Dissipative Light Bullets in Passively Mode-Locked Semiconductor Lasers

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We demonstrate the existence of stable three dimensional dissipative localized structures in the output of a laser coupled to a distant saturable absorber. These phase invariant light bullets are individually addressable and can be envisioned for three dimensional optical information storage. An effective theory provides for an intuitive picture and allows to relate their formation to the morphogenesis of static auto-solitons and cellular patterns. The complexity incurred by the widely different time scales present in the problem as well as by non-local couplings that stem from the material degrees of freedom is circumvented by the use of a multiple time-scale analysis. This provides a powerful model enabling to tackle effectively the three dimensional case.

The possibility of using light bullets (LBs), i.e. pulses simultaneously confined in the transverse and the propagation directions, has attracted a lot of interest in the last twenty years, both for fundamental and practical reasons. Optical confinement is ordinarily envisioned through a mechanism in which a Kerr self-focusing nonlinearity compensates the spreading effect of diffraction. Yet in this scenario, reminiscent of the conservative soliton theory, conservative LB are unstable and lead to a collapse [1] in three dimensions. Other confinement mechanisms were envisioned in systems far from equilibrium and LBs were predicted in semiconductor cavities [2, 3]. However, it was later shown that the proper consideration of the dynamics of the active material leads to collapse [4].

In this letter, we establish the existence of stable LBs in semiconductor laser cavities. Departing from previous approaches, the structuration mechanism exploits the existence of the active material temporal scales making our LB essentially multiple timescale objects. We present an intuitive theory that allows to understand the LBs as hybrids between the temporal localized structures described in [5] and the spatial diffractive autosolitons of [6].

Localized structures (LS) have been widely observed in nature [7–11] and can be interpreted —in the weak dissipative limit— as dissipative solitons [12, 13]. They may form when two homogeneous solutions coexist for the same values of the parameters [6, 14, 15]. These LS correspond to a locking between two opposed fronts connecting the two stable states. Another case consists in the coexistence of an homogeneous and a modulated solution [16] leading to the so-called cellular patterns [17].

One separates in optics the systems in which the LS are locked to an external injection beam from the ones that possess a phase invariance, see [18, 19] for a review. The former case leads to the so-called cavity solitons [20, 21] observed either in the transverse plane of broad area amplifiers [22] or in the temporal output of fibers [23, 24]. In the latter case, diffractive autosolitons were predicted either in situations where the cavity is composed of a gain medium coupled to a saturable absorber [6, 15] or to an external diffraction grating [25]. Because these lasing LS (LLS) appear in a phase invariant system [26], their properties are very different from phase-locked LS, leading for instance to optical vortices [27].

It was recently shown [5] that phase invariant temporal LLS can evolve from passive mode-locking (PML). The PML regime leads to the emission of temporal pulses much shorter than the cavity round-trip [28]. It is achieved by combining a laser amplifier providing gain and a nonlinear loss element, usually a saturable absorber (SA). The different dynamical properties of the two elements create a window for regeneration only around the pulse. Although many pulses may coexist within a PML cavity, they cannot be addressed individually. It is due to the fact that the dynamics cannot be reduced to the sole evolution of the field and while the pulses are typically short and may seem temporally localized, they may still strongly coupled trough the evolution of the gain that occurs on a much longer time scale.

For cavities with a large aspect ratio, as defined by the ratio of the gain recovery time and of the cavity round-trip time \( r = \tau/\tau_g \), the PML pulses may become under certain conditions individually addressable LLS coexisting with the off solution [5]. In this regime of temporal localization, the LLS were found to arise from a bifurcation scenario radically different to the one found in PML lasers. While PML occurs as a supercritical Andronov-Hopf bifurcation over a continuous wave solution, here the LLS are nascent from Saddle Node bifurcations of Limit cycles (SNL) and occur below the lasin threshold. As such, they coexist between themselves as well as with the zero intensity solution. The coexistence with the harmonic PML solution of maximal order hinted at a similarity with decomposable cellular patterns [17].

