Comparative analysis of optical-physical schemes of gyroscopes based on macroscopic quantum effects of superfluid helium isotopes ($^3$He & $^4$He)

V S Chernichenko, A I Bidenko, N I Krobka, N V Tribulev, A A Volyn'tsev
Scientific & Research Institute for Applied Mechanics named after academician V.I. Kuznetsov & Moscow State Technical University named after Bauman
E-mail: vsc13@mail.ru

Abstract. The first prototypes of superfluid gyroscopes were demonstrated in Saclay in 1996 [1] and in Berkeley in 1997 [2]. These gyroscopes were direct hydrodynamic analogues of electrodynamic rf-SQUIDs, based on superfluid $^4$He. Analogues of dc-SQUID based on $^3$He had been worked out in 2001 [3]. The first dc-SQUID on $^4$He was built in 2005 [4]. Comparing the different technical solutions we are gathering the world experience of superfluid gyro schemes to design own ones.

1. Introduction
Gyroscopes based on Sagnac effect have been employed with light as well as with atoms. A rotation with an angular velocity $\Omega$ induces a shift in the interference fringe by amount $\Delta \phi = 4\pi m \vec{A} \cdot \vec{\Omega} / h$, where $\vec{A}$ is the area vector of the interferometer loop, and $m$ is the mass of the interfering particle. The Sagnac phase shift in superfluid gyroscopes is typically $10^{10-11}$ times larger than it is in ring laser gyro or fibre-optic gyro with the same area A. This makes the matter-wave based interferometer a promising candidate for providing unprecedented sensitivities to rotation. The device can be used for rotation sensing with possible applications expected in geodesy, seismology, inertial navigation, and potentially testing fundamental theories such as general relativity theory.

2. Superfluid gyroscopes
2.1. Superfluid rf-SQUIDs
Rotation-induced velocity [5] was first measured with a $^4$He superfluid rf-SQUID by O. Avenel and E. Varoquaux [1]. The basic structure has two superfluid reservoirs shunted by a larger tube and coupled to a diaphragm to form a hydrodynamic resonator. An orifice is a $0.17 \times 2.8 \mu m$ slit in a $0.2 \mu m$ thick Ni foil connected to the parallel channel through a two-turn pickup loop, forming the equivalent of a torus with an effective area of $4 cm^2$. The Saclay team drove the soft membrane with a sinusoidal force at the resonant frequency of the oscillator, ramping up the amplitude and eventually driving phase slips in succession. The amplitude of the membrane at which the phase slips occur is directly proportional to the critical velocity. The observed critical velocity in the orifice is shifted by the rotation-induced backflow $v_{\text{eff}} = -2\vec{\Omega} \cdot \vec{A} / l_{\text{eff}}$. The critical velocity measurement is a rotation sensing method in this configuration. This experiment was carried out with superfluid $^4$He at $T \sim 12 mK$. An experiment with better accuracy has been reported by the same authors [6], and independent corroboration has been provided by Schwab et al [7]. Later work by Bruckner and Packard [8] employed a sensing area two

Published under licence by IOP Publishing Ltd
orders of magnitude larger than the work of Schwab et al, leading to $3 \times 10^{-6}$ (rad/s)-Hz$^{-1/2}$ sensitivity at $T \sim 300$ mK. Reported by Mukharsky et al [9] single junction $^3$He gyro has demonstrated the power spectrum of the noise $1.4 \times 10^{-3}$ (rad/s)-Hz$^{-1/2}$ an improvement of nearly three orders of magnitude over the gyro operating with superfluid $^4$He [10, 11]. With a loop area of 5.9 cm$^2$, this $^3$He gyro is nearly 20 times more sensitive than the large area $^4$He. The phase slip gyros discussed above are in the form of the rf-SQUID, where a loop of quantum coherent matter is interrupted by a single junction.

