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Revisiting kinematic fast dynamo in three-dimensional magnetohydrodynamic plasmas: dynamo transition from non-helical to helical flows

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Abstract

Dynamos wherein magnetic field is produced from velocity fluctuations are fundamental to our understanding of several astrophysical and/or laboratory phenomena. Though fluid helicity is known to play a key role in the onset of dynamo action, its effect is yet to be fully understood. In this work, a fluid flow proposed recently (Yoshida et al. 2017, Phys. Rev. Lett. 119, 244501) is invoked such that one may inject zero or finite fluid helicity using a control parameter, at the beginning of the simulation. Using a simple kinematic dynamo model, for the considered flow, we demonstrate unambiguously a strong dependency of short scale dynamo on fluid helicity. In contrast to conventional understanding, it is shown that fluid helicity does strongly influence the physics of short scale dynamo for the flow profiles considered. To corroborate our findings, late time magnetic field spectra for various values of injected fluid helicity is presented along with “geometric” signatures of the 3D magnetic field surfaces, which shows a transition from ‘twisted ribbon’ or ‘twisted’ sheet to ‘cigar’ like configurations. This work brings out, for the first time, the role of fluid helicity in the transition from ‘non-dynamo’ to ‘dynamo’ regime systematically. It is also shown that one of the most studied ABC dynamo model is not the “fastest” dynamo model for problems at lower magnetic Reynolds number.

1. Introduction

The theories of HydroDynamics (HD) [1, 2] and MagnetoHydroDynamics (MHD) [3–5] are often used to analyze the HD turbulence and magnetized plasma turbulence respectively, which are fundamental to our understanding of the behaviour of astrophysical plasmas present in the Sun or other young stars. The presence of small scale, mean or large scale magnetic fields are very much common in such astrophysical objects, planets, stars, interstellar medium, galaxy, accretion disks and also in the Sun [6–8]. Understanding the origin of these multi scale magnetic fields, which are in turn responsible for various complex phenomenon in the Universe, is of paramount importance. Naturally a relevant question arises as to, what are the sources of these multi scale magnetic energy? As first pointed out by Larmor[9] and later Parker[10], such magnetic fields are generated by the motion of conducting fluid which amplify a ‘seed’ magnetic field, leading to a transfer of kinetic to magnetic energy, that is, via Dynamo action.

Depending on the length scales involved, dynamos are broadly classified into two categories: Small Scale Dynamo (SSD) and Large Scale Dynamo (LSD) or mean field dynamo. For LSD, it is believed that a lack of reflectional symmetry (or in other words, nonzero fluid helicity) to be a crucial element, whereas for SSD, no correlation has been shown to exist between onset and sustenance of SSD on fluid helicity [11]. Depending on the time scale, dynamos may further divided into two categories: Fast dynamos (growth rate remain finite in the limit $R_m \rightarrow \infty$) and Slow dynamos (magnetic diffusion plays a significant role) [11]. In a fast dynamo model,
below a certain value of resistivity, the growth rate becomes insensitive to the magnitude of resistivity, i.e. becomes independent of resistivity [12]. A popular example of fast dynamo is the Solar dynamo [13]. Depending on the feedback strength of the magnetic field on to the fluid motion, dynamos are categorized as linear or nonlinear. A linear dynamo is one in which the magnetic field dynamics does not ‘back react’ with the velocity field and the velocity field is either given or it obeys the NS equation [11], whereas a self-consistent dynamo or a nonlinear dynamo tends to change the flow - as the magnetic field becomes large enough - to further impede magnetic field growth. That is, the flow and the B-field ‘back react’ on each other, leading to a nonlinear saturation [11].

According to Zel’dovich anti-dynamo theorem [11, 14], a planar flow cannot excite dynamo instability. Hence, it is now well acknowledged that, to achieve fast dynamo, a careful selection of plasma flow profile is necessary, as the key mechanism behind the fast dynamo action is that of stretching, twisting and folding of magnetic field lines, ‘advected’ in the fluid flow. Hence the flow is expected to have ‘enough dynamics’ to amplify the magnetic field exponentially.

Among various astrophysical flows addressed, the Arnold-Beltrami-Childress (ABC) flow is a well-known prototype for fast dynamo action for its stretching ability and chaotic transport properties. A detailed dynamo study using ABC flow has been reported by various authors [15–21]. Along with the exponential growth of magnetic energy, the magnetic energy iso-surfaces are known to exhibit special signature called ‘cigar’ like [22] or ‘ribbon’ like [20, 22] structure. For ABC flow profile, the vorticity field is parallel to the velocity direction, i.e. $\mathbf{\nabla} \times \mathbf{u}$ has a finite parallel component to $\mathbf{u}$ at all times. This naturally indicates that, finite fluid helicity $(\int_V \mathbf{u} \cdot (\mathbf{\nabla} \times \mathbf{u}) dV)$ is present in the flow. The fluid helicity effect on dynamo action is often expressed as an effect, and studied extensively using ABC flow family [23].

