Hints of Grand Unification in Neutrino Data

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Abstract

There are strong indications for neutrino masses and mixings in the data on solar neutrinos, the observed deficit of muon neutrinos from the atmosphere as well as from discussions of the dark matter of the universe after COBE data. It is argued that an SO(10) grand unified theory has the right symmetry breaking properties needed to accommodate the neutrino masses and mixings suggested by these experiments. The minimal version of the model in fact leads to a complete prediction for the neutrino masses and mixings which can accommodate the observations partially, making the theory testable in near future. If the model is supplemented by an $S_4$ horizontal symmetry, it leads to a highly degenerate light neutrino spectrum which is the only way fit all data with the three known light neutrinos.

I. Introduction:

One of the strongest indications of new physics beyond the standard model is in the arena of neutrinos where there are experimental results, which can be understood most easily if the neutrinos are assumed to have nonvanishing masses and mixings. The experimental results are: i) the deficit of solar neutrinos now observed in four different experiments[1] compared to the calculations based on the standard solar model[2] ii) the depletion of atmospheric muon neutrinos observed in three different experiments[3] compared to calculations[4]; and iii) the apparent need for some hot dark matter in the Universe[5]. In this talk, I will first argue that the masses and mixings for neutrinos required by the above data are very strongly suggestive of an SO(10) grand unified theory beyond the standard model; then I present a recent

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work with K. S. Babu[6] which showed that the solar neutrino results are most easily accounted for by the minimal SO(10) grandunified theory where the constraints of grandunification and a realistic charged fermion spectrum allow a complete prediction for both the neutrino masses and mixings upto an overall scale. This minimal model however cannot accommodate the atmospheric neutrino data in conjunction with the solar neutrino data. Caldwell and I [7] have argued that the only way to accommodate all the above neutrino observations with only three light neutrinos is to have a degenerate spectrum for them, a property that can emerge in an $SO(10)$ model, if it is supplemented by extra horizontal symmetries[7,8,9].

Let us first summarize the values for neutrino masses and mixings required to understand the data on the basis of simple two neutrino mixing.

1.1 Solar Neutrino Deficit:

In the two neutrino mixing approximation, the following choice of masses and mixings is consistent with present data[10]: i) The small angle non-adiabatic MSW solution[11]: $\Delta m_{\nu_e,\nu_i}^2 \simeq (.3 - 1.2) \times 10^{-5} eV^2$ and $sin^2 2\theta \simeq (.4 - 1.5) \times 10^{-2}$ ii) Large angle MSW solution: $\Delta m_{\nu_e,\nu_i}^2 \simeq (.3 - 3) \times 10^{-5} eV^2$ and $sin^2 2\theta \simeq .6 - .9$ iii) Vacuum oscillation solution: $\Delta m_{\nu_e,\nu_i}^2 \simeq (.5 - 1.1) \times 10^{-10} eV^2$ and $sin^2 2\theta \simeq (.8 - 1)$

1.2 Atmospheric Neutrino Puzzle:

A straightforward way to understand the deficit of the muon neutrinos is to assume that $\nu_\mu$ oscillates to another light neutrino. Assuming the latter to be the tau neutrino, the data can be fitted with[4] the values of $\Delta m_{\nu_\mu,\nu_\tau}^2 \simeq .5 - .005 eV^2$ and $sin^2 2\theta \simeq .5$. We do not consider the alternative possibility that atmospheric neutrino anomaly could be resolved via $\nu_\mu-\nu_e$ oscillation. Although strictly this is not ruled out[4,12], it would imply distortion in the observed $\nu_e$ spectrum in the underground experiments for which there seems to be no evidence.

