Event-shape and Multiplicity dependence of Chemical freeze-out Parameters in Proton+Proton Collisions at $\sqrt{s} = 13$ TeV using PYTHIA8

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(Dated: June 11, 2019)

The event-shape and multiplicity dependence of chemical freeze-out temperature ($T_{ch}$), freeze-out radius ($R$) and strangeness saturation factor ($\gamma_s$) are obtained by studying the particle yields obtained from PYTHIA8 event generator in proton+proton (pp) collisions at $\sqrt{s} = 13$ TeV. Spherocity is one of the transverse event-shape techniques to isolate jetty and isotropic events in high-energy collisions and helps in looking into various observables in more differential manner. In this study, spherocity classes are divided into three categories, namely (i) Spherocity integrated, (ii) Isotropic, and (iii) Jetty. The chemical freeze-out parameters are extracted using a statistical thermal model as a function of spherocity classes and charged particle multiplicity in canonical, strangeness canonical and grand canonical ensembles. A clear observation of multiplicity and spherocity class dependence of $T_{ch}$, $R$ and $\gamma_s$ is observed. This study plays an important role in understanding the particle production mechanism in high-multiplicity pp collisions at the Large Hadron Collider energies.

PACS numbers: 13.85.Ni, 25.75.Dw

I. INTRODUCTION

Quark Gluon Plasma (QGP), a deconfined state of quarks and gluons, is believed to be produced in heavy-ion collisions at Relativistic Heavy-Ion Collider (RHIC) and Large Hadron Collider (LHC). However, the observations of QGP signatures like the strangeness enhancement\textsuperscript{1} and double ridge structure\textsuperscript{2} in high multiplicity pp collisions at the LHC, indicate a possible formation of QGP droplets in pp collisions. These discoveries have important consequences on whether to use the pp collisions as a baseline to understand a medium formation in heavy-ion collisions. Thus, a closer look at the underlying physics mechanisms in pp collisions has become very essential. Many of the QGP-like behaviours have been successfully explained by the phenomena such as multipartonic interactions, string fragmentation, color reconnection, rope hadronization etc., which are incorporated in PYTHIA8 event generator\textsuperscript{3}. Unlike the lower collision energies, where pp has been used as a reference measurement to study heavy-ion collisions, the pp collisions at the LHC energies has brought up new challenges and opportunities in terms of their high-multiplicity environment to study many newly emergent phenomena. The study of particle abundances after the chemical freeze-out and hence, characterizing the system using the freeze-out parameters is a requirement. In this direction, one uses the recently introduced transverse spherocity to separate jetty and isotropic events in pp collisions, as the production dynamics for both are different. When the jetty events involve high-$p_T$ phenomena and are described by pQCD, the isotropic events are mostly dominated by soft-physics (low-$p_T$). It would be interesting to look into the dependence of chemical freeze-out parameters on event-shape and multiplicity. In order to do that, we use a statistical thermal model in different tunes of ensembles and the method of transverse sphericity.

The thermal statistical model has been quite successful in describing the composition of stable particles in high-energy pp, p-Pb, Xe-Xe and Pb-Pb collisions at the LHC\textsuperscript{4, 5}. In general, the grand-canonical ensemble is used for heavy-ion collisions as the volume of the produced system is large enough so that it holds the relation $VT^3 > 1$. However for small collision systems, where the number of particles are small, one has to explicitly conserve the charges of the Quantum Chromodynamics (QCD) namely, Baryon Number (B), Strangeness (S) and electric charge (Q) in the small volume. Such a canonical treatment of particle production results in suppression of particles carrying non-zero quantum numbers as it has to be created in pairs. In this framework, yields of non-strange particles in small systems are expected to be very similar to those observed in large collision systems, whereas the strangeness production is expected to be suppressed in smaller systems. But, recent multiplicity dependent study of production of strange and multi-strange particles relative to pions in pp collisions at the LHC shows an enhancement and for high multiplicity pp collisions, the obtained values are similar to Pb-Pb collisions\textsuperscript{1}. This observation raised one of the key questions whether high multiplicity pp collision has reached a thermodynamical limit where both the canonical and grand canonical ensembles are equivalent. This is addressed in Ref.\textsuperscript{4, 5}, where it is indeed observed that the chemical freeze-out parameters obtained in high multiplicity pp collisions are similar to the peripheral Pb+Pb collisions.

