Application of logistic cycle model to determine high water content reservoir reasonable injection-production ratio

HU Qihao\textsuperscript{1}, WANG Hongmei\textsuperscript{1}, WANG Xia\textsuperscript{1}, YANG Huipeng\textsuperscript{1}, DU Jiande\textsuperscript{1}, YANG Kai\textsuperscript{1}

\textsuperscript{1}The Exploration and Development Research institute of Petrochina Huabei Oilfield Company, Renqiu city, Hebei province, China

hqh59085@163.com; yjy_WXIA@petrochina.com.cn

Abstract. In view of the actual situation of Chaheji oilfield steps into the high water content development phase after several times of encryption adjustment. C30 fault block of Chaheji oilfield is taken as the research object. Combined logistic cycle theory with the principle of material balance to determine the reasonable injection-production ratio. Firstly, the quantitative relationship between the annual oil production quantity and the annual water injection quantity of different water content ratio is determined by the logistic cycle theory. Secondly, the relationship between the pressure recovery rate and the difference of underground injection-production fluid volume is determined by using the material balance principle. Thirdly, the relationship of injection-production ratio with water content ratio and pressure recovery rate is studied. Finally, this method is applied to determine the reasonable injection-production ratio of C30 fault block in current development situation. And the field practice shows that the method can guide oilfield development and adjustment better.

1. Preface
At present, most oilfields domestic and abroad mainly adopt the way of water drive development and achieve good beneficial result \cite{1, 2}. Reasonable injection-production ratio is important guarantee of keeping oilfield efficient development. Especially the oilfield of high water content stage, keeping the reservoir pressure, stabilizing the output, controlling the rising speed of water content ratio are the focus work. The determination of reasonable injection-production ratio is the key to achieve the above goal. But determining the reasonable injection-production ratio is always a difficult problem of oilfield development \cite{3-5}. Some oilfields mainly rely on experience to allocate injection. And the human influence factor is larger. Now the method of oilfield injection allocation mainly includes logistic cycle model \cite{6}, water drive curve method \cite{7} and stage water storage chart method. The logistic cycle model is mainly applicable to oilfield with 50\% cumulative production capacity of recoverable reserves \cite{8}, which production is decreasing. The water drive curve method is mainly used in the production situation of oilfield in accordance with the water drive curve. The stage water storage chart method is suitable for the production situation of obvious water storage rate change law.

The research object of this article is high water content reservoir, which is in the late development. The oilfield reasonable injection-production ratio in condition of certain water content ratio and certain production is determined by means of combining logisitic cycle theory with material balance principle. The practical application shows that the method is in good practicability and operability. It can be used as the theoretical basis for optimizing the development index of oilfield water injection.
2. Logistic cycle model determines the injection-production ratio

2.1. Method principle
The mathematical model of logistic cycle model is:

\[ X = \frac{D}{1 + Ae^{\alpha t}} \]  

In the formula, \( X \) is the system; \( t \) is the development time or process of the system; \( D \) is the empirical constant of the life process; \( A, B \) are the fitting coefficients. In condition of \( B < 0 \), the cycle model represents the process from the rise of system to \( X \lim_{t \to \infty} X \to D \).

2.2. Formula derivation
The cumulative water consumption is an important index to evaluate the economic effect of oilfield development, and indicating the amount of water injection required to produce 1t crude oil.

\[ W_w' = \frac{W_w}{N_p} \]  

The cumulative water/oil ratio indicates the amount of water production required to produce 1t crude oil.

\[ F_{w/o} = \frac{W_p}{N_p} \]  

With the increasing development time of water flooding oilfield, the cumulative water consumption, cumulative water/oil ratio increases and the comprehensive water content rises. When the water content ratio is approaching the limit water content ratio, the quantitative relationship of comprehensive water content ratio with cumulative water consumption and comprehensive water content with cumulative water/oil ratio can be established by the logistic cycle model, and its mathematical model is:

\[ f_w = \frac{f_w \lim}{1 + A_1e^{(-B_1f_w)}} \]  

\[ f_w = \frac{f_w \lim}{1 + A_2e^{(-B_2f_w)}} \]  

In the formula: \( f_w \) block comprehensive water content ratio; \( f_w \lim \) ultimate water content ratio, usually \( f_w \lim = 0.98 \); \( W_w \) the amount of block cumulative water injection, \( 10^4 \) m³; \( W_p \) the amount of block cumulative water production, \( 10^4 \) m³; \( N_p \) the amount of block cumulative oil production, \( 10^4 \) t; \( A_1, B_1, A_2, B_2 \) fitting coefficient.

