Research Article

Regarding the Charmed-Strange Member of the $2^3S_1$ Meson State

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Received 17 August 2013; Accepted 8 September 2013

1. Introduction

Charmed spectroscopy becomes an active field with many new states observed in the experiment in the last few years [1–8]. The $D_{sJ}(2632)$ (with the mass $2632.5 \pm 1.7$ MeV and decay width $\Gamma_{\text{tot}} \leq 17$ MeV) was reported in the two decay modes $D^*_s \eta$ and $D^0 K^+$ by the Selex collaboration [5]. In [9–11], $D_{sJ}(2632)$ was interpreted as the first radical excitation of $D_s^+(2112)$. However, the narrow and dominated $D^*_s \eta$ decay mode implies that the assignment is problematic. On the other hand, the mass of the $D_{sJ}(2632)$ is lower than the value of the tradition potential model predictions. Therefore, the $D_{sJ}(2632)$ is also interpreted as a tetraquark, a hybrid, a diquark-antiquark bound state, and so forth.

Recently, Belle collaboration observed a new $c\bar{s}$ state $D_{sJ}(2700)$ with a mass of $2708 \pm 9_{10}^{+11} \pm 19$ MeV and width $108 \pm 23^{+31}_{-30} \text{MeV}$ [8]. Based on its observed decay channel $B^+ \rightarrow \overline{D}^0 D_{sJ} \rightarrow \overline{D}^0 D^0 K^+$, the $D_{sJ}(2700)$ resonance is interpreted as a charmed-strange meson state.

Firstly, we reviewed the assignment of the $2^3S_1$ meson state in the $q\bar{q}$ quark model (see Table 1). According to the new edition of Particle Data Group (PDG) [12], the states $\omega(1420)$ and $\phi(1680)$ have been well established as the isoscalar member in the $2^3S_1$ meson state. $K^*(1410)$ is assigned as isodoublet member however, it is still based on very weak experimental signals. Till now, the $K^*(1410)$ has been reported by two experiments $K^- p \rightarrow K^- \pi^+ n$ (with mass $1380 \pm 21 \pm 19$ MeV and width $176 \pm 52 \pm 22$ MeV) and $K^- p \rightarrow \overline{K}^0 \pi^+ \pi^- n$ [12] (with mass $1420 \pm 7 \pm 10$ MeV and width $240 \pm 18 \pm 12$ MeV). The assignment of isodoublet of the $2^3S_1$ meson state was investigated in our previous work [13]. For the heavy-light meson, the charmed-strange member of the $2^3S_1$ meson state has attracted more attention, recently. Both the $D_{sJ}(2632)$ and the $D_{sJ}(2700)$ are probably interpreted as charmed-strange candidate of the $2^3S_1$ meson state [9–11, 14]. The nonstrange partner of $D_{sJ}$ has not been observed in the experiment.

In this work, based on the isoscalars states can mix to form the physical states in the $q\bar{q}$ quark model and the linear Regge trajectory; we establish the new mass relations which relate the mass spectrum of meson state and constituent quark masses. Inserting the corresponding constituent quark masses and the following well-established states, we reexamine the mass spectrum of the $2^3S_1$ meson state. The results could be a useful comparison with the experiment data in the new experiment.

2. Mass Matrix and Regge Trajectory

In the quark model, the two isoscalar states with the same $J^{PC}$ will mix to form the physical isoscalar states. We can establish
the mass-squared matrix in the $ss$ and $N = (u \bar{u} + d \bar{d})/\sqrt{2}$ basis [25] as follows:

$$M^2 = \left( \begin{array}{cc} M^2_{\rho(1450)} + 2A_{nn} & \sqrt{2}A_{ns} \\ \sqrt{2}A_{ns} & 2M^2_{K^*(1410)} - M^2_{\rho(1450)} + A_{ss} \end{array} \right),$$

(1)

where $M_{\rho(1450)}$ and $M_{K^*(1410)}$ are the masses of isovector and isodoublet states of the $2^3S_1$ meson nonet, respectively, and $A_{nn}, A_{ns},$ and $A_{ss}$ are the mixing parameters which describe the $q \bar{q} \leftrightarrow q' \bar{q}'$ transition amplitudes. In order to reduce the number of parameters, we adopt the similar expression of the transition amplitudes in the $q \bar{q} \leftrightarrow q' \bar{q}'$ process which is widely used in [26–28] as follows:

$$A_{nn} = \frac{\Lambda}{m_n m_n},$$
$$A_{ns} = \frac{\Lambda}{m_n m_s},$$
$$A_{ss} = \frac{\Lambda}{m_s m_s},$$

(2)

where $\Lambda$ is a phenomenological parameter. Based on the isospin symmetry, we have $m_n = m_{\bar{n}} = m_d = m_{\bar{d}}, m_s = m_{\bar{s}}$ ($m_n, m_s, m_d, m_{\bar{d}}$ denote the mass of light quark $u, d, s)$.

