Investigation of critical behaviour from nuclear fragment yield ratios

R. Tripathi1, A. Bonasera1,2, S. Wuenschel1,3, L. W. May1,3, Z. Kohley1,3, G. A. Souliotis1,4, S. Galanopoulos1,#, K. Hagel1, D. V. Shetty1,##, K. Huseman1, S. N. Soisson1,3, B. C. Stein1,3, and S. J. Yennello1,3

1Cyclotron Institute, Texas A&M University, College Station, 77843, TX, USA
2Laboratori Nazionali del Sud-INFN, v. S. Sofia 64, 95123 Catania, Italy
3Chemistry Department, Texas A&M University, College Station, 77843, TX, USA
4Laboratory of Physical Chemistry, Department of Chemistry, National and Kapodistrian University of Athens, Athens, Greece

E-mail: rtripathi@comp.tamu.edu

Abstract. Nuclear fragment yield data has been analyzed using the Landau free energy description to investigate the critical phenomena in the fragmentation of quasiprojectile in the reactions 78,86Kr+58,64Ni at beam energy of 35 MeV/nucleon. The data on mirror nuclei yield ratio for A=3 and 7 showed an exponential dependence on the isospin asymmetry of the fragmenting source provided a reasonable estimate of the nuclear symmetry energy, and showed a systematic decrease with increasing excitation energy of the quasiprojectile.

1. Introduction

Investigation of the nuclear liquid-gas phase transition is currently one of the important research objectives of heavy-ion collisions in the Fermi energy domain. Various signatures have been employed to investigate the critical phenomena in nuclear systems [1-11]. Recently, Bonasera et al. [11,12] used fragment yield data from multiple reactions to investigate the nuclear phase transition using the Landau free energy approach [13,14], which is applicable to the systems in the vicinity of a critical point. In this work, the isospin degree of freedom was identified as an additional order parameter in nuclear phase transition. In this approach, the free energy per nucleon \( F \) of a fragment is related to an order parameter \( m \) as given by the following equation

\[
\frac{F}{T} = \frac{1}{2} am^2 + \frac{1}{4} bm^4 + \frac{1}{6} cm^6 - \frac{H}{T} m
\]  

(1)

where \( m = (N-Z)/A \), \( N, Z \) and \( A \) are the neutron, proton and mass numbers of the fragment respectively. The quantity \( m \), which is a measure of the isospin asymmetry of the fragment, can be defined as an order parameter if \( m = -\partial F/\partial H \), where \( H \) is its conjugate variable [13,14]. The coefficients \( a, b \) and \( c \) are fitting parameters [11,13] and \( T \) is the temperature of the fragmenting source. The
quantity $H$ acts as an external field, arising from the isospin asymmetry of the source, which shifts the critical value of $m$ (corresponding to $F=0$) away from zero to $m_s$, as observed in the studies of Ref. [11]. Thus, Eq (1) may also be written as [12]

$$\frac{F}{T} = \frac{1}{2}a(m - m_s)^2 + \frac{1}{4}b(m - m_s)^4 + \frac{1}{6}c(m - m_s)^6$$

(2)

where $m_i$ is the isospin asymmetry of the fragmenting source providing the external field. Comparing Eq (1) and (2), we get

$$\frac{H}{T} = am_s(1 + \text{'Higher order terms in' } m \text{ and ' } m_s)$$

(3)

Since fragment yields are proportional to $A^\tau e^{F/T/A}$, where $A$ is the fragment mass number and $\tau$ is the critical exponent, Landau approach can be used to explain the fragment yields in terms of their dependence on the isospin asymmetry of the fragmenting source. In an earlier work [11], the critical exponent $\tau$ was determined as 2.3 from the power law dependence of mass yields, which is a signature of critical behaviour. Using Eq (1)-(3), it can be shown that the ratio of the yields for a pair of mirror nuclei depends exponentially on the isospin asymmetry ($m_s$) of the fragmenting source while the power law dependence cancels out exactly. This can be tested by measuring the mirror nuclei yield ratios from the fragmentation of sources spanning a wide range of $m_s$ values.

