Thermomagnetic effect on the propagation of Rayleigh waves in an isotropic homogeneous elastic half-space under initial stress

Rajeev Ghatuary and Nilratan Chakraborty
Thermomagnetic effect on the propagation of Rayleigh waves in an isotropic homogeneous elastic half-space under initial stress

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Abstract: Rayleigh wave propagation in an isotropic homogeneous initially stressed thermoelastic half-space under the effect of magnetic field has been studied using Green–Lindsay (GL) theory of generalized thermoelasticity. The frequency equation has been obtained for Rayleigh waves. The Rayleigh wave velocity is computed numerically for different values of initial stress parameter, magnetic pressure number, thermoelastic coupling parameter, and wave number for aluminium material, and the results obtained are compared graphically.

1. Introduction

The classical-coupled thermoelasticity theory predicts an infinite speed of heat propagation but this is physically impossible. To make necessary amendments in the theory predicting infinite speed of heat propagation, Lord and Shulman (1967) developed a new theory based on a modified Fourier’s law of heat conduction with one relaxation time. Since the heat equation formulated in this theory is of wave type, it obviously explains finite speed of propagation for the heat and elastic waves. Later, a more rigorous theory of thermoelasticity was formulated by Green and Lindsay (1972) introducing two relaxation times or the theory of temperature-rate-dependent thermoelasticity. These nonclassical theories are often regarded as the generalized dynamic theory of thermoelasticity. According to these theories, heat propagation should be viewed as a wave phenomenon rather than

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PUBLIC INTEREST STATEMENT

In this study, the effect of electromagnetic field, thermal field, and initial stress on the velocity of Rayleigh wave propagation in an elastic solid has been investigated which is of great importance in various disciplines. In geophysics, it helps to understand the effect of the Earth’s magnetic field on seismic wave propagation. The result of this paper can be used (1) to study damping of acoustic waves in a magnetic field, (2) in various problems in biomedical engineering involving thermal stress, (3) in emissions of the electromagnetic radiations from nuclear devices, (4) for development of a highly sensitive super conducting magnetometer, and (5) in electrical power engineering, plasma physics and many more.

ABOUT THE AUTHOR

Rajeev Ghatuary is a research fellow enrolled in the Department of Physics, Kolhan University, working towards a PhD degree. The research work is mainly the study of “Magneto-thermo-elastic effect on the propagation of seismic waves in different media, under initial stress using different models of generalized thermoelasticity”.

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a diffusion phenomenon. This wave-like thermal disturbance is referred to as “second sound” by Chandrasekhar (1986). These theories have been developed using different concepts and one cannot be obtained as a special case of the other. Various problems have been investigated using these theories and the studies reveal some interesting phenomenon. Brief reviews of different theories used in thermoelasticity have been reported by Chandrasekhar (1998).

Rayleigh waves in an isotropic thermoelastic media have been studied by Nayfeh and Nemat-Nasser (1971), Dawn and Chakraborty (1988) and many others under different context of the variables and different physical conditions using the effect of thermal field only where the effect of initial stress was not considered. Initial stress has a major influence on the mechanical response of the materials, and has important applications in geophysics, engineering structures and in the study of soft biological tissues. It is therefore of great interest to study the effect of initial stress on the propagation of elastic waves. Biot (1965) showed that presence of initial stress in the medium considerably influences the propagation of elastic waves. Based on Biot’s theory, Montanaro (1999) made an in-depth study on the isotropic linear thermoelasticity with initial stress. Since the development of the theory of initial stress, numerous problems on wave propagation in an initially stressed medium have been discussed by many authors. Dey, Roy, and Dutta (1984) studied the velocities of $P$ and $S$ waves in a medium under initial stress and under gravity field. Ahmed (2000) discussed the effect of initial stress on the propagation of Rayleigh waves in a thermoelastic granular medium. Addy and Chakraborty (2003) studied the propagation of Rayleigh waves in a thermoelastic half-space under initial hydrostatic stress, with variable rigidity and density. Addy and Chakraborty (2005) also studied the propagation of Rayleigh waves in a viscoelastic half-space under initial hydrostatic stress in presence of temperature field. Abd-Alla, Abo-Dahab, and Al-Thamali (2012) investigated the propagation of Rayleigh waves in a rotating orthotropic material elastic half-space under initial stress and gravity. Not only wave propagation but problems of reflection and transmission of thermoelastic plane waves in presence of initial stress have also been discussed by many authors, such as Othman and Song (2007), Chakraborty and Singh (2011), etc. But these authors have not considered the effect of magnetic field along with thermal field and initial stress.