In the $2^3S_1$ meson nonet, we assume that the physical states $\omega(1420)$ and $\phi(1680)$ are the eigenstates of mass-squared matrix and the masses square of $M^2_{\omega(1420)}$ and $M^2_{\phi(1680)}$ are the eigenvalues, respectively. The physical states $\omega(1420)$ and $\phi(1680)$ can be related to the $ss$ and $N = (u \bar{u} + d \bar{d})/\sqrt{2}$ by

$$\begin{bmatrix} |\omega(1420)\rangle \\ |\phi(1680)\rangle \end{bmatrix} = U \begin{bmatrix} |N\rangle \\ |S\rangle \end{bmatrix},$$

(3)

and the unitary matrix $U$ can be described as

$$UM^2U^+ = \begin{pmatrix} M^2_{\omega(1420)} & 0 \\ 0 & M^2_{\phi(1680)} \end{pmatrix}.$$  

(4)

According to relations (1), (2), (3), and (4), we will obtain

$$2 \frac{\Lambda}{m_n^2} + 2M^2_{K^*(1410)} + \frac{\Lambda}{m_s^2} = M^2_{\omega(1420)} + M^2_{\phi(1680)},$$

(5)

$$2 \frac{\Lambda}{m_n^2} + 2 \frac{\Lambda}{m_n^2} \left( 2M^2_{K^*(1410)} - M^2_{\rho(1450)} + \frac{\Lambda}{m_s^2} \right) - 2 \frac{\Lambda}{m_s^2} = M^2_{\omega(1420)} + M^2_{\phi(1680)}.$$

In relation (5), the masses of $K^*(1410)$ and $\phi(1680)$ are related with constituent quarks mass and phenomenological parameter $\Lambda$.

Regge theory is cornered with the particle spectrum, the forces between particles, and the high energy behavior of scattering amplitudes. Because the Regge trajectories can offer an effective way for the assignment and the classification of meson states, it also become an active field with many new particles and resonances being observed in the experiment in the last decade. In [29], the authors investigated the mass of different meson multiplets and suggested that the quasilinear Regge trajectories could describe the meson mass spectrum. Khruschov [30], using the phenomenology formulae deprive from the Regge trajectories, predicted the masses of excited meson states. Anisovich et al. [31] show that meson states can fit to the quasi-linear Regge trajectories with good accuracy.

According to the hadron with a set of given quantum numbers belonging to a quasilinear trajectory, we will have the following relation [7]:

$$J = \alpha_i^q(0) + \alpha_j^{\bar{q}} M^2_{\gamma^*}(0),$$

(6)

where $i\gamma^*$ refers to the quark (antiquark) flavor, $J$ and $M^2_{\gamma^*}$ are, respectively, the spin and mass of the $i\gamma^*$ meson. The parameters $\alpha_i^q$ and $\alpha_j^{\bar{q}}$ (0) are, respectively, the slope and intercept of the trajectory. The intercepts and slopes can be described by [15, 29]

$$\alpha_i^q(0) + \alpha_j^{\bar{q}}(0) = 2\alpha_i^q(0),$$
$$\frac{1}{\alpha_i^q} + \frac{1}{\alpha_j^{\bar{q}}} = \frac{2}{\alpha_{i\gamma^*}}.$$  

(7)

Relation (7) is satisfied in two-dimensional QCD [32], the dual-analytic model [33], and the quark bremsstrahlung model [34]. Relation (8) is derived from the topological and the $q\bar{q}$-string picture of hadrons [35]. According to available data of meson states, Burakovsky constructed a slope formula (9) for all quarks flavors [36] as follows:

$$\frac{\pi}{4} \alpha_i^{(i+1)} + \frac{\pi}{4} \sqrt{\alpha^2 m_i^2 + m_j^2} - \alpha_{ji} = \alpha',$$

where $m_i$ and $m_j$ are the corresponding constituent quark masses and the $\alpha' = 0.88 \text{ GeV}^{-2}$ is the standard Regge slope in the light quark sector.

