Dissipation by ideal quantum gas

Janos Polonyi

Strasbourg University, CNRS-IPHC, 23 rue du Loess, BP28 67037 Strasbourg Cedex 2, France

Abstract

The effective Lagrangian of a test particle, interacting with an ideal gas, is calculated with in the Closed Time Path formalism in the one-loop and the leading order of the particle trajectory. The expansion in the time derivative is available for slow enough motion and it uncovers diffusive effective forces and decoherence for the coordinate and the momentum. A pure Newtonian friction force and an anisotropic coordinate decoherence are found for zero temperature ideal gas of fermions.

Keywords: friction, decoherence, closed time path formalism

1. Introduction

The time reversal invariance of effective forces is a nontrivial issue: Even if the dynamics of a closed system is time reversal invariant, the effective interaction between a subsystem and the rest, its environment, breaks time reversal invariance due to the environment boundary conditions in time. A further layer of complications is found in infinite systems where the effective forces may be dissipative. Quantum systems raise an additional question about the decoherence, another irreversible process, generated by the effective interactions. These issues had already attracted much attention and the emergence of irreversibility has been demonstrated in a physically appealing way, by coarse graining [1]. In a similar manner, decoherence can be generated by the large, highly degenerate environment [2, 3]. Such a general picture naturally leaves open the details about the actual loss of information, realized by the use of a restricted set
of observables, in a given microscopic model.

The simplest context in which these issues may arise is the Brownian motion, the effective dynamics of a particle, interacting with a gas. This problem has already thoroughly been studied: The dissipative forces, experienced by a tracer particle which interacts with a classical ideal gas have been established by the linearized Boltzmann equation [4] and similar results have been found in the quantum case [5], too. The dephasing of the test particle state due to the continuous scattering off a gas has been described [2], followed by the inclusion of relaxation [4, 7, 8]. A simpler but more formal way to establish dissipative forces is by the construction of an exactly solvable model [9]. Some elements to render this model more realistic may come from comparing it with the Brownian limit of the quantum linearized Boltzmann equation where the gas particles are light [10].

The effective dynamics of a particle, interacting with an ideal quantum gas, is sought within the Closed Time Path (CTP) formalism in this work. This formalism has been introduced in quantum field theory [11] and has since that time successfully applied in different area of Condensed Matter Physics [12, 13] and Particle Physics [14]. The fact that the interaction with an ideal gas may give rise to dissipative forces is not surprising since the leading order, one-loop contribution to the transport coefficients which is equivalent with Kubo’s formulas [15] can be interpreted as the contribution of an ideal gas. To relate such a derivation of the effective dynamics to well known models the effective Lagrangian is presented in the case of the harmonic model [9], as well, which has already been thoroughly studied in the CTP formalism [16].

The collision-based and the CTP effective action descriptions have different starting points but share some assumptions. The strategy of the Boltzmann equation assumes a dilute gas to truncate the hierarchy equations, requires short range interactions to rely on scattering processes and needs high enough temperature to render the effective dynamics Markovian. Finally, the Brownian limit is carried out by letting the ratio of the gas particles and test particle mass go to zero to arrive at a simpler equation of motion for the reduced density matrix.
These assumptions are mirrored and partially softened in the effective action approach where the effective action is usually constructed within the framework of the Landau-Ginzburg double expansion. In fact, the expansion in the amplitude is reminiscent of the assumption of weak interactions in a dilute gas. The expansion in the gradient, the assumption that the effective interactions are local, corresponds to the use of scattering processes in the Boltzmann equation strategy. The calculation of the higher order contributions is not exceedingly difficult in the effective action, leaving open the way to a systematic inclusion of multi-particle correlations and the non-Markovian effects. Furthermore the light gas particle limit is not necessary for the calculation of the effective action.

The effective action of a particle which interacts with an ideal gas has been calculated since some time ago by using the traditional effective action approach in imaginary time \cite{17} and by means of the CTP formalism \cite{18}. Though our procedure is similar to the one, followed in these works, the final form of the effective action is different. This is because one has to go beyond the traditional action formalism to recover dissipative forces and the separation of the conservative and the dissipative forces was properly done. Another novel elements of this work is that the decoherence of both the coordinate and the momentum has been monitored and the comparison with the harmonic model of ref. \cite{9} is made.

