Towards an exact evaluation of the supersymmetric $O(\alpha_s \tan \beta)$ corrections to $\overline{B} \to X_s \gamma$

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ABSTRACT

The charged-Higgs contributions to the decay $\overline{B} \to X_s \gamma$ are discussed in the minimal supersymmetric standard model at large $\tan \beta$. These contributions receive two-loop $O(\alpha_s \tan \beta)$ corrections by squark-gluino subloops, which are nondecoupling in the limit of heavy superpartners and possibly large. Their leading parts are already known and were evaluated by using an effective two-Higgs-doublet Lagrangian. Subleading corrections coming from higher-dimension operators in the effective Lagrangian were ignored, although this is not, a priori, justified when $m_{H^\pm}$ is not much smaller than the typical supersymmetric mass $M_{\text{SUSY}}$. Here, we calculate all subleading terms of the $O(\alpha_s \tan \beta)$ corrections up to $O((m_t^2, m_{H^\pm}^2/M_{\text{SUSY}}^2)^2)$, as well as all the exact two-loop diagrams with squark-gluino subloops, beyond the effective-Lagrangian approximation. Comments are made on the size of these corrections.

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Abstract

The charged-Higgs contributions to the decay $B \to X_s \gamma$ are discussed in the minimal supersymmetric standard model at large $\tan \beta$. These contributions receive two-loop $O(\alpha_s \tan \beta)$ corrections by squark-gluino subloops, which are nondecoupling in the limit of heavy superpartners and possibly large. Their leading parts are already known and were evaluated by using an effective two-Higgs-doublet Lagrangian. Subleading corrections coming from higher-dimension operators in the effective Lagrangian were ignored, although this is not, a priori, justified when $m_{H^\pm}$ is not much smaller than the typical supersymmetric mass $M_{\text{SUSY}}$. Here, we calculate all subleading terms of the $O(\alpha_s \tan \beta)$ corrections up to $O((m_t^2, m_{H^\pm}^2/M_{\text{SUSY}}^2)^2)$, as well as all the exact two-loop diagrams with squark-gluino subloops, beyond the effective-Lagrangian approximation. Comments are made on the size of these corrections.

1 Introduction

The inclusive width of the radiative decays of the $B$ mesons, $B \to X_s \gamma$, is well described by the short-distance processes $b \to s\gamma$ and $b \to sg$, since nonperturbative hadronic corrections are small and well under control. The partonic processes have been evaluated up to the next-to-leading order in QCD. See Ref. [1] for a review listing the steps that brought to this achievement, after it was noted that QCD plays a particularly important role for the decay $B \to X_s \gamma$ [2]. The Standard Model (SM) prediction for the branching ratio $\text{BR}(B \to X_s \gamma)$ is, up to today [3],

$$\text{BR}(B \to X_s \gamma)(\text{SM}) = (3.54 \pm 0.49) \times 10^{-4}. \quad (1)$$

This average includes results obtained in the various papers of Ref. [4]. The comparison of the SM result with the world average [5] of the inclusive branching ratio from recent experiments at Belle [6], CLEO [7], and BABAR [8] detectors,

$$\text{BR}(B \to X_s \gamma)_{\text{exp.}} = (3.34 \pm 0.38) \times 10^{-4}, \quad (2)$$

is rather satisfactory. In the SM, the process $b \to s\gamma$, as well as $b \to sg$, occurs through loops with $W^\pm$ and $t$-quark exchange, i.e. at the same level in perturbation theory at which physics beyond the SM may contribute. The agreement between experimental and SM results for $\text{BR}(B \to X_s \gamma)$, therefore, is already good enough to constrain exotic contributions. Indeed, it is already routinely used to check whether particular directions of parameter space of supersymmetric extensions of the SM are viable or not.

In the minimal supersymmetric standard model (MSSM), new loop contributions to the two radiative decays of the $B$ meson, $b \to sg$ and $b \to s\gamma$, come [9] from the charged-Higgs boson, charginos, gluino and...
neutralino. Their contributions are often comparable to or even larger than the SM one. The inclusion of QCD corrections to these contributions \[10\], however, is far from having reached the level of precision already achieved in the SM, at least for generic regions of the supersymmetric parameter space. The latest development in this direction has been the observation that the SUSY-QCD corrections by squark-gluino subloops can be significant when \(\tan\beta\) is very large \[11\] \[12\].

