Effect of Finite Granularity of Detector on Anisotropy Coefficients

Sudhir Raniwala\textsuperscript{1}, Marek Idzik\textsuperscript{2}, Rashmi Raniwala\textsuperscript{1}, Yogendra Pathak Viyogi\textsuperscript{2,*}

\textsuperscript{1} Physics Department, University of Rajasthan, Jaipur 302 004, India
\textsuperscript{2} Faculty of Physics and Applied Computer Science, AGH University of Science and Technology, Krakow, Poland
\textsuperscript{3} Variable Energy Cyclotron Centre, Kolkata 700 064, India
\textsuperscript{*} now at Institute of Physics, Bhubaneswar 751 005, India

Abstract

The coefficients that describe the anisotropy in the azimuthal distribution of particles are lower when the particles are recorded in a detector with finite granularity and measures only hits. This arises due to loss of information because of multiple hits in any channel. The magnitude of this loss of signal depends both on the occupancy and on the value of the coefficient. These correction factors are obtained for analysis methods differing in detail, and are found to be different.

1 Introduction

Azimuthal anisotropy in particle emission in ultra-relativistic heavy ion collisions was proposed as an important probe of the dynamics of the system \cite{1}. Subsequently, various methods have been proposed to obtain this anisotropy \cite{2, 3, 4}. In the more commonly used method the azimuthal distributions are expanded in a Fourier series where the coefficients of expansion are the measures of different orders of anisotropy \cite{2}. For small values of these coefficients, the first two terms describe an elliptic shape. The first order anisotropy $v_1$, the directed flow, measures the shift of the centroid of the distribution and is the coefficient of the first term in the expansion. The second order anisotropy $v_2$, the elliptic flow, measures the difference between the major and minor axes of the elliptic shape of the azimuthal distribution and is the coefficient of the second term in the expansion. The elliptic flow, $v_2$, probes the early stages of expansion of the interacting system and has been measured by large number of experiments for different particle species in different kinematic domains for a variety of colliding systems and a range of center of mass energies \cite{5, 6, 7, 8, 9, 10, 11, 12, 13}. These measurements have provided new perspectives on the observed mass dependence of the elliptic flow \cite{14}.

In detector sub systems where $p_T$ or energy is not measured, the anisotropy coefficients are determined from the azimuthal distribution of the number of particles. Some detectors measure the distribution of hits\textsuperscript{1}. If a cell is hit by more than one particle, information is lost because the cell is still registered as one hit. The $(\Delta \eta \times \Delta \Phi)$ size of each cell in Silicon Pad Multiplicity Detector in WA98 experiment is about 0.07 $\times$ 2$^\circ$ \cite{15}, whereas the corresponding size in the silicon strip detector in NA50 experiment is about 0.014 $\times$ 10$^\circ$ \cite{16}. For the same occupancy, both detectors will lose comparable number of particles by measuring hits. Therefore, the anisotropy coefficients describing distribution of hits are expected to be smaller than coefficients describing distribution of particles; $v_n^{\text{hits}} < v_n$ and there is need to determine an appropriate correction factor.

\textsuperscript{1}One activated cell is counted as $N_{\text{hits}} = 1$ irrespective of the number of tracks activating it.
which will depend on the granularity of the detector. Methods to estimate the effect of finite granularity have been discussed in [17]. In the present work the effect of finite granularity on the standard methods of analysis is investigated by folding the detector geometry in the simulated data.

In Section 2, an approximate expression for the ratio \( \frac{v_{\text{hits}}}{v} \) is obtained as a function of occupancy. Section 3 describes the simulation and the different methods of analysis. The Results are discussed in Section 4.

