SLD-resolution without occur-check, 
an example

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Abstract

We prove that the occur-check is not needed for a certain definite clause logic program, independently from the selection rule. First we prove that the program is occur-check free. Then we consider a more general class of queries, under which the program is not occur-check free; however we show that it will be correctly executed under Prolog without occur-check.

The main result of this report states that the occur-check may be skipped for the cases in which a single run of a standard nondeterministic unification algorithm does not fail due to the occur-check. The usual approaches are based on the notion of NSTO (not subject to occur-check), which considers all the runs. To formulate the result, it was necessary to introduce an abstraction of a “unification” algorithm without the occur-check.

Keywords: logic programming, Prolog, unification, occur-check, NSTO

1 Introduction

Programming language Prolog implements SLD-resolution, employing an unsound implementation of unification without the occur-check. In practice this creates no problems. Programmers know that they do not need to care about it, unless they deal with something like difference lists. Then one should be careful at tasks like checking a difference list for emptiness\footnote{In the important Prolog textbook by Sterling and Shapiro \textit{SS94}, the occur-check is mentioned (in the context of actual programs) only when dealing with checking a difference list for emptiness (p. 298–300).}

As the occur-check is so unimportant in practice, one would expect that it should be easy to establish formally that it can safely be skipped. For instance, two sufficient conditions for occur-check freeness are presented in the textbook

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They are however applicable only for LD-resolution (SLD-resolution under the Prolog selection rule). Moreover they turn out inapplicable for some simple programs, like the $n$ queens program by Frühwirth [Fri91]. They are based on the notion of NSTO (not subject to occur-check) [DFT91, Apt97], which requires that the occur-check never succeeds within each possible run of a non-deterministic unification algorithm, for the input of interest.

In this paper we prove that for the $n$ queens program the occur-check is not needed. First, in Section 3, we apply an ad hoc approach, and prove that the program with the intended queries is occur-check free [Apt97], for any selection rule. (This means all the unifications in the SLD-derivations are NSTO).

Then, in Section 4, we show that even when the unification cases are not NSTO the occur-check may be not necessary. We provide an appropriate sufficient condition. It makes possible showing that the occur-check may be omitted also for some programs/queries which are not occur-check free in the usual sense. In particular, we show that the $n$ queens program may be executed without the occur-check for a wider class of queries than those considered in Section 3. Moreover the proof is simpler.

Preliminaries. We use definitions, theorems etc from [Apt97] (and we do not repeat them here). We refer to two unification algorithms, the Nondeterministic Robinson Algorithm [Apt97, p. 28], and the (also nondeterministic) Martelli-Montanari algorithm (MMA) [Apt97, p. 32]. In particular we refer to the action numbers of MMA used there. (The same algorithm is presented in [NM95], however actions 5 and 6 are numbered, respectively 5b and 5a.) As in [Apt97], the two usages of “=” (as a syntactic symbol in equations, and as the meta-language symbol for equality) are to be distinguished by the context.

We need to generalize some definitions from [Apt97], in order not to be limited to LD-resolution. We will say that unification of $A$ and $H$ is available in an SLD-derivation (or SLD-tree) for a program $P$, if $A$ is the selected atom in a query of the derivation (tree), and $H$ is a standardized apart head of a clause from $P$, such that $A$ and $H$ have the same predicate symbol. (A more formal phrasing is “equation set $\{A = H\}$ is available”.) If all the unifications available in an SLD-derivation are NSTO [Apt97, Def. 7.1] then the derivation is occur-check free. We say that a program $P$ with a query $Q$ is occur-check free if, under a given selection rule, all SLD-derivations for $P$ with $Q$ are occur-check free.

By an expression we mean a term, an atom, a tuple of terms (or atoms), or an equation. An expression, a set of equations, or a substitution is linear when no variable occurs in it more than once. As in Prolog, each occurrence of _ in an expression (or an equation set, or a substitution) will stand for a distinct variable. Otherwise variable names begin with upper case letters.

