THE PHYSICAL ORIGIN OF SCALE-DEPENDENT BIAS IN COSMOLOGICAL SIMULATIONS

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ABSTRACT

Using a large-scale hydrodynamic simulation with heuristic criteria for galaxy formation, we investigate how the galaxy field is related to physical parameters such as the mass density and the gas temperature. In our flat cold dark matter model with \( \Omega_0 = 0.37 \), we find that the relation between the galaxy and mass density fields is a function of scale. The bias \( b(R) = \sigma_r(R)/\sigma_g(R) \), where \( \sigma_r(R) \) is the variance of galaxy counts in spheres of radius \( R \), is scale dependent, ranging from 2.6 at 1 \( h^{-1} \) Mpc to 1.2 at 30 \( h^{-1} \) Mpc. Including the dependence of the galaxy density on local gas temperature as well as on local mass density can fully account for this scale dependence. Galaxy density depends on temperature because gas that is too hot cannot cool to form galaxies; this causes scale dependence of \( b(R) \) because local gas temperature is related to the gravitational potential and thus contains information about the large-scale density field. We show that temperature dependence generally causes \( b(R) \) to vary on quasi-linear and nonlinear scales, indicating that scale dependence of bias may be a generic effect in realistic galaxy formation scenarios. We find that the relationship between the galaxy and mass density fields is also a function of galaxy age. On large scales, the older galaxies are highly biased (\( b \approx 1.7 \) and highly correlated \( \langle r \equiv \delta g \rangle / \sigma_g \approx 1.0 \) with the mass density field; younger galaxies are not biased (\( b \approx 0.8 \)) and are poorly correlated \( \langle r \approx 0.5 \) with the mass. We argue that linear bias is inadequate to describe the relationship between galaxies and mass. We present a more physically based prescription that better fits our results and reproduces the scale dependence of the bias: \( \rho_g/\langle \rho_g \rangle = L(\rho/\langle \rho \rangle)^{0.66}(1 + T/40,000 \text{ K})^{4} \), where \( L = 1.23, M = 1.9, \) and \( N = -0.66 \).

Subject headings: galaxies: formation — hydrodynamics — large-scale structure of universe

1. MOTIVATION

Imminent large-scale galaxy redshift surveys such as the Sloan Digital Sky Survey (SDSS; Gunn & Weinberg 1995) and the Two-Degree Field (2DF; Colless 1998) will probe the galaxy density field of the universe with unprecedented precision. Were galaxies accurate tracers of the mass density field, the results of these surveys would put severe constraints on cosmological models. If the cold dark matter (CDM) picture for the linear-theory power spectrum is correct, these surveys could in fact measure cosmological parameters such as the mass density, \( \Omega_0 \), the vacuum density, \( \Lambda \), and the baryon density, \( \Omega_b \) (de Laix & Starkman 1998; Tegmark et al. 1998; Goldberg & Strauss 1998; Wang, Spergel, & Strauss 1999). However, the visible matter in galaxies is only a small percentage of the baryons in the universe, which in turn is a small percentage of the mass in the universe (Fukugita, Hogan, & Peebles 1998; Cen & Ostriker 1998b). Moreover, the process of galaxy formation is complex and nonlinear, including a complicated interplay between gravitational fields, hydrodynamics, microphysics, and star formation. Does this complicated process produce a population of galaxies whose number density field traces the mass density field perfectly? In this introductory section, we argue that observations already suggest that it does not, that the relationship between the density fields of galaxies and mass is biased, scale dependent, and nonlinear, as well as dependent on morphological type. This discussion provides the observational motivation for asking the theoretical question: how is the galaxy density field related to that of the mass?

The crucial element of our argument is that different morphological types of galaxies have different density fields (Hubble 1936; Oemler 1974). Consider the observed differences between the clustering strengths of galaxies of different types. Various authors have compared elliptical and spiral galaxies, generally finding that the fluctuation amplitude of ellipticals is stronger than that of spirals by a factor of 1.3–1.5 (Davis & Geller 1976; Giovanelli, Haynes, & Chincarini 1986; Santiago & Strauss 1992; Loveday et al. 1996; Hermit et al. 1996; Guzzo et al. 1997). Similarly, a comparison of the galaxy distribution in the IRAS redshift survey (Strauss et al. 1992b) with those in the Center for Astrophysics redshift survey (CIA; Huchra et al. 1983) and in the Optical Redshift Survey (ORS; Santiago et al. 1995) shows that optically selected galaxies are clustered more strongly than infrared-selected galaxies by a similar factor (Davis et al. 1988; Babul & Postman 1990; Strauss et al. 1992a). These differences in the amplitude of clustering can be accounted for by invoking a deterministic linear bias prescription:

\[
\delta_g(r) = b \delta(r) ,
\]

where \( \delta_g(r) \equiv \rho_g(r)/\langle \rho_g \rangle - 1 \) is the galaxy overdensity and \( \delta(r) \equiv \rho(r)/\langle \rho \rangle - 1 \) is the mass overdensity, smoothed on some scale. To explain the observations, one must assume that different galaxy populations are “biased” by different factors \( b \); clearly, all but one of these bias factors must differ from unity. Furthermore, there are differences between the shapes of correlation functions of galaxies of different types, at least at small scales. For instance, the ratio of the correlation functions of ellipticals and spirals found by Hermit et al. (1996) and Guzzo et al. (1997) declines with scale over the range between 1 and 10 \( h^{-1} \) Mpc. This scale dependence cannot result from a deterministic linear bias. Therefore,
there must exist a more complicated relation between the density fields of different morphological types.

Alternatively, consider the density-morphology relation, quantified by Dressler (1980), Postman & Geller (1984), and Whitmore, Gilmore, & Jones (1993). In the field, spirals comprise about 70% of all galaxies, and ellipticals and lenticulars comprise the rest; in the cores of rich clusters the situation is reversed, and ellipticals and lenticulars account for 90% of all galaxies. The relationship between spiral density and the density of all galaxies is extremely nonlinear in the densest cluster regions.

These differences among different morphological types suggest that the relationship between all galaxies and mass is comparably complicated. After all, it would be a coincidence if the overdensity field of all galaxies exactly traced the full mass overdensity, despite the fact that the different morphologies formed at different times and with different efficiencies. In any case, the selection effects of redshift surveys (color, surface brightness, luminosity, etc.) will cause any catalog to contain a mix of morphological types that differs from the mix in a volume-limited sample. Since the overdensity fields of different morphologies have different density fields, the results of every survey are "biased" to some degree. It is thus interesting to explore theoretically how the mass density in the universe might be related to the galaxy density and to the density of different morphological types.

One approach is to express the galaxy density as a local transformation of the dark matter density (or other variables), the simplest version of which is the deterministic linear bias of equation (1). Peaks biasing (Bardeen et al. 1986) and threshold biasing (Kaiser 1984) were the first suggestions along these lines. Indeed, Davis et al. (1985) used the peaks-biasing scheme to reconcile an unbiased linear bias of equation (1). Peaks biasing (Bardeen et al. 1985) and threshold biasing (Kaiser 1984) were the first and threshold biasing (Kaiser 1984) were the first formalisms to address the question of galaxy bias; Gnedin (1996a, 1998b, Cen & Ostriker (1992b, 1998a), and this paper use Eulerian methods for the same purpose.

