On the lepton-nucleon neutral and charged current deep inelastic scattering cross sections

Xing-Long Li\textsuperscript{a,\ast}, Yu-Liang Yan\textsuperscript{a}, Xiao-Mei Li\textsuperscript{a}, Dai-Mei Zhou\textsuperscript{b}, Xu Cai\textsuperscript{b}, Ben-Hao Sa\textsuperscript{a,b,\ast}

\textsuperscript{a}China Institute of Atomic Energy, P. O. Box 275 (10), Beijing, 102413 China. 
\textsuperscript{b}Key Laboratory of Quark and Lepton Physics (MOE) and Institute of Particle Physics, Central China Normal University, Wuhan 430079, China.

Abstract

Based on the requirement in the simulation of lepton-nucleus deep inelastic scattering (DIS), we construct a fortran program LDCS 1.0 calculating the differential and total cross sections for the unpolarized charged lepton-unpolarized nucleon and neutrino-unpolarized nucleon neutral current (charged current) DIS at leading order. Any set of the experimentally fitted parton distribution functions could be employed directly. The mass of incident and scattered leptons is taken into account and the boundary conditions calculating the single differential and total cross section are studied. The calculated results well agree with the corresponding experimental data which indicating the LDCS 1.0 program is good. It is also turned out that the effect of tauon mass is not negligible in the GeV energy level.

Keywords: cross section, lepton-nucleon deep inelastic scattering, neutral current, charged current.

PROGRAM SUMMARY

Manuscript Title: On the lepton-nucleon neutral and charged current deep inelastic scattering cross sections

Authors: Xing-Long Li, Yu-liang Yan, Xiao-Mei Li, Dai-Mei Zhou, Xu Cai, Ben-Hao Sa

Program Title: LDCS version 1.0
The total cross section of the lepton-nucleon neutral current and charged current deep inelastic scattering (DIS) is required in the lepton-nucleus DIS simulations (e.g. in the PACIAE 2.2 model [1]) and in the design of DIS experiments. The incident and scattered lepton masses are usually neglected [2] in the lepton-nucleon DIS calculations. However, Ref. [3] has pointed out that the tauon mass can not be neglected in the GeV energy level.

Solution method:
The mass of the incident and scattered lepton are considered in the lepton-nucleon DIS double differential cross section at leading order and in the integral region calculating the single differential and total cross section. Correspondingly, a FORTRAN program LDCS 1.0 is constructed referring to the HERAfitter-1.0.0 [4]. The LHAPDF 5.9.1 parton distribution function (PDF) set [5] is used.

Summary of revisions:

Restrictions:
LDCS 1.0 can only be used calculating the unpolarized charged lepton- unpolarized nucleon and the neutrino-unpolarized nucleon DIS double differential, single differential, and total cross sections at leading order.

Unusual features:
Since the lepton mass is included, the LDCS 1.0 program is able to consider all 12
kinds of lepton-nucleon DIS, provided the corresponding PDF set is at hand.

Additional comments:

Running time:
Calculating the unpolarized electron-unpolarized proton neutral current DIS total cross section at $\sqrt{s}=318.7$ GeV with HERAPDF1.5 LO PDF set [6] takes 3.4 second. Calculating the neutrino-unpolarized iron charged current DIS total cross section at $\sqrt{s} = 20.0$ GeV with HKNlo PDF set [7] takes 0.46 second.

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1. Introduction

The lepton-nucleon deep inelastic scattering (DIS) has been one of the most important experiments in the high energy physics. It plays crucial role in the investigation of the electroweak and strong interactions (Quantum Chromodynamics, QCD) as well as the hadronic structure.

On the other hand, the lepton-nucleon (lepton-nucleus) DIS is significant in the studies of time evolution of the hadronization and final hadronic
state (final hadronic rescattering). In the lepton-nucleus DIS simulations the lepton-nucleon DIS total cross section is required in order to decide which nucleon among the nucleons distributed randomly in the target nucleus sphere, can collide with the incident lepton and how that DIS is evolved in the nuclear medium.

The messages of lepton-nucleon DIS cross section are always embedded in the complex parton distribution function (PDF) fitting packages, such as HERAfitter [1]. However, a simple but self-consistent program calculating the lepton-nucleon DIS differential and total cross sections based on the experimentally fitted PDF set is of benefit to the lepton-nucleus DIS simulations.

