On relating multiple M2 and D2-branes

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Abstract: We discuss the difficulties to find Lagrangian theories for multiple M2-branes. By investigating the conditions needed for the existence of maximally supersymmetric field equations we are led to introduce a new weaker form of the fundamental identity, called the weak fundamental identity. For totally antisymmetric \( f^{ABCD} = f^{ABC} e^{D} h^{ED} \) these identities coincide. We also discuss the extra conditions needed to construct a Lagrangian. We exemplify the discussion concerning the field equations by presenting a non-antisymmetric \( f^{ABC} D \) constructed from the structure constants \( f^{abc} \) of an arbitrary gauge group. Although our choice of \( f^{ABC} D \) does not admit an obvious Lagrangian description, it does reproduce the correct SYM theory for a stack of \( N \) D2-branes to leading order in \( g^{-1}_{YM} \) upon reduction and, moreover, it sheds new light on the centre of mass coordinates for multiple M2-branes.

Keywords: String theory, M-theory, Branes

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1. Introduction

Finding a model for the dynamics of multiple M2-branes is a central problem in the quest for a better understanding of M-theory. Building on previous work [1, 2, 3], a new class of maximally supersymmetric field equations, proposed to describe multiple M2-branes at the conformal fixed point, was recently constructed by demanding closure of the supersymmetry transformations [4, 5, 6]. These field equations are parameterised by a four-index tensor $f^{ABC\, D}$, antisymmetric in the first three indices. One finds that in order for the supersymmetry transformations to close, and a Lagrangian to exist, these structure constants have to satisfy the so called fundamental identity

$$f^{ABC\, G} f^{EFG\, D} = 3 f^{EF[\, A\, G} f^{BC]G\, D}.$$  \hspace{1cm} (1.1)

When constructing the Lagrangian one also needs to introduce a metric, $h^{AB}$, in order to be able to write down a scalar with respect to the indices appearing on $f^{ABC\, D}$. In addition, one is forced to assume that $f^{ABCD} = f^{[ABCD]}$, i.e. that the four-index tensor is antisymmetric [5]. Despite the growing attention that the Bagger-Lambert-Gustavsson theory is receiving [7]-[13], it has proved very difficult to find solutions to the fundamental identity using an antisymmetric $f^{ABCD}$, and so far only one solution is known, namely $f^{ABCD} = \epsilon^{ABCD}$ [3], usually referred to as the $A_4$ theory. There is by now evidence mounting for the interpretation that this theory describes two coincident M2-branes [8, 12, 13].

In a recent paper by Mukhi and Papageorgakis [8], a mechanism for relating the proposed multiple membrane theory to a maximally supersymmetric Yang-Mills (SYM) theory living on D2-branes was presented. Previous attempts in this direction was made in [4, 7]. The mechanism consists of making a particular choice for $f^{ABC\, D}$, and then in an elegant way compactify and Higgs the theory by giving a vev to one of the scalars leading to the promotion of a non-dynamical gauge field to a dynamical one. To obtain the SYM theory with gauge group $SU(2)$ these authors used the $f^{ABCD} = \epsilon^{ABCD}$ mentioned above. For gauge group $SU(N)$, i.e. for a stack of $N > 2$ branes, it was assumed in [8] that their choice of an antisymmetric $f^{ABCD}$, which does not solve the fundamental identity per se, could be completed with terms not relevant to leading order in $g^{-1}_{YM}$ in such a way that the resulting $f^{ABCD}$ does solve the fundamental identity. However, one can check that the $f^{ABCD}$ used to get a SYM theory with gauge group $SU(3)$ can not be completed in this fashion. It is therefore not clear how the procedure will work in detail for $SU(N)$, with $N > 2$. However, as will be described below, by using a new set of non-antisymmetric structure constants, we are able to apply their proposed Higgs mechanism at the level of the field equations and find the infrared limit of the SYM theory for a stack of $N$ D2-branes to leading order in $g^{-1}_{YM}$.

