Lepton $U_1(1)$ symmetry and interaction of neutrinos with deuterons at high energies

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Abstract

We discuss first the lepton $U(1)$ symmetry and its relation to the presence of the neutrino mass term in the Hamiltonian. Then we consider the possibility of the measurement of neutrino-deuteron reactions with high energy neutrinos at SNO by using neutrinos from a neutrino factory. Next we report on a construction of weak axial one-boson exchange currents for the Bethe-Salpeter equation, starting from chiral Lagrangians. These Lagrangians are a natural extension of the non-linear $\sigma$ model and they serve to describe processes at energies far from the threshold. It is shown that the currents fulfil the Ward-Takahashi identities and the matrix element of the full current between the two-body solutions of the Bethe-Salpeter equation satisfies the PCAC constraint exactly. Consistent calculations based on the proposed formalism would give more confidence in conclusions about the neutrino-deuteron processes and consequently, about the neutrino oscillations.

1 Introduction

The physics of neutrino represents now a vast interdisciplinary area overlapping elementary particle-, nuclear-, astrophysics and cosmology. In the particle classification, the neutrino belongs to leptons. There exists a consensus that its mass should not be large, maybe it is zero.

* Talk given at the 16th IUPAP International Conference on Few–Body Problems in Physics, March 6–10, 2000, Taipei, Taiwan
With this in mind and in an analogy with the other unitary symmetries [1], one is tempted to assume for the world of leptons the unitary $U(1)$ symmetry

$$U(1) = e^{iL\theta},$$

(1)

where $L$ is the lepton number. If the Hamiltonian $H$ of the system is invariant under the transformation

$$UHU^+ = H,$$

(2)

then

$$[L, H] = 0,$$

(3)

$L$ is conserved and the phase $\theta$ is unobservable.

A more detailed analysis of the weak interaction Hamiltonian reveals [2,3] that the Dirac mass term can be written as

$$\mathcal{L}_D = -m\bar{\nu}\nu = -m(\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L),$$

(4)

where the left- and right-handed neutrinos are

$$\nu_L = \frac{1}{2}(1 - \gamma_5)\nu, \quad \nu_R = \frac{1}{2}(1 + \gamma_5)\nu.$$  

(5)

The mass term (4) is clearly invariant under the transformation (1). However, the problem is that the right-handed neutrinos are not observed experimentally.

On the other hand, one can construct for a Majorana neutrino [2,3]

$$\nu_M \equiv \nu_L + \nu_L^c, \quad \nu_L^c \equiv (\nu_L)^c, \quad \nu^c = C\bar{\nu}^T,$$

(6)

the mass term

$$\mathcal{L}_M = -m\bar{\nu}_M\nu_M = -m(\bar{\nu}_L^c\nu_L + \bar{\nu}_L\nu_L^c),$$

(7)

from $\nu_L$ alone, but this term is not invariant under the phase transformation (1) implying the lepton number non-conservation. However, the direct experimental verification of the Majorana nature of the neutrino is lacking: the neutrinoless double $\beta$-decay has not yet been observed, only lower bounds for the life time of some nuclei were established [4].
On the other hand, admitting that the lepton U(1) symmetry is not an exact one (in contrast to the electromagnetic U(1) symmetry), one is allowed to speculate on the presence of the Majorana neutrino mass terms in the Hamiltonian. Let us note that one is allowed to do it also in the Dirac neutrino case, because there does not exist experimental proof that the neutrino mass is zero: only upper bound in the direct neutrino mass search is known at present [5], with a value of about 2.5 eV for the electron neutrino mass. About the same upper limit for the Majorana neutrino mass is obtained in the new experiment [6] on the neutrinoless double $\beta$–decay of $^{116}$Cd.

