A Scaled Smart City for Experimental Validation of Connected and Automated Vehicles *

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Abstract: The common thread that characterizes energy efficient mobility systems for smart cities is their interconnectivity which enables the exchange of massive amounts of data; this, in turn, provides the opportunity to develop a decentralized framework to process this information and deliver real-time control actions that optimize energy consumption and other associated benefits. To seize these opportunities, this paper describes the development of a scaled smart city providing a glimpse that bridges the gap between simulation and full scale implementation of energy efficient mobility systems. Using this testbed, we can quickly, safely, and affordably experimentally validate control concepts aimed at enhancing our understanding of the implications of next generation mobility systems.

Keywords: Smart cities, connected and automated vehicles, vehicle coordination, cooperative merging control.

1. INTRODUCTION

In a rapidly urbanizing world, we need to make fundamental transformations in how we use and access transportation. Energy efficient mobility systems such as Connected and Automated Vehicles (CAVs) along with shared mobility and electric vehicles provide the most intriguing and promising opportunities for enabling users to better monitor transportation network conditions and make better operating decisions to reduce energy consumption, greenhouse gas emissions, travel delays and improve safety. As we move to increasingly complex transportation systems new control approaches are needed to optimize the system behavior resulting from the interactions between vehicles navigating different traffic scenarios.

Given this new environment, the overarching goal of this paper is to report on the development of the University of Delaware Scaled Smart City (UDSSC) that includes 35 robotic cars to replicate real-world traffic scenarios in a small and controlled ecosystem. UDSSC can serve as a testbed to explore the acquisition and processing of vehicle-to-vehicle and vehicle-to-infrastructure communication. It can also help us prove control concepts on coordinating CAVs in specific traffic scenarios, e.g., intersections, merging roadways, roundabouts, speed reduction zones, etc. These scenarios along with the driver responses to various disturbances are the primary sources of bottlenecks that contribute to traffic congestion; see Malikopoulos and Aguilar (2013); Margiotta and Snyder (2011). In 2015, congestion caused people in urban areas in the US to spend 6.9 billion additional hours on the road and to purchase an extra 3.1 billion gallons of fuel, resulting in a total cost estimated at $160 billion; see (Schrank et al., 2015).

CAVs can provide shorter gaps between vehicles and faster responses while improving highway capacity. Several research efforts have been reported in the literature proposing either centralized or decentralized approaches for coordinating CAVs in specific traffic scenarios. The overarching goal of such efforts is to yield a smooth traffic flow avoiding stop-and-go driving. Numerous approaches have been reported in the literature on coordinating CAVs in different transportation scenarios with the intention of improving traffic flow. Kachroo and Li (1997) proposed a longitudinal and lateral controller to guide the vehicle until the merging maneuver is completed. Other efforts have focused on developing a hybrid control aimed at keeping a safe headway between vehicles in the merging process, see Antoniotti et al. (1997); Kachroo and Li (1997); or developing three levels of assistance for the merging process to select a safe space for the vehicle to merge; see Ran et al. (1999). Some authors have explored virtual vehicle platooning for autonomous merging control, e.g., Dresner and Stone (2004); Lu et al. (2000), where a controller identifies and interchanges appropriate information between the vehicles involved in the merging maneuver while each vehicle assumes its own control actions to satisfy the assigned time and reference speed.

VanMiddlesworth et al. (2008) addressed the problem of traffic coordination for small intersections which commonly handle low traffic loads. Milanés et al. (2010) de-
signed a controller that allows a fully automated vehicle to yield to an incoming vehicle in the conflicting road or to cross, if it is feasible without the risk of potential collision. Alonso et al. (2011) proposed two conflict resolution schemes in which an autonomous vehicle could make a decision about the appropriate crossing schedule and trajectory to follow to avoid collision with other manually driven vehicles on the road. A survey of the research efforts in this area that have been reported in the literature to date can be found in Rios-Torres and Malikopoulos (2017a).

Although previous work has shown promising results emphasizing the potential benefits of coordination between CAVs, validation has been primarily in simulation. In this paper, we demonstrate coordination of scaled CAVs and quantify the benefits in energy usage. The contributions of this paper are: 1) the development of a 1:24 scaled smart city capable of testing coordination control algorithms on up to 35 Micro Connected and Automated Vehicles (MCAVs) and 2) the experimental validation of a control framework reported in Rios-Torres and Malikopoulos (2017b) for coordination of CAVs.

