Limit shocks of relativistic magnetohydrodynamics, Punsly’s waveguide and the Blandford-Znajek solution

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ABSTRACT
In this paper we examine various issues closely related to the ongoing discussion on the nature of the Blandford-Znajek mechanism of extraction of rotational energy of black holes. In particular, we show that switch-on and switch-off shocks are allowed by the shock equations of relativistic MHD and have similar properties to their Newtonian counterparts. Just like in Newtonian MHD they are limits of fast and slow shock solutions and as such they may be classified as weakly evolutionary shocks. The analysis of Punsly’s MHD waveguide problem shows that its solution cannot have the form of a traveling step wave and that both fast and Alfvén waves are essential for generating the flow in the guide. Causality considerations are used to argue that the Blandford-Znajek perturbative solution is in conflict with the membrane paradigm. An alternative interpretation is presented according to which the role of an effective unipolar inductor in the Blandford-Znajek mechanism is played by the ergospheric region of a rotating black hole. Various implications of this are discussed.

Key words: black hole physics – MHD – relativity – shock waves.

1 INTRODUCTION
In coordinate systems singular at the black hole horizon, like the popular Boyer-Lindquist coordinates, the horizon is inevitably turns into a rather peculiar boundary of spatial domain. According to the widely accepted “Membrane paradigm” the horizon, or rather somewhat less stringently defined “stretched horizon” which is placed somewhere just above the real horizon, may be identified with a rotating conducting sphere (e.g. Blandford 1979, Thorne et al. 1986). This makes magnetized black holes look analogous to magnetized neutron stars. For many years, beginning with (Punsly & Coroniti 1990), Brian Punsly have been criticizing this view and also the perturbative steady-state electro-magnetic wind solution for a force-free magnetosphere of a rotating black hole due to Blandford and Znajek (1977), the BZ solution, together with similar MHD models (e.g. Phinney 1982,1983) on the basis of causality arguments. Indeed, in the case of pulsars there is only an outgoing wind which passes first through the Alfvén critical surface and then through the fast critical surface and, thus, the neutron star can communicate with the wind by means of both fast and Alfvén waves (Since the gas pressure is dynamically insignificant in the tenuous magnetospheres of neutron stars and black holes, the slow waves seem to be irrelevant.) Black holes, however, must also develop an ingoing wind which passes through its own pair of critical surfaces before reaching the horizon (Takahashi et al. 1990). For the typical parameters of astrophysical black holes the inner fast surface is likely to be extremely close to the black hole horizon and one may argue that the stretched horizon can communicate with the outgoing wind by means of fast waves. On the other hand, the inner critical Alfvén surface may be rather distant from the black hole horizon. Thus, even the stretched horizon cannot communicate with the outgoing wind by means of Alfvén waves which makes black holes rather different from neutron stars.

Blandford (1979) proposed that the outgoing wind of black holes is established by means of fast waves alone (This seems to be the only way to reconcile the membrane paradigm with the BZ solution.) However, Punsly (1996,2001) argued that fast waves are completely irrelevant and Alfvén waves are solely responsible for creating the global system of poloidal electric currents of such winds and adjusting the angular velocity of magnetic field lines and suggested that the steady-state BZ solution is unstable and, hence, nonphysical. In particular, Punsly criticized Znajek’s horizon boundary condition (Znajek 1977) used to determine the wind constants in the BZ electrodynamic solution and in the MHD analysis of Phinney (1982,1983) as a condition imposed in a region causally disconnected from the outgoing wind.

However, Znajek’s condition is not really a boundary condition as it simply prohibits infinitely strong electromagnetic field as measured by a physical observer, e.g. a free falling observer (e.g. Phinney 1983). Recently, Beskin and
Kuznetsova (2000) stressed this point once more and argued that, though the interpretation of the stretched horizon as a unipolar inductor is misleading, there is nothing wrong with causality of the BZ-like MHD models. This conclusion is strongly supported by the results of recent time-dependent electromagnetic simulations (Komissarov 2003) which indicate asymptotic stability of the BZ solution (Znak-jek’s boundary condition was not imposed in these simulations.)

In this paper we continue the discussion of the causality paradox a bit further and attempt to clarify the nature of BZ mechanism. At first sight, the study presented here may appear rather unfocused but a closer look reveals strong connections between its sections. In Sec.2 we study limit shocks of relativistic MHD. In Sec.3 we use these results to analyse the Punsly’s waveguide problem that provides important insights into the problem of relativistic MHD and electrodynamic winds. In Sec.4 we propose a modification of the membrane paradigm that does not conflict with causality.

