Dynamic dislocation drag near to a point of martensitic transformation

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The influence of coherent interface on dissipations of mechanical energy of driven dislocations near to a point of martensite type phase transition is considered. The expressions for dynamic braking of dislocations, owing to losses of energy on excitation of deformations-type phase transition are received.

Intensive experimental and theoretical study of dislocations dynamics [1-3] has resulted in creation of correct classification of mechanisms of energy dissipations of dislocation which drives in isotropic body [4-7]. In the same time the questions connected to dynamic braking dislocations in a crystal containing dot defects were considered [8,9]. It is known, that on plastic and relaxations property of materials, essential influence renders domain structure: its static and dynamic characteristics. Constructed in works [10-11] the dynamic theory of coherent interphase boundaries allows at a qualitatively new level to consider questions of interaction of elastic fields created by dislocations and domain walls.

In present work the dynamic braking of dislocations in a crystal with coherent interphase boundaries is investigated. As against earlier works on this theme, self-consistent dynamic theory of interface here is used, considering them as independent objects, which have of internal degrees of freedom.

Let’s proceed from the following expression for dissipations of energy $D$ [12]

$$D = 2\pi \int \frac{dq_{||}}{(2\pi)^2} \int \frac{d\omega}{2\pi} |f^{ext}(q_{||},\omega)|^2 \text{Im} g(q_{||},\omega), \quad (1)$$

where

$$f^{ext}(q_{||},\omega) = -\{\sigma_{ik}^{ext}\} [S_{ik}] \quad (2)$$

- is configuration force acting on the part of moving dislocations in a direction of boundary,
$q$ and $\omega$ wave vector and frequency of the elastic vibrations,
$q_{||}^2 = q_x^2 + q_y^2$,
$\sigma_{ik}^{ext}$ - stress tensor created by an external source,
$[S_{ik}] = 1/2(m_i S_k + m_k S_i)$ plastic deformation tensor of interface,
$m_i$ unit vector of the normal to the habit planes,
$S_k$ plastic displacement vector,
$\{\ldots\}$ half-sum of the values of the braced quantity on both sides of the boundary,
$[\ldots]$ jump across the boundary.

Green function $g(q_{||},\omega)$ coherent interface we receive by solving the dynamics equations of elastic theory together with heat equation with the appropriate boundary conditions. (equality to zero of configuration force (2) in each point of a border surface) [11]

$$\rho \delta^2_{ik} u_i - \lambda \delta_{iqlm} \partial_k (\partial_l u_m - s_{lm}) = 0; \quad (3)$$

$$\partial_t T - \chi \partial^2_{kk} T = \frac{[H_\sigma]}{C_v} \partial_t \zeta(\mathbf{r}_||, t) \delta(z); \quad (4)$$

