The imprint of pulsar parameters on the morphology of Pulsar Wind Nebulae

Rolf Bühler, Matteo Giomi

DESY, Platanenallee 6, 15738 Zeuthen, Germany

ABSTRACT
The morphology of young Pulsar Wind Nebulae (PWN) is largely determined by the properties of the wind injected by the pulsar. We have used a recent parametrization of the wind obtained from Force Free Electrodynamics simulations of pulsar magnetospheres to simulate nebulae for different sets of pulsar parameters. We performed axisymmetric Relativistic Magnetohydrodynamics simulations to test the morphology dependence of the nebula on the obliquity of the pulsar and on the magnetization of the pulsar wind. We compare these simulations to the morphology of the Vela and Crab PWN. We find that the morphology of Vela can be reproduced qualitatively if the pulsar obliquity angle is $\alpha \approx 45^\circ$ and the magnetization of the wind is high ($\sigma_0 \approx 3.0$). A morphology similar to the one of the Crab Nebula is only obtained for low magnetization simulations with $\alpha \gtrsim 45^\circ$. Interestingly, we find that Kelvin-Helmholtz instabilities produce small scale turbulences downstream of the reverse shock of the pulsar wind.

Key words: instabilities – MHD – shock waves – pulsars: general – pulsars: individual: Crab, Vela – ISM: supernova remnants

1 INTRODUCTION
Most of the rotational energy lost by a pulsar is transferred to a relativistic particle wind. These particles are by numbers predominantly electrons and positrons (referred to together as electrons in the following). The wind is thought to be cold, meaning that its thermal energy is much less than its bulk kinetic and magnetic energies. When the wind interacts with ambient material, the particles become isotropised and radiate (Arons 2012). This is what is then seen as a Pulsar Wind Nebula (PWN) in the sky. To date, around 100 of these systems have been found (Kargaltsev et al. 2015). In the X-rays, several of them show a torus morphology, with a jet emerging perpendicular to it (Kargaltsev & Pavlov 2008). Young PWN, with an age smaller than $\approx 10000$ yrs, have not yet been disturbed by the reverse shock of the stellar explosion (Gaensler & Slane 2006; Kargaltsev et al. 2015). Their morphology is still closely related to the properties of the pulsar wind. Particularly in two cases, the Crab and Vela PWN, the plasma outflow can be resolved observationally in great detail, down to spatial scales of $\Delta r \lesssim 0.03$ ly. These systems therefore provide a test bed to study the behaviour of relativistic plasma, which is also of relevance for other non-thermal sources as Active Galactic Nuclei (Netzer 2014; Massaro et al. 2015) or Gamma-ray Bursts (Gehrels & Mészáros 2012; Berger 2014).

The properties of the pulsar wind – as its magnetic field, particle and velocity distributions – are not known with certainty today. However, over the past years there has been great progress in this respect. Several groups have performed Force Free Electrodynamics (FFE) (Spitkovsky 2006; Kalapotharakos et al. 2012; Tchekhovskoy et al. 2013) and Particle in Cell (PIC) simulations of pulsar magnetospheres (Philipppov & Spitkovsky 2014; Cerutti et al. 2016). These simulations allow one to trace the properties of the wind out to several light cylinder radii $r_{lc}$. Due to the relativistic speed of the wind, it is expected that the wind does not have time to re-arrange itself on larger scales afterwards. In particular, the latitude dependence of its energy flux is expected to remain unchanged as the wind moves out into the nebula (Tchekhovskoy et al. 2016).

Recently, the first analytic parametrization of the latitude dependent luminosity of the pulsar wind has been derived from FFE simulations (Tchekhovskoy et al. 2016). The main parameter determining the wind properties is the obliquity angle between the pulsar spin axis and its magnetic moment $\alpha$. The latter can not be measured directly. Constraints from pulsar light curve modelling usually differ vastly between pulsar emission models (Pierbattista et al. 2015). The other important unknown parameter is the mag-
netization $\sigma$ of the wind. It is thought that most of the wind energy is in its magnetic fields ($\sigma \gg 1$) at $r_{LC}$. When and where this energy is transferred to kinetic particle energy is not known. FFE simulations of pulsar magnetospheres do not include non-thermal particle acceleration, this question cannot be addressed by them.

In this paper we will study the dependence of the nebula morphology on $\alpha$ and $\sigma$. Both of these parameters strongly affect the forces acting on the wind plasma. They are therefore expected to shape the morphology of the resulting nebula. We performed Relativistic Magnetohydrodynamic (RMHD) simulations to scan the parameter space of different obliquity angles and a high and low magnetization of the wind. RMHD simulations of PWN performed in the past have primarily focused on the Crab nebula (Hester 2008; Bühler & Blandford 2014). Qualitatively, the toroidal structure and the jet are well reproduced in 2D axisymmetric simulations (Komissarov & Lyubarsky 2004; Del Zanna et al. 2004, 2006; Volpi et al. 2009; Bucciantini et al. 2011). Dynamically, the motion of thin filaments, so called “wisps” is also reproduced (Camus et al. 2009). Recently, the first 3D simulations of the Crab nebula showed that axisymmetric simulations overpredict the strength of the jet (Porth et al. 2013, 2014). In addition, compared to 2D simulations, significant turbulence emerges far downstream of the wind termination shock in 3D. This potentially enhances the magnetic dissipation, allowing for larger magnetizations of the wind to reproduce the Crab’s morphology.

