Semiclassical Effects in Color Flavor Locked Strange Stars

Guilherme Lorenzatto Volkmer · Dimiter Hadjimichef

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Abstract
Strange stars in the color flavor locked phase, as described by a nonlinear generalization of the phenomenological MIT bag model proposed in the context of color superconductivity, are studied through a formalism motivated by the two semiclassical solutions for hydrostatic equilibrium. The semiclassical effects in the model are represented by a negative pressure fluid that might take place in the collapse of ultradense matter. Results show that within this framework it is possible to find ultracompact configurations situated between regular compact stars and black holes.

Keywords Semiclassical gravity · Strange stars · Ultracompact stars · Quark superconductivity

1 Introduction

General relativity, the currently accepted theory of the gravitational field, was proposed by Einstein more than a hundred years ago and has been confirmed by all experiments and astrophysical observations so far. Despite the success, general relativity is a completely classical theory, therefore it inevitably imposes a formal discontinuity between gravitational and quantum principles. In fact, Einstein himself was aware that quantum effects would demand modifications in his theory [1].

Considering that the subject of quantum gravity remains highly speculative, semiclassical gravity offers a less ambitious approach from which new insights are feasible. In this framework quantum fields interact with a classical unquantized spacetime metric. Such procedure, as John Earman described, can be seen as a “shot-gun marriage” between general relativity and quantum field theory, the two pillars of modern physics [2]. However, due to the character of its foundations, it is unlikely that such a program reflects an exact description of nature since it combines interactions between quantum fields which are treated in probabilistic terms, with a classical gravitational field relying on well-determined values [3]. Even though some interesting results were derived using semiclassical gravity. The most remarkable example is Hawking’s demonstration that black holes can evaporate through a quantum induced thermal radiation, reducing its mass, and consequently its surface area.

Studies have shown that semiclassical effects could be relevant even in relativistic stars, challenging the common view that the influence of gravity on quantum fields is negligible except in the most extreme environments [4–6]. In particular, it has been shown that, due to a semiclassical mechanism, for a compact object with radius \( R \approx 10 \) km it could only take a few milliseconds for the vacuum energy density to become dominant over classical matter at densities as high as \( 10^{14} - 10^{17} g/cm^3 \) [5]. It has also been proposed that the phenomenon known as quantum vacuum polarization in the presence of a gravitational field (where the vacuum state is changed due to the influence of the classical background) may be a new stabilizing ingredient in horizonless ultracompact objects, like black stars, but explicit models for such objects are not currently available [7]. In addition, the current gravitational wave era in astronomy is renewing the interest in theoretical scenarios where new classes of highly compact objects could emerge. For instance, it has been argued that the low mass companion (about \( 2.6 M_\odot \)) of the black hole in the source of GW190814 could be a strange star [8, 9].

Strange stars are motivated by the so-called strange matter hypothesis, developed independently by Bodmer and
Witten, which asserts that the true ground state of the strong interaction is strange quark matter, composed of an approximately equal proportion of up, down, and strange quarks [10, 11]. Since Witten’s work the MIT bag model with unpaired strange quark matter has been widely used to study strange stars. The model considers a gas of free relativistic quarks where confinement is achieved through a vacuum pressure, called the bag constant $B$.

More sophisticated quark dynamics can be introduced by applying the BCS mechanism to quark matter. The strong interaction among quarks is very attractive in some channels and quarks are expected to form Cooper pairs easily (which in quark matter always implies color superconductivity) [12–14]. Although many pairing schemes have been proposed, if central densities in compact stars are sufficient to support quark matter, it will probably manifest itself through the most symmetrical state, namely the color flavor locked (CFL) phase. In this phase all three quarks composing strange quark matter are paired on an approximately equal footing and form a color condensate.

Quark matter in the CFL phase has been applied in different astrophysical contexts, including the possibility of supporting exotic structures like wormholes [15]. When applied to strange stars, it has been shown that the CFL state affects the mass-radius relationship considerably, allowing configurations with large maximum masses [16]. Although CFL strange stars may not be as extreme as the hypothetical models for black hole mimickers available in the literature (like black stars), they still offer a high density environment combined with a solid theoretical background, avoiding the problems usually found in more extreme proposals [7].