2 The program

We will deal with the core fragment of the $n$ queens program [Fri91], we will call it NQUEENS [Dra19].
The typical initial query is \( Q_0 = \text{pqs}(n,q_0,-,-) \), where \( q_0 \) is a list of distinct variables, and \( n \) a natural number represented as \( s^i(0) \). We only mention that the sufficient conditions from \([\text{Apt97}]\) and \([\text{AL95}]\) for occur-check freeness are inapplicable for \text{nqueens} with such query. Note that:

**Lemma 1** If the first argument of any predicate symbol in an initial query \( Q_0 \) is ground, then in any query of any SLD-derivation of \text{nqueens} starting with \( Q_0 \), the first argument of any predicate symbol is ground.

## 3 Occur-check freeness of \text{nqueens}

Here we prove that \text{nqueens} is occur-check free (for the intended initial queries). The proof is based on the fact that all the atoms appearing in the queries of the SLD-derivations of interest are linear.

**Lemma 2** Unification of a linear pair of atoms \( A, H \) results in a linear atom.

If each of two pairs \( A, H \) and \( B, H \) is linear, and \( \theta \) is an mgu of \( A \) and \( H \) then \( B\theta \) is linear.

**Proof** The Nondeterministic Robinson Algorithm \([\text{Apt97}]\) p.28, produces not only an mgu, but also the result of the unification. While unifying \( A \) and \( H \) it maintains a current substitution \( \theta \), and two instances \( A\theta \) and \( H\theta \). We show by induction that at each step, each of \( A\theta \) and \( H\theta \) is linear, and if a variable \( Y \) occurs in both of them then \( Y \) does not occur within a disagreement pair of \( A\theta \) and \( H\theta \); moreover, \( B\theta \) is linear, and if \( Y \) occurs in both \( B\theta \), \( H\theta \) then \( Y \) does not occur within a disagreement pair of \( A\theta \) and \( H\theta \).

Obviously this holds at the beginning, with \( \theta = \epsilon \).

Assume that the property holds for \( \theta, A\theta, H\theta, B\theta \). The new current substitution is \( \theta' = \theta[X/t] \), where \( X, t \) (or \( t, X \)) is a disagreement pair. So \( X \), and each variable from \( \text{Var}(t) \) occurs exactly once in \( A\theta, H\theta \). Moreover, \( t \) is a subterm of that atom from \( A\theta, H\theta \) in which \( X \) does not occur. So replacing \( X \) by \( t \) in \( A\theta \) or in \( H\theta \) results in a linear atom. (The other atom is unchanged by applying \( \{X/t\} \) to it.) Thus each \( A\theta' \) and \( H\theta' \) is linear.

Each variable of \( t \) occurs once in \( A\theta' \) and once in \( H\theta' \), but this is not within any disagreement pair of \( A\theta' \) and \( H\theta' \). If a variable \( Y \) occurs in both \( A\theta' \) and \( H\theta' \) then it occurs in \( A\theta \) and in \( H\theta \), or in \( t \). In both cases it does not occur within a disagreement pair of \( A\theta \) and \( H\theta \).

Let us now consider \( B\theta' \). If \( X \) does not occur in \( B\theta \) then \( B\theta' \) is \( B\theta \), hence linear. Otherwise \( B\theta' \) is obtained by replacing the single occurrence of \( X \) in \( B\theta \) by a linear term \( t \). Variable \( X \) does not occur in \( H\theta \) (otherwise \( X \) does not occur in a disagreement pair of \( A\theta \) and \( H\theta \), contradiction). So \( t \) is a subterm of \( H\theta \). As \( t \) occurs within a disagreement pair of \( A\theta, H\theta \), any
variable of \( t \) does not occur in \( B\theta \), hence it occurs exactly once in \( B\theta' \). Now \( \text{Var}(B\theta') \setminus \text{Var}(t) = \text{Var}(B\theta) \setminus \{X\} \), each of these variables occurs exactly once in \( B\theta \) and thus exactly once in \( B\theta' \). Hence \( B\theta' \) is linear.

Consider a variable \( Y \) that occurs in both \( B\theta' \) and \( H\theta' \). Similarly to the previous case, \( Y \) occurs in both \( B\theta \) and \( H\theta \), or in \( t \). In both cases it does not occur within a disagreement pair of \( A\theta' \) and \( H\theta' \).

