DISTANCES AND LENSING IN COSMOLOGICAL VOID MODELS

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ABSTRACT

We study the distances and gravitational lensing in spherically symmetric inhomogeneous cosmological models consisting of inner and outer homogeneous regions that are connected by a single shell or double shells at redshift \( z_1 \sim 0.067 \). The density and Hubble parameters in the inner region are assumed to be smaller and larger, respectively, than those in the outer region. It is found that at the stage \( z_1 < z < 1.5 \) the distances from an observer in the inner voidlike region are larger than the counterparts (with equal \( z \)) in the corresponding homogeneous Friedmann models, and hence the magnitudes for the sources at this stage are larger. This effect of the voidlike low-density region may explain the deviations of the observed (magnitude-redshift) relation of supernovae (SNe) Ia from the relation in homogeneous models, independently of the cosmological constant. When the position of the observer deviates from the center, moreover, it is shown that the distances are anisotropic, and the images of remote sources are systematically deformed. The above relation at \( z \gtrsim 1.0 \), and this anisotropy will observationally distinguish the role of the above voidlike region from that of the positive cosmological constant. The influence on the time delay measurement is also discussed.

Subject headings: cosmology: observations — cosmology: theory — large-scale structure of universe

1. INTRODUCTION

In recent cosmological observations, the following three remarkable phenomena have been discovered, which may contradict the homogeneity of the universe. One of them is the large-scale bulk flows in the region at distance \(< 150 \, h^{-1} \, \text{Mpc} \) (\( H_0 = 100 \, h \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1} \)) without the associated large cosmic microwave background (CMB) dipole anisotropy (Hudson et al. 1999; Willick 1999). Second we have the inhomogeneity of the observed Hubble constant, whose values are smaller in the measurements for remoter sources (Branch 1998; Freedman 1997; Sandage & Tammann 1997; Blandford & Kundic 1997). The last one is the magnitude-redshift relation of supernovae (SNe) Ia, in which the observed magnitudes of sources are larger than those expected in the homogeneous Friedmann models without the cosmological constant. For the model fitting the positive cosmological constant that brings an “accelerating” universe was considered a necessary quantity (Riess et al. 1998; Schmidt et al. 1998; Garnavich et al. 1998).

For the explanation of the first phenomenon we considered spherically symmetric inhomogeneous models in a previous paper (Tomita 2000, hereafter Paper I). They consist of inner and outer homogeneous regions connected with a single shell, double shells, or an intermediate self-similar region (Tomita 1995, 1996), and it is assumed that the density and Hubble parameters in the inner region are smaller and larger, respectively, than those in the outer region. Then it was shown that the observed situation of bulk flows and CMB dipole anisotropy can be reproduced if the radii of the boundary of the two regions and the observer’s position from the center are about \( 200 \, h^{-1} \, \text{Mpc} \) and \( 40 \, h^{-1} \, \text{Mpc} \), respectively. The consistency with the observed bulk flows (Hudson et al. 1999; Willick 1999; Dale, Giovanelli, & Haynes 1999; Giovanelli et al. 1998) was shown in Paper I. These models are also found to be consistent with the observed inhomogeneity of the Hubble constant.

The inhomogeneity of the Hubble constant has already been discussed by various workers (e.g., Turner, Cen, & Ostriker 1992; Bartlett et al. 1995). The local void region with higher Hubble constant was studied independently by Zehavi et al. (1998) as the local Hubble bubble, which has the scale \( \sim 70 \, h^{-1} \, \text{Mpc} \) and is bordered by the dense walls such as the Great Attractor. They analyzed the statistical relation between the distances and the local Hubble constants derived from SN Ia data and found the existence of a void region with a local Hubble constant larger than the global Hubble constant. The relation to SN Ia data on the scale \( \sim 150 \, h^{-1} \, \text{Mpc} \) will be discussed in this paper from our standpoint.

In the present paper the behavior of the distances is investigated in the models in the previous paper with similar model parameters. In § 2, we treat the distances from a virtual observer who is in the center of the inner voidlike region in models with a single shell and derive the magnitude (m)–redshift (z) relation. This relation is compared with its counterpart in the homogeneous models. Then the relation in the present models is found to deviate from that in the homogeneous models with \( \Lambda = 0 \) at the stage of \( z < 1.5 \). It is partially similar to that in the nonzero-\( \Lambda \) homogeneous models, but the remarkable difference appears at the high-redshift stage \( z > 1.0 \). In § 3, we consider a realistic observer who is in a position deviated from the center and calculate the distances from him. The distances depend on the direction of incident light, and the area angular diameter distance is different from the linear angular diameter distances. It is shown as the result that the m–z relation is anisotropic, but the relation averaged with respect to the angle is very near to the relation by the virtual observer. Comparison with the observed relation for SNe Ia is also discussed. In § 4 we derive the lens effects, such as the convergence and shear of the images caused by the above anisotropic nature of distances, and in § 5 we discuss the influence on the time delay from a remote double quasar. Section 6 is dedicated to concluding remarks. In the Appendix, the derivation of
2. DISTANCES FROM THE CENTER OF THE INNER REGION

