Dendritic flux avalanches and nonlocal electrodynamics in thin superconducting films

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We present numerical and analytical studies of coupled nonlinear Maxwell and thermal diffusion equations which describe nonisothermal dendritic flux penetration in superconducting films. We show that spontaneous branching of propagating flux filaments occurs due to nonlocal magnetic flux diffusion and positive feedback between flux motion and Joule heat generation. The branching is triggered by a thermomagnetic edge instability which causes stratification of the critical state. The resulting distribution of magnetic microavalanches depends on a spatial distribution of defects. Our results are in good agreement with experiments performed on Nb films.

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Penetration of magnetic flux in a type-II superconductor can result in nonequilibrium pattern formation, such as magnetic macroturbulence \cite{1}, kinetic front roughening \cite{2}, dendritic flux diffusion \cite{3}, and dendritic structures \cite{4}. Dendritic flux penetration has been revealed by magneto-optical imaging (MOI) on multiple scales $\sim 1 - 100\mu m$ much greater than intervortex spacings in YBa$_2$Cu$_3$O$_7$\cite{5,6}, Nb\cite{7}, Nb$_3$Sn\cite{8}, and MgB$_2$\cite{9}. Similarity of these dynamic flux patterns in different materials indicates a generic collective behavior of vortices.

Recently it has been shown both experimentally and theoretically that dendritic flux penetration is due to a positive feedback between moving flux and the Joule heating coupled by a highly nonlinear voltage-current characteristic \cite{10,11}. The resulting thermal bistability of current-carrying superconductors gives rise to switching waves between a cold superconducting phase and a hot resistive phase self-sustained by the Joule heating \cite{11}. Dendritic flux penetration in superconductors has analogs in the theory of pattern formation out of equilibrium \cite{12} and instability of solidification fronts \cite{13}. Dendritic flux dynamics in superconductors was observed in numerical analysis of nonisothermal magnetic diffusion in a slab in a parallel field, when flux penetration was triggered by a local heat pulse \cite{14}. However, experiments have been mostly done on films in a perpendicular ramping magnetic field in which case magnetic flux diffusion becomes strongly nonlocal due to long-range interaction of vortices \cite{15}. In this Letter we calculate dendritic flux penetration controlled by nonlocal magnetic flux diffusion coupled to thermal diffusion in thin films. We report a novel nonlocal mechanism of flux branching, which captures salient features of dendritic flux penetration in superconducting films.

We consider a thin film strip of the width $w$ along the $y$-axis and thickness $d \ll w$ in the $xy$ plane perpendicular to the magnetic field $H_0$. Distributions of the magnetic induction, $\mathbf{B}(\mathbf{r}, t)$, and temperature $T(\mathbf{r}, t)$ are described by the Maxwell equation coupled to the heat diffusion:

$$C\partial_t T = \nabla \kappa \nabla T - (T - T_0) h/d + J \mathbf{E}(\mathbf{J}, T),$$

$$\partial_t \mathbf{B} = -\nabla \times \mathbf{E} (\mathbf{J}, T), \quad \nabla \times \mathbf{H} = \mathbf{J} \delta(z).$$

Here $C(T)$ is the heat capacity, $\kappa(T)$ is the thermal conductivity, $h(T, T_0)$ is the heat transfer coefficient to the coolant or substrate held at the temperature $T_0$, and $\mathbf{E} = J \mathbf{E}(\mathbf{J}, T)/J$ is the electric field, which strongly depends on $T(\mathbf{r}, t)$ and the sheet current density $\mathbf{J}(\mathbf{r}, t)$.

The $E(\mathbf{J}, T, B)$ characteristic accounts for a resistive flux flow state with $E = (J - J_c) \rho_F$ for $J > J_c$ and a low-resistive flux creep state with $E = E_c \exp(J - J_c)/J_1$ for $J < J_c$, where $J_c(T, B)$ is the critical current density. We use the following interpolation formula expressed in terms of observable parameters \cite{10}:

$$E = \rho_F J_1 \ln[1 + \exp(J - J_c)/J_1],$$

where $J_1(T)$ is logarithmic flux creep rate ($J_1 \ll J_c$ below the irreversibility field $B < B^*$), and $\rho_F(T) = \rho_0 B/B_{c2}$ is the flux flow resistivity.

