CHIRAL SYMMETRY IN SUPERSYMMETRIC THREE DIMENSIONAL QUANTUM ELECTRODYNAMICS

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Abstract
We describe the investigation of spontaneous mass-generation and chiral symmetry breaking in supersymmetric QED3 using numerical solutions of the Dyson-Schwinger equation together with the CJT effective action and supersymmetric Ward identities. We find that, within the quenched, bare vertex approximation, the chirally symmetric solution is favoured.

1 Introduction
Any realistic attempt to unify gravity with the other fundamental forces of nature must almost certainly be supersymmetric. Whilst perturbative results are often easier to find in supersymmetric field theories than in ordinary gauge theories, especially with the use of superfields, the development of non-perturbative tools for supersymmetric theories has not yet reached maturity. In this paper we apply a nonperturbative tool which has proved to be effective in the study of strong interactions, namely Dyson-Schwinger equations (DSEs), to one of the simplest supersymmetric gauge theories.

We choose as our gauge theory supersymmetric quantum electrodynamics in three space-time dimensions (SQED3). In its nonsupersymmetric form, three dimensional quantum electrodynamics (QED3) with four component spinors exhibits the interesting nonperturbative phenomena of confinement and, at least for small numbers of flavours, chiral symmetry breaking. Much of what is known about the behaviour of QED3 has been learnt from numerical DSE or lattice calculations. Considering the success of DSE techniques in non supersymmetric theories, there is a surprising scarcity of applications of the method to supersymmetric theories in the literature, and an almost total absence of numerical DSE calculations in particular.

The version of SQED3 which we consider is the four-component fermion version first proposed by Pisarski, who approached the model by dimensional reduction from SQED4. Herein, we develop SQED3 using the Wess Zumino construction. Our construction produces supersymmetry multiplets which
differ slightly from those of Pisarski because of an extra degree of freedom in the charge conjugation matrix. Pisarski’s approach was an analytic one based on a $1/N_{\text{flavor}}$ expansion. His analysis indicates the existence of a dynamical mass generating solution in the large $N_{\text{flavor}}$ limit.

Koopmans and Steringa\textsuperscript{6} have analysed SQED3 with 2-component fermions using DSEs and claim the critical number $N_c = 3.24$ of flavours below which spontaneous mass generation occurs. This result should be treated with some caution however as it ignores the possibility of spontaneous parity violation\textsuperscript{7} and the generation of a photon mass via a Chern-Simons term when the number of two-component fermions is odd. Furthermore, there is no indication whether spontaneous mass generation corresponds to the stable vacuum of the theory.

The above treatments use a component formalism to study SQED3. By contrast, Clark and Love\textsuperscript{8} develop DSEs for SQED4 using a superfield formalism. While more elegant as a mathematical formalism, the superfield approach has the disadvantage that each DSE contains an infinite number of terms. This is dealt with by truncating from the chiral multiplet self energy diagrams containing seagull and higher order $n$-point vertices. They find that the effective mass contains a prefactor which vanishes in Feynman gauge and conclude that there can be no spontaneous mass generation in SQED4.

The work of Clark and Love has been criticised by Kaiser and Selipsky\textsuperscript{9} on two grounds. Firstly they argue that the truncation of seagull diagrams is too severe as it ignores contributions even at the one-loop level. Secondly they point out that infinities arising from infrared divergences which plague the superfield formalism can counter the vanishing prefactor and allow spontaneous mass generation. These criticisms highlight some of the dangers of attempting to extract phenomenological consequences of supersymmetric DSEs by working solely with the superfield formalism. In fact, analyses in the literature of SQCD\textsuperscript{10} have generally found the component formalism to be the most efficient way to proceed.

In this paper we present a first detailed numerical analysis of the chiral multiplet DSE for a supersymmetric gauge theory. We work with the component formalism rather than the superfield formalism to avoid the problems encountered by Clark and Love. By choosing to work in three dimensions, and with the component formalism, we find that all integrals encountered are infrared and ultraviolet convergent.

Section 2 describes the four-component fermion Clifford algebra of QED3 and gives the supersymmetry multiplets. In section 3 we present the SQED3 lagrangian and chiral multiplet propagator DSEs. We explain how supersymmetric Ward Identities can be used to simplify what would otherwise be an
arduous problem and present our numerical solutions for quenched SQED3
in the bare vertex approximation. We present both chirally symmetric and
asymmetric solutions and use the Cornwall-Jackiw-Tomboulis (CJT) effective
potential to determine that the massless solution is dynamically favoured.