The effective Lagrangian is calculated below for a small amplitude motion, in leading order of the particle trajectory. The order $\mathcal{O} \left( \partial^2_t \right)$ real part of the Lagrangian displays Newton’s friction force and a mass renormalization and the imaginary part describes the decoherence. The result simplifies considerably when an ideal gas of fermions is considered at vanishing temperature: Both the friction force and the strengths of decoherence stem from the same source and are proportional to the particle velocity. Furthermore the decoherence shows a characteristic anisotropy, namely it is six time stronger in the direction of the velocity than in the perpendicular plane.
2. Toy model

To introduce the method of calculating the effective Lagrangian we consider first the simple model of linearly coupled harmonic oscillators [9], given by the action

\[ S = S_s + S_e, \]

\[ S_s = \int_t \left( \frac{m}{2} \dot{x}^2 - \frac{m_0^2}{2} x^2 \right), \]

\[ S_e = \sum_n \int_t \left( \frac{m}{2} \dot{y}_n^2 - \frac{m_0^2}{2} y_n^2 - g_n x y_n \right), \]

(1)

where \( \int_t = \int_{t_i}^{t_f} dt \). It is more advantageous to separate the system and environment in a way that the system potential reflects clearer the effective dynamics for small \( y_n \) and \( \dot{y}_n \).

\[ S_s = \int_t \left[ \frac{m}{2} \dot{x}^2 - \left( \frac{m_0^2}{2} - \sum_n \frac{g_n^2}{2m_0^2} \right) x^2 \right], \]

\[ S_e = \sum_n \int_t \left\{ \left[ \frac{m}{2} \dot{y}_n^2 - \frac{m_0^2}{2} \left( y_n + \frac{g_n x}{m_0^2} \right)^2 \right] \right\}, \]

(2)

We are interested in the effective dynamics of the system, represented by \( x \), generated by its environment, parametrized by \( y_n \). The effective action for the particle can be constructed by the help of the generator functional for the connected Green function for the system coordinate, \( W[\hat{j}] \), a functional of two sources, \( \hat{j} = (j^+, j^-) \), defined by the equation

\[ e^{i\bar{\hbar} W[\hat{j}]} = \Tr \left[ T[e^{-\frac{\hbar}{2} \int dt [H(t) - j^+(t)x(t)]}] \rho_i T^*[e^{\frac{\hbar}{2} \int dt [H(t) + j^+(t)x(t)]}] \right] \]

(3)

in the Heisenberg representation. Here \( H(t) \) denotes the Hamiltonian and \( \rho_i \) stands for the initial density matrix, assumed to be \( \rho_i = |0\rangle \langle 0| \). The path integral representation of \( W[\hat{j}] \) is written as

\[ e^{i\bar{\hbar} W[\hat{j}]} = \int D[\hat{x}] D[\hat{y}] e^{\frac{\bar{\hbar}}{2} S[x^+, y^+] - \frac{\hbar}{2} S[x^-, y^-] + \bar{\hbar} \int_t \hat{j}(t) \hat{x}(t)}, \]

(4)

where the integration is over the CTP doublets \( \hat{x} = (x^+, x^-), \) etc., which satisfy the boundary condition, \( x^+(t_f) = x^-(t_f) \), etc., representing the trace in the generator functional [3].
The CTP action can be written in a condensed notation where the trajectory is treated as a vector and the scalar product denotes the time integration as

\[
S[\hat{x}, \hat{y}] = \frac{1}{2} \hat{x} \hat{D}^{-1} \hat{x} + \frac{1}{2} \hat{y} \hat{G}^{-1} \hat{y} - \hat{x} \sigma g \hat{y},
\]

where \( \hat{D} \) and \( \hat{G} \) denote the system and environment Green functions, respectively. The Green functions possess a characteristic CTP block structure,

\[
\hat{D} = \begin{pmatrix} D^n + i D^i & -D^f + i D^i \\ D^f + i D^i & D^n + i D^i \end{pmatrix},
\]

where \( D^n \) and \( D^f \) can be called the near and the far field Green function by analogy with electrodynamics. The CTP matrix

\[
\hat{\sigma} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]

denotes the metric tensor of the symplectic structure of the CTP formalism.

