A GL Double Layer EM Cloaks In Broad Frequency Band And Reciprocal Law

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In this paper, we propose a novel GL double layer EM cloak in the broad frequency band. In short, we call it as GLWF double EM cloak. The GLWF double layer cloak (GLWF cloak) consists of two sphere annular layers, two type cloak materials are proposed and installed in its each layer, respectively. The outer layer of the GL cloak has the invisible function in broad frequency band, while its inner layer has the fully absorption and rapid delay function. The outer layer cloaks the Local concealment from the Global exterior EM field; The inner layer cloaks the Global free space region from the Local field excited inside the concealment. The GLWF double layer cloak overcomes the following difficulties of the single layer PS cloak. (1) There exists no EM wavefield can be excited inside the concealment of the PS cloak, the concealment of the PS cloak is blind. Our GL double layer cloak recovered the normal EM environment in its concealment, such that there exists no EM wavefield in our GL double layer cloak. (2) There is exceeding light speed physical violation in PS cloak, its invisibility only in very narrow frequency band. Our GLWF double layer cloak corrects the violation. (3) The reciprocal law is satisfied in our GL double cloak media. However, the PS cloak damaged the reciprocal law. The simulations and comparisons of the EM wave field propagation through the GLWF double cloak, GL double cloak and GL double layer with PS outer layer are presented to show the advantages of the GLWF double layer EM cloak. The GL double layer cloaks are proposed by our GL EM modeling and inversion. The 3D GL EM modeling simulations for the double layer cloak are presented. The copyright and patent of the GLWF double layer cloak materials and GL EM modeling and inversion in this paper are reserved by authors in GL Geophysical Laboratory.

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I. INTRODUCTION

We have proved that there exists no Maxwell wavefield can be excited by sources inside the single layer cloaked concealment with normal materials [1]. For covering this difficulty, we proposed a GL double layer cloak [2]. Its outer layer has invisibility from the exterior light and EM wave field, never disturb the exterior field, and cloak the Local inner layer and concealment from the Global exterior wavefield. Its inner layer cloak absorbs the internal wave field, such that the internal wave field, which is excited from the sources inside the concealment, can not propagate outside of the inner layer. Our GL double layer cloak is proposed by the GL EM modeling and inversion [3]. The GL double layer cloak overcomes the following difficulties of the single layer PS cloak by Pendry [4]. (1) The PS cloak damaged the EM environment of its concealment, such that there exists no EM wavefield can be excited inside the concealment of the PS cloak, the concealment of the PS cloak is blind. Our GL double layer cloak recovered the normal EM environment in its concealment, such that the EM wave field can be excited inside the concealment of the GL double layer cloak. (2) The two sources reciprocal law is very important principle in the electromagnetic theory and application. The reciprocal law is satisfied in our GL double cloak media. However, the reciprocal law is damaged by the PS cloak. The PS cloak is strong dispersive cloak material, in which the \( \varepsilon_1 \varepsilon_\theta = R_3^2 (r - R_2)^2 / r^2 / (R_3 - R_2)^2 \), when \( r \to R_2 \varepsilon_1 \varepsilon_\theta \approx (r - R_2)^2 \to 0 \). such that the EM wave velocity exceed the light speed in some part of the PS cloak that is violated to the physical principle. The PS cloak has invisibility only in very narrow frequency band. In the GL outer layer cloak [2] [5] [6], the weak dispersion is obtained, when \( r \to R_2 \varepsilon_1 \varepsilon_\theta \approx (r - R_2)^{1.5} \to 0 \). In this paper, we obtain very significant progress to overcome the difficulty. very perfect weak dispersive and degenerative rate, \( \varepsilon_1 \varepsilon_\theta \approx 1 / \log(r - R_2) \to 0 \), is obtained. Chen et al proposed an analytical method for analysis of the PS cloak [7].

