Deep Reinforcement Learning for a Two-Echelon Supply Chain with Seasonal Demand

Francesco Stranieri∗, Fabio Stella

Department of Informatics, Systems, and Communications
University of Milano-Bicocca
Viale Sarca 336, 20126 Milan, Italy

Abstract

This paper leverages recent developments in reinforcement learning and deep learning to solve the supply chain inventory management problem, a complex sequential decision-making problem consisting of determining the optimal quantity of products to produce and ship to different warehouses over a given time horizon. A mathematical formulation of the stochastic two-echelon supply chain environment is given, which allows an arbitrary number of warehouses and product types to be managed. Additionally, an open-source library that interfaces with deep reinforcement learning algorithms is developed and made publicly available for solving the inventory management problem. Performances achieved by state-of-the-art deep reinforcement learning algorithms are compared through a rich set of numerical experiments on synthetically generated data. The experimental plan is designed and performed, including different structures, topologies, demands, capacities, and costs of the supply chain. Results show that the PPO algorithm adapts very well to different characteristics of the environment. The VPG algorithm almost always converges to a local maximum, even if it typically achieves an acceptable performance level. Finally, A3C is the fastest algorithm, but just like the VPG, it never achieves the best performance when compared to PPO. In conclusion, numerical experiments show that deep reinforcement learning performs consistently better than standard inventory management strategies, such as the static \((s, Q)\)-policy. Thus, it can be considered a practical and effective option for solving real-world instances of the stochastic two-echelon supply chain problem.

Keywords: machine learning, inventory management, deep reinforcement learning

∗Corresponding author: francesco.stranieri@unimib.it
1. Introduction

Seoul, South Korea, from 2016 March 9th to 15th, Lee Sedol, the eighteen times Go world champion, was defeated in a five-game match by AlphaGo (Silver et al., 2017) in what is known as the Google DeepMind Challenge Match. In just a week, AlphaGo, a computer program based on deep reinforcement learning (DRL), won all but one game. Three years later, on 2019 November 19th, Lee Sedol, national hero of South Corea, announced his retirement from professional play stating the following ”Even if I become the number one, there is an entity that cannot be defeated.” Fortunately, later on in December 2019, he accepted to play a three-game match against the HanDol AI system, which was developed by Korean NHN Entertainment Corporation. Since then, DRL has been applied to solve many challenging problems in various fields, including robotics, video games, medicine and healthcare, finance, transportation systems, and industry 4.0, to mention just a few (Li, 2017). Recently, DRL was also investigated to tackle complex sequential decision-making problems in operations research, such as the virtual machine packing or the asset allocation problem (Hubbs et al., 2020). However, the significant number of structural design choices to be made for developing a DRL algorithm, together with its computationally intensive nature, i.e., parameters evaluation and tuning, and the lack of publicly available and open access software environments, are major hurdles for practical application of DRL algorithms by researchers and practitioners in both academy and industry.

Supply chain inventory management (SCIM) is a complex sequential decision-making problem consisting of determining the optimal quantity of products to produce and distribute to warehouses over a given time horizon. As evidenced by the helpful roadmap of Boute et al. (2021), DRL algorithms are rarely applied to this field, although they can be used to develop near-optimal policies that are difficult, or impossible at worst, to achieve using traditional methods. Indeed, the uncertain and stochastic nature of products demand, as well as lead times, represent significant obstacles for mathematical programming approaches to be effective, with specific reference to those cases where the number of considered SCIM’s entities, i.e., number of products type and warehouses, is realistic and not too small.

Recently, some DRL algorithms have been studied and applied to tackle the SCIM problem (Peng et al., 2019; Alves & Mateus, 2020; Hubbs et al., 2020; Oroojlooyjadid et al., 2021; Gijsbrechts et al., 2022). These approaches are interesting and show how DRL algorithms can be both effective and efficient in solving the SCIM problem. However, they suffer the following limitations: i) given a supply chain structure (i.e., divergent two-echelon), no DRL algorithm has been deeply tested with respect to different topologies (i.e., by changing the number of warehouses); ii) no extensive experiments have been performed on the same supply chain structure by varying different configurations (e.g., demands, capacities, and costs); iii) as suggested by more than one article, no extension has been proposed for comparing different DRL algorithms to determine which one is more appropriate for a particular supply chain topology.
and configuration. Furthermore, relevant aspects of the SCIM problem have not yet been addressed efficiently, for example: i) the sequence of events required to reproduce and validate a simulation model is not always well defined or given. As a result, making available a consistent and universal open-source SCIM environment can improve reusability and reproducibility, especially when implemented with standard APIs (like those of OpenAI Gym \cite{brockman2016gym}). In this way, it is also possible to import DRL algorithms from reliable libraries and to focus solely on their fine-tuning, instead of developing them from scratch; ii) performances achieved by DRL algorithms are typically compared to those achieved by some standard static reorder policies. However, they are not compared to the performance achieved by an oracle (i.e., a baseline who knows the optimal action to take a priori), thus making it difficult to evaluate their true effectiveness in real-world environments; iii) none of the DRL papers available in the specialized literature considers a multi-product approach, whereas it has been considered in relation to other solution methods \cite{shervais2003decentralized, sui2010multi, cimen2013optimization}. Considering more than one product increases the dimensionality and complexity of the problem, consequently requiring an efficient implementation of the SCIM environment and DRL algorithms.

This paper makes the following contributions to the SCIM decision making problem:

• design and formulation of a stochastic two-echelon SCIM environment under seasonal demands, which for the first time allows an arbitrary number of warehouses and products to be managed;

• comparison of a set of state-of-the-art DRL algorithms in terms of their capability to find an optimal policy, i.e., a policy which maximizes the SCIM’s profit as achieved by an oracle;

• evaluation of performances achieved by state-of-the-art DRL algorithms and comparison to a static reorder policy, i.e., an \((s, Q)\)-policy, whose optimal parameters have been set through adaptive and data-driven Bayesian optimization (BO);

• design and run of a rich experimental plan involving different SCIM topologies and configurations as well as values of hyperparameters associated with DRL algorithms;

• design and development of an open-source library for solving the SCIM problem (available on \url{https://github.com/frenkowski/SCIMAI-Gym}), thus embracing the open science principles and guaranteeing reproducible research.

The rest of the paper is organized as follows: Section 2 is devoted to introducing and describing the SCIM problem, providing main definitions and notation, and highlighting how reinforcement learning (RL) approaches have dealt with this problem. DRL is introduced and described in Section 2.1 in this section,
we also describe the state-of-the-art DRL algorithms that we used to solve the SCIM problem. Section 4 describes the main methodological contributions of this paper, while the results of the rich numerical experiments plan are reported in Section 5. Finally, conclusions and directions for further research are given in Section 6.

2. Background Theory

According to Slack & Lewis (2002), a supply chain is a network of interrelated organizations participating in activities that lead to value creation. The task of managing a supply chain is called supply chain management and focuses on the integrated management of product flows through the whole supply chain, in order to guarantee that the appropriate products are delivered in the correct quantity and at the proper locations at the right time (Christopher, 2016). Supply chain inventory management is a critical challenge faced by companies whose supply chain typically consists of a factory and several warehouses. More in detail, SCIM consists of deciding how many products should be produced at the factory and how many stocks, of such products, should be sent from the factory to the warehouses. While higher inventory stocks allow companies to meet customers' demands better, they come at a cost. Hence, the goal of SCIM is to find a balance between the customers' demand satisfaction and the cost of maintaining inventory stocks in such a way as to maximize the supply chain profit and, at the same time, ensure market competitiveness.

SCIM is often formulated as a constrained optimization problem. In practice, this problem is generally approached in terms of static reorder policies, that is, simple rules that determine when to replenish inventories as well as the amount of replenishment. However, a crucial issue is represented by the uncertain and stochastic nature of the product’s demand, which can be characterized by seasonality patterns and thus, may require warehouses to preserve stocks in advance. Coordination between supply chain participants is thus essential for increasing efficiency throughout the entire process, i.e., from when an order is placed to when products are dispatched. Coordination can also prevent the Forrester effect, also known as the bullwhip effect (Forrester, 1997; Lee et al., 1997), a phenomenon that demonstrates how small fluctuations in the final demand by customers can result in more significant fluctuations at previous stages of the supply chain, negatively impacting performances and costs.

In this context, operations research methods, and in particular mathematical programming models, have been shown to be inapplicable and ineffective in finding optimal policies with regard to different SCIM topologies and configurations. In fact, the analytic intractability makes closed-form expression of the optimal policies practically impossible to derive (Boute et al., 2021). To overcome these limitations, mathematical models often rely on assumptions and simplifications, making them simultaneously more difficult to relate to real-world scenarios. In addition, establishing an optimal policy can be intractable even for basic supply chain structures owing to the curse of dimensionality (de Kok et al., 2018). A
A viable alternative is represented by heuristics, which are used to find a near-optimal solution, e.g., the optimal parameters for a given static reorder policy, in a limited amount of time. Although, heuristics are also based on assumptions and simplifications, and they are generally problem-dependent as well as their effectiveness is (Gijsbrechts et al., 2022). RL techniques can therefore be used to develop near-optimal policies for a wide range of supply chain topologies and configurations that would otherwise be difficult, or impossible in the worst of cases, to achieve using traditional operations research methods.

