Performance Analysis of OTFS Modulation with Receive Antenna Selection

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Abstract—In this paper, we analyze the performance of orthogonal time frequency space (OTFS) modulation with antenna selection at the receiver, where \( n_s \) out of \( n_r \) receive antennas with maximum channel Frobenius norms in the delay-Doppler (DD) domain are selected. Single-input multiple-output OTFS (SIMO-OTFS), multiple-input multiple-output OTFS (MIMO-OTFS), and space-time coded OTFS (STC-OTFS) systems with receive antenna selection (RAS) are considered. We consider these systems without and with phase rotation. Our diversity analysis results show that, with no phase rotation, SIMO-OTFS and MIMO-OTFS systems with RAS are rank deficient, and therefore they do not extract the full receive diversity as well as the diversity present in the DD domain. Also, Alamouti coded STC-OTFS system with RAS and no phase rotation extracts the full transmit diversity, but it fails to extract the DD diversity. On the other hand, SIMO-OTFS and STC-OTFS systems with RAS become full-ranked when phase rotation is used, because of which they extract the full spatial as well as the DD diversity present in the system. Also, when phase rotation is used, MIMO-OTFS systems with RAS extract the full DD diversity, but they do not extract the full receive diversity because of rank deficiency. Simulation results are shown to validate the analytically predicted diversity performance.

Index Terms—OTFS modulation, receive antenna selection, diversity, MIMO-OTFS, space-time coded OTFS.

I. INTRODUCTION

Orthogonal time frequency space (OTFS) modulation is a two-dimensional (2D) modulation scheme proposed in the recent literature to tackle the doubly-dispersive nature of mobile radio channels, caused by multipath propagation environments [1], [2], [8]. Conventional multicarrier modulation schemes such as orthogonal frequency division multiplexing (OFDM) embed information symbols in the time-frequency (TF) domain to mitigate inter-symbol interference (ISI) caused by time dispersion. However, the Doppler shifts encountered in high-mobility channels destroy the orthogonality among subcarriers in OFDM. This results in degraded performance of OFDM systems in time-varying channels [4]. OTFS, on the other hand, places the information symbols in delay-Doppler (DD) domain which result in 2D convolution of the information symbols with the channel in the DD domain. OTFS has been found to perform better than OFDM in high-Doppler communication scenarios, such as high-speed trains and vehicle-to-vehicle/vehicle-to-infrastructure communications. Since the signaling in OTFS is done in the DD domain rather than in the TF domain, the interaction of information symbol and rapidly time-varying channel appear as almost time invariant in the DD domain. Also, because of the constant DD channel gain experienced by a OTFS frame, design of equalizers and channel estimation in DD domain is easy. One more advantage of OTFS is that it can be implemented using existing multicarrier modulation schemes, such as OFDM, with additional pre-processing and post-processing modules [15].

Several papers in the recent literature have investigated many key issues in OTFS such as low-complexity signal detection [5]-[13], channel estimation [14]-[16], peak-to-average power ratio (PAPR) and pulse shapes [17]-[20], and multiple access [21]-[24]. In terms of performance analysis, an asymptotic diversity analysis for OTFS has been carried out in [25]. It established that the asymptotic diversity order achieved in single-input single-output OTFS (SISO-OTFS) is one for ideal biorthogonal waveforms. In other words, OTFS in its basic form does not extract the diversity present in the DD domain. It also explored a phase rotation scheme using transcendental numbers to extract full diversity in the DD domain. It has also reported diversity orders of \( n_r \) and \( n_r P \) for multiple-input multiple-output OTFS (MIMO-OTFS) without and with phase rotation, respectively, where \( n_r \) and \( P \) denote the number of receive antennas and the number of resolvable paths in the DD domain, respectively. The analysis in [26] on the effective diversity of OTFS using rectangular waveforms and a two-path channel has shown that the number of signal pairs that prevent the achievability of full rank is very small for sufficiently large frame sizes. The analysis in [27] for space-time coded OTFS (STC-OTFS) with Alamouti code with two transmit antennas has reported diversity orders of \( 2n_r \) and \( 2n_r P \) for STC-OTFS without and with phase rotation, respectively. Because of the good diversity slopes in the finite signal-to-noise ratio (SNR) regime even with small frame sizes, STC-OTFS was suggested to be suited for low-latency applications.

Antenna selection techniques allow the use of fewer radio frequency (RF) chains than the number of antenna elements. This reduces the RF hardware complexity and cost. In this regard, it is of interest to analyze the performance of OTFS with antenna selection, and such an analysis has not been reported so far. Our new and novel contributions in this paper can be highlighted as follows. First, we analyze and establish the diversity orders achieved by different multi-antenna OTFS systems with antenna selection at the receiver, where \( n_s \) out of \( n_r \) receive antennas are selected. Second, in rapidly time-varying channels, devising suitable antenna selection metric is a crucial issue. We address this issue by proposing the Frobenius norm of the channel matrix in the DD domain as
the antenna selection criterion. This is novel and attractive because it takes advantage of the simplicity of DD channel estimation in OTFS due to the sparsity and slow variation of rapidly time-varying channels when viewed in the DD domain.

In our analysis, we consider the diversity performance of single-input multiple-output OTFS (SIMO-OTFS), MIMO-OTFS, and STC-OTFS systems with receive antenna selection (RAS). Our diversity analysis results show that, with no phase rotation, SIMO-OTFS and MIMO-OTFS systems with RAS are rank deficient, and therefore they do not extract the full receive diversity as well as the diversity present in the DD domain. Also, Alamouti coded STC-OTFS system with RAS and no phase rotation extracts the full transmit diversity, but it fails to extract the DD diversity. On the other hand, SIMO-OTFS and STC-OTFS systems with RAS become full-ranked when phase rotation is used, because of which they extract the full spatial as well as the DD diversity present in the system. Also, when phase rotation is used, MIMO-OTFS systems with RAS extract the full DD diversity, but they do not extract the full receive diversity because of rank deficiency. A summary of the diversity orders achieved in different multi-antenna OTFS systems with RAS are presented in Table I in Sec. III. In the later sections, we will present analytical derivations for the diversity orders in Table I and supporting simulation results that verify the analytically predicted diversity orders.

The rest of the paper is organized as follows. The considered multi-antenna OTFS systems with receive antenna selection are presented in Sec. III. The diversity analyses of these systems for full rank and rank deficient are presented in Sec. III. Numerical results and discussions are presented in Sec. IV. Conclusions are presented in Sec. V.

Notations: Capital boldface letters denote matrices, lower case boldface letters denote vectors, diag\{x_1, \cdots, x_n\} denotes a diagonal matrix with \{x_1, \cdots, x_n\} as its diagonal entries, and \|X\| denotes the Frobenius norm of matrix X. Transpose and Hermitian operators are denoted by (\cdot)^T and (\cdot)^H, respectively. |c| and |S| denote the magnitude of the complex scalar c and size of the set S, respectively. E[\cdot] and Tr[\cdot] denote the expectation and trace operations, respectively. \mathcal{C}N(a, b) denotes complex Gaussian distribution with mean a and variance b.

II. MULTI-ANTENNA OTFS SYSTEMS WITH RAS

In this section, we present the basic OTFS modulation scheme and the system models corresponding to different multi-antenna OTFS systems. The analyses that follow in Sec. III are for integer Dopplers/delays, and the case of fractional Doppler/delays will be analyzed in the Appendix.

A. Basic OTFS modulation

The OTFS modulation scheme consists of cascaded structures of two 2D transforms at the transmitter and the receiver. The block diagram of the basic OTFS modulation scheme is shown in Fig. 1. At the transmitter, information symbols in the DD domain are mapped to TF domain using inverse symplectic finite Fourier transform (ISFFT) followed by windowing. The TF symbols are then converted to time domain using Heisenberg transform for transmission over the channel. At the receiver, Wigner transform (inverse of Heisenberg transform) is performed to get TF symbols. Using windowing and symplectic finite Fourier transform (SFFT), TF symbols are mapped back to DD domain for demodulation.

