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A novel Covid-19 model with fractional differential operators with singular and non-singular kernels: Analysis and numerical scheme based on Newton polynomial

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Abstract To capture more complexities associated to the spread of Covid-19 within a given population, we considered a system of nine differential equations that include a class of susceptible, 5 sub-classes of infected population, recovered, death and vaccine. The mathematical model was suggested with a lockdown function such that after the lockdown, the function follows a fading memory rate, a concept that is justified by the effect of social distancing that suggests, susceptible class should stay away from infected objects and humans. We presented a detailed analysis that includes reproductive number and stability analysis. Also, we introduced the concept of fractional Lyapunov function for Caputo, Caputo-Fabrizio and the Atangana-Baleanu fractional derivatives. We established the sign of the fractional Lyapunov function in all cases. Additionally we proved that, if the fractional order is one, we recover the results Lyapunov for the model with classical differential operators. With the nonlinearity of the differential equations depicting the complexities of the Covid-19 spread especially the cases with nonlocal operators, and due to the failure of existing analytical methods to provide exact solutions to the system, we employed a numerical method based on the Newton polynomial to provide exact solutions to the system, we employed a numerical method based on the Newton polynomial to derive numerical solutions for all cases and numerical simulations are provided for different values of fractional orders and fractal dimensions. Collected data from Turkey case for a period of 90 days were compared with the suggested mathematical model with Atangana-Baleanu fractional derivative and an agreement was reached for alpha 1.009.

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1. Introduction

Since December 2019, a date when the world first witnessed the breakout of the novel Covid-19 that started in Wuhan, a China
city, mathematicians as well as many others researchers in many academic disciplines have focused their attention in modelling a dynamic spread of Covid-19 [1–18]. The literature is now full of new mathematical models, of course all these mathematical models have their limitations, advantages and disadvantages. It is important to note that, there is no yet any mathematical model suggested in the literature that have taking into account all complexities connected the dynamic spread of this fatal disease. It is worth noting that, to really replicate the observed facts into mathematical model, a clear translation from real world situation to mathematical models should be performed accurately. However, in the case of Covid-19, there are many uncertainties around the spread of the fatal disease. For example, it was suggested that the spread is not high in hot places, however, in center African countries, including Cameroon and Nigeria, we have seen a spread of Covid-19, many other misunderstandings of the spread dynamic can be named. With all these uncertainties, one should expect that no suggested mathematical model will be sufficient enough to replicate accurately the spread of Covid-19 among mankind, thus, many new mathematical models are welcome. Nevertheless, since mathematical models are the tools used by mankind for the moment to foresee the spread of the fatal Covid-19 among mankind, a suggested mathematical model should include as many information as possible. As for example, very recently, a complex mathematical model comprising nine components was suggested by Atangana and Seda [3]. In their model, they considered a total population that can be divided in 9 classes including: \( S(t) \) is the class of individuals that are susceptible to contact Covid-19 at time \( t \); \( I(t) \) is the class of individuals that are susceptible to contacted Covid-19, but have no symptoms and have not been tested. \( I_1(t) \) is the class of individuals that have some symptoms but not tested yet. \( I_2(t) \) is the class of individuals that have contacted Covid-19, have been tested positive, but no symptoms. \( I_R(t) \) is the class of individuals that have contacted Covid19, have been tested positive and have symptoms. \( R(t) \) is the class of individuals that have contacted Covid-19 and one is critical condition. \( R(t) \) is the class of recovered individuals at time \( t \). \( D(t) \) is the number of death at time \( t \); \( V(t) \) is the class of individuals that have been vaccination, while their model includes parameters, they assumed no transmission from class of individuals that have contacted Covid-19 and one is critical condition to susceptible class. In this work, we assume that there is transmission between these two classes. Additionally, we will introduce the concept of fractional Lyapunov function.

Many mathematical models about Covid-19 spread were investigated to understand, analyze and interpret the dynamics spread of the disease. In [8], Ghanbari has aimed to predict the number of infected individuals in the second wave of Covid-19 spread in Iran. Mishra et. al. have considered a mathematical Covid-19 model which consist of susceptible, exposed, infected, asymptotic, quarantine/isolation and recovered classes in [12]. Alkahtani and Alzaid presented a detailed investigation of stability and numerical simulations for a mathematical model depicting the spread of Covid-19 epidemic in Italy in [15]. The authors of [16] have considered a fuzzy mathematical model of corona virus with Atangana-Baleanu fractional derivative. In [17], Ullah and Khan dealt with a mathematical model to analyze the transmission dynamics of the Covid-19 pandemic in Pakistan. For this model, they give a detailed analysis including stability analysis, optimal control for this model.

The organization of this paper is as follows. In Section 2, some definitions of differential and integral operators are presented. In Section 3, the comprehensive mathematical model presented in [5] has been modified with the addition of the lockdown function. In Section 4, the equilibrium points for both disease-free and endemic are obtained. In Section 5, we calculate the reproductive number by using next generation technique. In Section 6, the global asymptotic stability for disease-free equilibrium is presented considering the Lyapunov function for classical differentiation and fractional derivatives. In Section 7, local and global stability of the endemic equilibrium is investigated using Lyapunov function with classical and fractional derivatives. In Section 8, we present error analysis for numerical method based on the Newton polynomial interpolation [22] and solve our model by this method. In Section 9, the numerical simulations are performed for the considered model for different values of fractional order and fractal dimension. In Section 10, we compare the numerical solution of the mathematical model with experimental data in Turkey to show efficiency of the model where is used Atangana-Baleanu fractional derivative.

2. Differential and integral operators with singular and non-singular kernels

To accommodate researchers that are not acquainted to the concept of non-local differential operators with singular and non-singular kernels, also, to differential and integral operators with fractal dimension and fractional orders, we present in this section, some definitions of differential and integral operators starting with Caputo fractional derivative [26–28]

\[ 0^C D^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{d}{dt} f(\tau)(t-\tau)^{-\alpha} d\tau, \]

Caputo-Fabrizio fractional derivative

\[ 0^CF D^\alpha f(t) = \frac{M(z)}{1-z} \int_0^t \frac{d}{dt} f(\tau) \exp \left[ -\frac{z}{1-z} (t-\tau) \right] d\tau, \]

Atangana-Baleanu fractional derivative

\[ 0^{ABC} D^\alpha f(t) = \frac{AB(z)}{1-z} \int_0^t \frac{d}{dt} f(\tau) E_{\alpha} \left[ -\frac{z}{1-z} (t-\tau)^\alpha \right] d\tau. \]

The fractal fractional derivative with power-law kernel, exponential decay and Mittag-Leffler kernel are given by:

\[ 0^{FFP} D^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t f(\tau)(t-\tau)^{-\alpha} d\tau, \]

\[ 0^{FPP} D^\alpha f(t) = \frac{M(z)}{1-z} \int_0^t f(\tau) \exp \left[ -\frac{z}{1-z} (t-\tau) \right] d\tau, \]

\[ 0^{FFM} D^\alpha f(t) = \frac{AB(z)}{1-z} \frac{d}{dt} \int_0^t f(\tau) E_{\alpha} \left[ -\frac{z}{1-z} (t-\tau)^\alpha \right] d\tau, \]

where

\[ \frac{df(t)}{d\beta} = \lim_{\epsilon \to 0} \frac{f(t+\epsilon) - f(t)}{\epsilon^{\beta}}, \]

The fractal-fractional integral with power-law, exponential decay and Mittag-Leffler kernel are as below;
\[ \frac{d}{dt} T^\beta f(t) = \frac{1}{\Gamma(1-\beta)} \int_0^t (t-\tau)^{1-\beta} f(\tau) d\tau, \]
\[ \frac{d}{dt} S^\beta f(t) = \frac{1}{\Gamma(1-\beta)} \int_0^t (t-\tau)^{1-\beta} f(\tau) d\tau, \]
\[ \frac{d}{dt} S_{FM}^\beta f(t) = \frac{1-2}{AB(z)} \Gamma(1-\beta) \int_0^t (t-\tau)^{1-\beta} f(\tau) d\tau. \]

### 3. Mathematical model

Very recently, Atangana and Seda suggested a mathematical model to depict the spread of Covid-19 in a given population. Their model considered a total population of nine classes, which takes into account susceptible, five sub-classes of infected populations, recovered, deaths and vaccinated classes. However, in their model, they assumed that the class of individuals that have contacted and one is critical condition were not in contact with susceptible. This assumption could be argued as there is interaction between these classes and medical staffs, thus, a possibility of transmission from these classes to susceptible. This assumption could be taken into account susceptible, five sub-classes of infected classes and medical staffs, thus, a possibility of transmission from these classes to susceptible belonging to medical staffs. In this section, we modified their model by introducing transmission between the class of individuals that have contacted and one is critical condition with susceptible.

\[
\begin{align*}
\dot{S} &= \Lambda - \delta(t)(\sigma I - w(\beta R + \gamma I + \delta I + \xi I) + \gamma + \mu) S - (\varepsilon + \zeta + \lambda + \mu) I \\
\dot{I} &= \delta(t)(\sigma I - w(\beta R + \gamma I + \delta I + \xi I) + \gamma + \mu) S - (\varepsilon + \zeta + \lambda + \mu) I + \gamma I \\
\dot{R} &= \varepsilon I + \rho I - (\sigma + \tau + \mu) I \\
\dot{V} &= \gamma I + \rho I - (\sigma + \tau + \mu) I - (\Phi + \mu) V.
\end{align*}
\]

Here, \( \delta(t) \) is the contact rate of \( I(t) \) and \( S(t) \), and we take as \( \delta(t) = 1 \). For example, medical staff that are in charge of the patients of class \( I(t) \). We consider \( \delta(t) \) to be function of time as the containment rule become law at time \( t_0 \) thus, according to [21], we can take

\[ \delta(t) = \begin{cases} 
\delta_0 & \text{for } t \leq t_0 \\
\delta_0 (1 - \varepsilon) e^{-\frac{t_0}{\varepsilon}} & \text{for } t > t_0 
\end{cases} \]

where \( \varepsilon \) is a fractional number accounting for asymptotic decrease of infection rate afforded by the lockdown measures \( \varepsilon \in [0, 1] \). The descriptions of the parameters for the considered model are given in Table 1.

