Circuit QED: generation of two-transmon-qutrit entangled states via resonant interaction

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Abstract We present a way to create entangled states of two superconducting transmon qutrits based on circuit QED. Here, a qutrit refers to a three-level quantum system. Since only resonant interaction is employed, the entanglement creation can be completed within a short time. The degree of entanglement for the prepared entangled state can be controlled by varying the weight factors of the initial state of one qutrit, which allows the prepared entangled state to change from a partially entangled state to a maximally entangled state. Because a single cavity is used, only resonant interaction is employed, and none of identical qutrit–cavity coupling constant, measurement, and auxiliary qutrit is needed, this proposal is easy to implement in experiments. The proposal is quite general and can be applied to prepare a two-qutrit partially or maximally entangled state with two natural or artificial atoms of a ladder-type level structure, coupled to an optical or microwave cavity.

Keywords Qutrit · Transmon qutrit · Entangled state · Cavity · Circuit QED

1 Introduction

Entanglement is one of the most striking aspects of quantum mechanics. A system is entangled if its quantum state cannot be described as a direct product of states of its subsystems. Entanglement has many applications in quantum communication and
quantum information processing (QIP) [1–5]. In the last decade, quantum entangle-
ment engineering has been widely studied with various physical systems. For instance,
tanglement creation has been theoretically proposed with superconducting (SC)
qubits coupled via Josephson junctions [6] or capacitors [7–9]. The focus of this work
is on entanglement creation with a circuit-QED system, which has been considered
as one of the leading candidates for QIP [10–17]. For the past years, based on circuit
QED, much process has been made on entanglement generation with SC qubits. For
instance, using SC qubits coupled to a single cavity or multiple resonators (hereafter,
the terms cavity and resonator are used interchangeably), a number of theoretical pro-
posals have been presented for realizing entangled states of SC qubits [10–12,18–23].
Moreover, various two-qubit or three-qubit entangled states have been experimentally
demonstrated with superconducting qubits based on circuit QED [24–29]. Recently,
using SC qubits coupled to a single resonator, a Greenberger–Horne–Zeilinger entan-
gled state of ten SC qubits has been demonstrated in experiments [30]. With solid-state
platforms, this is the largest number of entangled qubits reported so far, after the pre-
vious experimental demonstration of entanglement with five superconducting qubits
via capacitance coupling was reported [31].

A significant experimental challenge for entanglement engineering is how to create
quantum entanglement with high-dimensional quantum systems. Compared with a
qubit (two-level system), a qutrit (three-level system) has a larger Hilbert space and
thus can be used to encode more information in QIP and communication. As is well
known, high-dimensional (HD) quantum entanglement is not only of great interest in
providing an additional way for the fundamental tests of quantum nonlocality, but also
central for HD-based quantum computation, quantum communication, quantum error
correction, and quantum simulation. Over the past years, based on cavity or circuit
QED, a large number of theoretical methods have been presented for creating two-qubit
or multi-qubit entangled states with various physical systems (e.g., atoms, nitrogen-
vacancy centers, quantum dots, and SC devices) [10–12,18–23,32–39]. However, after
a deep search of the literature, we found that based on cavity QED or circuit QED,
how to create entangled states of qutrits or higher-dimensional quantum systems with
natural or artificial atoms has been rarely investigated.

Motivated by the above, in this work we will consider a physical system, which
consists of two superconducting transmon qutrits coupled to a single microwave cavity
or resonator. Note that transmon devices have been considered as one of the best SC
information carriers because of their relatively longer coherence times compared with
other type of SC devices (e.g., flux, phase, and charge qubits/qutrits). In the following,
we will propose an approach to prepare the two transmon qutrits in an entangled state
(either a partially entangled state or a maximally entangled state). To the best of our
knowledge, our proposal is the first one to show how to create entangled states of
qutrits with natural or artificial atoms based on cavity or circuit QED.

As shown below, this proposal has the following advantages: (i) because only qutrit–
cavity resonant interaction and qutrit-pulse resonant interaction are employed, the
entanglement creation can be completed within a short time; (ii) there is no requirement
on the identical qutrit–cavity coupling constants; thus either non-uniformity in the
qutrit device parameters (resulting in nonidentical qutrit level spacings) or non-exact
placement of qutrits in the cavity is allowed by this proposal; (iii) since only a single
microwave cavity is used and no auxiliary qutrit is needed, the setup is simple; (iv) there is no need of measurement on the qutrit states or the cavity state; thus, the entangled states can be created deterministically; and (v) the degree of entanglement for the prepared entangled state is adjustable by varying the weight factors of the initial state of one qutrit; thus, this proposal can be used to create the two-qutrit entangled state varying from a partially entangled state to a maximally entangled state. This proposal is quite general, which can be extended to create a two-qutrit entangled state with a wide range of physical systems, such as two natural atoms (with a ladder-type three-level structure) coupled to an optical cavity, or two artificial atoms of a ladder-type three-level structure (e.g., superconducting phase qutrits, Xmon qutrits, nitrogen-vacancy centers, quantum dots) coupled to a microwave cavity or resonator.

