The Statistical Interpretation of Entangled States

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Abstract

Entangled EPR spin pairs can be treated using the statistical ensemble interpretation of quantum mechanics. As such the singlet state results from an ensemble of spin pairs each with an arbitrary axis of quantization. This axis acts as a quantum mechanical hidden variable. If the spins lose coherence they disentangle into a mixed state. Whether or not the EPR spin pairs retain entanglement or disentangle, however, the statistical ensemble interpretation resolves the EPR paradox and gives a mechanism for quantum “teleportation” without the need for instantaneous action-at-a-distance.

Keywords: Statistical ensemble, entanglement, disentanglement, quantum correlations, EPR paradox, Bell’s inequalities, quantum non-locality and locality, coincidence detection

1. Introduction

The fundamental questions of quantum mechanics (QM) are rooted in the philosophical interpretation of the wave function\(^1\). At the time these were first debated, covering the fifty or so years following the formulation of QM, the arguments were based primarily on gedanken experiments\(^2\). Today the situation has changed with numerous experiments now possible that can guide us in our search for the true nature of the microscopic world, and how The Infamous Boundary\(^3\) to the macroscopic world is breached. The current view is based upon pivotal experiments, performed by Aspect\(^4\) showing that quantum mechanics is correct and Bell’s inequalities\(^5\) are violated. From this the non-local nature of QM became firmly entrenched in physics leading to other experiments, notably those demonstrating that non-locally is fundamental to quantum “teleportation”.

Yet as firmly as the evidence appears to be for quantum non-locality, it contains a troubling feature: instantaneous action-at-a-distance. In the past, in other fields of physics, such a notion was at one time invoked, only to be later repudiated and replaced with more palatable mechanisms. The reluctance to accept this notion without first ruling out other possible interpretations initially motivated this work.

The question then comes down to the interpretation of the wave function, two possible ones are the following. Does the wave function describe the possible states of a single entity, or does it describe a statistical ensemble of similarly prepared systems? In the former interpretation, the wave function describes all we can know about a system, such as an electron, photon etc. Upon measurement, the system collapses into one specific state. Alternately, as first stated by Einstein\(^6\), and later formulated more concretely by others\(^1\), the wave function describes a statistically large number of events,
either one particle measured many times, or many particles measured once. In this paper the latter view is promoted.

The arguments presented here do not use any new mathematics or new formulation of QM. No hidden variable theories are invoked and the mathematical rigor of Bell’s inequalities is not questioned. It is also not questioned that QM can violate Bell’s inequalities. Rather, using a statistical ensemble interpretation of the wave function, it is shown that the need for instantaneous action-at-a-distance evaporates and violation of Bell’s inequalities by QM takes on a different meaning. The key process used in this approach was first introduced by Schrödinger in 1935 to formulate the EPR paradox. He called this process, disentanglement which is reviewed here and used to resolve the EPR paradox as expressed by a pair of entangled spins of magnitude \( \frac{1}{2} \) that form a singlet state. In particular the statistical ensemble interpretation of the entangled singlet state is crucial. Following this the experimental results of the Aspect-type experiments are examined as well as three experiments in the area of quantum “teleportation”. Even though there is need for more accurate experiments to distinguish between whether EPR pairs remain entangled or disentangle into a mixed state, this is not the essential point. Even if the states are entangled, the statistical ensemble view of the wave function explains the experimental results without recourse to instantaneous action-at-a-distance.

2. The Statistical Singlet state.

The singlet state of a pair of spins as introduced by Bohm and Aharanov is adopted here. In that case the two states of a spin \( \frac{1}{2} \) are given by

\[
|+\rangle_{\theta, \phi} = \begin{pmatrix} \cos \frac{\theta}{2} \\ \sin \frac{\theta}{2} e^{i\phi} \end{pmatrix} \quad \text{and} \quad |-\rangle_{\theta, \phi} = \begin{pmatrix} -\sin \frac{\theta}{2} e^{-i\phi} \\ \cos \frac{\theta}{2} \end{pmatrix}
\]

where \( i = 1 \) or \( 2 \). The angles \( \theta, \phi \) define an axis along which the spin is quantized which we denote by the unit vector \( \hat{\mathbf{P}} \) and these are relative to a basis defined with respect to the \( z \) axis, \( \theta = 0, \phi = 0, |\pm\rangle_{0,0} \). These spin states are eigenstates of the Pauli spin operator, \( \sigma^i_{\theta, \phi} \), with eigenvalues,

\[
\sigma^i_{\theta, \phi} |\pm\rangle_{\theta, \phi} = \pm |\pm\rangle_{\theta, \phi}
\]

Consider now a state formed from the two spin as follows,

\[
|\Psi^i_{12, \theta_1, \phi_1, \theta_2, \phi_2}\rangle = \frac{1}{\sqrt{2}} \left( |+\rangle_{\theta_1, \phi_1} |\pm\rangle_{\theta_2, \phi_2} - |-\rangle_{\theta_1, \phi_1} |\pm\rangle_{\theta_2, \phi_2} + |\pm\rangle_{\theta_1, \phi_1} |-\rangle_{\theta_2, \phi_2} \right)
\]
Although this state is entangled, it is not a singlet state since the angles are unequal, \( \theta_1 \neq \theta_2 \) and \( \phi_1 \neq \phi_2 \). In the following, it is assumed, for simplicity that \( \theta = \theta_1 = \theta_2 \).

