Parametric optimization of the thermal processing of foam glass on basis of heat transfer models

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Abstract. The foam glass is one of the most prospective materials in the market of vitro-crystalline building materials. To achieve optimal physical and technical parameters of foam glass, the development of technology that will regulate the processes of heat treatment of the raw material mixture to produce foam glass without reducing its performance is required. Foaming process is an important parameter in the foam glass technology. The temperature of the raw material mixture changes from the surface layers to the center of the backfill during foaming, the activation reaction of the gasifier thermal decomposition proceeds similarly, which can result in the material uneven air-entraining. Mathematical model presented in this paper makes modeling the distribution of temperature fields at the contact boundaries of the metal mold for foaming and the raw material mixture for the foam glass production as well as prediction of the further temperature changes in the raw material mixture, which is the key factor in the formation of a uniformly porous final product, possible.

1. Introduction

Analysis of the market of modern thermal insulation materials has reviled that foam glass [1], which is a fire-and environmentally friendly material, is one of the most efficient. It is distinguished from the other materials by low thermal conductivity and chemical and biological resistance [2].

To date, powder method is the most common method used for production of the foam glass. The given method makes obtaining the finished product with different properties depending on the feed raw material mixture composition and percentage possible [3]. A schematic diagram of the technological process for the foam glass production by means of the powder method is presented in Figure 1.

However, despite the fact that the foam glass possess higher thermal insulation characteristics as compared to other materials, it remains in short demand due to its high cost. The foam glass production technology is quite energy-intensive, so the scientists’ special attention is attracted by mathematical modeling of the heat treatment processes in the foam glass production [4] aimed at selecting rational temperature conditions to reduce energy consumption without compromising the product quality. In this case, the processes occurring with the gradual heating of the foam glass mixture are to be defined, since the temperature in the furnace chamber is close to the values at which the melting of glass grains is initiated, at that the surface layers being in direct contact with the metal...
faces of the foaming mold start melting first and the layer heated due to the thermal conductivity starts melting later (in time). The process of subsurface foam glass mixture melting occurs – the central layers of the material are still not heated (due to the low thermal conductivity of the surrounding material). As a result, gas-emission sources "do not work" in these pores while the surrounding this center working mixture material is being foamed and the pores radius continue to increase. Thus, the working mixture material is formed unevenly by pore-formation, which affects the final product thermophysical properties. However, in cases when the foaming time significantly exceeds the glass grains melting time, fusing of the foam glass mixture subsurface layers occurs, since the gas-emission sources are burnt out completely by heating, and the glass viscosity decreases, at that the surface stress does not allow to detain the emission gas phase in the formed pore spheres fed into the furnace chamber for foaming, and thus, the central layer of the foam glass mixture becomes more porous than the subsurface layers.

Thus, we assume that the distribution of temperature fields in the glass foam mixture is transmitted from the subsurface layers of the mixture to the center.

Analysis of works [5, 6, 7] devoted to mathematical modeling of the foam glass production technological processes has revealed that the problem of uniform heating of the raw mixture remains less than fully investigated. In this regard, an issue to improve the foam glass heat treatment process by developing various mathematical models of heat transfer is relevant.

2. Research Methods
Statement and solution of the problems of heat transfer dynamics in the raw material mixture fed into a metal form are considered in the paper.

The system under consideration element is presented schematically in Figure 2.
Figure 2. Raw Material Mixture (2) – Metal Mold (1) Model

Temperature of the mold wall and the raw material mixture is assumed to be identical at the initial moment and equal to:

$$t(x, \tau)|_{\tau=0} = t_{no}$$  \hspace{1cm} (1)

With gaseous medium temperature in the furnace chamber changes (increases or decreases) due to convective heat exchange, the metal form starts heating up (cooling down), and the heating kinetics can be characterized by the following correspondence:

$$t_n(\tau) = f_n(\tau)$$  \hspace{1cm} (2)

Accordingly, it can be assumed that the temperature of the raw material mixture layer adjacent to the metal form will change in accordance with the following law:

$$t(x, \tau)|_{x=0} = f_n(\tau)$$  \hspace{1cm} (3)

where $x$ is the coordinate from the metal form to the layers of the raw material mixture; $x=0$ is the coordinate of the contact zone.

