Progress towards a lattice determination of (moments of) nucleon structure functions

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Using unimproved and non-perturbatively $O(a)$ improved Wilson fermions, results are given for the three lowest moments of unpolarised nucleon structure functions. Renormalisation, chiral extrapolation and the continuum limit of the matrix elements are briefly discussed. The simulations are performed for both quenched and two flavours of unquenched fermions. No obvious sign of deviation from linearity in the chiral extrapolations are found. (This is most clearly seen in our quenched unimproved data, which remains too large, so it seems to be necessary to reach smaller quark masses in numerical simulations. Possible quenching effects also seem to be small. The lowest moment thus remains too large, so it seems to be necessary to reach smaller quark masses in numerical simulations.

1. INTRODUCTION

Much of our knowledge about QCD and the structure of the nucleus has been derived from Deep Inelastic Scattering (DIS) experiments, either $lN \to lX$ ($l = e^-, \mu^-$) via the exchange of a photon or $\nu N \to lX, \bar{\nu} p \to lX$ via $W^+$ or $W^-$ respectively. The cross section is determined by the structure functions $F(x, Q^2)$ via the structure functions by a convolution.

Considering non-singlet ($NS$ or $F^p - F^n$) combinations only to avoid any extra gluon terms, the operator product expansion relates moments of these structure functions to nucleon matrix elements $v_{n;NS}$ as

$$\int_0^1 dx x^{-2} F^{NS}(x, Q^2) = E^{RGI}_{F;v_{n;NS}}(Q) v^{RGI}_{n;NS},$$

where $E^{RGI}_{F;v_{n;NS}}$ is the Wilson coefficient and $v^{RGI}_{n;NS}$ is proportional to the matrix element of $O^{(a)} - O^{(d)}$. $O^{(a)}$ is a quark bilinear form, involving a $\gamma$-matrix and $n - 1$ covariant derivatives (see \cite{footnote1} for our conventions). This ‘renormalisation group invariant’ (RGI) form gives a clean separation between a non-perturbative number $v_{RGI}$ (which can be computed on the lattice) and a function $E_{F;v_{n;NS}}^{RGI}(Q)$ (which is perturbatively known). This is a possible direct comparison between the experimental result and the lattice result. More practical at present, however, is to use parton density functions (e.g. MRS, \cite{footnote2}) determined from global fits and related to the structure function by a convolution. Finally, in a scheme $S$ at scale $M$ then $v_{n;NS}^{RGI} \equiv \Delta Z_s v_{n;NS}^S(M)$ with

$$[\Delta Z_s^S(M)]^{-1} \equiv \left[ 2 b_0 \alpha_s(M) \right]^2 \frac{x_{QCD}}{\frac{4\pi}{\alpha_s(M)}} \times \exp \left\{ \int_0^{g_s^S(M)} d\xi \left[ \frac{\gamma^{\xi}(\beta_s^\xi)}{\beta_s^\xi} + \frac{dQ_{0}}{b_0 \xi} \right] \right\},$$

which in the $\overline{MS}$ scheme with $\Lambda_{\overline{MS}}$ from \cite{footnote3} at $M = 2$GeV gives for quenched 0.732(9), 0.596(10), 0.534(13) ($n = 2, 3, 4$ respectively) while for unquenched, $[\Delta Z_{\overline{MS}}^{RGI}]^{-1} \sim 0.695(10)$.

2. LATTICE DETAILS

The euclideanisation and form of the $O^{(a)}$ has been described in \cite{footnote4}. $v_{2b}$ can be deter-
minded with nucleon momentum $\vec{p} = \vec{0}$, while $v_{2a}$, $v_3$ and $v_4$ need a three-momentum with one non-zero component. By considering the NS term the difficult to compute one-quark-line-disconnected terms cancel.

A given operator can mix with three different classes of operators (with the same quantum numbers under the hypercubic group): $[A]$ of higher dimension; $[B]$ the same dimension; $[C]$ lower dimension. At present the $O(a)$ improvement operators (class $[A]$) are only known for $v_2$. The associated improvement coefficients are not completely known, $\Box$. For $v_3$ and $v_4$ there are additionally two mixing operators belonging to class $[B]$ and for $v_4$ a further operator in class $[C]$.