Unfortunately, performing several 3D simulations to scan the phase space of pulsar wind parameters is still computationally too expensive. We therefore performed 2D axisymmetric simulations. In contrast to most previous studies, we simulate both hemispheres. As was shown by Porth et al. (2014), this enhances the magnetic dissipation also in the axisymmetric case in the equatorial regions. Nevertheless, we will keep the axisymmetric limitation of our simulations in mind and will come back to it in the discussion of the simulation results in section 3.

We chose the length scales and spin-down power of the pulsar to values appropriate for the Vela PWN (Pavlov et al. 2003; Durant et al. 2013). To our knowledge this system has not been simulated in RMHD to date. We expect that apart from scaling factors, the PWN morphology does not depend strongly on this choice. Qualitatively, we expect the simulated morphologies to be similar also in other young PWN. We will use cgs units throughout, except for length scales, which for convenience will be given in light years.

## 2 PULSAR WIND NEBULA SIMULATIONS

In our simulations, a pulsar wind is injected into a homogeneous ambient medium at rest. The later has only the role to confine the pulsar wind. The regions of interest for the high-energy emission are in the inner nebula, close to the reverse shock of the pulsar wind. Once the pulsar wind has blown a sufficiently large bubble inside of its surrounding medium, the properties of the inner region do not depend strongly on the ambient environment. Instead, the inner nebula morphology is predominantly determined by the wind properties. As mentioned before, the latter are determined by the obliquity angle of the pulsars and by the wind magnetization. We performed six simulations with different combinations of these parameters shown in table 1. The simulation setup will be described in more detail in the following.

### 2.1 Simulation setup

The simulations were run with the RMHD module of the PLUTO\(^1\) code version 4.2 (Mignone et al. 2007). The RMHD equations were evolved in time using an HLLC solver. We assumed a polytropic equation of state with an adiabatic index of $4/3$. We applied spherical coordinates, with a linear binning in the polar angle $\theta$ and a logarithmic binning in radius $r$. Adaptive Mesh Refinement (AMR) was used with four refinement levels, with a factor two in cell sizes between the levels (Mignone et al. 2012). The first grid was divided in 88 radial bins, from $r_{\text{min}} = 0.0002$ ly to $r_{\text{max}} = 1.4$ ly, and 32 polar angle bins. At a typical distance of the reverse shock of $r_{\text{shock}} = 0.05 − 0.2$ ly the resolution is $\Delta r_{\text{shock}} \approx (3 − 13) \times 10^{-4}$ ly at the highest AMR level. To speed up the simulations, AMR is not activated in the unshocked wind region for $r > 0.001$ ly. Reflective boundary conditions were applied along the symmetry axis and continuous outflow boundary conditions were applied at $r_{\text{max}}$.

More details on the simulation parameters can be found in the PLUTO configuration files shown in Appendix A.

The pulsar wind was injected into an ambient medium at $r_{\text{min}}$. In order to speed up the simulations, we used a relatively thin ambient medium with a density of $\rho_{\text{am}} = 10^{-28}$ g cm$^{-2}$. The work required to blow the pulsar wind bubble into the surrounding medium is thereby reduced. In order to further accelerate the simulations, the region within $r < 0.06$ ly was filled with the unperturbed pulsar wind parameters at the start of the simulations.

We setup the pulsar wind parameters following the prescriptions given in Porth et al. (2014). The total energy flux density of the wind, $f_{\text{tot}}$, as a function of the polar angle $\theta$ and the obliquity angle of the pulsar is given by:

$$f_{\text{tot}}(r, \theta, \alpha) = \frac{L}{U^2 \pi} \times (g(\theta, \alpha) \times \sin^2 \theta + d).$$

The angular dependence of the wind is given by the function $g(\theta, \alpha)$, and $d = 0.02$ is added for numerical reasons to avoid vanishing energy flux at the poles. $L$ is a scaling factor that normalizes the total energy flux to the spin-down luminosity of the Vela pulsar $L = 6.9 \times 10^{36}$ ergs s$^{-1}$ (Manchester et al. 2003; Durant et al. 2013). To our knowledge this system has not been simulated in RMHD to date. We expect that apart from scaling factors, the PWN morphology does not depend strongly on this choice. Qualitatively, we expect the simulated morphologies to be similar also in other young PWN. We will use cgs units throughout, except for length scales, which for convenience will be given in light years.

