CCD Photometry of the Globular Cluster M5. I. The Color-Magnitude Diagram and Luminosity Functions

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ABSTRACT

We present new $BV_I$ photometry for the halo globular cluster M5 (NGC 5904 = C1516+022), and examine the $B$- and $I$-band luminosity functions (LFs), based on over 20,000 stars — one of the largest samples ever gathered for a cluster luminosity function. Extensive artificial star tests have been conducted to quantify incompleteness as a function of magnitude and cluster radius. We do not see evidence in the LF of a “subgiant excess” or of a discrepancy in the relative numbers of stars on the red-giant branch and main sequence, both of which have been claimed in more metal-poor clusters.

Enhancements of $\alpha$-element have been taken into account in our analysis. This improves the agreement between the observed and predicted positions of the “red-giant bump”. Depending on the average $\alpha$-element enhancement among globular clusters and the distance calibration, the observed discrepancy between the theoretical and observed position for a large number of clusters (Fusi Pecci et al. 1990) can be almost completely removed.

The helium abundance of M5, as determined by the population ratio $R$, is found to be $Y = 0.19 \pm 0.02$. However, there is no other indication that the helium abundance is different from other clusters of similar metallicity, and values calculated for other helium indicators are consistent with $Y \approx 0.23$.

The relative ages of M5, Palomar 5, M4, NGC 288, NGC 362, NGC 1261, NGC 1851 and NGC 2808 are derived via the $\Delta V_{HB}$ method using M5’s horizontal branch (HB) as a bridge to compare clusters with very different HB morphology. We conclude that at the level of $\sim 1.5$ Gyr these clusters of comparable metallicity are the same age with the possible exception of NGC 288 (older by $3.5 \pm 1.5$ if the reddest NGC 288 HB stars are on the zero-age horizontal branch) and Palomar 5 (which may be marginally younger). Even with NGC 288 set aside, there is a large range in HB morphology between the remaining clusters which appears to eliminate age as the sole second parameter determining HB morphology in the case of constant mass loss between RGB and HB (although a Reimers’ mass-loss relation weakens this statement considerably).

We are unable to choose between the two competing values for M5’s (absolute) metallicity: $[\text{Fe/H}] = -1.40$ (Zinn & West 1984) and $-1.17$ (Sneden et al. 1992) based on recent high-dispersion spectroscopy. This level of discrepancy has a significant effect on the derivation of the distance modulus and absolute age of M5. From subdwarf fitting to the main sequence of the cluster, we find an apparent distance modulus $(m - M)_V = 14.41 \pm 0.07$ for $[\text{Fe/H}]_{M5} = -1.40$, and $14.50 \pm 0.07$ if $[\text{Fe/H}]_{M5} = -1.17$. From comparisons with theoretical isochrones and
luminosity functions, we find an absolute age for M5 of 13.5 ± 1 Gyr (internal error, assuming perfect models and no [M/H] error) for the Zinn & West abundance scale and 11 ± 1 Gyr for the higher abundance value.

Subject headings: globular clusters: general — globular clusters: individual (M5) — stars: luminosity function — stars: abundances — stars: distances — stars: interiors

1. Introduction

Stellar population studies of Galactic globular clusters (GGC) have played a critical role in the last 40 years of efforts to understand stellar evolution for ~ 1M⊙ stars in the metal-poor regime. The luminosity function (LF) for evolved stars gives a direct measure of the time stars spend in each phase of post-main-sequence evolution, so that the LF is an expression of physical processes occurring deep in the interior of the stars. Renzini & Fusi Pecci (1988) give an excellent comprehensive review of the critical role accurate LFs for GGC have to play in evaluating the adequacy of the canonical evolution models and in deriving values for key quantities such as helium abundance, the extent of convective overshooting, neutrino cooling rates, and mass loss on the giant branches.

However, the challenges to observers presented in Renzini & Fusi Pecci have not yet been met. Until very recently, the only means of measuring large samples of evolved stars in GGC was via photographic plates. The difficulties of accurate photometry in crowded fields with a non-linear detector are sufficiently large that LFs including both evolved stars and main sequence stars have been rare. The first LF studies covered the clusters M92 (Tayler 1954), M3 (Sandage 1957), and M13 (Simoda & Kimura 1968) down to about the main sequence turnoffs. The typical precision of the photometry for even the bright giants was such that the separation of the asymptotic giant branch and first ascent giant branch was difficult, and the effects of blending and completeness on the observed LFs were very difficult to quantify with photographic data (although see Buonanno et al. 1994 for a state-of-the-art photographic study of M3).

Early CCD-based studies had the potential to increase the dynamic range of LFs enormously, but because of the small field size of early CCD cameras, the number of evolved stars was typically far too small to be useful in testing evolution models. Attempts to mosaic small-field CCD images and piece together LFs (Bergbusch 1993, Bolte 1994), meshing photographic- and CCD-based studies (Bergbusch & VandenBerg 1992; hereafter BV92), and deriving composite LFs combining the results from CCD studies of several clusters (Stetson 1991) have produced some intriguing results (see discussion below). However, there are caveats that accompany all of these techniques, and the results require verification.

The wide-field CCDs now in use can cover much larger areas than the first generation of CCDs, so that for the first time it is possible to derive accurate LFs for large samples of stars in all phases of evolution. It is still not a trivial task to measure these LFs because of the computing requirements and various large and subtle effects resulting from extreme crowding in typical GGC, but the way is clear for LF investigations of stars in a large number of GGC.

In addition to the “classical” tests of stellar evolution, two unexpected observations of the past few years require further investigation. In a LF formed from the combination of the CCD-based LFs of the three metal-poor clusters M68 (NGC 4590 = C1236-264), NGC 6397 (C1736-536), and M92 (NGC 6341 = C1715+432), Stetson (1991) found an excess of stars on the subgiant branch (SGB) just above the main-sequence turnoff (MSTO). Bolte (1994) observed the metal-poor cluster M30 (NGC 7099 =
C2137-174) using a mosaic of small-field CCD images and found a similar excess of SGB stars. This excess is intriguing as it could be the observable result of an unusually extended isothermal core in MSTO stars, as would be produced by the actions of “WIMPs” (Faulkner & Swenson 1993). (The SGB is often defined differently by different authors. We will take it to be the transitional region between the main sequence turnoff and the base of the red giant branch at the point of maximum curvature.)

Another unexpected observation involving the LFs is the mismatch between theoretical predictions and the observed size of the “jump” dividing the main sequence (MS) and the red giant branch (RGB). When a theoretical LF is normalized to the MS, there is an excess of giants observed relative to MS stars (Stetson 1991, BV92, Bolte 1994). These results might be explained by the action of core rotation (Larson, VandenBerg & DePropris 1995). Because of the potential importance of non-standard physics in stars, and because of the caveats associated with earlier LF studies, the most productive next step is to derive better LFs in a number of GGC.

M5 is ideal for these types of investigations. It is one of the most massive clusters having moderate central density ($\log(M/M_\odot) = 5.6$ and $\log (\rho_0/(M_\odot/p^3)) = 4.0$; Pryor & Meylan 1993), is fairly close ($\sim 8$ kpc; Peterson 1993), and is at high galactic latitude ($b = 46^\circ.8$). Thus, it is possible to measure a large sample of post-MS stars with minimal field star contamination. Because the well-studied globular cluster M3 (NGC 5272 = C1339+286; Buonanno et al. 1994, and references therein) has physical characteristics that very much resemble those of M5, a comparison of the two data sets should be quite valuable. In addition, M5 is part of a set of clusters (with NGC 288 = C0050-268 and NGC 362 = C0100-711) showcasing the “second parameter” effect on HB morphology. One additional fact makes M5 unusual: according to Cudworth & Hanson (1993), M5 has the largest space velocity of the globular clusters studied to date, with a very eccentric orbit that takes it far from the plane of the galaxy. What impact its orbit may have on the intrinsic properties of the cluster is not clear.

In the next section, we describe our observations of the cluster. In §3, we describe the reduction and calibration of the program frames. In §4, we discuss the features observed in the color-magnitude diagram. In §5, we present the results of artificial star experiments that were executed to determine incompleteness in the sample, and present the LFs. Finally, in §6, we discuss the constraints that can be put on the global parameters of the cluster — metallicity, distance, and age.

2. Observations

The data used in deriving the $B$- and $I$-band LFs of M5 were taken on June 14/15, 1993 at the Cerro Tololo Inter-American Observatory (CTIO) 4 m telescope. In all, five exposures of 120 s and one exposure of 10 s were made in $B$, and six exposures of 60 s and one exposure of 6 s were made in $I$. All frames were taken using a 2048 $\times$ 2048 pixel “Tek #3” CCD chip having a sampling of about 0\'\'48 per pixel, and a field 16\'3 on a side. The exposure times were chosen so that stars were observed on all of the frames if they had magnitudes placing them on the horizontal branch (HB), or on the principal sequence, fainter than the level of the HB and brighter than about two magnitudes below the turnoff. Our five 120 s $B$-band exposures were combined into one master $B$ frame, and the six 60 s $I$-band exposures were averaged together to create two master $I$-band frames, with the exposures being divided between a good-seeing and a bad-seeing set. These longer exposures were averaged together to reduce the effects of background noise. The two short exposures (one $B$ and one $I$) were kept separate. Stars that were saturated on the long exposures were only measured on the short exposure frames, and stars up to two magnitudes brighter than the detection limit.
were only measured on the long exposure frames. All of the frames were centered approximately on the
cluster center, giving us a total radial coverage of about 8′.

The night of these 4 m observations was not photometric. In order to set the observations on a
standard photometric system, we used observations made at the CTIO 0.9 m telescope on two photometric
nights (June 16/17 and 19/20, 1993). The CCD used was the “Tek #1” 1024 × 1024 chip, having a size
of 0″396 per pixel, for a total sky coverage of 6″75 on a side. Landolt (1992) and Graham (1982) standard
star observations were used to calibrate a secondary field that overlapped the 4 m field. The secondary field
corresponds to the M5 West region used by Stetson & Harris (1988) to calibrate their M92 data. On the
two photometric nights, a total of 4 B, 7 V, and 8 I exposures were taken. A sample of 118 stars having
13.0 < V < 18.5 and −0.06 < (B − V) < 1.26 was calibrated as secondary standards in this way.

Supplementary observations of M5 were made in V band on July 7/8, 1994 on the CTIO 4 m telescope.
Three exposures of 10 s and three exposures of 60 s were taken. Again, the shortest exposures were left
separate, while the longer exposures were combined into one frame. The “Tek #4” 2048 × 2048 CCD chip
was used during these observations. The pixels had a sky coverage of 0″44 each. In addition to the smaller
field of view for these observations, the frame center was offset west from the center of the previous CTIO
4m observations by approximately 2″3.

3. Data Reduction

3.1. Primary Standard Calibration Fields

All of the primary standard observations were made using the CTIO 0.9 m telescope, and only the first
and last of those four nights of observing were photometric, as determined from later analysis of aperture
photometry results. For the present discussion, we will concentrate on issues related to the reduction of
photometric data that will be used to calibrate the 4 m observations.

For the 0.9 m frames, the bias level and pattern was removed by subtracting both a fit to the overscan
region and a master zero-level frame. We found that the twilight flats removed the remaining trends on
the frames (especially low-frequency variations) in the best manner, and so they were used exclusively in
flat-fielding the object frames.

3.1.1. Aperture Photometry

Primary standard stars were observed on the two photometric nights and on one nonphotometric night
(June 18/19, 1993). The standard star observations were taken at a range of airmasses spaced throughout
the nights in order to determine the extinction coefficient, and the fields themselves were chosen to give
good color coverage. Data from the nonphotometric night was only used to help constrain the color terms
in the transformation equations. Aperture photometry was performed using the program DAOPHOT II
(Stetson 1987). Using the aperture photometry data, growth curves were constructed for each frame, in
order to extrapolate from the flux measurements over a circular area of finite radius to the total flux
observable for the star. For this purpose, the program DAOGROW (Stetson 1990) was employed.

The aperture magnitudes and the known standard system magnitudes of Landolt (1992) and Graham
(1982), with some updated values from Stetson & Harris (1988), were then used to derive coefficients of the
transformation equations, which are as follows:

\[ b = B + a_0 + a_1 \cdot (B - V) + a_2 \cdot (X - 1.25) + a_3 \cdot t \]
\[ v = V + b_0 + b_1 \cdot (B - V) + b_2 \cdot (X - 1.25) + b_3 \cdot t \]
\[ i = I + c_0 + c_1 \cdot (V - I) + c_2 \cdot (X - 1.25) + c_3 \cdot t, \]

where \( b, v, \) and \( i \) are observed aperture photometry magnitudes, \( B, V, \) and \( I \) are the standard system magnitudes, \( X \) is airmass, and \( t \) is Universal Time (UT) of observation. The time terms were only added after an initial calibration using the zero point, color, and airmass terms showed a very slight trend in the residuals with this variable. As the strength of a color term is a partial reflection of the mismatch between the transmission curve of the filter used and that of the standard system filter, a \((V - I)\) term was used in the \(I\)-band transformation equation, instead of a \((B - V)\) term. The primary standards used in the calibration covered a color range \(-0.3 < (B - V) < 1.3\). The color-dependent term was also determined on the nonphotometric (third) night by a similar method, with the photometric zero point of each frame allowed to vary independently of the others, so as to remove the effects of variable extinction during the night. As a result, neither airmass nor time terms were used in the transformation equations for that night.

To improve the evaluation of the color terms (which should be fairly consistent from night to night), some measurements were eliminated. The stars 110-502 and T Phe D (Landolt 1992) were removed due to their extremely red colors. As these were the only stars observed with \((B - V) > 1.5\), their inclusion would have required several additional color terms to model. The largest effect of this choice is on \(B\) band, which matches the standard filter transmission most poorly. Higher order color terms would have been necessary to effectively calibrate this band because of the rapidly falling intensity distribution from red stars in this wavelength band — the measured \(B\)-band magnitude would be abnormally sensitive to temperature changes. With the linear term used here we get consistent values from night to night. The systematic nature of magnitude errors for \(B\)-band measurements for red stars should be kept in mind though — stars with \((B - V) > 1.5\) could be measured too faint relative to standard values by approximately 0.04 mag, if we can judge from the residuals of 110-502 and T Phe D.

The color terms derived for the three nights on which standard stars were observed were then combined in an average weighted by the calculated errors in the terms. These color terms are presented in Table 1a, along with the error in the mean. The derived values for the color coefficients were fixed in a redetermination of the coefficients of the other terms for the photometric nights. The values of these coefficients are shown in Table 1b. The time term was only found to be significant on the first night.

While it has been found that there have been reports of shutter delays of several tens of milliseconds on the telescopes at CTIO, none of our exposure times on the 4 m or 0.9 m telescopes were shorter than 6 s. As a result, the shutter corrections are 0.005 mag at most. We have opted not to make any attempt to compensate for this.

With the coefficients determined, the observed magnitudes of the standard stars were run through the transformation equations, and multiple measurements were combined to get a file of observed values of the primary standard magnitudes. A comparison with the published values was made to determine the average residuals. (In this and all subsequent comparisons, the residuals are calculated in the sense of ours – theirs.) The average residuals for these stars are expected to be low; the comparison for the sample of 38 stars is shown in Figure 1, and the average residuals are given in Table 3.
3.2. Secondary Standard Calibration

The M5 secondary standard stars were calibrated from the observations on June 16/17 and 19/20 via a method similar to that used for the primary standards. The unsaturated stars found in all frames with magnitude placing them above the turnoff were included in the first list. Stars which appeared to fall in the RR Lyrae gap were excised, as were stars that fell too far from the fiducial lines of the cluster, and stars that had neighbors within about 10 pixels. Three blue horizontal branch stars were returned to the list even though they were not present on all frames, so as to improve the color coverage of the sample. Once the list was finalized, all other stars were subtracted from the frames.

Aperture photometry was taken for the stars on these secondary standard frames. Repeated observations were combined with weights based on the square of the measurement error in order to create the final library of magnitude values. The values for a total of 118 stars are provided in Table 2, along with cross identifications to M5 calibration standards in Stetson & Harris (1988). The field is approximately 9′ west of the cluster center — a finding chart is provided in Figure 2. A comparison of values for 34 stars is shown in Figure 3, and the average residuals are given in Table 3. Values from Stetson & Harris (1988) were not used in the comparison, however, as it was found that there was a trend in residuals with Y coordinate (approximately 0.07 mag in B over 500 pixels, and 0.04 mag in V). Newer values provided by Stetson (1994b) did not show this effect. There is still a trend in the residuals with color that would account for the difference in V magnitudes. This may be the result of our smaller primary standard color range compared to Stetson & Harris. Since our sample is optimized for the range of color in which all of the comparison stars fall, we will continue to use our library of secondary standards to calibrate the CTIO 4 m data.

The secondary standard field covers the majority of the faint star field from Arp (1962), and to a lesser extent the middle field from Richer & Fahlman (1987; hereafter RF). The secondary standards that RF calibrated were fainter than any we used, so no comparison will be made here.

3.3. Object Frames

3.3.1. Profile Fitting Photometry

The CTIO 4 m data were reduced using the standard suite of programs developed by Peter Stetson: DAOPHOT/ALLSTAR/ALLFRAME (Stetson 1987, 1989b, 1994a). Stars were chosen on each of the master B, V, and I frames in order to determine the point-spread function (PSF), and its variation with position on the frame. Each frame was divided into 25 bins, and the 15 brightest stars from each subsection were put into a list. The average FWHM for these stars was calculated, and stars which deviated from this value by more than 3σ were discarded. The radial profiles of the stars were then viewed to beyond the radius to be included in the PSF to eliminate stars with contaminants, such as faint companions. Typically about 150 stars were used in determining the PSF and its spatial variation on each frame. There was a definite bias against crowded portions of the field in the selection of PSF stars. However, because the frames were centered on the cluster, PSF variations across the frame were mapped well.