This paper is organized as follows. In Sect. 2, we review the basic theory of a transmon qutrit resonantly interacting with a single cavity or a classical microwave pulse. In Sect. 3, we explicitly show how to prepare two transmon qutrits in an entangled state based on circuit QED. In Sect. 4, we give a brief discussion on the experimental feasibility. A concluding summary is given in Sect. 5.

2 Basic theory

A transmon qutrit has a ladder-type three-level structure [40]. In this work, we consider the three lowest levels of a transmon qutrit, which are denoted as |0⟩, |1⟩ and |2⟩ (Fig. 1). For a transmon qutrit, the transition between the two levels |0⟩ and |2⟩ is much weaker compared to the transition between the two levels |0⟩ and |1⟩ and the transition between the two levels |1⟩ and |2⟩ [40]. As shown in next section, the entanglement generation requires two types of resonant interaction, namely the resonant interaction between a cavity and the |0⟩ ↔ |1⟩ transition of the qutrit, and the resonant interaction between a classical pulse and the |1⟩ ↔ |2⟩ transition of the qutrit. To make our presentation given in next section clear, in the following we will give a brief review on the state evolutions for these two kinds of interactions.
2.1 Qutrit coupled to a single cavity

Consider a transmon qutrit coupled to a single-mode cavity. Assume that the cavity is resonantly coupled to the transition between the two lowest levels $|0\rangle$ and $|1\rangle$ while highly detuned (decoupled) from the transitions between other levels of the qutrit. Under this consideration, the interaction Hamiltonian in the interaction picture, after making the rotating-wave approximation, can be written as

$$H_{I1} = \hbar g a^\dagger |0\rangle \langle 1| + \text{h.c.},$$

where $a^\dagger (a)$ is the creation (annihilation) operator of the cavity and $g$ is the coupling constant between the cavity and the $|0\rangle \leftrightarrow |1\rangle$ transition of the qutrit.

Under the Hamiltonian $H_{I1}$, one can obtain the following state evolution:

$$|0\rangle|0\rangle_c \rightarrow |0\rangle|0\rangle_c,$$

$$|1\rangle|0\rangle_c \rightarrow -ie^{-i\phi} \sin g t |1\rangle + \cos g t |0\rangle_c,$$

$$|0\rangle|1\rangle_c \rightarrow \cos g t |0\rangle|1\rangle_c - ie^{-i\phi} \sin g t |0\rangle_c,$$  \hspace{1cm} (1)

where $|0\rangle_c$ is the vacuum state of the cavity, while $|1\rangle_c$ is the single-photon state of the cavity.

As shown in next section, the entanglement preparation requires that a cavity interacts with each of two qutrits 1 and 2. In reality, it is a challenge to have the qutrit–cavity coupling constants to be identical for both of the qutrits. Thus, we replace the coupling constant $g$ with $g_1$ for qutrit 1 while $g_2$ for qutrit 2.

2.2 Qutrit driven by a classical pulse

Let us consider a transmon qutrit driven by a microwave pulse. If the pulse is resonantly coupled to the $|0\rangle \leftrightarrow |1\rangle$ transition but highly detuned (decoupled) from the transitions between other levels of the qutrit, then the interaction Hamiltonian in the interaction picture is given by $H_{I2} = \hbar (\Omega_{10} e^{i\phi} |0\rangle \langle 1| + \text{h.c.})$, where $\phi$ is the initial phase of the pulse, while $\Omega_{10}$ is the Rabi frequency of the pulse. From the Hamiltonian $H_{I2}$, it is easy to show that a pulse of duration $t$ results in the following state evolution:

$$|0\rangle \rightarrow \cos \Omega_{10} t |0\rangle - ie^{-i\phi} \sin \Omega_{10} t |1\rangle,$$

$$|1\rangle \rightarrow -ie^{i\phi} \sin \Omega_{10} t |0\rangle + \cos \Omega_{10} t |1\rangle.$$  \hspace{1cm} (2)

Similarly, when the pulse is resonantly coupled to the $|1\rangle \leftrightarrow |2\rangle$ transition but highly detuned (decoupled) from the transitions between other levels of the qutrit, the interaction Hamiltonian in the interaction picture is given by $H_{I3} = \hbar (\Omega_{21} e^{i\phi} |1\rangle \langle 2| + \text{h.c.})$, where $\Omega_{21}$ is the Rabi frequency of the pulse. Based on this Hamiltonian, we can obtain the following state rotation:

$$|1\rangle \rightarrow \cos \Omega_{21} t |1\rangle - ie^{-i\phi} \sin \Omega_{21} t |2\rangle,$$

$$|2\rangle \rightarrow -ie^{i\phi} \sin \Omega_{21} t |1\rangle + \cos \Omega_{21} t |2\rangle.$$  \hspace{1cm} (3)

Finally, it should be mentioned that when the cavity is highly detuned (decoupled) from the $|1\rangle \leftrightarrow |2\rangle$ and $|0\rangle \leftrightarrow |2\rangle$ transitions of the qutrit, there is neither coupling
between the cavity and the qutrit nor transition between the levels \(|2\rangle\) and \(|1\rangle\) or \(|0\rangle\) induced by the cavity. As a result, the states \(|2\rangle |0\rangle_c\), and \(|2\rangle |1\rangle_c\) remain unchanged during the cavity on resonance with the \(|0\rangle \leftrightarrow |1\rangle\) transition of the qutrit. For a similar reason, the state \(|2\rangle (|0\rangle)\) of the qutrit is unaffected during applying the pulse resonant with the \(|0\rangle \leftrightarrow |1\rangle\) \((|1\rangle \leftrightarrow |2\rangle)\) transition of the qutrit. These results, together with the results given in Eqs. \((1–3)\), will be employed for the entanglement generation, as shown in the next section.