In this paper, we examine the galaxy density field produced by the large-scale hydrodynamic simulations of Cen & Ostriker (1998a). With recent improvements in computational methods and computer hardware, the dynamic range in these simulations is approaching the level at which one can examine the formation of galaxies in a cosmological context. In § 2, we present the formalism of Dekel & Lahav (1998) for expressing the relation between galaxies and mass, and review some previous theoretical results. In § 3, we present details of the numerical simulations of Cen & Ostriker (1998a). In § 4, we examine the relation between the distributions of galaxies and mass in these simulations, as a function of smoothing scale and age. In § 5, we show that one can explain the properties of the galaxy density field more completely by allowing for its dependence on gas temperature as well as on mass density. We present an analytic fit to our results that describes the galaxy density field well. In § 6, we show that the dependence on gas temperature explains the scale dependence of the bias. Finally, we introduce a toy model that reproduces some of the salient properties of the galaxy density field and shows how scale dependence could be a generic property of galaxy formation. We discuss some directions for future work in § 7.

2. Formalism

In this section, we present a general formalism developed by Dekel & Lahav (1998) for expressing the present-day Eulerian relation between the galaxy and mass density fields smoothed on a local scale, \( R_0 \). Such an approach, which considers only the density fields at \( z = 0 \), of course does not account for the fact that two regions that have similar properties now may have had significantly different histories. We relate this formalism to the simple approximation that the galaxy and mass densities form a bivariate Gaussian distribution. Finally, we describe how certain complications allowed by this formalism, namely, nonlinearity and stochasticity, can cause the relation between the two density fields to be a function of scale. This formalism is not a theoretical model in its own right, but rather gives a useful framework for quantifying all relevant aspects of galaxy bias.

We begin by defining \( P(\delta_g | X_i) \) as the conditional probability of a certain overdensity of galaxies, \( \delta_g \equiv \rho_g / \langle \rho_g \rangle - 1 \), given the set of local conditions expressed by \( X_i \), where the \( X_i \) can be mass density, temperature, velocity shear, or any other present-day parameter thought to be relevant to galaxy formation. We use this probabilistic formalism despite the fact that galaxy formation is a deterministic process, for two reasons. First, two regions with similar properties at \( z = 0 \) may have had quite different histories; these differences may make it impossible to find a perfectly deterministic relation between \( \delta_g \) and other Eulerian variables. Second, even if such a relation did exist, one might not be able to identify the correct set of Eulerian variables with which to express it. Choosing a wrong or incomplete set of \( X_i \) would cause the relation to have large scatter.
Because this resulting stochasticity would in fact have a physical basis, it would be likely to have interesting statistical properties, such as spatial correlations. Thus, one of our goals will be to find a set of $X_i$ that minimizes this scatter.

Having defined the conditional probability, one can define the conditional mean,

$$\langle \delta_g | X_i \rangle = \int d\delta_g \delta_g P(\delta_g | X_i) .$$

(2)

The variance of the scatter about this mean is

$$\sigma^2_g \equiv \langle \epsilon^2 \rangle = \langle \delta_g - \langle \delta_g | X_i \rangle \rangle^2 ,$$

(3)

where, as indicated, $\epsilon$ represents the residuals of the conditional mean galaxy density at scale $R_o$. The quantity $\sigma_g/\sigma_g$, where $\sigma^2_g \equiv \langle \delta^2_g \rangle$, expresses the degree of stochasticity in the relation between galaxies and the variables $X_i$. Typically, investigators in this field have assumed that the most important (if not the only) local condition worth considering is the mass overdensity, $X_i = \delta \equiv \rho/\langle \rho \rangle - 1$ (an exception being Narayanan, Berlind, & Weinberg 1998). In this case, one considers the conditional probability, $P(\delta_g | \delta)$, and the corresponding conditional mean, $\langle \delta_g | \delta \rangle$, which is meant to summarize the relation between galaxies and mass on the given scale $R_o$.

If the joint distribution of $\delta$ and $\delta_g$ is a bivariate Gaussian at scale $R_o$, an appropriate approximation in the linear regime, the joint probability can be expressed as

$$P(\delta, \delta_g) = \frac{1}{2\pi \sqrt{\sigma^2_g \sigma^2_{\delta}}} \exp \left( - \frac{1}{2} \frac{\sigma^2_g \delta^2 - 2 \langle \delta \delta_g \rangle \delta \delta_g + \sigma^2_{\delta} \delta^2_g}{\sigma^2_g \sigma^2_\delta - \langle \delta \delta_g \rangle^2} \right) .$$

(4)

We will refer to this special case as “linear bias.” Such a distribution can be completely characterized by three quantities: $\sigma^2_g \equiv \langle \delta^2 \rangle$, $b \equiv \sigma_g/\sigma$, and $r \equiv \langle \delta \delta_g \rangle/\sigma_g \sigma_\delta$, and indeed, this motivates the definitions of these quantities for arbitrary $P(\delta, \delta_g)$. The quantity $b$, the bias, compares the rms amplitude of the galaxy overdensity to that of the mass overdensity. The quantity $r$, the correlation coefficient, expresses how closely the galaxy density field traces the mass density field. That is, if $r = \pm 1$, the relation between mass and galaxies is deterministic; if $r = 0$ the galaxies are distributed independently of the mass. Note that the Gaussian assumption implies that

$$\langle \delta_g | \delta \rangle = b \delta ;$$

$$\sigma_g / \sigma_\delta = \sqrt{1 - r^2} \quad \text{(Gaussian model)} .$$

(5)

In the nonlinear regime, the density field is far from Gaussian; however, calculating the second moments $b$ and $r$ will still give useful information on how the galaxy and mass density fields relate. The quantity $br$ for any given pair of galaxy and mass density fields is the slope of a linear regression of $\delta_g$ on $\delta$. Similarly, $b/r$ is the slope of the linear regression of $\delta$ on $\delta_g$. The quantity $\sqrt{1 - r^2}$ is a measure of the scatter around either regression. The scatter can occur either because nonlinearities make a straight line a poor approximation to $\langle \delta_g | \delta \rangle$, or because of stochasticity. As Dekel & Lahav (1998) note, the ratio $\sigma_g/\sigma_g(1 - r^2)^{1/2}$ measures the contribution of stochasticity (as opposed to nonlinearity) to the total scatter around the linear regression $br$.

In the context of the formalism presented here, let us examine how scale dependence in the relation between galaxies and mass may arise. Most work to date assumes that the deterministic linear bias of equation (1) holds (i.e., $r = 1$). Such a relation is scale independent; the same factor $b$ applies at all scales. However, $\langle \delta_g | \delta \rangle$ may in general be a nonlinear function of $\delta$. In fact, on scales at which $\sigma \gg 1$, this nonlinearity inevitably results from the condition $\delta > -1$ (as long as $\langle \delta_g | \delta \rangle$ does not exactly equal $\delta$). A simple approach is to expand $\delta_g$ in a Taylor series around $\delta$ (Fry & Gaztañaga 1993):

$$\langle \delta_g | \delta \rangle = b_1 \delta + \frac{b_2}{2} (\delta^2 - \langle \delta^2 \rangle) + \frac{b_3}{6} (\delta^3 - \langle \delta^3 \rangle) + \ldots .$$

(6)

The introduction of nonlinearity opens the door to scale dependence; if $\delta_g$ and $\delta$ are smoothed on a scale above $R_o$, the coefficients in equation (6) may change. On the other hand, Scherrer & Weinberg (1998) show that for hierarchical clustering, even in the presence of nonlinear bias on small scales, $b(R)$ is independent of $R$ on large scales.

