Fuzzy Discrete Problems with Summed Objective Function and their Crisp Bi-Objective Modifications

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Abstract. Discrete optimization problem with fuzzy input parameters having the form of triangular fuzzy numbers is considered. Objective function is given by the sum of some subset of fuzzy input data. Crisp model with two criteria: objective function and its membership function is formulated. We propose an algorithm which constructs the Pareto set of this bi-criteria problem. We consider particular fuzzy discrete problems, such as scheduling problems, routing problems, problems of selecting a subset of elements, maximum matching problems, where the obtained results could be applied.

1. Introduction

Usually input data of optimization problems could have uncertain nature, e.g. we could not certain evaluate processing time and cost of some operation, or due dates are not rather crisp. To operate with uncertainty Zadeh set up theory of fuzzy sets \cite{1}. Since that time fuzziness came to different areas of research, including discrete optimization problems.

In \cite{2} Ekel et al discussed how deal with uncertainty in optimization problem: fuzzy constraints are approximated by the set of deterministic (nonfuzzy) constraints of greater dimension, and then problem with fuzzy objective function and nonfuzzy constraints is solved by some kind of gradient method.

Chanas and Kuchta in their chapter \cite{3} gave a survey of fuzziness of classical discrete optimization problems, such as network optimization problems, integer transportation problems, assignment problem, network planning problem, scheduling problems, set covering problem, knapsack problem, 0-1 linear problems. Here, weights of arcs, coefficients of objective functions, the coefficients and/or free term of constraints are fuzzy interval numbers depending on the certain problem. In particular triangular and trapezoidal fuzzy numbers are used. Also a notation of fuzzy covering is considered for set covering problem. Analysis and algorithms on particular problems could also be found in the following references: fuzzy scheduling problems \cite{4–6}, fuzzy instances of transportation problems \cite{7, 8}.

In \cite{9} Sakawa considered various not only single but also multiobjective problems: fuzzy 0-1 programming problem, fuzzy nonlinear programming problem, fuzzy job-shop scheduling problem, including genetic algorithms and some applications.

In our paper firstly we recall main definitions and notations of fuzzy sets theory. Secondly, we consider discrete optimization problem, which objective function is a summed value over fuzzy input data. We use triangular fuzzy numbers. Then we transform fuzzy single-objective
problem into crisp bi-objective problem, where vector criterion consists of objective function and its membership function. Algorithm constructing the Pareto set of this bi-objective problem is proposed. We consider particular discrete problems, which objective function has the form of sum over values of fuzzy input parameters: scheduling problems, routing problems, problems of selecting a subset of elements.

2. Elements of Fuzzy Set Theory
Consider some definitions of fuzzy set theory according to [1, 10].

Let \( U \) be a non-empty set of elements (the so-called universal set). A fuzzy set \( A \) in \( U \) is defined as the set of ordered pairs \( \{ u, \mu_A(u) \} \) for any \( u \in U \), where \( \mu_A(\cdot) \) is called a membership function which associates with each element in \( U \) a real number in the interval \([0, 1]\). The value of \( \mu_A(u) \) indicates the grade of membership of element \( u \) in the set \( A \). When a membership function takes only values 0 and 1, we have a set \( A \) in the ordinary sense (crisp set), i.e. element does or does not belong to the set \( A \).

Fuzzy sets could be defined by enumerating elements with its grade of membership or analytically by formula, figure, etc.

The height of a fuzzy set \( A \) is the number \( h(A) = \sup_{u \in U} \mu_A(u) \). If the height of fuzzy set equals 1, then it is called normal, otherwise fuzzy set is subnormal.