We consider weak Joule heating, for which the most essential temperature dependence comes from $E(T)$, while other parameters may be taken at $T = T_0$. The relation between current and the $z$-component $\mathbf{B}_z$ in a film is given by the non-local Biot-Savart law. Expressing $J_x = \partial_y g$ and $J_y = -\partial_x g$ in Eq. (2) in terms of the current stream function $g(x, y, t)$, we obtain the equations for $g$ and the dimensionless temperature $\theta$:

$$\tau \dot{g} = \hat{K}[\partial_x [r(\mathbf{j}, \theta) \partial_x g + \partial_y [r(\mathbf{j}, \theta) \partial_y g] - \tau H_0(t)],$$

$$\dot{\theta} = \nabla^2 \theta - \pi j^2 r(\mathbf{j}, \theta).$$

Here we define the operator $\hat{K}$ in the Fourier space, $\hat{K} = \sum_{\mathbf{k}} \sin(k_x x) \sin(k_y y) g_k/k$, where $k_x = \pi n/L$ and...
\[ k_y = \pi m/w \] with integer \( m \) and \( n \) to ensure zero normal component of \( \mathbf{J} \) at edges of a rectangular film of width \( w \) and length \( L \). Furthermore, \( \theta = (T - T_0)/(T - T_H) \), \( \theta_c(T^*) = 0 \), \( j_c = J/(2\pi) \), \( H_z(x, y) = \sum_k \exp(-dk - i\mathbf{k}\cdot\mathbf{r})g_k/2 + H_0(t) \), the factor \( \exp(-dk) \) accounts for a finite film thickness, the derivatives in Eqs. 14 and 15 are taken over normalized time \( t/t_h \) and coordinates \( r/L_h \), and the nonlinear resistivity \( \rho \) is obtained from Eq. 13. Here the thermal length \( L_h = (\kappa/\rho C)^{1/2} \) and time \( t_h = C_d/h \) define the spatial scale and the cool down time of \( T(r, t) \) at a frozen \( \mathbf{J}(r) \). Hereafter we take \( j_c(\theta) = j_0(1 - \theta) \) for \( \theta < 1 \), and \( j_c(\theta) = 0 \) for \( \theta > 1 \), and \( J_1(T) = \text{const} \), assume that a uniform magnetic field \( H_0(t) = H_0 \) is ramped up with the rate \( H_0 \), and neglect the field dependence of \( J_c \). Evolution of \( \theta(r, t) \) and \( g(r, t) \) is controlled by two dimensionless parameters:

\[
\tau = \frac{\mu_0 \sqrt{\kappa h}}{2\rho FC}, \quad \alpha = \frac{\rho J^2 c}{h(T' - T_0)}. \tag{6}
\]

Here \( \tau = t_m/t_h \) is the ratio of magnetic and thermal diffusion times, and \( \alpha \) quantifies the Joule dissipation. Magnetic nonlocality strongly reduces \( \tau = \tau_0 d/L_h \) in a film as compared to \( \tau_0 = \mu_0 \kappa/\rho F C \) in the bulk. Indeed, flux diffusion over a distance \( L_h \) along a film takes \( t_m \sim dL_h/D_m \), while thermal diffusion takes \( t_h \sim L_h^2/D_h \), where \( D_m = \mu_0/\rho F \) and \( D_h = \kappa/C \). For Nb films with \( d = 0.5 \mu m \), at 42K, \( \kappa \approx 0.2W/cmK, h \approx 1W/cm^2K, \rho_n \approx 3 \times 10^{-7}\Omega cm \), and \( C \approx 2 \times 10^{-3}J/cm^3K \), we obtain \( L_h = (d/L)^{1/2} \approx 0.03mm, t_h = C_d/h \approx 10^{-7}s, d/L_h \approx 10^{-2}, \tau \approx 0.1 \), with \( \tau \) decreasing as \( t_0 \) increases.