In its turn, the presence of the neutrino mass term in the Hamiltonian can give rise to the phenomenon of neutrino oscillations [2,3,7]. Actually, these oscillations are rather convincingly seen in experiments with the solar and atmospheric neutrinos such as Homestake, SuperKamiokande, GALLEX and SAGE [8]. In terrestrial experiments, only the result of LSND [9] is interpreted in favour of the oscillations.

## 2 Neutrino oscillations and SNO experiment

The clean experimental investigation of the neutrino oscillations is offered by exploiting the ratio of charged (CC) and neutral (NC) cross sections for the neutrino-deuteron reactions,

\[
\begin{align*}
\nu_l + d &\rightarrow l^- + p + p, \quad (CC) \\
\nu_x + d &\rightarrow \nu_x + n + p, \quad (NC) \\
\nu_x + e^- &\rightarrow \nu_x + e^-,
\end{align*}
\]

where $\nu_x$ refers to any active flavour of the neutrino.

These reactions are important for studying the solar electron neutrino oscillations ($l=e$) and they are the main object of the SNO detector [10] at present. The aim of the detector is to compare the flux of the electron neutrinos produced in the sun presumably by the reaction

\[
^8B \rightarrow ^8Be^* + e^+ + \nu_e, \quad E_{\nu} \leq 15 MeV,
\]

\[ (11) \]

to the flux of all active flavours of the neutrinos. Deviation of this ratio from the prediction based on the Standard Model with the ansatz of zero neutrino mass would provide a strong support for the massive neutrinos, but not for the violation of the lepton U(1) symmetry. As mentioned above, the fate of this symmetry is strongly correlated with the experimental evidence of the nuclear neutrinoless double $\beta$-decay.
Among other experimental activities, SNO intends to measure the flavour composition of the atmospheric neutrino flux [10]. The energies of these neutrinos can be much larger than for the solar neutrinos (up to 10 GeV and more).

Another investigative potential of SNO is related to the nowadays discussed neutrino factory [12–14]. The idea is to create muon storage rings which would produce the neutrino beam by allowing muons to decay in the straight section of a storage ring. The obtained beam would have a precisely known composition depending only on the muons which decay,

\[
\begin{align*}
\mu^+ &\rightarrow e^+ + \nu_e + \bar{\nu}_\mu, \\
\mu^- &\rightarrow e^- + \bar{\nu}_e + \nu_\mu.
\end{align*}
\]

(12) \hspace{1cm} (13)

In comparison with the low-energy solar neutrinos, the neutrinos from reactions (12), (13) can be produced at much higher energy interval (up to 50 GeV at present), thus giving another powerful method to investigate the transition probability \(\nu_e (\bar{\nu}_e) \rightarrow \nu_\mu (\bar{\nu}_\mu)\) by searching for leptons with the opposite sign ("wrong-sign" leptons). Besides reactions (8)-(10), we now also have similar reaction for the antineutrinos,

\[
\begin{align*}
\bar{\nu}_l + d &\rightarrow l^+ + n + n, \\
\bar{\nu}_x + d &\rightarrow \bar{\nu}_x + n + p, \\
\bar{\nu}_x + e^- &\rightarrow \bar{\nu}_x + e^-.
\end{align*}
\]

(14) \hspace{1cm} (15) \hspace{1cm} (16)

The estimated neutrino flux is \(\sim 10^{10} - 10^{11}\ \bar{\nu}_\mu\ m^{-2}\ \text{year}^{-1}\) [12] from the positive muons stored with the momenta \(p = 20 - 50\ \text{GeV/c}\). In such a factory, \(\nu_e\) are produced with about the same flux [12]. It is to be noted that the flux of the atmospheric neutrinos at the terrestrial surface is \(\sim 3 \times 10^{10}\ m^{-2}\ \text{year}^{-1}\) [15], which is close to the estimated flux of the neutrinos from the neutrino factory.

