The background expansion is given by
\[ \dot{\Gamma} = 3 \Omega_a a / a^* \]
from which it is not difficult to show that \( \dot{\Lambda} = \frac{H_0^2}{c^2} \Gamma \). Constant \( \Gamma \) is equivalent to take \( \Lambda = 2 \Gamma \) vacuum term \( \Lambda, a \) from which we can derive the matter density evolution
\[ \rho_m = 3 H_0^2 \left[ \Omega_{m0} a^{-3} + (1 - \Omega_{m0}) \Omega_{m0} a^{-3/2} \right]. \]

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I. INTRODUCTION

Although the great success of the standard ΛCDM cosmological model in describing most observations, we are still distant from the full understanding of the cosmic dynamics. Apart from the so called small scale problems in the dark matter sector, there is also a huge discussion about dark energy properties. At the same time, the efficient approach given by a cosmological constant \( \Lambda \) still seems to face some challenges, mainly from the theoretical point of view.

Among the viable alternatives, it has been argued that a model with the mechanism of dark matter particle production at a constant rate \( \Gamma \) (implying in a dynamical vacuum term \( \Lambda(t) \)) is also capable to produce a concordance model [1], although with a larger current matter fraction \( \Omega_{m0} \approx 0.45 \).

If the total energy is given by \( \rho = \rho_m + \Lambda \) and the pressure of the vacuum contribution sets \( p_\Lambda = -\Lambda \), a constant \( \Gamma \) is equivalent to take \( \Delta = 2 \Gamma H_0 \), where \( H = \dot{a}/a \) is the Hubble rate[2]. Taking today’s values one finds \( \Delta = 3 \Omega_{m0} H_0 / 2 \). The matter dynamics corresponds to the balance
\[ \ddot{\rho}_m + 3 H \dot{\rho}_m = -\dot{\Lambda}, \]

from which it is not difficult to show that \( \dot{\Lambda} = -\Gamma \rho_m \).

The background expansion is given by
\[ H = H_0 \left( 1 - \Omega_{m0} + \Omega_{m0} a^{-3/2} \right), \]

Note that, as in the flat ΛCDM model, there are here also only two free parameters. The concordance of this Λ(t) cosmology with \( \Omega_{m0} \approx 0.45 \) has been verified via many different observational data at both the background [2,3] and perturbative [4,6] levels.

Concerning structure formation, the full CMB spectrum has not yet been obtained for this model, but CMB physics can also be accessed via ISW studies, i.e., how time-varying gravitational potential wells change the temperature of the CMB photons as they cross structures [7]. The expansion of an Einstein-de Sitter universe compensates the clustering of structures, producing no ISW effect, i.e., in a matter dominated universe there is no “late time” ISW effect[3]. Dark energy modifies the background expansion and leads to a net contribution to \( (\Delta T/T)_{\text{ISW}} \). Then, it is expected that the modified background and perturbative expansion of the Λ(t)CDM model leaves a distinct imprint on the ISW signal.

Since the ISW is a secondary CMB temperature effect its detection occurs only via the cross-correlation with other large scale probes like galaxies and quasars surveys [8]. Ref. [9] used the cross-correlation technique to probe the Λ(t)CDM model, finding an increase of the ISW-galaxy spectrum \( (C_l^\text{ISW}) \) in comparison to the ΛCDM model. Our aim in this paper is to perform this analysis taking care with some peculiarities of the model. In particular, it is important to differentiate the evolution of the baryonic and dark matter components. At the background level, baryons are included in the conserved part of \( \dot{\Lambda} \). Concerning perturbations, the observed mat-
ter power spectrum $P(k) = |\delta(k)|^2$ is a measurement of the baryonic clustering, which is sourced by the gravitational potential. In the ΛCDM model there is almost no difference between the evolution on large scales of such components. The same does not happen in the Λ(t)CDM model, where one observes a late-time suppression in the dark matter and total matter contrasts owing to dark matter production, a suppression not observed in the baryonic contrast.

Our interest is also justified by a possible tension between the theoretical ΛCDM predictions and the observed ISW signal. Some analysis have reported a cross-correlation signal $1\sigma - 2\sigma$ above that expected in the standard cosmology [1] (see [11] for a critical review on this issue). Recent studies claim a better statistical concordance though the observed signal is still higher than the theoretical one [12, 13]. Also, the inferred stacking of CMB data on the position of superstructures is about 5 times larger than the ΛCDM prediction [14]. This result has recently been confirmed by the 2015 release of the Planck CMB data [15].