The interaction between electromagnetic field, thermal field, and elastic field in an elastic solid is of great importance in various disciplines. In geophysics, it will help to understand the effect of the Earth’s magnetic field on seismic wave propagation. Other applications are damping of acoustic waves in a magnetic field, biomedical engineering (problems involving thermal stress), emissions of the electromagnetic radiations from nuclear devices, development of a highly sensitive superconducting magnetometer, electrical power engineering, plasma physics, etc. The development of the theory of interaction of electromagnetic field, thermal field, and elastic field is available in the papers of Sherief and Ezzat (1996), Ezzat and Othman (2000), Ezzat and El-Karamany (2003), Ezzat and Youssef (2005), Othman and Song (2008), Abo-Dahab and Singh (2009), Kumar and Devi (2010), etc.

Some problems on wave propagation in thermoelastic solids in presence of magnetic field have been studied by many authors such as Abd-Alla, Hamad, and Abo-Dahab (2004), Mahmoud (2011), Singh, Kumar, and Singh (2012a, 2012b), etc. but these authors did not consider the wave propagation in an isotropic generalized magneto-thermoelastic medium in presence of initial stress.

In the present paper, we have discussed the effect of initial stress, magnetic field, and thermal field on Rayleigh wave propagation in an isotropic homogenous elastic solid half-space in the context of Green–Lindsay theory. The frequency equation for the Rayleigh wave velocity has been obtained. The numerical values of Rayleigh wave velocity are calculated from the frequency equation. The numerical results are shown graphically to show the effect of initial stress parameter, magnetic pressure number, thermoelastic coupling parameter, and wave number on the velocity of Rayleigh waves.
2. Geometrical description of the problem

We consider a homogeneous, isotropic, and perfectly conducting thermoelastic solid half-space in the undisturbed state at uniform temperature \( T_0 \), as shown in Figure 1.

3. Formulation of the problem

We consider a rectangular Cartesian coordinate system \( oxyz \) and the origin “\( o \)” of the coordinate system is taken at any point on the plane surface; the \( x-z \) plane which is assumed to be stress free.

We choose \( x \)-axis in the direction of wave propagation. The solid medium is under initial compressive stress \( P \) along the \( x \)-axis and under constant magnetic field \( H_0 = (0, 0, H_3) \) acting along \( z \)-axis. The problem under consideration being two-dimensional, we restrict our analysis to plane strain parallel to \( x-y \) plane only. Therefore, all the field quantities will be independent of coordinate \( z \), and depend on variables \( x \) and \( y \) besides time.

3.1. Basic equations

(i) The dynamical equations of motion under initial compressive stress \( P \) in the \( x \)-direction, in absence of heat source, are given by Biot (1965),

\[
\frac{\partial s_{11}}{\partial x} + \frac{\partial s_{12}}{\partial y} - \rho \frac{\partial^2 \omega}{\partial y} + F_1 = \rho \frac{\partial^2 u}{\partial t^2}
\]

\[
\frac{\partial s_{12}}{\partial x} + \frac{\partial s_{22}}{\partial y} - \rho \frac{\partial^2 \omega}{\partial x} + F_2 = \rho \frac{\partial^2 v}{\partial t^2}
\]

where \( s_{11}, s_{12}, \) and \( s_{12} \) are incremental stress components. The first two are principal stress components along \( x \)-axis, \( y \)-axis, respectively, and the last one is shear-stress component in the \( xy \)-plane.

\( \omega = \frac{1}{2} \left( \frac{\partial u}{\partial x} - \frac{\partial w}{\partial y} \right) \) is the magnitude of local rotation. \( F_1 \) and \( F_2 \) are the Lorentz force components along \( x \)- and \( y \)-axes, respectively.