Inserting Eq.(2.1) into (2.3) gives, therefore

\[
|\psi_{12,\theta,\phi,-}\rangle = \frac{1}{\sqrt{2}} \left[ \begin{array}{c}
\cos \frac{\theta}{2} \sin \frac{\theta}{2} (e^{-i\phi} - e^{-i\phi_2}) \\
\cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} e^{-i(\phi + \phi_2)} \\
-\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} e^{-i(\phi - \phi_2)} \\
\cos \frac{\theta}{2} \sin \frac{\theta}{2} (e^{i\phi} - e^{i\phi_2})
\end{array} \right] \quad (2.4)
\]

Clearly this state is anisotropic and depends upon the axes of quantization of the two spins. However, no matter what axes of quantization exists, if the two exactly coincide (i.e. \( \theta_1 = \theta_2 \) and \( \phi_1 = \phi_2 \)) for a given spin pair, the singlet state results,

\[
|\psi_{12,\theta,\phi,-}\rangle \equiv |\psi_{12}\rangle = \frac{1}{\sqrt{2}} \left[ \begin{array}{c}
0 \\
1 \\
-1 \\
0
\end{array} \right] \quad \text{for all } \theta, \phi \quad (2.5)
\]

This state is, of course, completely isotropic and arises for all possible values of \( \theta \phi \). No matter what axis of quantization exists between the two spins, Eq.(2.1), the single state in Eq.(2.5) is always isotropic. Expressing this well known result in terms of state vectors gives

\[
|\psi_{12}\rangle = \frac{1}{\sqrt{2}} \left[ |+\rangle_P |\rangle_P^2 - |\rangle_P^1 \langle -|_P \langle +|_P^2 \right]. \quad (2.6)
\]

so it is immediately evident that the RHS depends upon the choice of axis of quantization, but the LHS does not. To this extent, one can envision the axis of quantization as a hidden variable. The hidden variable theory for a spin \( \frac{1}{2} \), introduced by Bell, makes use of the axis of quantization. It is this axis, in the case of the singlet state, that provides the ‘element of reality’ for understanding experiments involving separated entangled particles. The statistical view of the EPR singlet state is that of an ensemble of spin pairs with each pair sharing a common axis of quantization \( \hat{\mathbf{P}} \). If the EPR source is isotropic, then an infinite number of quantization axes are possible, each equally probable, all of which result in isotropic singlet states that are indistinguishable from each other.

Obviously a small change in the axis of quantization for a given EPR pair in the ensemble, (for example if they lose phase coherence so that \( \phi_1 \neq \phi_2 \)), will cause the singlet
state to lose its isotropy. The ensemble then becomes a mixed state with the different axes of quantization being revealed for each spin. If, however, the EPR pairs only lose azimuthal coherence, \( \phi_1 \neq \phi_2 \), they still retain correlation by virtue of the fact that \( \theta_1 = \theta_2 \). If, finally, \( \theta_1 \neq \theta_2 \), then all spin correlation between the EPR pair is lost and one has an ensemble of spins, each characterized by its own unique axis of quantization, \( \hat{P} \).

The use of an ensemble here is not a construction for convenience. Rather it describes a physical system whereby the singlet state can be formed from any EPR pair with arbitrary angles, \( \theta \) and \( \phi \).

The other three Bell states, which form a triplet, are

\[
|\Psi_{12}^{0\phi,0\phi,\pm}\rangle = |\Psi^+_{12}\rangle = \frac{1}{\sqrt{2}} \left( |+\rangle_{0\phi}^1 - |\pm\rangle_{0\phi}^2 \right)
\]

\[
|\Phi_{12}^{0\phi,0\phi,\pm}\rangle = |\Phi^+_{12}\rangle = \frac{1}{\sqrt{2}} \left( |+\rangle_{0\phi}^1 + |\pm\rangle_{0\phi}^2 \right)
\]

(2.7)

To form the states Eqs.(2.7) and (2.8) a treatment similar to Eqs.(2.3) to (2.5) requires the specific quantization axis to be independent of the azimuthal angles (i.e. \( \theta = \theta_1 = \theta_2 = 0 \) for any values of \( \phi_1, \phi_2 \)). That is, these three Bell states can form from an ensemble of states with \( \theta = 0 \) and any value of \( \phi_1, \phi_2 \). Of course, \( \theta = 0 \) is completely arbitrary and depends on the choice of basis of the representation given in Eq.(2.1).

This treatment serves to show not only that the four Bell states are characterized by an ensemble of spins with different quantization axes but also the sensitivity of the Bell states to changes in the angles that define entanglement.

3. The process of disentanglement

In his analysis of the EPR problem, Schrödinger’s formulation\(^7\) was more general than that of EPR\(^8\). In his development he recognized that after particles interact they can become entangled. He considered a complete set of orthonormal states, \( |f_{n}\rangle \) for one (the second) particle and represented the wave function of the pair as

\[
|\Psi_{12}\rangle = \sum_n c_n |g_n\rangle |f_{n}\rangle
\]

(3.1)
where the states of the first particle, $|g_n\rangle$, are only required to be normalized, determined by the values of the constants, $c_n$. He stated that the property of entanglement is the characteristic difference between QM and classical mechanics.

