Neutrino scattering off a black hole surrounded by a magnetized accretion disk

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Abstract. We study the neutrino scattering off a rotating black hole with a realistic accretion disk permeated by an intrinsic magnetic field. Neutrino trajectories in curved spacetime as well as the particle spin evolution in dense matter of an accretion disk and in the magnetic field are accounted for exactly. We obtain the fluxes of outgoing ultrarelativistic neutrinos taking into account the change of the neutrino polarization owing to spin oscillations. Using the conservative value of the neutrino magnetic moment and realistic radial distributions of the matter density and the magnetic field strength, we get that these fluxes are reduced by several percent compared to the case when no spin oscillations are accounted for. In some situations, there are spikes in the neutrino fluxes because of the neutrino interaction with the rotating plasma of an accretion disk. Taking into account the uncertainties in the astrophysical neutrino fluxes, the predicted effects turn out to be quite small to be observed with the current neutrino telescopes.

Keywords: astrophysical black holes, neutrino astronomy, magnetic fields, accretion

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1 Introduction

After numerous successful neutrino oscillations experiments (see, e.g., refs. [1–3]), we believe that neutrinos are massive particles and there is a mixing between different neutrino types. These neutrino characteristics are the indications to physics beyond the standard model. It is also known [4] that neutrino interaction with external fields can modify the process of neutrino oscillations. For example, the neutrino interaction with background matter results in the amplification of the transition probability of neutrino flavor oscillations, which, in its turn, is the most plausible solution to the solar neutrino problem [5]. Neutrino interaction with an electromagnetic field results in transitions between neutrino states belonging to different helicities, i.e. in spin and/or spin-flavor oscillations [6].

In the present work, we study neutrino spin oscillations, i.e. the transitions $\nu_L \rightarrow \nu_R$ within the same neutrino flavor under the influence of external fields. Neutrinos are produced as left polarized particles in the standard model. If the neutrino helicity changes while a particle propagates, a detector registers the reduced neutrino flux. As a rule, neutrino spin oscillations happen in a magnetic field owing to the presence of a nonzero magnetic moment [6]. Thus, we suppose that neutrinos are Dirac particles. Despite the significant experimental progress (see, e.g., ref. [7]), the question on the neutrino nature is still open [8].

The gravitational interaction also can induce the change of the polarization of a spinning particle [9], including that of neutrinos [10]. The evolution of a fermion spin in curved spacetime under the various external fields was examined in multiple works (see, e.g., refs. [11, 12]). We studied neutrino spin oscillations in curved spacetime in refs. [13, 14] applying different approaches. In refs. [13, 14], we considered the neutrino spin evolution both in static gravitational backgrounds, like that of a black hole (BH), and in time dependent gravitational fields, such as gravity waves.

In the present work, we discuss spin effects in the neutrino scattering off BH. This problem is important since we can fix neutrino helicities for in- and out-states because they are in the flat spacetime asymptotically. Thus, spin oscillations in the neutrino gravitational scattering is a more realistic problem compared to the case when a neutrino is captured gravitationally by BH [11, 13]. Note that the spin-flip of neutrinos in their gravitational scattering was studied in ref. [15] using the weak field approximation.
The study of the present work is motivated by the recent observation of the event horizon silhouette of a supermassive BH (SMBH) [16]. An accretion disk, which typically surrounds BH, is a source of photons, which form its visible image. The same disk can emit neutrinos (see, e.g., ref. [17]), which are gravitationally lensed and potentially observed by a neutrino telescope. One expects not only the gravitational lensing of these neutrinos, but also their spin precession in strong external fields in the vicinity of BH. The latter effect decreases the observed neutrino flux. It should be noted that the neutrino emission by an accretion disk is effective if the matter density is high [17]: \( \rho \sim (10^{11}–10^{12}) \text{ g} \cdot \text{cm}^{-3} \). This scenario is implemented neither for SMBH in our galaxy (Sgr A*) nor in the center of M87. It can happen for BH with a stellar mass entering in a binary system.

Moreover, we can imagine that a supernova explodes in our galaxy. The flux of emitted neutrinos is lensed gravitationally by SMBH in the center of the Galaxy before particles arrive to the Earth. Spin oscillations of these neutrinos will modify the neutrino flux observed in a neutrino telescope. Although this situation is very improbable, the possibility of lensing of such neutrinos is discussed in ref. [18].

This work is organized in the following way. In section 2, we derive the main equations which the neutrino spin obeys while a particle interacts with external fields in curved spacetime. We adapt them for the problem of the neutrino scattering off BH. The effective Schrödinger equation for neutrino spin oscillations is derived. Then, in section 3, we fix the parameters of a neutrino and SMBH, which correspond to a realistic situation. Finally, in section 4, we present the neutrino fluxes, which are measured by a terrestrial detector, obtained in the numerical solution of the Schrödinger equation derived in section 2. These fluxes account for the neutrino interaction with an accretion disk and the magnetic field in curved spacetime. The derivation of the electromagnetic field in the vicinity of BH is briefly outlined in appendix A.

