Characteristics of bent terahertz antiresonant reflecting pipe waveguides

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Abstract: Bending characteristics of the terahertz (THz) pipe waveguides are numerically investigated. Numerical results reveal that the inherent periodic feature of the loss spectrum, resulting from the antiresonant reflection guiding mechanism, is nearly unaffected under bending. However, attenuation constant of the fundamental (HE11) mode becomes polarization dependent for the bent pipe waveguide, and the polarization perpendicular to the bending plane experiences less bending losses. Moreover, unlike the straight case where a larger air-core diameter leads to a smaller attenuation constant, increasing core diameter of the bent pipe waveguide is unable to reduce attenuation constant effectively if the propagation mode is a whispering gallery mode. Finally, behavior of the bent pipe waveguide connected to a straight one is also examined in this work.

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1. Introduction

Terahertz (THz) radiation has received significant attentions due to its increasing applications in biology and medicine, environment monitoring, security, imaging, and communications [1–5]. To facilitate THz science and technologies, it is essential to develop low-loss and low-cost THz waveguides [6–16]. The pipe waveguide is a recently proposed hollow structure for THz waveguiding [17,18]. It is a simple pipe with a large air core surrounded by a thin dielectric cladding layer, and its guiding mechanism is based on the antiresonant reflection guiding [19]. A commercially available 3-m long Teflon pipe was demonstrated to achieve very low propagation losses, with the attenuation constant well below 0.005 cm$^{-1}$, under straight condition [17]. Owing to the structure simplicity and excellent performance, after the pioneering Teflon pipe, various pipe waveguides have been reported for THz transmission, being made of silica [20], polypropylene (PP) [21], chalcogenide glass [22], and polymethylmethacrylate (PMMA) [23].

In practical applications, waveguides often suffer bending. Hence it is important to investigate the transmission characteristics of the THz pipe waveguide under bent condition. Attempts have been made experimentally [23,24] in that bending losses of the bent THz pipe waveguides were measured. However, because the experimental setups comprised both straight and bent sections, according to the waveguide theory, the except the bending loss due to a uniform curvature, there would be an additional power loss incurred at the straight-to-bent interface, owing to the mode mismatch between the straight and bent waveguides [25].
Therefore, the previously measured results are not “pure” bending losses and thus the bending characteristics of the THz pipe waveguide require further investigation.

In this paper, we aim to numerically investigate the bending characteristics of the THz pipe waveguides. For the study of waveguide bends, one widely used technique is the conformal transformation [26], where the curved waveguide is transformed into an approximate equivalent straight waveguide (ESW) with a modified index profile. An alternative approach is to employ a rigorous formulation derived in a cylindrical coordinate system (CCS) [27–30]. It has been shown that the CCS approach is more accurate than the ESW one, especially when the bending radius is small [29]. In this work, we use the full-vectorial finite-difference frequency-domain (FDFD) mode solver in a local CCS [30] to perform the bending analysis. To effectively deal with the leaky modes, the perfectly matched layer (PML) absorbing boundary condition based on the complex coordinate stretching [31] is also incorporated.

2. Structure for numerical modeling

Structure of the bent THz pipe waveguide in the local CCS is shown in Fig. 1(a). The bending plane is assumed to be the x-z plane and the bending radius $R$ is measured from the bending center to the point where $x = 0$. Cross-section of the pipe waveguide is shown in Fig. 1(b). It consists of a large air core with refractive index $n_1 = 1$ and a uniform dielectric cladding layer with refractive index $n_2$. The thickness $t$ of the cladding layer is much smaller than the diameter $D$ of the air core.

![Diagram of the bent THz pipe waveguide](image)

Fig. 1. (a) Structure and (b) cross-section of the bent THz pipe waveguide.

Guiding mechanism of the THz pipe waveguide can be described by treating its cladding layer as a Fabry–Perot etalon [18]. At the resonant frequencies, transmittance of the cladding is near unity and no THz waves would be confined in the air-core region, causing a large amount of losses. While at the antiresonant frequencies, strong reflection occurs at the core–cladding interface, allowing THz waves to bounce back and forth inside the air-core region with relatively low propagation losses. This guiding mechanism is similar to that of the antiresonant reflecting optical waveguide (ARROW) [19], but is realized with a single-layer cylindrical hollow waveguide structure. Resonant frequencies of the cladding are given by [18]

$$f_w = \frac{mc}{2n_1t\sqrt{(n_2/n_1)^2 - 1}}, \quad m = 1, 2, 3, \ldots$$

where $c$ is the speed of light in vacuum and $m$ is an integer.

