Evolutionary-based optimization of steel moment frames using direct analysis method

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Abstract. Steel moment frames are widely used in multi-story buildings due to their advantages such as robust as well as flexibility in architectural planning. This paper proposes a design method of steel moment frames based on an optimization algorithm called Differential Evolution. In the optimization problem, the design variables are the cross-sections of the steel members while the objective function is the total weight of the frame. The frame is designed based on the direct analysis method which is specified in the ANSI/AISC 360-10 “Specification for Structural Steel Buildings”. A program, called FrameOpt, is developed using Visual Basic for Application language to implement the optimization algorithm. This program connects to the SAP2000 analysis program for the steel design through the open Application Programming Interface feature. An example is conducted to demonstrate the applicability of the proposed method.

1. Introduction

In recent years, the number of multi-story buildings is rapidly growing. Two commonly used materials for multi-story buildings are reinforced concrete (R.C) and steel, in which steel constructions have several good attributes such as high strength, lightweight, and speedy construction. However, the price of steel material is extremely higher than R.C. one. Therefore, the weight optimization of steel frames is necessary. Among there available steel structural systems for multi-story buildings, moment frames are preferred because they provide a highly efficient way to carry both vertical and horizontal loads while ensuring an optimal useable space. A moment frame typically comprises of orthogonally beams and columns which are connected through moment-resisting connections. The lateral load resistance of a moment frame is provided by the bending strength of its beams and columns. Moment frames were developed since the late 1800s and the number of moment frame constructions grew rapidly from the early 1900s to the present. According to F. R. Khan [1], moment frames are efficient for buildings up to 30 stories. For taller buildings, the moment frame solution is not efficient yet due to its large lateral deformation.

Over the years, many researchers have successfully applied optimization algorithms to design moment frames. For example, A. Kaveh published a study on optimization of planar steel frames using an improved ant colony optimization in 2010 [2]. E. Dogan and M. P. Saka used particle the swarm optimization (PSO) algorithm to optimize unbraced frames [3]. In 2017, M. R. Maheri et al. employed a metaheuristic algorithm, called Enhanced Honey Bee Mating optimization algorithm, to optimize side sway steel frames [4]. Most recently, H. G. Arab and his colleagues proposed an improved
evolutionary algorithm for the optimization of planar moment frames in 2018 [5]. In all the above-mentioned studies, steel frames were designed using the effective length method (ELM).

The ELM approach determines the critical buckling loads of columns by isolating the considered column within a frame and evaluating the rotational and translational stiffness of its end restraints. The ELM is the most popular method for stability checking, which was described in all current steel design codes such as ANSI/AISC 360-05 [6], EN 1993-1 [7], SP 16.1330.2017 [8]. However, the application of ELM is limited. First of all, the effective length factor namely K-factor is normally determined for every column situation by using the alignment charts. Consequently, the ELM is not suitable for automatic designs. Secondly, ELM cannot apply for stability sensitive structures where the ratio of second-order to first-order effects is greater than 1.5 [6]. Therefore, a sophisticated method called the direct analysis method (DAM) has been proposed. The DAM offers many advantages as eliminating the need to calculate the effective length factors and increasing the accuracy of the internal forces at the ultimate limit state. Since 2010, the ELM is officially replaced by the DAM in AISC/AISC 360-10 [9] and the DAM is currently offered by many commercial analysis programs like CSI SAP2000. The purpose of this study is to integrate the DAM into the optimization process.

In detail, Differential Evolution (DE), a robust metaheuristic algorithm, is used to optimize the weight of moment frames. In the constraint evaluation step, the steel frame is designed based on the DAM which is described in Chapter C of [9] with the support of the CSI SAP2000 analysis program. In [10], a Visual Basic for Application (VBA) language program, called FrameOpt, was developed to optimize truss frames. This program connects to SAP2000 software through the open Application Programming Interface (oAPI) feature. In the current study, the FrameOpt program is improved for the application of DAM. SAP2000 and VBA are chosen because they are two familiar tools for design engineers. A well-known example of a multi-story moment frame is designed using the proposed procedure to demonstrate its applicability.

