Compact parallel-plate waveguide half-Luneburg geodesic lens in the Ka-band

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Abstract

A parallel-plate waveguide half-Luneburg geodesic lens is designed and experimentally validated in the Ka-band. The geodesic lens profile is modulated using spline functions to reduce its height, while the symmetry halving the lens enables reduction of its in-plane dimensions. This design provides high gain, equivalent to that of a full-Luneburg lens, over a reduced angular range of $\pm 20^\circ$ to $\pm 30^\circ$ in azimuth. The specific design reported here has a maximum realized gain of 23.3 dBi in measurement, compared with 23.8 dBi for a full-Luneburg lens twice the size of this design. Very wideband operation is demonstrated both in simulations and measurements. This design is of interest for applications having limited space like millimetre-wave systems on board cubesats and small satellites.

1 | INTRODUCTION

In recent years, there has been a regain of interest for line-source antennas fed by planar lens beamformers, such as Luneburg lenses [1], Rotman lenses [2] and pillbox antennas [3]. While early applications were mostly radar and surveillance systems, current developments mostly target communication systems in the millimetre-wave range, benefiting from the wideband operation of such beamforming techniques to provide higher capacity and wider service areas. There have been developments in 5G terrestrial communication systems [4–7] as well as ground terminals for communication satellites [8,9]. Similar developments were reported for automotive radars in the sub-millimetre-wave range [10–12]. Quasi-optical beamformers are also of interest for space applications. Due to the harsh environment and extreme temperatures, fully metallic solutions are usually preferred and a few designs were reported in the literature [13–15], including some promising evolutions of the Rinehart–Luneburg lens [16,17] proposed by the authors in Ref. [18–22] as an extension of previous works on conformal Luneburg lenses [23,24]. Although it has been demonstrated that the angular range of a Rotman lens may be extended by an adequate design of its focal curve [25], it usually does not exceed $\pm 50^\circ$. Luneburg-lens-based antenna solutions may cover an extended angular range (up to $\pm 75^\circ$) but at the expense of a larger beamforming device, as its inherent rotational symmetry leads to a beamformer length equal to the projected line-source aperture, corresponding to the lens diameter. Note that all solutions discussed above have a beamforming length typically equal or slightly smaller than the aperture size. Implementations of hemispherical Luneburg lenses are reported for applications with a reduced scanning range (typically around $\pm 30^\circ$) [26–29]. They benefit from the use of a ground plane cutting a spherical Luneburg lens in two, thus reducing significantly the size of the antenna while maintaining its focusing properties albeit over a reduced angular range. To the best of the authors’ knowledge, such a design has never been reported in the case of a parallel-plate waveguide (PPW) geodesic lens implementation. A PPW half-Luneburg lens design based on metallic posts to synthesize the graded index has been reported recently [30], with only three beams covering a limited field of view ($\pm 15^\circ$).

A thorough discussion of PPW half-Luneburg geodesic lenses is provided, supported by a design and experimental validation in the Ka-band using the water drop geodesic lens concept introduced by the authors in Ref. [18–22]. This is the first time that such a thorough discussion of a half-Luneburg geodesic lens is reported, including a detailed comparison with its full-Luneburg lens counterpart.
2 | THE HALF-LUNEBURG LENS

The refractive index profile \( n(r) \) of a planar rotationally symmetric Luneburg lens (see Figure 1a) is defined as follows in the cylindrical coordinate system \((r, \phi, z)\) with its origin \(O\) set at the centre of the lens [1]:

\[
n(r) = \begin{cases} 
\sqrt{2-r^2}, & r \leq 1 \\
1, & r > 1 
\end{cases}
\]

where \(r\) is the normalized radius of the lens.

