The relativistic quantum channel of communication through field quanta

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(Dated: September 7, 2009)

Setups in which a system Alice emits field quanta which a system Bob receives are prototypical for wireless communication and have been extensively studied. In the most basic setup, Alice and Bob are modelled as Unruh-DeWitt detectors, i.e., as point-like two-level systems that interact through a scalar quantum field. Our aim is to construct and study the information-theoretic quantum channel. We calculate the classical channel capacity as a function of the spacetime separation and we confirm that the classical as well as the quantum channel capacity are strictly zero when the detectors’ spacetime separation. We then show, to all orders in perturbation theory, that causality is manifest in the channel. In Sec. III we provide a perturbative expansion for it in terms of the detectors’ spacetime separation. We show that this channel can be used to entangle Alice and Bob instantaneously. Alice and Bob are shown to extract this entanglement from the vacuum through a Casimir-Polder effect.

The setup in which two quantum systems, Alice and Bob, communicate using bosonic field quanta can be viewed as a prototype for wireless communication. Numerous aspects of this general setup have been studied in the literature, see e.g. [1]. Here, we focus on the most basic case, where Alice and Bob are modelled as Unruh-DeWitt detectors, i.e., as point-like two-level quantum systems that interact through a scalar quantum field. Our aim is to construct and study the information-theoretic quantum channel, $\xi$, i.e., the completely positive trace preserving map between the input density matrix $\rho$, in which Alice prepares her detector for the emission, and the output density matrix $\rho' = \xi(\rho)$ of Bob’s detector at a later time. This model captures the communicating of individual q-bits and allows us to study how communication and entanglement are impacted by both relativity and by the unavoidable noise that is due to the quantum fluctuations of the field.

Concretely, we construct the quantum channel and provide a perturbative expansion for it in terms of Feynman-like diagrams. We also calculate the classical channel capacity of the quantum channel as a function of the detectors’ spacetime separation. We then show, to all orders in perturbation theory, that both the classical and the orders channel capacities are strictly zero when Bob and Alice are spacelike separated. The impossibility of superluminal signalling has of course been discussed before, see e.g. [2]. What is new here is that we prove the impossibility of superluminal signalling information-theoretically by constructing and studying the quantum channel. We will then discuss how Alice and Bob can use the quantum channel to extract entanglement from the vacuum. It has been known that Alice and Bob when coupled to a quantum field can have non-trivial entanglement dynamics, see e.g. [3]. It is also known that, due to the entanglement of the vacuum [4, 5], or the exchange of virtual photons [6], two detectors can become entangled even at spacelike separations, and the speed with which this can happen has been discussed. Here, we will show that Alice and Bob can naturally and instantaneously become entangled through the Casimir-Polder effect.

To begin, let us denote the overall Hilbert space by $\mathcal{H} = \mathcal{H}^{(1)} \otimes \mathcal{H}^{(2)} \otimes \mathcal{H}^{(3)}$, where the first two Hilbert spaces belong to the detectors of Alice and Bob respectively and where the third Hilbert space is that of the field. Whenever necessary to avoid ambiguity we will denote operators $\mathcal{O}$ or states $|\psi\rangle$ which live in the Hilbert space $\mathcal{H}^{(j)}$ by a superscript $(j)$, for example, $\mathcal{O}^{(j)}$ and $|\psi^{(j)}\rangle$ with $j \in \{1,2,3\}$. Also, when such operators occur tensored with identity operators, such as $I^{(1)} \otimes I^{(2)} \otimes \mathcal{O}^{(3)}$, we will often abbreviate this as, for example, $\mathcal{O}^{(3)}$.

The Hamiltonian of the system is

\begin{align}
H &= H_F + H_D + H_{\text{int}} \\
H_F &= \int d^3x \left( \frac{1}{2} \pi^2(x) + \frac{1}{2} (\nabla \phi(x))^2 + \frac{1}{2} m^2 \phi^2(x) \right) \\
H_D &= \sum_{j=1}^{2} E_j |\epsilon^{(j)}\rangle \langle \epsilon^{(j)}| + E_g |\phi^{(j)}\rangle \langle \phi^{(j)}| \\
H_{\text{int}} &= \sum_{j=1}^{2} \alpha_j \eta \left( |\epsilon^{(j)}\rangle \langle \phi^{(j)}| + |\phi^{(j)}\rangle \langle \epsilon^{(j)}| \right) \phi(x_j)
\end{align}

where $H_F$ is the Hamiltonian of a free field, $H_D$ is the Hamiltonian of the two detectors, $H_{\text{int}}$ is the interaction Hamiltonian between the field and the detectors, $\alpha_j$ is the coupling constant of the $j$th detector ($j \in \{1,2\}$), $\phi(x_j)$ is the field at the point of the $j$th detector, and $m^{(j)} := \left( |\epsilon^{(j)}\rangle \langle \phi^{(j)}| + |\phi^{(j)}\rangle \langle \epsilon^{(j)}| \right)$ is the monopole matrix of the $j$th detector. The function $\eta(t)$ will be used to describe the continuous switching on and off of the detectors within some finite time interval. The use of suitably smooth switching functions allows one to avoid certain divergences associated with hard on and off switches, [2]. For simplicity we will always choose the same switching function $\eta(t)$ for both detectors. We note that the type of interaction term between the detector and the field that we use in Eq(1) has been extensively studied in the field of quantum field theory in curved space [8].

The paper is organized as follows. In Sec. II we show that causality is manifest in the channel. In Sec. III we
study the properties of the channel and derive a Kraus representation, and in Sec. IV we compute explicitly the classical channel capacity of the channel. In Sec. V we present a perturbative expansion of the channel. In Sec. VI we show that the channel can extract entanglement from the vacuum and in Sec. VII we compare our channel with similar models which were analyzed in the quantum optics framework. In the last section we propose extensions. We work with the natural units $\hbar = c = 1$.