To the end, we are devoted to construct a simple but self-consistent program calculating the lepton-nucleon neutral current (NC) and charged current (CC) DIS cross sections at leading order based on the fitted PDF. As new features, the mass of the incident and scattered leptons are taken into account and all 12 kinds of leptons are covered in the constructed program LDCS 1.0.

The cross sections can only be calculated provided the PDF is known. Unfortunately, the PDF cannot be calculated in first principle and just can be parameterized via fitting the measured lepton-nucleon differential cross section to the corresponding theoretical calculation iteratively. For example, the HERA1 and ZEUS groups measured the electron-proton DIS cross sections at DESY [1], these data are then used to extract the PDF set of HERAPDF 1.5 LO [2] by HERAfitter program [1, 3].

In the section 2, we briefly introduce the theory of lepton-nucleon NC and CC DIS at leading order. We compare the calculated differential and total cross section to the corresponding experimental data in the section 3. Here the LDCS 1.0 is employed calculating the unpolarized charged lepton-unpolarized proton DIS cross sections based on the HERAPDF 1.5 LO PDF set [2] and the neutrino-unpolarized iron DIS cross sections based on the HKNlo iron PDF set [4]. A brief summary is given in the section 4. The LDCS 1.0 program is described briefly in Appendix A.

2. Neutral and charged current cross section

Figure 1 gives the Feynman diagram for the electron-proton NC (left panel) and CC (right panel) DIS at leading order (Born approximation). Correspondingly, the Kinematic variables defined in the nucleon rest frame
(adopted later on) is given in Tab. 1. We note that DIS is restricted to the lepton-nucleon inelastic scattering process with $Q^2 \gg M^2$ and $W^2 \gg M^2$.

\[ \sum_j \eta_j L^{\mu\nu}_j W_{\mu\nu} \]

Figure 1: Feynman diagram of the electron-proton NC ($ep \rightarrow eX$, left panel) and CC ($ep \rightarrow \nu X$, right panel) at leading order (Born approximation).

In the lowest-order perturbative theory, the cross section of lepton-nucleon DIS can be expressed as

\[
\frac{d^2\sigma}{dx dy} = \frac{2\pi y\alpha^2}{Q^4} \sum_j \eta_j L^{\mu\nu}_j W_{\mu\nu}
\]

where $j = \gamma, Z, \gamma Z$ for NC and $j = W^\pm$ for CC. $\alpha$ is the fine-structure constant. $\eta_j$ is defined by

\[
\eta_\gamma = 1 \quad , \quad \eta_{\gamma Z} = \left( \frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha} \right) \left( \frac{Q^2}{Q^2 + M_Z^2} \right),
\]

\[
\eta_Z = \eta_{\gamma Z}^2 \quad , \quad \eta_W = \frac{1}{2} \left( \frac{G_F M_W^2}{4\pi\alpha} \frac{Q^2}{Q^2 + M_W^2} \right)^2
\]

where $M_Z$ and $M_W$ are, respectively, the mass of $Z$ and $W$, and $G_F$ is the Fermi constant given by

\[
G_F = \frac{e^2}{4\sqrt{2}\sin^2\theta_W M_W^2}
\]

The lepton tensor $L^{\mu\nu}$ is associated with the coupling of the exchange boson to the lepton. For charged lepton ($e = \pm 1$, helicity $\lambda = \pm 1$), $L^{\mu\nu}$
is given by

\[ L_{\gamma}^{\mu\nu} = 2 \left( k^\mu k'^\nu + k'^\mu k^\nu - (k \cdot k' - m_i^2) g^{\mu\nu} - i\lambda \varepsilon^{\mu\nu\alpha\beta} k_\alpha k'_\beta \right) , \]

\[ L_{\gamma Z}^{\mu\nu} = \left( g_{\gamma V}^{cl} + e\lambda g_{\gamma A}^{cl} \right) L_{\gamma}^{\mu\nu} , \]

\[ L_{Z}^{\mu\nu} = \left( g_{\gamma V}^{cl} + e\lambda g_{\gamma A}^{cl} \right)^2 L_{\gamma}^{\mu\nu} , \]

\[ L_{W}^{\mu\nu} = (1 + e\lambda)^2 L_{\gamma}^{\mu\nu} \]  