2 LIMIT SHOCKS OF RELATIVISTIC MHD

It is well known that shock equations of Newtonian MHD allow compressive solutions that have non-vanishing tangential component of magnetic field, $B_t$, only on one side of the discontinuity. These are called switch-on and switch-off shocks, e.g. (Jeffrey and Taniuti 1964). Both shocks propagate with Alfvén speed relative to the state with non-vanishing tangential component of the magnetic field. Relative to the other state, a switch-on shock is super-fast and a switch-off shock is sub-slow. It is also known that any switch-on solution can be considered as a fast shock in the limit $B_t \rightarrow 0$ and any switch-off solution as a similar limit of a slow shock solution and this is why these shock are often called limit shocks (Jeffrey and Taniuti 1964).

Strictly speaking, switch-on shocks are not evolutionary simply because it is impossible to ensure that $B_t$ is exactly zero upstream of the shock. An infinitesimally small perturbation of the upstream state resulting in an infinitesimally small upstream tangential field will generally turn this shock into a fast shock followed by an Alfvén discontinuity. This fast shock will be infinitesimally close to the original switch-on shock in all respects except the direction of the downstream tangential component of magnetic field. Moreover, both the fast and the Alfvén waves will have infinitesimally close wave speeds. Only a finite amplitude perturbation can result in a finite split of a switch-on shock. In this sense, switch-on shocks are similar to fast shocks but rather distinct from intermediate shocks of MHD which are genuinely non-evolutionary and split as a result of interaction with waves of infinitesimally small amplitude (e.g. Landau & Lifshitz 1959, Falle and Komissarov 2001). Similar arguments apply to switch-off shocks. All these specific properties of limit shocks explain why they should be considered as physically meaningful solutions and why Jeffrey and Taniuti (1964) called them weakly evolutionary.

Shock solutions of relativistic MHD have been a subject of rigorous analysis beginning with the pioneering paper by Hoffman and Teller (1950). The results have been summarized in two rather comprehensive monographs by A.Lichnerowicz (1967) and and A.M.Anile (1989). In Lichnerowicz (1967) it was apparently shown that limit shocks do not exist in relativistic MHD. In Anile (1989) and Majorana and Anile 1987 the analysis of limit shocks is not presented and readers are referred to the work by Lichnerowicz. It has to be stressed that, as it has been explained in Lichnerowicz (1967), the non-existence of limit shocks in relativistic MHD would make this system qualitatively different from Newtonian MHD. However, the author of this paper has recently found such shocks in numerical solutions of certain Riemann problems and realized that there must be a flaw in the analysis of Lichnerowicz.

Since the style adopted in these monographs is a bit too mathematical we shall follow the more traditional approach of Hoffman and Teller (1950). First, we construct the limit shock solutions, then we show that there exist evolutionary shock solutions in the neighborhood of these solutions, and point out the error in the analysis of Lichnerowicz.

Let us define more precisely what is meant by the limit shocks in relativistic MHD. The first condition is that, in the rest frame of the fluid on one side of the shock the magnetic field is parallel to the shock normal. This allows us to construct a shock frame that is moving with respect to the original fluid frame along the magnetic field. In this frame the shock is at rest, the electric field is vanishing and the fluid velocity is parallel to the magnetic field on both sides of the shock. This frame was first introduced by Hoffman and Teller (1950) to simplify the analysis of oblique MHD shocks. By construction, in this frame the magnetic field on one side of a limit shock is normal to the shock front. The second condition is a non-vanishing tangential component of magnetic field on the other side. Thus, the shock can be described as either switching on or switching off the tangential component of magnetic field in the Hoffman-Teller frame.

