Anomalous Global Currents and Compensating Fields in the BV Formalism

Ricardo Amorim\textsuperscript{a}, Nelson R. F. Braga\textsuperscript{a} and Marc Henneaux\textsuperscript{b,c}

\textsuperscript{a} Instituto de Física, Universidade Federal do Rio de Janeiro, Caixa Postal 68528, 21945-970 Rio de Janeiro, Brazil

\textsuperscript{b} Faculté des Sciences, Université Libre de Bruxelles, Campus Plaine C.P. 231, B–1050 Bruxelles, Belgium

\textsuperscript{c} Centro de Estudios Científicos de Santiago, Casilla 16443, Santiago 9, Chile

Abstract

We compute the anomalous divergence of currents associated with global transformations in the antifield formalism, by introducing compensating fields that gauge these transformations. We consider the explicit case of the global axial current in QCD but the method applies to any global transformation of the fields.

PACS: 03.70.+k, 11.10.Ef, 11.15.-q

amorim@if.ufrj.br; braga@if.ufrj.br; henneaux@ulb.ac.be
1 Introduction

It is well known that quantum corrections can modify the expectation values of the divergence of global currents [1, 2]. In particular, a classically vanishing divergence of a global current can acquire a non-vanishing expectation value at the quantum level. It was shown by Fujikawa [3] that these quantum contributions can be calculated by path integral methods if one appropriately regularizes the functional measure.

The Batalin Vilkovisky (BV), or field-antifield formalism, is an extremely powerful procedure for the quantization of gauge theories [4, 5, 6]. The occurrence of local anomalies in this formalism has been discussed in [7], where they have been related to the non-invariance of the measure under (rigid) BRST transformations. The purpose of this letter is to develop a method for computing the anomalous divergence of currents associated with global (as opposed to local) transformations. To that end, we introduce pure gauge “compensating fields” [8] that couple to the divergence in question. We then turn to the master equation and show that quantum corrections are needed in order to fulfill the quantum master equation. These quantum corrections to the solution of the master equation do exist (no gauge anomaly) and turn out to be crucial for our purposes. Indeed, they precisely generate the quantum corrections to the divergence of the global current. This is easily seen by choosing appropriately the gauge for the new gauge freedom and using the standard Fradkin-Vilkovisky theorem of the antifield formalism. [Rigid symmetries have been discussed from a different point of view in the antifield formalism in [9]. See also [10].]

In reference [11] a procedure for calculating anomalous divergences in the particular case of Abelian global symmetries was presented. In that paper the original symmetry content of the action was enlarged by the introduction of extra gauge fields that could be trivially removed by a gauge fixing. At the quantum level the symmetries introduced in that way are apparently broken but can be trivially restored with the introduction of appropriate counterterms. This could be done because of the cohomological triviality of the field extension, which was easily proved once the new fields and the corresponding ghosts are combined in BRST doublets. The anomalous divergences of the currents associated with the enlarged symmetries are then calculated by using the independence of the path integral with respect to the gauge fixing.

In the present paper the treatment of [11] is generalized in several non
trivial ways. First, we are considering non Abelian global transformations in the context of an original theory that presents itself a non Abelian local symmetry. The gauging procedure, necessary for the calculation of the anomalous current divergences, mix non-trivially both kinds of symmetries. We show in section (2) that the resulting gauge structure becomes actually a semi-direct product of $SU(N)$ with itself instead of a direct product. This non trivial algebraic structure reflects itself in the process of quantization. For instance, the cohomological triviality of the extension can only be proved in a much more elaborated way (see section (4) ) since the new fields only form BRST doublets in the Abelian limit. Also, the form of the counterterms is not a trivial generalization of the one of reference [11] but relies heavily on peculiar aspects of the Lie algebra cohomology of non-abelian (semi-simple) Lie groups.