The Green function of the \( n \)-the environment coordinate, \( \hat{G}_n(t, t') = \hat{G}(t - t', \omega_n)/m \), is given by the Fourier integral

\[
\hat{G}(t, \Omega) = \int \omega e^{-i\omega t} \hat{G}_\omega(\Omega),
\]

where \( \int_\omega = \int d\omega/2\pi \) and

\[
\hat{G}_\omega(\Omega) = \begin{pmatrix} \frac{1}{\omega^2 - \Omega^2 + i\epsilon} & 2\pi i \delta(\omega^2 - \Omega^2) \Theta(-\omega) \\ 2\pi i \delta(\omega^2 - \Omega^2) \Theta(\omega) & -\frac{1}{\omega^2 - \Omega^2 + i\epsilon} \end{pmatrix}.
\]

The simplest parametrization of the model is given in terms of the spectral function,

\[
\rho(\Omega) = \sum_n \frac{g_n^2}{2m\omega_n} \delta(\omega_n - \Omega),
\]

rather than the coupling constants and frequencies. One finds

\[
\hat{G}(\omega) = \frac{1}{m} \int_0^\infty d\Omega 2\Omega \rho(\Omega) \hat{G}(\omega, \Omega),
\]

in particular

\[
G^n_\omega = \frac{2}{m} P \int_0^\infty d\Omega \frac{\Omega \rho(\Omega)}{\omega^2 - \Omega^2},
\]
where $P$ denotes the principal value prescription, $G_\omega^f = -i\pi \text{sign}(\omega)\rho(|\omega|)$ and 
$G_\omega^i = -\pi \rho(|\omega|)$.

Once the generator functional, $W[\hat{j}]$, is known the effective Wilsonian action, $S_{eff}[\hat{x}]$, is defined by

$$e^{i\hat{x}W[\hat{j}]} = \int D[\hat{x}] e^{iS_{eff}[\hat{x}]+i\hat{j}\hat{x}}. \quad (13)$$

The comparison of this expression with eq. (4) yields $S_{eff} = S_s + S_{infl}$, where the second term is the influence functional $S_{infl}$, defined by

$$e^{iS_{infl}[\hat{x}]} = \int D[\hat{y}] e^{iS_s[\hat{x},\hat{y}]-i\hat{x}\hat{y}}. \quad (14)$$

The elimination of a variable by its classical equation of motion is exact for a Gaussian integral and the effective action

$$S_{eff}[\hat{x}] = \frac{1}{2} \hat{x}(\hat{D}_0^{-1} - \sigma \hat{\Sigma})\hat{x} \quad (15)$$

with the self energy $\hat{\Sigma} = g\hat{G}g$ follows, where the influence functional, given by the self energy piece of the effective action, is

$$S_{infl}[\hat{x}] = -\frac{1}{2} \sum_{\sigma\sigma'} \sigma\sigma' \int_{t\omega} e^{-i\omega(t'-t)} \Sigma_{\sigma\sigma'} x^\sigma(t') x^{\sigma'}(t). \quad (16)$$

We assume that the self energy is a real, analytic function of $i\omega$ and write the influence functional after a partial integration as

$$S_{infl}[\hat{x}] = -\frac{1}{2} \sum_{\sigma\sigma'} \sigma\sigma' \sum_{\ell=0}^\infty \frac{(-1)^\ell}{\ell!} \partial_{t\omega}^{\ell} \Sigma_{\sigma\sigma'} x^\sigma(t') x^{\sigma'}(t). \quad (17)$$

