A comparative investigation of meshing characteristics of anti-backlash single- and double-roller enveloping hourglass worm gears

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Abstract

The new precision transmission system-Anti-backlash roller enveloping hourglass worm gears are now widely employed in many industrial sectors such as robot joint design, packaging production line, and CNC (computer numerical control) machine tools. Therefore, it is of great interest to have knowledge of their corresponding transmission principles that are seldom discussed in the literature. Using the theories of differential geometry and gear meshing, the objective of this paper is to analyze and compare meshing characteristics of anti-backlash single- and double-roller enveloping hourglass worm gears in terms of their transmission principle, engagement equation, induced normal curvature, lubrication angle, autorotation angle, entrainment velocity, helix angle, and distribution of contact curves. The major differences between these two roller enveloping hourglass worms are theoretically investigated in this work. Our results show that the anti-backlash single-roller enveloping hourglass worm (ASEHW) gears provide better performance with respect to gear meshing and transmission than the anti-backlash double-roller enveloping hourglass worm (ADEHW) gears. In addition, compared with the ADEHW gears, the ASEHW gears are easier to fabricate, install, and calibrate. However, the ADEHW gears are more auto-adjustable in eliminating gear backlash. Our study puts forward a theoretical background for futuristic design, application, and promotion of the anti-backlash roller enveloping hourglass worm gears.

Keywords: Anti-backlash single-roller enveloping hourglass worm (ASEHW) gear, Anti-backlash double-roller enveloping hourglass worm (ADEHW) gear, Tooth contact analysis (TCA), Backlash, Gear engagement

1. Introduction

Compared to conventional worm gears, anti-backlash roller enveloping hourglass worm gears offer advantages such as a higher horsepower-to-weight ratio, a higher contact ration, greater compactness, lower levels of noise, higher operating efficiencies, and longer life time (Wang et al., 2010; Wang et al., 2014; Rong et al., 2010). Because of the roller interactions with the hourglass worm gear, the system alleviates the problem wherein the meshing contact does not accommodate rolling movement around itself. Moreover, as the name suggests, the anti-backlash roller enveloping hourglass worm drive has minimized or even eliminated gear backlash through the application of the roller shapes. There are currently two types of anti-backlash roller enveloping hourglass worm drives, the ASEHW gears achieved through the displacement of single-row rollers (Fig. 1a) and ADEHW gears that are characterized by mismatched arrangements of the double-row rollers (Fig. 1b).
The idea of using rollers to reduce the backlash in gear transmission was first put forward by Motohashi et al. (1986). In their study an effective zero-backlash mechanism was designed by using a pair of a globoidal cam and its mating roller wheel. Since then there have been very few researchers continued to investigate the significant effect of the rollers on backlash reduction. In 2009, Deng and Wang started to explore the possibility of introducing rollers in the worm gear to achieve a novel design of worm drive with no backlash and proposed the first design of the ADEHW gear in 2008 (Wang et al., 2010). In the proposed ADEHW gear design, a mismatched arrangement of the double-row rollers was introduced to remove the backlash and the application of the rollers turned the sliding friction to rolling friction during worm gear engagement. Compared to the traditional worm gears, the proposed design effectively improved the transmission efficiency and precision of worm drive. The initial design was then modified and optimized to further reduce the volume of the worm gear system and improve its transmission ratio (Wang et al., 2014; Rong et al., 2010). Follow-up studies were conducted to investigate the effects of different meshing rollers of the ADEHW gears (roller, rolling cone, and ball) on their meshing characteristics (Deng et al., 2012, 2013) and discuss the methods of manufacturing and testing such gears (Jiang and Deng, 2017), the material microstructure effects are also discussed (Gao et al., 2019).

It had been argued that the replacement of the trapezoidal screw with the rollers would weaken the load-bearing capacity of the worm gear thereby affecting the applicability of the ADEHW gears. In order to maintain its load-bearing capacity, Deng and Wang conducted several parametric studies to optimize the worm roller structure and eventually achieved a final design of the ADEHW gears with satisfied load-bearing capacity (Zheng et al., 2017; Deng et al., 2017, 2019). The optimized gears have been used to build worm gear reducers (Fig. 1a) and the reducers have already been tested and commercialized.