Our method applies to any transformation of the fields, even those that are not symmetries of the classical action. We shall develop the formalism by considering the explicit case of an $SU(N)$ Yang-Mills theory with fermions in the fundamental representation. The non abelian chiral transformation is not a symmetry of the action, and the corresponding currents are covariantly conserved (rather than conserved in the strict sense). We shall compute their covariant divergence in the quantum theory by following the method outlined above, and show how the standard anomalous term [12, 13] arises in that approach.

2 Compensating Fields and Conservation Laws

Our starting point is the Yang-Mills action

$$S_0 = \int d^k x \left( -\frac{1}{4} Tr (F^{\mu \nu} F_{\mu \nu}) + i \bar{\psi} \gamma^\mu (\partial_\mu - i A_\mu) \psi \right)$$

where $A_\mu$ is a $SU(N)$-connection and the fermions are taken in the fundamental representation. We assume the spacetime dimension $k$ to be even in order to have non-trivial chirality transformations. The action is invariant under the local transformations

$$\psi' = \Lambda \psi, \; \bar{\psi}' = \bar{\psi} \Lambda^{-1}, \; A'_\mu = \Lambda A_\mu \Lambda^{-1} - i (\partial_\mu \Lambda) \Lambda^{-1}$$

where $\Lambda \in SU(N)$. 

2
The chiral infinitesimal SU(N) transformations are

$$
\psi' = (1 - i \epsilon P_+ ) \psi, \quad \bar{\psi}' = \bar{\psi} (1 - i \epsilon P_-), \quad A'_\mu = A_\mu
$$

(2.3)

where $P_\pm = \frac{1}{2} (1 \pm \gamma_5)$. Here, $\epsilon = \epsilon^a T^a$ is a constant element of the $SU(N)$ algebra. If the connection does not vanish, the transformations (2.3) are not symmetries of the action. However, it is straightforward to verify that the associated chiral current $J^\mu_+ = \bar{\psi} \gamma_\mu T^a P_+ \psi$ is covariantly conserved,

$$
(D_\mu J^\mu_+ )^a \equiv \partial_\mu J^\mu_+ + f^{abc} A^b_\mu J^\mu_+^c = 0,
$$

(2.4)

where $[T^a, T^b] = i f^{abc}$ defines the structure constants of the algebra. Of course, if $A_\mu = 0$, this relation reduces to $\partial_\mu J^\mu_+ = 0$, in agreement with the fact that the chiral transformations are then symmetries of the action.

It is possible to enlarge the gauge symmetry content of a field theory by introducing compensating fields which, as discussed in [8], may lead to a different representation for the same theory, where some calculations become simpler. In the present case, we will make the chiral transformations of (2.3), with $\epsilon = \epsilon(x)$, become gauge symmetries of the action by introducing pure gauge compensating group elements of SU(N) denoted as $g(x)$. Being pure gauge, the group element will have no independent equation of motion. Rather, its equation of motion will be precisely the covariant conservation law (2.4) (at $g = 1$).

The constructive way to derive the action with the compensating field included is to replace the fermionic field $\psi$ by $(P_- + P_+ g) \psi$. If one does so, one gets the extended action

$$
S_1 [\psi, \bar{\psi}, A_\mu, g] = \int d^4 x \left( - \frac{1}{4} Tr (F^{\mu \nu} F_{\mu \nu}) + i \bar{\psi} \gamma^\mu (\partial_\mu - i \tilde{A}_\mu) \psi \right)
$$

(2.5)

where $\tilde{A}_\mu$ stands for $\tilde{A}_\mu = \tilde{A}_\mu [A_\mu, g] = P_- A_\mu + P_+ B_\mu$ with

$$
B_\mu = g^{-1} A_\mu g + i g^{-1} \partial_\mu g.
$$

(2.6)

By construction, the action (2.5) is invariant under the local transformations

$$
\delta \psi = i (\eta(x) - \epsilon(x) P_+) \psi, \quad \delta \bar{\psi} = - i \bar{\psi} (\eta(x) - \epsilon(x) P_-),
$$

$$
\delta A_\mu = \partial_\mu \eta(x) + i [\eta(x), A_\mu], \quad \delta g = i (g \epsilon(x) + [\eta(x), g])
$$

(2.7)
which include both the original symmetry and the “gauged” chiral transformations. The complete gauge group of (2.3) is the semi-direct product of $SU(N)$ with itself.

