Expulsion of Magnetic Flux Lines from Growing Superconducting Core of a Quark Star and The Possibility of Mullins-Sekerka Interface Instability

Somenath Chakrabarty

† Department of Physics, University of Kalyani
West Bengal, India 741 235
E-Mail:somenath@klyuniv.ernet.in

and

Inter-University centre for Astronomy & Astrophysics
Post Bag 4, Ganeshkhind
Pune 411 007, India

Abstract

The expulsion of magnetic flux lines from a growing superconducting core of a quark star has been investigated using the idea of impurity diffusion in molten alloys. The possibility of Mullins-Sekerka normal-superconducting interface instability has also been studied.

PACS NO.:24.85+p, 97.60Jd, 74.20Hi, 64.70-p

† Permanent Address
If the matter density at the core of a neutron star exceeds a few times normal nuclear density (e.g. \( > 3n_0 \), where \( n_0 = 0.17\text{fm}^{-3} \), the normal nuclear density), a deconfining phase transition to quark matter may take place. As a consequence a normal neutron star will be converted to a hybrid star with an infinite cluster of quark matter core and a crust of neutron matter. If the speculation of Witten [1] that a flavour symmetric quark matter may be the absolute ground state at zero temperature and pressure is true, there is a possibility that the whole star will be converted to a star of strange quark matter (SQM) known as strange star. In normal quark matter the strange quarks are produced through flavour non-conserving weak processes which ultimately lead to a dynamical chemical equilibrium among various constituents.

From the observed features in the spectra of pulsating accreting neutron stars in binary system, the strength of surface magnetic field of a neutron star is found to be \( \sim 10^{12}\text{G} \). At the core region of a newly born neutron star it probably reaches \( \sim 10^{18}\text{G} \) [2]. In a recent publication [3] we have shown that if the magnetic field intensity exceeds some critical value which is the typical strength of magnetic field at which the cyclotron lines begin to occur or equivalently at which the cyclotron quantum is of the order of or greater than the rest mass of the particle considered or the de Broglie wave length is of the order of or greater than the Larmor radius of the particle, there can not be nucleation of a single quark matter bubble in the metastable neutron matter. The surface as well as the curvature energies diverge in this case. As a consequence the new stable (quark matter) phase can not be thermodynamically favourable over metastable (neutron matter) phase. Therefore to achieve a first order deconfining transition initiated by the nucleation of quark droplets at the core of neutron star, we assume that the strength of magnetic field throughout the star is much less than the corresponding critical value. In the case
of electron of mass 0.5MeV, this critical field is $\sim 4.4 \times 10^{13}$G, for light quarks of current mass 5MeV, it is $\sim 10^{15}$G, whereas for $s$-quark of current mass 150MeV, it is $\sim 10^{20}$G [4-6].

Now for a many body fermion system, the microscopic theory of superconductivity suggests [7] that if the interaction favours formation of pairs at low temperature, the system may undergo a phase transition to a superfluid state. This is expected to occur in the dense neutron matter present in neutron star [8,9]. On the other hand, if the particles carry charges, the paired state will be superconducting. In the case of an electronic system BCS theory applied to study the superconducting properties [7]. One electron of momentum $\vec{k}$ and spin $\vec{s}$ combines with another one of momentum $-\vec{k}$ and spin $-\vec{s}$ and form a Cooper pair. The coupling is mediated by the electron-phonon interaction in the lattice. In the case of SQM, the basic quark-quark interaction is attractive at large distance and consequently the BCS pairing mechanism is also applicable here. For a highly degenerate system, which is true in strange star, the pairing takes place near the fermi surface. The other condition that must be satisfied to form Cooper pair is that the temperature ($T$) of the system should be much less than the superconducting energy gap ($\Delta$), which is a function of the interaction strength and the density of the system. This is the most important criterion for the occurrence of superconducting transition. In the case of strange star, only quarks can form Cooper pairs. The electrons, whose density is extremely low, form highly degenerate relativistic plasma, are unlikely to form Cooper pairs. The kinetic energy of the electronic part dominates over its attractive potential energy, and as a result the corresponding superconducting transition temperature is extremely low and may not be achieved in a strange star. The relativistic theory of superfluidity and superconductivity for a fermion system was given by Bailin and Love [10]. Recently, Horvath et al [11] and also we have
studied [12] the superconductivity of quark matter using the results of ref. [10]. We have also studied qualitatively the magnetic properties of quark matter in ref. [12] for massive quarks.

