Arbitrary rotation and entanglement of flux SQUID qubits

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We propose a new approach for the arbitrary rotation of a three-level SQUID qubit and describe a new strategy for the creation of coherence transfer and entangled states between two three-level SQUID qubits. The former is succeeded by exploring the coupled-uncoupled states of the system when irradiated with two microwave pulses, and the latter is succeeded by placing the SQUID qubits into a microwave cavity and using adiabatic passage methods for their manipulation.

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I. INTRODUCTION

It has been realized over the last few years that solid state systems that make use of the Josephson effect could play an important role in the area of quantum computation\textsuperscript{1}. A series of successfully performed interesting experiments\textsuperscript{2,3,4,5,6,7,8,9,10,11,12} have confirmed the applicability of these systems. Among superconducting systems performing quantum computations, emphasis has been given to the study of schemes based on magnetic flux states in superconducting quantum interference devices (SQUID)\textsuperscript{1,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20}.

Zhou \textit{et al.}\textsuperscript{15} have recently proposed a three-level Λ-type rf-SQUID qubit. Here, the states of the qubit are the two lower flux states |0⟩ and |1⟩ of the SQUID system that reside in two distinct potential valleys, see figure \textbf{1}. The manipulation of the qubit is done with two microwave fields that couple the lower states to an upper state |e⟩ in a Λ configuration. As the coupling matrix elements corresponding to the transitions |0⟩ ↔ |e⟩ and |1⟩ ↔ |e⟩ are larger than that of the |0⟩ ↔ |1⟩ transition, the three-level SQUID qubit has been shown to be more favorable than the conventional two-level SQUID qubit. Amin \textit{et al.}\textsuperscript{16} have latter shown that the approach of Zhou \textit{et al.} was incomplete and have proposed a more general method for producing an arbitrary qubit rotation using the three-level SQUID qubit. In addition, more recently Yang and Han\textsuperscript{19} have shown that large detuning of the driving fields from the upper state could be favorable for implementing single qubit rotation in the three-level SQUID qubit. Finally, Yang \textit{et al.}\textsuperscript{20,21} have proposed two different strategies using three-level SQUID qubits in a microwave cavity for entanglement, logical quantum gates and quantum information transfer.

In this article our goal is two-fold. First, we propose a new approach for the arbitrary rotation of a three-level SQUID qubit and present a rotation method based on Rabi oscillations. As we are in the microwave domain, the sufficiently precise control of the pulse area is possible, unlike in the optical domain. Second, we apply a new strategy for the realization of coherence transfer and for the creation of entangled states between two three-level SQUID qubits. This is succeeded by placing the qubits into a microwave cavity and by using adiabatic passage methods for their manipulation. Both of our approaches are fundamentally different from the ones that have been proposed so far in the literature as they are based on the exploration of the coupled and uncoupled states of the system. Moreover, the latter method is an adiabatic method, and such methods are robust with respect to fluctuations of several experimental parameters\textsuperscript{22}. We will discuss this issue for our case below.

The article is organized as follows. In the next section we briefly summarize the model for the three-level Λ-type SQUID qubit\textsuperscript{15} and present our method for arbitrary qubit rotation. The general method is also specialized in two simple cases. Then, in section III we study the interaction of two three-level SQUID qubits in a microwave cavity and show that using adiabatic methods both robust quantum information transfer and entanglement are possible. Finally, we summarize our results in section IV.

II. ROTATION OF A THREE-LEVEL SQUID QUBIT

The model qubit consists of an rf-SQUID which interacts with two microwave fields. The rf-SQUID is made of a superconducting ring interrupted by a Josephson tunnel junction. The system’s generalized coordinate is the total magnetic flux in the ring Φ, subjected to the potential\textsuperscript{22}

$$\hat{U}(\Phi) = \frac{(\Phi - \Phi_x)^2}{2L} - E_J \cos \left(2\pi \frac{\Phi}{\Phi_0}\right). \quad (1)$$

Here, $L$ is the ring inductance, $\Phi_x$ is an externally applied magnetic flux to the SQUID, $E_J = L \Phi_0 / 2\pi$ is the maximum value of the Josephson energy, with $I_c$ being...
the critical current of the junction, and $\Phi_0 = h/2e$ is the flux quantum. The Hamiltonian of the system can be written as

$$\hat{H}_0 = \frac{Q^2}{2C} + U(\Phi),$$  \hspace{1cm} \text{(2)}$$

where $Q = -i\hbar \partial / \partial \Phi$ is the charge on the junction capacitance $C$. The flux $\Phi$ and the charge $Q$ are canonically conjugate operators satisfying the commutation relation $[\Phi, Q] = i\hbar$. A typical potential of this form is shown in Fig. [1].

