GROUND STATE CORRELATIONS AND FINAL STATE INTERACTIONS IN TWO-NUCLEON EMISSION PROCESSES OFF FEW-NUCLEON SYSTEMS*

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The two nucleon emission process from \(^3\text{He}, ^3\text{He}(e, e'N_2N_3)N_1\), has been theoretically analyzed using realistic three-nucleon wave functions and taking the final state interaction into account. Various kinematical conditions have been considered in order to clarify the question whether the effects of the final state interaction could be minimized by a proper choice of the kinematics of the process.

1. GROUND STATE CORRELATIONS IN NUCLEI

The investigation of Ground State Correlations (GSC) in nuclei, in particular those which originate from the most peculiar features of the Nucleon-Nucleon (NN) interaction, i.e. its strong short range repulsion and complex state dependence (spin, isospin, tensor, etc), is one of the most challenging aspects of experimental and theoretical nuclear physics and, more generally, of hadronic physics. The results of sophisticated many-body calculations in terms of realistic models of the NN interactions, show that the complex structure of the latter generates a rich correlation structure of the nuclear ground state wave function. The experimental investigation of the nuclear wave function or, better, of various density matrices, \(\rho(1), \rho(1, 1'), \rho(1, 2), \text{etc}\), is therefore necessary in order to ascertain whether the prediction of the Standard Model of nuclei (structureless non-relativistic nucleons interacting via the free NN interaction) is indeed justified in practice, or other phenomena, e.g.:

1. effective single-particle mean field,
2. relativistic effects,
3. many-body forces,
4. medium modification of nucleon properties,
5. explicit sub-nucleonic degrees of freedom (quark and gluons),

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have to be advocated in order to describe ground-state properties of nuclei at normal density and temperature.

Unfortunately, whereas the one-body density matrix (charge density) is experimentally well known since many years (see e.g. [1]), the present knowledge of those quantities, e.g. the non-diagonal one-body (momentum distributions) and two-body (two-nucleon correlation function) density matrices, which could provide more reliable information on GSC, is still too scarce. One the reasons for that should be ascribed to the effects of the final state interaction (FSI), which very often compete with the effects generated by GSC. The present situation is therefore such that the long-standing question:

- **Does FSI hinder the investigation of GSC?**

has not yet been clearly answered. Moreover, due to the difficulty to treat consistently GSC and FSI within a many-body approach, the answer to the above question was in the past merely dictated by philosophical taste rather than by the results of solid calculations and by unambiguous experimental data. To-day the answer could probably be provided in a more reliable way, particularly in the case of few-body systems, for which accurate ground state wave functions are available and FSI effects can also be calculated in a satisfactory way (see e.g. [2], [3]). In this paper we will discuss the process of two-nucleon emission off the three-nucleon systems, which is being experimentally investigated (see e.g. [4], [5]), with the aim of providing another attempt at answering the above long-standing question. The results of some theoretical calculations, using realistic three-nucleon wave function, and taking into account FSI effects will be presented, and future perspectives in the field briefly discussed.

2. Two-nucleon emission off the three-nucleon systems: kinematics and cross section

We will consider the absorption of a virtual photon $\gamma^*$ by a nucleon bound in $^3$He, followed by two-nucleon emission. In the rest of this paper the photon four momentum transfer will be denoted by $Q^2 = q^2 - \nu^2$, the momenta of the bound nucleons, before $\gamma^*$ absorption, by $k_i$, and the momenta in the continuum final state by $p_i$. Momentum conservation requires that

$$\sum_{i=1}^{3} k_i = 0 \quad \sum_{i=1}^{3} p_i = q$$

and energy conservation that

$$\nu + M_3 = \sum_{i=1}^{3} (M^2 + p_i^2)^{1/2}$$

where $M$ and $M_3$ are the nucleon and the three-nucleon system masses, respectively.