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collisions. To address this behavior, one can also use strangeness canonical ensemble where the exact conservation of strangeness is required while the baryon and charge content is treated grand-canonical. Studying the chemical freeze-out parameters (CFO) such as chemical freeze-out temperature ($T_{ch}$), strangeness saturation factor ($\gamma_s$) and freeze-out radius ($R$) using multiplicity alone could be an incomplete study as in each multiplicity classes we can have jetty as well as isotropic events. To separate the jetty and isotropic events from the average shaped events, one should look into the geometrical shape of events using event shape observables.

Transverse spherocity, one of the event-shape observables has given a new direction for underlying events in pp collisions to have further differential study along with multiplicity. Recently, the kinetic freeze-out scenario and system thermodynamics is studied using event shape and multiplicity in pp collisions at $\sqrt{s} = 13$ TeV using PYTHIA8 [6, 7]. It would be interesting to look into the spherocity dependence of chemical freeze-out parameters, which would provide a complete picture of formation of QGP droplets and possible hadronic phase in high multiplicity px collisions. In this work, we perform a double differential study of chemical freeze-out parameters using spherocity and final state charged-particle multiplicity in pp collisions at $\sqrt{s} = 13$ TeV using PYTHIA8.

![Jetty and Isotropic Events](image)

**TABLE I: V0M multiplicity classes and the corresponding charged particle multiplicities.**

| V0M class | I | II | III | IV | V | VI | VII | VIII | IX | X |
|-----------|---|----|-----|----|---|----|-----|------|----|---|
| $N_{ch}$  | 50-140 | 42-49 | 36-41 | 31-35 | 27-30 | 23-26 | 19-22 | 15-18 | 10-14 | 0-9 |

It is worth noting here that although there are many tunes of physics processes in PYTHIA used to explain observable specific physics findings, our aim in this paper, is to see the event-shape and multiplicity dependence of CFO parameters.

Transverse spherocity is defined for an unit vector $\hat{n}(n_T,0)$ which minimizes the following quantity [11]:

$$S_0 = \frac{\pi^2}{4} \left( \frac{\sum_i \vec{p}_{T_i} \times \hat{n}}{\sum_i p_{T_i}} \right)^2,$$

(1)

where $p_{T_i}$ is the transverse momentum of $i^{th}$ particle. The distinct configuration of events if they are isotropic or jetty in transverse plane are coupled to the extreme

II. EVENT GENERATION AND ANALYSIS METHODOLOGY

For this analysis, we have used PYTHIA8 event generator to simulate ultra-relativistic proton+proton collisions. It is a blend of many models and theory relevant for physics like parton distributions, hard and soft interactions, initial and final-state parton showers, fragmentation, multipartonic interactions, color reconnection and decay [8]. We use 8.235 version of PYTHIA, which includes multi-partonic interaction (MPI) which is crucial to explain the underlying events and multiplicity distributions, along with other observables. Also, this version includes flow-like patterns in terms of color reconnection. The detailed description of the physics processes in PYTHIA8.235 can be found in Ref. [3]. We have implemented the inelastic, non-diffractive component of the total cross-section for all soft QCD processes using the switch (SoftQCD : all = on). Around 250 million events were generated at $\sqrt{s} = 13$ TeV with Monash 2013 Tune (Tune:14) [9]. We use MPI based scheme of color reconnection (ColorReconnection:mode(0)). One should note that for our analysis, the minimum bias events are those events where no selection on charged particle multiplicity and spherocity is applied. For the generated events, all the resonances were allowed to decay except the ones used in our study (HadronLevel:Decay = on). In our analysis the event selection criteria is such that only those events were chosen which have at-least 5 tracks (charged particles) and the charged particle multiplicities ($N_{ch}$) have been chosen in the acceptance of V0 detector with pseudo-rapidity range of V0A ($2.8 < \eta < 5.1$) and V0C ($-3.7 < \eta < -1.7$) [10] to match with experimental conditions in ALICE. The events generated using these cuts are then further divided in ten multiplicity (V0M) classes, each class containing 10% of total events. Table I shows all ten charged particle multiplicity classes.
III. USE OF ENSEMBLES IN THERMAL MODEL

In this study, $(\pi^+ + \pi^-)/2$, $(K^+ + K^-)/2$, $(p + \bar{p})/2$, $\phi$ and $(\Lambda + \bar{\Lambda})/2$ yields at mid-rapidity obtained from PYTHIA8 is used to extract the CFO parameters in THERMUS [12] based on canonical, strangeness canonical and grand canonical ensembles. For convenience, here onwards $(\pi^+ + \pi^-)/2$, $(K^+ + K^-)/2$, $(p + \bar{p})/2$ and $(\Lambda + \bar{\Lambda})/2$ are denoted as $\pi$, $K$, $p$ and $\Lambda$ respectively. Let us now discuss about these three ensembles in details.

