Producing the event-ready two photon polarization entangled state with normal photon detectors

Wang Xiang-Bin*†

Imai Quantum Computation and Information project, ERATO, Japan Sci. and Tech. Corp.
Daini Hongo White Bldg. 201, 5-28-3, Hongo, Bunkyo, Tokyo 113-0033, Japan

Abstract

We propose a scheme to produce the maximally two photon polarization entangled state (EPR state) with single photon sources and the passive linear optics devices. In particular, our scheme only requires the normal photon detectors which distinguish the vacuum and non-vacuum Fock number states. A sophisticated photon detector distinguishing one-photon state and two-photon state is unnecessary in the scheme.

The resource of maximally entangled state (EPR state) plays a fundamentally important role in testing quantum laws related to the non-locality [1] and in many tasks of quantum information processing [2,3] such as the quantum teleportation [4,5], quantum dense coding [4] the entanglement based quantum key distribution [6] and quantum computation [3,7]. So far, it is generally believed that the two photon polarization EPR state is particularly useful in quantum information processing.

In quantum teleportation, initially two remotely separated parties Alice and Bob share an entangled pair of particle 2 and 3. Alice is offered with particle 1 which is in an unknown state $|u\rangle$. Alice’s task here is to produce the unknown state $|u\rangle$ in Bob’s side without sending

*email: wang@qci.jst.go.jp

†The initials of the author’s name are X.B.
particle 1 to Bob or taking any action to observe the state information of particle 1. To do so, she takes a joint measurement on particle 1 and 2 in the Bell-state basis. By observing the measurement result she does not know any information of the original state of particle 1, however, she knows that by what unitary transformation the state of particle 3 at Bob’s side can be transformed to the unknown state $|u\rangle$. In such a task, the resource of pre-shared (event-ready) entanglement between Alice and Bob is crucial for a deterministic quantum teleportation [8]. In the quantum dense coding, with the help of pre-shared event-ready entanglement, Alice may send 2 bits information to Bob by only sending him one quantum bit (a two-level-state particle) [4].

The observation of EPR pairs has been carried out by many experiments (for example, ref [9]). However, those entangled polarized photon pairs were only produced randomly among vacuum states since there is no way to know whether a polarization EPR pair is generated without destroying the state itself. To make sure which states are indeed EPR states normally we have to observe them and destroy them. That is to say, we are sure which ones had been in EPR states only after we have destroyed the states. We call such a case as post-selection entanglement. The post-selection property in the EPR state generation is not a serious drawback in some quantum tasks such as the testing of the violation of Bell inequality. However, in many other tasks, such as the deterministic quantum teleportation and quantum dense coding [4], the resource of event-ready entanglement is a must. (Event-ready EPR means that we are sure certain pair is in EPR state and the state is not destroyed.)

The study of event-ready entanglement can be dated back to 1993 [10]. Recently, some proposals are raised to make the event-ready two photon polarization EPR state or the photon number entangled state. (One may refer to Ref [11] for a complete description on the theoretical condition to produce event-ready EPR pair through the parametric down conversion.) Among all the proposals (see, e.g., Ref. [11–14]) to produce the entangled states in either polarization space or the photon number space, most of them demand both single photon sources and sophisticated photon detectors which can distinguish one-photon Fock state and two-photon Fock state. Both the single photon sources and the sophisticated photon
detectors are difficult techniques and they are thought to be the main barriers to produce
the event-ready polarization EPR pairs with currently existing technology in linear optics
[13]. Although both the single photon source [15–19] and the imperfect sophisticated photon
detector [20] have been demonstrated already however, it is generally believed that both of
them are rather difficult technologies. On the other hand, so far a successful combination
of these two techniques in one experiment has never been reported. Therefore it should be
interesting to seek new schemes which do not depend on either of these two sophisticated
techniques. Very recently, Sliwa and Banaszek [21] proposed a scheme not demanding the
single photon source but still demanding a sophisticated photon detector. So far, all pro-
posals for event-ready polarizatin EPR pairs with passive linear optics devices depend on
the sophisticated photons detectors to distinguish one-photon state and two-photon state,
although normal photon detectors with very good detection efficiency are in principle enough
for producing the event-ready entanglement in photon number space [22,23]. Normally, a
sophisticated detector cannot be replaced by a cascaded system of many normal photon
detectors unless the efficiency of the normal photon detectors are impractically high [24].
In this paper, we propose a totally new scheme. Our new scheme requires the single pho-
ton sources but only uses normal photon detectors which only distinguish the vacuum and
non-vacuum Fock number states. Moreover, our result is insensitive to detection efficiency
of those photon detectors.

