HOW NEUTRAL IS THE INTERGALACTIC MEDIUM AT $z \sim 6$?

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ABSTRACT

Recent observations of high-redshift quasar spectra reveal long gaps with little flux. A small or no detectable flux does not by itself imply that the intergalactic medium (IGM) is neutral. Inferring the average neutral fraction from the observed absorption requires assumptions about clustering of the IGM, which the gravitational instability model supplies. Our most stringent constraint on the neutral fraction at $z \sim 6$ is derived from the mean Ly$\beta$ transmission measured from the $z = 6.28$ Sloan Digital Sky Survey quasar of Becker and coworkers; the neutral hydrogen fraction at mean density has to be larger than $4.7 \times 10^{-4}$. This is substantially higher than the neutral fraction of $(3.5) \times 10^{-3}$ at $z = 4.5$–5.7, suggesting that dramatic changes take place around or just before $z \sim 6$, even though current constraints are still consistent with a fairly ionized IGM at $z \sim 6$. These constraints also translate into constraints on the ionizing background, subject to uncertainties in the IGM temperature. An interesting alternative method to constrain the neutral fraction is to consider the probability of having many consecutive pixels with little flux, which is small unless the neutral fraction is high. It turns out that this constraint is slightly weaker than the one obtained from the mean transmission. We show that while the derived neutral fraction at a given redshift is sensitive to the power-spectrum normalization, the size of the jump around $z \sim 6$ is not. We caution that the main systematic uncertainties include spatial fluctuations in the ionizing background and the continuum placement. Tests are proposed. In particular, the sight line–to–sight line dispersion in mean transmission might provide a useful diagnostic. We express the dispersion in terms of the transmission power spectrum and develop a method to calculate the dispersion for spectra that are longer than the typical simulation box.

Subject headings: cosmology: theory — intergalactic medium — large-scale structure of universe — quasars: absorption lines

1. INTRODUCTION

Recent spectroscopic observations of $z \geq 4.5$ quasars discovered by the Sloan Digital Sky Survey (SDSS) have opened up new windows into the study of the high-redshift intergalactic medium (IGM) (Fan et al. 2000, 2001; Zheng et al. 2000; Schneider et al. 2001; Anderson et al. 2001; Becker et al. 2001; Djorgovski et al. 2001). In particular, Becker et al. (2001) observed Gunn-Peterson troughs (Gunn & Peterson 1965) in the spectrum of a $z = 6.28$ quasar, which were interpreted as suggesting that the universe was close to the reionization epoch at $z \sim 6$.

That the absorption increases quickly with redshift is not by itself surprising: ionization equilibrium tells us that the neutral hydrogen density is proportional to the gas density squared, which is proportional to $(1 + z)^6$ at the cosmic mean. The evolution of the ionizing background and gas temperature modifies this redshift dependence, but the rapid evolution of absorption remains a robust outcome. What is interesting, as Becker et al. (2001) emphasized, is that the observed mean transmission at redshift $z \sim 6$ is lower than what one would expect based on an extrapolation of the column density distribution and its redshift evolution [number density of clouds scaling as $\sim (1 + z)^{2.3}$] from lower redshifts. On the other hand, the popular gravitational instability theory of structure formation provides detailed predictions for how the IGM should be clustered and how this clustering evolves with redshift, which has been shown to be quite successful when compared to observations at $z \sim 2$–4 (see, e.g., Cen et al. 1994; Zhang, Anninos, & Norman 1995; Reisenegger & Miralda-Escude 1995; Hernquist et al. 1996; Miralda-Escude et al. 1996; Muecket et al. 1996; Bi & Davidsen 1997; Bond & Wadsley 1997; Hui, Gnedin, & Zhang 1997; Croft et al. 1998; Theuns et al. 1999; Bryan et al. 1999; McDonald et al. 2000). These predictions allow us to directly infer the neutral fraction of the IGM from the observed absorption (the relation between the two depends on the nature of the clustering of the IGM) and so can further inform our interpretations of the recent $z \sim 6$ results.

How neutral is the IGM at $z \sim 6$, and how different is the neutral fraction compared to lower redshifts? These are the questions we would like to address quantitatively, making use of the gravitational instability model of the IGM.

The paper is organized as follows. First, we start with a brief description of the gravitational instability model for the IGM and the simulation technique in § 2. In § 3.1 we derive the neutral hydrogen fraction $X_{\rm H_1}$, and equivalently, the level of ionizing flux $J_{\lambda 13}$, at several different redshifts leading up to $z \sim 6$ from the observed mean Ly$\alpha$ transmission. This exercise using the Ly$\alpha$ spectrum is similar to the one carried out in McDonald & Miralda-Escudé (2001), except for the addition of new high-redshift data. 4 We then

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4 This part of the calculation involving the matching of the mean Ly$\alpha$ transmission is also similar to a number of earlier papers in which the primary quantity of interest is the baryon density (e.g., Rauch et al. 1997; Weinberg et al. 1997; Choudhury, Srianand, & Padmanabhan 2001; Hui et al. 2002). Here, we fix the baryon density and study the ionizing background or the neutral fraction instead (see § 2).
examine in § 3.2 the constraints on the same quantities $X_{\mathrm{H}_1}$ and $J_{\mathrm{H}_1}$ from the observed mean Ly$\beta$ transmission, Ly$\beta$ being particularly useful at high-Ly$\alpha$ optical depth, because the Ly$\beta$ absorption cross section is a factor of ~5 smaller than the Ly$\alpha$ cross section. The goal here is to use Ly$\beta$ absorption to obtain constraints on $X_{\mathrm{H}_1}$ and $J_{\mathrm{H}_1}$ that are as stringent as possible. In § 3.2 we also examine the sensitivity of our conclusions to the power-spectrum normalization.

An intriguing question is: Instead of focusing on the mean transmission, can one make use of the fact that the observed spectrum at $z \sim 6$ contains a continuous and long stretch (~200–300 Å) with little or no detected flux to obtain more stringent limits on the neutral fraction or $J_{\mathrm{H}_1}$? The idea is that since the IGM gas density naturally fluctuates spatially, it seems a priori unlikely to have no significant upward fluctuation in transmission for many pixels in a row, unless of course, the neutral fraction $X_{\mathrm{H}_1}$ is indeed quite high. We show in § 3.3 that this provides constraints that are slightly weaker than those obtained using the mean transmission.

In all the simulations discussed in this paper, the ionizing background is assumed to be spatially uniform, just as in the majority of high-redshift IGM simulations. A natural worry is that as the universe becomes more neutral at higher redshifts, the ionizing background may become more non-uniform. One way to test this is to use several lines of sight, available at $z \sim 5.5$, and compare the observed line-of-sight scatter in mean transmission against the predicted scatter based on simulations with a uniform background. We discuss this in § 4, estimate the level of ionizing background fluctuations, and make predictions for the scatter at $z \sim 6$. Here, we also introduce a technique to handle the problem of limited box size.