Three passes were made using a combination of DAOPHOT’s FIND routine and ALLSTAR to derive a star list for each frame. These star lists were combined according to filter of observation, and then the filter master lists were combined into a master star list. Stars were only kept if they were detected on at least one frame in each filter. The master list of stars, and the frames and their PSFs were supplied as input into ALLFRAME (Stetson 1994a), which simultaneously reduces stars on all of the frames at once in order to
derive consistent positions.

The correcting lens used in taking the V frames on the CTIO 4 meter telescope had the effect of modifying the coordinate system of the V-band frames. While linear coordinate transformations did a reasonable job in mapping between frames from the two runs, there was a scale change in the sky coverage of the pixels with distance from the center of the chip. This should not affect the photometry (as the PSF variations should easily take this into account), but we resorted to running ALLFRAME on the V frames separately. The BVI overlap sample was derived by matching the V and BI samples together.

3.3.2. Calibration

Two different samples were created from the B, V, and I frames, and each was calibrated separately. Because the B and I frames were better centered and had better seeing, the sample was larger by about 25%. 54 of the secondary standards observed in the CTIO 0.9 m sample also fell on these 4 m frames. The transformation equations used in the calibration were:

\[
\begin{align*}
    b &= B + a_{0,k} + a_1 \cdot (B - I) + a_2 \cdot (B - I)^2 \\
    i &= I + c_{0,k} + c_2 \cdot (B - I)^2.
\end{align*}
\]

The coefficients of the color terms in this case were \( a_1 = -0.0349 \), \( a_2 = 0.0349 \), and \( c_2 = -0.0095 \). A weakness of this calibration is the fact that only one extremely blue (\((B - I) < 0\)) and three extremely red stars (\((B - I) > 2\)) fell in the section of overlap.

The V-band frames from the 1994 run were taken with the cluster off center, resulting in a smaller overlap with the earlier B and I frames. So, a smaller sample consisting of all stars detected in B, V, and I was also calibrated using the equations:

\[
\begin{align*}
    b &= B + a_{0,k} + a_1 \cdot (B - V) + a_2 \cdot (B - V)^2 \\
    v &= V + b_{0,k} + b_1 \cdot (B - V) + b_2 \cdot (B - V)^2 \\
    i &= I + c_{0,k} + c_2 \cdot (V - I)^2.
\end{align*}
\]

The color terms are shown in Table 1c.

The size of the CTIO 4 m frames means that most other surveys of M5 overlap the program area at least partially. Table 3 provides a summary of several of the zero-point offsets for comparisons with several of these studies. The field of the Buonanno, Corsi, & Fusi Pecci (1981; hereafter BCF81) survey is completely included on all 4 m frames. We clearly see scale differences (a systematic trend in the residuals with magnitude), color-dependent residuals, and zero-point differences between our photometry and their B- and V-band data of BCF81, as is typical for the other photographic surveys.

The frames also completely enclose the inner field of RF. They tabulate values for 12 stars in common with BCF81, and for the 7 secondary standards they used in this field. There does seem to be a small systematic trend of \( \Delta(B - V) \) with \((B - V)\) across the entire range of color in our comparison with RF. We have also made a larger comparison with the photometry of Storm, Carney & Beck (1991; hereafter SCB) for a region corresponding to field IV of Arp and BCF81, as shown in Figure 4. We have 1584 stars in common, but we only show comparisons for the 384 stars with \( V > 17.5 \) as fainter stars in SCB’s photometry have much larger uncertainties. A trend in the color residuals with color also seems to be
present, in the same sense as in the RF comparison. We are inclined to believe our results due to more extensive primary and secondary calibrations.

Calibrated $I$-band data for the cluster is rather sparse. The best source of comparison is with the photoelectric photometry of Lloyd Evans (1983). Figure 3 shows the comparison with his sample of 22 stars from the Arp (1962) sample, with two additional stars near the red-giant tip. This comparison is of some importance, as the position of the upper RGB in $V$ and $I$ band is often used as a metallicity indicator (Da Costa & Armandroff 1990; Sarajedini 1994). There does appear to be systematic zero point offsets in $V$ and $I$ bands of approximately 0.02 mag, in the sense that our values are brighter. (It should be noted that there is good agreement between the average zero-point offsets in $V$ band for the comparisons with RF, SCB, and Lloyd Evans (1983) data.) The zero-point differences are small enough that they should not present a problem in using the upper RGB to determine the metallicity. The star TLE1, which is near the tip of the RGB, has the largest absolute residual of any star in the comparison sample. The large residual for this star may be partly the result of our calibrations procedure, as the color of the star $(V = 12.056, (B - V) = 1.653, (V - I) = 1.746)$ puts it outside of the range of colors over which the calibration was optimized. However, it is also the star of Lloyd Evans’ sample that is in the most crowded region, and would be most likely to be affected by background considerations. The agreement with the photoelectric value is reassuring.

4. The Color-Magnitude Diagram

Table 4 presents the fiducial lines determined from the CMD of the cluster. We also include the number of stars used in the calculation of the fiducial line for each bin. Due to the poorer quality of the $V$-band data, special care had to be taken to ensure that the effects of blends and crowding did not overly influence the values derived. Fiducial points for the MS and lower RGB were determined by finding the mode of the color distribution of the points in magnitude bins. Blends of stars produce significant redward biases in the fiducial determination, particularly on the MS, so that the use of the mode is preferred over a mean. A cut was made on projected radius ($r > 350$ pix = 2.8′) to restrict the sample to the best measured stars. The position of the SGB was determined by finding the mode of the star distribution in color bins. This was done because the SGB is close to horizontal in the CMD, so that mode finding in magnitude bins was affected by the blend sequence.

The fiducial line on the RGB was determined by finding the mean color of the stars in magnitude bins. Once a mean was determined, stars falling more than $5\sigma$ from the fiducial point were discarded (so as to eliminate AGB and HB stars, as well as blends and poorly measured stars), and the mean redetermined. This procedure was iterated until the star list did not change between iterations. The lower portion of the AGB was also measured in this way. At the tips of the RGB and AGB, the positions of individual stars were included as fiducial points if they appeared to be continuations of the mean fiducial line. The fiducial line for the HB was determined by determining mean points in magnitude bins for the blue tail, and in color bins for the horizontal part of the branch. It should be stressed that the HB fiducial points represent the most populated portion of the HB, and not the zero-age line. For the RGB, AGB and HB, a color error cut ($\sigma_{B-V} < 0.015$) was used to restrict the sample to the best measured stars. All sequences were checked for continuity in regions of the CMD where fiducial points could be derived by different methods. No smoothing has been applied.

Figure 3 shows a comparison between the fiducial sequence of the present data set and those of BCF81.
and RF. Also included is BCF81's determination of the fiducial lines of the RGB and AGB from Simoda & Tanikawa's (1970) data. There is fairly good agreement among the data sets. For example, our fiducial points fall directly between the RF inner and outer field values at the turnoff, and also match the SGB quite well. Our fiducial is bluer than either set of RF points lower on the MS by as much as 0.04 mag. This is probably not surprising, as RF determined their fiducial points by taking means in magnitude bins. The mean would tend to be redder than the mode on the main sequence due to the inclusion of unresolved blends or binaries. However, our fiducial points for the lower RGB also lie to the blue of the RF points. The upper RGB fiducial points show some systematic disagreement with those derived from the photographic studies, but this is not significant in light of the systematic trends known to exist in the photometry from those studies (see §3.3.2). The AGB points agree very well with those of BCF81.

The BI fiducial was easier to determine due to the larger sample, better seeing for the frames (which improved the photometry in the cluster center), and the better sensitivity of the \((B - I)\) color to surface temperature differences (color errors are smaller relative to the total color range, compared to \((B - V)\)). Only a radial cut was applied \((r > 200\ \text{pix} = 1'6)\). Mean RGB points are also tabulated for \((I, B - I)\) in order to allow us to compare the position of the giant branch with those of other clusters (Da Costa & Armandroff 1990).

In Figure 7, we plot the total samples for both the \(BV I\) (28339 objects) and \(BI\) (42456 objects) sample. It is apparent that the quality of the \((I, B - I)\) data is quite superior to the \((V, B - V)\) data. Both samples cover the center of the cluster. Several conclusions can be gleaned from the \((I, B - I)\) CMD. First, it should be possible to make distinctions between stars on different evolutionary branches once the most crowded regions are removed. Second, there is a fair number of galaxies to be found on the frames. This is the reason for the population seen well to the red of the MS. Third, there is a hint of a blue straggler star sequence between the MS turnoff and the blue end of the HB (which becomes obvious for more restrictive cuts on the sample).

Figure 8 presents plots of restricted subsets of the data plotted in Figure 7. The total \(BV I\) and \(BI\) samples were sorted according to CHI value (CHI is related to the quality of the object image; see Stetson 1989a). Less restrictive cuts were used for the bright stars (above the SGB) in order to maintain the definition of the branches in the CMD. In the \((V, B - V)\) diagram, for stars with \(V < 17.4\) only stars with \(\text{CHI} < 1.3\) were kept, whereas for \(V > 17.4\), only stars with \(\text{CHI} < 1.07\) were kept. In the \((I, B - I)\) diagram, \(\text{CHI} < 1.6\) for \(I < 16.5\), and \(\text{CHI} < 1.09\) for \(I > 16.5\).

5. Determination of the LF

5.1. Artificial Star Tests

A particularly crucial component of the calculation of an accurate LF is the determination of incompleteness corrections. For the data at hand, crowding is the primary source of incompleteness. Because of the large range of crowding conditions present in the observations, it is necessary to determine these corrections as a function of position (projected radius) as well as magnitude. To this end, we carried out extensive artificial star tests to correct the data for biases of various sorts.

The bias-subtracted and flat-fielded \(B\)- and \(I\)-band frames were the only ones used in this portion of the data reduction (although the long exposure \(I\) frame with poorer seeing was discarded in order to reduce the amount of computer time used in the calculations). The PSFs we used were simply those
determined by the procedure outlined in §3.3.1. The output of the reduction of these frames using DAOPHOT/ALLSTAR/ALLFRAME without artificial stars is considered the “control” run. A virtually identical procedure was followed in doing the artificial star experiments. The starting data were: the $B$ and $I$ frames, the PSFs for each frame, the fiducial lines of the cluster and an initial luminosity function. A theoretical luminosity function was initially used to give the probability that an artificial star would have a particular magnitude. The set of artificial stars was thus weighted toward the faint end. Initially, artificial stars were only allowed to be brighter than about two magnitudes below the turnoff in $I$ band, so as to allow the program to create an adequate number of artificial red giant stars for analysis. This procedure has the advantage of allowing us to mimic the LF of the cluster so that we can get a fair estimate of the amount of photometric scatter created by blending.

The fiducial lines of the cluster were determined in instrumental magnitudes from the control run. Once a magnitude was picked in one band ($I$-band in our runs), the magnitude in the other band was chosen so as to put the artificial star on the fiducial. Positions were chosen randomly, so that they had an even distribution on the frame. Each star was given a consistent position on all frames for which the star fell in the field of view. Given the previously determined form of the PSFs for each of the frames, the artificial star images were placed on the frame using the DAOPHOT ADDSTAR routine. The resulting frames were put through a reduction procedure identical to that for the control run (starting after the PSF determination) to ensure that the artificial stars were treated exactly like the real stars. No more than 3000 artificial stars were put on the frames at any one time so that the artificial stars did not significantly affect the completeness at any radius. In 20 artificial star runs, a total 48,058 artificial stars were added and reduced.

The output from each artificial star run was a list of stellar positions and magnitudes for all stars that were classified as “detected.” It is not a trivial matter to determine which detected stars should be identified as input artificial stars. Faint artificial stars that happened to be placed on bright real stars would not be detected as such. If you simply look at the positions at which you know artificial stars were placed, your search would identify the bright real star as the artificial star. If you decide to discard the star in the above example, you would be throwing away data on the extreme limit of the blending problem — how the photometry of a cluster of stars is affected by the imperfect resolution of the instrument.

In keeping with this philosophy, a simple positional search was conducted on the artificial star run photometry, and the input magnitudes were recorded, along with the final magnitudes if any object was found at the input position (to within a pixel). Limited computer time forces us to try to decouple the effects of blending and crowding, even though they are clearly linked to increases in stellar image density on images. Crowding effects can be corrected by determining sample completeness versus magnitude and projected radius. A correct simulation of blending requires adding stars several magnitudes fainter than the range of magnitudes in which we are interested. If a real star is blended with the faintest artificial star we added, the resultant star will be brighter by a certain magnitude increment, which will be nonzero. In practice, the fainter the added star the more negligible the increment.

It is then necessary to reanalyze the artificial star sample so that artificial stars that were blended with much brighter stars are labeled as undetected and so will count toward incompleteness. One pass was made through the artificial star data to make preliminary calculations of the external errors in the photometry $\sigma_{\text{ext}}$ by looking at the scatter in the differences between the input and output magnitudes. A second pass was then made, and any artificial star that was detected more than 1.0 magnitude brighter than its input magnitude or more than $5\sigma_{\text{ext}}$ from the fiducial was labeled as undetected. The resulting sample of recovered artificial stars was then used to calculate the following quantities in bins sorted by projected
radius and magnitude: 1) median magnitude \((B\) or \(I\)), 2) median color \((B-I)\), 3) median internal error estimates \((\sigma_B\) or \(\sigma_I\), and \(\sigma_{B-I}\)), 4) median magnitude and color biases \((\delta_B \equiv \text{median}(B_{\text{output}} - B_{\text{input}})\) or \(\delta_I\), and \(\delta_{B-I}\)), 5) median external error estimates \((\sigma_{\text{ext}}(B) \equiv \text{median}(|\delta_B - \text{median}(\delta_B)|/0.6745\) or \(\sigma_{\text{ext}}(I)\), and \(\sigma_{\text{ext}}(B-I)\)), and 6) total recovery probabilities \((F(B)\) or \(F(I)\) — the fraction of stars added to the bin that were recovered with any magnitude). The width of each magnitude bin was adjusted so as to put equal numbers of recovered artificial stars in each, in order to try to maximize the accuracy of the median for quantities 1) - 5). The binning for the total recovery probabilities was done somewhat differently — bins containing equal numbers of recovered stars were used along with the unrecovered stars in the bin to give \(F\). This method loses sensitivity near the detection limit. To characterize this region, bins containing equal numbers of unrecovered stars were formed, and the number of recovered stars in the bin was used to give \(F\). For magnitudes for which larger numbers of artificial stars were used, several bins were combined so as to reduce the noise in the determination of the incompleteness quantities. The result was very well-determined curves for \(F\) in the different radial bins.

The runs of these quantities naturally show noise, and do not cover the entire range of detected real stars. In order to smooth out noise and to enable reasonably accurate extrapolation beyond the magnitude range of the artificial stars, we looked for functions that would fit the data. The functional forms chosen for accuracy and stability were: 1) magnitude and color error estimates:

\[
\sigma(B) = \sigma_0(B) + \exp[a_1 + a_2 B]
\]

The magnitude errors are well fit by a simple exponential — the errors go up substantially at the completeness limit as the typical size of the noise peaks becomes comparable to the peak size of the stars under consideration. 2) magnitude biases:

\[
\delta(B) = \delta_0(B) + b_3 B - \exp[b_1 + b_2 B]
\]

The magnitude bias tends to become more negative with increasing magnitude in crowded regions and at the completeness limit. In other words, the artificial star is measured brighter, whether due to contamination from the light of other stars, or due to noise spikes that create a detection bias for stars that fall on positive spikes. The linear term was included to take account of low-level trends with magnitude seen in the outermost radial bins. These trends are not visible in plots in similar studies (Figures 21 and 41 of Stetson & Harris 1988, Figure 16 of Bergbusch 1993) due to scale. These trends only amount to 0.002 magnitudes of bias per magnitude, so it was not necessary to correct for this in the CMD. At the completeness limit there is real scatter in the median points. 3) recovery probabilities \((0 \leq F \leq 1)\):

\[
F(B) = 1 - \exp[c_1 + c_2 B - \begin{cases} 
0 & \text{if } B < B_{\text{crowd}} \\
(c_3 + c_4 B) & \text{if } B \geq B_{\text{crowd}} 
\end{cases}
\]

where \(B_{\text{crowd}} = -c_3/c_4\). There are three magnitude regimes inherent in this formulation. The first is the complete sample — for the brightest stars in the cluster, there are not enough stars of equal or greater brightness to allow them to get lost. The linear portion of the curve models the effects of crowding (see Figure 3 of Stetson 1991), and essentially just reflects the growing number of stars bright enough to create blends with a star of the given magnitude and significantly modify the measured brightness. Finally, the exponential cutoff fits the detection limit of the frames, probably reflecting the fact that noise spikes are less and less likely to be able to move a given faint star above the detection threshold. In our sample, the linear regime begins approximately at the cluster turnoff for the outermost radial bins, due to the rapidly rising number of stars. It begins brighter in the innermost radial bin due to the background of unresolved stars.
This represents the practical limit to our ability to find all of the stars in the frame of a given brightness under the seeing conditions in the atmosphere and with the resolution capabilities of the telescope.

Errors on each of these quantities were estimated using a similar procedure. The points in each radial bin were adjusted upwards and downwards by a fraction of the errors on the points, and the adjusted points were fitted using the same functional forms. The fractional adjustment was chosen so that the adjusted curves encompassed the majority of the points between them. The constraint that $F$ approach 1 for bright stars was retained in all cases, with the result that the error in $F$ at the bright end is effectively zero. The results of these calculations for $2'$ radial bins are shown in Figures 9-12.

