Can the Pioneer anomaly be of gravitational origin? A phenomenological answer

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Abstract

In order to satisfy the equivalence principle, any non-conventional mechanism proposed to gravitationally explain the Pioneer anomaly, in the form in which it is presently known from the so-far analyzed Pioneer 10/11 data, cannot leave out of consideration its impact on the motion of the planets of the Solar System as well, especially those orbiting in the regions in which the anomalous behavior of the Pioneer probes manifested itself. In this paper we, first, discuss the residuals of the right ascension $\alpha$ and declination $\delta$ of Uranus, Neptune and Pluto obtained by processing various data sets with different, well-established dynamical theories (JPL DE, IAA EPM, VSOP). Second, we use the latest determinations of the perihelion secular advances of some planets in order to put on the test two gravitational mechanisms recently proposed to accommodate the Pioneer anomaly based on two models of modified gravity. Finally, we adopt the ranging data to Voyager 2 when it encountered Uranus and Neptune to perform a further, independent test of the hypothesis that a Pioneer-like acceleration can also affect the motion of the outer planets of the Solar System. The obtained answers are negative.

Keywords: gravity tests; modified theories of gravity; Pioneer anomaly

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1 Introduction

1.1 The Pioneer anomaly

The so-called Pioneer anomaly (Anderson et al. 1998; 2002a) consists of an unexpected, almost constant and uniform acceleration directed towards the Sun

$$A_{\text{Pio}} = (8.74 \pm 1.33) \times 10^{-10} \text{ m s}^{-2}$$

(1)
detected in the so-far analyzed data of both the spacecraft Pioneer 10 (launched in March 1972) and Pioneer 11 (launched in April 1973) after they passed the threshold of 20 Astronomical Units (AU; 1 AU is slightly less than the average Earth-Sun distance and amounts to about 150 millions kilometers), although it might also have started to occur after 10 AU only, according to a recent analysis of the Pioneer 11 data (Nieto and Anderson 2005). Latest communications with the Pioneer spacecraft, confirming the persistence of such an anomalous feature, occurred when they reached 40 AU (Pioneer 11) and 70 AU (Pioneer 10). A thorough re-analysis of all the available data sets of both the Pioneer spacecraft (Turyshev et al. 2006a; 2006b) is currently ongoing and should be completed within next year.

1.2 Gravitational explanations of the Pioneer anomaly and planetary motions

This anomalous effect recently attracted considerable attention because of the possibility that it may be a signal of some failure in the currently accepted Newton-Einstein laws of gravitation; indeed, at present no convincing explanations of it in terms of either known gravitational effects or some non-gravitational forces peculiar to the spacecraft themselves were yet found. A review of some of the proposed mechanisms of gravitational origin see, e.g., (Anderson et al. 2002a; Dittus et al. 2005). However, Murphy (1999) and Katz (1999) suggested non-gravitational mechanisms which, in their intentions, would be able to explain the Pioneer 10/11 anomalous behavior; see (Anderson et al. 1999a; 1999b) for replies. Interesting considerations about the energy transfer process in planetary flybys and their connection with the Pioneer anomaly can be found in (Anderson et al. 2007).

If the Pioneer anomaly is of gravitational origin, it must, then, fulfill the equivalence principle, which is presently tested at a $10^{-12}$ level (Will 2006) and lies at the foundations of the currently accepted metric theories of gravity. In its weak form, it states that different bodies fall with the same acceleration in a given external gravitational field. As a consequence, whatever gravitational mechanism is proposed to explain the investigated effect, it must also act, in general, on the Solar System bodies and, in particular, on those planets whose orbits reside in the region in which the Pioneer anomaly manifested itself, according to what we presently know about it.

In this framework, Jaekel and Reynaud (2005a; 2005b) put forth an ingenious gravitational mechanism able to accommodate the Pioneer anomaly; it is based on a suitable metric linear extension of general relativity which
yields, among other things, an acceleration only affecting the radial component of the velocity of a test particle. In (Jaekel and Reynaud 2006a) a further, non-linear extension of such a model was proposed and used. The last attempt by Jaekel and Reynaud (2006b) to find a non-conventional explanation of gravitational-type of the Pioneer anomaly was based on an extra-potential quadratic in distance. Brownstein and Moffat (2006), instead, adopted a Yukawa-like, explicit analytical model for an extra-acceleration acting upon a test particle involving four free parameters, and fitted it to all the presently available Pioneer 10/11 data points.

In this paper we will focus on such proposals by explicitly deriving theoretical predictions of some dynamical orbital effects, and by comparing them with the latest determinations (Section 2 and Section 3). The Russian astronomer E.V. Pitjeva (Institute of Applied Astronomy, Russian Academy of Sciences) recently processed almost one century of data of all types in the effort of continuously improving the EPM2004 planetary ephemerides (Pitjeva 2005a). Among other things, she also determined anomalous secular, i.e. averaged over one orbital revolution, advances of the perihelia $\Delta \dot{\varpi}_{\text{det}}$ of the inner (Pitjeva 2005b) and of some of the outer (Pitjeva 2006a; 2006b) planets as fit-for parameter$^1$ of global solutions in which she contrasted, in a least-square way, the observations (ranges, range-rates, angles like right ascension $\alpha$, and declination $\delta$, etc.) to their predicted values computed with a complete suite of dynamical force models including all the known Newtonian and Einsteinian features of motion$^2$. Thus, any unmodelled force, as it would be the case for a Pioneer-like one if present in Nature, is entirely accounted for by the determined perihelia extra-advances. In regard to the outer planets, Pitjeva was able to determine the extra-advances of perihelion for Jupiter, Saturn and Uranus (see Table 1 for their relevant orbital parameters) because the temporal extension of the used data set covered at least one full orbital revolution just for such planets: indeed, the orbital periods of Neptune and Pluto amount to about 164 and 248 years, respectively. For the external regions of the Solar System only optical observations were used, apart from Jupiter (Pitjeva 2005a); they are, undoubtedly, of poorer accuracy with respect to those used for the inner planets which also benefit of radar-ranging measurements, but we will show that they are accurate

\footnote{1}The perihelia, as the other Keplerian orbital elements, are not directly observable. \footnote{2}Only the general relativistic, gravitomagnetic Lense-Thirring effect and the Newtonian force due to the Kuiper belt objects (in the case of the inner planets) were not modelled. In regard to the other dynamical accelerations, the general relativistic gravitoelectric field and the Newtonian effect due to the Sun’s oblateness were included by keeping the PPN parameters $\beta$ and $\gamma$ fixed to 1 and $J_2 = 2 \times 10^{-7}$, respectively.
Table 1: Semimajor axes $a$, in AU, eccentricities $e$ and orbital periods $P$, in years, of the outer planets of the Solar System. Modern data sets covering at least one full orbital revolution currently exist only for Jupiter, Saturn and Uranus.

|        | Jupiter | Saturn | Uranus | Neptune | Pluto |
|--------|---------|--------|--------|---------|-------|
| $a$    | 5.2     | 9.5    | 19.19  | 30.06   | 39.48 |
| $e$    | 0.048   | 0.056  | 0.047  | 0.008   | 0.248 |
| $P$    | 11.86   | 29.45  | 84.07  | 163.72  | 248.02|

enough for our purposes. In regard to Uranus and Neptune, in Section 4 we will use certain short-period, i.e. not averaged over one revolution, effects of their semimajor axes $a$ and the ranging distance measurements to them performed at Jet Propulsion Laboratory (JPL), NASA, during their encounter with the Voyager 2 spacecraft (Anderson et al. 1995).

