Comment on "Parity Doubling and SU(2)_L × SU(2)_R Restoration in the Hadronic Spectrum" and "Parity Doubling Among the Baryons"

In a recent paper [1] the authors arrived at the following result: In a world with spontaneous symmetry breaking and massless pions, the chiral symmetry realized in a linear way gives no relations among the properties of hadron states, such as masses and couplings. Such predictions would hold only if certain chirally invariant operators are dynamically suppressed. The final conclusion was that restoration of SU(2)_L × SU(2)_R cannot occur in the presence of massless pions at zero temperature and chemical potential. Needless to say that such a conclusion, if true, has a fundamental character. Subsequently Comment [2] appeared where it was emphasized that paper [1] "may create a false impression about the nature of the problem" and that suppression of offending operators occurs naturally high in the spectrum. While we agree with the conclusion about "false impression", we did not find in [2] a clear explanation why the suppression of chirally invariant operators must be natural (in the cited papers some general quasiclassical arguments were used to justify the chiral symmetry restoration, but how the quasiclassics suppresses the chirally invariant operators of [1] was not shown). We would like to comment on the result of [1] using another arguments.

It has been known for long ago (see, e.g., [3]) that non-linearly realized chiral symmetry (i.e. the Nambu-Goldstone realization) is a dynamic symmetry. Unlike algebraic symmetries, as, e.g., the linearly realized chiral symmetry (the Wigner-Weil realization), dynamic symmetries alone do not lead to any algebraic relations. The result of [1] is just a demonstration of this fact in the language of effective field theory. If one applies such a description to hadron excitations, of course, one inevitably arrives at the conclusion in [1], nothing surprising. As was stressed in the same seminal paper [3], algebraic consequences of non-linearly realized chiral symmetry follow from restrictions on asymptotic behavior of amplitudes at high energy. The famous Weinberg’s sum rules [4] and resulting (using KSFR relation) formula $m_{a_{1}} = \sqrt{2}m_{\rho}$ can be considered as classical examples of such relations, which were derived from asymptotic restrictions on high energy behavior of two-point correlators of vector (V) and axial (A) currents. All this was known before QCD. Now we know from the asymptotic freedom of QCD that these restrictions are governed by the perturbation theory, i.e. they are chirally symmetric in a usual algebraic sense. If one saturates the V.A correlators by "one resonance + perturbative continuum" one obtains different masses of states because pion contributes significantly to the A-channel due to PCAC. But the radial excitations of $a_{1}$-meson do not have this contribution due to generalized PCAC. To correctly define the high meson excitations one needs the large-$N_c$ limit. The number of excitations then must be infinite in order to reproduce the perturbative asymptotics of correlators. Thus, saturating the correlators by infinite number of resonances, the relative role of pion contribution, in a sense, becomes infinitely small. Since the resonances reproduce the perturbative asymptotics, the spectrum should "repeat" the chiral invariance of perturbation theory. In this sense the algebraic chiral symmetry is restored in the spectrum of radial excitations. Generalization of this idea to arbitrary channels is straightforward. To our knowledge, first it was proposed in [1]. Subsequently it was widely used in different models (see, e.g., [5]) and, moreover, it was directly derived in various papers within QCD sum rules [6]. In some sum rule analyses [5] the (masses)$^2$ of chiral partners have a constant shift at any energy. In this case one has an effective restoration of algebraic chiral symmetry in the sense that the corresponding masses tend to degeneracy (since they grow), but with a slower rate [7]. Even in that case there are algebraic relations between hadronic parameters.