The data derived from the artificial star tests were used to derive corrections to the observed luminosity function according to the procedure outlined in Bergbusch (1993), which was based on Stetson & Harris (1988), and ultimately on Lucy (1974). This method uses the error distribution, magnitude bias, and recovery probability to predict the form of the observed luminosity function $\eta(m)$, given an initial guess for the “true” luminosity function $\phi(m_t)$:

$$\eta(m) = \int_{-\infty}^{+\infty} \phi(m_t) P\{m \mid m_t\} dm_t$$

where $P\{m \mid m_t\}$ is the probability that a star of true magnitude $m_t$ is measured at magnitude $m$. Following Stetson & Harris, we will assume that this probability distribution has the form

$$P\{m \mid m_t\} = \frac{F(m_t)}{\sqrt{2\pi}\sigma(m_t)} \exp\left(-\frac{[m - m_t - \delta(m_t)]^2}{2\sigma^2(m_t)}\right),$$

or in other words, a set of stars of true magnitude $m_t$ will be measured with a Gaussian distribution around the biased magnitude $m_t + \delta(m_t)$. The initial guess for the true LF can be corrected iteratively by comparing the observed LF to the predicted LF and adjusting the bins that contribute to discrepant predicted bins according to

$$\phi^{r+1}(m_t) = \frac{\int_{-\infty}^{+\infty} [\tilde{\eta}(m)/\eta(m)] \phi^r(m_t) P\{m \mid m_t\} dm}{\int_{-\infty}^{+\infty} P\{m \mid m_t\} dm}$$

where $\tilde{\eta}(m)$ is the observed LF. This procedure matches the observed LF best in just a few iterations, even though a calculation indicates that $\chi^2$ is still decreasing (Lucy 1974).

A few comments should be made about the usefulness of this procedure. Clearly it is designed to account for magnitude biases that would cause stars to move in magnitude from bin to bin. This effect is especially important in regions where crowding effects are important and near the detection limit, as can be seen from Figure 7. This procedure, however, cannot correct for blending effects explicitly. This can be done most simply and effectively by restricting the sample to those regions of the cluster in which the effect is least important — uncrowded regions in the outer parts.

Once this procedure is completed, the completeness fraction $f$ can be calculated simply by taking the ratio of the predicted number of observed stars to the true number of stars (taken from the derived “true” LF). The calculated values of the completeness fraction were also fitted using the functional form used for the total recovery probability $F$. The value of $f$ for a star of any magnitude can then be calculated by interpolation in the values at various radii. To calculate the total LF, the real observed stars can just be read in (multiplied by a factor of $f^{-1}$ to account for incompleteness) and binned. No star was included if the completeness factor $f$ was less than 0.3. This condition defines a projected radius cut-off for each magnitude bin.
One additional step was then taken to try to increase the accuracy of the LF values for the RGB, where Poisson statistical noise dominates over incompleteness. For the faintest magnitude bin considered, we cover the minimum amount of cluster area fairly completely ($f > 0.3$). For the brightest stars in the sample, we are essentially assured of detecting them anywhere on the frame — even into the core of the cluster. Because of this, we decided to count all of the bright stars in the sample. Following Bolte (1994), we scale the total number of stars detected to the number detected within the area for which the faintest bin in the LF was more than 30% complete. In other words, the corrected number of stars in a bright bin is $N'_i = sN_i$, where $N_i$ is total number of stars detected in the cluster that fell in magnitude bin $i$. The normalization factor is

$$s = \frac{\sum_{i=n_1}^{n_2} N_i(r > r(n_{tot}))}{\sum_{i=n_1}^{n_2} N_i},$$

where $n_{tot}$ is the faintest bin in the LF. $r(n_{tot})$ is the projected radius at which stars in the faintest bin are 30% complete. Bins in the range $n_1 \leq i \leq n_2$ must be 100% complete, or already corrected for the incompleteness as a function of radius. By definition, $s \leq 1.0$.

The scale factor $s$ was determined for a magnitude range on the lower RGB, and the value was applied to all of the magnitude bins included in this bright sample. We found that the value of $s$ remained quite constant on the lower RGB, but increased within a couple of magnitudes of the RGB tip. This is an indication that there may be a deficiency of giants on the upper RGB in the center of the cluster (a larger fraction of the stars in those bins were from the outskirts of the cluster). The constant value found on the lower RGB is more likely to reflect a population that has not been affected by dynamical processes in the cluster. By using this value of $s$ for all bright magnitude bins, we are able to get “global” values for the LF in these bright bins, incorporating the deficiencies of RGB stars. The deficiency of upper RGB stars in the core of the cluster is reflected in the population of the innermost annulus in Table 7.

The LFs determined via the procedure outlined in §5 are given in Table 5. A total of 22044 and 21388 detected stars were used in the calculation of the cluster’s true LFs in $B$ and $I$ bands respectively. The faint portion of the sample was restricted to projected radii greater than 450 pixels from the center of the cluster for $B > 17.8$ (and for $I > 16$ in its LF) in order to diminish the effects of blends. The normalization regions for the two LFs were $21 < B < 21.5$ and $19.6 < I < 20.1$.

### 5.2. Image Blending

The effects of optical blending on the photometry of cluster stars must also be properly gauged and eliminated in order to ensure the validity of the star counts. The effects of blending are quite obvious in CMDs of dense regions of globular clusters. Blends involving MS or RGB stars can be seen in a swath bounded on the faint side by the fiducial line of the cluster, and on the bright side by the same fiducial offset by about 0.75 mag. Blends are most important on the main sequence and subgiant branch because of the sheer numbers of stars that can potentially be blended — if one of the stars being blended is too faint relative to the other, it will not change the luminosity of the blended star far from that of the brighter component. On the main sequence, blended stars can potentially be mistaken for binary stars. On the subgiant branch, they can add erroneously to brighter bins. Figure 13 shows the full effect of blending on the $B$-band LF when no restrictions are made.

In the turnoff region and at the base of the RGB, the blend problem is especially insidious because two stars of approximately the same brightness create a blended star that falls very near the cluster fiducial
because the fiducial is vertical at that point in the CMD. Such stars will then pass any tests that reject stars based on distance from the cluster fiducial. An additional effect is created by the horizontal nature of the SGB in $B$-band. Blended SGB stars move more nearly perpendicular to the fiducial line of the cluster, and are therefore much more likely to be eliminated by cuts based on proximity to the fiducial. In the case shown in Figure 13, when crowded regions are included in the sample, the LF can appear to reflect that of a more metal-poor or older cluster (see discussion in §6.5.2).

One can potentially use Monte Carlo simulations to derive statistical corrections for blending. However, the simplest method of reducing the problem is to measure large samples of stars in relatively sparse regions of a massive cluster. In this way, the effect of blending can be minimized, and a comparison of the LF derived at large radii with an LF including crowded regions gives an estimate of the size of the effect.

The choice of filter band can also change the impact of blended stars on the LF. In $V$ band, the subgiant branch has a relatively flat slope in the CMD. As a result, the entire subgiant branch falls within 0.75 mag of the turnoff, and so is especially susceptible to blending. In the survey at hand, we have used both $B$ and $I$ magnitudes to construct luminosity functions. In $B$ band, the subgiant branch covers a smaller range of magnitudes than in $V$ band, and so the LF will have the same problems as a $V$ band LF. In $I$ band, the SGB has a much steeper slope in the CMD. As a result, the blend sequence will make contributions over a wider range of magnitudes, and will be diluted in proportion.

Careful attention was paid to the upper RGB in an attempt to push the LF as bright as possible. It is difficult to distinguish between individual AGB and RGB stars near the tip, although the fiducial sequences can be followed. We found that our procedure for eliminating outliers in the CMD was too conservative on the upper RGB. To circumvent this, we varied the threshold value of the external error used to make the cut until we found an optimal value that seemed to keep RGB stars and eliminate AGB stars. There will undoubtedly be contamination to an extent, and this will be the dominant source of systematic error.

We will defer the remainder of the discussion of the LFs until §6.4.

6. Discussion

Because there are a large number of parameters (most importantly, chemical composition, distance, and age) that influence the photometry of a globular cluster, we now discuss the ways in which each of these parameters can be constrained, and the expected reliability of the results we derive. In the discussion that follows, we will generally assume $E(B - V) = 0.03 \pm 0.01$.

6.1. Metallicity

Sneden et al. (1992) provide a summary of many of the metallicity determinations for M5 made in the past 20 years. Among medium- and high-resolution spectral analyses of giants using techniques similar to those by Sneden et al., the $[\text{Fe/H}]$ values range from $-1.09$ to $-1.5$. The authors found that they were able to reconcile four out of the five studies with their measured value of $<[\text{Fe/H}]>= -1.17 \pm 0.01$ from 13 giants. Sneden et al. also briefly discuss four determinations based on low-resolution spectrophotometric methods that were calibrated using stars having high-resolution data. Three out of four of these studies also returned metallicity values at the high end of the range, with the Zinn & West (1984; hereafter ZW84) study being at the low-metal end. The ZW84 data were calibrated using high-resolution data from Cohen
(1983), which returned $[\text{Fe/H}] = -1.4$.

While the majority of the determinations are consistent, the absolute zero points for the metallicity scales in the various investigations are certainly offset relative to each other. This is an important issue because the absolute metal content does play a role in the choice of isochrones and also in our determination of the distance modulus via subdwarf fitting. Of the high-resolution spectroscopic determinations, the Sneden et al. result is to be preferred in an absolute sense for their use of improved input physics and spectra. The results of the Lick-Texas group (see Kraft et al. 1995 for the latest in this series) also form an excellent relative abundance system.

If the Lick-Texas group results are directly compared with ZW84 measurements, there is a noticeable nonlinearity - in the sense that the metal-rich and metal-poor ends of the range are systematically higher in the ZW84 scale, and systematically lower at intermediate metallicities (around that of M5). With a sample of 21 clusters having high dispersion spectroscopy, Carretta & Gratton (1996) find that a second order fit is preferred to a linear fit. These studies provide additional reason for considering metallicity scales beside that of ZW84. We will continue to regard the absolute metallicity of M5 as uncertain at the 0.2 dex level, and we will attempt to propagate the this range through the following analysis.

Several methods of checking the relative metallicity are open to us using the BVI data set. Three indicators are $\Delta V_{1,4}$ (Sandage & Wallerstein 1960), $(B - V)_{0,g}$ (Sandage & Smith 1966), and RGB position in $(M_I, (V - I)_0)$ (Da Costa & Armandroff 1990).

Figure 14 shows the CMD for the stellar sample used to calculate $(B - V)_{0,g}$ and $\Delta V_{1,4}$. From the CMD, it is apparent that there is a nonzero slope to the horizontal branch across the RR Lyrae strip. From 20 stars nearest the blue edge of the RR Lyrae strip we get $V_{HB,be} = 15.079 \pm 0.029$, and from 18 stars near the red edge of the strip we get $V_{HB,rc} = 15.105 \pm 0.021$. From 11 RR Lyrae variables for which we have intensity-weighted mean $V$ magnitudes (SCB), we get $V_{RR} = 15.057 \pm 0.028$. There appears to be a slight offset between our data and that of SCB, so we will just adopt the value $V_{HB} = 15.092 \pm 0.017$ in order to remain consistent with the rest of our photometry.

From the fiducial line of the cluster, we can derive the colors of the RGB at the level of the HB: $(B - V)_g = 0.855 \pm 0.002$ and $(V - I)_g = 0.966 \pm 0.003$. The quoted errors reflect the error in the determination of $V_{HB}$ and the slope of RGB. The adopted reddening gives us $(B - V)_{0,g} = 0.83 \pm 0.01$. The magnitude at which $(B - V)_0 = 1.4$ is found to be $V_{1,4} = 12.422 \pm 0.022$, which gives a value of $\Delta V_{1,4} = 2.66 \pm 0.02$. The primary source of error in $V_{1,4}$ and $(B - V)_{0,g}$ is the uncertainty in the reddening, which we take to be $\pm 0.01$.

We can also evaluate the same quantities for other clusters of similar metallicity. We include NGC 288 (Bergbusch 1993), NGC 362 (Harris 1982), NGC 1261 (Ferraro et al. 1993), NGC 1851 (Walker 1992a), NGC 2808 (Ferraro et al. 1990), M4 (NGC 6121; Kanatas et al. 1995), and Palomar 5 (C1513+000; Smith et al. 1986), and give the calculated values in Table 6. Comparison of $\Delta V_{1,4}$ values indicates that the total scatter among the seven clusters corresponds to about 0.35 dex in [Fe/H] according to the linear relationship in ZW84 (0.2 dex if M4 is left out). By the $(B - V)_{0,g}$ measure, the total scatter corresponds to a scatter of about 0.3 dex in [Fe/H]. M5 does appear to have a metallicity consistent with this group of clusters. We must keep in mind that the RGB above the level of the HB may be abnormally blue, or the HB may be abnormally faint, since the $\Delta V_{1,4}$ measurement seems secure. As ZW84 noted, M3 (Buonanno et al. 1994) and M5 have very similar $\Delta V_{1,4}$ values, despite having metallicities that appear to differ by 0.27 dex by other measures.
We can also compare the upper RGB as measured in \((I, V - I)\) with those of M15, NGC 6397, M2, NGC 6752, NGC 1851, and 47 Tucanae, as presented in Da Costa & Armandroff (1990). (As a reminder, the metallicities in that study use the ZW84 zero point.) A disadvantage of using this method is that it also requires a distance modulus — we use the value derived in §6.3.1 via subdwarf fitting to the MS, corrected for the difference between \(A_V\) and \(A_I\) (using the reddening law given by Cardelli, Clayton & Mathis 1989). (In deciding not to use a distance modulus derived from fitting the HB, we are breaking with Da Costa & Armandroff’s distance scale because of the possibility that M5’s HB may be abnormally faint.) A comparison is shown in Figure 15. The positions of the RGBs indicate that M5 has a metallicity slightly higher than those of NGC 6752 \([\text{Fe/H}] = -1.54; \text{Armandroff} & \text{Zinn} 1988\) and M2 \([\text{Fe/H}] = -1.58\).

Alternately we can apply a lower value of the reddening \((E(V - I) = 0.03, \text{corresponding to } E(B - V) = 0.02)\) with the corresponding distance modulus. When this is done, we find that the metallicity appears to be closer to that of NGC 1851 \([\text{Fe/H}] = -1.29; \text{Armandroff} & \text{Zinn} 1988\), The agreement of the RGBs in slope is better in this case. Using the RGB fiducials as reference points, we can constrain M5’s metallicity to the range \(-1.5 \gtrsim \text{[Fe/H]} \gtrsim -1.3\), which nicely brackets the ZW84 value of -1.40.

On the basis of these considerations, we find that M5 is one of the most metal-poor members of this metallicity group (listed in Table 6) in a relative sense.

6.1.1. Abundances of \(\alpha\)-Elements

M5, like other globular clusters, is observed to have enhancements in the abundances of the \(\alpha\)-elements (Pilachowski, Olszewski & Odell 1983; Gratton, Quarta & Ortolani 1986; Sneden et al 1992). The overall enhancement of \(\alpha\)-element abundances makes scaled-solar abundance isochrones appear, to good accuracy, slightly more metal-rich, or in other words, redder and fainter (Chieffi, Straniero & Salaris 1991; Chaboyer, Sarajedini & Demarque 1992) This shift is primarily due to increased envelope H\(^-\) opacity due to electron contributions from low ionization-potential \(\alpha\)-elements. To first order, it is possible to remove the effects of \(\alpha\)-element enhancement from consideration except when we must discuss characteristics that differentiate between \([\text{Fe/H}]\) and \([M/H]\), the overall metal content relative to the sun.

Figure 16 shows a comparison of a preliminary \(\alpha\)-enhanced isochrone \(([\alpha/\text{Fe}] = +0.3; \text{VandenBerg} 1995)\) with oxygen-enhanced isochrones (BV92). There are several features to notice here. First, to very good accuracy, the MS of the \(\alpha\)-enhanced isochrone parallels those of the oxygen-enhanced isochrones. Second, the RGB of the \(\alpha\)-enhanced isochrone initially parallels the oxygen-enhanced ones, but becomes slightly steeper toward the RGB tip. This probably results from the recombination of electrons with certain of the \(\alpha\)-elements at the lower temperatures, which helps to reduce the H\(^-\) opacity at the surface. Oxygen does not contribute to the surface opacity on the RGB due to its relatively high ionization potential. For surface temperatures near that of the turnoff, oxygen does make a contribution fairly near the surface, making the turnoff redder (Rood 1981, Rood & Crocker 1993). In the figure, the \(\alpha\)-enhanced and oxygen-enhanced isochrones remain parallel through the turnoff. (For the most metal-poor BV92 isochrones, the oxygen enhancement continues to increase, meaning that the oxygen-enhanced isochrones will tend to get redder at the turnoff relative to constant \(\alpha\)-enhancement isochrones. This does not affect the metallicities we will be dealing with.) At the same time, the increased energy generation from the CNO cycle actually drives a slight decrease in luminosity on the subgiant branch (Rood 1981). As a result, the main shape differences between the oxygen-enhanced and \(\alpha\)-enhanced isochrones occur only in the luminosity of the subgiant branch and the color of the upper RGB. Based on these details we will make our analysis using the complete set of
oxygen-enhanced isochrones, keeping in mind the model differences.

The effect of an $\alpha$-enhanced composition on the LF can be seen in Figure [1]. The enhancement has the effect of shifting the SGB fainter and decreasing the luminosity of the RG bump. With the exception of the luminosity of the RG bump, and the exact height of the SGB jump, the $\alpha$-enhanced LF matches an oxygen-enhanced LF 0.3 dex more metal-rich. The different heights of the SGB jump indicate a slight difference in the slope of the SGB in the CMD (see § 6.4.1). Each of these effects is consistent with making the $\alpha$-enhanced models appear much like an oxygen-enhanced model of higher metallicity.