II. CAUSALITY

The so-called Fermi problem arises in any system that is analogous to two atoms communicating via the electromagnetic field, and it has been studied extensively, see e.g. [9]. Consider, in the vacuum, the probability, $P_{\text{Fermi}}$, that a photon is emitted by atom 1 followed by the absorption of a photon by atom 2. In our model, it is the probability if starting with the state $|e^{(1)}\rangle|g^{(2)}\rangle|0\rangle$ to end in the state $|g^{(1)}\rangle|e^{(2)}\rangle|0\rangle$. Using the perturbative expansion of the evolution operator $U(t_f,t_i)$ in the interaction picture, one obtains the transition probability

$$P_{\text{Fermi}} = \left|\langle e^{(1)}|g^{(2)}\rangle|0\rangle U(t_f,t_i)|g^{(1)}\rangle|e^{(2)}\rangle|0\rangle\right|^2 = \left|\alpha_1 \alpha_2 \int_{t_i}^{t_f} dt_1 \int_{t_i}^{t_f} dt_2 n(t_1)n(t_2) \times e^{i\Delta E(t_2-t_1)} D_F(x_1(t_1), x_2(t_2))\right|^2 + O(\alpha^6)$$

(2)

where $D_F(x-y) := \langle 0|T\phi(x)\phi(y)|0\rangle$ is the Feynman propagator and where we defined $\Delta E := E_e - E_g$. By choosing the separation between the two detectors $|x_1 - x_2|$ and a time interval $t_f - t_i$ in which both detectors are on we can choose the spacetime windows for emission and absorption to be time-like or spacelike (or mixed) relative to another. The Fermi problem is the fact that this probability amplitude, from Eq. (2), is non-vanishing even in the case of spacelike separation. Technically, this is due to the non-vanishing tail of the Feynman propagator outside the lightcone. Hegerfeld and Feynman showed that in fact no Feynman propagator can identically vanish outside the lightcone, [10] [11].

This reinforces the need to clarify the reason for the non-vanishing of the Fermi probability in the spacelike separated case. As was pointed out in [9], the key to resolving the puzzle is to take into account that measurements on the detectors are local measurements. Namely, Bob performs a measurement only of his detector 2, he does not measure Alice’s detector, nor does he measure the field. This means that Fermi’s probability amplitude is the amplitude for just one of several processes that Bob cannot distinguish. What should actually vanish for spacelike separations is the sum of the probability amplitudes for all processes that depend on the state of Alice.

Here, our first aim is to make this argument explicit within the information-theoretic framework of quantum channels. To this end, we notice that Bob’s ignorance of Alice’ and the field’s state at the late time $t_f$ means that at $t_f$ both the state of Alice’s detector and the state of the field are to be traced over. These traces perform the sum over the probability amplitudes for processes that Bob cannot distinguish. We therefore naturally arrive at the description of a quantum channel $\xi : \rho^{(1)} \mapsto \xi(\rho^{(1)}) = \rho^{(2')}$. Here, the input is the initial density matrix $\rho^{(1)}$ of Alice at $t_i$ and the output of the channel is Bob’s density matrix $\rho^{(2')}$ at $t_f$.

We assume that the system starts in the state $\rho(t_i) = \rho^{(1)}\rho^{(2)}\rho^{(3)}$, where the initial state of Alice’ detector, $\rho^{(1)}$, is arbitrary, the initial state of the Bob’s detector, $\rho^{(2)}$, is the ground state and the initial state of the field, $\rho^{(3)}$, is the vacuum. The full density matrix evolves according to $\rho(t_f) = U(t_f,t_i)\rho(t_i)U^\dagger(t_f,t_i)$, where in the interaction picture $U(t_f,t_i) = Te^{-i\int_{t_i}^{t_f} dt' H_{\text{int}}(t')}$. As always, the time evolution can be formulated in terms of an infinite series of commutators [12]:

$$\rho(t_f) = \rho^{(1)}\rho^{(2)}\rho^{(3)} + \sum_{j=1}^{\infty} \left( \prod_{i=j}^{j} \left[ i \int_{t_i}^{t_{i-1}} dt_i \ldots \int_{t_i}^{t_1} dt_j \times \left[ \ldots \left[ \left[ \rho^{(1)}\rho^{(2)}\rho^{(3)},H_{\text{int}}(t_n) \right], \ldots, H_{\text{int}}(t_1) \right] \right] \right) \right).$$

(3)

Then, the trace over detector 1 and the field, which we will denote $\text{Tr}_{(1,3)}$, gives the final state $\rho^{(2)}(t_f) = \xi(\rho^{(1)})$ of Bob’s detector:

$$\xi(\rho^{(1)}) = \rho^{(2)} + \sum_{j=1}^{\infty} \left( \prod_{i=j}^{j} \left[ i \int_{t_i}^{t_{i-1}} dt_i \ldots \int_{t_i}^{t_1} dt_j \times \text{Tr}_{(1,3)} \left[ \ldots \left[ \rho^{(1)}\rho^{(2)}\rho^{(3)},H_{\text{int}}(t_n) \right], \ldots, H_{\text{int}}(t_1) \right] \right) \right).$$

(4)

To prove causality from this starting point, we will use the following simple lemmas:

I) Traces are cyclic and $\text{Tr}([A,B]) = 0$.

II) $([A^{(1)}B^{(2)}], [D^{(1)}E^{(2)}]) =$ \left\{ \left[ A^{(1)}, D^{(1)} \right] \left[ B^{(2)}, E^{(2)} \right] + \left[ D^{(1)}, A^{(1)} \right] \left[ B^{(2)}, E^{(2)} \right] \right\} C^{(3)}.

III) $\exists \left\{ R_k, S_k, T_k \right\}$ such that $\left[ \ldots \left[ [A^{(1)}B^{(2)}], C^{(3)} \right], \ldots, F^{(1)} \} G^{(3)} = \sum_k R_k S_k T_k$.