2 Spin evolution in the neutrino scattering

In this section, we derive the effective Hamiltonian for the spin evolution of a neutrino scattered off a rotating BH surrounded by an accretion disk and embedded to the external electromagnetic field \( F_{\mu\nu} \), which is defined in world coordinates. The electromagnetic interaction is owing to the presence of a nonzero neutrino magnetic moment. The neutrino electroweak interaction with background matter is in the forward scattering approximation. We consider only the equatorial neutrino motion.

The metric of spacetime of a rotating BH has the form [19],

\[
ds^2 = g_{\mu\nu}dx^\mu dx^{\nu} = \left(1 - \frac{r_g}{\Sigma}\right)dt^2 + 2\frac{r_g a}{\Sigma}dtd\phi - \frac{\Sigma}{\Delta}dr^2 - \Sigma d\theta^2 - \frac{\Xi}{\Sigma} \sin^2 \theta d\phi^2, \tag{2.1}\]

where

\[
\Delta = r^2 - r_g a^2, \quad \Sigma = r^2 + a^2 \cos^2 \theta, \quad \Xi = \left(r^2 + a^2\right)\Sigma + r_g a^2 \sin^2 \theta. \tag{2.2}\]

We use the Boyer-Lindquist coordinates \( x^\mu = (t, r, \theta, \phi) \) in eqs. (2.1) and (2.2). We suppose that the Newton constant equals unity. In this units, the mass of BH is \( M = r_g/2 \) and its angular momentum \( J = Ma \). Here \( r_g \) is the Schwarzschild radius. The parameter \( a \leq r_g/2 \).

According to refs. [13, 20], the evolution of a neutrino spin is defined in the locally Minkowskian frame \( x^a = e^a_\mu x^\mu \), where \( e^a_\mu = \partial x^a / \partial x^\mu \), \( a = 0, \ldots, 3 \), are the vierbein vectors.
These vectors are chosen in such a way to diagonalize the metric in eq. (2.1), \( g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab} \), where \( \eta_{ab} = \text{diag} (1, -1, -1, -1) \) is the Minkowski metric tensor. One can check by the direct calculation that the following vectors:

\[
e^0_\mu = \left( \sqrt{\frac{\Xi}{\Sigma \Delta}}, 0, 0, \frac{a r r_g}{\sqrt{\Delta \Sigma \Xi}} \right), \quad e^1_\mu = \left( 0, \sqrt{\frac{\Xi}{\Sigma}}, 0, 0 \right),
\]

\[
e^2_\mu = \left( 0, 0, \frac{1}{\sqrt{\Sigma}}, 0 \right), \quad e^3_\mu = \left( 0, 0, 0, \frac{1}{\sin \theta} \sqrt{\frac{\Sigma}{\Xi}} \right),
\]

satisfy the relation \( \eta_{ab} = e^a_\mu e^b_\nu g_{\mu\nu} \).

The four vector of the spin \( s^a = e^a_\mu S^\mu \), where \( S^\mu \) is the spin vector in world coordinates, in the locally Minkowskian frame for a neutrino interacting with the gravitational and electromagnetic fields, as well as with a background matter, obeys the equation \([21]\),

\[
d s^a \frac{dt}{d} = \frac{1}{U_f} \left[ G^{ab} s_b + 2 \mu \left( f^{ab} s_b - u^a u_b f^{bc} s_c \right) + \sqrt{2} G_f \varepsilon^{abcd} g_{bu} u_{cd} \right],
\]

where \( G_{ab} = \gamma_{abc} u^c \) is the antisymmetric tensor accounting for the gravitational interaction of neutrinos, \( \gamma_{abc} = \eta_{ad} e^d_\mu e^b_\nu e^c_\rho \) are the Ricci rotation coefficients, the semicolon stays for the covariant derivative, \( g^a = e^a_\mu G^\mu = (g^0, g) \) is the effective potential of the neutrino matter interaction in the locally Minkowskian frame, \( f_{ab} = e^a_\mu e^b_\nu F^\mu = (e, b) \) is the electromagnetic field tensor in the locally Minkowskian frame, \( u^a = (u^0, u) = e^a_\mu U^\mu \), \( U^\mu = (U^t, U^r, U^\theta, U^\phi) = dx^\mu / ds \) is the neutrino velocity in the world coordinates, \( \varepsilon^{abcd} \) is the absolute antisymmetric tensor in Minkowski spacetime having \( \varepsilon^{0123} = +1 \), \( \mu \) is the neutrino magnetic moment, and \( G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2} \) is the Fermi constant.

We suppose that the background matter is an electroneutral hydrogen plasma with \( n_e = n_p \), where \( n_{e,p} \) are the number densities of electrons and protons. It is assumed to be unpolarized and move as a whole. In this case \([22]\), \( G^\mu = n_e U^\mu_f \), where \( U^\mu_f \) is the macroscopic four velocity of plasma defined in the world coordinates. Here \( n_e \) is the invariant electron number density given in the rest frame of plasma.