There is some uncertainty in the absolute amount of $\alpha$-element enhancement, and this should be kept in mind. According to Chieffi et al. (1991), an enhancement in $\alpha$-element abundance by a factor $f_\alpha$ changes the effective metal content according to the formula

$$[M/H] = [\text{Fe}/H] + \log(0.638 f_\alpha + 0.362).$$

For the $\alpha$-enhancement observed in M5 by Sneden et al. ($[\alpha/\text{Fe}] \sim +0.2$), a correction of −0.14 is needed to bring the $[M/H]$ value to an $[\text{Fe}/H]$ scale, if we use the relation given by Salaris, Chieffi & Straniero (1993). The Sneden et al. result is consistent with an average enhancement of $[\alpha/\text{Fe}] = +0.3$ seen for a number of globular clusters (see Kraft et al. 1995 for references). From here on, when we consider the ZW84 metallicity scale, we will use $[\text{Fe}/H] = −1.40$ and $[M/H] = −1.19$, and when we consider the Lick-Texas group scale (Sneden et al. 1992), we will use $[\text{Fe}/H] = −1.17$ and $[M/H] = −1.03$.

6.2. Helium Abundance

In the following sections, we re-examine the helium abundance indicators for M5 and for other clusters of similar metallicity.

6.2.1. Population Ratios and the $R$ Indicator

In post-main-sequence phases of evolution, numbers of stars are directly proportional to the evolutionary timescale, and as a result, the ratios of populations in different phases reflect the efficiency of energy generation and mixing processes. The most used ratios are

$$R = N_{\text{HB}}/N_{\text{RGB}}, \quad R' = N_{\text{HB}}/(N_{\text{RGB}} + N_{\text{AGB}}), \quad R_1 = N_{\text{AGB}}/N_{\text{RGB}}, \quad R_2 = N_{\text{AGB}}/N_{\text{HB}}, \quad \text{and} \quad R_{HB} = (N_{\text{BHB}} - N_{\text{RHB}})/(N_{\text{BHB}} + N_{\text{RR}} + N_{\text{RHB}}),$$

where $N_{\text{BHB}}$, $N_{\text{RR}}$, and $N_{\text{RHB}}$ are the numbers of blue horizontal branch, RR Lyrae variable, and red horizontal branch stars respectively. The ratios $R$ and $R'$ have been used as an indicator of helium content $Y$ (Iben 1968, Buzzoni et al. 1983). This primarily reflects the dependence of the progress of the hydrogen-burning shell on the hydrogen content of the envelope material being fed in (and to a lesser extent, the change of helium core mass at helium flash, which affects the HB luminosity). $R_1$ and $R_2$ have been used to confirm the existence of semiconvective zones outside the helium-burning core of HB stars (Renzini & Fusi Pecci 1988), as such zones feed extra helium into the core, prolonging the HB phase. The values we calculate for $R_1$ and $R_2$ fall within the range of values published for clusters over a range of metallicities. This indicates that HB stars develop semiconvective zones during their evolution that mix extra helium into the core, prolonging their stay on the HB. The ratio $R_{HB}$ (Lee 1989) was intended to be a quantitative indication of HB morphology.

Because the stars in these phases are bright, it is possible to calculate these population ratios for any sample of stellar photometry covering the entire HB, and the samples used are more likely to be complete
even into the center of the cluster. We have chosen to use the \((I, B - I)\) dataset because of the larger sample, better seeing, and the superior temperature resolution of \((B - I)\). This allows us to use stars over a wider range of radii, as even stars whose measurements have been somewhat contaminated can be identified with their true evolutionary phase with high confidence.

Photometric scatter due to crowding effects was significant enough in the innermost 30″ of the CTIO 4 m data that this region was excluded from consideration. Known field stars (Cudworth 1979, Rees 1993) have also been removed from the data. The results of population counts and ratios are shown in Table 7. The RR Lyrae sample was composed of those stars known to be variable (Sawyer Hogg 1973; Kravtsov 1988, 1991; Kadla et al. 1987), and stars determined to be variable to good probability (see §6.2.2). The RGB sample is defined as the stars brighter than the average bolometric luminosity of the RR Lyrae variables. To define the cutoff magnitude for our sample, we used the HB models of Dorman (1992b) to determine the average luminosity and absolute \(V\) magnitude for the RR Lyraes (defined as where \(\log T_{\text{eff}} = 3.85\)), and the isochrones of BV92 to determine the corresponding quantities for the RGB. These two theoretical studies form a set of evolutionary tracks with consistent physics, so that the values derived should be correct in a differential sense. Using the \([\text{Fe/H}] = -1.26\) tracks, we find \(\log(L/L_\odot)_{\text{HB}} = 1.645, M_{V,\text{HB}} = 0.626,\) and \(M_{V,\text{RGB}} = 0.891\), giving us a bolometric correction of 0.265 mag in \(V\). (A similar calculation produces 0.23 mag for M3’s metallicity, in good agreement with the bolometric correction of 0.22 mag used by Buonanno et al. (1994).) From the \((V, B - V)\) dataset, we determined \(V_{\text{HB}}\), and applied the bolometric correction.

So, based on this, we used \(B_{\text{RGB}} = 16.21\) for the faint end of the RGB population.

To reduce the number of misidentifications among RGB and AGB stars, we compared with samples taken from uncalibrated data for the core of M5 from the Canada-France-Hawaii Telescope (CFHT) with the High-Resolution Camera in April, 1993. The higher resolution allows us to make nearly unambiguous decisions as to which population a bright giant star belongs to. In the comparison, we found no stars misidentified as RGB in the CTIO data set and AGB in the CFHT dataset, but found 8 stars given as AGB in the CTIO set and RGB in the CFHT data. This sort of bias is understandable, as the effect of blending is to increase the measured brightness of a star, so that it is easier to make AGB-like objects from RGB stars than vice versa. We have corrected for these misidentified stars in our population ratios, but the effect is undoubtedly present to some small extent in our data (and the datasets of other studies), as the field of view of the CFHT frames was small enough (2.2′ × 2.2′) that we were unable to correct the whole core region for this kind of effect. The CFHT frames are roughly centered on the cluster center, so that we are getting the region most affected.

We compare our samples to those of BCF81 and Brocato et al. (1995; hereafter BCR). BCF81’s numbers have been corrected for two RR Lyraes from Sawyer Hogg (1973) that were included in their list: I-104 (V21, identified as a blue HB star) and IV-96 (V76, identified as a red HB star). The overall agreement is quite good — there were some changes that resulted from improved photometry of several stars. The majority of the stars that we moved were actually red giants. This partly explains the large discrepancy between the numbers of RGB stars. Most of the remaining difference stems from the larger bolometric correction we used: 0.27 mag instead of the “average” bolometric correction for clusters of 0.15 mag (Buzzoni et al. 1983). The larger bolometric correction is more valid because the theoretical calibrations of the \(R\) method implicitly assume that the RGB cutoff is equal to the mean luminosity of the RR Lyraes. If we use the smaller bolometric correction, our RGB sample of BCF81 stars would be 149. A higher metallicity would result in a larger bolometric correction. We also include our sample for the region for which BCR tabulated data. It appears that BCR are significantly incomplete toward the center of the cluster, since we find many more RGB, HB, and AGB stars than they do. The surpluses even show up
when we compare with only the CTIO 4 m data set over the radial range $30'' < r < 120''$. This result is surprising since such bright stars should be visible into the center of the cluster (even though they may be difficult to identify with the correct phase of evolution). Curiously though, our sample also shows high values of $R$ and $R_1$ over this range of radii relative to the rest of the cluster. This should indicate that there are fairly large fluctuations in the sky distribution of the different populations, and that this kind of fluctuation can change the calculated population ratios substantially. We did not see signs of significant radial trends in the population ratios in our sample, however.

In re-examining the determinations of $R$ for other clusters in the same metallicity bin, we have found that the vast majority of calculations have used the “average” bolometric correction of 0.15 mag in $V$ in determining the faint end of the RGB sample. This value was used in the original Buzzoni et al. (1983) paper, but only because uncertainties of various sorts made the correction unimportant. However, the correction is a function of metallicity, and it is applied to the faint end of the RGB sample, which means this can introduce a significant systematic error. In Table 6, we attempt to correct the situation for the clusters in this metallicity bin. We include values for NGC 288 (Buonanno et al. 1984), NGC 362 (Harris 1982), NGC 1261 (C0310-554; Ferraro et al. 1993), NGC 1851 (C0512-400; Walker 1992a), NGC 2808 (C0911-646; Ferraro et al. 1990), and M4 (NGC 6121 = C1620-264; Cudworth & Rees 1990). Once this correction has been made, all of the $R$ values are consistent with each other to within the errors. They are also low with respect to that of M3 (Buonanno et al. 1994), and slightly high compared to that of 47 Tuc (Lee 1977). This could reflect a metal-dependent trend.

The values we derive for $R$ and $R'$ are quite a bit lower than typically quoted values for other clusters (Buzzoni et al. 1983; Buonanno, Corsi & Fusli Pecci 1985). However, the helium abundance derived for M5 in the Buzzoni et al. study ($Y = 0.20$) was also the lowest found for their sample of clusters. Using equation 11 of Buzzoni et al. for our total CTIO sample, we get $Y = 0.18 \pm 0.02$. For the combined CTIO and CFHT sample, we get $Y = 0.20 \pm 0.02$. (The error refers only to the statistical error in the population counts.) Identical results are derived from equation 12 of Buzzoni et al. for $R'$, and no significant change occurs if we use the calibration of Caputo, Martinez Roger & Paez (1987) for canonical assumptions about the mass of the helium cores in HB stars. (The absolute calibration of helium is somewhat uncertain, as it is unclear whether non-canonical assumptions, such as enhanced He-core mass for HB stars, are important.) For the M5 $R$ value to match the mean value of 1.35 found for clusters tabulated by Buzzoni et al. (1983), we would have to have missed 112 HB stars, or counted 83 RGB stars too many — requiring a change of over 17% in the sample for either case separately. (If we use the smaller bolometric correction, the RGB sample decreases from 494 to 452 stars, still requiring changes of approximately 10% to match the M3 ratio.) As can be seen from our incompleteness calculations for the LF, we should be 100% complete for stars this bright.

Figures 18 and 19 should also make it clear that misidentification is not a possible explanation. Figure 18 plots all stars that have been included in the RGB, HB (excluding RR Lyrae stars), and AGB populations, and that also have calibrated magnitudes from the CTIO 4 m data. For the majority of the stars plotted, the population was assigned based on position in the CMD. Also included are stars that were measured in both the CTIO 4 m and CFHT data. The population assignment for these stars was based on position in the CFHT CMD, while the magnitude and color plotted correspond to the CTIO 4 m measurements. This is why, for example, a number of blue HB stars appear in the instability strip, and why some RGB stars appear closer to the AGB. Misidentification is only a problem between the RGB and AGB, and for the overwhelming majority of these stars, photometric scatter or blending effects do not make any difference. Figure 19 plots the CFHT data (12739 objects total in the frames) for the cluster core.
The sequences in this data are extremely tight, and misidentification is not a factor. (A population of blue stragglers can be seen in the figure as well).

To examine the possibility that individual populations have been affected differently by cluster dynamics, we plot the cumulative radial distributions for the RGB, AGB, and HB stars used in the determination of the population ratios in Figure 20. This is the most extensive radial distribution published to date, as it completely covers 8′ in projected radius. All three distributions agree extremely well even into the center of the cluster, although the AGB stars may be slightly less centrally concentrated compared to the others. A K-S test indicates that the AGB stars belong to the same radial distribution as the RGB and HB stars at 66% and 64% probabilities respectively. (For comparison, the RGB and HB stars have an 80% probability.) The sample of AGB stars is large enough that statistical fluctuations are relatively unimportant. AGB star samples have in general not been large enough to make significant comparisons with RGB or HB stars, even in post-core collapse (PCC) clusters. There is some evidence for deficiencies of RGB stars compared to HB stars in the PCC cluster M30 (Djorgovski & Piotto 1993), but only in the innermost 20″ or so. As M5 has a King-model surface brightness profile (Trager et al. 1995), it is not surprising that there are no obvious dynamical effects evident in the distributions. None of the clusters of similar metallicity with good data seem to be in a post-core collapse phase, with the possible exception of NGC 362 (Trager et al. 1995). The clusters used in these calculations do cover a wide range of stellar densities though.

### 6.2.2. RR Lyrae Stars and the A Indicator

M5 is one of the most populous globular clusters in terms of RR Lyrae stars, and yet very few of these stars have been studied extensively enough to understand M5’s contribution to the Sandage period-shift effect (Sandage 1982). Although our photometry is not extensive enough to measure light curves, we did conduct a search for unidentified variables. As M5 is one of the nearest clusters and has moderate central density, our spatial resolution is enough to examine even the core of the cluster.

Coordinates from the list of Sawyer Hogg (1973) were transformed to the system of our frames, and stars were matched with the list based on this position, with a check to verify that the magnitudes agreed roughly. The 87 listed variable stars that fell within the frames were successfully identified (although apparently two digits in the Y coordinate for V39 were transposed — the correct value is −250.2). Two of the 87 variables are not RR Lyraes. In addition, the stars V102 and V103 that were listed as potential variables in Sawyer Hogg’s list have been confirmed as RR Lyrae variables by Kravtsov (1992).

For HB stars with V measurements, we determined the ratio of external magnitude error (determined from scatter in the measured magnitude values) to internal error (determined from deviations of the stellar profile from the point-spread function for the frames). Stars for which \( \sigma_{\text{ext}} / \sigma_{\text{int}} > 3.5 \) were identified as variable. This level was chosen based on the maximum background level for red giant stars of comparable magnitude. From these cuts, we identified 25 RR Lyrae star candidates — their coordinates on the system of Sawyer Hogg are included in Table 8. Most of these stars had been discovered previously by Kadla et al. (1987) or Kravtsov (1988, 1991). We have included stars from these three studies that we have not detected, so as to continue the numbering of Sawyer Hogg and these studies. Virtually all of these stars are in the innermost 1′ of the cluster — a region practically unreachable by the early photographic studies. Judging by the accuracy of the transformation from the Sawyer Hogg coordinate system to our own for the known variables, we estimate that these positions should be accurate to better than an arcsecond.
Based on the number of previously identified RR Lyraes that satisfied our cuts, we can calculate the completeness of this new sample, assuming that nondetection is due only to stars having unfortunate combinations of period and phase. Of the 80 RR Lyraes from the Sawyer Hogg list that had positions that put them in the $V$ frames and magnitudes that fell in the selected range, 61 were detected by the $\sigma_{ext}/\sigma_{int}$ cut, giving us a completeness of 76%. Of the samples of RR Lyrae candidates in Kadla et al. (1987) and Kravtsov (1988), we only recover 54% and 51% respectively, which may indicate that some of their candidates are not intrinsically variable. From what we have been able to understand of the two papers, not all of the variables have measured periods. However, we are unable to comment further on the reliability of their lists.

The helium indicator $A$ (Caputo, Cayrel & Cayrel de Strobel 1983) is related to the mass-luminosity relationship for stars inside the instability strip:

$$A = \log(L/L_\odot) - 0.81 \log(M/M_\odot).$$

$A$ has a relatively small sensitivity to helium abundance ($\partial A/\partial Y = 1.2$; Caputo et al. 1983), and also requires periods and pulsation amplitudes for cluster variables, if there are any. We can calculate values of $A$ for RR Lyrae stars in M5 measured by SCB, and using equation 16 of Carney, Storm & Jones (1992) (a relation between effective temperature, period, B-band amplitude, and metallicity) and the period relation of van Albada & Baker (1971). We restrict ourselves to tabulated RRab variables (V7, V8, V18, V19, V28, V30 and V32) due to the better determined effective temperature relation. We find $A = 1.80 \pm 0.02$, where the error is just the error in the mean. Comparing with values tabulated in the HB models of Dorman (1992b), we find that this value is slightly low, but consistent with $Y \approx 0.23$ to within the errors. We can also calculate values for RRab variables in NGC 1261 (10 variables; Wehlau & Demers 1977), NGC 1851 (15 variables; Wehlau et al. 1978, Wehlau et al. 1982), NGC 2808 (1 variable; Clement & Hazen 1989), M4 (15 variables; tabulated in Sandage 1990), M14 (40 variables; Wehlau & Froelich 1994), M3, and M15 (36 and 25 variables respectively; tabulated in Sandage 1990). The values are again shown in Table 6. (For M15 we find 1.87 ± 0.02.) Among the clusters tabulated here, there appears to be a trend in $A$ with $[\text{Fe/H}]$, which is not expected. In any case, we have no evidence from this indicator that the helium abundance in M5 is smaller than the norm for the globular cluster system.

