Abstract

We propose an inclusive analysis of a stransverse mass \( m_{T2} \) using a hemisphere method for supersymmetry studies at the LHC. The hemisphere method is an algorithm to group collinear and high \( p_T \) particles and jets, assuming that there are two of such groups in an event. The \( m_{T2} \) is defined as a function of the unknown LSP mass, two hemisphere momenta, and missing transverse momentum. The kinematical end point of the \( m_{T2} \) distribution provides information on the squark and gluino masses. We perform a Monte Carlo simulation to study the inclusive \( m_{T2} \) distribution at the LHC. We show that the end point of the inclusive \( m_{T2} \) distribution has a cusp-like structure around the true LSP mass. The knowledge of the expected kinematical behavior near the end point for true events is important to establish the end point of the inclusive distribution. We find that the inclusive analysis is useful to obtain the information on the heaviest of the squark/gluino.
1 Introduction

While the particle interactions at low energy are described correctly by the standard model (SM), the mechanism of electroweak symmetry breaking by Higgs boson suffers from the fine turning problem. In addition, the SM is not successful to describe the dark matter in our Universe.

We expect to obtain information on the physics beyond the standard model at ATLAS and CMS experiments at the LHC, which is scheduled to start this year (2008). Among the various proposals, the phenomenology of the models with quark and gauge partners with multiplicatively conserved parity, such as supersymmetric models with conserved R parity, Little Higgs models with T parity and and universal extra dimension models, get much attention. In the supersymmetric models, quark and gluon partners (squark and gluino) are pair produced at the LHC, and subsequently decay into the SM particles and the lightest supersymmetric particles (LSP’s). The signature at the LHC will be the high $p_T$ jets and leptons with significant missing transverse momentum which arises from the LSP escaping from detection. By the end of the LHC experiment, the squark and gluino in minimal supergravity model will be searched up to $\sim 2.5$ TeV [1, 2].

Interests in the new physics go beyond the discovery. Many studies have been carried out to find out the possible clues to study “the nature of the new physics”, such as the masses, spins and interactions of the new particles. The progress has been made especially in the exclusive channels. The end points of the invariant mass distributions constrain the sparticle masses and for some cases nearly all sparticle masses can be measured. The end point study is extremely successful when the decay involves many leptons [1, 2, 3, 4, 5, 6].

Other important variables are transverse masses. The peak of the effective mass distribution, which is the sum of the transverse momenta of jets, leptons and $E_T^{\text{miss}}$, is related to the sum of the sparticle masses produced by $pp$ collision. The quantity is inclusive and would be useful in the early stage of the LHC experiment. More sophisticated quantity is the $m_{T2}$ variable [7, 8]. This can be calculated from the two visible objects, and the missing momentum of the events and a test LSP mass. Recently, the quantity acquires much of attention because this function has a cusp at the correct LSP mass when the squark/gluino undergoes three body decays. Some exercises have been carried out for
some model points where only gluino-gluino production can be observed, or for several other decay patterns without specifying the selection processes [9, 10, 11, 12].

In this paper, we propose an inclusive study of $m_{T2}$ variables using a hemisphere method. Namely we group jets into two "visible objects" and calculate $m_{T2}$ variable based on them. The grouping algorithm is called a hemisphere method and discussed in earlier works [13, 14]. The motivation is to collect the cascade decay products from a squark or a gluino enough probability to see the $m_{T2}$ end point, and obtain the information on their masses without going into exclusive analysis. If it works, the LSP mass also can be obtained in the early stage of the experiment.

We found the correspondence between the reconstructed $m_{T2}$ end point and the mass of squark or gluino is good. We recognized that $m_{T2}$ is sensitive to $\max(m_{\tilde{g}}, m_{\tilde{q}})$ as $m_{T2}$ is defined from the maximum of transverse mass of the visible objects for the test LSP momenta consistent with $E_T^{\text{miss}}$. Not surprisingly, the probability to reconstruct the correct object is rather low and the end point is smeared, however, we find the event-wise response of the $m_{T2}$ to the test LSP mass mentioned in Refs. [9, 10, 11, 12] is useful to ensure the correctness of the end point.

