THE RADIO WAVELENGTH TIME DELAY OF GRAVITATIONAL LENS 0957 + 561

D. B. Haarsma, J. N. Hewitt, J. Lehar, and B. F. Burke

ABSTRACT

The gravitational lens 0957 + 561 was monitored with the Very Large Array from 1979 to 1997. The 6 cm light-curve data from 1995 to 1997 and the 4 cm data from 1990 to 1997 are reported here. At 4 cm the intrinsic source variations occur earlier and are twice as large as the corresponding variations at 6 cm. The VLBI core and jet components have different magnification factors, leading to different flux ratios for the varying and nonvarying portions of the VLA light curves. Using both the Press, Rybicki & Hewitt Q (PRHQ) and dispersion statistical techniques, we determined the time delay, core flux ratio, and excess nonvarying B image flux density. The fits were performed for the 4 and 6 cm light curves, both individually and jointly, and we used Gaussian Monte Carlo data to estimate 68% statistical confidence levels. The delay estimates from each individual wavelength were inconsistent given the formal uncertainties, suggesting that there are unmodeled systematic errors in the analysis. We roughly estimate the systematic uncertainty in the joint result from the difference between the 6 and 4 cm results, giving $409 \pm 30$ days for the PRHQ statistic and $397 \pm 20$ days for the dispersion statistic. These results are consistent with the current optical time delay of $417 \pm 3$ days, reconciling the long-standing difference between the optical and radio light curves and between different statistical analyses. The unmodeled systematic effects may also corrupt light curves for other lenses, and we caution that multiple events at multiple wavelengths may be necessary to determine an accurate delay in any lens system. Now that consensus has been reached regarding the time delay in the 0957 + 561 system, the most pressing issue remaining for determining $H_0$ is a full understanding of the mass distribution in the lens.

Subject headings: distance scale — gravitational lensing — quasars: individual (0957 + 561) — radio continuum: stars

1. INTRODUCTION

The time delay between multiple gravitationally lensed images can be used to measure the distance of high-redshift objects and thus is a useful estimator of the Hubble parameter, $H_0$. After many years of monitoring the lens 0957 + 561, the time-delay estimates for this system are finally converging on an accepted value. Groups monitoring 0957 + 561 at optical wavelengths have detected a sharp variation in each image and have found the optical delay to be $417 \pm 3$ days (Kundic et al. 1995, 1997; Oscoz et al. 1997; Schild & Thomson 1997). Given the long controversy over the value of the delay (for a history see Table 1 of Haarsma et al. 1997, hereafter Paper I), it is important that the optical measurement be confirmed at radio wavelengths. The MIT radio astronomy group has monitored the source at radio wavelengths from 1979 to 1997, and the final light-curve data and time-delay results are reported here.

2. OBSERVATIONS

Observations have occurred monthly at the National Radio Astronomy Observatory (NRAO) Very Large Array radio telescope (VLA) since 1979 at 6 cm and since 1990 at 4 cm. The monitoring ended in 1997 December. All of the data were reduced in the manner described in Paper I and Lehar et al. (1992). To determine the flux densities of the point images, it was necessary to subtract the extended structure in the field. At both 6 and 4 cm this subtraction was difficult in the most compact VLA array, D; thus there are gaps in the light curves for 4 months of every 16 month cycle. In addition, some observations in other VLA arrays were excluded due to bad weather or poor subtraction of the extended structure. When the observations were made in a combination or nonstandard array configuration, the data were analyzed according to the next largest standard configuration (A, B, or C).

The 6 cm data through 1994 December were presented in Paper I. The remaining 6 cm data and all of the 4 cm data are given in Tables 1 and 2 and plotted in Figure 1. There are a total of 147 points in the 6 cm light curve and 58 points in the 4 cm light curve. At 6 cm the flux density of the B image increased in 1995, following the A image increase in 1994. The current 6 cm feature has lasted longer than the similar feature around 1989—1991, but the A image is now declining. At 4 cm the quasar is twice as variable as at 6 cm (as a percentage of average flux density). Also, the variations in the 4 cm light curves occur earlier than the corresponding features at 6 cm. Both of these characteristics are consistent with multiwavelength models and other observations of active galactic nucleus (AGN) variability (e.g., Marscher & Gear 1985; Stevens et al. 1996). The well-sampled increase and decrease at 4 cm in 1994—1997 has helped significantly in determining the radio time delay.

