Orbital Stability of Earth-Type Planets in Binary Systems

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Abstract. About half of all known stellar systems with Sun-like stars consist of two or more stars, significantly affecting the orbital stability of any planet in these systems. Here we study the onset of instability for an Earth-type planet that is part of a binary system. Our investigation makes use of previous analytical work allowing to describe the permissible region of planetary motion. This allows us to establish a criterion for the orbital stability of planets that may be useful in the context of future observational and theoretical studies.

1. Introduction

Observational evidence for the existence of planets in stellar binary (and higher order) systems has been given by Patience et al. (2002), Eggenberger et al. (2004), Eggenberger & Udry (2007), and others. Eggenberger & Udry presented data for more than thirty systems, mostly wide binaries, as well as several triple star systems, with separation distances as close as 20 AU (GJ 86). These observations are consistent with the finding that binary (and higher order) systems occur in high frequency in the local Galactic neighborhood (Duquennoy & Mayor 1991; Lada 2006; Raghavan et al. 2006; Bonavita & Desidera 2007). The fact that planets in binary systems are now considered to be relatively common is also implied by the recent detection of debris disks in various main-sequence stellar binary systems using the Spitzer Space Telescope (e.g., Trilling et al. 2007).

In the last few decades, significant progress has been made in the study of stability of planetary orbits in stellar binary systems. Most of these studies focused on S-type systems, where the planet is orbiting one of the stars with the second star to be considered a perturbator. Recently, David et al. (2003) investigated the orbital stability of an Earth-mass planet around a solar-mass star in the presence of a companion star and determined the planet’s ejection time for systems with a variety of orbital eccentricities and semimajor axes.

In our previous work (Stuit 1995; Musielak et al. 2005), we studied the stability of both S-type and P-type orbits in stellar binary systems, and deduced orbital stability limits for planets. These limits were found to depend on the mass ratio between the stellar components. This topic was recently revisited by Cuntz et al. (2005) and Eberle et al. (2007), who used the concept of Jacobi’s integral and Jacobi’s constant (Szebehely 1967; Roy 2005) to deduce stringent criteria for the stability of planetary orbits in binary systems for the special case of the “coplanar circular restricted three-body problem”. In this paper, we present case studies of planetary orbital stability for different stellar mass ratios and different initial planetary distances from its host star.
Figure 1. Model simulations for $\mu = 0.2$ and 0.5, and different values of $\rho_0$. Each panel shows the primary (large dot) and secondary (small dot) star, the planetary orbit (solid line), the five Lagrange points, and the “zero velocity contours” (dash-dotted lines). The Lagrange points are denoted as L2, L1, L3, respectively, from left to right along the line connecting the two stars, and as L4 (top) and L5 (bottom) apart from this line. For $\mu = 0.2$, the critical values $\rho_0^{(1)}$, $\rho_0^{(2)}$, and $\rho_0^{(3)}$ are given as 0.353, 0.420, and 0.692, respectively, and for $\mu = 0.5$, they are given as 0.251, 0.442, and 0.442, respectively.
2. Methods and results

In the so-called coplanar circular restricted three-body problem the two stars are assumed to orbit each other in circles and their masses are much larger than that of the planet. In our case, it is assumed that the planetary mass is $1 \times 10^{-6}$ of the mass of the star it orbits; also note that the planetary motion is constrained to the orbital plane of the two stars. In addition, it is assumed that the initial velocity of the planet is set for an initially circular orbit, and that it is in the same direction as the orbital velocity of its host star. This star shall be the more massive of the two stars. Furthermore, the starting position of the planet is to the right of its host star along the line joining the binary components (3 o'clock position). The mass ratio $\mu$ of the two stars is defined as $\mu = M_2/M$ with $M = M_1 + M_2$, where $M_1$ and $M_2$ are the masses of the primary and secondary star, respectively. Additionally, $\rho_0$ denotes the planet’s relative initial distance $\rho_0 = R_0/D$ from its host star, with $D$ as distance between the two stars and $R_0$ the initial planetary distance from the primary star.