In this paper, we especially compare the mixed modulus anomaly mediation (MMAM) model to the supergravity (SUGRA) model. The MMAM predicts a degenerate mass spectrum in some parameter region of the model, where the sparticles are heavy while available $p_T$‘s of the daughter particles are small. This is the model where the kink at the LSP mass should appear in a rather high value. This point may be compared with model points in the mSUGRA (minimal Supergravity) model, where the gluino mass is lighter than the corresponding MMAM point, but the squark mass is much heavier than the gluino so that the total squark/gluino production cross sections are same. We find that the $m_{T2}$ end point is useful to extract the squark mass for this case, therefore the MMAM and the SUGRA models can be distinguished.

This paper is organized as follows. In Section 2 we review the $m_{T2}$ variable and the cusp structure appearing in the endpoint of the $m_{T2}$ distribution as the function of the test LSP mass. In Section 3 we describe the inclusive $m_{T2}$ and perform a Monte Carlo simulation to study the distribution. Section 4 is devoted to the conclusion.
2 Transverse mass ($m_{T2}$)

In hadron collisions, squarks and gluinos are produced in pair and these SUSY particles decay subsequently into the final states including jets, leptons, and two LSP’s. The LSP is usually the lightest neutralino. With $R$-parity conservation, the LSP is a neutral and stable particle and it escapes from detection. There are two LSP’s in the final state and one cannot measure each LSP momentum experimentally while the total transverse momentum can be measured. The transverse mass $m_{T2}$ is defined as follows:

$$m_{T2}^2(m_\chi) \equiv \min_{p_{T1}^{\text{vis}}+p_{T2}^{\text{miss}}=p_T^{\text{miss}}} \left[ \max \{ m_T^2(p_{T1}^{\text{vis}}, p_{T1}^{\text{miss}}), m_T^2(p_{T2}^{\text{vis}}, p_{T2}^{\text{miss}}) \} \right],$$

(1)

where $p_{T1}^{\text{vis}}$ is the transverse momentum of a “visible object” from a squark/gluino decay, which is defined as the sum of visible particle momenta. The $p_{T}^{\text{miss}}$ is the total missing transverse momentum. The minimization is taken with respect to the unknown LSP momenta $p_{T1}^{\text{miss}}, p_{T2}^{\text{miss}}$ under the constraint $p_{T1}^{\text{miss}} + p_{T2}^{\text{miss}} = p_T^{\text{miss}}$. The transverse mass, $m_T^2$, is defined as

$$m_T^2(p_{T1}^{\text{vis}}, p_{T1}^{\text{miss}}) = (m_{\chi i}^{\text{vis}})^2 + m_\chi^2 + 2 (E_{T1}^{\text{vis}} E_{T1}^{\text{miss}} - p_{T1}^{\text{vis}} \cdot p_{T1}^{\text{miss}}),$$

(2)

where $E_{T1} = \sqrt{p_{T1}^2 + m_\chi^2}$. It should be noted that the true LSP mass ($m_{\chi 0}$) is unlikely to be known in advance, so $m_{T2}$ is regarded as a function of a test LSP mass ($m_\chi$).

One of the important features is that the $m_{T2}$ is smaller than the parent gluino/squark masses if a test LSP mass is set equal to the true value.

$$m_{T2}(m_{\chi 0}) \leq \max(m_{\tilde{q}}, m_{\tilde{g}}).$$

(3)

From the upper end point of $m_{T2}$ ($m_{T2}^{\text{max}}$), one can obtain the information on the mass of the parent particle. Without knowledge of the true LSP mass, $m_{T2}^{\text{max}}$ provides a one-dimensional constraint between the masses of the squark/gluino and the LSP.

Recently, it is pointed out that the $m_{T2}^{\text{max}}(m_\chi)$ function has a kink structure at which $m_\chi$ is the true LSP mass unless the squark/gluino decays directly into the LSP through a two body decay. An analytic expression of $m_{T2}^{\text{max}}$ is derived in Refs. [11, 12]. If one considers events in which the squark and the gluino are produced in pair with a vanishing
total transverse momentum, $m_{T2}^{\max}(m_{\chi})$ is given as follows.

$$m_{T2}^{\max}(m_{\chi}) = \begin{cases} F_{\leq}^{\max}(m_{\chi}) & \text{for } m_{\chi} < m_{\chi,0}^1, \\ F_{\geq}^{\max}(m_{\chi}) & \text{for } m_{\chi} > m_{\chi,0}^1, \end{cases} \quad (4)$$

where

$$F_{\leq}^{\max}(m_{\chi}) = F(m_{\text{vis}}^1 = m_{\text{min}}^{\text{vis}}, m_{\text{vis}}^2 = m_{\text{min}}^{\text{vis}}, \theta = 0, m_{\chi}),$$

$$F_{\geq}^{\max}(m_{\chi}) = F(m_{\text{vis}}^1 = m_{\text{max}}^{\text{vis}}, m_{\text{vis}}^2 = m_{\text{max}}^{\text{vis}}, \theta = 0, m_{\chi}). \quad (5)$$

