Resistance Analysis of Low Reynolds number flow in porous media

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Abstract: At present, some researchers believe that Darcy's law may not be valid, which may be the root cause of complex porous media problems. Porous media flow model is generally regarded as pipe flow, but in theory, it is necessary to consider it as bypass flow. However, there are few studies in this area. According to the seepage characteristics, we established the model of flowing around a single sphere in porous media and derived the resistance correlation of Darcy velocity (Re < 1). However, the Darcy experimental data analysis shows that the predicted value of the flow around the sphere is far lower than the experimental value, which proves that the assumption of the model of flowing around a single sphere is unreasonable. Even for the seepage of the extremely low Reynolds number, the flow interference around the sphere should be taken into account. Therefore, when the interference coefficient is 45, our formulas are superior to the classical Kozeny-Carman equation and Ergun equation in the prediction of all Darcy's experimental data and Charles Ritter's experimental data. Ergun equation has the largest error.

1. Introduction
In 1856, Henry Darcy, a French engineer, studied the seepage resistance in a homogeneous sand layer in a circular pipe by using the tap water of a hospital, and obtained the universally recognized Darcy's law. Now the law has become the theoretical basis of porous media. Table 1 [1-3] is the experimental data obtained by Darcy in that year. The second and sixth columns in the table are the flow rate, the third and seventh columns are the total pressure drop measured by the experiment, and the fourth and eighth columns are the pressure drop per unit bed height measured by the experiment. The calculated parameters in the table are as follows: the inner diameter of the bed section is 0.35m; the height of the seepage bed is 0.58M in the first group, 1.14M in the second group, 1.31M in the third group and 1.70m in the fourth group; the porous medium is sand passing through 0.77mm sieve in the experiment, and the diameter of the seepage section is 35 cm.
Table 1. Darcy original data on pressure drop and discharge

| Mean Discharge (L/min) | Mean Pressure (m) | Mean Pressure (Pa/m) | Mean Discharge (L/min) | Mean Pressure (m) | Mean Pressure (Pa/m) |
|------------------------|-------------------|----------------------|------------------------|-------------------|----------------------|
| 3.60                   | 1.11              | 18755.17             | Second series          | 2.66              | 2.60                 | 22350.88            |
| 7.65                   | 2.36              | 39875.86             |                        | 4.28              | 4.70                 | 40403.51            |
| 12.00                  | 4.00              | 67586.21             |                        | 5.26              | 7.71                 | 66278.95            |
| 14.28                  | 4.90              | 82793.10             |                        | 8.60              | 10.34                | 88887.72            |
| 15.20                  | 5.02              | 84820.69             |                        | 8.90              | 10.75                | 92412.28            |
| First series           |                   |                      |                        |                   |                      |                     |
| 21.80                  | 7.63              | 128920.69            | Third series           | 10.40             | 12.34                | 106080.70           |
| 23.41                  | 8.13              | 137368.97            |                        | 2.13              | 2.57                 | 19225.95            |
| 24.50                  | 8.58              | 144972.41            |                        | 3.90              | 5.09                 | 38077.86            |
| 27.80                  | 9.86              | 166660.00            |                        | 7.25              | 9.46                 | 70769.47            |
| 29.40                  | 10.89             | 184003.45            |                        | 8.55              | 12.35                | 92389.31            |
| Fourth series          |                   |                      |                        |                   |                      |                     |
| 21.80                  | 7.63              | 128920.69            | Fourth series          | 5.25              | 6.98                 | 40237.65            |
| 23.41                  | 8.13              | 137368.97            |                        | 7.00              | 9.95                 | 57358.82            |
| 24.50                  | 8.58              | 144972.41            |                        | 10.30             | 13.93                | 80302.35            |
| 27.80                  | 9.86              | 166660.00            |                        |                   |                      |                     |
| 29.40                  | 10.89             | 184003.45            |                        |                   |                      |                     |

According to Darcy’s law, Kozeny and Carman [4] obtained Kozeny-Carman equation (Formula 1) through a large number of experimental data and theoretical research, taking into account the effects of porosity, particle equivalent diameter, specific surface area and tortuosity.

\[
\frac{\Delta P}{L} = \frac{180 \mu (1 - \varepsilon)^2}{\Phi^2 D_p^2 \varepsilon^3} v
\]

(1)