Finding and exploration is inverse problem; Hiding and cloakning is other inverse problem. They have close relationship. Based on the 3D GL EM modeling simulations[3] and GL Metre Carlo inversion [5], We propose a novel GL double layer cloak in the broad frequency band.

The President Professor Yuesheng Li in Sun Yat-Sen University very concerns and encourages our research works on the GL EM modeling method and the GL double cloak. Our paper is for celebrating his great scientific works on the GL EM modeling method and the GL double layer with PS outer layer are presented to show the advantages of the GLWF double layer EM cloak. The GL double layer cloaks are proposed by our GL EM modeling and inversion. The 3D GL EM modeling simulations for the double layer cloak are presented. The copyright and patent of the GLWF double layer cloak materials and GL EM modeling and inversion in this paper are reserved by authors in GL Geophysical Laboratory.

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propagation simulations in our wide frequency band GL double layer cloak are presented in Section 3. The important reciprocal law in the cloak media is presented in Section 4. In Section 5, we will conclude our paper.

II. THE GL DOUBLE LAYER CLOAK MATERIALS IN BROAD FREQUENCY BAND

For overcoming the weakness of the single layer cloak, we proposed the GL double layer cloak in broad frequency band, in short it is called GLWF, which consists of the inner layer cloak and outer layer cloak

A. GLWF Inner layer Cloak Anisotropic Material

On the inner sphere annular layer domain, $\Omega_{GLI} = \{r : R_1 \leq r \leq R_2\}$, by the GL EM modeling and inversion $[3][5]$, we propose an anisotropic metamaterial as follows,

$$
[D]_{GLI} = \text{diag} \left[ \varepsilon_r, \mu_r \right], \varepsilon_r = \text{diag} \left[ \varepsilon_r, \varepsilon_0, \varepsilon_\phi \right], \mu_r = \mu_0, \\
\varepsilon_r = \left( \frac{R_2^2 - R_1^2}{R_2^2 - R_1^2} \right) \varepsilon_r, \\
\varepsilon_\phi = \varepsilon_0, \mu_0 = \mu_0 \sqrt{\frac{R_2^2 - R_1^2}{R_2^2 - R_1^2}}.
$$

The $\Omega_{GLI}$ is called as GL inner layer cloak, the materials, $[D]_{GLI} = \text{diag} \left[ \varepsilon_r, \mu_r \right]$ in (1), are the anisotropic GL inner layer cloak metamaterials. Where the subscript GLI means the GL inner layer, the symbol $\text{diag}$ denotes the diagonal matrix, $[D]_{GLI}$ is 6x6 diagonal matrix, $\varepsilon_i$ is 3x3 dielectric diagonal matrix in the inner layer, the subscript $i$ denotes the inner layer, $\mu_i$ is 3x3 magnetic permeability diagonal matrix in the inner layer, $\varepsilon_\theta,i$ is the relative dielectric cloak metamaterial which is formulated by the fourth sub equation in (1), the subscript index $r, i$ denotes the dielectric is in $r$ direction and in the inner layer, $\varepsilon_\theta,i$ is the relative dielectric cloak metamaterial in $\theta$ direction and in the inner layer which is formulated by the fifth sub equation in (1). The $\mu_r,i$ is the relative permeability cloak metamaterial which is formulated by the fourth sub equation in (1), $\varepsilon_b$ is the basic dielectric in free space, $\varepsilon_b = 8.8541878176 \times 10^{-11}$ F/m, $\mu_b$ is the basic magnetic permeability in free space, $\mu_b = 1.25663706 \times 10^{-6} \text{kg} \cdot \text{s}^{-2} \cdot \text{A}^{-2}$, other symbols in (1) have similar explanation.

The EM wavefield of sources located inside inner layer or the cloaked concealment is completely absorbed by the inner layer and never reaches the outside boundary of the inner layer, also, the EM wavefield excited in concealment is not disturbed by the cloak. The inner layer metamaterial, in equation (1), cloaks outer space from the local field excited in the inner layer and concealment, which can also be useful for making a complete absorption boundary condition to truncate infinite domain in numerical simulation.

