Strong washout approximation to resonant leptogenesis

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Received July 3, 2014
Accepted August 21, 2014
Published September 18, 2014

Abstract. We show that the effective decay asymmetry for resonant Leptogenesis in the strong washout regime with two sterile neutrinos and a single active flavour can in wide regions of parameter space be approximated by its late-time limit \( \varepsilon = X \sin(2\varphi)/(X^2+\sin^2\varphi) \), where \( X = 8\pi \Delta/(|Y_1|^2 + |Y_2|^2) \), \( \Delta = 4(M_1 - M_2)/(M_1 + M_2) \), \( \varphi = \arg(Y_2/Y_1) \), and \( M_{1,2}, Y_{1,2} \) are the masses and Yukawa couplings of the sterile neutrinos. This approximation in particular extends to parametric regions where \( |Y_{1,2}|^2 \gg \Delta \), i.e. where the width dominates the mass splitting. We generalise the formula for the effective decay asymmetry to the case of several flavours of active leptons and demonstrate how this quantity can be used to calculate the lepton asymmetry for phenomenological scenarios that are in agreement with the observed neutrino oscillations. We establish analytic criteria for the validity of the late-time approximation for the decay asymmetry and compare these with numerical results that are obtained by solving for the mixing and the oscillations of the sterile neutrinos. For phenomenologically viable models with two sterile neutrinos, we find that the flavoured effective late-time decay asymmetry can be applied throughout parameter space.

Keywords: leptogenesis, baryon asymmetry

ArXiv ePrint: 1406.4190
1 Introduction

Resonant enhancement from mass degeneracies is a way of obtaining sizeable charge-parity \((CP)\) violating effects, that would be strongly suppressed by powers of small couplings otherwise. Depending on the ratio of the mass splitting to the decay rate in a system of mixing particles, it may either be more advantageous to describe the \(CP\)-violating effects as a time-dependent phenomenon due to mixing and oscillations of the almost mass-degenerate states, or, further away from the mass degeneracy, in terms of a time-independent effective decay asymmetry [1]. The important role that resonant \(CP\)-violation assumes in many systems that can be tested in the laboratory has lead to the idea that a resonantly enhanced decay asymmetry for sterile neutrinos may have been of importance for Leptogenesis in the Early Universe [2–7].

Standard Leptogenesis calculations typically rely on a time-independent effective asymmetry \(\epsilon\), which may be resonantly enhanced or not. It isolates the \(CP\)-violating loop effects from the leading-order out-of-equilibrium dynamics, that may be described in terms of tree-level rates, see e.g. refs. [8, 9]. While this separation approach brings along some caveats and pitfalls, most notably the necessity of a subtraction of real intermediate states (RIS) in order to comply with the consequences of the combined charge-, parity- and time-reversal symmetry [10], it has proved very useful for practical phenomenological calculations as well as for the conceptual description of the dynamics of the generation and the freeze out of the lepton asymmetry.

A more unified approach to Leptogenesis, starting from first principles, is provided by the Closed-Time-Path (CTP) method [11–13], that is formulated in terms of Green functions and leads to kinetic equations that readily encompass the crucial higher-order corrections [14–23]. No ad hoc subtraction of RIS is needed here. For the present context, we note that
in the appropriate limiting cases, we recover either the description of resonant Leptogenesis from mixing and oscillations or in terms of a time-independent decay asymmetry \( \varepsilon \) [24]. Both regimes overlap, such that suitable calculational methods for all parametric configurations are available. A general formulation that spans from the regime where the sterile neutrinos are fully relativistic to the case when these are non-relativistic, which is of relevance for strong washout and that accounts for the expansion of the Universe, is developed in refs. [24, 25], that are a main basis for the present work. The derivation in refs. [24, 25] relies on Green functions in Wigner-space (where the two-point functions are Fourier transformed with respect to the relative coordinate). Alternative approaches also based on the CTP method employ Green functions in the coordinate representation [26–29], but the results agree with those obtained in Wigner space, which is most evident when comparing refs. [24] and [28], where consistent effective evolution equations for the sterile neutrinos and for the final freeze-out asymmetry are obtained.

We also note that mixing and oscillations can be treated within a density matrix approach, that is typically applied to Leptogenesis in the fully relativistic regime, see refs. [30–37]. More recently, the density matrix method has also been applied to Leptogenesis in the non-relativistic strong washout regime [38].

While the CTP formulation of resonant Leptogenesis is rederived and confirmed in ref. [28], an important point concerning approximate solutions is added there: Since by definition of the strong washout regime, the relaxation rate \( \Gamma \) of the sterile neutrinos exceeds the Hubble rate \( H \), neglecting time-derivatives acting on the non-equilibrium distributions of the sterile neutrinos should only incur an error that is of order \( H/\Gamma \). This allows for a quasi-static solution for the right-handed neutrino distributions and their off-diagonal correlations, from which an effective late-time decay-parameter \( \varepsilon \) can be constructed, even when their mass splitting is smaller than their decay rate.

Based on above developments, we present here the following points that are of relevance for resonant Leptogenesis in the strong washout regime:

- We show how the non-relativistic approximations and simplifications, that are of relevance in the strong washout regime, follow from the general treatment of refs. [24, 25].

- We define the effective decay asymmetry \( \varepsilon \) as the lepton asymmetry that results on average from the decay of one out-of-equilibrium sterile neutrino. When compared to the decay asymmetry introduced in ref. [28], this definition resembles more closely the expressions that are usually employed in Leptogenesis calculations, such that it leads to a simple and straightforward way of obtaining the lepton asymmetry. We present the relevant equations that determine the freeze-out asymmetry as well as example solutions.

- We give an expression for the decay asymmetry taking account of active lepton flavours and their possible correlations. We emphasise that flavour effects should be phenomenologically relevant throughout the parameter space. Again, we illustrate the use of this effective asymmetry with numerical examples.

- Since it is crucial for resonant Leptogenesis to treat the decay rate \( \Gamma \) of the sterile neutrinos as matrix-valued, the criterion \( H/\Gamma \ll 1 \) for the applicability of the approximation in terms of an effective decay asymmetry can only be of schematic meaning. For a simplified scenario with one active lepton flavour only, we determine the smallest eigenvalue associated with the linear differential equation that governs the evolution of
the sterile neutrino densities and their flavour-off-diagonal correlations. By comparison with the Hubble rate, this eigenvalue can be used in order to assess whether the approximation in terms of the effective decay asymmetry $\varepsilon$ is applicable.

- For a phenomenological scenario with two sterile neutrinos, that explains the observed oscillations of active neutrinos, we find that the use of the effective late-time decay-asymmetry can be justified for all regions of parameter space. This conclusion is also based on comparing the eigenvalues of the equations that govern the mixing and the oscillations of the sterile neutrinos with the Hubble expansion rate prior to the freeze out of the lepton asymmetry.

2 Relativistic resonant leptogenesis

We consider the usual see-saw model for neutrino masses that is given by the Lagrangian

$$\mathcal{L} = \frac{1}{2} \bar{N}_i (i\partial \! - M)_{ij} N_j + \bar{\ell}_a \partial \! \ell_a + (\partial^\mu \phi^\dagger)(\partial_\mu \phi) - Y_{ia}^* \bar{\ell}_a \epsilon_{SU(2)} \phi P_R N_i - Y_{iu} \bar{N}_i P_L \phi^\dagger \epsilon_{SU(2)}^\dagger \ell_a.$$  

(2.1)

Here, the $N_i$ are the sterile neutrinos, that observe the Majorana condition $N^c_i = N_i$, where the superscript $c$ stands for charge conjugation. The Higgs doublet is given by $\phi$ and $\epsilon_{SU(2)}$ is the antisymmetric, SU(2)-invariant tensor with $\epsilon_{SU(2)}^{12} = 1$. The Standard Model (SM) lepton doublets are given by $\ell_a$, where $a = e, \mu, \tau$. When considering the single-flavour model, we drop the index $a$ on the fields $\ell$ as well as the on Yukawa couplings $Y$. We make use of the freedom of field redefinitions in order to choose the symmetric matrix $M$ to be real and diagonal, and we refer to the diagonal elements as $M_i \equiv M_{ii}$.

We describe the generation of the comoving lepton charge density $q_{ab}$ in terms of a source term $S_{ab}$ and a washout term $W$ as [24, 25]

$$q_{ab}' = g_w S_{ab} - \frac{1}{2} [W, q_{ab}].$$  

(2.2)

The charge density accounts for the gauge multiplicity, hence we include here the factor $g_w = 2$. Moreover, as mentioned in the Introduction, we allow for the possibility of correlations of the SM lepton flavours. The expansion of the Universe is accounted for through the metric in conformal coordinates $g_{\mu\nu} = a(\eta) \eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowski metric, $a(\eta)$ is the scale factor and $\eta$ is conformal time. A prime denotes a derivative with respect to $\eta$.