From Maxwell equations it follows that in the Hoffman-Teller frame the electric field vanishes on both sides of the shock and, thus, the fluid velocity vector is parallel to the magnetic field vector (Hoffman & Teller 1950):

$$u^i = A^i = B^i,$$

where $u^i$ are the space components of the 4-velocity vector. Following (Hoffman & Teller 1950) we introduce Cartesian coordinates such that the $x$-axis is along the shock normal and $B^i = 0$ on both sides of the shock. Then the only shock equations we have to analyse are

Continuity equation:

$$[D] = 0,$$  \hspace{1cm} (2)

Energy equation:

$$[T^{i\xi}] = 0,$$  \hspace{1cm} (3)

Momentum equations:

$$[T^{i\xi}] = 0,$$  \hspace{1cm} (4)

$$[T^{i\rho}] = 0,$$ \hspace{1cm} (5)

where for any quantity $A$

$$[A] = A_2 - A_1,$$

$$D = \rho u^i,$$  \hspace{1cm} (6)

$$T^{i\xi} = w u^i u^\xi,$$ \hspace{1cm} (7)
\[ T^{ij} = w u^i u^j + (p + B^2/2) g^{ij} - B^i B^j. \]  

(8)

Here \( \rho \) is the rest mass density, \( w \) is the relativistic enthalpy, \( p \) is the thermodynamic pressure, \( u^\nu \) is the fluid 4-velocity, and we are using units such that \( 4\pi \), the magnetic permeability and the speed of light do not appear in the equations. From (8) one has

\[ [(w s^2 - 1) B^x B^y] = 0. \]  

(9)

Thus if \( B^y = 0 \) only on one side of the shock then on the other side

\[ s^2 = 1/w. \]  

(10)

Combining (8) with (9) one obtains

\[ (u^x)^2 = (w + B^2)/w, \]  

(11)

\[ (u^y)^2 = (B^x)^2/w, \]  

(12)

and, finally,

\[ (v^x)^2 = e_a^2 = (B^x)^2/(w + B^2). \]  

(13)

The last equation tells us that relative to the state with non-vanishing tangential field the shock propagates with
Figure 3. Splitting of a switch-on shock into a fast shock and Alfvén wave as the result of interaction with a small perturbation of magnetic field introduced upstream of the initial solution. The top panels show the initial switch-on solution, where $p$ is the gas pressure. The shock is stationary relative to the computational grid. Its internal structure is due to artificial viscosity. The bottom panels show the result of interaction with a small perturbation of magnetic field ($B_z = 0.05$ for $x > 0.5$) upstream of the initial shock. The fast shock is located at $x \simeq 0.05$ and the Alfvén wave in $-0.3 < x < -0.1$.

Alfvén speed, $c_a$, e.g. (Komissarov 1999). Substitution of these results into (2–4) leads to

$$B^2 + 2p = 2T^{xx}, \quad (14)$$

$$w - 2p = \left( \frac{T^{xx}}{B^2} \right)^2 - 2T^{xx}, \quad (15)$$

and

$$\frac{\rho^2}{w} = \left( \frac{D}{B^2} \right)^2, \quad (16)$$

where the shock invariants $D$, $T^{xx}$, and $T^{xx}$ may be evaluated given the parameters of the state with vanishing $B^y$. Combined with the equation of state, (14–15) allow us to determine the thermodynamic parameters on the other side of the shock. In the following we use index “1” for the state with vanishing $B^y$ and index “2” for the state with non-vanishing $B^y$.

It is easy to verify that if $u_{x1} = c_{a1}$ then the shock vanishes. Consider weak shocks with

$$u_{x2}^2 = u_{a2}^2 (1 + \alpha), \quad |\alpha| \ll 1, \quad (17)$$

where $u_a = c_a / \sqrt{(1 - c_a^2)}$ may loosely be called the “Alfvén 4-velocity”. Substituting this into (14–16) and retaining only terms linear in $\alpha$ one obtains
\[ B_{z_2}^2 + 2[p] = \alpha(2w_1u_{a_1}^2), \]  
\[ [w] - 2[p] = \alpha w_1, \]  
\[ [\rho^2/w] = \alpha(\rho_1^2/w_1). \]  
From these and the second law of thermodynamics one has  
\[ [p] = \alpha w_1u_{a_1}^2, \]  
\[ B_{z_2}^2 = 2w_1\alpha(u_{a_1}^2 - u_{a_2}^2), \]  
where \( u_a = a/\sqrt{1 - a^2} \) is the “sound 4-velocity”, \( a \) is the sound speed. From (22) one can see that weak limit shocks exist only if  
\[ \alpha(c_{a_1} - a_1) > 0. \]  
This can only be satisfied in the following two cases  
- If \( c_{a_1} > a_1 \) and \( |v_1^x| > c_a \), then \( p_2 > p_1 \), this is a switch-on compressive shock,  
- If \( c_{a_1} < a_1 \) and \( |v_1^x| < c_a \), then \( p_2 < p_1 \), this is a switch-off compressive shock.