In what follows, the two nucleons which are detected will be denoted by $N_2$ and $N_3$ and the third one by $N_1$. In one-photon exchange approximation the cross section of the process, depicted in Fig. 1, reads as follows

$$\frac{d^{12}\sigma}{d\nu d\Omega d^4p_1 d^4p_2 d^4p_3} = \sigma_{Mot} \cdot \sum_{i=1}^{6} v_i \cdot W_i \cdot \delta(q - \sum_{i=1}^{3} p_i) \delta(\nu + M_3 - \sum_{i=1}^{3} (M^2 + p_i^2)^{1/2})$$
where \( v_i \) are well known kinematical factors, and \( W_i \) the response functions, which have the following general form

\[
W_i \propto |\langle \Psi_f^- | (p_1, p_2, p_3) | \hat{\mathcal{O}}_i(q) | \Psi_i(k_1, k_2, k_3) \rangle|^2
\] (4)

In Eq. (4) \( |\Psi_f^- (p_1, p_2, p_3) \rangle \) and \( |\Psi_i(k_1, k_2, k_3) \rangle \) are the continuum and ground state wave functions of the three body system, respectively, and \( \hat{\mathcal{O}}_i(q) \) is a quantity depending on proper combinations of the components of the nucleon current operator \( \hat{j}^\mu \) (see e.g. [1]).

If FSI is fully disregarded, two nucleon emission originated by NN correlations can occur because of two different processes:

1. in the initial state \( N_2 \) and \( N_3 \) are correlated and \( N_1 \) is far apart; \( \gamma^* \) is absorbed by \( N_1 \) and \( N_2 \) and \( N_3 \) are emitted in the continuum;

2. in the initial state \( N_1 \) is correlated with either \( N_2 \) or \( N_3 \) (e.g. \( N_2 \)); \( \gamma^* \) is absorbed by \( N_1 \) and \( N_2 \) and \( N_3 \) are emitted in the continuum.

In both cases momentum conservation reads as follows

\[
p_1 = k_1 + q \quad p_2 = k_2 \quad p_3 = k_3
\] (5)

Thus, if the uncorrelated nucleon was at rest in the initial state, in case \( a) \) the correlated nucleons \( N_2 \) and \( N_3 \) are emitted back-to-back, and in case \( b) \) \( N_2 \) is emitted with momentum \( p_2 = q - p_1 \) and \( N_3 \) with momentum \( p_3 = 0 \).

The above picture is distorted by the final state interaction. In what follows we will investigate process 1., in particular we will investigate how FSI will distort the simple picture described above.

3. The Final State Interaction in the process \( ^3He(e, e'N_2N_3)N_1 \)

The various processes, in order of increasing complexity, which contribute to the reaction \( ^3He(e, e'N_2N_3)N_1 \) are depicted in Fig. 2.

Let us introduce the following quantities:

\[
\begin{align*}
Q^2 & \rightarrow p_1 \\
p_2 & \rightarrow p_2 \\
p_3 & \rightarrow p_3
\end{align*}
\]
Figure 2. The various processes contributing to the reaction $^3$He($e,e'N_2N_3$)$N_1$: (a) will be called No FSI, (b) will be called the NN rescattering, (c) will be called the three-body rescattering. Note that in this paper, following Ref. [6], the sum of a) and b) is called The Plane Wave Impulse Approximation (PWIA), whereas in Ref. [2] PWIA is the same as our No FSI (process b)).

