Comments to the
"Theory of Thomson scattering in inhomogeneous media"

V. V. Belyi

a IZMIRAN, Russian Academy of Sciences, Troitsk, Moscow, 108840, Russia

Abstract

In this short communication, we draw the readers’ attention to the inconsistency in the derivation of the Thomson scattering spectrum in inhomogeneous plasma, which leads to violation of the Fluctuation-Dissipation Theorem and a substantial deviation from the results of the rigorous kinetic theory. Moreover, the self-consistent kinetic theory predicts the asymmetry of the spectral lines in inhomogeneous plasma.

1 Introduction

On 12 April 2016 the Journal of Scientific Reports published the article: P.M. Kozłowski, B.J.B. Crowley, D.O. Gericke, S.P. Regan and G. Gregori, ”Theory of Thomson scattering in inhomogeneous media” [1]. The main result of paper has been based on the Eq.:

\[ S(k,\omega) = \frac{S(k,\omega)_{id}}{|\epsilon(k,\omega)|^2}, \] (1.1)

where \( S(k,\omega) \) is the total dynamic structure factor, \( S(k,\omega)_{id} \) is the dynamic structure factor for an ideal (noninteracting gas), and the dielectric (screening) function \( \epsilon(k,\omega) \) in the denominator of Eq. (1.1) in the first order gradient expansion over slow time and space is:

\[ \epsilon(k,\omega) = 1 + \chi(k,\omega) = 1 + (1 + i \frac{\partial}{\partial \omega} \frac{\partial}{\partial \mu t} - i \frac{\partial}{\partial \mu r} \cdot \frac{\partial}{\partial k}) \chi_{eq}(k,\omega, \mu t, \mu r), \] (1.2)

\( \chi_{eq} \) is the susceptibility of the ideal Coulomb plasma. The index ”eq” labels the susceptibility for a homogeneous system in thermodynamic equilibrium [1]. Herewith the structure factor
\(S(k, \omega)^{id}\) in Eq. (1.1) remains unchanged (sic!). Regretfully, instead of constructing a self-consistent kinetic theory of Thomson scattering in a non-uniform plasma, the authors, on an ad hoc basis, expanded in a small parameter of inhomogeneity and nonstationarity the dielectric permeability in the denominator of the well-known expression for the spectral function of a stationary and uniform plasma \([2]\). This caused two consequences: First, the obtained result is nonphysical, since it contradicts the basic physical principle – the Fluctuation-Dissipation Theorem (FDT) in the local equilibrium state. FDT for a local equilibrium state was proved by Balescu \([3]\). In the local equilibrium state the parameters of a system can be changed adiabatically on a scale greater than the particle mean free path. Inhomogeneity and nonstationarity in the plasma fluctuations is manifested via a non-local dependence upon coordinates and time (Non-Markov) \([4]\). FDT for a non-local plasma was given in our paper \([5]\). Second, the obtained correction due to the inhomogeneity is erroneous for the Langmuir oscillations, especially for small wave numbers \(k < k_D\), which usually occurs in experiments. And last but not least: the rigorous kinetic theory predicts \([5]\) the asymmetry of the spectral lines in inhomogeneous plasma. Such asymmetry has been indeed detected in experiments \([6, 7]\), but does not appear in the incorrect approach of \([1]\).

**Results**

For a correct description of non-local effects in inhomogeneous plasma the kinetic approach should be applied. Indeed, using Klimontovich-Langevin method \([8]\), for describing kinetic fluctuation, as well as a multiple space and time scale analysis we have shown \([9]\) that the spectral function of electrostatic field fluctuations \((\delta E \delta E)_{\omega, k}\) for the local equilibrium state is determined by the following expression:

\[
(\delta E \delta E)_{\omega, k} = \sum_a \frac{8\pi \Theta_a \text{Im} \chi_a(k, \omega)}{\omega_a |\epsilon(k, \omega)|^2}.
\]  

(1.3)

Here

\[\epsilon(k, \omega) = 1 + \sum_a \chi_a(k, \omega);\]  

(1.4)

\[\chi_a(k, \omega) = (1 + i\mu \frac{\partial}{\partial \omega} \frac{\partial}{\partial \mu t} - i\mu \frac{\partial}{\partial \mu r} \cdot \frac{\partial}{\partial k}) \chi^{leq}_a(k, \omega, \mu t, \mu r),\]  

(1.5)

\(\chi_a^{leq}\) - is the local equilibrium susceptibility. \(\omega_a = \omega - kV_a, \Theta_a\) is the temperature in energy units. Eq. (1.3) satisfies the FDT, when \(V_a = 0\) and \(\Theta_a = \Theta\). Imaginary part of the susceptibility \(\chi(k, \omega)\) determines the width of the spectral line \((\delta E \delta E)_{\omega, k}\) near the resonance:

\[\gamma = \frac{(Im \chi^{leq} + \mu \frac{\partial}{\partial \omega} \frac{\partial}{\partial \mu t} Re \chi^{leq} - \mu \frac{\partial}{\partial \mu r} \cdot \frac{\partial}{\partial k} Re \chi^{leq})}{\partial \omega} Re \chi^{leq}.\]  

(1.6)

