Supertubes in reduced holonomy manifolds

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Abstract

We show that the supertube configurations exist in all supersymmetric type IIA backgrounds which are purely geometrical and which have, at least, one flat direction. In other words, they exist in any spacetime of the form $\mathbb{R}^{1,1} \times \mathcal{M}_8$, with $\mathcal{M}_8$ any of the usual reduced holonomy manifolds. These generalised supertubes preserve 1/4 of the supersymmetries preserved by the choice of the manifold $\mathcal{M}_8$. We also support this picture with the construction of their corresponding family of IIA supergravity backgrounds preserving from 1/4 to 1/32 of the total supercharges.

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1 Introduction and Results

The fact that D-branes couple to background fluxes can allow, under the appropriate circumstances, a collection of D-branes to expand into another brane of higher dimension. Also the inverse process is observed, where higher dimensional D-branes collapse into smaller dimensional ones or even into fundamental strings. Non-supersymmetric examples of such configurations are the expansion of Born-Infeld strings [1], the dielectric branes [2] and the matrix string theory calculations of [3,4]. More recent supersymmetric cases have also been constructed, like the giant gravitons in AdS spaces [5,6]. All these configurations share the handicap that the perturbative quantisation of string theory is still not possible due to the presence of Ramond-Ramond fluxes.

Supertubes [7] are very different from the former cases because they are expanded configurations that live in a completely flat space, with all other background fields turned off. They correspond to a bound state of D0-branes and fundamental strings that expand into a D2 with tubular shape due to the addition of angular momentum. Remarkably, they also preserve 1/4 of the 32 supersymmetries of the Minkowski vacuum, unlike some other similar (but non-supersymmetric) configurations that were constructed in [8]. Furthermore, the simplicity of the background allowed for a perturbative string-theoretical study of the supertube, beyond the probe or the supergravity approximations [9].

The purpose of this paper is to show that it is possible to generalise the construction of the original supertube configurations to other purely geometrical backgrounds, while still preserving some supersymmetry. This generalisation consists on choosing a type IIA background of the form $\mathbb{R}^{1,1} \times \mathcal{M}_8$, with $\mathcal{M}_8$ a curved manifold. Since we do not turn on any other supergravity field, supersymmetry restricts $\mathcal{M}_8$ to be one of the usual manifolds with reduced holonomy [10]:

| $\mathcal{M}_8$          | Fraction of the 32 supersymmetries preserved |
|--------------------------|---------------------------------------------|
| $\mathbb{R} \times CY_2$ | 1/2                                         |
| $CY_2 \times CY_2$       | 1/4                                         |
| $\mathbb{R}^2 \times CY_3$ | 1/4                                      |
| $CY_4$                   | 1/8                                         |
| $\mathbb{R} \times G_2$  | 1/8                                         |
| Spin(7)                  | 1/8                                         |
| Sp(2)                    | 3/8                                         |

We will show that it is possible to supersymmetrically embed the supertube in these backgrounds in such a way that its time and longitudinal directions fill the $\mathbb{R}^{1,1}$ factor, while its compact direction can describe an arbitrary curve $\mathcal{C}$ in $\mathcal{M}_8$.

The problem will be analysed in two different descriptions. In the first one, we will perform a worldvolume approach by considering a D2 probe in these backgrounds with the mentioned embedding and with an electromagnetic worldvolume gauge field corresponding to the threshold bound state of D0/F1. With the knowledge of some general properties of the Killing spinors of the $\mathcal{M}_8$ manifolds, it will be shown, using its $\kappa$-symmetry, that
the probe bosonic effective action is supersymmetric. As in flat space supertubes, the only charges and projections involved correspond to the D0-branes and the fundamental strings, while the D2 ones do not appear anywhere. This is why, in all cases, the preserved amount of supersymmetry will be 1/4 of the fraction already preserved by the choice of background.

Note that, in particular, this allows for configurations preserving a single supercharge, as is shown in one of the examples of this work. In the other example that we present, we exploit the fact that the curve $C$ can now wind around the non-trivial cycles that the $M_8$ manifolds have, and construct a supertube with cylindrical shape $\mathbb{R} \times S^1$, with the $S^1$ wrapping one of the non-trivial $S^2$ cycles of an ALE space. In the absence of D0 and F1 charges, $q_0$ and $q_s$ respectively, the $S^1$ is a collapsed point in one of the poles of the $S^2$. As $|q_0q_s|$ is increased, the $S^1$ slides down towards the equator. Unlike in flat space, here $|q_0q_s|$ is bounded from above and it acquires its maximum value precisely when the $S^1$ is a maximal circle inside the $S^2$.

The second approach will be a spacetime description, where the back-reaction of the system will be taken into account, and we will be able to describe the configuration by means of a supersymmetric solution of type IIA supergravity, the low-energy effective theory of the closed string sector. Such solutions can be obtained from the original ones, found in [11], by simply replacing the 8-dimensional Euclidean space that appears in the metric by $M_8$. We will show that this change is consistent with the supergravity equations of motion as long as the various functions and one-forms that were harmonic in $E_8$ are now harmonic in $M_8$. It will also be shown that the supergravity solution preserves the same amount of supersymmetry that was found by the probe analysis.

Physically, the construction of these generalised supertubes is possible because the cancellation of the gravitational attraction by the angular momentum is a local phenomenon. By choosing the worldvolume electric field $E$ such that $E^2 = 1$, and an arbitrary non-zero magnetic field $B$, the Poynting vector automatically acquires the required value to prevent the collapse at every point of $C$. This remains true even after the replacement of the space where $C$ lives from $E_8$ by a curved $M_8$.

This paper is organised as follows: in section 2 we analyse the system where the D2-supertube probes the $\mathbb{R}^{1,1} \times M_8$, and prove that the effective worldvolume action for the D2 is supersymmetric using the $\kappa$-symmetry. In section 3 we perform the Hamiltonian analysis of the system. We show that the supersymmetric embeddings minimise the energy for given D0 and F1 charges, showing that gravity is locally compensated by the Poynting vector. In section 4 we give to examples in order to clarify and illustrate these constructions. Section 5 is devoted to the supergravity analysis of the generalised supertubes. We prove there the supersymmetry from a spacetime point of view. Conclusions are given in section 6.