1.3 Previous planetary data analysis and the Pioneer anomaly

The idea of looking at the impact of a Pioneer-like acceleration on the orbital dynamics of the Solar System bodies was put forth for the first time by Anderson et al. (1998; 2002a) who, however, considered the motion of the Earth and Mars finding no evidence of any effect induced by an extra-acceleration like that of eq. (1) (see also Section 4). More interestingly, Anderson et al. (2002b) preliminarily investigated the effect of an ever-present, uniform Pioneer-like force on the long-period comets. Wright (2003) got another negative answer for Neptune from an analysis of the ranging data of Voyager 2 (see Section 4). Page et al. (2005) proposed to use comets and asteroids to assess the gravitational field in the outer regions of the Solar System and thereby investigate the Pioneer anomaly. The first extensive analysis involving all the outer planets can be found in (Iorio and Giudice 2006). In it the time-dependent patterns of the true observable quantities $\alpha \cos \delta$ and $\delta$ induced by a Pioneer-like acceleration on Uranus, Neptune and Pluto were compared with the observational residuals determined in (Pitjeva 2005a) for the same quantities and the same planets over a time span of about 90 years from 1913 (1914 for Pluto) to 2003. While the former ones exhibited well defined polynomial signatures yielding shifts of hundreds of arcseconds, the latter ones did not show any particular patterns, being almost uniform strips constrained well within $\pm 5$ arcseconds over the data set time span which includes the entire Pioneer 10/11 lifetimes. An analogous conclusion can also be found in (Tangen 2006), although a different theoret-
ical quantity was used in the comparison with the data. It should also be
remarked that the very same conclusion could already have been obtained
long time ago by using the residuals of some sets of modern optical obser-
vations (1984-1997) to the outer planets processed by Morrison and Evans
(1998) with the NASA JPL DE405 ephemerides (Standish 1998): indeed,
the residuals of $\alpha \cos \delta$ and $\delta$, shown in Figure 4 of (Morrison and Evans
1998), are well within $\pm 0.4$ arcseconds. Analysis of residuals obtained with
even older ephemerides would have yielded the same results. For example,
Standish (1993) used JPL DE200 ephemerides (Standish 1982) to process
optical data of Uranus dating back to 1800: the obtained residuals of $\alpha$ and
$\delta$ do not show any particular structure being well constrained within $\pm 5$
arcsseconds. Gomes and Ferraz-Mello (1987) used the VSOP82 ephemerides
(Bretagnon 1982) to process more than one century (1846-1982) of optical
data of Neptune getting no anomalous signatures as large as predicted by
the presence of a Pioneer-like anomalous force. In regard to Pluto, Gemmo
and Barbieri (1994) and Rylkov et al. (1995) used the JPL DE200 and JPL
DE202 ephemerides (Standish 1990) in producing residuals of $\alpha$ and $\delta$: no
Pioneer-type signatures can be detected in them.

2 The Jaekel and Reynaud models

2.1 The linear model

In order to find an explanation of gravitational origin for the anomalous
acceleration experienced by the Pioneer 10/11 spacecraft, Jaekel and Reyna-
d (2005a; 2005b) proposed, as a first attempt, to use a suitable metric
linear extension of general relativity with two potentials $\Phi_N$ and $\Phi_P$. In the
gauge convention of the PPN formalism its space-time line element, written
in isotropic spherical coordinates, is (Jaekel and Reynaud 2005b)

$$ds^2 = g_{00}c^2dt^2 + g_{rr}[dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)],$$

with

$$\begin{cases}
g_{00} = 1 + 2\Phi_N, \\
g_{rr} = -1 + 2\Phi_N - 2\Phi_P.
\end{cases}$$

In order to accommodate the Pioneer anomaly, the following simple model

$$\Phi_j(r) = -\frac{G_j M}{c^2 r} + \frac{\zeta_j M r}{c^2}, j = N, P,$$

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In order to accommodate the Pioneer anomaly, the following simple model

$$\Phi_j(r) = -\frac{G_j M}{c^2 r} + \frac{\zeta_j M r}{c^2}, j = N, P,$$
was used (Jaekel and Reynaud 2005a; 2005b). It is determined by four constants: the Newtonian constant $G_N$ and the three small parameters $G_P$, $\zeta_N$ and $\zeta_P$ which measure the deviation from general relativity. In the intentions of Jaekel and Reynaud, their theory should be able to explain the occurrence of the Pioneer anomaly a) without violating either the existing constraints from the planetary motions b) or the equivalence principle. The latter goal was ensured by the metric character of the proposed extension of general relativity. In regard to a), they, first, focussed their attention to the modification of the Newtonian potential. By using the orbits of Mars and the Earth they got an upper bound $|\zeta_N M| \approx 5 \times 10^{-13}$ m s$^{-2}$ (Jaekel and Reynaud 2005b) which excludes that $\zeta_N M r/c^2$ is capable to account for the anomalous Pioneer acceleration. The key point of their line of reasoning in explaining the Pioneer anomaly without contradicting our knowledge of the planetary orbits consisted in considering from the simple expression of eq. (4) for $\Phi_P$ the following extra-kinetic radial acceleration

$$A_{JR} = 2\zeta_P M v_r^2/c^2,$$  

(5)

where $v_r$ is the radial component of the velocity of the moving body, in identifying it with the source of the Pioneer anomalous acceleration by getting\(^3\) $\zeta_P M = 0.25$ m s$^{-2}$ and in claiming that eq. (5) cannot affect the planetary motions because circular. Conscious of the fact that independent tests are required to support their hypothesis and since no accurate and reliable data from other spacecraft are available to this aim, Jaekel and Reynaud (2005b) proposed to perform light deflection measurements because $\Phi_P$ affects the motion of electromagnetic waves as well. A re-analysis of the Cassini (Bertotti et al. 2003) data was suggested (Jaekel and Reynaud 2005b).

In this Section we will show that, in fact, the extra-kinetic acceleration of eq. (5) does also affect the orbital motions of the planets in such a way that it is possible to compare the resulting features of motion with the latest data from planetary ephemerides, thus performing right now a clean and independent test of the hypothesis that eq. (5) is able to accommodate the Pioneer anomaly. We will also discuss the feasibility of the proposed light deflection measurements in view of the results obtained from the perihelia test.

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\(^3\)The contribution of $G_P$ was found to be negligible.

