Governance of Social Welfare in Networked Markets
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Abstract—This article aims to investigate how a central authority (e.g., a government) can increase social welfare (SW) in a network of markets and firms. In these networks, modeled using a bipartite graph, firms compete with each other à la Cournot. Each firm can supply homogeneous goods in markets which it has access to. The central authority may take different policies for its aim. In this article, we assume that the government has a budget by which it can supply some goods and inject them into various markets. We discuss how the central authority can best allocate its budget for the distribution of goods to maximize SW. We show that the solution is highly dependent on the structure of the network. Then, using the network’s structural features, we present a heuristic algorithm for our target problem. Finally, we compare the performance of our algorithm with other heuristics with experimentation on real datasets.

Index Terms—Cournot competition, governance, networked markets, optimization, social welfare (SW).

I. INTRODUCTION

COURNOT competition in the single-market setting has been vastly studied in the literature. For instance, refer to [1], [2], [3], and [4]. In this oligopolistic model, each firm decides the quantity of the homogenous good it is willing to supply into the market. Then, according to the inverse demand function, the market-clearing price is determined based on the aggregate supply in the market. However, with the emergence of diverse and complicated economic scenarios, single-market models are inadequate for studying reality. In many settings, firms can compete in different markets, whether or not the good is identical in those markets. Typically, this situation is modeled using a bipartite graph in which one side of nodes represents firms and the other side depicts various markets. Each market is characterized by an inverse demand function. Multimarket competition is found abundantly in industries such as natural gas, water, electricity, airlines, cement, and healthcare; see [5], [6], [7], [8].

One question that arises naturally in the presence of strategic agents is the means by which it is possible to raise welfare measures like the ones used in [9] and [10]. One such measure is social welfare (SW), which captures the aggregate well-being in the environment, as discussed in [11] and [12]. In this case, it is typically the government that seeks higher SW. While there have been many studies on interactions among firms and equilibria in networked markets (see [13], [14], [15]), to the best of our knowledge, there is little work on how to govern and control SW in networked markets. The prevalence of networked markets in real-life experiences motivates us to study SW in this model. Our article takes one step forward toward this objective.

We consider a limited intervention budget for the government in the pursuit of higher SW. Therefore, we assume that the government is able to have a small amount of supply into every market. This small intervention setting enables us to use some techniques for the estimation of SW in terms of government’s supplies. The simple structure of the approximation leads to a strategy for the government. However, it is good to note that the actions taken by the government are specified by the structure of the network.

This problem arises from a real-world scenario in every country (especially in third-world countries) controlling their inflation by directly setting prices. Such intervention is absolutely incorrect and conflicts with the free-market economy. The framework offered in this article gives a natural way for governing the markets and controlling the prices (maximizing SW) without violating the free-market economy, which concurrently makes firms happy.

In this article, first, we mathematically model the target problem using, which is typically, an optimization problem. Then we propose a heuristic for solving this optimization problem and then compare the performance of our heuristic with other heuristics one may propose for solving this problem. The experiments are evaluated on a real-world dataset gathered by the research team and are one of the contributions of this article.

A. Related Works

Our work is in essence related to several categories of papers. First, there have been many attempts in studying the strategic behavior of firms and equilibria in the competition; for example, refer to [14], [16], [17], and [18]. One such study, which has been our first step-stone, is done by Bimpikis et al. [14], where they present a “characterization of the production quantities at the unique equilibrium of the resulting game for any given network,” in terms of supply paths in the network. Furthermore, they introduce the price–impact matrix which enables them to explore the effect of the changes in network structure on firms’ profits and consumer welfare. These changes include entering of a firm in a new market.
and also merging of two firms. Their results challenge the standard beliefs in Cournot oligopoly that more competition necessarily leads to higher welfare. Relatedly, [15] turn their focus on finding algorithms that compute pure Nash equilibria in Cournot competitions in networks. Moreover, [13] study the impact of monopolies on SW in a certain model of Bertrand network competition.

