Loading Capacity Prediction of Rubberized Reinforced Concrete Continuous Deep Beams

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Abstract. Results of fourteen two-span continuous concrete deep beams were presented and discussed in this research. One sample was designed as a reference to be an ordinary reinforced concrete beam (ORC) while thirteen samples were made of rubberized concrete (RRC). The investigated parameters were the shear span/overall depth ratio (a/h) and the rubber replacement ratio with the fine and coarse aggregate. The volumetric replacement ratios were 0, 5%, 10%, 15%, and 20% of the fine aggregate while the coarse aggregate content was kept constant. The same replacement ratios were used for the coarse aggregate. Two shear span/overall depth ratios were used of 1.66 and 1.33. All beams showed the main crack formatted diagonally between the intermediate support and the applied load, indicating a shear failure. The excluded results were compared with those calculated by the strut and tie methods (STM) of the American and Euro codes. It was found that the predicted results of the American and European standards were conservative with mean values of experimental/calculated results of 1.29 and 1.24, respectively. Additionally, the effectiveness factors of the STM suggested by different researchers have been examined. It was found that those approaches were incapable to reflect the effects of the size and amount of rubber used in this study. Therefore, another effective factor was recommended to meet the requirements investigated in the current study. A respectable convergence between the predicted and the experimental results was found in which the mean and coefficient of variation values were 1.004 and 1.158%, respectively.

Keywords: Plasticity theory; Continuous deep beams; Effectiveness factor; Strut-and-tie model; Rubberized concrete.

1. Introduction

One of the important members in the field of constructions is the reinforced concrete deep beams (RCDB) that can be utilized to dispense the use of columns in some positions. Considerable attention has been given to RCDBs as a result of its wide applicability in various structural applications. Reinforced concrete deep beams are used in many structural applications such as, folded plates, transport girders and piles caps.

On the other hand, many scientists and researchers are in search of developing alternative construction materials that are environmentally friendly and contribute towards sustainable development. Rubber waste tires were chosen due to the huge amount produced daily which creates many environmental problems of waste disposal resulting from it.

Many previous researchers show that tire-rubbered concrete (Rubcrete) has a higher ductility and a lower density relative to ordinary concrete. Thus, waste tire rubber is an elastic material having less specific gravity and more energy absorption that can be used as aggregates partial alternative material to obtain rubberized concrete. The drop in the hardened properties of concrete (compressive and tensile strength values) and the difficulty of the prediction load capacity of members made from rubberized
concrete are considered as the main difficulties in using this type of rubcrete mixes. Therefore, if these unwanted properties are controlled, we will obtain a new composite of cementitious rubber materials with better ductility which have useful usability in concrete structures.

In practice fields (design and construction), the use of reinforced concrete continuous deep beams (RCCDB) is more than simply supported reinforced concrete deep beams. However, many researchers studied the simply supported deep beams behavior more than RCCDB. The shear capacity of both continuous and simply support deep beams increased due to decreasing shear-span/depth ratio. But the behavior of RCCDB act in a different way compared to both continuous support shallow beam and simply supported deep beam.

Various design codes such as ACI 318M-19 and EC2-2004 proposed the use of the strut and tie method (STM) to design structural parts that had discontinuity spots such as RCCDBs. The basic concept of the STM depends on the plasticity theory (equilibrium or balance theory and yielding states). According to many codes, the disconnection regions (also known as regions D) are the beam’s regions with length equal or less than twice of beam depth and lies between supports and applied loads, as shown in Figure 1. Elasticity concept applicable before a crack is formed in deep beams. However, after cracks formed, deep beams showed a non-linear stress behavior that was difficult to accurately describe through elastic analysis [1]. Therefore, various methods were established to analyze the shear capacity of the deep beams. ACI Building and Euro code suggested that deep beams must be designed using either STM or using a non-linear analysis that takes the effect of non-linear strain distribution [2,3].

The STM model has been defined in various design codes as a truss model containing struts and ties that meet concentrated load in a specific joint called node zone. Struts transferred the compression force, while ties transferred the tension one. The tension tie consists of the main tension reinforcing bars bounded by part of the concrete. The forces calculation in struts and ties can be done by applying equilibrium theories with the applied loads at the nodes. Ashour and Morley [4] showed that the difference amongst the various proposed STMs in design codes is only the effectiveness factor that was introduced to overcome the disadvantages of using plasticity theory to analyses the behaviors of reinforced concrete and to take into account the limitations of concrete ductility. Yang et al. [5] used the STM based on the (ACI 318-05) recommendation for predicting the load capacity of RCCDBs. They conclude that analytical results according to ACI 318-05 were conservative for RCCDBs with a/d ratio greater than 1.0 and/or with a lower amount of web reinforcement and longitudinal reinforcement. The RCCDBs made of hybrid concrete with normal strength concrete (NSC) and high strength concrete (HSC) those fail in shear can be designed according to (ACI-318C-14) because the mechanical behavior and design of deep beams are governed by shear and its load-carrying capacity depends on the strength of compressive strut that joins the loading and reaction points (Tied-Arch Action) [6,7].

