Final state strong interaction constraints on weak $D^0 \rightarrow K^0_S \pi^+ \pi^-$ decay amplitudes

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Abstract Weak decay tree and annihilation - t-channel $W$- exchange amplitudes for the $D^0 \rightarrow K^0_S \pi^+ \pi^-$ process are calculated using quasi two-body QCD factorization approach and unitarity constraints. Final state strong $K\pi$ and $\pi\pi$ interactions in $S$, $P$ and $D$ waves are described through corresponding form factors including many resonances. Preliminary results compare well with the effective mass distributions of the Belle and BABAR Collaboration analyses.

Keywords QCD factorization · Final state interactions · Unitary form factors

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1 Introduction

Why should one study the weak decays $D^0 \rightarrow K^0_S \pi^+ \pi^-$? First, the recent measurements of the $D^0$-$\bar{D}^0$ mixing parameters for this self-conjugate reaction by the BABAR [1] and Belle [2] Collaborations could show the presence of new physics contribution beyond the standard model. Second, the Cabibbo-Kobayashi-Maskawa, CKM, angle $\gamma$ can be evaluated from the analyses of the $B^\pm \rightarrow D^0 K^\pm, D^0 \rightarrow K^0_S \pi^+ \pi^-$ decays. Third, one can learn about the final state meson-meson interactions, the meson resonances decaying into different meson-meson pairs and their interferences in the Dalitz plot. One can also perform a partial wave analysis of decay amplitudes. In
addition, constraints from quasi two-body QCD factorization \[3\], QCDF, approach will allow to test theoretical models of form factors entering in the decay amplitudes.

**Quasi two-body factorization** Following a program devoted to the understanding of the rare three-body $B$ decays (see e.g. Ref. \[4\]) the presently available $D^0 \rightarrow K^0_S \pi^+ \pi^-$ data are analyzed in the framework of QCDF. Neglecting the small $CP$ violation in $K^0$ decays, it will be assumed that $\{K^0_S\} = \{\overline{K}^0\}/\sqrt{2}$. The three-meson final states $\overline{K}^0 \pi^+ \pi^-$ are approximated (quasi two-body approximation) as being created by a meson-meson state, $[\overline{K}^0 \pi^+]_{S,P,D}$ or $[\overline{K}^0 \pi^-]_{S,P,D}$ or $[\pi^0 \pi^-]_{S,P,D}$ in a $S$, $P$ or $D$ wave created by a $q\overline{q}$ pair and the remaining meson, $\pi^-$, $\pi^+$ or $\overline{K}^0$, respectively. Amplitudes are derived from the weak effective Hamiltonian, $H_{eff}$ which is a superposition of left-handed quark current-current operators. For instance one has the operator $O_1 = j_1 \otimes j_2$ where $j_1 = \bar{s}_a \gamma^\mu (1 - \gamma^5) c_a \equiv \langle \bar{s}c \rangle_{V-A}$ and $j_2 = \bar{u}_b \gamma^\nu (1 - \gamma^5) d_b \equiv \langle \bar{u}d \rangle_{V-A}$, $\alpha$ and $\beta$ being color indices. Applying QCDF to the part of the amplitude proportional to the product of the CKM quark mixing matrix elements $V_{cs} V_{ud} \equiv A_1$ one has,

$$
\langle \overline{K}^0 \pi^- \pi^+ | H_{eff} | D^0 \rangle \approx \frac{G_F}{\sqrt{2}} A_1 \left\{ a_1 \langle \pi^+ | j_2 | 0 \rangle \langle \overline{K}^0 | \pi^- | S,P,D | j_1 | D^0 \rangle + a_2 \langle \overline{K}^0 | j_2 | 0 \rangle \langle \pi^+ \pi^- | S,P,D | j_1 | D^0 \rangle + a_3 \langle \overline{K}^0 | \pi^- | S,P,D | j_2 | 0 \rangle \langle \pi^+ \pi^- | S,P,D | j_1 | D^0 \rangle \right\}
$$

