Fokker-Plank entropic force model for galactic rotation curves

VS Morales-Salgado,1 H. Martínez-Huerta,2 and P. I. Ramírez-Baca3

1 CEDIP-Cámara de Diputados, H. Congreso de la Unión 66, El Parque, Venustiano Carranza, 15960, Ciudad de México, México
2 Department of Physics and Mathematics, Universidad de Monterrey, Av. Morones Prieto 4500, 66238, San Pedro Garza García NL, México
3 Facultad de Ciencias, Universidad Autónoma de San Luis Potosí Campus Pedregal, Av. Parque Chapultepec 1610, Col. Privadas del Pedregal, San Luis Potosí, SLP, 78217, Mexico
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We investigate the possible entropic nature of the force responsible for the discrepancy between the observed galactic rotation curves and those expected from the distribution of visible matter in the galaxy. Observations from the Spitzer Photometry and Accurate Rotation Curves (SPARC) data base are used to study the adequacy of the proposed models. A concrete model derived from a simple solution of the Fokker-Plank equation is used to fit SPARC data, resulting in strong agreement with observations compared to the popular dark matter NFW mass profile. We also show that correlations exist between a parameter of the proposed model and the flat velocity as well as the Luminosity at 3.6 µm of the sample of galaxies.

I. INTRODUCTION

Current models framing astronomical observations demand the presence of exotic components introduced to explain the difference between how objects in the sky ought to move, according to some preconceived notion, and how they are actually observed to move [1]. However, the dark sector of the universe, as has been coined colloquially, still eludes detection in a plethora of experiments and has no clear origin within the standard model of fundamental particles [2]. This compels us to consider alternative explanations for the observed motion of celestial bodies.

One of the earliest motivations to introduce the concept of dark matter is the so-called galactic rotation curves problem. A rotation curve describes the rotational velocity of a test mass as a function of the radial distance to the galactic center. Starting with the seminal work of Rubin and Ford [3], numerous studies have been performed to obtain rotation curves of a vast sample of galaxies. However, there exist discrepancies between astrophysical observations and rotational velocities modeled according to predicted mass profiles. This is because, while a classical macroscopic treatment of gravity predicts a fall of orbital speeds like \( r^{-1/2} \) at large radii, observations show that velocity remains constant with respect to distance far from galactic centers [4].

In this work we use observations compiled by the Spitzer Photometry and Accurate Rotation Curves (SPARC) project. This is a major database of high quality rotation curves spanning a broad range of morphologies, luminosities, and surface brightnesses [5]. Studies using this data set with a variety of models can be found in literature, see for instance [6–10], although the true nature of the discrepancies remains unsolved.

Most proposals to solve the galaxy rotation curve problem, including dark matter, take the general approach of adding an extra term \( F_E \) to the balance producing the centripetal force necessary to explain the observations, yielding the equation

\[
F_C = F_G + F_E, \tag{1}
\]

where \( F_C = m v^2 / r \) is the centripetal force exerted on a test mass \( m \), and \( F_G = -m \frac{d\Phi}{dr} \) is the gravitational attraction exerted by the baryonic matter in the galaxy on the same test mass \( m \), whose gravitational potential is given by \( \Phi(r) \). What changes from one proposal to another is the form and nature of \( F_E \). For example, in the case of dark matter models, the extra force \( F_E = -m \frac{d\Phi}{dr} \)...
is of gravitational nature and its source is identified with a mass distribution, usually in the shape of a halo surrounding the galaxy. An alternative to dark matter is the so-called Modified Newtonian Dynamics (MOND), which proposes to modify Newton’s law of gravitation in the galactic regime \[11–13\]. This can also be written in the form of eq. (1).

In this work we consider an extra force of entropic nature emerging from statistical considerations. Statistical physics is a suitable framework to study the way macroscopical phenomena, e.g. entropic forces, may emerge in systems with a large number of interacting constituents, such as galaxies. Indeed, galactic constituents (stars, dust, radiation) and processes (stellar aggregation, dissipation, diffusion) provide a richer environment than the usual highly idealized considerations employed in their study \[14\]. With this in mind, we investigate here the possible entropic nature of the extra force \(F_E\) in eq. (1) from the point of view of its adequacy to described observed rotation curves.

