Nonlinear control and estimation in induction machine using state estimation techniques

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In this paper, several techniques are addressed for both estimation and control to be integrated into a unified closed-loop or feedback control system that is applicable for a general family of nonlinear control structures. The estimation techniques include the extended Kalman filter (EKF), unscented Kalman filter (UKF), and particle filter (PF). Specifically, two comparative studies are performed. In the first comparative study, the state variables are estimated from noisy measurements of these variables, and the various estimation techniques are compared by computing the estimation root mean square errors with respect to the noise-free data. In the second comparative study, the state variables as well as the model parameters are simultaneously estimated. In this case, in addition to comparing the performances of the various state estimation techniques, the effect of the number of estimated model parameters on the accuracy and convergence of these techniques is also assessed. The results of both comparative studies show that the UKF provides a higher accuracy than the EKF, due to the limited ability of EKF to accurately estimate the mean and covariance matrix of the estimated states through linearization of the nonlinear process model. The results also show that the PF provides a significant improvement over the UKF and EKF and can still provide both convergence as well as accuracy-related advantages over other estimation methods. This is because the covariance is propagated through linearization of the underlying nonlinear model, when the state transition and observation models are highly nonlinear.

Keywords: states and parameters estimation; particle filter; induction machine; nonlinear control

1. Introduction

Parameter and states estimation in nonlinear systems is an important issue in control. However, due to the difficulty of, or cost associated with, obtaining these measurements, state and/or parameter estimators are often used to overcome this problem. A typical feedback control system with performance requirement consists of two critical tasks, estimation and control. For highly nonlinear systems, finding explicit controller and estimators is extremely important.

In process of control, sometimes it is challenging to measure some of the key variables, such as nonlinear control of induction machines on the basis of a three-order electrical model, and so on. In such cases, estimates of these variables can be obtained using state estimation. Also, this is the case in modeling the drive system where several parameters need to be estimated. Estimating these parameters usually requires several experimental setups that can be challenging and expensive. Hence, estimating such parameters using state estimation techniques can be of a great value.

The estimation problem that is addressed here can be viewed as an optimal filtering problem, in which the posterior distribution of the unobserved state, given the sequence of observed data and the state evolution model, is recursively updated (Mansouri, Mohamed-Seghir, Nounou, Nounou, & Abu-Rub, 2014b; Matthies, Kanade, & Szeliski, 1989; Newlands et al., 2014). Several techniques for prediction and modeling in power systems are developed and used in practice. These techniques include classical and GGE (G, genotype + GEI, genotype-by-environment interaction) biplot methods (Fruutos, Galindo, & Leiva, 2014), the extended Kalman filter (EKF) (Simon, 2006), the ensemble Kalman filter (Shu, Kembowski, & McKee, 2005), the unscented Kalman filter (UKF) (Mansouri, Dumont, & Destain, 2013), the particle filter (PF) (Mansouri, Dumont, & Destain, 2013), the variational Bayesian filter (Mansouri, Dumont, Leemans, & Destain, 2014) and more recently the improved particle filter (Mansouri, Mohamed-Seghir, Nounou, Nounou, & Abu-Rub, 2014a).

Several contributions have been proposed to resolve the problem of states and parameters’ estimation for power. Lindenmeyer, Donnel, Moshef, and Kundur (2001) have proposed to estimate a motor parameters from manufacturer data such as name plate data and motor performance characteristics. This proposed technique is based on a nonlinear optimization routine and is not suitable for

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to be embedded on a low-performance microprocessor. Ben-Brahim and Tadakuma (1998) has described a newly developed speed sensorless drive based on neural networks. A backpropagation neural network is used to provide real-time adaptive identification of the motor speed. The estimation objective is the sum of squared errors between a target trajectory and the neural network model output.

Wlas, Krzeminski, Guzinski, Abu-Rub, and Toliyat (2005), Guzinski, Abu-Rub, Diguet, Krzeminski, and Lewicki (2010), and Abu-Rub, Guzinski, Krzeminski, and Toliyat (2003) have proposed a new speed estimation method of an induction motor. In the new approach, a multilayer Neural Network (NN) with one hidden layer is used. The speed value is obtained at the NN output and the feedback is connected between the output and the input node. Many closed-loop observers utilize some kind of feedback, such as model reference adaptive system (MRAS) (Levi & Wang, 2002; Peng & Fukao, 1994). These MRAS schemes require pure integrators in their reference models, and are affected by stator resistance thermal variations. These problems have limited the applications of speed sensorless vector control in high-performance and low-speed drives.

The EKF technique (Shi, Chan, Wong, & Ho, 2002) has been proposed to improve the estimation performance. In these closed-loop estimators, EKF is the most attractive one because of its better performance. A Kalman filter (Atkinson, Acarnley, & Finch, 1991) that liberalizes about the current mean and covariance using Taylor series expansions is referred to as an EKF. EKF has been applied extensively in the field of state estimation of nonlinear systems and almost has become a standard technique for deterministic approaches, where the structure of the standard Luenberger observer for a linear system is enhanced to permit the simultaneous estimation of the rotor flux and speed (Kubota, Matsuse, & Nakano, 1993; Li, Xu, & Zhang, 2005).

