Grand Unified Theories
and
Lepton-Flavour Violation

C.S. Lim

Department of Physics, Kobe University, Nada, Kobe 657, Japan

Abstract

Lepton-flavour violating processes, such as $\mu \to e\gamma$, are studied in ordinary (non-SUSY) SU(5) and SUSY SU(5) grand unified theories. First given are some introductory argument on the mechanism of U.V. divergence cancellation in flavour changing neutral current processes and on the decoupling of particles with GUT scale masses. We next see that such general argument is confirmed by an explicit calculation of the amplitude of $\mu \to e\gamma$ in ordinary SU(5), which shows that logarithmic divergence really cancels among diagrams and remaining finite part are suppressed by at least $1/M_{GUT}^2$. In SUSY SU(5), flavour changing slepton mass-squared term get a logarithmic correction, as recently claimed. However, when the effect of flavour changing wave function renormalization is also taken into account such logarithmic correction turns out to disappear, provided a condition is met among SUSY breaking soft masses. In SUGRA-inspired SUSY GUT, such condition is not satisfied. But the remaining logarithmic effect is argued not to be taken as a prediction of the theory associated with an irrelevant operator. We find that the log-correction should not exist in models with gauge mediated SUSY breaking.

\footnote{Talk given at the XVI Autumn School and Workshop on Fermion Masses, Mixing and CP Violation, Institute Superior Tecnico, Lisboa, Portugal, 6-15 October 1997}

\footnote{e-mail:lim@oct.phys.kobe-u.ac.jp}
This paper is based upon the talk given at the XVI Autumn School and Workshop on Fermion Masses, Mixing and CP Violation, Institute Superior Tecnico, Lisboa, Portugal, 6-15 October 1997. The former part is aimed to give some introductory argument on the basic properties of flavour changing neutral current processes, especially on the mechanism of U.V. divergence cancellation and the decoupling of heavy particles with GUT or SUSY breaking mass scales. The latter part is devoted to the lepton flavour violating processes in ordinary (non-SUSY) SU(5) and SUSY SU(5), which is extensively based on the recent original work with B. Taga [1].

1. Introduction

1.1 Flavour violation and flavour changing neutral currents

So far we do not have any definitive argument which reveals the origin of flavour. We do not know how quarks of the same charge have different masses and how the mixing among these different flavours stems.

Provided all quark masses are degenerate, the system of quarks has a flavour symmetry, i.e. global “horizontal symmetry” U(3) among three generations. The global symmetry leads to conserved charges, \(N_d, N_s\), etc., ‘down number’, strangeness, etc.. Hence flavour changing neutral current (FCNC) processes, such as \(s \to d \gamma\), are strictly forbidden. For non-degenerate masses, the flavour symmetry U(3) is broken, but there still remain U(1) \(^3\) as the sub-symmetry, which again forbids the FCNC processes. The sub-symmetry is eventually broken by flavour mixing described by the CKM mixing matrix. The violation of flavour symmetry (“flavour violation” for short) now leads to FCNC.

FCNC processes have long served as good thinking ground and good experimental probe of “new physics” in each stage of the development. They are caused in general by quantum effects of all possible intermediate states, therefore being suitable places to search for new unknown particles, predicted by new physics. They are also closely related with CP violation. Typical examples to show these facts are the introduction of charm quark by GIM [2], the following prediction of the mass by Gaillard and Lee by use of the rate of \(K^0 \leftrightarrow \bar{K}^0\) [3], and the Kobayashi-Maskawa model [4] in order to explain CP violation in FCNC processes of neutral kaon system. Let us note that in the Kobayashi-Maskawa model any CP violating observable should be handled by \(\text{det}[M_u^\dagger M_u, M_d^\dagger M_d] = (m_u^2 - m_c^2)(m_c^2 - m_t^2)(m_t^2 - m_u^2)(m_d^2 - m_s^2)(m_s^2 - m_b^2)(m_b^2 - m_d^2) \times J\), with \(J\) being Jarlskog’s parameter, which clearly suggests the necessity of flavour violation or FCNC processes due to the violation of U(3) by mass differences and flavour mixing.

Nowadays rare FCNC processes are still playing important roles in searching for physics beyond the standard model, whose typical examples are grand unified theories (GUT) and supersymmetric (SUSY) theories. Flavour violation in the lepton sector, “lepton flavour
violation (LFV)\textsuperscript{\textcopyright}, is of special interest to us. FCNC processes due to LFV are strictly forbidden in the Standard Model, even though the masses of charged leptons are all different. This is because neutrinos are massless in the model; because of the absence of right-handed neutrinos and lepton number conservation, neither Dirac-type nor Majorana-type mass is possible. For degenerate neutrino masses, in the sense of \( m_\nu = 0 \), flavour mixing has no physical meaning, as we can always perform a unitary transformation to move to the basis of neutrino states where there is no flavour mixing. Thus LFV is a very clean signal of new physics, if it ever exists. The typical examples of LFV process are \( \mu \to e\gamma \) and \( \mu \to ee\), where both of electron number and muon number, \( N_e \) and \( N_\mu \), change by one unit. Our main interest here is \( \mu \to e\gamma \), whose experimental bound is \( Br(\mu \to e\gamma) < 4.9 \times 10^{-11} \).

The standard model, by construction, has a nature that FCNC processes are forbidden at the tree level, thanks to the idea of GIM. In that sense, we may say that FCNC is naturally suppressed in the standard model. Let us note that when the differences of Yukawa couplings vanish, the global flavour symmetry U(3) is enhanced in the theory, and therefore small FCNC is naturally preserved, according to the argument of ’t Hooft \[5\]. At the quantum level FCNC processes are induced due to the flavour mixing in charged current interaction handled by the CKM matrix.

