AutoInit: Analytic Signal-Preserving Weight Initialization for Neural Networks

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Abstract

Neural networks require careful weight initialization to prevent signals from exploding or vanishing. Existing initialization schemes solve this problem in specific cases by assuming that the network has a certain activation function or topology. It is difficult to derive such weight initialization strategies, and modern architectures therefore often use these same initialization schemes even though their assumptions do not hold. This paper introduces AutoInit, a weight initialization algorithm that automatically adapts to different neural network architectures. By analytically tracking the mean and variance of signals as they propagate through the network, AutoInit appropriately scales the weights at each layer to avoid exploding or vanishing signals. Experiments demonstrate that AutoInit improves performance of convolutional, residual, and transformer networks across a range of activation function, dropout, weight decay, learning rate, and normalizer settings, and does so more reliably than data-dependent initialization methods. This flexibility allows AutoInit to initialize models for everything from small tabular tasks to large datasets such as ImageNet. Such generality turns out particularly useful in neural architecture search and in activation function discovery. In these settings, AutoInit initializes each candidate appropriately, making performance evaluations more accurate. AutoInit thus serves as an automatic configuration tool that makes design of new neural network architectures more robust. The AutoInit package provides a wrapper around TensorFlow models and is available at https://github.com/cognizant-ai-labs/autoinit.

1 Introduction

Proper weight initialization is crucial to achieve high performance with deep networks. A common motif in such networks is repeated layers or building blocks. Thus, if a given layer amplifies or diminishes the forward or backward propagation of signals, repeated applications of that layer will result in exploding or vanishing signals, respectively (Hochreiter 1991; Hanin 2018). This phenomenon makes optimization difficult, and can even exceed machine precision. The issue persists regardless of whether the weights are sampled to be uniform, normal, or orthogonal (Saxe, McClelland, and Ganguli 2013; Hu, Xiao, and Pennington 2020).

While many initialization strategies have been proposed in the past, these strategies apply only to neural networks with specific activation functions, topologies, or layer types. Thus, researchers designing new models or activation functions have two options. The first option is to derive weight initialization strategies manually for every architecture considered, which is generally difficult and time consuming. The second option is to use existing initialization strategies in new settings, where they may be incorrect and therefore misleading: A candidate model may appear poor when it is the suboptimal initialization that makes training difficult.

To overcome this problem, this paper proposes AutoInit, an algorithm that automatically calculates analytic mean- and variance-preserving weight initialization for neural networks. Since AutoInit is algorithmic, it relieves the researcher from a difficult but consequential step in model design. It is no longer necessary to use existing weight initialization strategies in incorrect settings: AutoInit provides an appropriate default initialization automatically, resulting in better and more reliable performance.

2 Related Work

Weight initialization strategies attempt to solve the following problem: How should weights be initialized so that the neural network does not suffer from vanishing or exploding signals? This section reviews previous research in neural network weight initialization, which has focused on stabilizing signals by accounting for specific components of neural networks such as the activation function, topology, layer types, and training data distribution.

Activation-Function-Dependent Initialization As is common in the literature, \( \text{fan}_\text{in} \) and \( \text{fan}_\text{out} \) refer to the number of connections feeding into and out of a node, respectively. LeCun et al. (2012) recommend sampling weights from a distribution with mean zero and standard deviation \( \sqrt{\text{fan}_\text{in}} \). This initialization encourages propagated signals to have variance approximately one if used with an activation function symmetric about the origin, like \( 1.7159 \tanh(\frac{2}{3}x) \) or \( \tanh(x) + \alpha x \) for some small choice of \( \alpha \). The standard sigmoid \( f(x) = 1/(1 + e^{-x}) \) induces a mean shift and should not be used in this setting.

Glorot and Bengio (2010) proposed one initialization strategy to ensure unit variance in the forward-propagated signals and another to ensure unit variance for the backward-propagated gradients. As a compromise between
the two strategies, they initialized weights by sampling from 
\( \mathcal{U} \left( \frac{\sqrt{\pi}}{\text{fan}_\text{in} + \text{fan}_\text{out}} \sqrt{\pi}, \frac{\sqrt{\pi}}{\text{fan}_\text{in} + \text{fan}_\text{out}} \right) \). They also avoided sigmoid, and instead chose symmetric functions with unit derivatives at 0, such as tanh or Softsign \( x = \frac{x}{1 + |x|} \).

