The supercluster-void network V.

Alternative evidence for its regularity

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Received 1999 / Accepted ...

Abstract. We analyse the distribution of Abell clusters of galaxies to study the regularity of the supercluster-void network. We apply a new method sensitive to the geometry of the location of clusters, and measure the goodness of regularity of the network. We find that the supercluster-void network resembles a cubical lattice over the whole space investigated. The distribution of rich superclusters is not isotropic: along the main axis of the network it is periodic with a step of length ≈ 130 h⁻¹ Mpc, whereas along the diagonal of the network the period is larger, as expected for a cubical lattice. This large-scale inhomogeneity is compatible with recent CMB data.

Key words: cosmology: observations – cosmology: large-scale structure of the Universe

1. Introduction

The basic assumption in the standard Friedman-Robertson-Walker cosmology is that the Universe is homogeneous and isotropic on large scales (Peebles 1980). Clusters of galaxies and galaxy filaments form superclusters (Abell 1958); examples are the Local supercluster (de Vaucouleurs 1956, Einasto et al. 1984) and the Perseus-Pisces supercluster (Jöeveer et al. 1978, Giovanelli 1993). Superclusters and voids between them form a continuous network of high- and low-density regions which extends over the entire part of the Universe studied in sufficient detail up to a redshift of z ≈ 0.13 (Einasto et al. 1994, 1997b, hereafter Paper I). According to the conventional paradigm the location of superclusters in this network is random, as density perturbations on all scales are believed to be randomly distributed (Feldman et al. 1994).

There exists growing evidence that the supercluster-void network has some regularity. Broadhurst et al. (1990) have measured redshifts of galaxies in a narrow beam along the direction of the northern and southern Galactic poles and found that the distribution is periodic: high-density regions alternate with low-density ones with a surprisingly constant interval of 128 h⁻¹ Mpc (here h is the Hubble constant in units of 100 km s⁻¹ Mpc⁻¹). The three-dimensional distribution of clusters shows clear signs of regularity (Paper I). One method to characterize this regularity is a correlation analysis. Kopylov et al. (1984, 1988), Fetisova et al. (1993), Mo et al. (1992), and Einasto & Gramann (1993) have found evidence for the presence of a secondary peak of the correlation function of clusters of galaxies at 125 h⁻¹ Mpc. The secondary peak has been interpreted as the correlation of clusters in superclusters located on opposite sides of large voids. Recent studies show that the correlation function oscillates with a period equal to that of the periodicity of the supercluster-void network (Einasto et al. 1997b, hereafter Paper II). An oscillation with very low amplitude is seen also in the correlation function of LCRS galaxies (Tucker et al. 1997). An oscillating correlation function corresponds to a peaked power spectrum (Einasto et al. 1997, hereafter Paper III). Peacock (1997) and Gaztañaga & Baugh (1998) determined the three-dimensional power spectrum from the projected distribution of APM galaxies; the Abell cluster power spectrum was derived by Einasto et al. (1997b), and Retzlaff et al. (1998) and Tadros et al. (1998) calculated the power spectrum for APM clusters. All these recent studies find the peak or turnover of the power spectrum near a wavelength of λ₀ = 120 h⁻¹ Mpc or wavenumber k₀ = 2π/λ₀ = 0.05 h Mpc⁻¹. In addition, in the line-of-sight correlation function of Lyα-break galaxies with redshifts z ∼ 3 secondary peaks were found at a redshift separation of Δz ≈ 0.22 ± 0.02 (Broadhurst & Jaffe 1999). This separation corresponds to the comoving scale 120 (1 + z)⁻¹ Mpc if a flat ΛCDM isotropic cosmological model with Ω_m = 1 − Ω_Λ ≈ 0.4 ± 0.1 is assumed. A low-density model dominated by a cosmological term fits nicely to many other recent data, in particular, those from high-z supernovae (Perlmutter et al. 1998).

