Adaptive Finite-time Synergetic Control of Delta Robot based on Radial basis Function Neural Networks

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Adaptive finite-time synergetic control of delta robot based on radial basis function neural networks

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Abstract: This paper proposes a novel robust proportional derivative adaptive non-singular synergetic control (PDATS) for the delta robot system. A proposal radial basis function approximation neural networks (RBF) compensates for external disturbances and uncertainty parameters. To counteract the chattering noise of the low-resolution encoder, a second-order sliding mode (SOSM) observer in the feedback loop showed the ability to obtain the angular velocity estimations. The stability of the PDATS approach is proven using the Lyapunov stability theory. Both the simulation and experiment result effectiveness and performances of the PDATS controller in trajectory; pick and place operations of a parallel delta robot. The characteristics of the controller demonstrate that the proposed method can effectively reduce external disturbance and uncertainty parameters of the robot by a convergent finite-time, and provide higher accuracy in comparison with finite-time synergetic control and PD control.

Keywords Adaptive synergetic control, dynamics delta robot, radial basis function approximation, second-order sliding mode observer.

Abbreviations

\( m_p \) Mass of the travelling plate

\( m_L \) Mass of external load

\( m_{fa} \) Mass of each forearm

\( r \) Optimal distribution of the three forearms mass

\( L_c \) Position of the center of arm mass

\( m_c \) Mass of the elbow
\( m_{ua} \) Mass of the each upper arm
\( I_{ai} \) Inertia of the each upper arm
\( L_A \) Length of upper arm
\( m_e \) Mass of elbow
\( \dot{X}_p \) Cartesian velocity vector
\( \ddot{X}_p \) Cartesian acceleration vector
\( \dot{q} \) Angular velocity vector
\( \ddot{q} \) Angular acceleration vector
\( J \) Jacobian matrix
\( J^T \) the transpose of Jacobian matrix
\( \dot{J} \) the derivative of Jacobian matrix
\( \psi \) the manifold variable
\( \lambda \) a positive-definite, diagonal constant matrix with \( \lambda \in R^{3 \times 3} \)
\( e \) Tracking error vector in joint space with \( e(t) \in R^{3 \times 1} \)
\( q_d \) Desired trajectory in joint space
\( q \) Actual joint position
\( \beta \) Quality of convergence
\( \phi_{ij}(.) \) Gaussian function of neural net \( j \)th.
\( W_i \) Weight vector of joint \( i \)th
\( W_i^* \) Update weight vector of joint \( i \)th.
\( j \) Hidden neural net number in the hidden layer,
\( h_i \) Output vector of hidden layer.
\( i \) Number order of joint
\( E_i \) Input vector of RBF NNs
\( c_{ij} \) Center of \( j \)th hidden node
\( W_{ij} \) Weight between the \( j \)th hidden node and the \( i \)th output node
\( \Gamma = [\gamma_1 \, \gamma_2 \, \gamma_3]^T \) Adaption speed of the control gains.
\( M \) Actual mass matrix of delta robot
\( \bar{M} \) Nominal mass matrix of delta robot
\( C \) Actual centrifugal and Coriolis term
\( \dot{C} \) Nominal centrifugal and Coriolis term

\( \tau_g \) Actual gravitational vector

\( \hat{\tau}_g \) Nominal gravitational vector

\( f_c \) Cutoff frequency

## 1 Introduction

Delta parallel robot is uncontrovertibly the most successful commercial parallel robot to date. In general, a parallel manipulator has the advantages of high precision trajectory combined with high speed and acceleration\cite{1},\cite{2}. Many industrial robots employ the type of parallel robot, consists of three arms connected to universal joints at the base. The robot is a multivariable, multi-parameter coupling, multi-degree-of-freedom nonlinear system. Many studies showed the remarkable effect of the kinematics \cite{3}\cite{4}\cite{5} and dynamics model \cite{6, 7} directly on precise trajectory performance. There are many control strategies for the robot. In the studies \cite{8},\cite{9}, the authors improved the performance via PD feedforward control based on the nominal dynamics robot system. In \cite{10}, A dynamic feedforward decoupling control scheme aimed at a compensated feedforward gravity and an independently adjustable three PID control loop, corresponding to the position, velocity, and acceleration control in simulation. The simulation results showed that the scheme was capable of controlling each motion factors effectively. But it is hard to practically implement because it can meet noise with the loop feedback signals. In addition to this, the closed-loop system did not include the influences of uncertainty parameters and unknown external disturbance.

To solve robust and high speed trajectory problems with the reduced influences of external disturbance, a sliding mode control (SMC) approach based on delta robot dynamics is introduced in \cite{11}, which is beneficial to remain the system trajectories on the sliding surface. Another SMC improvement is terminal sliding mode (TSMC) control \cite{12} applied to robotic manipulators for finite-time stability. Thanks to Lyapunov function analysis, these controllers verified the ability to superior performance. But both SMC and TSMC techniques have a drawback in that they produce the high-frequency oscillations of the controller output known as chattering.
To eliminate chattering phenomenon, another new methodology for modern technologies and complex systems led to the introduction of synergetic control (SC) theory [13]. SC employs a state-space method providing the object motion stability by the Analytical design of Aggregated Regulators (ADAR) method. It simplifies the original nonlinear mathematical model without losing its dynamical properties to make engineering implementation easier. Similar to sliding mode control properties, the advantages of synergetic control are well-suited for order reduction of the controlled system, robustness to external disturbance, and better control of the off-manifold dynamics. In many studies about synergetic literature[14–16], the synergetic-based controllers are characterized chattering-free dynamics compared to conventional sliding mode controls. In [17, 18] a finite-time synergetic control (FTS) scheme employed the combination between the synergetic theory and a terminal attractor technique for controlling series robot manipulators. The new synergetic approach which allowed to perform well in many areas of synthesis of multiply connected systems of continuous, discontinuous, discrete-time, terminal, and adaptive control for nonlinear dynamic objects such as robots [18][19], electric drives[20], and flying apparatus[21].

