Model-independent analysis of dark energy:
supernova fitting result

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Abstract
This paper uses supernova data to explore the property of dark energy by
some model-independent methods. We first Taylor expand the scale factor \( a(t) \)
and the luminosity distance \( d_L \) to the fifth order to find that the deceleration
parameter \( q_0 < 0 \). This result just invokes the Robertson–Walker metric. So
the conclusion that the universe is expanding with acceleration is more general.
Then we discuss several different parametrizations used in the literature. We
also propose two modified parametrizations. We find that \( \omega_{DE0} \) is less than \(-1\)
almost at \( 1\sigma \) level from all the parametrizations used in this paper. We also
find that the transition redshift from deceleration phase to acceleration phase is
\( z_T \sim 0.3 \).

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

The type Ia supernova (SN Ia) observations suggest that dark energy contributes \( 2/3 \) to the
critical density of the present universe [1–3]. SN Ia observations also provide evidence of a
decelerated universe in the recent past with the transition redshift \( z_T \sim 0.5 \) [4–6]. The cosmic
microwave background (CMB) observations favour a spatially flat universe as predicted by
inflationary models [7, 8]. There are many dark energy models proposed in the literature. For
a review of dark energy models, see, for example, [9, 10] and references therein. However,
the nature of dark energy is still unknown. It is not practical to test every single dark energy
model by using the observational data. Therefore, a model-independent probe of dark energy
is one of the best ways to study the nature of dark energy.

The type Ia supernovae (SNe Ia) as standard candles are used to measure the luminosity
distance–redshift relationship \( d_L(z) \). So we can model the luminosity distance \( d_L \) to study
the property of dark energy. Melchiorri et al first found that dark energy may be a phantom
type by combining different observational data to probe the behaviour of dark energy [11]. Huterer and Turner modelled the luminosity distance by a simple power law $d_L(z) = \sum_{i=1}^{N} c_i z^i$ [12]. Saini et al. used a more complicated function to model the luminosity distance [13]. Another way to probe the nature of dark energy is to parametrize the dark energy equation of state parameter $\omega_{DE}$. The simplest parametrization is the constant equation of state $\omega_{DE} = \text{constant}$. Several authors modelled $\omega_{DE}$ as $\omega_{DE} = \sum_{i=0}^{N_i} \omega_i z^i$ [14–16]. Apparently this parametrization is not good for high $z$. Recently, a stable parametrization $\omega_{DE} = \omega_0 + \omega_a z/(1+z)$ was used in [17–20]. By fitting the model to SN Ia data, we find that $\omega_0 + \omega_a > 0$, so this parametrization is not good at high $z$ either. Jassal, Bagla and Padmanabhan modified this parametrization as $\omega_{DE} = \omega_0 + \omega_a z/(1+z)^2$ and the problem was solved because $\omega_{DE} = \omega_0$ at present and at high $z$ [21]. More complicated functional forms for $\omega_{DE}(z)$ were also proposed in the literature [22–25]. We can also model the dark energy density itself. For example, a simple power law expansion $\Omega_{DE} = \sum_{i=0}^{N_i} A_i z^i$ was used to investigate the nature of dark energy [26–31]. There are other parametrizations, like the piecewise constant parametrization [32–35].

This paper is organized as follows. In section 2, we first use a Taylor expansion to expand the scale factor, then we fit the model to the whole 157 gold sample of SNe Ia compiled by Riess et al. in [6]. By expanding the scale factor, the fitting parameters have physical meanings. In section 3, we analyse the dark energy parametrization proposed by Alam et al. [26]. In section 4, we first study the parametrization $\omega_{DE} = \omega_0 + \omega_a z/(1+z)$ and point out that this parametrization is not good at high $z$. Then we study the parametrization $\omega_{DE} = \omega_0 + \omega_a z/(1+z)^2$. In section 5, we first investigate the parametrization proposed by Wetterich [25], then we propose two modified parametrizations. In section 6, we give some discussions.