In this section we treat the models with a single shell (Fig. 1), in which the inner homogeneous region $V^I$ and the outer homogeneous region $V^{II}$ are connected with a shell and the line element is expressed as

$$ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu}$$

$$= -c^2(dt)^2 + \left[a(t')\right]^2(d\chi')^2 + \left[f(x')\right]^2d\Omega^2,$$  
(1)

where $j (= 1$ or $II)$ represents the regions $f(x') = \sin x', x'$, and $\sin x'$ for $k = 1, 0, -1$, respectively, and $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$. The model parameters are expressed as $H_0 (= 100h_0)$, $\Omega_0^I$, and $\chi'_0$. Here negative curvature is assumed in all regions. In Paper I, we showed the Einstein equations in both regions and the junction conditions at the boundary shell, which is given as $x' = \chi'_0$ and $d^2x'/dx'^2$, and derived the equations of light rays in both regions.

Using the latter equations we obtained CMB dipole and quadrupole anisotropies and found that the influence of motions of the shell on the light paths is negligibly small, so that the approximation of the comoving shell is good for the treatment of light rays. In this paper this approximation is assumed throughout in all cases, and equations in Paper I are cited as equation (1.1), equation (1.1A), and so on.

The angular diameter distance $d_A$ between a virtual observer at the center $O$ (in the inner region $V^I$) and the sources $S$ is given by

$$d_A = a(\tilde{\eta}^I_S) \sinh (\tilde{\chi}^I_S)$$

(2)

if $S$ is in $V^I$, where $(\tilde{\eta}^I_S, \tilde{\chi}^I_S)$ are the coordinates of $S$. Here and in the following, bars are used for the coordinates along the light paths to the virtual observer, as in Paper I. If $S$ is in $V^{II}$, we have

$$d_A = a(\tilde{\eta}^{II}_S) \sinh (\tilde{\chi}^{II}_S) - a(\tilde{\eta}^I_S) \sinh (\tilde{\chi}^I_S)$$

$$= a(\tilde{\eta}^{II}_S) \sinh (\tilde{\chi}^{II}_S),$$

(3)

where equation (1.2) for $R$ was used. When $S$ is in $V^I$, $(\tilde{\eta}^I_S, \tilde{\chi}^I_S)$ are related to $(\tilde{\eta}^I_O, 0)$ by

$$\tilde{\eta}^I_S - \tilde{\eta}^I_O = \tilde{\chi}^I_S,$$

(4)

where $\tilde{\eta}^I_O$ is given by equations (1.13) and (1.14) with $y' = 1$, and $\tilde{\chi}^I_S$ is related to the source redshift $z^I_S$ by

$$1 + z^I_S = 2(1 - \Omega^I_0)\Omega^I_0/(\cosh \tilde{\eta}^I_O - 1).$$

(5)

For a given $z^I_S$, we obtain $\tilde{\eta}^I_S$ and $\tilde{\chi}^I_S$ and hence $d_A$ from equation (2).

When $S$ is in $V^{II}$, we have

$$\tilde{\eta}^{II}_S - \tilde{\eta}^{II}_O = \tilde{\chi}^{II}_S,$$

(6)

and

$$\tilde{\eta}^{II}_O - \tilde{\chi}^{II}_S = \tilde{\chi}^{II}_S,$$

(7)

where $\tilde{\eta}^{II}_S$ is related to $z^I_S$ by

$$1 + z^I_S = 2(1 - \Omega^I_0)\Omega^I_0/(\cosh \tilde{\eta}^{II}_O - 1),$$

(8)

and so $\tilde{\chi}^{II}_S$ is also related to $z^I_S$ using equation (6). Coordinates at the shell, $(\tilde{\eta}^I_S, \tilde{\chi}^I_S)$ and $(\tilde{\eta}^{II}_S, \tilde{\chi}^{II}_S)$, are connected by equations (1.53) and (1.54). On the other hand, $\tilde{\eta}^{II}_S$ is related to $z^{II}_S$ by

$$1 + z^{II}_S = \cosh \tilde{\eta}^{II}_S - 1,$$

(9)

where $z^{II}_S$ is equal to $z^I_S$ under the approximation of the comoving shell (cf. eq. [1.6]). Accordingly, $d_A$ is uniquely determined for given $z^I_S (= z^{II}_S)$ and $z^{II}_S$.

