Ultra high quality factor resonators for kinetic inductance detectors

G Hammer, S Wuensch, K Ilin and M Siegel
Institute for Micro- and Nanoelectronic Systems, University of Karlsruhe, Karlsruhe, Germany
Email: g.hammer@ims.uni-karlsruhe.de

Abstract. The sensitivity of a kinetic inductance detector is determined by the performance of the cavity circuit and therefore a high quality factor and low phase noise are required. We developed superconducting niobium coplanar waveguide resonators for kinetic inductance detectors at a resonance frequency of 6 GHz. The length of the resonators is adjusted to a quarter of a wavelength which corresponds to $\ell \approx 5.3$ mm in our geometry. Different layouts with coplanar waveguide transmission line topology for the resonator were designed and simulated. Some parameters of the device fabrication process were varied. The niobium thin films were deposited by magnetron sputtering on sapphire substrates kept at ambient temperature. The resonators were patterned by photolithography and reactive ion etching and measured at helium temperatures. Because of the quarter wave geometry with one shorted end the reflection parameter was measured. We have studied the influence of the coupling capacitances on quality factor. The optimization includes microwave simulations and implementation in different resonator structures. The measurements of optimized devices showed a very high quality factor above one million ($10^6$). The obtained results and perspective on further development of resonators will be discussed.

1. Introduction
In the field of sensor development in radio astronomy, civil security and medical applications, there is a great demand for high sensitive detectors especially in the new field of THz detectors and THz imaging. Besides development of sensitive single pixel detectors, multi-pixel arrays will become more important for further progression [1]. Kinetic inductance detectors (KID) represent an emerging sensor technology which uses superconducting resonators to readout the energy of absorbed photons. Incoming radiation breaks Cooper pairs creating quasiparticle excitations in the KID. The raise of quasiparticles density will increase the intrinsic kinetic inductance in the detector which is part of the resonator. A change of kinetic inductance is detuning the cavity circuit and results in a shift of the resonance frequency. Both, frequency and phase shift can be read out by additional electronic circuits. A cavity circuit system offers the possibility of a sensitive read-out for very small values of the kinetic inductance. The shift of resonance frequency is also very small and depends on the loaded quality factor, $Q_L$, of the resonator. In contrary to a cavity circuit with a small $Q_L$, a resonator with a higher $Q_L$ will show a bigger shift of the resonance frequency and phase for a certain change in quasiparticle density and therefore a more sensitive read-out is possible. Resonators with a high loaded quality factor $Q_L$ are also essential for the development of large sensor arrays with this technology [2]. Higher values of $Q_L$ mean a smaller 3dB-bandwidth of the resonance which allows more cavity circuits per frequency range thus increasing the number of detector pixels in an array.
2. Material Approach

Material parameters have a strong influence on the quality factor of resonators. Therefore a short overview of the influences of individual parameters on the device characteristics is given. Substrate and conductor properties are introduced and the transmission line technique in respect of resonator application is discussed.

2.1. Substrate

The dielectric constant, $\varepsilon_r$, and a low loss tangent, $\tan \delta$, are important properties for the design of resonators with small dimensions and a high quality factor. It is required to have an almost temperature independent behavior of the dielectric constant as seen in sapphire substrate ($\text{Al}_2\text{O}_3$). Therefore unpredictable behavior of the structure at cryogenic temperatures can be reduced. Sapphire is an anisotropic substrate with a dielectric constant $\varepsilon_r$ between 9.4 and 11.6. A special sapphire substrate in the m-cut plane i.e. is homogenous. The exact value of $\varepsilon_r$ depends on the angle between transmission line direction and substrate orientation which can vary from parallel to orthogonal. Our resonator structures were fabricated on r-cut sapphire substrates. In the case of a coplanar waveguide, transmission lines can change their direction on the substrate for wiring purposes. Separate calculations were necessary for the different orientations with appropriate dielectric constants. Therefore the dielectric constant can be assumed to be isotropic [3] with a uniform value $\varepsilon_r = 10.06$. Sapphire is well known as an excellent microwave substrate because of the very low loss tangent $\tan \delta = 10^{-5}$ @ 4.2 K. The substrate thickness was $h = 430 \ \mu m$.