The equation is suitable for laminar flow in porous media with Reynolds number less than 1 and has been widely used in many fields. In-depth analysis of a large number of data, which is made by Robert P. Chapuis and Michel Aubertin [5] (2003), shows that Kozeny-Carman equation is correct for most soils, the problem occurs in the case of anisotropic or unsaturated porous media, or when the specific surface area of porous media is not accurately measured. To solve this problem, Hasan A. Nooruddin and M. Enamul Hossain [6] (2012) modify the tortuosity and then the equation is well used in rocks. However, Darcy’s law is based on the small Reynolds number, which is not applicable when the Reynolds number is large. Xiong Dao-kun (2004) pointed out that Darcy’s law is an experimental law and not rigorous in theory in his article “Improvement of Newton’s Law of Internal Friction and Darcy’s Law”. Xiong Daokun found that the greater the hydraulic gradient, the greater the deviation between Darcy’s law and experimental data. He believed that the force required for fluid movement was proportional to the pressure, regardless of the friction force produced by fluid movement or the destructive force of fluid deformation. Therefore, he improved Newton’s law of internal friction for porous media flow, and eventually improved Darcy’s law (Formula 2).

\[
v = k \frac{P_0}{P} J
\]

(2)

Where, J denotes hydraulic gradient.

Wan Junwei [8] (2013) analyzed Darcy experimental data and found that the permeability coefficient K of porous media was not equal to a constant and further confirmed that the seepage resistance does not obey the linear Darcy’s law through experiments. Shi Tian [9] (2017) revised Darcy’s law based on the seepage equation of soft soil mixed with combined and free water, and established a seepage model of soft soil of low permeability considering pore size distribution. Zheng Kuncan [10] and Wang Tong [11] (2017) deduced new resistance formulas from the traditional and fractal theory respectively based on the new pore throat model of staggered stacking porous media. It was found that Ergun equation and new equation had larger prediction errors under the condition of low Reynolds number flow in porous media. From this point of view, the flow mechanism of porous media is not only unclear at large Reynolds number, but also unclear at low speed. Therefore, the study of porous media
still needs to be concentrated on Darcy’s law.

2. Theoretically deriving the resistance of porous media

2.1. Establishment of Resistance Model of Flowing around

There are many porous media in nature, such as chemical fillers, coal, coke, activated carbon, sinter, soil and so on. Moreover, other porous media with solid skeleton can also be seen as the accumulation of particles of different sizes. Therefore, the flow in porous media can be seen as the flow around numerous accumulated particles respectively. Therefore, we assume that porous media are composed of spherical particles of equal diameter, and fluid flow through porous media can be regarded as the flow around these spheres separately. The model is shown in Figure 1, where the equivalent particle diameter is \( d \), and a sphere is chosen as the unit model of flowing around. When the velocity is small enough, the flow around the sphere should be close to Stokes resistance region. At the same time, it is considered that there is no interference between the particles, so the resistance flowing through porous media can be regarded as the sum of the resistance of flowing around a single spherical particle.

2.2. Deducing porous media resistance of Darcy flow rate

The formula for calculating resistance of flowing around in fluid dynamics is as shown in Equation 3.

\[
D = C_d A \frac{\rho \nu^2}{2}
\]  
(3)

Where \( D \) denotes the resistance of flow in porous media; \( C_d \) denotes the dimensionless coefficient of resistance; \( A \) denotes the projection area of the object on the plane vertical to flow; and \( \nu \) denotes the actual average velocity of flow.

The fluid resistance in Formula 3 is expressed by pressure drop as follows.

\[
\Delta P = \frac{D}{A} = \frac{C_d A \rho \nu^2}{2A} = C_d \frac{\rho \nu^2}{2}
\]

When Reynolds number is less than 1, Stokes formula 5 holds.

\[
C_d = \frac{24}{\text{Re}}
\]  
(5)
Equation 5 is substituted into Equation 4 to obtain Equation 6.

\[ C_d = \frac{24}{\rho \nu d} = \frac{24 \mu}{\rho \nu d} \]  \hspace{1cm} (6)

So

\[ \Delta P = \frac{24 \mu \rho \nu^2}{\rho \nu d} \cdot \frac{2}{d} = \frac{12 \mu \nu}{d} \]  \hspace{1cm} (7)

In this model, the average length \( L \) of fluid flow is the equivalent diameter of the sphere, and the drag loss per unit length of fluid in porous media is:

\[ \frac{\Delta P}{L} = \frac{12 \mu \nu}{d^2} \]  \hspace{1cm} (8)

The actual velocity is converted to the apparent velocity \( V \) to obtain:

\[ \frac{\Delta P}{L} = \frac{12 \mu \nu}{d^2 \phi} \]  \hspace{1cm} (9)

