Multiple knapsack problem for racking selection model

Danan Anbiyaa¹, Erlangga Bayu Setyawan² and Humiras Hardi Purba¹

¹ Master of Industrial Engineering Program, Mercu Buana University, Jakarta, Indonesia
² Department of Industrial Engineering, Telkom University, Bandung, Indonesia
*anbiyaad@gmail.com

Abstract. Choosing the right racking selection in a warehouse is important, because in addition to increasing warehouse capacity and utility, it will also speed up the picking process. Racking system design is used to choose the type of rack that best matches the characteristics of the warehouse and can increase capacity with a low investment cost. In addition, the use of racking can utilize empty space in the vertical part of the warehouse so that warehouse utilization can increase. We used the study case in a warehouse chemical in oil and gas companies. The warehouse is a push system to serve the production process. In this study we conducted racking selection by adopting a model of multiple knapsack problem. We use tree criteria in combining racking systems, the capacity, investment cost and cross aisle cost.

1. Introduction

In Supply Chain, warehouse has an important role to increase business success in the level of cost and customer service. Warehousing is one of the most important and critical activities in logistics in the industrial system and also services. Warehouse is divided into several types according to its warehouse role, namely raw material warehouse, semi-finished warehouse, finished good warehouse, distribution warehouses and distribution centers, fulfillment warehouses and fulfillment centers, and local warehouses [1].

In this study, we will study the warehouse of raw materials in an oil and gas company. The warehouse is a push system and serves to supply chemicals as raw material for the production process at one of the refineries. Push System is an operating system that occurs only as a response to scheduling for each operation without considering the real-time status of the operation concerned. The purpose of this system is to operate scheduling. To deal with the possibility of changes in consumer demand, companies provide stock or inventory at each work station. This is to prevent changes in the entire schedule of each production line when there is a change in consumer demand [2]. This system often results in the formation of unbalance stock between processes, which results in the emergence of dead stock, addition of handling equipment, and the addition of people to maintain inventory.

The problem faced by the company is that the available capacity is unable to handle the throughput needed for the production process. Block stack capacity that cannot meet the pallet's position every month results in overloaded blocks, so that the product is placed outside the storage location or called out of block. This causes the activity in the warehouse to be disturbed and cause danger, considering the type of product stored is a chemical product.
To solve these problems, we develop mathematical models by adopting the multiple knapsack problem method. Knapsack problem is determining the number of items from each object that must be inserted into the knapsack (container) so that the total value obtained is as maximum as possible, where each object has a certain weight. This study aims to determine the number of shelf combinations in a storage area (warehouse) so that the total value obtained is as optimal as possible (in this study minimizing investment costs and cross-aisle costs).

2. Related Works

Multiple Knapsack Problem (MKP) is a generalization on the standard Knapsack Problem (KP) which has the objective to assign each item to at most one of the knapsacks such that none of the capacity constraints are violated and the total profit of the items put into knapsacks is maximized with (possibly) different capacities [3]. This MKP is often applied to the context such as capital budgeting, cargo loading, cutting stock problems, and asset-backed securitization [4]. Basically, MKP is a knapsack method for specifying subset problems of items \( n \) for different \( m \) bin (knapsack) such that each item has a profit \( p(i) \) and a size \( s(i) \), and each bin \( j \) has a capacity \( c(j) \), so that the total profits obtained can be maximized without exceeding the capacity of the container provided. This paper discusses the use of MKP to optimize layouts in three-dimensional warehouses by considering determinants of lane depth, number of storage levels, lateral depth and longitudinal width. Optimization of three-dimensional warehouses by designing a racking system match the characteristics of the chemical warehouse and can increase capacity but have low investment costs. In this study, the knapsack or capacity in the warehouse is divided into two zones, with a combination of two types of racks that have low investment costs and security factors that relate to goods handled in warehouses are chemicals that require special handling. This study aims to fulfill the pallet position needs of the warehouse by considering the investment costs and cross-aisle costs use the formulation of a linear programming mathematical model. Some references used to conduct this research can be seen in Figure 2.

Research that uses MKP aims to find a subset of items that have a feasible packing in sets of bin (knapsack) and maximizes the profit of packing [4],[5],[6]. In the previous research, the use of MKP was not applied in warehouse optimization. Based on the criteria found in MKP, in determining the maximum capacity consider the size of the knapsack used. The consideration used for optimization of the three dimensional warehouse is lane depth, number of storage levels, lateral depth and longitudinal width [7][8].