B. GLWF Outer layer Cloak Anisotropic Material

We propose a new novel GLWF outer layer cloak in this section. Let the outer sphere annular layer domain $\Omega_{GLO} = \{r : R_2 \leq r \leq R_3\}$ be the GL outer layer cloak with the following anisotropic GLWF outer layer cloak metamaterials,

$$
[D]_{GLWFO} = \text{diag} \left[ \varepsilon_r, \mu_r \right], \\
\varepsilon_r = \text{diag} \left[ \varepsilon_r, \varepsilon_0, \varepsilon_\phi \right], \\
\mu_r = \mu_0, \\
\varepsilon_r = \left( \frac{R_2^2 - R_1^2}{R_2^2 - R_1^2} \right) \varepsilon_r, \\
\varepsilon_\phi = \varepsilon_0, \mu_0 = \mu_0 \sqrt{\frac{R_2^2 - R_1^2}{R_2^2 - R_1^2}}.
$$

The GL outer layer cloak $[2]$ is presented as follows,

$$
[D]_{GLO} = \text{diag} \left[ \varepsilon_r, \mu_r \right], \\
\varepsilon_r = \text{diag} \left[ \varepsilon_r, \varepsilon_0, \varepsilon_\phi \right], \\
\mu_r = \mu_0, \\
\varepsilon_r = \left( \frac{R_2^2 - R_1^2}{R_2^2 - R_1^2} \right) \varepsilon_r, \\
\varepsilon_\phi = \varepsilon_0, \mu_0 = \mu_0 \sqrt{\frac{R_2^2 - R_1^2}{R_2^2 - R_1^2}}.
$$

We chose the PS cloak $[4]$ as outer layer cloak

$$
[D]_{PSO} = \text{diag} \left[ \varepsilon_{\phi,PSO}, \mu_{\phi,PSO} \right], \\
\varepsilon_{\phi,PSO} = \varepsilon_\phi, \\
\mu_{\phi,PSO} = \mu_\phi, \\
\varepsilon_{\phi,PSO} = \left( \frac{R_2^2 - R_1^2}{R_2^2 - R_1^2} \right) \varepsilon_{\phi,PSO}, \\
\mu_{\phi,PSO} = \mu_\phi.
$$

Where the subscript GLWF outer layer, $[D]_{GLWFO}$ is 6x6 diagonal matrix, $\varepsilon_{\phi,PSO}$ is 3x3 dielectric diagonal matrix in the outer layer, the subscript $o$ denotes the outer layer, $\mu_{\phi,PSO}$ is 3x3 magnetic permeability diagonal matrix in the outer layer, $\varepsilon_{\phi,PSO}$ is the relative dielectric cloak metamaterial which is formulated by the fourth sub equation in (2), the subscript index $r, glwfo$ denotes the dielectric is in $r$ direction and in the outer layer, $\varepsilon_\phi,glwfo$ is the relative dielectric cloak metamaterial in $\phi$ direction and in the outer layer which is formulated by the fifth sub equation in (2). The $\mu_{\phi,glwfo}$ is the relative permeability cloak in outer layer and in $r$ direction which is formulated by the fourth sub equation in (2). The $\varepsilon_{\phi,glwfo}$ is the relative dielectric cloak in outer layer and in $\phi$ direction which is formulated by the fifth sub equation in (2). Similar explanation for GL outer layer cloak GLO and PS cloak as outer layer PSO, The outer layer cloak provides invisibility, does not disturb exterior EM wave field, and cloaks the local concealment from the global exterior EM wavefield.
C. GLWF Double Layer Cloak