Now the quarks of same fermi energy can only combine to form Cooper pairs. Since the $u$ and $d$ current masses are equal and also their chemical potentials are almost identical, whereas $s$ quark is much heavier than $u$ and $d$ quarks and also its chemical potential is different, we can have only $uu$, $dd$, $ud$ and $ss$ Cooper pairs in the system. For iso-spin 1/2 flavours, the contribution may come either from iso-scalar or iso-vector channels. It was shown in ref. [10] that the pairing of a $u − d$ system will be favoured by iso-scalar combination rather than iso-vector channel. On the other hand the $s − s$ combination is a triplet state with $J^P = 1^+$. Now from the conclusions of refs. [10-12] we know that if a normal SQM system undergoes a superconducting phase transition, the newly produced SQM phase will be a type I superconductor. We have also seen that the critical magnetic field for such pairing is $\sim 10^{16}G$ for $n \sim 2 - 3n_0$, which is indeed much larger than the typical pulsar magnetic field. The corresponding critical temperature is $10^9 - 10^{10}K$, this can possibly be achieved in quark star, which is expected to be an extremely cold object. Since the quark star magnetic field is less than the corresponding critical field for the destruction of superconductivity, during such phase transition, the magnetic flux lines from the superconducting quark sector of the strange star will be pushed out towards the normal crust region. Unlike a small type I superconducting laboratory sample placed in an external magnetic field ($j$critical value) for which the expulsion of magnetic field takes place almost instantaneously, in this particular case the scenario may be completely different.

The aim of the present note is to investigate using the idea of impurity diffusion in molten alloys, the expulsion of magnetic flux lines from growing super-
conducting core of a strange star. We have also studied the possibility of Mullins-Sekerka normal-superconducting interface instability [13] in quark matter. This is generally observed (i) in the case of solidification of pure molten metals at the solid-liquid interface, if there is a temperature gradient. The interface will always be stable if the temperature gradient is positive and unstable otherwise, (ii) during solidification of molten alloys. In alloys, the criteria for stable / unstable behaviour is more complicated. It is seen that, during the solidification of an alloy, there is a substantial change in the concentration ahead of the interface. Here solute diffusion as well as the heat flow effects must be considered simultaneously. As we will see, the particular problem we are going to investigate is analogous to solute diffusion during solidification of an alloy.

It has been assumed that the growth of superconducting quark bubble has started from the centre of the star and we use the nomenclature controlled growth for such phenomenon. If the magnetic field strength and the temperature of the star are much less than their critical values, the normal SQM phase is thermodynamically unstable relative to the corresponding superconducting one. Then due to fluctuation, a droplet of superconducting quark matter bubble may be produced in metastable medium. If the size of this superconducting bubble is greater than the corresponding critical value, it will act as the nucleating centre for the growth of superconducting quark core. The critical radius can be obtained by minimizing the free energy and is given by \( r_c = 16\pi\alpha/B_m^{(c)^2}[1 - (B_m/B_m^{(c)})^2] \), where \( \alpha \) is the surface tension, which is greater than zero for a type I superconductor-normal interface, \( B_m^{(c)} \) is the critical magnetic field. In presence of a magnetic field \( B_m < B_m^{(c)} \), the normal to superconducting transition is first order in nature. As the superconducting phase grows continuously, the magnetic field lines will be pushed out into the normal quark matter crust. This is the usual Meissner effect,
observed in type I superconductor. We compare this phenomenon of magnetic flux expulsion from a growing superconducting SQM core with the diffusion of impurities from a molten metal. The formation of superconducting zone is compared with the solidification of molten metal. It is known from simple thermodynamic calculations that if the free energy of molten phase decreases in presence of impurity atoms, then during solidification they prefer to recide in the molten phase. In this particular case the magnetic field lines play the role of impurity atoms, the normal quark matter phase plays the role of molten metal, whereas the superconducting phase can be compared with the frozen solid phase. (This idea was recently applied to baryon number transport during first order quark-hadron phase transition in the early Universe, where baryon number replaces impurity, quark phase replaces molten metal and hadronic matter replaces that of solid metal [14]). The magnetic flux lines prefer to recide in the normal phase. Then the Meissner effect can be restated as the solubility of magnetic flux lines in the superconducting phase is zero (of course, there is a finite penetration depth).

