STREAMER PROPAGATION IN MAGNETIC FIELD

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Abstract

The propagation of a streamer near an insulating surface under the influence of a transverse magnetic field is theoretically investigated. In the weak magnetic field limit it is shown that the trajectory of the streamer has a circular form with a radius that is much larger than the cyclotron radius of an electron. The charge distribution within the streamer head is strongly polarized by the Lorentz force exerted perpendicular to the streamer velocity. A critical magnetic field for the branching of a streamer is estimated. Our results are in good agreement with available experimental data.

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Recent experiments [1] on gas breakdown near an insulating surface in a high magnetic field $\mathbf{B}$ have shown new remarkable properties of such discharges. The channel of discharge in a magnetic field appears to have a circular form with radius $R_s$ several orders of magnitude
larger than the electronic cyclotron radius. It decreases with increasing $B$ and reaches $R_s \sim 1 \text{cm}$ at $B \sim 7 \text{T}$. At higher magnetic field the discharge has a branched structure. These experiments have shown that the streamer propagation cannot be treated as the motion of a charge particle in crossed external electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields.

Interest in the theory of streamers is usually associated with investigation of gas breakdown phenomena. On an insulating surface near a point electrode, in the region with strong electric field, the discharge has a filamentary structure. The tip of the filament moves with high velocity $v_0$ ($v_0 \sim 10^8 \text{cm/s}$), that exceeds the drift velocity $v_d$ of electrons in the streamer head field $\mathbf{E}_s$. The increase of external electric field $\mathbf{E}$ results in penetration of some filaments (so called "leaders") deep into the surrounding gas. Although the plasma parameters in streamers are different from those in leaders, their propagation is associated with the same physical processes and is defined mainly by the parameters of the streamer head. Since we are interested in the behavior of the streamer front only, we will not distinguish here between a streamer and a leader.

The streamer propagation mechanism was suggested by Raether, Loeb, and Meek [2]-[4], and was further developed by other authors [5]-[7]. According to this theory the charged head induces in its vicinity a strong electric field. This field leads to the increase of the electron density ahead of the streamer front due to impact ionization. The charge is displaced from this region via Maxwell relaxation. It is assumed that the free electron density ahead of the streamer front is not zero due, e.g. to absorption of the streamer head radiation produced, for example, by the streamer head radiation. The simple model which takes into account only these main processes was considered by M.I. D’yakonov and V.Y. Kashovskii [8]-[9]. They have estimated theoretically the streamer parameters and have shown that the streamer velocity $v_0$ and radius $r_s$ change smoothly with the external electric field $\mathbf{E}$, such that the propagation of the streamer head can be treated as a quasistationary process.

In the present paper we generalize the streamer model [8]-[9] to include an external magnetic field. It is assumed that the plasma filaments propagate in a plane perpendicular to the external magnetic field and the streamer parameters do not change in the direction parallel
to the magnetic field. It is shown that, in the weak magnetic field limit, a quasistationary streamer in the frame of reference rotating with a constant angular velocity \( \omega_s = v_0/R_s \), proportional to the head charge density, can be considered as a streamer in the absence of the magnetic field. We estimate the main parameters of a streamer head, and show that the obtained value for the radius of curvature is in close agreement with experimental data [1].

The influence of the magnetic field on the charge distribution within the streamer head is discussed, and a critical magnetic field for the onset of branching is estimated and compared to the experimental data.

Since the energy relaxation time of electrons is much larger than the electron-ion relaxation time we ignore the gas heating processes. Thus the concentration of atoms changes smoothly on the distance of the order of the streamer head size and is assumed to be constant inside the head. We neglect also the ion drift velocity in comparison with electron drift velocity \( v_d \) and streamer velocity \( v_0 \).

The system of equations for the electron density \( n \), the ion density \( N \), and the electric field \( \vec{E} \) is

\[
\frac{\partial n}{\partial t} + \text{div}(n \vec{v}_d) = \beta(E) n
\]

\[
\frac{\partial N}{\partial t} = \beta(E) n
\]

\[
\text{div} \vec{E} = 4\pi \rho(x), \quad \text{rot} \vec{E} = 0
\]

(1)

where \( \rho(x) = e(N - n) \) is the charge distribution, \( \beta(E) = v_d \alpha(E) \). The impact-ionization coefficient \( \alpha(E) \) increases very sharply with the field and saturates at some field value \( E_0 \)

\[
\alpha(E) = \alpha_0 e^{-E_0/E}
\]

(2)