In addition to being nonlinear, the relation can be stochastic, such that $\sigma_g \neq 0$. Scherrer & Weinberg (1998) have found that $b$ is independent of scale in this case as well. However, they assumed that the residual field $\epsilon$ about $\langle \delta_g | \delta \rangle$ is spatially uncorrelated. However, the scatter $\epsilon$ may have a physical basis; thus, it is possible that $\epsilon$ correlates with the large-scale density field. If such a correlation existed, $b(R)$ would be a function of scale. To illustrate this possibility, assume for the moment that at some small smoothing scale $R_o$, the joint $\delta_g, \delta$ distribution is a bivariate Gaussian (eq. [4]), with a bias of $b(R_o)$ and correlation coefficient $r(R_o)$. We define $\epsilon = \delta_g - b \delta$ on this scale. We can subsequently smooth $\delta_g, \delta$, and $\epsilon$ over a large scale $R \gg R_o$. In this case,

$$b^2(R) = b^2(R_o) r^2(R_o) + \frac{\langle \epsilon^2 \rangle_R + 2b(R_o)r(R_o)\langle \delta \epsilon \rangle_R)}{\sigma^2(R) .}$$

(7)

As $R \rightarrow R_o$, by definition we have $\langle \delta \epsilon \rangle_R \approx 0$ and $\langle \epsilon^2 \rangle_R = \sigma^2_g[1 - r^2(R_o)]$, so that $b(R \rightarrow R_o) = b(R_o)$, as necessary. If $\langle \delta \epsilon \rangle_R$ or $\langle \epsilon^2 \rangle_R$ varies on larger scales, clearly $b(R)$ will vary as well. We show below that this variation can result from fairly simple physical considerations; indeed, the effects are strong in the simulations considered here.

In the following sections, we will describe the numerical simulations of Cen & Ostriker (1998a) and present the results in the context of the formalism presented here. As the independent variable $X_i$, we at first use the traditional dark matter overdensity $\delta$, and find that the dependence on $\delta$ cannot completely characterize the galaxy density field. We will find that using the description $P(\delta_g | \delta, T)$, where $T$ is the local gas temperature, gives a much more satisfactory description of the galaxy density field. In particular, accounting for the dependence on $T$ (or its counterpart, the dark matter velocity dispersion $\langle v^2 \rangle$) also accounts for most of the scale dependence of the bias. Thus, temperature dependence causes stochasticity in the galaxy-mass relation, and this stochasticity is correlated with the mass distribution over large scales, causing $b(R)$ to vary with scale.
### TABLE 1

AGE AND REDSHIFT RANGES OF GALAXY PARTICLE QUARTILES OF EQUAL TOTAL MASS

| Quartile          | Age Range (Gyr) | Redshift Range |
|-------------------|-----------------|----------------|
| Oldest.............| 9.6–12.7        | 1.9–∞          |
| Second-oldest......| 7.8–9.6         | 1.1–1.9        |
| Second-youngest....| 5.7–7.8         | 0.6–1.1        |
| Youngest...........| 0–5.7           | 0–0.6          |

3. SIMULATIONS

For these simulations, the work of Ostriker & Steinhardt (1995) motivated the choice of a flat cold dark matter cosmology with $\Omega_m = 0.37$, $\Omega_{\Lambda} = 0.63$, and $\Omega_b = 0.049$. Recent observations of high-redshift supernovae have lent support to the picture of a flat, low-density universe, although great uncertainty still remains (Perlmutter et al. 1997; Garnavich et al. 1998). The Hubble constant was set to $H_0 = 100 h$ km s$^{-1}$ Mpc$^{-1}$, with $h = 0.7$. The primordial perturbations were adiabatic and random-phase, with a power-spectrum slope of $n = 0.95$ and amplitudes such that for the dark matter at $z = 0$, at which time the age of the universe is 12.7 Gyr. We use a periodic box $100 h^{-1}$ Mpc on a side, with $512^3$ grid cells and $256^3$ dark matter particles. Thus, the mass resolution is about $5 \times 10^9 M_\odot$ and the grid cell size is $\sim 200 h^{-1}$ kpc. The smallest smoothing length we consider is a $1 h^{-1}$ Mpc radius top hat, which is considerably larger than a cell size. On these scales and larger, the relevant gravitational and hydrodynamical physics are correctly handled. On the other hand, subgrid effects, such as the fine-grain structure of the gas and star formation, may influence large-scale properties of the galaxy distribution. As described below, we handle these effects using plausible, though crude, rules.

Cen & Ostriker (1998a) describe the hydrodynamic code in detail; it is similar to but greatly improved over that of Cen & Ostriker (1992a). The simulations are Eulerian on a Cartesian grid and use the total variation diminishing method with a shock-capturing scheme. In addition, the code accounts for cooling processes and incorporates a heuristic galaxy formation criterion, whose essence is as follows: if a cell's density is high enough, if the cooling time of the gas in it is shorter than its dynamical time, if it contains greater than the Jeans mass, and if the flow around that cell is converging, it will have stars forming inside of it. The code turns a fraction of the baryonic fluid component into collisionless stellar particles (hereafter “galaxy particles”), which subsequently contribute to metal production and the background ionizing UV radiation. The masses of these galaxy particles range from about $10^6$–$10^9 M_\odot$. Thus, many galaxy particles are contained in what would correspond to a single galaxy in the real universe.

Instead of grouping the particles into galaxies, we simply define a galaxy mass density field from the distribution of

![Image](image_url)

**Fig. 1.** Slice through our simulation 50 $h^{-1}$ Mpc on a side (one-half the total box length). As labeled, the quadrants show the fractional overdensity $\delta$ in dark matter, in the galaxies, in the young galaxies, and in the old galaxies, in clockwise order. The stretch is logarithmic and is the same in all quadrants. The fields are smoothed with a $1 h^{-1}$ Mpc Gaussian filter. The scale bar indicates 10 $h^{-1}$ Mpc. Note the large voids in the galaxy distribution and the reduction in the fraction of young galaxies in the large overdensity near the center.
galaxy particles themselves. Thus, our results will not be directly comparable to observations of the density field based on galaxy counts.

