Generation of a macroscopic Schrödinger cat using vortex light

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Introduction -The quantum world is puzzling. A particle can be simultaneously in several states or several different identities at once. This nonlocality originates from quantum superposition, a feature so amazing that Erwin Schrödinger and independently Albert Einstein and collaborators presented now-famous paradoxes, i.e., "Schrödinger cat" (SC) and "EPR", respectively [1–4]. In analogy with this famous Gedankenexperiment, the modern notion of the SC state is a superposition state of two macroscopically distinguishable states (e.g., the alive and dead states of the cat) in a certain degree of freedom [5]. Such states lies at the heart of quantum mechanics, not only featuring in the theoretical interpretation of microscopic and macroscopic worlds, but also serve as information carriers for quantum information processing, such as quantum computation and metrology [6–8]. Over the past few decades, the generation and enlargement of this type of cat states have been studied widely. Various approaches have been demonstrated in the photon-number representation |n⟩, including photon-subtracted squeezed vacuum state [17–19], ancilla-assisted photon subtraction, time-separated two-photon subtraction [30–32]. Unfortunately, most of the preparations is "kitten", and then many enlargement-schemes are proposed [29, 30]; however it is unwieldy to realize in laboratory. Because the manipulation of cat state is difficult in photon-number representation, exploring their generations in other representation are valuable. And some delightful advances have been made in several systems, e.g., the excitation a different Rydberg states to fit the cat state [27]; the generation of the cat states by timing ultrafast laser pulses [28]; the formation of the cat states by two Bose-Einstein condensates [33]; and others [34, 35]. The attempt to manipulate cat state in above is to find a suitable Hilbert space where it can live.

There always exist a criterion or physical space to characterize the macroscopic quantum superposition [21–26], specifically, phase space based on Wigner function W(x, p) [18], energy space [20], position space [20] ('alive cat' locating at one place, and the superposition state, 'dead cat' living in another place), [27, 28], among others. In quantum optics, a general approximate realization of SC state, and most popular, is the superposition of two harmonic oscillator coherent states |α⟩. This type of cat state has the form of |Cat⟩ = N(|α⟩Alive + eιθ|−α⟩Dead). Employing tomography of Wigner function, the dynamic of SC state with two wave packets can be observed in the phase space [29]:

\[ W_{\text{cat}} = W_0(x-x_0,p)+W_0(x+x_0,p)+2\cos(4x_0p+\psi)W_0(x,p) \]

Where \( W_0(x\pm x_0,p) \) represent a moved Gaussian wave packet as the original wave packet \( W_0(x,p) \). Over the last few decades, the generation and enlargement of this type of cat states have been studied widely. Various approaches have been demonstrated in the photon-number representation |n⟩, including photon-subtracted squeezed vacuum state [17–19], ancilla-assisted photon subtraction, time-separated two-photon subtraction [30–32]. Unfortunately, most of the preparations is "kitten", and then many enlargement-schemes are proposed [29, 30]; however it is unwieldy to realize in laboratory. Because the manipulation of cat state is difficult in photon-number representation, exploring their generations in other representation are valuable. And some delightful advances have been made in several systems, e.g., the excitation a different Rydberg states to fit the cat state [27]; the generation of the cat states by timing ultrafast laser pulses [28]; the formation of the cat states by two Bose-Einstein condensates [33]; and others [34, 35]. The attempt to manipulate cat state in above is to find a suitable Hilbert space where it can live.

Recently, orbital angular momentum (OAM) of the photon, a remarkable photonic freedom that inherently has infinite dimensions, obtained increasing attention and rapid development [36, 37]. Quantum mechanically, an arbitrary pure quantum state can be represented by a set of OAM eigenstates |L⟩ in the corresponding Hilbert
space (see in Fig. 1(c)). Because it is surprisingly simple to manipulate experimentally OAM states of light, the OAM degree of freedom in photons has been identified as a useful and valuable platform for realizing various multilevel quantum systems\cite{37,38}, and testing several fundamental quantum theories\cite{39,40}, and other quantum information applications\cite{40}.

In this letter, a ‘Schrödinger cat’ is engineered in the Hilbert space of orbital angular momentum (OAM) of photon. In this representation \( |L \rangle \), the cat state will be naturally divided into two spatially distinct Gaussian packets in position space. The behaviors are closely analogous to Schrödinger’s cat (see Fig. 1(a)): the ‘alive cat’ living in a location, and the other component of the superposition ‘dead cat’, the corresponding Gaussian packet being in the other distinct location. This degenerate phenomenon also appears in other systems, such as Bose-Einstein condensates (BECs) of atoms\cite{35}. In the laboratory, although the two partially localized wavepackets can be generated simply using a beam splitter, there are many differences between them such as interference and rotation. We clarify these differences in the supplementary material. This type of SC state has many advantages than previous SC realizations. The SC state can simply be from a ‘kitten’ to ‘cat’, and the coherence of the prepared cat state is more robust than generations in other systems. Different from the usual cat states constructed with simple harmonic oscillators, this type cat state, encoded in the OAM degree of photon, is not sensitive to the absorptions in the process of manipulations.