Another group of studies relevant to ours are the ones that analyze interactions in the networks and their impact on aggregate measures, e.g., [19], [20], [21], [22]. Most relatedly, Acemoglu et al. [20] have proposed a framework that paves the way to examine equilibria in such interdependent agents’ setup and discover the influence of small microeconomic shocks on the economy’s aggregate performance. Acting as our main inspiring study, they use Taylor expansion to acquire insights on the impact of shocks. Their examinations yield different results about the economy’s ex ante aggregate performance in the case of linear and nonlinear worlds. To understand how the structure of the network shapes the performance, they demonstrate that the Bonacich centrality measure can capture this effect when the nature between the firms is linear. Such analysis is prevalent in economics. For example, the general notion of production networks demands consideration of dependencies and network effects [23].

Finally, following the connections found by [14] and [24] between Bonacich centrality and network effects in network Cournot competition, studies about controlling centrality measures in networks can be considered related to ours. Generally, with an established relationship between centrality measures and SW in our setting, one might use these methods to change the structure of the competition such that SW increases. As such, [25], [26], and [27] model the centrality control problem as an optimization problem and present an algorithm to solve it.

II. PROBLEM FORMULATION

Consider a network game $G$ which consists of $n$ firms $F = \{f_1, \ldots, f_n\}$ and $m$ markets $M = \{m_1, \ldots, m_m\}$ in which the firms compete. Each firm has access to a set of markets, meaning that it can supply the good only in those certain markets. For firm $f_i$, let $M_i$ be the set of those markets.

Similarly, let $F_j$ denote the set of firms that have access to market $m_j$. The amount of good that firm $f_i$ supplies in $m_j$ is denoted by $q_{ij}$. Moreover, firm $f_i$ would incur the production cost $C_i(q)$ ($q$ is the vector of all $q_{ij}$s). Following the framework used by [14], we consider the inverse demand functions of the markets as affine. More specifically, the price of the good in market $m_j$, which we denote by $P_j(q)$, is governed by the relation

$$P_j(q) = \alpha_j - \beta_j \sum_{f_i \in F_j} q_{ij}. \tag{1}$$

In addition, we assume

$$C_i(q) = c_i \cdot \left( \sum_{m_j \in M_i} q_{ij} \right)^2. \tag{2}$$

For the sake of simplicity of our formulas, we suppose that for all $m_i \in M$, $\alpha_i = \alpha$, and $\beta_i = \beta$ and for all $f_j \in F$, $c_j = c$, where $\alpha, \beta, c > 0$. We model this economy with a bipartite graph $G = (V, E)$. An example of this graph can be seen in Fig. 1.

It is essential to note that the structure of the cost functions $C_i$s is what determines whether the analysis of different markets can be done separately. If the cost functions $C_i$s are additive in terms of $q_{ij}$s, then there is no need for studying these interdependencies. However, with general cost functions, decision in different markets is coupled. Our considered quadratic form brings out the role of the underlying network structure. It is worth mentioning that while our approach is generally applicable on many other parametric assumptions, this form makes the calculations easier to follow.

Briefly speaking, we can consider firm $f_i$’s profit as a combination of the aforementioned components

$$\pi_i(q) = \sum_{m_j \in M_i} q_{ij} \cdot P_j(q) - C_i(q). \tag{3}$$

Given a set of network graph $G$, each $f_i$ in competition with other firms solves the following optimization problems for computing its best response

$$\max_{q_i} \pi_i(q_i, q_{-i}) \tag{4}$$

$$\text{s.t.} \quad q_{ik} \geq 0 \quad \text{for} \quad m_k \in M_i \quad (4a)$$

$$q_{ik} = 0 \quad \text{for} \quad m_k \notin M_i \quad (4b)$$

where $q_i$ and $q_{-i}$ denote the vector of production quantities of $f_i$ and its competitors, respectively.

Bimpikis et al. [14] have focused on the equilibrium analysis of this model, and their main result about existence and characterization of the unique Nash equilibrium of this game is adopted as the foundation of this research.