The mathematical model of the cumulative water consumption and the cumulative water/oil ratio is taken logarithm on both sides, and the equation can be obtained.

\[ \ln\left(\frac{f_w \lim}{f_w} - 1\right) = \ln A_1 - B_1W_w \]  

\[ \ln\left(\frac{f_w \lim}{f_w} - 1\right) = \ln A_2 - B_2F_{w/o} \]  

The equation coefficients \( \ln A_1, B_1, \ln A_2 \) and \( B_2 \) can be obtained by substituting the actual production data into the mathematical model.

Making the above two equations equal:

\[ W_w = \frac{\ln A_1 - \ln A_2}{B_1} + \frac{B_2}{B_1}F_{w/o} \]
The definition equation of cumulative water consumption and cumulative water/oil ratio is substituted into equation (8), then equation (9) can be obtained.

\[ W_j = \frac{ln A_i - ln A_p}{B_i} N_o + \frac{B_o}{B_i} W_o \tag{9} \]

The equation (9) is differentated and then the equation of the water content and the water/oil ratio is substituted into the differential equation (9).

\[ Q_j = \frac{ln A_i - ln A_p}{B_i} + \frac{B_o}{B_i} \left( \frac{f_w}{1 - f_w} \right) Q_o \tag{10} \]

In the formula, \( Q_j \) is the annual water injection amount of the block, 10^4m^3; \( Q_o \) is the annual oil production amount of the block, 10^4t; \( Q_r \) is the annual water production amount of the block, 10^4m^3.

According to the actual production dynamic data of the oilfield and the variable requirements of formula (6) and (7), the linear fitting of ln(0.98/f_w-1) with \( H_{cum} \) and ln(0.98/f_w-1) with \( WOR_{cum} \) is performed respectively, then the coefficients are substituted into the formula (10). Formula (10) is quantitative relationship between annual oil production and annual water injection in different water content development stages of oilfield. It can be used to predict and evaluate water injection of different periods.

For a specific reservoir, the quantitative relationship between the annual water injection and the annual oil production in different water content stage can be established by using the formula (10) through the numerical fitting of actual development data. The reservoir reasonable water injection of quota oil quantity in different water content period can be obtained. Evaluation and analysis can be carried out to predict the reasonable water injection in the future.

The injection-production ratio in the process of oilfield development is changing regularly. According to the method of determining the reasonable water injection quantity, the reasonable injection-production ratio of different water content can be predicted. And the formula is as follows:

\[ R_{ip} = \frac{\rho_o}{f_w} \left( \frac{ln A_i - ln A_p}{B_i} + \frac{B_o}{B_i} \left( \frac{f_w}{1 - f_w} \right) \right) \tag{11} \]

The reservoir reasonable injection-production ratio of different water content ratio can be obtained by formula (11).

3. Material balance equation determines the injection-production ratio

3.1. Method principle

In condition of artificial water flooding development, based on the reservoir material balance equation, the following equations can be obtained:

\[ \frac{N_o B_o}{\rho_o} = N B_o C_i (p_i - p) + \left( W_f - W_r \right) B_o \tag{12} \]

In the formula, \( N_o \) - geological reserves of the block, 10^4t; \( N_{o_e} \) - block cumulative oil production, 10^4t; \( p_e \) - original formation pressure, Mpa; \( p_i \) - current formation pressure, Mpa; \( \rho_o \) - ground degassing crude oil density, dimensionless; \( C_i \) - reservoir comprehensive compression coefficient, dimensionless; \( B_{o_e} \) - original crude oil volume coefficient, dimensionless; \( B_{o_e} \) - current volume coefficient of crude oil, dimensionless; \( B_{w_e} \) - formation water volume coefficient, dimensionless; \( W_f \) - block cumulative water injection, 10^4m^3; \( W_r \) - block cumulative water production, 10^4m^3.