The information symbols \(x[k, l]\)s are multiplexed on an \(N \times M\) DD grid, given by

\[
\Gamma = \{(k, l), k = 0, \cdots, N-1, l = 0, \cdots, M-1\}, \quad (1)
\]

where \(1/N\) and \(1/M\Delta f\) denote the bin sizes in the Doppler domain and delay domain, respectively, and \(N\) and \(M\) denote the number of Doppler and delay bins, respectively. The DD domain symbols \(x[k, l]\)s are mapped to symbols in the TF domain \(X[n, m]\)s using ISFFT. Assuming rectangular windowing, the TF signal can be written as

\[
X[n, m] = \frac{1}{\sqrt{MN}} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} x[k, l] e^{j2\pi (\frac{n}{N} - \frac{k}{M})}, \quad (2)
\]

This TF signal is converted into a time domain signal \(x(t)\), using Heisenberg transform and transmit pulse \(g_{tx}(t)\), as

\[
x(t) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} X[n, m] g_{tx}(t-nT)e^{j2\pi m\Delta f(t-nT)}. \quad (3)
\]

The transmitted signal \(x(t)\) passes through the channel, whose complex baseband channel response in the DD domain, denoted by \(h(\tau, \nu)\), is given by

\[
h(\tau, \nu) = \sum_{i=1}^{P} h_i \delta(\tau - \tau_i) \delta(\nu - \nu_i), \quad (4)
\]

where \(P\) is the number of paths in the DD domain, and \(h_i\), \(\tau_i\), and \(\nu_i\) denote the channel gain, delay, and Doppler shift, respectively, associated with the \(i\)th path. The received time domain signal \(y(t)\) at the receiver is then given by

\[
y(t) = \int_{\tau} h(\tau, \nu)x(t-\tau)e^{j2\pi \nu(t-\tau)}d\tau + v(t), \quad (5)
\]

where \(v(t)\) denotes the additive white Gaussian noise.

At the receiver, the received signal \(y(t)\) is matched filtered with a receive pulse \(g_{rx}(t)\), yielding the cross-ambiguity function \(A_{g_{rx}, y}(t, f)\), given by

\[
A_{g_{rx}, y}(t, f) = \int g_{rx}^*(t' - t)y(t')e^{-j2\pi f(t'-t)}dt'. \quad (6)
\]
The pulses \(g_{tx}(t)\) and \(g_{rx}(t)\) are chosen such that the biorthogonality condition is satisfied, i.e., \(A_{g_{tx},g_{rx}}(f, t)|_{nT, m\Delta f} = \delta(m)\delta(n)\). Sampling \(A_{g_{tx},y}(f, t)|_{t = nT, f = m\Delta f}\) gives

\[
Y[n, m] = A_{g_{tx},y}(t, f)|_{t = nT, f = m\Delta f}.
\]

(7)

This received TF domain signal \(Y[n, m]\) is mapped to the corresponding DD domain signal \(y[k, l]\) using SFFT as

\[
y[k, l] = \frac{1}{\sqrt{MN}} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} Y[n, m] e^{-j2\pi \left( \frac{nk}{N} + \frac{ml}{M} \right)}.
\]

(8)

From (5), (8), the input-output relation in the DD domain can be written as (6)

\[
y[k, l] = \sum_{i=1}^{P} h_i x[(k - \beta_i)N, (l - \alpha_i)M] + v[k, l],
\]

(9)

where \(h_i = e^{-j2\pi \nu_i \tau_i}, \alpha_i\) and \(\beta_i\) are assumed to be integers corresponding to the indices of the delay tap and Doppler frequency associated with \(\tau_i\) and \(\nu_i\), respectively, i.e., \(\alpha_i = \frac{\alpha_i}{M\Delta f}\) and \(\nu_i = \frac{\beta_i}{N\Delta f}\). \(Y\) denotes the modulo \(N\) operation, and \(v[k, l]\) denotes the additive white Gaussian noise. Vectorizing the input-output relation in (9), we can write

\[
y = \mathbf{H} \mathbf{x} + \mathbf{v},
\]

(10)

where \(\mathbf{H} \in \mathbb{C}^{MN \times MN}, \mathbf{x}, \mathbf{y}, \mathbf{v} \in \mathbb{C}^{MN \times 1}\), the \((k + Nl)\)th entry of \(\mathbf{x}, x_{k+Nl} = x[k, l], k = 0, \cdots, N-1, l = 0, \cdots, M-1\) and \(x[k, l] \in \mathbb{A}\), where \(\mathbb{A}\) is the modulation alphabet (e.g., quadrature amplitude modulation (QAM) or phase shift keying (PSK)). Likewise, \(y_{k+Nl} = y[k, l]\) and \(v_{k+Nl} = v[k, l], k = 0, \cdots, N-1, l = 0, \cdots, M-1\). It is assumed that the \(h_i\)'s are i.i.d and are distributed as \(CN(0, 1/P)\), assuming uniform scattering profile.

**An alternate form of input-output relation (10):** The vectorized form of input-output relation in (10) can be written in an alternate form which is essential for our diversity analysis. This alternate representation is also useful in writing the system model for STC-OTFS systems. Towards this, it is observed that there are only \(P\) non-zero entries in each row and column of the equivalent channel matrix \(\mathbf{H}\) because of the modulo operations in (9), i.e., there are only \(MN\) non-zero entries in \(\mathbf{H}\). Also, among the non-zero entries there are only \(P\) unique values, since each transmitted symbol experiences the same channel gain as can be seen in (9). With this, the relation in (10) can be written in an alternate form as (25)

\[
y^T = \mathbf{h}'^T \mathbf{X} + \mathbf{v}^T,
\]

(11)

where \(y^T = 1 \times MN\) received vector, \(\mathbf{h}' = 1 \times P\) vector whose \(i\)th entry is given by \(h'_i = h_i e^{-j2\pi \nu_i \tau_i}\), \(\mathbf{v}^T = 1 \times MN\) noise vector, and \(\mathbf{X} = P \times MN\) matrix whose \(i\)th column \(\mathbf{X}[i], i = k + Nl, k = 0, \cdots, N-1, l = 0, \cdots, M-1\) is given by

\[
\mathbf{X}[i] = \begin{bmatrix}
x_{(k-\beta_1)N + (l-\alpha_1)M} \\
x_{(k-\beta_2)N + (l-\alpha_2)M} \\
\vdots \\
x_{(k-\beta_P)N + (l-\alpha_P)M}
\end{bmatrix},
\]

(12)

This representation allows us to view the matrix \(\mathbf{X}\) in the form of \(P \times MN\) symbol matrix.

**B. MIMO-OTFS with receive antenna selection**

The input-output relation of MIMO-OTFS system with \(n_r\) receive antennas and \(n_t\) transmit antennas can be written as

\[
\begin{bmatrix}
\mathbf{y}_1 \\
\vdots \\
\mathbf{y}_{n_r}
\end{bmatrix} = \begin{bmatrix}
\mathbf{H}_{11} & \cdots & \mathbf{H}_{1n_t} \\
\vdots & \ddots & \vdots \\
\mathbf{H}_{n_r1} & \cdots & \mathbf{H}_{n_rn_t}
\end{bmatrix} \begin{bmatrix}
\mathbf{x}_1 \\
\vdots \\
\mathbf{x}_{n_t}
\end{bmatrix} + \begin{bmatrix}
\mathbf{v}_1 \\
\vdots \\
\mathbf{v}_{n_r}
\end{bmatrix},
\]

(13)

or equivalently

\[
\tilde{\mathbf{y}} = \mathbf{H} \tilde{\mathbf{x}} + \mathbf{v},
\]

(14)

where \(\tilde{\mathbf{y}} \in \mathbb{C}^{n_r \times MN} \times 1\) is the received signal vector, \(\tilde{\mathbf{H}} \in \mathbb{C}^{n_r \times MN \times n_t \times MN}\) is the overall equivalent channel matrix with \(\mathbf{H}_{ij}\) being the \(MN \times MN\) equivalent channel matrix between the \(j\)th transmit antenna and \(i\)th receive antenna, \(\tilde{\mathbf{x}} \in \mathbb{C}^{n_t \times MN}\) is the OTFS transmit vector, and \(\mathbf{v} \in \mathbb{C}^{n_r \times MN}\) is the noise vector. Perfect DD channel knowledge is assumed at the receiver. The receiver selects \(n_s\) out of the \(n_t\) antennas with the largest Frobenius norms of the channel in the DD domain, i.e., selects the \(n_s\) antennas whose Frobenius norms among those of all the \(n_t\) antennas, given by

\[
\sum_{j=1}^{n_t} \|\mathbf{H}_{ij}\|^2, \quad i = 1, 2, \cdots, n_r,
\]

(15)

are the largest. Observing that each \(\mathbf{H}_{ij}\) contains only \(PM\) non-zero elements with \(P\) unique elements using the definition of Frobenius norm, the selection metric in (15) can be written as

\[
\sum_{k=1}^{P} \sum_{j=1}^{n_t} |h_{ij}^{(k)}|^2, \quad i = 1, 2, \cdots, n_r,
\]

(16)

where \(h_{ij}^{(k)}\) are the unique non-zero entries of \(\mathbf{H}_{ij}\). Therefore, with antenna selection, the input-output relation of the MIMO-OTFS system can be written as

\[
\begin{bmatrix}
\mathbf{y}_1' \\
\vdots \\
\mathbf{y}_{n_r}'
\end{bmatrix} = \begin{bmatrix}
\mathbf{H}_{11}' & \cdots & \mathbf{H}_{1n_t}' \\
\vdots & \ddots & \vdots \\
\mathbf{H}_{n_r1}' & \cdots & \mathbf{H}_{n_rn_t}'
\end{bmatrix} \begin{bmatrix}
\mathbf{x}_1 \\
\vdots \\
\mathbf{x}_{n_t}
\end{bmatrix} + \begin{bmatrix}
\mathbf{v}_1' \\
\vdots \\
\mathbf{v}_{n_r}'
\end{bmatrix},
\]

(17)

or equivalently

\[
\tilde{\mathbf{y}}' = \tilde{\mathbf{H}}' \tilde{\mathbf{x}} + \tilde{\mathbf{v}}',
\]

(18)

where \(\tilde{\mathbf{y}}' \in \mathbb{C}^{n_t \times MN \times 1}, \tilde{\mathbf{H}}' \in \mathbb{C}^{n_t \times MN \times n_t \times MN}\) is the equivalent channel matrix with antenna selection, \(\tilde{\mathbf{x}} \in \mathbb{C}^{n_t \times MN}\) is the OTFS transmit vector, and \(\tilde{\mathbf{v}}' \in \mathbb{C}^{n_t \times MN}\) is the noise vector. Figure 2 shows the block diagram of MIMO-OTFS with receive antenna selection.

