Observation-Assisted Heuristic Synthesis of Covert Attackers Against Unknown Supervisors

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Abstract  In this work, we address the problem of synthesis of covert attackers in the setup where the model of the plant is available, but the model of the supervisor is unknown, to the adversary. To compensate the lack of knowledge on the supervisor, we assume that the adversary has recorded a (prefix-closed) finite set of observations of the runs of the closed-loop system, which can be used for assisting the synthesis. We present a heuristic algorithm for the synthesis of covert damage-reachable attackers, based on the model of the plant and the (finite) set of observations, by a transformation into solving an instance of the partial-observation supervisor synthesis problem. The heuristic algorithm developed in this paper may allow the adversary to synthesize covert attackers without having to know the model of the supervisor, which could be hard to obtain in practice. For simplicity, we shall only consider covert attackers that are able to carry out sensor replacement attacks and actuator disablement attacks. The effectiveness of our approach is illustrated on a water tank example adapted from the literature.

Keywords  cyber-physical systems · discrete-event systems · covert attack · partial-observation · supervisor synthesis · unknown model

1 Introduction

The security of cyber-physical systems, modelled in the abstraction level of events [1], has drawn much research interest from the discrete-event systems community, with most of the existing works devoted to attack detection and security verification [2][7], synthesis of covert attackers [8][10] and synthesis of resilient supervisors [16][22]. Intuitively, the covertness property says that the attacker cannot reach a situation where its presence has been detected by the monitor but no damage can be inflicted [13][14].
Thus, the covertness property is a safety property for the attacker. In this paper, we focus on the synthesis of covert attackers in a more practical setup than those of \[8–16\].

The problem of covert sensor attacker synthesis has been studied extensively \[8–10,16\]. In [16], it is shown that, under a normality assumption on the sensor attackers, the supremal covert sensor attacker exists and can be effectively synthesized. In [8–9], a game-theoretic approach is presented to synthesize covert sensor attackers, without imposing the normality assumption. Recently, based on the game arena of [8–9], [10] develops an abstraction based synthesis approach to improve the synthesis efficiency of [8–9]. The problem of covert actuator attacker synthesis has been addressed in [11] and [12], by employing a reduction to the (partial-observation) supervisor synthesis problem [12]. With the reduction based approach, the more general problem of covert actuator and sensor attacker synthesis has also been addressed [13–15].

The synthesis approach developed in the existing works is quite powerful, in the sense that maximally permissive covert attackers can be synthesized from the model of the plant and the model of the supervisor. While it is natural to assume the model of the plant to be known, it seems a bit restrictive to also assume the model of the supervisor to be known, which could limit the usefulness of the existing covert attacker synthesis procedures in practice. In this paper, we relax the assumption that the model of the supervisor is known. We assume the adversary has recorded a (prefix-closed) finite set of observations of the runs of the closed-loop system, which can be used for assisting the synthesis of covert attackers. In this new setup, a covert attacker needs to be synthesized, if it is possible, based (solely) on the model of the plant and the given set of observations. The synthesized attacker needs to ensure damage-infliction and covertness against all the supervisors which are consistent with the given set of observations. The difficulty of this synthesis problem lies in the fact that there is in general an infinite number of supervisors that are consistent with the observations, rendering the synthesis approach developed in the existing works ineffective.

In this work, we consider covert attackers whose attack mechanisms are restricted to sensor replacement attacks and actuator disablement attacks. For simplicity, we only address the problem of synthesis of covert damage-reachable attackers. The main contributions of this work are listed as follows.

- We consider a new, but more challenging, setup where covert attackers need to be synthesized solely based on the model of the plant and a (prefix-closed) finite set of observations of the runs of the closed-loop system. This effectively removes the assumption that the model of the supervisor is known (a priori) to the adversary, which is assumed in [8–16] that address the covert attacker synthesis problem.
- We provide a heuristic algorithm for the synthesis of covert attacker, based solely on the model of the plant and the given set of observations. The solution methodology is to formulate the covert attacker synthesis problem in this new setup as an instance of the (partial-observation) supervisor synthesis problem, and it follows that we can employ the existing supervisor synthesis solvers [23–25] to synthesize

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1 From the adversary’s point of view, any supervisor that is consistent with the given set of observations may have been deployed.

2 It is worth mentioning that sensor replacement attacks can help achieve both of the damage-infliction goal and the covertness goal; actuator disablement attack can help achieve the covertness goal.
covert attackers, without knowing the model of the supervisor. The effectiveness of our approach is illustrated on a water tank example adapted from [16].

• We explain how the correctness of the synthesis approach can be reasoned based on the model of the attacked closed-loop system adapted from [13], [14], [22].

This paper is organized as follows. In Section 2, we recall the preliminaries which are needed for better understanding this work. In Section 3, we then introduce the system setup, present the model constructions, the proposed synthesis solution as well as the correctness proof. To establish the correctness of the synthesis solution, we recall the model of the attacked closed-loop system, which is adapted from [13], [14], [22], in Section 3. We also provide a brief discussion on the time complexity of our heuristic synthesis algorithm in Section 3. Finally, in Section 4, conclusions and future works are discussed.

2 Preliminaries

In this section, we introduce some basic notations and terminologies that will be used in this work, mostly following [1][26][27].

For any two sets $A$ and $B$, we use $A \times B$ to denote their Cartesian product and use $A - B$ to denote their difference. For any relation $R \subseteq A \times B$ and any $a \in A$, we define $R[a] := \{ b \in B \mid (a, b) \in R \}$.

A (partial) finite state automaton $G$ over alphabet $\Sigma$ is a 5-tuple $(Q, \Sigma, \delta, q_0, Q_m)$, where $Q$ is the finite set of states, $\delta : Q \times \Sigma \rightarrow Q$ is the (partial) transition function, $q_0 \in Q$ the initial state and $Q_m \subseteq Q$ the set of marked states. We shall write $\delta(q, \sigma)$ to mean $\delta(q, \sigma)$ if $\delta(q, \sigma)$ is defined. When $Q_m = \emptyset$, we also write $G = (Q, \Sigma, \delta, q_0)$. As usual, for any $G_1 = (Q_1, \Sigma_1, \delta_1, q_{1,0}, Q_{1,m})$, $G_2 = (Q_2, \Sigma_2, \delta_2, q_{2,0}, Q_{2,m})$, we write $G := G_1 \parallel G_2$ to denote their synchronous product. We have $G = (Q := Q_1 \times Q_2, \Sigma := \Sigma_1 \cup \Sigma_2, \delta := \delta_1 \parallel \delta_2 : q_0 := (q_{1,0}, q_{2,0}), Q_m := Q_{1,m} \times Q_{2,m})$, where the (partial) transition function $\delta$ is defined as follows: for any $q = (q_1, q_2) \in Q$ and any $\sigma \in \Sigma$,

$$\delta(q, \sigma) := \begin{cases} (\delta_1(q_1, \sigma), q_2), & \text{if } \sigma \in \Sigma_1 - \Sigma_2 \\ (q_1, \delta_2(q_2, \sigma)), & \text{if } \sigma \in \Sigma_2 - \Sigma_1 \\ (\delta_1(q_1, \sigma), \delta_2(q_2, \sigma)), & \text{if } \sigma \in \Sigma_1 \cap \Sigma_2 \end{cases}$$

For each sub-alphabet $\Sigma' \subseteq \Sigma$, the natural projection $P_{\Sigma'} : \Sigma^* \rightarrow \Sigma'^*$ is defined, which is extended to a mapping between languages as usual [1]. Let $G = (Q, \Sigma, \delta, q_0)$. We abuse the notation and define $P_{\Sigma'}(G)$ to be the finite automaton $(2^Q, \Sigma, \Delta, UR_{G, \Sigma - \Sigma'}(q_0))$, where the unobservable reach $UR_{G, \Sigma - \Sigma'}(q_0) := \{ q \in Q \mid \exists s \in (\Sigma - \Sigma')^*, q = \delta(q_0, s) \} \in 2^Q$ of $q_0$ with respect to the sub-alphabet $\Sigma - \Sigma' \subseteq \Sigma$ is the initial state (of $P_{\Sigma'}(G)$), and the partial transition function $\Delta : 2^Q \times \Sigma \rightarrow 2^Q$ is defined as follows.

1. for any $\emptyset \neq \mathcal{O} \subseteq Q$ and any $\sigma \in \Sigma'$, $\Delta(\mathcal{O}, \sigma) = UR_{G, \Sigma - \Sigma'}(\delta(\mathcal{O}, \sigma))$, where we define $UR_{G, \Sigma - \Sigma'}(\mathcal{O}') := \bigcup_{q \in \mathcal{O}} UR_{G, \Sigma - \Sigma'}(q)$ for any $\mathcal{O}' \subseteq Q$.\footnote{As usual, we also view the partial transition function $\delta : Q \times \Sigma \rightarrow Q$ as a relation $\delta \subseteq Q \times \Sigma \times Q$.}
2. for any $\emptyset \neq Q' \subseteq Q$ and any $\sigma \in \Sigma - \Sigma'$, $\Delta(Q', \sigma) = Q'$.

