Quark matter and quark stars at finite temperature in Nambu–Jona-Lasinio model

Peng-Cheng Chu¹,a, Xiao-Hua Li²,3,b, Bin Wang¹, Yu-Min Dong¹, Yu-Yue Jia¹, Shu-Mei Wang¹, Hong-Yang Ma¹

¹ School of Science, Qingdao Technological University, Qingdao 266000, China
² School of Nuclear Science and Technology, University of South China, Hengyang 421001, China
³ Cooperative Innovation Center for Nuclear Fuel Cycle Technology and Equipment, University of South China, Hengyang 421001, China

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Abstract  We extend the SU(3) Nambu–Jona-Lasinio (NJL) model to include two types of vector interaction. Using these two types of vector interaction in NJL model, we study the quark symmetry free energy in asymmetric quark matter, the constituent quark mass, the quark fraction, the equation of state (EOS) for β-equilibrium quark matter, the maximum mass of QSs at finite temperature, the maximum mass of proto-quark stars (PQSs) along the star evolution, and the effects of the vector interaction on the QCD phase diagram. We find that comparing zero temperature case, the values of quark matter symmetry free energy get larger with temperature increasing, which will reduce the difference between the fraction of u, d, and s quarks and stiffen the EoS for β-equilibrium quark matter. In particular, our results indicate that the maximum masses of the quark stars increase with temperature because of the effects of the quark matter symmetry free energy, and we find that the heating(cooling) process for PQSs will increase (decrease) the maximum mass within NJL model.

1 Introduction

Investigating the properties of strong interaction matter plays a central role in understanding the nuclear structures and reactions, and the critical issues in astrophysics, which is one of the fundamental issues in modern nuclear physics, astrophysics, and cosmology. Quantum chromodynamics (QCD) is widely accepted as the fundamental theory for the strong interaction. In terrestrial laboratories, high energy heavy ion collisions (HICs) provide a unique tool to explore the strong interaction matter. In astrophysics, compact stars can provide a way to explore the strong interaction matter at high baryon density and low temperature [1,2]. Neutron stars (NSs) have been shown to provide the natural testing grounds of our knowledge about the equation of state (EOS) of neutron-rich nuclear matter [3,4]. In the core of NSs, the baryon density can even be larger than about six times the normal nuclear matter density $n_0 = 0.16$ fm$^{-3}$, so there might exist quark matter. Then NSs may be converted to quark stars (QSs) [5,6], which consists of deconfined u, d, and s quark matter(with some leptons) in β-equilibrium condition. The reliable existence of the QSs, whose hypothesis cannot be conclusively ruled out, is one of the most intriguing part of modern astrophysics and has important implications for investigating the strong interaction physics, especially the EoS of strange quark matter (SQM) that essentially determine the structure of QSs [7–13]. Phenomenological models have been built to study the properties of SQM, for instance, the MIT bag model [13–15], the Nambu–Jona-Lasinio (NJL) model [16,17], the pQCD approach [18–20], the Dyson–Schwinger approach [21–24], confined density-dependent quark mass model [25–28].

In these years, two massive neutron stars have been measured. One is the radio pulsar J1614-2230 [29] with the mass of $1.97 \pm 0.04 M_\odot$ by using the general relativistic Shapiro delay, and the other is J0348+0432 [30] with mass $2.01 \pm 0.04 M_\odot$. Due to the EoS for quark matter is soft, these high mass compact stars seem to rule out conventional QS models, although some other models can still calculate the large mass pulsar [15,31–42]. In QSs, the isovector properties of SQM may play an important role, because the star matter has large u–d quark asymmetry, i.e., isospin asymmetry [43–48]. Therefore, it is interesting and important to investigate the isovector properties of quark matter, which can be further used to understand the properties of QSs and

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¹ e-mail: kyois@126.com
² e-mail: lixiaohuaphysics@126.com
the QCD phase diagram, and the isospin effects of partonic dynamics in high energy HICs. In Refs. [49–51], the authors’ results indicate that increasing the quark matter symmetry energy in CIDDM model can significantly stiffen the EoS for SQM and enhance the maximum mass of static QSs at zero temperature. In order to investigate the thermodynamical properties for asymmetric quark matter at finite temperature, one should consider the symmetry free energy in $u$–$d$–$s$ quark matter, whose value is identical to that of the quark matter symmetry energy at zero temperature. In this work, we intend to study the properties of quark matter at finite temperature, and it is of great interest and importance to find if the effects of quark matter symmetry free energy can still stiffen the EoS for SQM and enhance the maximum mass of QSs at finite temperature, as what the effects of the quark matter symmetry energy perform at zero temperature in Ref. [49].

