Restoring Classical Energy Conditions At Microscopic Level

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Abstract

We propose a mechanism for restoring classical energy conditions at quantized level based on quantum inequalities and quantum interest conjecture. More concretely, we assume for each quantum state in our visible universe to have anti-state trapped at quantum gravity scale (or true vacuum in inflation terminology). The proposed mechanism answers many outstanding questions related to the arrow of time problem, possible interpretation of the weakness of gravity and hierarchy problem, the probabilistic nature of quantum mechanics, matter-antimatter asymmetry. Moreover, the proposed mechanism gives a strong endorsement to inflation as a possible solution of flatness and horizon problems.

1 Introduction

The main purpose of physics is to solve questions related to the beginning, evolution and fate of our universe. To answer these vital questions, we need precise description to all phenomena in nature from “tiny” particles to “giant” black holes. There has been many attempts in the past under the name of “quantization of gravity”, “unified physics”..etc to tackle these questions. Probably the first decent attempt to quantize gravity using canonical quantization was done by Bronstein in 1935 [1]. Unfortunately his treatment was limited to weak gravitational fields. Apparently there exist many models in the literature which attempt to describe the reality at microscopic level including gravity either by changing the number of spacetime dimensions or gravity itself. One important question to ask “what makes a specific model true?”. This question would be answered gently if we could have true description of the whole universe but at microscopic level. Instead of attempting to quantize gravity and other forces at Planck scale (or any sub-atomic scale) to make a description we will start our analysis by formulating general quantum-based conditions that holds for large-scale objects.

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Quantum field theory allows for energy density to be locally negative for specific period and magnitude determined by the so-called “Quantum Energy Inequalities” (QEI) [2, 3, 4, 5]. This fact violates the Weak-Energy Condition (WEC) $T_{\mu \nu} u^\mu u^\nu \geq 0$ where $T_{\mu \nu}$ is the renormalized stress-energy tensor and $u^\mu$ is arbitrary timelike vector. By continuity, WEC holds for null vectors too. Weak energy condition has been used in proving singularity theorems for gravitational collapse, open universes and inflationary cosmologies [6, 7, 8, 9]. Besides WEC, we can impose further conditions on the stress-energy tensor such as the Dominant- Energy Condition (DEC) and the Strong-Energy Condition (SEC) [4]. At cosmological scale, SEC is violated sometime between the epoch of galaxy formation and the present time as shown in [10, 11]. This implies that no possible combination of ordinary matter is capable of fitting the observational data.

The strong-energy condition requires

$$\left( T_{\mu \nu} - \frac{1}{2} g_{\mu \nu} T \right) u^\mu u^\nu \geq 0,$$  \hspace{1cm} (1)

for all causal vectors $u^\mu$. The strong-energy condition played crucial role in proving the well-known Hawking-Penrose singularity theorem [9]. However many massive field theories such as massive scalar fields violate the strong energy condition. To see this, consider massive scalar field $\phi$ in Minkowski spacetime, its stress-energy tensor is

$$\left( T_{\mu \nu} - \frac{1}{2} g_{\mu \nu} T \right) u^\mu u^\nu = \left( \partial_\mu \phi u^\mu \right)^2 + \frac{1}{2} m^2 \phi^2 u^\mu u^\nu$$  \hspace{1cm} (2)

If the scalar field or its second derivative in 2 is sufficiently large, then the absolute value of the second term will be larger than the first term and since $u^\mu u^\nu \leq 0$, the strong energy condition is violated. As shown by Bekenstein, the violation of strong energy condition may have implications on nuclear matter [12].

It is important to note that we defined all aforementioned classical energy conditions locally. However as shown by Tipler [13], it is possible to define the same energy conditions over complete timelike or null geodesics.

Over a complete timelike geodesic, the Averaged Weak-Energy Condition (AWEC) reads

$$\int_{-\infty}^{\infty} T_{\mu \nu} u^\mu u^\nu d\tau \geq 0,$$  \hspace{1cm} (3)

where $u^\mu$ is the tangent vector to the timelike geodesic and $\tau$ is the observer’s proper time [14]. Analogously we can define similar condition for null geodesics.
According to topological censorship principle, the possibility of traversable wormholes requires violation of the averaged null energy condition (ANEC)

$$\int d\lambda T_{\mu\nu} \kappa^\mu \kappa^\nu \geq 0$$ (4)

for any null geodesic with associated affine parameter $\lambda$ and tangent vector $\kappa^\mu$ [15]. Classical energy conditions violation corresponds to local negative energy densities that appears in the Casimir effect [16] and the squeezed state of light [17]. The existence of negative energy fluxes in large amounts and for significant duration may give rise to traversable wormholes and time machines at macroscopic level [18, 19, 20, 21, 22, 23, 24].

