Novel Mathematical Modeling and Motion Analysis of a Sphere Considering Slipping

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ABSTRACT

Many mobile robots that use spherical locomotion employ friction-drive systems because such systems offer omnidirectional locomotion and are more capable of climbing steps than omni-wheel systems. One notable issue associated with friction-drive systems is slipping between the sphere and the roller. However, previously established sphere kinematics models do not consider slipping. This study proposes a mathematical model that allows for slipping and can be broadly applied to a variety of mobile robots in a range of situations.

1. INTRODUCTION

Spherical wheels can be used in mobile and omnidirectional robots. For example, the ball wheel [1] comprises a sphere with a circular-roller ring arranged on the outer circumference of the sphere. The propulsive force is generated by the rotation of the oblique-rotation axis. Further, there is no binding force in the direction orthogonal to the propulsion force as there is with an omni-wheel [2]. The omni-ball [3] solves the constraint of the ball wheel [1] as it enables omnidirectional driving via the passive rotation of a pair of hemispheres. It offers similar driving performance as the omni-wheel [2] and superior step-climbing ability.

In mobile robots, Active-Caster [4] is composed of an upper sphere and a lower sphere, each of which is in contact with two driving rollers. The upper sphere uses driving rollers to transmit dynamic motion to the lower sphere. This mechanism is used for driving and steering of the caster and enables omnidirectional motion. The balanced-ball robot [5] achieves omnidirectional locomotion by spherical driving using three omni-wheel arranged on the upper hemisphere of an equilateral triangle. The CPU-ball robot [6] has four-omni-wheel arranged on the upper hemisphere of a regular quadrilateral to achieve spherical driving and realize omnidirectional locomotion.

The RoboCup middle-size-league soccer robot utilizes a ball-dribbling mechanism to control the rotation of the ball. Most of the RoboCup teams, such as the Turtles [7], implemented two constraint rollers on the upper half of the ball. Due to makes strong friction force and enhanced ball-holding ability, most designs use slip-roller arrangements as are determined heuristically via experiments in the absence of suitable mathematical models.

In a previous study, we developed a non-slip omnidirectional-locomotion kinematics model that accounts for the sphere kinematics and roller arrangement [8]. However, this model cannot be used for mobile robots as they also undergo slip locomotion. In this study, we modify the previously developed kinematics model and present a novel mathematical model of sphere rotational motion by two constraint rollers that allows for slipping.

In this paper, the outline of the section is as follows: Section 2 considers discusses the existence space of angular velocity vector and the sphere kinematics by two roller. Section 3 conducted simulation. Finally, we present the summary and discuss future tasks.

2. THE SPHERE KINEMATICS BY CONSTRAINT ROLLERS

In this section, we introduce the angular velocity vector of the sphere to algebraically model the sphere rotational motion.

2.1. The Existence of Angular Velocity Vector of the Sphere by Single-constraint Roller

As shown in Figure 1, the center O of a sphere with radius r is fixed as the origin of the coordinate system Σ = xyz. Table 1 shows the variables related to the sphere kinematics. The ith constraint roller is in point contact with the sphere at a position vector Pi and is arranged such that the center of mass of the roller Pi and O are on the same line. ηi denotes the angular velocity vector of the sphere. ηi denotes the unit vector along the rotational axis of constraint roller.

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Table 1 | The variables related to the sphere kinematics