In ref. [24], it is shown that the source term for resonant Leptogenesis through the lepton-number violating Majorana mass can be computed by first solving for the flavour correlations of the oscillating sterile neutrinos, similar to the standard calculations for $CP$-violation in mixing meson systems [1] or to the lepton-number conserving source in the scenarios that are usually referred to as Leptogenesis from neutrino oscillations [25, 30–37]. The result of ref. [24] is generalised to include flavour correlations in ref. [25] and then reads

$$S_{ab} = - \sum_{i,j} Y_{ia}^* Y_{jb} \int \frac{d^4k}{(2\pi)^4} \text{tr} \left[ P_R i\delta S_{Nij}(k) 2P_L \hat{\Sigma}_N^A(k) \right],$$  

(2.3)

where $\hat{\Sigma}_N^A(k)$ is the reduced spectral self-energy of the sterile neutrinos as defined in ref. [25]. The correlations of the sterile neutrinos are described by $i\delta S_{Nij}(k)$. Besides the indices $i, j$
for the sterile neutrino flavours, this function corresponds to a rank two tensor in terms of Dirac spinors. It satisfies Kadanoff-Baym equations and the solutions can be decomposed as

\[ i\delta S_N = \sum_{h=\pm} i\delta S_{Nh}, \quad -i\gamma^0\delta S_{Nh} = \frac{1}{4}(1 + \hbar k^\sigma) \otimes \rho^g_{gh}, \]  

(2.4)

where \(\sigma\) and \(\rho\) are Pauli matrices. In the resonant regime \(|M_i - M_j| \ll \bar{M}\), the different components may be written as \([24]\)

\[ g_{abij}(k) = 2\pi\delta(k^2 - a^2\bar{M}^2)2k^0\delta f_{abij}, \]  

(2.5)

where \(\bar{M} = (M_i + M_j)/2\). Moreover, the Kadanoff-Baym equations also imply the relations \([24]\)

\[ \delta f_{1bij}(k) = \delta f_{3bij}(k)a\frac{M_i + M_j}{2h|k|}, \quad \delta f_{1bij}(k) = \delta f_{0bij}(k)a\frac{M_i + M_j}{2k^0}. \]  

(2.6)

In view of the non-relativistic approximation below, the \(a = 0\) component is of particular interest. The function \(\delta f_{0bij}\) may be interpreted as the distribution function of the sterile neutrinos and of their flavour correlations. Using the decomposition (2.4) and the relations (2.6), the source term (2.3) can be expressed as \(S_{ab} = \int \frac{d^3k}{(2\pi)^3} S_{ab}(k)\), where

\[ S_{ab}(k) = \sum_{i,j} \sum_{\substack{i,j \neq j \cdots \pm 2}} Y_{ia}^* Y_{jb} \left\{ \frac{k \cdot \hat{\Sigma}_N^a(k)}{k^0} \left[ \delta f_{0bij}(k) - \delta f_{3bij}(k) \right] + h\frac{k \cdot \hat{\Sigma}_N^a(k)}{k^0} \right\}_{k^0 = \omega(k)}, \]  

(2.7)

\[ \omega(k) = \sqrt{k^2 + aM^2}, \quad \tilde{k} = (|k|, \tilde{k}^0/|k|) \text{ and } \delta f^{*a}_{0bij}(k^0) = \delta f_{3bij}(-k^0). \]

The Kadanoff-Baym equations imply that the sterile neutrino distributions and their correlations satisfy \([24, 28]\)

\[ \delta f_{0}^{a} + \frac{a^2(\eta)}{2k^0} \left[i[M^2, \delta f_{0}] + f^{*a} - \frac{\Im[Y^* Y]}{k^0} \right] = \frac{\Re[Y^* Y]}{k^0} \left[i \cdot \hat{\Sigma}_N^a, \delta f_{0} \right], \]  

(2.8)

where \(f^{*a}\) is the equilibrium Fermi-Dirac distribution of the sterile neutrinos. One may alternatively derive this equation using a more heuristic approach in terms of a density matrix instead of the two-point function of the sterile neutrinos. The solution may be substituted back into the source term (2.3) and eventually into the equation for generating the lepton charge-density (2.2) in order to obtain predictions for the freeze-out asymmetry.

When comparing eq. (2.8) with the corresponding expressions in e.g. ref. [39] (for oscillations of scalar particles derived in the CTP framework) or [40] (for neutrino oscillations using a density-matrix approach) one notices that the commutator term in these references involves a matrix of frequencies \(\omega\) rather than \(M^2\). The different forms are consistent in the resonant regime where \(M_i^2 - M_j^2 \ll \bar{\omega}^2 = k^2 + \bar{M}^2\) because there is agreement to leading order in \((M_i^2 - M_j^2)/\bar{\omega}^2\): \(\omega = \sqrt{k^2 + M^2} = \bar{\omega} + \delta M^2/(2\bar{\omega}) + O\left(\left[(M_i^2 - M_j^2)/\bar{\omega}^2\right]^2\right)\), where we have written \(M^2 = \bar{M}^2 + \delta M^2\). The commutator, of course, only depends on the non-diagonal terms, such that \([2\bar{\omega}^2 + \delta M^2, \cdot] \equiv [\delta M^2, \cdot] \equiv [M^2, \cdot].\] While the derivation in ref. [39] relies on approximations up to \(O\left(\left(M_i^2 - M_j^2)/\bar{\omega}^2\right)^2\right)\), it is demonstrated in ref. [41] using the
CTP approach that the form with $\omega$ in the commutator indeed corresponds to the correct kinetic term to all orders. However, one should be aware of the fact that the collision term on the right hand side of eq. (2.8) is evaluated to order $[(M_i^2 - M_j^2)/\bar{\omega}^2]^0$ only. Extending to higher orders requires a gradient expansion of the convolution of Wigner functions, which is formally worked out also in ref. [41], but leads to considerable complications. In conclusion, the present form of the commutator term is not only sufficiently accurate approximation for the present purposes, but a consistent treatment to higher orders in $[(M_i^2 - M_j^2)/\bar{\omega}^2]$ would also imply a considerably more complicated form of the collision term. This has neither been worked out yet in the context of resonant Leptogenesis, nor is this necessary in order to obtain results to leading accuracy.

3 Non-relativistic approximations

Now, we consider a situation, where $\bar{M} \gg T$ (and all sterile neutrinos are assumed to be close together in mass, $|M_i - M_j| \ll \bar{M}$), as it is of relevance in strong washout scenarios around the time of freeze out. The main simplification arises here due to the fact that modes that do not satisfy $|k| \ll a\bar{M}$ are strongly Maxwell suppressed, such that we may approximate the four momenta as

$$k^\mu = (k^0, \mathbf{k}) \approx (\pm a\bar{M}, 0), \quad \tilde{k}^\mu \approx (0, k^0/|k|).$$

(3.1)

Due to the same reason, we can neglect the thermal contributions to the spectral self-energy of the sterile neutrinos, such that it takes its vacuum form

$$\left(\Sigma^A_N\right)^\mu = \text{sign}(k^0) \frac{k^\mu}{32\pi}.$$  

(3.2)

For the terms involving $\hat{\Sigma}_N^A$ that appear in eq. (2.8), this implies that we can take the approximate forms

$$k \cdot \hat{\Sigma}_N^A = \text{sign}(k^0) \frac{a^2 M^2}{32\pi}, \quad \tilde{k} \cdot \hat{\Sigma}_N^A = 0.$$  

(3.3)

Then, we integrate that equation with the result

$$\delta n_{0h}^{\pm} = \frac{a}{2M} i [M^2, \delta n_{0h}^\pm] + n_{eq}^\prime = - \frac{g_w a\bar{M}}{32\pi} \left\{ \text{Re}[Y^* Y^t], \delta n_{0h}^\pm \right\},$$

(3.4)

where we have defined

$$\delta n_{0h}^\pm = \int \frac{d^3k}{(2\pi)^3} \delta f_{0h}(\pm \omega(\mathbf{k}), \mathbf{k}).$$

(3.5)

This is the comoving non-equilibrium number density of sterile neutrinos, $\delta n_{0hij}^\pm = \delta n_{0hji}^\pm$, which is of the form of a Hermitian matrix. The comoving equilibrium number density is denoted by $n_{eq}$. The Majorana nature of the sterile neutrinos implies that $\delta n_{0hij}^+ = \delta n_{0hji}^-$, a property that is directly inherited from the distribution $\delta f_{0h}(\pm \omega, \mathbf{k})$ and that is derived in ref. [24]. Note that in the non-relativistic limit, the solutions for the sterile neutrino densities are helicity independent. The relativistic generalisation that accounts for helicity is worked out in ref. [24].
In order to substitute these results into the source term (2.3), we use the relations (2.6) that imply a vanishing axial density \( \delta f_{3h_{ij}} \) in the non-relativistic limit. Note moreover that the Dirac trace in eq. (2.3) selects then contributions from \( \delta f_{0h} \) only. The result for the flavoured source term in the non-relativistic approximation then is

\[
S_{ab} = \frac{aM}{16\pi} \sum_{i \neq j} Y_{ia}^* Y_{jb} \left( \delta n_{0hij}^+ - \delta n_{0hij}^- \right). \tag{3.6}
\]

Note that we do not sum over \( h \) here and make use of the fact that in the non-relativistic limit, we can approximate \( n_{\pm 0}^+ = n_{\pm 0}^- \).

