Heterotic String Theory on Anti-de-Sitter Spaces

Karim Benakli

Center for Theoretical Physics, Texas A&M University, College Station, Texas 77843

Abstract

We suggest that compactifications on Anti-de-Sitter (AdS) spaces of type IIA, IIB, heterotic strings and eleven dimensional vacuua of M-theory are related by a combination of T and strong/weak dualities. Maldacena conjecture relates then all these vacuua to a conformal theory on the boundaries. Furthermore acting with discrete groups on part of the internal spaces of these theories leads to further dual theories with less or no supersymmetry.
The last four years high energy physics has known a lot of excitement as for the discovery of a set of dualities in certain class of theories where quantum behavior is under control (supersymmetric and finite theories). These dualities state that models which seemed to be arising from totally different theories might describe the same physical system. Sometimes these dualities are symmetries, but often they are not but correspond to expansions around different values of the parameters of the theory. By studying these different limits, one hopes to reconstruct (and later understand) the full (non-perturbative) picture of the behavior of the system. In this short note we would like to consider dualities among different vacua of string and M-theory on Anti-de-Sitter (AdS) spaces. A detailed study of many related technical issues will be presented elsewhere.

Supersymmetric compactifications on AdS spaces often involve compact internal spheres. The cases of interest to us are those where either Type IIA or Type IIB superstrings internal space contains an $S^3$ factor. A well known mathematical fact is that three spheres can be regarded as Hopf fibration of $U(1)$ on $CP^1 \equiv S^2$

\[ ds_{S^3}^2 = ds_{S^2}^2 + R^2(dz + A)^2 \]  

where $A$ is a one-form on $CP^1$ and $R$ is the radius of the sphere. In contrast with supergravity theories that we believe have a cut-off at the Planck scale, M-theory (and its stringy vacua) make sense even for very small radii. Following, one can perform a $T$-duality on the $U(1)$ Hopf fiber. Because of the non-trivial 3-forms appearing in the Type IIA and IIB compactifications it was shown in that $T$-duality maps $S^3$ vacua of Type IIA (IIB) to Type IIB (IIA) on $S^2 \times S^1$:

\[ ds_{S^3}^2 = ds_{S^2}^2 + \frac{1}{R^2}dz^2 . \]  

The fiber is untwisted by the Hopf $T$-duality transformation. Combining this with strong/weak coupling duality will allow us to connect AdS vacua of type IIA, IIB, eleven dimensional M-theory and heterotic strings.

Our first example is Type IIB string theory on $AdS_3 \times S^3 \times K3$. Performing $T$-duality on the $S^3$ factor this theory is mapped to IIA string theory on $AdS_3 \times S^2 \times S^1 \times K3$ where now the radius of the $S^1$ factor is inversely proportional to the one of the $AdS$ space. Strong/weak coupling duality relates Type IIA on $K3$ to heterotic strings on $T^4$ or M-theory on $S^3 \times K3$. We assume that the strong/weak coupling duality between IIA on $K3$ and heterotic on $T^4$ strings remains valid when both spaces are compactified to $S^2 \times S^1$ then we can relate the
following set of vacua:

(1b) IIB string theory on $AdS_3 \times S^3 \times K^3$,
(1a) IIA string theory on $AdS_3 \times S^2 \times S^1 \times K^3$,
(1m) M-theory on $AdS_3 \times S^2 \times T^2 \times K^3$,
(1h) Heterotic on $AdS_3 \times S^2 \times T^5$.

Notice that although the $AdS$ radius is function of the string coupling constant, it is possible to choose the other parameters large so that the volume of the $AdS$ space remains large compare to the string scale in all these theories. This is necessary as it is meaningless here to speak of $AdS$ geometry if the scales and curvature are smaller than the Planck length.

The supergravity limit of model (1b) has attracted a lot of interest recently. Starting from a configuration of parallel $D5$-branes wrapped around $K^3$ and $D$-strings one can construct a self-dual string solution in six dimensions. Looking at the brane world-volume theory and taking the limit where gravity decouples from the brane dynamics leads to the large $N$ limit of a conformal Yang-Mills theory in $1+1$ dimensions. For the space-time geometry (as a solution of the Type IIB equations of motion) the same limit gives rise to Type IIB supergravity on $AdS_3 \times S^3$. Following these observations, it was conjectured that supergravity on $AdS_3 \times S^3 \times K^3$ is dual to a $(4,0)$ supersymmetric theory in $1+1$ dimensions with an $SO(2,2)$ conformal symmetry. While evidence for the conjecture has been exhibited only in the supergravity limit it is natural to extend it to the level of the full string theory. The precise map between the two theories follows a holography principle and has been explained in.

