Stiffness selection in synthesis of mechatronic discrete systems

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Abstract. In the paper, the known algorithm of designing of mechatronic discrete systems has been decomposed. As a result, detailed analysis of stiffness selection, during the process of distribution of dynamical characteristics functions, has been done. Based on synthesized one degree of freedom system that utilize piezostack actuator, detailed constrains related to the stiffness and their impact for mechanical, dimensionless and mechatronic parameters, have been investigated. The work extends the known problem of vibration control in discrete mechatronic systems.

1. Introduction

Knowing the details related with stiffness selection, vibration isolation in mechatronic discrete systems, by use of synthesis method, can be deeply investigated and significantly improved. Utilization of piezo stack actuators for vibration control is well known [1]. Mathematical model of passive vibration with piezo transducers was shown in [2]. Different types of damping, respectively passive and active type in discrete mechatronic systems was confronted in [3].

Possibility to use negative value elements, in parallel with piezo has been introduced in [4], while using negative capacitance circuit [5]. Based on [4, 5] and works related with negative stiffness [6] authors introduced the problem of usage of negative elements in synthesis of mechatronic discrete systems [7].

The paper is verification and the continuation of Authors [1,3,7] works focused on stiffness selection. The work results in investigations related with stiffness impact for mechanical, dimensionless and mechatronic parameters in the mechatronic one degree of freedom cascade system. Based on the presented example, the paper leads to improvements in the efficiency of physical application of mechatronic systems, that can be easily synthesized and designed for given requirements in form of resonant and anti-resonant frequencies [7], in vibration control area.

2. Synthesis of mechatronic discrete systems

Synthesis of mechatronic systems is well known [1,3,7] and in case of cascade systems, is done based on dynamical characteristics function distribution into continuous fractions. The study on influence of main stiffness selections to all parameters has been done based on 1 degree of freedom (DOF) mechatronic structure, and in 2 DOF of system in mechanical replacement model. Two types of configurations have been in this case applied: LC and LRC, fig. 1.

The slowness function has been written as:
\[ U(s) = H \frac{d_1 s^l + d_2 s^{l-2} + \ldots + d_l}{c_1 s^k + c_2 s^{k-2} + \ldots + c_k s} \]  

(1)

where: \( l \) – odd degree of numerator, \( k \) – degree of denominator, \( l-k = 1 \), \( H \) – any positive real number.

The dimensionless equation of presented in fig. 1. mechatronic structure in LRC configuration, and its mechanical replacement models is written as:

\[
\begin{bmatrix}
1 & 0 & x_1' \\
0 & 0 & x_2' \\
0 & \lambda & x_2 \\
\end{bmatrix} + \begin{bmatrix}
0 & 0 & x_1 \\
0 & 2D_{LRC} & x_2 \\
1 + \gamma & -\gamma & x_1 \\
-1 & 1 + \delta & x_2 \\
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0 \\
\end{bmatrix}
\]  

(2)

where:

\( \lambda \) – nondimensional parameter which describes the relations of inertial elements,
\( \gamma, \delta \) – nondimensional parameters which describes the stiffness elements,
\( 2D_{LRC} \) – dimensionless parameter which describes damping element.

\[ F(t) \]

\[ L_x \]

\[ C_x \]

\[ c_1 \]

\[ m_1 \]

\[ c_2 \]

\[ c_3 \]

\[ m_2 \]

\[ c_1 \]

\[ m_1 \]

\[ d_f \]

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3. Mechanical parameters

Stiffness selection, in considered cases (c1), is the first step to determine the structure of designed mechatronic systems. Normally it should be taken from required range:

\[ \left( 0, H \frac{d_1}{c_1} \right) \]  

(3)

However it’s possible to take it out of the range as well, causing negative value of the stiffness, and therefore impacting also on the other parameters in the system. This selection results in mechanical replacement model (fig. 1). The impact on inertial element \( m_2 \) and stiffness elements \( c_2 \) and \( c_3 \) are shown respectively in the fig. 2 and fig. 3.

For the visualization, requirements in form of poles and zeros have been set in the range of 1000 – 2500 Hz, and for this values following graphs are created.

The value of \( m_2 \), is always positive and rapidly increases after taking the \( c_1 \) out of required range defined by the equation (2).
Opposite to inertial element $m_2$, stiffness elements might have negative value while taking the $c_1$ above required range. This cause also indirectly the impact on capacitance in external electric network that is connected to piezostack actuator. In this case, negative capacitance might be realized by the system described in [5].

![Graph showing influence of $c_1$ selection for $m_2$.](image)

**Figure 2.** Influence of $c_1$ selection for $m_2$

Influence for dimensionless inertial element is shown in fig. 4. The value of $\lambda$ is positive, independently from selection of stiffness $c_1$.

![Graph showing influence of $c_1$ selection for remaining in the system stiffness elements $c_2$ and $c_3$.](image)

**Figure 3.** Influence of $c_1$ selection for remaining in the system stiffness elements $c_2$ and $c_3$

4. **Dimensionless parameters**

After reception of mechanical replacement model, during synthesis process, in next step this model is transformed to dimensional model, so the mechanical and electrical constrains related to piezoelectric effect and external electric network can be combined [7]. The system is analyzed in dimensional time:

$$\tau = \omega_m t,$$

where: $\omega_m$ - frequency connected with piezoelement position in the structure.

