NAE-SAT-based probabilistic membership filters

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Abstract. Probabilistic membership filters are a type of data structure designed to quickly verify whether an element of a large data set belongs to a subset of the data. While false negatives are not possible, false positives are. Therefore, the main goal of any good probabilistic membership filter is to have a small false-positive rate while being memory efficient and fast to query. Although Bloom filters are fast to construct, their memory efficiency is bounded by a strict theoretical upper bound. Weaver et al. introduced random satisfiability-based filters that significantly improved the efficiency of the probabilistic filters, however, at the cost of solving a complex random satisfiability (SAT) formula when constructing the filter. Here we present an improved SAT filter approach with a focus on reducing the filter building times, as well as query times. Our approach is based on using not-all-equal (NAE) SAT formulas to build the filters, solving these via a mapping to random SAT using traditionally-fast random SAT solvers, as well as bit packing and the reduction of the number of hash functions. Paired with fast hardware, NAE-SAT filters could result in enterprise-size applications.

Keywords: Satisfiability, Set Membership Filters, NAE-SAT

1 Introduction

The set membership problem is ubiquitous. It appears in many industrial \(^1\), computer science \(^2\), and security applications, and finds applicability across many fields of research. The problem is simple to pose: Given a pool of subjects \(D\), and a set of interest \(Y \subseteq D\), determine if an element \(x \in D\) belongs to \(Y\). In real-world applications the subset \(Y\) is finite, however, it can be very large. Therefore, determining if \(x\) is a member of the subset \(Y\) can be a computationally expensive task.

A simple example is the following: Let \(D\) all travelers crossing a country’s border in a year and \(Y\) be a terrorist watch list. The set membership problem is then to determine if a randomly-screened traveler \(x \in D\) is also a member of the watch list \(Y \subseteq D\). For a country with few travelers \(|Y|\) crossing the border, this task is easily accomplished by listing all members of \(Y\) and testing if \(x\) is one of
them. However, for a large country where millions of travelers cross the border each year, verifying that \( x \in Y \) can be a time-consuming task. It is therefore desirable to develop a filter that quickly verifies if a particular traveler \( x \) is on the watch list \( Y \). In turn, membership filters are not Boolean in nature. If an element \( x \in D \) is sent through a filter, it will return either \textit{maybe} or \textit{no}. While \textit{no} here is a definite no, \textit{maybe} is interpreted as a possible presence of \( x \) in \( Y \).

This means that membership filters have a finite false-positive rate. However, the storage needed to store the filter is considerably smaller than the storage needed to store the set \( Y \). Furthermore, the query time is (ideally) faster than exhaustively searching for \( x \) in the set \( Y \). Within the traveler example, this would mean that when travelers are screened using a probabilistic membership filter, a query returning \textit{no} means \( x \not\in Y \). However, should this not be the case, then \( x \) would be sent to secondary screening.

An ideal probabilistic set membership filter should therefore be fast, have a small memory footprint, and a low false-positive rate. Traditional work horses are Bloom filters \cite{Bloom1970}. These are fast and easy to implement. However, there is an information theoretical upper bound on their memory efficiency. More recently, Weaver et al. \cite{Weaver2005}, introduced random satisfiability-based membership filters that significantly improved the efficiency. At the core of the filter lies the solution of a complex satisfiability (SAT) formula \cite{Bengtsson2002} needed to construct the actual filter. In this work we present a variation of Weaver et al.’s SAT filter approach with a focus on improving the filter building, as well as query times. Our approach is based on using not-all-equal (NAE) SAT formulas to build the filters, as well as bit packing and reduction of the hash functions to reduce the query times. We show that NAE-SAT filters have excellent memory efficiency and are fast, therefore ideally suited for deployments on large-scale applications.

2 Preliminaries

Before outlining our implementation of SAT-based probabilistic membership filters based on NAE-SAT, we remind the reader of traditionally-used Bloom filters, as well as outline traditional random \( k \)-SAT and the special case of not-all-equal (NAE) SAT.