The remainder of the paper proceeds as follows. In Section II, we introduce the configurations of UDSSC. In Section III, we review a decentralized control framework for coordination of CAVs in merging roadways. Experimental results in Section IV illustrate the effectiveness of the proposed solution in the scaled smart city environment. We draw concluding remarks in Section V.

2. UNIVERSITY OF DELAWARE SCALED SMART CITY (UDSSC)

UDSSC is a testbed that can replicate real-world traffic scenarios in a small and controlled environment and help us formulate the appropriate features of a “smart” city. It can be used as an effective way to visualize the concepts developed using CAVs and their related implications in energy usage. UDSSC is a fully integrated smart city (Fig. 1) incorporating realistic environmental cues, scaled MCAVs, and state-of-the-art, high-end computers supporting standard software for system analysis and optimization for simulating different control strategies and distributing control inputs to as many as 35 autonomous vehicles.

Fig. 1. Birdseye view of the University of Delaware’s Scaled Smart City.

Fig. 2. SolidWorks provides a convenient workspace for representing fully dimensioned roadways including labeling for easy identification of each road segment.

2.1 Physical Design I: Map

The UDSSC spans over 400 square feet and includes one lane intersections, two lane intersections, roundabouts, and a highway with entrance and exit ramps. Using SolidWorks to accurately maintain 1:24 scale, a fully dimensioned two-dimensional blueprint (Fig. 2) was designed forming a cohesive roadway representative of real world road scenarios. Labels act as unique identifiers distinguishing between straight line and arc segments and containing path parameters. The blueprint is then exported into Adobe Illustrator where seamless texturing and environmental cues are added for realism. Special care has been taken to include three-dimensional aspects including, trees, houses and even humans so future work focusing on lane tracking will include realistic visual tracking elements. Using a HP DesignJet z5200 photo printer the entire layout is printed on twelve 44”x120” sheets of wear resistant HP Artist Matte Canvas that can be easily replaced to either reconfigure or repair sections of the city independently. Double sided carpet tape holds each map section to over 120 2’x2’x7/8” DRIcore sub-floor panels helping to distribute floor level variations while providing a smooth mounting surface. Eight Vicon Vantage V16 cameras are used to localize the map within a globally recognized coordinate system. Critically, Vicon markers are placed atop each map subsection to locate a centralized origin and abate the effect of slight offsets and stretching in the canvas material. Once a map frame is initialized the dimensions of the road sections are known exactly and are easily referenced using the original two-dimensional blueprints.

2.2 Physical Design II: Cars

Scaled MCAVs have been designed using easily assembled off-the-shelf components coupled with several 3D printed parts (Fig. 3). At the core of each platform is a 75.81:1
geared, differentially driven Pololu Zumo, offering dual H-bridge motor drivers, \( n_e = 12 \) counts per revolution (CPR) encoders and an on-board Atmega 32U4 microcontroller. Additionally the Zumo contains an embedded set of sensors including a buzzer, IMU, line-following and infrared proximity sensors which can provide feedback to each MCAV. Although each Zumo is originally equipped with tracks, they are replaced with rubberized wheels with radius \( r = 1.6 \) cm mounted directly to each gear motor output shaft and separated by \( d = 9 \) cm to roughly mimic the 1:24 scale width of full sized cars/trucks. The MCAV platform (not including its car-shaped shell) measures 13 cm x 10.5 cm x 4.5 cm (l/w/h), keeping a low profile due to their analytical representation. Given\( \dot{x} \),\( \dot{y} \), and overshoot. A length and width are also specified such that larger values cause more dramatic convergence in the direction of the road and \( c_w \) is a tuning parameter.\( \cos \),\( \sin \), creating a region in which the vector field is active.\( \mathbf{d} \) is a unit vector for a user friendly interface into the Robot Operating System (ROS) architecture. Generally control of each MCAV can be broken into merging, lane and reference tracking controllers, the latter two of which enforce realistic road behaviors (i.e. staying in the center of the road and respecting speed limits.)

**Lane tracking:** The roads of UDSSC are encoded as sequences of tangent arcs and straight line segments. A general representation of each of these road types is then encoded as a potential field directing a particle placed anywhere on the first segment and will follow along the road until its battery is depleted.