**Principle.** The prepared SC state \((|\alpha_L \rangle \pm |\!\!\alpha_L \rangle)\) encoded in the OAM space is a coherent superposition of the two opposite-amplitude distinct states \(|\pm \alpha_L \rangle\). They are engineered by infinite OAM eigenstate, which have following form,

\[
|\alpha_L \rangle = e^{-\frac{i\alpha^2}{2}} \sum_{L=0}^{\infty} \frac{\alpha^L_L}{\sqrt{L!}} |L\rangle
\]

Where \(\alpha_L (=|\alpha_L |e^{i\varphi})\), a complex number, determines the position and the size of the OAM-superposition state (see supplementary). Because the OAM-based superposition state \(|\alpha_L \rangle\) is similar to the expression of usual coherent state, we rename it to high-dimensional coherent states (HD-CS) for similarity. Therefore, the superposed cat state, can be named as high-dimensional SC states (HD-SCS). In particular, \(|\alpha_L \rangle = |0\rangle\) of \(\alpha_L = 0\) show that the HD-CS is degraded into a Gaussian beam, or named as high-dimensional vacuum states (HD-VS)\cite{41}. The OAM eigenstate \(|L\rangle\) used in our experiment, is the Laguerre Gauss mode\cite{30}. In a cylinder-coordinate representation, the intensity distribution of cat state, calculated in the supplementary material, has such a form,

\[
|\langle Cy|\text{Cat} \rangle |^2 = G(|\alpha_L \rangle)+G(|\!\!\alpha_L \rangle)+2\cos(\theta-2\alpha_L \sin(\Phi))G(0)
\]

Where G(0) represent the Gaussian wave packet, G(\(\alpha_L\)) and G(\(\!\!\alpha_L\)) depict a Gaussian packet moving to the right and left as the input referenced beam G(0); the third term gives the interference between ‘alive’ and ‘dead’ cats, which is similar to the Wigner function of Eq(1) that is a suitable space to describe the behaviors of cat state.

Fig. 1(d) and (e) present the intensity distributions of two cats with \(|\alpha_L \rangle = \sqrt{1.5}\) and \(|\!\!\alpha_L \rangle = \sqrt{3.5}\), here the ‘kitten’ and ‘cat’, respectively. Usually, the cat states can be classified as even and odd based on the occupation number of photons. Here, our HD-SCS can also be constructed based on their occupation modes. For example, an even HD-SCS has only even number of OAM modes: \(|\text{Cat}_{\text{even}} \rangle = c_0 |0\rangle + c_2 |2\rangle + c_4 |4\rangle + \ldots\), likewise the odd HD-SCS. The experimental details, and cartoon schematic for engineering HD-SCS are shown in Fig. 2.

**Results.** The normalized spatial spectra from experiments and theories are shown in Fig. 3 for SCs with \(\alpha_L = \sqrt{6.5}, \alpha_L = \sqrt{1.5}\), and \(\alpha_L = \sqrt{3.5}\), respectively. The HD-SCS keeps growing with the separation of two corresponding opposite-amplitude coherent states. The overlap of the two separated Gaussian wave packets approaches zero when the size \(|\alpha|\) is equal to \(\sqrt{3.5}\), which shows that the two cats are simultaneously alive and dead in two distinct locations, and hence the cat is an ideal SC state (see supplementary).

To demonstrate the coherence property of the HD-SCSs, the interferences between the sum of Gaussian
Figs. 2, 3, and 4. Schematic of experimental setup for engineering the high-dimensional Schrödinger cat state (HD-SCS). a: Generation and measurement of HD-SCS. The laser source, from a Ti:sapphire laser (MBR110, Coherent), pumps a spatial light modulator (SLM,G) with the binary-gray phase to generate HD-SCS. SLM,M, located on the focal plane of the 4f systems consisting of two lenses (f=350 mm), is used to measure the HD-SCS. With the help of the SLM,M, a fiber power meter measures the coupled powers to ensure the spatial spectra of the HD-SCS and to reconstruct quantum state in the high-dimensional space. b: The cartoon schemes of HD-SCS. c: The intensity-distributions for even cat states from a 3D perspective.

