The ultraviolet and rapidity divergences of transverse-momentum dependent parton distribution functions with lightlike and transverse gauge links is studied, also incorporating a soft eikonal factor. We find that in the light-cone gauge with $q^-$-independent pole prescriptions extra divergences appear which amount, at one-loop, to a cusp-like anomalous dimension. We show that such contributions are absent when the Mandelstam-Leibbrandt prescription is used. In the first case, the soft factor cancels the anomalous-dimension defect, while in the second case its ultraviolet-divergent part reduces to unity.

Keywords: Transverse-momentum dependence; Wilson lines; renormalization group.

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1. Introduction

In recent years, with the observation of observables related to transversity, there has been renewed theoretical attention to the gauge invariance of QCD operators in the definition of unintegrated, i.e., transverse-momentum dependent (TMD), parton distribution functions (PDFs). The core components of such quantities are gauge links (also called Wilson lines) which are, in general, gauge-contour dependent and have, therefore, more complicated renormalization properties ensuing from their contour obstructions: end-points, cusps, and self-intersections. In fact, as we pointed out in Refs. 7, 8, the gauge-invariant definition of TMD PDFs...
which includes gauge links in the transverse direction, inevitably involves gauge contours with a cusped-like junction point. The renormalization effect on this point induces extra ultraviolet (UV) divergences which give rise to an anomalous dimension that in one-loop order coincides with the corresponding term of the universal cusp anomalous dimension.12

We argued in Refs. 7, 8 that in order to counter this problem, the fully gauge-invariant definition of the TMD PDFs has to include an additional soft (eikonal) factor13,14,15 along a special cusped gauge contour (see Sec. 4) which extends to the light-cone infinity in the transverse direction and serves to compensate the spurious contribution to the anomalous dimension peculiar to the light-cone gauge in connection with the advanced, retarded, or principal-value prescription. On the other hand, we have demonstrated16 that adopting instead the Mandelstam-Leibbrandt (ML) pole prescription17,18 for the gluon propagator, such contributions are absent so that the anomalous dimension of the TMD PDF coincides with the result in covariant gauges, while the UV-divergent part of the soft factor reduces to unity.

2. Gauge links in TMD PDFs

The study of inclusive processes, e.g., deeply inelastic scattering (DIS), involves integrated parton distribution functions which have the following gauge-invariant definition ($i$ labels the sort of the parton in hadron $h$)

$$f_{i/h}(x, Q^2) = \frac{1}{2} \int \frac{dk_-}{2\pi} e^{-ik^+} \langle h(P) | \bar{\psi}_i(\xi^-) \left[ \gamma^+ \psi_i(0^-) \right] | h(P) \rangle \bigg|_{\xi^+, \xi_-=0}$$  \hspace{1cm} (1)

and have renormalization properties controlled by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation. Gauge invariance is ensured via the gauge link

$$[\xi^-, 0^-] = \mathcal{P} \exp \left[ -ig \int_{0^-}^{\xi^-} dz^\mu A_\mu^a(0^+, z^-, 0_\perp) t^a \right]$$  \hspace{1cm} (2)

along a gauge contour on the light cone.

On the other hand, the study of semi-inclusive processes, such as semi-inclusive deeply inelastic scattering (SIDIS), or the Drell-Yan (DY) process—where one more final or initial hadron is detected and its transverse momentum is observed—requires the introduction of more complicated quantities, viz., distribution or fragmentation functions depending on the parton’s intrinsic transverse momentum $k_\perp$, which is, therefore, kept unintegrated. In that case, the imposition of the light-cone gauge $A^+ = 0$ is not sufficient to exhaust the gauge freedom in the TMD PDF.9 One can still perform $x^-$-independent gauge transformations without changing the gauge condition. The way out is to include into the definition of the TMD PDF also a gauge link in the transverse direction off the light cone.9 Then, in order to eliminate both gauge links, that along the light cone and the transverse one, one has to adopt the light-cone (LC) gauge $A^+ = 0$ together with the advanced boundary condition.
Renormalization-group anatomy of TMD PDFs in QCD