Straight line roads can be described by,

\[
\begin{align*}
\dot{x} &= dX + p(x_o - x - \left((x_o - x)dX + (y_o - y)dY\right)dX) \\
\dot{y} &= dY + p(y_o - y - \left((x_o - x)dX + (y_o - y)dY\right)dY),
\end{align*}
\]

where \( (x_o, y_o) \) is the initial point, \( \mathbf{dX, dY} \) is a unit vector in the direction of the road and \( p \) is a tuning parameter such that larger values cause more dramatic convergence and overshoot. A length and width are also specified creating a region in which the vector field is active.

Arcs are described by,

\[
\begin{align*}
\dot{x} &= r(y - y_c)cw - 4p(x - x_c)((x - x_c)^2 + (y - y_c)^2 - r^2) \\
\dot{y} &= -r(x - x_c)cw - 4p(y - y_c)((x - x_c)^2 + (y - y_c)^2 - r^2),
\end{align*}
\]

where \( (x_c, y_c) \) designate the center of the arc and \( r \) the radius. Similarly to the straight line equation, a tuning
parameter drives faster convergence at higher values. A parameter \( cw = 1 \) if the rotational direction is clock-wise, otherwise \( cw = -1 \).

Fig. 5. Line (a) and arc (b) vector fields with parameter \( p=2.0 \) and \( p=0.2 \) respectively.

Transition regions are constructed by specifying half-planes at the intersection point between two road segments such that transition from the center of one road places a reference point in the center of the next. Although an MCAV can use lane tracking directly to follow along a road, while in the control region merging requires a carefully maintained forward velocity profile with respect to the lane offsets. From the centerline of the lane using lane tracking results in noisy forward velocity measurements, however a virtual robot can track the center of each lane exactly. For situations where more careful velocity profiles are required, such as in merging control, instead of controlling each MCAV using the potential field directly a virtual robot is simulated within the vector field and used as a reference point that is tracked by the real robot.

Reference tracking: As long as a reference point tracks a sequence of road segments using lane tracking the MCAV can be controlled by reference tracking. Knowing each MCAV is differentially driven a virtual robot with the same unicycle dynamics can be used for tracking with inputs \( u = [v, \omega] \). A simple state tracking method as described in Giuseppe and Vendittelli (2002) is used (2), approximately linearizing the error dynamics of the MCAV’s local frame with respect to a reference trajectory. Based on the original control input we can write,

\[
v = v_d \cos(\theta_d - \theta) + k_1((x_d - x) \cos(\theta) + (y_d - y) \sin(\theta))
\]

\[
w = w_d + k_2 \text{sign}(v_d)((y_d - y) \cos(\theta) - (x_d - x) \sin(\theta)) + k_3(\theta_d - \theta),
\]

where \( v_d, \omega, \theta_d \), given by the reference trajectory, are the desired velocity, angular velocity and orientation respectively. The gains of the controller \( k_1, k_2, k_3 \), are chosen as in Giuseppe and Vendittelli (2002),

\[
k_1 = k_2 = 2\sqrt{\omega^2_d(t) + b^2 v_d(t)}, \quad k_3 = bv_d(t),
\]

and \( \zeta \in (0,1) \), \( b > 0 \). For the Zumo based MCAVs described here, \( \zeta = 0.8, b = 70 \) are chosen as appropriate values.

Merging scenario: There have been several approaches for automated vehicle merging as reported in Rios-Torres and Malikopoulos (2017a). In this paper, we consider the decentralized control approach described in Section 3 with the Main Frame tracking the positions of each MCAV in order to determine when vehicles enter merging control regions. Practically a ROS service updates an ordered list as new cars enter a region and the boundaries of each region are indicated by a transition similarly to how transitions join road segments. Once inside a control region a virtual robot tracks the desired velocity profile exactly and reference tracking is used to mimic this behavior by the associated MCAV. The software is structured such that new control methods can be easily interchanged for comparison and testing highlighting another advantage of the UDSSC testbed.

\[
\text{Low level control: On-board encoders enable low-level control from inputs } u = [v, \omega], \text{ that are converted with two relationships,}
\]

\[
v = R \frac{\dot{\phi}_R + \dot{\phi}_L}{2} \quad \omega = R \frac{\dot{\phi}_R - \dot{\phi}_L}{2},
\]

where \( \dot{\phi}_R, \dot{\phi}_L \) are the right and left wheels angular velocity respectively and \( R \) is wheel radius. High frequency control, however, results in noisy measurements due to low-resolution encoders. The Atmega 32U4 measures encoder pulses at a frequency of 2 KHz then smooths the velocity estimate by managing a running 25 measurement queue. Depending on the queue length, noise in the velocity measurement can be attenuated at the cost of increased settling time. A proportional controller adjusts the PWM duty cycle depending on the error between measured and desired velocity, saturating at 0.7m/s ± 0.1m/s depending on transmission friction and slight mechanical variations between vehicles.

