A phenomenological analysis of antiproton interactions at low energies

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Abstract

We present an optical potential analysis of the $\bar{p}p$ interactions at low energies. Our optical potential is purely phenomenological, and has been parametrized on data recently obtained by the Obelix Collaboration at momenta below 180 MeV/c. It reasonably fits annihilation and elastic data below 600 MeV/c, and allows us for an evaluation of the elastic cross section and $\rho$–parameter down to zero kinetic energy. Moreover we show that the mechanism that depresses $\bar{p}$–nucleus annihilation cross sections at low energies is present in $\bar{p}p$ interactions too.

1 Introduction

Recently data on $\bar{p}$ annihilations on light nuclei (H, D and $^4$He) have become available at very small $\bar{p}$ momenta (down to 45 MeV/c)\cite{1–4}. A new data on $^{20}$Ne at 57 MeV/c is also available now\cite{5}. Together with previously available data (for a review see e.g. ref.\cite{6}), and with data on antiprotonic atoms\cite{7,8}, the full set presents some interesting features, that we will try and correlate in this work. As far as a qualitative physical understanding is concerned, the unifying feature is a mechanism that we call “inversion”, i.e. a repulsion-dominated low energy $\bar{p}p$ interaction. From a practical point of view, we will widely rely on the possibility of reproducing the available elastic and annihilation $\bar{p}p$ data below 600 MeV/c via an energy independent optical potential.

Let us initially discuss some relevant points of the phenomenology:

1) Annihilation $\bar{p}p$ data show, in a log-log plot, a series of roughly rectilinear behaviors (see fig.1). These can be approximately identified with regions where different angular momentum components are dominant, with the S-P transition at about 100 MeV/c. At 50 MeV/c it is possible to assume S-wave
dominance and estimate the imaginary part of the scattering length $\alpha$[9]. The real part, is extracted from the widths and shifts of the levels of antiprotonic Hydrogen atoms[7], together with an independent measure of $\Im(\alpha)$. Elastic $\bar{p}p$ data at values of the laboratory $\bar{p}$ momentum $k$ in the 200-500 MeV/c range were reproduced by Brückner et al[10] with a phenomenological optical potential. These authors left a wide range of uncertainty for the suggested potential parameters. We have noticed that the same potential, with a finer tuning of the parameters, can fit all the annihilation data which have been later measured at smaller $k$, down to 30 MeV/c, by the Obelix Collaboration[1,3,4] (see fig.2, and section 3 for details). It can also calculate the real and imaginary parts of the scattering length. Optical potential analysis, partial wave analysis and atomic data agree on $\Re(\alpha) \approx -\Im(\alpha) \approx 0.7\div0.8$ fm, with positive sign.

Fig. 1. Annihilation cross sections (mb) vs laboratory (left) and center of mass (right) momentum. Stars correspond to $\bar{p}p[1,3,4,11]$ circles to $\bar{p}D[2,12]$, crosses to $\bar{p}^3He[2,13,14]$ and triangles to $p^{20}Ne[15,5]$.

2) The $\rho$ parameter, i.e. the ratio between the real and imaginary part of the forward scattering amplitude, can be measured at zero or near zero energy exploiting $\rho(0) = \Re(\alpha)/\Im(\alpha)$, which means $\rho \approx -1$. At larger energy, it must be extracted by a very delicate (and partly model dependent) analysis of the elastic $\bar{p}p$ angular distributions. Despite the behavior of $\rho$ is still unclear in the region 100-200 MeV/c[16], an overview of the experimental data[17–21,7] suggest that $\rho$ is small but positive (0.1–0.3) at projectile momenta over some value which lies somewhere around 500 MeV/c, smaller (with uncertain sign) in the region 180-500 MeV/c, and tends to some negative value $\sim -1$ at zero energy. As better described in the following, we have applied the optical potential (whose parameters have been fine-tuned on the $\bar{p}p$ annihilation data at 30-100 MeV/c), to predict the $\rho$ behavior. The results agree with the large and the zero energy data, and suggest that $\rho$ varies monotonously in the less
known intermediate momentum region (see fig.3).