2.1 Reminder — probabilistic Bloom filters

Probably the widest used probabilistic membership filters are Bloom filters \cite{Bloom1970}. Let \( D \) be any set and \( Y \subseteq D \) with \( m = |Y| \). We assume that the available memory for the filter \( B_Y \) is \( n \) bits. Select a hash function \( h: D \rightarrow \mathbb{Z} \) that maps the elements in \( D \) to the range \([0:n]\) uniformly and randomly. After having chosen the hash function, the bits of \( B_Y \) must be initialized to 0. Then, for an element in \( y \in Y \), we set the bit at \( h(y) \) to 1, i.e., \( B_Y[h(Y)] \equiv 1 \). To store all \( y \in Y \), store all elements of \( Y \) one by one. In most implementations there are multiple hash functions \( h_1, h_2, \ldots, h_k \). In that case \( h_1(y), h_2(y), \ldots, h_k(y) \) must be computed first. Once that is completed, the bits of \( B_Y \) are all set to 1 at the
respective locations (for a pseudo-code version of the full Bloom filter algorithm, see, for example, Ref. [4]).

To query the filter with an element \( x \in D \), simply verify that the bits at all the locations \( h_1(x), h_2(x), \ldots, h_k(x) \) are set to 1. Only when the bits at those locations are 1, will the filter return a *maybe*, otherwise the filter returns a definite *no*. In the latter case \( x \not\in Y \).

While Bloom filters are relatively fast, there is a information-theoretical limit to their memory efficiency [4]. Therefore, fast probabilistic membership filters with a better memory footprint are desirable.

2.2 Reminder — random \( k \)-SAT and NAE SAT

The problem of determining if there exists an assignment of Boolean variables that satisfies a Boolean formula, i.e., such that the Boolean formula evaluates to *true* is known as the satisfiability problem (SAT). If the formula evaluates to *true* it is called *satisfiable*, otherwise *unsatisfiable*. Random \( k \)-SAT [5] is a special case where finite-domain constraint-satisfaction problems [6,7] are encoded as Boolean functions. They are usually expressed in a conjunctive normal form (CNF) formula, i.e.,

\[
C_1 \land C_2 \cdots \land C_m,
\]

where \( \land \) represents conjunction (logical AND) and each \( C_i \) \((1 \leq i \leq m)\) represents a *clause*, i.e., an expression of the form

\[
l_{i,1} \lor \cdots \lor l_{i,k_i}.
\]

Here, \( \lor \) represents a logical disjunction (logical OR) and each \( l_{r,s} \) is called a *literal*. Note that literals are Boolean variables that can also appear negated. A pair of literals is said to be complementary if both are the same variable but have opposite signs.

An assignment is a function \( f \) from the set of variables to the set \( \{0, 1\} \). An assignment satisfies \( x_i \) if \( f(x_i) = 1 \) and satisfies \( \neg x_i \) if \( f(x_i) = 0 \). If at least one of the literals in a clause is satisfied, we say that this clause is satisfied. Only if all the clauses in the CNF are satisfied, the CNF is said to be *satisfied*.

If there are \( k \) different literals (not including complementary pairs) in every clause, then the CNF is called a \( k \)-SAT instance. In random \( k \)-SAT instances, the clause-to-variable ratio \( \alpha = m/n \) (where \( m \) is the number of clauses \( C_i \) and \( n \) the number of variables) plays an important role. Interestingly, there is a complexity phase transition in the solvability of an instance as a function of \( \alpha \). For small \( \alpha \), satisfied instances are easy to find. For large \( \alpha \) instances can almost never be satisfied and for a critical \( \alpha = \alpha_c \) a phase transition [8] occurs. For \( k < 3 \), random \( k \)-SAT formulae can be solved in polynomial time and, for example, \( \alpha_c(k = 2) = 1 \). However, for \( k \geq 3 \) random \( k \)-SAT problems fall into the NP-complete complexity class. This means there are no polynomial-time algorithms (to date) that can solve these. In particular, \( \alpha_c(k = 3) \approx 4.17 \) [9].