### Multiple Knapsack Problem

| Author          | Year | Description                                                                                                                                 |
|-----------------|------|--------------------------------------------------------------------------------------------------------------------------------------------|
| Khanna, 2006    |      | Find a subset of item that has a feasible packing in set of bin (knapsack) and maximizes the profit of the packing.                           |
| This paper has a problem with different bin capacities.  |
| So in this paper we resolve the approximability of MKP by obtaining a PTAS for it.                        |
| Angelelli, Mamini, & Grazia Speranza, 2010 |      | The result of the paper is to develop new concepts for solving the MKP using ILP-based and memetic algorithms.                           |
| Yu, Gao, & Lei, 2012 |      | This paper describes a new model for facility location capacity problems by considering radius serve and economic serve benefits.          |
| The objective function in the proposed model is to maximize the total return investment.                |
| However, the objective for multiple knapsack problem is to maximize the total profit.                   |
| The Model propose a hybrid harmony search algorithm, which incorporate harmony search algorithm with greedy algorithm. |

### Layout Optimization

| Author          | Year | Description                                                                                                                                 |
|-----------------|------|--------------------------------------------------------------------------------------------------------------------------------------------|
| Rakesh & Adil, 2015 |      | The objective of this paper to develop an algorithm that determines lane depth, number of storage levels, lateral depth and longitudinal width of a three-dimensional order picking warehouse minimizing space and material handling costs. |
| Anbiya & Setyawan, 2018 |      | The objective function in the proposed model is to optimize the number of rack combinations by considering the minimum investment and cross-aisle costs use the formulation of MKP Model by a linear programming mathematical model. |

Figure 1. Related Works
3. Discussion

3.1. Racking System Selection
The planning of racking systems begins by identifying the type of rack to be used. There are several types of racks used in warehouse racking system designs such as selective, double-deep, drive-in, gravity and others. The use of shelves depends on the needs of the users. The leading company is a company engaged in the oil and gas sector so the safety becomes number one priority. The characteristics of the product to be stored are in the form of chemical so that special storage is needed. Based on Table I, types of racking system that are suitable only selective and double-deep rack because of the characteristics of the material handled by the warehouse are chemical so that the warehouse requires special treatment such as storage that has certain specifications. The new warehouse design considers material characteristics that are chemical in nature so that the security factor is the main thing, therefore the new warehouse will be divided into four areas:

1. Area 1 is an area for SKU in a safe category.
2. Area 2 for SKU with liquid category.
3. Area 3 for SKU is a solid flammable category.
4. Area 4 for products is in reactive category.

Warehouse area distribution is based on safety factors so there is a barrier or safe area between 1,2 and 3,4 areas. In addition, there is a boundary provision, namely the placement of shelves between area 1 and 2, and area 3 with 4 may not coincide (min 5 meters of safe distance).

| Racking Type          | Benefit                                      | Weakness                                      |
|-----------------------|----------------------------------------------|-----------------------------------------------|
| Single-deep (Selective) Rack | Simple, support fast operation, flexible [9]   | Requires excessive aisle space                 |
| Double-deep Rack      | Need a little space for aisle                | Slow, accessed only in one direction           |
| Drive-in Rack         | Has a high density, on average has 5-10 lane depth, and 3-5 levels | Need high-skilled operators because MHE enters the rack, so it doesn't match the characteristics of the warehouse chemical (can harm the operator), accessed only in one direction |

3.2. Determine Number of Rack and Rack Dimension
Determining the number of rack levels is determined based on the height of the warehouse as high as 12m, in the calculation of the warehouse can accommodate up to the number of rack levels as much as 7 levels. Determination of the number of levels based on the height of the rack to be designed. Rack height can be determined by adding the height of the pallet to the product height and the allowance provided between the product and the next shelf [7]. Allowance is given 0.15m to make it easier for forklift operators to carry out warehouse activities. The highest product of 0.94m and the height of the pallet used is 0.14m so the total shelf height is 1.34m. Figure 3 shows the calculation of the shelf height (in units of m) in the front of the shelf.

![Figure 2. Rack Design (Length).](image1)

![Figure 3. Rack Design (Length).](image2)
Length calculation for selective and double-deep shelves uses a length of one bay on average 2 pallets with allowance between 0.15m pallet and allowance between pallet and rack of 0.15m, total rack length of 2.85m. Figure 4 is the calculation of the shelf length (in units of m).

