Spin pumping and magnetization-precession trajectory in thin film systems

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Abstract. The spin pumping, generation of spin currents from magnetization precession, has been investigated in terms of the trajectory of magnetization precession in thin film systems. By using the Landau-Lifshitz-Gilbert equation combined with the model of the spin pumping, we found that the magnitude of the spin current generated by the spin pumping is determined by the elliptical orbit area of magnetization precession, which is maximized when the external magnetic field is applied oblique to the film plane.

1. Introduction
The generation, manipulation, and detection of a spin current, a flow of electron spins, are the main challenges in the field of spintronics, which involves the study of active control and manipulation of the spin degree of freedom in solid-state systems [1, 2, 3]. A spin current interacts with magnetization by exchanging the spin-angular momentum, enabling the direct manipulation of magnetization without using magnetic fields [4, 5].

The interaction between spin currents and magnetization provides also a method for spin-current generation from magnetization precession, which is the spin pumping [6, 7]. The spin pumping is a method for inducing spin currents in a paramagnetic metal attached to a ferromagnetic metal; the precessing magnetization in the ferromagnetic metal pumps a spin current into the paramagnetic metal. The spin pumping was observed electrically in a simple Ni₈₁Fe₁₉/Pt film using the strong inverse spin-Hall effect (ISHE) in the Pt layer; in the Ni₈₁Fe₁₉/Pt film, when the external magnetic field and the frequency of the applied microwave fulfill the ferromagnetic resonance (FMR) condition, a spin current is injected into the Pt layer by the spin pumping, giving rise to an electric voltage via the ISHE in the Pt layer [8, 9, 10, 11].

In thin film systems, when the external magnetic field is applied oblique to the film plane, the magnetization-precession trajectory, the trajectory of the point of a magnetization vector is distorted due to a demagnetization field. Since the spin pumping generates spin currents from magnetization precession, the precession trajectory is expected to strongly affect the amplitude of the generated spin currents. In this paper, we investigate the amplitude of the spin current generated by the spin pumping in terms of a magnetization-precession trajectory based on the Landau-Lifshitz-Gilbert (LLG) equation combined with the model of the spin pumping.
2. Phenomenological model

The dynamics of magnetization $\mathbf{M}(t)$ under an effective magnetic field $\mathbf{H}_{\text{eff}}$ is described by the LLG equation,

$$\frac{d\mathbf{M}(t)}{dt} = -\gamma \mathbf{M}(t) \times \mathbf{H}_{\text{eff}} + \frac{\alpha}{M_s} \mathbf{M}(t) \times \frac{d\mathbf{M}(t)}{dt}.$$  

Here, $\gamma$, $\alpha$, and $M_s$ are the gyromagnetic ratio, the Gilbert damping constant, and the saturation magnetization, respectively. We consider the magnetization $\mathbf{M}(t)$ precession around the $z$ axis in a ferromagnetic film attached to a paramagnetic film as shown in figure 1, where $\mathbf{M}(t) = \mathbf{M} + \mathbf{m}(t)$. $\mathbf{M}$ and $\mathbf{m}(t)$ are the static and the dynamic components of the magnetization $\mathbf{M}(t)$, respectively. Here, we consider a soft ferromagnetic thin film, e.g. Ni$_{81}$Fe$_{19}$, and we neglect the magnetocrystalline anisotropy. Taking into account the external magnetic field $\mathbf{H}$, the static demagnetizing field $\mathbf{H}_m$ induced by $\mathbf{M}$, the dynamic demagnetization field $\mathbf{H}_m(t)$ induced by $\mathbf{m}(t)$, and the external ac field $\mathbf{h}(t)$ as the effective magnetic field $\mathbf{H}_{\text{eff}}$: $\mathbf{H}_{\text{eff}} = \mathbf{H} + \mathbf{H}_m + \mathbf{H}_m(t) + \mathbf{h}(t)$, and using the ferromagnetic resonance condition [9]:

$$\left( \frac{\omega}{\gamma} \right)^2 = \left| H_{\text{FMR}} \cos(\theta_H - \theta_M) - 4\pi M_s \cos 2\theta_M \right| H_{\text{FMR}} \cos(\theta_H - \theta_M) - 4\pi M_s \cos^2 \theta_M,$$

we obtain the dynamic components of the magnetization $\mathbf{m}(t)$ at ferromagnetic resonance (FMR) condition as

$$m_x(t) = \frac{4\pi M_s h \gamma}{8\pi \alpha \omega \sqrt{(4\pi M_s)^2 \gamma^2 \sin^4 \theta_M + 4\omega^2}} \left[ 2\alpha \omega \cos \omega t + \left( 4\pi M_s \gamma \sin^2 \theta_M + \sqrt{(4\pi M_s)^2 \gamma^2 \sin^4 \theta_M + 4\omega^2} \right) \sin \omega t \right],$$

$$m_y(t) = -\frac{4\pi M_s h \gamma \cos \omega t}{4\pi \alpha \sqrt{(4\pi M_s)^2 \gamma^2 \sin^4 \theta_M + 4\omega^2}}.$$  

where $\theta_M$ and $\theta_H$ are the magnetization and the external magnetic field angle to the normal vector of the film plane, respectively [see figure 1]. Here, $\omega = 2\pi f$, $f$ is the microwave frequency. $H_{\text{FMR}}$ is the resonance field.

In the model of the spin pumping [6], the dc component of a spin current density $j_s$ is expressed as

$$j_s = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} \left. \frac{d\mathbf{M}(t)}{dt} \right|_z dt,$$

where $h$ and $g_{\gamma}^{\dagger}$ are the Dirac constant and the real part of the mixing conductance, respectively. Here, $[\mathbf{M}(t) \times d\mathbf{M}(t)/dt]_z$ is the $z$ component of $\mathbf{M}(t) \times d\mathbf{M}(t)/dt$. $z$ axis is directed along the magnetization-precession axis [see figure 1]. Using equations (3) and (4), we find the phenomenological expression of $j_s$ as

$$j_s = \frac{g_{\gamma}^{\dagger} \gamma^2 h^2 \hbar}{8\pi \alpha^2 ((4\pi M_s)^2 \gamma^2 \sin^4 \theta_M + 4\omega^2)} \left( 4\pi M_s \gamma \sin^2 \theta_M + \sqrt{(4\pi M_s)^2 \gamma^2 \sin^4 \theta_M + 4\omega^2} \right).$$

3. Discussion

In order to characterize a magnetization-precession trajectory, we define the ellipticity $A$ of a magnetization-precession trajectory as $A \equiv |m_y|/|m_x|$, where $|m_x|$ and $|m_y|$ are the major
and minor radiuses of the trajectory, respectively. Using equations (3) and (4), we obtain the ellipticity of a magnetization-precession trajectory as

\[ A = \frac{2\omega}{4\pi M_s\gamma \sin^2 \theta_M + \sqrt{(4\pi M_s)^2 \gamma^2 \sin^4 \theta_M + 4\omega^2}}. \]  

(7)

As described in equation (7), in thin film systems, the magnetization precesses in an elliptical orbit due to the demagnetization field except at \( \theta_H = \theta_M = 0 \). When \( \theta_H = \theta_M = 0 \), the magnetization precesses in a circular orbit \( (A = 1) \).

