Suppression of elliptic-flow-induced correlations in an observable of possible local parity violation
Adam Bzdak
Phys. Rev. C 85, 044919 — Published 19 April 2012
DOI: 10.1103/PhysRevC.85.044919
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Adam Bzdak
RIKEN BNL Research Center
Brookhaven National Laboratory*

Abstract

We show that fluctuations in elliptic anisotropy in peripheral heavy-ion collisions can be used to significantly reduce the contribution of transverse-momentum conservation, and of all background effects independent on the orientation of the reaction plane, from an observable of the chiral magnetic effect. We argue that for a given impact parameter, the magnetic field is approximately independent of the fluctuating shape of the fireball.

1. The main feature of the chiral magnetic effect (CME) [1] is the existence of an electric current parallel (or anti-parallel) to the direction of magnetic field produced in heavy-ion collisions [2], see also [1], [3]. Consequently, charge separation can be observed in the direction perpendicular to the reaction plane. Recently, evidence for the CME was found in lattice QCD calculations [4]. A detailed discussion of this effect was given in Ref. [5].

As discussed in Ref. [6], such charge separation can be observed via the following two-particle correlator

\[ \gamma = \langle \cos(\phi_1 + \phi_2 - 2\Psi_{RP}) \rangle, \]  

(1)

*Address: Upton, NY 11973, USA; email: abzdak@bnl.gov
where $\Psi_{RP}$, $\phi_1$, and $\phi_2$, respectively, denote the azimuthal angles of the reaction plane, and the two charged particles produced. Alternative observables were proposed in Refs. [7, 8] (see last section). Recently, the STAR collaboration measured $\gamma$ and their result [9, 10] is qualitatively consistent with the CME expectation; however, the interpretation of these data still are debatable, see e.g., [11, 12, 13, 14, 15, 16, 17, 18]. The interpretation of experimental results is not simple because practically all two-particle correlations contribute to $\gamma$, as seen from (here $\Psi_{RP} = 0$
)

$$\gamma = \langle \cos(\phi_1) \cos(\phi_2) \rangle - \langle \sin(\phi_1) \sin(\phi_2) \rangle.$$ 

(2)

Indeed, in the presence of elliptic anisotropy [19], both terms can differ even though the correlation mechanism itself is not directly sensitive to the orientation of the reaction plane.

In this paper, we discuss this problem in detail. In the next Section, we undertake explicit calculations to quantify the contribution of elliptic anisotropy to $\gamma$. Next, we argue that in peripheral heavy-ion collisions (where the CME is considered to have a maximum strength [1]) we expect large fluctuations in elliptic flow ($v_2$) such that it allows us to deduct the $v_2$-driven background from $\gamma$. We also show that the changing shape of the fireball, at a given impact parameter, does not change the contribution of the CME to $\gamma$. Finally, we present a Monte Carlo model, wherein we evaluate the contribution of transverse-momentum conservation to $\gamma$ at vanishing elliptic anisotropy ($v_2 \to 0$). In the last section we give our comments and conclusions.

2. By definition

$$\gamma = \frac{\int \rho_2(\phi_1, \phi_2, x_1, x_2, \Psi_{RP}) \cos(\phi_1 + \phi_2 - 2\Psi_{RP})d\phi_1 d\phi_2 dx_1 dx_2}{\int \rho_2(\phi_1, \phi_2, x_1, x_2, \Psi_{RP})d\phi_1 d\phi_2 dx_1 dx_2},$$ 

(3)

where $\rho_2$ is the two-particle distribution at a given angle of the reaction plane, $\Psi_{RP}$. To make our notation shorter, we denote $x = (p_t, \eta)$ and $dx = p_t dp_t d\eta$, where $p_t$ is the absolute value of transverse-momentum, while $\eta$ is pseudorapidity. The distribution $\rho_2$ can be expressed via the correlation function $C$

$$\rho_2(\phi_1, \phi_2, x_1, x_2, \Psi_{RP}) = \rho(\phi_1, x_1, \Psi_{RP}) \rho(\phi_2, x_2, \Psi_{RP}) [1 + C(\phi_1, \phi_2, x_1, x_2)],$$ 

(4)
with the single-particle distribution

\[
\rho(\phi, x, \Psi_{RP}) = \frac{\rho_0(x)}{2\pi} \left[ 1 + 2v_2(x) \cos (2\phi - 2\Psi_{RP}) \right],
\]