In the newly born compact stars (PQSs), neutrinos are produced by electron capture in SQM and trapped by their short mean free paths from leaving the star on dynamical timescales, and then at the very beginning of the birth of a PQS, the number of leptons per baryon with these trapped neutrinos is approximately 0.4, which depends on the efficiency of electron capture reactions during the gravitational collapse of the progenitor star, and the entropy per baryon is about one. In the following 10–20 s, the star matter is heated by the diffusing neutrinos, and the entropy density increases to two, while the neutrino ratio decreases almost down to zero. Following this heating stage, PQSs begins cooling down, and finally forms into cold QSs. Along the evolution line from a PQS to a cold QS, people usually take the snapshots to study how the star evolves as (I) $S/n_B = 1, Y_l = 0.4$; (II) $S/n_B = 2, Y_{\mu} = 0$; (III) $S/n_B = 0, Y_{\mu} = 0$ [52–56], where $S$ is the entropy density for star matter, $n_B$ is the baryon density, and $Y_l$ and $Y_{\mu}$ stand for the lepton fraction and neutrino fraction, respectively.

This paper is organized as follows. In Sect. 2, we describe the SU(3) Nambu–Jona-Lasinio (NJL) model with two types of vector interaction: (1) the flavor-dependent repulsion among different flavors of quarks with the coupling constant $G_V$, and (2) the universal repulsion and the vector/scalar–isovector interaction among different flavors of quarks with the coupling constants $g_{1V}, G_{1V}$, and $G_{1S}$. In Sect. 3, we study the quark symmetry free energy in asymmetric quark matter, the constituent quark mass, the quark fraction, the EoS in SQM, the maximum mass of quark stars at finite temperature, the maximum mass of proto-quark star, and the $M$–$T$ and $\mu$–$T$ plane for the properties of the phase diagram by using NJL model. The conclusion and discussion is given in Sect. 4.

### 2 The theoretical formulism

#### 2.1 The three-flavor NJL model with two types of vector interactions

The lagrangian density for SU(3) NJL model is written as

$$L = \bar{\psi}(i\gamma_\mu \partial^\mu - \hat{M} + \hat{\mu}\gamma_0)\psi + L_{\text{det}} + L_S + \left[ L_{V_a} + L_{V_b} \right],$$

(1)

where the form of the scalar part $L_S$ is

$$L_S = G_S \sum_\alpha [ (\bar{\psi}_\alpha^a \gamma^a \psi) + (\bar{\psi} V^a \gamma^a \psi)^2 ].$$

(2)

Here $G_S$ is the coupling constant in the scalar channel, $\lambda_\alpha (a = 1, \ldots, 8)$ are the Gell-Mann matrices and the generators of the SU(3) flavor groups, and $\lambda_0 = \sqrt{2}/3 I$ with $I$ the $3 \times 3$ unit matrix. The 't Hooft term $L_{\text{det}}$ takes the form

$$L_{\text{det}} = -K \{ \det_f[\bar{\psi}(1 + \gamma_5)\psi] + \det_f[\bar{\psi}(1 - \gamma_5)\psi] \},$$

(3)

which breaks the axial $U(1)_A$ symmetry. $\psi = (u, d, s)^T$ represents the three-flavor quark field, and the bare quark mass matrix is defined as $\hat{m}_c = \text{diag}(m_u, m_d, m_s)$.

In this paper, we use two types of vector interaction, “type A” ($L_{V_a}$) and “type B” ($L_{V_b}$), in the lagrangian as

$$L_{V_a} = -G_V \sum_\alpha [ (\bar{\psi}^a \gamma^\mu \lambda^a \psi)^2 + (\bar{\psi}^a i\gamma^\mu \gamma_5 \lambda^a \psi)^2 ],$$

(4)

$$L_{V_b} = -g_{1V}(\bar{\psi} \gamma^\mu \psi)^2 - G_{1V}[ (\bar{\psi}^a \gamma^\mu \tau^a \psi)^2 + (\bar{\psi}^a i\gamma^\mu \gamma_5 \tau^a \psi)^2 ] - G_{1S}(\bar{\psi} \tau^a \psi)^2 + (\bar{\psi} i\gamma^\mu \gamma_5 \tau^a \psi)^2 ],$$

