The role of three-body collisions in $\phi$ meson production processes near threshold

H.W. Barz and B. Kämpfer
Forschungszentrum Rossendorf, Pf 510119, 01314 Dresden, Germany

Abstract:
The amplitude of subthreshold $\phi$ meson production is calculated using dominant tree-level diagrams for three-body collisions. It is shown that the production can overwhelmingly be described by two-step processes. The effect of the genuine three-body contribution (i.e. the contribution which cannot be factorized) is discussed. The production rate of $\phi$ mesons is presented for proton induced reactions on carbon.
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1 Introduction
Currently there is much interest in subthreshold production of heavy mesons in relativistic nuclear collisions. It is expected that measurements will reveal the change of the properties of mesons in dense and hot nuclear matter. Such predictions are made from theoretical reasons [1] and, indeed, hints to a dropping mass of anti-kaons have been found by comparing recent measurements [2] with calculations based on transport models [3]. For $\phi$ meson production there are only a few measurements [4] in the reactions of Ni+Ni
at 1.93 A·GeV and Ru+Ru at 1.69 A·GeV. Although only a limited amount of the phase space was accessible in these experiments, extrapolations to the full phase space point to a surprisingly large production cross section.

First estimates using transport models \[5\] based on two-particle cross sections, derived in a simple meson exchange model \[6\], seem to underestimate the $\phi$ multiplicity. One should however take into account the lack of precise knowledge concerning elementary $\pi\Delta$, $\Delta N$ and $\Delta\Delta$ collisions. In addition multiparticle processes could contribute. Experimentally, such multi-step processes can be investigated by accompanying cluster emission in meson production \[4\].

In this work we study whether three-body collisions could remarkably contribute to the production of $\phi$ mesons in proton-nucleus and heavy-ion collisions. At threshold such a mechanism could be dominant because less bombarding energy is required if the projectile nucleon interacts with two target nucleons instead with a single one. For 'free' nucleons the threshold reduces from 2.6 GeV to 1.8 GeV.

The consideration of $\phi$ production is interesting since also the $K^-$ production proceeds considerably through an intermediate $\phi$ meson. In the near-threshold proton-proton collision roughly half of the $K^-$ mesons come from the $\phi$ decay \[8\]. $K^-$ mesons have been measured by the KaoS collaboration at the heavy-ion synchrotron SIS of GSI Darmstadt in proton-Au collisions for a bombarding energy of 3.5 GeV \[9\], and the ANKE collaboration envisages the $K^-$ measurements for collisions of protons on various light targets at the cooler synchrotron COSY of FZ Jülich \[10\]. For a firm understanding of these reactions a necessary prerequisite is the theoretical understanding of the role of the $\phi$ channels.

We are going to calculate the cross section for the reaction $p+2N \rightarrow \phi +$
3N from the invariant amplitude based on tree-level diagrams. In refs. [11, 12] several diagrams were explored thoroughly for the case of the two-nucleon \( \phi \) production, i.e. \( N+N \rightarrow \phi+2N \). As shown there, the available data can be described by diagrams with internal meson conversion in a \( \pi \rho \phi \) vertex. Based on this investigation we restrict ourselves to such types of diagrams as illustrated in Fig. [1]. Three nucleons in states \( N_1, N_2, N_3 \) act together via boson exchanges. Literally in the first diagram, after the interaction of particles 1 and 2, the intermediate fermion \( X \) or pion \( \pi_2 \) has gathered sufficient energy to produce the \( \phi \) meson interacting with the third nucleon. In the right hand diagram the time ordering is exchanged.

Similar processes are described within the current transport models by sequential two-step processes which also allow to accumulate the energy of several nucleons. In this case, however, the intermediate particles are real, i.e. they are moving on-shell. Recently also effort is made to implement properly the particles’ off-shell propagation (see ref. [13]). In the diagrams displayed in Fig. [1] the intermediate particles \( X \) and \( \pi_2 \) can be either on shell or off shell depending on the initial and final momenta of the particles. There are subspaces in the phase space, where the intermediate particles move on shell. It is our aim to investigate these two effects and to relate them to the standard treatment as sequential two-step processes.

