Decentralized multisensor estimation of motion parameters of an object moving along a complex trajectory

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Abstract. The paper addresses the problem of multisensor estimation of the motion parameters of an object moving along a complex trajectory which consists of parts of the uniform linear and circular motion subject to noisy measurements. To solve the problem, we describe the object motion as a set of linear stochastic models responsible for different parts of the trajectory and use a decentralized multisensor algorithm for estimating the object state vector based on the information form of the Kalman filter. The results of numerical experiments confirm the applicability of the proposed approach.

1. Introduction

Nowadays, the problem of adaptive estimation of the parameters of an object motion along a complex trajectory under noisy measurements and unforeseen changes in the motion mode of an object is relevant because of the importance of its practical applications. Examples of such applications are tasks of target tracking, robotics, signal processing from scanning range finders. This paper proposes the development of the idea of modeling and estimating the object motion along a complex trajectory using a hybrid stochastic model, which is a set of discrete linear stochastic models responsible for different parts of the trajectory [1, 3]. To calculate the estimates of the motion parameters of the object and detect the moment of change in the motion mode, we suggest using multisensor networks.

The paper considers two fundamentally different multisensor estimation schemes: a centralized Kalman filter (CKF) and a decentralized Kalman filter (DKF). The Centralized Kalman filter (CKF) refers to the standard algorithm of discrete Kalman filtering with vector processing of measurements. The latter means that all measurements available at the current time moment are treated as one composite measurement vector $z(k)$ [4]. Separate measurements can be obtained from different sensors, but they are all transmitted to the central processor for processing in order to calculate the state vector estimates. Such estimates are called global estimates. The disadvantage of the CKF is that the failure of one of the sensors can lead to incorrect estimates of the entire state vector. The second major drawback is that the failure of
the central processor automatically leads to a complete loss of the efficiency of the estimation algorithm.

In contrast to the CKF, the decentralized Kalman filter (DKF) refers to an estimation algorithm that calculates estimates of the state vector locally in each of the many sensors located at the nodes of the multisensor network [5]. Such estimates are called local. The sensors then exchange local estimates with each other in order to calculate the global estimate of the state vector. Thus, there is no need for a central processor since, at each discrete time moment, each node has its own “copy” of the global estimate. In this case, the failure of one of the sensors will not lead to the loss of efficiency of the estimation algorithm. Currently, many different decentralized filtering algorithms have been developed. In [4] the comparative analysis of some popular schemes for the problem of heat transfer is presented. A detailed bibliographic review of the methods of distributed Kalman filtering can be found in [6].

In this paper, we propose to use the DKF based on the information form of the Kalman filter [5, 7] to calculate estimates of the motion parameters of an object moving along a complex trajectory. To detect an unforeseen change in the motion mode, we supplemented the communication and assimilation step of the DKF with the ability to calculate the signal scalar function based on the well-known property of the measurement residual in the optimal filter to be a white Gaussian sequence. Such addition of DKF equations allows controlling the optimality of the estimation algorithm in each sensor. The loss of the optimality of the filter indicates that there was a change in the motion mode and the object began to move along the next part of the trajectory. The proposed new expression for calculating the scalar signal function is obtained in terms of only those quantities that are directly available in the decentralized scheme.

Numerical experiments were carried out using Matlab. A comparison was made of the estimated motion parameters of the object obtained using the CKF and DKF and their identity was demonstrated. The plots of the values of the signal function are presented for the cases when there is no change in the motion mode and when the motion mode changes from a uniform linear motion to uniform circular motion. It is shown that when the motion mode changes, the signal function exceeds the upper threshold value. Thus, the results of numerical experiments confirm the feasibility of the proposed approach to solving the problem of estimating the parameters of the object motion along a complex trajectory.

2. Motion model

Suppose that the trajectory of an object can be divided into separate sufficiently long parts, on each of which its motion can be presented by a linear stochastic model describing either a uniform linear motion (straight motion) or a uniform circular anticlockwise/clockwise motion (left/right turn) with a given radius.

Consider three such models. Then the motion of an object along the entire trajectory can be described by a hybrid stochastic model:

\[ x(k) = \Phi^p x(k-1) + B^p + Gw(k-1), \quad p \in \mathbb{Z}, \]

where \( k \) is a discrete time moment, \( p \) is the number of the motion mode, \( x = [x_1, x_2, x_3, x_4]^T \in \mathbb{R}^4 \) is the vector of motion parameters of the object, in which \( x_1 \) is the coordinate of the object along the axis \( Ox \) (\( m \)), \( x_2 \) is the velocity \( v_x \) along the axis \( Ox \) (\( m/s \)), \( x_3 \) is the coordinate of the object along the \( Oy \) axis (\( m \)), \( x_4 \) is the velocity \( v_y \) along the \( Oy \) axis (\( m/s \)).

