Quantitative characterization of collapse and fracture pressure uncertainty based on Monte Carlo simulation

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Abstract
The complex geological conditions of drilling, the difficulty of formation collapse and fracture pressure prediction in South Sichuan work area lead to the complex drilling and frequent failure, which seriously restricts the safe and efficient development of shale gas. In view of this problem, this paper has carried out relevant research. First of all, the existing calculation model of formation collapse and fracture pressure is established and improved; on this basis, the sources of uncertainty in the calculation model of collapse and fracture pressure are analyzed, mainly the in-situ stress and rock mechanics parameters, which have a lot of uncertainties; then, the uncertainty of rock mechanics parameters and in-situ stress is analyzed, and its probability is determined. Finally, based on Monte Carlo simulation, the quantitative characterization method of formation collapse and fracture pressure uncertainty is established. The prediction result of collapse and fracture pressure is no longer a single curve or value, but an interval, which is more practical for drilling in complex geological environment. The results of this study are helpful to better describe the collapse and fracture pressure of complex formation and can provide more valuable reference data for drilling design.

Keywords Shale gas drilling · Collapse and fracture pressure uncertainty · Monte Carlo simulation · Wellbore stability evaluation

Introduction
Formation collapse pressure and formation fracture pressure are the upper and lower limits of safe drilling fluid density window to maintain wellbore stability. How to accurately predict and describe formation collapse and fracture pressure are an important content to avoid wellbore instability risk (Tinggen and Zhichuan 2000; Ottesen et al. 1999). With the development of drilling to deep well complex formation and deep water, the geological environment encountered in the process of drilling is becoming more and more complex. In the actual drilling engineering, due to the particularity of the drilling engineering construction, as well as the uncertainty of geological conditions, the complexity of operating environment factors, the variability of construction methods and design parameters, etc., many uncertain factors will be encountered from time to time in the process of drilling construction. It is increasingly difficult to accurately predict formation collapse and fracture pressure. The rock mechanics parameters and in-situ stress input in the existing calculation model of formation collapse and fracture pressure are all treated according to the fixed value, and the prediction results of formation collapse and fracture pressure are all single value results. This method ignores the error between the prediction result and the actual result due to the uncertainty of the input parameters of the calculation model of collapse and fracture pressure (Guangfu et al. 2019; Kolawole et al. 2018; Moos and Peska 2003). The in-situ stress and rock mechanics parameters in the calculation model need to be calculated based on the indirect formula according to the seismic or logging data, and there is uncertainty (Mostafavi et al. 2011; Zhide, 2004; Limin et al. 2017). The design based on the inaccurate prediction results may lead to the risk of wellbore instability. In order to solve this problem, this paper proposes a quantitative characterization method of formation collapse and fracture pressure uncertainty based on Monte Carlo simulation.
pressure uncertainty based on Monte Carlo simulation. First of all, established and improved the existing calculation model of formation collapse and fracture pressure. On this basis, analyzed the sources of uncertainty in the calculation model of formation collapse and fracture pressure, mainly the in-situ stress and rock mechanics parameters, which are usually obtained through indirect mathematical model calculation based on seismic or logging data, so there are a lot of uncertainties. After that, the uncertainty of rock mechanical parameters and in-situ stress is analyzed, and its probability distribution is determined. Finally, based on Monte Carlo simulation, the quantitative characterization method of the uncertainty of formation collapse and fracture pressure is established. The prediction result of formation collapse and fracture pressure is not a single curve or value, but an interval value, which is more practical for drilling in complex geological environment.

Methods

Quantitative calculation model of collapse and fracture pressure

Formation collapse and fracture pressure are the upper and lower limits of safe drilling fluid density window to maintain wellbore stability (Yi et al. 2019). How to accurately predict and describe formation collapse and fracture pressure are an important content to avoid wellbore instability risk. In the calculation of formation collapse and fracture pressure, many key calculation parameters affect the results, mainly including in-situ stress and rock mechanics parameters, which can be calculated based on logging data or seismic interpretation data.