Figure 3 shows the switch-on solutions for the polytropic equation of state with \( \gamma = 4/3 \), \( \rho_1 = 1 \), \( p_1 = 1 \) and \( k = u_{a_1}/u_{a_3} = 1.1, 2.3, 10 \). Like in Newtonian MHD, a relativistic switch-on shock turns into a pure gas dynamical shock propagating along the magnetic field if the shock speed exceeds a certain critical value.

Figure 4 shows the switch-off solutions for the polytropic equation of state with \( \gamma = 4/3 \), \( \rho_1 = 1 \), \( p_1 = 1 \) and \( k = u_{a_1}/u_{a_3} = 0.8, 0.7, 0.5, 0.2 \). For \( k = 0.8 \) the switch-off shocks turn into a pure gas dynamical shock. For other values of \( k \) the shock curve terminates as the gas pressure of the upstream state vanishes.

The evolutionary conditions for shock solutions are of very general nature and apply to relativistic shocks in exactly the same manner as to Newtonian shocks. Thus, evolutionary compressive relativistic MHD shocks must be either super- or sub-Alfvénic on both sides of the shock (Komissarov 2001). Let us show that in the neighborhood of limit shocks there exist evolutionary shock solutions. From (1) one can see that if \( B^x \) has the same sign on both sides of the shock then so does  
\[ \mu = ws^2 - 1. \]  
From (1) one obtains  
\[ v_x^2 = f(\mu) = (1 + \mu)B_x^2/(w + (1 + \mu)B^2). \]  
Since \( f'(\mu) > 0 \), one immediately concludes that  
- If \( \mu > 0 \) then \( v_x^2 > c_x^2 \) on both sides of the shock,  
- If \( \mu < 0 \) then \( v_x^2 < c_x^2 \) on both sides of the shock,  
and, thus, the shock is evolutionary. For a switch-on shock \( \mu_2 > 0 \), thus, if we introduce an infinitesimally small \( B_{z_2}^2 \) of the same sign as \( B_{z_1}^2 \) the new shock solution will be a fast shock, just like in Newtonian MHD (Jeffrey and Taniuti 1964). Similarly, a switch-off shock turns into a slow shock.

Due to the general nature of evolutionary conditions splitting of not strictly evolutionary shocks must proceed in the same fashion regardless of relativistic or classical nature of governing equations. Figure 5 shows the effect of a small perturbation of the upstream magnetic field on a switch-on shock. The initial solution  
Left state: \[ B = (3.149, 2.749, 0), \quad E = 0, \quad \rho = 3.454, \quad p = 6.743, \quad \rho = 1, \quad p = 1. \]
describes a stationary switch-on shock. It internal structure seen in fig. 6 is entirely due to artificial viscosity. The upstream state is then perturbed by introducing tangential magnetic field with \( B_z = 0.05 \) upstream of the shock. The eventual interaction splits the switch-on shock into a fast shock and Alfvén wave in the same manner as it has been described in Sec.1. In these simulations we used polytropic equation of state with the ratio of specific heats \( \gamma = 4/3 \). These simulation were carried out using the upwind numerical scheme described in Komissarov (1999).

Finally, we find necessary to explain the error in the analysis of Lichnerowicz (1967) that eventually led to the incorrect conclusion on non-existence of the limits shock solutions (p.161). On page 151 of the book, equation (47-1), a space-like 4-vector \( h^\alpha \) is decomposed into a sum of two 4-vectors, one parallel and the other one normal to a unit space-like vector \( n^\alpha \):  
\[ h^\alpha = t^\alpha - \eta n^\alpha, \quad t^\alpha n_\alpha = 0. \]
Then it is claimed that \( t^\alpha \) is always space-like. However, this is not true. The reader can easily verify that if \( \alpha = (0, h, 0, 0) \) and \( n^\alpha = (\sqrt{3}, 2, 0, 0) \) then \( t^\alpha \) is time-like.

3 PUNSLY’S WAVEGUIDE PROBLEM

In order to demonstrate the exceptional role of Alfvén waves, Punsly proposed to consider a much simpler problem involving a Faraday wheel connected to a cylindrical waveguide uniformly filled with cold tenuous plasma and a strong axial magnetic field (Sec.2.9.4 in Punsly 2001). For this problem he claimed to have constructed a step Alfvén wave solution of MHD equations corresponding to an instantaneous spinning up of the disc. Downstream of the wave front the magnetic field has only axial and azimuthal components, the electric field is radial and the electric current is axial (see figure 6). The return current flows over the surface of the guide and the global current closure is ensured by the displacement current of the leading front. Such simple step wave solution, however, is impossible for the following simple reason.

