Nonlinear Robust Control of a Stewart Parallel Robot

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Abstract. In this paper, the control problem of a Stewart parallel robot is investigated, and a sliding mode control method based on self-tuning sliding mode disturbance compensator and double power reaching law is presented. The presented control method can realize fast regulation of a Stewart parallel robot system, and has good robustness and adaptability towards the system uncertainties. The design of a self-tuning sliding mode disturbance compensator is included in the presented control method. The designed sliding mode disturbance compensator does not need the upper bound information of the system uncertainties, and can accurately estimate the system uncertainties. The double power reaching law can not only ensure fast convergence of the system states, but also maintain the continuity of the sliding mode controller. Lyapunov stability theory is adopted to prove the stability of the closed-loop control system, and the validity of the presented control method is verified by a numerical simulation test.

1. Introduction

Recently, there has been considerable research interest in the area of Stewart parallel robots. Compared with the widely used serial robots, Stewart parallel robots has the distinctive advantages of strong bearing capacity, stable structure, high pose accuracy and so on. Based on the above advantages of Stewart parallel robots, Stewart parallel robots are widely used in motion simulators [1], tire testing machines [2], parallel machine tools [3], precision positioning platforms [4], operators [5] and entertainment occasions [6].

The schematic diagram of a Stewart parallel robot is shown in Figure 1. A Stewart parallel robot consists of a lower platform, six legs, an upper platform, and twelve spherical hinges or universal joints. The movement of each leg is individually controlled. Since a Stewart parallel robot has six degrees of freedom, each leg can expand and contract freely. As a result, in order to produce regular movement of a Stewart parallel robot, it is necessary to coordinate the movement of the six legs.

A Stewart parallel robot is a complex dynamic system, which is composed of six legs and two platforms, and has multiple inputs and multiple outputs. There is a complex coupling relationship between the robot parts, which produces serious nonlinearities in the robot dynamics. The actuating mechanism of a Stewart parallel robot is influenced by uncertain factors, such as model structural perturbations, time-varying parameters and unpredictable external disturbances, so the traditional control system design method is difficult to meet the control requirements of a Stewart...
parallel robot. The research on the robust control strategies of Stewart parallel robots has become one of the most important topics in the area of Stewart parallel robots.

![Figure 1. Schematic diagram of a Stewart parallel robot.](image)

The design of Stewart parallel robots’ motion controller can be divided into two categories: joint space based control and workspace based control. Joint space based controller design mainly depends on the kinematic relationship of the parallel robot and the dynamic model of the driving device, without considering the dynamic model of the whole parallel robot. The joint space based control transforms the controller design task of a Stewart parallel robot into a series of controller design of single axis servo system. While workspace based controller design needs the dynamic analysis of the parallel robot, based on which the driving forces required to move the parallel robot according to the expected configuration is calculated. It has been shown in the literature that workspace based control can provide better performance than that of joint space based control [7]. For workspace based control of Stewart parallel robots, many researchers have put forward different control strategies to solve the control problem of Stewart parallel robots. The proposed control strategies include inverse dynamic compensation based control [7], neuro-fuzzy adaptive control [8], linear control [9], super-twisting sliding mode control [10], modal space controller [11], and higher order sliding mode control [12].

In this paper, the workspace based control problem of a Stewart parallel robot is addressed, and a sliding mode control algorithm based on self-tuning sliding mode disturbance compensator and double power reaching law is proposed. Based on the Lagrange dynamics model of the whole system and the defined system sliding surface, a self-tuning sliding mode disturbance compensator is designed to accurately estimate the system uncertainties with unknown upper bounds. The sliding mode control law is derived based on the double power reaching law to realize fast regulation of the robotic system. The stability of the closed-loop control system is proved by using Lyapunov stability theory, and the numerical simulation test is provided to verify the effectiveness of the proposed control algorithm.