2 Derivation of quantum inequalities

In [3, 25], Ford suggested an inequality that restricts the magnitude and duration of negative energies seen by inertial observers. These quantum energy inequalities are uncertainty-principle type inequalities imposed on the duration and magnitude of negative energy fluxes. The general shape of the averaged quantum inequalities for massless fields is

$$\int_{-\infty}^{\infty} S(t) \langle T_{\mu\nu} \mu^\mu \mu^\nu \rangle dt \geq -\frac{C}{\tau^d},$$ (5)

where $T_{\mu\nu} \mu^\mu \mu^\nu$ is the normal-ordered energy density operator which is classically non-negative, $t$ is the observer’s time, and $S(t)$ is the sampling function with characteristic width $\tau$. From the right hand side of (4) we notice that quantum inequalities are inversely proportional to $\tau^d$ where $d$ is the spacetime dimension number. The numerical constant $C$ varies according to the sampling function [4]. The generic form of the constant $C$ written in term of dimensionless variable $\frac{t}{t_0}$ in two (Flanagan optimal-bound) [26] and four-dimensional Minkowski spacetime [27] are

$$C = \frac{1}{24\pi} \int_{-\infty}^{\infty} \frac{|G(z)|^2}{G(z)} dz,$$ (6)

$$C = \frac{1}{16\pi^2} \int_{-\infty}^{\infty} [G''(z)]^2 dz,$$ (7)

where $G(z) = t_0 S(t)$.

We start our analysis with free massive scalar field in flat spacetime with dimensions $d$. Our metric sign convention is ($-,,+,,+$) with $\hbar = c = G = 1$ ($G$ being the gravitational constant). The wave equation for the scalar field $\phi$ is

$$\left(\Box - m^2\right) \phi = 0,$$ (8)
where \( \square = g^\mu \nu \partial_\mu \partial_\nu = -\frac{\partial^2}{\partial t^2} + \sum_{i=1}^{d-1} \frac{\partial^2}{\partial x_i^2} \) is the d’Alembert operator in Minkowski spacetime with metric \( g_{\mu \nu} \). As usual we expand the field operator in term of annihilation and creation operators

\[
\phi = \sum_k \left( a_k \hat{f}_k + a_k^\dagger \hat{f}_k^* \right),
\]

where the mode function is taken over a normalization volume \( V \) as

\[
f_k = \frac{i}{\sqrt{2\omega V}} e^{i(k \cdot x - \omega t)}.
\]

with \( \omega = \sqrt{|k|^2 + m^2} \) and \( m \) being the rest-mass. The stress tensor for massive scalar field is

\[
T_{\mu \nu} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \eta_{\mu \nu} \left( \partial_\alpha \phi \partial^\alpha \phi + m^2 \phi^2 \right).
\]

The renormalized expectation value of the energy density for arbitrary state \( |\psi\rangle \) in the reference frame of an internal observer at an arbitrary spatial point \( x = 0 \) is

\[
\langle T_{00} \rangle = \frac{\text{Re}}{2V} \sum_{k,k'} \frac{(\omega' + k' \cdot k)}{\sqrt{\omega' \omega}} \left[ \langle a_{k'}^\dagger a_k \rangle e^{i(\omega' - \omega)t} + \langle a_{k'} a_k \rangle e^{-i(\omega' + \omega)t} \right] \quad (12)
\]

\[
+ \frac{\text{Re}}{2V} \sum_{k,k'} \frac{m^2}{\sqrt{\omega' \omega}} \left[ \langle a_{k'}^\dagger a_k \rangle e^{i(\omega' - \omega)t} - \langle a_{k'} a_k \rangle e^{-i(\omega' + \omega)t} \right].
\]

where \( t \) is the proper time of observer. Following \([25, 28, 29]\), we multiply \( \langle T_{00} \rangle \) by a Lorentzian sampling function with characteristic width \( t_0 \). Then the integrated energy density becomes

\[
\dot{\rho} = \frac{t_0}{\pi} \int_{-\infty}^{\infty} \frac{\langle T_{00} \rangle dt}{t_0^2 + t^2} = \frac{\text{Re}}{2V} \sum_{k,k'} \frac{(\omega' + k' \cdot k)}{\sqrt{\omega' \omega}} \left[ \langle a_{k'}^\dagger a_k \rangle e^{i(\omega' - \omega)t_0} + \langle a_{k'} a_k \rangle e^{-i(\omega' + \omega)t_0} \right] \quad (13)
\]

\[
+ \frac{\text{Re}}{2V} \sum_{k,k'} \frac{m^2}{\sqrt{\omega' \omega}} \left[ \langle a_{k'}^\dagger a_k \rangle e^{i(\omega' - \omega)t_0} - \langle a_{k'} a_k \rangle e^{-i(\omega' + \omega)t_0} \right].
\]