1.2 Ultraviolet divergences and FCNC processes

As was discussed above, FCNC does not exist at the tree (classical) level. Generally speaking, if some observables are forbidden at the classical level (by some symmetry reason), the quantum corrections to them will be described in the form of effective lagrangian \( L_{\text{eff}} \) with higher dimensional \((d > 4)\) gauge invariant operators, “irrelevant operators”, including Higgs field, in general. (In gauge theories any quantum correction can be described by a gauge invariant operator when Higgs field is included, inspite of the spontaneous gauge symmetry breaking.) The typical examples, besides FCNC, are the oblique corrections \( S,T,U \) \((\epsilon_1, \epsilon_2, \epsilon_3)\) \[6\]. For instance, the irrelevant operator, responsible for the \( T \)-parameter, is

\[
L_{\text{eff}} = C_T[(H^\dagger D_\mu H)(H^\dagger D_\mu H) - \frac{1}{3}(H^\dagger D_\mu D_\mu H)(H^\dagger H)],
\]

where \( H \) denotes the Higgs doublet and \( D_\mu \) stands for a covariant derivative. The coefficient function \( C_T \) summalizes the quantum effects due to the intermediate states, and is related to the \( T \)-parameter as \( T = -(\sin^2 \theta_W M_W^2/3\pi\alpha)C_T \). The coefficient \( C_T \) has a mass dimension \( d = -2 \), and should be finite, as there is no counterterm in the original lagrangian to cancel the divergence even if it appears while the theory is renormalizable. In the same way as the case of \( T \), the ultraviolet (U.V.) divergences in the quantum corrections to FCNC processes should automatically disappear.

The above statement is rather trivial. But since the issue plays a central role in the forthcoming discussions, let us study the mechanism of the divergence cancellation more carefully. Let us focus on the FCNC of quark sector in the standard model. Diagrams can
be divided into two types depending on whether FCNC stems from “soft” or “hard” flavour violation. In the (1-loop) diagrams with \( W^\pm \) exchange, the flavour symmetry is softly broken. This is because the gauge coupling is independent of flavour, and the only source to break the flavour symmetry is quark mass(-squared) in the quark propagator. The breaking is soft, in the sense that it is due to a quantity with positive mass dimension (quark mass(-squared)), and can be neglected in higher energies (U.V. region). Hence this type of diagrams does not have any U.V. divergence. Or, one can directly confirm this by noting the unitarity of CKM matrix, e.g.,

\[
\sum_{i=u,c,t} V_{bi}^\dagger V_{is} \ln \Lambda = (V_{bi}^\dagger V_{is}) \ln \Lambda = 0. \tag{2}
\]

In the language of the gauge invariant operator, the quantum effects should be summarized by an irrelevant operator \( (d = 6) \) with two Higgs doublets (to provide quark mass-squared after spontaneous symmetry breaking), whose coefficient should be finite. On the other hand in the diagrams where an unphysical scalar (“would be Nambu-Goldstone boson”), \( \phi^\pm \), is exchanged, the flavour symmetry is hardly broken by the dimensionless non-degenerate Yukawa-couplings of \( \phi^\pm \) with quarks. Each of this type of diagrams, therefore, may give a contribution with U.V. divergence to a marginal \( (d = 4) \) operator. More precisely, both of flavour changing self-energy diagram (e.g. \( \bar{b}i\partial_\mu \gamma^\mu b \)) and flavour changing vertex diagram (e.g. \( \bar{s}\gamma^\mu b A_\mu \), with \( A_\mu \) denoting photon field) have U.V. divergences. What we find \[7\], however, is that there is an exact cancellation of U.V. divergence between the diagram where a photon is attached to the external legs of the flavour changing self-energy diagram, “external leg correction” diagram shown in Fig.1 (a), (b), and the proper diagram for flavour changing vertex shown in Fig.1 (c), (d). Thus the final result is finite, in spite of the presence of the hard flavour violation.

Fig.1: The diagrams with logarithmic divergences contributing to the amplitude of \( d \rightarrow s\gamma \) transition in the standard model. (a), (b) are called external leg corrections and (c), (d) are proper diagrams for flavour changing vertex. \( \phi^\dagger \) denotes a would-be N.-G. boson.

One may feel uncomfortable with the diagrams where a photon is attached to the external legs of the flavour changing self-energy diagram, since these diagrams are 1-particle reducible. If we wish we may add the contribution of counterterm for flavour changing self-energy, so that the external leg correction disappears. The net effect of flavour changing counterterm contributions, however, turns out to vanish exactly \[8\]. Let \( \psi_0 \) and \( \psi \) be the column vectors of bare and renormalized down type quarks, respectively: \( \psi_0^t = (d_0, s_0, ...) \) and \( \psi^t = (d, s, ...) \). They are related not only by rescaling but also by unitary transformation among generations,
which is necessary to get flavour changing counterterms;

\[ \psi_0 = (Z_L^{1-\gamma_5} + Z_R^{1+\gamma_5})\psi \]

where renormalization constants \( Z_L \) and \( Z_R \) for each chirality are \( 3 \times 3 \) matrices. The obtained counterterms are written as

\[ L_c = \bar{\psi}i\partial_{\mu}\gamma^{\mu}(A \frac{1 - \gamma_5}{2} + B \frac{1 + \gamma_5}{2})\psi - \frac{e}{3}\bar{\psi}\gamma_{\mu}(A \frac{1 - \gamma_5}{2} + B \frac{1 + \gamma_5}{2})\psi A_{\mu}, \]

where \( A = Z_L^T Z_L - I, B = Z_R^T Z_R - I \). Let us note that the wave function renormalization for the photon field \( A_{\mu}, Z_3 \), does not contribute to flavour changing counterterm (at the 1-loop level), and the counterterms for the quark self-energy and photon vertex are not independent (Ward identity, \( Z_1 = Z_2 \)). This Ward identity is responsible for the vanishing net contribution of the counterterms, irrespectively of the choice of \( A \) and \( B \) [8]. In other words, we may say that in terms of the renormalized field \( \psi \), defined such that their kinetic terms are flavour diagonal, the flavour changing vertex correction should be finite. If the flavour changing self-energy counterterm is chosen so that it cancels the corresponding 1-loop contribution, the external leg correction just vanishes. In terms of such defined renormalized field \( \psi \), the sum of the 1-loop proper diagram for vertex correction and the contribution of the counterterm for flavour changing vertex should be finite, as the sum of all 1-loop diagrams and counterterm contributions should be finite.

