An optimization model of variable size
two-dimensional nesting with stacked sheets and
usable leftovers for the CNC routing process

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Abstract. The research is motivated by a problem in the aviation industry, where nesting is
performed to allocate and arrange parts in metal sheets for a CNC router machine. In addition to
the usual objective to minimize material waste, the nesting problem has a few characteristics,
i.e., variable sheet size, stacked sheet routing, and usable leftovers. The three aspects are
integrated into an optimization model developed from the fundamental two-dimensional bin
packing problem. A simple numerical example is given to show the capability of the model and
the characteristics of the results

1. Introduction

CNC router machine is used to cut parts from metal sheets. Many elements, possibly of different
part types that use the same material type and thickness, can be grouped and cut from the same
metal sheet. Nesting is the process of allocating parts to sheets and arranging the details such
that the elements do not overlap with each other and the waste material is as small as possible.

The object of this research is the routing process in an airplane manufacturer industry. In
this shop, the sheets are variable in size. Therefore the width and length of the metal sheets
may differ from each other. This particular router machine can cut several stacked sheets at
a time, provided that the sheets have the same cutting pattern. The stacked routing can save
setup time and cutting time because the stacked sheets are processed in a batch, and all sheets
in the stack are processed at the same time.

The layout of parts in a sheet constitutes a pattern. The allocation and the placement of
components determine how much material is left after the process. The remaining material may
need to be scrapped. Therefore the pattern must be designed in such a way as to minimize it.
However, in the real system that we consider in this research, it can be used again for the next
cutting if the leftover is still large enough. To be usable, the length of the fragment must be
more than a certain minimum width.

The nesting problem, and the more general bin-packing problem, is a popular topic of research
with many branches. The optimization of nesting in a two-dimensional object, such as the CNC
router case, is analogous to the 2D bin packing problem (2DBPP). The basic model [1] used
a single bin size, but bin-packing with variable bin size is also considered (e.g., in [2, 3]). The
router machine can cut sheets in a non-guillotine manner, in contrast to the guillotine cutting
(e.g., in [2]) used in other types of processes.
The consideration of leftover is also important because the materials remaining after the routing process may still have some value. It can be used for the next cutting process of smaller parts or even used for other functions. Andrade et al. [5] proposed a bi-level mathematical programming model for the residual bin-packing problem that is further reformulated as a single-level mixed-integer programming model. Viegas et al. [8] developed heuristics for a three-dimensional steel cutting with usable leftovers. Relatively recent research [9] developed a diving heuristics for a 2D guillotine cutting-stock problem with portions.

In many research works on bin-packing, the stacked sheet aspect has not been investigated to the best of our knowledge. However, we believe that the explicit consideration of stacked sheets in the nesting optimization of CNC router is beneficial, especially if the nesting is integrated with batching and scheduling since stacking in the nesting (determining patterns for stacked sheets) would affect the processing time of parts. Further, it may affect the flow time of cuts in a shop and also the completion time. If CNC involves a large data scale, it needs to be reduced first. One of the dimensional reduction methods is PCA. Researchers [6, 7] have reviewed four-dimensional reduction methods for large-scale data.

In this paper, we propose an optimization model formulation for the case of two-dimensional nesting with consideration of stacked sheet routing and usable leftover. Section 2 explains the model developed in detail from the basic model of 2DBPP. In Section 3, we provide a numerical example and a discussion of the result. Finally, we conclude with remarks and recommendations in Section 4.

2. Model Development

2.1. The basic model

The basic model is the two-dimensional bin packing Integer Linear Programming model. In the context of the CNC routing process, the problem can be stated as follows. Several parts of different widths and lengths are to be nested into some sheets. All sheets have the same width and length. The term ‘width’ is to the direction of the X-axis, and ‘length’ is to the order of the machine coordinate system’s Y-axis. Each part is assumed to be a rectangle with a certain width and length, and rotation is not allowed. It is desired to minimize the number of used sheets. The following researchers have developed CNC algorithms for scientific advancement [10–19]. The notations used are as follows.