1.3 Hot Dark Matter Neutrinos:

Data on the extent of structure in the universe available on a wide range of distance scales together with the COBE results on the anisotropy of the cosmic microwave background radiation, galaxy-galaxy angular correlation, large scale velocity fields, and correlations of galactic clusters can all be fit[5] by a model of the universe containing 70% cold dark matter and 30% hot dark matter. (But perhaps an admixture in the ratio 90% to 10% of CDM to HDM may not be inconsistent). Although, there are other possibilities such as using the cosmological constant in conjunction with CDM, tilted spectrum plus CDM etc., the mixed dark matter scenario has its own appeal since the already known neutrino with mass in the appropriate range of 7eV to 2eV could be the HDM.
1.4 Possible signal of an eV Majorana mass in $\beta\beta_0\nu$ decay

More recently the data from $^{76}\text{Ge}$[13] and $^{130}\text{Te}$[13] neutrinoless double beta decay($\beta\beta_0\nu$) experiments have led to the possibility that the existence of an effective Majorana mass, $<m_\nu> \approx 1 - 2eV$ can either be confirmed or ruled out in very near future.

There is of course a tentativeness to some of the data under consideration. Nevertheless, we believe it is not premature to discuss what their implications are for neutrino mass matrices and physics beyond the standard model.

II. Neutrino Mass Matrices Suggested by Data:

The kind of neutrino spectrum and their mass matrices that would be required to fit the above observations has been the subject of two recent papers by D.Caldwell and this author[7]. We have found two possible scenarios, which fit all the above constraints; but the most economical one that uses only the three known light neutrinos has a very intriguing structure that we give below. The $\nu_e$, $\nu_\mu$ and $\nu_\tau$ are all nearly degenerate with mass around 2 eV. The mass differences are appropriately arranged so that $\nu_\mu$-$\nu_\tau$ oscillations explain the atmospheric neutrino problem and similarly $\nu_e$ - $\nu_\mu$ mass differences as well as mixings are so arranged that they can explain the solar neutrino deficit via the MSW mechanism using the small angle non-adiabatic solution. The simplest mass matrix, which can achieve this is:

$$M = \begin{pmatrix}
m + \delta_1 s_1^2 & -\delta_1 s_2 c_1 c_2 & -\delta_1 c_1 s_1 s_2 \\
-\delta_1 s_2 c_1 c_2 & m + \delta_1 c_1^2 c_2^2 + \delta_2 s_2^2 & (\delta_1 - \delta_2) s_2 c_2 \\
-\delta_1 c_1 s_1 s_2 & (\delta_1 - \delta_2) c_2 s_2 & m + \delta_1 s_2^2 + \delta_2 c_2^2
\end{pmatrix}$$

(1)

In eq.(1), $m \simeq 2eV$; $\delta_1 \simeq 1.5 \times 10^{-6}eV$; $\delta_2 \simeq .2$ to $.002eV$; $s_1 \simeq .05$ and $s_2 \simeq .35$. It is worth repeating that Majorana mass for $\nu_e$ of this magnitude will be tested by the current generation of neutrinoless double beta decay experiments. Obviously, hot dark matter in thus case is, distributed between the three active species of neutrinos almost equally.

In view of the tentative nature of some of the data at the moment, first we explore the theoretical implications of a non-vanishing neutrino mass in the simplest
grand unified model based on the group SO(10) so that the simplest model can be exposed to tests via neutrino experiments. Then I discuss what modifications are needed to fit all data in the SO(10) framework.

III. Massive Neutrinos, Local B-L Symmetry and SO(10) Grandunification:

Let us start by reminding the reader that in the standard model, the neutrinos are massless because only the left-handed neutrinos appear in the the spectrum and $B-L$ is an exact symmetry of the Lagrangian. In order to obtain massive neutrinos, one must therefore include the right-handed neutrino in the spectrum. It however turns out that as soon as this is done, in the theory there appears a completely triangle anomaly free generator, the $B-L$. This symmetry is then a gaugeable symmetry and it would be rather peculiar if nature chooses not to gauge a symmetry which is gaugeable. If following this line of reasoning, we use $B-L$ as a gauge symmetry, the most natural gauge group turns out to be the Left-Right symmetric group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [14], which breaks at some high scale to the standard model group. The quarks and leptons in this model are assigned in a completely left-right symmetric manner, i.e. if we define $Q \equiv (u,d)$ and $\psi \equiv (\nu,e)$, then $Q_L(2,1,1/3)$ and $Q_R(1,2,1/3)$ are assigned in a left-right symmetric manner and similarly, $\psi_L(2,1,-1)$ and $\psi_R(1,2,-1)$. The Higgs sector of the model that leads naturally to small neutrino masses in this model consists of the bi-doublet field $\phi \equiv (2,2,0)$ and the triplet fields $\Delta_L \equiv (3,1,+2)$ and $\Delta_R \equiv (1,3,+2)$ [15]. The Yukawa couplings of the model are:

$$L_Y = h_1 Q_L \phi Q_R + h'_1 Q_L \tilde{\phi} Q_R h_\ell \tilde{\psi}_L \phi \psi_R + h'_\ell \tilde{\psi}_L \tilde{\phi} \psi_R + f \psi_L^T C^{-1} \tau_2 \Delta_L \psi_R + L \to R + h.c.$$ (2)

The gauge symmetry breaking is achieved in two stages: in the first stage, the neutral component of $\Delta_R$ multiplet acquires a vev $\langle \Delta_R^0 \rangle = v_R$, thereby breaking the gauge symmetry down to the $SU(2)_L \times U(1)_Y$ group of the standard model; in the second stage, the neutral components of the multiplet $\phi$ acquire vev breaking the standard model symmetry down to $U(1)_{em}$. At the first stage of symmetry breaking, $W_R$ and $Z'$ acquire masses of order $gv_R$ and in the second stage the familiar $W_L$ and $Z_L$ acquire masses. The near maximality of parity violation at low energies is due to the masses of $W_R$ and $Z'$ being bigger than those of the $W_L$ and the $Z$ boson. At the first stage of symmetry breaking, the $f$-terms in the Yukawa coupling give
nonvanishing masses to the three right-handed neutrinos of order $f \nu_R$ keeping all other fermions massless. At the second stage, quarks, charged leptons as well as the neutrinos acquire Dirac masses. The $\nu_L$-$\nu_R$ mass mass matrix at this stage is a $6 \times 6$ mass matrix of the following see-saw form[16]:

$$M_{\nu} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}$$

(3)

As is well known this see-saw form leads to three light eigen-values generically of order

$$m_{\nu_i} \simeq - \left( m_D M_R^{-1} m_D^T \right)$$

(4)

The typical values of $m_D$ are expected to be of order of the charged fermion masses in the theory whereas the $M_R$ corresponds to the scale of $B-L$ breaking which is a very high scale, thereby explaining the smallness of the neutrino masses. The specific value of $m_D$ is however model-dependent and depending on what the value of $m_D$ is, the spectrum of the light left-handed Majorana neutrinos will be of the eV : keV : MeV type or of the micro : milli : eV type. The former type of spectrum can be tested in the double beta decay as well as the conventional beta decay end-point experiments whereas the second spectrum can be tested in the solar neutrino as well as the long base-line neutrino experiments.

In view of the discussion of the previous section, the micro-milli-eV spectrum for the light neutrinos is of great current interest. In the simple see-saw models that naturally emerge in the left-right symmetric models, one generically has $m_D \simeq m_f$, where $f =$ leptons or quarks; so if we want $m_{\nu_\mu} \simeq 10^{-3}$eV, then the mass $M_R$ must be of order $10^{10} - 10^{12}$GeV. This would suggest grandunification models of type $SO(10)$ or some higher group containing it. The $SO(10)$ possibility is the most exciting because all its symmetry breaking scales i.e. the GUT scale $M_U$ and the $B-L$ breaking scale $M_R$ are predicted by the LEP data and amazingly enough, they are precisely in the above mentioned range for the non-SUSY versions of the model[17]. Such an intermediate scale is also required for adequate cosmological baryogenesis[18] and the tau neutrino being the hot dark matter of the Universe. This constitutes enough circumstantial evidence to take the $SO(10)$ model seriously and study its detailed predictions so that it can be subjected to experimental testing.
IV. Minimal SO(10) GUT and Predictions for Neutrino Masses and Mixings:

As we saw in the previous section, the simple see-saw model predicts a scale of B-L symmetry breaking near $10^{11}$ GeV or so if it is to solve the solar neutrino puzzle. Both the see-saw formula as well as a large $B - L$ symmetry scale emerge naturally from the SO(10) models. The minimal SO(10) model without supersymmetry leads to a two step breaking of SO(10) down to the standard model. There are four possibilities, two corresponding to the case where the discrete $Z_2$ local subgroup (called D-parity)[19] is broken and two where D-parity survives down to the $B - L$ breaking scale. In the D-parity broken case, we have the intermediate symmetry group to be $SU(2)_L \times SU(2)_R \times G_c$ where $G_c$ is $SU(4)_C$ (denoted as case (A)) or $SU(3)_c \times U(1)_{B-L}$ (denoted as case (B)). The advantage of this case is that it makes the conventional see-saw formula natural[20]. Use of Higgs multiplets belonging to $210$ and $45+54$ representations to break SO(10) leads to such scenarios (A) and (B) respectively. It however turns out that in order to realize the degenerate neutrino spectrum, one needs to preserve D-parity down to the scale of $B - L$ symmetry breaking where one has to use the second two possibilities. Depending on whether the color gauge subgroup below GUT scale is $SU(4)_C$ or $SU(3)_c \times U(1)_{B-L}$; we denote these cases as case (C) and case (D) respectively.

A very important point worth emphasizing here is that inputting the LEP data for the three gauge couplings for the standard model leads to unique predictions for the unification scale $M_U$ and the intermediate scale $M_I$. These predictions for non-SUSY version of the model have been studied including two-loop and threshold corrections in ref.17 and 21 and the results are:

$$Model(A) : \quad M_U = 10^{15.26^{+1.13}_{-1.24} \pm 0.25} \text{GeV} \quad M_I = 10^{10.7^{+2.65}_{-0.7} \pm 0.02} \text{GeV}$$ (5)

$$Model(B) : \quad M_U = 10^{16.42^{+0.18}_{-0.25}} \text{GeV} \quad M_I = 10^{9^{+0.69}_{-3}} \text{GeV}$$ (6)

$$Model(C) : \quad M_U = 10^{15.02^{+0.48}_{-0.25}} \text{GeV} \quad M_I = 10^{13.64^{+0.88}} \text{GeV}$$ (7)

$$Model(D) : \quad M_U = 10^{15.55^{+0.43}_{-0.20}} \text{GeV} \quad M_I = 10^{10.16^{+0.57}} \text{GeV}$$ (8)

First , we note that the values of the intermediate scale are in the range required by the see-saw formula to give the neutrino masses which can play a role in the understanding of the various anomalies described in the introduction. Whether they really do or not depends of course on the various mixing angles. We will see that in the minimal models, the mixing angles are in the right range (contrary to a common
belief in some quarters that the neutrino mixing angles should mirror the quark CKM mixing angles).

Secondly, we also have prediction for the proton life-time in non-SUSY $SO(10)$ models for these cases:

$$\tau_p = 1.44 \times 10^{37.4 \pm 1.0 \pm 5} \text{ years} \quad \text{Model(A)}$$

$$\tau_p = 1.44 \times 10^{37.7 \pm 9 \pm 2.0} \text{ years} \quad \text{Model(B)}$$

$$\tau_p = 1.44 \times 10^{32.1 \pm 1.0 \pm 1.9} \text{ years} \quad \text{Model(C)}$$

$$\tau_p = 1.44 \times 10^{34.2 \pm 1.8 \pm 1.7} \text{ years} \quad \text{Model(D)}$$

Some of these predictions are within the reach of the Super-Kamiokande experiment[22], which should therefore throw light on the non-SUSY version of the $SO(10)$ model.