| Symbol | Definition |
|--------|------------|
| $\Sigma$ | Three-dimensional coordinate system fixed the sphere |
| $\langle a, b \rangle$ | Inner product with respect to $a$ and $b$ |
| $||a||$ | Norm of vector $a$ |
| $\text{span}(X, Y)$ | The existence space of $\omega$ allow for slip |
| $O$ | The sphere center |
| $P_i$ | The Position vector of sphere |
| $\eta$ | The unit vector along the rotational axis of the constraint-roller |
| $\omega$ | The angular velocity vector of the sphere |
| $\nu_i^a$ | The orthogonal projection of $\omega$ with respect to $\text{span}(P_1, P_2)$ |
| $\nu_i^b$ | The orthogonal vector of $\omega$ with respect to $P_i \times P_2$ |
| $\nu_i^s$ | The velocity vector of constraint roller |
| $\nu_i^e$ | The velocity vector of the sphere |
| $\nu_i^e$ | Slip velocity of the sphere with respect to $\nu_i^a$ |
| $e$ | The unit normal vector along $\nu_i^a$ |
| $e_{\phi}$ | Upper unit normal vector of $\text{span}(P_1, P_2)$ |
| $V$ | Mobile velocity of sphere on $xy$-plane |
| $X\epsilon$ | Normal orthogonal bases on tangent plane of the sphere at $P_i$, $\eta$ can be represented as $\text{Equation (5)}$. |
| $l(v_i)$ | The location of end point of $\omega$ |
| $l(v_i)$ | Set that exists of end point of $\omega$ |
| $l(v_i)$ | The orthogonal projection of $l(v_i)$ with respect to $\text{span}(P_1, P_2)$ |
| $V_i$ | Peripheral speed of constraint roller |
| $r$ | The sphere radius |
| $\alpha$ | The roller arrangement angle between $\eta$ and $\text{span}(P_1, P_2)$ |
| $\phi$ | Sphere direction |
| $\rho$ | Angle of sphere rotational axis |

$\nu_i$ denotes the peripheral speed of the constraint roller. Hence, the velocity vector of the sphere $\nu_i^a$ with respect to $P_i$ can be represented by $\text{Equation (1)}$:

$$\nu_i^a = \omega \times P_i$$  \hspace{1cm} (1)

$e_i \in \text{span}(P_i, \eta)$ is the unit normal vector along $\nu_i^s$. Using $\nu_i = \langle \nu_i^a, e_i \rangle$ ($\nu_i^s = \nu_i^a$: non-slip condition) and $\text{Equation (1)}$, $\nu_i$ is represented as follows:

$$\nu_i = \langle \nu_i^a, e_i \rangle = \langle \omega \times P_i, e_i \rangle$$  \hspace{1cm} (2)

Thus, $\omega$ can be satisfied as $\text{Equation (3)}$.

$$\langle \eta_i, \omega \rangle = \frac{-\nu_i}{r}$$  \hspace{1cm} (3)

Further, from the property of the constraint roller (i.e., that slip does not occur in side direction of the roller), $\omega$ must be on $\text{span}(\eta_i, P_i)$. Thus, $\text{Eq. (3)}$ indicates that $\omega$ is constructed as the sum of $\nu_i \eta_i$ and $P_i$ (see Figure 1). Thus, $\omega$ cannot be uniquely determined using a single roller. However, the end point set of $\omega$ can be represented as a line set as follows:

$$l(v_i) = \left\{ \omega \left( -\frac{\nu_i}{r} \right) \eta_i + kP_i, k \in \mathbb{R} \right\}$$  \hspace{1cm} (4)

2.2. The Existence of Sphere Angular Velocity Vector of the Sphere by Two-constraint Rollers

Let the roller arrangement the angle $\alpha$ ($-90^\circ \leq \alpha \leq 90^\circ$) between $\eta$ and $\text{span}(P_1, P_2)$. Using the normal orthogonal base $\langle X, e \rangle$ on tangent plane of the sphere at $P_i$, $\eta$ can be represented as $\text{Equation (5)}$.

$$\eta_i = X_i \cos \alpha + e \sin \alpha$$  \hspace{1cm} (5)

where

$$X_i = r^{-1}e \times P_i, e = \frac{P_i \times P_2}{||P_i \times P_2||}$$  \hspace{1cm} (6)

In this section, we consider location between $l(v_i)$ and $l(v_i)$ which depend on parameter $v_i, v_i$ (when $\alpha_i, \alpha_i$ are fixed) and determined the rotational axis of the sphere.

Using $\text{Equation (4)}$ (as $i = 1, 2$), as shown in Figure 2a, if a pair of $v_i, v_i$ exists such that $l(v_i)$ and $l(v_i)$ have points in common, the end point of $\omega_i$ can be uniquely determined. Using $\text{Equation (4)}$ (as $i = 1, 2$), $\omega_i$ must be on $\text{span}(P_1, \eta_i) \cap \text{span}(P_2, \eta_i)$. On the other hand, as shown in Figure 2b, if a pair of $v_i, v_i$ exists such that $l(v_i)$ and $l(v_i)$ have no points in common, slip can occur.