Our scheme is schematically shown in figure 1. In this scheme, our task is to observe the
following coincident event:

**Coincidence**: Both detectors D$_3$ and D$_2$ are clicked; or both D$_1$ and D$_4$ are clicked.

If the above coincident event is observed, then we believe that beam 2’ and 4’ are in the
singlet EPR state:

$$|\Psi^-\rangle_{2'4'} = \frac{1}{\sqrt{2}}(|H\rangle_{2'}|V\rangle_{4'} - |V\rangle_{2'}|H\rangle_{4'})$$

(1)

The 4 input beams, beam 1,2,3 and 4 are from single photon sources. The polarization of
both beam 1 and beam 3 deviate a little bit from the vertical polarization while polarization
of both beam 2 and beam 4 deviate a little bit from the horizontal one. We assume that we have tuned the optical paths very carefully so that to each beam splitter (BS) or polarizing beam splitter (PBS), the two input beams reach it simultaneously. A detailed study on the time window related to the quantum coherence had been shown in the seminal work by Zukowski, Zeilinger, Horne and Ekert [10]. A further study can be seen e.g., in [25].

Mathematically, the total input state is

$$|V'\rangle_1|H'\rangle_2|V'\rangle_3|H'\rangle_4,$$

(2)

where the subscripts indicate the different beams, $|H'\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}} (|H\rangle + \epsilon|V\rangle)$ and $|V'\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}} (\epsilon|H\rangle - |V\rangle)$, $|\epsilon| < < 1$ and $|H\rangle$ and $|V\rangle$ are horizontally and vertically polarized states respectively. A PBS has the property to transmit the horizontal polarization and reflect the vertical polarization as shown in figure 2. After the four input beams reached the two PBS (PBS1 and PBS2), beams 1' and 2' are in the following state:

$$|\chi\rangle_{1'2'} = \frac{1}{1+|\epsilon|^2} (\epsilon|H\rangle_{1'} - |V\rangle_{2'})(\epsilon|V\rangle_{1'} + |H\rangle_{2'})$$

(3)

Similarly, beam 3' and 4' are in the following state

$$|\chi\rangle_{3'4'} = \frac{1}{1+|\epsilon|^2} (\epsilon|H\rangle_{3'} - |V\rangle_{4'})(\epsilon|V\rangle_{3'} + |H\rangle_{4'}).$$

(4)

Beam 2' and 4' are now the outcome beams and they are a good entangled pair if 1' and 3' are collapsed to the singlet state $|\Psi^-\rangle_{1'3'}$. This can be shown by mathematically recasting the product state $|\chi\rangle_{1'2'} \otimes |\chi\rangle_{3'4'}$. This product state is

$$\frac{1}{(1+|\epsilon|^2)^2} [X_0 + \epsilon X_1 + \epsilon^2 (A + B + C) + O(\epsilon^3)]$$

(5)

where $X_0 = |V\rangle_{2'}|H\rangle_{2'}|V\rangle_{4'}|H\rangle_{4'}$;

$X_1 = (|V\rangle_{1'}|V\rangle_{2'} - |H\rangle_{1'}|H\rangle_{2'})|V\rangle_{4'}|H\rangle_{4'} + (|V\rangle_{3'}|V\rangle_{4'} - |H\rangle_{3'}|H\rangle_{4'})|V\rangle_{2'}|H\rangle_{2'}$;

$A = |H\rangle_{1'}|V\rangle_{1'}|V\rangle_{4'}|H\rangle_{4'}$, $B = |V\rangle_{2'}|H\rangle_{2'}|H\rangle_{3'}|V\rangle_{3'}$, and

$C = (|H\rangle_{1'}|H\rangle_{2'} - |V\rangle_{1'}|V\rangle_{2'})(|H\rangle_{3'}|H\rangle_{4'} - |V\rangle_{3'}|V\rangle_{4'})$.