**Theorem 1 (Adopted From [14]):** The unique Nash equilibrium of the game is given by

$$q^* = \left( I + \gamma W \right)^{-1} \gamma \tilde{a} \tag{5}$$

where $\gamma = (1/2(c + \beta))$, $\tilde{a}$ is a $|E| \times 1$ vector such that for every edge $(i, k) \in E$ we have $\tilde{a}_{ik} = a_{ik}$, and $W$ is an
The matrix \([I + \gamma W]^{-1}\) is called the Leontief inverse. We assume that the matrix \([I + \gamma W]\) is invertible. For a given economy, this assumption is shown to be true by [28] and [29]. In this article, we propose and formalize the problem of governance of the aforementioned networked markets (networked Cournot competition) with the objective of maximizing the SW. SW in Cournot competitions is defined as the sum of consumer surplus (CS) and firms’ profits. The CS in the Nash equilibrium is computed by the following formula (see [14], [30]):

\[
\text{CS} = \sum_{m_i \in M} \frac{(a_k - P_k(q^*))^2}{2\beta_k}.
\]

Therefore, SW’s formula is

\[
\text{SW} = \sum_{f_i \in F} \pi_i(q^*) + \text{CS}
\]

\[
= \sum_{f_i \in F} \left[ \sum_{m_i \in M} q_{ik} \cdot P_k(q^*) - C_i(q^*) \right]
+ \sum_{m_i \in M} \frac{1}{2\beta_k}(a_k - P_k(q^*))^2.
\]

Now, assume that an additional firm node is added to the network, such that it is owned by the government and has access to all the markets. This node’s target is to maximize the weighted SW. It is provided with a budget \(B\) and can use this budget to provide shocks to each market. In this article, each shock to market \(m_k\) is defined as provisioning some quantity of the good (\(q^* \leq q\)) to this market alongside the competing firms. \(q^*\) is a threshold that is forced by external entities such as law or social pressure and is the maximum amount that the government can intervene in a market. Firms compete until they reach an equilibrium. The equilibrium can be computed by the following theorem.

**Theorem 2:** The unique Nash equilibrium of the game \(G\) in the presence of shocks \(\{\epsilon_i\}_{i=1}^n\) is given by

\[
q^* = [I + \gamma W]^{-1} \gamma \left( \bar{\alpha} - \bar{p}\epsilon \right)
\]

where \(W\) and \(\bar{\alpha}\) are defined as in Theorem 1, and \(\bar{p}\epsilon\) is a \(|E| \times 1\) vector such that for every edge \((i, k) \in E\) we have \(\beta, \epsilon_k = \beta_k \cdot \epsilon_k\).

**Proof:** In the presence of shocks, we can rewrite the firms’ utility functions

\[
\pi_i(q, \epsilon) = \sum_{m_i \in M_i} q_{ik} \cdot P_k(q) - C_i(q)
\]

\[
= \sum_{m_i \in M_i} q_{ik} \left[ a_k - \beta_k \left( \sum_{f_j \in F_k} q_{jk} + \epsilon_k \right) \right]
- c_i \cdot \left( \sum_{m_i \in M_i} q_{ik} \right)^2.
\]

Assume that we have a new game \(G'\) where everything is the same as the previous setting \((G)\) except that the values of \(a_k\)'s have changed to \(a_k - \beta_k \epsilon_k\). By Theorem 1, we will have the following formula for the Nash equilibrium point:

\[
q^* = [I + \gamma W]^{-1} \gamma \left( \bar{\alpha} - \bar{p}\epsilon \right)
\]

Equilibria in \(G\) in the presence of shocks are equal to equilibria of \(G'\) because \(\pi_i\)'s computed by the above formula is exactly what must be for \(G'\). So by following the method used in [14] for proving Theorem 1, our desired target will be achieved.