Influence for dimensionless inertial element is shown in fig. 4. The value of $\lambda$ is positive, independently from selection of stiffness $c_1$. 

![Graph showing influence of $c_1$ selection for remaining in the system stiffness elements $c_2$ and $c_3$.](image)
Influence of $c_1$ selection for parameter $\lambda$

In relation to given range (2) it’s increasing until upper limit and then above defined ratio decrease. The impact $c_1$ for parameter $\gamma$ is shown in fig. 5.

Here selection from or out of the required range is neutral for the value. Similarly to parameter $\lambda$, $\gamma$ which describe stiffness elements in mechanical replacement model is positive, but it’s decreasing with increase of the value of $c_1$.

5. Mechatronic parameters

Retransformation of dimensionless model to mechatronic structure is the last step in synthesis of considered discrete systems process [7]. Following the general piezoelectric equations it’s possible to determine parameters of the piezostack actuator, and calculate the values of elements inside the external electric network.

To refer results of investigations to requirements which means to check how the requirements could also affect the values of considered parameters, three different resonant and antiresonant frequencies
has been evaluated (from range of 1000 – 2500 Hz). The bigger number in index of investigated parameter, the bigger value of pole and zero is applied.

The impact on selection of \(c_1\) for capacitance in the external electric circuit is shown in fig. 6.

![Figure 6](image)

**Figure 6.** The impact of \(c_1\) for capacitance \(C_x\) for different requirements

In all three cases, above the required range of selection of \(c_1\), the value of capacitance in external electric network is negative. The influence of considered stiffness for resistance in electric circuit is shown in fig 7.

In this case, with increase of \(c_1\), the resistance is also increasing. Another parameter that is affecting resistance in electric circuit is selection of parameter \(d_{p}\), which is in this case proportional to stiffness \(c_3\) and was described inter alia in [7].

![Figure 7](image)

**Figure 7.** Influence of \(c_1\) selection for resistance in external electric network \(R_x\) studied for different requirements

Finally, the impact of selection of \(c_1\) for inductance in the final mechatronic system, is shown in fig. 8. Opposite to capacitance and resistance, inductance is decreasing with the increase of \(c_1\).
6. Summary

In the paper, study on selection of stiffness elements, which is the key point during synthesis process, has been done. Based on example of cascade system with 1 DOF in mechatronic system, investigations related for verification of the impact for mechanical, dimensionless and mechatronic parameters of the system has been shown. Following the results and presented graphs, increase of the value of $c_1$ decreases the value of inductance and increase the value of $R_x$ and $C_x$. Over the upper limit for $c_1$ selection, described by (3), $C_x$ gets negative value, which is then also increasing with the increase of the value of the stiffness $c_1$.

It’s also possible to select stiffness to obtain different damping functions of the systems: passive damping, where all parameters has positive values and contains passive elements ($L_x$, $R_x$), and adaptive-passive ($C_x$). That leads to negative values of capacitance or resistance elements which are another options for vibration elimination in considered systems that must comply with given requirements in form or resonant and anti-resonant frequencies.

Additionally, detailed graphs related to major constrains have been presented for the extension of possible applications of considered systems in vibration control as dampers that have been built from piezostack actuator and connected $LRC$ network with/without negative values elements. Potential applications of the systems can be found in the areas of micro-positioning, optics elements and precise devices.

In further research works, detailed study on stability conditions and physical realization related to available in the market piezoceramic elements and their parameters like piezoelectric capacitance will be studied and constrained.

References

[1] Białas K, Buchacz A, Gałąziowski D 2013 Modelowanie dyskretnych układów mechatronicznych ze względu na funkcję tłumienia, Modelowanie Inżynierskie, 16, 47, pp 31-35,

[2] Buchacz A, Placzek M, Wrobel A 2014 Modelling of passive vibration damping using piezoelectric transducers – the mathematical model, Maintenance and Reliability, 16, 2, pp 301-306.

[3] Białas K, Buchacz A, Gałąziowski D 2015 Passive and active vibration isolation methods in discrete mechatronic systems, Solid St Phen., 220-221, pp 15-20.
[4] Han X, Neubauer M, Wallaschek J 2004 Improved piezoelectric switch shunt damping technique using negative capacitance, J. Sound and Vib., 332, 11, 2013, pp7-16.

[5] Fukada E, Date M, Kimura K and others 2004 Sound Isolation by Piezoelectric Polymer Films Connected to Negative Capacitance Circuits, IEEE Transactions on Dielectrics and Electrical Insulation, 11, 2;

[6] Kashdan L, Conner Seepersad C, Haberman M, Wilson PS 2012 Design, fabrication and evaluation of negative stiffness elements using SLS, Rapid Prot. J., 18, 13, pp 194-200,

[7] Buchacz A, Gałęziowski D 2015 Designing of discrete mechatronic vibration systems with negative value parameters, Mechanical Systems and Signal Processing, dx.doi.org/10.1016/j.ymssp.2015.02.003.

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