CAN THE JET STEEPEN THE LIGHT CURVES OF GAMMA-RAY BURST AFTERGLOWS?

D. M. Wei1,2 AND T. Lu3,4,5

Received 1999 September 19; accepted 2000 April 24

ABSTRACT

Beaming of relativistic ejecta in gamma-ray bursts (GRBs) has been postulated by many authors in order to reduce the total GRB energy; thus it is very important to look for the observational evidence of beaming. Rhoads has pointed out recently that the dynamics of the blast wave, which is formed when the beamed ejecta sweep the external medium, will be significantly modified by the sideways expansion caused by the increased swept-up matter. He has claimed that shortly after the bulk Lorentz factor (\( \Gamma \)) of the blast wave drops below the inverse of the initial opening angle (\( \theta_0 \)) of the beamed ejecta, there will be a sharp break in the afterglow light curves. However, some other authors have performed numerical calculations and shown that the break of the light curve is weaker and much smoother than the one analytically predicted. In this paper we reanalyze the dynamical evolution of the jet blast wave and calculate the jet emission analytically; we find that the sharp break predicted by Rhoads will actually not exist, and for most cases the afterglow light curve will be almost unaffected by sideways expansion unless the beaming angle is extremely small. We demonstrate that only when \( \theta_0 < 0.1 \) may the afterglow light curves be steepened by sideways expansion, and in fact there cannot be two breaks as claimed before. We have also constructed a simple numerical code to verify our conclusion.

Subject heading: gamma rays: bursts

1. INTRODUCTION

The BeppoSAX results have revolutionized our understanding of gamma-ray bursts (GRBs) by opening a window on X-ray, optical, and radio afterglows (e.g., Costa et al. 1997; Piro et al. 1998; van Paradijs et al. 1997). The decaying power-law long-wavelength afterglows are explained as the emission from a relativistic blast wave that decelerates when sweeping up interstellar medium. The dynamical evolution of GRB fireballs and the emission features have been studied by many authors (e.g., Sari 1997, 1998; Mészáros, Rees, & Wijers 1998; Wei & Lu 1998a, 1998b; Sari, Piran, & Narayan 1998), most of whom considered the fireball to be isotropic.

The discovery of GRB afterglows shows that GRBs are at cosmological distances. If so, the total energy for a typical GRB event is about \( 10^{52} \) ergs. This year, an extraordinary event, GRB 990123, was detected, which was the brightest burst ever detected by BeppoSAX and is in the top 0.3% of all bursts (Feroci et al. 1999). The detection of the redshift showed that the burst appears at \( z \geq 1.6 \), with its gamma-ray fluence of \( \sim 5 \times 10^{-4} \) ergs cm\(^{-2}\); the total energy of this source is \( \geq 1.6 \times 10^{54} \) ergs if the emission is isotropic (Andersen et al. 1999; Kulkarni et al. 1999). This energy is so large that it gives a great challenge to the popular models. For models involving stellar mass central engines it is necessary to assume that the ejecta are beamed in order to explain such a huge amount of energy.

Now the main uncertainty of bursts' energy is whether the bursts radiate isotropically or are beamed into a small solid angle. As shown above, the extreme large energy value favors the emission being beaming. Then there is one question: how can we find out whether the energy is beaming or not? Rhoads (1997, 1999) has shown that the lateral expansion of the shocked, relativistic plasma requires that at some moment the surface of the blast wave starts to increase faster than would be caused by the cone outflow alone; then the blast wave begins to decelerate faster than it would without the sideways expansion, since more interstellar medium has been swept up by the blast wave. Rhoads claimed that this effect will produce a sharp break in the GRB afterglow light curves. Such a break is claimed to be present in the light curves of GRB 990123 and GRB 990510 (Kulkarni et al. 1999; Harrison et al. 1999). Sari, Piran, & Halpern (1999) speculate that afterglows with very steep light curves are highly beamed.

However, some other authors (Panaitescu & Mészáros 1999; Moderski, Sikora, & Bulik 2000) have performed numerical calculations of dynamical evolution of the blast wave, and have shown that the break of the light curve is weaker and much smoother than the one analytically predicted. Thus there are two opposite conclusions about the jet emission: the analytical treatment predicts a sharp break, while the numerical calculation shows no such sharp break.