A. Grand canonical ensemble (GCE)

The GCE is generally used in applications to heavy-ion collisions. In this ensemble, energy and number of particles along with their quantum numbers are enforced to conservation laws on an average by temperature and chemical potential. The partition function for a system with $N$-hadrons within a volume $V$ with temperature $T$, and chemical potential, $\mu$ is given by,

$$\ln Z^{GCE}(T, V, \{\mu_i\}) = \sum_i \frac{g_i V}{(2\pi)^3} \int d^3p \exp \left( -\frac{E_i - \mu_i}{T} \right).$$  

$$\ln \left( 1 \pm e^{-\left( E_i - \mu_i \right)/T} \right)^{\pm 1}. \hspace{1cm} (2)$$

Here, $g_i$ is the spin-isospin degeneracy factor and $\mu_i$ is the chemical potential of $i^{th}$-species. $T$ is the temperature and the “+” and “−” signs in the distribution functions refer to bosons and fermions, respectively. Here, the individual particle numbers are not conserved while the quantum numbers $B$, $S$, and $Q$ are conserved. For $i^{th}$-hadron, the chemical potential is given by,

$$\mu_i = B_i \mu_B + S_i \mu_S + Q_i \mu_Q. \hspace{1cm} (3)$$

Here, $B_i$, $S_i$ and $Q_i$ are the baryon number, strangeness and electric charge of the $i^{th}$-hadron, respectively. Applying the Boltzmann approximation to partition function,

$$\ln Z^{GCE}(T, V, \{\mu_i\}) = \sum_i \frac{g_i V}{(2\pi)^3} \int d^3p \exp \left( -\frac{E_i - \mu_i}{T} \right).$$  

Hence, the particle multiplicity in GCE is given by,

$$N_i^{GCE} = \frac{g_i V}{(2\pi)^3} \int d^3p \exp \left( -\frac{E_i - \mu_i}{T} \right). \hspace{1cm} (5)$$

At the LHC energies, the $\mu \approx 0$ and the expression for particle multiplicities will become,

$$N_i^{GCE} = \frac{g_i V}{(2\pi)^3} \int d^3p \exp \left( -\frac{E_i}{T} \right). \hspace{1cm} (6)$$

The particle yields measured by detectors in ultra-relativistic collisions also include the feed down from the heavier hadrons and resonances. This has significant contribution for lighter particles like pions. Thus the final yields is given by,

$$N_i^{GCE} (\text{total}) = N_i^{GCE} + \sum_j Br(j \rightarrow i) N_i^{GCE}. \hspace{1cm} (7)$$

Here, $Br(j \rightarrow i)$ is the number of $i^{th}$-species into which a single particle of species $j$ decays.

B. Strangeness canonical ensemble (SCE)

The SCE, also called as mixed canonical ensemble in THERMUS terminology, requires the exact conservation of strange quantum number while the baryon and charge content are treated grand-canonically. Thus, the partition function is given by,

$$Z_{SC} = \frac{1}{(2\pi)} \int_0^{2\pi} d\phi e^{-iS\phi} Z_{GCE}(T, \mu_B, \lambda_S) \hspace{1cm} (8)$$

Here, we will only consider the case where overall strangeness is zero, $S = 0$. The chemical potential of $i^{th}$-hadron is given by,

$$\mu_i = B_i \mu_B + Q_i \mu_Q, \hspace{1cm} (9)$$

and the fugacity factor is replaced by,

$$\lambda_S = e^{i\phi}. \hspace{1cm} (10)$$

As previously done for the case of GCE, the decays of resonances have to be added to the final yield. Thus, the expression for particle multiplicity is given by,

$$N_i^{SCE} (\text{total}) = N_i^{SCE} + \sum_j Br(j \rightarrow i) N_i^{SCE}. \hspace{1cm} (11)$$
C. Canonical ensemble (CE)

In CE, the conservation of quantum numbers corresponding to B, S and Q are exactly enforced. The partition function is given by,

\[ Z^{CE} = \left( \frac{1}{2\pi} \right)^{3N} \int_0^{2\pi} d\phi e^{-iQ\phi} \int_0^{2\pi} d\psi e^{-iQ\psi} \int_0^{2\pi} d\alpha e^{-iB\alpha} \int_0^{2\pi} d\beta e^{-iS\beta} Z_{GCE}(T, \lambda_B, \lambda_Q, \lambda_S). \]