It should be noted that no such feature is seen in the power spectra of some galaxy catalogs, in particular, in...
the recent IRAS PSC redshift survey (Sutherland et al. 1999). However, we suppose that this may be the result of IRAS galaxies avoiding overdense regions like rich clusters (see more detailed explanation of this point by Einasto et al. 1999).

Quantitative methods used so far to describe the regularity of the matter distribution are insensitive to the directional and phase information, and thus characterise the regularity only indirectly. In this paper we shall use a new geometric method to investigate the distribution of clusters of galaxies, sensitive to the regularity and the isotropy of the distribution.

2. Observational data

As in previous papers of this series we use the 1995 version of the compilation by Andernach & Tago (1998) of all published galaxy redshifts towards galaxy clusters in the catalogue of rich clusters of galaxies by Abell (1958) and Abell et al. (1989) (hereafter Abell clusters). Individual galaxies were associated to a given Abell cluster if they lay within a projected distance of \( \leq 1.5 \, h^{-1} \) Mpc (1 Abell radius) and within a factor of two of the redshift estimated from the brightness of the clusters 10-th brightest galaxy, using the photometric estimate of Peacock & West (1992). For the present analysis we used a sample of all rich clusters (richness class \( R \geq 0 \) and excluding clusters from ACO’s supplementary list of S-clusters) in this compilation with redshifts up to \( z = 0.12 \). The sample contains 1304 clusters, 869 of which have measured redshifts for at least two galaxies. Distances of clusters without measured redshifts and of clusters with only 1 galaxy measured have been estimated on the basis of the apparent magnitude of the 10-th brightest galaxy of the cluster.

As representatives of high-density regions in the Universe we used rich superclusters from the list of superclusters presented in Paper I. Superclusters were identified using the friend-of-friends algorithm by Zeldovich et al. (1982) with a neighbourhood radius of \( 24 \, h^{-1} \) Mpc. In this way all clusters of a supercluster have at least one neighbour at a distance not exceeding the neighborhood radius. To illustrate the distribution of clusters in high-density regions we plot in Fig. 1 only clusters in rich superclusters, while in the quantitative analysis below we shall use all clusters. The sheets plotted are \( 300 \, h^{-1} \) Mpc thick, thus some superclusters overlap in projection. The sheet in the left panel of Fig. 1 crosses the majority of cells of the supercluster-void network present in our cluster survey; the sheet in the right panel has the full depth of the sample in the southern Galactic hemisphere. Fig. 1 shows clearly the quasi-regular network of superclusters interspersed with voids. The three-dimensional distribution of all Abell and APM clusters in rich superclusters in the whole space within a limiting radius \( 350 \, h^{-1} \) Mpc around us is shown in the home page of Tartu Observatory (http://www.aai.ee).

For comparison we plot in Fig. 1 also the distribution of APM clusters of galaxies in rich superclusters. We see that
the APM cluster sample covers a much smaller volume in space which makes it difficult to investigate the regularity of the distribution of high-density regions on large scales. The APM cluster sample is defined only in the southern Galactic hemisphere, and even here the APM sample containing clusters with measured redshifts covers only three high-density regions defined by very rich superclusters with at least 8 member-clusters. These are the Sculptor (SC9), Pisces-Cetus (SC10), and Horologium-Reticulum (SC48) superclusters of the catalogue in Paper I. To investigate the regularity of the supercluster-void network, the sample volume must exceed the period of the network at least several times. For this reason we used in the following analysis only Abell clusters.

3. Method

To investigate the regularity of the distribution of clusters of galaxies we shall use a novel geometric method. The idea of the method is taken from the periodicity analysis of time series of variable stars. To derive the period of a variable star the time series of brightness measurements is folded using a certain trial period, i.e. individual measurements are stacked to yield a combined light curve for a single period. If the trial period is wrong then the distribution of data-points in the combined curve is almost random. In contrast, if the trial period is correct and the star under study is a periodic variable, then one has a clear mean light curve of the star.