In Lee, Lee, Yoon, Choy, and Song (2005), the rotor speed and flux are estimated on the basis of a fourth-order electrical model of the induction motor; the resulting scheme is independent of the current value of the rotor speed, which is in contrast with existing methods mentioned in the previous paragraph.

The above-mentioned techniques usually require large amounts of computational resources and time. One aspect of concern is that these methods do not sufficiently take the special structures of biological system models into consideration. However, the propositions in Wu (2007) and Jia (2009) have shown that consideration of the model structure may simplify the parameter estimation problem. For example, a nonlinear model can be linearized to a linear one and the EKF can be applied to estimate the unknown sensitive parameters of the model. In EKF, the model describing the system is linearized at every time sample (in order to estimate the mean and covariance matrix of the state vector), and thus the model is assumed to be differentiable. Unfortunately, for highly nonlinear or complex models, the EKF does not usually provide a satisfactory performance. On the other hand, instead of linearizing the model to approximate the mean and covariance matrix of the state vector, the UKF uses the unscented transformation to improve the approximation of these moments. In the unscented transformation, a set of samples (called sigma points) are selected and propagated through the nonlinear model, which provides more accurate approximations of the mean and covariance matrix of the state vector, and thus results in a more accurate state estimation.

In this work, the objective is to compare the performances of various state-of-the-art state estimation techniques in estimating the states of the power process model representing the nonlinear control of induction machines (NCIMs) (i.e. the rotor speed, the rotor flux, the stator flux, the rotor resistance and the magnetizing inductance) and their abilities to estimate some of the key system parameters, which are needed to define the NCIM process model.

The rest of the paper is organized as follows. The problem statement formulation and state estimation technique will be described in Section 2. Section 3 presents the model of NCIM that will be used as a case study in the analysis and gives some numerical results. Finally, Section 4 concludes the paper.

2. Problem formulation

In this section, the state and parameter estimation problem is formulated, and then a comparative performance analysis of states estimation using PF will be conducted for a NCIM modeling.

2.1. Problem statement

Here, the estimation problem of interest is formulated for a general system model. Let a nonlinear state space model be described as follows:

$$\dot{x} = g(x, u, \theta, v),$$
$$\dot{y} = l(x, u, \theta, v),$$

(1)

where $x \in \mathbb{R}^n$ is a vector of the state variables, $u \in \mathbb{R}^p$ is a vector of the input variables, $\theta \in \mathbb{R}^q$ is an unknown parameter vector, $y \in \mathbb{R}^m$ is a vector of the measured variables, and $g$ and $l$ are nonlinear differentiable functions.
Discretizing the state space model (1), the discrete model can be written as follows:

\[ x_k = f(x_{k-1}, u_{k-1}, \theta_{k-1}, w_{k-1}), \]
\[ y_k = h(x_k, u_k, \theta_k, v_k), \]

which describes the state variables at some time step \( k \) in terms of their values at a previous time step \( k - 1 \).

Let the process and measurement noise vectors have the following properties: \( E[w_k] = 0, E[w_k w_k^T] = Q_k, E[v_k] = 0 \) and \( E[v_k v_k^T] = R_k \).

Since in this problem we are interested in estimating the state vector, \( x_k \), as well as the parameter vector, \( \theta_k \), let us assume that the parameter vector is described by the following model:

\[ \theta_k = \theta_{k-1} + \gamma_{k-1}. \]

In other words, the parameter vector model (3) corresponds to a stationary process, with an identity transition matrix, driven by white noise. We can define a new state vector \( z_k \) that augments the state vector \( x_k \) and the parameter vector \( \theta_k \) as follows:

\[ z_k = \begin{bmatrix} x_k \\ \theta_k \end{bmatrix} = \begin{bmatrix} f(x_{k-1}, u_{k-1}, w_{k-1}, \theta_{k-1}) \\ \theta_{k-1} + \gamma_{k-1} \end{bmatrix}, \]

where \( z_k \in \mathbb{R}^{n+q} \) is assumed to follow a Gaussian model as \( z_k \sim \mathcal{N}(\mu_k, \lambda_k) \), where at any time instant \( k \) the expectation \( \mu_k \) and the covariance matrix \( \lambda_k \) are both constants. Also, defining the augmented noise vector as

\[ \epsilon_{k-1} = \begin{bmatrix} w_{k-1} \\ \gamma_{k-1} \end{bmatrix}, \]

the model (2) can be written as

\[ z_k = \mathcal{F}(z_{k-1}, u_{k-1}, \epsilon_{k-1}), \]
\[ y_k = \mathcal{H}(z_k, u_k, v_k), \]

where \( \mathcal{F} \) and \( \mathcal{H} \) are differentiable nonlinear functions. The objective is to estimate the augmented state vector \( z_k \), given the measurements vector \( y_k \).

Next, the estimation techniques will be described.

2.2. Description of state and parameter estimation techniques

The EKF and UKF algorithms do not always provide a satisfactory performance, especially for highly nonlinear processes, because linearizing the process model does not necessarily provide good estimates of the mean of the state vector and the covariance matrix of the estimation error which are used in state estimation. These issues are addressed by the PF. Here, the estimation technique of interest (PF) is described.