Let us now discuss the predictions for neutrino masses in the minimal $SO(10)$ model. This necessitates detailed knowledge of the Dirac neutrino mass matrix as well as the Majorana neutrino mass matrix. Luckily, it turns out that in $SO(10)$ models, the charge $-1/3$ quark mass matrix is related to the charged lepton matrix and the neutrino Dirac mass matrix is related to the charge $2/3$ quark matrix at the unification scale. However, prior to the work of ref.6, no simple way was known to relate the heavy Majorana matrix to the charged fermion observables. This stood in the way of predicting the light neutrino spectrum. It was however shown in ref.[6] that in a class of minimal $SO(10)$ models, in fact, not only the Dirac neutrino matrix, but the Majorana matrix also gets related to observables in the charged fermion sector. This leads to a very predictive neutrino spectrum. We use a simple Higgs system with one (complex) $10$ and one $126$ that have Yukawa couplings to fermions. The $10$ is needed for quark and lepton masses, the $126$ is needed for the see–saw mechanism. Crucial to the predictivity of the neutrino spectrum is the observation that the standard model doublet contained in the $126$ receives an induced vacuum expectation value (vev) at tree–level. In its absence, one would have the asymptotic mass relations $m_b = m_r$, $m_s = m_\mu$, $m_d = m_e$. While the first relation would lead to a successful prediction of $m_b$ at low energies, the last two are in disagreement with observations. The induced vev of the standard doublet of $126$ corrects these bad relations and at the same time also relates the Majorana neutrino mass matrix to observables in the charged fermion sector, leading to a predictive neutrino spectrum.

We shall consider non–Susy $SO(10)$ breaking to the standard model via the $SU(2)_L \times SU(2)_R \times SU(4)_C \equiv G_{224}$ chain as well as Susy-$SO(10)$ breaking directly
to the standard model. The breaking of $SO(10)$ via $G_{224}$ is achieved by a $210$ of Higgs which breaks the discrete $D$–parity. The second stage of symmetry breaking goes via the $126$. Finally, the electro–weak symmetry breaking proceeds via the $10$. In Susy–$SO(10)$, the first two symmetry breaking scales coalesce into one.

In the fermion sector, denoting the three families belonging to $16$–dimensional spinor representation of $SO(10)$ by $\psi_a$, $a = 1 - 3$, the complex $10$–plet of Higgs by $H$, and the $126$–plet of Higgs by $\Delta$, the Yukawa couplings can be written down as

$$L_Y = h_{ab} \psi_a \psi_b H + f_{ab} \psi_a \psi_b \Delta + H.C.$$ \hspace{1cm} (9)

Note that since the $10$–plet is complex, one other coupling $\psi_a \psi_b \overline{H}$ is allowed in general. In Susy–$SO(10)$, the requirement of supersymmetry prevents such a term. In the non–Susy case, we forbid this term by imposing a $U(1)_{PQ}$ symmetry, which may anyway be needed in order to solve the strong CP problem.

The $10$ and $126$ of Higgs have the following decomposition under $G_{224}$: $126 \rightarrow (1, 1, 6) + (1, 3, 10) + (3, 1, \overline{10}) + (2, 2, 15)$, $10 \rightarrow (1, 1, 6) + (2, 2, 1)$. Denote the $(1, 3, 10)$ and $(2, 2, 15)$ components of $\Delta(126)$ by $\Delta_R$ and $\Sigma$ respectively and the $(2, 2, 1)$ component of $H(10)$ by $\Phi$. The vev $<\Delta_R^0 > \equiv v_R \sim 10^{12}$ GeV breaks the intermediate symmetry down to the standard model and generates Majorana neutrino masses given by $fv_R$. $\Phi$ contains two standard model doublets which acquire vev’s denoted by $\kappa_u$ and $\kappa_d$ with $\kappa_{u,d} \sim 10^2$ GeV. $\kappa_u$ generates charge $2/3$ quark as well as Dirac neutrino masses, while $\kappa_d$ gives rise to $-1/3$ quark and charged lepton masses.