We used Eqs. 13, 14 to study flux penetration in a film with periodic boundary conditions (b.c.) along \( x \) and \( \partial_x \theta = g = 0 \) at \( y = 0, w \). Eqs. 13, 15 were solved numerically by a quasi-spectral method based on the Fast Fourier Transform; up to 1024 \( \times \) 512 harmonics were used. To implement non-periodic b.c. in the \( y \)-direction we used the domain of doubled length with the condition \( g(x, y) = -g(x, 2w - y) \). Calculated steady-state distributions of \( B_z(x) \) are very close to those of the Bean model. In the majority of numerical runs for \( \tau \ll 1 \), we observed spontaneous avalanches induced by ramping magnetic field \( H_0(t) = H_0t \), starting from a zero field cooled state. We also took into account randomly-distributed macroscopic defects modelled by \( j_c(r) = j_0(1 - \theta - \sum_i q_i \cosh^{-1}|(r - r_i)/\xi_0|) \) where \( q_i, r_i \) and \( \xi_0 \) determine the strength, the position and the radius of the i-th defect. Extended current-blocking defects have been commonly revealed by the MOI.

We start with flux penetration in a film with no macro-defects, Fig.3a and Movie 1 in [17]. The ramping magnetic field first caused penetration of a stable cold flux front. Then an instability causing periodic modulations of temperature and propagation of hot magnetic filaments over the preceding smooth magnetic flux distribution develops at the film edge. Once the first wave of magnetic

\[ \lambda = 1 + q^2 + k^2 - \beta(\lambda_1 \sqrt{k^2 + q^2 + q^2 + \frac{2\kappa}{\rho C}}^2 + q^2 \beta(1 + s) = 0, \tag{7} \]

where \( \lambda \) and \( (k, q) \) are measured in \( t_h^{-1} \) and \( L_h^{-1} \), respectively. The dissipation control parameter \( \beta = (d/\kappa)\partial E/\partial T \) depends on \( E_0 \): if \( E(J) = E_c \exp((J - J_c)/J_1) \) then \( \beta \approx E_0 J_1d/\partial J_0 \). The parameter \( \tau_1 = \mu_0 \sqrt{\kappa}/2\rho C \) is similar to that in Eq. 13.
Eq. \(\text{(7)}\) describes coupled thermal and magnetic diffusion modes, the factor \(\sqrt{k^2 + q^2}\) accounting for the magnetic nonlocality. Positive eigenvalues \(\lambda(k, q)\) correspond to unstable modes resulting in spontaneous thermomagnetic structures \(\text{[13]}\). For slow flux diffusion \(\tau \gg 1\), thermal perturbations with \(\lambda \approx \beta - 1 - k^2 - q^2\) are unstable above the thermal runaway threshold, \(\beta > 1\). In this case \(\lambda\) is maximum at \(k = q = 0\) so no periodic structures are expected. A completely different situation occurs for fast flux diffusion \(\tau \ll 1\) for which Eq. \(\text{(7)}\) yields

\[
\lambda = \beta - 1 - q^2 - k^2 - \frac{q^2\beta(1 + s)}{q^2 + sk^2}. \tag{8}
\]

The spectrum of \(q\) is determined by the full set of boundary conditions in the theory of flux jumps \(\text{[13]}\). For further qualitative analysis we take \(q \approx \pi/2b\), where \(\delta J_c(x, b) = 0\), \(b = |1 - 1/(\cosh(B_0/B_p))|/2\) is the width of the flux penetrated critical state region, and \(B_p = \mu_0 J_c/\pi \text{[20]}\). For a given \(q\), the increment \(\lambda(q, k)\) passes through a maximum at the wave vector \(k_m\), which defines the period \(\ell = 2\pi/k_m\) of the fastest growing thermomagnetic structure along the film. Here \(sk_m^2 = \frac{1}{2}q^2\beta(1 + s)^{1/2} - q^2\), thus \(\lambda(k, q)\) is maximum at the finite \(k_m\) if \(\beta > \beta_c = q^2/s(1 + s)\), or \(B_0 > B_1\). For \(s \ll 1\), we obtain \(B_1\) and \(\ell\) in normal units:

\[
B_1 = \frac{\pi^2\kappa}{4b^2|\partial J_c/\partial T|}, \quad \ell^2 \approx \frac{16b^2s}{(B_0/B_1)^{1/2} - 1}. \tag{9}
\]