3. COORDINATION OF CONNECTED AND AUTOMATED VEHICLES

3.1 Modeling Framework

We consider a merging roadway (Fig. 6) consisting of main and secondary roads. The region that potential lateral collision between vehicles can occur is called merging zone and has a length of \( S \). On each road, there is a control zone inside of which all vehicles can communicate with each other and with a coordinator. Note that the coordinator is not involved in any decision for any CAV and only enables communication of appropriate information among CAVs. The distance from the entry of the control zone to the entry of the merging zone is \( L \). The value of \( L \) depends on the coordinator’s communication range capability with...
the CAVs, while \( S < L \) is the physical length of a typical merging zone. 

Let \( N(t) \in \mathbb{N} \) be the number of CAVs inside the control zone at time \( t \in \mathbb{R}^+ \) and \( N(t) = \{1, \ldots, N(t)\} \) be a queue which designates the order in which these vehicles will be entering the merging zone. Thus, letting \( t_i^m \) be the assigned time for vehicle \( i \) to enter the merging zone, we require that 
\[
t_i^m \geq t_{i-1}^m, \quad \forall i \in N(t), \quad i > 1.
\] (4)

There are a number of ways to satisfy (4). For example, we may impose a strict First-in-First-Out (FIFO) queuing structure, where each vehicle must enter the merging zone in the same order it entered the control zone. More generally, however, \( t_i^m \) may be determined for each vehicle \( i \) at time \( t_i^0 \) when the vehicle enters the control zone and \( N(t_i^0) = \{1, \ldots, i-1\} \). If \( t_i^m > t_{i-1}^m \), then the order in the queue is preserved. If, on the other hand, there exists some \( j \in N(t_i^0) \), where \( j < i-1 \), such that \( t_j^m > t_i^m > t_{i-1}^m \), then the order is updated so that CAV \( i \) is placed in the \( j \)-th queue position. The policy through which the sequence is specified may be the result of a higher level optimization problem as long as the condition \( t_i^m \geq t_{i-1}^m \) is preserved in between CAV arrival events at the control zone. In what follows, we will adopt a specific scheme for determining \( t_i^m \) (upon arrival of CAV) based on our problem formulation, without affecting \( t_1^m, \ldots, t_{i-1}^m \), but we emphasize that our analysis is not restricted by the policy designating the order of the vehicles within the queue \( N(t) \).

We adopt the optimization framework proposed in Rios-Torres and Malikopoulos (2017b) for coordinating the merging of CAVs. The dynamics of each vehicle \( i \in N(t) \) are represented by a double integrator, 
\[
p_i(t) = v_i(t), \quad \dot{v}_i = u_i(t),
\] (5) 

where \( t \in \mathbb{R}^+ \) denotes the time and \( p_i(t) \in P_i \), \( v_i(t) \in V_i \), and \( u_i(t) \in U_i \) denote position, speed and acceleration/deceleration (control input) of each vehicle \( i \in N(t) \) inside the control zone. Let \( [p_i(t), v_i(t)]^T \) denote the state of each vehicle \( i \), with initial value \([0, 0]^T\). The state space \( P_i \times V_i \) is closed with respect to the induced topology, thus, it is compact.

For any initial state \([p_i(t^0), v_i(t^0)]^T\), where \( t^0 \) is the time that the vehicle \( i \) enters the control zone, and every admissible control \( u_i(t) \), the double integrator has a unique solution on some interval \([t^0, t_i^m]\), where \( t_i^m \) is the time that vehicle \( i \in N(t) \) enters the merging zone. In our framework we impose the following state and control constraints:
\[
u_{i,min} \leq u_i(t) \leq u_{i,max}, \quad \text{and} \quad 0 \leq v_{i,min} \leq v_i(t) \leq v_{i,max}, \quad \forall t \in [t_i^0, t_i^m].
\] (6)

where \( u_{i,min} \) and \( u_{i,max} \) are the minimum and maximum control inputs (maximum acceleration/deceleration) for each vehicle \( i \in N(t) \), and \( v_{i,min} \) and \( v_{i,max} \) are the minimum and maximum speed limits respectively. For simplicity, in the rest of the paper we consider no vehicle diversity, and thus, we set \( u_{i,min} = u_{min} \) and \( u_{i,max} = u_{max} \).