In order to arrive at a complete orthonormal set to describe the first particle, he undertook a program of measurement on the second particle from which he deduces his non-invariance theorem, i.e. the set of states that describes the first particle depend upon the program of measurement on the second. If $x$ and $y$ denote the coordinates of the first and second particles respectively, then the relationship that leads to disentanglement is Schrödinger’s Eq.(3)

$$c_k g_k(x) = \int f'_k(y)\Psi^{12}(x, y)\,dy$$  \hspace{1cm} (3.2)

Applying the above ideas to a singlet state starts with the density operator defined by

$$\rho_{\text{EPR}} = \frac{1}{4} \left( I^1 I^2 - \vec{\sigma}^1 \cdot \vec{\sigma}^2 \right).$$  \hspace{1cm} (3.3)

There are an infinite number of ways of disentangling $\rho_{\text{EPR}}$ even though a spin of $\frac{1}{2}$ has only two states available to it. This is due to the infinity of different orientations available for the quantization axis for the pair. It is clear from Eq.(2.5), however, that for an EPR pair, the axis of quantization must be the same for both spins. With this proviso, therefore, one way that the EPR density operator can disentangle depends upon $\hat{P}$. Carrying out a similar procedure as suggested in Eq.(3.2), leads immediately to

$$\rho_{\theta \phi}^{12}(\pm) = \frac{1}{2} \left( \pm \frac{1}{\omega} \rho_{\text{EPR}} \right).$$  \hspace{1cm} (3.4)

In order to conserve angular momentum, the second spin of the disentangled EPR pair must have its state opposite to the first,

$$\rho_{\theta \phi}^2(\mp) = \frac{1}{2} \left( \pm \frac{1}{\omega} \rho_{\text{EPR}} \right).$$  \hspace{1cm} (3.5)

After disentanglement, the two single particle density operators are equivalent up to an arbitrary phase, $\phi$. Therefore the disentangled EPR state is given by an ensemble composed of pairs of particles each with density operator,

$$\rho_{\theta \phi, \theta \phi, D}^{12} = \frac{1}{2} \left( \rho_{\theta \phi}^1(+) \rho_{\theta \phi}^2(-) + \rho_{\theta \phi}^1(-) \rho_{\theta \phi}^2(+) \right).$$  \hspace{1cm} (3.6)

Disentanglement is therefore a process of partial reduction of the state when particles have separated which leads to the destruction of the quantum interference terms between them yet retains correlation due to their past interaction. In the above treatment,
the correlation is determined by the separated particles having opposite states, “+” or “-”, and by virtue of their common polar angle, $\theta$.

When he considered disentanglement as depending on a program of measurement Schrödinger’s did not consider one as better than the other: “after all we are not actually experimenting, but sitting at our desk”. He later conjectured that measurement was not indeed necessary for disentanglement to occur, but might well take place during the “process of separation”. Guided by the treatment in section 2, at least three different cases can be envisioned where disentanglement occurs.

Firstly disentanglement can occur during a process of separation. This case is common during chemical reactions, e.g. from the chemical kinetics of simple molecules to the complex enzyme catalysis of biochemical processes. A system that is initially interacting undergoes a process of separation into two subsystems. The original system and each subsystem produced after separation is described by QM. This means that forces are present within these systems that mediate their evolution but once separated the two subsystems move beyond the range of such forces. When disentanglement occurs all effects from quantum interference are lost and the absence of any force between them means that no new quantum correlations can develop between the two systems. The two subsystems, however, retain correlation between them due to symmetry. This is commonly observed in such experiments. A simple example is the conversion of pure para-hydrogen (a singlet nuclear state) in the presence of a catalyst at high temperatures into a statistical mixture of ortho-para-hydrogen in a ratio of 3:1. The singlet state first disentangles into separated hydrogen atoms and then the atoms recombine in the observed statistical ratio.

Secondly particles, such as an entangled pair of photons, might disentangle due to the loss of phase coherence after they have separated. Such a case can be envisioned from Eqs. (2.5) and (2.4) whence an initially entangled state loses phase coherence so that $|\Psi_{12}\rangle \rightarrow |\Psi_{12}^{(6,6)}\rangle$. Processes which produce entangled pairs of photons, such as parametric down-conversion\(^{19}\), start out with phase coherence, $\phi_1 = \phi_2$, but might well lose it thereafter by the passage through polarizers, beam-splitters, fibre optics, etc. The quantum interference terms can become phase randomized therefore destroying (disentangling) the originally prepared singlet state.

Thirdly, it is possible that entangled particles can persist as, for example, in the successful retention of phase coherence between initially entangled photon pairs in experiments such as parametric down-conversion. In this case, every member of the ensemble is represented by a pure state independent of the axes of quantization of its constituents. Upon measurement, however, the singlet state is resolved into a specific state. In the process, the axes of quantization are revealed so that each spin measured on the left has a partner on the right with a common quantization axis and the opposite spin component. Whatever the choice of analyzer settings, measurement of a sub-ensemble of disentangled EPR pairs enables the state of its distant partner to be deduced by virtue of its correlation due to the common origin of the EPR pair since they share a common axis of quantization.
4. The EPR paradox

By adopting a statistical ensemble interpretation of the singlet state, the EPR paradox is resolved. Consider the following experiment. Prepare a singlet state composed of an ensemble of EPR pairs that all have the same axis of quantization, $\hat{P}$. In other words, even though the singlet state is insensitive to such a choice of axis, (see Eq.(2.5)), the actual ensemble is assumed prepared so that it contains only spin pairs that have a specific quantization axis $\hat{P}$ and none other. Then arranging the production of EPR pairs so that one spin moves left in the “+” state and the other moves right in the “−” state leads to a density operator for all members of the ensemble expressed by (disentanglement, subscript D),

$$\rho_{D,\phi_a,\phi_b}^{12} = \rho_{\phi_a}^{1}(+)\rho_{\phi_b}^{2}(-)$$

(4.1)