2.2.1. Particle filter

A PF is an implementation of a recursive Bayesian estimator (Gustafsson et al., 2002). Bayesian estimation relies on computing the posterior \( p(z_k | y_{0:k}) \), which is the density function of the unobserved state vector, \( z_k \), given the sequence of the observed data \( y_{0:k} \equiv \{y_0, y_1, \ldots, y_k\} \).

However, instead of describing the required posterior distribution in a functional form, in this PF scheme, it is represented approximately as a set of random samples of the posterior distribution. These random samples, which are called the particles of the filter, are propagated and updated according to the dynamics and measurement models (Arulampalam, Maskell, Gordon, & Clapp, 2002). The advantage of the PF is that it is not restricted by the linear and Gaussian assumptions, which makes it applicable in a wide range of applications. The basic form of the PF is also very simple, but may be computationally expensive. Thus, the advent of cheap, powerful computers over the last 10 years has been a key to the introduction and utilization of PFS in various applications. More details about PFS can be found in Arulampalam et al. (2002), Doucet and Johansen (2009), and Doucet, Godsill, and Andrieu (2000).

For a given dynamical system describing the evolution of the states and parameters that we wish to estimate, the estimation problem can be viewed as an optimal filtering problem (Andrews, Yi, & Iglesias, 2006), in which the posterior distribution, \( p(z_k | y_{0:k}) \), is recursively updated. Here, the dynamical system is characterized by a Markov state evolution model, \( p(z_k | z_{1:k-1}) = p(z_k | z_{k-1}) \), and an observation model, \( p(y_k | z_k) \). In a Bayesian context, the task of state estimation can be formulated as recursively calculating the predictive distribution \( p(z_k | y_{1:k-1}) \) and the filtering distribution \( p(z_k | y_{0:k}) \) as follows:

\[ p(z_k | y_{1:k-1}) = \int_{\Theta} p(z_k | z_{k-1}) p(z_{k-1} | y_{1:k-1}) \, dz_{k-1}, \]

and

\[ p(z_k | y_{0:k}) = \frac{p(y_k | z_k) p(z_k | y_{1:k-1})}{p(y_k | y_{1:k-1})}, \]

where

\[ p(y_k | y_{1:k-1}) = \int_{\Theta} p(y_k | z_k) p(z_k | y_{1:k-1}) \, dz_k. \]

The state vector \( z_k \) is assumed to follow a Gaussian model, \( z_k \sim \mathcal{N}(\mu_k, \lambda_k) \), where at any time instant \( k \), the expectation \( \mu_k \) and the covariance matrix \( \lambda_k \) are both constants. Thus, the marginal state distribution is obtained by integrating over the mean and covariance matrix as follows:

\[ p(z_k | z_{k-1}) = \int \mathcal{N}(z_k | \mu_k, \lambda_k) p(\mu_k, \lambda_k | z_{k-1}) \, d\mu_k \, d\lambda_k, \]

where the integration with respect to the covariance matrix leads to the known class of scale mixture distributions introduced by Barndorff-Nielsen (1977) for the scalar case.
The nonlinear nature of the system model leads to intractable integrals when evaluating the marginal state distribution, \( p(z_k | z_{k-1}) \). Therefore, Monte Carlo approximation is utilized, where the joint posterior distribution, \( p(z_{0:k} | y_{0:k}) \), is approximated by the point-mass distribution of a set of weighted samples (particles) \( \{\hat{z}_{0:k}^{(i)}, \hat{\ell}_k^{(i)}\}_{i=1}^{N} \), i.e. (Doucet & Johansen, 2009):

\[
\hat{p}_N(z_{0:k} | y_{0:k}) = \frac{\sum_{i=1}^{N} \hat{\ell}_k^{(i)} \delta_{z_{0:k}}(d z_{0:k})}{\sum_{i=1}^{N} \hat{\ell}_k^{(i)}}, \quad (10)
\]

where \( \delta_{z_{0:k}}(d z_{0:k}) \) denotes the Dirac function and \( N \) is the total number of particles. Based on the same set of particles, the marginal posterior probability of interest, \( p(z_k | y_{0:k}) \), can also be approximated as follows (Doucet & Johansen, 2009):

\[
\hat{p}_N(z_k | y_{0:k}) = \frac{\sum_{i=1}^{N} \hat{\ell}_k^{(i)} \delta_{z_k}(d z_k)}{\sum_{i=1}^{N} \hat{\ell}_k^{(i)}}, \quad (11)
\]

In the Bayesian importance sampling (IS) method, the particles \( \{\hat{z}_{0:k}^{(i)}\}_{i=1}^{N} \) are sampled according to a distribution (Doucet & Johansen, 2009),

\[
\pi(z_{0:k} | y_{0:k}) = p(z_k | z_{k-1}) = \int \mathcal{N}(z_k | \mu_k, \lambda_k) p(\mu_k, \lambda_k | z_{k-1}) \, d\mu_k \, d\lambda_k. \quad (12)
\]