3. FREE PARAMETERS IN THE LIGHT CURVES

When fitting for the time delay between the images, the difference in magnification between them must be properly

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4 The National Radio Astronomy Observatory is operated by Associated Universities, Inc., under cooperative agreement with the National Science Foundation.

5 The light-curve data are also available electronically through http://space.mit.edu/RADIO/papers.html.
The 4 cm A image has been shifted up by 8% to avoid overlap with the core. The flux ratio of the jet (which is constant in time) and the core (which has a strong correlation in the 4 cm light curves, so that statistic which were described in Paper I. We used linear units (mJy) rather than the logarithmic units defined in Paper I. The discrete correlation function (Lehár et al. 1992) did not find a strong correlation in the 4 cm light curves, so that statistic was not used here. Gaussian Monte Carlo data sets were used to estimate the 68% confidence intervals on the results for the real light curves, where the fitted $c$ values were compared to the input param-

\begin{equation}
\begin{aligned}
    c &= B_{\text{DC}} - A_{\text{DC}} \\
    &= B_{\text{jet}} + B_{\text{core,DC}} - (A_{\text{jet}} + A_{\text{core,DC}}),
\end{aligned}
\end{equation}

(Press & Rybicki 1998), where $R_{\text{AC}}$ is the flux ratio of the variable component. It is useful to write $c$ in terms of the core and jet components of the radio images as

\begin{equation}
    c = \frac{B_{\text{jet}}}{R_{\text{core}}} - \frac{1}{R_{\text{jet}}},
\end{equation}

where the DC core components cancel out. The value of $c$ can thus be estimated from the values of $B_{\text{jet}}$, $R_{\text{core}}$, and $R_{\text{jet}}$. Since in the case of 0957 + 561 we have $R_{\text{jet}} < R_{\text{core}}$ (Conner et al. 1992), the value of $c$ must be negative; i.e., the A curve has a larger amount of constant flux than the core-ratio corrected B curve.

The values of several of the above parameters can be estimated from observations without doing time-delay fitting. Garrett et al. (1994) compiled the information on the core flux ratio from VLBI and optical observations and found the weighted average of these estimates to be $R_{\text{core}} = 0.75 \pm 0.02$. Also, the faintest portions of the VLA light curves set upper limits on the jet flux density, i.e., $B_{\text{jet}} \leq 21$ mJy at 6 cm, and $B_{\text{jet}} \lesssim 15$ mJy at 4 cm. A better estimate of $B_{\text{jet}}$ can be obtained by comparing coincident VLBI and VLA observations. The VLBI observations give the core flux density at a particular epoch, which can be subtracted from the VLA flux density to obtain the VLA jet flux density. Campbell et al. (1995) report VLBI observations at 6 cm on 1987 September 28 and 1989 September 26, and by comparing these to VLA observations occurring on the same days we find $B_{\text{jet}} = 11.1 \pm 0.4$ mJy and $R_{\text{jet}} = 0.63 \pm 0.03$. The values for $R_{\text{core}}$, $R_{\text{jet}}$, and $B_{\text{core}}$ can be combined using equation (3) to find $c_0 = -2.7 \pm 0.8$ mJy.

The above estimates are all for the 6 cm light curves. At 4 cm there are no coincident VLBI/VLA observations, so we cannot make similar estimates. The value of $c$ is different at 6 and 4 cm due to the difference in $B_{\text{jet}}$; note that the ratios $R_{\text{jet}}$ and $R_{\text{core}}$ are the same for the two bands. For a synchrotron spectrum, $B_{\text{jet}}$ will be smaller at 6 cm than 4 cm, and thus we expect $|c_4|$ to be smaller than $|c_6|$.