Using the following lemma

\[
\sum_{k,k'} \frac{k' k}{\sqrt{\omega' \omega}} \langle a_{k'}^\dagger a_k \rangle e^{i(\omega' - \omega)t_0} \geq \sum_{k,k'} \frac{k' k}{\sqrt{\omega' \omega}} \langle a_{k'}^\dagger a_k \rangle e^{i(\omega' - \omega)t_0}
\]

where \( i = x, y, z \ldots \) are the spatial coordinates, we obtain

\[
\dot{\rho} \geq \frac{\text{Re}}{2V} \sum_{k,k'} \frac{(\omega' + k' \cdot k)}{\sqrt{\omega' \omega}} e^{-i(\omega' + \omega)t_0} \left[ \langle a_{k'}^\dagger a_k \rangle + \langle a_{k'} a_k \rangle \right] \quad (14)
\]

\[
+ \frac{\text{Re}}{2V} \sum_{k,k'} \frac{m^2}{\sqrt{\omega' \omega}} e^{-i(\omega' + \omega)t_0} \left[ \langle a_{k'}^\dagger a_k \rangle - \langle a_{k'} a_k \rangle \right].
\]

The last equation can be simplified further using the following lemma \([29]\)

\[
2\text{Re} \sum_{k,k'} C_{k'} C_k \left[ \langle a_{k'}^\dagger a_k \rangle + \langle a_{k'} a_k \rangle \right] \geq - \sum_k (C_k)^2. \quad (15)
\]
We finally obtain
\[ \hat{\rho} \geq -\frac{1}{2V} \sum_k \omega e^{-2\omega t_0}. \] (16)

In the continuum limit, we let \( V \to \infty \) and \( \sum_k \to \frac{V}{(2\pi)^{d-1}} \int_{-\infty}^{\infty} d^{d-1}k \), then the inequality (16) becomes
\[ \hat{\rho} \geq -\frac{1}{2(2\pi)^{d-1}} \int_{-\infty}^{\infty} d^{d-1}k \omega e^{-2\omega t_0} \] (17)

Using \( d^{d-1}k = \frac{(d-1)\pi^{\frac{d-1}{2}}}{\Gamma\left(\frac{d-1}{2}+1\right)} k^{d-2} dk \) and \( |k| = \sqrt{\omega^2 - m^2} \) with fixed \( m \), we find
\[ \hat{\rho} \geq -\frac{(d-1)\pi^{\frac{d-1}{2}}}{2(2\pi)^{d-1}\Gamma\left(\frac{d-1}{2}+1\right)} \int_{m}^{\infty} (\omega^2 - m^2) \frac{d-3}{2} \omega^2 e^{-2\omega t_0} d\omega \] (18)

We make change of variables in the last inequality, the final result can be written as
\[ \hat{\rho} \geq -\frac{(d-1)\pi^{\frac{d-1}{2}}}{2(2\pi)^{d-1}\Gamma\left(\frac{d-1}{2}+1\right)(2t_0)^d} \int_{y}^{\infty} (x^2 - y^2)^{\frac{d-3}{2}} x^2 e^{-x} dx, \] (19)

where \( x = 2\omega t_0 \) and \( y = 2m t_0 \). We define the scaling integral as
\[ G(y, d) = N \int_{y}^{\infty} (x^2 - y^2)^{\frac{d-3}{2}} x^2 e^{-x} dx \] (20)

where \( N = \lim_{y \to 0} \int_{y}^{\infty} (x^2 - y^2)^{\frac{d-3}{2}} x^2 e^{-x} dx \) is the normalization constant. The scaling integral should approach 1 when \( y \to 0 \) in order not to have contradiction with the massless energy inequalities i.e., \( \lim_{y \to 0} [G(y, d)] = 1 \). In other words, (19) should gives the massless quantum energy inequality when \( m \to 0 \). Moreover, \( \lim_{y \to \infty} [G(y, d)] = 0 \). For \( d = 2 \), \( G(y, 2) = y^2[K_0(y) + K_2(y)] \) where \( K_\nu(y) \) is the modified Bessel function of the second kind whereas for \( d = 4 \), it equals to \( G(y, 4) = \frac{1}{4} \int_{y}^{\infty} dx \sqrt{x^2 - y^2} x^2 e^{-x} \). In two-dimensions the QI-bound is not optimal for massive case and can take slightly a value greater than 1 for small masses \[29\]. Similar behavior are present in the case of quantum inequality in the four-dimensional Einstein universe \[30\].