\(^4\)The almost constant value $v_r = 1.2 \times 10^4$ m s$^{-1}$ was used for both the Pioneer spacecraft.
2.1.1 The orbital effects of the kinetic acceleration and comparison with the latest data

In order to make a direct comparison with the extra-rates of perihelia determined by Pitjeva (2005b), we will now analytically work out the secular effects induced by the extra-kinetic acceleration of eq. (5) on the pericentre of a test particle. To this aim, we will treat eq. (5) as a small perturbation of the Newtonian monopole. In order to justify this assumption, we will, first, evaluate the average of eq. (5) and, then, we will compare it with the the Newtonian mean accelerations throughout the Solar System. To this aim, we must evaluate eq. (5) onto an unperturbed Keplerian ellipse by using

\[ v_r = \frac{nae \sin f}{\sqrt{1 - e^2}}, \]  

where \( n = \sqrt{GM/a^3} \) is the (unperturbed) Keplerian mean motion and \( f \) is the true anomaly. Subsequently, the average over one orbital period \( P = 2\pi/n \) has to be performed. It is useful to adopt the eccentric anomaly \( E \) by means of the relations

\[
\begin{align*}
\frac{dt}{n} & = \frac{(1-e \cos E)}{n} dE, \\
\cos f & = \frac{\cos E - e}{1 - e \cos E}, \\
\sin f & = \frac{\sin E \sqrt{1 - e^2}}{1 - e \cos E}.
\end{align*}
\]

With

\[
\int_0^{2\pi} \frac{\sin^2 E}{1 - e \cos E} dE = \frac{2\pi}{e^2} \left( 1 - \sqrt{1 - e^2} \right),
\]

we get

\[
\langle A_P \rangle = \frac{2\zeta P M n^2 a^2}{e^2} \left( 1 - \sqrt{1 - e^2} \right).
\]

Eq. (9) can, now, be compared with the averaged Newtonian monopole acceleration

\[
\langle A_N \rangle = \frac{GM}{a^2 \sqrt{1 - e^2}},
\]

The results are in Table 2 from which it clearly turns out that the use of the perturbative scheme is quite adequate for our purposes. The Gauss equation for the variation of \( \varpi \) under the action of an entirely radial perturbing acceleration \( A_r \) is

\[
\frac{d\varpi}{dt} = -\frac{\sqrt{1 - e^2}}{nae} A_r \cos f.
\]
Table 2: Average Pioneer and Newtonian accelerations for the Solar System planets, in \( m\,s^{-2} \). For \( \langle A_P \rangle \) the expression of eq. 9 was used with \( \zeta_P M = 0.25\, m\,s^{-2} \).

| Planet  | \( \langle A_P \rangle \) | \( \langle A_N \rangle \) |
|---------|-----------------|-----------------|
| Mercury | \( 2 \times 10^{-10} \) | \( 4 \times 10^{-2} \) |
| Venus   | \( 1 \times 10^{-13} \) | \( 1 \times 10^{-2} \) |
| Earth   | \( 6 \times 10^{-13} \) | \( 6 \times 10^{-3} \) |
| Mars    | \( 1 \times 10^{-11} \) | \( 2 \times 10^{-3} \) |
| Jupiter | \( 1 \times 10^{-12} \) | \( 2 \times 10^{-4} \) |
| Saturn  | \( 8 \times 10^{-13} \) | \( 6 \times 10^{-5} \) |
| Uranus  | \( 2 \times 10^{-13} \) | \( 1 \times 10^{-5} \) |
| Neptune | \( 6 \times 10^{-15} \) | \( 6 \times 10^{-6} \) |
| Pluto   | \( 4 \times 10^{-12} \) | \( 4 \times 10^{-6} \) |

After being evaluated onto the unperturbed Keplerian ellipse by using eq. 6, eq. 5 must be inserted into eq. 11; then, the average over one orbital period has to be taken. By means of

\[
\int_0^{2\pi} \frac{\sin^2 E (\cos E - e)}{(1 - e \cos E)^2} dE = \frac{2\pi}{e^3} \left(-2 + e^2 + 2\sqrt{1 - e^2}\right),
\]

(12)

it is possible to obtain

\[
\langle \frac{d\varpi}{dt} \rangle = -\frac{2\zeta_P M a \sqrt{1 - e^2}}{e^2 e^2} \left(-2 + e^2 + 2\sqrt{1 - e^2}\right).
\]

(13)

Note that eq. 13 is an exact result, not based on approximations for \( e \). It may be interesting to note that the rates for the semimajor axis and the eccentricity turn out to be zero; it is not so for the mean anomaly \( M \), but no observational determinations exist for its extra-rate. We will now use eq. 13 and \( \zeta_P M = 0.25\, m\,s^{-2} \), which has been derived from eq. 5 by imposing that it is the source of the anomalous Pioneer acceleration, to calculate the perihelion rates of the inner planets of the Solar System for which estimates of their extra-advances accurate enough for our purposes exist (Pitjeva 2005b). The results are summarized in Table 3.

It clearly turns out that the determined extra-advances of perihelia are quite different from the values predicted in the hypothesis that eq. 5 can explain the Pioneer anomaly. In Table 4 we show the values of \( \zeta_P M \) which can be obtained from the determined extra-advances of perihelia (Pitjeva 2005b); as can be noted, all of them are far from the value which would be
Table 3: (P): predicted extra-precessions of the longitudes of perihelia of the inner planets, in arcseconds per century, by using eq. (13) and $\zeta_P M = 0.25$ m s$^{-2}$. (D): determined extra-precessions of the longitudes of perihelia of the inner planets, in arcseconds per century. Data taken from Table 3 of (Pitjeva 2005b). It is important to note that the quoted uncertainties are not the mere formal, statistical errors but are realistic in the sense that they were obtained from comparison of many different solutions with different sets of parameters and observations (Pitjeva, private communication 2005).

|            | Mercury | Venus | Earth | Mars  |
|------------|---------|-------|-------|-------|
| **(P)**    | 1.8323  | 0.001 | 0.0075| 0.1906|
| **(D)**    | $-0.0036 \pm 0.0050$ | $0.53 \pm 0.30$ | $-0.0002 \pm 0.0004$ | $0.0001 \pm 0.0005$ |

Table 4: Values of $\zeta_P M$, in m s$^{-2}$, obtained from the determined extra-advances of perihelia (Pitjeva 2005b). After discarding the value for Venus, the weighted mean for the other planets yields $\zeta_P M = -0.0001 \pm 0.0004$ m s$^{-2}$. The Pioneer anomalous acceleration is, instead, reproduced for $\zeta_P M = 0.25$ m s$^{-2}$.

|            | Mercury | Venus | Earth | Mars  |
|------------|---------|-------|-------|-------|
| $\zeta_P M$ | $-0.0005 \pm 0.0007$ | $91 \pm 51$ | $-0.006 \pm 0.013$ | $0.0001 \pm 0.0006$ |
required to obtain the correct magnitude of the anomalous Pioneer acceleration. The experimental intervals obtained from Mercury, the Earth and Mars are compatible each other; Venus, instead, yields values in disagreement with them. This fact can be explained by noting that its perihelion is a bad observable due to its low eccentricity ($e_{Venus} = 0.00677$). By applying the Chauvenet criterion (Taylor 1997) we reject the value obtained from the Venus perihelion since it lies at almost $2\sigma$ from the mean value of the distribution of Table 4. The weighted mean for Mercury, the Earth and Mars is, thus, $⟨\zeta_PM⟩_w = −0.0001$ m s$^{-2}$ with a variance, obtained from $1/\sigma^2 = \sum_i (1/\sigma_i^2)$, of 0.0004 m s$^{-2}$.