We consider plasma forming an accretion disk, in which particles move around BH on circular orbits with the radius \( r \). In this case, \( U^r_f = 0 \) and \( U^\phi_f = 0 \). Using the results of ref. \([23]\), one gets for the remaining nonzero components,

\[
U^t_f = \frac{\sqrt{2} x^{3/2} + \lambda z}{2 x^3 - 3 x^2 + 2 \lambda \sqrt{2} x^{3/2}}, \quad U^\phi_f = \frac{1}{r_g} \frac{\lambda}{\sqrt{2 x^3 - 3 x^2 + 2 \lambda \sqrt{2} x^{3/2}}},
\]

where \( x = r / r_g \) and \( z = a / r_g \). The parameter \( \lambda \) corresponds to a disk, which corotates (\( \lambda = +1 \)) or counter-rotates (\( \lambda = -1 \)) BH.

Instead of the four vector \( s^a \), we deal with the invariant three vector \( \zeta \) of the neutrino spin, which describes the polarization in the neutrino rest frame. These spin vectors are related by the following expression:

\[
s^a = \left( (\zeta \cdot u), \zeta + \frac{u (\zeta \cdot u)}{1 + u^0} \right).
\]

Using eqs. (2.4) and (2.5), we get the equation for \( \zeta \) in the form,

\[
\frac{d\zeta}{dt} = 2 (\zeta \times \Omega),
\]
where

\[
\Omega = \frac{1}{U_t} \left\{ \frac{1}{2} \left[ b_g + \frac{1}{1 + w_0^2} (e_g \times u) \right] + \frac{G_F}{\sqrt{2}} \left[ u \left( \frac{g^0}{1 + w_0^2} - g \right) \right] + \mu \left[ u^0 b - \frac{u(u b)}{1 + w_0^2} \right] \right\}.
\]  

(2.8)

Here \(e_g\) and \(b_g\) are the components of the tensor \(G_{ab}\): \(G_{ab} = (e_g, b_g)\).

The neutrino characteristics depend on the particle helicity, i.e. the projection of the particle spin on the neutrino velocity: \(h = (\zeta \cdot u)/|u|\). Hence, despite accounting for the neutrino spin evolution in eq. (2.7), we have to track the evolution of \(u\) in the neutrino scattering [20], \(\frac{du}{dt} = \frac{1}{U_t} G^{ab} u_b\). If a particle moves in the equatorial plane of BH, then the asymptotic values of \(u\) are [24], \(u_{\pm \infty} = (\pm \sqrt{E^2 - m^2}/m, 0, 0)\), where ‘+’ stays for an outgoing neutrino and ‘−’ for an incoming one, \(E\) is the neutrino energy, which is the integral of motion in the metric in eq. (2.1), and \(m\) is the neutrino mass.

Instead of solving eqs. (2.7) and (2.8), we can consider the evolution of the effective spinor \(\psi^T = (\psi_1, \psi_2)\), with the components being responsible for the particular neutrino polarizations. It obeys the effective Schrödinger equation. The coordinate \(r\), or its dimensionless analogue \(x = r/r_g\), is more convenient than \(t\) in describing the neutrino evolution in the particle scattering off BH. Using the results of refs. [24, 25], we get that

\[
\frac{i}{\hbar} \frac{d\psi}{dx} = \hat{H}_x \psi, \quad \hat{H}_x = -\mathcal{U}_2 (\sigma \cdot \Omega_x) \mathcal{U}_2^T,
\]  

(2.9)

where \(\sigma = (\sigma_1, \sigma_2, \sigma_3)\) are the Pauli matrices, \(\Omega_x = r_g \Omega \frac{dt}{dr}\), and \(\mathcal{U}_2 = \exp(i\sigma_2/4)\). One should account for the matrix \(\mathcal{U}_2\) in \(\hat{H}_x\) since the spin is quantized along the first\(^1\) rather than the third axis.

Equation (2.9) should be supplied with the initial condition. If incoming neutrinos are left polarized, i.e. \(h_{- \infty} = -1\) at \(t \to -\infty\), then \(\psi^T_{- \infty} = (1, 0)\). The transition \(P_{LR}\) (i.e., when scattered neutrinos are right polarized with \(h_{+ \infty} = +1\) and survival \(P_{LL}\) probabilities in the wake of the scattering can be found on the basis of the asymptotic solution of eq. (2.9), \(\psi^T_{+ \infty} = (\psi_{+ \infty, 1}, \psi_{+ \infty, 2})\) at \(t \to +\infty\). They are\(^2\)

\[
P_{LR} = |\psi_{+ \infty, 1}|^2 \quad \text{and} \quad P_{LL} = |\psi_{+ \infty, 2}|^2.
\]

In the following, we consider only ultrarelativistic neutrinos moving in the equatorial plane of BH. In general, there are nonrelativistic cosmic neutrinos (see, e.g., ref. [26]). However, the study of spin oscillations of such particles in questionable since one can hardly form an initial beam of left polarized nonrelativistic neutrinos in natural conditions.

We suppose that BH is embedded in the electromagnetic field which asymptotically, at \(r \to \infty\), approaches to the constant magnetic field parallel to the BH rotation axis. Realistic electromagnetic fields in the vicinity of BHs can have a complicated structure. The magnetic field in a disk has both poloidal and toroidal components. The poloidal field is generated mainly by the plasma circular motion in an accretion disk. The toroidal magnetic field results from the differential rotation of a disk. The toroidal component was mentioned in ref. [27] to be dominant in many cases. However, the main contribution to the neutrino spin-flip arises when a particle is at the minimal distance to BH: \(x = x_m\). In these moments, the toroidal component is along the neutrino velocity and does not change the neutrino polarization.