Figure 1. $^3$He rf-SQUID reported by Mukharsky et al [9]

2.2. Superfluid dc-SQUID
In 2005 E. Hoskinson made the discovery that an array of nano-apertures in $^4$He exhibited the same kind of “quantum whistle” as that seen earlier in $^3$He [10]. To observe this phenomenon the helium must be a few millikelvin below the transition temperature, 2.17K. E. Hoskinson and Y. Sato demonstrated a dc-SHeQUID, which successfully measured the Earth’s rotation rate. T intrinsic phase sensitivity $\sim 3 \times 10^{-2}$ rad Hz$^{-1/2}$ which corresponds to angular velocity resolution of $\sim 2 \times 10^{-7}$ (rad/s)-Hz$^{-1/2}$ [11]. Since the high (compared to $^3$He) operating temperature of this device is achievable using a mechanical cryocooler, one could for the first time envision superfluid interferometers being practical probes of rotation. The first dc-SQUID (shown schematically in figure 2 consists of two arrays within tubes filled with $^4$He that form an interferometer loop.

Figure 2. Scheme of the dc-SQUID with two junctions [4]

Figure 3. Scheme of the dc-SQUID with four junctions [13]
Each array contains 4225 apertures, nominally 90 nm in diameter, spaced on a 3 μm square lattice in a 50-nm-thick silicon nitride membrane. The enclosed sense area of the loop is nominally 10 cm². The arrays are positioned equidistant from a diaphragm displacement transducer that functions as a microphone to detect the oscillating currents. Using a combination of electrostatic forces applied to the diaphragm and power applied to a heater just below the diaphragm it is possible to create and control hydrodynamic parameters. If there is a well-defined phase difference \( \Delta \Phi \equiv \Delta \Phi_1 - \Delta \Phi_2 \) between the arrays, then by superposition the signal amplitude of the total current \( I_t = I_1 + I_2 \) detected at the microphone can be written as \( I_m = 2I_c |\cos(\Delta \Phi/2)| \). Here the two arrays are assumed to have equal flow oscillation amplitudes \( I_c \). This is the typical behavior of an interferometer: a phase difference between two paths modulates the combined signal. The external phase shift may be due to a rotation-induced Sagnac effect. Configured as a gyroscope the angular velocity sensitivity of the interferometer improves with several variables including the number of apertures, the number of “turns” in the interferometer loop, and the enclosed area. If the aperture array contains a factor of 10 more apertures, if the loop diameter is increased by a factor of 4, and if the loop geometry is reconfigured as a ten turn helix, the extrapolated resolution of this interferometer surpasses the reported resolution of the best optical and cold-atom Sagnac gyroscopes. The sensitivity of the device is proportional to the slope of the interference pattern at its steepest point. This sensitivity can be increased by placing more than two arrays in parallel thus narrowing the peaks in the interference pattern. The figure 4 shows the modulation of the amplitude of Josephson frequency oscillations as a function of rotation flux in the case of two and four (left and right plot) junctions respectively [4, 13]. The slope at the steepest part of the interference pattern for this grating is found to be \( \approx 4.3 \) times larger than that of a superfluid \(^4\)He double-path interferometer operating at the same temperature. For example, that a grating structure with 10 weak links should give phase change sensitivity \( \approx 20 \) times greater than that of a double-path interferometer.

![Figure 4](image)

\textbf{Figure 4.} Mass-current oscillation amplitude versus \( \Delta \Phi \) in the case of two (a) and four (b) junctions respectively [4, 13]

For dc-SQUID-like superfluid devices, the acquisition rate can be three orders of magnitude higher as it is only limited by Josephson frequency. On the other hand, the inductance of the rf-SQUID type devices (and thus the loop area) can be made larger than that of the dc-SQUID type devices. Therefore, the comparison of the ‘ultimate’ performances of these devices is a rather complex issue. See a review on gyroimeters by Avenel \textit{et al} [14].