### 6.2.3. The $\Delta$ Indicator

The indicator $\Delta$ (Caputo, Cayrel & Cayrel de Strobel 1983) is defined simply as the magnitude difference between the MS at $(B - V)_0 = 0.7$ and the HB at the instability strip. (Caputo et al. originally defined the HB point to be at the blue edge of the instability strip. We have chosen to revise this definition to make it more easily calculable theoretically and observationally. The various sensitivities of the indicator are unchanged by this revision.) As an indicator, it has the advantage of being fairly sensitive to the helium abundance ($\partial \Delta/\partial Y = 5.8$ mag; Caputo et al. 1983), and the disadvantages of having a definite metallicity dependence ($\partial \Delta/\partial [\text{Fe/H}] = -1$ mag / dex) and of requiring photometry from the HB to well below the MS turnoff. As found in [6.3], $V_{HB,be} = 15.079 \pm 0.029$, and from the fiducial line we find that $V_{0.7} = 20.88 \pm 0.03$. Thus, $\Delta = 5.80 \pm 0.04$. We can calculate theoretical values of $\Delta$ using a self-consistent set of isochrones (BV92) and HB models (Dorman 1992b) having $Y \approx 0.236$ in order to check the value in M5. For metallicities $[M/H] = -0.47, -0.65, -0.78, -1.03, -1.26, -1.48, -1.66, -1.78, -2.03$, and $-2.26$, we find values for $\Delta$ of 5.17, 5.38, 5.53, 5.76, 5.92, 6.07, 6.14, 6.21, 6.30, and 6.38 respectively. Regardless of the value of $[M/H]$ chosen for M5, the helium abundance indicated by $\Delta$ is consistent with the values
used in the theoretical models to within the errors. We also calculate this quantity for some additional clusters: NGC 288, NGC 1851, 47 Tucanae, M3, and M15 (Durrell & Harris 1993; $\Delta = 6.29 \pm 0.05$). These values are tabulated in Table 6. With the exception of NGC 1851, these values agree very well with the theoretical predictions (for M15, $[M/H] \approx -1.89$, for M3, $[M/H] \approx -1.45$, and for 47 Tuc, $[M/H] \approx -0.6$ are appropriate). There does not appear to be an obvious trend with $[\text{Fe/H}]$ among the clusters discussed here.

6.2.4. Other Indicators

A relative indication of $Y$ is found in the slope of the blue end of the HB (Caputo, Natta & Castellani 1973). The slope of the blue end of the ZAHB is extremely insensitive to metallicity, as can be seen in Dorman’s (1992b) models. The slope does vary significantly with the initial helium content given to the stars, in the sense that helium-deficient clusters should have more gently sloping (bluer) blue ends, and should have a fainter HB overall (Sweigart & Gross 1976; Dorman 1992a). In practice though, blue HB slope comparisons are untrustworthy due to the scatter in photometry (making it a difficult task to derive an accurate HB locus) and color calibrations (typically just a few blue stars are used to calibrate color terms for globular cluster observations, making color-dependent errors a real possibility). With these pitfalls in mind, we make several comparisons in Figure 21. The best comparison that can be made is with the “second-parameter” cluster NGC 288. Bergbusch (1993) derives a helium abundance $Y = 0.232^{+0.020}_{-0.036}$ for NGC 288 via the $R$-method using a much smaller sample of stars ($N_{\text{RGB}} = 56, N_{\text{AGB}} = 11$, and $N_{\text{HB}} = 77$).

If we shift the CMD using offsets derived from the relative age technique of VandenBerg, Bolte & Stetson (1990; hereafter VBS), we can look for the effects of different relative $Y$ values. The slope of the blue end is slightly smaller for M5, although not significantly so. Additional comparisons can be made with NGC 1851 (Walker 1992a) and M3 (Buonanno et al. 1994) on the blue side of the HB. NGC 1851 shows a marginally steeper slope, while M3 shows a shallower slope (although we must question why the overall slope of the HB is so much different from the other clusters examined). This test does not seem to indicate a significant difference in helium abundance.

From pulsational theory (Deupree 1977), it is expected that the width of the instability strip should increase with decreased helium content. Restricting ourselves to the stars with the lowest color errors, we find that the observed RR Lyrae gap for M5 lies between $(B-V) = 0.24 \pm 0.02$ and $0.47 \pm 0.01$. The width derived from M3 is virtually identical: $\Delta(B-V)_{\text{RR}} = 0.24 \pm 0.03$. The comparison with M3 does not indicate any difference in helium abundance. NGC 1851 cannot be used in this comparison as its HB appears to have a bimodal distribution that does not include the edges of the instability strip (Walker 1992a), while NGC 288 and NGC 362 do not have HB stars on both sides of the gap.

We conclude that despite the well-determined fact that M5 has an $R$ value indicating it is deficient in helium, there is no other evidence in the photometry that would support this. The helium abundances as derived from the other indicators appear consistent with determinations from other sources (see Boesgaard & Steigman 1985 for a review of older studies). For example, Campbell (1992) finds $(\text{He/H}) = 0.0759 \pm 0.014$ (corresponding approximately to $Y_P = 0.233 \pm 0.003$) from a sample of H II galaxies, while Olive & Steigman (1995) derive $Y_P = 0.232 \pm 0.003$ from helium abundances in metal-poor extragalactic H II regions, extrapolated to zero metallicity. At the same time, observations of D and $^3$He abundances provide a lower limit $Y_P \geq 0.238$ (Walker et al. 1991). As mentioned earlier, an unexpected metal-dependent trend in the $R$ values may be present in the limited sample examined here. An examination of BV92 isochrones and Dorman (1992b) HB tracks shows that the metallicity dependence of $R$ is negligible as Buzzoni et al.
(1983) indicate. So, any metallicity dependence is indirect at best. These possibilities should be examined
using a larger sample of clusters. However, the value of $R$ as a helium indicator is limited by the total
stellar populations in individual clusters, and little improvement can be expected beyond determinations
like the one for M5 presented in this paper. The indicator $\Delta$ is likely to provide a better handle on helium
abundance in the future.

6.3. The Distance Modulus

An accurately determined distance modulus for the cluster, combined with a known reddening value,
allows the fiducial line to be placed in a diagram of absolute magnitude versus color, and compared directly
with theoretical isochrones. In principle, the age of the cluster could be determined simply from this
comparison. However, any distance determination requires the assumption of a value for the metallicity.

6.3.1. Subdwarf Fitting

Subdwarfs having known metallicities and well-measured trigonometric parallaxes provide a calibration
of the brightness of the main sequence fiducial of the cluster. Our sample of 23 potentially-useful subdwarfs
(along with 5 subgiant stars) is provided in Table 9. The first 9 subdwarfs (which we call the “classical”
subdwarfs) are taken from Table II of Laird, Carney & Latham (1988). Due to M5’s high metallicity, we
 opted to include the star HD23439A ([Fe/H] = –1.02). Each of these stars has relative parallax errors of
$\sigma_\pi/\pi \leq 0.12$. We have supplemented this list with other subdwarfs identified from the literature as having
relatively small errors in their parallaxes ($\sigma_\pi/\pi \leq 0.30$). These stars are identified as halo population dwarfs
by their metallicities ([Fe/H] $\gtrsim$ –1.0). Five subgiants that pass the cut on relative parallax error are also
included.

The $V$ magnitudes and $(B - V)$ colors of these stars were taken from the Hipparcos Input Catalogue
(HIC; Turon et al. 1993). The values for the classical subdwarfs agree with those tabulated in Laird et al.
In any case, the accuracy of these values is certainly not the major contribution to the error in this method
— parallax error is. The parallaxes were taken from the General Catalogue of Trigonometric Parallaxes
(van Altena, Truen-liang Lee & Hoffleit 1991). The parallaxes of 15 stars used the values given in Dahn
(1994), as they often represented significant improvements, as judged by the agreement with theoretical
isochrones.

The absolute magnitudes were calculated using Lutz-Kelker corrections (Lutz & Kelker 1973) derived
by the method of Hanson (1979). These corrections are intended to rectify a systematic bias in samples
selected by proper motion or trigonometric parallax — stars with parallaxes measured too high are more
likely to be included in the sample. The original Lutz-Kelker corrections were derived with the assumption
that the sample stars have a uniform space density. This assumption implies particular parallax and proper
motion distributions (a probability distribution $P(\pi) \propto \pi^{-4}$ and a number distribution $N(\mu) \propto \mu^{-3}$, where
$\mu$ is proper motion). As a different spatial distribution would change the values of the corrections, following
Hanson, we examined the proper motion distribution of our subdwarf sample using values from Luyten
(1976) and found $N(\mu) \propto \mu^{-1.2}$. This result implies that proper motion and luminosity (or Malmquist)
selection biases are present in the sample in addition to the parallax selection bias. These effects act to
reduce the Lutz-Kelker corrections below the constant-density relation, since proper motion and luminosity
selection biases tend to discriminate against distant stars — an effect opposite to the parallax selection
bias. Because subdwarfs are selected largely on the basis of high proper motions, this bias is probably the more important. As a result of this finding, we have used Hanson’s relation for \( n = 2 \) (where \( P(\pi) \propto \pi^{-n} \) and \( N(\mu) \propto \mu^{-(n-1)} \)) to calculate the magnitude corrections. For the classical subdwarfs, these corrections are very small \((-0.08 \leq \Delta M_{L_K} \leq 0)\).

The metallicities for the subdwarfs were taken from Carney et al. (1994) when available, from Laird et al. (1988), Ryan & Norris (1991), and from literature values as compiled in Cayrel de Strobel et al. (1992) in other cases. The Ryan & Norris metallicity data were calibrated using values measured by Laird et al., so we can be reasonably assured that any systematic differences are small. We did change the metallicity value for only one star (HD 201891) that had a value in the Carney et al. survey, and only because all other measurements indicated a metallicity that was higher by 0.2 - 0.4 dex. For the subgiant stars (excluding HD 140283), we take the metallicities from Pilachowski, Sneden & Booth (1993) with a correction of –0.35 to account for measured zero-point differences.

There is a large body of evidence (see Lambert 1989) that indicates the \( \alpha \)-element abundances of the subdwarfs are enhanced in a manner similar to those of the globular clusters. In order to check how well the colors of the subdwarfs are predicted, we corrected the metallicities of the subdwarfs for an enhancement \([\alpha/Fe] \sim +0.3\), which corresponds to \([M/H] = [Fe/H] + 0.21\). Using a polynomial interpolation between the oxygen-enhanced 14 Gyr isochrones of BV92, we calculated the colors corresponding to \([M/H]\) for the subdwarf and to the value of \([M/H]\) for M5 for the absolute V magnitude of each subdwarf. The corrected colors and deviations of the measured colors from the relevant isochrone \((\delta B - V)\) are presented in Table 9, while the best fit of the M5 fiducial to these stars is shown in Figure 22. The agreement of the theoretical with the observed colors is quite good for the classical subdwarfs: \( \delta(B - V) = 0.010 \pm 0.015 \). About half of the other candidates have error bars that overlap with the predicted position. The fit for the subgiant HD140283 is reasonably good considering the controversy over its distance. The subgiant HD211998 also matches the predictions quite well. Although age differences among the stars could be problematic, these stars are potentially valuable in assessing whether the isochrones can predict giant branch colors in addition to those for the main sequence. With the release of Hipparcos parallax data in the near future, a set of well-measured subgiants could potentially provide a useful check of the variation of the mixing-length parameter \( \alpha \) with metallicity.

There appears to be a systematic color offset between a number of the subdwarfs and the theoretical predictions, as shown in Figure 23. The offset is approximately +0.08, and is in the sense that the subdwarfs are redder than their predicted positions. This difference cannot be explained by reddening (only one of the stars for which we have reddenings has \( E(B - V) \) as large as 0.02). A systematic shift in the metallicity scale by about 0.6 dex would be required to explain the difference, which is unlikely given that the quoted errors in the metallicities are approximately 0.2 dex. Small errors in the parallax could cause relatively large differences in the derived color for stars near the turnoff. This would not explain some of the fainter subdwarfs though. Several of these deviant subdwarfs have among the best measured parallaxes, which seems to rule out overestimation of the Lutz-Kelker corrections. The shape of the \( \alpha \)-enhanced isochrones in the region of the turnoff is probably different from the oxygen-enhanced isochrones we used, but this would only affect subdwarfs near the turnoff. Potential age differences between the subdwarfs and the isochrones are very unlikely to be large enough to explain the discrepancy as main sequence stars more than half a magnitude below the turnoff do not evolve enough in color (Figure 15 in BV92). In addition, comparisons of field subdwarf colors with turnoff colors of globular clusters indicate that the field subdwarfs have ages approximately equal to those of the halo globular clusters (Figure 5 in Gilmore, Kuijken & Wyse 1989; Figures 3-5 in Carney, Latham & Laird 1989). Some of the stars might be unresolved binaries, which could
explain many of the deviations. As a result of this unresolved problem, we decided to use only the classical subdwarfs (BD +66°268, HD 23439A, HD 25329, HD 64090, HD 103095, HD 134439, HD 134440, HD 194598, and HD 201891) in our determination of the distance modulus, as they agreed fairly well with the theoretical isochrones without any additional correction.

Given the metallicity of the subdwarf and M5, we computed a color shift for each subdwarf. The fiducial of M5 was then shifted in magnitude to match each subdwarf individually. The distance modulus values derived for each subdwarf in the sample were then combined in an average weighted by the squares of the error estimates for the absolute magnitudes. This error estimate includes the error resulting from the uncertainty in the parallax, the reddening, and in the metallicity (taken to be 0.2 dex for each subdwarf). The final error estimate is simply the error in the weighted mean. For \([\text{Fe/H}] = -1.40\), the derived apparent distance moduli for each of the classical subdwarfs are: BD+66°268, 14.49 ± 0.27; HD 23439A, 14.45 ± 0.21; HD 25329, 14.59 ± 0.56; HD 64090, 14.43 ± 0.28; HD 103095, 14.41 ± 0.10; HD 134439, 14.37 ± 0.18; HD 134440, 14.36 ± 0.19; HD 194598, 14.52 ± 0.43; and HD 201891, 14.38 ± 0.42.

With these subdwarfs, we derive an apparent distance modulus \((m - M)_V = 14.41 ± 0.07\), assuming M5’s metal content is \([\text{M/H}] \sim -1.19\) (\([\text{Fe/H}] \sim -1.40\)). The largest uncertainty is due to potential errors in the metallicity. If we redo the analysis for \([\text{M/H}] = -1.03\) (the Lick-Texas value), –1.26, and –1.48, we derive apparent moduli \((m - M)_V = 14.39 ± 0.07\) for \([\text{M/H}] = -1.19\). If the reddening is reduced to \(E(B-V) = 0.02\), the apparent modulus changes to 14.40. In any case, the apparent distance modulus is somewhat larger than that derived by RF (14.30 ± 0.20), primarily as a result of the systematic color offset between our photometry and theirs on the MS. If we correct for absorption, the true distance modulus is then \((m - M)_V = 14.32 ± 0.07\). This distance modulus agrees well with Baade-Wesselink distances to two RR Lyraes: \((m - M)_V = 14.37 ± 0.13\) (Storm, Carney & Latham 1994).

### 6.3.2. SGB Fitting

Because the subgiant branch “jump” is a very sharp feature in \(B\)-band magnitude, a determination of the distance modulus from a theoretical LF fit (shifting the theoretical LF to match the magnitude position of the feature) provides a consistency check on the subdwarf distance — the estimated distance modulus increases with increasing metallicity for subdwarfs, and decreases with increasing metallicity for a SGB fit. The magnitude position has a moderate sensitivity to age (0.07 mag / Gyr at this metallicity in \(B\)-band; see Figure 32) and a dependence on details of the chemical composition (see 6.1.1), and so should not be considered a primary distance indicator. Figure 24 shows the results of this fit for models with an age of 14 Gyr. For the metal content of –1.19, we would derive an apparent modulus of \((m - M)_B ≈ 14.4\), which is slightly lower than the subdwarf value. A smaller age (by about 3 Gyr) would be needed to bring the results for \([\text{M/H}] = -1.03\) into consistency.

### 6.3.3. HB Fitting and RR Lyrae \(M_V - [\text{Fe/H}]\) Relations

We may attempt to fit theoretical HB models to the observed HB sequence as another indication of the distance modulus. The best fits using models from Dorman (1992b) are shown in Figure 25. Fitting by eye with the \([\text{Fe/H}] = -1.26\) models, we get an apparent distance modulus of 14.52 ± 0.05. A difference
of metallicity of 0.2 dex only changes the distance modulus fit by about 0.05 mag. This value is in good agreement with the distance modulus used by Da Costa & Armandroff (1990). The overall fit is not as satisfactory — the slope of the blue end of the HB is not matched. However, this may be a result of our color calibration. (The red end is best fit by the [Fe/H] = −1.48 tracks.) The disagreement between the two distance moduli we have derived cannot be explained by the color calibration, however. If the difference between the subdwarf and HB fitting values for the distance modulus is real, this would mostly explain the earlier disagreement of the value of $\Delta V_{1.4}$ for M5 with those of other clusters of the same metallicity. Alternately, the distance modulus from HB fitting would match the value from subdwarf fitting at a metallicity between the values given by ZW84 and the Lick-Texas group.

The linear dependence of $M_V$ on [Fe/H] for RR Lyraes is also an important means of determining the distance modulus for distant old populations via photometric means. The slope and zero-point of the relation still remain in dispute, however. Recent analyses seem to indicate that a low value for the slope $\left(\frac{\partial M_V}{\partial \text{[Fe/H]}} \approx 0.15\right)$ is favored over a value that would explain the Sandage period shift effect (0.39; Sandage & Cacciari 1990). The zero-point is more in doubt. Some of recent determinations of this relation are: Carney et al. (1992) from a reevaluation of earlier studies

$$< M_V(\text{RR}) >= (0.15 \pm 0.01)[\text{Fe/H}] + (1.01 \pm 0.08);$$

from field RR Lyrae stars, Jones et al. (1992)

$$< M_V(\text{RR}) >= (0.16 \pm 0.03)[\text{Fe/H}] + (1.02 \pm 0.03)$$

and Skillen et al. (1993)

$$< M_V(\text{RR}) >= (0.21 \pm 0.05)[\text{Fe/H}] + (1.04 \pm 0.10);$$

Lee (1990) using theoretical models for $Y = 0.23$

$$< M_V(\text{RR}) >= 0.17[\text{Fe/H}] + 0.79;$$

and Walker (1992b) using the Carney et al. slope and the distance to Large Magellanic Cloud (LMC) cluster variables

$$< M_V(\text{RR}) >= (0.15 \pm 0.01)[\text{Fe/H}] + (0.73 \pm 0.10).$$

If the distance modulus for the LMC is revised from 18.50 to 18.37 ± 0.04 as suggested by a reanalysis of data from the light rings of SN1987A (Gould 1995), this zero-point would become 0.86 ± 0.01.