### 2. Model-independent method

In a homogeneous and isotropic universe, the Friedmann–Robertson–Walker (FRW) spacetime metric is

$$ds^2 = -dr^2 + a^2(t) \left[ \frac{dr^2}{1-kr^2} + r^2 d\Omega \right].$$  

(1)

For a null geodesic, we have

$$\int_{t_1}^{t_0} \frac{dt}{a(t)} = \int_{r_1}^{r_0} \frac{dr}{\sqrt{1-kr^2}} = f(r_1),$$  

(2)

where

$$f(r_1) = \begin{cases} \sin^{-1} r_1, & k = 1, \\ r_1, & k = 0, \\ \sinh^{-1} r_1, & k = -1. \end{cases}$$  

(3)

From equation (2), we get the luminosity distance $d_L = a_0(1+z)r_1$ by Taylor expansion [36],

$$H_0d_L = z + \frac{1}{2}(1-q_0)z^2 + \frac{1}{2}(q_0 + 3q_0^2 - 1 - j_0 - \Omega_k)z^3 + \frac{1}{12}(2 - 2q_0 - 15q_0^2 - 15q_0^3 + 5j_0 + 10q_0j_0 + s_0 + 2\Omega_k + 6\Omega_kq_0)z^4 + O(z^5),$$  

(4)

where the redshift $z$ is defined as $1 + z = a_0/a(t)$, the subscript 0 means that a variable is evaluated at the present time, the Hubble parameter $H(t)$, the deceleration parameter $q(t)$, the jerk parameter $j(t)$ and the snap parameter $s(t)$ are defined as

$$H(t) = \dot{a}/a = \frac{1}{a} \frac{da}{dt},$$  

(5)
where \( \sigma_i \) and \( H \) are determined by minimizing
\[
\chi^2 = \sum_i \frac{[\mu_{\text{obs}}(z_i) - \mu(z_i)]^2}{\sigma_i^2},
\]
where \( \sigma_i \) is the total uncertainty in the SN Ia observation and the extinction-corrected distance modulus \( \mu(z) = 5 \log_{10}(d_L(z)/\text{Mpc}) + 25 \). In the fitting process, we use the SN Ia gold sample only and we consider a flat universe with \( \Omega_k = 0 \). Because we use Taylor expansion to get the luminosity distance, this expansion may break down at high \( z \). Therefore, we first use the full 157 gold sample SNe, then we use those 148 SNe with \( z \leq 1.0 \). The best fit parameters to the whole 157 gold sample SNe are \( (q_0, j_0, s_0) = (-1.1, 6.4, 39.5) \) with \( \chi^2 = 174.2 \). The best fit parameters to the 148 gold sample SNe with \( z \leq 1.0 \) are \( (q_0, j_0, s_0) = (-1.7, 14.4, 149.4) \) with \( \chi^2 = 160.8 \).

If we expand the luminosity distance \( d_L \) to the third order only, i.e., we only consider the parameters \( q_0 \) and \( j_0 \) in equation (4), then we find that the best fit parameters to the whole 157 gold sample SNe are \( q_0 = -0.64^{+0.25}_{-0.26} \), \( j_0 = 1.2^{+1.5}_{-1.4} \) and \( \chi^2 = 176.1 \). At 99.5% confidence level, \( q_0 = -0.64^{+0.50}_{-0.56} \), so we conclude that the expansion of the universe is accelerating with 99.5% confidence. From equation (10), we get \( \chi_T = q_0/(q_0 + 2q_0^2 - j_0) = 0.595^{+1.89}_{-0.39} \). The best fit parameters to the 148 gold sample SNe with \( z \leq 1.0 \) are \( q_0 = -1.0 \pm 0.4 \), \( j_0 = 4.7^{+1.1}_{-3.1} \) and \( \chi^2 = 161.3 \). At 99.5% confidence level, \( q_0 = -1.0^{+0.9}_{-1.0} \), so we conclude again that the expansion of the universe is accelerating with 99.5% confidence. With the best fit parameters, we find that \( \chi_T = q_0/(q_0 + 2q_0^2 - j_0) = 0.295^{+0.14}_{-0.09} \). The contour plot for \( q_0 \) and \( j_0 \) is shown in figures 1 and 2.