To simulate the behaviors of the bent THz pipe waveguide, the full-vectorial FDFD mode solver in the local CCS is applied [30]. As shown in Fig. 1(a), the computational window is located in the x-y plane, with $(x, y) = (0, 0)$ being the waveguide center. Essentially, the FDFD mode solver is to solve an eigenvalue matrix equation in the form:
where $E_x$ and $E_y$ are transverse components of the electric field and $\beta$ is the complex propagation constant. Once Eq. (2) is solved, bending loss, in terms of attenuation constant, can be obtained as $-2\text{Im}(\beta)$. In subsequent simulations, the following parameters are assumed: $D = 9 \text{ mm}$, $t = 0.5 \text{ mm}$, and $n_2 = 1.4$. We primarily investigate the fundamental (HE$_{11}$) mode in this work.

3. Numerical results

3.1 General characteristics

We first calculate the attenuation spectrum, i.e., attenuation constant as a function of frequency, of the bent THz pipe waveguide. Calculated results for the polarization parallel to the bending plane ($x$-polarized) and the polarization perpendicular to the bending plane ($y$-polarized) are both shown in Fig. 2. The bending radius is $R = 200 \text{ cm}$. Attenuation spectrum of the straight pipe waveguide is also shown for comparison, where the polarization is not defined for the straight case, because attenuation constants of the $x$- and $y$-polarized HE$_{11}$ modes are the same owing to cylindrical symmetry. In Fig. 2, it is noted that two discontinuities (around 300 and 600 GHz, respectively) occur in the attenuation spectrum of the straight pipe waveguide. These discontinuity frequencies are the resonant frequencies predicted by Eq. (1), which can be confirmed by substituting $m = 2$ and 3 into Eq. (1) and the resultant resonant frequencies are 306 and 612 GHz, respectively. As previously stated, near the resonant frequencies, the cladding is almost transparent and THz waves could hardly be confined in the air-core region. Thus local maximum losses occur. These resonant frequencies make the attenuation spectrum periodic, and such a periodicity is a unique feature of the ARROW-like waveguides. Obviously, the bent pipe waveguide exhibits the same periodic behavior as that of the straight waveguide, in that the two discontinuities in the attenuation spectrum of the bent pipe waveguide also coincide with the resonant frequencies. Hence, simulation results in Fig. 2 reveal that the inherent periodic characteristics of the THz pipe waveguide is nearly unaffected under bending. In addition, as expected, attenuation constants of the bent pipe waveguide are larger than those of the straight one because of the radiation losses caused by bending. However, it is found that the $y$-polarized mode suffers less bending losses than the $x$-polarized one does. The polarization effect on bending will be discussed later.

![Fig. 2. Attenuation spectra of the bent and straight THz pipe waveguides. For the bent case, the bending radius $R = 200 \text{ cm}$.
](#)
We next examine the modal field distributions of the bent and straight THz pipe waveguides. Figure 3(a) shows the intensity distributions of the \(y\)-polarized mode. Since there are three pass bands in the attenuation spectra shown in Fig. 2, we choose one frequency from each band and the results are calculated at 200, 500, and 800 GHz, respectively. Clearly, owing to the bending, field profile of the THz wave not only shifts toward the outer cladding (toward the + \(x\) direction) but also narrows substantially. The shift of the THz wave from the air-core center increases as the bending radius decreases. It also increase with frequency. According to the simulations, the field shift is polarization independent, i.e., it is the same for both \(x\)- and \(y\)-polarized modes. One simulated example is illustrated in Fig. 3(b), where the intensity distributions and the electric field vector distributions of both polarizations are displayed. They are calculated at 800 GHz and \(R = 75\) cm. If the shift ratio is defined as: (the shift of intensity peak from the core center) / (the core radius) \(\times 100\%\), for the case shown in Fig. 3(b), the shift ratios are 67\% and 66\% for the \(x\)- and \(y\)-polarized modes, respectively. From Fig. (3), it is noted that even though the field profiles suffer considerable deformation, THz waves are still well-confined in the air-core region. This suggests that the THz pipe waveguide may sustain severe bending without suffering excessive bending losses.