One motivation for studying the relation between D2 and M2-branes only at the level of the field equations, and (at least for the moment) ignore the problem of
not being able to find a Lagrangian formulation, is the great difficulty, mentioned above, of finding Lagrangians for the supersymmetric field equations supposedly describing multiple membranes. Furthermore, there is no \textit{a priori} reason why the infrared conformal fixed points of nonconformal $\mathcal{N} = 8$ SYM theory should allow for a Lagrangian formulation.

In this context, we note that the constraints on $f^{ABC}_D$ coming from demanding that the supersymmetry transformations close are not given by the fundamental identity but by a weaker condition, which we will call the \textit{weak fundamental identity}\footnote{Note the similarity between this formula and the Jacobi identity for Lie algebras, i.e. $f^{[ab}_d f^{c]d}_e = 0$.}

\begin{equation}
   f^{[ABC}_G f^{D]GE}_F = 0, \tag{1.2}
\end{equation}

further discussed in section \footnote{This has also been noticed by Gustavsson in \cite{Gustavsson}.}. For antisymmetric $f^{ABCD}$ the weak fundamental identity and the fundamental identity are equivalent, but for a non-antisymmetric $f^{ABC}_D$, like the one we employ in the construction below, this is no longer the case. Throughout the paper we will be careful in stating what extra assumptions we make in addition to requiring closure of the supersymmetry algebra as some of these assumptions might be possible to relax.

We present an explicit non-antisymmetric choice for $f^{ABC}_D$ in order to exemplify the above discussion. This gives rise to field equations that, after using to the Higgs mechanism in \cite{Higgs} in combination with a consistent truncation of the vector fields, correspond to the SYM theory for a stack of $N$ D2-brane with an arbitrary (compact and semi-simple) gauge group. Thus, although this choice of $f^{ABC}_D$ inserted into the field equations give equations which do not admit an obvious Lagrangian description, it does reproduce the infrared limit of the nonconformal D2-brane SYM theory.

Furthermore, in contrast to previous attempts, the $U(1)$ degrees of freedom we find on the D2-branes satisfy free field equations to all orders in $g^{-1}_{YM}$ and is thus a natural candidate for the centre of mass multiplet for multiple M2-branes. It is interesting to note that in our approach the centre of mass coordinates arise already at the level of the M2-branes\footnote{This has also been noticed by Gustavsson in \cite{Gustavsson}.}, thereby shedding new light on the problem with the M-theory translation invariance discussed in \cite{M-theory}. However, although the centre of mass modes obey free dynamics, they do appear in the other field equations, thereby explaining why a Lagrangian, if it exists, is not easily constructed.

The paper is organised as follows. In section \footnote{This has also been noticed by Gustavsson in \cite{Gustavsson}.} we reanalyse the conditions that the structure constants must satisfy in order to obtain closure of the supersymmetry transformations. We then discuss the extra conditions needed for a straightforward construction of a Lagrangian. A set of non-antisymmetric structure constants relating D2 and M2-branes for any number of branes at the level of the field equations is given in section \footnote{Note the similarity between this formula and the Jacobi identity for Lie algebras, i.e. $f^{[ab}_d f^{c]d}_e = 0$.}. Finally, section \footnote{This has also been noticed by Gustavsson in \cite{Gustavsson}.} contains some closing comments.
2. Supersymmetric field equations

A new class of supersymmetric field equations was recently constructed in [4] and [5], using different but equivalent formulations [6], by demanding closure of the supersymmetry transformations, which we briefly review below. In this paper we will use the notation and conventions of [5].

