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Comparative Analysis of Naira/US Dollar Exchange Rate Volatility Using GARCH Variant Modeling

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Abstract

This paper employed the GARCH variance models to examine the return volatilities of official bank, interbank and Bureau de change. Using the monthly exchange rate of Naira/USD from January 2004 to September 2020 (2004:1-2020:9), it was observed that the returns were not normally distributed and were stationary at level. The power statistics of Ljung-Box Q and Ljung-Box Q\(^2\) transformed, using the powers 0.25, 0.5 and 0.75 for conditional heteroscedasticity and lags of 6, 12 and 20 to indicate conditional heteroscedasticity in all returns. The study also found exchange rate volatility in official, interbank and Bureau de change, observing that exchange rate returns were persistent. However, Bureau de change return was relatively more persistent while official exchange rate return was the least persistent. Also, it can be said that leverage effect exists in all the three exchange rate returns; while asymmetric model was the best model to estimate the exchange rate, IGARCH was not a suitable model to estimate the exchange rate return in Nigeria. There is also a need to incorporate the impact of news when developing an exchange rate policy by the monetary authority in Nigeria.

**Keywords:** exchange rate volatility, GARCH variant, leverage effects, Naira/USD, persistency

**JEL Code:** G, G1, G12

Introduction

Nigeria is an open economy with trading partners worldwide. The stability of its exchange rate or otherwise has far reaching implications on Nigeria’s current and capital account, foreign direct investment and polio investment. Also, the stability of the country’s currency plays an important role on the cross border currency transactions especially when the investor usually weighs the risk associated with the exchange rate when making international investments while also assessing the political risks involved. While a country may think that depreciation of its

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currency is an opportunity to increase exports, it could adversely affect the domestic output, especially when intermediate inputs are imported into the country for production activities. Foreign investor weighs the exchange rate volatility against the anticipated profit before investing in a particular economy. Furthermore, export-import activities are significantly affected by the volatility of the exchange rate, because following the depreciation/appreciation of exchange rates the invoicing currency has an important implication on the importer’s cost especially in terms of credit trade.

Foreign exchange market in Nigeria is divided into three markets with distinct rates which are operational side by side. For instance, the official foreign exchange market is operated by the Central bank of Nigeria (CBN) as the buyer and seller of foreign exchange to banks through the weekly Wholesale Dutch Auction system and Bureau de change operators. It’s also serving as the regulator in the foreign exchange market. The interbank market is the market where foreign exchange is bought and sold between banks in Nigeria, multinational oil company (IOC), Nigeria national petroleum company (NNPC) and other companies dealing in foreign trade. The last segment of foreign market is the Bureau de change; which was established in 1989 to cater to the end user of foreign exchange in Nigeria. It provides services such as personal travel allowance, school fee to students studying abroad, medical bills and credit card payment among others.

The above arrangement was meant to ensure stability in foreign exchange market in Nigeria by providing foreign exchange to those who need foreign currencies. However, foreign exchange rate continues to be volatile with unprecedented rates different from those in the markets. For instance, Emenike (2016) compared volatility persistent in official, interbank and bureau de change and found bureau de change market volatility was explosive while Oyinlola (2018) examined the impact of past volatility on current volatility in interbank and bureau de change and found past volatility played a significant role in the current volatility in interbank and Bureau de exchange. The study examines three foreign exchange markets in this present study. However, there is a need to account for recent developments in the foreign exchange rate market, thus the impact of structural breaks in these rates cannot be overemphasized. This is the gap, this current study has identified to address in this research.

Following the introduction is the literature review, the next section deals with the methodology, followed by analysis and discussion of the result while the last section provides the concluding remarks and recommendations.
Literature review

Economic literature is replete with studies that examine the effect of exchange rate volatility on economic growth and determine the exchange rate volatility persistency between fixed and floating exchange rate system in Nigeria. For instance, Ehikioya (2019) examined exchange rate volatility in Nigeria, using monthly data for the period of January 1980 to December 2019. The study found that the exchange rate volatility of Naira against US Dollar is persistent during the period of analysis and has a negative impact on the economy of Nigeria. In the same vein, Musyoki et al. (2012) used monthly data and employed GARCH and generalized moment method to study the volatility of Kenya’s exchange rate, for the period of January 1993 to December 2009. They found Kenya’s exchange rate volatility was persistent throughout the period and thus had a negative impact on its economic growth.

Kuhe et al. (2018) examined exchange rates returns of Naira vis-à-vis Euro, UK Pound Sterling, CFA, US Dollar and West African Unit of Account (WAUA) as well as Japanese Yen, using daily data for the period of 11th December 2001 to 13th April 2018. They employed symmetric and asymmetric GARCH methods with non-Gaussian errors. The result from EGARCH (1.1) found CFA and US Dollar has the highest and least volatility among the exchange rate returns respectively. They also found the presence of volatility clustering and shocks were persistent in all the six exchange rate returns. They also found evidence of leverage effects in all return series. In a single country study, Oyinlola (2018) examined exchange rate return volatility persistent and asymmetric of Naira against US dollar exchange rate for interbank and Bureaux de exchange (BDC) using monthly data from January 2004 to November 2017. The study employed Threshold GARCH [T-GARCH (1.1)] and Exponential GARCH [E-GARCH (1,1)] as well as Bai-Parron (2003) unit root with break to capture the impact of structural break on the returns volatility. The study found two break dates in 2014 and 2015 and explosive volatility in BDC while the interbank was high but not explosive. Also, it was found that symmetric model is best for interbank return while asymmetric appears the best in BDC market respectively.