Since in the unperturbed state the Alfvén speed in the direction of magnetic field is uniform across the waveguide, an Alfvén front launched from the surface of the disc would indeed stay normal to the guide axis. The problem is that such wave would have no effect on the state of plasma, one could call it a ghost wave. Indeed, a frame propagating with Alfvén speed in the axial direction is the Hoffmann-Teller frame, as in this frame \( E = 0 \) upstream. However, in the Hoffmann-Teller frame the amplitude of the tangential component of magnetic field remains unchanged by an Alfvén shock (Komissarov 1997) just like it does in Newtonian MHD. Since in this case the tangential component is zero upstream it must be vanishing downstream as well. It is easy to verify that the Lorentz transformation to the original frame of the waveguide preserves this result. Thus, the
Alfvén shock has to be ruled out as well as the claim that the solution to this problem involves only Alfvén waves.

The tangential component of magnetic can be generated by a switch-on fast compression wave including a switch-on shock. However, a switch-on shock cannot not remain plane given the properties of the “solution” downstream of the discontinuity. Indeed, for a constant shock speed relative to the unperturbed state the tangential component of the downstream magnetic field would have the same amplitude everywhere (see eq. [3.12]) whereas in the guide it must vanish along the symmetry axis. Thus, the solution to the Punsly’s problem in the framework of relativistic MHD cannot have the simple form of a step wave altogether.

Although we have not been able to find an analytical solution to the waveguide problem, the following basic arguments suggest that both fast and Alfvén waves are important in this problem. When the disc rotation is switched on, the induced velocity shear generates the azimuthal magnetic field $B_{\phi} \propto r$ at the disc surface. Since the force balance is broken a switch-on MHD shock is driven into the guide. Initially this shock is plane but since the shock speed depends on $r$ it eventually becomes curved. In the local Hoffman-Teller frame of such curved discontinuity the upstream tangential component of magnetic field will generally have a direction which is different from the one of the downstream tangential component and, thus, the discontinuity will no longer satisfy the MHD shock equations. As explained in Sec.2 it will split mainly into a fast shock followed by an Alfvén wave. Thus, the disc ultimately emits both fast and Alfvén waves. The same evolution would be observed in the case of Newtonian MHD. The role of fast waves is to create and amplify the non-axial field whereas the Alfvén waves ensure that it becomes azimuthal.

In the limit of force-free degenerate electrodynamics (Komissarov 2002, Blandford 2002) the wavespeed of the fast wave tends to the speed of light and this makes a step wave solution possible. Indeed, it is easy to verify that equations of degenerate electrodynamics allow the following traveling wave solution:

$$B_r = 0, \quad B_{\phi}(t, z) = -\Omega(t - z)rB_0, \quad B_z = B_0,$$
$$E_r(t, z) = B_{\phi}(t, z), \quad E_{\phi} = 0, \quad E_z = 0,$$  \hspace{1cm} (25)

where the components of vectors are given in the orthonormal basis of cylindrical coordinates and $\Omega(t)$ is the angular velocity of the disc. One could be tempted to interpret the discontinuity of such step-wave solution as a limit of a switch-on shock and call it a fast wave. However, the eigensystem of degenerate electrodynamics degenerates in the axial direction (in general, in the direction of $E \times B \pm B\sqrt{B^2 - E^2}$ (Komissarov 2002)), that is both the Alfvén and the fast linear waves propagate with the same speed, which is the speed of light, and are no longer distinct. Moreover, both modes are linearly degenerate and, thus, their shock solutions have the same properties as the small amplitude waves. This makes it quite impossible to tell whether this solution represents a fast or an Alfvén wave of degenerate electrodynamics. In fact, it should be regarded as a mixture of both modes. Indeed, in the waveguide which is not perfectly cylindrical this degeneracy will be removed and the solution will split into two distinct waves in the way similar to splitting of a switch-on shock discussed above.