A fuzzy set \( A \) in linear space \( U \) is convex if its membership function \( \mu_A \) satisfies the inequality
\[
\mu_A(\lambda u + (1 - \lambda)w) \geq \min\{\mu_A(u), \mu_A(w)\}
\]
for any \( u, w \in U, \lambda \in (0, 1) \). A fuzzy number is convex normal fuzzy set in the set of real numbers \( \mathbb{R} \). One of the most commonly used fuzzy number is the triangular fuzzy number (TFN)

\[
\mu_A(x) = \begin{cases} 
0 & \text{if } x < a, \\
\frac{x-a}{b-a} & \text{if } a \leq x < b, \\
\frac{c-x}{c-b} & \text{if } b \leq x < c, \\
0 & \text{if } x \geq c,
\end{cases}
\]

which plot has the form of triangle and is denoted by triplet \((a, b, c)\). Another frequently considered fuzzy number is the trapezoidal fuzzy number.

3. Bi-criteria Problem
3.1. Problem Formulation
We consider a discrete optimization problem with the set of feasible solutions \( X \) and objective function \( f \). Objective function is going to be maximized. \( Y = f(X) \) is the set of feasible outcomes. The problem has fuzzy input parameters represented by TFNs and denoted by triplet \((p_j - \delta_j, p_j, p_j + \varepsilon_j)\), \( \varepsilon_j, \delta_j > 0 \), \( j \in J \) (\( J \) is the set of parameters indices). We suppose that the objective function \( f \) is summed value of some set of input parameters values. It is well known that the sum of two TFNs will also be a TFN \((a_1, b_1, c_1) + (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2)\). According to the extension principle of Zadeh [10] the membership function \( \mu_f \) of function \( f \) will be the following:

\[
\mu_f(y) = \sup_{x \in f^{-1}(y)} \{ \alpha \in [0, 1] \mid \mu(x) = \alpha \}, \quad \text{if } f^{-1}(y) \neq \emptyset,
\]

\[
0, \quad \text{otherwise.}
\]

Thus, the membership function \( \mu_f(\cdot) \) will be the upper bound of TFNs union, where each \( y \in Y \) assign the TFN that has the form of triplet \((y - \delta(y), y, y + \varepsilon(y))\) (see, e.g. bold line on Fig. 1).

Here \( y = \sum_{j \subseteq J} \delta_j \), \( \delta(y) = \sum_{j \subseteq J} \delta_j, \varepsilon(y) = \sum_{j \subseteq J} \varepsilon_j \) for some subset \( S \subseteq J \).
In particular, the aforementioned could be applied to the following problems: fuzzy single-machine scheduling having fuzzy durations and release times of jobs as input parameters and total flow time as objective function, fuzzy set covering problem with fuzzy weights of elements, and fuzzy travelling salesman problem with fuzzy lengths of arcs.

![Figure 1. Example of membership function \( \mu_f \) of objective function \( f \)](image)

Noghin [11] proposed to modify optimization problem with fuzzy \( m \)-objective vector function to crisp \( m + 1 \)-criteria problem as follows. Objective functions and their membership functions, all of them should be maximized, compose the criteria of crisp problem. Maximization of membership function has intention to get solutions with higher grade of membership/confidence. In our case we have single-objective function and thus we get the crisp bi-criteria problem \( < X, g > \) with the set of feasible solutions \( X \) and criteria \( g = (f, \mu_f) \), such that \( g(x) = (f(x), \mu_f(f(x))) \) for all \( x \in X \).

Recall [12], that the set of pareto-optimal solutions (alternatives) of the set \( X \) with respect to criteria \( f = (f_1, \ldots, f_m) \) is \( P_f(X) = \{ x \in X \mid \not\exists x^* \in X : f(x^*) \geq f(x) \} \). Also the Pareto set \( P(Y) = f(P_f(X)) \), \( P(Y) = \{ y \in Y \mid \not\exists y^* \in Y : y^* \geq y \} \). Here, \( y^* \geq y \) means that \( y^* \neq y \) and \( y_i^* \geq y_i \), for all \( i = 1, \ldots, m \).

In multicriteria optimization the Pareto set is usually considered as generalization of optimum of single criteria problem [12, 13]. In further section we provide the construction of the Pareto set of the problem \( < X, g > \).