2.1. Reinforcement Learning

According to Sutton & Barto (2018), reinforcement learning is learning what to do, i.e., how to map situations to actions, in such a way that a given numerical reward is maximized. The learning agent, or simply the agent, is not told which actions to take, but instead must discover which actions yield the highest reward by trying them. The most interesting and challenging scenario occurs when the action chosen influences the immediate reward as well as the next situation, thereby influencing all subsequent rewards. Therefore, the main features of RL are trial-and-error search and delayed reward. Indeed, rather than being instructed through the correct actions, the agent uses training data to evaluate the effectiveness of the actions performed and, thus, to learn how to maximize the reward; this requires an active exploration and is the basis of indicative feedback, which reflects the effectiveness of the action adopted but not whether it was the best or the worst possible one.

RL also trades off between exploration and exploitation. To maximize a reward, an agent must exploit actions that it has already performed and identified as effective. However, to discover effective actions, the agent must explore actions that were not previously selected; this means that the agent has to exploit what it has already experienced to gain reward, but it also has to explore to select better actions in the future. As a strategy, the agent has to deal with various actions and gradually prefer the best ones (in a stochastic environment where the reward is not a fixed quantity, this implies that each action must be attempted several times to obtain an accurate estimate of its expected reward).

Markov Decision Processes (MDPs) are designed to provide a concise mathematical framework for addressing the challenge of learning through interactions to achieve a goal. Therefore, they are well suited to represent RL problems. In fact, RL adopts the MDP framework to represent the interactions between a learning agent and its environment in terms of states, actions, and rewards. The agent is the learner and decision-maker, while the environment is the entity it interacts with. These two components are continuously in interaction: the agent selects actions, and the environment reacts to them by presenting new states. Simultaneously, the environment generates numerical rewards that the agent seeks to maximize over time through its action selections. Inevitably, the specific states and actions can differ significantly between different learning tasks, and their accurate representation can dramatically impact the performance achieved by the agent.
As shown in Fig. 1, at each time step $t$, the agent receives a representation of the state of the environment, $S_t \in S$, and selects an action, $A_t \in A(S_t)$, based on that representation. One time step later, i.e., at $t + 1$, and partly due to its action, the agent obtains a numerical reward, $R_{t+1} \in \mathcal{R} \subset \mathbb{R}$, finding itself in state $S_{t+1}$.

Figure 1: Agent-environment interface in an MDP (Sutton & Barto, 2018).

Consequently, a trajectory is being generated:

$$S_0, A_0, R_1, S_1, A_1, R_2, S_2, A_2, R_3, \ldots$$

If we denote the succession of rewards received after time step $t$ as $R_{t+1} + R_{t+2} + R_{t+3} + \ldots$, then we want to maximize the expected discounted return:

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots = \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k,$$

where the discount rate $\gamma$ ($0 \leq \gamma \leq 1$) is a parameter used to calculate the value of future rewards. For example, if $\gamma = 0$, the agent is myopic since it concentrates solely on maximizing immediate rewards, i.e., choosing $A_t$ in a way that maximizes just $R_{t+1}$. Conditioning on state $s$ and action $a$ at time step $t - 1$, the probability for random variables $s'$ and $r$ at time $t$ is given by:

$$p(s', r \mid s, a) = \Pr\{S_t = s', R_t = r \mid S_{t-1} = s, A_{t-1} = a\},$$

for all $s', s \in \mathcal{S}, r \in \mathcal{R}$, and $a \in A(s)$.

For MDPs, we can define the value of a state $s$ under a policy $\pi$, denoted $v_{\pi}(s)$, as the expected return when starting in $s$ and continuing following policy $\pi$. A deterministic policy is a mapping from perceived states of the environment to actions to be performed in those states, $\pi : \mathcal{S} \rightarrow \mathcal{A}$. In general, policies may also be stochastic, $\pi : \mathcal{A} \times \mathcal{S} \rightarrow [0, 1]$, where $\pi(a \mid s)$ denotes the probability of taking an action $a$ in a state $s$. Similarly, we can define the value of performing an action $a$ in state $s$ under a policy $\pi$, denoted $q_{\pi}(s, a)$, as the expected return starting in $s$, performing $a$, and then continuing following the policy $\pi$.

Almost all RL algorithms incorporate a mechanism for estimating the state-value function for policy $\pi$:

$$v_{\pi}(s) = \mathbb{E}_{\pi} [G_t \mid S_t = s] = \mathbb{E}_{\pi} [R_{t+1} + \gamma G_{t+1} \mid S_t = s] = \sum_a \pi(a \mid s) \sum_{s', r} p(s', r \mid s, a) [r + \gamma v_{\pi}(s')].$$
or the action-value function for policy $\pi$:

$$q_\pi(s, a) = \mathbb{E}_\pi [G_t \mid S_t = s, A_t = a]$$

$$= \mathbb{E}_\pi [R_t + \gamma G_{t+1} \mid S_t = s, A_t = a]$$

$$= \sum_{s', r} p(s', r \mid s, a) [r + \gamma \sum_a \pi(a \mid s) q_\pi(s', a')]$$.

These are functions of states, or state-action pairs, that estimate how desirable it is for the agent to be in a particular state, or how desirable it is for the agent to select a specific action in a particular state, in terms of expected reward. Clearly, the agent’s future rewards are dependent upon the actions it takes. As a consequence, value functions are expressed in terms of policies. At least one policy is always better than or equal to all other policies. We denote all the optimal policies as $\pi^\ast$. They all share the same state-value function, called optimal state-value function, and defined as:

$$v^\ast(s) = \max_{\pi} v_\pi(s),$$

or, analogously, they all share the same optimal action-value function, defined as:

$$q^\ast(s, a) = \max_{\pi} q_\pi(s, a).$$

Hence, RL methods define how an agent’s policy is adjusted in response to its experience.

The fact that value functions satisfy recursive relationships is a crucial property. In detail, the Bellman optimality equation for $v_\pi$ expresses a relationship between the value of a state and the value of its successor:

$$v_\pi(s) = \max_{a \in A(s)} q_\pi(s, a)$$

$$= \max_a \mathbb{E}_{\pi_\ast} [G_t \mid S_t = s, A_t = a]$$

$$= \max_a \sum_{s', r} p(s', r \mid s, a) [r + \gamma v_\pi(s')]$$.

Similarly, the Bellman optimality equation for $q_\pi$ is defined as:

$$q_\pi(s, a) = \mathbb{E} \left[ R_{t+1} + \gamma \max_{a'} q_\pi(s_{t+1}, a') \mid S_t = s, A_t = a \right]$$

$$= \sum_{s', r} p(s', r \mid s, a) \left[ r + \gamma \max_{a'} q_\pi(s', a') \right].$$

Although, with $q_\pi$, it is not even required to perform a one-step-ahead search. For any state $s$, we can simply find any action that maximizes Eq. (1).

Solving explicitly the Bellman optimality equations is a way for determining an optimal policy and, therefore, for solving an RL problem. This solution is analogous to an exhaustive search in which all possibilities are explored. At the heart of this strategy, there are three basic assumptions (Sutton & Barto, 7).
i) we have complete knowledge about the dynamics of the environment; ii) we have the computational power necessary to calculate the solution; iii) the Markov property is respected. Unfortunately, this approach cannot always be implemented due to the violation of various combinations of these assumptions. Therefore, several RL methods can be interpreted as approximate solutions to the Bellman optimality equations, as they rely on experienced transitions rather than on the knowledge of expected transitions.

Memory availability is also a constraint, as we must physically store approximations of value functions and policies. For tasks with small and finite state and action spaces, these approximations can be implemented using tabular methods, which involve tables with one item for each state or each state-action pair. However, there may be too many states in many practical scenarios for storing them in a table, thus requiring alternative methodologies. In these cases, policies and value functions can be approximated using a compact parameterized function representation, such as a neural network.

2.2. RL for Inventory Management

Reinforcement learning has recently achieved remarkable results in the field of artificial intelligence (AI), mainly when applied to video games and gaming in a more general sense. Nevertheless, there are still few use-cases in industrial applications, even if RL proved to be effective in solving complex sequential decision-making problems. RL proceeds by interacting with a dynamic environment in order to maximize an overall reward over a given time horizon, and that makes it a promising technique for tackling the SCIM problem. Indeed, RL methods can define, through simulation, a dynamic reorder policy that is dependent exclusively on the current state of the system. In this way, no prior knowledge or restrictive assumptions are required, making it possible to overcome the limits of traditional mathematical models and associated solution procedures.

