Mathematical modelling of a single supersonic jet impingement with a flat obstacle

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Abstract. In this paper the mathematical modeling of the interaction of a single supersonic jet with a flat obstacle in a rarefied atmosphere and Mach numbers at the nozzle exit 4 are considered. Mathematical modeling using free OpenFOAM-Extended software and authors’ program are done. Comparison of calculation methods: PISO algorithm, Rusanov and Godunov schemes was carried out. The comparison results show a qualitative coincidence in the shape of the supersonic jet, wall jet and the system of shock waves. But the numerical results are quantitatively different, pressure on flat obstacle varied within 7-14 kPa. The peak of pressure is on the periphery for results of sonicFoam and dbnsFoam. For all solvers average pressure at axis of jet is 8 kPa and the first barrel of wall jet is started about at 0.4 meter from axis was obtained.

1. Introduction

Now organizations and institutes involved in the design of rocket and space equipment components are carrying out experimental and theoretical studies of the physical processes at the launch of rockets, descent and landing of spacecraft. A large number of experimental and theoretical works to the study of jet flows and their interaction with obstacles are devoted [1-12]. In experimental studies, the main study was the leakage of jets (Mach numbers at the nozzle section M<3) onto flat (inclined) obstacles under atmospheric conditions. Complication of experiments (modeling the conditions of other planets) requires a significant increase in the scope of work, its cost and the solution of a number of additional technical problems (creation of specialized test benches). In theoretical studies, cycles of work on the simulation of single and multi-block turbulent jets, and their interaction with flat obstacles when varying various parameters (Mach numbers at the nozzle section, number of jets, degrees of non-calculation, distances to obstacles, oblique inclination angles, etc.) were carried out.

The aim of the work is a study of the interaction of a supersonic jet with obstacles in a wide range of applicability, namely with a rarefied atmosphere and Mach numbers at the nozzle exit 4-5; as well as to investigate the shock-wave structure of the impingement jet and its force action on obstacle using deference solver and schemes.

2. Physical and mathematical model

The supersonic jet emanates from the nozzle and flows onto a flat, impenetrable barrier. The jet leakage, the main gas flow is inhibited and spreads around the periphery. At high pressure ratio, a fan jet appears. Several cases of jet interaction with a flat obstacle are possible. First, when the maximum pressure falls on the jet axis. The second, when the maximum pressure is formed at the periphery, in this case, the formation of self-oscillatory modes.
For the mathematical description of the interaction of supersonic jet with a flat obstacle, the turbulent flow of a viscous compressible gas in the three-dimensional approximation is considered [13-16]. The system of equations in a rectangular Cartesian coordinate system \(x_1, x_2, x_3\) for nonstationary gas flow has the form:

The continuity equation

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} \left[ \rho u_j \right] = 0
\]  

(1)

where \(\rho\) – density, kg/m\(^3\); \(t\) – time, s; \(u_j\) – velocity, m/s; \(x_j\) – coordinate, m.

The momentum equation

\[
\frac{\partial}{\partial t} \left( \rho u_j \right) + \frac{\partial}{\partial x_k} \left[ \rho u_k u_j + p \delta_{jk} - \tau_{kj} \right] = 0
\]  

(2)

where \(P\) – pressure, Pa; \(\delta_{jk}\) – Kronecker delta.

The viscous stress tensor has the following form

\[
\tau_{ij} = 2\mu \left[ \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{1}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \right]
\]  

(3)

The energy equation

\[
\frac{\partial}{\partial t} \left( \rho E \right) + \frac{\partial}{\partial x_j} \left[ \rho u_j E + u_j p + q_j - u_i \tau_{ij} \right] = 0
\]  

(4)

where \(E\) – total internal energy, J; \(q_j\) – heat-flux, J/(m\(^2\)·s).

The ideal gas equation

\[
\rho = \rho R T
\]  

(5)

where \(T\) – temperature, K; \(R\) – gas constant, J/(kg·K), indexes \(i, j, k = 1,2,3\).

