BPS Domain Walls in Large $N$ Supersymmetric QCD

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Abstract

We explicitly construct BPS domain walls interpolating between neighboring chirally asymmetric vacua in a model for large $N$ pure supersymmetric QCD. The BPS equations for the corresponding $Z_N$ symmetric order parameter effective Lagrangian reduce to those in the $A_N$ Landau-Ginsburg model assuming that the higher derivative terms in the Kähler potential are suppressed in the large $N$ limit. These BPS domain walls, which have vanishing width in the large $N$ limit, can be viewed as supermembranes embedded in a (3+1)-dimensional supersymmetric QCD background. The supermembrane couples to a three-form supermultiplet whose components we identify with the composite fields of supersymmetric QCD. We also discuss certain aspects of chromoelectric flux tubes (open strings) ending on these walls which appear to support their interpretation as D-branes.
I. INTRODUCTION

Supersymmetric gauge theories provide opportunities to gain analytic control over certain complicated non-perturbative phenomena in (3+1)-dimensional strongly coupled gauge models \(^1\). The closest supersymmetric cousin of QCD, \(\mathcal{N} = 1\) supersymmetric \(SU(N)\) gluodynamics (pure supersymmetric QCD), is believed to be a confining gauge theory with a mass gap and chiral symmetry breaking \(\mathcal{N} = 1\). One can therefore expect that a better understanding of pure supersymmetric QCD (SQCD)\(^1\) might shed some light on non-perturbative phenomena in conventional Quantum Chromodynamics.

Chiral symmetry breaking in SQCD is due to gluino condensation. The composite operator \(\text{Tr}(\lambda^a\lambda^a) \equiv \lambda\lambda\) acquires a non-zero vacuum expectation value (VEV) \(\langle \lambda\lambda \rangle_k = N \exp(2\pi i k/N)\Lambda^3\), where \(\Lambda\) is the dynamically generated scale of SQCD. Thus, there are \(N\) inequivalent vacua related by \(\mathbb{Z}_N\) discrete symmetry and labeled by the phase of the gluino condensate \(k = 0, \ldots, N - 1\). In each particular vacuum state the discrete \(\mathbb{Z}_N\) symmetry is broken. As a result, there should exist domain walls separating pairs of distinct vacua.

In \(\mathcal{N} = 1\) supersymmetry (SUSY) algebra admits a nontrivial central extension if BPS saturated domain walls are present in the theory. Assuming that such BPS walls exist, their tension can be calculated exactly on the basis of the SUSY algebra alone \(\mathcal{N} = 1\). Whether the BPS saturated domain walls are indeed present in a given model is a dynamical issue. The strong coupling nature of SQCD, however, makes it difficult to study this problem in terms of colored fields. In order to circumvent this difficulty, one can use an order parameter effective Lagrangian relevant for the chiral symmetry breaking. Thus, in the large \(N\) SQCD one could attempt to solve the corresponding BPS equations in terms of colorless composite variables (order parameters). This approach, as we will discuss later, can be argued to be reliable in the limit of infinitely large \(N\). This is the route we are going to follow in the present work. In particular, we will explicitly construct BPS saturated domain walls interpolating between neighboring chirally asymmetric vacua in a model for large \(N\) SQCD. The width of these walls is vanishing in the large \(N\) limit. We argue that this is precisely the property which gives rise to the wall tension consistent with the prediction of the SUSY algebra \(\mathcal{N} = 1\).

Having found the BPS walls in the large \(N\) limit, we discuss more general questions related to physics of extended objects in SQCD. In fact, we address the issue whether the lowest-spin states of SQCD can be viewed as excitations of such extended objects. Our discussion is motivated by the relation of the large \(N\) gauge models and string theories. Indeed, in \(\mathcal{N} = 1\) it was shown that in the large \(N\) limit the gauge theory diagrams are organized in terms of Riemann surfaces with boundaries and handles, and addition of each extra handle on the surface corresponds to suppression by a factor of \(1/N^2\). This observation leads to the hope that the large \(N\) gauge theory might be described by some kind of string theory. Then the large \(N\) expansion of gauge theories would be mapped to the string expansion in terms of properly weighted world-sheets of various topologies.

\(^1\)Throughout this paper “SQCD” refers to the \(\mathcal{N} = 1\) supersymmetric \(SU(N)\) gauge theory without matter, \textit{i.e.}, pure supersymmetric gluodynamics.
The first concrete realization of this idea in the context of weakly coupled gauge theories (i.e., when the effective gauge coupling $\lambda = N g_M^2$ is fixed at a value $\lambda \lesssim 1$) was given in [11] in the framework of three-dimensional Chern-Simons gauge theory where the boundaries of the string world-sheet are "topological" D-branes. More recently, in [13] string expansion was shown to precisely reproduce 't Hooft's large $N$ expansion for certain four dimensional (super)conformal gauge theories.

In strongly coupled gauge theories, however, the story appears to be much more involved. In particular, it is unknown what is the string theory that governs the dynamics of the QCD string which is expected to arise as an effective description in the large $N$ limit. This string theory is expected to be non-critical [14], which makes it difficult to study. Nonetheless, in [16] Witten has made an interesting observation that there might be a connection between SQCD strings and domain walls in the large $N$ limit. In particular, based on the assumption that the SQCD domain walls are BPS saturated [9], Witten argued that in the large $N$ limit the domain walls connecting two vacua labeled by $k$ and $k' = k + 1$ appear to be objects that look like D-branes [17] on which the SQCD string should be able to end. Such D-brane-like domain walls have also appeared in [18] in the context of large $N$ non-supersymmetric QCD.

We argue in this work that in the large $N$ limit the BPS domain walls with the vanishing width can be viewed as supermembranes embedded in an SQCD background. Moreover, the component fields of the three-form supermultiplet which couples to the supermembrane can be identified in terms of the composite fields of SQCD. We also point out that some of the lowest-spin states of SQCD can be viewed as excitations of a closed string which propagates in the vacua separated by the supermembranes. A BPS supermembrane can be regarded as a D-brane in the large $N$ limit if there are open strings ending on it. We discuss certain aspects of chromoelectric flux tubes ending on the domain walls [19] which appear to support the D-brane interpretation of these objects.

The rest of this paper is organized as follows. In section II we discuss the large $N$ behavior of a generic SQCD configuration and point out why a BPS state can exist in the large $N$ limit. Then, we briefly discuss the $\mathbb{Z}_N$ symmetric order parameter effective Lagrangian for SQCD (see [20,21] and [22]), and also give a general discussion of domain walls in $\mathcal{N} = 1$ supersymmetric theories. In section III we explicitly construct BPS domain walls using a model $\mathbb{Z}_N$ symmetric order parameter effective superpotential [22], by solving the corresponding BPS equations in the large $N$ limit. Note that large $N$ BPS domain walls have been constructed in [23]. The idea of [23] is to avoid certain complications arising in SQCD by taking an indirect route. In [23] it was argued that in the large $N$ limit the problem effectively reduces to the question of finding BPS solitons in the large $N$ A$_N$ Landau-Ginsburg theory. In this paper we construct the BPS domain walls directly within the framework of the $\mathbb{Z}_N$ symmetric model of large $N$ SQCD. We show that the corresponding system of BPS equations indeed reduces to that in the large $N$ A$_N$ Landau-Ginsburg model provided that higher derivative terms in the Kähler potential are suppressed in the large $N$ limit. This way we derive the results of [23] which were obtained via an indirect construction.

\[\text{For recent developments in these directions, see [12].}\]

\[\text{This was subsequently generalized to include unoriented world-sheets in [14].}\]
In section IV we elucidate some relations to other works on the subject. In section V we present our discussions and conclusions.

II. $Z_N$ SYMMETRIC VACUA AND DOMAIN WALLS

In this section we review some facts about the large $N$ limit of $\mathcal{N} = 1$ supersymmetric QCD. We argue that the SQCD domain wall configurations can saturate the BPS bound if the width of the walls vanishes in the large $N$ limit. We will also discuss the $Z_N$ symmetric order parameter effective superpotential of $[22]$ which effectively encodes dynamics relevant for describing large $N$ domain walls.

A. Gaugino Condensation and Domain Walls in SQCD

Consider $\mathcal{N} = 1$ supersymmetric QCD with $SU(N)$ gauge group and no matter. Let $\Lambda$ be the dynamically generated scale of the model. In this theory the gluino condensate is given by $[4]$:

$$\langle \lambda \lambda \rangle_k = N \exp\left(\frac{2\pi i k}{N}\right) \Lambda^3.$$  \hspace{1cm} (1)

Here $k = 0, \ldots, N - 1$, that is, there are $N$ inequivalent vacua corresponding to $N$ different phases for the gaugino condensate. Note that the overall factor of $N$ in (1) follows from the fact that the gaugino condensate $\langle \lambda \lambda \rangle$ involves a trace over the gauge indices, and the resulting VEV should contain the corresponding second Casimir factor.