He et al. (2015) introduced the PReLU activation function and a variance-preserving weight initialization to be used with it that samples weights from \( \mathcal{N}(0, \sqrt{2/\text{fan}_\text{in}}) \). Similarly, Klambauer et al. (2017) introduced SELU, an activation function with self-normalizing properties. These properties are only realized when SELU is used with the initialization scheme by LeCun et al. (2012).

The above weight initialization strategies attempt to solve the same fundamental problem: How can weights be scaled so that repeated applications of the activation function do not result in vanishing or exploding signals? While these approaches solve this problem in a few special cases, the issue is more general. Manually deriving the correct scaling is intractable for complicated activation functions. One approach for an arbitrary function \( f \) is to sample Gaussian inputs \( x \) and adjust the weights according to the empirical variance \( \text{Var}(f(x)) \) (Brock, De, and Smith 2021). This paper proposes an alternative and potentially more accurate approach: integration by adaptive quadrature. The result is a weight initialization strategy that is compatible with any integrable activation function. Indeed, previous activation-function-dependent initializations are special cases of the AutoInit algorithm.

**Topology-Dependent Initialization** The activation-function-dependent initializations discussed above were designed for neural networks composed of convolutional or dense layers. After the introduction of residual networks (ResNets; He et al. 2016a,b), new weight initialization schemes had to be developed to account for the effect of shortcut connections and various types of residual branches.

Taki (2017) analyzed signal propagation in plain and batch-normalized ResNets. They developed a new weight initialization to stabilize training, but did not consider modifications like using deeper residual blocks or reordering components like the activation function or batch normalization layers. In contrast, AutoInit is topology-agnostic: It adapts to any of these changes automatically.

Zhang, Dauphin, and Ma (2019) introduced Fixup, an initialization method that rescales residual branches to stabilize training. Fixup replaces batch normalization in standard and wide residual networks (Ioffe and Szegedy 2015; He et al. 2016a,b; Zagoruyko and Komodakis 2016) and replaces layer normalization (Ba, Kiros, and Hinton 2016) in transformer models (Vaswani et al. 2017). The disadvantages of this scheme are that it only applies to residual architectures, needs proper regularization to get optimal performance, and requires additional learnable scalars that slightly increase model size.

Arpit, Campos, and Bengio (2019) proposed a new initialization scheme for weight-normalized networks (Salimans and Kingma 2016) that relies on carefully scaling weights, residual blocks, and stages in the network. Like related approaches, this technique improves performance in specific cases, but imposes design constraints, like requiring ReLU activation functions and a specific Conv \( \rightarrow \text{ReLU} \rightarrow \text{Conv} \) block structure.

Just as tanh-inspired weight initialization does not stabilize training of ReLU networks, initialization schemes designed for non-residual networks fail with ResNets (Hanin and Rolnick 2018; Bachlechner et al. 2020; Brock, De, and Smith 2021). This observation suggests that future classes of neural networks will again require developing new weight initializations. Additionally, practitioners with models that do not fit neatly within the restricted settings of existing weight initialization research are left to derive their own initialization or use a suboptimal one. For example, many initialization schemes assume that the activation function is ReLU (He et al. 2015; Taki 2017; Arpit, Campos, and Bengio 2019; Zhang, Dauphin, and Ma 2019; De and Smith 2020). Indeed, ReLU is currently the most popular activation function (Nwankpa et al. 2018; Apicella et al. 2021), but it is not the best choice in every case. ReLU prevents dynamical isometry (Saxe, McClelland, and Ganguli 2013; Pennington, Schoenholz, and Ganguli 2017), weakens adversarial training (Xie et al. 2020), and results in poorer accuracy compared to other activation functions in certain tasks (Bingham and Miikkulainen 2022). A general weight initialization strategy that does not impose architectural constraints and achieves good performance in diverse settings is needed. AutoInit is designed to meet this challenge.

**Layer-Dependent Initialization** Hendrycks and Gimpel (2016) noted that dropout layers (Srivastava et al. 2014) also affect the variance of forward-propagated signals in a network. To stabilize training properly, it is necessary to take dropout layers and the specific dropout rate into account in weight initialization. In fact, pooling, normalization, recurrent, padding, concatenation, and other layer types affect the signal variance in a similar way, but current initialization schemes do not take this effect into account. AutoInit is designed to adapt to each of these layer types dynamically, and can be extended to include new layer types as they are introduced in the future.