(5)

where the term proportional to $g_{1V}(>0)$ in $L_{V_a}$ gives a flavor-dependent vector repulsion among different flavors of quarks, while the one proportional to $g_{1V}(>0)$ in $L_{V_b}$ shows the universal vector repulsion which cannot be distinguished in different flavors. The terms proportional to $G_{1V}(>0)$ and $G_{1S}$ in Eq. (5) indicate the vector/scalar–isoscalar interactions among different flavors of quarks, and $\tau$ is for the Pauli matrices. Here we consider the extra term for the isovector channels($G_{1V}$ and $G_{1S}$ terms) in $L_{V_b}$ in order to distinguish the isoscalar and isovector effects for the vector channel, because the $G_{1V}$ term interaction is flavor dependent while $g_{1V}$ term is flavor independent.

Using the mean field approximation, one can calculate the lagrangian density for quarks as

$$L = \bar{\psi} \left[ \gamma_\mu i\partial^\mu - \hat{M} + \hat{\mu}\gamma_0 - 4G_{1V}\theta - 2g_{1V}\gamma_0 \sum_{i=u,d,s} n_i \right. \left. -2G_{1S}(n_u - n_d) \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & n_d & 0 \\ 0 & 0 & n_s \end{array} \right) \psi \right].$$

(6)
where \( \Omega \) is the total baryon number density, \( \delta = 3(n_d - n_u)/(n_u + n_d) \) is the isospin asymmetry for asymmetric quark matter, \( n_i = u, d, s \) is the number density for the \( i \)th flavor of quarks, and \( E_0(n_B, n_s) = E(n_B, \delta = 0, n_s) \) is the binding energy per baryon number in three-flavor \( u-d-s \) quark matter with an equal fraction of \( u \) and \( d \) quarks. The quark matter symmetry energy is expressed as

\[
E_{\text{sym}}(n_B, n_s) = \left. \frac{1}{2!} \frac{\partial^2 E(n_B, \delta, n_s)}{\partial \delta^2} \right|_{\delta=0}. 
\]

In order to investigate the thermodynamical properties for asymmetric quark matter at finite temperature case, one should consider the symmetry free energy in \( u-d-s \) quark matter instead of the symmetry energy, because symmetry energy is customarily used to investigate the properties of quark matter at zero temperature condition, classically. In our own perspective, using a similar expression to symmetry energy is a feasible way of defining the quark symmetry free energy, then the quark matter symmetry free energy \( F_{\text{sym}} \) can be expressed as

\[
F_{\text{sym}} = \left. \frac{1}{2!} \frac{\partial^2 F}{\partial \delta^2} \right|_{\delta=0},
\]

where \( F \) is the free energy per baryon for quark matter. The symmetry free energy and symmetry energy possess the same value at zero temperature because of the entropy density is zero. Since the isovector properties of quark matter is poorly known at finite baryon density and finite temperature, it is of great interest and importance to find if the effects of quark matter symmetry free energy can still stiffen the EoS for SQM and enhance the maximum mass of QSs at finite temperature compared to the zero temperature case.

### 2.2 The quark matter symmetry free energy

The EOS of quark matter consisting of \( u, d, \) and \( s \) quarks, being defined by its binding energy per baryon number, can be expanded in isospin asymmetry \( \delta \), which is similar to the case of nuclear matter [57], as

\[
E(n_B, \delta, n_s) = E_0(n_B, n_s) + E_{\text{sym}}(n_B, n_s)\delta^2 + O(\delta^4),
\]

with (\( i, j, k \) = \( u, d, s \)) and \( \tau_{3i} \) being the isovector quantum number for quarks: \( \tau_{3u} = 1, \tau_{3d} = -1 \) and \( \tau_{3s} = 0 \). The chiral condensate is determined as

\[
\sigma_f = \langle \bar{\psi}_f \psi_f \rangle = -i \int \frac{d^4p}{(2\pi)^4} \text{tr} \left( \frac{1}{\slashed{p} - M_f + i\epsilon} \right).
\]