According to (1), there are $N$ inequivalent vacua with spontaneously broken discrete symmetry in SQCD. Hence, there should exist domain walls separating these different vacua $[9]$. On dimensional grounds we expect the tension of such a wall to be of order $\Lambda^3$. It is, however, possible to compute the tension exactly in terms of the gluino condensate provided that they are BPS saturated $[9]$. Consider the domain wall separating the vacua with $\langle \lambda \lambda \rangle_k = N \exp\left(\frac{2\pi i k}{N}\right) \Lambda^3$ and $\langle \lambda \lambda \rangle_{k'} = N \exp\left(\frac{2\pi i k'}{N}\right) \Lambda^3$. The tension of the wall (provided that it is BPS saturated) is defined by the central charge $Q_{kk'}$ in the corresponding central extension of the $\mathcal{N} = 1$ superalgebra. The central charge is proportional to the absolute value of the difference between the values of the superpotential in the two vacua:

$$Q_{kk'} \propto |\mathcal{W}_k - \mathcal{W}_{k'}|.$$ \hspace{1cm} (2)

We therefore have the following tension for the BPS domain walls:

$$T_{kk'} = \frac{N^2}{4\pi^2} \left| \sin\left(\frac{\pi (k - k'N)}{N}\right) \right| \Lambda^3.$$ \hspace{1cm} (3)

It is, however, difficult to explicitly construct these domain walls (or even check that they are BPS saturated) in SQCD: the order parameter $\lambda \lambda$ is a composite operator, and we are

A number of interesting properties of domain walls in various models were discussed in $[24]$. $[35]$. 

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dealing with a strong coupling regime. Instead, we will be able to explicitly find BPS domain walls in large $N$ SQCD using the $Z_N$ symmetric order parameter effective superpotential \[22\].

The presence of BPS saturated domain walls in large $N$ SQCD has important implications \[16\]. Strongly coupled SQCD in the large $N$ limit is believed to be described by a string theory with the string coupling $\lambda_s \sim 1/N$. Extended solitons in this string theory are expected to have tension which goes as $1/\lambda_s^2 \sim N^2$. Let us, however, consider BPS domain walls with $k' = k + 1$. In the large $N$ limit their tension goes as $\sim N \sim 1/\lambda_s$. In \[16\] it was suggested that such domain walls can be viewed as D-branes \[17\] rather than solitons in the SQCD string context. Open SQCD strings then can end on these D-branes \[16\].

B. Domain Walls at Large $N$

Let us consider the domain walls with $k' = k + 1$ in the large $N$ limit. According to (3) the tension of such a domain wall (provided that it is BPS saturated) is given by

$$T_D = \frac{N}{4\pi} \Lambda^3.$$  \hspace{1cm} (4)

The fact that $T_D$ scales as $N$ in the large $N$ limit might appear a bit puzzling if one thinks of such BPS domain walls as SQCD solitons. Indeed, the number of degrees of freedom (gluons and gluinos) in SQCD scales as $N^2$ in the large $N$ limit, so naively one expects the energy density of a generic SQCD configuration to scale not as $N$ but as $N^2$. If this argument were precise, it would imply that the large $N$ SQCD domain walls with $k' = k + 1$ could not be BPS saturated. However, in the following we will argue that the $N$ dependence of the energy density $T_D$ given by (4) is indeed correct for the corresponding BPS saturated SQCD solitons

In the large $N$ limit the effective Lagrangian is expected to have the following form:

$$\mathcal{L} = N^2 \mathcal{L}_0(\Phi, \nabla \Phi, \nabla^2 \Phi, \ldots),$$  \hspace{1cm} (5)

where $\Phi$ collectively denotes composite colorless bound states of gluons and/or gluinos. The corresponding fields $\Phi$ are rescaled in such a way that all the leading $N$ dependence in (4) is in the overall factor of $N^2$. In particular, the equations of motions for $\Phi$ do not contain any factors of $N$. Thus, in the large $N$ limit the volume energy density of any solitonic configuration $\epsilon \sim N^2 \Lambda^4$. This implies that the tension of a domain wall $T \sim \epsilon d$ is proportional to $N^2 \Lambda^3$ if the “width” of the domain wall $d \sim 1/\Lambda$. However, if $d \sim 1/N \Lambda$, then we have $T \sim N \Lambda^3$. This indicates that the domain walls with $k' = k + 1$ should have vanishingly small width $d \sim 1/N \Lambda$ in the large $N$ limit \[23\]. In section III we will explicitly show that this is indeed the case. Here, however, we will give a simple explanation of why $d \sim 1/N \Lambda$ for such domain walls.

Since all the fields $\Phi$ in $\mathcal{L}_0$ are such that $|\Phi| \sim 1$ in the large $N$ limit, solitonic solutions with $d$ depending on $N$ might at first seem impossible to accommodate. Thus, naively one

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\[5\] We are grateful to M. Shifman for useful conversations on the issues discussed in this subsection.
might expect that $\mathcal{L}_0$ has no “knowledge” of $N$. This, however, is not the case provided that the number of inequivalent vacua described by $\mathcal{L}_0$ scales as $N^{19}$, which is precisely what happens in SQCD. This way the Lagrangian $\mathcal{L}_0$ does “know” about $N$. As a simple toy example of such a system consider a single chiral superfield $\Phi$ with the scalar potential $V(\Phi) = N^2|1 - \Phi|^2$. There are $N$ inequivalent vacua described by $\mathcal{L}_0$; $k = 0, 1, \ldots, N - 1$. Thus, even though $|\Phi_k| \equiv 1$ in $V$, we have $|\Phi_{k+1} - \Phi_k| \propto 1/N$ in the large $N$ limit.

In the following we will show that the model effective Lagrangian for supersymmetric gluodynamics [22] possesses precisely the properties discussed above. In fact, among other things it describes $N$ inequivalent vacua with the broken chiral symmetry. In the large $N$ limit it admits BPS solitons with the width proportional to $1/N$. These solitons are the BPS domain walls separating neighboring chirally asymmetric vacua of the theory.

### C. The Order Parameter Effective Action

The $N$ chirally asymmetric vacua in SQCD are defined by the VEV of the order parameter $\lambda\lambda$, the gluino bilinear. To construct the domain walls interpolating between different vacua we can try to write down an effective Lagrangian for this order parameter. The full effective Lagrangian (5) would contain infinite number of fields corresponding to higher dimensional operators of the theory and higher derivatives as well. However, in the large $N$ limit we expect certain simplifications, in fact, the higher dimensional operator contributions should be suppressed by extra powers of $N$. More precisely, we expect this to be the case for domain walls interpolating between neighboring vacua; in this case the relative change in the gluino condensate $\langle \lambda\lambda \rangle$ is $O(1/N)$, so that we expect the adequate description in the large $N$ limit to be in terms of a truncated effective Lagrangian containing a finite number of fields. On the other hand, if we consider domain walls at finite $N$ the relative change in the gaugino condensate $\langle \lambda\lambda \rangle$ is no longer small, so that truncating the full effective action (5) to a finite number of lowest dimensional operators may no longer be justified. A similar remark applies to the case of large $N$ domain walls with $|k' - k| \sim N$. In the following, therefore, we will focus on the large $N$ limit where we will confine our attention to the domain walls with $k' = k + 1$.

Thus, we are going to be interested in deducing the effective Lagrangian adequate for studying large $N$ domain walls. It is reasonable to assume [20] that the order parameter effective Lagrangian should be constructed in terms of the chiral superfield

$$
S \equiv \langle \text{Tr}(W\alpha W^\alpha) \rangle = \langle \text{Tr}(\lambda^\alpha\lambda_\alpha) \rangle + \ldots \equiv \langle \lambda\lambda \rangle + \ldots,
$$

where $S$ is regarded as a classical superfield, and the matrix elements are defined in the presence of an appropriate background source (super)field (for detail see [8]). Note that the gluino bilinear $\lambda\lambda$ is the lowest component of the chiral superfield $\text{Tr}(W\alpha W^\alpha)$. (Here

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6By “truncated” we mean that the corresponding effective action is obtained from the full effective action (5) by keeping a finite number of fields $\Phi$ and neglecting the rest of the fields in the large $N$ limit.
\( W_\alpha \) is the usual gauge field strength chiral superfield.) Then, the order parameter effective superpotential reproducing all the anomalies of the model is given by [20]:

\[
W_{VY} = NS \left[ \ln \left( \frac{N^3}{S} \right) + 1 \right].
\]  

(6)

The corresponding scalar potential describes spontaneous chiral symmetry breaking with a non-zero gaugino condensate [20].