We define the normalized spin current density \( \tilde{j}_s \) as \( \tilde{j}_s = j_s/j_s^{A=1} \), where \( j_s^{A=1} \) is the spin-current density when the magnetization precesses in a circular orbit (the external magnetic field is applied perpendicular film plane: \( \theta_H = \theta_M = 0 \)). Using equation (6), we find \( \tilde{j}_s \) as

\[ \tilde{j}_s = \frac{2\omega \left( 4\pi M_s\gamma \sin^2 \theta_M + \sqrt{(4\pi M_s)^2 \gamma^2 \sin^4 \theta_M + 4\omega^2} \right)}{(4\pi M_s)^2 \gamma^2 \sin^4 \theta_M + 4\omega^2}. \]  

(8)

The relation between the spin current density \( \tilde{j}_s \) and the ellipticity \( A \) is readily obtained by combining equations (7) and (8), which is expressed as

\[ \tilde{j}_s = \frac{4A}{(1 + A^2)^2}. \]  

(9)

This simple expression indicates that the spin current density is maximized when the precession trajectory is distorted: \( A = 1/\sqrt{3} \). Here, the magnetization angle \( \theta_M \) at which \( \tilde{j}_s \) is maximized is given as

\[ \sin \theta_M = 3^{-1/4} \sqrt{\frac{2\omega}{4\pi M_s\gamma}}, \]  

(10)

showing that the spin current density is maximized when the external magnetic field is applied oblique to the film plane.

A magnetization-precession trajectory is characterized also by the elliptical area of a magnetization-precession trajectory, which is defined as \( S \equiv |m_x||m_y| \). We define the normalized area of a magnetization-precession trajectory as \( \tilde{S} = S/S^{A=1} \), where \( S^{A=1} \) is the area of the magnetization-precession trajectory when the magnetization precesses in a circular orbit. Using equations (3) and (4), we obtain \( \tilde{S} \) as

\[ \tilde{S} = \frac{2\omega \left( 4\pi M_s\gamma \sin^2 \theta_M + \sqrt{(4\pi M_s)^2 \gamma^2 \sin^4 \theta_M + 4\omega^2} \right)}{(4\pi M_s)^2 \gamma^2 \sin^4 \theta_M + 4\omega^2}. \]  

(11)

This expression of \( \tilde{S} \) is exactly the same as that of the spin current density \( \tilde{j}_s \) in equation (8): \( \tilde{j}_s = \tilde{S} \). This shows that the spin current density \( \tilde{j}_s \) is determined by the elliptical area of a
Figure 2. (a) The magnetization angle $\theta_M$ dependence of the normalized area of a magnetization-precession trajectory $\tilde{S}$ for ferromagnetic films with the saturation magnetization $4\pi M_s$. Here, $\tilde{S} \equiv S/S^M$ and $S \equiv \pi |m_x||m_y|$. (b) The magnetization angle $\theta_M$ dependence of the dimensionless precession amplitude $m/M_s$. Here, $\omega = 5.93 \times 10^{10} s^{-1}$, $\gamma = 1.86 \times 10^{11} T^{-1} s^{-1}$.

magnetization-precession trajectory $\tilde{S}$, which is maximized when the magnetization precession axis is oblique to the film plane as shown in figure 2(a). In figure 2(b), we show the averaged precession amplitude $m$, which is defined as $m = (\omega/2\pi) \int_0^{2\pi/\omega} |\mathbf{m}(t)| dt$. The variation of $m$ is different from that of $\tilde{S}$, showing that the spin pumping is dominated by the elliptical area of a magnetization-precession trajectory not by the precession amplitude.

4. Summary
In summary, we calculated a spin current density generated by the spin pumping in terms of a magnetization-precession trajectory. The spin current density was found to be determined by the elliptical orbit area of the magnetization precession. Since the spin pumping enables the spin-current injection into a wide range of systems, this result will be useful for developing spin-current technologies.

5. Acknowledgments
This work was supported by a Grant-in-Aid for Scientific Research in Priority Area “Creation and control of spin current” (19048028) from MEXT, Japan, a Grant-in-Aid for Scientific Research (A) from MEXT, Japan, a Grant for Industrial Technology Research from NEDO, Japan, and Fundamental Research Grant from TRF, Japan.

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