Now in Eq. (4), the terms that have a dependence on the input $\rho^{(1)}$ must have at least one $m^{(1)}\phi(x_1)$ which multiplies $\rho^{(1)}$ since otherwise we simply have $\text{Tr}(\rho^{(1)}) = 1$. In addition, since the trace of commutators vanishes (I), the non-vanishing terms which have a dependence on $\rho^{(1)}$
need to be interacting with at least one \( m^{(2)} \phi(x_2) \), such that all the terms dependent on \( \rho^{(1)} \) will be of the form

\[
f_n(\rho^{(1)}) = \text{Tr}_{(1,3)} \left( [[[\rho^{(1)}(\rho^{(2)} \rho^{(3)}), m^{(j)} \phi(x_j)], ...], m^{(r)} \phi(x_r)] \right)
\]

where at least one of the indices \( \{ j...r \} \) is equal to 1 and at least one of the indices is equal to 2, and \( n \) is the number of commutators \( (n \geq 2) \). Note that the time dependence is implicit in this formulation, each \( \phi(x) \) is integrated over time such that the time difference between two \( \phi(x) \) is at most \( t_f - t_i \). If the last index in Eq. (5) is 1, using (III) for everything before the last commutator, and (II) to expand the last commutator, \( f_n(\rho^{(1)}) \) would simplify to:

\[
f_n(\rho^{(1)}) = \sum_k \text{Tr}_{(1,3)} \left( [R_k^{(1)} S_k^{(2)} T_k^{(3)}, m^{(1)} \phi(x_1)] \right)
= \sum_k S_k^{(2)} \left\{ \text{Tr}_{(3)} \left( T_k^{(3)} \phi(x_1) \right) \text{Tr}_{(1)} \left( [R_k^{(1)}, m^{(1)}] \right) + \text{Tr}_{(3)} \left( [T_k^{(3)}, \phi(x_1)] \right) \text{Tr}_{(1)} \left( m^{(1)} R_k^{(1)} \right) \right\}
= 0.
\]

Thus the non-vanishing contributions of \( f_n(\rho^{(1)}) \) must come from commutators for which the very last index is 2. Now, let us consider the rightmost occurrence of index 1 and let us apply (III) to the commutators to the left of it:

\[
f_n(\rho^{(1)}) = \sum_k \text{Tr}_{(1,3)} \left( [[[R_k^{(1)} S_k^{(2)} T_k^{(3)}, m^{(1)} \phi(x_1)], m^{(2)} \phi(x_2)],...], m^{(2)} \phi(x_2)] \right).
\]

We can expand the most inner commutators with (II) to obtain:

\[
[R_k^{(1)} S_k^{(2)} T_k^{(3)}, m^{(1)} \phi(x_1)] = [R_k^{(1)}, m^{(1)}] \left( S_k^{(2)} T_k^{(3)} \phi(x_1) \right) + m^{(1)} R_k^{(1)} \left( S_k^{(2)} [T_k^{(3)}, \phi(x_1)] \right).
\]

Notice that when the first term is back in Eq. (6), it forms an expression of the form:

\[
f_n(\rho^{(1)}) = \sum_k \text{Tr}_{(1,3)} \left( [[[R_k^{(1)} S_k^{(2)} T_k^{(3)}, \phi(x_1)], m^{(2)} \phi(x_2)],...], m^{(2)} \phi(x_2)] \right)
\]

which implies that after the tracing out of detector 1 this term is always absent. Notice also that when the second term is back in Eq. (6), it gives an expression of the form:

\[
f_n(\rho^{(1)}) = \sum_k \text{Tr}_{(1,3)} \left( [m^{(1)} R_k^{(1)} \times [[[S_k^{(2)} T_k^{(3)} \phi(x_1)], m^{(2)} \phi(x_2)]...], m^{(2)} \phi(x_2)] \right).
\]

Therefore, the term \( [T_k^{(3)}, \phi(x_1)] \) will be multiplied on each side by some powers of \( \phi(x_2) \), so there exists a set of operators \( V_{k,i,j}^{(2)} \) such that:

\[
f_n(\rho^{(1)}) = \sum_{k,i,j} \left\{ V_{k,i,j}^{(2)} \text{Tr}_{(1)} \left( [m^{(1)} R_k^{(1)} \times \text{Tr}_{(3)} \left( \phi^i(x_2) [T_k^{(3)}, \phi(x_1)] \phi^j(x_2) \right) \right) \right\}.
\]

Using cyclicity of the trace (I), this expression can be simplified to:

\[
f_n(\rho^{(1)}) = \sum_{k,i,j} \left\{ V_{k,i,j}^{(2)} \text{Tr}_{(1)} \left( [m^{(1)} R_k^{(1)} \times \text{Tr}_{(3)} \left( [T_k^{(3)}, \phi(x_1)] \phi^{i+j}(x_2) \right) \right) \right\}.
\]

Note that all the information about \( \rho^{(1)} \) is contained in the operators \( R_k^{(1)} \). Causality in the channel therefore follows directly from microcausality in quantum field theory [13], namely from the fact that \( \langle \phi(x), \phi(y) | (x-y)^2 > 0 = 0 \) (where \( (x-y)^2 = -(x^0 - y^0)^2 + (\vec{x} - \vec{y})^2 \)). If the two detectors are spacelike separated during the entire interaction, \( \rho^{(2)}(t_f) \) does not depend on the state \( \rho^{(1)} \), i.e., Bob’s detector 2 is not sensitive to the state in which Alice prepared detector 1.