These considerations make theoretical studies of high energy neutrinos with the deuterons interesting. The cross sections for the neutrino-deuteron reactions up to energies 170 MeV have been already calculated [11] within the framework of the standard nuclear physics calculations at low energies, however, with the static part of the weak exchange current of the pion range included.
At high energies, relativistic effects both in the potential kernel and transition operator (current) should be considered within the same formalism. This can be done from the beginning within the framework of relativistic equations. As a step in this direction, we have recently carried out [16] an investigation of the structure of the weak axial meson exchange currents (MEC) in conjunction with the Bethe-Salpeter (BS) equation considered in the ladder approximation. Both the OBE potentials $\hat{V}_B$ and the one- and two-nucleon current operators were constructed from chiral Lagrangians of the $N\Delta(1236)$-$\rho\ a_1\omega$ system and from the associated one-body currents using the technique of Feynman diagrams. These Lagrangians are an extension of the standard non-linear $\sigma$ model [2], which is invariant under the transformations from the global chiral group $SU(2)_L \times SU(2)_R$. This model contains nucleons and pions only.

One of the employed Lagrangians [17] represents the approach developed earlier, in which the heavy meson fields $\rho$ and $a_1$ are introduced as massless Yang-Mills (YM) compensating fields. These fields belong to the linear realization of the local chiral $SU(2)_L \times SU(2)_R$ symmetry. Actually, this symmetry is violated by the heavy meson mass terms introduced by hands. Also the external electroweak interactions should be introduced by hand, since there are no other charges available, which could be associated with them. Inspite of this internal inconsistency, this approach turned out to be succesful in describing the nuclear phenomena in the region of low and intermediate energies [18].

The defects of the approach of the massive YM fields are removed in the approach of hidden local symmetries (HLS) [19,20]. In this method, a given global symmetry group $G_g$ of a system Lagrangian is extended to a larger one by a local group $H_l$ and the Higgs mechanism generates the mass terms for gauge fields of the local group in such a way that the local symmetry is preserved. For the chiral group $G_g \equiv [SU(2)_L \times SU(2)_R]_g$ and $H_l \equiv [SU(2)_L \times SU(2)_R]_l$, the gauge particles are identified [19,20] with the $\rho$- and $a_1$ mesons. An additional extension by a local group $U_l(1)$ allows one to include the isoscalar $\omega$ meson as well [21]. Moreover, external gauge fields, which are related to the electroweak interactions of the Standard Model, are included by gauging the global chiral symmetry group $G_g$. Lagrangian constructed within the HLS scheme and suitable for constructing the exchange currents is given in [22].

The physical content of our YM and HLS Lagrangians differs only due to the different choice of higher order terms in the Lagrangian correcting the high energy behaviour of elementary amplitudes.

The Lagrangians contain all necessary vertices which together with the as-
associated one-body currents can be used in constructing many body current operators. We restrict ourselves to the one- and two-nucleon weak axial MEC.

In our approach, the operator of the one-nucleon weak axial current for the \( i \)th nucleon is

\[
\hat{J}_a^{\mu}(1,i) = \frac{g_A}{2} m_{a_1}^2 \Delta_{a_1}^{\mu \nu}(q) \left( \gamma_\nu \gamma_5 \tau^a \right)_i - g f_\pi \Delta_\pi^\mu(q^2) g^\mu \left( \gamma_5 \tau^a \right)_i .
\] (17)

Here \( g_A = -1.26 \) and \( \Delta_{a_1}^{\mu \nu}(q) \left( \Delta_\pi^\nu(q^2) \right) \) is the propagator of the \( a_1 (\pi) \) meson.