The paper is structured as follows. The design of steel frames according to the DAM is presented in Section 2. Section 3 describes the optimization problem of moment frames and briefly introduced the DE algorithm. The procedure for the optimum design of moment frames using the DAM and the DE is also proposed in Section 3. A design example is conducted in Section 4. Finally, some conclusions are pointed out in Section 5.

2. Design of steel frames using the direct analysis method

The DAM is firstly mentioned in Chapter C of the specification AISC 2005 edition [6] as an alternative method besides the ELM as the main method. The DAM is described in more detail in Appendix 7 of [6]. In the 2010 edition [9], the DAM is considered as the primary method in Chapter C with no limitations on its application while the ELM and the first-order analysis method (FOAM) are still permitted for structures that satisfy the constraints specified in Appendix 7. The design procedures of both the ELM and the DAM are presented in Figure 1 for the comparison.

![Figure 1. Comparison of ELM and DAM.](image-url)
Generally, the stability analysis must consider some effects including second-order effects; geometric imperfections; stiffness reductions due to residual stresses; and flexural, shear and axial member deformations. In the ELM approach, the second-order effects can be considered in two ways: using second-order analysis without stiffness reduction or using first-order analysis where the second-order effects are taken into account by the moment amplifications. The residual stress effect and the member geometric imperfections are compensated in the member design equations with the effective length factor \( K > 1 \). The ELM does not transparently separate the residual stress effect and the imperfection effect. In the DAM approach, the residual stress effect is directly introduced to the structural analysis by the stiffness reduction. The second-order analysis is strictly required for the DAM. Therefore, the internal forces obtained by the DAM approach are more accurate than those obtained by the ELM approach. Consequently, the available strength of the members can be determined with no further consideration of the overall stability of the structure. The DAM approach uses the same member design equations of the ELM approach but the effective length factors of all members are taken as \( K = 1.0 \). In the following sections, the DAM approach is presented in detail.

2.1. Second-order effects

The second-order effects refer to the additional internal forces applied to the structure due to the lateral displacement. The second-order effects consist of two components: \( P-\Delta \) effect and \( P-\delta \) effect. The \( P-\Delta \) effect is associated with the displacement of the member ends while the \( P-\delta \) effect is associated with the local deformation of the member. Two types of second-order effects are illustrated in Figure 2. The software package used in this study (CSI SAP2000) supports the nonlinear \( P-\Delta \) analysis. To capture the \( P-\delta \) effect, the elements must be divide into many segments as the recommendation of CSI [11].

![Figure 2. Second-order \( P-\Delta \) and \( P-\delta \) effects.](image)

2.2. Initial imperfection

The initial geometric imperfections are the combination of the manufacturing imperfections (out-of-straightness of individual members) and the assembling imperfections (out-of-plumbness of columns). The out-of-straightness of members is accounted in the member design equations and it does not need to be considered in the analysis. In contrast, the out-of-plumbness of columns must be considered in the analysis. There are two approaches to take into account the effect of the out-of-plumbness. The first approach is to introduce the initial imperfection directly into the geometry of the model. According to the recommendation in [9], the pattern of the initial imperfections is similar to the first buckling mode shape while the amplitude of the initial displacements is based on permissible construction tolerances. Alternatively, the effect of the initial imperfection could be treated by applying equivalent lateral forces, called notional loads, to the frames with its nominal geometry. The magnitude of the notional loads is determined by the following formula.

\[
N_i = 0.002aY_i
\]
where: \( N_i \) is the notional load at level \( i \); \( Y_i \) is the gravity load applied at level \( i \); factor \( \alpha = 1 \) for the Load and Resistance Factor Design (LRFD) method or 1.6 for the Allowable Stress Design (ASD) method. The notional load coefficient of 0.002 is based on the allowable initial story out-of-plumbness ratio of 1/500.