4 Strong washout regime

In the radiation-dominated Universe, \( a(\eta) = a_R \eta \). A particularly convenient choice is \( \eta = 1/T \), what requires \( a_R = m_{\text{Pl}} \sqrt{45/(4g_\ast \pi^3)} \equiv T^2/H \). Moreover, one can then easily define the parameter \( z = M/T = M \eta \), that is often used in Leptogenesis calculations.

We investigate under which circumstances the maximal enhancement of the decay asymmetry can be attained. For this purpose, we solve the eq. (3.4) in the form that is obtained when using above parametrisation in terms of \( z \):

\[
\tilde{M} \frac{d}{dz} \delta n_{0h}^\pm(z) + \frac{iaR z}{2M^2} [M^2, \delta n_{0h}^\pm] + a_R z \frac{1}{2} \tilde{\Gamma} \{ \text{Re}[Y^* Y^\dagger], \delta n_{0h}^\pm \} + \tilde{M} \frac{d}{dz} n_{\text{eq}} = 0, \tag{4.1}
\]

where

\[
n_{\text{eq}} = 2^{-\frac{3}{2}} \pi^{-\frac{3}{2}} \frac{1}{2} z^2 e^{-z} a_R^3 \times \text{diag}(1,1) \tag{4.2}
\]

and \( \tilde{\Gamma} = 1/(8\pi) \). Since larger entries of \( Y \) correspond to larger washout, it is proposed in ref. [28] to obtain a simplified approximation in the strong washout regime by neglecting the first term of eq. (4.1). To put this more precisely, note that out of the first three terms of eq. (4.1), which are the homogeneous terms, the second and the third grow with \( z \). Therefore, neglecting the first term corresponds to taking the late-time limit of the solution. If the late time-limit applies before the freeze-out of the lepton asymmetry, that occurs for \( z = z_f \), it leads to a valid approximation of the freeze-out asymmetry.

It is conceptually interesting to include also thermal masses for the sterile neutrinos in addition to the Majorana masses within eq. (4.1). In the non-relativistic regime, the thermal mass squares are of order \( YY^\dagger T^2 \), which is to be compared with \( M \) times the width of the sterile neutrinos, what is of order \( YY^\dagger M^2 \). We therefore neglect this effect in the present context where we can assume that \( M \gg T \) and refer to ref. [29], where details on how to include thermal masses of the sterile neutrinos are worked out.

The evolution of the lepton asymmetry is governed by the equation

\[
-\tilde{M} \frac{d}{dz} \Delta_{ab} = g_w S_{ab} - \frac{1}{2} \{ W, \phi \}_{ab} - \frac{1}{2} W_{ab} \phi - \Gamma_{\ell ab}^q \tag{4.3}
\]

\[
\equiv 4 \epsilon_{ab}(z) \tilde{M} \frac{d}{dz} n_{\text{eq}} - \frac{1}{2} \{ W, \phi \}_{ab} - \frac{1}{2} W_{ab} \phi - \Gamma_{\ell ab}^q,
\]

where the last equality defines the time-dependent effective decay asymmetry \( \epsilon_{ab}(z) \), in consistency with eq. (4.7) below. In view of flavour effects, we have written this in terms of
the asymmetries $\Delta_{aa} = B/3 - q_{\ell aa}$ that are conserved by SM interactions and where $B$ is the baryon number density. Off-diagonal flavour-correlations can be accounted for by $\Delta_{ab} = -q_{\ell ab}$ for $a \neq b$, if necessary. Moreover, $q_\phi$ stands for the charge density in Higgs bosons, that is present in general. We have also expressed eq. (4.3) in a way that defines the decay asymmetry $\varepsilon$ as the lepton asymmetry that results from one sterile neutrino that initially drops out of equilibrium as a mass eigenstate. Note that the factor of four in front of $\varepsilon_{ab}$ arises because of the two helicity eigenstates of to the two sterile neutrinos. In addition, this equation includes the crucial washout term $W$ in its flavoured variant, that is derived in ref. [21], see also refs. [38, 42]. In the present context, we are interested in the situation where the sterile neutrinos are non-relativistic, such that the washout matrix can be approximated by

$$W = Y^\dagger Y \frac{3a_R}{2\pi^2} z^\frac{2}{5} e^{-z}.$$ (4.4)

Lepton-flavour violating interactions mediated through SM Yukawa-couplings are described by the term $\Gamma_{\ell ab}^{fv}$, that is defined and explained in ref. [21]. In the fully flavoured approximation, one assumes that these interaction delete the off-diagonal correlations in $q_\ell$ and $\Delta$. Effectively, one may then just set the off-diagonal elements to zero and ignore $\Gamma_{\ell ab}^{fv}$.

Solving eq. (4.1) when neglecting the derivatives acting on $\delta n_{0hij}$ yields for the off-diagonal correlations ($i \neq j$) of the sterile neutrinos

$$\delta n_{0hij} = \frac{\bar{M}}{2D} ([YY^\dagger]_{ij} + [Y^*Y^\dagger]_{ij}) ([YY^\dagger]_{ii} + [YY^\dagger]_{jj})$$

$$\times [\bar{M}^2 \Gamma([YY^\dagger]_{ii} + [YY^\dagger]_{jj}) - i(M_i^2 - M_j^2)] \times \frac{\bar{M}^2}{a_R z \frac{d}{dz} n^{eq}}.$$ (4.5)

where

$$D = [YY^\dagger]_{11}[YY^\dagger]_{22}(M_1^2 - M_2^2)^2$$

$$+ \bar{M}^4 \Gamma^2([YY^\dagger]_{11} + [YY^\dagger]_{22})^2 ([YY^\dagger]_{11}[YY^\dagger]_{22} - \text{Re}([YY^\dagger]_{12})^2).$$ (4.6)

To obtain simple analytic results, we have specialised here on a case when only two sterile neutrinos are dynamically relevant. For three and more sterile neutrinos in the game, one may still approximate eq. (4.1) by an algebraic equation when neglecting derivatives, but one does not find closed forms for the solutions as simple as in a situation that can be described by two sterile flavours only.

Comparing with eqs. (3.6) and (4.3), we identify the time-dependent effective decay-asymmetry

$$\varepsilon_{ab}(z) = \frac{1}{16\pi} \frac{a_R z}{\bar{M}} \sum_{i,j} \sum_{i \neq j} Y^a_i Y^b_j \left( \delta n_{0hij}^+ - \delta n_{0hij}^- \right) \left( \frac{d}{dz} n^{eq} \right)^{-1}.$$ (4.7)

It can be straightforwardly interpreted as the asymmetry yield per sterile neutrino that drops out of equilibrium. This quantity differs from the $CP$-violating parameter defined in ref. [28], that quantifies the yield in terms of the out-of-equilibrium neutrinos that are present at a

\footnote{Here, we define it in a different manner such that it is larger by a factor of two compared to its form in ref. [21].}
given point in time. The discrepancy is due to the time delay in the transition from diagonal out-of-equilibrium densities to off-diagonal correlations due to oscillations. We write the late-time limit of the decay asymmetry \((4.7)\) by dropping the argument \(z\), e.g. \(\varepsilon \equiv \varepsilon(\infty)\), for which we find when using eq. \((4.5)\)

\[
\varepsilon_{ab} = \frac{M\Gamma}{D} (M_1^2 - M_2^2) \bar{M} \left( [YY^\dagger]_{11} + [YY^\dagger]_{22} \right) \mathcal{Y}_{ab}, \quad (4.8)
\]

where

\[
\mathcal{Y}_{ab} = -\frac{i}{2} \left( Y_{a1}^\dagger [YY^\dagger]_{12} Y_{2b} - Y_{a2}^\dagger [YY^\dagger]_{21} Y_{1b} + Y_{a1}^\dagger [Y^*Y^\dagger]_{12} Y_{2b} - Y_{a2}^\dagger [Y^*Y^\dagger]_{21} Y_{1b} \right). \quad (4.9)
\]

Provided the strong washout approximation holds, it is then easy to solve eq. \((4.1)\) numerically. In the fully flavoured regime, \(q_{\ell ab}\) can be reduced to its diagonal components and the flavoured asymmetry can be calculated in straightforward generalisation (see e.g. refs. \[43, 44\]) of the methods for the single-flavour case \[9, 45\].

