Magneto-transport Properties near the Superconductor-Insulator Transition in 2D

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We analyze here the behavior near the 2D insulator-superconductor quantum critical point in the presence of a perpendicular magnetic field. We show that with increasing field $H$, the quantum disordered and quantum critical regimes, in which vortex degrees of freedom are suppressed, crossover to a new magnetically activated (MA) regime, where the correlation length $\xi \sim 1/\sqrt{H}$. In this regime, we show that the conductivity decreases monotonically as opposed to the anticipated saturation predicted from hyperuniversality arguments. This discrepancy arises from the lack of commutativity of the frequency and temperature tending to zero limits of the conductivity. In the low-field regime such that $\sqrt{H} \ll \Delta$, and in the absence of Ohmic dissipation, where $\Delta$ is a measure of the distance from the quantum critical point, the resistivity saturates to the Bose metal value found previously for Cooper pairs lacking phase coherence.

From the earliest experiments \[1\,2\] on superconducting thin metal alloy films, a simple picture emerged for the magnetic-field tuned transition to the insulating state. Namely, the transition is continuous with a well-defined critical field, $H_c$, below which superconductivity occurs and above which an insulator obtains. At the critical field, the resistivity is independent of temperature. Within the dirty boson model \[3\] in an applied magnetic field, these results follow naturally as a direct consequence of the inherent duality between Cooper pairs and vortices: above $H_c$, vortices condense while Cooper pairs remain localized but below $H_c$ it is the Cooper pair condensation and vortex localization that affords the zero-resistance state. At criticality, Cooper pairs on the brink of forming a phase coherent state diffuse with the quantum of conductance $\sigma_Q = 4e^2/h$ for charge $2e$ bosons. However, more recent experiments have shown that this simple picture breaks down dramatically at and below $H_c$. In particular, a metallic phase \[4\] emerges below $H_c$ with the zero-resistance state occurring only at a much smaller field $H_M \ll H_c$. On phenomenological grounds, it has been suggested that the metallic phase arises from the quantum tunneling of field-induced vortices in the presence of dissipation \[5\].

As the metallic phase has been observed at zero-field \[10\] in the disorder-tuned insulator-superconductor transition as well, we reinvestigated the conductivity of the standard quantum-disordered regime in zero field. We found that quite generally Cooper pairs lacking phase coherence form a Bose metal phase \[11\]. This result arises from the fact that in the quantum-disordered regime the elementary excitations obey Boltzmann statistics. Hence, their concentration $n$ is exponentially suppressed. As a consequence, so is the inverse scattering time $1/\tau$ that is determined by the probability that two quasiparticles collide. But because the conductivity is a product of the density and the scattering time, the exponentials cancel one another, thereby leading to a finite conductivity as $T \to 0$. In the current study, we focus on the fate of the Bose metal phase in a magnetic field. Near the quantum critical point where vortex dynamics are subdominant, the relevant physics can be accessed by the standard $O(M)$ Ginzburg-Landau (GL) theory \[13\]. In this work, we utilize this approach for a system near the IST in the presence of a perpendicular magnetic field.

We establish that at low temperature $T$ and close to the quantum critical point, $\Delta \ll 1$, the application of a field $H$ drives the system to the high-field magnetically activated (MA) regime once $\sqrt{H} \gg \Delta, T$, thereby compounding the analysis of the transport properties in the quantum disordered regime. In this regime, the energy levels of the bosonic quasiparticle excitations are quantized and have a gap $m \sim \sqrt{H}$ that is inversely proportional to the correlation length $\xi$. Working within the large $M$ approximation, we discuss the crossovers and limiting forms of the scaling functions for the correlation length $\xi = \xi(H, T, \Delta)$ and the static conductivity $\sigma$ in the vicinity of the critical point. We present the qualitative form for the conductivity in the presence of dissipation based on the standard Kubo formula neglecting the effects of frustration. The dc conductivity is regularized by inclusion of the weak coupling to an Ohmic heat bath $\eta$ as well by quasiparticle scattering $1/\tau$ due to the quartic non-linearity in the GL action. Collectively, internal and external dissipation can be treated jointly with an effective coupling constant $\tilde{\eta} = \eta + 1/\tau$. In the absence of the bath and at a small constant magnetic field such that $\sqrt{H} \ll \Delta$, the magnetic energy scale is irrelevant. Consequently, the Bose metal \[11\] obtained previously in the quantum disordered regime is recovered. We show that in the MA regime, the conductivity $\sigma$ is directly, rather than inversely, proportional to the total dissipation $\tilde{\eta}$. Further, in contrast to the low-field limit, the conductivity in the MA regime is found to be a decreas-
ing function of temperature and inversely related to the field.