Within this minimal picture, if $\kappa_u$, $\kappa_d$ and $v_R$ are the only vev’s contributing to fermion masses, in addition to the $SU(5)$ relations $m_b = m_\tau$, $m_s = m_\mu$, $m_d = m_e$, it will also lead to the unacceptable relations $m_u : m_c : m_t = m_d : m_s : m_b$. Moreover, the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix will be identity. It was however shown in ref.6, that in this model there exist new contributions to the fermion mass matrices which are of the right order of magnitude to correct these bad relations. To see this, note that the scalar potential contains, among other terms, a crucial term $V_1 = \lambda \Delta \overline{\Sigma} \Delta H + H.C$. Such a term is invariant under the $U(1)_{PQ}$ symmetry. It will be present in the Susy $SO(10)$ as well, arising from the $210$ $F$–term. This term induces vev’s for the standard doublets contained in the $\Sigma$ multiplet of $126$. The vev arises through a term $\overline{\Delta_R} \Delta_R \Sigma \Phi$ contained in $V_1$. The magnitudes of the induced vev’s of $\Sigma$ (denoted by $v_u$ and $v_d$ along the up and down directions) can be estimated using the survival hypothesis : $v_{u,d} \sim \lambda (v_R^2/M_\Sigma^2) \kappa_{u,d}$. Suppose $M_U \sim 10^{15}$ GeV, $M_I \sim 3 \times 10^{12}$ GeV and $M_\Sigma \sim 10^{14}$ GeV, consistent with survival hypothesis, then $v_u$ and $v_d$ are of order 100 MeV, in the right range for correcting
the bad mass relations. We emphasize that there is no need for a second fine-tuning to generate such induced vev’s. In the Susy version with no intermediate scale, the factor \((v_R^2/M_P^2)\) is not a suppression, so the induced vev’s can be as large as \(\kappa_{u,d}\).

We are now in a position to write down the quark and lepton mass matrices of the model:

\[
M_u = h\kappa_u + fv_u \\
M_d = h\kappa_d + fv_d \\
M_D^\nu = h\kappa_u - 3fv_u \\
M_l = h\kappa_d - 3fv_d \\
M_\nu^M = fv_R. 
\] (10)

Here \(M_D^\nu\) is the Dirac neutrino matrix and \(M_\nu^M\) is the Majorana mass matrix. Let us ignore CP-violation, which has been taken into account in [6]. Note that, there are 12 parameters in all, not counting the superheavy scale \(v_R\): 3 diagonal elements of the matrix \(h\kappa_u\), 6 elements of \(fv_u\), and three vev’s. These are completely determined by the charged fermion sector, viz., 9 fermion masses, 3 quark mixing angles. The light neutrino mass matrix is then completely predicted up to the overall scale \(v_R\). In making the predictions, we have been careful to take into account the renormalization extrapolation of the relations in eq. to the weak scale. Below, we present results for the non–Susy \(SO(10)\) model with the \(G_{224}\) intermediate symmetry. We fix the intermediate scale at \(M_I = 10^{12} \text{ GeV}\). We find that there are essentially three different solutions. The one that can fit the solar neutrino data is the one below.

Input : \(m_u(1 \text{ GeV}) = 3 \text{ MeV, } m_c(m_c) = 1.22 \text{ GeV, } m_t = 150 \text{ GeV} \)

\(m_b(m_b) = -4.35 \text{ GeV, } r_1 = -1/51, \ r_2 = 0.2 \)