(1)

where $G_F = 1.166 \times 10^{-5}$ GeV$^{-2}$ is the Fermi coupling constant, $a_{1,2}$ are effective QCD Wilson coefficients and $j_1 = \langle \bar{u}c \rangle_{V-A}$, $j_2 = \langle \bar{s}d \rangle_{V-A}$ derived from $j_{1,2}$ via a Fierz transformation. In Eq. (1) it appears that the cross product of bilinear quark currents in $O_1$ factorizes, after the introduction of the vacuum state, into the product of two matrix elements. The first ones are proportional to the $\pi^+, \overline{K}^0, D^0$ decay constants $f_\pi, f_{K^0}, f_{D^0}$ since $\langle \pi^+ | j_2 | 0 \rangle = i f_\pi p_\pi$, $\langle \overline{K}^0 | j_2 | 0 \rangle = i f_{K^0} p_{K^0}$, $\langle \pi^+ \pi^- | j_1 | D^0 \rangle = -i f_{D^0} p_{D^0}$, $p_\pi$, $p_{K^0}$ and $p_{D^0}$ being the $\pi^+$, $\overline{K}^0$ and $D^0$ four momenta, respectively. The second ones are transition matrix elements or form factors as with $M_i = \overline{K}^0 \pi^+$, $j = j_{1,2}$ or $j_1 j_2$ one has, $\langle M_1 M_2 | S,P,D | j | D^0 \rangle = \langle D^0 | M_1 M_2 | S,P,D | j | 0 \rangle$ and $\langle M_1 M_2 | S,P,D | M_3 | 0 \rangle = \langle D^0 | M_1 M_2 | S,P,D | M_3 | 0 \rangle$. It can be shown from field theory and using dispersion relations \[3\] that these form factors can be calculated exactly if one knows the $D^0$, $[M_1 M_2]_{S,P,D}$ or $M_3 \overline{M}_3$ strong interactions at all energies.

### 2 Decay amplitudes

**Different type of amplitudes** Amplitudes with $c \rightarrow s d u$ transition are proportional to $V_{cs} V_{ud} \approx \cos \theta_C \approx 0.975$, $\theta_C$ being the Cabibbo angle. There are 7 such allowed tree amplitudes: for the $\pi^+ [K^0 \pi^-]_{S,P,D}$, $\pi^+ [K^0 \pi^-]_{S,P,D}$ and $1$ for the $K^0 \omega (\omega \rightarrow [\pi^+ \pi^-]_p$ by $G$-parity violation) final states. Amplitudes with $c \rightarrow d u s$ transition, proportional to $\sin^2 \theta_C \approx (0.225)^2$, are doubly Cabibbo suppressed. There are 6 of them as the $W$ meson cannot couple to the $[K^0 \pi^-]_D$ final state. There are also 7 allowed tree annihilation or $t$-channel $W$-exchange amplitudes corresponding to the $c\bar{u}$ annihilation into $s d$ and 7 doubly Cabibbo suppressed annihilation amplitudes from the $c\bar{u}$ annihilation into $d s$. Altogether, the quasi two-body QCD approach leads to 27 non-zero - 13 tree and 14 annihilation - amplitudes,
Transition matrix elements} Several meson resonances can decay into the two meson final states in $S,P$ or $D$ wave. For the kaon-pion subsystems, the scalar resonances, $K_S^0(800)^\pm$ or $K_L^0(1440)^\pm$ decay into $[K^0\pi^\pm]_S$, the vector resonances, $K^*(892)^\pm, K_1(1410)^\pm, K^*(1680)^\pm$ into $[K^0\pi^\pm]_P$ and the tensor resonances $K_2^0(1430)^\pm$ into $[K^0\pi^\pm]_D$. For the pion-pion subsystems, the scalar $f_0(600)$ or $\sigma$, $f_0(980)$, $f_0(1400)$ decay into $[\pi^+\pi^-]_S$, the vector, $\rho(770)^0, \omega(782), \rho(1450)^0, \rho(1700)^0$ into $[\pi^+\pi^-]_P$ and the tensor $f_2(1270)$ into $[\pi^+\pi^-]_D$. This leads to a rich interference pattern in the Dalitz plot.