Given these uncertainties, we instead calculate $M_V(\text{HB})$ for M5 using distance moduli calculated from subdwarf fitting, and compare with values for various $M_V(\text{RR})$ relations. Using $[M/H] = −1.19$ and $V_H = 15.092 \pm 0.02$, we find that $M_V(\text{HB}) = 0.68$, which should correspond to the mean magnitude level of the RR Lyraes. The Carney et al., Jones et al. and Skillen et al. relations all predict HB magnitudes that are fainter than observed by 0.10 - 0.15 mag, predicting an apparent distance modulus $(m - M)_V \approx 14.3$. The Lee theoretical models and the Walker relation predict an HB that is too bright by about 0.10 mag and 0.13 mag respectively. (The error in these distance moduli is approximately 0.12 mag.) For this metal content, the value of $M_V(\text{HB})$ does not favor either the larger or smaller value for the zero-point.

If we use $[M/H] = −1.03$, $M_V(\text{HB}) = 0.54$. In this case, Carney et al., Jones et al. and Skillen et al. relations all predict HB magnitudes that are too faint by about 0.3 mag. In contrast, the Lee theoretical models are too faint by only 0.07 mag, and the Walker relation is only faint by 0.03 mag.
6.3.4. AGB Fitting

The theoretical HB evolutionary tracks (Dorman 1992b) also allow us to examine the constraints that the asymptotic giant branch provides. None of the sets of tracks adequately reproduce all of the features of the lower AGB. The \([\text{Fe/H}] = -1.03\) tracks are marginally red enough to explain the color of the lower AGB, but the other two sets are too blue. All of the sets seem to overestimate the luminosity of the AGB clump as well. Dorman’s models would imply an apparent distance modulus of approximately 14.63. The position of this feature should correspond to the range of magnitudes in which the evolutionary track turns fainter for a time. (The cause is the envelope adjustment to the recently-formed helium-burning shell.) The helium-burning models of Castellani, Chieffi & Pulone (1991) (their Figure 9) seem to match the M5 data from BCF81 for both the color and level of the AGB clump for an apparent modulus of 14.45 and a metallicity \([\text{Fe/H}] = -1.13\). This metallicity is consistent with M5’s metal content if \(\alpha\)-element enhancement is taken into account.

6.4. The Luminosity Functions

Figures 26 and 27 plot the \(B\)- and \(I\)-band LFs along with theoretical LFs derived from BV92 LFs. The theoretical LFs use distance moduli derived from subdwarf fitting to the main sequence (see \(\S\) 6.3.1). The theoretical LFs were also degraded to the finite resolution of the bins in the observed LF.

6.4.1. The Subgiant Branch

In several metal-poor clusters, so-called “WIMP bumps” (or subgiant excesses) have been observed near this location in the LF. In \(B\) band, the peak \((B \approx 18.5)\) and dip \((B \approx 18.7)\) features seen in the SGB region of the observed LF in Figure 23 (about 0.4 mag brighter than the turnoff) are affected by the presence of significant numbers of blends in the photometry. However, we do not believe that blends could be responsible for the unexplained subgiant excesses. Blends do not seem to be capable of creating bumps in the LF, but can only reduce the number of counts in the bins corresponding to the most horizontal portion of the SGB in the CMD (in the “peak”). Despite the fact that the blend sequence crosses the the fiducial sequence of the cluster at approximately this position, no excess of stars appears there in the LF when crowded regions of the cluster are included in the sample.

Because the SGB is closest to horizontal in the color-magnitude diagram when \(B\)-band is used, any potential subgiant excess would be confined to the smallest number of magnitude bins, and would thus be viewed at its greatest significance. We find only one bin exceeding the theoretical models, with a significance of only about 1\(\sigma\). In \(I\) band, the SGB is much less horizontal in the CMD, so we are able to examine the branch in finer detail. The observed counts follow the theoretical predictions very well, and there is no hint of a multiple bin enhancement to the counts that would be present if some physical process was slowing the movement of stars across the CMD. There also does not appear to be evidence for any excess (or of significant blending effects) in the \(V\)-band LF of NGC 288 (Bergbusch 1993), a cluster with metallicity similar to that of M5, but with a much smaller central density. We conclude that there is no subgiant excess in the LF for M5, and that this extends to other clusters of similar metallicity.

The subgiant branch is also the portion of the LF most affected by the specifics of the chemical composition. The helium abundance exerts a great effect on the SGB (Simoda 1972), while leaving the
relative numbers of stars on the RGB and MS mostly unchanged (Ratcliff 1987). A decrease in helium abundance causes the slope of the SGB in the CMD to become shallower. The SGB fits into fewer bins, enhancing the peak in the LF. Thus, the peak we see (mimicked by an increase in [Fe/H] in Figure [20]) may be an indication that the helium abundance in M5 is low.

6.4.2. The MS-RGB Discrepancy

The discrepancy between the relative numbers of stars observed on the MS and RGB and the predicted numbers for metal-poor clusters is notably unresolved. The absolute number of stars on the MS in the LF is determined by the total number of stars in the cluster and the mass function from which they are taken. The number of RGB stars is primarily a function of the evolutionary timescale for those stars. So, if the discrepancy observed in metal-poor clusters like M92 (Stetson 1991) and M30 (Bolte 1994) is due to errors in theoretically-determined RG evolutionary timescales, there is a potential impact on spectral synthesis models, as RGB stars provide a majority of the integrated light in old stellar populations in V-band and at longer wavelengths.

In the case of M5 though, there does not appear to be a significant discrepancy. For the B-band LF, the agreement with theory is quite good all of the way up the RGB. The MS and SGB for the I-band LF also match quite well, but the slope of the RGB portion may deviate from the theoretical prediction. Without the additional evidence provided by the upper RGB, this deviation might not have been noticed. The slope difference shows up in the raw data for the cluster, and it is not due in any way to the procedures used in calculating the LF. As our photometric calibration showed, we do not appear to have any magnitude-dependent trends in our photometry in I-band this high on the RGB (see §3.3.2). Possibly this results from problems in the conversion from bolometric luminosity to I-band magnitude in the models. In the majority of the fits using “reasonable” ages and metallicities though, the MS and RGB are matched simultaneously. This match is unaffected by our use of rescaled counts for RGB stars.

The V-band LF of the cluster NGC 288 (Bergbusch 1993) also shows good agreement between the relative levels of the MS and the RGB. Taken together with the evidence for metal-poor clusters like M92 and M30, this may be an indication of a metal-dependent trend. A few input parameters can change the relative levels (Ratcliff 1987). The initial mass function (IMF) exponent that is used in the theoretical LF can change the normalization of the MS, but it also changes the shape of the MS portion of the LF. We have chosen values of the exponent that match this shape best, and when this is done we do not see a significant discrepancy. There is no evidence that the slope of the MS for the metal-poor clusters is poorly matched.

Helium abundance and metallicity have effects on the relative levels, but change the morphology of the SGB bump at the same time. Increased metallicity increases the size of the difference between the MS and RGB levels, and steepens the “jump” at the SGB. Decreased helium abundance has similar effects.

6.4.3. The RGB Bump

The position of the bump in luminosity directly relates to the point in a star’s evolution at which the hydrogen-burning shell source passes through the former base of the convection zone (Thomas 1967, Iben 1968). The increase in the hydrogen content of the material being fed into the burning shell causes the star to readjust its structure somewhat, creating a pause in the evolution. For a sample of 13 clusters, Fusi Pecci
et al. (1990) found that the observed bumps were fainter than predicted by approximately 0.4 magnitudes. This fact does hold some interest — it tells us that either the hydrogen burning shell has moved outward more quickly than expected, or that convection penetrated more deeply than expected at the base of the RGB. The former is unlikely to be true, as we have already seen that the MS-RGB agreement gives no indication that the red giants are evolving differently than predicted. Thus, the discrepancy between the observed and predicted positions of the bump may reflect the effects that non-standard (but physically motivated) processes like convective overshooting have on the hydrogen profile.

From 107 stars in the $BV$ sample judged to be in the bump, we derive $V_{\text{bump}} = 14.964 \pm 0.007$ ($\Delta V_{\text{bump}}^H = V_{\text{bump}} - V_H = -0.13 \pm 0.02$), and $B_{\text{bump}} = 15.833 \pm 0.007$. Figure 28 shows the $B$-band LF in expanded detail near the observed RG bump. If we were to simply use $[\text{Fe/H}] = -1.40$ for M5 (ZW84), we would find that the bump is fainter than predicted by approximately 0.3 mag in $B$ (0.25 mag in $V$), using the subdwarf distance modulus. A fainter-than-predicted RGB bump can potentially be explained by a higher cluster age, a higher metal content, or a lower helium abundance than assumed. From BV92 and Fusi Pecci et al. 1990, we can estimate some partial derivatives for the approximate parameters of M5 (age 14 Gyr, $[\text{Fe/H}] = -1.15$, and helium abundance $Y = 0.23$):

\[
\frac{\partial M_V^{\text{bump}}}{\partial \text{age}} \approx 0.05 \text{ mag / Gyr}
\]

\[
\frac{\partial M_V^{\text{bump}}}{\partial [\text{Fe/H}]} \approx 0.9 \text{ mag / dex}
\]

\[
\frac{\partial M_V^{\text{bump}}}{\partial Y} \approx -5 \text{ mag}
\]

Typically quoted errors for globular cluster ages ($\pm 3$ Gyr) can only explain about 60% of the RGB bump discrepancy here, given a systematic favorable offset. An error in distance modulus could also be invoked to explain part of the difference in some previous studies. However, this explanation is unlikely in this case, since we can judge the position of the RGB bump relative to the MS turnoff. We find that the difference in magnitude between bump and turnoff ($\Delta V_{\text{TO}} = 3.61 \pm 0.05$) is also smaller than theory predicts ($\Delta V_{\text{TO}}^{\text{bump}} = 3.88$ for $[\text{Fe/H}] = -1.40$ and age 14 Gyr), independent of distance modulus. ($\Delta V_{\text{TO}}^{\text{bump}}$ also has only a small age dependence of about 0.02 mag per Gyr, so that a large error in our age assumption would be still be necessary.)

A helium deficiency on the order of 0.04 could explain most of the disagreement. However, the derivative above fails to take into account the increased turnoff mass implied by a decreased helium content for a fixed age (Alongi et al. 1991). An increased turnoff mass causes the bump to become brighter, which means that less than half of the disagreement could be explained by this kind of helium deficiency. An error in the zero-point of the metallicity scale as discussed earlier could help to remove part of the difference. The Sneden et al. (1992) value $<[\text{Fe/H}]>= -1.17 \pm 0.01$ could just be able to account for all of the difference ($\Delta V_{\text{TO}}^{\text{bump}} \approx 3.65$).

However, the effects of $\alpha$-element enhancement certainly must be accounted for. These elements are not as important to the energy generation for the stars as they are to the envelope opacity. Thus, the LF would presumably be unaffected by $\alpha$-enhancement except for the SGB (as discussed earlier) and the position of the RGB bump relative to the turnoff of the cluster. Using the formula given by Chieffi et al. (1991) for determining the enhancement of metal content $\alpha$-elements, we find that the movement of the
bump goes like

\[
\left( \frac{\partial M_b^{\text{bump}}}{\partial [\alpha/Fe]} \right) \approx 0.6 \text{ mag / dex.}
\]

An enhancement of +0.3 dex in the \(\alpha\)-element abundances would remove most or all of the discrepancy. For M5, the discrepancy is reduced to approximately 0.06 mag for \(M_V\) and \(\Delta V_{bump}^{HB}\) if we use the ZW84 metallicity zero-point ([M/H] = −1.19). An additional adjustment of the metallicity zero-point could make the agreement perfect, or cause the theoretical predictions to become fainter than the observed bump.

The theoretical prediction for \(\Delta V_{bump}^{TO}\) for the metal content of M5 is then reduced to 3.75. This measure of the bump’s position may be useful, as theoretical predictions can be made without needing to make assumptions about the HB models. By this comparison, we see that the bump discrepancy may not be completely resolved — relative to the turnoff, the observed bump is still about 0.14 mag too faint. This may be another indication that M5’s HB is abnormally faint by about 0.1 mag (which would artificially improve the agreement of \(\Delta V_{bump}^{HB}\) with theory), or that the zero-point of the metallicity scale needs to be closer to the Lick-Texas group value. However, the magnitude level of the turnoff is also notoriously difficult to measure, so that part of the discrepancy could be due to mismeasurement.

Regardless of the case with M5 however, observed \(\alpha\)-element enhancements do improve the agreement of the magnitude of the RG bump with theory for the globular cluster system as a whole. In so doing, this decreases the need for non-standard mixing processes like convective overshooting.

6.5. Age Indicators

Age determination for M5 is of particular interest because M5’s horizontal branch morphology is intermediate between those of the “second parameter” pair NGC 288 and NGC 362 (\(R_{HB} = 0.35\) for M5, versus 0.95 for NGC 288 and −0.87 for NGC 362; Lee 1989). In the scenario of Lee, Demarque & Zinn (1994) (with a metallicity [Fe/H] = −1.40 from ZW84), M5 would have an age approximately 1 Gyr older than the constant age line in their [Fe/H] versus HB Type diagram. The exact value of such an age offset is highly sensitive to the input parameters (such as mass loss prescription and chemical composition) used in the calculations (Catelan & de Freitas Pacheco 1993). For example, if the metal abundance of M5 is that given by the Lick-Texas group, the age difference would increase to about 3 Gyr.

In the following analysis, we have chosen not to use the indicator \(\Delta(B - V)\) (Sarajedini & Demarque 1990; VBS) because the fiducial lines of the other clusters in this metallicity group are not determined to high enough accuracy, and color term calibration is not carried out in a uniform manner for all of the studies.

6.5.1. \(\Delta V_{bump}^{TO}\)

The magnitude difference between the turnoff (\(V_{TO} = 18.57 \pm 0.05\) for M5) and the HB is a widely-used indicator of relative age. From our photometry, \(\Delta V_{bump}^{TO} = 3.47 \pm 0.06\), which agrees quite well with the value of 3.52 ± 0.09 derived for M3 by Buonanno et al. (1994), and the average of 3.55 ± 0.09 for a sample of 19 clusters examined by Buonanno, Corsi & Fusi Pecchi (1989). For the other clusters in Table 6, we have chosen to use studies with the best statistics on both the MS and HB, so that the quantity is determined as well as possible. We used the following studies toward this end: NGC 288, Bolte (1992), Bergbusch (1993);
NGC 362, Harris (1982), Bolte (1994); NGC 1261, Ferraro et al. (1993); NGC 1851, Walker (1992a); NGC 2808, Alcaino et al. (1990); M4 (NGC 6121), Kanatas et al. (1995); and Palomar 5, Smith et al. (1986). (Our value for NGC 2808 is smaller than that stated by Alcaino et al. because their photometry supports a larger value for $V_{HB}$ than the value they assumed.)

Because of the color extent of the HB for M5, we can use this dataset as a “bridge” between clusters with widely varying morphology, enabling us to extend the use of the indicator $\Delta V_{TO}^{HB}$. We can calculate $\Delta V_{TO}^{HB}$ using another point on the HB for clusters that do not populate the vicinity of the RR Lyrae gap, and then do the same for M5 to make a relative age comparison possible. If we use the red end of NGC 288’s HB to define the magnitude level of the HB, we derive $\Delta V_{TO}^{HB} = 3.47 \pm 0.07$ for M5, and $\Delta V_{TO}^{HB} = 3.73$ for Bolte’s and Bergbusch’s NGC 288 photometry (approximately $\pm 0.09$ for both). For the color of the blue end of NGC 362’s HB, we derive $\Delta V_{TO}^{HB} = 3.46 \pm 0.07$ for M5 and $\Delta V_{TO}^{HB} = 3.41 \pm 0.12$ using the bright star photometry of Harris (1982) and the smoothed fiducial of VBS for NGC 362. In light of the kind of zero-point differences between photographic and CCD photometry in general, the $\Delta V_{TO}^{HB}$ value derived for NGC 362 should be used with caution. The data for the other clusters, however, are definite improvements over the same quantities as derived by Buonanno et al. (1989) simply because the photometry for the MS and HB of each cluster now comes from a single study.

From the values of $\Delta V_{TO}^{HB}$ alone, there does appear to be a significant age difference between NGC 288 and M5 (although see the following discussion). An age difference between NGC 288 and NGC 362 is slightly less secure due to the possibility of large zero-point differences between the bright and faint samples used for NGC 362. However, a large and systematic error ($\sim +0.1$ mag) in the zero points of the bright-star and MS photometry studies of NGC 362 would be needed to simply throw doubt on the age ordering. Unmeasured metallicity differences almost certainly do not play a role, since the effect on $\Delta V_{TO}^{HB}$ is quite minor. Even if the most unfavorable and unrealistic $M_{VB}^{HB} - [Fe/H]$ relation is assumed (constant $M_{VB}$; Buonanno et al. 1989), and M5 in reality has a metallicity 0.3 dex less than that of NGC 288, this only explains approximately 0.1 mag of the difference in $\Delta V_{TO}^{HB}$. For a realistic $M_{VB}^{HB} - [Fe/H]$ relation, only 0.05 mag could be explained by such a metallicity error.

Another influence that must be considered is helium abundance variations, as an increase in MS helium abundance results in an increase in the luminosity of the HB, while the effect on the turnoff is comparatively small ($\partial V_{TO}/\partial Y = -1.4$ mag from VandenBerg & Bell isochrones). Our earlier consideration of the helium abundances now becomes important. From the values of $R$ and $\Delta$, we find that there is no evidence for differences in helium abundance greater than 0.02 among M5, NGC 288, and NGC 362. (It is particularly interesting to note that the values of $\Delta$ for these three clusters agree much more accurately than the values of $\Delta V_{TO}^{HB}$. The only real difference between the definitions of the two indicators is the position of the fainter reference point.) We estimate that $\Delta V_{TO}^{HB}$ should increase by approximately 0.04 mag for a 0.01 increase in $Y$. So, even if M5 has a helium mass fraction that is 0.02 lower than NGC 288, we can only explain about 0.08 mag of the difference. In combination with a systematic unfavorable metallicity error, we can at most explain about half of the difference.