Let us write down all the matrices of model (1).

- Uniform linear motion (the number of the motion mode \( p = 0 \)):

\[
\Phi^0 = \Phi^0(\tau) = \begin{bmatrix} \Phi_l & 0 \\ 0 & \Phi_l \end{bmatrix}, \Phi_l = \begin{bmatrix} 1 & \tau \\ 0 & 1 \end{bmatrix},
\]
\[ B^0 = [0 \ 0 \ 0 \ 0]^T, \]

where \( \tau = t_k - t_{k-1} \) is the sampling interval.

- Uniform circular anticlockwise motion with a given radius \( r \) (the number of the motion mode \( p = 1 \)) or a uniform circular clockwise motion with a given radius \( r \) (the number of the motion mode \( p = 2 \)):

\[
\Phi^{1,2} = \Phi^{1,2}(x_s, r, \tau) = \begin{bmatrix} \Phi_c & 0 \\ 0 & \Phi_c \end{bmatrix}, \quad \Phi_c = \begin{bmatrix} \cos \omega \tau & \omega^{-1} \sin \omega \tau \\ -\omega \sin \omega \tau & \cos \omega \tau \end{bmatrix},
\]

\[
B^1 = B^1(x_s, r, \tau) = \begin{bmatrix} (x_{1,s} - \omega^{-1} x_{4,s})(1 - \cos \omega \tau) \\ (\omega x_{1,s} - x_{4,s}) \sin \omega \tau \\ (x_{3,s} + \omega^{-1} x_{2,s})(1 - \cos \omega \tau) \\ (\omega x_{3,s} + x_{2,s}) \sin \omega \tau \end{bmatrix},
\]

\[
B^2 = B^2(x_k, r, \tau) = \begin{bmatrix} (x_{1,s} + \omega^{-1} x_{4,s})(1 - \cos \omega \tau) \\ (\omega x_{1,s} + x_{4,s}) \sin \omega \tau \\ (x_{3,s} - \omega^{-1} x_{2,s})(1 - \cos \omega \tau) \\ (\omega x_{3,s} - x_{2,s}) \sin \omega \tau \end{bmatrix},
\]

where \( r \) is the given radius, \( \omega = |v_s|/r > 0 \), \( v_s = \begin{bmatrix} x_{2,s} \\ x_{4,s} \end{bmatrix} \) is the velocity vector at the point with coordinates \( (x_{1,s}, x_{3,s}) \) at the moment of changing the motion mode.

- For all motion modes, the transfer matrix of discrete white noise \( w(k) \sim N(0, Q) \) is

\[ G = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T. \]

Hybrid stochastic model (1) allows us to model the movement of an object along a complex trajectory using the algorithm described in [1].

Provided that only spatial coordinates of the object are measured, the corresponding measurement model can be written as follows:

\[
z(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(k) + v(k), \quad (2)
\]

where \( v(k) \) is the measurement error vector, \( v(k) \sim N(0, R) \).

3. Multisensor estimation algorithms

Consider two fundamentally different multisensor estimation schemes: a centralized Kalman filter and decentralized Kalman filter.

**Centralized Kalman filter**

The CKF covariance form is described by the following equations:

**Time update**

\[
\dot{x}^- (k) = \Phi P \dot{x}^+ (k-1) + B^p u(k-1), \quad (3)
\]

\[
P^- (k) = \Phi P^+ (k-1) \Phi^T + G Q G^T, \quad (4)
\]

**Measurement update**

\[
K(k) = P^- (k) H^T [H P^- (k) H^T + R]^{-1}, \quad (5)
\]

\[
\nu(k) = z(k) - H \dot{x}^- (k), \quad (6)
\]

\[
\dot{x}^+ (k) = \dot{x}^- (k) + K(k) \nu(k), \quad (7)
\]

\[
P^+ (k) = (I - K(k)) P^- (k), \quad (8)
\]
where \( P(k) \) is the estimation error covariance matrix, the signs "+" and "−" mean, respectively, a priori and a posteriori estimates of the state vector and the corresponding covariance matrices of estimation error.