The flavoured expression \((4.8)\) for the decay asymmetry in resonant Leptogenesis is of importance throughout the parameter space. If the sterile neutrino mass is below \(10^9\) GeV, the usual treatment of flavoured Leptogenesis should apply, i.e. \(\varepsilon_{ab}\) can be reduced to its diagonal components, because interactions mediated by SM-lepton Yukawa-couplings effectively erase all coherence \[46, 47\]. (See however ref. \[38\] for a counterexample, where even Yukawa-suppressed correlations at low temperature are of importance, due to a special flavour alignment.) At higher temperatures, when the asymmetry results from the decay of one sterile neutrino only, it is sufficient to either deal with two (a linear combination of \(e\) and \(\mu\)) or one single flavour (a linear combination of \(e, \mu\) and \(\tau\)) only. Once the decay of more than one neutrino contributes, as it is the case for resonant Leptogenesis, there will be decay asymmetries in different linear combinations \[42, 48\] that in general cannot be aligned simultaneously. It then appears simplest to take the full expression for \(\varepsilon_{ab}\), including the off-diagonal correlations, and compute their evolution following ref. \[21\] (see also ref. \[38\]).

5 Applicability of approximations

The effective decay asymmetry \((4.8)\) and the equation for the evolution of the lepton asymmetry \((4.3)\) offer a simple way of accurately calculating the freeze-out asymmetry even in the resonant regime, where approximations based on the mass splitting of the sterile neutrinos being larger than their width are not applicable. In order to describe the parametric range of validity of neglecting derivatives acting on \(\delta n_{\ell h}^\pm\) in eq. \((4.1)\) more precisely, we first take the simplifying assumption of a single lepton flavour only. The effective decay asymmetry can then be expressed in the simple form

\[
\varepsilon = \frac{X \sin(2\varphi)}{X^2 + \sin^2(\varphi)}, \quad (5.1)
\]

where \(X\) is a dimensionless parameter defined as

\[
X = \frac{\Delta}{\Gamma(y_1^2 + y_2^2)}, \quad (5.2)
\]

and where \(\Delta = \frac{M_1^2 - M_2^2}{M_2^2}\) is the normalised mass difference, \(y_{1,2} = |Y_{1,2}|\) and \(\varphi\) is the relative phase of the Yukawa couplings, \(\varphi = \arg(Y_2/Y_1)\). Note that the solutions to eq. \((4.1)\) remain
unaltered as a function of $z$, provided we leave the ratios $\bar{M} : \Delta : Y^2$ invariant. Therefore, such a rescaling leaves $\epsilon(z)$ and the late-time solutions unchanged as well. This invariance can also be explicitly observed in the late-time asymmetry (5.1).

The late-time asymmetry (5.1) can also be constructed from the solutions given in ref. [28], such that we note agreement with the results of that work. However, our definition for $\epsilon$ differs from the $CP$-violating parameter proposed in ref. [28]. Our choice is motivated by the fact that the result (5.1) quantifies the yield of lepton asymmetry in a transparent manner and that it allows for a straightforward calculation of the final asymmetry, provided the late-time limit is a good approximation at the time of freeze out, what we illustrate in the remainder of this section.

The expression for the late-time decay asymmetry (5.1) only leads to an accurate approximation for the process of Leptogenesis, provided the solutions to eq. (4.1) reach their late-time form, where the derivatives acting on $\delta n_{0h}^\pm$ may be neglected, prior to the freeze-out of the asymmetry. Based on this requirement, we derive a more precise analytical condition that allows to identify the parametric regions where neglecting the derivatives of $\delta n_{0h}^\pm$ is indeed justified. Since $\delta n_{0h}^\pm$ are Hermitian two by two matrices and moreover, $n_{0h}^\pm = n_{0i}^\tau$, eq. (4.1) corresponds to a coupled set of four real differential equations. The smallest eigenvalue\(^2\) in vicinity of the parametric points where $\epsilon$ is close to unity [cf. eq. (5.7)] is given by $\epsilon = \epsilon_{R2}$, which is presented explicitly by eq. (B.1), or alternatively by

$$
\epsilon = \frac{a_{R} z}{2 \bar{M}} \left[ 2 y^2 \bar{\Gamma} - \frac{1}{\sqrt{2}} \left( - \Delta^2 + 4 y^4 \bar{\Gamma}^2 - 4 y_1^2 y_2^2 \bar{\Gamma}^2 \sin^2 \varphi \right) \right. \\
+ \left. \left( \Delta^4 + (4 y^4 - 4 y_1^2 y_2^2 \sin^2 \varphi) \bar{\Gamma}^2 + 2 \Delta^2 \bar{\Gamma}^2 \left( 4 y^4 + 4 y_1^2 y_2^2 (\sin^2 \varphi - 2) \right) \right) \right]^{1/2},
$$

(5.3)

where $y^2 = (y_1^2 + y_2^2)/2$. Notice also that $\epsilon$ is invariant when keeping the ratio $\bar{M} : \Delta : Y^2$ fixed. This is more easily seen in the democratic case $y_1 = y_2$, where the smallest eigenvalue is given by

$$
\frac{\epsilon}{\bar{\epsilon}} = 1 - \vartheta (\cos^2 \varphi - X^2) \sqrt{\cos^2 \varphi - X^2},
$$

(5.4)

where $\vartheta$ is the Heaviside step function and where we have defined $\bar{\epsilon} = (a_{R} z / \bar{M}) y^2 \bar{\Gamma}$. Since $(dn^{a \tau}/dz)/n^{a \tau} = O(1)$ around freeze out, one should require $\epsilon \gg 1$ in order to neglect derivatives acting on $\delta n_{0h}^\pm$. [A condition that amounts to requiring that the slowest eigenmode of eq. (4.1) is faster than the Hubble expansion rate.] This also implies that $\epsilon \gg \bar{\epsilon}/\epsilon$. The quantity $\bar{\epsilon}/\epsilon$ therefore is of phenomenological interest, because it indicates how strong the washout must at least be such that we can justify the neglect of the derivatives of $\delta n_{0h}^\pm$. In order to relate to the parameters that are typically employed in calculations on Leptogenesis, note that $\bar{\epsilon}/z = \bar{K} = (K_1 + K_2)/2$, where the $K_i = y_i^2 M\bar{\Gamma} / H |_{T = \bar{M}}$ are the usual washout parameters [9]. In order to satisfy $\epsilon \gg 1$ at the time of freeze-out, that occurs for $z = z_f = O(10)$, it follows that we must require

$$
\bar{K} \gg (1 / z_f) (\bar{\epsilon} / \epsilon).
$$

(5.5)

We can therefore use the ratio $\bar{\epsilon}/\epsilon$ in order to infer the minimal washout strength that is necessary for consistently neglecting the derivatives of $\delta n_{0h}^\pm$.

\(^2\)The eigenvalues presented in this work are for notational simplicity understood as minus one times the actual eigenvalues of eq. (4.1). The latter have negative real parts, because the equation describes the relaxation of $\delta n_{0h}^\pm$ toward zero.
Figure 1. The ratio $\bar{\epsilon}/\epsilon$ of the diagonal relaxation rate of the sterile neutrinos to the smallest eigenvalue, with $\varphi$ given by eq. (5.6). In order for the derivatives of $\delta n_{0h}$ to be negligible, the washout strength should satisfy relation (5.5).

Note that the washout strength $\bar{K}$ can also be employed as an expansion parameter for a series approximation that generalises the truncation of the derivative of $\delta n_{0h}^\pm$ in eq. (4.1) in a systematic manner. Details of this are worked out in appendix A.

It is interesting to consider the situation where, for a given value of $X$, the phase $\varphi$ maximises the decay asymmetry (5.1). This occurs for $\varphi = \varphi_M$, where

$$\varphi_M = \arctan \frac{X}{\sqrt{1 + X^2}},$$

and where the asymmetry is then given by

$$\epsilon = \frac{1}{\sqrt{1 + X^2}}.$$  \hspace{1cm} (5.7)

For $X \rightarrow 0$, the decay asymmetry attains its maximum value $\epsilon \rightarrow 1$. Curiously, in this case the CP-violating phase tends to be vanishing, $\varphi_M \rightarrow 0$. The exact limit can however not be reached because for such an alignment scenario, it takes infinitely long for the off-diagonal correlations in $\delta n_{0h}^\pm$ to build up. In particular, this does not occur before freeze-out. In the examples below, we observe however that it is possible in practice to obtain asymmetries that are at least close to maximal.

For comparison, we also comment the opposite regime, where $X \gg 1$ (which may still allow for $\Delta \ll 1$). In that case the asymmetry is maximal when $\varphi_M(X \gg 1) = \pi/4$.