**An alternate form of MIMO-OTFS with antenna selection:**

The input-output relation in (13) can be written in an alternate form similar to that in (11), by observing that each \(\mathbf{H}_i'\) in (17) contains only \(P\) unique non-zero elements and hence \(\tilde{\mathbf{H}}'\) in (13) contains only \(Pn_s n_t\) unique non-zero elements with each row having only \(Pn_s\) unique non-zero elements and each
respectively. Following the development of the system model, \( \bar{y}_{ij} \) denotes the OTFS transmit matrix in the \( X \) corresponding to matrices. An STC-OTFS codeword matrix \( \bar{x} \), uses the structure of the well known Alamouti code, generalized to matrices. That is, the \( X \) transmit symbol matrix, and \( \bar{v} \) is \( n \times P \) unique non-zero elements. Therefore, the compact form of the channel matrix with \( h_{ij} \in \mathbb{C}^{1 \times P} \), \( P \) containing \( X \) transmit antennas at the receiver can be written in the form

\[
\bar{y} = \bar{H} \bar{x} + \bar{v},
\]

where \( \bar{y} \in \mathbb{C}^{n \times P} \) with its \( i \)th row corresponding to the received signal in the \( i \)th selected receive antenna, \( \bar{H} \in \mathbb{C}^{n \times n \times P} \) is the channel matrix, \( \bar{v} \in \mathbb{C}^{n \times P} \) is the noise matrix.

C. STC-OTFS with antenna selection

Figure 3 shows the block diagram of STC-OTFS with receive antenna selection. In this subsection, we develop the system model for Alamouti code based STC-OTFS with receive antenna selection.

1) Alamouti STC-OTFS: Alamouti code based STC-OTFS uses the structure of the well known Alamouti code, generalized to matrices. An STC-OTFS codeword matrix \( \bar{x} \) is an \( n \times M \times T \times M \times N \) block matrix. Each block in this matrix is an \( M \times M \) OTFS transmit matrix; e.g., the block \( x_{kt} \) in \( \bar{x} \) denotes the OTFS transmit matrix in the \( t \)th frame from \( k \)th transmit antenna. If \( \bar{x} \) contains \( Z \) independent OTFS symbol matrices which are transmitted over \( T' \) frame uses, then the code rate is \( Z/T' \) symbols per channel use. A delay-Doppler channel which is quasi-stationary over \( T' \) frame duration is assumed. A \( 2M \times 2M \times N \) Alamouti STC-OTFS codeword matrix with \( n_t = T' = 2 \) is given by

\[
\bar{x} = \begin{bmatrix} X_1 & -X_2^H \\ X_2 & X_1^H \end{bmatrix},
\]

where \( X_1 \) and \( X_2 \) are the symbol matrices. That is, the OTFS transmit vectors corresponding to \( X_1 \) and \( X_2 \) are transmitted from the 1st and 2nd antennas, respectively, in the first frame. In the second frame, the vectors corresponding to \( -X_2^H \) and \( X_1^H \) are transmitted from the 1st and 2nd antennas, respectively. Following the development of the system model without receive antenna selection in (22), the input-output relation for Alamouti STC-OTFS with selection of \( n \) antennas at the receiver can be written in the form

\[
\begin{bmatrix} y_{i1} \\ \vdots \\ y_{in} \\ \end{bmatrix} = \begin{bmatrix} H_{i1} \\ \vdots \\ H_{iN} \\ \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_2 \\ \end{bmatrix} + \begin{bmatrix} v_{i1} \\ \vdots \\ v_{iN} \\ \end{bmatrix},
\]

where \( y_{ij} \in \mathbb{C}^{MN \times 1} \) is the received signal at the \( i \)th receive antenna in the \( j \)th time slot with \( \hat{y}_{ij} = P y_{ij} \), where \( P \) is \( M \times M \) permutation matrix given by

\[
P = P_M \otimes P_N,
\]

where \( \otimes \) denotes the Kronecker product, and \( P_M \) and \( P_N \) are left circulant matrices, which are given by

\[
P_M = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \quad \text{and} \quad P_N = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}. \]

(24)

The equivalent channel matrix between \( i \)th selected receive antenna and \( j \)th transmit antenna, and \( x_i \in \mathbb{C}^{MN \times 1} \) is the transmitted OTFS vector. The compact form of (22) is given by

\[
\bar{y}' = \bar{H}' \bar{x} + \bar{v}',
\]

where \( \bar{y}', \bar{v}' \in \mathbb{C}^{2n \times MN} \), \( \bar{H}' \in \mathbb{C}^{2n \times MN} \), and \( \bar{x} \in \mathbb{C}^{2MN \times 1} \).

An alternate form of Alamouti STC-OTFS with antenna selection: The input-output relation in (25) can be written in
D. OTFS with phase rotation

In this subsection, we present OTFS modulation with phase rotation. In OTFS with phase rotation, the OTFS vector \( \mathbf{x} \) is pre-multiplied by a phase rotation matrix \( \Phi \), which is of the form

\[
\Phi = \text{diag}\{\phi_0, \phi_1, \cdots, \phi_{M-1}\}.
\]

That is, \( \mathbf{x}' = \Phi \mathbf{x} \) is the phase rotated OTFS transmit vector. It has been shown in \([25]\) that SISO-OTFS with the above phase rotation achieves the full diversity available in the DD domain when \( \phi_i = e^{j\theta_i}, i = 0, 1, \cdots, M-1 \), are transcendental numbers with \( \theta_i \) being real, distinct and algebraic. We consider this phase rotation scheme for multi-antenna OTFS systems, where the OTFS vector in each transmit antenna is pre-multiplied by the phase rotation matrix \( \Phi \).

E. Rank of multi-antenna OTFS systems

In the next section, we carry out the diversity analysis for multi-antenna systems for full rank and rank deficient cases. In this subsection, we identify the rank of the considered multi-antenna OTFS systems without and with phase rotation.

1) MIMO-OTFS, SIMO-OTFS: Consider MIMO-OTFS \((n_t \geq 2)\) without phase rotation. Let \( \mathbf{\tilde{x}}_i \) and \( \mathbf{\tilde{x}}_j \) be two distinct symbol matrices defined in \([20]\). The minimum rank of \( (\mathbf{\tilde{x}}_i - \mathbf{\tilde{x}}_j) \) is \( 1 < \min(n_t P, MN) \) \([25]\). Therefore, MIMO-OTFS system with phase rotation is also rank deficient.

SIMO-OTFS can be viewed as a special case of MIMO-OTFS with \( n_t = 1 \). Therefore, for SIMO-OTFS without phase rotation, the minimum rank of \( (\mathbf{\tilde{x}}_i - \mathbf{\tilde{x}}_j) \) is \( 1 < \min(P, MN) \). Therefore, SIMO-OTFS system without phase rotation is rank deficient for \( P > 1 \). For \( P = 1 \), the dimension of \( (\mathbf{\tilde{x}}_i - \mathbf{\tilde{x}}_j) \) is \( 1 \times MN \) and the minimum rank is 1, so it is full rank. For SIMO-OTFS with phase rotation, the minimum rank of \( (\mathbf{\tilde{x}}_i - \mathbf{\tilde{x}}_j') \) is \( P = \min(P, MN) \). Since \( P \leq MN \) and minimum rank is \( P \), and so it is full rank.

2) STC-OTFS: Consider Alamouti STC-OTFS without phase rotation. Let \( \mathbf{\tilde{x}}_i \) and \( \mathbf{\tilde{x}}_j \) be the two distinct symbol matrices defined in \([27]\). The minimum rank of \( (\mathbf{\tilde{x}}_i - \mathbf{\tilde{x}}_j) \) is \( 2 < \min(2P, 2MN) \) \([27]\). Therefore, for \( P > 1 \) Alamouti STC-OTFS is rank deficient, and for \( P = 1 \) it is full rank with rank 2. For Alamouti STC-OTFS with phase rotation, the minimum rank of \( (\mathbf{\tilde{x}}_i - \mathbf{\tilde{x}}_j') \) is \( 2P \leq \min(2P, 2MN) \) \([27]\). Therefore, Alamouti STC-OTFS with phase rotation is full rank with rank \( 2P \).