Note that for each \(m_k \in M\), we must have \(\epsilon_k < (\alpha/\beta)\), because if not, the price function \(P_k\) will be negative which is not acceptable. Now, we can write the formulation of SW. The SW function in the presence of shocks can be recalculated as follows:

\[
\text{SW} = \sum_{f_i \in F} \pi_i(q^*, \epsilon) + \text{CS}
\]

\[
= \sum_{f_i \in F} \left[ \sum_{m_i \in M_i} q_{ik} \cdot P_k(q^*, \epsilon) - C_i(q^*) \right]
+ \sum_{m_i \in M} \frac{1}{2\beta_k}(a_k - P_k(q^*, \epsilon))^2
\]

\[
= \sum_{m_i \in M} \left[ \sum_{f_i \in F_k} q_{ik} \left( a_k - \beta_k \sum_{f_j \in F_k} q_{jk} - \beta \epsilon_k \right) - c \left( \sum_{m_i \in M_i} q_{ik} \right)^2 \right]
+ \sum_{m_i \in M} \frac{1}{2\beta_k} \left[ \beta \sum_{f_j \in F_k} q_{jk} + \beta \epsilon_k \right]^2
\]

\[
= \sum_{m_i \in M} \left( \sum_{f_j \in F_k} q_{ik} a_k \right)
- \beta/2 \cdot \sum_{m_i \in M} \left( \sum_{f_j \in F_k} q_{ik} \right)^2
- c \sum_{m_i \in M} \left( \sum_{f_j \in F_k} q_{ik} \right)^2
+ (\beta/2) \cdot \epsilon_k^2.
\]

Therefore, we have the following vectorized formula:

\[
\text{SW} = q^* \alpha - (\beta/2 + c) q^T q^* - (1/2)q^T W q^* + (\beta/2)\epsilon^T \epsilon
\]
where $\bar{\epsilon}$ is a $m \times 1$ vector whose $k$th component is equal to $\epsilon_k$, and other variables are defined as before (see Theorems 1 and 2). By the above formulation, the problem of governing SW with market shocks can be modeled by the optimization problem described in Definition 1.

Definition 1: The problem of governing (maximizing) SW with shocks in a networked market $G$ [Max$SW(G)$] is defined as the following optimization problem:

Maximize $q^* = \pi q^T - (\beta/2 + c)q^* q^* - (1/2)q^T Wq^*$

s.t. $q^* \leq [I + \gamma W]^{-1} \gamma (\bar{\alpha} - \bar{\beta} \epsilon)$

$c \cdot \left( \sum_{m_k \in M} \epsilon_k \right) \leq B$

$0 \leq \epsilon_k \leq q^* \quad \forall m_k \in M.$

The above formula is not concave or convex, because both convex and concave expressions appear in it. This makes the convex optimization framework ineffective. In the next section, we devise a heuristic algorithm for this optimization problem. This is done by proposing a linear estimation for SW and an optimization algorithm for maximizing it. Then, in Section IV, good performance of this heuristic algorithm is demonstrated by experimentation on real and synthetic data.

### III. Solution Estimation

In this section, we provide some insights into the SW function and by linearizing it with the Taylor expansion, we propose an algorithm called the linear heuristic for the Max$SW(G)$ problem. More precisely, we propose a metric that can be computed using the network structure and we analytically show that picking the markets with larger amounts of this metric can (approximately) maximize the SW. In Section IV, by running experiments on real and synthetic datasets, we will show the superiority of this approach over others.

The main idea is to use the first-order multivariate Taylor expansion [31] to create a linear approximation for the SW function. This approximation leads to a linear combination of $\epsilon_i$'s:

$$SW(\epsilon) = SW(0) + \zeta_1 \epsilon_1 + \zeta_2 \epsilon_2 + \cdots + \zeta_m \epsilon_m. \quad (15)$$

Keeping in mind that the government has a limited budget for its interventions, we can consider the shocks small ($\sum_{m_k \in M} \epsilon_k \leq q^*$), so that this approximation may be valid. The coefficient of each $\epsilon_i$ ($\zeta_i$) can be considered as the aforementioned metric. Since SW is a differentiable function, we can write SW as follows:

$$SW(\epsilon) \approx SW(0) + \epsilon \cdot \nabla SW(0)$$

Thus, the amount of SW added by the shocks is a linear combination of $\epsilon_i$'s whose coefficients are $\zeta_i = (\partial SW/\partial \epsilon_i)_{\epsilon=0}$. Therefore, to maximize SW, markets should be targeted for supplies in order of their $\zeta_i$'s. The details of this algorithm can be seen in Algorithm 1. Now, we discuss how to calculate the coefficients.

