VISCOELASTIC MODELS OF TIDALLY HEATED EXOMOONS

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ABSTRACT

Tidal heating of exomoons may play a key role in their habitability, since the elevated temperature can melt the ice on the body even without significant solar radiation. The possibility of life has been intensely studied on solar system moons such as Europa or Enceladus where the surface ice layer covers a tidally heated water ocean. Tidal forces may be even stronger in extrasolar systems, depending on the properties of the moon and its orbit. To study the tidally heated surface temperature of exomoons, we used a viscoelastic model for the first time. This model is more realistic than the widely used, so-called fixed \(Q\) models because it takes into account the temperature dependence of the tidal heat flux and the melting of the inner material. Using this model, we introduced the circumplanetary Tidal Temperate Zone (TTZ), which strongly depends on the orbital period of the moon and less on its radius. We compared the results with the fixed \(Q\) model and investigated the statistical volume of the TTZ using both models. We have found that the viscoelastic model predicts 2.8 times more exomoons in the TTZ with orbital periods between 0.1 and 3.5 days than the fixed \(Q\) model for plausible distributions of physical and orbital parameters. The viscoelastic model provides more promising results in terms of habitability because the inner melting of the body moderates the surface temperature, acting like a thermostat.

Key words: planets and satellites: general

1. INTRODUCTION

No exomoons have been discovered yet, but such measurements are expected in the next decade. Bennett et al. (2014) present a candidate which has been detected via the MOA-2011-BLG-262 microlensing event. The best-fit solution for the data implies the presence of an exoplanet hosting a sub-Earth-mass moon; however, an alternate solution is more likely. Nevertheless, this measurement indicates that the era of exomoon detections is about to begin.

The most favorable method for exomoon discovery is photometry. An exoplanetary transit may enlighten the presence of a moon in the light curve. The details of this method are thoroughly discussed in the literature (Simon et al. 2007, 2010, 2012; Kipping 2009a, 2009b; Kipping et al. 2012).

In addition, the habitability of exomoons is also under examination (see, e.g., Kaltenegger 2010; Heller & Barnes 2013; Heller et al. 2014). Hinkel & Kane (2013) investigated the influence of eccentric planetary orbits on moons, and concluded that a moon with sufficient atmospheric heat redistribution may sustain a suitable temperature for life on its surface even if it orbits a planet that moves temporarily outside of the habitable zone (HZ) during each orbital period.

Solar system analogs may serve as useful examples for different exomoon types. The satellites in the solar system are diverse and life on them is a puzzling question. The icy surfaces of Europa and Enceladus probably cover water oceans, which may provide suitable environments for life (Carr et al. 1998; Kargel et al. 2000; Collins & Goodman 2007; Less et al. 2014). Tidal and radiogenic heat keeps the interior of the body warm, and hence maintain the water in a liquid state. In fact, these internal heat sources drive the eruption of plumes on Enceladus, and a similar phenomenon was discovered on Europa as well (Porco et al. 2006; Roth et al. 2014).

The idea of a circumplanetary, tidally heated HZ has emerged and was investigated by several authors (e.g., Reynolds et al. 1987; Scharf 2006; Heller & Barnes 2013). For the first time, we apply a viscoelastic model to study tidal heating in exomoons. This work aims to provide a detailed study of the circumplanetary tidal temperate zone (TTZ), and discusses the differences with other models.

2. VISCOELASTIC MODEL

2.1. Advantages

The tidal heat rate of a moon is usually calculated using the following expression (e.g., Reynolds et al. 1987; Meyer & Wisdom 2007):

\[
\dot{E}_{\text{tidal}} = \frac{21}{2} k_2 \frac{G M_p^2 R_m^5 n e^2}{\alpha^6},
\]

where \(G\) is the gravitational constant, \(M_p\) is the mass of the planet, and \(R_m\), \(n\), \(e\), and \(\alpha\) are the radius, mean motion, eccentricity, and semimajor axis of the moon, respectively. \(Q\) is the tidal dissipation factor and \(k_2\) is the second-order Love number:

\[
k_2 = \frac{3/2}{1 + \frac{19 \mu}{2 \rho R_m}}.
\]

where \(\mu\) is the rigidity, \(\rho\) is the density, and \(g\) is the surface gravity of the satellite. This calculation method is called the fixed \(Q\) model because \(Q\), \(\mu\), and \(k_2\) are considered to be constants.

