Radiative Origin of All Quark and Lepton Masses through Dark Matter with Flavor Symmetry

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Abstract

The fundamental issue of the origin of mass for all quarks and leptons (including Majorana neutrinos) is linked to dark matter, odd under an exactly conserved $Z_2$ symmetry which may or may not be derivable from an $U(1)_D$ gauge symmetry. The observable sector interacts with a proposed dark sector which consists of heavy neutral singlet Dirac fermions and suitably chosen new scalars. Flavor symmetry is implemented in a renormalizable context with just the one Higgs doublet $(\phi^+, \phi^0)$ of the standard model in such a way that all observed fermions obtain their masses radiatively through dark matter.
In the standard model (SM) of particle interactions, the origin of mass is the electroweak Higgs doublet \((\phi^+, \phi^0)\). With the recent discovery \([1, 2]\) of the 126 GeV particle at the Large Hadron Collider (LHC), this is apparently confirmed as the one physical neutral Higgs boson, i.e. \(h = \sqrt{2}Re(\phi^0)\) predicted by the SM, leaving the other three degrees of freedom, i.e. \((\phi^\pm, \sqrt{2}Im(\phi^0))\) as the longitudinal components of the observed massive electroweak vector gauge bosons \((W^\pm, Z^0)\). On the other hand, the existence of dark matter and the observed flavor structure of quarks and leptons remain unexplained.

In this paper, it is proposed that these three fundamental issues (mass, dark matter, and flavor) are interconnected in the context of a theoretical framework for the radiative generation of all SM fermion masses.

Consider first the origin of charged-lepton masses. Under the \(SU(2)_L \times U(1)_Y\) gauge symmetry of the SM, there are left-handed doublets \(L_iL = (\nu_i, l_i)_L\) and right-handed singlets \(l_iR\). The one Higgs doublet \(\Phi = (\phi^+, \phi^0)\) of the SM connects them through the invariant Yukawa terms

\[
\mathcal{L}_Y = f_i\bar{L}_iLl_iR\Phi + H.c. = f_i(\bar{\nu}_iL\phi^+ + \bar{l}_iL\phi^0)l_iR + H.c. \tag{1}
\]

As \(\phi^0\) acquires a vacuum expectation value \(\langle \phi^0 \rangle = v\), charged leptons become massive with \(m_i = f_i v\).

Suppose now there exists a flavor symmetry which forbids Eq. (1). As a concrete example, consider the non-Abelian discrete symmetry \(A_4\) \([3, 4, 5, 6]\), which is also the symmetry group of the tetrahedron. It has four irreducible representations \(1, 1', 1'', 3\), with the multiplication rule

\[
3 \times 3 = 1 + 1' + 1'' + 3 + 3. \tag{2}
\]

Let \(L_{iL} \sim 1, 1', 1''\), \(l_{iR} \sim 3\), \(\Phi \sim 1\), then the usual SM Yukawa couplings are forbidden. There are two ways now for the charged leptons to acquire mass. (1) Nonrenormalizable interactions of the form \(\bar{L}_{iL}l_{jR}\Phi \chi_k\) are postulated, where the flavor structure is carried by
the scalar singlets $\chi_k$. For $A_4$, $\chi_k \sim 3$ works. This scenario requires an ultraviolet completion, which may involve many new fields and parameters, and often raises more questions than the ones it attempts to answer. (2) Renormalizability of the theory is maintained by extending the scalar sector to include more doublets which should be observable. For $A_4$, $\Phi_{1,2,3} \sim 3$ works, as proposed in the original papers [3, 4, 5, 6].

Given that the recently discovered 126 GeV particle [1, 2] at the LHC is very likely to be the one Higgs boson of the SM, it is time to consider how a renormalizable theory of flavor may be sustained with just the one Higgs boson of the SM. The solution is actually very simple. Let all known fermion masses be generated as quantum corrections from a dark sector which also carries flavor. This notion links the origin of SM fermion masses with flavor and dark matter which could also be self-interacting, an idea increasingly relevant to present astrophysical observations [7].