3) In the laboratory frame $\bar{p}$ total annihilation cross sections (TPA from now on) on Deuteron and $^4$He are almost equal, and both are smaller than TPA on Hydrogen. The $^{20}$Ne datum is larger, but not so large as one could expect[5]. See fig.1 for a general view of the data. Taking into account that a large enhance of reaction cross sections is predicted at low energies because of charge effects[22], this phenomenon is surprising. According with the notations used in previous works[23,24] we will call this behavior “inversion”. Actually, if the data are represented in the center of mass frame, TPA on D and $^4$He are slightly larger than TPA on Hydrogen, however the dependence of the TPA on the mass number is still much smaller than any geometrical expectance (see later for a discussion of the “geometrical expectance” and of the role of the center of mass). For $k >> 100$ MeV/c this phenomenon is not observed and the ratio between $\bar{p}p$ and $\bar{p}^4$He annihilation rates is qualitatively what one would expect. The inversion behavior is confirmed by an analysis of antiprotonic atoms[8], where it is found that $|Im(\alpha)|$ is smaller in antiprotonic Deuterium than in antiprotonic Hydrogen.

4) From an overview of the available $\bar{p} - nucleus$[6] and $\bar{n} - nucleus$[25] annihilation data below 600 MeV/c it appears that: (i) Where many partial waves dominate the $\bar{p}$–nucleus interaction the cross sections relative to different nuclear species are parallel, and agree with a law $\sigma \propto \sigma_o A^{2/3}$. (ii) Where only a few partial waves are supposed to dominate, a convergency (for decreasing energies) between the different TPA is clearly visible. In a log-log plot, the extrapolations of the different TPA seem to aim at some common intersection point somewhere at $k_{cm} \sim 1$ MeV/c (see fig.1).

2 General theoretical background.

To better understand the significance of the previous nuclear data some considerations are useful. Both the “inversion” and the convergency behavior contradict the geometrical predictions. Assuming that the imaginary part of the scattering length is roughly equal to the nuclear size $R \approx 1.3A^{1/3}$ fm, and exploiting the traditional estimation of the Coulomb focusing effect[22,26], one has TPA $\sim ZA^{1/3}/k^2$ at very small momenta. At larger momenta the semiclassical expectance is TPA $\propto A^{2/3}$ (well verified for $\bar{p}$[6] and $\bar{n}$[25] annihilations at any $k_{lab} > 180$ MeV/c). Since for most nuclei $ZA^{1/3} \approx 0.5A^{4/3}$, one should naively expect that TPA on different nuclei increase their separation when momenta decrease below 100 MeV/c, while exactly the opposite is seen. In addition, at any precise lab or c.m. momentum below $k_{lab} = 100$ MeV/c, the $A$–dependence of the known TPA is below both the $A^{2/3}$ and the $ZA^{1/3}$ prediction.
Regarding the question whether the TPA on different nuclei must be compared at the same laboratory or center of mass momenta, the answer is model-dependent. In Impulse Approximation inspired models, the annihilation process only involves one of the nucleons in the target nucleus, which has average momentum equal to zero in the laboratory. It is then reasonable to compare data at the same laboratory momentum. In compound-nucleus inspired models, the collision process directly transfers momentum from the projectile to the full target. In this case, data taken on different targets should be compared at the same c.m. momentum.

![Optical potential fits to \( \bar{p}p \) annihilation data. The continuous line fitting the low energy points corresponds to the potential described in the text. The upper continuous line corresponds to the same potential modified by decreasing the imaginary strength from 8000 to 1000 MeV. Dotted lines show the S- and P-wave contributions for the former potential, dashed lines show the same for the latter. Data are taken from Brückner et al\[11\], and from the Obelix collaboration\[1,3,4\].](image)

The key point is the generalization of the concept of low energy “inversion”. On the ground of general quantum principles it is possible to demonstrate\[27,23\] that, in presence of a very effective exothermic hadronic reaction mechanism and in conditions of S-wave dominance: (i) the reaction cross section must
stay much below geometrical expectations, and is largely independent on the target nucleus size; (ii) most attempts to increase those model parameters which supposedly should enhance the annihilation rate (e.g. strength or radius of a potential) lead to the opposite or to no result; (iii) a strong non-diffractive elastic scattering accompanies the reaction at low energies, and this scattering has repulsive character (i.e. $\text{Re}(\alpha) > 0$). So, with “inversion” we will refer to the presence of these three features. We have previously demonstrated[23] that strong inversion must be expected whenever disappearance of the projectile S-wave wavefunction $\Psi_S$ (at the nuclear surface) is produced within a range much smaller than $\Delta r \approx 1$ fm. Then, regularity conditions on $\Psi_S$ at the nuclear surface necessarily produce a large flux reflection and a $\Psi_S$ which is similar to the one produced by a repulsive potential with little absorption. For this reason it is not proper to consider the scattered flux as “diffractive”, although it is a by-product of absorption. It is a refractive process, as in elastic potential scattering. Now we can better specify the above required condition of “very effective reaction mechanism” (since at low energies it is not so effective): It means that (i) the reaction is esothermic, (ii) it produces large reaction rates at large energies, (iii) at any energy its free mean path in nuclear matter can be estimated to be shorter than 1 fm. We remark that the described behavior is experimentally confirmed by the fact that for the $\bar{p}p$ scattering length $\alpha$ we have $\text{Re}(\alpha) \approx -\text{Im}(\alpha) > 0$, or equivalently $\rho(0) \approx -1$. And by the fact that $\bar{p}$ annihilation rates on nuclei are not that large.