For regular 3-SAT, if an assignment satisfies all the literals in a clause, the clause is considered as satisfied. The special not-all-equal (NAE) 3-SAT case,
however, requires at least one literal to be true and at least one literal to be false. Therefore, the case where all three literals are true is not allowed.

2.3 Reminder — SAT filters

There are two ways to build a SAT filter. Here we only discuss the single-instance filter. For details on how to build a filter with more than one instance see Ref. [4]. The following steps are needed to build a probabilistic membership filter based on SAT formulas:

- **Build a CNF** — We use a set of hash functions \( h_1, h_2, \ldots, h_k \) to create a random clause \( C_Y \) with \( k \) literals for each \( y \in Y \) with \(|Y| = m\). This means there are \( m \) clauses, which make a random \( k \)-SAT instance (CNF) \( X_Y \). During the process, it is important to ensure that all literals are different in each clause. Furthermore, the clause-to-variable ratio \( m/n \) should not be too high such that the CNF is not in the unsatisfied regime.

- **Find solutions to the CNF** — Once the CNF has been constructed, multiple uncorrelated solutions are needed to construct a good filter. It is important to use SAT formulas for which efficient solvers are known to speed up this step of the filter building process.

- **Store the filter** — After finding a number of (ideally uncorrelated) solutions, these are stored in an array as the filter. Note that the storage requirements for the filter are considerably smaller than the original data.

Note that SAT filters do not allow insertions after they have been built, i.e., adding a new element to \( D \) will require a new filter to be constructed.

To query the filter, the first step is to generate a clause \( C_x \) from the elements \( x \in D \) using the hash functions used to generate the random clause in the first step of the filter construction process. Then one has to verify if \( C_x \) is satisfied by the filter, i.e., **all** solutions stored. If so, the filter returns *maybe*. However, if \( C_x \) is not satisfied by any one of the solutions, the filter returns a definite *no*. A filter is characterized by its false positive rate, which should be as low as possible, its memory efficiency, as well as ideally short build and query times. We discuss both for the case of SAT filters in what follows.

**False positive rate** — Probabilistic membership filters have a finite false positive rate (FPR). This means that for an element \( x \) that is not in the set of interest \( Y \) the filter might still return a *maybe* result. The FPR for SAT filters is equivalent to the probability that a random \( k \)-SAT clause can be satisfied by a specific solution. For a random \( k \)-SAT clause, the probability that the clause can definitely not pass one regular solution is \( 2^{-k} \). Therefore, for a single solution to the CNF, the FPR is \((1 - 2^{-k})\). This can be improved, by using \( s > 1 \) solutions to the CNF, i.e., for \( s \) solutions

\[
p_{\text{SAT}} = (1 - 2^{-k})^s. \tag{3}
\]
Using $s > 1$ solutions reduces the FPR, but increases both query times and storage requirements by a factor $s$. Note that if the solutions are correlated, then the FPR might not be reduced by increasing the number of solutions. Therefore, it is imperative to use a SAT solver that produces as uncorrelated solutions as possible (i.e., with a large hamming distance). We note that for the special case of building SAT filters with NAE-SAT formulas Eq. (3) changes to

$$p_{\text{NAE}} = (1 - 2^{-k+1})^s. \quad (4)$$

**Efficiency** — The memory efficiency of a probabilistic membership filter is defined as the number of filter bits required per keyword item. As introduced in Ref. [4], the memory efficiency $\xi$ for a SAT filter is given by

$$\xi = -\frac{\log_2 p}{n/m}. \quad (5)$$

Here, $n$ is the number of memory bits and $m = |Y|$. For a SAT filter that uses $s$ solutions, one needs $sn$ memory bits and therefore the efficiency is given by

$$\xi_{\text{SAT}} = -\frac{\log_2 ps_{\text{SAT}}}{sn/m} = -\frac{\log_2 (1 - 2^{-k})}{n/m}. \quad (6)$$

Again, for the special case that uses NAE-SAT solutions, Eq. (6) changes to

$$\xi_{\text{NAE}} = -\frac{\log_2 (1 - 2^{-k+1})}{n/m}. \quad (7)$$

It can be shown rigorously [4] that SAT filters can achieve a theoretical efficiency of 1 (i.e., 100%). This is to be contrasted to Bloom filters that have an information-theoretical upper bound for the efficiency, namely $\xi_{\text{Bloom}} \leq \log 2 \approx 0.698$. This is the main reason why SAT filters are more desirable than Bloom filters.

