An experimental test of gravity at high energy

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Abstract. Gravitational lensing of very high energy photons has recently been observed in the JVAS B0218+357 strong lensing system. This observation opens the possibility of performing a test of gravity at high energy by comparing the difference in propagation time of high energy photons over different travel paths. The time delay is computed in the framework of a LIV (Lorentz Invariance Violation) extension of the equations of motion of photons in the field of a massive object. However, the method obtained can be transposed to other models of gravity at high energy. The potential for constraining high energy gravity with future observations of JVAS B0218+357 is discussed. The bounds on the LIV energy scale will not be competitive with other astrophysical bounds such as those coming from AGN and GRB flares. However, these bounds are free of any assumption on the emission process.

Keywords: gamma ray experiments, gravitational lensing, quantum gravity phenomenology

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1 Introduction

Observing gravitational interactions of high energy particles is especially challenging, since these observations must involve at the same time large mass objects such as galaxies acting as lenses and high energy cosmic rays. High and very high energy photons have been observed only in two strong lensing systems: PKS 1830-211 and JVAS B0218+357. The evidence for lensing at high energy in PKS 1830-211 has been disputed [1, 2] and no lensing signal has been observed at very high energy [3]. The situation is different for JVAS B0218+357, for which lensing of high energy [4] and very high energy [5] photons has been observed. The high energy time delay between the two compact components of JVAS B0218+357 has been measured and is compatible with radio measurements [6].

There is no universally accepted model for gravitational interactions at high energy and very few experimental constraints. One of the most popular tests of models consists in searching for energy dependent photon propagation from distant sources. These searches often assume a violation of Lorentz invariance and the results are expressed as bounds on the Lorentz Invariance Violation (LIV) energy scale. Very constraining results on the LIV energy scale have been obtained by looking at a variety of sources such as AGN or GRB (see e.g. [7] and references therein). Photon deflection by massive objects and gravitational time delays have also been investigated in the context of the Standard Model Extension (SME) [8, 9] and rainbow gravity [10, 11]. These computations generally use the Schwarzschild metric and they give constraints based on solar tests. An early study of lensing with LIV was published by Biesiada and Piorkowska [12]. These authors extrapolate the formula by Jacob and Piran [13] to lensing delays by introducing a geometric “LIV comoving distance”. They estimated that the observation of 20 TeV photons in the HST 14176+5226 lensing system would provide delays of tens of nanoseconds if the LIV scale is the Planck scale. The formula by Jacob and Piran used in reference [12] is suited to the propagation of light in a cosmological context with some caveats [14]. But it is not adapted to lensing studies, since the lensing time delay comes mostly from the local distortion of photon trajectories in the vicinity of the lens. Photon trajectories and time delays could be further affected by a possible energy dependence of the Schwarzschild metric or Newton’s constant [15].

In the first section of the paper, the lensing time delay and gravitational deflection of high energy photons is recalculated by an Hamiltonian method. The Hamiltonian used is a special case of the general LIV Hamiltonians with spherical symmetry given by Barcaroli et al. [16]. The formula obtained is then applied in the next section to future observations of
JVAS B0218+357. The sensitivity to the LIV energy scale is discussed and equivalent limits in the case of the rainbow metric are given.

2 Single particle Hamiltonian with LIV in the field of a massive object

The Hamiltonian of particle motion in a spherical potential with Lorentz Invariance Violation (LIV) has the form

\[ H = -\frac{\vec{p}^2}{f(r)} + \left(1 + \epsilon l_P \left(\frac{p_t}{\sqrt{f(r)}}\right)\right) \left[f(r)p_r^2 + \frac{1}{r^2} w^2\right]. \] (2.1)

where \( w^2 = p_\theta^2 + \frac{p_\phi^2}{\sin^2 \theta}, p_t, p_r, p_\theta, p_\phi \) are the usual momenta conjugate to \( t, r, \theta, \phi \) and \( \epsilon = \pm 1 \) depending whether the particle motion is super- or supraluminic. It depends on the unknown energy scale of LIV, which is denoted by \( l_P \) and is of the order of the inverse of the Planck mass. This Hamiltonian is a special case of a more general Lorentz Invariance Violating Hamiltonian given by Barcaroli et al. [16] which depends only on \( l_P p_t \) and not on \( l_P p_r \).