Readers who are not interested in details can skip to § 5, in which we summarize the constraints obtained. We also discuss in § 5 the issue of continuum placement, and how the associated uncertainties can be estimated. While the work described in this paper was being carried out, several papers appeared that investigate related issues (Barkana 2002; Razoumov et al. 2002; Cen & McDonald 2002; Gnedin 2001; Fan et al. 2002). Where there is overlap, our results are in broad agreement with these papers. We present a comparison with other authors at the end of § 3.2. Our approach here is most similar to that of Cen & McDonald (2002). In addition to obtaining constraints on the ionizing background from the Ly$\alpha$ and Ly$\beta$ transmission, as was considered by Cen & McDonald, we consider the possible constraint from the Gunn-Peterson trough itself, examine the dependence on power-spectrum normalization, and develop a method to predict the scatter in mean transmission by relating it to the power spectrum, which might be of wider interest. We also place a slightly stronger emphasis on the neutral fraction, which is more robustly determined compared to the ionizing background or photoionization rate.

2. THE GRAVITATIONAL INSTABILITY MODEL FOR THE IGM

The Ly$\alpha$ optical depth is related to the IGM density, assuming ionization equilibrium, via

$$\tau_{\alpha} = A_{\alpha} (1 + \delta)^{2.07(\gamma - 1)} t, \quad (1)$$

where $\delta$ is the gas overdensity [$\delta = (\rho - \bar{\rho})/\bar{\rho}$, where $\rho$ is the gas density and $\bar{\rho}$ its mean], $\gamma$ is the equation-of-state index for the IGM, and $A_{\alpha}$ is given by (see, e.g., Hui et al. 2002, and references therein)

$$A_{\alpha} = 51 \left( \frac{X_{\mathrm{H}_1}}{1.6 \times 10^{-4}} \right) \left( \frac{\Omega_{b} h^2}{0.02} \right) \left( \frac{0.65}{\bar{H} \bar{H}_0} \right) \times \left( \frac{1 + z}{7} \right)^7 \frac{11.7}{\bar{H}(z)/\bar{H}_0}, \quad (2)$$

where $X_{\mathrm{H}_1} = n_{\mathrm{H}_1}/n_{\mathrm{H}_0}$ ($\rho_{\mathrm{H}_1}$ is the total density of neutral and ionized hydrogen, and $n_{\mathrm{H}_0}$ is the neutral hydrogen density) is the neutral hydrogen fraction at mean density ($\delta = 0$). Here $H(z)$ is the Hubble parameter at redshift $z$, $\bar{H}_0$ is the Hubble parameter today, $\bar{H}_0 = 100 h \, \text{km s}^{-1} \, \text{Mpc}^{-1}$, and $\Omega_{b}$ is the baryon density in units of the critical density. The value of 11.7 for $H(z)/\bar{H}_0$ above corresponds to that appropriate for a cosmology with $\Omega_m = 0.4$ and $\Omega_{\Lambda} = 0.6$ at $z = 6$, where $\Omega_m$ and $\Omega_{\Lambda}$ are the matter and vacuum densities in units of the critical density today.

The neutral fraction $X_{\mathrm{H}_1}$ is related to the ionizing background by

$$X_{\mathrm{H}_1} = 1.6 \times 10^{-4} \left( \frac{\Omega_{b} h^2}{0.02} \right) \left( \frac{2.55 \times 10^{-2}}{J_{\mathrm{H}_1}} \right) \times \left( \frac{T_0}{2 \times 10^4 \, \text{K}} \right)^{-0.7} \left( \frac{1 + z}{7} \right)^3, \quad (3)$$

where the dimensionless quantity $J_{\mathrm{H}_1}$ is related to the photoionization rate $\Gamma_{\mathrm{H}_1}$ by

$$\Gamma_{\mathrm{H}_1} = 4.3 \times 10^{-12} J_{\mathrm{H}_1} \, \text{s}^{-1}. \quad (4)$$

The quantity $J_{\mathrm{H}_1}$ provides a convenient way of describing the normalization of the ionizing background without specifying the exact spectrum, in a way that is directly related to the physically relevant $\Gamma_{\mathrm{H}_1}$ (e.g., Miralda-Escudé et al. 1996). It is related to the specific intensity at 912 A, $I_{912}$, by $J_{\mathrm{H}_1} = I_{912}[3/(\beta + 3)]$, where $\beta$ is the slope of the specific intensity just blueward of 912 A ($j(\nu) \propto \nu^{\beta}$, where $\nu$ is frequency), and $I_{912}$ is measured in the customary units of $10^{-21}$ ergs s$^{-1}$ cm$^{-2}$ Hz$^{-1}$ sr$^{-1}$ (for a non–power law $j_{\nu}$, eq. [4] provides the exact definition for $J_{\mathrm{H}_1}$; see, e.g., Hui et al. 2002).

Two more ingredients should be mentioned to complete the specification of our model for the Ly$\alpha$ absorption (see, e.g., Hui et al. 1997 for details). First, the optical depth as a function of velocity is computed by taking the right-hand side of equation (1) in velocity space (i.e., taking into account peculiar velocities) and smoothing it with a thermal broadening window. Second, the gas density and velocity fields are predicted by some cold dark matter (CDM) cosmological model using numerical simulations.

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5 The photoionized IGM at an overdensity of a few or less is expected to follow a tight temperature-density relation of the form $T = T_0(1 + \delta)^{-1}$, where $T$ is the gas temperature and $T_0$ is its value at the cosmic mean density (see Hui & Gnedin 1997). We caution that close to reionization, these quantities may not be a function of $\delta$ alone. The IGM may be heated inhomogeneously, causing spatial fluctuations in $T_0$ and $\gamma$.

6 The neutral fraction at arbitrary $\delta$ is given by $X_{\mathrm{H}_1}(1 + \delta)^{-1}(\gamma - 1)$. Throughout this paper, whenever we quote values for $X_{\mathrm{H}_1}$, we refer to the neutral hydrogen fraction at the cosmic mean density $\delta = 0$.

This equation assumes that hydrogen is highly ionized and that helium is largely doubly ionized. If helium is only singly ionized, the relation between $J_{\mathrm{H}_1}$ and $X_{\mathrm{H}_1}$ is changed slightly: the right-hand side of eq. (3) is multiplied by 0.93.
There are obviously a number of free parameters in our model. Let us discuss each of them in turn.

Throughout this paper, we assume $\Omega_m h^2 = 0.02$, as supported by recent cosmic microwave background measurements (Netterfield et al. 2002; Pryke et al. 2002) and the nucleosynthesis constraint from primordial deuterium abundance (Burles, Nollett, & Turner 2001). We also assume throughout $h = 0.65$, $\Omega_m = 0.4$, and $\Omega_{\Lambda} = 0.6$. Variations of these parameters within the current bounds do not contribute significantly to the uncertainties of the constraints obtained in this paper (see Hui et al. 2002).