\[ Q^2 = -q^2 = 2M x y E_i \]  

\[ W^2 = (P + q)^2 = M^2 + 2M E_i y (1 - x) \]  

\[ s = (k + P)^2 = \frac{Q^2}{s} + M^2 + m_i^2 \]  

where \( cl \) refers to charged lepton. \( g_{\gamma V}^{cl} = -1/2 + 2\sin^2\theta_W \) is the weak vector coupling of the charged lepton to \( Z \). \( g_{\gamma A}^{cl} = -1/2 \) is the weak axial-vector coupling of the charged lepton to \( Z \). \( \theta_W \) is the Weinberg angle. \( g^{\mu\nu} \) is the metric tensor and \( \varepsilon^{\mu\nu\alpha\beta} \) is the completely antisymmetric unit tensor. Because the number of left-hand (\( \lambda = -1 \)) charged lepton is equal to the right-hand (\( \lambda = 1 \)) charged lepton, the unpolarized charged lepton-unpolarized nucleon DIS cross section is just the average of the left- and right-hand contributions.

For neutrino (\( e = 0, \lambda = \pm 1 \)), because \( \gamma \) exchange does not exist and only the left-handed neutrino (\( \lambda = -1 \)) as well as the right-handed anti-neutrino (\( \lambda = +1 \)) are observed so far, we have

\[ L_{\gamma Z}^{\mu\nu} = 2 \left( g_{\gamma V}^{nu} + g_{\gamma A}^{nu} \right)^2 \left( k^\mu k'^\nu + k'^\mu k^\nu - (k \cdot k' - m_i^2) g^{\mu\nu} - i\lambda \varepsilon^{\mu\nu\alpha\beta} k_\alpha k'_\beta \right) , \]
\[ L^{\mu\nu}_{W} = 8 \left( k^\mu k'^\nu + k'^\mu k^\nu - (k \cdot k' - m_i^2) g^{\mu\nu} - i\lambda\varepsilon^{\mu\nu\alpha\beta} k_\alpha k'_\beta \right) \]  

(5)

where \( g_{V}^{nu} = \frac{1}{2} \), \( g_{A}^{nu} = \frac{1}{2} \), and the superscript \( nu \) refers to neutrino. One has to point out here that the efforts searching for right-handed neutrino are in progress \([6]\), provided it is found eventually then the equation (5), of course, should be changed.

Neglecting CP-violating effect, the nucleon tensor \( W_{\mu\nu} \) can be expressed as

\[
W_{\mu\nu} = \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) F_1 (x, Q^2) + \frac{\hat{P}_\mu \hat{P}_\nu}{P \cdot q} F_2 (x, Q^2)
\]

\[
+ i\varepsilon_{\mu\nu\alpha\beta} \frac{q^\alpha P^\beta}{2P \cdot q} F_3 (x, Q^2)
\]

\[
- i\varepsilon_{\mu\nu\alpha\beta} \frac{q^\alpha (P \cdot q s^\beta - s \cdot q P^\beta)}{(P \cdot q)^2} g_2 (x, Q^2)
\]

\[
+ \frac{1}{P \cdot q} \left( \frac{1}{2} \left( \hat{P}_\mu \hat{s}_\nu + \hat{P}_\nu \hat{s}_\mu \right) - s \cdot q \frac{\hat{P}_\mu \hat{P}_\nu}{P \cdot q} \right) g_3 (x, Q^2)
\]

\[
+ \frac{s \cdot q}{P \cdot q} \left( \frac{\hat{P}_\mu \hat{P}_\nu}{P \cdot q} g_4 (x, Q^2) + \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) g_5 (x, Q^2) \right)
\]

(6)

\[ \text{[5, 7, 8].} \] Here \( q^\mu \) is the 4-momentum transfer, \( P^\mu \) and \( s^\mu \) are the 4-momentum and spin of nucleon, respectively. \( \hat{P}_\mu, \hat{s}_\mu \) are defined as follows

\[ \hat{P}_\mu = P_\mu - \frac{P \cdot q}{q^2} q_\mu, \quad \hat{s}_\mu = s_\mu - \frac{s \cdot q}{q^2} q_\mu. \]  

(7)

\( F_i (x, Q^2) \) \((i = 1, 2, 3)\) are unpolarized structure functions, and \( g_i (x, Q^2) \) \((i = 1, 2, 3, 4, 5)\) are polarized structure functions \([7, 8]\).

