Security of Authentication with a Fixed Key in Quantum Key Distribution

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Abstract. We study the security of a specific authentication procedure of interest in the context of Quantum Key Distribution (QKD). It works as follows: use a secret but fixed Strongly Universal 2 (SU2) hash function and encrypt the output tag with a one-time pad (OTP). If the OTP is completely secret, the expected time for an adversary to create a tag for a chosen message is exponential in the tag length. If, however, the OTP is partially known in each authentication round, as is the case in practical QKD protocols, then the picture is different; the adversary’s partial knowledge of the OTP in each authentication round gives partial information on the secret hash function, and this weakens the authentication in later rounds. The effect of this is that the lifetime of the system is linear in the length of the fixed key. This is supported by the composability theorem for QKD, that in this setting provides an upper bound to the security loss on the secret hash function, which is exponential in the number of authentication rounds. This needs to be taken into account when using the protocol, since the authentication gets weakened at each subsequent round and thus the QKD generated is key is not as strong as when the authentication is strong. Some countermeasures are discussed at the end of this paper.

Key words: Quantum key distribution, authentication, strongly universal hash functions, partially known key and composability.

1 Introduction

QKD is a provably secure key growing technique based on the laws of quantum physics. It was first introduced by Bennett and Brassard in 1984 [1], and uses a so-called quantum channel that obeys the laws of quantum physics, together with a public communication channel. A QKD round consists of five steps: raw key generation on the quantum channel, followed by sifting, error detection and reconciliation, privacy amplification, and authentication on the public channel; see [2–7] for the details of these steps. Practical implementations of QKD need a low-noise quantum channel but also an immutable public communication channel. Without
the latter, QKD can trivially be broken by a man-in-the-middle attack. Therefore, secure message authentication is indispensable for the security of QKD [8].

In the standard proposed QKD, authentication is achieved by using the Wegman-Carter approach [9,10], based on the idea of Universal hashing. The security of the Wegman-Carter authentication in the context of QKD was studied in [11], noting some problems arising from usage of a partially known key, and detailing some countermeasures.

1.1 Authentication with secret fixed hash function and OTP

The main goal of this paper is to study the security of authentication with a fixed key in the context of QKD. Namely, we study the security of an authentication procedure that works as follows: The legitimate communicating parties, Alice and Bob, share a secret but fixed hash function $f$ taken at random from a $SU_2$ hash function family and a short secret key to be used as OTP in advance. During the public discussion phase of each QKD round, Alice sends the classical message and tag pair $m + t$ with $t = f(m) \oplus K$, where $K$ is an OTP, to Bob. Upon receiving the message-tag pair $(m, t)$, Bob verifies whether the message $m$ did originate from Alice by comparing $f(m) \oplus K$ to $t$: If they are identical, then he accepts $m$ as authentic; otherwise, he rejects it.

This authentication primitive was originally proposed by Wegman and Carter in [10] with the intent to reduce the key consumption rate of authentication. Low key consumption is essential in QKD, since the key consumption rate of the used authentication directly influences the key growing rate. Wegman-Carter authentication using an $\varepsilon$-ASU$_2$ hash function family has a key consumption rate which is logarithmic in the message length, while using encrypted tags would reduce this; the rate is linear in the tag length as the round count increases.

Partial knowledge of the OTP key $K$ leaks information on secret but fixed $SU_2$ hash function $f$. In QKD, the privacy amplification step reduces the information leaked to Eve during each round, but not all the way to zero. Thus Eve may still have some partial knowledge of the OTP key used for authentication in the subsequent rounds. This information, $\varepsilon$, on the OTP key $K$ in each round leaks $\varepsilon$ information on the secret hash function $f$. Intuitively, the information on $f$ leaked to Eve is linear in the number of authentication rounds. In what follows, we show that this is really the case, and in fact Eve will eventually have enough knowledge of the hash function $f$ to enable her to create a tag for her forged message.
Furthermore, the composability theorem for QKD gives an exponential upper bound for the security loss of the system.