2 The algebra of SQED3

In 2+1 dimensions there are two inequivalent, irreducible representations
of the Clifford algebra, given in terms of $2 \times 2$ matrices. These two representations
differ by a minus sign $\gamma^1$, and have the undesirable property that either
one leads to a version of QED3 which is parity non-invariant. To circumvent
this property, it is common to consider a four-component version of QED3
incorporating Dirac matrices which are a direct sum of the two inequivalent
representations $\mathbb{1}$, $\gamma^4$. The Dirac matrix algebra we employ here is constructed as
follows $\mathbb{1}$.

The 4×4 matrices $\gamma_\mu$ satisfy $\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}$, $\eta_{\mu\nu} = \text{diag}(1, -1, -1)$ where $\mu$ takes the values 0, 1 and 2. We take the complete set of 16 matrices

$$\{\gamma_A\} = \{I, \gamma^4, \gamma^5, \gamma^45, \gamma_\mu, \gamma_\mu^4, \gamma_\mu^5, \gamma_\mu^45\},$$

$$\gamma_0 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}, \quad \gamma_{1,2} = -i \begin{pmatrix} \sigma_{1,2} & 0 \\ 0 & -\sigma_{1,2} \end{pmatrix},$$

$$\gamma_4 = \gamma^4 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma_5 = \gamma^5 = \begin{pmatrix} 0 & -iI \\ iI & 0 \end{pmatrix}, \quad \gamma_{45} = -i\gamma_4\gamma_5,$$

$$\gamma_{\mu4} = i\gamma_\mu\gamma^4, \quad \gamma_{\mu5} = i\gamma_\mu\gamma^5, \quad \gamma_{\mu45} = \gamma_\mu\gamma^45.$$

The matrices $I$, $\gamma^4$, $\gamma^5$, $\gamma^45$ are the Pauli matrices in block form and as such generate a $U(2)$ algebra. The parity and charge conjugation rules for four-component Dirac spinors are given by

$$\mathcal{P}\psi(x)\mathcal{P}^{-1} = \Pi\psi(x^0, -x^1, x^2), \quad \mathcal{P}\bar{\psi}(x)\mathcal{P}^{-1} = \bar{\psi}(x^0, -x^1, x^2)\Pi^{-1},$$

$$C\psi C^{-1} = C\bar{\psi}^T, \quad C\bar{\psi} C^{-1} = -\psi^T C^{-1},$$

where

$$\Pi = \gamma_{14}e^{i\phi_p\gamma^45}, \quad C = \gamma_{2e}^{i\phi_C\gamma^45},$$

and $0 \leq \phi_p, \phi_C < 2\pi$. A Majorana spinor is one for which $\psi = C\bar{\psi}^T$. The arbitrary phases $\phi_C$ and $\phi_p$ are important for classifying the bound states in QED3.
We have
\[
C^{-1} \begin{pmatrix} \gamma_4 \\ \gamma_5 \end{pmatrix} C = R_C \begin{pmatrix} \gamma_T^4 \\ \gamma_5 \end{pmatrix},
\]
(2.3)
\[
\Pi^{-1} \begin{pmatrix} \gamma_4 \\ \gamma_5 \end{pmatrix} \Pi = R_P \begin{pmatrix} \gamma_4 \\ \gamma_5 \end{pmatrix},
\]
where
\[
R_P = \begin{pmatrix} -\cos 2\phi_P - \sin 2\phi_P \\ -\sin 2\phi_P & \cos 2\phi_P \end{pmatrix},
R_C = \begin{pmatrix} -\cos 2\phi_C \sin 2\phi_C \\ \sin 2\phi_C & \cos 2\phi_C \end{pmatrix}.
\)
(2.4)