Study of dynamo instability using a 3D flows becomes numerically demanding and is perhaps relatively more complex. To reduce the complexity in numerical methods often, in the past, time dependent two dimensional flows ($\mathbf{u}(x, y, t)$) have been taken into consideration for the investigation of fast dynamo action. Naturally, the presence of time dependency introduces chaoticity in such flows. Galloway-Proctor flow, sometimes known as GP flow, is one such well-known flow [24]. In light of the fact that the flow profile is entirely independent of one spatial coordinate, in this case, the $z$-coordinate, it becomes possible to seek monochromatic solutions for the magnetic field of the form $\mathbf{B}(x, y, z, t) = \mathbf{B}(x, y, t) e^{i k_z z}$, so that the problem becomes two-dimensional for a given value of $k_z$. Recently GP flow [24] has been taken into consideration for studying large scale magnetic fields in the presence of velocity shear [25].

Kinematic fast dynamo action has previously been observed for 3-dimensional chaotic ABC flow and 2.5-dimensional time-dependent GP flow. In the present work, we have explored the possibility of dynamo action for the newly proposed YM flow. Some of the interesting and relevant questions are: what is the exact role of fluid helicity in the context of small-scale fast dynamo action? Is there flow field using which one can systematically inject fluid helicity in the system and clearly demonstrate a non-dynamo (meaning a decaying magnetic field and hence should show a decrease of magnetic energy in time i.e. a negative magnetic energy growth rate) to fully developed dynamo (where positive nonzero growth rate is seen at late times) transition, when the fluid flow transits from non-helical to helical flow? Importantly, does the fluid helicity affect short-scale dynamo action?

In past several authors [26, 27] examined the influence of the flow helicity in the context of kinematic fast dynamo action. For example, Hughes et al [27] show that the small scale, fast dynamo will always exist regardless of the flow helicity distribution using three different types of helicity distribution (non zero local and global helicity, non zero local helicity but zero global helicity, and zero local and global helicity). The flow considered by Hughes et al [27] is a time dependent 2-dimensional flow ($\mathbf{u}(x, y, t)$) [24]. Additionally, it was pointed out that as fluid helicity increases, the dynamo growth rate noticeably increases. However, these Authors quickly clarify that this is only coincidence [27].

By keeping these earlier ideas in mind, we study the fast dynamo action using a newly proposed Yoshida-Morrison flow or YM flow [28] to investigate the role of helicity on SSD. It is important to indicate that the whole class of YM flow depends upon all three spatial coordinates i.e. $\mathbf{u}(x, y, z)$ similar to ABC flow. Another interesting and useful aspect of YM flow is that, it is possible to inject finite fluid helicity $(\int_V \mathbf{u} \cdot (\mathbf{\nabla} \times \mathbf{u}) dV)$ in the system, by systematically varying certain physical parameter. More over this flow has a special property that, it establishes a topological bridge between quasi 2-dimensional class and 3-dimensional class of flows. Therefore, our present study focuses on exploring YM flows with varying kinetic energy, chaotic characteristics, and helicity on the onset and sustenance of SSD.

In the present work, we propose a new possible route that connects non-dynamo regime to dynamo regime via fluid helicity injection using Yoshida-Morrison (YM) flow as a prototype. Our spectral calculation shows unambiguously that fluid helicity does affect the dynamics of small-scale dynamos. Our findings systematically connect most of the previous works and brings in several new insights. To corroborate our findings, time dependent magnetic energy spectra, for various magnitudes of injected fluid helicity is calculated. We also show
that, how a ‘cigar’ like iso-surface, emerges naturally from a ‘twisted ribbon like’ iso-surface. Our numerical investigation further indicates that, for experiments with a lower magnetic Reynolds number \( R_m \), the regular ABC flow has a slower growth rate than class YM flows.

The organization of the paper is as follows. In section 2 we present the dynamic equations. Our numerical solver and simulation details are described in section 3. The initial conditions, parameter details are shown in section 4. Section 5 is dedicated to the simulation results on kinematic dynamo action that we obtained from our code and finally the summary and conclusions are listed in section 6.

2. Governing equations

The governing equations to study Kinematic Fast dynamo action for the single fluid MHD plasma are as follows,

\[
\frac{\partial \vec{B}}{\partial t} + \nabla \cdot (\vec{u} \otimes \vec{B} - \vec{B} \otimes \vec{u}) = \frac{1}{R_m} \nabla^2 \vec{B} \tag{1}
\]

\[
\nabla \cdot \vec{B} = 0 \tag{2}
\]

where, \( \vec{u} \), \( \vec{B} \) and \( R_m \) represent the velocity, magnetic fields and magnetic Reynolds number respectively. The magnetic Reynolds number \( R_m \) is defined as, \( R_m = \frac{\eta u_0}{\chi} \), where \( \eta \) is magnetic diffusivity and \( u_0 \) is a typical velocity scale. Time is normalized to Alfvén times and length to a typical characteristic length scale \( L \). The symbol ‘\( \otimes \)’ represents the dyadic between two vector quantities.

For solving the above set of equations at high enough grid resolution, one need a suitable scalable numerical solver.