\(^1\)We recall that only \(u_{\pm \infty, 1} \neq 0\). Thus, the neutrino spin should be projected on \(x^{\alpha = 1}\) axis asymptotically at \(r \to \infty\).

\(^2\)Note that the neutrino velocity changes its direction in the scattering: \(u_{\pm \infty} = -u_{- \infty}\).
Therefore, in our work, we consider the approximation when only a poloidal magnetic field is present. An electromagnetic field in such a system was studied in ref. [28]. The details of the derivation of $F_{\mu\nu}$ are present in appendix A.

Using eqs. (2.1)–(2.3), (2.5), (2.8), and (A.2), we can write down the components of $\Omega_x$ in eq. (2.9) as

$$
\Omega_{x1} = \frac{V_m x^{3/2} \sqrt{x^4 + z^2(x + 1)} \left[ \sqrt{2x^{3/2}} + \lambda(z - y) \right]}{[x^3 + z^2(x + 1) - yz] \sqrt{2x^3 - 3x^2 + 2\lambda \sqrt{2z} x^{3/2}}},
$$

$$
\Omega_{x2} = \pm \frac{1}{2\sqrt{R(x)}} \left\{ - \frac{V_B}{x^{3/2}} \left[ 2x^3 + z^2 - yz \right] + \frac{y}{2} \left[ x^4(3 - 2x) + xyz(3 - 4x) - xz^2(3 - 3x + 2x^2) - 2yz^3 + 2z^4 \right] \right\},
$$

$$
\Omega_{x3} = \pm \frac{V_m y x^{5/2} \sqrt{x(x - 1) + z^2} \left[ \sqrt{2x^{3/2}} + \lambda(z - y) \right]}{\sqrt{R(x)[x^3 + z^2(x + 1) - yz] \sqrt{2x^3 - 3x^2 + 2\lambda \sqrt{2z} x^{3/2}}}} \right\},
$$

(2.10)

where $y = b/r_g$, $b = L/E$ is the impact parameter for ultrarelativistic neutrinos, $L$ is the neutrino angular momentum, which is the integral of motion, $V_m = G_F n_e r_g/\sqrt{2}$ and $V_B = \mu B r_g$ are the dimensionless effective potentials of the neutrino interaction with matter of an accretion disk and with a magnetic field, and $R(x) = x^3 + (z^2 - y^2) x + (y - z)^2$ is the dimensionless effective potential, which defines the range of $x = r/r_g$ variation in the neutrino scattering. The maximal root $x_m$ of the equation $R(x) = 0$ gives the minimal distance which a neutrino approaches to BH. In eq. (2.10), the ‘−’ sign corresponds to an incoming neutrino and ‘+’ one for an outgoing one.

The derivation of eq. (2.10) is straightforward but quite lengthy. It is performed in refs. [21, 25]. Therefore we omit the details here.

### 3 Astrophysical applications

In this section, we choose the parameters of BH, its accretion disk, the magnetic field, and a neutrino which correspond to a realistic situation. These parameters are used in the numerical solution of eqs. (2.9) and (2.10).

Before we study a realistic case of BH surrounded by a magnetized accretion disk, we should consider the situation when a neutrino interacts only with the gravitational field of BH. For this purpose, we set $V_{m,B} = 0$ in eq. (2.10). In this situation, eq. (2.9) can be solved exactly. The probability of transitions $L \to R$ in the gravitational neutrino scattering is $P_{LR} = (1 + \cos \alpha_{+\infty}^{(g)})/2$, where

$$
\alpha_{+\infty}^{(g)} = 4 \int_{x_m}^{\infty} dx |\Omega_{x2}| = y \int_{x_m}^{\infty} dx \frac{x^4(3 - 2x) + xyz(3 - 4x) - xz^2(3 - 3x + 2x^2) - 2yz^3 + 2z^4}{\sqrt{R(x)[x(x - 1) + z^2][x^3 + z^2(x + 1)][x^3 + z^2(x + 1) - yz]}}.
$$

(3.1)

It turns out that $\alpha_{+\infty}^{(g)} = -\pi$ in eq. (3.1) for any $z \leq 1/2$ and $y > y_0 = 4 \cos^3 \left[ \frac{1}{3} \arccos(\mp 2z) \right] \pm z$. Double signs in the expression for the critical impact parameter $y_0$,3 correspond to the positive or negative projection of the neutrino angular momentum to the spin of BH.

3If $y \leq y_0$, an ultrarelativistic particle falls into BH [23].
The fact that $\alpha_{+}\approx -\pi$ in the purely gravitational neutrino scattering means that $P_{LR} = 0$, i.e. there are no spin oscillations of ultrarelativistic neutrinos. This fact is in agreement with the finding of ref. [29], where the same result was obtained in the weak field limit and considering the chiral neutrino eigenstates as incoming and outgoing wave functions. Note that the concepts of the helicity and chirality are not the same for an ultrarelativistic fermion in curved spacetime, as established in ref. [30]. We have proven the absence of spin oscillations of ultrarelativistic neutrinos meaning the transitions between the helicity states $|\mathbf{\zeta} \cdot \mathbf{u}| / |\mathbf{u}| = \pm 1$. Moreover, this fact holds true for the arbitrary particle motion in the allowed region in the gravitational scattering in the Kerr metric. This our finding corrects the statement of ref. [25], where $P_{LR} \neq 0$ was obtained for ultrarelativistic neutrinos scattered off a rotating BH.