$$[S_{jk}]\{\sigma_{jk}\} + [H_\sigma](T - T_0)/T_0 = 0; \quad (5)$$

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where  
\[ s_{ik} = [S_{ik}] \delta(z) \zeta(r||t) \] 
plastic deformation tensor of body,
\[ \lambda_{iklm} = \lambda \delta_{ik} \delta_{lm} + \mu(\delta_{il} \delta_{km} + \delta_{im} \delta_{kl}) \] 
elastic constants tensor in isotropic approximation,
\[ \lambda^s_{ik} = \lambda_{iklm} [S_{lm}] \]
Green function of equation (6) can be written as
\[ H_o \] 
enthalpy of the phase transformation,
\[ \zeta(r||t) \] 
displacement of boundary along the normal vector,
\[ C_v \] 
specific heat,
\[ \chi \] 
heat conductivity,
\[ T_o \] 
phase transitions temperature,
\[ \lambda, \mu \] 
Lame’s constants.
Equation (5) we shall write as
\[ [S_{ik}] \lambda_{iklm} (u_{lm} - \delta(z) \zeta(r||t) [S_{lm}]) \big|_{z=0} + [H_o] (T - T_o) / T_o = 0. \]
Solving the set of equations (3)-(5) in Fourier representation
\[ u_i (q, \omega) = -i \int \frac{dz}{2\pi} (G_{ik} \lambda^s_{kl} q_i \zeta(q||, \omega)), \]
\[ T (q, \omega) = \frac{[H_o]}{C_v} \int \frac{dz}{2\pi} \frac{(-i\omega/\chi)}{q^2 - i\omega/\chi} \zeta(q||, \omega), \]
\[ 0 = iq \lambda^s_{lm} u_m (q, \omega) - \int \frac{dz}{2\pi} \zeta(q||, \omega) \lambda^s + [H_o] (T - T_o) / T_o, \]
for the Fourier-transform of function \( \zeta(q||, \omega) \) we find
\[ \zeta(q||, \omega) \int \left( \lambda^s - G (q, \omega) + \frac{[H_o]^2}{T_o C_v} \frac{i\omega/\chi}{q^2 - i\omega/\chi} \right) \frac{dz}{2\pi} = 0. \]  
Green function of equation (6) can be written as
\[ g(q||, \omega) = \frac{1}{\int \left( G(0, 0, q||, 0) - G(q||, \omega) + \frac{[H_o]^2}{T_o C_v} \frac{i\omega/\chi}{q^2 - i\omega/\chi} \right) \frac{dz}{2\pi}}. \]
Here
\[ \lambda^s = [S_{ik}] \lambda_{iklm} [S_{lm}] = [S_{ik}] \lambda^s_{ik}, \]
\[ G = q_i \lambda^s_{ik} G_{kl} \lambda^s_{lm} q_m, \]
\[ G_{ij} = \frac{1}{\mu (q^2 - \omega^2 / c_t^2)} \left( \delta_{ij} - \frac{(1 - \gamma^2) q_i q_j}{q^2 - \omega^2 / c_t^2} \right) \]  
- Green function of dynamic equation of elastic theory,
\[ c_t, c_l \] 
sound velocity of transverse and longitudinal waves,
\[ \gamma = c_l / c_t. \]
If boundary orientation is \( S_i = (S_x, 0, S_z), n_i = (0, 0, n_z) \) then
\[ \lambda^s = \mu (S_x^2 + S_z^2 / \gamma^2), \]
\[ G(q||, \omega) = (\lambda q_j [S_{mm}] + 2\mu q_i [S_{ij}]) G_{jk} (\lambda q_k [S_{mm}] + 2\mu q_i [S_{ik}]) \]
\[ = (\lambda^2 q_j G_{jk} q_k + 4\lambda \mu q_3 G_{3k} q_k + 4\mu^2 G_{33} q_3^2) S_3^2 \]
\[ + (\lambda (q_1 G_{3k} q_k + q_3 G_{1k} q_k) + 2\mu [G_{31} q_3^2 + G_{33} q_1 q_3]) 2\mu S_1 S_3 \]
\[ + (2G_{31} q_1 + G_{33} q_3^2 + G_{31} q_3^2) \mu^2 S_3. \]
Using (8) can be written (10) as

$$G(q, \omega) = \left( \frac{\lambda}{\mu} + 2 \frac{q^2}{q^2 - c_l^2} \right)^2 \frac{\omega^2 \gamma^2}{\kappa^2} + 4 \frac{q^2}{q^2 - c_l^2} \frac{q^2}{\kappa^2} \mu S^2$$

$$+ \left( \frac{\lambda}{\mu} + 2 \frac{q^2}{q^2 - c_l^2} \right)^2 \gamma^2 \frac{q_1 q_3}{\kappa^2} + \frac{q_3 q_1 q_3 - q_3^3}{q^2 - c_l^2} \right) 4 \mu S_1 S_3,$$

$$+ \left( \frac{\gamma^2 (q_1^2 q_3^2 c_l^2 + q_1^2 q_3^2 - 4 q_3^2 q_3^2)}{\kappa^2} \right) \mu S^2_1,$$

corresponding for the Green function (7) we readily get

$$g^{-1}(q_\parallel, \omega) = \mu S^2 \int \left( \frac{4}{q_\parallel^2 - \omega^2/c_l^2} \left( \begin{array}{c} \frac{\omega^2/c_l^2}{(q_\parallel^2 - \omega^2/c_l^2)}(1 - \gamma^2) - \frac{\omega^2/c_l^2}{(q_\parallel^2 - \omega^2/c_l^2)} \end{array} \right) \right) \frac{dq_3}{2\pi}.$$ (11)