For absence of any rear-end collision of two consecutive vehicles traveling on the same lane, the position of the preceding vehicle should be greater than or equal to the position of the following vehicle plus a safe distance \( \delta(v_{ave}(t)) < S \), which is a function of the average speed of the vehicles inside the control zone. Thus, we impose the following rear-end safety constraint
\[
s_i(t) = p_k(t) - p_i(t) \geq \delta(v_{ave}(t)), \quad \forall t \in [t_i^m, t_i^m],
\] (7) 

where \( k \) denotes the vehicle that is physically located ahead of \( i \) in the same lane, and \( v_{ave}(t) \) is the average speed of the vehicles inside the control zone at time \( t \).

**Definition 1.** Each CAV \( i \in N(t) \) belongs to at least one of the following two subsets of \( N(t) \) depending on its physical location inside the control zone: 1) \( C_i(t) \) contains all CAVs traveling on the same road and lane as vehicle \( i \) and 2) \( \bar{C}_i(t) \) contains all CAVs traveling on a different road from \( i \) and can cause collision at the merging zone.

**Definition 2.** For each vehicle \( i \in N(t) \), we define the set \( \Gamma_i \) that includes only the positions along the lane where a lateral collision is possible, namely
\[
\Gamma_i = \left\{ t \mid t \in [t_i^m, t_i^f] \right\},
\] (8)

where \( t_i^f \) is the time that vehicle \( i \in N(t) \) exits the merging zone.

Consequently, to avoid a lateral collision for any two vehicles \( i, j \in N(t) \) on different roads, the following constraint should hold
\[
\Gamma_i \cap \Gamma_j = \emptyset, \quad \forall t \in [t_i^m, t_i^f], \quad j \in C_i(t).
\] (9)

The above constraint implies that only one vehicle at a time can be inside the merging zone. If the length of the merging is long, then this constraint may not be realistic since it results in dissipating space and capacity of the road. However, the constraint is not restrictive in the problem formulation and it can be modified appropriately.

In the modeling framework described above, we impose the following assumptions:

**Assumption 1.** The vehicles cruise inside the merging zone with an imposed speed limit, \( v_{srz} \). This implies that for each vehicle \( i \)
\[
t_i^f = t_i^m + \frac{S}{v_{srz}}.
\] (10)

This assumption is intended to enhance safety awareness, but it could be modified appropriately, if necessary.

### 3.2 Communication Structure of Connected and Automated Vehicles

We consider the problem of deriving the optimal control input (acceleration/deceleration) of each CAV inside the control zone (Fig. 6), under the hard safety constraints to avoid rear-end and lateral collision. By controlling the speed of the vehicles, the speed of queue built-up at the merging zone decreases, and thus the congestion recovery time is also reduced. The latter results in maximizing the throughput in the merging zone.

When a CAV \( i \) enters the control zone, it can communicate with the other CAVs that exist inside the control zone and with the coordinator. Note that the coordinator is not involved in any decision for any CAV and only enables communication of appropriate information among CAVs. The coordinator handles the information between the vehicles as follows: When a CAV reaches the control zone at some instant \( t \), the coordinator assigns a unique identity
to each vehicle \( i \in \mathcal{N}(t) \), which is a pair \((i, j)\), where \( i = \mathcal{N}(t) + 1 \) is an integer representing the location of the vehicle in a FIFO queue \( \mathcal{N}(t) \) and \( j \in \{1, 2\} \) is an integer based on a one-to-one mapping from \( \mathcal{L}_i(t) \) and \( \mathcal{C}_i(t) \) onto \( \{1, 2\} \). If the vehicles enter the control zone at the same time, then the coordinator selects randomly their position in the queue.

