Baryonia and near-threshold enhancements

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The baryon-antibaryon spectrum consisting of strange, charm and bottom quarks is studied in the color flux-tube model with a multi-body confinement interaction. Numerical results indicate that many low-spin baryon-antibaryon states can form compact hexaquark states and are stable against the decay into a baryon and an antibaryon. The multi-body confinement interaction as a binding mechanism plays an important role in the formation of the states. They can be searched in the e⁺e⁻ annihilation and charmonium or bottomonium decay if they really exist. The newly reported states, X(1835), X(2370), Y(2175), Y(4360) and Yb(10890), may be interpreted as NN, ΔΔ, ΛΛ, ΛcΛc and ΛbΛb states, respectively.

PACS numbers: 12.39.Jh, 13.75.Cs, 14.40.Rt

I. INTRODUCTION

The research of baryonia has a rather long history, which can date back to 1940s. Fermi and Yang proposed that the π-meson may be a composite particle of nucleon-antinucleon (NN) [1], in which a strong attractive force is assumed to bind them together because of the mass of π-meson is substantially smaller than twice the mass of a nucleon. Subsequently, Sakata extended Fermi and Yang’s idea by introducing a strange baryon Λ and its antiparticle. The strange baryon Λ, proton p and neutron n and their antiparticles were regarded as the fundamental building blocks to construct other mesons and baryons, which is the well known Fermi-Yang-Sakata (FYS) model [2]. The profound difficulty of the FYS model was the enormous binding energy for sticking a baryon and an antibaryon together to form a light meson, the FYS model was therefore replaced by quark models. Many researchers abandoned the FYS’s point of view and pioneered that NN states were no more associated with “ordinary” light mesons, but instead with new types of mesons with a mass near the NN threshold and specific decay properties [3].

In recent years, many near-threshold enhancements are observed in experiments, the pp enhancement is observed in the J/ψ → γp̅p̅, ψ' → π0p̅p̅ηp̅p̅, and B± → p̅p̅K± [4]; the ΛΛ enhancement is observed in the B+ → ΛΛK+ and e⁺e⁻ → ΛΛ [5]; the Λ⁺Λ⁻ enhancement is observed in the e⁺e⁻ → Λ⁺Λ⁻ [6], et al. Furthermore, many other resonances called XYZ particles were also observed in experiments. It is hard to accommodate some of them, such as X(3872) and Y(4260), into quark models due to their extraordinary properties, which goes beyond our anticipation, because it is taken for granted that the heavy mesons can be well described with quark models.

The appearance of XYZ particles forces us to propose various interpretations rather than a qq configuration to clarify the structure of the states. The recent progresses and a rather complete list of references on XYZ particles can be found in recent review and references therein [7]. In addition to the explanations of theses exotic states as tetraquark states, meson-meson molecular states, hybrid quarkonia and orbital excited states of conventional mesons, et al, baryonia or hexaquark states q3q3 are also possible interpretations. The pp enhancement was interpreted as a baryonium NN with quantum numbers JPC = 0−+ in the different theoretical frames by many authors [8]. The states Y(4260), Y(4361), Z±(4430) and Y(4664) were systematically embedded into an extended baryonium picture [9]. The Y(2175) was described as a bound state ΛΛ with quantum numbers 2S+1LJ=3S1 in the one-boson-exchange potential model [10].

Nonstrange hexaquark states q3q3 were systematically studied in a color flux-tube model with a six-body confinement potential instead of an additive two-body interaction in our previous work, and it was found that some ground states are stable against disintegrating into a baryon and an anti-baryon [11]. In the present work, the spectrum of the baryonia of the ground state consisting of strange, charm and bottom quarks is studied in the color flux-tube model. The work is not only a natural extension of the previous work to try to interpret the structure of the recently discovered resonances, but also provides a new insight into exploring the baryonium states. The research shows that many low-spin baryon-antibaryon states are compact hexaquark states and stable against direct decaying into a baryon and an antibaryon, a multi-body confinement interaction in the model plays an important role in the short-range domain. The dominant components of the new hadron states X(1835), X(2370), Y(2175), Y(4260) and Yb(10890) may be interpreted as NN, ΔΔ, ΛΛ, ΛcΛc and ΛbΛb bound states, respectively.

The paper is organized as follows: Section II is devoted to the descriptions of the color flux-tube model and gives the Hamiltonian of baryons and hexaquark systems. A
brief introductions of the constructions of the wave functions of baryons and hexaquark systems are given in Sec. III. The numerical results and discussions are presented in Sec. IV. A brief summary is given in the last section.

II. COLOR FLUX-TUBE MODEL AND HAMILTONIAN

Quantum Chromodynamics (QCD) is widely accepted as the fundamental theory to describe the strong interacting systems and has verified in high momentum transfer process. In the low energy region, such as hadron spectroscopy and hadron-hadron interaction study, the \textit{ab initio} calculation directly from QCD becomes very difficult due to the complication of nonperturbative nature. Although many nonperturbative methods have been developed, such as lattice QCD (LQCD), QCD sum rule, large-$N_c$ expansion, chiral unitary theory, et al, the QCD-inspired constituent quark model (CQM) is still a useful tool in obtaining physical insight for these complicated strong interacting systems. CQM can offer the most complete description of hadron properties and is probably the most successful phenomenological model of hadron structure [12].