Entropic forces are emergent phenomena occurring due to the tendency of a system to increase its entropy \[15–17\]. This means that fundamental interactions take place within galaxies that aggregate to produce an emergent phenomena. For example, in \[15, 18\] it is proposed that the very gravitational interactions are of entropic nature and the features of the resulting equations of motion are explored. Similarly, in \[19\], the authors study the anomalous galactic rotation curves as emergent from a entropy-area relation derived from a Schwarzchild blackhole metric.

In a different approach to entropic forces emerging in galactic contexts, here we start with very simple assumptions about the statistical system to obtain a simple model of an entropic force \(F_E\). Then, we proceed to test its adequacy to describe the observed rotation in galaxies. An appropriate performance of this simple model may shed light for further refinements in exploring the possible emergent nature of galactic dynamics.

Our initial assumptions include that the underlying statistical system is in equilibrium, with negligible fluctuations at macroscopical scales. We also assume null exchange of energy or particles with the environment. Thus, we can consider a microcanonical ensemble and a Boltzmann type of entropy. Even more, the probability associated to each state is given as a distribution \(P(r)\) over the galactic radius \(r\). Different sensible probability distributions may be investigated to possibly explain the behavior of galactic rotation curves. For example, one could demand that \(P(r)\) be a solution of an evolution equation, e.g. Fokker-Planck (F-P) equation, leading to its current distribution.

F-P equations have helped to study macroscopic properties of systems with many interacting particles \[20–24\]. These equations describe the evolution of the probability density of particles subject to drag interactions, as well as random interactions. Regarding galaxies as a class of such systems, the working hypothesis of this investigation is that entropic forces emerge from a probability distribution that is effectively described by a F-P equation. The aim of this article is to explore the viability of this description.

The article is organized as follows: in section II we review the expected rotation profiles resulting from a keplerian model of galaxies and their discrepancies with the observed rotation curves. Section III describes the proposal of our work. It presents an entropic force included in the balance of forces establishing the centripetal force that could explain the observed rotation curves. In section IV we use data from the SPARC database to investigate the idea of an entropic extra force in galaxies. Finally, in section V we discuss our findings and provide the final remarks of our work, including prospective topics to be investigated.

II. GALACTIC ROTATION CURVES

The aforementioned discrepancies in the rotation of galactic material derives from the expected notion that the centripetal pull \(F_C\) corresponds only to the gravitational force \(F_G\), i.e., \(F_C = F_G\), leading to the following equation:

\[
v(r) = \sqrt{-r \frac{d\Phi}{dr}}, \tag{2}
\]

that attempts (yet fails) to describe observed galactic rotation curves, i.e. the tangential speed of matter around the center of the galaxy as a function of the galactic radius \(r\). Observations of the distribution of baryonic mat-
ter yield a gravitational potential \( \Phi(r) \sim 1/r^2 \) for large radii. However, measurements of rotation curves show that, at large radii, the decay of the angular speed is not as steep as predicted by eq. (2) from the presence of baryonic matter only.

In the dark matter scenario, for instance, a halo of weakly interacting matter is added to each galaxy mass distribution. This modifies the expected rotation curve by adding an extra term in the potential \( \Phi \) in (2). While different dark matter potentials have been explored [2], the one suggested by Navarro, Frenk and White (NFW) is one of the most commonly used [25]. This mass profile yields the force:

\[
F_{\text{NFW}} = G \frac{M_{\text{vir}} m}{\ln(1 + c) - c/(1 + c)} \left( \frac{1}{r^2} \right) \\
\times \left[ \frac{r}{r + R_s} - \ln \left( 1 + \frac{r}{R_s} \right) \right],
\]  

(3)

where \( M_{\text{vir}}, \rho_0, R_s \) and \( c \) are parameters that depend on the galaxy studied. For a gravitational pull of this sort, a mass density profile \( \rho_{\text{NFW}}(r) = \rho_0 / \frac{\pi c}{1 + \frac{c}{R_s}} \) is considered. In this article we will explore an alternative description of the missing force responsible for the observed galactic rotation curves.

### III. ENTRepIC FORCES

The difference between the rotation predicted by the presence of baryonic matter in galaxies and its observed rotation may be explained by an extra term \( F_E \) in the balance of radial forces: \( F_C = F_G + F_E \). Thus, the rotation curve, eq. (2), becomes

\[
v(r) = \sqrt{r \left( -\frac{d\Phi}{dr} + \frac{1}{m} F_E \right)}.
\]  

(4)

Consider now an extra force \( F_E \) of entropic nature, given by the expression

\[
F_E = T \frac{\partial S}{\partial r},
\]  

(5)

where \( T^{-1} = \partial S/\partial E \) is the temperature parameter associated to an underlying statistical system.