Through those studies it was also found that the complicated double-roller structure could be reduced to a single-roller structure to simplify the design and make the entire worm gear system more compact. Based on that hypothesis, a novel design of the anti-backlash single-roller enveloping hourglass worm (ASEHW) gears has been proposed recently, whose parameters are adjustable to reduce and eliminate the backlash. In that design, both tooth surfaces of the roller are meshed with the thread of the worm and the relative rolling between the roller worm gear teeth and the worm threads will lead to thin lubricant films on the thread surfaces and achieve self-lubrication. A similar design is also presented by Sankyo, a Japanese company and use for their precision speed reducer (Fig. 1b) (Motohashi et al., 1986). Its transmission principle is the same as the one exhibited in our design, which is to achieve excellent precision, rigidity, and durability with rollers and the roller gear cam mechanism can provide high torque with good transmission efficiency.

Due to their high precision, efficiency, rigidity, and durability, the roller enveloping hourglass worm gears have found broad potential applications in robotics, agricultural machinery, and indexing units. It is of great significance to fully understand the transmission mechanisms and meshing characteristics of such worm gears. However, as the ADEHW gears and ASEHW gears were recently developed, no study to date has focused on evaluation and comparison of these two types of the roller enveloping hourglass worm drives. This paper fills this gap by presenting a comparison study to...
systematically analyze, compare, and assess these two types of worm gears. Based on the theories of differential geometry and gear meshing, this paper first discusses the difference in forming principles of worm tooth surface of the ADEHW and ASEHW gears and constructs engagement equations for these two roller worm gears based on theories of gear engagement. Next, Matlab is used to study and compare key design parameters of the two worm gears, including the induced normal curvature, lubrication angle, autorotation angle, entrainment velocity, helix angle, and distribution of contact curves. Finally, 3D models for these two worm gears are generated for TCA via finite element analysis (FEA), and the FEA results show that the ASEHW gears have better failure resistance than the ADEHW gears.

2. Theoretical background of roller enveloping hourglass worm drives

2.1 Engagement mechanism

Figures. 2a and 2b exhibit the ADEHW and ASEHW worms and Figs. 3a and 3b display the ADEHW and ASEHW worm wheels, respectively. The ADEHW drive employs the mismatched arrangement of double rows of rollers with one row of rollers meshing with the left worm tooth face and another with the right worm tooth face. The left and right faces are formed as the envelop of the double rollers (tool cutting edge), as indicated at c_2 in Figs. 2a and 3a. By adjusting the installation position of the worm gear, the rollers can maintain contact with the worm face, so as to achieve a zero-backlash transmission. In the ADEHW design, backlash is provided for each row of rollers, which allows for lubrication and enables the entire transmission to operate properly. For the entire system, the mismatched arrangement of the double rows of rollers eliminates the transmission error and improves smoothness and precision of the transmission. As reflected from Figs. 2b and 3b, in an ASEHW drive, both faces of the rollers have to remain contact with the worm tooth face during meshing to remove backlash; meanwhile, the rollers should be able to rotate in the worm tooth space to avoid transmission stuck and breaking in the worm wheel teeth.

2.2 Forming principle of worm tooth surface

The most difficult part in fabrication of the roller enveloping hourglass worms is the cutting and grinding of the worm tooth face, as shown in Fig. 4. As shown in that figure, for an ADEHW drive, its right face is enveloped by the lateral side face of a grinding roller located below the central plan at an offset of c_2. During the machining process, the worm blank is rotated about its own axis at an angular speed ω_1, the self-rotating grinding roller is rotated about the axis of the worm wheel at ω_2, and the ratio of ω_1/ω_2 is a constant. The helical surface machined following this approach becomes the worm tooth face. Formation of the left tooth is very similar to that of the right tooth face. The only difference is that the grinding roller is located above the central plan with the same offset of c_2 (Fig. 2a). Also, the rotation axes of the two grinding rollers make an angle of β and here we define α = β/2 as the circumferential angle of the worm gear.

The formation principle of the worm tooth surface of the ASEHW drives is similar to but much simpler than that of the ADEHW drives. For such drives, only one grinding roller is involved, whose rotation axis is perpendicular to the rotation axis of the worm, so that c_2 = 0 and α = 0.
Fig. 2  The arrangement of rows of rollers in (a) ADEHW drive and (b) ASEHW drive

Fig. 3  Double rows of rollers are arranged on the (a) ADEHW wheel in a mismatched manner while (b) ASEHW wheel only has a single row of rollers.