Many-body collisions have already been treated several times in the literature. In ref. [14] this mechanism was applied to analyse the cumulative backward emission of nucleons. In this region of the phase-space off-shell contributions become important. Many-body collisions were also incorporated in the framework of transport models [15, 16]. The results of these investigations show moderate effects; here we extend these studies to very subthreshold reactions for the \( \phi \) production.
2 Elementary cross section

To calculate the cross section for three-particle collisions we will apply the one-boson exchange model which is often used to parametrize Lorentz invariant cross sections. Our approach is strictly based on tree-level diagrams with effective parameters adjusted to experimental data. For our investigations we restrict ourselves to the diagrams displayed in Fig. 1. We also consider two further diagram types, where the $\rho$ and the $\pi_2$ mesons are interchanged. For the sake of simplicity in the present exploratory investigation we employ only the nucleon propagator for the intermediate particle $X$ and the pion exchange for the interaction between nucleons. Under these propositions the whole set of diagrams we are considering consists of $4 \times 36$ individual diagrams, where the second factor denotes the number of permutations of the respective three fermion lines in the initial and final states.

In ref. [5] the interaction Lagrangians for these diagrams have been presented which read in standard notation

\[ \mathcal{L}_{\pi NN} = - \frac{f_{\pi NN}}{m_\pi} \bar{\psi} \gamma^5 \gamma^\mu \tilde{\tau} \psi \partial_\mu \bar{\pi}, \]  \hfill (1)
\[ \mathcal{L}_{\rho NN} = - g_{\rho NN} \bar{\psi} \gamma^\mu \tilde{\tau} \psi \bar{\rho}_\mu - \frac{f_{\rho NN}}{2m_N} \bar{\psi} \sigma^{\mu\nu} \tilde{\tau} \psi \partial_\mu \bar{\rho}_\nu, \]  \hfill (2)
\[ \mathcal{L}_{\pi\rho\phi} = \frac{f_{\pi\rho\phi}}{m_\phi} \epsilon_{\mu\nu\alpha\beta} \partial^\mu \phi^\nu \partial^\alpha \bar{\rho}^\beta \bar{\pi}. \]  \hfill (3)

These equations contain the nucleon field $\psi$, the iso-vector meson fields $\bar{\pi}$, $\bar{\rho}^\mu$ and the iso-scalar field $\phi^\mu$. The Dirac matrices, the Levi-Civita symbol and the corresponding particle masses are denoted by $\gamma^\mu$, $\epsilon_{\mu\nu\alpha\beta}$ and $m_i$, respectively. Greek indices stand for Lorentz indices, and we put $\hbar = c = 1$. Furthermore, a formfactor is needed for each internal line of an exchanged meson $i$ coming from or entering a vertex $(jkl)$. We adopt here the standard monopole form factor as a function of the square of the transferred four-
momentum $q_i^\mu$ and a cut-off parameter $\Lambda_{ijkl}^i$:

$$F(q_i^2) = \frac{(\Lambda_{ijkl}^i)^2 - m_i^2}{(\Lambda_{ijkl}^i)^2 - q_i^2}. \quad (4)$$

The following values of the coupling constants and cut-off parameters have been used: $f_{\pi NN} = 0.989$, $g_{\rho NN} = 3.718$, $f_{\rho NN} = 6.1 g_{\rho NN}$, $f_{\rho \rho} = 1.04$; $\Lambda_{\pi NN}^\pi = 1.6$ GeV, $\Lambda_{\rho NN}^\rho = 1.3$ GeV, $\Lambda_{\pi \rho}^\pi = 0.95$ GeV, $\Lambda_{\pi \rho}^\rho = 1.2$ GeV. Most of these values are taken from refs. [5, 17] and have been successfully applied in the near-threshold region. For $\phi$ meson production at an energy of 82 MeV above the production threshold we obtain a cross section of 0.04 $\mu b$ using these parameters. This value is smaller than the recently measured value [8]. However, in ref. [12] it has been demonstrated that the agreement between data and calculation is improved by including in addition the $\phi$ meson bremsstrahlung from external nucleon legs and the final state interaction.