Substituting $\varphi = \varphi_M$ and the value of $X^2$ in terms of $\epsilon$ from relation (5.7) into eq. (5.4), we find

$$\frac{\epsilon}{\bar{\epsilon}} = 1 - \vartheta(\epsilon^2 - 2 + \sqrt{2}) \sqrt{\frac{-\epsilon^2 - \sqrt{2}}{\epsilon^2 - 2 + \sqrt{2}}}.$$  \hspace{1cm} (5.8)

This ratio vanishes as the asymmetry $\epsilon$ goes to 1, which reflects the fact that for large asymmetries, it takes a longer time to build the off-diagonal correlations in $\delta n_{0h}^\pm$, and the washout should be sufficiently strong in order for the late-time decay asymmetry $\epsilon$ to be a good approximation. The ratio $\bar{\epsilon}/\epsilon$ is presented in figure 1.

As an illustration for how to interpret the quantity $\bar{\epsilon}/\epsilon$, in figure 2, we show how the parameter $\epsilon(z)$ [as defined in eq. (4.7)] evolves in the case where it approaches the late-time value $\epsilon = 0.98$. We choose two washout strengths, where the weaker one violates
Figure 2. Upper panel: Evolution of the parameter $\varepsilon(z)$ toward the late-time limit $\varepsilon = 0.98$ (dotted) and the value (5.6) for $\varphi$ that maximises the asymmetry. We choose two different washout strengths, $\bar{K} = 5$ (solid) and $\bar{K} = 20$ (dashed). Lower panel: Lepton asymmetry $|Y_\ell| = |\Delta_\ell|/s$ obtained from eq. (4.3) and with the single-flavour simplifications explained in the text, obtained with the time-dependent solution for $\varepsilon(z)$ and $\bar{K} = 5$ (solid) and $\bar{K} = 20$ (dashed) and with with the late-time limit $\varepsilon = 0.98$ (dotted) (the cases $\bar{K} = 5$ and $\bar{K} = 20$ are distinguishable by their proximity to the solutions for $z$-dependent $\varepsilon(z)$).

the criterion (5.5) while the stronger one marginally complies with it. In order to obtain these results, we assume vanishing initial distributions for the sterile neutrinos and begin to integrate at $z = 0$. We observe indeed that when relation (5.5) holds, where $z_f = \mathcal{O}(10)$, a stationary form for $\varepsilon(z)$ corresponds to a good approximation. To see the effect on the freeze-out lepton asymmetry, we take both, the late-time value $\varepsilon$ and the time-dependent solution $\varepsilon(z)$, and solve eq. (4.3), where we assume one single flavour (and consequently suppress the flavour indices), set $q_\phi = 0$ for simplicity and take $q_\ell = -\Delta_\ell$. We express the result in terms of the ratio of the lepton-number to the entropy density $s$, $Y_\ell = -\Delta_\ell/s$ and use the value for $s$ with 106.75 relativistic degrees of freedom. For both washout strengths, we observe that initially, there is a substantial deviation between the solutions for $Y_\ell$ that are based on the time dependent $\varepsilon(z)$ and its late-time limit. While for the larger washout strength, the freeze-out asymmetries agree eventually up to about 40% accuracy, there is a discrepancy of about a factor of five for the smaller washout strength, that does clearly not satisfy relation (5.5).

Next, we again take $y_1 = y_2$ but impose fixed values of $\varphi$, in order to allow for a deviation from the relation (5.6). In figure 3, the ratios $\bar{\epsilon}/\epsilon$ are presented as functions of $\varphi$.
for various values of $\varepsilon$. The curves exhibit two branches, because for a given asymmetry $\varepsilon$ and phase $\varphi$, eq. (5.1) has two solutions for $X$. The two branches join at the point where there is only one root. It is easy to show, using eq. (5.1), that the condition for a unique root is $\varepsilon = \cos(\varphi)$, for which $X = \sin(\varphi)$. There are two more curves that we display in figure 3. First, we show the ratios of the eigenvalues when identifying $\varphi = \varphi_M$, what fixes $X$ through eqs. (5.6), and with eq. (5.8), we obtain

$$\frac{\epsilon}{\bar{\epsilon}} = 1 - \vartheta \left( \cos^2(\varphi_M) - \frac{\tan^2\varphi_M}{1 - \tan^2\varphi_M} \right) \sqrt{\cos^2(\varphi_M) - \frac{\tan^2\varphi_M}{1 - \tan^2\varphi_M}}. \tag{5.9}$$

Second, we determine the value of $\varphi$ that minimises the eigenvalue ratio, what defines the graph

$$\frac{\epsilon}{\bar{\epsilon}} = 1 - \cos(\varphi) \sqrt{1 - \sin^2(\varphi) \sec(2\varphi)}. \tag{5.10}$$

From figure 3, we observe asymptotic proximity between these two curves (5.9) and (5.10), and moreover, one can check that the junction points for the two solutions for $X$ are close to these curves as well. This implies that $\varphi = \varphi_M$ corresponds to a preferable choice for obtaining large asymmetries not only because it maximises $\varepsilon$ but also because at the same time, it minimises $\bar{\epsilon}/\epsilon$ and therefore the required washout strength.

Again, we present in figure 4 the evolution of the parameter $\varepsilon(z)$ and the lepton-number to entropy ratio $Y_\ell$ for two different washout strengths, what exemplifies the use of the criterion (5.5) for approximating the freeze-out asymmetry using the late-time decay asymmetry $\varepsilon$.

We now move from the simplifying single-flavour model to a more realistic scenario, where several flavours are present and where we take account of constraints from neutrino
Figure 4. Upper panel: Evolution of the parameter $\varepsilon(z)$ toward the late-time limit $\varepsilon = 0.9$ (dotted). We choose $\varphi = 0.4$ and two different washout strengths, $\bar{K} = 2$ (solid) and $\bar{K} = 5$ (dashed). Lower panel: Lepton asymmetry $|Y_\ell| = |\Delta\ell|/s$ obtained from eq. (4.3) and with the single-flavour simplifications explained in the text, obtained with the time-dependent solution for $\varepsilon(z)$ and $\bar{K} = 2$ (solid) and $\bar{K} = 5$ (dashed) and with the late-time limit $\varepsilon = 0.98$ (dotted) (the cases $\bar{K} = 2$ and $\bar{K} = 5$ are distinguishable by their proximity to the solutions for $z$-dependent $\varepsilon(z)$).

oscillation data. In order to avoid a proliferation of free parameters, we consider the case where there are only two sterile neutrinos or, alternatively, where a third sterile neutrino decouples. It follows that one of the masses $m_{1,2,3}$ of the observed light neutrino states vanishes, i.e. $m_1 = 0$ for a normal mass hierarchy, which is what we assume here. This leads to a simplified form of the Casas-Ibarra parametrisation of the Yukawa couplings [49]

$$Y^\dagger = \frac{\sqrt{2}}{v} U_\nu \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{m_2} & -\sin \varrho & \cos \varrho \\ 0 & -\cos \varrho & -\sin \varrho \end{pmatrix} \begin{pmatrix} \sqrt{M_1} & 0 \\ 0 & \sqrt{M_2} \end{pmatrix},$$

(5.11)

where $U_\nu$ is the PMNS matrix and $v = 246$ GeV is the vacuum expectation value of the Higgs field. Note that here, $Y$ is a $2 \times 3$ matrix. For the PMNS matrix and for the light neutrino masses, we take the best-fit parameters from the global analysis of ref. [50] (see also [51]), and for simplicity, we fix the Dirac and the Majorana phase therein to be zero. The parameter $\varrho$ is a complex angle, and its imaginary part acts here in absence of the PMNS phases as the only source of $CP$-violation. Moreover, this imaginary part largely controls the absolute value of $\cos \varrho$ and $\sin \varrho$, i.e. large imaginary parts imply a large washout strength.
For definiteness, we are considering this setup at temperatures of about $10^8$ GeV, where all second-generation but none of the first-generation Yukawa couplings are in equilibrium. The qualitative picture does not change when going to different temperatures, where other spectator fields give rise to $\mathcal{O}(10\%)$ corrections to the freeze-out asymmetries [52–54]. We can then relate

$$ q_\ell = A \Delta_\ell, \quad q_\phi = C_\phi \Delta_\ell, $$

where

$$ A = \frac{1}{1074} \begin{pmatrix} -906 & 120 & 120 \\ 75 & -688 & 28 \\ 75 & 28 & -688 \end{pmatrix}, $$

$$ C_\phi = -\frac{1}{179} \begin{pmatrix} 37 \\ 52 \\ 52 \end{pmatrix}. $$

Moreover, at temperatures below $10^9$ GeV, the off-diagonal correlations of the left-handed leptons are strongly suppressed due to the SM Yukawa interactions, such that we can neglect the off-diagonal elements of eq. (4.3) (see however ref. [38], where due to alignments of the Yukawa couplings $Y$ the off-diagonal correlations remain non-negligible at even smaller temperatures).