Fig. 4  Forming principle of ADEHW tool face
3. Comparison study between ADEHW and ASEHW gears

Mathematical descriptions of the two types of worm gears are established in this section, based on which the meshing characteristics of the ADEHW and ASEHW gears can be studied and compared. As described in above section, the ASEHW gears can be considered as a special case of the ADEHW gears, so we first establish parametric equations for the ADEHW gears and then develop the ones for the ASEHW gears.

Coordinate systems between the worm and worm wheel and between the worm and meshing roller is established in Fig. 5, including the fixed coordinate systems \( \mathbf{O}_{1i_1j_1k_1} \), \( \mathbf{O}_{2i_2j_2k_2} \) and the movable coordinate systems \( \mathbf{O}_{1'i_1'j_1'k_1'} \), \( \mathbf{O}_{2'i_2'j_2'k_2'} \), where \( \mathbf{k}_1 \) and \( \mathbf{k}_2 \) represent the rotation axes of the worm and the worm wheel, respectively. In addition, another fixed coordinate system \( \mathbf{O}_{0i_0j_0k_0} \) is established at the top center of the roller, that system is correlated with the worm wheel. The rotation axis of the roller is along the radial direction of the worm wheel and intersects the rotation axis of the worm wheel \( \mathbf{k}_2 \). The location of \( \mathbf{O}_0 \) in the \( \mathbf{O}_{2i_2j_2k_2} \) is defined as \((a_2, b_2, c_2)\). In Fig. 5, \( A \) is the central distance between the worm and the worm wheel; \( c_2 \) is the offset distance of the roller and \( \alpha \) is the circumferential angle of the worm wheel. \( \varphi_1 \) and \( \varphi_2 \) are the angles of rotation for the worm and worm wheel, respectively. The movable overlap with the fixed coordinate systems when \( \varphi_1 = \varphi_2 = 0 \).

A dynamic coordinate system \( \mathbf{O}_{pee'2n} \) is established at the contact point \( P \) (Fig. 6) so that the vector equation of the surface of the roller in \( \mathbf{O}_{0i_0j_0k_0} \) is given as

\[
\begin{align*}
    r_0 &= x_0 \mathbf{j}_0 + y_0 \mathbf{j}_0 + z_0 \mathbf{k}_0 \\
    x_0 &= R \cos \theta \\
    y_0 &= R \sin \theta \\
    z_0 &= u
\end{align*}
\]

where \( u \) and \( \theta \) are parameters of the cylindrical surface of the roller and \( R \) is its radius.
3.1 Engagement equations

According to the theory of gear meshing (Illes Dudas, 2004), the meshing equation gives the necessary and sufficient condition of existences of the ADEHW profile and can be obtained as

\[ \mathbf{v}_1^2 \cdot \mathbf{n} = 0 \]  

where \( \mathbf{v}_1^2 \) is the relative velocity vector of the worm with respect to the worm wheel on the contact point, and \( \mathbf{n} \) is the unit normal vector of the worm tooth surface with respect to the conjugate contact point. Eq. (2) means that the relative velocity at the contact point should be perpendicular to the normal direction at that point. During gear meshing, tooth surfaces of the meshing gears will remain contact only if Eq. (2) is satisfied.

As derived by Deng et al. (2012), the generic meshing function of the ADEHW gear can be mathematically developed by the following equations:

\[
\begin{align*}
\Phi &= \mathbf{v}_1^2 \cdot \mathbf{n} = M_1 \cos \phi_2 - M_1 \sin \phi_2 - M_1 = 0 \\
M_1 &= (c_2 + x_0) \sin \alpha \sin \theta + (a_2 - y_0 \sin \alpha - z_0 \cos \alpha) \cos \theta \\
M_2 &= -(c_2 + x_0) \cos \alpha \sin \theta + (b_2 + y_0 \cos \alpha - z_0 \sin \alpha) \cos \theta \\
M_3 &= i_{21} \sin \alpha \sin \theta (b_2 + y_0 \cos \alpha - z_0 \sin \alpha) + i_{21} (a_2 - y_0 \sin \alpha - z_0 \cos \alpha) \cos \alpha \sin \theta + A \cos \theta
\end{align*}
\]  