### Table 1. Parameters for the performed simulations: the pulsar obliquity angle $\alpha$ and the wind magnetization before ($\sigma_0$) and after ($\sigma$) the annihilation of the striped wind. The last column indicates which source the simulated morphology shown in figures 7 and 8 resembles qualitatively (see text).

| Sim. Nr. | $\alpha$ | $\sigma_0$ | $\sigma$ | Source |
|---------|----------|-----------|----------|--------|
| 1       | $10^5$   | 0.03      | 0.024    | Crab   |
| 2       | $10^6$   | 3         | 1.5      | Crab   |
| 3       | $45^\circ$ | 0.03      | 0.0043   | Crab   |
| 4       | $45^\circ$ | 3        | 0.12     | Vela   |
| 5       | $80^\circ$ | 0.03      | 0.00014  | Crab   |
| 6       | $80^\circ$ | 3        | 0.0008   | Crab   |

\(^1\) http://plutocode.ph.unito.it
The angular dependence of the wind is obtained by averaging the parametric solutions given by Tchekhovskoy et al. (2016) over the azimuth angle $\phi$:

\[ g(\theta, \alpha) = (w_1(\alpha) b_1(\theta, \phi) + w_2(\alpha) b_2(\theta, \phi))^2, \]

with \( w_1(\alpha) = 1 - 2\alpha/\pi \), \( w_2(\alpha) = 1 + 0.17|\sin 2\alpha - w_1(\alpha) \), \( b_1(\theta, \phi) = 1 + 0.02\sin \theta + 0.22(|\cos \theta| - 1) \)

\[ -0.07(|\cos \theta| - 1)^3 \times \text{sign} \cos \theta, \]

\[ \theta_m(\theta, \phi) = \arccos(\sin \alpha \sin \theta \cos \phi + \cos \theta \cos \alpha), \]

\[ b_2(\theta, \phi) = \cos(\phi - 3\phi) \sin \theta \]

The energy density flux is divided between a magnetic term \( f_m \) and a kinetic term \( f_k \):

\[ f_m(r, \theta, \alpha) = \sigma(\theta) f_{\text{int}}(r, \theta, \alpha) \frac{1 + \sigma(\theta)}{1 + \sigma(\theta)}, \]

\[ f_k(r, \theta, \alpha) = \frac{f_{\text{int}}(r, \theta, \alpha)}{1 + \sigma(\theta)}, \]

where \( \sigma(\theta) \equiv f_m / f_k \) is the wind magnetization. To assure that the Poynting flux vanishes at the poles, the magnetization saturates at:

\[ \bar{\sigma}(\theta) = \begin{cases} \left(\theta/10^5\right)^2 \times \sigma_0 & \theta \leq 10^5 \\ \sigma_0 & \theta > 10^5 \end{cases} \]

\[ \chi(\theta, \alpha) = \begin{cases} (2\phi(\theta, \alpha)/\pi - 1)^2 & \pi/2 - \alpha < \phi < \pi/2 + \alpha \\ 1 & \text{otherwise} \end{cases} \]

The magnetic field in the simulations is assumed to be purely toroidal, and inverts its direction in the equatorial plane:

\[ B_\phi = \pm \sqrt{4\pi f_m(r, \theta)/c} \]

Asymptotically one expects the poloidal and radial components of the magnetic field to decrease rapidly with distance with \( r^{-2} \), while the toroidal component decreases with \( r^{-1} \). The 

\[ B'_p = \sqrt{B^2 - D^2(B \cdot \mathbf{n})^2 + 2D^2D(B \cdot \mathbf{n})D(B \cdot \mathbf{B})/T} \]

\[ \epsilon \propto D^{\lambda+2}B^{\lambda+1}_p, \]

where the photon emissivity \( \epsilon \) depends primarily on the Doppler factor \( D = 1/\Gamma(1 - \beta n) \) and magnetic field \( B'_p \) perpendicular to the line of sight \( \mathbf{n} \) in the plasma rest frame:
Figure 1. Components of the energy flux density of the simulated pulsar wind injected by the pulsar as a function of the polar angle $\theta$. The distributions are shown for different obliquity angles $\alpha$ and are normalized to a distance of 1 ly.

Figure 2. Position of the reverse shock of the pulsar wind as a function of time for different obliquity angles $\alpha$ and magnetizations $\sigma_0$. The shock position has been averaged over $10^5$ around the equator.

Figure 3. Shape of the reverse shock of the pulsar wind as a function of time for different obliquity angles $\alpha$ and magnetizations $\sigma_0$. The shock position has been averaged over time from $t_{\text{sim}} = 8$ yrs to $t_{\text{sim}} = 10$ yrs.
3 RESULTS

In the following, we will focus our discussion on the snapshot at the end of the simulations at $t_{syn} = 10$ yrs. The nebula is already close to a self similar expansion at this time.