The inclusion of the term $\propto V_{B}$ to $\Omega_{e2}$ in eq. (2.10) makes the neutrino spin to precess even in the ultrarelativistic case. However, the approximation $V_{B} = \text{const}$ in eq. (2.10) leads to the divergent value of $\alpha_{+}\approx 0$. It results from the fact that the magnetic field at $r \to \infty$ is uniform and nonzero. Thus the neutrino spin makes an infinite number of revolutions with respect to its initial direction even when a particle is far away from BH. Therefore, the approximation $V_{B} = \text{const}$ is unphysical and we have to replace $B = \text{const}$ with some decreasing function of $x$.

As a rule, the electron number density $n_{e}$ also decreases with the radius. If we consider the neutrino scattering off SMBH with $M \sim 10^{8} \, M_{\odot}$, then $n_{e}(r) \propto n_{e}(0)^{-\beta}$, where the number density in the vicinity of SMBH is $n_{e}(0) \sim 10^{18} \, \text{cm}^{-3}$ [31]. Such a dense matter can exist in the inner part of an accretion disk in some active galactic nuclei (AGN). The index $\beta > 0$ is quite model dependent. We take $\beta = 3/2$, which corresponds to an advection dominated accretion flow [32].

The structure of the electromagnetic field in an accretion disk is quite complicated. Some models are reviewed in ref. [27]. We can assume that $B = B_{0} x^{-\kappa}$, where $\kappa > 1$. It corresponds to the magnetic field decreasing towards the edge of an accretion disk. The indexes $\kappa$ and $\beta$ are related by the formula, $2\kappa = \beta + 1$. Indeed, if we assume the equipartition of the energy of the magnetic field and the plasma accreting to BH [33], $B^{2} \propto n v^{2}$, one gets the required relation between $\beta$ and $\kappa$. Here $v$ is the velocity of plasma which scales as the Keplerian one, i.e. as $v \propto r^{-1/2}$. For $\beta = 3/2$, chosen above, one gets that $\kappa = 5/4$ [33]. We use this scaling law for the magnetic field.

The magnetic field $B_{0}$ in the vicinity of BH, $x \sim 1$, is constrained by the Eddington limit [27, pg. 184], $B_{\text{Edd}} = 10^{4} \, G (M/10^{9} \, M_{\odot})^{-1/2}$, which is the upper bound. Typically, $B_{0} \ll B_{\text{Edd}}$. Then, we adopt $B_{0} = 10^{-2} B_{\text{Edd}}$ in our analysis. For $M \sim 10^{8} \, M_{\odot}$, it gives one $B_{0} = 3.2 \times 10^{2} \, G$. Such magnetic fields are reported in ref. [34] to exist in the vicinity of some AGN. Magnetic fields near Sgr A* or M87 are weaker (see, e.g., ref. [35]).

We take the most conservative value for the neutrino magnetic moment $\mu = 10^{-14} \mu_{B}$, where $\mu_{B}$ is the Bohr magneton, which is the model independent constraint on the Dirac neutrino magnetic moment obtained in ref. [36]. Thus, for $M = 10^{8} \, M_{\odot}$, the dimensionless interaction potentials in eq. (2.10) are $V_{m}(x) = 10^{-1} \times x^{-3/2}$ and $V_{B}(x) = 2.7 \times 10^{-2} \times x^{-5/4}$ since $r_{g} = 2.95 \times 10^{13} \, \text{cm} = 1.5 \times 10^{27} \, \text{GeV}^{-1}$.

Now, we can discuss the general case of the neutrino scattering off a rotating BH surrounded by a magnetized accretion disk. We should also mention that, in practice, one cannot track a particular neutrino and measure the change of its polarization. A neutrino telescope can detect a flux of neutrinos scattered off BH. If neutrinos were scalar particles, the outgoing
flux could be defined as $F_0$. The measured flux of spinning neutrinos is $\propto P_{\text{LL}}F_0$ since a terrestrial detector is sensitive to left polarized ultrarelativistic neutrinos only.

Different neutrino trajectories contribute to the flux in a detector with a particular scattering angle $0 < |\chi| < \pi$. In general relativity, a particle can make multiple revolutions around BH before being scattered off. In our simulations, we account for up to four such revolutions. Moreover, the $\theta$-symmetry is broken in the Kerr metric. Thus, despite we consider the neutrino motion in the equatorial plane only, we can formally consider both positive and negative scattering angles $\chi$. Positive $\chi$ correspond to a detector inclined towards the BH rotation, whereas, when $\chi < 0$, it is inclined in the opposite direction. These two situations were explained in ref. [25] to result in the different scattering pictures. We call them the direct and retrograde scatterings.