We here shall emphasize that $P_{2\Sigma}(G)$ is over $\Sigma$, instead of $\Sigma'$, and there is no transition defined at the state $\emptyset \in 2^Q$.

For any finite state automaton $G = (Q, \Sigma, \delta, q_0, Q_m)$, we write $L(G)$ and $L_m(G)$ to denote the closed-behavior and the marked-behavior of $G$ \cite{1}, respectively.

3 System Setup, Model Constructions and Synthesis Solution

3.1 System Setup

We shall first introduce and present a formalization of the system components, mostly following \cite{11,14}. We adapt the water tank example of \cite{16} as a running example to illustrate the constructions and effectiveness of our approach.

**Plant:** The plant is given by a finite state automaton $G = (Q, \Sigma, \delta, q_0, Q_m)$. Let $Q_{bad} \subseteq Q$ denote the set of bad states for $G$. Without loss of generality, we shall assume each state in $Q_{bad}$ is deadlocked, since damage cannot be undone. Thus, we can merge all the $Q_{bad}$ states into an equivalent state $q_{bad} \in Q$. In the rest, we shall let $q_{bad}$ denote the unique bad state for $G$. Without loss of generality, we shall assume $q_{bad} \neq q_0$. We let $\Sigma_o \subseteq \Sigma$ denote the subset of observable events and $\Sigma_c \subseteq \Sigma$ denote the subset of controllable events for the supervisor. We shall refer to the tuple $(\Sigma_c, \Sigma_o)$ as a control constraint. As usual, let $\Sigma_{uc} = \Sigma - \Sigma_c$ denote the subset of uncontrollable events and $\Sigma_{uo} = \Sigma - \Sigma_o$ denote the subset of unobservable events.

**Example 1** The water tank system \cite{16} has a constant supply rate, a water tank, and a control valve at the bottom of the tank controlling the outgoing flow rate. We assume the valve can only be fully open or fully closed. The water level could be measured, whose value can trigger some predefined events that denote the water levels: low ($L$), high ($H$), extremely low ($EL$) and extremely high ($EH$). The model of the plant $G$ is shown in Fig. 1 and $S$ is the bad state $q_{bad}$ which is crossed. We assume all the events are observable to the supervisor, i.e., $\Sigma_o = \Sigma = \{L, H, EL, EH, open, close\}$; only the events of opening the valve and closing the valve are controllable to the supervisor, i.e., $\Sigma_c = \{open, close\}$.

**Supervisor:** In the absence of an attacker, a supervisor over the control constraint $(\Sigma_c, \Sigma_o)$ is often modelled as a finite state automaton $S = (X, \Sigma, \zeta, x_0)$, which satisfies the controllability and observability constraints \cite{28}:

- (controllability) for any state $x \in X$ and any uncontrollable event $\sigma \in \Sigma_{uc}$, $\zeta(x, \sigma)$!
- (observability) for any state $x \in X$ and any unobservable event $\sigma \in \Sigma_{uo}$, $\zeta(x, \sigma)$!

implies $\zeta(x, \sigma) = x$.

The control command issued by supervisor $S$ at state $x \in X$ is defined to be $\Gamma(x) := \{\sigma \in \Sigma \mid \zeta(x, \sigma)$!

We assume the supervisor $S$ will issue a control command to the plant whenever an observable event is received and when the supervisor is initiated at the initial state. Let $\Gamma := \{\gamma \subseteq \Sigma \mid \Sigma_{uc} \subseteq \gamma\}$ denote the set of all the possible control commands.
Example 2  We assume a supervisor \( S \) has been synthesized to control \( G \) in the water tank example. The model of the supervisor \( S \) is shown in Fig. 2. We shall remark that the supervisor \( S \) prevents the water level from becoming extremely high (respectively, extremely low), by opening (respectively, closing) the valve when the water level is high (respectively, low).

Observation Automaton: The adversary has recorded a (prefix-closed) finite set \( O \subseteq P_{\Sigma}(L(S \parallel G)) \) of observations of the runs of the closed-loop system \( S \parallel G \), where \( P_{\Sigma} : \Sigma^* \rightarrow \Sigma_o^* \) denotes the natural projection \([1]\). \( O \) is given by an automaton \( M_O = (U, \Sigma_o, \eta, u_0) \), i.e., \( O = L(M_O) \). We refer to \( M_O \) as an observation automaton. Without loss of generality, we shall assume there is exactly one deadlocked state \( u_\perp \in U \) in \( M_O \) and, for any maximal string \( s \in O \) (in the prefix ordering \([1]\) ), we have \( \eta(u_0, s) = u_\perp \). Any supervisor \( S' \) that can generate such observations \( O \), i.e., \( O \subseteq P_{\Sigma_o}(L(S' \parallel G)) \), is said to be consistent with \( O \).
Example 3 Let us continue with the water tank example. The observation automaton $M_O$ is given in Fig. 3. It is clear that $O \subseteq P_{\Sigma_o}(L(S\|G))$. Thus, $S$ is consistent with $O$.

![Fig. 3: The observation automaton $M_O$ for the water tank example]

**Monitor:** We assume there exists a monitor $M$ that records its observation $w \in \Sigma_o^*$ of the execution of the (attacked) closed-loop system and halts the system execution after the detection of an attacker [11–14]. It will conclude the existence of an attacker (at the first moment) when it observes some string $w \notin P_{\Sigma_o}(L(S\|G))$. We remark that some string $w \notin P_{\Sigma_o}(L(S\|G))$ has been generated if and only if $P_{\Sigma_o}(S\|G)$ reaches the $\emptyset \in 2^{X \times Q}$ state [12]. We here shall refer to $P_{\Sigma_o}(S\|G)$ as the monitor. That is, $M = P_{\Sigma_o}(S\|G) = (2^{X \times Q}, \Sigma, \Delta, UR_{S\|G}, \Sigma - \Sigma_o(x_0, q_0))$.

The monitor state $\emptyset \in 2^{X \times Q}$ and any plant state $q \neq q_{bad}$ together defines the covertness-breaking states for the attacker [12–14].

Example 4 Let us continue with the water tank example, with the model of the plant $G$ given in Fig. 1 and the model of the supervisor $S$ given in Fig. 2. Then, the monitor $M = P_{\Sigma_o}(S\|G)$ is given in Fig. 4, where $S4$ denotes the state $\emptyset \in 2^{X \times Q}$.

**Attacker:** In this paper, we assume $\Sigma_o$ is also the subset of plant events that can be observed by the attacker. Let $\Sigma_o^{tA} \subseteq \Sigma_o$ denote the subset of controllable events that can be compromised under actuator (disablement) attacks. Let $\Sigma_s^{tA} \subseteq \Sigma_s$ denote the subset of observable events that can be compromised under sensor attacks. We adopt a relation $R \subseteq \Sigma_s^{tA} \times \Sigma_s^{tA}$ to specify the sensor attack capabilities [13]. Intuitively, any event $\sigma \in \Sigma_s^{tA}$ executed in the plant $G$ may lead to the observation of some event $\sigma'$ in $R[\sigma] = \{ \sigma' \in \Sigma_s^{tA} | (\sigma, \sigma') \in R \}$ by the supervisor $S$, due to the attacker exercising the sensor replacement attack. Without loss of generality, we shall assume $\sigma \in R[\sigma]$ and $R[\sigma] - \{ \sigma \} \neq \emptyset$, for any $\sigma \in \Sigma_s^{tA}$. We shall refer to the tuple $\mathcal{T} = (\Sigma_o, \Sigma_s^{tA}, (\Sigma_s^{tA}, R))$ as an attack constraint.

In this paper, we assume the adversary knows the bad state $q_{bad} \in Q$, the model of $G$ and the control constraint $(\Sigma_s, \Sigma_o)$. On the other hand, we assume the model of the supervisor $S$, and thus the model of the monitor $M$, is unknown to the adversary.

Example 5 We assume all the water level events are compromised observable events to the attacker, i.e., $\Sigma_s^{tA} = \{ L, H, EL, EH \}$, and $R = \Sigma_s^{tA} \times \Sigma_s^{tA}$. We also assume all the controllable events are compromised, i.e., $\Sigma_o^{tA} = \Sigma_s^{tA} = \{ open, close \}$.
3.2 Model Constructions

Given the model of the plant $G = (Q, \Sigma, \delta, q_0)$, with the bad state $q_{bad} \in Q$, the observation automaton $MO = (U, \Sigma_o, \eta, u_0)$ and the attack constraint $T = (\Sigma_o, \Sigma_a, A, (\Sigma_s, A, R))$, in this paper we will construct the following four automata to perform the synthesis.