Delta parallel robot is highly nonlinear complexity system, so the design of the suitable control scheme with parametric and non-parametric uncertainties and external disturbance represent a significant challenge. Many pieces of research [22–24], Boudjedir and co-workers proposed the nonlinear controller to solve robust and precise trajectory problems using sliding mode or Nonlinear PD controller based on iterative learning. However, the scheme requires time consumption on some desired continuous repetitive or non-repetitive trajectories to adjust the PD parameters to improve precise performance and deal with the practical external disturbance. Additionally, an adaptive computed torque controller [25] was developed for trajectory tracking control of the 2-DOF parallel manipulator, and the adaptive law estimate the dynamics model includes the friction of active joints. The development of adaptive high-order sliding mode scheme [26] offer the alternative modulation gains to maintain the sliding in the presence of bounded and derivative bounded uncertainties and cancel chattering phenomenon. In [27], the authors present an adaptive sliding mode controller with switching gain which is adjustable real-time online by selecting the appropriate PID type sliding surface. Utilization of fuzzy logic system estimates the unknown nonlinear behavior of the
system and adapted successfully via synergetic control theory in some robots [28]. However, the choice of the linguistic rules and guaranteeing the system stability remain a challenging issue. A RISE (Robust Integral of the Sign Error) controller in [29] with a BSNN feedforward compensation was applied to a delta robot to regulate the trajectory tracking for a Pick and Place application. Since the addition of an intelligent compensation term may reduce the tracking error considerably and might cancel the steady-state error. However, only simulating validations is taken into account. In [30], a novel output feedback controller with a feedforward term based on the Radial Basis Function (RBF) neural network deals with compensating for uncertainties in the dynamics model of a robotic exoskeleton. This advanced control solution requires only states information for the RBF inputs. To deal with the discussed control challenges for the parallel manipulator, a first-time novel adaptive RBF neural network-based nonsingular finite time synergetic controller guarantees that all signal closed-loop systems remain bounded, and the tracking error converges to a small neighborhood of the origin. The uncertainty disturbance approximation is adapted online using the tracking error and its time derivative through manifold values on each trajectory in this paper.

The main contributions of this paper lie in the following aspects:

1. A systematic approach is developed to control a class of nonlinear systems with unknown uncertainty parameters in the nonlinear delta manipulator model and DC motor model.

2. An implement of high order sliding mode observer reduce the chattering of low resolution encoders for angular velocity estimation.

3. The unknown dynamic parameters and external disturbance of the system are estimated through online RBF 2-layer neural network systems.

4. The stability of the proposed control algorithm is analyzed and proven through Lyapunov stability theory.

5. The proposed controller is successfully applied to parallel delta robot in both simulation and practical experiments with different trajectories examples.

The rest of this paper is organized as follows. The nonlinear dynamic delta robot model and DC motor model are established and analyzed in sec 2. Then, the adaptive neural network synergetic control strategy is proposed in sec 3. Finally,
simulation in sec 4 and experiment in sec 5 results demonstrate the effectiveness of
the proposed method.

2 Nonlinear Dynamics of delta robot model

2.1 DC motor servo model

DC motor model is described by the electrical and mechanical differential
equations[31, 32]:

\[ u_A = Ri(t) + L \frac{di}{dt} + K_e \frac{d\theta}{dt} \tag{1} \]

\[ K_t i(t) = \tau_M(t) = J \frac{d^2 \theta}{dt^2} + f \frac{d\theta}{dt} + \tau_L + \tau_{EM} \tag{2} \]

Where \( u_a \) is input voltage applied to the motor’s armature, \( i(t) \) is the current in
the motor, and \( \omega(t) = \dot{\theta}(t) \) is the angular velocity of the motor shaft. The product
\( \tau_M(t) \) is the torque generated by the motor, and \( f \dot{\theta}(t) \) is the velocity dependent
friction in the motor. The back EMF voltage \( v_{emf} \) is proportional to angular
velocity of shaft by a constant factor \( K_e \).

Based on experimentation [31] and real data collection[33], using theoretical results
and simulation DC motor model to acquire the dynamic parameters of DC motor servo.

Table 1. Parameters of DC motor

| Symbol | Description       | Units | Values   |
|--------|-------------------|-------|----------|
| \( L \) | Motor Inductance  | H     | \( 1.8 \times 10^{-4} \) |
| \( R \) | Motor Armature resistance | Ω     | 3.4      |
| \( K_t \) | Torque constant  | Nm/A  | 1        |
| \( K_e \)  | Back EMF constant | Nm/A   | 1 |
|----------|------------------|--------|---|
| \( f \)  | Friction coefficient | Nm     | 0.01859 |
| \( J \)  | Motor inertia      | kgm²   | 0.0443 |

### 2.2 Delta robot manipulator

Consider the Delta robot illustrated in Fig. 2, consisting of three symmetric arms constrained in a kinematic manner by universal or spherical joint at the end-effector.

![Fig. 2 The delta robot in this article](image)

The inverse dynamic model of Delta robot developed in the work [8, 9] based on the Newton-Euler method with the following simplifying hypotheses:

- The rotational inertias of forearm are neglected.
- For analytical purposes, the masses of forearms are optimal separated into portions and places at their extremities: a two-third majority part at its upper extremity and the other part at its lower extremity, which is joined to the traveling plate mass.
- Fiction effects and elasticity are neglected

With the above mentioned simplifying hypothesis, the robot can be reduced to only 4 bodies: the travelling plate and the 3 upper arms. At the travelling plate, the total mass is calculated based on the hypothesis as

\[
m_{pt} = m_p + m_L + 3(1 - r) m_{fa}
\]

At the upper arm, the center of mass of the virtual arm based on the hypothesis is adjustable as
\[ L_c = L_A \left( \frac{1}{2} m_{ua} + m_c + rm_{fa} \right) \]