Khatab et al. [8] showed that the shear provisions of the ACI 318M-11 [9] reasonably predicted the shear resistance of normal and self-compact RCCDB, however, the STM models suggested by different design codes showed conservative results for all continuous deep beams. and there is underestimation behavior in the collected results for predicted load capacities of the specimens tested.
2. Research significance

In this research, the experimental results of the RCCDBs, ultimate failure load, will be compared with analytical prediction results from those calculated by applying the American and Euro codes recommendations for utilizing strut and tie methods (STMs) to calculate the loading capacity of RCCDBs. The ability and effectiveness of loading prediction for rubberized RCCDBs by using American and European codes in addition to other previous studies will be checked and evaluated against the experimental results.

3. Experimental Data

The full details of materials used in the experimental works, concrete mixtures, and the results of the hardened properties are fully explained in Kadhim and Al-Mutairee [10,11]. Figure 2 shows the overall geometrical dimensions and reinforcement details for all fourteen specimens. All beams had a total length of 2200 mm, height \( h = 300 \) mm, and width \( b = 100 \) mm. The investigation parameters were \( a/h \), amount, and types of rubber used. Crumb rubber was used to replace the fine aggregate in four different amount by volume 5%, 10%, 15%, and 20% in four samples at one load condition where \( a/h \) equals to 1.67, chip rubber was used to replace coarse aggregate in four different proportion by volume 5%, 10%, 15%, and 20% in eight samples at two load condition (\( a/h \) equals to 1.67 and 1.33). All the test specimens hadn’t horizontal or vertical shear reinforcement. Only one bar closed stirrup 6 mm diameter at each support has been provided to fix the longitudinal reinforcements. The ultimate failure load and compressive strength of each specimen are tabulated in Table 1.

![Figure 1. Description of discontinuity regions in deep beams: a- shear span (\( a < 2h \)); b- shear span (\( a = 2h \)), [1].](image-url)
Figure 2. Geometrical details and Steel Reinforcement of Specimens (all units in mm) [10].

Table 1. Failure and cracking load for test specimens

| items | Specimens code | $f'c'$ at test (MPa) | Chip rubber content as a volumetric replacement with coarse aggregate (%) | Crumb rubber content as a volumetric replacement with fine aggregate (%) | Failure load ($P_t$) kN | $a/h$ |
|-------|----------------|------------------------|------------------------------------------------------------------------|------------------------------------------------------------------------|-----------------------------|------|
|       |                |                         | G1                                                                     |                                                                        |                             |      |
| B1    | G1N            | 38.11                   | 0                                                                     | 0                                                                      | 340                         | 1.67 |
| B2    | G1CA5          | 32.24                   | 5                                                                     | 0                                                                      | 335                         | 1.67 |
| B3    | G1CA10         | 29.75                   | 10                                                                    | 0                                                                      | 310                         | 1.67 |
| B4    | G1CA15         | 25.17                   | 15                                                                    | 0                                                                      | 264                         | 1.67 |
| B5    | G1CA20         | 22.09                   | 20                                                                    | 0                                                                      | 231                         | 1.67 |
| B6    | G1FA5          | 31.00                   | 0                                                                     | 5                                                                      | 324                         | 1.67 |
| B7    | G1FA10         | 29.60                   | 0                                                                     | 10                                                                     | 310                         | 1.67 |
| B8    | G1FA15         | 25.00                   | 0                                                                     | 15                                                                     | 262                         | 1.67 |
| B9    | G1FA20         | 21.90                   | 0                                                                     | 20                                                                     | 229                         | 1.67 |
|       |                |                         | G2                                                                     |                                                                        |                             |      |
| B10   | G2N            | 38.70                   | 0                                                                     | 0                                                                      | 351                         | 1.33 |
| B11   | G2CA5          | 32.56                   | 5                                                                     | 0                                                                      | 346                         | 1.33 |
| B12   | G2CA10         | 30.05                   | 10                                                                    | 0                                                                      | 320                         | 1.33 |
| B13   | G2CA15         | 25.43                   | 15                                                                    | 0                                                                      | 273                         | 1.33 |
| B14   | G2CA20         | 22.10                   | 0                                                                     | 20                                                                     | 236                         | 1.33 |