**Data-Dependent Initialization** Mishkin and Matas (2015) fed data samples through a network and normalized the output of each layer to have unit variance. Krähenbühl et al. (2015) adopted a similar approach, but opted to normalize along the channel dimension instead of across an entire layer. Data-dependent weight initializations are most similar in spirit to AutoInit; they rely on empirical variance estimates derived from the data in order to be model-agnostic. However, data-dependent weight initializations introduce a computational overhead (Mishkin and Matas 2015), and are not applicable in settings where data is not available or its distribution may shift over time, such as online learning or reinforcement learning. The quality of the initialization is also dependent on the number of the data samples chosen, and suffers when the network is very deep (Zhang, Dauphin, and Ma 2019). AutoInit instead uses an analytic approach for greater efficiency and higher accuracy.
3 Neural Network Signal Propagation

AutoInit aims to stabilize signal propagation throughout an entire neural network. More precisely, consider a layer that shifts its input by $\alpha$ and scales the input by a factor of $\beta$. Given an input signal with mean $\mu_{\text{in}}$ and variance $\nu_{\text{in}}$, after applying the layer, the output signal will have mean $\mu_{\text{out}} = \alpha + \beta \mu_{\text{in}}$ and variance $\nu_{\text{out}} = \beta^2 \nu_{\text{in}}$. In a deep network in which the layer is applied $L$ times the effect is compounded and the signal at the final layer has mean and variance

$$
\mu_{\text{out}} = \beta^L \mu_{\text{in}} + \alpha \beta^{L-1} + \cdots + \beta + 1,
\nu_{\text{out}} = \beta^2 \nu_{\text{in}}.
$$

(1)

If $|\beta| > 1$, the network will suffer from a mean shift and exploding signals as it increases in depth:

$$
\lim_{L \to \infty} \mu_{\text{out}} = \infty, \quad \lim_{L \to \infty} \nu_{\text{out}} = \infty.
$$

(2)

In the case that $|\beta| < 1$, the network will suffer from a mean shift and vanishing signals:

$$
\lim_{L \to \infty} \mu_{\text{out}} = \alpha / (1 - \beta), \quad \lim_{L \to \infty} \nu_{\text{out}} = 0.
$$

(3)

AutoInit calculates analytic mean- and variance-preserving weight initialization so that $\alpha = 0$ and $\beta = 1$, thus avoiding the issues of mean shift and exploding/vanishing signals.

4 The AutoInit Framework

AutoInit is a general framework that adapts to different layer types. Its implementation is outlined in Algorithm 1. Given a layer in a neural network receives as its input a tensor $x$ with mean $\mu_{\text{in}}$ and variance $\nu_{\text{in}}$. After applying the layer, the output tensor has mean $\mu_{\text{out}} = E(layer(x))$ and variance $\nu_{\text{out}} = \text{Var}(layer(x))$. The function $g_{\text{layer}}$ maps input mean and variance to output mean and variance when the layer is applied:

$$
g_{\text{layer}} : (\mu_{\text{in}}, \nu_{\text{in}}) \mapsto (\mu_{\text{out}}, \nu_{\text{out}}).
$$

(4)

Note that $g$ in Equation 4 depends on the type of layer; e.g. $g_{\text{dropout}}$ and $g_{\text{ReLU}}$ are different functions. For layers with trainable weights, the mean and variance mapping will depend on those weights. For example, the function $g_{\text{Conv2D}, \theta}$ maps input mean and variance to output mean and variance after the application of a Conv2D layer parameterized by weights $\theta$. Deriving $g$ for all layers makes it possible to model signal propagation across an entire neural network. Thus, if $\mu_{\text{in}}$ and $\nu_{\text{in}}$ are known, it is natural to calculate initial weights $\theta$ such that the layer output will have zero mean and unit variance.

For example, for Conv2D layers, one possibility is

$$
\theta \sim \mathcal{N}\left(0, 1/\sqrt{\nu_{\text{in}}(\nu_{\text{in}} + \mu_{\text{in}}^2)}\right) \implies g_{\text{Conv2D}, \theta}(\mu_{\text{in}}, \nu_{\text{in}}) = (0, 1).
$$

(5)
Results Figure 1 shows the performance of the network with the default initialization and with AutoInit in these different settings. In sum, AutoInit improved performance in every hyperparameter variation evaluated. As Figure 2 shows, AutoInit is adaptive. It alters the initialization to account for different activation functions and dropout rates automatically. AutoInit is also robust. Even as other hyperparameters like learning rate and weight decay change, AutoInit still results in a higher performing network than the default initialization. The results thus suggest that AutoInit provides an improved default initialization for convolutional neural networks.

