Spin-dependent transport in multi-terminal Aharonov-Casher ring with quantum dot

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Abstract. The Aharonov-Casher (AC) effect is theoretically examined in a mesoscopic ring with an embedded quantum dot in a multi-terminal geometry. We examine a realistic model for such a system with spin-orbit interaction in the ring. A spin-polarized current can be generated in a three-terminal device even in the absence of magnetic field, whereas it cannot in a two-terminal device. The spin polarization is enhanced by the resonant tunneling around the Coulomb peak and Kondo effect in the Coulomb blockade regime. The efficiency for the spin polarization is discussed in realistic devices.

1. Introduction

A mesoscopic ring with an embedded quantum dot, so-called Aharonov-Bohm (AB) interferometer, is an important device to investigate the coexistence of the interference effect in a ring and single-electron transport through a quantum dot. It is known that the transport measurement of the phase shift through the quantum dot is not possible in a two-terminal setup because the conductance satisfies the relation of \( G(\phi) = G(-\phi) \) by the Onsager’s reciprocal theory, where \( \phi \) is the AB phase of the magnetic flux penetrating the ring [1], whereas it is possible in a multi-terminal setup [2,3].

In the present paper, we theoretically study the Aharonov-Casher (AC) effect in such a device, which is caused by the spin-orbit interaction in the ring. In this system, various transport phenomena were reported, e.g., control of the spin-polarized current using a magnetic field in a two-terminal setup [4], modulation of the Kondo effect by the AB and AC effects [5,6], and generation of spin-polarized current in a three-terminal setup [7]. We examine the spin polarization of the current in another geometry with three terminals, depicted in Fig. 1. We show that (i) a spin-polarized current can be generated in a three-terminal device even in the absence of magnetic field, whereas it cannot in a two-terminal device, and (ii) the spin polarization is enhanced by the resonant tunneling around the Coulomb peak and Kondo effect in the Coulomb blockade regime.

Note that a mesoscopic ring with an embedded quantum dot is usually examined using a simple model in which the quantum dot is connected to leads \( L \) and \( R \) by the tunnel coupling and an arm of the ring is described by the direct coupling \( W \) between the leads [8]. We examine a general model with \( W_{k',k} \) which depends on state \( k' \) in lead \( R \) and \( k \) in lead \( L \). Then we have a relevant parameter, \( p_\alpha (\alpha = L, R) \) defined in Eq. (4), which is the overlap integral between the conduction mode coupled to the quantum dot and that coupled to the upper arm of the ring.
in lead α [9]. The model in Ref. [8] corresponds to our model with \( p_α = 1 \) (when \( V_{R2,k''} = 0 \) and \( \pm \phi_{AC} \rightarrow 0 \) in Fig. 1), which is justified in the case of single channel in the leads. We evaluate the spin polarization with changing \( p_α \) to discuss the efficiency of realistic devices.

### 2. Model and calculation method

In the model depicted in Fig. 1, we assume a single level \( \varepsilon_d \) in the quantum dot and no magnetic field. The AC phase \( \phi_{AC} \) by the spin-orbit interaction in the ring is attached as \( W_{k',k}e^{i\phi_{AC}} \) for spin \( \sigma = \pm 1 \). The electro-chemical potentials in the leads are set to be \( \mu_{R1} = \mu_{R2} \) and \( \mu_L - \mu_{R1} = eV \) with applied bias \( V \). We have three parameters relevant to the transport, \( \pi \sum_k |V_{\alpha,k}|^2 \delta(\varepsilon_k - \varepsilon_F) = \Gamma_\alpha \), \( \pi \sum_k w_{\alpha,k} \delta(\varepsilon_k - \varepsilon_F) = x_\alpha \), and

\[
\pi \sum_k V_{\alpha,k} \sqrt{w_{\alpha,k}} \delta(\varepsilon_k - \varepsilon_F) = \sqrt{\Gamma_\alpha x_\alpha p_\alpha}
\]

\(|p_\alpha| \leq 1\) in lead \( \alpha \), where \( \varepsilon_F \) is the Fermi level in the leads. \( |p_\alpha| = 1 \) for the case of single channel in the leads, whereas \( |p_\alpha| < 1 \) for the case of more than one channel.

We express the current \( I_x \) for spin \( \sigma = \pm 1 \) from lead \( L \) to \( R1 \), where \( \mu_{R1} = \mu_{R2} \) and \( \mu_L - \mu_{R1} = eV \) (when \( p_\alpha \rightarrow \varepsilon_F \) as \( V \rightarrow 0 \)), in terms of retarded Green function of the quantum dot. The Green function is obtained analytically for \( U = 0 \) while it is evaluated using the Bethe ansatz exact solution [10,11] with Friedel sum rule for the Kondo effect with \( U \neq 0 \). The spin-dependent conductance, \( G_\sigma = dI_\sigma/dV |_{V=0} \), is calculated at temperature \( T = 0 \).
3. Calculated results

In the case of $U = 0$, we show the spin-dependent conductance $G_\sigma$ as a function of energy level $\varepsilon_d$ in the quantum dot (Fermi level in the leads is $\varepsilon_F = 0$). $p_L = p_{R1} = p$. $p = 1$ in panel (a) and 0.75 in panel (b) when the tunnel coupling to lead $R2$ is small ($\Gamma_R/\Gamma = 1/5$), where $\Gamma_L = \Gamma_R = \Gamma/2$). $G_+ = G_-$ when $\phi_{AC} = 0$ (solid line) and $\pi$ (broken line), whereas $G_+ \neq G_-$ when $\phi_{AC} = \pi/2$ (dotted and bold solid lines). Note that $G_+ = G_-$ always holds in the two-terminal geometry ($V_{R2,C} = 0$). We find an asymmetric Fano resonance for $G_\pm$ when $\phi_{AC} = 0$ and $\pi$ in panel (a) due to the large interference effect between the wavefunction through the upper arm of the ring and that through the quantum dot. The curve of $G_\pm$ becomes closer to the symmetric Breit-Wigner resonance in panel (b) since the interference is weaker with smaller $p$. For $U \neq 0$, the Fano-Kondo effect is seen when $p = 1$ [8], which changes to the conventional Kondo plateau as $p$ decreases (not shown here).

