Tunneling in heavy-fermion junctions
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In this paper I review recent theoretical and experimental advances in understanding of tunneling processes between normal metals and metals containing electrons which occupy partially filled \( f \)-orbitals. In heavy-fermion materials the effective mass of the quasiparticles far exceeds the bare electron mass due to strong hybridization between conduction and \( f \)-orbital states. Kondo lattices form a class of heavy-fermion systems in which an average occupation number of \( f \)-electron states is close to an integer. Therefore, the tunneling into a Kondo lattice necessarily involves co-tunneling process of a tip electron into an \( f \)-electron state of a Kondo lattice. This co-tunneling process is manifested in the Fano-lineshape of differential conductance as a function of an applied voltage, which has been routinely observed in recent experiments on various Kondo lattice systems. To illustrate these ideas, I discuss the problem of the tunneling junction when the single particle states in the tip are also a product of hybridization between conduction and \( f \)-states, i.e. tunneling between two heavy-fermion materials.

KEYWORDS: tunneling, Kondo lattice, correlated electrons

1. Tunneling into a Kondo lattice: overview

In complex materials with atoms containing unfilled \( f \)-orbitals, interaction between the conduction and \( f \)-electrons leads to the development of a novel electronic states of matter at very low-temperatures. One specific feature of these states is the large effective mass of the electronic excitations. A phenomenologically theory for the emergence of the heavy fermions has been proposed by Coqblin and Blandin\(^1\) and Sir Neville Mott:\(^2\) strong hybridization between conduction and \( f \)-electrons produces two bands separated by the hybridization gap, so that renormalized position of the chemical potentials crosses the lower band where the Fermi velocity is significantly reduced implying large effective mass of the electronic excitations. While this picture can also successfully account for the metal-insulator transition in a number of \( f \)-electron systems with mixed-valence of the \( f \)-ion - SmB\(_6\) and YbB\(_{12}\) being the two canonical \( f \)-orbital semiconductors - the emergence of the coherent band of heavy electrons in both mixed valence and Kondo lattice systems still remained not well understood.\(^3\)

Experimentally, one of the main challenges in probing the emergence of the heavy quasiparticles is in the lack of high resolution spectroscopic measurements. Remarkably, this challenge has been overcome in scanning tunneling microscopy measurements\(^4–8\) as well as in the point contact spectroscopy.\(^9,10,14\) Recent tunneling experiments have been convincingly able to trace the formation of the heavy quasiparticles. What is more, momentum and energy resolved tunneling spectra visualized not only the formation of the heavy quasiparticles, but also the formation of unconventional superconductivity in a prototypical Kondo lattice heavy-fermion superconductor CeCoIn\(_5\).\(^4,7\) Formation of the heavy-particles has also been successfully resolved in more itinerant systems, such as ‘hidden order’ compound UR\(_2\)Si\(_2\)\(^5\) and in a best candidate for correlated topological insulator SmB\(_6\).\(^8\)

Asymmetric or Fano lineshape of the differential tunneling conductance is the basic feature observed in tunneling experiments into Kondo lattice systems. The origin of the Fano lineshape has been explained in a number of recent theoretical papers.\(^11–14\) The basic idea for the understanding of the tunneling into a Kondo lattice originates from the earlier models developed by Appelbaum\(^15\) and Anderson\(^16\) of the tunneling in metal-insulator-metal (M-I-M) junctions with an insulating layer containing small concentration of magnetic impurities. As they have shown, the tunneling between two metals necessarily involves a process of co-tunneling: an electron from one metal can tunnel directly to another metal, but can also tunnel through a state on impurity by flipping its spin. Similarly, a tunneling process between the normal metal tip into a Kondo lattice, electron from a tip tunnels into a conduction orbitals as well as into a composite fermion state created by a strong hybridization between conduction and \( f \)-electrons of a Kondo lattice.\(^11\)

Interestingly, the observation of the Fano lineshape can actually be used as a fingerprint of strong hybridization between conduction and \( f \)-electrons even in heavy-fermion systems where Kondo screening competes with an onset of antiferromagnetic order as in CeAuS\(_2\), for example.\(^17\) It is important to note, however, that the Fano lineshape of the tunneling conductance appears only for the case when the \( f \)-electron level acquires a finite lifetime.\(^13\) Within the currently used mean-field theory approaches\(^18–22\) finite lifetime of the \( f \)-level can either be introduced on the phenomenological level or derived by taking into account the fluctuation corrections to the mean-field theory.\(^22\) In this paper, I will review these ideas by using the junction between the two heavy fermion metals as an example. I will derive an approximate tunneling Hamiltonian and by resorting to the mean-field approximation I calculate the differential tunneling conductance and discuss the various limiting cases.

