Optimal Distribution of Heat Collecting Area of Two Collectors in Solar Water Heating Systems

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Abstract. In view of the large seasonal temperature difference, and the large temperature difference between day and night, difficulty in supplying domestic hot water supply in high altitude area, and the contradiction between low efficiency and cost of single type solar collector system, a hybrid heat collection system based on two different types of solar collectors has been considered in this paper, and a multi-objective optimization model for the heat collecting area optimization of the hybrid solar collectors has also established to solve the above problems. Besides, a fast and effective multi-objective optimization method was proposed for solving of this multi-objective optimization model. This method is based on the fast non-dominated genetic algorithm and the multi-objective decision-making analytic hierarchy process, and was used to analyze the practical hot water supply in a farm in Guoluo zone. The research results shown that this proposed optimization algorithm can quickly and effectively complete the heat collecting area distribution of two different types of collectors in the combined heat collecting system given a certain weight of system cost and heat collecting area. Under the premise of guaranteeing sufficient hot water supply, the system cost is significantly reduced. Therefore, the problem of difficulty and high cost of hot water supply in the plateau region is solved, which has a good practical significance.

Keywords. Solar water heating system (SWHS); multi-objective optimization; optimization of heat collection area; NSGA-II; analytic hierarchy process (AHP).

1. Introduction

In recent years, with the increasingly maturity of solar energy technology and the growing demand for clean resources, solar collector systems have been increasingly favored. Especially in the plateau area, where the average sunshine time is relatively long and the temperature difference between day and night is large, which is very suitable for the use of solar collector systems. While single green energy technology normally can’t implement enough hot water supply for both domestic and rural area in the plateau, so considering the use of multiple heat sources combined with the heat collection mode may a new feasible way to solve the above problems. In the design of hybrid solar heat source hot water system, the differences between collectors are mainly reflected in thermal efficiency and system cost per unit area, and they are usually two contradictory quantities. The difference in thermal efficiency makes the required heat collection area different under certain hot water load requirements, and the system cost also varies with the heat collection area. Therefore, the optimal combination of two different collector not only determines the total heat collection area of the system, but also determines the total cost of the system. In summary, a rational allocation of the heat collecting area can not only effectively reduce the cost but also improve the use efficiency of the collecting area of the heat...
collecting system given the required cost of hot water, which plays an important role in the application and promotion of the composite solar collecting system.

At present, research on the optimization of solar water heating systems focuses on the optimization of design parameters in a single solar collector system, such as collector materials, heat collection area and water tank reserves. Guo et al. [1] analyzed the effect of collector area and storage on the thermal performance and economy of the system and given a simple program to determine the optimal system size; Saïf ed-Dîn Fertahi et al. [2] took the collector technology as a fixed research parameter, and studied other indexes conducive to the design and optimization of hot water system. These studies are concentrated on the consideration of a single solar heat source hot water system, and there is little research work on the heat collecting area optimization problem of two or more solar collector combinations. About the methods of optimization problems, the traditional calculation method includes the minimum and minimum value method for the target derivation [3] and the Hooke-Jeeves-based optimization method [4], which are generally inefficient. The genetic algorithm based on the comprehensive search of population to population has a high computational efficiency and can solve such complex multi-objective optimization problems completely and effectively.

In summary, currently no systematic calculation methods for the optimal combination of two collectors of solar water heating system. In this paper, the two collectors’ area is taken into consideration as the optimization objects, and the minimum construction area and cost are taken as the optimization targets. Hereby a dual-objective optimization model of the combined solar collector area of the two collectors is established. In this paper, the hot water supply in an agricultural and pastoral area in Guoluo area was used as an example to optimize using the fast non-dominated sorting genetic algorithm (NSGA-II) [5], and the Pareto optimal solution set of the heat collecting area of the two collecting systems was obtained. By analyzing the Pareto optimal solution set and selecting partial solution set as the design scheme, the hierarchical analysis method (AHP) [6] in multi-objective decision making is used to sort and optimize the scheme, and the optimal design parameters of the composite solar water heating system design were obtained.