Then, the estimate of the augmented state \( \hat{z}_k \) can be approximated by a Monte Carlo scheme (Doucet & Johansen, 2009):

\[
\hat{z}_k = \frac{\sum_{i=1}^{N} \hat{\ell}_k^{(i)} \hat{z}_k^{(i)}}{\sum_{i=1}^{N} \hat{\ell}_k^{(i)}} \quad (13)
\]

where \( \hat{\ell}_k^{(i)} \) are the corresponding importance weights (Doucet & Johansen, 2009):

\[
\hat{\ell}_k^{(i)} \propto \frac{p(y_{0:k} | \hat{z}_{0:k}^{(i)}) p(\hat{z}_{0:k}^{(i)})}{\pi(z_{0:k} | y_{0:k})}. \quad (14)
\]

A common problem with the sequential IS PF is the degeneracy phenomenon, where after a few iterations, all but one particle will have negligible weights. It has been shown (Yang, Tian, Jin, & Zhang, 2005) that the variance of the importance weights can only increase over time and, thus, it is impossible to avoid the degeneracy phenomenon. This degeneracy implies that a large computational effort is devoted to updating particles whose contribution to the approximation of \( p(z_k | y_{0:k}) \) is almost zero. A suitable measure of degeneracy of the algorithm is the effective sample size, \( N_{\text{eff}} \), introduced in Gustafsson et al. (2002) and Liu and Chen (1998) and defined as

\[
N_{\text{eff}} = \frac{1}{\sum_{i=1}^{N} (\hat{\ell}_k^{(i)})^2}, \quad (15)
\]

where \( \hat{\ell}_k^{(i)} \) is the normalized weight obtained using Equation (14).

The PF algorithm for state/parameter estimation is summarized in Algorithm 1.

**Algorithm 1: Particle filtering algorithm**

**Input:** \( y_k, \mu_0, \lambda_0 \)

**Output:** \( \hat{z}_k \)

**for** \( i = 1, 2, \ldots \) **do**

**IS step:**

Sample \( \hat{z}_k^{(i)} \sim \pi(\hat{z}_k | y_{0:k}, \hat{z}_{0:k}^{(i)}) \), according Equation (9), and set \( \hat{z}_{0:k}^{(i)} = \{\hat{z}_0^{(i)}, \hat{z}_k^{(i)}\} \);

Compute the approximated joint distribution, \( \hat{p}_N(z_{0:k} | y_{0:k}) \), using Equation (10);

Evaluate importance weights using Equation (14);

Normalize importance weights:

\[
\hat{\ell}_k^{(i)} = \frac{\hat{\ell}_k^{(i)}}{\sum_{j=1}^{N} (\hat{\ell}_k^{(j)})}. \quad (16)
\]

**Selection step:**

if \( N_{\text{eff}} = \frac{1}{\sum_{i=1}^{N} (\hat{\ell}_k^{(i)})^2} < N_{\text{threshold}} \) then

Resample with replacement \( N \) particles \( \{\hat{z}_{0:k}^{(i)}\}_{i=1}^{N} \) from the set \( \{\hat{z}_{0:k}^{(i)}\}_{i=1}^{N} \) according to the normalized importance weights, \( \hat{\ell}_k^{(i)} = \hat{\ell}_k^{(1)} \);

Compute the estimated state using Equation (13);

**end**

**end**

Return the estimated state vector \( \hat{z}_k \).

3. **Simulation results analysis**

Next, the model of NCIM, that will be used in our analysis, will be described.

3.1. **Nonlinear controller**

The main idea of system linearization using nonlinear feedback is shown in Figure 1 (Krzeminski, 1987) combining nonlinear feedback with the nonlinear model, and then making variable transformations, converts the highly nonlinear dynamic system, such as an induction motor, to a linear model (Krzeminski, 1987; Mansouri et al., 2013).
3.1.1. Linearization of IM fed by voltage controlled voltage source inverter

Nonlinear control with nonlinear feedback is used to obtain linear and fully decoupled control system of induction motors. The control signals are the voltage and its angular frequency using the induction motor model fed by voltage-regulated PVM. In nonlinear control, there are four state variables after differentiating them and substituting derivatives for the currents and fluxes from the motor model in the derivatives of new variables, taking into account the relationships of the new state variables, we obtain:

\[
\begin{align*}
\frac{dx_1}{dt} &= \frac{L_m}{JL_r} x_{12} - \frac{L_r}{JL_r} m_0, \\
\frac{dx_2}{dt} &= -\frac{1}{T_e} x_{12} - x_{11} \left( x_{22} + \frac{L_m}{w_5} x_{21} \right) + \frac{L_r}{w_5} u_1, \\
\frac{dx_{21}}{dt} &= -2 \frac{R_s}{L_s} x_{12} - 2 \frac{R_s}{L_s} \frac{L_m}{L_r} x_{22}, \\
\frac{dx_{22}}{dt} &= -\frac{1}{T_v} x_{12} - x_{11} x_{12} + \frac{R_s}{L_r} \frac{L_m}{L_r} x_{21} + \frac{R_s}{L_r} \frac{L_m}{L_r} x_{22} + \frac{L_r}{w_3} u_2,
\end{align*}
\]

where \( T_e = L_t w_5 / (R_s w_5 L_r + R_s L_r^2 + R_o L_m) \), \( w_5 = L_s L_r - L_m^2 \), and \( R_s \) is the stator resistance.