The thermodynamical potential \( \Omega_q \) for different flavors of quarks can be expressed as

\[
\Omega_q = \Omega_{in}^u + \Omega_{in}^d + \Omega_{in}^s + 2G_S(\sigma_u^2 + \sigma_d^2 + \sigma_s^2) - 4K_S\sigma_d \sigma_s - G_{IV}(\sigma_u - n_u - n_d)^2 + G_{IS}(\sigma_u - \sigma_d)^2,
\]

with the logarithmic contribution

\[
\Omega_{in}^f = -\frac{2N_C}{A} \int_0^\Lambda \frac{d^3p}{(2\pi)^3} \left\{ E_f + T \ln \left[ 1 + e^{-\beta(E_f - \tilde{\mu}_f)} \right] \right\} + T \ln \left[ 1 + e^{-\beta(E_f + \tilde{\mu}_f)} \right],
\]

where

\[
\tilde{\mu}_f = \mu_f - 4G_V n_f - 2G_V \sum_{i=u,d,s} n_i - 2G_{IV} \tau_3 f (n_u - n_d)
\]

is the chemical potential for each flavor of quarks, \( N_C = 3 \) is the color number, \( T = 1/\beta \) represents the temperature, \( \Lambda \) is the value of momentum cutoff, and \( E_f^2 = p^2 + M_f^2 \).

The pressure and entropy density can be derived with \( P = -(\partial \Omega_{\text{tot}}(T, \mu) - \partial \Omega_{\text{tot}}(0,0)) \) and \( S = \partial \Omega_{\text{tot}}/\partial T \), respectively, where \( \Omega_{\text{tot}} \) means the total thermodynamic potential for quark matter and \( \Omega_{\text{tot}}(0,0) \) is chosen to make pressure vanish in vacuum as customary. The free energy density and the energy density can be acquired by using \( F = \Omega_{\text{tot}} - \sum_i \mu_i \frac{\partial \Omega_{\text{tot}}}{\partial \mu_i} \) and \( \epsilon = \Omega_{\text{tot}} - T \frac{\partial \Omega_{\text{tot}}}{\partial T} + \sum_i \mu_i n_i \).

### 2.3 Properties of quark matter at finite temperature in beta-equilibrium

SQM is usually assumed to be composed of \( u, d, s \) quarks and leptons (\( e^-, \mu, \nu_e \) and \( \nu_\mu \)) with electric charge neutrality in beta-equilibrium. Then the weak beta-equilibrium condition can be expressed as

\[
\mu_u + \mu_e - \mu_{\nu_e} = \mu_d = \mu_s, \\
\mu_\mu = \mu_e \text{ and } \mu_{\nu_\mu} = \mu_{\nu_e}
\]
where $\mu_i (i = u, d, s, e^- \text{ and } \nu)$ is the chemical potential for all kinds of particles in SQM. The electric charge neutrality condition is

$$\frac{2}{3}n_u = \frac{1}{3}n_d + \frac{1}{3}n_s + n_e. \quad (16)$$

The number density for the $i$th particle can be written as

$$n_i = \frac{g_i}{2\pi^2} \int_0^\Lambda \left[ \frac{1}{1 + e^{(E_i - \mu_i)/T}} - \frac{1}{1 + e^{(E_i + \mu_i)/T}} \right] p^2 dp \quad (17)$$

where the value of the degeneracy factor $g_i$ is 6 for quarks, while $g_i = 2$ for leptons.

2.4 Properties of proto-quark stars

The Mass–radius relation of static QSs can be obtained by solving the Tolman–Oppenheimer–Volkov (TOV) equation [58]:

$$\frac{dM}{dr} = 4\pi r^2 \epsilon(r),$$

$$\frac{dp}{dr} = -\frac{G\epsilon(r)M(r)}{r^2} \left[ 1 + \frac{p(r)}{\epsilon(r)} \right],$$

$$\times \left[ 1 + \frac{4\pi p(r)r^3}{M(r)} \right] \left[ 1 - \frac{2GM(r)}{r} \right]^{-1}, \quad (19)$$

where $M(r)$ is the total mass inside the sphere of radius $r$, $\epsilon(r)$ is the corresponding energy density, $p(r)$ is the corresponding pressure, and $G$ is Newton’s gravitational constant.

A proto-neutron star (PNS) forms after the gravitational collapse of the core of massive star with the supernova explosion. For PQS, which cannot be conclusively ruled out to consider the symmetry free energy in SQM while playing important roles in QCD phase diagram.