1.2 Our contributions

In the case when the OTP key $K$ is completely secret to Eve, it behaves as an evenly distributed random variable to her (which is the reason for the upper-case $K$ notation). In this case, the best attack for Eve would be to guess the value of $t_E$, the tag value for her message $m_E$. Since all tag values are equally possible, the probability of each guess succeeding is one divided by the size of all possible tags $|\mathcal{T}|$. Furthermore, she can gain no knowledge about the secret hash function $f$ from guessing, because $K$ in the current round is independently distributed from previous rounds. The probability of a successful guess would in each round be $1/|\mathcal{T}| = 2^{-\log|\mathcal{T}|}$, which implies that the expected lifetime

$$n = |\mathcal{T}| = 2^{\log|\mathcal{T}|}$$

(1)

is exponential in the tag length $\log|\mathcal{T}|$.

We are interested in seeing how this exponential lifetime behavior would change if Eve has some knowledge of $K$ in each round. In the remainder of this paper we estimate how many rounds it takes for Eve to gain complete knowledge of the secret but fixed hash function $f$ (taken at random from an SU$_2$ family), under the assumption that the practical implementation of QKD protocol generates $\varepsilon$-perfect key in each run. We refer to [12] for the definitions of perfect and $\varepsilon$-perfect keys, and of ideal and $\varepsilon$-ideal protocols. Note that since the authentication primitive uses a fixed SU$_2$ hash function, the sequence of the security parameters for the key stream generated from the QKD protocols cannot be made a geometric sequence by increasing the protocol complexity at each run, as discussed in [12]. By fixing Eve’s partial knowledge of the OTP key in each authentication round, we derive an estimate for the lifetime of system which is linear in the length of the key identifying $f$ and proportional to her partial knowledge of OTP.

This is not in conflict with the composable security of QKD, which implies that the key generated by QKD can be used securely in classical cryptographic tasks such as authentication [12,13]. In this case, however, the authentication procedure itself degrades as the authentication round count increases. Below, we show that the composability theorem for QKD predicts that the security loss on the fixed secret hash function is exponentially upper bounded in the number of authenticaton rounds.
It should be pointed out that the attack needs a large computational capacity of the attacker. Usually, no bounds are imposed on the computational capacity of an eavesdropper attacking a QKD system. This is because QKD is provably secure based on laws of nature, rather than computational complexity as is usually the case for key-sharing systems. This large computational need of the attack will unfortunately limit simulations in this paper because of our bounded computational power.

1.3 Organization of the paper

The rest of the paper is organized as follows. In Section 2, we present an attack and estimate its effect on the system under simplifying assumptions, and also present simulations on a SU2 hash function family, followed by a modification to the attack that establishes the desired lifetime. Section 3 contains the theoretical upper bound for the security loss predicted by the composability theorem for QKD. Finally, Section 4 concludes the paper.

Notation. In what follows, $\mathcal{M}$ is the set of messages, $\mathcal{T}$ is the set of tags, $\mathcal{H}$ is a family of hash functions $f: \mathcal{M} \rightarrow \mathcal{T}$ with $|\mathcal{H}| = H + 1$, and $\mathcal{H}_i$ are integer-indexed subsets of $\mathcal{H}$. Logarithms are in base 2. The random variables used are $K$, $N$, and $X_i$, while lower-case $m$ denotes a message and $t$ a tag, $m_E$ and $t_E$ are Eve’s message and attempt at a tag.

2 Attack and lifetime estimate

Eve would like to perform an attack which is better than simply guessing the tag. Ideally, it should be better in two ways: it should succeed with high probability, and should not be detected easily. Eve wants in essence a good covert attack. The below description delineates an attack which achieves both goals: the expected number of rounds until success will be much lower than for the guessing attack, and in addition, the attack is covert, meaning that Eve only listens to the communication between Alice and Bob for a number of rounds, and only launches an attack when she is sure that it will succeed.