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1This is not in contradiction with the fact that the minimal spinors in 2+1 dimensions have 2 independent components since, because of the non-vanishing electromagnetic field, the theory on the worldvolume of the D2 is not Lorentz invariant.
2 Probe worldvolume analysis

In this section we will prove that the curved direction of a supertube can live in any of the usual manifolds with reduced holonomy, while still preserving some amount of supersymmetry. The analysis will be based on the $\kappa$-symmetry properties of the bosonic worldvolume action, and its relation with the supersymmetry transformation of the background fields.

2.1 The setup

As announced, we consider a general IIA background of the form $\mathbb{R}^{1,1} \times M_8$, with $M_8$ a possibly curved manifold. In the absence of fluxes, the requirement that the background preserves some supersymmetry\footnote{In [12], a first attempt to construct supertubes in curved spaces was performed. Their configurations are not supersymmetric because the backgrounds already destroy all supersymmetries.} implies that $M_8$ must admit covariantly constant spinors and, therefore, a holonomy group smaller than SO(8). The classification of such manifolds is well-known [10], and the only possible choices for $M_8$ are shown in the table of the introduction.

Let us write the target space metric as

$$ds^2_{IIA} = -(dx^0)^2 + (dx^1)^2 + e^i e^j \delta_{ij}, \quad e^i = dy^i e^i_j, \quad i, j = 2, 3, ..., 9,$$

(2.1)

where $e^i$ is the vielbein of a Ricci-flat metric on $M_8$. Underlined indices refer to tangent space objects. We will embed the supertube in such a way that its time and longitudinal directions live in $\mathbb{R}^{1,1}$ while its curved direction describes an arbitrary curve $C$ in $M_8$. By naming the D2 worldvolume coordinates $\{\sigma^0, \sigma^1, \sigma^2\}$, such an embedding is determined by

$$x^0 = \sigma^0, \quad x^1 = \sigma^1, \quad y^i = y^i(\sigma^2),$$

(2.2)

where $y^i$ are arbitrary functions of $\sigma^2$. The assignment of $\sigma^0$ and $\sigma^1$, i.e. the fact that $y^i$ is independent of $\sigma^0$ and $\sigma^1$, is a choice of parametrization.\footnote{In this sense, the apparently rotating supertubes considered in [13] are indeed equivalent, through a worldvolume reparametrisation, to the ordinary supertubes in flat space.}

Let us remark that, in general, the curve $C$ will be contractible in $M_8$. As a consequence, due to gravitational self-attraction, the compact direction of the D2 will naturally tend to collapse to a point.

Following [7], we will stabilise the D2 by turning on an electromagnetic flux in its worldvolume

$$F_2 = E \, d\sigma^0 \wedge d\sigma^1 + B \, d\sigma^1 \wedge d\sigma^2,$$

(2.3)

which will provide the necessary centrifugal force to compensate the gravitational attraction. In this paper we will restrict to static configurations.

The effective action of the D2 is the DBI action (the Wess-Zumino term vanishes in our purely geometrical backgrounds),

$$S = \int_{\mathbb{R}^{1,1} \times C} d\sigma^0 d\sigma^1 d\sigma^2 \mathcal{L}_{DBI}, \quad \mathcal{L}_{DBI} = -\Delta \equiv -\sqrt{-\det[g + F]},$$

(2.4)
where \( g \) is the induced metric determined by the embedding \( x^M(\sigma^\mu) \), and \( F_{\mu\nu} \) is the electromagnetic field strength. \( M \) denotes the spacetime components \( 0, 1, \ldots, 9 \), and \( \mu \) labels the worldvolume coordinates \( \mu = 0, 1, 2 \). The \( \kappa \)-symmetry imposes restrictions on the background supersymmetry transformation when only worldvolume bosonic configurations are considered. Basically we get \( \Gamma_\kappa \epsilon = \epsilon \) (see e.g. [14]), where \( \epsilon \) is the background Killing spinor and \( \Gamma_\kappa \) (see e.g. [15]) is a matrix that squares to 1:

\[
d^3\sigma \Gamma_\kappa = \Delta^{-1} [\gamma_3 + \gamma_1 \Gamma_\kappa \wedge F_2]. \tag{2.5}
\]

Here \( \Gamma_\kappa \) is the chirality matrix in ten dimensions (in our conventions it squares to one), and the other definitions are

\[
\begin{align*}
\gamma_3 &= d\sigma^0 \wedge d\sigma^1 \wedge d\sigma^2 \partial_0 x^M \partial_1 x^N \partial_2 x^P e_M e_N e_P \Gamma_{MNP}, \\
\gamma_1 &= d\sigma^\mu \partial_\mu x^M e_M \Gamma_M.
\end{align*}
\tag{2.6}
\]

where \( e_M \) are the vielbeins of the target space and \( \Gamma_M \) are the flat gamma matrices. We are using Greek letters for worldvolume indices and Latin characters for the target space.

We are now ready to see under which circumstances can the configuration (2.2), (2.3) be supersymmetric. This is determined by the condition for \( \kappa \)-symmetry, which becomes

\[
[\Gamma_{01} \gamma_2 + E \gamma_2 \Gamma_\kappa + B \Gamma_2 \Gamma_\kappa - \Delta] \epsilon = 0, \tag{2.7}
\]

where

\[
\Delta^2 = B^2 + y^i y^i (1 - E^2), \quad y^i = y^i e_i, \quad \gamma_2 = y^i \Gamma_i, \quad y^i := \partial_2 y^i. \tag{2.8}
\]

The solutions of (2.7) for \( \epsilon \) are the Killing spinors of the background, determining the remaining supersymmetry.

### 2.2 Proof of worldvolume supersymmetry

In this section we shall prove that the previous configurations always preserve 1/4 of the remaining background supersymmetries preserved by the choice of \( \mathcal{M}_8 \). We will show that the usual supertube projections are necessary and sufficient in all cases except when we do not require that the curve \( C \) is arbitrary and it lies completely within the flat directions that \( \mathcal{M}_8 \) may have. Therefore we first discuss the arbitrary case, and after that, we deal with the special situation.

