Unbraiding the Bounce: Superluminality around the Corner

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Abstract. We study a particular realization of the cosmological bounce scenario proposed recently by Ijjas and Steinhardt in [1]. First, we find that their bouncing solution starts from a divergent sound speed and ends with its vanishing. Thus, the solution connects two strongly coupled configurations. These pathologies are separated from the bouncing regime by only a few Planck times. We then reveal the exact structure of the Lagrangian, which reproduces this bouncing solution. This reconstruction allowed us to consider other cosmological solutions of the theory and analyze the phase space. In particular, we find other bouncing solutions and solutions with superluminal sound speed. These stable superluminal states can be continuously transformed into the solution constructed by Ijjas and Steinhardt. We discuss the consequences of this feature for a possible UV-completion.
1 Introduction and discussion

Since 2010 it is well known that minimally-coupled scalar-tensor theories with Kinetic Gravity Braiding [2] can dynamically violate the Null Energy Condition (NEC)\(^1\) and cross the phantom divide\(^2\) without developing ghost and gradient instabilities\(^3\), see [2, 10–12]. For a recent review of NEC violation see e.g. [13]. These *kinetically braided* theories extend the k-essence [14–17] by a braiding term and the “decoupling limit of DGP” [18, 19] or cubic Galileon [20] by higher nonlinear derivative interactions. The nonlinear interactions break the Galilean symmetry in the field space even in the Minkowski spacetime, but do not change the speed of propagation for the gravitational waves. For timelike field derivatives, *kinetically braided* theories represent imperfect (super-)fluids with the energy transport along the spatial gradients of the chemical potential [21]. The form of the braiding term defines the dependence of the transport coefficient on the chemical potential. In the cubic Galileon the transport coefficient is given by the square of the chemical potential. In the general *Kinetic Gravity Braiding*, the transport coefficient can be an arbitrary function of the chemical potential including the most natural case of a constant.

After the rediscovery [22, 23] of the general Horndeski theories [24] in 2011, these *kinetically braided* models are often referred to as “first two terms of Horndeski theories” or simply generalized / “cubic” Galileons. Since 2011 it is known [25, 26] that these theories can describe spatially-flat Friedmann universes evolving from contraction to expansion while being manifestly free of ghost and gradient instabilities around this cosmological bounce. Hence, these theories can realize a smooth “healthy” bounce. The possibility of the bounce in such systems was briefly mentioned in [11]. Furthermore, in [26] it was demonstrated that one can easily construct spatially-flat “healthy” bouncing universes with bouncing solutions of a non-vanishing measure. This reference established that there is a continuum of such minimally coupled healthy bouncing theories. The work provided sufficient conditions in the

\(^1\)For a system with energy-momentum tensor \(T^{\mu\nu}\), the NEC holds provided \(T^{\mu\nu}n^\mu n^\nu \geq 0\) for all null / light-like vectors \(n^\nu\).

\(^2\)This divide is physically impossible to cross [3–8] for k-essence, and theories with minimal coupling to gravity and without higher derivatives introducing the braiding between the derivatives of the metric and of the scalar field.

\(^3\)Despite the positivity of the energy density for linear perturbations, it is important to note that any theory violating the NEC necessarily has the energy density unbounded from below [9].
form of inequalities on the Lagrangian free functions to ensure a “healthy” bounce. *Kinetically braided* bouncing models also work in the (unavoidable) presence of normal matter, such as radiation, etc [26]. In particular, they allow for a smooth transition to the radiation dominated époque (“Hot G-Bounce”, see Fig 1, on page 10 [26]). In the same work, singularities in the gravitational metric and in the acoustic metric describing the cones of propagation for the scalar perturbations, have been discussed. These singularities can be at the beginning or at the end of the evolution, or both. Physically, these singularities correspond to (naively infinitely) strongly coupled configurations, where either the quasi-classical general relativity (GR) or quasi-classical description of the scalar field break down. Both theories are not renormalizable, have dynamical cones of influence / metric, and require a nontrivial ultraviolet (UV) completion. Under certain assumptions, the presence of these singularities was later proven for general *kinetically braided* theories in [27]. This proof was extended to general Horndeski theories in [28] and to theories interacting with another scalar field in [28, 29].

In this respect, *kinetically braided* theories, as well as more general Horndeski theories are not that different from regular GR, where the existence of singularities is a well established fact [32]. The only advantage is that one can relocate the initial cosmological singularity in the classical dynamical equations from the expanding to the contracting stage, even in a spatially-flat Friedmann universe. Hence the big bang could occur not at the beginning of the cosmological expansion, but at the onset of contraction. This crucial difference opens up new ways of thinking about initial conditions in the early universe. This is relevant for the initial conditions of inflation, which could now be preceded by a contraction, bounce or even the Minkowski space [11, 33–38]. There is plenty of theoretical and philosophical motivation to consider bounces and NEC violation in the early universe, for recent reviews see e.g. [13, 39, 40]. If the NEC can be violated by a physical system one can even consider such an exotic opportunity as a creation of a universe in a laboratory [41].

An interesting feature of *kinetically braided* theories is that the scalar perturbations around generic backgrounds propagate along an “acoustic” cone different from the light cone [2, 19, 42, 43]. This “acoustic” cone can be wider than the light cone or protrude outside of it just in some directions. In these cases the perturbations propagate faster than light see e.g. [42]. Contrary to k-essence where one can establish subluminality constraints on the form of the Lagrangian, in *kinetically braided* theories it seems that there are no such conditions. For a recent discussion on the dilatationally invariant subclass of these theories see [29]. There are examples [33, 44] of such theories where there is no superluminality for all cosmological configurations, for the proof see [44]. However, this only happens in an idealized universe without any external matter. An unusual property of the *kinetically braided* theories is that the value of the sound speed depends not only on the local state of the field, but also on the energy-momentum tensor of other matter fields present in the same point of spacetime. Even in the case of totally subluminal cosmological phase space [33, 44], an introduction of external matter sources instigates the superluminality at least for some regions of phase space [44].