III. Analysis of Multi-antenna OTFS with RAS

In this section, we analyze the performance of multi-antenna OTFS systems with RAS by deriving explicit upper bounds on pairwise error probability (PEP). We carry out the diversity analysis for full rank and rank deficient cases in the following subsections.

A. Full rank multi-antenna OTFS systems with RAS

Consider the case of full rank multi-antenna OTFS systems with receive antenna selection. Let \( \mathbf{\hat{x}}_i \) and \( \mathbf{\hat{x}}_j \) be two distinct symbol matrices. Assuming perfect DD channel knowledge and maximum likelihood (ML) detection at the receiver, the conditional PEP between the symbol matrices \( \mathbf{\hat{x}}_i \) and \( \mathbf{\hat{x}}_j \), assuming \( \mathbf{\hat{x}}_i \) to be the transmitted symbol matrix, is given by

\[
P(\mathbf{\hat{x}}_i \rightarrow \mathbf{\hat{x}}_j|\mathbf{\hat{h}}, \mathbf{\hat{x}}_i) = Q\left(\sqrt{\frac{\|\mathbf{\hat{h}}(\mathbf{\hat{x}}_i - \mathbf{\hat{x}}_j)\|^2}{2N_0}}\right),
\]

where \( Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-t^2/2} dt \). For convenience, the entries of \( \mathbf{\hat{h}} \) are normalized so that average energy per symbol time is one and the SNR, denoted by \( \gamma \), is given by \( \gamma = 1/N_0 \). Therefore, \( (29) \) can be written as

\[
P(\mathbf{\hat{x}}_i \rightarrow \mathbf{\hat{x}}_j|\mathbf{\hat{h}}, \mathbf{\hat{x}}_i) = Q\left(\sqrt{\frac{\|\mathbf{\hat{h}}(\mathbf{\hat{x}}_i - \mathbf{\hat{x}}_j)\|^2}{2}}\right).
\]

Averaging over the distribution of \( \mathbf{\hat{h}} \) and upper bounding using Chernoff bound, an upper bound on the unconditional PEP can be written as

\[
P(\mathbf{\hat{x}}_i \rightarrow \mathbf{\hat{x}}_j) \leq E_{\mathbf{\hat{h}}} \left[ \exp\left(-\frac{\gamma \|\mathbf{\hat{h}}(\mathbf{\hat{x}}_i - \mathbf{\hat{x}}_j)\|^2}{4}\right) \right].
\]
The distribution of $\tilde{H}$ is given by [29, 30]
\[
f_{\tilde{H}}(h_1', \ldots, h_n') = \frac{n_r!}{(n_r-n_s)!n_s!} \cdot \left(\sum_{l=1}^{n_s} \left[1 - e^{-P\|h_l'\|^2} \sum_{k=0}^{n_s-P-1} \frac{P^k\|h_l'\|^{2k}}{k!}\right]\right) \cdot I_{\tilde{H}_l}(h_1', \ldots, h_n') \cdot \frac{P^{n_s-n_P}}{n_s^{n_s-n_P}}e^{-P(\|h_1'\|^2 + \cdots + \|h_n'\|^2)},
\]
where $h_i'$ is the $i$th row of $\tilde{H}, I_{\tilde{H}_l}(h_1', \ldots, h_n')$ is the indicator function given by
\[
I_{\tilde{H}_l}(h_1', \ldots, h_n') = \begin{cases} 1 & \text{if } (h_1', \ldots, h_n') \in \tilde{H}_l \\ 0 & \text{else,} \end{cases}
\]
and the region $\tilde{H}_l$ is defined as $\tilde{H}_l = \{h_1', \ldots, h_n' : \|h_i'\| < \|h_k'\|, k = 1, \ldots, l-1, l+1, \ldots, n_s\}$. The PEP bound can be written as
\[
P(\tilde{X}_i \rightarrow \tilde{X}_j) \leq \sum_{l=1}^{n_s} \int_{H_l} e^{-P\|\tilde{H}(\tilde{X}_i - \tilde{X}_j)\|^2} \frac{n_r!}{(n_r-n_s)!n_s!} \cdot \left(1 - e^{-P\|h_l'\|^2} \sum_{k=0}^{n_s-P-1} \frac{P^k\|h_l'\|^{2k}}{k!}\right) \cdot \frac{P^{n_s-n_P}}{n_s^{n_s-n_P}}e^{-P(\|h_1'\|^2 + \cdots + \|h_n'\|^2)}dh_1' \cdots dh_n'.
\]
Letting $\sqrt{P}h_l' = s_l, l = 1, \ldots, n_s$, $S$ to be an $n_s \times n_P$ matrix whose $l$th row is $s_l$ and region $H_l = \{s_1, \ldots, s_{n_s} : \|s_l\| < \|s_k\|, k = 1, \ldots, l-1, l+1, \ldots, n_s\}$, we can write (34) as
\[
P(\tilde{X}_i \rightarrow \tilde{X}_j) \leq \sum_{l=1}^{n_s} \int_{H_l} e^{-\frac{P}{n_s}\|S(\tilde{X}_i - \tilde{X}_j)\|^2} \frac{n_r!}{(n_r-n_s)!n_s!} \cdot \left(1 - e^{-\frac{P}{n_s}\|s_l\|^2} \sum_{k=0}^{n_s-P-1} \frac{\|s_l\|^{2k}}{k!}\right) \cdot \frac{\sqrt{P}}{n_s^{n_s-n_P}}e^{-\left(\frac{\|s_l\|^2 + \cdots + \|s_{n_s}\|^2}{2}\right)} ds_1 \cdots ds_{n_s}.
\]
The term $||S(\tilde{X}_i - \tilde{X}_j)||^2$ in (35) can be simplified as
\[
||S(\tilde{X}_i - \tilde{X}_j)||^2 = Tr[S(\tilde{X}_i - \tilde{X}_j)(\tilde{X}_i - \tilde{X}_j)^H]S^H
\]
\[
= Tr[S(UA)(SU)^H]
\]
\[
= \sum_{k=1}^{n_P} \lambda_k ||c_k||^2,
\]
where (36) uses the eigenvector decomposition of $(\tilde{X}_i - \tilde{X}_j)(\tilde{X}_i - \tilde{X}_j)^H$. $U$ is the unitary matrix whose columns are the eigenvectors of $(\tilde{X}_i - \tilde{X}_j)(\tilde{X}_i - \tilde{X}_j)^H$, $A$ is the diagonal matrix containing its eigenvalues, and $c_k$ is the $k$th column of $SU$. Let $c_l'$ be the $l$th row of $SU$ so that $\tilde{H}_l = \{c_1', \ldots, c_{n_s}' : ||c_l'|| < ||c_k'||, k = 1, \ldots, l-1, l+1, \ldots, n_s\}$. Defining $K \triangleq n_lP$ and $\rho \triangleq (\sqrt{P})^{n_s-n_P}$, and changing variables in (35) by substituting $c_{ij} = s_{ij}$ for $i = 1, \ldots, n_s$ and $j = 1, \ldots, n_P$, we get
\[
P(\tilde{X}_i \rightarrow \tilde{X}_j) \leq \frac{\rho \cdot n_r!}{(n_r-n_s)!n_s!} \cdot \sum_{l=1}^{n_s} \int_{\tilde{H}_l} e^{-P\left(\lambda_1 ||c_{11}'||^2 + \cdots + ||c_{n_s}'||^2\right) + \cdots + \lambda_K ||c_{1K}'||^2 + \cdots + ||c_{n_s}'||^2}} \cdot \left(1 - e^{-\left(||c_{11}'||^2 + \cdots + ||c_{n_s}'||^2\right)} \sum_{k=0}^{K-1} \frac{\left(||c_{11}'||^2 + \cdots + ||c_{n_s}'||^2\right)^k}{k!}\right) \cdot \frac{1}{n_s^{n_s-n_P}}\Sigma_{j=1}^{K} ||c_{j1}'||^2 \cdot dc_{11} \cdots dc_{n_sK}.
\]
Evaluating the integral in (37) over the region is difficult. But because of symmetry of pdf it is possible to evaluate over the whole space which results in an upper bound. Because of the symmetry of the pdf, the integral over $H_l$ for each $l$ is same. The $l$th term in (37) can be rewritten using standard integration as
\[
I_l = \frac{\rho \cdot n_r!}{(n_r-n_s)!n_s!} \int_{0}^{\infty} \cdots \int_{0}^{\infty}
\cdot e^{-P\left(\lambda_1 ||c_{11}'||^2 + \cdots + ||c_{n_s}'||^2\right) + \cdots + \lambda_K ||c_{1K}'||^2 + \cdots + ||c_{n_s}'||^2}} \cdot \left(1 - e^{-\left(||c_{11}'||^2 + \cdots + ||c_{n_s}'||^2\right)} \sum_{k=0}^{K-1} \frac{\left(||c_{11}'||^2 + \cdots + ||c_{n_s}'||^2\right)^k}{k!}\right) \cdot \frac{1}{n_s^{n_s-n_P}}\Sigma_{j=1}^{K} ||c_{j1}'||^2 \cdot dc_{11} \cdots dc_{n_sK}.
\]
Changing the variables $c_{ij} = \sigma_{ij}e^{i\theta_{ij}}, i = 1, \ldots, n_s, j = 1, \ldots, K$ (with differential element $dc_{ij} = \sigma_{ij}d\sigma_{ij}d\theta_{ij}$), after evaluating integral w.r.t $d\theta_{ij}$ over $[0, 2\pi]$, we get
\[
I_l = \frac{\rho \cdot n_r!}{(n_r-n_s)!n_s!} \cdot \int_{0}^{\infty} \cdots \int_{0}^{\infty}
\cdot e^{-P\left(\lambda_1 ||c_{11}'||^2 + \cdots + ||c_{n_s}'||^2\right) + \cdots + \lambda_K ||c_{1K}'||^2 + \cdots + ||c_{n_s}'||^2}} \cdot \left(1 - e^{-\left(||c_{11}'||^2 + \cdots + ||c_{n_s}'||^2\right)} \sum_{k=0}^{K-1} \frac{\left(||c_{11}'||^2 + \cdots + ||c_{n_s}'||^2\right)^k}{k!}\right) \cdot \frac{1}{n_s^{n_s-n_P}}\Sigma_{j=1}^{K} \sigma_{j1}^2 \cdots \sigma_{n_sK} \cdot d\sigma_{11} \cdots d\sigma_{n_sK}.
\]
Substituting $\sigma_{ij}^2 = v_{ij}, i = 1, \ldots, n_s, j = 1, \ldots, K$, we get
\[
I_l = \frac{\rho \cdot n_r!}{(n_r-n_s)!n_s!} \cdot \int_{0}^{\infty} \cdots \int_{0}^{\infty}
\cdot e^{-P\left(\lambda_1 (v_{11} + \cdots + v_{n_s1}) + \cdots + \lambda_K (v_{1K} + \cdots + v_{n_sK})\right)} \cdot \left(1 - e^{-(v_{11} + \cdots + v_{n_s1})} \sum_{k=0}^{K-1} \frac{(v_{11} + \cdots + v_{n_s1})^k}{k!}\right) \cdot \frac{1}{n_s^{n_s-n_P}}\Sigma_{j=1}^{K} v_{j1} \cdot dv_{11} \cdots dv_{n_sK}.
\]
Now, (40) can be written as
\[
\mathcal{I}_l = \frac{e^{\lambda u} \sum_{i=1}^{K} \lambda_i v_{i_1}}{(n_r - n_s) } \int_0^\infty \cdots \int_0^\infty e^{-k \sum_{i=1}^{K} \lambda_i v_{i_1}} \cdot \left(1 - e^{-v_{i_1} + \cdots + v_{i_K}} \sum_{k=0}^{K-1} \left( \frac{v_{i_1} + \cdots + v_{i_K}}{k!} \right)^{n_r - n_s} \right) e^{-\sum_{i=1}^{K} \lambda_i v_{i_1}} dv_{i_1} \cdots dv_{i_K}.
\]
(41)