Emenike (2016) carried out a comparative analysis of the exchange rate volatility in official and interbank markets as well as the Bureau de exchange rate markets. The study employed GARCH (1, 1) and GJR-GARCH (1,1) for the period of January 1995 to December 2014. The study found past volatility in interbank and Bureaux de change rates to significantly influence their parent volatility and also observed that volatility clustering was present in both markets.
The study also found volatility persistent and clustering was more common in the Bureaux de change market than others markets. It also deduced that depreciation of exchange rate aggravates volatility in immediate future in both interbank and Bureau de change markets.

Ajayi et al. (2019) examined daily exchange rate returns of Naira against six currencies, such as Chinese Yuan, Indian Rupees, Spain Euro, UK Pound and US Dollar for the period of January 2012 to August 2019. The study employed GARCH (1,1), EGARCH (1,1), TGARCH (1,1) and GJR-GARCH(1,1) models. The study found high volatility and no leverage effect in all estimates without break and GJR-GARCH was the best model for all the exchange rate returns.

Bala and Asemota (2013) examined exchange rate volatilities of Naira against US Dollar and UK Pound for the period of January 1985 to July 2011 for Naira/US Dollar, January 2004 to January 2011 for Naira/British Pounds and Naira/Euro returns. The employed variant of GARCH models was examined with and without break. They used exogenous to determine break for US Dollar. The study found that volatility is persistent in all the three exchange rates and that all asymmetry models without break reject leverage effect; while models with break showed the presence of leverage effect in all the three currencies. The study further advocates the inclusion of break on the estimate of volatility in exchange rate returns as does the improved or reduced rate of volatility persistent. In a related analysis, Musa et al. (2014) examined daily exchange rate of Naira against US Dollar for the period of June 2000 to July 2011. They employed symmetry and asymmetry GARCH models. The study found significant asymmetry effects of exchange return and the loose function such as MAPE, MAE, RMAE while Theil inequality coefficient found T-GARCH model is the best model for forecast purposes. Also, Abdullah et al. (2017) examined daily exchange rate volatility for Naira against US Dollar for the period of 1st January 2008 to 30th April 2015. The study employed symmetry and asymmetry models. The study found in contrast to normal distribution, student t-distribution improved the model forecast performance and satisfied the diagnostics statistics. Afees (2011) examined the extend of Naira exchange rate volatility against US Dollar, using daily return series for the period of sustainable democracy based on sub-period of democratic transition of 05/29/1999-05/28/2003; 05/29/2003-05/28/2007; and 05/29/2007-05/28/2011 and employed variant of GARCH models. The study found exchange rate behavior change in short time, and that leverage and persistence vary over time.
Methodology

The paper employed GARCH, EGARCH, APARCH, IGARCH, TARCH and GARCH with structural break in volatility modeling; this is done to see whether structural break will improve our result.

The GARCH model is an extension of the ARCH, thus the GARCH model incorporates past conditional variances into current conditional variance equation.

The GARCH model is formulated as follows:

\[
\sigma_i^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i-1}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2
\]

Where \( p \geq 0, q > 0, \sigma > 0, \alpha_i \geq 0, \beta_j \geq 0, i=1,2...,p, j=1,2...,q. \)

Equation (1) is the GARCH \((p,q)\) model where \( p \) and \( q \) denote the lags terms of the squared error term and conditional variance respectively. This implies, the current conditional variance is the function of past shocks (ARCH term) and past variances (GARCH term). From equation (1) the trader predicts its current volatility by taking the weighted average of the long term mean (the constant), thus the information observed from previous period volatility (the ARCH term) and forecasted variance from the previous period (The GARCH). Where \( \sigma \) is the constant, \( \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i-1}^2 \) is the ARCH term and GARCH effect \( \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \) is the GARCH term.

Equation (1) will be stationary if the sum of the ARCH and GARCH \( \sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j \) is less than 1.

Equation (1) can be extended by adding an exogenous variable or dummy variable to account for structural break in the variance equation.

\[
\sigma_i^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i-1}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2
\]

Where \( \text{dum}_{1t}, \text{dum}_{nt} \) are dummy variables representing periods of key policy changes in the foreign exchange market and exogenous shocks (0 for normal periods and 1 for periods of high currency movements). We determined periods of high currency movements by detecting sudden jumps or outliers resulting from
Comparative Analysis of Naira/US Dollar Exchange Rate Policy Changes and Other Exogenous Shocks. Consequently, a higher order GARCH model, expressed as GARCH (p,q) is given by:

\[ \sigma_i^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{i-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{i-j}^2 \]  

Where p and q are lags order of ARCH term and GARCH term respectively and k lag order of dummy variables.

In addition, the integrated GARCH (p,q) or IGARCH(p,q) model is expressed as follows: Engle and Bollerslev (1986) extend a standard GARCH(1,1) model to an IGARCH(1,1) model by imposing the restriction that \( \alpha_1 + \beta_1 = 1 \). An IGARCH(p,q) is expressed thus:

\[ \sigma_i^2 = \sum_{i=1}^{p} \alpha_i \varepsilon_{i-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{i-j}^2 \]  

Such that  

\[ \sum_{i=1}^{q} \alpha_i \varepsilon_{i-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{i-j}^2 = 1 \]  