3 Results and discussions

The set of parameters for the current mass of particles in this paper we used are $\Lambda = 631.4 \text{ MeV}$, $m_u = m_d = 5.5 \text{ MeV}$, $m_s = 135.7 \text{ MeV}$, $GA^2 = 1.835$, $KA^4 = 9.29$, $m_e = 0.511 \text{ MeV}$, and $m_\mu = 105.7 \text{ MeV}$ [51]. The scalar–isovector channel $G_S$ is discussed in Sect. 3.5, and in Sects. 3.1–3.4 we only consider $G_V$, $G_{IV}$, and $g_V$, because the scalar–isovector channel $G_S$ has not much effects on the properties of EoS in SQM while playing important roles in QCD phase diagram.

3.1 The quark matter symmetry free energy

As customary, symmetry energy is often used to describe the properties of nuclear matter at zero temperature. In order to investigate the thermodynamical properties for asymmetric quark matter at finite temperature case, one should consider the symmetry free energy in $u\text{--}d\text{--}s$ quark matter. In Fig. 1, the quark matter symmetry free energy is calculated as a function of baryon density in asymmetric quark matter by using NJL model with two types of vector interaction at $T = 0$, $50 \text{MeV}$, and $100 \text{MeV}$, and one can see from Fig. 1 that (1) the symmetry free energy increases monotonically with the baryon density within all the parameter sets; (2) for the $G_V = g_V = G_{IV} = 0$ case in the upper left figure, in which we only consider the scalar channel and 6-point interaction, it can be seen that the quark matter symmetry free energy is obviously enhanced once finite temperature cases $T = 50 \text{MeV}$ and $T = 100 \text{MeV}$ are considered; (3) when the flavor-dependent repulsion coupling $G_V = 0.5 \text{ G}_S$ (“type A”) is considered, the value of quark matter symmetry free energy becomes larger than that of the $G_V = g_V = G_{IV} = 0$ case, indicating the contributions from flavor-dependent repulsion to enhance the symmetry free energy effects; (4) for the universal repul-
sion case \( (g_V = 0.5 G_S, G_V = G_{IV} = 0) \), one can find that the quark matter symmetry free energy is identical to that from \( G_V = g_V = G_{IV} = 0 \) case, implying that the universal repulsion has no effects on quark matter symmetry free energy within NJL model at finite temperature; (5) from the right panel of Fig. 1, we can see that flavor-dependent repulsion coupling and the vector–isovector coupling \( (G_V \) and \( G_{IV} \)) have an equal effect on the quark matter symmetry free energy within SU(3) NJL model. Following Ref. [49], the results indicate that the quark matter symmetry energy should be at least about twice than that of a free quark gas or normal quark matter within the conventional Nambu–Jona-Lasinio model at 1.5 fm\(^{-3}\) in order to describe PSR J1614.2230 and PSR J0348+0432 as quark stars by using CIDDM model. From the right panel of Fig. 1 in this paper, we can find that the quark matter symmetry energy can be as twice as that of normal quark matter within the conventional Nambu–Jona-Lasinio model at 1.5 fm\(^{-3}\) when \( G_V = 0.5 G_S \) or \( G_{IV} = 0.5 G_S \) is considered.

### 3.2 The constituent quark mass and quark fraction in SQM

As shown in Fig. 2, we draw the constituent mass of the \( u \), \( d \), and \( s \) quarks as functions of baryon density in SQM at \( T = 0 \), 50, and 100 MeV within (1) \( G_V = g_V = G_{IV} = 0 \), (2) \( G_V = 0.5 G_S, g_V = G_{IV} = 0 \), (3) \( g_V = 0.5 G_S, G_V = G_{IV} = 0 \), and (4) \( G_V = 0.5 G_S, g_V = G_{IV} = 0 \), respectively. From all the cases in this figure, one can find that the constituent masses for \( u \) and \( d \) quark all decrease when baryon density increasing, and there exists an isospin splitting between the \( u \) and \( d \) quark masses. At finite temperature cases, the values of the constituent mass for \( u \) and \( d \) quarks become larger with temperature for all the parameter sets, while the value constituent mass for \( s \) quark decrease at low baryon density then increase at high dens-

![Fig. 2 Constituent mass for \( u, d, \) and \( s \) quarks as functions of baryon density in SQM within different vector interaction at finite temperature](image1)