Here the function $F$ is given in Ref.[12] and $m_{\text{vis}}^i$ is kinematically bounded as follows,

$$m_{\text{min}}^{\text{vis}} \leq m_{\text{vis}}^i \leq m_{\text{max}}^{\text{vis}}. \quad (6)$$

Notice that the events at the end point satisfy $m_{\text{vis}}^1 = m_{\text{min}}^{\text{vis}}$ for $m_{\chi} < m_{\chi,0}^1$ while $m_{\text{vis}}^1 = m_{\text{max}}^{\text{vis}}$ for $m_{\chi} > m_{\chi,0}^1$. The kink structure of $m_{T2}^{\max}(m_{\chi})$ appears since the functional form of $m_{T2}^{\max}(m_{\chi})$ changes at $m_{\chi} = m_{\chi,0}^1$. In Ref.[11], it is shown that the kink structure appears even if the pair-produced squark and gluino have a non-vanishing transverse momentum. If one can identify the position of the kink from the LHC experiment, one can determine the masses of the squark/gluino and the LSP simultaneously. In Ref. [12], it is demonstrated that the masses of the squark/gluino and the LSP are determined using exclusive decay channel by performing Monte Carlo simulations. In particular, for the case that the gluino decay $\tilde{g} \rightarrow q\bar{q}\chi_1^0$ occurs through the off-shell squark exchange diagram, the $m_{T2}^{\max}$ from the gluino pair production has a very sharp kink structure and the masses are determined precisely.

3 Inclusive $m_{T2}$ Analysis

3.1 Hemisphere analysis and inclusive $m_{T2}$ parameter

In this section we argue that the kink method discussed in Sec[2] can be extended to an inclusive analysis. In case of exclusive analyses, one needs to specify a cascade decay chain. The branching ratio of the cascade decay chain would depend on the model parameters. On the other hand, inclusive distributions are rather insensitive to branching ratios. Therefore, if an inclusive quantity can be defined, it may be useful to determine
the squark and the gluino masses in the early stage of the LHC experiment. One disadvantage of inclusive approaches may be that all the production and decay modes should be taken into account simultaneously, and \( m_{T2} \) distribution may be smeared.

To define an inclusive \( m_{T2} \) distribution, we group the final particles into two “visible objects”. For this purpose, we adopt a hemisphere method in Ref. [13, 14]. For each event, two hemispheres are defined and high \( p_T \) jets, leptons, and photons are assigned into one of the hemispheres as follows;

1. Each hemisphere is defined by an axis \( p_{i}^{\text{vis}}(i = 1, 2) \), which is the sum of the momenta of high \( p_T \) objects belonging to hemisphere \( i \). We require \( p_T > 50 \) GeV for jets to reduce QCD backgrounds.

2. High \( p_T \) objects \( k \) belonging to hemisphere \( i \) satisfy the following conditions:

\[
d(p_i, p_i^{\text{vis}}) < d(p_k, p_j^{\text{vis}}),
\]

where the function \( d \) is defined by

\[
d(p_k, p_i^{\text{vis}}) = (E_i - |p_i^{\text{vis}}| \cos \theta_{ik}) \frac{E_i}{(E_i + E_k)^2}.
\]

Here \( \theta_{ik} \) is the angle between \( P_i \) and \( p_k \).

To find axises \( p_i^{\text{vis}} \), we adopted the algorithm discussed in Ref. [13, 14]. Once \( p_i^{\text{vis}} \)’s are determined, one can calculate \( m_{T2} \) by using Eq.(1).