The fixed \(Q\) model is broadly used in tidal calculations, but highly underestimates the tidal heat of the body (Ross & Schubert 1988; Meyer & Wisdom 2007). Moreover, both \(Q\)
and \( \mu \) are very difficult to determine and vary on a large scale for different bodies: from a few to hundreds for rocky planets, and tens or hundreds of thousands for giants (see e.g., Goldreich & Soter 1966). In addition, these parameters are not constants, since they strongly depend on the temperature (Fischer & Spohn 1990; Moore 2003; Henning et al. 2009; Shoji & Kurita 2014). As a consequence, the tidal heat flux also has a temperature dependency: it reaches maximum at a critical temperature \( T_c \), as can be seen in Figure 1. Between the solidus and the liquidus temperatures \( (T_s \) and \( T_l \) respectively), the material partially melts. Above the breakdown temperature \( T_b \) the mixture behaves as a suspension of particles. The dashed curve represents the convective heat loss of the body. Circles indicate equilibria; for example, the solid circle between \( T_s \) and \( T_b \) is a stable equilibrium point. If the temperature increases, convective cooling will be stronger than the heat flux, resulting in a cooler temperature. In the case of decreasing temperature, the tidal heat flux will be stronger, and hence the temperature increases, returning the system to the stable point. The stable equilibrium between the tidal heat and convection is not necessarily located between \( T_s \) and \( T_b \); in fact, there are cases when the two curves do not intersect at all (see Henning et al. 2009, Figure 6). In these cases, tidal heat is not strong enough to induce convection inside the body.

In contrast to the fixed \( Q \) model, viscoelastic models take into account the temperature dependency of the body, and hence are more realistic.

### 2.2. Description

In viscoelastic models, \( k_2/Q \) is replaced by the imaginary part of the complex Love number \( \text{Im}(k_2) \) which describes structure and rheology in the satellite (Segatz et al. 1988):

\[
\dot{E}_{\text{tidal}} = -\frac{21}{2} \text{Im}(k_2) \frac{R^5 n^5 e^2}{G}.
\]

Note that in this expression, the mass of the planet and the semimajor axis of the moon are eliminated by the mean motion \( (n = \sqrt{GM_p/a^3}) \).

Henning et al. (2009) gives the value of \( \text{Im}(k_2) \) for four different models (see Table 1 in their paper). In this work, we use the Maxwell model:

\[
-\text{Im}(k_2) = \frac{57\eta_0\omega}{4\rho g R_m^2 \left[ 1 + \left( 1 + \frac{49\mu}{2\rho g R_m} \right)^2 \eta_0^2 \omega^2 \right]^{3/2}},
\]

where \( \eta \) is the viscosity, \( \omega \) is the orbital frequency, and \( \mu \) is the shear modulus of the satellite.

The viscosity and the shear modulus of the body strongly depend on the temperature. Below \( T_s \), the shear modulus is constant, \( \mu = 50 \text{ GPa} \), and the viscosity follows an exponential function:

\[
\eta = \eta_0 \exp \left( \frac{E}{RT} \right),
\]

where \( \eta_0 = 1.6 \cdot 10^5 \text{ Pa s} \) (reference viscosity), \( E \) is the activation energy, \( R \) is the universal gas constant, and \( T \) is the temperature of the material (Fischer & Spohn 1990).

Between \( T_s \) and \( T_b \), the body starts to melt. The shear modulus changes by

\[
\mu = 10 \left( \frac{\mu_1 + \mu_2}{\phi} \right) \text{Pa},
\]

where \( \mu_1 = 8.2 \cdot 10^4 \text{ K} \) and \( \mu_2 = -40.6 \) (Fischer & Spohn 1990). The viscosity can be expressed by

\[
\eta = \eta_0 \exp \left( \frac{E}{RT} \right) \exp(-B\phi),
\]

where \( \phi \) is the melt fraction which increases linearly with the temperature between \( T_s \) and \( T_0 \) (0 \( \leq \phi \leq 1 \)), and \( B \) is the melt fraction coefficient (10 \( \leq B \leq 40 \); Moore 2003).

At \( T_b \) the grains disaggregate, leading to a sudden drop in both the shear modulus and the viscosity. Above this temperature, the shear modulus is set to a constant value: \( \mu = 10^{-7} \text{ Pa} \). The viscosity follows the Roscoe–Einstein relationship so long as it reaches the liquidus temperature (where \( \phi = 1 \); Moore 2003):

\[
\eta = 10^{-7} \exp \left( \frac{40,000 \text{ K}}{T} \right) (1.35\phi - 0.35)^{-5/2} \text{Pa s}. \]

Above \( T_1 \) the shear modulus stays at \( 10^{-7} \text{ Pa} \) and the viscosity is described by (Moore 2003)

\[
\eta = 10^{-7} \exp \left( \frac{40,000 \text{ K}}{T} \right) \text{Pa s}.
\]

In our calculations, rocky bodies are considered to be satellites, and for this reason we follow the melting temperatures of Henning et al. (2009), namely: \( T_s = 1600 \text{ K} \), \( T_1 = 2000 \text{ K} \). We assume that disaggregation occurs at 50% melt fraction, and hence the breakdown temperature will be \( T_b = 1800 \text{ K} \).

