SU(5) × SU(5) unification, see-saw mechanism and R-conservation

R.N. Mohapatra
Department of Physics and Astronomy
University of Maryland
College Park, MD 20742

Abstract

A supersymmetric grand unified model based on the gauge group SU(5) × SU(5) is discussed. This model has the new feature that the conventional see-saw mechanism for neutrino masses is embedded using the \((15, 1) + (1, T5)\) representations. This representation may have a better chance of arising from level two compactification of superstring theories than the \(126\)-dimensional representation used in the SO(10) grand unified models. The model also naturally suppresses all R-parity violating interactions.

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There is now a widespread belief that the next step beyond the standard model may very well contain a supersymmetric grand unified theory (SUSY GUT). This speculation is supported by the unification of gauge couplings that has been observed to occur \([1]\) at a scale around \(10^{16}\) GeV, with supersymmetry scale in the TeV region using the precise values of the low energy couplings measured at LEP and SLC. Meanwhile, the superstring theories, that have the potential to unify gravity with the strong and electroweak forces, have been shown to lead to a variety of such SUSY GUT groups via an appropriate compactification scheme. These facts have led to intensive investigations of many grand unification scenarios, the simplest and most notable ones being the \(SU(5)[4]\) and \(SO(10)[5]\) schemes. The detailed aspects of the models of course are dictated by low energy observations such as the spectrum of fermion masses, absence of baryon and lepton number violating interactions etc. In our investigation, we will focus on two properties, which we believe are desirable for any viable SUSY GUT model: (i) there must be a simple way to explain the smallness of neutrino masses; and (ii) R-parity violating interactions that can lead to uncontrollable baryon and lepton number violation must be absent or naturally suppressed. We would furthermore require that the model contain Higgs representations that have a chance of emerging from compactification of the heterotic string theory at some Kac-Moody level.

The simplest way to accommodate massive neutrinos in GUT theories is via the see-saw mechanism\([4]\), where the smallness of their masses is linked to the largeness of the \(B - L\) breaking scale. As far as automatic R-conservation is concerned, experience has shown that it depends very sensitively on the nature of the gauge symmetry and the Higgs content of the model\([4]\). Of course one may take the point of view that the process of string compactification may yield R-parity as a discrete symmetry. However, this property is much harder to demonstrate in a string theory. Furthermore, if R-parity arises as global symmetry, one still has to worry about possible Planck suppressed R-violating interactions of gravitational origin which can lead to undesirable and often catastrophic effects. It may therefore be preferable to rely on the gauge group and Higgs representations as a way to guarantee R-parity as an automatic symmetry of the theory.

Let us now come to specific models. In the simplest models based on the \(SU(5)\) group, \(B - L\) is not a gauged symmetry, and therefore, they are not suitable for understanding non-zero neutrino masses. Furthermore, the \(SU(5)\) theory also allows arbitrary strengths for R-violating interactions. This takes us to the next class of models based on the \(SO(10)\) group, where (i) the see-saw mechanism can be implemented by using the \((126 + \overline{126})\) representations to break the \(B - L\) symmetry, and (ii) also as noted in \([6]\), this model has the property that it leads to automatic suppression of all R-violating interactions, which satisfies our second re-
requirement above. However, it appears increasingly unlikely that the \((\mathbf{126} + \overline{\mathbf{126}})\) representations can emerge from the compactification of heterotic string models. Therefore such scenarios will be disfavored if one believes in superstring theories as the final theory of nature. The simplicity of the conventional seesaw mechanism and the requirement of automatic R-conservation are so appealing as phenomenological requirements that it is useful to seek alternative SUSYGUT frameworks which not only have both these properties but which also have a better chance of emerging from superstring models.

In this letter, we present a SUSY GUT model based on the gauge group \(SU(5) \times SU(5)\) which provides an alternative way to implement the see-saw mechanism using the representations \((\mathbf{15}, \mathbf{1}) + (\mathbf{1}, \mathbf{15})\). This may be more amenable to superstring embedding because, it has now been shown that for a single \(SU(5)\) group in a level two string compactification, there appear \(\mathbf{15}\)-dimensional representations and it is quite likely that for the \(SU(5) \times SU(5)\) case, the representations we use will appear. So far only level one \(SU(5) \times SU(5)\) models have been studied\(\footnote{Strictly the authors of Ref.\cite{7} have proved this for fermionic compactification schemes.}\) and perhaps the considerations of the present paper will motivate a study of these models at level two. At low energies, this model coincides with the usual minimal supersymmetric standard model (MSSM). Another property of our \(SU(5) \times SU(5)\) model is that the R-parity violating interactions are naturally suppressed.