(6)

Moreover, C is equivalent to
\[ C = |\Phi^+\rangle_{1'3'}|\Phi^+\rangle_{2'4'} + |\Phi^−\rangle_{1'3'}|\Phi^−\rangle_{2'4'} - |\Psi^+\rangle_{1'3'}|\Psi^+\rangle_{2'4'} - |\Psi^−\rangle_{1'3'}|\Psi^−\rangle_{2'4'} \]  

(7)

and \(|\Phi^\pm\rangle_{ij} = \frac{1}{\sqrt{2}}(|H\rangle_i|H\rangle_j \pm |V\rangle_i|V\rangle_j); \)|\Psi^\pm\rangle_{ij} = \frac{1}{\sqrt{2}}(|H\rangle_i|V\rangle_j \pm |V\rangle_i|H\rangle_j).\] Therefore the rest of our job is to distinguish \(|\Psi^−\rangle_{1'3'}\) from all other possible states of of beam 1',3'. Note that in our case, all other possible states of beam 1', 3' are orthogonal to state \(|\Psi^−\rangle\). To verify the state \(|\Psi^−\rangle_{1'3'}\), we have to make a collective measurement which can be carried out through using a beam splitter [5,26]. As it was shown in Ref [5], if beam 1' and 3' each contains one photon and 1" and 3" are proven to contain one photon in each beam, then 1' and 3' must be in the singlet state. However this is only true in the case that beam 1' and 3' each contains one photon. Before using this conclusion, we have to carefully study all possible states in beam 1' and 3' and the consequences of each of them. The state \(|\chi\rangle_{1'2'} \otimes |\chi\rangle_{3'4'}\) contains a number of components (i.e., \(X_0, X_1, A, B, C\) and higher order terms) with different probability amplitude, i.e., the total state is the linear superposed state of those components. Let’s first study what happens to each of those different components in the state \(|\chi\rangle_{1'2'} \otimes |\chi\rangle_{3'4'}\).

The component with largest probability is

\(|V\rangle_{2'}|H\rangle_{2'}|V\rangle_{4'}|H\rangle_{4'}\).

The prior probability (the probability before we make an observation on the photon detectors) for this component is \(P_1 = \frac{1}{(1+|\epsilon|^2)^2}\). For this component, there is no photon in beam 1’ or 3’, therefore no photon detector will be clicked. Such a component will be definitely ruled out by the conditions in our coincidence.

The component with the probability amplitude order of \(\epsilon\) is

\((|V\rangle_{1'}|V\rangle_{2'} - |H\rangle_{1'}|H\rangle_{2'})|V\rangle_{4'}|H\rangle_{4'} + (|V\rangle_{3'}|V\rangle_{4'} - |H\rangle_{3'}|H\rangle_{4'})|V\rangle_{2'}|H\rangle_{2'}\).

The prior probability for this component is \(P_2 = \frac{4|\epsilon|^2}{(1+|\epsilon|^2)^2}\). This means that there is only one photon altogether in both of beam 1’ and beam 3’. One can easily find this fact by checking each term of the above formula. Each term only allows one photon in beams 1’ and 3’. This component will cause only one detector to be clicked and all the other three are silent. Such
an event is obviously different from our required coincidence therefore the above component can be safely excluded once a coincident event is observed.

Now we consider the components with the probability amplitude order of $\epsilon^2$. These components are

$$A = |H\rangle_{1'}|V\rangle_{1'}|V\rangle_{4'}|H\rangle_{4'}, \quad B = |V\rangle_{2'}|H\rangle_{2'}|H\rangle_{3'}|V\rangle_{3'}$$

and

$$C = (|H\rangle_{1'}|H\rangle_{2'} - |V\rangle_{1'}|V\rangle_{2'})(|H\rangle_{3'}|H\rangle_{4'} - |V\rangle_{3'}|V\rangle_{4'})$$

The total prior probability for these three components is $P_3 = \frac{6\epsilon^4}{(1+|\epsilon|^2)^5}$. We now show that component $A, B$ will never cause the defined coincident event. Before we go into that, we first take a look at the properties of the beam splitter used in our scheme. The property of a balanced beam splitter is sketched in figure 3. A detailed study of the properties of a beam splitter can be seen e.g., in ref. [27,28]. For clarity, we use the Schrodinger picture here. The different modes are simply distinguished by the propagation directions. In our case, states of beam 1’ and 1” are of the same mode (we denote it as mode $a$) at different times, and beam 3’ and 3” are in another mode, mode $b$. Suppose the input beams (1’ and 3’) are in the state $|\text{input}\rangle$, then the output state (in beam 1” and 3”) is $|\text{output}\rangle = U_B|\text{input}\rangle$, where $U_B$ is the time evolution operator for the beam splitter. The unitary operator $U_B$ satisfies

$$U_B(a_H^\dagger, b_H^\dagger, a_V^\dagger, b_V^\dagger)U_B^{-1} = (a_H^\dagger, b_H^\dagger, a_V^\dagger, b_V^\dagger) \begin{pmatrix} H & O \\ O & H \end{pmatrix}$$

where $a^\dagger, b^\dagger$ are creation operators for mode $a$ and mode $b$ respectively, the subscripts $H, V$ indicate the horizontal and vertical polarizations respectively,