This method has been generalised to the 3-dimensional case by Toomet (1997). The whole space under study is divided into trial cells of side length \( d \). Then all objects in the individual cubical cells are stacked to a single combined cell, preserving their phases in the original trial cells. We then vary the side length of the trial cube to search for the periodicity of the cluster distribution. The method can be used for any form of tightly packed regular trial cells. We use cubical trial cells which have the simplest possible form, and because this form gives a satisfactory explanation of the distribution of clusters.

The next step is to calculate the goodness of regularity for this side length \( d \) of the trial cell. To do this we divide the combined stacked cell into small elementary sub-cells and count, for a particular data point (i.e. cluster of galaxies) within this sub-cell, the number of other data points which come from an original trial cell different from the one in which the given data point is located. The goodness of regularity is calculated as follows:

\[
k(d) = \frac{2}{N(N+1)} \frac{V(d)}{V_c} \sum_{i=1}^{N} \sum_{j=i+1}^{N} v(f_i, f_j).
\]

Here \( N \) is the total number of data points, i.e. individual Abell clusters; \( V \equiv d^3 \) is the volume of the trial cell of side length \( d \); \( f_i \) is the phase-vector of the cluster \( i \) in the combined cell (a spatial remainder-vector, the components of which are remainders, which arise if the original coordinates are divided by \( d \)); \( V_c \) is the elementary volume inside of the combined cell. We use an elementary cube (sub-cell) of side length \( 2d \). The counter \( v(f_i, f_j) \) is defined as follows: \( v_{ij} = 1 \), if the phase vector of the cluster \( j \) lies in the elementary volume \( V_c \) (the centre of which coincides with \( f_j \)), and if clusters \( i \) and \( j \) are located farther away from each other than \( d_x \). Otherwise we set \( v_{ij} = 0 \). Thus \( v_{ij} \) is unity only if the phases of clusters are close enough in the combined cube, but the clusters themselves are not members of the same supercluster (they lie in different trial cubes of the original space).

The sensitivity of the method and the rms error of the variable \( k(d) \) have been determined using mock samples of randomly and quasi-regularly located points. We use mock samples with two populations of clusters, in
one population all clusters are randomly distributed, in the second population they are located in superclusters which form a regular rectangular network with a step $130 \, h^{-1} \text{Mpc}$. Supercluster centre positions have random shifts $\pm 20 \, h^{-1} \text{Mpc}$ around corners of the regular network. Mock samples are described in more detail in Paper III. Here we are interested in the cosmic error due to variance of $k(d)$ in different realizations of the sample. This relative error can be expressed as $\sigma_k \approx b(d - 2d_e)/N$, where $N$ is the number of clusters in the sample, and $b \approx 0.5$ is a constant determined from the analysis of mock samples. The dependence of the error on $d$ comes from the fact, that, if $d = 2d_e$, then trial cells have the same size as elementary sub-cells. Thus all clusters of the trial cell are located in the elementary sub-cell and there is no variance of $k(d)$ between different realizations; for larger trial cell
size $d$ the error increases. The presence of a periodicity in the sample can be determined with confidence if the amplitude of the goodness near the maximum deviates from the Poisson value $k = 1$ by more than $2\sigma$.

Results of the analysis of mock samples are illustrated in Figure 3. We see in the right panel that the goodness curve has well defined maxima at integer multiples of $r_0 = 130$. In the mock sample shown in the left panel the fraction of clusters located in regular network is smaller and the amplitude of the maximum lies within the $2\sigma$ error corridor. This example shows that the period of a regular network of points can be determined if the population of regularly distributed points is sufficiently large (at least 10% of the total number of points) and if the total number of particles is at least several hundred.

Obviously, the method is sensitive to the direction of the axes of the trial cubes. If clusters form a quasi-rectangular cellular network, and the search cube is oriented along the main axis of the network, then the period is found to be equal to the side length of the cell. If the search cube is oriented at some non-zero angle $\theta$ with respect to the major axis of the network, then the presence of a periodicity and the period depend on the angle. If the angle $\theta \approx 45^\circ$, then the period is equal to the length of the diagonal of the cell. If the angle differs considerably from $\theta = 0^\circ$ and $45^\circ$, the periodicity is weak or absent. This property is also illustrated in Figures 3 and 4 for mock and real cluster samples, respectively.