The GL inner cloak $\Omega_{GLI}$ domain and GL outer cloak $\Omega_{GLWFO}$ domain are bordering on the sphere annular surface $r = R_2$. We assemble the $\Omega_{GLI}$ as the inner sphere annular domain and $\Omega_{GLWFO}$ as the outer sphere annular domain and make them coupling on their interface boundary annular surface $r = R_2$ as follows,

$$\Omega_{GLWF} = \Omega_{GLI} \cup \Omega_{GLWFO} = \{r : R_1 \leq r \leq R_2\} \cup \{r : R_2 \leq r \leq R_3\}$$

(5)

and situate the double layer anisotropic dielectric and magnetic permeability susceptibility tensors $[D]_{GLWF}$ on the $\Omega_{GLF}$ as follows,

$$[D]_{GLWF} = \begin{cases} [D]_{GLI}, r \in \Omega_{GLI} \\ [D]_{GLWFO}, r \in \Omega_{GLWFO}. \end{cases}$$

(6)

The GL cloak material $[D]_{GLI} = diag[\bar{\varepsilon}_i, \mu_i]$ in (1) on the inner layer domain $\Omega_{GLI}$ and GL outer layer cloak material $[D]_{GLWFO} = diag[\bar{\varepsilon}_{glwfo}, \mu_{glwfo}]$ in (2) on outer layer domain $\Omega_{GLWFO}$ are assembled into the GL anisotropic doubled layer cloak material on the domain $\Omega_{GLWF}$. The domain $\Omega_{GLWF}$ with the metamaterial $[D]_{GLWF}$ in (6) is called as the GL double layer cloak in broad frequency band, say GLWF double cloak. Similar explanations are for GL double layer cloak coupled by GLI in (1) and GLO in (3), and GLI - PSO double layer cloak coupled by GLI in (1) and PSO in (4).

The GL cloak means that the outer layer has invisibility, never disturb exterior wavefield and cloaks the Local concealment from the exterior Global field, while the inner layer cloaks outer Global space from interior wavefield excited in the Local concealment. The properties of the double layer material are studied by GL method simulations and analysis in next sections.

III. THE GL EM SIMULATIONS AND COMPARISON OF THE GLWF DOUBLE CLOAKS AND PENTRY CLOAK

A. The Simulation Model of The GLWFO, GLIO, GLI-PSO Double Layer Cloak

The simulation model: the 3D domain is $[-0.5m, 0.5m] \times [-0.5m, 0.5m] \times [-0.5m, 0.5m]$, the mesh number is $201 \times 201 \times 201$, the mesh size is 0.005m. The electric current point source is defined as

$$\delta(r - r_s) \delta(t) \vec{e},$$

(7)

where the $r_s$ denotes the location of the point source, the unit vector $\vec{e}$ is the polarization direction, the time step $dt = 0.3333 \times 10^{-10}$ second, the frequency band is from 0.05 GHz to 15 GHz, the largest frequency $f = 15GHz$, the shortest wave length is 0.02m. The inner layer $\Omega_{GLI}$ is denoted by (1). The EM GL double layer cloak $\Omega_{GLWF} = \Omega_{GLI} \cup \Omega_{GLWFO}$ in (2), $\Omega_{GLIO} = \Omega_{GLI} \cup \Omega_{GLO}$ in (3), and $\Omega_{GLI-PSO} = \Omega_{GLI} \cup \Omega_{PSO}$ in (4) are consist of the double spherical annular with the center in the origin and interir radius $R_1 = 0.21m$, meddle radius $R_2 = 0.31m$, and exterior radius $R_3 = 0.45m$. The cloak is divided into $90 \times 180 \times 90$ cells. The spherical coordinate is used in the sphere $r \leq R_3$, the Cartesian rectangular coordinate is used in outside $\Omega_{GL}$ to mesh the domain.