![Intensity distributions of the \(y\)-polarized mode](image)

![Electric field vector distributions of both polarizations](image)

Fig. 3. (a) Intensity distributions of the \(y\)-polarized mode. (b) Intensity distributions and electric field vector distributions of the \(x\)- and \(y\)-polarized modes for the bent case, which are obtained at 800 GHz and \(R = 75\) cm.

### 3.2 Polarization effect

Then, we investigate the polarization effect on bending loss. Calculated attenuation constants for \(x\)- and \(y\)-polarized modes as a function of bending radius are shown in Fig. 4. The results are calculated at 800 GHz. Clearly, attenuation constants for both polarizations are almost the same when the bending radius is large, e.g., \(R = 1000\) cm. Actually, if the bending radius approaches infinity, which is the case of straight pipe waveguide, there would be no difference between the attenuation constants of \(x\)- and \(y\)-polarizations because of the symmetrical geometry under straight condition. When the bending radius decreases, as expected, the attenuation constant of either polarization increases. However, it is found that the attenuation constant of the \(y\)-polarized mode is less than that of the \(x\)-polarized one, and the difference between the two polarizations is getting obvious with decreasing bending radius. This phenomenon is much different from that observed in the total-internal-reflection-based waveguides, such as the conventional step-index fiber. For the step-index fiber, bending
losses of the $x$- and $y$-polarized HE$_{11}$ modes are the same [32,33]. While for the antiresonant-reflection-based THz pipe waveguide studied here, the $y$-polarized HE$_{11}$ mode experiences less bending loss than the $x$-polarized one does. Similar polarization effect was also observed in the bent hollow-core Bragg fiber [34], but explanation was not provided. In the following, we explain the polarization-dependent phenomenon occurring in the bent pipe waveguide. The reason for the THz wave to be guided in the air-core region of the pipe waveguide is the partial reflection occurring at the core-cladding interface. The more THz wave the interface reflects, the less power loss the pipe waveguide suffers. As having been seen in Fig. 3, under bent condition, THz wave shifts toward the outer cladding of the pipe waveguide so that there is more amount of THz field existing near the outer cladding. Therefore, reflection occurring at the outer core-cladding interface plays a dominant role in determining the bending loss. When the THz wave is $x$-polarized (parallel to the bending plane), the wave close to the outer cladding is like a transverse magnetic one (TM-like), i.e., the electric field is perpendicular to the outer core-cladding interface, as shown in the left case of Fig. 3(b). On the contrary, for the $y$-polarized THz wave (perpendicular to the bending plane), the electric field is parallel to the outer core-cladding interface which makes the wave like a transverse electric wave (TE-like), as shown in the right case of Fig. 3(b). It is well known from the electromagnetic theory that reflection is larger for the TE wave than for the TM wave [35]. As a result, the bending loss for the $y$-polarized HE$_{11}$ mode is smaller than that of the $x$-polarized one. In other words, the $y$-polarization, i.e., the polarization perpendicular to the bending plane, would be more preferred for the THz pipe waveguides when bending is taken into consideration.

![Fig. 4. Attenuation constants as a function of bending radius. The frequency is 800 GHz.](image)

### 3.3 Influence of the air-core diameter

We further study the influence of the air-core diameter by examining the $y$-polarized mode. Intensity distributions of the bent pipe waveguides with various core diameters are shown in Fig. 5, where the cladding thickness is still fixed to 0.5 mm. The frequency is 500 GHz and the bending radius is $R = 200$ cm. Simulated results for the straight case are also shown for comparison. Clearly, under the same bending condition, the larger core diameter the pipe waveguide has, the more severe deformation the field profile suffers. In Fig. 5, shift ratios of the intensity peak are 3%, 13%, 34%, 51%, and 61% for $D = 3, 6, 9, 12,$ and 15 mm, respectively.