## 2 Multiple Hits and Azimuthal Anisotropy

It is possible to have an ideal setup for a simulation experiment for any conceivable granularity. However, an actual experiment has a finite number of detector cells \( N_{\text{cells}} \), which defines the coarseness of the granularity for a given acceptance of the detector. For a given average number of incident particles, \( < N_{\text{part}} > \), one can define the mean occupancy \( \mu_0 \)

\[
\mu_0 = \frac{< N_{\text{part}} >}{N_{\text{cell}}} \quad (1)
\]

Using the Poisson distribution for probability of \( n \) particles incident on any cell, one can deduce the average number of hits as

\[
\frac{< N_{\text{hits}} >}{< N_{\text{part}} >} = \frac{1 - e^{-\mu_0}}{\mu_0} = \frac{-x}{\ln(1-x)} \quad (2)
\]

where \( x = \frac{< N_{\text{hits}} >}{N_{\text{cell}}} \) is the hit-occupancy and is experimentally measurable. Mean occupancy can also be written as \( \mu_0 = -\ln(1-x) \). Since the total number of cells \( (N_{\text{cell}}) \) is the sum of the average number of occupied \( (< N_{\text{occ}} >) \) and unoccupied \( (< N_{\text{unocc}} >) \) cells, one can immediately obtain the expression for occupancy as in ref [13].

\[
\mu_0 = \ln(1 + \frac{< N_{\text{occ}} >}{< N_{\text{unocc}} >}) \quad (3)
\]

enabling its determination from experimentally measurable quantities.

The anisotropy in the azimuthal distribution of the number of incident particles is written as

\[
N_{\text{part}}(\phi) = < N_{\text{part}} > (1 + \sum 2v_n \cos n(\phi - \psi_n)) \quad (4)
\]

where \( \psi_n \) is the event plane angle. Using equations 1 and 2, one can write the azimuthal dependence of hits as

\[
N_{\text{hits}}(\phi) \propto 1 - e^{-\mu(\phi)} \quad (5)
\]

where \( \mu(\phi) \) denotes the azimuthal dependence of the occupancy. Since the intrinsic occupancy of cells increases with the increase in the number of incident particles, the occupancy will have the same azimuthal dependence as the incident particles and can be written as \( \mu(\phi) = \mu_0(1 + \sum 2v_n \cos n(\phi - \psi_n)) \). Substituting this in equation 5 and expressing \( N_{\text{hits}} \) as a Fourier series with coefficients \( v_{\text{hits}}^n \) enables a determination of the ratio \( v_{\text{hits}}^n/v_n \). To the first order, this ratio can be approximated as

\[
\frac{v_{\text{hits}}^n}{v_n} = \frac{1 - \mu_0 + \frac{\mu_0^2}{2} - f(v)}{1 - \frac{\mu_0^2}{2} + \frac{\mu_0^4}{6}} \quad (6)
\]
where \( f(v) = \mu_0 v^2 \) for \( n = 1 \), and \( = \frac{\mu_0 v^2}{2v_2} \) for \( n = 2 \).

The function \( f(v) \) contributes little for small values of occupancy and flow.

The ratio can also be approximated as

\[
\frac{v_n^{\text{hits}}}{v_n} = -\frac{1 - x}{x} \cdot \ln(1 - x)
\]

These results have been applied to the data recorded in the Silicon Pad Multiplicity Detector in the WA98 experiment [8]. The results from equation 6 and 7 are corroborated with results from simulations as described in the following.

3 Simulation and Analysis

For the present simulation experiments, the events have been generated with various values of charged particle multiplicity corresponding to different occupancies in the detector. Assuming a constant \( dN/d\eta \) and an exponential \( p_T \) distribution, the kinematic variables of each particle are generated with \( p_T \) in the region 0 to 6 GeV/c and \( \eta \) in an assumed region of acceptance of the detector. Typical ranges of \( \eta \) chosen in the present work vary between 0.5 and 1.0. Azimuthal angle of each particle is assigned according to the probability distribution [18]

\[
r(\phi) = \frac{1}{2\pi} \left[ 1 + 2v_1 \cos(\phi - \psi_R) + 2v_2 \cos 2(\phi - \psi_R) \right]
\]

where \( \psi_R \) is randomly generated once for each event. Events are generated for different granularities in \( \eta \) and \( \Phi \). A constant \( dN/d\eta \) distribution and cells of equal \( \Delta\eta \) intervals give a uniform intrinsic occupation probability for each cell. Detector geometry, flow and occupancy are varied for a systematic study.