4. TIME-DELAY ANALYSIS METHODS

To fit for the three parameters $\tau$, $R_{\text{core}}$, and $c$ (described in § 3), we used the PRHQ statistic (Press, Rybicki, & Hewitt 1992a, 1992b; Rybicki & Press 1992, incorporating the modifications of Rybicki & Kleyna 1994; Press & Rybicki 1998), and the dispersion statistic (Pelt et al. 1994, 1996), which were described in Paper I. We used linear units (mJy) rather than the logarithmic units defined in Paper I. The discrete correlation function (Lehár et al. 1992) did not find a strong correlation in the 4 cm light curves, so that statistic was not used here. Gaussian Monte Carlo data were made as described in Paper I, but now with the four physical parameters $\tau$, $R_{\text{core}}$, $R_{\text{jet}}$, and $B_{\text{jet}}$. Five hundred Gaussian Monte Carlo data sets were used to estimate the 68% confidence intervals on the results for the real light curves, where the fitted $c$ values were compared to the input param-

\begin{equation}
    c = B_{\text{DC}} - A_{\text{DC}}
\end{equation}

(Press & Rybicki 1998), where $R_{\text{AC}}$ is the flux ratio of the variable component. It is useful to write $c$ in terms of the core and jet components of the radio images as

\begin{equation}
    c = \frac{B_{\text{jet}} + B_{\text{core,DC}}}{R_{\text{core}}} - \left( A_{\text{jet}} + A_{\text{core,DC}} \right),
\end{equation}

and therefore

\begin{equation}
    c = B_{\text{jet}} \left( \frac{1}{R_{\text{core}}} - \frac{1}{R_{\text{jet}}} \right),
\end{equation}

where the DC core components cancel out. The value of $c$ can thus be estimated from the values of $B_{\text{jet}}$, $R_{\text{core}}$, and $R_{\text{jet}}$. Since in the case of 0957 + 561 we have $R_{\text{jet}} < R_{\text{core}}$ (Conner et al. 1992), the value of $c$ must be negative; i.e., the A curve has a larger amount of constant flux than the core-ratio corrected B curve.

The values of several of the above parameters can be estimated from observations without doing time-delay fitting. Garrett et al. (1994) compiled the information on the core flux ratio from VLBI and optical observations and found the weighted average of these estimates to be $R_{\text{core}} = 0.75 \pm 0.02$. Also, the faintest portions of the VLA light curves set upper limits on the jet flux density, i.e., $B_{\text{jet}} \leq 21$ mJy at 6 cm, and $B_{\text{jet}} \lesssim 15$ mJy at 4 cm. A better estimate of $B_{\text{jet}}$ can be obtained by comparing coincident VLBI and VLA observations. The VLBI observations give the core flux density at a particular epoch, which can be subtracted from the VLA flux density to obtain the VLA jet flux density. Campbell et al. (1995) report VLBI observations at 6 cm on 1987 September 28 and 1989 September 26, and by comparing these to VLA observations occurring on the same days we find $B_{\text{jet}} = 11.1 \pm 0.4$ mJy and $R_{\text{jet}} = 0.63 \pm 0.03$. The values for $R_{\text{core}}$, $R_{\text{jet}}$, and $B_{\text{core}}$ can be combined using equation (3) to find $c_0 = -2.7 \pm 0.8$ mJy.

The above estimates are all for the 6 cm light curves. At 4 cm there are no coincident VLBI/VLA observations, so we cannot make similar estimates. The value of $c$ is different at 6 and 4 cm due to the difference in $B_{\text{jet}}$; note that the ratios $R_{\text{jet}}$ and $R_{\text{core}}$ are the same for the two bands. For a synchrotron spectrum, $B_{\text{jet}}$ will be smaller at 6 cm than 4 cm, and thus we expect $|c_4|$ to be smaller than $|c_6|$.
eters using equation (3). The pseudo-jackknife test from Paper I was used to test the stability of the result to the removal of individual points.

