Earth Movement Optimization Model in Urbanistic Projects

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Abstract-- Urban areas comprising natural terrain are prepared empirically for the urbanistic development of cities, causing alteration to the natural relief and affecting notably the physical surroundings as result of deficient planning in the preparation of the terrain for urbanism works. This study seeks to formulate a rational solution to earth movements through two models, one having to do with simulation and the other being mathematical in contrast with the current empirical method applied in numerous works of engineering, where the budget has an important specific weight. In response to this, modeling of surfaces emerges, which describes the process of physical and artificial representation of the skin of the earth through a geometric model. This type of model permits designing the modification of a terrain’s topography to prepare it optimally for a given purpose, considering two fundamental variables: area and volume. As result of this research, an optimization model was obtained for the design of artificial earth movements applicable to the type of topography prevalent in the cities in the Colombian Eje Cafetero (coffee growing region). The study found that the variables of area, volume, cut and fill heights create a model which determines the optimal point and around such an uncertainty zone called design zone, achieving the greatest useful area in the construction with the least volume of earth to move.

Keywords: earth movement, simulation, cut heights, geometric model, modification of surfaces

1. Introduction

The empirical method used currently in modifying earth surfaces to implement urbanistic projects is based on measuring and representing on the terrain plan (topographic map), tracing the project over it, with the confrontation between data on the terrain’s elevation and that projected. Finally, cut and fill volumes are calculated and therein two questions emerge, what is the rationality of this process and how to optimize it. To answer these questions, several referents are proposed, among them those related with data capture, interpolation algorithms, interpretation of topographical information, the initial conditions for their application, graphic design of the project, and the calculation processes.

(Epps & Corey, 1990) developed procedures to calculate cut and fill volumes through cross sections using the average area method. Precision in calculating earth movements is a problem in many engineering applications: erosion studies, mining activities, and estimation of the material removed in construction projects (El-Nokrashy, Ragab, & Kamal, 2011). Design methods of artificial earth movements for purposes of topographic adaptation in urbanistic projects seek to study the alternatives of terrain preparation that respond to the project’s demands, permitting to select the one with the least environmental impact with lowest risk of generating natural disasters (Schofield, 2007), (Shawki, Kilani, & Gomaa, 2015)
Currently, part of this knowledge is widely used in Colombia to conserve the engineering tradition under the practicist concept of being faster and more economic, in keeping with the immediacy culture of our projects. According to (Liao, Petrou, & Zhao, 2008), worldwide there is a strong tendency to use information systems as a means to optimize resource through simulation processes. These define solution alternatives establishing their costs and finding an interval that is economically rational in terms of benefit; in this sense, diverse software applications have come into market that are based on calculating areas and volumes, producing alternatives that follow the trial and error method to find a solution that aims to be optimal. Anyhow, it is not the rational method with which we may model the behavior of data of areas and volumes established based on controllable variables, like cut and fill heights and relate it to the economic costs from differential alternatives and through which it would be viable to select the optimal (Chandra, 2005).

This study seeks to solve the problem of land imbalance between the cut and fill produced in urbanistic projects. To face this situation, Fractal Geometry and the Fractional Brownian Motion (FBM) were used to formulate a mathematical model, which uses simulation of topographic surfaces through the Newton Raphson solution. The resulting model was a precise and comfortable way of conducting designs of earth volumes in works of engineering. Its application is especially indicated in terrains with irregular topographic shapes.

2. Modeling of earth surfaces

Undeniably, engineering has generated a great deal of changes in nature, many producing big benefits for the community, while others are considered unsustainable from the environmental point of view. In the academy, classic geometry courses always coincide on the overwhelming leap from the plane onto space (from two to three dimensions). In geometric terms, this has been the Euclidean constant through its deductive axiomatic method—without discussing everything Euclid means to engineering—his geometry moves away from the forms of nature, it is there that a geometric approach becomes necessary with a broader vision through the observation of complex phenomena that make up our reality at the local level. What do the forms of the Earth’s surface have in common with a broccoli? The answer is that these are self-similar forms, if each of these forms is broadened a bit, it can be noted that each part belongs to the whole, that is, fractal forms that propagate at progressively reduced scales. If one manages to understand one of the parts, one will be able to understand the whole.