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix},$$

and $O$ is a $2 \times 2$ matrix with all elements being 0. Note that the evolution operator $U_B$ also satisfies

$$U_B|00\rangle_{\text{in}} = |00\rangle_{\text{out}}$$
due to the fact of no input no output. Here the subscripts \textit{in} and \textit{out} indicate the input beams and output beams respectively. In our case, the input beams are 1’ and 3’ the output beams are 1” and 3”, therefore \( |00\rangle_{\text{in}} = |00\rangle_{1'3'} \) and \( |00\rangle_{\text{out}} = |00\rangle_{1''3''} \).

With the above two equations, in general the state in the output beams and the state in the input beams are simply related by:

\[
|\text{output}\rangle_{\text{out}} = U_B f(a_H^\dagger, a_V^\dagger, b_H^\dagger, b_V^\dagger) U_B^{-1} \cdot U_B |00\rangle_{\text{in}} = \left( U_B f(a_H^\dagger, a_V^\dagger, b_H^\dagger, b_V^\dagger) U_B^{-1} \right) |00\rangle_{\text{out}} \quad (12)
\]

provided that \( |\text{input}\rangle_{\text{in}} = f(a_H^\dagger, a_V^\dagger, b_H^\dagger, b_V^\dagger) |00\rangle_{\text{in}} \). Note that \( U_B f(a_H^\dagger, a_V^\dagger, b_H^\dagger, b_V^\dagger) U_B^{-1} \) can be easily calculated by using eq.(9). In our treatment, all the creation operators are time independent since we are using the Schrodinger picture. Now we consider the component \( A \). Since \( A \) is a product state of different modes, we only consider evolution to the part in beam 1’ and beam 3’. There is no nontrivial change in beam 2’ and 4’. For the input of component \( A \), the output state of the BS here is

\[
|\text{output}\rangle_{1''3''} = B a_H^\dagger a_V^\dagger B^{-1} |00\rangle_{1'3'}. \quad (13)
\]

This is a direct consequence of eq.(12). Moreover, using eq.(9) one can easily obtain

\[
|\text{output}\rangle_{1''3''} = \frac{1}{2} [ (|HV\rangle_{3''} + |HV\rangle_{1''}) + (|H\rangle_{1''} |V\rangle_{3''} + |V\rangle_{1''} |H\rangle_{3''}) ]. \quad (14)
\]

The exact form of the term \( |HV\rangle_{3''} \) (or \( |HV\rangle_{1''} \)) is \( |0\rangle_{1''} |HV\rangle_{3''} \) (or \( |HV\rangle_{1''} |0\rangle_{3''} \)), it means beam 3” (or 1”) contains one horizontally polarized photon and one vertically polarized photon while beam 1” (or 3”) contains nothing. The term \( |HV\rangle_{3''} \) causes neither \( D_1 \) nor \( D_2 \) being clicked therefore it never causes our defined coincident event is. Similarly, the consequence of \( |HV\rangle_{1''} \) is that neither \( D_3 \) nor \( D_4 \) will be clicked therefore this term is also ruled out. The term \( |H\rangle_{1''} |V\rangle_{3''} + |V\rangle_{1''} |H\rangle_{3''} \) means that beam 1” and 3” each contain one photon. However, after the two half wave plates(HWP) the term is changed to

\[
|H\rangle_\alpha |H\rangle_\beta - |V\rangle_\alpha |V\rangle_\beta. \quad (15)
\]

One may easily check this result by using the time evolution operator of the HWP defined as
\[ U_H \left( \begin{array}{c} |H\rangle \\ |V\rangle \end{array} \right) = H \left( \begin{array}{c} |H\rangle \\ |V\rangle \end{array} \right). \] (16)

Note that \( H \) is defined by eq.(10). Obviously, the two detectors clicked by the state in equation(15) will be either \((D_1, D_3)\) or \((D_2, D_4)\), neither of them is the our defined coincidence. The coincident event will never happen with the state of eq.(15). Therefore component \( A \) is now totally ruled out.