The attack is as follows: Eve’s goal is to identify the used hash function $f$ among the $H + 1$ hash functions in $\mathcal{H}$, i.e., to eliminate $H$ functions from $\mathcal{H}$. In each QKD round, Eve intercepts a valid (classical) message-tag pair $m + t$, where $t = f(m) \oplus K$, from, say, Alice to Bob. The random variable $K$ (random to Eve) is not entirely evenly distributed because of Eve’s partial knowledge. We will, in what follows, assume that her knowledge
is such that she knows a few values of \( K \) that has probability 0. She uses this knowledge to identify possible candidates for \( f(m) \). This means that in each run, Eve can identify a subset \( \mathcal{H}_i \) out of all the possible hash functions in \( \mathcal{H} \) by eliminating the hash functions (in \( \mathcal{H} \)) that do not hash \( m \) to the set of possible candidates for \( f(m) \). The set \( \mathcal{H}_i \) will consist of the true match (the fixed secret hash function) and a number of false matches.

The set \( \mathcal{H}_i \) can in principle be of different size depending on the hash function family, which hash function is used, and the message, but here we are focusing on Strongly Universal hash function families and in this case, the inverse image of any tag has the same size, and each subset has the same size \( |\mathcal{H}_i| = h \). Therefore, Eve’s information on \( K \) in terms of min-entropy translates directly into the quantity \( -\log(h/H) \).

### 2.1 Bounds using simplifying assumptions

After \( i \) runs the set of possible hash functions will decrease to \( \cap_{j=1}^{i} \mathcal{H}_j \). In general, the remaining number of false matches in this intersection is a random variable

\[
X_i = |\cap_{j=1}^{i} \mathcal{H}_j| - 1. \tag{2}
\]

We are interested in the expected time it takes until Eve has identified the (no longer secret) true hash function, that is, the expectation of the (random) index \( N \) that is the earliest that gives \( X_N = 0 \) (such that \( X_{N-1} \geq 1 \)).

By assuming that that each round is independent of the former, and that each subset is exactly evenly distributed within the previous subset, we obtain \( X_i = X_{i-1} h/H \). This is only possible when the \( X_i \) are continuous variables; we will analyze the discrete (integer-valued) case below. With \( X_0 = k \) we obtain \( X_1 = kh/H, X_2 = k(h/H)^2, \ldots, X_l = k(h/H)^l \). Now, our demand \( (X_N = 0) \cap (X_{N-1} \geq 1) \) translates into \( (X_N < 1) \cap (X_{N-1} \geq 1) \), which in turn implies that \( N|X_0 = k \) is not random in this case, but is in fact equal to a number \( n_k \) for which

\[
k(h/H)^n_k < 1 \leq k(h/H)^{n_k-1}, \tag{3}
\]

which after some algebra simplifies to

\[
n_k - 1 \leq \frac{\log k}{\log h/H} < n_k, \tag{4}
\]

that is,

\[
n_k = \left\lceil \frac{\log k}{-\log h/H} \right\rceil. \tag{5}
\]
In particular, \( n_H = \lceil \log H / (-\log(h/H)) \rceil \), which means that the lifetime of the system would be directly proportional to the key length divided by the information on the OTP used in each step. This is what we would expect of a system in which there is a constant gain of information in each run.