We will first discuss the case of single SQUID qubit rotation. To achieve this the SQUID is driven by two microwave pulses. The microwave pulses are considered as linearly polarized electromagnetic fields with their magnetic fields perpendicular to the plane of the SQUID ring. We take the angular frequencies of the two microwave fields $\omega_0$ and $\omega_1$ to be near resonant with the transitions $|0\rangle \leftrightarrow |e\rangle$ and $|1\rangle \leftrightarrow |e\rangle$ respectively, where $|0\rangle$ and $|1\rangle$ are the states of the SQUID qubit and $|e\rangle$ is an excited state, as is shown in figure [1]. The resonant approximation can then be used to describe the dynamics of the system, i.e. we assume that only the states $|0\rangle$, $|1\rangle$, and $|e\rangle$ contribute to the dynamics of the system. This approximation has already been used successfully in several recent articles and the system has been termed three-level $\Lambda$-type SQUID qubits\textsuperscript{17,18,19,20,21}. In order to analyze our system further we use the interaction picture and apply the rotating wave approximation\textsuperscript{25,26}. Then, the Hamiltonian of the system can be expressed as

$$\hat{H} = \frac{\hbar}{2}\Omega_0(t)e^{-i\Delta_0 t}|0\rangle\langle e| + \frac{\hbar}{2}\Omega_1(t)e^{-i\Delta_1 t}|1\rangle\langle e| + \text{H.c.},$$  \hspace{1cm} \text{(3)}$$

where $\Delta_j = \omega_j - \omega_j - \tilde{\omega}_j$, with $j = 0, 1$ is the microwave field detuning from resonance with the $|j\rangle \leftrightarrow |e\rangle$ transition, where $\hbar \omega_j$ denotes the energy of the $j$th stationary state of the SQUID.

We assume that $\Delta_0 = \Delta_1$ such that the system is at two-photon resonance. Then, in the rotating wave picture\textsuperscript{25}, obtained by applying a unitary transformation to the Hamiltonian of Eq. (3), the Hamiltonian of the system is given by

$$\hat{H}' = \hbar \Delta |e\rangle\langle e| + \frac{\hbar}{2}(|0\rangle\langle 0|e| + \Omega_1|1\rangle\langle e| + \text{H.c.}).$$  \hspace{1cm} \text{(4)}$$

We require that the light pulses share the same time-dependence but their peak amplitudes can be different, and there can be a phase-difference between them. Hence, the Rabi frequencies in the Hamiltonian of Eq. (4) read

$$\Omega_0(t) = \Omega(t) \cos \varphi, \quad \Omega_1(t) = \Omega(t) e^{i\eta} \sin \varphi,$$  \hspace{1cm} \text{(5)}$$

where $\eta$ and $\varphi$ are fixed angles. These two pulses define a coupled state $|C\rangle$

$$|C\rangle = \cos \varphi |0\rangle + e^{i\eta} \sin \varphi |1\rangle,$$  \hspace{1cm} \text{(6)}$$

and an uncoupled state

$$|NC\rangle = -\sin \varphi |0\rangle + e^{i\eta} \cos \varphi |1\rangle,$$  \hspace{1cm} \text{(7)}$$

with respect to the microwave pulses\textsuperscript{27}. In this basis the initial state $|\psi_i\rangle$ of the SQUID under consideration is given by

$$|\psi_i\rangle = \langle NC|\psi_i\rangle|NC\rangle + \langle C|\psi_i\rangle|C\rangle,$$  \hspace{1cm} \text{(8)}$$

and the Hamiltonian Eq. (6) reads

$$\hat{H}' = \hbar \Delta |e\rangle\langle e| + \frac{\hbar}{2}(|0\rangle\langle 0|e| + \text{H.c.}).$$  \hspace{1cm} \text{(9)}$$