The information matrix \( I \) is the matrix inverse to the estimation error covariance matrix, i.e. \( I = P^{-1} \) [7]. A filter in which an information matrix is used instead of a covariance matrix is called an information filter. In the information filter, the measurement update step is determined by the following equations:

\[
\Delta s(k) = H^T R^{-1} z(k), \quad (9) \\
\Delta I = H^T R^{-1} H, \quad (10) \\
\hat{s}^+(k) = \hat{s}^-(k) + \Delta s(k), \quad (11) \\
I^+(k) = I^-(k) + \Delta I, \quad (12)
\]

where \( \Delta s \) and \( \Delta I \) are the updates of the information vector \( \hat{s}^+(k) \) and the information matrix \( I^+(k) \), respectively, where \( \hat{s}^+(k) = I^+(k) \hat{x}^+(k) \).

**Decentralized Kalman filter**

Consider a network of sensors with a fully connected topology consisting of \( N \) nodes, in which each node \( i \) has the ability to calculate its own estimates of \( \hat{x}_i(k) \) and corresponding estimation error covariance matrices \( P_i(k) \), see figure 1.

![Figure 1. Fully connected network topology.](image)

The measurements and estimates obtained in the nodes will be called local. Suppose that the model (1) is the same at each node, and the local measurements are described by the following model:

\[
z_i(k) = H_i x(k) + v_i(k). \quad (13)
\]

Suppose that the measurement noises at the nodes \( i \) and \( j \) are uncorrelated, i.e. the matrix \( R \) is block-diagonal.

Consider matrices \( H = [H_1^T H_2^T \ldots H_N^T]^T \) and \( R = \text{blockdiag}(R_1, R_2, \ldots, R_N) \). Then the corresponding global measurement model can be described as follows:

\[
z(k) = H x(k) + v(k), \quad (14)
\]
where \( v(k) \sim \mathcal{N}(0, R) \).

The key idea of the decentralized filter is the ability to express global updates of the information vector and information matrix through local [4, 5]:

\[
\Delta s(k) = H^T R^{-1} z(k) = \sum_{i=1}^{N} H^T_i R^{-1}_i z_i(k) = \sum_{i=1}^{N} \Delta s_i(k),
\]

\[
\Delta I = H^T R^{-1} H = \sum_{i=1}^{N} H^T_i R^{-1}_i H_i = \sum_{i=1}^{N} \Delta I_i.
\]

Local updates are calculated at each node and transmitted to all other nodes.

**Local time update**

\[
\dot{x}^-_i(k) = \Phi^p \dot{x}^+_i(k-1) + B^p u(k-1),
\]

\[
(P^-_i(k))^{-1} = (\Phi^p P^+_i(k-1) \Phi^p + G Q G^T)^{-1}.
\]

**Local measurement update**

\[
\Delta s_i(k) = H^T_i R^{-1}_i z_i(k),
\]

\[
\Delta I_i = H^T_i R^{-1} H_i.
\]

**Communication and assimilation**

\[
(P^+_i(k))^{-1} = (P^-_i(k))^{-1} + \sum_{j=1}^{N} \Delta I_j,
\]

\[
\dot{x}^+_i(k) = P^+_i(k) \left[ (P^-_i(k))^{-1} \dot{x}^-_i(k) + \sum_{j=1}^{N} \Delta s_j(k) \right].
\]

As shown in [5], the decentralized Kalman filter is equivalent to a centralized filter with a measurement model (14).

### 4. Control of the optimality of the estimation model in the decentralized algorithm

Suppose that at some discrete time moment there can be a change of the motion mode, i.e. the object begins to move along the next part of the trajectory. If the original motion mode is preserved, the filter used to estimate the motion parameters of the object will remain optimal, otherwise, it will become non-optimal and it will need to be replaced with a filter corresponding to the current motion mode.

In [8], an approach based on the cumulative sums method is considered to control the optimality of the estimation model in the central Kalman filter

\[
S_k = \sqrt{\frac{m}{2k}} \left( \frac{1}{m} \nu^T (k) C^{-1}(k) \nu(k) - 1 \right),
\]

where \( C(k) = H P^{-}(k) H^T + R \) is the covariance matrix of the residuals in the corresponding Kalman filter.

Denote \( \Sigma(k) \triangleq \nu^T(k) C^{-1}(k) \nu(k) \). In [9] it is shown that \( \Sigma(k) = ||\epsilon(k)||^2 \), where \( \epsilon(k) \) is the estimation error obtained during the measurement phase in square root information filter. Using this equality, we can show that

\[
\Sigma(k) = (\dot{x}^{-}(k))^T (P^{-}(k))^{-1} \dot{x}^{-}(k) - (\dot{x}^{+}(k))^T (P^{+}(k))^{-1} \dot{x}^{+}(k) + z^T(k) R^{-1} z(k).
\]
Since $z^T(k)R^{-1}z(k) = \sum_{j=1}^{N} z_j^T(k)R_j^{-1}z_j(k)$, then to control the optimality of the estimation model in each node at the communication and assimilation step we need to add the following equations:

$$\Sigma_i(k) = (\hat{x}_i^-(k))^T(P_i^-(k))^{-1}\hat{x}_i^-(k) - (\hat{x}_i^+(k))^T(P_i^+(k))^{-1}\hat{x}_i^+(k)$$

$$+ \sum_{j=1}^{N} z_j^T(k)R_j^{-1}z_j(k),$$

$$S_{k,i} = \sqrt{\frac{m}{2k}} \left( \frac{1}{m} \Sigma_i(k) - 1 \right) = \frac{1}{\sqrt{2km}}(\Sigma_i(k) - m),$$

where $m = \sum_{j=1}^{N} m_j$.

Thus, the calculation of the scalar value $S_{k,i}$ in each $i$-th node, $i = 1, \ldots, N$, allows to control in real time the optimality of the mode of the object motion. Note that the expression (25) (as opposed to (23)) is calculated in terms of the values available in the decentralized scheme.

5. Numerical Experiments

Consider a model of object motion (1) along a trajectory consisting of two consecutive parts of a straight motion and a left turn with a radius of $r = 4$, provided that there is no noise in the object (matrix $G = 0$):

$$x(k) = \Phi^0 x(k-1) + B^0, \quad k = 1, 2, \ldots, 100,$$

$$x(k) = \Phi^1 x(k-1) + B^1, \quad k = 101, 102, \ldots, 200.$$  

Let there be two sensor nodes, the first of which measures the $x$ coordinate of the object, and the second one measures the $y$ coordinate:

$$z_1(k) = [1 \ 0 \ 0 \ 0] x(k) + v_1(k),$$

$$z_2(k) = [0 \ 0 \ 1 \ 0] x(k) + v_2(k),$$

where $v_1(k) \sim \mathcal{N}(0, \sigma^2_1)$, $v_2(k) \sim \mathcal{N}(0, \sigma^2_2)$. This measurement model corresponds to a global measurement model (2) with the noise covariance matrix $R = \text{diag}(\sigma^2_1, \sigma^2_2)$.

Figures 2–5 show the results of estimating the components of the state vector of an object with a centralized Kalman filter (CKF) and a decentralized filter at each node (DKF$_1$, DKF$_2$). From the figures 2–5, it can be seen that all the estimates are the same.

Figures 6 and 7 show plots of $S_{k}$ in the absence and presence of a change in the motion mode, respectively. The time needed to determine the change in the motion mode was 39 discrete time moments with additional measurements. The thresholds of the decision rule were selected by the criterion of $3\sigma$.

6. Conclusion

In this paper, the problem of multisensor estimation of the motion parameters of an object moving along a complex trajectory was considered. The motion model is defined as a set of linear stochastic models for different parts of the trajectory: uniform linear motion and uniform circular motion when turning left or right. The decentralized multisensor algorithm for estimating the object state vector is based on the information form of the Kalman filter was used.

A new result is that we have expanded the decentralized filter by including an expression for calculating a scalar signal function that allows real-time control the optimality of the estimation algorithm. The difference of the new expression (25) for calculating the signal function $S_{k,i}$
Figure 2. Estimation of $x_1 = x$.

Figure 3. Estimation of $x_2 = v_x$.

from the previously proposed (see, for example, [8]) is that it is not calculated in the terms of measurement residuals, but in the terms of values directly available in the decentralized scheme. In case of an unexpected change in the object motion mode, the signal function allows detecting this change.

The performed computer simulation and results of numerical experiments confirm the applicability of the proposed approach to solving the problem of estimating the parameters
of the object motion along a complex trajectory using a decentralized scheme.

Advantages of the proposed approach are as follows:

(i) The hybrid stochastic model allows replacing a non-linear model of the object motion along a complex trajectory with a set of linear stochastic models.
(ii) With such an approach to modeling, there is no need to use nonlinear filtering algorithms,
Figure 6. The plot of $S_k$ in the absence of a change in the motion mode.

Figure 7. The plot of $S_k$ in the presence of a change in the motion mode.
and at each piece of the trajectory, the linear optimal discrete Kalman filter makes it possible to calculate optimal estimates of the motion parameters of the object.

(iii) A decentralized architecture is more stable to possible failures of sensor nodes than a centralized architecture with a single central node.

(iv) Extension of the functionality of the communication and assimilation step in the DKF gave the possibility to calculate the signal function which allows detecting unexpected changes in the object motion mode.

The disadvantage of this approach can be considered as a delay in detecting a change in the motion mode, which will depend on the specific parameters of the model.

The obtained results can be used in solving target tracking problems.

7. References
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