The strategy for the numerical solution of eqs. (2.9) and (2.10) is the following. First, we integrate eq. (2.9) from $x = \infty$ to $x = x_m$ with the initial condition $\psi = \psi_{-\infty}$. Then, we account for the change of the signs in $\Omega x^2, 3$ in eq. (2.10). We use the result of the first integration $\psi_m = \psi(\chi = 0)$ as the initial condition in the second numerical integration from $x = x_m$ to $x = \infty$. Finally, we find the spin state of outgoing neutrinos using $\psi_{+\infty} = \psi(x = \infty)$.

4 Results

In this section, we present the measured fluxes of neutrinos based on the numerical solution of eqs. (2.9) and (2.10) with the initial conditions and the parameters given in section 3.

First, we study the situation when only the magnetic field contribution is nonzero in eq. (2.10). In this case, only $\Omega x^2 \neq 0$ in eq. (2.10). Thus, eq. (2.9) can be solved in quadratures. We have already found in section 3 that solely the gravitational field does not contribute to the spin-flip of scattered neutrinos. The angle of the neutrino spin rotation owing to the magnetic field interaction reads

$$\alpha^{(B)}_{+\infty} = 2 V^{(0)}_B \int_{x_m}^{\infty} dx \frac{2 x^3 + z^2 - y z}{x^{1/4} \sqrt{R(x)}},$$

(4.1)

where $V^{(0)}_B = 2.7 \times 10^{-2}$. The survival probability is $P_{\text{LL}}^{(B)} = (1 - \cos \alpha^{(B)}_{+\infty})/2$.

We note that, unlike the solely gravitational interaction considered in section 3, $P_{\text{LL}}^{(B)} < 1$ for any $y > y_0$ and $z \leq 1/2$. It means that there is a neutrino spin-flip owing to the presence of the neutrino magnetic moment. Using $P_{\text{LL}}^{(B)}$, we calculate the flux of outgoing neutrinos basing on the flux $F_0$ of scalar particles. The calculation of $F_0$ is performed in the standard manner (see, e.g., ref. [37]).

In figure 1, we show the ratio $F_{\nu,d,r}^{(B)}/F_{0,d,r}$ for the direct and retrograde scatterings off a maximally rotating SMBH with $z = 1/2$ versus $\chi$. One can see that this ratio monotonically decreases from unity for the forward scattering, $\chi = 0$, to about 0.94 for the backward scattering, $\chi = \pi$. There is a negligible dependence of the fluxes on the type of the scattering, $F_{\nu,d}^{(B)}/F_{\nu,d} \approx F_{\nu,r}^{(B)}/F_{\nu,r}$. Therefore the flux of outgoing neutrinos is reduced by $\approx (6 \pm 1)\%$ compared to the case when spin effects are not accounted for.

Note that $F_0 \propto (d\sigma/d\Omega)_0$, which is the differential cross section for scalar particles. Here $d\Omega = 2\pi \sin \chi \, d\chi$, where $\chi$ is the scattering angle.

The value of $F_{\nu,d,r}^{(B)}/F_{0,d,r}$ is slightly less than 1 at $\chi = 0$. This deviation is caused by errors of numerical simulations.
Figure 1. The ratios of the fluxes $F_\nu(B)$ and $F_0$ versus the scattering angle $\chi$. The parameters of the system are $\mu = 10^{-14} \mu B$, $B_0 = 3.2 \times 10^2$ G, and $M = 10^8 M_\odot$. (a) Direct scattering; (b) retrograde scattering.

Now, we take into account the neutrino interaction with an accretion disk. In this case, all the components of $\Omega_x$ in eq. (2.10) are nonzero. It means that we have to solve eq. (2.9) numerically. We start with the situation of the disk which corotates SMBH, i.e. $\lambda = +1$ in eq. (2.10). In figures 2(c) and 2(d), we show the ratios $F_{\nu d,r}/F_{\nu(tot)}$ for the direct and retrograde scatterings. One can see that $F_{\nu(tot)}$ is $\approx 3\%$ greater than $F_{\nu(B)}$. The result that $F_{\nu(tot)} > F_{\nu(B)}$ stems from the fact that the nonzero components $\Omega_{\pi,1,3} \neq 0$ withdraw the system from the resonance condition implemented in the case when only $\Omega_{x2} \neq 0$.

In figures 2(a) and 2(b), we show the ratios $F_{\nu d,r}/F_{\nu 0,d,r}$ for the direct and retrograde scatterings. One can see that this ratio is minimal at $\chi = \pi$. The fluxes $F_{\nu(tot)}(\chi = \pi)$ are reduced $\approx 3\%$ compared to the scalar particles case. This value can be explained for if we account for figures 1, 2(c), and 2(d).

Figure 2 corresponds to $z = 1/2$. We analyzed the situations $z < 1/2$. The behavior of the neutrino fluxes $F_{\nu d,r}$ does not change qualitatively.

Now, we turn to an accretion disk which counter-rotates SMBH, i.e. $\lambda = -1$. The parameters of the disk and the neutrino are the same as in figure 2. The flux of scattered neutrinos, obtained from the numerical solution of eq. (2.9), for $z = 1/2$ and $\lambda = -1$ is shown in figure 3. The retrograde scattering, presented in figures 3(b) and 3(d), does not differ qualitatively from the case $\lambda = +1$, shown in figures 2(b) and 2(d).