To implement this interconnected scenario, new particles have to be introduced. As a simple example, three heavy Dirac singlet neutral fermions $N_{1,2,3}$ will be added [8]. Their masses will be the origin of all SM fermion masses (including those of the Majorana neutrinos). Consider first again the charged-lepton masses. A scalar doublet $(\eta^+, \eta^0)$ and a charged scalar singlet $\chi^+$ are added. These new particles may transform under a new $U(1)_D$ gauge symmetry, as well as $A_4$, i.e.

$$(\eta^+, \eta^0), \chi^+ \sim 1, \quad N_{iL} \sim \mathbf{3}, \quad N_{iR} \sim \mathbf{1}, \mathbf{1}', \mathbf{1}''.$$  \hspace{1cm} (3)

The allowed Yukawa couplings are then $f_i^{\prime} \tilde{N}_{iR}(l_i^+ \eta^+ - \nu_L \eta^0)$ and $f'' \tilde{l}_{iR} N_{iL} \chi^-$. Together with the invariant scalar trilinear coupling $\mu(\eta^+ \phi^0 - \eta^0 \phi^+) \chi^-$, a radiative charged-lepton mass matrix is obtained as shown in Fig. 1, where the $\tilde{N}_L N_R$ mass terms break $A_4$ explicitly but softly. Encoding of the flavor symmetry is thereby accomplished in a renormalizable theory, instead of the usual nonrenormalizable approach using $L_{LL} L_{RR} \Phi \chi$. Note that $U(1)_D$ may remain unbroken in the loop.
The nature of the soft breaking of $A_4$ is encoded in the $\tilde{N}_iLN_jR$ mass matrix. Let

$$\mathcal{M}_N = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix},$$

then a residual $Z_3$ symmetry exists [9] which protects it against arbitrary corrections. The charged-lepton mass matrix $\tilde{l}_iLl_jR$ is then given by

$$\mathcal{M}_l = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix},$$

where

$$m_{e,\mu,\tau} = \frac{f''f_1f_{1,2,3}}{16\pi^2} \sin \theta \cos \theta M_{1,2,3} \left[ \frac{m_{1,2}^2}{m_{1,2}^2 - M_{1,2,3}^2} \ln \frac{m_{1,2}^2}{M_{1,2,3}^2} - \frac{m_{1,2}^2}{m_{1,2}^2 - M_{1,2,3}^2} \ln \frac{m_{1,2}^2}{M_{1,2,3}^2} \right],$$

with $m_{1,2}^2$ the eigenvalues and $\theta$ the mixing angle of the mass-squared matrix

$$\mathcal{M}_{\eta\chi}^2 = \begin{pmatrix} m_\eta^2 & \mu v \\ \mu v & m_\chi^2 \end{pmatrix}.$$  

Note that $\mathcal{M}_l$ of Eq. (5) is the conjugate of the result obtained in the original $A_4$ proposal [3], using $\Phi_{1,2,3} \sim 3$ and $\langle \phi_1^0 \rangle = \langle \phi_2^0 \rangle = \langle \phi_3^0 \rangle = v/\sqrt{3}$.

At this stage, the theory is invariant under $Z_3$ and the massive charged leptons ($e, \mu, \tau$), the massless neutrinos ($\nu_e, \nu_\mu, \nu_\tau$), as well as the heavy dark fermion singlets ($N_e, N_\mu, N_\tau$) all transform as $(1, \omega, \omega^2)$ under $Z_3$. The next step is to obtain the radiative generation of

$$\text{Figure 1: One-loop generation of charged-lepton mass with } U(1)_D \text{ symmetry.}$$
Majorana neutrino masses. If $U(1)_D$ is replaced by $Z_2$, then the well-studied one-loop scotogenic model [10] may be used. If $U(1)_D$ is retained, then the recent one-loop proposal [11] with two scalar doublets $(\eta_{1,2}, \eta_{1,2}^0)$ transforming oppositely under $U(1)_D$ is a good simple choice. However, a two-loop realization may also be adopted, as shown in Fig. 2, which may preserve $U(1)_D$ as well. Under $Z_3$, $\nu_{e,\mu,\tau}, N_{e,\mu,\tau}, \rho_{1,2,3} \sim 1, \omega, \omega^2$, $(\phi^+, \phi^0), (\eta^+, \eta^0), \chi^0 \sim 1$. Under $U(1)_D$, $N, (\eta^+, \eta^0), \chi^0 \sim 1, \rho \sim 2$.