In the Schwarzschild metric, one has \( f(r) = 1 - \frac{r}{r_S} \) where \( r_S = \frac{2GM_L}{r} \) is the Schwarzschild radius and \( M_L \) is the mass of the lensing object. The light deflection angle and time delay with LIV in the Schwarzschild metric are derived in the appendix.

In this paper, we are interested in real lenses which are not well described by Schwarzschild lenses. For a more general astrophysical lens, one has \( f(r) = 1 + 2U(r) \) with \( U(r) \ll 1 \). The lens is still assumed to be spherically symmetric. To deal with these lenses, it is convenient to transform the Hamiltonian with a rescaling of the \( r \) variable and use isotropic coordinates. The Hamiltonian in isotropic coordinates is:

\[ H = -\frac{\vec{p}^2}{f(r)} + \left(1 + \epsilon l_P \left(\frac{p_t}{\sqrt{f(r)}}\right)\right) f(r) \vec{P}^2. \] (2.2)

with \( \vec{P}^2 = \sum_1^3 p_\alpha^2 \).

Hamilton's equation are obtained by derivating w.r.t. the affine parameter \( \lambda \) giving

\[
\frac{dp_t}{d\lambda} = -\frac{\partial H}{\partial t} = 0
\]

\[
\frac{dt}{d\lambda} = \frac{\partial H}{\partial p_t} = -2 \frac{p_t}{f(r)} + \left(\frac{\epsilon l_P}{\sqrt{f(r)}}\right) \vec{P}^2
\]

\[
\frac{dx_\alpha}{d\lambda} = \frac{\partial H}{\partial p_\alpha} = 2(f(r)p_\alpha) \left(1 + \epsilon l_P \left(\frac{p_t}{\sqrt{f(r)}}\right)\right)
\]

\[
\frac{dp_\alpha}{d\lambda} = -\frac{\partial H}{\partial x_\alpha} = -\partial_\alpha f(r) \left(\frac{p_t^2}{f(r)^2} + \vec{P}^2 \left(1 + \epsilon l_P \left(\frac{p_t}{2\sqrt{f(r)}}\right)\right)\right)
\]

The mass constraint relevant to photon motion is \( H = 0 \), which gives a relation between \( P, f(r) \) and \( p_t \)

\[
\frac{p_t}{P} = f \sqrt{1 + \epsilon l_P p_t \frac{1}{2\sqrt{f}}} \approx f \left(1 + \frac{\epsilon l_P p_t}{2\sqrt{f}}\right)
\] (2.3)

Hamilton’s equation can be transformed using the euclidian line element \( dt^2 = \sum_1^3 dx_\alpha^2 \) to give the evolution of \( t \) and \( p_\alpha \) with \( l \).
Taking
\[ \frac{dl}{d\lambda} = -2f(r)P \left( 1 + \epsilon_l P \left( \frac{p_t}{\sqrt{f(r)}} \right) \right) \]
one has
\[ \frac{dp_\alpha}{dl} = \partial_\alpha f(r)P \frac{\left( 2 + \epsilon_l P \left( \frac{3p_t}{2\sqrt{f(r)}} \right) \right)}{2f(r) \left( 1 + \epsilon_l P \left( \frac{p_t}{\sqrt{f(r)}} \right) \right)} \]

(2.4)
\[ \frac{dt}{dl} = \frac{p_t}{f^2(r)P} \left( 1 - \epsilon_l p_t \sqrt{f} \right) - \frac{\epsilon_l p_t P}{2\sqrt{f(r)}} \left( 1 - \frac{\epsilon_l p_t}{\sqrt{f}} \right) \]

(2.5)

