INTERRUPTED DISPERAL POPULATION MODEL WITH
ALMOST PERIOD PARAMETERS AND DISPERAL DELAYS

LONG ZHANG*, GAO XU AND ZHIDONG TENG
College of Mathematics and System Sciences, Xinjiang University
Urumqi 830046, China
(Communicated by Shigui Ruan)

ABSTRACT. We establish a class of interrupted bidirectional dispersal population models with almost periodic parameters and dispersal delays between two patches. The form of dispersal discussed in this paper is different from both continuous and impulsive dispersals, in which the dispersal behavior occurs either in a sustained manner or instantaneously; instead, it is a synthesis of these types. Dynamical properties such as permanence, existence, uniqueness, and globally asymptotic stability of almost periodic solutions are investigated by using Liapunov-Razumikhin type technique, using the comparison theorem, constructing a suitable Lyapunov functional, using almost periodic functional hull theory and analysis approach, etc. Finally, numerical simulations are presented and discussed to illustrate our analytic results, by which we find that interrupted dispersal systems are more complicated than continuous or impulsive dispersal systems.

1. Introduction. Migration, which we also refer to a bidirectional dispersal, is a prevalent phenomenon in ecosystems, and the evolutionary trajectories of migratory populations are of great interest. During the last couple of decades, single-species dynamics in a patchy environment has attracted great attention of ecologists and biomathematicians. Many significant papers and monographs about properties of population dynamics in spatial heterogeneous environment, such as permanence, extinction, and stability of positive periodic solutions, etc., have been published (see Allen [1], Beterra et al. [7, 8], Cui et al. [9, 10], Freedman et al. [13, 14], Hamilton and May [19], Kuang [23], Levin [25], Meng et al. [30], Muroya [32], Takeuchi et al. [40, 41], Teng and Lu [44, 45], Wang and Chen [46], Yang et al. [49], Zhang and Chen [50]).

Takeuchi et al. [40] considered a non-autonomous single-species model with dispersal time delay in patchy environment as follows:

\[ \dot{x}_i(t) = x_i(t)\left[a_i(t) - b_i(t)x_i(t)\right] + \sum_{j=1}^{n} \left[d_{ij}(t - \tau)x_j(t - \tau) - d_{ji}(t)x_i(t)\right], \quad (i, j = 1, 2, \ldots, n) \]  

2010 Mathematics Subject Classification. Primary: 92D25, 34D20; Secondary: 34D10.

Key words and phrases. Almost period, interrupted dispersal, global stability, delays, permanence.

This work was supported by the National Natural Science Foundation of P.R. China (11361059, 11271312,11401569), the Natural Science Foundation of Xinjiang Province of China (2014721014), and the Scientific Research Programmes of Colleges in Xinjiang (XJEDU2013I03).

* Corresponding author: Long Zhang.
where \(\tau\) is a positive constant, which represents the time for the species to disperse between two patches. The authors established criteria on permanence and extinction for the above system (1), in which bidirectional dispersal behavior of the modelled populations is occurring at every point in time and is occurring simultaneously between any two patches; i.e., it is continuous bidirectional dispersal.

In fact, dispersal behavior is often affected by environmental changes and human activities, it is usually discontinuous, and it is often the case that population dispersal among patches occurs during some transitory time period. When modelled, these short-time perturbations are often assumed to be in the form of impulses. Adequate mathematical models of such processes are provided by impulsive differential equations. The behaviors of impulsive dynamical systems have been extensively investigated (see Ahmad et al. [2, 3], Bainov et al. [4, 5], Ballinger and Liu [6], Fu et al. [15], Gopalsamy and Zhang [17] Hu [22], Lakshmikantham et al. [24], Liu and Ballinger [26], Liu and Chen [27], Liu et al. [28], Luo and Shen [29], Samoilenko and Perestyuk [34], Shen [35], Stanov et al. [36, 37, 38], Zhang et al. [51, 52, 53, 54]).

Zhang et al. [51] considered the following autonomous single species model with logistic growth and dissymmetric impulsive bi-directional dispersal:

\[
\begin{align*}
\dot{x}_1(t) &= r_1 x_1(t) \left(1 - \frac{x_1(t)}{k_1}\right), \\
\dot{x}_2(t) &= r_2 x_2(t) \left(1 - \frac{x_2(t)}{k_2}\right), \\
\Delta x_1(t) &= b_2 x_2(t^-) - a_1 x_1(t^-), \\
\Delta x_2(t) &= b_1 x_1(t^-) - a_2 x_2(t^-),
\end{align*}
\]

\[t \neq \tau_k, \quad t = \tau_k, \quad k = 1, 2, \ldots,\]

where \([\tau_k, \tau_{k+1})\) denotes the time period during which the population occupies one of the two patches, with diffusion between patches occurring only at every impulsive time \(\tau_{k+1}(k = 0, 1, 2, \ldots)\), and \(\tau_k < \tau_{k+1}\) with \(\lim_{k \to \infty} \tau_k = +\infty\). \(\Delta x_i(t) = x_i(t^+) - x_i(t^-)\), \(x_i(\tau_k^+) = \lim_{t \to \tau_k^+} x_i(t)\) represents the population density of species \(x\) after the \(k\)-th impulsive dispersal in the \(i\)-th patch, \(x_i(\tau_k^-) = \lim_{t \to \tau_k^-} x_i(t) = x_i(\tau_k)\) \((i = 1, 2)\) the population density of species \(x\) before the \(k\)-th impulsive dispersal in the \(i\)-th patch, \(b_i\) the immigration rate of species \(x\) from \(j\)-th patch to \(i\)-th patch, \(a_i\) the emigration rate of species \(x\) from \(i\)-th patch to \(j\)-th patch \((i, j = 1, 2, j \neq i)\). Criteria on the permanence, extinction, existence, uniqueness and global attractivity of positive periodic solutions for system (2) were obtained.

In real ecosystems, since the intrinsic physiological and environmental constraints on populations, such as seasonal effects, food supplies, breeding, spawning, etc., are naturally subject to fluctuations in time, it is usually assumed that the parameters of a system are periodic or almost periodic (see Ahmad and Stanov [3], Bainov et al. [4, 5], Feng [12], Freedman and Peng [14], Gopalsamy [16], He [20], Liu and Chen [27], Liu et al. [28], Meng et al. [30], Muchnik et al. [31], Stanov et al. [36, 37, 38], Takeuchi et al. [41], Tang and Kuang [42], Teng [43], Wang and Chen [46], Xiong and Wang [48], Yang et al. [49], Zhang and Chen [50], Zhang et al. [51, 52]). Moreover, there are many nondeterministic factors that can greatly impact and change the evolving trajectories of populations, such as the matching of periods of individual growth and seasonal alternation, the delay of emigration from the present patch or arrival to the new region, and natural disasters and man-made interventions. Therefore, there are hardly any purely periodic trajectories.
As a result, it is more realistic, important and general to consider almost periodic parameters than periodic parameters for a population system.

Meng et al. [30] investigated the following general non-autonomous delayed almost periodic single-species dispersal system in multiple patches:

\[
\dot{x}_i(t) = x_i(t)[a_i(t) - b_i(t)x_i(t) - b_{ii}(t)x_i(t - \tau_i(t)) - \int_{-\infty}^{0} k_i(t,s)x_i(t+s)ds] + \sum_{j=1}^{n} D_{ij}(t)(x_j(t) - x_i(t)), \quad i, j = 1, 2, ..., n.
\]

Conditions on the permanence, global asymptotic stability, existence and uniqueness of almost periodic solutions were determined.

Here we will restrict our meaning of ‘migration’ as the movement of individuals of a population back and forth between two spatial units (although this can be extended to multiple spatial units). As we know, the actual movement behavior of a migratory species between patches can have many facets, usually neither purely continuous nor purely impulsive in time. As a result, the whole dynamics of a meta-population can neither be fully studied in any single investigation, nor analyzed with a single type of meta-population model. Some of the environmental conditions in the landscape matrix between habitat patches may permit normal movement patterns between patches to occur only during certain intervals of time within seasons or within life-cycles, instead of allowing movement at all times, or at consistent times of the year. Environmental conditions (e.g., weather, occasional landscape impediments) may not allow animals to complete a migration over a short time period. For example, salmon can make amazing upstream journeys over hundreds of kilometers to reproduce, in which strong currents and rapids are hindrances that can prolong the journey (see Nislow et al. [33], Winemiller and Jepsen [47]). As another example, during their spring migration, because of certain types of food becoming available as snow melts, caribou may stop to take food during migration. In these types of cases, movement back and forth between two patches only occurs during certain time intervals that may vary, without covering whole seasons or life cycles. In fact, migration is a far from uniform process. As noted by Dingle [11]; within a population “Individuals differ not only in whether or not they migrate; when they do migrate, they vary in the distances travelled or the routes taken.” This is due to behavioral and physiological variation among individuals within a population, which affects the factors that trigger their choice of whether and when to migrate; such as seasonality, hunger, mating opportunities, etc. In other words, the movement can be intermittent, neither continuous at all times, nor impulsive at fixed time. Therefore, it is not reasonable to characterize the bidirectional movements of migration either with only impulsive diffusion models or only with continuous diffusion models. Instead, some sort of integration of the two types is required.