![Diagram](image)

**Figure 2:** Two-loop generation of Majorana neutrino mass with $U(1)_D$ symmetry.

The addition of $\chi^0$ and $\rho_1$ completes the two loops without breaking $U(1)_D$ or $Z_3$. Since $(\nu_e, \nu_\mu, \nu_\tau)$ transform as $(1, \omega, \omega^2)$ under the unbroken residual $Z_3$ at this stage, this would result in a Majorana neutrino mass matrix in the basis $(\nu_e, \nu_\mu, \nu_\tau)$ of the form

$$
\mathcal{M}_\nu = \begin{pmatrix}
A & 0 & 0 \\
0 & 0 & B \\
0 & B & 0
\end{pmatrix}.
$$

(8)

The further addition of $\rho_{2,3}$ together with the soft breaking of $Z_3$ using the trilinear $\chi^0 \chi^0 \rho_{2,3}^*$ couplings allows $\mathcal{M}_\nu$ to become

$$
\mathcal{M}_\nu = \begin{pmatrix}
A & C & C^* \\
C & D^* & B \\
C^* & B & D
\end{pmatrix},
$$

(9)

where $A$ and $B$ are real. Note that this pattern is protected by a symmetry first pointed out in Ref. [12], i.e. $e \rightarrow e$ and $\mu - \tau$ interchange with $CP$ conjugation, and obtained previously
Since $\theta$ is satisfied with a single $\nu_\mu - \nu_\tau$ mixing ($\theta_{23} = \pi/4$) and maximal $CP$ violation, i.e. $\exp(-i\delta) = \pm i$, whereas $\theta_{13}$ may be nonzero and arbitrary.

The mass matrix of Eq. (9) has six parameters: $A, B, C_R, C_I, D_R, D_I$, but only five are independent because the relative phase of $C$ and $D$ is unobservable. Using the conventional parametrization of the neutrino mixing matrix, the angle $\theta_{13}$ is given by

$$\frac{s_{13}}{c_{13}} = \frac{-D_I}{\sqrt{2}C_R}, \quad \frac{s_{13}c_{13}}{c_{13}^2 - s_{13}^2} = \frac{\sqrt{2}C_I}{A - B + D_R}. \quad (10)$$

The adjustable relative phase of $C$ and $D$ is used to allow the above two equations to be satisfied with a single $\theta_{13}$. The angle $\theta_{12}$ is then given by

$$\frac{s_{12}c_{12}}{c_{12}^2 - s_{12}^2} = \frac{-\sqrt{2}(c_{13}^2 - s_{13}^2)C_R}{c_{13}[c_{13}^2(A - B - D_R) + 2s_{13}^2D_R]}. \quad (11)$$

As a result, the three mass eigenvalues are

$$m_1 = \frac{c_{13}^2[c_{12}^2A - s_{12}^2B - s_{12}^2D_R] - s_{13}^2[(c_{12}^2 - s_{12}^2)B - D_R]}{(c_{13}^2 - s_{13}^2)(c_{12}^2 - s_{12}^2)}, \quad (12)$$

$$m_2 = \frac{c_{13}^2[-s_{12}^2A + c_{12}^2B + c_{12}^2D_R] - s_{13}^2[(c_{12}^2 - s_{12}^2)B + D_R]}{(c_{13}^2 - s_{13}^2)(c_{12}^2 - s_{12}^2)}, \quad (13)$$

$$m_3 = \frac{s_{13}^2A - c_{13}^2B + c_{13}^2D_R}{c_{13}^2 - s_{13}^2}. \quad (14)$$

Since $s_{13}^2 \approx 0.025$ is small, these expressions become

$$m_2 + m_1 \simeq A + B + D_R + s_{13}^2(A - B + D_R), \quad (15)$$

$$(c_{12}^2 - s_{12}^2)(m_2 - m_1) \simeq -A + B + D_R - s_{13}^2(A - B + D_R), \quad (16)$$

$$m_3 \simeq -B + D_R + s_{13}^2(A - B + D_R). \quad (17)$$

It is clear that a realistic neutrino mass spectrum with $m_2^2 - m_1^2 << |m_3^2 - (m_2^2 + m_1^2)/2|$ may be obtained with either $|m_1| < |m_2| < |m_3|$ (normal ordering) or $|m_3| < |m_1| < |m_2|$ (inverted ordering).

As for quarks, color triplet scalars are added, i.e. the electroweak doublet $(\xi^{2/3}, \xi^{-1/3})$ and singlets $\xi^{2/3}, \xi^{-1/3}$, all transforming as $-1$ under $U(1)_D$. The analogs of Fig. 1 are easily
obtained for $\mathcal{M}_{u,d}$ using again just $N_{1,2,3}$, as shown in Figs. 3 and 4. Under the symmetry $A_4$ again as an example,

