Measurement of the CKM angle \( \alpha (\phi_2) \) with \( B \to \rho \rho \) decays at Belle and B\( \bar{A} \)B\( \bar{A} \)\* 

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We overview recent measurements in \( B \to \rho \rho \) decays which are based on data samples collected at the PEP-II and KEKB asymmetric-energy \( e^+ e^- \) colliders with the B\( \bar{A} \)B\( \bar{A} \) and Belle detectors. Special emphasis is given to the determination of the \( C P \)-violating coefficients \( A \) and \( S \) from an analysis of \( B^0 \to \rho^+ \rho^- \) decays. The values of \( A \) and \( S \), branching fractions, and longitudinal polarization fractions of \( B \to \rho \rho \) decays are used to constrain the Cabibbo-Kobayashi-Maskawa phase \( \alpha (\phi_2) \) using an isospin analysis; the solution consistent with the standard model is \( 71^\circ < \alpha (\phi_2) < 113^\circ \) at 68\% C.L.

I. INTRODUCTION

\( C P \) violation in the Standard Model can be explained by the presence of an irreducible complex phase in the Cabibbo-Kobayashi-Maskawa \( (C K M) \) quark-mixing matrix. The unitarity of the CKM matrix leads to six triangles in the complex plane. One such triangle is given by the following relation among the matrix elements: \( V^*_{ub} V_{ud} + V^*_{cd} V_{cd} + V^*_{tb} V_{bd} = 0 \). The phase angle \( \phi_2 \), defined as \( \arg -(V_{ud} V^*_{tb})/|V_{ud} V^*_{ub}| \), can be determined by measuring a time-dependent \( C P \)-asymmetric decay distribution in \( b \to u \bar{d} \) decays such as \( B^0 \to \pi^+ \pi^- , \rho^+ \rho^- \), and \( \rho^0 \rho^0 \). The time-dependent decay rate for \( B \to \rho^+ \rho^- \) decays tagged with \( B^0(q = 1) \) and \( B^0(q = -1) \) mesons is given by

\[
\mathcal{P}_{\rho \rho}(\Delta t) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} \{1 + q[A_{\rho \rho} \cos(\Delta m \Delta t) + S_{\rho \rho} \sin(\Delta m \Delta t)]\},
\]

where \( \tau_{B^0} \) is the \( B^0 \) lifetime, \( \Delta m \) is the mass difference between the two \( B^0 \) mass eigenstates, \( \Delta t = t_{CP} - t_{tag} \), and \( A_{\rho \rho} \) (B\( \bar{A} \)B\( \bar{A} \)’s definition is \( C_{\rho \rho} = -A_{\rho \rho} \)) and \( S_{\rho \rho} \) are \( C P \) asymmetry coefficients to be obtained from a fit to the experimental data. If the decay amplitude is a pure \( C P \)-even state and is dominated by a tree diagram, \( S_{\rho \rho} = \sin(2\phi_2) \) and \( A_{\rho \rho} = 0 \). The presence of an amplitude with a different weak phase (such as from a "penguin" diagram) gives rise to direct \( C P \) violation and shifts \( S_{\rho \rho} \) from \( \sin(2\phi_2) \). However, the size of the loop amplitude is constrained by the branching fraction of \( B^0 \to \rho^0 \rho^0 \) [2], indicating that this effect is small.

The \( C P \)-violating parameters receive contributions from a longitudinally polarized state (\( C P \)-even) and two transversely polarized states (an admixture of \( C P \)-even and \( C P \)-odd states). Recent measurements of the polarization fraction by Belle and B\( \bar{A} \)B\( \bar{A} \) show that the longitudinal polarization fraction is approximately 100\% \( (f_L = 0.968 \pm 0.023 \) [5]). \( f_L \) can be extracted from a fit to helicity-angle distribution.

The angular decay rate \( d^2 \Gamma / (\Gamma d\cos\theta_+ d\cos\theta_-) \) is given by

\[
\frac{2}{3} \{ f_L \cos^2 \theta_+ \cos^2 \theta_+ + f_L (1 - f_L) \sin^2 \theta_+ \sin^2 \theta_- \},
\]

where, \( \theta_\pm \) is the angle between the direction of the \( \pi^0 \) from the \( \rho^\pm \) and the negative of the \( B^0 \) momentum in the \( \rho^\pm \) rest frame.