One of the most common approaches for solving the SCIM problem through RL algorithms turns out to be Q-learning. This approach is based on a tabular, model-free, and temporal-difference algorithm that learns how to determine the "quality" of an action $A_t$ in a state $S_t$, referred to as the Q-value, in accordance with the following update rule:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \delta_t,$$

where $\delta_t = [R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t)]$ is the temporal-difference error (or TD error in short), and $\alpha (0 \leq \alpha \leq 1)$ is the learning rate which indicates the extent to which Q-values are updated. The learned action-value function $Q$, also known as the Q-function, directly approximates $q_*$, the optimal action-value function. Q-values of each state-action pair are stored in a table, known as Q-table, where each state is represented by a row and each action by a column. They are initially set to an arbitrary value, such as 0. According to the Bellman equation, Q-values are then iteratively updated during the learning
process. Since Q-learning is a model-free algorithm, it only uses the environment model to simulate new sequences of states, actions, and rewards rather than to predict future states and rewards when performing an action (in this paper, we will discuss only model-free algorithm). If each action is performed infinite times in each state, and the learning rate $\alpha$ is decaying properly, then $Q$ converges with probability 1 to $q_*$ (Tsitsiklis, 1994; Jaakkola et al., 1994), i.e., the optimal action-value function. Consequently, when Q-values have almost reached convergence to their optimal values, it is reasonable for the agent to behave greedily, that is, to choose the action with the highest Q-value.

Chaharsooghi et al. (2008), authors of one of the most cited RL articles about SCIM, proposed an approach based on Q-learning to tackle the problem. In particular, they addressed (a variant of) the beer game problem (Forrester, 1958), which consists of a linear supply chain with four participants (i.e., supplier, manufacturer, distributor, and retailer), and is frequently discussed in academic contexts to demonstrate the bullwhip effect (defined in Section 2). We specify that, in a linear supply chain, each participant has one predecessor and one successor; in a divergent supply chain, each has one predecessor but multiple successors, while the opposite is true in a convergent supply chain. Finally, in a general supply chain, each participant has several predecessors and several successors. According to the beer game rules, each supply chain participant must submit an order quantity to its predecessor at each discrete time step, having only its local information available and with the aim of minimizing the overall cost. Indeed, standard rules assert that participants are not allowed to communicate their local stock levels or cost information to each other until the end of the game. The beer game assumes no transportation costs but only storage and penalty costs, i.e., costs incurred by storing unsold inventories and when the demand cannot be completely satisfied. Additionally, it also assumes deterministic lead times. If penalty costs exist only at the upper echelon, that is, only for the retailer, then the well-known algorithm by Clark & Scarf (1960) is able to determine the optimal parameters for the base-stock policy. Under a base-stock policy, each supply chain participant orders a quantity to bring its stocks equal to a fixed number, known as the base-stock level. Thus, the challenge is to find the corresponding optimal values.

To implement the Q-learning algorithm, Chaharsooghi et al. (2008) defined the current supply chain state as a vector consisting of the four inventory positions in terms of current stock levels. However, considering that inventory positions thus defined may take infinite values, applying this strategy appears unfeasible since the Q-table would be in turn infinite. Consequently, the authors discretized the state space into nine intervals. In this way, the possible state values amount to $9^4$. Regarding actions, their approach determines the number of products to order via the $d+x$ policy. Precisely, if a participant in the previous time step received a request for $d$ product units by the upstream stage, this policy requires to order $d+x$ units to the downstream stage in the current time step. The learning process’s objective is hence to determine the value of $x$ according to the given system state. For limiting the Q-table size, $x$ was constrained by the authors to belong to $[0, 3]$ so that the possible number of actions amounts
Lastly, the customer demand at the retailer was generated through a uniform distribution on $[0, 15]$, while the time horizon was set to $35$ (weeks). Obviously, by defining restricted state and action spaces, the resulting Q-table appears to be more manageable. Through the Q-learning procedure, Q-values associated with each state-action pair are estimated according to Eq. (2). In this way, a (near-)optimal policy can be easily obtained by identifying, for each state, the action with the highest Q-value. Other interesting articles that tackled the SCIM problem using Q-learning are those of Ravulapati et al. (2004); Sui et al. (2010); Mortazavi et al. (2015). However, analyzing the various RL studies, it becomes evident that the Q-tables implemented are typically huge and, thus, unscalable. For example, the Q-table adopted by Chaharsooghi et al. (2008) has a number of cells equal to $(9^4 \cdot 4^4 = 1,679,616$) (i.e., the number of states multiplied by the number of actions). Consequently, expanding the size of the state or action spaces might not be feasible, as the Q-tables can no longer be handled. Another critical point of reflection is given in Geevers (2020), where the author shows that the approach proposed by Chaharsooghi et al. (2008) is not able to learn an effective policy since the impact of the actions (restricted to take values on the $[0, 3]$ interval) is so limited that even random actions provide basically the same results.

In conclusion, tabular RL methods can only be applied to discretized or constrained state and action spaces. However, discretization leads to a loss of crucial information, in addition to being unfaithful to real-world scenarios; thus, we need other solutions to address the SCIM problem effectively. The next section presents specific applications of reinforcement learning combined with deep learning DL, which will be called deep reinforcement learning and promises to achieve improved performance and scalability, also concerning the SCIM problem.

### 3. Deep Reinforcement Learning

Deep learning enables reinforcement learning to scale to previously intractable decision-making problems, i.e., environments with high dimensional state and action spaces where representing value functions and policies using a table may be challenging. Along with memory and computational complexity, the challenge also concerns the fact that several states will almost certainly never be visited during the learning process. As a consequence, it becomes necessary to employ methodologies capable of generalizing from similar states. A well-accepted technique is to express value functions using function approximation rather than tables. In this way, function parameters $\theta$ are adjusted not just for a given state-action pair as the update also affects all other state-action pairs, thus allowing the necessary generalization.

DRL is rooted into artificial neural networks (ANNs), which are universal approximators, thus capable of providing an optimal approximation of highly nonlinear functions. An ANN consists of interconnected units called artificial neurons, or simply neurons. Fig. 2 illustrates a general feed-forward ANN, that is, a neural network without loops. The illustrated ANN consists of an input
layer composed of three input units, an output layer comprised of one output unit, and two hidden layers composed of neither input nor output units. Each link between neurons is assigned a real-valued weight; Thus, ANNs are parameterized by their weights. ANNs are commonly trained using the stochastic gradient descent procedure; this means that each weight is adjusted with the aim to improve the network’s overall performance, i.e., to maximize a given objective function.

![A two hidden layers feed-forward ANN.](image)

In supervised learning tasks, the objective function usually consists of the expected error, also known as loss, over a set of training examples. In reinforcement learning, ANNs can learn value functions by maximizing the expected reward (or, alternatively, by minimizing the TD error). Both instances require measuring the impact of the change in each link weight compared to the network’s overall performance.

Problems defined as MDPs can be solved using RL techniques of basically three types. Value-based methods estimate state or action values, $v_\theta(s) \approx v_\star(s)$ or $q_\theta(s,a) \approx q_\star(s,a)$, and then choose the best actions accordingly (like the already mentioned Q-learning algorithm). Without action-value estimates, their policies would not even exist. Another approach to handling large state and action spaces is to use policy-based methods, which learn a parameterized policy, $\pi_\theta(s) \approx \pi_\star(s)$, to select actions directly without consulting a value function. While a value function may still be used to learn policy parameters, it is no longer necessary to select actions. Lastly, there are hybrid actor-critic methods that combine value functions with policy search.

In particular, policy-based methods expose some crucial advantages. For example, they can learn specific probabilities for selecting actions. This implies that, unlike action-value methods, policy-based methods embed a natural mechanism of deriving an optimal stochastic policy, i.e., they can choose between different actions with specific probabilities. Moreover, a parameterized policy may also be simpler to represent than value functions for particular problems. As part of policy-based methods, policy gradient methods offer a considerable theoretical advantage through the policy gradient theorem. In fact, this theorem provides an exact formula for how policy parameters impact performance without considering state distribution derivatives, establishing the theoretical basis for all policy gradient algorithms. The vanilla policy gradient (VPG) algorithm (also known as REINFORCE (Williams, 1992)) is a natural result of
this theorem. In its update:

\[ \theta_{t+1} = \theta_t + \alpha G_t \nabla \theta \ln \pi (A_t \mid S_t, \theta_t), \]

each increment is proportional to the product of a return \( G_t \) and a vector that indicates the direction in parameter space which maximizes the probability of repeating the action \( A_t \) on subsequent visits to state \( S_t \). On average, actions that perform better will acquire a high probability of being chosen, thus improving the policy’s performance. Despite the considerable advantages, the high variance of gradient estimates usually results in policy update instabilities \cite{wu2018}; this is a significant disadvantage that leads it to converge slowly.

Even to mitigate this issue, \cite{schulman2015} proposed an (actor-critic) algorithm called trust region policy optimization (TRPO). In order to update the policy, this algorithm continuously optimizes a local approximation with a Kullback-Leibler (KL) divergence penalty. In this way, TRPO bounds the difference between the new and the old policy in a trust region, avoiding update instabilities and, hence, performance collapsing. Proximal policy optimization (PPO) \cite{schulman2017} shares the same background as TRPO, but it is easier to implement and has a better sample complexity. Instead of including a KL divergence, PPO uses a specific clipping in the objective function to prevent the new policy from deviating too far from the old one. PPO has demonstrated comparable or superior performance to other state-of-the-art DRL algorithms, while simultaneously being significantly simpler to implement and tune.