After linearization, new signals \( m_1 \) and \( m_2 \) are used in the feedback of the system. The two signals are defined to replace the nonlinearities in the above equations as below

\[
\begin{align*}
m_1 &= x_{11} \left( x_{22} + \frac{L_m}{w_5} x_{21} \right) + \frac{L_r}{w_5} u_1, \\
m_2 &= x_{11} x_{12} + \frac{R_s L_m}{L_r w_5} x_{21} + \frac{R_s L_m}{L_r} x_{22} + \frac{L_r}{w_5} u_2, \\
u_1 &= \frac{w_5}{L_r} \left[ x_{11} \left( x_{22} + \frac{L_m}{w_5} x_{21} \right) + m_1 \right], \\
u_2 &= \frac{w_5}{L_r} - x_{11} x_{22} - \frac{R_s L_m}{w_5 L_r} x_{21} - \frac{R_s L_m}{L_r} x_{22} + \frac{L_r}{L_m} x_{21}^2 + \frac{L_r}{L_m} x_{22}^2 + m_2,
\end{align*}
\]

In the control system, the control signals \( m_1 \) and \( m_2 \) are generated by the PI controller (see Figure 2, Krzeminski, 1987) of the state variables \( x_{12} \) and \( x_{21} \), respectively. Having these two signals generated, the signals \( u_1 \) and \( u_2 \) could be as follows (Krzeminski, 1987): the components’ vector’s \( u_{\alpha} \) and \( x_{\beta} \) (voltage stator) are defined as

\[
\begin{align*}
u_{\alpha} &= \frac{L_m}{\psi_1} u_2 - \frac{L_m}{\psi_2} u_1, \\
u_{\beta} &= \frac{\psi_1}{\psi_2} u_2 - \frac{\psi_1}{\psi_2} u_1
\end{align*}
\]

Using the above equations provides decoupling of the new motor model and two separate subsystems as follows:

**Mechanical subsystem:**

\[
\begin{align*}
\frac{dx_{11}}{dt} &= -\frac{L_m}{JL_r} x_{12} - \frac{1}{J} m_0, \\
\frac{dx_{12}}{dt} &= -\frac{1}{T_v} x_{12} + m_1
\end{align*}
\]

**Electromagnetic subsystem:**

\[
\begin{align*}
\frac{dx_{21}}{dt} &= -2 \frac{R_r}{L_r} x_{12} + 2 \frac{R_r}{L_r} \frac{L_m}{L_r} x_{22}, \\
\frac{dx_{22}}{dt} &= -\frac{1}{T_v} x_{22} + m_2
\end{align*}
\]

3.1.2. Control system

In the control system, the torque is produced by the induction motor, which is proportional only to the state variable \( x_{12} \). In the electromagnetic part of the system, the rotor flux linkage is stabilized and its command value may be described by assuming such a criterion as the minimization of energy losses in the system or the minimization of response time in the mechanical part of the system when commands are limited. The mechanical part consists of first-order delay connected with an integral element in series. Because of the necessity for torque limiting, it is appropriate to use
a cascade control structure for this subsystem (Figure 1, Krzeminski, 1987). The disturbance in the mechanical subsystem is the load torque. The electromagnetic subsystem consists of two first-order delay elements connected in series. To limit the square of rotor flux in this situation, it is also appropriate to use a cascade control structure. Hence in the control system when using the Multiscalar Model, cascaded controllers of the PI type may be applied with constant parameters tuned in accordance with the known control theory of linear systems.

Figure 3 shows the final control systems of an induction motor. The stator current components are controlled using current controllers. Instantaneous quantities of stator currents and voltages are used in the calculations and state estimation techniques block to compute the motor variables. Two PI controllers are used to control the variables $x_{12}$ and $x_{22}$. The command value for the $x_{12}$ controller is the output signal of the PI speed controller. The angular speed $x_{11}$ is measured or calculated and used in the control feedback. The command value for the $x_{22}$ controller is the output signal of the PI controller for the square of the rotor flux. In this system, the stator voltages are determined from the DC link voltage, with knowledge of the pulse width modulation algorithm or in the case of using voltage modulator directly from commanded voltage. Thus, this method simplifies the total drive system and decreases the cost. This structure of the control system presented in Figure 3 (Krzeminski, 1987) was chosen for realization in the simulation program.

Figures 4 and 5 present the results of the system shown in Figure 3, they shows the step changes of the rotor speed set values and of the load torque. Data of a 1.5 kW squirrel cage motor have been used in investigations. The response of the system has been observed after introducing a unit step of speed, motor load.