Our starting point is the $M$-component Ginzburg-Landau free energy [13, 14].

$$F[\psi] = \int d^2r \int dr \left\{ \left[ \nabla + \frac{i e^*}{\hbar} A_0(r) \right] \psi^*_n(r, \tau) \right\}$$

$$\cdot \left\{ \left[ \nabla - \frac{i e^*}{\hbar} A_0(r) \right] \psi_n(r, \tau) + |\partial_\tau \psi_n(r, \tau)|^2 + \delta |\psi_n(r, \tau)|^2 + \frac{U}{2M} |\psi_n(r, \tau)|^4 \right\}, \quad (1)$$

in the presence of a magnetic field, where $A_0(r) = \{0, Hx, 0\}$ is the vector potential due to the applied field, $e^* = 2e$, and $\delta$ is the $T = 0$ unrenormalized distance to the quantum critical point. For a time-independent magnetic field, it is convenient to expand the order parameter $\psi_n(r, \tau)$ in the corresponding eigenfunctions that diagonalize the Gaussian part of the free-energy density

$$\psi_n(r, \tau) = \sum_{\omega_m, p_y} e^{-ip_y y - i\omega_m \tau} \Psi_n(x - \hbar p_y / e^* H) c_a(n, \omega_m, p_y)$$

where $\Psi_n(x - \hbar p_y / e^* H)$ is the solution of the corresponding eigenequation, expressible in Hermite polynomials, and $c_a(n, \omega_m, p_y)$ are the expansion coefficients. The related eigenvalues are given by $\omega_H(n + 1/2)$ with $\omega_H = 2e^* Ha / \hbar = 4\pi f / a$, where $a$ is the lattice constant and $f = \Phi/\Phi_0$ is the magnetic frustration. Henceforth, we rescale $H$ such that $\omega_H \rightarrow H$. In the large $M$ (mean-field) treatment the bare $T = 0$ distance to the critical point $\delta$ is renormalized, and the equation for the quasiparticle excitation gap $m$,

$$m^2 = \delta + H^2 / 4\pi \sum_{\omega_m, p_y} \frac{1}{m^2 + Hn + \omega_n^2}, \quad (3)$$

must be solved to obtain the effective Gaussian action. Performing the summation over the Matsubara frequencies exactly and employing the Poisson formula for the sum over ‘Landau’ levels $n$, we obtain (assuming $m \ll 1$)

$$\Delta = 2T \ln \left( \frac{2 \sinh x}{x} \right) - \frac{H}{4m} \coth \frac{m}{2T} - \frac{H}{2} \int_0^{\infty} dx \frac{a_- \sinh(a_+/T) + a_+ \sinh(a_-/T)}{\sqrt{m^4 + H^2x^2} (\sinh^2(a_+/2T) + \sin^2(a_-/2T))}, \quad (4)$$

where

$$a_\pm = \frac{1}{\sqrt{2}} \left[ \sqrt{m^4 + H^2x^2} \pm m^2 \right]^{1/2}$$

and $\Delta = (4\pi / U) \delta + \Lambda$ (where $\Lambda$ is the upper momentum cutoff) is the renormalized distance to the zero-temperature critical point in the absence of the field.

