Measuring densities of cold atomic clouds smaller than the resolution limit.

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Experimental setup to study quantum magnetism on optical lattices

ULTRA COLD $^{87}\text{Sr}$ IN OPTICAL LATTICES
STUDYING QUANTUM MAGNETISM BEYOND SPIN $\frac{1}{2}$
Magnetism with Nuclear Spins

Fermi gas with 10 spin states $10^4$ atoms at $T/T_F \sim 0.2$
($T \approx 30 \text{ nK}, T_F \approx 150 \text{ nK}$)

$H_{Heis} = -J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$

P. Bataille and al. Phys. Rev. A 102, 013317 (2020)
Absorption imaging of objects smaller than the resolution limit

Beer Lambert Law

\[
\frac{dI(x, y, z)}{I(x, y, z)} = -n(x, y, z)\sigma_0 dz
\]

Resolution limited
Diffraction, aberrations, pixelation

Collected light over an area (pixel) matched to the resolution

\[
P(i, j) = \int_{D_{i,j}} I(x, y)dx dy
\]

Log of averaged transmission

\[
\log \left( \frac{P(i, j)}{P_0(i, j)} \right) \neq \frac{1}{a^2} \int_{D_{i,j}} \log \left( \frac{I(x, y)}{I_0(x, y)} \right) dx dy
\]

≠ Averaged density of atoms

\[
= -\tilde{n}(x, y)\sigma_0
\]

Error not only on peak density measurement but also on atom number count.

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Core principle:
The number of scattered photons per atom depends on the optical thickness, i.e. size, of an atomic cloud.

→ Shadowing Effect
Analysis of the scattered photons

Local ratio of missing photons $R_{ph}$ per pixel $(i,j)$

$$R_{ph}(i,j) = \frac{P_0(i,j) - P(i,j)}{P_0(i,j)}$$

Summation over pixel line for longitudinal profile

$$R_{ph}(j) = \sum_i R_{ph}(i,j)$$

Gauss ansatz: defines the transverse size $\sigma_x$ to be measured

$$n(x, y) = e^{-\frac{x^2}{2\sigma_x^2}} \tilde{n}(0, y)$$

Express the local density with respect to the absorbed photons. Depends on the parameter $\sigma_x$ only

$$R_{ph}(j) = \frac{\sigma_x}{a} F\left(\frac{\sigma_x}{\sigma_0} \tilde{n}(0, a) \right)$$

$$N_{at} = \frac{\sqrt{2\pi} a \sigma_x}{\sigma_0} \sum_j F^{-1}\left(\frac{a}{\sigma_x} R_{ph}(j) \right)$$

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Verification

Longitudinal distorted profile (resolved dimension)

\[ R_{ph}(j) = \frac{\sigma_x}{a} F(\sigma_0 \hat{n}(0, aj)) \]

Transverse unresolved dimension

Thermal gas:
Boltzmann distribution
Curve fitting of the recovered cloud shape.
Residuals show distorted shape initially.
Only noise after recover.

Fermi gas:
Degenerate Fermi gas at 0.2 T_F.
Expected distribution with no free parameter.
Residuals show that the recovered profile match the expected profile.

Prediction with independent measurement of the temperature and confinement frequencies

\[ \frac{1}{2} m \omega^2 \sigma^2_{x_{th}} = E_K \]