This model imposing restriction that \( \alpha_1 + \beta_1 = 1 \) and assuming the constant term is equal to zero, for detailed exposition see (Nelson, 1990) when \( \alpha_1 + \beta_1 > 1 \) and constant is greater than zero (\( \omega > 0 \)). Furthermore, Nelson's (1991) proposed an EGARCH model to allow for asymmetric effects between positive and negative shock to asset return. An EGARCH (p,q) model is expressed as:

\[ \log \sigma_i^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{i-i} - \frac{2}{\pi} \left( \sum_{j=1}^{q} \beta_j \log(\sigma_{i-j}) + \sum_{k=1}^{r} \gamma_k \frac{\varepsilon_{i-k}}{\sigma_{i-k}} \right) \]  

Where \( \omega, \alpha_i, \beta_j \) and \( \gamma_k \) are constant parameters. The EGARCH (p,q) model, unlike the GARCH (p, q) model, indicates that the conditional variance is an exponential function. The asymmetric effect of past shocks is captured by the \( \gamma \) coefficient, which is usually negative, that is, positive shocks generate less volatility than negative shocks (Longmore & Robinson, 2004). The leverage effect can be tested if \( \gamma < 0 \). If \( \gamma \neq 0 \) the news impact is asymmetric. Similarly, TGARCH Model also known as GJR-GARCH is employed related to the transformation to estimate the leverage effects on the conditional standard deviation. This model takes the following form:

\[ \sigma_i^2 = \omega + \sum_{i=1}^{p} (\alpha_i + \gamma_i N_{i-i}) \varepsilon_{i-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{i-j}^2 \]  


Where $N_{t-i}$ is an indicator of negative $\varepsilon_{t-i}$, that is;

$$N_{t-i} = \begin{cases} 1 & \text{for } \varepsilon_{t-i} < 0 \\ 0 & \text{for } \varepsilon_{t-i} \geq 0 \end{cases}$$

Or

$$\sigma_i = \sigma + \sum_{i=1}^{p} \alpha_i \varepsilon^2_{i-i} + \sum_{j=1}^{q} \beta_j \sigma^2_{i-j} + \sum_{k=1}^{r} \gamma_k \varepsilon^2_{i-k} I_{i-k}$$

Where $I_{i}$ is a dummy variable, 1 if $\varepsilon_{i} < 0$ and 0 otherwise. In the GJR-GARCH model, good news $\varepsilon_{t-i}>0$ and bad news, $\varepsilon_{t-i} < 0$, have differential effects on the conditional variance; good news has an impact of $\alpha_i$ while bad news has an impact of $\alpha_i + \gamma$. If $\gamma_i > 0$, bad news increases volatility, and there is a leverage effect for $i$-th order. If $\gamma \neq 0$, the news impact is asymmetric. Also, TS-GARCH model usually used to capture the information contained in the fat tails and is characterized to return distribution of speculative prices. The model is thus expressed as;

$$\sigma_i = \sigma + \sum_{i=1}^{p} \alpha_i |\varepsilon_{i-i}| + \sum_{j=1}^{q} \beta_j \sigma^2_{i-j}$$

The asymmetry power ARCH (APARCH) model developed by Ding et al. (1993) also, allows for asymmetric effects of shocks on conditional volatility. The APARCH ($p$, $q$) model is hereby expressed as follows:

$$\sigma^\delta_i = \sigma + \sum_{i=1}^{p} \alpha_i (|\varepsilon_{i-i}|)^\delta + \sum_{j=1}^{q} \beta_j \sigma^2_{i-j}$$

Where $\delta > 0$, $|\gamma_i| \leq 0$ for $i=\ldots,r$, $\gamma_r > 0$ for all $i>r$, and $r \leq p$ if $\gamma \neq 0$ shock impact is not asymmetrical. The power parameter of the standard deviation can be estimated rather than imposed, and $\gamma$ parameters are added to capture asymmetry of up to order $r$. The assumption of normality in modeling financial data, which restricts $d$ to either 1 or 2, is often a denial of reality due to significant skewness and kurtosis (Longmore & Robinson, 2004).

**Data Description and Source**

The data for the study consists of monthly exchange rate of Naira/USD from January 2004 to September 2020 (2004:1-2020:9) for official rate, interbank and Bureau de change exchange rates observations. The exchange rates were obtained from Central bank of Nigeria statistical bulletin. Here we employed continuously compounding returns due to its advantages over the simple net returns as well as
its attractive statistical properties. The returns are defined as \( r_t = \log(e_t / e_{t-1}) = \log(e_t) - \log(e_{t-1}) \), where \( r_t \) is the exchange rate return, \( e_t \) is the spot rate of Naira/USD at time \( t \) and \( e_{t-1} \) is the spot rate of Naira/USD exchange rate at time \( t-1 \).

**Data Analysis and Result Discussion**

**Table 1**

*Descriptive Statistics and Autocorrelation of Naira exchange rate (Raw)*

| Statistics          | Official Rate | Interbank rate | Bureau de change (BDC) |
|---------------------|---------------|----------------|------------------------|
| Mean                | 1.0003        | 1.0005         | 1.0007                 |
| Median              | 1.0000        | 0.9999         | 1.0000                 |
| Maximum             | 1.0293        | 1.0274         | 1.0303                 |
| Minimum             | 0.9936        | 0.9929         | 0.9839                 |
| Std. Dev.           | 0.0033        | 0.0038         | 0.0051                 |
| Skewness            | 6.1456        | 3.9111         | 1.9787                 |
| Kurtosis            | 51.1674       | 25.2299        | 13.8430                |
| Jarque-Bera         | 13694.42      | 3077.602       | 738.3367               |
|                     | (0.000)       | (0.000)        | (0.000)                |
| Observations        | 133           | 133            | 133                    |
| Ljung Box Q Statistics |              |                |                        |
| Q(1)                | 0.399**       | 0.488**        | 0.371**                |
|                     | (0.000)       | (0.000)        | (0.000)                |
| Q(5)                | 0.009**       | -0.026**       | 0.024**                |
|                     | (0.001)       | (0.000)        | (0.000)                |
| Q(10)               | -0.019**      | -0.063**       | -0.061**               |
|                     | (0.001)       | (0.000)        | (0.000)                |

