Measurement of differential polarizabilities at a mid-infrared wavelength in $^{171}\text{Yb}^+$

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Abstract.

An atom exposed to an electric field will experience Stark shifts of its internal energy levels, proportional to their polarizabilities. In optical frequency metrology, the Stark shift due to background black-body radiation (BBR) modifies the frequency of the optical clock transition, and often represents a large contribution to a clock’s uncertainty budget. For clocks based on singly-ionized ytterbium, the ion’s complex structure makes this shift difficult to calculate theoretically. We present a measurement of the differential polarizabilities of two ultra-narrow optical clock transitions present in $^{171}\text{Yb}^+$, performed by exposing the ion to an oscillating electric field at a wavelength in the region of room temperature BBR spectra. By measuring the frequency shift to the transitions caused by a laser at $\lambda = 7.17\text{\mu m}$, we obtain values for scalar and tensor differential polarizabilities with uncertainties at the percent level for both the electric quadrupole and octupole transitions at 436 nm and 467 nm respectively. These values agree with previously reported experimental measurements and, in the case of the electric quadrupole transition, allow a 5-fold improvement in the determination of the room-temperature BBR shift.

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1 Introduction

Singly-ionized $^{171}$Yb$^+$ has two ultra-narrow optical frequency transitions that have been selected as secondary representations of the SI second: an electric quadrupole (E2) transition and an electric octupole (E3) transition, respectively $4f^{14} 6s^2{^3}S_{1/2}(F=0, M_F = 0) \rightarrow {^2}D_{3/2}(F=2, M_F = 0)$ and $^2S_{1/2}(F=0, M_F = 0) \rightarrow {^2}F_{7/2}(F=3, M_F = 0)$ [1–4]. When realising a frequency standard based on these transitions, all sources of frequency shift that can perturb the natural atomic frequency must be characterized and either prevented or subtracted. This includes the frequency shift arising from the ion’s interaction with the electric field in the thermal background, i.e. the Stark shift due to environmental black-body radiation (BBR).

In the presence of an oscillating electric field $\vec{E}$, an energy level $\gamma$ in an atom is shifted by an amount $\delta W_{\gamma}$, given by

$$\delta W_{\gamma}(\vec{E}, \theta) = -\frac{1}{2} \alpha_\gamma(\theta) \langle \vec{E}^2 \rangle = -\frac{1}{2} \left[ \alpha_{\gamma}^{SC} + \frac{3M_F^2 - F(F+1)}{2F(2F-1)} (3\cos^2 \theta - 1) \alpha_{\gamma}^{TEN} \right] \langle \vec{E}^2 \rangle,$$

(1)

where $\theta$ is the angle between the electric field and the atom’s quantisation axis; $\alpha_\gamma$ is the polarizability of the state and $\alpha_{\gamma}^{SC}, \alpha_{\gamma}^{TEN}$ are the polarizability’s scalar and tensor components [5]. Note that the vector component of the polarizability is not present since both clock transitions involve states with $M_F = 0 \rightarrow 0$ [6,7].

This perturbation to the atomic energy levels results in an observed frequency shift $\delta \nu$ given by

$$h \delta \nu(\vec{E}, \theta) = \delta W_2 - \delta W_1 = -\frac{1}{2} \Delta \alpha_{21}(\theta) \langle \vec{E}^2 \rangle,$$

(2)

where $\Delta \alpha_{21} = \alpha_2 - \alpha_1$ is the differential polarizability of the transition between states 1 and 2 and is independent of the magnitude of the electric field. The differential polarizability of a transition is frequency dependent but, for electric fields in the frequency range typical of room temperature BBR spectra, the two clock transitions’ differential polarizabilities ($\Delta \alpha_{E2}$ and $\Delta \alpha_{E3}$) are smoothly varying and tend asymptotically to their dc values, as shown in Figure 1 [8].

The complex structure of the ytterbium ion makes theoretical calculations of these polarizabilities difficult [9,10] and, unlike in nearly temperature-insensitive clock species such as lutetium [11], the room-temperature BBR shift is large enough to make the accurate determination of these polarizabilities vital for clock operation with fractional frequency uncertainty at the $10^{-18}$ level. In this work, we therefore present direct measurements of the frequency shifts of the E2 and E3 transitions in $^{171}$Yb$^+$, induced by the electric field of a laser with a mid-IR wavelength of 7.17 µm, representative of
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Figure 1: Main plot The solid and dot-dashed lines show the differential polarizabilities of the E3 and E2 transitions respectively, calculated with oscillator strength corrected theory (see [8, 9]). The solid fill shows the spectrum of black-body radiation at 298 K in arbitrary units. The points show the values experimentally measured in this work.