As illustrated in Figure 1b, rays are mirrored using the \(x\)-axis as the symmetry axis (red line) in a half-Luneburg lens, thus reducing the size of the beamformer while maintaining its focusing properties albeit over a reduced angular range. Assuming that the feed angular position \(\phi_f\) may vary from \(90^\circ\) to \(180^\circ\), the corresponding plane wave will be formed in the angular direction \(\phi_w\) ranging from \(90^\circ\) to \(0^\circ\), respectively. In the case illustrated in Figure 1b, the feed position is located at \(135^\circ\), resulting in a plane wave propagating in the angular direction at \(45^\circ\) with respect to the \(x\)-axis. More generally, any axis passing through the centre of the lens can be an axis of symmetry due to the rotational symmetry of the lens. Assuming such an axis makes an angle \(\phi_\perp\) with respect to the \(x\)-axis, the angular direction of the plane wave produced by a feed on the surface of the lens and at the angular position \(\phi_f\) is \(\phi_w = \pi - \phi_f + 2\phi_\perp\). In the following, we will assume \(\phi_\perp = 0\) without loss of generality.

This half-lens geometry leads to a compact beamformer design, but also provides some undesired effects. For angular positions of the feed near \(90^\circ\), the electric field is reflected back towards the feed, resulting in degraded return losses for those ports. For angular positions of the feed near \(180^\circ\), a significant amount of the radiated field will not intercept the symmetry axis, resulting in degraded lens performance and spillover in the symmetric angular direction of the desired beam. The ray-tracing tool based on Snell–Descartes law of refraction, described in [22], is adapted here to assess the performance of the lens versus the feed position. The angular position of the feed \(\phi_f\) is introduced as a design parameter. The rotationally symmetric inhomogeneous lens is approximated by concentric layers of uniform dielectric material with an infinitesimal radial thickness. This is illustrated with a reduced number of layers in Figure 2. The symmetric spillover beam produced by the rays which are not reflected is clearly visible in that representation (rays in solid lines pointing downward). Starting from the feed position, each ray travels towards the centre of the lens, until it reaches an inflection point where the direction of propagation is orthogonal to the local radial direction. From that point onwards, the ray travels away from the centre of the lens, towards the periphery. As the ray travels through the lens, the phase delay is evaluated and used in an equivalent point source circular array for the evaluation of the radiation patterns, following an approach based on the Huygens–Fresnel principle. To halve the lens with \(\phi_\perp = 0\), we maintain the phase delays computed with the full-lens model and modify the circular array by mirroring all elements with respect to the \(x\)-axis, hence abscissas are unchanged while ordinates are replaced by their absolute values. This seemingly simplistic approach is found to provide an excellent evaluation of the focusing properties of the lenses under consideration.

![Figure 1](image1.png)  

**Figure 1**  (a) Design parameters of the rotationally symmetric Luneburg lens and (b) ray-tracing in a half-Luneburg lens

![Figure 2](image2.png)  

**Figure 2** Illustration of ray-tracing in a discretized half-Luneburg lens (solid lines) compared to ray-tracing in the equivalent full Luneburg lens (dashed lines)
The Luneburg lens configuration in [22] is used here for a fair comparison with the full-lens counterpart. The computations are performed at 30 GHz. The lens has a diameter of 150 mm, while the feeds are assumed to be H-plane rectangular horns with an aperture in the H-plane of 8.64 mm. The radiating aperture of the lens is assumed to be from 0° to 100°, meaning that rays falling outside of this angular range are not considered in the evaluation of the patterns with the circular array approximation. In a real multiple-beam lens implementation, those rays would either couple to other ports, produce some return loss or be reflected again within the cavity, producing multiple reflections. These secondary effects are neglected in the ray-tracing model. Radiation patterns are evaluated for feed positions varying from 172.5° to 105° with feeds located every 7.5°, corresponding to beams pointing at angles ranging from 7.5° to 75° with respect to the x-axis. The actual directivity of the beams cannot be evaluated with the ray-tracing tool because the patterns are only computed in the beam-forming plane, but a relative comparison is possible thus indicating achievable scanning performance with such a design. All patterns are normalized to the beam with the highest gain, which is the one pointing at 45°. The numerical results for the ideal half-Luneburg lens are reported in Figure 3a, confirming the undesired effects described above. In particular, the symmetric lobe of the main beam with respect to the x-axis, resulting from spillover, is well visible for the beams with the lowest azimuth angle. Its peak value reduces as spillover reduces too for larger azimuth angles. Considering that the beamformer has been halved, the reported results remain attractive. Within the angular range from 15° to 75°, scan losses are lower than 1.7 dB. For the beam at 7.5°, scan losses increase to 2.7 dB. This is mostly due to the large amount of energy in the symmetric beam at −7.5°. Part of this energy lost by spillover may be recovered by extending the symmetry plane but at the expense of increased size of the beamformer along the x-axis. From simple geometrical considerations, one can expect scan losses due to the reduction in projected aperture to be in the order of 3 dB at 0° and 90°. Hence, the values obtained with our ray-tracing model are in line with expectations. Similar results are reported for dielectric lens designs using in-plane transformation optics to reduce the length of the beamformer [30,31]. Section 3 discusses the design and experimental validation of a modulated half-Luneburg geodesic lens in the Ka-band.