The theory with compensating fields is clearly classically equivalent to the original theory. Indeed, one can gauge away the compensating field $g$ by using the new gauge freedom. For instance, if we choose the gauge $g = 1$, the action (2.5) reduces to its original form (2.1). The existence of a new gauge freedom implies further Noether identities. These identities relate the variational derivatives of the action with respect to the compensating fields to the other variational derivatives, and imply that the $g$-equations of motion are not independent. In fact, a straightforward calculation yields the interesting relation

$$\frac{\delta S}{\delta g^a} \bigg|_{\beta=0} \equiv -i \left( D^\mu J^\mu_a \right)^a,$$  (2.8)

where on has set $g = 1 + \beta$ (in the vicinity of the identity). Thus, one can say that the compensating field couples to the (covariant) divergence of the chiral current. This property will turn out to be crucial in the computation of the quantum corrections to $\left( D^\mu J^\mu_a \right)^a$.

The same procedure can be followed for any group of rigid transformations of any local action. One may introduce the group parameters as dynamical variables by parametrizing the fields $\phi^i$ as $\phi^i = \phi^i(g^{-1}, \phi')$ where $\phi^i(g, \phi')$ is the transformed of $\phi'$ under the transformation $g$. One takes as new variables $g$ and $\phi'$ (and drop the '). The action is invariant under the gauge transformations that shift in a spacetime-dependent way the group variable $g$ by arbitrary (left) multiplication on the group and transform $\phi'$ accordingly. One can use this symmetry to eliminate $g$ and recover the original action. In the extended formulation, the compensating field couples to the divergence of the current associated with the original rigid transformations. Indeed, the Euler-Lagrange equations for $g$ are equivalent to $\partial_\mu j^\mu = 0$ if the transformations are symmetries of the original action, where $j^\mu$ is the Noether current, which is conserved by Noether theorem. This is because the $g$’s are then “ignorable coordinates” of the extended action (the original action is invariant under constant transformations and thus only derivatives of $g$ can occur; see e.g. [14]). If the original transformations are not global symmetries of the original action, $g$ will couple to a generalized “covariant” divergence of $j^\mu$ rather than to the ordinary divergence, as in the specific
example given above.

3 Quantization

The action (2.1) has no gauge anomaly since there is no chiral fermion. The current \( J_{\mu}^a \) associated with the rigid transformation (2.3) has, however, an anomalous covariant divergence. This would seem to ruin the extended theory, since one might fear that the new gauge symmetry will become anomalous. If true, equivalence with the original model would be broken at the quantum level and potential inconsistencies could even arise. That the new gauge symmetry is not afflicted by anomalies was discussed in [8], where it was shown that “compensating fields also compensate for the anomaly”. This also follows from the general cohomological investigation of [15]. Apart from global cocycles related to the De Rham cohomology of the group manifold and presumably irrelevant in perturbation theory, the BRST cohomology group at ghost number one (related to the anomaly) has no cocycles involving the new ghosts. We shall verify this property explicitly in the context of the antifield formalism.

To that end, we first observe that the symmetries (2.7) close in an algebra, \([\delta_1, \delta_2] \phi^i = \delta_3 \phi^i\) for any field \(\phi^i\). The parameters of the transformation on the right hand side are given by \(\eta_3 = i[\eta_1, \eta_2]\) and \(\epsilon_3 = i(\epsilon_1, \epsilon_2) - [\epsilon_1, \epsilon_2]\). The BV action (at zero order in \(\bar{\hbar}\)) then follows by the standard procedure,

\[
S = S_1 + \int d^k x \left( i \bar{\psi}^*(c - bP_+)\psi - \bar{\psi}(c - bP_-)\psi^* + Tr\{ig^*(g b + [c, g]) \right. \\
\left. + \frac{i}{2} c^* [c, c] - \frac{i}{2} b^* ([b, b] - 2[c, b]) \right) \}
\]

(3.1)

where we have introduced the ghosts \(c\) and \(b\) corresponding to the parameters \(\eta\) and \(\epsilon\) respectively and also the antifields associated with each field.

