BCS-BEC crossover in p-wave resonance superfluids

M. Yu. Kagan, S. L. Ogarkov

1 P.L. Kapitza Institute for Physical Problems RAS, Kosygina street, 2, Moscow 119334, Russia
2 Moscow Engineering Physics Institute, Kashirskoe shosse, 31, Moscow 115409, Russia

E-mail: kagan@kapitza.ras.ru

Abstract. We consider 3D and 2D fully (100%) polarized resonance superfluid Fermi-gas with p-wave pairing in the regime of BCS-BEC crossover.

For low temperatures $T \ll T_{CP}$ ($T_{CP}$ is a p-wave critical temperature) we evaluate the contribution of the superfluid fermionic quasiparticles and bosonic collective waves (phonons) in the specific heat ($C_v$) and the normal density ($\rho_n$) in the BCS-regime for the chemical potential $\mu > 0$, in BEC-regime for $\mu < 0$ and in the interesting intermediate regime for $\mu \to 0$.

We discuss an experimental possibility to observe a chiral anomaly (which leads to mass-current non-conservation at $T = 0$) in 3D A1-phase in ballistic regime as well as a topological term with a charge $Q = 1$ in the 2D fully polarized axial phase in BCS-regime.

1. Introduction

The Feshbach resonance effect [1] yields us a unique possibility to study not only $s$-wave, but also p-wave BCS-BEC crossover in resonance Fermi-gases, thus providing a bridge between the physics of ultracold gasses and the physics of superfluid $^3$He.

Usually in the regime of Feshbach resonance only the $\uparrow\uparrow$ spins form the extended or local p-wave pairs at highest critical temperature, so throughout this paper we deal with fully polarized superfluid A1-phase in 3D and fully polarized superfluid axial phase in 2D.

2. 3D A1-phase

The spectrum of quasiparticles in 3D A1-phase reads: $E_p = \sqrt{(p^2_{2m} - \mu)^2 + \Delta_0^2\sin^2\theta\frac{p^2}{p_F^2}}$ for $T < T_{CP}$. The zeroes of the energy for the angles $\theta = 0$ and $\theta = \pi$ and for $\frac{p^2}{2m} = \mu$ guarantee the power-law contributions to specific heat $C_v \sim N(0)\frac{T^3}{\Delta_0^3}$ and normal density $\rho_n \sim n_\uparrow m\frac{\Delta_0^2}{\Delta_0^3}$ in the BCS-regime ($\mu > 0$), where $n_\uparrow = \frac{p_F^2}{6\pi^2}$ is a total density of "up" spins, $N(0) = \frac{mp_F}{2\pi^2}$ is a density of states on the Fermi level, $\Delta_0 \sim \varepsilon_F e^{-1/|\lambda_p|} \sim T_{CP}$ is a magnitude of a triplet superfluid gap, $\lambda_p$ is a coupling constant for p-wave pairing. In the same time in the BEC-regime ($\mu < 0$) the zeroes of the quasiparticle energy are absent and hence $C_v \sim e^{-|E_b|/2T}\frac{|E_b|^2}{4T^2}(2mT)^{3/2}; \rho_n \sim 2m(2mT)^{3/2}e^{-|E_b|/2T}$ behave in exponential fashion, where $\mu \approx -\frac{|E_b|}{2}$ and $E_b$ is a binding energy of a local triplet pair (molecule).

For the interesting point where the chemical potential $\mu \to 0$ and where some people discuss the possible phase transition between gapped and ungapped phase, namely for $|\mu| < T < \frac{\Delta_0^2}{\varepsilon_F}$. 

© 2009 IOP Publishing Ltd
\[ C_v \sim (2mT)^{3/2} \text{ and } \rho_n \sim m(2mT)^{3/2} \]. Note that in 3D A1-phase there are two branches of bosonic (collective) excitations in gauge-orbital sector [2]. First branch is a standard sound wave (phonon) one with a linear spectrum \( \omega = C_S k \), where \( C_S \) is a sound velocity. This branch also yields a power-law bosonic contribution to \( C_v^B \sim T^3/C_S^3 \) and \( \rho_n^B \sim T^4/C_S^5 \) both in BCS and BEC-regime (including a point \( \mu = 0 \)). Another branch corresponds to the so-called orbital waves. At low frequencies \( \omega < \Delta_0^2/\varepsilon_F \) it has a quadratic spectrum \( \rho_n \sim k^2 \) in BEC-regime and \((\rho - C_0) \omega \sim k^2 \) in BCS-regime, where \( \rho \) is a total mass density. The coefficient \( C_0 \) is a subject of long-lasting discussion in the physics of \(^3\)He - A. Namely it leads to the problem of chiral anomaly (mass current non-conservation) in the hydrodynamics of \(^3\)He - A at \( T = 0 \). The total current reads [2]:

\[
\overrightarrow{j} = \rho \overrightarrow{v}_s + \text{rot}(\frac{\rho \overrightarrow{T}}{2}) + C_0(\overrightarrow{T} \text{rot} \overrightarrow{T}) \overrightarrow{T},
\]