Most of the physical systems in nature and many human artefacts do not respond to regular geometric forms from Euclidean Geometry. Fractal geometry offers forms with almost unlimited description and measurement of these shapes and chaotic behaviors.

A fractal shape is a natural phenomenon or a mathematical set that exhibits a repetitive pattern shown at different scales. The fundamental concept of fractal geometry is associated with its dimension, which is an indicator of how much it occupies the space containing it (González & Guerrero, 2001), and may take continuous values in the space of real numbers between 0 and 3. A simplified explanation is given in Figure 1.

![Figure 1. Relationship between scale (r) and the number of elements (N)](image)
The concept of fractional dimension was introduced with fractals, which means that these have dimensions not represented with a whole number where each fractal has its own dimension. According to (Falconer, 1990), fractals have the following properties: (1) they have detail at any scale, (2) cannot be described by Euclidean geometry, and (3) have some type of self-similarity. Fractal geometry differs from Euclidean geometry in that it is a relatively new area in mathematical education, developed by Mandelbrot in the 1970s, and has grown considerably during the last 20 years. Its appearance provides a new scientific way of thinking on the nature of the world. Fractal geometry is determined by the Hurst Exponent \(0 \leq H \leq 1\).

The model to generate the fractal terrain involves the normal distribution and is based on the random displacement from the midpoint and the Fourier synthesis. The midpoint of a line segment moves randomly in y direction at a given vertical distance using recursive subdivision.

\[
x_{\text{new}} = \frac{1}{2}(x_i + x_{i+1}) \quad (1)
\]

\[
y_{\text{new}} = \frac{1}{2}(y_i + y_{i+1}) + P(x_{i+1} - x_i) \cdot R(x_{\text{new}}) \quad (2)
\]

Where \(P\) is given in function of the line length and \(R\) is a random number between 0 and 1 selected based on \(x_{\text{new}}\).

The FBM generates terrains through an initial square subdivided into four smaller ones.

\[
[x_0, y_0, f(x_0, y_0)], [x_0, y_1, f(x_1, y_0)], [x_0, y_1, f(x_0, y_0)], [x_1, y_1, f(x_1, y_1)]
\]

(3)

Then, a vertex is added in the middle denoted by \([x_{1/2}, y_{1/2}, f(x_{1/2}, y_{1/2})]\) where,

\[
X_{1/2} = \frac{1}{2}(x_0 + x_1)y_{1/2} \quad (4)
\]

\[
X_{1/2} = \frac{1}{2}(y_0 + y_1)f(x_{1/2}, y_{1/2}) \quad (5)
\]

\[
X_{1/2} = \frac{1}{4}(f(x_0 + y_0) + f(x_1, y_0) + f(x_0, y_1) + f(x_1, y_1)) \quad (6)
\]

The vertex added is changed in direction of the z coordinate by a random value denoted by \(\sigma_z\); this procedure is recursive and repeated for each sub-square, in the MFB random numbers, \(\sigma_z\), must be generated under normal distribution with \(\mu = 0\) and \(\sigma = 1\).

According to (Wood, 1996), in the generation of fractal surfaces, it is necessary to use the Box Müller algorithm to create a pair of random variables under normalized distribution.

\[
R = \sqrt{-2 \ln U_1}
\]

(7)
\[ \theta = 2\pi U_2 \]  
\[ X = R \sin \theta \]  
\[ Y = R \cos \theta \]

Where, \( U_1 \) and \( U_2 \) are two random numbers of uniform distribution.

Now is necessary the iteration that varies the position of \( X \) and \( Y \) and which is modified according to:

\[ \sigma_i^2 = \frac{1}{2^{2H(i+1)}} \sigma^2 \]  

Where \( H \) denotes the Hurst exponent \([0 \leq H \leq 1]\), also known as roughness exponent.

From equation 11, it is deduced that the first iteration has greater influence on the shape resulting from the surface than the other iterations. The method assigns random heights to the corners of the square, then calculates the heights of the intermediate points of each side through linear interpolation, and adds or subtracts a random value to this height. Thereafter, the square is shifted 45° and we proceed in the way already described, when the new point only has three neighbors, its median is calculated. It is again shifted 45°, proceeding in recursive iterated manner.