We may summarize this situation relying on operator language as follows. Because of the \( U(1)_{em} \) invariance, after the quantum corrections, the relevant marginal operator can be written in terms of the bare fields \( \psi_0 \) and a covariant derivative \( D_{\mu} \) as,

\[ \bar{\psi}_0 iD_{\mu} \gamma^{\mu} H \psi_0, \]

where the \( 3 \times 3 \) hermitina matrix \( H \) is generally allowed to have flavour changing off-diagonal elements. Hence, even if the original lagrangian is assumed to be flavour-diagonal, after the quantum correction the effective lagrangian is no longer flavour diagonal. However, we may move to the basis of the renormalized fields \( \psi \) so that their kinetic term becomes flavour diagonal, \( U^T H U = H_{diagonal} \). This unitary transformation, at the same time, diagonalizes the photon vertex coming from \( D_{\mu} \). We may further perform the rescaling of the fields so that the kinetic term is proportional to \( I \), a unit matrix. Then another unitary transformation becomes possible to make the mass term of the quarks flavour-diagonal, while keeping the form of the kinetic term. Thus again FCNC disappears from the whole relevant or marginal operator. There remain flavour changing irrelevant operators, but each of them is accompanied by finite coefficient. In fact, this freedom of unitary transformation and rescaling makes it possible to assume always that the original lagrangian to start with is flavour diagonal. The lesson here is the importance of redifinition or wave function renormalization of matter fields caused by the flavour changing self energies.

1.3 Decoupling

Another issue which also will play a central role in the later discussion is the concept of decoupling. Theories beyond the standard model, “new physics”, generally predict the
presence of unknown heavy particles. Then an important question is whether their presence can be searched for in low energies, $E \ll M$, with $M$ being a generic heavy particle mass. Concerning this, there is a well-known decoupling theorem by Appelquist and Carazzone [9]. The theorem is valid in vector-like (non-chiral) gauge theories without spontaneous gauge symmetry breaking, whose typical examples are QED and QCD. The theorem says that the effects of a heavy particle with mass $M$ in low energy processes are suppressed by the inverse power of $M$. More precisely, the quantum effects of the heavy particle may affect the coefficients of relevant or marginal operators, having mass dimension $d \leq 4$, behaving as $M^2$ or $\ln M$. But they just affect the relation between the bare and renormalized quantities, i.e. renormalization. On the other hand, quantum effects to irrelevant operators are genuine predictions of the theory. However, the coefficient functions, having $d < 0$, are suppressed by the power of $1/M$. Fortunately, in chiral theories with spontaneous gauge symmetry breaking, whose typical example is the standard model, there are a few observables known where the contributions of heavy particles are not suppressed: non-decoupling. Good example will be the $t$ quark contributions having as $m_t^2$ to the $\rho$-parameter [10] or to the FCNC processes [7] in the standard model.

Whether heavy particles decouple from low-energy processes or not depends on the nature of new physics, more specifically on the origin of the large mass scale $M$.

(i) The first to be considered is the case where $M$ is provided by a new large ($\gg M_W$) $SU(3) \times SU(2) \times U(1)$ singlet mass scale $M_s$. For instance, in SUSY theories $M_s$ should be the SUSY breaking scale $M_{SUSY}$, and in GUT theories $M_s$ should be taken to be $M_{GUT}$. In this case the contributions of heavy particles, such as super-partners or particles with GUT scale masses, are decoupled. The quantum effects of heavy particles are summarized by $L_{eff} = \sum C_i O_i$, where $O_i$ are gauge invariant irrelevant operators with dimension $d_i > 4$. The contribution of a heavy particle to the gauge invariant coefficient $C_i$ should behave as $C_i \propto 1/M_s^{d_i-4}$, leading to the decoupling. One example is the contribution of super-partners of light quarks, the doublet of first generation ($\tilde{u}, \tilde{d}$)$^t$, to the $\rho$-parameter. Writing the squark masses as $m_{\tilde{u}, \tilde{d}}^2 = m_{\tilde{u}, \tilde{d}}^2 + M_{SUSY}^2$, we find for $m_{\tilde{u}, \tilde{d}}^2 \ll M_{SUSY}^2$ (neglecting the left-right squark mixings for brevity) [11],

$$\Delta \rho \simeq \frac{g^2}{(4\pi)^2} \frac{1}{4} \frac{(m_u^2 - m_d^2)^2}{M_W^2 M_{SUSY}^2}.$$  

This result clearly shows the suppression by $1/M_{SUSY}^2$.

(ii) The second is the case where $M$ is provided by a large coupling with Higgs scalars, through spontaneous symmetry breaking. In this case there is no new mass scale, i.e. $M_s \sim M_W$, and non-decoupling phenomena are expected. One example is found in the contribution of $t$ quark to a FCNC process, $B_d \rightarrow \bar{B}_d$ mixing. The relevant effective lagrangian is given as

$$L_{eff} = G_F \frac{\alpha}{\sqrt{2} 4\pi \sin^2 \theta_W} (V_{td}^1 V_{td})^2 E(x) (\bar{b}_L \gamma_\mu d_L)^2.$$  


where \( x = m_t^2/M_W^2 \). For large \( m_t \), \( E(x) \) behaves as \( (1/4)x = (1/4)(m_t^2/M_W^2) \), thus leading to a non-decoupling effect of t quark [7]. The factor \( m_t^2/M_W^2 \) may be understood to come from a factor \( f_t^4/m_t^2 \sim g^2m_t^2/M_W^4 \), with \( f_t \) being the top Yukawa coupling appearing in the box diagram with \( \phi^\pm \) exchange.

1.4 Lepton flavour violation and grand unified theories

In the standard model lepton flavour violation (LFV) is strictly forbidden, as we have already seen. The situation is quite different when the standard model is embedded in GUT-type theories. The essential difference stems from the fact that in GUT there exist interactions which connect leptons with quarks through the exchange of leptoquark particles, and that flavour symmetry is broken in the quark sector. Then the difference between unitary transformations in charged lepton and quark sectors will result in LFV processes, such as \( \mu \to e\gamma \). We, however, expect that the rate of such LFV processes are extraordinarily small, being suppressed by a factor \( (\Delta m^2_q/M^2_{GUT})^2 \), where \( \Delta m^2_q \) denotes mass-squared differences of quarks. This suppression is suggested by the argument for the decoupling of GUT particles. Such suppression by \( 1/M^4_{GUT} \) is similar to what happens in the proton decay.