### 2.3. Internal Structure and Convection

The structure of the moon in the model is as follows: the body consists of an inner, homogenous part, which is convective, and an outer, conductive layer. If the tidal forces are weak, then the induced temperature will be low, resulting in a smaller convective region and a deeper conductive layer. However, in the case of strong tidal forces, the temperature will
be higher, and hence the convective zone will be larger with a thinner conductive layer.

To calculate the convective heat loss, we use the iterative method described by Henning et al. (2009). The convective heat flux can be obtained from

$$q_{BL} = k_{\text{therm}} \frac{T_{\text{mantle}} - T_{\text{surf}}}{\delta(T)},$$

where $k_{\text{therm}}$ is the thermal conductivity ($\sim 2$ W m$^{-1}$ K$^{-1}$), $T_{\text{mantle}}$ and $T_{\text{surf}}$ are the temperature in the mantle and on the surface, respectively, and $\delta(T)$ is the thickness of the conductive layer. We use $\delta(T) = 30$ km as a first approximation, and then for the iteration

$$\delta(T) = \frac{d}{2a_2} \left( \frac{\text{Ra}}{\text{Ra}_c} \right)^{1/4}$$

is used, where $d$ is the mantle thickness ($\sim 3000$ km), $a_2$ is the flow geometry constant ($\sim 1$), $\text{Ra}_c$ is the critical Rayleigh number ($\sim 1100$), and Ra is the Rayleigh number which can be expressed by

$$\text{Ra} = \frac{\alpha g \rho d^4 q_{BL}}{\eta(T) \kappa k_{\text{therm}}}.$$  

Here $\alpha$ is the thermal expansivity ($\sim 10^{-4}$) and $\kappa$ is the thermal diffusivity: $\kappa = \frac{k_{\text{therm}}}{(\rho C_p)}$ with $C_p = 1260$ J kg$^{-1}$ K$^{-1}$. For a detailed description, see the clear explanation of Henning et al. (2009).

Because of the viscosity of the material, the thickness of the boundary layer and the convection in the underlying zone changes strongly with temperature. The weaker temperature dependencies of density and thermal expansivity are neglected in the calculations. The iteration of the convective heat flux lasts until the difference of the last two values is higher than $10^{-10}$ W m$^{-2}$.

Calculations of tidal heat flux and convection are made for a fixed radius, density, eccentricity, and orbital period of the moon. We assume that with time the moon reaches the equilibrium state. Henning et al. (2009) showed that planets with significant tidal heating reach equilibrium with convection in a few million years. However, a change in the eccentricity can shift or destroy stable equilibria. After finding the stable equilibrium temperature, the tidal heat flux is calculated, from which the surface temperature can be obtained using the Stefan–Boltzmann law:

$$T_{\text{surf}} = \left( \frac{E_{\text{tidal}}}{4\pi R^2 \sigma} \right)^{1/4},$$

where $\sigma$ is the Stefan–Boltzmann constant. This is the first time using a viscoelastic model to obtain the tidal-heat-induced surface temperature on exomoons.

### 2.4. Results

The satellite’s surface temperature is calculated for different orbital periods and radii, at a fixed density and eccentricity. Stellar radiation and other heat sources are not considered, and have been neglected. The orbital period and radius of the moon varies between 2 and 20 days, and between 250 and 6550 km, respectively. It is common to consider Earth-mass moons in extrasolar systems when speaking of habitability, however, their existence has not been proven. In the solar system, the largest moon, Ganymede, has only 0.025 Earth mass. However, the mass of satellite systems is proportional to the mass of their host planet. Canup & Ward (2006) showed that this might be the case for extrasolar satellite systems as well, giving an upper limit for the mass ratio at around $10^{-4}$. This means that super-Jupiter planets may have Earth-mass satellites. Besides accretion, large moons can also form from collisions, as in the case of Earth’s Moon. Another possibility is the capturing of terrestrial-sized bodies through a close planetary encounter, as described by Williams (2013). For these reasons, we also take into account Earth-like moons.