This Hamiltonian is that of a two-level system. There are several analytically solvable models for two-level systems with pulsed excitation. The transfer matrix of the general solution can be parameterized as

$$\hat{U}(t_f, t_i) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \alpha & -\beta^* \\ 0 & \beta & \alpha^* \end{pmatrix},$$  \hspace{1cm} \text{(10)}$$

in the basis $\{|NC\rangle, |C\rangle, |e\rangle\}$. The columns of the matrix $\hat{U}(t_f, t_i)$ correspond to the state vector of the system at time $t_f$ if it was initially in states $|NC\rangle, |C\rangle$, and $|e\rangle$, respectively. Here, we need such a model that after the pulse has passed the coupled state $|C\rangle$ acquires a phase shift $-\delta$ and the excited state $|e\rangle$ is not populated. Hence the parameters of the transfer matrix $\hat{U}(t_f, t_i)$ should be given by

$$\alpha = e^{-i\delta}, \quad \beta = 0.$$  \hspace{1cm} \text{(11)}$$

Therefore, at the end of the process the state of the SQUID is given by

$$|\psi_f\rangle = \langle NC|\psi_i\rangle|NC\rangle + e^{-i\delta}\langle C|\psi_i\rangle|C\rangle.$$  \hspace{1cm} \text{(12)}$$

By inserting the explicit form of the scalar products $\langle NC|\psi_i\rangle$ and $\langle C|\psi_i\rangle$, and the definitions Eqs. (6), (7) into Eq. (12) we obtain

$$|\psi_f\rangle = e^{-i\delta/2} \hat{R}_n(\delta)|\psi_i\rangle,$$  \hspace{1cm} \text{(13)}$$

where $n = (\sin 2\varphi \cos \eta, \sin 2\varphi \sin \eta, \cos 2\varphi)$. Apart from a global phase $-\delta/2$ the states $|\psi_i\rangle$ and $|\psi_f\rangle$ are connected through the rotation $\hat{R}_n(\delta)$. The rotation $\hat{R}_n(\delta)$ is an element of the SU(2) group and describes a rotation about the axis $n$, through the angle $\delta$. If the qubit is isolated, then the global phase $-\delta/2$ is unimportant. If the qubit is part of a larger system, e.g. there are several qubits which form a quantum computer, then the global phase is clearly relevant, however, it may be incorporated into the algorithm being implemented on the quantum computer.

The simplest model for a two-level system that can be used to realize the dynamics described above is the Rabi model with rectangular pulse shape and constant detuning. Another possibility is the Rosen-Zener model\textsuperscript{28,29}. 
with hyperbolic-secant pulse shape and with constant detuning as well. In the case of the Rosen-Zener model the phases are given by rather involved formulae and we will not discuss it here. A further, and rather general model, is the one obtained under pulsed excitation in the case of far off-resonant Raman coupling, i.e. the case that \( \Delta \gg |\Omega(t)|/2 \). Below, we describe briefly the Rabi model and the off-resonant Raman model. In the Rabi model the elements of the transfer matrix [110] are given by

\[
\alpha = \left[ \cos \left( \frac{i}{2} \tilde{\Omega} T \right) + i \frac{\Delta}{\tilde{\Omega}} \sin \left( \frac{i}{2} \tilde{\Omega} T \right) \right] e^{-i\Delta T/2}, \tag{14a}
\]

\[
\beta = -i \frac{\Omega}{\tilde{\Omega}} \sin \left( \frac{i}{2} \tilde{\Omega} T \right) e^{-i\Delta T/2}, \tag{14b}
\]

where \( \tilde{\Omega} = \sqrt{\Omega^2 + \Delta^2} \). For pulse area \( \tilde{\Omega} T = 2\pi m \), with \( m \) being an integer, the transition amplitude \( \beta \) is zero, as is required in Eq. (11). Hence the rotation angle \( \delta \) is given by

\[
\delta = \left( \frac{\Delta}{\tilde{\Omega}} + 1 \right) m\pi. \tag{15}
\]

We note that the actual physical system is described by the Hamiltonian [1]. The two Rabi frequencies \( \Omega_0 \) and \( \Omega_1 \) are derived from the same pulse according to Eq. [1], which is described by the Rabi frequency \( \tilde{\Omega} \), detuning \( \Delta \), and duration \( T \) calculated above.