To determine whether neglecting the difference between the core and jet flux ratio caused an error in our previous analysis, we applied the two-dimensional fit (for $r$ and $R_{VLA}$, as in Paper I) to the Gaussian Monte Carlo data made with four parameters. The resulting “fitted-minus-true” values did not show a significant bias (to long or short delays, for example) but did show an increase in scatter about the true delay. We found that the error in the delay increased monotonically with $B_{\text{jet}}$, from roughly 20 days for $B_{\text{jet}} = 0$ mJy to roughly 100 days for $B_{\text{jet}} \sim 11$ mJy (the value for the real light curves) when using the PRH$_Q$ statistic and the 6 cm Monte Carlo data. The same test with the 4 cm Monte Carlo data, and with the dispersion statistic at both 4 and 6 cm, revealed a similar but somewhat milder effect, with the delay error at least doubling between small and large values of $B_{\text{jet}}$. This dependence on the amount of flux density in the jet component may be one cause of the inconsistency in delay estimates over the years, and we caution that fitting for only two parameters may introduce significant errors.

In addition to analyzing the two wavelengths individually, we also fitted for the parameters using both wavelengths at once. The dispersion and PRH$_Q$ statistics are both easily modified for this by minimizing the sum of the statistics from each wavelength (see Press et al. 1992b) and fitting for the parameters $r$, $R_{\text{core}}$, $c_6$, and $c_4$. Monte Carlo analysis was also done for the joint data, where the 6 and 4 cm Monte Carlo sets were constructed with the same set of $[r, R_{\text{core}}, R_{\text{jet}}, B_{\text{jet}}]$.

The covariance model (Paper I; Press et al. 1992a) used for the PRH$_Q$ statistic was found by an iterative procedure on the individual light curves. First, measurement errors of 2% were assumed and used to make point estimates for the structure function and then fitted to an exponential in the lag range of 100–700 days. This structure function was then used to determine the PRH$_Q^2$ value for the light curve, and the measurement errors were adjusted until PRH$_Q^2$ equaled the degrees of freedom. Then the process was repeated for the new measurement error value. Iterations stopped when the square root of (PRH$_Q^2$/degrees of freedom) changed by less than 1% when PRH$_Q^2$ was calculated with the measurement error of the previous iteration. At 6 cm the covariance model found was

$$V(T) = 1.673 \times 10^{-4} T^{1.606} \text{ mJy}^2,$$

with measurement errors $e_q = 1.82\%$ and $e_B = 2.34\%$, where $T$ is the time lag between two points on the curve. At 4 cm the fitted covariance model was