2. Model Introduction

2.1. Hot Water Load

Daily hot water load:

\[
Q_{hj} = c_p \rho q_i (T_L - T_c) \times 10^{-6}
\]  

(1)

In the formula, \(Q_{hj}\) —daily hot water load, MJ/d; \(c_p\) —water specific heat capacity, 4180 J/(kg·d); \(\rho\) —water density, 1000 kg/m³; \(q_i\) —daily hot water consumption, the system design does not consider the relationship between water consumption and the season. The daily hot water consumption is 30 m³/d; \(T_c\) —the hot water temperature used by the user, here the value is 45 ℃; \(T_L\) —the tap water temperature, 10 ℃.

The daily hot water load of the project can be calculated by using formula (1). According to the actual water use data of the farm in Guoluo area, the average daily hot water load value can be calculated as 4389 MJ.

2.2. Collecting Area Calculation

The heat collection area of the direct heating collector system is calculated according to equation (2) [7]:

\[
A_c = \frac{Q_{hj} f}{H \eta_r (1 - \eta_r)}
\]  

(2)

In the formula, \(A_c\) —collector heat collecting area; \(Q_{hj}\) —daily hot water load, according to formula (1) calculated result is 4389 MJ; \(f\) —domestic hot water guarantee rate, according to the solar radiation conditions of the project area and system requirements and other factors, the value is 0.5; \(H\) —daily
average solar radiation, query the solar radiation intensity of the local area to obtain the daily average solar radiation in the Guoluo is 19.57 MJ/m² [8]; $\eta_d$ — collector heat collecting efficiency, here reference to the actual project used flat plate collector system collector efficiency of 0.4 [9], Fresnel collector system collector efficiency of 0.5 [10]; $\eta_l$ — collector system heat loss rate, value of 0.2 [7].

2.3. Collector System Cost

The optimization of the heat collection area in this paper takes the initial investment as one of the goals. The initial investment is regarded as the total system cost of the two heat collecting systems, that is, the total system cost composed of the Fresnel collector system and the flat panel heat collecting system. Usually, the system cost of the heat collecting system is proportional to the heat collecting area. Through market research on the system cost of these two collector hot water systems, the market research results show that the system cost and heat collection area of the two collector hot water systems meet the following requirements (3) and (4) respectively. The quadratic function relationship shown by the formula:

$$C_1 = -1.32x^2 + 2268x + 4695$$

(3)

$$C_2 = -1.24x^2 + 3428x + 3426$$

(4)

where $x$ represents the required collector area; $C_1$ and $C_2$ represent the system cost of the Fresnel collector system and the plate collector system respectively.

3. Collecting Area Optimization and Model Solving

3.1. Multi-objective Optimization Model

3.1.1. Control Variables. The solar collector system mainly includes Fresnel solar collector system and flat-plate solar collector system. How to properly distribute the heat collection area of both affects the utilization rate of solar energy and affects the installation cost of the whole system. Therefore, the Fresnel solar heat collection area and the flat plate heat collection area are taken as the decision variables.

3.1.2. Objective Function. This paper is the design of the solar water heating system project in the farm in Guoluo area. In the actual project, there are strict requirements for system cost and total floor space. On the basis of meeting the hot water load, the system cost and total floor space are required to be the smallest. Therefore, the optimization target is selected as the system cost and heat collection area. According to the model established above, the optimization objective function can be obtained as shown in equation (5) (6):

$$\min f_1(x) = C_1 + C_2$$

(5)

$$\min f_2(x) = A_{c1} + A_{c2}$$

(6)

In the formula, $f_1(x)$ —the total system cost; $f_2(x)$ —the total heat collection area; $C_1$, $C_2$ —the system cost of the flat plate heat collecting system and the Fresnel collecting system respectively; $A_{c1}$ , $A_{c2}$ —heat collecting area of the flat plate collector and the Fresnel collector respectively. The relationship between the cost of $C_1$ and $C_2$ systems and the area of heat collection is shown in equations (3) and (4).