The relationship between parameters \( u \) and \( \theta \) (Eq. (1)) can be obtained as

\[
\begin{align*}
\Phi &= f(\theta, \varphi_2) = P_1 / P_2 \\
P_1 &= (i_{21}b_2 - c_2 \cos \varphi_2) \sin \alpha \sin \theta + (c_2 \sin \varphi_2 + i_{21}a_2) \cos \alpha \sin \theta + (b_2 \sin \varphi_2 - a_2 \cos \varphi_2 + A) \cos \theta \\
P_2 &= (\sin \alpha \sin \varphi_2 - \cos \alpha \cos \varphi_2) \cos \theta + i_{21} \sin \theta
\end{align*}
\]  

The contact curve defines instant contact between a meshing roller and the tooth profile of the ADEHW gear and has to satisfy the generic meshing function Eq. (3). In geometry, the contact curve is the intersection curve between the worm tooth surface and its enveloping surface. Thus, assuming \( \varphi_2 \) is a constant, the contact curve on the generating surface between the roller and the ADEHW gear can be obtained by combining Eqs. (3) and (4) as
\[
\begin{align*}
    & r_0 = x_0 i_0 + y_0 j_0 + z_0 k_0 \\
    & u = f(\theta, \varphi_2) = \frac{P_1}{P_2} \\
    & \varphi_2 = \text{const}
\end{align*}
\] (5)

Eq. (5) is defined in the coordinate system \( O_{0i0j0k0} \). When \( 0 \leq \theta \leq \pi \), Eq. (5) defines the contact curve between the meshing roller and the left tooth profile and when \( -\pi \leq \theta \leq 0 \), that equation defines the contact curve between the roller and the right tooth profile.

The contact curve on the worm tooth surface is a space curve on the cylindrical surface of the roller. In order to visually display this curve, we extend the cylindrical surface of the roller to a plane which is tangent to that surface at point \( \theta = 0 \) and make the contact curve a plane curve by projecting it to that plane. Fig. 7 plots the projected contact curves for the double roller and single roller, at that time the rollers of the worm wheel mesh with the left tooth profile of the worm and the worm wheel’s rotation angle \( \varphi_2 = 0 \). The coordinates in Fig. 7 are calculated as \( x_4 = \theta R \) and \( y_4 = u \).

![Fig. 7 Projected contact line](image)

It can be seen from Fig. 7 that both ADEHW and ASEHW gears have line contacts but the contact line of the ASEHW gear is longer than that of the ADEHW gear. According to the worm gear meshing theories (Illes Dudas, 2004), the load-bearing capacity of the ASEHW gear should be higher than that of the ADEHW gear.

During worm gear meshing or machining, the moving generatrix at different times will construct a one-parameter surface family, whose enveloping surface is the tooth profile of the worm. In another word, at any instant, the contact line does exist between the generatrix and the enveloping surface and the enveloping surface is composed of infinite number of the contact curves. Thus, through the transformation from \( O_{0i0j0k0} \) to \( O_{1i1j1k1} \), the equation of contact lines (Eq. (5)) can be mapped to the movable coordinate system of the worm \( O_{1i1j1k1} \) and let the angles of rotation \( \varphi_1 \) and \( \varphi_2 \) be two variables, the tooth profile equation for the double-roller enveloping hourglass worm can be obtained as
With the developed equation for the ADEHW drive, the equations for the ASEHW drive can be obtained by substituting \( \alpha = \beta/2 = 0 \), \( a_2 = da/2 \), and \( b_2 = c_2 = 0 \) into Eqs. (3), (4), and (6).