**Build & query times** — In Ref. [4] it was shown that while query times for SAT filters are short, they are still larger than for Bloom filters. Furthermore, the construction of a SAT filter requires multiple uncorrelated solutions to a CNF. While efficient SAT solvers exist, some tend to produce correlated solutions, which thus means that there is potentially a large overhead in finding as uncorrelated solutions (i.e., with a large hamming distance) as possible.

In what follows we demonstrate how filters designed using NAE-SAT formulas have considerably shorter build, as well as query times. Furthermore, we show how a NAE-SAT CNF can be reformulated into a SAT formula such that one can take advantage of state-of-the-art random $k$-SAT solvers. Finally, we show that only one hash function is needed, thus optimally speeding up build and query times.
3 Building a NAE-SAT filter

A low FPR is of utmost importance for probabilistic membership filters. In the case of SAT filters, the theoretical FPR [Eq. (3)] can only be reached if the solutions to the underlying SAT formula are uncorrelated. Because the solutions are from a single SAT formula, the probability that these are correlated is high, unless a typically large effort to find enough uncorrelated solutions is performed.

Here we work around this bottleneck by replacing standard SAT formulas with NAE-SAT formulas, because the solutions to NAE-SAT problems have large Hamming distances (typically around 50% of the number of variables) and are therefore far less correlated [10]. In our approach we randomly select the NAE-SAT solutions to build the filter. Because we never know which solution in the pair was selected, we can state statistically that the average hamming distances would be around 50% of the number of variables by construction. This saves considerable time when building the probabilistic filter. From now on, unless otherwise specified, we use NAE-SAT solutions.

As is the case for traditional SAT filters, it desirable to have a small value of k (number of literals per clause) to reduce query times. As we shall demonstrate, near-perfect efficiencies $\xi$ can be obtained for $k \lesssim 6$. The number of variables in the SAT instance $n$ should be chosen such that $\alpha = m/n$ is in a regime where it is not too hard to solve the CNF with a given SAT solver. Because the size of the data set is given ($m = |Y|$, $m$ the number of clauses per instance), this shall influence the choice of $n = m/\alpha$. Finally, because there is as tradeoff between the FPR $p$ and the memory efficiency, this is what dictates the value of $s$. In particular, the lower the amount of required storage, the higher the FPR. From Eq. (7) one can estimate the number of SAT instances $s$ needed to achieve a given FPR $p$.

Furthermore, we experiment with a SAT solver [11] that efficiently traverses the solution space, thus generating typically uncorrelated solutions. borealis — a method that works extremely well to solve both weighted and unweighted MAX-SAT problems — is based on parallel tempering Monte Carlo, a standard workhorse in the study of frustrated magnetic systems in statistical physics. The idea is to randomly propose variable changes using a simple Monte Carlo method. In addition to the local updates, the system is replicated at multiple temperatures [12,13,14]. Swaps between temperatures are allowed, therefore allowing the system to relax out of local minima and more efficiently sample the solution space. borealis is typically not faster than highly-tuned SAT solvers. However, it is a generic method that works relatively well for many SAT-type problems and can produce easily uncorrelated solutions.

We first analyze the performance of NAE-SAT filters using traditional random $k$-SAT solvers and then show results using borealis. Note that we use similar parameters as used in Ref. [4] to be able to perform a direct comparison between the results of Ref. [4] on traditional SAT filters and our NAE-SAT implementation shown here.
3.1 NAE-approach using traditional solvers