1. The transformed plant $G^T$, which reflects a) the bad state $q_{bad}$ of $G$ is the goal state for the attacker, b) before the damage is inflicted, i.e., before $q_{bad}$ is reached, any uncertainty that might cause the covertness to be broken is considered to be bad for the attacker, and c) nothing can be executed after the damage is inflicted.

2. The unconstrained sensor attack automaton $G_{SA}$, which specifies all the possible sensor replacement attacks that can be carried out, i.e., upon the receiving of some compromised observable event in $\Sigma_s$ from the plant, the attacker can issue some attacked copy in $\Sigma_s^\#$, as specified by $R$, to mislead the supervisor.

3. The attack-forcing automaton $G_{AF}$ that forces sensor replacement attacks, which ensures a) some sensor replacement attacks must be carried out in order to fulfill the damage-inflicting goal of the attacker, and b) covertness could be broken once some sensor replacement attacks have been performed.

4. The transformed observation automaton $M_O^T$, which reflects a) any (attacked) observation for the supervisor that falls within $O$ is not bad for the attacker, b) any (attacked) observation for the supervisor that falls outside $O$ is (considered to be) bad for the attacker, if the attacker has already carried out some sensor replacement attacks and the damage has not been inflicted, and c) the supervisor and the monitor receive attacked copies in $\Sigma_s^\#$ for events in $\Sigma_s$ executed by the plant.

The idea of the constructions is explained as follows.

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6 We here remark that actuator disablement attacks cannot cause the covertness to be broken, following [12]. Indeed, the supervisor is not sure whether some event $\sigma \in \Sigma_s$ has been disabled by an attacker, even if disabling $\sigma$ may result in deadlock, as the supervisor is never sure whether: 1) deadlock has occurred due to actuator attack, or 2) $\sigma$ will possibly fire soon (according to the internal mechanism of the plant), without an explicit timing mechanism.
1. Since the models of the supervisor $S$ and the monitor $M$ are not available, we do not have the model of the attacked closed-loop system $[13][14]$. Thus, we cannot use the set of covertness-breaking states of the attacked closed-loop system to perform the synthesis. To ensure the covertness of the synthesized attacker, the idea is to over-approximate the set of covertness-breaking states of the attacked closed-loop system, without using the models of $S$ and $M$. The over-approximation needs to work for any $S$ (and thus $M$) that is consistent with $O$. In this work, we use (a) the state $q \not= q_{bad}$ of $G^T$ where damage has not been inflicted, (b) the state of $G_{AF}$ where sensor replacement attacks have been carried out, and (c) the state of $M_D^T$ where an (attacked) observation that falls outside $O$ has been observed by the supervisor (and the monitor) to (together) over-approximate the set of covertness-breaking states. Indeed, if the monitor $M$ reaches the $\emptyset \in 2^{X \times Q}$ state (under attacks), then Conditions b) and c) above must be both satisfied.

2. We view $G^T, M_D^T, G_{SA}, G_{AF}$ as the components of the surrogate plant $G^T || M_D^T || G_{SA} || G_{AF}$ and view the attacker $A$ as a supervisor that controls the surrogate plant to i) avoid breaking the covertness, over-approximated with Conditions a), b) and c), and ii) ensure the damage-infliction, i.e., the reachability of the $q_{bad}$ state.

3. By construction, $G^T || M_D^T || G_{SA} || G_{AF} || A$ can be viewed as providing a (behavioral) upper bound for the attacked closed-loop system of $[13][14]$. If $G^T || M_D^T || G_{SA} || G_{AF}$ can be controlled to avoid reaching a set that contains all the covertness-breaking states by the attacker $A$, then the attacked closed loop system (induced by $A$) can also avoid reaching the covertness-breaking states.

4. The reachability of the state $q_{bad}$ in $G^T || M_D^T || G_{SA} || G_{AF} || A$ in general does not imply the reachability of the state $q_{bad}$ in the attacked closed-loop system. In particular, by construction, the $q_{bad}$ state in $G$ can already be reached in $G^T || M_D^T || G_{SA} || G_{AF}$, since we ignore the control effect of the supervisor that ensures the non-reachability of $q_{bad}$. That is, only some of the executions that lead to the state $q_{bad}$ in $G^T || M_D^T || G_{SA} || G_{AF} || A$ indeed exist in the attacked closed-loop system. In order to address this issue, we carefully design the state markings for the new plant $G^T || M_D^T || G_{SA} || G_{AF}$ to ensure that a marked state is reached in $G^T || M_D^T || G_{SA} || G_{AF} || A$ if and only if i) the bad state $q_{bad} \in O$ has been reached, ii) some sensor replacement attacks have been carried out (before the damage is inflicted). It turns out that the synthesized attacker can ensure the damage-reachability in the attacked closed-loop system if there exists a marked string $s \in L_m(G^T || M_D^T || G_{SA} || G_{AF} || A)$ such that $s$ is allowed by an attacked supervisor $S^A$ where $S^A$ under-approximates any supervisor that is consistent with $O$. It follows that $S^A$ can be used in the verification of damage-reachability or even in the synthesis for ensuring damage-reachability by construction.

We are now ready to present the model constructions.

**Transformed Plant:** We model the transformed plant as $$G^T = (Q \cup \{q^S\}, \Sigma \cup \Sigma^A \cup \{S\} \cup \Gamma, \delta^T, q_0, \{q_{bad}\}),$$ where $q^S \not\in Q$ is a newly added state, $\Sigma^A = \{\sigma^A \mid \sigma \in \Sigma_A\}$ is a relabelled copy of $\Sigma_A$, $S \not\in \Sigma \cup \Sigma^A \cup \Gamma$ is a newly added event and $\delta^T : (Q \cup \{q^S\}) \times (\Sigma \cup \Sigma^A \cup \{S\} \cup \Gamma) \rightarrow (Q \cup \{q^S\})$ is the partial transition function defined as follows.
1. for any $q \in Q$ and any $\sigma \in \Sigma$, $\delta^T(q, \sigma) = \delta(q, \sigma)$,
2. for any $q \in Q - \{q_{bad}\}$, $\delta(q, \$) = q^\$,
3. for any $q \in Q - \{q_{bad}\}$ and any $\sigma \in \Sigma^\$ \cup \Gamma$, $\delta(q, \sigma) = q$.

Intuitively, the event $\$ is used to denote that the following conditions hold simultaneously: a) damage has not been inflicted in the plant $G$, b) some sensor replacement attack has been performed, c) the (attacked) observation for the supervisor has fallen outside $O$. In particular, b) and c) implies that the presence of the attacker could have been discovered by the monitor; a), b) and c) together implies that the covertness of the attacker could have been broken. Thus, $\$ is an uncontrollable (and unobservable) “bad” event for the attacker that leads to the bad state $q^7$ in $G^T$. Rule 2) here contributes to Condition a) of the definition of $\$. Intuitively, the supervisor only receives the relabelled copies in $\Sigma^\$\_\_A$, while the events in $\Sigma\_\_A$ are executed in the plant. Rule 3) is added to ensure that nothing can be executed when the state $q_{bad}$ is reached. The state size of $G^T$ is $|Q| + 1$.

Example 6 For the water tank system, the transformed plant $G^T$ is provided in Fig. 5, where the event “bad” is used to represent $\$; the event pack (respectively, pack\_open, pack\_close\_open) denotes (the receiving of) the control command $\gamma = \Sigma_{ac}$ (respectively, $\gamma = \Sigma_{ac} \cup \{open\}$, $\gamma = \Sigma_{ac} \cup \{close\}$, $\gamma = \Sigma_{ac} \cup \{close, open\}$); the event $Lprime$ (respectively, $Hprime, ELprime, EHprime$) is then used to denote $L^\#$ (respectively, $H^\#, EL^\#, EH^\#$). In Fig. 5, S7 denotes the state $q^5$ and S5 is used to denote the state $q_{bad}$.

**Unconstrained Sensor Attack Automaton:** We model the sensor (replacement) attack capabilities by using a finite state automaton $G_{SA}$

\[ G_{SA} = (Q^SA, \Sigma \cup \Sigma^\$\_\_A, \delta^SA, q_{init}) \]

where $Q^SA = \{q^\sigma \mid \sigma \in \Sigma\_\_A\} \cup \{q_{init}\}$. $\delta^SA : Q^SA \times (\Sigma \cup \Sigma^\$\_\_A) \rightarrow Q^SA$ is the partial transition function defined in the following.