The dynamical equation for the flux expulsion can be obtained from the simplified model of sharp normal-superconducting interface. The expulsion equation is given by the well known diffusion equation [15]

$$\frac{\partial B_m}{\partial t} = D \nabla^2 B_m$$

(1)

where $B_m$ is the magnetic field intensity and $D$ is the diffusion coefficient, given by

$$D = \frac{c^2}{4\pi \sigma_n}$$

(2)

where $\sigma_n$ is the electrical conductivity of the normal SQM phase, for superconducting phase $B_m = 0$. Following ref. [16], the electrical conductivity of SQM for
\[ B_m = 0 \] is given by
\[ \sigma_n = 5.8 \times 10^{25} \left( \frac{\alpha_c}{0.1} \right)^{-3/2} T_{10}^{-2} \left( \frac{n}{n_0} \right) \] (3)
in sec\(^{-1}\), where \( \alpha_c \) is the strong coupling constant and \( T_{10} = T/10^{10} \)K, the value of this electrical conductivity in the case of strange quark matter is \( \sim 10^{26} \) sec\(^{-1}\).

We have used this expression to get an order of magnitude estimate of electrical conductivity of quark matter. In actual calculation one has to evaluate \( \sigma_n \) in presence of \( B_m \). In that case, \( \sigma_n \) can not be a scalar quantity. In particular, for extremely large \( B_m \), the components orthogonal to \( B_m \) tend to zero. The quarks can only move along the direction of magnetic field, across the field resistivity becomes infinity.

A solution of eqn.(1) with spherical symmetry (which may not be a valid assumption) can be obtained by Green’s function technique, and is given by (for a very general topological structure, no analytical solution is possible)
\[ B_m(r, t) = \frac{1}{2r(\pi Dt)^{1/2}} \int_0^\infty B_m^{(0)}(r') \left[ e^{-u_+^2} - e^{-u_-^2} \right] r' dr' \] (4)
where \( u_\pm = (r \pm r')/2(Dt)^{1/2} \) and \( B_m^{(0)}(r) \) is the magnetic field distribution within the star at \( t = 0 \), which is of course an entirely unknown function of radial coordinate \( r \). To obtain an idea of magnetic field diffusion time scale (\( \tau_D \)), we assume \( B_m^{(0)}(r) = B_m^{(0)} \) = constant (in reality, this is not possible inside the star). Then using the other approximate result for electrical conductivity (which is valid for zero magnetic field case), given by eqn.(3), we have \( \tau_D = 10 - 20 \) yrs. With this constant \( B_m^{(0)}(r) \), eqn.(4)
\[ B_m(r, t) = B_m^{(0)} \left[ \frac{1}{2} \{ \text{erf}(u_+) + \text{erf}(u_-) \} + \frac{1}{r} \left( \frac{Dt}{\pi} \right)^{1/2} \left\{ e^{-u_+^2} - e^{-u_-^2} \right\} \right] \] (5)
where
\[ \text{erf}(x) = \frac{2}{(\pi)^{1/2}} \int_0^x e^{-z^2} dz \]
is the well known error function.