In this talk, we focus on the contribution of the charged-Higgs boson \(H^\pm\) and analyze these two-loop SUSY-QCD corrections. So far, only the nondecoupling parts of these corrections have been evaluated, by using an effective two-Higgs-doublet (2HD) Lagrangian where squarks and gluino are integrated out \[11\] \[12\]. Subleading parts of these corrections, i.e. of order \((m_t^2, m_{H^\pm}^2/M^2_{\text{SUSY}})^n\), generated by higher-dimensional operators in the effective Lagrangian, were omitted in these studies, although they could give potentially large contributions when \(m_{H^\pm}\) is nonnegligible with respect to the typical supersymmetric mass \(M_{\text{SUSY}}\).

Here, we calculate some of these subleading corrections, i.e. up to \(O((m_t^2, m_{H^\pm}^2/M^2_{\text{SUSY}})^2)\). We also evaluate exactly all two-loop diagrams correcting at \(O(\alpha_s \tan\beta)\) the charged-Higgs contribution to \(b \to s\gamma\) and \(b \to sg\). This calculation, clearly, encompasses the effective-Lagrangian approximation and includes all subleading terms \((m_t^2, m_{H^\pm}^2/M^2_{\text{SUSY}})^n\). In this talk, after discussing the effective-Lagrangian formalism in Section 2 we show in Section 3 numerical results for the charged-Higgs contributions to the Wilson coefficients \(C_7\) and \(C_8\) at the matching scale, \(\sim M_W\). The evaluation of these same coefficients at low scale, \(\sim m_t\), and therefore of the BR(\(\overline{B} \to X_s\gamma\)) becomes, at this point, straightforward. We have, however, refrained from showing results for BR(\(\overline{B} \to X_s\gamma\)) and from extracting exclusion plots for the charged-Higgs boson mass, since the exact calculation of the \(O(\alpha_s \tan\beta)\) corrections to the W\(^\pm\) contribution, or at least the calculation of the subleading \(O(m_t^2/M^2_{\text{SUSY}})\) and \(O((m_t^2/M^2_{\text{SUSY}})^2)\) terms of these corrections, is not yet available. We comment on the outcome of our calculation in Section 4 and we conclude in Section 5.

2 \(H^\pm\) couplings to quarks

2.1 Tree-level couplings

The MSSM has two Higgs doublets \(H_D\) and \(H_U\), \(H_D = (H_D^0, H_D^-)\) and \(H_U = (H_U^+, H_U^0)\), which break the SU(2)\(\times U(1)\) gauge symmetry through the vacuum expectation values (VEVs) of their neutral components. The two VEVs are related to the W-boson mass as \(M_W^2 = g_2^2 v^2/2 \equiv g_2^2 (v_D^2 + v_U^2)/2\). Their ratio is conventionally called \(\tan\beta\), \(\tan\beta \equiv v_U/v_D\). They form the following mass eigenstates: two CP-even scalars, \((h^0, H^0)\); one CP-odd pseudoscalar, \(A^0\); the two states of a charged-Higgs boson, \(H^\pm\); and the unphysical Nambu-Goldstone modes, \((G^\pm, G^0)\). The charged scalars \((H^\pm, G^\pm)\) are related to the charged components of the gauge eigenfields \(H^\pm_{D,U}\) as

\[
\begin{pmatrix}
G^\pm \\
H^\pm
\end{pmatrix} = \begin{pmatrix}
\cos\beta & -\sin\beta \\
\sin\beta & \cos\beta
\end{pmatrix} \begin{pmatrix}
H^\pm_D \\
H^\pm_U
\end{pmatrix}.
\]