It is also interesting to discuss the radiative processes that lead to small corrections to the leading-order rates for the production of the sterile neutrinos and the washout of the lepton asymmetry that we employ in eqs. (4.1) and (4.3). The dominating corrections are due to the radiation of gauge bosons and top-quark Yukawa-interactions. In the context of resonant Leptogenesis, these are discussed in ref. [6]. There has been some recent progress in that the cancellation of soft and collinear divergences in the thermal background was shown for non-relativistic sterile neutrinos, leading to a consistent calculation of these rates in the strong washout regime [55–57]. The corrections are found to be at the few percent level [55], and therefore we do not include these in the calculations for the present numerical examples. We note also that the cancellation of soft and collinear divergences has recently been demonstrated as well for relativistic massive sterile neutrinos in refs. [58, 59]. Besides, it was pointed out in ref. [60], that radiative corrections (in particular the thermal masses) have a subleading effect on the spectator processes because they change the susceptibility relation between the chemical potentials and the charge densities.

The eigenvalues of the equation for mixing and oscillating sterile neutrinos (4.1) in terms of the Casas-Ibarra parametrisation are given in eq. (B.3). As the oscillatory contributions due to the mass splitting enter as an imaginary part and the damping contributions due to the Yukawa couplings as a real part, we can find a lower bound on the magnitude of these eigenvalues by setting $\Delta = 0$, what leads to a considerable simplification of the expressions:

$$ \epsilon_{C1}^{CI} C_{R1,2} / \epsilon_{C1}^{CI} = \frac{m_2 + m_3 \pm (m_3 - m_2) \text{sech}(2\text{Im}[\varrho])}{(m_2 + m_3)}, $$

Since the smallest ratio is $\epsilon_{R1}^{CI} / \epsilon_{C1}^{CI} \gtrsim 1/6$ for normal hierarchy, neglecting the derivatives on $\delta n_{\ell h}$ in eq. (4.1) is by the criterion (5.5) (assuming $z_f = \mathcal{O}(10)$) a good approximation everywhere in the strong washout regime of resonant Leptogenesis for the phenomenological model with two sterile neutrinos. Moreover, as washout is always strong in that scenario, what we show in appendix C, we can conclude that using the late-time asymmetry (4.8) is a valid approximation for any point in parameter space.
Figure 5. Time-dependent flavoured decay asymmetries from eq. (4.7) (solid) compared to their late-time limit $\varepsilon_{aa}$ from eq. (4.8) (dashed). We take the parameters $\delta = 0$, $\alpha = 0$, $\varphi = \pi/4 + 0.2i$, $\Delta/M = -2 \times 10^{-17} \text{GeV}^{-1}$. We also present the individual flavoured baryon-minus lepton asymmetries $|Y_{\ell aa}| = |\Delta_{\ell aa}|/s$ obtained from eq. (4.3), using the time-dependent decay asymmetry (solid) and the late-time limit (dashed). The quantities $\Delta_{\ell aa}$, $q_{\ell aa}$ and $q_{\phi}$ are related through eqs. (5.12).

For the phenomenological model specified above, we solve eq. (4.3) with the effective decay asymmetry (4.7) based on the full numerical solution to eqs. (4.1). This, we compare with the solution obtained when using the late-time limit for the decay asymmetry (4.8) for all times prior to freeze-out. Since by above arguments, there should be no points where the freeze-out asymmetries obtained by the two methods differ by substantial amounts, we show in figure 5 the evolutions of $\varepsilon_{aa}(z)$ from eq. (4.7) and the values of $\varepsilon_{aa}$ from eq. (4.8), along with the asymmetries $|Y_{\ell aa}| = |\Delta_{\ell aa}|/s$ obtained using the time-dependent and the
effective late-time decay asymmetries for a typical point in parameter space, for which the width dominates the mass splitting, $\Delta \ll (\text{tr}[YY^\dagger]|F|^2)$. As anticipated from the analysis of the eigenvalues, albeit the different time evolution at early stages, the freeze-out asymmetries agree very well.

6 Comparison with other regulators for the decay asymmetry

Due to the conceptual interest in the question of the behaviour of the decay asymmetry in the resonant regime, a number of terms have been suggested earlier to avoid a resonance catastrophe from the enhancement factor $1/(M_i^2 - M_j^2)$ in the degenerate limit $M_i \to M_j$. For the purpose of comparing with these results, we recast the asymmetry (4.8) to the form

$$\varepsilon_{ab} = \frac{\mathcal{Y}_{ab}}{8\pi} \left( \frac{1}{[YY^\dagger]_{11}} + \frac{1}{[YY^\dagger]_{22}} \right) \frac{\tilde{M}^2(M_i^2 - M_j^2)}{(M_i^2 - M_j^2)^2 + R},$$

(6.1)

where

$$R = \frac{\tilde{M}^4}{64\pi^2} \left( \frac{[YY^\dagger]_{11} + [YY^\dagger]_{22}}{[YY^\dagger]_{11}[YY^\dagger]_{22}} \right)^2 \left( (\text{Im}[YY^\dagger]_{12})^2 + \det YY^\dagger \right).$$

(6.2)

Moreover, while the results of refs. [6, 26, 61] do not include active lepton flavour effects, it is easy to supplement these with the flavour structure of the SM leptons, which we do here for the sake of clarity of the comparison.

We should emphasise once more that the decay asymmetry (4.8) [or its equivalent form (6.1) with the regulator (6.2)] applies only to the strong washout regime and when all damping rates in the linear differential equation (4.1) are large compared to the Hubble rate at the time of the freeze out of the asymmetry. In general, the decay asymmetries will depend on how the initial state in terms of the sterile neutrinos is prepared [i.e. for the formulae (4.8), (6.1), the out-of-equilibrium neutrinos appear due to the expansion of the Universe,], and there may be a time dependence, matters which should be familiar from systems of mixing neutral mesons [1]. However, the results of refs. [6, 61] were thought to be universally applicable, which is not the case according to the present work and other recent publications on resonant Leptogenesis [24, 26–28]. We remark that the asymmetry from ref. [6] is supplemented in ref. [38] by extra terms that describe the asymmetry from oscillations. While it would be interesting to compare both approaches in detail, one may find the path taken here, i.e. to attribute the entire asymmetry to oscillations of sterile neutrinos as described by eq. (4.1), more economical.

We now quote some of the most widely discussed previous expressions for the decay asymmetry for resonant Leptogenesis. In ref. [6], a regulator is obtained in the standard $S$-matrix formalism by using a resummed form for the intermediate propagator of the sterile neutrino that occurs in the wave-function diagram, such that the sum of the decay asymmetries of two sterile neutrinos is found to be

$$\varepsilon_{ab} = \frac{\mathcal{Y}_{ab}}{8\pi} \left( \frac{\tilde{M}^2(M_i^2 - M_j^2)}{(M_i^2 - M_j^2)^2 + \frac{[YY^\dagger]_{11}}{64\pi^2} \tilde{M}^4 \frac{[YY^\dagger]_{22}}{64\pi^2}} + \frac{\tilde{M}^2(M_i^2 - M_j^2)}{(M_i^2 - M_j^2)^2 + \frac{[YY^\dagger]_{11} \tilde{M}^4 [YY^\dagger]_{22}}{64\pi^2}} \right).$$

(6.3)

Subsequently, in ref. [61] it is argued that the resummation needs to take account of the mixing of both sterile neutrinos. This results in an expression of the form (6.1) (provided we
approximate $M_1 M_2 \approx M^2$), but with the regulator term

$$R = \frac{M^4}{64 \pi^2} \left( [YY^\dagger]_{11} - [YY^\dagger]_{22} \right)^2. \quad (6.4)$$

When solving the equations for the oscillating sterile neutrinos, one may recover the corresponding corrections for vanishing initial distributions for the sterile neutrinos that relax towards thermal equilibrium with leptons and Higgs particles [24, 26] (a setup typically not applicable to cosmological contexts), provided the mass separation is larger than the width of the sterile neutrinos. In the interesting degenerate regime (mass splitting smaller than the width), the regulator (6.4) is however not applicable.

A correct form for the regulator for vanishing initial distributions of almost mass-degenerate sterile neutrinos that relax to equilibrium is obtained instead in ref. [26]:

$$R = \frac{M^4}{64 \pi^2} \left( [YY^\dagger]_{11} + [YY^\dagger]_{22} \right)^2. \quad (6.5)$$

This analytic result relies on the assumption that $| (YY^\dagger)_{12} | \ll | (YY^\dagger)_{11,22} |$ (what necessarily requires the sterile neutrinos coupling to several flavours of active leptons), under which the result (6.2) of this work reduces to the same form.\footnote{We would like to thank M. Garny for pointing this out.} This observation may be explained by the fact that provided $| (YY^\dagger)_{12} | \ll | (YY^\dagger)_{11,22} |$, the time-derivatives acting on the off-diagonal correlations of the sterile neutrinos in the equations that describe the neutrino oscillations are negligible.

