NEW VELOCITY DISTRIBUTION FOR COLD DARK MATTER IN THE CONTEXT OF THE EDDINGTON THEORY

J. D. Vergados
Department of Physics, University of South Africa, P.O. Box 392, 0001 Pretoria, South Africa; vergados@cc.uoi.gr

AND

D. Owen
Department of Physics, Ben-Gurion University, P.O. Box 653, Beer-Sheva 84105, Israel; owen@bgumail.bgu.ac.il

Received 2002 March 19; accepted 2003 January 2

ABSTRACT

Exotic dark matter, together with the vacuum energy (associated with the cosmological constant), seems to dominate the universe. Thus, its direct detection is central to particle physics and cosmology. Supersymmetry provides a natural dark matter candidate, the lightest supersymmetric particle (LSP). One essential ingredient in obtaining the direct detection rates is the density and velocity distribution of the LSP. The detection rate is proportional to this density in our vicinity. Furthermore, since this rate is expected to be very low, one should explore the two characteristic signatures of the process, namely, the modulation effect, i.e., the dependence of the event rate on the Earth’s motion, and the correlation of the directional rate with the motion of the Sun. Both of these crucially depend on the LSP velocity distribution. In the present paper we study simultaneously density profiles and velocity distributions based on the Eddington theory.

Subject headings: cosmology: theory — dark matter — elementary particles — galaxies: kinematics and dynamics

1. INTRODUCTION

In recent years the consideration of exotic dark matter has become necessary in order to close the universe (Jungman, Kamionkowski, & Griest 1996). COBE data (Smoot et al. 1992) suggest that the cold dark matter (CDM) component is at least 60% (Gawiser & Silk 1988; Gross et al. 1998) of the total mass. On the other hand, evidence from two different teams, the High-z Supernova Search Team (Riess et al. 1998) and the Supernova Cosmology Project (Somerville, Primack, & Faber 2001; Perlmutter, Turner, & White 1999a; Perlmutter et al. 1999b, 1997), suggests that the universe may be dominated by the cosmological constant $\Lambda$. Thus, the situation can be adequately described by a baryonic component $\Omega_b = 0.1$, along with the exotic components $\Omega_{CDM} = 0.3$ and $\Omega_{\Lambda} = 0.6$. In a more detailed CDM analysis (Primack 2001) one finds

$$\Omega_b = 0.040 \pm 0.002 , \quad \Omega_m = \Omega_{CDM} = 0.33 \pm 0.035 , \quad \Omega_{\Lambda} = 0.73 \pm 0.08$$

(see also Turner 1990 and Einasto 2001). Since the non-exotic component cannot exceed 40% of the CDM (Jungman et al. 1996; Alcock et al. 1995), there is room for the exotic weakly interacting massive particles (WIMPs). In fact, the DAMA experiment (Bernabei et al. 1996) has claimed the observation of one signal in the direct detection of a WIMP, which, with better statistics, has subsequently been interpreted as a modulation signal (Bernabei et al. 1998, 1999).

In the most favored scenario of supersymmetry the lightest supersymmetric particle (LSP) can be simply described as a Majorana fermion, a linear combination of the neutral components of the gauginos and higgsinos (Jungman et al. 1996; Vergados 2002; Gómez & Vergados 2001a, 2001b; Gómez, Lazarides, & Pallis 2000a, 2000b; Nath & Arnott 1995; Arnott & Nath 1996; Bottino et al. 1997; Bednyakov, Klapdor-Kleingrothaus, & Kovalenko 1994).

Since this particle is expected to be very massive, $m_\chi \geq 30$ GeV, and extremely nonrelativistic, with average kinetic energy $T \leq 100$ KeV, it can be directly detected (Vergados 1996; Spira et al. 1995; Kosmas & Vergados 1997), mainly via the recoiling of a nucleus ($A, Z$) in elastic scattering.

In order to compute the event rate, one needs the following ingredients:

1. An effective Lagrangian at the elementary particle (quark) level obtained in the framework of supersymmetry as described in, e.g., Jungman et al. (1996), Nath & Arnott (1995), Arnott & Nath (1996), Bottino et al. (1997), and Bednyakov et al. (1994).

2. A procedure for going from the quark to the nucleon level, i.e., a quark model for the nucleon. The results depend crucially on the content of the nucleon in quarks other than $u$ and $d$. This is particularly true for the scalar couplings as well as the isoscalar axial coupling (Djouadi & Drees 2000; Drees & Nojiri 1993a, 1993b, 1993c; Cheng 1988; Cheng 1989).

3. Computations of the relevant nuclear matrix elements (Ressell et al. 1993; Divari et al. 2000) using as many reliable body nuclear wave functions as possible. The situation is a bit simpler in the case of the scalar coupling, in which case one only needs the nuclear form factor.

4. The LSP density and velocity distribution. Among other things, since the detection rates are expected to be very

\footnotesize

1 Permanent address: Theoretical Physics Division, University of Ioannina, Ioannina, Gr 451 10, Greece.

2 Visiting Astronomer: Theoretical Physics Division, University of Ioannina, Ioannina, Gr 451 10, Greece.
small, the velocity distribution is crucial in exploiting the
ccharacteristic experimental signatures provided by the
reaction, namely, (a) the modulation of the event rates due to the
Earth’s revolution around the Sun (Vergados 1998, 1999, 2000) and (b) the correlation of the rates with the
Sun’s direction of motion in directional experiments, i.e.,
experiments in which the direction of the recoiling nucleus is
observed (Vergados 2002; Lehner et al. 2001). To obtain the
right density and velocity distributions is the purpose of the
present paper.

In the past various velocity distributions have been con-
sidered. The most popular one is the isothermal Maxwell-
Boltzmann velocity distribution with \( \langle v^2 \rangle = (3/2) v_0^2 \), where
\( v_0 \) is the velocity of the Sun around the Galaxy, i.e., 220 km
\( \text{s}^{-1} \). Extensions of this Maxwell-Boltzmann distribution
were also considered, in particular those that were axially
symmetric with enhanced dispersion in the Galactocentric
direction (Drucker, Freeze, & Spergel 1986; Vergados 2000). In such distributions an upper cutoff \( v_{\text{esc}} = 2.84 v_0 \)
was introduced by hand.

Nonisothermal models have also been considered.
Among those we should mention the late infall of dark mat-
ter into the Galaxy, i.e., caustic rings (Sikivie 1999, 1998;
Vergados 2001b; Green 2001; Gelmini & Gondolo 2001),
and dark matter orbiting the Sun (Copi, Heo, & Krauss
1999).