Consider the fields $X^I_A$, $\Psi^A$, and $\tilde{A}^B_A$, where $X^I_A$ is an $SO(8)$ vector, $\Psi^A$ a chiral spinor and $\tilde{A}^B_A$ a non-dynamical gauge potential transforming as a vector under $SO(2,1)$. Indices $A, B, \ldots$ refer to an unspecified algebra defined by the structure constants $f^{ABC}$. The on-shell supersymmetry transformations are

$$
\delta X^I_A = i\bar{\epsilon} \Gamma^I \Psi^A ,
$$

$$
\delta \Psi^A = D_\mu X^I_A \Gamma^I \epsilon - \frac{1}{6} X^J_B X^K_D f^{BCD} A \Gamma_I J K \epsilon ,
$$

$$
\delta \tilde{A}^B_A = i\epsilon \Gamma_I X^I_C \Psi^D f^{CDB} A .
$$

Demanding that these supersymmetry transformations close into a translation and a gauge transformation, the following field equations are obtained

$$
0 = \Gamma^I D_\mu \Psi^A + \frac{1}{2} \Gamma_I J X^I_A X^J_B \Psi^D f^{CDB} A ,
$$

$$
0 = D^2 X^I_A - \frac{i}{2} \bar{\Psi}^C \Gamma^I J X^J_D \Psi^B f^{CDB} A + \frac{1}{2} f^{BCD} A f^{EFG} D X^J_B X^K_C X^I_D X^J_F X^K_G ,
$$

$$
0 = \tilde{F}^{B} A + \varepsilon_{\mu\nu\lambda}(X^J_C D^\lambda X^J_D + \frac{i}{2} \bar{\Psi}^C \Gamma^\lambda \Psi^D) f^{CDB} A ,
$$

where the covariant derivative and field strength are defined as

$$(D_\mu X)_A = \partial_\mu X_A - \tilde{A}_\mu^B A X_B ,$$

$$
\tilde{F}^{B} A = -2 \left( \partial_\mu \tilde{A}_\nu^B A + \tilde{A}_\mu^B A \tilde{A}_\nu^C A \right) .
$$

In addition, the Bianchi identity for the gauge field is satisfied

$$
\varepsilon^{\mu\nu\lambda} D_\mu \tilde{F}_{\nu\lambda}^{B} A = 0 .
$$

When deriving the above field equations the requirement that $[\delta_1, \delta_2] \tilde{A}^B_A$ closes on-shell implies that $f^{ABC} D$ satisfy the weak fundamental identity given in (1.2). We would like to stress that this is the only constraint imposed by the closure of the supersymmetry transformations. The identity (1.2) is not equivalent to the fundamental identity (1.1) but implied by (1.1) when antisymmetrised in $[ABEF]$, hence the choice of name. Note that the weak fundamental identity is equivalent to the fundamental identity if one also assumes the existence of a metric and that $f^{ABC} D$ is totally antisymmetric. Thus, if one chooses to work at the level of equations of motion, and not requires a straightforward Lagrangian, one only needs $f^{ABC} D$ to
satisfy the weak fundamental identity. This identity is also needed in order to show that the Bianchi identity is satisfied and when using supersymmetry to transform the field equations into each other.

The possibility of integrating the equations of motion to a Lagrangian clearly requires the existence of a metric $h^{AB}$ in order to form scalars. In addition, the only way to obtain a Lagrangian known so far is to also require $f^{ABCD}$ to be totally antisymmetric. In that case, the equations of motion above are obtained from the following Lagrangian \[5\]

$$
\mathcal{L} = -\frac{1}{2}(D_\mu X^A)(D^\mu X_A) + \frac{i}{2}\bar{\Psi}A_\Gamma^\mu D_\mu \Psi_A + \frac{i}{4}\bar{\Psi}_B \Gamma_{IJ} X^I_X^J \Psi_A f^{ABCD}$$

$$-V + \frac{1}{2} \varepsilon^{\mu \nu \lambda} \left( f^{ABCD} A_{\mu AB} \partial_\nu A_{CD} + \frac{2}{3} f^{CDA}_G f^{EFGB} A_{\mu AB} A_{\nu CD} A_{\lambda EF} \right),$$

where

$$V = \frac{1}{12} f^{ABCD} f^{EFG} D X^A X^B X^C X^D X^E X^F X^G.$$

After varying this Lagrangian the free gauge index must always sit on the last position of $f^{ABCD}$ in order to match with the field equations. Assuming $f^{ABCD}$ to be antisymmetric takes care of this problem.