We assume for simplicity that the electron drift velocity is proportional to electric field \( \vec{E} \), \( v_{di} = \sum_k \mu_{ik} E_k \). Without external magnetic field the mobility \( \mu_{ik} \) is a diagonal tensor,
\( \mu_{ik} = \mu_0 \delta_{ik} \). In a weak magnetic field the mobility \( \mu_{ik} \) is a function of \( \vec{B} \), which can be written as

\[
\mu_{ik} = \frac{\mu_0}{1 + \gamma^2} (\delta_{ik} + \gamma \varepsilon_{ik}),
\]

(3)

where \( \varepsilon_{ik} \) is the antisymmetric tensor in a plane perpendicular to \( \vec{B} \). The parameter \( \gamma = \omega_B \tau_{ea} \), where \( \omega_B = \frac{eB}{m_e c} \) is the cyclotron frequency and \( \tau_{ea} \) is the time of electron-atom collisions, is assumed to be small, \( \gamma \ll 1 \). In what follows we will be interested only in linear corrections in \( \gamma \) to the solution of Eq. (1). Since the parameters \( \alpha_0, E_0, \) and \( v_d \) depend on \( \gamma^2 \) [11] they are magnetic field independent, in our approximation.

Let us consider the streamer propagation equation (1) in the frame of reference with the origin at \( \vec{r}_h = \int \vec{r} \rho(\vec{r}, t) d\vec{r} \) and rotating with an angular velocity \( \omega(t) \). The resulting new coordinates are:

\[
\eta_i = \sum_k \Omega_{ik}(t) (x_k - r_{hk}(t))
\]

(4)

where \( \Omega_{ik}(t) \) is the rotation matrix for the angle \( \varphi = \int \omega(t) dt \) in the plane perpendicular to \( \vec{B} \):

\[
\Omega_{ik}(t) = \begin{pmatrix}
\cos \varphi & \sin \varphi \\
-\sin \varphi & \cos \varphi
\end{pmatrix}
\]

(5)

The transformation of derivatives are

\[
\frac{\partial}{\partial x_i} \rightarrow \sum_k \frac{\partial}{\partial \eta_k} \Omega_{ki}, \quad \frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial t} + \sum_{ikl} \frac{\partial}{\partial \eta_k} \left[ \Omega_{il} \Omega_{kl}^{-1} \eta_l - \Omega_{ki} \dot{r}_{hi} \right]
\]

(6)

Here the dot denotes time derivative. It follows from Eq. (6) that the streamer charge propagation is quasistationary if \( \sum_k \dot{\Omega}_{ik} \Omega_{kl}^{-1} = \varepsilon_{il} \dot{\omega} = const \) and \( \sum_i \dot{r}_{hi} = v_{0k} = const \). Thus the head of the quasistationary propagating streamer moves with constant velocity \( v_0 \) along a circle with radius \( R_s = v_0 / \omega \).

Rewriting Eq.(1) for quasistationary propagation in the rotating frame we obtain

\[
\sum_i \frac{\partial}{\partial \eta_i} n \left[ -v_{0i} + \omega \varepsilon_{il} \eta_l + \mu_0 (E_i + \gamma \varepsilon_{il} E_0 l) \right] = \beta(E) n
\]
\[-\sum_i v_0 \frac{\partial N}{\partial \eta_i} = \beta (E) n\]

\[\sum_i \frac{\partial E_i}{\partial \eta_i} = 4\pi \rho (\eta), \sum_{ik} \varepsilon_{ik} \frac{\partial E_i}{\partial \eta_k} = 0\]  \hspace{1cm} (7)

Eq. (7) should be solved with the following boundary condition

\[\rho_s v_0 = e n_s \mu_0 E_s,\]  \hspace{1cm} (8)

which follows from the charge conservation on the surface of the streamer front. Here \( n_s \) and \( \rho_s = e (N - n_s) \) are the electron and the charge densities on the front.

Appearing in the first order term with \( \gamma \), the electric field \( E_{0k} \) is the field of a streamer propagating in the absence of external magnetic field. It consists of two parts: the external field \( \vec{E} \) and field \( \vec{E}_\rho \) created by the head charge. Field \( \vec{E}_\rho \) is a symmetrical function with respect to the streamer axis. Usually \( |\vec{E}| \) is negligible in comparison with \( |\vec{E}_\rho| \), but near the electrode it can strongly influence the streamer propagation.