The initial conditions are given as
\[ N(0) = N_0, S(0) = S_0, I(0) = I_0, I_d(0) = I_d, I_a(0) = I_a, \]
\[ I_R(0) = I_R, I_T(0) = I_T, R(0) = R_0, D(0) = D_0, V(0) = V_0. \]

### 4. Equilibrium points: disease free and endemic

In this section, we derive the equilibrium points for both disease-free and endemic, but the death class is not considered in this analysis. The disease-free equilibrium is given as
\[ E^0 = \left( \frac{\Lambda}{\gamma_1 + \mu_1}, 0, 0, 0, 0, 0, 0, \frac{\Lambda \gamma_1}{\mu_1 (\gamma_1 + \mu_1)} \right). \]

The endemic equilibrium is obtained by solving the following system
\[ \begin{align*}
\dot{S} &= \delta_0 (\sigma I - \gamma I + \mu) S - (\varepsilon + \zeta + \lambda + \mu) I + \gamma I \\
\dot{I} &= \varepsilon I + \rho I - (\sigma + \tau + \mu) I \\
\dot{V} &= \gamma I + \rho I - (\sigma + \tau + \mu) I - (\Phi + \mu) V.
\end{align*} \]

For simplicity, we put
\[ l_1 = \gamma_1 + \mu_1, \]
\[ l_2 = \varepsilon + \zeta + \lambda + \mu_1, \]
\[ l_3 = \theta + \mu + \zeta + \mu_1, \]
\[ l_4 = \eta + \varepsilon + \mu_1, \]
\[ l_5 = \tau + \varepsilon + \mu_1, \]
\[ l_6 = \sigma + \tau + \mu_1, \]
\[ l_7 = \Phi + \mu_1, \]
\[ l_8 = -\mu_1. \]

Thus,
\[ S^* = \frac{\Lambda}{(l_1 + \kappa) + \gamma I + \mu I + \delta I + \xi I + \phi I + \sigma I + \varepsilon I + \rho I}, \]
\[ I^*_1 = \frac{\varepsilon I}{\gamma_1 + \mu_1}, \]
\[ I^*_2 = \frac{\sigma I}{l_3 l_4}, \]
\[ I^*_3 = \frac{\varepsilon I}{l_5 l_6}, \]
\[ I^*_4 = \frac{\gamma I}{l_7 l_8}, \]
\[ I^*_5 = \frac{\delta I}{l_1 l_2}, \]
\[ I^*_6 = \frac{\xi I}{l_3 l_4}, \]
\[ I^*_7 = \frac{\phi I}{l_5 l_6}, \]
\[ I^*_8 = \frac{\sigma I}{l_7 l_8}. \]
and
\[
\begin{bmatrix}
\delta_0 \left( z + w \beta \frac{\delta}{\Delta} + \gamma w \frac{\delta}{\Delta} + w \delta_1 \left( \frac{w + \delta}{\Delta} + \frac{\delta}{\Delta} \right) \Gamma \right) + \Gamma I_1 \\
\delta_2 \left( \frac{\delta}{\Delta} \right) + \delta_1 \left( \frac{w + \delta}{\Delta} + \frac{\delta}{\Delta} \right) \Gamma + I_4
\end{bmatrix} S' - I_5 \Gamma = 0.
\]\n
Therefore,
\[
S' = \left[ \delta_0 \left( z + w \beta \frac{\delta}{\Delta} + \gamma w \frac{\delta}{\Delta} + w \delta_1 \left( \frac{w + \delta}{\Delta} + \frac{\delta}{\Delta} \right) \Gamma \right) + \Gamma I_1 \right] \delta_2.
\]\n
Also,
\[
V' = \frac{\delta}{\mu_1} S' + \frac{\delta}{\mu_1} R' = \frac{\delta}{\mu_1} \left[ \delta_0 \left( z + w \beta \frac{\delta}{\Delta} + \gamma w \frac{\delta}{\Delta} + w \delta_1 \left( \frac{w + \delta}{\Delta} + \frac{\delta}{\Delta} \right) \Gamma \right) + \Gamma I_1 \right] \delta_2 + \frac{\delta}{\mu_1} \frac{\delta}{\mu_1} \Delta
\]

where \( \Delta = \delta_0 \left( z + w \beta \frac{\delta}{\Delta} + \gamma w \frac{\delta}{\Delta} + w \delta_1 \left( \frac{w + \delta}{\Delta} + \frac{\delta}{\Delta} \right) \right) \Gamma + \Gamma I_1 \)

However
\[
\Gamma = \frac{\Lambda - \gamma_1 + \mu_1}{l_2} = \frac{\Lambda - \gamma_1 + \mu_1}{l_2}.
\]

Thus, replacing \( \Gamma \), we get
\[
S' = \left[ \delta_0 \left( z + w \beta \frac{\delta}{\Delta} + \gamma w \frac{\delta}{\Delta} + w \delta_1 \left( \frac{w + \delta}{\Delta} + \frac{\delta}{\Delta} \right) \Gamma \right) + \Gamma I_1 \right] \delta_2
\]

and
\[
R' = \left[ \delta_0 \left( z + w \beta \frac{\delta}{\Delta} + \gamma w \frac{\delta}{\Delta} + w \delta_1 \left( \frac{w + \delta}{\Delta} + \frac{\delta}{\Delta} \right) \Gamma \right) + \Gamma I_1 \right] \delta_2.
\]

5. Reproductive number

We derive the reproductive number Here, using the next generation matrix [19]. To achieve this, we consider the following equation

\[
I = \delta(t) (z I + w (\beta I + \gamma I + \delta_1 I_R + \delta_1 I_T) S - (\epsilon + \delta + \lambda + \mu_1) I)
\]

\[
\dot{I}_I = \delta I - (\epsilon + \delta + \lambda + \mu_1) I
\]

\[
\dot{I}_R = \eta I_D + \theta I_A - (\epsilon + \delta + \lambda + \mu_1) I
\]

From the above, the following matrices are derived
\[
F = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\]

and
\[
V = \begin{bmatrix} I_2 I & -\frac{\delta}{\mu_1} I_4 & -\frac{\delta}{\mu_1} I_4 \\ -\frac{\delta}{\mu_1} I_4 & 0 & \frac{\delta}{\mu_1} I_4 \\ -\frac{\delta}{\mu_1} I_4 & \frac{\delta}{\mu_1} I_4 & 0 \end{bmatrix}
\]

Thus, we write
\[
\begin{bmatrix} \delta_0 & \frac{\delta}{\mu_1} & \delta_0 w S & \delta_0 w S & \frac{\delta}{\mu_1} w S & \frac{\delta}{\mu_1} w S \end{bmatrix}
\]

\[
F = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

Also, we have
\[
V^{-1} = \begin{bmatrix} I_2 & 0 & 0 & 0 & 0 \\ \frac{\delta}{\mu_1} & \frac{\delta}{\mu_1} & 0 & 0 & 0 \\ I_4 & 0 & 0 & 0 & 0 \\ \frac{\delta}{\mu_1} & \frac{\delta}{\mu_1} & 0 & 0 & 0 \\ I_4 & 0 & 0 & 0 & 0 \\ \frac{\delta}{\mu_1} & \frac{\delta}{\mu_1} & 0 & 0 & 0 \\ \frac{\delta}{\mu_1} & \frac{\delta}{\mu_1} & 0 & 0 & 0 \end{bmatrix}
\]

and
\[
F V^{-1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]
From above, we obtain reproductive number

\[
R_0 = \left\{ \begin{array}{l}
\frac{b S(I + w) (1 - I) + w L}{b S(I + w) (1 - I) + w L + \mu (I + w) (1 - I)} \\
\frac{b S(I + w) (1 - I) + w L}{b S(I + w) (1 - I) + w L + \mu (I + w) (1 - I)}
\end{array} \right.
\] (27)

**Lemma 1.** For the obtained reproductive number \( R_0 > 1 \), a unique equilibrium for endemic case \( E^* \) exists and there is no endemic equilibrium if \( R_0 = 1 \). However the disease is endemic for \( R_0 < 1 \).

\[
\delta(t)(xI + w(\beta I + \gamma A + \delta I + \delta I)) - l_2 I > 0,
\]

\[
I_A < \frac{1}{\beta} I,
\]

\[
I_B < \frac{1}{\beta} I,
\]

\[
I_R < \left( \frac{\mu}{\gamma} + \frac{\mu}{\delta} \right) I,
\]

\[
S' \delta(t) \geq \left( \frac{\mu}{\gamma} + \frac{\mu}{\delta} \right) I.
\]

Therefore, there exists a unique endemic equilibrium when \( R_0 > 1 \). The Jacobian matrix associated to disease-free equilibrium is given as

\[
J(E) = \begin{bmatrix}
-\delta_2 I_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -I_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\gamma}{\beta} & -I_3 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\delta}{\beta} & 0 & -I_4 & 0 & 0 & 0 & 0 \\
0 & \frac{\mu}{\gamma} & 0 & 0 & -I_5 & 0 & 0 & 0 \\
0 & \frac{\mu}{\delta} & 0 & 0 & 0 & -I_6 & 0 & 0 \\
\frac{\gamma}{\beta} & \frac{\delta}{\beta} & \phi & \frac{\gamma}{\beta} & \frac{\delta}{\beta} & \sigma & -I_7 & 0 \\
\frac{\gamma}{\beta} & \frac{\delta}{\beta} & \phi & \frac{\gamma}{\beta} & \frac{\delta}{\beta} & \sigma & -I_7 & 0 \\
\end{bmatrix}.
\] (29)

Thus,

\[
Tr(J(E')) = -(\delta_2 I_1 + l_2 + l_3 + l_4 + l_5 + l_6 + l_7) < 0
\] (30)

and

\[
Det(J(E')) = \delta_2 I_1 l_2 l_3 l_4 l_5 l_6 l_7.
\] (31)

### 6. Global asymptotic stability

In some extent the Lyapunov function can be viewed as an energy in the case of classical mechanical and structural systems. On the other hand the Lyapunov can be viewed as Hamiltonian system. Dynamical system are study within the framework of classical mechanical especially when using classical differential operators. In the last decades several authors studied fractional dynamical system for example epidemiological models. To study the stability of a fractional system, almost all the researchers use the classical Lyapunov function. This has been going on in many papers but the classical differentiation generate an energy with no memory effect. While the system with fractional derivatives generate an energy with accumulative information. It is perhaps unfair to evaluate a classical Lyapunov for a fractional system. In this section, we will introduce an analysis with fractional Lyapunov.

Now, we present the Lyapunov function for different cases when the model is with classical differentiation and fractional derivatives

\[
L = \frac{1}{l_2} I_1 + \frac{1}{l_3} I_4 + \frac{1}{l_4} I_D + \frac{1}{l_5} I_R + \frac{1}{l_6} I_T.
\] (32)

With fractional derivative, we consider the case Caputo derivative

\[
\frac{d^\alpha D_t^\alpha L}{dt} = \frac{1}{l_2} I_1 + \frac{1}{l_3} I_4 + \frac{1}{l_4} I_D + \frac{1}{l_5} I_R + \frac{1}{l_6} I_T.
\] (33)

We start with the classical case

\[
\frac{d}{dt}(\delta(t)(xI + w(\beta I + \gamma A + \delta I + \delta I)) - l_2 I) + \frac{1}{\beta} (\gamma - l_3 I) + \frac{1}{\beta} (\mu + v I) - \frac{1}{\beta} (\eta) + \frac{1}{\beta} (\mu I + v I).
\]

Dividing by \( (I + I_D + I_A + I_R + I_T) \), we have

\[
\frac{\delta(t)(xI + w(\beta I + \gamma A + \delta I + \delta I)) + \frac{1}{\beta} I + \frac{1}{\beta} I}{(I + I_D + I_A + I_R + I_T) - 1}.
\]

Thus,

\[
\frac{dL}{dt} \leq \left\{ \begin{array}{l}
\frac{\delta(t)(x + w(\beta I + \gamma A + \delta I + \delta I)) + \frac{1}{\beta} I + \frac{1}{\beta} I}{(I + I_D + I_A + I_R + I_T) - 1} \\
\frac{\delta(t)(x + w(\beta I + \gamma A + \delta I + \delta I)) + \frac{1}{\beta} I + \frac{1}{\beta} I}{(I + I_D + I_A + I_R + I_T) - 1}
\end{array} \right. (35)
\]

if \( R_0 < 1 \).