The CTP block structure, (6), results the influence Lagrangian,

$$L_{infl} = -\frac{1}{2}(x^\dagger \Sigma x + x\Sigma^\dagger x + x^\dagger \Sigma^\dagger x - x\Sigma x) + x^\dagger i\Sigma^\dagger x) \quad (18)$$

where the Keldysh parametrization $x^\pm = x \pm x^d/2$, is used and the arrow on

$$\Sigma_{\sigma\sigma'} = \sum_{\ell=0}^\infty \frac{(-1)^\ell}{\ell!} \partial_{t\omega}^{\ell} \Sigma_{\sigma\sigma'} \partial_{t\ell} \quad (19)$$

is a reminder that this a differential operator and it acts to right.
The expectation value, \( \langle x \rangle \), satisfies the classical equation of motion in the case of a harmonic system, therefore the variation equation for \( x^d \) at \( x^d = 0 \),

\[
m\langle \ddot{x} \rangle = -(m\omega_0^2 + \vec{\Sigma})\langle x \rangle, \tag{20}
\]

becomes the equation of motion for the coordinate expectation value. Up to order \( \mathcal{O} (\partial^2 t) \) it shifts the harmonic system potential, \( \omega_0^2 \rightarrow \omega_0^2 - \Delta \omega^2 \), with

\[
\Delta \omega^2 = \frac{2}{m^2} \int_0^\infty d\Omega \frac{\rho(\Omega)}{\Omega} \tag{21}
\]
to recover the system potential in (2), renormalizes the mass, \( m \rightarrow m + \delta m \), where

\[
\delta m = \frac{4}{m} \int_0^\infty d\Omega \frac{\rho(\Omega)}{\Omega^2}. \tag{22}
\]

Furthermore, a Newtonian friction force, \( F = -k\dot{x} \), with \( k = \pi \rho'(0) \) is generated. The imaginary part of the influence Lagrangian (37) yields a suppression factor, \( e^{-d_0 x^2} \) with \( d_0 = \pi \rho(0) \), in the system path integral (13) and generates decoherence in the coordinate diagonal basis.

3. Particle in a gas

Consider now the more realistic system of a point particle of mass \( M \), moving in a potential \( V(x) \) and interacting with a homogeneous ideal gas in equilibrium via a time-independent potential, \( U(x) \). The action, \( S = S_p + S_g + S_i \), is the sum of the terms

\[
S_p[x] = \int dt \left[ \frac{M}{2} \ddot{x}(t) - V(x(t)) \right],
\]

\[
S_g[\psi^\dagger, \psi] = \int dt d^3y \psi^\dagger(t, y) \left[ i\hbar \partial_t + \frac{\hbar^2}{2m} \Delta + \mu \right] \psi(t, y),
\]

\[
S_i[x, \psi^\dagger, \psi] = \int dt d^3y U(y - x(t)) \psi^\dagger(t, y) \psi(t, y). \tag{23}
\]

We shall use the condensed notation where space-time functions are represented as vectors and the scalar product is the space-time integration,

\[
\psi\phi = \int dt d^3y \psi(t, y)\phi(t, y), \tag{24}
\]
and write $S_t = \psi^\dagger \Gamma[x] \psi$.

The generator functional, (3), now contains the source $\hat{j} = (\hat{j}^+, \hat{j}^-)$,

$$e^{\hat{W}[\hat{j}]} = \int D[\hat{x}] D[\hat{\psi}] e^{\hat{S}_0[\hat{x}]+\hat{\psi}^\dagger (\hat{F}^{-1} + \hat{\Gamma}[\hat{x}]) \hat{\psi} + \frac{i}{\hbar} \int dt \hat{j}(t) \hat{x}(t)},$$

(25)

where the inverse of the quadratic form of the ideal gas action is

$$\hat{F}(t, y, t', y') = \int_{\omega} e^{-i\omega(t-t') + iq(y-y')} \hat{F}_{\omega q},$$

(26)

with $\int_q = \int d^3k/(2\pi)^3$, and

$$\hat{F}_{\omega q} = \begin{pmatrix} \frac{1}{\omega - \omega_q + i\epsilon} & 0 \\ -i2\pi\delta(\omega - \omega_q) & \frac{1}{\omega_q - \omega + i\epsilon} \end{pmatrix} - \xi n_q i2\pi\delta(\omega - \omega_q) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$  