Similarly, for the component \( B \), the output state of the BS is

\[ |\text{output}\rangle_{1'3'} = B b_H^\dagger b_V^\dagger B^{-1}|00\rangle_{1'3'} = \frac{1}{2} \left[ (|HV\rangle_{3''} + |HV\rangle_{1''}) - (|H\rangle_{1''}|V\rangle_{3''} + |V\rangle_{1''}|H\rangle_{3''}) \right]. \] (17)

It’s easy to see that component \( B \) should be also ruled out due to the same arguments used in the case of component \( A \).

Now the only component with the same magnitude order of probability amplitude \( \epsilon^2 \) is the component \( C \):

\[ C = |H\rangle_{1'}|H\rangle_{3'}|H\rangle_{2'}|H\rangle_{4'} + |V\rangle_{1'}|V\rangle_{3'}|V\rangle_{2'}|V\rangle_{4'} - |\Psi^+\rangle_{13'}|\Psi^+\rangle_{24'} - |\Psi^-\rangle_{13'}|\Psi^-\rangle_{24'} \] (18)

As it is well known, to a beam splitter, if each of the input beam contains one photon and the total input polarization state is symmetric, one output beam must be vacant. For the component \( C \), each of beam 1’ and 3’ always contains one photon. The first three terms are all symmetric states. Given these three terms as the input, one output beam of the BS must be vacant, i.e., either beam 1” or beam 3” must be empty. Consequently, given any of the first three terms in component \( C \) as the input, one will observe that either both \((D_1, D_2)\) or both \((D_3, D_4)\) are silent. This definitely violates the conditions of our required coincidence therefore the first three terms of the right hand side in eq.(18) are excluded for a coincident event. However, the last term in eq.(18) exactly satisfies the conditions of our required coincidence. For the input state \( |\Psi^-\rangle_{13'} \), the output state of BS is still a singlet state, i.e. \( |\Psi^-\rangle_{13'} \) [5,26]. This state is invariant under the transformation of two separate HWPs. Therefore finally, after the beams pass through the two separate HWPs, the two
photons are still in the state $|\Psi^-\rangle$ consequently the required coincidence is observed because the polarizing beam splitters PBS3 and PBS4 evolve the state $|\Psi^-\rangle$ into

$$\frac{1}{\sqrt{2}}(|x\rangle|w\rangle - |y\rangle|z\rangle).$$

(19)

Here $|x\rangle, |w\rangle, |y\rangle$ and $|z\rangle$ represent for the state of one photon in beam $x, w, y$ and $z$, respectively. As we have shown, in all terms with the same probability amplitude order, state $|\Psi^-\rangle_{1'3'}|\Psi^-\rangle_{2'4'}$ is the only term that causes our defined coincident event. Therefore once a coincident event is observed, beam 2’ and 4’ must be in the state $|\Psi^-\rangle_{2'4'}$ with a probability close to 1 [29].

In our scheme, the total probability that a coincident event takes place is around $|\epsilon|^4$. In the above study, we have ignored the effects of those components with a probability amplitude order higher than $|\epsilon|^2$. Whenever a coincident event is observed, although the state $\rho_{2'4'}$ for the outcome beams is very close to the singlet state $|\Psi^+\rangle_{2'4'}$, it is not the perfectly pure singlet state because the outcome beams could be a single photon state or a vacuum state with a very small probability (the magnitude order of probability amplitude is $\epsilon^3$). To calculate the fidelity between the produced state $\rho_{2'4'}$ and the perfect singlet state $|\Psi^-\rangle$, we need calculate the post probability (the probability after the observation of the coincident events) of $\rho_{2'4'}$ being the singlet state. In the case that a coincident event takes place, the component $A, B$, the first 3 terms in component $C$ and all of the terms with the probability amplitude order lower than $|\epsilon|^2$ are excluded. The prior probability of all those excluded states is

$$P_{im} = \frac{1}{(1 + |\epsilon|^2)^4(1 + 4|\epsilon|^2 + 5|\epsilon|^4)}.$$  

(20)