If, now, two Stern-Gerlach filters are oriented in the direction $a$ on the left and $b$ on the right, the probabilities for detecting spins moving left in either the $|+\rangle_a$ or $|−\rangle_a$ states are:

$$P_{+,a}^{1} \equiv \left| \frac{1}{\phi_a} \langle + | + \rangle_a \right|^2 = \frac{1}{2} \cos^2 \left( \frac{\theta_a}{2} \right)$$

and $$P_{+,b}^{1} \equiv \left| \frac{2}{\phi_b} \langle + | + \rangle_b \right|^2 = \frac{1}{2} \sin^2 \left( \frac{\theta_b}{2} \right)$$

(4.2)

and likewise on the right as,

$$P_{−,a}^{2} \equiv \left| \frac{1}{\phi_a} \langle − | − \rangle_a \right|^2 = \frac{1}{2} \sin^2 \left( \frac{\theta_a}{2} \right)$$

and $$P_{−,b}^{2} \equiv \left| \frac{2}{\phi_b} \langle − | − \rangle_b \right|^2 = \frac{1}{2} \cos^2 \left( \frac{\theta_b}{2} \right)$$

(4.3)

The factors of $\frac{1}{2}$ are a result of normalization. The sum of the four probabilities is unity. The angles are those between the direction of spin quantization axis for this ensemble and the direction of the magnetic field orientations,

$$\cos \theta_a = a \cdot \hat{P} \quad \text{and} \quad \cos \theta_b = b \cdot \hat{P}$$

(4.4)

These results are independent of the choice of azimuthal phases, $\phi_a$ and $\phi_b$.

Alternately, if the singlet state were a result of a uniform distribution of quantization axes, rather than the sub-ensemble containing only one, ensemble averaging of these would lead to no correlation between the spins on the left and right and all four probabilities would then be equal to $\frac{1}{4}$. This ensemble averaged disentangled case then agrees with the calculation using pure entangled states Eq.(3.3).

The resolution of the EPR paradox in terms of a statistical ensemble of singlet states is that an instantaneous action-at-a-distance is not required to ensure that conservation of angular momentum is maintained. That is, the choice of measurement on the left does not determine the outcome on the right but rather measurement of one spin is correlated to the other by virtue of the fact that they belong to the same sub-ensemble that
retains the necessary correlation. That correlation is determined at the source by virtue of the common axis of quantization and not at the time of measurement.

In contrast, if the wave function were assumed to describe the possible states of a single EPR spin pair rather than an ensemble, the measurement of one spin state would require its distant partner to collapse into a specific state which is consistent with the measured particle.

5. Spin correlations

In this section it is shown that correlations exist between separated EPR pairs even when the singlet state disentangles into a product state devoid of entanglement. The functional form of the correlation from both entangled and disentangled spin pairs, is identical, being \( \cos \theta_{ab} \), and only differs by a constant factor. In other words, long range correlation is predicted from disentangled EPR pairs even though these correlations obey Bell’s inequalities and are local in origin.

The calculations presented in the previous section resolve the EPR paradox by virtue of the spin pairs maintaining the same axis of quantization as they separate. Experimentally, coincidence detection is required in order to ensure that the spin pairs come from the same EPR pair. Moreover, spins moving left and right can be in either of the two states, “±”. If the two remain entangled, the calculations are well known and are given here for reference. Since the entangled EPR density operator, \( \rho_{EPR}^{12} \), is isotropic, one pure state describes every spin pair in the ensemble, it is not necessary to perform an ensemble average. The results are,

\[
\begin{align*}
\langle a \cdot \sigma^1 \sigma^2 \cdot b \rangle_E &= a \cdot \left[ Tr_{12} \rho_{EPR}^{12} \sigma^1 \sigma^2 \right] \cdot b = -a \cdot b = -\cos \theta_{ab} , \\
\langle a \cdot \sigma^1 \rangle_E &= a \cdot \left[ Tr_{12} \rho_{EPR}^{12} \sigma^1 \right] = 0 .
\end{align*}
\]

(5.1)

so the correlation function is defined and given by

\[
E(a, b) = \langle a \cdot \sigma^1 \sigma^2 \cdot b \rangle - \langle a \cdot \sigma^1 \rangle \langle \sigma^2 \cdot b \rangle
\]

(5.2)

The angle is that between the two filters pointing in the \( a \) and \( b \) directions, \( a \cdot b = \cos \theta_{ab} \), independent, as expected, of the axis of quantization. The four probabilities for coincident detection are:

\[
P_{EPR}^{x+y} (a, b) = \frac{1}{4} (1 + \cos \theta_{ab}) = \frac{1}{2} \cos^2 \frac{\theta_{ab}}{2}
\]

(5.3)

and
\[ P_{\pm}^E(a, b) = \frac{1}{4} \left( 1 - \cos \theta_{ab} \right) = \frac{1}{2} \sin^2 \frac{\theta_{ab}}{2} \]  
so that:

\[ E_E(a, b) = P_{++}^E(a, b) - P_{+-}^E(a, b) - P_{-+}^E(a, b) + P_{--}^E(a, b) = -\cos \theta_{ab} . \]  

The sum of the four probabilities is unity.