We will represent the total sets of fields and antifields respectively as \(\{\varphi^f\}, \{\varphi^*\}\). Defining the BRST transformation of any quantity \(X\) as: \(sX = (X, S)\), where \((X, Y)\) is the standard antibracket [9], it is not difficult to see that \(s^2X = 0\) and that the classical master equation \((S, S) = 0\) is valid. The BRST transformations of the fields are given by (2.7) with
the gauge parameters replaced by the ghosts, as well as $sc = icc, sb = -ibb + i[c, b]$. It is useful to introduce the invariant forms $\sigma = -ig^{-1}sg = b - c + g^{-1}cg$. These fulfill the Maurer-Cartan Eqs. $s\sigma = isi^2$. The related form $\sigma_\mu = -ig^{-1}\partial_\mu g$ transforms as $s\sigma_\mu = \partial_\mu \sigma - i[\sigma, \sigma_\mu]$. Furthermore, $s(g^{-1}A_\mu g) = i[c - b, g^{-1}A_\mu g] + g^{-1}\partial_\mu cg$ and as $B_\mu = -\sigma_\mu + g^{-1}A_\mu g$, one gets $sB_\mu = \partial_\mu(c - b) + i[c - b, B_\mu]$. This equation shows explicitly that the ghost associated with the connection $B_\mu$ is $c - b$.

4 One-loop order master equation

The BV vacuum functional is defined as

$$Z_{\overline{\Psi}} = \int [d\varphi^I] \delta[\varphi_I^* - \partial_\varphi I] \exp(i\overline{\hbar}W)$$  (4.1)

with $W = S + \hbar M_1$. Properly speaking, one should include the non-minimal sector. This will be done below, but since these variables do not affect the cohomological considerations (they form trivial pairs [6]), they will not be written explicitly here. The BV vacuum functional is independent of the choice of gauge fermion $\overline{\Psi}$ if, besides the classical master equation, the one loop order master equation is also satisfied

$$(M_1, S) = i\Delta S,$$  (4.2)

where $\Delta \equiv \frac{\delta}{\delta\varphi^I} \frac{\delta}{\delta\varphi^*_I}$. This equation is undefined unless we regularize the action of the $\Delta$ operator. Using a Pauli Villars (PV) regularization, with usual mass terms for the PV fermionic fields, the four dimensional case ($k = 4$) to which we shall restrict our attention from now on, can be written in the form [7]

$$\Delta S = \alpha tr \int d^4x \left( \partial_\mu (c - b) \Delta_\mu - \partial_\mu c \Delta_\mu \right).$$  (4.3)

Here,

$$\Delta_\mu = \epsilon^{\mu\nu\rho\sigma}(A_\nu\partial_\rho A_\sigma - \frac{i}{2}A_\nu A_\rho A_\sigma)$$  (4.4)

and $\Delta_\mu$ is given by a similar expression, with the replacement $A_\mu \to B_\mu$ given by (2.6). In the above expression, $\alpha = -\frac{1}{24\pi^2}$. We are assuming.
that the measure for the $g$ sector is BRST invariant (we are taking the Haar measure).