In general cases, heat transfer boundary value problems can be presented by nonlinear inhomogeneous partial differential equations of parabolic type [8]:

$$\rho(u, t) c(u, t) \frac{\partial t(x, \tau)}{\partial \tau} + \frac{\partial}{\partial x} \left[ \lambda(u, t) \frac{\partial t(x, \tau)}{\partial x} \right] = 0$$  \hspace{1cm} (4)

where $\rho(u, t), c(u, t), \lambda(u, t)$ is the raw material mixture thermophysical properties (density, heat capacity, thermal conductivity) depending on moisture content and temperature in general cases.

- initial condition:

$$t(x, \tau)|_{\tau=0} = t_0(x)$$  \hspace{1cm} (5)

- limiting conditions:

$$t(x, \tau)|_{x=0} = f_n(\tau)$$  \hspace{1cm} (6)

$$\frac{\partial t(x, \tau)}{\partial x} \bigg|_{x=L} = 0$$  \hspace{1cm} (7)
The initial condition (5) demonstrates that at the moment taken as the reference point, generalized temperature distribution along the coordinate is taken.

The limiting condition (6), as already noted, reflects the fact that temperatures of the metal and the raw material mixture in the contact zone are equal. Condition (7) demonstrates that the problem can be considered as a symmetric one.

Solution of boundary value problems (4) – (7) by modern analytical methods of mathematical physics is problematic in general cases.

Application “microprocessing” method [9] allows these boundary value problems to be restricted to the linear homogeneous equations with constant coefficients.

Let us work in non-dimensional variables:

\[
T(\tilde{x}, \tilde{F}_0) = \frac{t(x, \tau) - t_0}{t_n - t_0}; \quad \tilde{F}_0 = \frac{a \tau}{(L/2)^2}; \quad \tilde{x} = \frac{x}{(L/2)}
\]  

At that the thermal conductivity boundary value problem will take the following form:

\[
\frac{\partial T(\tilde{x}, \tilde{F}_0)}{\partial \tilde{F}_0} = \frac{\partial^2 T(\tilde{x}, \tilde{F}_0)}{\partial \tilde{x}^2}; \quad \tilde{F}_0 > 0; \quad 0 \leq \tilde{x} \leq 1
\]

\[
T(\tilde{x}, \tilde{F}_0) = \frac{t(x, \tau) - t_0}{t_n - t_0} = T_0(\tilde{x})
\]

\[
T(\tilde{x}, \tilde{F}_0)\bigg|_{\tilde{x}=0} = \frac{t_n - t_0}{t_n - t_0} = 1
\]

\[
\frac{\partial T(\tilde{x}, \tilde{F}_0)}{\partial \tilde{x}}\bigg|_{\tilde{x}=1} = 0
\]

We will solve the thermal conductivity boundary value problem by the Laplace integral transformation method [10]. In the field of Laplace images, the solution of equation (9) with regard to the initial condition (10) will be as follows [9]:

\[
T(\tilde{x}, s) = A \cdot ch\left(\sqrt{s} \tilde{x}\right) + B \cdot sh\left(\sqrt{s} \tilde{x}\right) - \frac{1}{\sqrt{s}} \int_0^\tilde{x} T_0(\xi) \cdot sh\sqrt{s}(\tilde{x} - \xi) d\xi
\]

The limiting conditions (11) and (12) in the field of Laplace images are presented as follows:

\[
T(\tilde{x}, s)\bigg|_{\tilde{x}=0} = \frac{1}{s}
\]

\[
\frac{\partial T(\tilde{x}, s)}{\partial \tilde{x}}\bigg|_{\tilde{x}=1} = 0
\]

Let us substitute the solution (13) in the limiting condition (14):

\[
\frac{1}{s} = A \cdot ch\left(\sqrt{s} x\right)\bigg|_{x=0} + B \cdot sh\left(\sqrt{s} x\right)\bigg|_{x=0}
\]

Hence, for constant A it follows that:
Let us differentiate the solution (13) on \( x \) according to (17):

\[
\frac{\partial T(\tilde{x}, x)}{\partial x} = \frac{1}{s} \sqrt{s} \cdot sh(\sqrt{s} \tilde{x}) + B \sqrt{s} \cdot ch(\sqrt{s} \tilde{x}) - \frac{1}{\sqrt{s}} \int_0^1 T_0(\xi) \cdot \sqrt{s} \cdot ch\sqrt{s}(\tilde{x} - \xi) d \xi
\]  