1. for any $\sigma \in \Sigma\_\_A$, $\delta^SA(q_{init}, \sigma) = q^\sigma$,
2. for any $q^\sigma \in Q^SA - \{q_{init}\}$ and for any $\sigma' \in R[\sigma]$, $\delta^SA(q^\sigma, \sigma'^\#) = q_{init}$,
3. for any $\sigma \in \Sigma - \Sigma\_\_A$, $\delta^SA(q_{init}, \sigma) = q_{init}$.

$G_{SA}$ specifies all the possible attacked copies in $\Sigma^\$\_\_A$ which could be received by the supervisor, due to the sensor replacement attacks, for each compromised observable event $\sigma \in \Sigma\_\_A$ executed in the plant. The state $q^\sigma$, where $\sigma \in \Sigma\_\_A$, is used to denote that the attacker has just received the compromised observable event $\sigma$, with Rule 1). Rule 2) then forces the attacker to (immediately) make a sensor attack decision, upon receiving each compromised observable event. Rule 3) is added such that $G_{SA}$ is over $\Sigma^\$\_\_A \cup \Sigma$ and no attack could be performed when $\sigma \in \Sigma - \Sigma\_\_A$ is executed. The state size of $G_{SA}$ is $|\Sigma\_\_A| + 1$, before automaton minimization.

Example 7 We now continue with the water tank example. The unconstrained sensor attack automaton $G_{SA}$ is shown in Fig. 6. We note that the automaton $G_{SA}$ has been minimized and all the $q^\sigma$ states, where $\sigma \in \Sigma\_\_A$, have been merged into an equivalent state, i.e., S1.
**Transformed Observation Automaton:** We model the transformed observation automaton as

\[ M^T = (U \cup \{u^1, u^5\}, (\Sigma_o - \Sigma_{rA}) \cup \Sigma_{rA}^\# \cup \{\$\}, \eta^T, u_0, U \cup \{u!\}) \]

where \(u^1, u^5 \notin U\) and \(\eta^T : (U \cup \{u^1, u^5\}) \times ((\Sigma_o - \Sigma_{rA}) \cup \Sigma_{rA}^\# \cup \{\$\}) \to (U \cup \{u^1, u^5\})\) is the partial transition function defined as follows.

1. for each \(u \in U\) and each \(\sigma \in \Sigma_o - \Sigma_{rA}\), if \(\eta(u, \sigma)!\), then \(\eta^T(u, \sigma) = \eta(u, \sigma)\),
2. for each \(u \in U\) and each \(\sigma \in \Sigma_o - \Sigma_{rA}\), if \(\neg \eta(u, \sigma)!\), then \(\eta^T(u, \sigma) = u^1\),
3. for each \(u \in U\) and each \(\sigma \in \Sigma_{rA}\), if \(\eta(u, \sigma)!\), then \(\eta^T(u, \sigma^\#) = \eta(u, \sigma)\),
4. for each \(u \in U\) and each \(\sigma \in \Sigma_{rA}\), if \(\neg \eta(u, \sigma)!\), then \(\eta^T(u, \sigma^\#) = u^1\).

Recall that \(q_{bad}\) is the goal state for the attacker.
5. for each \( \sigma \in \Sigma_o - \Sigma_s \), \( \eta^T(u^i, \sigma) = u^i \).
6. for each \( \sigma \in \Sigma_s \), \( \eta^T(u^i, \sigma^b) = u^i \).
7. \( \eta^T(u^i, \$) = u^5 \).

Intuitively, the state \( u^i \) here means that the (attacked) observation of the supervisor has fallen outside \( O \) and the attacker may be in the risk of exposing itself. Upon the execution of the “bad” event \( \$ \) for the attacker, the bad state \( u^5 \) of \( M^T_O \) can be reached from \( u^i \). In particular, Rule 7) contributes to Condition c) of the definition of \( \$ \). The state size of \( M^T_O \) is \(|U| + 2\). We remark that \( L_m(M^T_O) = ((\Sigma_o - \Sigma_s) \cup \Sigma_{sA})^* \).

Example 8 We now continue with the water tank example. The transformed observation automaton \( M^T_O \) is given in Fig. 7. S4 denotes the state \( u^i \) and S5 denotes the state \( u^5 \).

**Attack-Forcing Automaton:** We model the attack-forcing automaton as

\[
G_{AF} = (Q^{AF}, \Sigma_{sA} \cup \Sigma_{sA}^b \cup \{\$\}, \delta^{AF}, q_0^{AF}, \{q^{AF}_1\}),
\]

where \( Q^{AF} = \{q^{AF}_\sigma \mid \sigma \in \Sigma_{sA}\} \cup \{q_0^{AF}, q^{AF}_1, q^{AF}_b\} \) and \( \delta^{AF} : Q^{AF} \times (\Sigma_{sA} \cup \Sigma_{sA}^b \cup \{\$\}) \to Q^{AF} \) is the partial transition function defined as follows.

1. for any \( \sigma \in \Sigma_{sA} \), \( \delta^{AF}(q_0^{AF}, \sigma) = q^{AF}_\sigma \).
2. for any \( \sigma \in \Sigma_{sA} \), \( \delta^{AF}(q^{AF}_\sigma, \sigma^b) = q_0^{AF} \).
3. for any \( \sigma, \sigma' \in \Sigma_{sA} \) with \( \sigma \neq \sigma' \), \( \delta^{AF}(q^{AF}_\sigma, \sigma^b) = q^{AF}_1 \).
4. $\delta^{AF}(q^{AF!}, S) = q^{AF!}$.

The state $q^{AF!}$, where $\sigma \in \Sigma_A$, is used to denote that the attacker just receives the compromised observable event $\sigma$, with Rule 1). Rule 2) states that the attacker does not effectively carry out sensor replacement attacks and thus returns to the initial state $q_0^{AF}$. Rule 3) captures the situation that the attacker has effectively carried out some sensor replacement attacks. Rule 4) says that, upon the execution of the “bad” event $\sigma$ for the attacker, the bad state $q^{AF!!}$ in $G_{AF}$ can be reached from $q^{AF!}$. In particular, Rule 4) here contributes to Condition b) of the definition of $S$. The state size of $G_{AF}$ is $|\Sigma_A| + 3$.

Example 9 We continue with the water tank example. The attack-forcing automaton $G_{AF}$ is given in Fig. 8. Here, S4 denotes $q^{AF!}$ and S5 denotes $q^{AF!!}$.

![Fig. 8: The attack-forcing automaton $G_{AF}$](image)

**Attacker:** An attacker over $\mathcal{T} = (\Sigma_0, \Sigma_{uA}, (\Sigma_tA, R))$ is modelled by a supervisor $A = (Y, \Sigma \cup \Sigma^#_A \cup \{\$\} \cup \Gamma, \beta, y_0)$ over the control constraint $(\Sigma_{uA} \cup \Sigma^#_A, \Sigma_{o} \cup \Sigma^#_{tA})$. Intuitively, the attacker $A$ can only control events in $\Sigma_{uA} \cup \Sigma^#_A$ and can only observe events in $\Sigma_{o} \cup \Sigma^#_{tA}$. In this paper, we assume the attacker cannot observe the control commands issued by the supervisor.

3.3 Synthesis Solution

As we have discussed before, we could view $G^T, M^T_D, G_{SA}, G_{AF}$ as the components of the surrogate plant $G^T || M^T_D || G_{SA} || G_{AF}$ and view the attacker $A$ as a supervisor over the
control constraint \((\Sigma_{nA} \cup \Sigma_{oA}^o, \Sigma_{n} \cup \Sigma_{o}^o)\) which controls the surrogate plant. We can then synthesize (maximally permissive) safe attackers for \(G^T \parallel [M^0_a] \parallel [G_{SA}] \parallel [G_{AE}]\) by using existing synthesis tools \([23], [24], [25]\), where \(\text{BAD} := \{(q^S, u^T, q^{SA},q^{AF}), | q^{SA} \in Q^{SA}\}\) denotes the set of bad states for the attacker \(A\) in \(G^T \parallel [M^0_a] \parallel [G_{SA}] \parallel [G_{AE}]\). Let \(A''\) denote any non-empty (maximally permissive) safe attacker, if it exists, synthesized by using any existing synthesis tool. We still need to ensure the correctness of \(A''\) for the attacked closed-loop system \([13]\). To that end, we need to introduce the model of the attacked closed-loop system, induced by \(A''\), which is \(G^T \parallel [BT(S)]^A \parallel [M^A] \parallel [G_{SA}] \parallel [G_{CE}] \parallel [A''\] (see Fig. 9), where \(BT(S)^A\) is the attacked supervisor (with an explicit control command sending phase), \(M^A = P_{\Sigma, S}(G)^A\) is the attacked monitor and \(G_{CE}\) is the command execution automaton (which transduces control command \(\gamma\) from the supervisor into event \(\sigma \in \Sigma\) executed in the plant). Thus, we still need to introduce the three components \(BT(S)^A, G_{CE}\) and \(M^A\) from \([13], [22]\).