The inertia matrix of the upper arms is a diagonal matrix \( I_a = diag(I_{a1}, I_{a2}, I_{a3}) \) in joints space, where \( I_{a1} = I_{a2} = I_{a3} = I_{ai} \) is given by:

\[ I_{ai} = \frac{1}{3} m_{fa} L_A^2 + m_c L_A^2 \]  \hspace{1cm} (5)

| Parameter                          | Value   |
|------------------------------------|---------|
| Length of upper arm (m)            | 0.175   |
| Length of forearm (m)              | 0.25    |
| Radius of moving plate (m)         | 0.09    |
| Radius of fixed base (m)           | 0.07    |
| Mass of upper arm (kg)             | 0.046   |
| Mass of forearm (kg)               | 0.082   |
| Mass of connector (kg)             | 0.048   |
| Mass of moving plate (kg)          | 0.317   |

There are two kinds of forces act on the travelling plate: the gravity force and the inertia force. They are respectively given by:

\[ G_p = m_{pt} \begin{bmatrix} 0 & 0 & -g \end{bmatrix}^T \]  \hspace{1cm} (6)

\[ F_p = m_{pt} \ddot{x}_p \]  \hspace{1cm} (7)

We can transform into generalized angular coordinates by Jacobian, the description of Jacobian matrix are discussed in more detail in [1] and [3] so

\[ \tau_{Fp} = J^T F_p = J^T m_{pt} \ddot{x}_p \]  \hspace{1cm} (8)

\[ \tau_{Gp} = J^T G_p = J^T m_{pt} \begin{bmatrix} 0 & 0 & -g \end{bmatrix}^T \]  \hspace{1cm} (9)
According to the virtual work principle, We define $\tau_{Ga}$ is the torque produced by the gravitational force of the arms, so the torque contributions of delta robot applying to the motors is given by

$$
\tau_\Delta = I_a \ddot{q} + \int^T_m p_t (f \ddot{q} + \dot{j} \dot{q}) - f^T G_p - \tau_{Ga}
$$

(10)

The relationships between velocities $\dot{X}_p$, $\dot{q}$ and accelerations $\ddot{X}_p$, $\ddot{q}$ is described as

$$
\dot{X}_p = J \dot{q}
$$

(11)

$$
\ddot{X}_p = J \ddot{q} + \dot{j} \dot{q}
$$

(12)

From (10), we state the inverse dynamic model in function produced by the mass matrix $M(q)$, the Coriolis and centrifugal matrix $C(q, \dot{q})$ and the gravity contribution $\tau_g$ as equation below

$$
\tau_\Delta = M(q) \ddot{q}(t) + C(q, \dot{q}) \dot{q} + \tau_g
$$

(13)

Where:

$$
M = I_a + m_n J^T J
$$

(14)

$$
C(q, \dot{q}) = m_p J^T \dot{j}
$$

(15)

$$
\tau_g = -f^T m_n \begin{bmatrix} 0 & 0 & -g \end{bmatrix} - \tau_{Ga}
$$

(16)

$$
\tau_{Ga} = m_a L_c g \begin{bmatrix} \cos q_1 & \cos q_2 & \cos q_3 \end{bmatrix}^T
$$

(17)

3 The proposed controller

3.1 The finite time synergetic controller (FTS) for Delta manipulator

Rewriting the nominal class of nonlinear dynamics delta robot model (13) as follows:

$$
\dot{x} = f(x) + b(x) \tau_\Delta
$$

(18)
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -M(q)^{-1} \left( C(q, \dot{q})\dot{q} + \tau_g(q) \right) + M(q)^{-1} \tau_\Delta
\end{align*}
\]  
(19)

Where \( x_1 = [q_1 \quad q_2 \quad q_3]^T \) with \( x_1 \in \mathbb{R}^{3 \times 1} \); \( x_2 = [\dot{q}_1 \quad \dot{q}_2 \quad \dot{q}_3]^T \) with \( x_2 \in \mathbb{R}^{3 \times 1} \); \( f(x) \in \mathbb{R}^{3 \times 1} \); \( b(x) \in \mathbb{R}^{3 \times 3} \) and \( \tau_\Delta \in \mathbb{R}^{3 \times 1} \).

Step 1: The design procedure of the controller starts by choosing a macro-variable \( \psi \) which is generally a linear combination of the state variables.

\[
M_1 = \{ \varepsilon : \psi = s(\varepsilon) = 0, s(\varepsilon) \in \mathbb{R}^{3 \times 1} \}
\]  
(20)

Where \( \varepsilon = e = q_d - q \) and \( \psi = [\psi_1, \psi_2, \psi_3]^T \). The control law forces the trajectories to operate on the manifold \( \psi(\varepsilon) = 0 \) and to move toward this manifold exponentially according to the following equations:

\[
\dot{\psi} = \dot{e} + \lambda e
\]  
(21)

\[
\ddot{\psi} = \ddot{q}_d - \dot{q} + \lambda \dot{e}
\]  
(22)

\[
= \ddot{q}_d + \lambda \dot{e} - M^{-1} \tau_\Delta - M^{-1} \left( C(q, \dot{q})\dot{q} + \tau_g(q) \right)
\]  
(23)

Step 2: The proposed controller will be designed here such that it will force the states to approach the manifold \( M \) smoothly at finite time with a new evolution constraint according to the following constraint equation:

\[
\beta \dot{\psi}_p^r + \psi = 0 \implies \dot{\psi} = (-\beta^{-1} \psi)^T
\]  
(24)

Where \( \dot{\psi}_p^r = \left[ \psi_{p_1 r_1}, \psi_{p_2 r_2}, \psi_{p_3 r_3} \right]^T \), \( p_i \) and \( r_i \) are positive odd numbers which satisfy the condition \( 1 < \frac{p_i}{r_i} < 2, i = 1, 2, 3 \). This constraint will drive the macro variable \( \psi \) and its derivative \( \dot{\psi} \) to zero at finite time.