Here, the fugacity factor is replaced by

\[ \lambda_B = e^{i\alpha}, \quad \lambda_Q = e^{i\psi}, \quad \lambda_S = e^{i\phi}. \]

The feed downs from resonances have to be added to the final yield similar to GCE,

\[ N_i^{CE}(\text{total}) = N_i^{CE} + \sum_j Br(j \rightarrow i) N_j^{CE}. \]

These ensembles are implemented in the THERMUS program and we refer to [12] for more detailed steps and their implementation in THERMUS.

The number density, energy density and pressure of each hadron species in the Boltzmann approximation within the canonical ensemble differ from the grand canonical ensemble by a multiplicative factor with all the chemical potentials set to zero. This multiplicative correction factor depends on thermal parameters and the quantum numbers of the particle. For large-systems one approaches the grand-canonical ensemble while the extension of thermal model to elementary collisions requires an additional parameter known as strangeness saturation tension of thermal model to elementary collisions requires an additional parameter known as strangeness saturation tension of thermal model to elementary collisions.

Here, \(|s_i|\) is the number of valence strange quarks and anti-quarks in \(i^{th}\) species.

Keeping this in mind and considering above three ensembles, we proceed with our calculation of CFO parameters in different multiplicity and sphericity classes in the next section using the identified particle yields obtained from PYTHIA event generator.

IV. RESULTS AND DISCUSSION

The grand canonical, strangeness canonical and canonical ensembles are considered for pp at \(\sqrt{s} = 13\) TeV using PYTHIA8 Monash tune for three sphericity classes namely, sphericity integrated, isotropic and jetty events. We have considered \(\pi, K, p, \phi\) and \(\Lambda\) yields in THERMUS for our study. The simulated results in high multiplicity pp collisions at \(\sqrt{s} = 13\) TeV for different sphericity classes allow us to access into very high multiplicity events in small collision systems and further investigate about the applicability of these three ensembles. Looking into the particle-antiparticle symmetry at the LHC energies, the net baryon number and strangeness numbers are set to zero. Hence the canonical ensemble differs from the grand-canonical ensemble by a multiplicative factor as mentioned in the previous section. But, in the thermodynamic limit they must be equivalent.

The particle yields in pp collisions are fitted using THERMUS [12] for the grand canonical, strangeness canonical and canonical ensembles with three free parameters namely, \(T_{ch}, \gamma_s\) and \(R\). In case of SCE, we have an additional free parameter, known as the canonical radius. The extracted canonical radius is found to be similar to the freeze-out radius. The comparison between model and data for sphericity integrated class for highest multiplicity class is shown in Fig. 2. In the top panel, the black bar shows the simulated data points obtained using PYTHIA8. The blue solid circles are obtained for the grand-canonical ensemble, black solid stars are for strangeness canonical ensemble and red rectangular boxes are for the canonical ensemble. The bottom panel shows the standard deviation.
Standard Deviation = $\frac{\text{PYTHIA8} - \text{Model(M)}}{\sigma_E}$. (16)

Here $\sigma_E$ are the statistical errors in particle yields.

Similarly the comparison of particle yields in thermal model and data for the isotropic and jetty events for high multiplicity class are shown in Figs. 3 and 4, respectively. The fitting of thermal model to simulated data points is performed as a function of charged particle multiplicity and sphericity classes for all the ensembles. The goodness of fit, $\chi^2$/NDF values for different multiplicity and sphericity classes in pp collisions at $\sqrt{s} = 13$ TeV for different ensembles are given in Table II, III and IV. The obtained $\chi^2$/NDF is better for the higher charged particle multiplicity. The extracted thermodynamic parameters are shown in Fig. 5, 6 and 7 as a function of charged particle multiplicity for three different sphericity classes.

In Fig. 5, we show the fireball radius $R$ as a function of charged particle multiplicity for different sphericity classes in pp collisions at $\sqrt{s} = 13$ TeV by considering the grand-canonical, strangeness canonical and canonical ensembles. The freeze-out radius increases with charged particle multiplicity for all three ensembles. However, the radius obtained for the canonical ensemble is lower compared to other two ensembles for lower charged particle multiplicity. The radius in canonical ensemble approaches towards the strangeness canonical ensemble for above charged particle multiplicity around 20. For the strangeness canonical ensemble, the value of the radius is lying between the canonical and grand canonical ensembles. For isotropic events, the obtained radius is higher compared to the jetty and the sphericity integrated ones.
as seen by ALICE [14]. It is observed that the fireball radius is lower for the jetty like events. In all the three ensembles, the observed behaviour is similar.