Figs. 3. Spatial spectra and intensity of HD-SCS for $|\alpha_L|=\sqrt{0.5}, \sqrt{1.5}$ and $|\alpha_L|=\sqrt{3.5}$. In each plot, the orange and green bars represent measured and theoretical calculations, respectively. The x-axis is the inverse of the phase of the OAM eigenstates loaded in SLM,M, and the y-axis is the normalized power measured by the fiber power meter. The red lines and blue dots are theoretical and experimentally measured spectral distributions, respectively. The imaging acquired by CCD in each subfigures is the corresponding intensity of cat state.

Figs. 4. Coherence of high-dimensional SC states. a1 and a2: The theoretical and measured interference pattern between HD-SCS ($|\alpha_L|=\sqrt{3.5}$) and a reference plane waves (Gaussian beam). a3: The Bloch-sphere representations by HD-SCS of $|\alpha_L|=\sqrt{1.5}$. b: The interference of the even (blue line) and odd (red line) HD-SCS ($|\alpha_L|\pm|\alpha_L|$) and the rotated HD-SCS ($|\alpha_L|e^{i\theta}|\pm\alpha_L|$) in the equator plane of Bloch-sphere. c: Interference patterns of cat state ($|\alpha_L|=\sqrt{3.5}$) for different attenuation-levels from 0 to 9dB. d: Visibility and deviations for kitten-, cat- states (blue bars) and attenuation-levels (orange bars).

Wave packets of HD-SCS and a referenced plane wave are studied. The theoretical and experimental interference patterns for cat ($|\alpha_L|=\sqrt{3.5}$) state are presented in Fig. 4(a1) and (a2), respectively, and detailed calculations and realizations are presented in supplementary. The off-axis interferogram is created by an input field and a reference plane wave with a small incident angle. This type of interference illustrates that the two separated wave packets are two genuine Gaussian wave packets. Next, we study the interference of the input HD-SCS (in SLM,G, $|\alpha_L|+|\alpha_L|$) and a rotated state (in SLM,M, $|\alpha_L|e^{i\theta}|\pm\alpha_L|$) in the equator plane of Bloch-sphere (Fig. 4(a3), $|\alpha_L|=\sqrt{1.5}$). This type of conjugate interference can directly evaluate the coherence property of generated HD-SCS. Fig. 4(b) presents the corresponding interference curves of odd and even cat ($|\alpha_L|=\sqrt{3.5}$) states, and the calculated average visibility ($V=P_{\text{Max}}-P_{\text{Min}}/P_{\text{Max}}+P_{\text{Min}}$) are 98.45±0.03% (others in Fig. 4(d), blue bars). The clear interference pattern and high visibility show that the coherence of HD-SCS is tolerable and performable. Furthermore, we measured a group of interference under different transmission losses (Fig. 4(c)). The visibility for attenuations 0, 3, and 9dB are shown in Fig. 4(d) (orange bars). The high visibility illustrates the coherence property of HD-SCS is maintained, and it is robust to the attenuation of channel.

In addition, we evaluate the HD-SCSs by the quantum state tomography with mutually unbiased measurements, which is an exhaustive and reliable methods to describe a quantum state. These MUBs can be generated by a discrete Fourier transformation in a dimension of prime order (see supplementary). Several groups have realized the quantum reconstruction of high-dimensional photonic quantum states from $d=2$ to 6 in the OAM space. Using this reliable strategy, we performed quantum-state reconstruction on the kitten ($|\alpha_L|=\sqrt{0.5}$) and cat ($|\alpha_L|=\sqrt{3.5}$) states for $d=7$ and
17 in the OAM space, respectively. The corresponding 3D-bars and 2D-planforms are shown in Fig. 5(a1)-(a4) and Fig. 5(b1)-(b4). The reconstructed matrices yield similarities of $S=0.9907$ and 0.9533 for kitten and cat states, respectively. The high similarities between them directly indicate that the generations in our systems are reliable and closely similar to the ideal SC state.

Conclusion. Towards generating a controllable macroscopic Schrödinger cat (SC) state, we took the first step to engineer its that lives in Hilbert space of orbital angular momentum (OAM) of photon. In the photonic OAM space, this high-dimensional SC state (HD-SCS) is easy to generate, engineer, and measure, as well as having good coherence, and it provides an important resource to understand quantum superposition. In the foregoing, the scheme proposed for generating HD-SCS has many advantages: i) the HD-SCS are easy to prepare in the laboratory using linear optical elements, and ii) the size of SC states can be expanded by adjusting the acquired phase in the SLM. One problem remaining is the generation efficiency (see supplementary), which needs to be improved. iii) By our scheme, we can directly observe two distinct Gaussian packets for HD-SCS with two opposite-
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