In equation (2.4) \( \partial_\alpha f(r) = 2 \partial_\alpha U(r) \) is a first order quantity in \( U \). To first order in \( U \), the evolution of momenta is thus given by
\[ \frac{dp_\alpha}{dl} = 2 \partial_\alpha U(r) \left( 1 - \epsilon_l P \left( \frac{p_t}{4} \right) \right) \]

(2.6)

The right hand side of equation (2.5) is the sum of two terms. The first one is
\[ I_1 = \frac{p_t}{f^2(r)P} \left( 1 - \epsilon_l p_t \sqrt{f} \right) - \frac{\epsilon_l p_t P}{2\sqrt{f(r)}} \left( 1 - \frac{\epsilon_l p_t}{\sqrt{f}} \right) \]
\[ = (1 - 2U) \left( 1 - (1 - U) \frac{\epsilon_l p_t}{2} \right) \]
\[ = \left( 1 - \frac{\epsilon_l p_t}{2} \right) - 2U \left( 1 - \frac{3}{2} \epsilon_l p_t \right) + O(U^2) \]

and the second is
\[ I_2 = -\left( \frac{\epsilon_l P}{2f(r)^{1/2}} \right) \left( 1 - \epsilon_l p_t \sqrt{f} \right) - \left( \frac{\epsilon_l p_t}{2f(r)^{3/2}} \right)^2 \]
\[ = -\left( \frac{\epsilon_l p_t}{2f(r)^{3/2}} \right) + O(p_t^2) \]
\[ = -\left( \frac{\epsilon_l p_t}{2} \right) (1 - 3U) \]

Summing the 2 contributions:
\[ \frac{dt}{dl} = (I_1 + I_2) = (1 - \epsilon_l p_t) - 2U \left( 1 - \frac{3}{2} \epsilon_l p_t \right) \]

(2.7)

The rest of this section follows closely the formalism described in [17]. The thin lens approximation is used, photons move on a straight lines until they get deflected toward the observer. The distance to the lens, located at redshift \( z_L \) is \( D_{OL} \), the distance from the lens to the observer is \( D_{LO} \). Coordinates are taken in the plane perpendicular to the line of sight (the lens plane). The projected source position on the lens plane is at \( \eta \), and the impact parameter of the particle trajectory at \( \zeta \).

A popular lens model is the Singular Isothermal Lens (SIL) lens model, characterized by \( U(r) = 2\sigma_v^2 \ln r \). \( \sigma_v \) is the velocity dispersion of stars in the lens galaxy. \( \sigma_v^2 \) is proportional
to Newton’s constant $G$. Using equation (2.7) and following the steps outlined in [17], the
travel time from source to observer in a SIL model is

$$T_{OS} = (z_L + 1) \left( \frac{1}{2} (1 - \epsilon l_p p_t) \left( \frac{1}{D_{OL}} + \frac{1}{D_{LS}} \right) \right) \left( \zeta - \eta \right)^2 - 2 \pi \sigma_v^2 \left( \frac{3}{2} \epsilon l_p p_t \right) |\zeta| + T_0$$

(2.8)

The absolute value of the deflection angle is obtained from equation (2.6) (see [17] for
details).

$$\alpha = 2 \pi \sigma_v^2 \left( 1 - \epsilon l_p \left( \frac{p_t}{4} \right) \right) \text{sgn}(\zeta).$$

(2.9)

The lens equation is:

$$\zeta - \eta = \text{sgn}(\zeta) l_E,$$

(2.10)

with the Einstein length $l_E$ defined by

$$l_E = \frac{4 \pi \sigma_v^2 \left( 1 - \epsilon l_p \left( \frac{p_t}{4} \right) \right) D_{OL} D_{LS}}{D_{OL} + D_{LS}}$$

(2.11)

Equation (2.10) has 2 solutions for $|\eta| \leq l_E$. These solutions are

$$\zeta_+ = l_E + \eta$$

(2.12)

$$\zeta_- = \eta - l_E$$

(2.13)