3.1 The plasma flow

The global plasma flow patterns of the different simulations are shown in figure 4. Common patterns are observed for all simulations: the pulsar wind is decelerated abruptly at an oblate reverse shock. Plasma at higher latitudes is transported to the polar regions due to the magnetic hoop stress and a jet emerges. In the equatorial region, plasma continues to move radially outward, creating a torus in the equatorial plane. The torus region is highly turbulent and the radial flow pattern is lost. The polar flow becomes stronger compared to the equatorial one with increasing average wind magnetization. As expected, the most relevant parameter is $\bar{\sigma}$ and not $\sigma_0$. This can be seen clearest for the $\alpha = 80^\circ$ case, where the flow patterns are very similar for $\sigma_0 = 0.03$ and $\sigma_0 = 3.0$.

A zoom in on the flow pattern in the region close to the reverse shock is shown in figure 5. The region before the reverse shock, referred to as “wind region” in the following, becomes smaller with increasing $\bar{\sigma}$. This can also be seen in the shock position at different latitude angles shown in figure 3. The wind region also becomes more oblate with increasing $\bar{\sigma}$. This is expected, as the increased hoop stress results in increasing pressure in the polar regions (Lyutikov et al. 2016b). Despite these dependencies, the flow patterns are similar. The exception is the simulation of high magnetizations. The jet is also clearly visible in this case.

It is apparent from figure 5 that the degree of plasma turbulence increases with decreasing $\bar{\sigma}$. The flow is very regular for the $\sigma_0 = 3.0$ and $\alpha = 10^\circ$ simulation. There is also a regular plasma flow in the downstream of the reverse shock for $\sigma_0 = 10^\circ$. These are the two simulations with the largest $\bar{\sigma}$. In contrast, all other simulations show a high degree of turbulence, which emerges almost directly downstream of the termination shock.

Interestingly, loop like patterns from magnetic Kelvin-Helmholtz (KH) instabilities are observed downstream of the shear flow regions. An example is shown in figure 6. As the shear flow is reduced for high magnetizations, KH loops are predominantly observed in the low-sigma simulations. KH instabilities in PWN have been studied by Bucciantini & Del Zanna (2006) for local features within nebulae. To our knowledge, KH instabilities have not been reported in previous publications of global PWN simulations. However, its emergence had already been seen in the simulations discussed in the PhD thesis of Camus (2009). The reason that most previous simulations did not reveal this instability is likely that their spatial resolution was a factor $> 5$ larger compared to the ones presented here (Del Zanna et al. 2006; Camus et al. 2009; Bucciantini et al. 2011). As will be discussed in more detail in section 3.4, the presence of the KH instability could lead to interesting radiative signatures.

3.2 Synchrotron emission maps

In this section, we will look at the morphology expected from the synchrotron emission of the plasma flow described in the previous section. The emission of the inner regions of PWN is very anisotropic due to the strong Doppler boosting. The viewing angle therefore plays an important role in determining the observed morphology. For our discussion, we will assume a viewing angle of $\theta_{view} = 120^\circ$. This is approximately the angle under which we observe the Vela and Crab nebulae from Earth (Weisskopf et al. 2000; Helfand et al. 2001; Ng & Romani 2004; Weisskopf et al. 2012). These sources will be discussed in more detail in sections 3.3 and 3.4. Maps for viewing angles of $\theta_{view} = 90^\circ$ and $\theta_{view} = 150^\circ$ are shown in appendix B.

The synchrotron maps for simulations with $\sigma_0 = 0.03$ are shown in figure 7. All of them show a torus, which comes from the equatorial region of the PWN. The synchrotron maps for $\alpha = 45^\circ$ and $\alpha = 80^\circ$ are qualitatively similar, with wider rings than the ones seen in simulation with $\alpha = 10^\circ$. The reason for this is that in the latter case more of the emission comes from higher latitudes. This is a result of the higher average magnetization and higher wind energy flux for $\alpha = 10^\circ$ (see figure 1).

Figure 8 shows the synchrotron maps calculated for the simulations done with $\sigma_0 = 3.0$. In this case, the difference between different obliquities is stronger. The main reason for this is that for $\alpha = 10^\circ$ and $\alpha = 45^\circ$ the average magnetization is close to unity. This results in elongated synchrotron nebulae. In the case of $\alpha = 10^\circ$, the torus is barely visible anymore as the emission is dominated by the higher latitudes. The jet is also clearly visible in this case.

Before we continue to confront these simulations with observations, we would like to recall two caveats: (1) Our simulations are axisymmetric. 3D simulations have shown that axisymmetry is a good approximation close to the wind reverse shock, in particular in the equatorial regions (Porth et al. 2014). However, at high latitudes and further away from the reverse shock, 3D simulations show significantly different emission patterns. It follows from this that emission associated to the equatorial region in the inner nebula can be expected to be more robust. Polar regions on the other hand, and in particular the jet, are less trustworthy and will therefore not be a focus of discussion. (2) The emission model we apply here ignores spatial differences in the particle distribution function. The lack of strong spectral variation in the emission from the inner regions of PWN indicates that this is likely a good approximation (Mori et al. 2004). Nevertheless, this is certainly oversimplified; e.g. individual structures in the Crab nebula are known to be visible only at particular wavebands (Hester et al. 2002). Keeping these caveats in mind, we will proceed to compare our simulation results to the Vela and Crab nebulae.

