Controlled electron acceleration in the bubble regime by optimizing plasma density

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New Journal of Physics 12 (2010) 045010 (9pp)
Received 27 August 2009
Published 30 April 2010
Online at http://www.njp.org/
doi:10.1088/1367-2630/12/4/045010

Abstract. Improvement of the quality of the monoenergetic electron bunch generated in the laser wakefield is investigated. The electrostatic field is more intense near the back of the bubble than at other locations in the bubble. By optimizing the density gradient of background plasma, the local dephasing problem can theoretically be overcome and the electron bunch can be stably accelerated at the back of the bubble so that the accelerated electrons experience nearly the same electric field. Three-dimensional simulations were performed. Compared with the standard wakefield acceleration schemes, a better-quality electron bunch, with narrower energy spread and higher energy, is obtained with a shorter acceleration distance.

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1. Introduction

Laser wakefield acceleration has become important because of its huge acceleration rate, and thus its potential for making compact accelerators [1]–[3]. It has been demonstrated theoretically and experimentally that in the so-called ‘bubble’ regime of wakefield acceleration, high-quality electron beams can be generated [4]–[13].

Only when a laser pulse with power $P > (\tau_L [\text{fs}] \lambda_L [\mu \text{m}])^2 \times 30 \text{GW}$ propagates in plasma of appropriate density, can the ‘bubble regime’ be easily realized [14]. Here $\tau_L$ and $\lambda_L$ are the duration and wavelength of the driver laser, respectively. The bubble is basically a three-dimensional (3D) wake plasma wave generated behind an intense laser pulse propagating through the plasma. Plasma electrons are expelled by the laser ponderomotive force, leaving behind it an electron-free cavity. After the laser pulse has passed, the electrons are pulled back by the ions, forming the bubble and the wake oscillations behind it. The bubble propagates in the same direction of laser propagation with a phase velocity near the group velocity of the laser pulse. The bubble represents a highly nonlinear 3D plasma wave with an intense charge-separation electric field, which corresponds to a potential well moving at high speed. Electrons trapped at the bottom of the latter can thus be accelerated with narrow energy spread.

There are several methods for injecting electrons into the bubble [15]–[26]. Electron injection can also be realized by using a nanowire as the triggering disturbance [27]. When the electron bunch is injected into the accelerating field, it can only be accelerated in the back half of the bubble. The electron bunch can then slip into the decelerating phase of the bubble potential well and lose energy. This limits the energy gain of the electron bunch in the bubble and is referred to as electron dephasing. The limitation caused by the dephasing can be overcome by employing a proper plasma density [28]. The latter can be accurately modulated by varying the pressure of the gas jet and the shape of the gas nozzle [29]. Pukhov [30] proposed control of the phase of wakefield accelerated particles by tailoring the initial plasma density profile. With appropriate laser and plasma parameters, phase control in the bubble regime can be realized.

In this paper, we realize the control of the electrons’ phase in the bubble by optimizing the plasma density gradient. The scheme is demonstrated using the 3D particle-in-cell (PIC) simulation code VORPAL [31].

2. Theory model

For convenience, we define $v_f$ and $v_b$ as the velocities of the front and back of the bubble, and $v_e$ as the velocity of the electron bunch (see figure 1). Although the electrostatic force in
Figure 1. Structural diagram of the bubble. Here, \( v_f \) and \( v_b \) are defined as the velocities of the front and back of the bubble, and \( v_e \) is defined as the velocity of the electron bunch.

The bubble is much more intense than the electrostatic repulsive force among electrons of the accelerated electron bunch, the gradient of the bunch field can be larger than that of the bubble field and can broaden the energy spread of the bunch. When an electron bunch is accelerated in a bubble, its velocity \( v_e \) is larger than that of the bubble. The injected electrons at first have nearly the same energy. As the electron bunch moves forward from the back of the bubble, the longitudinal bubble electric field in the bunch becomes weaker and its gradient smaller. When the gradient of the electron-bunch Coulomb field becomes larger than that of the bubble electric field, the tail electrons gain relatively less energy from the field. To prevent this, a positive plasma density gradient is induced to make the back of the bubble move faster. With a positive plasma density gradient, the length of the bubble decreases. The group velocity, the same as \( v_f \), of the laser pulse in the plasma also becomes smaller. However, the increase of \( v_b \) can balance the limitation caused by the resulting dephasing.