It should be emphasized that there is no clear criteria to establish when semiclassical corrections start being relevant, but some perspective of how extreme a compact star is can be given in terms of the compactness parameter $C$ (which in $G = c = 1$ units is just the ratio between gravitational mass and radius). For instance, general relativity introduces significant corrections in newtonian gravitation for $C \approx 0.1$ [17]. A CFL strange star described as in general relativity has a compactness parameter which is more than half of a black hole for all nineteen parametrizations proposed in Ref. [16].

All things considered, it seems pertinent to evaluate the impact of semiclassical effects in CFL strange stars and the rest of the paper is organized as follows: Sect. 2 is devoted to the equation of state of CFL quark matter and the set of parametrizations that will be adopted further. In Sect. 3 basic aspects regarding semiclassical hydrostatic equilibrium are discussed and some appropriate limits for dealing with strange stars are established. In Sect. 4 it will be hypothesized that the environment produced by CFL strange stars is able to ignite a semiclassical correction, absent in general relativity, that will be associated with a generic linear equation of state with negative pressure. This is inspired in Gliner’s idea that certain gravitational collapses might reach an ultradense “vacuum-like” state, that could be described phenomenologically by a negative pressure equation of state [18]. Section 5 presents the results of the proposed model and compares them with those obtained via general relativity. Section 6 closes the article with the final remarks.

## 2 The Color Flavor Locked Equation of State

The CFL equation of state is a nonlinear generalization of the unpaired version of the MIT bag model, proposed in the context of color superconductivity. Different parametrizations are possible depending on the values of $B$ (the bag constant), $\Delta$ (the gap of the QCD Cooper pairs) and $m_s$ (the strange quark mass), which are not accurately known and are taken as free parameters. All results in Sect. 5 refer to the set of parametrizations presented in Ref. [16], which are also displayed in Table 1. It is customary to use a semi-empirical model in which the thermodynamic potential to order $\Delta^2$ can be expressed as [19]

$$\Omega_{\text{CFL}} = \Omega_{\text{free}} - \frac{3}{\pi^2} \Delta^2 \mu^2 + B,$$

where $\Omega_{\text{free}}$ represents the non-paired state and $\mu$ is the chemical potential. From the above equation, pressure and energy density can be analytically expressed to order $m_s^2$ as

$$p_{\text{CFL}} = \frac{3\mu^4}{4\pi^2} + \frac{9\alpha \mu^2}{2\pi^2} - B,$$

where $\alpha$ is a parameter.

### Table 1 Parametrizations originally presented in Ref. [16] which are also adopted in this work

| Parametrization | $B$(MeV/fm$^3$) | $\Delta$(MeV) | $m_s$(MeV) |
|----------------|----------------|--------------|-------------|
| CFL1           | 60             | 50           | 0           |
| CFL2           | 60             | 50           | 150         |
| CFL3           | 60             | 100          | 0           |
| CFL4           | 60             | 100          | 150         |
| CFL5           | 60             | 150          | 0           |
| CFL6           | 60             | 150          | 150         |
| CFL7           | 80             | 100          | 0           |
| CFL8           | 80             | 100          | 150         |
| CFL9           | 80             | 150          | 0           |
| CFL10          | 80             | 150          | 150         |
| CFL11          | 100            | 50           | 0           |
| CFL12          | 100            | 100          | 0           |
| CFL13          | 100            | 100          | 150         |
| CFL14          | 100            | 150          | 0           |
| CFL15          | 100            | 150          | 150         |
| CFL16          | 120            | 100          | 0           |
| CFL17          | 120            | 150          | 0           |
| CFL18          | 120            | 150          | 150         |
| CFL19          | 140            | 150          | 0           |
\[ \epsilon_{CFL} = \frac{9\mu^4}{4\pi^2} + \frac{9\alpha\mu^2}{2\pi^2} + B, \]  
with
\[ \alpha = -\frac{m^2}{6} + \frac{2\Delta^2}{3}. \]  

It is straightforward to express pressure and energy density as a one parameter equation of state of the form \( \epsilon(p) \), namely
\[ \epsilon_{CFL} = 3p_{CFL} + 4B - \frac{9\alpha\mu^2}{\pi^2}, \]
with the chemical potential expressed by
\[ \mu^2 = -3\alpha + \left[ \frac{4\pi^2}{3}(B + p_{CFL}) + 9\alpha \right]^\frac{3}{2}. \]