B. Point Source $S_1$ In The Concealment And Other Point Source $S_2$ In The Free Space

The two point sources $S_1$ and $S_2$ are used to excite the EM wave propagation through the GLWF, GLO, and GL-PSO double layer cloaks. The first point current source $S_1$ is located inside of the center sphere concealment at $(-0.18m, 0.0, 0.0)$, the excited EM wave excided by $S_1$ is named as First electric wave, $E_{xx,1}$. The second point current source $S_2$ is located in free space at $(0.73m, 0.0, 0.0)$ where is located the right side outside of the whole GL double layer cloaks. The EM wave excited by the $S_2$ is named as Second electric wave, $E_{xx,2}$.
C. Comparison Between EM Wave Propagation
Through The GLWF, GLO,And GLI-PSO Double Layer Cloaks

The GL modeling simulations of the EM wave excited by above two point sources \( S_1 \) and \( S_2 \) propagation through the GLWF, GLO, and GLI-PSO double layer cloaks are presented in the Figures 1-3 at 48th time step, in Figures 4-6 at 74th time step, in Figures 7-9 at 128th time step, respectively. For comparison, we arrange Figures 1-3, Figures 4-6, and Figures 7-9 as four figure group rows. At time step 48\( dt \), front of electric wave \( E_{xx,2} \) inside of the outer layer of GLWF, GLO outer layer cloak, and PS outer layer, \( R_2 \leq r \leq R_3 \) are presented in Figures 1-3 respectively. In Figure 1, \( E_{xx,2} \) inside of the outer layer of GLWF, \( R_2 \leq r \leq R_3 \), propagates no faster than light speed, it is faster than the wave speed in figure 2 and in figure 3. In Figure 2 electric wave \( E_{xx,2} \) inside of the outer layer of GL outer layer cloak \( R_2 \leq r \leq R_3 \), propagates no faster than light speed. It is slower than \( E_{xx,2} \) in figure 1 and faster than \( E_{xx,2} \) in figure 3. In Figure 3, front of \( E_{xx,2} \) inside of the outer layer of PS outer layer cloak, \( R_2 \leq r \leq R_3 \), propagates no faster than light speed. It is slower than \( E_{xx,2} \) in figure 1 and figure 2. The wave front of the \( E_{xx,1} \), propagates inside the inner layer, \( R_1 \leq r \leq R_2 \). At time step 74\( dt \), front of electric wave \( E_{xx,2} \) inside of the outer layer of GLWF, GLO outer layer cloak, and PS outer layer, \( R_2 \leq r \leq R_3 \) are presented in Figures 4-6 respectively. In Figure 4, front of \( E_{xx,2} \) inside of GLWF outer layer cloak \( R_2 \leq r \leq R_3 \), propagates no faster than light speed, it is slower than the wave speed in figure 5 and in figure 6. In Figure 5, front of \( E_{xx,2} \) inside of GL outer layer cloak \( R_2 \leq r \leq R_3 \) propagates little faster than light speed. It is faster than \( E_{xx,2} \) in GLWF in figure 4 and slower than \( E_{xx,2} \) in PS in figure 6. In Figure 6, front of \( E_{xx,2} \) inside of PS outer layer cloak \( R_2 \leq r \leq R_3 \) propagates faster than light speed. It is faster than \( E_{xx,2} \) in GLWF in figure 4 and GLO in figure 5. At time step 128\( dt \), front of electric wave \( E_{xx,2} \) inside of the outer layer of GLWF, GLO outer layer cloak, and PS outer layer, \( R_2 \leq r \leq R_3 \) are presented in Figures 7-9 respectively. In Figure 7, front of \( E_{xx,2} \) inside of GLWF outer layer cloak \( R_2 \leq r \leq R_3 \) propagates slower than light speed, it is slower than the wave speed in figure 8 and in figure 9. In Figure 8, front of \( E_{xx,2} \) inside of GL outer layer cloak \( R_2 \leq r \leq R_3 \) propagates little faster than light speed. It is faster than \( E_{xx,2} \) in GLWF in figure 7 and slower than \( E_{xx,2} \) in PS in figure 9. In Figure 9, front of \( E_{xx,2} \) inside of PS outer layer cloak \( R_2 \leq r \leq R_3 \) propagates more faster than light speed. It is faster than \( E_{xx,2} \) in GLWF in figure 7 and GLO in figure 8. The wave front of the \( E_{xx,1} \), propagates inside the inner layer, \( R_1 \leq r \leq R_2 \).