$$Q_{iL} = (u,d)_{iL} \sim 1, 1', 1'' \quad u_{iR}, \quad d_{iR} \sim 3,$$

$$\xi^{2/3}, \xi^{-1/3}, \quad \zeta^{2/3}, \zeta^{-1/3} \sim 1.$$ 

in complete analogy to the charged-lepton sector, resulting also in arbitrary quark masses with a residual symmetry $Z_3$ under which $(u,c,t)$ and $(d,s,b)$ transform as $(1, \omega, \omega^2)$. (Note that to get a realistic $m_t$ using Eq. (6) with Yukawa couplings of order unity, $M_3$ and $m_{1,2}$ should all be of order 10 TeV, with no cancellation between the $m_{1,2}$ terms. A variant of this scheme is to use a flavor symmetry such that only $t$ couples to $\Phi$ at tree level, which may be possible with $\Delta(27)$ for example, because it has 9 inequivalent one-dimensional representations.) The quark matrices are thus both diagonal, so there is no mixing in the quark sector in this symmetry limit. In other words, there is now a theoretical understanding
of why the actual mixing angles are small. They are the result of breaking this $Z_3$ symmetry
in the soft terms of the Dirac mass matrix of the singlet $N$’s.

The $\bar{q}_i L q_j R$ mass matrix is of the form

$$ \mathcal{M}_q = \begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_3 \end{pmatrix} U^L_M \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix} (U^R_M)^\dagger. $$ (20)

If the unitary matrices $U^L_R M$ are the identity, then $Z_3$ is not broken and the quark mixing
matrix $V_{CKM}$ is also the identity. Let $U^L_M$ be approximately given by

$$ U^L_M \simeq \begin{pmatrix} 1 & -\epsilon_{12} & -\epsilon_{13} \\ \epsilon_{12}^* & 1 & -\epsilon_{23} \\ \epsilon_{13}^* & \epsilon_{23}^* & 1 \end{pmatrix} $$ (21)

then

$$ \mathcal{M}_q U^L_M \mathcal{M}_q^\dagger \simeq \begin{pmatrix} f_1^2 M_1^2 & f_1 f_2 \epsilon_{12} (M_1^2 - M_2^2) & f_1 f_3 \epsilon_{13} (M_1^2 - M_3^2) \\ f_1 f_2 \epsilon_{12}^* (M_1^2 - M_2^2) & f_2^2 M_2^2 & f_2 f_3 \epsilon_{23} (M_2^2 - M_3^2) \\ f_1 f_3 \epsilon_{13}^* (M_1^2 - M_3^2) & f_2 f_3 \epsilon_{23}^* (M_2^2 - M_3^2) & f_3^2 M_3^2 \end{pmatrix}. $$ (22)

Let $m_d \simeq f_1^d M_1$, $m_s \simeq f_2^d M_2$, $m_b \simeq f_3^d M_3$, $m_u \simeq f_1^u M_1$, $m_c \simeq f_2^u M_2$, $m_t \simeq f_3^u M_3$, then

$$ V_{CKM} \text{ is approximately given by} $$

$$ V_{ud} \simeq V_{cs} \simeq V_{tb} \simeq 1, \quad V_{us} \simeq \left( \frac{m_d}{m_s} - \frac{m_u}{m_c} \right) \epsilon_{12} \left( \frac{M_2^2 - M_1^2}{M_2 M_1} \right), $$ (23)

$$ V_{ub} \simeq \left( \frac{m_d}{m_b} - \frac{m_u}{m_t} \right) \epsilon_{13} \left( \frac{M_3^2 - M_1^2}{M_3 M_1} \right), \quad V_{cb} \simeq \left( \frac{m_s}{m_b} - \frac{m_c}{m_t} \right) \epsilon_{23} \left( \frac{M_3^2 - M_2^2}{M_3 M_2} \right). $$ (24)

There are many realistic solutions of the above. The simplest is to set $f_1^d = f_2^d = f_3^d$, in
which case $V_{CKM} \simeq (U^L_M)^\dagger$. In other words, the soft breaking of $Z_3$ which generates $U^L_M$ is
directly linked to the observed $V_{CKM}$.

In this grand scheme, all SM fermions owe their masses not just to the one and only one
Higgs boson, but also to the invariant masses of the three neutral singlet fermions $N_{1,2,3}$.
Flavor structure is carried by the $\bar{N}_i L N_j R$ mass matrix and transmitted to the quarks and
leptons through new observable scalars. These scalars as well as $N_{1,2,3}$ may have their own
\(U(1)_D\) gauge interactions, and the lightest \(N\) is a dark-matter candidate. The \(U(1)_D\) gauge symmetry may be broken to or replaced by an exact residual \(Z_2\) symmetry which maintains the stability of dark matter. A specific \(A_4\) model of flavor has been presented which shows how a predictive neutrino mass matrix may be generated in two loops, and how a realistic \(V_{CKM}\) matrix is obtained using again only \(N_{1,2,3}\). There are no flavor-changing neutral-current (FCNC) processes at tree level. They may appear in one loop through the dark sector, but they are suppressed by the flavor symmetry. The interconnection between mass, flavor, and dark matter is the key.