2.2. Conductor

The conductor material consists of a thin sputtered niobium film with a thickness of 250 nm. Quality ratings of the conductor can be made by measurement of the critical temperature $T_C$ and the residual resistance ratio ($\text{RRR} = R_{300K}/R_{10K}$). For the used niobium films $T_C = 9.2–9.3$ and $\text{RRR} = 4.5$. The surface resistance $R_s$ is calculated using formula (1) with the vacuum permeability $\mu_0$, London penetration depth $\lambda_L$, angular frequency $\omega$ and the real part of the complex conductivity $\sigma_1$ for niobium [4] at 4.2 K. This results in a value of $R_s = 11.56 \ \mu \Omega$ @ 6 GHz. Calculations with the program PICOP [5], [6] show that kinetic inductance effects can be neglected for the chosen transmission line cross section dimensions at 4.2 K.

$$R_s = \frac{1}{2} \mu_0^2 \cdot \lambda_L^3 \cdot \omega^2 \cdot \sigma_1 \quad (1)$$

2.3. Transmission line technique

A symmetrical coplanar waveguide transmission line (CPW) was chosen for the circuits’ topology. The main benefit of CPW compared to other techniques like microstrip topology is an easily designable characteristic impedance of $Z_L = 50 \ \Omega$ by the relation of inner line width $w$ to gap width $s$, nearly independent of a given substrate thickness $h$. This increases the potential of miniaturization of the structures in further developments. In addition a single metallization layer is easier and faster to fabricate. For coplanar waveguide topology the values for inductance per unit length $L'$, capacitance per unit length $C'$ and resistance load per unit length $R'$ can be calculated by (2) with the characteristic impedance $Z_L$, the effective dielectric constant $\varepsilon_{r,\text{eff}}$, the vacuum speed of light $c_0$ and the attenuation constant $\alpha$.
3. Quarter Wavelength Resonators

In principle, our quarter wavelength resonators (distributed cavities) are realized as CPW structures as shown in figure 1 working as kind of a notch filter. The cavity circuit consists of a transmission line with a length \( \ell \) corresponding to the resonance frequency \( f_0 \) and a capacitive coupling element at the open side to the feed-line which is realized as coupling gap \( s_c \). A shortcut to the ground plane determines the second end of the structure. The transmission line length \( \ell \) can be calculated with the wavelength \( \lambda \) or in more detail with the vacuum speed of light \( c_0 \), the effective permittivity \( \varepsilon_{\text{r,eff}} \) and the resonance frequency \( f_0 \) by

\[
\frac{\lambda}{4} = \frac{1}{4} \cdot f_0 \cdot (\varepsilon_{\text{r,eff}})^{\frac{1}{2}}.
\]

(3)

In a CPW the effective dielectric constant depends on the ratio \( w/(w+2s) \) and \( s/h \) of the inner line width \( w \), gap width \( s \) and substrate thickness \( h \). For thick substrates, \( h \) can be assumed infinite, therefore the value of \( \varepsilon_{\text{r,eff}} \) saturate to \( \varepsilon_{\text{r,eff}} = (\varepsilon_r + 1)/2 \). Finally all values for resonator length \( \ell \) and cross section parameter \( w \) and \( s \) of the CPW for 50 \( \Omega \) characteristics are calculated.

3.1. Simulation

A lumped element equivalent circuit of a quarter wavelength resonator consists of a parallel circuit with an inductance \( L_p \), a capacitance \( C_p \) and a resistance \( R_p \) as shown in figure 2. The shorted end is directly connected to ground and the open end is capacitive coupled to the excitation port via capacitance \( C_c \). The cavity circuit elements \( L_p, C_p \) and \( R_p \) can be calculated by (4) with resonator length \( \ell \), inductance per unit length \( L' \), capacitance per unit length \( C' \), characteristic impedance \( Z_0 \), and attenuation constant \( \alpha \). Furthermore the resonance frequency \( f_r \) and the intrinsic quality factor \( Q_i \) of the parallel cavity circuit (without effects of the coupling capacitance) can be determined.

\[
L = \frac{8\ell}{\pi^2} \cdot L', \quad C = C' \cdot \frac{\ell}{2}, \quad R = \frac{Z_0}{\alpha \cdot \ell}, \quad f_r = \frac{1}{2\pi \sqrt{LC'}}, \quad Q_i = 2\pi f C_p R_p
\]

(4)
Intrinsic quality factor values of $Q_i = 3 \cdot 10^6$ can be calculated using design values from section 3.2. Simulations were made in Microwave Office [7] including material parameters of metal and substrate. This lumped model based simulation method allows fast simulation results of the whole resonator structure including coupling capacitances. The complete simulation model contains model based parts for the transmission line sections and a lumped element coupling capacitance $C_c$ for connection to the feed line. To improve the quality of simulation results, a part of the resonators was simulated with the EM simulation software Sonnet [8] in more detail e.g. for material parameters of the superconducting niobium films. Slight changes in these capacitances cause large effects on the resonance frequency of the circuit. Accurate simulations of the coupling gaps $s_c$ were done followed by extraction of $s$-parameter sets and calculation of the corresponding coupling capacitance values using a π-equivalent circuit. Several resonators with different values for $C_c$ were simulated to achieve high quality factors $Q_L$. A coupling capacitance of about $C_c = 1.3$ fF show best results in respect of $Q_L$. Stronger coupling as well as weaker coupling shows diminution of the values because of very strong and very weak influence on the cavity circuit respectively.