For the lepton-unpolarized nucleon DIS, the nucleon tensor can be simplified as

\[
W_{\mu\nu} = \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) F_1 (x, Q^2) + \frac{\hat{P}_\mu \hat{P}_\nu}{P \cdot q} F_2 (x, Q^2)
\]

\[
+ i\varepsilon_{\mu\nu\alpha\beta} \frac{q^\alpha P^\beta}{2P \cdot q} F_3 (x, Q^2)
\]

(8)
In the naive quark-parton model \[9\], \(F_1\) and \(F_2\) are approximately related by Callan-Gross relation

\[
F_1 = \frac{1}{2x} \left( 1 + 4 \frac{x^2 M^2}{Q^2} \right) F_2. \tag{9}
\]

Inserting Eqs. \((2), (4), (5)\) and \((8)\) into Eq. \((1)\) we obtain

\[
\frac{d^2 \sigma^I}{dx dy} = 8 \pi \alpha^2 M E_i \left( c_1 F_1^I + c_2 F_2^I + c_3 x F_3^I \right),
\]

\[
c_1 = xy^2 - \left( \frac{m_i^2 - m_o^2}{8xM^2E_i^2} \right) - \frac{y(y-4) + 4M^2x^2y}{4ME_i},
\]

\[
c_2 = 1 - y + \left( \frac{m_i^2 - m_o^2}{16x^2M^2E_i^2} \right) - \frac{(m_i^2 - m_o^2)(y-4) + 4M^2x^2y}{8ME_i},
\]

\[
c_3 = \frac{\lambda y(y-2)}{2} - \frac{\lambda y(m_i^2 - m_o^2)}{4ME_i} \tag{10}
\]

for the lepton-unpolarized nucleon DIS. In the Eq. \((10)\) \(I = clNC, nuNC, clCC,\) and \(nuCC\) \((c=\pm, \mu, \tau; \nu = \nu_e, \nu_\mu, \nu_\tau, \overline{\nu_e}, \overline{\nu_\mu}, \overline{\nu_\tau}).\)

In the quark-parton model, the structure functions are related to the parton distribution function \((q(x, Q^2))\). For neutral current DIS, we have

\[
[F_2^\gamma, F_2^{\gamma Z}, F_2^Z] = x \sum_q [e_q^2, 2g_V^g, g_V^g + g_A^g]\,(q + \overline{q}),
\]

\[
[x F_3^\gamma, x F_3^{\gamma Z}, x F_3^Z] = x \sum_q [0, 2g_A^g, 2g_V^g g_A^g]\,(q - \overline{q}) \tag{12}
\]
where $e^q$ is the charge of $q$ quark. $g^q_V$ as well as $g^q_A$ are, respectively, the weak vector coupling between $q$ and $Z$ as well as the weak axial-vector coupling between $q$ and $Z$. They are

$$q = u, c, t : \quad g^q_A = \frac{1}{2}, \quad g^q_V = g^q_A - \frac{4}{3} \sin^2 \theta_W;$$

$$q = d, s, b : \quad g^q_A = -\frac{1}{2}, \quad g^q_V = g^q_A + \frac{2}{3} \sin^2 \theta_W.$$  (13)

For charged current DIS, we have

$$F^W_2 = 2x \left( u + d + c + \tau + t + b \right),$$

$$xF^W_3 = 2x \left( u - d + c - \tau + t - b \right)$$  (14)

for incident lepton of $e^-, \mu^-, \tau^-, \nu_e, \nu_\mu$ and $\nu_\tau$, as well as

$$F^W_2 = 2x \left( \bar{\tau} + d + \bar{c} + s + \bar{t} + \bar{b} \right),$$

$$xF^W_3 = 2x \left( \bar{u} - d + \bar{c} + s - \bar{t} + \bar{b} \right)$$  (15)

for incident lepton of $e^+, \mu^+, \tau^+, \nu_e, \nu_\mu$, and $\nu_\tau$.