We will now consider a particular example where there is no obvious Lagrangian formulation based on the following structure constants, where we have split the gauge indices according to $A = \{a, \phi\}$,

$$f^{\phi ab} = f_{\phi ab}, \quad f^{ab} = f^{abc} = f^{abc} = f^{abc} = 0,$$

where $f^{ab} c$ are the structure constants of a (compact and semi-simple) Lie algebra, see appendix \[A\] for details. Note in particular that this $f^{ABCD}$ in non-antisymmetric (without a metric to raise the $D$ index there is not even a notion of total antisymmetry). This form of $f^{ABCD}$ solves both the weak and the original fundamental identity\(^3\), and has also been considered in \[7, 14\]. In the next section we will discuss the implications of this $f^{ABCD}$ at the level of equations of motion.

3. Relating multiple M2 and D2-branes

We will in this section investigate the physics of the explicit $f^{ABCD}$ introduced in \[2.7\] and, in particular, study the possible relation to D2-brane physics using the Higgs mechanism introduced recently by Mukhi and Papageorgakis \[8\]. The investigation here will be done at the level of equations of motion and consistency, with respect to the truncation we perform, is checked. In addition to focusing on the field equations,
the main difference of our analysis compared to that of [8] is that we choose $f^{ABC \, D}$ as given in (2.7) which, even for the $SU(2)$ case, is different from the one used in [9].

This choice of $f^{ABC \, D}$ has several special features. The condition $f^{ABC \phi} = 0$ implies that the interaction terms for $X^I \phi$ and $\Psi \phi$ in (2.2) vanish. To also get the covariant derivatives acting on $X^I \phi$ and $\Psi \phi$ to reduce to ordinary derivatives we need to make the truncation $\tilde{A}^A \phi = 0$. That this truncation is consistent follows from the field equations (2.2) together with $f^{ABC \phi} = 0$ and the definition of the field strength (2.3). Then $X^I \phi$ and $\Psi \phi$ obey free dynamics and these fields will, as explained below, give rise to the $U(1)$ centre of mass degrees of freedom in the SYM theory. But, as these fields obey free dynamics already before we apply the Higgs mechanism and break the $SO(8)$ covariance, they could also be interpreted as the centre of mass degrees of freedom for multiple M2-branes. Note, however, that $X^I \phi$ and $\Psi \phi$ will appear in the equations of motion for the other fields in (2.2), thereby obstructing the possibility of obtaining a Lagrangian in a straightforward manner.

Alternatively, if we start by requiring the correct number of degrees of freedom suitable for matching to a SYM theory we need to impose the constraint [5, 8]

$$\tilde{A}^A \phi = A_{\mu CD} f^{CD \mu \phi}, \quad (3.1)$$

where $A_{\mu \phi a}$ will be promoted to a dynamical SYM gauge potential through the Higgs mechanism. In the supersymmetry transformations and field equations only $\tilde{A}^B \phi$ appears, and the constraint (3.3), together with the choice of $f^{ABC \, D}$ is equivalent to setting $\tilde{A}^A \phi = 0$. This means that constraint (3.1) implies a truncation which, in fact, is the same one as we considered above and verified the consistency of. Thus, the choice we make for $f^{ABC \, D}$ in (2.7) implies that there is a centre of mass multiplet obeying free dynamics and moreover, that the truncation to the SYM degrees of freedom is consistent.