In the discrete case, the analysis is more complicated. We extend to a random draw of hash functions, but keep the assumption that each round is independent of the former. This means that the probability of drawing a hash function present in \( \cap_{j=1}^{i-1} H_j \) in run \( i \) only depends on \( X_{i-1} \), which corresponds to a random draw of \( h \) elements without replacement from \( H \), where there are two types of elements: those in \( \cap_{j=1}^{i-1} H_j \) (\( X_{i-1} \) of them), and those outside the set. In other words, the number of hash functions in \( \cap_{j=1}^{i} H_j \) given \( X_{i-1} \) is hypergeometrically distributed, so that,

\[
p_{jk} := P(X_i = j \mid X_{i-1} = k) = \frac{\binom{k}{j} \binom{H-k}{h-j}}{\binom{H}{h}}. \tag{6}
\]

The expected lifetime time when \( k \) false hash functions remain is

\[ n_k = E(N \mid X_0 = k). \tag{7} \]

Then, \( n_0 = 0 \) and

\[
n_k = \sum_{j=0}^{k} E(N \mid X_1 = j) P(X_1 = j \mid X_0 = k) = \sum_{j=0}^{k} \left( E(N \mid X_0 = j) + 1 \right) P(X_1 = j \mid X_0 = k) = 1 + \sum_{j=0}^{k} p_{jk} n_j. \tag{8}
\]

Solving for \( n_k \) gives

\[
n_k = \frac{1 + \sum_{j=0}^{k-1} p_{jk} n_j}{1 - p_{kk}}, \tag{9}
\]

and since \( p_{jk} \), \( j = 0, 1, \cdots, k \), are given explicitly above, the \( n_k \) can be calculated from this equation, although the expressions are complicated.

The goal is to prove a logarithmic bound on \( n_k \) in terms of \( k \) as in (5) in general. Splitting the sum of (8) at the point \( s \) (just before the index \( \lceil s \rceil \)) gives

\[
n_k = 1 + n_{\lceil s \rceil - 1} P(X_i < s \mid X_{i-1} = k) + n_k P(X_i > s \mid X_{i-1} = k). \tag{10}
\]

\(^1\) Here, the length of the key identifying the secret hash function is actually \( \log(H+1) \).
And now solving for \( n_k \) gives

\[
 n_k \leq \frac{1}{1 - P(X_i > s | X_{i-1} = k)} + n_{\left\lfloor s \right\rfloor - 1}.
\] (11)

If the probability in the denominator does not grow too fast when \( s \) decreases from \( k-1 \), we can use a value \( s \) sufficiently far from \( k \) to establish logarithmic growth of \( n_k \) in \( k \). The one-sided Chebyshev inequality implies

\[
P\left( X_i > s \bigg| X_{i-1} = k \right) \leq \frac{V(X_i | X_{i-1} = k)}{(s - E(X_i | X_{i-1} = k))^2 + V(X_i | X_{i-1} = k)},
\] (12)

so that

\[
\frac{1}{1 - P(X_i \geq s | X_{i-1} = k)} \leq \frac{V(X_i | X_{i-1} = k)}{(s - E(X_i | X_{i-1} = k))^2 + V(X_i | X_{i-1} = k)}
= 1 + \frac{k h H}{s - E(X_i | X_{i-1} = k)^2} \leq 1 + \frac{k h H}{s - k h H^2}.
\] (13)

This implies that

\[
 n_k \leq 1 + \frac{k h H}{s - k h H^2} \left( 1 - \frac{h}{H} \right) + n_{\left\lfloor s \right\rfloor - 1}.
\] (14)

Note that even if \( \left\lfloor s \right\rfloor \) and \( k \) coincide, the indices above do not. Now let us prove using induction that

\[
 n_k \leq a + b \log k
\] (15)

with the appropriate \( a \) and \( b \). A simple starting point is \( n_1 \):

\[
a = n_1 = \frac{1}{1 - \frac{h}{H}}.
\] (16)

Now, we assume (15) holds for \( k \) less than \( p \geq 2 \) which implies

\[
n_{\left\lfloor s \right\rfloor - 1} \leq a + b \log \left( \left\lfloor s \right\rfloor - 1 \right) \leq a + b \log s = b \log \frac{s}{k} + a0 + b \log k,
\] (17)
so that
\[ n_p \leq 1 + \frac{k \frac{h}{H} \left( 1 - \frac{h}{H} \right)}{(s - k \frac{h}{H})^2} + b \log \frac{s}{k} + a + b \log k. \] (18)