To go around the poles of the gluon propagator in the complex \((\text{Re}\, q^+, \text{Im}\, q^+)\) (or \(q^0\)) plane, the gluon propagator in the \(A^+ = 0\) gauge has the form

\[
D_{\mu\nu}^{LC}(q) = -\frac{i q^2}{q^2 - \lambda^2 + i\epsilon} \left( g_{\mu\nu} - \frac{q_{\mu} n_{\nu} - q_{\nu} n_{\mu}}{|q^+|} \right) \tag{3}
\]

and has to be evaluated according to one of the following pole prescriptions

\[
\left| \frac{1}{|q^+|} \right|_{\text{Ret/Adv}} = \frac{1}{q^+ \pm i\eta}, \quad \left| \frac{1}{|q^+|} \right|_{\text{PV}} = \frac{1}{2} \left[ \frac{1}{q^+ + i\eta} + \frac{1}{q^+ - i\eta} \right], \tag{4}
\]

where \(\eta\) has the dimension of mass and is kept small but finite, while the collinear poles are controlled by the quark virtuality \(p^2 < 0\), and the infrared (IR) singularities are regularized by the auxiliary gluon mass \(\lambda\). None of these regulators will appear in the final expressions for the anomalous dimensions.

Hence, one has the following operator definition

\[
f_{ijh}(x, k_\perp; \mu^2) = \frac{1}{2} \int \frac{d\xi^+ d^2\xi_\perp}{2\pi(2\pi)^2} e^{-ik^+\xi^- + ik_\perp \cdot \xi_\perp} \langle h|\bar{\psi}(\xi^-; \xi_\perp)\psi(\xi^-; \xi_\perp)|\infty - i\eta \rangle \times \left[ 1, \xi_\perp; \infty^-; \xi_\perp \right] \gamma^+ \left[ 1, \infty^-; \infty^-; 0 \right] \psi_i(0^-; 0_\perp)|h\rangle, \tag{5}
\]

where gauge invariance is ensured by means of the path-ordered gauge links

\[
[\infty^-, z_\perp; z^-; z_\perp] \equiv \mathcal{P} \exp \left[ ig \int_{0}^{\infty} d\tau n^- A_a^\mu t^a (z + n^- \tau) \right] \text{ lightlike link,}
\]

\[
[\infty^-, 0^-; \xi_\perp] \equiv \mathcal{P} \exp \left[ ig \int_{0}^{\infty} d\tau \cdot l \cdot A_a t^a (\xi_\perp + l\tau) \right] \text{ transverse link.} \tag{6}
\]

Note that the two-dimensional vector \(l\) is arbitrary with no influence on the (local) anomalous dimensions (i.e., it drops out from the final results). Physically, the advanced prescription means that all final-state gluon interactions between the struck quark and the spectators have been reshuffled from the final to the initial state and have been absorbed into the corresponding wave function.

\[
\text{Fig. 1. One-loop gluon radiative corrections (curly lines) contributing UV divergences to the TMD PDF with gauge links (double lines). Diagrams (a), (b), and (c) appear in covariant gauges, while in light-cone gauges only diagrams (a) and (d) contribute. The “mirror” (Hermitian conjugate) diagrams—abbreviated by (h.c.)—are not shown.}
\]

The one-loop diagrams (see Fig. 1) in the light-cone gauge with the advanced prescription, contributing to the TMD distribution of a quark in a quark, have been
calculated in Refs. 9, 19. There is a concealed assumption in these calculations. This is that the junction point of the two transverse gauge links at light-cone infinity is such that the two gauge (i.e., integration) contours are joined smoothly. However, we have shown 7, 8 by explicit calculation of the UV divergences of these diagrams that extra contributions appear that generate an anomalous dimension equal—at this loop order—to the universal cusp anomalous dimension. 12 [Whether this finding remains valid in higher loop orders has not been proved yet.] This indicates that the transverse gauge links off the light cone have a cusp-like junction point whose renormalization gives a nontrivial contribution to the anomalous dimension of the TMD PDF.