The first step in the extension of QED3 to supersymmetry is the construction of chiral multiplets. A reasonable tentative suggestion is
\[
\delta a = -i\bar{\zeta}\psi,
\]
\[
\delta b = \bar{\zeta}\gamma_5\psi,
\]
\[
\delta \psi = (f + i\gamma_5g) + \bar{\theta}(a + i\gamma_5b)\zeta,
\]
\[
\delta f = \bar{\zeta}\gamma_5\bar{\theta}\psi,
\]
\[
\delta g = i\bar{\zeta}\gamma_5\bar{\theta}\psi,
\]
(2.5)
where \(a, b, f\) and \(g\) are real and \(\zeta\) and \(\psi\) are Majorana. If the superalgebra is to hold then the commutator of two of these transformations must obey
\[
[\delta_1, \delta_2]X = 2\bar{\zeta}_2\gamma^\mu\zeta_1\partial_\mu X,
\]
(2.6)
where \(X\) is any component of the multiplet. For the case of \(a\) we have
\[
[\delta_1, \delta_2]a = \bar{\zeta}_2\gamma^\mu\zeta_1\partial_\mu a - i\bar{\zeta}_2\gamma_5\gamma^\mu\zeta_1\partial_\mu b - (1 \leftrightarrow 2).
\]
Now, since \(\zeta\) is Majorana
\[
\bar{\zeta}_1\gamma^\mu\zeta_2 = -\bar{\zeta}_2\gamma^\mu\zeta_1,
\]
(2.7)
as required, but using equations (2.3) and (2.4)
\[
-i\bar{\zeta}_1\gamma_5\gamma^\mu\zeta_2 = \bar{\zeta}_2(\gamma_5\cos 2\phi_C + \gamma_4\sin 2\phi_C)\gamma^\mu\zeta_1,
\]
(2.8)
which does not cancel \(\bar{\zeta}_2\gamma_5\gamma_5\gamma^\mu\zeta_1\). A similar situation arises if \(a\) is replaced by any other member of the multiplet. To stop this ‘blowing out’ of terms we could simply set \(\phi_C = 0\). This would be unfortunate though as the angular freedom \(\phi_C\) in the matrix \(C\) would be lost. As stated earlier, this angle is important for classifying the bound states in QED3 so we would like to preserve it if possible to see what effects it has (if any) in the supersymmetric theory. To
this end we define the rotated Dirac matrices. This can be done by making the substitution
\[
\begin{pmatrix} \gamma_4 \\ \gamma_5 \end{pmatrix} \rightarrow \begin{pmatrix} \gamma_P \\ \gamma_W \end{pmatrix} = \begin{pmatrix} \cos \phi_C - \sin \phi_C \\ \sin \phi_C \cos \phi_C \end{pmatrix} \begin{pmatrix} \gamma_4 \\ \gamma_5 \end{pmatrix} = M \begin{pmatrix} \gamma_4 \\ \gamma_5 \end{pmatrix},
\]
(2.9)
in the Clifford algebra. (Note that \(-i\gamma_P\gamma_W = \gamma_{45}\) so the matrices \(I, \gamma_P, \gamma_W, \gamma_{45}\) again generate an \(SU(2)\) algebra.) Then
\[
C^{-1} \begin{pmatrix} \gamma_P \\ \gamma_W \end{pmatrix} C = \begin{pmatrix} -\gamma_P^T \\ \gamma_W^T \end{pmatrix},
\]
(2.10)
since
\[
MR_C M^{-1} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}.
\]
(2.11)
The rotated matrices can be used to define a supersymmetry transformation consistent with Eq. (2.6). In order to do this, all terms except \(\bar{\zeta}_2 \gamma_5 \bar{\psi} \delta_1 \delta_2 \chi\) generated by \(\delta_1 \delta_2 \chi\) must be symmetric under interchange of \(\zeta_1, \zeta_2\), i.e.
\[
\bar{\zeta}_2 \gamma_5 \gamma_5 \zeta_1 = \bar{\zeta}_1 \gamma_5 \gamma_5 \zeta_2.
\]
(2.12)
In this sense \(\gamma_5\) is well-behaved but \(\gamma_P\) and \(\gamma_{45}\) are problem matrices because
\[
\bar{\zeta}_2 (\gamma_P, \gamma_{45}) \gamma_5 \zeta_1 = -\bar{\zeta}_1 (\gamma_P, \gamma_{45}) \gamma_5 \zeta_2.
\]
(2.13)
Making the substitution (2.9) gives the 2+1 dimensional chiral multiplet
\[
\begin{align*}
\delta a &= -i \bar{\zeta} \psi \\
\delta b &= \bar{\zeta} \gamma_W \psi \\
\delta \psi &= (f + i \gamma_W g) + i \gamma_5 \partial_\mu (a + i \gamma_W b) \zeta \\
\delta f &= \bar{\zeta} \partial \bar{\psi} \\
\delta g &= i \bar{\zeta} \gamma_W \partial \bar{\psi},
\end{align*}
\]
(2.14)
which is the analogue of the standard chiral multiplet in 3 + 1 dimensions.