In the present work, the granularities that are chosen are fairly arbitrary but commensurate with the coarseness of certain detectors [15, 16]. More specifically, simulations are performed for \( \Delta\eta = 0.07, \Delta\Phi = 2^\circ; \Delta\eta = 0.014, \Delta\Phi = 10^\circ; \Delta\eta = 0.00875, \Delta\Phi = 10^\circ \). The results are based on an analysis of \( 10^6 \) events in each case. For low flow values \( (v_n = 0.02) \), the number of generated events is \( 2.5 \cdot 10^6 \).

3.1 Different Data-Sets

The number of particles simulated and the number of cells activated \( (N_{\text{hits}}) \) are known for each event. It is assumed that one incident particle does not activate more than one cell.

The following information is stored as three different data-sets from the simulated events.

(a) the number of particles and the azimuthal angle of each particle. This corresponds to a measurement in an ideal detector of infinite granularity. Anisotropy coefficients measured thus are labeled as \( v_n^{\text{ideal}} \).

(b) the number of particles and the azimuthal angle of each hit cell. This corresponds to the case when azimuthal angle of each particle is known to an accuracy determined by the azimuthal size of the cell and the number of particles can be determined using the pulse height information. This is equivalent to randomly adding (or subtracting) \( \delta\phi \) \( (\leq \frac{\Delta\Phi}{2}) \) to each \( \phi \) where \( \Delta\Phi \) is the azimuthal size of
each cell. The anisotropy coefficients obtained using these are called $v^{ch}_n$ and can be written as 

$$\frac{v^{ch}_n}{v^{ideal}_n} = \sin \frac{n \Delta \Phi}{2}$$

(c) the number of hits and azimuthal angle of each hit cell. This is the information recorded by the detectors that produce only a binary (hit/no-hit) signal for each cell and the corresponding anisotropy coefficients are called $v^{hits}_n$.

### 3.2 Methods of Analysis

Fourier coefficients of $n^{th}$ order can be determined from the azimuthal distribution of the particles with respect to the event plane angle of order $m$, provided $n$ is an integral multiple of $m$, by fitting to the following equations [2]

$$\frac{dN}{d(\phi - \psi'_m)} \propto 1 + \sum_{n=1}^{\infty} 2v'_{nm} \cos nm(\phi - \psi'_m)$$

The event plane angle is given by

$$\psi'_m = \frac{1}{m} \left( \tan^{-1} \frac{\sum w_i \sin m \phi_i}{\sum w_i \cos m \phi_i} \right)$$

where the summation is over all particles $i$ and the weights $w_i$ are all set to 1. The average deviation of the estimated event plane from the true event plane due to multiplicity fluctuations can be determined experimentally and is termed as the resolution correction factor (RCF). Experimentally, RCF is obtained using the sub-event method described in reference [2]. Here every event is divided into two sub-events of equal multiplicity and the event plane angle $\psi'_m$ is determined for each sub-event. This enables determination of a parameter $\chi_m$ directly from the experimental data using the fraction of events where the correlation of the planes of the sub-events is greater than $\pi/2$ [2, 20]:

$$\frac{N_{events}(m | \psi'^a_m - \psi'^b_m | > \pi/2)}{N_{total}} = \frac{e^{-\chi^2_m}}{2}$$

where $N_{total}$ denotes the total number of events, $\psi'^a_m, \psi'^b_m$ are the observed event plane angles of the two sub-events (labeled a and b) and the numerator on the left denotes the number of events having the angle between sub-events greater than $\pi/2m$. The parameter $\chi_m$ is used to determine RCF$_{nm} = \langle \cos(nm(\psi'_m - \psi'^{true}_m))\rangle$, where $\psi'^{true}_m$ is the true direction of the event plane, and the average is over all events. The RCF can be determined from $\chi_m$ by the following relation in reference [2].