Motivated by the above considerations, in this paper we propose the following non-autonomous almost periodic single species model with intermittent dispersal...
and dispersal delays between two patches:

$$
\begin{align*}
\dot{x}_1(t) &= x_1(t)[a_1(t) - b_1(t)x_1(t)], \\
\dot{x}_2(t) &= x_2(t)[a_2(t) - b_2(t)x_2(t)], \\
\dot{x}_1(t) &= x_1(t)[\tilde{a}_1(t) - \tilde{b}_1(t)x_1(t)] \\
&\quad + D_{12}(t)(x_2(t) - x_1(t)), \\
\dot{x}_2(t) &= x_2(t)[\tilde{a}_2(t) - \tilde{b}_2(t)x_2(t)] \\
&\quad + D_{21}(t)(x_1(t) - x_2(t)), \\
x_1(\tau_{k+1}) &= D_{11}x_1(\tau_{k+2}), \\
x_2(\tau_{k+1}) &= D_{22}x_2(\tau_{k+2}).
\end{align*}
$$

(4)

This system is composed of two patches. When \( t \in [\tau_{2k}, \tau_{2k+1}) \), the species \( x \) inhabits patch \( i \) (\( i = 1, 2 \)) and does not disperse. When \( t \to \tau_{2k+1} \), the intrinsic relationship of species \( x \) in each of the two patches will change, as the channels between the two patches are open, which permits species \( x \) disperse bi-directionally from one patch to another. The parameter \( d_i \) represents survival rate during the switching from stage 1 (without dispersal movement) to stage 2 (dispersal movement). Then the dispersal movement between the two patches will continue during the time interval \( t \in [\tau_{2k+1}, \tau_{2k+2}) \), when \( t \to \tau_{2k+2} \), the gate of dispersing for species \( x \) between the two patches will close, and the species \( x \) will stop dispersing and occupy patch \( i \), where \( D_i \) is the survival rate of switching from stage 2 to stage 1, \( a_i(t) \) presents the intrinsic growth rate of population in the \( i \)th patch over the time interval \([\tau_{2k}, \tau_{2k+1})\), \( \tilde{a}_i(t) \) is the intrinsic growth rate of population in the \( i \)th patch over the time interval \([\tau_{2k+1}, \tau_{2k+2})\), \( b_i(t) \) is the density-dependent coefficient of population in the \( i \)th patch over the time interval \([\tau_{2k}, \tau_{2k+1})\), \( \tilde{b}_i(t) \) is the density-dependent coefficient of population in the \( i \)th patch over the time interval \([\tau_{2k+1}, \tau_{2k+2})\), \( \tau_i \) are positive constants, represents the time for the population to disperse from patch \( j \) to \( i \) (\( i \neq j, i, j = 1, 2 \)).

Our main purpose in this paper is to analyze the dynamical properties of intermittent dispersal, such as permanence, existence, uniqueness and global stability of almost periodic solutions. Furthermore, we try to explore and determine the differences in dynamics among continuous dispersal, impulse dispersal and intermittent dispersal.

This paper is organized as follows: In Section 2, some preliminaries, assumptions and useful lemmas are presented. In Section 3.1, we obtain the sufficient conditions on existence, uniqueness of positive almost periodic solutions and global asymptotic stability of the auxiliary system for system (4). In Section 3.2, criteria on permanence, global asymptotic stability, existence and uniqueness of positive almost periodic solutions of system (4) are established. Finally, a brief discussion and numerical simulations are illustrated in section 4.

2. Preliminaries. Let \( R \) denote the set of real numbers, \( R^2 \) the 2-dimensional Euclidean linear space equipped with the norm \( \| x \| = \max_{i=1,2} | x_i | \), \( R_+ = \)


(0, +∞), \( R_- = (−∞, 0] \), \( R^2_+ = \{ x ∈ R^2 : x_i ≥ 0, i = 1, 2 \} \) and \( \text{int} R^2_+ = \{ x ∈ R^2 : x_i > 0, i = 1, 2 \} \). Let \( \{ τ_k \} \) be a time sequence, satisfying \( τ_k < τ_{k+1} \), and \( τ_k → ∞ \) as \( k → ∞ \). Define \( τ = \max(τ_1, τ_2) \), \( PC([t − τ, t], R^2) = \{ ϕ : [t − τ, t] → R^2 \} \) \( ϕ = (φ_1, φ_2) \) is continuous everywhere except at \( t = τ_k ∈ [t − τ, t] \), \( φ(τ^+_k) \) and \( φ(τ^-_k) \) exist with \( φ(τ^+_k) = φ(τ^-_k) \). Define \( BPC = \{ ϕ ∈ PC([R_-, R^2]) : ϕ \text{ is bounded} \} \) the norm of \( ϕ \) is defined by \( \| ϕ \| = \sup_{θ ∈ [−τ, 0]} | φ(θ) | \). Let \( BPC_+ = \{ ϕ = (φ_1, φ_2) ∈ BPC : φ_1(θ) ≥ 0 \text{ for all } θ ∈ R_- \text{ and } φ_i(0^+) > 0 \text{ for } i = 1, 2 \)\\In this paper, we assume that all solutions of system \( (4) \) satisfy the following initial conditions
\[
x_i(s, φ) = φ_i(s), \ x_i(0^+, φ) = φ_i(0^+), s ∈ [−τ, 0].
\]\\Where \( φ = (φ_1, φ_2) ∈ BPC_+ \). By the fundamental theory of impulsive functional differential equation \(([4, 15, 24])\), system \( (4) \) has a unique solution \( x(t, φ) = (x_1(t, φ), x_2(t, φ)) \) satisfying the initial conditions \( (5) \). Obviously, the solution \( x(t, φ) \) is positive in its maximal interval of existence.

**Definition 2.1.** \(([34])\) The set of sequences \( \{ τ^+_k | τ^-_k = τ_k+i − τ_k, k, i ∈ Z, τ_k ∈ B \} \) is said to be uniformly almost periodic if for arbitrary \( ε > 0 \), there exists a relatively dense set in \( R \) of ε− almost periods common for all of the sequences, here \( B = \{ τ_k | τ_k ∈ R, τ_k < τ_{k+1}, k ∈ Z, \lim_{k → ±∞} τ_k = ±∞ \} \).

**Definition 2.2.** \(([34])\) The function \( φ ∈ PC([t − τ, t], R^2) \) is said to be almost periodic if the following conditions hold

(a) The set of sequences \( \{ τ^+_k \} \) is uniformly almost periodic, \( k, i ∈ Z \).

(b) For any \( ε > 0 \) there exists a real number \( δ > 0 \) such that if the points \( t’ \) and \( t'' \) belong to the same interval of continuity of \( φ(t) \) and satisfying \( | t’ − t'' | < δ \), then \( | φ(t’) − φ(t'') | < ε \).

(c) For any \( ε > 0 \) there exists a relatively dense set \( T \) such that if \( σ ∈ T \), then \( | φ(t + σ) − φ(t) | < ε \) for all \( t ∈ R \) satisfying \( | t − τ_k | > ε, k ∈ Z \). The elements of \( T \) are called \( ε \)– almost periods.

**Definition 2.3.** \(([20])\) The hull function \( H(f(t)) \) of \( f(t) \) is the set of real function \( g(t) \) such that for any \( g(t) ∈ H(f(t)) \), there exists a sequence \( \{ t_n \} \) satisfying \( \lim_{n → +∞} f(t + t_n) = g(t) \) uniformly on \( R \).

**Definition 2.4.** \(([23])\) System \( (4) \) is said to be permanent if there exists a compact region \( Ω = \{ (x_1(t), x_2(t)) | m ≤ x_i(t) ≤ M, i = 1, 2 \} ⊂ \text{int} R^2_+ \), such that every solution \( x(t) = (x_1(t), x_2(t)) \) of system \( (4) \) with initial conditions \( (5) \) eventually enters and remains in the region \( Ω \).

**Definition 2.5.** \(([30, 43])\) Suppose \( x(t) = (x_1(t), x_2(t)) \) is any solution of system \( (4) \), \( x(t) \) is said to be a strictly positive solution if for \( t ∈ R \) and \( i = 1, 2 \) such that
\[
0 < \inf_{t ∈ R} x_i(t) ≤ \sup_{t ∈ R} x_i(t) < ∞.
\]

**Definition 2.6.** \(([23])\) System \( (4) \) is said to be globally asymptotically stable if for any two positive solutions \( x(t) \) and \( y(t) \) of system \( (4) \) with initial conditions \( (5) \) such that
\[
\lim_{k → ∞} | x_i(t) − y_i(t) | = 0, i = 1, 2.
\]
Throughout this paper, we shall use the following notations.
Notation. If $f(t), t \in R$, is an almost periodic function, we define
\[
    f^u = \sup_{t \in R} f(t), \quad f^l = \inf_{t \in R} f(t).
\]
\[
    m(f) = \lim_{t \to +\infty} \frac{1}{T} \int_t^{t+T} f(t) d(t),
\]
where $T$ is a positive constant. We let $\sigma_i(t) = t - \tau_i$, $\sigma_i^{-1}(t)$ is the inverse function of function $\sigma_i(t)$.

We set
\[
    G_k = (\tau_k, \tau_{k+1}) \times R^n, k \in Z; \quad G = \bigcup_{k=-\infty}^{\infty} G_k.
\]

Let $V_0 = \{V \in C[G, R^+]\}$, for any $V \in V_0$, we define the right-hand derivative $D^+V(t, x(t))$ along the solution $x(t_0, x_0)$ by
\[
    D^+V(t, x(t)) = \lim_{\delta \to 0^+} \sup_{t \in \delta} \{V(t + \delta, x(t + \delta)) - V(t, x(t))\}.
\]

In this paper, for system (4) we always suppose that the following assumptions hold for each $i, j = 1, 2$, and $i \neq j$.

(H1) The bounded almost periodic functions $a_i(t), \tilde{a}_i(t), b_i(t), \tilde{b}_i(t)$, and $D_{ij}(t)$ are continuous for all $i \in R$, and $\tilde{b}_i^i \geq 0, \tilde{b}_i^j \geq 0, 0 < d_i, D_i \leq 1$.

(H2) The set of sequences $\{\tau_i\}, k, i \in Z$, is uniformly almost periodic, and
\[
    \inf_{k \in Z} |\tau_{k+1} - \tau_k| > 0.
\]

(H3) $m(a_i(t)) > 0, m(\tilde{a}_i(t)) > 0, m(b_i(t)) > 0, m(\tilde{b}_i(t)) > 0$.

(H4) There exists a constant $\omega > 0$, such that for any $t \geq 0$,
\[
    \int_t^{t+\omega} \left[\frac{(-1)^k + 1}{2} a_i(s) + \frac{(-1)^{k+1} + 1}{2} \tilde{a}_i(s)\right] ds
    + \sum_{t < \tau_{k+1} \leq t + \omega} \ln\left(\frac{(-1)^k + 1}{2} d_i + \frac{(-1)^{k+1} + 1}{2} D_i\right) > 0.
\]

(H5) There are constants $\eta_i > 0 (i = 1, 2)$, such that for all $t \geq 0$,
\[
    \sum_{t < \tau_{k+1} \leq t + \omega} \ln\left(\frac{(-1)^k + 1}{2} d_i + \frac{(-1)^{k+1} + 1}{2} D_i\right) \leq \eta_i, s \in [0, \omega].
\]

(H6) $m(\frac{(-1)^k + 1}{2} a_i(t) + \frac{(-1)^{k+1} + 1}{2} \tilde{a}_i(t) - \frac{(-1)^{k+1} + 1}{2} D_{ij}(t)) > 0$.