CQM is formulated under the assumption that the hadrons are color singlet non-relativistic bound states of constituent quarks with phenomenological effective masses and interactions. The effective interactions include one gluon exchange (OGE), one boson exchange (OBE) and a confinement potential. Traditional CQM includes the typical Isgur-Karl model and chiral quark model [13, 14], in which the confinement potential can be phenomenologically described as the sum of two-body interactions proportional to the color charges and $r_{ij}^k$,

\[ V^C = -a_c \sum_{i>j}^n \lambda_i \cdot \lambda_j r_{ij}^k \]  

(1)

where $r_{ij}$ is the distance between two interacting quarks and $k$ usually takes 1 or 2. The traditional models can describe the properties of ordinary hadrons ($q^2$ and $qq$) well. However, the traditional models lead to power law van der Waals forces between color-singlet hadrons and the anti-confinement in a color symmetrical quark or antiquark pair. The problems are related to the fact that the traditional CQM does not respect local color gauge invariance.

The color flux-tube structures of ordinary hadrons are unique and trivial, many important low-energy QCD information may be absent in the descriptions of these objects, such as quark pair in color symmetrical $6(6)$ representation. Multiquark systems, if they really exist, have various color flux-tube structures in the intermediate- and short-distance domains, which may contain abundant low-energy QCD information and affect the properties of multiquark systems [18 19]. The mixing effect of the color flux-tube structures can provide the intermediate-range attractive force coming from the $\sigma$ meson or $\pi \pi$ exchange [20]. The traditional CQM is hard to describe various color flux-tube structures of multiquark systems.

LQCD calculations of ordinary hadrons, tetraquark and pentaquark states reveal various color flux-tube structures [21]. Within the color flux-tube picture, the confinement potential of multiquark states is a multibody interaction and can be simulated by a potential which proportional to the minimum of the total length of all color flux tubes which connects the quarks (antiquarks) to form a multiquark system [21]. Based on the traditional CQM and the LQCD picture, the color flux-tube model has been developed to study multiquark systems, in which a multibody confinement interaction is employed, and a sum of the square of the length of flux tubes rather than a linear one is assumed to simplify the calculation [15 19]. The approximation is justified because of the following two reasons: one is that the spatial variations in separation of the quarks (lengths of the flux tube) in different hadrons do not differ significantly, so the difference between the two functional forms is small and can be absorbed in the adjustable parameter, the stiffness. The other is that we are using a nonrelativistic dynamics in the study. As was shown long ago [22], an interaction energy that varies linearly with separation between fermions in a relativistic first order differential dynamics has a wide region in which a harmonic approximation is valid for the second order (Feynman-Gell-Mann) reduction of the equations of motion. The comparative studies also indicated that the difference between the two type confinement potentials is very small [18 19].

The description of the properties of ordinary hadrons is the starting point of the phenomenological investigation of multiquark systems. The Y-shaped color flux-tube structure, the LQCD picture of a baryon [23], is shown in Fig.1, in which $r_i$ represents the spatial position of the $i$-th quark denoted by a black dot and $y_0$ denotes a junction where three color flux tubes meet.

In the color flux-tube model with quadratic confinement potential, the three-body potential can be written as

\[ V^C(3) = K \left( (r_1 - y_0)^2 + (r_2 - y_0)^2 + (r_3 - y_0)^2 \right) \]  

(2)

the position of the junction $y_0$ can be fixed by minimizing the energy of baryons, then we get

\[ y_0 = \frac{r_1 + r_2 + r_3}{3} \]  

(3)

the minimum of the confinement potential for baryons has therefore the following forms

\[ V_{min}^C(3) = K \left( \left( \frac{r_1 - r_2}{\sqrt{2}} \right)^2 + \left( \frac{2r_3 - r_1 - r_2}{\sqrt{6}} \right)^2 \right) \]  

(4)

the above equation can also be expressed as the sum of
three pairs of two-body interactions,

\[
V_{\text{min}}^C(3) = \frac{K}{3} \left[ (r_1 - r_2)^2 + (r_2 - r_3)^2 + (r_1 - r_3)^2 \right]
\]  

(5)

It can be seen that the three-body quadratic confinement potential of a baryon is totally equivalent to the sum of the two-body one, see the \(\Delta\)-shaped structure in Fig.1, although the equivalence is only approximately valid for the linear confinement potential.