**The \(SU(5) \times SU(5)\) model:**

We assume the gauge group to be \(SU(5)_A \times SU(5)_B\) with the associated gauge couplings denoted by \(g_A\) and \(g_B\) respectively. We will see later that phenomenologically reasonable unification scale consistent with the low energy precision measurement of the standard model gauge couplings will require that at the GUT scale the two couplings are unequal. Such a scenario requires that the discrete symmetry that transforms one \(SU(5)\) group to the other is broken at string scale.

We will assign the matter superfields to transform as \((\overline{\mathbf{5}}, \mathbf{10}, \mathbf{1}) + (\mathbf{1}, \mathbf{5} + \overline{\mathbf{10}})\) for each generation. This implies that we must have extra fermions beyond those present in the standard model. We denote them by \((U, U^c, D, D^c, E, E^c)\); of these the \((U, U^c, D, D^c)\) are the heavy vector like analogs of the familiar up and down quarks respectively (and therefore have obvious \(SU(3)_c\) color transformation properties) whereas the \(E, E^c\) are color singlet singly charged heavy fermions. These extra fermions also come three varieties corresponding to the three generations of

\footnote{A different class of \(SU(5) \times SU(5)\) string-embeddable GUT model has recently been proposed in Ref.\cite{8}; our model is very different from this model not only in terms of the fermion content but also symmetry breaking as well as of course in its implementation of the see-saw mechanism.}
known quarks and leptons. Note that there is no heavy vectorlike analog of the neutrinos. The assignment of these fermions to the representations of the gauge group are given below:

\[ \psi = \begin{pmatrix} D_1^c \\ D_2^c \\ D_3^c \\ e^- \\ \nu \end{pmatrix}; \chi = \begin{pmatrix} 0 & U_3^c & -U_2^c & u_1 & d_1 \\ -U_3^c & 0 & U_1^c & u_2 & d_2 \\ U_2^c & -U_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & E^+ \\ -d_1 & -d_2 & -d_3 & -E^+ & 0 \end{pmatrix}; \quad (1) \]

Similarly, the fermions in \((1, 5 + \overline{10})\) denoted by \(\psi^c\) and \(\chi^c\) can be written as:

\[ \psi^c = \begin{pmatrix} D_1 \\ D_2 \\ D_3 \\ e^+ \\ \nu^c \end{pmatrix}; \chi^c = \begin{pmatrix} 0 & U_3 & -U_2 & u_1^c & d_1^c \\ -U_3 & 0 & U_1 & u_2^c & d_2^c \\ U_2 & -U_1 & 0 & u_3^c & d_3^c \\ -u_1^c & -u_2^c & -u_3^c & 0 & E^- \\ -d_1^c & -d_2^c & -d_3^c & -E^- & 0 \end{pmatrix}. \quad (2) \]

Note that below the scale where the heavy vectorlike fermions (i.e., \(U, D, E\)) become massive, the fermion content is same as in the left-right symmetric models. To make this point transparent, we discuss the symmetry breaking of the GUT group to the standard model gauge group below. We assume the Higgs fields of the model to transform as follows: there are two sets of multiplets belonging to \(\mathbf{5}, \overline{5} + (\overline{5}, \mathbf{5})\) representations (denoted by \(H_{1,2} + \overline{H}_{1,2}\)); one set belonging to \((\mathbf{24}, \mathbf{1}) + (\mathbf{1}, \mathbf{24})\) (denoted by \(\Phi_A + \Phi_B\)) and another set transforming as \((\mathbf{15}, \mathbf{1}) + (\mathbf{1}, \mathbf{15})\) (denoted by \(S_A + S_B\)) and \((\overline{\mathbf{15}}, \mathbf{1}) + (\mathbf{1}, \mathbf{15})\) (denoted by \(\overline{S}_A + \overline{S}_B\)). The first point to note is that all these Higgs representations have good chance of arising from level two fermionic compactification of the heterotic string theory as already mentioned earlier since both the representations \(\mathbf{15}\) and \(\mathbf{24}\) have already been shown to appear in level two \(SU(5)\) string GUT models.