1.5 Lepton flavour violation in supersymmetric grand unified theories

Next we will focus on LFV in supersymmetric grand unified theories (SUSY GUT), which is of great current interest. What is new in SUSY theories concerning LFV? There seems to be two new sources of LFV even in ordinary SUSY models without grand unification, which will be discussed below.

(a) “Sizable” (larger than we usually expect) rates of LFV become possible, once R-invariance is relaxed. Unless we impose R-invariance, superpotential \( W \) may have new source of flavour violation,

\[
W = c_1 \bar{u}d\bar{s} + c_2 e\bar{u}d + c_3 \mu\bar{u}d + \ldots,
\]

in addition to ordinary R-invariant couplings of Higgs scalar. The term with coefficient \( c_1 \) violates baryon number and strangeness, \(|\Delta B| = |\Delta S| = 1\). The terms with coefficients \( c_2 \) and \( c_3 \) violate lepton number and electron or muon number, \(|\Delta L| = 1\), \(|\Delta N_e| \) or \(|\Delta N_\mu| = 1\). Thus the combined effect of these couplings yields \( \mu \to e\gamma \) via \( \tilde{u} \)-exchange, for instance.

(b) Another possible source of LFV is soft SUSY breaking terms. For instance, gauge invariant LFV slepton mass term,

\[
M^2_{SUSY} \tilde{e}_R\tilde{\mu}_R,
\]

leads to \( \Delta N_e = -\Delta N_\mu = 1 \) processes, including \( \mu \to e\gamma \). However, in supergravity (SUGRA) inspired MSSM R-invariance is assumed and the soft breaking terms themselves induced by SUGRA interactions are assumed to be universal and not to break flavour symmetry at least at the tree level. Thus these new features of SUSY theories seem not to play a crucial role in LFV processes.
In this context, a very interesting claim has been made by Barbieri - Hall and collaborators that in SUSY GUT models “sizable” rates of $\mu \to e\gamma$ are expected, contrary to ordinary expectation \cite{12}. Such claim has been followed by many works calculating the rate in a few GUT theories \cite{13}. The crucial observation there is that even in R-conserving SUGRA-inspired SUSY GUT models, with flavour symmetric universal SUSY breaking slepton masses, the large flavour violation due to top quark Yukawa coupling combined with GUT interaction give sizable non-universal or LFV renormalization group effects on the SUSY breaking slepton masses. They cause the discrepancy between mass matrices of charged leptons and their superpartners. Thus super-GIM mechanism \cite{14} is no longer valid and the resultant term in Eq.(8) and photino-exchange, for instance, leads to $\mu \to e\gamma$ (one of the possible diagrams is shown in Fig.2), at the rates which is not so far from the present experimental upper bound.

\begin{equation}
\begin{array}{c}
\mu \\
\mu' \\
\gamma \\
\gamma \\
e \\
e'
\end{array}
\end{equation}

Fig.2 : A 1-loop diagram for $\mu \to e\gamma$ decay, caused by the lepton flavour changing mass-squared term denoted by a blob.

While their result \cite{12} is quite impressive, it seems to be different from what we expect according to our argument given above. Namely,

(i) The reason they got sizable rates is the presence of aforementioned LFV slepton masses due to renormalization, which are proportional to $ln \frac{\Lambda}{M_{GUT}}$, instead of $\frac{m^2_\mu}{M_{GUT}^2}$. The difference is outrageous! The U.V. cutoff $\Lambda$ was taken to be the Planck scale $\Lambda = M_{Pl}$. This, however, in turn mean that logarithmic divergence remains in the quantum effects on LFV slepton masses in SUSY GUT, and seems to contradict with what we have seen above in the standard model and what we expect in non-SUSY SU(5) discussed below.

(ii) Next, the $ln \frac{\Lambda}{M_{GUT}}$ contribution clearly shows that GUT particles do not decouple from the low energy process, in contradiction with our analysis based on general argument that GUT particle contributions should decouple.

The purpose of our work \cite{1} is to clarify whether the interesting features claimed in Ref.\cite{12} are natural consequences of SUSY GUT theories. We will compare LFV in ordinary non-SUSY SU(5) GUT and in SUSY SU(5) GUT, in order to see how SUSY can be essential in getting the sizable effects. A special attention will be paid on the effect of lepton-flavour changing wave function renormalization, which played a central role in the cancellation of U.V. divergences in the standard model.

2. Lepton flavour violation in ordinary non-SUSY SU(5) GUT

The content of this section is based on the master thesis of B.Tag\cite{15}. As we have
discussed in the introduction, even if neutrinos are massless GUT interactions connecting leptons with quarks make LFV possible, though the rates are expected to be quite strongly suppressed by $1/M^4_{\text{GUT}}$. The purpose here is to confirm this expectation by actual calculations. As LFV becomes possible due to GUT interactions, we may focus only on the diagrams with exchanges of heavy particles with GUT mass scale $M_{\text{GUT}}$. More precisely, the exchanges of $Y_\mu$ (lepto-quark gauge boson which connects charged leptons and up-type quarks), $h'$ (color triplet Higgs) and $\Sigma'_Y$ (Nambu-Goldstone mode for $Y_\mu$). (The prime of $h'$ and $\Sigma'_Y$ denotes a small admixture of color triplet components of 5-plet and adjoint Higgs fields). One remark here is that another lepto-quark gauge boson $X_\mu$, which connects charged leptons with down-type quarks, does not contributes to the LFV. This is simply because in the GUT scheme the mass matrices of charged lepton and of down-type quarks are the same (at tree level). We find that out of 14 Feynman diagrams, only $h'$-exchange diagrams give numerically important contributions. Let $L_{\text{eff}}$ be an effective lagrangian, which is responsible for the $\mu \rightarrow e \gamma$ decay,

$$L_{\text{eff}} = c_{\text{LFV}} \times \overline{e} \sigma_{\mu \nu} (m_\mu \frac{1 - \gamma_5}{2} + m_e \frac{1 + \gamma_5}{2}) \mu \times F_{\mu \nu},$$

where $F_{\mu \nu}$ is the photon field strength. The contribution of each type of diagrams to the coefficient function $c_{\text{LFV}}$ is given as