Design parameters used for the presented ADEHW and ASEHW gears are listed in Table 1 where \( A \) is the central distance between the worm and the worm wheel; \( i_{12} \) is the gear ratio that is defined as the ratio of the angular speed of the worm \( \omega_1 \) and that of the worm wheel \( \omega_2 \) around their own axis: \( i_{12} = \omega_1 / \omega_2 \); \( R \) is the radius of the roller (see Fig. 6); \( \alpha \) is the worm gear circumference angle that is shown in Fig. 5; \( c_2 \) is the offset distance of the roller and \( m \) is the module of gear. It is worth mentioning that in the design of the ASEHW gear, \( \alpha \) and \( c_2 \) are not considered, this is the major difference in the design of ADEHW and ASEHW drives.

| Key parameters | \( A \) | \( i_{12} \) | \( R \) | \( \alpha \) | \( c_2 \) | \( m \) |
|----------------|--------|--------|-----|------|-------|-----|
| Double roller  | 210 mm | 36     | 8 mm| 0.6° | 11 mm | 9.63 mm |
| Single roller  | 210 mm | 36     | 8 mm| 0    | 0     | 9.62 mm |

Substituting above parameters into Eq. (6), contact curves for the ADEHW and ASEHW drives are obtained and plotted in Fig. 8.

Fig. 8  Contact curves for (a) double-roller and (b) single-roller worm gear

The contact curves displayed in Fig. 8 are imported into a 3D design software tool to generate the corresponding
single- and double-roller worm gears, as depicted in Fig. 9.

In Fig. 9, the ADEHW drive is shown in red and the ASEHW drive is shown in black. From that figure it can be seen that the tooth profile of the ADEHW drive is rendered as a trapezoid and that of the ASEHW is close to a rectangular.

### 3.2 Lubrication angle

The lubrication angle is the angular separation between the contact curve and the relative velocity and can be used as an important indicator to evaluate the lubrication conditions between two conjugated surfaces. A comparatively large lubrication angle will allow the generation of a hydrodynamic lubricant film in an arc-surface worm drive.

As illustrated in Fig. 10, when an object \( A \) moves along another object \( B \) at a relative speed of \( V_A \), the angle between \( V_A \) and the contact line (CD) between the object \( A \) and \( B \) is defined as the lubrication angle \( \mu \). The more the \( \mu \) is close to 90°, the more likely that the hydrodynamic lubricant film will be generated. On the contrary, when the \( \mu \) is small, the lubricant film can hardly be generated.

For regular cylindrical worm drives, the lubrication angle \( \mu \) is relative small, so it is very hard to create the hydrodynamic lubricant film between the contacting teeth surfaces, leading to semi-dry or even dry friction between the surfaces of the meshing teeth. As calculated in Deng et al. (2012), the lubrication angle for the proposed ADEHW drive is comparatively large, which makes it easy for the lubricant film to generate between the teeth surfaces, therefore evidently enhances the lubrication condition, betters the meshing quality, and improves the load-bearing capacity.

As derived in Deng et al. (2012), the lubrication angle for the ADEHW drive, which is an acute angle, is mathematically described as
\[
\mu = \arcsin \left( \frac{\sigma \cdot v^{12}}{\|\sigma\| \cdot \|v^{12}\|} \right) = \arcsin \left( \frac{v^{12}_{1} \left( \frac{v^{12}_{1}}{R - \omega^{12}_{2}} \right) + v^{12}_{2} \omega^{12}_{2}}{\sqrt{v^{12}_{1} \left( \frac{v^{12}_{1}}{R - \omega^{12}_{2}} \right)^2 + \left( \omega^{12}_{2} \right)^2 \left( v^{12}_{1} \right)^2 + \left( v^{12}_{2} \right)^2}} \right)
\] 

where \( \sigma \) is a vector along the normal direction of the contact curve and is given as

\[
\sigma = \left( \frac{v^{12}_{1}}{R - \omega^{12}_{2}} \right) e_{1} + \omega^{12}_{2} e_{2}
\]

Fig. 11 elucidates the relationship between the rotation angle and the lubrication angle for both single- and double-roller worm drives. As shown in that figure, for the single-roller worm drive, the lubrication angle is less affected by the rotation angle while for the double-roller worm drive, the lubrication angle changes acutely. In another word, for the ADEHW gear, its lubrication condition significantly differs at different contact points.

### 3.3 Autorotation angle of the cylindrical roller

In the roller enveloping hourglass worm gears, the replacement of the traditional worm gear teeth with the rollers that can rotate about their own axes effectively turns sliding contact to rolling contact during gear meshing, thereby reducing the friction and increasing the transmission efficiency. Thus, self-rotation property of the rollers directly affect the performance of transmission of the entire worm drive.