Choosing
\[ b = -\left( 1 + \frac{k \frac{h}{H} \left( 1 - \frac{h}{H} \right)}{(s - k \frac{h}{H})^2} \right) / \log \frac{s}{k} > 0 \] (19)
gives the desired
\[ n_p \leq a + b \log p. \] (20)

By induction we obtain that the lifetime \( n_k \) is bounded by
\[ n_k \leq \frac{1}{1 - \frac{h}{H}} + \left( 1 + \frac{k \frac{h}{H} \left( 1 - \frac{h}{H} \right)}{(s - k \frac{h}{H})^2} \right) \frac{\log k}{-\log \frac{s}{k}}. \] (21)

If \( s \) is chosen proportional to \( k \), the first term in the parenthesis will dominate at large values of \( k \), and the proportionality constant appears in \( \log s/k \). Choosing \( s = k \sqrt{h/H} \), then with similar calculations as above we obtain
\[ n_k \leq \frac{1}{1 - \frac{h}{H}} + \left( 1 + \frac{1 + \sqrt{\frac{h}{H}}}{k(1 - \sqrt{\frac{h}{H}})} \right) \frac{2 \log k}{-\log \frac{s}{k}}, \] (22)
where the coefficient in front of the logarithm decreases to 1 when \( k \) increases. The bound for \( n_H \) is logarithmic in \( H \) and slightly larger than the one in (5), which is natural taking the broadening of the distribution into account. This is similar as in the previous more simplified situation; the lifetime of the system is linear in the key length rather than exponential in the tag length. We now need to check the remaining assumption that each round is independent of the former: does the random draw in each round follow a hypergeometric distribution?

### 2.2 Simulations for an SU\(_2\) Family

We want to simulate an authentication system with a secret fixed hash function from an SU\(_2\) hash function family, where the tag is OTP encrypted with a partially known key. Here, we restrict ourselves to a specific hash function family from [9] as follows. Let \( \mathcal{M} \) and \( \mathcal{T} \) be finite sets of messages and tags, respectively. Let \( p \) be smallest prime number greater than \( |\mathcal{M}| \).
For each integer $0 < q < p$ and $0 \leq r < p$, define a hash function $f_{(q,r)} : \mathcal{M} \to \mathcal{T}$ by

$$f_{(q,r)}(m) \equiv ((mq + r) \mod p) \mod |\mathcal{T}|.$$ (23)

Then, $\mathcal{H}_1 = \{f_{(q,r)} : q \in \mathbb{Z}_p \setminus \{0\} \text{ and } r \in \mathbb{Z}_p\}$ is an SU$_2$ hash function family. This family was introduced as "$\mathcal{H}_1$" in [9] (the index is not used in the same way as in this paper), and is not quite SU$_2$, in Wegman and Carter’s own words: it is "close".

The parameters chosen for our simulations will admittedly be very restrictive and somewhat unrealistic when compared to a full-blown QKD system. The reason for this is our bounded computational capabilities; as already mentioned, no bound is usually imposed on an attacker in QKD, but this does unfortunately not apply to authors of scientific papers. The largest hash function family we will use will be of size $2^{28}$, and our attack uses the equivalent of round-by-round exhaustive search, by keeping track of eliminated keys at each round, and this gives a high computational demand. The hash function family size will not be kept fixed in the different simulations. We use a set $\mathcal{T}$ of tags with size $2^7$, and message sets $\mathcal{M}$ with a varying size from $2^9$ through $2^{13}$. For each pair of $\mathcal{M}$ and $\mathcal{T}$, there is a corresponding hash function family $\mathcal{H}$. We set Eve’s partial information on the OTP key $K$ to 10%, again an unrealistically high number, but this is chosen to show the results qualitatively while still bounding the lifetime of the system, see below.