Since entropic forces emerge from the tendency of a statistical system to maximize its entropy [10], a possible entropic nature of \( F_E \) implies the existence of more fundamental phenomena. Notice that, to describe the observed galactic rotation curves, only the radial dependence of the probability distribution and its corresponding entropy is needed.

To illustrate the change in paradigm when considering an emergent entropic force instead of a gravitational pull due to undetectable matter content in galaxies, consider the NFW profile of the form \( \text{[3]} \). The same profile can be obtained by means of an entropic force, eq. (5), using an entropy of the form

\[
S(r) = - \frac{A}{r} \ln \left( 1 + \frac{r}{B} \right).
\]  

(6)

To obtain the extra force found in the usual presentation of the NFW model one must consider \( A = GM_{\text{vir}} (1 + c)/(rT[(1 + c) \ln(1 + c) - c]) \) and \( B = R_s \).

Many other functions apart from the NFW model can be considered in the investigation of the galactic rotation curves; nonetheless, let us start by considering simple cases. For example, for a Boltzmann type of entropy \( S(r) = -k_B \ln P(r) \), the corresponding entropic force is given by \( F_E(r) = -k_B T \frac{\partial}{\partial r} \ln[P(r)] \), yielding a rotation curve profile described by

\[
v(r) = \sqrt{F_G - k_B T \frac{r}{m} \frac{\partial}{\partial r} \ln[P(r)]},
\]  

(7)

that now depends on a choice of a probability distribution \( P(r) \).

As an initial approximation, we assume galaxies to be isolated in particle and energy exchange from the rest of the universe. This assumption is reasonable for the timescales of current observations. Thus, we may proceed to regard galaxies as thermodynamic systems where microstates are equally probable and employ a Boltzmann entropy function.

Once the expression for the entropy in terms of a probability distribution \( P(r) \) has been fixed, we make assumptions precisely on such a probability distribution. For the sake of concreteness, and as a sensible initial investigation, we shall assume that \( P(r) \) is a solution of the three-dimensional generalization of the usual one-dimensional F-P equation:

\[
\frac{\partial}{\partial t} P = \nabla^2 (\epsilon P) - \nabla \cdot (\vec{\mu} P),
\]  

(8)
where $\epsilon$ is the diffusion coefficient and $\vec{\mu}$ the drift vector. In the most general case the diffusion and drift coefficients depend on both space and time. Nonetheless, for the case of galaxies considered here, we can assume that the probability is steady: $\frac{\partial}{\partial t} P = 0$. Therefore, the probability distributions to consider are given by solutions of the equation

$$\nabla^2 (\epsilon P) - \nabla \cdot (\vec{\mu} P) = 0.$$  

(9)

This allows us to commence exploring explicit functional forms of entropic extra forces to describe the observed rotation curves.

A simple entropic force model

To obtain a simple initial model, let us consider a radial-only drift vector $\vec{\mu} = \mu_r \hat{r}$, where $\hat{r}$ is the unitary spherical radial vector. Considering $\epsilon$ constant and $\mu_r = \mu_r(r)$, the F-P equation is only radial and reduces to

$$\epsilon \partial_r (r^2 \partial_r P) - \partial_r (r^2 \mu_r P) = 0,$$  

(10)

whose general solution is given by

$$P(r) = e^{\int_1^r \frac{\mu_r(\xi)}{\epsilon} d\xi} \left( \int_1^r \frac{C_1}{\epsilon \xi^2} e^{-\int_1^\xi \frac{\mu_r(\zeta)}{\epsilon} d\zeta} d\xi + C_2 \right).$$

(11)

Furthermore, if we substitute an inverse $r$-squared drift, $\mu_r = k/r^2$, in (10), subject to the boundary condition $\lim_{r \to \infty} P(r) = 0$, we obtain the following radial probability distribution

$$P(r) \propto \exp \left( -\frac{k \epsilon}{r} \right) - 1,$$  

(12)

that in turn yields an entropic force

$$F_r(r) = \frac{k/\epsilon}{\beta r^2 \exp (k/\epsilon r) - 1}.$$  

(13)

Parameters $k/\epsilon$ and $\beta^{-1} = k_B T > 0$ characterize each galaxy. Qualitative properties of these functions are depicted in Figure 1.