I have organized this paper as follows. In the next Section I will discuss the problem of tunneling between a tip and a host both of which contain states with partially filled \( f \)-orbitals. In deriving the effective tunneling hamiltonian for that problem I will review the main ideas which went into recently developed
2. Tunneling between the two Kondo lattices

In this Section we discuss the features in differential conductance which would appear in the experiment involving the tunneling contact between the two heavy-fermion metals. At first sight it may seem that as soon as electron leaves a heavy-fermion tip it looses all its mass. In what follows I first show that the quasiparticle coherence factors are sufficiently long ranged and a quasiparticle from a tip retains its heavy mass as it reaches the Kondo lattice. Then I proceed with the derivation of the effective tunneling Hamiltonian within the mean-field approximation and obtain analytic expression for the differential conductance. At the end of this Section I also discuss the effects of the fluctuations beyond the mean-field theory on the tunneling conductance.

2.1 general discussion

In a heavy-fermion metal single particle states \(|\sigma\rangle = \hat{p}^\dagger_{\sigma}(0)|0\rangle\) in the tip are formed by the superposition of the conduction and localized \(f\)-states. Within the mean-field theory approximation controlled by the parameter \(1/N\) where \(N\) is given by the degeneracy of the \(f\)-orbital multiplet, it follows,\(^{18-24}\)

\[
\hat{p}_{\sigma} = \sum_l \left( u_{il} \hat{f}_{l\sigma} + v_{il} \hat{d}_{l\sigma} \right). \tag{1}
\]

Here \(\hat{d}_{l\sigma}, \hat{f}_{l\sigma}\) are an annihilation operators for conduction and \(f\)-electrons on a site \(l\) in the tip and \(u_{il}, v_{il}\) are heavy-fermion coherence factors:

\[
\begin{bmatrix}
    u(r_{ij}) \\
    v(r_{ij})
\end{bmatrix} = \sum_{k} \left( \frac{u_k}{v_k} \right) e^{-i\vec{k} \cdot \vec{r}_{ij}}, \quad \vec{r}_{ij} = \vec{r}_i - \vec{r}_j. \tag{2}
\]

Consequently, the momentum dependence of the coherence factors is determined by the spectrum of the conduction electrons \(\epsilon_k\), \(f\)-electron single particle energy \(\epsilon_f\) and hybridization between them \(V_i\). In the simplest tight-binding approximation the spectrum of the conduction electrons is given by \(\epsilon_k = -2t_c \sum_{i=x,y,z} \cos k_i - \mu\) where \(t_c\) is the hopping amplitude and \(\mu\) is the chemical potential. The expressions for the coherence factors are

\[
u_k^2 = \frac{1}{2} \left( 1 + \frac{\epsilon_k - \epsilon_f}{R_k} \right), \quad \nu_k^2 = \frac{1}{2} \left( 1 - \frac{\epsilon_k - \epsilon_f}{R_k} \right), \tag{3}
\]

\[R_k = \sqrt{(\epsilon_k - \epsilon_f)^2 + 4V_i^2}.
\]

The heavy-electron, when it tunnels from a tip into a Kondo lattice, will retain its heavy effective mass due to the relatively slow decay of the coherence factors with distance. From the analysis of the momentum integrals (3) it is clear that both \(u(r_{ij})\) and \(v(r_{ij})\) will decay as \(1/r_{ij}^0\) since functions \(u_k\) and \(v_k\) are analytic functions of momentum. The values of \(\mu, \epsilon_f\) and \(V_i\) can be found by employing slave boson mean-field theory.\(^{18}\) In order to compute the spatial dependence of the coherence factors, I have solved the mean-field equations\(^{24}\) assuming that \(f\)-orbital multiplet is sixfold degenerate (\(N = 6\)). The results of the calculation are shown on Fig. 1. As one can see, the coherence factors extend on the distances of the order of several lattice spacings. Thus, for sufficiently small separation between a tip and a host, there is a finite probability for the composite fermion excitations to tunnel.