(1) Rock mechanics parameters

Rock mechanics parameters can be divided into mechanical properties and elastic properties (Xiangjun and Pingya 1999; Min et al. 2009).

a. Rock elastic properties

Table 1 Calculation formula of dynamic elastic parameters

| Dynamic elastic parameters | Calculation formula |
|----------------------------|---------------------|
| Poisson’s ratio /μ_d       | μ_d = (0.5ΔT_p^2 − ΔT^2_p) / (ΔT^2_s − ΔT^2_p) |
| Young’s modulus /E_d       | E_d = ρ(3ΔT_s^2 − 4ΔT_p^2) / (ΔT^2_s − ΔT^2_p) |
| Shear modulus /G           | G = E_d / (2(1 + μ_d)) |
| Bulk modulus of elasticity /K_b | K_b = E_d / (3(1 − 2μ_d)) |
| Volume compression coefficient /C_b | C_b = 1 / K_b |
| Compressibility coefficient/C_r | C_r = 1 / (ρ(1/ΔT_ρm^2 − 4/(ΔT^2_s − 1))) |
| Effective stress coefficient/α | α = 1 − C_r / C_b |

P-wave velocity of rock:

V_p = \frac{E_d(1 − μ_d)^{0.5}}{[ρ_d(1 + μ_d)(1 − 2μ_d)]^{0.5}} = \frac{1}{ΔT_p} \quad (1)

S-wave velocity of rock:

V_s = \frac{E_d^{0.5}}{[2ρ_b(1 + μ_d)]^{0.5}} = \frac{1}{ΔT_s} \quad (2)

In formula, ΔT_p—Rock P-wave time difference, us/m. ΔT_s—Rock S-wave time difference, us/m. ρ_d—Rock density, g/cm³. E_d—Dynamic Young’s modulus, MPa. μ_d—Dynamic Poisson’s ratio, dimensionless.

The transformation relationship between each rock elastic parameters:

G = \frac{E}{2(1 + μ_d)} K_b = \frac{E}{3(1 − 2μ_d)} C_b = \frac{1}{K_b} \quad (3)

In formula, E—Young’s modulus, MPa. G—Shear modulus, MPa. K_b—Bulk modulus of elasticity, MPa. C_b—Volume compression coefficient, dimensionless.

The elastic parameters of rock can be divided into dynamic and static parameters, which can better reflect the real elastic characteristics of rock. In practical application, the dynamic elastic parameters should be converted into static elastic parameters.

① Dynamic elastic parameters

According to formula (1), formula (2) and formula (3), the elastic parameter formula of dynamic rock mechanics is derived, as shown in Table 1

② Conversion of dynamic and static elastic parameters

The transformation relationship between dynamic and static elastic parameters (Min et al. 2009):

μ_s = A_1 + K_1 μ_d E_s = A_2 + K_2 E_d \quad (4)
In formula, $A_1 = a_{11} + a_{12} \log(\sigma_1 - \sigma_2) \\setminus A_2 = a_{21} + a_{22} \log(\sigma_1 - \sigma_3) \\setminus k_1 = k_{11} + k_{12} \log(\sigma_1 - \sigma_3) \\setminus k_2 = k_{21} + k_{22} \log(\sigma_1 - \sigma_3)$, $\sigma_1$ and $\sigma_3$ are the maximum and minimum principal stresses, respectively. $a_{11} \setminus a_{12} \setminus a_{21} \setminus a_{22} \setminus k_{11} \setminus k_{12} \setminus k_{21} \setminus k_{22}$ are regression coefficients. The above transformation relationship needs to be obtained according to the core laboratory experiments.

b. Rock mechanical properties

The calculation formula of rock strength parameters is shown in Table 2.

In Table, $\rho$—Rock density, g/cm$^3$. $V_{cf}$—Shale content, dimensionless. $\phi$—Internal friction angle, $^\circ$. $M = a - b \times C$. $a \setminus b$—The coefficients related to rock properties are obtained by inverse calculation of core test experiments.

(2) In-situ stress

The commonly used calculation formula of in-situ stress is (Mian et al. 2008):

$$\sigma_v = \int G_d dH$$

$$\sigma_H = \frac{\beta_1 E + 2 \mu(\sigma_v - a G_p)}{2(1 - \mu)} + \frac{\beta_2 E}{2(1 + \mu)} + a G_p$$

$$\sigma_h = \frac{\beta_1 E + 2 \mu(\sigma_v - a G_p)}{2(1 - \mu)} - \frac{\beta_2 E}{2(1 + \mu)} + a G_p$$