In the lepton-nucleon DIS, because of $Q^2 \gg m_i^2$ and $Q^2 \gg m_o^2$, one may assume $m_i^2 = m_o^2 = 0$. Therefore Eq. (10) can be simplified to

$$\frac{d^2 \sigma}{dxdy} = \frac{4\pi \alpha^2}{xyQ^2} \left( xyF_1^I + \left( 1 - y - \frac{x^2y^2M^2}{Q^2} \right) F_2^I - \lambda \left( y - \frac{y^2}{2} \right) xF_3^I \right).$$  (16)

Note, a relation of

$$\frac{d^2\sigma}{dxdQ^2} = \frac{y}{Q^2} \frac{d^2\sigma}{dxdy}$$  (17)

is always required in the argument transformation from $dxdy$ to $\frac{Q^2}{y}dxdQ^2$.

The total cross section can be expressed by the differential cross section (Eq. (10)) as follows

$$\sigma = \int_{x_{\min}}^{x_{\max}} \int_{y_{\min}}^{y_{\max}} \frac{d^2\sigma}{dxdy} dxdy$$  (18)

where the integral limits should be decided by the scattering kinematics. In order that the direction of incident lepton is set on the $z$ axis and the lepton
scattering angle is denoted as $\theta$, one then has

$$
k = \left( E_i, 0, 0, \sqrt{E_i^2 - m_i^2} \right),
$$

$$
k' = \left( E', 0, \sqrt{E'^2 - m_o^2 \sin \theta}, \sqrt{E'^2 - m_o^2 \cos \theta} \right),
$$

$$
2 M E_i x y = Q^2 = -(k - k')^2,
$$

$$
y = \frac{E_i - E'}{E_i}. \tag{19}
$$

Further one can deduce

$$
\cos^2 \theta = \frac{(m_i^2 + m_o^2 + 2 E_i^2 y - 1 + 2 E_i M x y)^2}{4 (E_i^2 - m_i^2) (E_i^2 (y - 1)^2 - m_o^2)} \tag{20}
$$

from Eq. (19). Together with the facts of $\cos^2 \theta \leq 1$, $0 \leq x \leq 1$ and $0 \leq y \leq 1$, the $x$ and $y$ kinematic limits are obtained

$$
\max \left( 0, \frac{m_o^2 - m_i^2}{2 M (E_i - m_o)} \right) \leq x \leq 1,
$$

$$
A - B \leq y \leq A + B,
$$

$$
A = \frac{2 M x E_i^2 + E_i (m_i^2 - m_o^2) - M x (m_i^2 + m_o^2)}{2 E_i (M^2 x^2 + 2 E_i M x + m_i^2)},
$$

$$
B = \frac{\sqrt{(E_i^2 - m_i^2) \left( 4 M^2 x^2 (E_i^2 - m_o^2) + 4 E_i M x (m_i^2 - m_o^2) + (m_i^2 - m_o^2)^2 \right)}}{2 E_i (M^2 x^2 + 2 E_i M x + m_i^2)}. \tag{21}
$$

Meanwhile, the $x$ and $y$ integral limits must keep with the DIS characters of $Q^2 \gg M^2$ and $W^2 \gg M^2$, which lead to

$$
\frac{Q^2_{\min}}{2 M E_i} \leq x \leq 1 - \frac{W^2_{\min} - M^2}{2 M E_i}, \tag{22}
$$

and

$$
\max \left( \frac{Q^2_{\min}}{2 M E_i x}, \frac{W^2_{\min} - M^2}{2 M E_i (1 - x)} \right) \leq y \tag{23}
$$

where $0 \leq y \leq 1$ is employed.
Combining Eq. (21), (22) and (23), we obtain $x$ and $y$ integral limits eventually

$$\max \left( \frac{m_o^2 - m_i^2}{2M (E_i - m_o)} \frac{Q_{\min}^2}{2ME_i} \right) \leq x \leq 1 - \frac{W_{\min}^2 - M^2}{2ME_i},$$

(24)

$$\max \left( A - B, \frac{Q_{\min}^2}{2ME_i x}, \frac{W_{\min}^2 - M^2}{2ME_i (1-x)} \right) \leq y \leq A + B.$$ (25)

3. Results

Hereafter, we will mainly use the LDCS 1.0 program (the equation (10)) to calculate the cross sections for the various lepton-nucleon DIS and compared with the corresponding experimental data in order to prove the correction of the LDCS 1.0 program.