### 3 Generation of two-transmon-qutrit entangled states

Let us now consider two transmon qutrits 1 and 2 embedded in a single-mode cavity. Initially, the two qutrits are decoupled from the cavity, qutrit 1 is in the state \(\alpha |0\rangle + \gamma |1\rangle + \beta |2\rangle\), qutrit 2 is in the ground state \(|0\rangle\), and the cavity is in the vacuum state \(|0\rangle_c\). The initial state of qutrit 1 here can be prepared by applying classical microwave pulses. The initial state of the whole system is thus given by 

\[
|\psi\rangle_{in} = (\alpha |0\rangle_1 + \gamma |1\rangle_1 + \beta |2\rangle_1) |0\rangle_2 |0\rangle_c .
\]

Hereafter, the subscripts 1 and 2 represent qutrits 1 and 2, respectively.

To begin with, we define \(\omega_{20}(\omega_{21})\) as the \(|0\rangle \leftrightarrow |1\rangle\) \((|1\rangle \leftrightarrow |2\rangle)\) transition frequency of each qutrit and \(\Omega_{10}(\Omega_{21})\) as the pulse Rabi frequency of the coherent \(|0\rangle \leftrightarrow |1\rangle\) \((|1\rangle \leftrightarrow |2\rangle)\) transition. In addition, the frequency, initial phase, and duration of a microwave pulse are denoted as \(\omega, \phi, t'\) in the remaining of the paper.

The operations for creating entangled state \((3)\) are described below:

**Step 1:** Apply a microwave pulse of \(\{\omega_{10}, -\pi/2, \pi/(4\Omega_{10})\}\) and then a microwave pulse of \(\{\omega_{21}, -\pi/2, \pi/(2\Omega_{21})\}\) to qutrit 2 (Fig. 2a). According to Eqs. \((2)\) and \((3)\), the pulse application results in the state transformation \(|0\rangle_2 \rightarrow (|0\rangle_2 + |2\rangle_2) / \sqrt{2}\) (see Appendix). Therefore, the initial state \(|\psi\rangle_{in}\) of the system becomes

\[
\frac{1}{\sqrt{2}} (\alpha |0\rangle_1 + \gamma |1\rangle_1 + \beta |2\rangle_1) (|0\rangle_2 + |2\rangle_2) |0\rangle_c . \tag{4}
\]

**Step 2:** Bring the \(|0\rangle \leftrightarrow |1\rangle\) transition of qutrit 1 to resonance with the cavity for an interaction time \(\pi / (2g_1)\) (Fig. 2b). According to Eq. \((1)\), we have \(|1\rangle_1 |0\rangle_c \rightarrow -i |0\rangle_1 |1\rangle_c\). Thus, state \((4)\) changes to

\[
\frac{1}{\sqrt{2}} \left[ (\alpha |0\rangle_1 + \beta |2\rangle_1) (|0\rangle_2 + |2\rangle_2) |0\rangle_c - i \gamma |0\rangle_1 (|0\rangle_2 + |2\rangle_2) |1\rangle_c \right] . \tag{5}
\]

**Step 3:** Bring the \(|0\rangle \leftrightarrow |1\rangle\) transition of qutrit 2 to resonance with the cavity for an interaction time \(\pi / (2g_2)\) (Fig. 2c), resulting in \(|0\rangle_2 |1\rangle_c \rightarrow - |0\rangle_2 |1\rangle_c\) according to Eq. \((1)\). State \((5)\) thus changes to

\[
\frac{1}{\sqrt{2}} \left[ (\alpha |0\rangle_1 + \beta |2\rangle_1) (|0\rangle_2 + |2\rangle_2) |0\rangle_c - i \gamma |0\rangle_1 (- |0\rangle_2 + |2\rangle_2) |1\rangle_c \right] . \tag{6}
\]
Fig. 2 (Color online) Illustration for the change of the level structure of two transmon qutrits during the entanglement preparation. In a–d, figures on the left represent the level structure for qutrit 1, while figures on the right represent the level structure for qutrit 2. a Decoupling of qutrits 1 and 2 from the cavity during applying microwave pulses to qutrit 2. The pulse with a Rabi frequency $\Omega_{21}$ is applied after the pulse of a Rabi frequency $\Omega_{10}$ is completed. b Decoupling of qutrit 2 from the cavity during bringing the $|0\rangle \leftrightarrow |1\rangle$ transition of qutrit 1 to resonance with the cavity. $g_1$ is the coupling constant. c Decoupling of qutrit 1 from the cavity during applying a microwave pulse to qutrit 1 and bringing the $|0\rangle \leftrightarrow |1\rangle$ transition of qutrit 2 to resonance with the cavity. $g_2$ is the coupling constant. d Decoupling of qutrits 1 and 2 from the cavity during applying microwave pulses to each qutrit (for the sequence of the applied pulses, see the operation of step 5 in the text). In b, c during the cavity on resonance with the $|0\rangle \leftrightarrow |1\rangle$ transition of qutrit 1 (2), the cavity is highly detuned (or decoupled) from the $|1\rangle \leftrightarrow |2\rangle$ transition of qutrit 1 (2) can be made by having the $|1\rangle \leftrightarrow |2\rangle$ transition frequency highly detuning from the cavity frequency. Note that the coupling or decoupling of the qutrits with the cavity can be achieved by adjusting the level spacings of the qutrits [14,40–43]