6 Stability in Deep ResNets

This section expands the experimental analysis of AutoInit to residual networks, focusing on preactivation residual networks of various depths (He et al. 2016b). The training setup is standard unless explicitly stated otherwise (Appendix B). In particular, the initialization is “He Normal” (He et al. 2015), where weights are sampled from \( \mathcal{N}(0, \sqrt{2/fan_{in}}) \).

Visualizing Signal Propagation Figure 3 shows how the signal variance changes with depth. With ResNet-56, the variance increases where the shortcut connection and residual branch meet, and the variance drops whenever ReLU is applied. Although the variance increases exponentially with the default initialization and linearly with AutoInit (note the log scale on the y axis), training is still stable because batch normalization layers return the signal to variance 1.0. Without batch normalization, the signal variance never stabilizes under the default initialization. In contrast, removing batch normalization is not an issue with AutoInit; the signal variance remains stable with depth.

With the deeper ResNet-164 and ResNet-812 networks, the conclusions are similar but more pronounced. In the case of ResNet-812 without batch normalization, the signals explode so severely that they exceed machine precision. AutoInit avoids this issue entirely.
Stable Initial Learning Rates Exploding or vanishing signals make optimization difficult because they result in gradient updates that are too large to be accurate or too small to be meaningful. This phenomenon can be observed when the network does not exceed chance accuracy. Therefore, a simple way to quantify whether a weight initialization is effective is to observe a network’s performance after a few epochs.

Using this metric, AutoInit was compared against the default initialization by training unnormalized versions of ResNet-56, ResNet-164, and ResNet-812 for five epochs with a variety of learning rates. With the default initialization, ResNet-56 requires a learning rate between $10^{-8}$ and $0.5 \times 10^{-3}$ to begin training, but training was not possible with ResNet-164 or ResNet-812 because of exploding signals (Figure 4a). AutoInit stabilizes training for all three networks, and its effect does not diminish with depth. The networks remain stable with higher learning rates between $10^{-4}$ and $0.05$. Such rates speed up learning, and also correlate with better generalization performance (Jastrzebski et al. 2017; Smith et al. 2018; Smith and Le 2018).

Full ResNet Training In the third residual network experiment, ResNet-164 was trained to completion on CIFAR-10 with different learning rate schedules. All schedules included a linear warm-up phase followed by a decay to zero using cosine annealing (Loshchilov and Hutter 2016).

Figure 4b displays the performance with a variety of such schedules. When the best learning rate schedules are used, ResNet-164 achieves comparable performance with the default initialization and with AutoInit. However, when a suboptimal schedule is used, performance degrades more quickly with the default initialization than it does with AutoInit. Without batch normalization, the network requires proper weight initialization for stability. In this case, ResNet-164 with the default initialization fails to train regardless of the learning rate schedule, whereas AutoInit results in high accuracy for the majority of them.

Together, the experiments in this section show that AutoInit is effective with deep networks. It prevents signals from exploding or vanishing, makes it possible to use larger learning rates, and achieves high accuracy, with and without batch normalization.

7 High-Resolution Images with Transformers

This section extends AutoInit to transformer architectures and applies them to high-resolution image classification. Specifically, AutoInit is applied to CoAtNet, a model that combines convolutional and attention layers (Dai et al. 2021). The model is trained on Imagenette, a subset of 10 classes from the ImageNet dataset (Howard 2019; Deng et al. 2009). Imagenette allows evaluating AutoInit in a high-resolution setting.

| CoAtNet       | GELU | ReLU | SELU | Swish | No BN |
|---------------|------|------|------|-------|-------|
| Default Init. | 89.38| 89.22| 86.09| 88.69 | -     |
| Glorot Norm.  | 91.44| 91.54| 87.59| 90.42 | 85.89 |
| Glorot Unif.  | 91.16| 91.18| 88.25| 90.06 | 85.73 |
| He Normal     | 88.48| 88.05| 86.11| 88.36 | -     |
| He Uniform    | 88.66| 87.87| 86.37| 88.41 | -     |
| LeCun Norm.   | 91.11| 90.57| 87.80| 90.83 | -     |
| LeCun Unif.   | 90.55| 90.65| 87.67| 90.57 | -     |
| AutoInit      | 92.48| 92.15| 86.80| 92.28 | 85.73 |