**Arbitrary Curve:** If we demand that the configuration is supersymmetric for any arbitrary curve in \( \mathcal{M}_8 \), then all the terms in (2.7) that contain the derivatives \( y^i(\sigma^2) \) must vanish independently of those that do not contain them. The vanishing of the first ones (those containing \( \gamma_2 \)) give

\[
\Gamma_{01} \Gamma_\kappa \epsilon = -E \epsilon \quad \implies \quad E^2 = 1, \quad \text{and} \quad \Gamma_{01} \Gamma_\kappa \epsilon = -\text{sign}(E) \epsilon, \tag{2.9}
\]
which signals the presence of fundamental strings in the longitudinal direction of the tube. Now, when $E_2^2 = 1$, then $\Delta = |B|$, and the vanishing of the terms independent of $y^\alpha(\sigma^2)$ in (2.7) give

$$\Gamma_2 \Gamma_\alpha \epsilon = \text{sign}(B) \epsilon,$$

(2.10)

which signals the presence of D0 branes dissolved in the worldvolume of the supertube. Since both projections, (2.9) and (2.10), commute, the configuration will preserve 1/4 of the background supersymmetries as long as they also commute with all the projections imposed by the background itself.

It is easy to prove that this will always be the case. Since the target space is of the form $\mathbb{R}^{1,1} \times M_8$, the only nontrivial conditions that its Killing spinors have to fulfil are

$$\nabla_i \epsilon = \left( \partial_i + \frac{1}{4} w_{ijk} \Gamma_{jk} \right) \epsilon = 0,$$

(2.11)

with all indices only on $M_8$ (which in our ordering, means $2 \leq i \leq 9$). If one prefers, the integrability condition can be written as

$$[\nabla_i, \nabla_j] \epsilon = \frac{1}{4} R_{ij}^{kl} \Gamma_{kl} \epsilon = 0.$$

(2.12)

In either form, all the conditions on the background spinors involve only a sum of terms with two (or none) gamma matrices of $M_8$. It is then clear that such projections will always commute with the F1 and the D0 ones, since they do not involve any gamma matrix of $M_8$.

To complete the proof, one must take into account further possible problems that could be caused by the fact that the projections considered so far are applied to background spinors which are not necessarily constant. To see that this does not change the results, note that (2.11) implies that all the dependence of $\epsilon$ on the $M_8$ coordinates $y^i$ must be of the form

$$\epsilon = M(y) \epsilon_0,$$

(2.13)

with $\epsilon_0$ a constant spinor, and $M(y^i)$ a matrix that involves only products of even number of gamma matrices on $M_8$ (it may well happen that $M(y) = 1$). Now, any projection on $\epsilon$ can be translated to a projection on $\epsilon_0$ since

$$P \epsilon = \epsilon, \quad \text{with} \quad P^2 = \mathbb{1}, \quad \text{Tr} \ P = 0, \quad \Rightarrow$$

$$\tilde{P} \epsilon_0 = \epsilon_0, \quad \text{with} \quad \tilde{P} \equiv M^{-1}(y) P M(y), \quad \tilde{P}^2 = \mathbb{1}, \quad \text{Tr} \ \tilde{P} = 0.$$

(2.14)

The only subtle point here is that, if some of the $\epsilon_0$ have to survive, the product of $M^{-1}(y) P M(y)$ must be a constant matrix\(^4\). But this is always the case for all the projections related to the presence of $M_8$, since we know that such spaces preserve some Killing spinors. Finally, it is also the case for the F1 and D0 projections, since they commute with any even number of gamma matrices on $M_8$.

The conclusion is that, for an arbitrary curve in $M_8$ to preserve supersymmetry, it is necessary and sufficient to impose the F1 and D0 projections. In all cases, it will preserve

\(^4\)Note that it is not necessary that $P$ commutes with $M(y)$.\]
1/4 of the background supersymmetry. We will illustrate this with particular examples in section 3.

**Non-Arbitrary Curve:** If we now give up the restriction that the curve must be arbitrary, we can still show that the F1 and D0 projection are necessary and sufficient, except for those cases in which the curve lies entirely in the flat directions that \( M_8 \) may have. Of course, the former discussion shows that such projections are always sufficient, so we will now study in which cases they are necessary as well.

In order to proceed, we need to prove an intermediate result.

**Lemma:** If the velocity of the curve does not point in a flat direction of \( M_8 \), then the background spinor always satisfies at least one projection like

\[
P \epsilon = Q \epsilon, \quad \text{such that} \quad [P, \gamma_2] = 0, \quad \{Q, \gamma_2\} = 0,
\]

with \( P \) and \( Q \) a non-vanishing sum of terms involving only an even number of gamma matrices, and \( Q \) invertible.

To prove this, we move to a point of the curve that lies in a curved direction of \( M_8 \), i.e. a point where not all components of \( R_{ij}^{ab} \) are zero. We perform a rotation in the tangent space such that the velocity of the curve points only in one of the curved directions, e.g.

\[
y'^a \neq 0, \quad y'^a = 0, \quad a = 2, \ldots, 8, \quad R_{ij}^{a9} \neq 0,
\]

at least one choice of \( i, j \) and \( a \), and where we use the definitions of (2.8). With this choice, \( \gamma_2 \) becomes simply \( \gamma_2 = y'^a \Gamma_a \). Therefore, at least one of the equations in (2.12) can be split in

\[
(R_{ij}^{ab} \Gamma_{ab} + R_{ij}^{a9} \Gamma_{a9}) \epsilon = 0,
\]

with the definitions

\[
P = R_{ij}^{ab} \Gamma_{ab}, \quad Q = -R_{ij}^{a9} \Gamma_{a9}.
\]

The assumption (2.16) implies that \( Q \) is nonzero and invertible, as the square of \( Q \) is a negative definite multiple of the unit matrix. This implies that also \( P \) is non-zero since, otherwise, \( \epsilon \) would have to be zero and this is against the fact that all the listed \( M_8 \) manifolds admit covariantly constant spinors. It is now immediate to check that \( \gamma_2 \) commutes with \( P \) while it anticommutes with \( Q \), which completes the proof.