Now, we introduce several lemmas which will be useful in the proofs of the main results. We consider the following vector impulsive differential equation
\[
    \begin{aligned}
        \dot{x}(t) &= f(t, x(t)), & t \neq \tau_k + 1, \\
        x(\tau_k^+) &= I(x(\tau_k + 1)), & t = \tau_k + 1,
    \end{aligned}
\]
where $f(t, x) \in C(R_+ \times R^n, R^2)$ is quasi-non-decreasing in $x$, $I(x) \in C(R^2, R^2)$, is a non-decreasing function, we have the following comparison theorem for system (6).

Lemma 2.7. (24) Let $x(t)$ be a solution of system (6) existing on $[t_0, \infty)$ and satisfying
\[
    \begin{aligned}
        \dot{u}(t) &\leq (\geq) f(t, u(t)), & t \neq \tau_k + 1, \\
        u(\tau_k^+) &\leq (\geq) I(u(\tau_k + 1)), & t = \tau_k + 1,
    \end{aligned}
\]
then $u(t_0) \leq (\geq) x(t_0)$ implies $u(t) \leq (\geq) x(t)$ for all $t \geq t_0$. 

The following lemma will be used in the proof of the global asymptotic stability for the almost periodic system (4).

**Lemma 2.8.** ([20]) Let function \( f(t) \) be continuous and the right upper Dini derivative exist on \([0, +\infty)\). Assume that there exists a sequence \( \{\tau_k\} \) with \( \tau_k \to \infty \) as \( k \to \infty \) and a positive constant \( M \), such that \( \frac{df(t)}{dt} \) exists for all \( t \in \mathbb{R}^+ \) and \( t \neq \tau_k \), \( k \in \mathbb{N} \) and \( |D^+ f(t)| < M \), then \( f(t) \) is uniformly continuous on \([0, +\infty)\).

Let \( a_i^*(t) \in H(a_i(t)), \ b_i^*(t) \in H(b_i(t)), \ c_i^*(t) \in H(c_i(t)), \ d_i^*(t) \in H(d_i(t)), \ D^*_{ij}(t) \in H(D^*_{ij}(t)), \) \( \tau_k \in H(\tau_k) \), we have the following hull equations of system (4),

\[
\begin{align*}
\dot{x}_1(t) &= x_1(t)[a_1^*(t) - b_1^*(t)x_1(t)], \\
\dot{x}_2(t) &= x_2(t)[a_2^*(t) - b_2^*(t)x_2(t)], \\
\dot{x}_1(t) &= x_1(t)[a_1^*(t) - b_1^*(t)x_1(t)], \\
\dot{x}_2(t) &= x_2(t)[a_2^*(t) - b_2^*(t)x_2(t)], \\
\dot{x}_1(t) &= x_1(t)[\tilde{a}_1^*(t) - \tilde{b}_1^*(t)x_1(t)] + \tilde{D}_{12}(t)(x_2(t - \tau_1) - x_1(t)), \\
\dot{x}_2(t) &= x_2(t)[\tilde{a}_2^*(t) - \tilde{b}_2^*(t)x_2(t)] + \tilde{D}_{21}(t)(x_1(t - \tau_2) - x_2(t)), \\
\dot{x}_1(t) &= D_1 x_1(\tau_{2k+1} - \tau_{2k}), \\
\dot{x}_2(t) &= D_2 x_2(\tau_{2k+2} - \tau_{2k}), \\
\end{align*}
\]

About system (7), we have the following result (refer to [30, 43]):

**Lemma 2.9.** If system (4) satisfies \((H_1)-(H_6)\), then the hull equations (7) also satisfy \((H_1)-(H_6)\).

For the convenience to present our results, we rewrite system (4) as follows:

\[
\begin{align*}
\dot{x} &= f(t, x), \\
x(t \tau_{k+1}) &= h_k x(t \tau_{k+1}), \quad t = \tau_{k+1}, \\
\end{align*}
\]

where \( x = (x_1, x_2), \ f(t, x) = (f_1(t, x), f_2(t, x)), \ h_k = (h_{1k}, h_{2k}), \) and

\[
\begin{align*}
f_1(t, x) &= \frac{(-1)^k + 1}{2} x_1(t)[a_1(t) - b_1(t)x_1(t)] + \frac{(-1)^{k+1} + 1}{2} x_1(t)\tilde{a}_1(t) - \tilde{b}_1(t)x_1(t) + \frac{(-1)^{k+1} + 1}{2} \tilde{D}_{12}(t)(x_2(t - \tau_1) - x_1(t)), \\
f_2(t, x) &= \frac{(-1)^k + 1}{2} x_2(t)[a_2(t) - b_2(t)x_2(t)] + \frac{(-1)^{k+1} + 1}{2} x_2(t)\tilde{a}_2(t) - \tilde{b}_2(t)x_2(t) + \frac{(-1)^{k+1} + 1}{2} \tilde{D}_{21}(t)(x_1(t - \tau_2) - x_2(t)), \\
h_{1k} &= \frac{(-1)^k + 1}{2} d_1 + \frac{(-1)^{k+1} + 1}{2} D_1, \\
h_{2k} &= \frac{(-1)^k + 1}{2} d_2 + \frac{(-1)^{k+1} + 1}{2} D_2.
\end{align*}
\]

**Remark 1.** The intrinsic rules of species \( x \) in system (4) are divided into 2 parts, two regarding the inhabiting process in patch 1 and patch 2, i.e., equations 1 and 2, and two regarding the migration process between patch 1 and patch 2, i.e., equations
Lemma 2.10. Assume that system (8) satisfies \((H_1)-(H_6)\), if each of hull equations (7) has a unique strictly positive solution, then system (8) has a unique strictly positive almost periodic solution.

Proof. From \((H_1), (H_2)\), it follows that \(f(t, x)\) is an almost periodic function with respect to \(t \in R\), obviously, \(\{h_k\}\) is an almost periodic sequence with respect to \(k \in Z\). Let \(\phi(t)\) be a strictly positive solution of (7) for \(t \in R\). There exist sequences of real values \(\alpha'\) and \(\beta'\), which have common subsequences \(\alpha \subset \alpha'\) and \(\beta \subset \beta'\) such that \(T_{\alpha+\beta}f(t, x) = T_\alpha T_\beta f(t, x)\), for \(t \in R\) and \(x \in R^2\). Then \(T_{\alpha+\beta}\phi(t)\), \(T_\alpha T_\beta\phi(t)\) are solutions of the following common hull equations of system (7)

\[
\begin{aligned}
\dot{x} &= f^{\alpha+\beta}(t, x), \quad t \in [\tau_k^{\alpha+\beta}, \tau_{k+1}^{\alpha+\beta}], \\
x(\tau_{k+1}^{\alpha+\beta}) &= h_k^{\alpha+\beta}x(\tau_{k+1}^{\alpha+\beta-}), \quad t = \tau_{k+1}^{\alpha+\beta}.
\end{aligned}
\]

Therefore, we have \(T_{\alpha+\beta}\phi(t) = T_\alpha T_\beta\phi(t)\), thus according to Lemma 2, \([37]\), \(\phi(t)\) is an almost periodic solution of system (8). The proof is complete. \(\square\)

Lemma 2.11. (Liapunov-Razumikhin Type Theorem [29, 35, 39]) If there exist \(W_1, W_2, W_3 \in K_0 = \{g \in C(R_+, R_+)|g(0) = 0, g(s) > s \text{ for } s > 0, g \text{ is strictly increasing in } s \text{ and } g(s) \to \infty \text{ as } s \to \infty\}\) and \(V(t, x)\), such that

(i) \(W_1(\|x\|) \leq V(t, x) \leq W_2(\|x\|)\); 
(ii) \(V(\tau_{k+1}^+, +\Delta x(\tau_{k+1})) \leq (1 + b_k)V(\tau_{k+1}^-, x), \) where \(b_k \geq 0 \) with \(\sum_{k=1}^{\infty} b_k < \infty\), and \(k = 0, 1, 2, ...\);

(iii) There exists some constant \(H > 0\) such that for any solution \(x(t)\) of (4)

\[D^+V(t, x(t)) \leq -W_3(\|x(t)\|)\]

whenever \(\|x(t)\| \geq H\) and \(P(V(t, x(t))) > V(s, x(s))\) for \(s \in [t - \tau, t]\), where \(P \in C(R_+, R_+)\) and \(P(s) > M_0 s\) for \(s > 0\), in which \(M_0 = \prod_{k=1}^{\infty}(1 + b_k)\), then system (4) is uniformly ultimately bounded.

3. Results.

3.1. Auxiliary theorems. In this subsection, we first consider the following auxiliary system (9), and establish a series of criteria on the existence, uniqueness and globally asymptotical stability of positive almost periodic solutions for system (9), which will be used in the next subsection.

We consider the following auxiliary system

\[
\begin{aligned}
\dot{x}_i(t) &= x_i(t)[a_{ik}(t) - b_{ik}(t)x_i(t)], \quad t \in [\tau_k, \tau_{k+1}), \\
x_i(\tau_{k+1}) &= h_{ik}x_i(\tau_{k+1}), \quad t = \tau_{k+1},
\end{aligned}
\]

where

\[
\begin{aligned}
a_{ik}(t) &= \frac{(-1)^k + 1}{2} a_i(t) + \frac{(-1)^{k+1} + 1}{2} \hat{a}_i(t); \\
b_{ik}(t) &= \frac{(-1)^k + 1}{2} b_i(t) + \frac{(-1)^{k+1} + 1}{2} \hat{b}_i(t); \\
h_{ik} &= \frac{(-1)^k + 1}{2} d_i + \frac{(-1)^{k+1} + 1}{2} \hat{D}_i, \quad i = 1, 2.
\end{aligned}
\]

About system (9), we have the following result.
Theorem 3.1. Assume that system (9) satisfies \((H_1)-(H_5)\), then system (9) has a unique positive almost periodic solution.