In the off-resonant Raman model [111] state \(|e\rangle\) is eliminated adiabatically and

\[
\alpha \approx \exp \left[ \frac{i}{4\Delta} \int_{t_i}^{t_f} |\Omega(t)|^2 dt \right], \tag{16}
\]

\[
\beta \approx 0. \tag{17}
\]

Therefore, the rotation angle reads

\[
\delta = -\frac{1}{4\Delta} \int_{t_i}^{t_f} |\Omega(t)|^2 dt. \tag{18}
\]

We have performed simulations for a realistic SQUID system. We used the same parameters for the SQUID as in the work of Zhou et al. [117], i.e. \( L = 100 \) pH, \( C = 40 \) fF, \( I_c = 3.95 \mu A \) and \( \Phi_0 = -0.501 \Phi_0 \). For this SQUID the dissipation time could exceed 1 \( \mu \)sec. The qubit rotation time was found to be dependent on the model of interaction that we used and varied from sub-nanosecond times to about 30 nsec for moderate Rabi frequencies values of maximum strength of 1-5 GHz. An example of inversion from state \(|0\rangle\) to state \(|1\rangle\) using the Rabi model is shown in Fig. 2. We note that shorter qubit rotation times can be achieved by increasing the microwave field intensities. Of course, this cannot happen arbitrarily as after a certain limit the rotating wave approximation will not be appropriate for describing the system and the effect of other states, that have been omitted here, will have to be taken into account.

### III. Quantum Information Transfer and Creation of Entangled States

We will now present a strategy for achieving entanglement and also information (coherence) transfer between two A-type SQUID qubits. We place both SQUIDs (we denote them by \( A, B \)) in a microwave cavity, see Fig. 3 and assume that the transitions \(|0\rangle_j \rightarrow |e\rangle_j\), with \( j = A, B \) and where \(|e\rangle_j\) are excited states different from those used for the single qubit rotation, are coupled to the same cavity mode, with coupling constant \( g \), which is assumed real. The other transitions \(|0\rangle_j \rightarrow |e\rangle_j\), with \( j = A, B \) are coupled with external laser fields, with Rabi frequency \( \Omega_j \), with \( j = A, B \), such that each microwave field addresses individually only one SQUID. We assume again that the cavity field and the external microwave fields are at two photon resonance. The Hamiltonian for the two SQUID system in the rotating wave picture and in the rotating wave approximation is given by

\[
\hat{H}_{AB} = \hbar \sum_{j=A,B} \Delta |e\rangle_j \langle e'| + \frac{\hbar}{2} \sum_{j=A,B} \left( \Omega_j |0\rangle_j \langle e' | + g |1\rangle_j \langle e'| \hat{b}^+ + \text{H.c.} \right)
\]

where \( \hat{b}^\dagger \) is the creation operator of a cavity photon. We are interested in the uncoupled states of the two SQUID system, which involves the vacuum cavity state \(|0\rangle_c\). These are

\[
|\psi^{I} \rangle = \mathcal{N} \left[ (\Omega_A(t)g|0,1,0\rangle + \Omega_B(t)g|0,1,1\rangle)|0\rangle_c \right.
\]

\[
-\Omega_A(t)\Omega_B(t)|1,1,1\rangle), \tag{20a}
\]

\[
|\psi^{II} \rangle = |1,1,0\rangle, \tag{20b}
\]

where \( \mathcal{N} = 1/\sqrt{(|\Omega_A|^2 + |\Omega_B|^2)g^2 + |\Omega_A|^4|\Omega_B|^2} \). These states are uncoupled because

\[
\hat{H}_{AB} |\psi^q \rangle = 0, \quad q = I, II \tag{21}
\]

The state of Eq. (20a) is constant, it doesn’t change with time. The other uncoupled state Eq. (20a) may vary with time as the Rabi frequencies \( \Omega_A(t) \) and \( \Omega_B(t) \) can be time dependent.