**Fig. 9: The diagram of the attacked closed-loop system**

**Attacked Supervisor:** The attacked supervisor \(BT(S)^A\) is constructed from \(S\) as follows. Let

\[
BT(S)^A = (X \cup X_{com} \cup \{x^i\}, \Sigma \cup \Sigma_{oA}^o \cup \Gamma, \xi^{BT}_A, x_{0,com}),
\]

where \(X_{com} = \{x_{com} \mid x \in X\}\) is a relabelled copy of \(X\), with \(X \cap X_{com} = \emptyset, x_{0,com} \in X_{com}\) is the relabelled copy of \(x_0 \in X\) and \(x^i \notin X \cup X_{com}\) is a newly added state. The partial transition function \(\xi^{BT}_A\) is defined as follows.

1. For any \(x \in X\), \(\xi^{BT}_A(x_{com}, \Gamma(x)) = x\).
2. For any \(x \in X\) and any \(\sigma \in \Sigma_{n},\) if \(\zeta(x, \sigma)\), then \(\xi^{BT}_A(x, \sigma) = \zeta(x, \sigma) = x\).
3. For any \(x \in X\) and any \(\sigma \in \Sigma_{oA}\), if \(\zeta(x, \sigma)\), then \(\xi^{BT}_A(x, \sigma) = \zeta(x, \sigma)_{com}\), with \(\zeta(x, \sigma)_{com}\) denoting the relabelled copy of \(\zeta(x, \sigma)\).
4. For any \(x \in X\) and any \(\sigma \in \Sigma_{nA}\), if \(\zeta(x, \sigma)\), then \(\xi^{BT}_A(x, \sigma^o) = \zeta(x, \sigma)_{com}\).
5. For any \(x \in X\) and any \(\sigma \in \Sigma_{oA}\), if \(\zeta(x, \sigma)\), then \(\xi^{BT}_A(x, \sigma^A) = x^i\).
6. For any \(x \in X \cup X_{com}\) and any \(\sigma \in \Sigma_{nA}\), \(\xi^{BT}_A(x, \sigma) = x\).

Intuitively, each \(x_{com}\) is the control state corresponding to \(x\), which is ready to issue the control command \(\Gamma(x)\). In Rule 1), the transition \(\xi^{BT}_A(x_{com}, \Gamma(x)) = x\) represents the event that the supervisor sends the control command \(\Gamma(x)\) to the plant. Each \(x \in X\) is a reaction state which is ready to react to an event \(x \in \Sigma\) executed by the plant \(G\).
For any $x \in X$ and any $\sigma \in \Sigma$, the supervisor reacts to the corresponding $\sigma$ transition (fired in the plant) if $\zeta(x, \sigma)$! If $\sigma \in \Sigma_{\text{in}}$, then the supervisor observes nothing and it remains in the same reaction state $\zeta(x, \sigma) = x \in X$, as defined in Rule 2; if $\sigma \in \Sigma_{\text{o}}$, then the supervisor proceeds to the next control state $\zeta(x, \sigma)_{\text{com}}$ and is ready to issue a new control command. Since the supervisor reacts to those events in $\Sigma_{\text{A}}$ instead of the events in $\Sigma_{\text{A}^\#}$, we need to divide the case $\sigma \in \Sigma_{\text{in}}$ into the case $\sigma \in \Sigma_{\text{in}} - \Sigma_{\text{A}}$ and the case $\sigma \in \Sigma_{\text{A}}$ with Rule 3) and Rule 4), respectively. Thus, Rules 1)-4) together captures the control logic of the supervisor $S$ and makes the control command sending phase explicit. Rule 5) specifies the situation when the sensor replacement attacks can lead to the state $x'$, where the existence of attacker is detected (based on the structure of the supervisor alone). Rule 6) is added here such that $BT(S)^4$ is over $\Sigma \cup \Sigma_{\text{A}^\#} \cup \Gamma$ and no event in $\Sigma \cup \Sigma_{\text{A}^\#} \cup \Gamma$ can be executed at state $x'$, where the system execution is halted.

**Attacked Monitor:** The attacked monitor $M^A$ is constructed from the monitor $M$ as follows. Let

$$M^A = Pr_{\Sigma}(S)[G]^A = (2^{X \times Q}, \Sigma \cup \Sigma_{\text{A}^\#} \cup \{S\} \cup \Gamma, \Delta^A, URS_{\Sigma - \Sigma_{\text{in}}} (x_0, q_0), 2^{X \times Q}),$$

where $D^S \subseteq 2^{X \times Q}$ is the distinguished bad state of $M^A$ and $\Delta^A : (2^{X \times Q} \cup \{D^S\}) \times (\Sigma \cup \Sigma_{\text{A}^\#} \cup \{S\} \cup \Gamma) \rightarrow (2^{X \times Q} \cup \{D^S\})$ is the partial transition function defined as follows.

1. for any $D \subseteq X \times Q$ and for any $\sigma \in \Sigma - \Sigma_{\text{A}}$, $\Delta^A(D, \sigma) = \Delta(D, \sigma)$,
2. for any $D \subseteq X \times Q$ and for any $\sigma \in \Sigma_{\text{A}}$, $\Delta^A(D, \sigma^\#) = \Delta(D, \sigma)$,
3. $\Delta^A(\emptyset, S) = D^S$,
4. for any $\emptyset \neq D \subseteq X \times Q$ and for any $\sigma \in \Sigma_{\text{A}} \cup \Gamma$, $\Delta^A(D, \sigma) = D$.

Rule 1) and Rule 2) states that those (and only those) $\sigma \in \Sigma_{\text{A}}$ transitions of $M$ are relabelled with their attacked copies $\sigma^\# \in \Sigma_{\text{A}^\#}$ in $M^A$. Rule 3) states that it is bad for the attacker if the system execution has been halted and damage has not been inflicted. Rule 4) is added so that $M^A$ is over $\Sigma \cup \Sigma_{\text{A}^\#} \cup \{S\} \cup \Gamma$ and no event in $\Sigma \cup \Sigma_{\text{A}^\#} \cup \Gamma$ can be executed at state $\emptyset \in 2^{X \times Q}$. By construction, there is an outgoing transition labelled by each event in $\Sigma \cup \Sigma_{\text{A}^\#} \cup \Gamma$ at each non-empty state $\emptyset \neq D \subseteq X \times Q$ of $M^A$.

**Command Execution Automaton:** The command execution automaton $[13], [14], [29], [30]$ is given by the 4-tuple $G_{CE} = (Q^{CE}, \Sigma \cup \Gamma, \delta^{CE}, q_0^{CE})$, where $Q^{CE} = \{q^F \mid \gamma \in \Gamma\} \cup \{q_{\text{wait}}^F\}$ and $q_0^{CE} = q_{\text{wait}}^F$. The partial transition function $\delta^{CE} : Q^{CE} \times (\Sigma \cup \Gamma) \rightarrow Q^{CE}$ is defined as follows.

1. for any $\gamma \in \Gamma$, $\delta^{CE}(q_{\text{wait}}^F, \gamma) = q^F$,
2. for any $q^F$, if $\sigma \in \Sigma_{\text{in}} \cap \gamma$, $\delta^{CE}(q^F, \sigma) = q^F$,
3. for any $q^S$, if $\sigma \in \Sigma_{\text{in}} \cap \gamma$, $\delta^{CE}(q^S, \sigma) = q_{\text{wait}}$.

Intuitively, at the initial state $q_{\text{wait}}^F$, the command execution automaton $G_{CE}$ waits for the supervisor to issue a control command. Once a control command $\gamma$ has been received, it transits to state $q^F$, recording this most recent control command. At state $q^F$, only those events in $\gamma$ are allowed to be fired. If $\sigma \in \Sigma_{\text{in}}$ is fired, then the command execution automaton returns to the initial state $q_{\text{wait}}^F$ and waits for a new control command. If $\sigma \in \Sigma_{\text{in}}$ is fired, then temporarily no new control command will be issued.
by the supervisor and the command execution automaton self-loops \( \sigma \) as if \( \sigma \in \Sigma_{uo} \) has never occurred. We can view \( G_{CE} \) as transducing from control command \( \gamma \) issued by the supervisor into event \( \sigma \in \Sigma \) executed in the plant.

