Mathematical simulation of vibration signature of ball bearing defects in a rotor bearing system

J Prabu*, Pavuluri Uday1, M Solairaju2, C Shravankumar1, K Jegadeesan1 and T V V L N Rao1

1Department of Mechanical Engineering, SRM Institute of Science and Technology, Kattankulathur, Tamil Nadu, India.
2Combat Vehicles Research and Development Establishment, Avadi, Tamil Nadu, India.

E-mail: prabu_ja@srmuniv.edu.in

Abstract. This paper presents the analytical model of a ball bearing with localized defect in the outer race. The outer raceway defect is modelled based on the reduction in the contact force due to localised increase in the clearance. The contact forces are modelled based on Hertzian contact deformation theory. The geometry of the defect is considered as a half sinusoidal wave. The contact between the ball and the outer raceway is modelled using non-linear springs, acted upon rotor mass. The system governing equations of motion are obtained and represented in state space form. Numerical simulations are carried out using MATLAB environment to study the effect of outer race defect on vibration responses. Time and Frequency domain characteristics of the simulated vibration signals are obtained.

1. Introduction
Rolling element bearing is a machine element which is used to rotate and enhance the free rotation of an element around fixed axis. It is used to reduce the friction between the moving parts. It supports the shaft and transmits the rotary power. Any defects in the bearing should be diagnosed at early stage. Any serious defects may cause catastrophic failure of machinery. Pits, cracks and spalls come under the category of localized defect. Surface roughness, waviness, off size rolling elements and misaligned races comes under the category of distributed defects. Condition monitoring of the bearing is essential to predict the possible occurrence of these defects in the rolling element. Vibration based condition monitoring is the widely used method. When the defects are present in any part of the bearing, changes in vibration characteristics are generated due to that defect. A theoretical model proposed by Tandon and Choudhury [1] predicted the amplitude of the spectral components due to waviness. The distributed defects has been considered as the outer race waviness, inner race waviness, and off size rolling elements. For each order of waviness, the discrete spectrum with specific frequency has been predicted. The solutions are obtained by using the mechanical impedance method. Vibration signals are emitted by the rolling element defect includes frequencies in order of bearing geometry, defect plane and speed of both the races in the bearing. If there is defect in bearing, then the frequencies will be emitted when the ball gets impact on the defect. A laboratory rolling test rig has been developed by Roque [2] to predict the envelope spectrum, harmonics and the defect frequency of the element. In this, four equal rolling bearings have been used and different single defect has been introduced in each bearing. The data acquisition system has been used for the analysis of global values, frequency
spectrum, time signals and the shaft rotation speed. Cheng [3] in his paper presented a model of the bearing in which the non-linear dynamic analysis are carried out. The model consists of the defect and the responses are obtained due to vibration. The experimental work is conducted and the results are compared. A theoretical model was developed by Patil [4] with defect and the size of the defect and its effects are studied in the paper. Due to the defect, some impulses are created but the author instead using the periodical repeated impulse function, the defect itself has been modelled a circumferential sinusoidal wave. The experimental work is also conducted and the results are compared in the paper. Kong [5] in his paper has mentioned some of the disadvantages of the traditional approach and presented a new approach. The author has developed a new model in which the contact forces between the rolling elements and the defect are analyzed. The experimental work is conducted with the defect ball bearing using a bearing test rig. The results are compared. The present work focuses on the development of ball bearing model using mathematical method. The defect is modelled and the vibration responses are obtained. The defect model is further integrated with the equation of motion of a rotor bearing system and the transient responses are obtained.

2. Dynamic model of bearing
Theoretically, when the bearing experiences a radial load, the ball element and bearing raceway experiences point contact in between them. If there is a point contact, then the stress produced will be infinite. But practically, there is a slight elastic deformation that happens between the rolling element and the raceways. The Hertzian theory explains about this theory of elasticity by pointing out that the elastic deformation which gets produced in between the ball and raceway contacts gives rise to a non-linear relation between the force and the deflection. The non-linear relation between the load, \( F \) and the deflection \( \delta_r \) is given by Eqn. (1)

\[
F = K \delta_r^n
\]  

Where \( K \) is the load-deflection factor which depends on geometry of contact, \( \delta_r \) is the radial deflection and \( n = 3/2 \) for ball bearing.