At the tree-level, the couplings of the Higgs doublets \(H_i (i = D, U)\) to quarks obey the selection rule of the 2HD model of Type II. That is to say, \((d_R)_i = (d, s, b)_R\) couple only to \(H_D\), whereas \((u_R)_i = (u, c, t)_R\) couple only to \(H_U\), as shown by the interaction Lagrangian in the gauge eigenbasis:

\[
\mathcal{L} = \bar{d}_R Y^d q_L \cdot H_D - \bar{u}_R Y^u q_L \cdot H_U + \text{(h.c.)},
\]

in which the SU(2)-invariant multiplication of doublets was adopted \((A \cdot B \equiv \epsilon_{ij} A_i B_j, \text{ with } \epsilon_{12} = -\epsilon_{21} = 1 \text{ and } \epsilon_{11} = \epsilon_{22} = 0\). This constraint is a consequence of supersymmetry.
After diagonalization of the Yukawa matrices $Y^q$, $Y^q \to \text{diag}(h_q)$, and the breaking of $SU(2) \times U(1)$, the $b$- and $t$-quark acquire masses

$$m_b = h_b v_D = h_b \bar{v} \cos \beta, \quad m_t = h_t v_U = h_t \bar{v} \sin \beta,$$

and the $H^+$ couplings to the $t$- and $b$-quarks become

$$\mathcal{L} = V_{tb} h_b \sin \beta H^+ \bar{t}_L b_R + V_{tb} h_t \cos \beta H^+ \bar{t}_R b_L + (\text{h.c.}).$$

Here, $V_{tb}$ is a element of the CKM matrix. Of the two couplings

$$g(H^+ \bar{t}_L b_R) = V_{tb} h_b \sin \beta = V_{tb} \frac{m_b}{\bar{v}} \tan \beta,$$

$$g(H^+ \bar{t}_R b_L) = V_{tb} h_t \cos \beta = V_{tb} \frac{m_t}{\bar{v}} \cot \beta,$$

the first is greatly enhanced for large $\tan \beta$.

### 2.2 Couplings up to $O(\alpha_s \tan \beta)$

After supersymmetry breaking, loop corrections depending on soft supersymmetry-breaking parameters generate effective couplings of quarks $d_{IR}$ to $H_U^-$ and of $u_{IR}$ to $H_D^+$, which are forbidden at the tree-level. Squark and gluino loops inducing the couplings $\bar{b}_R u_L H_U^-$ and $\bar{d}_L t_R H_D^+$, are shown explicitly in Fig. 1.

Figure 1: $\bar{b}_R u_L H_U^-$ and $\bar{d}_L t_R H_D^+$ vertices generated by squark-gluino loop, shown in the gauge eigenbasis of squarks.

At momentum scales sufficiently smaller than the typical supersymmetric mass $M_{\text{SUSY}}$, these contributions can be expressed in terms of an effective 2HD Lagrangian (not of Type II anymore)

$$\mathcal{L}^{\text{eff}} = \bar{d}_{R} Y_{d} q_{L} \cdot (H_{D} - \Delta_{d_{R,q}} H_{D}^0) - \bar{u}_{R} Y_{u} q_{L} \cdot (H_{U} + \Delta_{u_{R,q}} H_{D}^0) + (\text{h.c.}),$$

where $H_D^0 = (H_D^+, H_D^-)$ and $H_U^0 = (H_U^+, H_U^-)$, respectively, and where the index $q$ in $\Delta_{d_{R,q}}$ and $\Delta_{u_{R,q}}$ is understood to denote a left-handed quark. The effective couplings $\Delta_{d_{R,q}}$ and $\Delta_{u_{R,q}}$ induced by SUSY-QCD loops (as well as Higgsino-squarks loops) are of order

$$\Delta_{d_{R,q}}, \Delta_{u_{R,q}} = O\left(\alpha_s \frac{|\mu m_{\tilde{g}}|}{M_{\text{SUSY}}^2}\right) = O(\alpha_s M_{\text{SUSY}}^0).$$

i.e. do not decouple [13,14,15] in the limit of heavy superpartners, $M_{\text{SUSY}} \to \infty$. Although sufficiently smaller than unity, they can induce large corrections to the $H^\pm$ couplings to quarks for large $\tan \beta$.