The superluminality (with hyperbolic equations of motion for perturbations) per se does not necessarily cause any causal paradoxes, see e.g. [45–50]. Nevertheless, one can always construct nontrivial non-cosmological configurations, where closed causal curves (CCC) can be formed at the level of classical dynamics [42, 43, 51]. However, similarly to GR where we have the chronology protection conjecture due to Hawking [52], quantum field theory (QFT)
may protect the system from forming CCC in all such theories with dynamical cones of
influence, see e.g. [45, 47, 53].

On the other hand, there are powerful arguments that effective field theories (EFT) with
at least one configuration permitting superluminality do not allow for a standard Wilsonian
UV-completion in terms of local, Lorentz-invariant and weakly coupled fields or strings [42].
In order to apply these arguments, the superluminal configuration should belong to the same
EFT, which UV-completion is investigated. The latter condition is rather nontrivial, as
different semiclassical states can be isolated by regions with ghosts, regions with strong cou-
pling, or other features where the EFT breaks down. In these cases each separated region
corresponds to a different EFT. The way out is provided by a recent conjecture concerning a
possible Wilsonian UV-completion in such nonstandard theories. It was conjectured [54, 55]
(see also [56]), that a theory can UV-complete itself by forming classicalons - extended field
configurations playing the role of elementary quantum excitations, hence, the term classical-
ization. These classicalons appear as intermediate long-lived states and slowly decay into a
large number of soft IR elementary excitations. Later, it was argued that this UV-completion
by classicalization takes place, only provided some configurations allow for the superluminal
propagation [57–59]. The extended elementary excitations may induce a non-locality of the
UV completion of these theories [60].

Following this discussion, it is expected that general bouncing cosmologies can only
be realized in theories equipped with superluminality around some configurations. Conse-
quently, these theories cannot be UV-completed in the standard way. Only classicalization,
or maybe some other yet unknown construction, can UV-complete such bouncing models.

In 2016, Ijjas and Steinhardt (IS) proposed in [1] an interesting “inverse” method. The
method allows one to find particular realizations of the cosmological bounce scenario in a
specific subclass of kinetically braided theories. Specifically, they found a kinetically braided
model for a given cosmological evolution $H(t)$, where $H$ is the Hubble parameter. Kineti-
cally braided theories have two free functions, $K(\phi, \partial \phi)$ and $G(\phi, \partial \phi)$. Thus, there is enough
freedom to choose not only $H(t)$, but also a time-dependence for one of the two coefficients
in the quadratic action for curvature perturbations. The advantage of this method is that
it allows to construct a theory for a given evolution while keeping a direct control over per-
turbations. This procedure enables one to find $\phi(t)$ and to specify different free functions
in the Lagrangian as functions of time. Hence, this method yields an implicit construction
of the Lagrangian of the model realizing the bounce. Using the inverse method, IS found a
particular theory, which accommodates the bounce free of ghosts and gradient instabilities.
For convenience we denote this realization as IS-bounce. The IS-bounce was not only claimed
to be free from ghost and gradient instabilities, but also be exempt from superluminal prop-
agation of perturbations. The reconstructed solution also included healthy stages before and
after NEC violation. In this way, the system could enter the NEC violating bouncing stage
and leave it without encountering any problem for stability or UV-completion. These findings
are illustrated by explicit numerical calculations and plots corresponding to two sets of five
independent free parameters.

Overview of the paper In this paper we analyze the system introduced by IS in [1]. First
in section (2) we derive and discuss main formulas for the dynamics of cosmological solutions
and perturbations in braided theories under consideration. In section (3) we briefly discuss acoustic geometry for cosmological perturbations, convenient variables and their relations by different gauge and conformal transformations. Then in section (4) we uncovered the explicit structure of the Lagrangian by deriving the functions $k(\phi)$ and $q(\phi)$ of the theory, see equations (4.11)-(4.14). We then used these results to study the IS-bounce for the first set of parameters, see (1a) and (1b). We found that IS-bounce solution starts with a divergent sound speed around 15 Planck times, $t_{pl}$, before the NEC violation starts. The sound speed is still superluminal less than $10 t_{pl}$ before the onset of the NEC violation. Moreover, the system enters into the strongly coupled regime with vanishing sound speed and consequently loses predictive power in just 15 $t_{pl}$ after the exit from the bouncing stage with the NEC violation. From the classical perspective the IS trajectory begins with a singularity of the acoustic metric and ends in its another singularity. This evolution of the universe is evidently less appealing, than that during the “Hot G-Bounce” scenario mentioned above. If the system were in a standard weakly coupled vacuum, this would imply that short-wavelength curvature perturbations evolve from a state with infinite quantum fluctuations of canonical momentum to a state with infinite quantum fluctuations of the conjugated canonical field. Clearly this picture is unphysical, and this implies that the system is strongly coupled in these singular states. It is a challenge to modify the theory in such a way that preserves the required evolution, $H(t)$, but changes the dynamics on these ultra short time scales. To make a proper comparison, it is worth noting that gravity quantum strong coupling scale (and ultimate EFT cutoff) depends on the number of degrees of freedom $N$ as $M_{pl}/\sqrt{N}$, [61], while already in the Standard Model there are around 100 degrees of freedom. This implies that the pathologies of the IS-bounce are not really separated in a distinguishable way from the desired semiclassical evolution.