The temperature $T_0$ and equation-of-state index $\gamma$ at the redshifts of interest in this paper are somewhat uncertain. There are no direct measurements of the thermal state of the IGM at our redshifts of interest, $z \gtrsim 4$. Measurements at $z \lesssim 4$ yield values consistent with $T_0 = 2 \times 10^4$ K and $\gamma = 1$ (McDonald et al. 2001; Ricotti, Gnedin, & Shull 2000; Zaldarriaga, Hui, & tegmark 2001). Scayle et al. (2000), however, measure a slightly lower temperature. Given that the temperature right after reionization is expected to be about 25,000 K with $\gamma = 1$ (with some dependence on the hardness of the ionizing spectrum; see, e.g., Hui & Gnedin 1997), which is not too different from the measurements at $z \lesssim 4$, we assume throughout this paper, when making use of equation (3) to infer $J_{\text{HI}}$, that $T_0 = 2 \times 10^4$ K and $\gamma = 1$. Note that while the theoretically allowed range for $\gamma$ is from 1 to 1.6 (Hui & Gnedin 1997), what matters for our purpose is $2 - 0.7(\gamma - 1)$ (eq. [1]), which only ranges from 1.58 to 2 and does not significantly affect our results. It is also important to emphasize that the inference of $X_{\text{HI}}$ from observations, unlike the case for $J_{\text{HI}}$, is not directly subject to uncertainties in the temperature $T_0$. This is because observations constrain $A_\alpha$, from which we can obtain $X_{\text{HI}}$ without knowing $T_0$ (see eq. [2]).

To generate realizations of the density and velocity fields for a given cosmology, we run hydro-particle-mesh (HPM) simulations (Gnedin & Hui 1998). The HPM algorithm is essentially a particle-mesh code, modified to incorporate a force term due to gas pressure in the equation of motion. For the initial power spectrum, we use a CDM-type transfer function, as parameterized by Ma (1996), which is very similar to the commonly used Bardeen et al. (1986) transfer function. For the primordial spectral slope, we adopt $n = 0.93$ (Croft et al. 2000; McDonald et al. 2000). For the linear power-spectrum normalization, we employ the range suggested by measurements from the Ly$\alpha$ forest of Croft et al. (2000): $\Delta^2(k) \equiv 4\pi k^3 P(k)/(2\pi)^3 = 0.74^{+0.30}_{-0.16}$ at $z = 2.72$ at a velocity scale of $k = 0.03$ s km$^{-1}$. However, we caution that the error bar given is somewhat dependent on the assumed error of the mean transmission measurements, which is sensitive to the accuracy of the continuum-fitting procedure (see, e.g., Zaldarriaga et al. 2001 for a slightly different assessment of the error bar). The power spectrum in this model has a similar shape to that of favored cosmological models, but a slightly lower amplitude (Croft et al. 2000). In 3.2 we demonstrate that our main conclusion, that the neutral fraction increases dramatically near $z \sim 6$, is insensitive to our assumptions about the amplitude of the power spectrum. In practice, we examine models with different normalizations by running a simulation with outputs at several different redshifts: each redshift then corresponds to a different power-spectrum normalization, and linear interpolation is performed to reach any desired normalization. Our simulations have a box size of 8.9 Mpc $h^{-1}$, with a 256$^3$ grid. McDonald & Miralda-Escudé (2001) found this box size and resolution to be adequate for IGM studies up to $z \sim 5$. We have verified that the same is true up to $z = 6$, in the sense that the transmission probability distribution has converged for our choice of simulation size and resolution.

Finally, we should say a few words about simulations of the Ly$\beta$ region. In regions of the quasar spectrum that are between (973 Å)/(1 + $z_{\text{em}}$) and (1026 Å)/(1 + $z_{\text{em}}$), where $z_{\text{em}}$ is the redshift of the quasar, two kinds of absorption can exist: one is Ly$\beta$ due to material at redshift 0.948(1 + $z_{\text{em}}$) < 1 + $z$ < 1 + $z_{\text{em}}$, and the other is Ly$\alpha$ due to material at redshift 0.800(1 + $z_{\text{em}}$) < 1 + $z$ < 0.844(1 + $z_{\text{em}}$). In other words, in such a region, the observed optical depth would be given by $\tau = \tau_\beta + \tau_\alpha$, where $\tau_\beta$ and $\tau_\alpha$ arise at different redshifts. The Ly$\alpha$ optical depth can be computed as before. The Ly$\beta$ optical depth $\tau_\beta$ can be computed using equation (1), except that $A_\alpha$ is replaced by $A_\beta$:

$$A_\beta = \frac{1}{5.27} A_\alpha .$$

The factor of 5.27 reflects the fact that the Ly$\beta$ transition has a cross section that is 5.27 times smaller than for Ly$\alpha$.

3. CONSTRAINTS ON THE NEUTRAL HYDROGEN FRACTION AND THE IONIZING BACKGROUND

3.1. Constraints from the Ly$\alpha$ Mean Transmission

Using equations (1) and (2), we compute $X_{\text{HI}}$, which also fixes $J_{\text{HI}}$ (eq. [3]), necessary to match the observed Ly$\alpha$ mean transmission ($e^{-\tau_\alpha}$) at $z = 4.5$–6 (see Table 1 for a summary of the measurements). The results of our calculation are presented in Figure 1. This plot also contains a point at $z = 6.05$, which is the result of matching the mean transmission in the Ly$\beta$ forest, as we describe in 3.2.

Also shown in the figure is a dotted line that shows $X_{\text{HI}} \propto (1 + z)^2$, which appears to be a good fit to the data from $z = 4.5$ to 5.7. From equation (3), one can see that

11 This normalization corrects an error in an earlier draft of Croft et al. (2000) (R. Croft 2001, private communication).

12 We do not vary the primordial spectral index $n$ here. Quantities such as the mean transmission that we are interested in here are generally sensitive to power on only a small range of scales. Varying $n$ is therefore largely degenerate with varying $\Delta^2$. 

8 The above statement is subject to two small caveats. First, the optical depth given in eq. (1) has to be smoothed with a thermal broadening window whose width depends on $T_0$. We find that in practice, the exact width of the thermal broadening kernel does not very much affect quantities such as the mean transmission, which is what we are interested in. Second, $T_0$ also affects the gas dynamics via the pressure term in the equation of motion. As we discuss below, the effect of varying $T_0$ also appears to be small in this regard.

9 The temperature-density relation has to be specified as a function of redshift in the HPM code to compute the pressure term. We follow McDonald & Miralda-Escudé (2001) and linearly interpolate from $T_0 = 19,000$ K and $\gamma = 1.2$ at $z = 3.9$ to $T_0 = 25,000$ K and $\gamma = 1$ at the redshift of reionization $z_{\text{reion}}$. We find that assuming $z_{\text{reion}} = 7$ vs. $z_{\text{reion}} = 10$ results in a negligible difference in our results, in particular concerning the mean decrement and the probability distribution of transmission. All results in this paper are quoted from the $z_{\text{reion}} = 7$ HPM simulations. Note that in inferring $X_{\text{HI}}$ and $J_{\text{HI}}$ from eqs. (2) and (3), we always use $T_0 = 20,000$ K and $\gamma = 1$ for simplicity, as mentioned before.
such a trend for the neutral fraction is equivalent to assuming a constant $J_{H_i}$ (or more accurately, a constant $J_{H_i} T_0^{0.7}$; see eq. [3]).