Let us now perform the Higgsing procedure along the lines of [8], by giving a vev to the scalar $X^8 \phi$,

$$X^8 \phi = < X^8 \phi > + x^8 \phi = g_{YM} + x^8 \phi. \quad (3.2)$$

From the last equation in (2.2) with $\tilde{F}_{\mu \nu} \phi$ we can algebraically solve for $\tilde{A}_\mu \phi$:

$$\tilde{A}_\mu \phi = \frac{1}{X^I \phi} \left( \frac{1}{2 \lambda} \epsilon_{\nu}^{\mu \nu \rho} \tilde{F}_{\nu \rho} \phi + X^J J_\mu X^J - X^J J_\mu X^J + i \bar{\Psi} \phi \Gamma_{\mu} \Psi \phi \right) \quad (3.3)$$

where $\lambda$ is defined in (A.1). By substituting this expression back into the field equations, rescale the fields $X$ and $\Psi$ according to their canonical dimension in the world volume theory of the D2-branes $(X, \Psi) \to (X/g_{YM}, \Psi/g_{YM})$ and keeping terms
to leading order in $g_{YM}^{-1}$, we get
\[ 0 = \Gamma^\mu \partial_\mu \Psi^\phi, \]
\[ 0 = \Gamma^\mu \nabla_\mu \Psi_a + \Gamma_i X^i_b \Psi^e f_{bc} a + \mathcal{O} \left( \frac{1}{g_{YM}^2} \right), \]
\[ 0 = \partial^2 X^I_\phi, \] (3.4)
\[ 0 = \nabla^2 X^i_a - \frac{i}{2} \tilde{\psi}_b \Gamma^i \Psi^c f^{bc} a - f^{cd} g^{b_a} X^i_c X^j_d + \mathcal{O} \left( \frac{1}{g_{YM}^2} \right), \]
\[ 0 = \nabla^\mu F_{\mu
u a} - \frac{1}{2} \left( X^i_c \nabla_\nu X^j_d + \frac{i}{2} \tilde{\psi}_c \Gamma_\nu \Psi^d \right) f^{cd} a + \mathcal{O} \left( \frac{1}{g_{YM}^2} \right), \]
where $X^I_\phi = (X^i_\phi, x^8_\phi)$ with $i = 1, ..., 7$, the covariant derivative $\nabla^\mu$ is defined as
\[ (\nabla^\mu X)_a = \partial^\mu X_a - \tilde{A}^b_\mu X_b = \partial^\mu X_a - 2A_{\mu \phi} X^f f^{bc} a \] (3.5)
and
\[ F_{\mu
u CD} := -2 \left( \partial_\mu A_{\nu CD} - A_{\mu|EF} f^{EF} [CA_{\nu|G|D}] \right), \] (3.6)
satisfying $\tilde{F}_{\mu
u B A} = F_{\mu
u CD} f^{CD} B A$. The fact that $F_{\mu
u CD}$, which satisfies the previous relation and is consistent with (2.3), can be defined follows from the weak fundamental identity (1.2). Note that the Higgs mechanism has transformed the algebraic equation for $\tilde{A}_\mu \phi a$ into a dynamical equation for the gauge potential $A_{\mu \phi} a$ [8].

The leading terms in (3.4) can be integrated to the Lagrangian
\[ \mathcal{L} = \frac{1}{g_{YM}^2} \left( \mathcal{L}_{\text{decoupled}} + \mathcal{L}_0 \right), \] (3.7)
where
\[ \mathcal{L}_{\text{decoupled}} = -\frac{1}{2} \partial^\mu X^I_\phi \partial^\mu X^I_\phi + \frac{i}{2} \tilde{\psi}_\phi \Gamma^\mu \partial_\mu \psi_\phi \] (3.8)
and $\mathcal{L}_0$ is the standard $2 + 1$ dimensional SYM Lagrangian
\[ \mathcal{L}_0 = -\frac{1}{4} F_{\mu a} F^{\mu a} - \frac{1}{2} \nabla^\mu X^{ai} \nabla^\mu X^i_a + \frac{1}{4} \left( f_{abc} X^{ai} X^{b_j} \right) \left( f_{de} X^{di} X^{ej} \right) + \frac{i}{2} \tilde{\psi}_a \nabla \Psi_a + \frac{i}{2} \tilde{f}_{abc} \tilde{\psi}_c \Gamma^i X^{bi} \Psi^c, \] (3.9)
where we have here denoted $F_{\mu a} = 4F_{\mu \phi a}$.

