Estimating the synodic period of moon by meridian transit method and analyzing the cause of variations

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Abstract The study about synodic period of the Moon during 1983 - 2030 by Ephemeris. The Moon’s transit time at observer’s meridian were estimated consequently and can be plotted with Julian date to compare with The transit times of the sun in the same graph, therefore the difference between two intersected points were estimated to each synodic period of the Moon. The result was shown that there were 594 numbers of synodic period in 1983-2030 and tended to variate from the 5 effects of orbit between Sun-Earth and Moon.

1. Introduction
The synodic period of the Moon was investigated from the Moon’s orbit with respect to the configuration joining with the Sun and Earth. The average duration is 29.530587981 days (29d 12h 44m 2.8016s), however the angular speed of the Moon and Earth in their orbits are not constant value because the shape of their orbits are ellipse. However, the synodic period varies from about 29.18 to about 29.93 days in the envelope. McCurdy (2001) studied their variation and represented that it caused effect by the combination of the eccentricity of the Moon and Earth [1].

In this study, we would estimat this long-term value of 1983-2030 synodic periods by our technique, meridian transit method [2], there were 594 number of them and plotted them by the time of Julian date. After we obtained the data of variation curve, they would be fit by Fourier Fitting Analysis to saperate the various frequencies that made the curve be and analyzed their frequencies of the next.

2. Methodology
2.1. Synodic period of Moon The estimation about synodic period of the Moon started with the Moon’s equatorial coordinate data from Ephemeris and the calculated Greenwich’s sidereal time on each day from Jan 1, 1983 to Dec 31, 2030. Therefore, the hour angle of the Moon in 17,530 days is represented by equation (1).

\[ HA_{moon} = GST - RA_{moon} \] (1)
The Moon’s transit times were estimated consequently by \( HA_{moon} \) and can be plotted with their Julian date. The plot could show the 17,530-day of Moon’s transit time in the range of 0-24 hours. Next, the equation of time (EoT) from the Sun, in each day during 1983-2030 for Greenwich’s observer was obtained. In addition, the transit times of the Sun were calculated by equation (2).

\[
(Transit \ time)_{\odot} = 12:00:00 - EoT \tag{2}
\]

The Sun’s transit times were plotted together with the Moon’s transit time in the same Julian date. There were 595 points of intersection between them, which the synodic period of the Moon, for indicated, could be analyzed by measuring the difference between two adjacent intersected points.

The intersected points of the Moon and Sun’s transit times were shown. The Moon and the Sun could be interpreted to locate at observer’s meridian above Greenwich’s sky. This event occurs when the Sun, the Moon and Earth are in the same alignment. This phenomenon is called ‘new moon’. Therefore, the difference between two intersected points in Julian date is the period of certain new moon to the next. This period is called synodic period. Finally, the 594 values of synodic period from 1983-2030 were estimated and plotted with the time of Julian date. It was distributed in the range of 29.1-29.9 days. In exactly, the maximum value of synodic period in this long range was about 29.90275175 days, the minimum one was about 29.0981884 days and the average of all values was about 29.53090161 days. However, it seemed that all points were scattered periodically overview.

2.2. Fourier Curve Fitting Method. In figure 1, The scattered points in long-term synodic period looks like the superposition of many sine wave. The software Period04 [3] was used to analyze this plot to split of each possible sine wave, then they would be considered consequently. The algorithm of software could be generated the coefficient elements of each wave from equation (3).

\[
f(t) = Z + \sum_i A_i \sin(2\pi (F_i t + \phi_i)) \tag{3}
\]

Where \( F_i, A_i \) and \( \phi_i \) is frequency, amplitude and angle difference of the i-th wave which are the components of the variations of the synodic period of the Moon.

3. Results and Discussion

From Meridian transit method, the variations of synodic period of the Moon could be entered to the Fourier curve fitting as figure 1 to analyze the composition of this plot. This fitting could represent the 5-main-sub sine waves which could superpose to this variations of synodic period. It could express that the long-term of synodic period was resulted from 5-main-mechanisms in the Sun-Moon-Earth orbits as shown in table 1.