In Fig. 6, the chemical freeze-out temperature $T_{ch}$ is shown as a function of charged particle multiplicity for different spherocity classes in pp collisions at $\sqrt{s} = 13$ TeV by taking grand-canonical, strangeness canonical and canonical ensembles. In general, the freeze-out temperature obtained for the canonical ensemble is higher compared to the GCE and SCE. The trend of freeze-out temperature shows a slight increase with charged particle multiplicity for GCE and SCE while CE shows a decreasing behaviour as a function of charged-particle multiplicity. Beyond charged-particle multiplicity of around 20, the freeze-out temperature for CE is similar to SCE. In line with the previous observation [4], this threshold number of charged particles in the final state appears to be a thermodynamic limit. Contrary to the radius of the fireball, the $T_{ch}$ is highest for jetty events and lowest for isotropic events. CE shows a larger separation of $T_{ch}$ in spherocity classes compared to other ensembles.

As mentioned in the previous section, the strangeness saturation factor $\gamma_s$, is responsible for the degree of deviation from the strangeness chemical equilibrium in the strangeness sector. Thus, it is important to understand the strange particle production in smaller collision systems as a function of charged particle multiplicity and spherocity classes. Fig. 7 shows the strangeness saturation factor $\gamma_s$ as a function of charged particle multiplicity for different spherocity classes in pp collisions at $\sqrt{s} = 13$ TeV by considering grand-canonical, strangeness canonical and canonical ensembles. The strangeness saturation factor $\gamma_s$ increases with multiplicity in pp collisions and reaches unity for the higher charged particle

| Multiplicity Classes | Strangeness Canonical | Canonical | Grand Canonical |
|----------------------|-----------------------|-----------|-----------------|
| V0M1                 | 23.325/1              | 30.284/2  | 33.681/2        |
| V0M2                 | 23.789/1              | 30.092/2  | 33.711/2        |
| V0M3                 | 24.278/1              | 30.432/2  | 34.209/2        |
| V0M4                 | 24.905/1              | 31.062/2  | 34.935/2        |
| V0M5                 | 25.572/1              | 31.463/2  | 36.2792         |
| V0M6                 | 26.521/1              | 32.629/2  | 36.2792         |
| V0M7                 | 27.856/1              | 33.829/2  | 36.88/2         |
| V0M8                 | 29.731/1              | 35.686/2  | 38.361/2        |
| V0M9                 | 33.148/2              | 38.961/2  | 41.379/2        |
| V0M10                | 40.044/1              | 44.328/2  | 46.803/2        |

| Multiplicity Classes | Strangeness Canonical | Canonical | Grand Canonical |
|----------------------|-----------------------|-----------|-----------------|
| V0M1                 | 18.591/1              | 25.978/2  | 30.078/2        |
| V0M2                 | 19.158/1              | 26.429/2  | 30.891/2        |
| V0M3                 | 19.384/1              | 26.408/2  | 30.889/2        |
| V0M4                 | 19.99/1               | 26.848/2  | 31.085/2        |
| V0M5                 | 21.105/1              | 28.074/2  | 32.199/2        |
| V0M6                 | 22.319/1              | 29.232/2  | 32.961/2        |
| V0M7                 | 24.322/1              | 31.153/2  | 34.477/2        |
| V0M8                 | 26.650/1              | 33.297/2  | 36.325/2        |
| V0M9                 | 30.909/1              | 36.997/2  | 39.691/2        |
| V0M10                | 38.791/1              | 43.634/2  | 46.007/2        |