Sub plots Close-up views of the (a) E2 and (b) E3 differential polarizabilities, where the blue lines are those from the main plot, the red line results from a least-squares fit to measurements taken in the near-IR for the E3 transition (calculated and reported in [3]) and the points show the measurements taken in this work at $\lambda = 7.17\mu m$.

a typical room-temperature thermal environment. From this we deduce values for the differential polarizabilities of both transitions that are of direct use in determining the BBR shifts for a $^{171}$Yb$^+$ frequency standard.

2 Experimental method

In our experimental setup, as shown in Figure 2, a single ion of $^{171}$Yb$^+$ is trapped in an rf Paul trap. Two ultra-stable lasers at 436 nm and 467 nm are used to drive the ultra-narrow quadrupole (E2) and octupole (E3) optical clock transitions (see, for example, [4, 12]).

In addition to this, a quantum cascade laser (QCL) [13] oscillating with $\lambda = 7.17\mu m$ was mounted to a NanoMover translation stage [14] and aligned onto the ion along the z-axis. The vacuum chamber of the system was fitted with a window made from MgF$_2$, chosen for its transmissivity at this wavelength [15]. The laser was linearly polarised with electric field always in the y-direction.

The 7-µm laser’s electric field induces a frequency shift of the clock transition as described by Equation 2. This shift was measured by running two interleaved frequency locks to the atomic transition: one with the 7-µm light incident on the ion and another with it blocked by a shutter. All other experimental conditions were kept identical between the two servos so we attribute any observed frequency difference as entirely due to the Stark shift of the infrared light. Figure 3 shows data taken in this arrangement, as the translation stage was moved to scan the position of the 7-µm laser beam over the ion. The laser was typically in a given position for a period of 30 s to allow frequency
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Figure 2: A trapped $^{171}\text{Yb}^+$ ion is interrogated by an ultra-stable probe laser at either 436 nm or 467 nm. The transition frequency is perturbed by the electric field created by an incident laser beam of $\lambda = 7\ \mu\text{m}$. The laser is mounted on a computer-controlled translation stage to allow the full intensity profile of the beam to be explored. Total power in the beam is continuously monitored for stability, and can be measured with a calibrated carbon nanotube-based detector.

Figure 3: Typical data taken as the 7-\(\mu\text{m}\) laser beam is translated over the ion’s position (upper axis). Servo 1 shows the clock laser frequency in the absence of perturbation, while servo 2 records the Stark shift induced by the 7-\(\mu\text{m}\) radiation.

data to be acquired, before stepping to the next position.

Control of the ion’s quantisation axis was achieved by the application of an external magnetic field to set or vary the angle $\theta$ in Equation 1.

3 Polarizability ratios

Without characterising the absolute strength of the electric field at 7-\(\mu\text{m}\), information can already be obtained about the relative polarizabilities between different transitions or between different orientations of the ion, by making measurements in a constant electric field. For these measurements the intensity maximum of the 7-\(\mu\text{m}\) beam was positioned onto the ion by maximizing shifts to the clock transitions, increasing our signal-to-noise ratio, while also minimising sensitivity to small excursions in the beam position.

By monitoring the power at the beam pick-off shown in Figure 2, we determined
that the total power of IR light produced by the 7-µm laser in the plane of the ion was
stable at the $\delta P/P \leq 6 \times 10^{-5}$ level over time-scales of $10^3$ to $10^4$ seconds. Note that
this stability refers to the total power in the beam, not the intensity experienced by
the ion which is subject to further uncertainties such as beam pointing which will be
addressed below.