An analysis involving the perihelia of Mars only can be found in (Jaekel and Reynaud 2006a). In it Jaekel and Reynaud presented a nonlinear generalization of their model, and an explicit approximate expression of the perihelion rate different from eq. (13) can be found; it is calculated with $\zeta_PM = 0.25$ m s$^{-2}$ yielding a value for the Martian perihelion advance which is about one half of our value in Table 6. Even in this case, the results by Pitjeva (2005b) for Mars would rule out the hypothesis that the Pioneer anomaly can be explained by the proposed nonlinear model. By the way, in (Jaekel and Reynaud 2006a) no explicit comparison with published or publicly available data was presented.

All the previous considerations are based on the simple model of eq. (4), with $\zeta_P$ constant over the whole range of distances from the radius of the Sun to the size of the Solar System. Jaekel and Reynaud (2005a; 2005b; 2006a), in fact, left generically open the possibility that, instead, $\zeta_P$ may vary with distance across the Solar System, but neither specific empirical or theoretical justifications for such a behavior were given nor any explicit functional dependence for $\zeta_P(r)$ was introduced. By the way, as already noted, they explicitly applied their explicit model, with $\zeta_PM = 0.25$ m s$^{-2}$, to the Mars’ perihelion (Jaekel and Reynaud 2006a).

2.1.2 The deflection of light

The results for $\zeta_PM$ from the determined extra-rates of the perihelia of the inner planets allow us to safely examine the light deflection measurements originally proposed by Jaekel and Reynaud as independent tests of their theory; indeed, the values of Table 4 certainly apply to the light grazing

\footnote{More precisely, in (Jaekel and Reynaud 2006a) an explicit expression for the adimensional perihelion shift after one orbital period, in units of $2\pi$, i.e. $(\tilde{\pi}P)/2\pi$, is present; a direct comparison with our results can be simply done by multiplying their formula by $n$ and making the conversion from s$^{-1}$ to arcseconds per century.}
the Sun, also in the case of a hypothetical variation of $\zeta_P(r)$ with distance. Jaekel and Reynaud (2005a) found the following approximate expression for the deflection angle induced by $\zeta_P$

$$\psi_P = -\frac{2\zeta_P M \rho}{c^2} L,$$  \hfill (14)

where $\rho$ is the impact parameter and $L$ is a factor of order of unity which depends logarithmically on $\rho$ and on the distances of the emitter and receiver to the Sun. For $\rho = R_\odot$, $L \sim 1$, and $\zeta_P M = -0.0001 \text{ m s}^{-2}$, eq. (14) yields a deflection of only -0.3 microarcseconds, which can be translated into an equivalent accuracy of about $2 \times 10^{-7}$ in measuring the PPN parameter $\gamma$ with the well-known first-order Einsteinian effect (1.75 arcseconds at the Sun’s limb). Such a small value is beyond the presently available possibilities; indeed, the Cassini test (Bertotti et al. 2003) reached a $10^{-5}$ level, which has recently been questioned by Kopeikin et al. (2006) who suggest a more realistic $10^{-4}$ error. Instead, it falls within the expected 0.02 microarcseconds accuracy of the proposed LATOR mission (Turyshev et al. 2006), which might be ready for launch in 2014. Also ASTROD (Ni 2002) and, perhaps, GAIA (Vecchiato et al. 2003), could reach the required sensitivity to measure such an effect. However, because of technological and programmatic difficulties, the launch of an ASTROD-like mission is not expected before 2025. GAIA is scheduled to be launched in 2011 (http://gaia.esa.int/science-e/www/area/index.cfm?fareaid=26).

### 2.2 The quadratic model and its confrontation with the observationally determinations

In (Jaekel and Reynaud 2006b) a further development of the post-Einsteinian metric extension of general relativity proposed by such authors is presented. It, among other things, amends previous versions (Jaekel and Reynaud 2005a; 2005b) and yields another possible explanation of the behavior experienced by the Pioneer probes. Indeed, it was found that a roughly constant anomaly is produced when a second, extra-potential $\delta \Phi_P(r)$, quadratic in $r$, is introduced in the range of Pioneer distances. The choice by Jaekel and Reynaud (2006b) was

$$\delta \Phi_P(r) = c^2 \chi r^2, \quad \chi \simeq 4 \times 10^{-8} \text{ AU}^{-2},$$  \hfill (15)

where $c$ is the speed of light in vacuum. The resulting acceleration

$$A_P(r) = -2c^2 \chi r,$$  \hfill (16)
Figure 1: Anomalous acceleration, in nm s\(^{-2}\), induced by \(\delta \Phi_P = c^2 \chi r^2\), with \(\chi = 4 \times 10^{-8}\) AU\(^{-2}\), according to Jaekel and Reynaud (2006b).
Table 5: First row: determined extra-precessions of the longitudes of perihelia \( \varpi \) of Jupiter, Saturn and Uranus, in arcseconds per century (Pitjeva 2006a; 2006b). The quoted uncertainties are the formal, statistical errors re-scaled by a factor 10 in order to get realistic estimates. Second row: predicted anomalous extra-precessions of the perihelia for Jupiter, Saturn and Uranus, in arcseconds per century, according to eq. (17).

|       | Jupiter | Saturn | Uranus |
|-------|---------|--------|--------|
| (D)   | 0.0062 ± 0.036 | -0.92 ± 2.9 | 0.57 ± 13.0 |
| (P)   | -18.679 | -46.3  | -132.3 |

in units of nm s\(^{-2}\), is plotted in Figure 1. Without investigating how well such a model fits, in fact, all of the currently available data of the Pioneer 10/11 data, here we are going to derive theoretical predictions for the secular perihelion advance induced by eq. (16). The standard methods of perturbative celestial mechanics yield

\[
\left\langle \frac{d \varpi}{dt} \right\rangle = -3c^2 \sqrt{\frac{a^3(1-e^2)}{GM}}.
\]

Note that eq. (17) is an exact result. The comparison among the anomalous advances for Jupiter, Saturn and Uranus predicted with eq. (17) and the determined perihelia rates is in Table 5.

As can be noted, even by re-scaling by a factor 10 the formal errors released by Pitjeva (2006a; 2006b), the discrepancy among the predicted and the determined values amounts to 519, 15 and 10 sigma for Jupiter, Saturn and Uranus, respectively.