Turning to symmetry breaking and fermion masses, we assume that \(H_1\) has GUT scale vev with the pattern \(\langle H_1 \rangle = \text{diag}(V,V,V,0,0) = \langle \overline{H}_1 \rangle\) so that the gauge group breaks down to \(SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}\). At the next stage, we assume that \(S_B\) has a vev along the \(\nu^c \nu^c\) direction i.e. \(\langle S_{B,55} \rangle = V_R\) so that the gauge group below \(V_R\) is the standard model group. As we will show the see-saw mechanism for neutrino masses arises at this stage. The final breaking of the
standard model is achieved by giving vev to the $H_2$ as $\langle H_2 \rangle = \text{diag}(0,0,\kappa,\kappa')$. We also assume that $\overline{H}_2$ has a similar vev pattern. In principle, the seesaw scale $V_R$ could be lower than the GUT scale here for simplicity, we will focus on a scenario with $V = V_R$.

Having provided the general outline and the symmetry breaking of the model, let us now discuss the decoupling of the heavy fermions and the generation of the various light fermion masses in the theory. The two crucial vev’s for this discussion are those of $H_{1,2}$ given above. The heavy fermion (i.e. $U, D, E$) masses arise from the following couplings in the superpotential: $\psi H_1 \psi^c$ and $\chi H_1^c \chi^c / M$ where $M$ could either be a scale corresponding to new physics between $M_{\text{GUT}}$ and the Planck scale or the Planck scale itself i.e. $M = M_{\text{Pl}} / \sqrt{8\pi} \approx 10^{18} \text{ GeV}$. It is easy to check that these two couplings give masses of order $M_{\text{GUT}}$ and $M_{\text{GUT}}^2 / M$ to the $D$ and $U$ colored fermions respectively. If we choose $M_{\text{GUT}} \approx 10^{16} \text{ GeV}$, we could choose $M \approx 10^{17} \text{ GeV}$ or so that $M_U \approx 10^{15} \text{ GeV}$. The mass of the heavy lepton $E$ arises from the non-renormalizable coupling $\varepsilon^{ijklm} \chi_{ij}^H H_{kp}^l H_{qr}^m \chi_{st}^c \psi^{pqrs} / M^2$. We get $M_E \approx 10^{14} \text{ GeV}$. Thus, the vector like fermions decouple from the low energy spectrum.

Let us now turn to the light fermion masses. Since they arise from the breaking of the standard model gauge group, they will involve the vev of $H_2$ given above. The relevant terms in the superpotential that give rise to these masses are $h_e \psi H_2 \psi^c$, $h_q \chi H_1 \chi^c / M$ where $h_e,q$ are $3 \times 3$ matrices in the generation space; the $h_e$ term gives rise to the charged lepton masses and the Dirac masses for the neutrinos and the $h_q$ term gives rise to the mass matrices for the up and the down quark masses. Note that since the up and down quark masses at this stage are proportional to each other, the CKM mixing angles vanish. We also note that the 33 element of $h_q$ must be of order 3 to 10 in order to understand why the top quark is so heavy. In order to generate quark mixing angles, we include next higher terms in the superpotential of the form $\chi H_1 \chi^c H_1 \chi^c / M^3$. The coupling matrix in front of the above term leads to a misalignment between the up and down sectors leading to non-vanishing CKM angles. Moreover, this being a higher order term in $M_{\text{GUT}} / M$, also naturally explains why the mixing angles between the quarks must be small.

An important point to note here is that in our model there is no mixing between the superheavy vector like quarks and leptons and the known light fermions. In this respect our model differs from several recent works [10], who had very similar fermion assignment to ours but had the heavy-light fermion mixing as an essential ingredient.

Let us now discuss the implementation of the see-saw mechanism in our model. Crucial for this purpose is the symmetric $(\textbf{15}, \textbf{1}) + (\textbf{1}, \overline{\textbf{15}})$ representations. In our superpotential we include their coupling to the fermions given by $f (\psi \psi S_A + \psi^c \psi^c S_B)$. As already mentioned, the vev $\langle S_{B,55} \rangle = V_R$ breaks the $B-L$ symmetry and in
that process gives Majorana masses to the right-handed neutrinos of magnitude \( fV_R \). Since we already showed that neutrinos have Dirac masses from their \( \psi H_2 \psi^c \) coupling, we now have all the ingredients of the conventional see-saw mechanism leading to the usual see-saw formula for neutrino masses i.e. \( m_{\nu_i} \simeq m_{\nu D}^2 / f_i V_R \). The gauge symmetry allows a nonrenormalizable term of the form \( S_A H_2^2 S_B / M \), which induces the vev for \( S_{A,55} \) of order \( \kappa^2 / M \), which is of order \( 10^{-5} \) eV and hence too small to effect the usual see-saw formula. The symmetric \( [15] \)-dimensional multiplet plays exactly the same role as the \( [126] \) multiplet in the \( SO(10) \) models except that it has a chance to arise from level two compactification of heterotic string models unlike the \( [126] \).