Hence, the function \( L \) is the Lyapunov function on a largest compact \( \Delta \) invariant set in \( \{ S, I, I_D, I_A, I_R, I_T \} \). Thus, using Lasalle’s invariance principle all solution of the system with initial condition in \( \Delta \) tends \( E' \) when \( t \to \infty \) only if \( R_0 < 1 \).

We consider a general fractional derivative, we have the following

\[
\frac{dL}{dt} = 0 \text{ if } R_0 < 1.
\]

Hence, the function \( L \) is the Lyapunov function on a largest compact \( \Delta \) invariant set in \( \{ S, I, I_D, I_A, I_R, I_T \} \). Thus, using Lasalle’s invariance principle all solution of the system with initial condition in \( \Delta \) tends \( E' \) when \( t \to \infty \) only if \( R_0 < 1 \).

We consider a general fractional derivative, we have the following
\( \phi^c D_t^c L = \frac{1}{\lambda} \phi^c D_t^1 I + \frac{1}{\lambda^2} \phi^c D_t^1 J \lambda + \frac{1}{\lambda^4} \phi^c D_t^1 D \lambda + \frac{1}{\lambda^6} \phi^c D_t^1 R \)
\[ + \frac{1}{\lambda^8} \phi^c D_t^1 T. \]  

Replacing each fractional class by its value, we obtain again
\( \phi^c D_t^c L = \frac{1}{\lambda} (\delta (t + w (\beta D + \gamma D + \delta I + \delta I) - \delta D)) (S - I) \)
\[ + \frac{1}{\lambda} (\zeta I - I_4) + \frac{1}{\lambda} (\zeta I - I_4) + \frac{1}{\lambda} (\eta I_4 + \theta I_4) \]
\[ + \frac{1}{\lambda^6} (\mu I_4 + \nu I_4 - I_4). \]

Therefore, following the routine presented earlier, we obtain
\( \phi^c D_t^c L \leq (R_0 - 1) (1 + I_4 + I_4 + I_4 + I_4). \)

So, \( \phi^c D_t^c L \leq 0 \) if \( R_0 \leq 1 \) and \( \phi^c D_t^c L > 0 \) if \( R_0 > 1 \).

7. Local and global stability of the endemic equilibrium

We compute first the Jacobian matrix of the Covid-19 model for endemic equilibrium case
\[
JE_c = \begin{bmatrix}
-\delta' - (\gamma_1 + \mu_1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\delta' & -I_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \zeta & -I_2 & 0 & 0 & 0 & 0 & 0 \\
0 & \varepsilon & 0 & -I_4 & 0 & 0 & 0 & 0 \\
0 & 0 & \theta & \eta & -I_4 & 0 & 0 & 0 \\
0 & 0 & \mu & 0 & \nu & -I_4 & 0 & 0 \\
0 & \lambda & \zeta & \varphi & \zeta & \sigma & -I_6 & 0 \\
\gamma_1 & 0 & 0 & 0 & 0 & 0 & 0 & \Phi - \mu_1
\end{bmatrix}.
\]

We now construct a characteristic equation associated to this model
\[ P = \det [I_m \lambda - JE_c] = 0 \]

where \( I_m \) is the 8 \times 8 unit matrix. Then, we have
\[
\begin{bmatrix}
\lambda + I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\alpha' & \lambda + I_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \zeta & \lambda + I_2 & 0 & 0 & 0 & 0 & 0 \\
0 & \varepsilon & 0 & \lambda + I_4 & 0 & 0 & 0 & 0 \\
0 & 0 & \theta & \eta & \lambda + I_4 & 0 & 0 & 0 \\
0 & 0 & \mu & 0 & \nu & \lambda + I_5 & 0 & 0 \\
0 & \lambda & \zeta & \varphi & \zeta & \sigma & \lambda + I_6 & 0 \\
\gamma_1 & 0 & 0 & 0 & 0 & 0 & 0 & \Phi \lambda + \mu_1
\end{bmatrix}
\]

From the above, we obtain the following polynomial
\[
K(\lambda) = \lambda^8 + k_1 \lambda^7 + k_2 \lambda^6 + k_3 \lambda^5 + k_4 \lambda^4 + k_5 \lambda^3 + k_6 \lambda^2 + k_7 \lambda + k_8.
\]

The Hurwitz matrix for the characteristic polynomial \( K(\lambda) \) is written as
\[
H = \begin{bmatrix}
k_1 & k_3 & k_5 & k_7 & 0 & 0 & 0 & 0 \\
k_2 & k_4 & k_6 & k_8 & 0 & 0 & 0 & 0 \\
0 & k_1 & k_3 & k_5 & k_7 & 0 & 0 & 0 \\
0 & 1 & k_2 & k_4 & k_6 & k_8 & 0 & 0 \\
0 & 0 & k_1 & k_3 & k_5 & k_7 & 0 & 0 \\
0 & 0 & 1 & k_2 & k_4 & k_6 & k_8 & 0 \\
0 & 0 & 0 & 1 & k_2 & k_4 & k_6 & k_8
\end{bmatrix}.
\]

Then, we have
\[
H_1 = k_1 > 0
\]
\[
H_2 = k_1 k_2 - k_3 > 0
\]
\[
H_3 = -k_1^2 k_4 + k_1 k_2 k_3 + k_1 k_5 - k_3^2 > 0
\]
\[
H_4 = -k_1^2 k_2 k_4 - k_1^2 k_2^2 - k_1 k_2 k_3 k_5 + k_1 k_2 k_3 k_4
\]
\[
- k_1 k_2 k_7 - k_1 k_4 k_6 + 2 k_1 k_3 k_5
\]
\[
+ k_1 k_3 k_5 - k_1^2 k_3 k_4 + 2 k_1 k_3 k_5
\]
\[
H_5 = k_1^2 k_4 k_6 - k_1^2 k_2^2 - k_1 k_2 k_3 k_5 - k_1^2 k_2 k_3 k_4
\]
\[
+ 2 k_1 k_2 k_3 k_6 + k_1 k_2 k_3 k_6
\]
\[
+ k_1 k_2 k_3 k_6 - k_1 k_2 k_3 k_6
\]
\[
+ k_1 k_2 k_3 k_6 + 2 k_1 k_2 k_3 k_6
\]
\[
- k_1 k_2 k_3 k_6 - 3 k_1 k_2 k_3 k_6
\]
\[
+ 2 k_1 k_2 k_3 k_6 + k_1 k_2 k_3 k_6
\]
\[
+ k_1 k_2 k_3 k_6 - k_1 k_2 k_3 k_6
\]
\[
- k_1 k_2 k_3 k_6 + 2 k_1 k_2 k_3 k_6
\]
\[
- k_1 k_2 k_3 k_6 - k_1 k_2 k_3 k_6
\]
\[
H_6 = -k_1^2 k_2 k_4 + 2 k_1 k_2 k_4 k_6 - k_1^2 k_6
\]
\[
+ 2 k_1 k_2 k_3 k_6 + 2 k_1 k_2 k_3 k_6
\]
\[
+ 3 k_1 k_2 k_3 k_6 + k_1 k_2 k_3 k_4
\]
\[
+ k_1 k_2 k_3 k_4 - k_1 k_2 k_3 k_4
\]
\[
- k_1 k_2 k_3 k_5 - 2 k_1 k_2 k_3 k_5 + 2 k_1 k_2 k_3 k_5 + k_1 k_2 k_3 k_5
\]
\[
- k_1 k_2 k_3 k_6 + 2 k_1 k_2 k_3 k_6
\]
\[
+ k_1 k_2 k_3 k_6 + 2 k_1 k_2 k_3 k_6
\]
\[
- 2 k_1 k_2 k_3 k_6 + 2 k_1 k_2 k_3 k_6 + 3 k_1 k_2 k_3 k_6 - 3 k_1 k_2 k_3 k_6
\]
\[
+ k_1 k_2 k_3 k_4 - 3 k_1 k_2 k_3 k_4
\]
\[
- k_1 k_2 k_3 k_5 + 3 k_1 k_2 k_3 k_5 - k_1 k_2 k_3 k_5 + k_1 k_2 k_3 k_5
\]
\[
+ 2 k_1 k_2 k_3 k_6 - 3 k_1 k_2 k_3 k_6
\]
\[
- k_1 k_2 k_3 k_5 + 3 k_1 k_2 k_3 k_5 - 3 k_1 k_2 k_3 k_6
\]
\[
+ 2 k_1 k_2 k_3 k_6 + 3 k_1 k_2 k_3 k_6
\]
\[
- 2 k_1 k_2 k_3 k_6 + 2 k_1 k_2 k_3 k_6
\]
\[
- 2 k_1 k_2 k_3 k_6 - 2 k_1 k_2 k_3 k_6
\]
\[
- k_1 k_2 k_3 k_5 + k_1 k_2 k_3 k_5 - 3 k_1 k_2 k_3 k_6
\]
\[
- 2 k_1 k_2 k_3 k_6 + 3 k_1 k_2 k_3 k_6
\]
\[
- k_1 k_2 k_3 k_5 + k_1 k_2 k_3 k_5 - 3 k_1 k_2 k_3 k_6
\]
\[
- 2 k_1 k_2 k_3 k_6 + 3 k_1 k_2 k_3 k_6
\]
A novel Covid-19 model with fractional differential operators with singular and non-singular kernels

We apply a fractional differential derivative on both sides, in particular without loss of generality, we use the Caputo derivative

\[
{\cal D}_t^\alpha L(S', \Gamma, \Gamma_r, \Gamma_D, R, V', \Gamma_r') = \left( I - \Gamma + \Gamma \log \frac{\alpha}{\Gamma} \right) (R - R' + \Gamma \log \frac{\alpha}{\Gamma}) + \left( I - \Gamma + \Gamma \log \frac{\alpha}{\Gamma} \right) + \left( I - \Gamma + \Gamma \log \frac{\alpha}{\Gamma} \right)
\]

