Nonlinear Control of Wind Turbines with Hydrostatic Transmission Based on Takagi-Sugeno Model

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Abstract.
A nonlinear model-based control concept for wind turbines with hydrostatic transmission is proposed. The complete mathematical model of a wind turbine drive train with variable displacement pump and variable displacement motor is presented. The controller design takes into consideration the nonlinearity of the aerodynamic maps and hydrostatic drive train by an convex combination of state space controller with measurable generator speed and hydraulic motor displacement as scheduling parameters. The objectives are the set point control of generator speed and tracking control of the rotor speed to reach the maximum power according to the power curve in the partial-load region.

1. Introduction

Wind turbines with hydrostatic transmission have been considered as an alternative to conventional drive-train concepts. Over the rated power range between 1.5 and 10 MW, the existing gearless direct-drive concepts cause an increase of weight around 25 percent and a cost increase of around 30 percent [1]. The conventional gear boxes of modern wind turbines at the MegaWatt (MW) level of rated power are highly stressed by different load cases, where wind gusts and turbulence lead to misalignment of the drive train and a gradual failure of the gear components. This failure interval creates a significant increase in the capital and operating costs and downtime of a turbine.

By contrast, the hydrostatic transmission allows mechanically decoupled operation over a wider range of wind speeds as a consequence of the omission of mechanical gearboxes and power converters. It permits the use of synchronous generators with low numbers of poles, which are cheaper than the double fed induction generators and multi-pole synchron generators commonly used in variable speed machines today. According to the investigation in [2], hydrostatic transmissions also have a positive impact on power quality, since small rotor speed fluctuations due to wind gusts are absorbed.

Up to now, hydraulic transmissions are mainly used in construction and agricultural equipment. For these kind of applications, relatively simple transmission controllers are sufficiently because the controller in the combustion engine provides a desired input torque to the transmission and supports the operation of the whole drive train. This is of course not the case in wind turbine applications. However, the variable aero-dynamic torque and rotor speed need a much higher...
performance of the controller in a large operating space. In [4] a linearized model for a wind turbine with a hydraulic transmission system and a linear pressure controller is represented. The transmission consists of a variable displacement pump and six fixed displacement radial piston hydraulic motors. In [5] a control-oriented model of a wind turbine drive train with a hydraulic transmission in an extended linearized form is proposed and used to design a gain scheduled linear quadratic regulator (LQR). In this case the low speed shaft is connected with fixed displacement pump, the continuous adjustment is obtained by a variable displacement motor.

In this paper, a nonlinear controller design based on Takagi-Sugeno (TS) model for wind turbines with hydrostatic transmissions is presented. In contrast to [4] and [5] a transmission with variable displacement pump and variable displacement motor is proposed. The hydraulic pump regulates the rotor speed and simultaneously the variable volume of the motor adjusts the generator speed. Hence, the proposed controller concept and design method takes into account the nonlinear coupling between the hydraulic pressure in the circuit and generator speed, the nonlinearity of aerodynamics, and structured model uncertainties such as leakages flows in the transmission and the wind speed as an external input.

This paper is organized as follows: In section 2 the control-oriented model of a horizontal axis wind turbines with hydrostatic transmission is proposed. This model describes the dynamics of the transmission with a variable displacement pump and a variable displacement motor [10] and a wind turbine model with four degree of freedom [6] in Takagi-Sugeno form. The modeling steps from the ordinary differential equations to the state-space Takagi-Sugeno form are detailed presented. In section 3 the controller structure is introduced and the controller design is briefly explained.

2. Control-oriented modeling of wind turbines with hydrostatic transmission

2.1. Description and operation principle of hydrostatic transmission

In contrast to wind turbines with conventional drive trains, the inertia power in hydrostatic transmissions is transmitted by static oil pressure and flow rate. One advantage is that the gear ratio is continuously adjustable. The entire drive train consist of the low speed shaft (LSS), the high speed shaft (HSS) and the hydrostatic transmission line. In its simplest form, the hydrostatic transmission consists of a hydraulic pump and motor, of which at least one must have a variable displacement. A configuration with both a variable pump and a motor is illustrated in Figure 1. Other configurations are investigated in [4], with a variable displacement pump and fixed motors and in [5] with a fixed pump and a variable displacement motor.