2.2 approximate tunneling Hamiltonian

To discuss the tunneling between two heavy-fermion metals, I consider the following model Hamiltonian:

\[
\hat{H} = \hat{H}_{AL}^{(1)} + \hat{H}_{AL}^{(2)} + \hat{H}_t, \tag{4}
\]

Here \(\hat{H}_{AL}^{(a)}\) describe the electrons in the tip \((a = 1)\) and in the host \((a = 2)\) correspondingly. We choose them to have the following form of the Anderson lattice model \(\hat{H}_{AL}^{(a)} = \hat{H}_0^{(a)} + \hat{H}_{V}^{(a)}\):

\[
\hat{H}_0^{(a)} = \sum_{k\sigma} \xi_{a\sigma} \hat{d}_{a\sigma}^\dagger \hat{d}_{a\sigma} + \sum_{k\sigma} \varepsilon_{f\sigma} \hat{f}_{a\sigma}^\dagger \hat{f}_{a\sigma} + \frac{U_f}{2} \sum_i \hat{f}_{a\uparrow}^\dagger \hat{f}_{a\uparrow} \hat{f}_{a\downarrow}^\dagger \hat{f}_{a\downarrow}, \tag{5}
\]

\[
\hat{H}_V^{(a)} = V_a \sum_{k\sigma} \left( \hat{d}_{a\sigma}^\dagger \hat{f}_{a\sigma} + h.c. \right),
\]

where the first term in \(\hat{H}_0\) describes conduction electrons and the remaining two terms describe the \(f\)-electrons, while \(\hat{H}_V\) accounts for the hybridization between conduction and \(f\)-electrons. Few comments are in order. To simplify our subsequent discussion here I consider the Kramers doublets for the ground state of the \(f\)-electrons and label them the same way as the spin state of the conduction electrons, \(\sigma = \uparrow, \downarrow\). In (5) we have also ignored that fact that conduction electrons orbitals usually have \(l = 0, 1, 2\) orbital number, which makes the hybridization matrix element with \(f\)-electrons \((l = 3)\) nonlocal. Lastly, the third term in (5) accounts for the tunneling events. In accord with our general discussion above, the non-local form of the coherence factors allows for the tunneling.
We note that tunneling amplitudes between the conduction electron in the f-orbitals is much smaller than the conduction electrons density of states \( \rho_f \) \( \delta \sigma_\alpha \) the main subject of this Section - the Hubbard repulsion between the f-electrons (5) is the largest energy scale of the problem. When both \( |\varepsilon_{fa}| \) and \( \varepsilon_{fa} + U_f \) are much larger than the conduction electrons density of states \( \rho_F \) times the square of the hybridization amplitude, \( |\varepsilon_{fa}|, \varepsilon_{fa} + U_f \gg \max[\rho_F|V|^2, \rho_F|t_{af}\beta|^2] \), the doubly occupied states on f-sites can be integrated out by means of the Schrieffer and Wolff transformation. Specifically, the effective tunneling Hamiltonian can be obtained by the unitary transformation \( \hat{H} = e^\hat{S} \hat{H} e^{-\hat{S}} = \hat{H}_0 + \hat{H}_t \) and the anti-hermitian operator \( \hat{S} \) must be determined from

\[
[\hat{S}, \hat{H}_0^{(1)} + \hat{H}_0^{(2)}] = -\hat{H}_V^{(1)} - \hat{H}_V^{(2)} - \hat{H}_{co-tun}. \tag{8}
\]

We note, that \( \hat{S} \), Eq. (8), will depend on hybridization amplitudes in the tip and the host Anderson lattices as well as the tunneling amplitudes between the conduction electron in the tip and an f-electron in the host and visa versa. As a result of this transformation, we obtain an effective Hamiltonian \( \hat{H}_{eff} \) by retaining terms up to the second order in \( V \) and/or \( t_{af\beta} \). Naturally, \( \hat{H}_{eff} \) will be given by the sum of the Kondo lattice Hamiltonians for both the tip and the host electrons:

\[
\hat{H}_{KL}^{(a)} \approx \sum_{\alpha\beta} \xi_{af\alpha} \hat{d}_{af\alpha}^{\dagger} \hat{d}_{af\alpha} \quad + J_K^{(a)} \sum_{i\alpha\beta} \hat{d}_{ai\alpha}^{\dagger} \left( \hat{S}_{ai} \cdot \hat{S}_{\alpha\beta} \right) \hat{d}_{ai\beta}^{\dagger} \tag{9}
\]

and the effective tunneling Hamiltonian which we write down as a sum of the two terms \( \hat{H}_{tun} = \hat{H}_{tun}^{(d)} + \hat{H}_{tun}^{(f)} \), where:

\[
\hat{H}_{tun}^{(d)} = \sum_{i\sigma} \left( t_d \hat{d}_{1i\sigma}^{\dagger} \hat{d}_{2i\sigma} + t_d \hat{d}_{2i\sigma}^{\dagger} \hat{d}_{1i\sigma} \right) \delta_{i,0} \tag{10}
\]

with \( t_d = T_{dd} \) describes the tunneling between the conduction states in the tip and the conduction states in the host. Consequently, the second term in the tunneling Hamiltonian

\[
\hat{H}_{tun}^{(f)} = \left[ \begin{array}{c} J_{12} \sum_{i\alpha\beta} \hat{d}_{1i\alpha}^{\dagger} \left( \hat{S}_{1i} \cdot \hat{S}_{\alpha\beta} \right) \hat{d}_{2i\beta}^{\dagger} \end{array} \right] \delta_{i,0} + \text{h.c.}, \tag{11}
\]

where \( \hat{S}_{ai} = \frac{1}{2} \hat{f}_{ai\alpha}^{\dagger} \hat{S}_{\alpha\beta} \hat{f}_{ai\beta} \) are local moments in a tip and a host, \( J_{12} \) and \( J_{21} \) are corresponding exchange coupling constants proportional to the tunneling matrix elements \( T_{dd} \). A crucial difference with the models considered earlier is the presence of the co-tunneling terms proportional to \( J_{21} \), which account for the tunneling of the composite electrons in the tip into conduction electron orbitals of the host. Before we proceed with the calculation of the tunneling current, we note that in deriving an effective Hamiltonian we have ignored the tunneling events between the predominantly localized f-electrons as well as other terms generated by the Schrieffer-Wolf transformation, which in principle could affect the tunneling current. However, it is known that these terms can be safely ignored in the problem of the tunneling from normal metal into a Kondo lattice since the model Hamiltonian \( \hat{H}_{eff} \) with \( J_{21} = 0 \), Eqs. (10,11), provides more than adequate description of the available experimental data. At the same time, for the analysis of tunneling experiments into a superconducting Kondo lattice these terms may actually be important, especially for probing unconventional Cooper pairing mechanisms.

### 2.3 Tunneling Current

The tunneling current is defined by the rate of change in the number of conduction electrons in a tip, \( I(V) = |e| \langle \dot{N}_{tip} \rangle \) with \( \dot{N}_{tip} = \sum_{i\sigma} \hat{d}_{1i\sigma}^{\dagger} \hat{d}_{1i\sigma} \), and averaging is performed in the grand canonical ensemble with full Hamiltonian \( \hat{H}_{eff} \). Clearly, the nonzero value for the current is furnished by the presence of the tunneling terms (10,11) in the Hamiltonian. Moreover, the substantial progress can be made by adopting the large-N limit approximation for the Kondo lattice. Within this approximation, the composite fermion operators entering into the expression for tunneling current can be expressed in terms of the f-electrons operators, which simplifies the expression for tunneling current:
pressed as a single fermionic operator according to: \[ \sum_{\beta} \left( \vec{S}_{1\beta} \cdot \vec{\sigma}_{\alpha\beta} \right) \alpha \beta \rightarrow \frac{\tilde{t}_f}{\tilde{J}_{21}} \tilde{f}_{1\alpha}, \]
\[ \sum_{\beta} \left( \vec{S}_{2\beta} \cdot \vec{\sigma}_{\alpha\beta} \right) \alpha \beta \rightarrow \frac{\tilde{t}_f}{\tilde{J}_{12}} \tilde{f}_{2\alpha}. \]  

In what follows, without loss of generality we take \( t_f \approx \tilde{t}_f \). The rest of the calculation employs the standard methods and we will not provide the details here. The resulting expression for the current reads:

\[ I(V) = I_{\text{tun}}(V) + \delta I(V), \]
\[ I_{\text{tun}}(V) = \frac{2\pi e}{h} \int d\omega [n_F(\omega - eV) - n_F(\omega)] \]
\[ \times \sum_{\alpha=1,2} \Pi_{\alpha,\beta \neq \alpha}(\omega - eV, \omega), \]
\[ \delta I(V) = \frac{2\pi e}{h} \int d\omega [n_F(\omega - eV) - n_F(\omega)] \]
\[ \times \delta \Pi(\omega - eV, \omega), \]