II. MEASUREMENTS

The common features of \( B \to \rho \rho \) analyses are: \( 1 \) relatively small signal yields; the branching fractions of \( B \to \rho \rho \) decays are in the order of \( 10^{-6} \to 10^{-5} \), \( 2 \) large width of \( \rho \) mesons results in the large background; the fraction of signal events in most analysis is less than 1\%, \( 3 \) there are several background sources: \( e^+ e^- \to q\bar{q} (q = u, d, s, c) \) continuum, \( b \to c \), and \( b \to u \) backgrounds, \( 4 \) significant amount of events with multiple reconstructed \( B \) candidates, \( 5 \) various variables are used in the likelihood functions to distinguish among signal and backgrounds.

Both Belle and B\( \bar{A} \)B\( \bar{A} \) analyses identify \( B \to \rho \rho \) decays using the beam-energy constrained mass \( M_{bc} = \sqrt{E_{beam}^2 - p_{T B}^2} \) (called as beam-energy-substituted mass, \( m_{ES}, \) in B\( \bar{A} \)B\( \bar{A} \)) and energy difference \( E = E_B - E_{beam} \), where \( E_{beam} \) is the beam energy, and \( E_B \) and \( p_{T B} \) are the energy and momentum of the reconstructed \( B \) candidate, all evaluated in the center-of-mass (CM) frame.

The dominant background originates from \( e^+ e^- \to q\bar{q} (q = u, d, s, c) \) continuum events. To separate \( q\bar{q} \) jet-like events from spherical-like \( B\bar{B} \) events, Belle uses event-shape variables, specifically, modified Fox-Wolfram moments combined into a Fisher discriminant [3], and \( \theta_B \), the polar angle in the CM frame between the \( B \) direction and the beam axis. The Fisher discriminant and \( \theta_B \) are used to form a ratio of signal and background likelihoods \( R \). In the \( B^\pm \to \rho^\pm B^0 \) analysis Belle also requires \( |\cos \theta_T| < 0.8 \), where \( \theta_T \) is the angle between the thrust axis of the candidate tracks and that of the remaining tracks in the event. In the B\( \bar{A} \)B\( \bar{A} \) analyses \( q\bar{q} \) background is suppressed by requiring \( |\cos \theta_T| < 0.8 \) and making use of a neural network discriminant \( N \) which is based on several event-shape variables. Bellow we describe \( B \to \rho \rho \) measurements in detail.

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The naming convention: angles \( \alpha \), \( \beta \), and \( \gamma \) used in B\( \bar{A} \)B\( \bar{A} \) are referred to as \( \phi_2 \), \( \phi_1 \), and \( \phi_3 \) in Belle.
B**0 → ρ⁺ρ⁻** are reconstructed by combining two oppositely charged pion tracks with two neutral pions. The ρ± mesons are selected combining π± with π⁰ candidates. The π⁰ candidates are reconstructed from γγ pairs. Main event reconstruction requirements are listed in Table I. A flavor of the π candidate is selected based on the π⁰ mass, i.e. minimizing \( \sum_{\pi^\pm,\pi^0} (m_{\gamma\gamma} - m_{\pi^0})^2 \). The fraction of signal decays which have at least one π ± track incorrectly identified but pass all selection criteria is 13.8% and 6.5% for **BABAR** and Belle, respectively. These are referred to as “self-cross-feed” (SCF) events. The following components are distinguished in the analyses: signal and ρτπ non-resonant decays, SCF events, continuum background (q̅q), charm B background (b → c), and charmless (b → u) background. The b → u background is dominated by B→(μπ, a₂π, a₁ ρ, ρ± ρ⁰) decays.

**BABAR** obtains the signal yield, \( f_L \), and CP-violating parameters \( A_{ρ^+\rho^-} \) and \( S_{ρ^+\rho^-} \) from an unbinned extended maximum-likelihood (ML) fit to \( m_{ES}, \Delta E, \Delta t, m_{π^±π^0}, \cos θ_{±,0} \), and \( N \) distribution of 33902 events [7]. The Belle analysis is organized in two main steps: (a) we first determine the yields of signal and background components from an unbinned extended ML fit to the three-dimensional \( m_{bc}, \Delta E, R \) distribution. (b) we perform a fit to the \( \Delta t \) distribution of 18004 events to determine the CP parameters \( A_{ρ^+\rho^-} \) and \( S_{ρ^+\rho^-} \). The fit results are presented in Figures 1 and 2 and are listed in Table 11. The fraction of longitudinal polarization and fraction of non-resonant events (6.3 ± 6.7%) were measured in our previous analysis [8]. The branching fraction, \( f_L \), and CP asymmetries measured by Belle are similar to those obtained by **BABAR**. The values \( A_{ρ^+\rho^-} \) and \( S_{ρ^+\rho^-} \) are also consistent with no CP violation (\( A = S = 0 \)).