### III. Noise Structure of the Channel

Let us now calculate the precise quantum channel for both time-like and spacelike separations. Since the evolution of the full system is unitary, our channel is necessarily described by a CPTP map [14]. Then, as we will show, assuming detector 2 starts in the ground state, \( \rho^{(2)} = |g^{(2)}\rangle \langle g^{(2)}| \), we can write the channel map in the following way, in the basis \( |e^{(2)}, g^{(2)}\rangle \),

\[
\xi \left( \begin{array}{c} e \\ e^{\beta} \\ g \\ g^{\beta} \end{array} \right) = \left( \begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right) + \theta \left( \begin{array}{cccc} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{array} \right)
\]

where we use \( \theta + \beta = 1. \) All terms are space-time scalars. Note that \( A, B, C \) and \( D \) are causal terms in the sense that they depend on the input density matrix \( \rho^{(1)} \). In contrast, \( P_e \) represents noise in the quantum channel.
since its presence does not depend on the input $\rho^{(1)}$. To prove Eq. (11), we will use the following properties which are easy to verify ($k \in \mathbb{Z}$):

i) $Tr(\rho^{(3)} \phi^{2k+1}) = 0$.

ii) $m^{2k+1}$ has no diagonal elements, and therefore $Tr(m^{2k+1}M_d) = 0$ where $M_d$ is any diagonal matrix.

iii) $m^{2k}$ has only diagonal elements, and therefore $Tr(m^{2k}M_{nd}) = 0$ where $M_{nd}$ is any matrix with no diagonal elements.

In a series expansion of the non-causal terms, each order has the form $\rho^{(2)}m^{(2)k}Tr(\rho^{(3)}\phi(x_2)^k)$. Thus, because of (i) the non-vanishing terms will be proportional to $\rho^{(2)}m^{(2)2k}$, and because of (iii) we know that these are diagonal. Therefore, because we have trace preservation and because detector 2 starts initially in the ground state, there cannot be a more general expression for the non-causal terms of Eq. (11). For the causal terms, each order in a series expansion have the form $\rho^{(2)}m^{(2)k}Tr(m^{(1)}\rho^{(1)})Tr(\rho^{(3)}\phi(x_1)^j\phi(x_2)^k)$. Now consider the case where the input density matrix $\rho^{(1)}$ is diagonal, then because of (ii) the non-vanishing terms will have $j$ even. Using (i), this also means we need $k$ to be even, hence $\rho^{(2)}m^{(2)k}$ is diagonal following (iii). A similar argument can show that an input density matrix with no diagonal elements cannot have diagonal elements at the output. Finally, trace preservation, hermiticity and linearity of the channel are sufficient properties to prove the validity of Eq. (11).

From this analysis, we can find a Kraus representation by imposing $\xi(([\alpha, \beta]^T)) = \sum_{k=1}^4 \alpha_k^* \beta_k E_k$ and $\sum_{k=1}^4 E_k^\dagger E_k = I$ where we use $E_k = \begin{pmatrix} a_{1k} & a_{2k} \\ a_{3k} & a_{4k} \end{pmatrix}$. Solving this nonlinear system of equations is relatively straightforward as we have more unknowns than equations, so for simplicity we try to have as many zero matrix elements as possible. We arrive at a simple representation, in the basis $|e(2)\rangle\langle e(1)|, |e(2)\rangle\langle g(1)|, |g(2)\rangle\langle e(1)|, |g(2)\rangle\langle g(1)|$:

$$E_1 = \begin{pmatrix} \sqrt{1-\frac{P_e+B}{P_e-B}} & 0 \\ 0 & \sqrt{1-\frac{P_e-B}{P_e+B}} \end{pmatrix}$$

$$E_2 = \begin{pmatrix} \sqrt{P_e+A} - \frac{|C|^2}{\sqrt{1-\frac{P_e-B}{P_e+A}}} & 0 \\ 0 & 0 \end{pmatrix}$$

$$E_3 = \begin{pmatrix} 0 & \sqrt{1-\frac{P_e-B}{P_e+A}} \\ \sqrt{1-\frac{P_e-B}{P_e+A}} & 0 \end{pmatrix}$$

$$E_4 = \begin{pmatrix} 0 & \sqrt{1-\frac{P_e+B}{P_e-A}} \\ 0 & 0 \end{pmatrix}.$$  \hspace{1cm} (12)

There exists no representation with a smaller number of Kraus operator since we verified that the rank of the matrix $\langle I^{(Q)} \otimes e(x) | \beta^{(Q,1)} \rangle \langle \beta^{(Q,1)} |$ is equal to 4.

IV. CHANNEL CAPACITY

The classical channel capacity $C$ (often called the product state capacity) of a quantum channel $\xi$ is equal to [14]

$$C(\xi) = \max_{p_j, p_2} \left[ S \left( \xi \left( \sum_j p_j \rho_j \right) \right) - \sum_j p_j S(\xi(\rho_j)) \right]$$  \hspace{1cm} (13)

where $S$ is the Von Neumann entropy $S(\rho) := -Tr(\rho \ln \rho)$. This quantity corresponds to the amount of reliable classical bit we can send through the quantum channel per use of the channel.

Let us first maximize over the input state to obtain: $\rho_1^{(1)} = |e(1)\rangle\langle e(1)|$ and $\rho_2^{(1)} = |g(1)\rangle\langle g(1)|$. The maximization over the probability $p_1$ gives:

$$p_1 = \frac{2^{w} - P_e - B}{A - B}$$

$$w - \ln(1 - 2^w) = \frac{H(P_e + B) - H(P_e + A)}{A - B}$$ \hspace{1cm} (14)

where we use the binary entropy $H(p) := -p \ln p - (1-p) \ln (1-p)$. We finally arrive at the classical channel capacity $C$, which we divide by $t_f - t_i$ to get $R$, namely the amount of bits/time which can be sent reliably:

$$R = \frac{1}{t_f - t_i} \left[ H(P_e + p_1 A + (1-p_1) B) - p_1 H(P_e + A) - (1-p_1) H(P_e + B) \right].$$  \hspace{1cm} (15)

As expected the classical channel capacity is zero for spacelike interactions since in that case $A = B = 0$.