From the eqn.(5), using the approximate results for electrical conductivity, given by eqn.(3) (which is valid for $B_m = 0$) one can get an estimate of time scale for the expulsion of magnetic lines of force and is $\sim 10 - 20$ yrs. Latter we shall see that such a time scale can also be obtained from stability analysis of planer normal-superconducting interface. Again the time dependence of the electrical conductivity profile for normal quark matter is not known. This is another uncertainty in obtaining exact solution for the diffusion equation.

Therefore, almost nothing can be said about the growth of superconducting zone and the expulsion of magnetic flux lines from this region by solving eqn.(1). We shall now study the morphological instability of normal-superconducting interface of quark matter in the star using the idea of solute diffusion during solidification of alloys. The motion of normal-superconducting interface is extremely important in this case and has to be taken into consideration. Then instead of eqn.(1) which is valid in the rest frame, an equation expressed in a coordinate system which is moving with an element of the boundary layer is the correct description of such superconducting growth, known as Directional Growth, and the equation is called Directional Growth Equation, and is given by

$$\frac{\partial B_m}{\partial t} - v \frac{\partial B_m}{\partial z} = D \nabla^2 B_m$$

(6)

where the motion of the interface is along the $z$-axis and $v$ is the velocity of the front. This diffusion equation must be supplemented by the boundary conditions at the interface. The first boundary condition is obtained by combining Ampere’s and Faraday’s laws at the interface, and is given by

$$B_m v \big|_s = -D(\nabla B_m) \cdot \hat{n} \big|_s$$

(7)

where $\hat{n}$ is the unit vector normal to the interface directed from the normal phase
to the superconducting phase. This is actually the continuity equation for magnetic flux diffusion. The rate at which excess magnetic field lines are rejected from the interior of the phase is balanced by the rate at which magnetic flux lines diffuses ahead of the two-phase interface. This effect makes the boundary layer between superconducting-normal quark matter phases unstable due to excess magnetic field lines present on the surface of the growing superconducting bubble. Local thermodynamic equilibrium at the interface gives (Gibbs-Thompson condition)

\[ B_{m} \approx B_{m}^{(c)} \left( 1 - \frac{4\pi \alpha}{RB_{m}^{(c)^{2}}} \right) = B_{m}^{(c)} (1 - \delta c) \]  

where \( \delta \) is called capillary length with \( \alpha \) the surface tension, \( c \) is the curvature \( = 1/R \) (for a spherical surface), and \( B_{m}^{(c)} \) is the thermodynamic critical field.

To investigate the stability of superconducting-normal interface, we shall follow the original work by Mullins and Sekerka [13], and consider a steady state growth of superconducting core, then the time derivative in eqn.(6) will not appear. Introducing \( r_{\perp} = (x^{2} + y^{2})^{1/2} \) as the transverse coordinate at the interface, we have after rearranging eqn.(6)

\[ \left[ \frac{\partial^{2}}{\partial r_{\perp}^{2}} + \frac{1}{r_{\perp}} \frac{\partial}{\partial r_{\perp}} + \frac{\partial^{2}}{\partial z^{2}} + \frac{v}{D} \frac{\partial}{\partial z} \right] B_{m} = 0 \]  

Following the most common approximation which is made in the case of freezing of molten solid is that the solidification is occurring under steady state condition, which in this particular case is the normal to superconducting phase transition, and that, therefore, the concentration of magnetic flux lines and normal-superconducting interface morphology are independent of time. The principal disadvantage of this assumption is that no evolution of the interface shape can occur. The result of this constraint is that the solution to the basic diffusion problem is indeterminate and a whole range of morphologies is permissible from
the mathematical point of view. In order to distinguish the solution which is the most likely to correspond to reality, it is necessary to find some additional criteria. Examination of the stabilities of a slightly perturbed growth form is probably the most reasonable manner in which to treat this situation. In the following we shall investigate the morphological instability of normal-superconducting interface following eqn. (9). Assuming a solution of this equation expressed as the product of separate functions of $r_{\perp}$ and $z$ and setting the separation constant equal to zero and using the boundary condition given by eqn. (8), we have for an unperturbed boundary layer moving along $z$-axis

$$B_m = B_m^{(s)} e^{-zv/D} = B_m^{(s)} e^{-2z/l} \quad (10)$$

where $l = 2D/v$ is the layer thickness, which is very small for small $D$. Mathematically, the thickness of this layer is infinity. For practical purpose an effective value $l$ can be taken. The order of magnitude estimates or limiting values for the three quantities $D$, $v$ and $l$ can be obtained from the stability condition of planer interface, which will be discussed latter.