So far our analysis uses the FRW metric only, we have not specified any gravitational theory yet. The above results are applicable to a wide range of theories. For example, \( q_0 = (\Omega_{\text{cdm}} - 2\Omega_\Lambda)/2 \) and \( j_0 = \Omega_{\text{cdm}} + \Omega_\Lambda \) for the \( \Lambda \)-CDM model. If we expand the luminosity distance to the fifth order with the crackle parameter \( c(t) = (aH^5)^{-1} \) \( d^5a/dt^5 \), then we need to add to equation (4) the following correction:
\[
- \frac{1}{120} (6 + 14q_0 - 61q_0^2 - 160q_0^3 - 105q_0^4 + 110q_0j_0 + 105q_0^2j_0 + 15q_0s_0 + 27j_0 - 10j_0^2 + 11s_0 + c_0) z^5.
\]

(12)
The best fit parameters to the whole 157 gold sample SNe are \((q_0, j_0, s_0, c_0) = (-1.1, 7.6, 55.6, 676.6)\) with \(\chi^2 = 173.4\). The correction to \(H_0d_L\) at \(z = 1.5\) is about \(-1.3\) which is around 34%. The best fit parameters to the 148 gold sample SNe with \(z \leq 1.0\) are \((q_0, j_0, s_0, c_0) = (-1.5, 11.4, 101.2, 1484.2)\) with \(\chi^2 = 160.8\). The correction to \(H_0d_L\) at \(z = 1.5\) is about 0.19 which is around 8.5%. Therefore, the introduction of the fifth-order correction changes the value of \(q_0\) a little. We still have \(q_0 < 0\). It is clear that the kinematic determination of the cosmological parameters is better suited for low redshift SNe Ia.
However, from the observational data, \((z, \mu(z)) = (1.4, 45.09)\), \((z, \mu(z)) = (1.551, 45.3)\) and \((z, \mu(z)) = (1.755, 45.53)\), we see that \(\Delta \mu(z) = 0.21\) when \(\Delta z = 0.151\) and \(\Delta \mu(z) = 0.23\) when \(\Delta z = 0.205\). Theoretically, we know that \(d\mu(z) = (5/\ln(10)) (d\chi(z)/d\chi(z) dz)\). From equation (4), we get \(d\mu(z) = (5/\ln(10)) (n/z) dz\) if the luminosity distance is dominated by the higher term \(z^n\). Combining the above analysis, we find that \(n \sim 1\). Therefore, in this case, the higher term may not be the dominant term.

We conclude that \(q_0 < 0\) with 99.5% confidence. In other words, we conclude that the universe is expanding with acceleration.

3. ‘Taylor expansion’ of dark energy density

In this section, we parametrize the dark energy density as [26]

\[
\Omega_{\text{DE}}(z) = A_0 + A_1(1 + z) + A_2(1 + z)^2,
\]

where \(\Omega_{\text{DE}}(z) = 8\pi G \rho_{\text{DE}}(z)/(3H_0^2)\). \(\Omega_{m0} = 8\pi G \rho_{m0}/(3H_0^2)\) and \(A_0 = 1 - \Omega_{m0} - A_1 - A_2\). This parametrization is equivalent to equation (9) with \(\Omega_{m0} = -(s_0 + q_0 h_0)/3\). The relationship between \(\omega_{\text{DE}}\) and \(z\) is

\[
\omega_{\text{DE}} = \frac{1 + z}{3} \left( A_0 + A_1(1 + z) + A_2(1 + z)^2 \right) - 1.
\]

With the above parametrization, we find that \(\Omega_{\text{DE}} \ll \Omega_m\) and \(\omega_{\text{DE}} \approx -1/3\) when \(z \gg 1\). Combining the above two equations, we find that the transition redshift \(z_T\) satisfies

\[
\Omega_{m0}(1 + z)^3 - A_1(1 + z) - 2A_0 = 0.
\]

The best fit to the whole 157 gold sample SNe gives \(A_1 = -5.79\), \(A_2 = 2.9\) and \(\Omega_{m0} \sim 0\) with \(\chi^2 = 174.05\). If we use a Gaussian prior \(\Omega_{m0} = 0.3 \pm 0.04\) [41], then we get the best fit parameters \(A_1 = -4.2^{+1.4}_{-1.1}\) and \(A_2 = 1.7^{+2.2}_{-1.8}\) with \(\chi^2 = 174.21\). Substituting these parameter values into equation (14), we find that \(z_T = 0.35\). The evolution of the dark energy density and \(\omega_{\text{DE}}\) is shown in figure 3. Alam et al showed that the SNe Ia data favoured an evolving dark energy model by using the above reconstruction [26, 27]. They also showed that \(z_T \sim 0.4\). Our results are consistent with those analyses.