With an increase in tunnel coupling to lead $R2$, the difference between $G_+$ and $G_-$ becomes large until $\Gamma_R/\Gamma = 5/3$. At this critical value of $\Gamma_R$, the conductance is plotted in Fig. 2(c) when $p = 1$. $G_+ = 0$ if $\varepsilon_d$ matches the Fermi level in the leads (resonant tunneling). This means that the perfect spin polarization can be realized by tuning the parameters to this condition in the case of $p = 1$.

To evaluate the spin-polarized current, we introduce the spin polarization ratio, $\eta = (G_+ - G_-)/(G_+ + G_-)$. For $U = 0$, we obtain the analytical expression

$$\eta = \frac{4\sqrt{2}\Gamma_R p^2 \sin \phi_{AC}}{2[2\sqrt{2}\Gamma \varepsilon_d \cos \phi_{AC} + 4x\Gamma R_2 + x(x + 5)\Gamma^2]p^2 - \Gamma^2[\frac{x^2}{4} + (1 + x)^2] - 4x\varepsilon_d^2 + (\Gamma + \Gamma_R)^2]}.$$ 

From this expression, we find $\eta = 0$ in the two-terminal set-up ($\Gamma_R = 0$) or $\phi_{AC} = 0$, $\pi$. We also find that $\eta = \pm 1$ when $\Gamma_R/\Gamma = 5/3$, $\phi_{AC} = \mp \pi/2$, and $p = 1$ only. We plot $\eta$ in Fig. 3(a) when $\Gamma_R/\Gamma = 5/3$ and $\phi_{AC} = \pm \pi/2$. $\eta = \pm 1$ at the resonance for $p = 1$ (curves a), but $|\eta| < 1$ in realistic situations with $|p| < 1$ (curves b and c).

Finally, we examine the spin polarization by the Kondo effect in the Coulomb blockade regime. We plot $\eta$ when $U = 24\Gamma$ ($U = 9\Gamma$ with level broadening $\Gamma = \Gamma_L + \Gamma_{R1} + \Gamma_{R2}$) in Fig. 3(b). The other parameters are the same as in Fig. 3(a). We find a plateau of $\eta = \pm 1$ in the Coulomb blockade regime ($\varepsilon_d < \varepsilon_F < \varepsilon_d + U$) when $p = 1$. This is because the resonance level is fixed at the Fermi level by the Kondo effect. The absolute value of $\eta$ at the plateau becomes smaller for smaller $p$. 

![Figure 2](image-url).

**Figure 2.** Conductance $G_\pm$ for spin $\sigma = \pm 1$ as a function of energy level $\varepsilon_d$ in the quantum dot. The Fermi level in the leads is set to be $\varepsilon_F = 0$. $\Gamma_L = \Gamma_R = \Gamma/2$, $x_L = x_{R1} = 0.3$, and $p_L = p_{R1} = p$. (a) $\Gamma_R/\Gamma = 1/5$ and $p = 1$. (b) $\Gamma_R/\Gamma = 1/5$ and $p = 0.75$. (c) $\Gamma_R/\Gamma = 5/3$ and $p = 1$. The AC phase is $\phi_{AC} = 0$ (solid line; $G_+ = G_-$), $\pi/2$ ($G_+$ by dotted line and $G_-$ by bold solid line), and $\pi$ (broken line; $G_+ = G_-)$.
Figure 3. Spin polarization ratio $\eta = (G_+ - G_-)/(G_+ + G_-)$, as a function of energy level $\varepsilon_d$ in the quantum dot. The Fermi level in the leads is set to be $\varepsilon_F = 0$. $\Gamma_L = \Gamma_{R1} = \Gamma/2$, $\Gamma_{R2}/\Gamma = 5/3$, and $x_L = x_{R1} = 0.3$. (a) $U = 0$ and (b) $U = 24\Gamma$ (Kondo effect). Bold solid (dotted) lines correspond to the AC phase of $\phi_{AC} = -\pi/2$ ($\pi/2$), whereas solid lines of $\eta = 0$ to $\phi_{AC} = 0$ and $\pi$. $p \equiv p_L = p_{R1} = 1$ (curve a), 0.75 (curve b), and 0.5 (curve c).

4. Conclusions
The generation of spin-polarized current has been studied theoretically using a mesoscopic ring with an embedded quantum dot in a multi-terminal geometry, considering the AC effect by the spin-orbit interaction in the ring. We have examined a realistic model with a parameter $p$ for the overlap integral between the conduction mode coupled to the quantum dot and that coupled to the upper arm of the ring. We have shown that a spin-polarized current is generated in a three-terminal device even in the absence of magnetic field, whereas it cannot in a two-terminal device. The spin polarization is enhanced by the resonant tunneling around the Coulomb peak and Kondo effect in the Coulomb blockade regime. In an ideal situation with single conduction mode in the leads ($|p| = 1$), the perfect spin polarization can be realized by tuning the parameters in the device, whereas the spin polarization is less in realistic devices with $|p| < 1$.

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