(ii) The stress–strain relations with incremental isotropy are given by Biot (1965),

\[
\begin{align*}
        s_{11} &= (\lambda + 2\mu + P) e_{xx} + (\lambda + P) e_{yy} - \gamma \left( 1 + t_1 \frac{\partial}{\partial t} \right) T \\
        s_{22} &= \lambda e_{xx} + (\lambda + 2\mu) e_{yy} - \gamma \left( 1 + t_1 \frac{\partial}{\partial t} \right) T \\
        s_{12} &= 2\mu e_{xy}
\end{align*}
\]  

where \( e_{xx}, e_{yy} \) are the principal strain components, \( e_{xy} \) is the shear strain component, and \( t_1 \) is the second relaxation time.

(iii) The incremental strain components are given by Biot (1965),

\[
\begin{align*}
        s_{11} &= (\lambda + 2\mu + P) e_{xx} + (\lambda + P) e_{yy} - \gamma \left( 1 + t_1 \frac{\partial}{\partial t} \right) T \\
        s_{22} &= \lambda e_{xx} + (\lambda + 2\mu) e_{yy} - \gamma \left( 1 + t_1 \frac{\partial}{\partial t} \right) T \\
        s_{12} &= 2\mu e_{xy}
\end{align*}
\]
\[ e_{xx} = \frac{\partial u}{\partial x}, \quad e_{yy} = \frac{\partial v}{\partial y} \quad \text{and} \quad e_{xy} = \frac{1}{2} \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \quad (3) \]

(iv) The modified heat conduction equation is

\[ \kappa \nabla^2 T = \rho C_e \left( 1 + t_0 \frac{\partial}{\partial t} \right) \frac{\partial T}{\partial t} + \gamma T_0 \left( 1 + t_0 \delta \right) \frac{\partial}{\partial t} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \quad (4) \]

where \( t_0 \) is the first relaxation time and \( \delta \) is Kronecker delta.

(v) The linearized Maxwell equations governing the electromagnetic field for a slowly moving solid media of perfect electrical conductivity, taking into account the absence of displacement current are

\[ \text{curl} \, h = J, \quad \text{curl} \, E = -\mu \frac{\partial}{\partial t} h, \quad \text{div} \, h = 0, \quad \text{div} \, E = 0 \quad (5) \]

where

\[ h = \text{curl} (u \times H_0) \quad (6) \]

The equation of Lorentz force is

\[ F = \mu (J \times H_0) \quad (7) \]

The components of Lorentz force can be obtained from Equations 5 and 7 in the forms

\[ F_1 = \mu H_0^2 \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} \right), \quad F_2 = \mu H_0^2 \left( \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2} \right) \quad (8) \]

The Maxwell’s electromagnetic stress tensor can be given in the form as

\[ \tau_{ij} = \mu [H_i h_j + H_j h_i - (H_i h_j) \delta_{ij}], \quad \text{where} \ i, j, k = 1, 2, 3 \quad (9) \]

Equation 9 gives

\[ \tau_{12} = 0, \quad \tau_{22} = \mu H_0^2 \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \quad (10) \]

3.2. Solution of the problem

Using Equations 2, 3, and 8 in Equation 1, we get

\[ \left( \lambda + 2\mu + P + \mu e H_0^2 \right) \frac{\partial u}{\partial x} + \left( \lambda + \mu + \frac{\rho}{2} + \mu e H_0^2 \right) \frac{\partial^2 u}{\partial y \partial y} + \left( \mu + \frac{\rho}{2} \right) \frac{\partial^3 u}{\partial y^3} = \frac{\partial^2 u}{\partial t \partial x} + \gamma \left( 1 + t_1 \right) \frac{\partial^2 u}{\partial y \partial x} \quad (11a) \]

\[ \left( \lambda + 2\mu + \mu e H_0^2 \right) \frac{\partial v}{\partial y} + \left( \lambda + \mu + \frac{\rho}{2} + \mu e H_0^2 \right) \frac{\partial^2 v}{\partial y \partial x} + \left( \mu - \frac{\rho}{2} \right) \frac{\partial^3 v}{\partial x^3} = \frac{\partial^2 v}{\partial t \partial y} + \gamma \left( 1 + t_1 \right) \frac{\partial^2 v}{\partial y \partial x} \quad (11b) \]