The simulations are done as follows: a hash function $f$ is taken at random from the appropriate SU$_2$ family. In each round, a message $m_i$ is randomly drawn, and the tag $t_i$ is calculated using $f$. This tag is entered into the set $\mathcal{T}_i$, and more tags are randomly chosen from $\mathcal{T}$ to make $|\mathcal{T}_i| = h|\mathcal{T}|/|\mathcal{H}|$, which corresponds to a situation where Eve can use the OTP-encrypted tag $t_i \oplus K$ together with her partial knowledge of $K$ to identify the set $\mathcal{T}_i$. She then uses this set to identify the set of possible hash functions $\mathcal{H}_i$, and she forms the intersection $\bigcap_{j=1}^i \mathcal{H}_j$. When the intersection has been reduced to just one hash function, Eve has identified $f$, and this is repeated many times to estimate the lifetime of the system, the results can be seen in Fig. 1.

As we can see, the lifetime is not as was estimated in (22). It now increases exponentially as the key length increases, contrary to our earlier linear estimate. The reason for this is that the rounds are not independent, at least not for this hash function family. This is especially pronounced when there are few hash functions left: most of the increase occurs when waiting for the last few false matches to disappear. Recall
that the hash functions are eliminated by using the inverse image, for one message in each round from a set of “possible” tags, to a set of “possible” hash functions. And hash functions that have not been eliminated already have a lower probability to be eliminated than they would in the case of independent rounds.

However, Eve’s goal is not really finding the secret hash function \( f \). Eve’s objective is to be able to generate the correct tag for her (forged) message, to breach security of the authentication. So far, our focus has been on finding the secret hash function \( f \). We note that even if the remaining set \( \cap_{i=1}^{j} H_j \) contains more than one hash function, Eve can generate the correct (unencrypted) tag for her message if all the remaining hash functions map her message to the same value (say, \( t_E \)). Eve can check for this event, by comparing tags for her message for the different remaining hash functions. When there are few hash functions remaining, and they have a low probability to be eliminated, the probability is high that a random message is mapped to the same tag by all remaining hash functions. This means that the probability for Eve’s message to be mapped to the same tag is high.

Eve also needs to identify the OTP to encrypt her tag. She can do that when the remaining hash functions in \( \cap_{j=1}^{t} H_j \) also map Alice’s message to the same value \( t \) (possibly different from \( t_E \)). Using the value of \( t \), Eve can identify the OTP key \( K \) used, and use that to encrypt her tag. At this point the system is broken. Changing the simulation so that Eve checks

Fig. 1: The number of rounds until the secret hash function \( f \) is found when it is taken at random from the family \( H_1 \).
for this event, gives a linear lifetime in the key length, as can be seen from Fig. 2. The simulated lifetime is slightly shorter than the estimated value, but Eve is solving a simpler task by not trying to identify the correct hash function $f$ but instead a subset that has the desired properties.

### 3 Upper bound to security loss

This is not in conflict with composable security of QKD [12,13]. Moreover, the composability can be used to provide an upper bound to the security loss on the fixed secret hash function. The composability theorem for QKD states that if an $\varepsilon_1$-ideal QKD protocol is composed with an $\varepsilon_2$-ideal cryptographic application, e.g., $\varepsilon_2$-ideal authentication, the whole system is $\varepsilon_1 + \varepsilon_2$-ideal. So, if an $\varepsilon_1$-ideal QKD is composed with an $\varepsilon_2$-ideal authentication, then the whole system becomes $\varepsilon_1 + \varepsilon_2$-ideal and generates an $\varepsilon_1 + \varepsilon_2$-perfect key. It was argued in [12] that the security parameter for the key stream generated from the repeated use of QKD can be made arbitrarily small by increasing the communication complexity of the protocol; that is, by making the sequence of security parameters for QKD-generated keys a geometric sequence. This unfortunately is not possible with the authentication under consideration here. The present authentication procedure uses a fixed secret $SU_2$ hash function, and this

Fig. 2: The number of rounds until $f$ is found under the uniform and hypergeometric assumptions, and the number of rounds until Eve gains enough information to generate the valid tag for her forged message.
fixes the length of the message that can be authenticated. Thus, it is reasonable to assume that the practical implementation of the QKD is $\varepsilon_1$-ideal at a constant $\varepsilon_1$ in each round.