Table 1: CoAtNet top-1 accuracy on Imagenette, shown as median of three runs. The first four experiments vary the activation function, while the fifth removes all normalization layers from the architecture. A “−” indicates that training diverged. AutoInit produces the best model in three of the five settings, and remains stable even without normalization layers.
| Number of samples $S$ used for LSUV initialization | Test Accuracy |
|--------------------------------------------------|---------------|
| 0.0                                              | 0.75          |
| 0.2                                              | 0.80          |
| 0.4                                              | 0.85          |
| 0.6                                              | 0.90          |
| 0.8                                              | 0.95          |

Table 2: ResNet-50 top-1 and top-5 validation accuracy on ImageNet. AutoInit improves performance, even with large and challenging datasets.

Figure 5: Mean CIFAR-10 test accuracy for AutoInit vs. LSUV with different numbers of samples $S$. Each evaluation is repeated 10 times; the shaded area shows the max and min accuracy among all trials. AutoInit is consistent, but LSUV struggles when $S$ is small or the network is deep.

resolution image classification task with a $132 \times$ smaller carbon footprint than the full ImageNet dataset would (Appendix F). As Table 1 shows, AutoInit outperforms six commonly used initialization schemes as well as the default initialization, which initializes convolutional layers from $\mathcal{N}(0, \sqrt{2/fan\_out})$ and fully-connected layers from $\mathcal{U}\left(-\frac{\sqrt{\sigma}}{\sqrt{fan\_in+fan\_out}}, \frac{\sqrt{\sigma}}{\sqrt{fan\_in+fan\_out}}\right)$. Furthermore, AutoInit stabilizes the network even when normalization layers are removed, suggesting that it is a promising candidate towards developing normalization-free transformer architectures. Full experiment details are in Appendix B.

### 8 Scaling up to ImageNet

In order to compliment the results from Section 7 and demonstrate that AutoInit can scale to more difficult tasks, ResNet-50 was trained from scratch on ImageNet with the default initialization and with AutoInit. As Table 2 shows, AutoInit improves top-1 and top-5 accuracy in this task as well. Full training details are in Appendix B.

### 9 Contrast with Data-Dependent Initialization

The layer-sequential unit-variance (LSUV) algorithm is the most natural data-dependent initialization comparison to AutoInit because both approaches aim to scale the weights appropriately in an architecture-agnostic way. LSUV pre-initializes the weights with an existing approach, feeds $S$ training samples through the network, and adjusts the scale of the weights so that each layer’s output variance is approximately one (Mishkin and Matas 2015).

Data-dependent initialization is time-consuming for large $S$ (indeed, even $S = 1$ is used in practice (Kingma and Dhariwal 2018)). However, if $S$ is too small, the samples may not reflect the statistics of the dataset accurately, leading to poor initialization. Figure 5 demonstrates this phenomenon. In some training runs LSUV matches the performance of AutoInit, but in many instances the randomly selected samples do not accurately reflect the overall dataset and performance suffers. Since AutoInit is not data-dependent, it does not have this issue. Details of this experiment are in Appendix B.

### 10 Enabling Neural Architecture Search

Sections 5 through 9 demonstrated that AutoInit works well for convolutional, residual, and transformer networks with a variety of hyperparameter values and depths. In this section, the results are extended to a broader variety of network topologies and types of tasks, for two reasons. First, whereas custom weight initialization may be developed by hand for the most popular machine learning benchmarks, it is unlikely to happen for a variety of architectures and tasks beyond them. Second, as new types of neural network designs are developed in the future, it will be important to initialize them properly to reduce uncertainty in their performance. This section evaluates the generality of AutoInit by applying it to the variety of networks generated in a neural architecture search process with five types of tasks.