More in detail, actor-critic methods learn approximations to both policy and value functions. Specifically, the actor learns a policy, while the critic learns a value function (usually a state-value function) in order to evaluate the learned policy; this means that the actor receives a state from the environment and selects an action accordingly. Concurrently, the critic obtains the same state together with the reward resulting from the previous interaction. At this point, the critic updates itself and the actor using the TD error computed on the basis of these inputs. This strategy seeks to speed up the whole learning process, minimizing at the same time the variance. The most performing actor-critic methods implement an ANN for both the actor and the critic, even if this leads them to be hyperparameter sensitive \cite{alves2020}.

Asynchronous advantage actor-critic (A3C) \cite{mnih2016} is one of the available state-of-the-art actor-critic algorithms. Its core idea is to have different agents that interact with different representations of the environment, each with its own parameter. The agents periodically update a global ANN that incorporates shared parameters. The updates are not occurring synchronously, hence the term asynchronous. This allows A3C to learn faster, as more agents are training in parallel, but also allows it to reach a more diversified training experience, as each agent has its independent exploration. An in-depth and more rigorous discussion on the various DRL algorithms can be found in \cite{francois2018}.
3.1. DRL for Inventory Management

To the best of the authors’ knowledge, only five papers have implemented DRL algorithms to solve the SCIM problem, despite some restrictions. More in detail, an extension of deep Q-network (DQN) has been proposed in Oroojlooyjadid et al. (2021) to solve the beer game problem while observing standard rules, in particular the one concerning decentralized information. The authors show that a DQN agent, which involves learning an ANN instead of a Q-table to return the Q-value for a state-action pair, can learn a near-optimal policy when its co-players follow a base-stock policy. Indeed, in their formulation, only one agent depends on the DQN, while the others are controlled by simple formulas. Because Q-learning requires a restricted action space cardinality, in Oroojlooyjadid et al. (2021) the authors performed numerical experiments using a \( d+x \) policy to choose the optimal reorder quantity, with \( x \) constrained to one of the following intervals: \([-2, +2]\), \([-5, +5]\), and \([-8, +8]\).

Alternatively, Peng et al. (2019) proposed a DRL approach based on a policy-based algorithm, more precisely the VPG, to address a two-echelon supply chain with stochastic and seasonal demand. In their work, the supply chain state is represented as a vector, which includes current stock levels concatenated with the last two demand values (resuming the idea of Kemmer et al. (2018)). Due to storage capacity constraints, the authors designed a dynamic action space. As a result, the number of products shipped takes into account the actual stock levels present in the warehouses. In order to evaluate the VPG performance, numerical experiments on three different cases are described, and the results showed that the VPG agent outperformed the \((s, Q)\)-policy employed as a baseline in all three cases.

Using the same supply chain structure but with ten warehouses and a normal distribution, Gijsbrechts et al. (2022) applied and tuned the A3C algorithm for two different numerical experiments (taken from Van Roy et al. (1997)). The authors restricted the action space by implementing a state-dependent base-stock policy. In particular, they simplified the action space by restricting it to two dimensions: one for the factory’s base-stock level (restricted in turn to take on 6 possible values set by the authors) and the other one for all the warehouses’ base-stock levels (restricted to 9 and 11 values, depending on the experiments). The paper shows that A3C is able to achieve a performance comparable to state-of-the-art heuristics and approximate dynamic programming algorithms, despite the initial tuning of the A3C algorithm remaining computationally intensive.

A SCIM problem with a linear four-echelon supply chain is considered in Hubbs et al. (2020), where the authors introduced and implemented the PPO algorithm. The input state is defined by stock levels at each time step concatenated with previously taken actions, thus ensuring that both available and in transit stocks are captured. Actions instead correspond to the reorder amount at each stage of the supply chain and ensure that capacity constraints are preserved. The simulation was run for 30 steps (days), while the demand was sampled from a Poisson distribution. This paper compared different operations research methods with a DRL approach in two different environments, i.e., without and with backlog (which means that unsatisfied demands will be fulfilled at
Numerical experiments show that the PPO algorithm outperforms the (static) base-stock policy in both environments.

Finally, in the experimental scenario analyzed by Alves & Mateus (2020), a general four-echelon supply chain with two nodes per echelon is presented. The system state consists of product quantity currently available and in transit across the supply chain, plus future customer demands. Actions consist of product quantity that factories produce and that each node submits to its predecessor. In this paper, the authors proposed the PPO algorithm to deal with the optimization problem, while an agent based on linear programming (LP) was employed as a baseline. In order to design the LP agent, an LP model for a deterministic formulation of the problem (i.e., considering deterministic demand) was solved. Results of numerical experiments show that PPO achieves satisfactory results.

4. Environment Formulation

The SCIM environment we propose is primarily motivated by what is presented and discussed in Kemmer et al. (2018); Peng et al. (2019). Inspired by these works, we designed a divergent two-echelon supply chain that includes a single factory, multiple distribution warehouses, and multiple product types over a fixed number of time steps; an example of this structure is shown in Fig. 3. At each time step, the agent is asked to find the number of products to be produced and preserved at the factory, as well as the number of products to be shipped to different distribution warehouses. To make the supply chain more realistic, we set capacity constraints on warehouses (and consequently, on how many units to produce at the factory), along with storage and transportation costs. We then simulate a stochastic and seasonal demand for each distribution warehouse and product type. If the demand for a specific warehouse cannot be satisfied, then a penalty cost is incurred until the specific warehouse is able to satisfy it. Furthermore, we let the demand exceed the warehouses’ capacity; in this way, the agent is driven to learn the demand curves’ seasonality in order to build up stocks as efficiently as possible.

Figure 3: A two-echelon supply chain consisting of a factory, its warehouse, and three distribution warehouses.
4.1. Mathematical Formulation

This subsection introduces the mathematical formulation of the SCIM environment, which consists of a factory, a factory warehouse, and J distribution warehouses. We assume that the factory produces I different product types. For each product type i, the factory decides, at every time step t, its respective production level \(a_{i,0,t}\) (i.e., how many units to produce), considering a fixed production cost of \(z_{i,0}\) per unit. Moreover, the factory warehouse is associated with a maximum capacity of \(c_{i,0}\) units for each product type i; this means that its total overall capacity is given by the sum of the maximum capacities across all product types, i.e., the overall capacity is given by \(\sum_{i=0}^{I} c_{i,0} = c_0\). The cost of storing one unit of product type i at the factory warehouse is \(z_{S,i,0}\) per time step, while the corresponding stock level at time t equals \(q_{i,0,t}\).

At every time step t, \(a_{i,j,t}\) units of product type i are shipped from the factory warehouse to the distribution warehouse j, with an associated transportation cost of \(z_{T,i,j}\) per unit. For each product type i, each distribution warehouse j has a maximum capacity \(c_{i,j}\) (\(\sum_{i=0}^{I} c_{i,j} = c_j\)), a storage cost of \(z_{S,i,j}\) per unit, and a stock level at time t equal to \(q_{i,j,t}\). The demand for product type i at warehouse j for time step t is equivalent to \(d_{i,j,t}\) units, while each product type i is sold to retailers at sale price \(p_i\) (which is identical across all warehouses).

Products are non-perishable and supplied in discrete quantities. Additionally, we assume that the factory is legally obligated to fulfill all orders submitted by retailers. Consequently, if the demand for a certain time step exceeds the associated stock level, a penalty coefficient per unsatisfied unit is applied (the related penalty cost \(z_{P,i,j}\) is obtained by multiplying the penalty coefficient by the sale price value). Unsatisfied demands are also maintained over time, and we design them as a negative stock level (this corresponds to back-ordering).

It is worth mentioning that sale prices and production costs may vary between product types, while maximum capacities, as well as storage and transportation costs, may differ according to different product types and warehouses. Table 1 provides a summary of the formulation parameters.

| Parameter | Explanation | Parameter | Explanation |
|-----------|-------------|-----------|-------------|
| I         | Number of Products Types | \(q_{i,j,t}\) | Stock Level (units) |
| J         | Number of Warehouses | \(c_{i,j}\) | Storage Capacity (units) |
| \(a_{i,0,t}\) | Production Level (units) | \(z_{S,i,j}\) | Storage Cost (per unit) |
| \(z_{i,0}\) | Production Cost (per unit) | \(z_{T,i,j}\) | Penalty Coefficient (per unit) |
| \(a_{i,j,t}\) | Shipped Stocks (units) | \(p_i\) | Sale Price (per unit) |
| \(z_{i,j}\) | Transportation Cost (per unit) | \(d_{i,j,t}\) | Demand (units) |

Table 1: The considered SCIM parameters with relative explanation (and units of measure).