*Note.* figure in parentheses are p-value ** indicates significant at 5 percent level

Source: Authors’ computation

Table 1, shows the descriptive statistics of Naira/USD exchange rate, Bureau de change has the highest mean while official rate has less mean value, the official rate and Bureau de change has the highest median value of 1.000 while interbank rate has the least median value (0.999). The maximum or the highest value was for Bureau de change 1.03 while interbank rate has the least maximum value (1.02). Also, Official rate has the highest minimum rate (0.993) while interbank has the least minimum rate (0.992). Standard deviation which measures the volatility of the rate showed that Bureau de change was the most volatile
while official rate was the least volatile of the rates observed. The skewness of the rates further showed that all the rates were positively skewed as against the normal distribution (0 indicates skewness for the normal distribution of rates), an indication of asymmetry distribution and Kurtosis were far greater than 3 for a normal distribution of all the rates. Skewness also indicates a non-normal distribution and the large kurtosis series are leptokurtic, providing evidence of fat tails. The JB test further confirms the non-normality of distribution with a probability of (0.000) for all rates. The Ljung Box Q statistics for lags of 1, 5 and 10 considered were significant at 5 percent, indicating autocorrelation (serial correlation) in the rates for all exchange rate return. The Q-Q plot for official rate, interbank rate and Bureau de change exchange rate returns and diagrams clearly show a marked departure from the normality graphs.

Having found that our series are non-normal, the usual method of testing conditional homoscedasticity by testing autocorrelation in squared return series is inappropriate. As opined by Mckenzie (1997) volatility clustering is not unique to squared returns of assets price. Absolute changes in an assets price usually exhibit volatility clustering, hence, inclusion of power term amplified relative period of tranquility and volatility by identifying outliers in the returns.

Again, we perform conditional homoscedasticity by testing for autocorrelation of power transformed for the exchange rate returns of the following: official, interbank and Bureau de change using powers 0.25, 0.5 and 0.75. The Ljung-Box $Q^20.25$, $Q^20.5$ and $Q^20.75$ statistics for the three exchange rate returns at 5 percent critical value are significant for all the lags and powers implying the presence of conditional heteroscedasticity.

**Figure 1**

*Volatility Clustering of Official Exchange Rate Return*

![Graphs showing volatility clustering of Official Exchange Rate Return](image)
Figure 2
Volatility Clustering of Interbank Exchange Rate Return

Figure 3
Volatility Clustering of Bureau de Change Exchange Rate Return

Figure 1, 2 and 3 clearly show the presence of volatility clustering, where periods of high volatility are followed by periods of high volatility while period of low volatility are followed by period low volatility. The official return tends to be more clustered with spike in 2009 while Bureau de change is relatively less clustered of all the returns with spike in 2008.
Table 2

Autocorrelation of Power Transformed Return Series Using Powers of 0.25, 0.5 and 0.75

| Ljung-Box Q^{0.25} statistics | Official rate | Interbank Rate | Bureau de change |
|-------------------------------|--------------|----------------|------------------|
| Box Q^{0.25}(6)               | 0.09402*     | -0.004*        | -0.117*          |
|                               | (0.001)      | (0.000)        | (0.000)          |
| Box Q^{0.25}(12)              | -0.076*      | -0.039*        | 0.110*           |
|                               | (0.012)      | (0.000)        | (0.000)          |
| Box Q^{0.25}(20)              | -0.010*      | 0.081*         | -0.113*          |
|                               | (0.017)      | (0.000)        | (0.000)          |

| Ljung-Box Q^{0.5} statistics |
|-------------------------------|--------------|----------------|------------------|
| Box Q^{0.5}(6)               | 0.020*       | -0.004*        | -0.116*          |
|                               | (0.001)      | (0.000)        | (0.000)          |
| Box Q^{0.5}(12)              | -0.075*      | 0.0399*        | 0.110*           |
|                               | (0.012)      | (0.000)        | (0.000)          |
| Box Q^{0.5}(20)              | 0.002*       | 0.081*         | -0.113*          |
|                               | (0.017)      | (0.000)        | (0.000)          |

| Ljung-Box Q^{0.75} statistics |
|-------------------------------|--------------|----------------|------------------|
| Box Q^{0.75}(6)               | -0.116*      | -0.004*        | -0.116*          |
|                               | (0.000)      | (0.000)        | (0.000)          |
| Box Q^{0.75}(12)              | 0.111*       | -0.039*        | 0.111*           |
|                               | (0.000)      | (0.000)        | (0.000)          |
| Box Q^{0.75}(20)              | -0.013*      | 0.081*         | -0.013*          |
|                               | (0.000)      | (0.000)        | (0.000)          |

Note. figure in parentheses are p-value * indicates significant at 5 percent level
Source: Authors’ computation

Table 3 displayed unit root test result which shows all returns were stationary at level, this is discernable by comparing the ADF and PP test statistics with critical value of 1%, 5% and 10% were greater than respective critical value at level implying returns are integrated of order zero I(0).
### Table 3