Due to excess magnetic flux lines at the interface, the form of the planer normal-superconducting interface described by the equation $z = 0$ is assumed to be changed by a small perturbation represented by the simple sine function

$$z = \epsilon \sin(\vec{k} \cdot \vec{r}_{\perp}) \quad (11)$$

where $\epsilon$ is very small amplitude and $\vec{k}$ is the wave vector of the perturbation. Then the perturbed solution of the magnetic field distribution near the interface can be written as

$$B_m = B_m^{(s)} e^{-vz/D} + A\epsilon \sin(\vec{k} \cdot \vec{r}_{\perp})e^{-bz} \quad (12)$$

where $A$ and $b$ are two unknown constants. Since the solution should satisfy the
To evaluate $A$, we utilise the assumption that $\epsilon$ and $\epsilon \sin(\vec{k}.\vec{r}_\perp)$ are small enough so that we can keep only the linear terms in the expansion of exponentials present in eqn.(12). Then after some straightforward algebraic manipulation, we have

$$A = -\frac{v}{D} B_m^{(s)}$$

(14)

The expression describing the magnetic field distribution ahead of the slightly perturbed interface then reduces to

$$B_m = B_m^{(s)} \left[ e^{-vz/D} - \frac{v}{D} \epsilon \sin(\vec{k}.\vec{r}_\perp) e^{-bz} \right]$$

(15)

Now from the other boundary condition (eqn.(8)) we have

$$B_m^{(s)} = B_m^{(c)} - \frac{4\pi \alpha B_m^{(c)}}{B_m^{(s)} c}$$

(16)

where $c = z''/(1 + z'^2)^{3/2}$ is the curvature at $z = \epsilon \sin(\vec{k}.\vec{r}_\perp)$ and prime indicates derivative with respect to $r_\perp$.

Neglecting $z'^2$, which is small for small perturbation, we have

$$B_m^{(s)} = B_m^{(c)} + \Gamma k^2 S$$

(17)

where $\Gamma = 4\pi \alpha B_m^{(c)} / B_m^{(s)}$ and we have replaced $\epsilon \sin(\vec{k}.\vec{r}_\perp)$ by $S$. Since the amplitude of perturbation $\epsilon$ is extremely small, the quantity $S$ is also negligibly small.

Now the eqn.(17) is also given by

$$B_m^{(s)} = B_m^{(c)} + GS$$

(18)

where

$$G = \frac{dB_m}{dz} \bigg|_{z=S} = -\frac{v}{D} \left( 1 - \frac{vS}{D} \right) B_m^{(s)} - bAS(1 - bS)$$

(19)
Combining these two eqns., we have

\[ k^2 \Gamma + \frac{v}{D} \left(1 - \frac{vS}{D}\right) B^{(s)} - \frac{bv}{D} B^{(s)} S (1 - bS) = 0 \]  \hspace{1cm} (20)

This expression determines the form (values of \( k \)) which the perturbed interface must assume in order to satisfy all of the conditions of the problem. To analyse the behaviour of the roots we replace right hand side of eqn.(20) by some parameter \(-P\). (We have taken \(-P\) in order to draw a close analogy with the method given in ref.[13]). Rearranging eqn.(20), we have

\[ -k^2 \Gamma - \frac{v}{D} \left(1 - \frac{vS}{D}\right) B^{(s)} + \frac{bvB^{(s)} S}{D} (1 - bS) = P \]  \hspace{1cm} (21)