The distance ($d^I_A$) in a homogeneous Friedman model is

$$d^I_A = a(\tilde{\eta}^I_S) \sinh \tilde{\chi}^I_S,$$

(10)

which is defined in $V^I$ for arbitrary $z^I_S$, assuming that $V^I$ covers every region of the model.

The luminosity distances $d_L$ and $d_L'$ are defined in terms of $d_A$ and $d_A'$ as $d_L = (1 + z)^2d_A$ and $d_L' = (1 + z)^2d_A'$. Accordingly the ratio of luminosity distances is equal to the ratio of angular diameter distances. In Figures 2 and 3 the behavior of 5 log $d_L$ as a function of $z (= z^I)$ is shown in the case of $z^I = 0.067$ for the parameters $(\Omega_0^I, \Omega^I_0, \Omega^I_0, \Omega^I_0, h^I/\Omega^I_0 = (0.2, 0.56, 0.7, 0.82)$ and $(0.2, 0.88, 0.7, 0.82)$, respectively, which are appropriate to describe the bulk flow in Paper I. The lines in the homogeneous models with $(\Omega_0^I, \lambda_0^I) = (0.2, 0)$ and $(0.2, 0.8)$ are also shown for comparison. It is found that 5 log $d_L$ is larger than that in the model with $(0.2, 0)$ for $z < 1.5$, 0.8 for $0.1 < z < 0.3$, and it is intermediate for $0.1 < z < 1.0$ between the two models with $(0.2, 0)$ and $(0.2, 0.8)$. The difference between the present shell model and the nonzero $\lambda_0$ model is remarkable for $z > 1.0$. According to recent data for high-redshift SNe Ia (Riess et al. 1998; Schmidt et al. 1988; Garnavich et al. 1998), it is at epochs of $z \sim 0.4$ that their data on $\chi$ deviate conspicuously from the values in the homogeneous models with varying cosmological constant. From the comparison with their data, it seems that the present models can explain their data as well as the nonzero $\lambda_0$ model.

In Figure 4 the behavior of 5 log $d_L$ (by the observer at C) in the double-shell models is shown for $z^I = 0.05$ and $z^I = 0.1$, where the distances in the double-shell case are derived in the Appendix. It is found that the general trend is the same as that in the single-shell case.

In Figures 5 and 6, the magnitude differences $\Delta m$ [\(= 5 \log d_L - 5 \log (d_L)/\text{homog}_C\)] are shown in the single-shell
3. Distances from a Noncentral Observer O in the Inner Region

Next let us derive the angular diameter distance $d_A$ from an observer $O$ whose position is deviated from the center. When the source $S$ is in $V^1$, $d_A$ is equal to $d_A^0$ in the homogeneous Friedmann model. In the following we consider the case where $S$ is in the outer regions. In this case we have two linear angular diameter distances (the longitudinal linear distance $d_{A_L}$ and the transverse linear distance $d_{A_T}$) for the angles in the $\phi$ and $\theta$ directions, respectively, and the area angular diameter distance $d_{A_A} = [\sqrt{(d_{A_L}^2 + d_{A_T}^2)}]^{1/2}$ (cf. Fig. 7).

Let us consider a single-shell model and assume that, in the plane $\theta = \text{const}$, two light rays start from $S$ at $(\eta^0_S, \chi_S, \phi_S)$ and $(\eta^0_S, \chi_S, \phi_S + \Delta \phi_S)$ and reach $O$ at $(\eta^0_O, \chi_O, 0)$ with the angles $\phi$ and $\phi + \Delta \phi$, respectively. If the angular diameter distance from $S$ to the center $C$ is $d_A(\eta^0_S, \chi_S)$, the proper interval of the two rays in $S$ is equal to $d_A(\eta^0_S, \chi_S)d\phi_S$, so that the longitudinal linear distance is defined by

$$d_{A_L} = d_A(\eta^0_S, \chi_S) \frac{\partial\phi_S}{d\phi} [\cos (\phi - \phi_S)] ,$$