**Unit Root Test Result**

| Variables           | Statistics  | ADF Critical Value | PP Critical Value |
|---------------------|-------------|--------------------|-------------------|
|                     |             | 1%                 | 5%                | 10%              | 1%       | 5%       | 10%       |
| Official Rate       | -14.831*    | -4.949             | -4.443            | -4.193           | -7.343*  | 0.000    | -3.480    | -2.883    | -2.579    |
|                     | (0.01)      |                    |                   |                  |          |          |          |          |          |
| Interbank Rate      | -11.839*    | -4.949             | -4.443            | -4.193           | -7.800*  | 0.000    | -3.463    | -2.876    | -2.575    |
|                     | (0.01)      |                    |                   |                  |          |          |          |          |          |
| Bureau de change    | -10.237*    | -4.949             | -4.443            | -4.193           | -9.646*  | 0.000    | -3.466    | -2.876    | -2.575    |
|                     | (0.01)      |                    |                   |                  |          |          |          |          |          |

*Note.* figure in parentheses are p-value * indicates significant at 5 percent level

Source: Authors’ computation

### Table 4

**Estimates of GARCH Models Official Rate Return, January 2004 – September 2020**

|               | GARCH | GJR-GARCH | EGARCH | APARCH | IGARCH | TS-GARCH |
|---------------|-------|-----------|--------|--------|--------|----------|
| Mean equation |       |           |        |        |        |          |
| C             | 0.999 | 0.999     | 0.999  | 0.999  | 0.999  | 0.999    |
|               | (2.280)| (1.890)   | (2.790)| (2.550)| (1.905)| (2.490)  |
| Variance Equation |      |           |        |        |        |          |
| ϑ             | 2.750 | 2.130     | -3.271 | 1.950  | 1.310  |          |
|               | (1.310)| (1.250)   | (0.561)| (1.060)| (1.610)|          |
| α             | 0.701 | 0.812     | 0.762  | 0.811  | 0.061  | 0.712    |
|               | (0.025)| (0.054)   | (0.041)| (0.009)| (0.026)| (0.036)  |
| β             | 0.148 | 0.024     | 0.201  | 0.069  | 0.311  | 0.116    |
|               | (0.054)| (0.001)   | (0.038)| (0.042)| (0.026)| (0.073)  |
| γ             | 5.941*| -1.535*   | 0.226* |       |        | 6.994    |
|               | (3.102)| (0.885)   | (0.110)|        |        | (5.340)  |
| δ             |       |           |        | 0.146  |        |          |
|               |       |           |        | (0.100)|        |          |
| V             | 2.406 | 2.208     | 2.012  | 2.084  | 2.677  | 2.138    |
|               | (0.193)| (0.935)   | (0.013)| (0.139)| (0.130)| (0.177)  |
| LL            | 827.679| 835.758   | 9344.422| 842.539| 805.763| 842.857  |
|                | GARCH | GJR-GARCH | EGARCH | APARCH | IGARCH | TS-GARCH |
|----------------|-------|-----------|--------|--------|--------|----------|
| Pers.          | 0.849 | 0.836     | 0.963  | 0.880  | 0.372  | 0.828    |
| AIC            | -12.371 | -12.463 | -11.697 | -12.564 | -12.072 | -12.584  |
| SC             | -12.262 | -12.457 | -11.567 | -12.412 | -12.006 | -12.453  |
| HQC            | -12.326 | -12.521 | -11.645 | -12.502 | -12.045 | -12.531  |
| N              | 133    | 133       | 133     | 133     | 133     | 133      |

*Notes.* Standard errors are in parentheses. * indicates significant at the 5% level. LL, AIC, SC, HQC and N are the maximum log-likelihood, Akaike information Criterion, Schwarz Criterion, Hannan-Quinn criterion and Number of observations respectively. Source: Authors’ computation.

### Table 5

*Estimates of GARCH Models Interbank Rate Return, January 2004-September 2020*

|                | GARCH | GJR-GARCH | EGARCH | APARCH | IGARCH | TS-GARCH |
|----------------|-------|-----------|--------|--------|--------|----------|
| **Mean equation** |       |           |        |        |        |          |
| C              | 1.000 | 1.000     | 0.999  | 1.000  | 1.000  | 1.000    |
|                | (0.000) | (0.002) | (0.000) | (0.000) | (6.570) | (0.001)  |
| **Variance Equation** |       |           |        |        |        |          |
| σ              | 4.120 | 3.206     | -2.977 | 1.690  | 4.620  |          |
|                | (2.640) | (1.514) | (0.0521) | (0.000) | (6.740) |          |
| α              | 0.209 | 0.890     | 0.521  | 0.156  | -0.009 | 0.950    |
|                | (0.561) | (0.403) | (0.206) | (0.316) | (0.002) | (0.140)  |
| β              | 0.511 | 0.021     | 0.219  | 0.656  | 1.001  | -0.160   |
|                | (0.022) | (0.019) | (0.046) | (0.134) | (0.012) | (0.007)  |
| γ              | 2.613* | -4.160*   | 0.057* | 1.312  |        |          |
|                | (1.215) | (2.434) | (0.010) | (2.388) |        |          |
| δ              | 0.441 |          |        |        |        |          |
|                | (0.049) |        |        |        |        |          |
| V              | 2.139* | 2.145*    | 2.005* | 2.223* | 2.261* | 2.123*   |
|                | (0.134) | (0.407) | (0.006) | (0.154) | (0.047) | (0.204)  |
| LL             | 1083.049 | 1099.647 | 1016.967 | 1077.605 | 977.8761002.049 |          |
| Pers.          | 0.720 | 0.911     | 0.740  | 0.812  | 0.992  | 0.790    |
| AIC            | -10.834 | -10.020 | -10.161 | -10.759 | -9.797 | -10.016  |
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|        | GARCH | GJR-GARCH | EGARCH | APARCH | IGARCH | TS-GARCH |
|--------|-------|-----------|--------|--------|--------|----------|
| SC     | -10.751 | -9.919 | -10.061 | -10.644 | -9.748 | -9.917 |
| HQC    | -10.801 | -9.985 | -10.120 | -10.712 | -9.778 | -9.976 |
| N      | 199 | 199 | 199 | 199 | 199 | 199 |