The period \(\ell\) decreases as \(1/s\) and \(B_0\) increase, in agreement with our numerical results \(\text{[17]}\). The branching instability with \(k_m > 0\) and \(\text{Re} \lambda(\beta_c, k_m, q) > 0\) occurs at \(\beta = \beta_c\) in Eq. \(\text{(5)}\), that is, \(B_0 > B_c = hJ_c/\partial J_c/\partial T|\), where

\[
\beta_c^{1/2} = (1 + 2q^2)^{1/2} + q\sqrt{1 + 1/s}. \tag{10}
\]

For \(\tau_i \ll 1\), Eq. \(\text{(10)}\) gives \(\beta_c > \beta_i\) for all \(q(t) = \pi/2b(t)\), so a thermomagnetic structure with the finite period \(\ell(\beta_i) \ll b\) develops as the width \(b(t)\) exceeds \(b_c\), where \(B_0 = \hat{B}_c(b_c)\). Notice that Eq. \(\text{(7)}\) defines a region \(\tau_1 < \tau < \tau_2\) in which \(\lambda\) is complex, which manifests itself in temporal oscillations of growing flux structures.

Next we consider dendritic flux penetration initiated by macroscopic defects, both at the film edge and in the bulk. Such defects can trigger local flux jumps even if the critical state in the bulk is stable \(\text{[21]}\) and cause branching instability of flux filaments in a slab in a parallel field \(\text{[10]}\). Selected results of flux patterns in a film are shown in Fig. \(2\). For \(j_0 = 1/s = 20\) in Fig. \(2a\), edge defects produce flux fingers superimposed with a smooth flux front. This behavior is characteristic of any superconductor with a highly nonlinear \(E(J)\), for which a defect of size \(\xi_0\) produces a much larger disturbance \(\approx \xi_0/s\) across the current flow \(\text{[22]}\). The flux fingers widen and split at the ends as they collide with the central line where magnetization currents change direction \(\text{[11]}\). As \(j_0\) increases, hot flux filaments in Figs. \(2b\) and \(2c\) get thinner and start branching even before they reach the central line. Then new fingers start growing between the defects due to the edge instability considered above, see animations for more details \(\text{[17]}\). Moreover, in wider samples (Fig. \(2c, d\)) the finger undergoes multiple branching giving rise to the characteristic flux dendrites. This new branching mechanism, which was not observed in simulations of flux patterns in a slab \(\text{[11]}\), is principally due to nonlocal flux diffusion in films. Thinning the filaments as \(s\) decreases follows from Eq. \(\text{(6)}\), while the branching shape instability is facilitated by magnetic nonlocality, which results in a weaker damping of short-wave electromagnetic modes \(\lambda_m \propto k^2\) as compared to local flux diffusion for which \(\lambda_m \propto k^2\). Another new effect at higher values of \(j_0\) is a “giant” flux avalanche in Fig. \(2c\), which starts propagating from the region with no surface defects after the first wave of smaller flux filaments reached the center.

Fig. \(2d\) shows flux patterns in a film with randomly-distributed bulk defects. In addition to the branching due to magnetic nonlocality, propagating flux filaments can undergo splitting caused by local transient heat spikes as they collide with defects. This results in local shape instability of the filaments and their subsequent branching similar to that obtained for local magnetic flux diffusion \(\text{[10]}\). With the decrease of \(j_0\) (which is equivalent to an increase of \(T_0\) or \(H_0\) in the experiment) flux filaments become wider and eventually start overlapping, forming a continuous flux front. However, even in this case a significant front roughness still persists both due to a size distribution of individual filaments and due to local heat releases as the flux front collides with defects \(\text{[17]}\).