The relative difference between the arbitrary phases \(\phi_P\) and \(\phi_C\) is fixed by the imposition of supersymmetry. Indeed, from equations (2.3) and (2.9) we have that
\[
\Pi^{-1} \begin{pmatrix} \gamma_P \\ \gamma_W \end{pmatrix} \Pi = MR_P M^{-1} \begin{pmatrix} \gamma_P \\ \gamma_W \end{pmatrix}
\]
where
\[
MR_P M^{-1} = \begin{pmatrix} -\cos 2(\phi_P + \phi_C) - \sin 2(\phi_P + \phi_C) \\ -\sin 2(\phi_P + \phi_C) \cos 2(\phi_P + \phi_C) \end{pmatrix}.
\]
(2.15)
If the form of the chiral multiplet transformation Eq. (2.14) is to be maintained under parity transformations, the off-diagonal terms in this matrix must be set to zero. \( \phi_P \) must therefore be set to one of

\[
\phi_P = -\phi_C, \pi - \phi_C
\]

and we choose the former. The Clifford algebra to be used is now

\[
\gamma_A = \{I, \gamma_P, \gamma_W, \gamma_{45}, \gamma_\mu, \gamma_\mu_\nu, \gamma_\mu_\nu_{45} \}
\]

\[C = \gamma_2 e^{i\phi_C}, \Pi = \gamma_P.\]

With the Clifford algebra and chiral multiplets established we now look for a general multiplet. This is very similar to the standard general multiplet in 3+1 dimensions except for an extra scalar field \( K \), which makes up the bosonic degree of freedom lost when the vector field is taken from 3+1 to 2+1 dimensions. Our general multiplet \( V \) is defined by the following fields and transformations:

\[
\begin{align*}
\delta C &= \bar{\zeta} \gamma_W \chi \\
\delta \chi &= (M + i\gamma_W N)\zeta + i\gamma^\mu (A_\mu + i\gamma_W \partial_\mu C)\zeta - \gamma_P K \zeta \\
\delta M &= \bar{\zeta} (\bar{\theta} \chi + i\lambda) \\
\delta N &= i\zeta \gamma_W (\bar{\theta} \chi + i\lambda) \\
\delta A_\mu &= \bar{\zeta} \gamma_\mu \lambda - i\zeta \partial_\mu \chi \\
\delta K &= -i\zeta \gamma_P \lambda \\
\delta \lambda &= \frac{1}{2} (\gamma^\mu \gamma^\nu - \gamma^\mu \gamma^\nu) \partial_\mu A_\nu \chi + i\gamma_W D \zeta + i\gamma_P \bar{\theta} K \zeta \\
\delta D &= i\bar{\lambda} \gamma_W \bar{\theta} \lambda.
\end{align*}
\]

### 3 Solving the DSEs of SQED3

Repeating the method of Wess and Zumino\(^5\) yields the SQED3 lagrangian

\[
L = |f|^2 + |g|^2 + |\partial_\mu a|^2 + |\partial_\mu b|^2 - \bar{\psi} \not\partial \psi - m(a^* f + af^* + b^* g + bg^* + i\bar{\psi} \psi)
\]

\[
-ieA^\mu (a^* \not\partial_\mu a + b^* \not\partial_\mu b + \bar{\psi} \gamma_\mu \psi) + eK \bar{\psi} \gamma_P \psi
\]

\[
-e[\bar{\lambda}(a^* + i\gamma_W b^*) \psi - \bar{\psi} (a + i\gamma_W b) \lambda]
\]

\[
+i e D(a^* b - ab^*) - e^2 (K^2 - A_\mu A^\mu)(|a|^2 + |b|^2)
\]

\[
-\frac{1}{4} F_{\mu\nu} F_{\mu\nu} - \frac{1}{2} \bar{\lambda} \not\partial \lambda + \frac{1}{2} \not\partial \mu K \not\partial_\mu K + \frac{1}{2} D^2
\]

(3.1)
which becomes that found by Pisarski by dimensional reduction of SQED4 when $\phi_C$ is set to zero and the scalar fields are trivially redefined.