For $v_3$ we have found that the numerical values of the improvement terms are much smaller than the operator itself; thus dropping them will cause only an insignificant error.

Renormalisation can be considered in the MOM scheme – both perturbatively and non-perturbatively, $\Box$, by defining $Z_O^{\text{MOM}}$ from $\langle q(p)|O^{\text{MOM}}|q(p)\rangle = \langle q(p)|O^{\text{BORN}}|q(p)\rangle$, at $M^2 = p^2$. Perturbation theory gives

$$Z_O^{\text{MOM}}(p, a) = 1 + g_0^2 [d_{O,a} \ln(a p) - B_{O,\text{MOM}}(c_{sw})] + O(g_0^4),$$

where we have now computed $B_{O,\text{MOM}}(c_{sw})$ for a general value of $c_{sw}$, $\Box$. The perturbative renormalisation constant for the first mixing term for $v_3$ has also been computed; it turns out to be very small. Numerically this is also smaller than the $v_3$ matrix element so we shall also drop it. Problems arise with the remaining mixing terms though: the Born term (between quark states) vanishes, making their determination using only quark states impossible. At the present stage of development we shall simply ignore these extra terms.

Practically for the operator renormalisation we shall use a variant of ‘tadpole improved, renormalisation group improved, boosted perturbation theory’ (TI-RGI-BPT), $O^{\text{RGI}}$

$$O^{\text{RGI}} \equiv Z_O^{\text{RGI}} \mathcal{O}(a) \equiv \Delta Z_O^{\text{MOM}} Z_O^{\text{MOM}} \mathcal{O} \equiv \Delta Z_O^{\text{RGI}}(a) \mathcal{O}(a),$$

$\Delta Z_O^{\text{RGI}}(a) = u_0^{1-n_D} \left[ 2 b_0 g_0^2 \right]^{d_{O,a}} \times$

$$\left[ 1 + \frac{b_1}{b_0} g_0^2 \right]^{2 b_0 g_0^2 (1-n_D)} + \frac{p_1 b_0}{1} (1-n_D),$$

(3.5)

where $n_D$ is the number of derivatives in the operator, $u_0 = (s \text{Tr} U^D)$, $g_0^2 = g_s^2 / u_0^2$ and $p_1$ is the first perturbative coefficient in $u_0^1$. $d_{O,a}$ in the ‘□’ scheme may be found from the known $d_{O,\text{MOM}}^2$ (from $O^{\text{RGI}}$). Results for unimproved quenched fermions are shown in Fig. 1 and compared with the non-perturbative method, $\Box$. Reasonable agreement is seen.

$3$. RESULTS

There have recently been suggestions $[8–11]$ for the behaviour of $v_n$ close to the chiral limit,

$v_n^{\text{RGI}} = C_n (r_0 m_{ps})^2 + B_n \left[ 1 - d_n (r_0 m_{ps})^2 \ln \left( \frac{(r_0 m_{ps})^2}{(r_0 m_{ps})^2 + (r_0 m_{ps})^2} \right) \right]$

(rather than a linear form: $C_n (r_0 m_{ps})^2 + B_n$). We check this using quenched unimproved results (because this data extends down to lighter quark mass) in Fig. 3. Although not conclusive, it would seem that presently linear fits are adequate and any possible non-linearities can only show up at rather small quark mass outside the present range of data.
We first consider the continuum limit using quenched $O(a)$ improved fermions (at $\beta = 6.0, 6.2, 6.4$) after taking the chiral limit. In Fig. 3 we show the results. The data for the higher moments is unfortunately rather noisy.

Finally we consider unquenched results. In Fig. 4 we use a fit function ansatz

$$\nu_{2b;NS}^{RGI} = A_2 (a/r_0)^2 + B_2 + C_2 (r_0 m_{ps})^2,$$

for $O(a)$ improved fermions with 7 data sets.

In conclusion the results presented here would seem to indicate that we have to be much closer to the chiral limit in order to be able to perceive the partonic properties of the nucleon. Further details will be given in [5].

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