In this paper we will first give an analytical treatment of the dynamical evolution of the jet blast wave and its emission features, and will demonstrate that the sharp break will not actually exist; we may observe the steepening of the light curve only when the jet angle is extremely small, i.e., \( \theta_0 < 0.1 \). We then perform a simple numerical calculation to confirm our results. In the next section we discuss the dynamical evolution of the blast wave, in § 3 we calculate the jet emission analytically, and finally we present some discussion and conclusions.

2. DYNAMICAL EVOLUTION OF THE JET

Now we consider an adiabatic relativistic jet expanding in the surrounding medium. For energy conservation, the
evolution equation is

$$\Gamma^2 V = \text{constant}, \quad (1)$$

where $\Gamma$ is the bulk Lorentz factor and $V$ is the jet volume, $V = 2\pi r^3 (1 - \cos \theta_j)/3 \approx r^2 \theta_j^2$ for $\theta_j \ll 1$, and $\theta_j = \theta_0 + \theta = \theta_0 + c_s t_{\text{exp}}/ct$, where $\theta_0$ is the initial jet opening half-angle, $\theta$ describes the lateral expansion, $c_s$ is the expanding velocity of ejecta material in its comoving frame, and $t (t_{\text{co}})$ is the time measured in the burster frame (comoving frame). For relativistic expanding material it is appropriate to take $c_s$ to be the sound speed $c_s = c/\sqrt{3}$ (Rhoads 1997, 1999, and Rhoads 1999) has given $t_{\text{co}}/t = 2/5T$. Since the jet expands relativistically, there is the relation $T \propto r^{2/3}$, where $T$ is the time measured in the observer frame and $r = ct$ is the radial coordinate in the burster frame. From the above relations, we have

$$\Gamma (1 + \Gamma^2 \Gamma_0^{1/4}) \approx T^{-3/8}, \quad (2)$$

where $\Gamma_0 = \sqrt{\frac{c}{\theta_0}}$. In this paper all quantities with the subscript $\beta$ refer to the point at which $c_s = r_0$, which means that after that time the sideways expansion begins to dominate the radial divergence. Since $T \propto r^{1/2}$, so $\Gamma_0 / \Gamma \approx (T/T_0)^{1/2}$. Therefore we have

$$\Gamma \propto \left\{ \begin{array}{ll}
T^{-3/8} [1 + (T/T_0)^{3/8}]^{-1/4} & \text{if } T < T_0, \\
T^{-3/8} [1 + (T/T_0)^{1/2}] & \text{if } T > T_0.
\end{array} \right. \quad (3)$$

It is obvious that $\Gamma \propto T^{-3/8}$ for $T \ll T_0$ and $\Gamma \propto T^{-1/2}$ for $T \gg T_0$. The rapid decrease with time of the Lorentz factor $\Gamma$ is due to the fact that larger amounts of surrounding matter have been swept up by ejecta (Rhoads 1997, 1999).

In the following we calculate the value of $T_0$. According to the fireball model, the decelerating radius of the ejecta is

$$r_0 = \left( \frac{E}{\pi \theta_0^2 \Gamma_0^3 m_p c^2} \right)^{1/3}, \quad (4)$$

where $E$ is the burst energy, $\Gamma_0 = E/M_0 c^2$, and $M_0$ is the initial baryon mass. Rhoads (1999) has given $r_0 = [75T_0^3 m_p / 8(c_s/\theta_0)^3]^{1/3}$. The relation between $T$ and $r$ is $T = r/c \Gamma c$, where the numerical value of $\zeta$ lies between $\sim 3$ and $\sim 7$ depending on the details of the hydrodynamical evolution and the spectrum. Sari (1998) and Waxman (1997) have shown that the typical value of $\zeta$ is about 4; then the time is

$$T_0 \approx \frac{r_0}{4T_0^2 c} = \frac{70}{c_s (c_s/\sqrt{3})} \frac{\theta_0^2}{0.1} \frac{1}{E_{3.2}^{1/3} n_{-1}^{-1/3}} \text{ days}. \quad (5)$$

We see that the break time $T_0$ is very large for typical parameters, which means that the transition from $\Gamma \propto T^{-3/8}$ to $\Gamma \propto T^{-1/2}$ is usually very slow and smooth.