In accordance with [8], the scalars $X^8_a$ behave as Goldstone bosons giving a mass to $\tilde{A}_\mu \phi a$. Moreover, the scalar $x^8_\phi$ in (1.2), corresponding to the fluctuation around the vev, can be dualised to an abelian gauge field and will, together with the centre of mass modes $X^I_\phi$ for the D2-branes and the superpartners $\Psi_\phi$, form a free $U(1)$ vector multiplet. In addition to the fact that this $U(1)$ centre of mass multiplet is decoupled to lowest order in $g_{YM}^{-1}$, in our construction it actually obeys free dynamics to all orders in $g_{YM}^{-1}$. 


4. Conclusions

In light of the fact that it seems to be exceedingly difficult to find solutions to the fundamental identity based on a totally antisymmetric $f^{ABCD}$, which is required for a Lagrangian description, we work instead at the level of field equations. This leads to new possibilities of finding solutions since the constraints on $f^{ABCD}$, imposed only by supersymmetry closure, are then weaker. Also, since there is no a priori reason to expect a Lagrangian formulation for the theories which describe conformal fixed points of the nonconformal $\mathcal{N} = 8$ SYM theories it seems relevant to not rule out this possibility.

Putting potential problems that could accompany this approach aside for the moment, we use a choice of $f^{ABCD}$ that satisfies both the weak and the original fundamental identity. With this choice we are able to obtain, using the Higgs mechanism of [3], the SYM field equations for any (compact and semi-simple) gauge group, to leading order in $g_{YM}^{-1}$. A novel feature of this construction is that the $U(1)$ centre of mass multiplet for the D2-branes obeys free dynamics, even before applying the Higgs mechanism and breaking the $SO(8)$ covariance. This means that we have identified a candidate for the centre of mass multiplet also for multiple M2-branes. We believe that these results indicate that this approach could be consistent despite the lack of a Lagrangian, but this of course needs further investigation. Note that, the leading order terms, corresponding to the ordinary SYM D2-theory, of course do have a Lagrangian formulation.

An interesting feature of this construction is that the choice of $f^{ABCD}$ in (2.7) seems rather unique if we want to make contact with D2-brane physics. In order to have a $U(1)$ centre of mass multiplet obeying free dynamics we need to set $f^{ABC} \phi = 0$, which is also important in order to be able to make a consistent truncation to the SYM degrees of freedom. Then, out of the remaining components we need to embed the Lie algebra structure constants, implying that $f^{\phi abc} \sim f^{abc}$. The only remaining components, $f^{abc}$, are forced to vanish by the weak fundamental identity (1.2). Thus, since the choice of $f^{ABCD}$ seems to be so tightly constrained, it would be interesting to also investigate what this proposed D2-M2 correspondence implies to higher order in $g_{YM}^{-1}$.

Note added: As this paper was being prepared, a preprint appeared [15] which has some overlap with our paper.

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A. Useful formulæ

In this appendix we collect some useful formulæ needed for the computations related to the Higgs mechanism. We restrict our attention to compact and semi-simple Lie algebras, which means that the generators can be chosen to satisfy

\[ \text{tr}(T^a T^b) = \lambda \delta^{ab}, \]  

(A.1)

where

\[ (T^a)^b_c = -i f^{ab}_c, \]  

\[ [T^a, T^b] = i f^{ab}_c T^c \]  

(A.2)

and the structure constants $f^{ab}_c$ are totally antisymmetric. In these conventions the Jacobi identity is

\[ f^{[ab}_d f^{c]}_d = 0. \]  

(A.3)

From the constraint $\tilde{A}^B_A = A_{\mu CD} f^{CDB}_A$, and the above Lie algebra properties, we find that

\[ \tilde{F}_{\mu \nu}^{~d} c f^e_d f^b_e a = -\lambda \tilde{F}_{\mu \nu}^{~b} a, \]  

(A.4)

which is needed for the Higgsing. Note that in the final formulæ (3.4), the normalisation constant $\lambda$ drops out.

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