**Definition 3.** For each CAV \( i \) entering the control zone, we define the information set \( Y_i(t) \), which includes all information that each vehicle shares, as

\[
Y_i(t) \triangleq \left\{ p_i(t), v_i(t), Q, t_{m}^i \right\}, \forall t \in [t_0^i, t_{m}^i],
\]

where \( p_i(t), v_i(t) \) are the position and speed of CAV \( i \) inside the control zone, \( Q = \{ \mathcal{L}_i(t), \mathcal{C}_i(t) \} \) is the subset assigned to CAV \( i \) by the coordinator, and \( t_{m}^i \) is the time targeted for CAV \( i \) to enter the merging zone, whose evaluation is discussed next.

The time \( t_{m}^i \) that the vehicle \( i \) will enter the merging zone is restricted by the imposing rear-end and lateral collision constraints. Therefore, to ensure that (7) and (9) are satisfied at \( t_{m}^i \) we impose the following conditions which depend on the subset that the vehicle \( i - 1 \) belongs to. If \( i - 1 \in \mathcal{L}_i(t) \),

\[
t_{m}^i = \max \left\{ \min \left\{ t_{m}^{i-1} + \frac{L}{v_{min}}, \frac{L}{v_i(t_{m}^i)}, 1 \right\}, \frac{L}{v_{max}} \right\},
\]

and if \( i - 1 \in \mathcal{C}_i(t) \),

\[
t_{m}^i = \max \left\{ \min \left\{ t_{m}^{i-1} + \frac{S}{v_{srz}}, \frac{L}{v_{min}}, \frac{L}{v_i(t_{m}^i)}, \frac{L}{v_{max}} \right\}, \right\}
\]

where \( v_{srz} \) is the imposed speed inside the merging zone (Assumption 1), and \( v_i(t_{m}^i) \) is the initial speed of vehicle \( i \) when it enters the control zone at \( t_{m}^i \). The conditions (12) and (13) ensures that the time \( t_{m}^i \) each vehicle \( i \) will be entering the merging zone is feasible and can be attained based on the imposed speed limits inside the control zone. In addition, for low traffic flow where vehicle \( i - 1 \) and \( i \) might be located far away from each other, there is no compelling reason for vehicle \( i \) to accelerate within the control zone to maintain a distance \( \delta(v_{ave}(t)) \) from vehicle \( i - 1 \), if \( i - 1 \in \mathcal{L}_i(t) \), or a distance \( S \) if \( i - 1 \in \mathcal{C}_i(t) \), at the time \( t_{m}^i \) that vehicle \( i \) enters the merging zone. Therefore, in such cases vehicle \( i \) can keep cruising within the control zone with the initial speed \( v_i(t_{m}^i) \) that entered the control zone at \( t_{m}^i \).

The recursion is initialized when the first vehicle enters the control zone, i.e., it is assigned \( i = 1 \). In this case, \( t_{m}^1 \) can be externally assigned as the desired exit time of this vehicle whose behavior is unconstrained. Thus the time \( t_{m}^i \) is fixed and available through \( Y_1(t) \). The second vehicle will access \( Y_1(t) \) to compute the times \( t_{m}^2 \). The third vehicle will access \( Y_2(t) \) and the communication process will continue with the same fashion until the vehicle \( N(t) \) in the queue access the \( Y_{N(t)-1}(t) \).

### 3.3 Optimal Control Problem Formulation for Connected and Automated Vehicles

Since the coordinator is not involved in any decision on the vehicle coordination we can formulate \( N(t) \) sequential decentralized control problems that may be solved on-line:

\[
\min_{u_i} \frac{1}{2} \int_{t_0}^{t_f} u_i^2(t) \, dt,
\]

subject to: (5) and (6),

with initial and final conditions: \( p_i(t_0^i) = 0, p_i(t_f^i) = L, t_0^i, v_i(t_0^i) = L, v_i(t_f^i) = 0 \). In the problem formulation above, we have omitted the rear end (7) and lateral (9) collision safety constraints. As mentioned earlier, (9) implicitly handled by the selection of \( t_{m}^i \) in (13). Eq. (7) is omitted because it has been shown Malikopoulos et al. (2017) that the solution of (14) guarantees that this constraint holds throughout \([t_0^i, t_f^i]\). Thus, (14) is a simpler problem to solve on-line.