The direct scattering, depicted figures 3(a) and 3(c), on its turn, reveals the spike in the total flux. This feature happens when the total survival probability, accounting for both matter and the magnetic field, is great, $P_{LL}^{(tot)} \approx 1$. It takes place at a certain impact parameter.

We can see the analogous behavior of $F_{\nu(tot)}$ for $z = 0.1$ and $\lambda = -1$ shown in figure 4. However, firstly, there is the downward spike in this case. Secondly, it appears in the retrograde neutrino scattering depicted in figures 4(b) and 4(d). This feature results from the enhancement of the transition probability $P_{LR}^{(tot)}$ for ultrarelativistic neutrinos with some impact parameter. Performing numerical simulations, we have obtained that the border between the qualitatively different cases, shown in figures 3 and 4, is at $z \approx (0.12 - 0.13)$. 
Figure 2. The outgoing fluxes $F^{\text{tot}}_\nu$ based on the numerical solution of eq. (2.9) versus the scattering angle for SMBH with the maximal spin $z = 1/2$ surrounded by a corotating accretion disk ($\lambda = +1$). The magnetic parameters are the same as in figure 1 and $n_e^{(0)} = 10^{18}$ cm$^{-3}$. (a) and (c) Direct scattering; (b) and (d) retrograde scattering.

The explanation of the features in figures 3(a) and 4(b) is not obvious. Unlike neutrino oscillations in constant external fields, where a resonance in oscillations can be revealed just from the expression of an effective Hamiltonian, here the external fields in eq. (2.10) are coordinate dependent. Moreover, this effect is integral, i.e. we have to make a neutrino to pass on the whole trajectory and, then, obtain the state of the polarization of the particle. Therefore, in our case, one should rely mainly on the numerical simulations rather than on the naive analysis of the effective Hamiltonian in eq. (2.9).

Nevertheless, the appearance of spikes in figures 3(a) and 4(b) can be accounted for qualitatively by the neutrino interaction with moving matter. Plasma of an accretion disk moves with relativistic velocities on circular orbits; see eq. (2.5), where $U_\phi \neq 0$. In the neutrino gravitational scattering, there are situations when the neutrino velocity has a nonzero angle with respect to the plasma velocity. Hence, the term $\propto G_F g$ in $\Omega$ in eq. (2.8) has a component perpendicular to the neutrino velocity. It is this term, which causes the neutrino spin-flip. Analogous effect in neutrino (flavor) oscillations was considered in ref. [38].
Figure 3. The outgoing fluxes $F^{\text{tot}}_{\nu}$ based on the numerical solution of eq. (2.9) versus the scattering angle for SMBH with maximal spin $z = 1/2$ surrounded by a counter-rotating accretion disk ($\lambda = -1$). The parameters of the system are the same as in figure 2. (a) and (c) Direct scattering; (b) and (d) retrograde scattering.

5 Conclusion

We have studied spin effects in the neutrino scattering off a rotating SMBH. This SMBH is supposed to be surrounded by a realistic magnetized accretion disk. We assume that neutrinos are ultrarelativistic particles. These neutrinos are supposed to interact with dense matter of the accretion disk by the electroweak forces. We use the forward scattering approximation in the neutrino interaction with matter. It means that the matter contribution is in the invariant four vector effective potential $G^\mu$. The neutrino interaction with the magnetic field in a disk is owing to the presence of a nonzero diagonal neutrino magnetic moment, i.e. we assume that a neutrino is a Dirac particle. Of course, we exactly account for the effects of curved spacetime on the propagation and spin oscillations of neutrinos.

Considering the equatorial motion of neutrinos in section 2, we have derived the effective Schrödinger equation for the evolution of the neutrino polarization in the particle scattering. Initially all ultrarelativistic neutrinos are taken to be left polarized. A detector is also
sensitive to left polarized neutrinos. Thus, the measured flux is the product of the survival probability of spin oscillations and the outgoing flux of scalar particles. The latter flux is computed by the standard technique (see, e.g., ref. [37]).

In section 3, we have chosen the parameters of the accretion disk, such as the radial distributions of background matter and the magnetic field, and the value of the neutrino magnetic moment. These parameters are realistic and do not violate current constraints. We have also found in section 3 that solely gravitational interaction does not contribute to the spin-flip in the scattering of ultrarelativistic neutrinos in the Kerr metric. This our result is in agreement with the finding of ref. [29]. However, we generalized the statement of ref. [29] by considering arbitrary neutrino trajectories.

The outgoing fluxes of neutrinos based on the numerical solution of eq. (2.9) are presented in section 4. If we neglect the contribution of the accretion disk, eq. (2.9) can be solved in quadratures. The corresponding fluxes for the direct and retrograde scatterings are shown
in figure 1. Spin effects are negligible for the forward neutrino scattering: $F_\nu(\chi = 0) = F_0$. The maximal deviation of the flux is for the backward neutrino scattering at $\chi = \pi$. It can reach about 6%.

Then, we have taken into account the neutrino interaction with matter. We have considered accretion disks which corotate and counter-rotate SMBH. The case of a corotating disk differs from the magnetic case only quantitatively; cf. figures 1 and 2. However, if a disk counter-rotates ($\lambda = -1$), there are spikes in the total outgoing fluxes. These spikes appear either in the direct scattering, see figure 3(a), or in the retrograde scattering, see figure 4(b), depending on the angular momentum of BH $J = 2M^2z$. One can treat these spikes, especially that shown in figure 4(b), as the appearance of a quasi-resonance in spin oscillations.