3. One-loop radiative corrections of TMD PDFs

3.1. $q^-$-independent pole prescriptions

Let us now look at the diagrams in Fig. 1 in more detail. First some general remarks. Diagrams (a), (b), and (c) contribute in covariant gauges, whereas in the light-cone gauge only diagrams (a) and (d) contribute. Diagram (d) is associated with the transverse gauge link and, hence, it is peculiar to the light-cone gauge. Note that the Hermitian-conjugate diagrams (labeled h.c.) are generated by “mirror” diagrams which are not shown explicitly.

First, we consider the imposition of $q^-$-independent pole prescriptions, like those in Eq. (4). In that case, the transverse component of the gauge field reads

$$ \mathbf{A}^\perp(\infty^-; \xi^\perp) = \frac{g}{4\pi} C_\infty \nabla^\perp \ln \Lambda |\xi^\perp|, $$

(7)

where the numerical constant $C_\infty$ depends on the pole prescription to treat the light-cone divergences in the notation of Ref. 9

$$ C_\infty = \begin{cases} 
0, & \text{Advanced} \\
-1, & \text{Retarded} \\
-\frac{1}{2}, & \text{Principal Value} 
\end{cases} $$

(8)

and $\Lambda$ is an IR regulator that does not enter the final results.

Using dimensional regularization within the $\overline{\text{MS}}$ scheme, one can now compute the contributions of diagrams (a) and (d) in Fig. 1 $\Sigma^{(a)}$ and $\Sigma^{(d)}$. Diagram (a) contains UV divergences which translate into a single pole $1/\epsilon$. In addition, it contains divergences which are related to the poles of the gluon propagator in the $A^+ = 0$ gauge and are controlled by the employed pole prescription—embodied in the factor $C_\infty$. Moreover, it also contains a UV-finite (i.e., $\epsilon$-independent) part, which has, however, rapidity divergences—i.e., a term proportional to $C_\infty$. The explicit expressions for all these terms can be found in Ref. 8.

On the other hand, diagram (d), which we repeat, is absent in covariant gauges, contains rapidity divergences driven by $C_\infty$ (see in Ref. 8 for the explicit expression and further details). It turns out that—including the mirror diagrams of (a) and (d)—this contribution exactly cancels all terms proportional to $C_\infty$ in $\Sigma^{(a)}_{\text{UV}}$ and
also in $\Sigma_{\text{finite}}^{(a)}$, rendering the total expression pole-prescription independent. What remains reads

$$\Sigma_{\text{UV}}^{(a+d)}(p, \mu, \alpha_s; \epsilon) = -\frac{\alpha_s}{\pi} C_F \frac{1}{\epsilon} \left[ \frac{1}{4} - \frac{\gamma^+ \hat{p}}{2p^+} \left( 1 + \ln \frac{\eta}{p^+} - i\pi \frac{\eta}{p^+} - i\pi C_{\infty}^{(a)} + i\pi C_{\infty}^{(d)} \right) \right],$$

(9)

where $C_F = \frac{(N_c^2 - 1)}{2N_c} = 4/3$ and the superscripts $(a)$ and $(d)$ on $C_{\infty}$ serve to indicate the diagram from which the corresponding contribution originates. The above expression can be further evaluated by virtue of $\gamma + \hat{p} \gamma + \hat{p}^2 = \gamma + \hat{p}$ to obtain

$$\Sigma_{\text{UV}}^{(a+d)}(\alpha_s, \epsilon) = 2\frac{\alpha_s}{\pi} C_F \left[ \frac{1}{\epsilon} \left( \frac{3}{4} + \ln \frac{\eta}{p^+} \right) - \gamma_E + \ln 4\pi \right].$$

(10)