The results can be seen in Figure 2 where the density of the moon is that of Io and its eccentricity is set to 0.1. Different colors indicate different surface temperatures. In the white region, there is no stable equilibrium between tidal heat and convective cooling. In other words, tidal heat is not strong enough to induce convection. For comparison, a few solar system moons are plotted that have densities similar to Io’s.
Yellow contour curves denote 0°C and 100°C. The green area between these curves indicates that water may be liquid on the surface of the moon (atmospheric considerations were not applied). We define this territory as the TTZ.

Interestingly, the location of the TTZ strongly depends on the orbital period and less on the radius of the moon. Low radii are less relevant, since smaller bodies are less capable of maintaining significant atmospheres.

The dependence on the eccentricity can be seen by comparing Figures 2 and 3. In the case of the latter figure, the moon’s eccentricity is 0.01. For most of the orbital period–radius pairs, there is no solution (white area). Due to this drastic difference, Europa analogs get out of equilibrium for smaller eccentricities, and the TTZ becomes narrower and shifts to shorter orbital periods. Note that radiogenic heat is not considered in the model, which could push the moon back into a state of equilibrium, and would result in a higher surface temperature.

Similar calculations were made for the density of the Earth and Titan (left and right panels of Figure 4, respectively). The densities do not have a high influence on the tidally induced surface temperature; however, the TTZ slightly shifts to lower orbital parameters for higher densities. (The densities of Earth, Io, and Titan are 5515, 3528, and 1880 kg m⁻³, respectively.)

In the left panel of Figure 4, an example Earth-like moon is plotted inside the TTZ. This hypothetical body has the same mean surface temperature (288 K), radius (6370 km), and density as the Earth, and hence its orbital period is 2.06 days. In the right panel, a few solar system satellites are plotted that have densities similar to that of Titan.

The stellar flux for moons with ambient temperatures of ~100 K (which is similar to the case of the Galilean and Saturnian moons in the solar system) is about one percent of the tidal flux in the TTZ. For this reason, stellar insolation may be safely ignored if the planet–moon system orbits the star at a far distance, or if they are free floating. For moon systems in which stellar irradiation alone is sufficient to heat the surface to levels of the order of the melting temperature or higher, the models presented here would need to be replaced by more complex hybrid models to account for both sources of heat and their very different spatial distributions on and within the moon.

3. COMPARISON TO THE FIXED Q MODEL

3.1. Method

It is clear from the results that the viscoelastic model does not provide a solution in the case of small tidal forces. In other words, the amount of heat produced by tidal interactions is insufficient to induce convective movements inside the body, and for this reason there is no equilibrium between them. In contrast, the fixed Q model provides solutions both for weak and strong tidal forces. However, the viscoelastic model describes the tidal heating of the body more realistically than the fixed Q model, due to the temperature dependence of the Q and μ parameters. How are the results of the two models related to each other?

To compare the results of the two kinds of models, we use the expression of Equation (7) from Peters & Turner (2013) for the fixed Q calculation:

\[
T_{\text{surf}} = \left( \frac{392 \pi^5 G^5}{9747 \sigma^2} \right)^{1/2} \left( \frac{R_m \rho \beta}{\mu \mathcal{Q}} \right)^{9/2} \left( \frac{e^2}{\beta^{15/2}} \right)^{1/4},
\]

where \(T_{\text{surf}}\) is the surface temperature of the moon induced by tidal heating, \(G\) is the gravitational constant, \(\sigma\) is the Stefan–Boltzmann constant, \(R_m\) is the radius, \(\rho\) is the density, \(\mu\) is the elastic rigidity, and \(\mathcal{Q}\) is the dissipation function of the moon, while \(e\) is the eccentricity of the moon’s orbit and \(\beta\) is expressed with the semimajor axis \((a)\) and the mass of the planet \((M_p)\):

\[
a = \beta a_R = \beta \left( \frac{3 M_p}{2 \pi \rho} \right)^{1/3},
\]

where \(a_R\) is the Roche radius of the host planet. These equations can be used to calculate the surface temperature of the moon heated solely by tidal forces.

The viscoelastic model is described in detail in Sections 2.2 and 2.3. The satellite’s mean motion can be expressed from \(\beta\) by

\[
n = \sqrt{\frac{2 \pi G}{3 \beta^3}},
\]
which facilitates comparison of the two models.