2.3. Stiffness reduction

The axial stiffness and flexural stiffness should be adjusted following the below formulas.

\[
EA^* = 0.8EA \\
EI^* = 0.8\tau_b EI
\]

where: \( \tau_b \) is the additional factor that is used to take into account the contribution of the flexural stiffnesses to the stability of the structure. \( \tau_b = 4(\alpha P_r/P_y)[1-(\alpha P_r/P_y)] \) when \( \alpha P_r/P_y > 0.5 \); otherwise \( \tau_b = 1.0 \); \( P_r \) is the required axial compressive strength; \( P_y \) is the axial yield strength.

2.4. Effect of flexural, shear and axial deformations

The contributions of flexural, shear and axial deformations to the displacements of the structure are already considered in many advanced finite element analysis programs such as CSI SAP2000.

2.5. Implementation of DAM in SAP2000

The DAM approach was supported since version 11.0 of SAP2000 [11]. The notional loads are automatically generated by setting as shown in Figure 3(a). All load combinations should be converted to nonlinear load cases for considering the second-order effects. The steel design according to the DAM is easily done with the setup as illustrated in Figure 3(b).

(a) Define notional loads  (b) Define steel frame design using AISC-DAM

Figure 3. Implementation of the DAM in SAP2000

3. Optimization-based design of steel moment frames

In the following section, the weight optimization problem of steel moment frames is described. To solve this problem, an optimization algorithm called Differential Evolution is employed. After presenting the detail of the DE algorithm, a procedure for optimizing steel moment frames based on the combination of the DAM and the DE is proposed.

3.1. Optimization problem

Generally, the weight optimization problem of steel moment frames can be described as the following.
Find
\[ \mathbf{x} = [x_1, x_2, \ldots, x_n] \text{ where } x_i^L \leq x_j \leq x_i^U \]  
(4)

To minimize
\[ f(\mathbf{x}) = \sum_{i=1}^{n} \rho A_i L_k \]  
(5)

Subject to:
Strength constraints
\[ R_u \leq \phi R_n \]  
(6)

Serviceability constraints
\[ \Delta \leq [\Delta] \]  
(7)

where: \( \mathbf{x} \) is a \( D \)-dimensional vector of design variables; \( x_i^L \) and \( x_i^U \) are the lower and upper bound of the design variable \( x_i \), respectively; \( f(\mathbf{x}) \) is the objective function, for the weight optimization problem, the objective function is the weight of the frame; \( n \) is the total number of members in the frame; \( \rho \) is the steel density; \( A_i \) is the cross-sectional area of the member \( k \); \( L_k \) is the length of the member \( k \); \( R_u \) and \( \phi R_n \) are the required strength and the design strength, respectively; \( \Delta \) and \([\Delta]\) are the displacement and the allowable displacement, respectively.

The steel frames are designed according to the LRFD method of the AISC specification [9]. Thus, the strength constraints (6) can be expressed as the following formulas.

Shear strength constraint:
\[ V_u \leq \phi V_n \]  
(8)

P-M interaction constraint:
\[ \begin{aligned}
\frac{P_r}{\phi P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi M_{ux}} + \frac{M_{uy}}{\phi M_{uy}} \right) &\leq 1 \quad \text{when} \quad \frac{P_r}{\phi P_n} \geq 0.2 \\
\frac{P_r}{2\phi P_n} + \frac{M_{ux}}{\phi M_{ux}} + \frac{M_{uy}}{\phi M_{uy}} &\leq 1 \quad \text{when} \quad \frac{P_r}{\phi P_n} < 0.2
\end{aligned} \]  
(9)

where: \( V_u \) is the required shear strength; \( \phi V_n \) is the design shear strength; \( P_r \) is the required axial strength; \( \phi P_n \) is the design axial strength; \( M_{ux}, M_{uy} \) and \( \phi M_{ux}, \phi M_{uy} \) are the required flexural strength and the design flexural strength for the strong axis bending, respectively; \( M_{ux}, M_{uy} \) and \( \phi M_{ux}, \phi M_{uy} \) are the required flexural strength and the design flexural strength for the weak axis bending, respectively.