3. THE EMISSION FROM THE JET

Now we calculate the emission flux from the jet. Here we adopt the formulation and notations of Mao & Yi (1994). In our model the ejecta is flowing outward relativistically (with Lorentz factor $\Gamma$) in a cone with opening half-angle $\theta_j$. For simplicity, we assume that the radiation is isotropic in the comoving frame of the ejecta and has no dependence on the angular positions within the cone. The radiation cone is uniquely defined by the angular spherical coordinates $(\theta, \phi)$ of its symmetry axis; here $\theta$ is the angle between the line of sight (along the $z$-axis) and the symmetry axis, and $\phi$ is the azimuthal angle. Because of cylindrical symmetry, we can assume that the symmetry axis of the cone is in the $y$-$z$ plane. In order to see more clearly, let us establish an auxiliary coordinate system $(\chi, \gamma, z)$ with the $z'$-axis along the symmetry axis of the cone and the $\chi'$ parallel to the $x$-axis. Then the position within the cone is specified by its angular spherical coordinates $\theta'$ and $\phi'$ ($0 \leq \theta' \leq \theta_j, 0 \leq \phi' \leq 2\pi$). It can be shown that the angle $\theta$ between a direction $(\theta', \phi')$ within the cone, and the line of sight satisfies $\cos \Theta = \cos \theta \cos \theta' - \sin \theta \sin \theta' \sin \phi'$. Then the observed flux is

$$F(v, \theta) = \int_0^{2\pi} d\phi \int_0^\pi \sin \theta' d\theta' D^2 \Gamma(vD^{-1}) \frac{r^2}{d^2}, \quad (6)$$

where $D = [\Gamma(1 - \beta \cos \Theta)]^{-1}$ is the Doppler factor, $\beta = (1 - \gamma^{-2})^{-1/2}$, $v = Dv'$, $\Gamma(v)$ is the specific intensity of synchrotron radiation at $v'$, and $d$ is the distance of the burst source. Here the quantities with a prime are measured in the comoving frame. For simplicity we have ignored the relative time delay of radiation from different parts of the cone.

For the expanding jet, we have $r = DT \theta_j \Gamma \propto DT \propto DT$, the magnetic field strength $B \propto \Gamma$, the peak frequency of synchrotron radiation $v_m = Dv' \propto DT$, and the emission spectrum $\Gamma(v') \propto v^{-\kappa}$, then $\Gamma'(v) = \Gamma(v_m)v'/v_m = \Gamma(v_m)\theta_j / \theta_j = \Gamma(v_m)^{v^{-\kappa} - 2}$. Therefore we have the flux

$$F(v, \theta) \propto v^{-2(\kappa - 1)} T^3 \Gamma(\theta, \Gamma, \alpha), \quad (7)$$

where

$$\theta(\Gamma, \alpha, x) = \int_0^{2\pi} d\phi' \int_0^\pi \sin \theta' d\theta'(1 - \beta \cos \Theta)^{-(6 + 4\kappa)}. \quad (8)$$

In general, the value of $\alpha$ can only be calculated numerically. However, here we consider the case $\theta_j < 1$ and $\theta_j \ll 1$; then $\cos \Theta \approx \cos \theta \cos \theta'$. In this case we can calculate the value of $\alpha$ analytically under certain conditions. After complicated calculation we find $g \approx \theta_2 \Gamma^2 (\theta_2 + 2\theta_2^{-1} \Gamma_2)^{-1/2} \Gamma^{-1} < \theta_j$, $g \approx \theta_2 \Gamma^2 (\theta_2 + 2\theta_2^{-1} \Gamma_2)^{-1} \Gamma^{-1} < \theta_j$, and $\theta > \theta_j, g \approx \theta_2 \Gamma^2 (\theta_2 + 2\theta_2^{-1} \Gamma_2)^{-1} \Gamma^{-1} < \theta_j$, and $g \approx \theta_2 \Gamma^2 (\theta_2 + 2\theta_2^{-1} \Gamma_2)^{-1} \Gamma^{-1} < \theta_j$. Therefore we have the results

$$F(v, \theta) \propto \left\{ \begin{array}{ll}
v^{-2\theta_2 \theta_2 - 2(\kappa - 1)} T^3 & \text{for } \Gamma^{-1} < \theta < \theta_j, \\
v^{-2\theta_2 \theta_2 - 2(\kappa - 1)} T^3 & \text{for } \Gamma^{-1} \theta_j. \quad (9)$$