**Gauge coupling unification:**

Let us now turn to gauge coupling unification in the model and the mass scales for the symmetry breaking at various stages. It was noted in \([10]\), that if we assume the gauge couplings for the two \( SU(5) \) groups to be equal (i.e. require an exact discrete symmetry that transforms \( SU(5)_A \) to \( SU(5)_B \)), then the model predicts very small value for \( \sin^2 \theta_W = 3/16 \) at the GUT scale and since \( \sin^2 \theta_W \) decreases in general at lower scales, such a scenario is in gross disagreement with observations. On the other hand in string inspired models it is conceivable that the discrete symmetry that guarantees that the two \( SU(5) \) couplings are equal, is broken below the string scale. This could for instance happen if there are singlet fields odd under the above discrete symmetry, in which case, non-renormalizable couplings involving this field and the two \( SU(5) \) gauge fields could also lead to splitting between the two gauge couplings. We will therefore work with a scenario where this happens (i.e. where \( g_A \neq g_B \) at the GUT scale). If we denote the ratio of the two fine structure constants for the two \( SU(5) \) groups by \( y \equiv (\alpha_A / \alpha_B) \), then we get \( \sin^2 \theta_W (M_{\text{GUT}}) = \frac{3}{8(1+y)} \); if we then chose \( y \ll 1 \), then the GUT scale value for \( \sin^2 \theta_W \) approaches that of the \( SU(5) \) or \( SO(10) \) prediction for it and one can obtain a value for it at the scale \( M_Z \) in agreement with observations. Below we present an example of such a scenario.

The equations relating the gauge couplings at \( M_Z \) to those at the GUT scale are given by:

\[
\alpha^{-1}(M_Z) = \frac{5}{13} \alpha_{A}^{-1} + \frac{8}{13} \alpha_{B}^{-1} + b_1 U + c_1 R \tag{3}
\]

\[
\alpha_2^{-1}(M_Z) = \alpha_{A}^{-1} + b_2 U + (b_2' - b_2) R \tag{4}
\]

\[
\alpha_3^{-1}(M_Z) = \frac{1}{2}(\alpha_{A}^{-1} + \alpha_{B}^{-1}) + b_3 U + (b_3' - b_3) R \tag{5}
\]
In the above equations, we have denoted $U = \frac{1}{2\pi} \ln(M_{GUT}/M_Z)$ and $R = \frac{1}{2\pi} \ln(M_{GUT}/M_R)$, with $M_R$ denoting the scale at which the symmetry $SU(2)_R \times U(1)_{B-L}$ breaks down to $U(1)_Y$ and $M_{GUT}$ as already noted stands for the unification scale. The coefficients in front of $U$ and $R$ are model dependent and represent the way the gauge couplings evolve in different models. We have assumed only a single intermediate scale below $M_{GUT}$ corresponding to the symmetry $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ (denoted by $G_{3221}$) above $M_R$ and also that the $SU(2)_L$ group is embedded only in $SU(5)_A$ group. In this case we have: $b_1 = \frac{3}{13} \Sigma \left( \frac{Y}{2} \right)^2$, $c_1 = \frac{3}{13} b_{2R} + \frac{10}{13} b_{BL} - b_1$; $b_{2L}$ and $b'_{2L}$ are $SU(2)_L$ beta function coefficients above and below the scale $M_R$ and $b_3$ and $b'_3$ are the $SU(3)_c$ beta function coefficients above and below $M_R$ (their values are half the usual $SU(3)_c$ coefficients due to the fact that $SU(3)_c$ arises as the diagonal sum of the two $SU(3)$'s in the $SU(5)_{A,B}$) and $b_{BL} = \left( \frac{3}{13} \right) \Sigma \left( \frac{B-L}{2} \right)^2$.