1. the relative momentum of the detected pair

$$t = \frac{p_2 - p_3}{2}$$

(6)

2. the Center-of-Mass momentum of the pair

$$P = p_2 + p_3$$

(7)

3. the missing momentum

$$p_m = p_1 - q = -(p_2 + p_3)$$

(8)

As already stated, we will consider the process $^3$He($e,e'p_1p_2)n$ ($^3$He($e,e'p_1n)p_2$), in which $\gamma^*$ interacts with the neutron (proton) and the two protons (proton-neutron) correlated in the initial state are emitted and detected. Let us disregard, for the time being, the interaction of the hit neutron(proton) with the emitted proton-proton (proton-neutron) pair, but take into account the final state rescattering between the two detected nucleons. Then the processes contributing to the cross section are a) and b) of Fig. 2.
By integrating the cross section (Eq. 3) over $\mathbf{P}$ and the kinetic energy of $N_1$, and taking $q \parallel z$, one obtains

$$\frac{d^8 \sigma}{d\epsilon d\Omega d\Omega_{N_1} dt d\Omega_t} = \mathcal{K} \cdot M(\mathbf{p}_m, t)$$

(9)

where $\mathbf{p}_m = -\mathbf{k}_1$, $\mathcal{K}$ incorporates all kinematical factors, and

$$M(\mathbf{p}_m, t) = M(\mathbf{k}_1, t) = \frac{1}{2} \sum_{M_3, \sigma} \sum_{s_f, \mu_f} \left| \int \exp(i \mathbf{p}_m \mathbf{\rho}) \chi_{\frac{1}{2}}(\mathbf{\rho}) \Phi_{s_f \mu_f}(\mathbf{r}) \Psi_{3M_3}(\mathbf{r}, \mathbf{\rho}) \right|^2$$

(10)

where $\chi_{\frac{1}{2}}$ represents the Pauli spinor for the hit particle, $\Phi_{s_f \mu_f}(\mathbf{r})$ is the two-nucleon wave function in the continuum, i.e.

$$\Phi_{s_f \mu_f}(\mathbf{r}) = \sum_{l, m, J} \sum_{M_J} \langle l m s_f \mu_f | J M_J \rangle Y_{l m}(\hat{\mathbf{r}}) R_{l s_f \lambda}(r) Y_{l' \lambda}(r) T_f$$

(11)

$T_f$ is the isospin function of the final pair, and the other notations are self-explaining. In the rest of the paper we will omit, for ease of presentation, all explicit summations over the quantum numbers and will denote the continuum two-nucleon wave function simply by $\Phi_{N_2 N_3}$. Equation (10) can then be cast in the following simple form

$$M(\mathbf{p}_m, t) = M(\mathbf{k}_1, t) = \left| \int \exp(i \mathbf{p}_m \mathbf{\rho}) I_{N_2 N_3}^t(\mathbf{\rho}) d^3 \mathbf{\rho} \right|^2$$

(12)

where $I_{N_2 N_3}^t(\mathbf{\rho})$ is the overlap integral between the three-nucleon ground state wave function and the two-nucleon continuum state, i.e.

$$I_{N_2 N_3}^t(\mathbf{\rho}) = \int \Phi_{N_2 N_3}^{t(-)}(\mathbf{r}) \Psi_{3M_3}(\mathbf{r}, \mathbf{\rho}) d^3 r$$

(13)

where $\mathbf{r}$ and $\mathbf{\rho}$ are usual Jacobi coordinates.

If also the NN rescattering is disregarded (No FSI approximation (process a)), one has

$$\Phi_{N_2 N_3}^{t(-)} \propto e^{i t \mathbf{r}}$$

(14)

$$I_{N_2 N_3}^t(\mathbf{\rho}) = \int e^{i t \mathbf{r}} \Psi_{3M_3}(\mathbf{r}, \mathbf{\rho}) d^3 r$$

(15)

and

$$M(\mathbf{k}_1, t) = \left| \int \exp(i \mathbf{k}_1 \mathbf{\rho}) \exp(i t \mathbf{r}) \Psi_{3M_3}(\mathbf{r}, \mathbf{\rho}) d^3 r d^3 \mathbf{\rho} \right|^2$$

(16)

which represents nothing but the three-nucleon wave function in momentum space. Thus, if FSI is fully disregarded, the process $^3\text{He}(e, e'N_2N_3)N_1$ would be directly related to the three-nucleon wave function in momentum space.