3 | MODULATED PPW HALF-LUNEBURG GEODESIC LENS IN THE KA-BAND

3.1 | Design considerations

The modulated geodesic lens described in [22] is used here as the starting point for the analyses. The lens profile is defined using spline functions, enabling reduction of the height of the lens by a factor 4 when compared to the reference geodesic lens. Using the ray-tracing model described in the previous section, the radiation patterns are evaluated and reported in Figure 3b for comparison with the reference half-Luneburg lens. A slight increase in scan losses is observed. Within the angular range from 15° to 75°, scan losses are lower than 2.8 dB, while a scan loss of 3.7 dB is obtained at 7.5°. This is a degradation of about 1 dB when compared to the ideal case. One can also note a degradation of about 2–3 dB in side-lobe levels. Considering the significant size reduction of the beamformer, these are still considered good results.

The lens design was also analysed using the frequency domain solver of ANSYS HFSS [32]. The full-wave model of the lens is illustrated in Figure 4. The model includes coaxial-to-waveguide transitions for testing purposes, which were optimized as a stand-alone component. A picture of the two parts forming the prototype is provided in Figure 5. The flare horn has a length of 25 mm and an aperture height of 15 mm. It is important to note that the side of the flare horn along the
x-axis is closed with metal, thus extending slightly the ground plane of the half-Luneburg geodesic lens along the x-axis (see inset in Figure 5). This is expected to have a positive effect on beams with the lowest beam-pointing angles. Electric field distributions provided by the full-wave model at 24 and 30 GHz for three different feeding ports are reported in Figure 6. They clearly illustrate the wideband operation of the lens, as well as the interference patterns within the PPW section, resulting from the axis symmetry. The increase in port-to-port coupling is clearly visible for feeds closer to the y-axis.

### 3.2 Experimental validation

Some selected S-parameters, representative of the overall performance, are reported in Figure 7, both simulations and measurements. The nominal operation frequency band considered in this work ranges from 27.5 to 31 GHz (highlighted with a solid black line in Figure 7), while all results are investigated from 24 to 34 GHz. The worst results appear to be on return loss, indicating that a combined optimization of the lens and the coaxial-to-waveguide transition is required for applications extending all the way up to 75° in pointing direction (corresponding to port 1). From port 3 onwards, all return loss values are better than 14 dB over a very wide frequency bandwidth. Port-to-port coupling is also better than −14 dB over the nominal operation band. In measurement, an overall degradation of the performance is observed. Yet, the wideband operation remains, with most S-parameters (from port 3 onwards) being lower than −10 dB over most of the investigated frequency band. Overall, a reasonably good agreement is found between simulations and measurements, indicating that a prototype manufactured with higher precision is reasonably expected to provide better correlation. This degradation was not observed in the case of the full lens reported in [22], although the same manufacturing technique was used. This seems to indicate that the manufacturing of the
Symmetry plane is quite critical, and alternative manufacturing techniques may be considered to ensure good alignment between the two half plates in that part of the lens, such as 3D printing with a monolithic approach. A comparison between simulation and measurement results is also provided for what concerns radiation patterns. Realized gain patterns at 30 GHz are reported in Figure 8 for all the feeding ports. Realized gain is used here to combine in the same plot information about all S-parameters and their impact on the actual energy radiated. The antenna prototype was measured in a far-field set-up in the anechoic chamber of the Division of Electromagnetic Engineering at KTH. All unused ports were connected with 50-Ω loads. Overall, a good agreement is found between simulated and measured patterns. The maximum directivity is 23.5 and 23.3 dBi (beams pointing around 45°) in simulations and measurements, respectively. Simulated realized gains at 15° and 75° are 21.1 and 21.5 dB, respectively, indicating scan losses in the range of 2.4 dB. Interestingly, scan losses are reduced to 1.7 dB when considering directivity only, in line with the ray-tracing model presented in Section 2.1. In the measurements, the realized gains at 15° and 75° are 21.6 and 20.4 dB, respectively, indicating some imbalance in the scan losses with a value of 1.7 dB towards smaller angles and 2.9 dB towards larger angles. The realized gain at 7.5° is 20.2 and 21.2 dB for simulation and measurement, respectively. To verify the impact of misalignment between the two halves of this prototype, analyses were performed assuming a displacement of 0.2 mm along the y-axis. The resulting realized gain patterns are reported in Figure 9. This displacement seems to be responsible for the degradation of the side lobes for the most scanned beams, with simulated values well in line with measurement data. These results also confirm the scan loss
degradation in that angular range. Interestingly, this displacement seems to also be the explanation for the slight gain degradation at 45°, visible both in simulated and measured data. However, this displacement does not justify the discrepancy in scan losses for lower values of $\phi$. This may result from other manufacturing errors and misalignment. Measurement errors may also contribute in part to the discrepancy, considering that the beams are quite wide in the plane orthogonal to the lens, which often results in some gain ripples in measurement.