2. Dynamics model of a Stewart parallel robot

As shown in Fig. 1, a Stewart parallel robot is a 6-DOF mechanism with two platforms connected by six extensible legs. The inertial coordinate frame \( \Sigma_0 \{X, Y, Z\} \) is fixed at the base platform, and the body frame \( \Sigma_p \{x, y, z\} \) is attached to the mass centroid of the moving platform. The configuration of the robot system can be determined by the following generalized coordinate vector

\[
q = \left[ x, y, z, \alpha, \beta, \gamma \right]^	op
\]

where \((x, y, z)\) represent the linear motions of the moving platform, and \((\alpha, \beta, \gamma)\) denote the Euler angles of the rotational motions of the moving platform around \(x\)-axis, \(y\)-axis and \(z\)-axis, respectively.

A straightforward calculation of the kinetic energy and potential energy of the upper moving platform and the six legs, and then using the Lagrange dynamics equation, one can obtain the dynamics model of a Stewart parallel robot as follows [7, 13]
$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + D(t) = J^TF$  \hspace{1cm} (2)

where $M(q) \in \mathbb{R}^{6\times6}$ is the inertial matrix, which can be directly obtained by the expression of the system kinetic energy $T(q, \dot{q})$; $C(q, \dot{q})\dot{q} \in \mathbb{R}^{6\times1}$ is the Centripetal force and Coriolis force term; $G(q) \in \mathbb{R}^{6\times1}$ is the gravity term, which can be obtained by the partial derivative of the system potential energy, i.e., $G(q) = \frac{\partial P(q)}{\partial q}$; $D(t) \in \mathbb{R}^{6\times1}$ represents bounded unknown system uncertainties, which includes parameters perturbations, unmodelled dynamics and external disturbances; $J \in \mathbb{R}^{6\times6}$ is the Jacobian matrix; $F \in \mathbb{R}^{6\times1}$ is the driving force vector exerted upon the legs.

Due to the complexity of the parallel robot’s dynamics model, the detailed mathematical expressions of the robot motion equation will not be presented here, the detailed calculation results of the mathematical expressions in the robot dynamics mentioned above are referred to [7, 13].

3. Design of sliding mode control based on sliding mode compensator

The aim of workspace based control of a Stewart parallel robot is to find a feedback control law $F$ such that the robot configuration $q$ are accurately regulated to a given desired configuration $q^d$ for any initial condition $q(0)$. In this section, a sliding mode controller based on self-tuning sliding mode disturbance compensator and double power reaching law is designed. The designed sliding mode disturbance compensator does not need the upper bound information of the system uncertainties, and can accurately estimate the system uncertainties. The double power reaching law can not only ensure fast convergence of the system states, but also maintain the continuity of the sliding mode controller.

3.1. Design of system sliding surface

According to the control objective of workspace based control of the parallel robot, the system sliding surface is defined as

$$S(t) = \dot{e}(t) + \Lambda e(t)$$  \hspace{1cm} (3)

where $e(t) = q(t) - q^d$ is the error vector, $q^d$ denotes the desired vector of the coordinate variable vector $q(t)$, $\Lambda$ is a positive diagonal matrix. The error vector $e(t)$ and matrix $\Lambda$ are given by

$$e(t) = [e_1(t), e_2(t), e_3(t), e_4(t), e_5(t), e_6(t)]^T$$

$$\Lambda = \text{diag}(c_1, c_2, c_3, c_4, c_5, c_6)$$  \hspace{1cm} (4)

where $e_i(t) = q_i(t) - q^d_i$, $q^d_i$ represents the desired value of the $i$-th coordinate variable $q_i(t)$, $c_i$ is a positive constant.

Note that the inertial matrix $M(q)$ is positive definite and thus invertible. From (2), we can obtain

$$\ddot{q}(t) = M^{-1} \left( J^TF - C\dot{q} - G - D \right)$$  \hspace{1cm} (5)

Differentiating (3) and using (5), we can obtain

$$\dot{S}(t) = \dot{\dot{e}}(t) + \Lambda \dot{e}(t) = BF - H - \ddot{q} + \Lambda \dot{e}$$  \hspace{1cm} (6)

where $B = M^{-1}J^T$, $H = M^{-1}(C\dot{q} + G)$, $D = M^{-1}D$.

