Analysis on Degree of Freedom of Mechanism Based on Characteristics Description of Joint Motion Domain

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Abstract. The concepts includes input and output degrees of freedom of the mechanism are distinguished and redefined, The motion domain characteristics of translation and rotation joints can be described by planar six-dimensional topological graph, the output kinematic characteristics of RRPRR series branch chain and 3-RRPRR parallel mechanism are obtained by an example analysis, this new theory and method not only realize the quantitative display of the instantaneous degree of freedom of mechanism for the first time historically. In addition, the detailed process and form of the comprehensive transformation between the input and output motions of the mechanism are revealed historically for the first time and presented in the form of formulas. Theoretical definition and computational analysis method based on the six elements description of moving scope of joint newly proposed, which provides a set of scientific, universal, intuitive and effective theory and method for DOF analysis of arbitrary serial and parallel mechanisms.

Keywords: Degree of Freedom; Joint Motion Domain; Whole Basic Unit of Translation/Rotation; Six Elements Description; Planar Six-Dimensional Topological Graph.

1. Introduction
The calculation of mechanism degree of freedom (DOF for short) is the most basic problem in mechanism analysis. According to IFToMM terminology standard[1], degree of freedom is defined as the number of independent parameters required to determine the position of mechanism or kinematic chain, that is, the number of independent parameters required to determine the position of all components of a mechanism or kinematic chain in a certain position. The Grubler-Kutzbach formula (G-K formula for short) has been used for a long time to recognize the degrees of freedom of almost all planar mechanisms and some spatial mechanisms, and it is only based on the most basic arithmetic operations. The G-K formula has made outstanding contributions to the development of mechanical science and the progress of civilization of human society, and all the people in the world who study and pay attention to the DOF of mechanism are indispensable. However, there are many counter-examples in history, such as American Professor Suh[2] in 1978, Shigley and Uicker[3] in 1980, American Professor Sandor and Erdman[4] in 1984, Mabie and Reinholtz[5] in 1987, Eckhardt[6] in 1998, Professor Tsai of University of Maryland[7] in 1999, French Professor Merlet[8] in 2000, and internationally renowned scholars Waldron and Kinzel[9] in 2004, they all pointed out in the corresponding works that the G-K formula cannot adapt to the examples and situations of certain mechanism, so this formula cannot be considered as a universal formula. It has been more than 150 years to find the general formula of freedom of mechanism, in 2011, Professor Huang Zhen of...
Yanshan university published a monograph on the degree of freedom of mechanism, that is General Formula for The Degree of Freedom Which Have Been Found for 150 Years[10], there are three situations in this book, general application, classical mechanism and parallel mechanism, and the formula for calculating the degree of freedom of the specified rod in the mechanism, therefore, the formula is highly adaptable and scientific. The known theories and methods do not effectively distinguish the input DOF from the output DOF of the mechanism, and do not find the inherent connection and difference between the motion elements of input and the motion elements of output, without the basic physical representation and motion of the motion. Therefore, the theoretical and computational analysis methods which embody the physical mathematics essence of DOF can’t be obtained. Based on the above considerations, this work introduces the principal-parasitic motion theory and the feature description of joint motion domain and the analysis of the output kinematic characteristics of the branched chain/mechanism to study the DOF of the mechanism, and obtains the general theory and method of the analysis and calculation of the DOF, and proves its validity by an example.

2. Analysis of Motion Characteristics on 3-RRPRR

A 3-RRPRR parallel mechanism is constructed as shown in Fig.1. In the figure, the | red line represents the rotation axis, and the | blue line represents the translation axis. (In colorless graphs, the | black line can be used to represent the rotation axis, and the T line with the horizontal cut-off line segment is used to represent the translation axis.)

Set the values of the parameters as follows: \( A_1B_1 = B_1C_1 = C_1A_1 = 1000 \text{mm} \), \( A_2A_2 = B_2B_2 = C_2C_2 = 200 \text{mm} \), \( A_3A_3 = B_3B_3 = C_3C_3 = 300 \text{mm} \), \( A_4A_4 = B_4B_4 = C_4C_4 = 200 \text{mm} \), \( A_5A_5 = B_5B_5 = C_5C_5 = 500 \text{mm} \). Based on the theory of principal-parasitic motion, the planar six-dimensional topological graph is analyzed by using the six elements description of the joint motion domain and the ability of motion characteristics, and then the degree of freedom of the mechanism is analyzed by using the plane six-dimensional topological graph.