Integral from cross component \(S_1 S_3\) is equal to zero and we receive

$$(\mu S^2 g)^{-1} = \frac{S_1^2}{S_2^2} \left( \frac{q_\parallel^2 - \omega^2/c_l^2}{q_\parallel^2 - \omega^2/c_l^2} + \frac{q_1^2}{q_\parallel^2 - \omega^2/c_l^2} \left( \sqrt{q_\parallel^2 - \omega^2/c_l^2} - \sqrt{q_\parallel^2 - \omega^2/c_l^2} \right) \right)$$

$$+ 4 \frac{S_3^2}{S_2^2} \left( \frac{q_\parallel^2 - \omega^2/c_l^2}{\omega^2/c_l^2} \right) \left( \frac{1}{\sqrt{q_\parallel^2 - \omega^2/c_l^2}} - \frac{1}{\sqrt{q_\parallel^2 - \omega^2/c_l^2}} - \frac{\omega^2/4c_l^2}{\kappa^2} \right)$$

$$+ \frac{i \omega \delta/\chi}{\sqrt{q_\parallel^2 + \omega^2/c_l^2}} \left( \sqrt{q_\parallel^2 + \omega^2/c_l^2 + q_1^2} \frac{\sqrt{q_\parallel^2 + \omega^2/c_l^2 + q_1^2}}{2} + i \sqrt{q_\parallel^2 + \omega^2/c_l^2 + q_1^2} \right).$$ (12)

where \(\delta = \frac{[H_o]^2}{\mu S^2 T_C C_v}\) factor describing specific losses of energy of unit of the boundary area on formation of a new phase at its movement.

If \(\omega = \xi_c q_\parallel\) then we obtain the dispersion law of coherent interphase boundary:

$$\cos^2 \theta \left( \sin^2 \varphi - \xi^2 \right) \left( \frac{\kappa}{\sqrt{1 - \xi^2}} \right) + \frac{4 \cos^2 \varphi}{\kappa} \left( \sqrt{1 - \gamma^2 \xi^2 - \sqrt{1 - \xi^2}} \right)$$

$$+ \sin^2 \theta \sqrt{1 - \xi^2} \left( \frac{-\xi^2}{\sqrt{1 - \xi^2}} + \frac{4}{\kappa} \left( \sqrt{1 - \gamma^2 \xi^2 - \sqrt{1 - \xi^2}} \right) \right)$$

$$= \frac{\delta \xi_c/\chi q_\parallel}{\sqrt{1 + \xi^2 (c_1/\chi q_\parallel)^2}} \left( \sqrt{1 + \xi^2 (c_1/\chi q_\parallel)^2 - 1} + i \sqrt{1 + \xi^2 (c_1/\chi q_\parallel)^2 + 1} \right).$$ (13)

where \(\theta = \arctan \frac{S_y}{S_x}\) is the dilatation angle, \(\varphi = \arctan \frac{q_y}{q_x}\) direction of wave distribution located near to a boundary.

Basic feature of moving coherent interfaces is the obligatory radiation or absorption by it of the latent heat of phase transformation \([H_o]\). In this connection the most essential channel of energy losses of border oscillation is becomes dissipations caused by absorption of latent heat. That allows to neglect attenuation, caused by viscosity, heat conductivity of a crystal etc., considered in all earlier works on this theme.
Let’s consider dislocations, moving in a crystal with constant speed $V$. Then elastic stress created by it is possible to present as

$$
\sigma_{ik}^{\text{ext}}(r, t) = \int \frac{dq}{(2\pi)^3} \sigma_{ik}^{\text{ext}} e^{iqr - i\Omega t},
$$

where $\Omega = Vq$. In view of this expression, for dissipations of energy (1) we shall receive

$$
\overline{D} = \int \frac{dq}{(2\pi)^2} |f_{\Omega}(q_\parallel)|^2 \text{Im} \langle q_\parallel, \omega \rangle.
$$

Taking into account, that on structure of created sheared stress of considered defects their interaction is possible only by means of transverse elastic fields, that for the given orientation of coherent interface from every possible orientations of a dislocations line for us essential appear:

a) for screw dislocations

$$
\sigma_{xx} = \frac{\mu b}{2\pi(1 - \nu)} \frac{y - Vt}{(y - Vt)^2 + z_0^2},
$$

if $b \parallel Ox$, $\tau \parallel Ox$, where $\tau$ is unit vector tangent to dislocation line, $b$ the Burgers vector of the dislocation, $z_0$ distance up to boundary.

b) for edge dislocations

$$
\sigma_{xx} = \frac{\mu b}{2\pi(1 - \nu)} \frac{(x - Vz_t) ((x - Vz_t)^2 - z_0^2)}{((x - Vz_t)^2 + z_0^2)^2}
$$

if $b \parallel Ox$, $\tau \parallel Oy$.

Let’s consider these cases separately.