**Condition:** 
"The magnitude of quantum inequalities for massive quantum fields are always less than the quantum inequalities for the same quantum fields with zero mass computed using the same sampling function." In other words, as we increase the mass, the corresponding quantum inequality decrease and vice versa.

In case of electromagnetic field (photons) in four-dimensional Minkowski spacetime, the corresponding quantum inequality in the Coulomb gauge and zero scalar potential is
\[ \hat{\rho} \geq -\frac{1}{16\pi^2} \int_{0}^{\infty} d\omega \omega^4 e^{-2\omega t_0}. \] Performing the integral gives us the desired result \( \hat{\rho} \geq -\frac{1}{16\pi^2 t_0} \) in the Lorentzian sampling function \[29\].
We may define similar inequalities for other vector fields and in principle any higher-rank tensor quantum fields.

According to Quantum Interest Conjecture “a positive energy pulse must overcompensate the negative energy pulse by an amount which is a monotonically increasing function of the pulse separation” [31]. As shown by Pretorius in [32] quantum interest conjecture holds for massive as well as massless scalar fields for arbitrary sampling functions (distributions) in the four-dimensional Minkowski spacetime. The same conclusion applies for Dirac, electromagnetic fields and any quantum field in general.

As we shall see later in this article, the idea of treating any change or movement in masses or energy pulses in our visible universe or any dynamical system as the attempt of this system for reducing the quantum interest can solve many longstanding questions in physics. Quantum interest conjecture introduced in [31] and proven to hold in (3+1)-Minkowski spacetime using generalized version of Simon’s theorem [33, 34]. The main idea is to treat the quantum interest as an eigenvalue problem of multi-harmonic time-dependent Schrödinger equation in even \( d \)-dimensional spacetime.

In curved spacetimes, quantum energy inequalities assume some corrections in comparison with their counterparts in Minkowski spacetime. Consider, for example, a scalar field \( \phi \) in a curved spacetime with metric tensor \( g_{\mu\nu} \), the Lagrangian density is [35, 36]

\[
\mathcal{L}_g[\phi] = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - (\mu^2 + \zeta R) \phi^2,
\]

where \( R \) and \( \zeta \) are the Ricci scalar and the dimensionless coupling constant respectively. The case \( \zeta = 0 \) is known as minimal coupling case. The conformal coupling case corresponds to \( m = 0 \) and \( \zeta = \frac{d-2}{4d-4} \). At conformal coupling, the Lagrangian [21] is invariant under the following conformal transformations i.e. \( \mathcal{L}_\Omega^\Omega = \mathcal{L}_g[\phi] \)

\[
\overline{g}_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu},
\]

\[
\overline{\phi} \rightarrow \Omega^{1-\frac{d}{4}} \phi,
\]

for any smooth function \( \Omega \). We have to modify the scaling function \( G(y, 4) \) in curved spacetime. Consider for example a scalar field at minimal coupling in open universe, the corresponding quantum inequality is given by \( \hat{\rho} \geq -\frac{3}{32\pi^4 t_0} G(2t_0 \sqrt{\mu^2 + \frac{1}{4}}, 4) \). The scaling function in this case can be written in terms of the modified Bessel function \( K_\nu(y) \) as \( G(y, 4) = \frac{1}{4} [y y^3 K_3(y) - y^2 K_2(y)] \) [37].
3 Restoring classical energy conditions

The Robertson-Walker metric is

$$ds^2 = -dt^2 + a^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2)\right\}$$  \hspace{1cm} (24)

and the Friedmann equations are \[38\]

$$\ddot{a} = -\frac{4\pi}{3}G(\rho + 3p)a,$$ \hspace{1cm} (25)

$$H^2 = \frac{8\pi}{3}G\rho - \frac{k}{a^2},$$ \hspace{1cm} (26)

$$\dot{\rho} = -3H(\rho + p),$$ \hspace{1cm} (27)

where \(k = 0, \pm 1\), \(\rho\) is the energy density, \(p\) is the pressure and \(G\) is the universal gravitational constant and \(H = \frac{\dot{a}}{a}\). Equation 25 shows that in order to have accelerated universe, the pressure must be negative and this in turn correspond to some form of repulsive gravity \[39\]. Furthermore, we assume the energy density \(\rho\) to be approximately constant, then the late-time asymptotic solution of the differential equation 25 is simply exponential expansion of the scale factor i.e \(a(t) \sim e^{\chi t}\) where \(\chi = \sqrt{\frac{8\pi G \rho}{3}}\) \[40\].

