Superconducting properties of corner-shaped Al microstrips

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The electrical transport properties of corner-shaped Al superconducting microstrips have been investigated. We demonstrate that the sharp turns lead to asymmetric vortex dynamics, allowing for easier penetration from the inner concave angle than from the outer convex angle. This effect is evidenced by a strong rectification of the voltage signal otherwise absent in straight superconducting strips. At low magnetic fields, an enhancement of the critical current with increasing magnetic field is observed for a particular combination of field and current polarity, confirming a recently theoretically predicted competing interplay of superconducting screening currents and applied currents at the inner side of the turn.

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The ability of superconductors to carry electricity without resistance holds in a restricted current density range \( j < j_{\text{max}} \). Several physical mechanisms can be identified as responsible for limiting \( j_{\text{max}} \) such as the motion of vortices, the formation of phase slip centers or eventually when the pair-breaking current, \( j_{\text{pb}} \), is reached.

In principle, it is possible to attain the ultimate limit \( j_{\text{max}}=j_{\text{pb}} \) by properly choosing the dimensions of the superconducting strip. Indeed, if the width \( w \) of the strip is such that \( w < 4.4\xi \), where \( \xi \) is the coherence length, vortices cannot fit into the sample and therefore \( j_{\text{max}} \) cannot be limited by a vortex depinning process. In 1980 Kupriyanov and Lukichev were able to determine theoretically \( j_{\text{pb}} \) for all temperatures, by solving the Eilenberger equations, and only two years later their predictions were experimentally confirmed by Romijn et al. using straight Al strips. These works focused on the case where \( w \ll \Lambda \), with \( \Lambda = 2\lambda^2/d \) the Pearl length, \( \lambda \) the London penetration depth, and \( d \) the thickness of the superconductor.

Recently a renewed interest for understanding the limiting factors of \( j_{\text{max}} \) in non-straight strips has arisen, partially motivated by the ubiquitous presence of sharp turns in more realistic architectures as those used in the superconducting meanders for single photon and single electron detectors.

Early theoretical calculations by Hagerdorn and Hall showed that a sharp bend in a superconducting wire leads to current crowding effects at the inner corner of the bend, which in turn reduces the total critical current when compared to a straight wire. Not only sharp angles along the superconducting bridge, but any sudden change in the cross section of the wire, can lead to a reduction of the critical current. For instance, it has been pointed out in Ref. 10 that a sudden increase in the cross section of a transport bridge leads to severe modifications of the voltage-current characteristics rendering unreliable those measurements performed in cross-shaped geometries. More recently, Clem and Berggren have theoretically demonstrated that sudden increases in the cross section of a transport bridge lead to current crowding effects and the consequent detriment of the critical current, similarly to right-angle bends. These predictions have been independently confirmed experimentally by Hortensius et al. and by Henrich et al. in submicron scale samples of NbTiN and NbN, respectively, and found to be also relevant in larger samples.

The effect of a magnetic field applied perpendicularly to the plane containing the superconducting wire with a sharp turn has been discussed in Ref. 10 and Ref. 13. Strikingly, in Ref. 13 it is theoretically predicted that due to compensation effects between the field induced streamlines and the externally applied current at the current crowding point, the critical current of thin and narrow superconducting bridges (\( \xi \ll w \ll \Lambda \)) should increase with field for small fields values and for a particular polarity of the applied field.

In this work we provide experimental confirmation of the theoretical predictions of Ref. 13 and show that current crowding leads also to a clearly distinct superconducting response for positive and negative fields (or currents), making these asymmetric superconducting nanocircuits potentially efficient voltage rectifiers.

The samples investigated were all co-fabricated on the same chip and consist of electron-beam lithographically defined Al structures of thickness \( d = 67 \pm 2 \) nm, de-
posed by rf sputtering on top of a Si/SiO$_2$ substrate. We focus on two different geometries. Sample S90 consist of a 3.3 $\mu$m wide transport bridge with a 90$^\circ$ corner equidistant from two voltage probes separated 9.6 $\mu$m from the inner angle of the sharp bend. Similarly, S180 is a conventional straight transport bridge 3.7 $\mu$m wide and with voltage probes separated by 20.9 $\mu$m. These dimensions depart from the nominal values and were obtained via atomic force microscopy as shown in Figure 1(a)-(b).