Since $(\zeta_+ - \eta)^2 = (\zeta_- - \eta)^2 = l_E^2$, only the second term in equation (2.8) contributes to the
time delay between images. This term depends on

$$|\zeta_+| - |\zeta_-| = 2 \eta.$$

The time delay between the $\zeta_\pm$ images is thus:

$$\Delta T = -4(z_L + 1) \pi \sigma_v^2 \left( 1 - \frac{3}{2} \epsilon l_p p_t \right) \eta = \Delta T(p_t = 0) \left( 1 - \frac{3}{2} \epsilon l_p p_t \right)$$

(2.14)

The time difference in equation (2.14) is somewhat similar (up to multiplicative terms
of order 1) to the lensing time delay quoted in [12], but is obtained by a completely different
method. The expression for the Einstein length (equation (2.11)) differs from that obtained
in [12].

Since $\Delta T$ scales linearly with $\sigma_v^2$, hence with $G$, an equation similar to equation (2.14)
would be obtained in gravity models with an energy-dependent $G$, for instance a lensing
gravity rainbow metric expressed with energy dependent coordinates (equation 43 of refer-
ence [15]). Introducing the “rainbow function” $g$, one has

$$\Delta T = \Delta T(p_t = 0) \frac{G(p_t)}{G(p_t = 0)} \frac{\Delta T(p_t = 0)}{g(p_t)} = \frac{\Delta T(p_t = 0)}{g(p_t)}.$$

(2.15)

The $g$ function can be parametrized as

$$g(E) = \frac{1}{(1 - \frac{E}{M_P})},$$

where $M_P = 1.22 \times 10^{19}$ GeV is the Planck mass.
Figure 1. Number of events versus date in a simulated lensing event. The original flare (solid line) has a gaussian time profile (dotted line). Photons from the delayed flare (dashed line) have an energy-dependent delay described by equation (2.14) with $l_P = 3.3 \times 10^{-4}$ GeV$^{-1}$. The constant part of the delay has been subtracted for clarity.

3 Application to high energy AGN flares

As mentioned in the introduction, two lensed AGN have been observed in high and very high energy: PKS 1830-211 and JVAS B0218+357. While the situation of PKS 1830-211 is still controversial, delayed lensing flares from JVAS B0218+357 have been detected in the high energy regime by the Fermi-LAT collaboration and in the very high energy regime by the MAGIC array of imaging Cerenkov telescopes. In addition, lens models of JVAS B0218+357 are available [18]. The JVAS B0218+357 lens is well approximated by a SIL model. More than 200 photons were detected in the range 65-175 GeV during the MAGIC observation of a delayed flare of JVAS B0218+357 in 2014. The photon number density was well described by a powerlaw

$$\frac{dN}{dE} \propto E^{-\gamma}$$

with a photon index $\gamma = 3.8 \pm 0.6$. In this paper, the sensitivity of future high energy detections to $l_P$ is studied with a simulation. The numbers used are slightly more optimistic than the MAGIC observations, since the forthcoming CTA observatory [19] will provide observations with a better sensitivity over a larger energy range. Thousand flaring events with $N_\gamma = 300$ detected photons both in the initial and delayed flare were simulated for a range of $l_P$ values. The energy range of the detected photons is 30 to 300 GeV. The simulated photons have a photon index of 3.8 and the initial flare has a gaussian luminosity profile in time (figure 1). The initial and delayed flare time profile are compared with a Kolmogorov-Smirnov test. When $l_P$ is large enough, the delayed flare has a distorted shape compared to the initial flare. The observable is the Kolmogorov distance $D$. Figure 2 shows the distribution of the scaled distance $\sqrt{\frac{N_\gamma}{l_P}}D$ for values $l_P = 0, 1.3$ and $3.3 \times 10^{-4}$ GeV$^{-1}$. A single observation could easily provide a limit on $l_P$ of several TeV$^{-1}$. Simultaneous observations (e.g. with the Fermi-LAT instrument and ground based Imaging Cerenkov arrays) would further improve the constraints. These limits on the LIV energy scale $l_P$ would be totally independent of the flare emission model, but more than 15 order of magnitude lower than the Planck mass.
As noted in the end of section 2, the constraint on $l_P$ is equivalent to a constraint on the $\alpha$ parameter of the “rainbow function” $g(E)$. Future observations of high energy photon lensing would give upper limits on $\alpha$ at the level of $10^{15}$, improving on the existing bounds [20, 21] by several order of magnitudes.