The size \( \lambda_{\text{bubble}} \) of the bubble is a function of plasma density. Here, \( \lambda_{\text{bubble}} = \sqrt{\gamma_a \lambda_p} = \sqrt{(\gamma_a n_c/n)\lambda_L} \), where \( n \) is the plasma density, \( n_c \) is the critical density, \( \gamma_a = \sqrt{1 + a_0^2} \) has been averaged over the electron distribution and \( a_0 = eA_0/mc^2 \) is the normalized amplitude of the laser pulse. Hence the bubble will shrink as it propagates along the positive plasma density gradient. The shrink speed \( v_{sh} \), which can be used to denote the time-dependent shrinkage of the bubble, is

\[
v_{sh} = -\frac{d\lambda_{\text{bubble}}}{dr} = -\lambda_L \frac{d\sqrt{\gamma_a n_c/n}}{dx} v_b,
\]

where \( x = \int_0^t v_b \, dt \) gives the position of the bubble.

The bubble-front velocity is the same as that of the laser front \([32]\). For the high nonlinear wake bubble, the velocity of the leading edge of the pulse is the linear group velocity minus the velocity of the local pump depletion, so that the front velocity is

\[
v_f \approx \left( 1 - \frac{1}{2} \frac{n}{\gamma_a n_c} \right) c - \frac{n}{\gamma_a n_c} c = \left( 1 - \frac{3}{2} \frac{n}{\gamma_a n_c} \right) c.
\]

The bubble-back velocity \( v_b \) is the sum of \( v_f \) and \( v_{sh} \), and can be written as

\[
v_b = v_f + v_{sh} = v_f - \lambda_L \frac{d\sqrt{\gamma_a n_c/n}}{dx} v_b,
\]
from which we obtain

\[ v_b \approx \frac{1 - (3n/2\gamma_a n_c)}{1 + \lambda_L (d\sqrt{\gamma_a n_c/n/dx})} c. \] (4)

Here, the electron bunch can be assumed to have vacuum light speed \(c\) [1] (see also figure 4). From equation (4), one finds that \(v_b\) can be larger than \(v_e\), even though \(v_f\) becomes smaller in a plasma with positive density gradient.

The relative position of the electron bunch trapped in the bubble is \(\xi = ct - x\) (see figure 1). From equation (4), one finds that \(v_b\) can be larger than \(v_e\), even though \(v_f\) becomes smaller in a plasma with positive density gradient.

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The phase of the bunch in the bubble is given by \(\Phi_1 = 2\pi \xi/\lambda_{\text{bubble}}\), so that we have

\[ \frac{\text{d}}{\text{d}x} \left( \frac{\Phi}{\omega} \sqrt{\gamma_a n_c/n} \right) = \frac{1}{v_b} - \frac{1}{c}. \] (5)

The density gradient of the plasma can be optimized to match the movement of the bunch in the bubble or to keep the phase \(\Phi_1\) constant at \(\Phi = \Phi_0\). The solution of equation (5) for phase synchronism is

\[ n = \frac{n_0}{(1 - x/L)^{2/3}}, \] (6)

\[ L = \frac{2}{9} \left( \lambda_L - \frac{c\Phi_0}{\omega} \right) \left( \frac{\gamma_a n_c}{n_0} \right)^{3/2}, \] (7)

where \(n_0 = n(x = 0)\).

### 3. Simulation and discussion

In order to demonstrate the effect of the proposed density-gradient tailoring, we carried out two 3D simulations (using the PIC code VORPAL [31]) for comparison. In the simulations, we consider a circularly polarized laser pulse with wavelength \(\lambda_L = 0.8\ \mu\text{m}\) injected from the left boundary of the simulation box. The normalized amplitude of the laser pulse is \(a_0 = \sqrt{2}\).

The laser pulse has a transverse Gaussian distribution with a full-width at half-maximum 15.24 \(\mu\text{m}\) and a longitudinal sinusoidal waveform with duration 30 fs. For electron injection at \(x = 200\ \mu\text{m}\) and \(z = 6.35\ \mu\text{m}\), we also use the same trigger wire as in [27].