To separate the dilatational and rotational components of strain, we introduce displacement potentials \( \psi \) and \( \varphi \) by relations

\[ u = \frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = \frac{\partial \varphi}{\partial y} + \frac{\partial \psi}{\partial x} \quad (12) \]
With the help of Equations 11a and 12, we get

\[ V^2 \varphi = \frac{\rho}{(\lambda + 2\mu + P + \mu_e H_3^2)} \frac{\partial^2 \varphi}{\partial t^2} + \frac{\gamma}{(\lambda + 2\mu + \mu_e H_3^2)} \left( 1 + t_1 \frac{\partial}{\partial t} \right) T \]  

(13a)

\[ V^2 \psi = \frac{\rho}{(\mu + \frac{P}{2})} \frac{\partial^2 \psi}{\partial t^2} \]  

(13b)

With the help of Equations 11b and 12, we get

\[ V^2 \varphi = \frac{\rho}{(\lambda + 2\mu + \mu_e H_3^2)} \frac{\partial^2 \varphi}{\partial t^2} + \frac{\gamma}{(\lambda + 2\mu + \mu_e H_3^2)} \left( 1 + t_1 \frac{\partial}{\partial t} \right) T \]  

(13c)

\[ V^2 \psi = \frac{\rho}{(\mu - \frac{P}{2})} \frac{\partial^2 \psi}{\partial t^2} \]  

(13d)

Equations 13a and 13b as above are obtained first by putting Equation 12 in 11a and after simplification they are integrated partially with respect to \( x \) and \( y \), respectively. Proceeding exactly in a similar way, we get Equations 13c and 13d putting Equation 12 in 11b and partially integrating with respect to \( y \) and \( x \), respectively. Equation 13a being obtained by partial integration with respect to variable \( x \) and involves scalar potential \( \varphi \) which represents dilatational wave propagating along \( x \)-axis. Similarly, Equation 13d involving vector potential \( \psi \) is obtained by partial integration with respect to the variable \( x \) which represents rotational wave propagating along \( x \)-axis. The other two Equations 13b and 13c involving vector potentials \( \psi \) and \( \varphi \), respectively, being obtained by partial integration with respect to variable \( y \), and they represent rotational wave and dilatational wave propagating along \( y \)-axis, respectively. Since we consider propagation along \( x \)-axis, we proceed with Equations 13a and 13d, because they represent the dilatational and rotational waves propagating along \( x \)-axis.

Equations 13a and 13d can be rewritten as

\[ V^2 \varphi = \frac{1}{C_1^2 (1 + R_H)} \frac{\partial^2 \varphi}{\partial t^2} + \frac{\gamma}{\rho C_1^2 (1 + R_H)} \left( 1 + t_1 \frac{\partial}{\partial t} \right) T \]  

(14a)

\[ V^2 \psi = \frac{1}{C_2^2} \frac{\partial^2 \psi}{\partial t^2} \]  

(14b)

Here \( R_H = \frac{C_\psi}{C_\varphi}, \ C_1^2 = \frac{\mu H_3^2}{\rho}, \ C_2^2 = \frac{\mu + 2\mu + P}{\rho} = \frac{2\mu}{\rho} \left( 1 + \zeta + \frac{1}{2}\zeta \right), \ C_1^2 = \frac{\mu - 2\mu}{\rho} = \frac{1}{2}\mu(1 - \zeta). \)

where \( R_H, \ C_\varphi, \ C_\psi, \) and \( C_1 \) represent magnetic pressure number, Alfvén wave velocity, isothermal dilatational, and rotational wave velocities, respectively, and \( \zeta = \frac{\mu}{2\rho} \) is the initial stress parameter in dimensionless form.

Using Equation 12 in 4, we get,

\[ \kappa V^2 T = \rho C_c \left( 1 + t_3 \frac{\partial}{\partial t} \right) \frac{\partial T}{\partial t} + \gamma T_0 \left( 1 + t_3 \frac{\partial}{\partial t} \right) \frac{\partial}{\partial t} (V^2 \varphi) \]  

(15)
We use Green–Lindsay theory (i.e. $t_1 > t_0 > 0$, $\delta_y = 0$) for the solution of this problem.