Let \(\mathcal{I}_l^{(1)}\) denote the first integral and \(\mathcal{I}_l^{(2)}\) denote the second integral in the above expression. Evaluating \(\mathcal{I}_l^{(1)}\) using \(\int_0^\infty e^{-\lambda u} du = \frac{1}{\lambda u}\), we get
\[
\mathcal{I}_l^{(1)} = \frac{1}{(1 + \frac{\lambda u}{4\pi})^{n_r - n_s}}.
\]
and \(\mathcal{I}_l^{(2)}\) as
\[
\mathcal{I}_l^{(2)} = \int_0^\infty \cdots \int_0^\infty e^{\lambda u} \sum_{i=1}^{K} \lambda_i v_{i_1} \cdot \left(1 - e^{-v_{i_1} + \cdots + v_{i_K}} \sum_{k=0}^{K-1} \left( \frac{v_{i_1} + \cdots + v_{i_K}}{k!} \right)^{n_r - n_s} \right) e^{-\sum_{i=1}^{K} \lambda_i v_{i_1}} dv_{i_1} \cdots dv_{i_K}.
\]
(42)

Let \(g(u) = 1 - e^{-u} \sum_{m=0}^{K-1} \frac{u^m}{m!}\) be the incomplete Gamma function satisfying \(g(u) \leq \frac{u^K}{4\pi^2}\) for \(u > 0\). Upper bounding the RHS of (43) by \(\frac{u^K}{4\pi^2}\) with \(u = v_{i_1} + \cdots + v_{i_K}\) in (43), we can write
\[
\mathcal{I}_l^{(2)} \leq \frac{1}{(K!)^{n_r - n_s}} \int_0^\infty \cdots \int_0^\infty e^{\lambda u} \sum_{i=1}^{K} \lambda_i v_{i_1} \cdot \left(1 - e^{-v_{i_1} + \cdots + v_{i_K}} \sum_{k=0}^{K-1} \left( \frac{v_{i_1} + \cdots + v_{i_K}}{k!} \right)^{n_r - n_s} \right) e^{-\sum_{i=1}^{K} \lambda_i v_{i_1}} dv_{i_1} \cdots dv_{i_K}.
\]
(43)

We observe that
\[
(v_{i_1} + \cdots + v_{i_K})^{K(n_r - n_s)} = \sum_{i_1=1}^{K} \cdots \sum_{i_K(n_r - n_s)+1}^{K} v_{i_{1:K}} \cdot \left( \frac{v_{i_{1:K}}}{\lambda_{i_{1:K}}} \right)^{n_r - n_s}.
\]
(45)

where index \(i_k\) in \(v_{i_k}\) takes values from the set \(\zeta = \{1, \ldots, K\}\) with \(k \in \{1, \ldots, K(n_r - n_s)\}\). Let the index \(j\) appear \(m_j\) times among the subscripts of the term \(v_{i_{1:K}}\) in (45). Then,
\[
\prod_{j=1}^{K} (v_{i_j})^{m_j} = \prod_{j=1}^{K} (v_{i_j})^{m_j}
\]
(46)

such that \(\sum_{j=1}^{K} m_j = K(n_r - n_s)\). Using (45) and (46) in (44) and changing the order of summation and integration, we get
\[
\mathcal{I}_l^{(2)} \leq \frac{1}{(K!)^{n_r - n_s}} \sum_{i_1=1}^{K} \cdots \sum_{i_K(n_r - n_s)+1}^{K} \frac{m_1! \cdots m_K!}{\lambda_{i_{1:K}}!} \left( \frac{\lambda_{i_{1:K}}}{4\pi} \right)^{-K(n_r - n_s)}.
\]
(47)

Using \(\int_0^\infty x^ne^{-ax} dx = \frac{n!}{a^{n+1}}\), (47) can be written as
\[
\mathcal{I}_l^{(2)} \leq \frac{1}{(K!)^{n_r - n_s}} \sum_{i_1=1}^{K} \cdots \sum_{i_K(n_r - n_s)+1}^{K} \frac{m_1! \cdots m_K!}{\lambda_{i_{1:K}}!} \left( \frac{\lambda_{i_{1:K}}}{4\pi} \right)^{-K(n_r - n_s)}.
\]
(48)

Using (48) and (42) in (41), \(\mathcal{I}_l\) can be written as
\[
\mathcal{I}_l \leq \frac{\rho \cdot n_r!}{(n_r - n_s)!} \frac{1}{\left( \frac{\lambda_{i_{1:K}}}{4\pi} \right)^{n_r - n_s}} \sum_{i_1=1}^{K} \cdots \sum_{i_K(n_r - n_s)+1}^{K} \frac{m_1! \cdots m_K!}{\lambda_{i_{1:K}}!} \left( \frac{\lambda_{i_{1:K}}}{4\pi} \right)^{-K(n_r - n_s)}.
\]
(49)

The above bound is independent of \(l\). Therefore, substituting (49) in (37), we can write
\[
P(\tilde{X}_i \rightarrow \tilde{X}_j) \leq \frac{(n_r - n_s)!}{(n_s - 1)!} \left( \frac{K!}{\lambda_{i_{1:K}}!} \right)^{n_r - n_s} \frac{1}{\left( \frac{\lambda_{i_{1:K}}}{4\pi} \right)^{n_r - n_s}} \sum_{i_1=1}^{K} \cdots \sum_{i_K(n_r - n_s)+1}^{K} \frac{m_1! \cdots m_K!}{\lambda_{i_{1:K}}!} \left( \frac{\lambda_{i_{1:K}}}{4\pi} \right)^{-K(n_r - n_s)}.
\]
(50)