Proof. For any \(a_{ik}(t) \in H(a_{ik}(t))\), \(b_{ik}(t) \in H(b_{ik}(t))\) and \(\tau_k^* \in H(\tau_k)\), we have the following hull equations of Eq.(9):

\[
\begin{align*}
\dot{x}_i(t) &= x_i(t)[a_{ik}(t) - b_{ik}(t)x_i(t)], \\
\quad x_i(\tau_k^*) &= h_{ik}x_i(\tau_k^*),
\end{align*}
\]

for all \(t \in [\tau_k^*, \tau_k^* + 1), t = \tau_k^* + 1\). (10)

By conditions \((H_3)-(H_5)\) and Lemma 2.9, we obtain that there are positive constants \(\delta\) and \(\omega\) such that

\[
\int_t^{t+\omega} a_{ik}(s) ds > \delta, \int_t^{t+\omega} b_{ik}(s) ds > \delta,
\]

for all \(t \in R\), and \(i = 1, 2\). Hence, there are positive constants, \(k_1, k_2, \varepsilon_0\) and \(k_1 > k_2\) such that

\[
\int_t^{t+\omega} (a_{ik}(s) - b_{ik}(s)k_1) ds + \sum_{t < \tau_k^* + 1 \leq t + \omega} \ln h_{ik} < -\varepsilon_0, \tag{11}
\]

\[
\int_t^{t+\omega} (a_{ik}(s) - b_{ik}(s)k_2) ds + \sum_{t < \tau_k^* + 1 \leq t + \omega} \ln h_{ik} > \varepsilon_0, \tag{12}
\]

for all \(t \geq 0\), and \(i = 1, 2\). Let \(A_j = \sup\{a_{ik}^*(t) + b_{ik}(t)k_j : t \in R\} \) for \(i, j = 1, 2\), then \(A_j > 0\). For any integer \(m > 0\), consider the solution \((x_{im}, x_{2m})\) of Eq.(10) with initial value \(x_{im}(-m) \in (k_2, k_1)\), we first prove that

\[
x_{im}(t) \leq k_1 \exp(A_1 \omega + \eta_i), \tag{13}
\]

for all \(t \geq -m\).

It is easy to show that the inequality (13) does not hold, then by \(x_{im}(-m) < k_1 < k_1 \exp(A_1 \omega + \eta_i)\), \(0 < d_i, D_i \leq 1\), there must exist \(t_1\) and \(t_2\) satisfying \(t_2 > t_1 > 0\), such that \(x_{im}(t_1) > k_1 \exp(A_1 \omega + \eta_i)\), \(x_{im}(t_1^*) \geq k_1, x_{im}(t_1) = k_1\) and \(x_{im}(t) > k_1\) for all \(t \in (t_1, t_2]\). Choosing an integer \(p \geq 0\) such that \(t_2 \in [t_1 + p\omega, t_1 + (p + 1)\omega]\), then integrating Eq.(10) from \(t_1\) to \(t_2\), by (11) and (12), we have

\[
k_1 \exp(A_1 \omega + \eta_i) < x_{im}(t_2) = x_{im}(t_1) \exp\left(\int_{t_1}^{t_2} \left( a_{ik}(s) - b_{ik}(s)x_{im}(s) \right) ds + \sum_{t_1 < \tau_k^* + 1 \leq t_2} \ln h_{ik} \right) \leq k_1 \exp\left(\int_{t_1}^{t_2} \left( a_{ik}(s) - b_{ik}(s)k_1 \right) ds + \sum_{t_1 < \tau_k^* + 1 \leq t_2} \ln h_{ik} \right) \leq k_1 \exp(-p\varepsilon_0 + A_1 \omega + \eta_i) \leq k_1 \exp(-2\varepsilon_0 + A_1 \omega + \eta_i)
\]

which leads to a contradiction, therefore, (13) is true. We now prove

\[
x_{im}(t) \geq k_2 \exp(-A_2 \omega - 2\eta_i), \tag{14}
\]

for all \(t \geq -m\).
If the inequality (14) is not true, then from \( x_{im}(-m) > k_2 > k_2 \exp(-A_2 \omega - 2 \eta_i) \), there are \( t_3 \) and \( t_4 \) satisfying \( t_4 \geq t_3 > -m \), such that \( x_{im}(t_4) < k_2 \exp(-A_2 \omega - 2 \eta_i) \), \( x_{im}(t_3) \geq k_2 \), \( x_{im}(t_3) \leq k_2 \).

For \( t_4 \), there are the following two cases.

**Case 1.** \( t_3 = t_4 \).

**Case 2.** \( t_3 < t_4 \).

For case 1, \( t_3 \) is an impulsive time. Hence, there is an integer \( k > 0 \) such that \( t_3 = \tau_{k+1}^* \). From \((H_3)\), we can obtain that

\[
\begin{align*}
k_2 \exp(-A_2 \omega - 2 \eta_i) &> x_{im}(t_3) \\
&= h_{ik} x_{im}(\tau_{k+1}^*) \\
&\geq k_2 \exp(\ln h_{ik}) \\
&\geq k_2 \exp(-\eta_i) \\
&\geq k_2 \exp(-A_2 \omega - 2 \eta_i)
\end{align*}
\]

which leads to a contradiction.

For case 2, we have \( x_{im}(t_4) < k_2 \exp(-A_2 \omega - 2 \eta_i) \), \( x_{im}(t_3) \leq k_2 \), \( x_{im}(t_3) \geq k_2 \), \( k_2 \exp(-A_2 \omega - 2 \eta_i) \leq x_{im}(t_3) \leq k_2 \), and \( x_{im}(t) < k_2 \) for all \( t \in (t_3, t_4] \). Choosing an integer \( p \geq 0 \) such that \( t_4 \in [t_3 + p \omega, t_3 + (p + 1) \omega) \), then integrating Eq.\((10)\) from \( t_3 \) to \( t_4 \), by \((11)\) and \((12)\), we have

\[
\begin{align*}k_2 \exp(-A_2 \omega - 2 \eta_i) &> x_{im}(t_4) \\
&= x_{im}(t_3) \exp \left( \int_{t_3}^{t_4} (a_{ik}^*(s) - b_{ik}^*(s)) x_{im}(s) \, ds \right) \\
&\quad + \sum_{t_3 < \tau_{k+1}^* \leq t_4} \ln h_{ik} \\
&\geq h_{ik} k_2 \exp \left( \int_{t_3}^{t_4} (a_{ik}^*(s) - b_{ik}^*(s)) \, ds \right) \\
&\quad + \sum_{t_3 < \tau_{k+1}^* \leq t_4} \ln h_{ik} \\
&\geq k_2 \exp \left( \int_{t_3}^{t_4} (a_{ik}^*(s) - b_{ik}^*(s)) \, ds \right) \\
&\quad + \sum_{t_3 \leq \tau_{k+1}^* \leq t_4} \ln h_{ik} \\
&\geq k_2 \exp \left( \int_{t_3}^{t_3 + p \omega} (a_{ik}^*(s) - b_{ik}^*(s)) \, ds \right. \\
&\quad + \left. \sum_{t_3 < \tau_{k+1}^* \leq t_3 + p \omega} (a_{ik}^*(s) - b_{ik}^*(s)) \, ds \right) \ln h_{ik} \\
&\geq k_2 \exp(p \omega - A_2 \omega - 2 \eta_i) \\
&\geq k_2 \exp(-A_2 \omega - 2 \eta_i)
\end{align*}
\]

which is a contradiction. Thus we finally have

\[
k_2 \exp(-A_2 \omega - 2 \eta_i) \leq x_{im}(t) \leq k_1 \exp(A_1 \omega + \eta_i),
\]

for all \( t \geq -m \), and \( i = 1, 2 \).

Consider the sequence \((x_{im}(t), x_{2m}(t))\) of solutions of Eq.\((10)\). The inequality \((13)\) and \((14)\) imply that there exists a positive constant \( M \) such that

\[
| \dot{x}_{im}(t) | \leq M,
\]
for all $t \geq -m$, and $i = 1, 2$. Hence, for any integer $p > 0$, the sequence $\{x_{im}(t) : m \geq p\}$ is uniformly bounded and equi-continuous on interval $[-p, p]$. Applying the Ascoli-Arzela Lemma to sequence $\{x_{im}(t) : m \geq p\}$, we can obtain that the sequence $\{x_{im}(t)\}$ has a subsequence $\{x_{im}^{(1)}(t)\}$ that uniformly converges on $[-1, 1]$, $\{x_{im}^{(2)}(t)\}$ has a subsequence $\{x_{im}^{(2)}(t)\}$ that uniformly converges on $[-2, 2]$, and so on, we have $\{x_{im}^{(m)}(t)\}$ uniformly converges on $[-p, p]$ for any integer $p > 0$. Let $\bar{x}_i^*(t)$ be the limit function of $\{x_{im}^{(m)}(t)\}$, it is easily to see that $\bar{x}_i^*(t)$ ($i=1,2$) are defined on $R$ what satisfy Eq.(10) and

$$k_2 \exp(-A_2\omega - 2\eta_i) \leq \bar{x}_i^*(t) \leq k_1 \exp(A_1\omega + \eta_i),$$

for all $t \in R$, and $i = 1, 2$. Hence, $(\bar{x}_1^*(t), \bar{x}_2^*(t))$ is a strictly positive solution of Eq.(10).

Next, we prove the uniqueness of strictly positive solution of Eq.(10). Suppose $(\bar{y}_1^*(t), \bar{y}_2^*(t))$ is also a strictly positive solution of Eq.(10). Let

$$V^*(t) = \sum_{i=1}^{2} c_i \mid \ln \bar{x}_i^*(t) - \ln \bar{y}_i^*(t) \mid,$$

where $c_i$ are positive constants for $i = 1, 2$.

For $t = \tau_{k+1}^*$, we have

$$V^*(\tau_{k+1}^*) = \sum_{i=1}^{2} c_i \mid \ln \bar{x}_i^*(\tau_{k+1}^*) - \ln \bar{y}_i^*(\tau_{k+1}^*) \mid$$

$$= \sum_{i=1}^{2} c_i \mid \ln(h_{ik}\bar{x}_i^*(\tau_{k+1}^*)) - \ln(h_{ik}\bar{y}_i^*(\tau_{k+1}^*)) \mid$$

$$= \sum_{i=1}^{2} c_i \mid \ln \bar{x}_i^*(\tau_{k+1}^*) - \ln \bar{y}_i^*(\tau_{k+1}^*) \mid$$

then $V^*(t)$ is continuous for all $t \in R$.