Because it is possible that \(\omega_{\text{DE}} < -1\), so we consider another two parameter representation of dark energy

\[
\Omega_{\text{DE}}(z) = B_0 + B_1(1 + z) + B_{-1}/(1 + z),
\]

where \(B_0 = 1 - \Omega_{m0} - B_1 - B_{-1}\). With this parametrization, we get

\[
\omega_{\text{DE}} = \frac{1}{3} \left( B_1(1 + z)^2 - B_{-1} \right) - 1.
\]

The above equation tells us that \(\Omega_{\text{DE}} \ll \Omega_m\) and \(\omega_{\text{DE}} \approx -2/3\) when \(z \gg 1\). Combining the above two equations, we find that the transition redshift \(z_T\) satisfies

\[
\Omega_{m0}(1 + z)^3 - B_1(1 + z) - 2B_0 - \frac{3B_{-1}}{1 + z} = 0.
\]

The best fit to the whole 157 gold sample SNe Ia gives \(B_{-1} = 6.87\), \(B_1 = 6.14\) and \(\Omega_{m0} \sim 0\) with \(\chi^2 = 173.2\). If we use a Gaussian prior \(\Omega_{m0} = 0.3 \pm 0.04\), then we get the best fit parameters \(B_{-1} = 4.1^{+3.7}_{-4.2}\) and \(B_1 = 2.8^{+3.4}_{-3.0}\) with \(\chi^2 = 173.65\). Substituting the best fit parameters into equation (16), we get \(z_T = 0.30\). The evolution of \(\omega_{\text{DE}}\) and \(\Omega_{\text{DE}}\) is shown in figure 4.
Figure 3. The best fit to the 157 gold sample SNe Ia with the prior $\Omega_m^0 = 0.3 \pm 0.04$. The upper panel shows $\omega_{DE}(z)$, the dotted dashed lines are the 1$\sigma$ regions. The lower panel shows $\Omega_m(z)$ and $\Omega_{DE}(z)$.

Figure 4. The best fit to the 157 gold sample SNe Ia with the prior $\Omega_m^0 = 0.3 \pm 0.04$. The upper panel shows $\omega_{DE}(z)$, the dotted dashed lines are the 1$\sigma$ regions. The lower panel shows $\Omega_m(z)$ and $\Omega_{DE}(z)$.
4. Stable parametrization

In this section, we first consider the parametrization [17, 18]

$$\omega_{DE} = \omega_0 + \frac{\omega_a z}{1+z}.\quad (17)$$

When $z \gg 1$, we have $\omega_{DE} \sim \omega_0 + \omega_a$. The dark energy density is

$$\Omega_{DE} = \Omega_{DE0}(1+z)^{3(\omega_0+\omega_a)}\exp\left(-\frac{3\omega_az}{1+z}\right).$$

Combining the above two equations, we find that $z_T$ satisfies

$$\Omega_{m0} + (1-\Omega_{m0})\left(1 + 3\omega_0 + \frac{3\omega_a z}{1+z}\right)(1+z)^{3(\omega_0+\omega_a)}\exp\left(-\frac{3\omega_az}{1+z}\right) = 0.\quad (18)$$

The best fit to the whole 157 gold sample SNe Ia gives $\omega_0 = -2.5$, $\omega_a = 3.7$ and $\Omega_{m0} = 0.46$ with $\chi^2 = 173.5$. If we use a Gaussian prior $\Omega_{m0} = 0.3 \pm 0.04$, then we get the best fit parameters $\omega_0 = -1.6^{+0.6}_{-0.8}$ and $\omega_a = 3.3^{+1.4}_{-2.7}$ with $\chi^2 = 173.92$. Substitute the best fit parameters into equation (18), we get $z_T = 0.35$. The evolution of $\omega_{DE}$ and $\Omega_{DE}$ is shown in figure 5.