According to the rules and conclusions shown in Table 1, the following data can be obtained:

\[
\begin{align*}
A_1B_1 & = B_1C_1 = C_1A_1 = 1000 \text{mm} , \\
A_2A_2 & = B_2B_2 = C_2C_2 = 200 \text{mm} , \\
A_3A_3 & = B_3B_3 = C_3C_3 = 300 \text{mm} , \\
A_4A_4 & = B_4B_4 = C_4C_4 = 200 \text{mm} , \\
A_5A_5 & = B_5B_5 = C_5C_5 = 500 \text{mm} , \\
A_6A_6 & = B_6B_6 = C_6C_6 = 409.57 \text{mm} .
\end{align*}
\]

In the subsequent analysis, the relevant coordinates and radius parameters can be obtained by simple drawing method, so that they will not be described again.

2.1. Analysis of Output Kinematic Capability Characteristic of Branch \( A_1A_2A_3A_4A_5E \)

The previous known parameter representations are substituted into the formula for calculation, and the results are as follows.

\[
\begin{align*}
\text{ID}(&\text{IR}_{A_1}) + \text{ID}(&\text{IR}_{A_2}) + \text{ID}(&\text{IP}_{A_3}) + \text{ID}(&\text{IR}_{A_4}) + \text{ID}(&\text{IR}_{A_5}) \\
= \hat{P}_s & W_{13} + \frac{\sqrt{3}}{2} \hat{R}_s (W_{12} \cup W_{14}) + \frac{1}{2} \hat{R}_s (W_{12} \cup W_{14}) + \hat{R}_s (W_{11} \cup W_{13}) + \hat{P}_s W_{16} + \hat{P}_s W_{17} + \hat{P}_s W_{18} \tag{1}
\end{align*}
\]

\[
\{ \hat{R}_s (W_{13} \hat{W}_{17})_s + \hat{R}_s (W_{13} \hat{W}_{14})_s \}
\]

Among them, \( W_{16} = \frac{1}{2} (W_{12} \hat{W}_{14})_s \), \( W_{17} = \frac{\sqrt{3}}{2} (W_{12} \hat{W}_{14})_s \), \( W_{18} = \frac{\sqrt{3}}{2} (W_{12} \hat{W}_{14})_s \). The symbol \( \hat{A} \) represents the translation operation of the motion element domain, the symbol \( \hat{\Lambda} \) represents the rotation operation of the motion element domain, and the symbol \( \cup \) represents the union operation of the motion element domain. The specific rules of operation \( \hat{A} \) and \( \hat{\Lambda} \) and \( \cup \) are as follows.

**Definition 1:** Define the formula \( a \hat{P}_s W_{16} + a \hat{P}_s W_{17} = a \hat{P}_s (W_{16} \cup W_{17}) \), \( a \hat{P}_s W_{16} + b \hat{P}_s W_{18} = \hat{P}_s (a W_{16} \cup b W_{18}) \).
Fig. 2 shows the joint motion domain analysis of the branch chain $A_1A_2A_3A_4A_5E$, the motion domain of joint $A_1$ is the spatial arc field of air starting angles $-90^\circ,180^\circ$, which passes through point E and is parallel to the horizontal coordinate plane, as shown by the purple dotted line in Fig. 6.

Among them, the specific operation rules of $X$ are as follows, taking $W_{11}$ as an example.

\[
W_{11} = (x_{A1}, y_{A1}, z_{A1} + 800, -90^\circ, 180^\circ, A_1E) = (288.68, -500, 800, -90^\circ, 180^\circ, 577.35),
\]

\[
W_{12} = (x_{A2}, y_{A2}, z_{A2}, 0^\circ, 360^\circ, A_2E) = (188.68, -326.79, 0^\circ, 360^\circ, 891.08),
\]

\[
W_{14} = (x_{A4}, y_{A4}, z_{A4}, 0, 360^\circ, A_4E) = (88.68, -153.59, 300^\circ, 360^\circ, 530.52),
\]

\[
W_{15} = (x_{A5}, y_{A5}, z_{A5}, -90^\circ, 180^\circ, A_5E) = (88.68, -153.59, 800, -90^\circ, 180^\circ, 177.35).
\]

**Definition 2:** Define the union operation $\cup$ in $W_{11} \cup W_{13}$ and $W_{12} \cup W_{14}$, select the point closest to the base point of the fixed coordinate system as the rotation center point, select the starting angle of the rotation joint angle of the rotation center point as the starting angle, and select the radius corresponding to the rotation center point and the maximum of the sum of the subsequent other joint radii as the radius.
Therefore, $W_{11} \cup W_{15} = W_{11}$, $W_{12} \cup W_{14} = W_{12}$.