Finally, according to, the fractal dimension \( D \), of a topographic surface is obtained through:

\[ D = 3 - H \]

3. Materials and Methods

The study area is in the central western part of the Colombian Andean region, covering the departments of Risaralda, Caldas, and Quindío, with latitudes from 04°01′N to 05°43′N and longitudes from 75°55′W to 74°37′W, covering nearly 13,800 Km². This zone is characterized for having elevations ranging between 155 and 5200 meters above mean sea level, with vast hydric wealth, besides housing xerophytic vegetation and flora- and fauna-rich ecosystems. In 2011, UNESCO declared the zone World Heritage because of its Coffee Cultural Landscape. The geomorphology of the Colombian Andean region has infinite forms in zones of different sizes, one of the most recurrent is characterized by wavy topography, with a convex upper segment and a lower concave segment.

A. Methodological approach

To systematize knowledge and the research process, a deductive method was established by implementing an experimental model made up of three phases, as indicated in Figure 2: simulation of earth surfaces, statistical modeling, and mathematical modeling.
By using equation 13, described in (Mendenhall, Beaver, & Beaver, 2010), sensitivity was determined through simple random sampling, taking the variables that possibly explain the phenomenon: fractal dimension, slope, entropy, and standard deviation, considered the most representative in this study. The slope was used as descriptor of the earth surface, given that it is a variable sensitive to the change of the topographical characteristics in the region, as inferred in Figure 3.

\[ n = \frac{Z^2 \cdot S^2}{d^2} \]  

(13)

Where \( n \) is the number of samples, \( Z \) is the confidence level, \( S \) is the variance, and \( d \) is the expected error in the sampling.

B. Simulation of earth surfaces

Considering that indicated by (Goodchild & Mark, 1987), earth topographic surfaces are in a fractal dimension interval \([2.1 \leq DF \leq 2.4]\), that is, from a flat vision of the terrain to a completely mountainous vision and that the \( H \) exponent by Hurst for these fractal ranges is between 0.6 and 0.9, the midpoint displacement algorithm was developed to simulate the shapes of 724 fractal mountains through digital elevation models.
C. Statistical modeling

With variables from the simulation, data were normalized with the form \( \frac{X}{\Delta x} \) to avoid problems associated with their units and dimensions. A Pearson correlation analysis was performed to define the intensity between variables. Thereafter, a multiple regression analysis was conducted permitting the selection of predictors (stepwise) where a dependent variable, \( \hat{Y} \), is related with other independent variables \( X_1, X_2, X_3, \ldots, X_n \) through a linear equation in the form:

\[
\hat{Y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots
\]  \hspace{1cm} (14)

Where \( \beta_0, \beta_1, \text{ and } \beta_2 \) are the regression coefficients.

The stepwise regression procedure is a statistical technique of variable selection based on the successive adjustment of models through input or output of predictors. The process starts with a model that detects a predictor so that if it is included in the model, it causes a maximum reduction of error. This set of operations is repeated until it is not possible to include any predictor, or until, within a given level of the process, any variable previously incorporated does not fulfill the requisites to remain in the model, which will lead to its elimination.

D. Mathematical modeling

As takes place in the professional practice to calculate volumes, Figure 5 projects cut planes (dotted lines on the vertical plane) from the vertex to the base, at intervals \( h(h_1, h_2, \ldots, h_n) \) projecting the section of areas \( a(a_1, a_2, \ldots, a_n) \) on the horizontal plane.

Figure 4. Midpoint displacement
This surface representation in parabolic form of the convex segment of the wavy topography is representative of the concave form and that of the topography in the study zone.

Based on the statistical results, the prediction was formulated of a response variable from another prediction variable, where their relation was modeled as an $n$-order polynomial function, seeking to optimize the volume in earth movements within a context that protects the environment, producing the least amount of volume and generating the largest amount of useful area according to the needs of the project and surface conditions.

$$h(x) = p(x) = a_4 \cdot x^3 + a_3 \cdot x^2 + a_2 \cdot x + a_1 + a_0$$

where $h_i, a_i$ are the variables defining the model.

### 4. Results

Upon calculating the sensitivity of the sample through equation 13, a size of 724 topographic units was obtained. Backed by principles of fractal geometry, FBM, and midpoint displacement algorithms, an application was designed in C++ to randomly produce the simulation of the 724 surfaces, using equations from 1 to 11 and producing terrains with different fractal dimensions, as those indicated in Figures 6 and 7.