| Multiplicity Classes | Strangeness Canonical | Canonical | Grand Canonical |
|----------------------|-----------------------|-----------|-----------------|
| V0M1                 | 21.808/1              | 28.850/2  | 32.449/2        |
| V0M2                 | 21.984/1              | 28.679/2  | 32.636/2        |
| V0M3                 | 22.177/1              | 28.698/2  | 32.745/2        |
| V0M4                 | 22.564/1              | 29.015/2  | 33.006/2        |
| V0M5                 | 23.153/1              | 29.608/2  | 33.436/2        |
| V0M6                 | 24.051/1              | 30.515/2  | 34.024/2        |
| V0M7                 | 25.576/1              | 32.055/2  | 35.167/2        |
| V0M8                 | 27.699/1              | 34.113/2  | 36.912/2        |
| V0M9                 | 31.452/1              | 37.473/2  | 39.976/2        |
| V0M10                | 38.964/1              | 40.147/2  | 46.069/2        |
multiplicity for grand-canonical and strangeness canonical ensembles. This indicates the strangeness chemical equilibrium for high multiplicity pp collisions at $\sqrt{s} = 13$ TeV. For higher multiplicity classes, $\gamma_s$ is similar for grand-canonical and strangeness canonical ensembles. However, when we study the yields with exact conservations of all the three quantum numbers i.e, for canonical ensemble, the $\gamma_s$ is below one for the higher charged particle multiplicity classes. In case of spherocity dependence study of $\gamma_s$, it is higher for the jetty events followed by spherocity integrated and isotropic events although the separation is not very significant except for the lowest multiplicity class.

As we observe significant dependence of CFO parameters with sphericity and multiplicity classes, to compliment/confront our results, it will be interesting to look into the particle yields in pp collisions at $\sqrt{s} = 13$ TeV once the sphericity dependent experimental data points are available.

V. SUMMARY AND CONCLUSION

Recent heavy-ion like properties such as enhanced production of strange particles with respect to pions [1], degree of collectivity [15], hardening of particle spectra with multiplicity [16, 17] etc. are seen in high-multiplicity events in pp collisions at the LHC energies and have drawn considerable interest in the research community. In this manuscript, we have extracted CFO parameters such as the chemical freeze-out radius (R), temperature ($T_{ch}$) and strangeness saturation factor ($\gamma_s$) by studying the particle yields in a thermal model for pp at $\sqrt{s} = 13$ TeV in different multiplicity and sphericity classes with primary focus on the dependence of CFO parameters with geometrical shape of an event. Here, three types of ensembles namely, grand-canonical, strangeness canonical and canonical have been used to look into their applicability in small collision systems. This study has been performed by considering $\pi$, $K$, $p$, $\phi$ and $A$ yields in THERMUS. The important findings are summarised below:

- The freeze-out radius increases with charged particle multiplicity and highest value is observed for the grand canonical ensemble followed by strangeness canonical and canonical ensemble.

- For higher charged particle multiplicity, the radii obtained in canonical ensemble are similar to the strangeness canonical value.

- While studying as a function of sphericity classes, the obtained radius is found to be higher for the isotropic events followed by spherocity integrated and jetty events. As the process of isotropization takes more time, the radii being higher make sense and points towards a lower freeze-out temperature, which is in fact observed in our case for all ensembles under study.

- The chemical freeze-out temperature $T_{ch}$ increases with charged particle multiplicity for grand canonical and strangeness canonical ensembles whereas canonical ensemble shows a reverse trend.

- As seen in the case of radius parameter, the chemical freeze-out temperature is similar for strangeness canonical and canonical ensemble at higher charged particle multiplicities.

- The strangeness saturation factor, $\gamma_s$ shows a clear evolution as a function of charged particle multiplicity and reaches the value one for GCE and SCE, which indicates complete strangeness chemical equilibrium.

- The strangeness saturation factor, $\gamma_s$ is higher for the grand canonical ensemble and minimum for the canonical ensemble.

- A final state multiplicity of around $N_{ch} \sim 20$, appears to be a thermodynamic limit, where the freeze-out parameters become almost independent of ensembles.

- The sphericity dependence of CFO parameters clearly show significant separation for different geometrical shape of events.

In this study the grand-canonical and strangeness canonical ensembles converge for the higher charged particle multiplicity classes. For high multiplicity pp collisions $\gamma_s$ reaches unity, which shows a full strangeness
chemical equilibrium. It will be more interesting to look into the high multiplicity pp events in different sphericity classes in future when experimental data are available to get more information about the strangeness chemical equilibrium and to understand the particle production mechanism in smaller collision systems.

Acknowledgement

The authors acknowledge the financial supports from ALICE Project No. SR/MF/PS-01/2014-IITI(G) of Department of Science & Technology, Government of India. RR and ST acknowledge the financial support by DST-INSPIRE program of Government of India. The authors thankfully acknowledge the helps of Ashish Bisht, MSc student of IIT Indore for PYTHIA data generation. The authors further acknowledge the usage of resources of the LHC grid computing facility of VECC, Kolkata.

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