The correct approach in our view is to consider the
Eddington approach (Eddington 1916), i.e., to obtain both the
density and the velocity distribution from a mass
distribution, which depends on both the velocity and the
gravitational potential. This approach has been extensively
studied by Merritt (1985) and recently applied to dark matter by Ullio & Kamionkowski (2001).

2. DENSITY PROFILES

As we have seen in § 1, the matter distribution can be
given as follows:

\[
dM = 2\pi f[\Phi(r), v_r, v_t] dx \, dy \, dz \, dv_r \, dv_t \, dv_r ,
\]

(1)

where the function \( f \) is the distribution function, which
depends on \( r \) through the potential \( \Phi(r) \) and the tangential
and radial velocities \( v_t \) and \( v_r \), respectively (we assume axial
symmetry in velocity space). Thus, the density of matter \( \rho \)
satisfies the equation

\[
d\rho = 2\pi f[\Phi(r), v_r, v_t] dv_r \, dv_t .
\]

(2)

It is more convenient to use the total energy \( E \) and the angu-
lar momentum \( J \) instead of the velocities via the equations

\[
J = v_t r , \quad 2E = v_r^2 + J^2 / r^2 + 2\Phi(r) .
\]

(3)

The use of these variables, which are constants of motion, is
very useful when one wants to study equilibrium states. We
thus find

\[
\rho = \frac{2\pi}{r^2} \int \int \frac{f(E, J) J}{\sqrt{2[E - \Phi(r)] - J^2 / r^2}} dJ \, dE .
\]

(4)

The limits of integration for \( E \) are from \( \Phi \) to 0, and those for
\( J \) are from 0 to \( \{2r^2[E - \Phi(r)]\}^{1/2} \).

Following Eddington, we choose a distribution function of the form

\[
f(E, J) = K_\lambda (-2E)^\lambda \]

(5)

(\( E \) is negative for a bound system), where \( \lambda \) is a parameter
that depends on the type of matter and \( K_\lambda \) is a normalization
constant that is related to the density at some point. With
this choice of the distribution function, it is quite straight-
forward to find the relationship between the density \( \rho \)
and the potential. The result is

\[
\rho = K_\lambda \frac{\lambda^{3/2} \pi[\Phi(r)]^{\lambda + 3/2} \beta(\lambda + 1, \frac{3}{2})}{(3/2)} ,
\]

(6)

with

\[
\beta(a, b) = \frac{\Gamma(a) \Gamma(b)}{\Gamma(a + b)} .
\]

(7)

Equation (6) can be cast in the simple form

\[
\rho = \rho_\lambda(0) \left[ \chi(x) \right]^{\lambda + 3/2} ,
\]

(8)

with

\[
\chi(x) = \frac{\Phi(x)}{\Phi_0} , \quad x = \frac{r}{r_S} ,
\]

(9)

where \( \Phi_0 = \Phi(0) \) and \( r_S \) is the Galactic radius (position of
the Sun). The constant \( K_\lambda \) is related to the density at the
origin via the relation

\[
K_\lambda = \frac{\rho_\lambda(0)}{\pi(2[\Phi_0])^{\lambda + 3/2} \beta(\lambda + 1, \frac{3}{2})} .
\]

(10)

The above distribution function is isotropic. Following
Merritt (1985), we can introduce an anisotropy by modifying
the distribution function as follows:

\[
\rho = K_\lambda \frac{\rho_\lambda(0) J^2}{(r_S v_m)^2} .
\]

(11)

Instead of the parameter \( r_S \), we find it convenient for our
applications later on (see below) to adopt the more recent
conventions and write the above equation as follows:

\[
\rho = \rho_\lambda(0) \left[ 1 + \alpha_s \frac{J^2}{(r_S v_m)^2} \right] ,
\]

(12)

where \( v_m \) is the maximum velocity allowed by the potential,
to be specified below, and \( \alpha_s \) is the asymmetry parameter.
Proceeding as above, we find that this induces a correction
to the density of the form

\[
\Delta \rho = K_\lambda \frac{4}{5} 2^{\lambda + 3/2} \pi[\Phi(r)]^{\lambda + 5/2} \beta(\lambda + 1, \frac{5}{2}) \alpha_s \frac{J^2}{v_m^2} .
\]

(13)

Combining equations (6) and (13), we get

\[
\rho_\lambda(r) = \rho_\lambda(0) \psi_\lambda(x) ,
\]

(14)

\[
\psi_\lambda(x) = \left[ \chi(x) \right]^{\lambda + 3/2} \left[ 1 + \frac{4}{5} a \frac{\beta(\lambda + 1, \frac{5}{2})}{\beta(\lambda + 1, \frac{3}{2})} \chi(x) \right] ,
\]

(15)

with \( a = \alpha_s \Phi_0 / v_m^2 \).
Since the scale of the potential appears only via the parameter $a$, one, in principle, could have two $a$ parameters, one for matter ($a_\text{m}$) and one for dark matter ($a_\text{dm}$). In the present work we assume that they are equal. We remind the reader that $\alpha_s$ is the asymmetry parameter to be treated phenomenologically.

### 3. ALLOWED DENSITY FUNCTIONS

The central question is to specify density $\rho_\text{s}(x)$, the potential $\chi(x)$, and the mass density distribution entering equation (1). One can adopt one of two procedures:

1. Start with a given density, obtained, e.g., phenomenologically, and find the potential by solving Poisson’s equation. This way one obtains, at least parametrically, e.g., with the radial coordinate as a parameter, the proper relation between the density and the potential. This approach has been adopted by a lot of researchers (see, e.g., Widrow 2000; Evans 1994; Henriksen & Widrow 1995; Ullio & Kamionkowski 2001). From this one can obtain the mass density function $f(\Phi, v)$. This approach leads, in general, to a nonanalytic, i.e., complicated, velocity distribution, hard to implement in dark matter calculations (Ullio & Kamionkowski 2001), especially for non–spherically symmetric mass distributions.

2. Use the above simple relation between the density and the potential and solve the differential equation resulting from Poisson’s equation and thus obtain each one of them. One hopes that this way one will get a density distribution that describes adequately dark matter distribution globally, via fitting the rotational curves, and in our vicinity. This approach can easily deal with asymmetric density distributions. The simplicity of the relation between the density and the potential thus yields the bonus that the obtained velocity distribution is analytic and can be easily implemented in obtaining the event rates for direct dark matter detections. It can also be extended to include more than one power in $\lambda$. In this paper we follow this approach and leave it for the future to use semianalytic mass distributions obtained in approach 1.