As a reference, the maximum realized gain of the full lens reported in [22] is 23.8 dBi at 30 GHz. With a measured maximum realized gain of 23.3 dBi, this half-Luneburg lens confirms its potential for applications with reduced scanning range and requiring compact designs without compromising significantly the performance. Note that a similar increase in the side lobes was reported in [22] for the case of the full lens manufactured with the same modulated profile and same manufacturing technique, confirming that this solution requires a good alignment between the two halves or a monolithic manufacturing technique. We provide measured realized gain over frequency for selected beams in Figure 10. These results confirm the good stability of focusing properties over a broad frequency band within the angular range $\pm 15^\circ$, and still reasonably acceptable performance up to $\pm 30^\circ$. A comparison of the realized gain over frequency for various lens designs is provided in Figure 11, with an indication of the corresponding aperture efficiency and including simulated and measured data.

The performance of the full-Luneburg lenses is evaluated for the on-axis beam, while the performance of the half-Luneburg lenses is evaluated for the beam at 45°. The reference aperture to evaluate the aperture efficiency is defined as a rectangular aperture having as width the lens diameter and the same height as the actual flare aperture. This corresponds to the projected aperture of the full-Luneburg lens, also taken as the reference aperture in [22]. Results of the reference and modulated full-

![Figure 9](image9.png)

**Figure 9** Simulated realized gain patterns of the modulated half-Luneburg geodesic lens at 30 GHz in the beamforming plane assuming a displacement of 0.2 mm between the two halves.

![Figure 10](image10.png)

**Figure 10** Measured realized gain patterns of the modulated half-Luneburg geodesic lens over frequency in the beamforming plane.

![Figure 11](image11.png)

**Figure 11** Compared realized gain performance of full- and half-Luneburg geodesic lenses over frequency and associated aperture efficiency (simulated results in solid lines and measured results in dashed lines).

Luneburg lens are reported from [22] for comparison, as well as the measured data of the modulated full-Luneburg lens. These results are compared to the realized gain of the half-Luneburg lens described here. They confirm that the half-Luneburg lens performs quite well over a wide frequency range, with an aperture efficiency around 80% in simulation and ranging from 70% to 75% in measurements. There are some oscillations over frequency in the measured data, stronger than those anticipated with simulation and which were also observed with the full-Luneburg lens design. These are likely due to the test set-up and measurement uncertainties, as they are quite similar for both lenses. Interestingly, we note that the gap between the full- and half-Luneburg lenses reduces as the frequency increases. This could be expected as spillover effects...
reduce at higher frequencies, minimizing the undesired effects resulting from halving the lens.