Let us expand \( E_{0k} (\eta) \) near the central point \( \eta_i = 0 \)

\[E_{0k} (\eta) = E_{0k} (0) + a \sum_i \delta_{ki} \eta_i + \sum_i b_{ki} \eta_i + D_k\]  \hspace{1cm} (9)

where \( a = 2\pi \rho (0) \), \( b_{kl} \) - symmetrical matrix with \( Sp (b) = 0 \). The term \( E_{0k} (0) + \sum_i b_{ki} \eta_i \) corresponds to the potential field, which satisfies the Laplace equation and can be absorbed into \( E_k \) as a correction. The field \( D_k \), defined by Eq.(9), is proportional to the deviation of the charge distribution from the uniform one. It is small in the central region and becomes large near the surface of the streamer head. Assuming that the streamer propagation is determined mainly by the central region, one can discard the terms with \( D \) while determining the streamer trajectory.

The second term in (9) leads to the streamer curving. At \( \omega = -2\pi \rho (0) \gamma \mu_0 \) the equations (7) turn to a system of equations describing the quasistationary streamer propagation without magnetic field. Thus streamers moving from cathode or anode will curve in opposite directions with frequency \( \omega_s = |\omega| = |2\pi \rho (0) \gamma \mu_0| \).
Introducing Maxwell relaxation time $\tau_m^{-1} = 4\pi \mu_0 e n_s$, one obtains $\omega_s \tau_m = \frac{\gamma \rho(0)}{2en_s}$. If the streamer radius $r_s$ is of the same order of magnitude as the characteristic distance of the increase of electric field from internal region to the front, one can estimate $r_s$ as $r_s \simeq \tau_m v_0$. So we have

$$\frac{r_s}{R_s} = \frac{\gamma \rho(0)}{2en_s}$$

This value is very small, since $\gamma \ll 1$ and $\frac{\rho(0)}{en_s} \sim \frac{\mu E_s}{v_0} \ll 1$. The last inequality can be obtained from Eq. (8) at $\rho(0) \simeq \rho_s$. Parameter $\gamma$ is proportional to magnetic field $B$, so the streamer radius decreases as $1/B$. This form of $R_s (B)$ is somewhat different from the experimentally observed field dependence reported in [1], i.e. $R_s (B) \sim 1/B^{\alpha}$, where $\alpha \sim 1.3 - 1.5$. Such a disagreement is connected probably with the approximate description of the ionization coefficient and mobility by formulas (2), (3). It must be especially noticeable at high magnetic field.

To evaluate the streamer head charge we will consider the one dimensional streamer equations [4]. Such approximation holds if the width $\delta$ of the streamer front is much smaller than the head size $r_s$: $\delta \ll r_s$. This is the case if the electron density in front of the streamer is much smaller than inside [4]. Equations [4] at $B = 0$ have a simple analytical solution. Assuming that the streamer moves along the $x$-axis and choosing the boundary conditions as $n (\infty) = n_\infty$, $E (\infty) = 0$, and $n (E = E_s) = 0$ we can easily obtain the relation between the equilibrium electron density $n_\infty$ and the electric field $E_s$ on the front

$$\frac{n_\infty}{n_0} = \frac{1}{1 - \frac{\mu_0 E_s}{v_0}} \int_0^{E_s/E_0} e^{-1/x} dx$$

Here $n_0 = \frac{\alpha_0}{4\pi e}$. This solution describes a plane wave with narrow front if $\mu_0 E_s \ll v_0$. In the opposite case $\mu_0 E \gg v_0$, one can neglect time derivatives in [4] and obtain the stationary solution.

The equilibrium electron density $n_\infty$ and the propagation velocity $v_0$ are defined by the conditions of the streamer formation. This stage of the discharge development should be described by essentially nonstationary equations. Their solution depends on external
electric field and parameters of initial "seed". On the quasistationary stage of the streamer propagation the electron density \( n_\infty \) can be estimated with the help of the relation

\[
r_s \simeq \frac{v_0}{4\pi \varepsilon \mu_0 n_s}
\]

Here it is supposed that \( n_\infty \simeq n_s \) in agreement with \( \rho(0) \sim \rho_s \ll en_s \). Equation (8) allows to relate the charge density \( \rho_s \) with \( E_s \). Note that \( E_s \) and \( \rho_s/en_s \) have logarithmic dependence on the density \( n_s \) and radius \( r_s \). Thus, the experimental error in \( r_s \) gives rise to small logarithmic correction to the relative charge \( \rho_s/en_s \).