We can now apply this lemma and rewrite one of the conditions in (2.12) as an equation of the kind (2.15). We then multiply the \( \kappa \)-symmetry condition (2.7) by \( P - Q \). Clearly only the first two terms survive, and we can write

\[
0 = [\Gamma_{01} - E \Gamma_s] (P - Q) \gamma_2 \epsilon = -2 [\Gamma_{01} - E \Gamma_s] \gamma_2 Q \epsilon = -2 \gamma_2 Q [\Gamma_{01} + E \Gamma_s] \epsilon.
\]

Since \((\gamma_2)^2 = y'^a y'^a\) cannot be zero if the curve is not degenerate, we just have to multiply with \( Q^{-1} \gamma_2 \) to find again (2.9). Plugging this back into (2.7) gives the remaining D0 condition (2.10).

Summarising, the usual supertube conditions are always necessary and sufficient except for those cases where the curve is not required to be arbitrary and lives entirely in flat space;
then, they are just sufficient. For example, one could choose $\mathcal{C}$ to be a straight line in one of the $\mathbb{R}$ factors that some of the $\mathcal{M}_8$ have, and take a constant $B$, which would correspond to a planar D2-brane preserving $1/2$ of the background supersymmetry.

3 Hamiltonian analysis

We showed that in order for the supertube configurations (2.2), (2.3) to be supersymmetric we needed $E^2 = 1$, but we found no restriction on the magnetic field $B(\sigma^1, \sigma^2)$. We shall now check that some conditions must hold in order to solve the equations of motion of the Maxwell fields. We will go through the Hamiltonian analysis which will enable us to show that these supertubes saturate a BPS bound which, in turn, implies the second-order Lagrange equations of the submanifold determined by the constraints. We will restrict to time-independent configurations, which we have checked to be compatible with the full equations of motion. The Lagrangian is then given by (2.4)

$$\mathcal{L} = -\Delta = -\sqrt{B^2 + R^2(1 - E^2)}, \quad (3.1)$$

where we have defined $R^2 = y^I y_I^2$ and $R > 0$. To obtain the Hamiltonian we first need the displacement field,

$$\Pi = \frac{\partial \mathcal{L}}{\partial E} = \frac{ER^2}{\sqrt{B^2 + (1 - E^2)R^2}}, \quad (3.2)$$

which can be inverted to give

$$E = \frac{\Pi R}{\sqrt{B^2 + R^2 + \Pi^2}}, \quad \Delta = R \sqrt{\frac{B^2 + R^2}{R^2 + \Pi^2}}. \quad (3.3)$$

The Lagrange equations for $A_0$ and $A_2$ give two constraints

$$\partial_1 \Pi = 0, \quad \partial_1 \left(\frac{B R}{\sqrt{B^2 + R^2}} \sqrt{\frac{R^2 + \Pi^2}{B^2 + R^2}}\right) = 0, \quad (3.4)$$

the first one being the usual Gauss law. Together, they imply that $\partial_1 B = 0$, i.e., the magnetic field can only depend on $\sigma^2$. Finally, the equations for $A_1$ and $y^i$ give, respectively,

$$\partial_2 \left(\frac{B}{R} \sqrt{\frac{R^2 + \Pi^2}{B^2 + R^2}}\right) = 0, \quad \partial_2 \left[2y^i \frac{R^4 - \Pi^2 B^2}{R^2 \sqrt{(R^2 + \Pi^2)(R^2 + B^2)}}\right] = 0. \quad (3.5)$$

The Hamiltonian density is given by

$$\mathcal{H} = E\Pi - \mathcal{L} = \frac{1}{R} \sqrt{(R^2 + \Pi^2)(B^2 + R^2)}. \quad (3.6)$$

In order to obtain a BPS bound [10], we rewrite the square of the Hamiltonian density as

$$\mathcal{H}^2 = (\Pi \pm B)^2 + \left(\frac{\Pi B}{R} \mp R\right)^2, \quad (3.7)$$
from which we obtain the local inequality

\[ \mathcal{H} \geq |\Pi \pm B|, \]  

(3.8)

which can be saturated only if

\[ R^2 = y'^2 y'' = \pm \Pi B \quad \Leftrightarrow \quad E^2 = 1. \]  

(3.9)

It can be checked that the configurations saturating this bound satisfy the remaining equations of motion (3.5).

Note that the Poynting vector generated by the electromagnetic field is always tangent to the curve \( C \) and its modulus is precisely \(|\Pi B|\). We can then use exactly the same arguments as in [9]. Equation (3.9) tells us that, once we set \( E^2 = 1 \), and regardless of the value of \( B(\sigma^2) \), the Poynting vector is automatically adjusted to provide the required centripetal force that compensates the gravitational attraction at every point of \( C \). The only difference with respect to the original supertubes in flat space is that the curvature of the background is taken into account in (3.9), through the explicit dependence of \( R^2 \) on the metric of \( \mathcal{M}_8 \).

Finally, the integrated version of the BPS bound (3.8) is

\[ \tau \geq |q_0 \pm q_s|, \quad \text{with} \quad \tau \equiv \int_C d\sigma^2 \mathcal{H}, \quad q_0 \equiv \int_C d\sigma^2 B, \quad q_s \equiv \int_C d\sigma^2 \Pi. \]  

(3.10)

and the normalisation \( 0 \leq \sigma^2 < 1 \). Similarly, the integrated bound is saturated when

\[ L(C) = \int_C d\sigma^2 \sqrt{g_{22}} = \int_C d\sigma^2 \sqrt{y'^2 y''} = \int_C d\sigma^2 \sqrt{|\Pi B|} = \sqrt{|q_s q_0|}, \]  

(3.11)

where \( L(C) \) is precisely the proper length of the curve \( C \), and the last equality is only valid when both \( \Pi \) and \( B \) are constant, as will be the case in our examples.

4 Examples

After having discussed the general construction of supertubes in reduced holonomy manifolds, we shall now present two examples in order to illustrate some of their physical features.