The cross section for $\phi$ meson production by an incoming nucleon with momentum $p_1 = p_{\text{lab}}$ hitting a nucleon with momentum $p_2$ which is surrounded by another nucleon with momentum $p_3$ in a medium of density $n_0$ reads

$$\sigma = \frac{1}{4m_N p_{\text{lab}}} \frac{n_0}{2m_N} \frac{1}{3!} \frac{1}{(2\pi)^8} \frac{1}{8} \sum_{\text{projections}} \int |dL_{\text{inv}}| \sum_{\text{diagrams}} T |^2. \quad (5)$$

The first quotient contains the incoming flux with the laboratory momentum $p_{\text{lab}}$. The second quotient is related to the density $n_0$ of the second collision partner; the fact that we calculate the cross section at a single nucleon accounts for the following factor 1/2. The factorial 3! occurs because the outgoing nucleons are indistinguishable. The first sum (including 1/8) averages over the initial spin-projection quantum-numbers and runs over all outgoing spin and isospin states while the second sum contains the $4 \times 36$
diagrams mentioned above which are numerically evaluated including all interference terms. The integration in Eq. (3) is carried out over the Lorentz invariant phase space spanned by the momenta of the outgoing particles, 
\[ dL_{inv} = \prod_f d^3 p_f / (2p_0^f) \delta^4(\sum p_f - \sum p_i). \]

### 3 On-shell problem

In some regions of the phase space the intermediate particle \( X \) can become on-shell and the propagator gets singular. In the following we discuss first the situation for the pion propagator \( \pi \) in Fig. 1 which in a large part of the phase space becomes on shell. The standard procedure \[6, 14, 16, 18\] to avoid the singularity is to introduce into the propagator an imaginary part \( im\Gamma \). That means that one writes the matrix element of Fig. 1 as
\[ T = \bar{T}_1 \frac{1}{p^2 - m^2 + im\Gamma} \bar{T}_2, \]
where the quantities \( \bar{T}_1 \) and \( \bar{T}_2 \) describe the two subprocesses \( 1+2 \rightarrow 4+5+\pi \) and \( \pi+3 \rightarrow 6+\phi \), which are connected by the propagator of the pion with mass \( m = m_\pi \) and four-momentum \( p^\mu \). The physical reason for the occurrence of the imaginary part is that the intermediate particle does not propagate in vacuum but in a dense medium instead. Then it can move only a finite distance of the mean free path \( \lambda \) during its life time \( \tau \) which is related to the imaginary part in the denominator via \( \Gamma = 1/\tau \) (see refs. \[14, 16\]). The same method is applied if unstable particle are treated in the entrance channels \[6, 18\].

However, the on-shell propagation is in fact a process which is already treated in standard transport models as two consecutive two-step processes, where the particle \( X \) is treated as a real on-shell particle. Now we try to
separate the genuine three-body processes out of the contributions given by
the whole propagator in Eq. (3).

If the particles were in vacuum the amplitude $\mathcal{T}$ would also be given
by Eq. (3), but the quantity $m\Gamma$ would be replaced by $\varepsilon$ with Feynman’s
prescription $\varepsilon \to +0$. The square of this matrix element diverges since it is
proportional to the square of the function $\delta(p^2 - m^2)$ in the limit $\varepsilon \to 0$.
Since the origin (see e.g. [19]) of the $\delta$ function is the integration over the
time the pion moves, one can replace it with $\tau \delta(p^2 - m^2)/(2\pi m)$, where the
time $\tau$ is connected with $\varepsilon$ via $\tau = m/\varepsilon$. Thus, the transition probability
is proportional to the proper time $\tau$ in the rest system of the intermediate
particle. This treatment is analogous to the standard derivation [19] of the
cross section (3) which becomes proportional to the space-time volume.

Due to the terms coming from the exchange of the initial and final mo-
menta the matrix $\mathcal{T}$ becomes on shell in different regions of the phase space
spanned by the four final-state momenta. In general there are $3 \times 3$ sub-
spaces defined by $(p_a - p_b)^2 = m^2$, where $p_a$ and $p_b$ are the total momenta
of the three possible initial pairs $a = [(12),(23),(31)]$ and three final pairs $b =
[(45),(56),(64)]$, respectively.

In the near neighbourhood of each subspace we divide the $\mathcal{T}$ matrix in a
smooth part and a singular one,

$$\mathcal{T} = \mathcal{T} + T_1 \frac{1}{p^2 - m^2 + i\varepsilon} T_2.$$  \hspace{1cm} (7)

Contrary to Eq. (3) the amplitudes $T_1$ and $T_2$ describe now subprocesses with
free (on-shell) particles. In calculating the phase space integral we treat the
square of the second term as

$$\sigma' = \sigma_{\text{two-step}} + \sigma'_{\text{three}}.$$  \hspace{1cm} (8)
\begin{align*}
\sigma_{\text{two-step}} &= C \int dL_{\text{inv}} \frac{\pi \tau}{m} \delta(p^2 - m^2) |\hat{T}|^2, \\
\sigma'_{\text{three}} &= C \lim_{\varepsilon \to 0} \int dL_{\text{inv}} |\hat{T}|^2 \left[ \frac{(p^2 - m^2)^2}{[(p^2 - m^2)^2 + \varepsilon^2]^2} - \frac{\pi}{2\varepsilon} \delta(p^2 - m^2) \right],
\end{align*}
where \( \hat{T} = T_1 T_2 \). The quantity \( C \) comprises the factors standing in front of the integral in Eq. (\( 5 \)). These expressions arise by splitting the propagator in Eq. (7) into the \( \delta \) function and its principal value, while the last term \( \sigma'_{\text{three}} \) is constructed such that it is finite as we will see below.