Ideally, we would like to study the properties of $\delta_g$ for different galaxy morphologies. Our simulations clearly do not have the resolution necessary to determine the morphology of individual galaxies based on their internal structure. However, there is a rough correlation between a galaxy's morphology and its star formation history (Roberts & Haynes 1994; Kennicutt 1998). Keeping this in mind, we can examine the simulations for an age-density relation in analogy to the morphology-density relation of Dressler (1980) and Postman & Geller (1984). Cen & Ostriker (1993) did so, finding qualitatively that the oldest galaxies had fallen into clusters and that the younger galaxies were forming in the lower density regions. In addition, Cen & Ostriker (1998a) have examined the dependence of the power spectrum on galaxy age. Here we visit the problem again, now looking in more detail at the joint distribution of the mass density and the density of each galaxy population. We split the galaxy particles into four age quartiles, defined such that the total mass of each age quartile is the same. These age quartiles are not meant to be taken as literally corresponding to different morphologies, since there are other variables besides age that determine galaxy type; however, their differences should at least be indicative of the differences between different galaxy morphologies. The ages and redshift ranges of the quartiles are given in Table 1. As we examine the properties of the full galaxy mass density field, we do the same for the density fields of each age quartile.

Figures 1 and 2 show a slice through the galaxy and dark matter density fields $50 \, h^{-1} \text{Mpc}$ on a side, at $1 \, h^{-1} \text{Mpc}$ and $10 \, h^{-1} \text{Mpc}$ smoothing, respectively. From top left, in clockwise order, we show the quantity $\delta$ for the dark matter, all of the galaxy particles, the youngest galaxy quartile, and the oldest galaxy quartile. For each of these, $\delta$ is normalized to the mean density of the sample in question, so that in the absence of biasing, these plots would be identical. Evidently, the galaxy distribution follows the dark matter distribution well, except in the underdense regions, which are completely empty voids in the galaxy particle distribution. It is apparent from these pictures that the youngest galaxy particles are distributed quite differently from the mass. At small smoothing scales the effect is obvious only in the clusters; otherwise, the young galaxies follow the dark matter. At large smoothing scales, however, the young galaxies are underdense in the clusters, and their density fields peak along the filaments. On the other hand, the oldest galaxies follow the mass distribution well on all scales, and are quite obviously biased. In the rest of the paper, we will quantify the differences among these various density fields.

### 4. SINGLE-VARIABLE BIAS

In this section, we study the relation between the galaxy and mass density fields, expressed by $P(\delta_g | \delta)$ and $\langle \delta_g | \delta \rangle$. 

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![Figure 2](image-url)
We begin by smoothing the galaxy density field over several different scales and showing that the relationship between galaxies and mass is a function of scale. Then we show that it is the properties of the scatter about $\langle \delta_g | \delta \rangle$ at small scales, and not the form of $\langle \delta_g | \delta \rangle$ itself, that causes this scale dependence.

4.1. Galaxy Density versus Mass Density

First we directly compare the density field of dark matter to that of galaxies. We do so by plotting in Figure 3 the conditional probability $P(1 + \delta_g | 1 + \delta)$ using top-hat smoothing filters of six different radii: 1, 2, 5, 8, 16, and 30 $h^{-1}$ Mpc. We use $1 + \delta$ here simply for convenience in plotting the results. The gray scale in this figure is a logarithmic stretch of $P(1 + \delta_g | 1 + \delta)$; the gray scale for each column is normalized separately. Note that for small smoothing scales, the discreteness of the dark matter particles limits our measurement of $\delta$ in voids. The vertical dashed line is the density corresponding to about 50 particles within one smoothing length, below which this effect becomes important. Note further that there is structure in the histograms at large smoothing scales, since there are many bins in the histogram but only a few truly independent values in the periodic volume. We plot $\langle \delta_g | \delta \rangle$ for each smoothing scale as solid lines; the $1\sigma$ deviations from this mean line are shown as dotted lines. It is immediately apparent that this function is nonlinear and that there is large scatter about it.

The same comparison can be made for the galaxies in each age quartile separately, as is shown in Figures 4 and 5 for top-hat smoothing filters of two radii, 1 and 30 $h^{-1}$ Mpc. The overdensity $\delta_g$ for each quartile is defined by normalizing to the mean density of that quartile. Note that there is a tight and highly biased relation between the distributions of older galaxies and the dark matter. On the other hand, the relation between the youngest galaxies and
the mass is quite stochastic, even at the largest scales. At small scales, \langle \delta_g | \delta \rangle for young galaxies is not monotonic; as in the real universe, young galaxies rarely live in the highest density regions. This trend with age is easily understood. The densest regions of the universe are in the deepest potential wells, so they have the hottest gas with the longest cooling times; thus, this gas stopped forming galaxies some time ago. As a consequence, on average, the galaxies in the densest regions are older than galaxies elsewhere.

One can quantify the relation between galaxies and mass by calculating second moments of the galaxy-mass distribution. Thus, at each smoothing scale, we calculate the parameters \( b \equiv \sigma_g/\sigma \) and \( r \equiv \langle \delta_g | \delta \rangle / \sigma \sigma_g \) and list their values in Table 2. Note that \( b \) declines strongly with scale, from 2.6 at 1 \( h^{-1} \) Mpc to 1.2 at 30 \( h^{-1} \) Mpc; this behavior is consistent with the work of Cen & Ostriker (1992b). Meanwhile, \( r \sim 0.9 \) almost independent of scale, meaning that galaxies are well correlated with mass. Similarly, we can calculate \( \sigma_b \), the variance of the scatter about \( \langle \delta_g | \delta \rangle \), and find that \( \sigma_g/\sigma_g \sim 0.3 \sim 0.4 \). For comparison to \( r \), we list the ratio \( \sigma_g/\sigma_g (1 - r^2)^{1/2} \), which is \( \sim 0.8 \sim 0.9 \) at all scales, indicating that stochasticity, rather than nonlinearity, dominates the scatter around the linear regression slopes \( br \) and \( b/r \).

The parameters \( b \) and \( r \) will also clearly depend on what age galaxies one considers, as Figures 4 and 5 indicate. In Table 3 we list \( b \) and \( r \) for each of the quartiles and smoothing scales used in Figures 4 and 5. Again, the dependence on scale is evident. However, more prominent are the differences between galaxies of different ages. Older galaxies are much more highly biased than young galaxies at all scales. In addition, older galaxies are more correlated with the mass distribution; \( r \sim 0.9 \sim 1.0 \) for the oldest galaxies, while \( r \sim 0.5 \) for the youngest galaxies. These results agree with observations that early-type galaxies are biased relative to late-type galaxies. The low correlation coefficient for young galaxy particles means that fluctuations in their density field are poorly correlated with fluctuations in the mass density field, as one can see from Figures 1 and especially 2; that \( b \) is near unity on large scales merely indicates that the fluctuations in the two fields are of similar amplitudes.