This requirement can be fulfilled by studying the fragmentation of the projectile-like source (quasiprojectile) formed in peripheral and mid-peripheral collisions. With improved 4π-multi-detector systems it is possible to reconstruct such events by measuring the charge, mass and momentum of the detected particles/fragments. Reconstruction of the quasiprojectile leads to a better characterization of events and, in turn, a better control over the $m$ value. Furthermore, thermodynamic properties (such as the temperature) of the fragmenting source can also be determined. In the present work, the yield ratios of mirror nuclei pairs for $A=3$ ($^3$H, $^3$He) and $A=7$ ($^7$Li, $^7$Be) formed in the fragmentation of the quasiprojectiles in the reactions $^{78,86}$Kr+$^{58,64}$Ni at a beam energy of 35MeV/nucleon have been analyzed.

The main reason for choosing these isotopes was the available statistics of the data. Moreover, with increasing $Z$ of the fragment, Coulomb effects may become significant and complicate the analysis.

2. Experimental details and data analysis

The experiments utilized the Texas A&M University K500 superconducting cyclotron. Charged particles were measured using the NIMROD-ISiS array [15,16]. Neutrons were detected with the TAMU neutron ball [15] surrounding the NIMROD-ISiS array. The details of the experiment can be found in [17,18]. The quasiprojectiles sources, formed in peripheral and mid-peripheral reactions, were reconstructed by selecting events with the condition that the longitudinal velocity of the fragments with $Z=1$, 2, 3 be, respectively, in the range of ±65%, 60% and 40% of the velocity of the heaviest fragment in the event [18]. Further, the total $Z$ of the measured fragments was selected to be in range $30\leq Z\leq 40$ encompassing the projectile $Z$ of 36. With the four reaction systems, the yield ratios of mirror nuclei were determined over a wide range of $m_i$ from -0.03 to 0.21. This range of $m_i$ was divided into bins of width 0.02. The $m_i$ values were calculated after correcting for free neutrons emitted by the quasiprojectile using free neutron data from TAMU neutron ball [17,18].

3. Results and discussion

Based on the exponential dependence of the fragment yield on the quantity $F/T$, the yield ratio of a mirror nuclei pair with mass $A$ can be written as
\[
\ln \left( \frac{Y_2}{Y_1} \right) = -\frac{F_2 - F_1}{T} A = \frac{\Delta F}{T} A
\]

The subscripts ‘1’ and ‘2’ refer to the neutron deficient and neutron rich members of the pair of mirror nuclei, respectively. As evident from Eq (1), only the last (odd) term will contribute in the calculation of \( \Delta F \). By substituting the value of \( \frac{H}{T} \) from Eq (3) (ignoring the higher order terms) in Eq (4) we get

**Figure 1.** (a) Plot of ‘0.5 \( \ln\left(\frac{Y_2}{Y_1}\right) \)’ for \( A=3 \) and 7 as a function isospin asymmetry (\( m_s \)) of the quasiprojectile source for the reactions \(^{78,86}\text{Kr}^{+}\text{Ni}^{64}\) at beam energy of 35 MeV/nucleon. Different colours/symbols correspond to different reaction systems. Filled and Open symbols correspond to \( A=3 \) and \( A=7 \) respectively. The subscripts ‘1’ and ‘2’ refer to the neutron deficient and neutron rich members of the mirror nuclei pair. (b) Plot of ‘0.5 \( \ln\left(\frac{Y_2}{Y_1}\right) \)’, averaged over different reaction systems for \( A=3 \) and 7 as a function of \( m_s \). Solid and dashed lines are linear fit to the data for \( A=3 \) and 7 respectively.
Substituting \((m_2-m_1)=2/A\) for mirror nuclei ratio gives
\[
\frac{1}{A} \ln \left( \frac{Y_2}{Y_1} \right) = a m_s (m_2 - m_1)
\]
(5)