For Green–Lindsay theory, Equation 15 reduces to

$$\kappa \nabla^2 T = \rho C_v \left(1 + t_0 \frac{\partial}{\partial t} + \gamma T_0 \frac{\partial}{\partial t} (\nabla^2 \varphi) \right)$$ \hspace{1cm} (16)

Eliminating “T” between Equations 16 and 14a, we get a fourth-order equation satisfied by $\varphi$ in the form

$$\nabla^4 \varphi \left[ \frac{1}{c_1^2} + \frac{\varepsilon}{c_1^2 c_2^2} \right] \frac{\partial}{\partial t} + \left[ \frac{t_0}{c_1^2} + \varepsilon \frac{t_1}{c_1^2 c_2} + \left( \frac{1}{c_1^2 c_2^2} + \frac{1}{c_1^2} \right) \frac{\partial}{\partial t} \right] \nabla^2 \varphi = 0$$ \hspace{1cm} (17)

where $c_3^2 = \frac{\kappa}{\rho C_v}$ and $\varepsilon = \frac{\gamma t_0}{\rho C_v}$ is the thermoelastic coupling parameter.

For plane harmonic waves traveling along x-axis with velocity “c”, the solution of 14a and 14b can be taken in form

$$\varphi = f(y) \exp \left[i \alpha (x - ct) \right]$$ \hspace{1cm} (18a)

$$\psi = g(y) \exp \left[i \alpha (x - ct) \right]$$ \hspace{1cm} (18b)

$$T = h(y) \exp \left[i \alpha (x - ct) \right]$$ \hspace{1cm} (18c)

where $\alpha$ is the wave number.

Using Equation 18a in 17, we get

$$\frac{d^2 f(y)}{dy^2} + \left[-2a^2 + \frac{a^2 c_1^2}{c_2^2 c_1^2} \left( (1 + R_H) t_0 + \varepsilon t_1 + \frac{c_1^2}{c_2^2} \right) + \frac{i \alpha c_2}{c_1^2 c_2} \left( 1 + R_H + \varepsilon \right) \right] \frac{d^2 f(y)}{dy^2} = 0$$ \hspace{1cm} (19)

The above equation can be reduced to the form

$$\left( \frac{d^2}{dy^2} - \lambda_1^2 \right) \left( \frac{d^2}{dy^2} - \lambda_2^2 \right) f(y) = 0$$ \hspace{1cm} (20)

where $\lambda_1^2, \lambda_2^2$ are the roots of Equation 19, and

$$\lambda_1^2 + \lambda_2^2 = a^2 \left[ 2 - \frac{c_1^2}{c_1^2 c_2^2} \left( (1 + R_H) t_0 + \varepsilon t_1 + \frac{c_1^2}{c_2^2} \right) \right] - \frac{ic_2}{ac_1^2 c_2} \left( 1 + R_H + \varepsilon \right)$$ \hspace{1cm} (21a)

$$\lambda_1^2 \lambda_2^2 = a^4 \left[ 1 - \frac{c_1^2}{c_1^2 c_2^2} \left( (1 + R_H) t_0 + \varepsilon t_1 + \frac{c_1^2}{c_2^2} \right) \right] + \frac{c_2^4}{c_1^2 c_2^2} \left( 1 + R_H + \varepsilon - \frac{c_1^2}{c_2^2} \right)$$ \hspace{1cm} (21b)
where \( L_1 = \frac{c^2}{c(1+R_H)} \left\{ \left( 1 + R_H \right) t_0 + \epsilon t_1 + \frac{c^2}{c^2} \right\} \), \( L_2 = \frac{c^2}{c(1+R_H)} \left\{ \frac{c^2}{c^2} \left( 1 + R_H + \frac{c^2}{c^2} \right) \right\} \), and \( L_3 = \frac{c}{1+R_H} \left\{ 1 + R_H + \frac{c^2}{c^2} \right\} \).

