Cyclic Family Symmetry and Lepton Hierarchy in Supersymmetry

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Abstract

A cyclic symmetry among the left-handed doublets of the three families is proposed. This symmetry can naturally result in a realistic hierarchical pattern of the fermion masses within the framework of supersymmetry with nonvanishing sneutrino vacuum expectation values.
Within the framework of R-parity violating supersymmetry [1], in a previous paper [2], the authors thought that, in addition to the Yukawa interactions, the trilinear lepton number violating interactions also contribute masses to the leptons if the sneutrinos have nonvanishing vacuum expectation values (vevs). This might be helpful to understand the hierarchical pattern of the fermion masses. However, the form of the trilinear interactions was too arbitrary to give some definite predictions. In this paper, we will reconsider this problem from the point of view of symmetry among the fermion families. And we will also discuss some phenomenological aspects of this idea on neutrino physics.

It will be interesting if there is some symmetry among the three families. Such symmetry can break spontaneously after the three sneutrinos get different vevs [2], hence no exact symmetry appears explicitly in the fermion spectra. From the general fact that the third family of fermions is much heavier than the other two families, the fermion mass matrix is usually assumed to be the form of the democratic family mixing [3] which exhibits an $S_{3L} \times S_{3R}$ symmetry in some basis of eigenstates of gauge interactions. Actually one of the $S_3$ groups, $S_{3L}$ or $S_{3R}$, is enough to reflect this fact. In the case of the $S_{3L}$ symmetry, the fermion mass matrix has the form:

\[ M = \begin{pmatrix} a & b & c \\ a & b & c \\ a & b & c \end{pmatrix} \]  

Furthermore, the above matrix can be obtained under the cyclic symmetry which is just a subgroup of $S_{3L}$. This order 3 cyclic group is also called $Z_3$. In the following, we will assume that there is a cyclic symmetry among the left-handed doublets of the three families, which is denoted as $Z_{3L}$. Of course, this $Z_{3L}$ symmetry has to be slightly violated because the mass matrix (1) is still of rank-one. This violation can be achieved through choosing appropriate scalar potential to make the sneutrinos have different vevs.
In this paper we still exploit the low energy supersymmetric model of Ref. [2], and focus on the lepton sector. In stead of the R-parity, the baryon number conservation is adopted. The trilinear R-parity violating interactions are introduced. The supersymmetric gauge interactions can be found in text books. With the left-handed chiral lepton superfields and their SU(2) × U(1) quantum number \( L_i(2, -1) \) and \( E^c_i(1, 2) \), the Higgs superfields \( H_u(2, 1), H_d(2, -1) \) and \( X(1, 0) \), where \( i \) denotes the family index, the superpotential of our model which is invariant under the cyclic family symmetry \( Z_{3L} \) is

\[
W = g_Y \left( \sum_i L_i^a H_d^b E^c_j \epsilon_{ab} + \lambda_j (L_1^a L_2^b + L_2^a L_3^b + L_3^a L_1^b) E^c_j \epsilon_{ab} + \lambda' X (H_u^a H_d^b \epsilon_{ab} - \mu^2) \right),
\]

with \( a \) and \( b \) being the SU(2) indices. In addition, the soft supersymmetric breaking terms which include gaugino and scalar mass terms and trilinear scalar interactions should be added in the Lagrangian. It can be seen that the Yukawa couplings only depend on the flavor of the SU(2) singlet leptons. Such Yukawa couplings will produce the mass matrix (1) after the Higgs fields getting nonvanishing vevs.

This \( Z_{3L} \) symmetry breaks when the sneutrino fields obtain different vevs. It has been shown straightforwardly that this is indeed the case for the scalar potential with appropriate parameters [2]. All these vevs can be taken real for our present purpose. As for the fermion spectra, the physical neutrinos are massless at tree level. The mass matrix of the charged leptons from the superpotential (2) involves the contributions of both the Yukawa and the R-parity violating interactions:

\[
M = \begin{pmatrix}
g_Y v_d + \lambda_1 (v_2 - v_3) & g_Y v_d + \lambda_2 (v_2 - v_3) & g_Y v_d + \lambda_3 (v_2 - v_3) \\
g_Y v_d + \lambda_1 (v_3 - v_1) & g_Y v_d + \lambda_2 (v_3 - v_1) & g_Y v_d + \lambda_3 (v_3 - v_1) \\
g_Y v_d + \lambda_1 (v_1 - v_2) & g_Y v_d + \lambda_2 (v_1 - v_2) & g_Y v_d + \lambda_3 (v_1 - v_2)
\end{pmatrix},
\]

where \( v_d \) and \( v_i \) are the vevs of the Higgs field and the sneutrino fields, respectively. As we have expected, it is the differences of the vevs of the sneutrino fields that violate the \( Z_{3L} \) symmetry. These differences should be much smaller than the vevs of the Higgs fields, this is also required by the lepton universality. From the mass matrix (3), we
obtain the following masses for the charged leptons:

\[ m_\tau \simeq \sqrt{3}(g_{Y_1}^2 + g_{Y_2}^2 + g_{Y_3}^2)^{1/2} v_d , \]
\[ m_\mu \simeq a[(v_1 - v_2)^2 + (v_2 - v_3)^2 + (v_3 - v_1)^2]^{1/2} , \]  \hspace{1cm} (4)
\[ m_e = 0 , \]

where

\[ a = \left[ \frac{(g_{Y_1} \lambda_2 - g_{Y_2} \lambda_1)^2 + (g_{Y_3} \lambda_3 - g_{Y_2} \lambda_2)^2 + (g_{Y_3} \lambda_1 - g_{Y_1} \lambda_3)^2}{g_{Y_1}^2 + g_{Y_2}^2 + g_{Y_3}^2} \right]^{1/2} . \]  \hspace{1cm} (5)