2.3. Verification and Correction of the Formula 9

Darcy’s experiment is greatly influenced by water hammer because it is connected with hospital tap water. In order to make up for the shortcomings of the last three groups of experiments, Charles Ritter carried out new tests on the above experimental devices on February 17 and February 18, 1856. The test data are shown in the first column of Table 2, “mean pressure”. This data has little external influence. Therefore, this set of data is first used to verify whether the flow model correctly estimates the flow resistance of porous media. The calculated value of formula 9 is shown in the column "our equation 9" in Table 2. Compared with the experimental data, the calculated value of formula 9 is much lower than the experimental value, and there is a gap of tens of times. Under the same conditions, it can be seen from Table 2 that the maximum error of Kozeny-Carman equation is only 16.2%, Ergun equation is larger, but the maximum error is only 29.8%. This shows that the traditional single sphere flow theory is not suitable for laminar flow in porous media. Therefore, although seepage flow is very slow, the interference between flows is still very large and can not be easily ignored. So we consider a disturbance coefficient \( E \), and try it again and again with the experimental value. When the final value is 45, the error between the predicted value and the experimental value is the smallest, the maximum error is only 7.1%, which is better than the prediction of Kozeny-Carman equation. In this way, we get a revised formula for calculating the drag around the flow (Formula 10).

\[ \frac{\Delta P}{L} = \frac{540 \mu \nu}{d^2 \phi} \]  \hspace{1cm} (10)

| Pressure (Pa/m) | Kozeny-Carman (Pa/m) | Ergun equation (Pa/m) | our equation 9 (Pa/m) | our equation (10) (Pa/m) |
|----------------|---------------------|----------------------|----------------------|------------------------|
| Darcy experiment | equation value | error | equation value | error | compute value | error |
| 116530.91 | 102717.86 | 0.119 | 86647.88 | 0.256 | 2572.40 | 115758.00 | 0.007 |
| 114749.09 | 99986.00 | 0.129 | 84316.24 | 0.265 | 2503.99 | 112679.33 | 0.018 |
| 112076.36 | 98346.89 | 0.123 | 82917.97 | 0.260 | 2462.94 | 110832.13 | 0.011 |
It is necessary to mention that the original text of Darcy does not clearly specify the particle diameter. It only shows that the sand is filtered through a 0.77 mm sieve. It can only be inferred that the average particle size of the particles is less than 0.77 mm. As we know, the effect of particle diameter on drag calculation is generally inverse square ratio. It can be said that different particle diameter will eventually lead to a great difference between drag calculation and experimental value. Therefore, in order to determine the average diameter of the particles, we use the widely applicable Kozeny-Carman equation and the Ergun equation to try different particle diameters until the calculated values of the two equations are very close to the experimental values. The particle diameter was finally obtained to be 0.2 mm, which was adopted in the paper.

2.4. Darcy experimental data test and further comparative analysis with classical formula

To investigate the suitability of Equation 10, we used four sets of data from the Darcy experiment for analysis. The Kozeny-Carman equation and the Ergun equation, which are currently the most widely used, are here used for comparison. The experimental values, the calculated values of the three equations, and the errors are all shown in Table 3. It can be seen from the table that the Kozeny-Carman equation has the best prediction for the first set of data of the Darcy experiment. The maximum error is 12.7%, and the Ergun equation is the worst. Except for the first two data errors of about 18%, other data of our equation 10 are in good agreement. For Darcy's second, third and fourth sets of data, the error is generally larger for all equations, but we know that the error of our equation is the smallest, followed by Kozeny-Carman equation, Ergun equation is the worst. Therefore, combining four sets of Darcy data and the data of Charles Ritter in Table 2, we can see that the adaptability of our equation is obviously better than that of the Kozeny-Carman equation and the Ergun equation. It is also confirmed once again that Ergun equation has a large deviation in predicting small Reynolds number flow.

| Mean Pressure (Pa/m) | Kozeny-Carman (Pa/m) | Ergun equation (Pa/m) | our equation (10) (Pa/m) |
|----------------------|----------------------|-----------------------|-------------------------|
| Darcy experiment value | equation value | error | equation value | error | equation value | error |
| First series | 18755.17 | 19669.38 | 0.049 | 19707.87 | 0.051 | 22166.43 | 0.182 |
| | 39875.86 | 41797.43 | 0.048 | 35004.99 | 0.122 | 47103.66 | 0.181 |
| | 67586.21 | 65564.59 | 0.030 | 55064.82 | 0.185 | 73888.09 | 0.093 |
| | 82793.10 | 78021.86 | 0.058 | 65623.83 | 0.207 | 87926.82 | 0.062 |
2.5. Verification of near Darcy flow regime

For providing a more general example we have added 2 sets of additional experimental data in Darcy regime published by Özer Bağcı [12] in the Journal of Transport in Porous Media in 2014. The experimental parameters are as follows: the inner diameter of the bed section is 51.4 mm. The diameter of the steel ball are 1.14 mm and 3.03 mm, respectively, and their corresponding porosity is 35.01% and 35.58%. The viscosity of water is 0.001 Pa.s, and the density of water is 1000 kg/m³. In the following, only the speed and pressure drop experimental data of the near Darcy regime are selected for comparison.

