The effect of impact velocity on rebound height after impact interaction

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Abstract. Using the mechanorheological viscoelastic plastic model, the effect of impact velocity of a spherical body on rebound height after impact interaction was studied. With an increase in impact velocity, the energy losses of impact interaction increase. Under the elastic deformations, energy dissipation depends on the deformation rate and increases with increasing impact velocity. During plastic deformations, energy dissipation increases with increasing deformations. Plasticity of the material significantly reduces rebound height. The more ductile the material, the lower the rebound height due to large plastic deformations. In case of an elastic impact, a decrease in elasticity leads to a decrease in the rebound height, since the viscoelastic model becomes more viscous, and the viscous resistance to deformations increases compared to the elastic resistance. This increases energy losses and decreases the rebound height. For the elastic-plastic model, the relationship is reverse. A decrease in elasticity leads to an increase in the rebound height. This is due to the fact that an increase in elasticity of the material leads to an increase in its stiffness. Elastic deformation resistance increases. An increase in the force causes an increase in plastic deformations and an increase in energy losses. An increase in the viscous resistance to deformations leads to an increase in energy losses and a decrease in the rebound height. Thus, the rebound height depends on impact interaction velocity. To improve reliability of dynamic process simulation, it is necessary to take into account these factors when studying the operation of equipment in the conditions of impact interaction of structural elements.

1. Introduction

Machinery and equipment used in various industries operate under dynamic loading. For these loading conditions, mechanical shock and dynamic interaction of bodies are typical. In this case, some part of the impact energy is spent on the deformation of interacting bodies, another part is dissipated during impact interactions and energy becomes dissipated. The energy dissipation plays a positive role when it is necessary to reduce deformations and stress. In other cases, energy losses can play a negative role when the energy of impact is used to complete useful work. In any case, it is necessary to calculate energy losses and study factors affecting the energy distribution during the dynamic interaction of bodies and mechanical systems. Therefore, the tasks of physical and mathematical modeling of dynamic processes are crucial; many scientific works are devoted to these problems [1-23].

When developing theoretical models of dynamic interaction of bodies, the mechanorheological approach is often used. Mechanical schemes of models are based on it [23-28]. Describing the dynamics of the mechanical system of the model using differential equations and developing an algorithm for its functioning, one can create a mathematical model of the dynamic process.
Thus, dynamic processes often cause dangerous deformations, forces, and stresses. They can destroy parts of equipment. Research results show that models are based on the mechanorheological approach [23-28].

2. Problem statement

For the effective operation of equipment, it is necessary to ensure reliability, performance, durability, vibration protection of equipment and solve some other tasks. Therefore, theoretical and experimental studies of shock and vibration processes allow us to explain the dynamics of interaction of solids and mechanical systems and find optimal design solutions for real structures.

The problems can be solved by modelling dynamic processes; therefore, many scientific studies are devoted to these problems [19-23]. At the same time, further development of computational models and algorithms is required. Further development is possible by modeling dynamic processes in mechanical systems.

Researchers of Irkutsk National Research Technical University are developing mathematical models of shock processes [23-25,29-34]. The results of studies on the influence of body size on the rebound height after impact interaction were described in [29]. It was found that the rebound height depends on the size of a striking body. The influence is exerted by the body mass and the radius of surface curvature.

The paper presents results of studies on the influence of impact velocity on the rebound height after impact interaction. The impact interaction velocity is the most important impact parameter. It determines the dynamics of the process, including energy losses during impact interaction. Therefore, these factors should be taken into account.

Thus, the aim is to study the influence of impact velocity and other factors on the rebound height of a spherical body. The viscoelastic plastic parameters were changed in order to assess their influence on the dynamics of impact interaction.

3. Theory

The viscoelastic plastic mechanorheological model developed by Irkutsk National Research Technical University was used. To analyze research results, it is necessary to describe how the model works.

The model consists of two consecutive rheological blocks with inertial elements \( m_1 \) and \( m_2 \) (figure 1). It simulates the impact interaction of a spherical body with the surface of a material. The viscoelastic block \( K_1 - C_1 \) describes elastic deformations of the interacting bodies. The elastic-plastic block \( K_2 - f_2 \) describes plastic deformations of the surface. The spherical body is considered elastic, not subject to plastic deformations. The mass of the spherical body is inertial element \( m_1 \). The mass of a part of the surface displaced as a result of plastic deformations is inertial element \( m_2 \). It is small in comparison with the mass of the spherical body.

At the initial stage, only elastic deformations \( y_e \) occur. Only the viscoelastic block \( K_1 - C_1 \) is deformed. Using damper \( C_1 \), the energy losses (energy dissipation) caused by dynamic elastic deformations are measured. When the impact force reaches \( F_{ST} \), the elastoplastic block \( K_2 - f_2 \) comes into operation. It models the development of plastic (residual) deformations \( y_{pl} \). When \( F_{ST} \) varies, the ratio between elastic and plastic deformations changes. When unloading the model, the process is described by the viscoelastic block, which reflects the disappearance of elastic deformations.