![FIG. 1. Phase diagram for the field-tuned transition in the presence of quantum fluctuations, $\delta$. The zero-temperature quantum critical point studied here corresponds to $\delta = \delta_c$. Quantum disordered (QD), quantum critical (QC) and magnetically activated (MA) regimes correspond to the limiting cases $\Delta \gg \sqrt{H}, T$, $T \gg \sqrt{H}, \Delta$ and $\sqrt{H} \gg T, \Delta$, respectively.](image)
finite \( T \), finite \( H \) transition, transport is determined by the motion of vortices as illustrated schematically in Fig. 1. The finite-size scaling considerations suggest that the correlation length near the transition is a universal function of two arguments: \( \xi = T^{-1} f(\Delta/T, \sqrt{H}/T) \) and assumes the appropriate limiting forms if one of the parameters tends to zero. The \( M = \infty \) form of \( m = \xi^{-1} \) as a function of \( \sqrt{H}/T \) obtained from the numerical solution of Eq. (1) for different values of \( \Delta/T \) is presented on Fig. 2. As is evident, a crossover occurs to \( \sqrt{H} \) dependence as the field increases.

The linear conductivity (per flavor) near the IST can be obtained from the GL action with the help of the Kubo formula [13]

\[
\sigma_{\alpha \beta}(i \omega_n) = -\frac{\hbar}{\omega_n} \int d^2r \int d\tau \frac{\delta^2 \ln Z}{\delta A_\alpha(\tau, \mathbf{r}) \delta A_\beta(0)} e^{i \omega_n \tau}.
\]

While calculating the averages in the partition function we expand \( \psi \) using Eq. (2) which yields for the longitudinal conductivity [13]

\[
\sigma(\omega_\nu) = \frac{(e^*)^2}{2\hbar \omega_\nu} T H^2 \sum_{n=0}^{\infty} \frac{n+1}{2} G(\omega_m, n) - G(\omega_m + \omega_\nu, n+1) - G(\omega_m, n+1) G(\omega_m + \omega_\nu, n)
\]

The experimentally observable finite temperature conductivity must be calculated once dissipative mechanisms are taken into account [13]. Weak dissipation, \( \tilde{\eta} = \eta + 1/\tau \), is accounted for by introducing the term \( \tilde{\eta} |\omega_m| \psi(\mathbf{r}, \omega_m)|^2 \) into the free-energy density [13]. Its inclusion changes the analytical properties of the Green functions in Eq. (1) [13]. Performing the analytical continuation, one finds that the static conductivity is given by

\[
\sigma = \frac{(e^*)^2 H^2}{4\pi \hbar} \sum_{n=0}^{\infty} \frac{n+1}{2} \frac{dx}{\sinh^2 x} \frac{8\tilde{\eta}^2 T^2 x^2}{(\epsilon_n - 4T^2 x^2)^2 + 4\eta^2 T^2 x^2} \\
\times \frac{1}{(\epsilon_{n+1} - 4T^2 x^2)^2 + 4\eta^2 T^2 x^2}.
\]

The analysis of Eq. (6) simplifies if the dissipation is weak compared to the quasiparticle energies, implying \( \tilde{\eta} \ll m \). Asymptotic analytical expressions are readily obtained for \( m \gg T \), which define QD and MA regimes. In these cases, there are two competitive contributions to the integral over \( x \). One comes from the minima of the denominator at \( x_{n+1}^2 \approx (\epsilon_n^2 - \tilde{\eta}^2/2)/4T^2 \) and \( x_{n+1}^2 \approx (\epsilon_{n+1}^2 - \tilde{\eta}^2/2)/4T^2 \) and the second from the region near \( x = 0 \). Performing the approximate integration over \( x \), and analyzing subsequently the sums over \( n \), we arrive at the following limiting cases.

In the QD regime, \( m = \Delta \gg \sqrt{H} \), and the conductivity is given by \( \sigma = \sigma^{(1)} + \sigma^{(2)} \), where

\[
\sigma^{(1)} = \frac{4e^2}{\pi \hbar} \left( \frac{\pi \tilde{\eta} T}{3\Delta^2} \right)^2
\]

and

\[
\sigma^{(2)} = \frac{4e^2}{\pi \hbar} \left( \frac{2\pi T}{\tilde{\eta}} e^{-\Delta/T} + \frac{8\pi T \Delta^2}{H^2} e^{-\Delta/T} \right).
\]

If we are in the MA regime with \( \sqrt{H} \sim m \),

\[
\sigma = \frac{4e^2}{\hbar} \left( \frac{\tilde{\eta} e^{-A \sqrt{\pi}/T}}{T} + \frac{2m}{3} \frac{(\tilde{\eta} T/H)^2}{\Delta^2} \right).
\]

is a rather large numerical prefactor, arising from the \( M = \infty \) limit. These expressions describe qualitatively the crossover from the QD to MA regime, provided the quasiparticles are well defined-- that is, dissipation is sufficiently weak. As the field increases, the conductivity becomes directly proportional to dissipation, as dictated by Eq. (4). This can be attributed to the splitting of the minima in the denominator of the general formula, Eq. (6).