In this manuscript we discuss in which conditions the transverse profile of these temporal LLS can self-organize yielding to a new robust LB formation scenario. We describe the PML laser using the generic delayed differential equation model presented in [29] which can describe both the pulsating regimes and the steady solutions. We work in the limit of low gain \((G)\) and saturable absorption \((Q)\) as to justify a first order approximation to the single pass evolution of the field profile. In addition, we assume that the transverse section is sufficiently broad for the effect of diffraction at each round-trip to be small. This allows the lumping and the commuting of the various nonlinear elements. In this uniform field limit, the equation for the field amplitude \( E(r_\perp, t) \) reads
where $\gamma$ is the bandwidth of the spectral filter, $\Delta_\perp = \partial_x^2 + \partial_y^2$ is the transverse Laplacian, $\kappa$ is the fraction of the power remaining in the cavity after each round-trip and $\alpha$ and $\beta$ are the linewidth enhancement factors of the gain and absorber sections, respectively. In Eq. (1), the transverse space variables $r_\perp = (x,y)$ have been normalized to the diffraction length. As such, the domain size $L_\perp$ representing the dimension of the broad area laser becomes a bifurcation parameter: we foresee that with $L_\perp \ll 1$, one may only find a uniform spatial state while localized patterns may occur when $L_\perp \gg 1$.

In addition to the two reversible transverse spatial dimensions $r_\perp$ (found setting $r_\perp \rightarrow -r_\perp$), the delayed values of the variables $(E,G,Q)$ render Eq. (1) infinite-dimensional along the propagation direction. The time delay ($\tau$) describes the spatial boundary conditions for a loop cavity and governs the fundamental repetition rate of the PML laser. The carrier equations read

$$\partial_t G = \Gamma G_0 - G \left( 1 + |E|^2 \right) + \mathcal{D}_g \Delta_\perp G,$$

$$\partial_t Q = Q_0 - Q \left( 1 + s |E|^2 \right) + \mathcal{D}_q \Delta_\perp Q,$$

with $G_0$ the pumping rate, $\Gamma = \tau_g^{-1}$ the gain recovery rate, $Q_0$ is the value of the unsaturated losses which determines the modulation depth of the SA and $s$ the ratio of the saturation energy of the gain and of the SA sections while the scaled diffusion coefficients are $\mathcal{D}_g$ and $\mathcal{D}_q$. In Eqs. (1-3) time has been normalized to the SA recovery time. The slow temporal scales associated with $G$ and $Q$ prevent their adiabatic elimination which renders the dynamics along the propagation (time) axis irreversible, notwithstanding the presence of dissipative terms.

The lasing threshold above which the off solution, defined as $(E,G,Q) = (0,G_0,Q_0)$, becomes unstable is $G_{th} = 2/\sqrt{\kappa} - 2 + Q_0$. We study the dynamical system defined in Eqs. (1-3) in the newly found regimes of LLS [5], where the cavity round-trip time is much longer than the gain recovery $r = \tau/\tau_g \gg 1$ and for values of the gain below threshold, i.e. $G_0 < G_{th}$. In this regime, LLS occur essentially as multiple timescales objects due to the widely different dynamics of the light and of the matter components.