Notes. Standard errors are in parentheses. * indicates significant at the 5% level. LL, AIC, SC, HQC and N are the maximum log-likelihood, Akaike information Criterion, Schwarz Criterion, Hannan-Quinn criterion and Number of observations respectively. Source: Authors’ computation

Table 6

Estimates of GARCH Models for Bureau de Change Rate Return, January 2004-September 2020

|        | GARCH | GJR-GARCH | EGARCH | APARCH | IGARCH | TS-ARCH |
|--------|-------|-----------|--------|--------|--------|---------|
| Mean equation |
| C      | 0.999 | 1.000 | 0.999 | 0.999 | 0.999 | 0.999 |
|        | (0.002) | (0.001) | (0.000) | (0.007) | (0.000) | (0.051) |
| Variance Equation |
| Π      | 0.000 | 4.130 | -1.762 | 5.580 | 0.000 |
|        | (0.013) | (6.690) | (0.317) | (0.004) | (0.014) |
| A      | 0.417 | 0.952 | 0.022 | 0.801 | 0.618 | 0.802 |
|        | (0.272) | (0.267) | (0.107) | (0.410) | (0.024) | (0.340) |
| B      | 0.355 | -0.107 | 0.859 | 0.026 | 0.361 | -0.024 |
|        | (0.081) | (0.059) | (0.025) | (0.007) | (0.024) | (0.003) |
| Γ      | 0.027* | 0.579* | -0.667* | 1.520* |
|        | (0.000) | (0.178) | (0.213) | (0.206) |
| Δ      | 0.470 |
|        | (0.078) |
| V      | 2.001* | 2.340* | 2.349* | 2.223* | 3.397* | 2.001* |
| Model      | GARCH | GJR-GARCH | EGARCH | APARCH | IGARCH | TS-ARCH |
|------------|-------|-----------|--------|--------|--------|---------|
|            |       | 0.118     | 0.315  | 0.256  | 0.154  | 0.241   | 0.105   |
| LL         | 862.857 | 864.341   | 853.070| 871.769| 839.634871.570|
| Pers.      | 0.772  | 0.845     | 0.881  | 0.827  | 0.979  | 0.756   |
| AIC        | -8.622 | -8.749    | -8.513 | -8.691 | -8.408 | -8.699  |
| SC         | -8.538 | -8.580    | -8.413 | -8.575 | -8.358 | -8.599  |
| HQC        | -8.588 | -8.675    | -8.473 | -8.644 | -8.388 | -8.659  |
| N          | 199    | 199       | 199    | 199    | 199    | 199     |

Notes. Standard errors are in parentheses. *indicates significant at the 5% level. LL, AIC, SC, HQC and N are the maximum log-likelihood, Akaike information Criterion, Schwarz Criterion, Hannan-Quinn criterion and Number of observations respectively. Source: Authors’ computation

Table 4 shows the sum of α and β in the GARCH, GJR-GARCH, EGARCH, APARCH, IGARCH model were less than 1, indicates the variance process are mean reverting and that shocks to volatility will die down slowly, thus the variance process revert slowly to their mean, except for IGARCH that has a rapid mean reversion process to it mean. In table 5, the sum of α and β for GJR-GARCH and IGARCH were close to 1 which is an indication of slow mean reverting process, implying that shock to volatility will die down slowly while GARCH, EGARCH, APARCH and TS-GARCH has fast mean reverting process and shock to their variance means that it will revert quickly to their mean. In the same vein, table 6 shows the sum of α and β were all less than 1, indicating mean reverting process and shock to volatility will die down relative slowly for GARCH, GJR-GARCH, EGARCH, APARCH and TS-GARCH. However, IGARCH is close to 1 implying a very sluggish mean reverting process and shock to volatility will die down rather slowly. In a nutshell, bureau de-change volatility was most persistent, followed by the official and interbank rate which were the least volatile of the three returns examined within the period.

Table 4, 5 and 6 present γ coefficients, which measure symmetry and leverage effects, in table 4, two were positive and statistically significant at 5% level in GJR-GARCH and APARCH models and negative while significant in EGARCH model. Leverage effect exists, if γ > 0 in the GJR-GARCH and APARCH models and γ < 0 in EGARCH. In view of the above, we cannot reject null hypothesis of leverage effect for GJR-GARCH, APARCH and EGARCH models, this implies
that negative shock to volatility exerts more impact on volatility than positive shock of equivalent magnitude. Table 5, also shows γ coefficients with positive effect and significant in GJR-GARCH and APARCH and negative and significant in EGARCH model. We cannot reject the null hypothesis of leverage effect in GJR-GARCH, APARCH and EGARCH models; this implies that negative shock exerts more impact on the interbank exchange return than positive shock of equivalent magnitude. Furthermore, table 6, shows γ coefficients were positive and significant in GJR-GARCH, EGARCH, APARCH, TS-GARCH and positive and significant in EGARCH model. Since EGARCH is positive and we reject the null hypothesis: because we need the negative significant for leverage effect to exist, hence, we reject the leverage effect in EGARCH model but cannot reject the null hypothesis of leverage effect in GJR-GARCH, APARCH and TS-GARCH models. It implies that negative shock exerts more impact on Bureau de change return than positive shock of equivalent magnitude. As expected bureau de change return was the most volatile followed by official rate and the interbank return being the least volatile. As seen in preliminary investigation in table 1, the returns were not normally distributed, hence, we employed student t to estimate our models and degree of freedom represented by V coefficients were statistically significant at 5 percent level in all models as presented in tables 4, 5 and 6, thus validating the use of student t instead of normality assumption.