As one can see, ignoring for now the Ly$\beta$ point, the neutral fraction does appear to have a modest jump around $z \sim 6$: it increases by a factor of $\sim 4.0$ from $z = 5.7$ to $6.05$, while it changes by at most $\sim 1.9$ from $z = 4.5$ to $5.7$. A similar trend (but opposite in sign) can be seen in the ionizing flux $J_{H_i}$. The 1 $\sigma$ error bar here takes into account the measurement error in the mean transmission and the range of power-spectrum normalization stated in $\S$ 2. As we have explained in $\S$ 2, while $X_{H_i}$ is not sensitive to the assumed temperature of the IGM ($T_0$), our constraints on $J_{H_i}$ are directly influenced by it. As emphasized before, we assume $T_0 = 2 \times 10^4$ K. In other words, our constraints on $J_{H_i}$ are really constraints on the quantity $J_{H_i}(T_0/2 \times 10^4 K)^{0.7}$ (see eq. [3]). It would therefore be straightforward to rescale our constraints on $J_{H_i}$ if the temperature were a little bit different.$^{12}$ It is an interesting question to ask whether the apparent jump in the ionizing flux can instead be attributed to a jump in the temperature. In general, the temperature is expected to evolve slowly with redshift after reionization (Hui & Gnedin 1997).

Regarding the measurement error, we should also emphasize that the 2 $\sigma$ error of Becker et al. (2001) actually includes the possibility of having zero transmission at $z = 6.05$. This means that at 2 $\sigma$, we would only have a lower limit on $X_{H_i}$, or an upper limit on $J_{H_i}$, for the highest redshift point in Figure 1, allowing the possibility that the IGM is neutral at $z \sim 6$, $X_{H_i} = 1$.

### 3.2. Constraints from the Ly$\beta$ Mean Transmission

In this section we consider the constraints placed by the measurement of Becker et al. (2001) of the mean transmission in the Ly$\beta$ region. Absorption in the Ly$\beta$ region has two components: $\tau = \tau_0 + \tau_3$, where the Ly$\alpha$ optical depth $\tau_0$ and the Ly$\beta$ optical depth $\tau_3$ originate at different redshifts. Ly$\beta$ absorption due to material at $z = 6$ coincides in wavelength with Ly$\alpha$ absorption due to material at $z = (1 + 6)(1026/1216) - 1 = 4.9$. Because the points of origin are so widely separated, they can be effectively treated as statistically independent, i.e., $\langle e^{-\tau} \rangle = \langle e^{-\tau_0} \rangle \langle e^{-\tau_3} \rangle$. Becker et al. (2001) measured $\langle e^{-\tau_3} \rangle$ at $z \sim 6$ by dividing the net mean transmission $\langle e^{-\tau} \rangle$ in the Ly$\beta$ region by the mean transmission in Ly$\alpha$ at $z \sim 5$. They obtained $\langle e^{-\tau_3} \rangle = -0.002 \pm 0.020$. Clearly, this measurement is consistent with a completely neutral IGM. However, the interesting question is: What kind of lower limit does it set on the neutral fraction, and does it improve on the lower limit from the mean Ly$\alpha$ absorption?

We carry out a calculation that is analogous to what is described in $\S$ 3.1, except for the key difference that in computing $\tau_3$, we use $A_3$, which is a factor of $5.27$ smaller than $A_0$ (see eqs. [1] and [5]). The results of our calculation are shown in Figure 1 as the highest redshift points in the plot, which have error-bar arrows pointing toward a completely neutral IGM and a vanishing ionizing background. It can be seen that the $(1 - \sigma)$ lower limit on $X_{H_i}$ is $X_{H_i} > 4.7 \times 10^{-4}$. This is a factor of $\sim 3$ larger than the neutral fraction required to match the upper limit on the mean transmission in Ly$\alpha$ for our fiducial density, and a slightly stronger constraint than that obtained in $\S$ 3.1, including the uncertainty in power-spectrum normalization. Similarly, the upper limit on $J_{H_i}$ is $J_{H_i} < 9.0 \times 10^{-3}$. The moral here is that because the Ly$\beta$ absorption cross section is a factor of $5.27$ smaller than the Ly$\alpha$ cross section, Ly$\beta$ offers a more sensitive probe of the neutral fraction, especially when the Ly$\alpha$ optical depth is high.

The neutral fraction at $z \sim 6$ is thus a factor of $\sim 10$ higher than that at redshift $z \sim 5.7$, where it is $X_{H_i} = 4.9 \times 10^{-5}$. This dramatic change in the neutral fraction is suggestive, probably indicating that the reionization epoch is nearby.

Furthermore, this conclusion is not sensitive to our assumptions about the amplitude of the power spectrum.
Although the neutral fraction at redshift \( z = 6.05 \) is itself sensitive to the amplitude of the power spectrum, we find that the factor by which the neutral fraction increases from \( z = 5.7 \) to 6.05 depends only weakly on the amplitude. In Figure 2 we plot both the neutral fraction at \( z = 6.05 \) and the jump in the neutral fraction for a range of different power-spectrum normalizations. The jump is defined as the ratio \( X_{\text{HI}}(z = 6.05)/X_{\text{HI}}(z = 5.7) \). Here \( X_{\text{HI}}(z = 6.05) \) is the lower limit resulting from the 1 \( \sigma \) error in the mean transmission in Ly\( \alpha \) at \( z = 6.05 \), and the error bars in the jump arise from the 1 \( \sigma \) error in the mean transmission in Ly\( \alpha \) at \( z = 5.7 \). As one can see in the plot, the lower limit on the neutral fraction at \( z = 6.05 \) varies from \( X_{\text{HI}} > 3 \times 10^{-4} \) to \( X_{\text{HI}} > 9 \times 10^{-4} \) as \( \Delta^2(k = 0.03 \text{ km}^{-1}, z = 2.72) \) varies from 0.5 to 1.3. The neutral fraction itself varies significantly with power-spectrum normalization, scaling approximately as \( X_{\text{HI}} \propto |\Delta^2(k = 0.03 \text{ km}^{-1}, z = 2.72)|^{1.1} \) for this range of normalizations. The jump, however, changes only slightly over a large range of normalizations. As \( \Delta^2(k = 0.03 \text{ km}^{-1}, z = 2.72) \) varies from 0.5 to 1.3, the jump only changes from \( \sim 9.7 \) to \( \sim 11.1 \). Our conclusion that the neutral fraction of the IGM increases dramatically near \( z \sim 6 \) seems robust.