The key feature of inflation is the existence of metastable or local minimum in the potential energy known as false vacuum. Classically any state in this false vacuum can not decay into the true vacuum (global minimum of the potential energy). However quantum mechanically it is possible by tunneling to penetrate the potential barrier and decay into the true vacuum which has global minimum. Several types of inflation models has been suggested such as the original inflationary, the new inflationary and the chaotic inflation models \[40, 41, 42, 43\].

As we have seen earlier, classical energy conditions are severely violated at quantum level. This corresponds to local negative energies which are deemed to decay and overcompensated by positive energy pulses according to quantum interest conjecture and considerations related mainly to the second law of thermodynamics.

In order to restore classical energy conditions we assume the existence of anti-particles with negative energies trapped in true vacuum. These anti-particles are the same introduced by Dirac \[44\] and observed experimentally in case of anti-electrons by Anderson \[45\]. However the mass of these antiparticles are larger since they are trapped in a curved spacetime not the usual Minkowski spacetime. Moreover the time direction of these trapped antiparticles is reversed with respect to the time direction of their corresponding particles in our reference frame. This assure the stability of these negative energy states since in our
reference frame, with usual time direction, any negative energy state is deemed to collapse. This assumption is in agreement with the Feynman-Stückelberg interpretation of anti-particles.

If we look at antiparticles as a “loan” then this loan must re-paid from the extracted positive energy states with “interest” according to quantum interest conjecture during fixed interval of time. Since the loan should be paid in a uniform manner, time must go in one-direction since each time slice represents a specific event with a certain installment quantity. This explains why time as a universal parameter must follow one direction. Thus, adopting this explanation solves the arrow of time problem and gives strong endorsement to chronology protection conjecture of Hawking [46]. Any positive state in our universe with its “own” anti-state trapped in the true vacuum forms closed entity from which all the dynamics of it can be deduced and accurately calculated. Furthermore, we may describe order in our universe using this model and the restrictions imposed on massless states at some stage of the universe to become massive and reduce the interest rate.

The proposed setup in this work is in agreement with Sakharov conditions for successful Baryongenesis [47]. The Baryon asymmetry of the visible universe \( \eta = \frac{n_B - n_{\bar{B}}}{n_B - n_{\bar{B}}} \) where \( n_B \) and \( n_{\bar{B}} \) are the baryon and antibaryon number was of order \( 10^{-10} \) in the past [48]. This is in agreement with the assumptions that most of antiparticles were trapped in the true vacuum with curved spacetime while our ordinary particles evolved rapidly and formed the galaxies .. etc.

To this end we must accomplish the following:

- Each state in our visible universe has anti-state trapped in quantum gravity scale (true vacuum if we use inflation terminology ) with negative energy, reverse spin and charge. This negative state has the same mass of its corresponding positive state in the same spacetime type. However since they are trapped in a curved spacetime (most likely de Sitter spacetime in the case of our universe), its mass is greater than the mass of the positive state in Minkowski spacetime.

- According to quantum interest conjecture, negative energy states can be considered as a loan and the evolution of positive energy states as the attempt of re-paying this loan with interest.

- Any dynamical system in our universe follows the fastest trajectory available to re-pay the loan with lowest interest. This assumption prevents macroscopic violation of the second law of thermodynamics by restricting the extent of negative energy densities at quantum level.

- We require the trapped anti-states in the true vacuum with negative en-
ergies to have reversed arrow of time and to vary slowly in the opposite
direction with respect to us (in flat spacetime).

We apply the above assumptions to the case of complex scalar field \( \phi = (\phi_1 + i\phi_2) / \sqrt{2} \) with its anti-state trapped in the quantum gravity level \( \phi^{(g)_\dagger} = (\phi_2^{(g)} - i\phi_1^{(g)}) / \sqrt{2} \) described by the action \( S[\phi, \phi^{(g)}] = \int d^4x' \eta \partial \mu \phi \partial \nu \phi^{\dagger} - m^2 \phi \phi^{\dagger} \) (28)

\[
S[\phi, \phi^{(g)}] = \int d^4x' \eta \partial \mu \phi \partial \nu \phi^{\dagger} - m^2 \phi \phi^{\dagger} 
- \int d^4x' \eta \partial \mu \phi^{(g)} \partial \nu \phi^{(g)} \dagger + (\mu^2 + \zeta R) \phi^{(g)} \phi^{(g)} \dagger, 
\]

The integral of (28) should respect the integral measure for each metric.