Further we considered the phase space in this model, see Fig. (3) and Fig. (4). We found other stable bouncing trajectories. Then we identified stable regions, where superluminality is present. In parts of these regions the NEC holds, while in others it is violated. Moreover, there is a region of phase space, where the NEC is broken, but the sound speed is subluminal. This shows again that the superluminality is not directly linked to stability of the Phantom stage, c.f. [62]. In particular, we found superluminality just around the corner - in the regions very close to the IS-bounce. These regions are well within the field range corresponding to the NEC violation phase. Clearly a source or simple interaction can continuously deform these states into the IS-bounce trajectory. Hence, these states belong to the same EFT.

Thus it is impossible to avoid this type of superluminality by modifying functions $k(\phi)$ and $q(\phi)$ in the Lagrangian outside of the needed field range. In order to attempt escaping superluminality one has to modify either the desired evolution $H(t)$ or the structure of the theory or both.

Other interesting findings include the following. The IS-bounce is a separatrix, see Fig. (4). This solution goes through the singularity of the equation of motion, similarly to the singular trajectories found in [3]. For the second choice of the parameters used in [1] to obtain their Fig. 3, we could not reproduce the IS claims. It seems that below Fig. 3 from [1] there is a typo somewhere either in the set of parameters or in the form of the functions.

To conclude, we think it is interesting to understand the consequences of the possible bounces in the early universe. Though, so far, this nonstandard option for the early universe seems to be inseparable from superluminality and a nonstandard UV-completion with classicalization as the only current candidate for the latter.
2 Model and main equations

The IS-bounce uses a class of Kinetic Gravity Braiding theories with explicitly strongly broken shift-symmetry \( \phi \rightarrow \phi + c \)

\[
S = \frac{1}{2} \int d^4x \sqrt{-g} \left( k(\phi) (\partial \phi)^2 + \frac{1}{2} q(\phi) (\partial \phi)^4 + (\partial \phi)^2 \Box \phi \right), \tag{2.1}
\]

where

\[
(\partial \phi)^2 \equiv g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \equiv 2X, \quad \Box \phi \equiv g^{\mu\nu} \nabla_\mu \nabla_\nu \phi, \tag{2.2}
\]

where \( \nabla_\mu \) is the usual Levi-Civita connection. The scalar field is supposed to be minimally coupled to gravity. Hence, the theory is defined by two free functions \( k(\phi) \) and \( q(\phi) \). In notation of \([2]\) where generic theories of the type

\[
S = \int d^4x \sqrt{-g} [K(X, \phi) + G(X, \phi) \Box \phi], \tag{2.3}
\]

were introduced, we have\(^6\)

\[
K(X, \phi) = k(\phi) X + q(\phi) X^2, \quad G(X, \phi) = X. \tag{2.4}
\]

This identification allows us to directly use all necessary formulas derived in \([2]\) (see also \([10]\)) for arbitrary \( K(X, \phi) \) and \( G(X, \phi) \) for the background dynamics and perturbations in the spatially-flat Friedmann universe

\[
ds^2 = dt^2 - a^2(t) dx^2. \tag{2.5}
\]

Below, instead of rederiving formulas for the particular case \((2.4)\), as it was done by IS, we use general results from \([2]\). In particular, the pressure is

\[
P(\phi, X, \dot{\phi}) = K - 2XG,\phi - 2XG,X \ddot{\phi} = kX + qX^2 - 2X\ddot{\phi}, \tag{2.6}
\]

the charge density with respect to shifts \( \phi \rightarrow \phi + c \)

\[
J = \dot{\phi} \left( K_{,X} - 2G,\phi + 3\dot{\phi}HG,X \right) = \dot{\phi} \left( k + 2qX + 3\dot{\phi}H \right), \tag{2.7}
\]

where \( H = \dot{a}/a \) is the Hubble parameter. The variation of the action \((2.3)\) with respect to the field \( \phi \) gives an equation of motion, which in terms of the charge density takes the following elegant form

\[
\dot{J} + 3HJ = P,\phi. \tag{2.8}
\]

The kinetic braiding with gravity reveals itself in the presence of the \( \dot{H} \) in this equation.

The general expression for the energy density

\[
\varepsilon(\phi, \dot{\phi}, H) = \dot{\phi}J - P + 2XG,X \ddot{\phi} = 2X \left( K_{,X} - G,\phi + 3\dot{\phi}HG,X \right) - K, \tag{2.9}
\]

\(^5\)Further we use: the standard notation \( \sqrt{-g} \equiv \sqrt{-\det g_{\mu\nu}} \) where \( g_{\mu\nu} \) is the metric, the signature convention \((+,-,-,-)\) (contrary to \([1]\)), and the units \( c = \hbar = 1 \), \( M_{Pl} = (8\pi G_N)^{-1/2} = 1 \).