3.2. Construction of the Pareto Set

We state the crisp bi-criteria problem \( < X, g > \), denote \( G = g(X) \). The Pareto set of any multicriteria problem in terms of maximization represents a north-east border of the set of feasible outcomes. Thus, as the set \( G \) is the upper bound of union of TFNs, the Pareto set \( P(G) \) is the most right polygonal line \( L \), which is constructed by the right sides of TFNs corresponding to the maximum and near-maximum values of the function \( f(x) \) (see, Fig. 2).

The algorithm of construction of the Pareto set \( P(G) \) consists of three stages.

The first stage. Iterating through all set of feasible solutions \( X \) we find maximum value of function \( g_1(x) = f(x) \). Let \( y^* = \max_{x \in X} f(x) \).

The second stage. Iterating through all set of feasible solutions \( X \) we find the set of nearby values \( Y^*_\varepsilon \) to the value \( y^* \), such that \( Y^*_\varepsilon = \{ y + \varepsilon(y) \mid y^* + \varepsilon(y') < y + \varepsilon(y), y = f(x) \ \forall x \in X \} \).

Here the value \( \varepsilon(y) \) is the third (the right) parameter of triplet of TFN corresponding to the value \( y = f(x) \) for some \( x \in X \). Suppose there exist such \( y' + \varepsilon(y'), y'' + \varepsilon(y'') \in Y^*_\varepsilon \) that \( y' + \varepsilon(y') = y'' + \varepsilon(y'') \), but \( y' \neq y'' \). Then we exclude the value \( \bar{y} + \varepsilon(\bar{y}) \) from the set \( Y^*_\varepsilon \), where \( \bar{y} = \min\{y', y''\} \), because it does not influence on polygonal line \( L \).

The third stage. Let \( Y^*_\varepsilon = \{ y_1 + \varepsilon(y_1), \ldots, y_l + \varepsilon(y_l) \} \). Since all values are distinct we could assume that \( y_1 + \varepsilon(y_1) < y_2 + \varepsilon(y_2) < \ldots < y_l + \varepsilon(y_l) \). Denote the nodes of polygonal line \( L \) in the sequence from left to right as follows: \( (\hat{y}_1, \lambda_1), (\hat{y}_2, \lambda_2), \ldots, (\hat{y}_p, \lambda_p), \hat{y}_1 < \ldots < \hat{y}_p, \lambda_1 > \ldots > \lambda_p \).
Number of points \( p \) could be less than \( l + 1 \). Obviously \((\hat{y}_1, \lambda_1) = (y_1, 1), (\hat{y}_p, \lambda_p) = (y_l + \varepsilon(y_l), 0)\).

The line passing through the pair of points \((y_i, 1), (y_i + \varepsilon(y_i), 0)\) we call the line \(l_i\). The intersection of the lines \(l_i\) and \(l_j\) for some \(i, j \in \{1, \ldots, l\}, i \neq j\) gives the point \((\hat{y}, \lambda)\) with components

\[
\hat{y} = \frac{\varepsilon(y_j)y_i - \varepsilon(y_i)y_j}{\varepsilon(y_j) - \varepsilon(y_i)}, \quad \lambda = \frac{y_j + \varepsilon(y_j) - y_i - \varepsilon(y_i)}{\varepsilon(y_j) - \varepsilon(y_i)}.
\]

Now, we describe how to find the values of points \((\hat{y}_2, \lambda_2), \ldots, (\hat{y}_{p-1}, \lambda_{p-1})\), and thus construct the polygonal line \(L\). The procedure consists of \((l - 1)\) steps. Denote by \(I_L\) the ordered set of lines indices that determine the polygonal line \(L\) in order of the construction. Also by \(I_L(i)\) we denote the index of the line at position \(i\). At each step we could overwrite and delete some points, obtained at previous steps, modify the set \(I_L\) and also specify the number \(p\). The final step \(l\) gives all points of the polygonal line \(L\) and the set \(I_L\). Initially we put \(I_L = \{1\}, (\hat{y}_1, \lambda_1) = (y_1, 1)\).