Step 3: From the constraint equation and substituting (24) into nonlinear dynamics robot model, one can obtain the resulting control law associated with delta manipulator can be expressed as

\[
(-\beta^{-1} \psi)^T = \ddot{q}_d + \lambda \dot{e} - M^{-1} \tau_\Delta + M^{-1} \left( C\dot{q} + \tau_g \right)
\]  
(25)
\[ \tau_\Delta = \tilde{M}(q) \left( \ddot{q}_d + \left( \frac{\psi^r}{\beta} \right)^P + \lambda \dot{\phi} \right) + \tilde{C}(q, \dot{q}) \dot{q} + \tilde{g}(q) \]  

(26)

**Lemma 1.** [12, 34] Suppose that a continuous, positive-definite function satisfies the following inequality

\[ \dot{V}(t) \leq -\alpha V^\eta(t), \forall t \geq 0, \quad V(t_0) \geq 0 \]  

(27)

Where \( \alpha > 0, \ 0 < \eta < 1 \) are constants. Then, for any given \( t_0 \), \( V(t) \) satisfies the following inequality:

\[ \frac{V(t) - V(t_0)}{t - t_0} \leq -\alpha V^\eta(t) \]  

(28)

\[ V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - \alpha(1 - \eta)(t - t_0), \quad t_0 < t < t_1; \]  

(29)

With \( t_1 \) is given by

\[ t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{\alpha(1 - \eta)} \]  

(30)

**Theorem 1:** Consider the nonlinear with control law above, the tracking error will converge to zero in the finite time with the convergence rate depending on the selected parameter \( \beta \)

Proof: To demonstrate the asymptotic stability of the system, we define a Lyapunov candidate function \( V = \frac{1}{2} \psi^T \psi \), and we have its derivative as

\[ \dot{V} = \psi^T \dot{\psi} \]  

(31)

Substituting \( \beta \dot{\psi}^P + \psi = 0 \) into the equation above, and then

\[ \dot{V} = \psi^T \left( -\beta^{-1} \psi \right)^P = -\beta^{-1} \left( \psi^P \right)^P \]  

(32)

\[ = -\beta^{-1} \left| \psi \right|^P \frac{p+r}{p} \leq -\psi^{-2} \frac{p+r}{2p} \sqrt{\psi^2} \frac{p+r}{2p} \]

\[ \leq -\beta^{-1}2^{p+r} \left( \frac{1}{2} \psi^2 \right)^{p+r} \]
\[
\dot{V} \leq -\beta_1 V(t) \frac{p+r}{2p} \frac{1 < p < 2}{r \rightarrow \dot{V} \leq 0}
\] (33)

Where \( \beta_1^{-1} = \beta^{-1} 2 \frac{p+r}{2p} \)

Compare with (28). The synergetic manifold \( \psi \) can converge to zero in the finite time, which depends on \( r, p \) given by

\[
t_1 = V^{\left[1 - \frac{p+r}{2p}\right]}(0) \frac{\beta_1^{-1}}{\left(1 - \frac{p+r}{2p}\right)}
\] (34)

3.2 The PD adaptive finite time nonsingular synergetic controller based on radial basis function neural network (PDAFTS)

The general equations of nonlinear dynamics Delta robot and DC motor servo, that including external disturbance is presented as follows

\[
\begin{cases}
\dot{x}_1 = x_2 \\
\dot{x}_2 = -M(q)^{-1}(C(q, \dot{q})\dot{q} + \tau_g + \tau_M + \tau_D) + M(q)^{-1}\tau \\
q = x_1
\end{cases}
\] (35)

To control the system model as (35), this paper proposes a PD-AFTS controller

\[
\tau = \tau_{FTSC} + \tau_{PD} + \dot{\tau}_D
\] (36)

**Assumption 1.** A proportional derivative control (PD) to estimate the torque matching with dynamics motor model. To guarantee stability of angular motor position without load, we choose \( K_P = \lambda K_D \):

\[
\tau_{PD} = \dot{\tau}_M \\
= K_P e + K_D \dot{e} \\
= \lambda K_D e + K_D \dot{e} = K_D \psi
\] (37)

**Assumption 2.** A finite time synergetic contributed torque is calculated on computer to overcome the nominal class of nonlinear dynamics delta robot

\[
\tau_{FTSC} = \dot{M}(q) \left( \dot{x}_d + \frac{(\psi \dot{p})}{\dot{p}} + \lambda \dot{e} \right) + \dot{\psi}(q, \dot{q})\dot{q} + \dot{\tau}_g(q)
\] (38)

Where \( \dot{M}(q), \dot{\psi}(q, \dot{q}), \dot{\tau}_g(q) \) are estimated as section 2.2
Assumption 3. The unknown parameter and external disturbance of practical nonlinear delta $\hat{\tau}_D = [\hat{\tau}_{d1}, \hat{\tau}_{d2}, \hat{\tau}_{d3}]^T$ with $\hat{\tau}_D \in \mathbb{R}^{3 \times 1}$, is estimated online by three RBF neural networks [30], and designed on computer as

$$\hat{\tau}_D(t) = \hat{W}^T h(q, \dot{q})$$

(39)

The adaptive law is designed as

$$\dot{\hat{W}}(t) = \Gamma h(q(t), \dot{q}(t)) \psi(t)$$

(40)

In the control system, if we use RBF to approximate $\hat{\tau}_D(t) = [\hat{\tau}_{1D}; \hat{\tau}_{2D}; \hat{\tau}_{3D}]$, the system states is often chosen as the input of RBF networks. The RBF algorithm is described as [35][36], we can choose the tracking error and its derivative value as the input vector

$$h_{ij}(t) = \phi_{ij}([\|E_i(t) - c_{ij}\|], b_j)$$

$$= \exp \left(- \frac{\|E_i(t) - c_{ij}\|^2}{b_j^2}\right)$$

(41)

By the use of the gradient descent, the weight are adapted as

$$W_i(t) = W_i(t - \Delta t_s) - \hat{W}_i(t)$$

$$W_i^* = W_i - \hat{W}_i$$

(42)