The serviceability constraints include the lateral drift requirement of the frame, the deflection requirement of the flexural members, as well as the vibration requirement of the slabs. The AISC specification [9] refers to the ASCE/SEI 7 Minimum Design Loads for Buildings and Other Structures [12] for the serviceability requirements. Normally, the allowable drift ratio for steel moment frames is taken as 1/300.

3.2. Differential Evolution algorithm

The DE algorithm was firstly introduced by R. Stone and K. Price in 1997 [13]. Some previous studies have been proved the DE as one of the most powerful optimization algorithms. Consequently, the DE algorithm has received a lot of attention from researchers and many improved versions of the DE algorithm have been proposed. However, the original version of the DE algorithm with the “DE/rand/1/exp” mutation strategy is used in this study. The algorithm consists of four main steps.

Initially, a population of \( N_p \) \( D \)-dimensional vectors is randomly generated using the following formula:
\[ x_{ij}(0) = x_{ij}^{\min} + \text{rand}(0, 1)(x_{ij}^{\max} - x_{ij}^{\min}) \]  
(10)

where: \( \text{rand}(0, 1) \) is a uniformly distributed random number between 0 and 1; \( x_{ij}^{\min} \) and \( x_{ij}^{\max} \) are the lower and upper bound for the \( j \)-th component of the \( i \)-th vector.

Secondly, \( N_p \) mutant vectors are created using the following formula.
\[ \mathbf{v}_i = \mathbf{x}_i + F \times (\mathbf{x}_r - \mathbf{x}_s) \]  
(11)
in which: \( r_1 \neq r_2 \neq r_3 \) are randomly selected between 1 and \( Np; F \) is the scaling factor.

In the next step, a trial vector \( u_i \) is created by the cross-over between the mutant vector \( v_i \) and the target vector \( x_i \).

\[
u_{ij}^{(t)} = \begin{cases} v_{ij}^{(t)} & \text{if } j = K \text{ or } \text{rand}[0,1] \leq Cr \\ x_{ij}^{(t)} & \text{otherwise} \end{cases}
\]

(12)

where: \( u_{ij}^{(t)}, v_{ij}^{(t)} \) and \( x_{ij}^{(t)} \) are the \( j^{th} \) component of the trial vector, the mutant vector, and the target vector, respectively; \( K \) is any random number in range 1 to \( D; Cr \) is the crossover rate.

The new population is then produced for the next iteration based on the selection mechanism as described below.

\[
x_{i}^{(t+1)} = \begin{cases} u_{i}^{(t)} & \text{if } f(u_{i}^{(t)}) \leq f(x_{i}^{(t)}) \\ x_{i}^{(t)} & \text{otherwise} \end{cases}
\]

(13)

where: \( f(u_i) \) and \( f(x_i) \) are the objective function value of the trial vector and the target vector, respectively.

For constrained optimization problems, the penalty function is normally used instead of the objective function. The penalty function is calculated by adding a weighted sum of constraint violations into the value of the objective function. By this way, a constrained problem is converted into an unconstrained one. In this study, an effective approach for handling constraints proposed by J. Lampien [14] is used. In general, the constraints can be expressed by the formula:

\[ g_k(x) \leq 1 \]  

(14)

The formula (13) is replaced by the following formula:

\[
x_i = \begin{cases} u_i & \text{if} \\
\left\{ \forall k \in \{1, \ldots, m\}: g_k(u_i) \leq 1 \land g_k(x_i) \leq 1 \\
\land \left\{ f(u_i) \leq f(x_i) \lor \left( \forall k \in \{1, \ldots, m\}: g_k(u_i) \leq 1 \land \left( \forall k \in \{1, \ldots, m\}: g_k(x_i) > 1 \lor \left( \forall k \in \{1, \ldots, m\}: g_k(u_i) > 1 \land g_k(x_i) \leq g'_k(x_i) \right) \right) \right) \right\} & \text{otherwise} \end{cases}
\]

(15)