To close the system of equations (1)-(5) for viscous gas \(SST k-\omega\) turbulent model is used [17].

In figure 1 computational domain and boundaries are shown. At the inlet section of the nozzle (boundary BC1), the gas flows into the computational domain at a subsonic speed. The following boundary conditions are set on the boundary of BC1: \(p = p_o\), \(T = T_o\), \(k = k_o\), \(\omega = \omega_o\), \(u_x = 0\), \(u_z = 0\). For the boundaries BC2 and BC5, the following conditions are specified: for velocity, the sticking condition \(u_x = u_y = u_z = 0\), for temperature \(\nabla T = 0\). For the kinetic energy of turbulence and the specific dissipation of the kinetic energy of turbulence, the wall functions are used. On the boundaries of BC3 and BC4 exhibited “soft” boundary conditions.

To implement the physical and mathematical model (1)-(5) and conduct numerical research free software OpenFOAM-Extended [18-19] and authors’ computational program [20] was used. For viscous gas sonicFoam solver of second order accuracy was used, where pressure and velocity as dependent variables through the PISO method.
For inviscid gas density based solver dbnsFoam and authors program of first order accuracy was used. In dbnsFoam solver was the approximate solution of the Riemann problem according to the Rusanov scheme, and in authors program Godunov method. System of linear of algebraic equations was solved using the Gauss-Seidel method. The time sampling was carried out by a four-step Runge-Kutta method. Numerical investigation were done using SKIF Cyberia supercomputer of Tomsk State University.

### 3. Numerical results

The geometric characteristics of the nozzle were as follows: nozzle throat diameter 0.03613 m, nozzle diameter 0.19395 m, and the angle of the semi-solution of the nozzle is 10 degrees. For numerical calculations at the initial time unmoved environment was assumed. The ambient pressure was 650 Pa, and temperature was 250 K. The parameters at the nozzle inlet was: pressure – 0.28 MPa, temperature – 1180 K, adiabatic index – 1.33719. The distance from nozzle exit to flat obstacle was 0.5 meter.

The results of calculations are shown in figure 2 of Mach number and in figure 3 the distribution of pressure on flat obstacle for calculation results of sonicFoam, dbnsFoam solvers and of authors’ program.

The qualitatively pattern of the distribution of Mach numbers are coincide. Local maximum of Mach numbers for sonicFoam – 6.5, dbnsFoam – 5.4, and authors’ program – 5.9. Due to second order accuracy of sonicFoam solver, it gave system of shock waves.
Figure 3. Distribution of pressure on flat obstacle

For dbnsFoam and authors’ program blurred shock wave patterns. From distribution of pressure on flat obstacle in axis of jet maximum for sonicFoam – 9 kPa, dbnsFoam – 7.5 kPa, and authors’ program – 8.2 kPa are obtained. For sonicFoam and dbnsFoam peak pressure is on the periphery: sonicFoam – 14 kPa, and dbnsFoam – 8.4 kPa. For all solvers wall jet started about at 0.4 meter. The second barrel of wall jet for sonicFoam and authors’ program is observed. The difference in the results to the applicable scheme of calculations and orders of accuracy is related.

4. Conclusion
Mathematical modeling of the interaction of a single supersonic jet with a flat obstacle in rarefied atmosphere and Mach number at the nozzle exit 4 are done. Comparison of numerical results of sonicFoam, dnsFoam, and authors’ program are done, which are used PISO algorithm, Rusanov scheme, and Godunov method. A qualitative coincidence was obtained in the shape of the supersonic jet, wall jet and the system of shock waves, but the results are quantitatively different. The average pressure at axis of jet is 8 kPa, for sonicFoam and dbnsFoam peak of pressure is on the periphery, and the first barrel of wall jet for all solvers is start about at 0.4 meter from axis was obtained.

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