In totally, the proposal controller of the nonlinear delta robot is proposed as

$$\tau = M(q) \left( \ddot{x}_d + \left( \frac{\dot{y}^T}{\beta} \right) \frac{\tau}{\beta} + \lambda \ddot{e} \right) + \hat{C}(q, \dot{q}) \dot{q} + \hat{\tau}_g(q) + K_D \psi + \hat{\tau}_D$$

(43)

The system errors and error rate will converge to zero in finite time with the rate convergence depending on the parameter $\beta$, $p$, and $r$ if the control law is selected as (39). The adaptive control law $\hat{\tau}_D$ in (36) is used to estimate the external disturbance and model errors torque that can bring the tracking to the...
synergetic manifold surface. In the following, we will prove the asymptotic stability of the parallel manipulator system controlled by the PD-AFTS controller

**Assumption 4.** The uncertainty parameters and external disturbance are unknown but bounded. The characteristics of the rigid robotic manipulator, associated with (14)-(16) is used in the stability analysis and is described as follows, more detail please refer in [37, 38]:

i. The inertia matrix $M(q)$ is symmetric positive definite symmetric matrix.

ii. The matrix $\dot{M}(q) - 2C(q, \dot{q})$ is a skew symmetric matrix

**Theorem 2.** Assume that the desired trajectories $q_d$, $\dot{q}_d$, $\ddot{q}_d$ are bounded. Let the control input be given by (43) with (37), (39). Then, we ensure that $\psi$ is uniformly ultimately bounded.

**Proof:** If a Lyapunov candidate function is defined as

$$V = 0.5\psi^T M\psi + \frac{1}{2} \Gamma^{-1} \ddot{\tau}_D^T \ddot{\tau}_D$$

then we can have its derivative as

$$\dot{V} = \frac{1}{2} \psi^T \dot{M}(q)\psi + \psi^T M(q)\dot{\psi} + \Gamma^{-1} \ddot{\tau}_D^T \ddot{\psi}$$

Consider the structural property (ii) in **Assumption 4**, then one can have $\psi^T [\dot{M}(q) - 2C(q, \dot{q})] \psi = 0$ and:

$$\dot{V} = \psi^T C(q, \dot{q}) \psi + \psi^T M(q) \dot{\psi} + \Gamma^{-1} \ddot{\tau}_D^T \ddot{\psi}$$

From (21) and substituting (35) into (22), we have the equation below

$$M\ddot{\psi} = M(q)(\ddot{q}_d + \lambda \dot{e}) + C(q, \dot{q})(\dot{q}_d + \lambda e) + \tau_g(q)$$

$$-C(q, \dot{q})\dot{\psi} + \tau_M + \tau_D - \tau$$

Defining the following reference states as $q_r, \dot{q}_r$ and require torque $f_r(q, \dot{q}, \ddot{q}_r, \dot{q}_d)$ to control the general delta robot system are described by equations (48)-(50):

$$q_r = \dot{q}_d + \lambda e$$

(48)
\[ \dot{q}_r = \dot{q}_d + \lambda \dot{e} \quad (49) \]

\[ f_r(q, \dot{q}, q_r, \dot{q}_r) = M(q)\dot{q}_r + C(q, \dot{q})q_r + \tau_g(q) + \tau_M + \tau_D \quad (50) \]

Therefore, formula (47) can be simplified.

\[ M\dot{\psi} = f_r - C\psi - \tau \quad (51) \]

Substituting (51) into (46), we can get the following formula

\[ \dot{V} = \psi^T (f_r - \tau) + \Gamma^{-1}\dot{\tau}_D(\psi)^T \dot{\psi}(\psi) \quad (52) \]

We define the computed torque vector \( \hat{f}_r(q, \dot{q}, q_r, \dot{q}_r) \) from (43) as

\[ \hat{f}_r = \hat{f}(q, \dot{q}, \dot{q}_r, \dot{q}_r) + \hat{W}h = \hat{M}(q)(\dot{q}_r) + \hat{C}(q, \dot{q})(q_r) + \hat{\tau}_g - \hat{C}(q, \dot{q})(\psi) + K_d\psi + \hat{\tau}_D \quad (53) \]

With the ideal torque to control the delta robot as \( f_r = \hat{f} + \tau_D^* \); the computed torque \( \hat{f}_r = \hat{f} + \hat{\tau}_D \), for any \( \psi \), we have the difference between \( f_r \) and \( \hat{f}_r \) is caused by model errors and external disturbances[11, 39], denoted as \( \hat{\tau}_D \), which is set as follows

Since the disturbance are always bounded. Assume that \( \tau_D^* = W^* h(e, \dot{e}) + \varepsilon \) with (39), the error uncertainty parameters \( \hat{\tau}_D \) has an upper bound \( \varepsilon \), which can be written as

\[ \|\hat{\tau}_D\| = \|f_r - \hat{f}_r\| = \|\tau_D^* - \hat{\tau}_D\| \leq \varepsilon, \varepsilon > 0 \quad (54) \]

\[ \hat{\tau}_D = \tau_D^* - \hat{\tau}_D = \hat{W}^T h(e, \dot{e}) + \varepsilon \text{ where, } \hat{W} = W^* - \hat{W} \]

From (52)-(54), we have

\[ \dot{V} = \psi^T \left( -\hat{M}(q) \left( \frac{\psi}{\beta} \right)^T \right) + \hat{W}_D^T h(e, \dot{e})\psi + \Gamma^{-1}\hat{W}_D^T \hat{\dot{W}}_D, \quad (55) \]

\[ \dot{V} = \psi^T \left( -\hat{M}(q) \left( \frac{\psi}{\beta} \right)^T \right) + \hat{W}_D^T [h(e, \dot{e})\psi - \Gamma^{-1}\hat{W}] \quad (56) \]

Substituting the adaptive law (40) into (56), yields
\[ \dot{V} = \psi^T \left( -\ddot{M}(q) \frac{\psi}{\beta} \right) \leq 0 \]  

(57)

According to the Lyapunov theory, we prove that the proposed PDAFTS is stable for the tracking control of the delta robot. The equality \( \dot{V} = 0 \) is satisfied if and only if \( e = \dot{e} = 0 \). Since the Lyapunov function \( V \) is positive definite and \( \dot{V} \) is negative definite, it is concluded that the proposed PDAFTS is globally asymptotically stable from the Lyapunov stability criterion.