After the breaking of $SU(2) \times U(1)$, masses and $H^\pm$-couplings of the $t$- and $b$-quarks become, at this order,

$$\mathcal{L}^{\text{eff, int}} = -h_b \bar{v} \cos \beta \left[1 + \Delta_{b_{R,b}} \tan \beta\right] \bar{b} b - h_t \bar{v} \sin \beta \left[1 + \Delta_{t_{R,t}} \cot \beta\right] \bar{t} t$$

$$+ V_{tb} h_b \sin \beta \left[1 - \Delta_{b_{R,b}} \cot \beta\right] H^+ \bar{t}_L b_R + V_{tb} h_t \cos \beta \left[1 - \Delta_{t_{R,t}} \tan \beta\right] H^+ \bar{t}_R b_L + (\text{h.c.}).$$
where CP-violating phases in supersymmetric parameters were dropped for simplicity. Notice that the $\Delta_{bR,q}$ and $\Delta_{tR,q}$ of Eq. (5) are now split into $\Delta_{bR,b}$, $\Delta_{bR,t}$ and $\Delta_{tR,b}$, $\Delta_{tR,t}$. The elements of each of these two pairs differ by SU(2)$\times$U(1) breaking effects. This splitting is achieved by including in the effective lagrangian of Eq. (3) additional higher-dimensional operators. See for example $\bar{b}_R(q_L \cdot H_U^0)H_U^0H_U$ and $\bar{b}_R(q_L \cdot H_D)H_U^0H_U$.

The first line of Eq. (10) denotes the running $b$-quark mass within the SM, that is to say:

$$m_b(SM) = h_b v \cos \beta \left[ 1 + \Delta_{bR,b} \tan \beta \right].$$

The corrections $\Delta_{bR,b} \tan \beta$, of $O(\alpha_s \tan \beta)$ are potentially large [13] for $\tan \beta \gtrsim (\Delta_{bR,b})^{-1}$. As a result, when parametrized by $m_b(SM)$, the coupling $H^+\bar{t}_Lb_R$ may significantly deviate [14] [15] from the (renormalization group improved) tree-level result,

$$g\left(H^+\bar{t}_Lb_R\right)(\text{eff}) \sim V_{tb} m_b(SM) \frac{\tan \beta}{v} \frac{1}{1 + \Delta_{bR,b} \tan \beta},$$

where the second term in parentheses can very well be of the same order of magnitude of the first.

For momentum scales nonnegligible with respect to the superpartner masses, the above effective Lagrangian has to be enlarged to incorporate additional higher-dimensional operators that have momentum dependence. For example, the diagram in Fig. 2 generates effective operators of dimension six, such as

$$\tilde{\gamma}_L H_D \gamma_\mu (\partial_\mu q_L) \cdot H_U,$$

and in particular, $H_D^0 \tilde{d}_L \gamma_\mu (\partial_\mu t_L) H_U^0$, where $H_U^0$ gets a vacuum expectation value. This gives rise to a new set of couplings $H^+\tilde{t}_L \gamma_\mu d_L$, which in the large $\tan \beta$ limit are of size

$$g\left(H^+(q)\tilde{t}_L(p)\gamma_\mu d_L(q-p)\right)(\text{eff}) \sim V_{td} p^\mu \frac{m_t(SM)}{M^2_{\text{SUSY}}} \Delta_{\tilde{t}_L,d} \sin \beta \sim V_{td} p^\mu \frac{m_t(SM)}{M^2_{\text{SUSY}}} \Delta_{\tilde{t}_L,d}.$$

These operators are suppressed by inverse powers of $M_{\text{SUSY}}$ and decouple in the limit of heavy superpartners.

### 3 $H^+$ contributions to $b \to s\gamma$ and $b \to sg$ up to $O(\alpha_s \tan \beta)$

It is well known that $b \to s\gamma$ and $b \to sg$ are very sensitive to $\tan \beta$ [16], at times even dangerously so [17]. However, at the one-loop level, the charged-Higgs contributions to these decays remains fairly independent from this parameter, for $\tan \beta \gtrsim 3$. In this regime, the dominant part of the $H^+$ contribution to the decays
$b \rightarrow s\gamma$ and $b \rightarrow sg$, comes from the diagrams in Fig. 3, where the photon and gluon have still to be attached in all possible ways. In this Figure, it is shown explicitly how the factor $\tan \beta$ for the vertex $\bar{t}_L b_R H_D^+$ is cancelled by the mixing between $H_D$ and $H_U$, which brings in a suppression factor $\sin 2\beta \sim 1/\tan \beta$. Phrased in an equivalent way, the decay amplitude for this diagram becomes insensitive to $\tan \beta$, due to the cancellation of the factor $\tan \beta$ between the $\bar{t}_L b_R H_D^+$ and $\bar{s}_L t_R H_D^-$ vertices.