Step 4: Bring the $|0\rangle \leftrightarrow |1\rangle$ transition of qutrit 1 to resonance with the cavity for an interaction time $3\pi/(2g_1)$ (Fig. 2b). According to Eq. (1), we have $|0\rangle_1|1\rangle_c \rightarrow i|1\rangle_1|0\rangle_c$. Hence, state (6) changes to

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\[
\frac{1}{\sqrt{2}} \left[ (\alpha |0\rangle_1 + \beta |2\rangle_1) (|0\rangle_2 + |2\rangle_2) + \gamma |1\rangle_1 (-|0\rangle_2 + |2\rangle_2) \right] |0\rangle_c .
\] (7)

Step 5: Apply microwave pulses of \{ω_{10}, −π/2, π/ (2Ω_{10})\}, \{ω_{21}, π/2, π/(4Ω_{21})\}, \{ω_{10}, π/2, 3π/ (4Ω_{10})\}, and then \{ω_{21}, π/2, π/(2Ω_{21})\} to qutrit 2 in turn (Fig. 2d). According to Eqs. (2) and (3), the pulse application leads to \((-|0\rangle_2 + |2\rangle_2) / \sqrt{2} \rightarrow |1\rangle_2\) but nothing to the state \((|0\rangle_2 + |2\rangle_2) / \sqrt{2}\) of qutrit 2 (see Appendix). Meanwhile, apply a microwave pulse of \{ω_{21}, π/2, π/(2Ω_{21})\} to qutrit 1 (Fig. 2d), resulting in \(|2\rangle_1 \rightarrow |1\rangle_1\) and \(|1\rangle_1 \rightarrow −|2\rangle_1\) according to Eq. (3). Therefore, state (7) becomes

\[
\left[ \frac{1}{\sqrt{2}} \alpha (|0\rangle_1 + \beta |1\rangle_1) (|0\rangle_2 + |2\rangle_2) − \gamma |2\rangle_1 |1\rangle_2 \right] |0\rangle_c .
\] (8)

Step 6: Bring the \(|0\rangle \leftrightarrow |1\rangle\) transition of qutrit 1 to resonance with the cavity for an interaction time \(\frac{π}{ (2g_1)}\) (Fig. 2b). As a result, we have \(|1\rangle_1 |0\rangle_c \rightarrow −i |0\rangle_1 |1\rangle_c\). Thus, state (8) becomes

\[
\frac{1}{\sqrt{2}} \left[ \alpha |0\rangle_1 (|0\rangle_2 + |2\rangle_2) |0\rangle_c − i\beta |0\rangle_1 (|0\rangle_2 + |2\rangle_2) |1\rangle_c \right] − \gamma |2\rangle_1 |1\rangle_2 |0\rangle_c .
\] (9)

Step 7: Bring the \(|0\rangle \leftrightarrow |1\rangle\) transition of qutrit 2 to resonance with the cavity for an interaction time \(\frac{π}{g_2}\) (Fig. 2c), resulting in \(|0\rangle_2 |1\rangle_c \rightarrow −|0\rangle_2 |1\rangle_c\) and \(|1\rangle_2 |0\rangle_c \rightarrow −|1\rangle_2 |0\rangle_c\). Meanwhile, apply a microwave pulse of \{ω_{21}, π/2, π/Ω_{21}\} to qutrit 1 (Fig. 2c), resulting in \(|2\rangle_1 \rightarrow −|2\rangle_1\). Thus, state (9) becomes

\[
\frac{1}{\sqrt{2}} \left[ \alpha |0\rangle_1 (|0\rangle_2 + |2\rangle_2) |0\rangle_c − i\beta |0\rangle_1 (−|0\rangle_2 + |2\rangle_2) |1\rangle_c \right] − \gamma |2\rangle_1 |1\rangle_2 |0\rangle_c .
\] (10)

Step 8: Bring the \(|0\rangle \leftrightarrow |1\rangle\) transition of qutrit 1 to resonance with the cavity for an interaction time \(3π/ (2g_1)\) (Fig. 2b). As a result, we have \(|0\rangle_1 |1\rangle_c \rightarrow i |1\rangle_1 |0\rangle_c\). Therefore, state (10) changes to

\[
\left\{ \frac{1}{\sqrt{2}} \left[ \alpha |0\rangle_1 (|0\rangle_2 + |2\rangle_2) + \beta |1\rangle_1 (−|0\rangle_2 + |2\rangle_2) \right] − \gamma |2\rangle_1 |1\rangle_2 \right\} |0\rangle_c .
\] (11)