For a spherically symmetric potential, Poisson’s equation leads to a differential equation of the type

$$x\chi''(x) + 2\chi'(x) = -\Lambda x[\chi(x)]^n \left[ 1 + \frac{2a}{n+1} x^2 \chi(x) \right],$$

with $n = \lambda + \frac{1}{2}$ and

$$\Lambda = -\frac{4\pi G N2\rho_\text{s}(0)}{\Phi_0},$$

where $G_N$ is Newton’s gravitational constant. The dimensionless quantity $\Lambda$ is assumed to be positive (attractive potential). Since the asymmetry parameter is assumed to be small, the first term on the right-hand side dominates. Introducing the variables $\xi = \sqrt{\Lambda} x$ and $\chi(x) = u(x\sqrt{\Lambda})$, we arrive at

$$\xi u''(\xi) + 2u'(\xi) = -\xi[u(\xi)]^n \left[ 1 + \frac{2b}{n+1} \xi^2 u(\xi) \right], \quad b = \frac{a}{\Lambda}.$$  

The last equation is a nonlinear differential equation, which must be solved for $\xi > 0$, with the conditions $u(0) = 1$ and $u'(0) = 0$. We also demand that the solution remains positive; i.e., the solution drops from unity to zero. For $\alpha_s = 0$, the density is a rapidly decreasing function of $\xi$.

The above equation can be solved analytically in the special case $\alpha = 0$ and when $\lambda = -\frac{1}{2}$ ($n = 1$) or $\lambda = 7/2$ ($n = 5$). In these cases the solutions are

$$\lambda = -\frac{1}{2} \rightarrow u(\xi) = \frac{\sin \xi}{\xi}$$

and

$$\lambda = \frac{7}{2} \rightarrow u(\xi) = \frac{1}{(1 + \xi^2/3)^{3/2}}.$$  

The case $\lambda = -\frac{1}{2}$ may not have physical meaning, but it will serve as an illustrative example for the realistic cases to be discussed below.

We should stress that the last solution associated with $\lambda = 7/2$, admissible over all space, can be considered to be providing an adequate description of ordinary matter. In the absence of asymmetry no constraint is imposed on the parameter $\Lambda$ by the positivity and finiteness condition of the solution. The solution given by equation (19) is constrained in the range $0 \leq \xi \leq \pi$. If we demand that the range of the potential for dark matter extends far out in the halo, e.g., up to $x_r = r/r_s = 20$, we must have $\Lambda = (\pi/x_r)^{2} = 0.025$. This gives a useful constraint between $\Phi_0$ and $\rho_\text{s}(0)$.

The differential equation (18) can, in general, be solved only numerically. For $\lambda = \frac{1}{2}$, we find that there is a singularity at $\xi = 12.43$, while the first zero of the potential occurs at $\xi = 4.35$. This leads to the constraint $\Lambda = (4.35/20)^2 = 0.047$. For $\lambda = 1$, the corresponding value is $\Lambda = 0.065$.

The above results are significantly modified when the asymmetry parameter is turned on. One now can see that, depending on the parameters $\Lambda$ and $\alpha_s$, the shape of the density is significantly modified.

We examine the special case of $\lambda = 7/2$, which corresponds to the Eddington solution for ordinary matter. As we have seen in the absence of asymmetry, the solution can be found exactly. In the presence of asymmetry we distinguish two cases:

1. **Positive values of $\alpha_s$.**—For small values of $b$, the solution has no roots. Beyond a critical value of $b$ the solution attains the value zero. Then, farther out for still larger $\xi$, it becomes negative. At some point it becomes singular. The extracted values of $\Lambda$ range between 1.5 and 0.12.

2. **Negative values of $\alpha_s$.**—In this case when the absolute value of $b$ becomes sufficiently large, the solution goes through zero. Again, a value of $\Lambda$ can be extracted. The situation is quite unstable. Generally speaking, for arbitrary values of $b$, the solution blows up to infinity without going through zero. One cannot extract values of $\Lambda$ in this case.

In any case, these observations are intended as illustrations of what is expected for dark matter when the

---

3 The designation of $\lambda = 7/2$ as representing ordinary matter is due to the work of Eddington (1916), in which he points out that this value leads to the Plummer distribution. Furthermore, this distribution provides an adequate description of the Galaxy (of ordinary matter) he was concerned with. A discussion of some of the thermodynamical aspects of the $\lambda = 7/2$ distribution is also given in this paper.
exact solution cannot be obtained. In the case of ordinary matter we do not consider \( \alpha_s \) different from zero, since the range of the solution is small (the density is assumed to vanish for \( r \geq r_s \)).

Let us return to equation (12) and express it in terms of the velocity in our vicinity. We obtain

\[
f(v) = N \left[ -2\Phi(r_s) - \nu^2 \right] \left( 1 + \alpha_s \frac{v^2}{v_m^2} \right),
\]

where \( N \) is a normalization constant that depends on \( \lambda, \alpha_s, \) and \( v_m \). From this we see that the maximum velocity in our vicinity, \( v_m \), depends on the value of the potential, i.e.,

\[
v_m^2 = -2\Phi(r_s) = -2\Phi_0 u(\sqrt{\lambda}).
\]

This means that

\[
b = \frac{a}{\Lambda} = \frac{\alpha_s}{2\Lambda u(\sqrt{\lambda})}.
\]

Our strategy is clear:

1. Vary \( b \) so that the solution has its first zero at some value \( \xi_0 \) and is monotonically decreasing up to that point.
2. Determine \( \Lambda \) so that \( \xi_0 \) corresponds to a range \( x_r \) of the potential \( \chi(x) \).
3. From equation (23) obtain an acceptable (self-consistent) value of \( \alpha_s, \alpha_s = 2\Lambda u(\sqrt{\lambda}) \).
4. The parameter \( v_m \) is obtained via the relation

\[
v_m^2 = -2\Phi_0 u(\sqrt{\lambda}) = 8\pi G_N r_s^2 \frac{u(\sqrt{\lambda}) \rho_0(0)}{\lambda}
\]

or, equivalently,

\[
v_m = \nu_0 \sqrt{\eta_\lambda} h_v(n, a, \Lambda),
\]

\[
h_v(n, a, \Lambda) = \frac{u(\sqrt{\lambda})}{(n+1) u(\sqrt{\lambda}) \left( 1 + [2a/(n+1)] u(\sqrt{\lambda}) \right)},
\]

\[
n = \lambda + \frac{3}{2},
\]

where \( v_0 \) is our velocity of rotation around the center of our Galaxy (220 km s\(^{-1}\)). In obtaining the last equation, we used our solution to relate the density \( \rho_\lambda(0) \) to the density of dark matter in our vicinity, \( \rho_0 \), via the equation

\[
\rho_\lambda(0) = \frac{\rho_0}{(n+1) u(\sqrt{\lambda}) \left( 1 + [2a/(n+1)] u(\sqrt{\lambda}) \right)}, \quad n = \lambda + \frac{3}{2}.
\]

The parameter \( \eta_\chi \) is defined by \( \eta_\chi = \rho_0/(0.3 \text{ GeV cm}^{-3}) \); \( \eta_\chi \) is normally taken to be unity (Jungman et al. 1996).