As shown in Fig. 2, we draw the constituent mass of the \( u \), \( d \), and \( s \) quarks as functions of baryon density in SQM at \( T = 0 \), 50, and 100 MeV within (1) \( G_V = g_V = G_{IV} = 0 \), (2) \( G_V = 0.5 G_S, g_V = G_{IV} = 0 \), (3) \( g_V = 0.5 G_S, G_V = G_{IV} = 0 \), and (4) \( G_V = 0.5 G_S, g_V = G_{IV} = 0 \), respectively. From all the cases in this figure, one can find that the constituent masses for \( u \) and \( d \) quark all decrease when baryon density increasing, and there exists an isospin splitting between the \( u \) and \( d \) quark masses. At finite temperature cases, the values of the constituent mass for \( u \) and \( d \) quarks become larger with temperature for all the parameter sets, while the value constituent mass for \( s \) quark decrease at low baryon density then increase at high dens-

![Fig. 3 Quark fraction as a function of the baryon density in SQM within the NJL model with different vector interaction](image2)

We can also obtain similar results in Fig. 1 from the work [47], in which the authors find that the chiral restoration for \( s \) quark is suppressed due to not only the increasing temperature but also the high lepton fraction case being considered.

In Fig. 3, we show the \( u \), \( d \), and \( s \) quark fraction as functions of the baryon density in SQM at \( T = 0 \), 50, and 100 MeV within (1) \( G_V = g_V = G_{IV} = 0 \), (2) \( G_V = 0.5 G_S, g_V = G_{IV} = 0 \), (3) \( g_V = 0.5 G_S, G_V = G_{IV} = 0 \), and (4) \( G_V = 0.5 G_S, g_V = G_{IV} = 0 \). As Fig. 1 shown, the symmetry free energy increases with the increment of temperature, and a larger symmetry free energy effect will reduce the isospin asymmetry, then the difference among \( u \), \( d \), and \( s \) quark fractions become smaller as temperature increasing. Since the universal repulsion has no effects on quark matter symmetry free energy within NJL model at finite temperature, one can find in the case that \( g_V = 0.5 G_S, G_V = G_{IV} = 0 \) that the \( u \), \( d \), and \( s \) quark fractions are identical to the lines in case (1) at \( T = 0 \), 50, and 100 MeV.

### 3.3 The EOS for SQM

Figure 4 shows the pressure density of quarks as a function of free energy density for SQM at \( T = 0 \), 50, and 100 MeV within SU(3) NJL model, and we can find that the EOS gets stiffer when temperature increases for all the cases in this figure, which is consistent with the results in Fig. 1 and demonstrates the effects of the symmetry free energy on the EOS. We can also get the similar results from the left panel of Fig. 1 in the work [47] that considering temperature is important in deciding the stiffness of EOS, and the authors also acquire that the lepton fraction shows more importance in changing the stiffness of EOS in SQM. In the upper left figure, we set \( G_V = g_V = 0 \) and vary the vector–isovector coupling
We first present the results for the properties of quark stars at different temperature. In this treatment, we do not consider the isentropic stages for PQS, only considering the properties of quark matter in β-equilibrium condition. The main purpose for calculating the properties of QSs at different temperature is to dig out the relation between the EOS of SQM and the maximum mass of QS at finite temperature, which is very important to reveal the temperature influence over the maximum mass of QSs precisely.

Shown in Fig. 5 is the maximum mass for static QSs in SU(3) NJL model with \( G_V = g_V = G_{IV} = 0 \) and \( g_V = 1.1 G_S \), \( G_{IV} = 0.5 G_S \) at different temperature. From Fig. 5, one can see that the maximum mass of the static QS is \( 1.65 M_\odot \) at zero temperature with \( G_V = g_V = G_{IV} = 0 \), while for the parameter set \( g_V = 1.1 G_S \), \( G_{IV} = 0.5 G_S \), the maximum mass of the static QS is \( 2.01 M_\odot \) for the zero temperature case, which is able to describe PSR J1614.2230 and PSR J0348+0432 as quark stars at zero temperature with the quark matter symmetry energy being approximately as twice as that of normal quark matter within the conventional Nambu–Jona-Lasinio model (\( G_V = g_V = G_{IV} = 0 \)). It is obvious that the maximum mass of static QSs with the two sets of parameters both increase with temperature because of the effects of symmetry free energy. For the parameter set \( G_V = g_V = G_{IV} = 0 \), when temperature reaches 50, 80, and 100 MeV, the maximum mass of the static QSs will increase to 1.54 \( M_\odot \), 1.97 \( M_\odot \), and 2.30 \( M_\odot \). For the parameter set \( g_V = 1.1 G_S \), \( G_{IV} = 0.5 G_S \), when temperature rises to 50, 80, and 100 MeV, the maximum mass of QSs will reach 2.13 \( M_\odot \), 2.46 \( M_\odot \), and 2.71 \( M_\odot \), respectively. From Fig. 5, the results indicate that the maximum mass for QSs in NJL model increases with the increment of the temperature, because the effects of symmetry free energy increase and the EoS gets stiffer when temperature increases. The sound velocity square is defined as \( C_s^2 = \frac{d\varepsilon}{d\rho} \). In Fig. 6, we set the sound velocity square as a function of energy density for SQM at \( T=0 \), 50MeV and 100 MeV within \( G_V = g_V = G_{IV} = 0 \) and \( g_V = 1.1 G_S \), \( G_{IV} = 0.5 G_S \). One can find from Fig. 6 that the sound velocity does not exceed the velocity of light for all the cases, which implies that the parameter sets for Fig. 5 is physical.