![Figure 5: Level and profile curves of a dome](image)

**Figure 5.** Level and profile curves of a dome

![Figure 6: Topographic surfaces F.D.: 2.1, 2.2](image)

**Figure 6.** Topographic surfaces F.D.: 2.1, 2.2
Thereafter, the variables derived from the models were normalized — variation coefficient, standard deviation, fractal dimension, slope, roughness, orientation, entropy, cut height, volume, area, and variance — and the Pearson correlation analysis was performed, as referred to in Table 1, where those with the highest correlation were volume, area, and height.

### Table 1. Pearson correlation among variables

|       | VOLU  | AREA | AS | COEFF. | F. | DS | ENTR | MEAN | SLOPE | VOL./A | VARIA |
|-------|-------|------|----|--------|----|----|------|------|-------|--------|-------|
| VOLU  | 1.000 | 0.8  | 0.0 | -0.527 | -  | 0.2 | 0.110| 0.88 | 0.01  | 0.869  | 0.256 |
| ME    | 0.885 | 0.05 | 0.0 | -0.704 | 0.0| 25 | 0.019| 0.5  | 0.02  | 0.588  | 0.027 |
| AREA  | 0.006 | 1.0  | 0.0 | -0.028 | 56 | 0.0| -0.011| 0.91 | -0.05 | -0.005 | -0.032|
| ASP   | -0.527| 0.09 | 1.0 | 0.0   | 1.0| 21 | 0.300| 2.0  | 0.02  | -0.170 | 0.516 |
| COEFF. | -0.056| 0.0  | 1.0 | 0.073 | 0.0| - | 0.005| 0.02 | 3.0   | -0.040 | 0.042 |
| VAR.  | 0.255 | 0.0  | 0.0 | 0.534 | 37 | 0.0| 0.638| 4.0  | -0.0  | 0.533  | 0.990 |
| F.D.  | 0.110 | 0.0  | 0.0 | 0.300 | 0.0| 26 | 1.000| -    | 0.04  | 0.263  | 0.625 |
| DS    | 0.880 | 0.7  | 0.0 | -0.808 | 12 | 0.5| 0.000| 0.80 | 6.0   | 0.628  | 0.004 |
| ENTRO | 0.015 | 0.28 | 0.0 | 0.168 | 0.0| 34 | 0.136| 8.0  | 0.16  | 0.085  | 0.228 |
| PY    | 0.869 | 0.0  | 0.0 | -0.170 | 73 | 0.0| 0.263| -    | 8.0   | 1.000  | 0.535 |
| MEAN  | 0.256 | 0.0  | 12  | 0.516 | 1.0| 41 | 0.625| 0.05 | 0.19  | 0.535  | 1.000 |
| SLOPE | 37    | -    | 1.0 | -     | 0.0| 1.0| -    | 7.0  | 5.0   | -      | -      |
| VOL./A| 0.0   | 0.0  | 0.0 | 0.0   | 0.0| 0.0| -    | 0.27 | 2.0   | -      | 0.004 |
| REA   | 21    | 26   | 0.0 | 0.6   | 41 | 0.0| 0.01 | 2.0  | 0.13  | 0.00   | 6.0    |
| VARIA | 0.0   | 0.0  | 0.0 | 3.0   | 0.0| 38 | -    | 0.0  | 0.00  | 0.00   | 0.00   |
| NCE   | 0.9   | 0.0  | 11  | 0.0   | 0.0| 13 | -    | 0.0  | 0.00  | 0.00   | 0.00   |
|       | 12    | 0.0  | 0.0 | 0.0   | 57 | 0.2| -    | 1.00 | 0.02  | -      | -      |
|       | -     | 24   | 0.0 | 0.0   | 0.0| 33 | -    | 5.0  | 0.08  | 0.00   | 0.00   |
|       | 8     | 0.0  | 0.0 | 0.0   | 0.0| 33 | -    | 5.0  | 0.08  | 0.00   | 0.00   |
|       | 0.0   | 25   | 0.0 | 0.0   | 0.0| 33 | -    | 5.0  | 0.08  | 0.00   | 0.00   |
|       | 0.0   | 0.0  | 0.0 | 0.0   | 0.0| 33 | -    | 5.0  | 0.08  | 0.00   | 0.00   |
|       | 27    | 0.0  | 0.0 | 0.0   | 0.0| 33 | -    | 5.0  | 0.08  | 0.00   | 0.00   |
|       | 0.0   | 32   | 0.0 | 0.0   | 0.0| 33 | -    | 5.0  | 0.08  | 0.00   | 0.00   |