4.1 Supertubes in ALE spaces: 4 supercharges

Let us choose \( \mathcal{M}_8 = \mathbb{R}^4 \times CY_2 \), i.e. the full model being \( \mathbb{R}^{1,5} \times CY_2 \). We take the \( CY_2 \) to be an ALE space provided with a multi-Eguchi–Hanson metric [17]

\[ d\tilde{s}_4^2 = V^{-1}(\tilde{y})d\tilde{y} \cdot d\tilde{y} + V(\tilde{y}) \left( d\psi + \tilde{A} \cdot d\tilde{y} \right)^2, \]

\[ V^{-1}(\tilde{y}) = \sum_{r=1}^{N} \frac{Q}{|\tilde{y} - \tilde{y}_r|}, \quad \tilde{\nabla} \times \tilde{A} = \tilde{\nabla} V^{-1}(\tilde{y}), \]  

(4.1)
with $\vec{y} \in \mathbb{R}^3$. These metrics describe a $U(1)$ fibration over $\mathbb{R}^3$, the circles being parametrized by $\psi \in [0, 1]$. They present $N$ removable bolt singularities at the points $\vec{y}_r$, where the $U(1)$ fibres contract to a point. Therefore, a segment connecting any two such points, together with the fibre, form (topologically) an $S^2$. For simplicity, we will just consider the two-monopoles case which, without loss of generality, can be placed at $\vec{y} = \vec{0}$ and $\vec{y} = (0, 0, b)$. Therefore, the complete IIA background is

$$ds^2_{IIA} = -(dx^0)^2 + (dx^1)^2 + \ldots + (dx^5)^2 + ds^2_{(4)},$$

with

$$V^{-1}(\vec{y}) = \frac{Q}{|\vec{y}|} + \frac{Q}{|\vec{y} - (0, 0, b)|}.$$  

Let us embed the D2 supertube in a way such that its longitudinal direction lies in $\mathbb{R}^5$ while its compact one wraps $S^1$ inside the $S^2$ that connects the two monopoles. More explicitly,

$$X^0 = \sigma^0, \quad X^1 = \sigma^1, \quad \psi = \sigma^2, \quad y^3 = \text{const.,} \quad y^1 = y^2 = 0.$$  

Since any $S^1$ is contractible inside an $S^2$, the curved part would tend collapse to the nearest pole, located at $y^3 = 0$ or $y^3 = b$. As in flat space, we therefore need to turn on a worldvolume flux as in (2.3), with $E$ and $B$ constant for the moment.

According to our general discussion, this configuration should preserve 1/4 of the 16 background supercharges already preserved by the ALE space. In this case, the $\kappa$-symmetry equation is simply

$$\left( \Gamma_{01\psi} + ET_{\psi} \Gamma_\star + B \Gamma_\star \Gamma_\star - \Delta \right) \epsilon = 0,$$  

where $\epsilon$ are the Killing spinors of the background (4.2). They can easily be computed and shown to be just constant spinors subject to the projection

$$\Gamma_{y^1 y^2 y^3} \epsilon = -\epsilon.$$  

Then, the $\kappa$-symmetry equation can be solved by requiring (2.9) and (2.10), which involve the usual D0/F1 projections of the supertube. Since they commute with (4.6), the configuration preserves a total of 1/8 of the 32 supercharges.
It is interesting to see what are the consequences of having $E^2 = 1$ for this case. Note that, from our general Hamiltonian analysis, we saw that, for fixed $D_0$ and $F_1$ charges, the energy is minimised for $E^2 = 1$. When applied to the present configuration, (3.11) reads

$$V(y^3) = |q_0 q_s|.$$  

which determines $y^3$, and therefore selects the position of the $S^1$ inside the $S^2$ that is compatible with supersymmetry. Since $V(y^3)$ is invariant under $y^3 \leftrightarrow (b - y^3)$, the solutions always come in mirror pairs with respect to the equator of the $S^2$. The explicit solutions are indeed

$$y^3 = \frac{b}{2} \left( 1 \pm \sqrt{1 - \frac{4Q}{b} |q_0 q_s|} \right).$$  

Note that a solution exists as long as the product of the charges is bounded from above to

$$|q_0 q_s| \leq \frac{b}{4Q}.$$  

The point is that this will always happen due to the fact that, contrary to the flat space case, the $S^1$ cannot grow arbitrarily within the $S^2$. As a consequence, the angular momentum acquires its maximum value when the $S^1$ is precisely in the equator. To see it more explicitly, setting $E^2 = 1$ and computing $q_0$ and $q_s$ for our configuration gives

$$|q_0 q_s| = V(y^3) \leq V(y^3 \to \frac{b}{2}) = \frac{b}{4Q},$$

which guarantees that (4.9) is always satisfied.

Finally, note that we could have perfectly chosen, for instance, a more sophisticated embedding in which $y^3$ was not constant. This would be the analogue of taking a non-constant radius in the original flat space supertube. Again, by the general analysis of the previous sections, this would require the Poynting vector to vary in order to locally compensate for the gravitational attraction everywhere, and no further supersymmetry would be broken.

### 4.2 Supertubes in CY$_4$ spaces: 1 supercharge

The purpose of the next example is to show how one can reach a configuration with one single surviving supercharge in a concrete example. One could take any of the $1/8$-preserving backgrounds of the $\mathcal{M}$ Table. Many metrics for these spaces have been recently found in the context of supergravity duals of non-maximally supersymmetric field theories. Let us take the CY$_4$ that was found in [18,19] since the Killing spinors have been already calculated explicitly [20]. This space is a $C^2$ bundle over $S^2 \times S^2$, and the metric is

$$ds^2_{\text{(CY}_4\text{)}} = A(r) \left[d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2 + d\theta_2^2 + \sin^2 \theta_2 d\phi_2^2 \right] + U^{-1}dr^2 + \frac{r^2}{4} \left(d\theta^2 + \sin^2 \theta d\phi^2 \right) + \frac{1}{4} U r^2 \left(d\psi + \cos \theta d\phi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2 \right)^2,$$  