![Figure 1](image1.png) | The existence of sphere angular velocity vector in case of single-constraint roller.

![Figure 2](image2.png) | The location of $l(v_i)$ and $l(v_i)$. (a) A pair of $v_i, v_i$ exists such that $l(v_i)$ and $l(v_i)$ have points in common. (b) A pair of $v_i, v_i$ exists such that $l(v_i)$ and $l(v_i)$ have no points in common.
Because the sphere rotational axis is defined with respect to the arbitrary parameters \( \nu_i \) and \( \nu_j \), a \( Q \in \mathbb{R}^3 \) can be determined such that the sum of the squared distances between \( Q \) and \( l(\nu) \) (\( i = 1, 2 \)) is minimized. Then, \( Q \) corresponds to the midpoint between \( l(\nu) \) and \( l(\nu) \) [see Appendix (A)]. Using \( X, P \), orthogonal projection of \( l(\nu) \) with respect to \( \text{span}\{P_i, P_j\} \) is represented as Equation (7).

\[
l_1(\nu) = \left\{ \omega \left( -\frac{\nu_i \cos \alpha_i}{r} \right) X_i + kP_i, k \in \mathbb{R} \right\}
\]

The end point of \( \omega \) (the orthogonal projection of \( \omega \) with respect to \( \text{span}\{P_i, P_j\} \)) is represented as common point of \( l_1(\nu) \) and \( l_j(\nu) \) [see Appendix (B)].

\[
\omega_j = \frac{1}{\|P_i \times P_j\|} [\nu_j \cos \alpha_j] P_i - (\nu_j \cos \alpha_j) P_j
\]

A heights \( l(\nu) \) from \( \text{span}\{P_i, P_j\} \) \((i = 1, 2)\), is represented as follows:

\[
\omega_i = -\frac{\nu_i \sin \alpha_i}{r}, -\nu_j \sin \alpha_j
\]

Using Equation (9), \( \omega \) (the orthogonal projection of \( \omega \) with respect to \( P_1 \times P_2 \)) is represented as mean of these.

\[
\omega_i = -\frac{\nu_i \sin \alpha_i + \nu_j \sin \alpha_j}{2r} P_1 \times P_2
\]

Thus, \( \omega \) can be represented with respect to \( \nu_i \) and \( \nu_j \) \( \in \mathbb{R} \).

\[
[\omega_i, \omega_j, \omega_1] = \omega_i + \omega_j
\]

By rearranging Equation (11), following equation can be obtained as linear combination of \( X \) and \( Y \).

\[
\omega = r \omega X + r \nu Y
\]

Specifically, when \( \alpha_i = 0^\circ \), the second term of the right-hand side of Equation (11) vanishes. Thus, for all \( \nu_i, \nu_j, \omega \in \text{span}\{P_1, P_2\} \). In other words, the sphere has omnidirectional locomotion without slip in the expanded form of kinematics model [8].

### 2.3. The Sphere Kinematics by Two-constraint Rollers

#### 2.3.1. Forward kinematics

\( \phi (0^\circ \leq \phi < 360^\circ) \) is the angle from x-axis. Mobile velocity of sphere \( V \) (the center velocity of sphere) is on \( xy \)-plane and represented as Equation (13).

\[
V = \|V\| [\cos \phi, \sin \phi, 0]^T
\]

\( \omega \) is perpendicular to \( V \), the angle of sphere rotational axis \( \rho (-90^\circ \leq \rho \leq 90^\circ) \) is the angle between \( \omega \) and \( xy \)-plane. Therefore, using \( \omega = [\omega_x, \omega_y, \omega_z]^T \), forward kinematics is given as follows:

\[
\|V\| = r \left[ \frac{\omega_x}{\cos \alpha} \right] (\omega_x < 0)
\]

\[
\phi = \begin{cases} \cos^{-1} \left( \frac{\omega_y}{\sqrt{\omega_x^2 + \omega_y^2}} \right) & (\omega_x < 0) \\ 360^\circ - \cos^{-1} \left( \frac{\omega_y}{\sqrt{\omega_x^2 + \omega_y^2}} \right) & (\omega_x \geq 0) \end{cases}
\]

\[
\rho = \tan^{-1} \left( \frac{\omega_z}{\sqrt{\omega_x^2 + \omega_y^2}} \right)
\]