We now formalize the RL problem as an MDP. More precisely, we introduce and define the main components of the SCIM environment that we propose in this paper: the state vector, the action space, and the reward function. The state vector includes all current stock levels for each warehouse and product type, plus the last \(\tau\) demand values, and is defined as follows:

\[
s_t = (q_{0,0,t}, \ldots, q_{I,j,t}, d_{t-\tau}, \ldots, d_{t-1}),
\]
where \( d_{t-1} = (d_{0,t-1}, \ldots, d_{I,J,t-1}) \). It is worth noticing that the actual demand \( d_t \) for the current time step \( t \) will not be known until the next time step \( t+1 \). This implementation choice ensures that the agent may benefit from learning the demand pattern so as to integrate a sort of demand forecasting directly into the policy. Additionally, we include the last demand values in order to enable the agent to have limited knowledge of the demand history and, consequently, to gain a basic comprehension of its fluctuations (similar to what was made originally by Kemmer et al. (2018)). In our SCIM implementation, we let the agent access demand values of the last five time steps, even if preliminary results suggest that comparable performances are obtained when demand values are known for the last three or four time steps.

Regarding the action space, this can be discrete or continuous in an RL problem. With a discrete action space, the agent chooses from a finite action set which specific action to perform (this implies that each action must be representable as a single vector). Hence, each ANN output node generates the probability of performing a specific action. For example, in Chaharsooghi et al. (2008), the number of output nodes matches the number of rows in the Q-table, i.e., 4^4. Conversely, with a continuous action space, the ANN generates the action value directly. In Chaharsooghi et al. (2008), the number of output nodes would be then 4, which corresponds to the number of participants for which we want to take an action.

Since it is more scalable and can also be applied to wider action spaces, we have chosen to implement a continuous action space consisting of production and shipping controls:

\[
a_t = (a_{0,0,t}, \ldots, a_{I,J,t})
\]

For each product type \( i \), the agent can thus specify the factory’s production level and the number of units to deliver to each warehouse \( j \). In literature, a relatively small and identical upper bound is typically adopted for all the action values in order to reduce computational effort. However, the drawback is that this might lead to a significant drop in terms of performance. Indeed, if the upper bound is set too small, the agent may select an inefficient action given that the optimal one is outside of the admissible range. Otherwise, if the upper bound is set too high, the agent may repeatedly choose an incoherent action, i.e., one that falls within the permissible range but exceeds a specified maximum capacity, consequently slowing down the training process.

Our implementation thus provides a continuous action space with an independent upper bound for each action value in order to find a trade-off between efficiency and performance. In practical terms, the dimension of the action space is equal to the number of warehouses (including the factory warehouse) multiplied by the number of products. The lower bound for each coordinate is simply zero. In fact, it would be illogical to produce or ship negative quantities of products. Conversely, the upper bound of each distribution warehouse corresponds to its maximum capacity with respect to each product type (by referring to Eq. (4), 0 ≤ \( a_{i,j,t} \leq c_{i,j} \)). To guarantee that the factory can adequately handle the various demands, its upper bound instead amounts to the sum of all ware-
houses’ capacities with regard to each product type \((0 \leq a_{i,0,t} \leq \sum_{j=0}^{I} c_{i,j})\). Accordingly, we now have a well-defined and coherent action space. We expect to improve both efficiency and performance with this intuition, as the action space is bounded (and hence restricted) but contains only coherent (and possibly optimal) actions.

For the sake of simplicity, we assume that there are no lead times, both for production and transportation (or to refer to literature, we assume constant lead times equal to 0). To evaluate the agent, we simulate a seasonal behavior by representing the demand as a co-sinusoidal function with a stochastic component, defined according to the following equation:

\[
d_{i,j,t} = \left\lfloor \frac{d_{\text{max}}}{2} \left( 1 + \cos \left( \frac{4\pi(2ij + t)}{T} \right) \right) + \mathcal{U}(0, d_{\text{var},i}) \right\rfloor,
\]

where \(\left\lfloor \cdot \right\rfloor\) is the floor function, \(d_{\text{max},i}\) is the maximum demand value for product type \(i\), \(\mathcal{U}\) is a random variable assumed to be distributed according to a uniform distribution on the support \((0, d_{\text{var},i})\), and \(T\) is the final time step of the episode. At each time step \(t\), the demand may vary for each warehouse \(j\) and product type \(i\) while maintaining the same behavior, as can be seen in Fig. 4.

\(\text{(a)}\) An instance of the demand profile for the 1P1W scenario.

\(\text{(b)}\) An instance of demand profile for the 1P3W scenario.

\(\text{(c)}\) An instance of demand profile for the 2P2W scenario.

Figure 4: Some instances of demand behavior for different topology and configurations of the SCIM problem generated according to Eq. (5): \(\text{(a)}\) the scenario consisting of one product type and one distribution warehouse (1P1W), \(\text{(b)}\) the scenario consisting of one product type and three distribution warehouses (1P3W), and \(\text{(c)}\) the scenario consisting of two product types and two distribution warehouses (2P2W).
Finally, the *reward function* is defined for each time step \( t \) as follows:

\[
\begin{align*}
    r_t &= \sum_{j=1}^{J} \sum_{i=0}^{I} p_i \cdot d_{i,j,t} \quad (6a) \\
    &- \sum_{i=0}^{I} z_{i,0} \cdot a_{i,0,t} \quad (6b) \\
    &- \sum_{j=1}^{J} \sum_{i=0}^{I} z_{i,j}^T \cdot a_{i,j,t} \quad (6c) \\
    &- \sum_{j=0}^{J} \sum_{i=0}^{I} z_{i,j}^S \cdot \max(q_{i,j,t}, 0) \quad (6d) \\
    &+ \sum_{j=0}^{J} \sum_{i=0}^{I} z_{i,j}^P \cdot \min(q_{i,j,t}, 0). \quad (6e)
\end{align*}
\]

The first term \( (6a) \) represents revenues, the second one \( (6b) \) production costs, while the third one \( (6c) \) corresponds to transportation costs. The fourth term \( (6d) \) is the overall storage costs. The function \( \max \) is implemented to avoid negative inventories (i.e., back-ordering) from being counted. Then, the last term \( (6e) \) denotes the penalty costs, which is introduced with a plus sign because stock levels would already be negative in the eventuality of unsatisfied demands. Hence, the agent’s goal is to *maximize the supply chain profit* as defined in the reward function.

Regarding the dynamics of the SCIM environment, the following *sequence of events* occurs for each time step \( t \):

1. starting from the current state of the supply chain (Eq. (3)), the agent determines, for each product type \( i \), the number of units to be produced and shipped to each warehouse \( j \) (following Eq. (4)). Due to capacity constraints, there is an upper bound to the number of units that the factory can physically store (i.e., \( q_{i,0,t} + a_{i,0,t} - \sum_{j=1}^{J} a_{i,j,t} \leq c_{i,0} \)). Actually, the available stocks are not explicitly considered when the agent chooses an action. However, producing or shipping a number of stocks that it is not possible to store leads to costs and, therefore, an *implicit penalty* for the agent;

2. the distribution warehouses receive products. At this point, the customer demand for each product \( i \) is satisfied at each warehouse \( j \) on the basis of the available stocks (i.e., \( q_{i,j,t} + a_{i,j,t} - d_{i,j,t} \leq c_{i,j} \)), where \( d_{i,j,t} \) is generated according to Eq. (5)). Surplus stocks and unsatisfied demands are transferred over the next time step; this means that the decisions taken at the current time step \( t \) will affect subsequent time steps. When surplus stocks are generated, a storage cost is imposed. Otherwise, a penalty cost is considered for unsatisfied demands;
after computing the total reward (following Eq. (6)) and generating the 
next state of the environment (according to Eq. (7)), these two elements 
will be returned to the agent at the next time step \( t + 1 \).

Formally, the state’s updating rule is defined as follows:

\[
s_{t+1} = \min\left(\left[q_0,0,t + a_{0,0,t} - \sum_{j=1}^{J} a_{0,j,t} \right], c_{0,0}\right), \\
\cdots, \\
\min\left(\left[q_{I,J,t} + a_{I,I,t} - d_{I,I,t} \right], c_{I,J}\right), \\
d_t + 1 - \tau, \\
\cdots, \\
d_t\right).
\]

This implies that, at the beginning of the next time step, the factory’s stocks are 
equal to the initial stocks, plus the units produced, minus the stocks shipped. 
Similarly, the warehouses’ stocks are equal to the initial stocks, plus the units 
received, minus the current sales. Lastly, the demand values included in the 
state vector are also updated, discarding the oldest value and concatenating the 
most recent one.

4.2. Baselines

To assess and compare performances achieved by the adopted DRL algo-
rithms, we implement a static reorder policy known in the specialized literature 
as the \((s, Q)\)-policy. This policy can be expressed by a rule, which can be sum-
marized as follows: at each time step \( t \), the current stock level for a specific 
warehouse and product type is compared to the reorder point \( s \). If the stock 
level falls below the reorder point \( s \), then the \((s, Q)\)-policy orders \( Q \) units of 
product; otherwise, it does not take any action. Typically, the \((s, Q)\)-policy 
generates a sawtooth pattern caused by its way of acting. In our SCIM imple-
mentation, we opted to make reordering decisions independently; this means 
that the \((s, Q)\)-policy parameters, \( s \) and \( Q \), can differ for each warehouse and 
product type. However, finding optimal values for \( s \) and \( Q \) is a tough challenge. 
Indeed, a SCIM environment consisting of a single factory, two warehouses, and 
two product types requires to set twelve parameter values, i.e., six \((s, Q)\) pairs. 
Generally, parameters value must be adjusted to balance storage and penalty 
costs against an uncertain demand. Moreover, the reorder points must be cho-

en to appropriately absorb demand shocks, i.e., sudden or unexpected events 
that dramatically increase or decrease the demand.

