 1.0 \) |
|---|---|---|---|---|
|   | \( n \) | \( c_1 \) | \( c_2 \) | \( m \) | \( P_\alpha(p_1) \) | \( n \) | \( c_1 \) | \( c_2 \) | \( m \) | \( P_\alpha(p_1) \) | \( n \) | \( c_1 \) | \( c_2 \) | \( m \) | \( P_\alpha(p_1) \) |
| 0.25 | 2 | 23 | 1 | 4 | 2 | 0.9915 | 10 | 1 | 3 | 2 | 0.9876 | 5 | 1 | 2 | 1 | 0.9745 |
|   | 4 | 11 | 0 | 1 | 3 | 0.9977 | 5 | 0 | 1 | 2 | 0.9976 | 3 | 0 | 2 | 1 | 0.9971 |
|   | 6 | 11 | 0 | 1 | 3 | 0.9998 | 5 | 0 | 1 | 2 | 0.9998 | 3 | 0 | 2 | 1 | 0.9997 |
|   | 8 | 11 | 0 | 1 | 3 | 1.0000 | 5 | 0 | 1 | 2 | 1.0000 | 3 | 0 | 2 | 1 | 0.9999 |
|   | 10 | 11 | 0 | 1 | 3 | 1.0000 | 5 | 0 | 1 | 2 | 1.0000 | 3 | 0 | 2 | 1 | 1.0000 |
| 0.10 | 2 | 31 | 1 | 11 | 3 | 0.9671 | 14 | 1 | 2 | 1 | 0.9583 | 7 | 1 | 3 | 1 | 0.9550 |
|   | 4 | 19 | 0 | 10 | 2 | 0.9960 | 8 | 0 | 7 | 2 | 0.9949 | 4 | 0 | 3 | 2 | 0.9901 |
|   | 6 | 19 | 0 | 10 | 2 | 0.9996 | 8 | 0 | 7 | 2 | 0.9995 | 4 | 0 | 1 | 1 | 0.9993 |
|   | 8 | 19 | 0 | 10 | 2 | 0.9999 | 8 | 0 | 7 | 2 | 0.9999 | 4 | 0 | 1 | 1 | 0.9999 |
|   | 10 | 19 | 0 | 10 | 2 | 1.0000 | 8 | 0 | 7 | 2 | 1.0000 | 4 | 0 | 1 | 1 | 1.0000 |
| 0.05 | 2 | 38 | 1 | 11 | 2 | 0.9560 | 18 | 1 | 4 | 1 | 0.9559 | 11 | 2 | 5 | 1 | 0.9805 |
|   | 4 | 24 | 0 | 10 | 2 | 0.9938 | 10 | 0 | 2 | 2 | 0.9922 | 5 | 0 | 2 | 1 | 0.9921 |
|   | 6 | 24 | 0 | 10 | 2 | 0.9994 | 10 | 0 | 2 | 2 | 0.9992 | 5 | 0 | 2 | 1 | 0.9992 |
|   | 8 | 24 | 0 | 10 | 2 | 0.9999 | 10 | 0 | 2 | 2 | 0.9999 | 5 | 0 | 2 | 1 | 0.9999 |
|   | 10 | 24 | 0 | 10 | 2 | 1.0000 | 10 | 0 | 2 | 2 | 1.0000 | 5 | 0 | 2 | 1 | 1.0000 |
| 0.01 | 2 | 67 | 2 | 3 | 1 | 0.9535 | 29 | 2 | 12 | 2 | 0.9555 | 17 | 3 | 8 | 1 | 0.9807 |
|   | 4 | 36 | 0 | 3 | 2 | 0.9866 | 16 | 0 | 2 | 1 | 0.9899 | 7 | 0 | 2 | 1 | 0.9850 |
|   | 6 | 36 | 0 | 3 | 2 | 0.9987 | 16 | 0 | 2 | 1 | 0.9999 | 7 | 0 | 2 | 1 | 0.9985 |
|   | 8 | 36 | 0 | 3 | 2 | 0.9998 | 16 | 0 | 2 | 1 | 0.9998 | 7 | 0 | 2 | 1 | 0.9997 |
|   | 10 | 36 | 0 | 3 | 2 | 0.9999 | 16 | 0 | 2 | 1 | 1.0000 | 7 | 0 | 2 | 1 | 0.9999 |
| $\theta$ | $t_{\alpha}/C$ | $\alpha = 0.5$ | | $\alpha = 0.7$ | | $\alpha = 1.0$ | |