We will first discuss the ratio of the differential scalar polarizabilities between the
E3 and E2 clock transitions, $\Delta \alpha_{SC}^{E3}/\Delta \alpha_{SC}^{E2}$. The frequency shifts from the incident 7-µm
light were measured for each clock transition in turn, in a set of orthogonal magnetic
field directions chosen to simultaneously maximise the three Rabi frequencies for the
clock probe-beam’s fixed direction. The average of the frequency shifts across the
three orthogonal orientations of the ion revealed the contribution from the differential
scalar polarizability component for each transition, with the tensor component averaging
to zero over the three orthogonal directions [5]. Taking the ratio of these average
frequency shifts for the two transitions, recorded in equal 7-µm electric field strengths,
gives the ratio $\Delta \alpha_{SC}^{E3}/\Delta \alpha_{SC}^{E2}$, as shown in Table 1. Evaluating the uncertainty required
consideration of both statistical uncertainties in the measured frequency shifts, and also
non-orthogonalities in the applied magnetic field directions. Both these uncertainties
dominated over any changes in the field strength or direction of the 7-µm electric field
during the measurements.

Statistical uncertainties due to quantum projection noise were readily obtained
from the fluctuations in the measured frequency shifts and then propagated through
the averaging process and into the ratio. Uncertainties in the magnetic field directions,
however, were assessed through a combination of measurements and simulations. In
the experimental setup, specific magnetic fields are created by passing currents through
three orthogonal pairs of coils around the vacuum chamber. The ion itself can be used
as a sensor, with Zeeman splittings of magnetically sensitive internal states revealing the
magnitude of applied magnetic fields. Each pair of coils was calibrated to determine the
change in current needed to effect a specific change in magnetic field, and also the current
needed to null the component of background magnetic field in that direction. These
“null currents” were checked before and after every measurement to detect any changes
in background field. A small additional background field would, in general, modify
the magnitudes and directions of all applied fields, and lead to a non-orthogonal set of
ion orientations being used, resulting in incomplete cancellation of the tensor shift in
Equation 1. As the ion’s Zeeman splittings do not give information about field direction,
it is necessary to test for any changes in the background field by checking for any changes
in field magnitudes over a wide range of applied magnetic field angles. Typically three
independent sets of three orthogonal field directions were used for this testing. The scale
of any deviations from the intended field magnitudes was then used as an input into a
simulation to assess the resulting uncertainties on the frequency shifts. The simulation
built a grid of all possible magnetic field errors that would satisfy the observed deviations
in field magnitudes. These field errors were then used to compute the full range of
possible errors in the frequency shifts and their averages. The resulting histogram of
Measurement of differential polarizabilities at a mid-infrared wavelength in $^{171}$Yb$^+$. The ratio of differential scalar to tensor polarizabilities was also measured for both the E2 and E3 clock transitions, $\Delta \alpha_{E2}^{SC} / \Delta \alpha_{E2}^{TEN}$ and $\Delta \alpha_{E3}^{SC} / \Delta \alpha_{E3}^{TEN}$. As above, the frequency shift arising from the differential scalar polarizability was determined by averaging over the frequency shifts recorded in three orthogonal directions. For the E2 transition, with its large tensor polarizability, the frequency shifts in the orthogonal directions were well separated and it was possible to derive $\Delta \alpha_{E2}^{SC} / \Delta \alpha_{E2}^{TEN}$ directly from these shifts, using Equation 1. Once again, simulations based on observed deviations in the magnitudes of test magnetic fields were used to derive the uncertainties arising from non-orthogonality in the fields of the experiment. These frequency uncertainties, arising from uncertainties in direction, dominated over the statistical uncertainties for the E2 transition.

For the E3 transition however the scalar and tensor shifts were both much smaller leading to a higher fractional uncertainty for the statistical noise. To resolve the tensor shift more clearly, the frequency shift in $\vec{B}_y$ was measured, corresponding to $\theta = 0$ in Equation 1 to give an extreme value for the induced tensor shift. The tensor shift in $\vec{B}_y$ and the scalar shift from the average over three orthogonal directions were therefore used in Equation 1 to establish $\Delta \alpha_{E3}^{SC} / \Delta \alpha_{E3}^{TEN}$.

| Ratio                              | Frac. Uncertainty ($10^{-3}$) |
|------------------------------------|-------------------------------|
| Quadrupole : Octupole              |                               |
| $\Delta \alpha_{E2}^{SC} / \Delta \alpha_{E3}^{SC}$ | 9.127                         |
| Scalar : Tensor                    |                               |
| $\Delta \alpha_{E3}^{SC} / \Delta \alpha_{E3}^{TEN}$ | -4.131                        |
| $\Delta \alpha_{E2}^{SC} / \Delta \alpha_{E2}^{TEN}$ | -0.656                        |
| Scalar : Probe field ($\vec{B}_z$) |                               |
| $\Delta \alpha_{E3}^{SC} / \Delta \alpha_{E2}^{\vec{B}_z}$ | 0.4706                        |