We prove thus using the idea of a fractional Lyapunov function. We start by defining the Lyapunov function associated the system

\[
\begin{aligned}
L(S', \Gamma, \Gamma_r, \Gamma_D, R, V', \Gamma_r') &= (S - S' + S' \log \frac{\alpha}{S'}) + (I - \Gamma + \Gamma \log \frac{\alpha}{\Gamma}) \\
&+ \left( I - \Gamma + \Gamma \log \frac{\alpha}{\Gamma} \right)
\end{aligned}
\]

\[
\text{Theorem 1. If } R_0 \geq 1, \text{ then the endemic point } E^* \text{ is globally asymptotically stable.}
\]

\[
\text{Proof.}
\]

\[
0^C D_t^\alpha L \geq \frac{1}{\Gamma(1 - 2z_t)} \int_0^t \left\{ \begin{array}{l}
\left( \frac{\alpha}{S'} \right) \Lambda - \delta(\tau) \\
\phi(I - \Gamma) + w\beta(I_D - I_D') \\
+ \gamma w(I_D - I_D') \\
+ \omega \delta_1(I_R - I_R') \\
+ \omega \delta_2(I_T - I_T') \\
+ \left( \frac{I_T - I_T'}{\gamma} \right) \left( \frac{\alpha}{S} \right) \left( I - \Gamma \right) - l \left( I_A - I_A' \right) \\
+ \left( \frac{I_T - I_T'}{\gamma} \right) \left( \frac{\alpha}{S} \right) \left( I - \Gamma \right) - l \left( I_A - I_A' \right) \\
+ \left( \frac{I_T - I_T'}{\gamma} \right) \left( \frac{\alpha}{S} \right) \left( I - \Gamma \right) - l \left( I_A - I_A' \right) \\
+ \left( \frac{I_T - I_T'}{\gamma} \right) \left( \frac{\alpha}{S} \right) \left( I - \Gamma \right) - l \left( I_A - I_A' \right) \\
\end{array} \right\} dt
\]

\[
\frac{\alpha}{S} \left( I - \Gamma \right) - l \left( I_A - I_A' \right) + \left( \frac{I_T - I_T'}{\gamma} \right) \left( \frac{\alpha}{S} \right) \left( I - \Gamma \right) - l \left( I_A - I_A' \right) + \left( \frac{I_T - I_T'}{\gamma} \right) \left( \frac{\alpha}{S} \right) \left( I - \Gamma \right) - l \left( I_A - I_A' \right)
\]

\[
\text{and}
\]

\[
\left( \frac{\alpha}{S} \right) \left( I - \Gamma \right) - l \left( I_A - I_A' \right) + \left( \frac{I_T - I_T'}{\gamma} \right) \left( \frac{\alpha}{S} \right) \left( I - \Gamma \right) - l \left( I_A - I_A' \right) + \left( \frac{I_T - I_T'}{\gamma} \right) \left( \frac{\alpha}{S} \right) \left( I - \Gamma \right) - l \left( I_A - I_A' \right)
\]

\[
\left( \frac{\alpha}{S} \right) \left( I - \Gamma \right) - l \left( I_A - I_A' \right) + \left( \frac{I_T - I_T'}{\gamma} \right) \left( \frac{\alpha}{S} \right) \left( I - \Gamma \right) - l \left( I_A - I_A' \right) + \left( \frac{I_T - I_T'}{\gamma} \right) \left( \frac{\alpha}{S} \right) \left( I - \Gamma \right) - l \left( I_A - I_A' \right)
\]

\[
\left( \frac{\alpha}{S} \right) \left( I - \Gamma \right) - l \left( I_A - I_A' \right) + \left( \frac{I_T - I_T'}{\gamma} \right) \left( \frac{\alpha}{S} \right) \left( I - \Gamma \right) - l \left( I_A - I_A' \right) + \left( \frac{I_T - I_T'}{\gamma} \right) \left( \frac{\alpha}{S} \right) \left( I - \Gamma \right) - l \left( I_A - I_A' \right)
\]
Thus, in the case of ABC derivative, we obtain
\[ \frac{d^\alpha(t)}{dt^\alpha} L(t) = \frac{d^\alpha D^\nu_c L(t)}{dt^\alpha} + \frac{d^\alpha D^\nu_f L(t)}{dt^\alpha} \] (53)
and Caputo-Fabrizio, we have
\[ \frac{d^\alpha(t)}{dt^\alpha} L(t) = \frac{d^\alpha C^\nu_c L(t)}{dt^\alpha} + \frac{d^\alpha C^\nu_f L(t)}{dt^\alpha} \] (54)
Therefore, for all three cases the fractional Lyapunov
\[ \frac{d^\alpha(t)}{dt^\alpha} L(t) > 0 \text{ if } \frac{d^\alpha D^\nu_c L(t)}{dt^\alpha} > 0 \] (55)
\[ \frac{d^\alpha(t)}{dt^\alpha} L(t) < 0 \text{ if } \frac{d^\alpha D^\nu_f L(t)}{dt^\alpha} < 0 \] (56)
It is worth noting that when the fractional order \( \alpha_L = 1 \), we recover the case of classical model.

8. Numerical approximation for Covid-19 spread

Due to the complexity and the nonlinearity of the suggested model, analytical methods cannot be used to provide exact solutions to the system the nonlinear equations. Especially as we consider, in this section the model with fractional differential operators with non-singular and singular kernels [23,24]. Additionally, to these kernels, we considered the case where the differential operators has a fractal dimension and fractional orders. These differential and integral operators have been introduced very recently and have been recognized to be powerful mathematical operators able to capture memory and self-similarities. To solve models with these differential operators, we use a newly numerical method which is based on the Newton polynomial interpolation [22]. Before giving numerical scheme for this model, we present error analysis to show accuracy of the suggested method.

8.1. Error analysis for a general Cauchy problem

In this section, we present error analysis for a general Cauchy problem for some fractional differential operators which are Riemann-Liouville, Caputo-Fabrizio and Atangana-Baleanu fractional operators.
We start with a general Riemann-Liouville case
\[
\begin{align*}
\left\{ \begin{array}{l}
D_t^\alpha y(t) = f(t, y(t)) \\
y(0) = y_0.
\end{array} \right.
\end{align*}
\]
(58)

We convert this equation as follows;
\[
y(t) = \frac{1}{\Gamma(\alpha)} \int_0^t f(\tau, y(\tau))(t-\tau)^{\alpha-1} \, d\tau.
\]
(59)

For derivation numerical solution based on the Newton polynomial, we write
\[
y(t_{n+1}) = \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{n} \int_{t_j}^{t_{j+1}} (t_{n+1} - \tau)^{\alpha-1} \left[ P_j(\tau) + R_j(\tau) \right] \, d\tau
\]
where
\[
R_j(\tau) = \frac{(\tau - t_{j})(\tau - t_{j+1})(t_{n+1} - \tau)^{-1} \left[ \frac{\partial^2}{\partial \tau^2} f(\tau, y(\tau)) \right]_{\tau = t_{n+1}}}{2!}.
\]
(60)

Therefore, the error is given as
\[
E = \frac{1}{2 \Gamma(\alpha)} \sum_{j=0}^{n} \int_{t_j}^{t_{j+1}} (t_{n+1} - \tau)^{\alpha-1} \left[ \frac{\partial^2}{\partial \tau^2} f(\tau, y(\tau)) \right]_{\tau = t_{n+1}} \, d\tau.
\]
(61)

We define
\[
\| \varphi \|_\infty = \sup_{t \in [a, b]} |\varphi(t)|.
\]

Then, we next consider the absolute value of $E$
\[
|E| \leq \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{n} \int_{t_j}^{t_{j+1}} (t_{n+1} - \tau)^{\alpha-1} \left| \frac{\partial^2}{\partial \tau^2} f(\tau, y(\tau)) \right|_{\tau = t_{n+1}} \, d\tau.
\]
(62)

\[
\leq \frac{1}{\Gamma(\alpha)} \sup_{t \in [a, b]} \left| \frac{\partial^2}{\partial \tau^2} f(\tau, y(\tau)) \right| \sum_{j=0}^{n} \int_{t_j}^{t_{j+1}} (t_{n+1} - \tau)^{\alpha-1} \, d\tau
\]
\[
\leq \frac{1}{\Gamma(\alpha)} \sup_{t \in [a, b]} \left| \frac{\partial^2}{\partial \tau^2} f(\tau, y(\tau)) \right| \sum_{j=0}^{n} \int_{t_j}^{t_{j+1}} (t_{n+1} - \tau)^{\alpha-1} \, d\tau
\]

whereas within $[0, t_{n+1}]$ the function $(t_{n+1} - \tau)$ is positive therefore
\[
\int_{t_j}^{t_{j+1}} (t_{n+1} - \tau)^{\alpha-1} \, d\tau \leq \int_{t_j}^{t_{j+1}} t^{\alpha-1} \, d\tau \leq \frac{t^{\alpha-1}}{\alpha-1} B(3, \alpha).
\]
(63)

Also
\[
\int_{t_j}^{t_{j+1}} tf_j(t_{n+1} - \tau)^{\alpha-1} \, d\tau = tf_j \left[ (t_{n+1} - t_j)^{\alpha-1} - \frac{(t_{n+1} - t_{j+1})^{\alpha-1}}{\alpha} \right]
\]
\[
= j(\alpha - 1) \left( \frac{\alpha}{\alpha + 2} \right) \left[ (n - j + 2)^\alpha - (n - j + 1)^\alpha \right]
\]
and
\[
\int_{t_j}^{t_{j+1}} tf_j(t_{n+1} - \tau)^{\alpha-1} \, d\tau \leq j(\alpha - 1)(j + 1) \left( \frac{\alpha}{\alpha + 2} \right) \left( \frac{(t_{n+1} - t_j)^\alpha}{\alpha} \right)
\]
\[
\leq \frac{2^\alpha}{\alpha + 2} \left( \frac{\alpha}{\alpha + 2} \right)^{\alpha-2} \alpha.
\]
(64)

Therefore, we have
\[
|E| < \frac{1}{\Gamma(\alpha)} \sup_{t \in [a, b]} \left| \frac{\partial^2}{\partial \tau^2} f(\tau, y(\tau)) \right| \sum_{j=0}^{n} \int_{t_j}^{t_{j+1}} (t_{n+1} - \tau)^{\alpha-1} \left[ P_j(\tau) + R_j(\tau) \right] \, d\tau
\]
\[
< \frac{1}{\Gamma(\alpha)} \sup_{t \in [a, b]} \left| \frac{\partial^2}{\partial \tau^2} f(\tau, y(\tau)) \right| \left\{ (n+1)^{\alpha-1} B(3, \alpha) + \frac{2^\alpha}{\alpha + 2} \right\}.
\]
(66)

Therefore, the error is bounded as
\[
|E| < \frac{1}{\alpha} \sup_{t \in [a, b]} \left| \frac{\partial^2}{\partial \tau^2} f(\tau, y(\tau)) \right| \Pi(n, x).
\]
(67)

With Atangana-Baleanu derivative, we consider the following problem
\[
\begin{align*}
\left\{ \begin{array}{l}
0 AB_t^\alpha y(t) = f(t, y(t)) \\
y(0) = y_0.
\end{array} \right.
\end{align*}
\]
(68)

Integrating the above equation, we can have the following
\[
y(t) = \frac{1 - x}{AB(\alpha)} f(t, y(t)) + \frac{x}{AB(\alpha)\Gamma(\alpha)} \int_0^t f(\tau, y(\tau))(t-\tau)^{\alpha-1} \, d\tau.
\]
(69)