The calculation has two steps when the EPR pair has undergone, at some stage, disentanglement. The disentangled density operator, Eq.(3.6), is first used to evaluate the same expectation values as for entanglement, Eq.(5.1) to (5.4). In order to compare the results, the normalization of Eq.(3.6) must be changed. As seen from the calculations of the single spin correlations in the previous section, Eqs.(4.2) and (4.3), the sum of the four probabilities is unity. In this section, Eqs.(5.3) and (5.4), the joint probabilities are also normalized to unity. In order to directly compare single spin probabilities with pair probabilities Eqs.(3.4) and (3.5) are renormalized to

\[ \rho^{\pm\pm}_{\pm\pm}(\pm) = \left| \pm \right>_{\pm\pm} \left< \pm \right| \text{ and } \rho^{\mp\mp}_{\mp\mp}(\mp) = \left| \mp \right>_{\mp\mp} \left< \mp \right| \text{.} \]  

Since spin correlation between distant particles is now at issue it is necessary that the spin quantization axes of the two disentangled EPR spins exactly coincide. This requires \( \theta_1 = \theta_2 \) and \( \phi_1 = \phi_2 \). Since the spins have already disentangled, a non-unitary process, requiring the azimuthal angles to match does not restore entanglement to the pair.

Secondly, the results must be ensemble averaged over all random values of \( \hat{P} \). The four probabilities before performing the ensemble average are given by

\[ P^{D}_{\pm\pm}(a, b) = \frac{1}{4} \left( 1 - \cos \theta_a \cos \theta_b \right) \]  

\[ P^{D}_{\pm\mp}(a, b) = \frac{1}{4} \left( 1 + \cos \theta_a \cos \theta_b \right) \]  

The product of angles occurring in Eqs.(5.7) and (5.8) can be expressed in terms of the axis of quantization, \( \hat{P} \), so that the spin correlation for the sub-ensemble before averaging is

\[ E^{D}_{\hat{P}}(a, b) = -\cos \theta_a \cos \theta_b = -a \cdot \hat{P} \hat{P} \cdot b \]  

The ensemble average over all possible quantization axes depends upon the system under study. If the system is isotropic in all three Cartesian directions, then the ensemble average gives,

\[ \left< a \cdot \sigma^3 \sigma^2 \cdot b \right>_{\text{disentangled}} = -a \cdot \hat{P} \hat{P} \cdot b = -\frac{1}{3} a \cdot b = -\frac{1}{3} \cos \theta_{ab} . \]
In contrast photon helicity is quantized along the direction of photon propagation. If this axis is defined as \( \hat{z} \), then the ensemble average is performed in the \( \hat{x}\hat{y} \) plane perpendicular to the direction of propagation. In that plane, isotropy implies that \( x^2 = y^2 \) leading to an ensemble averaged result appropriate for photons (except that the angle must be doubled),

\[
\overline{P_{\pm,\pm}^{D}(a,b)} = \frac{1}{4} \left(1 - a \cdot \mathbf{1} \cdot \mathbf{1} \cdot b\right) = \frac{1}{4} \left(1 - \frac{1}{2} a \cdot \left(\hat{x}\hat{x} + \hat{y}\hat{y}\right) \cdot b\right)
\]

\[
= \frac{1}{4} \left(1 - \frac{1}{2} a \cdot b + \frac{1}{2} a \cdot \hat{z}\hat{z} \cdot b\right) = \frac{1}{4} \left(1 - \frac{1}{2} \cos \theta_{ab}\right)
\]

(5.11)

The ensemble averages over the individual spins is the same as from entanglement, cf. Eq.(5.1),

\[
\overline{\langle a \cdot \sigma^1 \rangle_F} = 0
\]

(5.12)

\[
\overline{\langle b \cdot \sigma^2 \rangle_F} = 0
\]

(5.13)

The final results can be compared with those from entanglement, Eqs.(5.2) to (5.5)

\[
\overline{P_{\pm,\pm}^{D}(a,b)} = \frac{1}{4} \left(1 - \frac{1}{2} \cos \theta_{ab}\right)
\]

(5.14)

\[
\overline{P_{\pm,\pm}^{D}(a,b)} = \frac{1}{4} \left(1 + \frac{1}{2} \cos \theta_{ab}\right)
\]

(5.15)

and from Eq.(5.5), the disentangled correlation function is

\[
E^{D}(a,b) = -\frac{1}{2} \cos \theta_{ab}
\]

(5.16)

When the Aspects\(^4\) coincidence detection experiments on photons were performed they showed, as did repeats\(^9\) of those experiments, a sinusoidal dependence between the two detectors as a function of the angle between the two filters, \( \theta_{ab} \). In such experiments, a sinusoidal dependence was deemed sufficient to show the persistence of long range correlation, yet also violate Bell’s inequalities. In these experiments, the sinusoidal dependence for disentangled particles was not considered, but the only difference between entanglement and disentanglement is the factor of \( \frac{1}{2} \) seen in Eq.(5.16). The factor \( \frac{1}{2} \) between the two cases simply signifies whether the quantum interference terms are present or not. Of course the entangled result can lead to violation of Bell’s Inequalities whereas the disentangled result always satisfies them.

The detection loophole refers to the low detection rates that are encountered in coincident photon experiments. In these experiments, a large number of events are
recorded that are disregarded. The relationship between the probabilities from fully entangled pairs and disentangled pairs is,

\[
P_{\pm}(a,b) = \frac{1}{8} + \frac{1}{2} P^E_{\pm}(a,b) \tag{5.17}
\]

The constant term corresponds to random coincidences due to the loss of correlation that arises from the quantum interference terms. It is not clear if the experiments are able to distinguish the factor of 1/8. However, Aspect subtracted the random coincidences before analysis. It has been argued that such coincidences should not be subtracted. Equation (5.17), predicts a constant, 12.5%, non-sinusoidal, dependence which, if random non-coincidences are retained, appears to support the presence of disentangled photons. In the repeat of the experiments by Weihs et al, it appears that no constant dependence is observed, supporting the presence of entangled photons only. More experiments are needed to distinguish the two cases. Should the disentangled prediction be verified, then this would give strong support for the statistical ensemble treatment presented here.