### 3.2 Surface Temperature

For comparison of the fixed \(Q\) and viscoelastic models, see Figures 5–7 which show the surface temperature of a moon calculated with both the viscoelastic (red solid curve) and the fixed \(Q\) models (green dashed curve) as functions of the eccentricity, radius, and orbital period of the satellite. For the density of the moon we used \(5515\ \text{kg m}^{-3}\), which is the density of the Earth, and for the fixed \(Q\) model we used \(Q = 280\) and \(\mu = 12 \cdot 10^9\ \text{kg m}^{-1}\ \text{s}^{-2}\) in each case (Peters & Turner 2013 Table 1). The radius, orbital period, and eccentricity of the satellite are set to that of the Earth, Io, and 0.03, respectively, except that one of these parameters is varied in each figure (horizontal axes). The horizontal, dashed light blue lines indicate 0\(^\circ\)C and 100\(^\circ\)C (marking the boundaries of the TTZ), and the solid blue lines denote the minimum and maximum temperatures (−20\(^\circ\)C and 60\(^\circ\)C) which are the probable limits of habitability on an Earth-like body (Sullivan & Baross 2007, chapter 4). In salty solutions, the lower limit for microbial activity is around −20\(^\circ\)C, and the upper limit for complex eukaryotic life is 60\(^\circ\)C. The latter temperature is also approximately the runaway greenhouse limit for Earth. These limits are only used for Earth-like bodies \((\rho = \rho_{\text{Earth}}\) and \(R_m \approx R_{\text{Earth}}\)). The vertical lines show a few examples from the solar system for different eccentricities, radii, and orbital periods.

It is notable that the red curve is less steep than the green curve, and a larger portion of it is located inside the TTZ, especially in Figures 5 and 6. It shows that the viscoelastic model stabilizes the surface temperature compared to the fixed \(Q\) model. These are just a few examples indicating that the viscoelastic model is less sensitive to these parameters, and that there are huge differences in the results of the models. In the next section, the volume of the TTZ is investigated more thoroughly.

### 3.3 Occurrence Rate of “Habitable” Moons

Habitability on extraterrestrial bodies is an exciting, but complex question. Here, we only consider the tidally induced surface temperature of a hypothetical moon. We were curious about the occurrence rate of moons with suitable surface temperature for life. For this reason, we mapped the phase space evenly with hypothetical moons that have different radii (between 250 and 6550 km) and eccentricities (between 0.001 and 0.1), and their densities are that of the Earth. We used both the viscoelastic and fixed \(Q\) models to calculate the surface temperature of these bodies, and then calculated the percentage of those that have a suitable surface temperature, i.e., located inside the TTZ \((0 \leq T_{\text{surf}} \leq 100\,^\circ\text{C})\). The calculation was made for different orbital periods between 0.1 and 3.5 days, and for each value there were 63,100 hypothetical moons distributed in the radius–eccentricity phase space. The result can be seen in Figure 8. The solid red and dashed green curves indicate the fraction of the grid of parameters that correspond to moons inside the TTZ for the viscoelastic and fixed \(Q\) models, respectively. The dotted blue curve shows the percentage of those cases that do not give a result for the viscoelastic model. The top axis shows the \(\beta\) parameter, which is the ratio of the moon’s semimajor axis and the planet’s Roche radius. It can be clearly seen that the red and green curves have a peak, which means that the probability of having a suitable surface temperature has a maximum at a certain orbital period. The viscoelastic model predicts much more efficient heating than the fixed \(Q\) model, i.e., a much larger fraction of the hypothetical moons have their surface temperature between 0\(^\circ\)C and 100\(^\circ\)C. The ratio of the integral under the red curve to that under the green curve is 2.8, meaning that 2.8 times more exomoons are predicted in the TTZ with the viscoelastic model. For the viscoelastic model, the maximum percentage appears around a 1 day orbital period, and here the probability of the moon being inside the TTZ is almost 80\%. For higher orbital periods, this probability falls rapidly, which is in contrast to the fixed \(Q\) model. The latter has its peak around 1.5 days and has less than 20\% chance of satellites being in the TTZ. Despite the high probabilities achieved by the viscoelastic model for small orbital periods, the fixed \(Q\) model provides more promising results for those moons that have orbital periods of 2 days or more.

For a more detailed study, the 0\(^\circ\)C and 100\(^\circ\)C temperature contours were plotted in the radius–eccentricity plane for a few specific orbital periods, namely, \(P = 0.5\) days (top panel), \(P = 1\) day (middle panel), and \(P = 1.5\) days (bottom panel; see Figure 9). Again, the red and green colors represent the viscoelastic and fixed \(Q\) models, respectively. Between the contour curves, the region of the TTZ is filled with light red and light green shading. The result shows that the viscoelastic model mostly favors the small moons, especially at high eccentricities, but also some large moons at small eccentricities over the fixed \(Q\) model. This suggests that the viscoelastic model is less sensitive to the parameters of the moon, and holds the temperature more steady than the fixed \(Q\) model. This is due to the melting of the inner material of the moon that leads to a less elevated temperature, as discussed by Peters & Turner (2013). On the other hand, the lower temperature implies that the total irradiated flux of the moon will also be lower, hence making the detection of the moon more difficult.