The term $\text{tr} \int d^4x \partial_\mu c \Delta^\mu_A$ is the standard ABBJ (Adler-Bardeen-Bell-Jackiw) anomaly for the gauge field $A_\mu$. It can be rewritten in form notations as $\text{tr} \int dc(AdA - (i/2)A^3) \equiv \int a^A_{ABBJ}$ and is well known to be a solution of the Wess-Zumino consistency condition. Similarly, the term $\text{tr} \int d^4x \partial_\mu (c - b) \Delta^\mu_B$, which can be rewritten $\text{tr} \int d(c - b)(BdB - (i/2)B^3) \equiv \int a^B_{ABBJ}$ is the ABBJ anomaly with $B_\mu$ instead of $A_\mu$. It also solves the Wess-Zumino consistency condition. Consequently, for the full $\Delta S$, one has $s \Delta S = (\Delta S, S) = 0$.

The quantity $\Delta S$ represents the variation of the path integral measure under BRST transformations. If this variation cannot be compensated by the variation of some local counterterm then the theory would be anomalous, and this would be a priori a disaster since it is a gauge anomaly. So, the important point now is to find out if there is a local counterterm, whose BRST variation cancels the candidate anomaly $\Delta S$. It was shown in [11] that such a counterterm exists in the Abelian case. We extend this result here to the non-abelian case.

It is rather easy to see that $\Delta S$ is $s$-exact in the space of local functionals, and thus that $M_1$ exists. Indeed, it is well known that the ABBJ anomaly is related to the invariant $\text{tr} F^3$ in 2 dimensions higher, i.e., here, in six dimensions through a chain of descent equations (see e.g. [9]). Explicitly, one has $\text{tr} F^3_A = dQ^5_{A,0}$ where $Q^5_{A,0}$ is the Chern-Simons 5-form constructed out of $A$, and $sQ^5_{A,0} + dQ^4_{A,1} = 0$, where $Q^4_{A,1}$ is the ABBJ anomaly $a^A_{ABBJ}$. In $Q^{i,j}$, the first superscript is the form-degree, while the second superscript is the ghost number. Similarly, one gets $\text{tr} F^3_B = dQ^5_{B,0}$ where $Q^5_{B,0}$ is the Chern-Simons 5-form constructed out of $B$, and $sQ^5_{B,0} + dQ^4_{B,1} = 0$, where $Q^4_{B,1}$ is the ABBJ anomaly $a^B_{ABBJ}$. Now, because $A$ and $B$ are related by a gauge transformation (Eq. [2,3]), they have field strengths related as $F_B = g^{-1}F_Ag$ and thus $\text{tr} F^3_B = \text{tr} F^3_A$. This implies $d(Q^5_{B,0} - Q^5_{A,0}) = 0$, i.e., $Q^5_{B,0} - Q^5_{A,0} = dM^{4,0}$ for some 4-form $M^{4,0}$. Substituting this result in the next descent equation yields then $d(Q^4_{B,1} - Q^4_{A,1} - sM^{4,0}) = 0$, i.e.,

$$a^B_{ABBJ} - a^A_{ABBJ} = Q^4_{B,1} - Q^4_{A,1} = sM^{4,0} + dM^{3,1} \quad (4.5)$$

for some 3-form $M^{3,1}$. This implies, upon integration, that $\Delta S$ is indeed exact,

$$i\Delta S = i\alpha \int (a^B_{ABBJ} - a^A_{ABBJ}) = sM_1, \quad M_1 = i\alpha \int M^{4,0}. \quad (4.6)$$
The explicit form of the counterterm $M_1$ may be found either by following the above procedure, or by using a perturbative expansion in the number of fields. One gets

$$M_1 = \alpha \left( \int_M d^5 \varepsilon_{\mu \nu \rho \omega} (i \frac{1}{10} Tr (\sigma_\mu \sigma_\nu \sigma_\rho \sigma_\omega \sigma_\lambda) + \int_{\partial M} d^4 \varepsilon_{\mu \nu \rho \sigma} Tr \left\{ \partial_\mu g g^{-1} (A_\nu \partial_\rho A_\sigma - i \frac{1}{2} A_\nu A_\rho A_\sigma) - \frac{1}{4} \partial_\mu g g^{-1} A_\nu \partial_\rho g g^{-1} A_\sigma + \frac{1}{2} \partial_\mu g g^{-1} \partial_\nu g g^{-1} A_\rho A_\sigma - \frac{i}{2} \partial_\mu g g^{-1} \partial_\nu g g^{-1} \partial_\rho g g^{-1} A_\sigma \right\} \right)$$ (4.7)