### 3.3 The velocity second order sliding mode observer

To implement the PDAFTS defined in (43), the measurement of angular position and velocity must be known. However, in the experimental delta robot system (35), we only use the encoder to measure joint position of robot arm. Hence, a velocity estimator is essential for practical implementation of the controller. Popularly, the velocity signals can be calculated by backward differentiator (BD) technique,

\[ \dot{q}_i(t) = \frac{q_i(t) - q_i(t - \Delta t_s)}{\Delta t_s} \]  

(58)

However, this approach provides less estimation due to the differentiating of the measured angular position signal, which does not only contain quantization noise due to the employed low resolution encoder but also the disturbance sampling time in iterations. To effectively estimate the velocity in real experiment, a combination of low pass filter (LPF) and second order exact differentiation via sliding mode (SOSM) [40, 41] is introduced in the experiment results of this paper. The algorithm of velocity estimator is presented as the equations (59) below

\[ \dot{q}_{SOSM}(t) = v(t) \]

\[ v(t) = v_1(t) - \eta|q(t) - q_d(t - \Delta t_s)|^{1/2} \text{sgn}(q(t) - q_d(t - \Delta t_s)) \]  

\[ \dot{v}_1(t) = -\alpha \text{sgn}(\dot{q}(t) - \dot{q}_d(t - \Delta t_s)) \]  

\[ \dot{q}_{LFP}(t) = \frac{\dot{q}_{SOSM}(t) + \frac{1}{f_c \Delta t_s} \dot{q}_{LFP}(t - \Delta t_s)}{\frac{1}{f_c \Delta t_s} + 1} \]  

(59)

(60)
Remark 1:
Using the PD controller is very realistic in the experiment, where the PD controller is implemented directly on the microcontroller and stabilizes the robot in the short term. And the complex computation is performed on the computer, then transfer data to guarantee the accuracy and robustness of the robot in the long term control.

Remark 2:
The proposed control guarantees the exponential convergence of the tracking error with the manifold sliding surface and eliminates the chattering phenomenon from the sign function.

Remark 3:
In the developed FTS in (38), the control system has a finite time convergence with the experience of external disturbance. Fortunately, the proposed PDAFTS based on RBF approximation can compensate for the effects of the disturbance torque or unknown parameters.

Remark 4:
It can be noticed from equation (59) that the velocity estimator using SOSM can achieve finite time error convergence no matter what the controller input. So the proposed controller and observer can be designed separately. In practical application, the encoder signal is always quantized and contaminated by noise, and the control effort leads to chattering. A combination of SOSM and LPF can reduce the noise and chattering effects.

4 The simulation results
In this section, helix and pick and place characteristics of the dynamic delta robot model are calculated. For reducing the robot vibration, the velocities and accelerations of the moving plate must become zero at the initial point and the final point. Meaningful, the planning trajectories followed by a quintic function of time will generate smoothing velocities and accelerations and meets the reducing vibration requirement. The fifth polynomial is as below, describes the quintic function.
\[ x_H(u) = R \cos(k2\pi u) \]
\[ y_H(u) = R \sin(k2\pi u) \]
\[ z_H(u) = hz_i u \]  \hspace{1cm} (61)

\[ x_p(u) = x_i + (x_f - x_i)u \]
\[ y_p(u) = y_i + (y_f - y_i)u \]
\[ z_H(u) = z_i - 4hu^2 + 4hu \]  \hspace{1cm} (62)

\[ u(t) = 10 \frac{t^3}{t_f^3} - 15 \frac{t^4}{t_f^4} + 6 \frac{t^5}{t_f^5}, (0 \leq t \leq t_f) \]  \hspace{1cm} (63)

Where \( x_H, y_H, z_H \) and \( x_p, y_p, z_p \) are coordinates of helix and parabolic curves at the determined time. The coordinates as \( x_i, y_i, z_i \) and \( x_f, y_f, z_f \) are initial and final position. In specifically, \( R \) is the radius and \( k \) is the number circle of the helix, whereas \( h \) is the height of parabolic on z axis. \( u(t) \) is the quintic function of time \( t \), and \( t_f \) is the duration of the movement, respectively. The information of these results are referenced values to evaluate the performance of the proposed controller. The characteristic curves of the nominal robot model in Fig. 4 and Fig. 5 provide the positions in Cartesian system, angular system, velocities, accelerations and desired joint torques for the delta manipulator. The parameters are calculated via inverse kinematics [1, 3] and the equation from (11) - (13), respectively.

Fig. 4. Helix trajectory characteristics: position transitions in workspace (a), versus time (b); in joint space (c), angular velocities (d); angular accelerations (e); torques of delta manipulator (f)
For simulating the dynamic delta robot model, which is a combination of dynamic DC motor from (1), (2), and the delta manipulator (13), described as Fig. 6. The control signal vector $\tau = [\tau_0, \tau_1, \tau_2]^T$ is the sum of the computed torques of ATFS and PD controller, calculated as the equations (41)
\[ RMSE_i = \sqrt{\frac{1}{n} \sum_{k=1}^{n} (q_{di_k} - q_{i_k})^2} \]  

(64)

And the control signal of the proposed controller is validated to comparison to the suggested other approaches. The controller parameters are chosen constantly in the two cases. Moreover, the adaptability of RBF neural networks is evaluated by the assumption of external disturbance as (65) into the delta robot model.

\[
\tau_{D1} = 0.4 \sin(\dot{q}_1) \\
\tau_{D2} = 0.8 \sin(\dot{q}_2) \\
\tau_{D3} = 1.1\sin(\dot{q}_3)
\]

(65)

The dynamic model of delta robot parameter are selected as Table 1 and Table 2. The parameters of controllers (43) in detail as (37), (38) and (39) are selected as \(K_P = diag(18,18,18), K_D = diag(1.8,1.8,1.8)\), the constraint in (24) takes the parameters as \(\beta^{-1} = 2000, r = 5, p = 7\). Otherwise, the manifold in (21) takes \(\lambda = 10\). The RBF NNs contains 7 nodes with the center vector is chosen as \(c_{qi} = [-0.03, -0.02, -0.01, 0, 0.01, 0.02, 0.03]\) and \(c_{\dot{q}i} = [-0.3, -0.2, -0.1, 0, 0.1, 0.2, 0.3]\). The initial neural network weights \(\hat{W}_i\) vector are selected as zeros. The updated weight law is given by (40) and (42) with the adaptation rates \(\Gamma = 0.005I\), where \(I\) is the positive unit diagonal matrix. These simulations are implemented with the interval time is \(\Delta t_s = 10^{-3} s\) in both cases.