For $T \ll T_0$, the evolution is about $\Gamma - \Gamma_0 (r/r_0)^{-3/2}$. Here we define the Lorentz factor, $\Gamma_0 \equiv \Gamma(r_0) = \theta_j^{-1}$, then $r_j = (\Gamma_0 \theta_j)^{1/2} r_0$, and the corresponding timescale is $T_j \approx r_j / c \simeq 1.3(\theta_0 / 0.1)^2 E_{3.2}^{1/3} n_{-1}^{-1} (\theta_0 / 0.1)^{3/2}$ day. Then the observed flux at fixed frequency is

$$F \propto \left\{ \begin{array}{ll}
T^{-3/2} & \text{for } T < T_j, \\
T^{-3/2 - 3/4} & \text{for } T_j < T < T_b, \\
T^{-2a - 1} & \text{for } T > T_b. \quad (10)$$

From the above it seems that there should be two temporal index breaks in light curves. However, if we compare the
values of $T_J$ and $T_b$, we will find that $T_J \ll T_b$, i.e., the time interval between $T_J$ and $T_b$ is very large; the beaming break is much earlier than the break due to sideways expansion. We know that in order to see the steepening of the light curve, $T_J$ or $T_b$ must be small, so in fact we can only see one temporal break. In addition, in the Rhoads treatment the effect of the sideways expansion on the $\Gamma$ evolution was ignored when $T < T_b$; however, in fact, there is still some sideways expansion during this phase, so the evolution of $\Gamma$ must be affected by sideways expansion when $\Gamma \approx \theta_0^{-1}$. Therefore, we expect that the evolution of $\Gamma$ is continuous, and the transition from $\Gamma \propto T^{-3/8}$ to $\Gamma \propto T^{-1/2}$ is much smoother than previously claimed.

In order to test our conclusion, we make a simple numerical calculation. We assume that the blast wave evolution is adiabatic and ignore cooling of the swept-up particles. We take the following initial parameters: $\Gamma_0 = 300$, the electron distribution index $p = 2.5$. In Figure 1 we present the light curve for different initial opening angles: the solid, dotted, and dashed lines represent the cases where $\theta_0 = 0.1$, $0.0174$, and $0.01$, respectively. We show that when $\theta_0 = 0.1$, the light curve is nearly unaffected by sideways expansion, while when $\theta_0 = 0.0174$ and $0.01$, the light curves are clearly steepened, confirming our analytic conclusion.

For comparison, we also calculate the afterglow light curves of a blast wave with no spreading. We know that, if the blast wave is highly radiative, the internal energy of the blast wave will be low, since it is converted to photons and radiated away, so the lateral expanding velocity will be very small, $c_s \ll c$; in this case, the sideways expansion is unimportantly small, and the blast wave can be regarded as without spreading. For highly radiative evolution, it has been shown that the Lorentz factor is given by $\Gamma \propto T^{-3/7}$ (Wei & Lu 1998a). Taking the parameters as above, we calculate the afterglow light curves under this situation, Figure 2 gives our results; the solid, dotted, and dashed lines also correspond to the cases $\theta_0 = 0.1$, $0.0174$, and $0.01$, respectively. It is obvious that, when $T < T_c \left[ T_c \sim T_0(\Gamma_0 \theta_0)^{-3/7} \right]$, the light curves decay as a simple power law; when $T > T_c$, the light curves deviate from the simple power law and show a steepening, and a clear break occurs at about $T_c$. Also, we note that only when $\theta_0 < 0.1$, the steepening is obvious. Therefore we suggest that if we observe the sharp break in the GRB afterglow light curves, then it may indicate that the blast wave is highly radiative rather than adiabatic.