In the high SNR regime, with some algebraic manipulations, we can write
\[
P(\tilde{X}_i \rightarrow \tilde{X}_j) \leq \frac{(n_r - n_s)!}{(n_s - 1)!} \left( \frac{K!}{\lambda_{i_{1:K}}!} \right)^{n_r - n_s} \frac{1}{\left( \frac{\lambda_{i_{1:K}}}{4\pi} \right)^{n_r - n_s}} \sum_{i_1=1}^{K} \cdots \sum_{i_K(n_r - n_s)+1}^{K} \frac{m_1! \cdots m_K!}{\lambda_{i_{1:K}}!} \left( \frac{\lambda_{i_{1:K}}}{4\pi} \right)^{-K(n_r - n_s)}.
\]
(51)

Finally, substituting \(\sum_{i=1}^{K} m_i = K(n_r - n_s)\) in (51), we get
\[
P(\tilde{X}_i \rightarrow \tilde{X}_j) \leq \frac{(n_r - n_s)!}{(n_s - 1)!} \left( \frac{K!}{\lambda_{i_{1:K}}!} \right)^{n_r - n_s} \frac{1}{\left( \frac{\lambda_{i_{1:K}}}{4\pi} \right)^{n_r - n_s}} \sum_{i_1=1}^{K} \cdots \sum_{i_K(n_r - n_s)+1}^{K} \frac{m_1! \cdots m_K!}{\lambda_{i_{1:K}}!} \left( \frac{\lambda_{i_{1:K}}}{4\pi} \right)^{-K(n_r - n_s)}.
\]
(52)
Note that the inequality (52) implies that a diversity order of \(n_r K = n_r n_t P\) is achieved in a full rank multi-antenna OTFS system when \(n_s\), antennas are selected at the receiver. We can now specialize the above diversity result for the considered multi-antenna OTFS systems which are full rank as follows.

- **SIMO-OTFS systems without phase rotation for** \(P = 1\) 
  and with phase rotation for \(P > 1\) are full rank. 
  Therefore, in these cases, full spatial and DD diversity of \(n_r P\) is achieved when \(n_s\) receive antennas are selected.

- **STC-OTFS systems with Alamouti code without phase** 
  rotation for \(P = 1\) and with phase rotation for \(P > 1\) 
  are also full rank. Therefore, in these cases, full spatial 
  and DD diversity of \(2n_r P\) is achieved when \(n_s\) received 
  antennas are selected.

The above diversity results have been summarized in Table I.

### B. Rank deficient multi-antenna OTFS systems with RAS

Consider the case of rank deficient multi-antenna OTFS systems with receive antenna selection. Let \(\tilde{X}_i\) and \(\tilde{X}_j\) be two distinct symbol matrices. Let \(r < K\) be the minimum rank of \((\tilde{X}_i - \tilde{X}_j)\). For rank deficient case, the diversity analysis follows from (37)-(49), except now \(\lambda_1, \ldots, \lambda_r > 0\), \(\lambda_{r+1} = \ldots = \lambda_K = 0\). Therefore, in the high SNR regime, the average PEP between \(\tilde{X}_i\) and \(\tilde{X}_j\), assuming \(\tilde{X}_i\) to be the transmitted symbol matrix, is given by

\[
P(\tilde{X}_i \rightarrow \tilde{X}_j) \leq \frac{\rho \cdot n_r!}{(n_r - n_s)!((n_s - 1)!(K)!^{n_r-n_s}} \cdot \frac{1}{(\prod_{i=1}^{n_r} \lambda_i)^{n_s}} \cdot \sum_{i_1 = 1}^{K} \ldots \sum_{i_K = 1}^{K} m_{i_1}! \ldots m_{i_K}! \frac{(\gamma)}{4P}^{-\sum_{i=1}^{r} m_i} \frac{1}{\lambda_{i_1}^m \ldots \lambda_{i_K}^m} \frac{(\gamma)}{4P}^{-m_{n_s}}
\]

(53)

Since \(\sum_{i=1}^{K} m_i = K(n_r - n_s)\), it follows that \(0 \leq \sum_{i=1}^{r} m_i \leq K(n_r - n_s)\). It is observed that the square brackets is function of \(\frac{1}{4P}\) and there exist terms \(i_1 \cdots i_K(n_r - n_s)\) such that \(\sum_{i=1}^{r} m_i = 0\). Regrouping the terms in (53), we can write

\[
P(\tilde{X}_i \rightarrow \tilde{X}_j) \leq \frac{\rho \cdot n_r!}{(n_r - n_s)!((n_s - 1)!(K)!^{n_r-n_s}} \cdot \frac{1}{(\prod_{i=1}^{n_r} \lambda_i)^{n_s}} \cdot \sum_{j=0}^{r} \psi_j \frac{(\gamma)}{4P}^{-j} \frac{1}{\lambda_{i_1}^m \ldots \lambda_{i_K}^m} \frac{(\gamma)}{4P}^{-m_{n_s}}
\]

(54)

where \(j = \sum_{i=1}^{r} m_i\) and \(\psi_j\) is the sum of the terms multiplying \(\frac{1}{\lambda_{i_1}^m \ldots \lambda_{i_K}^m}\) with the same exponents. For sufficiently high SNRs, the term \(\frac{1}{\lambda_{i_1}^m \ldots \lambda_{i_K}^m}\) vanishes for \(\sum_{i=1}^{r} m_i > 0\). Thus, we have

\[
P(\tilde{X}_i \rightarrow \tilde{X}_j) \leq \frac{\rho \cdot n_r!}{(n_r - n_s)!((n_s - 1)!(K)!^{n_r-n_s}} \cdot \frac{1}{(\prod_{i=1}^{n_r} \lambda_i)^{n_s}} \cdot \psi_0 \frac{(\gamma)}{4P}^{-m_{n_s}}
\]

(55)

The above expression shows that a diversity order of \(n_r P\) is achieved for a rank deficient multi-antenna OTFS system when \(n_s\) antennas are selected at the receiver. We specialize the

### Table I: Summary of diversity order results for multi-antenna OTFS systems with RAS

| OTFS system                  | # ant. selected | # DD paths | Diversity order |
|-----------------------------|-----------------|------------|-----------------|
| SIMO-OTFS, \(n_r \geq 1\)   | \(n_s \geq 1\)  | \(P = 1\)  | \(n_r P\)       |
| MIMO-OTFS, \(n_r \geq n_t\) | \(n_s \geq n_t\)| \(P = 1\)  | \(n_r n_t P\)   |
| STC-OTFS (Alamouti), \(n_t = 2\), \(n_r \geq 1\)| \(n_s \geq 1\)| \(P = 1\)  | \(2n_s\) \(2n_r P\) |

In addition to the simulated BER plot, upper bound and lower bounds on the bit error performance are also plotted.

### IV. Simulation results

In this section, we present simulation results on the bit error performance that validate the analytical diversity results derived in the previous section. We evaluate the bit error rate (BER) of the considered multi-antenna OTFS systems without and with phase rotation for \(P = 1, 2, 4\) and \(n_s \geq 1\). The simulation parameters used are listed in Table I.

**SIMO-OTFS (without phase rotation) for** \(P = 1\): Figure 4 shows the simulated BER performance of SIMO-OTFS without phase rotation for \(P = 1, M = N = 2, n_s = 1, n_r = 1, 2, 3, 4, BPSK,\) and ML detection. A carrier frequency of 4 GHz, subcarrier spacing of 3.75 kHz, and a maximum speed of 506.2 km/h are considered. The considered carrier frequency and maximum speed correspond to a maximum Doppler of 1.875 kHz. The DD channel model is as per (4) and the DD profiles for different values of \(P\) are presented in Table I. The considered system is full rank and the analytically predicted diversity order is \(n_r\) (refer Table I and Sec. III-A). The BER plots in Fig. 4 show that the system indeed achieves first, second, third, and fourth order diversity slopes for \(n_r = 1, 2, 3,\) and 4, respectively, corroborating the analytically predicted diversity orders.