Table 1: Observed ratios between the scalar and tensor differential polarizabilities of the two clock transitions in $^{171}$Yb$^+$ measured at $\lambda = 7 \mu m$. E2 and E3 refer to the electric quadrupole and octupole clock transitions in $^{171}$Yb$^+$ respectively. Also included is the ratio of the E3 scalar differential polarizability vs. the E2 differential polarizability in a particular magnetic field, $\vec{B}_z$, used in Section 4 to establish an absolute value for the differential polarizability.

all possible offsets in the frequency average formed a normal distribution from which the standard deviation was determined. This error arising from non-orthogonal field directions could then be taken into account alongside the statistical uncertainty when computing the total uncertainty on the ratio shown in Table 1.
4 Absolute polarizabilities

In order to determine the absolute value of a transition’s differential polarizability, knowledge of the strength of electric field from the 7-µm laser at the ion is necessary. Since the electric field $\vec{E}$ from the applied laser was position dependent across the spatial extent of the beam ($S$), the induced shift $\delta \nu(\theta, x, y)$ was measured in a 2D grid of positions, $\vec{x} = (x, y)$, across the beam.

Starting from Equation 2

$$h \delta \nu(\vec{x}, \theta) = - \frac{1}{2} \Delta \alpha_{21}(\theta) \langle \vec{E}(\vec{x})^2 \rangle$$

$$h \int \delta \nu(\vec{x}, \theta) dS = - \frac{1}{2 \epsilon_0 c} \Delta \alpha_{21}(\theta) \int I(\vec{x}) dS,$$

(3)

where $I(\vec{x})$ is the position-dependent intensity of the 7-µm radiation. The beam produced by the 7-µm QCL contained high-order components making a Gaussian fit untenable, demonstrated by the 1D slices through the frequency shift profile shown in Figure 4. For this reason, we chose to measure and sum the frequency shifts over the entire extent of the beam. By approximating the LHS integral in Equation 3 by a sum over pixels each of area $A$, and noting that the RHS integral is simply equal to the total power in the beam, $P$,,

$$\Delta \alpha_{21}(\theta) = - \frac{2 \epsilon_0 \hbar c}{P} \sum_{x,y} A \delta \nu(\vec{x}, \theta).$$

(4)

The polarizability can thus be derived by summing the frequency shifts measured across an entire grid, such as that shown in Figure 5. Having measured this quantity for one transition in a known quantisation direction $\theta$, the value of any other differential polarizability component can be deduced by means of the ratios given in Section 3.

The profiles obtained using this method showed good agreement with optical profiles taken of the beam by scanning over a pinhole and measuring a chopped signal with a photo-detector and lock-in amplifier. Unfortunately however, uncertainty of the ion’s distance from the focal point prevented a quantitative comparison.

4.1 Setup

As shown by Equation 4, knowledge of four quantities is essential in order to evaluate any component of the ion’s differential polarizability: the area of each pixel in a 2D scan, the total power in the beam used to perturb the clock transition, the sum of the resultant shifts across the whole beam, and the ratio linking the polarizability component in the measured direction to the direction of interest. The following sections will detail how each of those quantities was controlled and measured.
4.1.1 Sum of frequency shifts over laser profile: \( \sum_{x,y} \delta \nu(x, \theta) \)

The spatial profile of the beam was mapped out by translating the 7-\( \mu \)m beam across the ion and recording the frequency shifts induced at each \((x, y)\) position, as shown in Figure 5. This was done using the E2 transition since its polarizability is \( \sim 10\times \) greater than that of the E3 transition, allowing the shift to be resolved more quickly. The large number of points required to cover the entire transverse beam profile meant that each scan could take around two days. By operating in a magnetic field such that \( \theta = \pi/2 \) (field \( \vec{B}_z \) in Figure 2) we minimised our sensitivity to changes in the background field affecting the tensor shift during this time.

Figure 4: Shift induced by the 7-\( \mu \)m beam for various axial positions of the translation stage. Note that as \( z \) nears the focal point of the optical setup (blue) the peak shift increases and the power far from beam-centre (visible in these slices as deviations from the Gaussian lineshape) is reduced.