3.3. DAM-DE optimization procedure

The optimization procedure that combines the steel design according to the DAM approach and the optimization using the DE algorithm is presented in Figure 4. A VBA language program named FrameOpt was previously developed for the optimization of truss frames. This program is currently improved for the proposed procedure. Firstly, a model where the initial imperfections and the stiffness reduction are fully introduced is developed in SAP2000. After conducting the second-order nonlinear analysis, the design constraints are evaluated using the steel design feature of the program SAP2000. During the optimization process, the data such as the cross-section areas as well as the design results are transferred between FrameOpt and SAP2000 through oAPI. The cycle process including mutation, crossover, selection and constraint evaluation is repeated until the termination criterion is satisfied.
4. Numerical examples
In this section, a two-bay, three-story frame is designed based on the proposed procedure to demonstrate its efficiency. This is a well-known benchmark problem which was already studied by Pezeshk et al. [15] using Genetic Algorithm (GA), by Camp et al. [16] using Ant Colony Optimization (ACO), by Carraro et al. [17] using Search Group Algorithm (SGA), by Maheri et al. [18] using a modified multiple-deme Genetic Algorithm (MMDGA).

Figure 4. DAM-DE optimization procedure.

The topology and loading are presented in Figure 5. The material used for the frame is the American steel ASTM A36 with the modulus of elasticity $E=29,000$ ksi ($200,000$ N/mm$^2$) and the yield strength $f_y=36$ ksi ($250$ MPa). The frame members are fabricated from hot rolled wide flange profiles (W-shapes), in which beams’ sections are selected from a list of all W-shape profiles (273 options) while columns’ sections are limited to W10 profiles (18 options). The DE algorithm was originally developed for the continuous design variables, which is not suitable for this problem when the cross-sections of members are selected from a list of available profiles. To deal with this issue, the
cross-section is treated by an integer number corresponding to the sequence number of the considered cross-section in the profile list.

The members are grouped into three groups: (1) outer columns; (2) inner columns and (3) beams. All members of a group are assigned to the same cross-section. Columns are assumed unbraced along their length and the lateral brace points are arranged at every one-sixth span length of beams. The serviceability constraints are neglected in this study.

The frame is designed according to the DAM-DE optimization procedure with the following parameters: the scaling factor $F=0.8$; the crossover probability $Cr=0.7$, the number of individuals in one population $N_p=10\times D=30$. The optimization is terminated when the number of iterations reaches $n=50$.

Figure 6. Convergence history of the optimization process.

The convergence history of an optimization run is shown in Figure 6. The results for 30 independent runs are presented in Table 1. The results of other algorithms in the literature are also summarized in Table 1 for the comparison. It can be seen that the results obtained from the proposed procedure are similar to those from previous studies.

Table 1. Design results for the two-bay, three-story frame.

| Steel design method | [15] | [16] | [17] | [18] | This study |
|---------------------|------|------|------|------|------------|
| Optimization algorithm | ELM  | ELM  | ELM  | ELM  | DAM        |
| (1) Outer columns   | W10x60 | W10x60 | W10x60 | W10x60 | W10x60    |
| (2) Inner columns   | W10x60 | W10x60 | W10x60 | W10x60 | W10x60    |
| (3) Beams           | W24x62 | W24x62 | W24x62 | W24x62 | W24x62    |
| Best weight (lbs)   | 18,792 | 18,792 | 18,792 | 18,792 | 18,792    |
| Worst weight (lbs)  | 40,968 | -     | -     | 19,615 | 20,581    |
| Average weight (lbs)| 22,080 | 19,163 | 19,011 | 19,032 | 19,136    |
| Standard deviation (lbs)| 5,818 | 1,693 | 385 | 533 | 517 |

5. Conclusions
In this study, an optimization procedure, called DAM-DE, is proposed. It combines the new design method and the powerful optimization algorithm for the optimum design of steel frames. To implement the proposed procedure, a program called FrameOpt is developed. This program connects to the commercial program SAP2000 for the steel design because the direct analysis method is well supported in SAP2000. An example of a two-bay, three-story frame is designed using the proposed procedure and the results prove its applicability. The developed program can be applied for practical design.
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