For the analytical solution and real-time implementation of the control problem (14), we apply Hamiltonian analysis. In our analysis, we consider that when the vehicles enter the control zone, none of the constraints are active. To address this problem, the constrained and unconstrained arcs need to be pieced together to satisfy the Euler-Lagrange equations and necessary condition of optimality. The analytical solution of (14) without considering state and control constraints was presented in Ntousakis et al. (2016); Rios-Torres et al. (2015); Rios-Torres and Malikopoulos (2017b) for coordinating in real time CAVs at highway on-ramps and Zhang et al. (2016) at two adjacent intersections. When the state and control constraints are not active, the optimal control input (acceleration/deceleration) as a function of time is given by

\[
u_i^e(t) = a_i t + b_i,
\]

and the optimal speed and position for each vehicle are

\[
u_i^0(t) = \frac{1}{2} a_i t^2 + b_i t + c_i
\]

\[
p_i^0(t) = \frac{1}{6} a_i t^3 + \frac{1}{2} b_i t^2 + c_i t + d_i,
\]

where \( a_i, b_i, c_i \) and \( d_i \) are constants of integration. These constants can be computed by using the initial and final conditions. Since we seek to derive the optimal control (15) in real time, we can designate initial values \( p_i(t_0^i) \) and \( v_i(t_0^i) \), and initial time, \( t_0^i \), to be the current values of the states \( p_i(t) \) and \( v_i(t) \) and time \( t \), where \( t_0^i < t \leq t_{m}^i \). Similar results to (15)-(16) can be obtained when the state and control constraints become active Malikopoulos et al. (2017).

### 4. EXPERIMENTAL RESULTS

To evaluate the effectiveness of the efficiency of the proposed approach, a total number of 10 MCAVs are set up in a merging scenario [video available in IDS (2017)]. Five MCAVs cruise on the main road in UDSSC, while the other five MCAVs cruise on the secondary road with the intention to merge into the main road (as shown in Fig. 7).

We considered the following two scenarios: 1) all MCAVs will be controlled by the decentralized control algorithm; and 2) all MCAVs will behave based on a simple lane following model, based on which the MCAVs cruising on the secondary road will have to yield to MCAVs of the main road to avoid potential lateral collision inside the merging zone (baseline scenario).
Vehicle Position Trajectory: The position trajectories of the MCAVs under the first scenario are illustrated in Fig. 8. The dashed line represents the reference trajectory for each vehicle commanded by the control algorithm, while a dense scatter plot represents the point measured along the actual trajectory achieved by each MCAV. To separate the MCAVs on two roads, the trajectories are flipped over Y-axis. Thus, in Fig. 8, the red dots stand for the trajectory points of the MCAVs of main road, and the blue dots stand for the trajectory points of the MCAVs of the secondary roads. The MCAVs are able to follow the optimal trajectory and manage to merge successfully without stop-and-go driving with only marginal errors. The position trajectories of the MCAVs cruising without the optimal control (baseline scenario) are shown in Fig. 9. Since the gaps between the mainline cars are not large enough for the merging cars to safely merge into the roadway, the merging cars need to stop until all the leading mainline vehicles traverse the merging zone, resulting in a queue built up on the merging roadway segment. For comparison, the merging maneuver for all the ten cars is completed in 16.5 sec with the optimal control algorithm, whereas it takes 20.3 sec for all the cars to pass the merging point (i.e., an 18.7% travel time savings is achieved with the proposed optimal control algorithm).

State-of-Charge of the Battery: To quantify the benefits of vehicle coordination, we compare the battery state of charge (SOC) of MCAVs. Under both scenarios, the MCAVs loop around the merging zone following a predefined trajectory as shown in Fig. 10. SOC is recorded for 4-minute run. The estimated battery efficiencies for MCAVs under the two scenarios are illustrated in Fig. 11. From the final SOC of each MCAV (Fig. 12), it is clear that coordination of MCAVs improves the efficiency of the battery in the merging scenario of MCAVs due to the elimination of the stop-and-go driving.
UDSSC is a small-scale “smart” city that can replicate real-world traffic scenarios in a small and controlled environment. This testbed can be an effective way to visualize the concepts developed in real world traffic scenarios using CAVs in a quick, safe and affordable way. The UDSSC helps bridge the gap between theory and practical implementation by providing a means of simultaneously testing as many as 35 MCAVs. We used UDSSC to validate experimentally a control framework reported in Rios-Torres and Malikopoulos (2017b) for coordination CAVs. The results demonstrate that coordination of CAVs can improve the battery efficiency due to elimination of the stop-and-go driving.

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Fig. 12. Final state of charge of the battery for each robotic car.