To calculate the lower bound of the fidelity, we assume the worst situation that all omitted higher order terms will cause the coincident events. In such a situation, when a coincident event is observed, the fidelity between $\rho_{2'4'}$ and the singlet state is

$$\langle \Psi^-|\rho_{2'4'}|\Psi^-\rangle \geq \frac{|\epsilon|^4/(1 + |\epsilon|^2)^4}{1 - P_{im}} = \frac{|\epsilon|^4}{(1 + |\epsilon|^2)^4 - (1 + 4|\epsilon|^2 + 5|\epsilon|^4)} = 1 - 4|\epsilon|^2.$$  

(21)
This is to say, if we set $\epsilon = \frac{1}{20}$, we can make a singlet state in beam 2' and 4' with a purity larger than 99%, once the coincident event is observed. Note that this is the lower bound of the fidelity, since we have assumed all the terms with probability amplitude order higher than $|\epsilon|^2$ will cause a coincident event wrongly. Actually, some of the higher order terms will not cause the required coincidence by our scheme and this will increase the actual fidelity. A detailed calculation shows that the actual fidelity is larger than 99.7%.

In general, the efficiency of a photon detector is far from perfect. In our scheme, if the efficiency is $\eta$, the total probability that a coincident event takes place is changed to $\eta^2|\epsilon|^4$. When a coincident event happens, the lower bound of the purity of the outcome beams is $1 - 4|\epsilon|^2\eta^{-2}$. This is to say, e.g. given the efficiency $\eta = 0.5$ and $\epsilon = \frac{1}{20}$, the fidelity between $\rho_{2'4'}$ and the singlet state is larger than 94%. Again, this value is only the lower bound of the fidelity. A detailed calculation shows that the actual fidelity will be larger than 99%.

In conclusion, by using the scheme as shown in figure 1, we can prepare a good EPR state in beam 2' and 4' conditionally on the observation of the coincident event that both $D_1$ and $D_4$ are clicked or both $D_2$ and $D_3$ are clicked. As far as we have known, so far this is the only passive linear optics scheme to produce EPR pairs with the normal photon detectors which only distinguish the vacuum and non-vacuum Fock state. It has already been shown [16] that the single photon state can be produced successfully with nearly 100% probability with the pump light repetition rate of $10^8$ per second, by using the quantum dot technique. Moreover, the single photon state can be mass produced by the robust electrically driven source [18]. The synchronization of two beams of single photon is a challenging task in practice. But it is not an unsolvable problem in principle. Actually, the indistinguishability and the interference of two single photon beams have been indeed experimentally observed recently [19] though the experimental efficiency there is low. As it has been calculated earlier, even imperfect normal photon detectors with an efficiency 50% are used, the fidelity between our outcome state and the perfect EPR state is still quite high. This is a bit different from the third order SPDC scheme given by Sliwa and Banaszek [21] where the result is seriously distorted by the low detection efficiency.
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The terms with probability amplitude order of $\epsilon^3$ and $\epsilon^4$ will distort the outcome a little bit.
FIG. 1. A schematic diagram for our scheme to produce the polarization entangled state with linear optics devices. The input beams 1, 2, 3, 4 are from independent single photon sources. Initially, the polarization of beam 1 and the polarization of beam 3 deviate a little bit from the vertical one; the polarization of beam 2 and the polarization of beam 4 deviate a little bit from the horizontal one. If the detector $D_1$ and $D_4$ are both clicked, or if the detector $D_2$ and $D_3$ are both clicked, a state $\rho$ which is very close to the singlet state $|\Psi^\rightarrow_{2',4'}\rangle$ has been prepared in beam $2'$ and $4'$. For all the beams as the input of a polarizing beam splitter (PBS) or a beam splitter (BS), the optical paths should be arranged carefully to make sure the two input beams reach a PBS or a BS in the same time.

![PBS Diagram](image)

FIG. 2. A schematic diagram for the property of a polarizing beam splitter (PBS). It transmits a horizontally polarized photon $H$ and reflects a vertically polarized photon $V$. 
FIG. 3. A schematic diagram for the beamsplitter operation. Both the input and the output are two mode states. The different mode is distinguished by the propagating direction of the field. If the input state is \( f(a_H^+, a_V^+, b_H^+, b_V^+)|00\rangle \), the output state is

\[
U_B f(a_H^+, a_V^+, b_H^+, b_V^+)|00\rangle = U_B f(a_H^+, a_V^+, b_H^+, b_V^+) U_B^+|00\rangle
\]

where \( U_B \) is the time evolution operator of the beam splitter.