$$V(T) = 3.174 \times 10^{-4} T^{1.633} \text{ mJy}^2,$$
| Calendar Date | Julian Day – 2,440,000.0 | Array | A Flux Density (mJy) | B Flux Density (mJy) |
|---------------|---------------------------|-------|----------------------|----------------------|
| 1990 Oct 4    | 8169.22                   | BnC   | 23.87                | 21.96                |
| 1990 Nov 1    | 8197.06                   | C     | 23.71                | 22.57                |
| 1990 Dec 13   | 8238.89                   | C     | 22.75                | 22.49                |
| 1991 Jul 10   | 8448.40                   | A     | 22.28                | 17.98                |
| 1992 Jan 6    | 8627.97                   | B     | 23.33                | 15.12                |
| 1992 Feb 4    | 8656.80                   | BnC   | 22.93                | 15.54                |
| 1992 Feb 29   | 8681.74                   | C     | 24.08                | 15.72                |
| 1992 Mar 7    | 8688.67                   | C     | 23.96                | 15.55                |
| 1992 Apr 18   | 8730.60                   | C     | 24.30                | 15.85                |
| 1992 May 3    | 8745.60                   | C     | 24.98                | 15.88                |
| 1992 Nov 11   | 8938.09                   | A     | 25.57                | 15.61                |
| 1992 Dec 10   | 8966.97                   | A     | 25.19                | 15.34                |
| 1993 Feb 5    | 9023.78                   | AnB   | 25.22                | 15.68                |
| 1993 Mar 21   | 9067.64                   | B     | 25.58                | 16.21                |
| 1993 Apr 9    | 9086.67                   | B     | 25.75                | 16.78                |
| 1993 May 18   | 9126.48                   | B → BnC | 25.55           | 16.54                |
| 1993 Jul 25   | 9194.21                   | C     | 27.82                | 17.13                |
| 1993 Aug 26   | 9226.26                   | C     | 28.02                | 16.65                |
| 1994 Mar 4    | 9415.73                   | A     | 31.34                | 17.71                |
| 1994 Apr 11   | 9453.68                   | A     | 31.14                | 17.17                |
| 1994 May 7    | 9479.63                   | A → AnB | 31.31           | 17.80                |
| 1994 Jun 25   | 9528.52                   | B     | 31.15                | 18.58                |
| 1994 Jul 6    | 9540.42                   | B     | 30.83                | 18.56                |
| 1994 Aug 18   | 9583.28                   | B     | 31.78                | 18.62                |
| 1994 Sep 8    | 9604.27                   | B     | 31.65                | 19.42                |
| 1994 Oct 10   | 9636.18                   | BnC   | 31.04                | 18.97                |
| 1994 Nov 7    | 9664.08                   | C     | 31.75                | 20.29                |
| 1994 Dec 8    | 9694.92                   | C     | 31.32                | 20.51                |
| 1995 Jun 23   | 9892.50                   | A     | 30.19                | 20.99                |
| 1995 Jul 8    | 9907.23                   | A     | 30.49                | 22.18                |
| 1995 Jul 21   | 9919.51                   | A     | 30.05                | 21.80                |
| 1995 Aug 7    | 9937.34                   | A     | 30.48                | 21.72                |
| 1995 Sep 1    | 9962.33                   | A     | 29.94                | 22.05                |
| 1995 Sep 9    | 9970.14                   | A → AnB | 29.91           | 21.02                |
| 1995 Sep 15   | 9976.10                   | AnB   | 30.76                | 21.78                |
| 1995 Sep 23   | 9984.17                   | AnB   | 29.45                | 21.73                |
| 1995 Sep 30   | 9991.17                   | AnB   | 29.19                | 22.54                |
| 1995 Oct 10   | 10001.17                  | B     | 29.41                | 22.83                |
| 1995 Oct 27   | 10018.20                  | B     | 29.95                | 22.51                |
| 1995 Nov 9    | 10031.06                  | B     | 28.55                | 21.81                |
| 1995 Dec 26   | 10077.95                  | B     | 28.34                | 21.56                |
| 1996 Jan 26   | 10108.83                  | BnC   | 27.02                | 22.52                |
| 1996 Feb 5    | 10118.77                  | BnC   | 27.19                | 21.96                |
| 1996 Feb 26   | 10139.67                  | C     | 27.95                | 23.03                |
| 1996 Mar 4    | 10146.68                  | C     | 28.11                | 22.79                |
| 1996 Apr 5    | 10178.56                  | C     | 27.52                | 22.89                |
| 1996 Apr 25   | 10198.66                  | C     | 27.84                | 23.15                |
| 1996 Oct 19   | 10376.05                  | A     | 25.49                | 20.67                |
| 1996 Nov 10   | 10397.95                  | A     | 24.19                | 20.64                |
| 1996 Dec 26   | 10443.98                  | A     | 24.73                | 20.05                |
| 1997 Jan 10   | 10458.89                  | A     | 25.32                | 20.13                |
| 1997 Feb 26   | 10505.82                  | B     | 24.53                | 19.44                |
| 1997 Mar 19   | 10526.69                  | B     | 24.79                | 19.56                |
| 1997 Apr 10   | 10548.65                  | B     | 25.23                | 20.10                |
| 1997 May 11   | 10579.65                  | B     | 24.95                | 18.65                |
| 1997 Jun 22   | 10622.38                  | BnC   | 24.71                | 18.32                |
| 1997 Jul 11   | 10641.43                  | C     | 26.58                | 18.31                |
| 1997 Sep 22   | 10714.17                  | C     | 26.97                | 18.02                |
with measurement errors $e_A = 1.67\%$ and $e_R = 2.19\%$.