The inclusive \( m_{T2} \) may be compared with MTGEN [15]. The MTGEN variable is a minimum of the \( m_{T2} \) variable for all possible choices of two subsets of particles \( \alpha \) and \( \beta \). The correct choice of subsets \( \alpha \) and \( \beta \) leads the heaviest sparticle mass as the end point, and the end point should be bounded from above by the mass, if the initial state radiation can be ignored. In our algorithm, we assume that the algorithm described above groups high \( p_T \) jets from the same cascade decay with enough probability. This approach is useful if the visible hemisphere mass is small compared with the the leading jet energies, which is expected especially for the events near the \( m_{T2} \) end point when the test mass is smaller than the LSP mass, see Eqs.(4), (5).
3.2 Model points

To perform a Monte Carlo analysis, we choose two sample points, A and B. The point A corresponds to the MMAM model [16, 17, 18, 19]. In the MMAM model, the mass spectrum is parametrized by the modular weights for matter fields $n_i$ and the gravitino mass ($m_{3/2}$) and $R \equiv m_{3/2} \langle (T + T^*)/F_T \rangle$ where $T$ and $F_T$ are a modulus field and its $F$-component, respectively. In general, the MMAM model predicts a degenerate SUSY spectrum compared with the mSUGRA model. If $\alpha = R/\ln(M_{pl}/m_{3/2})$ is large, the SUSY spectrum becomes more degenerate. In this analysis, we choose the point studied in Ref. [20], $n_i = 0(1)$ for squarks and sleptons (Higgs boson), $R = 20$, $\tan \beta = 10$ and the gravitino mass is determined so that $M_3 = 650$ GeV at the GUT scale. The point B corresponds to the mSUGRA with $m_0 = 1475$ GeV, $m_{1/2} = 561$ GeV, $A = 0$ and $\tan \beta = 10$.

The mass spectrum of SUSY particles is calculated using ISAJET [21] for each sample point. In Table 1, the relevant SUSY masses are listed. At point B, $m_{\tilde{q}} > m_{\tilde{g}}$ and the gluino undergoes three-body decay through the off-shell squark diagram. The total production cross section of SUSY events at the LHC is $\sigma = 0.13$ pb for both points. Squark-gluino coproduction is larger than squark-squark and gluino-gluino productions for both points.

The point B is chosen so that the $M_{\text{eff}}$ distribution of one lepton mode is very similar to that for point A, where $M_{\text{eff}}$ is defined from the sum of the $p_T$ of the first four jets and

|        | A: MMAM                              | B: mSUGRA                                |
|--------|--------------------------------------|------------------------------------------|
| $n_i = 0, R = 20, M_3(GUT) = 650$ | $m_0 = 1475, m_{1/2} = 561.2, A = 0, \tan \beta = 10$ |
| $\tilde{g}$ | 1491                                 | 1359                                     |
| $\tilde{u}_L$ | 1473                                 | 1852                                     |
| $\tilde{u}_R$ | 1431                                 | 1831                                     |
| $\tilde{d}_R$ | 1415                                 | 1830                                     |
| $\tilde{\chi}_1^0$ | 487                                  | 237                                      |

Table 1: Relevant SUSY mass parameters at points A and B. All the mass parameters are given in GeV.
a lepton and the missing transverse momentum as follows,

\[ M_{\text{eff}} = \sum_{i=1}^{4} p_{Ti} + p_{Tl} + E_{T}^{\text{miss}}. \]  

(9)

For the Monte Carlo analysis, we generate \(5 \times 10^4\) SUSY events by HERWIG 6.5 [22] for each sample point. To estimate event distributions measured by the LHC detector, we use AcerDET [23]. This code provides a simple detector simulation at the LHC.

In Fig.1(a), the \(M_{\text{eff}}\) distribution is shown for one lepton channel. Here we require the following cut.

1. \(n_{\text{jet}}(p_T > 100 \text{ GeV}) \equiv n_{100} \geq 1\) and \(n_{\text{jet}}(p_T > 50 \text{ GeV}) \equiv n_{50} \geq 4\) within \(|\eta| < 3\).
2. \(E_T^{\text{miss}} > 0.2M_{\text{eff}}\) and \(E_T^{\text{miss}} > 100 \text{ GeV}\) and \(S_T > 0.2\).
3. There is one isolated lepton with \(p_T > 20 \text{ GeV}\).

The solid (dashed) histogram is the distribution for point A (B) and the \(M_{\text{eff}}\) distributions roughly agree.

Although there is not much difference in \(M_{\text{eff}}\) distribution defined in Eq.(9), there are more high \(p_T\) jets on average at point B compared with point A. This is because the squark-gluino coproduction is dominant, and a squark decaying into a gluino leads additional high \(p_T\) jets in the events. If one sums all jets with \(p_T > 50 \text{ GeV}\), then the distribution of point B is significantly higher than that of point A. In Fig.1(b), the \(M_{\text{eff}}\) distribution summing up all jets with \(p_T > 50 \text{ GeV}\) is shown for one lepton channel. We will see in the next subsections that inclusive \(m_{T2}\) analyses give us a more quantitative measure to the difference of the two points.