For $t \neq \tau_{k+1}$, calculating the Dini upper right derivative of $V^*(t)$, we have

$$D^+ V^*(t) \leq \sum_{i=1}^{2} -c_i b_{ik}^*(t) \mid \bar{x}_i^*(t) - \bar{y}_i^*(t) \mid$$

$$= \sum_{i=1}^{2} -c_i b_{ik}^*(t) \xi \mid \ln \bar{x}_i^*(t) - \ln \bar{y}_i^*(t) \mid$$

$$\leq -b \xi V^*(t)$$

where

$$\xi = \inf_{t \in R} \{\bar{x}_i^*(t), \bar{y}_i^*(t), i = 1, 2\}, b = \min\{b_{1k}^*, b_{2k}^*\}.$$  

From this, for any $t \leq t_0$, where $t_0$ is any initial time, we further obtain that

$$V^*(t) \geq V^*(t_0) \exp(\int_{t}^{t_0} b \xi ds).$$

Since $\int_{-\infty}^{t_0} b \xi dt = \infty$ and $V^*(t)$ is a non-increasing and nonnegative bounded function on $R$, then $V^*(t_0) = 0$. That is, $\bar{x}_i^*(t_0) = \bar{y}_i^*(t_0)$ for $i = 1, 2$, by the uniqueness of solutions of the initial value problem of Eq.(10). Hence we have $\bar{x}_i^*(t) = \bar{y}_i^*(t)$ for all $t \in R$, and $i = 1, 2$.  

Finally, we can obtain that Eq.(10) has a unique strictly positive solution \((\bar{x}_1^*(t), \bar{x}_2^*(t))\), by Lemma 2.10, we have Eq.(9) has a unique positive almost periodic solution \(x^*(t) = (x_1^*(t), x_2^*(t))\). This completes the proof of the Theorem 3.1.

\[ \square \]

**Theorem 3.2.** System (9) is globally asymptotically stable if \((H_1) - (H_5)\) hold.

**Proof.** For two arbitrary positive solutions of system (9) \(x(t) = (x_1(t), x_2(t))\) and \(y(t) = (y_1(t), y_2(t))\), according to Theorem 3.1, we obtain that there are positive constants \(r\) and \(R\) such that

\[ r \leq x_i(t), y_i(t) \leq R \]

for all \(t \geq -m\) and \(i = 1, 2\). Choose a Liapunov function

\[ V(t) = \sum_{i=1}^{2} c_i | \ln x_i(t) - \ln y_i(t) |, \ i = 1, 2. \]

For \(t = \tau_{k+1}\), we have

\[ V(\tau_{k+1}) = \sum_{i=1}^{2} c_i | \ln x_i(\tau_{k+1}) - \ln y_i(\tau_{k+1}) | = V(\tau_{k+1}^-), \]

then \(V(t)\) is continuous for all \(t \in R\).

For \(t \neq \tau_{k+1}\), calculating the Dini upper right derivative of \(V(t)\), we have

\[ D^+ V(t) \leq \sum_{i=1}^{2} -c_i b_{ik}(t) | x_i(t) - y_i(t) | \]

\[ \leq \sum_{i=1}^{2} -c_i b_{ik}^* | x_i(t) - y_i(t) | \]

\[ \leq -\alpha \sum_{i=1}^{2} | x_i(t) - y_i(t) | \]

for all \(t \geq -m\) and \(i = 1, 2\), where constant \(\alpha \geq 0\) satisfying \(c_i b_{ik}^* \geq \alpha\), for all \(t \geq -m\) and \(i = 1, 2\). From this, for any \(t \geq -m\) we further obtain

\[ V(t) + \alpha \int_{-m}^{t} \sum_{i=1}^{2} | x_i(s) - y_i(s) | \ ds \leq V(-m). \]

Therefore, \(V(t)\) and \(\int_{-m}^{t} \sum_{i=1}^{2} | x_i(s) - y_i(s) | \ ds\) are bounded for all \(t \geq -m\). Then

\[ \int_{-m}^{\infty} | x_i(s) - y_i(s) | \ ds < \infty \]

for \(i = 1, 2\). By using (15) and Lemma 2.8, we have that \( | x_i(t) - y_i(t) | \) (\(i=1,2\)) are bounded and uniformly continuous, and by using the Barbalat’ Lemma (see [18] lemma 1.22), we obtain that

\[ \lim_{t \to \infty} | x_i(t) - y_i(t) | = 0, i = 1, 2. \]

This completes the proof of Theorem 3.2. \[ \square \]

3.2. **Main results.** In this subsection, we study the permanence, uniqueness and global asymptotic stability of almost periodic solutions for system (4).

We first discuss the permanence of system (4).
Theorem 3.3. Suppose that system (4) satisfies \((H_1)-(H_6)\), then system (4) is permanent.

Proof. We let
\[
d_{12k}(t) = \frac{(-1)^{k+1} + 1}{2} D_{12}(t), \quad d_{21k}(t) = \frac{(-1)^{k+1} + 1}{2} D_{21}(t),
\]
which on substituting into (4) becomes
\[
\begin{align*}
\dot{x}_i(t) &= x_i(t)[a_{ik}(t) - b_{ik}(t)x_i(t)] + d_{ijk}(t)[x_j(t - \tau_i) - x_i(t)], \quad t \in [\tau_k, \tau_{k+1}), \\
x_i(\tau_{k+1}) &= h_{ik}x_i(\tau_{k+1}^-), \quad t = \tau_{k+1}.
\end{align*}
\]
for \(k \in \mathbb{R}_+, i, j = 1, 2\), and \(i \neq j\). Suppose \(x(t) = (x_1(t), x_2(t))\) is any positive solution for system (16) with initial conditions (5).

We first prove that the system (16) is ultimately upper bounded, i.e., there is a constant \(M\) such that any positive solution of system (16) satisfies
\[
\limsup_{t \to \infty} x_i(t) \leq M, \quad (i = 1, 2).
\]

Define a Liapunov function
\[
V(t, x) = \sum_{i=1}^{2} x_i(t).
\]
By Lemma 2.11, we let \(W_1(s) = s\), \(W_2(s) = 2s\). For \(t \neq \tau_k\), calculating the derivative of \(V(t, x)\) along solution of (16), we have
\[
D^+ V(t, x) \leq \sum_{i=1}^{2} \{x_i(t)[(a_{ik}(t) - d_{ijk}(t)) - b_{ik}(t)x_i(t)] + d_{ijk}(t)[x_j(t - \tau_i)]
\leq \hat{a}(t) \sum_{i=1}^{2} x_i(t) - \hat{b}(t) \frac{\sum_{i=1}^{2} x_i(t)^2 + \hat{a}(t) \sum_{i=1}^{2} x_i(t - \tau_i)}{2} + \hat{d}(t)[V(t - \tau_1, x) + V(t - \tau_2, x)],
\]
where
\[
\hat{a}(t) = \max_{i=1,2} \{a_{ik}(t) - d_{ijk}(t)\}; \quad \hat{b}(t) = \min_{i=1,2} \{b_{ik}(t)\}; \quad \hat{d}(t) = \max_{i=1,2} \{d_{ijk}(t)\}.
\]
Let \(P(s) = M_0qs\), where for \(s > 0\) satisfying \(q > 1\), \(M_0 > 0\). We choose a sufficiently large \(H\), when \(\|x(t)\| \geq H\) and \(P(V(t, x(t))) > V(s, x(s))\) for \(s \in [t - \tau, t]\), we have
\[
D^+ V(t, x) \leq \hat{a}(t) V(t, x) - \frac{\hat{b}(t)}{2} V^2(t, x) + \hat{d}(t)[M_0qV(t, x) + M_0qV(t, x)]
\leq (\hat{a}(t) + 2M_0q\hat{d}(t))V(t, x) - \frac{\hat{b}(t)}{2} V^2(t, x).
\]
Then we obtain a positive constant \(\mu\) such that
\[
D^+ V(t, x) \leq -\mu V^2(t, x) \leq -\mu \|x\|^2 = -W_3(\|x(t)\|).
\]
Finally,
\[
V(\tau_{k+1}, x + \Delta x(\tau_{k+1})) = \sum_{i=1}^{2} (x_i(\tau_{k+1}) + \Delta x(\tau_{k+1}))
\leq \sum_{i=1}^{2} (h_{ik}x_i(\tau_{k+1})^2 + (h_{ik} - 1)x_i(\tau_{k+1}^-))
\]

This completes the proof of Theorem 3.4.

Therefore, by Lemma 2.11, we can see that there must exist positive constants $T$ and $M$ such that $x_i(t) \leq M$ ($i = 1, 2$) for all $t \geq T$. Next, we will prove that there is a constant $m > 0$ such that any positive solution of system (16) satisfies

$$\liminf_{t \to \infty} x_i(t) \geq m, \quad (i = 1, 2).$$

From the almost periodic system (16), and assumption $(H_1)$, we have

$$\begin{cases}
\dot{x}_i(t) \geq x_i(t)(a_{ik}(t) - d_{ijk}(t) - b_{ik}(t)x_i(t)), & t \in [\tau_k, \tau_{k+1}), \\
x_i(\tau_{k+1}) = h_{ik}x_i(\tau_{k+1}), & t = \tau_{k+1}.
\end{cases}$$

Consider the following auxiliary system

$$\begin{cases}
\dot{u}_i(t) = u_i(t)(a_{ik}(t) - d_{ijk}(t) - b_{ik}(t)u_i(t)), & t \in [\tau_k, \tau_{k+1}), \\
u_i(\tau_{k+1}) = h_{ik}u_i(\tau_{k+1}), & t = \tau_{k+1}.
\end{cases}$$

According to hypotheses $(H_1) - (H_6)$, Theorem 3.1, we can see that system (17) has a unique positive almost periodic solution $u(t) = (u_1(t), u_2(t))$ satisfying the initial condition (5), which is globally asymptotically stable. Therefore, there are positive constants $m$ and $T_1 > T$ such that $u_i(t) \geq m$ ($i = 1, 2$) for all $t \geq T_1$. Using Lemma 2.7, we obtain that for arbitrary positive solution $x(t) = (x_1(t), x_2(t))$ of system (16), $x_i(t) \geq m$, for all $t \geq T_1$, $i = 1, 2$. Finally, we have $m \leq x_i(t) \leq M$, for all $t \geq T_1$, $i = 1, 2$. Set

$$\Omega = \{(x_1(t), x_2(t))| m \leq x_i(t) \leq M, i = 1, 2\}.$$

This completes the proof of Theorem 3.3. \hfill \Box

Remark 2. Takeuchi et al. [40] showed that a nonautonomous single-species model with dispersal time delays in patchy environment is permanent if each “food-poor” patch is connected to at least one “food-rich” patch; population is permanent in “food-rich” patches in the sense that partial permanence ensures permanence. In this paper, the dispersal (or migration) movement of the population can’t happen all the time, i.e., when $t \in [\tau_{2k}, \tau_{2k+1})$, $d_{ijk} = 0$, species $x$ inhabits in two patches respectively and there exists no movement of dispersal between two patches; when $t \in [\tau_{2k+1}, \tau_{2k+2}), d_{ijk} > 0$, dispersal (or migration) between two patches occurs. Therefore, an interesting open question is whether the results in [40] are still true for system (4).