Figure 3: $b \rightarrow s\gamma$ by charged-Higgs exchange, with helicity flip on the internal fermion line. The Higgs doublet constituent of the charged-Higgs state is indicated explicitly.

However, since the vertex $H_D^+ \bar{s}_L t_R$ is generated at the one-loop level, it is possible to avoid this $\tan \beta$ cancellation, when considering two-loop contributions to $b \rightarrow s\gamma$ and $b \rightarrow sg$. The corresponding diagram, where again, the photon and the gluon are still to be attached in all possible ways, is shown on the left of Fig. 4. Since also the vertex $H_D^+ \bar{s}_L t_L$ gets generated at the one-loop level, there is another diagram in which it is possible to avoid the cancellation of $\tan \beta$ of the lowest order contribution. This is shown on the right side of Fig. 4. Note that, being the vertex $H_D^+ \bar{s}_L t_L$ of decoupling type, also the corresponding $b \rightarrow s\gamma$ and $b \rightarrow sg$ contributions decouple in the limit $M_{\text{SUSY}} \rightarrow \infty$. Both classes of contributions coming from the two diagrams in Fig. 4 are, nevertheless, of $\mathcal{O}(\alpha_s \tan \beta)$ with respect to the lowest order contribution. They are potentially large, without invalidating perturbation theory: their largeness derives from the suppression of the lowest order term.

Overall, potentially large $\mathcal{O}(\alpha_s \tan \beta)$ corrections to the charged-Higgs contribution to $b \rightarrow s\gamma$ (and $b \rightarrow sg$) may come from:

1. mass corrections, $\delta m_b$, to the vertex $H_D^+ \bar{t}_L b_R$ [12], see Ref. [17];
2. proper vertex corrections to the coupling $H_D^+ \bar{s}_L t_R$ [11] [12] as well as $H_D^+ \bar{s}_L t_L$, shown in Fig. 4;
3. corrections to the couplings $H_D^+ \bar{s}_L t_R \gamma$ and $H_D^+ \bar{s}_L t_L \gamma$, obtained from the two diagrams in Fig. 4 when the photon is attached to the $\bar{t}$- or the $\bar{s}$-squarks, and corrections to the couplings $H_D^+ \bar{s}_L t_R g$ and $H_D^+ \bar{s}_L t_L g$, obtained from the two diagrams in Fig. 4 when the gluon is attached to the $\bar{t}$- or the $\bar{s}$-squarks, or the gluino.

These corrections, induced by squarks and gluino subloops, have been so far calculated by using the
effective 2HD Lagrangian of Eq. (3), with all superpartners integrated out. This approach is justified as far as all the momenta of the fields in Eq. (3) are sufficiently smaller than the squarks and gluino. This condition is clearly satisfied for the counterterm $\delta m_b$ and its contribution to the $H^\pm t_L b_R$ vertex, see Eq. (12).

For the corrections to the $H^- s_L t_R$ vertex, the validity of the effective Lagrangian of Eq. (3) requires $m_t$ and $m_{H^\pm}$ to be smaller than $M_{\text{SUSY}}$. The condition $m_{H^\pm} \ll M_{\text{SUSY}}$, however, is often violated in the case of well-known candidates for the mechanism of supersymmetry breaking. Thus, in the resulting models, and in general when $m_{H^\pm} \gtrsim M_{\text{SUSY}}$, it is possible that the amplitude of the full two-loop diagrams deviates significantly from that obtained by making use of the effective 2HD Lagrangian of Eq. (3). One may try to handle the case of $m_{H^\pm}$ nonnegligible with respect to $M_{\text{SUSY}}$ by including in the effective Lagrangian higher-order operators, which are suppressed by inverse powers of $M_{\text{SUSY}}^2$. This is equivalent to making an expansion of the two-loop integrals corresponding to the diagrams in Fig. 4 with respect to $m_{H^\pm}^2/M_{\text{SUSY}}^2$ and $m_t^2/M_{\text{SUSY}}^2$. The operators inducing the vertex $H^- s_L t_R$ (see discussion at the end of Section 2.2), as well as those inducing the vertices $H^- s_L t_R \gamma$, $H^- s_L t_L \gamma$, and $H^- s_L t_R g$, $H^- s_L t_L g$, are already of decoupling type, i.e. suppressed by inverse powers of $M_{\text{SUSY}}^2$. Nevertheless, additional expansions of the two-loop integrals may still be needed.