3.1.3. Constraints. Taking the farmland in Guoluo as an example, this paper takes the local climate and the water consumption of the herdsmen, and establishes the constraint relationship between the heat collecting areas of the two heat collecting systems according to the energy balance principle. The specific constraint model is established as follows: For this system, the total heat generated by the flat
panel heat collecting system and the Fresnel collector system should meet the user's water demand, that is, the following expression (7) should be established:

\[ Q = Q_p + Q_f \]  \hspace{1cm} (7)

Where, \( Q_p \) —— the heat generated by the flat collector; \( Q_f \) —— the heat generated by the Fresnel collector.

Then, according to the heat collecting area formula of formula (1), the relationship between heat and heat collecting area is calculated as equation (8):

\[ Q = H \eta_{cd} (1 - \eta_c) A_c \]  \hspace{1cm} (8)

Therefore, the constraint relationship of the heat collecting area is established according to the formulas (7) and (10) as in the formula (9):

\[ 4A_{c1} + 5A_{c2} = 1400 \]  \hspace{1cm} (9)

Where, \( A_{c1} \) —— the area of the flat collector; \( A_{c2} \) —— the area of the Fresnel collector.

3.2. Genetic Algorithm Calculation Process and Analysis

According to the multi-objective optimization model established above, the genetic algorithm program is written in Python for optimization calculation. The population of the optimization algorithm is 100, the crossover probability is 0.8, the mutation probability is 0.2, and after the genetic iteration of genetic algorithm 200 times, it is obtained. The optimal Pareto solution set for target optimization is shown in figure 1:

![Figure 1. Pareto optimal solution calculated by NSGA-II.](image)

The result of multi-objective optimization is a set of solutions, called Pareto solution set. According to actual needs and project requirements, one or a set of solutions is selected from the Pareto solution set as the optimization result. The two objectives of optimization in this paper conflict with each other. As shown in figure 1, when the total heat collection area of the target is small, the system cost is high, and when the heat collection area is high, the system cost is low. The two objective functions conflict with each other, and the improvement of one target object must be at the expense of another target object. The optimization problem contains multiple solutions, so it is difficult to determine which solution is optimal from the Pareto solution set. If the economic cost is not considered, the solution set with the smallest total heat collection area is selected. If you do not consider the floor space, select the solution set with the lowest system cost. In the actual project, a variety of factors are considered in consideration of the appropriate amount. Here are some typical solutions for different total heat collection areas, as shown in table 1.

Table 1 is a partial solution set selected according to the optimization result. The optimal solution set can be divided into three parts. The first part has only one solution, that is, only the solution set of the Fresnel collector is used, the total heat collection area is the smallest, and the system cost is the
highest. The second part also has only one solution, that is, only the solution set of the flat panel collector is used, which has the largest total heat collection area and the lowest system cost. The third part is composed of Fresnel collector and flat collector. There are many solution sets in this part, which is also the main solution set of the optimal distribution of the two types of heat collecting methods. The analytic hierarchy process is used to analyze the solution set to obtain the optimal design.

### Table 1. Partial Pareto optimal solution.

| Total heat collection area (m²) | System cost (yuan) | Flat plate heat collection area (m²) | Fresnel heat collection area (m²) |
|---------------------------------|--------------------|-------------------------------------|----------------------------------|
| 280                             | 870745             | 0                                   | 280                              |
| 290                             | 869517             | 50                                  | 240                              |
| 300                             | 857721             | 100                                 | 200                              |
| 310                             | 835357             | 150                                 | 160                              |
| 320                             | 802425             | 200                                 | 120                              |
| 330                             | 758925             | 250                                 | 80                               |
| 340                             | 704867             | 300                                 | 40                               |
| 350                             | 640221             | 350                                 | 0                                |

### 3.3. Hierarchical Analysis Sorting and Selecting Calculation

Regarding the Pareto optimal solution set for optimal allocation of heat collecting area, it is necessary to consider selecting the appropriate solution set as the engineering design parameter in practical engineering. In this paper, the six optimal solutions of the third part of the Pareto optimal solution in Table 2 are used as the design scheme. The hierarchical structure method (AHP) in the target decision is used to carry out the multi-objective decision of the optimal allocation of the heat collecting area, and the optimal design is obtained. The decision goal of this paper is a typical hierarchical problem, which can be represented by Figure 2.