4 MEMBRANE PARADIGM AND THE BLANDFORD-ZNAJEK SOLUTION

The results of the previous section show that in Punsly’s MHD waveguide problem both fast and Alfvén waves are important. This has to be true in general including the case of black holes magnetospheres. In such magnetospheres Alfvén waves propagate only along the poloidal field lines (Appendix A). Thus, fast waves must play a special role in establishing the cross-field balance whereas Alfvén waves are particularly important for establishing the wind constants (They are constant along the poloidal field lines.)

Since neither of the waves can be ignored the causality paradox posed by Punsly and Coroniti (1990a) has to
be taken seriously. The resolution of the paradox proposed by Punsly and Coroniti involves rejection of both 1) the membrane paradigm or rather its part that identifies the stretched horizon with the rotating conducting surface of a unipolar inductor and 2) the Blandford-Znajek solution. They eventually tried to construct completely different models of black hole magnetospheres involving rotating dense shells and discs of accreting matter as material analogues of the Faraday disc (Punsly 2001). However there seems to exist a different resolution of this paradox which we discuss here.

In fact, the causality arguments directly hit the membrane paradigm and it cannot be saved. The analogy between the horizon and a rotating conducting sphere is not at all that complete as it is believed. It does allow a simplified presentation for an audience unfamiliar with general relativity but it does not really provide deep insights into the physics of black hole magnetospheres.

Things are different when it comes to the BZ solution. All what the causality paradox tells us is that this solution is inconsistent with the membrane paradigm! Historically, the BZ solution was used to build the paradigm and now it is widely considered that both are inseparable, but in fact the BZ solution clearly indicates the limitations of the paradigm. Clearly, there is no material analog of the Faraday disc in the BZ magnetosphere. If the horizon does not play its role then what is forcing the rotation of magnetic field lines? The answer can only lay in the properties of space-time outside of the black hole horizon as it has been suggested in (Blandford 2004). In fact, another energy extraction mechanism, namely the Penrose mechanism (Penrose 1969), operates only within the ergosphere it seems reasonable to consider the possibility that it is the ergospheric region of space-time that serves as a driving “force” of the BZ mechanism. In fact, this is consistent with the causality arguments.

The super-Alfvénic region of the ingoing wind cannot and does not have to communicate with the outgoing wind by means of Alfvén waves just like the super-Alfvénic region of the outgoing wind cannot communicate with the ingoing wind in such a way. However, the driving source responsible for both the outgoing and the ingoing winds must be able to communicate with the winds by means of both fast and Alfvén waves and, thus, it must be located between the Alfvén surfaces and, thus, well outside of the horizon (Similar conclusion was reached by Punsly (2001) and Beskin & Kuznetsova (2001).) The position of these critical surfaces, which merge with the light surfaces in the limit of degenerate electrodynamics (Appendix A), is not fixed as it depends on the angular velocity of magnetic field lines, $\Omega$, which depends on many factors including the interaction with the surrounding plasma (effective load of the black hole electric circuit). However, for all values of $\Omega$ consistent with extraction of energy of a black hole in the BZ solution the inner Alfvén surface is located inside the ergosphere (Appendix A). The only exception is the polar direction where the light surface, the horizon, and the ergosphere coincide, but the Poynting flux density vanishes along the symmetry axis and, thus, there is no outgoing wind as well. Thus, there always exists an outer region of the ergosphere which is causally connected to the outgoing wind of the BZ solution.

In order to verify this conjecture one could study the dynamical behaviour of magnetic field lines that do not penetrate the horizon. The lines that enter the ergosphere are expected to be forced into rotation whereas those that do not should remain nonrotating. Such study is under way.

Naturally, the driving force has to be the same both in electrodynamics and MHD models. In MHD approximation the wavespeeds of both fast and Alfvén waves are smaller then the speed of light and therefore the corresponding inner critical surfaces lay strictly outside of the horizon everywhere. Since the horizon and the ergosphere always coincide in the polar direction, both inner critical surfaces are situated outside of the ergosphere in the polar region. Thus, there exists a polar flux tube which is causally disconnected from the ergosphere. Within such a tube there can only be possible an accretion. Such conclusion is not entirely unexpected as a particle located on the symmetry axis is subject only to gravitational attraction.