We identify the first part (9) with the two-step process as it is proportional to the time of flight of the intermediate particle which can only be fixed by additional information on the collision geometry. This part can be transformed into an expression describing a sequence of two two-nucleon collisions. Introducing the identity \( \int d^4p \delta^4(p_1 + p_2 - p_4 - p) = 1 \) and using the definition of the two-particle cross sections we obtain

\begin{equation}
\sigma_{\text{two-step}} = \int dP_\pi \frac{d\sigma}{dP_\pi}(P_1, P_2) \sigma_\phi(P_\pi, P_3) (n_0 \hat{v} \tau).
\end{equation}

Now the cross section has been factorized in the cross sections \( d\sigma/dP_\pi \) for pion production and \( \sigma_\phi \) for \( \phi \) production via pion absorption by the nucleon 3. The sum over the final subspaces \( b \) has cancelled the factorials in the definitions of the cross sections. In principle one should sum Eq. (11) over the three subspaces \( a \), however, we have used only the first one in accordance to Eq. (5). The last factor in the round bracket contains the relative velocity \( \hat{v} = (p_\pi p_3) v_{\text{rel}}, \) \( v_{\text{rel}} = \sqrt{(1 - (m m_N/(p_\pi p_3))^2} \) between the intermediate particle and its partner 3. It originates from the flux factor in the definition of the production cross section and is proportional to the length \( L = \hat{v} \tau \) the pion travels.

A similar expression like Eq. (11) arises if the intermediate nucleon \( X \) moves on shell. Formally, the integral over the pion momentum in Eq. (11) is replaced by an integral over the scattering angle of the elastic nucleon-nucleon

\begin{align*}
\sigma_{\text{two-step}} &= C \int dL_{\text{inv}} \frac{\pi \tau}{m} \delta(p^2 - m^2) |\hat{T}|^2, \\
\sigma'_{\text{three}} &= C \lim_{\varepsilon \to 0} \int dL_{\text{inv}} |\hat{T}|^2 \left[ \frac{(p^2 - m^2)^2}{[(p^2 - m^2)^2 + \varepsilon^2]^2} - \frac{\pi}{2\varepsilon} \delta(p^2 - m^2) \right],
\end{align*}
where \( \hat{T} = T_1 T_2 \). The quantity \( C \) comprises the factors standing in front of the integral in Eq. (5). These expressions arise by splitting the propagator in Eq. (7) into the \( \delta \) function and its principal value, while the last term \( \sigma'_{\text{three}} \) is constructed such that it is finite as we will see below.

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\sigma_{\text{two-step}} = \int dP_\pi \frac{d\sigma}{dP_\pi}(P_1, P_2) \sigma_\phi(P_\pi, P_3) (n_0 \hat{v} \tau).
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scattering. To derive such an equation the on-shell part of the amplitude \( \hat{T} \), see Eq. (9), is transformed into a sum over the spin quantum numbers of the intermediate nucleon using the properties of the Dirac operator,

\[
\hat{T} = T_1(\gamma^\mu p_\mu + m_N)T_2 = \sum_n T_{1n}T_{2n} .
\] (12)

To arrive at the standard formula the following approximation is needed:

\[
\sum \sum |T_{1n}T_{2n}|^2 \approx \frac{1}{2} \sum |T_1|^2 \sum |T_2|^2
\]
which is fulfilled if spin-flip processes are not important.

If we apply Eq. (11) to a p+A collision of a nucleon on a nucleus of mass number \( A \) the cross section would be proportional to \( A^{4/3} \) in the subthreshold region as the length \( L \) is proportional to \( A^{1/3} \). However, if the reaction proceeds in a dense medium the length reduces to the mean free path, and the factor \( n_0\hat{v}\tau = 1/\sigma_{tot} \) gives the inverse total cross section. This brings us back to the prescription of Eq. (8) and the two-step formula employed, e.g., in [20]. In this case the cross section of a p+A reaction is proportional to \( A^{2/3} \) taking into account the fact that the incoming particle suffers rescattering processes too.