Figure 5 shows the shift induced at a range of positions of the translation stage for fixed \( z \)-position, large enough to account for the whole beam.

**Beam capture** The method of considering the sum of shifts in Equation 4 relies on probing the beam over its entire extent; power missed by the 2D grid would cause an underestimate of the differential polarizabilities. To judge how well all the frequency shifts have been captured, Figure 5 *inset* shows the cumulative sum of shifts within a given radius. At large radii, this reaches its asymptote, suggesting that the beam profile does not extend beyond the range of the 2D grid.

As another check, we considered the sum of shifts measured around the outer rim of the 2D grid, at maximal \( x \) and \( y \). If there were no power present in the 7-\( \mu \)m beam this far from its beam centre, these shifts would be expected to sum to 0 Hz. The total summed shift in this outer rim was \((26.3 \pm 37.9)\) Hz, consistent with zero to within the quantum projection noise of 38 Hz.
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Figure 5: Colour map of the induced frequency shift at a large range of $(x, y)$ positions. Each data-point represents the average shift observed in a 30s interleaved lock to both the shifted and unshifted quadrupole clock transition in Yb$^+$ with the ion’s quantization axis in the $\hat{z}$ direction and the electric field in the $\hat{y}$ direction. The scan was performed in a spiral manner outwards and then inwards.

*inset* Fractional cumulative shift against distance from beam-centre. At large values of $r$ this becomes constant, implying that this scan covers the whole beam.

**Summing uncertainty** In order to approximate the integral over all $(x, y)$ points in the beam, we used the simple method of summing all the measured shifts in a 2D grid, weighted by their corresponding grid square areas (described by Equation 4), since the inner region of the grid was scanned using smaller position steps than the outer.

Simulations of the error in the summed shift deduced by this simple method were performed, assuming a beam profile with either a Gaussian shape or an analytical function that closely approximated the observed profile. The results in both cases indicate that the penalty incurred by using the sum of pixelated shifts instead of the complete, smooth
beam integral is surprisingly small even for large (i.e. approaching the beam-waist) grid spacings and also for off-centre grids. The contribution of the grid summing method to our total uncertainty can therefore be neglected at the $10^{-4}$ level.

**Statistical** Each point in Figure 5 was typically scanned for 30 s. At this time-scale, the quantum projection noise of the interleaved E2 clock transition is around 3 Hz. Including this per-point uncertainty results in a fractional statistical uncertainty of 0.8% on the sum of shifts over the whole grid.

4.1.2 Area : $A$

To determine the area, $A$, of the grid squares used in Equation 4, a Michelson interferometer setup with $\lambda = 935$ nm was used to monitor the step size of the NanoMover translation stage and its repeatability.

The fringes produced by changes in one arm of the interferometer due to movement of the translation stage were measured by a photodiode and analog–digital converter to be analysed digitally.

We ran datasets in the same conditions as when probing the ion with the interferometer set up for monitoring either $x$ or $y$ step sizes: see Figure 6. The fractional instability in step sizes produced by the NanoMover was below $2 \times 10^{-3}$ over more than 8 hours: the time-scale of the 7-\(\mu\)m laser scans. We found that nominal 10\(\mu\)m steps in the $x$ and $y$ directions, as programmed in the control software, produced 9.95(2)\(\mu\)m and 9.78(2)\(\mu\)m steps respectively.
4.1.3 Beam power: \( P \)

In order to determine the beam power at the ion, knowledge of two elements is necessary. Firstly its power before entering the trap, and secondly the attenuation caused by the windows of the vacuum chamber.

**Power** A pick-off mirror in the 7-\( \mu \)m beam path enabled continuous monitoring of laser power stability and a NIST calibrated optical power transfer standard could be inserted into the main beam to measure its power with high accuracy. As optical power instrumentation suffers errors introduced by spectral non-uniformity, accurate measurement of laser power typically requires instrumentation calibrated against a primary standard at the wavelength to be studied. Due to power non-linearities, it is also necessary to calibrate near the power to be measured, typically within one-two decades depending on instrument performance. Direct calibrations of laser power instrumentation at the 20mW level at 7\( \mu \)m are not presently available from any calibration lab and, while spectrometer “bridging” measurements of the device or its coating are possible, this would introduce an unacceptably high uncertainty for the present application.