(11)

where $d_A(\eta^0_S, \chi_S)$ is given by

$$d_A(\eta^0_S, \chi_S) = d^0(\eta^0_S) \sinh (\chi_S) .$$

(12)
Next let us consider two rays with equal $\phi$ in planes of $\theta = 0$ and $\Delta \theta$, when $\Delta \theta \ll \pi$. Then the angle between two rays reaching $O (= \Delta \theta \sin \phi_0)$ is equal to the angle between two rays reaching $C$ multiplied by a factor $\sin \phi_0 / \sin \phi$. Therefore the transverse linear distance is

$$d_A = d_A(\eta^h, \chi^h) \sin \phi_0 / \sin \phi \, .$$

In the following we derive the relations between $(\eta^h, \chi^h, \phi_0)$ and the incident angle $\phi$. When we give the redshift $z^h_0$, the radial coordinate $\chi^h_1$ is fixed using equations (6) and (8).

In $V$ we have equation (I.40) for $\eta^I_1$ in the case of $\phi = \phi_1$ and $\pi - \phi_1$, and

$$h^I_0 = [1 + (z^I_0)^2]^{1/2}, \quad r^I = a_0 \chi^I_1 \sin \phi \, ,$$

where $\chi^I_0$ is fixed by giving the distance CD, that is,

$$l_0 = a_0 \chi^I_1 \, .$$

(14)

For $(\eta^h, \chi^h)$ we obtain from equation (I.41)

$$G(\chi^h) \equiv \cosh^{-1} \left( \frac{\cosh \chi^h}{h^h_0} \right) - \cosh^{-1} \left( \frac{\cosh \chi^h}{h^h_1} \right) = \eta^h - \eta^h_0 \, .$$

(16)

The definition (I.37) of $h^h_0$ and the junction conditions (I.46) and (I.47) lead to

$$h^h_0 = [1 + (\zeta^h)^2]^{1/2}, \quad \zeta^h = \frac{d_0}{d_0^h} \zeta^I \, ,$$

(17)

where $A_0$ ($j = I, II$) are given by equation (I.12). The coordinates at the boundary $(\eta^I_1, \chi^I_1)$ in equation (16) are related to $(\eta^I_1, \chi^I_1)$, using

$$a_0 \gamma(\eta^I_1) \sinh \chi^I_1 = a_0^I \gamma(\eta^I_0) \sinh \chi^I_0$$

(18)

and

$$a_0^I \int_0^{\eta^I_1} \gamma(\eta) \, d\eta = a_0^I \int_0^{\eta^I_0} \gamma(\eta) \, d\eta \, ,$$

(19)

which are derived from equation (I.2) for $R$ and equation (I.6) with $\gamma^I = \gamma^I$. Therefore, $\eta^h$ and $\chi^h$ are determined by specifying the values of $z^h_0, z^h_1$, and $\phi_1$.

Next let us derive $\phi$ by integrating the ray equations

$$\frac{d\phi}{d\chi^I} = \frac{(k^\gamma)^J}{(k^\gamma)^I} \, ,$$

(20)

where $(k^\gamma)^J$ and $(k^\gamma)^I$ for $j = I$ and $II$ are shown in equations (I.32) and (I.33). The solution of equation (20)
satisfying the conditions that $\varphi(\chi = \chi_0) = 0$ and $\varphi(\chi > 0) \to \phi$ for $\chi_0 \to 0$ is

\[
\varphi = \tan^{-1} \left\{ \frac{1}{\zeta} \left[ \sinh^2 \chi' + \cosh \chi' \sqrt{\sinh^2 \chi' - (\zeta')^2} \right] \right\} - \tan^{-1} \left\{ \frac{1}{\zeta} \left[ \sinh^2 \chi'_0 + \cosh \chi'_0 \sqrt{\sinh^2 \chi'_0 - (\zeta')^2} \right] \right\}.
\]

(21)

and

\[
\varphi = \tan^{-1} \left\{ \frac{1}{\zeta} \left[ \sinh^2 \chi' - \cosh \chi' \sqrt{\sinh^2 \chi' - (\zeta')^2} \right] \right\} - \tan^{-1} \left\{ \frac{1}{\zeta} \left[ \sinh^2 \chi'_0 - \cosh \chi'_0 \sqrt{\sinh^2 \chi'_0 - (\zeta')^2} \right] \right\}.
\]

(22)

Solutions (21) and (22) are applicable for $k' > 0$ and $< 0$, respectively.