Step 9: Apply a microwave pulse of \{ω_{21}, π/2, π/(2Ω_{21})\} and then a microwave pulse of \{ω_{10}, π/2, π/(4Ω_{10})\} to qutrit 2 (Fig. 2a), resulting in the state transformations \((|0\rangle_2 + |2\rangle_2) / \sqrt{2} \rightarrow |0\rangle_2\), \((−|0\rangle_2 + |2\rangle_2) / \sqrt{2} \rightarrow |1\rangle_2\), and \(|1\rangle_2 \rightarrow −|2\rangle_2\) (see Appendix). Hence, state (11) becomes

\[
(\alpha |0\rangle_1 |0\rangle_2 + \beta |1\rangle_1 |1\rangle_2 + \gamma |2\rangle_1 |2\rangle_2 ) |0\rangle_c .
\] (12)

The above operations are illustrated in Fig. 3. Result (12) shows that the two qutrits are prepared in an entangled state

\[
|\psi\rangle_{\text{ent}} = \alpha |0\rangle_1 |0\rangle_2 + \beta |1\rangle_1 |1\rangle_2 + \gamma |2\rangle_1 |2\rangle_2 ,
\] (13)
Fig. 3 (Color online) Circuit diagram for the two-qutrit entanglement preparation (the operation sequence follows the arrows). The subscript \(j\) of \(C_j\), \(p_j\), and \(P_j\) represents qutrit \(j\) (\(j = 1, 2\)) on which the operation is performed. \(C_1(g_1t)\), with \(g_1t = \pi/2\) or \(3\pi/2\), represents the operation that the cavity resonantly interacts with the \(|0\rangle \leftrightarrow |1\rangle\) transition of qutrit 1. \(C_2(g_2t)\), with \(g_2t = \pi\), represents the operation that the cavity resonantly interacts with the \(|0\rangle \leftrightarrow |1\rangle\) transition of qutrit 2. Here \(g_1\) or \(g_2\) is the qutrit–cavity coupling constant, while \(t\) is the qutrit–cavity interaction time. In addition, \(p_j(\phi, \Omega_{j2}t)\), with \(\phi = -\pi/2\) or \(\pi/2\) and \(\Omega_{j2}t = \pi/4, \pi/4\) or \(\pi\), represents the operation that a microwave pulse is resonant with the \(|1\rangle \leftrightarrow |2\rangle\) transition of qutrit \(j\) (\(j = 1, 2\)), while \(P_j(\phi, \Omega_{j1}t)\), with \(\phi = -\pi/2\) or \(\pi/2\) and \(\Omega_{j1}t = \pi/4, \pi/2\) or \(3\pi/4\), represents the operation that a microwave pulse is resonant with the \(|0\rangle \leftrightarrow |1\rangle\) transition of qutrit \(j\) (\(j = 2\)). Here, \(\phi\) is the pulse initial phase, \(\Omega_{j1}\) or \(\Omega_{j2}\) is the pulse Rabi frequency, while \(t\) is the pulse duration.

While the cavity returns to its original vacuum state after the entire operation. It can be seen that prepared entangled state (13) is different from the two-qubit entangled states \(\alpha|00\rangle + \beta|11\rangle / \sqrt{2}\) and \(\alpha|01\rangle + \beta|10\rangle\). It is well known that the two-qubit entangled states have been experimentally generated with various physical systems. Interestingly, it is noted that the procedure for entangling the two qutrits has nothing to do with \(\alpha\), \(\beta\), and \(\gamma\). Moreover, the degree of entanglement for prepared two-qutrit entangled states (13) is controllable by varying the weight factors \(\alpha\), \(\beta\), and \(\gamma\) involved in the initial state of qutrit 1. For \(\alpha = \beta = \gamma = 1/\sqrt{3}\), prepared entangled state (13) is a maximally entangled state \((|0\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2 + |2\rangle_1|2\rangle_2) / \sqrt{3}\) of the two qutrits. Otherwise, the two qutrits are prepared in a partially entangled state.

From description given above, one can see that this proposal does not require that the qutrit–cavity coupling constants \(g_1\) and \(g_2\) are identical. Thus, either non-uniformity in the qutrit device parameters or non-exact placement of qutrits in the cavity is allowed, which significantly reduces the experimental engineering difficulty. In addition, neither auxiliary qutrit nor measurement is needed for the two-qutrit entanglement production. Furthermore, as shown above, this proposal only requires
two different microwave pulse frequencies ($\omega_{10}$ and $\omega_{21}$), which are readily achievable in current experiments [41].

Before ending this section, several points are addressed as follows:

(a) During the operations of steps (2–4) and (6–8), the $|0\rangle \leftrightarrow |1\rangle$ transition of qutrit 1 or 2 is on resonance with the cavity. This can be achieved by adjusting the level spacings of the qutrits. For superconducting qutrits, the level spacings can be rapidly (within a few nanoseconds) adjusted by varying external control parameters (e.g., the magnetic flux applied to a superconducting loop of phase, transmon, Xmon, or flux qubits; see, e.g., [14,42,44,45]).