We rewrite (6) in the following form

$$\ddot{s}_i(t) = (BF)_{ii} - h_i - d_i - \ddot{q}_i + c_i \dot{e}_i$$  \hspace{1cm} (7)

where $s_i(t)$ represents the $i$-th element of the vector $S(t)$, $(BF)_{ii}$ represents the $i$-th element of the vector $BF$, $h_i$ represents the $i$-th element of the vector $H$, $d_i$ represents the $i$-th element of the vector $D$. 
Remark 1: Note the property that the inertial matrix $M(q)$ is bounded [14], it can be easily inferred that $M^{-1}(q)$ is also bounded. With the assumption that $D$ is bounded and from (6), it can be obtained that the uncertainty term $\tilde{D}$ is also bounded. This means that the element of the uncertainty term $d_i$ is upper bounded by some unknown constant, i.e.,

$$\|d_i(t)\| \leq K_i$$  \hspace{1cm} (8)$$

where $K_i$ is a positive constant.

3.2. Design of self-tuning sliding mode disturbance compensator

Since that there exist uncertainties in the parallel robot system, in order to reduce the adverse effect of the uncertainty on the system performance and to improve the control accuracy of the system, a self-tuning sliding mode disturbance compensator is designed to estimate the system uncertainty. The specific form of the self-tuning sliding mode disturbance compensator is defined as

$$\begin{align*}
\sigma_i(t) & = s_i(t) - z_i(t) \\
\dot{z}_i(t) & = (BF)_i - h_i - \tilde{q}^d + c_i \hat{e}_i - u_i \\
\hat{d}_i(t) & = u_i 
\end{align*}$$  \hspace{1cm} (9)$$

where $\sigma_i(t)$ is the auxiliary sliding surface, $z_i(t)$ is the internal state of the disturbance compensator, $\hat{d}_i(t)$ is the observation value of the system uncertainty $d_i(t)$, $u_i$ is the output injection term of the disturbance compensator and is designed as

$$u_i(t) = -\varepsilon_i \sigma_i - \rho_i(t) \text{sign} (\sigma_i)$$  \hspace{1cm} (10)$$

where $\varepsilon_i > 0$, the adaptation law of the switching gain $\rho_i(t)$ is defined as

$$\dot{\rho}_i(t) = \eta_i |\sigma_i|$$  \hspace{1cm} (11)$$

where $\eta_i > 0$, $\rho_i(0) > 0$.

Theorem 1: Consider the system uncertainty $d_i$ shown in (7), if the disturbance compensator output injection term $u_i$ shown in (10) is adopted, then the observation value of the system uncertainty $\hat{d}_i(t)$ converges to its true value $d_i(t)$ in a finite time.

Proof: A Lyapunov function candidate is defined as

$$V_i(t) = \frac{1}{2} \sigma_i^2 + \frac{1}{2 \eta_i} (\rho_i - K_i)^2$$  \hspace{1cm} (12)$$

From (7) and (9), we can obtain

$$\dot{\sigma}_i(t) = \dot{s}_i(t) - \dot{z}_i(t) = u_i(t) - d_i(t)$$  \hspace{1cm} (13)$$

From (13), the differentiation of (12) can be obtained as

$$\dot{V}_i(t) = \sigma_i \cdot \dot{\sigma}_i + \frac{\dot{\rho}_i(t)}{\eta_i} (\rho_i - K_i)$$

$$= \sigma_i (u_i - d_i) + |\sigma_i| (\rho_i - K_i)$$  \hspace{1cm} (14)$$

Note that $|d_i(t)| \leq K_i$. From (10), we can further obtain

$$\dot{V}_i(t) \leq -|\sigma_i| (K_i - |d_i|) - \varepsilon_i \sigma_i^2 < 0$$  \hspace{1cm} (15)$$
Therefore, the auxiliary sliding surface can converge to zero in a finite time and remain zero after that, i.e., after finite convergence time \( \sigma_i(t) = \dot{\sigma}_i(t) = 0 \). Furthermore, from (13), we can infer that the observation value of system uncertainty \( \hat{d}_i(t) \) can converge to its true value \( d_i(t) \) in a finite time.