3. Simulation details: GMHD3D solver

In this section, we discuss the details of the numerical solver along with the benchmarking of the solver carried out by us. In order to study the plasma dynamics governed by MHD equations described above, we have recently upgraded an already existing well bench-marked single GPU MHD solver [29], developed in house at Institute For Plasma Research to multi-node, multi-card (multi-GPU) architecture for better performance [30]. The newly upgraded GPU based magnetohydrodynamic solver (GMHD3D) is now capable of handling very large grid sizes. GMHD3D is a multi-node, multi-card, three dimensional (3D), weakly compressible, pseudo-spectral, visco-resistive magnetohydrodynamic solver [30]. It uses pseudo-spectral technique to simulate the dynamics of 3D magnetohydrodynamic plasma in a cartesian box with periodic boundary condition. By this technique one calculates the spatial derivative to evaluate non-linear term in governing equations with a standard \( \frac{1}{3} \) de-alising rule [31]. OpenACC FFT library (AccFFT library [32]) is used to perform Fourier transform and Adams-bashforth time solver, for time integration. For 3D iso-surface visualization, an open source Python based data converter to VTK (Visualization Tool kit) by ‘PyEVTK’ [33] is developed, which converts ASCII data to VTK binary format. After dumping the state data files to VTK, an open source visualization softwares, VisIt 3.1.2 [34] and Paraview [35] is used to visualize the data. As mentioned earlier, we have upgraded a well benchmarked single GPU solver to multi-node, multi-card (multi-GPU) architecture, for this present work, the new solver’s accuracy with the single GPU solver has been cross-checked and it is verified that the results match up to machine precision. Further several other benchmarking studies have been performed such as, the 3D kinematic dynamo effect [19, 20, 22], have been reproduced with ABC flow at grid resolution 64 \(^3\) (Details of these are not presented here.) As will be discussed in the coming section, numerical simulations reported here are performed in 256 \(^3\) grid size.

To study the kinematic fast dynamo action, an accurate selection of ‘drive’ velocity field is imperative, which we discuss in the section to follow.

4. Initial condition

The study of dynamo action or more precisely kinematic fast dynamo action in the presence of chaotic 3D Arnold-Beltrami-Childress (ABC) flow has been explored by various authors [16–21]. The velocity field for ABC flow is as follows:

\[
\begin{align*}
  u_x &= u_0[A \sin(k_0z) + C \cos(k_0y)] \\
  u_y &= u_0[B \sin(k_0x) + A \cos(k_0z)] \\
  u_z &= u_0[C \sin(k_0y) + B \cos(k_0x)]
\end{align*}
\] (3)
Recently Yoshida et al [28] proposed a new intermediate class of flow, that establishes a topological bridge between quasi-2D and 3D flow classes. The flow is formulated as follows:

\[
\vec{u} = u_0 \alpha \vec{u}_+ + u_0 \beta \vec{u}_-
\]  

with

\[
\vec{u}_+ = \begin{bmatrix}
B \sin(k_0 y) - C \cos(k_0 z) \\
0 \\
A \sin(k_0 x)
\end{bmatrix}
\]

and

\[
\vec{u}_- = \begin{bmatrix}
0 \\
C \sin(k_0 z) - A \cos(k_0 x) \\
-B \cos(k_0 y)
\end{bmatrix}
\]

so that,

\[
\begin{align*}
u_x &= \alpha u_0 [B \sin(k_0 y) - C \cos(k_0 z)] \\
u_y &= \beta u_0 [C \sin(k_0 z) - A \cos(k_0 x)] \\
u_z &= u_0 [\alpha A \sin(k_0 x) - \beta B \cos(k_0 y)]
\end{align*}
\]

where \(k_0, \alpha, \beta, A, B\) and \(C\) are arbitrary real constants. As indicated earlier, we dub this flow (equation (7)) as Yoshida-Morrison flow or YM flow. We consider the value of \(k_0, \alpha, A, B\) and \(C\) to be unity for this present study. The variation of \(\beta\) value in YM flow leads to new classes of flows.

For example, for \(\beta = 0\), Yoshida et al [28] classify this flow as EPI-2D flow (See figures 1(a) & (c)) which is given by:

\[
\begin{align*}
u_x &= u_0 [\sin(y) - \cos(z)] \\
u_y &= 0 \\
u_z &= u_0 [\sin(x)]
\end{align*}
\]

This flow (i.e. equation (8)) is dependent on all the 3 spatial coordinates (i.e. \(x, y, z\)), whereas only two flow components are nonzero. Thus EPI-2D flow is quasi-2D in nature.

Figure 1. Driving velocity contours of Yoshida-Morrison (YM) flow. The velocity contour visualization (Upper row: cross sectional view & Bottom row: 3D volume visualization) is shown for (a) & (c) \(\beta = 0.0\), (b) & (f) \(\beta = 0.2\), (c) & (g) \(\beta = 0.6\), (d) & (h) \(\beta = 1.0\). For increasing \(\beta\) values, the velocity contours become increasingly chaotic and visible separatrix forms emerge, which is a crucial ingredient of chaotic flows. Separatrices exist for all values of \(\beta\), however in figure 1(d) they are only depicted as points S1, S2, S3, S4, S5 and S6 for the case \(\beta = 1.0\).
Figure 2. Normalized fluid helicity as a function of $\beta$ for Yoshida-Morrison (YM) flow. Here $\beta$ is real constant. $\beta = 0.0$ corresponds to non-helical (zero normalized fluid helicity) flow and $\beta = 1.0$ corresponds to maximum helical (maximum normalized fluid helicity) flow. For detail calculation and normalization (See. appendix B).