We remark that the channel capacity as a function of the spacetime separation is a non-analytic function since it identically vanishes outside the lightcone but is a non-trivial function inside. Any analytic function that vanishes on a finite interval would of course vanish everywhere. The occurrence of this non-analyticity may seem surprising since our quantum channel is mapping between finite dimensional spaces and therefore appears to be a matter of mere linear algebra. The non-analyticity arises, of course, from the non-analyticity of the commutator $[\phi(x), \phi(y)]$ which originates in the fact that, in the full system, the field lives in an infinite dimensional Hilbert space. Conversely, if ultraviolet and infrared cutoffs are imposed on the quantum field theory so that its Hilbert space $\mathcal{H}^{(3)}$ becomes finite dimensional, this would reduce these calculations to linear algebra and will therefore yield some non-vanishing capacity outside the lightcone. Interestingly, this does not mean that the presence of a natural UV cutoff in nature would imply a violation of causality. This is because an ultraviolet cutoff implies that there is in effect a smallest resolvable length, which in turn means that the very boundaries of the lightcone become unsharp. The capacity should decay to essentially zero outside the lightcone at a distance from the
FIG. 1: Feynman diagrams involved in the quantum channel. $D_j$ stands for detector $j$, $j \in \{1, 2\}$.

lightcone that is about the size of the unsharpness scale induced by the UV cutoff. Any candidate quantum gravity theory has to reduce to quantum field theory in a limit and most come with a natural UV cutoff, see e.g., [10]. It should be interesting to check causality for such theories by calculating the channel capacity at distances close to the light cone.

Let us now also consider the quantum channel capacity, [17] [18], i.e., the amount of quantum information which can reliably be sent through the channel

$$Q(\xi) = \lim_{n \to \infty} \frac{I_c(\xi^n)}{n}$$

$$I_c(\xi) = \max_\rho \left[ S(\xi(\rho)) - S(\xi^C(\rho)) \right]$$

where $\xi^C$ is the complementary channel. For space-like separated detections, the quantum channel capacity is zero since the channel is then anti-degradable: there exists a channel $\Gamma$ such that $\Gamma(\xi^C(\rho)) = \xi(\rho)$ [19]. This confirms that superluminal propagation of classical or quantum information is not possible. For time-like separated detectors, the quantum channel capacity is extremely hard to compute because the channel is not degradable (a degradable channel is such that $I_c(\xi^n) = n I_c(\xi)$). Indeed, a theorem in [15] states that any channel with input and output of dimension 2 and with Choi rank (minimum number of Kraus operators) bigger than 2 cannot be degradable. Since the channel we consider has Choi rank equal to 4, it cannot be degradable. We therefore leave open the question of finding an explicit expression for the quantum channel capacity of our quantum channel.

V. PERTURBATIVE EXPANSION OF THE CHANNEL

Using perturbation theory, we can find explicit expressions for the terms $P_c, A, B, C$ and $D$ in the weak coupling regime ($\alpha_j \ll 1$). To this end we use the first orders of the perturbative expansion of equation (4) along with equation (11), and for simplicity we assume that the field starts in the vacuum $|0\rangle$:

$$P_c(\Delta E) = \alpha^2 \int_{t_1}^{t_f} dt_1 \int_{t_1}^{t_f} dt_2 \eta_1(t_2) |0\rangle \phi(x_2(t_2)) \langle 0| \phi(x_2(t_1)) |0\rangle e^{-i \Delta E (t_1 - t_2)} + O(\alpha^4)$$

$$A(\Delta E) = 2(\alpha_1 \alpha_2)^2 \int_{t_1}^{t_f} dt_1 \int_{t_1}^{t_f} dt_2 \int_{t_2}^{t_f} dt_3 \int_{t_2}^{t_f} dt_4 \left\{ \eta_1(t_1) \eta_2(t_3) \eta_3(t_4) \eta_4(t_4) \cos(\Delta E (t_1 - t_2)) \right\}$$

$$+ (t_1 \leftrightarrow t_2) + (t_2 \leftrightarrow t_3) + \sin(t_1) \eta_2(t_3) \eta_3(t_4) \sin(\Delta E (t_2 - t_3))$$

$$+ \int_{t_1}^{t_f} dt_1 \int_{t_1}^{t_f} dt_2 \int_{t_2}^{t_f} dt_3 \int_{t_2}^{t_f} dt_4 \left\{ \eta_1(t_1) \eta_2(t_2) \eta_3(t_3) \eta_4(t_4) \sin(\Delta E (t_1 - t_2)) \right\}$$

$$+ O(\alpha^6)$$

$$B(\Delta E) = A(-\Delta E)$$

$$C(\Delta E) = \alpha \alpha_2 \int_{t_1}^{t_f} dt_1 \int_{t_1}^{t_f} dt_2 \eta_1(t_2) \eta_2(t_2) e^{i \Delta E (t_1 - t_2)} |\phi(x_1(t_2)), \phi(x_2(t_2))\rangle + O(\alpha^4)$$

$$D(\Delta E) = -\alpha \alpha_2 \int_{t_1}^{t_f} dt_1 \int_{t_1}^{t_f} dt_2 \eta_1(t_2) \eta_2(t_2) e^{i \Delta E (t_1 + t_2)} |\phi(x_1(t_2)), \phi(x_2(t_2))\rangle + O(\alpha^4).$$

We can picture the perturbative expansion with Feynman diagrams [13], see Fig. 1 (the expressions of Eq. [17]-[21] are represented by the first diagram of their respective se-
A connection between the two detectors represents a photon emission/absorption process and a connection between a detector and itself (a loop) represents a quantum field fluctuation. The terms \{A, B\} have an even number of connections between the detectors while the terms \{C, D\} have an odd number of connections. The only distinction between \(A\) and \(B\) is the input state at detector 1: the excited state for \(A\) and the ground state for \(B\). Thus, the causal connections of \(A\) are resonant while the causal connections of \(B\) are not resonant. A similar argument is also true for \(C\) and \(D\), the connections of \(C\) are resonant while the connections of \(D\) are not resonant.