In the next section we develop the background expansion which will be used in this paper. In section III we explore the scalar perturbations of the model. The connection with the ISW effect is made via a direct calculation of the evolution of the gravitational potential (IIIB) and the ISW-LSS cross-correlation (IIIC). We conclude the final section.

II. Λ(t) BACKGROUND DYNAMICS

The brief description of the Λ(t)CDM dynamics given in equations [14] has been widely derived in the literature (see [11] and references therein). Note that as in the standard case the important quantity is the total matter density $Ω_{m0}$, the sum of the dark matter $Ω_{dm0}$ and baryons $Ω_{0b}$. The latter is constrained by nucleosynthesis and is well approximated by $Ω_{0b} = 0.05$. We assume hereafter this value. The baryonic sector is conserved and therefore we have the continuity equations

\[ \dot{\rho}_{dm} + 3H\rho_{dm} = -\dot{\Lambda} \]  \hspace{1cm} (4)

and

\[ \dot{\rho}_b + 3H\rho_b = 0. \]  \hspace{1cm} (5)

Adding these equations we obtain the total matter conservation equation (I). With $Λ = 2\Gamma H$, the dynamics presented in section II follows. That is, the background expansion is obtained after solving the system given by the continuity equation

\[ aH\rho_m' + 3H\rho_m = \Gamma \rho_m \]  \hspace{1cm} (6)

and the Friedmann equation

\[ 2aHH' + \rho_m = 0, \]  \hspace{1cm} (7)

where the prime means derivative with respect to the scale factor.

III. PERTURBATIVE DYNAMICS

A. Growth functions

When cosmological perturbation theory is used to study the large scale structure of the universe we are most interested in the evolution of scalar quantities like the density contrast and the gravitational potential. The Λ(t)CDM models have the particularity of suppressing the growth of dark matter contrast $δ_{dm}$. However, the observed structures reflect the behavior of the baryonic contrast, $δ_b = δ_{dm}/ρ_b$, instead. In the standard cosmology this quantity coincides with the dark matter contrast, and the total matter contrast $δ_m$ denotes both behaviors. For the Λ(t)CDM model, however, we should make a distinction between the different components. Using the fact that the vacuum fluctuations are negligible [5] and choosing the comoving synchronous gauge we obtain the set of equations

\[ \dot{δ}_m + \Gamma δ_m = \dot{\Lambda}, \]  \hspace{1cm} (8)

\[ \dot{δ}_b = \frac{\dot{h}}{2}, \]  \hspace{1cm} (9)

\[ \dot{h} + 2H\dot{h} = ρ_mδ_m, \]  \hspace{1cm} (10)

where $h$ is the trace of the spatial metric perturbation. Combining these equations and changing to the derivative with respect to the scale factor we find

\[ a^2H^2δ_m'' + aH(3H + aH' + \Gamma)δ_m' + 2\Gamma H δ_m = \frac{1}{2}ρ_m δ_m \]  \hspace{1cm} (11)

and

\[ a^2H^2δ_b'' + aH(3H + aH')δ_b' = \frac{1}{2}ρ_m δ_m. \]  \hspace{1cm} (12)

In Fig. 1 we plot the evolution of the density contrasts as calculated in [11] and [12]. For the ΛCDM model (black curves) we set $Ω_{m0} = 0.23$, the best-fit of LSS observations [17, 18]. We show the Λ(t)CDM evolutions for $δ_b$ and $δ_m$ in the red and blue curves, respectively. We have used initial conditions $δ_b(a = 0.001) = δ_{dm}(a = 0.001) = 10^{-5}$ and $δ_b'(a = 0.001) = δ_m'(a = 0.001) = 0$. As expected, $δ_b$ and $δ_m$ have the same evolution in the standard case. On the other hand, the plot shows a late-time growth suppression of $δ_m$ in the Λ(t)CDM model, a consequence of dark matter creation. The same suppression is not observed in the baryonic contrast though its amplitude achieves a plateau on late times which is slightly below the standard scenario.

\[ 4 \] A different approach to the baryonic sector can be found in [10].