It is clear that in our treatment for a given \( \lambda \) we have three independent parameters, \( b, x_r, \) and \( \rho_0 \). The obtained results for some interesting typical cases of \( \alpha_s \) are shown in Table 1. The value of the parameter \( h_v \) has the meaning of \( \chi_{esc} \) (see § 6), and it is rewarding that it is not far from the value of 2.84 deduced phenomenologically (Drucker et al. 1986).

The potentials obtained for the various such parameters are shown in Figures 1–4.

It is clear that the shapes of these potentials are similar.

The densities obtained with the same set of parameters are shown in Figures 5–8.

We observe that, even though the potentials are similar, the densities differ substantially. The asymmetry parameter has a big effect on the density, especially for \( \alpha_s \) greater than zero, when it changes substantially around \( x = 8 \). For negative \( \alpha_s \) there is small reduction of the density in the region of interest, but, as we have already mentioned, only special values of \( b \) are acceptable. A detailed, rigorous mathematical study of the behavior of the solutions for negative \( b \) is currently under study, and it will appear elsewhere.

![Fig. 1.—Potential function in units of \( \Phi_0 \) for \( \lambda = \frac{1}{2} \). The curves proceed with decreasing asymmetry parameter \( \alpha_s \) (from top to bottom): 0.200, 0.150, 0.100, 0.062, 0, -0.021, -0.038, and -0.080. The notation is the same for the potentials associated with the other values of \( \lambda \) as well as for all the dark matter densities \( \rho \).](image)
4. ROTATIONAL VELOCITIES

We are now in position to obtain the rotational velocity curves. The rotational velocity is given by

\[ v_\lambda(x) = \frac{v_0}{\sqrt{2}} \sqrt{g_\lambda(x)} \],

with

\[ g_\lambda(x) = \frac{1}{x} \int_0^x (x')^2 dx' \psi_\lambda(x') \].

We see that the scale of the rotational velocity is set by \( v_0/\sqrt{2} \), in agreement with the data (see, e.g., the recent review by Jungman et al. 1996). For dark matter, the above integrals can only be performed numerically. The obtained rotational velocities are shown in Figures 9, 10, and 11.

For ordinary matter (\( \lambda = 7/2 \)), we make the choice of \( \lambda = 3 \) to get the familiar solution

\[ \chi(x) = \frac{1}{(1 + x^2)^{1/2}} \], \[ \psi_{7/2} = \psi(x) = \frac{1}{(1 + x^2)^{1/2}} \]

(we drop the index 7/2 and use the index \( m \), for matter, when necessary). Thus, the rotational velocity due to ordinary matter is now given by

\[ v_m(x) = \frac{v_0}{\sqrt{2}} \sqrt{g_m(x)} \].

For \( x < 1 \), we write

\[ g_m(x) = g(x) \],

while for \( x > 1 \), we have

\[ g_m(x) = \frac{1}{x} g(1) \].

We now distinguish two cases:

1. **Spherical galaxy.**—Using the above density, we find

\[ g(x) = \frac{1}{3} \frac{x^2}{(1 + x^2)^{3/2}} \].

2. **Spiral galaxy (disk).**—In this case we use the same density as above. In the Eddington theory the equation, which relates the potential and the density, is no longer of the above simple form. One finds

\[ \frac{\Phi}{\Phi_0} = \frac{\Lambda}{12} \left( 2 \left[ \frac{\rho}{\rho(0)} \right]^5 - \ln \left( \frac{1 - \left[ \rho/\rho(0) \right]^5}{1 + \left( \rho/\rho_0 \right)^5} \right) \right) \].

This complexity is of no concern to us since we do not need to use this complicated equation. In the case of ordinary matter we do not need to obtain the velocity distribution.

---

**Fig. 2.**—Same as Fig. 1, but for \( \lambda = 1 \)

**Fig. 4.**—Same as Fig. 1, but for ordinary matter

**Fig. 3.**—Same as Fig. 1, but for \( \lambda = -\frac{1}{2} \)

**Fig. 5.**—Density in units of \( \rho(0) \) for \( \lambda = \frac{1}{2} \)
Thus, with the assumed density, we find

\[ g(x) = \frac{1}{3x} \left[ 1 - \frac{1}{(1 + x^2)^{3/2}} \right]. \]  

(35)

The above functions are plotted in Figure 12. In the graphs we show not only the physically interesting case, in which the density vanishes outside the radius of the Galaxy, but, for illustrative purposes, the case of the infinite extent of the same density function. We see that, since the density falls very fast, it makes very little difference which form we use.

By comparing the above graphs of dark matter with those of ordinary matter, we see that in the Eddington theory ordinary matter can make a significant contribution to the rotational velocities only if the density at the origin is much bigger compared to that of dark matter. As we have mentioned above, for ordinary matter, there is no constraint between \( \Phi_0 \) and \( \rho_0 \). So we simply rescale ordinary matter by a factor \( C_{\text{mdm}} \) and write

\[ g_{\text{matter}}(x) = C_{\text{mdm}} g_m(x); \]  

(36)

the scaling factor can be determined by a comparison to the experimental rotational curves.

Thus, the rotational velocity due to both matter and dark matter is given by

\[ v_\lambda(x) = \frac{v_0}{\sqrt{2}} \sqrt{g_\lambda(x) + g_{\text{matter}}(x)}. \]  

(37)

The obtained rotational velocities for two values of \( C_{\text{mdm}} \) are shown in Figures 13–16.

The larger value of \( C_{\text{mdm}} \) seems to be in better agreement with the data (see, e.g., the recent review by Jungman et al. 1996).