IV. DATA ANALYSIS

To test the model introduced in the previous section, let us compare it to rotation curves compiled in the SPARC project. The SPARC data base is a sample of 175 nearby galaxies with surface photometry at 3.6 $\mu$m and high-quality rotation curves from previous H$_1$/H$_\alpha$ studies. SPARC spans a broad range of morphologies (S0 to Irr), luminosities ($\sim$ 5 dex), and surface brightnesses ($\sim$ 4 dex). Figure 2 shows a typical galactic rotation curve as observed, as well as the contributions expected from each type of detectable (baryonic) matter.
In order to proceed, we define the variable

\[ \Delta v^2 = v^2_{\text{observed}} - (v^2_{\text{gas}} + \Upsilon v^2_{\text{star}}), \]  

(14)

where \( \Upsilon = M^* / L \) is the stellar mass-to-light ratio. Eq. (14) accounts for the discrepancy between the (squared) observed rotation speed and the expected one from the baryonic matter content in each galaxy [26]. Using eq. (7) for entropic forces, eq. (14) becomes

\[ \Delta v^2 = - \frac{r}{\beta m} \frac{d}{dr} \ln P(r). \]

(15)

We can note that for any given sensible choice of diffusion \( \epsilon \) and drift \( \mu \) in eq. (9), we can obtain a probability function \( P(r) \) that, in turn, yields a rotation curve profile to be compared against observed data. Two working assumptions are considered in what follows: the stellar mass-to-light ratio \( \Upsilon \) and the test mass \( m \) are considered constant for all radii in each rotation curve. Now, let us consider the simple probability distributions described in the previous section.

For the case of an extra force given by (13), corresponding to a simple Fokker-Plank entropic (FPE) force model, the discrepancy is

\[ \Delta v^2_{\text{FPE}} = \frac{k/\epsilon}{\beta m r \left[ \exp \left( \frac{k}{\epsilon} r \right) - 1 \right]} \frac{ab}{r} \left[ \exp \left( \frac{b}{r} \right) - 1 \right], \]  

(16)

where \( a^{-1} = \beta m \) and \( b = k/\epsilon \). While for the case of the NFW model, which we identify here with an entropy given by eq. (6), the squared discrepancy is

\[ \Delta v^2_{\text{NFW}} = \frac{A}{m} \left( \frac{1}{r} \right) \left[ \frac{r}{r + B} - \ln \left( 1 + \frac{r}{B} \right) \right]. \]

(17)

Using the SPARC data set, we select edge-on rotation curves of galaxies with an inclination greater than 30° [5], quality flag at least medium (1 and 2), and at least four data points in its rotation curve. This yields a set of 93 galactic rotation curves.

We perform a statistical test to investigate how well the FPE model, eq. (16), describes the data and we compare it to the well known NFW model, eq. (17). We use the python package lmfit [27] based on two free parameters: \( (a, b) \) for FPE and \( (A/m, B) \) for NFW. For each data set, model, and a given \( \Upsilon \) we find the least reduced chi-square,

\[ (\chi^2_{\text{red}})_{i, \Upsilon} = \frac{\chi^2_{i, \Upsilon}}{N - d} = \frac{1}{N - d} \sum_{i=1}^{93} \left( \frac{Y_{i, \text{obs}} - Y_{i, \text{fit}}}{\sigma(Y_{i, \text{obs}})} \right)^2, \]  

(18)

where \( i \) stands for the \( i \)-th value in the data set of each galaxy. \( Y_{i, \text{obs}} = \Delta v^2 \) is the squared velocity difference obtained from the data, as in eq. (14); \( Y_{i, \text{fit}} \) is the value of \( \Delta v^2 \) described by each model according to eq. (15); \( d = 2 \) is the number of free parameters; and \( N \) is the number of data points. Fig. 3 shows fit examples for each model for NGC3198.

Since the stellar mass-to-light ratio can take different values for each galaxy [28], we vary \( \Upsilon \) to find its most suitable value according to the models here studied and compare the results with those reported in literature using other methods. To do this, we span \( \Upsilon \) from 0 to 5 using steps of 0.01, and select the value that results in
the minimum \( (\chi^2)_{\text{min}} = \min((\chi^2)_{\text{red}}, \Upsilon) \). Fig. 4 shows the distribution of \( \Upsilon \) corresponding to the least reduced chi-square for the galaxies considered here. Note that the median values of \( \Upsilon \) for FPE and NFW models are 0.61 and 0.41, respectively, which is compatible with those reported in [29, 31].