From figure 5, we see that the dark energy density is greater than the matter density at high $z$ because $\omega_0 + \omega_a > 0$. So this stable parametrization may not be a good choice at high $z$. Recently, Jassal, Bagla and Padmanabhan considered the following parametrization [21]:

$$\omega_{DE} = \omega_0 + \frac{\omega_a z}{(1+z)^2}.\quad (19)$$

Figure 5. The best fit to the 157 gold sample SNe Ia with the prior $\Omega_{m0} = 0.3 \pm 0.04$. The upper panel shows $\omega_{DE}(z)$, the dotted dashed lines are the 1$\sigma$ regions. The lower panel shows $\Omega_{m}(z)$ and $\Omega_{DE}(z)$. 
When $z \gg 1$, we have $\omega_{DE} \sim \omega_0$. The dark energy density is

$$\Omega_{DE} = \Omega_{DE0}(1 + z)^{3(1 + \omega_0)} \exp\left(\frac{3\omega_a z^2}{2(1 + z)^2}\right).$$

Combining the above two equations, we find that $z_T$ satisfies

$$\Omega_{m0} + (1 - \Omega_{m0}) \left(1 + 3\omega_0 + \frac{3\omega_a z}{(1 + z)^2}\right)(1 + z)^{3\omega_0} \exp\left(\frac{3\omega_a z^2}{2(1 + z)^2}\right) = 0. \quad (20)$$

The best fit to the whole 157 gold sample SNe Ia gives $\omega_0 = -2.5$, $\omega_a = 7.6$ and $\Omega_{m0} = 0.42$ with $\chi^2 = 173.3$. If we use a Gaussian prior $\Omega_{m0} = 0.3 \pm 0.04$, then we get the best fit parameters $\omega_0 = -1.9^{+1.1}_{-0.4}$ and $\omega_a = 6.6 \pm 6.7$ with $\chi^2 = 173.41$. Substituting the best fit parameters into equation (20), we get $z_T = 0.30$. The evolution of $\omega_{DE}$ and $\Omega_{DE}$ is shown in figure 6. From figure 6, it is clear that the dark energy density did not dominate over the matter energy density at high $z$. Our result is consistent with that obtained in [21].

5. Wetterich's parametrization

In this section, we first consider the parametrization given in [25],

$$\omega_{DE} = \frac{\omega_0}{[1 + b \ln(1 + z)]^2}. \quad (21)$$

When $z \gg 1$, we have $\omega_{DE} \sim 0$. The dark energy density is

$$\Omega_{DE} = \Omega_{DE0}(1 + z)^{3\omega_0/[1 + b \ln(1 + z)]}.$$

Combining the above two equations, we find that $z_T$ satisfies

$$\Omega_{m0} + (1 - \Omega_{m0}) \left(1 + \frac{3\omega_0}{[1 + b \ln(1 + z)]^2}\right)(1 + z)^{3\omega_0/[1 + b \ln(1 + z)]} = 0. \quad (22)$$
The best fit to the whole 157 gold sample SNe Ia gives $\omega_0 = -1.84$, $b = 5.85$ and $\Omega_{m0} \sim 0$ with $\chi^2 = 173.09$. If we use a Gaussian prior $\Omega_{m0} = 0.3 \pm 0.04$, then we get the best fit parameters $\omega_0 = -2.5^{+1.3}_{-1.8}$ and $b = 4.0^{+1.1}_{-1.5}$ with $\chi^2 = 173.15$. Substituting the best fit parameters into equation (22), we get $z_T = 0.26$. The evolution of $\omega_{DE}$ and $\Omega_{DE}(z)$ is shown in figure 7. Because the best fit of the above parametrization gives $\Omega_{m0} \sim 0$, which is not physical, we first modify the above parametrization as

$$\omega_{DE} = \frac{\omega_0}{1 + b \ln(1+z)}. \quad (23)$$

When $z \gg 1$, we have $\omega_{DE} \sim 0$. The dark energy density is

$$\Omega_{DE} = \Omega_{DE0}(1+z)^3[1+b \ln(1+z)]^{3b_0/b}. \quad (24)$$

Combining the above two equations, we find that $z_T$ satisfies

$$\Omega_{m0} + (1 - \Omega_{m0}) \left(1 + \frac{3\omega_0}{1 + b \ln(1+z)} \right) [1+b \ln(1+z)]^{3b_0/b} = 0. \quad (24)$$

The best fit to the whole 157 gold sample SNe Ia gives $\omega_0 = -3.05$, $b = 36.8$ and $\Omega_{m0} \sim 0$ with $\chi^2 = 172.75$. If we use a Gaussian prior $\Omega_{m0} = 0.3 \pm 0.04$, then we get the best fit parameters $\omega_0 = -3.4^{+1.7}_{-1.1}$ and $\omega_a = 17.8^{+16.2}_{-16.4}$ with $\chi^2 = 172.91$. This modification does not solve the problem of $\Omega_{m0} \sim 0$. Substituting the best fit parameters into equation (24), we get $z_T = 0.25$. The evolution of $\omega_{DE}$ and $\Omega_{DE}$ is shown in figure 8.