Since the output injection term of the disturbance compensator \( u_i(t) \) contains discontinuous switching term \( \rho_i(t) \), the introduction of the observation value of the system uncertainty \( \hat{d}_i(t) \) into the control law may lead to the high frequency chattering of the controller. Therefore, the hyperbolic tangent function is used instead of the sign function in the output injection term of the disturbance compensator, i.e.,

\[
\begin{align*}
\lambda_i & = \varepsilon_i \sigma_i + \rho_i(t) \tanh(\lambda_i \sigma_i) \\
\tanh(\lambda_i \sigma_i) & = \frac{e^{\lambda_i \sigma_i} - e^{-\lambda_i \sigma_i}}{e^{\lambda_i \sigma_i} + e^{-\lambda_i \sigma_i}}
\end{align*}
\]

### 3.3. Design of sliding mode control based on double power reaching law

In order to ensure the fast convergence of the system states and to suppress the high frequency chattering of the controller, the sliding mode control law is derived based on double power reaching law proposed in [15].

The double power reaching law is described by

\[
\dot{s}_i(t) = -k_{i,1}|s_i|^\alpha \text{sign}(s_i) - k_{i,2}|s_i|^\beta \text{sign}(s_i)
\]

where \( k_{i,1} > 0 \), \( k_{i,2} > 0 \), \( 1 < \alpha < 2 \), \( 0 < \beta < 1 \). In the reaching law (17), when the system state is far away from the sliding mode \( s_i = 0 \), the reaching term \( -k_{i,1}|s_i|^\alpha \text{sign}(s_i) \) provides the main reaching effect. When the system state is close to the sliding mode \( s_i = 0 \), the reaching term \( -k_{i,2}|s_i|^\beta \text{sign}(s_i) \) provides the main reaching effect.

Since the observation value of the system uncertainty \( \hat{d}_i(t) \) can converge to its true value \( d_i(t) \) in a finite period of time, then after finite reaching time of the sliding surface \( s_i \), (7) can be expressed as

\[
\dot{s}_i(t) = (BF)_i - h_i - \dot{d}_i - \ddot{q}_i - c_i \dot{e}_i
\]

From (17) and (18), we can obtain

\[
(BF)_i = h_i + \dot{d}_i + \ddot{q}_i - c_i \dot{e}_i - k_{i,1}|s_i|^\alpha \text{sign}(s_i) - k_{i,2}|s_i|^\beta \text{sign}(s_i)
\]

We rewrite (19) in the following form

\[
BF = H + \dot{D} + \ddot{q} - \Lambda \dot{e} - \bar{K} \text{sign}(S)
\]

where

\[
\bar{K} = \text{diag}\left\{ k_{i,1}|s_i|^\alpha \text{sign}(s_i) + k_{i,2}|s_i|^\beta \text{sign}(s_i) \right\}
\]

\[
\text{sign}(S) = \left[ \text{sign}(s_i), \text{sign}(s_i), \text{sign}(s_i), \text{sign}(s_i), \text{sign}(s_i), \text{sign}(s_i) \right]^T
\]

Then, the sliding mode control law can be obtained as

\[
F = B^T \left[ H + \dot{D} + \ddot{q} - \Lambda \dot{e} \right] - B^T \bar{K} \text{sign}(S)
\]

**Theorem 2:** Considering the parallel robot system shown in (2), the sliding surfaces are designed according to (3), and the self-tuning sliding mode disturbance compensator is designed according to (9). If the sliding mode control law shown in (21) is adopted, then the sliding mode surface \( s_i(t) \) and its derivative \( \dot{s}_i(t) \) converge to zero in a finite time.
**Proof:** Define a Lyapunov function candidate as

\[ V_i(t) = \frac{1}{2} s_i^2 \]  

(22)

Substituting (19) into (18), we can get

\[ \dot{s}_i(t) = -k_{i,1} |s_i|^\alpha \operatorname{sign}(s_i) - k_{i,2} |s_i|^\beta \operatorname{sign}(s_i) + \hat{d}_i - d_i \]  

(23)

As has been proved that after finite convergence time of the auxiliary sliding surface \( \sigma_i \), the observation value of the system uncertainty \( \hat{d}_i(t) \) can accurately estimate its true value \( d_i(t) \), i.e., \( \hat{d}_i(t) - d_i(t) = 0 \). Then after finite reaching time of the auxiliary sliding surface \( \sigma_i \), we have

\[ \dot{s}_i(t) = -k_{i,1} |s_i|^\alpha \operatorname{sign}(s_i) - k_{i,2} |s_i|^\beta \operatorname{sign}(s_i) \]  