where the first term is the Wess-Zumino term which, as usual, is defined on an $k+1$ dimensional manifold $\mathcal{M}$, with boundary $\partial \mathcal{M}$ given by the four-dimensional space on which the theory is defined.

5 Anomalous divergence

We will now show how to get the anomalous divergence of the chiral current from the previous results. The quantum BV action $W$ is the sum of the counterterm from the last section with the BV classical action of equation (3.1). Let us introduce also a trivial pair of fields $\bar{\pi}, \bar{b}$, and the corresponding antifields $\bar{\pi}^*, \bar{b}^*$, in order to allow an appropriate gauge fixing of the extra symmetry. We should also introduce non minimal variables for the original gauge symmetry, but these will not be written explicitly. So we have $W = S + h M_1 + \int d^4 x Tr (\bar{\pi} \bar{b}^*)$. We choose a gauge fixing fermion of the form $\Psi = Tr (\bar{b} (g - 1 - \beta)) + \Psi$ where $\beta$ is an (infinitesimal) external function and where $\Psi$ does not depend on the fermionic variables or the extra fields $g, b, \pi$ and $\bar{b}$. This choice of gauge fixing fermion enforces the gauge $g = 1 + \beta$ and from what we have seen above, $\beta$ will appear as a source for the covariant divergence of the chiral current.

A direct calculation gives as gauge fixed quantum action

$$W_\Sigma = W[\phi^I, \varphi^*_I] = \frac{\partial \Psi}{\partial \varphi^I} = \int d^4 x \left( - \frac{1}{4} Tr (F^{\mu \nu} F_{\mu \nu}) \right)$$
Now we build up the vacuum functional $Z[\beta] = \int [d\phi^A] \exp\left\{ \frac{i}{\hbar} W_\Sigma \right\}$ where we are omitting the dependence on $\Psi$ in the notation. Then we integrate over the fields $\bar{\pi}$ and $g$. This amounts to substituting $g$ by $1 + \beta$. The fermionic term of the action becomes

$$i\bar{\psi}\gamma^\mu(\partial_\mu - i\bar{A}_\mu)\psi + \bar{\psi}\gamma^\mu P_+ \left( i\partial_\mu \beta + [A_\mu, \beta] \right) \psi.$$ (5.2)

The integration over $b$ and $\bar{b}$ is direct and yields one together with the Haar measure. The vacuum functional becomes therefore

$$Z[\beta] = \int [d\phi^A] \exp\left\{ \frac{i}{\hbar} W_\Sigma \right\}$$ (5.3)

where $\{\phi^A\} = \{A_\mu, c, \ldots\}$ (the dots refer to trivial pairs associated to the original gauge symmetry) and where $W_\Sigma$ is just the sum of the (extended) classical action and the counterterm of eq. (4.7) in the gauge $g = 1 + \beta$, plus non-minimal terms that do not involve $\beta$. As the master equation is satisfied, this object is independent of the external parameter $\beta$ ("Fradkin-Vilkovisky theorem"). Thus we have

$$\frac{\delta Z[\beta]}{\delta \beta^a}\bigg|_{\beta=0} = \frac{i}{\hbar} \langle \frac{\delta W_\Sigma}{\delta \beta^a}\rangle\bigg|_{\beta=0} = 0.$$ (5.4)

From the equation (4.7) we find

$$\frac{\delta M_1}{\delta \beta^a}\bigg|_{\beta=0} = -\alpha\epsilon^{\mu\nu\rho\sigma} Tr \left\{ \partial_\mu \left( A_\nu \partial_\rho A_\sigma - \frac{i}{2} A_\nu A_\rho A_\sigma \right) T^a \right\}.$$ (5.5)