3.3 Vela Pulsar Wind Nebula

The Vela PWN is embedded in the Vela Supernova Remnant, also known as G263.9-3.3. Several regions of non-thermal emission are known within this remnant. The brightest one in radio is labelled Vela-X. The Vela pulsar is located in Vela-X (Horns et al. 2006; de Jager et al. 2008; Abramowski et al. 2012; Grondin et al. 2013). It is one of
Figure 4. Streams of the plasma flow for different simulations at a time $t_{\text{sim}} = 10$ yrs. The colours show the plasma speed in units of $c$. The background image shows the logarithm of the magnetic field strength in a grey scale. The spatial extend of each panel is 1.6 ly on each side. A zoom of the region indicated by the yellow box in the lower left panel is shown in figure 5.
Figure 5. As figure 4, but zoomed in on the inner nebula region. The shown region is indicated in the lower left panel of figure 4. It extends from $x = -0.4$ ly to $x = 0$ ly and $z = -0.19$ ly to $z = 0.19$ ly.
the closest and brightest gamma-ray pulsar known to date. Its distance has been determined via parallax to 936$^{+62}_{-55}$ kpc (Caraveo et al. 2001; Dodson et al. 2003). Due to its proximity the Vela pulsar might contribute significantly to the local cosmic-ray electron flux (Hinton et al. 2011). In the region surrounding the pulsar, the Chandra X-ray Observatory revealed a double ring structure, which is likely related to the reverse shock of the pulsar wind (Helfand et al. 2001; Pavlov et al. 2003; Durant et al. 2013). It is this innermost emission which we aim to reproduce in the simulations here. Unfortunately, the observational data is restricted to the X-ray band, as the ring structure has not been detectable so far at other wavebands (Moran et al. 2014; Marubini et al. 2015).

The three dimensional structure of the Vela rings has been interpreted as two equal torii, which are on top of each other (Helfand et al. (2001), for an high contrast image of the Vela rings see figure 2 in Pavlov et al. (2003)). This results in a morphology which is more elongated along the symmetry axis compared to the Crab Nebula. The rings were also found to be closer to the pulsar as the innermost ring in the Crab nebula. It was suggested that this might be due to a larger wind magnetization of order unity (Helfand et al. 2001). Indeed, our simulations confirm that solutions with higher magnetization resemble the ring structure better. Solutions with a lower σ as all simulations with σ0 = 0.03 and both simulations with α = 80° are too wide and the tori are too narrow. Interestingly, several bright small-scale structures are found along the symmetry axis of the nebula. These features are related to the highly beamed right downstream of the reverse shock. Such features are also observed in the X-ray data (Levenfish et al. 2013).

The best morphology agreement is found for the α = 45° and σ0 = 3.0 simulation. In this simulation rings of similar size originate downstream of the reverse shock. One lower ring from the equatorial regions right behind the wind termination and an upper ring from the torus. In contrast, all other solutions result in rings of increasing size as one moves away from the pulsar. The exceptions are the simulations for α = 10°. However, for α = 3.0 the nebula is far too elongated and for the case of σ0 = 0.03 no double ring structure is obtained. The value of α = 45° agrees well with the value of α = 53° inferred from polarization measurements of the pulsar profile (Johnston et al. 2005). Models of the pulsed gamma-ray emission typically give higher values of α = 62° – 75° (Abdo et al. 2010).

### 3.4 Crab Pulsar Wind Nebula

The Crab is the most studied PWN. Its torus and jet structure can be observed in great detail from the radio to X-ray band (Hester 2008; Bühler & Blandford 2014). It has therefore been the primary target for RMHD studies of PWN. Several authors have qualitatively reproduced the morphology of the inner nebula in axisymmetric simulations assuming a low wind magnetization < σ > ≈ 0.01 (Komissarov & Lyubarsky 2004; Del Zanna et al. 2006; Volpi et al. 2009; Camus et al. 2009; Porth et al. 2014). We confirm this findings in our simulations. A good agreement is found for case of α = 45° and σ0 = 0.03 and for both simulations with α = 80°. Also in agreement with previous studies, a small bright feature is found just below the pulsar position (Lyutikov et al. 2016b; Yuan & Blandford 2015). This “inner knot” was proposed to be the site of the recently discovered gamma-ray flares (Buehler et al. 2012; Mayer et al. 2013). However, no observational evidence for this has been found to date (Rudy et al. 2015).