\(^6\)At the beginning the authors of \([1]\) also used \( G(X, \phi) = b(\phi) X \), however this additional free function \( b(\phi) \) can be eliminated by the simple field-redefinition: \( d\phi = b^{-1/3}(\phi) d\phi \).
reduces for the choice (2.4) to
\[\varepsilon = kX + 3qX^2 + 6\dot{\phi}HX.\] (2.10)

Further the first Friedmann equation reads
\[3H^2 = \varepsilon = kX + 3qX^2 + 6\dot{\phi}HX,\] (2.11)

while for the second equation we have
\[\dot{H} = \frac{1}{2} (\varepsilon + P) = XG_{,X}\dot{\phi} - \frac{1}{2} \dot{\phi}J.\] (2.12)

It is important that the energy density and the first Friedmann equation contain a term linear in \(H\). Therefore for kinetically braided systems the branches resulting from the first Friedmann equation do not correspond to expansion and contraction of the universe. Indeed, solving the quadratic equation we obtain
\[H_\pm = (XG_{,X}\dot{\phi} + \frac{1}{3} (2X (K_{,X} - G_{,\phi}) - K))^{\frac{1}{2}}.\] (2.13)

This relation implies that not all configurations \((\phi, \dot{\phi})\) with positive energy density are allowed, but only those satisfying an additional condition
\[6X (XG_{,X})^2 + 2X (K_{,X} - G_{,\phi}) - K \geq 0.\] (2.14)

Finally it is convenient to write an equation of motion for the scalar field (2.8) where \(\dot{H}\) is expressed through (2.12)
\[D\ddot{\phi} + 3J \left( H - \phi XG_{,X} \right) + \varepsilon_{,\phi} = 0,\] (2.15)

where general expression
\[D = K_{,X} + 2XK_{,XX} - 2G_{,\phi} - 2XG_{,X}\phi + 6\dot{\phi}H (G_{,X} + XG_{,XX}) + 6X^2G_{,XX}^2,\] (2.16)

reduces for the choice (2.4) to
\[D = k + 6Xq + 6\dot{\phi}H + 6X^2.\] (2.17)

Now we are prepared to write the formulas for the perturbations. We use the “unitary” gauge, and embed the spacelike hypersurface \(\delta \phi = 0\) into the perturbed Friedmann universe using the ADM decomposition
\[ds^2 = N^2dt^2 - a^2 e^{2R} \delta_{ik} \left( N^i dt + dx^i \right) \left( N^k dt + dx^k \right),\] (2.18)

which is useful to compare with the most general line element with scalar perturbations written in conformal time \(\tau\)
\[ds^2 = a^2 \left[ (1 + 2\varphi) d\tau^2 + 2B_i dx^i d\tau - [(1 - 2\psi) \delta_{ik} - 2E_{,ik}] dx^i dx^k \right].\] (2.19)

As it was pointed out in [10], the variable \(R\) is not a comoving curvature perturbation, because there is an energy flow \(T^i_j = \dot{\phi}^3 G_X \partial_i \delta N\) in the gauge (2.18). Expressing the lapse
fluctuation $\delta N$ through the longitudinal part of the momentum constraint (see (A.5) on page 36 [2])

$$
\left( H - \dot{\phi}XG,_{X} \right) \delta N = \dot{\mathcal{R}} , \tag{2.20}
$$

one obtains the quadratic action for curvature perturbations $\mathcal{R}$, \footnote{Note that [1] does not use the canonical normalization with $1/2$ in front of the action. Hence our coefficients $A(t)$ and $B(t)$ are twice larger than those in [1].} (see (A.8) on page 36 [2])

$$
S_c = \frac{1}{2} \int dt d^3x a^3 \left( A(t) \dot{\mathcal{R}}^2 - \frac{B(t)}{a^2} (\partial_\mathcal{R})^2 \right) . \tag{2.21}
$$

The formula for the normalization of the kinetic term is given by (A.9) page 36, [2]

$$
A = \frac{2XD}{(H - \dot{\phi}XG,_{X})^2} . \tag{2.22}
$$

Hence, it is the coefficient $D$, given by (2.16), in front of the second derivative in the reduced equation of motion (2.15) which determines ($D > 0$) whether the perturbations are free of ghosts. It is interesting to note that curves on phase space with $D = 0$ correspond to an infinitely-strong coupling of perturbations and to pressure-like curvature singularity [2], where GR breaks down, see (2.6) and (2.15). Clearly the reduced equation of motion is singular on these curves. The quantity $D$ corresponds to the determinant of the matrix in front of the second derivatives ($\ddot{\phi}, \ddot{a}$) in the equations (2.12) and (2.8).

The sound speed is given by the formula (A.11) page 36, [2]

$$
c_s^2 = \frac{\dot{\phi}XG,_{X} \left( H - \dot{\phi}XG,_{X} \right) - \partial_t \left( H - \dot{\phi}XG,_{X} \right)}{XD} , \tag{2.23}
$$

where we assumed that the field $\phi$ is the only source of energy-momentum. From this expression we can write

$$
\frac{1}{2} B(t) = \frac{\dot{\phi}XG,_{X} \left( H - \dot{\phi}XG,_{X} \right) - \partial_t \left( H - \dot{\phi}XG,_{X} \right)}{(H - \dot{\phi}XG,_{X})^2} . \tag{2.24}
$$

It is natural to introduce a quantity

$$
\gamma = H - \dot{\phi}XG,_{X} , \tag{2.25}
$$

in terms of which the expression for $B$ reads

$$
\frac{1}{2} B(t) = \frac{d}{dt} \gamma^{-1} + H \gamma^{-1} - 1 . \tag{2.26}
$$

The expression for $B(t)$ was written in this elegant form in [1] for a particular choice (2.4) of functions $K$ and $G$. Before that this variable (2.25) was used in [27] and [63]. The vanishing $\gamma$ corresponds to the change of the branch in the solution of the first Friedmann equation with respect to the Hubble parameter (2.13). In that case one cannot express the perturbation of the lapse $\delta N$ from the momentum constraint (2.20). Thus one has to use other dynamical
variables to describe the dynamics around this point. There is an interesting discussion [63–65] of gauge issues, choice of dynamical variables and slicing around $\gamma = 0$. This phenomenon is not special to *kinetically braided* theories. A spatially-flat Friedmann universe driven by a scalar field with canonical kinetic term and a negative potential can evolve from expansion to contraction, see e.g. [66]. At the turning point $\gamma = H = 0$, and one cannot exclude $\delta N$ from the action for perturbations.