Also the traditional view of the Coulomb focusing effect must be reconsidered. In a previous paper[24] we have already calculated and compared “charged” and “uncharged” annihilation rates on nuclei with finite size, and demonstrated that the traditional $Z/\beta$ Coulomb enhancement factor[22] is exaggerated. This factor is estimated with the two assumptions: (i) pointlike target (ii) completely independent action of Coulomb and strong forces. On the contrary, on one side the interplay between Coulomb and strong forces is not negligible, and on the other side finite size effects largely neutralize the Coulomb enhancement factor for intermediate and heavy nuclear targets. E.g., speaking in terms of target effective charge $Z_e$, we have $Z_e(^{4}\text{He})/Z_e(\text{H}) \approx 1$ (instead of 2), $Z_e(^{20}\text{Ne})/Z_e(^{4}\text{He}) \approx 2$ (instead of 5; comparisons are performed at the same laboratory momentum, but center of mass effects were included in the calculation[24]).

3 Optical potential fits on $\bar{p}p$ data

As previously anticipated, all the data on $\bar{p}p$ elastic and annihilation cross section below 600 MeV/c can be reasonably well fitted by the same potential,
with Woods-Saxon shape, used by Brückner *et al.*[10] to fit elastic $\bar{p}p$ data at 181, 287 and 505 MeV/c, after a finer tuning of the parameters. We have set the real and imaginary strength to -46 and -8000 MeV, the real and imaginary radius to 1.89 and 0.41 fm, and the diffuseness to the common value 0.2 fm. The fit on the annihilations is very good below 300 MeV/c and good within 10% at 600 MeV/c (the exact precision over 300 MeV/c depends on which set of data is chosen[6]), and the elastic distributions are still well reproduced. The total potential includes the Coulomb potential of a spherical charge distribution with radius 1.25 fm. In all the calculations center of mass corrections have been included. Together with the outcome of the above potential, in fig.2 we also show a curve corresponding to imaginary strength 1000 MeV. For both cases (strength 8000 and 1000 MeV) we also show the S- and P-wave contributions. Evidently the used potential does produce “inversion”, i.e. a larger annihilation potential produces a smaller annihilation rate. From the same figure it is obvious that this behavior is associated with the S-wave dominance, and is present only below an “inversion point” $k_{inv}$. In this case $k_{inv} \approx 200$ MeV/c.

In fig.3 we show the value of $\rho$ in the momentum range 0-600 MeV/c calculated with this potential. The change of sign of $\rho$ can be related with the transition from the dominance of the reaction-associated repulsion to the dominance of the direct potential attraction, at least in forward scattering. Indeed, at increasing momenta the Born approximation becomes progressively more reliable, and it permits to estimate $\rho \sim (V_R R_R^3)/(V_I R_I^3) \sim +0.2$, using as an effective radius the sum of the potential radius and diffuseness. The positive $\rho$ value at large momenta is thus directly due to the presence of a real attracting part in the potential. We notice that the “source” of the “direct” attraction will be the region where absolute value of the elastic potential is roughly equal to the kinetic energy, while the “source” of the reaction-induced repulsion will be the region where most annihilations take place, i.e. 0.5÷1 fm out of the edge of the annihilation core. This distance has been estimated in past years in analysis of both $\bar{p}p$[10] and $\bar{p}$–nucleus[28] interactions.