$$\langle \cos(nm(\psi'_m - \psi'^{true}_m))\rangle = \frac{\sqrt{\pi} \chi_m}{2\sqrt{2}} \exp(-\chi^2_m/4) \cdot \left[ I_{m+1}(\chi^2_m/4) + I_{m-1}(\chi^2_m/4) \right]$$

where $I_\nu$ are the modified Bessel functions of order $\nu$. The RCF can also be obtained by obtaining $\langle \cos(n(\psi'^a_n - \psi'^b_n))\rangle$, where $\psi'^{a,b}_n$ are the event plane angles of the two sub-events.
In the present work, Fourier coefficients $v'_{nm}$ are extracted for the case with event plane order equal to the order of the extracted Fourier coefficient, i.e. $v'_{nm} = v'_{nn}$ and is denoted here by $v'_n$. The values of $v_n$ have been obtained by the following methods:

**Method 1:** In this method, the sub-events for each event were formed by dividing the pseudorapidity range into two such that each sub-event has equal number of particles (hits). Then $v'_a = \langle \cos n(\phi_i^a - \psi'_b) \rangle$ and $v'_b = \langle \cos n(\phi_i^b - \psi'_a) \rangle$ are determined where $\phi_i^a$ represent the azimuthal angles of particles in sub-event $a$ and $\psi'_b$ is the event plane angle determined using particles in sub-event $b$. The averages are computed over all particles over all events. In the absence of non-flow correlations

$$v_n = \sqrt{\frac{v'_a v'_b}{\langle \cos n(\psi'_a - \psi'_b) \rangle}} \quad (14)$$

It is also possible to obtain $v'_n$ by fitting equation 10 to the $\phi_i - \psi'_n$ distribution. This distribution is a sum of the the distributions $\phi_i^a - \psi'_b$ and $\phi_i^b - \psi'_a$. This yields

$$v_n = \frac{v'_n}{\sqrt{\langle \cos n(\psi'_a - \psi'_b) \rangle}} \quad (15)$$

The denominator in both cases above is the event plane resolution correction factor when the event plane is determined for the sub-event with half of the complete event multiplicity and is approximately lower than the full event RCF by a factor $\sqrt{2}$. The values of $v_n$ obtained this way are termed as $v_{n\text{geom}}$.

**Method 2:** In this method, in each event, the sub-events were formed by randomly selecting one half of the particles. The $v'_n$ values are also extracted by fitting equation 10 to the $\phi_i - \psi'_n$ distribution, where $\psi'_n$ is obtained by excluding the particle (or hit) being entered in the distribution. The $v_n$ values are obtained from

$$v_n = \frac{v'_n}{RCF_n} \quad (16)$$

$RCF_n$ is the resolution correction factor for the full event plane and is obtained by the correlation between randomly divided sub-events of equal multiplicity and using equations 12 and 13. The values of $v_n$ obtained this way are termed as $v_{n\text{rand}}$.

### 4 Results and Discussion

A relation for the ratio of the anisotropy measured using hits to the actual anisotropy for different values of occupancy was obtained in section 2. These values are corroborated using simulations and the results are discussed in this section. The different methods of analysing the data discussed in section 3 are applied on simulated data to study the effect of finite granularity on the $v_n$ values. The simulated data are analysed

---

2The two sub-events were also formed by assigning particles (hits) to each from alternate segments of the azimuthally segmented detector.

3This avoids autocorrelations but also introduces a negative correlation, effectively decreasing the values of $v'_n$. 
for all the detector geometries described above. The results are discussed for the case \( \Delta \eta = 0.07, \Delta \Phi = 2^\circ \). The conclusions remain the same for the other geometries. For all simulations, \( N_{\text{hits}}/N_{\text{part}} \) is obtained for different values of \( x \) and is found to be consistent with results from equation 2.