In formula, $\sigma_v$—Vertical stress, MPa. $\sigma_H$—Maximum horizontal in-situ stress, MPa. $\sigma_h$—Minimum horizontal in-situ stress, MPa. $\mu$—Poisson’s ratio, dimensionless. $E$—Young’s modulus, MPa. $H$—Well depth, m. $G_p$—Pore pressure, MPa. $\beta_1 \setminus \beta_2$—Coefficient of tectonic stress:

$$\rho_c = \frac{\eta(3 \sigma_H - \sigma_h)}{(K^2 + \eta)} + \frac{\alpha \rho_p (K^2 - 1)}{(K^2 + \eta)}$$

In formula, $\rho_c$—Collapse pressure, g/cm$^3$. $H$—Well depth, m. $K = \cot(45^\circ - \phi/2)$. $\phi$—Internal friction angle, $^\circ$. $C$—Rock cohesion, MPa. $\rho_p$—Pore pressure, g/cm$^3$. $\sigma_H$—Maximum horizontal in-situ stress, g/cm$^3$. $\sigma_h$—Minimum horizontal in-situ stress, g/cm$^3$. $\eta$—Nonlinear correction coefficient of stress. $\alpha$—Biot coefficient.

(4) Fracture pressure

The commonly used calculation formula of fracture pressure is (Qining 1983):

$$\rho_f = 3 \sigma_h - \sigma_H - a \rho_p + S / 0.00981 \times H$$

In formula, $\rho_f$—Fracture pressure, g/cm$^3$. $S$—Uniaxial tensile strength, MPa.

### Analysis of uncertainty sources of collapse and fracture pressure

According to the calculation model of formation collapse and fracture pressure, the parameters in the calculation model can be divided into three categories: well trajectory parameters, in-situ stress and rock mechanical parameters. Among them, the well trajectory can be accurately obtained according to drilling design or MWD data. The in-situ stress and rock mechanical parameters are usually calculated by indirect mathematical model based on logging or seismic interpretation data, so there are a lot of uncertainties. For the drilled well, the in-situ stress and rock mechanics parameters can be obtained by various logging data or seismic data. However, it is difficult to obtain these parameters accurately before drilling, which
will lead to a large error or uncertainty in the pressure prediction results of the well to be drilled. Therefore, it is necessary to analyze the uncertainty of rock mechanical and in-situ stress parameters. Using the interpretation results of logging data to estimate the probability distribution of parameters, the interpretation results of formation collapse and fracture pressure with uncertainty are finally obtained.

**Quantitative characterization of collapse and fracture pressure uncertainty**

**Quantitative characterization of in-situ stress and rock mechanical parameters**

In order to analyze the uncertainty of rock mechanical parameters and in-situ stress and determine its probability distribution, it is necessary to establish the sample database of rock mechanical parameters and in-situ stress parameters. According to sequence stratigraphy (Yinye 2009): “under the same geological period and sedimentary conditions, the rocks have the same lithology, which will produce similar seismic or logging responses.” Therefore, we select the logging interpretation results of rock mechanics parameters and in-situ stress within a certain depth of the same formation as samples and build the sample database. It is assumed that there are \(2n+1\) log interpretation results of rock mechanical parameters and in-situ stress in the range of well depth \(\Delta H\). They are treated as a set of measurement samples, as shown in Fig. 1.

\(\Delta H\) is the sample interval, and its value is twice the range of theoretical variogram model in the sample formation group. Then, the probability distributions of rock mechanical parameters and in-situ stress are calculated by using normal information diffusion estimation theory (Yifeng et al. 2016; Shushen 2001). The probability density functions of rock mechanical parameters and in-situ stress \(X\) are assumed to be \(f(x)\). Finally, the normal information diffusion of \(f(x)\) is estimated as follows:

\[
\tilde{f}(x) = \frac{1}{\sqrt{2\pi(2n+1)h}} \sum_{i=n}^{i+n} \left\{ \exp \left[ -\frac{(x-x_i)^2}{2h^2} \right] \right\}
\]

(10)

In formula, \(h\)—Diffusion coefficient, m.

\[
h = \frac{\lambda(x_{\text{max}} - x_{\text{min}})}{2n}
\]

(11)

In formula, \(x_{\text{max}}\), \(x_{\text{min}}\)—The maximum and minimum value of rock mechanical parameters and in-situ stress \(X\) in the target formation. The coefficient \(\lambda\) can be obtained according to Table 3:

**Table 3** Corresponding relation between \(\lambda\) and sample number \((2n+1)\)

| \(2n+1\) | \(\lambda\)     | \(2n+1\) | \(\lambda\)     | \((2n+1)\) | \(\lambda\)     |
|---------|---------------|---------|---------------|-----------|---------------|
| 3       | 0.84932180   | 8       | 1.395189816  | 13        | 1.420698795  |
| 4       | 1.273982782  | 9       | 1.422962345  | 14        | 1.42069671   |
| 5       | 1.698643675  | 10      | 1.416278786  | 15        | 1.420693321  |
| 6       | 1.336252561  | 11      | 1.420835443  | 16        | 1.420692226  |
| 7       | 1.445461208  | 12      | 1.420269570  | 17        | 1.420693101  |

**Quantitative characterization of collapse and fracture pressure uncertainty**

The steps to quantify the uncertainty of collapse and fracture pressure based on Monte Carlo simulation (Sundar and Witt 1995; Junhu 2007) are as follows:

(1) Determination of the rock mechanical parameters and in-situ stress probability distributions. According to the
above method, the analysis sample database of model input parameters at any depth \( h \) position is constructed: 
\[ F_N \sim f(x_1), f(x_2), \cdots, f(x_n) \]

(2) Construction of random simulation sample sets. The random values are generated according to the probability distribution of the rock mechanical parameters and in-situ stress. The collapse and fracture pressure at any depth can be obtained by substituting them into the calculation models.

(3) Construction of collapse and fracture pressure sample sets. The probability distribution and cumulative probability distribution function of formation collapse and fracture pressure at any depth are obtained by selecting normal distribution form to fit the statistical analysis calculation results: 
\[ f_h(P_{tf}) \cdot F_h(P_{tf}) \]

(4) Quantitative characterization of collapse and fracture pressure uncertainty. Through the above methods, the cumulative probability of collapse and fracture pressure at different depths can be obtained, which can form the set:
\[
F(P_{tf}) = \{ F_{h_1}(P_{tf}), F_{h_2}(P_{tf}), F_{h_3}(P_{tf}),
\ldots, F_{h_i}(P_{tf}), \ldots, F_{h_n}(P_{tf}) \}\tag{12}
\]

In formula, \( (P_{tf})_{h_i} \) — The collapse and fracture pressure with cumulative probability \( j \) at the depth of \( h_i \). Take the same cumulative probability value \( j_0 \) to form the new set:
\[
(P_{tf})_{h_i} = \{ (P_{tf})_{h_{i,1}}, (P_{tf})_{h_{i,2}}, \ldots, (P_{tf})_{h_{i,j_0}}, \ldots, (P_{tf})_{h_{i,j_b}}, \ldots \}\tag{13}
\]

In formula, \( (P_{tf})_{h_{i,1}} (P_{tf})_{h_{i,2}} \) — The collapse and fracture pressure with cumulative probability \( j_1 \cdot j_2 \) \( (j_1 < j_2) \). The two curves constitute the interval of collapse and fracture pressure with a confidence of \( [j_1 - j_2] \times 100\% \), which indicates that the probability that the actual collapse and fracture pressure in \( [(P_{tf})_{h_{i,1}}, (P_{tf})_{h_{i,2}}] \) is \( [j_1 - j_2] \times 100\% \).

**Table 4** Probability distribution characteristic parameters of each parameter

| Parameters                          | Unit   | Distribution form | characteristic parameter |
|-------------------------------------|--------|------------------|-------------------------|
| Vertical stress—\( \sigma_v \)     | g/cm³  | Normal distribution \( N(u, \sigma^2) \) | \( u = 2.396, \sigma = 0.013 \) |
| Maximum horizontal in-situ stress—\( \sigma_H \) | g/cm³ | \( u = 2.25, \sigma = 0.042 \) |
| Minimum horizontal in-situ stress—\( \sigma_h \) | g/cm³ | \( u = 2.07, \sigma = 0.058 \) |
| Uniaxial tensile strength—\( S_u \) | MPa   | \( u = 1.72, \sigma = 0.157 \) |
| Pore pressure—\( \rho_p \)         | g/cm³ | \( u = 1.08, \sigma = 0.093 \) |
| Internal friction angle—\( \phi \)  | °      | \( u = 32.3, \sigma = 1.78 \) |
| Rock cohesion—\( C \)              | MPa   | \( u = 3.4, \sigma = 0.17 \) |

**Results and discussions**

Well XX is a deep shale gas exploration well in South Sichuan work area, taking XX as an example for analysis. Firstly, the probability distributions of rock mechanical parameters and in-situ stress at any depth are calculated. Table 4 shows the calculation results of parameter probability at the depth of 2000 m. Then, the probability distributions of collapse and fracture pressure at this depth are obtained, as shown in Fig. 2 and Fig. 3. Finally, the interval profile of collapse and fracture pressure with 90% confidence is obtained by programming calculation, as shown in Fig. 4.