From this expression we find

$$\gamma_{\text{LC}} = \frac{\mu}{2Z} \frac{1}{\partial\alpha_s} \partial Z = \frac{\alpha_s}{\pi} C_F \left( \frac{3}{4} + \ln \frac{\eta}{p^+} \right) = \gamma_{\text{smooth}} - \delta \gamma.$$  

(11)

Thus, the one-loop anomalous dimension in the LC gauge with $q^-$-independent pole prescriptions reads

$$\gamma_{\text{LC}} = \gamma_{\text{smooth}} - \delta \gamma, \quad \gamma_{\text{smooth}} = 3\frac{\alpha_s}{\pi} C_F + O(\alpha_s^2), \quad \delta \gamma = \frac{\alpha_s}{\pi} C_F \ln \frac{\eta}{p^+},$$

(12)

where the defect of the anomalous dimension $\delta \gamma$ encapsulates the deviation of the calculated quantity from the anomalous dimension of the two-quark operator with a gauge-connector insertion in a covariant gauge. Note that the only anomalous dimensions ensuing from the gauge connector along any smooth contour stem from its endpoints. Hence, the backfit logic in this finding is to modify the definition of the TMD PDF in Eq. (5) in such a way as to dispense with $\delta \gamma$. But to do so, we have first to understand the deeper meaning of the anomalous-dimension defect. To this end, write $p^+ = (p \cdot n^-) \sim \cosh \chi$ and observe that it defines an angle $\chi$ between the direction of the quark momentum $p^+$ and the lightlike vector $n^-$. In the large $\chi$ limit, $\ln p^+ \to \chi, \chi \to \infty$. Therefore, it appears natural to conclude that the defect of the anomalous dimension $\delta \gamma$ can be identified with the universal cusp anomalous dimension at the one-loop order:

$$\gamma_{\text{cusp}}(\alpha_s, \chi) = \frac{\alpha_s}{\pi} C_F (\chi \coth \chi - 1),$$

$$\frac{d}{d \ln p^+} \delta \gamma = \lim_{\chi \to \infty} \frac{d}{d \chi} \gamma_{\text{cusp}}(\alpha_s, \chi) = \frac{\alpha_s}{\pi} C_F.$$  

(13)

3.2. Mandelstam-Leibbrandt pole prescription

Let us now consider the application of the light-cone gauge in conjunction with a $q^-$-dependent pole prescription for the gluon propagator. Imposing the Mandelstam-Leibbrandt (ML) pole prescription $^{17,18}$ which depends on both variables $q^+$ and
\( q^-, \) gives rise to a more complicated structure of the gluon propagator in the complex \( q^0 \) plane, viz.,

\[
\frac{1}{[q^+]_{\text{ML}}} = \left\{ \begin{array}{ll}
\frac{1}{q^+ + i(\alpha s q^- + d)} & \\
\frac{1}{q^+ + i(\alpha s q^- + d)} & \text{for } q^+ > 0
\end{array} \right.,
\]

which is illustrated graphically in Fig. 2.

![Integration contour and poles of the gluon propagator in the (Re \( q^0 \), Im \( q^0 \)) plane.](image)

\[\text{Fig. 2. Integration contour and poles of the gluon propagator in the (Re \( q^0 \), Im \( q^0 \)) plane. The results of the ML-prescription (position 1) and those in a covariant gauge (position 2) belong to the same quadrants: second and fourth—not valid for the principal-value prescription (position 3).}\]

Our goal is to evaluate diagrams \( (a) \) and \( (d) \) in Fig. 1 using this pole prescription. In that case, the evaluation of \( \Sigma^{(a)} \) is more complicated and we refer for the technical steps to Ref. 16. Here we only sketch the main results. First, recall that \( \Sigma^{(a)} \) contains a part proportional to \( g_{\mu\nu} \)—Feynman term—which is pole-prescription independent. The rest, labeled \( \Sigma^{(a)}_{\text{ML}} \), can be computed analytically to yield

\[
\Sigma^{(a)}_{\text{ML}}(p, \alpha_s; \mu, \epsilon) = \Sigma^{(a)}_{\text{Feynman}} + \Sigma^{(a)}_{\text{ML}} = -\frac{\alpha_s}{4\pi} C_F \Gamma(\epsilon) \left( -4\pi \mu^2/\rho^2 \right)^{\epsilon} \frac{\Gamma^2(1-\epsilon)}{\Gamma(2-2\epsilon)} \left[ (1-\epsilon) - 4 \right].
\]