Alternatively, in a similar fashion \( p(\epsilon) \) is given by
\[ p_{CFL} = \frac{\epsilon_{CFL}}{3} - \frac{4B}{3} + \frac{3\alpha\mu^2}{\pi^2}, \]
where the chemical potential is expressed in terms of the energy density, namely
\[ \mu^2 = -\alpha + \left[ \frac{4\pi^2}{9}(\epsilon_{CFL} - B) + \alpha \right]^\frac{3}{2}. \]

The speed of sound can be obtained straightforwardly,
\[ c_{s,CFL}^2 = \frac{dp_{CFL}}{d\epsilon_{CFL}} = \frac{1}{3} + \frac{2\alpha}{3} \left( \frac{1}{\mu^2 + \alpha} \right). \]

The gap parameter plays a central role in the CFL phase because as the parameter increases the equation of state gets stiffer, allowing configurations with higher maximum masses when compared with regular strange stars [19]. The CFL matter is also significantly more bound than ordinary quark matter, being a candidate for the true ground state of hadronic matter for a much wider range of the parameters of the model than the state without any pairing [16, 20]. The goal in the rest of the paper is to revisit CFL strange stars through a semiclassically inspired model.

3 Semiclassical Hydrostatic Equilibrium

Aiming to go beyond the general relativistic picture regarding hydrostatic equilibrium, semiclassical gravity offers a conservative framework which should be valid in many situations where the vacuum polarization is small and the spacetime curvature is not comparable to the Planck length. So in what follows only situations where the fluctuations of the gravitational field are negligible will be considered. In this case a classical metric \( g_{\mu\nu} = \langle \bar{g}_{\mu\nu} \rangle \) is assumed just as in general relativity [21].

Now suppose that this classical spacetime is populated by a collection of quantum fields assumed to be in a given quantum state. In this context a semiclassical version of the Einstein field equations is proposed replacing the classical energy-momentum tensor by the expectation value of the energy-momentum tensor of the relevant quantized fields in the chosen state [22]. Particularly, using the \( 1/N \) expansion, one assumes that there are \( N \) non-interacting scalar fields present and the semiclassical Einstein equations can be expressed as [7]
\[ G_{\mu\nu} = 8\pi \left( GT_{\mu\nu} + hN\Theta Q_{\mu\nu} \right) + \mathcal{H}_{\mu\nu}, \]
where \( Q_{\mu\nu} \) represents the main contribution due to quantum effects and \( \Theta \) is a constant. The term \( \mathcal{H}_{\mu\nu} \) represents subdominant contributions and can be safely neglected for the present purposes [7].

The subsequent analysis is restricted to static and spherically symmetric spacetimes, with line element of the following form (hereafter Greek indices take four values and Latin indices only two)
\[ ds^2 = ds_{(2)}^2 + r^2 d\Omega^2 \]
\[ = g_{ab}(y)dy^a dy^b + r^2(y)d\Omega^2(\theta, \phi), \]
where \( d\Omega^2(\theta, \phi) \) denotes the angular line element on the 2-sphere and \( ds_{(2)}^2 \) is given by
\[ ds_{(2)}^2 = -e^{2\Phi(r)}dt^2 + e^{2\lambda(r)}dr^2, \]
with
\[ e^{2\lambda(r)} = \left( 1 - \frac{2Gm}{r} \right)^{-1}. \]

The classical source in Eq. (10) will be described by a perfect fluid mathematically represented by
\[ T_{\mu\nu} = (\epsilon + p)u_\mu u_\nu + pg_{\mu\nu}, \]
where \( p \) is the pressure, \( \epsilon \) is the energy density and \( u^\mu \) (satisfying \( u^\mu u_\mu = -1 \)) denotes the components of the four-velocity.

Regarding the semiclassical correction, it is useful to remember that in curved spacetime there is no preferred “vacuum state”. Fortunately, for a horizonless compact star of mass \( M \) in empty space, a more clear scenario rises and the appropriate vacuum state is known to be the Boulware vacuum state, a state with zero particle content for static observers and regular both inside and outside the star, also known as Schwarschild vacuum [23]. This vacuum state is approximately the same as the Minkowski vacuum at \( r \gg 2M \).