The sharp feature in the B image at 6 cm in spring 1990 is statistically inconsistent with the rest of the light curve (see Paper I), and, given the short delay found from the optical and 4 cm light curves, the A image shows that the feature is not intrinsic to the source. Thus we expect that the results with the points removed will be more accurate than the results for the full 6 cm curves. The 6 cm analysis was done both with and without the four points (1990 March 15, April 10, May 7, and May 23); the light curve without the points will be denoted 6* cm.

5. RESULTS

The main results of the time-delay analysis are shown in Tables 3 and 4. These and other aspects of the results are worth discussion in this section. First, the Monte Carlo test (see Paper I), and, given the short delay found from the optical and Contours start at $Q = 130$ and increase by 10 to $Q = 380$.

![Fig. 2.—PRH$Q$ for the 6* cm light curves, as a function of $c$ and $R_{core}$. The delay is fixed at 452 days. The minimum is $Q = 125.6$ at $c = -1.47$ and $R_{core} = 0.731$. Contours start at $Q = 130$ and increase by 10 to $Q = 380$.](image-url)
lengths found somewhat different values of $R_{\text{core}}$ and that $c_4$ is more negative than $c_6$, contrary to the predictions of § 3. The joint analysis of the two wavelengths, however, used all of the available information to constrain the fit, and the resulting $R_{\text{core}}$, $c_6$, and $c_4$ are in good agreement with the predictions of § 3.

Turning now to the results for the delay, it is of interest that all of the delay estimates in Tables 3 and 4 are much smaller than the value of 540 days found by Press et al. (1992b) using the first 80 points in the 6 cm light curve. As explained in Paper I, this change in the delay estimate is due entirely to the addition of new features to the light curve. If the first 80 points in the curve (with or without the four spring 1990 points) are fitted for the three parameters, the dispersion statistic finds a delay of roughly 550 ± 35 days, and the PRH$Q$ statistic (using the above covariance model, eq. [4]) finds a delay of roughly 525 ± 25 days (the confidence intervals were estimated from Monte Carlo analysis of 100 light curves). Therefore the change in the delay estimate between the first 80 points and the current 147 points occurs for both statistical methods, for fits with two or three parameters, and for a variety of covariance models. It does not, however, occur in our Gaussian Monte Carlo data. Comparing delay estimates from the first 80 points and the full curves in the Monte Carlo data, we find that 99% of the sets have a difference in delay less than the 75 day difference seen in the real light curves. Thus the Monte Carlo light curves must still be missing some characteristic of the real data.

A related issue is the difference in the delay estimates for the two wavelengths (for $\text{PRH}_Q$, $\tau_6 = 452^{+14}_{-14}$ days vs. $\tau_4 = 397^{+12}_{-12}$ days, a difference of about three confidence intervals). The time delay between lensed images should be completely independent of wavelength, and indeed the optical and radio estimates come remarkably close. This effect also does not occur in the Gaussian Monte Carlo data, which were constructed such that the 6 and 4 cm data have the same set of $[\tau, R_{\text{core}}, R_{\text{jet}}, B_{\text{jet}}]$. When applying the PH$Q$ statistic, we found that 99% of the Monte Carlo curves had a difference of $(|\tau_6 - \tau_4|)$ that was smaller than the 55 day difference in the real data; similarly, the dispersion statistic found that about 80% of the data sets had a smaller delay difference.