#### 2.3.2 Inverse kinematics

From Equations (13) and (14), \( \omega \) is represented as Equation (15).

\[
[\omega_x, \omega_y, \omega_z]^T = \left[ \frac{\|V\|}{r} \right] [\sin \phi, \cos \phi, \tan \rho]^T
\]

By rearranging Equation (11), following equation can be obtained as linear combination of \( X \) and \( Y \).

\[
\omega = r \omega X + r \nu Y
\]

Thus, for all \( \nu_i, \nu_j, \omega \in \text{span}\{X, Y\} \), two-dimensional-freedom existence space, which has the unit vector \( X \times Y \). From \( \omega \in \text{span}\{X, Y\} \), using Equation (15) and \( \langle \omega, X \times Y \rangle = 0, \rho \) is obtained as follows:

\[
\rho = \tan^{-1} \left( \frac{\langle X \times Y, X \rangle \sin \phi - \langle X \times Y, Y \rangle \cos \phi}{\langle X \times Y, X \rangle} \right)
\]

where \( \langle X \times Y, X \rangle, \langle X \times Y, Y \rangle \) and \( \langle X \times Y, Z \rangle \) are components of \( X \times Y \).

When Equation (18) is substituted in Equation (15), \( \omega \) is obtained as follows:

\[
[\omega_x, \omega_y, \omega_z]^T = \left[ \frac{\|V\|}{r} \right] [\sin \phi, \cos \phi, \tan \rho]^T
\]

And, from Equation (7), \( \omega \) is satisfied as

\[
\langle \omega, X \rangle = -\frac{\nu_i \cos \alpha_i}{r}
\]

Using Equation (6), \( \langle \omega, X \rangle \) is calculated as follows:

\[
\langle \omega, X \rangle = r \langle \omega, e \times P \rangle = -r \langle e, \omega \times P \rangle
\]

Thus, from Equations (20) and (21), \( \nu_i \) is obtained as follows:

\[
\nu_i = \frac{\langle e, \omega \times P \rangle}{\cos \alpha_i}
\]
2.4. Slip Velocity of the Sphere

Slip occurs when the roller velocity $v^i$ and the sphere velocity $v^k$ on the tangent plane span$[X, e]$ at the point $p_i$ are different. Slip velocity of sphere $\zeta_i$ which is relative speed with respect to $v^k$ can be represented as difference between $v^i$ and $v^k$ as Equation (23).

$$\zeta_i = v^i - v^k$$  \hspace{1cm} (23)

where

$$v^i = \omega \times p_i, v^k = v_i e_i$$  \hspace{1cm} (24)

Here, we substitute (24) for $v^i$ and $v^k$ in Equation (23).

$$\zeta_i = \omega \times p_i - v_i e_i$$  \hspace{1cm} (25)

Taking the inner product with $e$ on both sides of Equation (25) and using Equation (22) in the first term on the right-side and Equation (5) in the second term of the right-side, the following Equation (26) can be formulated.

$$\langle \zeta_i, e \rangle = \langle \omega \times p_i, e \rangle + v_i \langle e_i, e \rangle = v_i \cos \alpha - v_i \cos \alpha = 0$$  \hspace{1cm} (26)

Thus, $\zeta_i$ is parallel to $X_i$ (component vanished) and represented as

$$\zeta_i = S_i X_i$$  \hspace{1cm} (27)

where

$$S_i = \langle \zeta_i, X_i \rangle$$  \hspace{1cm} (28)