One can also consider the absorption in the Ly\( \gamma \) region, or even the higher Lyman series. In practice, the accumulated amount of absorption from Ly\( \alpha \) as well as Ly\( \beta \) at different redshifts makes it more difficult to measure the Ly\( \gamma \) transmission itself with good accuracy.

Our constraints on the neutral fraction and the intensity of the ionizing background are consistent with those found by other authors, given our different choices of power-spectrum normalization. Fan et al. (2002) found, from the mean Ly\( \beta \) transmission, that \( \Gamma_{-12} < 0.025 \), where \( \Gamma_{-12} \) is the photoionization rate of equation (4) in units of \( 10^{-12} \text{ s}^{-1} \). Although this constraint is somewhat stronger than the constraint implied by our fiducial model, \( \Gamma_{-12} < 0.039 \), we expect the difference because of our different power-spectrum normalizations. The constraint of Fan et al. (2002) comes from semianalytic arguments, shown consistent with a \( \Lambda \)CDM simulation with \( \Omega_m = 0.3 \), \( \Omega_{\Lambda} = 0.7 \), \( h = 0.65 \), \( \Omega_b h^2 = 0.02 \), and \( \sigma_8 = 0.9 \). This model has a substantially larger normalization, \( \Delta^2(k = 0.03 \text{ km}^{-1}, z = 2.72) = 1.25 \), than our fiducial model of \( \Delta^2(k = 0.03 \text{ km}^{-1}, z = 2.72) = 0.74 \). The difference in normalization reflects some tension between the normalization derived from the observed cluster abundance, which Fan uses, and that from the Ly\( \alpha \) forest that our model is based on (Croft et al. 2000). Fan et al. (2002) assume \( T_0 = 2.0 \times 10^4 \text{ K} \) in placing their constraint. Their limit, \( \Gamma_{-12} < 0.025 \), includes only uncertainties in the mean transmission and not additional uncertainties from the power-spectrum normalization. From Figure 2 we infer that the normalization of Fan et al. (2002) implies \( X_{\text{HI}} > 8.8 \times 10^{-4} \) in our cosmology. Rescaling this result from our assumed \( \Omega_m = 0.4 \) to an \( \Omega_m = 0.3 \) cosmology and using equations (3) and (4), we predict \( \Gamma_{-12} < 0.024 \), or \( X_{\text{HI}} > 7.6 \times 10^{-4} \), for the model of Fan et al. (2002). The constraint of Fan et al. (2002) is thus consistent with our constraint, given our different choice of normalization. Cen & McDonald (2002), using a model similar to that of Fan et al. (2002), obtained the constraint \( \Gamma_{-12} < 0.032 \) using the Ly\( \beta \) mean transmission. This constraint is slightly weaker than that of Fan et al. (2002) and our extrapolation to their normalization, because Cen & McDonald (2002) consider a larger upper limit to the observed mean transmission, including an estimate of the uncertainty due to sky subtraction. At slightly lower redshifts, we can also compare with the results of McDonald & Miralda-Escudé (2001) derived from matching the mean Ly\( \alpha \) transmission. For example, at \( z = 5.2 \), these authors found \( \Gamma_{-12} = 0.16 \) to match the observed mean transmission of \( (e^{-\tau}) = 0.09 \). McDonald & Miralda-Escudé (2001) consider a model whose normalization we infer to be \( \Delta^2(k = 0.03 \text{ km}^{-1}, z = 2.72) = 0.98 \). To match the same mean transmission with this normalization, we infer a somewhat higher photoionization rate, \( \Gamma_{-12} = 0.19 \). Part of the difference may be that the \( \Gamma_{-12} \) necessary to match a given mean transmission varies by \( \sim 5\% \) between two different realizations of the density field. The remaining difference may come from the procedure of linearly interpolating between outputs or from some modeling difference. At any rate, our results are roughly consistent with those of other authors given our different power-spectrum normalizations.

![Figure 2](image.png)

**Figure 2.** Top: Size of the jump in the neutral fraction, \( X_{\text{HI}}(z = 6.05)/X_{\text{HI}}(z = 5.7) \), as a function of power-spectrum amplitude. The amplitude is described by the value of \( \Delta^2(k) = 4\pi k^3 P(k)/(2\pi^2) \) at \( z = 2.72 \) and velocity scale \( k = 0.03 \text{ km}^{-1} \). The quantity \( X_{\text{HI}}(z = 6.05) \) corresponds to the lower limit arising from the 1 \( \sigma \) error in the mean transmission. The error bars come from the 1 \( \sigma \) uncertainty in the mean transmission at \( z = 5.7 \). Bottom: Neutral fraction itself at \( z = 6.05 \). The dotted line shows \( X_{\text{HI}} = 5.8 \times 10^{-4} (\Delta^2(k)/0.06)^{1.1} \), demonstrating how the neutral fraction scales with amplitude.

### 3.3. Constraints from the Gunn-Peterson Trough Itself: The Fluctuation Method

The fact that Becker et al. (2001) observed a Gunn-Peterson trough, where a long stretch of the spectrum contains little or no flux, can conceivably be used to further tighten the constraints obtained from the previous sections. Since the IGM is expected to have spatial fluctuations, the probability of having many pixels in a row turning up a very small transmission must be low, unless the neutral fraction is intrinsically quite high. The same reasoning can be applied to either the Ly\( \alpha \) or Ly\( \beta \) absorption. We discuss our method for Ly\( \alpha \) in detail below. The method for Ly\( \beta \) is a straightforward...
ward extension. For simplicity, we call this method the “fluctuation method.”

Becker et al. (2001) find from the spectrum of SDSS 1030+0524, the \( z = 6.28 \) quasar, that the \( \text{Ly} \alpha \) transmission is consistently below about 0.06 for a region that spans 260 Å, between 8450 and 8710 Å. The noise level per 4 Å pixel is \((n^2)^{1/2} \approx 0.02\), where \( n \) represents the photon noise fluctuation. The observed transmission \( F \) at a given pixel is \( F = e^{-\tau} + n \), where \( e^{-\tau} \) is the true transmission. The noise here should be dominated by Poisson fluctuations of the subtracted sky background (as well as perhaps readout error). Let \( P(F_1, F_2, \ldots, F_N) \) be the probability that \( N \) consecutive pixels have the observed transmission fall into the range \( F_1 \pm dF_1/2 \) to \( F_N \pm dF_N/2 \). In our case, \( N = 65 \) for a pixel size of 4 Å. The problem is then to find the probability \( \int_{-0.06}^{0.06} \cdots \int_{-0.06}^{0.06} P(F_1, \ldots, F_N) dF_1 \cdots dF_N \) as a function of \( J_{\text{HI}} \) and ask what maximal \( J_{\text{HI}} \) (or equivalently, minimal \( X_{\text{HI}} \)) would give an acceptable probability. By choosing the “acceptable probability” to be within 68% of the maximum likelihood (maximum likelihood is achieved when the neutral fraction is unity), we obtain the 1 \( \sigma \) upper limit on \( J_{\text{HI}} \), or 1 \( \sigma \) lower limit on \( X_{\text{HI}} \).