Step 1. We take the lines \(l_1\) and \(l_2\). Their intersection gives the point \((\hat{y}_2, \lambda_2)\) by formula (1). \(I_L = \{1, 2\}, p = 2\).

Step 2. We take the lines \(l_2\) and \(l_3\), evaluate the intersection by formula (1), denote the point by \((y', \lambda')\) and check whether \(y' \geq \hat{y}_2, \lambda' \leq \lambda_2\), and \((y', \lambda') \neq (\hat{y}_2, \lambda_2)\). If this condition is valid then we put \((\hat{y}_3, \lambda_3) = (y', \lambda'), I_L = \{1, 2, 3\}, p = 3\). Otherwise we overwrite the point \((\hat{y}_2, \lambda_2)\) by the value obtained as intersection of the lines \(l_1\) and \(l_3\). \(I_L = \{1, 3\}, p = 2\).

Step \(j\). Suppose at step \(j - 1\) we have got that \(p\) equals some \(p'\) and the point \((\hat{y}_{p'}, \lambda_{p'})\) was obtained by intersection of the lines \(l_{k'}, l_j\), \(k' < j\). It means that \(I_L(p - 1) = k'\), \(I_L(p) = j\). We consider the lines \(l_j\) and \(l_{j+1}\), which intersection gives the point \((y', \lambda')\) by formula (1). If \(y' \geq \hat{y}_{p'}, \lambda' \leq \lambda_{p'}\), and \((y', \lambda') \neq (\hat{y}_{p'}, \lambda_{p'})\), then we put \((\hat{y}_{p'+1}, \lambda_{p'+1}) = (y', \lambda')\), \(I_L = \{I_L(1), \ldots, I_L(p'), j + 1\}, p = p' + 1\). Thus we add new point \((y', \lambda')\) assigning it to \((\hat{y}_{p'+1}, \lambda_{p'+1})\), and increasing the value of \(p\) by one. Otherwise we find \(i^* = \min\{i \in \{2, \ldots, p\} | \).

Figure 2. The Pareto set of bi-criteria problem \((f, \mu_f)\)
\[ y' \leq \hat{y}_i, \lambda' \geq \lambda_i, (y', \lambda') = l_{i+1} \cap l_s, \quad s = I_L(i - 1). \]

Then we put \((\hat{y}_{i^*}, \lambda_{i^*}) = l_{i+1} \cap l_{s^*}, s^* = I_L(i^* - 1), I_L = \{I_L(1), \ldots, I_L(i^* - 1), j + 1\}, p = i^*\). Here we actually reevaluate the point \((\hat{y}_{i^*}, \lambda_{i^*})\) and remove the points with indices greater or equal to \(i^* + 1\). This leads to corresponding modifications of the set \(I_L\).

Step \(l\). We increase \(p\) by 1 and add point \((\hat{y}_p, \lambda_p) = (y_l + \varepsilon(y_l), 0), I_L\) is unchanged.

The asymptotic computational complexity of the presented algorithm is \(O(|X|^2)\), but the actual number of steps may be essentially smaller for individual instances.

Fig. 3 shows the example of evaluating the points of the polygonal line \(L\) (in bold on the figure). The number of steps is 4. Initially we put \(I_L = \{1\}\), \((\hat{y}_1, \lambda_1) = (a, 1), p = 1\).

Step 1. \(I_L = \{1, 2\}\), \((\hat{y}_1, \lambda_1) = (a, 1), (\hat{y}_2, \lambda_2) = (c, 0, 55), p = 2\).

Step 2. \(I_L = \{1, 2, 3\}\), \((\hat{y}_1, \lambda_1) = (a, 1), (\hat{y}_2, \lambda_2) = (c, 0, 55), (\hat{y}_3, \lambda_3) = (d, 0, 2), p = 3\).

Step 3. \(I_L = \{1, 4\}\), \((\hat{y}_1, \lambda_1) = (a, 1), (\hat{y}_2, \lambda_2) = (b, 0, 7), p = 2\).