We were also interested in “Earth twins” as satellites and in the probability of their “habitability.” For this reason, we performed similar calculations, but with the radius and density of the hypothetical moons set close to that of the Earth: \(R_m = 6378\ \text{km (±5\%)}\) and \(\rho = 5515\ \text{kg m}^{-3} (±5\%)\). The radius and density values were chosen randomly from these
The eccentricity was altered similarly to that in the previous case (uniformly between 0.001 and 0.1). Altogether, 200,000 cases were considered for each orbital period. The temperature limits were set to −20°C and 60°C, which are the probable limits for life on Earth. Figure 10 shows the results of this calculation. Note that the peaks of the red solid (viscoelastic model) and green dashed (fixed Q model) curves are shifted to higher orbital periods compared to Figure 8. This is caused in part by the changed temperature limits, and in part by the much shorter radius range. The maximum probabilities are also higher, which is especially visible in the case of the fixed Q model which reaches more than 40% at the curve’s peak (in the previous case it was less that 20%). As one would expect, this suggests that larger moons have a higher probability of maintaining warm surfaces. The ratio of the areas under the red and the green curves is 2.3.

In general, it can be concluded that the viscoelastic model is not just more realistic than the fixed Q model, but also gives more promising results for exomoons since a much larger fraction of the hypothetical satellites have been found in the TTZ. In those cases where the viscoelastic model does not provide a solution for the equilibrium temperature, one can use the fixed Q model instead; however, the values of $Q$ and $\mu$ are highly uncertain.

### 3.4. The Value of $Q\mu$

With the product of $Q$ and $\mu$, one can easily calculate the tidally induced surface temperature of a moon without using a complex viscoelastic model. Using Equation (14) is a fast way to obtain $T_{\text{surf}}$, but a good approximation is needed for the $Q\mu$ value. For such calculations, in the following, we give the $Q\mu$ values for hypothetical moons. Because of the large number of possible variations in the physical and orbital parameters of the moons, only a few solar-system-like bodies were considered. Since the $Q\mu$ varies by several orders of magnitude for different rocky bodies, a good estimate can serve almost as well as the
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exact value. The following examples can be used as a guideline for making such estimates. Note that the model used can be applied to rocky bodies, but for icy satellites, such as Enceladus or Europa, the results may be misleading because of the more complex structure and different behavior of the icy material.

From the surface temperature of the moon calculated from the viscoelastic model, the \( Q_H \) value was determined using Equation (14) for six orbital period–eccentricity pairs. In Table 1, the tidally induced surface temperatures and the logarithm of the \( Q_H \) values can be seen. The radius and density of the satellites are those of the corresponding solar system bodies (see the first column), and the values are from Murray & Dermott (1999). The eccentricities are set to 0.01 and 0.1, and the orbital periods to 1, 2, and 3 days. “n.a.” indicates that there was no solution (weak tidal forces).

3.5. Scaling the Galilean Satellite System

Since no satellite has so far been discovered outside the solar system, we used the Galilean system as a prototype for realistic calculations. Io, Europa, and Ganymede orbit in a 1:2:4 mean motion resonance that maintains their eccentricities, which play an essential role in continuously forcing their tidal heating. Ogihara & Ida (2012) investigated satellite formation in the circumplanetary disk of giant planets using N-body simulations including gravitational interactions with the circumplanetary gas disk. They have found that 2:1 mean motion resonances are almost inevitable in Galilean-like satellite systems, and based on their results they predict that mean motion resonances may be common in exoplanetary systems. For these reasons, the Galilean satellite system seems to be a candidate for realistic calculations, since the moons are in resonance and their scaled-up versions will probably also remain in resonance.

The test systems consist of a planet (Jupiter) and four moons (the Galilean satellites). Ninety-one cases are considered, one is the real Galilean system, and the others are the scaled-up versions: the masses of the planet and the moons were multiplied by the scale factor (scale = 1.0, 1.1, 1.2, ... 10.0), and the semimajor axes of the moons were altered with constant orbital periods for each scale value.

\[
P = 2\pi \sqrt[3]{\frac{a^3}{\text{scale} \cdot G(M_p + M_s)}},
\]

where \( a \) is the semimajor axis of the moon, and \( M_p \) and \( M_s \) are the masses of the planet and the moon, respectively. The fixed orbital periods guarantee that the satellites approximately stay in resonance. This calculation resulted in constant \( \beta \) values for all scale parameters.