4. STM for continuous reinforced concrete deep beams

The basic concept of the STM depends on the plasticity theory (lower bound theorem of plasticity), equilibrium or balance theory, and yielding states [12]. The proposed STM analyses discontinuous region in members like a truss where tension and compression forces are transported by ties and struts, respectively, as explained in Figure 3 (a). Strut and tie elements are connected at the nodes as shown in Figure 3 (b and c). Struts, ties, and nodes must be properly proportioned to resist the applied forces [1]. Using typical steps for truss analysis (joints methods) and by knowing the external forces, the internal forces in the ties and struts can be determined. The reactions at external support $VE$ and reactions in internal middle support $VI$ represent the shear strengths of the continuous deep beams and they can be calculated as follows [13][14]:

$$
F_E = v f'c' b W_{ES} \tag{1}
$$

$$
V_E = F_E \sin \theta \tag{2}
$$

$$
F_I = v f'c' b W_{IS} \tag{3}
$$

$$
V_I = F_I \sin \theta \tag{4}
$$

$$
\theta = \tan^{-1} ((h-c-c')/a) \tag{5}
$$

where ( $F_I$ and $F_E$ ) are interior and exterior struts loading capacities respectively, ($v$) represents the...
effectiveness factor of concrete, $b$ (beam width), $WIS$ and $WES$ represent the widths of interior and exterior the concrete compressive struts, respectively, the symbol $\theta$ is the angle between the exterior strut and the tie and must be not more than 25-degree according to ACI recommendations, ($c^*$) and ($c$) are the depth of concrete covers for the upper and lower main tension reinforcements, respectively, ($h$) represented the total depth of the member section and ($a$) is the distance between loading point and the support (shear span).

![Figure 3. STM for RCCDBs: a- RCCDB truss model in STM, b- Loading point joint, c- Intermediate support joint.](image-url)

The failure load calculated by using an average of lower and upper strut widths as follows [8]:

$$W_{ES} = \frac{(W_{ES} + W_{ES})}{2}$$

$$W_{ES} = \frac{(W_{ES}) + (W_{ES})}{2}$$

where ($W_{ES}$) and ($W_{ES}$) are the upper and lower exterior strut widths, respectively, ($W_{ES}$) and ($W_{ES}$) are the upper and lower interior strut width, respectively. The effective width of the strut depended on supporting plate length, tie depth, and $\theta$ as shown in the following equations.

$$W_{ES} = 0.5 \times LLP \sin \theta + Wtn \cos \theta$$

$$W_{ES} = LEP \sin \theta + Wb \cos \theta$$

$$W_{ES} = 0.5 \times LLP \sin \theta + Wtn \cos \theta$$

$$W_{ES} = 0.5 \times LIP \sin \theta + Wb \cos \theta$$

where (LEP) is the length of bearing plate at exterior support (100mm), (LIP) is the interior support bearing plate length (100mm), (LLP) is the bearing plate at loading points length (100mm), $Wtn$ is the upper tie depth which equal to $2c^*$, $Wb$ is the lower tie depth which equal to $2c$, $Pt$ is the total calculated load.

$$Pt = 2 \left( Vc + Vs \right)$$

where $VE$ and $VI$ are taken from Eqs.2 and 4, respectively.
5. Evaluation of STMs in ACI 318-19 and EC2-2004 Codes

By applying the STM described above for continuous deep beam according to the suggestion of ACI 318-19 and EC2-2004 codes, all steps of the solution were the same except the variance in effectiveness factor (v), it’s can be chosen from Table 2. To determine total shear capacity by ACI 318-19, Eq. 1 and 3 are multiplied 0.85 (strength reduction factor) to reduce the effect of sustained compression in concrete strength. Table 2. shows the effectiveness factors for STMs regarding ACI 318-19 and EC2-2004 codes. The effect of the ACI effectiveness factor depends basically on the existence of shear reinforcement (in the strut having bottle-shaped) or not, but doesn’t take the results of concrete strength and the ratio longitudinal reinforcement. However, the European code effectiveness factor takes account of concrete compressive strength value instant of the influences of web and longitudinal reinforcements. Figure 4 and Table 3 show the results of total loading capacities calculated by depending on the American and European codes and show the underestimation behavior in the collected results for predicted load capacities of all specimens.