![Fig. 7. The proposed control scheme.](image)
The desired trajectories for simulation use the information of helix and parabolic characteristics in Fig. 4 and Fig. 5 and generalized in Table 3. The objective of these cases is to validate the speed changes that affect the controllers’ stability.

Table 3. The parameters of helix and parabolic trajectory for delta robot

| Parameter                  | Helix trajectory | Parabolic trajectory |
|----------------------------|------------------|----------------------|
| Initial point $(x_0, y_0, z_0)$ (m) | $(0.05, 0, -0.35)$ m | $(-0.05, 0.05, -0.3)$ m |
| Final point $(x_f, y_f, z_f)$ (m) | $(0.05, 0, -0.25)$ m | $(0.05, -0.05, -0.3)$ m |
| Number of circles k | 4                | None                |
| Duration $t_f$ (s)     | 10               | 1                   |
| Maximum speed $v_{max}$ (m/s) | 0.24 m/s       | 0.4 m/s             |
| Maximum acceleration $a_{max}$ (m/s$^2$) | 1.2 m/s$^2$  | 2.8 m/s$^2$         |

For demonstrating different velocities and acceleration tracking along the generated paths, the fundamental two operations of the delta robot were performed with the external disturbance. The tracking error in joint space for the five controllers is illustrated in Fig. 8 for the helix case and Fig. 11 for pick and place object assumption. It is possible to see that the speed of joints is increasing causes the amplitude of the tracking error to the peaks. This effort is a reason for error increment for the higher maximum speed and acceleration in parabolic trajectory to compare that in helix trajectory. The RMSE values of the PDFTS and PDFTS controller in Table 4 and Table 5 are the lower values than those of the others, and the proposed controller provides the best performance.

The control signal of the controllers is presented in Fig. 9 and Fig. 12. In general, the control signals provided by the five controllers have the approximated transition, whereas the controllers based synergetic demonstrated the better strength signal of the control outline. The control signals from PDSM show the drawback with a chattering to track errors on the manifold surface.

In Fig. 10 and Fig. 13, it is noteworthy that the behavior of external torque estimation is similar to the added external torque. The effective action of RBF approximation leads the estimated disturbance torques to converge to the curve of the disturbance torques, which is described as the equation in (64). The accurate compensation to control signal leads to the highest performance among the simulated controller in both study cases.
Table 4. RMSE comparison among the controllers and PDAFTS in the helix simulation

| Joint 1 (Degree) | Joint 2 (Degree) | Joint 3 (Degree) |
|------------------|------------------|------------------|
| **PD**           | 0.0380           | 0.0420           | 0.0511           |
| **PDFF**         | 0.0237           | 0.0313           | 0.0388           |
| **PDSM**         | 0.0235           | 0.0301           | 0.0378           |
| **FTS**          | 0.0033           | 0.0043           | 0.0059           |
| **AFTS**         | $7.2 \times 10^{-4}$ | 0.0018           | 0.0022           |

Fig. 8. Tracking error comparison among the controllers and PDAFTS in the helix simulation

Fig. 9. Control signals generated by the controllers in the helix simulation
Fig. 10. Disturbance torques estimation and weight update of joints from RBF NNs in the helix simulation

Table 5. RMSE comparison among the controllers and PDAFTS in the parabolic simulation

|                | Joint 1 (Degree) | Joint 2 (Degree) | Joint 3 (Degree) |
|----------------|------------------|------------------|------------------|
| PD             | 0.0565           | 0.0716           | 0.0802           |
| PDFF           | 0.0509           | 0.0673           | 0.0800           |
| PDSM           | 0.0453           | 0.0615           | 0.0741           |
| PDFTS          | 0.0161           | 0.0195           | 0.0208           |
| PDAFTS         | 0.0123           | 0.0150           | 0.0136           |

Fig. 11. Comparison tracking error among the controllers and PD AFTS in parabolic simulation
5. Experiment

5.1 System description

In this section, the proposed control scheme is developed and evaluated on the experimental setup in Fig. 14. The experiment setup A system of the three motors is driven by the three MD01 high-power motor driver boards. The UNO R3 ATmega328 provides signal control to the motor driver. For the stability assurance of the robot, the PD controller is programmed directly on the UNO R3 and combines the AFTS control signal from a computer to generate the control signal. On the DC motor, the active joint angles are acquired by the mounted three encoders, and the accuracy of the Hall encoders has 0.367 degrees of rotation per pulse based on the specifications. Then the UNO R3 sends the feedback position signal to the computer for torque calculation in the next iteration. The desired joint positions of the delta robot and AFTS torque are verified on the computer, then transferred via serial port to the microcontroller board. We programmed the AFTS
controller and GUI control with C# Visual Studio, and the algorithm runs on a Ryzen 7 4800HS CPU, 16GB Ram, and GeForce GTX 1660Ti graphic card. The estimated sampling rate in the real-time system is 0.5ms on the UNO program, and the computer changes from 14ms to 20ms.

