EXPLORING AT MOST TWIN OUTER PERFECT DOMINATION NUMBER FOR GENERAL BINARY TREE AND SOME SPECIAL GRAPHS

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Abstract: A set $S \subseteq V(G)$ said to be an at most twin outer perfect dominating set if for every vertex $v \in V-S$, $1 \leq |N(v) \cap S| \leq 2$ and $< V-S>$ has at least one perfect matching. The minimum cardinality of at most twin outer perfect dominating set is called the at most twin outer perfect domination number and $\gamma_{atop}(G)$ denotes this number. This was initiated by G.Mahadevan.et.al., recently. Here we find this number for general Binary-tree, corona product of paths and cycles and lotus inside graph.

Key words Complementary perfect domination number, at most twin outer perfect domination number.

AMS Subject Classification: 05C69

1. Motivation
A set $S \subseteq V$ is called a complementary perfect dominating set, if $S$ is a dominating set of $G$ and the induced sub graph $< V-S >$ has a perfect matching. The minimum cardinality taken over all complementary perfect dominating sets is called the complementary perfect domination number and is denoted by $\gamma_{cp}(G)$ this was initiated by Paulraj Joseph et.al., [7]. A subset $S \subseteq V$ in a graph $G$ is a $[j,k]$-set if, $j \leq |N(V) \cap S| \leq k$ for every vertex, for any non-negative integer $j$ and $k$. this was first studied by Mustapha Chellali et.al.,[6]. Later, in [8], Xiaoqing Yang and Baoyindureng Wu, extended this study and initiated $[1,2]$- domination number. A vertex set $S$ in graph $G$ is $[1,2]$- set if, $1 \leq |N(V) \cap S| \leq 2$ for every vertex $v \in V-S$, that is, every vertex $v \in V-S$ is adjacent to either one or two vertices in $S$. The minimum cardinality of a $[1,2]$-set of $G$ is denoted by $\gamma_{[1,2]}(G)$ and it is said to be $[1,2]$ domination number of $G$. The above paper, stimulates us to do something using the condition of perfect matching in the complement of $[1,2]$ dominating set and we call this parameter as atmost twin outer perfect domination number of a graph. Motivated by above in [3], G.Mahadevan, et.al., introduced the concept of at most twin outer perfect domination number of a graph. In [4], the authors obtained many results of this parameter with graph colouring.
2. Introduction

Graphs considered here are simple and connected. The graph lotus inside circle is denoted by LIC, \( t \geq 3 \) and is defined as follows. Let \( S \) be the star graph with vertices \( x_0, x_1, x_2, \ldots, x_t \) whose center is \( x_0 \). Let \( C_t \) be the cycle of length \( t \) whose vertices are \( y_1, y_2, \ldots, y_t \). We join \( y_t \) with \( x_t \) and \( x_{t+1} \) for each \( 1 \leq i \leq t \) we join \( y_i \) with \( x_i \) and \( x_{i+1} \). The corona \( G_1 \odot G_2 \) is obtained by taking one copy of \( G_1 \) of order \( p \) and \( p \) copies of \( G_2 \) and then joining the \( i^{th} \) vertex of \( G_1 \) to every vertex in the \( i^{th} \) copy of \( G_2 \). A tree is a 2-array tree (binary tree) if every vertex has degree at most 3. If a root is appointed in the tree, then every vertex has at most 2 children. Throughout the paper, we find this number for lotus inside graph, corona product of two graphs and 2-array tree.

**Definition 2.1** A set \( S \subseteq V(G) \) said to be an at most twin outer perfect dominating set if for every vertex \( v \in V-S, 1 \leq \lfloor N(v) \cap S \rfloor \leq 2 \) and \( <V-S> \) has at least one perfect matching. The minimum cardinality of at most twin outer perfect dominating set is called the at most twin outer perfect domination number and \( \gamma_{atop}(G) \) denotes this number.