Figures 6(a)–6(c) show attenuation constants of bent pipe waveguides as a function of air-core diameter for 200, 500, and 800 GHz, respectively. Results for the straight case are also displayed, and it is apparent that the curve of attenuation constant of the straight pipe waveguide drops notably as the core diameter increases. One major reason is that, for the straight case, the distance between two consecutive reflections at the core-cladding interface increases with the core diameter. Hence, when the core diameter increases, the number of
Attenuation under the same waveguide length reduces so that the resultant loss from imperfect reflections decreases [18]. It has been shown that the attenuation constant is proportional to $1/D^4$ for the THz pipe waveguide under straight condition [17]. Then we turn to examine the influence of core diameter for the bent case. In Fig. 6, attenuation curves of the bent waveguides are shown for several bending radii. By using the curve of the straight pipe waveguide as a reference, each curve of the bent waveguides can be divided into two segments. One segment (appearing in the region of smaller core diameters) is sharper and coincides with the curve of the straight waveguide, while the other segment (appearing in the region of larger core diameters) is flatter and diverges from the curve of the straight waveguide. To distinguish the two segments with distinct characteristics, here they are referred to as the sharp segment and the flat segment, respectively. Clearly, the sharp segments for different bending radii all coincide with the curve of the straight pipe waveguide. In fact, according to the simulation, when the core diameter is small, attenuation constants of the bent pipe waveguides are slightly higher than that of the straight waveguide, and the smaller bending radius the pipe waveguide encounters, the larger attenuation constant the waveguide suffers. But they are very close in magnitude. Therefore, owing to the scale of the vertical axis, for fixed core diameters, attenuation constants of the straight and bent pipe waveguides all look the same in the figure, which causes the sharp segments of the bent pipe waveguides coincident with the curve of the straight waveguide. From Fig. 6, it is noted that behavior of the bent pipe waveguide is similar to that of the straight waveguide in the sharp segment, in that attenuation constant decreases significantly with increasing core diameter. However, in the flat segment, the bent pipe waveguide behaves differently and the attenuation constant is nearly unaffected by the core diameter.

Fig. 6. Intensity distributions of the straight and bent THz pipe waveguides for different air-core diameters. The frequency is 500 GHz and the bending radius is 200 cm.

Fig. 5. Intensity distributions of the straight and bent THz pipe waveguides for different air-core diameters. The frequency is 500 GHz and the bending radius is 200 cm.

Fig. 6. Attenuation constants as a function of core diameter for the straight and bent THz pipe waveguides. In the top axis, the core diameter is normalized with respect to the wavelength. (a) 200 GHz. (b) 500 GHz. (c) 800 GHz. The colored arrows indicate the critical diameters at which transition between the perturbed mode and the whispering gallery mode occurs. Each arrow corresponds to a curve of the same color.
The distinct behaviors of the bent THz pipe waveguides shown in Fig. 6 may be clarified by considering ray propagation inside the waveguide. According to the types of ray propagation, the propagation modes of the bent THz pipe waveguide can be grouped into two classes: the perturbed mode and the whispering gallery mode, and their propagation types are shown in Figs. 7(a) and 7(b), respectively. For the perturbed mode, as shown in Fig. 7(a), reflections occur at both inner and outer core-cladding interfaces. When the core diameter increases, the distance between two consecutive reflections also increases. As a result, the number of reflection inside the waveguide reduces and thus the loss resulting from the reflection decreases. Hence, similar to the behavior of the straight case, attenuation constant of the perturbed mode decreases significantly with increasing core diameter. While for the whispering gallery mode, as shown in Fig. 7(b), reflections only occur at the outer core-cladding interface. Since the inner interface will not be hit, increasing the core diameter is unable to effectively change the distance between consecutive reflections. Therefore, attenuation constant of the whispering gallery mode is almost independent of the core diameter. Following the accounts, it can be readily inferred that, for the attenuation curves of the bent pipe waveguides shown in Fig. 6, the sharp segments correspond to the perturbed mode while the flat segments correspond to the whispering gallery mode.