In the following, we illustrate the transition from stability to instability by progressively increasing the value of $\rho_0$ for binary systems with a fixed mass ratio $\mu$, given as $\mu = 0.2$ and $0.5$, respectively. For both $\mu = 0.2$ and $0.5$, we present the resulting planetary orbits for four different values of $\rho_0$ (see Fig. 1). Each panel shows the primary (larger dot) and secondary (smaller dot) star, the planetary orbit (solid line), as well as the zero velocity contours (dash-dotted lines). The upper four panels of Fig. 1 refer to the stellar mass ratio $\mu = 0.2$, whereas the lower four panels refer to $\mu = 0.5$.

Let us first focus on the case studies for $\mu = 0.2$ and $\rho_0 = 0.25$ and $0.323$. Both values of $\rho_0$ are smaller than the critical value of $\rho_0^{(1)} = 0.353$, indicating that the planetary orbits are stable. The fact that we restricted the time of the simulation to 50 yrs is inconsequential owing to the fact that the orbital stability of the planet is guaranteed by the analytical properties of the system, namely $\rho_0 < \rho_0^{(1)}$; see Cuntz et al. (2007) and Eberle et al. (2007) for a more extended discussion. Also note that the zero velocity contour changes between the two panels due to the increase in $\rho_0$ by getting closer to the Lagrange point L1, although its topology remains unaltered. Moreover, we show two cases of unstable orbits by choosing $\rho_0 = 0.41$ and $0.51$, respectively. Since both values exceed $\rho_0^{(1)}$, the zero velocity contour opens at L1, providing the possibility for the planet to be captured by the secondary star. For $\rho_0 = 0.51$, the zero velocity contour even opens at L2 because of $\rho_0 > \rho_0^{(2)}$. In case of $\rho_0 > \rho_0^{(3)}$ (not shown here), the contour would even open at L3, providing a further type of opportunity for the planet to escape from the binary system.

The results for $\rho_0 = 0.41$ show that the planetary orbit is unstable but still remains within the sphere of gravitational influence of the primary star. The situation is different for $\rho_0 = 0.51$ where the planet reaches the secondary star only after a few irregular orbits about the primary star have been completed. A more detailed analysis shows that the planet first encountered the secondary star after 39.6 yrs at a minimal distance of at most 0.1 AU. A second even closer encounter occurred after 41.2 yrs, when the simulation was stopped because the planet entered the Roche limit of the secondary star. The general behavior of the model with $\rho_0 = 0.51$ is due to the fact that $\rho_0$ exceeds both $\rho_0^{(1)}$ and $\rho_0^{(2)}$. 
which in principle allows the planet to escape from the binary system through the L2 point.

Our results for $\mu = 0.2$ demonstrate different cases of orbital stability and instability. Similar results are obtained for $\mu = 0.5$, albeit quantitative differences due to the different values of $\mu$, $\rho_0^{(1)}$, $\rho_0^{(2)}$, and $\rho_0^{(3)}$. We find again that orbital stability is obtained if $\rho_0 < \rho_0^{(1)}$, whereas for larger values of $\rho_0$ instability is expected to emerge. Highly unstable cases are found for $\rho_0 = 0.3$ and $0.45$.

3. Conclusions

For the special case of the “coplanar circular restricted three-body problem”, we applied stringent mathematical criteria that allow to precisely determine whether a planetary orbit in a stellar binary system is stable or unstable. This is accomplished by comparing the planet’s relative initial distance $\rho_0$ to the critical values $\rho_0^{(1)}$, $\rho_0^{(2)}$, and $\rho_0^{(3)}$, defined for a fixed stellar mass ratio $\mu$. An adequate way of demonstrating this different type of behavior is the assessment of the topology of the zero velocity contour, given by the $\mu$ and $\rho_0$ values of the system. In this case, planetary orbital stability is obtained if the contour is completely closed around the primary star and planet. All numerical case studies show a behavior consistent with this theoretical prediction. Note however that for generalized binary systems, other methods are required to determine the long-term stability of planetary orbits (e.g., Holman & Wiegert 1999; David et al. 2003). Important applications of our work include contesting numerically deduced stability limits for cases where analytically deduced results exist.

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