### 4.2. Scale Dependence

To study more carefully the scale dependence of the bias, we show \( b(R) \) in Figure 6 and \( r(R) \) in Figure 7. The solid lines show all the galaxies, and the dashed lines show each age quartile, as labeled. These curves show the same behavior found in Tables 2 and 3: the older galaxies are more

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**Table 2**

| Top-hat Radius \( h^{-1} \) Mpc | \( \sigma \) | \( b \equiv \sigma_g/\sigma \) | \( r \equiv \langle \delta_g | \delta \rangle / \sigma \sigma_g \) | \( \sigma_b/\sigma_g \) | \( \sigma_o/\sigma_g \sqrt{1 - r^2} \) |
|-------------------------------|---------|------------------|-----------------|--------------|------------------|
| 1.0                           | 4.77    | 2.61             | 0.886           | 0.420        | 0.905            |
| 2.0                           | 2.67    | 1.94             | 0.902           | 0.367        | 0.850            |
| 5.0                           | 1.14    | 1.52             | 0.923           | 0.314        | 0.817            |
| 8.0                           | 0.754   | 1.41             | 0.936           | 0.295        | 0.836            |
| 16.0                          | 0.401   | 1.27             | 0.941           | 0.312        | 0.920            |
| 30.0                          | 0.184   | 1.24             | 0.945           | 0.303        | 0.924            |

**Table 3**

| Top-hat Radius \( h^{-1} \) Mpc | Redshift Range | \( b \equiv \sigma_g/\sigma \) | \( r \equiv \langle \delta_g | \delta \rangle / \sigma \sigma_g \) | \( \sigma_b/\sigma_g \) | \( \sigma_o/\sigma_g \sqrt{1 - r^2} \) |
|-------------------------------|----------------|------------------|-----------------|--------------|------------------|
| 1.0                           | 1.9–\( \infty \) | 3.58             | 0.897           | 0.372        | 0.844            |
|                               | 1.1–1.9        | 3.29             | 0.896           | 0.394        | 0.886            |
|                               | 0.6–1.1        | 2.67             | 0.745           | 0.558        | 0.836            |
|                               | 0–0.6          | 2.13             | 0.524           | 0.687        | 0.807            |
| 30.0                          | 1.9–\( \infty \) | 1.65             | 0.990           | 0.139        | 0.981            |
|                               | 1.1–1.9        | 1.56             | 0.979           | 0.188        | 0.928            |
|                               | 0.6–1.1        | 1.25             | 0.882           | 0.437        | 0.929            |
|                               | 0–0.6          | 0.834            | 0.502           | 0.817        | 0.944            |
biased than the younger galaxies. Furthermore, the density field of the oldest quartile is extremely well correlated with the mass density field, while that of the youngest quartile is extremely poorly correlated. Meanwhile, bias declines with scale for all the galaxies and for each quartile. Other investigators have published comparable results, and it is appropriate here to address the similarities and differences between the current results and those of others. We first note that these results are qualitatively similar to the previous results found by Cen & Ostriker (1992b) using the same method but with different cosmologies. We also, for the purposes of this discussion, define $b_4(R) \equiv \varepsilon_{4}(R)/\varepsilon(R)$, the ratio of the galaxy and mass correlation functions on scale $R$. It is roughly, though not exactly, comparable to our definition $b \equiv \sigma_d/\sigma$.

The most recent N-body results, using the adaptive refinement tree (ART) method (Kravtsov, Klypin, & Khokhlov 1997), indicate, in fact, that there is antibias at small scales, and that the bias increases with scale (Kravtsov & Klypin 1998). This effect seems to be due to the merging and destruction of halos in dense regions. Klypin et al. (1999) claim that ART has sufficient dynamic range to avoid the “overmerging” problem that has plagued pure N-body simulations (White 1976; Frenk et al. 1988; van Kampen 1995; Summers, Davis, & Evrard 1995; Moore, Katz, & Lake 1996). Since we track stellar mass density, rather than galaxy number density, we cannot address the questions of mergers and destruction, and thus these effects are not apparent in our results.

On the other hand, the results of the SPH simulations of Carlberg & Couchman (1989), Katz et al. (1992), and Evrard et al. (1994) all show a slight dependence of $b_4(R)$ on $R$ between 1 and $10 h^{-1}$ Mpc, and a substantial increase of the bias on smaller scales ($\sim 0.3 h^{-1}$ Mpc). These results are in qualitative agreement with ours, and it is likely that the physical effects (described in §6) that cause the scale dependence in our simulations are also important in the SPH simulations, since the criteria for producing galaxies in those simulations are similar to those in ours.

Kauffmann, Nusser, & Steinmetz (1997) used semianalytic modeling techniques combined with N-body simulations to explore the relationship between the clustering of galaxies and mass. Their method has the advantage that it can efficiently explore parameter space; however, it cannot model the gas dynamics or the effects of environment. For galaxies with $M_g < -20$ in an open $\Omega_o = 0.2$ CDM model, they find a scale dependence of the bias similar to ours (their Fig. 6; note that their definition of “bias” is close to $br$, the regression of galaxy density on mass density, in our notation). For the $\Omega_o = 1$ CDM model, they find that the scale dependence is present but less acute. On the other hand, Kauffmann et al. (1998) shows $b_4 \equiv \varepsilon_{4}(r)/\varepsilon(r) < 1$ on small scales and increasing to larger scales for their $\Lambda$CDM cosmology, and $b$ slightly greater than 1 for the rCDM cosmology. Kauffmann et al. (1998) normalize their galaxy formation parameters somewhat differently than do Kauffmann et al. (1997), such that galaxies of the same luminosity are in lower mass halos in the later work; this accounts for part of the difference between their results. In sum, these semianalytic models, which have a vastly different model of galaxy formation from ours, can in some cases produce scale dependence similar to that found here.

We are interested in discovering exactly why bias decreases with scale in our results. Scale dependence can be due to two things only: the nonlinearity of the locally defined $\langle \delta_g | \delta \rangle$, or the properties of the field of residuals $\epsilon$ about that mean. Several theoretical forays suggest that the first possibility cannot be the case (Coles 1993; Scherrer & Weinberg 1998; Dekel & Lahav 1998). We can test these results by applying the $\langle \delta_g | \delta \rangle$ defined on small scales to the mass density field. Specifically, we calculate $\langle \delta_g | \delta \rangle$ for the galaxy and mass fields smoothed with a $1 h^{-1}$ Mpc radius top hat; then at each grid cell we check the value of $\delta$ and set the galaxy density in that cell to the appropriate value of $\langle \delta_g | \delta \rangle$. We refer to the resulting field as the “fake galaxy” density field.

We can now consider the statistic $b(R)$ for this fake galaxy field, which we determined using the mass-density field alone, as shown in the dotted lines in each panel of Figure 8. Here, $R$ takes into account the $1 h^{-1}$ Mpc smoothing already present in $\delta$. The solid lines show the actual $b(R)$ for the galaxies found in the simulations, from Figure 6. The top panel corresponds to all the galaxies, the middle panel to the oldest galaxies, and the bottom panel to the youngest galaxies. This procedure does not reproduce the behavior of bias as a function of scale, at least when we use this single-variable model, $\langle \delta_g | \delta \rangle$. There is some scale dependence, which is allowed by the results of Scherrer & Weinberg (1998) in the nonlinear regime. At small scales, the fake galaxy distribution has less power than the actual galaxy distribution, while at large scales the fake galaxy distribution remains much more biased than the actual galaxy distribution. Thus, the mean relation $\langle \delta_g | \delta \rangle$ between galaxies and mass at small scales does not contain all the information about $\delta_g$.