Using Eq. (17)-(19) along with Eq. (15), we can numerically evaluate the classical channel capacity as a function of time for inertial detectors in Minkowsky spacetime, for example, for a massless field, see Fig. (2). The arrow points to the threshold when the spacetime windows in which the detectors are switched on start to become partially time-like.

VI. CREATION OF ENTANGLEMENT IN THE CHANNEL

Two detectors that interact with a quantum field have access to a renewable source of entanglement. It has been shown in \[4,5\] that detectors coupled to a massless quantum field can become entangled even when spacelike separated. The entanglement was found to appear to propagate in quantum fields at a speed which depends on the switching functions \(\eta(t)\) and on the energy gap \(\Delta E\). The speed of propagation was found to be larger than the speed of light for suitable \(\eta(t)\) and \(\Delta E\). A related analysis was also conducted in an expanding spacetime \[20\]. In this section we follow up on these results by showing that the two detectors will in fact automatically and instantaneously become entangled, namely through what is essentially the Casimir effect. We find that the Casimir effect entangles significantly which is encouraging for experimental verification.

In this section, we switch from the interaction picture to the Schrödinger picture and we assume detectors at rest in Minkowsky spacetime separated by a fixed distance \(\bar{L} := \bar{x}_2 - \bar{x}_1\). This allows us to use perturbation theory for time-independent perturbations \[21\]. We obtain the new ground state

\[
|e_{g,\text{new}}\rangle = |e_g\rangle + \sum_{k \neq g} |e_k\rangle \frac{\langle e_k | H_{\text{int}} | e_g\rangle}{E_g - E_k} \\
+ \sum_{k \neq g} \sum_{l \neq g} |e_k\rangle \frac{\langle e_l | H_{\text{int}} | e_g\rangle}{(E_g - E_l)(E_g - E_k)} \\
- \frac{|e_g\rangle}{2} \sum_{k \neq g} \left| \langle e_k | H_{\text{int}} | e_g\rangle \right|^2 + O(\alpha^3) \tag{22}
\]

where \(|e_k\rangle\) are the eigenstates of the free Hamiltonian and we used the fact that in our case \(\langle e_g | H_{\text{int}} | e_g\rangle = 0\). To regularize the ultraviolet, we give a spatial extent to our detectors:

\[
H_{\text{int}} = \sum_{j=1}^{2} \alpha_j \left( |e^{(j)}\rangle \langle g^{(j)}| + |g^{(j)}\rangle \langle e^{(j)}| \right) \int d^3x f_j(\bar{x}) \phi(\bar{x}). \tag{23}
\]

Here, the functions \(f_j(\bar{x})\) describe the smearing of the detectors, and for simplicity we choose \(f_1(\bar{x}) = f_2(\bar{x} - \bar{L})\). Our initial ground state is \(|e_g\rangle = |g^{(1)}\rangle |g^{(2)}\rangle |0\rangle\), and using Eq. (22) the new ground state \(|e_{g,\text{new}}\rangle\) is

\[
|e_{g,\text{new}}\rangle = \left[ |g^{(1)}\rangle |g^{(2)}\rangle \left( 1 - \frac{\alpha_1^2 + \alpha_2^2}{2} \right) S(\Delta E) \right. \\
- \alpha_1 |e^{(1)}\rangle |g^{(2)}\rangle Q_1^{(3)}(\Delta E) \\
- \alpha_2 |g^{(1)}\rangle |e^{(2)}\rangle Q_2^{(3)}(\Delta E) \\
+ \alpha_1 \alpha_2 |e^{(1)}\rangle |e^{(2)}\rangle R(\Delta E, L) + \ldots \] \tag{24}
\]

where we use the definitions:

\[
Q_j^{(3)}(\Delta E) := \int \frac{d^3p}{(2\pi)^3} \int d^3x f_j(\bar{x}) e^{-i\bar{p} \cdot \bar{x}} a^\dagger_{p} \\
R(\Delta E, L) := \int \frac{d^3p}{(2\pi)^3} e^{i\bar{p} \cdot \bar{L}} \left| \int \frac{d^3x f_1(\bar{x}) e^{-i\bar{p} \cdot \bar{x}}}{2E_p + \Delta E} \right|^2 \tag{25}
\]

\[
S(\Delta E) := \int \frac{d^3p}{(2\pi)^3} \left| \int \frac{d^3x f_1(\bar{x}) e^{-i\bar{p} \cdot \bar{x}}}{2E_p + \Delta E} \right|^2. \tag{27}
\]

The resulting state is clearly entangled because it is a pure state which cannot be written in a tensor product form. Let us now ask whether this is indeed an entangled state from the point of view of the detectors. To see this

![Graph](image-url)
we need to trace out the field, leaving the remaining system in a mixed state \( \rho_{g,\text{new},d} := Tr_3(\langle e_{g,\text{new}}|e_{g,\text{new}}\rangle) \)

\[
\rho_{g,\text{new},d} = \begin{pmatrix}
0 & 0 & 0 & \alpha^2 R(\Delta E, L) \\
0 & \alpha^2 S(\Delta E) & \alpha^2 T(\Delta E, L) & 0 \\
\alpha^2 T(\Delta E, L) & 0 & \alpha^2 S(\Delta E) & 0 \\
\alpha^2 R(\Delta E, L) & 0 & 0 & 1-2\alpha^2 S(\Delta E)
\end{pmatrix} + O(\alpha^4)
\] (28)

where the matrix is written in the basis \( |e_1(1)e_2(2), e_1(1)g(2), e_1(2)g(2), g(1)g(2)\rangle \), we assumed for simplicity \( \alpha_1 = \alpha_2 = \alpha \) and we use the following definition:

\[
T(\Delta E, L) := \int \frac{d^3p}{(2\pi)^3} \frac{e^{i\vec{p}\cdot\vec{x}}}{2E_p(E_p + \Delta E)^2}.
\] (29)