There are two sources of dissipation for which we must account. Internal dissipation arises from mutual scattering of quasiparticles, leading to a finite quasiparticle scattering rate \( 1/\tau \). External Ohmic dissipation \( \eta \) may come from the resistive shunting of superconducting grains in a system of fabricated Josephson-junction arrays (JJA). The coefficient \( \eta \) is then inversely proportional to the resistivity of a shunting resistor, \( R_\eta \) [20], which may be itself a function of temperature and magnetic field. In addition to Ohmic dissipation, disorder can also serve as a dissipative channel as has been shown for static disorder in the presence of d-wave pairing [22]. In the QC regime, interactions alone lead to a large inverse scattering time, \( 1/\tau \sim T \sim m \). Hence, quasiparticles are poorly defined [13], and small \( \epsilon = 3 - d \) or \( 1/M \) expansions are necessary to determine the conductivity in a controlled way [13]. The implementation of this procedure leads to the universal value for the conductivity close to \( \sigma_Q = 4e^2/\hbar \) [13]. The weak external dissipation \( \eta \ll m \sim T \) is clearly subdominant in this regime, except at low temperatures [16]. On the contrary, in the QD regime, \( 1/\tau \sim T e^{-\Delta/T} \), (up to logarithmic corrections), as a result of the Boltzmann statistics of the quasiparticles. In the absence of
Ohmic dissipation, as follows from Eq. (8), metallic, rather than insulating behavior of the conductivity obtains provided that $\sqrt{H} \ll \Delta$. The persistence of the Bose metal phase for a finite range of magnetic field is possibly relevant to the recent experiments of Mason and Kapitulnik [1].

In the MA regime, quasiparticles also obey Boltzmann statistics. Based on dimensionality arguments similar to those valid for the QD regime, we suggest that in the MA regime, $1/\tau \sim Te^{-A\sqrt{H}/T}$, where the constant of proportionality is determined by a numerical coefficient that is of the first order in $1/M$. The calculation of the corresponding lifetime is complicated because of the discrete character of the quasiparticle spectrum. However, as follows from Eq. (9), the conductivity becomes directly, rather than inversely, proportional to dissipation for sufficiently large $H$. This means that inclusion of external dissipation $\eta$ and determines only how fast the conductivity decreases with increasing field. To illustrate this, we resort to a semi-phenomenological calculation in which we set $1/\tau = 2.3300502 \cdot Te^{-m/T}$ and $\eta = 0.1T$. The interpolation formula for $1/\tau$ is chosen to satisfy the behavior in all regimes near the critical point. The numerical prefactor in $1/\tau$ is taken in such a way, that for $\eta = 0$, $\sigma \approx \sigma_Q$ in the QC regime. The conductivity was then calculated numerically as a function of $\sqrt{H}/T$ for different values of $\Delta/T$, using Eqs. (8) and (9). The results that are depicted in Fig. support rather rapid decrease of $\sigma$ with increasing $\sqrt{H}/T$ but their dependence on $\eta/T$ is relatively weak. We conclude, then, based on the mean-field treatment, that the conductivity decreases when the crossover to the MA regime occurs even if $\eta = 0$. This behavior is in agreement with experiments on the magnetic field-tuned IST in disordered indium-oxide thin films [1]. However, these results are in disagreement with the suggestion of Kim and Weichman that the conductivity saturates with increasing field [14]. This discrepancy can be explained by the fact that for $\eta = 0$ in the general universal scaling function,