The remaining possibility is that the properties of the residual field $\epsilon$ about $\langle \delta_g | \delta \rangle$ cause the scale dependence. These residuals must correlate over large scales in such a way that $\epsilon < 0$ in large-scale overdensities and $\epsilon > 0$ in large-scale voids. In that case, $\langle \delta \epsilon \rangle_R$ will be negative on...
large scales, and, as equation (7) reveals, \( b(R) \) will decrease with scale. To test this possibility, we calculate \( \epsilon \) at 1 \( h^{-1} \) Mpc top-hat smoothing, and then smooth \( \epsilon \) and \( \delta \) on scale \( R \) in order to calculate \( \langle \delta \epsilon \rangle_b / \sigma^2(R) \). Figure 9 shows the behavior of this quantity. As one would predict, it is near zero when \( R \sim 1 h^{-1} \) Mpc and becomes strongly negative on larger scales. The same holds for the youngest and oldest quartiles, also shown in Figure 9. This result indicates that, indeed, the residual field tends to be negative in large-scale overdensities, and positive in large-scale voids.

In other words, the scatter about \( \langle \delta \epsilon \rangle_0 \) has interesting statistical correlations. It depends on other variables that are important to galaxy formation. The dependence on and the nature of these variables must be such as to reproduce the scale dependence found in this section. In the next two sections, we investigate the dependence of galaxy density on a number of other variables, and find that accounting for the dependence of galaxy density on local temperature can both reduce the scatter in the residual field and explain the scale dependence of \( b \).

5. TWO-VARIABLE BIAS

From the last section we learned that modeling the galaxy density field as a function of mass density alone was unsatisfactory. That is, spatial correlations in the scatter \( \epsilon \) are important. We would like to both reduce the scatter in our estimate of the galaxy density and explain the scale dependence of \( b(R) \). To do so, we can add an independent variable to our conditional probability and consider the function \( P(\delta | \delta, X_i) \). Note that to account for the scale dependence, \( X_i \) must have spatial correlations similar to those of \( \epsilon \); we discuss this point further in the next section. We try several \( X_i \) and find that the most successful is the local gas temperature \( T \), or, equivalently, the local dark matter velocity dispersion. This result is perhaps not surprising, since gas temperature is surely an important parameter in galaxy formation. That is, in order to form stars, gas must be able to cool efficiently and collapse, which cannot happen at very high temperatures. In addition, we will find in § 6 that accounting for this dependence on \( T \) also accounts for the dependence of \( b(R) \) on scale.

For each \( X_i \) we choose as a second independent variable, we want to know how much it reduces the stochasticity in the relation \( P(\delta | \delta, X_i) \). To quantify this, we calculate the ratio

\[
\frac{\sigma_{b,2}}{\sigma_b} = \frac{\langle (\delta - \langle \delta \rangle_0 | \delta, X_i) \rangle^2 \rangle^{1/2}}{\langle (\delta - \langle \delta \rangle_0 | \delta) \rangle^{1/2}},
\]

which expresses the scatter of \( \delta \) around \( \langle \delta \rangle_0 | \delta, X_i \) compared with the scatter around \( \langle \delta \rangle_0 | \delta \). Thus, if \( X_i \) is perfectly correlated with \( \delta \), or if it is perfectly uncorrelated with \( \delta \), then \( \sigma_{b,2}/\sigma_b \approx 1 \). In the next few paragraphs, we will substitute for the variable \( X_i \) the following: \( T \), the local temperature; \( \langle v^2 \rangle \), the local dark matter velocity dispersion; the protuteness and oblateness of the local dark matter density field; and the shear in the dark matter velocity field. Our results for \( \sigma_{b,2}/\sigma_b \) for these variables are given in Table 4.

The local temperature \( T \) is relevant to galaxy formation because gas that is too hot does not satisfy the Jeans criterion and cannot cool efficiently, and thus cannot collapse and form stars. Since this criterion is explicitly included in our conditions for the condensation of galaxy particles out of the gas, it is a reasonable second variable to investigate. Figure 10 shows a contour plot of galaxy density as a function of mass density and gas temperature; all fields in this
plot have been smoothed over 1 h⁻¹ Mpc radius top-hat spheres. First, note that local gas temperature and local dark matter density are clearly not independent variables; the upper left triangle in the figure is blank because there are no volume elements with high local density and low temperature on these scales. Nevertheless, one can see that galaxy density declines as the temperature increases at constant δ. Figure 11 shows δₜ as a function of T for δ = 20, corresponding to a horizontal slice of Figure 10. The strength of the temperature dependence is evident. From Table 4, we note that the inclusion of this extra variable reduces the variance by over a factor of 2, indicating that the dependence of galaxy density on mass density and temperature is identical, except for δ < 5, where galaxies are produced more efficiently in the higher resolution simulation. This difference has little effect on the quantities b and r, although the void probability function is affected, in the sense that voids are less likely in the higher resolution simulation.

A fairly good fit to the dependence of galaxy density on mass density and temperature on small scales is

\[
\frac{\rho_g}{\langle \rho_g \rangle} = L \left( \frac{\rho}{\langle \rho \rangle} \right)^M \left( 1 + \frac{T}{40,000 \, K} \right)^N. \tag{9}
\]

We show such a fit in Figure 12. As labeled, each pair of solid and dotted lines corresponds to a different value of δ. The solid lines show the results from Figure 10; the dotted lines show the fit of equation (9), using L = 1.23, M = 1.9, and N = -0.66. For N < 0, the factor involving T takes into account the fact that relatively fewer galaxies have formed in hotter regions; the form assumed here reflects the approximate power-law dependence in Figures 11 and 12. This effect is not important once the gas is as cold as 40,000 K, and thus we construct the temperature factor to have little effect in that regime.

We would like to be able to apply such a fit to N-body simulations. Doing so would allow us to explore changes of cosmological parameters more easily than the expensive hydrodynamical simulations allow. Although we cannot follow gas temperature in purely collisionless simulations, we can calculate the related quantity \( \langle v^2 \rangle \), the local dark matter velocity dispersion. The dependence on \( \langle v^2 \rangle \) should be similar to that on local gas temperature. After all, in virialized regions the velocity dispersion of dark matter particles is close to that of individual atoms. Indeed, from Table 4 it is clear that \( \sigma_{b,2}/\sigma_b \) is nearly the same for the velocity dispersion as it is for the temperature. Note, of course, that taking into account the dependence of galaxy density on both \( \langle v^2 \rangle \) and T would not improve on using

![Figure 10](image1.png)

**Figure 10.**—Dependence of galaxy density, \( 1 + \delta \), on dark matter density, \( 1 + \delta \), and temperature, T, evaluated at \( 1 \, h^{-1} \, \text{Mpc} \) smoothing. The gray scale is a logarithmic stretch of \( 1 + \delta \); the contours are in even logarithmic intervals.