(15)

Extracting the UV divergent terms in the MS-scheme, one gets (after adding the mirror diagrams):

\[
\Sigma^{(a)}_{\text{UV/ML}}(p, \alpha_s, \mu, \epsilon) = -\frac{\alpha_s}{4\pi} C_F \left[ \frac{1}{\epsilon} (1-\gamma_E + 4\pi) \right] = -\frac{3\alpha_s}{4\pi} C_F \left[ \frac{1}{\epsilon} - \gamma_E + 4\pi \right].
\]

(16)

Obviously, employing the ML-pole prescription in the light-cone gauge, both the UV-divergent part of the TMD PDF and also the finite one do not contain any extra terms of the form \( \ln p^+ \) which could be attributed to a cusped contour—in contrast to the findings of the previous subsection.

The next task is to evaluate diagram \( (d) \), which contains the cross-talk of the transverse gauge links \([\infty^-, 0^+; I(x)\tau], [\infty^-, 0^+; I(x)\tau + \xi \tau] \). To evaluate this expression, we have first to determine the transverse gauge field at light-cone infinity. This
calculation was performed in Ref. [16] from which we quote the result
\[ A^\perp(\infty^-; \xi^\perp) = -\frac{g}{4\pi} \nabla^\perp \ln \Lambda |\xi^\perp|. \] (17)

Then, we find
\[ \Sigma^{(d)}_{ML}(p, \mu, \gamma; \epsilon) = -g^2 C_F \mu^2 2\pi i \int \frac{d^2q}{(2\pi)^2} \delta(q^+) \left( \frac{\gamma^+(s - q)}{(p - q)^2 q^2} \right). \] (18)

Collecting the UV divergences of both diagrams (a) and (d), we obtain
\[ \Sigma^{(a+d)}_{ML/UV}(p, \mu, \alpha_s; \epsilon) = -\frac{4\pi}{\pi} \alpha_s \left( \frac{1}{4} - \frac{\gamma^+ + \gamma_E}{2p^+} \right) - \gamma_E + 4\pi \right), \] (19)

which finally yields \((\gamma^+ + \hat{p}^+ / 2p^+ = \gamma^+)\)
\[ \Sigma^{(a+d)}_{ML/UV}(p, \mu, \alpha_s; \epsilon) = -\frac{4\pi}{\pi} \alpha_s \left( \frac{1}{4} - \frac{\gamma^+ + \gamma_E}{2p^+} \right) - \gamma_E + 4\pi \right), \] (20)

After including the mirror contribution to graph (d) in Fig. 1, one arrives at the following expression
\[ \Sigma^{(a+d)}_{ML/UV}(p, \mu, \alpha_s; \epsilon) = \frac{4\pi}{\pi} \alpha_s \left( \frac{1}{4} - \frac{\gamma^+ + \gamma_E}{2p^+} \right) - \gamma_E + 4\pi \right), \] (21)

which does not contain an imaginary part and resembles the result one finds in covariant gauges.

Bottom line: Using the Mandelstam-Leibbrandt pole prescription to treat the rapidity divergences in the gluon propagator in association with the light-cone gauge, no anomalous-dimension defect appears, so that one gets the well-known expression
\[ \gamma_{ML} = -\frac{1}{2} \mu \frac{d}{d\mu} \ln \Sigma^{(a+d)}_{ML/UV}(\alpha_s, \epsilon) = \frac{3}{4} \frac{\alpha_s}{\pi} C_F + O(\alpha_s^2). \] (22)