**Example 10** For the water tank example, the command execution automaton \( G_{CE} \) is given in Fig. 10.

![Fig. 10: The command execution automaton \( G_{CE} \)](image)

The event \( $ \in G^T\|BT(S)^A\|M^A\|G_{SA}\|G_{CE}\|A^o \) means that a) the existence of the attacker has been detected by the monitor and the system execution has been halted, and b) damage has not been inflicted, which is different from the meaning of the event \( $ \in G^T\|BT(S)^A\|M^A\|G_{SA}\|G_{CE}\|A^o \). To show the correctness of \( A^o \), we need to ensure that \( $ \) cannot be executed in the attacked closed-loop system \( G^T\|BT(S)^A\|M^A\|G_{SA}\|G_{CE}\|A^o \), where \( $ \) is again uncontrollable and unobservable to the attacker \( A^o \), for any supervisor \( S \) that is consistent with \( O \).

**Proposition 1** No covertness-breaking state can be reached in the attacked closed-loop system \( G^T\|BT(S)^A\|M^A\|G_{SA}\|G_{CE}\|A^o \), induced by \( A^o \), for any supervisor \( S \) that is consistent with \( O \).

**Proof:** We need to show that no state in the set \( \{(q,x,D,q_{SA}^E,q_{CE}^E,y) \in (Q \cup \{q^A\}) \times (X \cup X_{com} \cup \{x^A\}) \times (2^{X \times \{q^A\}} \cup \{D^A\}) \times \sigma_{SA} \times \sigma_{CE} \times \gamma \mid q \in Q - \{q_{bad}\}, D = \emptyset \} \) can be reached in \( G^T\|BT(S)^A\|M^A\|G_{SA}\|G_{CE}\|A^o \), so that \( $ \) cannot be fired in \( G^T\|BT(S)^A\|M^A\|G_{SA}\|G_{CE}\|A^o \).

Suppose some such state \( (q,x,D,q_{SA}^E,q_{CE}^E,y) \), where \( q \in Q - \{q_{bad}\}, D = \emptyset \), can be reached in \( G^T\|BT(S)^A\|M^A\|G_{SA}\|G_{CE}\|A^o \) via some string \( s \in L(G^T\|BT(S)^A\|M^A\|G_{SA}\|G_{CE}\|A^o) \). Then, \( s \) can be executed in \( G^T, M^A \) and \( G_{SA} \), after we lift their alphabets to \( \Sigma \cup \Sigma_{uo}^0 \cup \{\$\} \cup \Gamma \). By construction, the string \( s \) can also be executed in \( M^T \) and \( G_{AF} \), since \( s \) can be executed in \( M^A \) and \( G_{SA} \) and also the event \( $ \) does not occur.
in the string $s$. Thus, $s$ can also be executed in $G^T \parallel M_0^T \parallel G_{SA} \parallel G_{AF} \parallel A^o$. Next, we examine what state is reached in $G^T \parallel M_0^T \parallel G_{SA} \parallel G_{AF} \parallel A^o$ via the string $s$. Clearly, state $q$ is reached in $G^T$ (as in the attacked closed-loop system $G^T \parallel BT(S)^A \parallel M^A \parallel G_{SA} \parallel G_{CE} \parallel A^o$) and state $u'$ is reached in $M_0^T$ (as $S$ is consistent with $O$) and state $q^{AF}$ is reached in $G_{AF}$ (as some sensor replacement attacks have been performed), via the string $s$. It follows that $A^o$ can be executed in $G^T \parallel M_0^T \parallel G_{SA} \parallel G_{AF} \parallel A^o$, which is a contradiction to the fact that $A^o$ is a safe attacker for $G^T \parallel M_0^T \parallel G_{SA} \parallel G_{AF}$. We thus can conclude that no covertness-breaking state can be reached in $G^T \parallel BT(S)^A \parallel M^A \parallel G_{SA} \parallel G_{CE} \parallel A^o$. $\square$

Proposition 1 states that $A^o$, synthesized for $G^T \parallel M_0^T \parallel G_{SA} \parallel G_{AF}$. is indeed a covert attacker for the attacked closed-loop system $G^T \parallel BT(S)^A \parallel M^A \parallel G_{SA} \parallel G_{CE} \parallel A^o$. We still need to ensure that $A^o$ is indeed damage-inflicting in $G^T \parallel BT(S)^A \parallel M^A \parallel G_{SA} \parallel G_{CE} \parallel A^o$, for any $S$ (and $M$) that is consistent with $O$. In general, this does not hold, as we have discussed before. But, it can be verified in a straightforward manner, which is shown in the following. Recall that $M_D = (U, \Sigma_o, \eta, u_0)$.

**Theorem 1** Let $S^l = (U, \Sigma, \eta^l, u_0)$ be a supervisor over the control constraint $(\Sigma, \Sigma_o)$, such that $\eta^l$ is the partial transition function defined as follows.

1. for any $u \in U$ and any $\sigma \in \Sigma_o$, $\eta^l(u, \sigma) = \eta(u, \sigma)$,
2. for any $u \in U$ and any $\sigma \in \Sigma_i \cap \Sigma_o$, $\eta^l(u, \sigma) = u$,
3. for any $u \in U$ and any $\sigma \in \Sigma_i \cap \Sigma_o$, if $\neg \eta(u, \sigma)$, then $\eta^l(u, \sigma) = u$.

If $L_m(G^T \parallel BT(S)^A \parallel P_{\Sigma_i}(S^l) \parallel G^A \parallel G_{SA} \parallel G_{CE} \parallel A^o) \neq \emptyset$, then $A^o$ is damage inflicting in $G^T \parallel BT(S)^A \parallel M^A \parallel G_{SA} \parallel G_{CE} \parallel A^o$, for any $S$ that is consistent with $O$.

**Proof:** $S^l$ is indeed a supervisor over the control constraint $(\Sigma, \Sigma_o)$. In particular, by construction, $S^l$ is an under-approximation of any supervisor $S$ that is consistent with $O$. Thus, if damage is reachable with $S^l$ by $A^o$, i.e., $L_m(G^T \parallel BT(S)^A \parallel P_{\Sigma_i}(S^l) \parallel G^A \parallel G_{SA} \parallel G_{CE} \parallel A^o) \neq \emptyset$, then damage is also reachable with any supervisor $S$ that is consistent with $O$ by $A^o$, since $S$ induces a (non-strictly) larger marked behavior. $\square$

**Example 11** For the water tank example, the attacked supervisor $BT(S)^A$, which is constructed from the supervisor $S^l$ defined in Theorem 1, is given in Fig. 11.

We here remark that $S^l$, defined in Theorem 1, disables all the events in $\Sigma_i \cap \Sigma_{\text{wo}}$. Thus, it introduces some pessimism in the verification of the damage-reachability in general and may result in the rejection of some synthesized covert damage-reachable attacker. That is, the damage-reachability verification of $A^o$ based on $S^l$ in Theorem 1 is sound but in general not complete. However, we shall note that there is a notable exception. $S^l$ is the least permissive supervisor that is consistent with $O$, not only an under-approximation of any supervisor that is consistent with $O$, when $\Sigma_i \subseteq \Sigma_o$, since we then have $\Sigma_i \cap \Sigma_{\text{wo}} = \emptyset$. When $\Sigma_i \subseteq \Sigma_o$, Rule 2) in Theorem 1 becomes: for any $u \in U$ and any $\sigma \in \Sigma_{\text{wo}}$, $\eta^l(u, \sigma) = u$.

**Proposition 2** $S^l$ is the least permissive supervisor that is consistent with $O$, when $\Sigma_i \subseteq \Sigma_o$.

**Proof:** We first observe the following two facts.
Suppose $\Sigma$ is a finite set of observations from the closed-loop system $\emptyset \subseteq P \subseteq O$. We have $\Sigma \subseteq \Sigma_o$, where $\Sigma_o \subseteq \Sigma_o$, we have $L(S) = P_{\Sigma}^{-1}(P_{\Sigma}(L(S))(\Sigma_{uc} \cap \Sigma_o)^*)$.

We first show that $S^i$ is consistent with $O$. Since $O$ records a finite set of observations from the closed-loop system $S \parallel G$, we have $O \subseteq P_{\Sigma}(L(S \parallel G))$, we then have $O \subseteq P_{\Sigma}(L(G))$. Thus, $P_{\Sigma}(L(S^i)(G)) = P_{\Sigma}^{-1}(O(\Sigma_{uc} \cap \Sigma_o)^*) \cap L(G)) = P_{\Sigma}(L(G)) \cap O(\Sigma_{uc} \cap \Sigma_o)^*) \subseteq O$.