In the first simulation (S-I) plasma density is separated into two parts, as shown in figure 2. In the first part the density remains homogeneous at \(n = 6 \times 10^{18}\ \text{cm}^{-3}\), corresponding to the area \([5\ \mu\text{m}, 300\ \mu\text{m}]\). At \(t = 1.07\ \text{ps}\), the laser pulse has just passed \(x_0 = 300\ \mu\text{m}\) and an electron bunch of length 6 \(\mu\text{m}\) has been injected and becomes stable, as shown in figure 3. As discussed in the above section, with a proper plasma density profile, the relative phase of the electron bunch can be maintained in the following acceleration process. The suitable phase \(\Phi_0\) of the electron bunch in the bubble at this instant can be obtained from the simulation result. Accordingly, plasma density is set at \(n = n_0/[1 - (x - x_0)/L]^{2/3}\), where \(n_0 = 6 \times 10^{18}\ \text{cm}^{-3}\) and \(x_0 = 300\ \mu\text{m}\), in the area \([300\ \mu\text{m}, 1700\ \mu\text{m}]\). Here, \(L\) can be obtained from equation (7).

For comparison, in our second simulation (S-II) the density is kept homogeneous at \(n = 6 \times 10^{18}\ \text{cm}^{-3}\) for the entire plasma region \([5\ \mu\text{m}, 3\ \text{mm}]\), the same as in [27].

Figure 4 shows the space and time evolution of charge density along the axis of the bubble obtained from the two simulations. The bubble’s front and back, and the accelerated electron bunch are marked by solid lines, and their velocities, \(v_f\), \(v_b\) and \(v_e\), are characterized by the gradients of the lines. We see that in case S-I, the position of the bunch in the bubble is
maintained at a relatively stable phase. In contrast, in case S-II $v_e$ is obviously larger than $v_l$ and $v_b$, resulting in rapid slipping of the electron bunch from the back half to the front half of the bubble.

In figure 5, the temporal evolution of the peak energy and energy spread of accelerated electron bunches in cases S-I and S-II is depicted. Initially, in both cases the energies of the bunch electrons increase and energy spreads reduce when the gradients of the electric fields of the bubbles are larger than those of the Coulomb fields from the bunch electrons themselves. Later, the energy spread in S-II becomes much larger than that in S-I. In this process, the electron bunch approaches the decelerating phase for the dephasing limit in S-II. Although the energy of the electron bunch is still increasing, the rate is reduced, leading to a maximum energy of about 227 MeV. Also, the electric field of the bubble becomes increasingly weak, and the Coulomb field in the electron bunch dominates, leading to wider energy spread. In contrast, the electron bunch in S-I is kept in the stable phase and the peak energy increases linearly with stable energy spread. However, the continuous acceleration is also limited, as will be discussed in the next section.

In the two simulation cases, different limits in the acceleration length exist, and therefore different plasma lengths are selected. The solid lines in figure 6 show the energy spectra of
Figure 4. Space and time evolution of charge density along the axis of the bubble for simulations S-I and S-II. The bubble’s front and back, and the accelerated electron bunch are marked by solid lines.

**Figure 5.** Evolution of the peak energy (——) and energy spread (· · · · ··) of accelerated electrons in S-I and S-II.

electron bunches before the latter leave the plasma, and the dashed lines show the energy spectra at $t = 4.67$ ps, when the energy is highest and the energy spread smallest. In order to show the spectrum of the relativistic electron bunch clearly, only the spectrum of electrons with energy larger than 20 MeV is shown. In both cases, electron bunches with total charge of about 9 pC are obtained. For case S-I, the peak energy is 273 MeV with energy spread 12% before the laser pulse leaves the 1.7 mm long plasma, corresponding to $t = 5.34$ ps, as shown in figure 6(a).

Here, energy spread is defined as $\Delta E/E_p$, where $E_p$ is the peak energy and $\Delta E$ is the full-width at $1/e$ of the energy spectrum. For case S-II, the peak energy reaches only 227 MeV, with energy spread 21% at $t = 10.01$ ps, as shown in figure 6(b). For S-I, at $t = 4.67$ ps the energy spread reduces to 4.5% with peak energy 236 MeV. Such an electron bunch with balanced peak energy and energy spread can have many potential applications. For comparison, we note that at the same instant in case S-II the peak energy is 167 MeV and the energy spread is 18%.
Figure 6. Energy spectra at different times for cases S-I and S-II. Solid lines show the energy spectra of electron bunches before the latter leave the plasma, and dashed lines show the energy spectra at $t = 4.67$ ps.