![FIG. 1: Three-body (left) and two-body (right) confinement potential](image)

With respect to a hexaquark system \(q^3\bar{q}^3\), four possible color flux-tube structures are listed in Fig.2, in which a black dot denotes a quark and a hollow dot denotes an antiquark. The structure (a) is a baryon-antibaryon molecule state: \([q^3]\bar{q}^3(1);\) The hexaquark state with the structure (b) is called a color octet baryon-antibaryon state: \([q^3][\bar{q}^3]^1\); In the case of the structure (c), it can be called a diquark-antidiquark hexaquark state: \([\bar{q}^2]q^2[\bar{q}^2]q^2(1);\) The last one is similar to a chemical benzene, and it is therefore called QCD benzene, this structure for a hexaquark state \(q^6\) was studied in our previous work [13]. Of course, the color flux-tube structures should include three color singlet mesons configuration: \([\bar{q}\bar{q}]_1[\bar{q}\bar{q}]_1[\bar{q}\bar{q}]_1\), which must be taken into account in the decay of a \(q^3\bar{q}^3\) system into three mesons, this task is left as our future work. The color flux-tube in hadrons should be very similar to the chemical bond in organic compounds. The same molecular constituents may have different chemical bond structure; those are called isomeric compounds. Therefore, the multiquark systems with the same quark content but different flux-tube structures are similarly called QCD isomeric compounds.

In general, a hexaquark system should be the mixture of all possible flux-tube structures. In order to avoid a too complicated calculation in the present work, only the first two structures in Fig.2 are considered. Within the color flux-tube model, the confinement potential for the structure (a) can be written as

\[
V_{\text{min}}^a(6) = K \left( \left( \frac{r_1 - r_2}{\sqrt{2}} \right)^2 + \left( \frac{2r_3 - r_1 - r_2}{\sqrt{6}} \right)^2 \right)
\]

\[
+ \left( \frac{r_4 - r_5}{\sqrt{2}} \right)^2 + \left( \frac{2r_6 - r_4 - r_5}{\sqrt{6}} \right)^2 \right)
\]

(6)

while the confinement potential for the structure (b) has the following form

\[
V_{\text{min}}^b(6) = K \left( (r_1 - y_1)^2 + (r_2 - y_1)^2 + (r_3 - y_2)^2 \right)
\]

\[
+ \left( r_6 - y_3 \right)^2 + (r_4 - y_4)^2 + (r_5 - y_4)^2 \right)
\]

\[
+ \frac{\kappa_{d_{12}} (y_1 - y_2)^2 + \kappa_{d_{23}} (y_2 - y_3)^2}{2}
\]

\[
+ \kappa_{d_{34}} (y_3 - y_4)^2 \right)
\]

(7)

In above equation, \(K\) is the stiffness constant of an elementary or color triplet flux-tube, while \(K\kappa_{d_{ij}}\) is other color flux-tube stiffness called as compound ones. The compound color flux-tube stiffness parameter \(\kappa_{d_{ij}}\) depends on the color dimension, \(d_{ij}\), of the string,

\[
\kappa_{d_{ij}} = \frac{C_{d_{ij}}}{C_3},
\]

(8)

where \(C_{d_{ij}}\) is the eigenvalue of the Casimir operator associated with the \(SU(3)\) color representation \(d_{ij}\) on either end of the string, namely \(C_3 = \frac{4}{3}, C_6 = \frac{16}{3}\) and \(C_8 = 3\). For the sake of simplicity, the average \(\kappa_d\) for \(\kappa_{d_{ij}}\) is used in numerical calculations.

For given quark (antiquark) positions \(r_i\), those junctions \(y_i\) are obtained by minimizing the confinement potential. By introducing the following set of canonical coordinates \(R_i\),

\[
R_1 = \frac{1}{\sqrt{2}} (r_1 - r_2), \quad R_2 = \frac{1}{\sqrt{2}} (r_4 - r_5)
\]

\[
R_3 = \frac{1}{\sqrt{12}} (r_1 + r_2 - 2r_3 + r_4 + r_5 - 2r_6)
\]

\[
R_4 = \frac{1}{\sqrt{33 + 5\sqrt{33}}} (r_1 + r_2 - \sqrt{3}r_3 - r_4 - r_5 + \sqrt{3}r_6)
\]

![FIG. 2: Four color flux-tube structures of a \(q^3\bar{q}^3\) system.](image)
\[ R_5 = \frac{1}{\sqrt{33 - 5\sqrt{33}}} (r_1 + r_2 + w_2 r_3 - r_4 - r_5 - w_2 r_6) \]

\[ R_6 = \frac{1}{\sqrt{6}} (r_1 + r_2 + r_3 + r_4 + r_5 + r_6). \] (9)

the minimum of the confinement potential takes the following form,

\[ V_{min}^b (6) = K \left( R_1^2 + R_2^2 + \frac{3\kappa_d}{2 + 3\kappa_d} R_3^2 \right) + \frac{2\kappa_d (\kappa_d + w_3)}{2\kappa_d + 2} R_4^2 + \frac{2\kappa_d (\kappa_d + w_4)}{2\kappa_d + 2} R_5^2 + \frac{2\kappa_d (\kappa_d + w_5)}{2\kappa_d + 2} R_6^2 \] (10)

where \( w_1 = \sqrt{33 + 5}, w_2 = \sqrt{33 - 5}, w_3 = 2 + \sqrt{33}, \) and \( w_4 = 2 - \sqrt{33}. \) Clearly this confinement potential is a multibody interaction rather than the sum of two-body one. When two clusters \( q^3 \) and \( \bar{q}^3 \) separate largely, a baryon and an antibaryon should be a dominant component of the system because other hidden color flux-tube structures are suppressed due to the color confinement. With the separation reducing, a hadronic molecule state may be formed if the attractive force between a baryon and an antibaryon is strong enough. When they are close enough to be within the range of confinement (about 1 fm), all possible flux-tube structures may appear due to the excitation and rearrangement of color flux tubes. In this case, the confinement potential of the system should at least be taken to be the minimum of the two flux-tube structures. It therefore reads

\[ V_{min}^c (6) = \min \{ V_{min}^a, V_{min}^b \} \] (11)