(5)

where \( \rho_0(x) \) does not depend on \( \phi \) and \( \Psi_{RP} \). We study only those correlations that do not depend on the reaction plane, i.e., \( C \) only depends on \( \Delta \phi = \phi_1 - \phi_2 \). Next we expand \( C \) in a Fourier series

\[
C(\Delta \phi, x_1, x_2) = \sum_{n=0}^{\infty} a_n(x_1, x_2) \cos (n\Delta \phi),
\]

(6)

where \( a_n(x_1, x_2) \) does not depend on \( \phi_1 \) and \( \phi_2 \). Substituting (6) and (4) into Eq. (3), we obtain

\[
\gamma = \frac{1}{2N^2} \int \rho_0(x_1) \rho_0(x_2) a_1(x_1, x_2) [v_2(x_1) + v_2(x_2)] dx_1 dx_2,
\]

(7)

where \( N = \int \rho_0(x) dx \) and we assume that \( 1 + a_n \approx 1 \). As seen from Eq. (7), all correlations with non-zero \( a_1(x_1, x_2) \) contribute to \( \gamma \), even if the underlying correlation mechanisms do not depend on the orientation of the reaction plane. This finding explains why transverse-momentum conservation [11, 14, 18], local charge-conservation [12], resonance- (cluster-) decay [15], and all other correlations with \( \Delta \phi \) dependence contribute to \( \gamma \). However, as pointed out recently in Ref. [20], those correlations can be removed from \( \gamma \) by taking only those events where \( v_2(x) \approx 0 \). We note that \( v_2(x) \) is defined solely through Eq. (5), and it can be positive or negative.

Taking the CME into account\(^2\)

\[
\rho_\chi(\phi, x, \Psi_{RP}) = \frac{\rho_0(x)}{2\pi} \left[ 1 + 2v_2(x) \cos (2\phi - 2\Psi_{RP}) + 2\chi d(x) \sin (\phi - \Psi_{RP}) \right],
\]

(8)

we obtain,

\[
\gamma = -\frac{1}{N^2} \left[ \int d(x) \rho_0(x) dx \right]^2 + \frac{1}{2N^2} \int \rho_0(x_1) \rho_0(x_2) a_1(x_1, x_2) [v_2(x_1) + v_2(x_2)] dx_1 dx_2,
\]

(9)

\(^1\)Higher \( v_n \) results in terms proportional to \( v_2v_4, v_4v_6 \) etc., and can be neglected, see Eq. (7).

\(^2\)The value of \( \chi \) flips between \(-1\) and \(+1\) so that \( \frac{1}{2} \sum_{\chi} \rho_\chi = \rho \), defined in Eq. (5).
wherein the first term represents the CME, and the second term is the elliptic-anisotropy-driven background.

3. We expected that elliptic anisotropy would be correlated with the participant eccentricity $\epsilon_2$ [21]. In the center of mass of the wounded nucleons, $\epsilon_2$ is given by

$$\epsilon_2 = \frac{\sqrt{\left(\sum_i r_i^2 \cos(2\phi_i)\right)^2 + \left(\sum_i r_i^2 \sin(2\phi_i)\right)^2}}{\sum_i r_i^2},$$

(10)

wherein the wounded nucleons are characterized by their radii, $r_i$, and their azimuthal angles, $\phi_i$.

In Fig. 1, we present the calculated $\epsilon_2$ distribution in Au+Au collisions at $\sqrt{s} = 200$ GeV with $b = 10$ fm (impact parameter). Accordingly, even at $b = 10$ fm, we obtain a broad range of $\epsilon_2$ (and $v_2$)$^4$ that can be used to significantly change the background present in the second term of Eq. (9).

![Figure 1: Normalized density distribution of the participant eccentricity $\epsilon_2$ at the impact parameter $b = 10$ fm in Au+Au collisions at $\sqrt{s} = 200$ GeV.](image)

However, we wanted to remove this background under the condition that the contribution from the CME is approximately unchanged. To verify this,

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3We use the Monte Carlo Glauber calculation with standard parameters [22].

4In 3D event-by-event hydrodynamics [23], $\langle v_2 \rangle = 0.08$ and $[\langle v_2^2 \rangle - \langle v_2 \rangle^2]^{1/2} = 0.04$ for 40 – 50% centrality ($b \approx 10$ fm) in Au+Au collisions.
we calculated the out-of-plane component of the magnetic field $B_y$ at $t = 0$ (time) as a function of $\epsilon_2$. As seen in Fig. 2, the magnetic field from wounded- and spectator- protons are approximately constant (in comparison to $v_2$ that scales linearly with $\epsilon_2$) in the broad range of $\epsilon_2$. We conclude that fluctuating $v_2$ in peripheral collisions will allow us to study the $v_2$ dependence of $\gamma$ at an approximately constant strength of the CME.