It is clear from these curves that the presence of asymmetry has a dramatic effect on the rotational velocities. Thus, in the context of the Eddington theory as discussed here, a large asymmetry is excluded by the data on rotational velocities. A detailed fitting of our input parameters to the rotational curves is not attempted here.

5. VELOCITY DISTRIBUTION WITH RESPECT TO THE GALACTIC CENTER

The above density via the Eddington formula leads to a velocity distribution of the form

\[ F(v, r) \propto [-2\Phi(r) - v^2]^{\lambda} \left( 1 + \alpha_s \frac{v^2}{v_m^2} \right). \]  

(38)

The above velocities and the distance \( r \) are defined with respect to the center of the matter distribution, i.e., to the
center of our Galaxy. We note that in the context of the Eddington theory the velocity distribution cannot be Maxwellian. For a given distance, it goes to zero at the boundaries of the corresponding ellipsoid. It is customary to consider the value of the above distribution in our vicinity, \( r = r_S \). This way it reduces to the product of the local density and the velocity distribution. The latter is given by equation (21), where \( N \) is a normalization constant that depends on \( \lambda \), \( \alpha_s \), and \( v_m \). The above notation was introduced to make the last equation coincide with the standard expression when the function \( f \) is chosen to be Maxwellian, i.e.,

\[
\exp \left\{ -\left( \frac{v^2 - \alpha_s v_0^2}{v_0^2} \right) \right\} \rightarrow \left[ 1 + \alpha_s \left( \frac{v^2}{v_0^2} \right) \right] \exp \left( -\frac{v^2}{v_0^2} \right)
\]

for sufficiently small \( \alpha_s \). In this limit we see that \( \alpha_s \) coincides with the parameter \( -\lambda \) of Vergados (2000) and Drucker et al. (1986) (in the present work \( \lambda \) is used for another purpose).

It is straightforward to find that the normalization factor \( \tilde{N} \) is given by

\[
\tilde{N}^{-1}(\lambda, \alpha_s, v_m) = 2\pi v_m^{2\lambda+3} \beta \left( \lambda + 1, \frac{3}{2} \right) \times \left[ 1 + \frac{4}{3} \alpha_s \beta(\lambda + 1, 5/2) \right], \tag{39}
\]

with \( v_m \) given by equation (22). In the special case of dark matter (\( \lambda = \frac{1}{2} \)) it becomes

\[
\tilde{N}^{-1}(\frac{1}{2}, \alpha_s, v_m) = \frac{\pi^2}{4} v_m^4 \left( 1 + \frac{1}{3} \alpha_s \right). \tag{40}
\]

From the above formulas we see that the velocity of dark matter with respect to the Galactic center ranges from 0 to a maximum speed \( v_m = \left[ 2\Phi(r_S) \right]^{1/2} \). Since the escape velocity is determined by the potential \( \Phi(r_S) \) due to all kinds of matter, the velocity \( v_m \) is not simply related to \( v_{esc} \). Thus, since the distribution function must remain positive, if \( \alpha_s < 0 \), its absolute value is bounded. This imposes a constraint, since in the traditional analysis with only axially symmetric Gaussian distribution it leads to negative \( \alpha_s \), i.e., enhanced dispersion in the Galactocentric direction, a phenomenologically preferred result (Drucker et al. 1986). The data on rotational curves may provide an additional constraint on the negative values of \( \alpha_s \). For positive values of \( \alpha_s \), the constraint coming from the rotational curves is, as we have seen, more stringent.

From then on one proceeds in the usual way to obtain the velocity distribution with respect to the laboratory.
6. VELOCITY DISTRIBUTION WITH RESPECT TO THE LABORATORY

For this transformation, one needs the velocity of the Sun around the Galaxy, \( v_0 = 220 \text{ km s}^{-1} \), a fraction of the escape velocity, which is \( v_{\text{esc}} = 625 \text{ km s}^{-1} = 2.84v_0 \) (Drucker et al. 1986).

It is convenient to choose coordinate system with its polar z-axis in the direction of the disk’s rotation, i.e., in the direction of the motion of the Sun, the \( x \)-axis in the outward radial direction and the \( y \)-axis perpendicular to the plane of the Galaxy. Since the axis of the ecliptic (Kosmas & Vergados 1997) lies very close to the \((y, z)\)-plane, the velocity of the Earth around the center of the Galaxy is given by

\[
v_E = v_0 \hat{z} + v_1 = v_0 \hat{z} + v_1 (\sin \alpha \hat{x} - \cos \alpha \cos \gamma \hat{y} + \cos \alpha \sin \gamma \hat{z}),
\]

where \( \alpha \) is the phase of the Earth’s orbital motion, \( \alpha = 2\pi(t - t_1)/T_E \), where \( t \) is the time of observation, \( t_1 = \) around June 2, and \( T_E = 1 \text{ yr} \). The magnitude of the Earth’s velocity is much smaller than that of the Sun, i.e., \( \delta_1 = 2v_1/v_0 = 0.27 \). The velocity of the Earth around its own axis is even smaller, and it is usually neglected.

One can now express the above distribution in the laboratory frame by writing \( \nu' = \nu + v_E \), where the prime indicates the velocity with respect to the center of the Galaxy. Indicating with \( y \) the velocity of the LSP in units of \( v_0 \), i.e., by defining \( y = \nu/v_0 \), we find

\[
f(y, \theta, \phi) = N(\lambda, \alpha_x, y_m) [y_m^2 - Y(y, \theta, \phi)]^\lambda 
\times \left\{ 1 + \alpha_x \left[ Y(y, \theta) 
- \left( y \sin \theta \cos \phi - \frac{\delta_1}{2} \sin \alpha \right)^2 \right] \right\}, \tag{42}
\]

where \( N(\lambda, \alpha_x, y_m) \) is given by equation (39), \( y_m = v_m/v_0 \), and

\[
Y(y, \theta, \phi) = 1 + \frac{\delta_1^2}{4} + y^2 + 2y \cos \theta + \delta_1 (y \cos \theta \cos \alpha \sin \gamma 
- y \sin \theta \cos \phi \cos \alpha \cos \gamma + y \sin \theta \cos \phi \sin \alpha). \tag{44}
\]

In the conventional axially symmetric Gaussian velocity distribution this function is given by

\[
f(y, \theta, \phi) = \frac{N(\alpha_x, y_{\text{esc}})}{\pi \sqrt{\pi}} \exp \left[ - (\alpha_x + 1)Y(y, \theta, \phi) 
+ \alpha_x \left( y \sin \theta \cos \phi - \frac{\delta_1}{2} \sin \alpha \right)^2 \right]. \tag{45}
\]

In this case \( 0 \leq y \leq y_{\text{esc}} \), but the upper cutoff is introduced here artificially. The normalization here is defined so that \( N(\alpha_x = 0, y_{\text{esc}} \to \infty) = 1 \) (Vergados 2000).