To be able to use traditional $k$-SAT solvers such as Dimetheus \[15\] ($\alpha \approx \alpha_c$) or WalkSATlm [16] ($\alpha \lesssim \alpha_c$) for NAE-SAT instances, we have modified the original NAE-SAT CNF into a SAT CNF such that by construction all solutions satisfy the NAE requirement. This is accomplished by adding a penalty clause to rule out the all-satisfied assignments to each clause in the original CNF. In the penalty clause all literals are complementary to the original clause. Then, as long as the solver finds a solution, the solution is a NAE solution to the original CNF. For example,

\[(x_3 \lor x_{18} \lor \overline{x_{12}} \lor x_5) \rightarrow (x_3 \lor x_{18} \lor \overline{x_{12}} \lor x_5) \land (\overline{x_3} \lor x_{18} \lor x_{12} \lor \overline{x_5}).\] (8)

The term on the right now satisfies the NAE constraint and can be handled by traditional random $k$-SAT solvers.

Build times — We first benchmark the build time of a NAE-SAT filter. As an example, we construct a filter with 75% memory efficiency and $2^{14}$ clauses. The results are shown in Table 1. Because $\alpha$ is small in this case, we use WalkSATlm for the case study. In Table 2 we quote for comparison the results of Ref. \[4\] for a regular $k$-SAT filter. The use of NAE-SAT solutions clearly reduces the filter build times in comparison to the $k$-SAT version of the filters. Although the used hardware is different, the difference cannot account for the large difference in filter build times.

Table 1. Build time in seconds, memory size in bytes and false-positive rate (FPR) in percent for the $k$-NAE-SAT filter case study using WalkSATlm. By design, the average Hamming distance is around 50%. Simulations were performed on a 2013 MacBook Pro with a 2.60 GHz processor. $s$ represents the number of NAE-SAT instances used to build the filter.

| Filter size | Build time (s) | Size (bytes) | FPR (%) |
|-------------|----------------|--------------|---------|
| $k = 4, s = 11$ | 0.6 | 44748 | 23.00% |
| $k = 5, s = 22$ | 1.0 | 44000 | 24.45% |
| $k = 6, s = 44$ | 1.2 | 44144 | 25.10% |

Efficiency vs False-positive rate — Figure 1 shows the memory efficiency $\xi_{\text{NAE}}$ vs the FPR $p_{\text{NAE}}$. Using formulas with $m/n = 10.1$ and increasing the filter size $m$ eventually has little effect on the efficiency. However, for increasing $m$ the FPR $p_{\text{NAE}}$ can be reduced to arbitrarily-low values; here below $10^{-5}$. The solid horizontal (green) line represents the optimal bound which can easily be achieved with little numerical effort. Note that the instances were generated using Dimetheus \[15\] because the ratio is close to the threshold.
Table 2. Build time in seconds, memory size in bytes and false-positive rate (FPR) in percent for the $k$-SAT filter studied in Ref. [4]. The average hamming distance is at least 50%. Simulations were performed on a 2009 MacBook Pro with a 3.06 GHz processor. $s$ represents the number of SAT instances used to build the filter.

| Filter size | Build time (s) | Size (bytes) | FPR (%) |
|-------------|----------------|--------------|---------|
| $k = 4$, $s = 22$ | 20802          | 44748        | 24.20%  |
| $k = 5$, $s = 44$ | 610            | 44000        | 24.86%  |
| $k = 6$, $s = 88$ | 643            | 44144        | 25.09%  |

Table 3. Build time in seconds, memory size in bytes and false-positive rate (FPR) in percent for the $k$-NAE-SAT filter case study using borealis. By design, the average Hamming distance is around 50%. Simulations were performed on a 2013 MacBook Pro with a 2.60 GHz processor. $s$ represents the number of NAE-SAT instances used to build the filter.

| Filter size | Build time(s) | Size(bytes) | FPR  |
|-------------|---------------|-------------|------|
| $k = 4$, $s = 11$ | 6.2           | 44748       | 23.00%|
| $k = 5$, $s = 22$ | 11.0          | 44000       | 24.45%|
| $k = 6$, $s = 44$ | 17.8          | 44144       | 25.10%|

3.2 Using physics-based solvers

Because borealis [11] is designed to tackle statistical physics problems, we need to cast the CNF of the NAE-SAT formula as a physical Hamiltonian (cost function). We use the number of unsatisfied clauses as a simple cost function for the $n$ Boolean variables in the CNF. If a cost of 0 is found (in physics, the ground state energy), the configuration represents a valid variable assignment to the CNF. Details on the algorithm and its implementation can be found in Ref. [11].