Now, we further discuss the global asymptotic stability of system (4) and introduce the following assumption.

$(H_T)$ There exist constants $\beta > 0$ and $c_i > 0$, $i = 1, 2$, such that

$$\liminf_{t \to \infty} B_i(t) = \liminf_{t \to \infty} [c_i(b_{ik}(t) - \frac{d_{ijk}(\sigma_j^{-1}(t))}{\tilde{m}})] \geq \beta,$$

where $\tilde{m} = \max\{1, [m]\}$, $[m]$ is the integer part of $m$.

Theorem 3.4. System (4) is globally asymptotically stable if $(H_1) - (H_T)$ hold.
Proof. Suppose \( x(t) = (x_1(t), x_2(t)) \) and \( y(t) = (y_1(t), y_2(t)) \) are any two positive solutions of system (4). From Theorem 3.3, we have
\[
m \leq x_i(t), y_i(t) \leq M
\]
for all \( t \geq T_1 \) and \( i = 1, 2 \). Construct the Liapunov function
\[
V(t) = \sum_{i=1}^{2} c_i[\ln x_i(t) - \ln y_i(t)] + \int_{t-t_j}^{t} \frac{d_{ijk}(\sigma^{-1}_j(s))}{\bar{m}} | x_i(s) - y_i(s) | ds,
\]
i, j = 1, 2, i \neq j. For \( t = \tau_{k+1} \), we have
\[
V(\tau_{k+1}) = \sum_{i=1}^{2} c_i[\ln x_i(\tau_{k+1}) - \ln y_i(\tau_{k+1})] + \int_{\tau_{k+1} - t_j}^{\tau_{k+1}} \frac{d_{ijk}(\sigma^{-1}_j(s))}{\bar{m}} | x_i(s) - y_i(s) | ds
\]
\[
= \sum_{i=1}^{2} c_i[\ln h_{ik} x_i(\tau_{k+1}^-) - \ln h_{ik} y_i(\tau_{k+1}^-)] + \int_{\tau_{k+1} - t_j}^{\tau_{k+1}} \frac{d_{ijk}(\sigma^{-1}_j(s))}{\bar{m}} | x_i(s) - y_i(s) | ds
\]
\[
= V(\tau_{k+1}),
\]
we can see that \( V(t) \) is continuous for all \( t \geq T \).

For \( t \neq \tau_{k+1} \), calculating the Dini upper right derivative of \( V(t) \) along the solutions of (4), we have
\[
D^+ V(t) = \sum_{i=1}^{2} c_i[sgn(x_i(t) - y_i(t))\left(\frac{\dot{x}_i(t)}{x_i(t)} - \frac{\dot{y}_i(t)}{y_i(t)}\right)
+ d_{ijk}(\sigma^{-1}_j(t)) \frac{\bar{m}}{\bar{m}} | x_i(t) - y_i(t) |
- d_{ijk}(\sigma^{-1}_j(t)) \frac{\bar{m}}{\bar{m}} | x_i(t - \tau_j) - y_i(t - \tau_j) |] + \sum_{i=1}^{2} c_i[-b_{ik}(t) | x_i(t) - y_i(t) |
+ sgn(x_i(t) - y_i(t))d_{ijk}(t) \frac{x_i(t - \tau_j) y_i(t) - y_i(t - \tau_j)x_i(t)}{x_i(t) y_i(t)}
+ d_{ijk}(\sigma^{-1}_j(t)) \frac{\bar{m}}{\bar{m}} | x_i(t) - y_i(t) |
- d_{ijk}(\sigma^{-1}_j(t)) \frac{\bar{m}}{\bar{m}} | x_i(t - \tau_j) - y_i(t - \tau_j) |] = \sum_{i=1}^{2} c_i[-b_{ik}(t) | x_i(t) - y_i(t) |
+ sgn(x_i(t) - y_i(t))d_{ijk}(t) \frac{x_i(t - \tau_j) y_i(t) - y_i(t - \tau_j)x_i(t)}{x_i(t) y_i(t)}
+ d_{ijk}(\sigma^{-1}_j(t)) \frac{\bar{m}}{\bar{m}} | x_i(t) - y_i(t) |
- d_{ijk}(\sigma^{-1}_j(t)) \frac{\bar{m}}{\bar{m}} | x_i(t - \tau_j) - y_i(t - \tau_j) |],
\]
(18)
We state
\[
\text{sgn}(x_i(t) - y_i(t))d_{ijk}(t)\frac{x_j(t - \tau_j)y_k(t) - y_j(t - \tau_j)x_i(t)}{x_i(t)y_i(t)} \leq d_{ijk}(t)\frac{|x_j(t - \tau_j) - y_j(t - \tau_j)|}{m}.
\] (19)

In fact, there are two cases as follows:

**Case 1.** If \(y_i(t) \leq x_i(t)\) \((i = 1, 2)\), then we obtain
\[
\text{sgn}(x_i(t) - y_i(t))d_{ijk}(t)\frac{x_j(t - \tau_j)y_k(t) - y_j(t - \tau_j)x_i(t)}{x_i(t)y_i(t)} \leq d_{ijk}(t)\frac{|x_j(t - \tau_j) - y_j(t - \tau_j)|}{y_i(t)} \leq d_{ijk}(t)\frac{|x_j(t - \tau_j) - y_j(t - \tau_j)|}{m}.
\]

**Case 2.** If \(y_i(t) > x_i(t)\) \((i = 1, 2)\), then we have
\[
\text{sgn}(x_i(t) - y_i(t))d_{ijk}(t)\frac{x_j(t - \tau_j)y_k(t) - y_j(t - \tau_j)x_i(t)}{x_i(t)y_i(t)} \leq -d_{ijk}(t)\frac{x_j(t - \tau_j) - y_j(t - \tau_j)}{y_i(t)} = d_{ijk}(t)\frac{|x_j(t - \tau_j) - y_j(t - \tau_j)|}{y_i(t)} \leq d_{ijk}(t)\frac{|x_j(t - \tau_j) - y_j(t - \tau_j)|}{m}.
\]

From (18), (19) and \((H_7)\), we obtain
\[
D^+V(t) \leq \sum_{i=1}^{2} c_i [-b_{ik}(t) | x_i(t) - y_i(t) | + \frac{d_{ijk}(t)}{m} | x_j(t - \tau_j) - y_j(t - \tau_j) | + \frac{d_{ijk}(\sigma^{-1}_j(t))}{m} \frac{\sigma^{-1}_j(t)}{m} | x_i(t) - y_i(t) | - \frac{d_{ijk}(t)}{m} | x_i(t - \tau_j) - y_i(t - \tau_j) |]
\]
\[
\leq \sum_{i=1}^{2} c_i [-b_{ik}(t) + \frac{d_{ijk}(\sigma^{-1}_j(t))}{m} \frac{\sigma^{-1}_j(t)}{m} | x_i(t) - y_i(t) | + \frac{d_{ijk}(t)}{m} | x_j(t - \tau_j) - y_j(t - \tau_j) |]
\]
\[
- \sum_{i=1}^{2} c_i \frac{d_{ijk}(t)}{m} | x_i(t - \tau_j) - y_i(t - \tau_j) |
\]
\[
\leq \sum_{i=1}^{2} c_i [-b_{ik}(t) + \frac{d_{ijk}(\sigma^{-1}_j(t))}{m} \frac{\sigma^{-1}_j(t)}{m} | x_i(t) - y_i(t) | + \sum_{j=1}^{2} c_j \frac{d_{ijk}(t)}{m} | x_j(t - \tau_j) - y_j(t - \tau_j) |]
\]
\[
+ \sum_{j=1}^{2} c_j \frac{d_{ijk}(t)}{m} | x_i(t - \tau_j) - y_i(t - \tau_j) | - \sum_{j=1}^{2} c_j \frac{d_{ijk}(t)}{m} | x_i(t - \tau_j) - y_i(t - \tau_j) |
\]
\[
\leq \sum_{i=1}^{2} c_i [-b_{ik}(t) + \frac{d_{ijk}(\sigma^{-1}_j(t))}{m} \frac{\sigma^{-1}_j(t)}{m} | x_i(t) - y_i(t) | + \sum_{j=1}^{2} c_j \frac{d_{ijk}(t)}{m} | x_j(t - \tau_j) - y_j(t - \tau_j) |]
\]
\[
- \sum_{i=1}^{2} c_i \frac{d_{ijk}(t)}{m} | x_i(t - \tau_j) - y_i(t - \tau_j) | \leq - \sum_{i=1}^{2} B_i(t) | x_i(t) - y_i(t) |.
\] (20)
for all $t \geq \bar{T} \geq T$. Integrating both sides of (20) on interval $[\bar{T}, t]$, we have
\[
V(t) + \int_{\bar{T}}^{t} \sum_{i=1}^{2} \beta \cdot |x_i(s) - y_i(s)| \, ds \leq V(\bar{T})
\]
for $i = 1, 2$, hence, $V(t)$ and $\int_{\bar{T}}^{t} \sum_{i=1}^{2} |x_i(s) - y_i(s)| \, ds$ are bounded for all $t \geq \bar{T}$. Then
\[
\int_{\bar{T}}^{\infty} \sum_{i=1}^{2} |x_i(t) - y_i(t)| \, dt < \infty \tag{21}
\]
for $i = 1, 2$. By using Lemma 2.8 and (21), we have $\sum_{i=1}^{2} |x_i(t) - y_i(t)|$ ($i = 1, 2$) are bounded and uniformly continuous. By using the Barbalat’s Lemma (see [18] lemma 1.22), we can obtain that
\[
\lim_{t \to \infty} |x_i(t) - y_i(t)| = 0
\]
for $i = 1, 2$. This completes the proof of Theorem 3.4.