A design parameter, autorotation angle, is introduced to indicate the self-rotation property of the rollers. This angle is defined as the angle between the relative velocity \( v^{12} \) and the axis of a roller \( k_{0} \), which can be calculated as

\[
\mu_{\alpha} = \arccos \left( \frac{k_{0} \cdot v^{12}}{|v^{12}|} \right) = \arccos \frac{|v^{12}|}{\sqrt{(v^{12}_{1})^2 + (v^{12}_{2})^2}}
\]

As a key meshing parameter, the closer the autorotation angle is to 90˚, the easier it is for the roller to self-rotate and the higher worm gear transmission quality can be achieved.

Fig. 12 displays the relationship between the rotation angle of the worm and the autorotation angle of the rollers. Similar to the lubrication angle, the autorotation angle is more influenced by the rotation angle in the single-roller worm drive. The tendency of variation in the autorotation angle with respect to the rotation angle as displayed in Fig. 12 is also similar to the tendency of variation in the lubrication angle exhibited in Fig. 11. Such similarity verifies a strong correlation between the autorotation angle of the roller and the lubrication angle. In fact, it is imaginable that when the self-rotation of the rollers is limited, the friction between the meshing teeth must increase and the lubrication condition is worsened.
3.4 Entrainment velocity

Entrainment velocity is an important parameter in gear analysis, which significantly affects the shock and vibration on a gear system and the generation of the lubricant film (Zhang and Yang, 2007). Here the entrainment velocity for the roller enveloping hourglass worm drive is developed based on gear meshing theories. At first, the linear velocity of the worm tooth profile at meshing point can be defined as \( v^1 = \omega_1 \times r_1 \). Substitute \( v^1 \) into the dynamic coordinate system \( O_p e_1 e_2 n \) and we can have

\[
\begin{align*}
 v^1 &= v^1_1 e_1 + v^1_2 e_2 + v^1_3 n \\
 v^1_1 &= -Q_1 \sin \alpha \cos \theta + Q_2 \cos \alpha \cos \theta - Q_3 \sin \theta \\
 v^1_2 &= Q_1 \cos \alpha - Q_2 \sin \alpha \\
 v^1_3 &= -Q_1 \sin \alpha \sin \theta + Q_2 \cos \alpha \sin \theta + Q_3 \cos \theta \\
 Q_1 &= (c_2 + x_0) \cos \varphi_2 \\
 Q_2 &= (c_2 + x_0) \sin \varphi_2 \\
 Q_3 &= (a_2 - y_0) \sin \alpha - z_0 \cos \alpha) \cos \varphi_2 - (b_2 + y_0 \cos \alpha - z_0 \sin \alpha) \cos \varphi_2 - A 
\end{align*}
\]

where \( x_0, y_0, z_0 \) is the coordinates of the meshing point P in \( O_p e_1 e_2 n \).

The velocity of the tooth surface of the worm wheel (which in our design is the cylindrical surface of the roller) at point P is \( v^2 = \omega_2 \times r_2 + \omega_0 \times k_0 \), where \( \omega_0 = \omega_0 \times k_0 \) is the angular velocity of the self-rotation of the roller. Substitute \( v^2 \) into \( O_p e_1 e_2 n \) and we have

![Fig. 12 Autorotation angles of rollers for the single- and double-roller worm drives](image-url)
According to the definition of the entrainment velocity, the relative entrainment velocity of the roller enveloping hourglass worm drive can be obtained as

\[ v_{\sigma} = \frac{1}{2\sigma}(v^1 + v^2) \cdot \sigma \]  

(12)

where \( \sigma \) is given in Eq. (8).