Using Theorem 2 and expanding the formula derived for $q^*$, we have

$$q^*_i = \frac{a - 2c \sum_{m_i \in M, m_i \neq m_k} q^*_i - \beta \sum_{f_i \in F} q^*_j}{2(\beta + c) - \sum_{(i,j) \in E(G)} (\gamma W)_{ik,j} q^*_j}. \quad (17)$$

If we define function $f(z) = \gamma a - \gamma z$ denotes the amount firm $f_i$ supplies in market $m_i$ in equilibrium under the presence of shock $\epsilon_i$. Moreover, from (13) with $h(x) = a x - ((\beta/2) + c) x^2$ and $u(q_{ij}, q_{k\ell}) = w_{ij, kr} q_{ij} q_{k\ell}$

$$SW = \sum_{i,j} h(q_{ij}) - \frac{1}{2} \sum_{i,j,k,\ell} u(q_{ij}, q_{k\ell}) + \sum_{m_i \in M} (\beta/2) \cdot \epsilon_i^2 \quad (19)$$

is SW under the circumstances discussed so far.

We start our analysis by calculating $(\partial q^*_i / \partial \epsilon_i)$. Exploiting the idea used by [20]

$$\frac{\partial q^*_i}{\partial \epsilon_i} = f'(q_{ij}^* + \gamma \beta \epsilon_i) \sum_{i,j,k,\ell} \left( \sum_{l} w_{ij, kr} \frac{\partial q^*_{ij}}{\partial \epsilon_i} + \gamma \beta \delta l \epsilon_i \right). \quad (20)$$

Evaluating this equation using the matrix form at point $\epsilon = (\epsilon_1, \ldots, \epsilon_{|E|}) = 0$, which is the absence of the government, yields

$$\frac{\partial q^*}{\partial \epsilon_i} \bigg|_{\epsilon=0} = -\gamma \beta (I + \gamma W)^{-1} \epsilon_i \quad (21)$$

where $\epsilon_i$ is a $|E| \times 1$ vector, with ones for edges connecting to market $m_i$, and zeros elsewhere. Thus, we have

$$\frac{\partial q^*_i}{\partial \epsilon_i} \bigg|_{\epsilon=0} = -\gamma \beta \lambda_{ij, kr} \quad (22)$$

where $\lambda_{ij, kr}$ is the corresponding element to edges $ij$ and $kr$ of matrix $(I + \gamma W)^{-1}$.

Setting our sights on SW, we have

$$\frac{\partial SW}{\partial \epsilon_i} = \sum_{i,j} h'(q^*_{ij}) \frac{\partial q^*_{ij}}{\partial \epsilon_i} - \frac{1}{2} \sum_{i,j,k,\ell} \left[ \frac{\partial u}{\partial q^*_{ij}} \frac{\partial q^*_{ij}}{\partial \epsilon_i} + \frac{\partial u}{\partial q^*_{k\ell}} \frac{\partial q^*_{k\ell}}{\partial \epsilon_i} \right]$$

$$+ \beta \epsilon_i. \quad (23)$$

Considering $h'(x) = a - (\beta+2c)x$ and $(\partial u(q_{ij}, q_{k\ell}) / \partial q_{ij}) = w_{ij, kr} q_{ij}$, we get

$$\frac{\partial SW}{\partial \epsilon_i} = \sum_{i,j} (a - (\beta+2c)q^*_{ij}) \frac{\partial q^*_{ij}}{\partial \epsilon_i} - \frac{1}{2} \sum_{i,j,k,\ell} \left[ w_{ij, kr} q_{ij} \frac{\partial q^*_{ij}}{\partial \epsilon_i} + w_{ij, kr} q_{ij}^* \frac{\partial q^*_{k\ell}}{\partial \epsilon_i} \right]$$