### Table 2. Total hierarchical ordering.

| G     | Target 1 weight $G_1$ | Target 2 weight $G_2$ | Hierarchical total ordering $V$ |
|-------|-----------------------|-----------------------|---------------------------------|
| $A_i$ | 0.0281                | 0.4378                | 0.192                           |
| $A_2$ | 0.0453                | 0.2608                | 0.1315                          |
| $A_3$ | 0.0811                | 0.147                 | 0.1075                          |
| $A_4$ | 0.147                 | 0.0811                | 0.1206                          |
| $A_5$ | 0.2608                | 0.0453                | 0.1746                          |
| $A_6$ | 0.4378                | 0.0281                | 0.2739                          |

![Figure 2. Hierarchical analysis structure.](image-url)
As can be seen from figure 2, the multi-objective decision problem has three levels: the target layer, the criteria layer, and the solution layer. Solving the finite order weights of each scheme relative to the target level, the scheme with the most weight is the best solution. First, the weight of the scheme layer to the criterion layer is calculated as following.

The indicator matrix $A$ is constructed according to the target value of the partial Pareto optimal solution set. The comparison matrix $C_j$ for each target is constructed using the three scale method according to the matrix $A$. The judgment matrix $B_j$ of each target can be constructed by using the favorable degree index $r_j$. Solve the eigenvalue $\lambda_{\text{max}}$ of the judgment matrix $B_j$ and the eigenvector $W_j$ and check whether the conformance requirements are met. The eigenvector $W_j$ is the weight of the target solution layer to the criteria layer. Find the weight of each scheme $1 \times [0.028, 0.045, 0.081, 0.147, 0.261, 0.438]$ in the target one, the weight of each scheme $2 \times [0.438, 0.261, 0.147, 0.081, 0.045, 0.028]$ in the target two, The eigenvalues of the two objective judgment matrices $\lambda_{\text{max}} = 6.115, CR = 0.023, CR < 0.1$, which meets the consistency requirement.

In this paper, the economic factors, constraints and feasibility of the actual project are considered. The weight between the objectives, that is, the weight of the criterion layer for the target layer is set to $[0.6, 0.4]$, and then the overall hierarchical order is performed. The results are shown in table 2.

The above calculation results show that the relative order of the six schemes considered is:

$$A_6 > A_5 > A_4 > A_2 > A_1 > A_3$$ (10)

3.4. Analysis of Results

In summary, the optimal result of sorting and selecting is scheme 6. Comparing the optimal scheme with the cost of a single Fresnel collector system, the optimal scheme is reduced by 19% in system cost; compared with the optimal scheme and single-plate collector system, the optimal scheme is reduced by 2.9% in the construction area.

4. Conclusion

This paper has established a complete multi-objective optimization model under the thorough consideration of the area consumption and system building cost, which focused on the optimal allocation of the heat collecting area of two different solar collectors. A systematic calculation method for the area allocation of two solar collectors is proposed, which based on the fast non-dominated genetic algorithm (NSGA-II) and the analytic hierarchy process of multi-objective decision making. After that this method has been used to task a practical hot water supply system based on solar collector for a farm in Guoluo area, finally an optimal area distribution of the two different heat collector, Fresnel heat collecting system and flat plate heat collecting system which constitute the hybrid solar water heating system is obtained. The results shown that this method can flexibly and conveniently be applied to the practical engineering optimization problem, such as hybrid solar water heating system combination optimization.

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