The detection rate in direct dark matter experiments is obtained by convoluting the relevant cross section with the above velocity distribution. If the dark matter candidate is the LSP, the \( \alpha \)-dependence of the above distribution, present only when \( \delta_1 \neq 0 \), gives rise to the modulation effect, i.e., the dependence of the rate on the Earth’s motion. This signal can be used to discriminate against background.
6.1. The Nondirectional Rate

The nondirectional differential event rate is given by

$$\frac{dR}{du} = \bar{R} \sqrt{\frac{2}{\pi}} T(u), \quad T(u) = a^2 |F(u)|^2 \Psi(a \sqrt{u})$$

(46)

for the coherent mode and

$$\frac{dR}{du} = \bar{R} \sqrt{\frac{2}{\pi}} T_{\text{spin}}(u), \quad T_{\text{spin}} = a^2 |F_{11}(u)| \Psi(a \sqrt{u}),$$

(47)

where $\bar{R}$ and $R_{\text{spin}}$ are the total rates for the coherent and the spin contributions associated with an average LSP velocity, respectively, and $\sqrt{(v^2)}^{1/2} = (3/2)^{1/2} v_0$. These parameters, which carry the dependence on the SUSY parameters, are the most important ones, but they are not of interest in our present calculation. $F(u)$ is the form factor, entering the coherent scattering, and $F_{11}(u)$ is the spin response function entering via the axial current (Vergados 2001a). The function $\Psi$ depends on the LSP distribution velocity employed and is a function of the energy $Q$ transferred to the nucleus:

$$u = \frac{Q}{Q_0}, \quad Q_0 = 4.1 \times 10^4 A^{-4/3} \text{ KeV},$$

(48)

where $A$ is the nuclear mass number and the parameter $a$ is given by

$$a = \left(\sqrt{2} \mu_c b v_0\right)^{-1},$$

(49)

where $\mu_c$ is the reduced mass of the LSP-nucleus system and $b$ is the (harmonic oscillator) size parameter.

The function $\Psi$, which is basic to us, is given by

$$\Psi(x) = \int_0^x dy \int_0^\pi \sin \theta \, d\theta \int_0^{2\pi} d\phi \, yf(y, \theta, \phi),$$

(50)

with $0 \leq x \leq y_m - 1 + (\delta_1/2) \cos \alpha \sin \beta$.

The total rate is given by

$$R = \int_{u_{\text{min}}}^{u_{\text{max}}} \frac{dR}{du} \, du,$$

(51)

where $u_{\text{min}}$ is determined from the cutoff energy of the detector and $u_{\text{max}} = (y_m/a)^2$.

From now on we specialize our results to the case $\lambda = \frac{1}{2}$, which yields an adequate description of the rotation curves in the case of dark matter. In the case of this velocity distribution, unlike the Gaussian one, one cannot approximate the distribution by a power series in $\delta_1$. The reason is that there may be threshold problems when the argument of the square root approaches zero. To simplify matters, we still make use of the fact that the velocity of the Earth around the Sun is much smaller than the velocity of the Sun around the Galaxy, $\delta_1 \ll 1$. So, if we expand the previous expression into a Fourier series with respect to the phase of the Earth, $\alpha$, only the lowest terms will become important. In other words, to leading order in $\delta_1$, it can be put in the form

$$R = \bar{R} R_0 + (R_1 \cos \alpha \sin \gamma - R_2 \cos \alpha \cos \gamma + R_3 \sin \alpha) \delta_1/2.$$

(52)

It turns out that the expansion coefficients $R_2$ and $R_3$ are zero. We can thus conveniently fit the rate with the formula

$$R = \bar{R} (1 + h \cos \alpha),$$

(53)

where $h$ is the modulation amplitude (the difference between the maximum and the minimum is equal to $2|h|$).

In the case of no modulation, $\delta_1 = 0$, the angular integrals can be done analytically to yield

$$\Psi(x) = 2\pi N \left(\frac{1}{2}, \alpha, y_m\right) \times \left[\frac{1 + \alpha y_m}{3} J_1(x, y_m) - \frac{\alpha y_m}{20} J_3(x, y_m)\right],$$

(54)

with $0 \leq x \leq y_m - 1$ and

$$J_n(x, y_m) = J_{\text{int}}(n, y_m, x - 1) - J_{\text{int}}(n, y_m, x + 1) + 2J_{\text{int}}(n, y_m, 1),$$

(55)

$$J_{\text{int}}(n, y_m, y) = \int_0^y (y_m - z)^{n/2} dz.$$

(56)

The above integral can be done analytically to yield

$$J_{\text{int}}(n, y_m, y) = y(y_m)^n 2F_1\left(\frac{1}{2}, -n, \frac{3}{2}, \frac{y^2}{y_m}\right),$$

(57)

where $2F_1$ is the usual hypergeometric function. For the cases of interest to us here, the hypergeometric function can be simplified to yield

$$J_{\text{int}}(3, y_m, y) = \frac{1}{7} \left[2y(y_m - y^2)^{3/2} + 3y^2(y_m^2 - y^2)^{1/2} + 3y^4 \sin^{-1}(y/y_m)\right],$$

(58)

$$J_{\text{int}}(5, y_m, y) = \frac{1}{21} \left[4y(y_m - y^2)^{5/2} + 5y^2(y_m^2 - y^2)^{3/2} + 15y^4(y_m^2 - y^2)^{1/2} + 15y^6 \sin^{-1}(y/y_m)\right],$$

(59)

$$J_{\text{int}}(7, y_m, y) = \frac{1}{105} \left[24y(y_m - y^2)^{7/2} + 28y^2(y_m^2 - y^2)^{5/2} + 35y^4(y_m^2 - y^2)^{3/2} + 105y^6(y_m^2 - y^2)^{1/2} + 105y^8 \sin^{-1}(y/y_m)\right].$$

(60)

The corresponding expressions for the Gaussian expressions for $\alpha_5$ cannot be done analytically. For the symmetric case, $\alpha_5 = 0$, one finds that

$$\Psi(x) = \frac{1}{4} \left[\text{erf}(x - 1) - \text{erf}(x + 1)\right] + 2\text{erf} 1,$$

(61)

with $\text{erf}$ the error function:

$$\text{erf} y = \frac{2}{\pi} \int_0^y e^{-t^2} dt.$$

(62)

The above functions $\Psi$ are plotted in Figures 17 and 18.