Furthermore, since there are no exact analytical expressions available for \( Q_{\mu\nu} \) in four dimensions, the s-wave Polyakov
approximation is also assumed [7]. The basic idea is that, under spherical symmetry, a decomposition through spherical harmonics allows the reduction from a four-dimensional theory to a set of two-dimensional theories specified by the angular-momentum. The dominant contribution in some spherically symmetric systems comes from the “s-wave sector”, that is, the \( l = 0 \) mode [24]. In physical terms it is responsible for neglecting quantum fluctuations that are not spherically symmetric and also effects of back-scattering [7]. In this approximation, the dimensional reduction is given by

\[
Q_{\mu\nu} = \frac{\delta a^a b^b}{4\pi r^2} \mathcal{O}_{ab}^{(2)},
\]

which is calculated with respect to \( d\zeta_{(2)} \). Fortunately, \( \mathcal{O}_{ab}^{(2)} \) can be expressed in a completely tensorial way as [7]

\[
\mathcal{O}_{ab}^{(2)} = \frac{1}{48\pi} \left( R^{(2)} g_{ab} + A_{ab} - \frac{1}{2} g_{ab} A \right),
\]

where \( A_{ab} = 4[\delta, |^{-1} \tilde{\nabla}_a \tilde{\nabla}_b \delta] \) for the Boulware state. It is possible to check that the semiclassical source in Eq. (15) is identically conserved [7]. It follows from the Bianchi identities that \( T_{\mu\nu} \) is conserved, providing the usual continuity equation

\[
p' = -(\epsilon + p) \Phi',
\]

where for any function a prime denotes a derivative with respect to \( r \).

Therefore hydrostatic equilibrium in semiclassical gravity is determined by solving the semiclassical Einstein equations as expressed in (10) for the sources described by Eqs. (14) and (15) under spherical symmetry, yielding [7]

\[
p' \left( 1 - \frac{p'}{2Y(r)} \right) = \Xi(r).
\]

For simplicity it was defined:

\[
\Xi(r) = -\frac{G (\epsilon + p)(m + 4\pi r^3 p)}{r - 2Gm},
\]

which represents the solution obtained in general relativity, and

\[
Y(r) = \frac{12\pi r (p + \epsilon)}{h\Theta N},
\]

carrying the semiclassical influence in hydrostatic equilibrium. Equation (18), being quadratic with respect to the pressure gradient, has two roots given by

\[
p'_{\pm} = Y(r) \left( 1 \pm \sqrt{1 - \frac{2\Xi(r)}{Y(r)}} \right).
\]

Exact solutions for \( p'_+ \) were analyzed in Ref. [7] by assuming that the quantity inside the square root is equal to a constant. The picture which emerged appears to be a nontrivial combination of two hypothetical compact objects, namely, black stars and gravastars [7]. Here a different approach will be adopted. First, observe that when \( h \to 0 \), \( p'_+ \) reduces to

\[
p'_- = \Xi,
\]

that is, at the classical limit the general relativistic picture for hydrostatic equilibrium is recovered [7].

Additionally, it can be shown that the semiclassical mass function adds a term proportional to \( h \) to the expression known from general relativity [7].

\[
\frac{2Gm}{r^2} = 8\pi Ge + \frac{h\Theta N}{12\pi^2} \left[ -\left( \frac{2Gm}{r} \right)^2 \Phi' + \left( 1 - \frac{2Gm}{r} \right) (2\Phi'' + \Phi'^2) \right].
\]

So it also recovers the classical expression when \( h \to 0 \), namely

\[
m' = 4\pi r^2 \epsilon.
\]

On the other hand, an alternative measure of classicality is more appropriate to analyze the \( p'_+ \) solution. The \( N \to \infty \) limit symbolically represents a quantum system when it becomes large [25]. With this in mind, observe that \( p'_+ \) can be rewritten as:

\[
\Theta hp'_+ = \frac{12\pi r (\epsilon + p)}{N} \left( 1 + \sqrt{1 - \frac{\Theta hN}{6\pi r (\epsilon + p)}} \Xi \right).
\]

Through a Taylor series about \( \hbar = 0 \) and matching both sides in powers of \( \hbar \) the above equation yields

\[
p'_+ = -\Xi(r) = \frac{G (\epsilon + p)(m + 4\pi r^3 p)}{r - 2Gm}.
\]

Since the number \( N \) is essentially arbitrary, the other terms in the expansion do not impose any restrictions, vanishing identically, once the limit \( N \to \infty \) with \( \Theta N \) fixed is taken in all of them [26]. This strategy, if applied to the minus root in Eq. (21), would give again as result Eq. (22).