### 4.1 \( v_n \) Using Known Event Plane: Actual Dilution

The dilution in the anisotropy coefficients due to finite granularity can be computed using the known event plane angle \((\psi_n)\) in simulation. The quantity \( v_n = \langle \cos(n(\phi_i - \psi_n)) \rangle \) is determined for the different data-sets described in Section 3.1 and yields the finite granularity effect on the anisotropies in the distribution.

1. \( v_n^{\text{ideal}} \) reproduces the input flow, as expected naively.

2. The dilution due to coarse information about the particle angle can be judged by plotting the ratio \( v_n^{\text{ch}}/v_n^{\text{ideal}} \). The result for the granularity \( \Delta \eta \times \Delta \Phi = 0.07 \times 2^\circ \) has been plotted in Fig. 1 for two different values of initial anisotropy. The correction factor due to coarse information of particle angle is very close to 1 for such a small azimuthal size of the cells. The results have been corroborated using simulations for values of \( \Delta \Phi \) up to 30\( ^\circ \), and agree with results from equation 9.

3. The anisotropy for the hits, \( v_n^{\text{hits}} \) is diluted both due to coarse information of particle angle and loss of particles because of the multiple hits. The resultant loss is best seen by plotting the ratio \( v_n^{\text{hits}}/v_n^{\text{ideal}} \) as a function of hit-occupancy for the simulated data. The results are shown in Fig. 1 for a \( \Delta \Phi = 2^\circ \) along with the estimates obtained using equations 6 and 7. For \( \Delta \Phi = 2^\circ \), results in (ii) above show that the coarse information of particle angle has very little effect, and the dilution in \( v_n^{\text{hits}} \) is primarily due to loss of particles because of multiple hits. Simulation results corroborate the analytical expression that include a weak dependence on anisotropy \( v \).

### 4.2 \( v_n \) Using Reconstructed Event Plane: Observed Dilution

In this section we investigate the results on dilution of anisotropy when the event plane and its resolution is determined from the data. Both the methods listed in Section 3 require a determination of (i) the uncorrected \( v'_n \) and (ii) the corresponding (sub) event plane resolution. The quantitative effect of finite granularity on each of these quantities is different, and hence the measured values of \( v_n \) are different from the initial values. The results of a systematic investigation are shown in figure 2 for varying hit-occupancies. The results from method 1 of section 3 are shown in the left column and those from method 2 are shown in the right column. For both methods, the results from data-sets (b) and (c) are scaled by the corresponding values obtained using data-set (a). The open circles show the results obtained using the data-set (b) for all charged particles and the mean angles of the cell positions. The filled circles show the results obtained using the data-set (c) for the hits and the corresponding angles. For both methods, analysis of data-set (a) reproduces the initial anisotropy, validating the methodology.
4.2.1 Division into sub-events based on geometry: $v_n^{geom}$

The anisotropy coefficients $v_n^{geom}$ determined using equations 14 and 15 yield identical results.

The event plane resolution correction factor and the uncorrected values of $v'_1$ are seen to decrease by different factors due to finite granularity effect, resulting in a reduced value of $v_1^{geom}$.

The results show that the value of anisotropy measured using a detector with 30% mean hit-occupancy is to be corrected by a factor of $\sim 1.2$ to obtain the actual value of anisotropy ($v_1$), clearly a significant effect. The results for the second order anisotropy are similar, with small quantitative difference as seen from equation 6.

The analysis described as Method 1 was repeated for the case when the two sub-events were formed by assigning particles(hits) to each from alternate segments of the azimuthally segmented detector. The results remain the same.