![Diagram]

**Fig. 3:** Composability diagram of QKD with authentication with fixed key followed by an OTP.

Now, let us look at the sequence of security loss on the secret but fixed hash function $f$ with help of the composability theorem (see Fig. 3).

- In the first round, the composed system of $\varepsilon_1$-ideal QKD and $\varepsilon_2$-ideal authentication produces an $\varepsilon_1 + \varepsilon_2$-perfect key. A portion of this $\varepsilon_1 + \varepsilon_2$-perfect QKD-generated key will be used as the OTP key for the $\varepsilon_2$-ideal authentication in the second round.

- In the second round, the composed system of $\varepsilon_1$-ideal QKD and $\varepsilon_1 + 2\varepsilon_2$-ideal authentication using an $\varepsilon_1 + \varepsilon_2$-perfect key produces an $2(\varepsilon_1 + \varepsilon_2)$-perfect key. A portion of this $2(\varepsilon_1 + \varepsilon_2)$-perfect key will be used as the OTP key for the authentication in the third round. Furthermore, the $\varepsilon_1 + \varepsilon_2$ information on the OTP key leaks $\varepsilon_1 + \varepsilon_2$ information on the fixed hash function, which makes the authentication $\varepsilon_1 + 2\varepsilon_2$-ideal.

- In the third round, the composed system of $\varepsilon_1$-ideal QKD and $\varepsilon_1 + 2\varepsilon_2$-ideal authentication using an $2(\varepsilon_1 + \varepsilon_2)$-perfect key produces an $4(\varepsilon_1 + \varepsilon_2)$-perfect key. A portion of this $4(\varepsilon_1 + \varepsilon_2)$-perfect key will be used as the OTP key for the authentication in the fourth round. Furthermore, the $2(\varepsilon_1 + \varepsilon_2)$ information on the OTP key leaks $2(\varepsilon_1 + \varepsilon_2)$ information
on the fixed hash function, which makes the authentication $3\varepsilon_1 + 4\varepsilon_2$-ideal.

- In the fourth and following rounds, the process continues, doubling the coefficient for each round.

The important property of this authentication scheme is that the information gained on the fixed hash function $f$ at the current round carries through to the next round. In other words, the information leakage on $f$ at each round can be combined. Therefore, after the $n$-th round, the information leaked to Eve on the secret but fixed hash function is $(2^{n-1}-1)\varepsilon_1 + 2^{n-1}\varepsilon_2$ so that the authentication becomes $(2^{n-1}-1)\varepsilon_1 + 2^{n-1}\varepsilon_2$-ideal. The attack in Section 2 only assumes that the QKD generated key in each round is equally strong; in other words, Eve’s knowledge of the QKD generated key in each round is the same.

4 Conclusions

In this paper, the security of a specific authentication primitive is studied, a primitive that uses a fixed secret hash function followed by a one-time-pad encryption on the tag. This is of interest in QKD because of its low consumption of secret key. We found that, by fixing Eve’s partial knowledge of the OTP key in each QKD round, the lifetime of the system is linear in the length of the fixed key. Moreover, using the composability theorem, we found that the leakage of information on the secret but fixed key is exponentially upper bounded in the number of authentication rounds. A suitable countermeasure would be to change the fixed secret key regularly, at an interval that ensures that Eve’s collected information on the fixed key does not become too large. This would make the key consumption rate again logarithmic in the message length, but at a rate much lower than the standard Wegman-Carter authentication that uses a new $\varepsilon$-ASU$_2$ hash function in each round.

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