It also has to fulfill the condition \( r > 2Gm \) at any point inside the star, thus forbidding the presence of a Schwarzschild black hole at any radius \( r \). Now observe that, for ordinary forms of matter with \( \epsilon > 0 \) and \( p > 0 \), Eq. (22) describes a negative pressure gradient and the pressure is a monotonically decreasing function of the radius which eventually vanishes. It is not difficult to realize that in order to have a similar picture regarding Eq. (26), an unconventional equation of state should be
employed. Consider, for instance, a linear equation of state with
negative pressure,
\[ \tilde{\rho} = -\omega \tilde{\varepsilon}, \]  
(27)
where \( \omega \) is a positive parameter smaller than unity. For con-
figurations where \( m > |4\pi r^3\rho| \) the pressure gradient is posi-
tive and assures that the absolute value of the pressure (and also the energy density) decreases as the radius increases.

The negative pressure fluid introduced by Eq. (27) is a subclass of what is sometimes called a \( \gamma \)-fluid, a relativistic perfect simple fluid which satisfies the \( \gamma \)-law equation of state [27]
\[ p = (\gamma - 1)\varepsilon, \]  
(28)
where \( \gamma \) is called the “adiabatic index” and lies in the range \([0, 2]\). Other important subclasses of the \( \gamma \)-law equation of state include stiff matter (\( \gamma = 2 \)), blackbody radiation (\( \gamma = 4/3 \)) and vacuum energy (\( \gamma = 0 \)). The thermody-
nical properties can easily be derived by considering \( \gamma \)-flu-
ids as a kind of generalized radiation. It can be shown, for
instance, that such fluids obey generalized versions of the usual Wien and Stefan-Boltzmann laws [27]. Regarding mechanical action, if fluids with \( \gamma < 1 \) could be somehow confined in a vessel, they would try to pull the walls inwards, instead of outwards as it happens with an ordinary gas.

This negative pressure subclass of the \( \gamma \)-law equation of state, although not forbidden by any law of nature, can only be realized with restricted stability. Therefore it is not pos-
sible to construct stable astrophysical configurations relying
only on \( \gamma \)-fluids with negative pressure. In a gravitational
collapse, where ordinary material elements are inevitably
present, the situation is quite different. It has been proposed
that such an effect may have a role in ultradense matter,
possibly reached through a gravitational collapse, provid-
ing environment is able to trigger a semiclassical correction,
via Eq. (22). It is then assumed that this quark superconduct-
ing additional attraction between material elements. This
is a consequence of the fact that negative pressure induces
attractive internal volume forces, while repulsion is the usual
outcome for an observational media consisting of particles
[18]. In other words, negative pressure states can attenuate
the usual repulsion of ordinary matter, thus allowing more
compact configurations.

All things considered, semiclassical gravity, having two
distinct ways of expressing hydrostatic equilibrium, opens
new possibilities regarding compact stellar models. The idea
in what follows is to construct a simple model which associates
to each hydrostatic equilibrium solution an appropriate source.
First, the CFL quark matter is treated as in general relativity
via Eq. (22). It is then assumed that this quark superconduct-
ing environment is able to trigger a semiclassical correction,
phenomenologically described by the Eq. (27), with a pres-
sure gradient obeys Eq. (26). In this sense, the semiclassical
solutions treated hereafter can be seen as describing \( \gamma \)-CFL
strange stars.

However, the proposed scenario ultimately demands a
decoupling of the gravitational sources and such simplifica-
tion may seem absurd due to the highly nonlinear structure
of the field equations. Fortunately this can be achieved, at
least for the spherically symmetric and static case, through a
technique called minimal geometric deformation, presented
in the next section [28].

4 Minimal Geometric Deformation

Before numerically implementing the model outlined in the
last section, it is worth to show how the CFL sector and the
semiclassical one can be decoupled, as long as the sources
interact only gravitationally [28]. Therefore, aiming to incor-
porate more intricate gravitational sources to CFL strange
stars, consider the field equations in the following form
\begin{equation}
G_{\mu\nu} = 8\pi \left( G_{\mu\nu}^{\text{CFL}} + \beta \Theta_{\mu\nu} \right),
\end{equation}
(29)
where \( \Theta_{\mu\nu} \) describes geometrical or physical contributions of
any additional source that may arise due to the presence of
extra interactions whose coupling to gravity is proportional
to the constant \( \beta \) [29]. Here it will describe a sector where
the semiclassical effects are dominant, namely
\begin{equation}
\Theta_{\mu\nu} = G^{\text{CFL}}_{\mu\nu} + h\pi Q_{\mu\nu} + \frac{\delta_{\mu} a^{\nu} \delta_{\nu} a^{\mu}}{8\pi r^2}.
\end{equation}
(30)
This new source includes a perfect fluid obeying Eq. (27),
represented by \( \tilde{T}_{\mu\nu} \). The last term in the right-hand side is a
geometrical factor (evaluated with respect to \( ds^2 \) in a similar
fashion to \( Q_{\mu\nu} \) introduced in order to preserve the spherically
symmetric form of the Einstein tensor (thus respecting the
Bianchi identities) in both sectors after the geometrical
derangement is performed [28].