As can be expected, for $\beta = 1$ equation (7) becomes well known ABC like flow (See figure 1(d) & 1(h)),

$$u_x = u_0[\sin(y) - \cos(z)]$$
$$u_y = u_0[\sin(z) - \cos(x)]$$
$$u_z = u_0[\sin(x) - \cos(y)]$$

(9)

[The only mathematical difference between the flow described in equation (9) and the ABC flow (equation (3)) is that the two terms are being subtracted in the latter while added in the former. Nevertheless, as will be shown below, the flow in equation (9) is identical in its properties to ABC flow (equation (3)).] As $\beta$ is varied from 0 to 1.0, a whole set of intermediate class of flows emerge, such that a normalized fluid helicity is exactly 0.0 for $\beta = 0$ and is maximum for $\beta = 1.0$ (i.e. ABC-like flows). For a comprehensive study, we explore various values of $\beta$ between 0 to 1, keeping all other parameters identical (See figures 1(b) & (f), (c) & (g)). In order to fully comprehend the structure of the flows, we have also employed a number of distinct visualization techniques for flow profiles (See. appendix A for details).

By increasing $\beta$ values we inject normalized fluid helicity in the system (See. appendix B for details). In figure 2, normalized fluid helicity is calculated, for YM flow. It is observed that for increasing $\beta$ values normalized fluid helicity also increases. It is also interesting to note from YM flow velocity contour visualization (See figure 1) that for increasing $\beta$ values, the velocity contours become increasingly chaotic and visible separatix forms emerge, which is a crucial ingredient of chaotic flows. ‘Separatrices’ are referred to the curve which separates different topological flow regions. For instance, for $\beta = 1$, examples of a separatric-like curves are pointed out as $S_1, S_2, S_3, S_4, S_5$ and $S_6$. Following Childress and Gilbert [12], Dorch [19] and Bouya et al [21], we have considered a periodic initial condition for magnetic field. This periodic initial condition is an eigenmode of equation (1) in the case of zero diffusivity. As a reassurance of the reliability of our numerical observation, we have also performed our numerical experiments with an initially random seed magnetic field as well as with a uniform magnetic field and found that the features of dynamo is essentially unaffected (See. appendix C). With these given velocity fields, we perform our simulation, the details of which is given next.

4.1. Parameter details
We evolve the above set of equations discussed in section 2, for class of YM flow profile, in a triply periodic box of length $L_x = L_y = L_z = 2\pi$ with time stepping $(dt) = 10^{-3}$ and grid resolution $256^3$. We have performed grid size scaling study (See appendix D) using Arnold-Beltrami-Childress (ABC) flow [16] for different magnetic Reynolds numbers. It is obvious that $256^3$ grid resolution is more than enough for this problem (See. Appendix D). With these initial conditions and parameter spaces we present our numerical simulation results.
An interesting and useful aspect of YM flow is that, it is possible to inject finite fluid helicity $(\int_V \hat{u} \cdot (\nabla \times \hat{u})dV)$ in the system (See figure 2), by systematically varying certain physically meaningful parameter, i.e. $\beta$.

We study the kinematic fast dynamo action for various $\beta$ value starting from 0.0 to 1.0. For $\beta = 0.0$ value, YM flow leads to EPI-2 dimensional flow [28]. EPI-2 dimensional flow makes a topological bridge between quasi-2D and 3D class of flows [28]. We perform numerical runs for a wide range of magnetic Reynolds numbers $R_m$ and calculate magnetic energy $(E_B = \frac{1}{2} \int_V (B_x^2 + B_y^2 + B_z^2) dx dy dz)$ growth rate $(\gamma = \frac{d}{dt} \ln (E_B(t)))$ at late times (e.g. $t \sim 200$ to 300) (See. Appendix E). From figure 3 it is observed that, for $\beta = 0.0$ there is no prominent dynamo action at late times, even at highest magnetic Reynolds numbers ($R_m = 5000$). Importantly at late times, magnetic energy curve is decaying with respect to time, indicating that the growth rate is negative. We classify this as a non-dynamo. From our earlier discussion it is already evident that, YM flow with $\beta = 0.0$ has zero fluid helicity and hence is not able to amplify the magnetic field in exponential manner i.e. not able to generate dynamo action at late times. This is in line with expectations of Zel’ dovich anti-dynamo theorem [14].

The key mechanism behind the fast dynamo action is believed to be stretching, twisting and folding (STF) of magnetic field lines [36], that generates exponential growth of magnetic energy and hence it is necessary that, the flow should be able to ‘twist’ (i.e. should contain enough chaoticity) and hence should contain nonzero fluid helicity to generate exponential growth of magnetic energy.

It is important to note that, there are examples of non-helical flows, that can produce fast dynamo action, which has been shown as due to cross-helicity effect or the so-called Yoshizawa effect [24, 37]. Also as discussed in the Introduction, several authors have reported dynamo action using 2-dimensional time dependent non-helical flows. For example, GP flow [24, 27], has been used to study kinematic fast dynamo action.