OGE and/or OBE are important and responsible for the mass splitting in the ordinary hadron spectra. The model with OGE and the model with OGE+OBE can all describe the ground state of baryons well, the differences of two models appear in the description of excited baryons [24]. The study on the ground states of non-strange hexaquark systems indicates that the difference of two models is small [13], only OGE is therefore taken into account in the present work. The complete Hamiltonian used here is listed as the following,

\[ H_n = \sum_{i=1}^{n} \left( m_i + \frac{p_i^2}{2m_i} \right) - T_C + \sum_{i<j}^n V_{ij}^G + V_{min}^C (n) \] (12)

\[ V_{ij}^G = \frac{1}{4} \alpha_s \lambda_i \cdot \lambda_j \left( \frac{1}{\delta_{ij}} - \frac{2\pi}{3} \delta (r_{ij}) \frac{\sigma_i \cdot \sigma_j}{m_i m_j} \right) \] (13)

The tensor forces and spin-orbit forces between quarks are omitted in the model, because our primary interest is in the lowest energies and their contributions to the ground states are small or zero. In the above expression of \( H_n, n = 3 \) or \( n = 6, T_C \) is the center-of-mass kinetic energy, \( m_i \) and \( p_i \) are the mass and momentum of the \( i \)-th quark, \( \lambda \) and \( \sigma \) are the \( SU(3) \) Gell-man and \( SU(2) \) Pauli matrices, respectively, note that \( \lambda \rightarrow -\lambda^* \) for antiquark, all other symbols have their usual meanings. An effective scale-dependent strong coupling constant is used here [25],

\[ \alpha_s (\mu) = \frac{\alpha_0}{\ln \left( \frac{\mu^2 + \mu_0^2}{\mu_0^2} \right)} \] (14)
For baryons with only two identical particles, such as $\Lambda$ and $\Sigma$, the spatial wave function has the following form,

$$\Psi_{LMT}(\mathbf{R}, \mathbf{r}) = [\phi_{lm}(\mathbf{r}_{ij})\phi_{LM}(\mathbf{R}_k)]_{LMT}$$

in which quarks $q_i$ and $q_j$ are identical particles. The spatial wave function of baryons with three different quarks is the same as Eq. (19), in which the quark $q_k$ is the heaviest one.

The relative motion wave functions $\phi_{lm}(\mathbf{r}_{ij})$ and $\phi_{LM}(\mathbf{R}_k)$ are the superpositions of Gaussian basis functions with different sizes,

$$\phi_{lm}(\mathbf{r}_{ij}) = \sum_{n=1}^{n_{max}} c_n N_{nl} r_{ij}^{l} e^{-\nu_n r_{ij}^2} Y_{lm}(\mathbf{r}_{ij})$$

$$\psi_{LM}(\mathbf{R}_k) = \sum_{N=1}^{N_{max}} c_N N_{NL} R_k^{l} e^{-\nu_N R_k^2} Y_{LM}(\mathbf{R}_k)$$

where $N_{nl}$ and $N_{NL}$ are normalization constants. Gaussian size parameters $\nu_n$ and $\nu_N$ are taken as geometric progression,

$$r_n = r_1 a^{n-1}, \quad \nu_n = \frac{r_{n_{max}}}{r_1}^{\frac{1}{n_{max} - n}}$$

$$R_N = R_1 A^{N-1}, \quad \nu_N = \frac{R_{N_{max}}}{R_1}^{\frac{1}{N_{max} - N}}$$

The numbers $n$ and $l$ ($N$ and $L$) specify, respectively, the radial and angular momenta excitations with respect to the Jacobi coordinate $\mathbf{r}$ ($\mathbf{R}$). The angular momenta $l$ and $L$ are coupled to the total orbital angular momentum $L_T$.

![Figure 3: Jacob ordinates for a $q^3q^3$ system.](image)

With regard to a hexaquark system $q^3\bar{q}^3$, the Jacobi coordinates are shown in Fig.3 and can be expressed as

$$\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j, \quad \mathbf{R}_k = \mathbf{r}_k - \frac{m_i \mathbf{r}_i + m_j \mathbf{r}_j}{m_i + m_j},$$

$$\mathbf{r}_{lm} = \mathbf{r}_l - \mathbf{r}_m, \quad \mathbf{r}_n = \mathbf{r}_n - \frac{m_i \mathbf{r}_i + m_m \mathbf{r}_m}{m_i + m_m},$$

$$\mathbf{X} = \frac{m_i \mathbf{r}_i + m_j \mathbf{r}_j + m_k \mathbf{r}_k}{m_i + m_j + m_k} - \frac{m_i \mathbf{r}_i + m_m \mathbf{r}_m + m_n \mathbf{r}_n}{m_i + m_m + m_n}.$$  

The model wave function with defined quantum numbers $I$ and $J$ can be expressed as,

$$\Psi_{IJ}^{q^3\bar{q}^3} = \sum_{\xi} c_\xi \left[ \Phi_{c_1I_1J_1}^{q^3} \Phi_{c_2I_2J_2}^{\bar{q}^3} \right]_I \Phi(\mathbf{X})$$

The Gaussian size parameter $\nu_N$ is taken the same form as $\nu_n$ or $\nu_N$.