Step 4. \(p = 3, (\hat{y}_3, \lambda_3) = (e, 0)\).

\[ \begin{array}{c}
\text{Figure 3. Example of construction of the points of the polygonal line } L \\
\end{array} \]

The equation of the line \(l_i\) is \(y - \varepsilon(y_i)\lambda - y_i - \varepsilon(y_i) = 0\) (\(y\) stands for \(g_1\), \(\lambda\) stands for \(g_2\)). Thus, we get analytical representation of the Pareto set

\[
P(G) = \bigcup_{j=1}^{p-1} \{(y, \lambda) \mid y - \varepsilon(y_j)\lambda - y_j - \varepsilon(y_j) = 0, \hat{y}_j \leq y \leq \hat{y}_{j+1}, \lambda_{j+1} \leq \lambda \leq \lambda_j, i = I_L(j)\}.
\]

We note that if the set \(Y^*_\varepsilon\) is empty, then the Pareto set is the right side of TFN corresponded to value \(y^*\), i.e. \(P(G) = \{(y, \lambda) \mid y - \varepsilon(y^*)\lambda - y^* - \varepsilon(y^*) = 0, y^* \leq y \leq y^* + \varepsilon(y^*), 0 \leq \lambda \leq 1\} \).
The obtained optimum $P(G)$ of the problem $<X, g>$ says that when the input data are fuzzy, the optimal value of the objective function $f$ becomes “uncertain”. If we have a crisp problem with the objective function $f$, everything is clear: we choose the alternative, which maximizes the function $f$ (in the case of maximization problem). But if we want to choose one alternative in fuzzy problem we should compromise between the goal and the level of membership of this goal, that leads us to nearby values of optimum $y^*$ of crisp problem. On one hand, we could aim at values of $f$ rather higher than $y^*$, in the scope of the Pareto set $P(G)$, but its grade of membership will be close to 0. On another hand, aiming at grade of membership close to 1 could give us values of function $f$ almost equal to $y^*$.

There exist various methods and techniques dedicated to the choice of an optimal alternative in multicriteria problem taking into account preferences of the decision maker [14]. For example [15], we could use axiomatic approach of the Pareto set reduction to choose some subset of the set $P(G)$ based on the compromise between the maximization of function $f$ and its grade of membership.

4. Fuzzy Discrete Problems with Criterion of Summing Input Data
The presented discrete problems arise in scheduling theory [16], routing problems [17] and applications of selecting a subset of elements [18].

Scheduling theory: a set of jobs is to be processed on a set of machines. Each job may be specified by one or several operations executed sequential or parallel, processing time and release time. Precedence constraints, resource and technological constraints, setup times may be given on a set of jobs or operations. We may consider a single machine, parallel identical, parallel uniform or unrelated machines. A feasible schedule is defined by starting or completion times of jobs, the optimization criterion is the mean flow time (the sum of completion times of jobs). Practical applications are open shop, job shop, flow shop systems and their flexible variants; customer order scheduling problems; scheduling single- and multi-processor jobs on parallel machines. Here we may incorporate fuzzy processing times, fuzzy release times and fuzzy setup times, which introduce the fuzzy completion times of jobs.

Routing problems: a set of vehicles must serve a set of clients. Vehicles may be uncapacitated or capacitated, homogeneous or heterogeneous. Time windows, demands, consistency service and splitting requirements, frequency and driver shift length constraints may be given for clients. The goal is to find a visiting schedule for each client and a set of routes for each vehicle that jointly service all clients with minimal sum of servicing completion times. In particular, the well known traveling salesman problem and its generalizations belongs to this class in the case of criterion minimizing the sum of completion times. For these problems we may investigate the fuzzy traveling times between clients locations (or between vertices).

Problems of selecting a subset of elements: a subset with prespecified properties and optimal sum of weights is required to select from a given set of elements. The considered elements may have some connections and structural characteristics. Examples are the following: set covering, set partition, set packing, hitting set problems and problems from the graph theory (vertex and edge covering, independent vertex set, clique, degree bounded connected subgraph, maximum matching). The fuzzy weights of elements may be involved in the considered context.