It can be seen from the analysis that the merger of the congener domain is to select a large-scale domain containing another domain, so there are:

$$\frac{\sqrt{3}}{2} (W_{12} \cup W_{14}) = \frac{\sqrt{3}}{2} W_{12} = (163.40,-283.0,0,0,360,771.68),$$

$$\frac{1}{2} (W_{12} \cup W_{14}) = \frac{1}{2} W_{12} = (94.34,-163.40,0,0,445.54),$$

$$W_{11} \cup W_{15} = W_{11} = (288.68,-500,800,-90,180,577.35).$$

**Definition 3:** define the operation $W_{12}\Delta W_{14}$ as the concomitant translation motion domain generated from the second joint rotation motion of branch chain 1 and the fourth joint rotation motion of branch chain 1. In the six elements of the concomitant translation motion domain, the second joint coordinate of the branch chain 1 nearest to the fixed coordinate system is taken as the center of a circle, the initial rotation angle of this joint is taken as the starting angle, and the rotation radius of this joint is taken as the rotation radius. In the six elements of $(W_{12}\Delta W_{14})_x$, the space are represented by $W_{12}\Delta W_{14}$ contains six parametric coordinates with the minimum and maximum values of the x-axis coordinates on the domain surface, these coordinates are arranged in the order of x-axis from small to large, left to right row. Similarly, in the six elements of $(W_{12}\Delta W_{14})_y$, the space are represented by $W_{12}\Delta W_{14}$ contains six parametric coordinates with the minimum and maximum values of the y-axis coordinates on the domain surface, these coordinates are arranged in the order of y-axis from small to large, left to right row. Similarly, in the six elements of $(W_{12}\Delta W_{14})_z$, the space are represented by $W_{12}\Delta W_{14}$ contains six parametric coordinates with the minimum and maximum values of the z-axis coordinates on the domain surface, these coordinates are arranged in the order of z-axis from small to large, left to right row.

Then, there are:

$$(W_{12}\Delta W_{14})_x = (-156.87,271.69,0,634.22,-1098.46,0),$$

$$(W_{12}\Delta W_{14})_y = (634.22,-1098.46,0,-156.87,271.69,0),$$

$$(W_{12}\Delta W_{14})_z = (188.68,-326.79,-891.08,188.68,-326.79,891.08).$$

Therefore:

$$W_{16} = \frac{1}{2} (W_{12}\Delta W_{14})_x = (-78.44,138.85,0,317.11,549.23,0),$$

$$W_{17} = \frac{\sqrt{3}}{2} (W_{12}\Delta W_{14})_y = (549.23,-951.27,0,-135.85,235.28,0),$$

$$(W_{12}\Delta W_{14})_z = (163.40,-283.00,-771.68,163.40,-283.00,771.68).$$

It can be seen from the analysis that the merger of the congener domain is to select a larger domain, so there is:

$$W_{18} = \frac{\sqrt{3}}{2} (W_{12}\Delta W_{14})_x \cup \frac{1}{2} (W_{12}\Delta W_{14})_z = \frac{\sqrt{3}}{2} (W_{12}\Delta W_{14})_z,$$

$$W_{18} = \frac{\sqrt{3}}{2} (W_{12}\Delta W_{14})_z = (163.40,-283.00,-771.68,163.40,-283.00,771.68).$$

**Definition 4:** define $W_{13}\Delta W_{17}$ operation as the second-order joint rotational motion domain generated by the third joint motion from branch chain 1 and the virtual (Formed by joint motion) seventh joint.
motion from branch chain 1. The determination of the six elements of the second-order joint rotational motion domain is divided into two categories. In the first category, when the two lines represented by the four points of the two translation motion domain are coplanar in space (parallel or intersect), the coordinates of the center point of the quadrilateral formed by the two linear translation motion domain are the center coordinates of the six elements, the $0^\circ$ as the initial rotation angle of the joint, the $360^\circ$ as the end angle of the joint, and the radius of the largest inscribed circle of the quadrilateral formed by the two translation joint motion domains is taken as the radius of rotation $r'$. In the second category, when the two lines of translation motion domain are not coplanar in space, there will be no parasitic motion exists in this condition.