The obtained values of the mean (m) and a coefficient of variation (COV) for ( Pt experimental / Pt calculated) ratio were 1.24, 4% and 1.29, 5.4% for ACI-318-08 and EC2-2004, respectively. The underestimation of the STM of ACI 318-19 was confirmed by Fouad B. et al [15] their conclusion depends on the evaluation of the results between the obtained experimental one and those predicted by ACI-318-08 code.

Results of Table 3 shows that both American, European codes STM’s effectiveness factors did not exactly reflect the effect of RCCDB essential parameters, especially rubberized deep beams so that the STM’s effectiveness factor needs to be improved.

| Codes name          | Effectiveness factor for struts (v) | Condition                                      | ACI 318-19 | (EC2-2004) EN 1992-1-1 |
|---------------------|------------------------------------|------------------------------------------------|------------|------------------------|
|                     | =0.85 (State of Bottle-shaped only) | Reinforcement satisfy the clues ACI 318-19     | =0.75      | =0.6 * (1 – \( \frac{\sigma_r}{250} \)) |
|                     |                                    | Reinforcement does not satisfy ACI 318-19      | =0.60      |                        |

Figure 4. Evaluation of STMs in EC-2004 and ACI codes for all specimens
Table 3. Difference between experimental and calculated results of failure load by using codes formula and the performance of proposed effectiveness factors

| Specimens | $f_{c'}$ | $P_{c, Euro}$ | $P_{c, ACI}$ | $P_{c, Exp}$ | $\frac{P_{c, Exp}}{P_{c, ACI}}$ | $\frac{P_{c, Exp}}{P_{c, Euro}}$ |
|-----------|--------|----------------|----------------|----------------|-------------------------------|-------------------------------|
| B1        | 38.11  | 302.11         | 302.97         | 340            | 1.122                         | 1.125                         |
| B2        | 32.24  | 262.66         | 256.30         | 335            | 1.307                         | 1.275                         |
| B3        | 29.75  | 245.14         | 236.52         | 310            | 1.311                         | 1.265                         |
| B4        | 25.17  | 211.71         | 200.13         | 264            | 1.319                         | 1.247                         |
| B5        | 22.09  | 188.35         | 175.61         | 231            | 1.315                         | 1.226                         |
| B6        | 31.00  | 253.99         | 246.45         | 324            | 1.315                         | 1.276                         |
| B7        | 29.60  | 244.07         | 235.32         | 310            | 1.317                         | 1.270                         |
| B8        | 25.00  | 210.44         | 198.75         | 262            | 1.318                         | 1.245                         |
| B9        | 21.90  | 186.89         | 174.11         | 229            | 1.315                         | 1.225                         |
| B10       | 38.70  | 309.11         | 310.86         | 351            | 1.129                         | 1.136                         |
| B11       | 32.56  | 267.62         | 261.56         | 346            | 1.323                         | 1.293                         |
| B12       | 30.05  | 249.84         | 241.38         | 320            | 1.326                         | 1.281                         |
| B13       | 25.43  | 215.87         | 204.24         | 273            | 1.336                         | 1.265                         |
| B14       | 22.10  | 190.39         | 177.52         | 236            | 1.329                         | 1.240                         |

Mean: 219.64, SD%: 7.076, COV %: 5.478

6. Effectiveness factors collected from the literature

A lot of research studies have been suggested various effectiveness factors ($\nu$) formulations as shown briefly in Table 4. Some of these factors, like those proposed by Rogowsky [15] and Marti [16] were taken as constant values. However, other effectiveness factors, such as those proposed by Nielsen [17] and Bergmeister et al. [18], depending on the main parameters that control the behavior of the RCCDBs, they took account of concrete compressive strength value as the main influences factor. Warwick and Foster suggested another formula which considers the influence of concrete compression resistance and the member cross-section of in term of $\alpha/d$ ratio.