4. DISCUSSION AND CONCLUSIONS

The GRB afterglows provide a very good opportunity to study whether and how much the GRB ejecta are beamed. Rhoads (1997, 1999) has pointed out that the beamed outflows should diverge from the cone geometry, and the sideways outflow of the shocked relativistic plasma would increase the front of the blast wave, which leads to a fast deceleration. He also predicted that the afterglow light curves should have a sharp break around $T_c$.

However, Moderski et al. (2000) have performed numerical calculations and shown that the break of the light curve is weaker and smoother than the prediction. Here we reanalyze the dynamical evolution of the jet blast wave and calculate the emission from the jet. Our calculations show that the main reason why the results of Moderski et al. are different from that of Rhoads is that the value of $T_c$ is very large when the parameters adopted by Moderski et al. are used (see eq. [5]). Our formula (eq. [3]) indicates that the evolution of the Lorentz factor $\Gamma$ with time $T$ is continuous, changing the slope from $-\frac{3}{7}$ to $-\frac{1}{2}$ smoothly. In particular, if the value of $T_c$ is large, then the transition is much smoother; in this case one expects that the sharp break will not exist. In order to test our analytic conclusion, we also make a simple numerical calculation; from Figure 1 it is shown that only when $\theta_0 < 0.1$ can we observe the steepening of the light curve, which is consistent with our analytic conclusion.

Our results are valid only if the remnant is still relativistic at time $T_c$. Since at this time the Lorentz factor $\Gamma_b \approx \frac{3}{2}c_s/c\theta_0^{-1}$, this condition reduces to $\theta_0 < 0.1$, i.e., the jet is very narrow. If $\theta_0 > 0.1$, then before the sideways expansion becomes important, the remnant has already become nonrelativistic. In fact, Dai & Lu (1999) have shown that even in the case of isotropic emission, the break in the light curve can appear during the transition from an ultrarelativistic to a nonrelativistic phase in the environment of dense material.
It should be emphasized that in our calculation we have ignored the relative time delay of radiation from different parts of the cone; if this effect is considered, then the slope of the light curve should be flatter than $T^{-2(a+1)}$ (Moderski et al. 2000). However, all these calculations are based on the assumption that the material is uniformly distributed across the blast wave. In fact, it is more likely that the lateral outflow can lead to a $\theta$-dependent structure, with the density of swept material and the bulk Lorentz factor decreasing with $\theta$; in this case the break in the light curve may become more prominent.

We thank the referee for several important comments that improved this paper. This work is supported by the National Natural Science Foundation (grants 19703003 and 19773007) and the National Climbing Project on Fundamental Researches of China.

REFERENCES

Andersen, M. I., et al. 1999, Science, 283, 2075
Costa, E., et al. 1997, Nature, 387, 783
Dai, Z. G., Lu, T. 1999, ApJ, 519, L155
Feroci, M., et al. 1999, IAU Circ. 7095
Harrison, F. A., et al. 1999, ApJ, 523, L121
Kulkarni, S. R., et al. 1999, Nature, 398, 389
Mao, S., & Yi, I. 1994, ApJ, 424, L131
Mészáros, P., Rees, M. J., & Wijers, R. 1998, ApJ, 499, 301
Moderski, R., Sikora, M., & Bulik, T. 2000, ApJ, 529, 151
Panaitescu, A., & Mészáros, P. 1999, ApJ, 526, 707
Piro, L., et al. 1998, A&A, 331, L41

Rhoads, J. E. 1997, ApJ, 487, L1
Sari, R. 1999, ApJ, 525, 737
Sari, R. 1998, ApJ, 494, L49
Sari, R., Piran, T., & Halpern, J. P. 1999, ApJ, 519, L17
Sari, R., Piran, T., & Narayan, R. 1998, ApJ, 497, L17
van Paradijs, J., et al. 1997, Nature, 386, 686
Waxman, E. 1997, ApJ, 491, L19
Wei, D. M., & Lu, T. 1998a, ApJ, 499, 754
——. 1998b, ApJ, 505, 252