Example:

![Figure 2.1](image)

In figure 2.1, the set \( S = \{v_1, v_6, v_7\} \) is the at most twin outer perfect dominating set \( \gamma_{atop}(G) = 3 \)

3. atop-number for lotus inside graph and corona product of a graph

**Theorem 3.1** For \( t \geq 4 \)

\[
\gamma_{atop}(LIC_t) = \begin{cases} 
\left\lceil \frac{t}{2} \right\rceil + 1 & \text{if } t \equiv 0, 3 \pmod{4} \\
\left\lceil \frac{t}{2} \right\rceil + 2 & \text{otherwise}
\end{cases}
\]

**Proof:** Consider the set \( S = \{x_0, x_1, x_2, \ldots, x_{t-1}\} \). \( x_0 \) dominates \( x_1, x_2, \ldots, x_p \) and \( x_1, x_3, \ldots, x_{t-1} \) dominates \( y_1, y_2, \ldots, y_t \) respectively.

\[
\mathcal{D} = \{s \cup \{x_i\} : s \in S \text{ and } i \equiv 0, 3 \pmod{4}\} \text{ if } t \equiv 0, 3 \pmod{4}
\]

is an atopd-set.

\[
|D| = \begin{cases} 
\left\lceil \frac{t}{2} \right\rceil + 1 & \text{if } t \equiv 0, 3 \pmod{4} \\
\left\lceil \frac{t}{2} \right\rceil + 2 & \text{otherwise}
\end{cases}
\]

Thus \( \gamma_{atop}(LIC_t) \leq |D| \).

As \( D \) is an atopd set implies that \( |V - D| \) is even, if there exists an atopd set \( T \) such that \( |T| < |D| \), then \( |V - T| \) will be odd implies that \( <V - T> \) does not have a perfect matching. Hence \( \gamma_{atop}(LIC_t) \geq |D| \). Therefore \( \gamma_{atop}(LIC_t) = |D| \).
In figure 3.1, Consider the graph LIC₄
S = \{x₀, x₁, x₃\} is an atopd-set. Hence |S| = 3
\[
\gamma_{atop}(LIC₄) = \left[ \begin{array}{c} 1 \\ 1 \\ 1 \end{array} \right] + 1 = 3
\]

In figure 3.2, Consider the graph LIC₇
S = \{x₀, x₁, x₃, x₅, x₇\} is an atopd-set. Hence |S| = 5
\[
\gamma_{atop}(LIC₇) = \left[ \begin{array}{c} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{array} \right] + 1 = 7
\]

In figure 3.3, Consider the graph LIC₅
S = \{x₀, x₁, x₃, x₄, x₅\} is an atopd-set. Hence |S| = 5
\[
\gamma_{atop}(LIC₅) = \left[ \begin{array}{c} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{array} \right] + 1 = 5
\]

In figure 3.4, Consider the graph LIC₆
S = \{x₀, x₁, x₃, x₄, x₆\} is an atopd-set. Hence |S| = 5
\[
\gamma_{atop}(LIC₆) = \left[ \begin{array}{c} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{array} \right] + 1 = 5
\]
\[ \gamma_{atop}(LIC_o) = \left\lceil \frac{n}{2} \right\rceil - 2 = 5 \]

**Theorem 3.2** For a corona product graph \( r \geq 2, s \geq 2 \),
\[ \gamma_{atop}(P_r \odot P_s) = \begin{cases} 2r & \text{if } s \text{ is odd} \\ r & \text{if } s \text{ is even} \end{cases} \]

**Proof** Let \( |V(P_r)| \) and \( |V(P_s)| \) be \( r \) and \( s \) respectively. Let \( P_r \odot P_s \) be the corona product of \( P_r \) and \( P_s \).
Here \( V(P_r \odot P_s) = \{u_i, v_j \mid 1 \leq i \leq r, 1 \leq j \leq s\} \)

**Case** is even
Consider the set \( S = \{u_1, u_2, \ldots, u_r\} \) be any atopd-set.
Since, every vertex \( u_i \in S \) is linked with one vertex in \( S \), this gives that minimum atopd-set in \( P_r \odot P_s \) is \( S \).

Here \( s \) is even the corona product of \( P_r \) and \( P_s \) has a perfect matching. Hence \( S \) is an atopd-set in \( P_r \odot P_s \). This gives \( \gamma_{atop}(P_r \odot P_s) \leq |S| = 2r \).