\[
K = \left[ \frac{1}{\left( \frac{1}{R_i} \right)^{n} + \left( \frac{1}{R_o} \right)^{n}} \right]^{\frac{1}{n}} 
\]  

(2)

Where \( K_i \) and \( K_o \) are the load-deflection coefficients for the inner and the outer raceways which are determined using the Harris equation given in Eqn. (3)

\[
K_p = 2.15 \times 10^5 \Sigma \rho \delta^4 \frac{1}{z} 
\]  

(3)

Here \( \Sigma \rho \) is the curvature sum which can be calculated using the radii of the curvature in a pair of plane passing through the point contact. \( \delta^4 \) is the dimensionless contact deformation. The unit of \( K_p \) is N/mm\(^{1.5}\).

The figure 1 represents the schematic diagram of a ball bearing with coordinate axis. \( x \) and \( y \) denoted radial deflection with respective axes. Thus, \( \delta_r \) at any ball angular position \( \theta_i \) is \( [(x \cos \theta_i + y \sin \theta_i) - C_r] \). Substituting \( \delta_r \) equation in Eqn. (1), we get,

\[
F = K[(x \cos \theta_i + y \sin \theta_i) - C_r]^\frac{1}{2} \cos \theta_i 
\]  

(4)

The total bearing forces \( F_x \) and \( F_y \) along \( X \) and \( Y \) directions are given in Eqn. 5 and Eqn. 6, where \( z \) is the number of ball elements.

\[
F_x = \sum_{i=1}^{z} K[(x \cos \theta_i + y \sin \theta_i) - C_r]^\frac{1}{2} \cos \theta_i 
\]  

(5)
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3. Bearing defect model
In this paper, an outer raceway defect in the form of a half-sinusoidal wave is modelled. The defect width is chosen as 0.27 mm, having a depth of 0.5 mm. When ball comes in contact with defect, the contact stress will get varied causes vibration. The restoring force for the presence of defect is given in Eqns. (7) and (8).

\[
F_{xD} = \sum_{i=1}^{n} K[(x \cos \theta_i + y \sin \theta_i) - (C_r + \Delta \sin \frac{\pi}{\varphi} (\theta_t - \theta_i))] \cos \theta_i
\]

\[
F_{yD} = \sum_{i=1}^{n} K[(x \cos \theta_i + y \sin \theta_i) - (C_r + \Delta \sin \frac{\pi}{\varphi} (\theta_t - \theta_i))] \sin \theta_i
\]

Here, \( \varphi \) is the defect arc length, \( \theta_t \) is the defect position, \( \theta_i \) is the angular position of element \( i \).

The ball sinking depth can be calculated using Eqn. (9).

\[
\Delta = R - \sqrt{R^2 - L^2}
\]

Where \( R \) is ball radius, \( L \) is defect length.

\[
\varphi = \frac{\text{Defect size}}{\text{Raceway radius}}
\]

\[
\theta_t = \omega_c t + \frac{2\pi}{Z(z-1)}
\]

Here \( i = Z \) to 1, \( \omega_c \) is the shaft speed, \( \omega_c \) is the cage speed.

The inner race of the bearing moves at a shaft speed (\( \omega \)) and the centre of the rolling element moves at the cage speed (\( \omega_c \)). The defect position is taken as 40\(^{\circ}\). The rolling element falls into the defect and give rise to the amplitude of the vibration signals. Vibration responses are obtained.

4. System equations of motion
In this section, a rotor system supported on ball bearings is analysed. The governing Equations of motion for such as system are presented in Eqns. (12) and (13).

\[
M\ddot{x} + C\dot{x} + F_x = 0
\]

\[
M\ddot{y} + C\dot{y} + F_y = W
\]

---

**Figure 1.** Schematic drawing of a ball bearing with coordinate axes.
Where $M$ is the mass of the system, $C$ is the damping coefficient, $F_x$ and $F_y$ are the bearing forces and $W$ is the reaction load acting on the bearing.

Using state space method, the second order differential equation are converted into first order. The state space equations are presented in Eqn. (14).