**Definition 5:** define the center of the space arc represented by $W_{13}\Delta W_{17}$ in the six elements of $(W_{13}\Delta W_{17})_1$ as the center of the circle, with $0^\circ$ as the starting rotation angle of the joint, and $360^\circ$ as the ending angle of the joint, and take the minimum projection value of $r'$ (Calculate the radius projection value by multiplying the radius with the cosine of the angle between the plane and the axis of rotation) in the fixed coordinate system plane $yoz$ as the rotation radius $r'$. Similarly, define the center of the space arc represented by $W_{11}\Delta W_{16}$ in the six elements of $(W_{11}\Delta W_{16})_1$ as the center of the circle, with $0^\circ$ as the starting rotation angle of the joint, and $360^\circ$ as the ending angle of the joint, and take the minimum projection value of $r''$ in the fixed coordinate system plane $yoz$ as the rotation radius $r''$.

The following conditions are known conditions:

$$W_{13} = (0,0,800,0,0,1000),$$

$$W_{16} = \frac{1}{2}(W_{12}\overline{W}_{14})_x = (-78.44,138.85,0,317.11,-549.23,0),$$

$$W_{17} = \frac{\sqrt{3}}{2}(W_{12}\overline{W}_{14})_y = (549.23,-951.27,0,-135.85,253.25,0),$$

Due to the reason that these two lines not coplanar, which means there will be no second level parasitic motion exists represented by null set as following, $W_{13}\Delta W_{17} = \emptyset$, $W_{11}\Delta W_{16} = \emptyset$. Therefore, $(W_{13}\Delta W_{17})_y = \emptyset$, $(W_{11}\Delta W_{16})_y = \emptyset$.

It is assumed that the coordinates of the six elements corresponding to each output domain of branch chain $A_1A_2A_3A_4A_5E$ are represented by $p_i$ respectively, and that the coordinate components corresponding to the $x, y, z$ axes are represented by $p_i^x, p_i^y, p_i^z$, therefore, there are $(p_1, p_2) = W_{16}$, $(p_3, p_4) = W_{17}, (p_5, p_6) = W_{18}$, $(p_9 | r_{p_9}) = \sqrt{\frac{5}{2}}(W_{12} \cup W_{14}), (p_{10} | r_{p_{10}}) = \frac{1}{2}(W_{12} \cup W_{14}), (p_{11} | r_{p_{11}}) = W_{13} \cup W_{15}$.

2.2. Analysis of Output Kinematic Capability Characteristic of Branch $B,B,B,B,B,E$

According to the previously known conditions, the following results can be obtained.

$$\begin{align*}
ID(1R_{a_k}) & + ID(1R_{a_k}) + ID(1P_{a_k}) + ID(1R_{a_k}) + ID(1R_{a_k}) \\
& = \{P, W_{26} + \frac{\sqrt{3}}{2}R_x(W_{22} \cup W_{24}) + \frac{1}{2}R_x(W_{22} \cup W_{24}) + R_y(W_{21} \cup W_{25}) \\
& + \{P, W_{26} + \frac{\sqrt{3}}{2}R_x(W_{22} \cup W_{24}) + \frac{1}{2}R_x(W_{22} \cup W_{24}) + R_y(W_{21} \cup W_{25})\} \}
\end{align*}$$

(2)

It is assumed that the coordinates of the six elements corresponding to each output domain of branch chain $B,B,B,B,B,E$ are represented by $q_i$ respectively, and that the coordinate components
corresponding to the x, y, z axes are represented by \( q_i^x, q_i^y, q_i^z \), therefore, there are \((q_1, q_2) = W_{23}, (q_3 | r_3) = \frac{\sqrt{3}}{2} (W_{22} \cup W_{24}), (q_4 | r_4) = \frac{1}{2} (W_{22} \cup W_{24}), (q_5 | r_5) \in W_{21} \cup W_{25}, (q_6, q_7) = W_{26}, (q_8, q_9) = W_{27}, (q_{10}, q_{11}) = W_{28} \).