The above effective superpotential, however, is not adequate for studying domain walls in SQCD. This can be seen as follows. First, the superpotential (6) does not respect the \( Z_N \) discrete symmetry [21]. This, in particular, implies that the use of (6) cannot be justified when describing domain walls interpolating between different chirally asymmetric vacua of the model [21]. Thus, the superpotential (6) is only adequate for describing a given chirally asymmetric vacuum. In fact, as it stands in (6), such a description is only appropriate for the vacuum with \( k = 0 \). This is related to the strong CP violation. Let us notice that there is no CP violation in SQCD since the CP odd \( \theta \) angle can always be canceled by chiral transformations of massless gluino fields. Let us go back to the effective Lagrangian and see how this property is realized there. As we mentioned above, the lowest component of the chiral superfield \( S \) is the gluino bilinear \( \langle \lambda \lambda \rangle \). The highest component of the superfield \( S \) is formed by the composite “glueballs”: \( \langle G_{\mu\nu}G^{\mu\nu} + iG_{\mu\nu}\tilde{G}^{\mu\nu} \rangle \). As usual, these latter fields are auxiliary components of the chiral superfield \( S \), and, therefore, do not appear in the on-shell scalar potential of the model [1]. Thus, the effective scalar potential of the model is a function of the gluino bilinear only. Let us now suppose that we would like to deal with the vacuum state of the model with a nonzero value of the imaginary phase of the gluino condensate. For such a vacuum we have

\[
\text{Im}(\langle \lambda \lambda \rangle_k) = N\Lambda^3 \sin \left( \frac{2\pi k}{N} \right) \neq 0.
\]  

(7)

The imaginary part of the gluino bilinear is a CP odd quantity. The non-zero vacuum value of this quantity signals a strong CP violation in the effective potential. This CP odd phase cannot be rotated away as there are no massless fermions in the effective Lagrangian. Thus, if the scalar potential is written in terms of the gluino bilinear only, then it cannot simultaneously describe the \( N \) different vacua and yet preserve the strong CP invariance of the model. We must therefore generalize the effective superpotential (6) by including additional fields such that the resulting effective superpotential respects the \( Z_N \) discrete symmetry, and is also consistent with the absence of the strong CP violation. This can be done by introducing the “glueball” order parameter fields in the effective Lagrangian. Thus,

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\[7\]These auxiliary fields can appear in the effective Lagrangian if higher superderivatives are present in the corresponding Kähler potential. In this case the F-terms become dynamical fields. However, including these fields as dynamical degrees of freedom is accompanied by problems with positive definiteness of the corresponding scalar potential (for details see [30]), and, therefore, it is assumed that the Kähler potential does not contain superderivatives. This is expected to be the case in the large \( N \) limit.
the observable effects of the CP odd phase in the gluino condensate should be compensated by another CP odd quantity, the non-zero vacuum value of the field $G_{\mu\nu}\tilde{G}^{\mu\nu}$. Moreover, this should also restore the discrete $Z_N$ symmetry.

The $Z_N$ symmetric effective superpotential has recently been proposed in [22]. This superpotential involves two chiral superfields $S$ and $\mathcal{X}$, where $S$ is the same as before, whereas $\mathcal{X}$ is a new superfield whose lowest component is related (in a rather complicated way) to the gluon bilinears $\langle GG \rangle$ and $\langle G\tilde{G} \rangle$ [37,22]. The corresponding scalar potential contains the gluino bilinear via the lowest component of the $S$ chiral superfield, and also “glueballs” via the lowest component of the $\mathcal{X}$ superfield. Indeed, the lowest component of $\mathcal{X}$ on mass-shell is related to the highest component of $S$, i.e., to the gluonic operators discussed above. Likewise, the F-component of $\mathcal{X}$ is related to the lowest component of $S$, i.e., to the gluino bilinear. The explicit relations between these fields are governed by an unknown Kähler potential in the on-shell formulation of the model. However, one can think of $\mathcal{X}$ as a superfield which introduces the necessary order parameters into the description and restores the $Z_N$ symmetry of the superpotential (6). The effective superpotential for the $S$ and $\mathcal{X}$ superfields is given by [22]:

$$W_{\text{eff}} = NS \left[ \ln \left( \frac{N\Lambda^3}{S} \right) + 1 - \mathcal{X} + \frac{1}{N} \sum_n c_n e^{nNX} \right].$$  \hspace{1cm} (8)

Here the coefficients $c_n$ are $O(1)$ in the large $N$ limit, and satisfy the following constraints: $\sum_n c_n = 0$, and $\sum_n nc_n = 1$. The additional (compared with (5)) terms in (8) containing the $\mathcal{X}$ superfield restore the discrete $Z_N$ symmetry as well as CP invariance of the effective action [22]. In particular, the $Z_N$ transformations read: $S \rightarrow S \exp(2\pi il/N)$, $\mathcal{X} \rightarrow \mathcal{X} - 2\pi il/N$, where $l$ is an integer. Thus, the superpotential (8) describes the $N$ chirally asymmetric vacua in SQCD [22]. These vacua are given by $(S, \mathcal{X}) = (S_k, \mathcal{X}_k)$, where $S_k = N\Lambda^3 \exp(-\mathcal{X}_k)$, and $\mathcal{X}_k = 2\pi ik/N$, $k = 0, 1, ..., N - 1$.

Here we would like to point out that the effective superpotential (8) can be regarded as the Veneziano-Yankielowicz superpotential (6) where the parameter $\Lambda^3$ has been promoted to the $\mathcal{X}$ dependent chiral superfield $\Lambda^3(\mathcal{X})$ [22]:

$$\Lambda^3(\mathcal{X}) \equiv \Lambda^3 \exp \left( -\mathcal{X} + \frac{1}{N} \sum_n c_n e^{nNX} \right).$$  \hspace{1cm} (9)

8The coefficients $c_n$ are not uniquely determined within the effective Lagrangian approach, but should in principle be fixed via the instanton calculus. If so, it is reasonable to expect that the sum over the integer $n$ in (8) should contain either only non-negative or only non-positive values (although this cannot be seen solely on the grounds of the effective Lagrangian arguments). Without loss of generality we can then assume that $n \leq 0$ in (8).

9The corresponding effective action is $Z_N$ symmetric as long as the Kähler potential depends on $\mathcal{X}$ and $\mathcal{X}^*$ via $Z_N$ symmetric combinations, such as $\mathcal{X} + \mathcal{X}^*$, $e^{nNX}$, $e^{nNX^*}$, and so on.

10Note that in the corresponding on-shell scalar potential ambiguities related to the different branches of the logarithmic function are canceled as well.
Then the dynamics of the $\mathcal{X}$ field defines the phase of $\Lambda^3(\mathcal{X})$ (or, equivalently, the phase of the gaugino condensate) in accordance with (2), and the superpotential (8) can be rewritten in the following useful form:

$$W_{\text{eff}} = NS \left[ \ln \left( \frac{N\Lambda^3(\mathcal{X})}{S} \right) + 1 \right].$$

(10)

The following notation will prove convenient in the subsequent discussions. Let

$$X \equiv Ne^{-\mathcal{X}}.$$  

(11)

The effective superpotential (8) can now be written as

$$W_{\text{eff}} = NS \left[ \ln \left( \Lambda^3 f(X) \right) + 1 \right].$$

(12)

Here $f(X)$ is given by

$$f(X) = X \exp \left[ \frac{1}{N} \sum_{n \geq 0} c_n \left( \frac{X}{N} \right)^{nN} \right],$$

(13)

and has the following properties:

$$f'(X_k) = 0, \quad f(X_k) = X_k \equiv N \exp(2\pi ik/N).$$

(14)

Note that $f(X)$ can be rewritten as a polynomial:

$$f(X) = N \sum_{n \geq 0} d_n \frac{X^{nN+1}}{nN+1},$$

(15)

where the sum is over non-negative integers, and the coefficients $d_n \sim 1$ are related to $c_n$'s and, as a result, satisfy the following constraints:

$$\sum_{n \geq 0} d_n = 0, \quad \sum_{n \geq 0} \frac{d_n}{nN+1} = 1.$$  

(16)

In particular, $d_0 = \exp(c_0/N) = 1 + \mathcal{O}(1/N)$.