IV. NUMERICAL RESULTS AND DISCUSSIONS

The spectrum of the ground states of baryons can be obtained by solving the three-body Schrödinger equation

$$(H_3 - E_{1J}) \Phi_{IJ}^{LM}(\mathbf{R}, \mathbf{r}) = 0$$

with Rayleigh-Ritz variational principle. The converged results are arrived by setting $r_1 = R_1 = 0.3 \text{ fm}$, $r_{n_{max}} = R_{n_{max}} = 2.0 \text{ fm}$ and $n_{max} = N_{max} = 5$. The model parameters are fixed by fitting the experimental data with the exception that the parameters $\Lambda_0$ and $\mu_0$ are taken from the paper $[25]$, $\Lambda_0 = 0.187 \text{ fm}$ and $\mu_0 = 0.113 \text{ fm}$. The values of model parameters and the masses of the ground baryon are listed in Table I and Table II, respectively. In general, the model describe the baryon spectrum well.

| Parameter: $m_{ud}$ | $m_u$ | $m_c$ | $m_\alpha$ | $\Lambda_0$ | $K$ | $\beta$ |
|---------------------|------|------|----------|---------|------|------|
| Units: MeV MeV MeV MeV ... MeV fm |
| Values: 313 545 1800 5140 5.41 400 0.47 |

The color flux-tube model with the model parameters listed in Table I are used to study the hexaquark systems $q^3\bar{q}^3$. It should be emphasized that no any new parameter is introduced in the calculation. The spectrum of hexaquark systems can be obtained by solving the six-body Schrödinger equation

$$(H_6 - E_{1J}) \Psi_{IJ}^{q^3\bar{q}^3} = 0.$$  

The converged numerical results can be obtained by setting $n_{max}=5$, $N_{max}=5$ and $N'_{max}=5$. The minimum and maximum ranges of the bases are 0.3 fm and 2.0 fm for coordinates $\mathbf{r}$, $\mathbf{R}$ and $\mathbf{X}$, respectively.

The binding energies, $\Delta E_J = E_{1J} - 2M_B$, of the ground states of the hexaquark systems $q^3\bar{q}^3$ are listed in Table III. It can be seen that the energies of many low-spin ($J \leq 1$) states lie below the threshold $2M_B$,...
the ground states are therefore stable against dissociation into a baryon and an antibaryon, while they can decay into three mesons. None of high-spin \((J \geq 2)\) states lie below the corresponding threshold. To check the rationality of the various color structures used, the spatial configurations of the bound states are calculated by using the wave functions obtained in solving the Schrödinger equation. The rms for \(r\), \(R\) and \(X\) of all possible bound states \(qq'\) with \(J^{PC}\) of \(0^{-+}\) and \(1^{--}\) are listed in Table IV, it can be seen that they are smaller than 1 fm in our model, so the introducing of hidden-color configuration is reasonable. The results also show that the dominant component of the bound states is not a loose baryon-antibaryon molecule state but a compact hexaquark state, which is formed by means of the multibody confinement potential originating from the color flux-tube picture. Compared with the early baryonia calculations in the traditional quark models \([28, 29]\), where no non-strange bound state was obtained, the multibody confinement interaction used in our model can globally give more attraction than the additive two-body one proportional to the color factor in the traditional quark models. Furthermore, the anti-confinement among a color symmetrical quark or antiquark pair does not shown up in the multibody confinement potential, because no color charge appeared \([15, 17]\). The similar string model with a multibody confinement potential was applied to study the stabilities of tetraquark, pentaquark and hexaquark states, and it was suggested that many compact multiquark states could exist \([30]\). In addition, the color-magnetic interaction \(\alpha_s \frac{g_s^2}{m_s m_b} \) in OGE can further depress the masses of low-spin states. The color-magnetic interaction was considered as a binding mechanism and play an important role in the formation of the famous H-particle \([31]\), which is tentatively below the threshold of \(\Lambda\Lambda\), although it is not confirmed by experiments so far. However, some researches of multiquark states indicated that the color-magnetic interaction as a unique binding mechanism encountered some difficulties \([32]\).