Another related issue is whether the approximation of degenerate electrodynamics (or magnetically dominated ideal MHD) brakes down somewhere near the horizon. It is impossible to get full answer to this question within the framework of degenerate electrodynamics. For example, if the condition $E \cdot B = 0$ is satisfied by the initial solution it will be preserved during the evolution though there may not be enough charged particles to ensure this condition. However, the preservation of the other condition $B^2 - E^2 > 0$, which is required for the hyperbolicity of degenerate electrodynamics, is not guaranteed (Komissarov 2002). As $B^2 \rightarrow E^2 \neq 0$ the drift velocity of plasma tends to the speed of light indicating that particle inertia may need to be taken into account. However, the BZ solution satisfies this condition all the way to the horizon and the inertial effects are unlikely to be important along the magnetic field lines threading the horizon. This is not so obvious in the case of magnetic field lines threading the equatorial plane of the ergosphere where the approximation of degenerate electrodynamics or ideal MHD has to break down in order to ensure the current closure condition. Punsly and Coroniti (1990) argue that particle inertia becomes important in the equatorial region as plasma is forced into rotation with almost speed of light relative to the zero angular velocity observers. However, they have not taken into account the Compton drag which may well lead to a much lower Lorentz factor of plasma rotation. This problem requires further investigation.

5 CONCLUSIONS

(i) Switch-on and switch-off shocks are allowed by the shock equations of relativistic MHD and have similar properties to their Newtonian counterparts. Just like in Newtonian MHD they are limits of fast and slow shock solutions and as such they may be classified as weakly evolutionary shocks.

(ii) Contrary to what is claimed in (Punsly 2001), the solution to Punsly’s MHD waveguide problem cannot have the form of a step-like traveling wave and the guide flow cannot be established by means of Alfvén waves alone. Even in the limit of degenerate electrodynamics where a step-wave solution exists it involves a mixture both fast and Alfvén waves. This suggests that both waves are important in the
problems of magnetically driven MHD and electrodynamic winds.

(iii) Blandford-Znajek solution contradicts to the membrane paradigm as the stretched horizon cannot play the role of a unipolar inductor. Causality arguments suggest that, just like in the case of the Penrose mechanism, the driving “force” of the Blandford-Znajek mechanism is the ergospheric region of space-time.

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REFERENCES

Anile A.M., 1989, “Relativistic Fluids and Magnetofluids”, Cambridge Univ. Press, Cambridge. 
Beskin V.S. and Kusnetsova I.V., 2000, Nuovo Cimento B, 115, 795.
Blandford R.D. 1979, in “Active Galactic Nuclei”, ed. C. Hazard and S. Mitton, Cambridge Univ. Press, Cambridge.
Blandford R.D. 2002, astro-ph/0202265.
Blandford R.D. and R.L. Znajek R.L., 1977, MNRAS, 179, 433.
Punsly B., 2002, astro-ph/0202265.
Komissarov S.S., 2001, MNRAS, 326, L41.
Komissarov S.S., 1997, Phys.Lett.A, 232, 435.
Falle S.A.E.G. and Komissarov S.S., 2001, J.Plasma Phys., 65, 29.
de Hoffmann F. and Teller E., 1950, Phys. Rev., 80, 692
Jeffrey A. and Taniuti T., 1964, “Non-linear wave propagation”, Academic Press, New York London.
Komissarov S.S., 1997, Phys.Lett.A, 232, 435.
Komissarov S.S., 1999, MNRAS, 303, 343.
Komissarov S.S., 2001, MNRAS, 326, L41.
Komissarov S.S., 2002, MNRAS, in press astro-ph/0202447.
Landau L.D. and Lifshitz E.M., 1959, “Fluid Mechanics”, Pergamon Press, Oxford.
Lichnerowicz A., 1967, “Relativistic Hydrodynamics and Magnetohydrodynamics”, , Benjamin, New York.
Majorana A. and Anile A.M., 1987, Phys.Fluids, 30, 3045.
Penrose R., 1969, Rev.Nuovo Cim., 1, 252.
Phinney E.S., 1989, in Ferrari A. and Pacholczyk A.G. eds, “Astrophysical Jets”, Reidel, Dordrecht
Phinney E.S., 1983, Ph.D. thesis, Univ.Cambridge.
Punsly B., 2001, “Black Hole Gravitohydromagnetics”, Springer-Verlag, Berlin.
Punsly B., 1996, ApJ., 467, 105.
Punsly B. and Coroniti F.V., 1990a, ApJ., 350, 518.
Punsly B. and Coroniti F.V., 1990b, ApJ., 354, 583.
Takahashi M., Nitta S., Tatenuma Y., and Tomimatsu A., 1990, ApJ., 363, 206.
Thorpe K.S., Price R.H., and Macdonald D.A., 1986, “The Membrane Paradigm”, Yale Univ. Press, New Haven.
Znajek R.L., 1977, MNRAS, 179, 457.