The double differential cross sections of the unpolarized electron-unpolarized proton DIS at $\sqrt{s}=318.7$ GeV are shown in Fig. 2 for different $x$ values. In this figure the solid and open triangles are experimental data measured by HERA1 and ZEUS [1], the solid and dashed lines are theoretical results calculated with HERAPDF1.5 LO PDF set [2]. One sees in this figure that the experimental data are well reproduced by the theoretical calculations.
Figure 2: Experimental and calculated double differential cross sections of the unpolarized electron-unpolarized proton DIS at $\sqrt{s}=318.7$ GeV. Solid and dashed lines are calculated with HERAPDF1.5 LO PDF set [2]. Solid and open triangles with error bars are experimental data measured by HERA1 and ZEUS [1].
We give the theoretical single differential cross sections $d\sigma/dx$ (solid curves) obtained by integrating the corresponding theoretical double differential cross sections (as shown in Fig.2) over $Q^2$ with $Q^2_{\text{min}} = 1000 \text{ GeV}^2$ and $y < 0.9$ in Fig.3. In this figure the solid and open circles with error bars are the corresponding experimental data taken from [10, 11]. One sees in this figure again that the theory well agrees with the experiment.

Figure 3: The experimental and calculated single differential cross sections $d\sigma/dx$ in the unpolarized electron-unpolarized proton DIS at $\sqrt{s} = 318.7 \text{ GeV}$. Solid lines are calculated with HERAPDF1.5 LO PDF set [2]. Solid and open circles with error bars are experimental data taken from [10, 11].

The unpolarized electron-unpolarized proton DIS total cross section $\sigma_{\text{DIS}}$ (NC+CC) as a function of $\sqrt{s}$ calculated with HERAPDF1.5 LO PDF set [2] and kinematical cuts of $Q^2_{\text{min}} = 1.0 \text{ GeV}^2$ and $W^2_{\text{min}} = 1.96 \text{ GeV}^2$ is shown in Fig.4 as black solid line. In this figure the red and blue circles are, respectively, the pp and $\gamma p$ total cross sections taken from [5]. We see in Fig.4 that the $e^- p$ DIS total cross section is a few order of magnitude smaller than pp at the range of $\sqrt{s} \leq 10^4 \text{ GeV}$. 

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Figure 4: The calculated unpolarized electron-unpolarized proton DIS total cross section and the pp as well as $\gamma p$ total cross sections (taken from [5]).

We compare the unpolarized electron-unpolarized proton DIS total cross section (solid line) with the corresponding $\mu^- p$ (crosses) and $\tau^- p$ (dashed line) DIS total cross sections in the Fig. 5. The difference among them stems from their mass.
Figure 5: The total cross sections of unpolarized electron-unpolarized proton, and the corresponding $\mu^- p$ as well as $\tau^- p$ DIS calculated with HERAPDF1.5 LO PDF set \cite{2} and kinematic cuts of $Q^2_{\text{min}} = 1.0 \ \text{GeV}^2$ as well as $W^2_{\text{min}} = 1.96 \ \text{GeV}^2$.

The solid circles and triangles as well as open circles and triangles in Fig. 6 present, respectively, the $\nu_{\mu} F e$ as well as $\bar{\nu}_{\mu} F e$ (here $F e$ is unpolarized) CC inclusive scattering total cross sections experimental data ($\sigma^{\exp}_{\text{cc}}$ as well as $\sigma^{\exp}_{\text{cc}}$). The triangle data are taken from \cite{12} and the circle data from \cite{13}. As mentioned in \cite{12} that their $\nu_{\mu} F e$ and $\bar{\nu}_{\mu} F e$ CC inclusive scattering cross sections were measured by the iron-scintillator detector with a 6.1% excess of neutron over proton. This was corrected by the NEUGEN3 cross section method \cite{14}. The correction is about -2% for neutrino and +2% for antineutrino. Therefore, the expected cross section of $\nu_{\mu} \left(\bar{\nu}_{\mu}\right)$ CC inclusive scattering off the isoscalar target is $(1-0.02)\sigma^{\exp}_{\text{cc}} \left((1+0.02)\sigma^{\exp}_{\text{cc}}\right)$. This cross section is quoted as the cross section of $\nu_{\mu} N \left(\bar{\nu}_{\mu} N\right)$ CC inclusive scattering in \cite{5}, $N$ here is refers to the nucleon (neutron and/or proton).