### 3.3 Monte Carlo analysis: Point A (MMAM)

First, let us consider point A. We require the following cut to select the events.

1. \(n_{100} \geq 2\) and \(n_{50} \geq 4\) within \(|\eta| < 3\).
2. The effective mass of the event must satisfy \(M_{\text{eff}} > 1200 \text{ GeV}\).
3. At least two jets in each hemisphere.
4. \(E_T^{\text{miss}} > 0.2M_{\text{eff}}\) and \(E_T^{\text{miss}} > 100 \text{ GeV}\).
Figure 1: (a). $M_{\text{eff}}$ distributions for one lepton channel. The solid (dashed) histogram is for point A (B). (b). $M_{\text{eff}}$ distributions for one lepton channel as in (a) but summing up all jets with $p_T > 50$ GeV. The solid (dashed) histogram is for point A (B).

5. There is no isolated lepton with $p_T > 20$ GeV.

With these cuts, the standard model backgrounds are expected to be reduced significantly, so we do not consider the SM background in this simulation.

Figure 2: The two-dimensional distribution in the $m_T^2 - R$ plane for point A. The test LSP mass is assumed as $m_\chi = 30$ GeV.

To check how well the hemisphere method works, let us consider the following ratio.

$$R(m_\chi) \equiv \frac{m_{T2}(m_\chi) - m_{T2}^{(p)}(m_\chi)}{m_{T2}^{(p)}(m_\chi)}. \quad (10)$$
where the parton level $m_{T2}^{(p)}$ is defined so that each visible momentum is the difference of a initially produced sparticle and the daughter LSP momentum, and $E_T^{\text{miss}}$ is the missing energy provided by AcerDET after smearing. If the hemisphere method works well, $R \sim 0$. In Fig.2, the two-dimensional distribution in the $m_{T2}^{(p)}$-R plane is shown for $m_\chi = 30$ GeV. The peak of the distribution appears around $R \sim 0$, but the deviation can be large. The reconstructed $m_{T2}$ tends to be smaller than $m_{T2}^{(p)}$. The main source of the deviation is the mis-grouping of the visible objects under the hemisphere method, and also neutrinos and jets with $p_T < 50$ GeV which are not included in the hemisphere.

Let us consider the $m_{T2}$ distribution for $m_\chi < m_{\chi_1}$. In Fig.3(a), the $m_{T2}^{(p)}$ distribution is shown for $m_\chi = 30$ GeV. There is an end point at $m_{T2}^{(p)} \approx 1250$ GeV. In Fig.3(b), the reconstructed $m_{T2}$ distribution is shown for $m_\chi = 30$ GeV. Compared with the $m_{T2}^{(p)}$ distribution, there is a long tail due to the mis-grouping of the hemisphere method, although there is some structure at $m_{T2} \sim 1250$ GeV.

![Figure 3](image)

Figure 3: (a). The $m_{T2} - m_\chi$ distribution at parton level for $m_\chi = 30$ GeV. (b). The reconstructed $m_{T2} - m_\chi$ distribution for $m_\chi = 30$ GeV. Fitting functions of the end points are also shown, see text.

Let us examine the events around the end point region in detail. Fig.4(a) shows max($m_{1\text{vis}}^{\text{vis}}, m_{2\text{vis}}^{\text{vis}}$) distribution for $m_{T2}(30) > 1000$ GeV. As discussed in Section 2, the true end point event is realized for $m_{i\text{vis}}^{\text{vis}} = m_{\text{min}}^{\text{vis}}$ and the events with large max($m_{1\text{vis}}^{\text{vis}}, m_{2\text{vis}}^{\text{vis}}$) are considered as fake events. To reduce them, the $m_{T2}$ distribution for max($m_{1\text{vis}}^{\text{vis}}, m_{2\text{vis}}^{\text{vis}}$) < 400 GeV is plotted in Fig.4(b). With the cut on the hemisphere mass, the long tail of $m_{T2}$ disappears and one can see a rather clear end point at $m_{T2} \sim 1350$ GeV.
Next, let us consider the \(m_{T2}\) distribution for \(m_\chi > m_\chi^0\). In Fig. 5 (a), the \(m_{T2}^{(p)} - m_\chi\) distribution is plotted for \(m_\chi = 900\) GeV. There is an end point at \(m_{T2}^{(p)} \sim 1900\) GeV. In Fig. 5 (b), the reconstructed \(m_{T2} - m_\chi\) distribution is plotted for \(m_\chi = 900\) GeV. The distribution has a long tail and one cannot see a clear end point.