Form factors. The $D$-pion [7] or pion-pion [4] systems in Eq. (2) are described in terms of the corresponding vertex functions, which are related to the $\bar{n}$-$\bar{p}$ transition matrix elements [see Eq. (1)] to two-meson states. As an example of application of Eq. (2) one can choose $M_1 \equiv K^0, [M_2M_3]_S \equiv [\pi^+\pi^-]_S$ and $R_S^{\pi^+\pi^-} \equiv f_0(980)$, so that,

$$\langle K^0(p_1)f_0(p_2+p_3)|\bar{s}d\rangle_{V-A}|0\rangle = -\frac{m_{D^0}^2-s_{23}}{p_{D^0}^2}m_{D^0}^{-1}p_{D^0}f_0^{K^0f_0}(980)(m_{D^0}^2)+1\text{ term},$$

where $p_{D^0} = p_1 + p_2 + p_3$. The $K^0$ to $f_0$ scalar transition form factor, $F_0^{K^0f_0}(m_{D^0}^2)$, related to the $K^0f_0$ interaction at $(p_K + p_{f_0})^2 = m_{D^0}^2$, is a complex number to be fitted. The extra “1 term” gives a null contribution when multiplied, in Eq. (4) by $\langle 0|\bar{n}c\rangle_{V-A}(D^0)$ [see Eq. (1)]. The vertex function $G_{f_0(980)}(s_{23}) \equiv \chi_2 F_0^{\pi^+\pi^-}(s_{23})$ where $\chi_2$ is a constant fitted to data and $F_0^{\pi^+\pi^-}(s_{23})$ is the complex pion scalar form factor [4] related to $\langle [\pi^+\pi^-]_S|\bar{u}u\rangle_{V-A}|0\rangle$. The other vertex functions for the kaon-pion [7] or pion-pion [4] systems in Eq. (2) are described in terms of the corresponding form factors. The $D$-wave meson-meson form factors are represented by relativistic Breit-Wigner formulae. In the transition form factors for $R_S^{M_1M_2}$ we choose $R_S^{K^0\pi^+\pi^-} \equiv K_S^0(1430), R_S^{K^0\pi^+\pi^-} \equiv K^*(892), R_S^{\rho\pi^+\pi^-} \equiv f_0(980)$ and $R_S^{\rho\pi^+\pi^-} \equiv \rho(770)^0$.

A selected amplitude} From Eqs. (1) and (3) one obtains for the $D^0 \rightarrow K_S^0\pi^+\pi^-$ $S$-wave annihilation amplitude with $[\pi^+\pi^-]_S$ subsystem in the final state,

$$A_{2S} = \frac{G_F}{2} A_1 \chi_2 \alpha_2 f_{D^0}(m_K^2-s_0) f_0^{K^0f_0}(m_{D^0}^2) f_0^{\pi^+\pi^-}(s_0),$$

where $s_\pm = (p_{\pi^+} + p_{\pi^-})^2, s_0 = (p_{\pi^+} + p_{\pi^-})^2$. Other amplitudes can be derived similarly, their expression will be given in a forthcoming paper, however some explicit formulae can be found in Ref. [6].

3 Experimental $D^0 \rightarrow K_S^0\pi^+\pi^-$ data

Isobar model and unitarity} The experimental Dalitz plot analyses [1][2] are performed within the isobar model to describe the final state meson-meson interactions and many free parameters are used (of the order of 2 per amplitude): the BABAR Collaboration.
model relies on 43 parameters and that of Belle on 40. Amplitudes are not unitary neither in the 3-body channels nor in the 2-body sub-channels. Two-body unitarity is incorporated in the present model: unitary form factors are used in the $K\pi$ S- and $P$-wave \cite{El-Bennich} and in the $\pi\pi$ S-wave \cite{Lešniak} amplitudes. The branching fraction of the sum of these amplitudes is larger than 80% of the total $D^0 \rightarrow K_S^0\pi^+\pi^-$ branching fraction.

4 Preliminary results and concluding remarks

The present model has 28 free parameters, mostly unknown transition form factors and a minimization procedure is used to reproduce the $K_S^0\pi^-, K_S^0\pi^+$ and $\pi^+\pi^-$ squared effective mass $m_\pm^2 = s_\pm$ and $m_0^2 = s_0$ projections of the experimental Dalitz plot analyses \cite{Zhang, Del Amo Sanchez}. Preliminary results are shown in Fig. 1 for the Belle Collaboration analysis \cite{Zhang}. Similar results are obtained with the BABAR analysis \cite{Del Amo Sanchez}.

**Conclusion** This theoretically constrained analysis might be useful to improve the determination of the $D^0$-$\bar{D}^0$ mixing parameters and of the CKM angle $\gamma$.

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