![Figure 11](image2.png)

**Figure 11.**—Dependence of \( \rho_g/\langle \rho_g \rangle \equiv 1 + \delta \) on T at fixed δ for all the galaxies (solid line) and each quartile (dashed lines, as labeled). Note that \( (1 + \delta) \) varies over an order of magnitude in this temperature range. Also note that the young galaxies have the strongest dependence on temperature and the oldest galaxies have the weakest.

| \( X_i \)                           | \( \sigma_{b,2}/\sigma_b \) |
|----------------------------------|--------------------------|
| Local temperature, T            | 0.70                     |
| Dark matter velocity dispersion, \( \langle v^2 \rangle \) | 0.68                     |
| Velocity shear, \( \Sigma H\delta(1 + \delta) \) | 0.96                     |
| Oblateness \( (1 \, h^{-1} \, \text{Mpc spheres}) \), \( \lambda_2 + \lambda_3 \) | 0.97                     |
| Prolateness \( (1 \, h^{-1} \, \text{Mpc spheres}) \), \( 2\lambda_2/(\lambda_2 + \lambda_3) - 1 \) | 0.97                     |

**TABLE 4**

*Reduction in the Standard Deviation \( \sigma_{b,2}/\sigma_b \) for Various Choices of \( X_i \)***
properties similar to those of the scatter around the conditional mean \( \langle \delta_j | \delta \rangle \)? We can repeat the exercise of § 4 and calculate a fake galaxy density field, this time using the two-variable mean \( \langle \delta_j | \delta, T \rangle \) of Figure 10. We then calculate \( b(R) \) as before and plot it as the short-dashed line in each panel of Figure 8. The temperature dependence indeed beautifully accounts for the variation of \( b(R) \) with scale, for all the galaxies as well as for each quartile. The fit to the density distribution given by equation (9) produces nearly identical results; it is shown by the long-dashed line in the upper panel.

What are the physical properties of the temperature field that cause this to happen? Essentially, local temperature reflects the gravitational potential and thus contains information about the large-scale density field. The temperature fluctuations \( \delta_T \equiv T/\langle T \rangle - 1 \) can be expressed in a simple way by considering some limits (D. N. Spergel 1998, private communication). First, in the nonlinear regions, gas is virialized and its temperature must scale as the local potential. From Poisson’s equation, we know that \( \phi(k) \propto k^{-2} \delta(k) \). Thus, on small scales, it must be that \( \delta_T(k) \propto k^{-2} \delta(k) \) as well. Second, on linear scales one can assume that the temperature fluctuations are dominated by the number density fluctuations of virialized halos, since gas in those areas is much hotter than that in the empty regions between halos. Thus, on large scales \( \delta_T(k) \propto \delta(k) \). These two limits may be combined:

\[
\delta_T(k) \propto \frac{\delta(k)}{1 + k^2 r_{nl}^2/(2\pi)^2},
\]

where \( r_{nl} \) is the transition scale between the linear and nonlinear regimes. Consequently, we expect the cross-spectrum of temperature and mass to be

\[
P_{Tm}(k) \propto \frac{P_{mm}(k)}{1 + k^2 r_{nl}^2/(2\pi)^2}.
\]

In Figure 13, we compare this simple model with the simulations, finding that it is a good fit for the choice \( r_{nl} = 16 \) \( h^{-1} \) Mpc. This value for \( r_{nl} \) agrees approximately with the scale on which nonlinear effects should become important. Thus, the temperature power spectrum peaks at large scales; furthermore, at those scales the temperature fluctuations are directly related to the mass density. This means that the largest contribution to the local temperature actually comes from large wavelength fluctuations, which follow the large wavelength fluctuations in mass density. The local gas temperature is therefore a direct indicator of the large-scale density field. Thus, accounting for the temperature dependence automatically accounts for the dependence on large-scale density, and consequently the dependence of bias on scale.

A simple model for the relation between galaxies, mass, and temperature reveals more explicitly how scale dependence and stochasticity enter the relation between galaxies and mass. Consider the fit given in equation (9). If one assumes that \( \delta_j \ll 1 \), \( \delta \ll 1 \), and \( \delta_T \ll 1 \), this relation becomes

\[
\delta_g = M' \delta + N' \delta_T,
\]

and one recovers the deterministic linear bias model if \( N' = 0 \) and \( M' = b \). This model is obviously highly unrealistic, especially at small scales. However, its simplicity will
shows the model, for cross-spectrum given in equation (12) with the cross-spectrum measured in dependent.

allow us to understand better how $b(R)$ becomes scale dependent.

We can perform a linear regression on $\delta$ and $\delta_T$ to determine $M'$ and $N'$. The results as a function of top-hat smoothing scale are shown in Figure 14. Note that $M'$ and $N'$ are approximately constant with respect to scale. This invariance indicates that the scale dependence is well accounted for by the temperature dependence. To examine this claim, let us assume that equation (13) holds at some small scale $R_0$. Now, if we smooth over a larger scale $R$, we find that

$$b^2(R) \equiv \frac{\sigma_T^2(R)}{\sigma^2(R)} = M'^2 + N'^2 \frac{\sigma_T^2(R)}{\sigma^2(R)} + 2M'N' \frac{\langle \delta_T \delta \rangle_R}{\sigma^2}.$$  (14)

Note that $\langle \delta T \delta \rangle_R/\sigma^2(R)$ and $\sigma_T^2(R)/\sigma^2(R)$ will depend on $R$, because equation (11) shows that $P_{TT}(k)/P_{\text{mm}}(k)$ and $P_{TT}(k)/P_{\text{mm}}(k)$ depend on $k$. Thus, $b(R) \equiv \sigma_T(R)/\sigma(R)$ will also depend on $R$. We can test this possibility directly by applying the deterministic two-variable linear model to this simulation and calculating $b(R)$. For this test we use the values $M' = 2.4$ and $N' = -0.4$, which are appropriate at $1 \, h^{-1} \, \text{Mpc}$; the resulting curve is shown by the dot-dashed line in the upper panel of Figure 8. At small scales, the linear approximation is (as expected) poor but of the right order; on large scales, it reproduces the value of $b(R)$ fairly well.

Thus, even this simple linear model reproduces the scale dependence. We do not claim that this model is a particularly good one for describing the galaxy density; the fit of equation (9) is much better. Rather, the linear model is merely a toy that illustrates the following point: given a dependence of galaxy density on local temperature, it is inevitable that $b(R)$ is a function of scale on scales smaller than about $r_{\text{nl}}$, simply because of the relationship between gas temperature and mass density.

7. DISCUSSION

Consider two regions of the universe, both having the same local mass overdensity, but with respect to the large-scale density field one being in an overdensity, the other in a void. The gas in these two regions will evolve similarly, forming galaxies with about equal efficiency, until the large-scale density field becomes nonlinear. At this point, the ambient gas around the first region, in the large-scale overdensity, will become too hot to accrete any longer, and galaxy formation will cease. Meanwhile, the gas around the second region, in the void, will remain cool enough to accrete onto old galaxies and continue to form new ones. This picture explains part of the scatter in the local relation between $\delta$ and $\delta_T$; it also explains why this scatter is such that in hot regions $\epsilon \equiv \delta_T - \langle \delta_T \rangle < 0$, and in cold regions $\epsilon > 0$. In turn, the correlation of temperature with large-scale density explains why $\epsilon$ also correlates with large-scale density, causing scale-dependent bias. In addition, it indicates qualitatively why spirals, which are caused by late-time accretion of gas, are relatively more abundant in the “field” and relatively underabundant in the rich clusters.

In this scenario, scale-dependent bias follows from only one rather robust assumption: that the ambient gas temperature affects the efficiency of galaxy formation. Since gas must be able to cool to form galaxies, this assumption is well-motivated theoretically. Furthermore, observations of the star formation rate as a function of local density indicate that indeed star formation is reduced in the hot cluster environments, even at a fixed morphological type (Young et al. 1996; Hashimoto et al. 1997; Balogh et al. 1998). Once one concedes that local temperature is an indicator of the efficiency of galaxy formation, the arguments given in § 6 lead one to directly conclude that $b(R)$ should depend on scale, at least in the quasi-linear and nonlinear regimes.
where $P_{T,0}(k)$ and $P_{T,1}(k)$ have shapes that differ from that of $P_{m,0}(k)$. In the simulations studied here, the effects are particularly strong.