3.1.3. Induction machine

The IM model is described in rectangular $xy$ coordinates rotating with reference speed $w_r$ and defined by a set of differential equations. IM model equations depend on the speed $w_r$ value and the state variables $\psi_{rx}, \psi_{ry}$, and $w_r$. IM model equations described in per-unit system are given by (Krzeminski, 2008)

$$\frac{di_{sx}}{d\tau} = -\frac{R_s L_i^2}{L_r w_r} i_{sx} + \frac{R_t L_m}{L_r w_r} \psi_{tx} + w_r i_{sy}$$

$$+ w_r \frac{L_m}{\partial w_r} \psi_{ry} + \frac{L_r}{\partial w_r} u_{sx},$$
Figure 4. System response after step change of speed.

Figure 5. System response after step change of speed.
\[
\frac{\partial i_s}{\partial \tau} = -\frac{R_e L_m^2}{L_s} i_s + \frac{R_e L_m}{L_s} \psi_y - w_s l_s
\]
\[
+ w_s L_m \frac{\partial}{\partial \omega_e} u_y', \tag{27}
\]
\[
\frac{\partial \psi_{xx}}{\partial \tau} = -\frac{R_e}{L_s} \psi_{xx} + (w_a - w_t) \psi_y + R_e \frac{L_m}{L_s} i_s,
\]
\[
\frac{\partial \psi_{yy}}{\partial \tau} = -\frac{R_e}{L_s} \psi_{yy} - (w_a - w_t) \psi_{xx},
\]
\[
\frac{\partial w_t}{\partial \tau} = -\frac{L_m}{L_s} (\psi_{xx} i_s - \psi_{yy} i_s) - \frac{1}{J} \tau,
\]
where
\[
w_s = \sigma L_t L_s = L_t L_s - L_m^2,
\]
and \(\sigma\) is the machine total leakage factor:
\[
\sigma = 1 - \frac{L_m^2}{L_t L_s} = 1 - \frac{1}{(1 + \sigma_1)(1 + \sigma_2)},
\]
as well as \(R_e, R_s, L_t, L_s, L_m\) are the machine’s equivalent circuit parameters, \(J\) is the machine’s inertia, \(\tau\) is per-unit time \((\tau = w_0 t)\), \(t_k\) is the machine’s load torque, and \(w_0\) is the machine’s voltage nominal pulsation. The IM model described in Equation (27) is named as the \((\psi_{xx}, i_s)\) model and is used for simulation purposes, where the stationary coordinates are noted as \(\alpha\beta\) and the reference speed is fixed to zero, i.e. \(w_0 = 0\). In addition, the electromagnetic relations for the IM are used in the derivation of the proposed observers’ dependencies. When selecting state variables as stator flux and stator current, the IM stator circuit model is expressed as (Holtz, 2002)
\[
\frac{\partial \psi_{xx}}{\partial \tau} = u_s - R_e i_s, \tag{28}
\]
and for \(\psi_{xx}, \psi_{yy}\) in the stationary frame, \(w_a = 0\), the stator circuit equation is (Holtz, 1995)
\[
\frac{\partial \psi_{xx}}{\partial \tau} = \frac{1}{\tau_s} (k_r \psi_r - \psi_{xx}) + u_s, \tag{29}
\]
where \(\tau_s = \sigma (L_s/R_s)\) is the stator time and \(k_r = L_s/R_s\) is the rotor coupling factor. The relations between fluxes and conerments are given by
\[
\psi_s = L_s i_s + L_{ml} i_t,
\]
\[
\psi_t = L_{ml} i_s + L_t i_t. \tag{30}
\]
It is assumed that only the stator currents and voltages are measurable, hence the IM model used for the estimation of the three states (the rotor flux \(\psi_{xx}\), the stator flux \(\psi_{yy}\) and the rotor speed \(w_t\)) is given by
\[
\frac{\partial \psi_{xx}}{\partial \tau} = -\frac{R_e}{L_s} \psi_{xx} + (w_a - w_t) \psi_{yy} + R_e \frac{L_m}{L_s} i_s,
\]
\[
\frac{\partial \psi_{yy}}{\partial \tau} = -\frac{R_e}{L_s} \psi_{yy} - (w_a - w_t) \psi_{xx},
\]
\[
\frac{\partial w_t}{\partial \tau} = -\frac{L_m}{L_s} (\psi_{xx} i_s - \psi_{yy} i_s) - \frac{1}{J} \tau.
\]
Discretizing the model using a sampling interval of \(\Delta t\) and incorporating random process noise (to account for any uncertainties in the IM process model), the model can be written as
\[
\psi_{xx,k} = \psi_{xx,k-1} + \left[ -\frac{R_e}{L_s} \psi_{xx,k} + (w_a - w_{t,k}) \psi_{yy,k} + R_e \frac{L_m}{L_s} i_{s,k} \right] \Delta t + w_{k-1},
\]
\[
\psi_{yy,k} = \psi_{yy,k-1} + \left[ -\frac{R_e}{L_s} \psi_{yy,k} - (w_a - w_{t,k}) \psi_{xx,k} + R_e \frac{L_m}{L_s} i_{yx,k} \right] \Delta t + w_{k-1},
\]
\[
w_{t,k} = w_{t,k-1} + \left[ -\frac{L_m}{L_s} (\psi_{xx,k} i_{s,k} - \psi_{yy,k} i_{s,k}) - \frac{1}{J} \tau \right] \Delta t + w_{k-1}, \tag{32}
\]
where the process noise is zero mean Gaussian noise, i.e. \(w_{k-1} \sim N(0, \sigma_w^2)\).