A systematic way of making these expansions ad simultaneously keep into account all needed higher-dimensional operators is provided by the Heavy Mass Expansion (HME) [10]. Using this technique, we find for example that, among the operators of dimension six to be added, it is necessary to include also the very same $O_7$ and $O_8$,

$$O_7(\mu) = \frac{e}{16\pi^2} m_b(\mu) s_L \sigma^{\mu\nu} b_R F_{\mu\nu}, \quad O_8(\mu) = \frac{g_s}{16\pi^2} m_b(\mu) s_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu},$$

which will be part of the effective Hamiltonian used for the calculation of amplitudes of radiative $b$ decays. $F_{\mu\nu}$ and $G^a_{\mu\nu}$ in these operators are the field strengths of the photon and the gluon, respectively. Details on the use of the HME are given in Ref. 20.

Through the HME technique we have evaluated terms up to $O((m_{H^\pm}^2, m_t^2/M_{\text{SUSY}}^2)^2)$. Notice that the first term of this expansion is, for small values of $m_{H^\pm}$, of the order of magnitude of the SU(2)$\times$U(1)-breaking corrections to the coefficients of the effective Lagrangian [3]; see Ref. 21. For $m_{H^\pm}$ closer to or even larger than $M_{\text{SUSY}}$, however, one may question even the validity of this expansion. Of course, the truncation up to the $O((m_{H^\pm}^2, m_t^2/M_{\text{SUSY}}^2)^2)$ can only be justified by comparing with the result of the complete two-loop calculation, which clearly goes beyond the effective-Lagrangian approach. We have, therefore, also calculated all two-loop diagrams exactly, using methods described in Ref. 22. In both cases, i.e. whether we make use of an expansion or not, we need to calculate the $O(\alpha_s \tan \beta)$ terms arising from both diagrams shown in Fig. 4. Nondecoupling corrections in the effective-Lagrangian approach come from the left diagram with photon/gluon emitted only from the t-quark or charged-Higgs boson. All other diagrams are decoupling in the $M_{\text{SUSY}} \rightarrow \infty$ limit, and have not been included in previous studies 11 12 23 24.

4 Numerical results

We present here results for the charged-Higgs contributions to the Wilson coefficients $C_7(\mu_W)$ and $C_8(\mu_W)$ of the operators $O_7$ and $O_8$ at the matching scale $\sim \mu_W$, which we have chosen to be $M_W$. We incorporate the corrections to $C_7(\mu_W)$ and $C_8(\mu_W)$ discussed in the previous Section. Our normalization of these Wilson coefficients is the conventional one, as follows from the definition of the effective Hamiltonian used for the calculation of amplitudes of radiative $b$ decays,

$$H_{\text{eff}} \supset -\frac{4G_F}{\sqrt{2}} V^*_{tb} V_{tb} (C_7(\mu) O_7(\mu) + C_8(\mu) O_8(\mu)),$$

\footnote{A similar procedure has been applied to collect higher order corrections of type $O(\alpha \tan \beta)$ in Ref. 13.}
Figure 5: Ratios $r_{7,8}(\mu_W)$, defined in Eq. (17), as functions of $m_{H^\pm}$. The supersymmetric spectrum considered here is the spectrum I, given in the text. The dotted lines show the goodness of the leading-order approximation of the two-loop calculation, the dashed and dot-dashed lines, the goodness of the approximation in which the first and the second subleading terms in $(m^2_{H^\pm}, m^2_t)/M^2_{\text{SUSY}}$ are included.

Figure 6: Same as in Fig. 5 for the spectrum II.

We show results in which the two-loop diagrams in Fig. 4 are calculated exactly, and results in which the off-shell vertices $H^- s_L t_R$ and $H^- s_L t_L$ are treated in an effective-Lagrangian approach, with inclusion of leading and subleading terms up to overall suppression factors $1/(M^2_{\text{SUSY}})^2$. We compare these approximated results, which we call $C_{7,8}(\mu_W)|_{\text{approx}}$, with the exact result, denoted by $C_{7,8}(\mu_W)|_{\text{exact}}$. To make this comparison more transparent, we plot in Figs. 5 and 6 the ratios

$$r_i(\mu_W) \equiv \frac{C_i(\mu_W)|_{\text{approx}} - C_i(\mu_W)|_{\text{exact}}}{C_i(\mu_W)|_{\text{exact}}} \quad (i = 7, 8),$$

as functions of $m_{H^\pm}$. In these ratios, the mass correction $\delta m_b$ to the vertex $H^+ t_L b_R$ cancels out. Therefore, we need to specify only a relatively small number of parameters.