For the Lagrangian given by (2.4) one obtains
\[ \gamma (t) = H - \dot{\phi} X, \] (2.27)
so that (2.24) (or (2.26)) yields
\[ B = \frac{2X \left( k + 2qX + 4H \dot{\phi} + 2\dot{\phi} - 2X^2 \right)}{(H - \dot{\phi} X)^2}, \] (2.28)
while (2.22) reads
\[ A = \frac{2X \left( k + 6qX + 6\dot{\phi} H + 6X^2 \right)}{(H - \dot{\phi} X)^2}. \] (2.29)

It is also useful to rewrite the action (2.21) in terms of the canonically-normalized Mukhanov-Sasaki variable
\[ v = z R, \] (2.30)
where we denoted
\[ z = a\sqrt{A} = a\sqrt{\frac{2XD}{\gamma^2}}. \] (2.31)
Then the action reads
\[ S_c = \frac{1}{2} \int d\tau d^3x \left( v'^2 - c_s^2 (\partial_i v)^2 + \frac{z''}{z} v^2 \right), \] (2.32)
where the prime denotes a derivative with respect to conformal time $\tau$, defined through $d\tau = dt/a$.

It is worth noting that vanishing $D$ generically corresponds to infinities of the square of the sound speed, (2.23) and to an infinitely strong coupling between canonically normalized perturbations $v$.

### 3 Conformal Transformations, Gauges and Acoustic Geometry

As this paper discusses superluminality, it is interesting to look at the effective (or acoustic) metric where the scalar perturbations propagate. It is possible to write the action (2.21) in an elegant way
\[ S_c = \frac{1}{2} \int d^4x \sqrt{-\mathcal{G}} \mathcal{G}^{\mu\nu} \partial_\mu R \partial_\nu R, \] (3.1)
where the *covariant acoustic* metric $\mathcal{G}^{-1}_{\mu\nu}$ for curvature perturbations is
\[ dL^2 = \mathcal{G}^{-1}_{\mu\nu} dx^\mu dx^\nu = z^2 c_s^2 \left( \frac{cs^2}{z^2} d\tau^2 - dx^2 \right). \] (3.2)
the contravariant metric is inverse to it and the notation is standard. This metric $G^{-1}_{\mu\nu}$ is singular for $\gamma = 0$, as $z \to \infty$. It seems that classically this singularity is not a problem \cite{63–65}. Though it is interesting to understand the quantum mechanical consequences of this singular behavior. As this study goes beyond the scope of this paper, we leave this for a future work.

The transformation to the canonical Mukhanov-Sasaki variable can be considered as a conformal transformation of the acoustic metric

$$G^{-1}_{\mu\nu} \rightarrow C^{-1}_{\mu\nu} = z^{-2}G^{-1}_{\mu\nu}, \quad \mathcal{R} \rightarrow v = z\mathcal{R}.$$  \hfill (3.3)

The gauge part of the metric follows the propagation of the fluctuations of the field $\delta \phi$. In the unitary gauge this is not obvious as $\delta \phi = 0$. However, one can perform a gauge transformation, see e.g. page 293 \cite{67}, so that $\tilde{\delta} \phi = -\phi'\xi^0$ and $\tilde{\psi} = \psi + \xi^0 a'/a = -\mathcal{R} + \xi^0 a'/a$. For example for $\xi^0 = \mathcal{R} a/a'$, the spatial metric becomes unperturbed, $\delta g_{ik} = 0$, and so in this “spatially flat” gauge

$$\delta \phi|_{\text{flat}} = -\frac{\phi}{H} \mathcal{R}.$$  \hfill (3.4)

Using analogy between $\gamma$ in braided models and $H$ in k-essence we can introduce the “$\gamma$-gauge”

$$\xi^0 = \frac{\mathcal{R}}{a\gamma},$$  \hfill (3.5)

which yields

$$\tilde{\phi} = \varphi - \frac{1}{a} \left(a\xi^0\right)' = \delta N - \frac{1}{a} \left(a\xi^0\right)' = \mathcal{R} \frac{\gamma'}{\gamma^2},$$  \hfill (3.6)

where we used the constraint (2.20), and

$$\delta \phi|_{\gamma} = -\frac{\phi}{\gamma} \mathcal{R}.$$  \hfill (3.7)

Using $\delta \phi|_{\gamma}$ the action (2.21) reads

$$S_c = \frac{1}{2} \int d^4x \sqrt{G} \ G^{\mu\nu} \partial_\mu \mathcal{R} \partial_\nu \mathcal{R} = \frac{1}{2} \int d^4x \sqrt{-C} \ C^{\mu\nu} \partial_\mu \delta \phi|_{\gamma} \partial_\nu \delta \phi|_{\gamma} + \ldots,$$  \hfill (3.8)

where the ellipsis stands for a “mass-like” term without derivatives of $\delta \phi|_{\gamma}$. The change of variable $\mathcal{R} \longleftrightarrow \delta \phi|_{\gamma}$ can be understood as another conformal transformation

$$G^{-1}_{\mu\nu} \rightarrow C^{-1}_{\mu\nu} = \omega^2 G^{-1}_{\mu\nu}, \quad \mathcal{R} \rightarrow \delta \phi|_{\gamma} = -\omega^{-1} \mathcal{R},$$  \hfill (3.9)

where $\omega = \gamma/\dot{\phi}$ so that

$$dC^2 = C^{-1}_{\mu\nu} dx^\mu dx^\nu = Dc_s a^2 \left(c_s^2 d\tau^2 - dx^2\right).$$  \hfill (3.10)