(b) During the cavity interacting with a qutrit, the other qutrit is decoupled from the cavity. During the pulse application, both qutrits are decoupled from the cavity, in order to avoid the effect of the unwanted qutrit–cavity interactions. In addition, during the cavity on resonance with the $|0\rangle \leftrightarrow |1\rangle$ transition of a qutrit, the $|1\rangle \leftrightarrow |2\rangle$ transition of this qutrit is highly detuned (or decoupled) from the cavity. In principle, these conditions can be satisfied by adjusting the level spacings of the qutrits [14,42,44,45].

(c) The ladder-type level structure is available in natural atoms or other artificial atoms such as nitrogen-vacancy (NV) centers, quantum dots, and various other SC qutrits (SC phase, charge, and Xmon qubits). For NV centers, the level spacings can be readily adjusted by changing the external magnetic field applied along the crystalline axis of each NV center [46,47]. For quantum dots and atoms, the level spacings can be adjusted by changing the voltage on the electrodes around each atom/quantum dot [48]. Thus, this proposal can be applied to create two-qutrit entangled state (13) in a wide range of physical systems.

(d) The method presented here is applicable to a 1D, 2D, or 3D cavity or resonator as long as the conditions described above can be met.

### 4 Discussion

The total operation time is

$$\tau = \frac{7\pi}{4\Omega_{10}} + \frac{13\pi}{4\Omega_{21}} + \frac{4\pi}{g_1} + \frac{2\pi}{g_2} + 8\tau_d,$$  \hspace{1cm} (14)$$

where $\tau_d$ is the typical time required for adjusting the qutrit level spacings. To reduce the effect of decoherence from the qutrits, the operation time $\tau$ should be much smaller than the energy relaxation time and the dephasing time of qutrits. In addition, to reduce the effect of dissipation from the cavity, $\tau$ should be much smaller than the lifetime of the cavity mode, which is given by $\kappa^{-1} = Q/(2\pi\nu_c)$ ($Q$ is the quality factor of the cavity, while $\nu_c$ is the cavity frequency). In principle, these requirements can be satisfied for the following reasons. First, the operation time $\tau$ can be shortened by increasing the coupling constants $g_1$ and $g_2$ (e.g., placing each qutrit at an antinode of the cavity magnetic field), increasing the pulse Rabi frequencies $\Omega_{10}$ and $\Omega_{21}$ (e.g., through increasing the pulse intensity), and rapidly adjusting the qutrit level spacings (e.g., $\tau_d \sim 1–3$ ns is the typical time for adjusting the level spacings of a
superconducting qutrit in experiments [41–43,49]). Second, one can choose qutrits with long decoherence times. Last, $\kappa^{-1}$ can be increased by employing a high-$Q$ resonator.

5 Possible experimental implementation

Let us now consider the experimental possibility of preparing two-qutrit entangled state (13) with two superconducting transmon qutrits embedded in a one-dimensional transmission line resonator (TLR) (Fig. 4).

When dissipation and dephasing are considered, the dynamics of the lossy system is determined by

$$\frac{d\rho}{dt} = -i[H_I, \rho] + \kappa L[a]$$

$$+ \sum_{j=1,2} \gamma_{10,j} L[\sigma^-_{10,j}] + \gamma_{21,j} L[\sigma^-_{21,j}] + \gamma_{20,j} L[\sigma^-_{20,j}]$$

$$+ \sum_{j=1,2} \{ \gamma_{\varphi 1,j} (\sigma_{11,j} \rho \sigma_{11,j} \rho / 2 - \rho \sigma_{11,j} \rho / 2) \}$$

$$+ \sum_{j=1,2} \{ \gamma_{\varphi 2,j} (\sigma_{22,j} \rho \sigma_{22,j} \rho / 2 - \rho \sigma_{22,j} \rho / 2) \},$$

where $H_I$ is the Hamiltonian $H_{I1}, H_{I2},$ or $H_{I3}$ above, the subscript $j$ represents qutrit $j \ (j = 1, 2)$; $\sigma^-_{10,j} = |0\rangle_j \langle 1|, \sigma^-_{21,j} = |1\rangle_j \langle 2|, \sigma^-_{20,j} = |0\rangle_j \langle 2|, \sigma_{11,j} = |1\rangle_j \langle 1|, \sigma_{22,j} = |2\rangle_j \langle 2|; \text{ and } L[\xi] = \xi \rho \xi^\dagger / 2 - \rho \xi \xi^\dagger / 2$, with $\xi = a, \sigma^-_{10,j}, \sigma^-_{21,j}, \sigma^-_{20,j}$. Here, $\kappa$ is the photon decay rate of the cavity or resonator. In addition, $\gamma_{10,j}$ is the energy relaxation rate for the level $|1\rangle$ of qutrit $j$, $\gamma_{21,j}$ ($\gamma_{20,j}$) is the energy relaxation rate of the level $|2\rangle$ of qutrit $j$ for the decay path $|2\rangle \rightarrow |1\rangle(|0\rangle)$, and $\gamma_{\varphi 1,j}$ ($\gamma_{\varphi 2,j}$) is the dephasing rate of the level $|1\rangle(|2\rangle)$ of qutrit $j \ (j = 1, 2)$.
Fig. 5 (Color online) Fidelity versus $T$ and $\kappa^{-1}$. The parameters used in the numerical simulation are referred to the text.