Table 4. A list of main collected effectiveness factors was used with the STM from references.

| Item | Reference | ($\nu$) Values |
|------|-----------|----------------|
| 1    | Rogowsky and MacGregor [15] | $\nu = 0.85$ |
| 2    | Marti [16] | $\nu = 0.6$ |
| 3    | Nielsen [17] | $\nu = 0.8 - (f_{c'}/200)$ |
| 4    | Bergmister et al. [18] | $\nu = (0.5 + 1.25 \sqrt{f_{c'}}) \ (f_{c'}) \ must \ be \ [20<f_{c'}<80]$ |
| 5    | Warwick and Foster [19] | $\nu = 1.25 - (f_{c'}/50) - 0.72 (f_{c'}) + 0.18 (f_{c'})^2 \ [\leq 1.0]$ |

All Compressive strength values ($f_{c'}$) are in MPa

The efficacy factors ($\nu$) obtained from Table 4 were used with the STM previously mentioned, namely equations 1 and 3, for estimating the overall load capacity of specimens in the current research. The performance of the collected effectiveness factors was evaluated in terms of the (m) and (COV) for (Pt. Experimental / Pt. Calculated) in Table 5. Among all researches, Nielsen’s equation for effectiveness factor gave more sensible predictions based on the values of the mean (m), the standard of deviation (SD), and the coefficient of variations (COV) those equal to 1.004, 3.740%, and 1.158%, respectively. Figure 5 shows the accuracy in predicted results by using Nielsen’s and Marti’s effectiveness factors. However, it is required more accurate effectiveness factor for two-span rubberized RCCDMs, that reflects directly the influence of rubber content in concrete in term of partial volumetric replacement with fine or coarse aggregate, with a better performance in values of (m), (SD), and (COV).
Table 5. Values of the mean, SD, and (P_t Experimental/ P_t Calculated) ratio by using the past studies effectiveness factors.

| Item | Reference                  | P_t Experimental/ P_t Calculated |
|------|----------------------------|----------------------------------|
| 1    | Rogowsky and MacGregor [15]  | Mean: 0.775                      |
|      |                            | SD %: 4.245                      |
|      |                            | COV %: 5.478                     |
| 2    | Marti [16]                 | Mean: 1.098                      |
|      |                            | SD %: 6.014                      |
|      |                            | COV %: 4.271                     |
| 3    | Nielsen [17]               | Mean: 1.004                      |
|      |                            | SD %: 3.740                      |
|      |                            | COV %: 1.158                     |
| 4    | Bergmister et al. [18]     | Mean: 0.895                      |
|      |                            | SD %: 3.824                      |
|      |                            | COV %: 3.724                     |
| 5    | Warwick and Foster [19]    | Mean: 0.659                      |
|      |                            | SD %: 3.609                      |
|      |                            | COV %: 0.027                     |

7. Proposed effectiveness factor for the STM

To take the effect of the brittle behavior of the concrete structure and to reduce the inaccurate due to the assumption of concrete is a rigid perfectly plastic material, the effectiveness factor, $\eta$, is applied [20]. Based on the experimental results of this study, two formulas were used to get the effectiveness factor for rubberized RCCDBs to reach extra correct predictions convergences with the experimental one. As already stated, the value of the efficiency factor depends mainly on the material properties. Therefore, the new formulas have been stated with terms of rubber content in the concrete mix as a percentage of volumetric replacement with fine or coarse aggregate as well as concrete compressive strength. The new present effectiveness factor was modified that recommended by Nielsen and Eurocode by adding the influences of replacement rubber with aggregates. The effects of concrete compressive strength and the rubber replacement percentage were considered in the final effectiveness factor formula as shown in Eq.13.

$$\eta = 0.6 \left(1 - \frac{f_{c}'}{750}\right)[1 + (RCP \times 10^{-9})^{0.1}]$$ (13)

Where $f_{c}'$ represented concrete compressive strength (MPa), RCP represented Rubber Content Percentage in concrete as a term of volumetric replacement with a fine or coarse aggregate range from (0-20) in the current study.
The same steps of the solution presented earlier are used to compute the Pt with the proposed efficacy factor Eq.13. Table 6 and Figure 6 showed the accuracy for estimating overall load capacity using the new proposed efficacy factor.

| Specimens | $f_{c'}$ Experimental | $f_{c'}$ Theoretical by using proposed effectiveness factor | $P_{t^{Exp}}/P_{t^{Cal}}$ |
|-----------|-----------------------|-----------------------------------------------------------|-----------------------------|
| B1        | 38.11                 | 338.33                                                    | 1.122                       |
| B2        | 32.24                 | 331.25                                                    | 1.307                       |
| B3        | 29.75                 | 309.57                                                    | 1.111                       |
| B4        | 25.17                 | 265.07                                                    | 1.319                       |
| B5        | 22.09                 | 234.59                                                    | 1.135                       |
| B6        | 31.00                 | 319.06                                                    | 1.131                       |
| B7        | 29.60                 | 308.07                                                    | 1.137                       |
| B8        | 25.00                 | 263.34                                                    | 1.131                       |
| B9        | 21.90                 | 232.63                                                    | 1.135                       |
| B10       | 38.70                 | 346.85                                                    | 1.129                       |
| B11       | 32.56                 | 337.86                                                    | 1.132                       |
| B12       | 30.05                 | 315.80                                                    | 1.126                       |
| B13       | 25.43                 | 270.49                                                    | 1.136                       |
| B14       | 22.10                 | 237.12                                                    | 1.129                       |
| Mean      |                       | 346.85                                                    | 1.129                       |
| SD%       |                       | 7.076                                                     | 0.995                       |
| CV %      |                       | 5.478                                                     | 1.158                       |