**SIMO-OTFS (without phase rotation) for** \(P > 1\): Figure 5 shows the simulated BER performance of SIMO-OTFS without phase rotation for \(P = 4, M = N = 2, n_s = 1, n_r = 1, 4, BPSK,\) and ML detection. Other simulation parameters are as given in Table I. In addition to the simulated BER plot, upper bound and lower bounds on the bit error performance are also plotted. The upper bound on the bit error probability
Table II: Simulation parameters.

| Parameter                        | Value                                      |
|----------------------------------|--------------------------------------------|
| Carrier frequency, $f_c$ (GHz)   | 4                                         |
| Subcarrier spacing, $\Delta f$ (kHz) | 3.75                                      |
| DD profile for $P = 1$ ($r_1$ (sec), $\nu_1$ (Hz)) | $(\frac{1}{M^2}, \frac{1}{NT})$          |
| DD profile for $P = 2$ & $M = 2$, $N = 2$ | $(0, 0), (\frac{1}{M^2}, \frac{1}{NT})$  |
| DD profile for $P = 2$ & $M = 4$, $N = 4$ | $(\frac{1}{M^2}, \frac{1}{NT}), (\frac{1}{M^2}, \frac{1}{NT})$ |
| DD profile for $P = 4$ & $M = 2$, $N = 2$ | $(0, 0), (0, \frac{1}{NT}), (\frac{1}{M^2}, \frac{1}{NT}), (\frac{1}{M^2}, \frac{1}{NT})$ |
| Maximum speed (km/h)             | BPSK, 16-QAM                               |

Fig. 4: BER performance of SIMO-OTFS without phase rotation for $P = 1$, $M = N = 2$, $n_s = 1$, and $n_r = 1, 2, 3, 4$, is obtained from PEP using union bound, as

$$P_b \leq \frac{1}{L n_t M N \log_2 |A_k|} \sum_{i=1}^{L} \sum_{j=1, j \neq i}^{L} P(\hat{X}_i \rightarrow \hat{X}_j),$$

where $L = |A_k|^{n_s M N}$. The lower bound is obtained based on summing the PEPs corresponding to all the pairs $X_i$ and $X_j$ such that the difference matrix $(X_i - X_j)$ has rank one [25]. The considered system is rank deficient and the analytically predicted diversity order is $n_s$ (refer Sec. II-B and Table I). Since the number of antennas selected is $n_s = 1$, the predicted diversity order is 1. We can make two key observations from Fig. 5. First, the diversity slope is one for both $n_t = 1$ and $n_r = 4$. Second, The upper bound, lower bound, and simulated BER almost merge at high SNRs. These observations validate the simulation results as well the analytically predicted diversity order.

SIMO-OTFS (without and with phase rotation) for $P > 1$: Figure 6 shows the BER performance of SIMO-OTFS without and with phase rotation for $P = 2$, $M = N = 4$, $n_s = 1$, $n_r = 1, 2, 3, 4$, BPSK, ML detection, and other parameters as in Table II. For $P > 1$, SIMO-OTFS without phase rotation is rank deficient and the analytical diversity order is $n_s$. With phase rotation, the system is full-ranked and it has a diversity order of $n_r P$ (refer Sec. II-B, Sec. II-A, and Table I). For the considered system, the predicted diversity orders are 1 and 4 for without and with phase rotation, respectively. The slopes in the BER plots in Fig. 6 are observed to be in line with the predicted diversity orders.

SIMO-OTFS (without and with phase rotation) for 16-QAM: Figure 7 shows the BER performance of SIMO-OTFS without and with phase rotation for 16 QAM, $P = 2$, $M = N = 2$, $n_s = 1, n_r = 1, 2$, ML detection, and other parameters as in Table II. For $P > 1$, the analytically predicted diversity orders for the considered SIMO-OTFS system without and with phase rotation are $1 (n_s)$ and $4 (n_r P)$, respectively. In Fig. 7, the diversity slopes are found to follow these diversity orders.

Alamouti STC-OTFS (without and with phase rotation) for $P > 1$: Figure 8 shows the BER performance of Alamouti STC-OTFS without phase rotation for $P = 2$, $M = N = 2$, $n_t = 2, n_s = 1, 2, n_r = 1, 2, 3$, BPSK, ML detection, and other parameters as in Table III. From Fig. 8, it is observed that the achieved diversity order is 2 for $n_s = 1$ and 4 for $n_s = 2$. This corroborates with the predicted diversity order of $2n_s$, the system being rank deficient. For the above Alamouti STC-OTFS system, Fig. 9 shows the performance with phase rotation. This system with phase rotation is full-ranked with a predicted diversity order of $2n_r P$. The diversity
slopes observed in Fig. 9 are in accordance with this analytical prediction.

**MIMO-OTFS (without and with phase rotation) for** $P > 1$: Figure 10 shows the BER performance of MIMO-OTFS without and with phase rotation for $P = 2$, $M = 4$, $N = 2$, $n_t = 2$, $n_s = 2$, and $n_r = 2, 3$. The considered systems are rank deficient, and the predicted diversity orders are $n_s$ and $n_s P$ for without and with phase rotation, respectively. It can be seen in Fig. 10 that, as predicted, MIMO-OTFS without phase rotation achieves 2nd order diversity slope and with phase rotation achieves 4th order diversity slope.

**V. Conclusions**

We analyzed the diversity performance of receive antenna selection in multi-antenna OTFS systems. Antennas were selected based on the maximum channel Frobenius norms in the DD domain. Our diversity analysis results showed that, with no phase rotation, SIMO-OTFS and STC-OTFS systems with RAS are rank deficient, and therefore they do not extract the full receive diversity as well as the diversity present in the DD domain. Also, Alamouti coded STC-OTFS system with RAS and no phase rotation was shown to extract the full transmit diversity, but it failed to extract the DD diversity. On the other hand, SIMO-OTFS and STC-OTFS systems with RAS become full-ranked when phase rotation is used, because of which they extracted the full spatial as well as the DD diversity present in the system. When phase rotation is used, MIMO-OTFS systems with RAS was shown to extract the full DD diversity, but they did not extract the full receive diversity because of rank deficiency. Detailed simulation results validated the analytically predicted diversity performance.

**Appendix**

**Analysis for Fractional Delays and Dopplers**

**A. Input-Output relation with fractional delays and Dopplers**

Considering the channel representation in DD defined in (4) with non-zero fractional delays and Dopplers, we have

$$\tau_i = \frac{\alpha_i + a_i}{M \Delta f}, \quad \nu_i = \frac{\beta_i + b_i}{NT}, \quad (57)$$
where \( \alpha_i = \lfloor \tau_i M \Delta f \rfloor \), \( \beta_i = \lfloor \nu_i N T \rfloor \), \( \lfloor \cdot \rfloor \) denotes the rounding operator (nearest integer), \( \alpha_i, \beta_i \) are assumed to be integers corresponding to the indices of the delay tap and Doppler frequency associated with \( \tau_i \) and \( \nu_i \), respectively, and \( a_i, b_i \) are the fractional delay and Doppler satisfying \( -\frac{1}{2} < a_i, b_i \leq \frac{1}{2} \). The DD channel with fractional delays and Dopplers, assuming rectangular window functions, can be written as

\[
h(\tau, \nu) = \sum_{i=1}^{P} h_i e^{-j2\pi\alpha_i \nu} G(\nu, \nu_i) F(\tau, \tau_i),
\]

where

\[
G(\nu, \nu_i) = \sum_{n'=0}^{N-1} e^{-j2\pi(\nu - \nu_i)n' T},
\]

\[
F(\tau, \tau_i) = \sum_{m'=0}^{M-1} e^{j2\pi(\tau - \tau_i)m' \Delta f}.
\]

The input-output relation with fractional delay-Doppler can be written as

\[
y[k, l] = \sum_{i=1}^{P} \sum_{q=0}^{M-1} \sum_{q'=0}^{N-1} \left( \frac{e^{-j2\pi(-q-a_i)} - 1}{Me^{-j2\pi(-q-a_i) - M}} \right) h_i e^{-j2\pi\nu_i}\cdot x\left[(k - \beta_i + q') N, (l - \alpha_i + q)M\right].
\]

Vectorizing the input-output relation in (60), we can write

\[
y = \mathbf{Hx} + \mathbf{v},
\]

where \( y \in \mathbb{C}^{MN \times 1} \) is the received signal vector, \( x \in \mathbb{C}^{MN \times 1} \) transmit signal vector, \( \mathbf{H} \in \mathbb{C}^{MN \times MN} \) is the equivalent channel matrix, and \( \mathbf{v} \in \mathbb{C}^{MN \times 1} \) is the noise vector.

Based on (60), the input-output relation with receive antennas selection in (13) can be extended to fractional delays and Dopplers, as

\[
y' = \tilde{\mathbf{H}}' \tilde{x} + \tilde{\mathbf{v}},
\]

where \( y' \in \mathbb{C}^{n_s MN \times 1} \) is the received signal vector, \( \tilde{\mathbf{H}}' \) is the channel matrix with antenna selection, \( \tilde{x} \in \mathbb{C}^{n_s MN \times 1} \) is the OTFS transmit vector, and \( \tilde{\mathbf{v}}' \) is the noise vector.

B. Diversity analysis for \( P = 1 \)

The selection rule in (15) and (16) are equivalent for \( P = 1 \). Therefore, for diversity analysis for \( P = 1 \), the input-output relation in (62) can be written in an alternate form as

\[
\tilde{y} = \tilde{\mathbf{H}} \tilde{x} + \tilde{\mathbf{v}},
\]

where \( \tilde{\mathbf{y}} \in \mathbb{C}^{n_s \times MN} \) with its ith row corresponding to the received signal in the ith selected receive antenna, \( \tilde{\mathbf{H}} \in \mathbb{C}^{n_s \times n_t} \) is the channel matrix whose \((i, j)\)th element is \( h_{ij} e^{-j2\pi\nu_j} \), \( \tilde{\mathbf{X}} \) is \( n_s \times MN \) symbol matrix whose ith column \( \tilde{\mathbf{X}}[i] \) is given by (64) shown at the top of next page, and \( \tilde{\mathbf{V}} \in \mathbb{C}^{n_s \times MN} \) is the noise matrix.