The evolutionary status of the bluest stars must also be considered, as the reddest HB stars in clusters with blue HB morphologies are likely to be evolving toward the AGB at higher luminosities than the ZAHB (Rood & Crocker 1993). In the extreme case that all of the HB stars with $(B - V)_0 < 0.01$ in NGC 288 have evolved to their locations from bluer on the HB, conventional evolution tracks predict the stars would be between 0.1 and 0.4 magnitudes brighter than the ZAHB, and $\Delta V_{TO}^{HB}$ would be larger by this amount even if NGC 288 was coeval with the other clusters. It is certainly true that the red portion of NGC 288’s HB does not follow the morphology of M5’s HB, while the blue ends do seem to match each other (along
with that of NGC 1851; see Figure [21]. In addition a gap appears in the distribution of stars in Bergbusch’s study at $(B - V)_0 \approx -0.01$ where the two HBs diverge.

However, a coherent well-populated structure is not to be expected from the evolutionary timescales in theoretical models. Given the 38 stars observed in most-populous portion of NGC 288’s HB ($(B - V) \approx -0.04$; Bergbusch 1993), Dorman’s models predict that at most two stars would be observed with $(B - V) < 0.01$, whereas 16 are seen. In addition, any stars evolving from the most-populous portion are predicted to evolve brighter than the observed red end of the HB by 0.2 mag or more. The difficulties are even greater for stars evolving from the blue end of the HB - they spend a smaller fraction of their time near the horizontal part of the HB, and evolve to the red (if at all) at even higher luminosity. A decreased helium abundance would constrain the HB evolutionary tracks to be closer to the zero-age HB, but would also tend to shift the HB stars as a group to the red. The helium indicators $R$ and $\Delta$ do not indicate that NGC 288’s helium is any different from the other clusters in this group.

If we assume the difference to be due to age alone, then we find that NGC 288 is approximately $3.7 \pm 1.5$ Gyr older than M5 (using $\partial M_V(TO)/\partial \text{age} \approx 0.07$ mag/Gyr from BV92 isochrones). We conclude that, to within the errors, six of the clusters in this metallicity group (NGC 362, NGC 1261, NGC 1851, NGC 2808, M4, and M5) appear coeval to an accuracy of about $\pm 1.5$ Gyr. Two other clusters may have different ages: NGC 288, which may be older, and Palomar 5, which may be younger. If we leave these two clusters out of consideration, we are still left with a large variation in HB morphology between M5 and either NGC 362 or 1261. Because the sensitivity of HB morphology to age depends on the unknown mass loss mechanism (Lee, Demarque & Zinn 1994), the viability of age as the second parameter varies. For a constant mass difference between helium flash red giants and zero-age HB stars, age differences of approximately 4 to 5 Gyr are required to explain the morphological variations — a case that can be ruled out by this data. If a Reimers’ mass loss relation holds, age differences of only about 2 to 3 Gyr are required. This possibility cannot be ruled out.

6.5.2. Isochrone and LF Comparison

The most direct absolute age determination that can be made involves comparing the absolute magnitude of the turnoff to models. Using the subdwarf distance modulus, we find $M_V(TO) = 4.16 \pm 0.08$, where the error refers only to the formal errors in the distance modulus and in the determination of the apparent magnitude of the turnoff. Figure [23] places this point in a comparison with turnoffs from the BV92 isochrones (which implicitly assume bolometric corrections from VandenBerg 1992). It should be noted that $\alpha$-element enhancements shift the theoretical relations to lower metallicity, and hence result in a slight ($\sim 1$ Gyr) age reduction for clusters on the whole. With $\alpha$-enhancement included, we find an empirical relationship

$$\log_{10} t_9 = -0.874 - 0.118 [M/H] + 0.446 M_V(TO),$$

which becomes

$$\log_{10} t_9 \approx -0.874 - 0.118 [Fe/H] - 0.08 [\alpha/Fe] + 0.446 M_V(TO).$$

Via this relation, we find an age of $14 \pm 1.2$ Gyr. Determinations of $M_V(TO)$ are also plotted for the clusters M3 (Buonanno et al. 1994), M15 (Durrell & Harris 1993), and M92 (Stetson & Harris 1988, as redetermined by Bolte & Hogan 1995). If the zero-point of the observed metallicity scale is actually that favored by the Lick-Texas group, then for M5, we derive $M_V(TO) = 4.07 \pm 0.09$, and an age of $11.6 \pm 1.1$ Gyr. On the ZW84 metal scale, the ages of these clusters are consistent with each other to within the
errors. However, the Lick-Texas scale would imply age differences: M92 has almost identical values in both studies, M15 is 0.15 dex lower, and M3 and M13 are about 0.2 dex higher on the Lick-Texas scale.

Finally, we compare the M5 fiducial with the isochrones of BV92. Although the model difficulties with predicting colors are well known (e.g. VandenBerg, Bolte & Stetson 1996), there are still valuable cross-checks to be made in comparing the observations to model isochrones throughout the CMD. Figure 30 shows 14 Gyr isochrones for a range of metal contents, with the M5 fiducial overlaid after being shifted in color to correct for the reddening ($E(B-V) = 0.03 \pm 0.01$), and in magnitude to remove the apparent distance modulus ($m - M)_V = 14.41 \pm 0.07$ as derived in §6.3.1). Not surprisingly the models and data agree very well along the unevolved main sequence — after all, the subset of subdwarfs that we used to calibrate the distance modulus had photometric values very close to the theoretical predictions. However, the shapes of the isochrones also match quite well for the 14 Gyr isochrones, up to the upper RGB, which is blue relative to the isochrones. The $\alpha$-enhanced isochrones shown in Figure 31 match the M5 fiducial better overall. (This seems to be evidence that $\alpha$-enhancements do have observable effects on the shape of the cluster fiducial line.) From the comparison to the $\alpha$-enhanced isochrones, we infer an age of between 12 and 14 Gyr. There is reason to hope that the external errors associated with this kind of fit have decreased in the past few years. As we saw in §6.1, there has been little change in the shape of the isochrones since the last set of oxygen-enhanced isochrones was released, and there have only been small shifts absolutely in color and magnitude.

As Renzini & Fusi Pecci (1988) pointed out, examining the union of CMD and LF data is the most effective way of making detailed comparisons with theoretical models — testing our ideas about stellar structure and timescales simultaneously. In considering age though, a LF comparison can be more sensitive than an isochrone comparison if the proper filter band is chosen. $B$-band is ideal for intermediate metal-poor clusters ($-1.4 < [\text{Fe}/\text{H}] < -0.9$), as the SGB has nearly constant magnitude in this range. Consequently, small changes in age (or chemical composition) result in slight changes in the slope of the SGB, but significant changes in the number of stars falling in the “SGB peak” feature in the LF. With a well-determined composition and distance modulus, this feature can provide excellent age discrimination. (In a similar vein, $V$-band LFs provide the best age sensitivity for more metal-rich clusters.)

Figure 32 shows the comparison of the $B$-band LF with the theoretical LFs in the vicinity of the SGB using the apparent distance modulus $(m - M)_V = 14.42$ for $[\text{M}/\text{H}] = -1.26$. The $[\text{M}/\text{H}] = -1.26$ model roughly matches the position of the subgiant branch “jump”, although it does not reproduce the star counts in the same range for an age of 14 Gyr. A small decrease in helium abundance would improve agreement between the theoretical and observed LFs, as helium changes affect only the SGB portion of the LF (see §6.4.1). Alternately, an age of approximately 13 Gyr would bring the models into better agreement with the observed LF in both magnitude position and size.

We must also consider the possibility of alternate metal contents for M5. For models having $[\text{M}/\text{H}] = -1.48$, it is not possible to produce a SGB peak of the observed size, even for ages as low as 10 Gyr. However, we have to consider seriously models having a metal content of $[\text{M}/\text{H}] = -1.03$ ($[\text{Fe}/\text{H}] = 1.17$ and $\alpha$-enhancements) for M5 as this is the value measured based on modern, high-dispersion spectroscopy. Models having a metal content of $[\text{M}/\text{H}] = -1.03$ are able to approximately reproduce the peak height with ages between about 9 and 15 Gyr. For this metal content and an age of 14 Gyr, the position of the SGB feature in magnitude would imply a distance modulus 0.3 mag smaller than derived from subdwarf fitting, as can be seen in Figure 29. An age of approximately 10 Gyr would be required to match the magnitude position of the SGB, and the height of the peak. This, however, is inconsistent with the color extent of the SGB in the theoretical isochrones. (Given the sensitivity of the SGB extent to the choice of mixing-length
parameter, this objection to the combination of low age and high metallicity could be weakened if the mixing-length theory is inadequate.) Independent of the distance modulus used, the size of the SGB peak strongly rules out ages of greater than 15 Gyr, and metal contents \([M/H] \lesssim -1.3\) ([Fe/H] \(\lesssim -1.3\)).

With regards to the age determination, the uncertainties in the the metallicity scale in the range around [Fe/H]=−1.3 is an important issue. For the Zinn & West scale, with [Fe/H] = −1.40, there is nearly complete consistency between the M5 CMD and LF observations and the BV92 models for an age between 13 and 14 Gyr with the LF providing the lower age. (An important exception is the different distance moduli derived from subdwarf and HB fitting.) Whether this is significantly different than the age of the well-studied more metal-poor M92 is difficult to answer. Bolte & Hogan (1995) use essentially the same subdwarf list and parameters to derive a distance to M92 and and age based on BV92 of 15.8 Gyr. However, if the metallicity scale of the Lick-Texas group, which is the same as Zinn & West for M92 but 0.23 dex more metal-rich for M5, is used, then the derived age for M5 is reduced to 11.5 Gyr. For the higher metallicity, low-age models, the consistency between the models and observations is not as complete, however. The inconsistencies are greatest in the areas where the models have weaknesses – prediction of the luminosity of the RGB bump, and the color extent of the SGB. The relative metallicities of M5 and other clusters of similar abundance are well established, as is the similarity in the ages of the clusters considered in §6.5.1. If the higher [Fe/H] values turn out to be the correct ones, a strong case can be made for an age-metallicity relationship in the halo globulars.

7. Conclusions

1. The population ratios derived from the bright star sample (complete into the center of the cluster) indicate that M5 stars have a semiconvective zone during their HB evolution. More puzzling though, is the indication that the \(R\) and \(R'\) ratios reflect a helium abundance of \(Y = 0.19 \pm 0.02\). This value is is seen at all cluster radii.

   We recalibrate the helium indicator \(\Delta\) and reexamine \(A\) using recent theoretical models. Overall, these indicators appear to show that the clusters in M5’s metallicity group (including M5) have helium abundances consistent with each other and with \(Y \approx 0.23\). By the indicator \(R\), the clusters are consistent with each other to within the errors, but appear to have values indicating low helium abundances. More globally, \(\Delta\) shows no apparent variation with [Fe/H], while \(R\) and \(A\) appear to increase with decreasing [Fe/H] for the limited sample of clusters examined. We recommend that \(\Delta\) be used as the helium indicator of choice for Galactic globular clusters with HB morphologies populating the instability strip, so as to overcome the statistical limitations inherent in \(R\) and the time-series photometry necessary to compute \(A\).

2. Extensive artificial star tests have been conducted to determine the incompleteness of our sample of stellar photometry as functions of magnitude and radius. Blending effects have been avoided by restricting the LF sample to stars well outside the core of the cluster. When blends are corrected for in the \(B\)-band LF, peak and dip features become apparent on the SGB. The \(I\)-band LF allows us to examine subgiant evolutionary time-scales in detail, and no significant deviations from theoretical predictions are found.

3. The relative star-count levels of the LF on the MS and RGB agree with theoretical predictions, in contrast to the case for metal-poor clusters like M30 and M92. In \(I\)-band, there may be a slope difference between the observations and theoretical predictions for the RGB. There is slight evidence for a deficit of the brightest giants in the core of the cluster relative to the outskirts.
4. The RG bump is observed to be fainter than the theoretical LFs indicate, in agreement with observations of the bump in several other clusters. However, once the effect of $\alpha$-element enhancement is taken into account, this discrepancy is significantly reduced. This decreases the need for non-standard mixing mechanisms (such as convective overshooting or meridional circulation) during first dredge-up on the RGB to force a match between theory and observations.

5. We identify 8 new RR Lyrae candidates in the core of M5 based on a series of $V$-band exposures, and the apparent scatter in their measured values on these frames.

6. We confirm that the metallicity of M5 is approximately the same as NGC 288 and NGC 362 (and other clusters compared listed in ZW84 as having $[\text{Fe}/\text{H}] \approx -1.35$), based on photometric indicators. However, we are unable to make a firm distinction between the competing values of the absolute metallicity: $[\text{Fe}/\text{H}] = -1.40$ (ZW84) or $-1.17$ (Sneden et al. 1992). With recent indications that the ZW84 scale may be nonlinear, this question should be pursued further. However, an enhancement in the $\alpha$-element abundances for the cluster definitely brings spectroscopic measurements of the metallicity into better agreement with indications from isochrone and LF fitting.

Isochrones and LFs for $\alpha$-enhanced compositions appear to fit the observed fiducial line and LFs better than oxygen-enhanced models. In addition, the colors of the “classical subdwarfs” with trigonometric parallaxes are predicted accurately when $\alpha$-element enhancements are taken into account.

7. The apparent distance modulus for M5 derived from subdwarf fitting of the MS is $(m - M)_V = 14.41 \pm 0.07$ for $[\text{Fe}/\text{H}] = -1.40$ ($(m - M)_V = 14.50 \pm 0.07$ for $[\text{Fe}/\text{H}] = -1.17$).

A model fit to the horizontal branch gives an apparent distance modulus $(m - M)_V = 14.52 \pm 0.05$. This appears to indicate a systematic difference between values taken from subdwarf fitting and from some HB model fits. The AGB (especially the AGB clump) is not well fit by the models of Dorman (1992b), but appears to be consistent with those of Castellani et al. (1991).

8. From isochrone and LF fitting considerations, M5 has an age of approximately 13.5 Gyr with an internal error of $\pm 1$ Gyr. Our age estimate would decrease by approximately 2 Gyr if the Lick-Texas zero-point of the metallicity scale is taken instead of the ZW84 value. M5 also appears to have approximately the same age as most of the other clusters of comparable metallicity, according to calculations of $\Delta V_{TO}^{HB}$. This constraint appears to be capable of eliminating age as the second parameter in HB morphology in the case of constant mass loss between RGB and HB (although a Reimers’ mass-loss relation may still be valid). NGC 288 is a possible exception, as it appears to be approximately 3.7 $\pm$ 1.5 Gyr older. Considerable differences between NGC 288’s metallicity or helium abundance and those of the other clusters considered cannot explain the differences in $\Delta V_{TO}^{HB}$. Evolution of the reddest HB stars away from the zero-age HB is ruled out by comparison with theoretical evolutionary tracks and timescales.