#### The CoDeepNEAT Architecture Search Method

Neural networks are evolved using CoDeepNEAT (Liang et al. 2019; Miikkulainen et al. 2019). CoDeepNEAT extends previous work on evolving network topologies and weights (Moriarty and Miikkulainen 1997; Stanley and Miikkulainen 2002) to the level of evolving deep learning architectures. Utilizing a cooperative coevolution framework (Potter and Jong 2000), CoDeepNEAT evolves populations of modules and blueprints simultaneously. Modules are small neural networks, complete with layers, connections, and hyperparameters. Blueprints are computation graphs containing only nodes and directed edges. To create a candidate neural network, CoDeepNEAT chooses a blueprint and replaces its nodes with selected modules. This mechanism makes it possible to evolve deep, complex, and recurrent structures, while taking advantage of the modularity often found in state-of-the-art models. In addition to the network structure, CoDeepNEAT evolves hyperparameters like dropout rate, kernel regularization, and learning rate. The network weights are not evolved, but instead trained with gradient descent. The generality of CoDeepNEAT helps minimize human design biases and makes it well-suited to
analyzing AutoInit’s performance in a variety of open-ended machine learning settings.

Tasks Using CoDeepNEAT, networks are evolved for their performance in vision (MNIST), language (Wikipedia Toxicity), tabular (PMLB Adult), multi-task (Omniglot), and transfer learning (Oxford 102 Flower) tasks (Appendix C).

Results Figure 7a shows how CoDeepNEAT discovers progressively better networks over time on the five tasks. Evolution often selects different weight initialization strategies for the different layers in these networks, so this scheme is already a flexible and powerful baseline. However, by accounting for each model’s unique topology and hyperparameters, AutoInit outperforms the baseline in four of the five tasks, and matches it in the fifth.

Beyond performance, three interesting phenomena can be observed. First, the mean population fitness varies greatly with the default initialization in each task, sometimes dropping significantly from one generation to the next (Figure 9 in Appendix C). Though some variation is natural in a stochastic evolutionary process like CoDeepNEAT, AutoInit makes the discovery process more reliable by stabilizing the performance of the entire population.

Second, hyperparameters play a large role in the final performance of the dense networks, in particular in the “Oxford 102 Flower” task. While CoDeepNEAT discovers good models with both initialization strategies, performance is consistently higher with AutoInit. This finding agrees with Section 5, where AutoInit was shown to be robust to different hyperparameter values.

Third, while many networks exhibit motifs popular in existing architectures, such as alternating convolution and dropout layers and utilizing residual connections, other phenomena are less common (Figure 7b). For example, the networks make use of different activation functions and contain several unique information processing paths from the input to the output. Because AutoInit provides effective initialization in each of these cases, it allows for taking full advantage of unusual design choices that might otherwise hurt performance under default initialization schemes.

The results in this section suggest that AutoInit is an effective, general-purpose algorithm that provides a more effective initialization than existing approaches when searching for new models.

11 Enabling Activation Function Discovery
As new activation functions are developed in the future, it will be important to adjust weight initialization to maintain stable signal propagation. Since AutoInit makes this adjustment automatically, it is well-suited to the task. Indeed, Figure 1 confirmed that AutoInit improves performance with several existing activation functions. This section presents a more challenging task. To simulate future research in activation function design, hundreds of novel activation functions were generated as arbitrary computation graphs and trained with a CNN. AutoInit’s ability to initialize each of these networks was then evaluated. The method for creating such activation functions is described first, followed by experimental details, and results on stability, performance, and generality.

Creating Novel Activation Functions An important area of automated machine learning (AutoML) is to discover new, better activation functions (Basirat and Roth 2018; Ramachandran, Zoph, and Le 2018; Bingham, Macke, and Miikkulainen 2020; Liu et al. 2020). Among existing approaches, PANGAEA (Bingham and Miikkulainen 2022) has the most flexible search space and is therefore used to generate new functions in this section.

PANGAEA represents activation functions as computation graphs containing unary and binary operators (Figure 6). Creating a novel activation function involves three steps. First, a minimal computation graph is initialized with randomly selected unary and binary operators. Second, the functional form of the activation function is modified by applying three random mutations to increase diversity. Third, the function is augmented with up to three learnable parameters. These parameters are analogous to those in other parametric activation functions, such as PReLU (He et al. 2015); they are initialized to one and learned during training by gradient descent. Through this process, it is possible to understand to what extent AutoInit can improve performance with activation functions that have yet to be discovered.