In Eq. (1.6) there appear additional terms at first order of the small parameter \(\mu\). It is important to note that the imaginary part of the dielectric susceptibility is now replaced by the real part, which in the plasma resonance may be greater than the imaginary part by the same factor \(\mu^{-1}\). Therefore, the second and third terms in Eq. (1.6) in the kinetic regime have an effect
comparable to that of the first term. At second order in the expansion in $\mu$ the corrections appear only in the imaginary part of the susceptibility, and they can reasonably be neglected. The width of the spectral lines Eq. (1.6) is affected by new nonlocal terms. They are not related to Joule dissipation and appear because of an additional phase shift between the vector of induction and the electric field. This phase shift results from the finite time needed to set the polarization in the plasma with dispersion. Such a phase shift in the plasma with space dispersion appears due to the medium inhomogeneity. For the case where the system parameters are homogeneous in space but vary in time, the correction to the width of the spectral lines in Eq. (1.6) is still symmetric with respect to the change of sign of $\omega$. However, when the plasma parameters are space dependent this symmetry is lost. The real part of the susceptibility $\chi^{leq}(k,\omega)$ in Eq. (1.6) is an even function of $\omega$. This property implies that the contribution of the space derivative to the expression for the width of the spectral lines is odd function of $\omega$. Besides this term gives rise to an anisotropy in $k$ space.

For the spatially homogeneous case there is no difference between the spectral properties of the longitudinal electric field and of the electron density, because they are related by the Poisson equation. This statement is no longer valid when considering an inhomogeneous plasma. Indeed the longitudinal electric field is linked to the particle density by the nonlocal relation:

$$\delta E(r,t) = -\frac{\partial}{\partial t} \sum_a e_a \int \frac{1}{|r - r'|} \delta n_a(r',t)dr'.$$

In the same approximation as in Eq. (1.3) the expression for the electron structure factor for a two-component ($a = e, i$) plasma has the form [5]:

$$S^e(k,\omega) = \frac{2n_e k^2}{\omega_e k_D^2} \left| \frac{1 + \tilde{\chi}_e(k,\omega)}{\tilde{\epsilon}(k,\omega)} \right|^2 \text{Im} \tilde{\chi}_e(k,\omega) + \left| \frac{\tilde{\chi}_e(k,\omega)}{\tilde{\epsilon}(k,\omega)} \right|^2 \frac{\Theta_i}{\omega_i k_D^2} \text{Im} \tilde{\chi}_i(k,\omega),$$

where $k_D$ is the inverse Debye length,

$$\tilde{\epsilon}(k,\omega) = 1 + \sum_a \tilde{\chi}_a(k,\omega),$$

$$\tilde{\chi}_a(\omega, k) = (1 + i\mu \frac{\partial}{\partial \omega} \frac{\partial}{\partial \mu t} - i\mu \frac{1}{k^2} \frac{\partial}{\partial \mu r_i} k_j \frac{\partial}{\partial k_i} k_j) \chi^{leq}_{a}(\omega, k, \mu t, \mu r).$$

The inhomogeneous correction in Eq. (1.10) ($\tilde{\chi}_a = \frac{\partial}{\partial \mu r_i} k_j \frac{\partial}{\partial k_j} \text{Re}\chi^{leq}_{a})$ is not the same as in Eq. (1.6) and for the plasma mode ($\omega = \omega_L$) is greater than the one in Eq. (1.6), particularly for $k < k_D$, which usually occurs in experiment, by the factor $3/2(1 + k_D^2/9k^2)$. The origin of this difference is that the Green functions for electrostatic field fluctuation and density particle fluctuations are not the same in inhomogeneous situation. We see (Fig 1) that the asymmetry of the spectral lines is present both for $S^e(k,\omega)$ and for ($\delta E\delta E$)$_{\omega,k}$. However, this effect is more pronounced in $S^e(k,\omega)$ than in ($\delta E\delta E$)$_{\omega,k}$. Such asymmetry has been detected in inhomogeneous plasma [6, 7]. The asymmetry of lines $S^e(k,\omega)$ can used as a new diagnostic tool to measure local gradients in the plasma by Thomson scattering.
Acknowledgements

Fruitful discussions with N. Brilliantov are gratefully acknowledged. Thanks to T. Beuermann for detecting a misprint in [5].

References

[1] P. M. Kozlowski, B. J. B. Crowley, D. O. Gericke, S. P. Regan and G. Gregori, Theory of Thomson scattering in inhomogeneous media, Sci. Rep., 6, 24283; doi: 10.1038/srep 24283 2016.

[2] S. Ishimaru, Basic Principles of Plasma Physics, Ch.7 Fluctuations, Addison-Wesley, 1973, pp. 251–255.

[3] R. Balescu, Equilibrium and Nonequilibrium Statistical Mechanics, John Wiley, New York, 1975.

[4] V. V. Belyi, Yu. A. Kukharenko and J. Wallenborn, Pair Correlation Function and Nonlinear Kinetic Equation for a Spatially Uniform Polarizable Nonideal Plasma, PRL 76, 3354 (1996).

[5] V.V. Belyi, Fluctuation-Dissipation Relation for a Nonlocal Plasma, PRL 88, 255001 (2002).

[6] V. M. Strunnikov, Spectroscopic Study of Plasma Flows in Magnetic Traps, Dissertation, Kurchatov Institute, Moscow, 1986.

[7] V. V. Belyi, A. A. Beschaposhnikov, V. B. Voronin, and V. M. Strunnikov, Diagnostics of Plasma Inhomogeneity by Thomson Scattering, Physics of Atomic Nuclei (submitted) (2017).

[8] Yu. L. Klimontovich, Statistical Physics, Harwood, New York, 1986.
Figure 1: The electron structure factor $S^e(k,\omega)$ (solid line) and the spectral function of electrostatic field fluctuations $(\delta E\delta E)_{\omega,k}$ (dashed line) as a function of frequency. $k_D = 3$; $k \frac{\partial n}{\partial r} = \frac{\nu_s n k^2}{2\omega_L}$ (The Knudsen number $Kn=1/9$).