Matrix Coding Technique on Sunflower Graphs with Edge Product Cordial Labeling

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Abstract. In this paper we develop a technique of coding a secret messages using Sun Flower graphs $SF_n$ by subdividing edges and applying edge product cordial labeling. The induced vertex labeling function defined by the product of labels of incident edges to each vertex is such that the number of edges with label 0 and 1 differ by atmost 1. Here, we discuss edge product cordial labeling for sunflower graphs subdivided edges and developed a coding technique of transforming plain text (Text message) into Cipher Text (Matrix code) and established algorithm. By recent development in coding theory involving programming concepts in any computer languages, this article will revisit without programming method, the matrix coding technique is developed.

1. Introduction
Graph Theory is too generous and prolific offering innumerable concepts with applications galore. Graph theory is one of the mathematic which growing rapidly and can be used to simplify the solution of a problem in day today life. Graph theory can be used to modeling a problem that can be easier to see and find the solution for the problem. one of the Graph subject is graph labeling topic. For more results on graph labeling can be found in \cite{2}. In this paper we transformed plain text in to cipher text in the form of Matrix and the key is clue for guessing the graph.Cryptography is the science containing methods
to transform an intelligible message into one that it is unintelligible and transforming the message back to its original form. The oldest types of ciphers (algorithm) was developed by Julius Caesar ([3], [4]).

This research work involving secret coding method through edge product cordial labeling. Computer scientists have now invented a way to hide secret messages in ordinary text by imperceptibly changing shapes of letters. Cryptography is the study of methods converting messages to a form unreadable except to one who knows how to decrypt them.

1.1. Literature Review:
The communication becomes very much limited between the sender and the receiver and not to be understood by others. The mind of man, then struck at the idea of coding languages ([I]). Sundaram et al., [8] have introduced product cordial labeling for some graphs. Vaidya and Barasara [14] introduced the concept of the edge product cordial labeling as an edge analogue of the product cordial labeling. Edge product cordial labeling of some cycle related graphs was developed by Udayan et al., [15]. A Graph Labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions. Graph labelings were first introduced in the mid 1960's. In the intervening 50 years over 200 graph labelings techniques have been studied in about 2500 papers. Most graph labeling methods trace their origin to one introduced by Rosa [5] in 1967, of one given by Graham and Sloane in 1980. Fascinated by a variety of graph labeling and their applications, the researcher made a deep study and selected a Edge product cordial labeling to work on.

Cryptography is the science containing methods to transform an intelligible message into one that it is unintelligible and transforming the message back to its original form. The oldest types of ciphers (algorithm) was developed by Julius Caesar. In the context of Cryptography the graph problems are usually trivial but sometimes by suitable generalization they can suggest concepts which are not only nontrivial but which may even be of some interest and few examples are given by R. C. Read [7]. The security of communication is a crucial issue on world wide web. It is about confidentiality, integrity, authentication during access or editing of confidential internal documents and are given by a research paper [9]. Alphabets shifting by three was implemented by Rizwan [6], coding through a two and three graph with super mean labeling is structured by Uma Maheswari et al.,[10],[11] and coding with Fibonacci web is established by Uma Maheswari et al.,[12]. Motivated by these work, we worked on sunflower graph $SF_4D_2$ and $SF_6D_1$ by applying Edge product cordial labeling and GMJ (Graph Message Jumbled) coding method and
hence this paper.

2. Definitions

Definition 2.1. For a graph $G$, the edge labeling function is defined as $f : E(G) \rightarrow \{0, 1\}$ and induced vertex labeling function $f^* : V(G) \rightarrow \{0, 1\}$ given $e_1, e_2, \ldots, e_n$ are all edges incident to the vertex $v$, then $f^*(v) = f(e_1)f(e_2)f(e_3)\cdots$. Let $v_f(i)$ be the number of vertices of $G$ having label $i$ under $f^*$ and $e_f(i)$ be the number of edges of $G$ having label $i$ under $f$ for $i = 0, 1$. Then $f$ is called an Edge Product Cordial Labeling of graph $G$ if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph $G$ is called Edge Product Cordial if it admits an Edge Product Cordial Labeling. The product cordial labeling the roles of vertices and edges are interchanged.

The Helm $H_n$ is the graph obtained from a wheel $W_n$ by attaching a pendant edge to each of the rim vertices. The flower graph $Fl_n$ is the graph obtained from a helm $H_n$ by joining each pendant vertex to the apex vertex of the helm $H_n$. Duplication of a vertex $v$ of a graph $G$ produces a new graph $G'$ by adding a new vertex $v'$ such that $N(v') = N(v)$. In other words, $v'$ is said to be a duplication of $v$ if all the vertices which are adjacent to $v$ in $G$ are also adjacent to $v'$ in $G'$.