The divergence of the current (17) is

\[
q_\mu \hat{J}_a^{\mu}(1,i) = \left[ \hat{e}_A(i), G_i^{-1} \right]_+ - g f_\pi m^2_\pi \Delta_\pi^\mu(q^2) \left( \gamma_5 \tau^a \right)_i .
\] (18)

Here the operator of the nucleon axial charge \( \hat{e}_A(i) \) and the inverse of the nucleon propagator \( G_i^{-1}(p) \) are defined as

\[
\hat{e}_A(i) = g_A \left( \gamma_5 \frac{\tau^a}{2} \right)_i , \quad G^{-1}(p) = \not{p} - M .
\] (19)

Generally, a MEC constructed in the conjunction with the BS contains various contact and mesonic terms, but it does not contain the nucleon Born terms, the contribution from which is actually generated when one calculates the matrix element of the one-nucleon current between the solutions of the BS equation. The general structure of the weak axial MEC operator is given in Fig. 1.

The details of the structure of the constructed weak axial MEC operators can be found in [16]. Here we discuss the essential features of their structure.

a) The currents derived from the YM type Lagrangian

For the B=\( \pi, \omega \) meson exchanges we have

\[
q_\mu \hat{J}_B^{a\mu}(ex) = \left[ \hat{e}_A(1) + \hat{e}_A(2), \hat{V}_B \right]_+ + i f_\pi m^2_\pi \Delta_\pi^\mu(q^2) \hat{M}_B^\mu(2) ,
\] (20)

where \( \hat{M}_B^\mu(2) \) is the two-body pion absorption amplitude.

In this model, the \( \rho \) and \( a_1 \) meson exchanges should be considered together and the resulting MEC operator satisfies the continuity equation

\[
q_\mu \hat{J}_{BS \rho+a_1}(ex) = \left[ \hat{e}_A(1) + \hat{e}_A(2), \hat{V}_\rho + \hat{V}_{a_1} \right]_+ + i f_\pi m^2_\pi \Delta_\pi^\mu(q^2) \hat{M}^\mu_{\rho+a_1}(2) .
\] (21)
The obtained MECs differ from those constructed for the on-shell nucleons by the presence of some additional terms which disappear for the on-shell nucleons. As an example, let us consider the contribution from one of the pion contact terms,

\[
\hat{J}_{c_1 \pi}^a \mu (\pi) \equiv i f_\pi q^\mu \Delta_F^\pi(q^2) \hat{M}_{c_1 \pi}^a,
\]

(22)

\[
\Delta \hat{J}_{c_1 \pi}^a \mu (\pi) \equiv i f_\pi q^\mu \Delta_F^\pi(q^2) \Delta \hat{M}_{c_1 \pi}^a,
\]

(23)

where the pion absorption amplitudes are

\[
\hat{M}_{c_1 \pi}^a = -\frac{i}{2} \left( \frac{g_M}{M} \right)^2 \varepsilon^{amn} \hat{a}_2 \tau_1^m \Delta_F^\pi(q_2^2) \hat{\Gamma}_2^n + (1 \leftrightarrow 2),
\]

(24)

\[
\Delta \hat{M}_{c_1 \pi}^a = -\frac{i}{4} \left( \frac{g_M}{M} \right)^2 \varepsilon^{amn} \hat{a}_1 \tau_1^m \Delta_F^\pi(q_2^2) \hat{\Gamma}_2^n + (1 \leftrightarrow 2),
\]

(25)

Besides the contact current (22), constructed earlier in [23] we have an additional term (23) which disappears for the on-shell nucleons. This can be seen from (25), where the right hand side is proportional to the momentum transfer \( \hat{q}_1 \) which provides zero when sandwiched between the spinors of the 1st nucleon.

b) For the currents derived from the HLS type Lagrangian, Eq. (20) holds for all considered exchanges.

The full BS weak axial current is defined as

\[
\hat{J}_{BS}^a \mu = \hat{J}_{IA}^a \mu + \hat{J}_{BS}^a \mu (ex),
\]

(26)

where the impulse approximation current is

\[
\hat{J}_{IA}^a \mu = i \hat{J}_a^\mu (1, 1) G_2^{-1} + i \hat{J}_a^\mu (1, 2) G_1^{-1},
\]

(27)

and the weak axial MEC \( \hat{J}_{BS}^a \mu (ex) \) is given by the contributions from the exchanges considered in [16].