where \( \overrightarrow{v}_s \) is a superfluid velocity, \( \overrightarrow{T} \) is a unit vector of the orbital momentum of a triplet pair. The presence of an anomalous term \( \overrightarrow{j}_{an} = C_0(\overrightarrow{T} \text{rot} \overrightarrow{T}) \overrightarrow{T} \) in the BCS-regime leads to mass-current nonconservation \( \frac{\partial \overrightarrow{j}_{an}}{\partial t} \neq -\frac{\partial}{\partial \overrightarrow{x}}(\Pi_{ik}) \). Note that for unpolarized or weakly polarized superfluid \(^3\)He-A (for magnetic fields \( H < T_{CP}/\mu_B \), where \( \mu_B \) is Bohr magneton) the problem of mass-current non-conservation for \( T = 0 \) has purely academic character since the anisotropic A-phase with zeroes of superfluid gap is only a local minimum at \( T = 0 \) (the global minimum of Ginzburg-Landau functional corresponds to isotropic B-phase [2]). However for fully polarized p-wave superfluid Fermi-gas in magnetic trap the B-phase is totally suppressed and A1-phase is a global minimum of energy already at \( T = 0 \). As early as 1987 A.F. Andreev and M.Yu.K. [3] proposed on the basis of supersymmetric hydrodynamics that even in BCS-case \( C_0 \sim \rho \) only in high-frequency collisionless (Knudsen) regime \( \omega \tau \gg 1 \), while \( C_0 \rightarrow 0 \) in the hydrodynamic regime \( \omega \tau \ll 1 \) (\( \tau (\varepsilon_F/\Delta_0) \) is a damping time defined by residual interaction between collective and one-particle excitations[3]), and thus superfluid hydrodynamics with the standard orbital momentum density \( \overrightarrow{T} = \rho \frac{\overrightarrow{l}}{\tau} \) and superfluid velocity \( \overrightarrow{v}_s \) is restored at very low frequencies. Volovick et al [4] and Combescot et al [5] have another point of view assuming that \( C_0 \sim \rho \) and \( \rho - C_0 \sim \rho \frac{\Delta_0 \overrightarrow{F}}{\varepsilon_F} \) at all frequencies due to topological considerations (spectrum asymmetry for zero mode in Bogolubov - de Gennes equations in the presence of the twisted texture of the l-vector) and the hydrodynamics in the conventional form of the conservation laws can be restored only by the inclusion of a normal velocity \( \overrightarrow{v}_n \) and a normal density \( \rho_n \) at \( T = 0 \). Note that a normal density \( \rho_n \) is connected with the statistical weight of fermionic quasiparticles near the zeroes of the gap where Landau criterion for superfluidity is violated. Note also that to observe experimentally a crossover from ballistic regime with the presence of chiral anomaly to the hydrodynamic regime with its absence we have to fulfill the obvious condition \( l < L_{samp} \), where \( l \sim v_F \tau \) is a mean-free path and \( L_{samp} \) is a sample size. This condition is also more easy to realize in the magnetic traps close to Feshbach resonance where the superfluid gap \( \Delta_0 \) becomes of the order of \( \varepsilon_F \) and thus \( \tau \) becomes smaller.

Later on the similar discussion connected with the possible anomalous contribution of the zero mode in the vortex core to Hall-Vinen friction coefficients was started by Volovick and Kopnin ([4],[6] and references therein).

3. 2D axial phase

In 2D the strong-coupling corrections (see Popov, Brusov [7]) provide that the global minima of Ginzburg-Landau-functional for fully polarized triplet superfluid Fermi-gas corresponds to the axial phase with a quasiparticle spectrum \( E_p = \sqrt{\left( \frac{p^2}{2m} - \mu \right)^2 + \Delta_0^2 \frac{p^2}{\varepsilon_F^2}} \) for \( T < T_{CP} \). There is only one zero in the spectrum here for \( \mu = 0 \) and \( p = 0 \). Hence \( C_v \) and \( \rho_n \) are exponential
\( \{ C_v, \rho_n \} \sim e^{-\Delta_0 / T} \) in BCS-regime as well as in BEC-regime, where \( \{ C_v, \rho_n \} \sim e^{-|E_b| / 2T} \). The power-law contribution is connected only with a region \(|\mu| < T < \Delta_0^2 / \varepsilon_F \), where \( C_v \sim n_1 T^2 / \Delta_0^3 \) and \( \rho_n \sim 2m \varepsilon_F T^3 / \Delta_0^4 \) (in 2D \( n_1 = \frac{\mathbf{p}_F^2}{4\pi} \)). Unfortunately the similar contributions to \( C_v \) and \( \rho_n \) yield here the collective bosonic excitations with the linear spectrum \( \omega = C_s / T^2 \) and \( C_v^B \sim T^2 / \Delta_0^2 \) (note that for \( \mu \to 0 \) the superfluid gap \( \Delta_0 \sim \varepsilon_F \) and the sound velocity \( C_s \sim v_F \) in 2D \([8]\)).