2.3. Analysis of Output Kinematic Capability Characteristic of Branch \( C_1C_2C_3C_4C_5E \)

According to the previously known conditions, the following results can be obtained.

\[
\begin{align*}
\text{ID}(\text{IR}_{C_1}) + \text{ID}(\text{IR}_{C_2}) + \text{ID}(\text{IR}_{C_3}) + \text{ID}(\text{IR}_{C_4}) + \text{ID}(\text{IR}_{C_5}) = & \{ P_{31}, W_{33} + \hat{R}_{32} (W_{31} \cup W_{33}) + \hat{R}_{34} (W_{32} \cup W_{34}) \\
+ & \{ P_{35}, W_{36} + \hat{P}_{37} + \hat{P}_{38} \} + \{ \{ \hat{R}_{31}, (W_{31} \Delta W_{37}) \}, \hat{R} (W_{32} \Delta W_{38}) \} \}
\end{align*}
\]

It is assumed that the coordinates of the six-elements corresponding to each output domain of branch chain \( C_1C_2C_3C_4C_5E \) are represented by \( s_j \) respectively, and that the coordinate components corresponding to the x, y, z axes are represented by \( s_i^x, s_i^y, s_i^z \), therefore, there are \((s_1, s_2) = W_{33}, (s_3 | r_3) \in W_{31} \cup W_{35}, (s_4 | r_4) \in W_{32} \cup W_{34}, (s_5, s_6) = W_{36}, (s_7, s_8) = W_{37}, (s_9, s_{10}) = W_{38} \).

![Figure 3. Analytical diagram of branched chain \( B_iB_iB_iB_i \) joint motion domain](image)

![Figure 4. Analytical diagram of branched \( C_1C_2C_3C_4C_5 \) joint motion domain](image)

2.4. Analysis of Output Kinematic Capability Characteristic of Parallel Mechanism

The calculation results of the output kinematic capability characteristics of the branches \( A_1A_2A_3A_4A_5E \).
$B_1B_2B_3B_4B_5E$ and $C_1C_2C_3C_4C_5E$ are mapped to the plane six-dimensional topological graph, as shown in Figure 5-7, where the black line domain is the main motion output domain and the red line domain is the first-order joint motion output domain, and the blue line domain is the second-order joint motion output domain. In the planar six-dimensional topological graph, the point coordinates under the horizontal line of the translation domain represent the component coordinates of the translation domain starting point, and the $(u|r)$ above the starting point of the rotating domain represent the rotation center point coordinates $u$ and the rotation radius $r$ of the rotating domain.

The output main motion is the output main degrees of freedom (M-DOF) of the corresponding branch chain, and the first-order joint output motion and the second-order joint output motion correspond to the output parasitic degree of freedom (P-DOF). The motion domain corresponding to M-DOF and P-DOF obtains the final motion domain of the branch chain or mechanism through the absorption principle and the supplement principle (Mathematical method is defined as union operation of all motion domains.). The number of corresponding motion domain non-empty description coordinates in the planar six-dimensional topological map in which the motion domain is the number of output degrees of freedom (O-DOF). The number of input degrees of freedom (I-DOF) is the number of input motion joints of the mechanism.

Figure 12-13 show the comprehensive analysis graphs of the three sets of branch chains of the parallel mechanism, which are the second-order joint output rotation around the $x$ axis and the main output rotation around the $y$ axis and the second order joint output rotation around the $y$ axis and the main output rotation around the $z$ axis. The red circle or red arc domain in the figures represent the circle of the rotation domains corresponding to each output branch chain (with its center point and radius), and the red domain must have a common intersection in order to synthesize the effect and obtain the corresponding rotation domain of the parallel mechanism output terminal. The blue circle or blue arc domain in the figures represent the rotation domain circle or arc obtained by the synthesize effect of the output end of the parallel mechanism output terminal.