It is puzzling, that the innermost ring of the Crab nebula observed in X-rays does not show a brightness profile as expected from Doppler beaming. The back side of the inner...
Figure 7. Emission maps calculated for simulations of low magnetization $\sigma_0 = 0.03$ in arbitrary units. The left panels show a slice through the $y = 0$ plane. The right panels show the integrated emission in the line of sight for an observed at a viewing angle $\theta_{\text{view}} = 120^\circ$, indicated by the dashed white line in the left panels. The spatial extend of each panel is 1.6 ly on each side.
Figure 8. As figure 7, but for simulations of high magnetization $\sigma_0 = 3.0$. 
ring has a brightness which is comparable to its front side. In RMHD simulations the ring is found to be much fainter than in observations (Porth et al. 2014). In addition, the ring is composed of a series of knots, in contrast to the smooth profiles found in the simulations. Our simulations show a similar disagreement. However, the observed KH instabilities at the shear flow downstream of the reverse shock lead to Doppler boosted emission regions also on the back side of emission rings close to the reverse shock. This results in a brighter emission from the receding part of the flow than expected in radial flow models. Our simulations show that this effect is not strong enough to result in a ring of equal brightness. However, the increased turbulence in the KH loops might trigger increased particle acceleration via magnetic reconnection (Cerutti et al. 2013; Sironi & Spitkovsky 2014) and magnetoluminescence (Blandford et al. 2014; East et al. 2015; Nalewajko et al. 2016; Yuan et al. 2016; Lyutikov et al. 2016a). Stochastic Fermi acceleration might also occur (Rieger et al. 2006). In either case, fresh injection of high energy particles in back-flowing plasma could lead to a decreased Doppler asymmetry only for the highest synchrotron frequencies. Indeed, the inner ring is only observed in X-rays and has not been detected at lower frequencies to date (Hester et al. 2002). As the development of the KH instability is reduced for higher plasma magnetization, the latter could be constrained if this interpretation is correct. Testing this idea quantitatively requires dedicated simulations which include particle acceleration, which is beyond the scope of this paper.

4 SUMMARY AND OUTLOOK

We have performed axisymmetric RMHD simulations of PWN to scan the parameter space for different pulsar wind properties. We have simulated the wind emerging for different pulsar obliquities, which has recently been derived from FFE simulations (Tchekhovskoy et al. 2016). In addition we have tested different wind magnetizations. In general, we find that the average wind magnetization is the most important parameter in determining the PWN morphology. The main effect of increasing obliquity angle is to increase the size of the striped wind region, where we have assumed a perfect dissipation of opposite magnetic field lines. We found that the wind region upstream of the reverse shock is smaller in size and becomes more oblate with increasing σ.

With the exception of the wind morphology for α = 10° and σ0 = 3.0, all simulations showed a torus in their emission maps, which emerges from the equatorial region downstream of the reverse shock. We have compared the morphologies of the different simulations to the Vela and Crab PWN. For Vela, we found that the simulation with the parameters α = 45° and σ0 = 3.0 gives the best match to the observed morphology. For the Crab nebula, all simulations with a low  overmatch the observed morphology (α = 10° and σ0 = 0.03 ; α = 45° and σ0 = 0.03 ; α = 80° for σ0 = 0.03 and σ0 = 3.0 ).

We found that, particularly for low magnetizations, KH instabilities develops at the downstream at the shear flow of the reverse shock. The KH loops have the effect to increase the emission of the receding side of the nebula compared from what is expected from Doppler boosting of a radial flow. We suggest that this effect might help to explain that the innermost ring observed in X-rays in the Crab nebula has almost constant brightness.

We have pointed out the caveat that these conclusions rely on axisymmetric simulations. It would be desirable to confirm these findings with 3D simulations in the future. For a more quantitative comparison of observations and simulations, it will also be important to include a model for particle acceleration in the simulations. From the observational side, the detection of the rings observed in the Vela PWN outside of the X-ray band would be crucial to constrain the electron energy distribution. Taken together these steps provide the prospects in understanding the plasma flow quantitatively in PWN. This would be the first time this is achieved for relativistic plasma flows and would likely have implications also for other sources as GRBs or AGN.