$$+ \beta \epsilon_i. \quad (24)$$
Thus, we evaluate (24) at $\epsilon = 0$

$$\frac{\partial SW}{\partial \epsilon_r}(\epsilon = 0) = \sum_{i,j} \left( \alpha - (\beta + 2c)q_{ij}^\ast \right) \frac{\partial q_{ij}^\ast}{\partial \epsilon_r}(\epsilon = 0)$$

$$- \frac{1}{2} \sum_{i,j,k,l} \left[ \sum_{i,j,k,l} \left( \frac{\partial q_{ij}^\ast}{\partial \epsilon_r}(\epsilon = 0) \right) + w_{ijkl} \right] \left( \sum_{i,j,k,l} \left( \frac{\partial q_{ij}^\ast}{\partial \epsilon_r}(\epsilon = 0) \right) \right)$$

$$+ w_{ijkl}q_{ij}^\ast \frac{\partial q_{ij}^\ast}{\partial \epsilon_r}(\epsilon = 0) \right) \right].$$

(25)

$q_{ij}^\ast$ has been studied in [14]. Based on their results, we can deduce the following in our setting:

$$q_{ij}^\ast(\epsilon = 0) = \gamma \alpha \sum_{k,l} \lambda_{ij,kl}.$$  

(26)

Using (22) and (26), we get

$$\frac{\partial SW}{\partial \epsilon_r}(\epsilon = 0) = -\gamma \beta \sum_{ij} \left( \alpha - \gamma \alpha (\beta + 2c) \sum_{k,l} \lambda_{ij,kl} \right) \sum_{k,l} \lambda_{ij,kl}$$

$$- \sum_{i,j} \left( \gamma \alpha \sum_{k,l} \lambda_{ij,kl} \sum_{k,l} \left( \sum_{i,j,k,l} \gamma \beta \lambda_{ij,kl} \right) \right).$$

(27)

Since we have derived a formula for computing $\zeta_r = (\partial SW/\partial \epsilon_r)(\epsilon = 0)$, we can state the final algorithm. This algorithm is shown in Algorithm 1. In the first four lines of this algorithm, $\zeta_r$'s are computed from the network structure and the Leontief matrix. After that, a set $T$ is initialized to the set of all markets and a variable $S$ is initialized to 0. In the next while loop, $T$ is to be the set of markets that have not been supplied with shocks so far and $S$ is defined as the total goods supplied by the government. Therefore, the loop execution will continue until either all the markets are supplied with shocks or the cost of shock supplies exceeds the budget intended for this purpose (B).

In each iteration of the while loop, the market with maximum $\zeta_r$ is chosen from $T$ and extracted from this set. Then the maximum possible shock is computed as $\epsilon_r$ and is added to $S$. Note that for computing $\epsilon_r$, three upper bounds must be considered.

1) $q_i$ is the upper bound defined in Definition 1.
2) $(\beta/\alpha)$ is the upper bound defined by the price function. If $\epsilon_r > (\beta/\alpha)$, the price will be negative at that market, which is not acceptable.
3) $(B/c_r)^{1/2} - S$ is the amount of goods that can be provided by the remaining budget.

IV. EMPIRICAL STUDY

We evaluate the performance of our proposed method on synthetic and real-world datasets of different pharmaceutical companies as our firms and different drugs as our markets. This dataset is collected by contacting 135 production companies which produce 603 drugs altogether, and after negotiation, we succeed in getting their data. Then we transform, clean, and integrate all their data to generate our desired dataset. For example, aspirin and its users define a market in which players are companies that produce this drug. In addition, we use identical parameters $\alpha$, $\beta$, $c$ for all the firms and markets, as we are considering the symmetric case. Using ordinary linear regression, these parameters are set in a way to be close to real-life values. The synthetic graph also has 603 markets and 135 firms, which has been chosen uniformly at random from all the bipartite graphs with the same numbers of markets, firms, and firm–market pairs. The characteristics of this dataset are shown in Table I. A subgraph of this network is shown in Fig. 2. The data associated with this subgraph are shown in Table II.