Build times — Table 3 lists our results for the same instances studied in Tab. 1. Although borealis is slower than Dimetheus for these instances, using NAE-SAT filters is still considerably faster than $k$-SAT filters (see Tab. 2). We do note, however, that borealis works reasonably well for a broad range of $\alpha$ values unlike traditional SAT solvers that are tuned to specific regimes of $\alpha$ values.

Efficiency vs false-positive rate — Figure 2 shows the efficiency $\xi$ vs FPR $p$ for a fixed filter size $m = 2^{14}$ and different values of $\alpha = m/n$ ($k = 5$). borealis performs reasonably well for a broad range of $\alpha$ values with $8 \lesssim \alpha \lesssim 19 < \alpha_c \approx 21.11$. There is a decrease of the efficiency $\xi$ for small FPRs. This is because $m$ is relatively small and therefore the number of uncorrelated solutions is accordingly small. As such, finding a set $s$ of many uncorrelated solutions is difficult. This problem is referred to as finite-size effect in physics and is easily
alleviated by increasing the filter size. For \( n/s \) large the finite-size effects become negligible. We expect that a better implementation of borealis might show a clear advantage over traditional solvers for larger filter sizes. We leave this study for a future publication.

### 3.3 Querying NAE-SAT filters

To speed up query times for the NAE-SAT filter we use bit packing. Given the 64-bit architecture of the benchmark machine, this means that 64 instances can be handled in parallel.

For the benchmarks, we use a single core 2013 MacBook Pro with a 2.60GHz processor and 8Gb RAM. We query \( 2^{17} \) 64-bit strings. The hash function used is MurmurHash3 [17]. Note that we deviate from the approach used in Ref. [4], because, as Fig. 3 shows, the difference between the FPR using one or two hashes is negligible. We did simply change the seed in MurmurHash3 to achieve these results. This is yet another advantage of our implementation that speeds up query times.

**Fig. 1.** Efficiency \( \xi_{\text{NAE}} \) as a function of false-positive rate \( p_{\text{NAE}} \) for a \( k \)-NAE-SAT filter. As \( m \) increases for fixed \( k \), the efficiency approaches the theoretical bound (solid horizontal line). Note that very low FPRs can be obtained. Instances generated with Dimetheus.
Figure 2. Efficiency $\xi_{\text{NAE}}$ as a function of false-positive rate $p_{\text{NAE}}$ for a $k$-NAE-SAT filter. Data computed with borealis for different values of $\alpha = m/n$. Finite-size effects (see text) are visible due to the small filter size used. However, borealis is an efficient solver for a broad range of $\alpha$ values.

Figure 4 shows that the query times are approximately similar for different values of $s$, as long as there are more than a certain number of solutions. The jump in the data might be due to buffering issues in the bit packing.

4 Summary

We have demonstrated that by using NAE-SAT problems for the construction of SAT filters as first mentioned in Ref. [4], filter build times can be considerably reduced. Using NAE-SAT formulas to build the filters has the advantage that, by design, the solutions tend to be uncorrelated. Furthermore, we show how the NAE-SAT constraint can be accommodated into a random SAT formula such that standard SAT solvers can be used. Query times in our implementation are reduced via bit packing and the use of a single hash function. Finally, by using physics-inspired algorithms such as borealis the filter construction can be further parallelized and improved further because the algorithm efficiently traverses the solution space.
Fig. 3. FPR $p$ as a function of the number of solutions $s$ using one, or two-hash functions in MurmurHash3 [17]. The difference between both approaches is $\sim 10^{-5}$, i.e., negligible.

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Fig. 4. Query time as a function of the number of used instances $s$ for a NAE-SAT filter with $k = 5$, $m = 2^{14}$, and $n = 16282$. Queried are $2^{17}$ 64-bit strings. Time $t$ is measured in seconds.

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