IV. THEORY OF RECIPROCAL LAW OF THE 
EM WAVE FIELD THROUGH THE CLOAKS

A. Theory Of The EM Wave Field Through The 
GL Double Layer Cloaks

We propose the theoretical analysis of the interaction between the EM wave and GL cloaks in this section.
Statement 1: Let domain $\Omega_{GL}$ in (3) and the metamaterial $D_{GL}$ in (4) be GL double layer cloak, and $\varepsilon = \varepsilon_0$, $\mu = \mu_0$ be basic permittivity and permeability, respectively, inside of the central sphere concealment $|\vec{r}'| < R_1$, the excited EM wave field inside of the concealment never be disturbed by the cloak; (2) provide the local source is located inside of concealment or inside of the inner layer of the GL double layer cloak, $|\vec{r}'| < R_2$, the EM wave field is vanished outside of the inner layer of GL cloak and is always propagating and going to the boundary $r = R_2$ and before $r = R_2$. (3) provide the
source is located outside of the GL double layer cloak, $|\vec{r}_s| > R_3$, the excited EM wave field propagation outside of the double layer cloak as same as in free space and never be disturbed by the double layer cloak; (4) provide the local source is located outside of double layer cloak or located inside of the outer layer of GL cloak, $|\vec{r}_s| > R_2$, the excited EM wave field never propagate into the inner layer of GL cloak and the concealment.

**Statement 2:** (1) In the domain consist of free space, single layer cloak and its cloaked concealment with normal material, the two sources reciprocal law is damaged. (2) In the domain consist of free space, single layer cloak and its cloaked concealment with some special double negative refractive index metamaterial, the two sources reciprocal law is recovered, but the cloak invisibility function is lose. (3) In the domain consist of free space, GL double layer cloak and its cloaked concealment with normal material, the two sources reciprocal law is satisfied, and the cloak invisibility function is complete and sufficient in wide frequency band.
C. There Exists No Maxwell EM Wavefield Can Be Excited By Nonzero Local Sources Inside Of The Single Layer Cloaked Concealment With Normal Materials

Statement 3: Suppose that a 3D anisotropic inhomogeneous single layer cloak domain separates the whole 3D space into three sub domains, one is the single layer cloak domain \( \Omega_{ctk} \) with the cloak material; the second one is the cloaked concealment domain \( \Omega_{con} \) with normal EM materials; other one is the free space outside of the cloak. If the Maxwell EM wavefield excited by a point source or local sources outside of the concealment \( \Omega_{con} \) is vanished inside of the concealment \( \Omega_{con} \), then there exists no Maxwell EM wave field can be excited by the local sources inside the cloaked concealment \( \Omega_{con} \) with normal materials.

The statement 2 is proved by the GL method in author’s paper [12].