After separating the two-step process from the total cross section the remaining parts form the genuine three-body cross section \( \sigma_{three} \). It contains the smooth terms of the \( \mathcal{T} \) matrix in Eq. (7) and the limit of the divergent contributions of Eq. (10). To see that the expression in the square brackets in Eq. (10) is finite we analytically integrate over \( p^2 \) around the singularity within the interval between \( p^2 = m^2 \pm mD \). Assuming that \( \hat{T} \) is a smooth function of \( p^\mu \) we obtain \(-2/mD\) for a small but finite value of \( D \). Dividing the integration into parts near and far the singularity we write the three-body cross section as

\[
\sigma_{three} = C \int dL_{inv} \left[ \Theta(|p^2 - m^2|) - mD \right] |\mathcal{T}|^2
\]
\[ + \delta(p^2 - m^2) \left( 2mD|\mathcal{T}|^2 + 2\pi \Im(\mathcal{T}^\dagger \mathcal{T}^\ast) - \frac{2}{mD} |\mathcal{T}|^2 \right) \]  

where \( \Theta \) denotes the step function. Terms of higher than first order in \( mD \) have been neglected.

4 In-medium effects

In Eq. (10) we have assumed that the intermediate particles move freely. Inside a medium we can employ a finite imaginary part as already inserted in Eq. (6). Recently great effort has been made to construct the in-medium propagators and corresponding spectral functions calculating the self-energies in one-loop order and higher approximations to couple the propagating particles to the constituents of the surrounding medium. However, for simplicity reasons we treat these effects phenomenologically as collision broadening [21] introducing in the propagator in Eq. (6) the collision width \( \Gamma_{\text{coll}} = n_0 \hat{v} \sigma_{\text{tot}} \)

\[ \Gamma = \frac{1}{\tau} = \Gamma_0(p^2) + \Gamma_{\text{coll}} \]  

We have already foreseen that the intermediate particle is a resonance with an energy dependent decay width \( \Gamma_0 \). Although the definitions in Eq. (10) can be used also for a finite value of \( \varepsilon \) after transforming back the \( \delta \) functions and their squares into the finite-epsilon representations, this procedure is ambiguous here since there is no singularity anymore. Thus, we define the genuine three-body cross section as the difference of the total cross sections and all possible two-step cross sections:

\[ \sigma_{\text{three}} = C \int dL_{\text{inv}} \left( |\mathcal{T}|^2 - \sum_c |\mathcal{T}_c|^2 \delta(p_c^2 - m_c^2) \frac{\pi}{\Gamma m_c} \right). \]  

The sum runs over all open channels \( c \) for two-step collisions, where resonances are included as long as their decay channels are open, e.g. intermediate \( \rho \) mesons with masses \( m_c \) being larger than the two-pion decay threshold.
Due to the surrounding medium the $T$ matrix looses its Lorentz invariance as the relative velocity refers to the medium’s rest frame which we fix to be the laboratory system. We mention that the value of $\hat{v}$ becomes very large for pions which create a $\phi$ meson. A pion needs a value of $\hat{v} \sim 11$ in a collision with a nucleon resting in the laboratory system, while its cross section amounts to about 25 mb. The corresponding relative velocity for a $\rho$ meson is considerably smaller due to its larger mass.

In our calculations we use the effective widths defined in Eq. (14) for those particles only which serve as intermediate particles, and treat particles which occur inside the diagrams of the two consecutive processes like free particles. This corresponds to the widely used standard method by which cross sections are calculated with free particle propagators.

5 Numerical results

Now we are going to discuss the cross section for a proton which hits another proton at rest embedded in a protonic surrounding with density $n_0=0.16$ fm$^{-3}$. We vary the bombarding energy $E_{\text{lab}}$ from the three-body threshold of 1.8 GeV for $\phi$ meson production up to the two-nucleon threshold of 2.6 GeV. In this special case of two particles without relative motion the nucleonic intermediate states cannot get on shell below the two-nucleon threshold while pions and $\rho$ mesons already allow two-step processes at an bombarding energy of about 0.04 GeV above threshold.

First we have calculated the cross section without the in-medium effect. The two-step cross section (9) is proportional to the flight time $\tau$ or the system size. On the other hand the genuine three-body cross section which is calculated according to Eq. (13) does not depend on the system size and
is presented in Fig. 2 by the thin full line.