3 The Brownstein and Moffat model

In order to explain the Pioneer anomaly, Brownstein and Moffat (2006), in the context of their STVG metric theory of gravitation (Moffat 2006a), considered a variation with distance of the Newtonian gravitational constant \( G(r) \) and proposed the following radial extra-acceleration affecting the motion of a test particle in the weak field of a central mass \( M \)

\[
A_{BM} = -\frac{G_0 M \xi(r)}{r^2} \left\{ 1 - \exp \left[ -\frac{r}{\lambda(r)} \right] \left[ 1 + \frac{r}{\lambda(r)} \right] \right\}.
\]

Here \( G_0 \) is the ‘bare’ value of the Newtonian gravitational constant. Lacking at present a solution for \( \xi(r) \) and \( \lambda(r) \), the following parameterization was
introduced for them:

\[
\begin{align*}
\xi(r) &= \xi_\infty \left[ 1 - \exp \left( -\frac{r}{d} \right) \right]^b, \\
\lambda(r) &= \lambda_\infty \left[ 1 - \exp \left( -\frac{r}{d} \right) \right]^{-b}.
\end{align*}
\] (19)

In eq. (19) \(d\) is a scale distance and \(b\) is a constant. The best fitted values which reproduce the magnitude of the anomalous Pioneer acceleration are (Brownstein and Moffat 2006)

\[
\begin{align*}
\xi_\infty &= (1.00 \pm 0.02) \times 10^{-3}, \\
\lambda_\infty &= 47 \pm 1 \text{ AU}, \\
d &= 4.6 \pm 0.2 \text{ AU}, \\
b &= 4.0.
\end{align*}
\] (20)

The ‘renormalized’ value \(G_\infty\) of the Newtonian gravitational constant-\(G\) in the following—which is measured by the usual astronomical techniques—is related to the ‘bare’ constant by (Brownstein and Moffat 2006)

\[
\frac{G_0}{G_\infty} = \frac{1}{1 + \sqrt{\xi_\infty}}.
\] (21)

With the fit of eq. (20) we have

\[
\frac{G_0}{G_\infty} = 0.96934.
\] (22)

The scope of Brownstein and Moffat (2006) was to correctly reproduce the Pioneer anomalous acceleration without contradicting either the equivalence principle or our knowledge of the planetary orbital motions. The first requirement was satisfied by the metric character of their theory. In regard to the second point, Brownstein and Moffat (2006) did not limit the validity of eq. (18) just to the region in which the Pioneer anomaly manifested itself, but extended it to the entire Solar System. Their model is not a mere more or less ad hoc scheme just to save the phenomena being, instead, rather

\footnote{In the notation of Brownstein and Moffat (2006) \(\xi(r)\) and \(d\) are \(\alpha(r)\) and \(\tau\), respectively. Note that there is an error in eq. (12) for \(\lambda(r)\), p. 3430 of (Brownstein and Moffat 2006): a \(-\) sign is lacking in front of \(b\). Instead, eq. (27) of (Moffat 2006b) gives the correct expression.}
‘rigid’ and predictive. It is an important feature because it, thus, allows for other tests independent of the Pioneer anomaly itself. This general characteristic will also be preserved in future, if and when more points to be fitted will be obtained by further and extensive re-analysis of the entire data set of the Pioneer spacecraft (Turyshev et al. 2006a; 2006b) yielding a modification of the fit of eq. (20). Brownstein and Moffat (2006) performed a test based on the observable

\[ \eta = \left[ \frac{G(a)}{G(a_{\oplus})} \right]^{1/3} - 1, \]  

(23)

where \( a \) and \( a_{\oplus} \) are the semimajor axes of a planet and of the Earth. The quantity \( \eta \) was related to the third Kepler’s law for which observational constraints existed from a previous model-independent analysis (Talmadge et al. 1988) for the inner planets and Jupiter. No observational limits were put beyond Saturn because of the inaccuracy of the optical data used at the time of the analysis by Talmadge et al. (1988). Brownstein and Moffat (2006) found their predictions for \( \eta \) in agreement with the data of Talmadge et al. (1988).

3.1 The confrontation with the observational determinations

We will now perform an independent test of the Brownstein and Moffat (2006) model by using the extra-rates of the perihelia of Jupiter, Saturn and Uranus determined by Pitjeva (2006a; 2006b). To this aim, it is important to note that the spatial variations experienced by the extra-acceleration of eq. (18) over the orbits of such planets are undetectable because they amount to just \( 0.1 - 0.01 \times 10^{-10} \text{ m s}^{-2} \); thus, we will assume the acceleration of eq. (18) to be uniform. From Table 2 it clearly turns out that the anomalous acceleration of eq. (18) can be considered as a small perturbation of the Newtonian monopole term which, indeed, is 6-4 orders of magnitude larger than it. In, e.g., (Iorio and Giudice 2006; Sanders 2006) it was shown that a radial, constant and uniform perturbing acceleration \( A_r \), induces a pericentre rate

\[ \frac{d\omega}{dt} = A_r \sqrt{\frac{a(1 - e^2)}{GM}}. \]  

(24)

We will use eq. (24) and the determined extra-advances of perihelion (Pitjeva 2006a; 2006b) in order to solve for \( A \) and compare the obtained values with those predicted by eq. (18) for Jupiter, Saturn and Uranus. The results are summarized in Table 6 and Figure 2. As can be noted, the gravitational
Figure 2: The continuous curve is the fit to the currently available Pioneer 10/11 data according to the model of eq. (18) by Brownstein and Moffat (2006). The anomalous accelerations experienced by Jupiter, Saturn and Uranus, obtained from the determined perihelion rates by Pitjeva (2006a; 2006b), are also shown.
Table 6: First row: determined extra-precessions of the longitudes of perihelia $\varpi$ of Jupiter, Saturn and Uranus, in arcseconds per century (Pitjeva 2006a; 2006b). The quoted uncertainties are the formal, statistical errors re-scaled by a factor 10 in order to get realistic estimates. Second row: predicted anomalous acceleration for Jupiter, Saturn and Uranus, in units of $10^{-10}$ m s$^{-2}$, according to the model of eq. (18) (Brownstein and Moffat 2006), evaluated at $r = a$. Third row: anomalous acceleration of Jupiter, Saturn and Uranus, in units of $10^{-10}$ m s$^{-2}$, from the determined perihelia precessions of the first row. The quoted uncertainties have been obtained by means of the re-scaled errors in the perihelia rates. Fourth row: discrepancy between the determined and predicted accelerations in units of errors $\sigma$.

|       | Jupiter | Saturn | Uranus |
|-------|---------|--------|--------|
| $\Delta \dot{\varpi}_{\text{det}}$ | $0.0062 \pm 0.036$ | $-0.92 \pm 2.9$ | $0.57 \pm 13.0$ |
| $A_{\text{BM}}(a)$ | $0.260$ | $3.136$ | $8.660$ |
| $A_{\text{det}}$ | $0.001 \pm 0.007$ | $-0.134 \pm 0.423$ | $0.058 \pm 1.338$ |
| $|A_{\text{det}} - A_{\text{BM}}(a)|/\sigma$ | $37$ | $7$ | $6$ |

solution to the Pioneer anomaly proposed by Brownstein and Moffat (2006), in the form of eq. (18) and with the fitted values of eq. (20), must be rejected. Note that the quoted errors for the perihelia rates are the formal uncertainties re-scaled by 10 in order to give conservative evaluations of the realistic ones. In the case of Jupiter even a re-scaling of 100 would still reject the value predicted by eq. (18).

Incidentally, let us note that the results by Iorio and Giudice (2006) for Uranus, Neptune and Pluto can be viewed as a further, negative test of the Brownstein and Moffat (2006) model because they are based on the use of a Pioneer-like acceleration assumed to be uniform in the regions crossed by such planetary orbits.

4 A test with the Voyager 2 ranging data to Neptune

Until now we used the secular perihelion advances of some of the outer planets determined from optical data only (apart from Jupiter). In this Section we will turn our attention to Neptune and to certain short-period dynamical effects. The ranging data from Voyager 2 will be used as well (Anderson et al. 1995).