Now let us consider another modification

$$\omega_{DE} = \omega_0 + \frac{\omega_a}{1 + \ln(1+z)}. \quad (25)$$
When $z \gg 1$, we have $\omega_{DE} \sim \omega_0$. The dark energy density is

$$\Omega_{DE} = \Omega_{DE0}(1 + z)^{3(1 + \omega_0)} [1 + \ln(1 + z)]^{3\omega_a}.$$  

Combining the above two equations, we find that $z_T$ satisfies

$$\Omega_{m0} + (1 - \Omega_{m0}) \left(1 + 3\omega_0 + \frac{3\omega_a}{1 + b \ln(1 + z)}\right)(1 + z)^{3\omega_a} [1 + \ln(1 + z)]^{3\omega_a} = 0. \quad (26)$$

The best fit to the whole 157 gold sample SNe Ia gives $\omega_0 = 2.2$, $\omega_a = -4.7$ and $\Omega_{m0} = 0.454$ with $\chi^2 = 173.47$. If we use a Gaussian prior $\Omega_{m0} = 0.3 \pm 0.04$, then we get the best fit parameters $\omega_0 = 2.4^{+2.6}_{-2.3}$ and $\omega_a = -4.1^{+1.3}_{-1.4}$ with $\chi^2 = 173.81$. Substituting the best fit parameters into equation (26), we get $z_T = 0.34$. The evolution of $\omega_{DE}$ and $\Omega_{DE}$ is shown in figure 9. Although this modification solves the problem of $\Omega_{m0} \sim 0$, it is not good at early times because the dark energy density dominates over the matter energy density at early times as shown in figure 9.

6. Discussions

The SN Ia data show that the expansion of the universe is accelerating. This conclusion derived from equations (4) and (12) does not depend on any particular model. We used the parametrizations (13), (15), (17), (19) and (21) proposed in the literature to discuss the properties of dark energy. We also proposed two modified parametrizations (23) and (25). By using the above parametrizations, we derived the equations satisfied by the transition redshift. In order to see the property of $\omega_{DE}(z)$, we re-plot $\omega_{DE}(z)$ for all the models considered in this paper together in figure 10. From figure 10, we see that (a) $\omega_{DE} < -1$. This is also true at
\[ \omega_{\text{DE}}(z) = \omega_0 + \omega_a / (1 + \ln(1+z)) \]

**Figure 9.** The best fit to the 157 gold sample SNe Ia with the prior \( \Omega_{\text{m0}} = 0.3 \pm 0.04 \). The upper panel shows \( \omega_{\text{DE}}(z) \), the dotted dashed lines are the 1σ regions. The lower panel shows \( \Omega_{\text{m}}(z) \) and \( \Omega_{\text{DE}}(z) \).

**Figure 10.** The evolution of \( \omega_{\text{DE}} \) for different parametrizations.

1σ level. So the current SN Ia data seem to marginally favour the dark energy metamorphosis suggested in [26, 27]. This does not mean that we can exclude the Λ-CDM model; (b) \( \omega_{\text{DE}}(z) \) increases when \( z \) increases. \( \omega_{\text{DE}}(z) \) changes more rapidly at low \( z \) than at high \( z \). This property
may be due to the choice of the parametrizations we made; (c) $z_T \sim 0.3$. We also see that the parametrization (19) is a good choice. It avoids the problem of the dark energy dominating the matter energy at early times and the best fit $\Omega_{m0}$ to the SN Ia data for this parametrization is not close to zero. The problem of $\Omega_{m0} \sim 0$ is not a serious problem because $\chi^2$ depends on $\Omega_{m0}$ weakly for all the models discussed in this paper. Daly and Djorgovski found that $z_T \sim 0.4$ by using a model-independent analysis [28, 29]. In our analysis, we used the Friedman equation and some priors to interpret the SN Ia data. As shown in [42], the interpretation of the observational data changes drastically if the priors are removed. We would like to stress that the results obtained in this paper are consistent with other model-independent analyses obtained in the literature [21, 26–30, 43, 32].

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