In $V^i$ we have for $\phi = \phi_1$

\[
\varphi = \tan^{-1} \left\{ \frac{1}{\zeta} \left[ \sinh^2 \chi' + \cosh \chi' \sqrt{\sinh^2 \chi' - (\zeta')^2} \right] \right\},
\]

(23)

and

\[
\varphi_1 = \varphi(\chi' = \chi'_1).
\]

(24)

For $\phi = \pi - \phi_1$

\[
\varphi = \tan^{-1} \left\{ \frac{1}{\zeta} \left[ \sinh^2 \chi' - \cosh \chi' \sqrt{\sinh^2 \chi' - (\zeta')^2} \right] \right\} - \tan^{-1} \left\{ \frac{1}{\zeta} \left[ \sinh^2 \chi'_0 - \cosh \chi'_0 \sqrt{\sinh^2 \chi'_0 - (\zeta')^2} \right] \right\},
\]

(25)

\[
\varphi_m = \varphi(\chi' = h\theta).
\]

(26)

in the interval $\chi_0 < \chi \leq \chi_m$, and

\[
\varphi = \varphi_m + \tan^{-1} \left\{ \frac{1}{\zeta} \left[ \sinh^2 \chi' + \cosh \chi' \sqrt{\sinh^2 \chi' - (\zeta')^2} \right] \right\} - \tan^{-1} \left\{ \frac{1}{\zeta} \left[ \sinh^2 \chi'_m + \cosh \chi'_m \sqrt{\sinh^2 \chi'_m - (\zeta')^2} \right] \right\}.
\]

(27)

\[
\varphi_1 = \varphi(\chi' = \chi'_1).
\]

(28)

in the interval $\chi_m < \chi \leq \chi_1$.

In $V^i$ we have

\[
\varphi = \varphi_1 + \tan^{-1} \left\{ \frac{1}{\zeta} \left[ \sinh^2 \chi'' + \cosh \chi'' \sqrt{\sinh^2 \chi'' - (\zeta'')^2} \right] \right\} - \tan^{-1} \left\{ \frac{1}{\zeta} \left[ \sinh^2 \chi''_0 + \cosh \chi''_0 \sqrt{\sinh^2 \chi''_0 - (\zeta'')^2} \right] \right\},
\]

(29)

\[
\varphi_s = \varphi(\chi'' = \chi''_s).
\]

(30)

Thus $\varphi_s$ was derived as a function of $z_s$, $\chi''_s$, and $\phi (= \phi_1$ or $\pi - \phi_1$). Values of $d_a$, $d_s$, and $d_s'$ depend on the angle $\phi$. Here we calculate the average value of $d_s'$ defined by

\[
\langle d_s' \rangle = \frac{1}{2} \int_0^{\pi/2} d_s' \sin \phi \, d\phi.
\]

(31)

The $z$-dependence of average values of the corresponding luminosity distance $d_l' \equiv (1 + z)^2 d_s'$ is shown in Figures 2 and 3 in comparison with the distance from the observer at C. It is found that, throughout almost the entire range of $z$, two lines indicating these two distances are overlapped and cannot be distinguished. Therefore, we can use the distances from the observer at C to approximate the average distances from the observer at O.

4. LENSING DUE TO THE INHOMOGENEITY

When the observer’s position (O) deviates from the center (C) of the inner region, the area distance from the observer is anisotropic and the two linear distances are not equal. This is a lens effect caused by the anisotropic and inhomogeneous matter distribution around the observer (O). Here we discuss the $\phi$ dependence of the area distance and the ratio of the two linear distances $d_a'/d_s'$.

4.1. Area Angular Diameter Distance

The angular diameter distance is largest and smallest in the directions of $\phi = 0$ and $\pi$, respectively. This is because light in the directions of $\phi = 0$ and $\pi$ spends the longest and shortest time in $V^i$, respectively. The behavior of $m$ is also similar. Here we consider $m$ for the distance and treat the magnitude difference $\Delta m = m - m_{\text{homog}}$, where $m_{\text{homog}}$ is the magnitude in the Friedmann model with the same $\Omega_0$. The $z$-dependence of the magnitude difference $\Delta m$ for the observer O was derived in the single-shell models. In Figures 8 and 9, the values averaged for $\phi < \pi/4$, $\pi/4 < \phi < 3\pi/4$, and $\phi > 3\pi/4$ are shown. The difference between $\Delta m(\phi < \pi/4)$ and $\Delta m(\phi > 3\pi/4)$ reaches $\sim 0.4$ mag at the epoch $z \sim 0.1$. This difference may represent large dispersions in $m$ around this epoch. To confirm observationally whether this anisotropy in $\Delta m$ actually exists is important to clarify the validity of the present models.
4.2. Ellipticity of Image Deformation