**Diagnostic Test**

Table 7, 8 and 9 show the diagnostic tests for the returns of official, inter-bank and bureau de change models. The Ljung-Box Q test statistics for autocorrelation of standardized residuals at 5 percent significant level shows that autocorrelation of standardized residuals are statistically insignificant for all lags. Hence, we cannot reject the null hypothesis of no autocorrelation in standardized residuals. The Ljung-Box Q²-statistics of squared standardized residuals in Tables 7, 8 and 9 are statistically insignificant at 5 percent significant level for all lags. Hence, we cannot reject null hypothesis of no autocorrelation in squared standardized residuals. The ARCH-LM test statistics presented in tables 7, 8 and 9 show that the standardized residuals did not exhibit ARCH effect anymore or that the ARCH effect has been adequately taken out. And Jarque-Bera statistics still indicates standardized residuals were non-normally distributed.
### Table 7
**Autocorrelation of Standardized Residuals, Autocorrelation of Squared Standardized Residuals and ARCH LM and Normality test for Official return**

|                  | Ljung-Box Q-Statistics | Ljung-Box Q-Statistics | ARCH LM | NML |
|------------------|------------------------|------------------------|---------|-----|
|                  | Q(6) Q(12) Q(20) Q^2(6) Q^2(12) Q^2(20) F N*R^2 JB |
| GARCH            | -0.009 -0.016 -0.009 -0.009 -0.009 -0.010 0.010 0.010 651 |
|                  | (1.000) (1.000) (1.000) (1.000) (1.000) (1.000) (0.918) (0.917) (0.000) |
| GJR-ARCH         | 0.010 0.015 0.009 -0.009 -0.010 -0.011 0.015 0.015 442 |
|                  | (1.000) (1.000) (1.000) (1.000) (1.000) (1.000) (0.900) (0.899) (0.000) |
| EGARCH           | 0.002 -0.029 0.000 -0.011 -0.011 -0.011 0.010 0.011 426 |
|                  | (1.000) (0.998) (1.000) (1.000) (1.000) (1.000) (0.917) (0.916) (0.000) |
| APARCH           | -0.010 -0.014 -0.008 -0.009 -0.010 -0.011 0.015 0.012 446 |
|                  | (1.000) (1.000) (1.000) (1.000) (1.000) (1.000) (0.901) (0.900) (0.000) |
| IGARCH           | -0.009 -0.010 -0.014 -0.010 -0.010 -0.011 0.019 0.020 347 |
|                  | (1.000) (1.000) (1.000) (1.000) (1.000) (1.000) (0.888) (0.887) (0.000) |
| TS-GARCH         | 0.010 0.014 0.009 -0.009 -0.010 -0.011 0.015 0.015 443 |
|                  | (1.000) (1.000) (1.000) (1.000) (1.000) (1.000) (0.900) (0.999) (0.000) |

*Note.* Figures in parentheses are p-values

**Source:** Authors’ computation

### Table 8
**Autocorrelation of Standardized Residuals, Autocorrelation of Squared Standardized Residuals and ARCH LM and Normality test for interbank return**

|                  | Ljung-Box Q-Statistics | Ljung-Box Q-Statistics | ARCH LM | NML |
|------------------|------------------------|------------------------|---------|-----|
|                  | Q(6) Q(12) Q(20) Q^2(6) Q^2(12) Q^2(20) F N*R^2 JB |
| GARCH            | -0.007 0.007 0.007 -0.006 -0.006 -0.006 0.007 0.007 208 |
|                  | (1.000) (1.000) (1.000) (1.000) (1.000) (1.000) (0.930) (0.929) (0.000) |
| GJR-ARCH         | 0.003 -0.019 -0.044 -0.016 -0.017 -0.016 0.021 0.021 209 |
|                  | (1.000) (1.000) (1.000) (1.000) (1.000) (1.000) (0.884) (0.883) (0.000) |
| EGARCH           | -0.011 -0.020 0.011 -0.012 -0.012 -0.012 0.026 0.027 516 |

*Note.* Figures in parentheses are p-values

**Source:** Authors’ computation
Table 9 presents the ranked model used in this study, based on Maximum Log-likelihood ratio, Akaike information criteria, Schwartz information criteria.
and Hannan-Quinn criterion. Table 10 shows EGARCH was the best model, followed by TS-GARCH, APARCH, GJR-GARCH, GARCH and I-GARCH models respectively. It implied EGARCH model is the best model for forecasting purposes in official exchange rate return market. In like manner, table 11 shows GARCH is the best model followed by APARCH, EGARCH, GJR-GARCH, TS-GARCH and I-GARCH models respectively, hence, GARCH is best model for forecasting purpose in inter-banks exchange rate return market while Table12, ranked shows GJR-GARCH is the best, followed by TS-GARCH, APARCH, GARCH, EGARCH and I-GARCH models respectively. It implies that GJR-GARCH model is the best for forecasting purpose in Bureau de change exchange rate return market. In summary, asymmetric models are best suited for exchange rate return estimate of volatilities in Nigeria foreign exchange market and IGARCH is the worst of all models.