Thus, if an mgu \( \theta \) of \( A, H \) is obtained, then \( A\theta \) is linear, and so is \( B\theta \). Any other mgu of \( A, H \) is \( \theta\gamma \) (where \( \gamma \) is a renaming). Hence both \( A\theta\gamma \) and \( B\theta\gamma \) are each linear. □

**Proposition 3** Let \( Q_0 = \text{pq}(n,t_1,t_2,t_3) \), where \( n \) is ground, be linear. In any SLD-derivation for NQUEENS and \( Q_0 \), each atom in each query is linear.

**Proof** by induction. The first query \( Q_0 \) of the derivation consists of one linear atom. Remember that the first argument of each atom in the derivation is ground (by Lemma 1).

Assume that each atom in a query \( Q_i \) is linear. Let \( Q_{i+1} \) be the next query in the SLD-derivation. If the clause applied in the resolution step has a linear head then, by Lemma 2, each atom of \( Q_{i+1} \) is linear (as such atom is obtained by applying the mgu to a linear atom from the body of a clause or from \( Q_i \)). This concerns clauses 1, 2, and 4.

For clause 3, the selected atom from \( Q_i \) is \( A = \text{pq}(m,u_1,u_2,u_3) \), where \( m \) is ground. It is unified with a standardized apart head \( H = \text{pq}(I,[I|\_],[I|\_],[I|\_]) \). \( A \) and \( H \) are unifiable iff \( E = \{I = m, u_1 = [I|\_], u_2 = [I|\_], u_3 = [I|\_]\} \) is unifiable. By Lemma 2.24 (Iteration) of [Apt97], \( E \) is unifiable iff \( E_2 = \{u_1 = [m|\_], u_2 = [m|\_], u_3 = [m|\_]\} \) is (as \( \{I/m\} \) is an mgu of \( I = m \), and \( u_1\{I/m\} = u_4 \) and \( [I|\_]\{I/m\} = [m|\_] \) for \( i = 1, 2, 3 \)). Moreover, an mgu of \( E \) is \( \{I/m\}\theta_2 \), where \( \theta_2 \) is an mgu of \( E_2 \). Note that \( \theta_2 \) is an mgu of \( A \) and \( H\{I/m\} \). As \( H\{I/m\} \) is linear, Lemma 2 applies, and we obtain that each atom of \( Q_{i+1} \) is linear. □

Note that the intended initial queries \( (Q_m) \) are of the form considered in Proposition 3. As unification of two variable disjoint atoms, one of which is linear, is not subject to occur-check (by Lemma 7.5 (NSTO) from [Apt97]), it immediately follows:

**Corollary 4** Program NQUEENS with a linear query \( Q_0 = \text{pq}(n,t_1,t_2,t_3) \), where \( n \) is ground, is occur-check free under any selection rule.

### 4 NSTO is a too strong requirement

NSTO states that each execution of MMA for a given equation set \( E \) does not require the occur-check. In this section we show a weaker condition under which the occur-check is not needed. As a motivating example, we first show that if MMA follows a certain left to right strategy, then the occur-check is not needed for NQUEENS with some queries for which the program is not occur-check free.

Then we formalize a notion of MMA without the occur-check. This is a basis to formulate the main result of this section: if some run of MMA does not involve
the occur-check then each run of MMA without the occur-check produces a correct result. Based on this, we show that nqueens can be correctly executed without the occur-check also for some queries for which nqueens is not occur-check free.

Now we show that the requirement of linearity from the previous section is not needed for executing nqueens without the occur-check. Assume that algorithm MMA selects equations from left to right. Using the notation from the proof of Proposition 3, let us consider unification of a possibly not linear atom $A = pq(m, u_1, u_2, u_3)$ (where $m$ is ground) with a standardized apart head $H$ of clause (3). After the first two steps we obtain equation set $E$ (as in the proof). Then $\{I/m\}$ is applied to $E \setminus \{I = m\}$. This results in $\{I = m\} \cup E_2$, which is NSTO (by Lemma 7.5 from [Apt97], as the tuple $m, [m|], [m|], [m|]$ of the right hand sides of the equations of $E_2$ is linear). As all the other clause heads are linear, all SLD-derivations starting from a query $Q' = pqs(n, t_1, t_2, t_3)$ where $n$ is ground (and $t_1, t_2, t_3$ are arbitrary) do not require the occur-check, provided MMA works as described above. So we showed the following property.