For Fig. 5 we have chosen a heavier superpartner spectrum, called here spectrum I, $(m_{\tilde{s_L}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}) = (700, 500, 450) \text{ GeV}$, the stop-sector mixing angle $\cos \theta_t = 0.8$, $\tan \beta = 30$, $m_{\tilde{g}} = 600 \text{ GeV}$, and $\mu = 550 \text{ GeV}$, whereas for Fig. 6 a lighter spectrum is considered: $(m_{\tilde{s_L}}, m_{\tilde{t}_1}, m_{\tilde{t}_2}) = (350, 400, 320) \text{ GeV}$, $\cos \theta_t = 0.8,$
\[ \tan \beta = 30, \quad m_\tilde{g} = 300 \text{ GeV}, \quad \mu = 450 \text{ GeV}. \] This is denoted as spectrum II. As for other input parameters, we have used \( m_t(\mu_W) = 176.5 \text{ GeV} \), which corresponds to a pole mass of 175 GeV, and \( \alpha_s(\mu_W) = 0.12 \).

In both sets of Figures, the dotted lines denote the ratios \( r_2(\mu_W) \) and \( r_8(\mu_W) \) in which only the leading-order approximation is used for the evaluation of the two-loop diagram, i.e. the nondecoupling contribution of Ref. [11]: the dashed (dot-dashed) lines denote the ratios in which the first (second) subleading terms in \( (m_1^2, m_2^2)/M_{\text{SUSY}}^2 \) are also included.

These Figures show explicitly that for \( m_{H^\pm} \) sufficiently smaller than \( M_{\text{SUSY}} \), the use of the HME, and therefore of the effective-Lagrangian approach, is indeed meaningful: the inclusion of an additional term of the expansion brings closer and closer to the exact result. For \( m_{H^\pm} \sim M_{\text{SUSY}} \), the improvement provided by additional terms stops being so evident, at least in the case of the coefficient \( C_8(\mu_W) \). Going further, it becomes quite clear that the results obtained from the HME, and from the effective-Lagrangian formalism, cannot be extended to values \( m_{H^\pm} \gtrsim M_{\text{SUSY}} \) (i.e. strictly speaking outside the range in which they had been derived), as one may have hoped. For these values of \( m_{H^\pm} \), the series of corrections in inverse powers of \( M_{\text{SUSY}}^2 \) does not seem to be convergent.

In the safe region \( m_{H^\pm} < M_{\text{SUSY}} \), (that is, safe for the HME), we find that the largest contribution to \( C_7(\mu_W) \) comes from the diagram on the left side of Fig. 4 with the photon emitted by the \( t \)-quark, for \( C_9(\mu_W) \) from the two diagrams still on the left side of Fig. 4 with the gluon emitted by the \( t \)-quark and the gluino. This is true for the exact calculation and for the approximations at different order in \( 1/M_{\text{SUSY}}^2 \). Above this range of values for \( m_{H^\pm} \), at different orders of approximation, the contributions from different diagrams tend to grow differently with \( m_{H^\pm} \), producing the crossing points of different lines, visible in Fig. 4 and partially in Fig. 5. For example, in the approximation in which all terms \( \mathcal{O}(m_1^2, m_2^2, M_{\text{SUSY}}^2) \) are retained, for a specific value of \( m_{H^\pm} \), all terms in \( 1/M_{\text{SUSY}}^2 \) cancel out and the value of the Wilson coefficient coincides with that in which only the nondecoupling terms are kept. Similar cancellations of terms happen in the other two crossing points. In any case, there is no particular meaning in these points, since they appear in a region in which the HME is not well behaved.