Figure 2 shows the compared result of the formulas with the two sets of experiment data (1.14 mm and 3.03 mm steel balls respectively). Figure 2(a) is for the steel balls of 1.14 mm of diameter and figure 2(b) 3.03 mm of diameter. In figure 2(a), Ergun equation has the maximum deviation from experimental value and Carman equation fit best with experiment. Equation (10) in the paper is in between and the average error is 15.2% with the experiment data. In figure 2(b), equation (10) is 7.5% of average error and best consistent with the experiment value. Carman equation is next and its average error is 15%. Ergun's average error is 18%. Consequently equation (10) has the better prediction to the two sets of experiment data when the Reynolds number is between 1 and 40. Combined with above Darcy’s experimental data, Equation (10) may be applied to the flow in the nature sand and artificial porous media paced with small steel balls. Its Reynolds number can extend to 40. This indicates the minor resistance model based on pore throat unit is reasonable to some extent in quality and in quantity. However, the formula is derived from the simplified hypothesis of minor resistance which is far away from the real porous media. It needs lots of experiments data and other ways to verify its Applicability in the future.

|                | 84820.69  | 83048.48  | 0.021   | 69893.22 | 0.176  | 93591.58 | 0.103 |
|----------------|-----------|-----------|---------|----------|--------|----------|-------|
|                | 128920.69 | 119109.01 | 0.076   | 100668.90| 0.219  | 134230.02| 0.041 |
|                | 137368.97 | 127905.59 | 0.069   | 108215.55| 0.212  | 144143.34| 0.049 |
|                | 144972.41 | 133861.04 | 0.077   | 113333.52| 0.218  | 150854.84| 0.041 |
|                | 166600.00 | 151891.30 | 0.088   | 128871.31| 0.226  | 171174.07| 0.027 |
|                | 184003.45 | 160633.25 | 0.127   | 136428.90| 0.259  | 181025.81| 0.016 |
|                | 22350.88  | 14533.48  | 0.350   | 12132.25 | 0.457  | 16378.53 | 0.267 |
|                | 40403.51  | 23384.70  | 0.421   | 19541.66 | 0.516  | 26353.42 | 0.348 |
|                | 66278.95  | 28739.15  | 0.566   | 24031.46 | 0.637  | 32387.61 | 0.511 |
|                | 88887.72  | 46987.96  | 0.471   | 39376.28 | 0.557  | 52953.13 | 0.404 |
|                | 92412.28  | 48627.07  | 0.474   | 40757.80 | 0.559  | 54800.33 | 0.407 |
|                | 106080.70 | 56822.65  | 0.464   | 47673.42 | 0.551  | 64036.34 | 0.396 |
|                | 19225.95  | 11637.71  | 0.395   | 9711.57  | 0.495  | 13115.14 | 0.318 |
|                | 38077.86  | 21308.49  | 0.440   | 17802.25 | 0.532  | 24013.63 | 0.369 |
|                | 70769.47  | 39611.94  | 0.440   | 33166.05 | 0.531  | 44640.72 | 0.369 |
|                | 92389.31  | 46714.77  | 0.494   | 39146.08 | 0.576  | 52645.26 | 0.430 |
|                | 40237.65  | 28684.51  | 0.287   | 23985.61 | 0.404  | 32326.04 | 0.197 |
|                | 57358.82  | 38246.01  | 0.333   | 32017.20 | 0.442  | 43101.38 | 0.249 |
|                | 80302.35  | 56276.27  | 0.299   | 47211.97 | 0.412  | 63420.61 | 0.210 |

Second series

Third series

Fourth series
3. Conclusion

(1) Studies show that the model assumption of flowing around a single sphere is unreasonable even at Darcy flow rate (Re < 1). It reveals that porous media seepage must take into account the interference of the flow inter-sphere in all cases.

(2) A new correlation of seepage resistance under Darcy velocity in porous media is deduced theoretically, and the interference coefficient is determined to be 45. For all the data of Darcy and Charles Ritter, the adaptability of our formula is obviously better than that of Kozeny-Carman equation and Ergun equation, which proves once again that Ergun equation has a large deviation in predicting small Reynolds number flow. The formula is as follows.

\[ \frac{\Delta P}{l} = \frac{540 \mu \nu}{d^2 \phi} \]

Acknowledgements
Project supported by National Natural Science Foundation of China (Grant No.1.51764046, Grant No.2.51768054, Grant No.3.51866013), Inner Mongolia University teaching reform fund (Gran No. 0302051601) and Natural Science Foundation of Inner Mongolia (Grant No.2018MS05035).

Conflict of Interest: The authors declare that they have no conflict of interest.

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