Progress in understanding colour confinement.
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New results from lattice are presented, which demonstrate that monopoles condense in the vacuum of the confined phase of QCD, which is thus a dual superconductor.

Monopoles defined by different abelian projections appear to be physically equivalent.

1. Introduction

An appealing mechanism for confinement of colour in QCD is dual superconductivity of the vacuum[1,2]. By dual Meissner effect the chromoelectric field acting between a \( q \bar{q} \) pair is confined into an Abrikosov flux tube, which in the ground state configuration has energy proportional to the distance: 

\[
V(r) = \sigma r
\]

\( \sigma \) is the string tension.

The idea that the strings appearing in hadron phenomenology could be Abrikosov flux tubes goes back to ref.[3], which was inspired by the Veneziano model[4].

Due to asymptotic freedom, it is very likely that QCD exists as a field theory, or that its euclidean version describes a statistical system for which a thermodynamical limit exists. For the same reason a significant sample of lattice configurations should be a good approximation to that limit and should identify the true vacuum, if the correlation length \( \lambda \) is much larger than the lattice spacing \( a \), and much smaller then the lattice size \( L (a \ll \lambda \ll L) \).

Dual superconductivity means condensation of monopole charges, or spontaneous breaking of the \( U(1) \) symmetry related to their conservation.

The colour deconfining transition is therefore, in this mechanism, a transition order-disorder, which can be investigated as a change of symmetry by use of an order parameter[5].

A sensible strategy to attack the problem is

a) identify the relevant magnetic \( U(1) \)

b) detect the deconfining transition by looking at the symmetry of the vacuum.

The question a) will be discussed in sect.2, the question b) in sect.3.

2. Monopoles in non abelian gauge theories.

The prototype of monopoles are t’Hooft-Polyakov monopoles, which were discovered as solitons of the Georgi Glashow model[6]

\[
\mathcal{L} = - \frac{1}{4} \tilde{G}_{\mu\nu} \tilde{G}^{\mu\nu} + D_\mu \tilde{\Phi} D^\mu \tilde{\Phi} - V(|\tilde{\Phi}|)
\]

(1)

\[
V(|\tilde{\Phi}|) = \frac{\lambda}{4} \left( \tilde{\Phi}^2 - \frac{m^2}{\lambda} \right)^2
\]

(2)

If \( m^2 > 0 \), the \( SO(3) \) symmetry of the model spontaneously breaks à la Higgs to \( U(1) \), and \( \tilde{\Phi}_0 \equiv \langle \tilde{\Phi} \rangle \neq 0 \).

A time independent ansatz of the form

\[
\tilde{\Phi}(\vec{r}) = f(r) \tilde{\Phi}_0 \hat{r} \quad A_0 = 0 \quad A_i = \frac{h(r)}{g} \varepsilon_{iak} \frac{r_k}{r}
\]

with \( f(r), h(r) \to 1 \) as \( r \to \infty \), admits a solution with finite energy, in which \( f(r), g(r) \) are practically equal to 1 everywhere except in a small radius \( a \sim 1/\mu \). The solution has the geometry of a monopole. This is explicitly seen by transforming to the gauge in which \( \tilde{\Phi} \equiv \tilde{\Phi} / |\tilde{\Phi}| \) is constant, say \( \tilde{\Phi} = (0, 0, 1) \): the corresponding gauge transformation \( U(\vec{r}) \) is called abelian projection. \( U(\vec{r}) \) is singular at \( \vec{r} = 0 \), where \( \tilde{\Phi} = 0 \).

The abelian gauge field of the residual \( U(1) \) symmetry in this gauge

\[
F_{\mu\nu} = \partial_\mu A^3_\nu - \partial_\nu A^3_\mu
\]

(3)
is the field of a Dirac monopole in the soliton configuration
\[ \vec{E} = 0 \quad \vec{H} \simeq r^{-2} \frac{\vec{r}}{g r^2} \text{ Dirac string} \quad (4) \]

In a covariant form
\[ \mathcal{F}_{\mu\nu} = \dot{\Phi} \tilde{G}_{\mu\nu} - \frac{1}{g} \dot{\Phi} (D_\mu \Phi \wedge D_\nu \Phi) \quad (5) \]

In terms of \( \mathcal{F}_{\mu\nu}^* = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} \mathcal{F}_{\rho\sigma} \) the magnetic current is defined by \( \partial_\mu \mathcal{F}_{\mu\nu} = j_\nu \) and is identically conserved:
\[ \partial_\mu j^M_\mu = 0 \quad (6) \]

A magnetic \( U(1) \) symmetry exists. Both \( \mathcal{F}_{\mu\nu} \) and the magnetic charge \( Q \) are colour singlets.