For \( m_{H^\pm} \) sufficiently smaller than \( M_{\text{SUSY}} \), the approximation in which only the nondecoupling terms of the two-loop diagrams are included, is not a bad approximation of the exact calculation, for the coefficient \( C_7(\mu_W) \). We observe, however, a 15\% deviation of the result of this approximation from that of the exact calculation for \( C_8(\mu_W) \) in the case of the lighter supersymmetric spectrum. In this case, these two results seem to have a similar behaviour in \( 1/m_{H^\pm}^2 \) for any value of \( m_{H^\pm} \), but they are split by terms \( (m_1^2/M_{\text{SUSY}}^2)^n \), resummed in the exact calculation, and, possibly, by intrinsic constants arising from the two-loop calculation.

What is the impact of this deviation, and of our exact calculation in general, for the BR(\( \bar{B} \rightarrow X_s \gamma \)) and for an exclusion plot of the charged-Higgs mass remains to be seen. We have not attempted to draw such a plot, since a calculation of the \( W^\pm \) contribution at the same precision on \( m_1^2/M_{\text{SUSY}}^2 \) is still not yet available. Moreover, in the region of supersymmetric parameter space in which the difference between our exact calculation and the approximated one of Ref. [11] is largest, one expect also a rather big contribution from the chargino-stop exchange. We postpone the presentation of such a plot to later work.

## 5 Conclusion

We have studied the SUSY-QCD corrections to the charged-Higgs contributions to the decay \( \bar{B} \rightarrow X_s \gamma \). They are induced by gluino-squark subloops of \( \mathcal{O}(\alpha_s \tan \beta) \) and are therefore potentially large in the large-\( \tan \beta \) regime.

The resulting two-loop diagrams had been dealt in the past in an approximate way. In particular, they had been treated in the approximation of an effective Lagrangian with two-Higgs doublets, in which only the
nondecoupling operators had been included. For the charged-Higgs contributions to $b \to s\gamma$ and $b \to s\gamma$, this means that terms of $\mathcal{O}(m^2_t, m^2_{H/}/M^2_{\text{SUSY}})$ or higher, (with $M_{\text{SUSY}}$ one of the typical squark/gluino masses) are neglected. Terms of this type are in general induced by higher-dimensional operators. A truncation of the basis of operators in the 2HD effective Lagrangian, is acceptable in particular directions of parameter space, in which $m_{H/} \ll M_{\text{SUSY}}$. This condition, however, is not generically supported by the most known and studied models of supersymmetry breaking. In these, the charged-Higgs mass tends to align along the gluino mass, in turn of the same order of the $\mu$ parameter. (There are, however, directions in which a special tuning among the different supersymmetric parameters allows considerably lower values of $m_{H/}$.)

For the charged-Higgs contributions to $b \to s\gamma$ and $b \to s\gamma$, we have, therefore included all subleading terms, up to $\mathcal{O}((m^2_t, m^2_{H/}/M^2_{\text{SUSY}})^2)$, obtained through the technique of Heavy Mass Expansion of multi-loop integrals. This implicitly takes into account the contribution of all relevant higher-dimensional operators that should be added to the 2HD effective Lagrangian. We have also performed the exact calculation of all two-loop diagrams correcting at order $\mathcal{O}(\alpha_s \tan \beta)$ the charged-Higgs contribution to $b \to s\gamma$ and $b \to s\gamma$. Thus, we have compared to this exact result the different approximations obtained in the HME, i.e.

1) that in which only the nondecoupling part of the two-loop integrals is retained; 2) that in which terms of $\mathcal{O}(m^2_t, m^2_{H/}/M^2_{\text{SUSY}})$ are also added; and finally 3) that in which terms up to $\mathcal{O}((m^2_t, m^2_{H/}/M^2_{\text{SUSY}})^2)$ are included.

We have found that for $H/\pm$ considerably lighter than the remaining supersymmetric particles, the result from the exact calculation and from the three approximations deviate very little one from the other, for the coefficient $C_7(\mu_W)$. For the coefficient $C_8(\mu_W)$ we find a deviation of 15% between the result of approximation 1) and that of the exact calculation, if the supersymmetric spectrum is not particularly heavy. The calculation presented here is a first step towards the exact evaluation of all supersymmetric contributions to $\overline{\tau} \to X_s\gamma$ at order $\mathcal{O}(\alpha_s \tan \beta)$. Only after the completion of such a program, will phenomenological analyses be performed and exclusion plots for the masses of the charged-Higgs and other supersymmetric particles be attempted.

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