To verify the aforementioned inference, it is essential to determine the condition in which the transition from the perturbed mode to the whispering gallery mode takes place. Actually, whether the propagation mode is a perturbed mode or a whispering gallery mode is dependent on the incident angle $\theta_i$ at the outer core-cladding interface, where the incident angle is defined with respect to the interface normal as shown in Figs. 7(a) and 7(b). At the point of transition, the reflected ray from the outer core-cladding interface must graze the inner core-cladding interface tangentially, as shown in Fig. 7(c). Let the incident angle satisfying this condition be denoted as $\theta_{WGM}$. From Fig. 7(c), one has

$$\theta_{WGM} = \sin^{-1} \left( \frac{OB}{OA} \right) = \sin^{-1} \left( \frac{R - (D/2)}{R + (D/2)} \right).$$

(3)

where $O$ is the point of bending center, and $A$ and $B$ are the points the ray touches the outer and inner core-cladding interfaces, respectively. For the bent pipe waveguide, if $\theta_i < \theta_{WGM}$, the propagation mode is a perturbed mode; otherwise, if $\theta_i > \theta_{WGM}$, it is a whispering gallery mode. From Eq. (3), $\theta_{WGM}$ would be smaller for a smaller $R$ or a larger $D$. In addition, $\theta_i$ would be larger at higher frequencies [18]. Therefore, the whispering gallery mode is more likely to appear with a smaller bending radius, a larger air-core diameter, or a higher operating frequency.

Acquiring the exact value of incident angle for the bent pipe waveguide is a complex problem. Here, we may provide a rough estimation by using the ESW model [26], and the
procedure is described in the Appendix. The estimated incident angles $\theta_i$ for the bent pipe waveguides operating at 500 GHz, with the bending radii of $R = 75, 200, 500, \text{and } 1000$ cm, are shown in Figs. 8(a)–8(d), respectively. The critical incident angles $\theta_{WGM}$ obtained according to Eq. (3) are also displayed in the figures. When $\theta_i = \theta_{WGM}$ transition between the perturbed mode and the whispering gallery mode will take place. From Fig. 8, it is found transitions occur at the core diameters $D = 4.3, 6.0, 8.1, \text{and } 10.3$ mm for $R = 75, 200, 500, \text{and } 1000$ cm, respectively. The results indicate that, taking $R = 75$ cm as an example, if the core diameter is less than 4.3 mm, $\theta_i < \theta_{WGM}$ thus the propagation mode is a perturbed mode; on the other hand, if the core diameter is larger than 4.3 mm, $\theta_i > \theta_{WGM}$ and it becomes a whispering gallery mode. The four critical diameters with which transitions occur are shown as colored arrows in Fig. 6(b) for the frequency of 500 GHz, where one color corresponds to one bending radius. From Fig. 6(b), it is apparent that the four critical diameters are consistent with the four points where the attenuation curves of bent pipe waveguides diverge from that of the straight one. That is to say, the critical diameter for some bending radius, e.g., $R = 75$ cm, agrees with the boundary point between the sharp and flat segments of the attenuation curve for that bending radius. Critical diameters for 200 and 800 GHz are also shown as colored arrows in Figs. 6(a) and 6(c), respectively, and agreement between the critical diameters and the boundary points can be clearly observed as well. Such an agreement exhibited in Fig. 6 provides a strong support of the aforementioned inference that the sharp segment corresponds to the perturbed mode, while the flat segment corresponds to the whispering gallery mode. Therefore, when the bent pipe waveguide is operated in the perturbed mode, increasing the air-core diameter can significantly reduce the attenuation constant; however, if the waveguide is in the whispering gallery mode, increasing the core diameter would be in vain for reduction of the attenuation constant.

![Graph](image-url)  
Fig. 8. Critical incident angles $\theta_{WGM}$ and estimated incident angles $\theta_i$ for the bent pipe waveguides. The frequency is 500 GHz. (a) $R = 75$ cm. (b) $R = 200$ cm. (c) $R = 500$ cm. (d) $R = 1000$ cm.

3.4 Considering the straight-to-bent interface

In practical applications, the pipe waveguide might contain a straight section as well as a bent section. When the HE$_{11}$ mode propagates from the straight one into the bent one, coupling loss will occur due to the mismatch between the field distributions of the straight and bent HE$_{11}$ modes. Moreover, higher order modes will be excited in the bent section. To study the problem with a straight-to-bent interface, one of the commonly used techniques is the coupled mode theory [34]. In the following, the coupled mode approach is utilized for further investigation.
It has been shown that the THz pipe waveguide is a multimode waveguide [18]. When an input field with a unity power is launched into the bent pipe waveguide, the excited modal power $P_i$ of the $i$-th mode can be calculated according to the following overlap integral:

$$P_i = \frac{\iint |E_{in} \cdot E_i|^2 dxdy}{\iint |E_{in}|^2 dxdy \iint |E_i|^2 dxdy},$$