On the transmission input side, the torque and speed of the rotor are converted by the hydraulic pump into a pressurised oil flow $q_P$. On the output side, the pressurised flow $q_M$ is converted back into mechanical torque and speed by the hydraulic motor. By varying the displacement of the hydraulic components, any desired transmission ratio can be adjusted. This can be illustrated by the following considerations: The fluid flow $q_P$ produced by the pump is proportional to its rotational speed $n_P$ and depends on the variable volume of the pump per revolution. Indeed, the volume is not constant and proportional to the normalised position $\tilde{x}_P$ of the displacement unit

$$\begin{align*}
q_P &= V_P \tilde{x}_P n_P \quad \text{with} \quad \tilde{x}_P = x_P / x_{P\text{max}} \quad (1)
\end{align*}$$

where $V_P$ is the maximum volumetric displacement per revolution of the pump. Similarly, the volume flow through the hydraulic motor is given by

$$q_M = V_M \tilde{x}_M n_M \quad (2)$$

where $\tilde{x}_M$ denotes the normalised position of the displacement unit, and $n_M$ the rotational speed of the motor shaft. Figure 1 shows that the pump feeds directly the motor. Neglecting
the compressibility of the fluid and hydrostatic losses it holds that \( q_P = q_M \) and with (1), (2) we obtain

\[
\frac{r}{n} = \frac{n_M}{n_P} = \frac{V_P}{V_M} \frac{\bar{x}_P}{\bar{x}_M}.
\]

That is, the continuously variable ratio \( r \) of the gear depends on the constant ratio of the maximum displacement of the pump/motor combination and the adjustable ratio of the position \( \bar{x}_P \) and \( \bar{x}_M \). The necessary high transmission ratio for wind turbines of the Megawatt class up to 2.5 MW can easily be reached by a suitably large ratio of the maximum displacements \( V_P/V_M \). By taking advantage of the second adjustable term \( \bar{x}_P/\bar{x}_M \), the transmission ratio can be varied in such a way that the generator operates at constant speed directly connected to the electric grid.

### 2.2. Dynamic model of hydrostatic transmissions

In the previous considerations a simple static model (3) was used to explain the basic principle of hydrostatic transmissions. This is not sufficient for the ensuing nonlinear analysis, control design, and simulation. Thus, the dynamic of the hydraulic pressure and flow have to be taken into consideration. Further, for consistent modeling of the overall system we introduce

\[
\bar{V}_P = \frac{V_P}{2\pi}, \quad \bar{V}_M = \frac{V_P}{2\pi}
\]

as the volumetric pump displacement \( \bar{V}_P \) and motor displacement \( \bar{V}_P \) per radian. The pressure dynamics in the main hydraulic circuit Figure 1 depends on the capacitance of the lines \( A, B \) and the flow rates \( q_P \) and \( q_M \). The hydraulic capacitance \( C_{A,B} \) for each line yields

\[
C_A = \frac{V_L}{\kappa_{oil}}, \quad C_B = \frac{V_L}{\kappa_{oil}},
\]
where \( \kappa \) is the effective bulk modulus of the line and the fluid, \( V_{LA,B} \) includes the volume of the lines and the dead volumes located in the pump and motor. The compressibility flows, which describe the flow resulting from pressure changes can be written

\[
q_{CA} = C_A \dot{p}_A, \quad q_{CB} = C_B \dot{p}_B .
\] (6)

Using the continuity equation to each line we get the pressure differential equations

\[
\dot{p}_A = \frac{1}{C_A} \left( V_P \ddot{x}_P \omega_r - \dot{V}_M \ddot{x}_M \omega_g - k_{\text{leak}} (p_A - p_B) \right),
\] (7)

\[
\dot{p}_B = -\frac{1}{C_B} \left( V_P \ddot{x}_P \omega_r - \dot{V}_M \ddot{x}_M \omega_g - k_{\text{leak}} (p_A - p_B) \right),
\] (8)

where \( k_{\text{leak}} \) denotes the lumped leakage coefficient. Assuming rigid shaft connections between hub and pump and motor and generator

\[
\omega_r = \omega_P, \quad \omega_g = \omega_M
\] (9)

where \( \omega_{M,P} \) denotes the angular frequency of the pump and motor shaft. In consideration of the symmetrical design of the hydraulic circuit with

\[
V_{LA} = V_{LB} \Rightarrow C_A = C_B =: C_H, \quad \Delta p := p_A - p_B ,
\] (10)

we get the first order differential equation of difference pressure

\[
\Delta \dot{p} = \dot{p}_A - \dot{p}_B = \frac{2}{C_H} \left( V_P \ddot{x}_P \omega_P - \dot{V}_M \ddot{x}_M \omega_M - k_{\text{leak}} \Delta p \right).
\] (11)