(24)

Differentiating (22) and using (24), we can obtain

\[ \dot{V}_i(t) = s_i \dot{s}_i \]

\[ = -k_{i,1} |s_i|^{\alpha+1} - k_{i,2} |s_i|^{\beta+1} \leq 0 \]  

(25)

From (25), we can infer that \( s_i(t) \) converges to zero in a finite time and remains zero thereafter. Further, from (24) we can obtain that when \( s_i(t) \) reduces to zero, \( \dot{s}_i(t) \) also converges to zero. This means that both \( s_i(t) \) and \( \dot{s}_i(t) \) converge to zero in a finite time.

### 4. Simulation test verification

In order to verify the effectiveness of the sliding mode control method proposed in this paper, the regulation control of a Stewart parallel robot is carried out by using MATLAB numerical simulation environment. In the numerical simulation test, the physical parameters of the parallel robot system are taken according to [7]. The initial conditions of the parallel robot system are set as

\[ \begin{bmatrix} x_0, y_0, z_0, \alpha_0, \beta_0, \gamma_0 \end{bmatrix}^T = \begin{bmatrix} 0, 0, 0.586, 0, 0, 0 \end{bmatrix} \]

\[ \begin{bmatrix} \dot{x}_0, \dot{y}_0, \dot{z}_0, \dot{\alpha}_0, \dot{\beta}_0, \dot{\gamma}_0 \end{bmatrix}^T = \begin{bmatrix} 0, 0, 0, 10, 10 \end{bmatrix}^T \]  

(26)

The desired configuration of the parallel robot system \( q^d \) is determined by

\[ q^d = \begin{bmatrix} x^d, y^d, z^d, \alpha^d, \beta^d, \gamma^d \end{bmatrix}^T = \begin{bmatrix} 0.1, 0.1, 0.6, 0.1, 0.1, 0.1 \end{bmatrix}^T \]  

(27)

To further verify the robustness of the sliding mode controller, an artificial uncertainty of \( \pm 10\% \) is added to the nominal model parameters of the Stewart parallel robot. It should be noted that the nominal model parameters are still used in the design of the sliding mode control law. In the numerical simulation, the parameters of the proposed sliding mode controller are selected as follows:

\[ c_i = 7, \quad k_{i,1} = 5, \quad k_{i,2} = 7, \quad \alpha_i = 1.5, \quad \beta_i = 0.5, \quad \lambda_i = 6, \quad \epsilon_i = 2.4, \quad \eta_i = 0.6, \quad \rho_i(0) = 0.5 \]  

(28)

![Image of simulation results](image-url)
The simulation results of the proposed sliding mode control method are shown in Figure 2 and Figure 3. As can be seen from Figure 2, under the effect of the proposed sliding mode controller, the closed-loop control system has good dynamic process. All the configuration variables of the robot system are regulated accurately to their values within a short finite time interval. Specifically speaking, the robot system is stabilized to its desired configuration at about 2.7s. From Figure 3 where $F_i$ represents the actuating force exerted upon the $i$-th leg, we can see that the sliding mode control effort is continuous and exhibits no chattering phenomenon. The numerical simulation results show that the sliding mode control method proposed in this paper not only can provide satisfactory control performance, but also has good robustness and adaptability towards system uncertainty.
5. Conclusion
In this paper, the control problem of a Stewart parallel robot is studied, and a sliding mode control scheme based on self-tuning sliding mode disturbance compensator and double power reaching law is proposed. The proposed control scheme can effectively realize fast regulation of Stewart parallel robot system, and has good robustness and adaptability for the system uncertainties. The proposed control scheme includes the design of a self-tuning sliding mode disturbance compensator. The designed sliding mode disturbance compensator does not need the upper bound information of the system uncertainties, and can accurately estimate the system uncertainties. The double power reaching law can not only ensure fast convergence of the system states, but also maintain the continuity of the sliding mode controller. Lyapunov stability theory is applied to prove the stability of the closed-loop control system, and the effectiveness of the proposed control scheme is verified by numerical simulation test.

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