Results of the period analysis of our cluster sample are shown in Figure 4. The axes of the search cubes were oriented at various angles with the coordinate axes of the supergalactic coordinate system. As seen from these figures, the period really depends on the orientation. The smallest value of the period, $P = 130 \ h^{-1} \ Mpc$, occurs when the cubes are oriented along supergalactic coordinates. If the trial cube is oriented at $\theta = 45^\circ$ with respect to supergalactic coordinates, then the period is $P = 190 \ h^{-1} \ Mpc$, as expected for a rectangular network of cells. In both cases there exist a second maximum of the curve of goodness of regularity at $d = 250 - 280 \ h^{-1} \ Mpc$ which is also well pronounced. If the trial cube is oriented at $22^\circ$, then the periodicity is much weaker, the first maximum at $d = 120 \ h^{-1} \ Mpc$ has a lower amplitude, and the second is absent. In principle there are more possibilities, for instance when the trial cells are oriented along the diagonal of the cubical lattice. However, in most additional cases the periodicity is weaker. Probably this can be due to the fact that the regularity is not very strong and there exist deviations from it. Much larger samples are needed to find limits of the regularity.

4. Discussion and conclusions

The periodicity analysis confirms the analysis made in Papers I and II using other methods. The cluster sample has a considerable fraction of clusters located in rich superclusters which form a rectangular lattice with a period of about $120 - 130 \ h^{-1} \ Mpc$. Our analysis shows that the supercluster-void network is oriented approximately along supergalactic coordinates. This confirms earlier results on the presence of a high concentration of clusters and superclusters towards both the Supergalactic Plane (Tully et al. 1992, and towards the Dominant Supercluster Plane, which are at right angles with respect to each other (Paper I). Supergalactic $Y$ axis is very close to the direction of Galactic poles, thus it is natural to expect a well-defined periodicity along Galactic poles as indeed observed by Broadhurst et al. (1990). The rectangular character of the distribution of rich clusters was also noticed by Tully et al. (1992). Recently Battaner (1998) has found that many known superclusters can be identified with the vertices of a octahedron network of the superstructure.

One principal result of our study is the direction dependence of the periodicity. This conclusion was not possible with the power spectrum or correlation function approach since they are not sensitive to directional information. A clear periodicity is observed only along supergalactic coordinates. This is in good agreement with results by Broadhurst et al. (1990) where a periodicity was observed in the direction of Galactic poles, i.e. almost exactly along the supergalactic $Y$ axis (supergalactic $X$ and $Z$ axes lie close to the galactic plane where clusters are invisible). In other directions the regularity is less pronounced (Guzzo et al. 1992, Willmer et al. 1994, Ettori et al. 1997). As noted already by Bahcall (1991), the nearest peaks of the Broadhurst et al. survey coincide in position and redshift with nearby rich superclusters. Thin deep slices, such as slices of the LCRS and the Century Survey, also show a weak periodicity signal (Landy et al. 1990, Geller et al. 1997). The scale length found in these studies is of the same order as derived in the present paper.