The acoustic metric $C^{-1}_{\mu\nu}$ differs from the metric given by the formula (3.15), \cite{2} by the normalization $D^2 c_s^2$. This conformal factor is not important for the propagation of the high frequency perturbations, and related stability studies, but is needed for a proper normalization of the action. This transformation provides a short explanation for the so-called “DPSV trick” discussed in \cite{68}. It is instructive to compare this acoustic metric with the one obtained for k-essence and cosmological perturbations \cite{45}, Appendix C. There it was demonstrated
that $\delta \phi \big|_{\text{flat}}$ propagate in the acoustic metric (3.10) with $D = \varepsilon, X = K, X + 2XK, XX$. There is a continuity in $G, X$ between “$\gamma$-gauge” and “flat gauge”.

The acoustic metric derived in [2] is generic and can be used to investigate the speed of propagation of fluctuations, gradient (in)stabilities and possible appearance of ghosts also around general inhomogeneous and anisotropic backgrounds. In particular, this check enables one to exclude wormholes [69, 70] and static semiclosed worlds [71]. The advantage of the acoustic metric is that it can be used for stability checks for high frequency perturbations without deriving the action for perturbations. The latter can be especially complicated in the presence of external matter.

Finally it is worth mentioning that one can express perturbations through gauge-invariant variables which coincide with the conformal Newtonian gauge (notation as in [67])

$$- \mathcal{R} = \Psi + \frac{H}{\phi} \delta \phi ,$$

and respectively

$$\delta \phi \big|_{\gamma} = \frac{H}{\gamma} \frac{\delta \phi}{\phi} + \frac{\dot{\phi}}{\gamma} \Psi .$$

4 Inverse method, finding the theory

The authors of IS-bounce postulated a fairly simple time-dependence of the Hubble parameter

$$H (t) = H_0 t \exp \left( -F (t - t_*)^2 \right) ,$$

where $H_0, F$ and $t_*$ are constants. They proposed an “inverse method” to find free functions $k (\phi)$ and $q (\phi)$ in (2.1) which can realize this cosmological evolution. The NEC is violated between $t_-$ and $t_+$ where

$$t_\pm = t_* \pm \sqrt{t_*^2 + 2F - 1}.\quad (4.2)$$

The bounce occurs at $t = 0$. For the bounce one has to start form $H_-$ branch of the solutions of the first Friedmann equation (2.13). The key observation of the “inverse method” proposed in [1] is that one can also independently postulate $\gamma (t)$ in (2.26). The IS-bounce postulates

$$\gamma = \gamma_0 \exp (3\theta t) + H (t) ,$$

where $\gamma_0$ and $\theta$ are additional constants with respect to already introduced $H_0, F$ and $t_*$. From (2.27) one can obtain

$$\phi_{IS} (t) = \phi_0 + \int_{t_0}^t dt' \left[ 2 (H (t') - \gamma (t')) \right]^{1/3} ,$$

where $\phi_{IS} (t_0) = \phi_0$. It is convenient to choose this initial value as $\phi_0 = \left( -2\gamma_0 / \theta \right)^{1/3} \exp (3t_0) ,$ so that the particular solution postulated in IS-bounce is

$$\phi_{IS} (t) = \phi_* \exp (\theta t) ,$$

where the field value at the bounce $\phi_*$ is given by

$$\phi_* = \left( \frac{-2\gamma_0}{\theta^3} \right)^{1/3} .$$
Then cosmological time is expressed on the IS-bounce as

\[ t = \frac{1}{\theta} \log \left( \frac{\phi_{IS}}{\phi_*} \right). \] (4.7)

The field values \( \phi_1 \) and \( \phi_2 \) corresponding to the beginning of the NEC violation and its restoration are given by

\[ \phi_1 = \phi_{IS} (t_-), \quad \phi_2 = \phi_{IS} (t_+). \] (4.8)

One has to rely on the reconstruction at least during the NEC violation, i.e. in this field range \( \phi_1 < \phi < \phi_2 \). Using the substitutions (4.1) and (4.3) one obtains the functions \( k(\phi) \) and \( q(\phi) \) as functions of time on the particular solution (4.5)

\[ k(t) = - \frac{2 \left( 2\dot{H} + 3H^2 + \dot{\gamma} + 3H\gamma \right)}{(2 (H - \gamma))^{2/3}}, \] (4.9)

and

\[ q(t) = \frac{4 \left( 2\dot{H} + \dot{\gamma} + 9H\gamma \right)}{3(2 (H - \gamma))^{4/3}}. \] (4.10)
Figure 2: The evolution of the coefficients in the quadratic action (2.21) for the scalar perturbations is shown here for the IS solution for the second choice of parameters (4.18). All quantities are in the Planck units. The dashed black vertical lines correspond to $t_- \simeq -84.3$ where the Phantom stage with NEC violation starts and to $t_+ \simeq 84.8$ where the NEC gets restored. All quantities are obtained using the same code as in the previous case. The caption of the Fig. 3 of [1] claims “all fundamental physical quantities including $H(t)$ and $c_s^2$ remain finite and positive” for this choice of parameters. On this plot one can clearly see that $A(t) > 0$, whereas $B(t) < 0$ during the whole Phantom stage so that $c_s^2 = B/A < 0$. It seems that there is a typo somewhere among the values of parameters (4.18).