It is not difficult to see that the superpotential (12) has $\mathbb{Z}_N$ discrete symmetry (with the transformations $S \to S \exp(2\pi il/N)$, $X \to X \exp(2\pi il/N)$), and the corresponding vacua are given by

$$S = S_k = X_k \Lambda^3, \quad X = X_k, \quad k = 0, 1, \ldots, N - 1.$$  

(17)

Thus, the superpotential (12) can be used to describe large $N$ BPS domain walls interpolating between two different chirally asymmetric vacua $(S, X) = (S_k, X_k)$ and $(S, X) = (S_{k'}, X_{k'})$ with $k' = k + 1$. Since the effective superpotential (12) is a truncation of the full effective Lagrangian (which unlike (12) contains an infinite number of fields), it is important to understand the applicability limits of this approach. Let the center of a domain wall solution (which depends only on the $z$ coordinate and is independent of the $x, y$ coordinates)
be located at a point \( z = z_0 \). Then, the effective Lagrangian approach should be applicable to study the domain wall at distances

\[
|z - z_0| \gg d \propto \frac{1}{N},
\]

where \( d \) stands for the “width” of the wall which is vanishing in the large \( N \) limit. This is the key simplification which allows us to explicitly construct BPS domain walls in the large \( N \) SQCD.

### D. BPS Domain Walls in \( \mathcal{N} = 1 \) Theories

Before we turn to finding BPS domain walls in the large \( N \) SQCD, in this subsection we would like to review some useful details concerning generic properties of domain walls in \( \mathcal{N} = 1 \) supersymmetric theories.

Consider an \( \mathcal{N} = 1 \) supersymmetric theory with one chiral superfield \( X \). Let us assume that the corresponding Kähler potential does not contain superderivatives. This is a reasonable assumption for an effective theory in the large \( N \) limit since higher derivative terms are expected to be suppressed by powers of \( 1/N \). Let \( W(X) \) be the superpotential in this theory such that the F-flatness condition \( W_X = 0 \) has a discrete set of non-degenerate solutions \( X = X_a \), \( a = 1, \ldots, N \).

In such a theory we expect presence of domain walls separating inequivalent vacua \( X = X_a \) and \( X = X_b \), \( a \neq b \). Let \( z \) be the spatial coordinate transverse to the wall. Then asymptotically we have \( X(z) \to X_a \) as \( z \to -\infty \), and \( X(z) \to X_b \) as \( z \to +\infty \). Such domain walls may or may not be BPS saturated.

The problem of finding BPS domain walls in four dimensional \( \mathcal{N} = 1 \) supersymmetric theories is really a two dimensional problem as the coordinates \( x, y \) along the wall play no role in the discussion. Thus, our problem effectively reduces to that of finding BPS solitons in two dimensional massive \( \mathcal{N} = 2 \) quantum field theories \[38\]. The corresponding BPS equation for the lowest component of the \( X \) superfield takes the following form:

\[
\partial_z X^*(z) = e^{i\gamma} F_X^*,
\]

where \( F_X \) denotes the F-component of the \( X \) superfield, and \( \gamma \) is some constant phase (whose precise value will be given in a moment). If the Kähler potential contains no superderivatives, then \( F_X^* \) is related to the derivative of the superpotential via the following equation of motion:

\[
g F_X^* = W_X, \quad \text{where} \quad g \equiv g(X, X^*) \text{ is the Kähler metric defined through the Kähler potential } K \text{ as } g = K_{XX^*}.\]

Combining the BPS equation and the equation of motion for \( F_X^* \) one finds: \( g \partial_z X^* = \exp(i\gamma) W_X \). Let us now turn to the calculation of the surface energy density of a domain wall which satisfies this relation. For a general supersymmetric theory with one chiral superfield and with no superderivatives in the Kähler potential the energy of a time-independent configuration is given by

\[
E_{ab} = \frac{1}{2} \int_{-\infty}^{+\infty} dz \left( g |\partial_z X|^2 + V \right),
\]

where \( V = g^{-1} |W_X|^2 \) is the scalar potential of the theory. Note that \( E_{ab} \) can be rewritten as follows:
\[ E_{ab} = \frac{1}{2} \int_{-\infty}^{\infty} dz g^{-1} |g \partial_z X^* - \exp(i\gamma)W_X|^2 + \text{Re} \left( \exp(i\gamma)(W_b - W_a) \right), \]  

(20)

where \( \exp(i\gamma) \) is an arbitrary constant phase. Note that \( E_{ab} \) is independent of \( \gamma \). For the choice \( \gamma = \gamma_{ba} \) we obtain the bound \( E_{ab} \geq |W_b - W_a| \) which is the BPS bound. Here

\[ \exp(-i\gamma_{ba}) = \frac{W_b - W_a}{|W_b - W_a|}. \]  

(21)

The BPS solutions (for which \( E_{ab} = |W_b - W_a| \)) are those that satisfy the following equation:

\[ g\partial_z X^* = \exp(i\gamma_{ba})W_X \]  

(22)

subject to the boundary conditions \( X(z) \to X_{a,b} \) as \( z \to \mp \infty \). Going back to the domain walls in four dimensions, we have the exact same BPS equation (22), and the tension of the domain wall is given by

\[ T_{ab} = \frac{1}{8\pi^2}|W_b - W_a|. \]  

(23)

Typically, one does not know the exact form of the Kähler metric \( g \). However, for the case of a single superfield \( X \) we are considering here one can still make certain statements about the corresponding soliton solutions. Thus, let \( X \) be a function of a new coordinate \( z' \) such that it satisfies the following equation

\[ \partial_{z'} X^* = \exp(i\gamma_{ba})W_X \]  

(24)

with the boundary conditions \( X(z') \to X_{a,b} \) as \( z' \to \mp \infty \). Suppose we are able to find the corresponding solution for \( X(z') \). Next, consider the following change of variables:

\[ \partial_{z'} z(z') = g(X(z'), X^*(z')) , \]  

(25)

where \( X(z') \) is the corresponding solution. Note that if we express \( X \) as a function of \( z \), then \( X(z) \) will satisfy the original BPS equation (22) with the corresponding boundary conditions. The change of variables (25) is simply a diffeomorphism which is one-to-one as long as the Kähler metric \( g \) is non-singular. Throughout this paper we will assume that the corresponding Kähler metric \( g \) is indeed non-singular. Moreover, we will always work in the coordinate system parametrized by \( z' \), but we will drop the prime for the sake of simplicity of the corresponding expressions. (This is effectively equivalent to the case where \( g = 1 \).) Note that as long as the Kähler metric \( g \) is non-singular, the solution of (24) (with the appropriate boundary conditions) implies that the corresponding solution of (22) exists. Moreover, this solution is BPS saturated.

In the next section we will see that the problem of finding BPS domain walls in the large \( N \) SQCD is reduced to that of finding BPS solitons in the so called \( A_N \) Landau-Ginzburg model. The solitons in Landau-Ginzburg theories have been studied in detail \[38-40\]. Many of these models are integrable. The general conditions for existence of BPS solutions were first formulated in \[41\], where exhaustive studies of related issues were performed. The superpotential in the \( A_N \) Landau-Ginzburg model (in our notations) is given by

\[ \mathcal{W} \propto \Lambda^3 \left( X - \frac{X^{N+1}}{N + 1} \right) . \]  

(26)

There are \( N \) distinct vacua in this model with \( X = X_k = \exp(2\pi i k/N) \), \( k = 0, \ldots, N - 1 \). The solitons interpolating between the vacua \( X_k \) and \( X_{k'} \) are BPS saturated \[39\].
III. LARGE $N$ DOMAIN WALLS

In this section we explicitly construct BPS domain walls in the large $N$ pure supersymmetric QCD using the model effective superpotential (12). We will be looking for BPS solutions interpolating between neighboring vacua with phases $k$ and $k' = k + 1$. Before we turn to the technical details of the corresponding equations, we would like to make a few observations which will lead to important simplifications, and will elucidate some features of the system we are dealing with.