3.2. Formula derivation

The volume coefficient of the formation crude oil can be regarded as a constant in the allowable range of formation pressure variation, namely \( B_o \) is approximately with \( B_{o_e} \). The equation (13) is derived from equation (12) after the derivative of time.
\[
\frac{dN_{p}}{dt} = NC_{i} \left( -\frac{dp}{dt} + \frac{B_{i} \rho_{o}}{B_{w}^i} \left( \frac{dW_{i}}{dt} - \frac{dW_{p}}{dt} \right) \right)
\]  

(13)

It is known that \(\frac{dN_{p}}{dt} = Q_{o}, \frac{dW_{i}}{dt} = Q_{i}, \frac{dW_{p}}{dt} = Q_{p}\), therefore:

\[
Q_{o} = NC_{i} \left( -\frac{dp}{dt} + \frac{B_{i} \rho_{o}}{B_{w}^i} \left( Q_{i} - Q_{w} \right) \right)
\]  

(14)

The annual pressure recovery rate is solved by formula (14):

\[
\frac{dp}{dt} = \frac{\rho_{o}}{NB_{w} \gamma_{i}} \left[ Q_{i} B_{w} - \left( \frac{Q_{i} B_{w}}{\rho_{o}} + Q_{p} B_{p} \right) \right]
\]  

(15)

\[
\frac{dp}{dt} = kAQ_{w}
\]  

(16)

In the formula, \(k\) - constant related to geologic reserves, rock and subsurface fluid property; \(AQ_{w}\) – the difference of subsurface annual injection and production fluid volume, 10^4 t/a.

Considering the influence of oilfield inefficient water injection and other factors, the relation of subsurface annual injection and production fluid quantity difference and annual pressure recovery rate is as follows:

\[
\frac{dp}{dt} = kAQ_{w} - b
\]  

(17)

According to the definition of injection-production ratio:

\[
R_{iw} = \frac{Q_{i} B_{w}}{\rho_{o}} + Q_{p} B_{p}
\]  

(18)

The injection-production ratio of different pressure recovery rate can be obtained by substituting the formula (18) into the formula (17), and the formula is:

\[
R_{iw} = \frac{1}{1 - \frac{\frac{dp}{dt} + b}{kAQ_{w}}}
\]  

(19)

The reservoir reasonable injection-production ratio of different pressure recovery rate can be obtained by formula (19).

4. Determination of C30 fault block reasonable injection-production ratio

4.1. The general situation of C30 fault block
The chaheji oilfield is a lithologic-tectonic reservoir based on shallow-water delta facies deposition. In 1982, the natural energy development was adopted to exploit chaheji oilfield. And in 1984, the water injection development was carried out by adopting triangle well network with 300m well distance. After years of well network encryption and comprehensive adjustment of injection-production, the C30 fault block has entered late stage oilfield development of high water content, the comprehensive water content ratio is 90%, and the degree of reserve recovery is 21.2%. At present, the annual injection-production ratio is 0.98 and the cumulative injection-production ratio is 0.95. The regional production volume is greater than the injection volume. There is irreconcilable contradiction between the increasing daily fluid volume by supplemental formation energy and the rising water content ratio.

4.2. Logistic cycle model method
Based on the actual development data of C30 fault block, the historical regression is fitted according to the formula (6) and (7), and the coefficients \(lnA_{i}=2.5955, B_{i}=1.476, lnA_{s}=1.1047\) and \(B_{s}=1.9853\) are obtained.
According to the parameters obtained by historical fitting, the relationship between annual water injection quantity and annual oil production quantity of the oilfield in different water content stages can be established as follows:

\[ Q_i = \left(1.01 + 1.345 \frac{t_i}{1 - t_i}\right)Q_o \]  \hspace{1cm} (20)

In condition of current water content ratio and annual oil production quantity, the reasonable annual water injection quantity is 53.24x10^4m^3, but the actual annual water injection is 50.65x10^4m^3. It is considered that, the annual water injection quantity of C30 fault block is low in the present situation, which can’t meet the annual oil production quantity of present water content ratio. And the annual water injection quantity of the oilfield should be appropriately improved.

According to the formula (11), the reasonable annual injection-production ratio is 1.04. But the actual annual injection-production ratio is 0.98, which is slightly smaller than the reasonable injection-production. Therefore, in order to ensure the annual oil production quantity of present water content ratio, the injection-production ratio should be raised appropriately.