The subset

\[
\mathcal{H}_0 = \{ |0,1,0\rangle, |e',1,0\rangle, |1,1,1\rangle, |1,e',0\rangle, |1,0,0\rangle \} \tag{22}
\]

of the basis states of the system is closed in the sense that the matrix elements of the Hamiltonian Eq. (19) between any other basis state and a state taken from this subset is zero. Here, \(|a,b,n\rangle = |a\rangle_A |b\rangle_B |n\rangle_c\), where \(|n\rangle_c\) denotes the state of the photons in the cavity. Since the uncoupled state Eq. (20a) is a superposition of the basis states from the subset Eq. (22), then the dynamics of this uncoupled state can be obtain in the subset \( \mathcal{H}_0 \). In the basis states
of $\mathcal{H}_0$ the Hamiltonian \[^{(19)}\] is given by

$$
\hat{H}_{AB}^\prime = \frac{\hbar}{2} \begin{bmatrix}
0 & \Omega_A & 0 & 0 \\
\Omega_A & 2\Delta' & g & 0 \\
0 & g & 0 & 0 \\
0 & 0 & 2\Delta' & \Omega_B \\
0 & 0 & 0 & 0
\end{bmatrix}.
$$

(23)

Let us consider first information transfer between SQUIDs $A$ and $B$. If the system is prepared initially in a state of the form

$$
|\psi(t_i)\rangle = (c_0|0\rangle_A + c_1|1\rangle_A)|1\rangle_B|0\rangle_c,
$$

(24)

with $c_0$ and $c_1$ being arbitrary coefficients satisfying $|c_0|^2 + |c_1|^2 = 1$, then it is said that the state of the qubit $A$ is transferred in qubit $B$ if after some interaction the state of the system is given by

$$
|\psi(t_f)\rangle = |1\rangle_A (c_0|0\rangle_B + c_1|1\rangle_B)|0\rangle_c.
$$

(25)

How can we realize this process in the two SQUID system? We observe that the uncoupled state Eq. (24a) is equal to the state $|0, 1, 0\rangle$, for $\Omega_B \gg \Omega_A$. Hence, let us choose the external pulses $A$ and $B$ such that at early times around $t_i$ the Rabi frequencies satisfy the condition $\Omega_B(t_i) \gg \Omega_A(t_i)$. Then the initial state of the system Eq. (23) can be expressed as

$$
|\psi(t_i)\rangle = c_0|\psi^t\rangle + c_1|\psi^f\rangle.
$$

(26)

In the opposite limit when $\Omega_B \ll \Omega_A$, the uncoupled state $|\psi^t\rangle$ coincides with the state $|1, 0, 0\rangle$. Let us choose the time dependence of the pulses $A$ and $B$ such that at early times we have $\Omega_B(t_i) \gg \Omega_A(t_i)$, then after a smooth variation, at late times we have $\Omega_B(t_f) \ll \Omega_A(t_f)$. We also require that the two pulses overlap. These conditions ensure that the uncoupled state $|\psi^t\rangle$ is well-defined throughout the whole time evolution and the state of the system follows adiabatically this uncoupled state\[^{(22)}\]. The pulse sequence described here resembles that of the stimulated Raman adiabatic passage (STIRAP), for reviews see\[^{(23)}\]. Hence, in the above adiabatic process the state $c_0|\psi^t\rangle + c_1|\psi^f\rangle$ will evolve smoothly to the final state given by Eq. (24a), and the required information transfer between SQUIDs $A$ and $B$ will be realized in this way. The adiabaticity conditions are given by

$$
|\langle\psi_k|\psi^t\rangle| \ll |\varepsilon_k|, \quad k = 1 \ldots 4,
$$

(27)

where $|\psi_k\rangle$ is an instantaneous eigenstate of the Hamiltonian \[^{(24)}\], and $\varepsilon_k$ is the corresponding non-zero eigenvalue

$$
\hat{H}_{AB}^\prime|\psi_k\rangle = \varepsilon_k|\psi_k\rangle.
$$

(28)