V. HISTORY AND DISCUSSIONS

A. History

A double layer cloth phenomenon to prevent the GILD inversion [6][8] detection has been observed in paper [9] in 2001 which is published in SEG online http://segdl.org/journals/doc/SEGLIB-home/dci/searchDCI.jsp. The double layer cloth to cloak fly from the exterior wave GILD detection is obvious around the fly which is shown in figure 10; the double cloth around the bar is shown in figure 11. We developed a novel and effective Global and Local field (GL) modeling and inversion[2][3][5] to study the meta materials, periodic photonic crystals and condense physics etc. wide physical sciences. 3D GL EM modeling and inversion [3][5] and computational mirage have been presented in P I E R S 2005 and published in proceeding of P I E R S 2005 in Hangzhou, which can be downloaded from http://piers.mit.edu/piersproceedings/piers2k5Proc.php please see the references of [2]. We developed 3D FEM for the elastic mechanics first in China in 1972[10] and discovered the superconvergence of the 3D cubic curve isoparameter element first in the world [11]. The 3D isoparameter element can be used for making arbitrary curve cloak [10]. We deeply to know the merits and drawbacks of FEM. The GL method overcomes the drawbacks of FEM and FD methods. The history of development of FEM and GILD and GL method has been described in [11] and reference of [2]. The 3D and 2D GL parallel software is made and patented by GLGEO. The GL modeling and its inversion [3][5] and GL EM quantum field modeling are suitable to solve quantization scattering problem of the electromagnetic field in the dispersive and loss metamaterials, cloaks and more wide anisotropic materials.

B. Advantages Of The GL Method

The GL EM modeling is fully different from FEM and FD and Born approximation methods and overcome their difficulties. There is no big matrix equation to solve in GL method. Moreover, it does not need artificial boundary and absorption condition to truncate the infinite domain. Born Approximation is a conventional method in the quantum mechanics and solid physics However, it is one iteration only in whole domain which is not accurate for high frequency and for high contrast materials. The GL method divides the domain as a set of small sub domains or sub lattices. The Global field is updated by the local field from the interaction between the global field and local subdomain materials successively. Once all subdomain materials are scattered, the GL field solution is obtained which is much more accurate than the Born approximation. GL method is suitable for all frequency and high contrast materials. When the size of the sub domain is going to zero, the GL method is convergent and has \( O(h^2) \) if the trapezoidal integral formula is used, moreover, is has super convergence \( O(h^4) \) if the Gaussian integral formula is used[10]. Chen et al proposed an analytical method for analysis of the PS cloak [7]. The GL method has double capabilities of the theoretical analysis and numerical simulations that has been shown in this paper.

VI. CONCLUSIONS

The simulations of the EM wave propagation through the GLWF. GLO, and GLI-PSO double layer cloaks and comparison between them show that the GLWF and GLO double layer cloak overcomes the following difficulties of the single layer PS cloak. (1) The PS cloak damaged the EM environment of its concealment, such that there exists no EM wavefield can be excited inside the concealment of the PS cloak, the concealment of the PS cloak is blind. Our GL double layer cloak recovered the normal EM environment in its concealment, such that the EM wave field can be excited inside the concealment of the GL double layer cloak. (2) The PS cloak is very strong dispersive and strong degenerative cloak material. The PS cloak has invisibility only in very narrow frequency band. There is exceeding light speed physical violation in PS cloak. Our GLWF double layer cloak corrects the violation. (3) The two sources reciprocal law is very important principle in the electromagnetic theory and application. The reciprocal law is satisfied in our GLWF and GLO double cloak media. However, the PS cloak damaged the reciprocal law. The following physical statements are described: (1) In the domain consist of free space, single layer PS cloak and its cloaked concealment with normal material, the two sources reciprocal law is damaged. (2) In the domain consist of free space, single layer cloak and its cloaked concealment with some special double negative refractive index metamate-
rial, the two sources reciprocal law is recovered, but the cloak invisibility function is lose. (3) In the domain consist of free space, GLWF and GLO double layer cloak and its cloaked concealment with any material, the two sources reciprocal law is satisfied, and the cloak invisibility function is complete, sufficient, moreover the GLWF has all advantages of the GL double layer cloak in broad frequency band. The GLWF and GLO double layer cloak materials and the 3D and 2D GL parallel algorithms and software are made by authors in GL Geophysical Laboratory and are patented by GLGEO and all rights are reserved in GLGEO.

The GL method is an effective physical simulation method. It has double abilities of the theoretical analysis and numerical simulations to study the cloak metamaterials and wide material and Field scattering in physical sciences.

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