Substitute Eqs. (10) and (11) into (12), the equation for calculating the relative entrainment velocity at the contact point between teeth profiles can be derived as

\[
\begin{align*}
  v_{\sigma}^{(1)} &= \frac{v^2 (v_{\sigma}^{(1)} / R - \omega_2^{(1)}) + v^2 \omega_2^{(1)}}{\sqrt{(v^2 / R - \omega_2^{(1)})^2 + (\omega_2^{(1)})^2}} \\
  v_{\sigma}^{(2)} &= \frac{v^2 (v_{\sigma}^{(2)} / R - \omega_2^{(2)}) + v^2 \omega_2^{(2)}}{\sqrt{(v^2 / R - \omega_2^{(2)})^2 + (\omega_2^{(2)})^2}}
\end{align*}
\]

(13)

Changes of the entrainment velocities for the ADEHW and ASEHW drives with respect to the rotation angle of the worm are exhibited in Fig. 13. From that figure it can be found that the arrangement of the rollers almost does not influence the entrainment velocity and the curves that reflect the variations of the entrainment velocities are close to a sine curve.
3.5 Engagement mechanism

Induced normal curvature is defined as the difference between the normal curvatures of the two meshing tooth surfaces along a common tangential direction at an instantaneous meshing point on an instantaneous contact line. This parameter indicates the proximity degree of the two tooth surfaces along their tangential direction and the curvature relation of the two line-conjugate surfaces. Induced normal curvature is an important concept for conjugate surfaces both in theory and practice, it is also essential in designing tooth surfaces, in the calculation of the strength of the gears, and the analysis of the contact regions of tooth surfaces, the load-bearing capacity of the lubricant file, the efficiency, and the expected operating life of a gear system. Luo and Wu (1992) illustrated the concept of induced normal curvature and Zhao and Zhang (2017) and Litvin and Fuentes (2005) proposed a method to compute that parameter.

According to previous literature, induced normal curvature can be calculated with

\[
 k_{n2}^{12} = k_{n1}^{12} = \frac{(\omega_{2}^{12} + v_{1}^{12} / R)^2 + (\omega_{1}^{12})^2}{\Psi}
\]  

(14)

where \( \Psi \) is a function and defined as

\[
 \Psi = \Phi + \omega_{2}^{12}v_{1}^{12} - \omega_{1}^{12}v_{2}^{12} - (v_{1}^{12})^2 / R
\]  

(15)

Using above equations, induced normal curvature is calculated for both single- and double-roller worm drives, as plotted in Fig. 14. From that figure it can be seen that the induced normal curvatures for the single- and double-roller worm drives are very close to each other, whose value varies from 0.1 to 0.145 mm\(^{-1}\). The small values of the calculated induced normal curvatures verify the good meshing characteristics acquired through the application of the anti-backlash roller enveloping hourglass worm gears.
3.6 Helix angle of the worm

For a worm, its helix is the shape formed by its threads; a helix angle (\(\lambda\)) is the constant angle between the tangent to the helix of the worm and a generator of the worm upon which the helix lies, as shown in Fig. 15.

![Fig. 15 Helix angle of a worm](image)

As derived by AGMA standard (2005), the helix angle for a worm can be calculated as

\[
\lambda = \arccos \left( \frac{\sqrt{A_1^2 + B_1^2}}{\sqrt{A_1^2 + B_1^2 + C_1^2}} \right)
\]  

(16)

A1, B1, and C1 are defined as
\[
\begin{align*}
A_1 &= n_{cy} n_{bz} - n_{cz} n_{by} \\
B_1 &= n_{cz} n_{bx} - n_{cx} n_{by} \\
C_1 &= n_{cx} n_{by} - n_{cy} n_{bx}
\end{align*}
\]  
(17)

where \( n_{ax1}, n_{ay1}, \) and \( n_{az1} \) denote the normal direction \( (n_a) \) of the worm tooth surface defined in \( O_1i_jk_1 \):

\[
\begin{align*}
n_{ax1} &= \cos \theta \sin \phi_1 + \sin \theta \cos \phi_1 \sin \phi_2 \\
n_{ay1} &= \cos \theta \cos \phi_1 + \sin \theta \sin \phi_1 \sin \phi_2 \\
n_{az1} &= -\sin \theta \cos \phi_2
\end{align*}
\]  
(18)

and \( n_{bx1}, n_{by1}, \) and \( n_{bz1} \) represent the normal direction \( (n_b) \) of a rotating surface of the worm that intersects the tooth surface at a contact point \( P \) (AGMA standard, 2005):

\[
\begin{align*}
n_{bx1} &= x_1 \left( 1 - A/\sqrt{x_1^2 + y_1^2} \right) \\
n_{by1} &= y_1 \left( 1 - A/\sqrt{x_1^2 + y_1^2} \right) \\
n_{bz1} &= z_1
\end{align*}
\]  
(19)