Using both the fixed \( Q \) and the viscoelastic models, the warmth of the tidal heat was investigated in each case. The tidal-heat-induced surface temperature can be seen in Figure 11 where the 91 cases are connected with solid curves for each satellite. One hundred and eighty-two other cases were calculated as well, and they are shown with dashed and dotted curves in the figure. These curves indicate that the densities (dashed curve) and eccentricities (dotted curves) of the satellites are doubled compared to their original values in the solar system. In the calculations, \( \mu \) and \( Q \) were set for all satellites to those of Io, namely, \( 10^{10} \text{ kg m}^{-1} \text{s}^{-2} \) and 36, respectively, except for Europa, which has the following parameters: \( \mu = 4.10^9 \text{ kg m}^{-1} \text{s}^{-2} \) and \( Q = 100 \) (Peters & Turner 2013). The densities of the moons are from Lodders & Fegley (1998), and the reference for the semimajor axis, eccentricity, and mass values is Murray & Dermott (1999, Appendix).

For Io, in the scale = 1 (solar system) case, the fixed \( Q \) and viscoelastic models give 60 and 160 K, respectively. The observed surface heat flux induced by tidal heat on Io is around 2 W m\(^{-2} \), which is a lower limit (Spencer et al. 2000). In other words, tidal forces produce at least 77 K heat on the surface of Io. The fixed \( Q \) model resulted in a lower value than this limit, but note that \( Q \) and \( \mu \) are very difficult to estimate. The viscoelastic model gave much a higher temperature than the observation, but keep in mind that the heat is concentrated in hotspots and is not evenly distributed on the surface of Io. The temperature of the warmest volcano, Loki, is higher than 300 K (Spencer et al. 2000).

The viscoelastic model provides solutions only for Io (orange curves) and Europa (light blue curves), but not for all scale values, as shown in Figure 11. In those cases where the densities are twice those in the solar system (dashed curves), the surface temperatures are only slightly higher. In fact, in the viscoelastic model, higher densities result in less tidal heat because of the imaginary part of the second-order Love number. Doubling the eccentricity instead of the density (dotted curves) makes the surface temperature higher in each case.

4. CONCLUSIONS

We have used, for the first time, a viscoelastic model to calculate the surface temperature of tidally heated exomoons. The viscoelastic model provides more reliable results than the widely used fixed \( Q \) model because it takes into account the fact that the tidal dissipation factor (\( Q \)) and rigidity (\( \mu \)) strongly depend on the temperature. Besides, these values are poorly known even for planets and satellites in the solar system. Using the viscoelastic model for exomoons helps to obtain a more realistic estimation of their surface temperature,
and to determine a circumplanetary region where liquid water may exist. This may be helpful for future missions in selecting targets for exomoon detections.

We have defined the TTZ, which is the region around a planet where the surface temperature of the satellite is between 0°C and 100°C. No sources of heat were considered other than tidal forces. Assuming that the planet–moon system orbits the star at a far distance or that the stellar radiation is low due to the spectral type, tidal heat can be the dominant heat source affecting the satellite. We have investigated such systems and found that the TTZ strongly depends on the orbital period, and less on the radius of the moon. For higher densities or eccentricities of the moon, the location of the TTZ is slightly closer to the planet.

Comparing this model to the traditionally used fixed $Q$ model revealed that there are huge differences in the results. Generally, the viscoelastic model is less sensitive to moon radius than the uniform $Q$ model, keeping the surface temperature of the body steadier. The reason for this is that higher tidal forces induce a higher melt fraction which results in a lower temperature than in the fixed $Q$ model. The viscoelastic model demonstrates the manner in which the partial melting of a moon can act as a thermostat and tends to fix its temperature somewhere near its melting point over a wide range of physical and orbital parameters. As a consequence, the statistical volume of the TTZ is much larger in the viscoelastic case, which is favorable for life. However, this lower temperature also means that the detectability of such moons is lower in the infrared. In addition, for low tidal forces there is no equilibrium with convective cooling; hence, only the fixed $Q$ model provides a solution. In these cases, the challenge is to determine the values of $Q$ and $\mu$.