In (Iorio and Giudice 2006) there are the analytical expressions of the
short-period shifts induced on the Keplerian orbital elements by a radial, constant perturbing acceleration $A_r$, whatever its physical origin may be. For the semimajor axis, we have

$$\frac{\Delta a}{a} = -\frac{2eA_r a^2}{GM} (\cos E - \cos E_0) = -\frac{2e}{\sqrt{1-e^2}} \langle A_r \rangle (\cos E - \cos E_0).$$ \tag{25}$$

In the following computation it will be useful to express the eccentric anomaly $E$ in terms of the mean anomaly $M$ as (Roy 2005)

$$E \sim M + \left( e - \frac{e^3}{8} \right) \sin M + \frac{e^2}{2} \sin 2M + \frac{3}{8} e^3 \sin 3M. \tag{26}$$

The reference epoch is customarily assumed to be J2000, i.e. JD=2451545.0 in Julian date. From eq. (25) it can be noted that, whatever the eccentricity of the orbit is,

$$\left\langle \frac{\Delta a}{a} \right\rangle = 0, \tag{27}$$

so that $\Delta a/a$ cannot tell us anything about the impact of an acceleration like $A_{\text{Pto}}$ for those planets for which data sets covering at least one full orbital revolution exist. As already pointed out, to date, only Neptune and Pluto have not yet described a full orbit since modern astronomical observations became available after the first decade of 1900. Incidentally, let us note that, according to eq. (25), $\Delta a/a = 0$ for $e = 0$.

The situation is different for Neptune since no secular effects can yet be measured for it. Thus, let us use eq. (25) and eq. (1) for $A_r$ getting

$$\left. \frac{\Delta a}{a} \right|_{\text{Nep}} = (-2.2882 \pm 0.3482) \times 10^{-6} (\cos E - \cos E_0). \tag{28}$$

Note that the anomalous acceleration predicted by Brownstein and Moffat (2006) experiences an un-appreciable variation of just $0.04 \times 10^{-10} \text{ m s}^{-2}$ over the Neptune’s orbit, so that it can safely be considered uniform. Thus, the use of eq. (28) can be considered as a test of the Brownstein and Moffat (2006) model as well, and of any other model capable of reproducing an extra-acceleration with the characteristics of eq. (1) acting upon a test particle which moves in the spatial regions crossed by the Neptune’s orbit. The predicted effect of eq. (28) can be compared with the latest available observational determinations. Pitjeva (2005a) used only optical data (Table

\footnote{Here and in the following we will assume $r \approx a$, neglecting the finite value of the eccentricities, quite small for Uranus and, especially, Neptune.}
3 of (Pitjeva 2005a)) for the outer planets (apart from Jupiter) obtaining a formal, statistical error \( \delta a = 478532 \) m for the Neptune’s semimajor axis (Table 4 of (Pitjeva 2005a)) at JD=2448000.5 epoch (Pitjeva 2006b). By re-scaling it by 10 – 30 times in order to get realistic uncertainty we get

\[
\frac{\delta a}{a}^{(\text{optical})}_{\text{Nep}} = (1 - 3) \times 10^{-6}. \tag{29}
\]

It must be compared with eq. (28) at JD=2448000.5 (\( E = 107.423 \) deg)

\[
\frac{\Delta a}{a}^{(\text{optical})}_{\text{Nep}} (\text{JD} = 2448000.5) = (-0.7413 \pm 0.1128) \times 10^{-6}. \tag{30}
\]

Such an effect would be too small to be detected.

In (Anderson et al. 1995) the ranging data of the Voyager 2 encounter with Neptune were used yielding a unique ranging measurement of \( a \) (Julian Date JD=2447763.67); eq. (28), evaluated at such epoch (\( E = 106.012 \) deg), predicts

\[
\frac{\Delta a}{a}^{(\text{ranging})}_{\text{Nep}} (\text{JD} = 2447763.67) = (-0.7954 \pm 0.1210) \times 10^{-6}. \tag{31}
\]

By assuming for \( \Delta a \) the residuals with respect to the DE200 JPL ephemerides used in Table 1 of (Anderson et al. 1995), i.e. 8224.0 \pm 1 km, one gets

\[
\frac{\Delta a}{a}^{(\text{ranging})}_{\text{Nep}} = (1.8282 \pm 0.0002) \times 10^{-6}. \tag{32}
\]

This clearly rules out the prediction of eq. (31).

The same analysis can also be repeated for Uranus (\( P = 84.07 \) yr) for which no modern data covering a full orbital revolution were available at the time of the Anderson et al. (1995) work; as for Neptune, one ranging distance measurement is available from the Voyager 2 flyby with Uranus (JD=2446455.25). The prediction of eq. (25), with eq. (1) for \( A_r \), for the flyby epoch \( (E = 8.860 \) deg) is

\[
\frac{\Delta a}{a}^{(\text{ranging})}_{\text{Ura}} (\text{JD} = 2446455.25) = (-3.3576 \pm 0.5109) \times 10^{-6}. \tag{33}
\]

\(^8\)For Neptune \( E_0 = 128.571 \) deg at JD=2451545.0.

\(^9\)For Uranus \( E_0 = 70.587 \) deg at JD=2451545.0.
Table 1 of (Anderson et al. 1995) yields for the DE200 residuals of the Uranus’ semimajor axis $\Delta a = 147.3 \pm 1$ km, so that

$$\frac{\Delta a}{a} \bigg|_{\text{ranging}}^{\text{Ura}} (\text{JD} = 2446455.25) = (0.0513 \pm 0.0003) \times 10^{-6}. \quad (34)$$

Also in this case, the effect which would be induced by $A_{\text{Pio}}$ on $\Delta a/a$ is absent. Since the variation of the acceleration predicted by Brownstein and Moffat (2006) over the Uranus orbit amounts to just $0.2 \times 10^{-10}$ m s$^{-2}$, the same considerations previously traced for Neptune hold for Uranus as well.

It may be interesting to note that the paper by Anderson et al. (1995) was used as a basis for other tests with the outer planets using different methods. E.g., Wright (2003) and Sanders (2006) adopted the third Kepler’s law. Basically, the line of reasoning is as follows. In the circular orbit limit, let us write, in general, $P = 2\pi a/v$; in particular, the third Kepler law states that $P = 2\pi \sqrt{a^3/K_p}$, where $K_p = GM_\odot$. If we assume that $K_p$ may vary by $\Delta K_p$, for some reasons$^{10}$ inducing a change in the orbital speed, then $\Delta v/v = (1/2)\Delta K_p/K_p$. In general, for an additional radial acceleration acting upon a test particle in circular orbit $\Delta A_r$, $\Delta A_r/A_r = 2\Delta v/v$: thus, we have

$$\frac{\Delta K_p}{K_p} = \frac{\Delta A_r}{A_r}. \quad (35)$$

Now, a measurement of the planet’s velocity is needed to get $\Delta K_p/K_p$ (or, equivalently, $\Delta A_r/A_r$): since $v = na$, where $n$ is the orbital frequency, this requires a measurement of both $a$ and $n$, while in our case we only use $a$. Moreover, the measurement of the orbital frequency pose problems for such planets which have not yet completed a full orbital revolution, as it was the case for Uranus and Neptune at the time of the analysis by Anderson et al. (1995). For Neptune, according to the last row of Table 2 of (Anderson et al. 1995), $\left(\frac{\Delta K_p}{K_p}\right)_{\text{meas}} = (−2.0 \pm 1.8) \times 10^{-6}$, while $A_{\text{Pio}}/A_N = (−133.2 \pm 20.3) \times 10^{-6}$. As can be noted, also in this case the answer is negative but the accuracy is far worse than in our test.