The motion equation of the low speed shaft is described by

\[
\dot{\omega}_r = \frac{1}{J_r} \left( T_r(\lambda, \beta) - d_r \omega_r - V_P \ddot{x}_P \Delta p \right)
\] (12)

with \( T_r \) as the aerodynamic torque extracted by the rotor from the wind, \( T_P = V_P \ddot{x}_P \Delta p \) as the pump torque and \( d_r \) as the bearing friction coefficient. The moment of inertia of the entire rotor, which contains the blades, hub and low speed shaft is denoted by \( J_r \). According to the torque balance at the high speed shaft the motion equation is

\[
\dot{\omega}_g = \frac{1}{J_g} \left( \dot{V}_M \ddot{x}_M \Delta p - d_g \omega_g - T_g(\delta) \right)
\] (13)

with \( T_M = \dot{V}_M \ddot{x}_M \Delta p \) as the hydro motor torque and \( d_g \) as the bearing friction coefficient of the high speed shaft. \( T_g(\delta) \) denotes the synchronous generator torque as a function of the torque angle \( \delta \).

The actuator dynamics of the electrohydraulic controlled displacement units, see Figure 1, are each considered by a first order lag element

\[
\begin{align*}
\dot{\ddot{x}}_P &= -\frac{1}{\tau_{up}} \ddot{x}_P + \frac{1}{\tau_{up}} u_P , \\
\dot{\ddot{x}}_M &= -\frac{1}{\tau_{um}} \ddot{x}_M + \frac{1}{\tau_{um}} u_M
\end{align*}
\] (14)

with \( \tau_{up} \) and \( \tau_{um} \) as the time constants of the displacement units and

\[
\begin{align*}
\tau_{up} \in [ -1, 1 ], \quad & u_P \in [ u_{M_{\text{min}}} , 1 ] , \quad u_{M_{\text{min}}} > 0
\end{align*}
\] (15)

as the control signals.
2.3. Wind turbine aerodynamics
The aerodynamic torque of the wind turbine is given by
\[ T_r = \frac{\rho}{2} \pi R^3 C_Q(\lambda, \beta) v^2 \] (16)
with \( v \) as the effective wind speed, \( \rho \) as the air density and \( R \) as the rotor radius, where \( C_Q(\lambda, \beta) \) denotes the torque coefficient which is a nonlinear function of the tip-speed ratio
\[ \lambda = \frac{\omega_r R}{v} \] (17)
and the blade pitch angle \( \beta \). In this study \( C_Q \) is given by an analytic expression which is taken from [6]:
\[ \tilde{C}_Q(\lambda, \beta) = c_1 \left( 1 + c_2 \beta \frac{1}{2} \right) + \frac{c_3}{\lambda} \left( \frac{c_4}{\lambda_i(\lambda, \beta)} - c_5 \beta - c_6 \beta^2 - c_8 \right) e^{-\frac{c_9}{\lambda_i(\lambda, \beta)}}, \] (18)
with
\[ \frac{1}{\lambda_i(\lambda, \beta)} = \frac{1}{\lambda + 0.08 \beta} - \frac{0.035}{c_{10} + c_{11} \beta^3} \quad (\lambda > 0). \]
To exclude unphysical negative terms, this expression is limited to values \( \geq 0 \):
\[ C_Q(\lambda, \beta) = \frac{\tilde{C}_Q(\lambda, \beta) + 1 + \text{sign}(\tilde{C}_Q(\lambda, \beta))}{2}. \] (19)
Hint: The coefficients \( c_i, i = 1, \ldots, 11 \) are calculable using curve fitting tools [6].

2.4. Permanent-magnet synchronous generator
The model of the permanent-magnet synchronous generator (PMSG) is restricted to the torque angle dynamics
\[ T_g = \sin(\delta) \frac{3 V_1 V_P(\omega_g)}{\omega_1 X_1}, \quad \delta = \int_0^t (\omega_1 - \omega_g) \, d\tau \] (20)
with \( \omega_1 = \frac{2\pi f_1}{p} \) as frequency of the rotating stator field (the synchronous frequency) and \( p \) as the number of pole pairs. \( \delta \) denotes the torque angle, \( V_1 \) denotes the stator voltage (line-to-neutral), and \( X_1 \) as stator reactance. The rotor voltage (line-to-neutral) \( V_P(\omega_g) = k_{PM} \omega_g \) of PMSG is proportional to the generator speed.