From our calculation, a tendency is apparent, the heavier the states, the deeper the binding and the smaller the size. For example, the binding energy of \(\Lambda\Lambda\) is -80 MeV, the distance between two clusters \(\sqrt{(X^2)}\) is 0.49 fm, while the binding energy of \(\Lambda_c\Lambda_c\) is -244 MeV with \(\sqrt{(X^2)} = 0.39\) fm, of \(\Lambda_b\Lambda_b\) is -363 MeV and \(\Lambda_b\Lambda_b\) with \(J^{PC} = 0^{-+}\). The masses obtained are 1832 MeV and 2384 MeV, respectively, which are very close to the experimental data of the \(X(1835)\) and \(X(2370)\).

| Baryons | Flavor | \(IJ^P\) | Calculated | Experimental |
|---------|--------|----------|------------|--------------|
| \(N\)   | \(nnn\) | \(\frac{3}{2}^+\) | 939        | 939          |
| \(\Lambda\) | \(nns\) | \(\frac{3}{2}^+\) | 1108       | 1116         |
| \(\Sigma\) | \(nns\) | \(\frac{3}{2}^+\) | 1213       | 1195         |
| \(\Xi\)  | \(nss\) | \(\frac{3}{2}^+\) | 1350       | 1315         |
| \(\Delta\) | \(nnn\) | \(\frac{3}{2}^+\) | 1232       | 1232         |
| \(\Sigma^*\) | \(nns\) | \(\frac{3}{2}^+\) | 1382       | 1385         |
| \(\Xi^*\) | \(nss\) | \(\frac{3}{2}^+\) | 1528       | 1530         |
| \(\Omega^-\) | \(sss\) | \(\frac{3}{2}^+\) | 1675       | 1672         |
| \(\Lambda_b^+\) | \(nnb\) | \(\frac{3}{2}^+\) | 2287       | 2285         |
| \(\Sigma_b^+\) | \(nnb\) | \(\frac{3}{2}^+\) | 2480       | 2455         |
| \(\Xi_b^+\) | \(nsc\) | \(\frac{3}{2}^+\) | 2533       | 2520         |
| \(\Omega_b^0\) | \(ssc\) | \(\frac{3}{2}^+\) | 2620       | 2466         |
| \(\Lambda_c^+\) | \(nnb\) | \(\frac{3}{2}^+\) | 2670       | 2645         |
| \(\Sigma_c^+\) | \(nnb\) | \(\frac{3}{2}^+\) | 2790       | 2695         |
| \(\Xi_c^+\) | \(nsc\) | \(\frac{3}{2}^+\) | 2819       | 2766         |
| \(\Omega_c^0\) | \(ssc\) | \(\frac{3}{2}^+\) | 5600       | 5620         |
| \(\Lambda_b^-\) | \(nnb\) | \(\frac{3}{2}^+\) | 5816       | 5808         |
| \(\Sigma_b^-\) | \(nnb\) | \(\frac{3}{2}^+\) | 5836       | 5830         |
| \(\Xi_b^-\) | \(nsc\) | \(\frac{3}{2}^+\) | 5948       | 5790         |
| \(\Xi_c^-\) | \(nsc\) | \(\frac{3}{2}^+\) | 5966       |              |
| \(\Omega_b^-\) | \(ssc\) | \(\frac{3}{2}^+\) | 6107       | 6071         |

TABLE II: The masses of the ground states of baryons, unit in MeV, in which \(n\) stands for a \(u\)- or \(d\)-quark.

### Table III: The binding energies of the ground states of hexaquark systems \(qq'q''\) with quantum numbers \(J^{PC}\) (unit: MeV), where \(\ldots\) means that the corresponding state does not exist.

| States | \(0^{-+}\) | \(1^{--}\) | \(2^{++}\) | \(3^{--}\) |
|--------|----------|----------|----------|----------|
| \(NN\) | -46      | \ldots   | \ldots   | \ldots   |
| \(\Lambda\Lambda\) | -80      | -46      | 0        | 0        |
| \(\Sigma\Sigma\) | -10      | \ldots   | \ldots   | \ldots   |
| \(\Xi\Xi\) | -8       | \ldots   | \ldots   | \ldots   |
| \(\Delta\Delta\) | -80      | 0        | 0        | 0        |
| \(\Sigma\Sigma^*\) | -20      | \ldots   | \ldots   | \ldots   |
| \(\Xi\Xi^*\) | -105     | -47      | \ldots   | \ldots   |
| \(\Omega^+\Omega^+\) | -8       | \ldots   | \ldots   | \ldots   |
| \(\Lambda_c^+\Lambda_c^-\) | -244     | -232     | \ldots   | \ldots   |
| \(\Sigma_c^+\Sigma_c^-\) | -144     | -132     | \ldots   | \ldots   |
| \(\Sigma_c^*\Sigma_c^*\) | -34      | \ldots   | \ldots   | \ldots   |
| \(\Xi_c^+\Xi_c^-\) | -207     | \ldots   | \ldots   | \ldots   |
| \(\Xi_c^*\Xi_c^*\) | -168     | -98      | \ldots   | \ldots   |
| \(\Omega_c^0\Omega_c^0\) | 0        | \ldots   | \ldots   | \ldots   |
| \(\Omega_c^0\Omega_c^*\) | 0        | \ldots   | \ldots   | \ldots   |
| \(\Lambda_b^+\Lambda_b^-\) | -363     | -360     | \ldots   | \ldots   |
| \(\Sigma_b^+\Sigma_b^-\) | -278     | -277     | \ldots   | \ldots   |
| \(\Sigma_b^*\Sigma_b^*\) | -58      | -10      | \ldots   | \ldots   |
| \(\Xi_b^+\Xi_b^-\) | -304     | -44      | \ldots   | \ldots   |
| \(\Xi_b^*\Xi_b^*\) | -266     | -200     | \ldots   | \ldots   |
| \(\Omega_b^0\Omega_b^+\) | 0        | \ldots   | \ldots   | \ldots   |
TABLE IV: Rms for \( r \), \( \mathbf{R} \) and \( \mathbf{X} \) of all possible bound states \( q\bar{q}^* \) with \( J^{PC} = 0^{-+} \) and \( 1^{-+} \), unit in fm.