Our simulation has a comoving box size of 8.9 Mpc \( h^{-1} \), corresponding to 42 Å for \( \text{Ly} \alpha \) at \( z = 6 \), which falls short of the wavelength range we need for this problem, which is 260 Å. In other words, the probability \( P(F_1, F_2, \ldots, F_N) \) can be estimated directly from our simulation only for \( N \leq 10 \). However, the mass correlation length scale at this redshift (\( \lesssim 1 \) Mpc \( h^{-1} \)) is actually a fraction of the box size, which means that one can treat fluctuations on scales beyond the box size as roughly uncorrelated. Assuming so, we estimate \( \int_{-0.06}^{0.06} \cdots \int_{-0.06}^{0.06} P(F_1, F_2, \ldots, F_{10}) dF_1 \cdots dF_{10} \) from the simulation and then take its sixth power, which would give us the probability that 60 consecutive pixels have a transmission below 0.06. This is slightly smaller than the number 65 that we need, but at least provides us conservative constraints on \( X_{\text{HI}} \) and \( J_{\text{HI}} \). We have also tested our approach by using fractions of the box size as a unit, and have found that our results do not change significantly (less than 10%).

Figure 3a (dotted curve) shows our estimate of the probability \( \int_{-0.06}^{0.06} \cdots \int_{-0.06}^{0.06} P(F_1, F_2, \ldots, F_N) dF_1 \cdots dF_N \) for \( N = 60 \) and pixel size 4 Å, as a function of \( J_{\text{HI}} \). Our simulated spectra have been convolved with the observation resolution (FWHM of 1.8 Å), rebinned into pixels of 4 Å each, and combined with Gaussian noise with a dispersion of 0.02. From the dotted curve in Figure 3a, applying a likelihood analysis, we obtain a 1 \( \sigma \) upper limit on \( J_{\text{HI}} \) of \( J_{\text{HI}} < 0.014 \), and a corresponding lower limit on \( X_{\text{HI}} \) of \( X_{\text{HI}} > 2.95 \times 10^{-4} \). This is for a model with a power-spectrum normalization of \( \Lambda^2(k = 0.03 \text{ s km}^{-1}, z = 2.72) = 0.74 \) (see §2). The mean \( \text{Ly} \alpha \) transmission constraints for the same model are \( J_{\text{HI}} < 0.028 \) and \( X_{\text{HI}} > 1.5 \times 10^{-4} \).

This means that considering \( \text{Ly} \alpha \) alone, the fluctuation method yields somewhat stronger constraints compared to simply using the mean transmission.

Figure 3b (dotted curve) shows the same methodology applied to the \( \text{Ly} \beta \) Gunn-Peterson trough. A new ingredient here is that one needs an additional simulation of the same model at redshift \( z = 4.9 \) to produce the \( \text{Ly} \alpha \) absorption that can be overlaid on top of the \( \text{Ly} \beta \) absorption from \( z = 6.05 \). This additional simulation should have different initial phases to mimic the fact that fluctuations at \( z = 4.9 \) and those at \( z = 6.05 \) should be uncorrelated. We obtain 1 \( \sigma \) limits of \( J_{\text{HI}} < 0.012 \) and \( X_{\text{HI}} > 3.4 \times 10^{-4} \). This is about 40% weaker than the constraints we obtain from the \( \text{Ly} \beta \) mean transmission. In other words, from \( \text{Ly} \beta \) absorption, the fluctuation method yields slightly weaker constraints compared to using the mean transmission. It is also only slightly stronger than the constraint obtained from the fluctuation method applied to \( \text{Ly} \alpha \).

It is an interesting question to ask how many sight lines one would need to improve the constraints by, for instance, a factor of 2. Our approach can be easily extended to multiple (uncorrelated) sight lines, and we find that about five sight lines (each containing a Gunn-Peterson trough of the same length and same signal-to-noise ratio) are necessary for such an improvement.

Part of the difficulty with obtaining stronger constraints, in addition to the small number of sight lines, is the dominance of noise. The bottom panel of Figure 4 shows the 1 pixel (4 Å) probability distribution function (PDF) of the true transmission \( e^{-\tau} \) (i.e., no noise added) for three different values of \( J_{\text{HI}} \) (the power-spectrum normalization is the same as that in Fig. 3). The top panel shows the corresponding PDFs of the observed transmission \( F \) [i.e., after convolving \( P(e^{-\tau}) \) with a Gaussian of dispersion 0.02]. As expected, noisy data make the PDFs more similar. Nonetheless, as we

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13 We estimate the noise per pixel from the error bar of Becker et al. (2001) in the mean transmission, which is \( \approx 0.003 \). This is estimated from a chunk of the spectrum that is 260 Å long, and so the dispersion per 4 Å pixel should be approximately \( \left( n^2 \right)^{1/2} \approx 0.03(65)^{1/2} \approx 0.2 \). Note that the actual dispersion varies across the spectrum, but this should suffice as a rough estimate. This estimate also agrees with an estimate of the error by comparing Figs. 1 and 3 of Becker et al.

14 Do not confuse these constraints, which are for the particular power-spectrum normalization mentioned above, to the constraints discussed in earlier sections, which include the uncertainty in the power-spectrum normalization. We focus on a single model in this section for simplicity.
pointed out above, with a sufficient number of sight lines, there might be a nonnegligible chance of seeing pixels with high transmission at the tail of the PDFs, hence allowing us to distinguish between the different levels of the ionizing background. Alternatively, one can try improving the signal-to-noise ratio per pixel. In Figure 3b we show with a dashed curve the corresponding probability if the noise per pixel is lowered by a factor of 4. The constraints improve by a little more than a factor of 2. We should emphasize, however, that systematic errors are likely important here; we discuss them in the next two sections.

4. THE VARIANCE OF THE MEAN TRANSMISSION

If, as is suggested by our discussion in § 3.2 (see Fig. 1), the IGM is close to the epoch of reionization at $z \sim 6$, one might expect large fluctuations in the ionizing background near that time. For instance, one line of sight might probe a region of the IGM where the ionized bubbles around galaxies or quasars have percolated, while another might probe the prepercolation IGM. As mentioned before, the simulations we employ do not take into account fluctuations in the ionizing background. (For simulations incorporating radiative transfer, see, e.g., Gnedin & Abel 2001; Razoumov et al. 2002.) One useful check would then be to predict the sight line–to–sight line scatter in mean transmission from our simulations and compare that against the observed scatter. At $z \sim 5.5$, four lines of sight are available for a measurement of the scatter. We examine this, as well as make predictions for the scatter at $z \sim 6$, which more high-redshift quasars in the future will allow us to measure.

Our estimate relies on simulation measurements of the transmission power spectrum. This is in contrast to an estimate of the same quantity made by Zuo (1993), who makes a prediction based on extrapolations of the column-density distribution and of the number of absorbing clouds per unit redshift (Zuo & Phinney 1993). Zuo also assumes that the clouds are Poisson distributed, while our measurement incorporates the clustering in the IGM via our numerical simulation.