It is worth noting that assuming that the demand is known in advance, the 
search for the optimal parameters values can be treated analytically. In this 
paper, we nonetheless propose an adaptive and data-driven approach based on 
Bayesian optimization. In this way, the solution method does not require to 
make any assumptions or simplifications, and hence it is no longer problem-
dependent; therefore, it can be applied to any SCIM topology and configuration.
just as it happens for DRL algorithms (they share, in fact, the same identical simulator). To compare DRL and BO approaches, we also implement an oracle, i.e., a baseline that knows the real demand value for each product type and warehouse in advance and can accordingly select the optimal action to take a priori.

Once the environment and the baselines have been specified, the DRL algorithms must be developed. We implemented the following three DRL algorithms: A3C, PPO, and VPG, which have been introduced in Section 2.1. In this respect, we rely on the implementations made available by Ray (Moritz et al., 2018), an open-source Python framework that is bundled with RLib, a scalable RL library, and Tune, a scalable hyperparameter tuning library. An advantage of Ray is that it natively supports OpenAI Gym as its environment interface. As a result, the simulator for the agents’ training process has been developed using the OpenAI Gym APIs.

5. Numerical Experiments

A rich set of numerical experiments have been designed and performed to compare the performances of DRL algorithms and BO. Numerical experiments concern the capacitated SCIM problem under three different scenarios. Each scenario is associated with different demand patterns with respect to each product type and warehouse, i.e., seasonal and stochastic fluctuations. Furthermore, each scenario has different capacities and costs for evaluating in-depth the adaptability and robustness of DRL algorithms.

One Product Type One Distribution Warehouse (1P1W)

Under the one product type and one distribution warehouse (1P1W) scenario, the supply chain is set to manage just one product type. Accordingly, it consists of one factory, a factory warehouse, and one distribution warehouse. This scenario consists of five experiments, as summarized in Table 1 of the supplementary material. In the first experiment, the demand value is (upper) bounded by 10 units, while, at each time step, it can stochastically grow up to 2 units (the maximum possible demand value for a specific time step is then 12 units). The second and third experiments allow the demand value to be as high as 5 with a stochasticity of 2, whereas, in the fifth experiment, the demand variation is increased to 3, thereby increasing the level of uncertainty. The rationale behind this latter experiment is that we expect that the performance of DRL algorithms decreases when the level of uncertainty increases. Indeed, when uncertainty grows, it becomes more and more challenging to make reliable forecasts about future products demands. Regarding instead the fourth experiment, the maximum demand value is higher and equal to 7, but with a lesser uncertainty equal to 1.

Under the 1P1W scenario, sale prices and costs are also manipulated so as to increase or decrease revenues and, consequently, the difficulty of the problem to be addressed. As a result, in the first experiment, we bounded the capacity of
the factory warehouse by half of the maximum demand value. This decision was made to study whether DRL algorithms are able to learn an efficient strategy, i.e., a strategy capable of predicting a growing demand and thus saving and shipping stocks in advance. Analogously, we expect a greater quantity of stocks to be stored and shipped when storage and transportation costs are low, like in the second experiment, while we expect the opposite when these costs are high, as in the fourth experiment. Finally, we generated multiple penalty coefficients to determine whether a hefty punishment forces DRL algorithms to be more or less effective, with particular attention to the more challenging experiments where low revenues and high costs are considered, like the first, the third, and the fifth experiments.

One Product Type Three Distribution Warehouses (1P3W)

The one product type three distribution warehouses (1P3W) scenario concerns a more complex supply chain configuration, consisting of a factory, a factory warehouse, and three distribution warehouses. Even in this scenario, the supply chain still manages a single product type. The design of the respective experiments follows that of the previous 1P1W scenario; however, a remarkable difference is found in storage capacities and costs, as depicted in Table 2 of the supplementary material.

In fact, under the 1P3W scenario, we set warehouses’ costs to be directly proportional to their corresponding capacities, i.e., the less storage space we have, the more expensive it is to store a product. This scenario is also designed to investigate the behavior of DRL algorithms strategy when capacities increase, given that the search space of optimal actions grows accordingly, as happens in the fourth and fifth experiments.

Furthermore, the 1P3W scenario allows us to study how DRL algorithms react when demand, with the related costs, becomes greater than actual capacities, like in the first and fourth experiments, considering that the supply chain now consists of three warehouses and, thus, the environment becomes certainly more challenging to be tackled.

Two Products Two Warehouses (2P2W)

In the two products type two distribution warehouses (2P2W) scenario, the supply chain consists of two product types, a factory, its associated warehouse, and two distribution warehouses. Due to computational time, we performed just three experiments under this scenario, as reported in Table 3 of the supplementary material.

Regarding the demand, we explored demand variations which can be different, like in the first and second experiments, or equal, as in the third experiment, on the basis of the specific product type.

Additionally, we thought of something different concerning storage capacities and, consequently, the search space. Indeed, in the third experiment, warehouses capacities for the first product type are designed in descending order, while for the second product type in ascending order; this implies that, for example, the
second distribution warehouse can store the minimum amount of stocks for the first product type and the maximum amount for the second product type. We expect that this *imbalance*, especially when combined with greater uncertainty, makes the SCIM problem more unpredictable and, thus, more difficult to be effectively solved.

5.1. Hyperparameters Selection

The task of selecting appropriate values of hyperparameters for ANNs is known to be non-trivial and time-consuming, other than broadly addressed in the specialized literature (Feurer & Hutter 2019; Yang & Shami 2020). Hyperparameters are particularly important concerning DRL algorithms because they can influence training and, hence, the relative performance (Boute et al. 2021). The hyperparameters we have chosen to tune have been selected following what was presented in the Ray documentation and discussed in the papers of Gijsbrechts et al. (2022) and Alves & Mateus (2020); they are reported in Table 4 of the supplementary material together with their corresponding values.

In our experimental plan, each DRL algorithm was trained for all possible assignments of hyperparameters values through a grid search. More in detail, a given number of instances for each DRL algorithm were run: 895 under the 1P1W scenario, 593 under the 1P3W scenario, and 593 for the third, for a total of 2383 instances. Each DRL algorithm instance is trained for a given number of episodes: 15,000 episodes for the 1P1W scenario and 50,000 for the 1P3W and 2P2W scenarios. However, running such a vast number of experiments is highly resource and time consuming. For this reason, we took advantage of using scheduling algorithms made available by Tune, which early stops those training sessions associated with bad hyperparameters configurations. To increase the reliability of such a procedure, we set for all DRL algorithm instances a grace period equal to one-tenth of the total number of episodes (this corresponds to the minimum number of episodes for which an instance cannot be stopped). In terms of implementation, we used the asynchronous successive halving algorithm (ASHA) (Li et al. 2018) as a scheduling algorithm. The asynchronous nature of ASHA allows overcoming the bottleneck inherent to synchronous promotions (i.e., to evaluate all configurations and keeps only the best). Indeed, ASHA attempts to maximize parallelism by promoting the best hyperparameters configurations whenever possible.

Regarding the BO approach, each numerical experiment uses a fixed number of training iterations: 50 under the 1P1W scenario and 100 under both the 1P3W and 2P2W scenarios, each one simulating 25 episodes. Consistency is ensured by replicating each experiment three times, for a total amount of 39 instances.

The results presented and discussed in the next section have been obtained by selecting, for each approach and experiment, the respective best instance.

5.2. Discussion

In order to evaluate the effective performances of the DRL algorithms, BO, and oracle, we simulated, for each scenario and experiment, 200 different
episodes. Each episode consists of 25 time steps, and we reported the *average cumulative profit* achieved, i.e., the sum of the per-step profits at the last time step $T$. All experiments were run on a machine equipped with an Intel® Xeon® Platinum 8272CL CPU at 2.6 GHz and 16 GB of RAM.