Whether applied to the cases where entanglement or disentanglement occurs, the statistical ensemble approach does not violate Einstein locality. Hence whether Bell’s inequalities are violated or not, from this view, quantum mechanics is retained as local theory. Detection of one spin component at one place does not imply instantaneous action-at-a-distance forcing its EPR partner at space-like separated distances to collapse into a state of definite polarization of one EPR pair. Rather an ensemble of EPR pairs is present and each spin is correlated to the other by their common origin which is carried by their common axis of quantization.

6. Quantum “teleportation”

Using the interpretation of the wave function that requires instantaneous action-at-a-distance, Bennett et al. developed a theory called quantum “teleportation”. In this section, without giving the details, an interpretation of this effect is given using the statistical interpretation of entangled states. The accepted explanation of this phenomenon is well documented, (see, e.g. reference (23)), although some comments are made on this.

It is not of importance for the statistical ensemble interpretation whether the photons pairs, (labeled 2 and 3), are disentangled or entangled. Rather whether these photons are entangled or disentangled only affects the detection rate. The mechanism for this three photon experiment is depicted in Figure 1. When Alice’s photon 1 encounters photon 2 it picks out from the ensemble of all possible polarizations, the one state that appropriately matches Alice’s photon properties. Thereby photons 1 and 2 become entangled. The conditions for this are given in Eqs. (2.4) to (2.5). That is, Alice’s photon can only entangle with a sub-ensemble of photons that have the same axis of quantization as hers. Since photon 3 also carries that axis of quantization, photons 1 and 3 are the same up to the well known transformations.
In other words, if Alice’s photon is polarized along an axis of quantization \( \hat{\mathbf{P}} \), then only the sub-ensemble of photons 2 and 3 that carry the same axis of quantization can lead to correlation between photons 1 and 3. It is essential that for this to be effective, the azimuthal angles match, i.e. \( \phi_1 = \phi_2 \). This interpretation of the quantum “teleportation” process is quite different from the current one that requires instantaneous action-at-a-distance.

![Figure 1](image)

Figure 1. Local statistical ensemble mechanism for teleportation: (a) A depiction of a statistical ensemble of EPR pairs that form a singlet state. Even though the axes of quantization are hidden, some are shown for clarity. The state remains isotropic, but any axis of quantization leads to an isotropic singlet state. (b) The EPR pair moves apart and remains entangled. For every photon moving to Alice in one state ("+" or ") there is a photon moving to Bob in the opposite state ("-" or ") with respect to the same axis of quantization. The component that can form a Bell state with Alice’s photon is highlighted as is its EPR partner at Bob’s location. (c) Alice’s photon can only form a Bell state with an EPR photon that has the same axis of quantization as her polarization state (see Eq.(2.6)). Therefore the photon at Bob’s location must have the same quantization axis as Alice’s. There is no wave function collapse and the effect is local and real.

Besides instrumental difficulties, disentanglement reveals two major causes for the low detection rates observed in the coincidence experiments. (1) In the statistical local theory, the requirement of sub-ensemble matching, whether the spins are entangled or not, obviously must lead to reduced detection rates since only that sub-ensemble has the correct properties to match Alice’s photon. (2) Entanglement may not be preserved between photons 2 and 3, thereby reducing correlation. In this case, the condition \( \phi_1 = \phi_2 \) is required. If, for purposes of illustration, these phases lie within \( 1^\circ \) of each other, a reduction of intensity by approximately \( 1/360 \) from the entangled results is expected.
In contrast, the mechanism that is based upon long range collapse of the wave function is illustrated in Figure 2. Here the singlet state, (represented in Figure 1a), due to its isotropy does not represent any specific axis of quantization and so, in order for photon 2 to form a new singlet state with Alice’s photon 1, photon 2 must collapse into a state with the same axis of quantization as that of Alice’s photon. Instantaneously photon 3 must collapse, even over space-like distances, into a state defined by the same axis of quantization as the other two spins. How this long range collapse occurs is exactly the point that is unexplained and which leads to the notion of “teleportation”. Such an interaction, or connection, between the photons 2 and 3 has been characterized as\textsuperscript{24}:

- Unattenuated, meaning that the interaction is the same, independent of distance between the two EPR particles.
- Discriminating, meaning that it operates between specific particles (e.g. EPR pairs), rather than between all particles
- Instantaneous, meaning that the interaction occurs instantaneously even over space-like distances, hence, faster than the speed of light.

To date, no satisfactory interpretation of how this non-local effect is achieved with most characterizing it as weird, magic and, of course, spooky. On the other hand, following Bell, quantum violations of his inequalities is interpreted as proof of quantum non-locality. Comments on this issue are reserved to the discussion. Simply stated here, none of the above three points apply to the statistical ensemble interpretation of quantum teleportation.

Figure 2. Non-local wave function collapse mechanism of teleportation: (a) Depiction of an isotropic singlet state as a sphere. The EPR pair is indicated by the up and down arrows but the orientation is
arbitrary since the wave function can collapse into a state with quantization axis in any direction. (b) The singlet state has separated, one photon moving towards Alice and the other towards Bob. They remain entangled over this distance. Alice’s photon can interfere with the left-moving photon to form a new entangled pair. (c) The singlet state collapses and forms a Bell state with Alice’s photon. Instantaneously, the photon arriving at Bob’s location collapses into the state determined by the polarization of Alice’s photon. The process is non-local.