Output : \(m_d(1 \text{ GeV}) = 5.6 \text{ MeV, } m_s(1 \text{ GeV}) = 156 \text{ MeV} \)

\[ (m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}) = R \left(7.5 \times 10^{-3}, 2.0, -2.8 \times 10^3\right) \text{ GeV} \]

\[
V_{KM}^{\text{lepton}} = \begin{pmatrix}
0.9961 & 0.0572 & -0.0676 \\
-0.0665 & 0.9873 & -0.1446 \\
0.0584 & 0.1485 & 0.9872 \\
\end{pmatrix}, 
\]

where \(R = v_u/v_R\).
Note that the pattern of mixing angles is very different from the quark sector and both the $\nu_e$-$\nu_\mu$ and $\nu_e$-$\nu_\tau$ mixing angles are in the range to be useful in understanding the solar neutrino puzzle. Moreover, the $\nu_\mu$-$\nu_\tau$ mixing angle is near $3|V_{c0}|$, so that the present neutrino oscillation data\cite{23} implies that $m_{\nu_\tau} \leq 2$ eV. From the $\nu_\tau/\nu_\mu$ mass ratio, which is $1.4 \times 10^3$ in this case, we see that $m_{\nu_\tau} \leq 1.5 \times 10^{-3}$ eV. This is just within the allowed range\cite{10} for small angle non–adiabatic $\nu_e - \nu_\mu$ MSW oscillation, with a predicted count rate of about 50 SNU for the Gallium experiment. Note that there is a lower limit of about 1 eV for the $\nu_\tau$ mass in this case. Forthcoming experiments (CHORUS and NOMAD\cite{24} at CERN and the Fermilab expt.) should then be able to observe $\nu_\mu - \nu_\tau$ oscillations. A $\nu_\tau$ mass in the (1 to 2) eV range can also be cosmologically significant, it can be at least part of the hot dark matter.

Three more sets of predictions for neutrino masses and mixings in this model have been found by Lavoura\cite{25}; none of them have features needed to accomodate both the solar and the atmospheric neutrino puzzle.

Before closing this section, let me make some comments on the SUSY-SO(10) model. First, if the minimal model discussed is supersymmetrized, the predictions for neutrino masses and mixings remain unchanged - with the difference that the $B - L$ scale which appears in the overall coefficient in the neutrino mass matrix is now same as the GUT scale. So, the more natural possibility here is to solve the solar neutrino puzzle via the $\nu_e$-$\nu_\tau$ oscillation since due to the high value of $v_R$, it is the tau neutrino mass which is more easily of order $10^{-3}$ eV. It is however possible that with the inclusion of threshold corrections, the $B - L$ symmetry breaking scale is somewhat lower than the GUT scale and the muon neutrino remains as milli-eV particle still allowing the $\nu_e$-$\nu_\mu$ oscillation solution to the solar neutrino puzzle. It is also important to point out that the SUSY SO(10) has the advantage that it automatically provides a cold dark matter candidate, the lightest supersymmetric particle (the LSP) due to the fact that R-parity is an automatic symmetry of the model. In the non-susy models we have to invoke perhaps an axion as the CDM\cite{26}. In both models, there appears to be no HDM candidate unless a two eV tau neutrino is considered adequate by cosmologists for the purpose.

V. An SO(10) model for a degenerate neutrino scenario:

In this section, we discuss the ingredients needed to build a model for degenerate neutrinos of the type discussed in section II in order to fit all the data summarized in sec.I. The basic strategy is to employ the fact that when the conventional see-saw
mechanism for neutrino-masses is implemented in gauge models such as SO(10) or the left-right symmetric models, it gets modified to the following form[15]

\[
\begin{pmatrix}
fv_L & m_{\nu D} \\
m^T_{\nu D} & fv_R
\end{pmatrix}
\]  

(12)

where \( v_L = \lambda \frac{v^2}{M^2} \), \( v_R \) is the scale of SU(2)_R-breaking and \( M_P \) is breaking scale of parity. Therefore, unless special care is taken to break parity symmetry at a scale higher than the SU(2)_R or U(1)_{B-L}, \( v_L \sim \lambda v^2_w/v_R \) (since \( v_R \sim M_P \)). The light neutrino masses are then given by:

\[
m_{\nu} \simeq f v_L - \frac{m_{\nu_0} f^{-1} m_{\nu D}^T}{v_R}.
\]  

(13)

Recall that the conventional see-saw formula omits the first term (which is justified only under special circumstances). We will however keep both the terms in the present discussion. Now notice that if due to some symmetry reasons, \( f_{ab} = f_0 \delta_{ab} \), then a degenerate neutrino spectrum emerges. This property has been used in several recent papers[7,8,9,27] to obtain a nearly degenerate spectrum for light neutrinos. In the rest of the paper, we discuss the model given in ref.8.