As such, a device coated with a Vertically Aligned Nanotube Array (VANTA) developed at NIST was used as the optical power standard. VANTA coatings show superior spectral uniformity in the VIS-FIR regime [16] and thereby permit a calibration performed against a primary laboratory standard at an arbitrary wavelength to serve as a standard at another arbitrary wavelength, with corrections for the small spectral variation and uncertainty contribution of the VANTA coating measurement.

Use of this device allowed the power in the 7-\( \mu \)m beam to be measured as 17.52(16) mW.

**Window attenuation** To know the power in the plane of the ion, it was necessary to measure the attenuation experienced by the 7-\( \mu \)m beam as it passed through the \( \text{MgF}_2 \) window of the vacuum chamber, chosen specifically for its transmissivity in the infra-red [15].

Since access to the in-vacuum side of the trap window was impractical, an identically manufactured window from the same batch as that mounted to the chamber was used to characterise absorption. The 7-\( \mu \)m beam was aligned such that it passed through the test window at the same angle and with the same beam waist as in the main experiment. The attenuation through the window was measured using an IR power meter. This test was then repeated for different beam positions on the window.

The attenuation caused by the window was found to be insensitive to position at the 0.2\% level. Taking this into account, the transmission of the \( \text{MgF}_2 \) window at 7\( \mu \)m was found to be 0.5101(14).
4.1.4 Ratio : $\Delta \alpha^{SC}/\Delta \alpha^{\vec{B}_z}_{E2}$

The previous sections have described the measurements undertaken to derive the absolute differential polarizability in the field $\vec{B}_z$ where $\theta = \pi/2$. To link this measurement to the scalar and tensor components of the two transitions requires knowledge of the ratio between these components and the differential polarizability in $\vec{B}_z$.

The determination of these ratios was subject to the same considerations as those required when evaluating the ratios presented in Section 3. For the scalar components of the E2 / E3 transition, the fractional uncertainty contribution was 2.0% / 1.3%. In order to derive the absolute values of the tensor shifts of the two transitions, the scalar : tensor ratio measurements previously presented in Table 1 were used.

4.2 Results

The results of the measurements described in Section 4 are shown in Table 2. These data, in combination with Equation 4, are used to calculate the absolute values of the scalar and tensor components of the differential polarizabilities of the two transitions.

| Contribution | Value  | Fractional uncertainty (10^{-3}) |
|--------------|--------|---------------------------------|
| Area of pixels / $\mu$m$^2$ | 24.3   | 3                               |
| Power at ion / mW            | 8.94   | 14                              |
| Sum of shifts / Hz           | -18712 | 8                               |
| $\Delta \alpha^{SC}_{E3}/\Delta \alpha^{\vec{B}_z}_{E2}$ | 0.4706 | 13                              |
| $\Delta \alpha^{SC}_{E2}/\Delta \alpha^{SC}_{E3}$ (used for E2 only) | 9.127  | 15                              |
| **Differential polarizability @ 7 $\mu$m** | | |
| scalar | 7.70(20) | 0.844(18) |
| tensor | -11.74(45) | -0.2042(67) |

Table 2: Uncertainty budget and result of the determination of the absolute differential polarizabilities at 7 $\mu$m of the two clock transitions. Tensor components are derived from the scalar components using the ratios presented in Table 1.

5 Estimating BBR frequency shift

The differential scalar polarizability at 7 $\mu$m allows the atomic frequency shift to be deduced close to the peak of the room-temperature black-body radiation spectrum. The radiation spectrum, however, extends to both higher and lower wavelengths, and so the total frequency shift from a black body at room temperature also depends on the polarizability’s wavelength dependence.
A basic theoretical model can be used to estimate the scalar polarizability $\alpha$ of state $|\gamma J\rangle$ at different wavelengths by summing over the oscillator strengths $f$ of all the electric dipole transitions connecting state $|\gamma J\rangle$ to states $|\gamma' J'\rangle$ using [17]:

$$\alpha(\gamma J) = \frac{e^2}{m_e} \sum_{\gamma' J'} \left( \frac{f(\gamma J; \gamma' J')}{\omega_{\gamma J, \gamma' J'}^2 - \omega_L^2} \right)$$  \hspace{1cm} (5)

where $\omega_{\gamma J, \gamma' J'}$ is the atomic transition frequency and $\omega_L$ is the frequency of the applied electric field. $e$ and $m_e$ are the electronic charge and mass. The oscillator strengths are taken from [8,9,18–20], except for the leading term in the summation for each state.