**Figure 6.** Evaluation of predicted results accuracy using the efficacy factors EC-2004, ACI codes, and proposed by this study.
The proposed $v$ in the current study was used to achieve reasonable predictions for the test samples rather than those indicated in the previous studies and code provisions. The mean and coefficient of the variance values are 1,004 to 1,158% respectively for $(Pt_{\text{experimental}} / Pt_{\text{calculated}})$ by new effectiveness factor, therefore the proposed effectiveness factor is more accurate to use with the calculation of load prediction capacity for ordinary and rubberized RCCDBs

8. Conclusions

According to the present analytical results of RCCDBs, the following conclusions can be drawn:

- The effectiveness factors identified by the ACI and EC-2 codes and those indicated by previous studies cannot represent the influence of rubber content in concrete in terms of replacement with fine or coarse aggregate, the new $(v)$ proposed for STM can be adopted by designers in the analysis of RCCDBs.
- Both American (ACI 318-19) and European (EC2-2004) codes give underestimated shear capacity.
- Comparing the practical results with the results calculated from the American (ACI 318-19) and European (EC2-2004) codes, it appears that the European code ostensibly gives results closer to reality.
- The positive features of the suggested effectiveness factor formula giving regard to taking the rubber material impact in concrete in term of volumetric replacement with fine or coarse aggregate and gave accurate predictions compare with American (ACI 318-19), European (EC2-2004) codes and the previous investigations formulas, with (m) and (COV) for the ratio $(Pt_{\text{experimental}} / Pt_{\text{calculated}})$ 1.004 and 1.158%, respectively. However, it's recommended for future studies to work on more related experimental tests to validate and recalibrate (if needed) the proposed effectiveness factor.
- The addition of the rubber in the concrete mix of RCCDBs causes an increase in the effectiveness factor values.

9. References

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10. Notations

| Symbol | Description                                      |
|--------|--------------------------------------------------|
| a      | Shear span:                                      |
| a/d    | Shear span-to-depth ratio                        |
| a/h    | Shear span-to-overall depth ratio                |
| b      | Beam width                                       |
| c      | Concrete cover of the bottom longitudinal reinforcements |
| c'     | Concrete cover of the top longitudinal reinforcements |
| COV    | Coefficient of variation                         |
| d      | The effective depth of the concrete section      |
| f_c    | Concrete compressive strength                    |
| F_E    | Load capacity of the exterior concrete struts     |
| f_r    | Modulus of Rupture                               |
| f_t    | Concrete tensile strength                        |
| f_y    | Yield strength of steel reinforcement             |
| h      | Overall beam depth                               |
| L      | Centre to the center of the beam span            |
| L_EP   | Width of the exterior support bearing plate      |
| L_IP   | Width of the interior support bearing plate       |
| L_LP   | Width of the loading plate                       |
| m      | Mean value                                       |
| P_t    | Total failure load                               |
| RCCDMs | Reinforced concrete continuous deep beams        |
| RCP    | Rubber content percentage                        |
| SD     | Standard of deviation value                      |
| STM    | Strut and tie method                             |
| v      | Strut and tie method effectiveness factor        |
| V_E    | External shear strength                          |
| V_I    | Internal shear strength of one span              |
| W_{bn} | Effective width of the bottom tie                |
| W_{ES} | Average width of the exterior compression struts  |
| (W_{ES})_b | Bottom width of the exterior strut               |
| (W_{ES})_t | Top width of the exterior strut                |
| W_{IS} | Average width of the interior compression struts |
| (W_{IS})_b | Bottom width of the exterior strut               |
| (W_{IS})_t | Top width of the interior strut                 |
| W_{m}  | Effective width of the top tie                   |
| W_{bn} | Effective width of the bottom tie                |
| \( \theta \) | Slope of the concrete strut                      |