From Figure 18 we see that both the value of the function $\Psi(x)$ and its range depend crucially on the parameters of the model. In the absence of asymmetry, the Gaussian model, with an appropriate escape velocity put in by hand, yields results that are almost 4 times larger than those of the Eddington theory. In the presence of asymmetry our results in the Eddington theory are substantially larger. This is due to the larger peak value attained as well as the larger allowed
range of $x$. On the other hand, we know that in the Gaussian model the total rate is not significantly affected by the asymmetry (Vergados 1999). The reason for the strong dependence of the results in the present model on the asymmetry is not, of course, the asymmetry per se, but the fact that all parameters change with it, in particular, the value of $y_m$ (see Table 1). In the Gaussian model the introduction of asymmetry did not affect the velocity distribution in any other way; e.g., it did not affect the cutoff value of the velocity distribution. The large upper values of $x$ in the function $\Psi(x)$, allowed in the present model for large $y_m$, are, of course, somewhat controlled by the nuclear form factor. In order to get a better feeling for such an effect on the rate, we also plot the function $T(u)$, which is proportional to the differential nondirectional rate. For illustration purposes, we have chosen to present results for the popular target $^{127}I$ and a typical LSP mass of 100 GeV (see Fig. 19). The results presented are for the coherent mechanism, but we expect very small changes when the spin contribution is considered. It is clear that the introduction of asymmetry has a profound effect on the rate. We have seen, however, that the large positive values of $\alpha$, can, in principle, be eliminated from the data on the rotational curves.

Effects like those discussed above may be more pronounced in the case of modulation, which is not studied in this work.

Fig. 17.—Function $\Psi(x)$ for dark matter in the case of the symmetric Gaussian distribution.

Fig. 18.—Function $\Psi(x)$ in the case of dark matter for the choice $\lambda = \frac{1}{4}$ of the Eddington theory. The graphs have been labeled as in Fig. 9.

6.2. The Directional Rate

The directional differential rate ($dR/du)_d$ is proportional to

$$T_d(u) = \frac{1}{2\pi} \mathcal{A}^2 |F(u)|^2 \Psi_d(a\sqrt{u}) ,$$

and

$$T^\text{spin}_d = \frac{1}{2\pi} \mathcal{A}^2 |F_1(u)| \Psi_d(a\sqrt{u}) ,$$

with

$$\Psi_d(x) = \int_0^x dy \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\phi \gamma(y, \theta, \phi)XH(X) ,$$

where $H(X)$ is the well-known Heaviside function, and $X$ is given by

$$X = \cos \Theta \cos \phi + \sin \Theta \sin \phi \sin \Phi \sin \phi + \cos \Phi \cos \phi ,$$

where $\Theta$ and $\Phi$ describe the direction of observation $\hat{e}$:

$$\hat{e} = \sin \Theta (\cos \Phi \hat{e}_x + \sin \Phi \hat{e}_y) + \cos \Theta \hat{e}_z ,$$

[there should be no confusion of the angle $\Phi$ used here with the potential $\Phi(r)$ used earlier]. Note the presence of the factor of $1/(2\pi)$, since the azimuthal integration of the recoiling nucleus is not present and we intend to use the same nucleon cross section in both the directional and the nondirectional case.

The total rate is proportional to

$$R_d = \int_{\theta_{\min}}^{\theta_{\max}} \frac{dR}{du} du .$$

Taking again the lowest Fourier components of the obtained rate as a function of the phase of the Earth, we get an expression similar to equation (52). Thus, to leading order in $\delta_1$, we can fit the total rate by an expression of the form

$$R_d = \frac{1}{2\pi} \mathcal{R}_d (1 + (h_1 - h_2) \cos \alpha + h_3 \sin \alpha) .$$

The parameters $t_d$ and $h_i$ ($i = 1, 2, 3$) are obviously
functions of the direction of observation, i.e., $\Theta$ and $\Phi$. If one observes in the direction of the Sun’s velocity, $h_2 = h_3 = 0$. Similarly, if one observes in a plane perpendicular to the Sun’s velocity, $h_1 = 0$. Instead of $t_{\text{dir}}$, it is best to use the ratio

$$\kappa = 2\pi \frac{R_{\text{dir}}}{R} = \frac{t_{\text{dir}}}{t}.$$  \hspace{1cm} (68)

The parameter $\kappa$ is essentially independent of the LSP mass, the nuclear parameters, and the asymmetry parameter $\alpha_s$. But it depends strongly on the direction of observation and is expected to correlate strongly with the angle between $\vec{e}$ and the Sun’s direction of motion. This correlation provides an experimental signature perhaps better than that of the modulation with the Earth’s motion in nondirectional experiments.

For $\delta_1 \neq 0$, the above integrals over $y$, $\theta$, $\phi$, especially in the directional case, can only be done numerically along the lines of the previous work (Vergados 2002).

7. CONCLUSIONS

In the present paper we studied the density and velocity distributions of CDM in the context of the Eddington theory, considering not only symmetric but axially symmetric distributions as well. In our approach we used standard simple distribution functions of the energy and angular momentum. This led us to simple relations between the density $\rho$ and the potential $\Phi$; $\rho$ and $\Phi$ were obtained by solving numerically Poisson’s equation in a suitable region of space. This procedure allowed us to determine the maximum permitted dark matter velocity. Then we were able to study both the rotational curves and the velocity distribution of dark matter in our vicinity. We should mention that this distribution is not Maxwellian and has an upper velocity cutoff built into it and not put in by hand, as in the traditional treatment with Gaussian distribution.

Our results depend on a minimum set of parameters, which were treated as free (see Table 1).

We saw that in the context of this theory the predicted rotational velocities (see Figs. 13–16), depending on the input parameters, can vary significantly in the presence of the asymmetry. This comes mainly from the factor $\lambda^2$ in the second term of equation (15), when the parameter $\alpha$ is reasonably large. By comparing our results to the observed rotational curves (Jungman et al. 1996), one may constrain the parameters of the model. The best choice seems to be the case with a small asymmetry parameter $\alpha_s$.