APPENDIX A:

The wavespeed of Alfvén waves of degenerate electrodynamics is given by

$$\mu_{\pm} = \frac{1}{B^2} \left( E \times B \pm B \sqrt{B^2 - E^2} \right)$$  \hspace{1cm} (A2)

where \( B \) is a unit vector normal to the wave front and

$$\mu_{\pm} \cdot n = 0$$  \hspace{1cm} (A3)

Consider the Kerr metric in the Boyer-Lindquist coordinate system and components of the metric tensor as

$$(A4)$$

$$K = \frac{\sqrt{\Omega_\alpha}}{\alpha} (\Omega_f - \Omega_T).$$  \hspace{1cm} (A6)

where \( \Omega_f \) is the angular velocity of the field lines and \( \Omega_T \) is the angular velocity of the FIDO, one has

$$E_\phi = 0, \quad E_\theta = KB_\phi, \quad E_\phi = -KB_\phi.$$

(A5)

where

$$K = \sqrt{\Omega_\alpha \Omega_T} (A6)$$

where \( B_\phi \) is the poloidal magnetic field. Thus, the condition $$(A3)$$ is satisfied if either

$$(B_\phi \cdot n) = 0,$$  \hspace{1cm} (A8)

or

$$\mu_{\pm} = 0.$$  \hspace{1cm} (A9)

$$(A8)$$ simply states that Alfvén waves propagate only along the poloidal magnetic field lines whereas $$(A9)$$ is the sought criticality condition. For $$(B_\phi \neq 0$$ this condition can be written in terms of \( \Omega_f \) and components of the metric tensor as

$$f(\Omega_f, r, \theta) = g_{\phi\phi} \Omega_f^2 + 2g_{t\phi} \Omega_f + g_{tt} = 0,$$  \hspace{1cm} (A10)

which is the well known equation of a light surface (Falasha et al. 1990).

Without any loss of generality we may assume that $$(B_\phi$$ is outgoing and consider only the northern hemisphere. In this case the condition

$$\mu_{+} = 0$$  \hspace{1cm} (A11)

corresponds to the inner critical surface whereas

$$\mu_{-} = 0$$  \hspace{1cm} (A12)

corresponds to the outer critical surface. From (Blandford & Znajek 1977) we find that in the case of outgoing energy flow

$$\frac{dE}{dt} = -B_T \Omega_f > 0,$$  \hspace{1cm} (A13)

where \( E \) is the energy flux within a flux tube of magnetic flux \( \Phi \) and

$$B_T = \sqrt{-g_{tt} \Omega_f} = \alpha \sqrt{\Omega_\alpha} B_\phi.$$  \hspace{1cm} (A14)

Thus, the energy is extracted from the black hole only if

$$B_\phi \Omega_f < 0.$$  \hspace{1cm} (A15)
Given this condition, \( A_7 \) and \( A_{11} \) show that at the inner critical surface

\[
\Omega_f (\Omega_f - \Omega_F) < 0. \tag{A16}
\]

For a black hole with positive angular velocity this means

\[
0 < \Omega_f < \Omega_F. \tag{A17}
\]

Thus, at the inner critical surface the magnetic field lines rotate slower than local FIDOs. Similarly one shows that at the outer critical surface the field lines rotate faster than local FIDOs

\[
0 < \Omega_F < \Omega_f. \tag{A18}
\]

On the surface of the ergosphere

\[
f(\Omega_f, r, \theta) = g_{\phi\phi} \Omega_f (\Omega_f - 2\Omega_F). \tag{A19}
\]

From this one can see that in the limit \( \Omega_f \to 0 \) the inner light surface coincides with the ergosphere, \( f \) being positive inside and negative outside. Since

\[
\frac{\partial f}{\partial \Omega_f} = 2g_{\phi\phi} (\Omega_f - \Omega_F) < 0 \quad \text{for } \Omega_f = 0, \theta \neq 0 \tag{A20}
\]

the inner critical surface moves inside the ergosphere as \( \Omega_f \) increases and must remain inside for all values satisfying \( A_{13} \) (The third factor in \( A_{14} \) vanishes only when the outer critical surface moves inside the ergosphere.)