### 1P1W Results

Results of numerical experiments under this scenario are summarized in Table 2. BO and PPO achieve an optimum profit in the first experiment, where the demand is greater than warehouses capacities, whereas A3C and VPG perform slightly worse. All DRL algorithms achieve comparable results in the second experiment, which involves a more simple configuration, with higher revenues and lower transportation and penalty costs. The third experiment faces a more complex problem to be solved, involving lower revenues and higher transportation costs and penalties. The optimal profit is relatively small in this case, but PPO tends to behave better than other DRL algorithms. The fourth experiment offers a more balanced configuration, increasing revenues and the maximum demand value but reducing uncertainty. The main difficulty here is represented by a wider search space (caused by greater storage capacities) and higher storage costs, especially for the factory. However, as shown in Fig. 5, BO, PPO, and A3C obtain satisfactory profits, while VPG seems to perform poorly. It is interesting to observe how A3C, in contrast to the other, produces and ships a constant number of stocks for all the time steps, also succeeding in not paying any storage cost for storing stocks in the factory warehouse. In the fifth and last experiment, the demand uncertainty increases, and it is more expensive to maintain stocks at the distribution warehouse rather than at the factory; in this case, all DRL algorithms achieve comparable and near-optimal results.

|                | A3C    | PPO    | VPG    | BO     | Oracle |
|----------------|--------|--------|--------|--------|--------|
| Exp 1          | 870 ± 67 | 1213 ± 68 | 885 ± 66 | 1226 ± 71 | 1474 ± 45 |
| Exp 2          | 1066 ± 94 | 1163 ± 66 | 1100 ± 77 | 1224 ± 60 | 1289 ± 68 |
| Exp 3          | 1066 ± 74 | 195 ± 43 | 12 ± 61  | 101 ± 50  | 345 ± 18  |
| Exp 4          | 1317 ± 60 | 1600 ± 62 | 883 ± 95 | 1633 ± 39 | 2046 ± 37 |
| Exp 5          | 736 ± 45  | 838 ± 58  | 789 ± 51 | 870 ± 67  | 966 ± 55  |

Table 2: Results covering the 1P1W scenario. It is possible to note how BO and PPO obtain near-optimal profits, while A3C and VPG seems slightly inferior.

### 1P3W Results

Table 3 summarizes the results for this scenario, which in design is similar to the 1P1W scenario. The first experiment is characterized by a high demand, especially if compared with the capacities of the factory and of the first distribution warehouse. With this setting, BO performs worse than DRL algorithms.

1 All the figures regarding 1P1W scenario and related to the behavior of DRL algorithms and BO are available on [https://github.com/frenkowski/SCIMAI-Gym](https://github.com/frenkowski/SCIMAI-Gym)
Cumulative Shipment, Production, Stock, Factory Profit

Figure 5: Different behaviors in terms of stock and production levels, shipping stocks, and per-step and cumulative profit concerning the fourth experiment of the 1P1W scenario and according to (a) A3C, (b) PPO, (c) VPG, and (d) BO.

However, BO achieves a nearly optimal profit in the second experiment, where a simpler configuration is investigated. In the third experiment, which is more challenging than the previous two, none of the algorithms achieves a profit greater than zero, with PPO achieving the worst one. As analyzed in Fig. 6, PPO pays many backlog penalties at the first time step of the episode, mainly because the stocks shipped are greater than those produced. The fourth experiment provides a more balanced configuration and is characterized by an increased search space together with higher storage costs. Even in this case, PPO outperforms A3C and VPG. Finally, BO and PPO achieved the best profits in the last experiment, where uncertainty and search space were increased, but fewer penalties were considered.

| Exp | A3C     | PPO     | VPG     | BO       | Oracle |
|-----|---------|---------|---------|----------|--------|
| 1   | 1606 ± 139 | 2319 ± 122 | 803 ± 154 | 486 ± 330 | 3211 ± 60 |
| 2   | 2196 ± 104 | 3461 ± 120 | 2568 ± 112 | 3193 ± 101 | 3848 ± 95 |
| 3   | -2142 ± 128 | -4337 ± 216 | -2638 ± 121 | -1682 ± 196 | 772 ± 21 |
| 4   | -561 ± 237 | 2945 ± 135 | 656 ± 149 | 1256 ± 170 | 4389 ± 64 |
| 5   | 1799 ± 306 | 2353 ± 131 | 1341 ± 79 | 2203 ± 152 | 2783 ± 91 |

Table 3: Results regarding the 1P3W scenario. BO performs worse on average than DRL algorithms, except in the third, and more challenging, experiment.

---

All the figures regarding 1P3W scenario and related to the behavior of DRL algorithms and BO are available on [https://github.com/frenkowski/SCIIMA-Gym](https://github.com/frenkowski/SCIIMA-Gym).
2P2W Results

Table 4 summarizes performances under this scenario. The first experiment provides a balanced configuration. Indeed, the demand for the first product type is upper bounded by 3 and allows a stochastic variation up to 2, whereas the demand for the second product type is bounded by 6 with a maximum variation of 1. Furthermore, revenues are particularly high for both products, while storage costs at the factory are greater than those at the two distribution warehouses. Under such a mix, PPO achieves a good profit, as it also does A3C, which overcomes BO. For the second experiment, sales prices are diminished and, accordingly, revenues reduce as well. Even storage and penalty costs are decreased, while penalties increase. PPO still achieves a nearly optimal result, and the same happens for VPG, while BO also behaves well. In the third and last experiment, the demand is decreased, while uncertainty and capacities are increased. In this experiment, we designed alternating storage costs; this means, for example, that maintaining stocks of the first product type at the factory warehouse is the most inexpensive option, while maintaining stocks of the second product type is the most expensive. The results allow us to conclude that PPO, followed by VPG, continues to perform successfully, whereas BO seems to suffer the most.

\[\text{(2P2W Results continued...)}\]

---

Footnote:

3 All the figures regarding 2P2W scenario and related to the behavior of DRL algorithms and BO are available on [https://github.com/frenkowski/SCIMAI-Gym](https://github.com/frenkowski/SCIMAI-Gym)
Table 4: Results concerning the 2P2W scenario. Results suggest that PPO behaves well, whereas BO seems to suffer more compared to the other DRL algorithms.

|       | A3C   | PPO   | VPG   | BO    | Oracle |
|-------|-------|-------|-------|-------|--------|
| Exp 1 | 2227±  | 2783±  | 1585±  | 2086±  | 3787±  |
|       | 178   | 139   | 184   | 173   | 102    |
| Exp 2 | 1751±  | 2867±  | 2329±  | 2246±  | 3488±  |
|       | 83    | 90    | 98    | 114   | 63     |
| Exp 3 | 1414±  | 2630±  | 2434±  | 552±   | 3549±  |
|       | 128   | 138   | 156   | 268   | 103    |

Table 5: Average DRL training times (in minutes) of the best instances for the three scenarios considered. From the results obtained, A3C proves to be the fastest DRL algorithm.

|       | A3C | PPO | VPG |
|-------|-----|-----|-----|
| 1P1W  | 5±1 | 13±3| 5±0 |
| 1P3W  | 21±1| 47±9| 24±2|
| 2P2W  | 24±2| 55±2| 25±0|

6. Conclusions

Results of numerical experiments show that the SCIM environment we propose is effective in representing states, actions, and rewards. Indeed, the DRL algorithms achieve nearly optimal solutions in all the three investigated scenarios. The PPO algorithm is the one that better adapts to different topologies and configurations of the SCIM environment. In fact, it achieves higher profit than other algorithms on average. However, the most challenging experiment of the 1P3W scenario shows that PPO can fail to reach a positive profit. Its’ behavior can be analyzed in more detail through Fig. 6, where it is noted that PPO suffers by paying many backlog penalties at the beginning of the episode. This compromises the agent from learning an appropriate strategy, and as a consequence, it incurs a negative profit across all the episode. A possible solution could be to consider penalty costs that are no longer stationary but otherwise dependent on a given time step. The VPG algorithm frequently appears to converge to a local maximum that seems slightly distant from PPO, especially in the second scenarios, but it still achieves acceptable results. Finally, the A3C algorithm is constantly the fastest DRL algorithm, as reported in Table 5, also due to the values of the hyperparameters relating to the best instances. However, just like the VPG algorithm, it is never the best-performing one, perhaps because of its heightened sensitivity to the complex task of hyperparameter tuning.

It is worthwhile to mention that the BO approach also shows remarkable results, especially when the search space is limited. When compared to DRL algorithms, the BO approach seems to suffer more when there are two product types or when the demand exceeds the capacities, as happens in the first experiments of the 1P3W scenario. This is mainly due to the static and non-dynamic nature of the (s, Q)-policy, which does not allow to develop an effective strat-

\footnote{All the figures related to the convergence of DRL algorithms and BO are available on \url{https://github.com/frenkowski/SCIMAI-Gym}}
egy, e.g., for saving stocks in advance, but, on the contrary, it culminates in a myopic behavior. Nevertheless, it usually achieves respectable results, and the absence of hyperparameters to be tuned offers a clear advantage.

6.1. Future Research

This paper can be extended and improved in many directions as:

- develop a more comprehensive SCIM environment, i.e., by considering all those dimensions mentioned in [de Kok et al. (2018)]. Additional supply chain structures, topologies, and configurations allow us to evaluate the robustness of the proposed approaches under more realistic experiments;
- taking into account the non-linearity of transportation costs, as well as non-zero leading times. In this respect, a significant issue is how to represent the system state as an MDP without violating the Markov property;
- alternative demand distributions (e.g., normal or Poisson) could be explored to assess how the proposed approaches react and adapt to different customer behaviors;
- real-world data could also be used to validate DRL algorithms and check whether they improve the performances of currently used supply chains management systems in practice.