An immediate problem presents itself: sight lines from which the mean transmission is measured are typically longer than the usual simulation box. We tackle this problem by expressing the variance of mean transmission in terms of the transmission power spectrum and making use of a reasonable assumption about the behavior of the power spectrum on large scales.

The mean transmission from one sight line is estimated using

$$F = \frac{1}{N} \sum_{i=1}^{N} F_i,$$

where $N$ is the number of pixels, $F_i$ is the observed transmission at pixel $i$, and $F_i = f_i + n_i$, where $f = e^{-r}$ is the true transmission and $n$ is the noise fluctuation. We use the symbol $\tilde{F}$ to represent the estimator and $\hat{f}$ to denote the true mean transmission. The variance of the estimated mean transmission is then

$$\sigma^2_{\tilde{F}} = \langle \tilde{F}^2 \rangle - \langle \tilde{F} \rangle^2 = \frac{1}{N^2} \sum_{i,j} [F_i F_j - \langle F_i \rangle \langle F_j \rangle]$$

$$= \frac{1}{N^2} \sum_{i,j} \xi_{ij} + \frac{1}{N} \sigma^2_n$$

$$= 2 \int_{k_0}^{\infty} \frac{dk}{2\pi} \left[ \frac{\sin(kL/2)}{kL/2} \right]^2 P_f(k) + \frac{\sigma_n^2}{N},$$

where $\sigma^2_n \equiv \langle n^2 \rangle$ is assumed to be roughly independent of position. $\xi_{ij}$ is the unnormalized two-point correlation of the transmission, i.e., $\xi_{ij} \equiv \langle f_i f_j \rangle - \langle f_i \rangle^2$, and $P_f(k)$ is its one-dimensional Fourier transform. The symbol $L$ denotes the comoving length of the spectrum from which the mean transmission is measured, and $k$ is the comoving wavenumber.

To evaluate $\sigma_n$, we need to know the transmission power spectrum on scales generally larger than the size of the typical simulation box. It is expected that the transmission power spectrum takes the shape (not the normalization) of the linear mass power spectrum on large scales (i.e., essentially linear biasing; see Scherrer & Weinberg 1998; Croft et al. 1998; Hui 1999). We therefore use this to extrapolate the simulation $P_f(k)$ to large scales (small $k$). We find that $P_f(k)$ is well approximated by $P_f(k) = B \exp(-ak^2) \int_{\Delta^2} dk / (2\pi^2) k P_{\text{mass}}(k)$, where $P_{\text{mass}}$ is the three-dimensional linear mass power spectrum.

Becker et al. (2001) gave an estimate of $\sigma_f \sim 0.03 \pm 0.01$, $f = 0.1$, at $z = 5.5$ using four different sight lines, each spanning $\Delta z = 0.2$, which corresponds to $L \sim 57$ Mpc $h^{-1}$. An estimate of the noise term is provided by the error in the mean transmission, $(\sigma^2_{\tilde{F}}/N)^{0.5} \sim 0.003$. For the $\Delta^2(k = 0.03 \; \text{s km}^{-1}, \; z = 2.72) = 0.74$ case, the fitting parameters are $B = 0.033$ and $a = 0.013$ Mpc$^2 h^{-2}$. Using equation (5), we then find $\sigma_f = 0.030$ for

The error for $\sigma_f$ is estimated assuming Gaussian statistics and that the four lines of sight are independent. Then, $\text{var}(\sigma_f) = \sigma_f^2 / 2n$ (see, e.g., Kendall & Stuart 1958).

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**Fig. 4.** Bottom: Plot of the 1 pixel (4 Å) PDF of the true noiseless transmission $e^{-r}$ (i.e., $P(e^{-r})$) for three different values of $J_{21}$: 0.004 (solid line), 0.012 (dotted line), and 0.028 (dashed line). Top: PDF of the noisy observed transmission $F$ for the same three values of $J_{21}$. The negative values for $F$ occur because of sky subtraction.
squares

transmission. We find that the dependence on normalization is larger at \( z \approx 5.7 \), though it changes by no more than a factor of about 2 from \( z = 4.5 \) to 5.7, suggests that \( z \approx 6 \) might be very close to the epoch of reionization. We emphasize that current constraints are still consistent with a highly ionized IGM at \( z \approx 6 \); it is the steep rise in \( X_{\text{HI}} \) that is suggestive of dramatic changes around or just before that redshift. We should also mention that the constraints on \( X_{\text{HI}} \) are less subject to uncertainties in the IGM temperature compared to those on \( J_{\text{HI}} \) (see §2).

2. The existence of a long Gunn-Peterson (Ly\( \alpha \) or Ly\( \beta \)) trough at \( z \approx 6 \), where little or no flux is detected, can also be used to obtain constraints on \( X_{\text{HI}} \) or \( J_{\text{HI}} \). This we call the fluctuation method: the fact that a long stretch of the spectrum exhibits no large upward fluctuations in transmission provides interesting information on the neutral fraction or ionizing background. The constraints obtained in this way turn out to be fairly similar to those obtained using the mean transmission. We estimate that a reduction in noise by a factor of 4, or an increase in the number of sight lines to five, would result in constraints that are 2 times stronger (§3.3).

3. We develop a method to predict the dispersion in mean transmission measured from sight lines that are longer than the typical simulation box (eq. [7] and Fig. 5). Our predicted dispersion is consistent with that observed at \( z = 5.5 \) (Becker et al. 2001). We also predict the scatter at redshift \( z = 6 \), which can be measured when more sight lines become available. Assuming a spatially homogeneous ionizing background, we predict a small scatter at \( z = 6 \), \( \sigma_t \approx \Delta X_{\text{HI}} \approx 10^{-3} \), neglecting photon noise. The dispersion provides a useful diagnostic of fluctuations in the ionizing background; close to the epoch of reionization, one expects large fluctuations from one line of sight to another depending on whether it goes through regions of the IGM where percolation of \( \text{H} \text{II} \) regions has occurred.

There are at least three issues that will be worth exploring. First, with more quasars at \( z \approx 6 \) or higher discovered in the future, applying some of the ideas mentioned above would be extremely interesting, such as the measurement of the line-of-sight scatter in mean transmission or the use of the Gunn-Peterson trough to obtain stronger constraints on the neutral fraction. Second, as we have commented on before, fluctuations in the ionizing background are expected to be important as we near the epoch of reionization. We have not discussed it here, but a calculation of the size of these fluctuations would be very interesting. Such a calculation will depend on both the mean free path of the ionizing photons as well as the spatial distribution of ionizing sources. The latter is probably quite uncertain, but useful estimates

expect large sight line–by–sight line variations. An observed scatter well in excess of what is predicted would be an interesting signature.