(in ref. [13] the parameter \( P \) is related to the time derivative \( \epsilon \) of the amplitude of small perturbation). If the parameter \( P \) is positive for any value of \( k \), the distortion of the interface will increase, whereas, if it is negative for all values of \( k \), the perturbation will disappear and the interface will be stable. In order to derive a stability criterion, one only needs to know whether eqn.(21) has roots for positive values of \( k \). If it has no roots, then the interface is stable because the \( P - k \) curve never rises above the positive \( k \)-axis and \( P \) is therefore negative for all wavelengths. We have used Decarte’s theorem to check how many positive roots are there. It is more convenient to express \( k \) in terms of \( b \) and then replacing \( b \) by \( \omega + v/D \), which gives

\[ -\omega^2 \left( \Gamma + \frac{vB^{(s)} S^2}{D} \right) - \omega \left( \Gamma + \frac{2vB^{(s)} S^2}{D} - B^{(s)} S \right) \frac{v}{D} - \frac{v}{D} B^{(s)} \left(1 - \frac{v}{D} S\right)^2 = P \]  \hspace{1cm} (22)

This is a quadratic equation for \( \omega \). The first and the third terms are always negative. The second term will also be negative if

\[ \Gamma + \frac{2vB^{(s)} S^2}{D} - B^{(s)} S > 0 \]  \hspace{1cm} (23)
Then it follows from Decart’s rule that if the condition (23) is satisfied, there can not be any positive root. Which implies that the small perturbation of the interface will disappear. Since the amplitude of perturbation is assumed to be extremely small, the quantity $S = \epsilon \sin(\vec{k}.\vec{r}_\perp)$ is also negligibly small. Under such circumstances the middle term of eqn.(23) is much smaller than rest of the terms. The Decart’s rule given by the condition (21) can be re-written as

$$\Gamma > B_m^{(s)} S$$

(24)

Which after some simplification gives the stability criterion for the plane unperturbed interface, given by

$$\alpha > \frac{B_m^{(s)} S}{4\pi B_m^{(c)}}$$

(25)

From the stability criterion, it follows that the normal-superconducting interface energy/area of quark matter has a lower bound, which depends on the interface magnetic field strength, critical field strength and also on the perturbation term $S$. An order of magnitude of this lower limit can be obtained by assuming $B_m^{(s)} = 10^{-3} B_m^{(c)}$. (Since the critical field $B_m^{(c)} \sim 10^{16} \text{G}$, and the neutron star magnetic field strength $B_m \sim 10^{13} \text{G}$, we may use this equality). Then the lower limit is given by

$$\alpha_L \approx 10^{-9} \text{ MeV/fm}^2 \left(\frac{S}{\text{fm}}\right)$$

(26a)

On the other hand for $B_m^{(s)} = 0.1 B_m^{(c)}$, we have

$$\alpha_L \approx 10^{-3} \text{ MeV/fm}^2 \left(\frac{S}{\text{fm}}\right)$$

(26b)

The approximate general expression for the lower limit is given by

$$\alpha_L \approx \hbar^3 \text{ MeV/fm}^2 \left(\frac{S}{\text{fm}}\right)$$

(26c)
where $h = B_m^{(s)}/B_m^{(c)}$. There for the maximum value of this lower limit is

$$\alpha_{L, \text{max}} \approx 1 \text{ MeV/fm}^2 \left( \frac{S}{\text{fm}} \right)$$  \hspace{1cm} (26d)

when the two phase are in thermodynamic equilibrium. Of course for such a strong magnetic field, as we have seen (see ref.[3]) that there can not be first order quark-hadron phase transition, and it should be some higher order one.