7. Some experimental evidence for “teleportation”

In this section three well known experiments are summarized in the light of the possibility that some of the photons become disentangled during the experiment. Both entangled and disentangled photons can lead to long range spin correlations as seen from Eqs.(5.5) and (5.16). These two equations are functionally the same although they differ by a factor of \( \frac{1}{2} \). Tests of teleportation have, to date, not considered the possibility that disentangled photons can lead to long range correlations. As a result the observation of such long range correlations is deemed sufficient evidence to conclude that teleportation occurs, the photons are entangled and the effect is a non-local one. However, long range correlations are also predicted from disentangled photons. One of the major differences between the two cases, entangled or disentangled photons, lies in the prediction of random coincidences in the disentangled case, (the factor 1/8 in Eq.(5.17). Since these are usually subtracted it is difficult to know exactly how to interpret the data. Nonetheless, the analyses presented below show that it is certainly likely that some of the correlation observed arises from disentangled photons.

Gisin et al. performed experiments where the measured state is \( |\Phi_{23}\rangle = |\Psi_{23}\rangle |\Psi_{3}(\beta)\rangle \) where \( \beta \) is an adjustable phase of photon 3. Assuming entanglement persists until photon 2 encounters Alice’s photon gives,

\[
\langle \Phi_{123} | \rho^{123} | \Phi_{123} \rangle = \frac{1}{8} (1 - \cos \beta) \quad (7.1)
\]

whereas if disentanglement occurs and after setting \( \phi_1 = \phi_2 \) gives,

\[
\langle \Phi_{123} | \rho^{1D} | \Phi_{123} \rangle = \frac{1}{8} \left(1 - \frac{1}{2} \cos \beta \right) \quad (7.2)
\]

In this experiment, the initial entangled state is, \( |\Phi_{23}^+\rangle = \frac{1}{\sqrt{2}} \left( |+,1\rangle_k + |-,1\rangle_k \right) \) rather than \( |\Psi_{23}^-\rangle \). Both predict a sinusoidal dependence on \( \beta \), however the difference between Eqs.(7.1) and (7.2) reveals a constant background of random coincidences. This background is observed in the data. Statistical analysis of how well the two functions, Eqs.(7.1) and (7.2), fit the data reveals a large experimental error. This makes it impossible to distinguish between Eqs.(7.1) and (7.2) based upon current data.
In a similar experiment\textsuperscript{13} in 1997, “teleportation” results using the Bell state $|\Psi_{12}\rangle$ were presented in which Alice’s photon is chosen to be initially polarized either as $+45^\circ$ or $-45^\circ$ whereas the source generates entangled photons described by $\rho_{EPR}^{23}$. A polarizing beam splitter at Bob’s location deflects the photons towards two detectors sensitive to photon 3 and having polarization oriented at $+45^\circ$ and $-45^\circ$. If Alice’s photon is initially polarized at $+45^\circ$ then “teleportation” is considered to have occurred if Bob’s detector at $+45^\circ$ records a coincidence while the $-45^\circ$ filter returns no response. The opposite results are expected if Alice’s initial state is $-45^\circ$. In this case the measured state is represented as $|\Phi_{123}\rangle = |\Psi^-_{12} \Psi_3 (\pm 45^\circ)\rangle$ then the theoretical predictions from entanglement give a value of $\frac{1}{4}$ for Alice’s state being $\rho^i = \rho^i (+45^\circ)$ and a value of zero for $\rho^i = \rho^i (-45^\circ)$. The theoretical results from disentanglement are,

$$\left< \Psi^-_{12} \Psi_3 (\pm 45^\circ) \right| \rho^i (\pm 45^\circ) \rho_{\text{disentangled}}^{23} \left| \Psi^-_{12} \Psi_3 (\pm 45^\circ) \right> = \frac{1}{8} \left( 1 + 2 \cos^2 \theta \sin^2 \theta \right) = \frac{3}{16} \quad (7.3)$$

and

$$\left< \Psi^-_{12} \Psi_3 (\pm 45^\circ) \right| \rho^i (\mp 45^\circ) \rho_{\text{disentangled}}^{23} \left| \Psi^-_{12} \Psi_3 (\pm 45^\circ) \right> = \frac{1}{8} \left( 1 - 2 \cos^2 \theta \sin^2 \theta \right) = \frac{1}{16} \quad (7.4)$$

Indeed a dip is observed but it does not go to zero as predicted from entanglement but rather drops to a value consistent with disentanglement\textsuperscript{25}, Eqs.(7.3) and (7.4).

As a final example, Kim\textsuperscript{14} \textit{et al.} used non-linear crystals in experiments that allow for all four of the Bell states to be measured at Alice’s location. In the disentanglement procedure the axis of quantization is oriented by angles $\theta$, $\phi_2$ and $\theta$, $\phi_3$. In this case, rather than requiring $\phi_2 = \phi_3$, the azimuthal angles are changed as photon 2 passes through non-linear crystals, from $\phi_2 \rightarrow \phi_2 + \pi$ and so the phase-matching requirement changes to from $\phi_2 = \phi_3$ to $\phi_2 + \pi = \phi_3$. An analyzer is placed in front of Bob’s detector so that the coincidences are measured as a function of the angle, $\phi$, of this analyzer within the range $0 < \phi \leq 2\pi$. If entanglement is preserved the expected correlation is

$$\left< \Phi_{123} (\phi) \right| \rho^i \rho_{EPR}^{23} \left| \Phi_{123} (\phi) \right> = \frac{1}{8} (1 \pm 2 \cos \phi \sin \phi) \quad (7.5)$$

The results from disentanglement after ensemble averaging are,

$$\left< \Phi_{123} (\phi) \right| \rho^i \rho_{\text{disentangled}}^{12} \left| \Phi_{123} (\phi) \right> = \frac{1}{8} (1 \pm \cos \phi \sin \phi) \quad (7.6)$$

Although the original experimental data was fitted only to the entangled result, Eq.(7.5), a better fit can be obtained if the disentangled result, Eq.(7.6), is included\textsuperscript{15}. The
statistical analysis shows the best fit occurs when 62% of the photons are disentangled and 38% are entangled. Random coincidences give a constant background that is experimentally observed consistent with the above two experiments and consistent with some of the photons undergoing disentanglement somewhere between the source and the detector.