The chiral limit is defined by taking $m \to 0$. In this limit the bare lagrangian is invariant with respect to a global $U(2)$ symmetry generated by $I$, $\gamma_4$, $\gamma_5$ and $\gamma_{45}$. In the non-supersymmetric chiral theory, spontaneous mass generation leads to a nonperturbative breaking of the chiral generators $\gamma_4$ and $\gamma_5$. Here we shall explore the possibility of chiral symmetry breaking in SQED3 by considering the chiral multiplet propagator DSEs. We shall use Witten’s result that supersymmetry remains unbroken.

Figure 1: The Dyson-Schwinger equation for the electron propagator in SQED3.

The electron propagator DSE in SQED3 is more complicated than that in the non-supersymmetric version since the electron interacts not only with the photon, but also with the photino, the $K$ and its own superpartners $a$ and $b$. The DSE for the electron is illustrated in Fig. 1, where a solid line indicates an electron propagator, dashed line an $a$ or $b$ propagator, a wavy line a photon propagator, an alternating dash-and-dot line a $K$ propagator and a wave with a solid line through it a photino propagator.

The DSE for the scalar partners is shown in Fig. 2. It would appear prima facie that we have two coupled DSEs to solve, the second considerably more complicated than the first. We are saved from this effort however, by the existence of supersymmetric Ward Identities (WI). The supersymmetric WI relating the scalar two point functions to the electron propagator is

$$\langle \psi \bar{\psi} \rangle = i \langle a^* f \rangle - i \rho \langle a^* a \rangle = i \langle b^* g \rangle - i \rho \langle b^* b \rangle. \quad (3.2)$$

Substituting in the fermion propagator Ansatz

$$S(p) = \langle \psi \bar{\psi} \rangle = \frac{-i}{A(p^2) + B(p^2)} \quad (3.3)$$
Figure 2: The Dyson-Schwinger equation for the $a$ and $b$ propagator in SQED3.

gives the scalar propagator

$$D(p^2) \equiv \langle a^*a \rangle = \langle b^*b \rangle = \frac{A(p^2)}{p^2 A^2(p^2) - B^2(p^2)}, \quad (3.4)$$

and the $a$ to $f$ amplitude

$$\langle a^*f \rangle = \langle b^*g \rangle = \frac{B(p^2)}{p^2 A^2(p^2) - B^2(p^2)} = \frac{B(p^2)}{A(p^2)} D(p^2). \quad (3.5)$$

If nonperturbative chiral symmetry breaking is allowed for, the two point function $\langle a^*f \rangle$ is potentially non-zero in the chiral limit even though perturbation theory predicts it should be identically zero.

Clearly it suffices to solve only the fermion DSE to determine the scalar functions $A(p^2)$ and $B(p^2)$. To do the calculations we perform a Wick rotation to Euclidean space. Our Euclidean conventions are the same as those set out in Appendix B of reference 12. Spacelike momenta satisfy $p^2 > 0$, and the fermion and boson propagators take the form

$$S(p) = \frac{1}{i \gamma \cdot p A(p^2) + B(p^2)}, \quad (3.6)$$

$$D(p) = \frac{A(p^2)}{p^2 A(p^2) + B^2(p^2)} \quad (3.7)$$
Figure 3: The chirally asymmetric solution for the propagator function $B(p^2)$ in Feynman gauge, $\xi = 1$. If chiral symmetry is not broken, $B$ is identically zero.

respectively. We use the chirally quenched approximation which is known to be compliant with $U(1)$ gauge invariance, that is, we take the propagators of the photon and its superpartners to be bare. The class of covariant gauges, for which the photon propagator is given by

$$D_{\mu\nu}(k) = (\delta_{\mu\nu} - k_{\mu}k_{\nu}/k^2) \frac{1}{k^2} + \xi \frac{k_{\mu}k_{\nu}}{k^4}$$

(3.8)

will be considered. The corresponding $K$ and photino propagators are

$$D_K(k) = \frac{1}{k^2},$$

(3.9)

$$D_\lambda(k) = \frac{1}{\gamma \cdot k},$$

(3.10)

respectively. We also employ the bare vertex or rainbow approximation

$$\Gamma_\mu(q,p) = ie\gamma_\mu,$$

(3.11)

$$\Gamma_P(q,p) = -ie\gamma_P,$$

(3.12)
Figure 4: The chirally asymmetric (solid curve) and symmetric (dashed curve) solutions for the propagator function $A(p^2)$ in Feynman gauge, $\xi = 1$.