In the following, we perform numerical runs and present simulation results with $\beta = 0.2$ for various magnetic Reynolds numbers $R_m$. The stretching, twisting and folding (STF) of magnetic field lines [36] doubles the magnetic field per cycle because of flux freezing. Presence of higher diffusivity in the system makes the flux freezing condition to be violated, which in turns results in, suppression of exponential growth of magnetic energy. Hence, the fast dynamo action gets hindered for a higher diffusive system. From figure 4(a) it is observed that, the YM flow ($\beta = 0.2$) efficiently amplifies the magnetic field in exponential manner for a wide range of $R_m$ value and exhibits dynamo action. It is important to note that, the magnetic energy growth rate $(\gamma)$ shows a non-monotonic nature with $R_m$ [See figure 4(a)]. In the past, numerous authors [21, 22] have reported a similar non-monotonicity in the growth rate curve for ABC flow. This non-monotonicity is believed to be highly influenced by the flow structure. A careful observation of magnetic energy iso-surface shows that, the energy is concentrated in long and twisted structures or ‘ribbon’ like structures (See figure 4(b) (multimedia view)). Although small scale structures are also present here, they appear to be more ‘organized’ to form special geometric patterns, which is reported earlier [38]. It is notable that, there is visible twisting in the ‘ribbons’ that indicates the presence of fluid helicity in the flow. With respect to earlier case, it is identified that by varying $\beta$
value from 0 to 0.2 we are able to produce fast dynamo action efficiently. For \( \beta = 0.2 \), there is fluid helicity present in the system and the presence of fluid helicity twists the ribbon which produces dynamo action.

For further higher \( \beta \) value say \( \beta = 0.6 \), it is observed that magnetic energy grows exponentially with time for a wide range of \( R_m \) values. We provide the results for magnetic Reynolds number \( R_m \) 50 to 5000 in small steps and observe that, for all the sets of numerical runs, there is prominent dynamo action (See figure 5(a)). This is due to the presence of fluid helicity and twisting ability in the system. The magnetic energy is seen to concentrate in elongated sheetlike or 'ribbon' like structures [See figure 5(b) (multimedia view)] reported earlier as well [38].

Finally, we explore the \( \beta = 1.0 \) case. As discussed earlier, YM flow with \( \beta = 1.0 \) regenerates the classical Arnold-Beltrami-Childress (ABC) flow. As in the earlier cases (viz. \( \beta = 0.0, 0.2, 0.6 \)), we provide runs for a wide range of magnetic Reynolds number \( R_m \) starting from \( R_m = 50 \). As is expected, for \( \beta = 1 \), for all the sets of numerical runs, we observe significant magnetic energy growth (See figure 6(a)). Figure 6(a) reveals a non-monotonic relationship between the magnetic Reynolds number \( R_m \) and the magnetic energy growth rate \( \gamma \).

Figure 4. Kinematic fast Dynamo effect using Yoshida-Morrison (YM) flow with \( \beta = 0.2 \). (a) Magnetic energy 
\[ E_B = \frac{1}{2} \int (B_x^2 + B_y^2 + B_z^2) \, dx \, dy \, dz \] 
growth rate \( \gamma = \frac{d}{dt} \left( \frac{1}{2} \ln E_B(t) \right) \) in log-linear scale (inset: linear-linear scale) for different magnetic Reynolds number \( R_m \). At late times (for example, \( t \sim 200.0 \) to 300.0), the growth or decay rate of magnetic energy \( \gamma \) has been computed. (b) The three-dimensional (3D) magnetic energy iso-surface [multimedia view:PSacdccfsupp1.mp4] (for magnetic Reynolds number \( R_m = 50 \)) is seen to be twisted 'ribbon' like nature. Note that the twisting indicates the presence of fluid helicity in the flow. Simulation details: grid resolution 256\(^3\), stepping time \( dt = 10^{-4} \). Time \( t \) is normalized to Alfvén time.

Figure 5. Kinematic fast Dynamo effect using Yoshida-Morrison (YM) flow with \( \beta = 0.6 \). (a) Magnetic energy 
\[ E_B = \frac{1}{2} \int (B_x^2 + B_y^2 + B_z^2) \, dx \, dy \, dz \] 
growth rate \( \gamma = \frac{d}{dt} \left( \frac{1}{2} \ln E_B(t) \right) \) in log-linear scale (inset: linear scale) for different magnetic Reynolds number \( R_m \). At late times (for example, \( t \sim 200.0 \) to 300.0), the growth or decay rate of magnetic energy \( \gamma \) has been computed. (b) The three-dimensional (3D) magnetic energy iso-surface (for magnetic Reynolds number \( R_m = 50 \)) is seen to be twisted 'ribbon' like nature [multimedia view:PSacdccfsupp3.mp4]. Simulation details: grid resolution 256\(^3\), stepping time \( dt = 10^{-4} \). Time \( t \) is normalized to Alfvén time.
As previously demonstrated, the structure of the flow profile has a significant impact on this non-monotonicity. The visualization of this maximum helical ($\beta = 1.0$) flow leads to visible separatix formation (See figure 1(d)), a signature of chaotic flow which drives the exponential energy growth for $\beta = 1.0$, as the flow becomes maximum helical similar like ABC flow. This exponential growth of magnetic energy using ABC flow via kinematic fast dynamo action is well explored by various authors [19–22]. We also produce identical magnetic energy growth like ABC flow starting from an initial negligible value, using YM flow with $\beta = 1$ (See figure 6(a)). The study of magnetic energy iso-surface also leads to an interesting observation. It is observed that, the exponentially growing nature of magnetic energy generates rigorous geometric structures or likely to say ‘cigar like’ structures (See figure 6(b) (multimedia view)), and it supported by literatures [19–22]. From our numerical experiments, we confirm that ‘cigar like’ structure is a stable structure.