Another interesting direction toward investigating DRL algorithms is to focus on transfer learning [Pan & Yang, 2009]. Indeed, it would be interesting to extend the work of [Oroojlooyjadid et al. (2021)], whose one of the central insight is that transferring knowledge between agents is more efficient than training them from scratch. Transfer learning may thus be applied when some configurations of the problem change, such as demands, capacities, or costs.

Lastly, even the BO approach could be extended to other static reorder policies, such as the base-stock policy, which has exactly half of the \((s, Q)\)-policy parameters. With such an extension, we expect to improve performances and convergence times, the latter currently comparable to those of DRL algorithms.

Acknowledgement

We would like to express our very great appreciation to Prof. Riccardo Melen for his valuable and constructive suggestions during the planning and development of this research work. His willingness to give his time so generously has been very much appreciated.

References

Alves, J. C., & Mateus, G. R. (2020). Deep reinforcement learning and optimization approach for multi-echelon supply chain with uncertain demands. In *International Conference on Computational Logistics* (pp. 584–599). Springer.
Boute, R. N., Gijsbrechts, J., van Jaarsveld, W., & Vanvuchelen, N. (2021). Deep reinforcement learning for inventory control: A roadmap. *European Journal of Operational Research*.

Brockman, G., Cheung, V., Pettersson, L., Schneider, J., Schulman, J., Tang, J., & Zaremba, W. (2016). Openai gym. *arXiv preprint arXiv:1606.01540*.

Chaharsoughi, S. K., Heydari, J., & Zegordi, S. H. (2008). A reinforcement learning model for supply chain ordering management: An application to the beer game. *Decision Support Systems, 45*, 949–959.

Christopher, M. (2016). *Logistics & supply chain management*. Pearson Uk.

Cimen, M., & Kirkbride, C. (2013). Approximate dynamic programming algorithms for multidimensional inventory optimization problems. *IFAC Proceedings Volumes, 46*, 2015–2020.

Clark, A. J., & Scarf, H. (1960). Optimal policies for a multi-echelon inventory problem. *Management science, 6*, 475–490.

Feurer, M., & Hutter, F. (2019). Hyperparameter optimization. In *Automated machine learning* (pp. 3–33). Springer, Cham.

Forrester, J. W. (1958). Industrial dynamics. a major breakthrough for decision makers. *Harvard business review, 36*, 37–66.

Forrester, J. W. (1997). Industrial dynamics. *Journal of the Operational Research Society, 48*, 1037–1041.

François-Lavet, V., Henderson, P., Islam, R., Bellemare, M. G., & Pineau, J. (2018). An introduction to deep reinforcement learning. *arXiv preprint arXiv:1811.12560*.

Geever, K. (2020). *Deep Reinforcement Learning in Inventory Management*. Master’s thesis University of Twente.

Gijsbrechts, J., Boute, R. N., Van Mieghem, J. A., & Zhang, D. J. (2022). Can deep reinforcement learning improve inventory management? performance on lost sales, dual-sourcing, and multi-echelon problems. *Manufacturing & Service Operations Management*.

Hubbs, C. D., Perez, H. D., Sarwar, O., Sahinidis, N. V., Grossmann, I. E., & Wassick, J. M. (2020). Or-gym: A reinforcement learning library for operations research problems. *arXiv preprint arXiv:2008.06319*.

Jaakkola, T., Jordan, M. I., & Singh, S. P. (1994). On the convergence of stochastic iterative dynamic programming algorithms. *Neural computation, 6*, 1185–1201.
Kemmer, L., von Kleist, H., de Rochebonêt, D., Tziortziotis, N., & Read, J. (2018). Reinforcement learning for supply chain optimization. In European Workshop on Reinforcement Learning. volume 14.

de Kok, T., Grob, C., Laumanns, M., Minner, S., Rambau, J., & Schade, K. (2018). A typology and literature review on stochastic multi-echelon inventory models. European Journal of Operational Research, 269, 955–983.

Lee, H. L., Padmanabhan, V., & Whang, S. (1997). Information distortion in a supply chain: The bullwhip effect. Management science, 43, 546–558.

Li, L., Jamieson, K., Rostamizadeh, A., Gonina, E., Hardt, M., Recht, B., & Talwalkar, A. (2018). A system for massively parallel hyperparameter tuning. arXiv preprint arXiv:1810.05934.

Li, Y. (2017). Deep reinforcement learning: An overview. arXiv preprint arXiv:1701.07274.

Mnih, V., Badia, A. P., Mirza, M., Graves, A., Lillicrap, T., Harley, T., Silver, D., & Kavukcuoglu, K. (2016). Asynchronous methods for deep reinforcement learning. In International conference on machine learning (pp. 1928–1937). PMLR.

Mnih, V., Kavukcuoglu, K., Silver, D., Rusu, A. A., Veness, J., Bellemare, M. G., Graves, A., Riedmiller, M., Fidjeland, A. K., Ostrovski, G. et al. (2015). Human-level control through deep reinforcement learning. nature, 518, 529–533.

Moritz, P., Nishihara, R., Wang, S., Tumanov, A., Liaw, R., Liang, E., Elibol, M., Yang, Z., Paul, W., Jordan, M. I. et al. (2018). Ray: A distributed framework for emerging {AI} applications. In 13th {USENIX} Symposium on Operating Systems Design and Implementation ({{OSDI} 18}) (pp. 561–577).

Mortazavi, A., Khamseh, A. A., & Azimi, P. (2015). Designing of an intelligent self-adaptive model for supply chain ordering management system. Engineering Applications of Artificial Intelligence, 37, 207–220.

Oroojlooyjadid, A., Nazari, M., Snyder, L. V., & Takáč, M. (2021). A deep q-network for the beer game: Deep reinforcement learning for inventory optimization. Manufacturing & Service Operations Management.

Pan, S. J., & Yang, Q. (2009). A survey on transfer learning. IEEE Transactions on knowledge and data engineering, 22, 1345–1359.

Peng, Z., Zhang, Y., Feng, Y., Zhang, T., Wu, Z., & Su, H. (2019). Deep reinforcement learning approach for capacitated supply chain optimization under demand uncertainty. In 2019 Chinese Automation Congress (CAC) (pp. 3512–3517). IEEE.
Ravulapati, K. K., Rao, J., & Das, T. K. (2004). A reinforcement learning approach to stochastic business games. *IIE Transactions, 36*, 373–385.

Schulman, J., Levine, S., Abbeel, P., Jordan, M., & Moritz, P. (2015). Trust region policy optimization. In *International conference on machine learning* (pp. 1889–1897). PMLR.

Schulman, J., Wolski, F., Dhariwal, P., Radford, A., & Klimov, O. (2017). Proximal policy optimization algorithms. *arXiv preprint arXiv:1707.06347*, .

Shervais, S., Shannon, T. T., & Lendaris, G. G. (2003). Intelligent supply chain management using adaptive critic learning. *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans, 33*, 235–244.

Silver, D., Schrittwieser, J., Simonyan, K., Antonoglou, I., Huang, A., Guez, A., Hubert, T., Baker, L., Lai, M., Bolton, A. et al. (2017). Mastering the game of go without human knowledge. *nature, 550*, 354–359.

Slack, N., & Lewis, M. (2002). *Operations strategy*. Pearson Education.

Sui, Z., Gosavi, A., & Lin, L. (2010). A reinforcement learning approach for inventory replenishment in vendor-managed inventory systems with consignment inventory. *Engineering Management Journal, 22*, 44–53.

Sutton, R. S., & Barto, A. G. (2018). *Reinforcement Learning: An Introduction*. MIT press.

Tsitsiklis, J. N. (1994). Asynchronous stochastic approximation and q-learning. *Machine learning, 16*, 185–202.

Van Roy, B., Bertsekas, D. P., Lee, Y., & Tsitsiklis, J. N. (1997). A neurodynamic programming approach to retailer inventory management. In *Proceedings of the 36th IEEE Conference on Decision and Control* (pp. 4052–4057). IEEE volume 4.

Vinyals, O., Babuschkin, I., Czarnecki, W. M., Mathieu, M., Dudzik, A., Chung, J., Choi, D. H., Powell, R., Ewalds, T., Georgiev, P. et al. (2019). Grandmaster level in starcraft ii using multi-agent reinforcement learning. *Nature, 575*, 350–354.

Watkins, C. J. C. H. (1989). *Learning from Delayed Rewards*. Ph.D. thesis King’s College, Cambridge United Kingdom.

Williams, R. J. (1992). Simple statistical gradient-following algorithms for connectionist reinforcement learning. *Machine learning, 8*, 229–256.

Wu, C., Rajeswaran, A., Duan, Y., Kumar, V., Bayen, A. M., Kakade, S., Mordatch, I., & Abbeel, P. (2018). Variance reduction for policy gradient with action-dependent factorized baselines. *arXiv preprint arXiv:1803.07246*, .

Yang, L., & Shami, A. (2020). On hyperparameter optimization of machine learning algorithms: Theory and practice. *Neurocomputing, 415*, 295–316.