### A. Screw dislocations

Substituting a Fourier-image (15):

$$
\sigma_{xx}(q_\parallel, \omega) = i(2\pi)^2 \frac{\mu b}{2} \delta(q_x) \delta(\omega - q_y V) e^{-|q_y|z_0}
$$

into expression for dissipation of energy (14), with the account (2) we shall receive

$$
D = (2\pi)^2 \left( \frac{\mu b}{2} \right)^2 S_l^2 \int dq_y q_y V \text{Im} \langle 0, q_y, Vq_y \rangle e^{-2|q_y|z_0}
$$

$$
= (\frac{\pi b V \cos \theta}{\chi})^2 \frac{\delta \mu}{\chi} \int \frac{dq_y}{q_y} \frac{\sqrt{1 + V^2/q_y^2 \chi^2} + 1}{2 (1 + V^2/q_y^2 \chi^2)} e^{-2|q_y|z_0}
$$

$$
= (\frac{\pi b V \cos \theta}{\chi})^2 \frac{\delta \mu}{\chi} \int \frac{V \delta}{q_y} \left[ \frac{\sqrt{1 + V^2/q_y^2 \chi^2} - 1}{2 (1 + V^2/q_y^2 \chi^2)} \right]^2 + \left( \frac{V \delta}{q_y} \right)^2 \frac{\sqrt{1 + V^2/q_y^2 \chi^2} + 1}{2 (1 + V^2/q_y^2 \chi^2)}
$$

where

$$
A = \sqrt{1 - V^2/c_1^2},
$$

$$
B = \frac{1 - V^2/c_1^2}{V^2/c_1^2} \left( \frac{1}{\sqrt{1 - V^2/c_1^2}} - \frac{1}{\sqrt{1 - V^2/c_1^2}} \right) - \frac{V^2/4c_1^2}{\sqrt{1 - V^2/c_1^2}}.
$$

Results of the numerical analysis of integral in (17) for concrete meanings $\gamma^2 = 0.3$ and $\theta = \pi/4$ show on figure. For screw dislocation, driven perpendicularly to vector of shift $S$ the maximum is necessary on $V^2/c_1^2 \approx 0.914$, i.e. near to a sound meaning of speed of movement dislocation.
B. Edge dislocations

Driven along an axis $Ox$ edge dislocations creates around of itself chopping off stress (16)

$$\sigma_{xz}(q, \omega) = i(2\pi)^2 \frac{\mu b}{(1-\nu)} \delta(q_y) \delta(\omega - q_x V) e^{-|q_y|z} (1 - q_x z_0),$$

by means of which there is its interaction with coherent boundary and outflow of energy on its excitation. In this case for dissipation of energy $D$ of unit of length of dislocations shall receive:

$$D = \left( \frac{\pi b V \cos \theta}{(1-\nu)} \right)^2 \frac{\delta \mu}{\chi} \int \frac{dq_x}{q_x} \frac{\sqrt{1 + V^2/q_x^2 \chi^2} + 1}{2 (1 + V^2/q_x^2 \chi^2)} e^{-2|q_x|z} (1 - q_x z_0)^2$$

$$+ \frac{V \delta}{\chi q_x} \left( A \cos^2 \theta + 4B \sin^2 \theta - \frac{V \delta}{\chi q_x} \sqrt{1 + V^2/q_x^2 \chi^2 - 1} \right) \left( \frac{V \delta}{\chi q_x} \sqrt{1 + V^2/q_x^2 \chi^2} + 1 \right)^2$$

(18)

$$+ \frac{V^2}{c_t^2} \sqrt{1 - V^2/c_t^2} - \frac{V^2}{c_t^2} \sqrt{1 - V^2/c_t^2}$$

and $B$ coincides with (17). The numerical analysis (18) qualitatively does not differ from results received for screw dislocations (17). However, here it is necessary to note displacement of a maximum of dynamic braking in the party of lower speeds, though and all the same, close to sound $V^2/c_t^2 \approx 0.985$.

The transition from (7) - (8) to usually measure experimentally size - factor of dynamic dislocation braking - is made by a standard equation:

$$B = D/V^2.$$ 

In summary we shall note, considered above mechanisms dissipation of energy, it is necessary to take into account obviously, and in crystals with more complex structure: ferroelectrics, ferroelectrics-ferroelastics etc.

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Dependence of dissipation of energy $D$ rectilinear screw dislocation from speed $V$ and factor $\alpha$, which describes losses of boundaries energy on formation of a new phase at her movement.
This figure "Dum1.gif" is available in "gif" format from:

http://arxiv.org/ps/cond-mat/0012062v1