By systematically varying the helicity parameter $\beta$ in the YM flow [28] or in the other words by injecting normalized fluid helicity in the system, we have shown how a twisted ‘ribbon’ like magnetic structures turn into a classical ‘cigar’ like structures. The emergence of these significant structures (‘ribbon’, ‘twisted ribbon’, ‘cigar’) due to the injected normalized fluid helicity are demonstrated via direct numerical simulation. Interestingly, though all these structures have been reported earlier, but the significant connection between these structures using a single fluid flow model, not explored so far. Here in this present study we connect all these different well known structures via normalized fluid helicity injection using YM flows.

Coming back to $\beta = 1.0$ case, as discussed, this class of flow is identical to the well known Arnold-Beltrami-Childress (ABC) flow. It is shown that, the fastest magnetic energy growth or the exponential energy growth for this kind of flow is associated with the generation of a special kind of structure called ‘cigar’ like structure. We study the magnetic Reynolds number ($R_m$) effect on the ‘cigar’ structure. Keeping all the parameters identical, we perform numerical runs for various magnetic Reynolds numbers ($R_m = 450.0, 1000.0, 3000.0, 4000.0$) and visualize the magnetic energy iso-surfaces (See figures 7(a), (b), (c), (d)). From figures 7(a), (b), (c) and (d) it is identified that, the thickness of a ‘cigar’ ($\delta$) decreases with the increase of magnetic Reynolds number ($R_m$). We also calculate the the ‘cigar’ thickness ($\delta$) for various intermediate $R_m$ and plot it (See figure 7(e)). From figure 7(e) it is observed that, the ‘cigar’ thickness ($\delta$) decreases with $R_m$ with a strong scaling of $\frac{1}{R_m}$. This $\frac{1}{R_m}$ scaling is also identified by various authors [20, 22] for ABC flow and suggests Sweet-Parker scaling. Here in this present study, we observe identical scaling for YM flow ($\beta = 1$). This is expected, because, mathematically speaking, the only difference between ABC flow and YM flow ($\beta = 1$) is a change in sign. To eliminate any doubt that YM flow ($\beta = 1$) and ABC flow are identical, we have performed this study as an important benchmark. It also demonstrates the numerical correctness of our approach.

We also calculate the magnetic energy growth rate ($\gamma = \frac{d}{dt} \left( \ln E_B(t) \right)$) at late times (e.g. $t \sim 200.0$ to $300.0$) for some other $\beta$ cases. For higher $\beta$ value, i.e. $\beta = 0.4, 0.6, 0.8$ it is noticed that, the magnetic energy growth rate increases as $R_m$ increases, after some critical value (See figure 8). From our numerical simulation, we observe that for a range of $\beta$ values and at lower magnetic Reynolds numbers ($R_m$), the growth rate of magnetic energy ($\gamma$) is
higher than that of ABC flow (i.e. $\beta = 1$ case) (See figure 8). This observation suggests that for the YM class of flows, maximum fluid helicity does not imply the fastest dynamo action at lower $R_m$ values.

For each of the cases mentioned above, we have determined the magnetic energy spectral density $|\hat{B}(k)|$ (such that $\int |\hat{B}(k, t)|^2 dk$ is the total energy at time $t$ and $k = (k_x^2 + k_y^2 + k_z^2)^{1/2}$). The spectra have a peak at a higher mode number, which is the hallmark of small scale dynamo (SSD), as can be seen in figure 9. Additionally, all of the spectra are truncated at the mode value ($k_{max} = 85$), and the peaks are in the range of $k = 15$ to 30, indicating that it is well resolved from a spectral perspective.

We have already discussed effect of magnetic Reynolds number ($R_m$) on different class of YM flow at different fluid helicity. We now present our results keeping a particular $R_m$ fixed, the effect of $\beta$ or the effect of fluid helicity on the flow. The rate of increase of magnetic energy ($\gamma$) for varying values of $R_m$ is shown in figure 10(a) & (b) as a function of $\beta$. For $R_m = 5000$, it is observed that, by varying $\beta$ value from 0 to 1 magnetic energy generation rapidly enhances. The reason behind this is explained via impact of fluid helicity on dynamo
instability. For $\beta = 0.0$ there is no fluid helicity in the system, whereas for $\beta = 1.0$ the flow is maximally helical, that generates dynamo action. The exponential energy growth for $\beta = 1.0$ flow or the ABC flow is well known so far, but here we identify a possible route that connects a non-dynamo (negative magnetic energy growth rate) regime ($\beta = 0.0$) to a fully developed dynamo (finite positive magnetic energy growth rate) regime ($\beta = 1.0$) via fluid helicity injection. From figure 10(a) it is verified that, the transition from non-dynamo to fully developed dynamo regime is not abrupt, but is a continuous transition that is achieved via fluid helicity injection in small steps.