$$c_{\text{LFV}}(h' - \text{exchange}) = -\frac{\sqrt{6}}{8} \frac{g^3}{(4\pi)^2} \frac{1}{M_h^2} (V_{KM}^\dagger)_{\mu j} (V_{KM})_{je} m_{\text{aj}}^2 M_W^2 (2\ln M_h^2 + 15 \frac{4}{4}),$$

$$c_{\text{LFV}}(Y_\mu, \Sigma'_Y - \text{exchange}) = O(g^2 m_{\text{aj}}^2 M_Y^2) (V_{KM}^\dagger)_{\mu j} (V_{KM})_{je},$$

where $M_h$ denotes the mass of $h'$ and the Kobayashi-Maskawa matrix $V_{KM}$ handles the LFV as well. The contributions of $Y_\mu$ and $\Sigma'_Y$-exchange diagrams are indistinguishable, as the gauge invariance is guaranteed when they are summed up. Thus only the $c_{\text{LFV}}(h' - \text{exchange})$ is important, though the resultant branching ratio is almost nothing being suppressed by $1/M^4_h \sim 1/M^4_{\text{GUT}}$. The difference of the suppression factors in $c_{\text{LFV}}(h' - \text{exchange})$ and $c_{\text{LFV}}(Y_\mu, \Sigma'_Y - \text{exchange})$ is understood as follows. In QED, $\mu \rightarrow e \gamma$ is described by $d = 5$ operator $\overline{e} \sigma_{\mu \nu} \mu \times F_{\mu \nu}$. In the standard model or GUT this operator cannot be gauge invariant as $e_L$ and $\mu_R$ have different quantum numbers. Thus a Higgs (doublet or 5-plet) should be included into the operator, which makes the dimension of the operator $d = 6$. This is why $c_{\text{LFV}}(h' - \text{exchange})$ is suppressed by $1/M^2_{\text{GUT}}$. In the case of $c_{\text{LFV}}(Y_\mu, \Sigma'_Y - \text{exchange})$ we need 2 more Higgs fields, as lepton flavour is only softly broken by the insertion of quark masses twice, thus making the operator $d = 8$. The coefficient is thus suppressed by $1/M^4_{\text{GUT}}$.

We have shown two things by explicit calculation, i.e., (i) the log-divergence cancels out when the sum of all $h'$-exchange diagrams is taken, and (ii) the decoupling of GUT particles also holds; after the cancellation of the log-divergence and constant terms, the remaining amplitude is suppressed at least by $1/M^2_{\text{GUT}}$. (Actually, the log-divergence cancellation has a
relevance only for the chirality preserving ‘charge radius’ term, not for the ‘magnetic moment’
term responsible for $\mu \to e\gamma$ at 1-loop level. Let us note, however, that what plays a central
role in $\mu \to e\gamma$ in SUSY SU(5) is chirality preserving flavour changing right-handed slepton
mass-squared term.)

3. Lepton flavour violation in supersymmetric SU(5) GUT

In Ref.[12] it has been claimed that the rate of $\mu \to e\gamma$ can be ‘sizable’ in SUSY GUT. We, however, have already seen in the previous section that such sizable effects do not appear in ordinary non-SUSY SU(5) theory. Why $\mu \to e\gamma$ is enhanced when the theory is made supersymmetric? Conceptually, SUSY itself is independent of flavour symmetry. More explicitly, we have the following questions raised concerning their results;
(i) Why does the logarithmic-divergence $\ln \frac{\Lambda^2}{M_{GUT}^2} = \ln \frac{M_{pl}^2}{M_{GUT}^2}$, implied by the claimed renormalization group effect, remain?
(ii)Why is the suppression by a factor $(\frac{M_{pl}^4}{M_{GUT}^4})$, i.e. the decoupling, absent?
The Taga’s thesis [15] has shown that these are not the case in non-SUSY SU(5) GUT.

Though we know that in SUSY GUT the soft SUSY breaking masses can be new source
of flavour violation, we still have the following questions;
(iii) In Ref.[12], soft SUSY breaking masses have been assumed to be universal, being flavour
independent at the tree level and cannot be a new source of LFV. Then why did such drastic
change as $(\frac{M_{GUT}^4}{M_{GUT}^4}) \to \ln \frac{M_{pl}^2}{M_{GUT}^2}$ become possible?
(iv) SUSY breaking terms are quite soft, i.e. $M_{SUSY} \ll M_{GUT}, M_{pl} = \Lambda$. Then how can
they drastically affect the U.V.-divergence?

Since LFV in SUSY GUT is so interesting and important issue, to settle the above
questions seems to be quite meaningful. We therefore try to reanalyze the issue with the help
of concrete computations. Our main interest is in the point whether or not the logarithmic
divergence naturally remains and whether the general argument for the decoupling of the
effects due to GUT interactions really breaks down. The model to work with is SUSY SU(5)
GUT with explicit soft SUSY breaking terms. According to Ref.[12], we assume that bare
SUSY breaking masses, the masses at the cutoff $\Lambda$, are flavour-independent. In particular,
for right-handed charged sleptons, they are given as

$$m_{0\ell}^2 (|\tilde{e}_R|^2 + |\tilde{\mu}_R|^2 + ...),$$

where $m_{0\ell}$ denotes a universal bare SUSY breaking mass. Namely SUSY breaking itself
cannot be the source of LFV. In MSSM, therefore, such flavour-independent masses will enable us to diagonalize both lepton and slepton mass matrices simultaneously (super GIM-
mechanism [14]), and for massless neutrinos there will be no LFV, just as in the standard
model. As was pointed out by Barbieri-Hall and collaborators, the situation changes in
SUSY SU(5), because of the presence of GUT interactions. The GUT interaction which
connects charged sleptons with stop \( \tilde{t} \), accompanied by large top Yukawa coupling \( f_t \), can be new source of LFV. When combined with SUSY breaking, such effect leads to LFV SUSY breaking masses like

\[
(f_t^2 \ln \frac{\Lambda^2}{M_{SUSY}^2}) \tilde{e}_L \tilde{\mu}_R.
\]

Let us note that this operator is a relevant operator having \( d = 2 \), and therefore the appearance of the logarithmic divergence just signals that such operator was possible to exist in the original lagrangian, though it was not included in our choice. The induced LFV slepton mass-squared term may make super-GIM mechanism invalid and may lead to \( \mu \rightarrow e\gamma \) through the ordinary MSSM interactions, for instance photino-exchange diagram, whose amplitude is suppressed just by the power of \( 1/M_{SUSY}^2 \), not by \( 1/M_{GUT}^4 \). Thus actually in this scenario the \( \mu \rightarrow e\gamma \) process is induced by the 2-loop effect. Let us note that if SUSY was not broken LFV will not get sizable effects, just as shown in Taga’s thesis [15] in non-SUSY SU(5) theory. The combined effects of GUT and SUSY breaking is crucial [12].