4 | CONCLUSIONS

A PPW half-Luneburg geodesic lens has been described and validated experimentally for the first time. Using a modulated spline profile, the design lens is also compact in height. This multiple-beam lens antenna provides gain values similar to a full lens albeit over a reduced angular range. Stable performance was demonstrated over an angular range of about ±20°. The operation of the lens may be extended to ±30° at the expense of increased scan losses. The symmetric lobes appearing at lower azimuth angles may be mitigated by extending the symmetry ground plane. For some applications, this dimension may be less critical than the length of the beamformer, which is halved with this design.

The ray-tracing tool developed to analyse modulated geodesic lenses has been adapted to evaluate the scanning performance of half-Luneburg lenses. Although it does provide a good indication of scan losses, further improvements are needed to better estimate the scan losses and side-lobe levels. Once upgraded, this tool may be used to perform a direct optimization of half-geodesic lenses to mitigate partly scan losses while maintaining its compact design.

The results reported also highlighted the importance of good alignment between the two halves of the mechanical design. Future works will investigate the feasibility of a monolithic implementation, using alternative manufacturing techniques such as additive layer manufacturing and metallized moulded plastics.

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REFERENCES
1. Luneburg, R.K.: Mathematical Theory of Optics. Brown University Press, Providence, RI (1944)
2. Rotman, W., Turner, R.F.: Wide-angle microwave lens for line source applications. IEEE Trans. Antenn. Propag. AP-11(6), 623–632 (1953)
3. Rotman, W.: Wide-angle scanning with microwave double-layer pillboxes. IRE Trans. Antenn. Propag. 6(1), 96–105 (1958)
4. Foglia Manzillo, F., Ettorre, M., Lahti, M.S., et al.: A multilayer LTCC solution for integrating 5G access point antenna modules. IEEE Trans. Microw. Theor. Tech. 64(7), 2272–2283 (2016)
5. Hong, W., Jiang, Z.H., Yu, C., et al.: Multibeam antenna technologies for 5G wireless communications. IEEE Trans. Antenn. Propag. 65(12), 6231–6249 (2017)
6. Quevedo-Teruel, O., Miao, J., Mattsson, M., et al.: Glide-symmetric fully-metallic Luneburg lens for 5G communications at Ka-band. IEEE Antenn. Wirel. Propag. Lett. 17(9), 1588–1592 (2018)
7. Chou, H-T., Yan, Z-D.: Parallel-plate Luneburg lens antenna for broadband multibeam radiation at millimeter-wave frequencies with design optimization. IEEE Trans. Antenn. Propag. 66(11), 5794–5804 (2018)
8. Gatti, R.V., Maraccioli, L., Sbarra, E., et al.: Flat array antenna for Ku-band mobile satellite terminals. In: Proceedings of the 5th European Conference on Antennas and Propagation (EuCAP), pp. 2618–2622. Rome (2011)
9. Ettorre, M., Foglia Manzillo, F., Casaletti, M., et al.: A compact and high-gain Ka-band multibeam continuous transverse stub antenna. In: 2015 IEEE International Symposium on Antennas and Propagation & USNC/URSI National Radio Science Meeting, pp. 671–672. IEEE, Vancouver (2015)
10. Xue, L., Fusco, V.F.: 24 GHz automotive radar planar Luneburg lens. IET Microw. Antenn. Propag. 13(3), 624–628 (2017)
11. Ettorre, M., Sauleau, R., Le Coq, L., et al.: Single-folded leaky-wave antennas for automotive radars at 77 GHz. IEEE Antenn. Wirel. Propag. Lett. 9, 859–862 (2010)
12. Saleem, M.K., Vettikaladi, H., Alkanhal, M.A.S., et al.: Lens antenna for wide angle beam scanning at 79 GHz for automotive short range radar applications. IEEE Trans. Antenn. Propag. 65(4), 2041–2046 (2017)