The field dependence of the superconducting-to-normal metal transitions, $T_c(H)$, determined as $0.95 R_N$, where $R_N$ is the normal state resistance, and using an ac-current of 1 $\mu$A, is basically the same for the two samples studied (see Figure 1(c)). This similarity of the phase boundaries allows us to make reliable and direct comparisons between the two samples without the necessity to work with reduced temperatures or field units. The critical temperature at zero field is $T_{c0} = 1.320 \pm 0.008$ K and the superconducting coherence length obtained from the Ginzburg-Landau approximation is $\xi(0) = 121 \pm 3$ nm. The BCS coherence length for Al of similar characteristics ($T_{c0}$ and $d$) as the one used here is $\xi_0 = 1320$ nm, indicating that our Al falls in the dirty limit $\ell \ll \xi_0$, with $\ell$ the electronic mean free path. Using the relation $\xi(0) = 0.855 \sqrt{\xi_0 \ell}$ we deduce $\ell \sim 15$ nm. An independent estimation of $\ell \sim 17$ nm can be obtained from the normal state resistivity $\rho = 2.0 \pm 0.1 \cdot 10^{-8}$ $\Omega$m, and taking $\rho \ell = 4 \cdot 10^{-16}$ $\Omega$m$^2$. In the dirty limit the magnetic penetration depth is given by $\lambda(0) = \lambda_L(0) \sqrt{\xi_0 / \ell} \approx 145$ nm, where $\lambda_L(0) = 16$ nm is the London penetration depth. For thin film geometry with a perpendicular external field we need to use the Pearl length $\Lambda = 2\lambda^2 / d$. In the considered samples $\Lambda > 2w$ for $T > 1.19$ K.

Let us now concentrate on the current-voltage characteristics, $V(I)$, of the considered systems. At zero external field, the $V(I)$ curves and, in particular the critical current, $I_c$, should be uniquely defined, irrespective of the direction of the applied current. This independence on the direction of the current persists at all fields for the S180 sample, but does not hold for the S90 sample. Indeed, on the one hand, the outer angle of the sharp corner has a larger surface nucleation critical field $H_{c3}$ (a factor $\sim 1.16$ higher for the S90) when compared to the critical field at the inner corner$^{12}$ thus making the outer corner a point of enhanced superconductivity. On the other hand, stream-lines of the applied current tend to conglomerate at the inner corner$^{12}$, depleting the order parameter at that place. Notice that both effects, larger surface nucleation field and lower applied current density at the sharper corner, share the same origin in the impossibility of both, screening or applied currents, to reach the tip of the bend.

The fact that current crowding at the inner corner leads to local depletion of the superconducting order parameter implies automatically a reduction of the surface barrier for vortex penetration$^{13}$ as long as the applied current is such that the Lorentz force pushes vortices from the inner towards the outer corner. However, if the current is reversed, vortices will not penetrate from the outer corner (where total current is nearly zero) but rather symmetrically from the straight legs of the bridge$^{13}$. As a consequence of this different nucleation position and nucleation condition for the two opposite current directions, it is predicted that such a simple corner shape wire will give rise to asymmetric $V(I)$ characteristics and therefore to a vortex ratchet effect.

In order to demonstrate the existence of vortex motion
rectification we submitted the samples to an ac current excitation of zero mean, $I_{ac}$, while measuring simultaneously the dc drop of voltage $V_{dc}$. The results of these measurements $V_{dc}(I_{ac})$ are presented in Figure 2 for both samples. The chosen temperature $T = 1.22$ K is such that $4.4\xi = 1.9\mu m < w = 3\mu m < \Lambda = 8.3\mu m$ ensuring the existence of vortices within the superconductor. There are several points that deserve to be highlighted here, (i) rectification effects are almost completely absent in the S180 sample, (ii) there is a very strong ratchet signal for the S90 sample, (iii) the ratchet signal changes polarity at zero field. Ideally, we expect no ratchet effect at all from the S180 sample, however, the fact that both voltage contacts are on the same side of the strip already impose a weak asymmetry in the system which can lead to asymmetric vortex penetration\cite{16,17}. In any case, the rectification signal obtained in the S180 sample is negligible in comparison to that observed in the sample with the sharp turn. The fact that the rectification signal is positive at positive fields for the S90 sample, and according to the sign convention depicted in Fig. 1(a), we conclude that the easy direction of vortex flow is from the inner corner towards the outer corner, in agreement with the theoretical findings\cite{13}. In Fig.3 we show how the ratchet signal progressively disappears as the temperature approaches 1.280 K. For temperatures above this value vortices cannot fit anymore in the bridge and consequently the difference between the two corners vanishes. Similar ratchet effects due to surface barrier asymmetry, have been recently reported\cite{18} in high-Tc superconducting asymmetric nanobridges, with one side straight and the other having a constriction with an angle of 90°.