Table 10

*Official return Models Ranking in Order of maximum log-likelihood, Akaike information Criterion, Schwarz Criterion, Hannan-Quinn criterion.*

|        | LL      | AIC     | SC      | HQC     | Ranking |
|--------|---------|---------|---------|---------|---------|
| GARCH  | 827.679 | -12.371 | -12.262 | -12.326 | 5<sup>th</sup> |
| GJR-GARCH | 835.758 | -12.463 | -12.457 | -12.521 | 4<sup>th</sup> |
| EGARCH | 9344.422 | -11.697 | -11.567 | -11.645 | 1<sup>st</sup> |
| APARCH | 842.539 | -12.564 | -12.412 | -12.502 | 3<sup>rd</sup> |
| IGARCH | 805.763 | -12.072 | -12.006 | -12.045 | 6<sup>th</sup> |
| TS-GARCH | 842.857 | -12.584 | -12.453 | -12.531 | 2<sup>nd</sup> |

*Source.* Authors’ computation

Table 11

*Interbank return Models Ranking in Order of maximum log-likelihood, Akaike information Criterion, Schwarz Criterion, Hannan-Quinn criterion.*

|        | LL      | AIC     | SC      | HQC     | Ranking |
|--------|---------|---------|---------|---------|---------|
| GARCH  | 1083.049 | 10.834  | 10.751  | 10.801  | 1<sup>st</sup> |
| GJR-GARCH | 1009.647 | 10.020  | 9.919   | 9.985   | 4<sup>th</sup> |
| EGARCH | 1016.967 | 10.161  | 10.061  | 10.120  | 3<sup>rd</sup> |
| APARCH | 1077.605 | 10.759  | 10.644  | 10.712  | 2<sup>nd</sup> |
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|                      | LL     | AIC   | SC     | HQC    | Ranking |
|----------------------|--------|-------|--------|--------|---------|
| IGARCH               | 977.876| 9.797 | 9.748  | 9.748  | 6th     |
| TS-GARCH             | 1002.049| 10.016| 9.917  | 9.976  | 5th     |

*Source.* Authors’ computation

**Table 12**

*Bureau de change return Models Ranking in Order of maximum log-likelihood, Akaike information Criterion, Schwarz Criterion, Hannan-Quinn criterion.*

|                      | LL     | AIC   | SC     | HQC    | Ranking |
|----------------------|--------|-------|--------|--------|---------|
| GARCH                | 862.857| -8.622| -8.538 | -8.588 | 4th     |
| GJR-GARCH            | 864.341| -8.749| -8.580 | -8.675 | 1st     |
| EGARCH               | 853.070| -8.513| -8.413 | -8.473 | 5th     |
| APARCH               | 871.769| -8.691| -8.575 | -8.644 | 3rd     |
| IGARCH               | 839.634| -8.408| -8.358 | -8.388 | 6th     |
| TS-GARCH             | 871.570| -8.699| -8.599 | -8.659 | 2nd     |

*Source.* Authors’ computation

**Conclusion**

The paper examined the foreign exchange market volatility of Naira/US Dollar for official rate, interbank rate and Bureau de change markets. Using monthly exchange rate of Naira/USD from January 2004 to September 2020 (2004:1-2020:9), the returns were not normally distributed and stationary at level. Ljung-Box Q statistic and Ljung-Box Q² statistics of power transformed using power 0.25, 0.5 and 0.75 for conditional heteroscedasticity for lags of 6, 12 and 20 indicates present of conditional heteroscedasticity in all returns.

The sum of $\alpha$ and $\beta$ in the GARCH, GJR-GARCH, EGARCH, APACRH model were less than 1, indicating that the variance process are the mean reverting and that shocks to volatility will die down slowly, thus the variance process reverts slowly to their mean, except for IGARCH that has a rapid mean reversion process. Also, the sum of $\alpha$ and $\beta$ for GJR-GARCH and IGARCH were close to 1, an indication of slow mean reverting process, shock to volatility will die down slowly while GARCH, EGARCH, APARCH and TS-GARCH has fast mean reverting process and shock to their variance reverts quickly to their mean in interbank return. In the same vein, the sum of $\alpha$ and $\beta$ were all less than 1,
indicating that mean reverting process and shock to volatility die down relatively slowly for GARCH, GJR-GARCH, EGARCH, APARCH and TS-GARCH. However, IGARCH is close to 1, and implied a very sluggish mean reverting process and indicating that shock to volatility will die down rather slowly in bureau de change. In sum, bureau de-change volatility was the most persistent, followed by official and interbank rates thus this was the least volatile of the three. Shocks to volatilities were asymmetric in the three exchange rate returns, that is, negative shock of the same magnitude has more impact on volatilities than positive shocks. Both Ljung-Box Q test statistics for autocorrelation of standardized residuals and Ljung-Box Q²-statistics of squared standardized residuals shows there were no autocorrelation in standardized and squared standardized residuals and no ARCH effect in residuals.

The ranks of the model show that EGARCH model is best for forecasting purposes in official exchange rate return market, whereas GARCH is best for forecasting purposes in inter-banks exchange rate return market while GJR-GARCH model is best for forecasting purpose in Bureau de change exchange rate return market. In summary, asymmetric models were best suited for the estimates of exchange rate return volatilities, IGARCH being the worst in Nigeria foreign exchange return market.

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