Following the methodology presented to derive numerical solution using the Newton polynomial, we have
\[
y(t_{n+1}) = \frac{1 - x}{AB(\alpha)} f(t_{n+1}, y(t_{n+1})) + \frac{x}{AB(\alpha)\Gamma(\alpha)} \sum_{j=0}^{n} \int_{t_j}^{t_{j+1}} (t_{n+1} - \tau)^{\alpha-1} \left[ P_j(\tau) + R_j(\tau) \right] \, d\tau.
\]
(70)

Here also, we write
\[
|E| \leq \frac{1}{\alpha} \sup_{t \in [a, b]} \left| \frac{\partial^2}{\partial \tau^2} f(\tau, y(\tau)) \right| \Pi(n, x).
\]
(71)

Finally, we present error analysis for Caputo-Fabrizio case
\[
\begin{align*}
\left\{ \begin{array}{l}
C^{RF}_t^\alpha y(t) = f(t, y(t)) \\
y(0) = y_0.
\end{array} \right.
\end{align*}
\]
(72)

Applying the Caputo-Fabrizio integral, we get
\[
y(t) = \frac{1 - x}{M(\alpha)} f(t, y(t)) + \frac{x}{M(\alpha)} \int_0^t f(\tau, y(\tau)) \, d\tau
\]
(73)

Using the Newton polynomial, we have
\[
y(t_{n+1}) = \frac{1 - x}{M(\alpha)} f(t_{n+1}, y(t_{n+1})) + \frac{x}{M(\alpha)} \int_{t_n}^{t_{n+1}} \left[ P_n(\tau) + R_n(\tau) \right] \, d\tau
\]
Here, we have
\[
E = \frac{x}{M(\alpha)} \int_{t_n}^{t_{n+1}} R_n(\tau) \, d\tau.
\]
Therefore, we evaluate the error
\[
\begin{align*}
|E| & \leq \frac{2}{M(z)} \int_{t_0}^{t_n} |R_0(t)| \, dt \\
& \leq \frac{2}{M(z)} \sup_{\tau \in [t,t_n]} \left| \frac{\partial^2 f(t,y(t))}{\partial t^2} \right| \int_{t_0}^{t_n} (t^2 - (t_n + t_n - 1) \tau + t_n \tau) \, dt \\
& \leq \frac{2}{M(z)} \sup_{\tau \in [t,t_n]} \left| \frac{\partial^2 f(t,y(t))}{\partial t^2} \right| \left( \frac{(t_n^3)}{3} - \frac{(t_n^3)}{3} - (2n - 1)(\Delta t)^3 \left( (n+1)^2 - \frac{2}{3} \right) \right) + n(n - 1)(\Delta t)^3 O(n^3).
\end{align*}
\]

8.2. Numerical scheme for Covid-19 model

Now, we present numerical scheme for our model. We shall start with the Caputo-Fabrizio fractional derivative, this will be followed Caputo and Atangana-Baleano fractional derivatives. Finally, we solve the models with fractal-fractional derivatives.

\[
\begin{align*}
_{\psi}D^\alpha S &= -\Lambda (\delta(t)(\alpha F + \omega B_F + \gamma N_F + \omega B_{Ft} + \omega B_{Ft}t) + \gamma_1 + \mu_1) S \\
_{\psi}D^\alpha I &= (\delta(t)(\alpha F + \omega B_F + \gamma N_F + \omega B_{Ft} + \omega B_{Ft}t) - S - (\alpha + \lambda + \mu_1)I \\
_{\psi}D^\alpha D &= \lambda I + \phi D + (\lambda + \sigma + \mu_t)D \\
_{\psi}D^\alpha R &= -\delta I + \delta R - (\delta + \mu_1)R \\
_{\psi}D^\alpha V &= \gamma_1 S + \Phi R - \mu_1 V.
\end{align*}
\]

For simplicity, we write above equation as follows;

\[
\begin{align*}
_{\psi}D^\alpha S &= S(t, S, I, A, I_d, I_b, I_r, R, D, V) \\
_{\psi}D^\alpha I &= I(t, S, I, A, I_d, I_b, I_r, R, D, V) \\
_{\psi}D^\alpha D &= D(t, S, I, A, I_d, I_b, I_r, R, D, V) \\
_{\psi}D^\alpha R &= R(t, S, I, A, I_d, I_b, I_r, R, D, V) \\
_{\psi}D^\alpha V &= V(t, S, I, A, I_d, I_b, I_r, R, D, V).
\end{align*}
\]

After applying fractional integral with exponential kernel and putting Newton polynomial into these equations, we can solve our model as follows

\[
\begin{align*}
S^{t+1} &= S^t + \frac{1 - \alpha}{M(z)} \left[ S^t (t_{s-1}, S^t, I^t, A^t, I_d^t, I_b^t, I_r^t, R^t, D^t, V^t) - S^t (t_{s-1}, S^{t-1}, I^{t-1}, A^{t-1}, I_d^{t-1}, I_b^{t-1}, I_r^{t-1}, R^{t-1}, D^{t-1}, V^{t-1}) + \frac{2}{M(z)} S^t (t_{s-2}, S^{t-2}, I^{t-2}, A^{t-2}, I_d^{t-2}, I_b^{t-2}, I_r^{t-2}, R^{t-2}, D^{t-2}, V^{t-2}) \Delta t \right] \\
& \quad + \frac{2}{M(z)} S^t (t_{s-3}, S^{t-3}, I^{t-3}, A^{t-3}, I_d^{t-3}, I_b^{t-3}, I_r^{t-3}, R^{t-3}, D^{t-3}, V^{t-3}) \Delta t \\
& \quad + \frac{2}{M(z)} S^t (t_{s-4}, S^{t-4}, I^{t-4}, A^{t-4}, I_d^{t-4}, I_b^{t-4}, I_r^{t-4}, R^{t-4}, D^{t-4}, V^{t-4}) \Delta t.
\end{align*}
\]

\[
\begin{align*}
I^{t+1} &= I^t + \frac{1 - \alpha}{M(z)} \left[ I^t (t_{s}, S^t, I^t, A^t, I_d^t, I_b^t, I_r^t, R^t, D^t, V^t) - I^t (t_{s-1}, S^{t-1}, I^{t-1}, A^{t-1}, I_d^{t-1}, I_b^{t-1}, I_r^{t-1}, R^{t-1}, D^{t-1}, V^{t-1}) + \frac{2}{M(z)} I^t (t_{s-2}, S^{t-2}, I^{t-2}, A^{t-2}, I_d^{t-2}, I_b^{t-2}, I_r^{t-2}, R^{t-2}, D^{t-2}, V^{t-2}) \Delta t \right] \\
& \quad + \frac{2}{M(z)} I^t (t_{s-3}, S^{t-3}, I^{t-3}, A^{t-3}, I_d^{t-3}, I_b^{t-3}, I_r^{t-3}, R^{t-3}, D^{t-3}, V^{t-3}) \Delta t \\
& \quad + \frac{2}{M(z)} I^t (t_{s-4}, S^{t-4}, I^{t-4}, A^{t-4}, I_d^{t-4}, I_b^{t-4}, I_r^{t-4}, R^{t-4}, D^{t-4}, V^{t-4}) \Delta t.
\end{align*}
\]

\[
\begin{align*}
R^{t+1} &= R^t + \frac{1 - \alpha}{M(z)} \left[ R^t (t_{s}, S^t, I^t, A^t, I_d^t, I_b^t, I_r^t, R^t, D^t, V^t) - R^t (t_{s-1}, S^{t-1}, I^{t-1}, A^{t-1}, I_d^{t-1}, I_b^{t-1}, I_r^{t-1}, R^{t-1}, D^{t-1}, V^{t-1}) + \frac{2}{M(z)} R^t (t_{s-2}, S^{t-2}, I^{t-2}, A^{t-2}, I_d^{t-2}, I_b^{t-2}, I_r^{t-2}, R^{t-2}, D^{t-2}, V^{t-2}) \Delta t \right] \\
& \quad + \frac{2}{M(z)} R^t (t_{s-3}, S^{t-3}, I^{t-3}, A^{t-3}, I_d^{t-3}, I_b^{t-3}, I_r^{t-3}, R^{t-3}, D^{t-3}, V^{t-3}) \Delta t \\
& \quad + \frac{2}{M(z)} R^t (t_{s-4}, S^{t-4}, I^{t-4}, A^{t-4}, I_d^{t-4}, I_b^{t-4}, I_r^{t-4}, R^{t-4}, D^{t-4}, V^{t-4}) \Delta t.
\end{align*}
\]

\[
\begin{align*}
V^{t+1} &= V^t + \frac{1 - \alpha}{M(z)} \left[ V^t (t_{s}, S^t, I^t, A^t, I_d^t, I_b^t, I_r^t, R^t, D^t, V^t) - V^t (t_{s-1}, S^{t-1}, I^{t-1}, A^{t-1}, I_d^{t-1}, I_b^{t-1}, I_r^{t-1}, R^{t-1}, D^{t-1}, V^{t-1}) + \frac{2}{M(z)} V^t (t_{s-2}, S^{t-2}, I^{t-2}, A^{t-2}, I_d^{t-2}, I_b^{t-2}, I_r^{t-2}, R^{t-2}, D^{t-2}, V^{t-2}) \Delta t \right] \\
& \quad + \frac{2}{M(z)} V^t (t_{s-3}, S^{t-3}, I^{t-3}, A^{t-3}, I_d^{t-3}, I_b^{t-3}, I_r^{t-3}, R^{t-3}, D^{t-3}, V^{t-3}) \Delta t \\
& \quad + \frac{2}{M(z)} V^t (t_{s-4}, S^{t-4}, I^{t-4}, A^{t-4}, I_d^{t-4}, I_b^{t-4}, I_r^{t-4}, R^{t-4}, D^{t-4}, V^{t-4}) \Delta t.
\end{align*}
\]
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\[ R^{+1} = R' + \frac{1}{\beta(t)} \left[ R'(t, S', I', E', P', P'_T, I'_D, I'_T, R', D', V') \right. \]
\[ + \frac{\alpha(t)}{\beta(t)} \sum_{i=2}^{\alpha(t)} [R'(t_i, S^{(i-1)}, I^{(i-1)}, E^{(i-1)}, P^{(i-1)}, P'^{(i-1)}_T, I'^{D(1-1)}, I'^{T(1-1)}, R^{(1-1)}, D^{(1-1)}, V^{(1-1)}_i) \Delta t] \]
\[ + \frac{\alpha(t)}{\beta(t)} \sum_{i=2}^{\alpha(t)} [R'(t_{i+1}, S^{(i+1)}, I^{(i+1)}, E^{(i+1)}, P^{(i+1)}, P'^{(i+1)}_T, I'^{D(i+1)}, I'^{T(i+1)}, R^{(i+1)}, D^{(i+1)}, V^{(i+1)}_i) \Delta t] \]
\[ D^{+1} = D' + \frac{1}{\beta(t)} \left[ D'(t, S', I', E', P', P'_T, I'_D, I'_T, R', D', V') \right. \]
\[ + \frac{\alpha(t)}{\beta(t)} \sum_{i=2}^{\alpha(t)} [D'(t_i, S^{(i-1)}, I^{(i-1)}, E^{(i-1)}, P^{(i-1)}, P'^{T(1-1)}, I'^{D(1-1)}, I'^{T(1-1)}, R^{(1-1)}, D^{(1-1)}, V^{(1-1)}_i) \Delta t] \]
\[ + \frac{\alpha(t)}{\beta(t)} \sum_{i=2}^{\alpha(t)} [D'(t_{i+1}, S^{(i+1)}, I^{(i+1)}, E^{(i+1)}, P^{(i+1)}, P'^{T(i+1)}, I'^{D(i+1)}, I'^{T(i+1)}, R^{(i+1)}, D^{(i+1)}, V^{(i+1)}_i) \Delta t] \]
\[ V^{+1} = V' + \frac{1}{\beta(t)} \left[ V'(t, S', I', E', P', P'_T, I'_D, I'_T, R', D', V') \right. \]
\[ + \frac{\alpha(t)}{\beta(t)} \sum_{i=2}^{\alpha(t)} [V'(t_i, S^{(i-1)}, I^{(i-1)}, E^{(i-1)}, P^{(i-1)}, P'^{T(1-1)}, R^{(1-1)}, D^{(1-1)}, V^{(1-1)}_i) \Delta t] \]
\[ + \frac{\alpha(t)}{\beta(t)} \sum_{i=2}^{\alpha(t)} [V'(t_{i+1}, S^{(i+1)}, I^{(i+1)}, E^{(i+1)}, P^{(i+1)}, P'^{T(i+1)}, R^{(i+1)}, D^{(i+1)}, V^{(i+1)}_i) \Delta t] \]