Notice that the ratchet effect here described results from the crowding of the applied current at the inner corner, and it would exist even if no screening currents were present. Let us now consider the additional effect of the screening currents. As it has been pointed out in Ref.\cite{13} based on both, London and Ginzburg-Landau theories, for a given direction of the applied current (as indicated in Fig.1(a)) a positive magnetic field will re-inforce the total current (i.e. applied plus screening) at the inner corner and therefore the critical current will decrease as the field intensity increases. On the contrary, a negative applied magnetic field will induce a screening current which partially compensates the applied current at the inner corner and a field dependent increase of the critical current is expected\cite{12}. We have experimentally confirmed this prediction by measuring the critical current using a voltage criterion of 1 $\mu$V as a function of field and current orientation. The results are presented in Figure 4(a) for three different temperatures and for the case where $\xi < w < \Lambda$. For positive current and field (as defined in Fig.1(a)), we observe a monotonous decrease of $I_c$. In contrast to that, for positive current and negative field, a clear enhancement of $I_c$ with field is observed for $H < H_{max}$, whereas for $H > H_{max}$ a monotonous decrease of $I_c$ is recovered as a consequence of antivortices induced by the magnetic field\cite{13} that start

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig2.png}
\caption{Color online) Contour plot of the dc voltage $V_{dc}$ as a function of magnetic field and ac current amplitude at $T = 1.220$ K, and frequency of 1 kHz for sample S180 (a) and sample S90 (b).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig3.png}
\caption{Color online) Contour plot of the dc voltage $V_{dc}$ as a function of temperature and ac current amplitude at $H = 0.1$ mT, and frequency of 1 kHz for the sample S90.}
\end{figure}
FIG. 4: The critical current $I_c$ of sample S90 as a function of applied magnetic field for both polarities of the applied current (a). Panel (b) shows the maximum field $H_{max}$ (normalized to the critical magnetic field $H_{c2}$) as a function of temperature. The inset in (b) shows for comparison the critical current of sample S180 versus magnetic field at positive applied currents.

to penetrate the sample. Reversing the applied current should lead to the opposite behavior, as indeed observed in Fig. 4(a). This double test for all polarities of current and field also permits us to accurately determine the value of zero external field at the point where both curves cross each other. This has been convincingly confirmed by independent measurement of the remanent field in the S180 sample. For the sake of comparison, in the inset of Fig. 4(b) we show the critical current for the S180 sample as a function of field. Notice that for this sample, the peak of maximum critical current is located at $H = 0$, in contrast to the behavior observed in sample S90. It is important to point out that in Ref. [13] the theoretical prediction of the curves in Fig. 4(a) corresponds to a sharp inverted-V shape according to the London model, whereas the Ginzburg-Landau calculations yield a rounded top, which becomes sharper the smaller the ratio of $\xi$ to $w$. This effect appears to be confirmed, at least qualitatively, in Fig. 4(a), in which the peaks become more rounded as the temperature increases and $\xi$ increases.

The compensation field $H_{max}$ is expected to depend on temperature since it is determined by the screening currents. In Fig. 4(b) we plot the temperature dependence of $H_{max}/H_{c2}(T)$ where it can be noticed that this compensation field $H_{max}$ is a small fraction of the upper critical field $H_{c2}(T)$ in agreement with the theoretical calculations).

To summarize, the superconducting properties of corner-shaped Al microstrips have been investigated. We show that sharp 90 degrees turns lead to asymmetric vortex penetration, being easier for vortices to penetrate from the inner side than from the outer side of the angle. We provide experimental confirmation of the predicted competing interplay of superconducting screening currents and applied currents at the inner side of the turn. We prove that current crowding leads to a distinctly different superconducting responses for positive and negative fields (or currents). These effects are evidenced also by a field dependent critical current enhancement and also by a strong rectification of the voltage signal, thus making these asymmetric superconducting nanocircuits efficient voltage rectifiers. Complementary measurements done in samples with 30° and 60° corners (not shown) reproduce the results presented here, i.e. ratchet signal and field-induced increase of critical current.

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The transport measurements have been done with the sample immersed in superfluid $^4$He for minimizing heating effects. Special care has been taken to avoid the high frequency noise signal (above $\sim 1$ MHz) by using a pi-filter.

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