2.5. Takagi-Sugeno model of the entire system
In the next step a Takagi-Sugeno model of the entire wind turbine system, i.e. from wind side to generator side, will be derived using the previous submodels (11), (12), (13), (14), (16), (19), and (20). Takagi-Sugeno (TS) model structure provides a way to obtain an exact representation of the full nonlinear model as a weighted combination of linear submodels, where the nonlinearities of the system are shifted into the weighting functions. For a system in TS model structure, stable controllers with desired performance can be designed by solving linear matrix inequalities (LMIs).
Starting point is the nonlinear state space model

\[
\begin{align*}
\dot{\omega}_r &= \frac{1}{J_r} \left( T_r(\lambda,\beta) \omega_r - d_r \omega_r - \hat{V}_P \hat{x}_P \Delta p \right), \\
\dot{\omega}_g &= \frac{1}{J_g} \left( \hat{V}_M \hat{x}_M \Delta p - d_g \omega_g - \frac{3V_1 k_{PM}}{\omega_1} \sin(\delta) \right) \\
\Delta \dot{p} &= \frac{2}{C_H} \left( \hat{V}_P \hat{x}_P \omega_r - \hat{V}_M \hat{x}_M \omega_g - k_{\text{leak}} \Delta p \right), \\
\dot{\delta} &= \omega_1 - \omega_g \\
\dot{\hat{x}}_P &= -\frac{1}{\tau_{uP}} \hat{x}_P + \frac{1}{\tau_{uP}} u_P \\
\dot{\hat{x}}_M &= -\frac{1}{\tau_{uM}} \hat{x}_M + \frac{1}{\tau_{uM}} u_M
\end{align*}
\]

with the inputs \( u = [u_P \ u_M]^T \), states \( x = [\omega_r \ \omega_g \ \Delta p \ \delta \ \hat{x}_P \ \hat{x}_M]^T \) and outputs \( y = [\omega_r \ \omega_g \ \Delta p]^T \). This sixth order nonlinear system (21) is now converted to an equivalent TS system for the purpose of control design in the partial-load region (\( \beta = 0 \)). First, the following functions are defined

\[
\begin{align*}
f_1(\lambda,\beta) &= \frac{T_r(\lambda,\beta)}{\omega_r}, \quad f_2(\hat{x}_P) = \hat{V}_P \hat{x}_P, \quad f_3(\hat{x}_M) = \hat{V}_M \hat{x}_M, \\
f_4(\delta) &= \sin(\delta), \quad f_5(\omega_r) = \hat{V}_P \omega_r, \quad f_6(\omega_g) = \hat{V}_M \omega_g.
\end{align*}
\]

such that (21) can be rearranged as

\[
\dot{x} = A(x,v)x + B(u)u
\]

where \( \gamma_P = \frac{3V_1 k_{PM}}{\omega_1} \). To avoid algebraic loops in the nonlinear TS controller the input matrix
\( \mathbf{B}(\mathbf{u}) \) in (23) is approximated by

\[
\bar{\mathbf{B}} = \begin{bmatrix}
0 & 0 & \frac{d_f \omega_1}{u_{M_{\min}}} \\
0 & 0 & 0 \\
0 & \frac{\omega_1}{u_{M_{\min}}} & 0 \\
\frac{1}{\tau_{uP}} & 0 & \frac{1}{\tau_{uM}}
\end{bmatrix}
\]  

(24)

To arrive a TS model structure, the functions (22) can be written as

\[
\begin{align*}
    f_1(\lambda, \beta) &= w_{11} \bar{f}_1 + w_{12} \bar{f}_1, \\
    f_2(\bar{x}_P) &= w_{21} \bar{f}_2 + w_{22} \bar{f}_2, \\
    f_3(\bar{x}_M) &= w_{31} \bar{f}_3 + w_{32} \bar{f}_3, \\
    f_4(\delta) &= w_{41} \bar{f}_4 + w_{42} \bar{f}_4, \\
    f_5(\omega_r) &= w_{51} \bar{f}_5 + w_{52} \bar{f}_5, \\
    f_6(\omega_g) &= w_{61} \bar{f}_6 + w_{62} \bar{f}_6,
\end{align*}
\]

where \( \bar{f}_j := \max f_j \) and \( \bar{f}_j := \min f_j \). The weighting functions \( w_{j1} \) and \( w_{j2} \) are defined by

\[
\begin{align*}
    w_{j1} &= \frac{f_j - \bar{f}_j}{\bar{f}_j - \bar{f}_j}, \\
    w_{j2} &= \frac{\bar{f}_j - f_j}{\bar{f}_j - \bar{f}_j},
\end{align*}
\]

which fulfill the convexity condition

\[
w_{j1}, w_{j2} \geq 0, \quad w_{j1} + w_{j2} = 1 \quad \text{for} \quad i = 1, \ldots, 6
\]