As discussed in Sec. 2, the cusp structure of \(m_{T2}^{\text{max}}\) appears since the functional form \(m_{T2}^{\text{max}}(m_\chi)\) changes at \(m_\chi = m_\chi^0\). The end point event for \(m_\chi < m_\chi^0\) is different from the one for \(m_\chi > m_\chi^0\) and these end point events are interchanged at \(m_\chi = m_\chi^0\). To confirm this, let us consider how the events near the end point for \(m_{T2}(30)\) behaves.
when \( m_\chi \) is large. In Fig.6(a), the \( m_{T2}(900) \) distribution is plotted for \( 1200 \, \text{GeV} < m_{T2}(30) < 1400 \, \text{GeV} \). There are two peaks in the distribution. The lower peak is smaller than the true end point \( m_{T2}(900) \simeq 1900 \, \text{GeV} \). These events are true end point events of \( m_{T2}(30) \) while the events around the higher peak are fake events. In Fig.6(b), the \( m_{T2}(900) \) distribution is plotted using events above the true end point \( m_{T2}(30) > 1400 \, \text{GeV} \). We find no peak lower than 1900 GeV as expected, because they are fake events for \( m_{T2}(30) \).

![Figure 6: (a). The \( m_{T2}(900) \) distribution for \( 1200 \, \text{GeV} < m_{T2}(30) < 1400 \, \text{GeV} \) (left).](image)

![Figure 6: (b). The \( m_{T2}(900) \) distribution for the fake events \( m_{T2}(30) > 1400 \, \text{GeV} \) (right).](image)

To find the end points of the \( m_{T2} \) distributions, we also show the fitting of the distribution in Figs.3 and 5. We fit the reconstructed \( m_{T2} \) distribution with a linear function which changes the slope at some \( m_\chi \). For comparison, we also show the fitting of \( m_{T2}^{(p)} \). We use a Gaussian smeared fitting function in Ref. [5] for that. \( \chi^2/\text{n.d.f} \) is not good for both fits, therefore our fits should be regarded as crude estimates. In addition, the end point for \( m_\chi = 900 \, \text{GeV} \) depends on the bins used for the fit. Note that the end point for \( m_\chi > m_\tilde{\chi}_1^0 \) is realized for the events with \( m_{\text{vis}} \sim m_{\text{vis}}^{\text{max}} \), while the efficiency to assign the particles correctly in hemisphere should be low in such case, see Fig.7.

In Fig.8 the end points of \( m_{T2} \) for various test LSP masses are plotted with solid lines. The end points of the \( m_{T2} \) are larger than \( m_{T2}^{(p)} \) by \( 150 - 200 \, \text{GeV} \), one can see a kink structure around \( m_\chi \sim 400 \, \text{GeV} \), which is close to the true LSP mass, \( m_{\tilde{\chi}_1^0} = 487 \, \text{GeV} \). At the kink, the end point value of \( m_{T2} \) is \( m_{T2} \sim 1650 \, \text{GeV} \). It should be noted that the
inclusive $m_{T2}$ distribution is dominated by the events from the squark-gluino coproduction since the production cross section is larger than those of gluino-gluino and squark-squark pair production. In such a situation, the end point of $m_{T2}$ distributions is sensitive to $\max(m_{\tilde{g}}, m_{\tilde{q}})$. At point A, the gluino is heavier than the squarks and the end point should be sensitive to the gluino mass, $m_{\tilde{g}} = 1491$ GeV. While the end point value is larger than the true gluino mass by about 150 GeV, we think the agreement between $m_{T2}$ and $m_{T2}^{(p)}$ is reasonable given the crudeness of our fit.

In the mSUGRA, the bino-like LSP mass is about 1/6 of the gluino mass. If we take the measured $m_{T2}$ end point at the kink as the gluino mass then the LSP mass assuming mSUGRA is around 270 GeV. The observed kink is clearly above 270 GeV, therefore we can say that the mass spectrum is the MMAM type for this case.
Figure 8: The end point of $m_{T2}(m_{\chi}) - m_{\chi}$ for various test LSP masses. The solid line is the $m_{T2}^{\text{max}}$ while the dashed line is the parton level $m_{T2}^{(p)\text{max}}$. 
3.4 Monte Carlo analysis: Point B (mSUGRA)

At point B, the squark is much heavier than the gluino and our interest is to measure the squark mass scale quantitatively using the inclusive $m_{T2}$ distributions. We require the following cuts to select the events.