Figure 2: The out-of-plane component of magnetic field at $b = 10$ fm in Au+Au collisions at $\sqrt{s} = 200$ GeV as a function of the participant eccentricity $\epsilon_2$. Contributions from the wounded- and spectator- protons are depicted separately.

In Figs. 1, 2 we made our calculations at a given impact parameter. In an actual experiment, we can select our peripheral collisions, e.g., by the number of particles produced at midrapidity. We consider that our conclusions also will hold in this situation.

4. As an example, we calculated $\gamma$ as a function of $v_2$ in a model with only transverse-momentum conservation and elliptic anisotropy.\(^7\) As shown in

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\(^5\)The details of our calculation are presented in Ref. [24].

\(^6\)Both sources that are expected to contribute differently during the evolution of the fireball [1]

\(^7\)The calculations presented in this section are for illustrative purposes only, and should not be compared with the STAR data.
[11, 14, 18], this effect contributes significantly to $\gamma$ and reasonably describes the $p_t$ and $\eta$ dependence.

We sampled $N_{all} = 50$ particles with $p_t$ according to the thermal distribution $p_t e^{-p_t/2T}$ with $2T = 0.45$ MeV, and $\phi$ according to $1 + 2v_2(p_t) \cos(2\phi)$. We took $v_2(p_t) = 0.14p_t$ for $p_t < 2$ GeV and $v_2(p_t) = 0.28$ for $p_t > 2$ GeV, so that we obtained the integrated $v_2 \approx 0.06$. Next, we imposed transverse-momentum conservation\footnote{We accepted only those events where the total (summed over $N_{all}$ particles) $|p_{t,x}| < 0.5$ GeV and $|p_{t,y}| < 0.5$ GeV.} and calculated $\gamma$ as a function of selected $v_2 = \frac{1}{N_{obs}} \sum_{i=1}^{N_{obs}} \cos(2\phi_i)$, where $N_{obs}$ is the number of observed particles (selected randomly from $N_{all}$). Owing to the statistical fluctuations, we obtained a broad range of $v_2$ that allows us to test Eq. (7).

\[
\gamma \sim -\frac{1}{N_{all}} \left[ 2\bar{v}_2,\Omega - \bar{v}_2,F \right],
\]  

\footnote{We accepted only those events where the total (summed over $N_{all}$ particles) $|p_{t,x}| < 0.5$ GeV and $|p_{t,y}| < 0.5$ GeV.}
where $\bar{v}_2 \sim \int \rho_0(x)v_2(x)p_t dx$, $\bar{v}_2 \sim \int \rho_0(x)v_2(x)p_t^2 dx$ and $x = (p_t, \eta)$, $dx = p_t dp_t d\eta$. The indexes $F$ and $\Omega$ indicate that integrations are performed over full phase-space ($F$) or the phase space wherein particles are measured ($\Omega$), respectively. When we calculate $\gamma$ for all produced particles, $\Omega = F$, then $v_{2,\Omega} = 0$ implies $v_{2,\Omega}(p_t, \eta) = 0$ and $\bar{v}_{2,\Omega} = \bar{v}_{2,F} = 0$. Consequently, $\gamma = 0$ at $v_{2,\Omega} = 0$. However, if we measure only a fraction of all particles, then for $v_{2,\Omega} = 0$ (and $\bar{v}_{2,\Omega} = 0$), $\bar{v}_{2,F}$ can differ from zero (positive) and $\gamma \sim \bar{v}_{2,F}/N_{all} > 0$, as seen from Eq. (11).

5. Several comments are warranted:

(i) Very recently, an observable related to $\gamma$ was studied as a function of elliptic anisotropy [20]. This finding argued that for mid-peripheral Au+Au collisions $\gamma < 0$ at $v_2 = 0$ for same-charge pairs, which is consistent with the CME.

(ii) Even if $\gamma < 0$ at $v_2 = 0$ for same-charge pairs, it does not imply the existence of the CME. It only indicates the presence of some correlation mechanism that explicitly depends on the orientation of the reaction plane. To measure the CME, a different observable is needed, e.g., the multiparticle charge-sensitive correlator [7] or direct measurements of the electric dipole [8].