Then, we show that $S^i$ is the least permissive supervisor that is consistent with $O$. Let $S$ be any supervisor that is consistent with $O$. We have $O \subseteq P_{\Sigma}(L(S \parallel G)) = P_{\Sigma}(L(G) \cap P_{\Sigma}^{-1}(P_{\Sigma}(L(S))(\Sigma_{uc} \cap \Sigma_o)^*)) = P_{\Sigma}(L(G)) \cap P_{\Sigma}(L(S))(\Sigma_{uc} \cap \Sigma_o)^*)$. We have $O \subseteq P_{\Sigma}(L(S))(\Sigma_{uc} \cap \Sigma_o)^*$ and $L(S^i) = P_{\Sigma}^{-1}(O(\Sigma_{uc} \cap \Sigma_o)^*) \subseteq P_{\Sigma}^{-1}(P_{\Sigma}(L(S))(\Sigma_{uc} \cap \Sigma_o)^*) = L(S)$.

We have the following characterization of the damage-reachability of $A^o$ when $\Sigma_o \subseteq \Sigma_o$.

**Theorem 2** Suppose $\Sigma_o \subseteq \Sigma_o$, then $L_m(G^T \parallel BT(S^i)^A \parallel P_{\Sigma}(S^i \parallel G)^A \parallel G_{SA} \parallel G_{CE} \parallel A^o) \neq \emptyset$ if $A^o$ is damage inflicting in $G^T \parallel BT(S)^A \parallel M^A \parallel G_{SA} \parallel G_{CE} \parallel A^o$, for any $S$ that is consistent with $O$.

**Proof:** This immediately follows from Theorem 1 and the fact that $S^i$ is the least permissive supervisor that is consistent with $O$ when $\Sigma_o \subseteq \Sigma_o$. 

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Fig. 11: $BT(S^i)^A$ for the water tank example
Theorem 1 is quite straightforward. But, there are two mild disadvantages. First of all, it involves the construction of $P_{\Sigma}(S \parallel G)^A$, which requires a subset construction. Secondly, most of the existing tools directly output $G^T \| M_O^T \| G_{SA} \| G_{AF} \| A^\sigma$, instead of $A^\sigma$ which requires another extraction step \cite{23}. In the following, we show how we can directly use $G^T \| M_O^T \| G_{SA} \| G_{AF} \| A^\sigma$ for the verification. We have the following useful result.

**Proposition 3** The following equality holds:

$$L_m(G^T \| M_O^T \| G_{SA} \| G_{AF} \| A^\sigma \| BT(S^i)^A \| G_{CE}^A) = L_m(G^T \| BT(S^i)^A \| P_{\Sigma}(S^i \parallel G)^A \| G_{SA} \| G_{CE} \| A^\sigma)$$

**Proof:** ($\subseteq$): Let $s$ be any string in $L_m(G^T \| M_O^T \| G_{SA} \| G_{AF} \| A^\sigma \| BT(S^i)^A \| G_{CE})$. It holds that $s$ can be executed in $G^T, BT(S^i)^A, G_{SA}, G_{CE}, A^\sigma, M_O^T, G_{AF}$, after we lift their alphabets to $\Sigma \cup \Sigma_A^u \cup \{S\} \cup \Gamma$.

We first observe that the $q_{bad}$ state is reached in $G^T$, some state $u \in U \cup \{u^1\}$ is reached in $M_O^T$ and the state $q_{AF}^1$ is reached in $G_{AF}$, via string $s$. $s$ must be of the form $s = s_1 \sigma$, for some $s_1 \in (\Sigma \cup \Sigma_A^u \cup \Gamma)^*$ and some $\sigma \in \Sigma$, such that the $q_{bad}$ state is reached in $G^T$ via the string $s_1 \sigma$ and the $q_{AF}^1$ state is reached in $G_{AF}$ via $s_1$. Thus, the $q_{bad}$ state has not been reached in $G^T$ via the string $s_1$. We conclude that some state in $U$ is reached in $M_O^T$ via $s_1$; otherwise $s_1 \sigma$ can be executed in $G^T \| M_O^T \| G_{SA} \| G_{AF} \| A^\sigma$, which is impossible. Thus, we conclude that some state $D \in 2^{\Sigma \times Q} - \{\varnothing\}$ is reached in $P_{\Sigma}(S^i \parallel G)^A$ via $s_1$ and some state $D' \in 2^{\Sigma \times Q}$ is reached in $P_{\Sigma}(S^i \parallel G)^A$ via $s = s_1 \sigma$. That is, $s$ can be executed in $P_{\Sigma}(S^i \parallel G)^A$.

It then follows that $s$ can be executed in $G^T \| BT(S^i)^A \| P_{\Sigma}(S^i \parallel G)^A \| G_{SA} \| G_{CE} \| A^\sigma$ and a marked state of $G^T \| BT(S^i)^A \| P_{\Sigma}(S^i \parallel G)^A \| G_{SA} \| G_{CE} \| A^\sigma$ is indeed reached via the string $s$. We can then conclude that $s \in L_m(G^T \| BT(S^i)^A \| P_{\Sigma}(S^i \parallel G)^A \| G_{SA} \| G_{CE} \| A^\sigma)$. Thus, we have

$$L(G^T \| BT(S^i)^A \| P_{\Sigma}(S^i \parallel G)^A \| G_{SA} \| G_{CE} \| A^\sigma) \subseteq L(G^T \| M_O^T \| G_{SA} \| G_{AF} \| A^\sigma)$$

and

$$L_m(G^T \| BT(S^i)^A \| P_{\Sigma}(S^i \parallel G)^A \| G_{SA} \| G_{CE} \| A^\sigma) \subseteq L_m(G^T \| M_O^T \| G_{SA} \| G_{AF} \| A^\sigma).$$

To see the first inclusion, we first remark that $\$ cannot be executed in $G^T \| BT(S^i)^A \| P_{\Sigma}(S^i \parallel G)^A \| G_{SA} \| G_{CE} \| A^\sigma$ or $G^T \| M_O^T \| G_{SA} \| G_{AF} \| A^\sigma$. We only need to show that for each component in the right hand side, it is lower bounded by some component in the left hand side, when their alphabets are lifted to $\Sigma \cup \Sigma_A^u \cup \{S\} \cup \Gamma$. For $M_O^T$, it is lower bounded by $P_{\Sigma}(S^i \parallel G)^A$; for $G_{AF}$, it is lower bounded by $G_{SA}$, as $\$ cannot be executed in $G_{AF}$ or $G_{SA}$.

To see the second inclusion, let $s \in L_m(G^T \| BT(S^i)^A \| P_{\Sigma}(S^i \parallel G)^A \| G_{SA} \| G_{CE} \| A^\sigma)$. We know that $s \in L(G^T \| M_O^T \| G_{SA} \| G_{AF} \| A^\sigma)$. It is clear that the state $q_{bad}$ is reached in $G^T$, some state $u \in U \cup \{u^1\}$ is reached in $M_O^T$, the state $q_{AF}^1$ is reached in $G_{AF}$. Thus, $s \in L_m(G^T \| M_O^T \| G_{SA} \| G_{AF} \| A^\sigma)$.\qed
With Theorem 1, Theorem 2 and Proposition 2, the following result is then immediate. It says that we only need to verify \( L_m(G^T || M^G_{OA} || G_{SA} || A^o || BT(S^i)^A || G_{CE}) \neq \emptyset \), instead of verifying \( L_m(G^T || BT(S^i)^A || P_{G_{SA} || G_{CE}} || G_{CE} || A^o) \neq \emptyset \).

**Corollary 1** If \( L_m(G^T || M^G_{OA} || G_{SA} || A^o || BT(S^i)^A || G_{CE}) \neq \emptyset \), then \( A^o \) is damage-inflicting in \( G^T || BT(S^i)^A || M^A || G_{SA} || G_{CE} || A^o \), for any \( S \) that is consistent with \( O \). Suppose \( \Sigma_c \subseteq \Sigma_m \), then \( L_m(G^T || M^G_{OA} || G_{SA} || A^o || BT(S^i)^A || G_{CE}) \neq \emptyset \) if \( A^o \) is damage-inflicting in \( G^T || BT(S^i)^A || M^A || G_{SA} || G_{CE} || A^o \), for any \( S \) that is consistent with \( O \).

Intuitively, the synthesized attacker \( A^o \) can ensure the damage-reachability in the attacked closed-loop system, for any supervisor \( S \) that is consistent with \( O \), if there exists a marked string \( s \in L_m(G^T || M^G_{OA} || G_{SA} || A^o) \) such that \( s \) is attacked by an attacked supervisor \( S^iA \) where \( S^i \) under-approximates any supervisor that is consistent with \( O \), i.e., \( s \) can be executed in \( BT(S^i)^A || G_{CE} \).