\section*{3. Interferometer size limitations}

All the phase shifts induced by external fields are proportional to the length of the SHEQUID loop, and one might imagine making ever larger devices to take advantage of that feature. However there is a
limit to such size increase. Analogous to superconducting weak links, a superfluid weak link kinetic inductance is given by \( L = (k/2\pi)(d\Phi/dI) \). In the weak-link limit near zero phase difference, this becomes \( L_j = k/2\pi I_c \). A simple tube of length \( l \) has a kinetic inductance given by \( L_t = l/\rho \sigma \) where \( \sigma \) is the tube’s cross-sectional area. If the length of the torus becomes too large, the loop inductance dominates the array inductance and the external phase shift is appreciably reduced along the tube rather than the apertures. This decreases the modulation depth of the interference pattern, diminishing its sensitivity. Probably the best compromise is for the loop inductance to match the array inductance. This sets a lower limit on the tubes internal radius which can be estimated to be \( r_{\text{min}} \approx (d\ln)^{1/2} \) where \( d \) is the aperture diameter and \( N \) the number of the apertures. With this estimate, an interferometer with loop circumference \( \sim 1 \text{m} \) requires a tube radius of \( \sim 1 \text{ cm} \). It is unknown at this time if the SHeQUID can be scaled to these (and even larger) dimensions. A superfluid interferometer with 53 cm path length and 225 cm² sensing area in counter-wound reciprocal geometry has been recently developed [15].

4. Applications

Why gyros on superfluid helium? Due to high sensitivity to absolute rotation the main fields of superfluid helium gyros applications are the problems of measuring the small rotation rates and their small variations.

Typical potential applications of superfluid helium gyros are following [16]:

1) Rotational Seismology (to detect rotational seismic waves caused by anisotropies in the crust inner core boundary);
2) Geodesy (to monitor small changes in the Earth’s rotation);
3) Very Long Baseline Interferometers of radio telescopes. Resolution is better than \( 10^9 \text{rad/s}\text{Hz}^{-1/2} \) with 24 hour averaging time (\( 4 \times 10^{12} \text{rad/s}\text{Hz}^{-1/2} \));
4) Tests of general relativity theory.

This work is partially supported by the state contract No. 02.740.11.0528 (March 15, 2010).

5. References

[1] Avenel O, Varoquaux E 1996 Czech. J. Phys. 46 3319
[2] Pereversev S V, Loshak A, Backhaus S, Davis J C and Packard R E 1997 Nature 388 449
[3] Simmonds R W, Marchenkov A, Hoskinson E, Davis J C Packard R E 2001 Nature 412 55
[4] Hoskinson E, Sato Y and Packard R E 2006 Phys. Rev. B 74 100509(R)
[5] Aarts R, Ihas G, Avenel O and Varoquaux E 1994 Physica B 194 493
[6] Avenel O, Hakonen P and Varoquaux E 1997 Phys. Rev. Lett. 78 3602
[7] Schwab K, Bruckner N and Packard R E 1997 Nature 386 585
[8] Bruckner N and Packard R E 2003 J. Appl. Phys. 93 1798
[9] Mukharsky Yu, Avenel O, Varoquaux E 2000 Physica B 280 287
[10] Hoskinson E, Parckard R E and Haard T 2005 Nature 433 376
[11] Hoskinson E, Sato Y and Packard R E 2006 Phys. Rev. B 74 100509(R)
[12] Pereversev S V, Loshak A, Backhaus S, Davis J C and Packard R E 1997 Nature 388 449
[13] Sato Y, Joshi A and Packard R E 2008 Phys. Rev. Lett. 101 085302
[14] Avenel O, Mukharsky Y and Varoquaux E 2004 J. Low Temp. Phys. 135 745
[15] Narayana S and Sato Y 2011 Phys. Rev. Lett. accepted
[16] http://www.physics.berkeley.edu/research/packard/talks/LBNL%20talk%20on%20gyroscopes.pdf