As a specific case, we assume the anti-states to be localized in a four-
dimensional de-Sitter spacetime at minimal coupling for simplicity. The cor-
responding metric in the static parametrization is

\[
ds^2 = -(1 - r^2 / \alpha^2) dt^2 + (1 - r^2 / \alpha^2)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \]

(29)

where there is a particle horizon at \( r = \alpha \) for an observer located at rest at \( r = 0 \). The coordinates assume the values \( 0 \leq r < \alpha, 0 \leq \theta < \pi \) and \( 0 \leq \phi < 2\pi \). The complex scalar field in the de-sitter spacetime is

\[
\phi^{(g)} = \left( \begin{array}{c} \phi_2^{(g)} \\ i\phi_1^{(g)} \end{array} \right) = \left( \begin{array}{c} \psi_{\omega, \ell, m}(t, r, \theta, \phi) \\ i\chi_{\omega, \ell, m}(t, r, \theta, \phi) \end{array} \right) 
\]

(30)

where \( \psi \) and \( \chi \) are two different real scalar fields but assume the same general formula of the type \( \psi(\text{or } \chi) \sim f_\ell(\chi)Y_{\ell m}(\theta, \phi)e^{-i\omega t} \) up to a normalization constant with different outputs. The averaged quantum weak energy condition (AQWEC) for the whole system (positive energy states in Minkowski spacetime + anti-states trapped in de-sitter spacetime with negative energies) is

\[
\hat{\rho} \geq -\frac{3}{32\pi^2 t^4} G(2\sqrt{2}t / \alpha, d = 4) + \frac{3}{32\pi^2 t^4} \left[ 1 + \frac{t^2}{\alpha^2} \right] 
\]

(31)

where we have considered the case \( m = \mu = \sqrt{2} \) for analytical reasons only (see Eq.4.130 from [37]). The second term in the right hand side is positive with respect to a measuring observer sitting inside ordinary matter. Moreover it is positive and greater in comparison with the first term since time \( t_0 \) varies very slowly comparing with \( t \) as we showed in [49]. Similar quantum inequalities are present in the case of planar mirrors i.e. see for example equation (4.91) from [37].

Apparently we can not have the case where states and their corresponding
anti-states (trapped in true vacuum) to be localized in the same curved spac-
time type because this case will give always quantum inequality equals to zero.
It means that no dynamics can be constructed from this system at quantized level. Inflation is the attempt of the visible universe to become flat (different from the negative energies trapped in quantum gravity scale) [40]. Moreover, this fact explains the weakness of gravity and the hierarchy problem since anti-states trapped in their true vacuum have high energy and trapped in a curved spacetime with energies close or equal to the Planck scale. This curved spacetime which hosts these states is different from the spacetime type that hosts the positive energy states in our universe (Minkowski spacetime).

Thus we conclude that: In any dynamical system, its quantum inequality should not equal to zero for massive case and has negative lower bound with respect to its own reference frame

\[ \rho_{\text{massive}} = \int \langle T_{00}(t, x) \rangle |S(t)|^2 dt \neq 0 \] (32)

where \( S(t) \) is arbitrary sampling function.

All Baryons and Leptons are massive, this is in coincidence with our considerations since increasing mass means lower interest. Neutrinos are massive and in See-saw mechanism which gives an interpretation for neutrino tiny mass, inclusion of a right-handed neutrino with larger mass [50]. According to our setup, these right-handed neutrinos are trapped in the true vacuum and thus have greater masses than left-handed neutrinos as we mentioned above.

4 Bimetric quantum mechanics

Quantum energy inequalities are also present in quantum mechanics [51]. One question to ask “Can quantum inequalities and quantum interest conjecture explain the many-world interpretation of quantum mechanics and the associated probabilistic nature?” [52, 53, 54, 55, 56, 57]. If we represent our universe quantum mechanically by a universal wavefunction \( |\psi\rangle \), then the quantum inequality associated with it will enforce this state to evolve and pay back the loan with interest. Since there always exist many choices for such quantum state to pay the loan. These different choices are the probabilistic energy eigenvalues options for the quantum state. This holds for any quantum wavefunction. Collectively the majority of quantum states follow the fastest and probably the easiest way in paying the loan. At quantum scale, the redundancy of choices is due to the possibilities available to any quantum state for evolution in a stable manner. This in turn implies the existence of our galaxies and any visible object. Following the longest way by all (or statistically large number) quantum states to re-pay the loan may cause accumulations of negative energies at large scale and thus macroscopic violation of the second law of thermodynamics.