4.2.2 Random division into sub-events: $v_n^{rand}$

The right column of Figure 2 shows the results for method 2 when the events are divided randomly into two sub-events, and the projection of particles/hits is taken on the event plane angle of the full event (after removing autocorrelations). This method works for the simulated data corresponding to data-sets (a) and (b) described in Section 3.1, and the values of data-set (b) scaled to the corresponding values from data-set (a) are shown as open symbols in the right column of Fig. 2. The values obtained using data-set (c) are much lower than the values obtained by analysing the same data using Method 1. The systematically lower values of the quantities for Method 2 arise due to multiple hits, due to removing autocorrelations and due to random division into sub-events. The combined effect results in much lower values of $v'_1^{rand}$. For data-set (b), the method of removing autocorrelation removes only one particle while the other neighbouring particles are used in determining the event plane. For data-set (c) comprising of hits, one detector cell is removed from the data that determines the event plane, effectively removing all particles within the azimuthal size of that cell, introducing a negative correlation, resulting in much lower values of $v'_1$. This holds true for all values of occupancy.

The decrease in the values of event plane resolution can be understood as follows: For an azimuthal distribution given by equation 8, the particle density is maximum along the direction of the reaction plane. On an event by event basis, the maximum loss of particles due to multiple hits will be along this direction. Consider there are $N_{corr}$ correlated particles in a region $\delta \phi$ about the reaction plane, where $N_{corr}/N_{total}$ is greater than $\delta \phi/2\pi$. When such an event is divided into two equal multiplicity sub-events, the correlation between the two sub-events will be maximum if $N_{corr}/2$ particles go into each sub-event. Though this is true on the average, on an event by event basis, only a certain number out of $N_{corr}$ fall into one sub-event. The correlation between sub-events for these events is less than the corresponding situation described above. The correlation will be weakest if all of these particles fall into one sub-event. In such a situation, the $v_n^{rand}$ values are likely to be much lower than the $v_n^{ideal}$. However, the probability of the random division into sub-events leading to this situation is $(1/2)^{N_{corr}-1}$, and is small.

The situation remains the same when hits are recorded instead of particles, and $N_{corr}$ is replaced by $N_{corr(hits)}$, and $N_{total}$ by $N_{hits}$. The probability of a random division with all $N_{hits}$ going into a sub event is $(1/2)^{N_{corr(hits)}-1}$. This probability is clearly
greater than the corresponding case where there is no loss of particles due to multiple hits. When this happens, the two sub-events show little correlation, causing both $v'_n$ and $RCF_n$ to decrease. The decrease in both quantities only partially compensate each other and the resultant values are observed to be lower, as shown in the third panel on the right column of figure 2.

The resultant effect is to produce a reduced value of $v_{rand}^n$, which would need a relatively larger correction factor to obtain the original value. Repeating the simulation for different values of anisotropy shows that the correction factor increases with decreasing values of anisotropy. Below a certain value of anisotropy, $v^{thres}$, the values of anisotropy extracted using this method yields $v_{rand}^n$ consistent with zero, putting a limit on the sensitivity of detecting anisotropy in the data. The values of $v^{thres}$ depend both on the granularity of the detector and on the order of the anisotropy being determined.

5 Conclusions

The anisotropy in the distribution of hits is shown to be lower than the anisotropy in the distribution of particles. Loss of particle information due to multiple hits (or two track resolution) contributes significantly to the dilution of the observed anisotropy values. The correction factor for the dilution has been obtained and confirmed by simulation experiments for different detector geometries. Different methods of event subdivision yield same results for an ideal detector. While it is known that the event subdivision obtained randomly does not work in the presence of non-flow correlations, the limitation of this method is shown here for the case where there are no non-flow correlations. The anisotropy values $v_{rand}^n$ obtained using the distribution of hits are much lower than the corresponding values of $v_{geom}^n$ and need a correction factor which is larger than the one obtained in Equation 6. The analysis and the discussion in the present work are suited for the case where the event plane is being determined from the same set of particles. The event plane resolution correction factor corrects only for fluctuations arising due to finite multiplicity. If the anisotropy parameters are determined with respect to an event plane determined from another set of particles measured using a detector with different granularity, then the correction factors need to be determined differently and their determination is outside the scope of the present work.

Acknowledgement

The financial support from the Department of Science and Technology and the Department of Atomic Energy of the Government of India is gratefully acknowledged.