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REFERENCES

Abdo A. A., et al., 2010, The Astrophysical Journal, 713, 154
Abramowski A., et al., 2012, Astronomy & Astrophysics, 548, A38
Amato E., Arons J., 2006, The Astrophysical Journal, 653, 325
Arons J., 2012, Space Science Reviews, 173, 341
Berger E., 2014, Annual Review of Astronomy and Astrophysics, 52, 43
Blandford R., Simeon P., Yuan Y., 2014, Nuclear Physics B (Proceedings Supplements), Volume 256, p. 9-22, 256, 9
Bucciantini N., Del Zanna L., 2006, Astronomy and Astrophysics, 454, 393
Bucciantini N., Arons J., Amato E., 2011, Monthly Notices of the Royal Astronomical Society, 410, 381
Buehler R., 2016, PWN Imprint webpage, http://www. rolfbuehler.net/notes/PWNimprint.html
Buehler R., et al., 2012, The Astrophysical Journal, 749, 26
Bühl R., Blandford R., 2014, Reports on Progress in Physics, 77, 066901
Camus N. F., 2009, Relativistic Magnetohydrodynamical Models of Pulsar Wind Nebulae, https://www1.maths.leeds.ac.uk/ applied/Research/afg/thesis/nicholas_camus.pdf
Camus N. F., Komissarov S. S., Bucciantini N., Hughes P. a., 2009, Monthly Notices of the Royal Astronomical Society, 400, 1241
Caraveo P. A., De Luca A., Mignani R. P., Bignami G. F., 2001, The Astrophysical Journal, 561, 930
Cerutti B., Werner G. R., Udalsky D. A., Begelman M. C., 2013, The Astrophysical Journal, Volume 782, Issue 2, article id. 104, 15 pp. (2014)., 782
Cerutti B., Philippov A., Spitkovsky A., 2016, Monthly Notices of the Royal Astronomical Society, 457, 2401
Del Zanna L., Amato E., Bucciantini N., 2004, Astronomy and Astrophysics, 421, 1063
Del Zanna L., Volpi D., Amato E., Bucciantini N., 2006, Astronomy and Astrophysics, 453, 621
Dodson R., Legge D., Reynolds J. E., McCulloch P. M., 2003, The Astrophysical Journal, 596, 1137
Durant M., Kargaltsev O., Pavlov G. G., Kropotina J., Levenfish K., 2013, The Astrophysical Journal, 763, 72
East W. E., Zrake J., Yuan Y., Blandford R. D., 2015, Physical Review Letters, Volume 115, Issue 9, id.095002, 115
Gaensler B. M., Slane P. O., 2006, Annual Review of Astronomy and Astrophysics, 44, 17
Gallant Y. A., Arons J., 1994, The Astrophysical Journal, 435, 230
Gehrels N., Mészáros P., 2012, Science (New York, N.Y.), 337, 932
Grondin M.-H., Romani R. W., Lemoine-Goumard M., Guillenmot L., Harding A. K., Reposeur T., 2013, The Astrophysical Journal, 774, 110
Helfand D. J., Gotthelf E. V., Halpern J. P., 2001, The Astrophysical Journal, 556, 380
Hester J. J., 2008, Annual Review of Astronomy and Astrophysics, 46, 127
Hester J. J., et al., 2002, The Astrophysical Journal, 577, L49
Hinton J. A., Funk S., Parsons R. D., Ohm S., 2011, The Astrophysical Journal, 743, L7
Horns D., Aharonian F., Santangelo A., Hoffmann A. I. D., Masterson C., 2006, Astronomy and Astrophysics, 451, L51
Johnston S., Hobbs G., Vigeland S., Kramer M., Weisberg J. M., Lynge A. G., 2005, Monthly Notices of the Royal Astronomical Society, 364, 1397
Kalapotharakos C., Contopoulos I., Kazanas D., 2012, Monthly Notices of the Royal Astronomical Society, 420, 2793
Kargaltsev O., Pavlov G. G., 2008, in AIP Conference Proceedings, AIP, pp 171–185, doi:10.1063/1.2900138
Kargaltsev O., Cerutti B., Lyubarsky Y., Striani E., 2015, Space Science Reviews, 191, 391
Komissarov S. S., Lyubarsky Y. E., 2004, Monthly Notices of the Royal Astronomical Society, 349, 779
Levenfish K. P., Bykov A. M., Durant M., Kargaltsev O. Y., Kropotina Y. A., Pavlov G. G., Krassilchtkov A. M., Uvarov Y. A., 2013, Memorie della Societa Astronomica Italiana, 84, 588
Lyutikov M., Sironi L., Komissarov S., Porth O., 2016a, p. 140
Lyutikov M., Komissarov S. S., Porth O., 2016b, Monthly Notices of the Royal Astronomical Society, 456, 286
Manchester R. N., Hobbs G. B., Teoh A., Hobbs M., 2005, The Astronomical Journal, 129, 1993
Marabini F., Tchekhovskoy A., Spitkovsky A., Li J. G., 2013, Monthly Notices of the Royal Astronomical Society, 435, L1
Mignone A., Bodo G., Massaglia S., Matsakos T., Tesileanu O., Zanni C., Ferrari A., 2007, The Astrophysical Journal Supplement Series, 170, 228
Mignone A., Zanni C., Tzeferacos P., van Straalen B., Colella P., Bodo G., 2012, The Astrophysical Journal Supplement Series, 198, 7
Moran P., Mignani R. P., Shearer A., 2014, Monthly Notices of the Royal Astronomical Society, 445, 835
Mori K., Burrows D. N., Hester J. J., Pavlov G. G., Shibata S., Tsunemi H., 2004, The Astrophysical Journal, 609, 186
Nalewajko K., Zrake J., Yuan Y., East W. E., Blandford R. D., 2016, eprint arXiv:1603.04850
Netzer H., 2014, Annual Review of Astronomy and Astrophysics, 53, 150619171245001
Ng C., Romani R. W., 2004, The Astrophysical Journal, 601, 479
Olm B., Del Zanna L., Amato E., Bandiera R., Bucciantini N., 2014, Monthly Notices of the Royal Astronomical Society, 438, 1518
Pavlov G. G., Teter M. A., Kargaltsev O., Sanwal D., 2003, The Astrophysical Journal, 591, 1157
Philippov A. A., Spitkovsky A., 2014, The Astrophysical Journal, 785, L33
Pierbattista M., Harding A. K., Grenier I. A., Johnson T. J., Caraveo P. A., Kerr M., Gonthier P. L., 2015, Astronomy & Astrophysics, 575, A3
Porth O., Komissarov S. S., Keppens R., 2013, Monthly Notices of the Royal Astronomical Society, 431, L48
Porth O., Komissarov S. S., Keppens R., 2014, Monthly Notices of the Royal Astronomical Society, 438, 278
Rieger F. M., Bosch-Ramon V., Duffy P., 2006, Astrophysics and Space Science, Volume 309, Issue 1-4, pp. 119-125, 309, 119
Rudy A., et al., 2015, The Astrophysical Journal, 811, 24
Sironi L., Spitkovsky A., 2011, The Astrophysical Journal, 741, 39
Sironi L., Spitkovsky A., 2014, The Astrophysical Journal Letters, Volume 783, Issue 1, article id. L21, 6 pp. (2014.), 783
Spitkovsky A., 2006, The Astrophysical Journal, 648, L51
Tchekhovskoy A., Spitkovsky A., Li J. G., 2013, Monthly Notices of the Royal Astronomical Society, 435, L1
Tchekhovskoy A., Philippov A., Spitkovsky A., 2016, Monthly Notices of the Royal Astronomical Society, 457, 3384
Volpi D., Del Zanna L., Amato E., Bucciantini N., 2009, preprint, 0903.4120
Weisskopf M. C., et al., 2000, The Astrophysical Journal, 536, L81
Weisskopf M. C., Elsner R. F., Kolodziejczak J. J., O’Dell S. L., Tennant A. F., 2012, The Astrophysical Journal, 746, 41
Yuan Y., Blandford R. D., 2015, Monthly Notices of the Royal Astronomical Society, 454, 2754
Yuan Y., Nalewajko K., Zrake J., East W. E., Blandford R. D., 2016, p. 23
de Jager O. C., Slane P. O., LaMassa S., 2008, The Astrophysical Journal, 689, L125