Since $d_A$ and $d'_A$ are different for the sources of $z > z_1^1$, their images are deformed and the degrees of deformation depend on $\phi$. Here we define the ellipticity $\varepsilon$ by

$$\frac{1 + \varepsilon}{1 - \varepsilon} = \frac{d_A}{d'_A} \quad \text{or} \quad \varepsilon = \frac{d_A - d'_A}{d_A + d'_A}. \tag{32}$$

The $z$-dependence of $\varepsilon$ was calculated in two model parameters for the study of its general behavior. The maximum and minimum of $\varepsilon$ are in $\phi \sim \pi/2$ and $\phi = 0, \pi$, respectively. To clarify the $\phi$ dependence of $\varepsilon$, we derived the average value for $0 < \phi < \pi$ and the values averaged for $\phi < \pi/4, \pi/4 < \phi < 3\pi/4$, and $\phi > 3\pi/4$. The results are shown in Figures 10 and 11. It is found that the ellipticity $\varepsilon$ increases abruptly directly after the epoch $z = z_1^1$ and decreases gradually with $z$. Accordingly, we should measure the images of the sources around $z = z_1^1$ to confirm the lens effect.

5. TIME DELAY FOR A REMOTE DOUBLE QUASAR

The time delay for a remote double source is basically caused by the geometrical length difference and gravitational redshift around the lens object, so the formula in the present situation is the same as that in the homogeneous models. It is expressed (Sasaki 1993; Blandford & Narayan 1986; Schneider, Ehlers, & Falco 1992) as

$$\Delta t = \Delta t_{\text{geom}} + \Delta t_{\text{grav}}, \tag{33}$$

where

$$\Delta t_{\text{geom}} = \mathcal{D} C_{\text{geom}}, \quad \mathcal{D} = \frac{D_L D_S}{D_{LS}}, \quad C_{\text{geom}} = \frac{1}{2} (1 + z_L)(\theta - \phi)^2, \tag{34}$$

$$\Delta t_{\text{grav}} = (D_L)^2 C_{\text{grav}}, \tag{35}$$

$$C_{\text{grav}} = -\frac{1}{\pi} (1 + z_L) \int d\theta' \Sigma(\theta') \ln(1 + z_L) (\theta - \theta'/\theta_+).$$

Angular diameter distances $D_L$, $D_S$, and $D_{LS}$ denote the distances between an observer O and a lens L, between O and a source S, and between L and S, respectively. The angle vector $\theta$ indicates the position of the ray relative to L in the lens plane, $\theta_+$ denotes the critical angle of the lens object, and $\Sigma(\theta)$ is the surface density. Another angle vector $\phi$ indicates the position of S relative to L. For a double quasar we have the difference of time delays $\delta\Delta t \equiv \mathcal{D}\delta C_{\text{geom}} + (D_L)^2 \delta C_{\text{grav}}$, where the differences of coefficients $\delta C_{\text{geom}}$ and $\delta C_{\text{grav}}$ are determined if the relative positions of L and S and the mass distribution in L are given.

If $z_0 > z_1^1$, both L and S are in $V^H$, so that in the above formulas $D_{LS}$ is given by $D_{LS} = D_A(\Omega_0^H, H_0^H, z_L, z_S)$, as in the
homogeneous models, where

\[ D_d(\Omega_0, H_0, z_l, z_h) \equiv \frac{2(c/H_0)}{\Omega_0(1 + z_l)(1 + z_h)} \times \left[ (2 - \Omega_0 + \Omega_0 z_l)(1 + \Omega_0 z_l)^{1/2} - (2 - \Omega_0 + \Omega_0 z_h)(1 + \Omega_0 z_h)^{1/2} \right]. \]

(36)

If we derive the effective Hubble constant \((H_0)_{\text{eff}}\) from the time delay measurement, we have the relation \((H_0)_{\text{eff}} \propto 1/\delta t\). For a homogeneous model with \(\Omega_0\) and \(H_0\), moreover, we have the relation \(H_0^2 \propto 1/\delta t_{\text{homog}}\). From these two relations we obtain

\[ \frac{H_0^2}{(H_0)_{\text{eff}}^2} = \frac{\delta t}{\delta t_{\text{homog}}} = \frac{\alpha_1 + \alpha_2 \beta}{1 + \beta}, \]

(37)

where \(\alpha_1 \equiv \mathcal{D}/\mathcal{D}_{\text{homog}}, \alpha_2 \equiv (D_L)^2/(D_L)_{\text{homog}}^2\), and \(\beta \equiv [\mathcal{D}/(D_L)^2]_{\text{homog}}\). By solving this equation, therefore, we can obtain \(H_0^2\) in the present single-shell models. Since the lensing is caused in \(V_0\), it is natural that we can get some information about \(H_0^2\) and \(\Omega_0\) from the time delay measurement.