### 7 Conclusions

We have studied the applicability of the late-time decay asymmetries $\varepsilon$ for sterile neutrinos in their multi-flavoured and single-flavoured forms (4.8) and (5.1) to computations of the freeze-out asymmetry in resonant Leptogenesis. This has been done by comparison with the results obtained from the time-dependent decay asymmetry (4.7) that is based on the solution to the evolution equation (4.1) for the mixing and oscillating sterile neutrinos. The evolution equation can be straightforwardly derived from its relativistic generalisation, that was first presented in ref. [24]. Following ref. [28], the approximations (4.8) and (5.1) are obtained by neglecting the time derivative acting on the non-equilibrium number densities and correlations in eq. (4.1).

In addition to the numerical comparisons, to gain analytical insight, we have derived expressions for the eigenvalues of the equation that governs the mixing of the sterile neutrinos and their deviation from equilibrium. This analysis reveals that $\varepsilon$ can reach its maximum value one provided $\Delta \to 0$ and $\varphi \to 0$ simultaneously. In that case however, also the smallest eigenvalue of the equation describing mixing and oscillations tends to zero, indicating that the approximation in terms of the late-time decay asymmetry is not valid in that limit. Nonetheless, the quantitative analysis (by studying the smallest eigenvalue as well as the numerical solution) reveals that the late-time asymmetry can be a good approximation already for moderately strong washout, even when $\varepsilon$ is close to one. To quantify this, cf. figures 1 and 3 in conjunction with the criterion (5.5). An increase of the washout strength generally leads to a better approximation.

While the derivation of the single-flavour decay asymmetry (5.1) makes use of the approximation proposed in ref. [28], its definition is different from the $CP$-violating parameter.
introduced in that work. We find the form that is suggested here somewhat more transparent, as it corresponds to the asymmetry yield per sterile neutrino that initially drops out of equilibrium through the Hubble expansion. Moreover, with its definition as in the present work, the parameter $\varepsilon$ can be employed in the same way the usual vacuum decay asymmetry is used in standard calculations on Leptogenesis \[8, 9, 54\]. We have exemplified this point by explicitly calculating the freeze-out lepton asymmetry in a phenomenological see-saw model that is consistent with the neutrino mixing and oscillation data.

We can draw the conclusion that the approximation proposed in ref. \[28\], which leads to the late-time asymmetries that we derive and study here, is applicable for Leptogenesis calculations in the strong washout regime of the single-flavour model, unless the $CP$ asymmetry and the mass splitting are very small simultaneously, cf. eqs. (5.3), (5.4), (B.1) and relation (5.5). For the phenomenological model with two sterile neutrinos that is consistent with the oscillations of active neutrinos, we find that the late-time asymmetries always lead to a good approximation for the freeze-out values of the lepton number densities. One potential caveat is that the early-time evolution of $\varepsilon(z)$ may strongly affect the asymmetry present within spectator fields, that in turn can have a substantial impact on the freeze-out lepton asymmetry \[63\]. It should also be noted, while the strong washout approximation always applies for resonant Leptogenesis with two sterile neutrinos, this does not need not to be the case when more of these are present. When the use of the late-time decay asymmetry cannot be justified, one should simply replace it with the time-dependent decay asymmetry (4.7) that is based on numerical solutions for the mixing and the oscillations of the sterile neutrinos. Methods for obtaining accurate quantitative results for Leptogenesis in the strong washout regime are therefore available throughout parameter space.

Acknowledgments

We would like to than M. Garny and A. Kartavtsev for valuable comments on the manuscript. This work is supported in parts by the Gottfried Wilhelm Leibniz programme of the Deutsche Forschungsgemeinschaft (DFG), by a DFG Research Grant, by the DFG cluster of excellence ‘Origin and Structure of the Universe’ and by the National Science Foundation under Grant No. NSF PHY11-25915. BG is grateful to the KITP at UC Santa Barbara for hospitality during completion of this work.

A Analytic expansion of the time evolution

When the derivative of the equilibrium distribution is neglected, eq. (4.1) becomes homogeneous, and can be solved exactly:

$$\delta n_{0h}^\pm (z) = e^{(\mp i\Omega - \frac{i}{2} \Gamma) z^2} \delta n_{0h}^\pm (z = 0) e^{(\mp i\Omega - \frac{i}{2} \Gamma) z^2},$$

where $\Omega$ is given by

$$\Omega = \frac{a_R}{M} \frac{M^2}{\bar{M}^2} = \bar{K} \begin{pmatrix} X & 0 \\ 0 & -X \end{pmatrix},$$

and $\Gamma$ by

$$\Gamma = \frac{a_R}{M} \text{Re} [Y^* Y^t] = \begin{pmatrix} K_1 \sqrt{K_1 K_2 \cos \varphi} \\ \sqrt{K_1 K_2 \cos \varphi} \end{pmatrix},$$
where $\bar{K} = (K_1 + K_2)/2$ and $X$ can in the single flavour case be identified with the parameter defined in eq. \ref{eq:K_def}.

To obtain the solutions, a matrix $\Xi$ is defined, similarly to the one in ref. \cite{24}:

\begin{equation}
\Xi = (\Gamma + i\Omega)/2. \tag{A.4}
\end{equation}

The solution \ref{eq:delta_n_0} can now be rewritten as:

\begin{equation}
\delta n_0^+(z) = e^{-\Xi z^2} \delta n_0^+(z = 0)e^{-\Xi^* z^2} \\
= U^{-1}e^{-\Xi z^2} U \delta n_0^+(z = 0)V^{-1}e^{-\Xi^* z^2} V, \tag{A.5}
\end{equation}

where the matrices $U$ and $V$ diagonalise $\Xi$ and $\Xi^*$. The corresponding eigenvalues $\Xi_D$ are:

\begin{equation}
\Xi_{D1,2} = \frac{\bar{K}}{2} \left(1 \pm \sqrt{\cos^2 \varphi + \Delta_K^2 \sin^2 \varphi - X^2 + 2i\Delta_K X}\right), \tag{A.6}
\end{equation}

where $\Delta_K = (K_1 - K_2)/(2\bar{K})$, which is zero in the democratic case. We define $\gamma$ and $\omega$ as the real and imaginary parts of the above root.

\begin{equation}
\gamma + i\omega = \sqrt{\cos^2(\varphi) + \Delta_K^2 \sin^2(\varphi) - X^2 + 2i\Delta_K X}. \tag{A.7}
\end{equation}

The transformation matrix $U$ is then given by:

\begin{equation}
U = c \begin{pmatrix}
\sqrt{1 - \Delta_K^2} \cos \varphi & -\gamma + i\omega + \Delta_K + iX \\
\gamma + i\omega + \Delta_K + iX & \sqrt{1 - \Delta_K^2} \cos \varphi
\end{pmatrix}. \tag{A.8}
\end{equation}

In the case of a symmetric matrix $\Xi$, if $c$ is chosen such that $\det(U) = 1$, the matrix inverse can be calculated as $U^{-1} = U^T$, and there is also the relation $V = U^*$. Rewriting eq. \ref{eq:n_1} in terms of $\Xi$ and $\Xi^*$, we can easily obtain the eigenmatrices:

\begin{equation}
e_{11} = U^{-1} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} V, \quad e_{12} = U^{-1} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} V, \tag{A.9}
e_{21} = U^{-1} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} V, \quad e_{22} = U^{-1} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} V,
\end{equation}

and the corresponding eigenvalues:

\begin{equation}
\lambda_{11} = \Xi_1 + \Xi_1^* = \bar{K}(1 - \gamma), \\
\lambda_{12} = \Xi_1 + \Xi_2^* = \bar{K}(1 - i\omega), \\
\lambda_{21} = \Xi_2 + \Xi_1^* = \bar{K}(1 + i\omega), \\
\lambda_{22} = \Xi_2 + \Xi_2^* = \bar{K}(1 + \gamma). \tag{A.10}
\end{equation}

It is important to notice here that $\lambda_{11}$ is equal to the smallest eigenvalue $\epsilon$ from eq. \ref{eq:eps_def} up to a factor of $z$.

\begin{equation}
\lambda_{11} = \epsilon \frac{z}{\bar{K}} = \bar{K}[1 - \theta(\cos^2 \varphi - X^2)\sqrt{\cos^2 \varphi - X^2}]. \tag{A.11}
\end{equation}
Now that we have the eigenvalues and eigenmatrices for the homogeneous system, we can find a solution for the inhomogeneous case. It can be constructed as:

\[
\delta n_{0h}^+(z) = -e^{-z^2/2} \left[ \int^z e^{z^2/2} \frac{dn_{eq}^1}{dz'} e^{z z'/2} dz' \right] e^{-z^2/2} = -U^{-1} e^{-z^2/2} \left[ \int^z e^{z^2/2} \frac{dn_{eq}^1}{dz'} V^{-1} e^{z^2/2} dz' \right] e^{-z^2/2} V. \tag{A.12}
\]