Experimental Setup An important insight in this domain is that in addition to modifying the variance of the signals
in a network, activation functions can induce mean shifts. Prior work encouraged stability by reparameterizing the weights to have zero empirical mean (Huang et al. 2017; Qiao et al. 2019; Brock, De, and Smith 2021). An alternative and more direct approach is to modify the activation function itself so that it does not cause a mean shift in the first place. Given an activation function $f$ with Gaussian mean $\mu_f = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)e^{-x^2/2}dx$, this goal can be accomplished with $\hat{f} := f - \mu_f$, which has zero Gaussian mean. To take advantage of this idea, a version of AutoInit called AutoInit++ was created for this domain, thus extending AutoInit slightly beyond weight initialization.

Thus, three initialization strategies were compared. With the default initialization, weights were sampled from $\mathcal{U} \left( -\frac{\sqrt{6}}{\sqrt{\text{fan} \text{ in} + \text{fan} \text{ out}}} , \frac{\sqrt{6}}{\sqrt{\text{fan} \text{ in} + \text{fan} \text{ out}}} \right)$ (Glorot and Bengio 2010). With AutoInit, the weights were sampled from $\mathcal{N} \left(0, 1/\sqrt{\text{fan} \text{ in} \mu_f}\right)$ to account for an arbitrary activation function $f$; the dropout adjustment (Section 5) was not used. Finally, AutoInit++ takes advantage of $\hat{f}$ as described above, but is otherwise identical to AutoInit.

For each initialization strategy, 200 activation functions were created using the PANGAEA process. Each activation function was used with the All-CNN-C architecture on the CIFAR-10 dataset following the standard training setup. To avoid overfitting to the test set when evaluating such a large number of activation functions, the accuracy with a balanced validation set of 5000 images is reported instead.

**Stability** Achieving better-than-chance accuracy is a useful metric of training stability (Section 6). As shown in Figure 8a, many activation functions result in chance accuracy regardless of how the network is initialized. This phenomenon is not surprising; since the activation functions are arbitrary computation graphs, many of them will turn out to be poor. With the default initialization strategy, 149 activation functions caused training to fail in this way. With AutoInit, the number of failed activation functions dropped to 130, and with AutoInit++, it further decreased to 117. AutoInit and AutoInit++ thus make training more stable, allowing it to succeed for a greater number of activation functions.

**Performance** Beyond training stability, a good weight initialization should also improve performance. As a baseline, when trained with ReLU and the default initialization, All-CNN-C achieved 89.10% test accuracy. Twenty-two of 200 activation functions from the PANGAEA search space outperformed this accuracy with the default initialization. With AutoInit, this number increased to 26, and with AutoInit++, to 50—a notable improvement. Thus, with the default initialization, one can naively create a randomly generated computation graph activation function and have roughly a one in nine chance of outperforming ReLU, but with AutoInit++, this probability increases to one in four.

Indeed, the Mann-Whitney U test (Mann and Whitney 1947) concludes that the distribution of accuracies induced by AutoInit++ is stochastically larger than that from AutoInit ($p < 0.05$) or the default initialization ($p < 0.01$). This result means that for any level of performance, it is always more probable to discover an activation function that achieves that level of performance when initializing with AutoInit++ versus AutoInit or the default initialization. The result implies that activation function researchers who properly initialize their networks are more likely to discover state-of-the-art activation functions, while staying with the default initialization may hinder that research effort. More detailed statistical significance analyses are included in Appendix D.

**Generality** Figure 8b plots several activation functions from the PANGAEA search space. Many discovered functions have similar shapes to existing functions. However, others are nonmonotonic, have discontinuous derivatives, or saturate to nonzero values. These properties are less common in existing activation functions. This observation suggests that AutoInit is a general approach that does not depend on a specific type of activation function; it may therefore serve as a useful tool in developing new such functions in the future.

### 12 Future Work

**Experiments in Other Domains** The experiments in this paper demonstrate that AutoInit can improve performance.
in a variety of settings, suggesting that it can be applied to other domains as well. For instance in reinforcement learning, good estimates of activation statistics are usually not available due to the online nature of the algorithm. It is not possible to stabilize training using e.g. batch normalization, but it may be possible to do it with AutoInit. Similarly, training of generative adversarial networks (Goodfellow et al. 2014) is often unstable, and proper initialization may help. Applying AutoInit to such different domains should not only make them more reliable, but also lead to a better understanding of their training dynamics.

**Accelerating Model Search** In Sections 10 and 11, AutoInit was shown to facilitate the discovery of better neural network designs and activation functions. This ability is possible because AutoInit is a general method, i.e. not restricted to a single class of models, and it could similarly augment other meta-learning algorithms (e.g. those reviewed by Elsken, Metzen, and Hutter 2019; Wistuba, Rawat, and Pedapati 2019).