Let $P_1 = x$, $P_2 = \dot{x}$, $P_3 = y$, $P_4 = \dot{y}$

$\dot{P}_1 = \dot{x} = P_2$, $\dot{P}_2 = \ddot{x}$, $\dot{P}_3 = \dot{y} = P_4$, $\dot{P}_4 = \ddot{y}$

Therefore,

$$ P = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix} \quad \text{and} \quad \dot{P} = \begin{bmatrix} \dot{P}_1 \\ \dot{P}_2 \\ \dot{P}_3 \\ \dot{P}_4 \end{bmatrix} = \begin{bmatrix} P_2 \\ \ddot{x} \\ P_4 \\ \ddot{y} \end{bmatrix} $$

(14)

5. Numerical Problem

Condition monitoring of bearing is essential which predicts the failure of a component before it occurs. The ball bearing is modelled analytically. The defect is located in the outer race of the bearing, while the inner race is made to rotate at the shaft speed, the rolling elements falls on the defect which leads to the vibrations giving a rise in amplitude of the vibration signals. The input values are given in tables 1 and 2 respectively. The flow chart diagram is given in figure 2.

![Flow chart diagram](image)

**Figure 2.** Flow chart diagram.

**Table 1.** Inputs for the model.

| Parameter | Value |
|-----------|-------|
| $W$       | 167.0255 N |
| $C_p$     | 0.007 |
| $Z$       | 9 |
| $N_x$     | 1200rpm |
| $\varphi$ | 0.2656 degrees |
| $\Delta$  | 0.5 mm |
6. Results and Discussions

Figure 3 and 4 shows the amplitude of the vibration signals increase at certain interval of time. This denotes that, when the ball comes and falls into the defect, the amplitude of the vibration signals increase at that interval of time. When ball crosses the defect, the amplitude reduce at that interval of time.

The defect depth is taken as 0.5mm. Sinking depth denotes the depth where the rolling element sink into the defect while rotating. Now, using the state space representation, the equations are solved using fourth order Runge-Kutta method and vibration responses are obtained. The amplitude of the vibration signals for a time span of 5 s is plotted in figure 5.

![Figure 3. Displacement vs Time.](image)

![Figure 4. Velocity vs Time.](image)

![Figure 5. Time and Frequency domain graph for displacement x and y.](image)

### Table 2. System parameters.

| Rotor system parameters |  |
|-------------------------|--|
| $C_{xx1}=5106.88$ N-s/m | $W_5=50$ rad/sec |
| $C_{yy1}=7460.24$ N-s/m | $I_d=4.30 \times 10^3$ |
| $K_{xx1}=5106.88 \times 10^3$ N/m | $I_p=8.6 \times 10^3$ |
| $K_{yy1}=7460.24 \times 10^3$ N/m | $e=30 \times 10^{-6}$ |
| $C_{xx2}=9585.10$ N-s/m | $K_{xxD}=2704.56 \times 10^3$ N/m |
| $C_{yy2}=14002.12$ N-s/m | $K_{yyD}=3909.84 \times 10^3$ N/m |
| $K_{xx2}=9585.10 \times 10^3$ N/m | $C_{xxD}=2704.56$ N-s/m |
| $K_{yy2}=14002.12 \times 10^3$ N/m | $C_{yyD}=3909.84$ N-s/m |
| $= 5106.88 \times 10^{-3}$ N/m |  |
| $= 7460.24 \times 10^{-3}$ N/m |  |
| $= 9585.10 \times 10^{-3}$ N/m |  |
| $= 14002.12 \times 10^{-3}$ N/m |  |
A Rotor system is mathematically modelled which is supported by one defected ball bearing and healthy ball bearing. The stiffness values of the bearing are calculated. The unbalance force is considered and the forced vibration responses of the system is obtained. The forced vibration is obtained at the frequency of an applied force. The forced frequency obtained is 49.85 rad/sec.

Figure 6. Time and Frequency domain graph for displacement $\theta_x$ and $\theta_y$.

7. Conclusions
The presented work mainly discusses obtaining the vibration response of the rotor bearing system. The bearing with defect is mathematically modelled and the system governing equation of motions are obtained and represented in state space form. The vibration responses are obtained. Then the rotor bearing system is mathematically modelled having the defected bearing at one end and the healthy bearing at the other end. The time domain characteristics and the fast Fourier transform responses are obtained. Simulations are done using MATLAB tool. Thus the vibration based condition monitoring is used in this paper and the change in the vibration characteristics are generated due to the presence of the defect.

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