Duplication of a vertex $v'$ of a graph $G'$ produces a new graph $G''$ by adding a new vertex $v''$ such that $N(v'') = N(v')$. In other words, $v''$ is said to be a double duplication of $v$ if all the vertices which are adjacent to $v$ in $G$ are also adjacent to $v''$ in $G''$ such that $N(v) = N(v') = N(v'')$.

3. Coding Method

By assigning numbers to the 26 alphabets of English in a different manner, choosing a suitable labeled graph with a given clue mathematical or non-mathematical, finding the number in the graph for each letter of each word of the given message and presenting the letter codes in a unique way in some form, writing it as a horizontal string or in any other way and creating a picture with the codes after shuffling the order of the letters in order to increase the secrecy of the coded message is named as GMJ coding method. An algorithm for transforming an intelligible message into one that is unintelligible by transposition and substitution methods is known as Cipher.

A Caesar cipher shifts the alphabet and is therefore called a shift cipher. Each letter is replaced by the letter three positions further down the alphabet. Caesar used a key 3 for his communication and it is known as Caesar Cipher. The original intelligible
message is known as Plain text, and the transformed message is known as Cipher text.

3.1. Numbering of Alphabets

(i) **PPST:** (product and powers of special triplet)

The triplet 1, 2, 3 is a special triplet as it is the only one triplet whose sum equals the product. Here the powers of 1, 2, 3 and the product of the powers of 2 and 3 are used for alphabets. So it is aptly named as PPST. The alphabets A, B, C are allotted the numbers 1, 2, 3 respectively. The alphabets from D to Z are divided into sets of 2 in order. Using powers of 2 and 3 alternately, the alphabets D to M are given the numbers $2^r$ and $3^r$ ($r = 2, 3, \cdots, 6$). From N to T, the alphabets are given the numbers $2^r \times 3$ ($r = 1, 2, 3, \cdots, 7$). From U to Z, $2 \times 3^r$ ($r = 1, 2, 3, \cdots, 7$) are given.

|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
| A | B | C | D | E | F | G |
| 1 | 2 | 3 | $2^2$ | $3^2$ | $2^3$ | $3^3$ |
| H | I | J | K | L | M | N |
| $2^4$ | $3^4$ | $2^5$ | $3^5$ | $2^6$ | $3^6$ | $2 \times 3$ |
| O | P | Q | R | S | T |
| $2^2 \times 3$ | $2^3 \times 3$ | $2^4 \times 3$ | $2^5 \times 3$ | $2^6 \times 3$ | $2^7 \times 3$ |
| U | V | W | X | Y | Z |
| $2 \times 3^2$ | $2 \times 3^3$ | $2 \times 3^4$ | $2 \times 3^5$ | $2 \times 3^6$ | $2 \times 3^7$ |

The function for encoding is given below:

\[
g(a_{2k}) = 2^k, \ k = 1, 2, 3, \cdots, 6
\]
\[
g(a_{2k+1}) = 3^k, \ k = 0, 1, 3, \cdots, 6
\]
\[
g(a_{13+k}) = 2^k \times 3, \ k = 1, 2, 3, \cdots, 7
\]
\[
g(a_{20+k}) = 2 \times 3^{k+1}, \ k = 1, 2, 3, 4 \cdots, 6
\]

(ii) **Coding a letter:**

Here all the spokes and the rim edges take the value 0. As 1 and powers of 2 and 3 are used for numbering the alphabets, the number of zeros and ones for LWE and RWE alone are considered. L(5, 0) represents the number of zeros of the fifth petal along the left whorls. Here L(5, 0) = 2. To get any power of 2, the number is written before L(5, 0) and R(2, 1) represents the number of ones on the right whorls of the second petal. Here R(2, 1) = 3. To get any power of 3, the number is written before
\( R(2,1) \). For example T corresponds to \( 2^7 \times 3 \) and so takes up the code \( 7L(5,0)R(2,1) \) and so on.

Before transmission, the sender transforms the plain text in to cipher text and this is known as encryption. When the cipher text is received the receiver must transform the cipher text back in to the plain text and this is known as decryption. To be secure, encryption and decryption must be require that the sender and receiver possess some secret information or key.

A Method of matrix coding technique with two illustrations by using two graphs are shaped by applying Edge product cordial labeling on a sunflower graphs. Numbering of alphabets and coding a letter is same for the two illustrations.

**Illustration 1:**

(i) **Message:** Meet captain on third Wednesday.

(ii) **clue:** Yellow fellow turning in sixth yearning.

(iii) **Graph:** The Sunflower graph \( SF_6D_1 \) is considered.

(iv) **Verification:**

Graph is obtained from \( SF_4 \) by duplicating of each vertices \( r_i, i=1,2,3,...,6 \) by new vertices \( s_i, i=1,2,3,...,6 \) also graph obtained from \( SF_6D_1 \) by subdividing the edges \( r_iv_i \) and \( r_iv_{i+1(modn)} \) for \( i=1,2,3,...,6 \) by a vertex \( u'_i \) and \( u''_i \) respectively for \( i=1,2,3,...,6 \) and subdividing the edges \( s_iv_i \) and \( s_iv_{i+1(modn)} \) for \( i=1,2,3,...,6 \) by a vertex \( o'_i \) and \( o''_i \) respectively for \( i=1,2,3,...,6 \) and it is edge product cordial labeling is shown in fig 1.