### 3.3. Material Handling Selection

Selection of MHE according to [10] based on material characteristics and material flow. Selection of MHE in a chemical warehouse using MHE forklift with pantograph type reach forklift truck. The physical properties of the material handled in the warehouse are solid, liquid and gas. Therefore, the authors propose a safer MHE part which is an additional part that can clamp the material is liquid. While the pantograph-reach with front parts such as forks are used to take products with solid physical properties and gas. The visualization of pantograph for forklift can be seen in Figure 5 and the specification of material handling can be show in Table 2.

**Table 2.** Material Handling Specifications.

| Lifting Speed (Laden) | 0.55 m/s |
|-----------------------|---------|
| Travel Speed (Laden)  | 12.1 Km/h |
| Horizontal Reach      | 2 Depth |
| Vertical Reach        | 10 m    |

### 3.4. Aisle Calculation

The size of the aisle is not right in the current state of the warehouse, so the proposal this time aisle considers the dimensions of the chosen MHE. The MHE used is a reach pantograph with a length of 3.1m and a width of 1.4m and diagonal 3.4m. Diagonal calculation using the formula:

\[ d = \sqrt{p^2 + l^2} \]

\[ d = \sqrt{3.1^2 + 1.4^2} = \sqrt{11.57} = 3.401 \]

The width of the aisle is determined by the MHE used, namely the pantograph reach which has a diagonal of 3.4 and an allowance of 0.1m so that the width of the aisle is 3.5m.

### 3.5. Racking Cost

The minimal investment cost is the objective of this study; therefore, the initial investment costs consist of investment. The investment cost of the shelf from the beginning of manufacture to installation because of the current condition of the warehouse does not have a shelf so it costs $70 / pallet for each type of rack but for the double-deep pallet type there is the addition of lower beam parts for $6 / set with a set of 2 pallets.

### 3.6. Cross Aisle Cost

Cross aisle costs are the operational costs required by material handling in carrying out activities in the warehouse. The calculation of the cross-aisle cost design begins with the calculation of material handling costs which is then used as a decision variable in the objective function so that the company can estimate the costs incurred in each alternative shelf design. Cross aisle cost calculation can be seen in Table 3.

**Table 3.** Cross Aisle Calculation.

| Rack Type         | Warehouse Length (m) | Warehouse Width (m) | Number of Aisle | Cross Aisle Cost ($/m) | Cross Aisle Cost ($/pallet) |
|-------------------|----------------------|---------------------|-----------------|------------------------|-----------------------------|
| Selective         | 66                   | 30                  | 10              | $7.47                  | $10.64                      |
| Double Deep       | 66                   | 30                  | 6               | $4.84                  | $6.89                       |
3.7. Racking Selection Model

To select the number of racking selection, we use a model of multiple knapsack problems. To adapt this model in determining the number of racking, the warehouse will be assumed as a large container which will be filled with several racking and racking will be considered as a sub-container. The optimization model used aims to minimize investment costs, which consist of capital expenses (racking system purchase costs) and operational expenses (cross aisle operational costs).

**Objective Function:**

\[
\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} p_j x_{ij}
\]  

(3)

Where, \( m \) = number of zones, \( n \) = number of slots of racking system, \( i \) = zones of racking system, \( j \) = type of racking system, \( p_j \) = considered parameter, \( x_{ij} \) = decision variables. While \( p_j \) is the parameter used in this study is the initial investment cost which consists of rack investment costs and cross-aisle cost design so that the parameters become:

\[
p_j = \sum_{i=1}^{n} [q_i + r_i]
\]  

(4)

Notation \( q_1 \) = selective racking system investment cost, \( q_2 \) = double-deep racking system investment cost, \( r_1 \) = cross aisle cost of selective racking system, \( r_2 \) = cross aisle cost of double-deep racking system. From equation (3) and (4), the objective function is transformed into notation (5).

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} [q_i x_{ij} + r_j x_{ij}]
\]  

(5)

**Constrains:**

*Pallet Position Requirements*

\[
x_{11} + x_{12} + x_{21} + x_{22} + x_{31} + x_{32} + x_{41} + x_{42} \geq 688
\]  

(6)

Equation (6) states the limits of the minimum number of position palettes that must be available in the warehouse. Calculation of the number of pallet needs obtained from the calculation of the average stock from January to December as many as 688 pallet positions.