In fig.4 the total annihilation and elastic cross sections are reported, compared with the corresponding cross sections calculated after turning off the electric charge. In the former case the contribution of the pure Coulomb forward peak and of the Coulomb-strong interference is excluded. Nevertheless, the elastic strong cross section is largely affected by Coulomb focusing effects. In particular, the figure shows that the ratio between the strong elastic and the annihilation total cross sections is completely dominated by the Coulomb effects. Without them, $\sigma_{el}/\sigma_a \to 0$ for $k \to 0$. With inclusion of the charge effect, approximately $\sigma_{el}/\sigma_a \to 1/6$. We have also calculated angular distributions at momenta between 25 and 100 MeV/c, but they are practically flat up to 50 MeV/c, and at 100 MeV/c present a 20% change between forward and backward scattering, so they are not very interesting. At 100 MeV/c the
Fig. 3. Continuous line: The optical potential prediction for $\rho$. Scattering data up to 650 MeV/c come from M.Cresti et al (1983)[18], H.Iwasaki et al (1985)[19], V.Ashford et al (1985)[20] W.Brückner et al (1985)[17] L.Linssen et al (1987)[21]. For the atomic data see e.g. C.J.Batty et al[7] and references therein, and also references contained in [8].

P-wave contributions are 1% in the total strong elastic cross section, and 10% in the annihilation. We remark that at such small momenta the Rutherford "forward" peak, which spreads at angles $\theta \propto 1/k$, becomes the most important source of elastic scattering at large angles too.

4 Annihilations on nuclei.

Up to now we did not succeed in fitting light nuclei data perfectly by energy-independent optical potentials (which take nuclear density distributions into account). In fact, at momenta below 100 MeV/c a certain energy dependence is introduced by the nontrivial energy dependence of the $\bar{p}n$ annihilation rate[29,30]). The study of the nuclear optical potential requires taking into
account nuclear structure details and $\bar{p}n$ interactions, so a more specific and longer work will be devoted to it in the next future. Qualitatively, it is evident that the energy dependence of the cross sections in the range 30-200 MeV/c is much slower in $\bar{p}$--nucleus than in $\bar{p}p$. This can be related to the change of sign of $\rho$ in $\bar{p}p$ interactions observing that if the $\bar{p}$--nucleon interaction is repulsive below a certain momentum of scale $\sim 100$ MeV/c, in a cluster of nucleons each single nucleon will contribute keeping the projectile far from itself and from all the other ones. In the language of the multiple scattering expansion this is an interference between single and double scattering processes, i.e. elastic scattering of $\bar{p}$ on one nucleon prevents annihilation on another one. This interpretations would confirm the suggestion given by Wycech et al in their analysis of antiprotonic deuterium[31]: they estimate single and double scattering amplitudes contributing to the $\bar{p}D$ interaction, and observe that the

Fig. 4. Continuous line: Total $\bar{p}p$ annihilation cross section calculated as in fig.2. Dashed line: Total strong elastic cross section (the Coulomb forward singularity is excluded). Dotted and dot/dashed line: Total annihilation and elastic cross sections calculated again after assuming zero charge for the projectile. At larger momenta the charge does not create large differences. At lower momenta $\sigma_{\text{ann}}^{\text{ch}} \sim 1/\beta^2$, $\sigma_{\text{el}}^{\text{ch}} \sim 1/\beta^2$, $\sigma_{\text{ann}}^{\text{neu}} \sim 1/\beta$, $\sigma_{\text{el}}^{\text{neu}} \sim \text{constant.}$
interference between them decreases the single scattering output. At the same time our calculations (still in progress) show that, in the case of light nuclei, nuclear structure details and $\bar{p}n$ features do affect the results.

5 Conclusions.

We have shown that the Obelix Collaboration data on $\bar{p}p$ annihilation in the range 30 to 180 MeV/c allow us for a finer tuning of the parameters of an optical potential, which was previously used by other authors to interpolate elastic differential cross sections at $k_{lab}$ 181, 287 and 505 MeV/c. Without the need of introducing any energy dependence of these parameters, the so-obtained potential can reproduce all the $\bar{p}p$ annihilation data between 30 and 600 MeV/c, the zero-energy value of the $\rho$ parameter together with its general trend at increasing energies, and the measured values of the scattering length (real and imaginary part) with correct sign. We have also used this potential to predict elastic cross sections and $\rho$ values in those regions where data are not available yet. We have also shown that the behavior of all the considered observables is largely affected by a mechanism that we call “inversion”: in presence of a very strong reaction mechanism the reaction cross sections become anomalously small at very low energies, while elastic interactions reverse from attractive to repulsive. We can’t make precise predictions for the $\bar{p}$–nucleus cross sections yet, but we stress that their smallness is closely related with the low-energy repulsive behavior of the $\bar{p}p$ interaction.

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