Using Equation 18b in 14b, we get

\[
\left( \frac{d^2}{dy^2} - \nu_1^2 \right) g(y) = 0
\]

(22)

where \( \nu_1^2 = \alpha^2 \left( 1 - \frac{c^2}{c^2} \right) \).

The requirement that the stresses and hence the functions \( \varphi \) and \( \psi \) vanish as \((x^2 + y^2) \to \infty\) leads to the following solutions of Equations 20 and 22:

\[
f(y) = A \exp \left( -\lambda_1 y \right) + B \exp \left( -\lambda_2 y \right) \quad \text{(23a)}
\]

\[
g(y) = C \exp \left( -\nu_1 y \right) \quad \text{(23b)}
\]

Substituting Equation 23a in 18a and equation 23b in 18b, we get

\[
\varphi = |A \exp \left( -\lambda_1 y \right) + B \exp \left( -\lambda_2 y \right)| \exp \left[ i\alpha(x - ct) \right]
\]

(24a)

\[
\psi = C \exp \left( -\nu_1 y \right) \exp \left[ i\alpha(x - ct) \right]
\]

(24b)

Using Equations 18a and 18c in 14a, we get the value of \( h(y) \) and using that value of \( h(y) \) in Equation 18c, we get

\[
T = \frac{\rho c^2_1}{\gamma \left( 1 - i\alpha ct \right)} \left[ \left\{ \lambda_1^2 - \alpha^2 \left( 1 - \frac{c^2}{c_1^2} (1 + R_H) \right) \right\} A \exp \left( -\lambda_1 y \right) \exp i\alpha(x - ct) \right]
\]

(24c)

4. Boundary conditions

We consider the boundary \( y = 0 \) is free from stresses and also thermally insulated. The boundary conditions are given by

\[
\Delta f_x = s_{12} + P_{xy} + r_{12} = 0 \quad \text{(25a)}
\]

\[
\Delta f_y = s_{22} + r_{22} = 0 \quad \text{(25b)}
\]

\[
\frac{dT}{dy} = 0 \quad \text{(25c)}
\]

where \( \Delta f_x \) and \( \Delta f_y \) are incremental boundary forces per unite initial area.

Using Equations 2, 3, 10, 12, 24a, and 24b in the boundary condition 25a, we get

\[
2i\alpha \left( 1 + \zeta \right) \lambda_1 A + 2i\alpha \left( 1 + \zeta \right) \lambda_2 B + (1 + \zeta) \left( \alpha^2 + \nu_1^2 \right) C = 0 \quad \text{(26a)}
\]

Using Equations 2, 3, 10, 12, 24a, 24b, and 24c in the boundary condition 25b, we get
\[
\left\{ a^2 (1 + \zeta) - \zeta \lambda_1^2 - \frac{\rho c^2 a^2}{2\mu} \right\} A + \left\{ a^2 (1 + \zeta) - \zeta \lambda_2^2 - \frac{\rho c^2 a^2}{2\mu} \right\} B - i\alpha v_1 C = 0
\]  
(26b)

Using Equation 24c in the boundary condition 25c, we get

\[
\lambda_1 \left[ \lambda_1^2 - a^2 \left\{ 1 - \frac{c^2}{c_1^2 (1 + R_H)} \right\} \right] A + \lambda_2 \left[ \lambda_2^2 - a^2 \left\{ 1 - \frac{c^2}{c_1^2 (1 + R_H)} \right\} \right] B = 0
\]  
(26c)

5. Frequency equation

Eliminating A, B, and C from Equations 26a, 26b, and 26c, we get

\[
\begin{vmatrix}
2ia (1 + \zeta) \lambda_1 & 2ia (1 + \zeta) \lambda_2 \\
\left\{ a^2 (1 + \zeta) - \zeta \lambda_1^2 - \frac{\rho c^2 a^2}{2\mu} \right\} & \left\{ a^2 (1 + \zeta) - \zeta \lambda_2^2 - \frac{\rho c^2 a^2}{2\mu} \right\}
\end{vmatrix}
\begin{vmatrix}
1 + \zeta
-i\alpha v_1
0
\end{vmatrix}
= 0
\]  
(27)