If \( S \) contains at least one vertex not on \( P_r \), then atopd-set is not possible, hence \( |S| \geq r \).

**Case** is odd
Let \( S = \{u_1, u_2, \ldots, u_r, v_{11}, v_{21}, \ldots, v_{r1}\} \) be any atopd-set.
Since, every vertex \( u_i \in S \) is linked with one vertex in \( S \), this gives that minimum atopd-set in \( P_r \odot P_s \) is \( S \) and also the corona product of \( P_r \odot P_s \) has a perfect matching. Hence \( S \) is an atopd-set in \( P_r \odot P_s \). This gives \( \gamma_{atop}(P_r \odot P_s) \leq |S| = 2r \).

If \( S \) contains all the vertices only on the path \( P_r \), then the corona product of \( P_r \odot P_s \) has no perfect matching. Hence \( S \) contains \( r \) times of vertices in the path \( P_r \) in addition to all the vertices in the path \( P_s \), hence \( |S| \geq 2r \). Hence the result.

**Observation 3.3** For a corona product graph \( r \geq 2, s \geq 2 \)
\[ \gamma_{atop}(P_r \odot C_o) = \begin{cases} 2r & \text{if } s \text{ is odd} \\ r & \text{if } s \text{ is even} \end{cases} \]

**Observation 3.4** For a corona product graph \( r \geq 2, s \geq 2 \)
\[ \gamma_{atop}(C \odot P_s) = \begin{cases} 2r & \text{if } s \text{ is odd} \\ r & \text{if } s \text{ is even} \end{cases} \]

4. *atop-number for complete 2-array tree.*

**Theorem 4.1** For any complete 2-array tree \( B \) of \( r \) th level, \( \gamma_{atop}(B_{gr}) = \frac{10r \times 2^{3r-3}}{r} \)

**Proof** Let \( v_0 \) be the root vertex.
Let \( v_{11}, v_{21} \) are adjacent to \( v_0 \).
Let \( v_{12}, v_{22} \) are adjacent to \( v_{11} \) and \( v_{32}, v_{42} \) are adjacent to \( v_{22} \).
In a similar manner,
Let \( v_{1r}, v_{2r} \) are adjacent to \( v_{1r-1} \). Let \( v_{3r}, v_{4r} \) are adjacent to \( v_{2r-1}, \ldots \), \( v_{2r}, v_{3r} \) are adjacent to \( v_{3r-1} \).
Let \( A_{0}, A_{1}, A_{2}, \ldots A_{3r} \) be the levels and \( v_i^j \) denotes the number of vertices in the respective level.
Consider the set \( A_{3r} = \{v_{1r}, v_{3r}, \ldots, v_{3r} \} \)
\[ |A_{3r}| = \sum_{i=1}^{r} v_{3r}^i = 2^{3r} \]
Let \( A_{3r-1} = \{v_{1r-1}, v_{2r-1}, \ldots, v_{3r-1} \} \)
\[ |A_{3r-1}| = U_{i=2,4,6} v_i^{3r-1} = 2^{3r-2} \]

Let \( A_{3r-2} = \emptyset \)

\[ |A_{3r-2}| = 0 \]

Let \( A_{3r-3} = \{ v_1^{3r-3}, v_2^{3r-3}, v_3^{3r-3}, \ldots, v_i^{3r-3} \} \)

\[ |A_{3r-3}| = \{ v_1^{3r-3}, v_2^{3r-3}, v_3^{3r-3}, \ldots, v_i^{3r-3} \} \]

\[ |A_{3r-4}| = U_{i=2,4,6} v_i^{3r-4} = 2^{3r-6} \]

Proceeding this manner,

Let \( A_3 = \{ v_1^3, v_2^3, v_3^3, \ldots, v_i^3 \} \)

\[ |A_3| = 2^3 \]

Let \( A_2 = \{ v_2^3, v_4^3 \} \)

\[ |A_2| = 2 \]

Let \( A_1 = \emptyset \)

\[ |A_1| = 0 \]

Let \( A_0 = v_0 \)

\[ |A_0| = 1 \]