(27)

Here $\xi$ denotes the exchange statics factor, $\xi = +1$ for bosons and $\xi = -1$ for fermions, $\epsilon_q = \hbar q^2/2m$ and $n_q$ denotes the occupation number. The temperature is supposed to be sufficiently high in the case of a bosonic ideal gas to suppress the condensate. The particle-gas interaction is described by the term $\hat{\psi}^\dagger \hat{\Gamma} \hat{\psi}$ in the action, where $\Gamma^{\sigma\sigma'}[\hat{x}] = \sigma\delta^{\sigma\sigma'}\Gamma[x^{\sigma}]$.

The influence functional is defined by the equation

$$e^{\hat{W}_{infl}[\hat{x}]} = \int D[\hat{\psi}] D[\hat{\psi}^\dagger] e^{\hat{\psi}^\dagger (\hat{F}^{-1} + \hat{\Gamma}[\hat{x}]) \hat{\psi}}.$$ 

(28)

in this case which gives

$$S_{infl}[\hat{x}] = -\frac{1}{2} \hat{j} \hat{\sigma} \hat{G} \hat{\sigma} \hat{j},$$

(29)

by ignoring the contributions, beyond $O(\Gamma^2)$ because we are looking for the leading order part of the effective action in the particle trajectory. The $O(\Gamma)$ tadpoles are canceled by introducing a fictitious, classical, homogeneous, neutralizing charge density. Here $j^\sigma(t, y) = U(y - x^\sigma(t))$ stands for the perturbation of the ideal gas, generated by the test particle and

$$G^{\sigma_1\sigma_2}(x_1, x_2) = i\xi \hbar n_x \hat{F}^{\sigma_1\sigma_2}(x_1, x_2) \hat{F}^{\sigma_2\sigma_1}(x_2, x_1)$$

(30)

denotes the density two-point function, given in terms of the Green functions.
\[ G_{\omega q} = G_{\omega q}^+ + G_{\omega q}^- \]

\[ G_{\omega q}^+ = \frac{-\xi n_s}{\hbar} \int \frac{n_q}{\omega - \omega q + \omega_q}, \]

\[ G_{\omega q}^- = -i\xi \frac{n_s}{\hbar^2} \int n_q(n_q + \xi) \delta(\omega - \omega q + \omega_q), \] (31)

where the spin is taken into account by the degeneracy factor, \( n_s = 2s + 1 \).

Since \( iG_{\omega q}^- = 2G_{\omega q}^- \) is the spectral function for the excited states of the gas \( iG_{\omega q}^- \geq 0 \) and \( G_{\omega q}^- = 0 \) for \( \omega < 0 \). The integrals (31) are analytic functions of the dimensionless variables \( x = \omega/|q|v_F \), \( y = |q|/k_F \) at finite temperature, here \( v_F = \frac{\hbar k_F}{m} \), \( k_F = \frac{3}{\sqrt{6\pi^2}}n/n_s \), \( n \) being the density of the gas.

The influence functional is non-local in time and renders the effective equation of motion an integro-differential equation. To reduce the complexity of the problem we restrict our attention to a slowly moving particle where the kernel can be expanded in the time derivative and assume \( |x| \ll 1 \). When the influence functional (29) is evaluated in the Fourier space then the dominant contribution comes from the region of \( \omega \) which is around the characteristic frequency of the particle motion, \( \omega \sim |q\dot{x}| \), indicating that the particle should be much slower than the characteristic velocity of the gas, \( |\dot{x}| \ll v_F \).