3. SIMULATION

This section presents the simulation results, including the trajectory of the end point of the angular velocity vector, the angle of the sphere rotational axis, the peripheral speed of the constraint roller, and $X_i$-component of the slip velocity in the given mobile speed of the sphere: $||V||=1 \text{ m/s}$. The conditions are as follows: $r=1 \text{ (m)}, \theta_1=215^\circ, \theta_2=325^\circ, \theta_1=60^\circ$, and $\alpha_i=\alpha$. Simulations were conducted at the four different angles, $\alpha_i$ [represented as $k$; curve color]. Further, $\omega_i, \rho_i, v_i$, and $S_{ik} (k = 0, 1, 2, 3)$ were indicated such as $\alpha_i = 0^\circ$ [red curve, $\alpha_i = 10^\circ$ [blue curve], $\alpha_i = 20^\circ$ [green curve], and $\alpha_i = 30^\circ$ [pink curve]. They are calculated from Equations (18), (19), (22), and (28), respectively. As shown in Figure 3a, ellipsoid trajectories $a_0 (k = 0, 1, 2, 3)$ are getting scarp in turn and have a common line parallel to the $x$-axis.

As shown in Figure 3b, $\rho_i (k = 0, 1, 2, 3)$ satisfy the inequality $|\rho_i| < |\rho_i| < |\rho_i| (i = 1, 2)$ for all $\varphi$. Specifically, when $\varphi = 90^\circ$ or $270^\circ$, $\rho_i = 0^\circ$. Thus, the sphere undergoes pure rotation (forward and backward movement).

As shown in Figure 3c, $v_{ik} (k = 0, 1, 2, 3)$ satisfy $|v_{ik}| < |v_{ik}| (< 180^\circ)$ for all $\varphi$. Specifically, when $\varphi = 0^\circ$ and $180^\circ$ (right and left side forward movement), $v_{ik} = -v_{ik}$ (opposite sign). Where $\varphi = 90^\circ$ and $270^\circ$, $v_{ik} = v_{ik}$ and $|v_{ik}| = 0.91, |v_{ik}| = 0.93, |v_{ik}| = 0.96, |v_{ik}| = 1.03 \text{ (m/s)}$.

As shown in Figure 3d, $S_{ik} (k = 0, 1, 2, 3)$ satisfy the inequality $|S_{ik}| < |S_{ik}| < |S_{ik}| (i = 1, 2)$ for all $\varphi$. From Equation (26), where $0^\circ < \varphi < 180^\circ$, $S_{ik}$ and $\zeta_{ik}$ are face-to-face. In contrast, $180^\circ < \varphi < 360^\circ$, $S_{ik}$ and $\zeta_{ik}$ are back-to-back. Specifically, when $\varphi = 0^\circ$ and $180^\circ$, the sphere slip speed is $||S_{ik}||=||\zeta_{ik}||=0 \text{ m/s}$. Specifically, when $\varphi = 90^\circ$ and $270^\circ$, $|S_{ik}|$ and $|\zeta_{ik}|$ are at their maxima ($|S_{ik}| = |S_{ik}| = 0$, $|S_{ik}| = |S_{ik}| = 0.16, |S_{ik}| = |S_{ik}| = 0.32, and $|S_{ik}| = |S_{ik}| = 0.52 \text{ m/s}$).

Further, when $\alpha_i = 0^\circ, |S_{ik}|=|S_{ik}|=0$ for all $\varphi$. Thus, in this case, the sphere has omnidirectional locomotion without slip. Moreover, the proposed model includes the previously developed model [8].

4. CONCLUSION

Herein, we consider the existence of an angular velocity vector for the sphere and propose a sphere kinematics model that allows
for slipping. In addition, we demonstrate the trajectory of the end point of the angular velocity vectors of the roller speed and slip speed of the spheres in simulations. This model includes the previously developed model [8] and is expected to be applicable to a wide range of mobile robots in a variety of situations.

In future studies, this model should be verified experimentally. Further, it could be applied to simulate the ball-dribbling mechanism.

CONFLICTS OF INTEREST

There is no conflicts of interest.

APPENDIX (A) CALCULATION OF MINIMAL POINT \( Q_0 \)

As shown in Figure 4a, \( d_i \) denote the distance between \( Q \in \mathbb{R}^3 \) and line \( l_i \) (\( i = 1, 2 \)). \( Q \) denote the points at which a perpendicular line intersects \( l_i \) (\( i = 1, 2 \)). Using follow

**Lemma.** Let \( k \) be the minimum value of \( d_i + d_j \) and \( M_k = \{x \mid d_i + d_j = k, k \leq k, k \in \mathbb{R}, X \in \mathbb{R}^3 \} \).