The orbital waves are gapped in 2D and the anomalous current \( \mathbf{j}_m = 0 \) (\( \mathbf{r} \) rot \( \mathbf{l} = 0 \), vector \( \mathbf{l} \) is perpendicular to 2D plane), so there are no problems with orbital hydrodynamics at \( T = 0 \).

The nontrivial topology in 2D is connected with a topological charge \([4,9]\):

\[
Q = \frac{\pi}{2} \varepsilon_{\alpha\beta} \oint d^2 \mathbf{p} \mathbf{n} \left[ \partial_\alpha \mathbf{n} \right] \left[ \partial_\beta \mathbf{n} \right],
\]

where the unit vector \( \mathbf{n} \) = \( \frac{1}{E_p} (\frac{\Delta_0}{p_F} p_x, \frac{\Delta_0}{p_F} p_y, \frac{p^2}{2m} - \mu) \). For the axial phase \( Q = \frac{1}{2}(1 + \frac{\mu}{|\mu|}) \) \([9]\) and hence \( Q = 1 \) in BCS-regime and \( Q = 0 \) in BEC-regime. Unfortunately in all the linear response theory, including solution of the Leggett equations \([10]\) for the gap \( \Delta_0 \), chemical potential \( \mu \), and compressibility \( \kappa^{-1} = C_s^2 \), there is no contribution from \( Q = 1 \) neither in BCS-regime nor close to a singular point \( \mu = 0 \) (where \( Q \) changes abruptly from 1 to 0 and where some people suspect in analogy with a 3D case the phase transition between gapped and ungapped phases).

4. Discussion

To observe experimentally the topological charges \( C_0 \) in ballistic regime in 3D and \( Q \) in 2D we probably have to study the anomalous mass-current in 3D or anomalous spin-current in 2D \([4]\) in the geometry of the Josephson effect with the triplet pairs tunnelling between two vacua with different \( C_0 \) in 3D and \( Q \) in 2D (instanton contribution). Magnetic trap is a convenient system for this type of experiment since it is possible to "cut" the triplet condensate by a laser beam on two parts with different orientations of \( \mathbf{l} \)-vector (different textures). Note, however, that it is an experimental project for the future because at present time the p-wave condensates in magnetic traps are rather short-living to fulfill this type of experiments \([9,11,12]\).

Another possibility to create different textures of \( \mathbf{l} \)-vector is connected with superfluid \( ^3\text{He} \) in aerogel. Finally the similar topological effects could be possibly observed in the textures associated with the different vortex cores in triplet 3D and 2D superfluids, including thin films and submonolayers of \(^3\text{He} \) (see \([4,6,9]\) for discussions).

5. Conclusions

We calculate the specific heat \( C_v \) and the normal density \( \rho_n \) in fully - polarized superfluid p-wave resonance Fermi-gas in 3D and 2D. We present the different points of view and possible connection between them on a very complicated problem of chiral anomaly existence in the BCS-regime of triplet A1-phase in 3D at \( T = 0 \). We discuss briefly the possible experiments to visualize the nontrivial topological effects (connected with anomalous current and topological charge) in 3D and 2D magnetic traps as well as in superfluid \( ^3\text{He} \) in aerogel and in different textures associated with the vortex cores in 3D and 2D triplet superfluids.

Acknowledgements

The authors acknowledge interesting discussions with A.F. Andreev, V. Gurarie, I.A. Fomin, G.E. Volovik, W. Halperin, L.P Pitaevskii and Yu. Kagan.

This work is supported by RFBR-grant No. 08-02-00224.
References

[1] Innoye S., Andrews M. R., Stengler J. et. al 1998 Nature 392 151
[2] D. Volhardt and P. Woelfle, The Superfluid Phases of Helium 3 (Taylor, London, 1990).
[3] Andreev A. F., Kagan M. Yu. 1987 JETP 66 504 ;Andreev A.F., Chubukov A.V., Kagan M. Yu. - unpublished
[4] Volovik G. E., Balatskii, and Konyshev 1986 JETP 63 1194; Volovik G. E., Solov’ev A., and Yakovenko V. M. 1989 JETP Lett. 49 65; G. E. Volovik, The Universe in a Helium Droplet (Oxford Univ. Press, Oxford, 2003).
[5] Combescot R. and Dombre T. 1986 Phys. Rev. B 33 79
[6] Kopnin N.B. and Salomaa M.M. 1991 Phys. Rev. B 44 9667; Kopnin N.B. 2000 Physica B 280 231 and references therein
[7] Brusov P. I. and Popov V. I. 1981 JETP 53 804
[8] Kagan M. Yu. and Ogarkov S. L. 2008 Laser Physics 18 509
[9] Gurarie V. and Radzihovsky L. 2007 Ann. Phys. 322 2
[10] Leggett A. J. 1980 Modern Trends in the Theory of Condenced Matter Physics (Berlin: Springer-Verlag)
[11] Ticknor C., Regal C.A., Jin D.S. et al. 2004 PRA 69 042712; Regal C.A., Ticknor C., Bohn J.L. et al. 2003 PRL 90 053201
[12] Schmuck C.H., Zwierlein M.W., Stan C.A. et al. 2005 PRA 71 045601