Scale dependence in the relation between galaxies and mass can affect the interpretation of future redshift and peculiar-velocity surveys. The most obvious example is that on small scales, the shape of the galaxy power spectrum will differ from that of the mass. Furthermore, as pointed out by Dekel & Lahav (1998), comparison of the observed galaxy density field to that inferred from observed peculiar velocities (Dekel 1994; Sigad et al. 1998) effectively perform a regression of $\delta$ on $\delta_g$ and thus measure the quantity $\beta = (r/b)f(\Omega)$, which we show here depends on both scale and the chosen galaxy sample. Thus, the current analyses, usually performed in the quasi-linear regime and using IRAS galaxies$^3$ to define the density field, may be sensitive to these effects. For instance, one’s estimate of $r/b$ will generally increase with scale, by about 20% between 5 and 30 $h^{-1}$ Mpc. Amusingly, although $r$ and $b$ vary by a factor of 2 between the youngest and oldest galaxies on large scales, the dependence of $r/b$ on galaxy age remains quite small; $r/b \sim 0.6$--0.7 for all four quartiles. In the real world, the regression of $\delta$ on $\delta_g$ will most likely not be this constant among the morphological types. The decrease of $\beta$ with scale could contribute to the differences between the results of Sigad et al. (1998) and analyses carried out with smaller smoothing scale, such as VELMOD (Willick & Strauss 1998); however, there are stark differences between these methods, and a more careful analysis is thus necessary. Indeed, we are interested in exploring the effects of nonlinear stochastic bias on a variety of large-scale structure statistics inferred from redshift surveys, including redshift-space distortions (Dekel & Lahav 1998) and the pairwise velocity dispersion as a function of local density (Strauss, Ostriker, & Cen 1998). A full treatment will require identifying individual galaxies from the galaxy particles in the simulations, which will require tackling the overmerging problem in the densest regions of the simulations.

We can address some of these issues without having to run expensive hydrodynamic simulations for a range of cosmological models. First, in order to examine in detail the effect of the relation between galaxy density, mass density, and temperature on all of these statistics, we plan to carry out $N$-body simulations of a larger dynamic range than is possible with the current hydrodynamical simulations. To characterize the galaxy distribution in these simulations, we will apply the model of equation (9), using $\langle v^2 \rangle$ as a proxy for temperature. Then one can explore the effect this type of bias can have on the various statistics discussed in the last paragraph. A second approach is to analyze observations, allowing for scale- and temperature-dependent bias of the character described here. In this vein, one could investigate the differences between galaxy types and see how they compare in detail with the differences we find between our age quartiles. Using the local galaxy velocity dispersion as a proxy for temperature, we could even directly investigate the dependence of the density of difference morphological types on temperature.

Of course, that temperature is the important causal variable in these simulations is based only on a post hoc (although physically plausible) argument in this paper. Any variable that probes the large-scale density field would serve just as well to reduce $\sigma_{h,0}/\sigma_b$ and explain the scale dependence of $b(R)$. However, as discussed above, temperature is a well-motivated quantity. Controlled tests of the effect of ambient temperature on star formation in these simulations might help clarify the matter. Another approach is to look at various output times and examine under what conditions in the simulations the galaxies actually form. We do so in a separate paper, which examines the time dependence of galaxy formation and bias in these simulations. That work makes it clear that the temperature dependence is a result of the Jeans mass and cooling criteria that the code uses to decide where galaxy particles condense out of the gas.

The alert reader will notice that some of our results contradict our opening statements in $\S$ 1. In particular, if one looks at our results on large scales, it does happen to be true that the distributions of old and young galaxies both differ considerably from the mass distribution, but in combination trace the mass quite well and are almost unbiased. We simply note here that the same would not be true were we to look at the results of these simulations at $z = 0.5$, for instance. In addition, Cen & Ostriker (1992b) found that for a hot dark matter universe (HDM), $b$ was significantly greater than unity on large scales, indicating that the level of bias in these simulations depends somewhat on the chosen cosmology. Thus, we ascribe little importance to this coincidence in the current simulations.

Note also that these results concern stellar mass density, not galaxy number density. Since the galaxies in the densest regions of the simulations overmerge, to what degree the bias described here affects the brightness, rather than the number, of galaxies is unknown. Only if the stellar mass function of galaxies is universal will the bias found here translate directly into galaxy number density bias. On the other hand, previous theoretical results hint that the most massive halos form preferentially in large-scale overdensities (Mo & White 1996). Meanwhile, these massive halos may form fewer stars per unit mass than less massive halos (White & Frenk 1991; Katz et al. 1992; Evrard et al. 1994; Kauffmann & Ostriker. 1998). Observationally, Bromley et al. (1998) find that early-type galaxies are fainter in dense regions than elsewhere. The temperature dependence found in our simulations would certainly explain the last two effects, which do affect the brightnesses, but not the numbers, of galaxies.

Several further notes of caution are in order concerning applying these results in detail. First, the simulations have limited resolution and do not probe physics on scales less than 200 $h^{-1}$ kpc. Thus, we cannot follow individual halos in dense regions. As a check, we performed the same analysis on a 50 $h^{-1}$ Mpc simulation of twice the resolution, finding consistent results in the regime $\delta > 5$ for the dependence of galaxy density on mass density and temperature, smoothed at 1 $h^{-1}$ Mpc. On the other hand, the 50 $h^{-1}$ Mpc simulations produced more galaxies in the regions with $\delta < 5$. Second, by necessity all the complications of the interstellar medium (ISM) and small-scale dynamics are ignored. Surely, the morphology of the ISM, its interaction with infalling gas, and its reaction to high star formation rates is important to the formation and evolution of galaxies. Third, even to the extent that the simulation is

$^3$ IRAS galaxies are typically young, although they are probably not as young as our youngest quartile. In particular, IRAS galaxies do not show the undersaturation in rich clusters that the youngest galaxies in the simulations do (Fig. 2), although they are less overdense than optically selected galaxies (Strauss et al. 1992a).
physically accurate, the dependence of the results on cosmological parameters is unknown. Finally, we have not experimented here with varying the galaxy formation and feedback parameters, although this has been done in previous work (Cen & Ostriker 1992b; Gnedin 1996a, 1996b). We plan future simulations of higher resolution both to probe variations in cosmology and to attempt to identify individual galaxies even in the denser regions of the simulation. In addition, we are working on methods of efficiently exploring the effects of varying the galaxy formation parameters.

The general result of the simulations is that the relationship between mass and galaxies is interestingly complicated. Given the precision and volume of upcoming redshift surveys, it is possible that we understand the mass distribution on small scales will be limited by our ignorance of the properties of galaxy formation and of the origin of different morphological types. We hope that cosmological simulations such as this one can help us understand in what ways, and possibly to what degree, the galaxy distribution can differ from that of the mass.

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