The following statements hold:

(i) \( k \neq k \leftrightarrow M_k \cap M_k = \emptyset \)

(ii) \( \cup_{k \neq k} M_k = \mathbb{R}^3 \)

Problem (A): Minimum of \( d_i + d_j \) such that \( (x, y, z) \in \mathbb{R} \) is equivalent to Problem (B): Minimum of \( d_i + d_j \) such that \( (d_i, d_j) \in \{ (d_i, d_j) \mid k \leq d_i + d_j \leq d_i \} \).

Thus, as shown in Figure 4b, when the line defined by \( d_i + d_j = k \) is tangent to the circle defined by \( d_i^2 + d_j^2 = k/2 \) at \( (d_i, d_j) = (k/2, k/2) \), \( d_i^2 + d_j^2 \) is minimized. Thus, \( Q_0 \in \mathbb{R}^3 \) is the midpoint of \( l_i \) and \( l_j \).

APPENDIX (B) CALCULATION OF \( \omega_i \)

As shown in Figure 5, following expressions are completed.

\[
\angle X_0P_2 = \angle P_1O_2 = -90^\circ \tag{B.1}
\]

and,

\[
\angle X_2OP_1 = 180^\circ - \angle X_1OP_2 \tag{B.2}
\]

\[
\sin \angle P_0OP_2 = r^{-1} \parallel P_1 \times P_2 \parallel \tag{B.3}
\]

Using Equations (B.1) and (B.3), \( \langle P_1, X_2 \rangle \) are represented as Equation (B.4).

\[
\langle P_1, X_2 \rangle = \| P_1 \| \| X_2 \| \cos \left( \angle P_0OP_2 - 90^\circ \right) = r \sin \left( \angle P_0OP_2 \right) = r^{-1} \parallel P_1 \times P_2 \parallel \tag{B.4}
\]

Using Equations (B.2) and (B.4), \( \langle P_1, X_2 \rangle \) are represented as Equation (B.5).

\[
\langle P_1, X_2 \rangle = -\langle P_2, X_1 \rangle = -r^{-1} \parallel P_1 \times P_2 \parallel \tag{B.5}
\]

\( \omega_i \) can be represented as shown in Equation (B.6).

\[
\omega_i = (C_1 P_1 + C_2 P_2), C_1, C_2 \in \mathbb{R} \tag{B.6}
\]

In both the sides of Equation (B.6), taking inner product with respect to \( X_i \), \( \langle \omega_i, X_i \rangle \) is represented as Equation (B.7).

\[
\langle \omega_i, X_i \rangle = C_1 \langle P_1, X_i \rangle + C_2 \langle P_2, X_i \rangle \tag{B.7}
\]

Using Equation (20), \( \omega_i \) is satisfied Equation (B.8).

\[
\langle \omega_i, X_i \rangle = -\frac{v_i \cos \alpha}{r} \tag{B.8}
\]

**Figure 4**  Minimum problem of sum of squares distance. (a) Problem (A): Minimum \( d_i + d_j \) such that \( (x, y, z) \in \mathbb{R} \). (b) Problem (B): Minimum \( d_i + d_j \) such that \( (d_i, d_j) \in D \).

**Figure 5**  The end point of \( \omega_i \) on \( \text{span}(P_1, P_2) \) is determined as common point \( l_i(v_i) \) and \( l_j(v_j) \).
Using $\langle P, X \rangle = 0$ and Equation (B.8), right side of Equation (B.7) is represented as Equation (B.9).

$$-\frac{v_1 \cos \alpha}{r} = C_1 \langle P_1, X_1 \rangle - \frac{v_2 \cos \alpha}{r} = C_1 \langle P_2, X_1 \rangle$$

(B.9)

Thus, using Equation (B.5), $C_1$ and $C_2$ are represented as Equation (B.10).

$$C_1 = \frac{v_1 \cos \alpha}{\|P_1 \times P_2\|}, \quad C_2 = -\frac{v_1 \cos \alpha}{\|P_1 \times P_2\|}$$

(B.10)

Equation (B.10) is substituted in Equation (B.6). Thus, it is given.

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