*Number of pallet positions on each alternative rack*

Racking systems have different dimensions, so that each area of the warehouse of leading company has a different capacity for each type of racking system. Equation (7) - (14) shows the limit of the capacity of each area for each rack.

\[
x_{11} \leq 228
\]  

(7)

\[
x_{12} \leq 252
\]  

(8)

\[
x_{21} \leq 192
\]  

(9)

\[
x_{22} \leq 204
\]  

(10)

\[
x_{31} \leq 264
\]  

(11)

\[
x_{32} \leq 324
\]  

(12)

\[
x_{41} \leq 192
\]  

(13)

\[
x_{42} \leq 204
\]  

(14)

**Number of pallet positions on each area**

Each area has a different number of SKUs. The constrains used in this mathematical model is the number of pallet positions in each area based on data in Table 4.

| Area | Number of SKU | Percentage |
|------|---------------|------------|
| Area 1 | 22            | 34%        |
| Area 2 | 14            | 22%        |
4. Result

To complete the mathematical model, we used Mixed Integer Linear Programming with the help of Lingo. The calculation results of the lingo software show that the rack combination is using double deep in area 1 as much as 236 pallet position, area 3 as much as 208 pallet position, and area 4 as much as 139 pallet position while for area 2 using selective rack as much as 105. In line with the objectives of this study, it is to choose a shelf with minimum investment and cross-aisle costs considering storage capacity. Double deep shelves have greater storage capacity than selective shelves. One of the factors chosen for double deep shelves is the cost of cross-aisle which is cheaper than selective because this type of rack can utilize the land to a minimum so that it can minimize the number of aisles. The results of the calculation of the rack combination indicate that storage can increase up to 688 pallet positions, if the storage increase percentage reaches 160% of the current condition of the warehouse. Table 5 show the detail of the improvement.

| Area 3 | 13 | 23% |
| Area 4 | 13 | 20% |

Calculation of the number of pallet positions in each area uses the multiplication between the percentage of SKUs and the total pallet requirement so that the limits used are shown in equation (15) - (18).

\[
x_{11} + x_{12} \geq 236 \\
x_{21} + x_{22} \geq 105 \\
x_{31} + x_{32} \geq 208 \\
x_{41} + x_{42} \geq 139
\]  

\[(15) \quad (16) \quad (17) \quad (18)\]

5. Conclusion

The selection of the right racking selection can increase the storage capacity in a warehouse. The right choice of racking, in addition to increasing storage capacity, will also minimize costs, both initial investment costs (racking system purchasing costs), and operational costs. In the case study that we used in this study, the problem faced by the leading company is that the company wants the right racking investment to increase capacity, however, with the least cost. To solve this problem, we developed a mathematical model that adopted multiple knapsack problems. The model is solved by mixed integer linear programming. The results obtained are capacity can increase by 160% so that it can meet the storage capacity requirements. Suggestions for further research, more consider many variables and multi-objectives in selecting the racking system.

6. References

[1] E. H. Frazelle, *World-Class Warehousing and Material handling*. Singapore: McGraw-Hill, 2012.
[2] E. B. Setyawan, “Integrated Railway Timetable Scheduling Optimization Model and Rescheduling Recovery Optimization Model: A Systematic Literature Review.”
[3] D. P. Kellerer Hans, Ulrich Pferschy, “Knapsack Problems,” 2004.
[4] E. Angelelli, R. Mansini, and M. Grazia Speranza, “Kernel search: A general heuristic for the multi-dimensional knapsack problem,” *Comput. Oper. Res.*, vol. 37, no. 11, pp. 2017–2026, 2010.
[5] H. Yu, L. Gao, and Y. Lei, “Introduction 2 Formulations for Capacitated,” pp. 1773–1776, 2012.
[6] S. Khanna, “Polynomial Time Approximation Scheme for The Multiple Knapsack Problem,” *Bell Labs Tech. J.*, vol. 2, no. 2, pp. 36–41, 2006.
[7] V. Rakesh and G. K. Adil, “Layout Optimization of a Three Dimensional Order Picking Warehouse,” IFAC-PapersOnLine, vol. 28, no. 3, pp. 1155–1160, 2015.

[8] F. Bindi, R. Manzini, A. Pareschi, and A. Regattieri, “Similarity-based storage allocation rules in an order picking system: An application to the food service industry,” Int. J. Logist. Res. Appl., vol. 12, no. 4, pp. 233–247, 2009.

[9] D. D. Damayanti, E. B. Setyawan, and L. Andrawina, “Warehouse Picking Model for Single Picker Routing Problem in Multi Dimensional Warehouse Layout using Genetic Algorithm Approach To Minimize Delay.”

[10] T. J. A., J. A. w., and Y. B., A., Facility Planning Fourth Edition. 2010.