From relations (6)–(9), we obtained the following relation:

$$\begin{align*}
M^2_{\rho(1450)} \alpha' + \frac{M^2_{\omega(25)} \alpha'}{2 + 2\sqrt{\alpha^2 m_n^2}} &= \frac{2M^2_{\gamma^*} \alpha'}{2 + \sqrt{\alpha^2 (m_n^2 + m_c^2)}}, \\
\frac{4M^2_{K^*(1410)} \alpha'}{2 + \sqrt{\alpha^2 (m_s^2 + m_n^2)}} + \frac{M^2_{\omega(25)} \alpha'}{1 + \sqrt{\alpha^2 m_c^2}} &= \frac{4M^2_{\gamma^*} \alpha'}{2 + \sqrt{\alpha^2 (m_s^2 + m_c^2)}},
\end{align*}$$

(10)

where $\alpha'$ is the standard Regge slope in the light quark sector.

Table 1: Assignment of the $2^3S_1$ meson state in PDG [12].

| $2^3S_1$ | $n\bar{n}$ | $n\bar{n}(s\bar{s})$ | $n\bar{s}$ | $c\bar{c}$ | $c\bar{n}$ | $c\bar{s}$ |
|----------|------------|---------------------|------------|-----------|-----------|-----------|
| $\rho(1450)$ | $\omega(1420)\phi(1680)$ | $K^*(1410)$ | $\psi(25)$ | $D$ | $D_s$ |
Using relations (5) and (10), we can obtain the masses of $K^*(1410)$, $D$, and $D_s$ in the $2^3S_1$ meson state. In this paper, we use the average values of the constituent quark mass as input parameters (see Table 2). The mass of $\omega(1420)$, $\phi(1680)$, and $\psi(2S)$ used are taken from the new edition of PDG [1]. Our results are presented in Table 3.

If $M_{K^* (1410)} = 1579.82$ MeV and $M_{\omega (1420)}$ and $M_{\phi (1680)}$ are assigned as the member of the $2^3S_1$ meson nonet, we can investigate the quarkonia content of isoscalar state. Based on the previous assumption that physical states $\omega(1420)$ and $\phi(1680)$ are the eigenvectors of mass-squared matrix, the unitary matrix $U$ can be described as

$$U = \begin{pmatrix} X_{\omega(1420)} & Y_{\omega(1420)} \\ X_{\phi(1680)} & Y_{\phi(1680)} \end{pmatrix}. \quad (11)$$

Inserting the masses of isoscalar states, we obtain the quarkonia content of $\omega(1420)$ and $\phi(1680)$ as follows:

$$|\omega(1420)\rangle = -0.997 |N\rangle - 0.074 |S\rangle, \quad (12)$$

$$|\phi(1680)\rangle = 0.074 |N\rangle - 0.997 |S\rangle.$$

3. The Results and Conclusions

In summary, based on the mass matrix and Regge trajectory, we investigate the masses of $K^*(1410)$, $D$, and $D_s$ in the $2^3S_1$ meson state. The mass of $K^*(1410)$ is determined to be 1579.82 MeV; the results is consistent with our previous work. For the heavy-light meson section, we predicted the mass of charmed-strange member to be 2700 MeV. The value is larger than $D_{s3}(2632)$ and in agreement with the $D_{s3}(2700)$. Moreover, on the basis of unusual decay modes and narrow width of the $D_{s3}(2632)$, we suggest that the $D_{s3}(2700)$ should be the first radial excitation of the $D_{s3}^*(2112)$. As a byproduct, the quarkonia content of $\omega(1420)$ and $\phi(1680)$ was offered, which implies that $\omega(1420)$ is pure $u\bar{u}+d\bar{d}$ state and $\phi(1680)$ is pure $s\bar{s}$ state. Our results should be useful for the assignment and the identification of the member of the $2^3S_1$ meson state in the new experiment.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

## Acknowledgments

This project was supported by Zhengzhou University of Light Industry Foundation of China (Grant nos. 2009XJ101 and 2012XJ0008) and the Key Project of Scientific and Technological Research of the Education Department of Henan Province (Grant no. 13B140332).

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