The most rigorous way to reach the rate of \( \mu \rightarrow e\gamma \) is to calculate all possible 2-loop diagrams, which we would like to avoid. Instead, we may take the following approach to analyze the effect. First, we perform the path-integral from the cutoff \( \Lambda \) to some scale \( \mu \), which satisfies \( M_W \ll \mu \ll M_{GUT} \) (“Wilsonian renormalization”), at the 1-loop level. We thus obtain SU(3)×SU(2)×U(1) invariant effective low-energy ( \( E \leq \mu \) ) lagrangian \( L_{\text{eff}} \) with respect to light (\( \ll M_{GUT} \)) particles, which should be identified with MSSM. Since \( \mu \ll M_{GUT} \), even if we let heavy GUT particles remain in \( L_{\text{eff}} \), their effects in low energies are suppressed by at least \( \mu^2/M_{GUT}^2 \) anyway, and these particles can be safely neglected in \( L_{\text{eff}} \). Then, by using the induced LFV masses for charged sleptons in \( L_{\text{eff}} \) the rate of \( \mu \rightarrow e\gamma \) can be calculated just as in MSSM. Thus our focus is on the point whether the non-decoupling logarithmic LFV quantum correction ever appears in \( L_{\text{eff}} \). The effective lagrangian can be decomposed into \( L_{\text{eff}} = L_{\text{rel}} + L_{\text{irrel}} \), where \( L_{\text{rel}} \) includes operators with \( d \leq 4 \), while \( L_{\text{irrel}} \) denotes the set of irrelevant operators with \( d > 4 \). Since LFV stems only from GUT particle exchanges, only their contributions to the LFV parts of \( L_{\text{eff}} \) are considered below. Some remarks are in order:

(a) Even if we get flavour changing slepton masses by the loop integral of the proper diagrams, it does not immediately lead to the presence of FCNC. As we have discussed in the introduction, the effects of flavour changing wave-function renormalization (self-energy) should also be taken into accounts. This point seems to have been missed in the previous analysis [12], [13].

(b) The GUT particle contributions to \( L_{\text{irrel}} \) will be suppressed by the inverse powers of \( M_{GUT} \). Thus only the contributions to \( L_{\text{rel}} \) will be considered below.

(c) Although they are “soft”, the SUSY breaking terms potentially affect operators with \( d \leq 3 \) in \( L_{\text{rel}} \), but only up to \( O(M_{SUSY}^2) \). For instance, the \( M_{SUSY}^2 \) insertion to the 1-loop diagram for LFV mass operator \( \tilde{e}_L \tilde{\mu}_R \) yields log-divergence, which plays important role in the discussion of Ref. [12]. One more insertion of \( M_{SUSY}^2 \) will make the diagram finite, being suppressed by the inverse powers of \( M_{GUT} \) (If we regard \( M_{SUSY}^2 \) as due to the VEV of some
spurious superfield’s F-component, the corresponding operator actually may be regarded as an irrelevant one).

(d) To get the flavour changing slepton masses, not only $M_{SUSY}^2$ but also flavour violation are necessary. Only hard flavour violation due to the Yukawa couplings via the exchange of colored Higgs superfield will be important. The soft flavour violation due to the mass-squared differences of up-type quarks contributes to the process only as an irrelevant operator, since $M_{SUSY}^2$ insertion together with the insertion of Higgs doublet twice to provide the up-quark masses makes the operator higher-dimensional. Thus the effects of the soft flavour violation will be suppressed by the inverse of $M_{GUT}^2$.

Hence our task is to calculate the quantum corrections due to the exchange of colored Higgs superfield to the slepton part of $L_{rel}$. (The quantum corrections to charged leptons are not independent of the supersymmetric terms of the slepton part, both being diagonalized simultaneously. Thus we need not calculate them independently.) The relevant part of the calculated $L_{eff}$ (in momentum space) takes the following form;

$$L_{eff} = \tilde{l}_i^*(p)[(\delta_{ij} + aH_{ij})p^2 - (\delta_{ij} + bH_{ij})m_{0l}^2]\tilde{l}_j(p) - e\tilde{l}_i^*(p)(p + p')_{\mu}(\delta_{ij} + cH_{ij})\tilde{l}_j(p') \cdot A^\mu(q), \quad (13)$$

where $\tilde{l}_i = \tilde{e}_R, \tilde{\mu}_R$ etc., and $q = p - p'$. $A^\mu$ denotes photon field. The $m_{0l}$ is the SUSY breaking mass for slepton, and $H_{ij} = f_{ti}f_{tj}^*$ with $f_{t\mu} = g(M_t/M_W)V_{ts}^{KM}$, for instance. Let us note that the large top Yukawa coupling gives contributions only to the operators with respect to right-handed sleptons. Thus terms in the above equation are all chirality preserving, and the SUSY breaking mass-squared term with $m_{0l}^2$ should be treated on an equal footing with the self energy term accompanied by $p^2$. The coefficients $a, b,$ and $c$ contain the results of 1-loop calculations. The log-divergent parts of these coefficients, of our main interest, are given as,

$$a = -3\Delta$$
$$b = 3 \cdot \frac{m_{0u}^2 + m_{0h}^2 + m_{03}^2}{m_{0l}^2} \cdot \Delta$$
$$c = a, \quad (14)$$

where $\Delta = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 + M_{GUT}^2)^2}$ with a generic GUT mass scale $M_{GUT}$. $m_{0l}^2, m_{0u}^2, m_{0h}^2$ are SUSY breaking masses for right-handed charged slepton, right-handed up-type squarks, and 5-plet Higgs, respectively, and $m_{03}$ is SUSY breaking trilinear coupling of $\tilde{l}\tilde{t}h$. (As far as the log-divergence is concerned, Wilsonian renormalization and full integration do not make any difference.) The third relation $c = a$ is the consequence of U(1)$_{em}$ symmetry, or Ward identity, as discussed in the introduction.
\[ \mu \rightarrow e \gamma \quad \text{(a)} \]
\[ \text{and} \quad \text{\(b\)} \]

\[ \tilde{\mu} \rightarrow \tilde{e} \gamma + \tilde{\mu} \rightarrow \tilde{\ell} \gamma + \ldots = 0 \]