One can operate the abelian projection
\[ U(\vec{r}) \Phi(\vec{r}) = (0, 0, 1) \quad (7) \]
on generic configurations also in the unbroken phase of the system, where monopoles do not exist as solitons. \( U(\vec{r}) \) will be singular at the sites where \( |\Phi| = 0 \) and the singularities will reflect in the topology of the gauge field, as a Dirac strings.

Monopoles are lumps in the Higgs phase, but they can also exist in the symmetric phase, where they condense.

In QCD there is no Higgs field. However any operator in the adjoint representation can play the role of \( \Phi \), and define monopoles which are exposed by the corresponding abelian projection.

On the lattice any closed path of parallel transport defines a \( \Phi \) in the adjoint representation. An open plaquette, an open Polyakov line, an open “butterfly”, \( \hat{\varepsilon}_{\mu\nu\rho\sigma} \Pi_{\mu\nu}(n) \Pi_{\rho\sigma}(n) \). There exist in fact a continuous infinity \( [1] \) of monopoles species. What monopoles do condense in the vacuum? A guess of ‘t Hooft \([1]\) is that all of them are physically equivalent: configuration by configuration the location and the number of the monopoles is different, but on the average they are indistinguishable.

There is a community of practitioners of the maximal abelian projection \([1]\) or of the Delambertian projection \([12]\), who believe that their choice is better than others because of abelian and monopole dominance \([13]\). The physical quantities, like the string tension, relevant to confinement, are indeed approximated within 20% by the corresponding quantities computed in the abelian projected \( U(1) \), or in terms of the monopole part of them, meaning that the projection identifies the degrees of freedom relevant to confinement.

3. The disorder parameter for dual superconductivity.

The basic idea is to construct an operator \( \mu \) which carries non zero monopole charge, and use its vacuum expectation value \( \langle \mu \rangle \) as a detector of symmetry. \( \langle \mu \rangle \neq 0 \) signals spontaneous breaking of magnetic \( U(1) \), and hence, under very general assumptions, dual superconductivity. In a theory in which electric charges are pointlike, magnetic monopoles have non trivial topology, due to Dirac string: moreover they are coupled with magnetic charge \( m = n/e \) due to Dirac quantization condition.

A similar feature is present in a variety of systems in statistical mechanics and is known as duality. The prototype is the 2d Ising model \([14]\), where the system can be described either in terms of local spin \( \sigma_i = \pm 1 \) or in terms of 1 dimensional kinks which are spins on the dual lattice. The partition function is the same with the correspondence \( \beta \rightarrow \sim 1/\beta \) which maps strong coupling regime of the model with weak coupling of its dual.

The basic idea to construct a creation operator for a topological configuration is to shift the field configuration by the classical topological configuration by the translation operator \([15,16] \).

In the same way as \( e^{i \rho \alpha} |x \rangle = |x + a \rangle \)
\[ \mu(\vec{y}, t) = \exp \left[ i \int \Pi(\vec{x}, t) \Phi_{cl}(\vec{x}, \vec{y}) d^3 x \right] \quad (8) \]
operates on a field configuration \( |\Phi(\vec{x}, t)\rangle \) as
\[ \mu(\vec{y}, t)|\Phi(\vec{x}, t)\rangle = |\Phi(\vec{x}, t) + \Phi_{cl}(\vec{x}, t)\rangle \]

Technical modifications are needed with compact theories, where the field cannot be shifted arbitrarily \([16]\).

What is measured on the lattice is the correlator
\[ \mathcal{D}(t) = \langle \bar{\mu}(\vec{0}, t) \mu(\vec{0}, 0) \rangle \quad (9) \]
which describes a monopole sitting at site \( \vec{x} = 0 \) and propagating from time \( x_0 = 0 \) to \( x_0 = t \).

At large values of \( t \)
\[
D(t) \approx A \exp(-Mt) + \langle \mu \rangle^2 \tag{10}
\]
\( \langle \mu \rangle \neq 0 \) means dual superconductivity.