8. Discussion

In the statistical ensemble interpretation of the singlet state, different sub-ensembles exist due to the infinity of different possible quantization axes. EPR spin pairs maintain entanglement if complete phase coherence persists between them as they separate. If phase coherence is lost then the pure singlet states becomes a statistical mixture of states wherein the particular axes of quantization are revealed.

To gain further insight into such a process, the origin of the correlation difference between entangled and disentangled EPR spin pairs can be traced. The fully isotropic entangled states, Eq.(5.5), involve only the angle between the two analyzers, \( \cos \theta_{ab} \). Comparing this with the correlation from a sub-ensemble of disentangled EPR pairs, Eq.(5.9), \( \cos \theta_a \cos \theta_b = \mathbf{a} \cdot \mathbf{P} \cdot \mathbf{b} \) with respect to a coordinate frame defined by \( \mathbf{P} \), and using the relationship for the angle between two vectors, \( \mathbf{a} \) and \( \mathbf{b} \) in that frame reveals

\[
\cos \theta_{ab} = \cos \theta_a \cos \theta_b + \sin \theta_a \sin \theta_b \cos(\phi_a - \phi_b)
\]

(8.1)

The only difference in calculation between the two is that the term \( \cos \theta_{ab} \) arises when the full isotropic EPR density operator is used, Eq.(3.3), while the \( \cos \theta_a \cos \theta_b \) term arises upon disentanglement using Eq.(3.6). In the former, quantum interference terms are present while in the latter they are not present. Therefore, the second term on the RHS of Eq.(8.1) arises from the quantum interference terms. The disentanglement treatment reveals a reduced level of correlation as evidenced by the factor of \( \frac{1}{2} \) difference between spin correlation for entangled, Eq.(5.5) and disentangled EPR pairs, Eq.(5.16). Experiments to confirm the factor of \( \frac{1}{2} \) difference between entangled and disentangled photons has not yet been undertaken.

In the CHSH\textsuperscript{26} form of Bell’s inequalities entanglement leads to a violation, \( 2 > 2\sqrt{2} \) while the disentangled approach gives no violation, \( 2 > \sqrt{2} \). Using the statistical approach violation of Bell’s inequalities, however, does not imply non-locality, but rather determines whether the system has disentangled or, stated otherwise, whether the quantum interference terms are present.

The treatment by EPR\textsuperscript{8} demonstrated that simultaneous reality exists for an EPR pair yet the wave function fails to describe this. EPR concluded that QM is, therefore, incomplete so that a hidden variable theory must, somehow, underpin QM. Since EPR
assumed non-local influences between the separated particles cannot arise, such a hidden variable theory envisioned by Einstein must be local in nature. Later Bell\(^5\) ruled out local hidden variable theories thereby showing Einstein’s conclusion that QM is incomplete, to be flawed. One the other hand, this paper, using a singlet pair of spins as a prototype, shows that locality is not an issue for hidden variable theories within the statistical ensemble approach. Like the trajectory equations in Bohmian mechanics\(^{27}\), the axis of quantization plays the same role as expected from a hidden variable. That is, the axis of quantization provides the element of reality needed to resolve the EPR paradox and, in addition, obviates the need for wave function collapse.

Bell’s theorem rules out local hidden variable theories leading to the conclusion that only non-local hidden variables can underpin quantum mechanics. Bohmian mechanics\(^{27}\) separates the wave function into a real part and a complex phase part. The equation of motion for the phase is obtained from the Schrödinger equation and the evolution of the particles in $3N$ phase space is governed by a non-local quantum potential. It is often remarked that, indeed, the Bohm hidden variable theory is non-local and therefore is consistent with Bell’s theorem. This non-locality is specifically in the $3N$ configuration space and it is not clear that this carries over to real $3D$ real space, in particular if the wave function is treated as a statistical ensemble.

The experiments that are currently possible have not been concerned with disentanglement primarily because entanglement and violation of Bell’s Inequalities are considered requirements for the long range correlations observed in EPR and teleportation experiments. This paper predicts long range sinusoidal correlations from disentangled EPR pairs and these always obey Bell’s inequalities. Whether or not, therefore, quantum mechanics is a non-local theory depends upon the interpretation of the wave function. If the wave function describes the possible states of a single EPR pair, then space-like separated particles must undergo instantaneous collapse. If, in contrast, the wave function is interpreted as an ensemble of EPR pairs, the EPR paradox is resolved, Einstein locality restored and quantum “teleportation” is a local phenomenon. Violation of Bell’s inequalities in this case does not mean that quantum mechanics is non-local but rather that the quantum interference terms are present.

As stated above, since experimental error in quantum “teleportation” is large, it is not yet possible to conclusively distinguish between entanglement and disentanglement. However, if the predictions of disentanglement are verified experimentally, it would be convincing confirmation that the statistical interpretation of the wave function is based on observation rather than on philosophical arguments alone.

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