4. Soft factor and TMD PDF redefinition

Aiming for a more suitable definition of TMD PDFs, we propose to refurbish Eq. (5) by including a soft factor \( R \)
\[ R \equiv \left[ 0 \right] \mathcal{P} \exp \left[ ig \int_{\Gamma_{cusp}} d\zeta^a t^a A^\mu_\mu(\zeta) \right] \mathcal{P}^{-1} \exp \left[ -ig \int_{\Gamma_{cusp}} d\zeta^a t^a A^\mu_\mu(\zeta + \zeta) \right] 0 \right), \] (23)

that contains eikonal lines, giving rise to an anomalous dimension which is equal in magnitude but opposite in sign to the defect of the anomalous dimension entailed by the cusped junction point of the gauge contours. This soft factor is calculated along the particular gauge contour shown in Fig. 3.

The involved gauge contours are defined as follows:
\[ \Gamma_{cusp} : \zeta_\mu = \{ [p^\mu_\mu s, -\infty < s < 0] \cup [n^\mu^- s', 0 < s' < \infty] \cup [l^\perp \tau, 0 < \tau < \infty] \} \]
\[ \Gamma'_{cusp} : \zeta_\mu = \{ [p^\mu_\mu s, +\infty < s < 0] \cup [n^\mu^- s', 0 < s' < \infty] \cup [l^\perp \tau, 0 < \tau < \infty] \}. \] (24)
Then, the TMD PDF can be redefined to read
\[ f_{i/h}^{\text{mod}}(x, k_\perp; \mu^2) = \frac{1}{2} \int d\xi_- \frac{d^2 \xi_\perp}{2\pi^2} e^{-ik_-\xi_- + i k_\perp \cdot \xi_\perp} \langle h | \bar{\psi}(\xi_-, \xi_\perp) [\xi_-^{-}, \xi_\perp] [\xi_-^{+}, \xi_\perp]| \lambda \rangle R(p^+, n^- | \xi_-^{-}, \xi_\perp). \]

The one-loop radiative corrections to \( R \) originate from diagrams analogous to those shown in Fig. 1 in which the thick quark line is replaced by a double line associated with the gauge link entering Eq. (23). Employing the light-cone gauge \( A_+ = 0 \) with one of the \( q^- \)-independent pole prescriptions \([\text{Adv}, \text{Ret}, \text{PV}]\) according to Eq. (4), we get for the UV part the following expression

\[ \Phi^{(a+d)}_{\text{UV/LC}}(\epsilon; \eta) = -\frac{\alpha_s}{\pi} C_F \frac{1}{\epsilon} \left( \ln \frac{\eta}{p^+} - i \frac{\pi}{2} \right). \]

Taking into account the mirror diagrams to (a) and (d) (with the thick quark line replaced by a double line for the gauge link), one obtains the total UV-divergent part of the soft factor at the one-loop order:

\[ \Phi^{1-\text{loop}}_{\text{UV/LC}}(\epsilon; \eta) = -\frac{\alpha_s}{\pi} C_F \frac{2}{\epsilon} \ln \frac{\eta}{p^+}. \]

One notes that this expression bears no dependence on the pole prescription, i.e., all \( C_\infty \)-dependent terms are absent. The only surviving contribution to the associated anomalous dimension is the cusp-related term \( \sim \ln p^+ \) which amounts to \( -\gamma_{\text{cusp}} \).

This completes the proof that the modified TMD PDF definition (25) contains no artifacts stemming from contour obstructions.

Consider now the case with the ML pole prescription. As we have already discussed, this pole prescription removes all undesirable light-cone singularities. Therefore, we should expect that the insertion of the soft factor \( R \) in that case does not destroy this property. Skipping details, let us remark that the UV singularities of the soft factor are generated by the self-energy of the lightlike gauge link and the
one-gluon exchanges between the lightlike and the transverse gauge link (analogous diagrams to (a) and (d) in Fig. 1). Thus, one has

$$\Phi_{\text{soft}}^{\text{LO/ML}} = \Phi_{\text{soft}}^{(0)} + O(\alpha_s^2),$$ \hspace{1cm} (28)