To measure the entanglement of the mixed state, we use the negativity \([22,24]\), which is twice the absolute value of the sum of the negative eigenvalues of the partial transpose of the density matrix. We find the negativity \( N \) for the density matrix \( \rho_{g,\text{new},d} \):

\[
N(\Delta E, L) = 2\alpha^2 \max(|R(\Delta E, L)| - S(\Delta E), 0).
\] (30)

For simplicity, we analyse this expression when the smearing functions are gaussian

\[
f_1(\vec{x}) = e^{-\frac{|\vec{x} - \vec{x}_0|^2}{(2\pi)^{3/2} \Delta X^3}}
\] (31)

so the size of the detectors is about \( \Delta X \). Such smearing functions could be physically implemented by putting the detectors in a quantum harmonic potential. Even if gaussian smearing functions have a finite probability for the detectors to overlap, we are only looking at the regime where \( L\Delta E \to 0 \), \( L\Delta m \to 0 \) and \( \frac{\Delta X}{\Delta m} \to \infty \), and in this regime the overlap is insignificant. In fact, in this regime all the smearing functions have the same effect, namely to create an effective momentum cutoff. Thus our results would not change for detectors which are delocalized within a region of space which has compact support. If \( \Delta E \gg m \) like for the case of a massless field, we arrive at

\[
N \approx \frac{\alpha^2}{2\pi^2} \max\left(\frac{\pi}{2L\Delta E} - \ln\left(\frac{1}{\Delta E\Delta X}\right), 0\right).
\] (32)

Similarly if \( \Delta E \ll m \) we have

\[
N \approx \frac{\alpha^2}{2\pi^2} \max\left(\frac{\pi}{2L\Delta E} - \ln\left(\frac{1}{m\Delta X}\right), 0\right).
\] (33)

We therefore see that the ground state of the interacting theory is entangled from the point of view of the detectors if \( L < \frac{\pi}{2\Delta E \ln(1/\Delta E \Delta X)} \) when \( \Delta E \gg m \) and if \( L < \frac{\pi}{2\Delta E \ln(1/m\Delta X)} \) when \( \Delta E \ll m \).

To estimate how long it takes to extract entanglement from the vacuum, we use the adiabatic theorem. We assume the system starts in the ground state of the free theory, \( |e_g\rangle = |g(1)\rangle |g(2)\rangle |0\rangle \). Then, the interaction Hamiltonian is smoothly turned on using the switching function \( \eta(t) \). For the system to remain in the ground state, we need \( \eta(t) \) to increase slowly enough such that the perturbation is adiabatic. Following the validity condition for adiabatic behaviour \([23,24]\), we need

\[
\max_t \left| \langle e_k|\hat{H}(t)|e_g\rangle \right| \ll \min_t |E_g(t) - E_k(t)|
\] (34)

to hold for any energy level \( E_k \). A rigorous use of the adiabatic theorem requires normalized eigenstates, so let us put our system in a large box of volume \( V = L_{1R}^3 \). This procedure creates an infrared cut-off and normalizes the eigenstates of the free Hamiltonian. Hence, in our case, if we retain only the dominant order, the adiabatic condition translates to:

\[
\max_t |\eta(t)| \ll \left[ m^2 + 3\left(\frac{2\pi}{L_{1R}}\right)^2 \right]^{1/4}/\alpha
\times \left( \sqrt{m^2 + 3\left(\frac{2\pi}{L_{1R}}\right)^2 + \Delta E} \right)^2.
\] (35)

Thus, for a massive field, it is always possible to adiabatically turn on the interaction, and since the ground state of the interacting theory is entangled, there will be an instantaneous creation of entanglement. If the field is massless, there still is instantaneous creation of entanglement, for any finite size of box to which we confine our system. Therefore, while Alice and Bob cannot exchange classical or quantum information faster than the speed of light, their ability to extract entanglement by interacting with the vacuum is not bounded by any finite speed.

From Eq.\([25]\) we notice that in order to obtain the full amount of entanglement from the ground state, the system needs an interval of time of the order of \( \max_t |\eta(t)|^{-1} \). This entanglement could either be used in computations or swapped to other quantum systems for distillation. After the entanglement is used up, the detector - field interaction may be switched off and the system can be put back in the ground state of the free theory \( |e_g\rangle = |g(1)\rangle |g(2)\rangle |0\rangle \), e.g., by cooling. Thus, Alice and Bob can extract entanglement by interacting with the field in a cyclic and therefore sustainable way. However, we also see that the extraction of a large amount of entanglement from the vacuum by this method will cost a large amount of time. Interestingly, the amount of time needed is determined in a similar way to how the speed of adiabatic quantum computation is determined. Recall that the closeness of eigenvalues determines how fast specific states such as the ground state can be reached via an adiabatic approach or through cooling, see e.g.\([25]\). The reason why the finite rate of entanglement extraction
does not lead to a finite speed of entanglement “propagation”, is that there is no threshold: negativity, indicating entanglement between the detectors, arises immediately as their interaction with the field is switched on.

We will now show that the underlying reason why Alice and Bob are entangled when in the ground state of the interacting theory is that this ground state is a state in which Alice and Bob are attracted to each other through the exchange of virtual photons. This exchange interaction is in effect the scalar field version of the Casimir-Polder force \[26\], which is known to be the relativistic generalization of the van der Waals force between atoms or molecules.