1. $n_{100} \geq 2$ and $n_{50} \geq 6$ within $|\eta| < 3$.
2. $M_{\text{eff}} > 1500$ GeV
3. At least two jets in each hemisphere.
4. $E_T^{\text{miss}} > 0.2 M_{\text{eff}}$ and $E_T^{\text{miss}} > 100$ GeV.
5. There is no isolated lepton with $p_T > 20$ GeV.

In Fig.9, the two-dimensional distribution in the $m_{T2}^{(p)}$-$R$ plane is plotted for $m_\chi = 30$ GeV. The peak of the distribution appears $R \sim -0.2$, and the misreconstruction rate is higher than point A. The low reconstruction efficiency may be understood as follows. At this point $m_{\tilde{q}} \sim 1850$ GeV, $m_{\tilde{g}} \sim 1360$ GeV and the squark decaying into the gluino gives high $p_T$ jets as discussed earlier. When this jet is misidentified to the other hemisphere, the reconstructed $m_{T2}$ may become much lower than the expected $m_{T2}^{(p)}$ value, because the squark is so much heavier than gluino. It can be as low as the order of gluino mass. Note that $(m_{\tilde{q}} - m_{\tilde{g}})/m_{\tilde{q}} \simeq 0.26$, roughly corresponds to the observed shift. Luckily, $m_{T2}^{(p)}$ strongly peaks near the end point, and the reconstructed event still makes visible end points.

In Fig.10(a), the $m_{T2}^{(p)}$ distribution is plotted for $m_\chi = 30$ GeV. There are two peaks in the $m_{T2}^{(p)}$ distribution. The higher peak corresponds to the squark while the lower peak corresponds to the gluino. At point B, the squark is much heavier than the gluino, so the end point is determined by the squark decay, $m_{T2}^{(p)} \sim 1850$ GeV. In Fig.10(b), the $m_{T2}$ distribution is shown for $m_\chi = 30$ GeV. The $m_{T2}$ distribution is smeared but one can still see the end point. The end point events are dominated by the events with small $m^{\text{vis}}$. There is again interchange of the events near the end point as we increase the test LSP mass, and we can see the two peak structure in $m_{T2}(m_\chi > m_{\chi_1}^0)$ distribution for the event near the end point of $m_{T2}(m_\chi < m_{\chi_1}^0)$ distribution, similar in Fig.6.
Figure 9: The two-dimensional distribution in \( m_{T2} - R \) plane for point B. The test LSP mass is assumed as \( m_\chi = 30 \) GeV.

Figure 10: (a). The \( m_{T2}^{(p)} \) distribution for \( m_\chi = 30 \) GeV. (b). The \( m_{T2} \) distribution for \( m_\chi = 30 \) GeV.

In Fig.11, the end points of \( m_{T2} \) for various test LSP masses are plotted with a solid line. The end points are determined as in point A. The end points of the \( m_{T2} \) is almost the same as the ones of \( m_{T2}^{(p)} \) within errors. One cannot see a clear kink structure around the true LSP mass, \( m_{\chi_1} = 237 \) GeV. While it is difficult to determine the squark mass from the kink method, the inclusive \( m_{T2} \) analysis is useful to obtain the information on the squark mass. To see whether the end point of \( m_{T2} \) correctly describes the squark mass for \( m_{\tilde{q}} > m_{\tilde{g}} \), we show the \( m_{T2} \) end point for \( m_\chi = 30 \) GeV at the mSUGRA points where the gaugino mass is kept the same as that of point B but the universal scalar mass \( m_0 \) is varied. In Fig.12 we plot the \( m_{T2} \) end point as the function of the squark mass and
find very good agreement from 1400 GeV to 1800 GeV.

Figure 11: The end point of $m_{T2}(m_\chi) - m_\chi$ for various test LSP masses. The solid line is the $m_{T2}^{\text{max}}$ while the dashed line is the parton level $m_{T2}^{(p)\text{max}}$.

