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Hadron structure within the point form of relativistic quantum mechanics

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Abstract. We present an outline of recent developments in the field of hadron form-factor calculations within constituent-quark models using the point form of relativistic quantum mechanics. Our method to calculate currents and form factors is exemplified by means of the weak $B \to D$ transition. We present results for weak $B \to D$ transition form factors in the space- and the time-like momentum-transfer region. We discuss how wrong cluster properties, which one has to deal with when employing relativistic quantum mechanics, affect these form factors and we estimate the role non-valence, $Z$-graph contributions may play for decay kinematics.

Keywords. Electroweak hadron structure · Constituent-quark model · Relativistic quantum mechanics · Point-form dynamics

1 Introduction

In Refs. [1; 2; 3; 4; 5] the point form of relativistic quantum mechanics has been advocated as an appropriate framework for calculating the electroweak structure of bound few-body systems, in particular of mesons and baryons within the scope of constituent-quark models. The route to derive electroweak form factors, pursued in these papers and followed also here, is to describe the physical process in which particular form factors are measured in a Poincaré invariant way, calculate the invariant $1\gamma$- or $1W$-exchange amplitude, extract the hadronic current from this amplitude, analyze its covariant structure and identify the form factors. Poincaré invariance is thereby guaranteed by employing the Bakamjian-Thomas construction [6]. The essence of the Bakamjian-Thomas construction in point form is that the 4-momentum operator factorizes into an interaction-dependent mass operator and a free 4-velocity operator

$$\hat{P}^\mu = \hat{\mathcal{M}} V^\mu_{\text{free}} = \left(\hat{M}_{\text{free}} + \hat{M}_{\text{int}}\right) V^\mu_{\text{free}}.$$  

(1)

The dynamics of the system is thus completely encoded in the mass operator. From Eq. (1) it is quite obvious that all four components of the 4-momentum operator are interaction dependent. The generators of Lorentz transformations, on the other hand, are interaction free. These are the properties which characterize the point form of relativistic dynamics. They make it comparably simple to boost and rotate wave functions and add angular momenta.

Since $\gamma$- and $W$-exchange are treated dynamically, one has to allow for particle emission and absorption. This is accomplished by using a coupled-channel framework with a matrix mass operator $\hat{\mathcal{M}}$ that acts on the direct sum of the pertinent multiparticle Hilbert spaces. The diagonal entries $\hat{M}_i$ of $\mathcal{M}$ are the sum of the relativistic kinetic energies of the particles in channel $i$. In addition, the $\hat{M}_i$
may contain instantaneous interactions between the particles, like the confinement potential between quark and antiquark. Off-diagonal entries of $\mathcal{M}$ are vertex operators $\hat{K}_{i\rightarrow j}$ and $\hat{K}_{j\rightarrow i} = \hat{K}_{i\rightarrow j}^\dagger$, which describe the absorption and emission of particles and hence the transition from one channel to the other. A most convenient basis to represent all these operators is formed by a complete set of velocity states $|\nu\rangle$. A velocity state is a multiparticle momentum state in the rest frame that is boosted to an overall 4-velocity $V (V_\mu V^\mu = 1)$ by means of a rotationless boost $B_\mu (V)$:

$$|V; k_1, \mu_1; k_2, \mu_2; \ldots; k_n, \mu_n\rangle := \hat{U}_{B_\mu (V)} |k_1, \mu_1; k_2, \mu_2; \ldots; k_n, \mu_n\rangle \quad \text{with} \quad \sum_{i=1}^n k_i = 0. \quad (2)$$

The $\mu_i$s are the spin projections of the individual particles. Using velocity states, matrix elements of vertex operators can be simply related to field theoretical interaction Lagrangean densities $\mathcal{L}$

$$\langle V'; k'_i, \mu'_i | \hat{K} | V; k_i, \mu_i \rangle \propto V^0 \delta^3 (V - V') \langle k'_i, \mu'_i | \mathcal{L}_{\text{int}} (0) | k_i, \mu_i \rangle. \quad (3)$$

At this point it is worthwhile to remark that conservation of the overall 3-velocity at interaction vertices is a specific feature of the Bakamjian-Thomas construction and does not hold, in general, for point-form quantum-field theories. It is this overall velocity-conserving delta function that leads to wrong cluster properties, an unwanted feature of the Bakamjian-Thomas construction which is observed in any form of relativistic dynamics and is not just specific to the point form $\mathcal{L}$. The physical consequences of wrong cluster properties in our case are that the gauge-boson-hadron vertices, which we analyze to obtain the transition form factors, may not only depend on the momenta attached to the vertex, but also on the lepton momenta. Formally such wrong cluster properties could be cured by means of, so called, "packing operators", but practically these are hard to construct. The strategy to obtain sensible results for the form factors adopted in Refs. [1; 2; 3; 4; 5] and also followed here is thus rather to look for kinematical situations in which those spurious dependencies are minimized or vanish.