Whereas the SM fermions are known to transform as \(5^*\) and \(10\) of \(SU(5)\), the new dark-sector scalars do so as well:

\[
(\eta^+, \eta^0), \zeta^{-1/3} \sim 5, \quad (\zeta^{2/3}, \zeta^{-1/3}), (\zeta^{2/3})^*, \chi^+ \sim 10.
\]  

(25)

In a supersymmetric context, these would be squarks and sleptons. Analogous diagrams to Figs. 1, 3, and 4 have been previously discussed \([13]\) in this regard. Instead of \(R\) parity, they are distinguished here by dark \(Z_2\). Note that \(U(1)_D\) is not compatible with this interpretation. The color triplet \(\zeta^{-1/3}\) has also been previously considered \([14]\). As for the singlets \(N_{1,2,3}, \rho_{1,2,3},\) and \(\chi^0\), they are also singlets under \(SU(5)\), although they may also have an \(SU(6)\) origin \([15]\).

The renormalization-group (RG) equations for the evolution of the SM gauge couplings are equally affected by the new particles because they form complete \(SU(5)\) multiplets, so there is no gauge-coupling unification as in the SM, but the simple addition of a few new particles will do the trick \([16]\) if desired.

The new scalar particles of Eq. (25) mimic those of supersymmetry, so they may be produced at the LHC. They also have similar signatures because the lightest \(N\) behaves as the LSP (lightest supersymmetric particle) of the MSSM (minimal supersymmetric standard
model). However, there is no gluino in this theory, so details of the quark-squark interactions will be different.

The lightest $N$ is the dark-matter candidate of this proposal. Since it is a Dirac fermion, its annihilation cross section to quark and lepton pairs through the exchange of scalar quarks and leptons are unsuppressed, unlike the case of the MSSM with the lightest Majorana neutralino as the LSP. Its couplings are also not constrained by the MSSM. Hence it has a much larger parameter space to be a viable dark-matter candidate and has greater discovery potential at the LHC. The phenomenology of such a Dirac fermion dark-matter candidate has been discussed in Ref. [8, 11]. Its relic density has been shown to be compatible with what is observed. Here there are more annihilation channels, but an overall acceptable parameter space is clearly available.

In summary, a unifying proposal has been made. (1) In addition to the SM particles, there exists a dark sector, odd under $Z_2$ which may be derived from an $U(1)_D$ gauge symmetry. The particles of this dark sector consist of three neutral singlet Dirac fermions $N_{1,2,3}$ and the scalars of Eq. (25) which are complete $SU(5)$ multiplets. The lightest $N$ is a possible dark-matter candidate. (2) A non-Abelian discrete symmetry such as $A_4$ exists, under which $N_{1,2,3}$ as well as the quarks and leptons of the SM transform nontrivially. (3) As a result of this flavor symmetry, the one and only one Higgs doublet $\Phi$ of the SM is forbidden to couple to $\bar{q}_L q_R$ and $\bar{l}_L l_R$. The nonzero vacuum expectation value of $\phi^0$ generates $W$ and $Z$ masses but not fermion masses. (4) The soft breaking of $A_4$ to $Z_3$ in the $3 \times 3$ Dirac mass matrix of $N_{1,2,3}$ allows $\Phi$ to couple to $\bar{q}_L q_R$ and $\bar{l}_L l_R$ in one loop. Thus all quarks and leptons owe their masses to dark matter in conjunction with $\Phi$. (5) The residual $Z_3$ symmetry maintains diagonal mass matrices for $u$ and $d$ quarks as well as charged leptons. This is an explanation of why the quark mixing matrix $V_{CKM}$ is nearly diagonal. (6) Further soft breaking of $Z_3$ allows a realistic $V_{CKM}$. A two-loop Majorana neutrino mass matrix is also obtained with
the addition of scalar singlets $\chi$ and $\rho_{1,2,3}$. (7) The resulting neutrino mass matrix may be imple-
mented with a generalized $CP$ transformation under $\mu-\tau$ exchange to obtain maximal $CP$ viola-
tion together with $\theta_{23} = \pi/4$ while allowing nonzero $\theta_{13}$. (8) The predicted scalars of Eq. (25) which connect the quarks and leptons to their common dark-matter antecedents, i.e. $N_{1,2,3}$, are possibly observable at the LHC. They may also change significantly the SM couplings of $\Phi$ [17].

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