(18)

Hence, for constant \( B \) it follows that:

\[
B = \frac{1}{\sqrt{s} \cdot ch\sqrt{s}} \left[ -\frac{1}{\sqrt{s}} sh\sqrt{s} + \frac{1}{0} T_0(\xi) ch\sqrt{s}(1 - \xi) d \xi \right]
\]

(19)

Substituting (17) and (19) in the equation (13), we obtain the final solution of the thermal conductivity problem in the image field:

\[
T(\tilde{x}, x) = \frac{1}{s} ch(\sqrt{s} \tilde{x}) + \frac{sh(\sqrt{s} \tilde{x})}{\sqrt{s} \cdot ch\sqrt{s}} \left[ -\frac{1}{\sqrt{s}} sh\sqrt{s} + \frac{1}{0} T_0(\xi) ch\sqrt{s}(1 - \xi) d \xi \right] -

\frac{1}{\sqrt{s}} \int_0^1 T_0(\xi) sh\sqrt{s}(\tilde{x} - \xi) d \xi
\]

(20)

In accordance with the second decomposition theorem, the transition from the image field to the original functions is performed for each summand in accordance with the following formula [9]:

\[
T(\tilde{x}, Fo) = L^{-1}[T(\tilde{x}, s)] = \frac{\varphi(s)}{\varphi'(s)}\bigg|_{s=0} + \sum_{n=1}^{\infty} \frac{\varphi(s_n)}{\varphi'(s_n)} \exp(s_n Fo)
\]

(21)

3. Research Results

After the step-by-step transformation of each summand and transition to the original functions [9], we obtain the final solution of the thermal conductivity boundary value problem:

\[
T(\tilde{x}, Fo) = 1 - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2(n-1)} \sin\left(\frac{\pi}{2} (2n-1) \tilde{x} \right) \cdot \exp\left[-\frac{\pi^2}{4} (2n-1)^2 Fo\right] +

+ 2 \sum_{n=1}^{\infty} \sin\left(\frac{\pi}{2} (2n-1) \tilde{x} \right) \cdot \int_0^1 T_0(\xi) \cdot \sin\left(\frac{\pi}{2} (2n-1) \xi \right) d \xi \cdot \exp\left[-\frac{\pi^2}{4} (2n-1)^2 Fo\right]
\]

(22)

Let us transform equation (22) into a dimensional form with regard to equation (10):

\[
t(x, \tau) = t_n - (t_n - t_0) \cdot \left( \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2(n-1)} \sin\left(\frac{\pi}{2} (2n-1) \cdot \tilde{x} \right) \cdot \exp\left[-\frac{\pi^2}{4} (2n-1)^2 Fo\right] +

+ 2 \sum_{n=1}^{\infty} \sin\left(\frac{\pi}{2} (2n-1) \cdot \tilde{x} \right) \cdot \int_0^1 T_0(\xi) \cdot \sin\left(\frac{\pi}{2} (2n-1) \cdot \xi \right) d \xi \cdot \exp\left[-\frac{\pi^2}{4} (2n-1)^2 Fo\right]\right)
\]

(23)

We simulate the heat treatment process at the temperature range from 20°C to 750°C, since most of the gasifiers used during the foam glass powder production method begin decomposition at the
temperature range of 680-800°C [11]. Thermal diffusivity values change in the range of simulated temperatures which affects the Fourier number, thus it is proposed to be calculated at each stage of the simulation according to the formula (8), while assuming that according to [12] thermal diffusivity \(a\) is changed according to the following law:

\[
a = \left( -3.1 \times 10^{-7} \cdot t_n^2 + 9.92 \times 10^{-4} \cdot t_n + 2.305 \right) \times 10^{-7},
\]

where \(t_n\) is the temperature on the material surface, °C.