Using this equation as well as (5.2) and (2.8) we finally get

$$\hbar \frac{\delta Z[\beta]}{\delta \beta^a}\bigg|_{\beta=0} = \langle \left( D_\mu J_\mu^a \right)^a - i\hbar \alpha\epsilon^{\mu\nu\rho\sigma} Tr \left\{ \partial_\mu \left( A_\nu \partial_\rho A_\sigma - \frac{i}{2} A_\nu A_\rho A_\sigma \right) T^a \right\} \rangle$$

$$= 0$$ (5.6)

where now the expectation values are taken in the original theory, with no extra variables. This reproduces the desired anomalous divergence.
6 Conclusion

We have shown that anomalous expectation values of currents associated with global transformations can be calculated by introducing compensating fields. We have performed the analysis in the antifield formalism. The present work extends the previous Abelian study of [11]. We have shown that the new gauge symmetries associated with the compensating fields are not obstructed at the quantum level since they do not change the cohomology of the theory. However, it is necessary to introduce quantum corrections to the BV action in order to fulfill the quantum master equation. These quantum corrections precisely generate the appropriate anomalous contribution to the divergence of the global current, although no gauge anomalies are present. Our procedure represents thus an interesting example where quantum corrections have a non trivial role.

Acknowledgment: This work is partially supported by CNPq, FINEP and FUJB (Brazilian Research Agencies). M. H. is grateful to Glenn Barnich for useful comments, as well as to CNPq and the physics departments of UERJ and UFRJ for kind hospitality.

References

[1] S. Adler, Phys. Rev. 177 (1969) 2426; J. Bell and R. Jackiw, Nuovo Cim. 60A (1969) 47.

[2] For a review, see R. Jackiw, in Lectures on Current Algebra and its Applications”, ed. S. Treiman et al., Princeton University Press, Princeton, NJ, 1972.

[3] K. Fujikawa, Phys. Rev. Lett. 42 (1979) 1195; Phys. Rev. D21 (1980) 2848.

[4] J. Zinn-Justin, J. Zinn-Justin, in Trends in Elementary Particle Physics, Lecture Notes in Physics 37, H. Rollnik and K. Dietz (Editors), Springer 1975.

[5] I. A. Batalin and G. A. Vilkovisky, Phys. Lett. B102 (1981) 27.
[6] M. Henneaux and C. Teitelboim, "Quantization of Gauge Systems", Princeton University Press 1992, Princeton, New Jersey.

[7] W. Troost, P. van Nieuwenhuizen and A. Van Proeyen, Nucl. Phys. B333 (1990) 727.

[8] B de Wit and M. T. Grisaru, "Compensating Fields and Anomalies" in Quantum Field Theory and Quantum Statistics, Vol.2, eds. I.A. Batalin, C.J. Isham and G.A. Vilkovisky, Adam Hilger, 1987.

[9] F. Brandt, M. Henneaux and A. Wilch, Nucl. Phys. B 510 (1998) 640.

[10] T. Hurth and K. Skenderis, "Quantum Noether Method", hep-th/9803030.

[11] R. Amorim and N. R. F. Braga, Phys. Rev. D57 (1998) 1225.

[12] W. A. Bardeen, Phys. Rev. 182 (1969) 1517.

[13] D. J. Gross and R. Jackiw, Phys. Rev. D6 (1972) 477.

[14] J. Zinn-Justin, "Quantum Field Theory and Critical Phenomena", Oxford Science Publications, Oxford, Third Edition, 1996.

[15] M. Henneaux and A. Wilch, “Local BRST Cohomology of the Gauged Principal Non-Linear Sigma Model”, hep-th/9802118.

[16] J. Mañes, R. Stora and B. Zumino, Commun. Math. Phys. 102 (1985) 157.