The tau lepton mass is determined mainly by the vev of the Higgs field \( H_d \). However at tree level, only the muon gets mass after the \( Z_{3L} \) symmetry breaking. The electron remains massless because the determinant of the matrix \( M \) is still zero. By choosing the values \( v_d \sim 100 \text{GeV} \), \( v_i - v_j \sim (10 - 30) \text{GeV} \), \( g_{Y_i} \sim 10^{-2} \), and \( \lambda_i \sim 10^{-2} - 10^{-3} \) as a numerical illustration, the \( \tau \) and \( \mu \) masses can be consistent with experimental measurements.

It is interesting to find that we have naturally obtained a hierarchical pattern for the leptons, that is \( m_\tau \gg m_\mu \gg m_e \), despite the fact that the electron is massless. To make this pattern to be realistic, we will consider the electron mass at the loop level.

Actually this model can generate nonvanishing electron mass at the loop level if we further assume the soft breaking of \( Z_{3L} \). In analyzing the neutrino magnetic moments, Refs. [4] and [5] discussed the generation of the electron mass by supersymmetric radiative corrections. In a similar way, the electron mass can be induced naturally through the one-loop diagram fig. 1 in this model, where \( \chi \) and \( l \) denote the neutral gauginos and the charged leptons, respectively. The mixing of the scalar leptons associated with different chiralities is due to the soft broken terms. The structure of these mixing has the same form as that shown in matrix (3), however, they are multiplied by a common typical supersymmetric mass parameter \( \tilde{m} \). Fig. 1 contributes to the lepton
mass matrix the following terms,

$$(\delta M)_{ij} = \sum_{\chi} \frac{g_{\chi}^2}{16\pi^2} \frac{m_{\chi}}{m_{\chi}^2 - m_{\tilde{l}}^2} \left( \frac{m_{\chi}^2}{m_{\chi}^2 - m_{i}^2} \ln \frac{m_{i}^2}{m_{i}^2 - m_{j}^2} + \frac{m_{j}^2}{m_{j}^2 - m_{i}^2} \ln \frac{m_{i}^2}{m_{i}^2 - m_{j}^2} \right) \tilde{m}M_{ij},$$

where $M_{ij}$ is the matrix element of Eq. (3). The mixing of the neutralinos have been simply neglected in writing the above equation. In general, the mass of the slepton $\tilde{l}_i$ receives two distinct contributions: one from the supersymmetric couplings, and the other from the soft breaking terms. Both of them violate the $Z_{3L}$ symmetry. The determinant of the mass matrix $(M + \delta M)$ determines the electron mass,

$$m_e = \frac{\det(M + \delta M)}{m_{\tau} m_{\mu}}.$$  

With suitable ranges for the parameters, realistic value of the electron mass can be generated, for instance, in the case of $m_{\chi} \sim 50\text{GeV}$, $m_{\tilde{l}} \sim 100\text{GeV}$, and $\tilde{m} \sim 150\text{GeV}$.

As a phenomenological application of above idea, let us turn to discuss the neutrino masses. In the superpotential (2), we have not introduced the term $L_i H_u$ which can be forbidden by some other discrete symmetry. Therefore, there is no tree-level mixing between the neutrinos and the neutralinos. The neutrinos are massless at tree level in spite of the large sneutrino vevs which have been taken in the numerical illustration. At the one-loop level, nonvanishing Majorana neutrino masses are produced in this model, in the same way as that given in Refs. [4-6]. Because the $Z_{3L}$ symmetry breaks softly in the slepton sector, there is no symmetry in the neutrino mass matrix, hence in general the masses of the three neutrinos are at the same order,

$$m_\nu \sim \frac{\lambda_i^2 \tilde{m}m_{\tilde{l}}^2}{16\pi^2 m_{\tilde{l}}^2},$$

where the sleptons have been taken to be degenerate and the squark’s contribution should have been included which we expect to be of the same order. With the above choices of the parameters, the neutrino mass is typically $m_\nu \sim (0.1 - 10)\text{eV}$. For the same reason, it seems that this pattern of neutrino masses cannot provide solution to the solar neutrino problem. However, it is cosmologically interesting that such
neutrinos could be the components of the hot dark matter \[7\]. It is also possible to solve the atmospheric neutrino problem provided the lepton mixing are appropriate. In addition, it is hopeful to test the prediction about the neutrino mass range directly in the double beta decay experiments in the near future.

In conclusion, if the sneutrinos have nonvanishing vevs which can be achieved through choosing appropriate scalar potential, it is possible to understand the fermion mass hierarchy within the framework of low energy supersymmetry. We have proposed that there is a cyclic symmetry among the left-handed doublets of the three families. This symmetry results in an interesting hierarchical pattern for the leptons, that is \( m_\tau \gg m_\mu \gg m_e \). After considering the quantum corrections, this pattern can be realistic. The phenomenology of this model is rich, however, it cannot be discussed thoroughly until the quark sector is included \[8\].

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Fig. 1. Supersymmetric generation of the lepton mass.
This figure "fig1-1.png" is available in "png" format from:

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