### 3.2. Simulation-based comparative performance analysis using EKF, UKF and PF

Here, we assume that the IM states of interest that we wish to estimate are the rotor flux \(\psi_{xx}\), the stator flux \(\psi_{yy}\) and the rotor speed \(w_t\). For simulation purposes, the simulation data are generated from the discretized model itself, given some pre-defined model parameters which are shown in Table 1. The sampling time used for discretization is 0.01 min. To perform the comparison between the various estimation techniques, the following estimation root mean square errors (RMSEs) criteria will be used for the estimated states with respect to the noise-free data:

\[
\text{RMSE} = \sqrt{E((x - \hat{x})^2)}, \tag{33}
\]
where \(x\) (resp. \(\hat{x}\)) is the true state/parameter vector (resp. the estimated state/parameter vector). In this section, we are interested in comparing the estimation performances of EKF, UKF and PF in estimating the three states (the rotor flux \(\psi_{xx}\), the stator flux \(\psi_{yy}\) and the rotor speed \(w_t\)). Hence, we consider the state vector that we wish to estimate, \(z_k = [\psi_{xx} \psi_{yy} w_t]^T\), and model parameters \(R_e\) and \(L_m\) are assumed to be constants.

The simulation results of estimating the three states \(\psi_{xx,k}, \psi_{yy,k}, w_{t,k}\) using EKF, UKF and PF are shown in Figure 6(a)–6(i), respectively. Also, the estimation RMSEs for the states (with respect to the noise-free data) are shown
in Table 2. It can be observed from Figure 6 and Table 2 that the PF shows improved estimation performance over UKF and EKF, even with abrupt changes in estimated states. The EKF and UKF estimations were not as accurate as the ones corresponding to PF due to the essential limitation of the UKF and EKF in nonlinear environments. Also, the estimation improvement of UKF over EKF is due to the fact that the UKF yields an improved utilization of model linearization.

| Technique | $\psi_{tx}$ | $\psi_{ty}$ | $w_r$ |
|-----------|-------------|-------------|-------|
| EKF       | 2.60E-05    | 3.32E-05    | 9.14E-06 |
| UKF       | 1.36E-05    | 1.42E-05    | 4.96E-06 |
| PF        | 2.98E-06    | 4.81E-06    | 5.20E-07 |

3.3. Simulation-based analysis of the effect of the number of estimated states on the performances of EKF, UKF and PF

In this section, we are interested in examining the effect of the number of estimated states and parameters on the estimation performances of EKF, UKF and PF in estimating the states and parameters of the IM model. Here, it is assumed that the state vector that we wish to estimate, $z_k$, includes the model states, $x_i = [\psi_{tx} \; \psi_{ty} \; w_r]^T$, as well as some, or all, of the following model parameters that are assumed to be unknown: $R_s$, $L_m$. Hence, the following equations are assumed to describe the evolution of model parameters:

$$\theta_{ik} = \theta_{i,k-1} + \gamma_{i,k-1}, \tag{34}$$

for $i = \{1, 2\}$, i.e. the parameters are a stationary process, with identity matrix, driven by white noise, $\gamma_{i,k}$. Incorporating the evolution of the parameters into model (32), it becomes

$$\psi_{tx,k} = \psi_{tx,k-1} + \left[-\frac{R_s}{L_r} \psi_{tx,k} + (w_a - w_{t,k}) \psi_{ty,k} + R_s \frac{L_m}{L_t} \frac{\partial}{\partial z_{t,k}} i_{tx,k} \right] \Delta t + \omega_{k-1},$$

Figure 6. Simulation-based estimation of state variables using various state estimation techniques – a comparative study.
\[
\psi_{ty,k} = \psi_{ty,k-1} + \left[ -\frac{R_{r,k}}{L_r} \psi_{ty,k} - (w_{a} - w_{r,k})\psi_{ty,k} \right. \\
+ R_{r,k} \frac{L_{m,k}}{\partial L_r} i_{sy,k} \left. \right] \Delta t + w_{k-1}, \\
\quad w_{r,k} = w_{r,k-1} + \left[ -\frac{L_{m,k}}{L_r} (\psi_{rx,k} i_{sy} - \psi_{ty,i_{sy}}) - \frac{1}{J} \omega \right] \Delta t + w_{k-1}. \\
\tag{35}
\]

Figure 7. Simulation-based estimation of the IM model parameters using EKF for the two cases – case 1: (a), case 2: (b), (c).