Thus, in the Landau approach, an exponential dependence of the yield ratio of mirror nuclei on the isospin asymmetry of the source is expected. Fig. 1(a) shows a plot of ‘0.5 \(\ln(Y_2/Y_1)\)’ for the four reactions systems as a function of mean \(m_s\) values. The error bars on the data are statistical errors. It can be seen in Fig. 1(a) that the mirror nuclei yield ratios indeed show a linear dependence, as predicted by Eq (6), for all four reaction systems. Linear fitting to the yield ratio data from different reaction systems gave slope values in close agreement as shown in Fig. 2. This observation suggests that once the experimental data are selected with a specific \(m_s\) value of the fragmenting source, they become independent of the reaction system. To further confirm this aspect, ordinate values from Fig. 1(a) for different reaction systems were averaged and subjected to linear fitting, as shown in Fig. 1(b). Filled and open symbols correspond to \(A=3\) and \(7\) respectively. The slope values obtained from the linear fitting of the average values were also in close agreement with those obtained from the individual reaction systems, as shown in Fig. 2 by solid lines. The observed linearity in Fig. 1 indicates that the condition \(m=-\partial F/\partial H\) is fulfilled and \(m\) is an order parameter [13]. This observation further suggests that the higher order terms in Eq (3) are not significant in the range of \(m_s\) values of the present study, however, some non-linearity starts appearing at the highest \(m_s\) value. It can be seen from Fig. 2 that the slope values for \(A=3\) and \(7\) are in reasonable agreement, suggesting that Coulomb...
effects are small, and the symmetry energy is the dominant contribution to the free energy. Since we are considering odd $A$ nuclei, pairing might be neglected if evaporation effects are not important, i.e. if the yields at freeze-out are not strongly modified due to secondary decays.

In the literature [11,12], a physical interpretation of the slope parameter ‘$a$’ can be obtained from the equivalence of the quantity $F/T$ with the symmetry energy per nucleon normalized with respect to the temperature of the source. In an approximate way, the coefficient ‘$a/2$’ of the first term in Eq (1) can be equated to $C_{sym}/T$, where $C_{sym}$ is the symmetry energy coefficient [17-22]. Thus, conversion of $a$ into $C_{sym}$ requires information about the temperature of the source. In the present study, the average excitation energy of the quasiprojectile was about 5 MeV/nucleon [17,18]. Details about the excitation energy calculation can be found in [17,18]. Based on the temperature compilation given in [8], an excitation energy of about 5 MeV/nucleon corresponds to a temperature of about 6 MeV. Using this temperature with the slope parameter $a$ as $\sim 6.9$ (corresponding to the fit to average yield ratios), we obtain $C_{sym}$ about 21 MeV. This value is lower compared to the value of $C_{sym}$ corresponding to the nuclear ground state as observed in other studies based on conventional isoscaling [17-22]. Further, in these studies [17-22], a systematic decrease in the $C_{sym}$ value with increasing excitation energy was observed.

In order to investigate this aspect in the Landau approach, the mirror nuclei yield ratio data for each $m_s$ bin was further divided into excitation energy bins of 0.6 MeV. Based on the fact that the mirror nuclei yield ratios become independent of the reaction system after sorting the data according to $m_s$, the data of all the four reaction systems were combined to improve the statistics. Fig. 3 shows a plot of the slope parameter $a$, extracted from plots similar to that in Fig. 2, as a function of excitation energy for $A=3$ and 7. It can be seen from this figure that $a$ values systematically decrease with increasing excitation energy of the fragmenting source as observed in recent isoscaling studies [17-22]. For comparison, $C_{sym}/T$ values obtained from the isoscaling studies for the same reaction systems from [17, 18] are also shown in Fig. 3. It can be seen from this figure that the $C_{sym}/T$ values obtained from the analysis of mirror nuclei yield ratios and isoscaling are in reasonable agreement. Here, we stress that $a$ in the Landau’s approach is the scale parameter of the distance from the critical point. The relation

![Figure 3](image-url)
between $a$ and $C_{\text{sym}}$ will be further investigated from an independent measurement of the temperature of the source [18].

4. Conclusions

In the present work, Fragment yield ratios have been analyzed using the Landau free energy approach. An exponential dependence of the mirror nuclei yield ratio on the $m$ value of the source was observed, as expected in the Landau approach. The relation between the conjugate field $H$ and the order parameter $m$ has been experimentally verified for the first time. The slope parameter ‘$a$’ gave a reasonable estimate of the symmetry energy coefficient $C_{\text{sym}}$ and showed a systematic decrease with increasing excitation energy of the source, consistent with recent observations from conventional isoscaling studies.

This work was supported by the U.S. DOE grant DE-FG03-93ER40773 and the Robert A. Welch Foundation grant A-1266.

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