4 Conclusion

In this paper, prospects for constraining the LIV energy scale with high energy observations of strong gravitational lenses have been discussed. Limits on the LIV energy scale of a few TeV can be reached. These limits would give constraints much weaker than those obtained with AGN or GRB flares, but obtained with a cleaner setup, with no assumption about the emission process. On the other hand, limits on gravity rainbow models do not rely on the assumption of an energy dependent speed of light. The observation of high energy photons lensed by systems such as JVAS B0218+357 could improve the existing limits on rainbow functions by several order of magnitude.

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A Light deflection and time delay for a Schwarzschild lens with LIV

This section contains the derivation of the LIV extension of the usual deflection and time delay of light in the field of a massive compact astrophysical object described by a Schwarzschild metric. Derivating Hamiltonian in equation (2.1) with respect to the affine parameter $\lambda$, one
obtains a set of equations

\[
\frac{dp_t}{d\lambda} = - \frac{\partial H}{\partial p_t} = 0
\]

\[
\frac{dt}{d\lambda} = \frac{\partial H}{\partial p_t} = -2 \frac{p_t}{f(r)} + \left( \frac{e \ell_p}{\sqrt{f(r)}} \right) \left[ f(r)p_t^2 + \frac{1}{r^2} p_\theta^2 \right]
\]

\[
\frac{dr}{d\lambda} = \frac{\partial H}{\partial p_r} = 2 f(r)p_r \left( 1 + \ell P_p \left( \frac{p_t}{\sqrt{f(r)}} \right) \right)
\]

\[
\frac{d\phi}{d\lambda} = \frac{\partial H}{\partial p_\phi} = 2 \frac{p_\phi}{r^2 \sin^2 \theta} \left( 1 + \ell P_p \left( \frac{p_t}{\sqrt{f(r)}} \right) \right)
\]

\[
\frac{dp_\phi}{d\lambda} = - \frac{\partial H}{\partial \phi} = 0
\]

\[
\frac{d\theta}{d\lambda} = \frac{\partial H}{\partial p_\theta} = 2 \frac{p_\theta}{r^2} \left( 1 + \ell P_p \left( \frac{p_t}{\sqrt{f(r)}} \right) \right)
\]

\[
\frac{dp_\theta}{d\lambda} = - \frac{\partial H}{\partial r} = \frac{d}{d\theta} \left( \frac{1}{2r^2 \sin^2 \theta} \right) p_\phi^2 \left( 1 + \ell P_p \left( \frac{p_t}{\sqrt{f(r)}} \right) \right)
\]

\[
\frac{dp_r}{d\lambda} = - \frac{\partial H}{\partial p_r} = \frac{f'(r)}{2f(r)^2} p_t^2 + \frac{1}{2} f'(r)p_r^2 - \frac{1}{r^2} \left( \frac{2 p_\theta^2 + 1}{\sin^2 \theta} \right) p_\phi^2
\]

In these equations, \( f(r) \) is given by \( f(r) = 1 - \frac{r}{r_s} \) where \( r_s = \frac{2GM_L}{r} \) is the Schwarzschild radius and \( M_L \) is the mass of the lensing object. \( p_t, p_\phi \) are constants of motion. If in addition \( \phi \) is constant, then \( p_\theta \) is also a constant of motion. The \( \lambda \) affine parameter is eliminated and the remaining variables \( t, \theta \) and \( p_r \) are written as functions of \( r \). The \( p_r \) variable can be further eliminated by the mass constraint \( H = 0 \), which gives

\[
f(r)p_r = \sqrt{\frac{p_t^2}{1 + \ell P_p \left( \frac{p_t}{\sqrt{f(r)}} \right)}} - \frac{f(r)}{r^2} p_\theta^2 \tag{A.1}
\]