To bring out the importance of our numerical findings, we examine magnetic energy growth rate with $R_m$ values 4000, 3500 and 3000 for all the class of YM flows (viz. $\beta = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0$). From figure 10(a) the expected transition from non-dynamo to fully developed dynamo regime is identified via fluid helicity injection, as before. From figures 10(a) & (b) it is observed that, for $R_m > 2500$, the $\gamma$($\beta$) curves are monotonic, however they are not for $R_m < 2500$. In figure 10(b), a sharp transition from the non-monotonic to the monotonic behavior is identified. The energy growth is higher for a flow which has lower fluid helicity for $R_m < 2500$. We believe that this non-monotonicity is highly influenced by the flow structure. To understand this non-monotonicity we further investigate for few lower magnetic Reynolds numbers ($R_m$). We plot the magnetic energy growth rate for $R_m = 1000$, $R_m = 450$ & $R_m = 120$ and observe that, the non-monotonicity is more
We have calculated the magnetic energy spectrum and it is observed that, the spectra contain a discernible maxima at a higher mode number, which is the distinguishing feature of small scale dynamo (SSD). This particular observation suggests existence of a non-monotonic relationship between flow field helicity and dynamo. Further investigation is required to completely understand this finding. For example, Lyapunov exponent measurement and cross-helicity dynamics may shed light on this observation. Such a study is beyond the scope of the present work and it would be interesting to explore these in greater detail, in the future.

It is important to indicate that, for this present study addressed so far, we have fixed $\alpha_0 = 1.0$ and changed $\beta$, which basically changes the kinetic energy of the driving flow. We have also considered another possible approach and performed test runs, in which we have kept the average kinetic energy ($\langle E \rangle$) constant (by increasing $\beta$ values and decreasing appropriately the $\alpha_0$ values) and normalized the magnetic Reynolds number using $\langle E \rangle$. Our numerical experiments have been conducted for various magnetic Reynolds numbers. The same unambiguous shift from the non-dynamo to the dynamo regime is seen in our numerical experiment using this initial condition, and the helical structure of the flow is also seen to influence SSD growth and the spectral nature (not shown here). Thus, the major findings are found to be unaffected.

6. Summary and conclusion

In this work, we have performed direct numerical simulations of 3-dimensional magnetohydrodynamic plasmas. We have analyzed kinematic fast dynamo model using YM flow recently proposed by Yoshida et al. [28]. An interesting and useful aspect of this flow is that, it is possible to inject finite fluid helicity $\langle \int \vec{u} \cdot (\nabla \times \vec{u}) dV \rangle$ in the system, by systematically varying certain physically meaningful parameter. Our major findings are:

- Using a simple kinematic fast dynamo model, we demonstrate, a fast dynamo action primarily due to normalized fluid helicity injection. By injecting normalized fluid helicity in the system systematically, we have explored a systematic route that connects ‘non-dynamo’ to ‘fully developed dynamo’ regime, via direct numerical simulation.

- We demonstrate as to, how a ‘twisted ribbon’ like iso-surface gets converted into classical ‘cigar’ like iso-surface with the injection of fluid helicity.

- We have calculated the magnetic energy spectrum and it is observed that, the spectra contain a discernible maxima at a higher mode number, which is the distinguishing feature of small scale dynamo (SSD).

- The conventional understanding is that a lack of reflectional symmetry (e.g. non-zero fluid helicity) is necessary for large scale dynamo action (LSD), whereas, for small scale dynamo (SSD), it is not. In our work, for the flow considered here, we find that helical structure of the flow appears to influence the SSD growth and spectral structure. This is one of the interesting findings of the present work.

- As an interesting aside, we also identify that the most studied ABC flow has lesser growth rate than class of YM flows for lower magnetic Reynolds number problems.

To conclude, with high enough resolution and at various magnetic Reynolds number, we investigate a systematic route that connects non-dynamo to dynamo regime via fluid helicity injection using YM flow as the underlying flow in our kinetic dynamo model. Our work also indicates that, the fluid helicity does affect the dynamics of small-scale dynamos. We believe that, this work brings out unambiguously, the role of fluid helicity on small-scale dynamos and on the transition from ‘non-dynamo’ to ‘dynamo’ regime, via direct numerical simulation. Our observation is seen to conform with the magnetic iso-surface dynamics. We have shown that, how a twisted ribbon like iso-surface slowly converts to cigar like fast dynamo iso-surface with injection of fluid helicity.

In our present work, the key role of fluid helicity in dynamo action is demonstrated. However, in the past, existence of non-helical flows (i.e. flows with zero fluid helicity) to be able to generate dynamo, in Kinematic Fast and Saturation (with magnetic feedback) dynamos models [24, 37], wherein cross-helicity (also called Yoshizawa effect) is shown to play a crucial role. However the investigation of the role of cross-helicity (also called Yoshizawa effect) on YM flow and others, in the context of various dynamo action is beyond the scope of the present work and will be addressed in future communication. The effect of fluid helicity injection in the context of recurrence [39] and quasi-recurrence [40] phenomenon is also an interesting study to investigate out, that will also be reported soon. The role of hydrodynamically unstable shear flow [41] at the largest scale is interesting to investigate in this context. We hope to attempt this problem in the near future.
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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Conflict of interest

The authors have no conflicts to disclose.