Because there exists for each state a corresponding antistate at quantum gravity scale \( |\psi^{(\bar{g})}\rangle \), each quantum state in our universe is connected to this state via a wormhole-like connection. Since particles energies in our visible
universe are very low comparing to the quantum gravity scale, we are unable to observe them. This fact explains the connection between quantum entanglement and wormholes proposed in [58].

What about the “multiverse” interpretation of quantum mechanics according to the proposed model?

At quantum level, it is possible to have many probabilistic outcomes prior to any measurement maintained by the uncertainty principle and probabilistic nature of quantum mechanics [55, 56, 57]. However, once the measurement is finalized, the meaning of wavefunction collapses according to Copenhagen interpretation. Our setup in this work strongly support this assumption. Before measurement, our choices are higher and the space of possible solutions is large. This has advantages in quantum computation. However, we do not think that these possible outcomes out of what we observe in our universe will establish their own universes. The probabilistic nature of quantum mechanics collapses when quantum interest associated with a quantum state is zero. Our observed reality is constructed from evolved states in a uniform manner. This restriction guarantees the stability and coherence of our observed universe. To conclude: *Quantum states are states with non-zero quantum inequalities, the probabilistic nature of quantum states is due to the many ways available for such a state to repay the loan with interest.*

The probabilistic nature of quantum mechanics is preserved according to the proposed mechanism in this work and all phenomena such as entanglement and quantum computing are valid [59, 60].

Each State with its anti-state trapped at quantum gravity level forms a closed entity and the future of each state can be known precisely by knowing the other state. The interaction between quantum states in our visible universe is due to the willingness of these states to increase their energies and be closer to their anti-states. At high critical energy when the state becomes very closer to its anti-state at quantum gravity level (in the true vacuum), the willingness of this state to make interactions with its surrounding states decline enormously.

We can develop new quantum mechanics with bimetric formulation. For each quantum state described by the wavefunction \( |\phi\rangle \) there exists anti-state with negative energy described by \( |\chi\rangle \). For example if \( |\phi\rangle \) represents electron with spin-up then \( |\chi\rangle \) would represent positron (with extra mass due to the localization in a curved spacetime) with spin-down and negative energy since time is reversed. Note that states \( |\phi\rangle \) and \( |\chi\rangle \) are entangled. Moreover they are connected by a wormhole since the time scale of these two states are different and thus no classical connection can be established between them. If one make measurement on the state \( |\phi\rangle \), then we immediately know the physical properties of \( |\chi\rangle \) such as charge, spin, energy eigenvalues....
In bimetric quantum mechanics, no particle is isolated since at least it must be entangled with its anti-state trapped at quantum gravity scale with negative energy.

All quantum computations on a quantum computer will correspond to classical computing performed on the corresponding entangled anti-states with negative energy. The probabilistic outcomes of a given quantum state will become deterministic after performing the measurement. Knowing the exact state of a quantum system will immediately tell us the exact state of the corresponding anti-state even though we do not measure them directly. Thus any quantum computer will have pure quantum operations performed on some quantum system and corresponding deterministic knowledge based on the fact that by knowing the correct state of our quantum system we immediately can record the state of the corresponding anti-states since they are highly entangled. This will open a new branch called computing with bimetric quantum mechanics and boost the field of quantum computation further by developing new algorithms [61].

The known canonical commutation relation in quantum mechanics \([x, p_x] = i \hbar\) will have corresponding relation for negative energy state given by \([x', p'_x] = -i \hbar\) where the minus sign is due to the fact that momentum is odd with respect to time reversal operation. Interestingly these commutation relations hold in curved spacetime \([R, P_R] = i \hbar\) where \(R\) is the position vector in curved spacetime and \(P_R\) is the corresponding linear momentum. If we define the grand position vector as \(X = x + x'\) and the grand momentum as \(P = p + p'\) then the uncertainty of measuring position and momentum will disappear. Since we are sitting in a place where we have no direct access to what is happening inside the trapped negative states our local measurements will have uncertainty according to quantum mechanics. For a universal observer who knows the states of our world plus its corresponding anti-states, the uncertainty principle collapses. We do not consider the many-world interpretation true since the quantum state will follow the fastest option in paying back the loan with interest according to [49] and that will eventually promote one state and "kills" the rest. However the probabilistic nature of quantum mechanics and its applications on quantum computation are valid as the observer who performs the calculations is sitting in the reference frame of our visible universe.