**Proposition 5** Consider a query $Q'_0 = pqs(n, t_1, t_2, t_3)$, where $n$ is ground. There exists a strategy of selecting actions in MMA, so that action (6) (halt on the occur-check) is not performed in any unification available in any SLD-derivation for nqueens with $Q'_0$.

However the program with such query is not occur-check free: under some other selection of actions in MMA action (6) may be performed. Consider for instance $A = pq(m, L, [L|], _)$ (with a ground $m$). A run of MMA on $A$ and $H$, as described above, does not employ the occur-check and halts with failure due to action (2). But some other run, which delays applying $\{I/m\}$, halts on the occur-check (action (6)). We are going to show that even in such cases MMA without the occur-check would produce correct results (by failing on action (2), after having produced infinite terms).

Tests under SICStus and SWI-Prolog confirm that Prolog behaves in this way. Prolog unification without the occur-check applied to $A$ and $H$ correctly results in failure. The same happens when the arguments of $A$ and $H$ are permuted (by the same permutation); we can assume that this changes the order of elementary actions of the Prolog unification. We may suppose that in the latter case infinite terms are created. A particular order of actions of the Prolog algorithm on $A$ and $H$ can be simulated by a Prolog query $(L, [L|], _)=((I|), (I|), (I|), s(0)\neq1$ (which fails). Posing only the first conjunct of this query shows, as expected, that the unsound Prolog algorithm “unifies” the two triples, producing a “unifier” containing infinite terms.

**MMA without the occur-check.** Here we introduce a version of the Martelli-Montanari algorithm without the occur-check. We will call it MMA'. It is a generic algorithm. To make sense, it has to be executed under a particular strategy of selecting equations and actions to be performed. This is needed, among others, to assure termination. It is not clear which (unsound) unification
algorithms are used by actual Prolog implementations. However we conjecture that the runs of such algorithms are sufficiently well abstracted by some runs of MMA\textsuperscript{−}.

MMA\textsuperscript{−} differs from MMA\textsuperscript{−} \[Apt97\] p. 32] by

(a) removing action (6),
(b) replacing condition $X \not\in \text{Var}(t)$ in action (5) by $X \neq t$,
(c) adding action

\[(5') \quad X = t \text{ where } X \neq t, \text{ and variable } X \text{ occurs elsewhere} \]

$\rightarrow$

replace some occurrences of $X$ by $t$ in the other equations,
(d) adding a requirement that action (5') can be performed only if (5) has been previously performed on $X = t'$, for some $t'$.

So action (5') is a generalization of (5) (which replaces all the occurrences of $X$).

Due to requirement (d), action (5') is performed only if the equation set $E$ dealt with is not unifiable (as if after the previous usage of (5) for $X = t'$ the variable $X$ occurs more than once, then $X \in \text{Var}(t')$). So, in a sense, (5') is applied only if we deal with equations representing infinite terms. Equation $X = t$ can be understood as describing such term, and the occurrences of $X$ in the other equations as references to the term. (Immediately after (5) each such occurrence occurs within a subterm $t$.)

It remains to describe the results of MMA\textsuperscript{−}.

**Definition 6** Consider a set of equations $E = \{X_1= t_1, \ldots, X_n= t_n\}$, and a relation $\succ_E$ on variables $\{X_1, \ldots, X_n\}$ defined by $X_i \succ_E X_k$ iff $X_k \in \text{Var}(t_i)$ (for $i,k \in \{1, \ldots, n\}$). Let $\succ^+_E$ be the transitive closure of $\succ_E$.

$E$ is semi-solved if $X_1, \ldots, X_n$ are distinct, no $t_i$ is $X_i$, and if $X_i$ occurs in $t_k$ then $X_i \succ^+_E X_i$, for any $i,k \in \{1, \ldots, n\}$.