\[
\begin{align*}
\Gamma_{\lambda\psi}(q,p) &= -ie, \\
\Gamma_{\psi\lambda}(q,p) &= ie,
\end{align*}
\]  

for the Euclidean electron–photon, electron–$K$ and electron–$a$–photino vertices respectively. The Euclidean space DSE with these approximations reduces after angular integration to the following coupled integral equations for $B(p^2)$ and $A(p^2)$:

\[
\begin{align*}
B(p^2) &= (\xi + 3) \frac{e^2}{4\pi^2 p} \int_0^\infty dq \frac{qB(q)}{q^2A^2(q^2) + B(q^2)} \ln \frac{|p + q|}{|p - q|} \\
A(p^2) &= (\xi - 1) \frac{e^2}{4\pi^2 p^2} \int_0^\infty dq \frac{qA(q^2)}{q^2A^2(q^2) + B(q^2)} \left( \frac{p^2 + q^2}{2p} \ln \frac{|p + q|}{|p - q|} - q \right) \\
&\quad + \frac{e^2}{2\pi^2 p} \int_0^\infty dq \frac{qA(q^2)}{q^2A^2(q^2) + B(q^2)} \ln \frac{|p + q|}{|p - q|} + 1
\end{align*}
\]  

We solve equations (3.15) and (3.16) numerically using the standard iterative procedure introduced by Applequist et al. The functions $A(p^2)$ and $B(p^2)$
are defined on a non-uniform grid of fifty-one points concentrated at small momenta where the function varies more rapidly. The integrand is interpolated using a cubic spline using an ultraviolet cut-off of $p = 1000e^2$.

We show in figures 3 and 4 both the chirally symmetric and asymmetric solutions to the massless ($m = 0$), Feynman gauge ($\xi = 1$) electron DSE of Fig. 1. The results are plotted in units with $e^2 = 1$. It is apparent in figure 3 that mass generation is remarkably suppressed and both graphs descend steeply to the values assumed when there is no dressing. We have tested the convergence of our numerical iteration procedure by varying the initial guess for $A$ and $B$ and the ultra-violet cutoff. These changes had no significant effect on the solution obtained.

The DSE is the first functional derivative with the respect to the electron propagator of the CJT effective potential and its solutions are therefore stationary points. To ascertain if the chirally asymmetric solution to the DSE
is dynamically favoured we make use of the CJT effective potential which, when evaluated at a stationary point, is given by

\[ V[S,D] = \int \frac{d^3p}{(2\pi)^3} \left\{ \text{tr} \ln[1 - \Sigma_S(p)S(p)] + \frac{1}{2} \text{tr} \Sigma_S(p)S(p) \right\} - 2 \int \frac{d^3p}{(2\pi)^3} \left\{ \text{tr} \ln[1 - \Sigma_D(p)D(p)] + \frac{1}{2} \text{tr} \Sigma_D(p)D(p) \right\}, \tag{3.17} \]

where \( \Sigma_S(p) = S(p)^{-1} - S^{-1}_{\text{bare}}(p) \) and \( \Sigma_D(p) = D(p)^{-1} - D^{-1}_{\text{bare}}(p) \). This formula neglects the contribution from the photon and its superpartners since it is calculated in the quenched approximation. Note that there is another term in Eq. (3.17), given by

\[ \int \frac{d^3p}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \left[ D_{\mu\nu}(q) + D_K(q) \right] D(p), \tag{3.18} \]

corresponding to the vacuum graphs giving rise to the tadpole contributions in the boson SDE. As we have written it, Eq. (3.17) only gives half of these particular diagrams. However, in the quenched approximation Eq. (3.18) is zero by dimensional reduction. In Table 1 we list the difference

\[ V_A[S,D] - V_S[S,D] = 2 \int \frac{d^3p}{(2\pi)^3} \left\{ \ln \left[ \frac{p^2A_S(p^2)^2}{p^2A_A(p^2)^2 + B_A(p^2)^2} \right] - \frac{p^2A_A(p^2)^2}{p^2A_A(p^2)^2 + B_A(p^2)^2} \right\} , \tag{3.19} \]

between the effective potentials calculated for the chirally asymmetric and symmetric solutions for a range of values of the gauge parameter \( \xi \). Our calculation confirms that the asymmetric solution is dynamically favoured in the nonsupersymmetric case. For SQED3, however, we find that chiral symmetry is not spontaneously broken, at least in the quenched, rainbow approximation.