5. DISCUSSION

Our findings are summarized as follows:

1. The most stringent (1 \( \sigma \)) lower limit on the neutral hydrogen fraction \( X_{\text{HI}} \) (eq. [3]) or upper limit on the ionizing background \( J_{\text{HI}} \) (eq. [4]) at \( z \approx 6 \) is obtained from the observed mean Ly\( \beta \) transmission: \( X_{\text{HI}} > 8.7 \times 10^{-4} \). A comparison of this limit versus constraints at lower redshifts is presented in Figure 1. The fact that the neutral fraction increases by a factor of \( \approx 10 \) from a redshift of 5.7 to 6, even though it changes by no more than a factor of about 2 from \( z = 4.5 \) to 5.7, suggests that \( z \approx 6 \) might be very close to the epoch of reionization. We emphasize that current constraints are still consistent with a highly ionized IGM at \( z \approx 6 \); it is the steep rise in \( X_{\text{HI}} \) that is suggestive of dramatic changes around or just before that redshift. We should also mention that the constraints on \( X_{\text{HI}} \) are less subject to uncertainties in the IGM temperature compared to those on \( J_{\text{HI}} \) (see §2).

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might be made (e.g., Razoumov et al. 2002). Finally, a main source of systematic error that we have not discussed is the continuum placement. The mean transmissions at various redshifts given by Becker et al. (2001) are all obtained by extrapolating the continuum from the red side of Lyα by assuming a power law of \( t^{-0.5} \). The continuum likely fluctuates from one quasar to another, and therefore, it would be very useful to apply exactly the same procedure to quasars at lower redshifts where the continuum on the blue side can be more reliably reconstructed. This will tell us how much scatter (and possibly systematic bias) the continuum placement procedure introduces to the measured mean transmission. This kind of error is especially important to quantify, given the limited number of quasars available for high-redshift measurements at the moment.

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REFERENCES

Anderson, S. F., et al. 2001, AJ, 122, 503
Bardeen, J. M., Bond, J. R., Kaiser, N., & Szalay, A. S. 1986, ApJ, 304, 15
Barkana, R. 2002, NewA, 7, 85
Becker, R. H., et al. 2001, AJ, 122, 2850
Bi, H., & Davidsen, A. F. 1997, ApJ, 479, 523
Bond, J. R., & Wadsley, J. W. 1997, in ASP Conf. Ser. 123, Computational Astrophysics, Proc. 12th Kingston Conf., ed. D. A. Clarke & M. J. West (San Francisco: ASP), 323
Bryan, G., Machacek, M., Anninos, P., & Norman, M. L. 1999, ApJ, 517, 13
Burles, S., Nollett, K. M., & Turner, M. S. 2001, ApJ, 552, L1
Cen, R., & McDonald, P. 2002, ApJ, 570, 457
Cen, R., Miralda-Escudé, J., Ostriker, J. P., & Rauch, M. 1994, ApJ, 437, L9
Choudhury, T. R., Srianand, R., & Padmanabhan, T. 2001, ApJ, 559, 29
Croft, R. A. C., Weinberg, D. H., Bolte, M., Burles, S., Hernquist, L., Katz, N., Kirkman, D., & Tytler, D. 2000, ApJ, submitted (astro-ph/0012524)
Croft, R. A. C., Weinberg, D. H., Katz, N., & Hernquist, L. 1998, ApJ, 495, 44
Djorgovski, S. G., Castro, S. M., Stern, D., & Mahabal, A. 2001, ApJ, 560, L5
Fan, X., et al. 2000, AJ, 120, 1167
———. 2001, AJ, 122, 2833
———. 2002, AJ, 123, 1247
Gnedin, N. 2001, MNRAS, submitted (astro-ph/0110290)
Gnedin, N., & Abel, T. 2001, NewA, 6, 437
Gnedin, N., & Hui, L. 1998, MNRAS, 296, 44
Gunn, J. E., & Peterson, B. A. 1965, ApJ, 142, 1633
Hernquist, L., Katz, N., Weinberg, D. H., & Miralda-Escudé, J. 1996, ApJ, 457, L51
Hui, L. 1999, ApJ, 516, 519
———. 1999, ApJ, 516, 519
Hui, L., & Gnedin, N. Y. 1997, MNRAS, 292, 27
Hui, L., Gnedin, N. Y., & Zhang, Y. 1997, ApJ, 486, 599
Hui, L., Haiman, Z., Zaldarriaga, M., & Alexander, T. 2002, ApJ, 564, 525
Kendall, M. G., & Stuart, A. 1958, The Advanced Theory of Statistics, Vol. 1 (New York: Hafner)
Ma, C.-P. 1996, ApJ, 471, 13
McDonald, P., & Miralda-Escudé, J. 2001, ApJ, 549, L11
McDonald, P., Miralda-Escudé, J., Rauch, M., Sargent, W. L. W., Barlow, T. A., & Cen, R. 2001, ApJ, 562, 52
McDonald, P., Miralda-Escudé, J., Rauch, M., Sargent, W. L. W., Barlow, T. A., Cen, R., & Ostriker, J. P. 2000, ApJ, 543, 1
Miralda-Escudé, J., Cen, R., Ostriker, J. P., & Rauch, M. 1996, ApJ, 471, 582
Muecket, J. P., Petitjean, P., Kates, R. E., & Riediger, R. 1996, A&A, 308, 17
Netterfield, C. B., et al. 2002, ApJ, 571, 604
Pryke, C., et al. 2002, ApJ, 568, 46
Rauch, M., Miralda-Escudé, J., Sargent, W. L. W., Barlow, T. A., Weinberg, D. H., Hernquist, L., Katz, N., Cen, R., & Ostriker, J. P. 1997, ApJ, 489, 7
Razoumov, A. O., Norman, M. L., Abel, T., & Scott, D. 2002, ApJ, 572, 695
Reisenegger, A., & Miralda-Escudé, J. 1995, ApJ, 449, 476
Ricotti, M., Gnedin, N. Y., & Shull, J. M. 2000, ApJ, 534, 41
Schaye, J., Theuns, T., Rauch, M., Efstathiou, G., & Sargent, W. L. W. 2000, MNRAS, 318, 817
Scherrer, R. J., & Weinberg, D. H. 1998, ApJ, 504, 607
Schneider, D. P., et al. 2001, AJ, 121, 1232
Songaila, A., Hu, E. M., Cowie, L. L., & McMahon, R. G. 1999, ApJ, 525, L5
Theuns, T., Leonard, A., Schaye, J., & Efstathiou, G. 1999, MNRAS, 303, L58
Weinberg, D. H., Miralda-Escudé, J., Hernquist, L., & Katz, N. 1997, ApJ, 490, 564
Zaldarriaga, M., Hui, L., & Tegmark, M. 2001, ApJ, 557, 519
Zhang, Y., Anninos, P., & Norman, M. L. 1995, ApJ, 453, L57
Zheng, W., et al. 2000, AJ, 120, 1607
Zuo, L. 1993, A&A, 278, 343
Zuo, L., & Phinney, E. S. 1993, ApJ, 418, 28