References

[1] J.Y. Ollitrault, Phys. Rev. D46 229 (1992).
[2] A.M. Poskanzer and S.A. Voloshin, Phys. Rev. C58 1671 (1998).
[3] N.Borghini, Phuong Mai Dinh and J.Y.Ollitrault Phys. Rev C64 054901 (2001)
[4] R.S.Bhalerao, N.Borghini and J.Y.Ollitrault Nucl.Phys.A727 373 (2003).
[5] E877 Collaboration, J.Barette et.al., Phys. Rev. Lett. 73 2532 (1994); Phys. Rev. Lett. 70 2996 (1993); Phys. Rev. C55 1420 (1997); Phys. Rev. C56 3254 (1997).
E802 Collaboration, L.Ahle et.al., Phys. Rev. C57 1416 (1998).
[6] WA93 Collaboration, M.M. Aggarwal et al., Phys. Lett. B 403, 390 (1997).

[7] NA49 Collaboration, H. Appelshäuser et al., Phys. Rev. Lett. 80, 4136 (1998); C. Alt et al., Phys. Rev. C 68, 034903 (2003).

[8] WA98 Collaboration, M.M. Aggarwal et al., Eur. Phys. J. C 41, 287 (2005).

[9] WA98 Collaboration, M.M. Aggarwal et al., Nucl. Phys. A 762, 129 (2005).

[10] CERES/NA45 Collaboration, J. Slivova et al., Nucl. Phys. A 715, 615c (2003).

[11] STAR Collaboration, K.H. Ackermann et al., Phys. Rev. Lett. 86, 402 (2001); C. Adler et al., Phys. Rev. Lett. 87, 182301 (2001).

[12] PHENIX Collaboration, K. Adcox et al., Phys. Rev. Lett. 89, 212301 (2002). S. Esumi et al., Nucl. Phys. A 715, 599c (2003).

[13] PHOBOS Collaboration, B.B. Back et al., Phys. Rev. Lett. 89, 222301 (2002). B. Back et al., nucl-ex/0406021.

[14] D. Molnar and S.A. Voloshin, Phys. Rev. Lett. 91, 092301 (2003).

[15] W.T. Lin et al., Nucl. Instrum. Meth. A 389, 415 (1997).

[16] B. Alessandro et al., Nucl. Instrum. Meth. A 493, 30 (2002).

[17] Rashmi Raniwala, Sudhir Raniwala, Yogendra P Viyogi, ALICE Internal Note; alice-int-2001-49.

[18] R. Raniwala, S. Raniwala, and Y.P. Viyogi, Phys. Lett. B 489, (2000) 9.

[19] Gang Wang, Ph.D. Thesis, Kent State University, 2006.

[20] J.Y. Ollitrault, e-print nucl-ex/0711003.
Figure 1: Open symbols show the ratio $v_{n}^{\text{ch}}/v_{n}^{\text{ideal}}$ for different values of $x$, where $x = <N_{\text{hits}}>/N_{\text{cell}}$ is experimentally measurable. Filled symbols show the ratio $v_{n}^{\text{hits}}/v_{n}^{\text{ideal}}$. Squares are for $v_{n} = 0.05$ and triangles are for $v_{n} = 0.02$. The top panel is for $v_{1}$ and the bottom panel is for $v_{2}$. The two dashed curves in each panel correspond to the different values of anisotropy $v_{n}$ and represent equation 6. The dotted curve represents equation 7. A horizontal line at the value of ratio equal to 1 is also drawn.
Figure 2: The three panels in the left column show uncorrected values of $v'_1$, $RCF_1$ and $v_{1}^{geom}$ for charged particles and hits for an initial $v_1 = 0.04$ for different values of hit-occupancy $x$. The open circles are for all particles and the filled circles are for hits, as described in the text. The values are scaled with corresponding values for the ideal case. The dashed curve represents equation 6. The panels in the right column show the corresponding results for method 2, the random subdivision of events with the bottom column showing the value of $v_1^{rand}$ scaled by the corresponding value for the ideal case.