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APPENDIX A: PLUTO CONFIGURATION FILE

The relevant parts of the pluto.ini file are listed below.

```
[Grid]
X1-grid 1 0.00002 88 l+ 0.14
X2-grid 1 0.0 32 u 3.14159265358979
X3-grid 1 0.0 1 u 1.0

[Chombo Refinement]
Levels 4
Ref_ratio 2 2 2 2
Regrid_interval 2 2 2 2
Refine_thresh 0.8
Tag_buffer_size 3
Block_factor 8
Max_grid_size 128
Fill_ratio 0.4

[Solver]
Solver hllc

[Boundary]
X1-beg userdef
X1-end outflow
X2-beg axisymmetric
X2-end axisymmetric
X3-beg outflow
X3-end outflow
```

The relevant parts of the definitions.h file are listed below.

```
#define PHYSICS RMHD
#define DIMENSIONS 2
#define COMPONENTS 3
#define GEOMETRY SPHERICAL
#define RECONSTRUCTION LINEAR
#define TIME_STEPPING RK2

#define EOSideal
#define ENTROPY_SWITCH CHOMBO_REGRID

#define SHOCK_FLATTENING MULTID
#define LIMITER DEFAULT
#define RECONSTRUCT_4VEL YES
#define CHOMBO_LOGR YES
#define RMHD_FAST_EIGENVALUES YES
```

APPENDIX B: ADDITIONAL MAPS
Figure B1. As figure 7, but for a viewing angle $\theta_{\text{view}} = 90^\circ$. 

$\alpha = 10^\circ$
$\sigma_0 = 0.03$

$\alpha = 45^\circ$
$\sigma_0 = 0.03$

$\alpha = 80^\circ$
$\sigma_0 = 0.03$
Figure B2. As figure 8, but for a viewing angle $\theta_{\text{view}} = 90^\circ$. 

$\alpha = 10^\circ$; $\sigma_0 = 3.0$

$\alpha = 45^\circ$; $\sigma_0 = 3.0$

$\alpha = 80^\circ$; $\sigma_0 = 3.0$
Figure B3. As figure 7, but for a viewing angle $\theta_{\text{view}} = 150^\circ$. 
Figure B4. As figure 8, but for a viewing angle $\theta_{\text{view}} = 150^\circ$. 