At finite temperature a direct measurement of \( \langle \mu \rangle \) can be done: then antiperiodic boundary conditions in time are needed.

Instead of \( D(t) \) it can prove numerically convenient to measure \( \rho(t) = \frac{1}{2} \frac{d}{d\beta} \ln D(t) \).

At large values of \( t \) again from eq.(10)
\[
\rho(t) \approx C \exp(-Mt) + \rho(0) \tag{11}
\]
with \( \rho = \frac{d}{d\beta} \ln \langle \mu \rangle \). Since \( \langle \mu \rangle = 1 \) for \( \beta = 0 \)
\[
\langle \mu \rangle = \exp(\int_0^\beta \rho(\beta')d\beta') \tag{12}
\]
The typical behaviour of \( \rho \) at the deconfining phase transition for \( SU(2) \) gauge theory is shown in fig.1, for different monopole species.

\[\text{Fig.1} \ \rho \text{ vs } \beta \text{ for different abelian projections in } SU(2). \text{ The negative peak signals phase transition.}\]

The main results are
a) Different monopole species have indistinguishable behaviour.

b) For \( \beta < \beta_c \) \( \rho \) tends to a finite limit as spatial volume goes to \( \infty \), showing that \( \langle \mu \rangle \neq 0 \) in the confined phase.

c) For \( \beta > \beta_c \) the regime is perturbative and
\[
\rho \sim -|c|L_S + c'
\]
with \( L_S \) the space extension of the lattice. This means that \( \langle \mu \rangle \to 0 \) as \( L_S \to \infty \), i.e. in the thermodynamical limit. The expectation for \( \langle \mu \rangle \) as a disorder parameter would be zero for \( \beta > \beta_c \). This can only happen in the thermodynamical limit: for finite volume \( \langle \mu \rangle \) is an analytic function of \( \beta \) for any finite size of the lattice, and cannot vanish identically for \( \beta > \beta_c \), except if it vanishes for all values of \( \beta \). Only in the infinite volume limit Lee Yang singularities develop and \( \langle \mu \rangle \) can become a real disorder parameter.

d) For \( \beta \sim \beta_c \) \( \rho \) has a sharp negative peak, which means an abrupt decrease of \( \langle \mu \rangle \) towards 0. In this region the correlation length goes large,
\[
\xi \sim (\beta_c - \beta)^{-\nu} \tag{13}
\]
with \( \nu \) a critical index. By dimensional arguments,
\[
\langle \mu \rangle = f(\frac{a}{\xi}, \frac{L}{\xi}) \sim f(0, \frac{L}{\xi}) \tag{14}
\]
or by eq.(13)
\[
\langle \mu \rangle = F(L^{1/\nu}(\beta_c - \beta)) \tag{15}
\]
whence the scaling law follows
\[
\rho/L^{1/\nu} = \Phi(L^{1/\nu}(\beta_c - \beta)) \tag{16}
\]
Scaling is obeyed only with the proper value of index \( \nu \), which can be then determined, together with \( \beta_c \). The quality of scaling is shown in fig.2 and the corresponding value of \( \nu \), determined by best fit procedure is
\[
\nu = 0.62 \pm 0.02 \tag{17}
\]
The expectation is that the transition is second order and that it belongs to the same class of universality as the 3d Ising model, or \( \nu = .631(1) \).
A similar analysis\cite{18} for SU(3) shows that there is no appreciable difference neither in the disorder parameters for the 2 possible choices of monopoles defined by a given abelian projection, nor between different abelian projections (plaque-tte, Polyakov line, butterfly), fig.3,fig.4.

Fig. 3 $\rho$ for the two different monopole species in the Polyakov projection. SU(3).

An attempt to determine an effective critical index for the transition, which is known to be weak first order gives $\nu \sim 0.6$.

The expectation for large enough values should be $1/3$, or $1/d$. Larger volumes are under investigation to check that.

Fig. 2 Finite size scaling eq. (16). SU(2).

Fig. 4 $\rho$ for different abelian projections. SU(3).

4. Conclusions.

Vacuum of SU(2), SU(3) non abelian gauge theories in the confined phase is a dual superconductor, independent of the abelian projection used\cite{10}. This evidence comes from direct investigation of the symmetry of the vacuum, by a disorder parameter detecting monopole condensation. The critical index for SU(2) deconfining phase transition can be determined and is compatible with 3d Ising model, as expected.

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