For a few characteristic cases, the product of the tidal dissipation factor and rigidity was calculated from the viscoelastic model in order to facilitate quick estimation of tidally heated exomoon surface temperatures. Since the

| Label   | $R_m$ (km) | $\rho$ (kg m$^{-3}$) | $e$ | $P$ (days) | $T_{surf}$ (K) | $\log_{10}(Q\mu)$ (Pa) |
|---------|------------|---------------------|-----|------------|----------------|-------------------------|
| Earth-like | 6378       | 5515                | 0.01| 1          | 281            | 14.0                    |
|          |            |                     |     | 2          | 213            | 13.0                    |
|          |            |                     |     | 3          | 180            | 12.4                    |
|          |            |                     | 0.1 | 1          | 378            | 15.5                    |
|          |            |                     |     | 2          | 291            | 14.4                    |
|          |            |                     |     | 3          | 249            | 13.8                    |
| Mars-like | 3394       | 3933                | 0.01| 1          | 256            | 12.5                    |
|          |            |                     |     | 2          | 194            | 11.5                    |
|          |            |                     |     | 3          | 163            | 10.9                    |
|          |            |                     | 0.1 | 1          | 342            | 14.0                    |
|          |            |                     |     | 2          | 263            | 13.0                    |
|          |            |                     |     | 3          | 225            | 12.4                    |
| Moon-like | 1738       | 3342                | 0.01| 1          | 232            | 11.1                    |
|          |            |                     |     | 2          | 176            | 10.1                    |
|          |            |                     |     | 3          | n.a.           | n.a.                    |
|          |            |                     | 0.1 | 1          | 313            | 12.6                    |
|          |            |                     |     | 2          | 240            | 11.5                    |
|          |            |                     |     | 3          | 205            | 10.9                    |
| Io-like   | 1821       | 3532                | 0.01| 1          | 235            | 11.2                    |
|          |            |                     |     | 2          | 177            | 10.2                    |
|          |            |                     |     | 3          | n.a.           | n.a.                    |
|          |            |                     | 0.1 | 1          | 315            | 12.7                    |
|          |            |                     |     | 2          | 242            | 11.7                    |
|          |            |                     |     | 3          | 207            | 11.1                    |

**Note.** The radii and densities are those of the corresponding solar system bodies. The reference for these values is Murray & Dermott (1999, their Appendix).

![Figure 11. Surface temperature of the Galilean moons as a function of the scale parameter, calculated by both the fixed $Q$ and the viscoelastic model. The dashed and dotted curves indicate that the density or the eccentricity of the moon is doubled compared to the solar system case, respectively.](image-url)
viscoelastic model is more realistic due to the inner melting and temperature dependence of the parameters, but the fixed $Q$ model is easier to use, these $Q$ values (along with the surface temperature) are provided in Table 1. By inserting $Q$ into Equation (14), one can obtain the estimate of the tidally induced surface temperature of a moon. The connection between the quality factor ($Q$) and the viscoelastic parameters (viscosity and shear modulus) was also given for the Maxwell model by Remus et al. (2012).

Earth-like bodies were also investigated as satellites, and in these cases the $-20^\circ$C and $60^\circ$C temperatures were used as the limits of habitability. The results are similar, but the volume of this HZ is larger than that of the TTZ for a wide range of satellite radii. This HZ includes atmospheric considerations of the moon, but stellar radiation was neglected in the calculations. In the case of significant radiation from other sources, the surface temperature of the moon will be higher. Additional heat sources (such as stellar radiation, radiogenic processes, and reflected stellar and emitted thermal radiation from the planet), and the effects of eclipses, or the obliquity of the satellite are thoroughly discussed by Heller & Barnes (2013).

To simulate realistic systems, the Galilean moons were used as a prototype. Their surface temperature was calculated with both models for different scaled-up masses. The mean motion resonance between the satellites helps to maintain their eccentricity, and consequently to maintain the tidal forces. By raising their masses, the temperatures of Io and Europa elevate less drastically in the viscoelastic model than in the fixed $Q$ model (see Figure 11). At scale = 5 (masses are five times as in the solar system case), the surface temperature of Europa is $\sim 150$ K, calculated from the viscoelastic model. Assuming that its density does not change, its radius will be approximately 0.25 Earth radii. In the case of additional 100–120 K heat (e.g., from stellar radiation), the ice would melt, and this super-Europa would become an “ocean moon,” covered entirely by a global water ocean. The used viscoelastic model might not be adequate and can be oversimplified for such bodies consisting of rocky and icy layers. Salty ice mixtures may also modify the results. The applied model ignores the structure, pressure, and other effects, and applies melting for the whole body. However, this provides a global picture of the tidally heated moon. Even with a more detailed viscoelastic model that describes Enceladus as a three layered body (rocky core, ocean, and ice shell), Barr (2008) have found that tidal heat is $\sim 10$ times lower than what was observed by the Cassini Composite Infrared Spectrometer. Similarly, Moore (2003) concluded that observed heat flux on Io is about an order of magnitude higher than what can be explained with a multilayered, viscoelastic model. These results suggest that tidal heat can be much more relevant than what is predicted by models.

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