|---|---|---|---|---|---|---|---|---|---|
| | $n$ | $c_1$ | $c_2$ | $m$ | $P_d(p_1)$ | $n$ | $c_1$ | $c_2$ | $m$ | $P_d(p_1)$ | $n$ | $c_1$ | $c_2$ | $m$ | $P_d(p_1)$ |
| 0.25 | 2 | 12 | 5 | 2 | 0.9665 | 7 | 2 | 6 | 3 | 0.9532 | 4 | 12 | 5 | 2 | 0.9641 | 3 | 0 | 2 | 1 | 0.9694 |
| | 4 | 7 | 0 | 1 | 2 | 0.9904 | 4 | 0 | 1 | 2 | 0.9902 | 3 | 0 | 2 | 1 | 0.9918 |
| | 6 | 7 | 0 | 1 | 2 | 0.9963 | 4 | 0 | 1 | 2 | 0.9962 | 3 | 0 | 2 | 1 | 0.9969 |
| | 8 | 7 | 0 | 1 | 2 | 0.9982 | 4 | 0 | 1 | 2 | 0.9982 | 3 | 0 | 2 | 1 | 0.9985 |
| | 10 | 7 | 0 | 1 | 2 | 0.9982 | 4 | 0 | 1 | 2 | 0.9982 | 3 | 0 | 2 | 1 | 0.9985 |
| 0.10 | 2 | 34 | 3 | 13 | 2 | 0.9550 | 20 | 3 | 13 | 2 | 0.9507 | 12 | 3 | 5 | 1 | 0.9536 |
| | 4 | 19 | 1 | 2 | 2 | 0.9904 | 11 | 1 | 4 | 2 | 0.9949 | 7 | 1 | 3 | 1 | 0.9953 |
| | 6 | 11 | 0 | 1 | 2 | 0.9775 | 7 | 0 | 1 | 1 | 0.9824 | 4 | 0 | 1 | 1 | 0.9810 |
| | 8 | 11 | 0 | 1 | 2 | 0.9911 | 7 | 0 | 1 | 1 | 0.9931 | 4 | 0 | 1 | 1 | 0.9926 |
| | 10 | 11 | 0 | 1 | 2 | 0.9957 | 7 | 0 | 1 | 1 | 0.9967 | 4 | 0 | 1 | 1 | 0.9965 |
| 0.05 | 2 | 46 | 4 | 14 | 2 | 0.9566 | 27 | 4 | 6 | 1 | 0.9557 | 16 | 4 | 6 | 1 | 0.9512 |
| | 4 | 23 | 1 | 4 | 2 | 0.9904 | 14 | 1 | 3 | 1 | 0.9928 | 8 | 1 | 3 | 1 | 0.9923 |
| | 6 | 15 | 0 | 1 | 1 | 0.9748 | 8 | 0 | 3 | 2 | 0.9696 | 5 | 0 | 2 | 1 | 0.9784 |
| | 8 | 15 | 0 | 1 | 1 | 0.9901 | 8 | 0 | 3 | 2 | 0.9879 | 5 | 0 | 2 | 1 | 0.9916 |
| | 10 | 15 | 0 | 1 | 1 | 0.9953 | 8 | 0 | 3 | 2 | 0.9942 | 5 | 0 | 2 | 1 | 0.9960 |
| 0.01 | 2 | 72 | 6 | 16 | 2 | 0.9548 | 42 | 6 | 9 | 1 | 0.9628 | 25 | 6 | 11 | 1 | 0.9638 |
| | 4 | 32 | 1 | 2 | 1 | 0.9650 | 19 | 1 | 4 | 1 | 0.9818 | 11 | 1 | 5 | 1 | 0.9789 |
| | 6 | 23 | 0 | 3 | 1 | 0.9614 | 13 | 0 | 3 | 1 | 0.9606 | 7 | 0 | 2 | 1 | 0.9599 |
| | 8 | 22 | 0 | 1 | 1 | 0.9795 | 13 | 0 | 3 | 1 | 0.9842 | 7 | 0 | 2 | 1 | 0.9840 |
| | 10 | 22 | 0 | 1 | 1 | 0.9901 | 13 | 0 | 3 | 1 | 0.9924 | 7 | 0 | 2 | 1 | 0.9923 |
The consumer’s risk is 0.10 if the actual median life is 800 hours and the producer’s risk is 0.05 if the actual median life is 1,600 hours. With these constraints the optimal parameters are selected from Table 4 as