We can have the following numerical scheme for Mittag-Leffler case:

\[ S^{+1} = 1 - \frac{\alpha(t)}{\beta(t)} S(t, S', I', E', P', P'_T, I'_D, I'_T, R', D', V') \]  \(78\)
\[ + \frac{\alpha(t)}{\beta(t)} \sum_{i=2}^{\alpha(t)} [S(t_i, S^{(i-1)}, I^{(i-1)}, E^{(i-1)}, P^{(i-1)}, P'^{T(1-1)}, I'^{D(1-1)}, I'^{T(1-1)}, R^{(1-1)}, D^{(1-1)}, V^{(1-1)}_i) \Delta t] \]
\[ + \frac{\alpha(t)}{\beta(t)} \sum_{i=2}^{\alpha(t)} [S(t_{i+1}, S^{(i+1)}, I^{(i+1)}, E^{(i+1)}, P^{(i+1)}, P'^{T(i+1)}, I'^{D(i+1)}, I'^{T(i+1)}, R^{(i+1)}, D^{(i+1)}, V^{(i+1)}_i) \Delta t] \]

\[ P^{+1} = \frac{\alpha(t)}{\beta(t)} P(t, S', I', E', P', P'_T, I'_D, I'_T, R', D', V') \]
\[ + \frac{\alpha(t)}{\beta(t)} \sum_{i=2}^{\alpha(t)} [P(t_i, S^{(i-1)}, I^{(i-1)}, E^{(i-1)}, P^{(i-1)}, P'^{T(1-1)}, R^{(1-1)}, D^{(1-1)}, V^{(1-1)}_i) \Delta t] \]
\[ + \frac{\alpha(t)}{\beta(t)} \sum_{i=2}^{\alpha(t)} [P(t_{i+1}, S^{(i+1)}, I^{(i+1)}, E^{(i+1)}, P^{(i+1)}, P'^{T(i+1)}, R^{(i+1)}, D^{(i+1)}, V^{(i+1)}_i) \Delta t] \]
\[ T^{(1)}_R = \frac{1}{\Delta t^R} P_T (t, S', \Gamma', \Gamma_T, \Gamma_D, \Gamma_P, R', D', V') \]
+ \[ \frac{\alpha}{\Delta t^R} \sum_{n=2}^{t} \left[ G (t, t_n, \Sigma t, S_n-1, F_n-1, F_T, F_P, R_n, D_n, V_n) \right] \]
+ \[ \frac{\alpha}{\Delta t^R} \sum_{n=2}^{t} \left[ \left( \Sigma - D' (t, t_n, S_n-1, F_n-1, F_T, F_P, R_n, D_n, V_n) \right) \right] \]

\[ R^{(1)} = \frac{1}{\Delta t^R} R (t, S', \Gamma', \Gamma_T, \Gamma_D, \Gamma_P, R', D', V') \]
+ \[ \frac{\alpha}{\Delta t^R} \sum_{n=2}^{t} \left[ R (t, t_n, \Sigma t, S_n-1, F_n-1, F_T, F_P, R_n, D_n, V_n) \right] \]
+ \[ \frac{\alpha}{\Delta t^R} \sum_{n=2}^{t} \left[ \left( \Delta - D' (t, t_n, S_n-1, F_n-1, F_T, F_P, R_n, D_n, V_n) \right) \right] \]

\[ D^{(1)} = \frac{1}{\Delta t^D} D' (t, S', \Gamma', \Gamma_D, \Gamma_P, R', D', V') \]
+ \[ \frac{\alpha}{\Delta t^D} \sum_{n=2}^{t} \left[ D' (t, t_n, S_n-1, F_n-1, F_T, F_P, R_n, D_n, V_n) \right] \]
+ \[ \frac{\alpha}{\Delta t^D} \sum_{n=2}^{t} \left[ \left( \Delta - D' (t, t_n, S_n-1, F_n-1, F_T, F_P, R_n, D_n, V_n) \right) \right] \]

where

\[ \Delta = \left( v-u+1 \right)^2 \left[ 2(v-u)^2 + (3x + 10)(v-u) \right] \]
+ \[ 2x^2 + 9x + 12 \]
+ \[ (v-u)^2 + (5x + 10)(v-u) \]
+ \[ 6x^2 + 18x + 12 \]

\[ \Sigma = \left( v-u+1 \right)^2 (v-u+3+2x) \]

Finally, we can have the following numerical approximation with Caputo derivative.
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\[ S^{i+1} = \sum_{\frac{\partial}{\partial t}} S^i(t_{i-1}, S^{i-1}, P^{i-1}, P_A^{i-1}, P_D^{i-1}, P_R^{i-1}, R^{i-1}, D^{i-1}, V^{i-1}) \] 
\[ S^i(t_{i-1}, S^{i-1}, P^{i-1}, P_A^{i-1}, P_D^{i-1}, P_R^{i-1}, R^{i-1}, D^{i-1}, V^{i-1}) \] 
\[ P^{i+1} = \sum_{\frac{\partial}{\partial t}} P^i(t_{i-1}, S^{i-1}, P^{i-1}, P_A^{i-1}, P_D^{i-1}, P_R^{i-1}, R^{i-1}, D^{i-1}, V^{i-1}) \] 
\[ P^i(t_{i-1}, S^{i-1}, P^{i-1}, P_A^{i-1}, P_D^{i-1}, P_R^{i-1}, R^{i-1}, D^{i-1}, V^{i-1}) \] 
\[ P_A^{i+1} = \sum_{\frac{\partial}{\partial t}} P_A^i(t_{i-1}, S^{i-1}, P^{i-1}, P_A^{i-1}, P_D^{i-1}, P_R^{i-1}, R^{i-1}, D^{i-1}, V^{i-1}) \] 
\[ P_A^i(t_{i-1}, S^{i-1}, P^{i-1}, P_A^{i-1}, P_D^{i-1}, P_R^{i-1}, R^{i-1}, D^{i-1}, V^{i-1}) \] 
\[ P_R^{i+1} = \sum_{\frac{\partial}{\partial t}} P_R^i(t_{i-1}, S^{i-1}, P^{i-1}, P_A^{i-1}, P_D^{i-1}, P_R^{i-1}, R^{i-1}, D^{i-1}, V^{i-1}) \] 
\[ P_R^i(t_{i-1}, S^{i-1}, P^{i-1}, P_A^{i-1}, P_D^{i-1}, P_R^{i-1}, R^{i-1}, D^{i-1}, V^{i-1}) \]
\[ R^{+1} = \frac{(\Delta t)^3}{3} \sum_{n+1} V^{(t_n, S_n, F_n, P_n)} P^{n-2}, R^{2, -}, D^{2, -}, V^{n-2}) \Pi \]
\[ + \frac{(\Delta t)^2}{2} \sum_{n+2} R^{(t_{n+1}, S_{n+1}, F_{n+1}, P_{n+1}, D_{n+1}, V_{n+1})} \]
\[ + \frac{(\Delta t)^2}{2} \sum_{n+2} \left[ \left( R^{(t_{n+1}, S_{n+1}, F_{n+1}, P_{n+1}, D_{n+1}, V_{n+1})} \right) \right] \]
\[ \Delta \]

\[ D^{+1} = \frac{(\Delta t)^3}{3} \sum_{n+1} D^{(t_n, S_n, F_n, P_n, D_n)} P^{n-2}, R^{2, -}, D^{2, -}, V^{n-2}) \Pi \]
\[ + \frac{(\Delta t)^2}{2} \sum_{n+2} D^{(t_{n+1}, S_{n+1}, F_{n+1}, P_{n+1}, D_{n+1}, V_{n+1})} \]
\[ + \frac{(\Delta t)^2}{2} \sum_{n+2} \left[ \left( D^{(t_{n+1}, S_{n+1}, F_{n+1}, P_{n+1}, D_{n+1}, V_{n+1})} \right) \right] \]
\[ \Delta \]

\[ V^{+1} = \frac{(\Delta t)^3}{3} \sum_{n+1} V^{(t_n, S_n, F_n, P_n)} P^{n-2}, R^{2, -}, D^{2, -}, V^{n-2}) \Pi \]
\[ + \frac{(\Delta t)^2}{2} \sum_{n+2} V^{(t_{n+1}, S_{n+1}, F_{n+1}, P_{n+1}, D_{n+1}, V_{n+1})} \]
\[ + \frac{(\Delta t)^2}{2} \sum_{n+2} \left[ \left( V^{(t_{n+1}, S_{n+1}, F_{n+1}, P_{n+1}, D_{n+1}, V_{n+1})} \right) \right] \]
\[ \Delta \]