### 4. Quadratic effective Lagrangian

The strategy, followed in the case of the toy model, gives now the influence functional

\[
S_{infl} = -\frac{1}{2} \sum_{\sigma\sigma'} \sigma\sigma' \int dt dt' U_q^2 e^{-i\omega(t'-t)+iq(x'(t')-x'(t))} G_{\omega q}^{\sigma,\sigma'}. \] (32)

The change of variables, \( t' \to t + u \), and the expansion in \( u \) yields the influence Lagrangian,

\[
L_{infl} = -\frac{1}{2} \sum_{\sigma\sigma'} \sigma\sigma' \int \omega_q du U_q^2 e^{-iu} e^{iq(x'-x')} \sum_{n=1}^{\infty} \frac{(-i)^n}{n!} x^{(n)} \delta^{(n)} G_{\omega q}^{\sigma,\sigma'}. \] (33)

where the notation \( x = x(t) \), \( x^{(n)} = dx^n/dt^n \) is applied. We assume that the potential \( V(x) \) is sufficiently deep to confine the particle close to its initial
position, $x = 0$ and retain the leading order, $\mathcal{O}(x^2)$ part only,
\begin{equation}
L_{\text{infl}} = \frac{1}{4} \sum_{\sigma \sigma'} \sigma \sigma' \int_q U_q^2 [q(x^\sigma - x'^\sigma + \Delta x^\sigma)]^2 G_{0q}^\sigma \sigma'.
\end{equation}

The derivatives in
\begin{equation}
\Delta x = \sum_{n=1}^{\infty} \frac{x^{(n)}}{n!} \partial_t^n
\end{equation}
act on the Green function, evaluated at vanishing frequency. The block structure allows us to write
\begin{equation}
L_{\text{infl}} = \frac{1}{24 \pi^2} \int_0^\infty dq q^2 U_q^2 [2 \Delta x \Delta x^d G^n_{0q} + (2 \Delta x \Delta x^d + 4 x^d \Delta x) G^f_{0q}]
\end{equation}
when the Keldysh parametrization is used. A more compact form, containing the terms up to $\mathcal{O}(\partial_t^3)$, is
\begin{equation}
L_{\text{infl}} = -k x^d \dot{x} + \delta M \dot{x}^d \dot{x} + id_0 x^d d^2 - id_2 \dot{x}^d d^2 + \mathcal{O}(\partial_t^3) + \mathcal{O}(x^3),
\end{equation}
with
\begin{align*}
k &= -\frac{1}{6 \pi^2 v_F} \int_0^\infty dq q^2 U_q^2 \partial_x G^f(x,y)|_{x=0}, \\
\delta M &= \frac{1}{12 \pi^2 v_F} \int_0^\infty dq q^2 U_q^2 \partial_x G^n(x,y)|_{x=0}, \\
d_0 &= -\frac{1}{12 \pi^2} \int_0^\infty dq q^4 U_q^2 G^f(x,y)|_{x=0}, \\
d_2 &= \frac{1}{24 \pi^2 v_F} \int_0^\infty dq q^2 U_q^2 \partial_x^2 G^f(x,y)|_{x=0}.
\end{align*}
The time reversal invariant part of the influence Lagrangian originates from $G^n$ and $G^f$, the dynamical breakdown of time reversal symmetry, irreversibility, is introduced by $G^f$.

The Euler-Lagrange equation for $x^d$ at $\langle x^d \rangle = 0,$
\begin{equation}
M_R(\dot{x}) = -\langle \nabla V(x) \rangle - k \langle x \rangle + \mathcal{O}(\partial_t^3) + \mathcal{O}(\langle x^3 \rangle),
\end{equation}
given by $\Re L_{\text{infl}},$ includes a mass renormalization, $M_R = M + \delta M,$ and a Newtonian friction force. In the calculation of ref. [18] $G^n$ was used in the place of
$G^n$ and no mass renormalization is found. The dissipative term of the equation of motion \[30\] has been found by the use of the linearized quantum Boltzmann equation, as well, the friction constant being expressed in terms of the $T$ matrix \[8\]. The imaginary part of $L_{\text{infl}}$ generates decoherence, a suppression factor $\exp -\Im S_{\text{infl}}$ in the system path integral. The terms involving $x^d_2$ and $\dot{x}^d_2$ control the decoherence in the coordinate and the momentum basis, respectively.