Figure 12: The end point of $m_{T2}$ for $m_\chi = 30$ at several mSUGRA points where the gaugino masses is the same as that of point B but $m_0$ is varied. Here the horizontal axis is the left-handed up-type squark mass.
4 Summary and Conclusion

In this paper, we have proposed an inclusive $m_{T2}$ analysis at the LHC to obtain information on squark and gluino masses by the hemisphere method. The hemisphere method is an algorithm to group collinear and high $p_T$ particles and jets, assuming that there are two of such groups in a event. The algorithm is to group the cascade decay products into two visible objects. To study the distributions, we perform the Monte Carlo simulation for two sample SUSY spectra from the MMAM and the mSUGRA models. The cascade decay products from a squark/gluino are grouped into a visible object with enough probability to see the parton level $m_{T2}$ end point. However, the end point of the $m_{T2}$ distribution is sometime smeared by mis-identification of hemispheres, which obscure the end point determination. We have fitted the $m_{T2}$ distribution near the end points. The end point determination suffers from various systematic uncertainties, such as a choice of the fitting function and the fitting region.

We have examined the events near the end point in detail. For $m_\chi < m_{\chi_1^0}$ the end point events has the minimum hemisphere mass, $m_{\text{vis}}^i = m_{\text{min}}^\text{vis}$. The fake end point events due to the mis-grouping of the hemisphere are reduced if we impose the cut on the hemisphere mass, without disturbing the correct end points. For $m_\chi > m_{\chi_1^0}$, the end point event is realized when the hemisphere mass is at maximum $m_{\text{vis}}^i = m_{\text{max}}^\text{vis}$. The end point event is interchanged at $m_\chi = m_{\chi_1^0}$ and the cusp structure of $m_{T2}^{\text{max}}$ appears. By checking the test mass behavior of the $m_{T2}$ variable for the events near the end point, we can prove if events near the end point obtained by the fit of $m_{T2}$ distribution is correctly reconstructed ones or not. We have shown that the true end point event for $m_\chi < m_{\chi_1^0}$ gives the $m_{T2}$ value smaller than $m_{T2}^{\text{max}}$ for $m_\chi \gg m_{\chi_1^0}$, while the fake end point event for $m_\chi < m_{\chi_1^0}$ gives $m_{T2}$ larger than $m_{T2}^{\text{max}}$ for $m_\chi > m_{\chi_1^0}$. From this observation, while there are various uncertainties for the end point determination, we conclude that the inclusive $m_{T2}$ distribution is useful to obtain the information of the masses of the gluino/squark and the LSP at the LHC experiment.

For both of the sample points, the main QCD production process of SUSY particles is squark-gluino coproduction, and the end point of $m_{T2}$ distribution is sensitive to max($m_{\tilde{g}}$, $m_{\tilde{q}}$). For the sample point in the MMAM model, $m_{\tilde{g}} > m_{\tilde{q}}$ and the end point should be
determined by $m_{\tilde{g}}$, while it should be determined by $m_{\tilde{q}}$ for the sample point in the mSUGRA model because $m_{\tilde{q}} > m_{\tilde{g}}$. From the Monte Carlo analysis, we have found that the end point is indeed determined by $\max(m_{\tilde{g}}, m_{\tilde{q}})$ for both of the sample points. For the MMAM sample point, we have found that there is a cusp-like structure of $m_{T2}^{\text{max}}(m_{\chi})$ at the true LSP mass and we can determine the gluino and the LSP mass simultaneously. For the mSUGRA sample points with $m_{\tilde{q}} > m_{\tilde{g}}$, we find the linearity between $m_{\tilde{q}}$ and $m_{T2}^{\text{max}}$. We have checked that the squark mass is reconstructed up to $m_{\tilde{q}} \sim 1.4m_{\tilde{g}}$ when $m_{\tilde{g}} \sim 1.4 \text{ TeV}$.

There have been different approaches in the LHC physics study. One of the direction is to study the inclusive quantities such as $M_{\text{eff}}$, $E_T^{\text{miss}}$, which do not require reconstruction and is useful to grab characters of the events. The other direction is to study quantities, which are specific to some processes, such as the end point measurements of the invariant mass and the $m_{T2}$ distribution. They are very powerful to determine the absolute sparticle masses. In this paper, we propose an inclusive $m_{T2}$, which is inclusive in the sense that we do not specify the decay channel. However, it bites the merit of exclusive analyses with helps of the new understandings of the stransverse mass function $m_{T2}(m_{\chi})$. While detailed analyses on systematical uncertainties are still needed, we hope that this quantity helps to determine the sparticle masses at the early stage of the LHC experiments.

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