Thus, Eq. (17) defines a line tangent to the helix of the worm at \( P \):

\[
n_a \times n_b = (n_{ay1} n_{bz1} - n_{az1} n_{by1})i + (n_{az1} n_{bx1} - n_{ax1} n_{by1})j + (n_{ax1} n_{by1} - n_{ay1} n_{bx1})k
\]  
(20)

Applying Eqs. (16-20), the helix angle for the ASEHW and ADEHW drives are calculated and plotted. Fig. 16 displays how the single- and double-roller designs affect the helix angle of the roller enveloping hourglass worm drive. Fig. 16a displays together the calculated helix angle values and the contact lines of the tooth surface of the worm to visually show the variation in the helix angle at different meshing positions on the worm. Fig. 16b only plots the helix angle values for ASEHW and ADEHW drives to zoom in the difference between the helix angles for the two types of roller worm drives. Our calculation results show that the value of helix angle at the center of the worm is clearly higher than that at both ends of the worm, the difference between the maximum and minimum helix angle reaches 3.85˚. The variation in the helix angle of the ASEHW drive is more evident than that of the ADEHW drive.

![Fig. 16](image-url)
3.7 FEA modeling and simulation

Using the parametric values listed in Table 1, 3D computer-aided design (CAD) models for the ASEHW drive and the ADEHW drive are created, as displayed in Figs. 17 and 18. Based on those CAD models, FEA models for the two drives are created within ABAQUS and the TCA of the two worm drives are conducted through computer simulations.

(a)                                           (b)

Fig. 17  3D models for the (a) ADEHW and (b) ASEHW drives

(a)                                           (b)

Fig. 18  3D models for the (a) ADEHW and (b) ASEHW gears
Fig. 19 shows a detailed FEA model for the ADEHW worm gear system, with boundary and loading conditions identified. That model is meshed using 10-node tetrahedral element (C3D10) with 4 integration points and consists of 160,846 C3D10 elements. During the contact stress analysis, the worm gear and worm shaft are fully constrained except that they can rotate around the axes X₁ and X₂, respectively. In the simulation, the worm shaft rotated around X₂ at a speed of 1500 rpm and the worm wheel was constrained by a moment of 50 N × m. The same boundary and loading conditions are assigned to both ADEHW and ASEHW system during FEA simulations. Both the worm gears and worm shafts are made of 40Cr alloy steel, whose Young’s modulus is 211 GPa, Poisson’s ratio is 0.28, and density is $7.9 \times 10^3$ kg/m$^3$. The dimensions of the FEA model are set to be as the same as those (given in Table 1) of the real worm gear system.

Simulation results are displayed in Figs. 20 and 21, where Fig. 20 shows the von Mises stress distribution along the worm shaft during the worm gear meshing and Fig. 21 compares the von Mises stress distribution on the ASEHW and ADEHW gear models. Fig. 22 plots the maximum von Mises stresses generated on the two models when the worm wheel rotated to different angles. From that figure it can be seen that the maximum von Mises stress on the ADEHW gear is always higher than those on the ASEHW gear during the entire meshing process. This result indicates that if made of the same material and under the same boundary and loading conditions, the ADEHW gear will reach the yield and failure point before the ASEHW will. It can therefore be deduced that failure resistance of the ASEHW is higher than that of the ADEHW.
4. Conclusions

This paper theoretically studies and compares the ASEHW drive and ADEHW drive through analyzing the key design parameters via a combined experimental and computational approach. Based on the obtained comparison results, following conclusions can be drawn:

(1) The ASEHW drive can be considered as a special case of the ADEHW drive.
(2) Compared to the ADEHW drive, the ASEHW drive offers better meshing and lubrication performance, and higher failure resistance.
(3) Compared to the ADEHW drive, the ASEHW drive has a simpler structure, is more compact, and therefore is easier to be fabricated.

In the future, more experimental study will be conducted to compare the zero backlash property of the ASEHW and ADEHW drives and investigate how the arrangement of the rollers affect that property during worm gear meshing. The results obtained from this study form a theoretical background to promote futuristic design, manufacturing, and application of the roller enveloping hourglass worm drives.
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