We also obtain the average \( \chi^2_{\text{red}} \) of 1.3 for FPE and 10.1 for NFW, with corresponding standard deviations of 1.9 and 19.2, respectively. Fig. 3 shows fit examples for each model including its corresponding \( \chi^2_{\text{red}} \). The cumulative distribution function (CDF) of \( \chi^2_{\text{red}} \) is plotted in Fig. 5, depicting lower \( \chi^2 \) values for FPE compared to NFW. This in turn implies a better fit for the FPE model.

To measure the compatibility of each model with the data, we follow Akaike’s information criterion (AICc) and Bayesian Information Criterion statistic (BICc) [32], given by

\[
AICc = N \ln(\chi^2/N) + 2N_{\text{varsys}},
\]

\[
BICc = N \ln(\chi^2/N) + \ln(N)N_{\text{varsys}},
\]

(19)

respectively. Lower AICc and BICc values correspond to greater model compatibility, that in turn implies a better description of the data set. The average value for FPE and NFW models result in \(-4 \text{ and } 29.6\), for AICc and \(-2.1 \text{ and } 31.5\) for BICc, respectively. Indeed, FPE model is more compatible with observations than the NFW one.

Once the galactic rotation curves are fitted, we also study the existence of possible relations between the parameters of the models and other properties of galaxies. The SPARC data base includes a list of these properties. After investigating correlations between the four parameters, two of FPE and two of NFW, we identify two combinations with significant measures of correlation: parameter \( a \) in FPE model vs both, the velocity along the flat part of the rotation curve, \( V_f \), and the total luminosity at 3.6 \( \mu \)m, \( L_{[36]} \).

On the one hand, a Spearman coefficient of 0.97 between \( \ln(a) \) and \( \ln(V_f) \) is found. This correlation can be appreciated as natural since one can write the model here proposed in the form \( v = \sqrt{c_1 a + c_0} \), where \( c_0 = (b/r)(e^{b/r} - 1) \) and \( c_1 = F_G \). Indeed, the correlation is non-linear, and its monotonicity and concavity correspond to the range of the parameters. We show this
correlation and a fit in the top panel of Fig. 6. For this analysis, we have not included ten rotational curves that report \( V_f = 0 \) nor UGC11914, which has an anomalously large value of \( a \); the latter is shown as the grey square.

On the other hand, a correlation can also be observed between \( a \) and \( L_{36} \). It is characterised by a Pearson coefficient of 0.92 in logarithmic scales for both variables. A linear regression in logarithmic scales is shown in the bottom panel of Fig. 6. As in the previous correlation, we omit UGC11914 due to its outlying nature. Since \( a \) is proportional to the temperature parameter of the underlying system from where the entropic force emerges, the correlation between \( L_{36} \) and \( a \) in the FPE model establishes, in turn, a correlation between such a temperature and the luminosity of the galaxies studied here.

V. DISCUSSION AND FINAL REMARKS

A possible entropic nature of an extra force producing the discrepancy of the expected and observed galactic rotation curves requires an appropriate expression of entropy in terms of the galactic radius. In this work we have investigated a Boltzmann type of entropy obtained from a probability distribution that is also a simple solution of the Fokker-Planck equation with constant diffusion coefficient. Despite the simplicity of the model, it performed satisfactorily in explaining the general behavior of the observed galactic rotation curves.

To test the idea of an entropic force contributing to the centripetal pull in galaxies, we fitted the observations in the SPARC database using a model that adds an entropic force to the one expected from the baryonic mass in each galaxy. Results show that the rotation curves in the sample have a strong compatibility with FPE model. The fitting also includes a selection of stellar mass-to-light ratio of \( \Upsilon \sim 0.61 \) for the FPE model. This is in agreement with the range provided in the literature for such quantity.

The same tests were conducted for the NFW model. However, NFW does not perform as good as the simple model presented here. Even more, the simplicity of FPE model leaves room for less strict conditions on the FP equation that in turn may describe the observed rotation curves more accurately.

Entropic forces, being an emergent statistical phenomena, model the cumulative behavior of a large number of interactions. These comprise the fundamental system underlying the entropic force. It is not clear what would be the nature of the fundamental phenomenon for entropic forces contributing to the centripetal forces in galaxies, although several authors have worked on such a problem. A study in this direction may shed light on the nature of the temperature parameter correlating to the luminosity of galaxies. This is beyond the scope of this work and will be left for further investigations.