Let \( S = \bigcup_{i=0}^{3r} A_i \) is an atopd-set of \( B_{3r} \)

\[ |S| = |A_0| + |A_1| + |A_2| + \ldots + |A_{3r}| \]

\[ = 1 + 0 + 2^3 + 2^4 + \ldots + 2^{3r-3} + 2^{3r-5} + 2^{3r-7} + 2^{3r-9} + \ldots + 2^3 + 2^5 + 2^7 + \ldots + 2^{3r-3} + 2^{3r-5} + 2^{3r-7} + 2^{3r-9} + \ldots + 2^3 + 2^5 + 2^7 + \ldots + 2^{3r-3} + 2^{3r-5} + 2^{3r-7} + 2^{3r-9} + \ldots \]

\[ = \left( \frac{1 - 2^{3r+1}}{1 - 2} \right) - \left( \frac{2^{3r+1} - 2}{2} \right) \]

\[ = \left( \frac{2^{3r+1} - 1}{2} \right) - \left( \frac{2^{3r+1} - 2}{2} \right) \]

\[ = \frac{4 \times 2^{3r-1} - 4}{2} \]

\[ = \frac{10 \times 2^{3r-6} - 3}{2} \]

Hence \( \gamma_{\text{atopp}}(B_{3r}) \leq |S| = \frac{10 \times 2^{3r-6} - 3}{2} \)

(i.e) \( \gamma_{\text{atopp}}(B_{3r}) \leq \frac{10 \times 2^{3r-6} - 3}{2} \)

Also, it is observed that \( \gamma_{\text{cp}}(G) \leq \gamma_{\text{atopp}}(G) \)

Hence \( \gamma_{\text{atopp}}(B_{3r}) \geq \gamma_{\text{cp}}(B_{3r}) = \frac{10 \times 2^{3r-6} - 3}{2} \)
Let $A_{3r-6} =$ 

hence $\gamma_{atop}(B_{3r}) = \frac{10 \times 2^{2r-3}}{7}$

Thus, $\gamma_{atop}(B_{3r}) = \frac{10 \times 2^{2r-3}}{7}$

**Illustration**

![Figure 4.1]

Consider the level $B_3$

The dark dot denotes atop-set and let it be denoted by $S$ then $|S|=11$

$\gamma_{atop}(B_{3r}) = \frac{3 \times 4 \times 2^{2r-3}}{7}$ where $r = 1$

$\gamma_{atop}(B_{3}) = \frac{10 \times 2^{2r-3}}{7} = 11$

**Theorem 4.2** For any complete 2-array tree $B$ of $r+1^{th}$ level, $\gamma_{atop}(B_{2r+1}) = \frac{28 \times 2^{2r+1}}{7}$

**Proof** Let $v_0$ be the root vertex.

Let $v_1^1, v_1^0$ are adjacent to $v_0$.

Let $v_1^3, v_2^1$ are adjacent to $v_1^1$ and $v_3^2, v_4^2$ are adjacent to $v_2^1$.

In a similar manner,

Let $v_i^{r+1}, v_i^{3r+1}$ are adjacent to $v_i^{3r}$. Let $v_1^{3r+1}, v_4^{3r+1}$ are adjacent to $v_2^{3r} \ldots \ldots v_i^{3r+1}, v_i^{3r+1}$ are adjacent to $v_i^{3r}$.

Let $A_0, A_1, A_2 \ldots \ldots A_{3r+1}$ be the levels and $v_i^r$ denotes the number of vertices in the respective level.