The fifth eigenstate is $|\psi^f\rangle$ belonging to the eigenvalue zero. The eigenvalues $\varepsilon_k$ are given by

$$
\varepsilon_k = \frac{\hbar \Delta}{2} \pm \frac{\hbar}{2} \sqrt{4\Delta^2 + 2\tilde{\Omega}^2 \pm 2\sqrt{(|\Omega_A|^2 - |\Omega_B|^2)^2 + 4g^4}},
$$

(29)

with $\tilde{\Omega}^2 = 2g^2 + |\Omega_A|^2 + |\Omega_B|^2$, and the eigenvectors read

$$
|\psi_k\rangle = \frac{1}{\sqrt{\lambda_k}} \begin{bmatrix}
g\varepsilon_k |\Omega_A|^2 \\
2g^2 \varepsilon_k \\
-g(|\Omega_A|^2 + 4\varepsilon_k(\Delta - \varepsilon_k)) \\
-2\varepsilon_k(g^2 + |\Omega_A|^2 + 4\varepsilon_k(\Delta - \varepsilon_k)) \\
-2\varepsilon_k(g^2 + |\Omega_B|^2 + 4\varepsilon_k(\Delta - \varepsilon_k))
\end{bmatrix}.
$$

(30)

By inserting Eqs. (24a), (24b), and (24c) into Eq. (24) into the fulfillment of adiabaticity can be verified for the chosen external Rabi frequencies, cavity coupling strength, and detuning; similarly to the standard three-level STIRAP, large pulse areas ($\Omega_{A,B}T, gT \gg 1$, with $T$ being the characteristic length of the pulses), and smooth, slowly varying pulse envelopes are required.

We now address the question of creating entanglement between the two SQUID systems. The entangled state is defined by

$$
|\psi(\theta, \xi)\rangle = (\cos \theta|1, 0\rangle + e^{-i\xi} \sin \theta|0, 1\rangle)|0\rangle_c,
$$

(31)

where $\theta$ and $\xi$ are fixed angles. In order to obtain this state, we apply the method of fractional STIRAP\[^{(34-37)}\]. We chose the initial state of the system to be $|\psi(t_i)\rangle = |0, 1, 0\rangle$ and the order of the pulses as before, i.e. at early times ($t \to t_f$) $\Omega_B(t) \gg \Omega_A(t)$, so that the uncoupled state $|\psi^t\rangle$ coincides with the initial state of the system. If the evolution is adiabatic and the two pulses are switched off such that $\Omega_A(t) = \Omega_B(t) \to \Omega_A(t)$, the entangled state Eq. (31) is created. Moreover, by means of the inversion of the state of SQUID A, the entangled state

$$
|\phi(\theta, \xi)\rangle = (\cos \theta|0, 0\rangle + e^{-i\xi} \sin \theta|1, 1\rangle)|0\rangle_c.
$$

(32)

can also be generated.

Our information transfer and entanglement scheme possesses the merits of both SQUID cavity QED quantum computing schemes\[^{(29)}\] and adiabatic evolution transfer schemes\[^{(33)}\]. The major advantages of SQUID cavity QED schemes are the simplicity of the coupling between the two SQUIDs via the cavity mode, the protection of SQUIDs from the interaction with the environment and thus the reduction of decoherence, and the technical simplicity of placing SQUID in cavities. The main advantage of adiabatic evolution transfer methods, such as STIRAP, is the robustness of the method for moderate fluctuations of the microwave pulse parameters. In addition, as long as the evolution remains adiabatic the excited states $|\epsilon\rangle_j$, with $j = A, B$ are minimally populated, while the cavity mode is only populated in the transient regime. The latter can be greatly suppressed, or completely avoided, by keeping the coupling $g$ greater than the Rabi frequencies $\Omega_j$, with $j = A, B$. Therefore dissipation and decoherence can be reduced using this scheme.

We have performed simulations for the SQUID presented in section II. For coupling strengths for the Rabi frequencies and the cavity coupling coefficient of the order of 1-5 GHz information transfer takes from 10-30
nsec while the creation of entangled states needs slightly larger times up to 50 nsec. A typical example for switching from state $|0, 1, 0\rangle$ to state $|1, 0, 0\rangle$ is shown in Fig. 4. We also present the creation of the entangled state $1/\sqrt{2}(|0, 1, 0\rangle + |1, 0, 0\rangle)$ in Fig. 5.