13. Legay, H., Tubau, S., Girard, E., et al.: Multiple beam antenna based on a parallel plate waveguide continuous delay lens beamformer. In: International Symposium on Antennas and Propagation (ISAP), pp. 118–119. Okinawa (2016)
14. Diao, C.D., Girard, E., Legay, H., et al.: All-metal Ku-band Luneburg lens antenna based on variable parallel plate spacing Fakir bed. In: 11th European Conference on Antennas and Propagation (EUCAP), pp. 1401–1404. Paris (2017)
15. Doucet, F., Fonseca, N.J.G., Girard, E., et al.: Analytical model and study of continuous parallel plate waveguide lens-like multiple beam antennas. IEEE Trans. Antenn. Propag. 66(9), 4426–4436 (2018)
16. Rinehart, R.F.: A solution of the problem of rapid scanning for radar antennas. J. Appl. Phys. 19(9), 860–862 (1948)
17. Rinehart, R.F.: A family of designs for rapid scanning radar antennas. Proc. IRE. 40, 686–688 (1952)
18. Liao, Q., Fonseca, N.J.G., Quevedo-Teruel, O.: Compact multibeam fully-metallic geodesic Luneburg lens antenna based on non-Euclidian transformation optics. IEEE Trans. Antenn. Propag. 66(12), 7383–7388 (2018)
19. Fonseca, N.J.G., Quevedo-Teruel, O.: Compact parallel plate waveguide geodesic lens for line sources with wide scanning range. In: 38th ESA Antenna Workshop, pp. 1–7. Noordwijk (2017)
20. Fonseca, N.J.G., Liao, Q., Quevedo-Teruel, O.: The water drop lens: a modulated geodesic lens antenna based on parallel curves. In: International Symposium on Antennas and Propagation (ISAP), pp. 1–2. Busan (2018)
21. Fonseca, N.J.G., Quevedo-Teruel, O.: The water drop lens: a low-profile geodesic parallel plate waveguide lens antenna for space applications. In: 13th European Conference on Antennas and Propagation (EuCAP). Krakow (2019)
22. Fonseca, N.J.G., Liao, Q., Quevedo-Teruel, O.: Equivalent planar lens ray-tracing model to design modulated geodesic lenses using non-Euclidian transformation optics. IEEE Trans. Antenn. Propag. 68(5), 3410–3422 (2020)
23. Mitchell–Thomas, R.C., Quevedo-Teruel, O., McManus, T.M., et al.: Lenses on curved surfaces. Opt. Lett. 39(12), 3551–3554 (2014)
24. Horsley, S.A.R., Hooper, I.R., Mitchell–Thomas, R.C., et al.: Removing singular refractive indices with sculpted surfaces. Sci. Rep. 4, 4876 (2014)
25. Fonseca, N.J.G.: A focal curve design method for Rotman lenses with wider angular scanning range. IEEE Antenn. Wirel. Propag. Lett. 16, 54–57 (2017)
26. Sanford, J.A: A Luneburg lens update. IEEE Antenn. Propag. Mag. 37(1), 76–79 (1995)
27. Cook, W.G.: Global Connectivity to Aerospace Forces via Satcom, Air Force Research Lab Report (1999). https://apps.dtic.mil/sti/citations/ADA458568
28. Thornton, J.: Wide-scanning multi-layer hemisphere lens antenna for Ka-band. IEE Proc. Microw. Antenn. Propag. 153(6), 573–578 (2006)
29. Wiley, A.R., Nikolic, N.: Dual-polarized planar feed for low-profile hemispherical Luneburg lens antennas. IEEE Trans. Antenn. Propag. 60(1), 402–407 (2012)
30. Lu, H., Liu, Z., Liu, Y., et al.: Compact air-filled Luneburg lens antennas based on almost-parallel plate waveguide loaded with equal-sized metallic posts. IEEE Trans. Antenn. Propag. 67(11), 6829–6838 (2019)
31. Mateo-Segura, C., Dyke, A., Dyke, H., et al.: Flat Luneburg lens via transformation optics for directive antenna applications. IEEE Trans. Antenn. Propag. 62(4), 1945–1953 (2014)
32. Su, Y., Chen, Z.N.: A flat dual-polarized transformation-optics beamscanning Luneburg lens antenna using PCB-stacked gradient index metamaterials. IEEE Trans. Antenn. Propag. 66(10), 5088–5097 (2018)
33. https://www.ansys.com/products/electronics/ansys-hfss

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