Let us now switch on the NN rescattering (process a) plus process b), i.e. the PWIA). To this end it is worth pointing out that if the overlap integral is integrated over the direction of $t$, the nucleon ($N_1$) Spectral Function is obtained, viz

$$\int M(\mathbf{k}_1, t) d\Omega_t = P_1(k_1, E^*)$$

(17)
Figure 3. The three-body channel neutron (proton) Spectral Function in $^3\text{He}$ ($^3\text{H}$) ($k \equiv k_1$). The dot-dashed line corresponds to the No FSI case, whereas the full line includes the neutron-neutron (proton-proton) rescattering. Three-nucleon wave function from [6]; Reid Soft Core interaction [7] (After Ref. [6]).

where

$$E^* = \frac{t^2}{M}$$

(18)

is the "excitation energy" of the spectator pair $N_2N_3$, which is related to the removal energy $E$ of nucleon $N_1$ by

$$E = E_3 + E^*$$

(19)

where $E_3$ is the (positive) binding energy of the three nucleon system.

Thus, by integrating Eq. 3 over $\Omega_i$, one gets

$$\frac{d^6\sigma}{dE'd\Omega'd\Omega_{N_1}dt} = K \cdot P(k_1, E^*)$$

(20)
If the Coulomb interaction is disregarded, the Spectral Function for the three-body channel is the same for both processes we are considering ($^3He(e, e'p_1p_2)n$ and $^3He(e, e'p_1n)p_2$). Therefore, in order to illustrate the effect of NN rescattering, we show in Fig. 3 the nucleon Spectral Function calculated with and without the NN rescattering [6]. It can be seen that there is a region where the FSI does not play any role. This is the so called two-nucleon correlation region, where the relation

$$E^* = \frac{t^2}{M} \simeq \frac{k_1^2}{4M}$$

holds (see e.g. Ref. [8]). The existence of such a region is a general feature of any Spectral Function, independently of the two-nucleon interaction and of the method to generate the wave function. This is illustrated in Fig. 4 where the Spectral Function corresponding to the variational wave function of Ref. [3] and the AV14 [9] interaction, is shown for several values of $k \equiv k_1$.

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**Figure 4.** The Nucleon Spectral Function as in Fig. 3 corresponding to the wave function of Ref. [3] and the AV14 interaction [9] ($k \equiv k_1$). The dashed line corresponds to the No FSI case, whereas the full line includes the neutron-neutron (proton-proton) rescattering (process b) in Fig. 3.
From the figures we have exhibited one expects that if the kinematics is properly chosen (i.e. $E^* = \frac{|t|^2}{M} \simeq \frac{k_1^2}{4M}$) the NN rescattering can be fully eliminated; on the contrary, if it is chosen improperly (in particular corresponding to an initial state characterized by $k_1 \simeq 0$), the two nucleon rescattering fully distorts the No FSI predictions.

\[ E^* = \frac{|t|^2}{M} = 50 \text{ MeV} \]

Figure 5. The quantity $M(p_m, |t|)$ (see (1)) calculated at fixed value of $E^* = 50 \text{ MeV}$, versus the angle $\theta_t$ between the relative momentum of the emitted nucleons (6) and the momentum transfer $q$. The full line includes the two nucleon rescattering and the dashed line represents the No FSI result. The three values of $k_1$ which have been chosen, correspond to three different regions of the Spectral Function (see text). Three-nucleon wave function from [3]; AV14 interaction.

Since the Spectral Function is related to the cross section, the same effects are expected to occur on the latter. This is indeed the case, as demonstrated in Fig. 5, where Eq.(12) is shown and compared with the No FSI approximation (Eq. (16)).
In this calculation, we have fixed the two-nucleon relative energy $E^* = t^2/M = 50\text{MeV}$ and have plotted, for a given values of $k_1$, the dependence of $M(p_m, t)$ upon the angle $\theta_t$ between $t$ and $q$, the latter being chosen along $k_1$. The three values of $k_1$ correspond to three relevant regions of the Spectral Function, viz

1. $E^* > \frac{k_1^2}{4M} (k_1 = 0.5\text{fm}^{-1})$;

2. $E^* \simeq \frac{k_1^2}{4M} (k_1 = 2.2\text{fm}^{-1})$, the correlation region;

3. $E^* < \frac{k_1^2}{4M} (k_1 = 3\text{fm}^{-1})$.