It can be identified by inspection that the combination
of the two fluids provide an effective energy density and an
effective pressure specified by the relations
\begin{equation}
\varepsilon = \varepsilon^{\text{CFL}} + \beta \tilde{\varepsilon},
\end{equation}
(31)
\begin{equation}
p = p^{\text{CFL}} + \beta \tilde{p}.
\end{equation}
(32)

The minimal geometric deformation can be introduced via a linear decomposition of the form [28]:
\begin{equation}
e^{-2\lambda(r)} = C(r) + \beta D(r).
\end{equation}
(33)
The linear decomposition splits the system into two sets.
The first, with \( \beta = 0 \), is obviously the CFL strange star
solution as obtained using general relativity. The other set
corresponds to the additional source, with \((t, t)\) and \((r, r)\) components obeying

\[
\begin{align*}
\frac{D - 1}{r^2} + \frac{D'}{r} &= 8\pi G T_t^t + 8\pi \hbar N \Theta Q_t^t, \\
\frac{D - 1}{r^2} + 2\frac{D'}{r} &= 8\pi G T_r^r + 8\pi \hbar N \Theta Q_r^r.
\end{align*}
\] (34)

These are simply the \((t, t)\) and \((r, r)\) components obtained through Eq. (10) under spherical symmetry, from which the semiclassical hydrostatic equilibrium is directly derived. Since this is the sector where semiclassical effects are assumed to be dominant, it is logical to choose Eq. (26) to represent its hydrostatic equilibrium.

5 Results and Discussion

The system of hydrostatic equilibrium equations, as expressed in Eqs. (22) and (26), will be applied respectively to CFL quark matter and to an additional \(\gamma\)-fluid with negative pressure, producing a \(\gamma\)-CFL strange star where both contributions interact only gravitationally. The mass function for both contributions is calculated using Eq. (24). Numerical solutions are obtained similarly to the standard procedure in general relativity, keeping in mind that here each fluid satisfies its own hydrostatic equilibrium equation. The system is solved from the center, with \(m(r = 0) = 0\) for both contributions, and the energy densities are needed as input. The integration stops when the effective pressure vanishes, defining the final mass (which is the total gravitational mass from its two contributions) and radius. It is worth mentioning that although the CFL strange star will be embedded in a negative pressure environment, all solutions considered have a non-negative effective pressure and energy density.

Figure 1 illustrates the impact of the \(\beta\) parameter on the effective equation of state for the CFL5 and CFL15 parametrizations. The slashed green line denotes the ultrarelativistic limit \(p = \epsilon/3\).

Since semiclassical stellar models are still in their early days, a detailed comparison with recent astronomical observations may not be a primary concern. Even though, in order to provide some perspective, a few comments are pertinent. In a combined effort by the North American Nanohertz Observatory for Gravitational Waves (NANOGrav) and the Canadian Hydrogen Intensity Mapping Experiment (CHIME)/Pulsar, the mass of the PSR J0740+6620 pulsar has been updated to the range \(M = 2.08 \pm 0.07M_\odot\), which is the highest reliably determined mass for a pulsar so far [30]. Also, by combining datasets from the X-ray telescopes NICER and XMM-Newton, in Ref. [31] the radius of PSR J0740+6620 has been constrained to be \(R = 12.39^{+1.30}_{-0.98}\) km.

Another important value for constraining equations of state is the radius of a 1.4\(M_\odot\) neutron star. The analysis made in Ref. [32] found \(R_{1.4} = 12.18^{+0.56}_{-0.76}\) km, adopting a model based on the speed of sound in a neutron star. These values were included in Figs. 2 and 3 as reference. However, the confrontation between those numbers and the strange star solutions presented here may not be immediate. For example, some works have discussed the idea that neutron and strange stars could coexist as two separate families, obviously resulting in distinct mass-radius relations [9]. Besides that, NICER data has also been used to study configurations at the threshold which marks the ultracompact regime (defined by \(C > 1/3\)) [33]. Although such stars are not currently an observational reality, they may become important in the near future.