However, this finding points to an even more promising idea. As model search techniques become more prevalent in real-world applications, it will be most worthwhile to derive general principles rather than specific instantiations of those principles. For example, past weight initialization strategies improved performance with specific activation functions through manual derivation of appropriate weight scaling (Section 2). In contrast, AutoInit is a general method, leveraging Gaussian quadrature for any activation function. Similarly, AutoInit resulted in better initialization than strategies discovered by CoDeepNEAT through evolution (Section 10). Further, AutoInit++ (Section 11), rather than producing a few high-performing activation functions, introduces the general property that activation functions with zero Gaussian mean \( \langle f \rangle \) tend to perform well. This property discovered a highly diverse set of powerful activation functions in the PANGAEA search space (Figure 8).

Thus, AutoInit is successful because it is not a single initialization strategy, but rather a mapping from architectures to initialization strategies. Such mappings, whether focused on initialization or some other aspect of model design, deserve increased attention in the future. They can lead to performance gains in a variety of scenarios. They also accelerate model search by focusing the search space to more promising regions. If one does not have to worry about discovering a good initialization, compute power can instead be used in other areas, like designing architectures and activation functions. Thus, general tools like AutoInit will save time and resources, and lead to better models as a result.

Further technical extensions to AutoInit are outlined in Appendix E, including variations on the core AutoInit algorithm, support for new layer types, and integration with deep learning frameworks.

**13 Conclusion**

This paper introduced AutoInit, an algorithm that calculates analytic mean- and variance-preserving weight initialization for neural networks automatically. In convolutional networks, the initialization improved performance with different activation functions, dropout rates, learning rates, and weight decay settings. In residual networks, AutoInit prevented exploding signals, allowed training with higher learning rates, and improved performance with or without batch normalization. In transformers, AutoInit was scaled up to high-resolution image classification, and improved performance with several activation functions with and without normalization. AutoInit also improved accuracy on the ImageNet dataset. The initialization is independent of data and is therefore efficient and reliable. AutoInit’s generality proved instrumental in two types of AutoML. In neural architecture search, new architectures were evaluated more accurately, resulting in better networks in vision, language, tabular, multi-task, and transfer learning settings. In activation function discovery, AutoInit stabilized training and improved accuracy with a large diversity of novel activation functions. Thus, AutoInit serves to make machine learning experiments more robust and reliable, resulting in higher performance, and facilitating future research in AutoML.

**Appendices**

The following appendices can be found online at https://arxiv.org/abs/2109.08958. A survey on modern trainable activation functions.

**A: Mean and Variance Estimation for Different Layer Types**

In Appendix A, the following layer types are covered by CoDeepNEAT through evolution (Section 10).

**B: Convolutional, Residual, and Transformer Network Experiment Details**

**C: Neural Architecture Search Experiment Details**

**D: Statistical Significance of Results in Activation Function Meta-Learning**

**E: Future Work**

**F: Computing Infrastructure**

In particular, Appendix A contains mean and variance mapping functions \( g \) for the following layer types:

**Weights** Conv\{1D,2D,3D\}, DepthwiseConv\{1D,2D\}, Dense

**Activation Functions** Activation, elu, exponential, gelu, hard sigmoid, LeakyReLU, linear, PReLU, ReLU, selu, sigmoid, softplus, softsign, swish, tanh, ThresholdedReLU

**Dropout** SpatialDropout\{1D,2D,3D\}, Dropout

**Pooling** AveragePooling\{1D,2D,3D\}, MaxPooling\{1D,2D,3D\}, GlobalAveragePooling\{1D,2D,3D\}, GlobalMaxPooling\{1D,2D,3D\}

**Normalization** BatchNormalization, GroupNormalization, LayerNormalization

**Arithmetic** Add, Average, Subtract, Multiply, tf.matmul, tf.reduce_mean, tf.reduce_sum

**Recurrent** GRU, LSTM, SimpleRNN

**Shape Adjustment** Flatten, Permute, Reshape, Upsampling\{1D,2D,3D\}, Cropping\{1D,2D,3D\}, tf.reshape, tf.split, tf.transpose

**Misc** Concatenate, ZeroPadding\{1D,2D,3D\}, InputLayer

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