(4.11)
where

\[ A(r) = \frac{3}{2} (r^2 + l^2), \quad U(r) = \frac{3r^4 + 8l^2r^2 + 6l^4}{6(r^2 + l^2)}, \quad C(r) = \frac{1}{4} U r^2. \]  \hspace{1cm} (4.12)

By writing the complete IIA background metric as

\[ ds^2_{IIA} = -(dx^0)^2 + (dx^1)^2 + ds^2_{(CY_4)}, \]  \hspace{1cm} (4.13)

and using the obvious vielbeins, with the order

\[ 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad \theta_1 \quad \theta_2 \quad \phi_2 \quad \phi_1 \quad r \quad \theta \quad \phi \quad \psi \]  \hspace{1cm} (4.14)

the corresponding Killing spinors are

\[ \epsilon = e^{-\frac{1}{2} \Gamma_7 8} \epsilon_0, \]  \hspace{1cm} (4.15)

with \( \epsilon_0 \) a constant spinor subject to

\[ \Gamma_{25} \epsilon_0 = \Gamma_{34} \epsilon_0, \quad \Gamma_{25} \epsilon_0 = \Gamma_{78} \epsilon_0, \quad \Gamma_{67} \epsilon_0 = \Gamma_{98} \epsilon_0. \]  \hspace{1cm} (4.16)

To analyse \( \kappa \)-symmetry, let us take the compact part of the supertube to lie along, say, the \( \phi_1 \) direction, while setting to constant the rest of the \( CY_4 \) coordinates. As in the previous example, this would have the interpretation of an \( S^1 \) embedding in one of the two \( S^2 \) in the base of the \( CY_4 \). Imposing \( \kappa \)-symmetry:

\[ (\Gamma_{015} + E \Gamma_5 \Gamma_+ + B \Gamma_2 \Gamma_+ - \Delta) \epsilon = 0. \]  \hspace{1cm} (4.17)

Now, the first projection of (4.16) happens to anticommute with the \( \gamma_2 \) defined in (2.8)

\[ \gamma_2 = y^i e_i \Gamma_+ = A^\frac{1}{2}(r) \sin \theta_1 \Gamma_5. \]  \hspace{1cm} (4.18)

In other words, this just illustrates a particular case of (2.15) for which the direction 5 plays the role of 9, and for which \( P = \Gamma_{34} \) and \( Q = \Gamma_{25} \). We can now follow the steps in section 2.2 and multiply (4.17) by \( P - Q \). This yields again the usual supertube conditions (2.9) and (2.10).

Since all the gamma matrices appearing in (4.16), (2.9) and (2.10) commute, square to one and are traceless, the configuration preserves only one of the 32 supercharges of the theory. Of course, this is not in contradiction with the fact that the minimal spinors in 2+1 dimensions have 2 components, since the field theory on the worldvolume of the D2 is not Lorentz invariant because of the non-vanishing electromagnetic field.
5 Supergravity analysis

In this section we construct the supergravity family of solutions that correspond to all the configurations studied before. We start our work with a generalisation of the ansatz used in [11, 9] to find the original solutions. Our analysis is performed in eleven dimensional supergravity, mainly because its field content is much simpler than in IIA supergravity. Once the eleven-dimensional solution is found, we reduce back to ten dimensions, obtaining our generalised supertube configurations.

The first step in finding the solutions is to look for supergravity configurations with the isometries and supersymmetries suggested by the worldvolume analysis of the previous sections. Then, we will turn to the supergravity field equations to find the constraints that the functions of our ansatz have to satisfy in order that our configurations correspond to minima of the eleventh dimensional action. Finally, we choose the correct behaviour for these functions so that they correctly describe the supertubes once the reduction to ten dimensions is carried on.

5.1 Supersymmetry analysis

Our starting point is the supertube ansatz of [11, 9]

\begin{align*}
      ds_{10}^2 &= -U^{-1}V^{-1/2} (dt - A)^2 + U^{-1}V^{1/2} dx^2 + V^{1/2} \delta_{ij}dy^i dy^j, \\
      B_2 &= -U^{-1} (dt - A) \wedge dx + dt \wedge dx, \\
      C_1 &= -V^{-1} (dt - A) + dt, \\
      C_3 &= -U^{-1} dt \wedge dx \wedge A, \\
      e^\phi &= U^{-1/2}V^{3/4},
\end{align*}

where the Euclidean space (E_8) coordinates are labelled by y^i, with i, j, \cdots = (2, \ldots, 9), V = 1 + K, A = A_i dy^i and B_2 and C_p are respectively, the Neveu-Schwarz and Ramond-Ramond potentials. V, U, A_i depend only on the E_8 coordinates.

To up-lift this ansatz, we use the normal Kaluza-Klein form of the eleven dimensional metric and three-form,

\begin{align*}
      ds_{11}^2 &= e^{-2\phi/3} ds_{10}^2 + e^{4\phi/3}(dz + C_1)^2, \\
      N_3 &= C_3 + B_2 \wedge dz,
\end{align*}

where N_3 is the eleventh dimensional three-form. The convention for curved indices is M = (\mu; i) = (t, z, x; 2, 3, \ldots, 9) and for flat ones A = (\alpha; a) = (t, z, x; 2, 3, \ldots, 9). The explicit form of the eleven-dimensional metric is given by,

\begin{align*}
      ds_{11}^2 &= U^{-2/3} [-dt^2 + dz^2 + K(dt + dz)^2 + 2(dt + dz)A + dx^2] + U^{1/3}ds_8^2, \\
      F_4 &= dt \wedge d(U^{-1}) \wedge dx \wedge dz - (dt + dz) \wedge dx \wedge d(U^{-1}A),
\end{align*}

where F_4 = dN_3. This background is a solution of the equations of motion in eleven dimensions derived from the action

\begin{align*}
      S_{11d} = \int \left[ R \ast 1 - \frac{1}{2} F_4 \wedge \ast F_4 + \frac{1}{3} F_4 \wedge F_4 \wedge N_3 \right],
\end{align*}

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when the two functions \( K \) and \( U \), as well as the one-form \( A_1 \), are harmonic in \( \mathbb{E}_8 \), i.e.,

\[
(d \ast_8 d) U = 0, \quad (d \ast_8 d) K = 0, \quad (d \ast_8 d) A_1 = 0,
\]

where \( \ast_8 \) is the Hodge dual with respect to the Euclidean flat metric on \( \mathbb{E}_8 \). It describes a background with an M2 brane along the directions \( \{ t, z, x \} \), together with a wave traveling along \( z \), and angular momentum along \( \mathbb{E}_8 \) provided by \( A_1 \).