5 Conclusions

In this paper we used the latest determinations of the secular extra-rates of the perihelia of some of the planets of the Solar System to test two recently proposed gravitational mechanisms to accommodate the Pioneer anomaly,$^{10}$ E.g. due to dark matter (Anderson et al. 1995).
in the form in which we presently know it, based on two models of modified
gavity. The cleanest test is for the Brownstein and Moffat (2006) model
which, by fitting a four-free parameters model to the presently available
data from the Pioneer spacecraft, yielded unambiguous predictions for an
extra-acceleration throughout the Solar System. The determined perihe-
lion rates of Jupiter, Saturn and Uranus neatly rule out such a nevertheless
interesting model. The linear model originally proposed by Jaekel and Rey-
naud (2005a; 2005b), along with its successive non-linear extensions (Jaekel
and Reynaud 2006a; 2006b), are, instead, disproved by the determined peri-
helion rates both of the inner planets of the Solar System, especially Mars,
and of the outer planets. We also used short-period effects on the semimajor
axis of Uranus and Neptune and the Voyager 2 ranging data to it to perform
another, negative test.

More generally, in regard to the impact of a Pioneer-like extra-acceleration
acting upon the celestial bodies lying at the edge of the region in which the
Pioneer anomaly manifested itself (∼20 − 70 AU), or entirely residing in it,
the present-day situation can be summarized as follows

• Uranus (a = 19.19 AU). 3 model-independent tests
  − Secular advance of perihelion (almost one century of optical data
    processed at IAA-RAS): negative
  − Right ascension/declination residuals (almost one century of opti-
    cal data processed with the ephemerides of IAA-RAS and JPL-
    NASA): negative
  − Short-period semimajor axis shift (1 ranging measurement at
    epoch JD=2446455.25 by JPL-NASA): negative

• Neptune (a = 30.08 AU). 2 independent tests
  − Right ascension/declination residuals (about one century or more
    of optical data processed with the ephemerides of IAA-RAS, JPL-
    NASA and VSOP82): negative
  − Short-period semimajor axis shift (1 ranging measurement at
    epoch JD=2447763.67 by JPL-NASA): negative

• Pluto (a = 39.48 AU). 1 test
  − Right ascension/declination residuals (almost one century of opti-
    cal data processed with the ephemerides of IAA-RAS; shorter
    data set analyzed with JPL-NASA ephemerides): negative
In all such tests the determined quantities—processed with the dynamical theories of JPL and IAA independently and without having the Pioneer anomaly in mind at all—were compared to unambiguous theoretical predictions based on the effects induced by a radial, constant and uniform acceleration with the same magnitude of that experienced by Pioneer 10/11, without making any assumptions about its physical origin.

In conclusion, it seems to us more and more difficult to realistically consider the possibility that some modifications of the current laws of Newton-Einstein gravity may be the cause of the Pioneer anomaly, at least in its present form, unless a very strange violation of the weak equivalence principle occurs in the outer regions of the Solar System (ten Boom 2005). By the way, the outcome of the currently ongoing re-analysis of the entire data-set of the Pioneer 10/11 spacecraft will be of crucial importance. Indeed, it may turn out that the characteristics of such an effect will be different from what we currently know about it, especially below 20 AU, and/or a satisfactorily non-gravitational mechanism will finally be found; in any case, any serious attempt to find a gravitational explanation for the Pioneer anomaly cannot leave out of consideration the orbital motions of the Solar System planets.

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References

[1] Anderson, J.S., Lau, E.L., Krisher, T.P., Dicus, D.A., Rosenbaum, D.C., and Teplitz, V.L., Improved Bounds on Nonluminous Matter in Solar Orbit, Astroph. J., 448, 885-892, 1995.

[2] Anderson, J.D., Laing, P.A., Lau, E.L., Liu, A.S., Nieto, M.M., and Turyshev, S.G., Indication, from Pioneer 10/11, Galileo, and Ulysses Data, of an Apparent Anomalous, Weak, Long-Range Acceleration, Phys. Rev. Lett., 81, 2858-2861, 1998.

[3] Anderson, J.D., Laing, P.A., Lau, E.L., Liu, A.S., Nieto, M.M., and Turyshev, S.G., Anderson et al. Reply:, Phys. Rev. Lett., 83, 1891-1891, 1999a.
[4] Anderson, J.D., Laing, P.A., Lau, E.L., Liu, A.S., Nieto, M.M., and Turyshev, S.G., Anderson et al. Reply, *Phys. Rev. Lett.*, 83, 1893-1893, 1999b.

[5] Anderson, J.D., Laing, P.A., Lau, E.L., Liu, A.S., Nieto, M.M., and Turyshev, S.G., Study of the anomalous acceleration of Pioneer 10 and 11, *Phys. Rev. D*, 65, 082004, 2002a.

[6] Anderson, J.D., Turyshev, S.G., and Nieto, M.M., Effect of the Pioneer Anomaly on Long-Period Comet Orbits, abstract no. 45.09 presented at the 201st American Astronomical Society Meeting; *Bull. Am. Astron. Soc.*, 34, 1172, 2002b.

[7] Anderson, J.D., Campbell, J.K., and Nieto, M.M., The energy transfer process in planetary flybys, *New Astron.*, 12, 383-397, 2007.

[8] Bretagnon, P., Theory for the motion of all the planets-The VSOP82 solution, *Astron. Astrophys.*, 114, 278-288, 1982.

[9] Brownstein, J.R., and Moffat, J.W., Gravitational solution to the Pioneer 10/11 anomaly, *Class. Quantum Grav.*, 23, 3427-3436, 2006.

[10] Dittus, H., Turyshev, S.G., Lämmerzahl, C., Theil, S., Förstner, R., Johann, U., Ertmer, W., Rasel, E., Dachwald, B., Seboldt, W., Hehl, F.W., Kiefer, C., Blome, H.-J., Kunz, J., Giuliani, D., Bingham, R., Kent, B., Summer, T.J., Bertolami, O., Páramos, J., Rosales, J.L., Cristophe, B., Foulon, B., Touboul, P., Bouyer, P., Reynaud, S., Brillet, A., Bondu, F., Samain, E., de Matos, C.J., Erd, C., Grenouilleau, J.C., Izzo, D., Rathke, A., Anderson, J.D., Asmar, S.W., Lau, E.E., Nieto, M.M., and Mashhoon, B., A mission to explore the Pioneer anomaly, *ESA Spec.Publ.*, 588, 3-10, 2005.