Changing variable to \( u = \frac{1}{r} \), one obtains

\[
\frac{dt}{du} = \frac{p_t}{u^2 f(u)^2 p_r \left( 1 + \ell P_p \left( \frac{p_t}{\sqrt{f(u)}} \right) \right)} \left( 1 - \left( \frac{e \ell_p p_t}{2 \sqrt{f(u)}} \left( 1 + \ell P_p \left( \frac{p_t}{\sqrt{f(u)}} \right) \right) \right) \right) \tag{A.2}
\]

\[
\frac{d\theta}{du} = \frac{p_\theta}{\sqrt{\frac{p_t^2}{1 + \ell P_p \left( \frac{p_t}{\sqrt{f(u)}} \right)}} - f(u)u^2 p_\theta^2} \tag{A.4}
\]
The calculation follows the steps outlined in the book by Wald [22]. The impact parameter is $\beta'$ defined by

$$\frac{1}{\beta'^2} = \frac{p_t^2}{p_0^2(1 + \ell \ell p_t)} \quad (A.5)$$

It is related to the solution $u_0$ of equation

$$\frac{p_t^2}{(1 + \ell \ell p_t \frac{p_t}{\sqrt{f(u)}})} - f(u)u_0^2 = 0 \quad (A.6)$$

by

$$\frac{1}{\beta'} = u_0 + \frac{r_S u_0^3 (1/2 \ell \ell p_t - 1)}{2} + O(r_S^2) \quad (A.7)$$

Keeping only the lowest order in $r_S$:

$$\frac{d\theta}{du} = -\frac{1}{\sqrt{u_0^2 - u^2 + r_S (u^3 - u_0^3) + \frac{\ell \ell p_t r_S u_0^2}{2} (u_0 - u)}}$$

$$\simeq -\frac{1}{\sqrt{u_0^2 - u^2}} - \frac{r_S}{2} \left( \frac{u^3 - u_0^3}{(u_0^2 - u^2)^{3/2}} + \frac{\ell \ell p_t u_0^2 (u_0 - u)}{2 (u_0^2 - u^2)^{3/2}} \right) \quad (A.8)$$

Integrating between 0 and $u_0$, the deflection angle between the source and the lens is

$$\theta = -\frac{\pi}{2} + \frac{r_S}{\beta'} \left( 1 - \frac{\ell \ell p_t}{4} \right) \quad (A.10)$$

Taking into account the deflection of light between the lens and the observer, the total observed deflection is:

$$\delta \theta = \frac{2r_S}{\beta'} \left( 1 - \frac{\ell \ell p_t}{4} \right) \quad (A.11)$$

which is the generalisation of the usual deflection angle formula to first order in $\ell \ell p_t$.

The calculation of time delay proceeds along similar lines. After a somewhat lengthy calculation, the time delay of a signal sent from Earth and reflected off a planet is found to be

$$\Delta T = 2 \left( 1 - \ell \ell p_t \right) \left( \sqrt{\frac{D_{LS}^2 - R_0^2}{D_{LS}^2}} + \sqrt{\frac{D_{OL}^2 - R_0^2}{D_{OL}^2}} \right)$$

$$+ r_S \left( 1 - \frac{3}{2} \ell \ell p_t \right) \left( \ln \frac{D_{LS} + \sqrt{D_{LS}^2 - R_0^2}}{R_0} + \ln \frac{D_{OL} + \sqrt{D_{OL}^2 - R_0^2}}{R_0} \right)$$

$$+ \frac{1}{2} \left( \sqrt{\frac{D_{LS} - R_0}{D_{LS} + R_0}} + \sqrt{\frac{D_{OL} - R_0}{D_{OL} + R_0}} \right) \quad (A.12)$$

where $R_0 = \frac{1}{u_0}$.
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