Also, an entropic extra force, as described here, is an alternative to the popular dark matter models. While dark matter models propose weakly interacting mass distributions in galaxies, entropic forces imply the existence of systems with states given by a probability distributions within galaxies. Nonetheless, dark matter is used to in-
investigate other unexplained observations in the universe too, astrophysical as well as cosmological. For example, gravitational lensing, galaxy distribution at large scales or the cosmic microwave background anisotropies. In this sense, entropic forces have been used to study other problems besides rotation curves. For example, the role that different formulations of entropic forces may have in the evolution of the universe [33], the possible answer to the dark energy problem [34] or its implications in a brane cosmology [35]. In the case of the simple model presented here, this is a work yet to be done.

Correlations between parameter \( a \) in FPE model, directly related to a temperature parameter, and \( V_f \) and \( L_{[36]} \) have also been established. While in the first case, the correlation derives from the functional form of the model, in the case of \( L_{[36]} \), we cannot but observe that the correlation resembles that depicted in the main sequence of a Hertzsprung–Russell diagram for stars. Further investigations in this regard and others about the nature of the discrepancy in predicted and observed rotation curves still remain.

We envision future work in two directions immediately related to the investigation presented here. On the one hand, other concrete models of entropic forces can be explored for a better fit of rotation curves, e.g. more refined solutions of the Fokker-Planck equation, as well as other sensible probability distributions and entropy functions. On the other hand, the nature of the fundamental system from where the entropic force emerges and the possibility for it to provide answers for other open problems may also be studied. There are already examples in the literature of works along these lines and we hope that this investigation and future one’s can contribute to such efforts.