To begin with, let us note that the following relation holds for the vacuum state labeled by the phase $k$: $S|_k = \Lambda^3 f(X)|_k$. This relation is nothing but the definition of the gluino condensate with the corresponding phase set by the vacuum value of the $X$ superfield. It is useful to introduce the chiral superfield $NS/\Lambda^3 f(X)$. In all the vacua this superfield takes the same value equal $N$. Let us now concentrate on interpolation between a pair of nearest-neighboring vacua. In this case the relative change in the gluino bilinear is of order $1/N$. Hence, the chiral superfield $NS/\Lambda^3 f(X)$ can only deviate from its vacuum value (which equals $N$) by a quantity of order 1. Thus, one can introduce the following parametrization:

$$\frac{NS}{\Lambda^3 f(X)} = N \left(1 - \frac{\Sigma}{N}\right), \quad (27)$$

with $\Sigma$ being a new chiral superfield. Thus, the $S$ superfield is defined via the $X$ and $\Sigma$ fields as far as the nearest-neighboring vacuum interpolation is concerned. Note that both real and imaginary parts of $\Sigma$ are at most of order 1 in the large $N$ limit. Substituting the expression (27) into the superpotential (12), one finds

$$W_{eff} = N\Lambda^3 f(X) \left[1 - \frac{\Sigma^2}{N^2} + O\left(\frac{1}{N^3}\right)\right].$$

This is the superpotential describing the variation of the order parameter during the transition between nearest-neighboring vacua. The superfield $\Sigma$ enters this superpotential in a subleading order in $1/N$. Thus, it is the superfield $X$ which is left in the leading order, and which should describe domain walls between the adjacent vacua in the large $N$ limit. Neglecting higher order terms, the superpotential is therefore given by:

$$W_{eff} \approx N\Lambda^3 f(X). \quad (28)$$

Before we go any further, notice that this description is valid only for a pair of adjacent vacua, and a priori the approximations made in (28) cannot be justified for a pair of vacua for which $|k - k'| \sim N$.

Let us now turn to the Kähler potential describing transitions between a pair of nearest-neighboring vacua. Generically, the Kähler potential is a function of the entire tower of SQCD composite fields and their superderivatives. It is reasonable to expect, however, that in the large $N$ limit the superderivative terms are suppressed by extra powers of $1/N$. Thus, we assume the following form for the Kähler potential $K = K(\Psi_i, \Psi_i^*, X, X^*)$. Here, $\Psi_i$ denote all the fields which are generically present in the theory, but which do not appear in the large $N$ effective superpotential (28) describing the nearest-neighboring transitions. Notice that this set of fields includes the superfield $\Sigma$ as well (in accordance with (27) one
of $\Psi_i$ can be defined as $\Psi_i \equiv N - \Sigma$). Here we are normalizing all the $\Psi_i$ fields so that in the large $N$ limit they scale as $\sim N$ (just as is the case for $S$ and $X$). This, in particular, implies that the corresponding components of the Kähler metric are (at most) of order 1 in the large $N$ limit. Using the Kähler potential and the superpotential (28) one can write down both the equations of motion and the BPS equations. Let us start with the equations of motion first. For simplicity of presentation we write down only some part of the system of equations of motion relevant for our purposes here. We must make sure, however, that eventually all the equations of motion are satisfied. Since there are no superderivatives in $K$, one can write the following equations of motion in the leading order in $1/N$:

$$
\partial (g_{\Psi_i \Psi_j} \partial \Psi_j^*) + \partial (g_{\Psi_i X} \partial X^*) \simeq 0,
$$

$$
g_{\Psi_i \Psi_j} F_{\Psi_j}^* + g_{\Psi_i X} F_X^* \simeq 0,
$$

$$
g_{XX} F_X^* + g_{X \Psi_i} F_{\Psi_i}^* \simeq W_X,
$$

(29)

where the sign "$\simeq$" denotes equality in the leading order of the large $N$ expansion. We are going to argue below that the fields $\Psi_i$ are irrelevant for the description of the domain walls interpolating between adjacent vacua. In other words, we will argue that the domain walls between the neighboring vacua are described by the superfield $X$ alone. To see how this comes about let us recall that the discrete $\mathbb{Z}_N$ symmetry is spontaneously broken in the theory. Thus, there are domain walls separating adjacent vacua. The question we are dealing with is whether these walls are BPS saturated or not. In other words, there should always exist a wall configuration which would satisfy the equations of motion. However, the very same configuration might or might not satisfy the corresponding BPS equations. Let us start with the equations of motion defined in (29). Given the boundary conditions $\partial \Psi(\pm \infty) = 0$ and $\partial X(\pm \infty) = 0$, and also taking into account non-singularity of the Kähler metric, one can rewrite the first equation in (29) as follows:

$$
g_{\Psi_i \Psi_j} \partial \Psi_j^* + g_{\Psi_i X} \partial X^* \simeq 0.
$$

(30)

While making a transition from one vacuum state to the neighboring one along the solution, the relative change in the fields $\Psi_i$ and $X$ should be at most of order $1/N$. We therefore will be looking for solutions of the form $\Psi_i = \Psi_{i0} + \Psi_i'$, where $\Psi_{i0}$ (which, if non-vanishing, scale as $\sim N$) denote some constants and $\Psi_i'$ set the coordinate dependence of the fields in the nearest-neighbor transitions. For instance, as we have already mentioned, the $\Sigma$ field enters the equations via $\Psi_i = N(1 - \Sigma/N)$. Likewise, the large $N$ behavior of the $X$ superfield is set by the expression $X = N(1 - \zeta/N)$, where both the real and imaginary parts of $\zeta$ are at most of order 1. Given these relations the coefficients of the Kähler metric, which also should be varying by an amount proportional to $1/N$, can be written in the following form:

$$
g_{\Psi_i \Psi_j} \big|_\text{solution} = C_{ij} + O \left( \frac{\Psi_i'}{N}, \frac{\zeta}{N} \right),
$$

$$
g_{\Psi_i X} \big|_\text{solution} = C_i + O \left( \frac{\Psi_i'}{N}, \frac{\zeta}{N} \right),
$$

(31)

where $C_{ij}$ and $C_i$ are some constants of (at most) order 1. Substituting these relations into (30) and taking into account the boundary conditions for the $X$ superfield as well as those
for the $\Psi'_i$ fields, $\Psi'_i(\pm\infty) = 0$, one finds that (30) admits no solutions but the trivial one: $\Psi'_i = 0$ provided that $g_{\Psi'_i X^*}|_{\text{solution}} \propto \mathcal{O}(\zeta/N)$, that is, $C_i \approx 0$. Note that since domain walls must exists (regardless of whether they are BPS saturated or not), the last condition on the Kähler potential must be satisfied. On the other hand, as we will show in a moment, once this condition is satisfied, the corresponding walls do saturate the BPS bound. Next, to solve for $\zeta$, or equivalently $X$, one should use the rest of the equations in (29). Let us turn to the second equation in (29). As we have shown above the off-diagonal components of the metric $g_{\Psi'_i X^*}$ should be of the subleading order on the solution. Thus, in the leading order of the large $N$ expansion the second equation in (29) turns into the relation $g_{\Psi'_i \Psi_j} F'_{\Psi_i}|_{\text{solution}} \approx 0$. Consider the trivial solution $F_{\Psi_i} \equiv 0$. This is precisely the solution we would like to consider for the purpose of finding BPS domain walls. Indeed, this solution implies that the set of BPS equations for $\Psi_i$, namely, $\partial \Psi_i \propto F_{\Psi_i}$ is automatically satisfied (recall that $\partial \Psi_i \equiv 0$ as a result of the first equation in (29)).

Finally, let us discuss the third equation in (29). Given the results of the first equation one finds in the leading order:

$$g_{XX^*} F'_{X} \simeq \mathcal{W}_X.$$  

(32)

Let us now turn to the corresponding BPS equation. In accordance with our discussions presented above the only nontrivial BPS equation is that for $X$, $\partial_z X^*(z) = \exp(i\gamma_{k+1,k}) F'_{X}$, where the phase $\gamma_{k+1,k}$ was defined in the previous section. As the next step, one can use the equation of motion (32) to derive:

$$g_{XX^*} \partial_z X^* \simeq \exp(i\gamma_{k+1,k}) \mathcal{W}_X.$$  

(33)

Below, we will assume that the corresponding Kähler metric $g_{XX^*}$ is non-singular. Then, as we reviewed in the previous section, we can absorb the Kähler metric into a redefinition of $z$. (More precisely, we will absorb $g_{XX^*}/\Lambda^2$ which is dimensionless.) In fact, in the large $N$ limit this procedure is equivalent to the absorption of a finite constant in the redefinition of the variable $z$, or, equivalently, to the redefinition of the width of the domain wall by a finite number. The corresponding BPS equation reads:

$$\partial_z X^* \simeq \exp(i\gamma_{k+1,k}) \Gamma f'(X),$$  

(34)

where we have introduced the inverse width

$$\Gamma = d^{-1} = N\Lambda.$$  

(35)