Here functions \( \Pi_{\alpha,\beta} \) and \( \delta \Pi \) are defined as follows:

\[ \Pi_{\alpha,\beta}(\omega, \omega') = \rho_{\alpha c}(\omega) \left[ \Gamma_{\alpha}^{2} \rho_{\beta c}(\omega') + 2t_f t_c \rho_{\beta m}(\omega') \right] + t_f^{2} \rho_{\beta f}(\omega'), \]
\[ \delta \Pi(\omega, \omega') = 2t_f^{2} \rho_{1m}(\omega) \rho_{2m}(\omega') - t_c^{2} \rho_{1c}(\omega) \rho_{2c}(\omega'). \]

with \( \rho_{\alpha \alpha}(\omega), (\alpha = 1, 2; \alpha = c, f, m) \) being determined by the single particle propagators of the corresponding Kondo lattices. Within the mean-field approximation we have:

\[ \rho_{\alpha c}(\omega) = -\frac{1}{\pi} \text{Im} \sum_{\mathbf{k}} \frac{1}{\omega^+ - \xi_{\alpha k} - \frac{V_{\alpha c}^{2}}{\omega^+ - \lambda_{\alpha}},} \]
\[ \rho_{\alpha f}(\omega) = -\frac{1}{\pi} \text{Im} \sum_{\mathbf{k}} \frac{1}{\omega^+ - \lambda_{\alpha} - \frac{V_{\alpha f}^{2}}{\omega^+ - \xi_{\alpha k}},} \]
\[ \rho_{\alpha m}(\omega) = -\frac{1}{\pi} \text{Im} \sum_{\mathbf{k}} \frac{V_{\alpha c}/(\omega^+ - \lambda_{\alpha})}{\omega^+ - \xi_{\alpha k} - \frac{V_{\alpha c}^{2}}{\omega^+ - \lambda_{\alpha}}}, \]

where \( \lambda_{\alpha} \) denotes renormalized position of the \( f \)-level and we have assumed that \( \omega^+ = \omega + i\delta \) in Eqs. (15) is a complex number with an infinitesimally small and positive imaginary part.

The first term in the expression for the current (13) has a simple physical interpretation: it describes the tunneling events between the conduction orbitals of the tip (host) into the conduction and composite fermion states of the host (tip). When one neglects the finite width of the \( f \)-electron level, the local density of states in the tip and the host has two peaks as a function of frequency. If we now include the corrections due to the fluctuations of the mean-field amplitude, which are proportional to \( 1/N \), where \( N \) is the degeneracy of the \( f \)-level, the \( f \)-electron energy acquires an imaginary part which depends both on momentum and frequency. To simplify our discussion, one can include the constant imaginary part, i.e. replace \( \lambda_{\alpha} \rightarrow \lambda_{\alpha} - i\gamma \), where \( \gamma \) is of the order of the Kondo lattice coherence temperature. Consequently, each of the two terms (second equation in (13)) contributing to the differential tunneling conductance \( g_{\text{tun}}(V) = dI/dV \) has an asymmetric shape as a function of voltage due to the cotunneling processes between the conduction and composite fermion states. Therefore, we can approximately write

\[ g_{\text{tun}}(\varepsilon) \approx \rho_{F1} \left( \frac{q_1 \Gamma_1 - \varepsilon - \epsilon_1}{(\varepsilon + \epsilon_1)^2 + \Gamma_1^2} \right) + \rho_{F2} \left( \frac{q_2 \Gamma_2 + \varepsilon - \epsilon_2}{(\varepsilon - \epsilon_2)^2 + \Gamma_2^2} \right). \]

Here \( q_0, \epsilon_0 \) and \( \Gamma_0 \approx 2T_K^{(a)} \) are corresponding parameters, which determine the Fano lineshape, while \( \rho_{F0} \) denotes the conduction band density of states in the tip \( (a = 1) \) and the host \( (a = 2) \). Moreover, the direct calculation shows that the second contribution \( \delta g(V) = d\delta I/dV \) to the tunneling conductance \( dI/dV \) (13) remains slightly asymmetric and does not have a characteristic Fano lineshape, see Fig. 3(c). Thus, we see that for the Kondo lattice materials with comparable host and conduction electron orbitals in the tip. Lastly, \( \delta g(V) = d\delta I/dV \) is an interference term between the co-tunneling events. All contributions to the differential tunneling conductance are plotted as a function of voltage in the units of the \( \sqrt{TK_1TK_2} \), where \( TK_{1,2} \) are the corresponding Kondo lattice coherence temperatures of the tip and the host.