In the two single-shell models, by the way, we numerically obtain the following ratios \((\alpha_1, \alpha_2)\) of \(\mathcal{D}\) and \((D_L)^2\) to the corresponding ones in the homogeneous models with

\[ (\Omega_0, \Omega_0, h, h^2/h) = (0.2, 0.56, 0.7, 0.82). \]

and for \((\Omega_0, \Omega_0, h, h^2/h) = (0.2, 0.88, 0.7, 0.82),

\[ \alpha_1 = 1.01, \quad \alpha_2 = 1.16, \]

(38)

and for \((\Omega_0, \Omega_0, h, h^2/h) = (0.2, 0.88, 0.7, 0.82),

\[ \alpha_1 = 1.14, \quad \alpha_2 = 1.25. \]

(39)

If we take \((H_0)_{\text{eff}} \approx 62 \text{ km s}^{-1} \text{ Mpc}^{-1}\) (Falco et al. 1997) and adopt \(H_0^2 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}\), the ratio (37) is \(H_0^2/(H_0)_{\text{eff}}^2 \approx 1.13\), which is consistent with the above values (38) and (39) for \(\beta \approx 1\).

6. CONCLUDING REMARKS

In this paper we derived the angular diameter distances from central and noncentral observers in the “cosmological void models,” for which we adopted the model parameters necessary to explain the bulk flow (derived in Paper I), and showed that the \(m-z\) relation due to these distances may explain the observed deviation of high-redshift supernovae (SNe Ia) from the relation in homogeneous Friedmann models, independently of the cosmological constant. This is possible because the voidlike low-density region with a high Hubble parameter gives some “acceleration” to light coming from the high-density region with a low Hubble parameter, as if we were in a universe dominated by the positive cosmological constant.

It was found that the remarkable difference between the relation in the present models and the relation in the
cosmological-constant–dominated model appears at epoch $z \approx 1.0$ or at the earlier stage. The observation of SNe Ia around this epoch is, therefore, important in order to discriminate between these two models observationally.

The unique property of the present models is that the $m$–$z$ relation is anisotropic, in contrast to the relation in homogeneous Friedmann models, and that the systematic image deformation appears for the sources with $z > z'_1$ ($\sim 0.067$).

The observations about the anisotropy and lens effect will also be useful to determine which of the two models is better.

The derivation of distances in models with the intermediate self-similar region was not treated here, but their behavior is basically similar to that in the double-shell models, though their analysis is somewhat more complicated.

APPENDIX

DISTANCES IN MODELS WITH DOUBLE SHELLS

A1. DISTANCES FROM THE CENTER OF THE INNER REGION

The line elements in the regions $V^I$, $V^II$, and $V^III$ are given by equation (1) with $j = I$, $II$, and $III$, respectively. When $S$ is in $V^I$, $V^II$, and $V^III$, the angular diameter distance $d_A$ is given by equation (2), equation (3), and

$$d_A = a(u^I S^I) \sinh z^III S,$$

(A1)

respectively, where the negative curvature was also assumed in $V^III$. When $S$ is in $V^I$ and $V^II$, $d_A$ has the same expressions (for given $z^1_1$ and $z^II_2$) as in the single-shell model. When $S$ is in $V^III$, we have for the coordinates of the second shell ($\eta^II_2$, $\chi^II_2$) and those of $S$, ($\eta^III_0$, $\chi^III_0$):

$$\eta^II_0 = \eta^II_0 = \chi^II_2 - \chi^II_1,$$

(A2)

$$\eta^II_1 - \eta^III_0 = \chi^III S - \chi^II_2,$$

(A3)

where $\eta^II_2$, $\eta^III_0$ are related to $z^II_2$, $z^III_0$ by

$$1 + z^II_2 = \cosh \eta^II_2 - 1,$$

(A4)

$$1 + z^III_0 = \cosh \eta^III_0 - 1.$$

(A5)

In the same way as $z^I_1 = z^II_1$ in the previous subsection, we have the equality of the shell redshifts $z^III_2 = z^III_2$. Moreover, coordinates ($\eta^III_0$, $\chi^III_0$) are connected with ($\eta^II_2$, $\chi^II_2$) by equations (I.A14) and (I.A15). Accordingly, $d_A$ is uniquely determined for given $z^III_1$ ($= z^III_1$, $z^II_2 = z^II_2$), and $z^III_2$.