As illustrated in Figs. 2 and 3, the main feature of introducing the semiclassical correction is the possibility of finding ultracompact configurations throughout all parametrizations without imposing significant deviations in the low mass-radius region. Besides that, all semiclassical solutions
present an upper bound to the mass of stable configurations. In Figs. 2 and 3, a gray line corresponding to $C = 1/3$ was added to demarcate the ultracompact region. Specifically, Fig. 2 confronts the mass-radius curve obtained using general relativity and semiclassical gravity for some parametrizations of the CFL equation of state, while the $\omega$ parameter is fixed in 0.05 (in $G = c = 1$ units where the parameter is dimensionless). The semiclassical configurations are more massive and smaller, crossing to the ultracompact region ($C = 1/3$)

Fig. 2 Comparison of the mass-radius relationship obtained in general relativity and semiclassical gravity for some parametrizations of the CFL equation of state. The rectangular region corresponds to mass and radius constraints for the pulsar PSR J0740+6620 [30, 31]. The green horizontal line denotes radius constraints for a $1.4 M_\odot$ neutron star [32]. The gray line demarcates the threshold for the ultracompact region ($C = 1/3$)

On the other hand, in Fig. 3 the $\omega$ parameter is varied while the parametrization is fixed. The goal is to illustrate that the maximum mass can not be increased indefinitely within the model. Due to the negative pressure associated with the component carrying the semiclassical corrections, after some value of $\omega$ that depends on the inputs for the central energy densities, the solutions become less massive. Figure 3 makes clear the possibility of producing solutions able to reach the ultracompact region which are also compatible with up-to-date observational data [30–32].

Also, the physically acceptable solutions found are restricted to the subset of Eq. (27) where $\omega \ll 1$. In another context, this equation of state has also been used to describe the total effect of a mixture of cold dark matter and dark energy [34].

Figures 4, 5, 6, 7, 8, and 9 compare some internal functions for the maximum mass configurations when using general relativity and semiclassical gravity with $\omega = 0.05$.

Fig. 3 Impact of the $\omega$ parameter on the mass-radius curve for the CFL3 parametrization. Other elements displayed are the same as those exhibited in Fig. 2

Fig. 4 Energy density profiles for different CFL parametrizations using general relativity

Fig. 5 Energy density profiles for different CFL parametrizations using semiclassical gravity ($\omega = 0.05$)
A subset of the parametrizations presented in Table 1 was chosen aiming for a better visualization.

The semiclassical solutions for the energy density and pressure (Figs. 5 and 7) preserve the characteristic monotonically decreasing behavior found in general relativity (Figs. 4 and 6), although the maximum mass configurations occur at higher central energy densities and pressures. Besides that, the energy density does not vanish at the surface, which is a well-known property of self-bound configurations. The pattern for the mass function inside the star, as illustrated in Figs. 8 and 9, is also preserved within the model. In both situations the mass continuously increases with increasing radius, but the \( y \)-CFL stars are about 1 km smaller and 0.3 \( M_\odot \) heavier.

In order to check that the causality constraint is not violated, in Fig. 10 the sound speed squared is plotted as a function of the pressure, for different values of the \( \omega \) parameter, using the CFL5 parametrization (which is the one with the highest sound speed [16]).

The sound speed squared associated with Eq. (27) is simply \( -\omega \) (strictly valid only for constant \( \omega \) [35]), therefore the sound speed of CFL matter inevitably constraints the \( \omega \) parameter, providing an upper limit, namely

\[
\omega < \frac{1}{3} + \frac{2\alpha}{3} \left( \frac{1}{\bar{\nu}^2 + \alpha} \right).
\]

The reason behind this condition is that the effective sound speed squared must be non-negative; otherwise, the composition of the compact star would be microscopically unstable [36].