Next, we generalise the ansatz above by replacing \( \mathbb{E}_8 \) by one of the eight dimensional \( \mathcal{M}_8 \) manifolds of the table, and by allowing \( K \), \( U \) and \( A_1 \) to have an arbitrary dependence on the \( \mathcal{M}_8 \) coordinates \( y^i \). We therefore replace the previously flat metric on \( \mathbb{E}_8 \) by a reduced holonomy metric on \( \mathcal{M}_8 \), with vielbeins \( \tilde{e}^a \). Hence, in (5.3), we replace

\[
U_1^3 \delta_{ij} dy^i dy^j \rightarrow U_1^3 \delta_{ab} \tilde{e}^a \tilde{e}^b.
\]

We use a null base of the cotangent space, defined by

\[
e^+ = -U^{-2/3}(dt + dz), \quad e^- = \frac{1}{2}(dt - dz) - \frac{K}{2}(dt + dz) - A,
\]

\[
e^x = U^{-1/3} dx, \quad e^a = U^{1/6} \tilde{e}^a.
\]

This brings the metric and \( F_4 \) into the form

\[
ds_{11}^2 = 2e^+ e^- + e^x e^x + \delta_{ab} e^a e^b, \quad F_4 = -U^{-1} dU \wedge e^x \wedge e^+ \wedge e^- - dA \wedge e^x \wedge e^+.
\]

As customary, the torsion-less condition can be used to determine the spin connection 1-form \( \omega_{AB} \). In our null base, the only non-zero components are

\[
\omega_{+ -} = -\frac{U_a}{3U} \tilde{e}^a, \quad \omega_{+ a} = \frac{1}{2} U^{1/2} \tilde{K}_a e^+ - \frac{1}{2} a_{ab} \tilde{e}^a, \quad \omega_{- a} = -\frac{U_a}{3U} e^+, \quad \omega_{xa} = -\frac{U_a}{6U} e^a - \frac{U_b}{6U} e^b + \tilde{\omega}_{ab} + \frac{1}{2} a_{ab} \tilde{e}^+,
\]

were we have defined various tensor quantities through the relations

\[
dU = U_a e^a, \quad dK = \tilde{K}_a e^a, \quad dA = \frac{1}{2} a_{ab} e^a \wedge e^b,
\]

and \( \tilde{\omega}^{bc} \) are the spin connection one-forms corresponding to \( \tilde{e}^a \), i.e. \( d\tilde{e}^a + \tilde{\omega}^a_{\ bc} e^b = 0 \).

We now want to see under which circumstances our backgrounds preserve some supersymmetry. Since we are in a bosonic background i.e. all the fermions are set to zero, we just need to ensure that the variation of the gravitino vanishes when evaluated on our configurations. In other words, supersymmetry is preserved if there exist nonzero background spinors \( \epsilon \) such that

\[
\left( \partial_A + \frac{1}{4} \omega_A^{BC} \Gamma_{BC} - \frac{1}{288} \Gamma_A^{BCDE} F_{BCDE} + \frac{1}{36} F_{ABCD} \Gamma^{BCD} \right) \epsilon = 0.
\]

\(^5\)For the components of \( p \)-forms we use the notations of [21].
We will try an ansatz such that the spinor depends only on the coordinates on $M_8$. It is straightforward to write down the eleven equations (5.11) for each value of $A = \{+,-,x,a\}$. The equation for $A = x$ is

$$\frac{U_a}{6U} \Gamma_a (\Gamma_x - \Gamma_{+ -}) \epsilon - \frac{a_{ab}}{12} \Gamma_{ab} \Gamma_- \epsilon = 0.$$  

(5.12)

Assuming that $a_{ab}$ and $\alpha_a$ are arbitrary and independent we find

$$\Gamma_- \epsilon = 0, \quad \text{and} \quad \Gamma_x \epsilon = -\epsilon.$$  

(5.13)

Using these projections, it is a straightforward algebraic work to see that the equation for $A = +$ and $A = -$ are automatically satisfied. Finally, the equations for $A = a$ simplify to

$$\nabla_i \epsilon \equiv \left( \partial_i + \frac{1}{4} \bar{\omega}_i^{\,bc} \Gamma_{bc} \right) \epsilon = 0.$$  

(5.14)

By the same arguments as in the previous sections, the projections (5.13) preserve 1/4 of the 32 real supercharges. On the other hand, (5.14) is just the statement that $M_8$ must admit covariantly constant spinors. Depending on the choice of $M_8$, the whole 11d background will preserve the expected total number of supersymmetries that we indicated in the table written in the introduction.

To reduce back to IIA supergravity, we first go to another flat basis

$$e^+ = -U^{-1/3} V^{-1/2} \left( e^0 + e^z \right), \quad e^- = \frac{1}{2} U^{1/3} V^{1/2} \left( e^0 - e^z \right),$$  

(5.15)

which implies that

$$\Gamma_- = U^{-1/3} V^{-1/2} \left( \Gamma_0 - \Gamma_z \right).$$  

(5.16)

We reduce along $z$, i.e. replace $\Gamma_z$ by $\Gamma_*$. The projections (5.13) become the usual D0/F1 projections, with the fundamental strings along the $x$-axis.

$$\Gamma_0 \Gamma_* \epsilon = -\epsilon, \quad \text{and} \quad \epsilon = -\Gamma_x \epsilon = \Gamma_{x0} \Gamma_* \epsilon.$$  

(5.17)