The boundary conditions for $X(z)$ are given by: $X(-\infty) = X_k, X(+\infty) = X_{k+1}$. Thus, the system of BPS equations and equations of motion for this particular case reduce formally to the system described by the superpotential

$$\mathcal{W} \simeq N\Lambda^3 f(X)$$  

(36)

amended by the $S$ field defining condition

$$S = \Lambda^3 f(X) \left(1 + \mathcal{O}(1/N^2)\right).$$

In addition, the values of the all other fields $\Psi'_i$ (including $\Sigma$) on the solution vanish in the leading order of the $1/N$ expansion. These relations are supposed to be satisfied on the particular BPS solution we are dealing with. In the following subsections we will discuss the solution to the BPS equation (34) and present the explicit form of the corresponding BPS domain wall.
A. A Simple Example

Before solving the BPS equation (34) for the most general form of \( f(X) \), in this subsection, for illustrative purposes, we will consider a simple example where we take the values of the numerical coefficients \( d_n \) as follows: \( d_0 = -d_1 = 1 + 1/N \), and all the other \( d_n \) \((n > 1)\) coefficients are \( O(1/N) \). Then in the leading order in \( 1/N \) one finds:

\[
f(X) \simeq d_0 \left( X - \frac{N}{N+1} \left( \frac{X}{N} \right)^{N+1} \right).
\] (37)

Note that the effective superpotential (36) in this case reduces to that of the \( A_N \) model discussed in the previous section.

Next, let us solve the BPS equation (34) in the large \( N \) limit for \( f(X) \) given by (37). (In this subsection we will closely follow the corresponding discussion in [23].) Let \( X = X_k Y \).

Then we can rewrite (34) in terms of \( Y \):

\[
N \partial_z Y^* \simeq -i \exp(-\pi i/N) \Gamma \left[ 1 - Y^N \right].
\] (38)

The boundary conditions on \( Y \) read:

\[ Y(z \to -\infty) = 1, \quad Y(z \to +\infty) = \exp(2\pi i/N). \] (39)

Thus, in the large \( N \) limit \( \text{arg}(Y) \) changes by a small amount, namely, from 0 to \( 2\pi/N \). It is convenient to parametrize \( Y \) as follows:

\[ Y = (1 - \rho/N) \exp(i(\phi + \pi)/N). \] (40)

Here \( \rho \) and \( \phi \) are real. The boundary conditions then read:

\[ \rho(z \to \pm\infty) = 0, \quad \phi(z \to \pm\infty) = \pm\pi. \] (41)

Then (38) becomes a system of the following first order differential equations:

\[
\partial_z \phi = \Gamma \left[ 1 + \exp(-\rho) \cos(\phi) \right],
\] (42)

\[
\partial_z \rho = -\Gamma \exp(-\rho) \sin(\phi).
\] (43)

Here we are taking the large \( N \) limit and only keeping the leading terms. In particular, we have taken into account that \( (1 - \rho/N)^N \to \exp(-\rho) \) as \( N \to \infty \).

The above system of differential equations can be integrated. Note that these equations do not explicitly contain \( z \). This implies that if \( \phi(z) \) and \( \rho(z) \) give a solution with the appropriate boundary conditions, so will \( \phi(z - z_0) \) and \( \rho(z - z_0) \) for any constant \( z_0 \). (This is simply the statement that the system possesses translational invariance in the \( z \) direction.)

The general solution is given by

\[
\cos(\phi) = (\rho - 1) \exp(\rho),
\] (44)

\[
\int_{\rho(z_0)}^{\rho(z)} d\xi \left[ \exp(-2\xi) - (1 - \xi)^2 \right]^{-\frac{1}{2}} = -\Gamma |z - z_0|,
\] (45)

where \( \rho(z_0) = \rho_0(\approx 1.278) \) is the solution of the following equation:
\[(\rho_0 - 1) \exp(\rho_0) = 1 \, .\]  
(46)

Note that \(z_0\) is the center of the domain wall. These solutions have characteristic for a domain wall behavior. The amplitude \(\rho\) changes from zero at \(-\infty\) to zero at \(+\infty\) with a bell-shaped extremum at \(z = z_0\). Likewise, the phase \(\phi\) changes monotonically from \(-\pi\) at \(-\infty\) to \(+\pi\) at \(+\infty\) going through the origin at \(z = z_0\). The width of this wall \(d \sim 1/NA\) vanishes in the large \(N\) limit.

**B. The General Case**

In this subsection we construct the BPS domain walls for the most general form of \(f(X)\). In fact, as we will see in a moment, the corresponding BPS equations are actually reduced to those discussed in the previous subsection. That is, for the most general effective superpotential \(W\) in (36) the problem of the large \(N\) domain walls can be reduced to that of finding BPS solitons in the large \(N\) \(A_N\) Landau-Ginsburg theory.

To begin with, let us simplify the form of the effective superpotential as follows. Let us make the following change of variables:

\[
f(X) = \tilde{f}(\tilde{X}) \equiv \tilde{X} - \frac{N}{N + 1} \left( \frac{\tilde{X}}{N} \right)^{N+1}. \tag{47}
\]

This transformation preserves the \(Z_N\) symmetry: in terms of the new variable \(\tilde{X}\) we still have \(N\) non-degenerate vacua \(\tilde{X} = X_k = N \exp(2\pi ik/N), \, k = 0, 1, \ldots, N - 1\). Also, the above non-linear change of variables is non-singular as long as the coefficients \(d_n\) satisfy constraints (16). This implies that the resulting Kähler metric \(\tilde{g}(\tilde{X}, \tilde{X}^*)\) is non-singular as well (provided that the original Kähler metric \(g(X, X^*)\) was non-singular). Therefore, as we reviewed in the previous section, we can absorb the Kähler metric into a redefinition of \(z\). (More precisely, we will absorb \(\tilde{g}/\Lambda^2\) which is dimensionless.) The corresponding BPS equation reads:

\[
\partial_z \tilde{X}^* = \exp(i\gamma_{k+1,k}) \Gamma \tilde{f}(\tilde{X}), \tag{48}
\]

where the phase \(\gamma_{k+1,k}\) is the same as in the previous subsection. The boundary conditions for \(\tilde{X}(z)\) are given by: \(\tilde{X}(\infty) = X_k, \, \tilde{X}(+\infty) = X_{k+1}\). Note that the BPS equation (18) is the same as that derived in [23] using a more indirect construction. In particular, it coincides with the BPS equation we just solved in the previous subsection for a simple example corresponding to the \(A_N\) superpotential. Thus, the problem of BPS domain walls in the large \(N\) SQCD is indeed reduced to that of finding BPS solitons in the large \(N\) \(A_N\) Landau-Ginsburg theory. In fact, this is a generic property of \(Z_N\) symmetric Landau-Ginsburg theories - they are all related via non-linear change of variables which is non-singular. This implies that if the Kähler metric is non-singular for one choice of variables, it is non-singular in the transformed variables as well. Since the induced Kähler metric can always be absorbed into the redefinition of \(z\), this allows us to solve the BPS equations exactly (in the large \(N\) limit) in the suitable coordinate “frame”.

For completeness we note that the shape of the gaugino condensate is given by
\[ S = \Lambda^3 f(X) = \Lambda^3 \tilde{f}(\tilde{X}) \simeq \Lambda^3 \tilde{X}. \]  

Thus, in the large \( N \) limit the gaugino condensate is controlled by the VEV of the chiral superfield \( \tilde{X} \). This important simplification is the key observation of [23]. In particular, as we explain in section IV, the additional singlet superfield used in [23] can be related to \( \tilde{X} \). This point is central to the fact that the indirect construction of [23] gives BPS domain walls in the large \( N \) SQCD, and not in some other theory.

The tension of the above domain walls is given by:

\[ T_D = \frac{N}{4\pi} \Lambda^3. \]  

The width of these domain walls is \( \sim \Gamma^{-1} \) which goes as \( \sim 1/N \). Thus, these domain walls are infinitely thin in the large \( N \) limit. Moreover, they are BPS saturated. Note that there are other domain walls in large \( N \) SQCD [23], namely, those with \( k' \neq k \pm 1 \). We refer the reader to [23] for details.

**C. Domain Walls and the Three-form Supermultiplet**

The large \( N \) SQCD domain walls found in the previous sections can be viewed as membranes which break half of SUSY generators and are embedded in a \( (3 + 1) \)-dimensional SQCD background. If so, a three-form supermultiplet [41] must couple to such a membrane to have world-volume \( \kappa \)-symmetry [42]. If the domain walls are indeed 2-branes (membranes), we must be able to identify the corresponding three-form supermultiplet with the appropriate \( (3 + 1) \)-dimensional “bulk” physical fields of SQCD. In fact, in the following we will argue that after gauge fixing the three-form superfield reduces to component fields, which in a given chirally asymmetric vacuum describe the corresponding gluon and/or gluino composites of SQCD.