As an example of a scenario, we assume $M_{GUT} = M_R$ and MSSM multiplet content below $M_{GUT}$ except that we require the Higgs fields in the representation $(1,3,\pm 2)$ and one color octet $(8,1,0)$ under the standard model group $SU(3)_c \times SU(2)_L \times U(1)_{Y}$ to have intermediate to low mass. Such a choice is completely compatible with low energy phenomenology. For this case, we find $M_{GUT} \approx 10^{16}$ GeV in the one loop approximation with $\alpha^{-1}_A \simeq 17$ and $\alpha^{-1}_B \simeq 1$ the triplet and octet fields with same mass $\approx 10^{10.5}$ GeV. Another scenario which also yields an $M_{GUT} = M_R \approx 10^{16}$ GeV is one with only a single $B-L$ neutral $SU(2)_L$ triplet with mass around $10^{11}$ GeV giving $\alpha^{-1}_A = 20.7$ and $\alpha^{-1}_B = 2.4$. The latter case is preferable from one loop perspective since both gauge couplings are in the perturbative region.

We emphasize that the spectra chosen in both these examples arise from the particle content of the theory by appropriate fine tuning. We expect that once two loop and the threshold corrections are included, the GUT scale could easily reach the range used in the fermion mass discussion of the paper.

It may be worth pointing out at this stage that, the light doublets needed for electroweak symmetry breaking arise in this model from the combination of terms $H_2 \Phi \overline{H}_2 + \mu H_2 \overline{H}_2$ by appropriate fine tuning of the parameter $\mu$. This would leave us with two pairs of standard model doublets; one of the pairs becomes very heavy due to the nonrenormalizable interaction $\overline{H}_{1,p} \overline{H}_{1,q} \overline{H}_{1,r} \overline{H}_{1,s} \varepsilon_{pqrst} \varepsilon_{ijklmn}/M^2$.

R-parity breaking, proton decay etc:

In the present model, R-parity is automatically conserved even in the presence of nonrenormalizable Planck scale suppressed terms. To see this, recall that in the
conventional $SU(5)$ SUSYGUT model, one source of R-parity breaking interactions is the term in the superpotential involving $10 \bar{5} \bar{5}$ terms (all fields are matter fields) which leads to terms of type $u^c d^c d^c$ and $QLd^c$. However in our case due to the assignment of heavy fermions, the analogous terms give rise to operators of type $QLD^c$, $U^c D^c D^c$ which do not lead to R-parity violation involving light fermions. Furthermore, there is no mixing between the heavy and light quarks in this model due to the existence of an exact global symmetry $(-1)^{B+H−L}$ where all quarks (both light and heavy) have the usual baryon number $B = 1/3$; the quantum number $H$ is +1 for heavy quarks and leptons and zero for light fermions. Thus R-parity is exactly conserved and as a result of this, the lightest supersymmetric particle (the LSP) is absolutely stable and can play the role of dark matter of the universe.

Coming to proton decay, again because there is no mixing between the heavy and light quarks, there are no leading order contributions to proton decay until we include non-renormalizable Planck scale induced terms. In the lowest order in $M_{Pl}^{-1}$, proton decay arises from the following operator: $\chi \chi \chi \psi$ which gives an operator of type $QQQL$ with coefficient $\lambda/M_{Pl}$. After gluino and wino dressing, it would lead to the four-fermion proton decay operator with strength $(\lambda g^2 M_{gaugino})/(16\pi^2 M_{sq}^2 M_{Pl})$. For $\lambda \approx 10^{-5}$ and other parameters being reasonable, this leads to proton lifetime long enough to be consistent with observations. Due to the presence of the unknown coupling parameters, it is not possible to make a more definitive statement about the proton lifetime.

In conclusion, we have pointed out that a SUSY GUT theory based on the $SU(5) \times SU(5)$ group has two properties highly desirable of a GUT model: (i) it can embed the see-saw mechanism for neutrino masses using multiplets that have a chance of emerging from the superstring theories; and (ii) the model has automatic R-parity conservation thus guaranteeing that the LSP (which is supposed to play the role of dark matter in the universe) is indeed truly stable without any extra theoretical assumptions. We have also analysed the coupling constant unification in this class of theories in the one-loop approximation and showed that realistic scales can emerge provided the two $SU(5)$ couplings are different from each other at the GUT scale.

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