Table 2 summarizes the results for the nineteen parametrizations of the CFL equation. The data corresponds to the maximum mass configurations. In order to interpret the

![Fig. 6](image6.png)  
**Fig. 6** Pressure profiles for different CFL parametrizations using general relativity

![Fig. 7](image7.png)  
**Fig. 7** Pressure profiles for different CFL parametrizations using semiclassical gravity (\( \omega = 0.05 \))

![Fig. 8](image8.png)  
**Fig. 8** Mass function profiles for different CFL parametrizations using general relativity

![Fig. 9](image9.png)  
**Fig. 9** Mass function profiles for different CFL parametrizations using semiclassical gravity (\( \omega = 0.05 \))
implications of such solutions is interesting to observe that semiclassical gravity and general relativity can produce, for different parametrizations, similar results for mass and radius (compare for example the CFL9 parametrization in general relativity and the CFL1 parametrization in semiclassical gravity). Probably a mass measurement would not be able to distinguish among such objects, even though they would be physically different. The ultracompact regime exhibits a variety of interesting visual effects when compared to regular compact stars [37, 38]. A critical distinctive feature is the presence of a photon sphere, in other words, the unstable circular null geodesic of the external Schwarzschild spacetime metric [36].

Ultracompact stars may also play an important role if future gravitational wave data confirm the phenomenon known as gravitational echoes, which are secondary pulses of gravitational radiation after the main burst of radiation related to the post-merger ringdown waveform. It has been argued that gravitational echoes are not, as commonly assumed, a unique prerogative of deviations from general relativity at the horizon scale [39, 40]. Similar signals may arise from ultracompact stars and could be a powerful physical property to distinguish different compact objects, possibly revealing a new branch of stable configurations. First, echoes imply that the remnant is more compact than a neutron star with an ordinary equation of state [40]. In addition, although the post-merger ringdown waveform of an ultracompact star is initially identical to that of a black hole, the echoes in the late-time ringdown would present significant differences [41]. Therefore at least in principle ultracompact stars could be distinguished from both ordinary neutron stars and black holes.

In Ref. [9] it was discussed the possible tension between the evidence of the existence of compact stars satisfying \( R \lesssim 11.6 \text{ km} \) at \( 1.4M_\odot \) (suggested by some analyses on thermonuclear bursts and X-ray binaries), and the possibility of very massive stars with \( M \sim 2.6M_\odot \). None of the parametrizations can accommodate, using general relativity, a family satisfying \( R_{1.4} \lesssim 11.6 \text{ km} \) and also stars with masses as high as \( 2.6M_\odot \). For the semiclassical solutions this is not necessarily the case. The CFL3 parametrization, for instance, predicts a radius of 11.40 km for a star with \( M = 1.4M_\odot \) (compatible with the sound speed model discussed in Ref. [32]) and a maximum mass of 2.725\( M_\odot \), being able to deal with both conditions at once.

### 6 Conclusion

The present work has explored the implications of incorporating semiclassical effects (represented by a \( \gamma \)-fluid with negative pressure) in a specific compact star model based on known physics, namely, CFL strange stars. The generalization studied here is in some sense in tune with Einstein’s doubts about the reality of his field equations in the face of quantum physics, particularly the right-hand side (a phenomenological representation of matter), while he believed that the left-hand side (obtained from first principles using
geometrical quantities) contained a deeper truth. To express this contrast he even used to say that the first was made of low grade wood and the second of fine marble.

Results have shown the possibility of horizonless ultracompact configurations, a class that has received much attention in the literature [7, 36, 40, 41]. These results were achieved without imposing drastic modifications on the low mass-radius regime. Within the semiclassical framework it is also possible to find families of stars satisfying $R_{1.4} \lesssim 11.6$ km and masses as high as $2.6M_\odot$, which is very difficult within regular neutron star or even strange star models [9].

Conceptually speaking, since any object that undergoes complete gravitational collapse passes through an ultracompact phase, any effect not taken into account in general relativity could lead to new configurations. Therefore the results presented here are also pleasing in the sense that the semiclassical analysis enabled an intermediate class of ultracompact stars between regular compact stars and black holes.

This also establishes an interesting link regarding physical properties, since a neutron star has neither a photon sphere or an event horizon and a black hole has both. Ultracompact stars could be an intermediate step presenting a photon sphere but no event horizon [37, 38].

Finally, events like the LIGO announcement of the discovery of an object inside the so-called mass gap, or even the tentative detection of echoes in the post-merger signal of the neutron star binary coalescence GW170817 (with $4.2\sigma$ significance level), are very stimulating, and new classes of compact stars may be soon a reality [8, 42, 43].

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Declarations

Conflict of Interest The authors declare no competing interests.

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