\[ 5 \] Using, instead, the ΛCDM concordance value $Ω_{m0} = 0.27$ does not change significantly our final results.
B. Evolution of the gravitational potential

The evolution of the gravitational potential $\Phi$ determines the integrated Sachs-Wolfe contribution to the total CMB spectrum via the formula

$$\frac{\Delta T}{T}^{ISW} = 2 \int_{a_d}^{1} \frac{\partial \Phi}{\partial a} da,$$  \hspace{1cm} (13)

where $a_d$ denotes the decoupling time, and $a = 1$ at present. Writing down the Poisson equation in the comoving synchronous gauge we find

$$k^2 \Phi = -\frac{a^2}{2} \rho_m \delta_m,$$ \hspace{1cm} (14)

where $k$ is the comoving wave-number. This expression allows us to calculate the ISW, which can be computed via the line of sight integration (13). It is useful to perform it in terms of the comoving distance

$$\chi(a) = \int_{a}^{1} \frac{da}{a^2 H(a)}.$$ \hspace{1cm} (15)

The ISW then reads

$$\frac{\Delta T}{T}^{ISW} = \frac{\Delta m(a = 1)}{a^2 H(a)} \int_{a_d}^{\chi_d} a^2 H(a) \frac{dQ_m(a)}{da} d\chi,$$ \hspace{1cm} (16)

where we have defined

$$Q_m(a) = a^2 \rho_m(a) D_m^+(a),$$ \hspace{1cm} (17)

with

$$D_m^+(a) = \frac{\delta_m(a)}{\delta_m(a = 1)}.$$ \hspace{1cm} (18)

The baryonic and total matter density contrasts can be directly calculated from the solution of Eqs. (11) and (12). Using Eq. (14) we compare in Fig. 2 the predictions for the $\Lambda$CDM and $\Lambda(t)$CDM gravitational potentials as functions of the redshift $z$. We access the observational predictions for the ISW effect via the cross-correlation of CMB maps and LSS surveys, to be done in next section.

C. Correlating CMB maps and galaxy surveys

In order to compute the cross-correlation between CMB and LSS we also need to describe the evolution of the observed galaxy contrast $\delta_g$ on the line of sight. This quantity depends on the survey design and is obtained from

$$\delta_g = \int_{0}^{\chi_d} b(z) \frac{dN}{dz} H(a) \delta_b(a) d\chi,$$ \hspace{1cm} (19)

where $\delta_b$ is the baryonic linear contrast, and $b(z)$ is the standard bias between the galaxy contrast and the linear one$^6$.

The redshift distribution of the observed galaxy sample is a model-independent quantity. Each survey has its own $dN/dz$ histogram function. We use in this work data from the NRAO VLA Sky Survey (NVSS)$^7$ and the Wide-field Infrared Survey (WISE)$^8$ catalogues. Such surveys are widely used for cross-correlation studies. For the former one has

$$b(z) \frac{dN}{dz} = b_{eff} \frac{\alpha^{\alpha+1}}{z_s^{\alpha+1} \Gamma(\alpha)} z^\alpha \exp \left[ -\frac{\alpha z}{z_s} \right],$$ \hspace{1cm} (20)

where the values $b_{eff} = 1.98$, $z_s = 0.79$ and $\alpha = 1.18$ were fitted in [19]. For the WISE catalogue $dN/dz$ can be obtained numerically from Ref. [20] and, following Ref. [19], we adopt a constant bias $b(z) = 1.41$. We will limit our results to constant bias models since the use of time-dependent $b(z)$ bias will not change our main conclusion.

$^6$ In the $\Lambda$CDM model is irrespective to use $\delta_b$ or $\delta_m$ in [19], but the same is not true for the $\Lambda(t)$ case, as can be seen in Fig. 1

$^7$ http://www.cv.nrao.edu/nvss/

$^8$ http://wise.ssl.berkeley.edu/
Combining \([16]\) and \([19]\), the multipole coefficients for the cross-correlation spectrum are given by

\[
C_T(l) = \int_1^{a_{eq}} \frac{W_T(a) W_g(a) P(k = l/\chi)}{a^2 H(a)} \frac{da}{l^2},
\]

where we have defined the weight functions

\[
W_T(a) = a^2 H(a) \frac{dQ(a)}{da},
\]

\[
W_g(a) = H(a) b(z) \frac{dN}{dz} D_b^+(a),
\]

with

\[
D_b^+(a) = \frac{\delta_b(a)}{\delta_b(a = 1)}. \tag{24}
\]

