Role of different scattering orders in the formation of intensity of light scattered by a cold atomic ensemble placed into static electric or magnetic field

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Abstract. The scattering of light by an optically dense cold atomic ensemble placed into the static electric or magnetic field is studied theoretically. The total intensity of light scattered into the given direction is considered. Analysis of the angular distributions of intensity of light for different scattering orders allows one to understand the formation of the total intensity.

1. Introduction

Nowadays atomic ensembles prepared in the magneto-optical traps (MOT) are objects of the strong scientific interest [1, 2]. In the experiments with the help of these traps the main ensemble characteristics such as shape, temperature, atomic density and populations of atomic energy levels can be manipulated. Therefore these ensembles are useful model objects in modern applications such as quantum information [3, 4], communication and metrology [5].

In above mentioned applications the light can play the role of a carrier of information which, for example, by means of control field in quantum memory protocols, can be stored and then reads from the atomic ensemble [6-11]. Another way to control over the properties of atomic ensembles is the use coherent population trapping and radio-optical resonance [12, 13]. Manipulation of the optical properties of ensemble, e.g. using external static fields [14-16], is one of the main ways to control information coding in light. Besides of application interest the cold atomic ensembles, because of their pure quantum nature, have a strong interest from the point of view of fundamental science. Here the light which probes ensemble and accompanied by interference effects, such as coherent backscattering of light (CBS) [17-24], play the role of instrument for investigation of the ensemble statistical properties [25, 26].

In our previews paper [27, 28] we considered the influence of the static electric and magnetic fields on the polarization properties, spectrum dependence and angular distribution of backscattered intensity of light scattered on the cold ensemble of two level atoms. All these scattered light characteristics were analyzed mainly for directions close to backscattering. It was shown that static fields can essentially modify properties of the scattered light and, in particular, strong affect the interference processes answered for CBS.

In this paper we consider the same atomic cloud as in [27, 28] and on the base of analysis of the angular distributions of light for different scattering orders we explain the formation of the total intensity for given direction. Note, that manifestation of particular scattering order in the total intensity can be recognized by beating spectroscopy [29].
2. Theoretical description of the light scattering

To describe the intensity of the scattered light under condition of the polarization and spectrum analysis one can define the following second order electric field correlation function

\[ \langle \hat{E}_{\mu_1}^{(i)}(\vec{r}_1, t_1) \hat{E}_{\mu_2}^{(i)}(\vec{r}_2, t_2) \rangle, \tag{1} \]

where \( \hat{E}_{\mu_1}^{(i)}(\vec{r}_1, t_1) \) and \( \hat{E}_{\mu_2}^{(i)}(\vec{r}_2, t_2) \) are negative and positive parts of the Heisenberg electric field operators which are associated with creation and annihilation field operators, respectively. The brackets \( \langle \ldots \rangle \) denote the quantum statistical averaging over the initial density matrix of the considered physical system. In this paper the correlation function (1) is analysed with the help of non-equilibrium Keldysh diagram technique [30-34]. Modification of this technique for the case of light scattering on the atomic ensemble in particular in some special conditions was considered by us in our previews works (see [27, 28] and reference there). Here we briefly describe the main points of this approach and used approximations.

We keep the following assumptions: the intensity of the scattering light is weak, so the saturation parameter is negligible; the atomic density is so that it allows neglecting all cooperative effects; the strong contribution into the total intensity related to coherent forward scattering do not considered here. In these assumptions the field correlation function (1) can be rewrite as a sum of two prevailing diagram expansions: so called ladder diagrams which answer for different orders of incoherent scattering and cyclic diagrams which describe the CBS effect.

Decoding of these diagrams allows one to write expression for differential cross section of \( N \)-order scattering in the analytical form through \( 6N \)-tuple integral with respect to the coordinates and velocities of all atoms. Integrand function consists on single-atom density operator, retarded photon Green function and scattering tensor.

The single-atom density operator describes the quantum state of atomic ensemble prepared in MOT. In our case of cold Gauss cloud of the two level atoms (transition \( |J_g = 0, M_g = 0\rangle \rightarrow |J_e = 1, M_e = 0, \pm 1\rangle \)) this operator reads

\[ \rho_{mn}(\vec{r}) = n_0 \exp\left( -\frac{r^2}{2r_0^2} \right), \tag{2} \]

where \( n_0 \) is a peak of atomic density and \( r_0 \) is a cloud radius and all atoms are prepared in the ground state \( |m\rangle \). We also assume that the cloud is so cold that we can ignore moving of atoms, but still the cloud is not a degenerate quantum gases.