[11] Gemmo, A.G., and Barbieri, C., Astrometry of Pluto from 1969 to 1989, *Icarus*, 108, 174-179, 1994.

[12] Gomes, R.S., and Ferraz-Mello, S., Comparison of Bretagnos’s VSOP82 theory with observations, *Astron. Astrophys.*, 185, 327-331, 1987.

[13] Iorio, L., and Giudice, G., What do the orbital motions of the outer planets of the Solar System tell us about the Pioneer anomaly?, *New Astron.*, 11, 600-607, 2006.
[14] Jaekel, M.-T., and Reynaud, S., Gravity tests in the solar system and the Pioneer anomaly, *Mod. Phys. Lett. A*, 20, 1047-1055, 2005a.

[15] Jaekel, M.-T., and Reynaud, S., Post-Einsteinian tests of linearized gravitation, *Class. Quantum Grav.*, 22, 2135-2158, 2005b.

[16] Jaekel, M.-T., and Reynaud, S., Post-Einsteinian tests of gravitation, *Class. Quantum Grav.*, 23, 777-798, 2006a.

[17] Jaekel, M.-T., and Reynaud, S., Radar ranging and Doppler tracking in post-Einsteinian metric theories of gravity, *Class. Quantum Grav.*, 23, 7561-7579, 2006b.

[18] Katz, J., Comment on “Indication, from Pioneer 10/11, Galileo and Ulysses Data, of an Apparent Anomalous, Weak, Long-Range Acceleration”, *Phys. Rev. Lett.*, 83, 1892-1892, 1999.

[19] Kopeikin, S., Schäfer, G., Polnarev, A., and Vlasov, I., The orbital motion of Sun and a new test of general relativity using radio links with the Cassini spacecraft, 2006. http://www.arxiv.org/abs/gr-qc/0604060

[20] Moffat, J.W., Scalar−tensor−vector gravity theory, *J. Cosmol. Astropart. Phys.*, 3, 4, 2006a.

[21] Moffat, J.W., A Modified Gravity and its Consequences for the Solar System, Astrophysics and Cosmology, paper presented at *International Workshop “From Quantum to Cosmos: Fundamental Physics in Space”*, 22-24 May, 2006, Warrenton, Virginia, USA, gr-qc/0608074, 2006b

[22] Morrison, L.V., and Evans, D.W., Check on JPL DE405 using modern optical observations, *Astron. Astrophys. Suppl. Ser.*, 132, 381-386, 1998.

[23] Murphy, E.M., A Prosaic Explanation for the Anomalous Accelerations Seen in Distant Spacecraft, *Phys. Rev. Lett.*, 83, 1890-1890, 1999.

[24] Ni, W.-T., ASTROD-An Overview, *Int. J. of Mod. Phys. D*, 11, 947-962, 2002.

[25] Nieto, M.M., and Anderson, J.D., Using Early Data to Illuminate the Pioneer Anomaly, *Class.Quant.Grav.* 22, 5343-5354, 2005.
[26] Page, G.L., Wallin, J.F., and Dixon, D.S., Minor Planets as a Probe of the Gravitational Field in the Outer Solar System, abstract no. 154.02 presented at the 207th American Astronomical Association meeting; Bull. Am. Astron. Soc., 37, 1414, 2005.

[27] Pitjeva, E.V., High-Precision Ephemerides of Planets-EPM and Determination of Some Astronomical Constants, Sol. Sys. Res., 39, 176-186, 2005a.

[28] Pitjeva, E.V., Relativistic Effects and Solar Oblateness from Radar Observations of Planets and Spacecraft, Astron. Lett., 31, 340-349, 2005b.

[29] Pitjeva, E.V., Limitations on Some Physical Parameters from Position Observations of Planets, paper presented at 26th meeting of the IAU, Joint Discussion 16, #55, 22-23 August 2006, Prague, Czech Republic, 2006a.

[30] Pitjeva, E.V., private communication, 2006b.

[31] Rylkov, V.P., Vityazev, V.V., and Dementieva, A.A., Pluto: an analysis of photographic positions obtained with the Pulkovo normal astrograph in 1939-1992, Astronomical and Astrophysical Transactions, 6, 265-281, 1995.

[32] Roy, A.E., Orbital Motion. Fourth Edition, Institute of Physics Publishing, Bristol, 2005.

[33] Sanders, R.H., Solar system constraints on multifield theories of modified dynamics, Mon. Not. Roy. Astron. Soc., 370, 1519-1528, 2006.

[34] Standish, E.M., Orientation of the JPL ephemerides, DE200/LE200, to the dynamical equinox of J2000, Astron. Astrophy., 114, 297-302, 1982.

[35] Standish, E.M., An approximation to the outer planet ephemerides errors in JPL’s DE200, Astron. Astrophy., 233, 272-274, 1990.

[36] Standish, E.M., Planet X: No Dynamical Evidence in the Optical Observations, Astron. J., 105, 2000-2006, 1993.

[37] Standish, E.M., JPL Planetary and Lunar Ephemerides, DE405/LE405, Interoffice Memorandum 312.F-98-048, 1998.
[38] Tangen, K., Could the Pioneer anomaly have a gravitational origin?, 2006. http://www.arxiv.org/abs/gr-qc/0602089.

[39] Talmadge, C., Berthias, J.-P., Hellings, R.W., and Standish, E.M., Model-independent constraints on possible modifications of Newtonian gravity, *Phys. Rev. Lett.*, 61, 1159-1162, 1988.

[40] Taylor, J.R., *An Introduction to Error Analysis. The Study of Uncertainties in Physical Measurements. Second Edition*, University Science Books, 1997.

[41] ten Boom, P.G., Reinterpreting the Pioneer anomaly and its annual residual, 2005. http://www.arxiv.org/abs/gr-qc/0505077.

[42] Turyshev, S.G., Shao, M., and Nordtvedt Jr., K.L., Science, Technology and Mission Design for the Laser Astrometric Test of Relativity, to appear in *Proceedings of the 359th WE-Heraeus Seminar on “Lasers, Clocks, and Drag-Free: Technologies for Future Exploration in Space and Tests of Gravity,”* ZARM, Bremen, Germany, May 30-June 1, 2005. http://www.arxiv.org/abs/gr-qc/0601035.

[43] Turyshev, S.G., Toth, V.T., Kellogg, L.R., Lau, E.L., and Lee, K.J., The Study of the Pioneer Anomaly: New Data and Objectives for New Investigation, *Int. J. Mod. Phys. D*, 15, 1-56, 2006a.

[44] Turyshev, S.G., Nieto, M.M, and Anderson, J.D., The Pioneer Anomaly and Its Implications, *EAS Publication Series* 20, 243-250, 2006b.

[45] Vecchiato, A., Lattanzi, M.G., Bucciarelli, B., Crosta, M., de Felice, F., and Gai, M., Testing general relativity by micro-arcsecond global astrometry, *Astron. and Astrophys.*, 399, 337-342, 2003.

[46] Will, C.M. *Living Rev. Relativity*, 9, 3, 2006. www.livingreviews.org/lrr-2006-3.

[47] Wright, E.L., Pioneer Anomalous Acceleration, http://www.astro.ucla.edu/~wright/PioneerAA.html, 2003.