(4)

where $E_{in}$ is the field distribution of the input, $E_i$ is the field distribution of the $i$-th mode in the bent pipe waveguide, and $\cdot$ represents the operation of complex conjugate. Since the eigen modes of the pipe waveguide are leaky, cut-off approximation [34] is adopted in Eq. (4) in that the integration is performed only in the finite region containing the air core and the cladding layer. Total output power of the bent pipe waveguide is the sum of the powers carried by individual modes at the output. It is given by

$$P_{out} = \sum_{i=1}^{N} P_i \exp(-\alpha_i L),$$

(5)

where $N$ is the number of modes in the bent pipe waveguide, $\alpha_i$ is the attenuation constant of the $i$-th mode, and $L$ is the length of the bent waveguide. The total power loss (in dB) after propagation through the bent pipe waveguide can be obtained as $-10 \log (P_{out})$.

We first examine the higher order modes of the bent pipe waveguide. Except the HE$_{11}$ mode, the next three higher order modes are the TE$_{01}$, HE$_{21}$, and TM$_{01}$ modes, respectively [18]. Figure 9 shows the intensity distributions and the electric field vector distributions of these modes in the bent pipe waveguide. The frequency is 500 GHz and the bending radius is $R = 500$ cm. Note that there are two orientations of the HE$_{21}$ mode shown in Fig. 9. They are denoted as HE$_{21o}$ and HE$_{21e}$ depending on whether they are odd or even with respect to the bending plane. In the straight case, the two orientations are identical except a rotation of 45°. They become different in the bent case, because the rotational symmetry is destroyed upon bending. Calculated attenuation constants under the assumed bending condition are 0.0034, 0.0063, 0.0072, and 0.0102 cm$^{-1}$ for the TE$_{01}$, HE$_{21o}$, HE$_{21e}$, and TM$_{01}$ modes, respectively. For the convenience of discussion, in what follows, the $y$- and $x$-polarized HE$_{11}$ modes in the bent pipe waveguide are denoted as HE$_{11y}$ and HE$_{11x}$. Under the same bending condition, attenuation constants of the HE$_{11y}$ and HE$_{11x}$ modes are 0.0018 and 0.0022 cm$^{-1}$, respectively.

![Fig. 9. Intensity distributions and electric field vector distributions of the straight and bent THz pipe waveguides. The frequency is 500 GHz and the bending radius is 500 cm.](image)
Then, we assume that the field distribution at the bend input is a \( y \)-polarized HE\(_{11} \) mode of a straight pipe waveguide. Taking \( R = 500 \) cm as an example, with the \( y \)-polarized input, excited modal powers of the HE\(_{11x} \), HE\(_{11x} \), HE\(_{21o} \), HE\(_{21e} \), and TM\(_{01} \) modes in the bent pipe waveguide calculated according to Eq. (4) are 0.978, 0, 0.018, 0.004, 0, and 0, respectively. Clearly, excited modal powers of the HE\(_{11x} \), HE\(_{21e} \), and TM\(_{01} \) modes are all zero. It is because the \( y \)-component of the electric field is absent for the HE\(_{11x} \) mode and antisymmetric with respect to the \( x \)-axis for the HE\(_{21e} \) and TM\(_{01} \) modes. Therefore, only the HE\(_{11y} \), TE\(_{01} \), and HE\(_{21o} \) modes can be excited by the \( y \)-polarized HE\(_{11} \) mode of the straight waveguide. Figures 10(a)–10(c) plot the total power losses of the bent pipe waveguide as a function of waveguide length \( L \) for \( R = 500, 200, \) and 75 cm, respectively. Two conditions are shown for comparison. One \( (N = 1) \) considers only the HE\(_{11y} \) mode in Eq. (5), and the other \( (N = 3) \) takes the HE\(_{11y} \), TE\(_{01} \), and HE\(_{21o} \) modes into account. Note that the power loss at \( L = 0 \) represents the coupling loss occurring at the straight-to-bent interface. As shown in Fig. 10, the coupling loss of the HE\(_{11y} \) mode \( (N = 1) \) increases with decreasing bending radius. The coupling losses are 0.1, 0.5, and 2.3 dB for \( R = 500, 200, \) and 75 cm, respectively, and correspondingly, 98\%, 88\%, and 59\% of the input power are transferred into the HE\(_{11y} \) mode. Hence, when there is a straight-to-bent interface, it should be enough to describe the bending behavior of the pipe waveguide by considering only the HE\(_{11y} \) mode for \( R = 500 \) and 200 cm, but insufficient for \( R = 75 \) cm. If the two higher order TE\(_{01} \) and HE\(_{21o} \) modes are included into simulation, the total coupling losses \( (N = 3) \) reduce to 0, 0, and 0.3 dB for the three bending radii. Thus the simulation can be improved by the inclusion of the higher order modes. It is noted that the coupling loss for \( R = 75 \) cm is not zero (0.3 dB) when three modes are considered, indicating that there might be additional higher order modes excited by the missing input power. In general, more higher order modes are necessary to have a more complete analysis, especially for the case of strong bending.