Shown in Fig. 7 is the mass–radius relations of PQS at three snapshots along a PQS evolution within NJL model with \( g_V = 1.1 G_S \), \( G_{IV} = 0.5 G_S \). Firstly, in the heating stage, the PQS expands with the decrement of the lepton fraction and the increment of entropy. When the trapped neutrinos are free, the star begins cooling stage. For the first stage of the evolution of PQS, when the entropy per baryon is 1 and the lepton fraction is 0.4, there exist a large number of trapped neutrinos, and the maximum mass of PQS is \( 2.027 M_\odot \). As the trapped neutrinos diffuse, the star continues heating up with the entropy per baryon reaching 2, and then the maximum mass of PQS in the second stage increases to \( 2.048 M_\odot \), which is the maximum mass case of all the three stages of evolution. Following the heating stages, the star begins

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**Fig. 4** Pressure as a function of free energy density for SQM within different vector interaction

**Fig. 5** The mass–radius relation for quark stars within SU(3) NJL model at different temperature

constant within \( G_{IV} = 0 \), \( G_S = 0.5 G_S \). It is observed that considering the vector–isovector interaction does not greatly change the equation of state in SQM, because the isospin asymmetry in SQM is not large enough. In the upper right figure and the lower left figure, we set \( G_{IV} = 0 \) and vary the coupling constant with \( G_V = 0.5 G_S \) and \( g_V = 0.5 G_S \). One can see that the repulsive vector interaction \( G_V \) and \( g_V \) both contribute much and stiffen the equation of state. The reason why we choose the set of parameter \( G_{IV} = 0.5 G_S \) and \( g_V = 1.1 G_S \) in this figure is that we can describe a 2.01 solar mass quark star as PSR J1614.2230 and PSR J0348+0432 by using this set of parameter in SU(3) NJL model at zero temperature in Fig. 5, with the quark matter symmetry energy being approximately as twice as that of normal quark matter within the conventional Nambu–Jona-Lasinio model (\( G_V = g_V = G_{IV} = 0 \)).

3.4 Properties of proto-quark star

We first present the results for the properties of quark stars at different temperature. In this treatment, we do not consider
cooling and the entropy per baryon reaches zero at the 3rd stage, which is the corresponding zero temperature case, and then the maximum mass of PQS at zero temperature in this snapshot is 2.01 $M_{\odot}$. These results indicates that the maximum mass of PQS at the heating stages is larger than that of PQS at zero temperature in SU(3) NJL model. One can acquire from the work [60] that during the early evolution the entropy in the central regions is moderately high, which corresponds to temperatures in the range of $T$ from 20 to 50 MeV. In this paper, we calculate that the temperatures at the central density for the maximum mass of the PQSs in Fig. 7 in the 1st and 2nd stages are 12.22 and 18.99 MeV, and one can find that the maximum mass for QSs cannot change largely at not high temperature cases.

3.5 Vector interaction effects on phase diagram

In this section, we calculate the vector interaction effects on phase diagram for NJL model by varying the value of quark chemical potential $\mu$ with the finite isospin chemical

$$\mu_I = 50 \text{ MeV}, \text{ where the quark chemical potential } \mu \text{ and the isospin chemical potential } \mu_I \text{ can be expressed, respectively, as}$$