The dendritic flux penetration can be regarded as avalanches of vortex bundles which do not trigger a global flux jump in the whole sample. Such avalanches produce local temperature spikes, partial flux penetration and a step on magnetization curves \(M(B)\) \(\text{[3, 4]}\). Nonlocal electrodynamics of films strongly facilitates branching flux propagation, which requires faster magnetic diffusion \(\tau = \tau_0\sqrt{dh/k} \ll 1\) characteristic of thin films with \(dh \ll k\). Notice that because \(b \sim w(B_0/B_p)^2\) for small \(B_0\), both critical ramp rates \(B_1 \propto B_0^{-6}\) and \(B_c \propto B_0^{-6}\) strongly depend on the applied field \(B_0\) for \(B_0 < B_p\) but level off for \(B_0 > B_p\) as \(b \sim w/2\). Furthermore, both \(B_1\) and \(B_c\) increase as \(T_0\) increases. Thus, for a given \(B_0\), the branching occurs at lower \(T\) above a certain field \(B_1(T)\), in agreement with many experiments \(\text{[4, 6, 8, 9]}\).

Fig. \(3\) shows MOI of flux penetration in two different Nb films in ramping fields. One \(9\text{mm}\times18\text{mm}\times0.5\mu\text{m}\) Nb film described in detail elsewhere \(\text{[8]}\) exhibits flux patterns similar to those in Figs. \(2a, b\) where dendritic flux
FIG. 2: Flux penetration in a film of $320L_h \times 40L_h$ with 30 random edge defects with $\xi_0 = 0.25L_h$, and amplitudes $\varphi_i$ uniformly distributed between 0 and 0.4, for $H_0 = 3J_1/t_h$, $\alpha = 0.08$, $\tau = 0.0025$ and $t = 25t_h$ for $j_0 = 20$ (a) and $j_0 = 80$. Black and white correspond to the Meissner and vortex phases, respectively; (c) Flux penetration in wide film of $600L_h \times 150L_h$ for $H_0 = 5J_1/t_h$, $j_0 = 60$, 20 edge defects. The “giant” avalanche develops in a defect free region; (d) Flux pattern in a film with 500 randomly distributed bulk defects at $j_0 = 80$. See also Movies 2-6 in [17].

FIG. 3: MOI of flux branching in Nb films of Ref. [7] at $T=6.2K$ and $B_0=31.2mT$ (a) and $4.7K$ and $B_0=36.5mT$ (b). “Giant flux avalanche” at 4.5K in Nb film of [2] (c). See also Movies 7-8 in [17].

penetration is initiated by surface defects. For this film ($\kappa = 0.2W/cmK$ and $h = 1W/cm^2K$), we obtain $dh/\kappa \sim 10^{-4}$ and the thermal length $L_h = (dh/h)^{1/2} = 0.03nm$, much smaller than the film width $w = 1.8nm$ (i.e. the film width is about 60$L_h$ as in Figs. 2a,b), in which case the magnetic nonlocality does play the key role. Fig. 3c shows MOI of an effectively wider (4mm×4mm×0.1µm) film [2] ($w \sim 10^2L_h$), in which in addition to small microavalanches near the film edge a “giant” avalanche, similar to that triggered by a laser pulse in YBa$_2$Cu$_3$O$_7$ [8] and those in Fig. 2c,d develops.

In conclusion, we proposed a mechanism of flux fragmentation in superconducting films caused by coupling of nonlocal flux diffusion with local thermal diffusion. This work was supported by the NSF MRSEC (DMR 9214707) (AG); US DOE, BES-Materials Sciences (# W-31-109-ENG-38) (IA,VV, VKV, UW); and by Stichting voor Fundamenteel Onderzoek der Materie, which is financially supported by Nederlandse Organisatie voor Wetenschappelijk Onderzoek (MW,RW).

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