Vertical cuts were made every 25 cm $h(h_1, h_2, ..., h_n)$ of all the terrains simulated, as indicated in Figure 5, obtaining their respective areas $a(a_1, a_2, ..., a_n)$ and volumes $v(v_1, v_2, ..., v_n)$. With these
vectors — height, area, volume — and supported on equation 15, two third-order polynomials were formed: $h_a$ related to cut height/area, and $h_v$ cut height/volume.

$$h_a(x) = p(x) = a_4 \cdot x^3 + a_3 \cdot x^3 + a_2 \cdot x^2 + a_1 \cdot x + a_0$$  \hspace{1cm} (16)

$$h_v(x) = p(x) = a_4 \cdot x^3 + a_3 \cdot x^3 + a_2 \cdot x^2 + a_1 \cdot x + a_0$$  \hspace{1cm} (17)

With data of areas and volumes, sensitivity analysis was performed according to the type of regression. The result was a polynomial; to determine its degree, the stepwise statistical analysis was conducted, obtaining that the polynomial that best explained the data was the third degree polynomial.

Subsequently, both polynomials were solved to obtain the solution $p(x) = h_a(x) - h_v(x)$, this polynomial had its roots calculated through Newton’s method, which was quite rapid and efficient because the convergence was of quadratic type (the number of significant figures is duplicated in each iteration). However, the convergence depends largely on the form that the function adopts near the iteration point. The roots of the polynomial solution $x_1, x_2, x_3, x_4, x_5$ were obtained; imaginary roots were discarded, along with those found beyond the study range, leaving two solutions: one whose value tends to zero and another with a higher value, discarding the one tending to zero because the area and volume are very small and do not correspond to a rational design. Thereby, a single solution was obtained corresponding to the optimal cut height, $h_0$. This value was replaced in the polynomials of equations 16 and 17 and the optimal design area and volume were obtained.

![Figure 8. Optimal height for design](image)

It was deemed convenient to express the estimation precision under a given probability, which meant providing the confidence limits within which its true value fluctuates; these were calculated based on the normal distribution. The standard deviation ($\sigma$) fixed the limits within which the measurements should be expected 68.27% of the times. The confidence coefficient and interval was 95% and these were given by the following expression:

$$\text{Confidence limits} = \bar{x} \pm \left( \frac{2\sigma}{\sqrt{n}} \right)$$  \hspace{1cm} (18)

Upon obtaining these limits, they were replaced in the polynomials of equations 16 and 17, finding the design range for area and volume.
5. Discussion
This article introduces a methodology to construct a rational model, in the design of earth movement, using fractal geometry as fundamental principle for stochastic simulation through the midpoint algorithm, satisfying the assumptions required to analyze spatial data.

According to (Baek & Seo, 2011), earth movements in urbanism processes use up 20% of the total cost of the project, which broadly justifies these studies; coincides in that the problem depends fundamentally on diminishing volumes in cut and/or fill, (Ji, Seipp, Borrmann, Ruzika, & Rank, 2010) proposed a mathematical model to optimize these movements through linear programming techniques in a roadway project, making efficient assignments to the heights in the change of state, from cut to fill. This coincides well with the results in this work, demonstrating that the area and its relationship with the height has a significant linear association with the volume.

Through the third order polynomial regression, the adjustment models were defined, which fixed the equilibrium criterion establishing the adequate ratio between the variables of area and volume in their intersection point. With this, uncertainty zones were set, offering theoretical elements for rational design in an earth movement whose object is exclusively to cut or fill.

6. Conclusions
From the results shown, their analysis, and discussion, it may be stated that a mathematical model has been formulated coherent with the rationality in the use of the earth as natural resource. This demands knowledge in Earth Science, modern information tools, and modeling systems, as requisiteto establish the technical-scientific foundations in the elaboration of a rational design of earth movement, finding a point of equilibrium where the largest area possible is obtained to urbanize with the least impact of artificial earth movement.

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