We show in Fig. 1 that Eqs. (1-3) can indeed support the emission of LBs as temporal LLS that are also confined in the transverse plane. Here the extent of the optical component of the LB normalized to the round-trip time is typically $\tau_g/\tau \sim 10^{-4}$. Due to the very large aspect ratio of our system, such LBs would be invisible in a spatio-temporal representation for the optical field intensity as it would corresponds to an extremely thin horizontal segment. It is why we represented in Fig. 1 the slower material variables $G$ and $Q$. Starting from the off solution, i.e. the solution with $N = 0$ LB, each additional LB was triggered individually by sending a perturbation in the form of an optical pulse. Since LBs are attractor, the details of the writing pulse are irrelevant as long as its energy remains above a certain threshold. After a transient of several tens of round-trips, the situation reaches an equilibrium and an additional LB is written. With respect to the existing LBs, a new LB can be written either at the same time but at a different location, like the two LBs represented in the white inset in Fig. 1b-e) the details of the LB at the lower and highest stable currents. In this case, then can even partially overlap spatially like the two leftmost LBs in Fig. 1 at coordinates $(x,t/\tau) = (-40,-0.75)$. Without this staggering, these two LBs could not coexist at the same instant.

In the case of a single LB, we performed a bifurcation diagram as a function of the gain parameter. Our results are depicted in Fig. 2 where we represent the pulse energy. Here one observes that the LBs occur below the lasing threshold. The square root behavior around the minimal gain value suggests a scenario based upon a SNL bifurcation. For high current, the LB spatial profile develops wings that oscillate at a low frequency signaling the onset of an Andronov-Hopf bifurcation. We depict in Fig. 2b-d) the details of the LB at the lower and highest stable currents.

We stress that the self-organization mechanism at work is not based upon a self-focusing Kerr nonlinearity. For instance, we obtained the results of Fig. 1 and Fig. 2 setting the values $\alpha = \beta = 0$. Exploiting the seminal work of New [30] and the fact that the LLS are com-

\[
(\gamma^{-1} \partial_t + 1 - i\Delta_\perp) E(r_\perp,t) = \sqrt{\kappa} \left[ 1 + \frac{1 - i\alpha}{2} G(r_\perp,t - \tau) - \frac{1 - i\beta}{2} Q(r_\perp,t - \tau) \right] E(r_\perp,t - \tau). \tag{1}
\]
posed of different variables evolving over widely different timescales, we find an effective equation for the transverse morphogenesis problem in presence of defraction. We assume that the field reads $E(r_{\perp}, t) = A(r_{\perp}, t) p(t)$ with $p(t + \tau) = p(t)$ a short normalized temporal pulse train and $A(r_{\perp}, t)$ a slowly evolving amplitude. Inserting this Ansatz in Eqs. (1-3), defining $\sigma = \gamma t$, we find that the equation governing the dynamics of the transverse profile reads

$$\partial_\sigma A = i A_{\perp} A + A f (|A|^2),$$

while the expression of the nonlinear function $f$ reads

$$f(I) = \sqrt{\kappa} \left[ 1 + \frac{1 - i \alpha}{2 I} G_0 (1 - e^{-I}) - \frac{1 - i \beta}{2 I} Q_0 (1 - e^{-s I}) \right] - 1.$$  

To obtain Eqs. (4,5), we used the fact that, during the emission of a LLS, the stimulated terms are dominant in Eqs. (2-3) which allows connecting the values of the material variables after the pulse impinging to the values before. In our case where $\tau \gg \tau_a$, the material variables perform a full recovery in-between the emission of a LLS.

We wrote Eq. (4) to make apparent the link between our approach and the work of [31, 32] for the case of static auto-solitons in bistable interferometers. In [31, 32], one assumes a monomode continuous wave emission along the longitudinal propagation direction, which allows, via the adiabatic elimination of the material variables, to find an effective equation for the transverse profile, yet with a different expression for the function $f$. This would corresponds in Eqs. (1-3) to taking the limit $\tau \to 0$ and setting $\partial_t G = \partial_t Q = 0$. It is striking that in the frame of a temporal train of short pulsating LLS, one finds exactly the same effective equation describing the transverse structuration dynamics. Following the method detailed in [32] in the case of a single transverse spatial dimension, we introduce in Eq. (4) a phase-amplitude decomposition and a spectral parameter $\omega$ as