Next, we isolate the integral

\[
\left[ \int^z e^{z^2/2} \frac{dn_{eq}^1}{dz'} V^{-1} e^{z^2/2} dz' \right]_{ij} = [UV^{-1}]_{ij} \int^z e^{\lambda_{ij} z^2/2} \frac{dn_{eq}^1}{dz'} dz', \tag{A.13}
\]

substitute \( \tau = z^2/2 \) and then integrate by parts, what results in the series

\[
\int z^2/2 e^{\lambda_{ij} \tau} \frac{dn_{eq}^1}{d\tau} d\tau = \lambda_{ij}^{-1} e^{\lambda_{ij} \tau} \frac{dn_{eq}^1}{d\tau} \int z^2/2 - \int z^2/2 e^{\lambda_{ij} \tau} \frac{d^2 n_{eq}^1}{d\tau^2} d\tau = e^{\lambda_{ij} z^2/2} \left[ \frac{1}{\lambda_{ij}} \frac{dn_{eq}^1}{d\tau} - \frac{1}{\lambda_{ij}^2} \frac{d^2 n_{eq}^1}{d\tau^2} + \frac{1}{\lambda_{ij}^3} \frac{d^3 n_{eq}^1}{d\tau^3} + \ldots \right] z^2/2. \tag{A.14}
\]

Using this result in the expression for the particular solution (A.12), we obtain:

\[
[d\delta n_{0h}^+(z)]_{hk} = U^{-1}_{hi}(UV^{-1})_{ij} V_{jk} e^{-z^2/2} e^{\lambda_{ij} z^2/2} e^{-z^2/2} \sum_{m=1}^{\infty} \left( -\frac{1}{\lambda_{ij}} \right)^m \frac{d^m n_{eq}^1}{d\tau^m}. \tag{A.15}
\]

\[
= U^{-1}_{hi}(UV^{-1})_{ij} V_{jk} \sum_{m=1}^{\infty} \left( -\frac{1}{\lambda_{ij}} \right)^m \frac{d^m n_{eq}^1}{d\tau^m}. \tag{A.16}
\]

As only off-diagonal terms enter the source, we only need to calculate \( \text{Im}(\delta n_{0h}^+) \):

\[
\text{Im}[\delta n_{0h,12}^+] = \sum_{m=1}^{\infty} \zeta_m \left( -\frac{1}{K} \right)^m \frac{d^m n_{eq}^1}{d\tau^m}, \tag{A.17}
\]

where we have introduced \( \zeta_m \), which can be obtained by multiplying the matrices in eq. (A.15). In the democratic case, it takes the form:

\[
\zeta_n = -\frac{X \cos \varphi}{2 (\cos^2 \varphi - X^2)} (2 - (1 - \gamma)^{-n} - (1 + \gamma)^{-n}) \tag{A.18}
\]

We show the first few coefficients \( \zeta_m \) in table 1. When neglecting terms of order \( \propto 1/K^2 \) and higher, one can easily obtain the late-time effective decay asymmetry (5.1).

### B Eigenvalues in the CI parametrization

In the single flavour case, the eigenvalues of eq. (4.1) for the mixing and oscillating sterile neutrinos are given by

\[
\epsilon_{R1,2} = \frac{\alpha R z}{2M} \left( (y_1^2 + y_2^2) \Gamma \pm \text{Re} \sqrt{D} \right) \quad \text{and} \quad \epsilon_{11,2} = \frac{\alpha R z}{2M} \left( (y_1^2 + y_2^2) \Gamma \pm i \text{Im} \sqrt{D} \right), \tag{B.1}
\]
\[
\begin{array}{|c|c|}
\hline
m & \zeta_m \\
\hline
1 & \frac{X \cos \varphi}{\sin^2 \varphi + X^2} \\
2 & \frac{-X \cos(\varphi) \left( \cos(2\varphi) - 2X^2 - 5 \right)}{2(\sin^2 \varphi + X^2)} \\
3 & \frac{X \cos(\varphi) \left( \cos(4\varphi) - 8(X^2 + 1) \cos(2\varphi) + 8(X^2 + 2)X^2 + 39 \right)}{8(\sin^2 \varphi + X^2)^3} \\
\hline
\end{array}
\]

Table 1. The first three coefficients \( \zeta_m \)

where

\[
D = (y_1^2 - y_2^2)^2 \bar{\Gamma}^2 + 4y_1^2 y_2^2 \bar{\Gamma}^2 \cos^2 \varphi + 2i \Delta (y_1^2 - y_2^2) \bar{\Gamma} - \Delta^2. \quad (B.2)
\]

Similarly, with the Casas-Ibarra parametrisation of the phenomenological model, we obtain the eigenvalues

\[
\epsilon_{R1,2}^{CI} = \alpha R \bar{\Gamma} \left( \Gamma M \left[ (m_3 - m_2) \Delta \cos(2 \Re[\rho]) + 4(m_2 + m_3) \cosh(2 \Im[\rho]) \right] \pm \Re \left[ \sqrt{D_{CI}} \right] \right), \quad (B.3a)
\]

\[
\epsilon_{I1,2}^{CI} = \alpha R \bar{\Gamma} \left( \Gamma M \left[ (m_3 - m_2) \Delta \cos(2 \Re[\rho]) + 4(m_2 + m_3) \cosh(2 \Im[\rho]) \right] \pm i \Im \left[ \sqrt{D_{CI}} \right] \right), \quad (B.3b)
\]

where

\[
D_{CI} = 4 \Gamma^2 M^2 \left( \frac{1 - \Delta^2}{16} \right) (m_3 - m_2)^2 \sin^2(2 \Re[\rho]) \quad (B.4)
\]

\[
- \left[ \Delta \left( \nu^2 + 1 \frac{\Gamma M}{2} (m_2 + m_3) \cosh(2 \Im[\rho]) \right) + 2i \bar{\Gamma} (m_3 - m_2) \cos(2 \Re[\rho]) \right]^2.
\]

In order to compare the magnitude of the individual eigenvalues, we define in addition and in analogy with the single-flavour model the parameter

\[
\bar{\epsilon}_{CI}^{\text{CI}} = \frac{\alpha R \bar{\Gamma}}{2M} \text{tr}[Y Y^\dagger] \bar{\Gamma} = \frac{\alpha R \bar{\Gamma}}{4M \nu^2} \left[ 4(m_2 + m_3) \cosh(2 \Im[\rho]) + (m_3 - m_2) \Delta \cos(2 \Re[\rho]) \right]. \quad (B.5)
\]

C Washout strength in resonant leptogenesis with two sterile neutrinos

As for the equilibration of the sterile neutrinos, we note that

\[
\text{tr}[Y Y^\dagger] \frac{M}{8\pi H |T = \bar{M}|} \approx 108 \cosh(2 \Im[\rho]), \quad (C.1)
\]

which can be inferred by substituting the observed neutrino masses (with \( m_1 = 0 \)) and mixing angles \([50]\) into eq. (B.5). Using the relations (5.14) or (B.3), it is clear that all eigenmodes are faster than the Hubble expansion rate \( H \) at the time when \( T = \bar{M} \), what characterises strong washout.
For normal hierarchy, the $e$ flavour couples most weakly to the sterile neutrinos. We find that

$$[Y^\dagger Y]_{ee} = \frac{2M}{v^2} \left( [m_3 \sin^2 \vartheta_{13} + m_2 \sin^2 \vartheta_{12} \cos^2 \vartheta_{13}] \cosh(2\text{Im}[\varphi]) 
- 2 \sin \vartheta_{12} \sin \vartheta_{13} \cos \vartheta_{13} \sqrt{m_2 m_3} \sin \left( \frac{\varphi_2}{2} + \delta \right) \sinh(2\text{Im}[\varphi]) \right),$$

where we parametrise the PMNS matrix as in ref. [25]. The washout strength has its global minimum along the curve where $\sin(\alpha_2/2 + \delta) = 1$, where it is given by

$$\frac{[Y^\dagger Y]_{ee}}{32\pi H|_{T=M}} \approx 0.89 \cosh(2\text{Im}[\varphi]) - 0.84 \sinh(2\text{Im}[\varphi])$$

which takes for $\text{Im}[\varphi] = 0.87$ its minimum value 0.31. Therefore, the $e$ flavour will always equilibrate sufficiently long before freeze out at $z_f = O(10)$.

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4The apparent numerical coincidence in this equation is also noted in refs. [37, 62]. If $\sin^2 \vartheta_{13} (\sin^2 \vartheta_{12})$ were taking values close to the upper (lower) observational bounds [50], this may lead to an even more effective suppression of the $e$ flavour from washout, with interesting consequences for Leptogenesis from oscillations of sterile neutrinos [25, 30–37].
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