Now, we consider our model with fractal-fractional operators. We start with Caputo-Fabrizio fractal-fractional derivative
\[ 0^{\text{ffe}} D^{t,S} = \left( t, S, I, I_{1}, I_{2}, I_{3}, I_{4}, I_{5}, R, D, V \right) \]
\[ 0^{\text{ffe}} D^{t,F} = \left( t, S, I, I_{1}, I_{2}, I_{3}, I_{4}, I_{5}, R, D, V \right) \]
\[ 0^{\text{ffe}} D^{t,I} = \left( t, S, I, I_{1}, I_{2}, I_{3}, I_{4}, I_{5}, R, D, V \right) \]
\[ 0^{\text{ffe}} D^{t,R} = \left( t, S, I, I_{1}, I_{2}, I_{3}, I_{4}, I_{5}, R, D, V \right) \]
\[ 0^{\text{ffe}} D^{t,V} = \left( t, S, I, I_{1}, I_{2}, I_{3}, I_{4}, I_{5}, R, D, V \right) \]
\[ \text{After applying fractal-fractional integral with exponential kernel, we have the following scheme for this model} \]
\[ S^{+1} = S^{0} + \frac{1 - \alpha}{M(x)} \left[ \begin{array}{c}
\frac{\Gamma^1}{\Gamma_1} \frac{\Gamma^1}{\Gamma_1}
\end{array} \right] \]
\[ \left( t_{n+1}, S_{n+1}, F_{n+1}, P_{n+1}, D_{n+1}, V_{n+1} \right) \]
\[ F^{+1} = F^{0} + \frac{1}{M(x)} \left[ \begin{array}{c}
\frac{\Gamma^1}{\Gamma_1} \frac{\Gamma^1}{\Gamma_1}
\end{array} \right] \]
\[ \left( t_{n+1}, S_{n+1}, F_{n+1}, P_{n+1}, D_{n+1}, V_{n+1} \right) \]
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\[ I^\alpha_D = I^\alpha_D + \frac{\alpha}{\Gamma(\alpha)} \begin{bmatrix}
I^{\alpha-1}D^\beta(t, S, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R,
I^\alpha_R, I^\alpha_F, R^\alpha_D, V) \\
-\lambda I^{\alpha-1}D^\beta(t, S, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R,
I^\alpha_R, I^\alpha_F, R^\alpha_D, V^{\alpha}) \\
I^{\alpha-1}D^\beta(t, S, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R,
I^\alpha_R, I^\alpha_F, R^\alpha_D, V^{\alpha})
\end{bmatrix} \]

\[ D^\alpha = D^\alpha + \frac{\alpha}{\Gamma(\alpha)} \begin{bmatrix}
I^{\alpha-1}D^\beta(t, S, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R,
I^\alpha_R, I^\alpha_F, R^\alpha_D, V) \\
-\lambda I^{\alpha-1}D^\beta(t, S, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R,
I^\alpha_R, I^\alpha_F, R^\alpha_D, V^{\alpha}) \\
I^{\alpha-1}D^\beta(t, S, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R,
I^\alpha_R, I^\alpha_F, R^\alpha_D, V^{\alpha})
\end{bmatrix} \]

\[ I^\alpha_R = I^\alpha_R + \frac{\alpha}{\Gamma(\alpha)} \begin{bmatrix}
I^{\alpha-1}R(t, S, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R,
I^\alpha_R, I^\alpha_F, R^\alpha_D, V) \\
I^{\alpha-1}R(t, S, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R,
I^\alpha_R, I^\alpha_F, R^\alpha_D, V^{\alpha}) \\
I^{\alpha-1}R(t, S, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R,
I^\alpha_R, I^\alpha_F, R^\alpha_D, V^{\alpha})
\end{bmatrix} \]

\[ V^\alpha = V^\alpha + \frac{\alpha}{\Gamma(\alpha)} \begin{bmatrix}
I^{\alpha-1}V(t, S, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R,
I^\alpha_R, I^\alpha_F, R^\alpha_D, V) \\
I^{\alpha-1}V(t, S, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R,
I^\alpha_R, I^\alpha_F, R^\alpha_D, V^{\alpha}) \\
I^{\alpha-1}V(t, S, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R,
I^\alpha_R, I^\alpha_F, R^\alpha_D, V^{\alpha})
\end{bmatrix} \]

For Mittag-Leffler kernel, we can get the following numerical scheme

\[ S(t, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R, R^\alpha_D, V^{\alpha}) = \frac{1}{AB(x)} \begin{bmatrix}
I^{\alpha-1}S(t, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R, R^\alpha_D, V^{\alpha}) \\
I^{\alpha-1}S(t, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R, R^\alpha_D, V^{\alpha}) \\
I^{\alpha-1}S(t, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R, R^\alpha_D, V^{\alpha})
\end{bmatrix} \]

\[ R^\alpha = R^\alpha + \frac{\alpha}{\Gamma(\alpha)} \begin{bmatrix}
I^{\alpha-1}R(t, S, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R,
I^\alpha_R, I^\alpha_F, R^\alpha_D, V) \\
I^{\alpha-1}R(t, S, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R,
I^\alpha_R, I^\alpha_F, R^\alpha_D, V^{\alpha}) \\
I^{\alpha-1}R(t, S, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R,
I^\alpha_R, I^\alpha_F, R^\alpha_D, V^{\alpha})
\end{bmatrix} \]

\[ \sum_{\alpha=1}^\infty \begin{bmatrix}
I^{\alpha-1}S(t, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R, R^\alpha_D, V^{\alpha}) \\
I^{\alpha-1}S(t, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R, R^\alpha_D, V^{\alpha}) \\
I^{\alpha-1}S(t, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R, R^\alpha_D, V^{\alpha})
\end{bmatrix} \]

\[ R^\alpha = R^\alpha + \frac{\alpha}{\Gamma(\alpha)} \begin{bmatrix}
I^{\alpha-1}R(t, S, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R,
I^\alpha_R, I^\alpha_F, R^\alpha_D, V) \\
I^{\alpha-1}R(t, S, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R,
I^\alpha_R, I^\alpha_F, R^\alpha_D, V^{\alpha}) \\
I^{\alpha-1}R(t, S, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R,
I^\alpha_R, I^\alpha_F, R^\alpha_D, V^{\alpha})
\end{bmatrix} \]

\[ \sum_{\alpha=1}^\infty \begin{bmatrix}
I^{\alpha-1}S(t, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R, R^\alpha_D, V^{\alpha}) \\
I^{\alpha-1}S(t, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R, R^\alpha_D, V^{\alpha}) \\
I^{\alpha-1}S(t, \Gamma, \Gamma_a, \Gamma_D, \Gamma_R, R^\alpha_D, V^{\alpha})
\end{bmatrix} \]
\[
F_{\alpha}^{1} = \frac{1}{4\Delta t} \sum_{n=2}^{N} \left( I_{u}^{n-1} F_{1} \left( t_{u}^{n-1}, S_{u}^{n-1}, P_{u}^{n-1}, F_{u}^{n-1}, P_{u}^{n-1}, F_{u}^{n}, R_{u}^{n}, D_{u}^{n}, V_{u}^{n} \right) + \frac{\Delta t}{\Delta x} \sum_{n=2}^{N} \left( I_{u}^{n-1} F_{1} \left( t_{u}^{n-1}, S_{u}^{n-1}, P_{u}^{n-1}, F_{u}^{n-1}, P_{u}^{n-1}, F_{u}^{n}, R_{u}^{n}, D_{u}^{n}, V_{u}^{n} \right) \right) \right)
\]

\[
F_{D}^{1} = \frac{1}{4\Delta t} \sum_{n=2}^{N} \left( I_{u}^{n-1} F_{D} \left( t_{u}^{n-1}, S_{u}^{n-1}, P_{u}^{n-1}, F_{u}^{n-1}, P_{u}^{n-1}, F_{u}^{n}, R_{u}^{n}, D_{u}^{n}, V_{u}^{n} \right) + \frac{\Delta t}{\Delta x} \sum_{n=2}^{N} \left( I_{u}^{n-1} F_{D} \left( t_{u}^{n-1}, S_{u}^{n-1}, P_{u}^{n-1}, F_{u}^{n-1}, P_{u}^{n-1}, F_{u}^{n}, R_{u}^{n}, D_{u}^{n}, V_{u}^{n} \right) \right) \right)
\]

\[
F_{R}^{1} = \frac{1}{4\Delta t} \sum_{n=2}^{N} \left( I_{u}^{n-1} F_{R} \left( t_{u}^{n-1}, S_{u}^{n-1}, P_{u}^{n-1}, F_{u}^{n-1}, P_{u}^{n-1}, F_{u}^{n}, R_{u}^{n}, D_{u}^{n}, V_{u}^{n} \right) + \frac{\Delta t}{\Delta x} \sum_{n=2}^{N} \left( I_{u}^{n-1} F_{R} \left( t_{u}^{n-1}, S_{u}^{n-1}, P_{u}^{n-1}, F_{u}^{n-1}, P_{u}^{n-1}, F_{u}^{n}, R_{u}^{n}, D_{u}^{n}, V_{u}^{n} \right) \right) \right)
\]
\[ D^{\alpha+1} = \frac{d}{dt} \int_{0}^{t} t^{\alpha-1} D^\alpha \left( t, S', F, \Gamma, \Gamma, \Gamma, \Gamma, \Gamma, R', D', V' \right) \]
\[ + \frac{d}{dt} \int_{0}^{t} t^{\alpha-2} D^\alpha \left( t, S', S'' - 2, F', R', D', V' \right) \]
\[ P^{\alpha+1} = \frac{d}{dt} \sum_{n=0}^{2} \int_{0}^{t} t^{\alpha-2} D^\alpha \left( t, S'' - 2, F', R', D', V' \right) \]
\[ + \frac{d}{dt} \sum_{n=0}^{2} \int_{0}^{t} t^{\alpha-2} D^\alpha \left( t, S'' - 2, F', R', D', V' \right) \]
\[ V^{\alpha+1} = \frac{d}{dt} \sum_{n=0}^{2} \int_{0}^{t} t^{\alpha-2} D^\alpha \left( t, S', F', \Gamma, \Gamma, \Gamma, \Gamma, R', R', D', V' \right) \]
\[ + \frac{d}{dt} \sum_{n=0}^{2} \int_{0}^{t} t^{\alpha-2} D^\alpha \left( t, S', S'' - 2, F', R', D', V' \right) \]
\[ + \frac{d}{dt} \sum_{n=0}^{2} \int_{0}^{t} t^{\alpha-2} D^\alpha \left( t, S', S'' - 2, F', R', D', V' \right) \]
\[ + \frac{d}{dt} \sum_{n=0}^{2} \int_{0}^{t} t^{\alpha-2} D^\alpha \left( t, S', S'' - 2, F', R', D', V' \right) \]
\[ \Gamma^{\alpha+1} = \frac{d}{dt} \int_{0}^{t} t^{\alpha-1} D^\alpha \left( t, S'' - 2, F', R', D', V' \right) \]
\[ + \frac{d}{dt} \int_{0}^{t} t^{\alpha-1} D^\alpha \left( t, S'' - 2, F', R', D', V' \right) \]
\[ + \frac{d}{dt} \int_{0}^{t} t^{\alpha-1} D^\alpha \left( t, S'' - 2, F', R', D', V' \right) \]
\[ + \frac{d}{dt} \int_{0}^{t} t^{\alpha-1} D^\alpha \left( t, S'' - 2, F', R', D', V' \right) \]