Since both of these effects (the significant change in the delay estimate as features are added to the curves and the significant difference between the two radio wavelengths) are not seen in the Monte Carlo data, there must still be some systematic effect that has not been taken into account in the creation of the Monte Carlo data or in our analysis. One source of the systematic error may be interstellar scintillation, which can cause variability at a level not much larger than our observational error of 2%, perhaps creating small features such as the discrepancy between the 6 cm A and B images in early 1985 (see Fig. 4). Although individual features of a few percent are difficult to identify, they may cause a significant bias in the delay estimate if they occur at crucial times in the light curves. We note that microlensing could cause similar low-level systematic effects in the optical light curves (Schild & Smith 1991; Schmidt & Wambsganss 1998). Since it is beyond the scope of this paper to model these systematic effects, we make a rough estimate from the difference between the 6 and 4 cm delay estimates, giving a systematic uncertainty of roughly ± 30 days for PH$Q$ and ± 20 days for the dispersion. Note that this uncertainty is still less than 10% and is only one factor contributing to the error in $H_0$ (see § 6). Note also that the optical time-delay estimate has a smaller error (only 1%; Kundic et al. 1997) primarily because the source varies much more rapidly at optical wavelengths than at radio wavelengths. We caution others monitoring gravitational lenses that in order to determine an accurate time delay it is preferable to have light curves with multiple features at multiple wavelengths, since the estimate of the time delay based on a single feature at a single wavelength could easily be corrupted by these low-level systematic effects (as the 0957 + 561 6 cm curves were after the first 80 observations). The 0957 radio light curves will continue to be a useful data set for studying systematic effects and time-delay analysis techniques.

Figure 3 shows the PH$Q$ and dispersion statistics as a function of delay for the joint analysis of the 4 and 6* cm curves, with the values of $c_6$, $c_4$, and $R_{\text{core}}$ fixed at the best-fit values. Figure 4 shows the aligned light curves at the two wavelengths with the PH optimal reconstruction (see Press et al. 1992a and Paper I).

6. CONCLUSIONS

Since our last report (Paper I), the B image has increased at 6 cm and the A image has entered a slow decline. The 4 cm curves, given here for the first time, are highly variable and give additional features to aid in determining the time delay. To take into account the difference in magnification of the core and jet components of each image, we have fitted for the delay, core flux ratio, and the excess flux density in the B image (as defined by Press & Rybicki 1998). The delay estimates found from the wavelengths individually disagree by a few confidence intervals, indicating that there are systematic effects not modeled in our analysis. The delay estimates found from the joint analyses of both wavelengths
were 409 ± 30 days for PRHQ and 395 ± 20 days for the dispersion, where the uncertainty is based on a rough estimate of the systematic error. Both results are consistent with the delay estimated from optical monitoring (417 ± 3 days; Kundić et al. 1997), and thus we now have good agreement for the value of the delay from both statistics in both the radio and optical light curves. Consensus has finally been reached on the value of the delay for gravitational lens 0957+561.

This measurement of the delay can now be used to answer cosmological questions. The Hubble parameter, however, depends not only on the delay but also on the lens model and the galaxy velocity dispersion. Using the SPLS model of Grogin & Narayan (1996a, 1996b), the recent Keck velocity dispersion measurement of 279 km s$^{-1}$ (Falco et al. 1997), and a time delay of 409 days, we obtain $H_0 = 67$ km s$^{-1}$ Mpc$^{-1}$. In fitting this model, Grogin & Narayan used the ground-based optical position of the lensing galaxy as a constraint, rather than the more precise VLBI position. Since then, the *Hubble Space Telescope* optical position of the lensing galaxy has been found to agree with the VLBI position (Bernstein et al. 1997). The modelers point out (Grogin & Narayan 1996b) that if the VLBI position is used as the model constraint, their model fit is very similar to that of Falco, Gorenstein, & Shapiro (1991): for the same delay and velocity dispersion, the Falco et al. (1991) model gives $H_0 = 41$ km s$^{-1}$ Mpc$^{-1}$. Thus, the change in the position of the lensing galaxy causes a change in the estimate of $H_0$ well beyond the statistical error, apparently in contradiction to the conclusions of Kundić et al. (1997) regarding the robustness of the $H_0$ determination.

New modeling work must be done that incorporates the improved galaxy position, the recent observations of the cluster mass distribution (Fischer et al. 1997), a careful treatment of the systematic errors in the velocity dispersion (Romanowsky & Kochanek 1999), and the recently reported structure at X-ray (Chartas et al. 1998), optical (Bernstein et al. 1997), and radio (Avruch et al. 1997; Harvanek et al. 1997) wavelengths. These new observations will allow an improved fit of the model to the data and provide a more accurate measure of the Hubble parameter.