2 Weak $B \rightarrow D$ transition form factors

2.1 Space-like momentum transfers

Our first goal is to derive weak $B \rightarrow D$ transition form factors for space-like momentum transfers as they can, in principle, be measured in $\nu_e B^- \rightarrow e^- D^0$ scattering. In order to account for dynamical exchange of $W$-bosons we set up a 4-channel problem that includes all states which occur during such a scattering process if considered within a valence-quark picture (i.e. $|\nu_e, b, \bar{u}\rangle$, $|e, W^+, b, \bar{u}\rangle$, $|e, c, \bar{u}\rangle$, $|\nu_e, W^-, c, \bar{u}\rangle$). An instantaneous confinement potential between quark and antiquark is included in the diagonal entries of the matrix mass operator. Using perturbation theory for the weak coupling we calculate the invariant 1$W$-exchange amplitude for $\nu_e B^- \rightarrow e^- D^0$ scattering. It is the sum of two time-ordered contributions which, as expected, is proportional to the contraction of a lepton with a meson current times the covariant $W$-boson propagator. This allows us to identify the weak meson current in a unique way. It is an overlap integral of $B$- and $D$-meson wave functions multiplied by the
weak quark current, a Wigner-rotation factor and a kinematical factor (see, e.g., Refs. [4; 5]). After replacing the CM momenta $k_B$ and $k_D$ by the respective physical particle momenta $p_B = B_i(V)k_B$ and $p'_D = B_i(V)k'_D$ we end up with a meson current that transforms like a 4-vector and has the covariant decomposition [4; 5] (with $q = p_B - p'_D$):

$$J^\mu(p'_D, p_B) = \left[(p_B + p'_D) - \frac{m_B^2 - m_D^2}{q^2} q\right]^\mu F_1(Q^2, s) + \frac{m_B^2 - m_D^2}{q^2} q^\mu F_0(Q^2, s).$$

The physical consequences of wrong cluster properties, inherent in the Bakamjian-Thomas construction, become obvious in this decomposition. The form factors cannot be chosen such that they are only absent) are shown for comparison (dashed lines). The differences may be considered as an estimate of this. This is done in Fig. 2 (solid lines). In this figure the results of a direct decay calculation [4; 5] (Z-graphs) are calculated in the BF and not in the IF (see also Ref. [12]). In order to avoid problems with Z-graphs leading to $B \to D$ subprocess. We consider two extreme cases, namely the minimum value of $s$ necessary to reach a particular $q^2 < 0$ and $s \to \infty$. The first choice corresponds to the Breit frame (BF), the second to the infinite-momentum frame (IF).

Using simple harmonic-oscillator wave functions with the oscillator parameters and masses taken from a front-form calculation of weak $B \to D$ transition form factors in the time-like region [10] we obtain the results that are plotted in Fig. 1. Whereas the differences between IF and BF are small for $F_1$, they can be sizeable for $F_0$. From a physical point of view the IF seems to be preferable: unlike the BF, so called “Z-graphs” are suppressed in the IF and it is thus not necessary to take them into account, the Mandelstam-$s$ dependence vanishes for $s \gg m_B^2$, and our IF results agree with front-form calculations in the $q^+ = 0$ frame.

### 2.2 Time-like momentum transfers

In the time-like momentum transfer region these form factors can be measured in semileptonic weak decay processes. Theoretically it is straightforward to adapt our relativistic multichannel approach such that one can deal with decay processes like $B \to D e^{-}\nu_e$. Working in the velocity-state representation the decaying $B$-meson has to be at rest. For this kinematical situation it is, however, known from front-form calculations [11] that non-valence contributions leading to Z-graphs may become important. As already mentioned, a similar observation can be made for space-like momentum transfers, if the form factors are calculated in the BF and not in the IF (see also Ref. [12]). In order to avoid problems with Z-graphs it is thus suggestive to take the form factor expressions obtained in the IF for space-like momentum transfers and continue them analytically to time-like momentum transfers by the replacement $Q \to iQ$. This is done in Fig. 2 (solid lines). In this figure the results of a direct decay calculation [4; 5] (Z-graphs absent) are shown for comparison (dashed lines). The differences may be considered as an estimate of the size of Z-graph contributions. From the considerations just made it is clear that the solid line should be closer to experiment than the dashed one. This is indeed the case. What one knows experimentally is the slope of $F_1$ at zero recoil as measured in $B \to D e^{-}\nu_e$ decays [13]. It agrees approximately with...
the value which we get from our analytic continuation, whereas the decay calculation provides a much smaller value.

3 Outlook

Having estimated the size of Z-graph contributions indirectly, our next task is now the explicit inclusion of Z-graphs in the decay calculation. One possible time ordering of a Z-graph contribution to the $B \to D e \bar{\nu}_e$ decay is shown in Fig. 3. It obviously requires the creation of a $c\bar{c}$-pair, which we plan to describe by means of a $^3P_0$ model \cite{Segovia}. Within our relativistic coupled-channel approach such a pair creation is easily accommodated by an additional channel and a vertex interaction obtained from the field-theoretical $^3P_0$ interaction Lagrangean \cite{Klink}. Assuming instantaneous confinement between the respective quark-antiquark pairs (indicated by blobs in Fig. 3) on end up with a simple, vector-meson-dominance-like physical picture. The Z-graph contribution to $B \to D$ decay can be understood as a $B \to D$ transition induced by the emission of a $B^*_c$ and, less important, excited ($b\bar{c}$) vector mesons, which subsequently decay into $e\bar{\nu}_e$ by means of a $W$. This simplifies the calculation of Z-graphs considerably, since the main part can be done on the hadronic level. Only the $B^*_cBD$-vertex (i.e. coupling strength and form factors) and the $B^*_c$ decay has to be determined on the quark level. Proceeding in this way we hope to achieve a more quantitative estimate of Z-graph contributions in weak meson decay form factors.

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