1) Full rank case: Let \( \tilde{\mathbf{X}}_i \) and \( \tilde{\mathbf{X}}_j \) be two distinct symbol matrices. The conditional PEP between \( \tilde{\mathbf{X}}_i \) and \( \tilde{\mathbf{X}}_j \), assuming perfect DD channel knowledge and ML detection, is given by

\[
P(\tilde{\mathbf{X}}_i \rightarrow \tilde{\mathbf{X}}_j | \tilde{\mathbf{H}}, \tilde{\mathbf{X}}_i) = Q\left(\frac{\sqrt{\mathbf{H}(\tilde{\mathbf{X}}_i - \tilde{\mathbf{X}}_j)\mathbf{H}^*}}{2N_0}\right).
\]

Upper bounding (65) using Chernoff bound and averaging over the distribution of \( \tilde{\mathbf{H}} \), the unconditional PEP can be written as

\[
P(\tilde{\mathbf{X}}_i \rightarrow \tilde{\mathbf{X}}_j) \leq \mathbb{E}_{\tilde{\mathbf{H}}} \exp\left(-\frac{\gamma}{4} \frac{\|\mathbf{H}(\tilde{\mathbf{X}}_i - \tilde{\mathbf{X}}_j)\|}{2N_0}\right).
\]

The distribution of \( \tilde{\mathbf{H}} \) is given in (32). Therefore, the PEP can be written as

\[
P(\tilde{\mathbf{X}}_i \rightarrow \tilde{\mathbf{X}}_j) \leq \sum_{l=1}^{n_s} \int_{\tilde{H}_l} e^{-\frac{\|\mathbf{H}(\tilde{\mathbf{X}}_i - \tilde{\mathbf{X}}_j)\|^2}{2N_0}} \frac{n_s!}{(n_s - n)! n!} \frac{1 - e^{-\|\tilde{\mathbf{H}}\|_2^2}}{k!} \cdot \frac{1}{\pi^{n_s n_t}} e^{-\sum_{r=1}^{n_s} \|\tilde{\mathbf{H}}\|_2^2)} d\tilde{H}_1 \cdots d\tilde{H}_n.
\]

Following the steps from (35)-(52) in Sec. III-A we can write PEP as

\[
P(\tilde{\mathbf{X}}_i \rightarrow \tilde{\mathbf{X}}_j) \leq \frac{n_s!}{(n_s - n)! (n_s - 1)! (n_t)!} \frac{1}{n_s} \cdot \frac{\prod_{i=1}^{n_s} \lambda_i}{n_s} \cdot \left(\frac{\gamma}{4}\right)^{-n_s n_r}.
\]

The above equation shows that, for fractional delay-Doppler also, diversity of \( n_s n_t \) is achieved when \( n_s \) antennas are selected at the receiver. Therefore, full spatial diversity is achieved for a full rank multi-antenna OTFS system. We can specialize the above generalized result for multi-antenna OTFS systems which are full rank for \( P = 1 \), as follows:

- SIMO-OTFS system for \( P = 1 \) is full rank. Therefore, for this system, full spatial diversity of \( n_s \) is achieved when \( n_s \) antennas are selected at the receiver.
- STC-OTFS system with Alamouti code for \( P = 1 \) is also full rank. Therefore, this system also achieves full spatial diversity of \( 2n_s \) when \( n_s \) receive antennas are selected.

2) Rank deficient case: Let \( \tilde{\mathbf{X}}_i \) and \( \tilde{\mathbf{X}}_j \) be two distinct symbol matrices. Let \( r \) be the minimum rank of \( (\tilde{\mathbf{X}}_i' - \tilde{\mathbf{X}}_j')' (\tilde{\mathbf{X}}_i' - \tilde{\mathbf{X}}_j') \). Following the diversity analysis for integer delay-Doppler in Sec. III-B, we can obtain the following expression as

\[
P(\tilde{\mathbf{X}}_i \rightarrow \tilde{\mathbf{X}}_j) \leq \frac{n_s!}{(n_s - n)! (n_s - 1)! (n_t)!} \frac{1}{\prod_{i=1}^{n_s} \lambda_i} \cdot \left(\frac{\gamma}{4}\right)^{-n_s n_r}.
\]

The above expression shows that, for the rank deficient case, diversity of \( n_s r \) is achieved when \( n_s \) antennas are selected at
the receiver. MIMO-OTFS system with $P = 1$ is rank deficient with minimum rank one. Therefore, diversity of $n_s$ is achieved when $n_s$ antennas are selected in MIMO-OTFS.

C. Simulation results

In this subsection, we present the simulation results for fractional delays and Dopplers. For all the simulation results presented in this subsection, the fractional delays and Dopplers are generated as follows. The Doppler shift corresponding to $i$th channel tap is generated using Jakes formula \( \nu_i = \nu_{\text{max}} \cos(\theta_i) \), where $\nu_{\text{max}}$ is the maximum Doppler shift and $\theta_i$ is uniformly distributed over $[-\pi, \pi]$. The delay corresponding to $i$th channel tap is generated as uniformly distributed over $[0, (M-1)T_s]$, where $T_s = 1/(M \Delta f)$ and $\Delta f$ is the subcarrier spacing. Exponential power delay profile and Jakes Doppler spectrum are considered [31].

Figure 11 shows the simulated bit error performance of SIMO-OTFS without phase rotation for $M = N = 2$, $P = 1$, $n_s = 1$, and $n_r = 1, 2, 3, 4$, with fractional delay and Doppler.

Fig. 11: BER performance of SIMO-OTFS without phase rotation for $M = N = 2$, $P = 1$, $n_s = 1$, and $n_r = 1, 2, 3, 4$, with fractional delays and Dopplers. For all the simulation results presented in this subsection, the fractional delays and Dopplers corresponding to $i$th channel tap is generated using Jakes formula [6] $\nu_i = \nu_{\text{max}} \cos(\theta_i)$, where $\nu_{\text{max}}$ is uniformly distributed over $[-\pi, \pi]$. The delay corresponding to $i$th channel tap is generated as uniformly distributed over $[0, (M-1)T_s]$, where $T_s = 1/(M \Delta f)$ and $\Delta f$ is the subcarrier spacing. Exponential power delay profile and Jakes Doppler spectrum are considered [31].

Figure 11 shows the simulated bit error performance of SIMO-OTFS without phase rotation for $M = 2, N = 2$, $P = 1$, $n_s = 1$, $n_r = 1, 2, 3, 4$, BPSK, and ML detection. The carrier frequency and subcarrier spacing are taken to be 5.9 GHz and 0.156 MHz, respectively. A frame size of 220 km/h (corresponding maximum Doppler of 1.2 kHz), $N = 12$, number of paths $P = 8$, $n_s = 1$, $n_r = 1, 2$, MMSE detection, and fractional delays/Dopplers.

Fig. 12: BER performance comparison between SIMO-OTFS with RAS and SIMO-OFDM with RAS for $M = 64, N = 12$, $P = 8$, $n_s = 1$, $n_r = 1, 2$, MMSE detection, and fractional delays/Dopplers.

Fig. 12: BER performance comparison between SIMO-OTFS with RAS and SIMO-OFDM with RAS for $M = 64, N = 12$, $P = 8$, $n_s = 1$, $n_r = 1, 2$, MMSE detection, and fractional delays/Dopplers.
SNR gain of about 11 dB compared to SIMO-OFDM with RAS.

Performance in LTE with rectangular pulse: Here, we present a performance comparison between MIMO-OTFS and MIMO-OFDM with RAS considering system parameters according to LTE standard as follows. The carrier frequency and subcarrier spacing are taken to be 4 GHz and 15 kHz respectively. A frame size of \( M = 12, N = 7, P = 5 \), and a maximum speed of 500 km/h (corresponding maximum Doppler of 1.85 kHz), and BPSK modulation are considered. Figure 13 shows the performance comparison between MIMO-OTFS with rectangular pulse and MIMO-OFDM for \( M = 12, N = 7, P = 5 \), and MP detection. From Fig. 13 we observe that MIMO-OTFS with RAS performs better than MIMO-OFDM with RAS. We further note that while the performance for \( P > 1 \) in Figs. 12 and 13 are observed through simulations, an analytical derivation of the diversity orders for \( P > 1 \) with RAS for the fractional delay-Doppler case is open for future investigation.

Fig. 13: BER performance comparison between MIMO-OTFS with RAS and MIMO-OFDM with RAS for \( M = 12, N = 7, P = 5 \), \( n_t = 2, n_s = 2, n_r = 2, 3 \), MP detection, and fractional delays/Dopplers.

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