Let us now compare our results with the experiment. For the streamer and plasma parameters in the absence of a magnetic field we have used the data from the paper of Dhali and Williams [12] for the streamer in \( N_2 \) at atmosphere pressure: \( v_0 = 2 \cdot 10^8 cm/s, r_s \simeq 10^{-2} cm \). Substituting these values to (11) and (12) one obtains

\[
n_s \simeq 3 \cdot 10^{13} cm^{-3},
\]

\[
E_s/E_0 \simeq 0.6, \rho_s/en_s \simeq \mu_0 E_s/v_0 \simeq 0.2.
\]

Estimating \( \gamma = 0.04 \) at \( B = 1T \) we have from Eq. (10) for the trajectory curvature radius \( R_s \simeq 0.5 cm \) at \( B = 5T \). The experimental value of \( R_s \) [1] for the same conditions is slightly larger, i.e. \( R_s^{ex} \simeq 1.2 cm \). This discrepancy can be explained e.g. by the growth of the charge density from external region towards the streamer front.

Let us consider the streamer in the limit of an infinitely narrow front in the system of reference where the streamer head is at rest. The electric field \( E \) inside the head is sufficiently small so that the ionization process can be safely neglected. The ions are assumed not to be affected by the electromagnetic field, i.e. \( N = \text{const.} \) The corresponding equations are

\[
\sum_{kl} \frac{\partial}{\partial \eta_k} \left[ n (-v_{0k} + \mu_{kl} E_l + \omega \varepsilon_{kl} \eta_l) \right] = 0
\]

\[
\sum_k \frac{\partial E_k}{\partial \eta_k} = 4\pi (N - n)
\]

(13)

For simplicity the streamer body will be represented as a cylinder with radius \( r_s \) and axis directed along the \( \eta_k \)-axis. Writing the content of the square brackets in Eq.(13) by:

\[
\sum_l n (-v_{0k} + \mu_{kl} E_l + \omega \varepsilon_{kl} \eta_l) = v_0 N \frac{\partial \Phi}{\partial \eta_k}
\]

(14)
the function $\Phi$ satisfies the Laplace equation $\Delta \Phi = 0$ inside the streamer body with boundary conditions

$$\frac{\partial \Phi}{\partial \eta_x} = 1, \quad \frac{\partial \Phi}{\partial \eta_y} = 0 \text{ at } \eta_x = 0; \quad \frac{\partial \Phi}{\partial \eta_k} = 0 \text{ at } \eta_y = r_s; \quad \Phi \to 0 \text{ at } \eta_x \to \infty$$  \hspace{1cm} (15)

Using $E_l$ from (14) and substituting it into Poisson equation we obtain the following equation for the normalized electron density $\overline{n} \equiv n/N$

$$\left( \nu_{ik} \frac{\partial \Phi}{\partial \eta_k} \right) \frac{\partial \overline{n}}{\partial \eta_i} = - \frac{\pi^2 (\overline{n} - 1)}{L_0} + \frac{2 \gamma \pi \overline{n}}{R_s}$$  \hspace{1cm} (16)

with the boundary condition

$$\overline{n} (\eta_x = 0, \eta_y = 0) = 1 + \Delta \overline{n}$$  \hspace{1cm} (17)

where $\nu_{ik} = \delta_{ik} - \gamma \varepsilon_{ik}$, $L_0 = \frac{\nu_{ik}}{4\pi \varepsilon_0} \mu$ is the characteristic length of the charge relaxation and $\Delta \overline{n} = \rho_s/eN$. Since $\gamma \ll 1$ and $L_0 \approx r_s \ll R_s$ the second term on the RHS of (16) may be neglected. In the case of small charge density $e \Delta \overline{n} \ll e \overline{n}$ the equation (16) has a simple analytical solution

$$\overline{n} (\eta_x, \eta_y) = 1 + \Delta \overline{n} \exp \left( - \frac{s (\eta_x, \eta_y)}{L_0} \right)$$  \hspace{1cm} (18)

where the effective path $s (\eta_x, \eta_y)$ is defined by an integral over a path from the point $(0, 0)$ to the point $\eta \equiv (\eta_x, \eta_y)$, i.e:

$$s (\eta_x, \eta_y) = \int_{(0,0)}^{(\eta_x, \eta_y)} \sum_{ik} \nu_{ik} \frac{\partial \Phi}{\partial \eta_k} d\eta_i \sum_{i} \sum_{k} \left| \nu_{ik} \frac{\partial \Phi}{\partial \eta_k} \right|^2$$  \hspace{1cm} (19)

According to (14) near the front surface $\frac{\partial \Phi}{\partial \eta_i} = \delta_{ix}$, so that to the first order with $\gamma$, $s (\eta) = \eta_x - \gamma \eta_y$ and

$$\overline{n} (\eta_x, \eta_y) = 1 + \Delta \overline{n} \exp \left( - \frac{\eta_x}{L_0} + \gamma \frac{\eta_y}{L_0} \right)$$  \hspace{1cm} (20)

The corresponding electric field is

$$E_{\eta_x} (\eta_x, \eta_y) = E_s \exp \left( - \frac{\eta_x}{L_0} + \gamma \frac{\eta_y}{L_0} \right)$$
\[ E_{ny}(\eta_x, \eta_y) = \gamma E_s \exp \left( \frac{-\eta_x}{L_0} + \gamma \frac{\eta_y}{L_0} \right) \] (21)

where \( E_s = e\Delta \tau \frac{\Delta \omega}{\mu_0} \).