The advance of the present work in comparison with refs. [24, 25] is the following. Firstly, we have accounted for the neutrino electromagnetic interaction with the magnetized accretion disk. Secondly, we have improved the description of the accretion disk. In refs. [24, 25], background matter was taken to be distributed isotropically around BH. Now, we suppose that particles of plasma rotate around BH on circular orbits, with the four velocity given in eq. (2.5). Hence we consider a realistic accretion disk. We have also taken the indexes in the matter distribution $\beta$ and the magnetic field radial dependence $\kappa$, which result from the equipartition of the energy between the magnetic field and the accreting plasma. Finally, we have corrected the statement of ref. [25] on the gravity contribution to the spin-flip of ultrarelativistic neutrinos scattering in the Kerr metric.

There is an assumption, which significantly simplifies the calculations in the present work: we have studied neutrinos moving in the equatorial plane only. Neutrinos in the vicinity of BH can be emitted inside its accretion disk in nuclear reactions between plasma particles. Then, these neutrinos are lensed, with their spin precessing in external fields in curved spacetime. In the present work, we have studied a relativistic accretion disk. It means that neutrinos are emitted mainly along the velocities of background fermions in an accretion disk, i.e. the transverse momentum of neutrinos is much less than the longitudinal one. Hence, the approximation of the equatorial neutrino motion is valid. Note that the arbitrary trajectories of massless particles (photons) emitted by an accretion disk of SMBH were studied in ref. [39].

In summary, we have studied the neutrino scattering off a rotating SMBH surrounded by a magnetized accretion disk with the realistic matter density distribution. Considering the conservative value of the neutrino magnetic moment $\mu = 10^{-14}\mu_B$ and the moderate magnetic field strength in the vicinity of SMBH $B_0 \sim 10^2 G$ [34], we have obtained that the outgoing neutrino flux can be reduced by $(3 - 6)\%$. Although the predicted phenomenon is beyond the sensitivity of the current neutrino telescopes, one expects that it can be observed in future (see, e.g., ref. [40]).

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A An electromagnetic field near BH

In this appendix, we briefly remind how to construct the electromagnetic field in the vicinity of a Kerr BH which asymptotically reaches the constant and uniform magnetic field $B$. Basically, we follow ref. [28].
The Kerr metric in eq. (2.1) has two Killing vectors: $\eta^\mu = (1, 0, 0, 0)$ and $\psi^\mu = (0, 0, 0, 1)$. They satisfy the relation, $\psi_{\mu\nu} + \psi_{\nu\mu} = 0$ and analogously for $\eta_{\mu}$. The one-form $\psi = \psi_{\mu}dx^\mu$ corresponding to the Killing vector $\psi^\mu$ generates the electromagnetic field $F_\psi \propto \int \! d\psi = \frac{1}{2}(\nabla_\mu \psi_\nu - \nabla_\nu \psi_\mu)dx^\mu \wedge dx^\nu$. One can check that this electromagnetic field obeys the Maxwell equation, $(F_\psi)_{\mu\nu} = 0$. Analogous contribution results from $\eta = \eta_\mu dx^\mu$.

Thus, the total electromagnetic field has the form,

$$F = a_\psi d\psi + a_\eta d\eta,$$

(A.1)

where the coefficients $a_\psi, a_\eta$ are uniquely fixed by the two conditions. Firstly, the total electric charge of BH equals zero, i.e. $f \ast F = 0$, where the integration is over the 2D surface with $t = \text{const}$ and $r \to \infty$. Accounting for the facts that $[-41]$ $f \ast d\psi = -16\pi Ma$ and $f \ast d\eta = 8\pi M$, we get that $a_\eta = 2a_\psi$.

Secondly, asymptotically, i.e. at $r \to \infty$, the components of the electromagnetic field $F$ in eq. (A.1) should be $F_{\phi \phi} = -F_{\phi r} = -Br^2 \sin^2 \theta$ and $F_{\phi \theta} = -F_{\theta \phi} = -Br^2 \sin \theta \cos \theta$. It corresponds to the uniform magnetic field $B$ along the rotation axis of BH. It gives one $a_\phi = B/2$ and $a_\eta = aB$.

We can obtain the vector potential of the electromagnetic field $F = dA = \frac{1}{2}(\nabla_\mu A_\nu - \nabla_\nu A_\mu)dx^\mu \wedge dx^\nu$ in the form, $A_\mu = B(\psi_\mu + 2a\eta_\mu) = B\left(\frac{g_{\mu \phi}}{2} + 2ag_{\mu \eta}\right)$. Using eq. (2.1), one finds that it has two nonzero components,

$$A_t = Br \left[1 - \frac{rr_g}{2\Sigma}(1 + \cos^2 \theta)\right], \quad A_\phi = -\frac{B}{2} \left[r^2 + a^2 - \frac{a^2rr_g}{\Sigma}(1 + \cos^2 \theta)\right] \sin^2 \theta,$$

(A.2)

which are used in section 2 to build the electromagnetic field in the world coordinates, $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$.

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