Consider the set $A_{3r+1} = \{v_1^{3r+1}, v_2^{3r+1}, \ldots, v_i^{3r+1}\}$

$|A_{3r+1}| = 1 |v_1^{3r+1}| = 2^{2r+1}$

Let $A_3 = \{v_2^{3r}, v_4^{3r}, \ldots, v_i^{3r}\}$

$|A_{3r}| = U_{l=2,4,6,8} v_l^{3r} = 2^{8r-1}$

Let $A_{3r} = \emptyset$

$|A_{3r+1}| = 1$

Let $A_{3r} = \{v_1^{3r}, v_2^{3r}, \ldots, v_i^{3r}\}$

$|A_{3r+2}| = U_{l=2,4,6,8} v_l^{3r+2} = 2^{8r-2}$

Let $A_{3r+2} = \{v_1^{3r+2}, v_2^{3r+2}, \ldots, v_i^{3r+2}\}$

$|A_{3r+3}| = 1 |v_1^{3r+3}, v_2^{3r+3}, \ldots, v_i^{3r+3}|$

$|A_{3r+4}| = U_{l=2,4,6,8} v_l^{3r+4} = 2^{8r-3}$

Let $A_{3r+4} = \{v_1^{3r+4}, v_2^{3r+4}, \ldots, v_i^{3r+4}\}$

$|A_{3r+5}| = 0$

Let $A_{3r+6} = \{v_1^{3r+6}, v_2^{3r+6}, \ldots, v_i^{3r+6}\}$
\[ |A_{3r}\| = U_{4-3+3r+1} h_{r-6} = 2^{3r-6} \]

Proceeding this manner,

Let \( A_4 = \{ v_1, v_2, \ldots, v_i \} \)
\[ |A_4| = 2^4 \]

Let \( A_3 = \{ v_2, v_4, v_6, \ldots, v_i \} \)
\[ |A_3| = 2^2 \]

Let \( A_2 = \emptyset \)
\[ |A_2| = 0 \]

Let \( A_1 = 2^1 \)

Let \( A_0 = 1 \)
\[ |A_0| = 1 \]

Let \( S = \bigcup_{i=0}^{3r+1} A_i \) is an atopd-set of \( B_{3r+1} \)
\[ |S| = |A_0| + |A_1| + |A_2| + \ldots + |A_{3r+1}| \]
\[ = 1 + 2 + 2^2 + 2^3 + \ldots + 2^{h_7} + 2^{h_6} + 2^{h_5} + 2^{h_4} + 2^{h_3} + 2^{h_2} + 2^{h_1} + 2^{h_0} \]
\[ = (1 + 2 + 2^2 + 2^3 + \ldots + 2^{h_0}) - (2 + 2^2 + 2^3 + \ldots + 2^{h_0}) \]
\[ = \frac{(2^{h_1} - 2^{h_0} - 1)}{2} \]
\[ \approx \frac{2^{3r} + 1}{2} \]

Hence \( y_{\text{atop}}(B_{3r+1}) \leq |S| = \frac{20 \times 2^{3r} + 1}{7} \)

(i.e) \( y_{\text{atop}}(B_{3r+1}) \leq \frac{20 \times 2^{3r} + 1}{7} \)

Also, it is observed that
\[ y_{\text{CP}}(G) \geq y_{\text{atop}}(G) \]

Hence \( y_{\text{atop}}(B_{3r+1}) \geq y_{\text{CP}}(B_{3r+1}) = \frac{20 \times 2^{3r} + 1}{7} \)

\[ \text{hence } y_{\text{atop}}(B_{3r+1}) \geq \frac{20 \times 2^{3r} + 1}{7} \]

Thus, \( y_{\text{atop}}(B_{3r+1}) = \frac{20 \times 2^{3r} + 1}{7} \)

**Illustration**

![Figure 4.2](image-url)
Consider the level $B_4$

The dark dot denotes atopd-set and let it be denoted by $S$ then $|S|=23$

$$\gamma_{\text{atop}}(B_{3r+2}) = \frac{20 \times 2^{3r+1}}{r}$$

where $r = 1$

$$\gamma_{\text{atop}}(B_4) = \frac{161}{7} = 23$$

Theorem 4.3 For any complete 2-array tree $B$ of $r+2^{th}$ level, $\gamma_{\text{atop}}(B_{3r+2}) = \frac{40 \times 2^{3r-6}}{r}$

Proof Let $v_0$ be the root vertex.

Let $v_1^1$, $v_2^1$ are adjacent to $v_0$. Let $v_1^2$, $v_2^2$ are adjacent to $v_1^1$ and $v_3^2$, $v_4^2$ are adjacent to $v_2^1$.