IV. CONCLUSION

In summary, we have presented schemes for basic state-manipulations of a single rf-SQUID and two rf-SQUID qubits. In the first part of the paper we have considered a scheme for arbitrary rotation of a three-level rf-SQUID qubit. The rotation is performed by irradiating the SQUID with two microwave pulses that share the same time dependence. These pulses define a coupled and an uncoupled state out of the two SQUID states forming the qubit, and the rotation of the qubit results from the phase-shift on the coupled state caused by the applied microwave pulses. The main advantage of our proposed scheme is that it can be implemented for a large variety of pulse shapes and detunings, according to the capabilities of the experimentalists.

In the second part of the paper we have proposed schemes for information transfer and creation of entangled states between two rf-SQUIDs. These processes are mediated by a microwave cavity, so that the SQUIDs communicate via photon exchange through a cavity mode. We have applied an adiabatic population transfer scheme, therefore our method is robust with respect to the moderate fluctuations of the experimental parameters. For proper choice of the external Rabi frequencies the excitation of the cavity field can be almost completely suppressed, hence the decoherence due to the imperfection of the cavity can be minimized.

In closing, we note that our schemes are quite general and can be applied to other areas of quantum computation where three-level qubits are used such as, for example, in atoms and trapped ions in cavities.35,39

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FIG. 1: The potential energy, the stationary states and the coupling configuration of the rf-SQUID system. The states $|0\rangle$, $|1\rangle$, and $|e\rangle$ are coupled my means of two microwave pulses with Rabi frequencies $\Omega_0, \Omega_1$ in a $\Lambda$ configuration.

FIG. 2: The time evolution of the populations in states $|0\rangle$ (solid curve), $|1\rangle$ (dashed curve) and $|e\rangle$ (dot-dashed curve) for parameters $\Omega = 2$ GHz, $\varphi = 5\pi/4$, $\eta = \pi$, $m = 2$, $\Delta = -2/\sqrt{3}$ GHz and $\delta = \pi$, using the Rabi model.
FIG. 3: A schematic representation of two rf-SQUIDs in a microwave cavity. The two SQUIDs are addressed individually by microwave pulses. They are also coupled to the cavity field with coupling strength $g$.

FIG. 4: (a) The Rabi frequencies $|\Omega_A(t)|$ (dashed curve) and $|\Omega_B(t)|$ (solid curve) for the case that $\Omega_A(t) = \Omega e^{-(t-\tau_A)^2/\tau_p^2}$, $\Omega_B(t) = \Omega e^{-(t-\tau_B)^2/\tau_p^2}$ for parameters $\Omega = -2$ GHz, $g = 3$ GHz, $\tau_A = 23$ ns, $\tau_B = 17$ ns, $\tau_p = 6.5$ ns. (b) The time evolution of the populations in states $|0,1,0\rangle$ (solid curve), $|1,0,0\rangle$ (dashed curve) and $|1,1,1\rangle$ (dot-dashed curve) for the above parameters.
FIG. 5: (a) The Rabi frequencies $|\Omega_A(t)|$ (dashed curve) and $|\Omega_B(t)|$ (solid curve) for the case that $\Omega_A(t) = \Omega \sin \theta e^{-(t-\tau_A)^2/\tau_p^2}$, $\Omega_B(t) = \Omega \left[ e^{-(t-\tau_B)^2/\tau_p^2} + \cos \theta e^{-(t-\tau_A)^2/\tau_p^2} \right]$ for parameters $\Omega = -2$ GHz, $g = 3$ GHz, $\tau_A = 38.5$ ns, $\tau_B = 25$ ns, $\tau_p = 10$ ns, $\theta = \pi/4$. (b) The time evolution of the populations in states $|0, 1, 0\rangle$ (solid curve), $|1, 0, 0\rangle$ (dashed curve) and $|1, 1, 1\rangle$ (dot-dashed curve) for the above parameters.