### 5.2 Equations of motion

Now that we have proved that the correct supersymmetry is preserved (matching the world-volume analysis), we proceed to determine the equations that $U$, $K$ and $A_1$ have to satisfy in order that our configurations solve the field equations of eleven-dimensional supergravity. Instead of checking each of the equations of motion, we use the analysis of [22] that is based on the integrability condition derived from the supersymmetry variation of the gravitino (5.11). The result of this analysis is that when at least one supersymmetry is preserved, and the Killing vector $K_\mu \equiv \bar{\epsilon} \Gamma_\mu \epsilon$ is null, all of the second order equations of motion are automatically satisfied, except for

1. The equation of motion for $F_4$, 

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2. The Einstein equation \( E_{++} = T_{++} \), where \( E_{++} \) and \( T_{++} \) are the Einstein and stress-energy tensors along the components ++ in a base where \( \mathcal{K}_\mu = \delta_\mu^+ \mathcal{K}_+ \). Let us explain why the above statement is correct. The integrability conditions give no information about the field equation for the matter content, therefore the equation of motion for \( F_4 \) has to be verified by hand. Also, in most cases all of the Einstein equations are automatically implied by the existence of a non-trivial solution of (5.11).

With (5.13) and in the base where the metric takes the form (5.8), and thus \( \Gamma_\mu \Gamma_- + \Gamma_- \Gamma_+ = 2 \), we have

\[
\mathcal{K}_\mu = \epsilon \Gamma_\mu \epsilon = \frac{1}{2} \epsilon \Gamma_\mu \Gamma_- \Gamma_+ \epsilon .
\]  
(5.18)

This vanishes for all \( \mu \) except \( \mu = + \), implying that our configuration falls into the classification of those backgrounds that admit a null Killing spinor and as a consequence the associated Einstein equations escape the analysis. We thus have to check the two items mentioned above.

Let us start with the equation for \( F_4 \), which is

\[
d \ast F_4 + F_4 \wedge F_4 = 0.
\]  
(5.19)

Using the fact that the Hodge dual of a p-form with respect to \( e^a \) is related to the one with respect to \( \tilde{e}^a \) by

\[
\ast_8 C_p = U^{(4-p)/3} \tilde{\ast}_8 C_p ,
\]  
(5.20)

where

\[
C_p = \frac{1}{p!} C_{a_1...a_p} \tilde{e}^{a_1} \wedge ... \wedge \tilde{e}^{a_p} \rightarrow \tilde{\ast}_8 C_p = \frac{1}{p!(8-p)!} C_{a_1...a_p} \varepsilon^{a_1...a_s} \varepsilon^{a_{p+1}} \wedge ... \wedge \varepsilon^{a_s} ,
\]  
(5.21)

it is easy to see that (5.19) becomes

\[
0 = (d \tilde{\ast}_8 d) U + (d t + d z) \wedge (d \tilde{\ast}_8 d) A .
\]  
(5.22)

This implies that \( U \) and \( A_I \) must be harmonic with respect to the metric of \( \mathcal{M}_8 \), i.e.,

\[
(d \tilde{\ast}_8 d) U = 0 , \quad (d \tilde{\ast}_8 d) A_I = 0 .
\]  
(5.23)

Finally, using (5.8) and (5.9), one can explicitly compute the \{+++\} components of the Einstein and stress-energy tensors, and obtain

\[
E_{++} = R_{++} = -\frac{1}{2} U^{1/3} (\ast_8 d \tilde{\ast}_8 d) K + \frac{1}{2} \ast_8 (d A \wedge \ast_8 d A) ,
\]

\[
T_{++} = \frac{1}{12} F_{+ABC} F_+^{ABC} = \frac{1}{2} \ast_8 (d A \wedge \ast_8 d A) ,
\]  
(5.24)

Therefore, the last non-trivial equation of motion tells us that also \( K \) must be harmonic on \( \mathcal{M}_8 \),

\[
(d \tilde{\ast}_8 d) K = 0 .
\]  
(5.25)
5.3 Constructing the supertube

In order to construct the supergravity solutions that properly describe supertubes in reduced holonomy manifolds, we reduce our eleven-dimensional background to a ten-dimensional background of type IIA supergravity, using (5.2) again. We obtain (5.1) with the replacement (5.6), and the constraints (5.23) and (5.25). At this point we have to choose $U$, $K$ and $A_1$ so that they describe a D2-brane with worldvolume $\mathbb{R}^{1,1} \times \mathcal{C}$, with $\mathcal{C}$ an arbitrary curve in $M_8$. As it was done in [11, 9], one should couple IIA supergravity to a source with support along $\mathbb{R}^{1,1} \times \mathcal{C}$, and solve the $M_8$ Laplace equations (5.23) and (5.25) with such a source term in the right hand sides. If this has to correspond to the picture of D0/F1 bound states expanded into a D2 by rotation, the boundary conditions of the Laplace equations must be such that the solution carries the right conserved charges. In the appropriate units,

$$q_0 = \int_{\partial M_8} \tilde{s}_8 dC_1, \quad q_s = \int_{\partial M_8} \tilde{s}_8 dB_2, \quad A_1 \partial M_8 \rightarrow L_{ij} y^i dy^j. \quad (5.26)$$

Here, as in [11, 9], $L_{ij}$ would have to match with the angular momentum carried by the electromagnetic field that we considered in the worldvolume approach.

The Laplace problem in a general manifold can be very complicated and, in most cases, it cannot be solved in terms of ordinary functions. We will not intend to do so, but rather we will just claim that, once $U$, $K$ and $A_1$ have been determined, they can be plugged back into (5.1), with (5.6), and the background will describe the configurations that we have been discussing in this paper. It will have the expected isometries, supersymmetries and conserved charges.

6 Conclusions

We have shown that the expansion of the D0/F1 system into a D2 can happen supersymmetrically in all the backgrounds of the form $\mathbb{R}^{1,1} \times M_8$, with $M_8$ the manifolds of the table. We have shown this in the worldvolume as well as in the supergravity setting. By a Hamiltonian analysis, we connected the result to a BPS bound on charges that are also well defined in the curved background. We remark that our research is different from [23], where it was shown that the supertube itself, after some T-dualities, can be described by a special Lorentzian-holonomy manifold in eleven dimensions.

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