It is convenient to introduce functions

$$W(\phi) = \exp \left[ \frac{F}{\theta^2} \left( \log \left( \frac{\phi}{\phi_*} \right) - \theta t_* \right)^2 \right],$$

and

$$\Omega(\phi) = W(\phi) \theta^3 (\theta \phi)^3 + H_0 \left[ \log \left( \frac{\phi}{\phi_*} \right) \left( 4F \left[ \log \left( \frac{\phi}{\phi_*} \right) - \theta t_* \right] + 2\theta (\theta \phi)^3 \right) - 2\theta^2 \right],$$

in terms of which the defining functions are

$$k(\phi) = -\frac{12H_0^2 \log^2 (\phi/\phi_*) - 3W(\phi) \left[ \Omega(\phi) - H_0 \theta (\theta \phi)^3 \log (\phi/\phi_*) \right]}{W^2(\phi) \theta^2 (\theta \phi)^2},$$

and

$$q(\phi) = \frac{12H_0^2 \log^2 (\phi/\phi_*) - 2W(\phi) \left[ \Omega(\phi) + H_0 \theta (\theta \phi)^3 \log (\phi/\phi_*) \right]}{W^2(\phi) \theta^2 (\theta \phi)^4}.$$ 

These expressions defining the theory, which should be related to the very origin of the universe, neither look well-motivated nor natural from any point of view. Neither these functions can be stable with respect to the quantum corrections. This is the price for the chosen simple exact solution (4.1), (4.5). We are left with the following five free parameters...
\( H_0, \theta, F, \gamma_0, t_s \) which specify the Lagrangian. The authors of [1] have chosen them below their Fig. 1 as

\[
H_0 = 3 \times 10^{-5}, \theta = 0.0046, F = 9 \times 10^{-5}, \gamma_0 = -0.0044, t_s = 0.5, \quad (4.15)
\]

all in reduced Planck units. The role of this unphysically tiny \( t_s \) remained an open question for us. For this choice of parameters the field value at the bounce is \( \phi_\ast \approx 44.88 \) and \( t_- \approx -74.286 \) while \( t_+ \approx 74.786 \). Now we can plot the coefficients \( A(t) \) and \( B(t) \) and the sound speed \( c_s^2 \) on the trajectory (4.5), see Fig. (1a) and Fig. (1b) respectively. On these plots we clearly see that less than 10 \( t_{pl} \) before the beginning of the Phantom stage the sound speed is superluminal. Moreover, just 15 \( t_{pl} \) after NEC is restored and the bouncing phase is finished the system enters into elliptic regime / regime with the gradient instability. When the system approaches the regime, where \( c_s^2 = 0 \) the quantum perturbations diverge and the system becomes strongly coupled. In that case the semiclassical equations completely lose predictability. On the other hand just some 15 \( t_{pl} \) before the beginning of the Phantom stage the coefficient \( A(t) \) vanishes and the sound speed becomes not only just superluminal, but simply divergent. Both divergent and vanishing sound speeds correspond to infinitely strongly coupled fluctuations. Indeed, in weakly coupled theories on short length scales \( \ell \), where \( (kc_s)^2 \gg |z''/z| \), one can use the uncertainty relation [7, 57] to find

\[
\delta v_\ell \cdot \delta v'_\ell \simeq \hbar \ell^{-3}. \quad (4.16)
\]

Further estimating \( \delta v_\ell \simeq \omega_\ell \delta v_\ell \simeq c_s \ell^{-1} \delta v_\ell \) we obtain

\[
\delta v_\ell \simeq \ell^{-1} \sqrt{\hbar/c_s}, \quad \delta v'_\ell \simeq \ell^{-2} \sqrt{\hbar c_s}. \quad (4.17)
\]

Hence larger values of \( c_s \) correspond to larger velocity fluctuation \( \delta v'_\ell \) on all short scales. Whereas vanishingly small \( c_s \) corresponds to a huge \( \delta v_\ell \). In these non-canonical theories both the field and the canonical momentum do enter the interaction vertices. Clearly very large (divergent) quantum fluctuations is a pathology. The only way to avoid these estimations is to assume that the theory is strongly coupled, so that the uncertainty relation is not saturated and that the fluctuation of momentum \( \delta v_\ell' \) is not related to the fluctuation of the field \( \delta v_\ell \) as it is in the quantum oscillator case. But then the theory is clearly strongly coupled in the quantum mechanical sense. In order to enter the unmodified Phantom bouncing stage and leave it without either starting or ending in these strongly-coupled regimes one has to modify dynamics on time-scales of 10 \( t_{pl} \) which is a clear challenge for the scenario.