The regularity found in this paper concerns high-density regions marked by clusters in rich superclusters. When we compare the distribution of galaxies in various regions of the supercluster-void network we see considerable individual differences. On small scales the distribution of galaxies and clusters is fractal, as shown by many studies (see Wu et al. 1994); and is well reproduced by models with randomly distributed density perturbations (see Feldman et al. 1994). On very large scales covering the whole volume of the Abell cluster sample we see in the distribution of clusters in extremely rich superclusters with $N > 30$ members no regularity. We come to the conclusion that a clear regularity is observed only near the wavelength $\lambda_0 \approx 120 \ h^{-1} \ Mpc$ of the peak of the power spectrum. This result can be expressed as follows: density perturbations on very small ($k \gg k_0$) and very large wavelengths ($k \ll k_0$) are uncorrelated and have random phases; however, near the wavelength $\lambda_0 = 2\pi/k_0$ they are correlated and have similar phases. The importance of the use of phase information was stressed by Szalay (1998).
The regularity of the large-scale structure of the Universe has been studied also using the distribution of centres of superclusters (Kalinkov et al. 1998, Kerscher 1998). Kalinkov et al. searched for the presence of a high-order clustering of superclusters using the correlation analysis, and found that superclusters are not clustered. Our analysis in Papers I and II and presented here suggests that the regularity is completely different: it consists of the presence of the supercluster-void network, not the clustering of superclusters (like galaxies concentrate to clusters). The supercluster-void regularity is expressed by the distribution of clusters themselves, not supercluster centres. Kerscher used a combination of the nearest neighbour distribution and the void probability function to measure the regularity of the structure. This method works well for small separations between superclusters (\(r \leq 60 \, h^{-1} \, \text{Mpc}\)), and yields results in good agreement with Paper I. However, the method does not characterise the supercluster-void network on larger scales.

It should be emphasized that this inhomogeneity in the Universe does not contradict the observed degree of isotropy of the cosmic microwave background (CMB) radiation. As follows from general expressions for \(r_m\) values of multipoles \(C_l\) of CMB angular temperature anisotropies (see, e.g., Starobinsky 1988), any additional localized excess of power in the Fourier spectrum of density perturbations introduced at a scale \(k_0 < 1/H_0\) affects only multipoles \(l < l_0 = k_0 R_H\) where \(R_H\) is the present horizon radius. This is valid for both Gaussian and non-Gaussian additional perturbations (assuming only that there is no correlation between perturbations with \(k \approx k_0\) and perturbations from the other, “regular" part of the spectrum). Moreover, multipoles with \(l = (0.7 - 0.9)l_0\) are mainly amplified. For the pure CDM cosmological model with \(\Omega_m = 1\), \(R_H = 2c/H_0\) and \(l_0 = 300\) mainly the first acoustic peak is enhanced (Atrio-Barandela et al. 1997; Eisenstein et al. 1998). Almost any additional localized amplification of multipoles \(l < l_0 \approx 500\) for this model, the other acoustic peak is enhanced (Atrio-Barandela et al. 1997; Eisenstein et al. 1998). Broadhurst & Jaffe 1999). However, in the ΛCDM model (which is strongly supported by numerous recent observational data) this enhancement shifts to larger values of \(l\). In particular, if \(\Omega_m = 1 - \Omega_\Lambda = 0.3\), then \(R_H = 3.305c/H_0\), and \(l_0 \approx 500\). For this model, the effect discussed in this paper mainly results in an increase of \(C_l\) in the valley between the first and second acoustic peaks of the CMB temperature anisotropy spectrum. In particular, the amplitude \((\Delta T/T)_l = \sqrt{C_l (l + 1)/2\pi}\) for \(l = 400\) will be \(2 \times 10^{-5}\), in agreement with recent CAT2 data (Baker et al. 1999), as well as with previous CAT1 results (Scott et al. 1999). A more detailed treatment of resulting CMB anisotropies in the presence of a cosmological constant will be presented in a separate paper.

The main conclusion we can draw from our study is that the Universe is not homogeneous and fully isotropic on scales \(\approx 120 \, h^{-1} \, \text{Mpc}\). High-density regions in the Universe form a quasi-rectangular lattice. The distribution of high-density regions depends on the direction: along the main axis (coinciding with the supergalactic Y axis) of the cellular system the regularity is well pronounced and has a period of \(120 - 130 \, h^{-1} \, \text{Mpc}\), while in other directions the distribution is less regular.

Acknowledgements. We thank Jaan Pelt for discussion and suggestions. The present study was supported by Estonian Science Foundation grant 2625. A.S. was partially supported by the grant of the Russian Foundation for Basic Research No. 99-02-16224. This paper was finished during his stay at the Institute of Theoretical Physics, ETH, Zurich.

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