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Electronic copies of the listings of the photometry for the $BI$ and $BVI$ samples are available on request to the first author.
REFERENCES

Alcaino, G., Liller, W., Alvarado, F., & Wenderoth, E. 1990, ApJS, 72, 693
Alongi, M., Bertelli, G., Bressan, A. & Chiosi, C. 1991, A&A, 244, 95
Armandroff, T. E. & Zinn, R. 1988, AJ, 96, 92
Arp, H. 1962, ApJ, 135, 311
Bergbusch, P. A. 1993, AJ, 106, 1024
Bergbusch, P. A., & VandenBerg, D. A. 1992, ApJS, 81, 163 (BV92)
Bolte, M. 1992, ApJS, 82, 145
Bolte, M. 1994, ApJ, 431, 223
Bolte, M. & Hogan, C. J. 1995, Nature, 376, 399
Boesgaard, A. M. & Steigman, G. 1985, ARA&A, 23, 319
Brocato, E., Castellani, V., & Ripepi, V. 1995, AJ, 109, 1670 (BCR)
Buonanno, R., Corsi, C. E., & Fusi Pecci, F. 1981, MNRAS, 196, 196 (BCF81)
Buonanno, R., Corsi, C. E., & Fusi Pecci, F. 1985, A&A, 145, 97
Buonanno, R., Corsi, C. E., & Fusi Pecci, F. 1989, A&A, 216, 80
Buonanno, R., Corsi, C. E., Fusi Pecci, F., Alcaino, G., & Liller, W. 1984, A&AS, 57, 75
Buonanno, R., Corsi, C. E., Buzzoni, A., Cacciari, C., Ferraro, F. R., & Fusi Pecci, F. 1994, A&A, 290, 69
Buzzoni, A., Fusi Pecci, F., Buonanno, R., & Corsi, C. E. 1983, A&A, 128, 94
Campbell, A. 1992, ApJ, 401, 157
Caputo, F., Cayrel, R. & Cayrel de Strobel, G. 1983, A&A, 123, 135
Caputo, F., Martinez Roger, C. & Paez, E. 1987, A&A, 183, 228
Caputo, F., Natta, A. & Castellani, V. 1973, Ap&SS, 22, 199
Cardelli, J. A., Clayton, G. A., & Mathis J. S. 1989, ApJ, 345, 245
Carney, B. W., Latham, D. W., & Laird, J. B. 1989, AJ, 97, 423
Carney, B. W., Latham, D. W., Laird, J. B., & Aguilar, L. A. 1994, AJ, 107, 2240
Carney, B. W., Storm, J. & Jones, R. V. 1992, ApJ, 386, 663
Carretta, E. & Gratton, R. G. 1996, in Formation of the Galactic Halo... Inside and Out, ed. H. Morrison and A. Sarajedini (San Francisco: ASP), 363
Castellani, V., Chieffi, A., & Pulone, L. 1991, ApJS, 76, 911
Catelan, M. & de Freitas Pacheco, J. A. 1993, AJ, 106, 1858
Cayrel de Strobel, G., Hauck, B., Francois, P., Thevenin, F., Friel, E., Mermilliod, M., & Borde, S. 1992, A&AS, 95, 273
Chaboyer, B., Sarajedini, A., & Demarque, P. 1992, ApJ, 394, 515
Chieffi, A., Straniero, O., & Salaris, M. 1991, in The Formation and Evolution of Star Clusters, ed. K. Janes (San Francisco: ASP), 219
Clement, C. M. & Hazen, M. L. 1989, AJ, 97, 414
Cohen, J. G. 1983, ApJ, 270, 654
Cudworth, K. M. 1979, AJ, 84, 1866
Cudworth, K. M. & Hanson, R. B. 1993, AJ, 105, 168
Cudworth, K. M. & Rees, R. 1990, AJ, 99, 1491
Da Costa, G. S. & Armandroff, T. E. 1990, AJ, 100, 107
Dahn, C. C. 1994, in Galactic and Solar System Optical Astrometry, ed. L. V. Morrison and G. F. Gilmore (Cambridge: Cambridge), 55
Deupree, R. G. 1977, ApJ, 214, 502
Djorgovski, S. & Meylan, G. 1993, in Structure and Dynamics of Globular Clusters, ed. S. G. Djorgovski and G. Meylan (San Francisco: ASP), 325
Djorgovski, S. & Piotto, G. 1993, in Structure and Dynamics of Globular Clusters, ed. S. G. Djorgovski and G. Meylan (San Francisco: ASP), 203
Dorman, B. 1992a, ApJS, 80, 701
Dorman, B. 1992b, ApJS, 81, 221
Durrell, P. R. & Harris, W. E. 1993, AJ, 105, 1420
Faulkner, J. & Swenson, F. J. 1993, ApJ, 411, 200
Ferraro, F. R., Clementini, G., Fusi Pecci, F., Buonanno, R., & Alcaino, G. 1990, A&AS, 84, 59
Ferraro, F. R., Clementini, G., Fusi Pecci, F., Vitiello, E., & Buonanno, R. 1993, MNRAS, 264, 273
Fusi Pecci, F., Ferraro, F. R., Crocker, D. A., Rood, R. T., & Buonanno, R. 1990, A&A, 238, 95
Gilmore, G., Kuijken, K., & Wyse, R. 1989, ARA&A, 27, 555
Gould, A. 1995, ApJ, 452, 189
Graham, J. A. 1982, PASP, 94, 244
Gratton, R. G., Quarta, M. & Ortolani, S. 1986, A&A, 169, 208
Hanson, R. B. 1979, MNRAS, 186, 875
Harris, W. E. 1982, ApJS, 50, 573
Iben, I. 1968, Nature, 220, 143
Jones, R. V., Carney, B. W., Storm, J. & Latham, D. W. 1992, ApJ, 386, 646
Kadla, Z. I., Gerashchenko, A. N., Yablokova, N. V., & Irkaev, B. N. 1987, Astron. Tsirk. 1502, 7
Kanatas, I. N., Griffiths, W. K., Dickens, R. J., & Penny, A. J. 1995, MNRAS, 272, 265
Kraft, R. P., Sneden, C., Langer, G. E., Shetrone, M. D. & Bolte, M. 1995, AJ, 109, 2586
Kravtsov, V. V. 1988, Astron. Tsirk. 1526, 6
——. 1991, Soviet Ast. Lett., 17, 455
——. 1992, Soviet Ast. Lett., 18, 246
Laird, J. B., Carney, B. W., & Latham, D. W. 1988, AJ, 95, 1843
Lambert, D. 1989, in Cosmic Abundances of Matter, ed. C. J. Waddington (New York: AIP), 168
Landolt, A. U. 1992, AJ, 104, 340
Larson, A. M., VandenBerg, D. A., & DePropris, R. 1995, BAAS, 27, 1431
Lee, S.-W. 1977, A&AS, 27, 381
Lee, Y.-W. 1989, Ph.D. thesis, Yale University
Lee, Y.-W. 1990, ApJ, 363, 159
Lee, Y.-W., Demarque, P., & Zinn, R. 1994, ApJ, 423, 248
Lloyd Evans, T. 1983, S. Afr. Astron. Obs. Circ. 7, 86
Lucy, L. B. 1974, AJ, 79, 745
Lutz, T. E. & Kelker, D. H. 1973, PASP, 85, 573
Luyten, W. J. 1976, A Catalogue of Stars with Proper Motions Exceeding 0′′.5 Annually (University of
Minnesota: Minneapolis, MN)
Olive, K. E. & Steigman, G. 1995, ApJS, 97, 49
Peterson, C. J. 1993, in Structure and Dynamics of Globular Clusters, ed. S. G. Djorgovski and G. Meylan
(San Francisco: ASP), 337
Peterson, R. C., Rood, R. T., & Crocker, D. A. 1995, ApJ, 453, 214
Pilachowski, C. A., Olszewski, E. W. & Odell, A. 1983, PASP, 95, 713
Pilachowski, C. A., Sneden, C., & Booth, J. 1993, ApJ, 407, 699
Pryor, C. & Meylan, G. 1993, in Structure and Dynamics of Globular Clusters, ed. S. G. Djorgovski and
G. Meylan (San Francisco: ASP), 357
Ratcliff, S. J. 1987, ApJ, 318, 196
Rees, R. F. 1993, AJ, 106, 1524
Renzini, A. & Fusi Pecci, F. 1988, ARA&A, 26, 199
Richer, H. B., & Fahlman, G. G. 1987, ApJ, 316, 189 (RF)
Rood, R. T. 1981, in Physical Processes in Red Giants, ed. I. Iben Jr. and A. Renzini (Dordrecht: Reidel),
51
Rood, R. T. & Crocker, D. A. 1993, unpublished
Ryan, S. G. & Norris, J. E. 1991, AJ, 101, 1835
Salaris, M., Chieffi, A. & Straniero, O. 1993, ApJ, 414, 580
Sandage, A. R. 1957, ApJ, 125, 422
Sandage, A. R. 1982, ApJ, 252, 553
Sandage, A. R. 1990, ApJ, 350, 603
Sandage, A. & Cacciari, C. 1990, ApJ, 350, 645
Sandage, A. R. & Fouts, G. 1987, AJ, 93, 74
Sandage, A. R. & Smith, L. L. 1966, ApJ, 144, 886
Sandage, A. R. & Wallerstein, G. 1960, ApJ, 131, 598
Sarajedini, A. 1994, AJ, 107, 618
Sarajedini, A. & Demarque, P. 1990, ApJ, 365, 219
Sawyer Hogg, H. 1973, Publ. David Dunlop Obs. 3, No. 6
Simoda, M. 1972, PASJ, 24, 13
Simoda, M. & Kimura, H. 1968, ApJ, 151, 133
Simoda, M. & Tanikawa, K. 1970, PASJ, 22, 143
Skillen, I., Fernley, J. A., Stobie, R. S., & Jameson, R. F. 1993, MNRAS, 265, 301
Smith, G. H., McClure, R. D., Stetson, P. B., Hesser, J. E. & Bell, R. A. 1986, AJ, 91, 842
Sneden, C. Kraft, R. P., Prosser, C. F. & Langer, G. E. 1992, AJ, 104, 2121
Stetson, P. B. 1987, PASP, 99, 191
———. 1989a, DAOPHOT II User’s Manual, private communication
———. 1989b, ALLSTAR User’s Manual, private communication
———. 1990, PASP, 102, 932
———. 1991 in The Formation and Evolution of Star Clusters, ed. K. Janes (San Francisco: ASP), 88
———. 1994a, PASP, 106, 250
———. 1994b, private communication
Stetson, P. B. & Harris, W. E. 1988, AJ, 96, 909
Storm, J., Carney, B. W., & Beck, J. A. 1991, PASP, 103, 1264 (SCB)
Storm, J., Carney, B. W., & Latham, D. W. 1994, A&A, 290, 443
Sweigart, A. V. & Gross, P. G. 1976, ApJS, 32, 367
Tayler, R. J. 1954, AJ, 59, 413
Thomas, H.-C. 1967, ZAp, 67, 420
Trager, S. C., King, I. R., & Djorgovski, S. 1995, AJ, 109, 218
Turon, C. et al. 1993, Bull. Inf. CDS 43, 5
van Albada, T. S. & Baker, N. 1971, ApJ, 169, 311
van Altena, W. F., Truen-liang Lee, J., & Hoffleit, D. 1991, The General Catalogue of Trigonometric Stellar Parallaxes (Yale University Observatory: New Haven, CN)
VandenBerg, D. A. 1992, ApJ, 391, 685
VandenBerg, D. A. 1995, private communication
VandenBerg, D. A. & Bell, R. A. 1985, ApJS, 58, 561
VandenBerg, D. A., Bolte, M., & Stetson, P. B. 1990, AJ, 100, 445
Walker, A. R. 1992a, PASP, 104, 1063
Walker, A. R. 1992b, ApJ, 390, 81
Walker, T. P., Steigman, G., Kang, H.-S., Schramm, D. M., & Olive, K. A. 1991, ApJ, 376, 51
Wehlau, A. & Demers, S. 1977, A&A, 57, 251
Wehlau, A. & Froelich, N. 1994, AJ, 108, 134
Wehlau, A., Liller, M. H., Demers, S. & Clement, C. C. 1978, AJ, 83, 598
Wehlau, A., Liller, M. H., Clement, C. C. & Wizinowich, P. 1982, AJ, 87, 1295
Zinn, R. 1980, ApJS, 42, 19
Zinn, R. & West, M. J. 1984, ApJS, 55, 45 (ZW84)
Fig. 1.— (a) Final magnitude residuals as a function of magnitude for the comparison of Landolt standards and measured values in this study. The residuals are in the sense of (us - Landolt). Stars plotted with $\times$ were not included in the transformation equation calibrations. (b) Magnitude residuals as a function of color. (c) Color residuals as a function of color.

Fig. 2.— $V$-band finding chart for calibrated secondary standards in M5.

Fig. 3.— Final residuals for the comparison of our M5 secondary standards with those of Stetson (1994b).

Fig. 4.— Residuals for the comparison between the CCD photometry of Storm, Carney & Beck (1991) and the CTIO 4m data.

Fig. 5.— Residuals for the comparison between the photoelectric photometry of Lloyd Evans (1983) and the CTIO 4m data.

Fig. 6.— A comparison of fiducial lines for several studies of M5. Crosses ($\times$) and circles (○) are derived from the inner and outer field photometry of Richer & Fahlman (1987), respectively. Triangles (△) come from Buonanno et al. (1981), and squares (□) from Simoda & Tanikawa (1970). The solid line is fiducial derived from the current dataset, and is unsmoothed.

Fig. 7.— The color-magnitude diagram for all objects in the total (a) BVI sample and (b) BI sample.

Fig. 8.— The color-magnitude diagram for samples restricted to stars having low CHI values. (a) The BVI sample. (b) The BI sample.

Fig. 9.— Internal magnitude errors versus instrumental $B$ magnitude. Circles indicate median calculations for magnitude bins, and the solid line shows the calculated fitting function. The dotted lines indicate the boundary curves used to estimate the uncertainty in the magnitude error.

Fig. 10.— Magnitude bias versus instrumental $B$ magnitude.

Fig. 11.— Total detection probability versus instrumental $B$ magnitude.

Fig. 12.— External magnitude errors versus instrumental $B$ magnitude.

Fig. 13.— The $B$-band luminosity function in the case of extreme crowding, compared to our best luminosity function (dashed line). The peak feature on the SGB has been erased as a result of blending, and an additional bump has formed on the lower RGB. A theoretical luminosity function of lower metallicity (solid line) is a better fit to this data.

Fig. 14.— The bright sample of M5, including RR Lyrae stars with intensity-weighted $V$ magnitudes and magnitude-weighted ($B - V$) colors from Storm, Carney & Beck (1991). The sample has been restricted to stars with projected radii greater than 100′′ from the cluster center.

Fig. 15.— A comparison of red giant branches in the ($M_I$, $V - I$) plane. From left to right at $M_I = -3.4$, the clusters picture are: M15, NGC 6397, M2, NGC 6752, NGC 1851, M5 (dotted line), and 47 Tucanae.

Fig. 16.— A comparison of oxygen-enhanced isochrones (solid lines; Bergbusch & VandenBerg 1992) with a preliminary $\alpha$-enhanced isochrone with [Fe/H] = −1.31 and [$\alpha$/Fe] = +0.3 (dotted line; VandenBerg 1995) for an age of 14 Gyr. From right to left, the metallicities of the oxygen-enhanced isochrones are: [Fe/H] = −0.78, −1.03, −1.26, −1.48, −1.66, −1.78, −2.03, and −2.26.
Fig. 17.— A comparison of luminosity functions for oxygen-enhanced compositions (solid lines; Bergbusch & VandenBerg 1992) with a preliminary luminosity function for an α-enhanced composition of [Fe/H] = -1.31 and [α/Fe] = +0.3 (dotted line; VandenBerg 1995) for an age of 14 Gyr. At the vertical subgiant branch feature from left to right, the metallicities of the oxygen-enhanced luminosity functions are: [Fe/H] = -1.48, -1.26, and -1.03. The luminosity functions all have the same mass function slope $x = -0.5$.

Fig. 18.— A plot of all stars included in the bright-star population study for which we have calibrated magnitudes from the CTIO 4m data. Circles are HB stars, squares are AGB stars, and triangles are RGB stars.

Fig. 19.— The color-magnitude diagram for the uncalibrated CFHT data.

Fig. 20.— The cumulative radial distribution for AGB stars (solid line), HB stars (dotted line), and RGB stars (dashed line). The sample used extends from the center of the cluster to the edge of the CTIO 4m frames.

Fig. 21.— Published fiducial lines for the horizontal branches of several clusters: M5 (solid line; this paper), NGC 288 (dotted line; Bolte 1992), NGC 362 (short dashed line; Harris 1982), NGC 1851 (long dashed line; Walker 1992a), and M3 (dot-dashed line; Buonanno et al. 1994). The fiducial lines for each cluster have been shifted by amounts equal to those used in the relative age method of VandenBerg, Bolte & Stetson (1990).

Fig. 22.— The subdwarf sample after correction to a common metallicity [Fe/H] = −1.19. The absolute magnitudes have been adjusted for Lutz-Kelker bias. “Classical” subdwarfs (•) and the extended list (✷) are plotted. Bergbusch & VandenBerg (1992) oxygen-enhanced isochrones (solid lines) for 14 Gyr are plotted for comparison, with the same metallicity spread as in Figure 16. The fit to the fiducial line of M5 (dotted line) is shown. Only the classical subdwarfs were used in the fit.

Fig. 23.— Color difference between photometry of subdwarf (corrected for α-element enhancement) and the prediction from the Bergbusch & VandenBerg (1992) isochrones, as a function of metallicity. The symbols are in the same sense as Figure 22.

Fig. 24.— The B-band luminosity function for M5, with theoretical luminosity functions having ages of 14 Gyr. The theoretical LFs have been adjusted in distance modulus for best fit to the subgiant jump.

Fig. 25.— The fit to the observed horizontal branch stars having projected radii greater than 100″ from the cluster center. The theoretical zero-age horizontal branches and horizontal branch tracks are from Dorman (1992b). From left to right, the panels show the models for [Fe/H] = -1.03, -1.26, and -1.48. The models have helium abundance $Y_{HB} \approx 0.25$, corresponding to $Y_{MS} \approx 0.236$.

Fig. 26.— The B-band luminosity function for M5, with theoretical luminosity functions having ages of 14 Gyr. The theoretical LFs have been adjusted in distance modulus to agree with values derived from subdwarf fitting.

Fig. 27.— The I-band luminosity function for M5. Theoretical models have been adjusted as in Figure 26.

Fig. 28.— The B-band luminosity function on the red giant branch, centered on the RGB bump. The distance moduli are derived from subdwarf fitting. The turnups at the bright ends of the theoretical LFs are caused by bolometric corrections, which force the giant branch to become horizontal. For $B$-band, this is near the tip of the RGB.
Fig. 29.— The age of M5 as derived from the position of the MS turnoff, along with the uncertainties. The solid line represents a fit to turnoff data from the isochrones of Bergbusch & VandenBerg (1992), with an additional correction for an $\alpha$-element enhancement of +0.3 dex. The fit represented by the dotted line does not include this correction. The open circle represents the value derived for the higher Lick-Texas value of [Fe/H] for M5.

Fig. 30.— A comparison of the M5 fiducial (*dotted line*) with oxygen-enhanced isochrones (*solid lines*) in the $(M_V, B - V)$ plane (VandenBerg 1995). From left to right, the 14 Gyr isochrones have [Fe/H] = −1.48, −1.26, and −1.03.

Fig. 31.— A comparison of the M5 fiducial (*dotted line*) with $\alpha$-enhanced isochrones (*solid lines*) in the $(M_V, B - V)$ plane (VandenBerg 1995). From left to right, the [Fe/H] = −1.31 ([$\alpha$/Fe] = +0.3) isochrones have ages 12, 14, and 16 Gyr. The apparent distance modulus used here is $(m - M)_V = 14.50$.

Fig. 32.— A comparison of the M5 $B$-band LF with oxygen-enhanced models (VandenBerg 1995). Each of the theoretical models has [Fe/H] = −1.26 and apparent distance modulus $(m - M)_B = 14.45$. 