The dynamics is described using second-order differential equations. For the viscoelastic model, the equation is as follows:

\[
\dddot{y}_1 + C_1 \dddot{y}_1 + K_1 y_1^{3/2} = -m_1 g
\]  
(1)

The dynamics of the viscoelastic plastic model is described by two differential equations:

\[
\dddot{y}_1 + C_1 (y_1 - y_2)(y_1 - y_2) + K_1 (y_1 - y_2)^{3/2} = -m_1 g ;
\]  
(2)

\[
\dddot{y}_2 + K_2 y_2^{3/2} + f_2 y_2 + C_1 (y_2 - y_1)(y_2 - y_1) + K_1 (y_2 - y_1)^{3/2} = -m_2 g + F_{ST} ,
\]  
(3)
Figure 1. The scheme of deformation of the viscoelastic plastic model

where: $y_1, y_2, v_1, v_2$ - displacement and mass velocity $m_1$ and $m_2$; $K_1$ is the stiffness coefficient of the elastic element of the viscoelastic block; $C_1$ is the viscosity coefficient of the viscous element of the viscoelastic block; $F_{ST}$ is the force corresponding to the occurrence of plastic deformations; $K_2$ is the stiffness coefficient of the elastic element of the elastoplastic block; $f_2$ is the shear coefficient of the elastoplastic block or the compliance coefficient of the material.

The transformation of the viscoelastic model into the viscoelastic plastic model occurs when the normal reaction force $N_1 = C_1 v_1 + K_1 y_1^{1/2}$ is $F_{ST}$.

To solve equations (1) - (3), it is recommended to use numerical methods, for example, the Runge-Kutta method.

4. Analysis of results

The impact process was modeled using steel spherical bodies of different diameters. As an example, the results of studies for a body with a diameter of 9.5 mm are presented. The radius of curvature of the spherical surface was 4.75 mm. The body weight was 3.5 grams. There were several design options [34]. Option 1: $F_{ST} = N_{\text{max}}$ - only elastic deformations. Option 2: $F_{ST} = 2N_{\text{max}}/3$. Option 3: $F_{ST} = N_{\text{max}}/3$. Option 4: $F_{ST} = 0$ - plastic deformations occurs at the beginning of the impact process. As a parameter of the shock interaction, we considered the relative rebound height of the spherical body after impact $h_1/h_2$ ($h_1$ is the rebound height, $h_2$ is the drop height). The drop height varied from 3 mm to 300 mm. This corresponded to a change in impact velocity from 0.24 m/s to 2.43 m/s.
Figures 2-5 show the dependence of rebound height on impact velocity at $F_{ST} = 0$ and $F_{ST} = N_{\text{max}}$. Parameters $K_1, K_2, f_2, C_1$ were changed within the specified limits. An analysis of the results allows us to draw the following conclusions.

With increasing impact velocity, the rebound height decreases. It depends on energy dissipation (energy losses) due to impact interaction and deformation of the bodies. With elastic deformations, energy dissipation depends on the deformation rate (impact velocity) and increases with increasing velocity. With plastic (residual) deformations, energy dissipation increases with increasing deformations.

Let us describe the influence of plasticity of the material (model parameter $f_2$) on the rebound height (figure 2). At $F_{ST} = N_{\text{max}}$, the elastic impact occurs, the elastoplastic block $K_2 - f_2$ does not come into operation. An increase in impact velocity leads to a decrease in the rebound height. At $F_{ST} = 0$, elastic and plastic deformations occur simultaneously at the beginning of the impact process. Both blocks come into operation. The plasticity reduces the rebound height. The more ductile the material ($f_2 \text{ - min}$), the lower the rebound height due to large plastic deformations. An increase in impact velocity leads to an increase in the deformation of bodies. In this case, the ratio of plastic deformations to elastic ones increases. As a result, the dissipation of impact energy increases significantly.

![Figure 2. Dependence of rebound height $dh$ on impact velocity upon changes in $f_2$.](image)

Let us describe the effect of elasticity of the material (model parameter $K_1$) on the rebound height (figure 3). At $F_{ST} = N_{\text{max}}$, the elastic impact occurs, the elastoplastic block $K_2 - f_2$ does not come into operation. An increase in impact velocity leads to a decrease in the rebound height. A decrease in elasticity ($K_1 \text{ - min}$) also leads to a decrease in the rebound height because the viscoelastic model becomes more viscous, viscous deformation resistance increases with respect to elastic resistance. This leads to an increase in energy losses and a decrease in the rebound height.

For the elastic-plastic model ($F_{ST} = 0$), the relationship is reverse. A decrease in elasticity ($K_1 \text{ - min}$) leads to an increase in the rebound height because an increase in elasticity ($K_1 \text{ - max}$) leads to an increase in rigidity of the material. Elastic deformation resistance increases. An increasing force causes an increase in plastic deformations and energy losses.
Let us describe the effect of elastic parameter $K_2$ on the rebound height (figure 4). This parameter allows us to adjust characteristics of the elastoplastic block in the area of hardening of the material and simulate the behavior of the material during plastic deformations. A decrease in $K_2$ ($K_2 - \text{min}$) leads to an increase in plastic deformations and a decrease in the rebound height.
The effect of viscous parameter $C$ on the rebound height is shown in figure 5. An increase in viscous resistance to deformations ($C_1 - \text{max}$) increases energy losses and decreases the rebound height.

**Figure 5.** Dependence of rebound height $d_h$ on impact velocity upon changes in $C$.

5. Conclusion

Thus, the results of studies on the impact process using the mechanorheological model show that the rebound height depends on impact interaction velocity. With an increase in impact velocity, the energy losses increase. Under elastic deformations, energy dissipation depends on impact velocity and increases with increasing impact velocity. With plastic (residual) deformations, energy dissipation increases with increasing deformations. To improve accuracy and reliability of modeling and research of various dynamic processes, it is necessary to take into account the influence of these factors on the dynamics of interaction processes.

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