5. Electron gas at vanishing temperature

As the temperature approaches zero in a fermionic ideal gas the Green function, $\hat{G}$, simplifies and develops singularities. The loop-integrals in eqs. \[31\] can easily be calculated for $n_q = \Theta(k_F - |q|)$ \[20\] and $G^\pm$, the Lindhard function, is found to be analytical for $|x| + y/2 < 1$. Within this domain we have $G^- = -k_F m \Theta(x)/2\pi\hbar^2$, yielding $G^f = -ik_Fmx/2\pi\hbar^2$ and $G^i = -k_F m |x|/2\pi\hbar^2$. The influence Lagrangian, using the same approximation as before, is now of the form

$$\Re L_{\text{infl}} = -k x^d \dot{x} + \delta M x^d \dot{x} + O (\dot{x}^4) + O (x^4), \quad (40)$$

with the friction constant

$$k = \frac{m^2}{12\pi^3\hbar^3} \int_0^{2k_F} dqq^2 U_q^2, \quad (41)$$

which is $O (k_F)$ as expected according to ref. \[8\], and $\delta M$, given by \[8\]. Observe that there are no higher order irreversible terms owing to $D^f(x, y) \sim x$.

In calculating $\Im L_{\text{infl}}$ the expansion of the exponent on the right hand side of eq. \[33\] must be carried out around $iu(qx - \omega)$, yielding

$$\Im L_{\text{infl}} = \frac{1}{4} \sum_{\sigma\sigma'} \sigma \sigma' \int_q U_q^2 (q(x^\sigma - x'^\sigma))^2 \Im G_{q xx^\sigma, x'^\sigma}. \quad (42)$$

Note that $\Delta x$, which now starts at the order $O (\dot{x}^2)$, can be ignored due to $G^i \sim |x|$. The block structure, \[6\], gives

$$\Im L_{\text{infl}} = -\frac{1}{4} \int_q U_q^2 (qx^d)^2 \left[ G_{q(\dot{x} + \dot{x})}(x + \dot{x}) + G_{q(\dot{x} - \dot{x})}(x - \dot{x}) \right]. \quad (43)$$
which can be replaced by

$$\Im L_{\text{in}} = \frac{k_F m}{4\pi\hbar^2}\int_q U_q^2 G^{i}_{q\hat{x},q}(q\hat{x})^2$$  \hspace{1cm} (44)

when the contributions beyond $\mathcal{O}(x^2)$ are neglected. The integration over the solid angle is straightforward and leads to a particular anisotropy,

$$\Im L_{\text{in}} = i\lambda|\dot{x}|(x_d^{d^2} + 6x_d^d)$$  \hspace{1cm} (45)

where $x^d$ was separated into longitudinal and transverse components, $x_d^{d^2}$, $x_d^d$, respectively, defined by $x^d = x_d^{d^2} + x_d^d$, $x_d^{d^2}\dot{x} = 0$ and

$$\lambda = \frac{m^2}{64\pi^3\hbar^3}\int_0^{2k_F} dq q^4 U_q^2,$$  \hspace{1cm} (46)

which is $\mathcal{O}(k_F)$, too.

It is easy to understand the origin of the factor $|\dot{x}|$ in (45). The dissipative processes and the decoherence are generated by an energy exchange with the environment and this is always possible if the gas is in a thermal equilibrium. For vanishing temperature the gas is in its ground state and the particle must possess some kinetic energy to interact with the gas. This condition is automatically satisfied by the friction force, being proportional to the velocity. But the decoherence would start in zeroth order if the particle is coupled to the gas by its density. The role of the factor $|\dot{x}|$ is to suppress the system-environment correlations for a particle at rest. The initial state of the toy model is not at the energy minimum which explains that the decoherence is present even for a static system.

The absence of further, higher order derivative dissipative forces and of the decoherence for a particle at rest remains valid when the higher powers of the coordinate are retained in the effective Lagrangian as long as the gas is eliminated in the one-loop approximation, i.e. eq. (29) applies. Actually both the friction force and the decoherence stem from the same source for linear system-environment coupling. In fact, friction and decoherence arise from the blocks $G^f$ and $iG^i$ of the Green function (6), respectively, and these blocks are identical up to a sign owing to $G_{\omega q} = 0$ for $\omega < 0$.  