Let us now derive the Casimir force between Alice and Bob for point-like detectors \[ f_1(x) = \delta(x - x_1) \]. To this end, we calculate the energy of the new ground state with time independent perturbation theory \[21\], and renormalize using \[ \delta \tilde{E}_g(L) := \delta E_g(L) - \lim_{L \to \infty} \delta E_g(L) \]. The result of the calculation is:

\[
\delta \tilde{E}_g(L, \Delta E) = \sum_{n \neq g} \sum_{k \neq g} \sum_{l \neq g} \frac{\langle \epsilon_g | H_{int} | \epsilon_n \rangle \langle \epsilon_n | H_{int} | \epsilon_k \rangle \langle \epsilon_k | H_{int} | \epsilon_l \rangle \langle \epsilon_l | H_{int} | \epsilon_g \rangle}{(E_g - E_n)(E_g - E_k)(E_g - E_l)} + O(\alpha^6)
\]

\[
= -2\alpha^4 \left[ \frac{1}{\Delta E} \int \frac{d^3p}{(2\pi)^3} e^{-i\vec{p} \cdot \vec{x}} \left( \frac{1}{E_{p1} + \Delta E} + \frac{1}{E_{p2} + \Delta E} \right)^2 \right] + O(\alpha^6).
\]

The ground state energy is lowered because of the interaction, causing Alice and Bob to attract each other with the Casimir force \[ F_C = -\frac{\delta \tilde{E}_g(L)}{\Delta E} \]. For a massless field, \( \delta \tilde{E}(L, \Delta E) \sim \frac{-\alpha^4}{L^3 \Delta E^2} \) in the limit \( L \Delta E \to \infty \). For comparison, the electromagnetic Casimir-Polder energy \[26\], scales as \( \sim -L^{-7} \) for large distances.

Note that so far we did not need to specify the detectors’ mass since we assumed their position to be fixed. Considering now the dynamics of Alice and Bob due to the Casimir force, it is clear that if their mass is small enough, their acceleration could be strong enough to become non-adiabatic. In this case, their motion would cause the system to evolve non-adiabatically and therefore to become excited. The Casimir force would therefore no longer be simply the derivative of the ground state energy, \( -\frac{\delta \tilde{E}_g(L)}{\partial L} \), because the state of the system would no longer be the ground state. To stay in the regime where the Casimir force is the derivative of the Casimir energy the detectors can move toward each other at a maximum speed \( v \) which needs to be small enough such that the perturbation is adiabatic. Thus, when our system is in a large box, the validity condition for adiabatic behaviour of Eq. (34) translates to \( v \ll \frac{\Delta E^{3/2}}{\alpha^{3/2}} \).
channel Glauber detectors instead of Unruh-DeWitt detectors. To this end, we use Eq.\cite{37} in Eq.\cite{4}. We see that then terms that are dependent on $\rho^{(1)}$ are no longer necessarily proportional to $|\phi(x_1), \phi(x_2)|$. Using the perturbative expansion of the channel in Sec.\ref{sec:V} shows that non-causal terms appear already in the $O(\alpha^2)$ order:

$$\xi(\rho^{(1)}) = |g^{(2)}(y)|g^{(2)}(x)| - \alpha_1 \alpha_2 \int_{t_1}^{t_1+\Delta t} \int_{t_2}^{t_2+\Delta t} d\eta(t_1) d\eta(t_2)
\times e^{i\Delta E(t_1-t_2)}|e^{(2)}(y)|g^{(2)}(y)|e^{(1)}(x)|g^{(1)}(x)|
\times D(x_2(t_2) - x_1(t_1)) + c.c. + O(\alpha^4).$$

(38)

Here, $D(x-y) := \langle 0|\phi^+(x)\phi^-(y)|0\rangle$. Since the correlator $D(x-y)$ is not vanishing outside the lightcone, detector 2 would indeed be influenced by detector 1 as soon as the interaction is turned on even if the detectors are spacelike separated. It may be interesting to see if similar effectively non-local detectors, such as the one in \cite{31}, behaves causally or non-causally under our channel picture.

\section{VIII. OUTLOOK}

The type of quantum channel that we here considered could be useful, for example, in the context of implementations of quantum networks, where photons carry quantum information in between atoms that possess effectively two levels, see e.g. \cite{32}. But it should also be straightforward to generalize our study to detectors with any number of energy levels. The number and spacing of the energy levels of the detectors should translate into an effective alphabet size. This should also allow one to generalize the results of \cite{33}, where it was first shown how quantum noise imposes a natural bound to the capacity of an otherwise noiseless bosonic channel. The analysis of \cite{33} employed the time-energy uncertainty principle to describe the limit to the distinguishability of photons of energy difference $\Delta E$ in an observation time $\Delta t$. It should be interesting to re-analyze these results within the present information-theoretic framework of the quantum channel in which all effects of quantum noise are built in from the start.

It should also be interesting to generalize our model to yield a new approach to analyzing the setup of \cite{34}, where Alice and Bob are inertial observers which are exchanging modes of a quantum field, while Eve is accelerating and tries to intercept the message. It was shown there that, because of the Unruh effect, it is always possible for Alice and Bob to communicate privately. To show this, the approach to the Unruh effect using Bogoliubov transformations was used. Generalizing our setup, one may use Unruh-DeWitt detectors, which are known to allow a more flexible description of the Unruh effect. For example, Eve would not have to accelerate uniformly and could indeed take an arbitrary trajectory.

The channel which we studied here should also be generalizable to curved spacetimes to study, for example, the impact of spacetime expansion and horizons. Finally, let us recall that, in the presence of a suitable natural ultraviolet cutoff, the density of degrees of freedom in quantum fields is finite, see e.g. \cite{35}. It should be interesting to investigate how this finite density of degrees of freedom translates into a finite information carrying capacity of quantum fields, in the concrete sense of the capacity of quantum channels. Indeed, the quantum channel that we investigated here can be interpreted as describing one detector which imprints information in a quantum field, and a second detector reading out this information. The approach therefore allows one to ask questions such as, how write and read cycles can be optimized, how much information is left in the field after a cycle, or how much quantum or classical information can maximally be written into and retrieved from a quantum field in some finite region of spacetime.

\section{Acknowledgments}

The authors wish to thank Ralf Schützhold for valuable comments and suggestions. M.C. acknowledges support from the NSERC PGS program. A.K. acknowledges support from CFI, OIT, the Discovery and Canada Research Chair programs of NSERC and is grateful for the very kind hospitality at the University of Queensland during the early stages of this work.

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