$$\mu = (\mu_u + \mu_d)/2, \quad \mu_I = (\mu_u - \mu_d)/2 \quad (23)$$

Figure 8 displays the constituent quark mass as functions of the temperature with different vector interaction ($G_V, g_{V}, G_{IV}$, and $G_{IS}$) in asymmetric quark matter within SU(3) NJL model. One can find from this figure that there is an obvious isospin splitting between the lines of $u$ and $d$ quark with $G_V = g_{V} = G_{IV} = 0$ by setting $\mu_I = 50 \text{ MeV}$, and this isospin splitting vanishes once $G_{IS} = G_{S}$, which shows that the scalar–isovector channel can significantly decrease the isospin splitting of $u$ and $d$ quark masses. As shown in the 3rd graph in this figure, the isospin splitting gets larger when $G_{IS} = -G_{S}$ is used, which indicates a proper way of enlarging the isospin splitting for $u$ and $d$ constituent quark masses. From all the graphs in this figure, one can find that the isospin splitting for $u$ and $d$ constituent quark masses decrease once the coupling constants $G_V, g_{V}, G_{IV}$, and $G_{IS}$ increase, and decreasing the value of $G_{IS}$ can significantly increase this isospin splitting.

In Fig. 9, we calculate the phase diagram in the $\mu$–$T$ plane for various quark chemical potential $\mu$ and coupling constants $G_V, g_{V}, G_{IV}$, and $G_{IS}$. One can find that the lines of the temperature for $u$ and $d$ quark increase when $G_V = G_S$ or $g_{V} = G_{S}$ is considered, which indicates that the repul-
sion vector channel can significantly change the phase diagram for asymmetric quark matter in NJL model, and we can also find that the effects of the universal repulsion contributes more than that of the flavor-dependent repulsion to increasing the temperature at a certain quark chemical potential. It can be seen that the isospin splitting for $u$ and $d$ constituent quark masses decrease when $G_{IS} = G_S$ or $G_{IV} = G_S$ is considered, while the masses increase when considering $G_{IS} = -G_S$, which matches the results from Fig. 8.

Furthermore, the high-density quark matter might be in a color superconducting phase, and the two-flavor color superconductor (2SC) [61–65], the color-flavor-locked phase (CFL) [41, 66–70], and the crystalline color superconductor can be found in the possible quark color superconducting phases [71]. In the present work, we have not considered color superconducting phases for simplicity, and it will be interesting to find how the color quark superconducting phases affect the properties of SQM and QSs at fine temperature within the SU(3) NJL model in our future work.

4 Conclusion and discussion

In this work, we investigate the properties of asymmetric quark matter and strange quark matter at finite temperature in the framework of the SU(3) Nambu–Jona-Lasinio (NJL) model. We study the quark matter symmetry free energy in asymmetric quark matter by using two types of vector interaction in SU(3) NJL model, and it is found that the symmetry free energy increases monotonically with the baryon density at a certain temperature, and the quark matter symmetry free energy can be obviously enhanced when temperature increases, which indicates that the isospin effects contributes more with the increment of temperature.

In order to investigate the properties of the strange quark matter, we study the constituent quark mass, quark fraction, and the equation of state. It is found that the values of the constituent mass for $u$ and $d$ quarks become larger with temperature for all the parameter sets, while the value constituent mass for $s$ quark decrease at low baryon density then increase at high density. For quark fraction, because the isospin asymmetry is reduced by the symmetry free energy when temperature increases, the difference among $u$, $d$, and $s$ quark fractions becomes smaller with temperature increasing. We also study the maximum mass for QSs at finite temperature in SU(3) NJL model, and the results show that the maximum mass for QSs increases with temperature, which shows the effects from the symmetry free energy increases and the EoS for quark matter gets stiffer when temperature increases.

Moreover, we calculate the maximum mass of proto-quark stars. Considering three different snapshots along the evolution line of the compact star, we have studied the properties of static PQSs. We have demonstrated that the maximum mass of PQS at the heating stages is always larger than that of PQS at zero temperature within SU(3) NJL model, which indicates that the heating process in the evolution will increase the maximum mass of PQS.

We further studied the QCD phase diagram for NJL model, and we find that the isospin splitting for the constituent quark mass of $u$ and $d$ quarks vanishes once $G_{IS} = G_S$ is considered, while this isospin splitting will get larger when $G_{IS} = -G_S$. We see that the constituent quark mass decreases slowly as the increment of temperature in $g_V = G_S$ and $G_V = G_S$ cases. We calculate the phase diagram in the $\mu$–$T$ plane for various quark chemical potential $\mu$ and coupling constants $G_V$, $g_V$, $G_{IV}$, and $G_{IS}$, and we find that the isospin splitting for temperature at a certain $\mu$ can be significantly influenced by considering $G_{IS}$.

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