Before we turn to this identification, let us make the following remark. Consider the domain wall separating two neighboring chirally asymmetric vacua. In each of these vacua there are SQCD fields with a well defined particle interpretation in terms of colorless composite bound states of gluons and/or gluinos. We would like to show that the SQCD fields in each of the two vacua consistently couple to the membrane world-volume. Note that the corresponding couplings for these two vacua are different. A consistent “sewing” of these couplings is guaranteed by the fact that the domain wall is a solution (interpolating between these two vacua) of the order parameter effective action (8). In the following we will therefore identify the components of the three-form supermultiplet with SQCD colorless fields in a given chirally asymmetric vacuum.

Let us start with the supersymmetric Green-Schwarz action for a 2-brane (membrane) embedded in the \( (3 + 1) \)-dimensional target space. The membrane action \( S_m \) includes the Wess-Zumino term corresponding to the three-form coupling to the world-volume of the membrane:

\[ \kappa \text{-symmetry eliminates half of the world-volume fermionic degrees of freedom, which is necessary for having a supersymmetric formulation of the membrane action.} \]
\[ S_m = T_D \int d^3 \sigma \left( -\frac{1}{2} \sqrt{g} g^{ab} E^a_i E^b_j \eta_{ij} + \frac{1}{2} \sqrt{g} + \frac{1}{3!} \epsilon^{abc} E^A_a E^B_b E^C_c B_{ABC} \right), \]  

where \( \sigma_a (a = 0, 1, 2) \) denote the membrane world-volume coordinates, and \( g_{ab} \) is the induced metric on the world-volume. Here the pull-back \( E^A_a = \partial_a Z^M(\sigma) E^A_M, \) \( Z^M \equiv (x^\mu, \theta^a) \) are the target superspace coordinates, and \( E^A_M \) denotes the supervielbein, where \( A = (i, \alpha) \) are the tangent superspace indices. The constraints \[ \square \] imposed by \( \kappa \)-symmetry on the three-form superfield \( B_{ABC} \) can be solved, and the remaining independent components of \( B_{ABC} \) can be combined into the following real tensor superfield:

\[ U = B + i \theta \chi - i \bar{\theta} \bar{\chi} + \frac{1}{16} \theta^2 A^* + \frac{1}{16} \bar{\theta}^2 A + \frac{1}{48} \theta \bar{\theta} \epsilon_{\mu \nu \lambda \rho} C^{\mu \nu \lambda \rho} + \frac{1}{2} \theta^2 \bar{\theta} \left( \frac{\sqrt{2}}{8} \bar{\Psi} + \bar{\sigma} \partial_\mu \chi \right) + \frac{1}{2} \bar{\theta}^2 \theta \left( \frac{\sqrt{2}}{8} \Psi - \sigma^\mu \partial_\mu \bar{\chi} \right) + \frac{1}{4} \theta \bar{\theta} \theta^2 \left( \frac{1}{4} \Sigma - \partial^2 B \right). \]  

We would like to point out that the supermembrane action by itself gives rise to the real tensor superfield \[ \text{Eq. (52)} \] only defined up to a shift by a linear supermultiplet \( U \) satisfying \( D^2 L = D^2 L = 0 \). If the membrane is considered in empty space, then this shift, \( U \rightarrow U - L \), can always be used to remove half of the components in \( \text{Eq. (52)} \). However, in our case the membrane is embedded in a space where composite colorless SQCD fields live. Some of these fields, as we are going to see shortly, couple to the membrane. Thus, the total classical action of the system is the sum of the membrane action \( S_m \) and the effective action for the bulk fields

\[ S = S_m + S_{\text{bulk}}. \]  

The action for the bulk fields \( S_{\text{bulk}} \) is a rather complicated object and generically consists of an infinite number of fields. However, it was shown in \[ \text{Eq. (57)} \] that lowest-spin SQCD excitations (spin-zero glueballs, gluino-gluino mesons and their fermionic superpartners) can be described by the components of the three-form supermultiplet \( U \). It was also argued in \[ \text{Eq. (57)} \] that the shift \( U \rightarrow U - L \) is not a symmetry of the bulk effective Lagrangian \( S_{\text{bulk}} \). Thus, all the components in \( \text{Eq. (52)} \) should be retained as physical ones. If so, one can identify the fields in \( \text{Eq. (52)} \) with the bulk composite massive fields of SQCD \[ \text{Eq. (57)}. \] Thus, the three-form field \( C_{\mu \nu \lambda} \) is just the “magnetic” dual of the Chern-Simons current of SQCD, and is related to the operator \( G \bar{G} \) via \( \epsilon_{\mu \nu \lambda \rho} \partial^\rho C^{\mu \nu \lambda} \propto G \bar{G} \). (Note that \( C_{\mu \nu \lambda} \) is a massive field in this approach with one physical pseudoscalar degree of freedom \[ \text{Eq. (57)}. \] ) The two Weyl fermion states \( \chi \) and \( \Psi \) in \( \text{Eq. (52)} \) correspond to \( (l, s) = (0, 1/2) \) and \( (l, s) = (1, 1/2) \) gluino-gluon bound states, respectively (where \( l \) and \( s \) denote the orbital momentum and spin respectively). The \( A \) field denotes the \( \lambda \lambda \) state, whereas the \( B \) field is related to the \( \Sigma \) field (on-shell) which corresponds to the scalar glueball given by the \( G^2_{\mu \nu} \) composite operator.

To summarize, there are four real massive scalar degrees of freedom corresponding to the scalar and pseudoscalar gluino-gluino composites, and the scalar and pseudoscalar glueballs. In addition, there are four real (two Weyl) fermionic degrees of freedom corresponding to the gluino-gluon composites with different orbital momenta. These states are present in the superfield \( \text{Eq. (52)} \) which necessarily appears in the supersymmetric formulation of the membrane action.

One might wonder now what is the relation between the bulk effective action \( S_{\text{bulk}} \) and the order parameter effective action for two chiral superfields \( S \) and \( X \) used previously. As we
mentioned above, $S_{\text{bulk}}$ depends in general on an infinite number of superfields corresponding to different spin-orbital excitations of SQCD. If one restricts consideration to the lowest-spin states only, then those states precisely fit in the supermultiplet $(52)_{[37]}$, so that $S_{\text{bulk}} = S_{\text{bulk}}(U, ...)$.

On the other hand, it has been known for some time [44,45] that the field content of the three-form supermultiplet $U$ can be rearranged into two chiral superfields. This property was used in [43,45] to rewrite the three-form supermultiplet $U$ in terms of two chiral superfields which in our case, in a given chirally asymmetric vacuum, correspond to the chiral superfield $S$ and some function of the chiral superfield $X$.

Before we turn to the next section we would like to comment on SQCD strings and some related phenomena. There are a number of indications, as we discussed in the introduction, that in the large $N$ limit QCD (or SQCD) can be viewed as some non-critical string theory. Thus, one might be able to describe all the SQCD states in the large $N$ limit as string excitations, without actually referring to the fundamental theory (SQCD). The question we address here is whether some of the SQCD states discussed above could indeed be identified with excitations of strings present in the “bulk”. In the string picture glueballs are identified with excitations of a closed non-critical string: the scalar glueball in the SQCD string theory would correspond to the string “dilaton” [45]. The pseudoscalar glueball is the analog of the “axion” and, by supersymmetry, a gluino-gluon bound state corresponds to the “dilatino”. These states, as we discussed above, along with the gluino-gluon bound states, are combined in one massive tensor supermultiplet, similarly to what can happen in superstring theories at the non-perturbative level (see discussions in the second reference in [45]). Note that the closed string itself, whose excitations we identify with the glueballs, can be viewed as a limit of a closed membrane with the topology of a torus. This is consistent with the results of [46] where a part of the low-energy glueball spectrum of QCD was calculated in a closed bosonic membrane model and a good agreement with the lattice QCD predictions was found. These observations might provide some hints toward understanding the string theory of strong interactions.