So e.g. $\{X=f(X), Y=X\}$ and $E_1 = \{X=f(Y), Y=f(X)\}$ are semi-solved, but $E_1 \cup \{X=a\}$ and $\{X=a, Y=f(X)\}$ are not. Note that $E$ is solved \[Apt97\] when $X_1, \ldots, X_n$ are distinct and $\succ_E$ is empty. Solved equation sets are results of MMA when it successfully terminates. Semi-solved ones play the same role for MMA\textsuperscript{−}.

Now we are ready to describe termination of MMA\textsuperscript{−}. The algorithm may terminate with failure or with success. **Termination with failure** is explicitly caused by action (2). **Termination with success** happens when the current equation set is semi-solved. Note that actions (5) or (5') may be applicable to such equation set, in such case the nondeterministic MMA\textsuperscript{−} may terminate or continue. Note also that if $E$ is not semi-solved then some action of MMA\textsuperscript{−} can be applied to $E$. Thus a finite run of MMA\textsuperscript{−} either terminates with failure, or with success.

We conjecture that it is sufficient to (a) not apply action (5) to $X = t$ if it has been previously performed on some $X = t'$, and (b) apply action (5') only to occurrences of $X$ that are left hand sides of equations of the form $X = t'$,
moreover where $|t| \leq |t'|$.

In some older Prolog systems the (unsound) implementation of unification did not terminate in some cases [MSS89]. It seems that it always terminates in the current systems. Also, MMA$^-$ may not terminate. We conjecture that it terminates with the restrictions from the previous paragraph on applying actions (5) and (5'). We expect that a termination proof similar to that of [CG82] can be constructed. E.g. take $E$ as $f = f_0(X)$, $X = f(f(X))$. Applying (5) to $X = f(X)$ and then (1) to $f(X) = f_0(X)$ results in $E$ again. Now there are two ways of applying (5') to a left hand side occurrence of $X$. (5') with $X = f(X)$ selected and then action (1) applied to $f(X) = f(f(X))$ results in $E$, which is semi-solved. (5') with $X = f(f(X))$ selected and then action (1) applied to $f(f(X)) = f(X)$ produces $E$ with the first equation reversed. This may be extended to an infinite loop. Note that the latter case violates one of the restrictions above; $t = f(f(X))$ replaces the left hand side of $X = f(X)$, however $|t| > |f(X)|$.

**Correct runs of MMA$^-$.** By an *occur-check free run* of MMA we mean a run of MMA in which action (6) is not performed. Note that an occur-check free run of MMA is also a run of MMA$^-$. Conversely, each run of MMA$^-$ in which whenever action (5) is applied to an equation $X = t$ then $X \notin \text{Var}(t)$ is an occur-check free run of MMA (as in such run of MMA$^-$ action (5') cannot be performed). So if $E$ is NSTO then each run of MMA$^-$ for $E$ is a run of MMA. The latter properties were the reason to introduce requirement (d) in MMA$^-$. We will say that a run of MMA$^-$ on an equation set $E$ is *correct*, if it produces the right result i.e. the run halts with failure if $E$ is not unifiable, and produces an mgu of $E$ otherwise. (Formally, producing an mgu means obtaining a solved equation set $E'$, such $E'$ uniquely represents the mgu.)

**Theorem 7** Consider an equation set $E$. Assume that there exists an occur-check free run of MMA on $E$. Then if a run of MMA$^-$ for $E$ terminates then it is correct.

For a proof we first introduce a few notions. We will consider possibly infinite terms ($i$-terms) over the given alphabet of function symbols, including constants. The corresponding generalization of the notion of substitution is called *i-substitution*. We will follow the ideas from the study of MMA in [Apt97], but we will employ $i$-terms. An $i$-substitution $\theta$ is an $i$-solution of an equation $t = u$ if $t\theta$ and $u\theta$ are the same expression; $\theta$ is a solution of a set $E$ of equations, if $\theta$ is a solution of each equation from $E$. Two sets of equations are $i$-equivalent if they have the same set of $i$-solutions.