Consider the breaking of SO(10) \( \rightarrow \) SU(2)_L \( \times \) SU(2)_R \( \times \) SU(4)_C \( \times \) P (denoted by \( G_{224P} \)) by means of a \{54\}-dim. Higgs multiplet. This symmetry is subsequently broken down to the standard model by a \{126\}-dim. Higgs multiplet. Detailed two-loop analysis of the mass scales in this model[21] leads to \( v_R \sim 10^{13.6} \) GeV. So that for \( f_0 \lambda \sim 1/2 \), we get \( f_0 v_L \sim 1 \) eV, as desired. We will supplement this model by a softly broken \( S_4 \) symmetry which restricts the Yukawa couplings in such a way that it not only leads to realistic charged fermion masses but also to the following predictions for the neutrino masses and mixings[8].

Writing \( m_{\nu_i} = m_0 + m'_{\nu_i} \), where \( m_0 \simeq 2 \) eV is the direct \( v_L \) contribution, we give a set of predictions for the masses and mixing angles which fit all known observations:

\[
(\ m'_{\nu_e}, m'_{\nu_\mu}, m'_{\nu_\tau}) = \frac{1}{f_{vR}} (-0.0000174665, -0.129248, -5759.27) GeV^2
\]

\[
V^T = \begin{pmatrix}
-.9982 & .05733 & .01476 \\
.05884 & .9334 & .3541 \\
-.006523 & -.3544 & .9351
\end{pmatrix}
\]  

(14)
Note that, for $v_R \simeq 10^{13.6} \text{ GeV}$ and $f \sim 3$, this predicts $|m_{\nu_\mu}^2 - m_{\nu_e}^2| \sim 4 \times 10^{-6} \text{ eV}^2$ for $m_0 = 2 \text{ eV}$, $|m_{\nu_e}^2 - m_{\nu_\mu}^2| \sim 0.2 \text{ eV}^2$, which are in the range required to solve both the solar and atmospheric neutrino deficit for the values of $\theta_{\nu_e,\nu_\mu}$ and $\theta_{\nu_\mu,\nu_e}$ given above. In particular, we wish to note the preference of theory for the small angle MSW solution to the solar neutrino problem.

VI. Summary and Conclusions:

In summary, I have argued in this report that if the present data on solar neutrinos and the C+HDM picture of the universe are taken seriously, then the most natural theoretical framework to understand their implications for neutrino masses and mixings is an SO(10) GUT models with the right-handed scale (or $B-L$ breaking scale) in the super-heavy range of $10^{11} \text{ GeV}$ or so. This result becomes more compelling, once one realizes that precisely such a value for the $B-L$ scale is implied by the low energy LEP data applied to a non-SUSY SO(10) model. Furthermore, in the minimal version of the SO(10) model, the values for neutrino masses and mixings are completely predicted and they fit the solar neutrino data rather beautifully and predict a tau neutrino mass around 2 eV. This is a bit low to be a good hot dark matter candidate but its role as a weak HDM may not be ruled out. The predictions for the mixing angle in the $\nu_e - \nu_\tau$ sector can be tested by the neutrino oscillation experiments such as CHORUS, NOMAD and the Fermilab experiments and proton decay searches to be carried out at SuperKamiokande. In fact, the present atmospheric neutrino data cannot be accomodated by the minimal SO(10) model; therefore if this data stands the test of time, a second minimal grandunified model will be ruled out by experiments and one may be forced into a degenerate neutrino scenario described in sec.V above. I then discuss, how the degenerate scenario may emerge in an SO(10) GUT framework.

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