5 Black holes and information paradox

All black holes are in reality connected to white holes forming a wormhole-structures. The trajectory of any quantum state will lead finally to this particle to be swallowed by a black hole at some stage of our universe. Before the entrance of a particle inside black hole horizon it is possible to figure out the history of this particle. However once the particle enters the black hole horizon it will be impossible to know the history of the falling particle. In brief
according to our setup, information is lost behind the horizon of black hole\textsuperscript{62}. Furthermore, we found our setup to be compatible with Page curve since we have increasing of entropy at the beginning and then decreasing of entropy due to the annihilation with negative energy states\textsuperscript{63}.

6 The weakness of gravity

Gravity is unique, there is no doubt in this. It is the weakest comparing with electromagnetic and nuclear forces. Interestingly, gravity plays other rule since it has the ability to change the geometry of spacetime which is necessary for having dynamical systems. So gravity is what makes system dynamical. Suppose the gravitational field strength of the trapped negative energy states is $H$ and let $H'$ to be the gravitational field strength of positive ordinary states. If $H = H'$ then no dynamics can be constructed from such system because the universe in this moment will vanish. In contrast if $H$ is very far from $H'$ in magnitude then we can have stable and evolving dynamical system that last longer as the difference is larger.

7 Origin and fate of Universe

Since the negative energy states are trapped in the true vacuum, they move slowly and thus they are colder than their positive states in our visible universe. This in support with the cold dark matter assumption in the $\Lambda$ – CDM model\textsuperscript{64} \textsuperscript{65}. In the interesting Maxwell’s demon Gedanken experiment, we have separation of the cold and hot particles in a way which violates the second law of thermodynamics\textsuperscript{66}. We believe something similar has happened during the creation of our universe since negative states (cold) which was trapped at the true vacuum are separated from the positive energy states (hot).

We have seen in earlier sections of this work and in \textsuperscript{49} that ordinary visible matter are evolving so that it assumes the same metric of the trapped negative energy states. At the moment when $g_1 = g_2$ the universe will be filled with black holes and it will disappear in accelerating way. So our universe has a beginning moment described as the transition from neutral entity to a highly negative and positive parts with two-time directions (the big bang) while the inflation can be seen as the attempt for positive energy states to expand faster and evolve in such a way its metric become identical to the almost frozen trapped negative energy states and annihilate. Time for an observer sitting inside the trapped negative states goes very slowly comparing to the time for an observer on Earth or any observer sitting in a flat spacetime but in opposite direction. Thus our universe will witness a "doomsday event" sometime in the future. By comparing the frozen trapped negative energy states with dark matter we see that both concepts play the same rule and this suggest to consider these states as a possible candidates for dark matter. In other hand, dark energy is
the trapping energy of these negative states which must be very large since the spacetime is very curved as we mentioned above. The current mathematical formalism of cosmology \cite{67} must be upgraded according to our setup in this article which is the topic for another works. Moreover since we have considered two directions of time it is worth to stress the reader attention to the following works \cite{68, 69}.

8 Conclusion

We proposed a model based on quantum inequalities and quantum interest conjecture to give a unified interpretation of classical and quantum worlds. Each quantum state must have anti-state at quantum gravity scale relative or identical to the Planck scale. These negative states are the ordinary anti-particles that we know from standard QFT courses but in opposite time direction with respect to us and with higher masses in general. Since the anti-particles are in opposite time direction their energies are negative. The dynamics of each state in our universe can be described as the attempt of this state to pay the loan with interest according to the quantum interest conjecture. Our model is not radical since what we assumed is the existence of anti-states trapped at quantum gravity scale with reversed arrow of time. Anti-states are always present at least in our ordinary quantum field theory as emphasized by Dirac. However, during the big bang moment states and anti-states deviated from each others. This can be seen in the pair production in black holes where matter and anti-matter share the same energy scale and thus see each others\cite{70}. Regarding time travel, our model suggests the fact that everything will happen with any state is determined since each state is connected with its anti-state at quantum gravity scale. It is possible to make the time for a given state to go faster by applying external energies. This will make the state closer to its anti-state. However due to the restrictions on negative energy densities, it is impossible to go back in time for large number of quantum states. Collectively objects in our universe follow one-direction of time enforced by considerations mentioned above related to the quantum interest conjecture and the laws of thermodynamics. So closed timelike curve (CTC) does not exist in our model for daily usage. In other words, our results supports Hawking’s chronology protection conjuncture\cite{46}. Moreover they explain why there is matter-antimatter asymmetry by assuming anti-states to be trapped away at quantum gravity scale with associated negative energies. One possible further direction is to study the renormalization and anomaly of field theories with fields and the corresponding anti-fields at quantum gravity scale with negative energies.

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