A. Competitor Benchmark

The essence of competitors we consider is that the government takes on a measure to rank the markets. Next, it supplies goods to markets in that order, as much as possible and as long as permissible. Naively, it is possible to choose the markets at random. No measure, to the best of our knowledge, has been presented for picking the markets yet. Nevertheless, centrality measures are natural candidates for us to use as benchmarks. As for the centrality measures we consider, we use the followings.

1) Degree: The simplest centrality measure is the degree of a node, which in our model is the number of firms competing in a market

$$d(m_i) = |F_i|.$$  

(28)

2) Betweenness: Generally speaking, betweenness centrality is a quantity for determining the impact of a node over the flow of information in a graph [32]. The betweenness of a node is an indicator for the fraction of shortest paths (with regard to the number of edges) in a graph that pass through this vertex. In our setting

$$b(m_i) = \sum_{f_j \in F} \frac{\sigma_{jk}(m_i)}{\sigma_{jk}}$$

(29)

where $\sigma_{jk}$ is the total number of shortest paths from firm $f_j$ to firm $f_k$, and $\sigma_{jk}(m_i)$ is the total number of those paths that include market $m_i$.

3) Closeness: Closeness centrality is an aggregate measure of a node’s proximity to other nodes. More precisely, closeness of node $v$ is defined as the inverse of sum of
Algorithm 1 Linear Heuristic for Solving MaxSW(\(G\))

**Input:** A network market \(G\) alongside parameters \(\alpha, \beta, \) and \(\gamma\)

**Output:** The amount of shocks \(\epsilon_1, \epsilon_2, \ldots, \epsilon_m\) which makes the maximum social welfare

1. Set \(\lambda_{ij,kr}\) equal to the corresponding elements to edges \(ij\) and \(kr\) of matrix \((I + \gamma W)^{-1}\)
2. for \(r = 1 \text{ to } m\) do
   4. \(\zeta_{r} \leftarrow -\gamma \beta \sum_{ij} (\alpha - \gamma a(\beta + 2c) \sum_{kr} \lambda_{ij,kr}) \sum_{ij} \lambda_{ij,kr} - \sum_{ij} (\gamma a \sum_{kr} \lambda_{ij,kr} (\sum_{kr} \alpha_{ij,kr} \sum_{ij} \gamma \beta \lambda_{ij,kr}))\)
3. end
4. \(T \leftarrow M; S \leftarrow 0\)
5. while \(T \neq \emptyset \) and \(c \cdot S^2 < B\) do
   8. Set \(r\) to the index of the market \(m_r \in T\) with maximum \(\zeta_r\)
   9. \(T \leftarrow T \setminus m_r\)
   10. \(\epsilon_r \leftarrow \min\{q'_r, \frac{n}{\sqrt{n}} \cdot \frac{B}{\epsilon} - S\}\)
   11. \(S \leftarrow S + \epsilon_r\)
6. end
7. return \(\epsilon_r, S\)

### Table I
**Summary of Dataset’s Characteristics**

| Characteristic | Drug Companies Dataset |
|---------------|------------------------|
| #Markets      | 603                    |
| #Firms        | 135                    |
| #Firm-Market pairs (edges) | 2049                  |