The designed quarter wavelength resonators are one port devices which can be characterized by measurement of the reflection parameter $|S_{11}|$. Simulations show a notch at resonance frequency where the loaded quality factor $Q_L$ is calculated by the resonance frequency $f_0$ and the 3dB-bandwidth $BW_{3\text{dB}}$ over the definition $Q_L = f_0/BW_{3\text{dB}}$. Table 1 shows the simulated quality factor $Q_L$ for different gap width $s_c$. An increase of $Q_L$ can be observed with increasing coupling gap width corresponding to decreasing coupling capacitances.

### 3.2. Structure

A first approach was realized as a single CPW resonator in a straight structure (figure 1) with an inner line width $w = 500 \, \mu m$, a gap width $s = 180 \, \mu m$ to the ground plane and a length $\ell = 5300 \, \mu m$ to acquire measurement data of achievable quality factors. The ratio of inner line width $w$ to gap width $s$ correspond to the chosen characteristic impedance $Z_L = 50 \, \Omega$ on the r-cut sapphire substrate (loss tangent $\tan \delta = 1 \cdot 10^{-5}$). For the chosen CPW geometry $\varepsilon_{\text{eff}}$ can be estimated to 5.53 using the dielectric constant $\varepsilon_r = 10.06$ of the sapphire. Resonator structures for the designed centre frequency of 6 GHz and feed-line are implemented on an area of 14 x 7 mm². Different resonators with variations of the coupling gap $s_c$ were written by direct laser writing for reliable dimensions, fabricated and measured.

### 3.3. Measurements

The resonator samples were housed in a gold-plated brass housing, connected by SMA-connectors with an Agilent E8361A network analyzer and cooled down to liquid helium temperatures. Calibrations of the measurement setup were done at ambient temperature. A broadband view and a narrow band view of one measurement data set are shown in figure 3. Measurement results for different coupling gap variations at 4.2 K are listed in table 1. The measured resonance frequencies are close to the simulated values where a relative deviation of about 2.2% can be observed for all samples.

| $s_c$ / µm | 830  | 885  | 900  | 910  |
|------------|------|------|------|------|
| $f_{r,\text{sim}}$ / GHz | 5.96 | 5.97 | 5.97 | 5.97 |
| $Q_{L,\text{sim}}$  | 8000 | 30000 | 45000 | 60000 |
| $f_{r,\text{meas}}$ / GHz | 6.09 | 6.10 | 6.10 | 6.10 |
| $Q_{L,\text{meas}}$ | 88000 | 108000 | 130000 | 266000 |
Figure 3. Measurement result of a quarter wavelength resonator with coupling gap $s_c = 910 \, \mu$m in a broadband view (left hand side) at 4.2 K. The picture on the right hand side shows a narrow band view of the same measurement.

Simulation results as well as measurements showed an increase of the quality factor with widening coupling gap $s_c$. Generally simulations show smaller values for resonance frequency and quality factor than measurements. This can be explained by the anisotropic behavior of $\varepsilon_r$ of sapphire described in section 2.1. The resonator structures were oriented in a defined angle to the $c$-axis of sapphire. A smaller dielectric constant as $\varepsilon_r = 10.06$ which was used in the simulations can explain the differences between simulation and measurement values. In this case the resonance frequency is shifted to higher values and the weaker coupling points to smaller values of $C_c$ which leads to higher values of the quality factor $Q_L$. This leads to the assumption that the angle orientation of resonator structure and sapphire substrate direction deviates from the simulation assumed values.