**Lemma 8** Each action of MMA$^-$ replaces an equation set by an $i$-equivalent one.

**Proof.** For actions (3), (4) the claim is obvious. The proof for (1) is as in the case of MMA [Apt97, Claim 2, p. 34]. Action (5) or (5') replaces an equation set $E = E_X \cup E_1$ by $E' = E_X \cup E_2$, where $E_X = \{X = t\}$. Consider a solution $\theta$ of $E_X$. So $X\theta, t\theta$ are the same term. As $E_2$ results from $E_1$ by replacing some

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2So (5') transforms $X = t'$ into $t = t'$, and the size of the/a larger term of an equation is preserved.
occurrences of $X$ by $t$, $E_1 \theta$ and $E_2 \theta$ are the same. Thus $\theta$ is a solution of $E$ iff $\theta$ is a solution of $E'$. □

Proof (of Theorem 7) Let $R_1$ be an occur-check free run of MMA on $E$, hence of $\text{MMA}^-$ on $E$. Let $R_2$ be an arbitrary run of $\text{MMA}^-$ on $E$. Let $S$ be the set of the i-solutions of $E$, and thus of every equation set $E'$ appearing in $R_1, R_2$ (by Lemma 8).

If $R_1$ is successful, then $E$ is unifiable, thus $S$ contains unifiers of $E$, and thus of any $E'$ appearing in $R_1, R_2$. Hence no equation in any equation set of $R_2$ is of the form $X = t$, where $X$ occurs in $t$ and $X \neq t$. Thus if action (5) is performed in $R_2$ for some $X = t$, then after this step each equation set contains exactly one occurrence of $X$. Thus action (5') is never performed in $R_2$. Hence $R_2$ is a run of MMA, so it produces an mgu of $E$.

Assume that $R_1$ halts with failure. This must be due to the last step performing action (2). So the last equation set contains $f(\ldots) = g(\ldots)$, hence $S$ is empty. So no semi-solved equation set appears in $R_2$ (as each such set has an i-solution). Thus $R_2$ does not terminate with success. If it terminates then it terminates with failure □

From Proposition 3 and Theorem 7 we immediately obtain:

**Corollary 9** Assume that $\text{MMA}^-$ is executed under a strategy which results in termination of $\text{MMA}^-$. Consider a query $Q_0' = \text{pq}(n,t_1,t_2,t_3)$, where $n$ is ground. $\text{MMA}^-$ is correct for any available unification in any SLD-derivation for nqueens and $Q_0'$.

In other words, the program with such queries will be correctly executed by Prolog, despite the lack of occur-check. This also holds for any modification of the Prolog selection rule.

Note that the proof of the latter result is simpler than that of Corollary 4, despite a more general class of queries dealt with. We just do not need to bother about three arguments of the atoms in the queries.

## 5 Conclusions

This report proposes a generalization of the usual notions of NSTO (not subject to occur-check) and occur-check free [DFT91, Apt97]. They consider each execution of a nondeterministic unification algorithm. We show that it is sufficient to consider a single execution. We prove that if an execution of the algorithm for an input $E$ does not require the occur-check then the occur-check may be dropped from any execution. To formulate this property, we described an abstract unification algorithm without the occur-check.

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3The proof of Proposition 5 is substantially shorter than the union of those of Lemma 2 and Proposition 3. Even if we exclude Lemma 2 from the comparison, as it does not deal directly with the program, the proof of Proposition 5 seems simpler than that of Proposition 3 alone.
The report is focused on an example, a simple logic program for which the standard criteria for occur-check freeness from [Apt97] and [AL95] are inapplicable. SLD-derivations under arbitrary selection rules are considered. First, a proof is presented that, for a class of queries, the SLD-derivations of the program are occur-check free. Then, based on the above-mentioned property, we prove that the program can be correctly executed without the occur-check, for a wider class of initial queries. The class also includes queries, for which the derivations are not occur-check free. Moreover, the latter proof turns out to be simpler than the former one.

The main result of this work is extending behind NSTO the class of cases for which unification without the occur-check works correctly.

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