The conclusion is as follows: the prediction result of collapse and fracture pressure is no longer a single curve or value, but an interval, which is more practical for drilling in complex geological environment. The results of this study are helpful to better describe the collapse and fracture pressure of complex formation and can provide more valuable reference data for drilling design.

**Conclusions and recommendations**

a. In the existing calculation model of collapse and fracture pressure, the input rock mechanical parameters and in-situ stress are all treated according to the fixed value, and the obtained pressure prediction results are all single value results. This method ignores the error between the predicted result and the actual result due to the uncertainty of the input calculation parameters.

b. In this paper, the existing calculation model of formation collapse and fracture pressure is established and improved, and the Monte–Carlo simulation method is used to characterize the collapse and fracture pressure, and the uncertainty quantitative description method of formation collapse and fracture pressure is established.
c. According to the method established in this paper, the predicted collapse and fracture pressure are no longer a single fixed value curve, but a pressure interval with probability information, which is more practical for drilling in complex geological environment. The results of this study are helpful to better describe the collapse and fracture pressure of complex formation and can provide more valuable reference data for drilling design.
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**References**

- Guangfu Z, Ming T, Shiming H (2019) Prediction of collapse pressure and optimization of borehole trajectory in coal measures [J]. Oil Drill Product Technol 41(4):405–411
- Junhu W (2007) Application of Monte Carlo method in system engineering [M]. Xi’an Jiaotong University Press, Xi’an
- Kolawole O, Federer G, Szabo I 2018 Formation Susceptibility to Wellbore Instability and Sand Production in the Pannonian Basin, Hungary[C]// 52nd U.S. Rock Mechanics/ Geomechanics Symposium, 17–20
- Limin L, Yanan S, Xiaopo L et al (2017) Prediction method of formation fracture pressure with credibility before drilling [J]. Fault Block Oil Gas Field 24(1):112–115
- Mian C, Yan J, Guangqing Z (2008) Mechanics of Petroleum Engineering [M]. Science Press, Beijing
- Min L, Zhanghua L, Shichun C et al (2009) Study on rock mechanics parameter test and formation fracture pressure prediction[J]. Oil Drill Product Technol 31(5):15–18
- Moos D, Peska P (2003) Comprehensive wellbore stability analysis utilizing quantitative risk assessment [J]. J Petrol Sci Eng 38(3):97–109
- Mostafavi S, Adnoy B, Hareland G (2011) Model based uncertainty assessment of wellbore stability analyses and downhole pressure estimations[J]. J Am Rock Mech 12(5):60–65
- Ottesen S. W., Zheng R. H., Mccann R. C. Borehole stability assessment using quantitative risk analysis[C]. SPE 52864, 1999.
- Qining F (1983) Formula and method of calculating formation fracture pressure with logging data[J]. J China Univ Pet (Nat Sci Ed) 11(3):41–48
- Shushen L (2001) Statistical processing and error analysis of test data [J]. Phys Chem Insp ect: Phys 37(2):88–91
- Sundar A, Witt F (1995) Probabilistic structural mechanics handbook [M]. Springer, Berlin
- Tinggen C, Zhichuan G (2000) Theory and technology of drilling engineering [M]. Petroleum University Press, Dongying, pp 51–54
- Xiangjun L, Pingya L (1999) The application and development of well logging in the study of wellbore stability[J]. Nat Gas Ind 19(6):33–35

Fig. 4 Collapse and fracture pressure interval with confidence level 90% of XX
Yifeng H, Xibin Li, Huanzhen Z (2016) Inference of probability distribution of large sample geotechnical parameters based on normal information diffusion principle [J]. Res Develop Sci Technol 38(2):249–253

Yinye W (2009) Principles of sequence stratigraphy [M]. Petroleum Industry Press, Beijing

Yi W, Renjun X, Shujie L et al (2019) The stability law of high temperature and high pressure vertical well wall considering temperature effect [J]. Fault Block Oil Gas Field 26(2):253–256

Zhide L 2004 Study on the evaluation method of wellbore stability logging in carbonate formation [D]. Southwest Petroleum Institute

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