$$\sigma = (4e^2/h)\Sigma(\omega/T, \Delta^2/T, \sqrt{H}/\Delta),$$

applicable for the conductivity near the $d = 2$ IST, the limits $T = 0$, $\omega \to 0$ and $\omega = 0$, $T \to 0$ do not commute [13]. The former is the collisionless limit, apparently considered in Ref. (14), in which the conductivity is likely to experience a crossover between two universal values depending on the ratio $\sqrt{H}/\Delta$ [15]. However, it is the collision-dominated limit that is experimentally relevant and hence the main subject of our treatment. It is straightforward to verify that Eq. (10) satisfies the general scaling relation (13) with $\omega = 0$ and $z = 1$, provided that $\eta = 0$ and the inverse scattering time $1/\tau$ obeys itself the scaling form $1/\tau = Tf(\Delta^2/T, \sqrt{H}/\Delta)$ in the critical region.

The results above were obtained for low enough magnetic fields, $\sqrt{H} \sim f \ll 1$ and hence, magnetic frustration can be neglected. In translationally periodic systems such as JJA, magnetic frustration plays an important role giving rise to peaks in the conductivity for integer values of frustration $f$ [24]. Moreover, the peaks corresponding to the rational values $f = \frac{1}{2}, \frac{1}{3}, \frac{3}{5}, \cdots$ may also be observed [24]. However, disorder disrupts the lattice periodicity smearing the peaks in the conductivity giving rise to a disappearance of frustration effects. This situation takes place in the disorder-tuned IST in thin films. Hence, we expect, that as long as Cooper pairs remain intact and the transition is described by Eq. (14), our calculations provide a qualitative picture for the behavior of the magneto-conductivity near the point of nominal the IST in thin films including the observation of a Bose metal phase.
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[1] A. F. Hebard and M. A. Paalanen, Phys. Rev. Lett. 65, 927 (1990).
[2] A. Yazdani and A. Kapitulnik, Phys. Rev. Lett. 74, 3037 (1995).
[3] For a review see A. M. Goldman and N. Markovic (1998) Physics Today 51 (11) 39-44.
[4] M. P. A. Fisher, Phys. Rev. Lett. 65, 923 (1990).
[5] Y. Liu and A. M. Goldman, Mod. Phys. Lett. 8, 277 (1994).
[6] N. Mason and A. Kapitulnik, Phys. Rev. Lett. 82, 534 (1999).
[7] D. Ephron, A. Yazdani, A. Kapitulnik, and M. R. Beasley, Phys. Rev. Lett. 76, 1529 (1996).
[8] N. Mason and A. Kapitulnik, cond-mat/0006138.
[9] E. Shimshoni, A. Auerbach, and A. Kapitulnik, Phys. Rev. Lett. 80, 3352 (1998).
[10] H. M. Yaeger, et. al., Phys. Rev. B 40, 182, (1989).
[11] D. Dalidovich and P. Phillips, cond-mat/0005119, to appear in PRB, 2001.
[12] Subir Sachdev, Quantum phase transitions (Cambridge University Press, New York, 1999).
[13] Min-Chul Cha, et. al. Phys. Rev. B 44, 6883, (1991).
[14] K. Kim and P. B. Weichman, Phys. Rev. B 43, 13583 (1991).
[15] K. Damle and S. Sachdev, Phys. Rev. B 56, 8714 (1997).
[16] D. Dalidovich and P. Phillips, Phys. Rev. Lett. 84, 737, (2000).
[17] A. Chubukov, S. Sachdev and J. Ye, Phys. Rev. B 49, 11919 (1994).
[18] A. van Otterlo, K.-H Wagenblast, R. Fazio, and G. Schön, Phys. Rev. B 48, 3316 (1993).
[19] A. O. Caldeira and A. J. Leggett, Phys. Rev. Lett. P46, 211 (1981).
[20] K. Wagenblast, A. van Otterlo, G. Schön, and G. Ziman, Phys. Rev. Lett. 79, 2730 (1997).
[21] H. M. Jaeger, D. B. Haviland, B. G. Orr, and A. M. Goldman, Phys. Rev. B 40, 182 (1989).
[22] I. F. Herbut, Phys. Rev. Lett. 85, 1532, (2000).
[23] R. A. Webb, et. al. Phys. Rev. Lett. 51, 690, (1983)
[24] H.S.J. van der Zant, et. al. Phys. Rev. B 54, 10081, (1996).