(iii) As argued in Ref. [25], central U+U collisions also can be used to distinguish between effects driven by elliptic anisotropy and the CME. In the present paper, we considered only peripheral collisions, where the CME and fluctuations in $v_2$ are expected to be the most visible. Both methods can be used independently to reduce the contribution of elliptic anisotropy.

(iv) In this paper, we proposed a way to remove the elliptic-flow-induced background from the correlator $\gamma$. However, we note that in Ref. [7] a new multiparticle charge-sensitive correlator $C_c$ was proposed that is insensitive to correlations due to (elliptic) flow, jets, or momentum conservation. The measurement of $C_c$ together with $\gamma$ (with removed background) could provide an important information about a possible signal of local parity violation.

In summary, we demonstrated that fluctuations in elliptic anisotropy in (mid-) peripheral heavy-ion collisions can be used effectively to reduce the contribution of $v_2$-induced correlations from the two-particle correlator (1). We showed that at a given impact parameter, the magnetic field produced in heavy-ion collisions depends weakly on the participant eccentricity, in contrast to the value of $v_2$. We also discussed the contribution of transverse-momentum conservation to $\gamma$ at a vanishing $v_2$. Preliminary experimental analysis [20] suggests the presence of a correlation mechanism that does not
scale with elliptic anisotropy.

**Acknowledgments**

We are grateful to Vladimir Skokov for numerous discussions. Correspondence with Scott Pratt and Sergei Voloshin is highly appreciated. We thank Ron Longacre for discussions about his recent paper. This investigation was supported by the U.S. Department of Energy under Contract No. DE-AC02-98CH10886 and by the grant N N202 125437 of the Polish Ministry of Science and Higher Education (2009-2012).
References

[1] D. E. Kharzeev, L. D. McLerran and H. J. Warringa, Nucl. Phys. A 803, 227 (2008)

[2] J. Rafelski and B. Muller, Phys. Rev. Lett. 36, 517 (1976).

[3] V. Skokov, A. Illarionov and V. Toneev, Int. J. Mod. Phys. A 24, 5925 (2009)

[4] P. V. Buividovich, M. N. Chernodub, E. V. Luschevskaya and M. I. Polikarpov, Phys. Rev. D 81, 036007 (2010)

[5] D. E. Kharzeev, Annals Phys. 325, 205 (2010)

[6] S. A. Voloshin, Phys. Rev. C 70, 057901 (2004)

[7] N. N. Ajitanand, R. A. Lacey, A. Taranenko and J. M. Alexander, Phys. Rev. C 83, 011901 (2011)

[8] J. Liao, V. Koch and A. Bzdak, Phys. Rev. C 82, 054902 (2010)

[9] B. I. Abelev et al. [STAR Collaboration], Phys. Rev. Lett. 103, 251601 (2009)

[10] B. I. Abelev et al. [STAR Collaboration], Phys. Rev. C 81, 054908 (2010)

[11] S. Pratt, S. Schlichting and S. Gavin, Phys. Rev. C 84, 024909 (2011)

[12] S. Schlichting and S. Pratt, Phys. Rev. C 83, 014913 (2011)

[13] A. Bzdak, V. Koch and J. Liao, Phys. Rev. C 81, 031901 (2010)

[14] A. Bzdak, V. Koch and J. Liao, Phys. Rev. C 83, 014905 (2011)

[15] F. Wang, Phys. Rev. C 81, 064902 (2010)

[16] B. Muller and A. Schafer, Phys. Rev. C 82, 057902 (2010)

[17] D. Teaney and L. Yan, Phys. Rev. C 83, 064904 (2011)

[18] G. -L. Ma and B. Zhang, Phys. Lett. B 700, 39 (2011)
[19] S. A. Voloshin, A. M. Poskanzer and R. Snellings, arXiv:0809.2949 [nucl-ex].

[20] R. Longacre, arXiv:1112.2139 [nucl-th].

[21] B. Alver et al. [PHOBOS Collaboration], Phys. Rev. Lett. 98, 242302 (2007)

[22] B. Alver, M. Baker, C. Loizides and P. Steinberg, arXiv:0805.4411 [nucl-ex].

[23] Private communication with Bjoern Schenke.

[24] A. Bzdak and V. Skokov, arXiv:1111.1949 [hep-ph].

[25] S. A. Voloshin, Phys. Rev. Lett. 105, 172301 (2010)