3. Conclusions

In this paper, I have reviewed some of the recent theoretical advances in the problem of tunneling between normal and heavy-fermion or Kondo lattice systems. The main feature in the tunneling conductance between normal and Kondo lattice systems gives the Kondo lattice coherence temperature.

![Figure 3](https://example.com/figure3.png)

*Figure 3.* (Color online) Three contributions to the differential tunneling conductance \( g(V) = dI/dV = g_{12}(V) + g_{21}(V) + \delta g(V) \), where \( I(V) \) is given by Eq. (13), together with \( g(V) \) (in arbitrary units) are shown. Here \( g_{12} \) is determined by the \( a = 1 \) term in the expression for \( I_{\text{tun}}(V) \) and is governed by the tunneling and cotunneling processes between conduction orbitals of the tip and conduction and \( f \)-electron orbitals in the host. Consequently, \( g_{21} \) is determined by the \( a = 2 \) contribution to \( I_{\text{tun}}(V) \) and describes the tunneling and cotunneling events between the conduction orbitals in the host and the conduction and \( f \)-electron orbitals in the tip. Lastly, \( \delta g(V) = d\delta I/dV \) is an interference term between the co-tunneling events.
lattice is the presence of the asymmetry well described by the Fano lineshape. The origin of the asymmetry lies in the tunneling processes of the uncorrelated electrons in the tip into the composite fermionic states created by the strong hybridization between the conduction and $f$-electron states in the Kondo lattice. In a junction between the two Kondo lattice the asymmetric features in the tunneling conductance are greatly suppressed compared to the normal metal-Kondo lattice junctions.

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1) B. Coqblin and A. Blandin, Adv. Phys. 17, 281 (1968).
2) N. F. Mott, Phil. Mag. 30, 403 (1974).
3) P. Coleman, Handbook of Magnetism and Advanced Magnetic Materials, edited by H. Kronmuller and S. Parkin (John Wiley and Sons, New York, 2007), Vol. 1, p. 95.
4) P. Aynajian et al., Proc. Nat. Acad. Sci. U.S.A. 107, 10383 (2010).
5) A. R. Schmidt et al., Nature 465, 570 (2010).
6) S. Ernst et al., Nature 474, 362 (2011).
7) P. Aynajian et al., Nature 486, 201 (2012).
8) X. Zhang, N. P. Butch, P. Syers, S. Ziemen, R. L. Green, J. P. Paglione, pre-print arXiv:1211.5532 (2012).
9) W. K. Park, J. L. Sarrao, J. D. Thompson, L. H. Greene, Phys. Rev. Lett. 100, 177001 (2008).
10) A. Sumiayama et al., J. Phys. Chem. Solids 69, 3018 (2008).
11) Marianna Maltseva, M. Dzero and P. Coleman, Phys. Rev. Lett. 103, 206402 (2009).
12) J. Figgins and D. K. Morr, Phys. Rev. Lett. 104, 187202 (2010).
13) P. Wölfle, Y. Dubi and A. V. Balatsky, Phys. Rev. Lett. 105, 246401 (2010).
14) M. Fogelström et al., Phys. Rev. B 82, 014527 (2010).
15) J. Appelbaum, Phys. Rev. Lett. 17, 91 (1966).
16) P. W. Anderson, Phys. Rev. Lett. 17, 95 (1966).
17) S. Seo et al., Phys. Rev. B 85, 205145 (2012).
18) N. Read and D. M. Newns, J. Phys. C 16, 3237 (1983).
19) P. Coleman, Phys. Rev. B 29, 3035 (1984).
20) Z. Tesanovic and O. Valls, Phys. Rev. B 34, 1918 (1987).
21) D. M. Newns and N. Read, Adv. Phys. 36, 799 (1987).
22) A. J. Millis and P. A. Lee, Phys. Rev. B 35, 3394 (1987).
23) A. C. Hewson, "The Kondo Problem to Heavy Fermions", (Cambridge University Press, 1993).
24) V. Barzykin, Phys. Rev. B 73, 094445 (2006).
25) C. Berthod and T. Giamarchi, pre-print arXiv:1102.3895 (2011).
26) J. R. Schrieffer and P. A. Wolff, Phys. Rev. 149, 491 (1966).
27) P. Coleman, pre-print arXiv:0206003 (2002).