The dashed and solid curves in Fig. 6 are the theoretical total cross sections of $\nu_{\mu} F e$ and $\bar{\nu}_{\mu} F e$ DIS calculated by Eq. (10) and with the iron PDF set (taken from \cite{4}), the kinematic cuts of $Q^2_{\text{min}} = 1.0 \ \text{GeV}^2$ and $W^2_{\text{min}} = 1.9 \ \text{GeV}^2$, as well as the correction factor of 0.98 for neutrino and 1.02 for antineutrino. One sees in Fig. 6 that the theory agrees with experiment fairly well at the
high incident lepton energy region \((E_i \geq 100 \text{ GeV})\), but the agreement is bad at the low energy region. This is because the theory is for the DIS process only but the experiment is for the inclusive scattering which is a combination of the quasi-elastic scattering, DIS, and resonance production. And the DIS is dominated at the high incident lepton energy region but at the low energy region the quasi-elastic scattering and resonance production are dominated \([5]\). Therefore, one may conclude either that the equation (10) (the LDCS 1.0 program) is also work for the neutrino induced DIS.

![Figure 6: The ratio of total cross section to the incident energy.](image)

Figure 6: The ratio of total cross section to the incident energy. The solid and open triangles as well as the solid and open circles are experimental data of \(\nu_\mu\text{Fe} \) and \(\bar{\nu}_\mu\text{Fe} \) CC inclusive scattering taken from \([12]\) as well as \([13]\), respectively. The solid and dashed curves are theoretical results of \(\nu_\mu\text{Fe} \) and \(\bar{\nu}_\mu\text{Fe} \) DIS calculated by Eq. (10) with the kinematical cuts of \(Q^2_{\text{min}} = 1.0 \text{ GeV}^2 \) and \(W^2_{\text{min}} = 1.96 \text{ GeV}^2 \) as well as the iron PDF set \([4]\).

Fig. 7 shows the lepton mass effect on the NC (panel (a)) and CC ((b)) unpolarized charged lepton-unpolarized proton DIS total cross sections as well as on the NC ((c)) and CC ((d)) neutrino-unpolarized iron DIS total cross sections. The proton PDF used in the panels (a) and (b) calculations is taken from \([2]\) and the iron PDF used in the panels (c) and (d) calculations is taken from \([4]\). We see in Fig. 7 that the mass effect on the NC lepton-proton
and neutrino-iron DIS total cross section is weak (cf. panels (a) and (c)). The effect on the CC lepton-proton DIS total cross section is also weak (cf. panel (b)) but on the CC neutrino-iron DIS is visible (cf. panel (d)).
Fig. 7: The lepton mass effect on the NC and CC unpolarized charged lepton-unpolarized proton DIS as well as on the NC and CC neutrino-unpolarized iron DIS. In the calculations the kinematic cuts are: $Q^2_{min} = 1.0 \text{ GeV}^2$ and $W^2_{min} = 1.96 \text{ GeV}^2$.

Fig. 8 shows the lepton mass effect on $x$ and $y$ integral region in the total cross section calculations for the unpolarized tauon-unpolarized proton CC DIS at different $\sqrt{s}$. Here we see that the integral region increases with increasing $\sqrt{s}$. We compare the lepton mass effect on the different unpolarized charge lepton-unpolarized proton (neutrino-unpolarized proton) NC and CC DIS at $\sqrt{s}=3.5 \text{ GeV}$ in the panels (a) and (c)((b) and (d)) of Fig. 9 respectively. One sees in the panels (a) and (c) ((b) and (d)) of Fig. 9 that the larger tauon mass leads to that the $x$ and $y$ integral region in tauon induced DIS is smaller than the ones in electron and/or muon induced DIS.
Figure 8: The $x$ and $y$ integral region in unpolarized tauon-unpolarized proton DIS total cross section calculations at different $\sqrt{s}$. $Q_{\text{min}}^2 = 1.0$ GeV$^2$ and $W_{\text{min}}^2 = 1.96$ GeV$^2$. 
Figure 9: The $x$ and $y$ integral region in different lepton-proton DIS total cross section calculations at $\sqrt{s}=3.5$ GeV. \( Q_{\text{min}}^2 = 1.0 \text{ GeV}^2 \) and \( W_{\text{min}}^2 = 1.96 \text{ GeV}^2 \)

### 4. Summary

The Monte Carlo simulation is one of methods investigating the lepton-nucleon and lepton-nucleus DIS. In the lepton-nucleus DIS, the nucleons in target nucleus are first randomly distributed in the target nuclear sphere according to the Woods-Saxon and $4\pi$ isotropic distributions. The collision possibility is then considered between the incident lepton and each one of the target nucleons, here the lepton-nucleon DIS total cross section is necessary.