We have also made a preliminary study of the effect of the new velocity distribution on the direct detection rates for CDM. For illustrative purposes, we have selected the case $\lambda = \frac{1}{2}$. We have seen that, in the context of the Eddington approach, the total rates, unlike the case of the Gaussian distribution (Vergados 1999, 2000), sensitively depend on the asymmetry parameter $\alpha_s$. This is due to the fact that, when the asymmetry changes, the upper value of the velocity distribution also changes.

It is thus not surprising that, in the Eddington theory, the total (nondirectional and nonmodulated) event rates for direct LSP detection may be substantially different from those of the phenomenological Gaussian distributions (compare Figs. 17 and 18). The strong dependence of the rate on the asymmetry parameter remains even after the nuclear form factor has been incorporated (see Fig. 19). Results of more detailed calculations of the event rates will appear elsewhere.

The dependence of the directional and/or modulated rates on the velocity distribution is currently under study. We expect this dependence to be more pronounced than that on the total rates.

J. D. V. would like to thank the physics department of UNISA and Professor S. Sofianos for their hospitality. D. O. appreciates the hospitality provided by the University of Ioannina.

REFERENCES

Alcock, C., et al. 1995, Phys. Rev. Lett., 74, 2867
Arnould, R., & Nath, P. 1996, Phys. Rev. D, 54, 2374
Bednarek, A., Donato, F., Mignola, G., Scopel, S., Belli, P., & Incicchitti, A. 1999, Phys. Rev. D, 60, 103512
Bonnabeau, R., et al. 1996, Phys. Lett. B, 389, 757
———. 1999, Phys. Lett. B, 440, 488
Bottino, A., Donato, F., Mignola, G., Scopel, S., Belli, P., & Incicchitti, A. 1997, Phys. Lett. B, 402, 113
Cheng, H.-Y. 1989, Phys. Lett. B, 219, 347
Cheng, T. P. 1988, Phys. Rev. D, 38, 2869
Copi, C., Heo, J., & Krauss, L. 1999, Phys. Lett. B, 461, 43
Divari, P. C., Kosmas, T. S., Vergados, J. D., & Skouras, L. D. 2000, Phys. Rev. D, 61, 054612
Djouadi, A., & Drees, M. 2000, Phys. Lett. B, 484, 183
Drees, M., & Nojiri, M. M. 1993a, Phys. Rev. D, 47, 376
———. 1993b, Phys. Rev. D, 47, 4226
———. 1993c, Phys. Rev. D, 48, 3483
Drucker, A., Freeze, A., & Spergel, D. 1986, Phys. Rev. D, 33, 3495
Eddington, A. S. 1916, MNRAS, 76, 572
Einasto, J. 2001, in Dark Matter in Astro- and Particle Physics, Proc. of Int. DARK 2000 Conf., ed. H. V. Klapdor-Kleingrothaus (Berlin: Springer), 3
Evans, N. W. 1994, MNRAS, 267, 333
Gawiser, E., & Silk, J. 1988, Science, 280, 1405
Gelmini, G., & Gondolo, P. 2001, Phys. Rev. D, 64, 023504
Gomez, M. E., Lazarides, G., & Pallis, C. 2000a, Phys. Rev. B, 487, 313
Gomez, M. E., Lazarides, G., & Pallis, C. 2000b, Phys. Rev. D, 61, 123512
Gomez, M. E., & Vergados, J. D. 2001a, Phys. Lett. B, 512, 252
Gomez, M. E., & Vergados, J. D. 2001b, in Proc. of Cairo Int. Conf. on High-Energy Astrophysics, ed. A. Kahil, Q. Shaﬁ, & H. Tallat (Princeton: Princeton Univ. Press), 248
Green, A. M. 2001, Phys. Rev. D, 63, 103003
Hess, M. A. K., Somerville, R. S., Primack, J. R., Holtzman, J., & Klypin, A. A. 1998, MNRAS, 301, 81
Henriksen, R. N., & Widrow, L. M. 1995, MNRAS, 276, 679
Jungman, G., Kamionkowski, M., & Griest, K. 1995, Phys. Rev., 267, 195
Kosmas, T. S., & Vergados, J. D. 1997, Phys. Rev. D, 55, 1752
Lehner, M. J. 2001, in Dark Matter in Astro- and Particle Physics, Proc. of Int. DARK 2000 Conf., ed. H. V. Klapdor-Kleingrothaus (Berlin: Springer), 390
Merritt, D. 1985, AJ, 90, 1027
Nath, P., & Arnould, R. 1995, Phys. Rev. Lett., 74, 4592
Perlmutter, S., Turner, M. S., & White, M. 1999a, Phys. Rev. Lett., 83, 670
Perlmutter, S., et al. 1997, ApJ, 483, 565
———. 1999b, ApJ, 517, 565
Primack, J. R. 2001, in Sources and Detection of Dark Matter and Dark Energy in the Universe, ed. D. Cline (Berlin: Springer), 3
Ressell, M. T., Aufderheide, M. A., Bloom, S. D., Griest, K., Mathews, G. J., & Resler, D. A. 1993, Phys. Rev. D, 48, 5519
Riess, A. G., et al. 1998, AJ, 116, 1009
Sikivie, P. 1999, Phys. Lett. B, 389, 757
Smoot, G. F., et al. 1992, ApJ, 396, L1

No. 1, 2003 NEW CDM VELOCITY DISTRIBUTION 27
Somerville, R. S., Primack, J. R., & Faber, S. M. 2003, MNRAS, in press (astro-ph/9806228)
Spira, M., Djouadi, A., Graudenz, D., & Zerwas, P. M. 1995, Nucl. Phys. B, 453, 17
Turner, M. S. 1990, Phys. Rep., 197, 69
Ullio, P., & Kamionkowski, M. 2001, J. High Energy Phys., 103, 49
Vergados, J. D. 1996, J. Phys. G., 22, 253
———. 1998, Phys. Rev. D, 58, 103001
———. 1999, Phys. Rev. Lett., 83, 3597
———. 2000, Phys. Rev. D, 62, 023519
———. 2001a, Part. Nucl. Lett., 106, 74
Vergados, J. D. 2001b, Phys. Rev. D, 63, 06351
———. 2002, in AIP Conf. Proc. 624, Cosmology and Elementary Particle Physics, ed. B. N. Kursonoglu, S. L. Mintz, & A. Perlmutter (Melville: AIP), 76
Widrow, L. M. 2000, ApJ, submitted (astro-ph/0003302)