The fidelity of the operations is given by

$$F = \sqrt{\langle \psi_{\text{id}} | \rho | \psi_{\text{id}} \rangle},$$  

(16)

where $|\psi_{\text{id}}\rangle$ is the output state of an ideal system without dissipation and dephasing, while $\rho$ is the final practical density operator of the system when the operation is performed in a realistic situation. As an example, consider $\alpha = \beta = \gamma = 1/\sqrt{3}$, for which prepared entangled state (13) is $|\psi_{\text{id}}\rangle = (|0\rangle_1 |0\rangle_2 + |1\rangle_1 |1\rangle_2 + |2\rangle_1 |2\rangle_2) / \sqrt{3}$ for the ideal case, which corresponds to the case that the initial state of the whole system is $|\psi\rangle_{\text{in}} = 1/\sqrt{3} (\alpha |0\rangle_1 + \gamma |1\rangle_1 + \beta |2\rangle_1) |0\rangle_2 |0\rangle_c$.

The parameters used in the numerical simulation are: (i) $\gamma_{10,j}^{-1} = 2T$, $\gamma_{20,j}^{-1} = 5T$, $\gamma_{21,j}^{-1} = T \mu s$, $\gamma_{\phi1,j}^{-1} = \gamma_{\phi2,j}^{-1} = T \mu s$, (ii) $g_1$, $g_2$, $\Omega_{10}$, $\Omega_{21} = 2\pi \times 100$ MHz. The coupling strengths and the pulse Rabi frequencies with the values chosen here are readily available in experiments because a coupling strength $g/2\pi \sim 220$ MHz has been experimentally demonstrated for a transmon device coupled to a TLR [50] and a microwave pulse Rabi frequency $\Omega/2\pi \sim 300$ MHz has been reported in experiments [51].

By solving master Eq. (15), we numerically calculate the fidelity versus $T$ and $\kappa^{-1}$, as shown in Fig. 5. Figure 5 illustrates that when $T = 30 \mu s$ and $\kappa^{-1} \geq 2.5 \mu s$, fidelity exceeds 98.03%. This result implies that a high fidelity can be obtained when the dissipation and dephasing are taken into account.

For $T = 30 \mu s$, we have $\gamma_{10,j}^{-1} = 60$, $\gamma_{20,j}^{-1} = 150$, $\gamma_{21,j}^{-1} = 30 \mu s$, and $\gamma_{\phi1,j}^{-1} = \gamma_{\phi2,j}^{-1} = 30 \mu s$, which are much longer than the total operation time $\sim 67$ ns (estimated based on the coupling constants and the pulse Rabi frequencies chosen above and for $\tau_d \sim 1.5$ ns). Note that energy relaxation time and dephasing time can be made to be on the order of $25-100 \mu s$ for state-of-the-art superconducting transmon devices at the present time [52–55]. It is worth noting that for an ideal transmon, the $|0\rangle \leftrightarrow |1\rangle$
transition is theoretically zero due to the selection rule [40,56], and in practice, there exists a weak transition between the two levels $|0\rangle$ and $|2\rangle$ [57]. Thus, $\gamma_{20,j}^{-1} \gg \gamma_{10,j}^{-1}, \gamma_{21,j}^{-1}$.

For a transmon qutrit, the typical transition frequency between the two lowest levels $|0\rangle$ and $|1\rangle$ can be made to be $3\text{–}10$ GHz. Thus, as an example, choose $\nu_c \sim 3$ GHz. For $\kappa^{-1} = 2.5$ µs, we have $Q \sim 4.7 \times 10^4$. The required resonator quality factor here is achievable in experiments because TLRs with a (loaded) quality factor $Q \sim 10^6$ have been experimentally demonstrated [58,59].

The analysis presented above shows that high-fidelity generation of the two-qutrit entangled state is feasible within present-day circuit QED.

Finally, for two superconducting qutrits located in a microwave resonator, they can be well separated, because the dimension of a superconducting qutrit is 10–100 micrometers, while the wavelength of the cavity mode for a microwave superconducting resonator is 1 to a few centimeters [60,61]. As long as the two qutrits are well separated in space (Fig. 4), the loop current of one qutrit affecting the other qutrit and the direct coupling between the two qutrits are negligible, which can be reached by designing the qutrits and the resonator appropriately.

6 Conclusion

We have presented a method to entangle two transmon qutrits based on circuit QED. Since only resonant interactions are employed, the entanglement can be produced within a short time (a few tens of nanoseconds), and thus, decoherence from the qutrits and the cavity are greatly suppressed. As shown above, this proposal does not require measurement on the states of the qutrits or the cavity-mode photons and only requires resonant qutrit–cavity interaction and resonant qutrit-pulse interaction for each step of the operation. Thus, this proposal is easy to implement in experiments. This proposal is quite general and can be applied to accomplish the same task with a wide range of physical systems. To the best of our knowledge, our work is the first to show how to create entangled states of qutrits with natural or artificial atoms based on cavity or circuit QED. We remark that the number of pulses can be reduced by employing dispersive qutrit–cavity interaction, but the operation time will become much prolonged due to the large detuning technique to be used. We hope that this work will stimulate the experimental activities in the near future.