Using the Ward-Takahashi identities for the one- and two-nucleon currents [16] yields for the divergence of the full BS current

\[
q_\mu \hat{J}_{BS}^a \mu = [ \hat{e}_A(1) + \hat{e}_A(2), \mathcal{G}^{-1}]_+ + if_\pi m_\pi^2 \Delta_F^\pi(q^2) \hat{M}^a,
\]

(28)

where the inverse Green function is

\[
\mathcal{G}^{-1} = \mathcal{G}_{BS}^{-1} + \hat{V},
\]

(29)
\( \mathcal{M}^a \) is the full pion absorption amplitude

\[
\mathcal{M}^a = i\Gamma_1 G_2^{-1} + i\Gamma_2 G_1^{-1} + \hat{\mathcal{M}}^a(2), \quad \hat{\Gamma}_i = i g (\gamma_5 \tau^a),
\]

(30)

the BS propagator in term of the single-particle propagators reads

\[
G_{BS} = -i G_1 G_2
\]

and the full potential is,

\[
\hat{V} = \hat{V}_\pi + \hat{V}_\rho + \hat{V}_{a_1} + \hat{V}_\omega.
\]

(31)

Because the two-body BS wave functions for both bound and scattering states satisfy the equation

\[
G^{-1}\psi = <\psi | G^{-1} = 0,
\]

(32)

the matrix element of the divergence of the full current (28) fulfil the standard PCAC constraint

\[
q_\mu <\psi | \hat{J}^a_{BS} | \psi > = if_\pi m^2_\pi \Delta_\pi(q^2) <\psi | \hat{M}^a | \psi > .
\]

(33)

The model dependence is given by effects coming from the difference between the MECs derived in considered schemes. More detailed checking [16] shows that they are either of short-range nature or momentum dependent.

4 Results and conclusions

We constructed the weak axial MECs of the \( \pi, \rho, a_1 \) and \( \omega \) range ready to use with the BS equation. Two different chiral schemes were chosen as a starting point to get a realistic set of currents. The full BS currents satisfy the Ward-Takahashi identity and the divergence of their matrix elements between the two-body BS wave functions satisfies the standard PCAC constraint.

The constructed currents can be immediately used to improve the recent calculations [24] of the solar proton burning process,

\[
p + p \longrightarrow d + e^+ + \nu_e,
\]

(34)

within the BS formalism. As the next step would be consistent covariant calculations of the cross sections for the neutrino-deuteron reactions (8),(9) and (14),(15) with energetic neutrinos coming from the atmosphere or from a neutrino factory. The consistent calculations would provide a possibility to
compare the covariant BS- and standard nuclear physics calculations, thus bringing them under control. Such calculations would give more confidence in conclusions about the neutrino oscillations being investigated at SNO.

Acknowledgments

The work of E. T. is supported by the grant GA ČR 202/00/1669 and by the grant of NSC ROC. Research of F. C. K. is supported in part by NSERCC.

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Fig. 1. The general structure of the weak axial MEC operators considered. The weak axial interaction is mediated by the meson B which is either $\pi$ or $a_1$ meson. The range of the current is given by the meson $B_2$. The graphs a, b, represent the current $\hat{J}_{\mu a}^a(N, B)$ with $N$ either for the nucleon N (nucleon Born terms) or for the $\Delta(1236)$ isobar. In conjunction with the BS equation, the nucleon Born terms do not enter the MEC. The graph c represents a contact current $\hat{J}_{\mu a}^a(B)$. Another type of the contact terms is given by the graph d, $\hat{J}_{B_1 B_2}^a(B)$, where the weak axial current interacts directly with the mesons $B_1$ and $B_2$. The graph e is for a mesonic current $\hat{J}_{B_1 B_2}^a(B)$. The associated pion absorption amplitudes correspond to the graphs where the weak axial interaction is mediated by the pion, but with the weak interaction wavy line removed.