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The main reconstruction features of the analysis are the same as those for the \( B^0 \rightarrow ρ^± ρ^0 \) decays. Event selection requirements are listed in Table II.
FIG. 4: Belle: the $\Delta t$ distribution and projections of the fit for high-purity tagged events: (a) $B^0$ tags, (b) $\bar{B}^0$ tags. The raw $CP$ asymmetry is shown in (c). The hatched region shows signal events.

FIG. 5: BABAR (a) $m_{ES}$, (b) $\Delta E$, (c) $\cos \theta$, and (d) $m_{\pi\pi\rho}$ distributions for a signal-enriched sample along with fit projections: the dashed lines show $q\bar{q}$ and $B\bar{B}$ backgrounds, the solid line is the total.

BABAR obtained the yields of $B^\pm \to \rho^\pm \rho^0$ and $B^\mp \to \rho^\mp f_0$ decays, polarization, and charge asymmetry $A_{\rho^\pm \rho^0} = (N_{B^+} - N_{B^-})/(N_{B^+} + N_{B^-})$ using an unbinned extended ML fit to $m_{ES}, \Delta E, \Delta t, m_{\pi\pi\rho}, m_{\pi\rho\rho}, \cos \theta_{\pm}, \cos \theta_0,$ and $N$ distribution of 74293 events [8]. The charmed $b \to u$ background is dominated by $\eta'\rho^\pm,$ $K^{*0}\rho^\pm,$ $a_1^0\pi^\pm,$ $a_1^\pm\pi^0,$ $a_1^\pm\rho^0,$ and $a_1^0\rho^\pm$ decays.

FIG. 6: Belle: $\Delta E$ (left) and $M_{bc}$ (right) distributions. The dashed curve is the sum of $B\bar{B}$ and continuum backgrounds, the dot-dashed curve is the signal, the hatched region shows $B\bar{B}$, and the solid curve is the total.

TABLE II: Reconstruction requirements used in $B^\pm \to \rho^\pm \rho^0$ analysis.

| Cut  | BABAR          | Belle          |
|------|----------------|----------------|
| $E_{c}$ (MeV) | 50             | 50 (barrel); 100 (endcap) |
| $M_{\gamma\gamma}$ (MeV/$c^2$) | 100 - 160, 118 - 150. |
| $M_{0}$ (MeV/$c^2$) | 0.396 - 1.146 | 0.65 - 0.89 |
| $M_{0}$ (MeV/$c^2$) | 0.520 - 1.146 | 0.65 - 0.89 |
| $\cos \theta_{\pm}$ | -0.9 - 0.95 | - |
| $\cos \theta_0$ | -0.95 - 0.95 | - |
| $\rho_{\gamma\gamma}^M$ (MeV/$c$) | > 0.5 |
| $M_{bc}$ (MeV/$c^2$) | 5.26 < 5.272 < |
| $\Delta E$ (GeV) | -0.15 - 0.15 | -0.4 - 0.4 |

FIG. 7: BABAR $m_{ES}$ and $\Delta E$ distributions and projections of the fit for a signal-enriched sample. The dashed curve is full background, the small solid curve is signal, the dotted curve is $B^+ \to \rho^0 f_0$, and the large solid is the total.

The signal yield in Belle analysis [9] is obtained from a fit to $\Delta E$ distribution. To measure the polarization, Belle bins in $\cos \theta_{\pm}$ and $\cos \theta_0$ and determine the signal yield for each bin from the fit to $\Delta E$ distribution. The polarization is obtained from a simultaneous fit to two background-subtracted helicity-angle distributions. The results of the fits are shown in Figures 6 and 7 and listed in Table III.