Thus, finally one has the asymptotically degenerate mass spectrum for chiral partners. The situation is indistinguishable from Wigner-Weyl realization of chiral symmetry. Although the large-$N_c$ limit usually works quite well, the argument might seem not compelling because in the real world $N_c = 3$. But what should be emphasized here is that such a phenomenon is expected to occur due to the asymptotic freedom of QCD, the latter was completely ignored in analysis [1]. Of course, one could speculate that it is asymptotic freedom which is responsible for the dynamical suppression of chirally invariant operators. But for us it looks more likely that the very problem of "chirally invariant operators" is in doubt. Low-energy pion-like effective Lagrangians (like those used in Ref. [1]) reflect only some features of QCD, their application is restricted and, hence, they seem not to be trustworthy for drawing very general statements which are valid at any energy.

Let us give a simple example. Let $\varphi_+ \text{ and } \varphi_-$ be fields of chiral partners having masses $m_+$ and $m_-$ correspondingly. In Lagrangian (9) of Ref. [1] the term breaking mass degeneracy is proportional to the factor $m_{a_{1}}$ and it describes different reactions with pions, e.g., the decay $\varphi_+ \to \varphi_- \pi$ (we assume $m_+ > m_-$.). Thus, such a language is relevant to the reality if this decay indeed can occur (together with decays like $\varphi_+ \to \varphi_- \pi \pi \pi$). This works well for the ground states. Let us consider, however, the radial excitations. Denote $m^2_+(n) - m^2_-(n) \equiv \delta(n)$ ($n$ is the radial quantum number). If $\delta(n)$ is not decreasing one has no genuine restoration of algebraic chiral symmetry. Even if we assume this variant, in the real world the masses grow with $n$ rapidly enough to ensure decreasing $m_+(n) - m_-(n) = \delta(n)/(m_+(n) + m_-(n))$ such that one usually has $m_+(n) - m_-(n) \lesssim m_\pi$ at $n \gtrsim 3$, i.e. this...
decay is impossible any more since some excitation. The conclusions for the real world inferred from the effective Lagrangian become just artefacts of chiral limit. On the other hand, this limit is expected to be a very good first approximation to the reality. To meet this expectation one should take \( n_1 = 0 \) for high enough resonances. But the chiral partners are then degenerate. The same can be repeated for the part involving the covariant derivative of the pion field, Eq. (12) in Ref. [1].

The authors of Ref. [1] argued in a companion paper that inclusion of finite width (i.e. taking into account next-to-leading order in the large-\( N_c \) counting) into QCD sum rules can, in principle, destroy the chiral symmetry restoration. Skipping some possible technical problems, we would like to note that unlike the string-like mass spectrum for light mesons, \( m^2(n) \sim n \), the string-motivated formula \( \Gamma(n) \sim \sqrt{n} \) is not supported experimentally. This can be straightforwardly checked by looking at Particle Data [11] and review [12]. Of course, may be this relation sets in at a very large \( n \), like in the \('t Hooft model (planar dim2 QCD), nevertheless there may be this relation sets in at a very large \( n \). But like in the first \( 't Hooft model (planar dim2 QCD), nevertheless there may be this relation sets in at a very large \( n \). But like in the

In Ref. [10] the analysis carried out in \( \mathbb{R}^3 \) was taken as an example of advocated ideology. We would like to remind that Ref. [8] deals with the ground states only. A result is that at any given helicity the mass matrix \( m^2 \) may be written as the sum of a chiral scalar \( m^2_0 \) and the fourth component of a chiral four-vector \( m^2_4 \) with respect to \( SU(2)_L \times SU(2)_R \) formed by the isospin \( T \) and the axial vector coupling matrix \( X \). The term \( m^2_4 \) appears due to existence of vacuum Regge trajectories and destroys the algebraic chiral symmetry. In order to extrapolate this conclusion to higher excitations, evidently, one has to apply the logic used in \( \mathbb{R}^3 \) to these states. However, from experiment (a strong suppression of reactions with one pion for many higher meson states) and generalized PCAC the matrix \( X \) is expected to be trivial resulting in \( m^2_4 = 0 \) (from Eq. (1.14) in Ref. [8]). This observation is only suggestive, just demonstrating that application of results from Ref. [8] to higher excitations needs a serious reanalysis.

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