4.3. Material balance equation method

Based on the actual development data of C30 fault block, the historical regression is fitted according to the formula (17). Then the relationship between underground injection and production fluid volume difference and the annual pressure recovery rate is obtained by formula (21).

\[ \frac{dp}{dt} = 0.0166 \Delta Q_t - 0.0472 \]  \hspace{1cm} (21)

According to formula (21), the value of \( k \) is 0.0166 and the value of \( b \) is 0.0472. By substituting the values of \( k \) and \( b \) into formula (19), the injection-production ratio of different pressure recovery rates can be calculated under the current production condition.

It is shown from table 1 that the injection-production ratio of the reservoir to maintain the current pressure level is 1.05. The current annual production quantity is guaranteed under the injection-production ratio of 1.05. And when the injection-production ratio is 1.05, the water content ratio is not increasing rapidly due to the high injection-production ratio. The reasonable injection-production ratio of logistic cycle model method is 1.04, the difference is slightly comparing with 1.05. Therefore, it is considered that the reasonable injection-production ratio of the reservoir in current development condition is 1.04.

| Pressure recovery rate | Annual water injection | Injection-production ratio |
|------------------------|------------------------|----------------------------|
| Mpa/a                  | 10^4m^3                |                           |

Table 1. Injection-production ratio of different pressure recovery rates
5. Conclusion

(1) According to the logistic cycle model, the relationship of reasonable water injection quantity, reasonable injection-production ratio with water content ratio is established. The reasonable water injection quantity and injection-production ratio of current development situation are forecasted and the water flooding development status is analysed and evaluated.

(2) The predictive relation of reasonable water injection quantity and reasonable injection-production ratio with water content ratio is established by taking chaheji oilfield as an example. Compared with the actual production data, the water injection quantity and injection-production ratio of C30 fault block are low. Therefore, it is necessary to improve the water injection quantity and injection-production ratio in current development situation.

(3) Combining the principle of material balance with logistic cycle theory, the reasonable injection-production ratio is determined in condition of certain water content ratio and certain oil production, which can effectively guide the development and adjustment of high water content period reservoir water flooding.

Acknowledgments

This work was financially supported by the Petrochina Huabei Oilfield Company major project “Research and Application of Huabei Oilfield exploration and development gordion technique” (2017E-15).

References

[1] Chen Y Q. Approach and application of determining economic limit water-cut using water drive curve methods[J]. Xinjiang Petroleum Geology, 2010, 31(2):158-162. (In Chinese)

[2] Fads R J. Oilfield Area Water Injection[M]. Beijing: Petroleum Industry Press, 1989:257-260. (In Chinese)

[3] Lerma M K. Analytical method to predict water flood performance[C]// SPE Western Regional: AAPG Pacific Section Joint Meeting, Long Beach, California. Dallas: Society of Petroleum Engineers, 2003:10-20. (10.2118/83511-MS)

[4] Rodriguez A, Dawood N J, Yinka Soremi, et al. Practical aspects of streamline application to water injection management of a giant carbonate reservoir[C]// SPE North Africa Technical Conference & Exhibition, Marrakech, Morocco. Dallas: Society of Petroleum Engineers, 2008:35-53. (10.2118/112899-MS)

[5] Lee K H, Ortega A, Amir Mohammad Nejad, et al. A method for characterization of flow units between injection-production wells using performance data[C]// SPE Western Regional and Pacific Section AAPG Joint Meeting, Bakersfield, California. Dallas: Society of Petroleum Engineers, 2008:60-79. (10.2118/114222-MS)

[6] Wu Q, Zhang W Z, Zhang C Q. Research on water injection rate prediction with Logistic cycle model[J]. Special Oil and Gas Reservoirs, 2011, 18(1):67-69. (In Chinese)

[7] Jiang M, Song F X. A method of determining the amount of injection allocation by using the production[J]. WTPT, 1998, 19(3):24-27. (In Chinese)

[8] Chen Y Q, Hu J G, Zhang D J. Derivation of Logistic model and its self-regression method[J]. Xinjiang Petroleum Geology, 1996, 17(2):150-155. (In Chinese)