Fig. 3: Diagrams contributing to the sub-process \( \tilde{\mu} \rightarrow \tilde{e} \gamma \) of \( \mu \rightarrow e \gamma \) at \( O(M_{\text{SUSY}}^2) \). When \( a = b \) holds the sum of these diagrams vanishes.

When a specific relation \( a = b = c \), which is equivalent to
\[ m_{0l}^2 + m_{0u}^2 + m_{0h}^2 + m_{03}^2 = 0, \] holds, the SUSY breaking term \((\delta_{ij} + b H_{ij})m_{0l}^2 \) has the same structure as other terms with coefficients \( a \) and \( c \), and these terms (and the kinetic term for charged leptons) can be diagonalized and rescaled simultaneously by a suitable wave-function renormalization. Thus all LFV effects go away in \( L_{\text{rel}} \), and the log-divergence \( \Delta \) does not remain. Let us note the photino vertex does not have any LFV either, as both lepton and slepton mass matrices are simultaneously diagonalized. This disappearance of LFV can also be checked diagramatically. In fact, if \( a = b \) holds the sum of “external-leg correction” diagrams, picking up the effect of either \( a \) or \( b \) at the first order of \( m_{0l}^2 \), just disappears (see Fig.3 ). Such cancellation mechanism does not seem to have been addressed in previous literature. Since right-handed charged lepton and right-handed up-type quark super-multiplets both belong to 10-plet repr. of SU(5), if we write \( m_{0l}^2 = m_{0u}^2 = m_{10}^2 \) and \( m_{0h}^2 = m_{5}^2 \), and neglect the trilinear coupling, the condition quoted above reduces to
\[ 10 \cdot m_{10}^2 + 5 \cdot m_{5}^2 = \text{Str}(M^2) = 0, \] where super-trace \( \text{Str} \) is over a anomaly free set of matter multiplets. This might suggest some theory which might have been hidden behind and provides a scheme of natural lepton flavour conservation.

In SUGRA-inspired SUSY GUT, however, the above condition is not chosen, and the logarithmically divergent LFV effect seems to remain. Then situation is quite different from what we got for non-SUSY SU(5), where no logarithmic divergence remained and the decoupling theorem, expected from a general argument, was valid. How should we interprete the remaining logarithmic divergence? The LFV SUSY breaking slepton mass-squared operator has \( d = 2 \) and should belong to the set of relevant operators, though we did not include it in \( L_{\text{rel}} \) at tree level. That is why the divergence appeared after the quantum correction. The divergence should be removed by the introduction of counterterms and is subject to a renormalization condition. Thus we are forced to make a conclusion that unfortunately the logarithmic correction cannot be taken as a prediction of the theory, at least as far as we work in the framework of SUSY GUT, as a renormalizable theory.
(calculable predictions of a theory should all come from the finite quantum corrections to irrelevant operators). Let us note that ignoring the LFV mass-squared operators,

$$m_{LFV}^2 \tilde{\epsilon}^* R \tilde{\mu} R, \quad m_{LFV}^2 (|\tilde{\mu} R|^2 - |\tilde{\epsilon} R|^2),$$

etc., does not enhance any symmetry of the theory, since SUSY has been explicitly broken by the flavour independent SUSY breaking masses and also the flavour symmetry has been broken by the large top Yukawa coupling, no matter the LFV mass-squared operators are present or not. In other words, there is no symmetry in the original lagrangian which guarantee small lepton flavour violation. We thus might have to say that “the theory does not lead to natural suppression of lepton flavour violation.”

One may wonder the situation discussed in Ref. [12] that slepton masses, whose boundary condition is set to be flavour independent at $\Lambda = M_{pl}$, get logarithmic LFV corrections by the renormalization group effect is similar to the well-known evolution of gauge couplings in ordinary SU(5) GUT, where three gauge couplings for SU(3), SU(2) and U(1), set to be all equal at $M_{GUT}$, deviate from each other in lower energies by renormalization group effect. In the case of the evolution of gauge couplings, however, the universal coupling at $M_{GUT}$ is naturally guaranteed by the symmetry of the theory, i.e. by SU(5). Thus the splitting of gauge couplings comes not from the correction to the $L_{rel}$ but from the appearance of a new irrelevant operator with adjoint Higgs field included. Thus the splitting is genuine prediction of the theory, and is independent of the choice of the cutoff $\Lambda$. On the other hand, in the case under consideration there will be no reason to expect that all SUSY breaking slepton masses evolve equally above $M_{pl}$ (though we are not sure what the ”above $M_{pl}$” means), even if they are once unified at $M_{pl}$. We should note that flavour symmetry is hardly broken by large top Yukawa coupling, while SU(5) symmetry is softly broken by the VEV of the adjoint Higgs in the issue of the gauge coupling evolution.