3.4. Temperature dependent quality factor

During cool down of the cavity circuits to 4.2 K dependences of the resonances on temperature can be observed. As shown in [9] the quality factor of the resonance improves for temperatures far below $T_C$ up to a saturation value that points to a limitation of the resonance and therefore $Q_L$ likely caused by a smaller $R_s$. Measurements of the cavity circuits show an unexpected behavior of the resonances in dependence of the temperature. While cooling the device from transition temperature down to liquid helium temperature, the measurement of $Q_L$ was performed. At a certain temperature the amplitude of the notch becomes minimal that indicates a maximum achievable value of $Q_L$. The temperature has been adjusted to reach quality factor values $Q_L > 1 \times 10^6$. Table 2 shows the corresponding values for temperature, center frequency and $Q_L$ when measurements of the resonance dip reaches maximum values for $Q_L$. This behaviour can be explained by kinetic inductance effects [10]. Near the transition temperature $T_C$ kinetic inductance is a very strong function of temperature. The current density becomes essentially uniform so that the kinetic inductance $L_k$ is dependent on a geometrical factor and the London penetration depth $\lambda_L$ (here $L_k \sim \lambda_L$). According to Ginzburg-Landau theory $\lambda_L$ depends on the inverse temperature difference between $T$ and $T_C$ that explains these temperature effects. Temperature changes show similar effects as excitation of quasiparticles by incoming radiation. The growing temperature increases the quasiparticle density and therefore the shift of the resonance. Table 2 shows the optimal conditions using the excitation effects of the kinetic inductance for resonance at approximately 6.12GHz which leads to the conclusion that the coupling gaps differ from the optimal combination of resonator and coupling gap at 4.2 K. Thus, measured quality factors approach related values and exceed the calculated intrinsic quality factor values for quarter wavelength resonators caused by kinetic inductance effects at temperatures above 4.2 K.
Table 2. Measurement results of the quarter wavelength resonators in respect of optimized quality factor $Q_L > 10^6$.

| $s_c / \mu m$ | 830 | 885 | 900 | 910 |
|-------------|-----|-----|-----|-----|
| $T / K$     | 6.1 | 5.8 | 5.4 | 4.6 |
| $f_{r,\text{meas}} / GHz$ | 6.11 | 6.12 | 6.12 | 6.11 |
| $Q_{L,\text{meas}} \times 10^6$ | 4.07 | 5.72 | 7.65 | 4.30 |

4. Conclusion

Different quarter wavelength resonators on a coplanar waveguide for kinetic inductance detectors were designed, implemented and measured using thin sputtered Nb films. The very high values of $Q_L$ above $10^6$ show the potential for improvement of the high sensitive read-out systems for kinetic inductance detector applications. In dependence of the coupling capacitance $C_c$ and the coupling gap $s_c$ the cavity circuits show very high quality factors $Q_L$ measured with the very sensitive reflection parameter $|S_{11}|$. Small mismatches in dimensions and fabrication parameters lead to strong influence on $Q_L$. An accurate implementation of the coupling capacitances in the simulation model and reduction of tolerances in the fabrication process were introduced to achieve a better agreement between simulations and measurements. The density of quasiparticles changes with temperature, especially near $T_C$ where Cooper pairs were excited to quasiparticles. Our measurements show these influences by detuning of the resonance frequency even for our large structures of cavity circuits. On the other hand the performance concerning the achievable quality factor of resonators is expected to increase with lower operation temperature in the mK temperature range because of decreasing surface resistance $R_s$. Further optimizations between resonator structure and coupling gap as discussed in section 3.4 have to be considered in future devices.

5. Acknowledgments

The authors would like to thank H. Wermund for preparation of niobium films, W. Loeffler for support in laser writing of structures and A. Stassen for lithography and assembly of samples. This work was supported partly by DFG Research Center of Functional Nanostructures.

References

[1] Day P K et al 2003 Nature 425 817–21
[2] Baselmans J et al 2005 Bulletin de la Société Royale des Sciences de Liège 74 5–6
[3] Kwok R S et al 1999 IEEE Trans. Microwave Theory and Techn. 5 47 586–90
[4] Hein M 1999 High-Temperature-Superconductor Thin Films at Microwave Frequencies (Springer tracts in modern physics no 155) (Berlin: Springer-Verlag)
[5] Benz G et al 1998 Cryogenics 6 38 697–700
[6] Benz G et al 1999 IEEE Trans. on Appl. Superconductivity 2 9 3046–49
[7] Applied Wave Research Inc. Manual of the program Microwave Office (United Kingdom: 2nd Floor, 24 Bucklersbury, Hitchin, Herts. SG5 1BG)
[8] Sonnet Software Inc. Manual of the program Sonnet (USA: 1020, Seventh North Street, Suite 210, Liverpool, NY 13088)
[9] Weber F M 2005 One dimensional resonators for enhanced quantum bit control master thesis (Stockholm: Royal Institute of Technology (KTH))
[10] Meservey R and Tedrow P M 1969 J. Appl. Phys. 5 40 2028–34