### Table II
**Sample of the Dataset Related to Fig. 2**

| Drug                | Company | Sale ($) | Sale ($) |
|---------------------|---------|----------|----------|
| Azithromycin        | KT      | 39,790   | 153,400  |
| Folic Acid          | JA      | 1,131,726| 4,369,600|
| Erythromycin        | HA      | 167,132  | 645,300  |
| Erythromycin        | KT      | 22,885,058| 88,359,300|
| Allopurinol         | HA      | 2,809,793| 8,058,700|
| Allopurinol         | KT      | 223,542  | 863,100  |
| Allopurinol         | JA      | 1,300,128| 5,019,800|
| Acetaminophen       | HA      | 45,079,908| 174,083,700|
| Acetaminophen       | KT      | 22,672,264| 87,537,700|
| Acetaminophen       | JA      | 12,809,311| 49,456,800|
| Acetaminophen       | AR      | 15,824,330| 61,097,800|
| Loratadine          | HA      | 3,447,220 | 6,894,400 |
| Loratadine          | AR      | 1,843,900 | 3,687,800 |

Table III. Then, the government begins supplying the amount \(q'\) of the commodity into the markets in the corresponding order, until the supplies violate the constraints in Definition 1. Thus, the government would supply up to \(\lfloor (B/C)^{1/2} \rfloor\), where \(B\) is the budget that inattends to the MaxSW(\(G\)) problem. Altering \(B\) gives us a trajectory for SW. Let SW\(_A(B)\) be the SW obtained by applying strategy \(A\) on MaxSW(\(G\)) with parameter \(B\). In Section IV-B, we compare trajectories SW\(_A(B)\) for policies in Table III.

**B. General Performance**

Our empirical results are indicated in Figs. 3(a) and (b), 4(a) and (b), and 5(a) and (b). For each policy \(A\) of Table III, we plot the difference between the amount of SW which can be obtained by our linear algorithm and the SW which can be obtained by the heuristic \(A\), i.e., SW\(_{Linear}(B) - SW_A(B)\). Moreover, in Fig. 6, SW obtained by implementing our algorithm with different budgets is depicted.

We observe that our proposed measure is strictly better than other mentioned quantities. For the random case, we use the average results over 50 different realizations to extract the expected performance. It is good to note that picking the markets according to ascending order of a centrality measure seems to be better than the reverse order. This is intuitive, since markets with lower centralities are generally more monopolized and government’s interventions have more impact on them. This observation is in accordance with the
The general idea in economics that more competition leads to higher SW. In addition, the difference in SW obtained by the linear heuristics and other methods tends to rise, until a certain point at which it drops. Since the endpoint of the plots suggests the government to supply in almost all the markets by each method, the last points have y coordinates close to zero. All the plots are strictly above the horizontal line $y = 0$, except beginning points which indicates the superiority of our proposed approach over other mentioned strategies.

The superiority of the presented algorithm compared with the competing algorithms is primarily due to the fact that the algorithm is really based on the optimization of the objective function, and its output is a metric that cannot be intuitively described based on structural features. In fact, the evaluation carried out in this article shows that the structural features of the network are not very suitable for choosing the best markets for intervention, and it is better to focus directly on the optimization of the objective function and designing better algorithms in this direction. Anyway, among the competing heuristics, the DescDegree algorithm has performed better. In fact, it can be concluded that if we are going to act only based on the structure, markets closer to other markets (in terms of distance on the graph) are more suitable for intervention.

In Fig. 6, we considered different budgets for the government and implemented the linear heuristic. Then, we plotted the SW obtained. We observe that the SW increases in the beginning, which is a result of increased consumers’ surplus. However, the SW peaks at a certain level and decreases from that point onward. This observation is due to the fact that the supply by the government decreases the producers’ surplus while it increases the consumers’ surplus. In the second half of the trajectory, the decrease in producers’ surplus dominates
the increase in the consumers’ surplus, which results in lower levels of SW.

V. CONCLUSION

In this article, we address the problem of increasing social welfare in networked markets by a central planner. The central planner has a limited budget for supplying the homogeneous good into different markets. Thus, it is of high importance how the planner should allocate its scarce resources, especially in the presence of strategic firms that compete à la Cournot. We devise a heuristic algorithm by the first-order-approximation of the obtained social welfare by any allocation. Our algorithm carefully allocates the resources by incorporating the network structure and market characteristics in its decision making. We empirically demonstrate the superiority of our method compared to several other heuristics on both a real-life and also a synthetic dataset.

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