The cross sections are considerably diminished if the rescattering effects of the intermediate particles are taken into account. We have used a total cross section of 25 mb for both pions and $\rho$ mesons. The two corresponding two-step processes (defined within the right hand site of Eq. (15)) are also displayed by the dashed and the dot-dashed lines in Fig. 2, respectively. These cross sections are considerably smaller than the original three-body cross section. The sum of these two quantities has to be compared to the total cross section calculated with the complete $T$ matrix of Eq. (15) which is presented by the thick full line. The difference should be the contribution of the genuine three-body cross section which, however, turns out to be negative (thick dashed line). The reason is that due to the large width of 200 MeV the pion propagator in the three-body matrix element reaches out very far in the phase space and picks up $T$ matrix elements which are smaller than in the on-shell region.

Finally, we calculate the cross section for the collision of a proton on a target nucleus consisting of $N$ neutrons and $Z$ protons. The production is now a superposition of primary reactions on pp and pn pairs. The production at a pn pair is strongly enhanced compared with that at a pp pair because the isospin coupling allows in the latter case only $\pi^0$ exchanges which have smaller vertex strengths than those of charged mesons. In more complete calculations which, e.g., include also the exchange of $\sigma$ mesons such a large difference may not occur. Furthermore, the nucleons have Fermi momentum which can be taken into account by using the spectral function $\delta(p^0 - m_N - \mathbf{p}^2/[2(A - 1)m_N + E_{sep})] \exp(-5\mathbf{p}^2/2k_{\text{Fermi}}^2)$ with a Fermi momentum of $k_{\text{Fermi}} = 0.27$ GeV and an average separation energy of $E_{sep} = 20$ MeV. The effect of the Fermi motion allows the production already at much
lower energy. In view of the exploratory nature of our calculations it is not worth using improved spectral functions which can be found in ref. [22]. Due to the effect of Fermi motion the production already begins at much lower energy. In Fig. 3 the cross sections $\sigma_{p-pp}$ and $\sigma_{p-pn}$ for $\phi$ production at a proton in the surrounding of protons or respectively neutrons are presented. They are used to calculate the $\phi$ production in the reaction of a proton with $^{12}\text{C}$ in accordance to $\sigma = [(Z(Z - 1)\sigma_{p-pp} + 2ZN\sigma_{p-pn} + N(N - 1)\sigma_{p-nn})/A$. We have used an effective particle number $A = 6$ as in ref. [21] which results from the fact that many target nucleons are screened due to scattering processes of the incoming proton. For comparison we have also included into Fig. 3 the $\phi$ production cross-section (dash-dotted line) calculated as superposition of primary two-nucleon collisions, $\sigma = Z\sigma_{p-p} + N\sigma_{p-n}$, which is smaller than the three-body production, in contrast to the results of ref. [20].

Finally we should keep in mind that we have constrained our Lagrangian of the nucleon-nucleon-meson interaction. For instance neglecting the intermediate $\Delta$ particles causes an underestimation of the pion production. Therefore, the actual $\phi$ production is expected to be larger than presented in Figs. 2 and 3.

6 Conclusion

It has been our aim to study the features and consequences of elementary three-body collisions. This is an important issue when calculating particle production near threshold for nucleon-nucleus and nucleus-nucleus collisions within the framework of transport models. Here, we have studied the $\phi$ production below the free nucleon-nucleon threshold. It turned out that genuine three-body processes are not important as most of the intensity can
be evaluated via consecutive two-nucleon reactions. In the case considered here the two-step processes slightly overestimate the cross section. However, it is necessary that all intermediate particles which can become on shell are treated properly within the transport codes as they contribute essentially to particle production.
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Figure 1: Dominating tree-level diagrams contributing to the three-body $\phi$ meson production. Two further diagrams are used in the text, where the $\pi_2$ and the $\rho$ lines are interchanged.
Figure 2: Cross section of $\phi$ meson production by a proton hitting a proton at rest embedded within a proton environment. The thin line gives the three-body cross section without intermediate rescattering whereas the other lines show the results in a medium with rescattering. For details see text.
Figure 3: Cross section for $\phi$ meson production for a proton on carbon collision (solid curve). Shown are also the cross sections for a proton colliding with a proton surrounded by protons (dotted line) or neutrons (dashed line) including Fermi motion. The dash-dotted line shows the effect of two-nucleon collisions.