We have also defined the crossed power spectrum

\[
P(k) = \frac{\delta_m(a = 1)}{\delta_b(a = 1)} P_b(k),
\]

where \(P_b(k)\) is the observed baryonic power spectrum, given by

\[
P_b(k) = P_0 k^{n_s} T^2(k), \tag{26}
\]

where \(P_0\) is a normalisation constant determined from observations \([4]\). This normalisation constant is not the same as in the \(\Lambda CDM\) model as can be seen by an inspection of figures \([1\) and \(2\). It can also be determined by writing \(P_b \delta_b^2(1) = P_0 \delta_b^2(1)\), where the barred quantities are the standard ones \([6]\). For \(T(k)\) we will use the BBKS transfer function \([21]\) which, for \(\Omega_d \ll \Omega_m\), can be approximated by

\[
T(x = k/k_{eq}) = \frac{\ln(1 + 0.171 x)}{(0.171 x)} \times \left[ 1 + 0.284 x + (1.18 x)^2 + (0.399 x)^3 + (0.490 x)^4 \right]^{-0.25}. \tag{27}
\]

Here, \(k_{eq}^{-1}\) is the comoving Hubble radius at the time of matter-radiation equality. From \([3]\) it is easy to show that it is given by \([16, 46]\)

\[
k_{eq} = 0.073 h^2 \Omega_m^{0.15} \text{ Mpc}^{-1},
\]

where, here, \(h = H_0/(100 \text{ km/s/Mpc})\) and we have set the present radiation density as \(\Omega_R^{0.1} = 4.15 \times 10^{-4} h^{-2}\). We will adopt \(h = 0.7\) and \(n_s = 1\).

The resulting cross-correlation spectrum is shown in Figs. 3 and 4. The \(C_l^{Tg}\) spectrum for the \(\Lambda(t)CDM\) model presents a slightly larger signal as compared to the standard model. However, given the large uncertainties in determining the observed \(C_l^{Tg}\) values, both models remain compatible with data. This seems to be a virtue of the particle creation model since we have learned from Refs. \([11, 14]\) that models with higher \(C_l^{Tg}\) power are desirable.

IV. CONCLUSIONS

The concordance particle creation model is a viable alternative to the standard \(\Lambda CDM\) paradigm, with the same free parameters, namely \(\Omega_m\) and \(H_0\). Since particle creation at a constant rate leads to a different background and perturbative dynamics, it is important to investigate specific signatures of this model in comparison to the standard cosmology. We provided in this work a direct computation of the evolution of perturbed scalar quantities like the matter and baryonic density contrasts and the gravitational potential for the \(\Lambda(t)CDM\) model. There is a clear scenario emerging in such a cosmology (see Fig. 4), in which the dark matter contrast is highly suppressed at late times while the baryonic contrast maintains a constant value, as in the standard case, but with a slightly smaller amplitude. This dynamics does indeed lead to a consistent description of the structure formation data \([3, 6]\). Our main goal here was to provide a comprehensive analysis of the ISW effect in such concordance particle creation cosmology. With the results for the perturbative dynamics calculated in section II we computed the CMB-LSS cross correlation spectrum. Our care in calculating the evolution of both baryonic and total matter contrasts in detail leads to a full
and safe analysis of the $C_{l}^{TT}$ spectrum.

Current efforts [11–13] in obtaining the observed $C_{l}^{TT}$ spectrum are sending a clear message: the CMB-LSS signal seems to slightly exceed the ΛCDM prediction. Therefore, it is timely to check the Λ(t)CDM outcomes for this cosmological observable. For both the NVSS (Fig. 3) and WISE (Fig. 4) data, the Λ(t)CDM model leads to a desirable excess of power in the $C_{l}^{TT}$ spectrum. It is worth noting that the available data is still insufficient for distinguishing cosmological models with a reliable statistical confidence. It is still not clear whether there is indeed a tension between the ΛCDM model and observations. Here we are still restricted to a qualitative comparison between the Λ(t) and standard cosmologies. Future data can improve the accuracy of the CMB-LSS cross-correlation technique and therefore specific features of different cosmological models concerning the ISW effect can be compared in more detail.

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