Fig. 14. The experiment setup

5.2 Velocity estimation results

For validating the estimator, the fifth polynomial reference position based on helix trajectory as (61) and (63) is implemented to the robot. The position measurements of Hall encoders suffer from quantization errors. Fig. 15 presents the velocity estimation using BD and the proposed approach. For comparison to the desired velocity, angular velocity estimations obtained by the BD technique show large spikes and not continuous states as the realistic velocity. In contrast, the SOSM + LPF estimator not only gives the fitting the desired values but also reacts with the spikes of the BD technique.
5.3 Experimental results

Experiment 1: Helix and parabolic trajectory without external load

In this section, the two cases experiments were conducted based on the desired trajectories in the simulation section. The experimental results in helix and parabolic trajectories were illustrated in Fig. 16, Fig. 17 and Fig. 20, Fig. 21, respectively. For comparing the simulation results, the tracking performance witnesses an accurate reduction in the RMSE in Table 6 and Table 7. And because of the quantization of feedback encoder values and disturbance of interval time, the tracking signals from these figures show slight fluctuations compared to those in simulations. However, the tracking errors of controllers based on the synergetic theorem are also noticeably under one tick resolution in RMSE in the first path and just over 2 degrees maximum error in the second path, smaller than those of the other approaches.

It is noteworthy that the behavior of the proposed controller in Fig. 18 and Fig. 22. is very similar to the torque produced by the remaining schemes. The speeding up velocity effort to the tracking performance and control signal consolidates the analyses in the simulation section. Although the SMC can guarantee stability, it is obtained clearly at the price of high control chattering in the control signal graph.
Meanwhile, the smoothing of control output from the synergetic algorithm demonstrated the advantage in maintaining the attractiveness of the boundary layer. The adaptive unknown torque approximation term in the proposed approach reduced quite well the effect of the disturbance, resulting in better dynamics performances than the other four methods. The updated weight in Fig. 19 and Fig. 23 presented the reaction with errors fluctuation and convergence to a constant value as soon as the system becomes more immovable. The power of torque estimations of the parabolic case witnesses a higher value than that of the helix case, so it is possible to track well the fast transition of external disturbances.

Fig. 16. Helix trajectory performance of joints (a, b, c) and tracking error of joints (d, e, f) of the experiment 1
Fig. 17. Helix tracking performance in workspace of the experiment 1

Table 6. RMSE comparison among the controllers and PDAFTS in the helix experiment 1

|               | Joint 1 (Degree) | Joint 2 (Degree) | Joint 3 (Degree) |
|---------------|------------------|------------------|------------------|
| **PD**        | 0.6115           | 0.6256           | 0.6648           |
| **PDFF**      | 0.5867           | 0.5918           | 0.6097           |
| **SMC**       | 0.4508           | 0.5808           | 0.4673           |
| **FTS**       | 0.3632           | 0.4157           | 0.4086           |
| **AFTS**      | 0.3221           | 0.3553           | 0.3720           |
Fig. 18. Control signals generated by the controllers in the helix experiment 1

Fig. 19. External disturbance approximation and weight update of joints from RBF NNs in the helix experiment 1

Table 7. RMSE comparison among the controllers and PDAFTS in the parabolic experiment 1

| Joint 1 (Degree) | Joint 2 (Degree) | Joint 3 (Degree) |
|------------------|------------------|------------------|
| PD               | 1.6516           | 1.5905           | 1.8641           |
| PDFF             | 1.5290           | 1.5471           | 1.5646           |
| PDSM             | 1.5417           | 1.6055           | 1.5207           |
Fig. 20. Parabolic trajectory performance in joint space (a, b, c) and tracing error (d, e, f) in experiment 1

Fig. 21. Parabolic trajectory performance in workspace
Experiment 2: Helix trajectory of delta robot with the external load

The productiveness of RBF networks in the proposed controller is confirmed by adding an object into the end effector of the delta robot in helix operation. The objective is to evaluate the adaptation and robustness of the proposed approach against model parameter variation. In this case, the end effector has added an external object with 0.2 kg. Despite an increasingly small number of errors, the PDAFTSC algorithm successfully overwhelms the external disturbance to achieve similar performance compared to others in the no-load case in Fig. 25 and Table 8. The control signal tends to widen the boundary when the joints increase the speed. In addition to this, Fig. 26 demonstrated a similar curve of disturbance torques from the RBF estimators. This action will create accurate signal compensation to catch up with the speed of tracking errors and keep the system in robustness. As a result, the adaptation of the proposed controller has been validated based on disturbance torque estimation and compared to no-load cases. Despite the influence of the external disturbances generated by joint friction or uncertainty parameters, the
system with RBF NNs attachment has presented remarkable stability compared with the others.

Table 8. RMSE comparison among the controllers and PDAFTS in the experiment 2

|                  | Joint 1 (Degree) | Joint 2 (Degree) | Joint 3 (Degree) |
|------------------|------------------|------------------|------------------|
| FTS (no load)    | 0.3632           | 0.4157           | 0.4086           |
| AFTS (no load)   | 0.3221           | 0.3553           | 0.3720           |
| FTS (load)       | 0.39             | 0.5155           | 0.4089           |
| AFTS (load)      | 0.3481           | 0.4613           | 0.3860           |

Fig. 24. Trajectory performance in joint space and tracking error in the experiment 2
5 Conclusions

In this work, a novel adaptive PDAFTS controller using synergetic theorem is the first time developed for dynamic delta robot model. To obtain results, various simulation and experiment are verified. Firstly, the delta robot generated by the combination of DC motor model and delta robot manipulator is developed for simulation. Then, the velocity estimator using SOSM and LPF to observe the angular velocity and reduce the chattering phenomenon, which then provides full states for the proposed controller. The proposed system stability is proved by Luyapunov method. Finally, the practicality of the PDAFTS controller is extracted from the experiment results with the unknown extended load. As a result, the proposed controller performs better than the other controllers in the tracking tasks. The comparison of the obtained results of the PDAFTS and the other control
algorithms show the superiority of the PDAFTS in the presence of disturbance and reduction of chattering phenomenon. Thanks to the RBF neural networks in the proposed technique, the unknown disturbance is estimated effectively and contribute to the control signals, which drive the robot more accuracy and robustness.

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