Although, the message of lepton-nucleon DIS cross section are always embedded in the complex PDF fitting packages such as HERAfitter [1]. However, for the benefit of lepton-nucleus DIS Monte Carlo simulation, a simple but self-consistent program calculating the lepton-nucleon DIS differential and total cross sections based on the experimentally fitted PDF set is highly required.

Therefore, we pick up the concerned messages (subprograms) from the
PDF fitting packages and compose a simple but self-consistent program (LDCS 1.0) for the calculation of the unpolarized charged lepton-unpolarized nucleon and the neutrino-unpolarized nucleon DIS differential and total cross sections. Before that, we have first briefly introduced the basic theory about the lepton-nucleon NC and CC DIS. The equation of (10) is then derived calculating the NC and CC differential cross sections for the lepton-unpolarized nucleon DIS at leading order with the incident and scattered lepton masses taken into account.

Then we have compared the calculated lepton-nucleon and/or lepton-iron double, single differential, and total cross sections to the corresponding experimental data. The good agreements between the theoretical results and the corresponding experimental data indicate that the program LDCS 1.0 works very well. Additionally, the integral region of Eq. (24) and (25) for the arguments of $x$ and $y$ are studied in detail. The investigations in both the cross section and the integral region lead to a conclusion that the mass of tauon can not be neglected in the GeV energy level.

Appendix A. A brief description for the program

The program LDCS 1.0, calculating the unpolarized charged lepton-unpolarized nucleon and neutrino-unpolarized nucleon NC and CC DIS cross sections, is mainly based on HERAfitter-1.0.0 program. The LHAPDF 5.9.1 packet is employed during running. The key subroutines in the program are defined in ‘ddcs.f’ and listed as follows:

- subroutine init
  input: none
  output: none
  function: define the values of some constants

- subroutine GetNCdds(x, Q2, y, NPT, M, lepin, xsec)
  input: x, Q2 (i.e. $Q^2$), y, NPT (the size of array x, q2, y and xsec), M, lepin (the number of incident lepton, 11: $e^-$, -11: $e^+$, 12: $\nu_e$, -12: $\bar{\nu}_e$, 13: $\mu^-$, -13: $\mu^+$, 14: $\nu_\mu$, -14: $\bar{\nu}_\mu$, 15: $\tau^-$, -15: $\tau^+$, 16: $\nu_\tau$, -16: $\bar{\nu}_\tau$)
  output: xsec ($d^2\sigma_{NC}/dx dy$)
  function: calculate the neutral current differential cross sections

- subroutine GetCCdds(x, Q2, y, NPT, M, lepin, xsec)
  input: x, Q2, y, NPT, M, lepin
output: \( \text{xsec} \left( \frac{d^2\sigma_{CC}}{dx dy} \right) \)
function: calculate the charged current differential cross sections

- subroutine getSF(SFD, x, Q2, SF, NPT)
  input: x, Q2, NPT, SFD (coefficients of the linear combination to calculate structure function from \( q(x, Q^2) \)).
  output: SF (the values of structure function \( F_2 \) or \( xF_3 \))
  function: calculate \( F_2 \) or \( xF_3 \) by LHAPDF 5.9.1

In “main.f”, the double differential cross sections \( \left( \frac{d^2\sigma}{dx dy} \right) \), single differential cross sections \( \left( \frac{d\sigma}{dx} \right) \), and the total cross section \( \sigma_{DIS} \) are calculated in subroutine of ‘ddsigma’, ‘dxsigma’ and ‘sigma’, respectively. The Simpson method is employed for the integrations. “SFdefine.inc” defines the coefficients of the linear combination to calculate structure function from quark distribution function of \( q(x, Q^2) \). The contribution of top quark is neglected. “couplings.inc” gives the declarations for some constants. “input.txt” sets the type and parameters for the calculation. “output.txt” saves the results. There are four examples for the input and output files given in folder “example”. “readme.txt” guides the program installation and running.

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