Abstract. We review the effects of electron–electron interactions on the ground-state spin and the transport properties of ultra-small chaotic metallic grains. Our studies are based on an effective Hamiltonian that combines a superconducting BCS-like term and a ferromagnetic Stoner-like term. Such terms originate in pairing and spin exchange correlations, respectively. This description is valid in the limit of a large dimensionless Thouless conductance. We present the ground-state phase diagram in the fluctuation-dominated regime where the single-particle mean level spacing is comparable to the bulk BCS pairing gap. This phase diagram contains a regime in which pairing and spin exchange correlations coexist in the ground-state wave function. We discuss the calculation of the tunneling conductance for an almost-isolated grain in the Coulomb-blockade regime, and present measurable signatures of the competition between superconductivity and ferromagnetism in the mesoscopic fluctuations of the conductance.

Key words: Metallic grains; Superconductivity; Ferromagnetism

1. Introduction

Superconductivity and ferromagnetism compete with each other. Pairing correlations lead to Cooper pairs of electrons with opposite spins and thus tend to minimize the total spin of the grain, while ferromagnetic correlations tend to maximize the total spin.

Nevertheless, it is well known that superconducting and ferromagnetic order can be present simultaneously in bulk systems when ferromagnetism is caused by localized paramagnetic impurities (Abrikosov and Gorkov 1960, 1961; Clogston 1962; Chandrasekhar 1962; Fulde and Ferrell 1964; Larkin and Ovchinnikov 1964, 1965). Recently, it was observed that both states of matter can coexist in high-Tc superconductors (Tallon et al. 1999; Bernhard et al. 1999) and in heavy fermion systems (Saxena et al. 2000; Pfleiderer et al. 2001; Aoki et al. 2001) even when the electrons that are
responsible for superconductivity and ferromagnetism are the same. This surprising observation led to the search of new theoretical models that can describe this coexistence.

In ultra-small metallic grains, in which the bulk pairing gap $\Delta$ is comparable to the single-particle mean level spacing $\delta$, a coexistence regime of superconductivity and ferromagnetism was predicted (Falci et al. 2003; Ying et al. 2006; Schmidt et al. 2007). The ground state of the grain is described by a state where a few single-particle levels around the Fermi energy are singly occupied while all other electron are paired. This coexistence regime is characterized by spin jumps and its size can be tuned by an external Zeeman field.

However, it is difficult to measure the ground-state spin of a grain, and a more directly measurable quantity is the tunneling conductance through the grain (von Delft and Ralph 2001). In addition, one has to take into account the mesoscopic fluctuations that are typical for chaotic grains (Alhassid 2000). Effects of exchange correlations on the conductance statistics in quantum dots, in which pairing correlations are absent, were studied in Alhassid and Rupp (2003). In Schmidt and Alhassid (2008) we identified signatures of the coexistence of pairing and exchange correlations in the mesoscopic fluctuations of the conductance through a metallic grain that is weakly coupled to leads.

The fabrication and control of nano-size metallic devices is a challenging task. The first conductance measurements in ultra-small metallic grains were carried out in the mid-nineties (Ralph et al. 1995; Ralph et al. 1997; Black et al. 1996). The grains were produced by breaking nanowires and their size was difficult to control. Coulomb blockade, discrete levels and pairing effects were observed in a single grain by measuring the tunneling conductance (von Delft and Ralph 2001). During the last decade numerous technological advances led to an increase in control and tunability of ultra-small metallic grains. Break junction techniques (Park et al. 1999) and electromigration (Bolotin et al. 2004) were used for gating and establishing precise contact between leads and grain. A particularly important recent development has been the use of monolayers of organic molecules as tunnel barriers, enabling control of the size and shape of the grain (Kuemmeth et al. 2008). New materials have been tested as well. Cobalt nanoparticles were used to investigate the effect of ferromagnetism (Deshmukh et al. 2001; Kleff et al. 2001). Spin-orbit coupling and non-equilibrium excitations were studied in gold grains (Bolotin et al. 2004; Kuemmeth et al. 2008; Gueron et al. 1999).

The recent discovery of superconductivity in doped silicon at atmospheric pressure and critical temperatures of a few hundred millikelvin (Bustarret et al. 2006) might further facilitate the development of mesoscopic superconducting devices. However, the competition between superconductivity and ferromagnetism has not been investigated so far.