III. $B^0 \to \rho^0 \rho^0$

BABAR finds evidence for $B^0 \to \rho^0 \rho^0$ decays and measures its branching fraction and polarization using a sample of about 348 million $B\bar{B}$ pairs [2]. Events are selected from the region $5.24 \text{ GeV}/c^2 < M_{bc} < 5.29 \text{ GeV}/c^2$, $|\Delta E| < 85 \text{ MeV}$, and are required to satisfy $0.55 < M_{\pi\pi} < 1.05 \text{ GeV}/c^2$, and $|\cos \theta_0| < 0.98$. In events with multiple $B$ candidates one is selected based on the the best $\chi^2$ of a four-pion vertex fit. Additional suppression of the dominant continuum background is achieved using the flavor tagging information. The data sample is divided into seven tag-quality intervals, $ctag$. The $B^0 \to \rho^0 \rho^0$ event yield and polarization $f_\perp$ are obtained from an unbinned extended ML fit to $m_{ES}, \Delta E, m_{\pi\pi} - 1.2, \cos \theta_{1,2}, N,$ and $ctag$ distribution of 65180 events. The fit also allows to obtain the yields for $B^0 \to \rho^0 f_0(990)$ and $B^0 \to f_0(980) f_0(980)$ decays. The charmed background is dominated by $B^0 \to a_1^\pm \pi^\mp$ events which number is a free parameter in the fit. Other $b \to u$ background modes include: $B \to (\rho^0 K^{*0}, \rho^0 \rho^0, \rho \pi)$, and $B^0 \to \rho^+ \rho^-$. The fit results are shown in Fig. 6 and listed in Table III.
### TABLE III: Summary of $B \rightarrow \rho \rho$ measurements.

| Decay | Quantity | BRAH | - | Value | $N_{\text{sig}}$ | $\mathcal{L}$(fb$^{-1}$) | Value | $N_{\text{sig}}$ | $\mathcal{L}$(fb$^{-1}$) | Value |
|-------|----------|------|---|-------|---------------|----------------|------|---------------|---------------|------|
| $B \rightarrow \rho^\pm \rho^\mp$ | $f_L$ | $23.5 \pm 2.2 \pm 4.1$ | 615 $\pm$ 57 | 316 | $22.8 \pm 3.8^{+3.3}_{-2.6}$ | 194 $\pm$ 32 | 253 | $23.1^{+4.2}_{-3.3}$ | 0.9680 $\pm$ 0.23 | |
| $A_{\rho^+ \rho^-}$ | $0.07 \pm 0.15 \pm 0.06$ | 615 $\pm$ 57 | 316 | $0.16 \pm 0.21 \pm 0.07$ | 372 $\pm$ 43 | 492 | $0.11 \pm 0.13$ | |
| $S_{\rho^+ \rho^-}$ | $-0.19 \pm 0.21^{+0.05}_{-0.07}$ | 615 $\pm$ 57 | 316 | $0.19 \pm 0.30 \pm 0.07$ | 372 $\pm$ 43 | 492 | $-0.06 \pm 0.18$ | |

**IV. CONSTRAINT ON $\alpha(\phi_2)$**

We constrain $\phi_2$ using an isospin analysis\([11]\), which allows one to relate six observables to six underlying parameters: five decay amplitudes for $B \rightarrow \rho \rho$ and the angle $\phi_2$. The observables are the branching fractions for $B \rightarrow \rho^e \rho^\mp$, $\rho^+ \rho^0$, and $\rho^0 \rho^0$ (listed in Table III), the $CP$ parameters $A_{\rho^+ \rho^-}$ and $S_{\rho^+ \rho^-}$; and the parameter $A_{\rho^0 \rho^0}$ for $B \rightarrow \rho^0 \rho^0$ decays. The branching fractions must be multiplied by the corresponding longitudinal polarization fractions (taken from Table III)\([5]\). We neglect possible contributions from electromagnet penguins and $I=1$ amplitudes\([11]\) to $B^0 \rightarrow \rho^+ \rho^-$. We follow the statistical method of Ref.\([12]\) and construct a $\chi^2(\phi_2)$ using the measured values and obtain a minimum $\chi^2$ (denoted $\chi^2_{\text{min}}$); we then scan $\phi_2$ from 0° to 180°, calculating the difference $\Delta \chi^2 \equiv \chi^2(\phi_2) - \chi^2_{\text{min}}$. We insert $\Delta \chi^2$ into the cumulative distribution function for the $\chi^2$ distribution for one degree of freedom to obtain a confidence level (C.L.) for each $\phi_2$ value. The resulting function $1-$C.L. (Fig. 8) has more than one peak due to ambiguities that arise when solving for $\phi_2$. Because $A_{\rho^0 \rho^0}$ is not yet measured, we allow this observable to float; this produces the “flat-top” regions in Fig. 8. The solution consistent with the Standard Model is $71^\circ < \phi_2 < 113^\circ$ at 68% C.L. or $67^\circ < \phi_2 < 116^\circ$ at 90% C.L. Recently, a different model-dependent approach to extract $\phi_2$ using flavor $SU(3)$ symmetry has been proposed\([13]\). This method would give more stringent constraints on $\phi_2$.

In summary, we present recent measurements in $B \rightarrow \rho \rho$ decays. These measurements are used to constrain the angle $\phi_2$ using an isospin analysis.

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