Expanding the determinant (27) and simplifying, we have the following frequency equation:

\[
-2a^2 \lambda_1 \lambda_2 v_1 (\lambda_1 + \lambda_2) + \left( a^2 + v_1^2 \right)
\begin{vmatrix}
\left\{ a^2 (1 + \zeta) - \frac{\rho c^2 a^2}{2\mu} \right\} & \left\{ a^2 (1 + \zeta) - \frac{\rho c^2 a^2}{2\mu} \right\}
\end{vmatrix}
\begin{vmatrix}
\left\{ a^2 (1 + \zeta) + \zeta \lambda_1 \lambda_2 - \frac{\rho c^2 a^2}{2\mu} \right\} & \left\{ 1 - \frac{c^2}{c_1^2 (1 + R_H)} \right\}
\end{vmatrix}
= 0
\]  
(28)

Substituting Equations 21a and 21b in 28 and equating the real parts only, we get

\[
-4v_1 a^2 \left\{ 1 - \left( \frac{3}{2} - \frac{L_3}{2} + \frac{\zeta^2 L_4}{4} \right) \frac{L_1}{4} + \left( 5 + \frac{L_1}{2} - \frac{L_2}{8} - \left( \frac{3}{2} - \frac{L_3}{2} + \frac{L_4}{4} \right)^2 \right) \right\}
\begin{vmatrix}
\left\{ a^2 (1 + \zeta) - \frac{\rho c^2 a^2}{2\mu} \right\} & \left\{ a^2 (1 + \zeta) - \frac{\rho c^2 a^2}{2\mu} \right\}
\end{vmatrix}
\begin{vmatrix}
\left\{ a^2 (1 + \zeta) + \zeta a^2 - \frac{\rho c^2 a^2}{2\mu} \right\} & \left\{ 1 - \frac{c^2}{c_1^2 (1 + R_H)} \right\}
\end{vmatrix}
= 0
\]  
(29)

6. Results and discussion

The Aluminium material is chosen for purpose of numerical computations, the physical data for such material are given as (Abd-Alla, Yahia, & Abo-Dahab, 2003):

\[
\lambda = 57.75 \times 10^8 \text{ Nm}^{-2}, \mu = 26.43 \times 10^8 \text{ Nm}^{-2}, \alpha_t = 23 \times 10^{-6} \text{ K}^{-1}, C_v = 900 \text{ J kg}^{-1} \text{ K}^{-1}, \rho = 2700 \text{ kg m}^{-3},
\]
\[
\kappa = 237 \text{ W m}^{-2} \text{ K}^{-1}.
\]

With the help of Equation 29, the Rayleigh wave velocity (c) is computed for different values of initial stress parameter (ζ), magnetic pressure number (R_H), thermoelastic coupling parameter ε, wave number (α), and wave length (L = \frac{2L_1}{3}) and the results obtained are shown in the following graphs:

6.1. Variation of Rayleigh wave velocity c with initial stress parameter ζ

From Figure 2, it is seen that in presence of magnetic field, Rayleigh wave velocity remains almost constant with increase in initial stress parameter.

6.2. Variation of Rayleigh wave velocity c with magnetic pressure number R_H

From Figure 3, we find that Rayleigh wave velocity c increases slowly in a linear fashion with increase in magnetic pressure number. For any particular value of magnetic pressure number magnitude of
Figure 2. Wave number ($\alpha = 0.20$) and thermoelastic coupling parameter ($\varepsilon = 0.03$) are kept constant, for different values of magnetic pressure number.

Figure 3(a). Thermoelastic coupling parameter is kept constant ($\varepsilon = 0.03$), for different values of wave number $\alpha$ in presence of initial stress ($\zeta = 0.5$).

Figure 3(b). Thermoelastic coupling parameter is kept constant ($\varepsilon = 0.03$), for different values of wave number $\alpha$ in absence of initial stress ($\zeta = 0$).

Figure 3(c). Wave number is kept constant ($\alpha = 0.20$), for different values of thermoelastic coupling parameter $\varepsilon$ in presence of initial stress ($\zeta = 0.5$).