It is clear from Table 1 that the calculated effective potential is dependent on the gauge parameter \( \xi \). This is an artifact of the rainbow approximation which, for dressed electron propagators, violates the \( U(1) \) Ward-Takahashi identity (WTI). Another useful indicator of gauge symmetry breaking is the \( U(1) \) invariant chiral condensate

\[ \langle \bar{\psi}\psi \rangle = \text{tr} S(x = 0) = \frac{2}{\pi^2} \int_0^{\infty} dp \frac{p^2 B(p^2)}{p^2 A^2(p^2) + B^2(p^2)}, \tag{3.20} \]

calculated in the asymmetric phase. In Figure 5 we plot the chiral condensate for quenched rainbow SQED3 and QED3. While the calculated condensate for
Table 1: The difference $V_A[S, D] - V_S[S, D]$ between the CJT effective potential of the chirally asymmetric solution and the chirally symmetric solution for QED3 and SQED3 at different values of the gauge parameter $\xi$.

| $\xi$ | SQED3       | QED3        |
|-------|-------------|-------------|
| 0     | $4.62 \times 10^{-3}$ | $-1.32 \times 10^{-5}$ |
| 0.5   | $4.29 \times 10^{-3}$ | $-6.02 \times 10^{-6}$ |
| 1     | $4.07 \times 10^{-3}$ | $-3.44 \times 10^{-6}$ |

SQED3 is surprisingly insensitive to the choice of gauge, the condensate for QED3 is, as expected, strongly gauge dependent.

For QED3 the invariance of the chiral condensate can be considerably improved by replacing the bare vertex with the minimal Ball-Chiu vertex Ansatz, which is specifically designed to respect the $U(1)$ WTI and be free of kinematic singularities. For comparison, the QED3 condensate obtained in this way in reference is plotted in Figure 5. Here we attempt a similar substitution for SQED3. Note that the photon’s supersymmetric partners $K$ and $\lambda$ are completely invariant under a gauge transformation and their vertices are not directly constrained by the WTI. Compliance with the WTI can therefore be achieved by replacing the bare photon-fermion vertex with the minimal Ball-Chiu Ansatz whilst the remaining vertices are kept bare. This method incurs the penalty of breaking supersymmetry. The resulting chiral condensate is plotted in Figure 5. Suprisingly the variation of the condensate with respect to the gauge parameter was found to be an order of magnitude greater than in the bare case. We attribute this to the violation of supersymmetry and conclude that any attempt to improve the vertex must remain supersymmetric.

4 Conclusions

We have used the method of Wess and Zumino to obtain a lagrangian for SQED3 with four-component fermions. The lagrangian derived in this way agrees with that obtained by dimensional reduction except that our form retains an angular degree of freedom associated with the definition of parity and charge conjugation in $(2 + 1)$ dimensions. This is effected by taking suitable linear combinations of the Dirac matrices $\gamma_4$ and $\gamma_5$. It was found that supersymmetry constrains the angles $\phi_P$ and $\phi_C$ associated with parity
and charge conjugation transformations in \text{QED}3 to be related by \( \phi_P = -\phi_C \) or \( \pi = \phi_C \).

The DSE of SQED3 was solved numerically in quenched, rainbow approximation after using supersymmetric \text{WIs} to relate scalar and fermion propagators within the chiral multiplet. Application of the CJT effective potential revealed that the chirally symmetric solution is dynamically preferred, in contrast to the situation in QED3. We conclude that chiral symmetry is not broken in SQED3 in the quenched, rainbow approximation.

Although the rainbow approximation violates the \text{U}(1) gauge symmetry, we find the chiral condensate to be relatively insensitive to the choice of gauge fixing parameter. We believe this to be fortuitous however, since replacing the bare vertex with an Ansatz which respects the \( \text{U}(1) \) \text{WTI} but not the supersymmetric \text{WIs} produces a chiral condensate which is significantly gauge dependent. Work on developing a vertex Ansatz which respects both the \( \text{U}(1) \) gauge and supersymmetric \text{WIs} is currently in progress.

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