The initial data and the calculations results are presented in Table 1. Time increment in the calculations was assumed as equal to 20 minutes.

| Seq No. | Temperature, \(t\), °C | Thermal diffusivity, \(a\times10^{-7}\), m²/s | Dimension, \(L/2\), m | Fourier Number, \(Fo\) |
|---------|----------------------|--------------------------------|-------------------|-------------------|
| 1       | 20                   | 2.32472                        | 0.05              | 0.112             |
| 2       | 50                   | 2.35383                        | 0.05              | 0.113             |
| 3       | 150                  | 2.44683                        | 0.05              | 0.117             |
| 4       | 250                  | 2.53363                        | 0.05              | 0.122             |
| 5       | 350                  | 2.61423                        | 0.05              | 0.125             |
| 6       | 450                  | 2.68863                        | 0.05              | 0.129             |
| 7       | 550                  | 2.75683                        | 0.05              | 0.132             |
| 8       | 650                  | 2.81883                        | 0.05              | 0.135             |
| 9       | 750                  | 2.87463                        | 0.05              | 0.138             |

Temperature fields when the raw material mixture heating up to the temperature of 750°C is calculated according to the equation (23). The value is assumed as \(T(0) = 20°C\) at the initial moment, and according to Table 1, \(Fo=0.112\). Distribution of temperature fields obtained at each stage of the calculation is approximated by degree 2 polynomials and assumed as the initial one at the next stage of the calculations.

Figure 3 demonstrates that the temperature in the center of the material will reach 452°C after 180 minutes of the raw material mixture heating at the rate of 5°C per minute. Further heating can result in
destructive processes, so we simulate the situation with the heat source shutdown for the uniform heating of the material throughout the volume and assume the surface temperature of 650 °C. Figure 4 demonstrates the distribution of temperature fields for different Fo values.

Figure 4. Graph of Distribution of the Temperature Fields after Cooling the Temperature on the Material Surface to 650°C:

1 – The last stage of the material heating calculation (Figure 3. curve 8); 2 – Fo = 0.007 (1 min); 3 – Fo = 0.034 (5 min); 4 – Fo = 0.068 (10 min); 5 – Fo = 0.135 (20 min); 6 – Fo = 0.271 (40 min); 7 – Fo = 0.507 (75 min); 8 – Fo = 1 (150 min);

Figure 4 demonstrates that with the temperature drops from 750 °C to 650 °C in short time ranges, curves illustrating the distribution of temperature fields with extreme points (Figure 4 curve 2) occur. This feature is explained by the fact that the heat transfer process in solids is inertial. Thus, with intensive cooling of the raw material mixture the subsurface layers of the backfill (0-0.1 cm) heated to the temperatures of more than 650°C cannot transfer the previously accumulated heat to the surface and less heated layers of the raw material mixture. The calculations have demonstrated that exposure at temperatures of 650 °C for more than 75 minutes is not efficient, since the final temperature equalization in the center of the material will occur after more than 2 hours. For this reason, the surface temperature is raised after 75 minutes to 700 °C and then to 750 °C with the time increment of 20 minutes as previously, without the fear of the material uneven air-entraining (Figure 5). The total estimated time of the foaming process initiation amounts to 255 minutes.

Figure 5. Graph of Distribution of the Temperature Fields at a Phased Heating to the Material Surface Temperature of 700°C, and then 750°C:

1 – Fo = 0.14 (20 min); 2 – Fo = 0.14 (20 min); 3 – Fo = 0.14 (20 min); 4 – Fo = 0.1 (15 min).

4. Conclusions

The developed mathematical model for calculating temperature fields in the raw material mixture in the foam glass production has allowed to determine patterns of the temperature fields distribution and calculate temperature values at any point and at any stage of the material heat treatment. However, the fact that the proposed model is two-dimensional and does not accurately reflect the real kinetics of the process should be noted, for this reason, the "superpositions" method to perform three-dimensional modeling is planned to be applied in the future.

Modeling the kinetics of the process of forming the porous structure of foam glass at various stages of heat treatment of the material makes it possible to predict the complex of macrophysical parameters of foam glass at the stages of the introduction of technological lines for its production, plan the budget
of organizations for energy resources. The development of mathematical methods that can allow three-dimensional modeling of the distribution of temperature fields in the raw material mixture for the production of foam glass, opens up a wide area of research in the field of optimization of high-temperature processing processes of foam glass. Modernization of technological methods in the production of foam glass in conjunction with three-dimensional modeling of high-temperature processing processes can give a good push towards optimization of the technological process of its production as an additional tool for controlling the cost of the final product, which is the direction of our further research.

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