Consider the classical set covering problem in the fuzzy statement. We have a set of elements $N = \{1, \ldots, n_e\}$, and a collection $M = \{M_1, \ldots, M_{m_s}\}$ of subsets of these elements. Let TFN $(c_j - \delta_j, c_j, c_j + \varepsilon_j)$ represent the fuzzy weight of subset $M_j$. A feasible subcollection $M_s \subseteq M$ is such that every element $i \in N$ is “covered” by (belongs to) at least one of the subsets in $M_s$, the set of all feasible subcollections is denoted by $\mathcal{M}$. The total sum of weights with the sign “minus” $y = f(M_s) = -\sum_{M_j \in M_s} c_j$ and the membership function $\mu_f(\cdot)$ are maximized over all feasible subcollections. Thus we have the bi-criteria problem $<\mathcal{M}, g>$ with the set of all feasible subcollections $\mathcal{M}$ (the set of feasible solutions) and vector criterion $g = (f, \mu_f)$. 


Present an illustrative instance. Let \( n_\epsilon = 12, m_\epsilon = 9, M_1 = \{1, 2, 3, 7, 8, 9\}, M_2 = \{4, 5, 6, 10, 11, 12\}, M_3 = \{1, 2, 7, 8\}, M_4 = \{3, 4, 9, 10\}, M_5 = \{5, 6, 11, 12\}, M_6 = \{1, 5, 9\}, M_7 = \{2, 6, 10\}, M_8 = \{3, 7, 11\} \) and \( M_9 = \{4, 8, 12\} \). We define the fuzzy weights as

\[
(c_j - \delta_j, c_j, c_j + \varepsilon_j) = (1 - 0.1, 1, 1 + 0.1), \quad j = 1, 2;
\]

\[
(c_j - \delta_j, c_j, c_j + \varepsilon_j) = (1 - 0.5, 1, 1 + 0.5), \quad j = 3, 4, 5;
\]

\[
(c_j - \delta_j, c_j, c_j + \varepsilon_j) = (1 - 0.7, 1, 1 + 0.7), \quad j = 6, 7, 8, 9.
\]

The set of pareto-optimal solutions consists of three collections \( M_{s_1} = \{M_1, M_2\}, M_{s_2} = \{M_3, M_4, M_5\} \) and \( M_{s_3} = \{M_6, M_7, M_8\} \). Its images with respect to objective function \( f \) are the following TFNs respectively

\[
y(M_{s_1}) - \delta(M_{s_1}), y(M_{s_1}), y(M_{s_1}) + \varepsilon(M_{s_1}) = (-2 - 0.2, -2, -2 + 0.2),
\]

\[
y(M_{s_2}) - \delta(M_{s_2}), y(M_{s_2}), y(M_{s_2}) + \varepsilon(M_{s_2}) = (-3 - 1.5, -3, -3 + 1.5),
\]

\[
y(M_{s_3}) - \delta(M_{s_3}), y(M_{s_3}), y(M_{s_3}) + \varepsilon(M_{s_3}) = (-4 - 2.8, -4, -4 + 2.8).
\]

The Pareto set \( P(G) \) is the polygonal line with three points, \( I_L = \{1, 3\}, p = 3 \)

\[
P(G) = \{(y, \lambda) \mid y - 0, 2\lambda + 1, 8 = 0, \quad -\frac{24}{13} \leq y \leq -2, \quad \frac{3}{13} \leq \lambda \leq 1\} \cup \{(y, \lambda) \mid y - 2, 8\lambda + 1, 2 = 0, \quad -1, 2 \leq y \leq -\frac{24}{13}, \quad 0 \leq \lambda \leq \frac{3}{13}\}
\]

All other feasible collections have \( y(M_\epsilon)  \geq -3 \) and \( y(M_\epsilon) + \varepsilon(M_\epsilon) \leq -1.8 \) and so do not belong to the Pareto optimal solutions set.

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