In the following section, we discuss the existence and uniqueness of almost periodic solutions of system (4).

For any $a_{ik}(t) \in H(a_{ik}(t))$, $b_{ik}(t) \in H(b_{ik}(t))$, $D_{ij}(t) \in H(D_{ij}(t))$, $h_{ik}^{*} \in H(h_{ik})$, and $\tau_{k}^{*} \in H(\tau_{k})$ ($i = 1, 2$), we consider the following hull equations of system (4):
\[
\begin{align*}
\dot{x}_i(t) &= x_i(t)[a_{ik}^{*}(t) - b_{ik}^{*}(t)x_i(t)] + d_{ijk}^{*}(t)[x_j(t - \tau_i) - x_i(t)], \quad t \in [\tau_{k}^{*}, \tau_{k+1}^{*}], \\
x_{i}(\tau_{k+1}) &= h_{ik}^{*}x_{i}(\tau_{k+1}^{*}), \quad t = \tau_{k+1}^{*}.
\end{align*}
\tag{22}
\]
According to Theorem 2.9, we can conclude that if system (4) satisfies $(H_1)$-$(H_7)$, then the hull equations (22) also satisfies $(H_1)$-$(H_7)$.

**Theorem 3.5.** Assume system (22) satisfies $(H_1)-(H_7)$, then there exists a unique strictly positive almost periodic solution which is global asymptotically stable for system (4).

**Proof.** By Lemma 2.10, we only need to prove that each hull equations of almost periodic system (4) has a unique strictly positive solution. We first prove the existence of strictly positive solution of any hull equations (22). Let $\{t_{n}^{*}\}$ be an arbitrary sequence of real numbers, then there exists a subsequence $\{t_{n}\}$, $t_{n} = t_{m}^{*}$, such that $a_{ik}(t + t_{n}) \rightarrow a_{ik}^{*}(t)$, $b_{ik}(t + t_{n}) \rightarrow b_{ik}^{*}(t)$, $d_{ijk}(t + t_{n}) \rightarrow d_{ijk}^{*}(t)$, uniformly for all $t \in R$ and the sequences $\{\tau_{k} - t_{n}\}$ ($k \in Z$), are convergent to the sequence $\{\tau_{k}^{*}\}$ uniformly with respect to $k \in Z$ as $n \to \infty$. Suppose $x(t) = (x_1(t), x_2(t))$ is any positive solution of (22). By the proof of Theorem 3.3, we have
\[
m \leq x_i(t) \leq M \tag{23}
\]
for all $t \geq T_1$ and $i = 1, 2$.

Obviously, $(x_1(t + t_{n}), x_2(t + t_{n}))$ satisfy the following auxiliary system:
\[
\begin{align*}
x_{i}^{*}(t) &= x_i(t)[a_{ik}^{*}(t + t_{n}) - b_{ik}^{*}(t + t_{n})x_i(t)] + d_{ijk}^{*}(t)[x_j(t - \tau_i) - x_i(t)], \quad t \in [\tau_{k}^{*}, \tau_{k+1}^{*}], \\
x_{i}(\tau_{k+1}) &= h_{ik}^{*}x_{i}(\tau_{k+1}^{*}), \quad t = \tau_{k+1}^{*}.
\end{align*}
\]
for all $t \geq -\tau - t_{n}$, and $i = 1, 2$, $n = 1, 2, 3, ...$. 

From (23), there exists a positive constant \( K \) which does not depend on \( n \) such that

\[ |\dot{x}_i(t + t_n)| \leq K, \]

for all \( t \geq -\tau - t_n \), and \( i = 1, 2 \). Then, for any positive integer \( p \), the sequence \( \{(x_1(t + t_n), x_2(t + t_n) : n \geq p\} \) is uniformly bounded and equi-continuous on interval \( [-\tau - t_n, \infty] \). Using the Ascoli-Arzela lemma, we obtain that there exists a subsequence \( \{t_k\} \subset \{t_n\} \) such that the sequence \( \{x_1(t + t_k), x_2(t + t_k)\} \) converges uniformly in \( t \) on any compact set of \( R \) as \( k \rightarrow \infty \). Let \( x^*_i(t) \) be the limit function of \( \{x_i(t + t_k)\} \), obviously, \( (x^*_1(t), x^*_2(t)) \) defined on \( R \) that satisfies Eq.(22) and

\[ m \leq x^*_i(t) \leq M, \]

for all \( t \geq T_1 \), and \( i = 1, 2 \). Hence, \( (x^*_1(t), x^*_2(t)) \) is a strictly positive solution of Eq.(22).

Next, we prove the uniqueness of a strictly positive solution of Eq.(22). Suppose \( (y^*_1(t), y^*_2(t)) \) is also a strictly positive solution of Eq.(22). Then we construct a Liapunov function \( V^*(t) \) on \( R \) as follows:

\[ V^*(t) = \sum_{i=1}^{2} c_i [\ln x^*_i(t) - \ln y^*_i(t)] + \int_{-\tau}^{t} \frac{d^2 y^*_i(t)}{dt^2} \bigg|_{\sigma_j^{-1}(s)} x^*_i(s) - y^*_i(s) | ds, \]

\[ i, j = 1, 2, i \neq j. \]

By the similar discussion in the proof of Theorem 3.4, for \( t = \tau_{k+1} \), we have

\[ V^*(\tau_{k+1}) = V^*(\tau_{k+1}^-), \]

then \( V^*(t) \) is continuous for all \( t \in R \). The rest of the proof of Theorem 3.5 is exactly the same as that of Theorems 3.1, 3.2, 3.4, so we omit it here.

Therefore, any hull equations of system (4) have a unique strictly positive solution. By Lemma 2.10 and Theorem 3.4, system (4) has a unique strictly positive almost periodic solution which is global asymptotically stable. This completes the proof of Theorem 3.5.

Remark 3. In this paper, we only discussed the case of two patches. However, in the real ecosystem, many migratory populations can move among many habitats during their whole migration cycle to inhabit, restore, mature, breed, etc. Therefore, it is more proper to consider the multi-patches cases for a long-distance migration populations. Thereby, we introduce the following dispersal model with \( n \) patches.

\[
\begin{align*}
\dot{x}_i(t) &= x_i(t)[a_i(t) - b_i(t)x_i(t)], \quad t \in [\tau_{2k}, \tau_{2k+1}), \\
x_i(\tau_{2k+1}) &= d_i x_i(\tau_{2k+1}), \quad t = \tau_{2k+1}, \\
\dot{x}_i(t) &= x_i(t)[\tilde{a}_i(t) - \tilde{b}_i(t)x_i(t)], \\
&+ \sum_{j=1}^{n} D_{ij}(t)(x_j(t) - \tau_i(t) - x_i(t)), \quad t \in [\tau_{2k+1}, \tau_{2k+2}), \\
x_i(\tau_{2k+2}) &= D_{ij} x_i(\tau_{2k+2}), \quad t = \tau_{2k+2}, \\
k = 0, 1, 2, \ldots, \quad i, j = 1, 2, \ldots, n, \quad i \neq j.
\end{align*}
\]

(24)

For system (24) we assume that the following assumptions hold for each \( i, j = 1, 2, \ldots, n \), and \( i \neq j \).

\begin{enumerate}
\item [(B1)] The bounded almost periodic functions \( a_i(t), \tilde{a}_i(t), b_i(t), \tilde{b}_i(t), \) and \( D_{ij}(t) \) are continuous for all \( t \in R \), and \( b_i(t) \geq 0, \tilde{b}_i(t) \geq 0, 0 < d_i, D_t \leq 1 \).
\item [(B2)] The set of sequences \( \{\tau_k^i \}, k, i \in Z \), is uniformly almost periodic, and \( \inf_{k \in Z} |\tau_{k+1} - \tau_k| > 0 \).
\end{enumerate}
Corollary 2. Assume that system (B) satisfies (B1)-(B6), then it is permanent.

Corollary 2. System (24) is globally asymptotically stable provided (B1)-(B7) hold.

Corollary 3. Assume that system (24) satisfies (B1)-(B7), then it has a unique strictly positive almost periodic solution which is globally asymptotically stable.

Remark 4. Based on the work of this paper, it is more realistic and interest to consider an open problem: how to establish the necessary and sufficient conditions on the extinction, permanence, existence, uniqueness and global asymptotical stability of positive almost periodic solutions for system (4). Moreover, we only discuss the finite delayed impulsive almost periodic population dynamical systems. An important and interesting open problem is whether the results obtained in this paper can be similarly extended to the case of infinite delays, we leave this for a future work.
4. Numerical simulations and discussion. In this paper, we have proposed a single-species almost periodic bidirectional dispersal system with impulses and time delays. Sufficient criteria on the permanence, existence, uniqueness and globally asymptotical stability of almost periodic solution are established. In order to validate our theoretical results for system (4), we perform some numerical simulations by using the values of parameters in Table 1.

Table 1: Parameter values used in the simulations of system (4).

| Parameter | Value |
|-----------|-------|
| $a_1(t)$  | 0.5 +0.2sin(1.2πt) +0.2sin(1.5πt) |
| $a_2(t)$  | 0.55+0.1sin(1.2πt)+0.1sin(1.5πt) |
| $b_1(t)$  | 0.25 +0.1cos(1.2πt) +0.1cos(1.5πt) |
| $b_2(t)$  | 0.3+0.04cos(1.2πt)+0.04cos(1.5πt) |
| $\tilde{a}_1(t)$ | 0.6 +0.14sin(1.2πt) +0.14sin(1.5πt) |
| $\tilde{a}_2(t)$ | 0.6+0.09sin(1.2πt)+0.09sin(1.5πt) |
| $\tilde{b}_1(t)$ | 0.4 +0.04cos(1.2πt)+0.04cos(1.5πt) |
| $\tilde{b}_2(t)$ | 0.4+0.05cos(1.2πt)+0.05cos(1.5πt) |
| $D_{12}(t)$ | 0.2 +0.05cos(1.2πt)+0.05cos(1.5πt) |
| $D_{21}(t)$ | 0.2+0.04cos(1.2πt)+0.04cos(1.5πt) |
| $d_1$     | 0.7   |
| $d_2$     | 0.8   |
| $D_1$     | 0.75  |
| $D_2$     | 0.85  |
| $\tau_1$  | 1.5   |
| $\tau_2$  | 1.8   |

Due to the system (4) being an almost periodic system, we will show the numerical simulation on the following ten intervals in Table 2.