In a similar manner,

Let $v_1^{3r-1}$, $v_2^{3r-1}$ are adjacent to $v_1^{3r-1}$. Let $v_3^{3r-2}$, $v_4^{3r-2}$ are adjacent to $v_2^{3r-2}$. $v_5^{3r-2}$, $v_6^{3r-2}$ are adjacent to $v_3^{3r-2}$.

Let $A_{0r}$, $A_1$, $A_2$,... $A_{3r+2}$ be the levels and $v_i^j$ denote the number of vertices in the respective level.

Consider the set $A_{3r+2} = \{v_1^{3r+2}, v_2^{3r+2}, \ldots, v_{3r+2}^{3r+2}\}$

$$|A_{3r+2}| = \bigcup_{i=1}^{r} v_i^{3r+2} = 2^{3r+2}$$

Let $A_{3r+1} = \{v_2^{3r+1}, v_3^{3r+1}, \ldots, v_{3r+2}^{3r+1}\}$

$$|A_{3r+1}| = \bigcup_{i=1}^{r} v_i^{3r+1} = 2^{3r+1}$$

Let $A_{3r+2} = \{v_2^{3r+2}, v_3^{3r+2}, \ldots, v_{3r+2}^{3r+2}\}$

$$|A_{3r+2}| = \bigcup_{i=1}^{r} v_i^{3r+2} = 2^{3r+2}$$

Let $A_{3r+3} = \emptyset$

Proceeding this manner,

Let $A_3 = \{v_1^5, v_2^5, v_3^5, \ldots, v_{3r}^5\}$

$$|A_3| = 2^3$$

Let $A_4 = \{v_2^6, v_3^6, \ldots, v_{3r}^6\}$

$$|A_4| = 2^3$$

Let $A_3 = \emptyset$

$$|A_3| = 0$$

Let $A_2 = 2^2$

$$|A_2| = 4$$

Let $A_1 = 1$

$$|A_1| = 1$$

Let $A_0 = \emptyset$

$$|A_0| = 0$$

Let $S = \bigcup_{i=0}^{3r+2} A_i$ is an atop-set of $B_{3r+2}$

$$|S| = |A_0| + |A_1| + |A_2| + \ldots + |A_{3r+2}|$$

$$= (1+2^3+2^3) + (1+2^3+2^3) + \ldots + (1+2^3+2^3)$$

$$= (1+2^3+2^3) - 2^{3r+2}$$

$$= \left(\frac{1-2^{3r+2}}{1-2}\right) - \left(\frac{(1-2^{3r+2})}{r}\right)$$
\[ \frac{7 \times 2^{3r+3} - 7 \times 2^{3r+4} + 2}{7} = \frac{2^{3r+4}(7-2)-2}{7} = \frac{4 \times 2^{3r+5}}{7} \]

Hence \( \gamma_{\text{atop}}(B_{3r+2}) \leq |S| = \frac{4 \times 2^{3r+5}}{7} \)

(i.e.) \( \gamma_{\text{atop}}(B_{3r+2}) \leq \frac{4 \times 2^{3r+5}}{7} \)

Also, it is observed that \( \gamma_{\text{cp}}(G) \leq \gamma_{\text{atop}}(G) \)

Hence \( \gamma_{\text{atop}}(B_{3r+2}) \leq \gamma_{\text{cp}}(B_{3r+2}) = \frac{42 \times 2^{3r+3}}{7} \)

hence \( \gamma_{\text{atop}}(B_{3r+2}) = \frac{42 \times 2^{3r+3}}{7} \)

Thus, \( \gamma_{\text{atop}}(B_{3r+2}) = \frac{42 \times 2^{3r+3}}{7} \)

**Illustration**

![Diagram](image)

*Figure 4.3*

Consider the level B_{5}
The dark dot denotes atopd-set and let it be denoted by S then |S|=45.

\( \gamma_{\text{atop}}(B_{3r+2}) = \frac{4 \times 2^{3r+5}}{7} \) where \( r = 1 \)
5. Conclusion
In this paper we explored the general result for atmost twin outer perfect domination number for 2-array tree, Lotus inside the flower and corona products of paths and cycles. The authors obtained this atmost twin outer perfect domination number for different various types products of graphs which will be reported in the subsequent papers.

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