For power-law kernel, we can get the following numerical scheme:

\[ S^{\alpha+1} = \frac{\left( \alpha \right)}{\Gamma \left( \alpha + 1 \right)} \sum_{n=0}^{2} \int_{0}^{t} t^{\alpha-2} S^\alpha \left( t, S'' - 2, F', R', D', V' \right) \]
\[ R^{\alpha+1} = \frac{\left( \alpha \right)}{\Gamma \left( \alpha + 1 \right)} \sum_{n=0}^{2} \int_{0}^{t} t^{\alpha-2} S^\alpha \left( t, S'' - 2, F', R', D', V' \right) \]
\[ + \frac{\left( \alpha \right)}{\Gamma \left( \alpha + 1 \right)} \sum_{n=0}^{2} \int_{0}^{t} t^{\alpha-2} S^\alpha \left( t, S'' - 2, F', R', D', V' \right) \]
\[ \Gamma^{\alpha+1} = \frac{\left( \alpha \right)}{\Gamma \left( \alpha + 1 \right)} \sum_{n=0}^{2} \int_{0}^{t} t^{\alpha-2} S^\alpha \left( t, S'' - 2, F', R', D', V' \right) \]
\[ + \frac{\left( \alpha \right)}{\Gamma \left( \alpha + 1 \right)} \sum_{n=0}^{2} \int_{0}^{t} t^{\alpha-2} S^\alpha \left( t, S'' - 2, F', R', D', V' \right) \]

The above equations represent a novel Covid-19 model with fractional differential operators with singular and non-singular kernels.
\[ F_R^{t+1} = \frac{\Delta t}{\Gamma(t+1)} \sum_{\nu=2}^{\infty} \int_{t_{\nu-2}}^{t_{\nu-1}} F_R(t_{\nu-2}, S_{\nu-2}^{-2}, R_{\nu-2}^{-2}, D_{\nu-2}^{-2}, V_{\nu-2}^{-2}) \Pi \\
+ \frac{\Delta t}{\Gamma(t+1)} \sum_{\nu=2}^{\infty} \left[ t_{\nu-1}^{-1} F_R(t_{\nu-1}, S_{\nu-1}^{-2}, R_{\nu-1}^{-2}, D_{\nu-1}^{-2}, V_{\nu-1}^{-2}) \right] \] 
\[ + \frac{\Delta t}{\Gamma(t+1)} \sum_{\nu=2}^{\infty} \left[ t_{\nu-1}^{-1} F_R(t_{\nu-1}, S_{\nu-1}^{-2}, R_{\nu-1}^{-2}, D_{\nu-1}^{-2}, V_{\nu-1}^{-2}) \right] \] 
\[ \Sigma \] 
\[ R^{t+1} = \frac{\Delta t}{\Gamma(t+1)} \sum_{\nu=2}^{\infty} \left[ t_{\nu-1}^{-1} R(t_{\nu-1}, S_{\nu-1}^{-2}, R_{\nu-1}^{-2}, D_{\nu-1}^{-2}, V_{\nu-1}^{-2}) \right] \] 
\[ + \frac{\Delta t}{\Gamma(t+1)} \sum_{\nu=2}^{\infty} \left[ t_{\nu-1}^{-1} R(t_{\nu-1}, S_{\nu-1}^{-2}, R_{\nu-1}^{-2}, D_{\nu-1}^{-2}, V_{\nu-1}^{-2}) \right] \] 
\[ \Delta \]

9. Numerical Simulation

In this section, using the numerical solutions obtained in the previous section, we present numerical method for all cases. The numerical simulations are depicted for different values of fractional order and fractal dimension as presented in Fig. 1–15.

\[ \sigma^{DM} D_0^+ S = \lambda - \beta (\sigma T + m T_0 + \gamma w T_2 + \alpha \beta F_0 + \theta \beta F_0 + \mu_0 + \mu_1) S \]
\[ \sigma^{DM} D_0^+ I = \lambda (\alpha T + m T_0 + \gamma w T_2 + \alpha \beta F_0 + \theta \beta F_0 + \mu_0 + \mu_1) I \]
\[ \sigma^{DM} D_0^+ T = \lambda (\alpha T + m T_0 + \gamma w T_2 + \alpha \beta F_0 + \theta \beta F_0 + \mu_0 + \mu_1) T \]
\[ \sigma^{DM} D_0^+ F = \lambda (\alpha T + m T_0 + \gamma w T_2 + \alpha \beta F_0 + \theta \beta F_0 + \mu_0 + \mu_1) F \]
\[ \sigma^{DM} D_0^+ R = \lambda (\alpha T + m T_0 + \gamma w T_2 + \alpha \beta F_0 + \theta \beta F_0 + \mu_0 + \mu_1) R \]

\[ \sigma^{DF} D_0^+ S = \lambda - \beta (\sigma T + m T_0 + \gamma w T_2 + \alpha \beta F_0 + \theta \beta F_0 + \mu_0 + \mu_1) S \]
\[ \sigma^{DF} D_0^+ I = \lambda (\alpha T + m T_0 + \gamma w T_2 + \alpha \beta F_0 + \theta \beta F_0 + \mu_0 + \mu_1) I \]
\[ \sigma^{DF} D_0^+ T = \lambda (\alpha T + m T_0 + \gamma w T_2 + \alpha \beta F_0 + \theta \beta F_0 + \mu_0 + \mu_1) T \]
\[ \sigma^{DF} D_0^+ F = \lambda (\alpha T + m T_0 + \gamma w T_2 + \alpha \beta F_0 + \theta \beta F_0 + \mu_0 + \mu_1) F \]
\[ \sigma^{DF} D_0^+ R = \lambda (\alpha T + m T_0 + \gamma w T_2 + \alpha \beta F_0 + \theta \beta F_0 + \mu_0 + \mu_1) R \]
Fig. 1  Numerical visualization for Covid-19 model with $\alpha = 0.01, \delta_0 = 0.99, \delta_1 = 0.5, \delta_2 = 0.4$ for different alphas.

Fig. 2  Numerical visualization for Covid-19 model with $\alpha = 0.01, \delta_0 = 0.99, \delta_1 = 0.5, \delta_2 = 0.4$ for different alphas.

Fig. 3  Numerical visualization for Covid-19 model with $\alpha = 0.01, \delta_0 = 0.99, \delta_1 = 0.5, \delta_2 = 0.4$ for different alphas.
Fig. 4 Numerical visualization for Covid-19 model with $\tau = 1$, $\alpha = 0.01$, $\delta_0 = 0.99$, $\delta_1 = 0.5$, $\delta_2 = 0.4$ for different alphas.

Fig. 5 Numerical visualization for Covid-19 model with $\tau = 1$, $\alpha = 0.1$, $\delta_0 = 0.99$, $\delta_1 = 0.5$, $\delta_2 = 0.4$ for different alphas.

Fig. 6 Numerical visualization for Covid-19 model with $\tau = 1$, $\alpha = 0.1$, $\delta_0 = 0.99$, $\delta_1 = 0.5$, $\delta_2 = 0.4$ for different alphas.
Fig. 7  Numerical visualization for Covid-19 model with $\alpha = 0.1, \delta_0 = 0.99, \delta_1 = 0.5, \delta_2 = 0.4$ for different alphas.

Fig. 8  Numerical visualization for Covid-19 model with $\alpha = 0.1, \delta_0 = 0.99, \delta_1 = 0.5, \delta_2 = 0.4$ for different alphas.

Fig. 9  Numerical visualization for Covid-19 model with $\alpha = 0.1, \delta_0 = 0.99, \delta_1 = 0.5, \delta_2 = 0.4$ for different alphas.
Fig. 10  Numerical visualization for Covid-19 model with $\varphi = 0.1, \delta_0 = 0.99, \delta_1 = 0.5, \delta_2 = 0.4$ for different alphas.

Fig. 11  Numerical visualization for Covid-19 model with $\varphi = 0.3, \delta_0 = 0.99, \delta_1 = 0.5, \delta_2 = 0.4$ for different alphas.

Fig. 12  Numerical visualization for Covid-19 model with $\varphi = 0.3, \delta_0 = 0.99, \delta_1 = 0.5, \delta_2 = 0.4$ for different alphas.
Fig. 13  Numerical visualization for Covid-19 model with $x = 0.3, \delta_0 = 0.99, \delta_1 = 0.5, \delta_2 = 0.4$ for different alphas.

Fig. 14  Numerical visualization for Covid-19 model with $x = 0.3, \delta_0 = 0.99, \delta_1 = 0.5, \delta_2 = 0.4$ for different alphas.

Fig. 15  Numerical visualization for Covid-19 model with $x = 0.3, \delta_0 = 0.99, \delta_1 = 0.5, \delta_2 = 0.4$ for different alphas.
where initial conditions are
\[ S(0) = 800000, I(0) = 3, I_A(0) = 0, I_D(0) = 0, I_R(0) = 0, \]
\[ I_T(0) = 0, R(0) = 0, D(0) = 0, V(0) = 0. \]

Also, the parameters are chosen as
\[ A = 810000, \alpha = 0.12, \gamma = 0.15, \alpha = 0.5, \tau = 0.4, \beta = 0.09, \beta = 0.75, \]
\[ \gamma_1 = 0.4, \mu_1 = 0.3, \xi = 0.161, \zeta = 0.015, \sigma = 0.015, \delta_0 = 0.99, \]
\[ \tau = 0.0199, \Phi = 0.015, \lambda = 0.0345, \phi = 0.0345, \delta_i = 0.5, \]
\[ \Delta t = 900, \delta_0 = 30, \delta_2 = 0.4, \omega = 0.4. \]

Fig. 16  Comparison of collected data with mathematical model for a period of 30 days.

Fig. 17  Comparison of collected data with mathematical model for a period of 60 days.
10. Comparison between mathematical model and experimental data in Turkey

In this section, we compare the numerical solution of the Atangana-Baleanu fractional derivative with experimental data from Turkey [25]. The data used are for 30 days from 11 March to 9 April, 60 days from 11 March to 9 May and finally from 11 March to 8 June 2020. Collected data represent the daily numbers of infection within Turkey. The comparison is performed between the suggested model with Atangana-Baleanu derivative and collected data in Fig. 16 for 30 days, in Fig. 17 for 60 days and finally in Fig. 18 for 90 days. In the first 30 days, the daily numbers of new infections followed an exponential spread, after 60 to 90 days the daily numbers followed a lognormal distribution. The comparison between collected data and mathematical model although not exactly in perfect agreement but most of the points are depicted by the mathematical models. This comparison was obtained for a fractional order 1.009. We stress on the fact that our model is the perfect one. However the model can be used to provide a trend of the spread in different countries.

11. Conclusion

Since the breakout of the novel Covid-19 in last December 2019, mathematical models have been suggested, all with their advantages and disadvantages, however, all of them aim at predicting the spread of Covid-19 in a given population. However, within the available literature there is mathematical model that can accurately depict the dynamic spread of the novel Covid-19. This perhaps due to the lack of understanding of the real world situation, thus, the conversion to mathematical model is also full of uncertainties and limitations. For example, very recently a complex system of linear and nonlinear equations with nine classes was introduced by Atangana and Seda, the model took into account many possibilities, however excluded the possibility of transmission between the class of individuals that have contacted and one is critical condition with susceptible. Thus, in this work, we included such effect and presented some important analysis. In particular, we introduced the global asymptotic stability using fractional Lyapunov. Numerical analysis using the numerical method based on Newton polynomial were used to solve the suggested model.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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