Table 2: Parameter values used in the simulations.

| No. | interval | $\tau_{k+1} - \tau_k$ | interval length | process |
|-----|----------|-----------------------|-----------------|---------|
| 1   | [0, 6)   | $\tau_1 - \tau_0$    | 6               | 1       |
| 2   | [6, 12)  | $\tau_2 - \tau_1$    | 6               | 2       |
| 3   | [12, 17) | $\tau_3 - \tau_2$    | 5               | 1       |
| 4   | [17, 23) | $\tau_4 - \tau_3$    | 6               | 2       |
| 5   | [23, 30) | $\tau_5 - \tau_4$    | 7               | 1       |
| 6   | [30, 36) | $\tau_6 - \tau_5$    | 6               | 2       |
| 7   | [36, 42) | $\tau_7 - \tau_6$    | 6               | 1       |
| 8   | [42, 48) | $\tau_8 - \tau_7$    | 6               | 2       |
| 9   | [48, 53) | $\tau_9 - \tau_8$    | 5               | 1       |
| 10  | [53, 60) | $\tau_{10} - \tau_9$ | 7               | 2       |

Therefore, we can consider almost periodic as periodic approximately and the $\omega$-period is 12.
Figure 1. (a): The time series of the permanence of species. (b): The portrait of the permanence of species $x$, here, we take the initial condition $\phi_i(s) = (\phi_1(s), \phi_2(s)) = (1.4, 1.5)$ for all $s \in [-6, 0]$.

By simple calculation we can easily verify that the assumptions $(H_1)$, $(H_2)$, $(H_3)$, $(H_4)$, $(H_6)$ hold, moreover, since

$$\int_0^{12} a_1(t) dt + \sum_{0 < \tau_k + 1 \leq 12} \ln d_1 \approx 5.3233 > 0,$$
$$\int_0^{12} \hat{a}_1(t) dt + \sum_{0 < \tau_k + 1 \leq 12} \ln D_1 \approx 6.1720 > 0,$$
$$\int_0^{12} a_2(t) dt + \sum_{0 < \tau_k + 1 \leq 12} \ln d_2 \approx 6.6503 > 0,$$
$$\int_0^{12} \hat{a}_2(t) dt + \sum_{0 < \tau_k + 1 \leq 12} \ln D_2 \approx 6.8915 > 0,$$

all the conditions required in Theorem 3.3 are satisfied. By numerical simulations, we obtain that system (4) is permanent (see Fig.1(a), Fig.1(b)).

By numerical calculation, we obtain that the lower boundedness of $(x_1(t), x_2(t))$ is $(1.1401, 1.3060)$ (see Fig.1(a), Fig.1(b)), we have $x_i(t) > 1$, $i = 1, 2$, so we further have $|m|=1$ in $(H_7)$, which means that assumption $(H_7)$ holds and all the conditions of Theorem 3.4 are satisfied. Therefore, the species $x$ in two patches has a uniquely positive almost periodic solution (see Fig.1(a), Fig.1(b)). When we take initial values $(1.4, 1.5)$, $(1.5, 1.4)$, $(1.6, 1.3)$, by numerical calculation, we can get that the minimum values of $(x_1(t), x_2(t))$ are $(1.1401, 1.3060)$, $(1.1401, 1.3060)$, $(1.1401, 1.2113)$ respectively. Similarly, there must be $x_i(t) > 1$ ($i = 1, 2$), such that $|m|=1$ in $(H_7)$, hence $(H_7)$ holds too. By numerical simulations (see Fig.2(a), Fig.2(b)), we can see that the almost periodic solution is globally stable.

Furthermore, we show the effect of time delays on the populations, here we take $\tau_1, \tau_2$ in five different cases and keep other parameters unchanged. The details are given in Table 3.
Table 3: Simulations of model (4).

| Case | $\tau_1$ | $\tau_2$ | $x_1$         | $x_2$         | Fig.  |
|------|----------|----------|---------------|---------------|------|
| 1    | 0        | 0        | Permanent     | Permanent     | Fig.3(a) |
| 2    | 1.5      | 1.8      | Permanent     | Permanent     | Fig.2(a) |
| 3    | 2        | 4        | Permanent     | Permanent     | Fig.3(b) |
| 4    | 6        | 6        | Permanent     | Permanent     | Fig.3(c) |
| 5    | 0        | 6        | Permanent     | Permanent     | Fig.3(d) |

The populations $x_1$ and $x_2$ are both permanent and globally asymptotically stable (see Fig.3(a)-Fig.3(d) and Fig.2(a)).

However, comparing Fig.3(a)-Fig.3(d) and Fig.2(a), the values of delays vary within half a $\omega$-period. We can see that the longer duration of the time delays, the more beneficial to the survival of the population $x$ in the second patch. No matter what the values of $\tau_1$ or $\tau_2$ are, they both have great impact on the balance of populations in two habitats. We realize that system (4) with time delays is more complicated than that without delays.

Moreover, we shall consider the following nonautonomous continuous dispersal single-species almost periodic system with time delays and ignore the influence of impulse on the population $x$ of system (4):

$$
\begin{align*}
\dot{x}_1(t) &= x_1(t)[\tilde{a}_1(t) - \tilde{b}_1(t)x_1(t)] + D_{12}(t)(x_2(t - \tau_1) - x_1(t)), \\
\dot{x}_2(t) &= x_2(t)[\tilde{a}_2(t) - \tilde{b}_2(t)x_2(t)] + D_{21}(t)(x_1(t - \tau_2) - x_2(t)),
\end{align*}
$$

we use the values of parameters in Table 1 and Table 2. The population $x$ has a uniquely positive almost periodic solution which is globally stable with or without delays (See Fig. 4(a), 4(b)). In order to illustrate the difference of the results between our mathematical models (4) and (25), we use the values of parameters in Table 4.
Figure 3. The time series of the globally stable positive almost periodic solution of species \( x \). Here, we take the initial condition \( \phi_i(s) = (\phi_1(s), \phi_2(s)) = (1.4, 1.5), (1.5, 1.4), (1.6, 1.3) \) for all \( s \in [-6, 0] \).

Figure 4. (a): The time series of dynamical behavior of system (5.1) with \( \tau_1 = 1.5, \tau_2 = 1.8 \). (b): The time series of dynamical behavior of system (5.1) with \( \tau_1 = 0, \tau_2 = 0 \).
Table 4: Parameter values used in the simulations of systems (4) and (25).

| Parameter | Value |
|-----------|-------|
| $a_1(t)$  | $0.5 + 0.2 \sin(1.2\pi t) + 0.2 \sin(1.5\pi t)$ |
| $a_2(t)$  | $0.55 + 0.1 \sin(1.2\pi t) + 0.1 \sin(1.5\pi t)$ |
| $b_1(t)$  | $0.25 + 0.1 \cos(1.2\pi t) + 0.1 \cos(1.5\pi t)$ |
| $b_2(t)$  | $0.3 + 0.04 \cos(1.2\pi t) + 0.04 \cos(1.5\pi t)$ |
| $\tilde{a}_1(t)$ | $0.01 + 0.01 \sin(1.2\pi t) + 0.01 \sin(1.5\pi t)$ |
| $\tilde{a}_2(t)$ | $0.01 + 0.02 \sin(1.2\pi t) + 0.02 \sin(1.5\pi t)$ |
| $\tilde{b}_1(t)$ | $6 + 0.05 \cos(1.2\pi t) + 0.05 \cos(1.5\pi t)$ |
| $\tilde{b}_2(t)$ | $5 + 0.15 \cos(1.2\pi t) + 0.15 \cos(1.5\pi t)$ |
| $D_{12}(t)$ | $0.5 + 0.05 \cos(1.2\pi t) + 0.05 \cos(1.5\pi t)$ |
| $D_{21}(t)$ | $0.5 + 0.04 \cos(1.2\pi t) + 0.04 \cos(1.5\pi t)$ |
| $d_1$ | 0.7 |
| $d_2$ | 0.8 |
| $D_1$ | 0.75 |
| $D_2$ | 0.85 |
| $\tau_1$ | 1.5 |
| $\tau_2$ | 1.8 |

We change some parameter values i.e., $\tilde{a}_1(t)$, $\tilde{a}_2(t)$, $\tilde{b}_1(t)$, $\tilde{b}_2(t)$, $D_{12}(t)$ and $D_{21}(t)$, and keep the other parameters unchanged, we can see some significant phenomena happen, the details are given in Table 5.

Table 5: Simulations of models (4) and (25).

| System | $x_1$ | $x_2$ | Fig. |
|--------|-------|-------|------|
| (4)    | Permanent | Permanent | Fig.5 a |
| (25)   | Extinction | Extinction | Fig.5 b |

This shows that intermittent bidirectional dispersal between two patches is beneficial to population $x$ in system (4) but harmful to population $x$ in continuous dispersal system (25), although both with the same parameters. Which means intermittent dispersal is more reasonable and a better choice for migratory populations in the real ecosystems than those with continuous or impulse dispersal movements.

In a word, systems with intermittent dispersal, impulse perturbations and time delays are more complicated than continuous dispersal systems, but are very consistent with real ecosystems, which help us better and more deeply read the ecosystems.

Acknowledgments. We are grateful to the handling editor and anonymous reviewers for their valuable comments and suggestions that greatly improved the presentation of this paper.

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Figure 5. (a): The time series of dynamical behavior of system (4) with \( \tau_1 = 1.5, \tau_2 = 1.8 \). (b): The time series of extinction of system (25) with \( \tau_1 = 1.5, \tau_2 = 1.8 \).

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Received April 2015; revised April 2016.

E-mail address: longzhang_xj@sohu.com
E-mail address: xianxiyaren@126.com
E-mail address: zhidong@xju.edu.cn