4. Conclusion

In conclusion, we have investigated the bending characteristics of the antiresonant-reflection-based THz pipe waveguides. Numerical results show that the inherent periodic feature of the loss spectrum is nearly unaffected under bending. However, attenuation constant of the fundamental mode (HE\(_{11} \) mode) is polarization dependent, and the polarization perpendicular to the bending plane experiences a smaller bending loss than that of the polarization parallel to the bending plane. Moreover, when the bent pipe waveguide is operated in the perturbed mode, like the straight case, increasing the air-core diameter can significantly reduce the attenuation constant; nevertheless, if the waveguide is operated in the whispering gallery mode, increasing the core diameter would be ineffective for reduction of the attenuation constant. Finally, bending behavior of the pipe waveguide with a straight-to-bent interface is examined by using the coupled mode theory. It is shown that simulation can be improved by considering the higher order modes, especially for the case of strong bending.
Appendix

In this appendix, the ESW model [26] is used to estimate the incident angle $\theta_i$ at the outer core-cladding interface for the bent pipe waveguide. Consider the straight case first, and Fig. 11(a) depicts the longitudinal cross-section of the straight pipe waveguide. It has been shown that the incident angle $\theta_i$ for the straight pipe waveguide is given by [18]:

$$\theta_i = \sin^{-1}\left(\frac{\text{Re}(\beta)}{n_1 k_0}\right).$$

where $\beta$ is the complex propagation constant, $n_1$ is the refractive index of the core, and $k_0$ is the free-space wavenumber.

![Fig. 11. (a) Longitudinal cross-section of the straight THz pipe waveguide. (b) Transformed index profile and field distribution of the bend THz pipe waveguide.](image)

By using the technique of conformal transformation [26], the bent pipe waveguide can be represented to its equivalent straight waveguide with a modified refractive index distribution:

$$n'(x, y) = n(x, y) \exp\left(\frac{x}{R}\right) \approx n(x, y)\left(1 + \frac{x}{R}\right),$$

where $n(x, y)$ is the original refractive index profile of the bent pipe waveguide, and $n'(x, y)$ is the transformed index profile of its equivalent straight waveguide. In Eq. (7), the latter approximation holds when $x \ll R$, i.e., the distance from the waveguide center is far smaller than the bending radius. The transformed index distribution $n'$ is tilted with respect to the original one, as shown in Fig. 11(b), where the corresponding field distribution for the bent pipe waveguide is illustrated as well. The index distribution and the field profile are shown along the $x$-axis ($y = 0$). It is noted that refractive index of the core of the equivalent straight waveguide, after conformal transformation, is no longer homogeneous. This prevents the direct usage of Eq. (6) to determine the incident angle, because the core index becomes position dependent. However, since most of the energy is around the peak of the field, it seems reasonable to represent the core index with the equivalent one $n_{eq}$, which is the refractive index in the position $x_{max}$ where the field peak takes place. The equivalent index $n_{eq}$ is given by

$$n_{eq} = n_1\left(1 + \frac{x_{max}}{R}\right).$$

By replacing $n_1$ in Eq. (6) with $n_{eq}$, the incident angle $\theta_i$ at the outer core-cladding interface of the bent pipe waveguide may be estimated as
\[
\theta_i = \sin \left( \frac{\text{Re}(\beta)}{n_{eq} k_0} \right).
\]

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