Figure 8. Simulation-based estimation of the IM model parameters using UKF for the two cases – case 1: (a), case 2: (b), (c).
Defining $\theta_k$ as the vector of the parameters to be estimated, which is described by model (3), we can write the model of IM as given in Equation (4). For example, if we are interested in estimating the three states, $\psi_{rx,k}, \psi_{ry,k}$ and $w_{k}$, and two other parameters (e.g. $R_t, L_m$), i.e. $\theta_k = [R_t \ L_m]^T$, then $z_k = [x_k^T \ \theta_k^T]^T = [\psi_{rx,k} \ \psi_{ry,k} \ \psi_{tx,k} \ \psi_{ty,k} \ \psi_{rx,k} \ \psi_{ty,k} R_t \ \psi_{tx,k} \ \psi_{ty,k} L_m]^T$.

Here, we will consider the following two cases, which are summarized below, where it is assumed that the three states (the rotor flux $\psi_{tx}$, the stator flux $\psi_{ty}$ and the rotor speed $w_r$) are measured.

1. **Case 1** The three states ($\psi_{tx}$, $\psi_{ty}$ and $w_r$) along with the first parameter $R_t$ will be estimated.
2. **Case 2** The three states ($\psi_{tx}$, $\psi_{ty}$ and $w_r$) along with the two parameters ($R_t$ and $L_m$) will be estimated.

The estimation of the state variables and parameter(s) for these two cases was performed using the four-state estimation techniques, EKF, UKF and PF, and the estimation results for the model parameters using these techniques are shown in Figures 7–9. For example, Figure 7(a) shows the estimation results of the parameter $R_t$ in case 1 using the EKF algorithm, and Figure 7(b) and 7(c) shows the estimation results of the parameters $R_t$ and $L_m$ in case 2 using the EKF algorithm. Also, Tables 3 and 4 compare the performances of the four estimation techniques for the two cases. For example, for case 1, Table 3 compares the estimation RMSEs for the three state variables $\psi_{tx}$, $\psi_{ty}$, and $w_r$ (with respect to the noise-free data) and the mean of the estimated parameter $R_t$ at steady state (i.e. after convergence of parameter(s)). Table 4 presents similar comparisons for case 2.

It can be seen from the results presented in Tables 3 and 4 that in all cases, the PF outperforms the EKF and UKF (i.e. provides smaller RMSE for the state variables). These results confirm the results obtained in the first comparative study, where only the state variables are estimated. The advantages of the PF over the EKF and UKF can also be seen through their abilities to estimate the model parameters. For example, EKF could estimate one parameter in case 1 (see Figure 7(a)), and it could not converge for the second parameter in case 2 (see Figure 7(b) and 7(c)), where it is used to estimate the two parameters. UKF could estimate one parameter in case 1 (see Figure 8(a)), but it took much longer time to estimate a second parameter in case 2 (see Figure 8(b) and 8(c)). The PF, on the other hand, could estimate the two parameters in both cases (cases 1 and 2), even though it took longer to converge in case 2, where the two parameters are estimated (see Figure 9).

The results also show that the number of estimated parameters affects the estimation accuracy of the estimated state variables. In other words, for all estimation techniques, the estimation RMSE of $\psi_{tx}$, $\psi_{ty}$ and $w_r$
increases from the previous comparative study shown in Section 3.2 (where only the state variables are estimated) to case 1 (where only one parameter, \( R_t \), is estimated) to case 2 (where the two parameters, \( R_t \) and \( L_m \), are estimated). For example, the RMSEs obtained using EKF for \( \psi_{r0} \) in the comparative study in Section 3.2 and cases 1 and 2 of the this comparative study are 2.60E−05, 6.75E−05, and 3.28E−04, respectively, which increase as the number of estimated parameters increases (refer to Tables 3 and 4). This observation is valid for the other state variables \( \psi_{ri} \) and \( w_i \) and for all other estimation techniques, UKF and PF.

### Table 4. Simulation-based RMSEs of estimated states for EKF, UKF and PF for case 2.

| Technique | \( \psi_{r0} \) | \( \psi_{ri} \) | \( w_i \) | \( R_t \) | \( L_m \) |
|-----------|----------------|----------------|---------|---------------|----------|
| EKF       | 3.28E−04       | 3.31E−04       | 7.42E−05 | Did not converge | Did not converge |
| UKF       | 2.87E−05       | 6.17E−05       | 3.67E−05 | 0.0559         | 2.171    |
| PF        | 5.61E−06       | 1.74E−05       | 2.75E−06 | 0.0559         | 2.171    |

4. Conclusions

In this paper, state estimation techniques are used to simultaneously estimate the state variables and model parameters of NCIMs on the basis of a third-order electrical model. Various state estimation techniques, which include the EKF, UKF and PF, are compared as they are used to achieve this objective. Two comparative studies have been conducted to compare the estimation performances of these three estimation techniques. In the first comparative study, EKF, UKF, and PF are used to estimate the four state variables (the rotor speed, the rotor flux and the stator flux) of the NCIMs model. In this second comparative study, the state variables and model parameters are simultaneously estimated, and the effect of the number of estimated parameters on the performances of the three estimation techniques is investigated. The simulation results of both comparative studies show that the PF provides a higher accuracy than the EKF and the UKF due to the limited ability of the EKF to deal with highly nonlinear process models. The results of the second comparative study show that, for all techniques, estimating more model parameters affects the estimation accuracy as well as the convergence of the estimated states and parameters.

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