$c$ is (1) greater for smaller wave number in presence of initial stress (Figure 3(a)) but same for different constant values of wave number in absence of initial stress (Figure 3(b)) and (2) greater for smaller temperature coupling factor in presence or absence of initial stress (Figures 3(c) and 3(d)).
6.3. Variation of Rayleigh wave velocity \( c \) with thermoelastic coupling parameter \( \varepsilon \)

From Figure 4, it is seen that Rayleigh wave velocity \( c \) decreases linearly and rapidly with increase in thermoelastic coupling parameter. For a particular value of thermoelastic coupling parameter, the wave velocity \( c \) is (1) more for higher magnetic pressure number in presence or absence of initial stress and difference increases as the thermoelastic coupling parameter increases (Figures 4(a) and 4(b)) and (2) less for higher value of wave number in presence of initial stress and the variations are almost parallel and equispaced (Figure 4(c)) but same for different constant values of wave number in absence of initial stress (Figure 4(d)).
6.4. Variation of Rayleigh wave velocity $c$ with wave number $\alpha$

From Figure 5, it is found that in presence of initial stress Raleigh wave velocity, $c$ decreases in an exponential fashion with an increase in wave number and the effect of variation in magnetic pressure number (Figure 5(a)) and thermoelastic coupling parameter (Figure 5(c)) on the wave velocity $c$ is almost negligible. However when initial stress is neglected, the Raleigh wave velocity $c$ remains

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Figure 4(d). Magnetic pressure number is kept constant ($R_H = 0.4$), for different values of wave number $\alpha$ in absence of Initial stress ($\zeta = 0$).

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Figure 5(a). Thermoelastic coupling parameter is kept constant ($\varepsilon = 0.03$), for different values of magnetic pressure number $R_H$ in presence of initial stress ($\zeta = 0.5$).

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Figure 5(b). Thermoelastic coupling parameter is kept constant ($\varepsilon = 0.03$), for different values of magnetic pressure number $R_H$ in absence of initial stress ($\zeta = 0$).

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Figure 5(c). Magnetic pressure number is kept constant ($R_H = 0.4$), for different values of thermoelastic coupling parameter $\varepsilon$ in presence of initial stress ($\zeta = 0.5$).
almost constant with increase in wave number. For a particular value of wave number, the wave velocity \( c \) is (1) more for higher magnetic pressure number (Figure 5(b)) and (2) less for higher value of thermoelastic coupling parameter (Figure 5(d)).

6.5. Variation of Rayleigh wave velocity \( c \) with wave length \( L \)

From Figure 6, we find that in presence of initial stress Rayleigh wave velocity \( c \) increases sharply and linearly with an increase in wave length \( L \) and in this case the effect of other parameters on the velocity is negligible (Figures 6(a) and 6(c)). However, when initial stress is neglected, the Raleigh wave velocity \( c \) remains almost constant with an increase in wave length. For a particular value of wave length, the wave velocity \( c \) is (1) more for higher magnetic pressure number (Figure 6(b)) and (2) less for higher value of thermoelastic coupling parameter (Figure 6(d)).

7. Conclusion

From graphical analysis, we conclude that thermoelastic coupling parameter and magnetic pressure number significantly influence the Rayleigh wave velocity while initial stress parameter in the presence of magnetic field and temperature field has negligible effect on the Rayleigh wave velocity. This analysis can be utilized in the field of seismology, earthquake science, geophysics, nuclear devices, etc.
**Nomenclature**

- $u, v$: displacement components along x-axis and y-axis respectively.
- $T$: incremental change of temperature from the initial state.
- $T_0$: initial temperature.
- $\lambda, \mu$: Lame's constants.
- $\alpha$: coefficient of linear thermal expansion of the medium.
- $\gamma$: a thermal parameter $\gamma = (3\lambda + 2\mu / \alpha)$.
- $\kappa$: thermal conductivity of the medium.
- $\rho$: density of the medium.
- $C_e$: specific heat per unit mass at constant strain.
- $\mu_e$: magnetic permeability.
- $J$: electric current density.
- $h$: induced magnetic field.
- $E$: induced electric field.
- $u$: displacement vector.

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