We thank the VLA staff for their assistance over the many years of this monitoring project. D. B. H. and B. F. B. have been supported in part by the National Science Foundation. J. N. H. acknowledges the support of a David and Lucille Packard Fellowship, a NSF Presidential Young Investigator Award, and NSF grant AST 96-17028. J. L. acknowledges the support of NSF grant AST 93-03527.

**REFERENCES**

Avruch, I. M., Cohen, A. S., Lehár, J., Conner, S. R., Haarsma, D. B., & Burke, B. F. 1997, ApJ, 488, L121

Bernstein, G., Fischer, P., Tyson, J. A., & Rhee, G. 1997, ApJ, 483, L79

Campbell, R. M., Lehár, J., Corey, B. E., Shapiro, I. I., & Falco, E. E. 1995, AJ, 110, 2566

Chartas, G., Chuss, D., Forman, W., Jones, C., & Shapiro, I. 1998, ApJ, 504, 661

Conner, S. R., Lehár, J., & Burke, B. F. 1992, ApJ, 397, L61

Falco, E. E., Gorenstein, M. V., & Shapiro, I. 1991, ApJ, 372, 364

Falco, E. E., Shapiro, I. I., Moustakas, L. A., & Davis, M. 1997, ApJ, 484, 70

Fischer, P., Bernstein, G., Rhee, G., & Tyson, J. A. 1997, AJ, 113, 521

Garrett, M. A., Calder, R. J., Porcas, R. W., King, L. J., Walsh, D., & Wilkinson, P. N. 1994, MNRAS, 270, 457

Grogin, N. A., & Narayan, R. 1996a, ApJ, 464, 92

Gondhalekar, K. Horne, & B. M. Peterson (San Francisco: ASP), 85

Kundić, T., Colley, W. N., Gott, J. R., III, Malhotra, S., Pen, U.-L., Rhoads, J. E., Stanek, K. Z., & Turner, E. L. 1995, ApJ, 455, L5

Lefèvre, R., et al. 1997, ApJ, 482, 75

Lehár, J., Hewitt, J. N., Roberts, D. H., & Burke, B. F. 1992, ApJ, 384, 453

Marscher, A. P., & Gear, W. K. 1985, ApJ, 298, 114

Oscoz, A., Mediavilla, E., Goicoechea, L. J., Serra-Ricart, M., & Buitrago, J. 1997, ApJ, 479, L89

Pelt, J., Hoff, W., Kayser, R., Refsdal, S., & Schramm, T. 1994, A&A, 286, 775

Pelt, J., Kayser, R., Refsdal, S., & Schramm, T. 1996, A&A, 305, 97

Press, W. H., & Rybicki, G. B. 1998, preprint (astro-ph/9803193)

Press, W. H., Rybicki, G. B., & Hewitt, J. N. 1992a, ApJ, 385, 404

Rybicki, G. B., & Press, W. H. 1992, ApJ, 385, 416

Romanowsky, A. J., & Kochanek, C. S. 1999, ApJ, submitted

Schild, R., & Thomson, D. J. 1997, AJ, 113, 130

Schmidt, R., & Wambsganss, J. 1998, A&A, 335, 379

Stevens, J. A., Litchﬁeld, S. J., Robson, E. I., Cawthorne, T. V., Aller, M. F., Aller, H. D., Hughes, P. A., & Wright, M. C. H. 1996, ApJ, 466, 158

Wilkinson, P. N. 1994, MNRAS, 270, 457

Schild, R. E., & Smith, R. C. 1991, AJ, 101, 813

Schmidt, R., & Wambsganss, J. 1998, A&A, 335, 379

Stevens, J. A., Litchfield, S. J., Robson, E. I., Cawthorne, T. V., Aller, M. F., Aller, H. D., Hughes, P. A., & Wright, M. C. H. 1996, ApJ, 466, 158

**Fig. 4.—Six and 4 cm light curves combined at τ = 409 days, R_{opt} = 0.753, c_o = -2.35, and c_d = -2.09, shifted to the time and flux density of the A image. The A image data are shown as open circles and the B image as solid circles. The 1 σ width of the PRH optimal reconstruction is shown as a pair of lines.**