Thermal Conductivity Temperature-Pressure Dependence of Rocks and Ceramics

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Abstract. We have proposed a low-parameter description of the thermal conductivity temperature-pressure dependence of natural and artificial composite materials describing the initial pressure region for the majority composites with great accuracy. It was found that the increase in pressure as usual leads to a decrease in the temperature dependence of the thermal conductivity. Moreover, if for some compounds the decrease for the power factor with pressure is linear (or quasi-linear) in the whole range of 0-400 MPa, then for others, a rapid change to 100 MPa takes place, after which the value of the power factor varies little.

1. Introduction

The results of laboratory studies of rocks under high pressure conditions in combination with high temperatures make it possible to ascertain the laws of thermodynamic and physicochemical processes occurring in the interior, and are used in a variety of both fundamental and applied problems of various fields of science.

Hydrostatic compression in the rock generates internal stresses of a complex nature, which cause a change in its volume and the elastic parameters of the lattice. In this case, in the rocks under the influence of internal stresses, longitudinal and transverse deformations develop. Experimental measurements of the effective thermal conductivity of dielectrics and rocks $\lambda(P,T)$ at high pressures and temperatures give fairly accurate information about the processes of propagation and scattering of thermal waves in complex, block and disordered solids, such as rocks (sandstone, granite, and others) and ceramics. It should be specially noted that such materials can contain chemical compounds with both crystalline and amorphous structure.

The current theory describing heat transfer processes in dielectrics shows that the temperature dependence of the thermal conductivity is a very sensitive parameter with respect to the crystal structure, compressibility and elastic constant. The heat transfer of ideal crystals is of a wave nature, described by the Eikine [1] and Debye models [2], and has temperature dependence $\lambda \sim T^{-1}$, while
the heat transfer of amorphous structures is of an activation nature, has temperature dependence \( \lambda \sim T^{0.5} \).

The experimental baric dependences of the thermal conductivity of semiconductors and rocks are, as a rule, nonlinear. This especially applies to the initial area of 0.1-100 MPa. At the same time, this (as well as the adjoining region) is of fundamental and applied interest.

2. Experiment
Measurements in the temperature range of 273-523 K and pressure from 0.1 to 400 MPa were carried out by an absolute steady-state method, the error of which did not exceed 3-4%. [3] The samples taken for the current analysis are decrypted in the correspondent references: marl [4], sandstone [5], granite [6] and polycrystalline CdGeAs₂ [7].

To verify and validate the method and measuring procedure, the thermal conductivity of standard materials (fused quartz, quartz ceramics) was also measured. The results turned out to be in good agreement (within 0.5%) with expected ones.

\[ f(P) = \frac{\lambda(T_0, P)}{\lambda(T_0, P_0)} \]

**Figure 1.** Dependences of \( f(P) \) (a) and \( n(P) \) (b) for: (1) marl [4], (2) sandstone [5], (3) granite [6] and (4) polycrystalline CdGeAs₂ [7].

3. Results and discussion
Applying the power law to both the variable \( T \) and the fixed temperature \( (T_0) \), and applying the dimensionless function \( f(P) = \frac{\lambda(T_0, P)}{\lambda(T_0, P_0)} \), we can proceed to dimensionless quantities:

\[ \lambda(T, P) = \lambda(T_0, P_0) \cdot f(P) \cdot \left( \frac{T}{T_0} \right)^{n(P)} \]

(1)
The power factor $n$ in equation (1) is an objective parameter that depends on the pressure and does not depend on the choice of temperature $T_0$.

To describe the function $f(P)$ throughout the investigated range we have taken $P_0^*=0$ and have found the following empirical law:

$$f(P) = \left(1 + \frac{P}{P_0^*}\right)^\beta.$$  

(2)

Here $\beta$ is a dimensionless constant, and $P_0^*$ is a constant having the dimensionality of pressure.

Being dimensionless, the function $f(P)$ is convenient for comparative analysis of various materials, the absolute values of the thermal conductivity of which can differ by more than an order of magnitude. We note that in the investigated range, the atmospheric pressure (0.1 MPa) can be regarded as a small correction with respect to the zero pressure.

Thus, from Eqn. (1)-(2) the temperature-pressure dependence of the thermal conductivity can be represented as:

$$\lambda(T, P) = \lambda_0 \left(1 + \frac{P}{P_0^*}\right)^\beta \left(\frac{T}{T_0}\right)^{n(P)}.$$  

(3)

The Eqn. (3) includes three constant parameters: $\lambda_0 = \lambda(T_0, 0)$, $P_0^*$, $\beta$, and the dependence $n(P)$. For a number of compounds, the dependence $n(P)$ is linear or at least quasi-linear, while for others, the dependence is more complicated. Values $\beta$ and $P_0^*$ found from the experimental points [4-7] by the least squares method, are presented in Table 1.

| $N$ | sample | $\beta$ | $P_0^*$, MPa | $n(0)$ | $\lambda_0$, W/m·K |
|-----|--------|--------|-------------|--------|-----------------|
| 1   | marl   | 0.078  | 10.9        | 0.39   | 1.63            |
| 2   | sandstone | 0.018  | 0.8         | -0.32  | 2.06            |
| 3   | granite | 0.054  | 28.3        | -0.72  | 3.61            |
| 4   | CdGeAs$_2$ | 0.020  | 11.7        | -0.91  | 14.75           |

The dependences of $n(T)$ (also found by the least squares method) and $f(P)$ (calculated from $\beta$ and $P_0^*$ from the Table 1) for the same materials are presented in the Fig. 1. The value and sign of the coefficient $n$ can serve as an indicator of the ordering of the composite compound, and the resulting empirical expression allows calculations with incomplete data.

It should be noted that a number of authors based on theoretical and experimental studies have shown that in solid bodies in the initial stage of pressure a reversible phase transition of the second kind arises that can occur by different mechanisms due to the smallest displacement of atoms in the crystal lattice and a slight relative change of an elementary volume [8-12].

As it was noted in [13], in the case of brittle solids the effect of pressure appears more distinctly, since these states acquire a new quality – plasticity, whereas in plastic solids only a more or less insignificant quantitative change is observed.

4. Conclusion

Thus, we proposed the analytical expressions, which describe the results of our experimental data on the effect of hydrostatic pressure up to 400 MPa on the thermal conductivity of a line of natural and artifice composite materials: rocks and ceramics. The measurements were provided in the temperature
range 273-523 K. The results of the study of other natural and artificial composite compounds and their comparative analysis are presented in the report and will be published soon.

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