(26)

The membership functions of the TS model are defined by

\[
\sum_{i=1}^{N_r} h_i(\mathbf{x}, v) = \prod_{j=1}^{N_l} \left( w_{j1} + w_{j2} \right)
\]

(27)

with \( N_r = 2^{N_l} \) and result from \( N_r \) combinations of the product

\[
h_i(\mathbf{x}, v) = w_{1k} \cdot w_{2k} \cdot w_{3k} \cdot w_{4k} \cdot w_{5k} \cdot w_{6k}, \quad k = 1, 2
\]

(28)

The corresponding \( \mathbf{A}_i \) matrices are

\[
\mathbf{A}_i = \begin{bmatrix}
\frac{1}{\tau} \left( \{ \bar{f}_1, \bar{f}_1 \} - d_r \right) & 0 & -\frac{1}{\tau} \{ \bar{f}_2, \bar{f}_2 \} & 0 & 0 & 0 \\
0 & -\frac{1}{\tau} \left( d_g - \gamma_P \{ \bar{f}_3, \bar{f}_3 \} \right) & 0 & -\frac{1}{\tau} \{ \bar{f}_4, \bar{f}_4 \} & 0 & 0 \\
0 & 0 & 0 & -\frac{d_H}{\tau} k_{leak} & 0 & -\frac{2}{\tau_{uP}} \{ \bar{f}_5, \bar{f}_5 \} \\
0 & -1 & 0 & 0 & 0 & -\frac{1}{\tau_{uP}} \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{\tau_{uM}}
\end{bmatrix}
\]

Finally, state-space model (23) with the common \( \bar{B} \) matrix (24) can now be written in TS form as

\[
\begin{align*}
    \dot{\mathbf{x}} &= \sum_{i=1}^{N_r} h_i(\mathbf{x}, v) \mathbf{A}_i \mathbf{x} + \bar{\mathbf{B}} \mathbf{u} \\
    \mathbf{y} &= \mathbf{C} \mathbf{x}.
\end{align*}
\]

(29)
3. Controller Structure
The overall control law is formulated as a weighted combination of linear state feedback controllers $K_{x_i}$ each with an additional integrator gain $K_{I_i}$

$$u = \sum_{j=1}^{N_r} h_i(x, \hat{v}) K_{x_i} x + \sum_{j=1}^{N_r} h_i(x, \hat{v}) K_{I_i} \left[ \int_0^t (\Delta p_d - \Delta p) \, dt \right]$$

$$+ \sum_{j=1}^{N_r} h_i(x, \hat{v}) \left[ \int_0^t (\omega_{g_d} - \omega_g) \, dt \right]$$

(30)

The desired difference pressure for the optimal tracking control of the rotor speed is calculated by

$$\Delta p_d = \frac{k_T}{\dot{V}_p} \bar{x}_p \omega_r^2$$

with

$$k_T = \frac{C_{Q_{opt}} \rho \pi R^5}{2 \lambda_{opt}^2}$$

(31)

where $\bar{x}_p = \dot{x}_p / 0$. The wind speed $\hat{v}$ in the control law is calculated by using a wind speed observer from [13], which estimates the rotor effective turbulent wind speed.

4. Conclusion and future work
In this paper, a nonlinear description of wind turbines with hydrostatic transmission based on Takagi-Sugeno (TS) model is presented. The model describes the dynamics of the transmission with a variable displacement pump and a variable displacement motor and a wind turbine model with four degree of freedom in Takagi-Sugeno form. The modeling steps from the ordinary differential equations to the state-space Takagi-Sugeno form were presented in detail. Based on the TS structure the overall control law is formulated as a PDC (parallel distributed controller) with an additional integrator gain.

In the future work the performance of the PDC law (30) is considered with the aero-elastic simulation code FAST by NREL [14] using the 5 MW reference wind turbine [15] extended by a hydrostatic transmission (Fig. 1).

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