Finally we will ask further a question whether there is some chance to get the logarithmic non-decoupling LFV effect as a natural prediction of some theory. More specifically, we will consider a renormalizable theory where the standard model is included into an “observable sector” of supersymmetric GUT, and spontaneous SUSY breaking is realized in a “hidden” sector. These two sectors are assumed to be almost deoupled, being indirectly connected only by some interaction which acts on the both sectors and is assumed not to transmit the SUSY breaking to the SUSY GUT sector at the tree level. We might think of a “gauge mediated SUSY breaking model”, for instance. At first glance the situation is very similar to that of the SUGRA theory with hidden sector, except that in SUGRA the SUSY breaking is transmitted to the observable sector already at the tree level via non-renormalizable (super-)gravitational interaction. However we will not get the non-decoupling effect in the renormalizable theory. The reason is that we cannot expect to get a log-divergent quantum correction to the LFV slepton masses in this case. The absence of the LFV slepton masses
in the lagrangian enhances supersymmetry, in contrast to the case of SUGRA where the absence does not enhance SUSY as there are universal SUSY breaking masses already at the tree lagrangian. Thus as long as the theory is renormalizable, there will not appear any log-divergent correction to the masses. (The spontaneous SUSY breaking at the hidden sector will not affect the divergence.) The LFV slepton masses should be described by a higher dimensional \((d > 4)\)irrelevant operator whose coefficient is finite. The operator should necessarily include \(F\), an auxiliary component of some chiral field, as it is caused by the spontaneous SUSY breaking due to \(< F >\). As \(d > 4\), the coefficient should behave as \(M_{\text{GUT}}^{d-4}\), provided \(M_{\text{GUT}}\) is the largest scale in the loop integral, since to get the LFV masses the GUT interaction is needed. Thus the LFV masses are expected to be suppressed at least by \(\frac{M_{\text{SUSY}}^2}{M_{\text{GUT}}^2}\), with \(M_{\text{SUSY}}\) being SUSY breaking mass scale coming from \(< F >\). Hence the rate of resultant \(\mu \rightarrow e\gamma\) is anticipated to be outrageously suppressed. It is interesting to note that the condition, Eq.(15), to cancel the log-divergence is trivially satisfied in this type of theories, as SUSY breaking masses, \(m_{\tilde{m}}^2\) etc., are all absent at the tree level.

4. Summary and discussions

After introductory and general (to some extent) argument on the mechanism of U.V. divergence cancellation in flavour changing neutral current processes and on the decoupling of particles with GUT scale masses, lepton-flavour violating processes, such as \(\mu \rightarrow e\gamma\), were studied in ordinary (non-SUSY) SU(5) and SUSY SU(5). We have seen that in the amplitude of \(\mu \rightarrow e\gamma\) calculated in ordinary SU(5) logarithmic divergence cancels among diagrams and remaining finite part are all suppressed by at least \(1/M_{\text{GUT}}^2\), in accordance with the general argument. In SUSY SU(5), flavour changing slepton mass-squared term does get a logarithmic correction, as claimed in the pioneering work [12]. However, we have stressed the importance of the effect of flavour changing wave function renormalization. It turned out that when such effect is also taken into account the logarithmic correction disappears, provided a condition is met among SUSY breaking soft masses. In SUGRA-inspired SUSY GUT, the condition is not satisfied. The remaining logarithmic effect, however, is argued not to be taken as a prediction of the theory associated with an irrelevant operator, in contrast to the case of the well-known splitting of three gauge couplings below \(M_{\text{GUT}}\) by renormalization group effect in SU(5). We find that the log-correction should not exist in the renormalizable models with gauge mediated SUSY breaking. It will be very challenging to search for a GUT-type model which has calculable sizable amplitudes of lepton flavour violating processes as a natural prediction of the theory, escaping the decoupling theorem.

Acknowledgment

The author would like to thank G. Branco, L. Lavoura and M.N. Rebelo for the very nice organization of the autumn school and warm hospitality during his stay in Lisboa. He is very grateful to T. Muta and T. Morozumi for their kind support by use of the international
collaboration project of Ministry of Education, Japan, No. 08044089, which made this visit possible.

The author also would like to thank F.J. Botella, T. Inami and B. Taga for very fruitful collaborations and valuable discussions in each stage, which this paper is extensively based upon. Thanks are also due to N. Haba for useful and informative arguments on the subject of gauge mediated SUSY breaking models.

This work has been supported by Grant-in-Aid for Scientific Research (09640361) from the Ministry of Education, Science and Culture, Japan.

References

[1] C.S. Lim and B. Taga, “Lepton-Flavour Violation in Ordinary and Supersymmetric Grand Unified Theories”, in preparation.

[2] S.L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D2 (1970) 1285.

[3] M.K. Gaillard and B.W. Lee, Phys. Rev. D10 (1974) 897.

[4] M. Kobayashi and T. Maskawa, Progr. Theor. Phys. 49 (1973) 652.

[5] G. ’t Hooft, The proceedings of Cargese Summer Inst. (1979) 135.

[6] M.E. Peskin and T. Takeuchi, Phys. Rev. Lett. 65 (1990) 964; G. Altarelli and R. Barbieri, Phys. Lett. B253 (1991) 161.

[7] T. Inami and C.S. Lim, Progr. Theor. Phys. 65 (1981) 297; E. Ma and A. Pramudita, Phys. Rev. D22 (1980) 214.

[8] F.J. Botella and C.S. Lim, Phys. Rev. D34 (1986) 301.

[9] T. Appelquist and J. Carrazone, Phys. Rev. D11 (1975) 2856.

[10] M. Veltman, Nucl. Phys. B21 (1970) 288.

[11] L. Alvarez-Gaume, J. Polchinski and M.B. Wise, Nucl. Phys. B221 (1983) 495; R. Barbieri and L. Maiani, Nucl. Phys. B224 (1983) 32; C.S. Lim, T. Inami and N. Sakai, Phys. Rev. D29 (1984) 1488.

[12] L.J. Hall, V.A. Kostelecky and S. Raby, Nucl. Phys. B267 (1986) 415; R. Barbieri and L.J. Hall, Phys. Lett. B338 (1994) 212; R. Barbieri, L. Hall and A. Strumia, Nucl. Phys. B445 (1995) 219.
[13] P. Ciafaloni, A. Romanino and A. Strumia, *Nucl. Phys.* **B458** (1996) 3; T.V. Duong, B. Dutta and E. Keith, *Phys. Lett.* **B378** (1996) 128; J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, *Phys. Lett.* **B391** (1997) 341; *Phys. Rev.* **D53** (1996) 2442.

[14] R. Barbieri and R. Gatto, *Phys. Lett.* **B110** (1982) 211; T. Inami and C.S. Lim, *Nucl. Phys. B207* (1982) 533.

[15] B. Taga, The master thesis submitted to Kobe University, Japan (1997), in Japanese.