**Example 12** For the water tank example, a non-empty safe attacker has been synthesized by using the Supremica \( m_{25} \), which is given in the form of \( G^T || M^G_{OA} || G_{SA} || A^o \) and shown in Fig. 12. Then, by applying Corollary 1, we can compute \( G^T || M^G_{OA} || G_{SA} || A^o || BT(S^i)^A || G_{CE} \), which is shown in Fig. 13. With a reachable marked state, i.e., \( S_{12}, \) Fig. [13] confirms that the synthesized attacker in Fig. 12 is indeed damage-reachable, in addition to being covert, against all the supervisors that are consistent with \( O \).

We now briefly discuss on the complexity of the above approach.

The size of the reachable state space of the surrogate plant \( G^T || M^G_{OA} || G_{SA} || A^o \) is no more than \( Z = (|Q| + 1)(|U| + 2)(|\Sigma_{s_A} + 1)(|\Sigma_{s_A} + 3) \). Depending on the choices of the adopted supervisor synthesis algorithms (see, for example, \( [1], [32], [33] \)), the complexity for the synthesis of non-empty safe attacker \( A^o \) could differ. For example, suppose we adopt the normality property based synthesis approach \( [1], [33] \), the time complexity for the synthesis is \( O(|\Sigma| + |\Sigma_{s_A}| + |\Gamma|)^2Z \), since we do not require the nonblockingness. To perform the damage-reachability verification, we need to verify \( L_m(G^T || M^G_{OA} || G_{SA} || A^o || BT(S^i)^A || G_{CE}) \neq \emptyset \).

The state size of \( G^T || M^G_{OA} || G_{SA} || A^o || BT(S^i)^A || G_{CE} \) is no more than \( Z2^2(2|U| + 1)(|\Gamma| + 1) \). Thus, the time complexity for damage-reachability verification is \( O(|\Sigma| + |\Sigma_{s_A}| + |\Gamma| + |\Gamma|)(|U| + |\Gamma|)|2^2Z^2 \). Thus, the total time complexity is \( O(|\Sigma| + |\Sigma_{s_A}| + |\Gamma|)|2^2Z^2 \), where \( Z = (|Q| + 1)(|U| + 2)(|\Sigma_{s_A} + 1)(|\Sigma_{s_A} + 3) \).

There remains one disadvantage of the above approach, that is, it requires a synthesis step followed by a verification step. Indeed, if the synthesized (maximally permissible) covert attacker \( A^o \) is verified to be not damage-reachable, then (in general) we may need to keep searching for another (maximal permissible) covert attacker and hope it is indeed damage-reachable. This is troublesome. In the following, we show how the synthesis can be carried out in such a way that damage-reachability is always ensured by construction. The key insight is to embed the verification step within the synthesis step.

\[^8\] Since the control constraint is \( (\Sigma_{s_A} \cup \Sigma_{s_A}^c, \Sigma_{s_A} \cup \Sigma_{s_A}^c) \) (see Section 3.3), the normality property based synthesis approach effectively only allows the attacker to control those events in \( (\Sigma_{s_A} \cap \Sigma_{s_A}^c) \cup \Sigma_{s_A}^c \).
Theorem 3 Any non-empty safe attacker for $G$ immediately have the following useful result.

Proof: We first observe that $L(G^T[M_D^T]G_S^A||G_{AF}||BT(S)^A||G_{CE}) = L(G^T[M_D^T]G_S^A||G_{AF})$ and also the sets of bad strings for $G^T[M_D^T]G_S^A||G_{AF}||BT(S)^A||G_{CE} and $G^T[M_D^T]$. For that end, we need to bring in the concept of automaton completion $[22]$. Formally, the completion of any (partial) finite-state automaton $P = (I, \Sigma, i_0, I_m)$ is a complete finite-state automaton $\overline{P} = (I \cup \{i_d\}, \Sigma, \overline{i}_0, I_m)$, where the distinguished state $i_d \notin I$ denotes the added dump state and $\overline{\pi} = \pi \cup (\{i_d\} \times \Sigma \times \{i_d\}) \cup \{(i, \sigma, i_d) \mid \pi(i, \sigma) \text{ is undefined, } i \in I, \sigma \in \Sigma\}$ denotes the transition function.

Let $BT(S)^A$ denote the completion of $BT(S)^A$ and let $G_{CE}$ denote the completion of $G_{CE}$. We now construct the surrogate plant $G^T[M_D^T]G_S^A||G_{AF}||BT(S)^A||G_{CE}$ and perform the synthesis of (maximally permissive) safe attacker on $G^T[M_D^T]G_S^A||G_{AF}||BT(S)^A||G_{CE}$ instead; as before, the set of bad states are again specified by (the combination of) the state $q^5$ in $G^T$, the state $\mu^3$ in $M_D^3$ and the state $q^4F, S$ in $G_{AF}$. We immediately have the following useful result.

\textbf{Theorem 3} Any non-empty safe attacker for $G^T[M_D^T]G_S^A||G_{AF}||BT(S)^A||G_{CE}$ is a covert damage-reachable attacker on the attacked closed-loop system $G^T[M_D^T]G_S^A||BT(S)^A||G_{CE}$.
\[ G_{SA} \parallel G_{AF} \] are the same. Thus, any safe attacker for \( G^T \parallel M^T_O \parallel G_{SA} \parallel G_{AF} \parallel BT(S^1)^A \parallel G_{CE} \) is also a safe attacker for \( G^T \parallel M^T_O \parallel G_{SA} \parallel G_{AF}, \) and vice versa.

Let \( A^o \) be any non-empty safe attacker for \( G^T \parallel M^T_O \parallel G_{SA} \parallel G_{AF} \parallel BT(S^1)^A \parallel G_{CE} \). By the above analysis, we know that \( A^o \) is a safe attacker for \( G^T \parallel M^T_O \parallel G_{SA} \parallel G_{AF}. \) Furthermore, we know that there exists some string \( s \in L_m(G^T \parallel M^T_O \parallel G_{SA} \parallel G_{AF} \parallel |BT(S^1)^A\parallel G_{CE}) = L_m(G^T \parallel M^T_O \parallel G_{SA} \parallel |A^o\parallel |BT(S^1)^A\parallel G_{CE}). \) Thus, by Proposition 1 and Corollary 1, we know that \( A^o \) is a covert damage-reachable attacker on the attacked closed-loop system \( G^T \parallel BT(S^1)^A \parallel M^A \parallel G_{SA} \parallel G_{CE} \parallel A \) induced by \( A, \) for any supervisor \( S \) that is consistent with \( O, \) i.e., \( O \subseteq P_2(L(S)|G)). \)

Intuitively, we only need to synthesize a non-empty safe attacker \( A^o \) for \( G^T \parallel M^T_O \parallel G_{SA} \parallel G_{AF} \parallel |BT(S^1)^A\parallel G_{CE}; \) and \( A^o \) is guaranteed to be covert damage-reachable on the attacked closed-loop system induced by any supervisor \( S \) that is consistent with \( O. \) Due to our use of over-approximation, the other direction of the implication in general does not hold for Theorem 3.

Next, we briefly discuss on the complexity of the approach based on Theorem 3.

The state size of \( G^T \parallel M^T_O \parallel G_{SA} \parallel G_{AF} \parallel |BT(S^1)^A\parallel G_{CE} \) is \( Z(2|U| + 2)(|\Gamma| + 2). \) Thus, the time complexity of the synthesis approach by using Theorem 3, if we adopt the normality property based synthesis approach, costs \( O((|\Sigma| + |\Gamma|)^2 Z(2|U| + 2)(|\Gamma| + 2)) \).
4 Conclusions and Future Works

This paper studies the covert attacker synthesis problem in the new setup where the model of the supervisor is unknown, but a (prefix-closed) finite set of observations of the runs of the closed-loop system is assumed to be available, to the adversary. We have proposed a heuristic synthesis algorithm, with a formal guarantee of correctness, for the synthesis of covert damage-reachable attackers, even without using the model of the supervisor. The solution methodology is to formulate the observation-assisted covert attacker synthesis problem as an instance of the partial-observation supervisor synthesis problem. It then follows that we can use the existing supervisor synthesis solvers to compute non-empty (i.e., damage-reachable) safe (i.e., covert) attackers.

There are several limitations of this work that need to be addressed. First of all, we only consider sensor replacement attacks and actuator disablement attacks; it is of interest to consider other attack mechanisms. Our approach seems capable of dealing with sensor insertion and deletion attacks, without much modification, but it has some difficulty in coping with actuator enablement attacks. Secondly, the synthesized covert damage-reachable attacker is in general not maximally permissive, due to the use of over-approximation in the surrogate plant. Thus, the approach has not resolved the decidability of the observation-assisted covert attacker synthesis problem. Lastly, we have not addressed the problem of synthesis of covert damage-nonblocking attackers, which seems to be more challenging. These and other issues are to be addressed in the future works.

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