Thus, an electric field \( E \) of the order of \( E_s \), in the \( \eta_x \)-direction at the streamer front stimulates its propagation in this direction. It is therefore reasonable to suggest that a sufficiently strong electric field component \( E_{ny} \sim E_s \), perpendicular to the streamer propagation, will lead to the breakdown of the streamer head and the formation of a new streamer deflected along the \( \eta_y \)-direction with respect to the original one. The new streamers will arise only from one side in the plane transverse to \( \vec{B} \). Substituting \( \eta_x = 0, \eta_y = r_s \simeq L_0 \) in (21) we conclude that the condition \( E_{ny} \sim E_s \) is fulfilled at \( \gamma e^\gamma \sim 1 \), i.e. \( \gamma \simeq 0.6 \). This value for the atmosphere discharge in \( N_2 \) corresponds to \( B = 12T \), which closely agree with the experimental result \( B^{ex} \simeq 7T \) [1].

In conclusion we have shown that the simple model taking into account only the main processes provides a reasonably good description of the streamer discharge in a magnetic field. The streamer head propagation is very similar to the movement of a free charged "particle". Without magnetic field this "particle" moves with a constant velocity \( v_0 = \text{const} \). In the presence of magnetic field the trajectory has a circular form. Such a simple picture occurs when the external electric field \( \mathcal{E} \) is negligible in comparison with the field \( E_s \) of the charged streamer head. Nevertheless, the role of the external field \( \mathcal{E} \) is very important not only for maintenance of the discharge, but also for the definition of the streamer parameters on the initial stage of the development. Strong electric field \( \mathcal{E} \sim \mathcal{E}_f \sim \mathcal{E} \) distorts the circular trajectory making it similar to the trajectory of a charged particle in crossed electric and magnetic fields. This phenomenon was observed in [1].

To estimate the "particle" mass density \( \rho_m \), we compare the expression for the radius \( R_s \) in the form \( R_s = \frac{\omega_{mc} \rho_m}{\rho_s B} \) with Eq. (10). We obtain the following expression for the ratio of the mass and the charge densities

\[ \frac{\rho_m}{\rho_s} = \frac{2}{\Delta n} \frac{\tau_m m_e}{\tau_{ea} e} \] (22)
Because of the large parameter \( \frac{2 m_e}{\Delta m \tau_{ca}} \gg 1 \) the streamer head turns in magnetic field more slowly than the particle with charge density \( \rho_s \) and mass density \( \rho_m \simeq m_e \rho_s / e \), whose radius does not depend on the plasma parameters of the streamer head.

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REFERENCES

[1] P.Uhlig, J.C.Maen, and P.Wyder, Phys. Rev. Lett. 63, 1968 (1989)

[2] M.Raether, Electron Analanches and Breakdown in Gases. (Butterworths, 1964)

[3] J.M.Meek, J.D.Craggs, Electron Breakdown of Gases, (Oxford, 1953)

[4] L.B.Loebe, Science 148, 1417 (1965)

[5] D.L.Turcotte, R.S.B.Ong, J. Plasma Physics 2, part 2, 145 (1968)

[6] G.A.Dawson, W.P.Winn, Zeittschrift für Physik 183, 159 (1965)

[7] R.Kligbell, D.A.Tidman, R.F.Fernsler, Physics of fluids 15, 1969 (1972)

[8] M.I.D’yakonov, V.Yu.Kachorovskii, Zh. Eksp. Teor. Fiz. 94, 321 (1988)

[9] M.I.D’yakonov, V.Yu.Kachorovskii, Zh. Eksp. Teor. Fiz. 95, 1850 (1989)

[10] A.von. Engel, Ionized Gases, (Oxford, 1965)

[11] Sanborn C.Brown, Introduction to Electrical Discharge in Gases, ( New York, 1966 )

[12] S.K.Dhali, P.F.Willians, J. Appl. Phys. 62, 4696 (1987)