| \( J^{PC} \)   | \( 0^{-+} \) | \( 1^{-+} \) |
|--------------|--------------|--------------|
| \( NN \)     | (\( r^2 \))<sub>t</sub> | (\( r^2 \))<sub>t</sub> | (\( r^2 \))<sub>t</sub> |
| \( \Delta \bar{\Delta} \) | 0.76 | 0.66 | 0.49 | 0.77 | 0.69 | 0.54 |
| \( \Sigma \bar{\Sigma} \) | 0.86 | 0.67 | 0.87 | ... | ... | ... |
| \( \Xi \Xi^* \) | 0.72 | 0.69 | 0.73 | ... | ... | ... |
| \( \Delta \Delta \) | 0.93 | 0.82 | 0.62 | ... | ... | ... |
| \( \Sigma \Sigma^* \) | 0.89 | 0.78 | 0.51 | ... | ... | ... |
| \( \Xi \Xi^* \) | 0.75 | 0.77 | 0.47 | ... | ... | ... |
| \( \Omega^{-} \bar{\Omega} \) | 0.75 | 0.70 | 0.45 | ... | ... | ... |
| \( \Lambda^+_c \bar{\Lambda}^{' -} \) | 0.75 | 0.62 | 0.39 | 0.75 | 0.62 | 0.40 |
| \( \Sigma^0_0 \bar{\Sigma}^0 \) | 0.85 | 0.65 | 0.42 | 0.85 | 0.66 | 0.41 |
| \( \Sigma^+ \bar{\Sigma}^0 \) | 0.88 | 0.70 | 0.50 | ... | ... | ... |
| \( \Xi^0 \bar{\Xi}^0 \) | 0.75 | 0.61 | 0.39 | ... | ... | ... |
| \( \Xi^0 \bar{\Xi}^0 \) | 0.81 | 0.62 | 0.40 | 0.83 | 0.62 | 0.40 |
| \( \Lambda^+_c \bar{\Lambda}^{' -} \) | 0.75 | 0.61 | 0.38 | 0.75 | 0.61 | 0.38 |
| \( \Sigma^0_0 \bar{\Sigma}^0 \) | 0.83 | 0.64 | 0.38 | 0.83 | 0.64 | 0.38 |
| \( \Xi^0 \bar{\Xi}^0 \) | 0.86 | 0.67 | 0.38 | 0.87 | 0.68 | 0.39 |
| \( \Xi^0 \bar{\Xi}^0 \) | 0.75 | 0.61 | 0.38 | 0.75 | 0.60 | 0.39 |
| \( \Xi^0 \bar{\Xi}^0 \) | 0.80 | 0.61 | 0.38 | 0.82 | 0.61 | 0.38 |

observed in the radiative decay of \( J/\psi \) by BES collaboration. Therefore the bound states \( NN \) and \( \Delta \Delta \) may be the dominant components of the \( X(1835) \) and \( X(2370) \), respectively. Our interpretation is consistent with many authors’ points of view in the different theoretical frameworks [8]. Alternatively, the \( X(2370) \) could also be explained as the bound state \( N(1440)\bar{N} \) or \( N(1440)\bar{N} \) with Bethe-Salpeter equation [35]. For the light strange hexaquark systems \( \pi s\bar{s}u\bar{n} \) and \( \pi s\bar{s}u\bar{u} \), several weakly bounded states, \( \Lambda \bar{\Lambda}, \Sigma \Sigma, \Sigma \Sigma^* \), \( \Xi \Xi^* \) and \( \Xi \Xi^* \) exist in the color flux-tube model. The mass of the bound state \( \Lambda \bar{\Lambda} \) with \( J^{PC} = 1^{-+} \), 2186 MeV, is close to the experimental value of the \( Y(2175) \). Therefore the dominant component of \( Y(2175) \) could be treated as a bound state \( \Lambda \bar{\Lambda} \) in our model. The interpretation of \( Y(2175) \) as the bound state \( \Lambda \bar{\Lambda} \) with \( J^{PC} = 1^{-+} \) is also proposed in other constituent models [10, 33]. The weakly bounded state of \( \Sigma \Sigma \) was also obtained in Ref. [31]. The states \( \Lambda \bar{\Lambda}, \Sigma \Sigma \) and \( \Xi \Xi \) were investigated in the framework of the Bethe-Salpeter equation with a phenomenological potential and similar conclusions were arrived [35].