Appendix A. Initial velocity profiles

We have also visualized the planer view of the flows (See figure 11).

Figure 11. Visualization in y-x (Upper Row: $z = \frac{\pi}{2}$, Middle Row: $z = \pi$, Bottom Row: $z = \frac{3\pi}{2}$) plane at time $t = 0$. Visible separatrices are marked out only for $\beta = 1.0$ case.
Appendix B. Fluid helicity calculation for Yoshida-Morrison (YM) flow

Fluid helicity is defined as,
\[ H_f = \int_V \mathbf{u} \cdot (\mathbf{\nabla} \times \mathbf{u}) dV. \]

Here,
\[ \mathbf{u} = \hat{\mathbf{u}}(\alpha u_0[\beta \sin(\mathbf{k} \cdot \mathbf{r})] - C \cos(\mathbf{k} \cdot \mathbf{r})) + \hat{\mathbf{j}}(\beta u_0[C \sin(\mathbf{k} \cdot \mathbf{r}) - A \cos(\mathbf{k} \cdot \mathbf{r})]) + \hat{\mathbf{k}}(\alpha A \sin(\mathbf{k} \cdot \mathbf{r}) - \beta B \cos(\mathbf{k} \cdot \mathbf{r})). \]

Taking curl and putting the real constant value, \( \alpha = A = B = C = 1.0, k_0 = 1.0 \), we get,
\[ \mathbf{\nabla} \times \mathbf{u} = \hat{\mathbf{i}}(\beta u_0[\sin(\mathbf{y} - \cos \mathbf{z})]) + \hat{\mathbf{j}}(u_0[\cos(\mathbf{x} - \sin \mathbf{z})]) + \hat{\mathbf{k}}(u_0[\beta \sin(\mathbf{x} - \cos \mathbf{y})]) \]

or,
\[ \mathbf{u} \cdot (\mathbf{\nabla} \times \mathbf{u}) = \beta u_0^2[(\sin(\mathbf{y} - \cos \mathbf{z})^2] - \beta u_0^2[(\cos(\mathbf{x} - \sin \mathbf{z})^2] + u_0^2[(\sin(\mathbf{x} - \beta \cos \mathbf{y})(\beta \sin(\mathbf{x} - \cos \mathbf{y})) \]

or,
\[ \mathbf{u} \cdot (\mathbf{\nabla} \times \mathbf{u}) = \beta u_0^2[(\sin(\mathbf{y}^2 + \cos^2 \mathbf{z} - 2 \sin(\mathbf{y} \cdot \cos \mathbf{z})) - \beta u_0^2[(\cos(\mathbf{x}^2 + \sin^2 \mathbf{z} - 2 \cos(\mathbf{x} \cdot \sin \mathbf{z})) + u_0^2[(\beta(\sin^2(\mathbf{x} - \sin(\mathbf{y} \cdot \cos \mathbf{y}) + \beta \sin(\mathbf{x} \cdot \cos(\mathbf{y})) \]

Integrating over total volume,
\[ \int_V \mathbf{u} \cdot (\mathbf{\nabla} \times \mathbf{u}) dV = 8\pi^2 \beta u_0^2 \]

Finally after simplification one obtains,
\[ \int_V \mathbf{u} \cdot (\mathbf{\nabla} \times \mathbf{u}) dV = 8\pi^2 \beta u_0^2 \]

Considering velocity to be normalized,
\[ H_f = \beta \]

So normalized fluid helicity for YM flow depends upon constant \( \beta \) value of the flow.

Appendix C. Robustness of initial condition

Numerical experiments have been performed (See figure 12) with uniform initial magnetic field (See figure 13), random seed magnetic field (See figure 14) and a periodic magnetic field (See figure 15). It is observed that, the variations in the magnetic field at the beginning have no discernible effect on the dynamics (See figure 12).
Figure 12. Normalized Magnetic energy versus time for different initial condition.

Figure 13. Dynamics of magnetic energy iso-surface starting from uniform initial magnetic field.

Figure 14. Dynamics of magnetic energy iso-surface starting from a random seed initial magnetic field.

Figure 15. Dynamics of magnetic energy iso-surface starting from a periodic initial magnetic field.
Appendix D. Grid size scaling study for ABC flow

Grid size scaling study have been performed using Arnold-Beltrami-Childress (ABC) flow (equation (3)) at different magnetic Reynolds numbers. It is seen that $256^3$ grid resolution is more than enough for this problem (See figures 16(a), (b)).

![Figure 16](image)

Figure 16. Fast Dynamo effect following Galloway et al [16] using Arnold-Beltrami-Childress (ABC) flow for magnetic Reynolds numbers (a) $R_m = 50.0$ (b) $R_m = 450.0$ at different grid resolutions.

Appendix E. Magnetic energy growth or decay rate calculation

The magnetic energy growth or decay rate $\left( \gamma = \frac{d}{dt} \left( \ln E_B(t) \right) \right)$ has been calculated at late times (e.g. $t \sim 200.0$ to $300.0$) for different cases (See figure 17).

![Figure 17](image)

Figure 17. Magnetic energy growth or decay rate calculation for for different $\beta$ values.

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