The scattering tensor is

\[ \tilde{e}_{pq}^{(nm)}(\omega) = -\sum_n \frac{1}{\hbar} \frac{\langle d_{p} \rangle_{mn} \langle d_{q} \rangle_{mn}}{\Delta_n + i\gamma/2} |n\rangle\langle n|, \tag{3} \]

where \( \Delta_n = \omega - \omega_{mn} \) is the detuning of the probe monochromatic light with frequency \( \omega \) from the atomic resonance with correction on Stark or Zeeman shifting. \( \gamma \) is the decay rate of the excited atomic levels. \( \langle d_{q} \rangle_{mn} = \tilde{d}_{mn} \cdot \bar{e}_{q} \) and \( \langle d_{p} \rangle_{mn} = \tilde{d}_{mn} \cdot \bar{e}_{p}^* \) are the transition dipole moments multiplied on polarization vector of incident \( \bar{e}_{q} \) and conjugated polarization vector of scattered \( \bar{e}_{p} \) photon, respectively.
The retarded photon Green function describes light propagation in the medium with susceptibility tensor \( \chi_q(r, \omega) \) between arbitrary two atoms in the cloud on which incoherent scattering occurs. In the laboratory frame, with the z axis directed along external static field, the susceptibility tensor in the basis of circular polarizations has a diagonal form and can be written as follows

\[
\chi_q(r, \omega) = -2\chi_{q}^{\nu} \frac{|d^{\nu}_{\mu\nu}(q)|^2}{h} P_{\mu\nu}(r) \Delta_q + i\gamma/2.
\] (4)

It is important to know how to write the tensor (4) in the frame cooperated with the light propagate direction. Such transformation one can find in our previous paper [27, 28].

The numerical calculation of the analytical expressions for differential cross section was provided with the help of Monte Carlo simulations [35].

3. Results

Atomic cloud will be characterized by optical depth \( b = \sqrt{2\pi \sigma_0 \rho_0} \) and radius \( r_0 \), where \( \sigma_0 = 6\pi\lambda^2/\lambda \) is the total cross section for resonant scattering of light with wavelength \( \lambda = 2\pi\lambda \) on our two level atom. The following two polarization schemes will be considered: \( H\parallel H \) - corresponds to analysing the scattering without any change in photon helicity and \( H \perp H \) - corresponds to measuring the intensity of the polarization component orthogonal to the initial one. The probe light is in resonant with corresponding transition. The values of the static fields will be given by the shift \( \Delta_{E,H} \) normalized on \( \gamma \) (low index \( E \) corresponds to Stark shift and index \( H \) - to Zeeman shift).

In the figure 1 the convergence of normalized total intensity of the scattered light for two given angles \( \theta = \pi/2, \pi \) and one polarization scheme \( H \perp H \) is analysed.

![Figure 1](image-url)

**Figure 1.** Total intensity of scattered light versus number N of incoherent scatterings included in numerical simulations. Intensity is normalized to its maximum value. \( H \perp H \), \( b = 5 \), \( r_0 = 35\lambda \). (a) \( \theta = \pi/2 \) (b) \( \theta = \pi \). Solid line - \( \Delta_{E,H} = 0 \), dashed line - \( \Delta_{E} = 8 \), dashed-dot-dot line - \( \Delta_{H} = 8 \).

From the figure 1 one can see that convergence of the total intensity is changed when the static field is switched on. It occurs because of applied static field removes degeneracy of the upper atomic energy level and makes highly unresonant some atomic transitions. Thus, for given strong value of the electric field \( \Delta_{E} \gg 1 \), the scattering channel through atomic transition \( |J_g = 0, M_g = 0\rangle \rightarrow |J_e = 1, M_e = 0\rangle \) is practically neglected. The strong magnetic field \( \Delta_{H} \gg 1 \),
which removes the energy level degeneracy completely, selects only one resonant scattering channels corresponding to transition $|J_e = 0, M_e = 0\rangle \rightarrow |J_e = 1, M_e = 1\rangle$. For further clarifying of the total intensity behavior let us consider the angular distribution of the light for different scattering orders.

In the figure 2 for the same parameters of the atomic cloud as in figure 1 and for two polarization scheme of detection, the normalized angular distribution of intensity of light for first and fifth orders of scattering are presented.

One can see that in the vicinity of angles $\theta = \pi$ for polarization channel $H \perp H$ and three considered cases $\Delta_{E,H} = 0$, $\Delta_{E,H} \neq 0$ the relative contributions into the total intensity of low and high orders of scattering are practically similar to each other. Thus we can see very similar behavior of the total intensity for all three cases (figure 1(b)). The small difference between curves corresponds to the fact that static field due to neglecting of some scattering channels provides increasing of contributions of the high orders of light scattering related to CBS effect into the total intensity.

For the scattering to the angle $\theta = \pi/2$ there is another situation. For this direction the applied static field decreases the contributions of the high order of light scattering into the total intensity (figure 2(b)) and convergence of the total intensity is defined by lower orders of scattering.

In the end let us note some more known features following from the figure 2: 1) the negligible contribution of the first order scattering into the total intensity of light for backscattering direction in the case of polarization scheme $\parallel H$; 2) suppression of the backscattering by means of strong magnetic field in the case of polarization scheme $\parallel H$; 3) fluctuations are enhanced with growing the number of ways for photon to leave the ensemble due to incoherent scattering.

4. Conclusion
In this paper we considered the formation of total intensity of light scattered by a cold optically dense atomic ensemble which was placed into the static electric or magnetic field. By analysing of angular dependence of intensity of light for different scattering orders the behaviour of total intensity was interpreted.
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