With respect to the heavy hexaquark systems with a \( c\bar{c} \) or \( bb \) pair, the states \( \Lambda^+_c \bar{\Lambda}^{' -} \), \( \Sigma^0_0 \bar{\Sigma}^0 \), \( \Xi^0 \bar{\Xi}^0 \), \( \Lambda^+_b \bar{\Lambda}^{' -} \), \( \Sigma_b \bar{\Sigma}_b \) and \( \Xi_b \bar{\Xi}_b \) can form baryonia with deep binding energies, while the binding energy of states \( \Sigma \Sigma^* \) and \( \Sigma \Sigma^* \) are several tens of MeVs. Concerning the states \( \Omega^{-} \bar{\Omega}^+ \), \( \Omega^0_0 \bar{\Omega}^0_0 \), \( \Omega^0_0 \bar{\Omega}^0_0 \) and \( \Omega^0_0 \bar{\Omega}^0_0 \), only the state \( \Omega^{-} \bar{\Omega}^+ \) has a shallow binding energy, about 8 MeV, the others are unbound. Compared with the state \( \Omega^{-} \bar{\Omega}^+ \), the states \( \Omega^0_0 \bar{\Omega}^0_0 \), \( \Omega^0_0 \bar{\Omega}^0_0 \) and \( \Omega^0_0 \bar{\Omega}^0_0 \) have a bigger quark mass, although it makes the kinetic energies lower, it can also reduce the color-magnetic interaction, which is disadvantaged to form a bound state in the model. The heavy baryonia with a \( c\bar{c} \) pair were systematically investigated within the framework of the one-boson-exchange \( \pi, \eta, \rho, \omega, \phi \) and \( \sigma \) model, it is suggested that the states \( \Lambda^+_c \bar{\Lambda}^{' -} \), \( \Sigma \Sigma^* \), \( \Xi \Xi^* \), and \( \Xi \Xi^* \) have deep attractive potentials in the short-distance domain [36], which is qualitatively consistent with our conclusions. It seems that the one-boson-exchange effect in the short-distance can be described by the coupling of different color flux-tube structures in our model, which is desired to be studied in the future work. The heavy partners \( \bar{\Lambda} \Lambda \) (with mass 4330 MeV) and \( \bar{\Lambda} \Lambda \) (with mass 10877 MeV) of the bound states \( \Lambda \Lambda \) may be used to explain the states \( Y(4260) \) or \( Y(4360) \), and \( Y(10890) \), respectively. The interpretation is consistent with the point of view in the paper [6, 37].

Those baryon-antibaryon bound states, if they really exist, can be observed in the corresponding baryon-antibaryon invariant mass spectrum when they are produced in the \( e^+e^- \) annihilation and charmonium or bottomonium decay processes, they can eventually decay into three color singlet mesons. Before the occurrence of this decay, the hidden color hexaquark states must change into several colorless subsystems by means of the rupture and recombination of color flux tubes because of a color confinement. This decay mechanism is similar to compound nucleus formation and therefore should induce a resonance, which is named as a “color confined, multiquark resonance” state in our models [38], it is different from all of those microscopic resonances discussed by S. Weinberg [39].

V. SUMMARY

The mass spectra of baryon-antibaryon states containing strange, charm and bottom quarks have been studied in the color flux-tube model with a multibody confinement interaction. A powerful numerical method with high precision, GEM is used in the calculation. The numerical results indicate that many low-spin states can not decay into a baryon and an anti-baryon but into three color singlet mesons by means of the rupture and recombination of color flux tubes, while the high-spin states cannot form bound states. The multi-body confinement interaction as a binding mechanism can globally give more attractions in the short-distance domain than the two-body one does in the study of the multiquark calculations, the effect seems to be equivalent to that of the \( \omega \) and \( \rho \) meson exchanges. In addition, the color-magnetic interaction can provide a further attraction for the low-spin states.

Our predicted bound baryon-antibaryon states \( \Sigma \Sigma, \Sigma \Sigma^*, \Xi \Xi^*, \Xi \Xi^*, \Xi \Xi^*, \Xi \Xi^* \), \( \Sigma \Sigma_0 \), \( \Sigma \Sigma_0 \), \( \Xi \Xi_0 \), and \( \Xi \Xi_0 \), if they really exist, can be observed in the corresponding baryon-antibaryon invariant mass spectrum when they are produced in the \( e^+e^- \) annihilation and charmonium or bottomonium decay processes. The dominant com-
components of the new hadron states, \(X(1835), X(2370), Y(2175), Y(4260)\) and \(Y_0(10890)\), may be interpreted as \(N\bar{N}, \Delta\bar{\Delta}, \Lambda\bar{\Lambda}, \Lambda_0\bar{\Lambda}_0\) bound states, respectively. The calculation of the decay properties of the states have to be invoked to justify the assignment, however, the present difficulty lies in the lack of the reliable knowledge of the rupture and recombination of color flux tubes, which is worth being studied in the future.

Acknowledgments

This research is supported partly by the National Science Foundation of China under Contract Nos. 11175088, 11035006, 11265017 and Chongqing Natural Science Foundation under the project No. cstc2013jcyjA00014.