The oscillator strengths of the leading terms for the excited and ground states are allowed to vary around their theoretical values§ and, for each combination of oscillator strengths, the differential polarizability at all wavelengths is calculated giving curves such as those shown in Figure 1. The validity of each combination of oscillator strengths is judged by comparing the predicted polarizability at 7 $\mu$m to our experimental observations for each clock state. Each polarizability curve is thus assigned a weight, calculated from its probability of lying in a normal distribution centred at our measurement with our experimental uncertainty. A weighted mean of all these curves gives our predicted differential scalar polarizability as a function of wavelength, with its uncertainty their standard deviation.

The BBR frequency shift can then be estimated by integrating, across all wavelengths, the product of the predicted differential scalar polarizability and the electric field squared. We derive the frequency shift from a black body at 298 K for the E2 transition to be $-334$ mHz with a fractional uncertainty of 3.5%, and for the E3 transition to be $-42.9$ mHz with a fractional uncertainty of 2.1%. For comparison with other work, the model also predicts the dc scalar differential polarizabilities (as shown in Table 3) to be $5.82(30) \times 10^{-40}$ J m$^2$ V$^{-2}$ for the E2 transition and $0.849(18) \times 10^{-40}$ J m$^2$ V$^{-2}$ for the E3 transition, and the dynamic corrections at 298 K to be $+0.137(23)$ and $-0.00249(16)$ for the E2 and E3 transitions respectively.

6 Conclusion

We have directly measured the differential polarizabilities for the E2 and E3 clock transitions in $^{171}$Yb$^+$ using a perturbing field at $\lambda = 7.17$ $\mu$m, a wavelength in the region typical of room temperature BBR spectra.

We presented ratios between the tensor and scalar components of the differential polarizabilities of both transitions and fully characterized the 7-$\mu$m beam’s intensity profile, allowing the absolute values of the differential polarizabilities to be deduced, all at the few percent level of uncertainty. A basic theoretical model was used to predict the polarizabilities at other wavelengths, thus allowing the BBR shifts to be calculated.

§ The range of these variations was chosen such that the weights assigned later were non-negligible or, for transitions with well-known, experimentally measured oscillator strengths, to cover a 3-sigma range.
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|                | Quadrupole $\Delta\alpha_{DC}^{SC}$/ J m$^2$ V$^{-2} \times 10^{-40}$ | Octupole $\Delta\alpha_{DC}^{SC}$/ J m$^2$ V$^{-2} \times 10^{-40}$ |
|----------------|--------------------------------------------------------------------------------|------------------------------------------------------------------|
| This work      | $5.82 \pm 0.30$                                                               | $0.849 \pm 0.018$                                               |
| Previous work  | $6.90 \pm 1.40$ [21]                                                         | $0.888 \pm 0.016$ [3]                                          |

Table 3: Values for the dc scalar differential polarizabilities of the E2 and E3 transitions extrapolated using the method described in Section 5, shown for comparison with previous work.

Further work with more sophisticated theoretical models may improve upon the values presented here. Nevertheless, our basic model already reveals that the polarizability measurements at 7 μm greatly reduce the uncertainty of the BBR frequency shift for the E2 transition from approximately 20% [21] to 3.5%. This is significant as the BBR shift dominates the uncertainty budget for the E2 transition frequency in $^{171}$Yb$^+$. We note that our predicted dc differential scalar polarizability for the E2 transition is significantly different from a recent ab initio theoretical calculation [10] that predicts $\Delta\alpha_{DC}^{SC} = 7.8(5) \times 10^{-40}$ J m$^2$ V$^{-2}$. Nevertheless, we note good agreement with totally independent experimental results for the dc differential scalar polarizabilities for both the E2 and E3 transitions, as shown in Table 3.

These new measurements show that limits on the fractional frequency uncertainty caused by room-temperature black-body radiation can be reduced to low parts in $10^{17}$ on the E2 transition and low parts in $10^{18}$ for the E3. These uncertainties assume a temperature uncertainty in the ion’s environment of 1 K or better, commonly achieved experimentally [15, 22].

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