**Figure 5.** Planar six dimensional topological graph of branch $A_1A_2A_3A_4A_5E$ output motion capability

**Figure 6.** Planar six dimensional topological graph of branch $B_1B_2B_3B_4B_5E$ output motion capability
Figure 7. Planar six dimensional topological graph of branch $C_2C_3C_4C_5E$ output motion capability

Figure 8. The analytical diagram of main output rotation around axis y

Figure 9. The analytical diagram of main output rotation around axis z

In Fig. 8, the output terminal point $E$ is rotated around the y-axis for main output rotation, and the rotation center coordinates $O_y$ and $p_{10}, q_4$ coincide with each other. The rotation radius $r_y$ is the same as $r_{p_{10}}(r_{q_4})$, and the rotation starting angles are $0^\circ, 90^\circ$, respectively. In Fig. 9, the output terminal point $E$ is rotated around the z-axis for main output rotation, and the rotation center coordinate $O_z$ coincides with coordinate origin $O$ of the fixed coordinate system. The rotation radius is $r_z = \min(r_{p_{11}}, r_{q_5}, r_{s_5}) - |p_{11}O_z|$, that is $r_z = 0^\circ$. This means that the output terminal point of the mechanism is self-rotating at this time, which is passing through the coordinate origin and taking the z-axis of the fixed coordinate system as the rotation axis, and the rotation starting angles are $0^\circ, 360^\circ$ respectively.

The mapping results of the planar six-dimensional topological graphs of the above three branch chains perform an intersection operation, and the mapping results of the planar six-dimensional topological
graphs of the output terminal motion characteristics of the parallel mechanism are obtained as follows:

![Figure 10](image)

**Figure 10.** Planar six-dimensional topological graph of output terminal motion characteristic of the 3-RRPRR parallel mechanism

According to the description of Fig. 10 above, the following results are known:

The number of output main degrees of freedom (M-DOF) corresponding to the output main motion of the 3-chain parallel mechanism is the number of black line domain, so M-DOF = 3.

The number of output parasitic degree of freedom (P-DOF) corresponding to the first-order joint output motion of the 3-chain parallel mechanism is the number of red line domain, so P-DOF = 3.

The number of output parasitic degree of freedom (P-DOF) corresponding to the second-order joint output motion of the 3-chain parallel mechanism is the number of blue line domain, so P-DOF = 3 + 0 = 3.

By using the motion absorption principle, it is known that the 3-chain parallel mechanism has a main motion translation output along the z-axis direction and a first-order joint motion translation output, so the degree of freedom is combined. The 3-chain parallel mechanism has a first-order joint motion translation output along the x and y-axis directions, so the degrees of freedom are supplemented, and the output freedom of the mechanism is O-DOF = 3 + 3 - 1 = 5.

The number of input degrees of freedom (I-DOF) of the three-chain parallel mechanism is the sum of the number of motion joints of the each mechanism branch, that is I-DOF = 5 + 5 + 5 = 15.

The above analysis process is to analyze the transient degree of freedom when the mechanism is in a certain position and orientation. This analysis conclusion clearly shows the motion domain characteristics of the output terminal point when the mechanism is in a certain position and orientation at a certain moment, which cannot be achieved by other analysis methods. As for the analysis of the whole degree of freedom, the result can be obtained by changing the position and orientation of the mechanism.

### 3. Conclusion

Based on the study of the existing methods for calculating and analyzing the degree of freedom of mechanisms, the shortcomings of these methods are found. The relationship between the input and output motions of mechanisms is studied by the principle of the principal-parasitic motion, the characteristic description of the joint motion domain and the analysis of output kinematic capability characteristic of the chain/mechanism. The following conclusions can be obtained:

1. The concepts includes input and output degrees of freedom of the mechanism are distinguished and redefined, the translation joint and rotation joint can be correspondingly defined as the whole basic unit of translation and the basic unit of translation $\mathbf{P}, \mathbf{P}_r, \mathbf{P}_z$, and the whole basic unit of rotation and the basic unit of rotation $\mathbf{R}, \mathbf{R}_r, \mathbf{R}_z$.

2. With the aid of planar six-dimensional topological graph, the output kinematic characteristics of RRPRR series branch chain and 3-RRPRR parallel mechanism are obtained by an example analysis, including the characteristics of the workspace and the instantaneous degree of freedom and other characteristics.

3. This new theory and method not only realizes the quantitative display of the instantaneous degree of freedom of mechanism for the first time historically, which provides a set of scientific, universal, intuitive and effective theory and method for DOF analysis of arbitrary serial and parallel mechanisms.
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