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Compliance with ethical standards

Conflict of interest All authors declare that they have no conflict of interest.
Appendix

We here give a derivation on the state transformations induced by the application of the pulses, for the operations of steps 1, 5, and 9 described above.

For step 1: A microwave pulse of \( \{\omega_{10}, -\pi/2, \pi/ (4\Omega_{10})\} \) and then a microwave pulse of \( \{\omega_{21}, -\pi/2, \pi/ (2\Omega_{21})\} \) were applied to qutrit 2 (Fig. 2a). The first pulse results in \( |0\rangle_2 \to (|0\rangle_2 + |1\rangle_2) / \sqrt{2} \) according to Eq. (2), while the second pulse leads to \( |1\rangle_2 \to |2\rangle_2 \) according to Eq. (3). After the two pulses, we thus have the state transformations:

\[
|0\rangle_2 \xrightarrow{p_1} (|0\rangle_2 + |1\rangle_2) / \sqrt{2} \xrightarrow{p_2} (|0\rangle_2 + |2\rangle_2) / \sqrt{2}, \quad (17)
\]

where the first transformation is obtained after applying the first pulse, while the second transformation is achieved after applying the second pulse. Note that the state \( |0\rangle_2 \) remains unchanged during the second pulse because the pulse is highly detuned (or decoupled) from the \( |1\rangle_2 \to |2\rangle_2 \) transition of qutrit 2.

For step 5: The microwave pulses of \( \{\omega_{10}, -\pi/2, \pi/ (2\Omega_{10})\}, \{\omega_{21}, \pi/2, \pi/ (4\Omega_{21})\} \), \( \{\omega_{10}, \pi/2, 3\pi/ (4\Omega_{10})\} \), and then \( \{\omega_{21}, \pi/2, \pi/ (2\Omega_{21})\} \) were applied to qutrit 2 in turn (Fig. 2d). According to Eqs. (2) and (3), the first pulse results in \( |0\rangle_2 \to |1\rangle_2 \), the second pulse results in \( (|1\rangle_2 + |2\rangle_2) / \sqrt{2} \to |1\rangle_2 \) and \( (|1\rangle_2 - |2\rangle_2) / \sqrt{2} \to |2\rangle_2 \), the third pulse results in \( |1\rangle_2 \to (|0\rangle_2 - |1\rangle_2) / \sqrt{2} \) but nothing to the state \( |2\rangle_2 \), and the last pulse leads to \( |1\rangle_2 \to - |2\rangle_2 \) and \( |2\rangle_2 \to |1\rangle_2 \). Based on these results, we can obtain the following state transformations:

\[
(|0\rangle_2 + |2\rangle_2) / \sqrt{2} \xrightarrow{p_1} (|1\rangle_2 + |2\rangle_2) / \sqrt{2} \xrightarrow{p_2} (|0\rangle_2 - |1\rangle_2) / \sqrt{2} \xrightarrow{p_3} (|0\rangle_2 + |2\rangle_2) / \sqrt{2},
\]

\[
(|-0\rangle_2 + |2\rangle_2) / \sqrt{2} \xrightarrow{p_1} (|-1\rangle_2 + |2\rangle_2) / \sqrt{2} \xrightarrow{p_2} (|0\rangle_2 - |1\rangle_2) / \sqrt{2} \xrightarrow{p_3} |2\rangle_2 \xrightarrow{p_4} |1\rangle_2, \quad (18)
\]

where the first, second, third, and last transformations are obtained after applying the first, second, third, and the last pulses, respectively.

For step 9: A microwave pulse of \( \{\omega_{21}, \pi/2, \pi/ (2\Omega_{21})\} \) and then a microwave pulse of \( \{\omega_{10}, \pi/2, \pi/ (4\Omega_{10})\} \) were applied to qutrit 2 (Fig. 2a). The first pulse results in \( |2\rangle_2 \to |1\rangle_2 \) and \( |1\rangle_2 \to - |2\rangle_2 \), while the second pulse leads to \( (|0\rangle_2 + |1\rangle_2) / \sqrt{2} \to |0\rangle_2 \) and \( (|0\rangle_2 - |1\rangle_2) / \sqrt{2} \to |1\rangle_2 \) but nothing to the state \( |2\rangle_2 \). Based on these results, one can have the following state transformations:

\[
(|0\rangle_2 + |2\rangle_2) / \sqrt{2} \xrightarrow{p_1} (|0\rangle_2 + |1\rangle_2) / \sqrt{2} \xrightarrow{p_2} |0\rangle_2, 
\]

\[
(|-0\rangle_2 + |2\rangle_2) / \sqrt{2} \xrightarrow{p_1} (|-0\rangle_2 + |1\rangle_2) / \sqrt{2} \xrightarrow{p_2} |1\rangle_2, 
\]

\[
|1\rangle_2 \xrightarrow{p_1} - |2\rangle_2 \xrightarrow{p_2} - |2\rangle_2, \quad (19)
\]

where the first transformation is obtained after the first pulse, while the second transformation is achieved after the second pulse.
Note that the $p_1$, $p_2$, $p_3$, and $p_4$ above represent the first, second, third, and fourth pulses, respectively.

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