The set of dualities presented above involves the exchange of coupling constant on the Type IIA side with radius of the $AdS_3 \times S^2 \times S^1$ on the heterotic side, while the volume of the $K3$ on the Type IIA side determines the coupling constant on the heterotic side. As a result under these set of dualities $AdS_3 \times S^2 \times T^5$ compactification of heterotic strings would be related to a $(0,4)$ superconformal theory in $1+1$ whose Kac-Moody level is related to the value of the heterotic string coupling (as it is at the level of the worldsheet sigma model).

A similar way one can start with Type IIA dyonic string solution in six dimensions

\[2\] I did not find reasons for a possible failure of this duality although all the evidence for it arises for cases with flat Minkowski spaces.

\[3\] In the case of Type IIB on $AdS_5 \times S^5$ the $SL(2,\mathbb{Z})$ strong/weak coupling symmetry is preserved and the radius of the near horizon Anti-de-Sitter space is kept large, this coresponds in \[3\] to have a large number of D3-branes.
The horizon geometry is given by $AdS_3 \times S^3$ whose physics, following the holography principle, will be encoded in a conformal field on the 1+1 dimensional boundary. Performing a Hopf T-duality on Type IIA on $AdS_3 \times S^3 \times K3$ followed by strong/weak coupling dualities leads to relate the following theories:

(2b) IIB string theory on $AdS_3 \times S^2 \times S^1 \times K3$,
(2a) IIA string theory on $AdS_3 \times S^3 \times K3$,
(2m) M-theory on $AdS_3 \times S^3 \times S^1 \times K3$,
(2h) Heterotic on $AdS_3 \times S^3 \times T^4$.

The first of these theories (2b) can be regarded as the near-horizon geometry of a black string in five dimensions. Thus these theories are also dual to a superconformal theory in 1+1 dimensions.

Other possibilities for which we may apply the set of dualities are compactifications of Type II strings on $AdS_2 \times S^3$ spaces. The physics of the $AdS_2$ space might be encoded in a quantum mechanical theory on its one-dimensional boundary if the simplest version of the holography principle applies. This supersymmetric quantum mechanics has an $SU(1,1)$ conformal symmetry and has been recently considered in [10]. If the same set of dualities applies it will relate:

(3b) IIB string theory on $AdS_2 \times S^3 \times S^1 \times K3$,
(3a) IIA string theory on $AdS_2 \times S^2 \times T^2 \times K3$,
(3m) M-theory on $AdS_2 \times S^2 \times T^3 \times K3$,
(3h) Heterotic on $AdS_2 \times S^2 \times T^6$.

The latter theory is interesting because of the $SL(2, Z)$ strong/weak coupling symmetry of heterotic strings on $T^6$ [2]. Exchanging the role of Type IIA and type IIB, we obtain the following set of dual compactifications:

(4b) IIB string theory on $AdS_2 \times S^2 \times T^2 \times K3$,
(4a) IIA string theory on $AdS_2 \times S^3 \times S^1 \times K3$,
(4m) M-theory on $AdS_2 \times S^3 \times T^2 \times K3$,
(4h) Heterotic on $AdS_2 \times S^3 \times T^5$.

One can generate a set of other dual theories by “dividing” the internal spaces in the examples above by their discrete subgroups [8]. A particular simple case of these are freely acting $Z_N$ on the $S^1$ factors. This $S^1$ might also be one of the $U(1)$ Hopf fibers [3]. This generates a set of theories with fewer or no supersymmetries. For example acting with
a $Z_k$ on circle corresponds to keeping only states which have charges multiple of $k$ under the $U(1)$ isometries of the circle. For generic $k$ this leads to to dualities between non-supersymmetric theories. This is due to the fact that the “orbifolding” does not act on the Anti-de-Sitter spaces, the theories are still dual to conformal theories on the boundaries with a corresponding number of supercharges [12]. Detailed results with many examples will be presented elsewhere [3].

If the dualities conjectured in this short note hold, they might provide helpful to study the CFT/AdS dualities on the heterotic side. These involve tori and spheres with known metrics instead of the complicate $K3$, some problems [13] of the Type IIB side might hopefully be easier to investigate.

While composing the bibliography we came across the work of [14] where $AdS_3 \times S^3 \times T^4$ of heterotic strings was discussed in a different contest and with different results.

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