\[ n = 12, c_1 = 3, c_2 = 5, \text{ and } m = 1 \text{ with values of } \nu = 1.728, t_q^0 = 800, \alpha = 0.05, \beta = 0.10, t_q/t_q^0 = 2 \]

at \( \alpha = 1.0. \) The multiple deferred state sampling plans is established as follows.

A sample of 30 observations will be selected at random for the submitted lot and put them on life test for 600 hours. If the number of failed units before 600 hours is less than or equal to 3 then the lot will be accepted and the lot will be rejected if the number of failed units is greater than 5.
There will be disposition of the present lot is deferred until the next successive lot will be tested in case of the number of failed units are in between 3 and 5.

4. Comparative study
Here, we compare the effectiveness of the proposed MDS sampling plan with existing sampling plans such as single sampling plan (SSP), group acceptance sampling plan based on resubmitted lots (RSP), etc. The ASN values of the respective sampling plans have been given in Table 5. Table 5 shows that the ASN of the proposed MDS sampling plan is smaller than that of the existing sampling plans such as SSP, RSP for different combinations of $\beta$, $t_q/t_A^0$, $\nu$ at $a = 1.0$. For example, the ASN of the proposed MDS sampling plan for $\beta = 0.10$, $t_q/t_A^0 = 2$, $\nu = 2$ at $a = 1.0$ is 12 whereas the ASN of the SSP, RSP for $\beta = 0.10$, $t_q/t_A^0 = 2$, $\nu = 2$ at $a = 1.0$ are 19 and 135, respectively. Therefore, we conclude that the proposed MDS sampling plan will be economical than the sampling plans SSP and RSP.

The Figure 2 shows the OC curve of the proposed MDS sampling plan along with SSP under exponentiated half logistic distribution with known shape parameter. In Figure 2 the MDS sampling plan parameters are considered as $n = 30$, $c_1 = 1$, $c_2 = 2$, and $m = 2$ and also for the SSP the parameters are $n = 30$ and $c = 1$. The two plans MDS and SSP are selected under the time-truncated life test with the same sample size. From Figure 2, we can observe that the probability of acceptance of the lot is more at lower failure probability under the MDS sampling plan than that of SSP and also when the failure probability increases the OC curve of the MDS sampling plan move toward the OC curve of the SSP. The probability of acceptance of the lot for the two plans MDS and SSP will be almost same at higher failure probability. So, we conclude that at higher quality level the MDS sampling plan gives protection to the producer as compared to SSP.

Similarly, Figure 3 shows the OC curve of the proposed MDS sampling plan along with SSP and RSP (Resubmitted sampling plan) under exponentiated half logistic distribution with known shape parameter. The MDS sampling plan parameters are considered as $n = 30$, $c_1 = 1$, $c_2 = 2$, and $m = 2$ and also for the SSP the parameters are $n = 30$ and $c = 1$ and for the RSP are $n = 30$, $c_1 = 1$, $c_2 = 2$.

The OC curves have constructed for the same sample size and acceptance number at $a = 1.0$. More specifically, we can observe from Figure 3 that the MDS sampling plan more appropriate than
the SSP and RSP and also it protects the producer at higher quality level and also safeguard to the consumer at lower quality level.

5. Conclusions
In this article, a multiple deferred state sampling plan under the assumption that the lifetime of the product follows an exponentiated half logistic distribution for truncated lifetime tests is developed. The optimal parameters of the proposed sampling plan are obtained using two points approach. A comparative study of the proposed MDS sampling plan have been performed using OC curves along with single sampling plan and group acceptance sampling plan based on resubmitted lots. The proposed MDS plan parameters are obtained by satisfying the respective consumer's and producer's risks simultaneously. We conclude that the proposed MDS sampling plan is more effective than the existing sampling plans like SSP, RSP, etc.in order to secure the consumer and prouder with less inspection. The limitation of this study is lifetime percentile quality characteristics should follow exponentiated half logistic distribution.

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Declaration of competing interest
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