Here we calculated the average values of $d_A$ and $d_L = (1 + z)^2$ defined by equation (31) and showed the $z$-dependence (see Fig. 4) of 5 log $d_L$ and $\Delta m$ (see Fig. 6).

A2. DISTANCES FROM A NONCENTRAL OBSERVER $O$ IN THE INNER REGION

When $S$ is in $V^I$ and $V^II$, $d'_A$, $d'_0$, and $\phi_S$ are the same as those in the single-shell model. When $S$ is in $V^III$, we have

$$d'_I = d_A(\eta^III S^I, \chi^III S^I) \hat{c} \phi_S/\hat{c} \phi/[\cos (\phi - \phi_S)],$$

(A6)

$$d'_0 = d_A(\eta^III S^I, \chi^III S^I) \sin \phi_S/\sin \phi,$$

(A7)

where

$$d_A(\eta^III S^I, \chi^III S^I) = a(u^III S^I) \sinh \chi^III S.$$

(A8)

The relation between ($\eta^III S^I$, $\chi^III S^I$, $\phi_S$) and ($\eta^II_0$, $\chi^II_0$, $0$) is given as follows by specifying $z^I_1$, $z^II_2$, $z^III_0$, and $\phi$. In the second shell we have

$$G(\chi^II_2) \equiv \cosh^{-1} \left( \frac{\cosh \chi^II_2}{h^II_0} \right) - \cosh^{-1} \left( \frac{\cosh \chi^I_1}{h^I_0} \right) = \eta^II_1 - \eta^II_2,$$

(A9)

and in $V^III$

$$G(\chi^III S) \equiv \cosh^{-1} \left( \frac{\cosh \chi^III S}{h^III_0} \right) - \cosh^{-1} \left( \frac{\cosh \chi^II_2}{h^II_0} \right)$$

$$= \eta^III_2 - \eta^III_0,$$

(A10)

where

$$h^III_0 = [1 + (\xi^III)^2]^{1/2}, \quad \xi^III = d^0_0 d^I_0 \xi^I.$$
The coordinates \((\eta_{II}^{III}, \chi_{II}^{III})\) are related to \((\eta_{II}, \chi_{II}^{II})\), using
\[
\begin{align*}
\lambda_{II}^{III}(\eta_{II}^{III}) \sinh \chi_{II}^{III} &= \lambda_{III}^{II} \sinh \chi_{II}^{III}, \\
\end{align*}
\]
and
\[
\begin{align*}
\lambda_{II}^{III} \int_{0}^{\eta_{II}^{III}} y(\eta) d\eta &= \lambda_{III}^{II} \int_{0}^{\eta_{III}^{III}} y(\eta) d\eta.
\end{align*}
\]
Moreover, \(\varphi_2\) is derived in \(V_{II}\) as
\[
\begin{align*}
\varphi_2 &= \varphi_1 + \tan^{-1} \left\{ \frac{1}{\zeta_{II}^{III}} \left[ \sinh^2 \chi_{II}^{III} + \cosh \chi_{II}^{III} \sqrt{\sinh^2 \chi_{II}^{III} - (\zeta_{II}^{III})^2} \right] \right\} \\
&\quad - \tan^{-1} \left\{ \frac{1}{\zeta_{II}^{III}} \left[ \sinh^2 \chi_{II}^{III} + \cosh \chi_{II}^{III} \sqrt{\sinh^2 \chi_{II}^{III} - (\zeta_{II}^{III})^2} \right] \right\},
\end{align*}
\]
and in \(V_{III}\) we have
\[
\begin{align*}
\varphi &= \varphi_2 + \tan^{-1} \left\{ \frac{1}{\zeta_{III}^{III}} \left[ \sinh^2 \chi_{III}^{III} + \cosh \chi_{III}^{III} \sqrt{\sinh^2 \chi_{III}^{III} - (\zeta_{III}^{III})^2} \right] \right\} \\
&\quad - \tan^{-1} \left\{ \frac{1}{\zeta_{III}^{III}} \left[ \sinh^2 \chi_{III}^{III} + \cosh \chi_{III}^{III} \sqrt{\sinh^2 \chi_{III}^{III} - (\zeta_{III}^{III})^2} \right] \right\},
\end{align*}
\]
\[
\varphi_{S} = \varphi(\chi_{III}^{III} = \chi_{S}^{III}).
\]
Thus \(\varphi_6\) was derived as a function of \(z_{II}^{I}, z_{III}^{I}, z_{II}^{III},\) and \(\phi (= \varphi_1 \text{ or } \pi - \varphi_1).\)

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