Finally, we would like to comment on the possibility of SQCD open strings ending on the domain walls. If the domain walls we found are to be interpreted as D-branes, there should be open strings ending on the brane [17]. Thus, in the D-brane picture, one should in principle be able to identify the $U(1)$ gauge field in the membrane world-volume action [17]. One might wonder how this $U(1)$ gauge field arises in the field theory context if the open strings we discuss are viewed as SQCD chromoelectric flux tubes. An example of a wall on which the flux tube can end was constructed in [9]. In that case, the corresponding theory is in the confining phase outside of the wall. Thus, all chromoelectric fields are squeezed into flux tubes outside of the wall. It is energetically favorable for the tubes to end on the wall and to spread the flux into the wall interior if the gauge theory inside the wall is in the Higgs phase [9]. Thus, the original gauge group of the confining theory, being Higgsed inside the defect, could support flux tubes ending on the wall. The Higgs phase inside the defect can support the residual unbroken $U(1)$ gauge group in the wall. In the large $N$ limit the wall becomes infinitely thin. Thus, the system of the flux tube and the wall can be regarded

\[12\text{In [46], because of computational difficulties, the spectrum of glueballs was actually calculated for a bosonic membrane with the topology of a sphere.}\]
in that limit as an SQCD string ending on a D-brane. The world-volume $U(1)$ field in that case can be interpreted as a collective solitonic excitation in the soliton picture for the wall, or, alternatively, as a mode of an open string ending on the D-brane, if the D-brane picture is adopted. Recently, some features of the system of $N$ such domain walls with flux tubes were discussed in [17]. It would be interesting to study these issues further.

IV. RELATIONS TO PREVIOUS STUDIES

Surprisingly enough, the solution of BPS equations as well as superpotential (37) coincide with the ones derived in [23] within a completely different approach. As we will argue below, this coincidence is not an accident. In [23] BPS domain wall solutions were found in the theory obtained by integrating out heavy states in SQCD with $N_f$ heavy quark flavors. The method of gaining an analytic control over BPS equations in pure SQCD by adding flavors and then integrating them out was used earlier in a number of papers, and, in that respect, was not new in [23]. However, there is a crucial point which makes conclusions of [23] so different from other approaches. Let us explore this difference in somewhat more detail. As is well known, the supersymmetric QCD with $N_f < N$ flavors possesses an anomaly free R-symmetry under which flavors carry the charge $\frac{N_f - N}{N_f}$. Adding a constant mass term for “mesons” $M$ in the supersymmetric Lagrangian

$$m \text{Tr}(M)$$

explicitly breaks this symmetry. However, in the massive theory there still is an anomaly free discrete $\mathbb{Z}_N$ symmetry. This group acts on the gaugino bilinear, and, in fact, coincides in that respect with the anomaly free $\mathbb{Z}_N$ symmetry of pure SQCD. Moreover, spontaneous breaking of this discrete symmetry by meson VEVs should produce domain walls which after taking the limit $m \to \infty$ are expected to transform continuously into the domain walls of pure SQCD. This was the method used in [31]. It was shown in [31] that within this approach the BPS walls do exist for sufficiently small values of $m$, however, above some critical $m_*$ the walls cease to exist. Thus, one would seemingly not have BPS domain walls in pure SQCD which is expected to be recovered in the limit $m \to \infty$. Although per sé there is nothing wrong with solving equations for fixed $m$ and then extrapolating them into the large $m$ region, it seems to us that the domain wall solutions obtained this way cannot be related to the walls in pure SQCD discussed in [3] and explicitly constructed in the present work. The reason for this is as follows. The heavy mesons which are being integrated out in this approach transform under the discrete symmetry which also acts on the degrees of freedom of the corresponding low-energy theory. If so, a straightforward decoupling of these fields cannot be justified for the problem of finding BPS domain walls of the corresponding low-energy theory. Indeed, it was shown by Kogan, Kovner and Shifman in [19] that when the heavy fields are charged under the discrete symmetry (which is subsequently broken spontaneously) the classical solutions that interpolate between different vacua cannot be adequately described by simply integrating heavy fields out and solving equations of the corresponding effective low-energy theory. This inadequacy arises due to “cusp singularities” which emerge in the low-energy theory once the fields charged under the discrete symmetry are integrated out [19]. Now, in SQCD with flavors and the regular mass term, mesons
necessarily transform under the remaining $\mathbb{Z}_N$ discrete symmetry group, and, thus, the “cusp crossing” will always occur once the mesons are integrated out. In this case, albeit one can consistently solve the BPS equations and find the wall solutions for a finite mass $m$ \[31\], this method would not allow one to account for the BPS walls of pure supersymmetric QCD which is reached in the $m \to \infty$ limit. In order to avoid this difficulty, the mesons which are being integrated out should not transform under the discrete symmetry. This is possible to accomplish if the mass term is not a constant, but rather is promoted into a dynamical field. This was the key observation of \[23\]. Consider SQCD with $N_f = N$ flavors. This theory has an anomaly free $U(1)_R$-symmetry under which mesons are neutral. To avoid the presence of “cusp singularities”, one can postulate that the mass term for “mesons” arises due to the VEV of a new chiral superfield $X$, and takes the form $\Lambda X \text{Tr}(M)$. In this case there is a discrete $\mathbb{Z}_N$ symmetry under which the $X$ field and gauginos transform but mesons are neutral. This group, in fact, is a subgroup of the anomaly free $U(1)_R$. Now, if the mesons acquire large masses due to the VEV of the $X$ superfield, then these mesons can be safely integrated out without encountering cusp singularities discussed above. Thus, one should be able to describe SQCD domain walls in this case. The necessary condition is that one should work in the region where $X$ is large. For this, one can add a small $\mathbb{Z}_N$-conserving term in the superpotential. The simplest one is $X^{N+1}/(N+1)$. In this case, after integrating mesons out, one recovers (37). Another indication of a deeper connection between $X$ in (12) and the one in \[23\] is the fact that in the above context $X$ essentially acts as a dilaton (plus axion) field. This can be seen in a number of ways. The simplest one is to point out that the VEV of $X$ spontaneously breaks an anomalous $U(1)$-symmetry (the Peccei-Quinn symmetry \[49\]) and, thus, its imaginary part is an axion field with a one-loop anomaly-induced coupling

$$\text{one loop factor} \times \text{Im}(X) F \tilde{F}. \tag{55}$$

The supersymmetric generalization of this coupling gives

$$\text{ln}(X)W_\alpha W^\alpha. \tag{56}$$

Thus, ln$(X)$ essentially sets the value of the inverse gauge coupling $1/g^2$. This precisely matches (12) and the form of the effective superpotential derived from gaugino condensation in the low-energy theory for large $X$. Indeed, for large $X$ the effective low-energy theory is pure Yang-Mills (plus $X$) with a gaugino condensate. The effective superpotential is simply $\langle \lambda \lambda \rangle \sim \Lambda^3$ where $\Lambda$ is a low-energy scale of the theory which from the one-loop matching of the gauge couplings at the scale $\Lambda$ is simply $\Lambda^3 \sim X \Lambda_{\text{QCD}}^3$. This is in agreement with the dilaton superpotential induced by the gaugino condensation

$$X \Lambda_{\text{QCD}}^3 \sim e^{-c/\sqrt{s}} \Lambda_{\text{QCD}}^3. \tag{57}$$

According to \[48\], the dilaton of QCD is related to $G^2_{\mu\nu}$ operator, which gives another way of deriving why $X$ should be interpreted as the field responsible for “gluonic” (that is, glueball) degrees of freedom in accordance with \[22\] and our analysis in this paper.

\section*{V. DISCUSSIONS AND CONCLUSIONS}

In this section we briefly summarize the main results of our discussions. Before we turn to the conclusions we would like to make the following comments.
Suppose we consider now domain walls at finite $N$. Can we use an effective superpotential (say, of the type we used here) to study them in this case? It appears that using any type of truncated effective superpotential may not be justified. The reason why is that the gluino condensate changes substantially inside of the domain walls, so that integrating out heavy fields, let us call them $Y_m$, might not be justified. Does this imply that SQCD domain walls are not BPS saturated at finite $N$? Strictly speaking one cannot draw this conclusion as the effective description in terms of $S$ and $X$ only may not be valid, and one might have to include all of the infinitely many fields $X, S, Y_m$ to obtain an adequate description. Practically, this means that the problem may not be easily tractable at finite $N$.

To summarize, we have constructed the BPS domain walls interpolating between neighboring vacua with broken chiral symmetry in large $N$ supersymmetric QCD. The tension of these walls, saturating the BPS bound, scales as $\sim N$, and the width of the walls vanishes as $\sim 1/N$ in the large $N$ limit. These walls can be interpreted as supermembranes embedded in a (3+1)-dimensional SQCD background. The components of the three-form supermultiplet that couples to the supermembrane were identified in terms of composite fields of SQCD.

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