Sets
\( K \) : a collection of sheets
\( I \) : a set of parts

Parameters
\( W, L \) : the width and length of a sheet
\( w_i, l_i \) : the width and length of part \( i \)

Variables
\( x_i, y_i \) : the origin of part \( i \) in a horizontal and vertical direction, respectively
\( u_{ij} \) : binary variable, equals 1 iff part \( i \) is on the left side of part \( j \), equals 0 otherwise
\( v_{ij} \) : binary variable, equals 1 iff part \( i \) is on the lower side of part \( j \), equals 0 otherwise
\( \delta_k \) : binary variable, equals 1 iff sheet \( k \) is used, equals 0 otherwise
\( a_{ik} \) : binary variable, equals 1 iff part \( i \) is assigned to sheet \( k \), equals 0 otherwise

We will restate the basic ILP model here as follows. Minimize

\[
F = \sum_{k \in K} \delta_k
\]

subject to:

\[
\sum_{k \in K} a_{ik} = 1 \quad \forall i \in I
\]
\[ a_{ik} \leq \delta_k \quad \forall i \in I, k \in K \quad (3) \]

\[ 0 \leq x_i \leq W - w_i, 0 \leq y_i \leq L - l_i \quad \forall i \in I \quad (4) \]

\[ x_i + w_i \leq x_j + W(1 - u_{ij}) \quad \forall i, j \in I \quad (5a) \]

\[ y_i + h_i \leq y_j + L(1 - v_{ij}) \quad \forall i, j \in I \quad (5b) \]

\[ u_{ij} + u_{ji} + v_{ij} + v_{ji} \geq a_{ik} + a_{jk} - 1 \quad \forall i, j \in I, i < j, k \in K \quad (6) \]

\[ u_{ij}, v_{ij} \in \{0, 1\} \quad \forall i, j \in I \quad (7a) \]

\[ a_{ik}, \delta_k \in \{0, 1\} \quad \forall i \in I, k \in K \quad (7b) \]

Equation (1) is the objective function to be minimized. Equation (2) states that each part can be assigned to only one sheet. Constraint (3) couples the two binary variables. A sheet is used if at least one part is assigned to it. Constraint (4) limits the origin point of details on the sheet. Regulations (5a) and (5b), along with condition (6), are to ensure that parts assigned to the same sheet do not overlap with each other. Constraints (7a) and (7b) define the domain for the binary variables.

### 2.2. The modeling of variable sheet size, stacked sheets, and usable leftovers

To model the stacked sheet routing, we now separate the definition of part and part item. Part \( p \) is the entity that is ordered to be produced in a required quantity \( R_p \). While part item \( i \) is the rectangle-sized part pattern to be used for nesting purposes, with stacked sheet routing, each part item in the model is no longer required to be assigned (cf. Constraint (2)). The possibility that the part’s production requirement may be satisfied by stacking the sheets containing a part item and cutting would be performed simultaneously. We use \( p(i) \) to denote the part \( p \) of part item \( i \).

In the same manner, because now sheets may have variable sizes, we separate the definition of sheet and sheet item. Sheet \( s \) is the entity provided in a limited number \( C_s \) for routing with a certain width \( W_s \) and length \( L_s \), while sheet item \( k \) is the sheet entity used for nesting, i.e., laying out part items. A few sheet items can be nested with the same pattern of part items and thus can be stacked for routing. The number of sheets stacked is denoted as \( m_k \). We use \( s(k) \) to denote the sheet \( s \) of sheet item \( k \).

To be able to utilize leftover, the remaining material’s width \( r_k \) must be greater than the required minimum width \( r_{MIN} \). Otherwise, the remaining material needs to be scrapped and cannot be used. Parameter \( SV \) is the percentage of the original material value that can be salvaged. The value is typically less than 1.

The benefit of stacked sheet routing comes in less processing time, consisting of cutting time and the related setups because sheets are now batched together in a stack. To capture that, another cost term, i.e., processing cost, is introduced to the objective function. In this model, the processing cost is assumed proportional to the number of batches.

### 2.3. The optimization model

All the considerations described in Section 2.2 result in the following optimization model.

Minimize

\[ F = (CM \cdot \sum_{k \in K} m_k \cdot L_{s(k)} \cdot (W_{s(k)} - SV \cdot \rho_k)) + (CP \cdot \sum_{k \in K} \delta_k) \quad (8) \]

subject to:

\[ \sum_{k \in K} a_{ik} = z_i \quad \forall i \in I \quad (9) \]
\[ z_i \leq 1 \quad \forall i \in I \]  
\[ a_{ik} \leq \delta_k \quad \forall i \in I, k \in K \]  
\[ 0 \leq x_i \leq \sum_k a_{ik} \cdot (W_s(k) - r_k) - z_i \cdot w_p(i) \quad \forall i \in I \]  
\[ 0 \leq y_i \leq \sum_k a_{ik} \cdot (L_s(k) - z_i \cdot l_p(i)) \quad \forall i \in I \]  
\[ x_i + w_p(i) \leq x_j + M(1 - u_{ij}) \quad \forall i, j \in I, i \neq j \]  
\[ y_i + h_p(i) \leq y_j + M(1 - v_{ij}) \quad \forall i, j \in I, i \neq j \]  
\[ u_{ij} + u_{ji} + v_{ij} + v_{ji} \geq a_{ik} + a_{jk} - 1 \quad \forall i, j \in I, i < j, k \in K \]  
\[ \rho_k = \begin{cases} r_k, r_k \geq r_{\min} \\ 0, \text{otherwise} \end{cases} \quad \forall k \in K \]  
\[ 0 \leq m_k \leq m_{\max} \quad \forall k \in K \]  
\[ \sum_{i \in I_p} \sum_k a_{ik} m_k = R_p \quad \forall p \in P \]  
\[ u_{ij}, v_{ij} \in \{0, 1\} \quad \forall i, j \in I, i \neq j \]  
\[ a_{ik}, \delta_k \in \{0, 1\} \quad \forall i \in I, k \in K \]  
\[ z_i \in \{0, 1\} \quad \forall i \in I \]  
\[ m_k \in \mathbb{Z}^+ \quad \forall k \in K \]  

The objective function in Equation (8) consists of material cost and processing cost. The material cost is the cost of the actual area of material used, subtracted by any usable leftovers. The processing cost is the cost for routing a process batch. The parameters $CM$ and $CP$ are material cost factor and processing cost factor, respectively. Constraint (9) and (10) replace regulation (2), noting that a part item may be unassigned. The consideration of leftovers in the width direction is included in Constraint (12a). The parameter $M$ in Constraints (13) is the Big M. Equation (15) calculates the usable leftover $\rho_k$. Constraint (16) bounds the number of stacked sheet items. Rule (17) is to ensure all the required number of parts are satisfied. $I_p$ is the set of all part items that belong to part $p$.

### 3. A numerical example and discussion

For model verification and to illustrate the characteristics of the model, we use it to solve the following small hypothetical example.

A CNC router machine receives jobs for three types of parts. Table 1 outlines the width, length, and required quantity for each part. The shop has two types of sheet, sheet 1 is 6 in width and 10 in length, while sheet 2 is 7 in width and 5 in length. There are many sheet items in each type that can be used for routing. Other parameters are as follows: processing cost factor $CP = 50$, material cost factor per area $CM = 1$, salvage value $SV = 0.8$, minimum usable width $r_{\min} = 2$, maximum number of stack $m_{\max} = 3$. The model is implemented and executed in Lingo software. The solver can find the optimal global solution in slightly under one minute in our development computer. The objective function is 472 that can be shown in Figure 1. The figure shows the nesting patterns in three sheet items. The first and second patterns
Table 1: Part data for the numerical example

| Part | Width | Length | Required Quantity |
|------|-------|--------|-------------------|
| 1    | 3     | 3      | 12                |
| 2    | 4     | 9      | 3                 |
| 3    | 7     | 2      | 2                 |

use sheet type 1 (6 × 10), while the third uses sheet type 2 (7 × 5). Each will be processed in a stack. The first pattern will be used by two sheets in the stack, the second pattern by three sheets, and the third pattern. It can be verified that the number of parts produced by this arrangement is the same as the required quantity of each part, as shown in Table 1. Numbers inside the rectangles indicate the index of the part and the part item. For example, ”1.6” means item 6 of part 1.

In this result, the usable leftover is shown in gray color in the second pattern. The size is 2 × 10. The first pattern also has quite large remaining items; however, because the model assumes that the leftover is in the width direction, it is not considered usable. The model’s weakness should be addressed later without adding too much complexity to the model.

![Figure 1: Nesting result of the numerical example](image)

4. Concluding remarks
This paper has outlined the initial development of an optimization model of two-dimensional nesting for the CNC routing process. The model development is described in detail from the basic 2DBPP model, and then three aspects are added to the basic model: variable sheet size, stacked sheet routing, and usable leftovers.

From this initial development, we can obtain the essential characteristics of the model. For further work, we plan to continue improving and streamlining the model. We also plan to develop heuristics to address the long calculation time, especially for larger practical problems. Another natural direction is to integrate the nesting model with batch scheduling models. The research results should be implemented to solve the real issues faced in the industry.

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