Figure 6. The quantity $M(p_m, t)$ (Eq. (12)) calculated in the kinematics where, in the initial state, nucleons $N_2$ and $N_3$ were correlated with momenta $k_2 = -k_3$ and $k_1 = 0$. The dashed line corresponds to the No FSI case, whereas the full line includes the two nucleon rescattering. Three-nucleon wave function from [3]; AV14 interaction.
It can be seen that in the first region the two nucleon rescattering is very large (cf. Figs. 3 and 4), whereas in the two other regions, it is very small.

We have also investigated the effect of NN rescattering on a particular kinematics, namely that which corresponds to the initial state in which $N_2$ and $N_3$ are correlated with momenta $k_2 = -k_3$ and $k_1 = 0$, so that, after $\gamma^*$ absorption, $N_1$ is emitted with momentum $q$ and $N_2$ and $N_3$ are emitted back-to-back with momenta $p_2 = -p_3$ ($p_m = 0$).

The results are presented in Fig. 6, where it can be seen that, as expected, the effect of the two-nucleon rescattering is large. We have repeated this calculation in the correlated region and found, obviously, that the rescattering, in this case, has negligible effects.

We have eventually considered the three-body rescattering, e.g. process $c$) of Fig. 2, by treating the rescattering of $N_1$ with the interacting pair $N_2N_3$ within an extended Glauber-type approach \cite{14}. The details of the calculation will be presented elsewhere \cite{11}. In Fig. 7 we show (preliminary results) the quantity (dot-dashed line)
\[ M^D(p_m, t) = \left| \int \Phi_{N_1N_2N_3}^p(r, \rho) R_{N_2N_3}^r(\rho) d^3r d^3\rho \right|^2 \]  \hspace{1cm} (22)

which is the generalization of Eq. 12 to take into account the rescattering of \( N_1 \) with the interacting pair \( N_1N_2 \), through the quantity \( \Phi_{N_1N_2N_3}^p(r, \rho) \).

In the figure, \( M^D(p_m, t) \) is plotted vs the missing momentum (\( p_m \neq k_1 \)) for a fixed value of \( t \); in the same Figure we also show the results corresponding to the case when only the NN rescattering is active (full line) and to the case when all FSI is switched off (dashed line). The kinematical variables were chosen such as to emphasize the correlation region of the Spectral Function. Moreover, we have considered high values of \(|q|\), such that the asymptotic values of those quantities which enter the calculation (e.g. the total NN cross section, the ratio of the imaginary to the real parts of the forward scattering amplitude, etc.) have been adopted. It can be seen that at high values of the missing momentum, the full FSI merely reduces to a change of the amplitude, without noticeably distorting the missing momentum distributions calculated taking into account only the NN rescattering contribution; since the latter can be reliably calculated with any realistic wave function and NN interaction, our result appears to be a very promising way to investigate the correlated part of the three-body wave function.

4. Conclusions

We have investigated the effects of the Final State Interaction in the process \( ^3He(e, e'N_2N_3)N_1 \) using realistic three nucleon wave functions which, being the exact solution of the Schrödinger equation, incorporate all types of correlations, in particular the short-range ones, generated by modern NN potentials. We have found that if the kinematics is chosen such as to emphasize the two-nucleon correlation sector of the wave function, Final State Interaction effects can be minimized.

In this paper, we have not discussed other effects competing with the Final State Interaction (Meson Exchange Currents, etc), which have to be analyzed before comparing theoretical results with experimental data.

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