with $\Phi_{\text{soft}}^{(0)} = 1$ and

$$\Phi_{\text{soft}}^{(1)} = \Phi_{\text{soft}}^{(1)} - \text{virt} + \Phi_{\text{soft}}^{(1)} - \text{real},$$ \hspace{1cm} (29)

where

$$\Phi_{\text{soft}}^{(1)} = 2ig^2\mu^2C_F\int_{0}^{\infty} d\sigma \int_{0}^{\sigma} d\tau \int \frac{d^2q}{(2\pi)^2} \frac{e^{-iq^- (\sigma-\tau)}}{2q^+q^- - q_\perp^2 + i0q^- + i0q^+}$$ \hspace{1cm} (30)

and the vector $u_\mu$ is being chosen to be lightlike: $u_\mu = (p^+, 0, 0, 0_\perp)$. Appealing to Fig. 2, we observe that the integral above vanishes by virtue of the location of the poles in the Feynman and ML denominators on the same side of the $q^+$-axis. Consequently, we have

$$\Phi_{\text{soft}}^{(a)\text{ML}} = 0.$$ \hspace{1cm} (31)

For the same reason, also the contribution of diagram (d) vanishes, entailing $\Phi_{\text{soft}}^{(d)\text{ML}} = 0$. On the other hand, the contribution arising from real gluons, $\Phi_{\text{soft}}^{(1)\text{real}}$, does not contain UV-singularities. Hence, the UV-divergent part of $\Phi_{\text{soft}}^{\text{LO/ML}}$ reduces to unity, validating Eq. (25) also for the case of the light-cone gauge with the ML-prescription.

5. Evolution of the TMD PDF

The evolution behavior of TMD PDFs is of particular theoretical and phenomenological importance. Theoretically, we have to verify whether the integrated PDF, obtained from Eq. (25), coincides with the standard one with no contour artifacts left over. Furthermore, we have to deal with the additional dependence on the scale $\eta$ by means of an additional evolution equation. Note that in our approach this mass parameter plays a role akin to the rapidity parameter $\zeta$ in the Collins-Soper evolution equation. We have discussed some of these issues and shown that the redefined TMD PDF satisfies the following renormalization-group equation

$$\frac{1}{2}\mu^2 \frac{d}{d\mu} \ln f_{q/q}^{\text{mod}}(x, k_\perp; \mu, \eta) = \frac{3}{4} \frac{\alpha_s}{\pi} C_F + O(\alpha_s^2) = \gamma_{f_{q/q}}.$$ \hspace{1cm} (32)

Taking logarithmic derivatives of $f_{q/q}^{\text{mod}}(x, k_\perp; \mu, \eta)$ with respect to both scales $\mu$ and $\eta$, we get

$$\mu \frac{d}{d\mu} \left[ \eta \frac{d}{d\eta} f_{q/q}^{\text{mod}}(x, k_\perp; \mu, \eta) \right] = 0,$$ \hspace{1cm} (33)

which establishes the formal analogy between our approach and the Collins-Soper one. We emphasize that only the modified definition via Eq. (25) satisfies such simple evolution equations.
6. Concluding Remarks

We have discussed an approach to TMD PDFs which takes into account the renormalization properties of the contour-dependent gauge links in terms of the anomalous dimensions ensuing from contour obstructions. We argued that supplementing the light-cone gauge with \( q^- \)-independent pole prescriptions for the gluon propagator, leads to the appearance of an anomalous dimension that can be associated with a cusped junction point of the transverse gauge contours. In contrast, we found that when the ML prescription is employed, which depends on both variables \( q^+ \) and \( q^- \), the defect of the anomalous dimension cancels out. In the first case, a nontrivial soft factor in the definition of the TMD PDF restores the correct anomalous dimension by compensating the renormalization effect on the cusp-like junction point of the contours. In the second case, the soft factor reduces to unity and one recovers the same result as in covariant gauges in which the transverse gauge links are absent.

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