The second example of IS was the set of parameters chosen below their Fig. 3 as

\[
H_0 = 3 \times 10^{-5}, \theta = 4.6 \times 10^{-6}, F = 7 \times 10^{-5}, \gamma_0 = -0.0044, t_s = 0.5. \quad (4.18)
\]

For the corresponding times we have \( t_- \approx -84.266 \) while \( t_+ \approx 84.766 \). Clearly the caption of the Fig. 3 of [1] claims “all fundamental physical quantities including \( H(t) \) and \( c_s^2 \) remain finite and positive” for this choice of parameters. However, we found that the sound speed is actually imaginary throughout the NEC-violating stage, see Fig. (2) for the coefficients \( A(t) \) and \( B(t) \). Clearly \( A(t) > 0 \) whereas \( B(t) < 0 \) during whole phantom stage. Hence the sound speed \( c_s^2 = B/A < 0 \). It seems that there is somewhere a typo in these values of parameters (4.18).
Figure 3: Here we plot different side views of the phase space hypersurface given by the constraint - the but first Friedmann equation (2.11) for the system defined by (2.1) and (4.13), (4.14). The parameters correspond to the choice of [1] below their Fig. 1, see (4.15). The red curve is the-IS bounce trajectory. The black curves correspond to $H = 0$ while the blue curves represent $\gamma = 0$. The purple dashed lines represent $\dot{\phi} = 0$. Each point on these lines on the hypersurface of the constraint is a fixed point. Therefore the self-crossing of the hypersurface does not cause any trouble.

5 Phase Space

The reconstruction of the Lagrangian through the identification of the functions (4.9) and (4.10) allows us to study the properties of other cosmological solutions in the system under consideration. A proper global analysis can follow the lines of [66]. One chooses dynamical variables $(\phi, \dot{\phi}, H)$ which evolution is given by the second Friedmann equation (2.12) and the equation of motion for the scalar field (2.15) written as a first order system. These dynamical variables are moving on a hypersurface given by the constraint - first Friedmann equation (2.11), see Fig. (3). In many cases this hypersurface cannot be uniquely projected onto $(\phi, \dot{\phi})$ plane, see discussion in [66] for a canonical scalar field and [26, 44] for kinetically braided theories. In the latter it was found that sometimes one can uniquely project the constraint hypersurface onto the space $(\dot{\phi}, H)$. To make a projection onto $(\dot{\phi}, H)$ plane, one has to choose the branch in the solution of the first Friedmann equation and substitute this branch $H_\pm$ into the reduced field equation (2.15). In that case, the dynamics are only present in the region of the phase space where condition (2.14) holds, i.e. where

$$\dot{\phi}^4 + q(\phi) \dot{\phi}^2 + \frac{2}{3} k(\phi) \geq 0.$$  \hspace{1cm} (5.1)

In the Fig. (4) we plot different regions in the remaining phase space. We found other stable bouncing trajectories, see the right plot on the Fig. (4). In this figure one can see stable regions where superluminality is present. In parts of these superluminal regions the NEC
Figure 4: Here we plot the phase space for the system defined by (2.1) and (4.13), (4.14). We pick the $H_-$ branch in (2.13). The parameters correspond to the choice of [1] below their Fig. 1, see (4.15). This plot is a projection of the hypersurface from the Fig. (3). The red line corresponds to the IS-Bounce. This bouncing trajectory is a separatrix which goes from a saddle point, see the plot on the right. In the yellow regions, the condition (2.13) or (5.1) is not fulfilled, so that there is no spatially-flat FRW geometry there. The phase space continues to the other branch of the Friedmann equation (2.13) through the borders of these regions where $\gamma = 0$. In the light brown / almond regions $c_s^2 < 0$, and the system has a gradient instability. The borders of these regions correspond to $c_s^2 = 0$ which causes an infinitely strong coupling of curvature perturbations. The burned orange / dark brown regions have ghosts, $D < 0$ there, see (2.16) and (2.17). The boundaries of these regions have $D = 0$ which implies an infinite pressure (2.6) and correspondingly an infinite curvature. These boundaries are also singularities of the background equations of motion (2.15). In the Congo pink / coral regions the sound speed is superluminal $c_s^2 > 1$, but the NEC holds. Light blue / lavender regions correspond to the NEC violation without superluminality and free of ghosts and gradient instabilities. Purple / blue bell regions have the NEC violation and superluminality, but are free of ghosts and gradient instabilities. These superluminal regions are located only slightly below the red IS-Bounce trajectory. Finally, four small white regions are rather boring as they are free of ghosts, gradient instabilities, superluminality and violation of the NEC. The IS-Bounce crosses two of these white regions. On the red bouncing trajectory (4.5) the NEC is broken between $\phi_1$ and $\phi_2$, (4.8). One has to rely on the reconstruction of the Lagrangian functions (4.13) and (4.14) at least between these two field values, between blue dashed lines. The black dashed curves correspond to $H = 0$. There are many trajectories above and below the red trajectory and on the right of the $\dot{\phi}_2$ which bounce and evolve through the black dashed curves. All trajectories start or end (or both) on a singularity or infinitely-strong coupling curves.
holds, while in others it is violated. Moreover, there is a region of phase space where NEC is broken but the sound speed is subluminal. This shows again that the superluminality is not directly linked to stability, c.f. [62]. The link is rather subtle. In particular one can find the superluminality just around the corner of the *IS-bounce*, in the neighborhood slightly below the trajectory. These superluminal stable regions are well within the field range corresponding to the NEC violation phase. Clearly a source or simple interaction can continuously deform these states into the *IS-bounce* trajectory. Hence, these states belong to the same EFT.

The origin of the trajectory is the ghosty region followed by a tiny superluminal region where NEC holds. Then it is followed by a vanishingly tiny white subluminal region where NEC holds. The *IS-bounce* trajectory leaves the ghosty region by going through the singularity of the equation of motion $D = 0$. This is also a pressure/curvature singularity. Thus this *IS-bounce* trajectory is clearly demonstrating the singular behavior similar to that crossing the phantom divide in k-essence models linear in $X$ [3]. It seems that around the point on the boundary of the central ghosty region the trajectories for a limiting cycle similarly to [7]. This is, however, an illusion as they start and end on the singularity approaching or leaving the boundary $D = 0$ vertically.

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