On the other hand if we do not have control on the interface energy, which can in principle be obtained from Landau-Ginzberg model, we can re-write the stability criteria in terms of interface concentration of magnetic field strength $B_m^{(s)}$, and is given by

$$B_m^{(s)} < \left[ \frac{4\pi \alpha B_m^{(c)}}{S \left( 1 - \frac{2v}{D} S \right)} \right]^{1/3}$$ \hspace{1cm} (27)

This is more realistic than the condition imposed on the surface tension $\alpha$. Now for a type I superconductor, the surface tension $\alpha > 0$, which implies $1 - 2vS/D > 0$. Therefore, we have $2vS/D < 1$, and for the typical value $\sigma_n \sim 10^{26} \text{ sec}^{-1}$ for the electrical conductivity of normal quark matter with zero magnetic field, the profile velocity $v < D/2S \sim 10^{-6} / S \text{ cm/sec} \sim 1 \text{ cm/sec}$ for $S \sim 10^{-6} \text{ cm}$. Therefore the interface velocity $< 1 \text{ cm/sec}$ for such typical values of $\sigma_n$ and $S$ to make the planer interface stable under small perturbation. Now the thickness of the layer at the interface is $l = 2D/v > 10^{-6}$ for such values of $D$ (or $\sigma_n$) and $v$. Here $S$ is always greater than 0, otherwise, the magnetic field strength at the normal-superconductor interface becomes unphysical. As before, if the second term of eqn.(23) is negligibly small compared to other two terms, we have

$$B_m^{(s)} < \left[ \frac{4\pi \alpha B_m^{(c)}}{S} \right]^{1/3}$$ \hspace{1cm} (28)

Therefore we may conclude by saying that if a superconducting transition can take place in a quark star, the magnetic properties of such bulk object are entirely
different from that of a small laboratory superconducting sample. Expulsion of magnetic flux lines from the superconducting zone is not instantaneous. The typical time scale is $10^{-15}$ yrs. Due to the presence of excess magnetic flux lines at the interface, which is possibly true if the diffusion rate of magnetic lines of forces in the normal phase is less than the rate of growth of the superconducting zone, the characteristic of normal-superconducting boundary layer may change significantly. Of course, it depends on the magnitude of surface tension $\alpha$. It may take dendritic shape instead of planer structure. The stability of planer interface also depends on the strength of interface magnetic field, if we do not have control on the interface energy and are given by eqns. (27) and (28). How to get an experimental evidence for such an unusual shape is a matter of further study.
References

1. E. Witten, Phys. Rev. D30, 272 (1984).

2. J. Trumper et al, Ap. J. 219, L105 (1978); W. A. Wheaton et al, Nature 272, 240 (1979); D. E. Gruber et al, Ap. J. 240, L127 (1980); T. Mihara et al, Nature 346, 250 (1990).

3. S. Chakrabarty, Phys. Rev. D51, 4591 (1995).

4. S. Chakrabarty, Astrphys. and Space Sci., 213, 121 (1994).

5. S. Chakrabarty and A. Goyal, Mod. Phys. Lett. A9, 3611 (1994).

6. S. Chakrabarty and P. K. Sahu, Phys. Rev. D (in press 1996); S. Chakrabarty, Phys. Rev. D (submitted).

7. A.L. Fetter and J. D. Walecha, Quantum Theory of Many Particle System, McGraw Hill Book Company, New York, 1971.

8. G. Baym, C. Pethick and D. Pines, Nature 224, 224 (1969).

9. M. Baldo et al, Nucl. Phys. A536, 349 (1992).

10. D. Bailin and A. Love, Phys. Rep. 107, 325 (1984).

11. J. E. Horvath et al, Mod. Phys. Lett. A7, 995 (1992).

12. S. Chakrabarty, Can. J. Phys. 71, 488 (1993).

13. W.W. Mullins and R. F. Sekerka, Jour. Appl. Phys. 34, 323 (1963); 35, 444 (1964).

14. F. C. Adams, K. Freese and J. S. Langer, Phys. Rev. D47, 4303 (1993); Marc Kamionkowski and K. Freese, Phys. Rev. Lett. 69, 2743 (1992).

15. J. S. Langer, Rev. Mod. Phys. 52, 1 (1980).

16. P. Haensel and A. J. Jerzak, Acta Phys. Pol. B20, 141 (1989); P. Haensel, Nucl. Phys. B24 (proc. of the Int. Workshop on Strange quark Matter in Physics and Astrophysics, Univ. of Aarhus, Denmark, May 20-24, 1991), 23 (1991).