From table 1, frequency, amplitude and angle difference of each 5 waves were simulated by Fourier curve fitting method. Each of frequency was transformed to period of each sub-wave in unit of day. Then we interpreted the meaning of sub-wave by the approximation value of period which could describe us the stories of them.
Figure 1. the using Fourier curve fitting method to fit and analyse the waves which disturbed on.

Table 1. The coefficient elements of sub-wave possibly ripple the long-term synodic period of the Moon.

| Sub-wave | Frequency (day$^{-1}$) | Period (day) | Amplitude (hour) | Angle difference (day) | Interpretation |
|----------|------------------------|--------------|------------------|------------------------|----------------|
| 1        | 0.00242983321          | 411.5508817  | 4.35249         | 0.41758                | the synchronous between phase (position of Sun-Moon-Earth) and perigee revolution |
| 2        | 0.00274105744          | 364.8227087  | 2.05948         | 0.36173                | the synchronous between the orbit of the Moon and the orbit of Earth |
| 3        | 0.00562202303          | 177.8719146  | 2.05781         | 0.73554                | the synchronous between phase and the half orbit of Earth |
| 4        | 0.00577335224          | 173.2095944  | 0.33716         | 0.38958                | the synchronous between the orbit of nodes of the Moon orbit-Earth |
| 5        | 0.00485966642          | 205.7754409  | 0.29977         | 0.55848                | the lunar tidal cycle (straight-line configuration of Sun-Moon-Earth in the same direction) |

The first three sub-wave, which had larger amplitudes may be the three main mechanisms that made the forming of variations on lunar synodic period. The first one was the 411-day [4] period of resultant orbit which the same phase of the moon was synchronized with the direction in lunar orbit i.e. when full moon synchronize with the perigee direction in orbit of the Moon, apparent supermoon on Earth, that occurred again in every 411 days. This sub-wave could mainly disturb on the value of the long-term synodic period by the amplitude about 4.35249 hours which spread over and under the mean value of synodic period that was the main mechanism of variations. The second sub-wave that may disturbed on was the 364-day [4] period’s wave which harmonized between the position of the Moon in lunar orbit acted on the position of the Earth around the Sun. This synchronization of these was occur in every 364 days. And the Third one was 177-day [5] period from the simultaneity between the same phase of the Moon and the half orbit of the Earth around the Sun, in the simple way, the duration of lunar eclipse to the next one. It occurred 177 days apart between the same-year lunar eclipses. According to table 1, the second and third sub-wave had the same amplitude about 2.06 hours which spread together the long-term synodic period over and under the mean value. The fourth disturbed sub-wave was the 173-day [6] period’s tiny wave that composed with the periodic motion of lunar line of nodes (the intersected line
between lunar and ecliptic planes around the Earth. This ripple had less effective than the first three results by the amplitude about 0.337 hours which closed to the effect of the fifth sub-wave (0.29976 hours). This fifth one was the 206-day [7] period of resultant orbit which be the lunar tidal cycles or period of the same straight-line configuration between Sun-Moon-Earth. All of these could principally form the variations on the long-term synodic period. In addition, the Fourier curve fitting method could fit for the sixth, seventh, eighth sub-wave but these amplitudes of them were very small and continuously decrease therefore these additional term could be left from these variations.

4. Conclusions
We obtained the equatorial coordinate of the Moon and the equation of time of the Sun in during 1983-2030 and used them to estimate 594 data of synodic period of the Moon by meridian transit method. It revealed the variations for the long-term of them. However, it seems that all points are scattered periodically. Therefore, we used the method of Fourier curve fitting to separate some sub-waves which could probably spread and ripple the data of them.

This study concludes that There are 3 main mechanisms to spread the data in the range of 29.1-29.9 days and 2 mechanisms to distort the data by analyzing the period and amplitude of sub-wave. First main mechanism is the resultant orbit which the same phase of the moon is synchronized with the direction path in the ellipse of lunar orbit. Second main one is the synchronous between the position of the Moon in lunar orbit acted on the position of the Earth around the Sun and the third is the simultaneity between the same phase of the Moon and the half orbit of the Earth around the Sun. Moreover, the 2 mechanisms that ripple the long-term synodic period of the Moon is the periodic motion of lunar line of nodes around the Earth and the period of the same straight-line configuration between Sun-Moon-Earth.

In next study, we are looking into ways to improve our results by obtaining more data to estimate the precise of long-term synodic period of the Moon later.

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