$$A(x, \sigma) = \rho(x) \exp \{ i [\phi(x) - \omega \sigma] \},$$

and are left searching for static spatial LLS as heteroclinic and homoclinic orbits of

$$\partial_\sigma \rho = k \rho, \quad \partial_\sigma q = -2qg + \Re [f (\rho^2)],$$

$$\partial_\sigma k = -\omega + q^2 - k^2 - 3 \Re [f (\rho^2)].$$

Figure 2. (Color online). Bifurcation diagram for the spatially integrated peak power of the LB intensity ($P$) as a function of the gain $G_0$. The single LB regime occurs as a SNL bifurcation around $G_{SN} = 0.686 G_{th}$ and loses its stability via an Andronov-Hopf bifurcation at $G_{H} \sim 0.694 G_{th}$. In both cases, the spatio-temporal profile of the field is represented in panels b,d) and c,e) with $G_0 = G_{SN}$ and $G_0 = G_{H}$, respectively. The others parameters are like in Fig. 1.

Figure 3. Bifurcation diagram of the reduced model Eq. (4) as a function of the gain parameter $G_0$. The panels a,b) represent the evolution of the spectral parameter $\omega$ while the spatially integrated intensity $E$ is depicted in panels c,d). The strong multistability of solutions is depicted in panel e). Usually, only the transverse LS of higher intensity is stable. The main solution is stable in the vicinity of the Saddle-Node bifurcation that occurs at $G_{cns} = 0.606 G_{th}$ and loses its stability via and Andronov-Hopf bifurcation at $G_{s} = 0.63 G_{th}$.

We represent the oscillations of the transverse profile slightly above $G_0 = G_{th}$ in panel f). Parameters as in Fig. 1.
where we defined \( k = \rho^{-1} \partial_x \rho \) and \( q = \partial_x \phi \). Upon continuation as a function of a parameter one is able to reconstruct a full bifurcation diagram and our results are shown in Fig. 3. Here, the spiral shape of the single LS branch is reminiscent of the results of [32] in the case of static auto-solitons. The stability of the solution branch was found via the reconstruction of the linear evolution operator along the lines of [33, 34]. As detailed in [32], Eq. (4) exhibits three zero eigenvalues related to the phase, translational and Galilean invariance. This reduced model bifurcation diagram is in good qualitative agreement with one of the full model. The extent of the stable LB region is well reproduced as agreement with one of the full model. The extent of the reduced model bifurcation diagram is in good qualitative

\[
\partial_t G = \Gamma G_0 - G \left( T + |E|^2 \right) + D_g \Delta \perp G,
\]

(10)

\[
\partial_t Q = Q_0 - Q \left( 1 + s \left| E \right|^2 \right) + D_q \Delta \perp G,
\]

(11)

with \( T \) a slow time scale. The Eqs. (9-11) clarify the composite nature of the LBs as one notices the presence of diffusion along the longitudinal dimension and diffraction in the transverse dimensions. Although Eqs. (9-11) may seems a complication with respect to Eqs. (1-3) since the system is now 4D, it allows to cut the domain along the \( z \)-axis to a box of the size of the optical pulse and neglect the long tail of the gain recovery. The reason for doing so is that the gain material losses entirely its memory at the next round-trip, so that one can always set the boundary condition \((G,Q) (z = 0, r_\perp, T) = (G_0, Q_0)\). One can appreciate in Fig. 4 the results of this approach and notice the excellent agreement with the results of the full model. Finally, this approach can readily be extended to the study of 3D LB as depicted in Fig. 5.

In conclusion, we have shown that broad area PML lasers with a large temporal aspect-ratio can display addressable Lasing Light bullets. We presented an intuitive theory that allows to understand these multiple time scales object as hybrids between the temporal localized structures [5] and spatial diffractive auto-solitons [6]. A multiple time scale analysis allows reducing the size of the longitudinal domain by several orders of magnitude rendering the three dimensional analysis feasible.

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