4. Discussion

As shown in figure 5(b), electron acceleration in the homogeneous plasma has a limitation on energy gain because of dephasing. As the electron bunch is accelerated, its velocity $v_e$ increases and approaches the speed of light, $v_e \rightarrow c$. If the phase velocity of the bubble is a constant $v_f < c$, electrons will outrun the plasma wave and move into the decelerating-phase region of the bubble. This electron dephasing limits the energy gain of electrons in the plasma wave. The dephasing length $L_d \simeq \gamma_p^2 \lambda_{\text{bubble}}$ can be obtained from [1]. The resulting maximum energy gain is approximately $W_{\text{max}} \simeq eE_{\text{max}} L_d \simeq 260$ MeV. However, the acceleration field is not always at its maximum value in the acceleration process. As a result, in the simulation the energy attains only a maximum value 227 MeV, which is somewhat smaller than the theoretical maximum value. After that, the electron bunch can no longer gain energy from the wave and it starts to lose energy.

Although the dephasing limitation is inhibited in case S-I, another limitation, namely self-reinjection, appears in the acceleration process. When plasma density increases to the wave breaking threshold, self-injection occurs again and many more electrons are continuously injected, and the highly relativistic electron bunch will no longer be accelerated. Figure 6(a) shows the energy spectra at $t = 4.67$ and 5.34 ps, and self-reinjection occurs during this period. We can also estimate the energy of the electron bunch theoretically for case S-I. From Pukhov and Meyer-ter-Vehn [4], we have $E_{\text{max}} = (1/\gamma_p)(a_0^2/\sqrt{1 + a_0^2})$. The electron bunch here has a length of about 6 $\mu$m (see figure 7(a)) so that the average accelerating field $\langle E_{\text{max}} \rangle$ acting on the bunch can only be about 180 GV m$^{-1}$ instead of $E_{\text{max}}$. Until $t = 5.34$ ps, the maximum particle energy $\int e \langle E_{\text{max}} \rangle dt$ is about 289 MeV, which is also near the peak energy obtained for case S-I. Figures 7(b) and (d) show the longitudinal electric fields along the axes of the bubbles for S-I and S-II. We find that the field in the area of the bunch remains strong in S-I, resulting in the linear acceleration shown in figure 5(a). However, in S-II, the field in the area of the bunch becomes smaller as $t$ increases, and the rate of increase of the electron energy slowly decreases to zero (figure 5(b)).
With our tailored plasma density profile, the energy spread is reduced by about five times. We believe the energy spread is dominated by a space-charge field. The Coulomb field of the electron bunch itself has an opposite gradient to the space-charge electric field of the bubble. The accelerating field that the electrons experience is the sum of the two fields. In S-I, since the plasma density increases in the direction of laser propagation, the bubble shrinks as the laser propagates in the plasma. In this case, the velocity of the bubble’s back becomes higher, and can match the velocity of accelerated electrons. Therefore, the electron bunch remains near the back of the bubble, experiencing a stable high accelerating field in the whole process, as shown in figures 7(a) and (b), and the tail electrons obtain nearly the same energy as that of the center electrons. This makes the energy spectrum in S-I narrower than that in S-II.

In S-II, the energy spread increases after the energy of the electron bunch has reached the relatively high value of 100 MeV. Figures 7(c) and (d) show the electrons trapped in the first bubble and the total electric field at different times for case S-II. As \( t \) increases, the bubble’s longitudinal electrostatic field around the electron bunch becomes weaker and the tail electrons in the bunch cannot balance the intense Coulomb field, resulting in the relatively wide energy spread in S-II.

5. Summary

In summary, we have demonstrated that the dephasing limitation can be inhibited by a proper plasma density distribution. By introducing an optimized density profile, a more energetic electron bunch with lower energy spread is obtained. The acceleration length is limited by the self-reinjection.
Acknowledgments

We thank Dr M Y Yu for reviewing the manuscript. This work was supported by the National Natural Science Foundation of China (Project Nos. 10675155, 60921004 and 10834008), the Program of Shanghai Subject Chief Scientist (09XD1404300) and the 973 program (No. 2006CB806004).

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