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[1] R. H. Sanders, The Dark Matter Problem: A Historical Perspective. Cambridge University Press, 2010.
[2] L. Baudis, “Dark matter detection,” Journal of Physics G: Nuclear and Particle Physics, vol. 43, no. 4, p. 044001, 2016.
[3] V. C. Rubin and W. K. Ford, Jr., “Rotation of the Andromeda Nebula from a Spectroscopic Survey of Emission Regions,” Astrophys. J., vol. 159, pp. 379–403, 1970.
[4] M. Persic, P. Salucci, and F. Stel, “The Universal rotation curve of spiral galaxies: 1. The Dark matter connection,” Mon. Not. Roy. Astron. Soc., vol. 281, p. 27, 1996.
[5] F. Lelli, S. S. McGaugh, and J. M. Schombert, “SPARC: Mass Models for 175 Disk Galaxies with Spitzer Photometry and Accurate Rotation Curves,” Astron. J., vol. 152, p. 157, 2016.
[6] J. J. Ancona-Flores, A. Hernández-Almada, and M. A. García-Aspeitia, “Testing noncommutativity-like model as a galactic density profile,” Galaxies, vol. 9, no. 1, 2021.
[7] T. Bernal, L. M. Fernández-Hernández, T. Matos, and M. A. Rodríguez-Meza, “Rotation curves of high-resolution LSB and SPARC galaxies with fuzzy and multistate (ultralight boson) scalar field dark matter,” Mon. Not. Roy. Astron. Soc., vol. 475, no. 2, pp. 1447–1468, 2018.
[8] A. Hernández-Almada and M. A. García-Aspeitia, “Multistate scalar field dark matter and its correlation with galactic properties,” Int. J. Mod. Phys. D, vol. 27, no. 03, p. 1850031, 2017.
[9] Q. Li and L. Modesto, “Galactic Rotation Curves in Conformal Scalar-Tensor Gravity,” Grav. Cosmol., vol. 26, no. 2, pp. 99–117, 2020.
[10] A. O. F. de Almeida, L. Amendola, and V. Niro, “Galaxy rotation curves in modified gravity models,” JCAP, vol. 08, p. 012, 2018.
[11] M. Milgrom, “A Modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis,” Astrophys. J., vol. 270, pp. 365–370, 1983.
[12] M. Milgrom, “A Modification of the Newtonian dynamics: Implications for galaxies,” Astrophys. J., vol. 270, pp. 371–383, 1983.
[13] M. Milgrom, “A modification of the Newtonian dynamics: implications for galaxy systems,” Astrophys. J., vol. 270, pp. 384–389, 1983.
[14] J. Hjorth and J. Madsen, “Statistical mechanics of galaxies,” Monthly Notices of the Royal Astronomical Society.
[15] E. P. Verlinde, “Emergent Gravity and the Dark Universe,” SciPost Phys., vol. 2, no. 3, p. 016, 2017.
[16] N. Roos, “Entropic forces in brownian motion,” American Journal of Physics, vol. 82, p. 1161–1166, Dec 2014.
[17] R. M. Neumann, “Entropic approach to Brownian movement,” American Journal of Physics, vol. 48, pp. 354–357, May 1990.
[18] T. Jacobson, “Thermodynamics of spacetime: The einstein equation of state,” Physical Review Letters, vol. 75, p. 1260–1263, Aug 1995.
[19] I. Díaz-Saldaña, J. C. López-Domínguez, and M. Sabido, “On Emergent Gravity, Black Hole Entropy and Galactic Rotation Curves,” Phys. Dark Univ., vol. 22, pp. 147–151, 2018.
[20] A. Fokker, “Die mittlere Energie rotierender elektrischer Dipole im Strahlungsfeld,” Ann. Phys., vol. 348, pp. 810–820, 1914.
[21] M. Planck, “Über einen Satz der statistischen Dynamik und seine Erweiterung in der Quantentheorie,” Sitzungsberichte der Preussischen Akademie der Wissenschaften zu Berlin, vol. 24, pp. 324–341, 1917.
[22] P. L. J. A. Cañizo, J. A. Carrillo and J. Rosado, “The Fokker–Planck equation for bosons in 2D: Well-posedness and asymptotic behavior,” Nonlinear Analysis, vol. 137, pp. 291–305, 2016.
[23] R. Q. J. Guiñez and A. Rueda, “Calculating Steady States For A Fokker-Planck equation,” Acta Math. Hungar., vol. 91, pp. 311–323, 2001.
[24] B. Hao, “Steady State Solutions of the Fokker-Planck Equations,” Commun. in Theor. Phys., vol. 8, pp. 153–166, 1987.
[25] J. F. Navarro, C. S. Frenk, and S. D. M. White, “The Structure of cold dark matter halos,” Astrophys. J., vol. 462, pp. 563–575, 1996.
[26] H. Katz, F. Lelli, S. S. McGaugh, A. D. Cintio, C. B. Brook, and J. M. Schombert, “Testing feedback-modified dark matter haloes with galaxy rotation curves: estimation of halo parameters and consistency with CDM scaling relations,” Mon. Not. R. Astron. Soc., vol. 466, pp. 1648–1668, 2016.
[27] M. Newville, T. Stensitzki, D. B. Allen, M. Rawlik, A. Ingargiola, and A. Nelson, “Lmfit: Non-linear least-square minimization and curve-fitting for python,” Astrophysics Source Code Library, pp. ascl–1606, 2016.
[28] P. Li, F. Lelli, S. McGaugh, and J. Schombert, “A comprehensive catalog of dark matter halo models for SPARC galaxies,” vol. 247, p. 31, mar 2020.
[29] S. S. McGaugh and J. M. Schombert, “COLOR-MASS-TO-LIGHT-RATIO RELATIONS FOR DISK GALAXIES,” vol. 148, p. 77, sep 2014.
[30] S. E. Meidt et al., “Reconstructing the stellar mass distributions of galaxies using S^4G IRAC 3.6 and 4.5 μm images: II. The conversion from light to mass,” Astrophys. J., vol. 788, p. 144, 2014.
[31] J. Schombert, S. McGaugh, and F. Lelli, “The mass-to-light ratios and the star formation histories of disc galaxies,” Monthly Notices of the Royal Astronomical Society, vol. 483, pp. 1496–1512, 12 2018.
[32] H. Akaike, “A new look at the statistical model identification,” IEEE Transactions on Automatic Control, vol. 19, no. 6, pp. 716–723, 1974.
[33] N. Komatsu and S. Kimura, “Evolution of the universe in entropic cosmologies via different formulations,” Phys. Rev. D, vol. 89, p. 123501, Jun 2014.
[34] S. Basilakos and J. Solà, “Entropic-force dark energy reconsidered,” Phys. Rev. D, vol. 90, p. 023008, Jul 2014.
[35] Y. Ling and J.-P. Wu, “A note on entropic force and brane cosmology,” Journal of Cosmology and Astroparticle Physics, vol. 2010, pp. 017–017, aug 2010.