12
6. Conclusion

A simple scheme was presented to calculate the effective Lagrangian of a particle which performs a slow, small amplitude motion and interacts with an ideal gas by a potential. The calculation, performed on the one-loop level, is based on the Landau-Ginzburg double expansion in the particle trajectory and the time derivative. A Newtonian friction force and a mass renormalization were found in the linearized equation of motion, considered in the $\mathcal{O}(\partial_t^2)$ order. Furthermore the decoherence of the particle coordinate and momentum has been established, as well. In a fermion gas at vanishing temperature the higher derivative dissipative forces and the decoherence of the particle momentum are absent in the one-loop effective dynamics.

It is instructive to compare the dissipative forces with the Abraham-Lorentz force, the “friction force” of the electromagnetic radiation field. The Newtonian friction force, discussed in this work in the context of the electron gas can be considered as the radiational reaction force, generated by the emission of particle-hole pairs. The Newtonian friction force allows the measurement of the velocity with respect to the environment and it may appear if the system loses either the Galilean- or the Lorentz-boost invariance due to the coupling to its environment. If the electromagnetic field is boost invariant in the initial state then the diffusive part of the radiational reaction force must contain at least the third time derivative of the charge trajectory.

There are obvious questions, related to the higher order terms in the equation of motion. They were neglected in the present work but are not difficult to calculate some of them. How can these be identified in the framework of the linearized quantum Boltzmann equation? Though the coefficients of the linearized equation of motion are parametrized by different quantities in the two schemes the relations among these coefficients should be universal and comparable. Such a parallel calculation may show two different faces of the same physical phenomenon. If the non-locality is retained, rather then breaking it up into higher order derivative terms, then the memory effect should provide a non-
trivial, physically motivated example of non-Markovian dynamics [21] which is inherently consistent by construction.

References

[1] R. Zwanzig, *Nonequilibrium Statistica mechanics* (Oxford University, Oxford) (2001).

[2] E. Joos, H. D. Zeh, Z. Phys. B59, 223 (1985).

[3] W. H. Zurek, Phys. Rev. D24, 1516 (1981).

[4] C. Cercignani, *Theory and application of the Boltzmann equation* Scottish Academic Presss, Edinburgh (1975).

[5] B. Vacchini, K. Hornberger, Phys. Rep. 478, 71 (2009).

[6] L. Diosi, Europhys. Lett. 30, 63 (1995).

[7] K. Hornberger, Phys. rev. Lett. 97, 060601 (2006).

[8] B. Vacchini, Phys. Rev. E63, 066115 (2001).

[9] A. O. Caldeira, A. Leggett, Phys. Rev. Lett. 46, 211 (1981); Phys. Rev. Lett. 48, 1571 (1982); Ann. Phys. (N.Y.) 149, 374 (1983).

[10] B. Vacchini, K. Hornberger, Eur. Phys. J. Special Topics 151, 59 (2007).

[11] J. Schwinger, J. Math. Phys. 2, 407 (1961).

[12] L. V. Keldysh, Zh. Eksp. Teor. Fiz. 47, 1515 (1964) (Sov. Phys. JETP 20, 1018 (1965)).

[13] A. Kamenev, *Many-body theory of non-equilibrium systems*, arXiv:cond-mat/0412296.

[14] E. A. Calzetta, B. L. A. Hu, *Nonequilibrium Quantum Field Theory*, (Cambridge University Press) 2008.
[15] R. Kubo, M. Toda, N. Hasitsume, *Statistical Physics II* (Springer-Verlag, Berlin) 1978.

[16] H. Grabert, P. Schram, G.L. Ingold, Phys. Rep. **168**, 115 (1988).

[17] F. Guinea, Phys. Rev. Lett. **53**, 1268 (1984).

[18] P. Hedegård A. O. Caldeira, Phys. Scr. **35**, 609 (1987).

[19] R. P. Feynman, F. L. Vernon, Ann. Phys. **24**, 118 (1963).

[20] J. Polonyi, Phys. Rev. **D77**, 125018 (2008).

[21] H. P. Breuer, J. Phys. **B45**, 154001 (2012).