(a) The spoke edges 6, the rim edges 6 and the LWE and RWE between \( v_4 \) to \( v_6 \) and to \( v_1 \) of the inner petal with subdivision edges totally 30 are assigned the number 0.

\[
1
\]

(b) The LWE and RWE of the two whorls(Inner and outer with subdivisions) from \( v_1 \) to \( v_4 \) totally 22 and the LWE and RWE of the outer whorls between \( v_4 \) to \( v_6 \) and \( v_6 \) to \( v_1 \) totally 30 edges are assigned the number 1 .

\[
2
\]

From (1) and (2), \( |v_f(0) - v_f(1)| = 1 \leq 1 \) is satisfied.

Therefore it is a Edge product cordial labeling.
Figure 1 shows the Pictorial representation for Sunflower graph with Edge Product Cordial Labeling.

Figure 1. Sunflower graph $SF_6D_2$

(v) **Coding:** (word wise)

Meet  

$- 6L(4, 0)R(5, 1)2L(5, 0)R(6, 1)2L(6, 0)R(4, 1)7(L(1, 1)/2)R(5, 1)$

captain  

$- L(6, 0)R(5, 1)L(4, 1)R(6, 1)3(L(3, 1)/2)R(5, 0)7(L(2, 1)/2)R(6, 0)L(4, 1)L(4, 1)3R(5, 0)(L(3, 1)/2)R(4, 0)$

on  

$- 2(L(2, 1)/2)R(5, 0)(L(1, 1)/2)R(5, 0)$

third  

$- 7(L(2, 1)/2)R(6, 0)4(L(1, 1)/2)R(5, 0)4L(4, 0)R(5, 1)5(L(1, 1)/2)R(6, 1)2(L(1, 1)/2)R(6, 0)$

wednesday  

$- (L(1, 1)/2)4R(6, 0)2L(5, 0)R(6, 1)2(L(2, 1)/2)R(5, 1)(L(2, 1)/2)R(5, 0)2L(3, 1)/2)R(5, 1)L(5, 1)R(4, 0)(L(1, 1)/2)6R(5, 0)$

(vi) **Presenting the letter codes:**

There are 6 petals in the sunflower taken. Divide the letters in the message into sets of 6 letters. The left part of these letters are written horizontally first, followed by the right part seperated by a ‘∗’. The matrix code omitting $L$ and $R$ is sure to trigger off the secrecy. The Horizontal string is written and the string is given below:
(vii) **Horizontal string:**

\[ 6L(4,0)2L(5,0)2L(6,0)7(L(1,1)/2)L(6,0)L(4,1)*R(5,1)R(6,1)R(4,1) \]
\[ R(5,1)R(5,1)R(6,1)*2(L(2,1)/2)*R(5,0)R(6,0)R(4,1)*2(L(2,1)/2) \]
\[ R(5,0)R(6,0)R(5,0)*R(5,0)R(6,0)R(5,0)* (L(1,1)/2) \]
\[ 7L(2,1)/2)7(L(2,1)/2)L(6,1)L(4,1)(L(3,1)/2) \]
\[ 2(L(2,1)/2)*R(5,0)R(6,0)R(4,1)3R(5,0)R(4,0)R(5,0)*R(5,1) \]
\[ R(4,0)6R(5,0) \]

(viii) **Matrix coding :**

\[
\begin{pmatrix}
6(4,0)2(5,0) & 2(6,0)7((1,1)/2) & (6,0)(4,1) \\
(5,1)(6,1) & (4,1)(5,1) & (5,1)(6,1) \\
3((3,1)/2)7((2,1)/2) & (6,1)(4,1) & ((3,1)/2)2((2,1)/2) \\
(5,0)(6,0) & (4,1)3(5,0) & (4,0)(5,0) \\
((1,1)/2)7((2,1)/2) & 4((1,1)/2)4(4,0) & 5((1,1)/2)2((1,1)/2) \\
(5,0)(6,0) & (5,0)(5,1) & (6,1)(6,1) \\
((1,1)/2)2(5,0) & 2((2,1)/2)((2,1)/2) & 2(6,0)6((2,1)/2) \\
4(6,0)(6,1) & (5,1)(5,0) & (5,1)(6,0) \\
2((3,1)/2) & (5,1) & ((1,1)/2) \\
(5,1) & (4,0) & (6,5,0) \\
\end{pmatrix}
\]

**Illustration 2:**

(i) **Message:** Secret bag in your hand.

(ii) **clue:** Yellow fellow turning in fourth yearning.

(iii) **Graph:** The Sunflower graph \( SF_4D_2 \) is considered.

(iv) **Verification:**

Graph is obtained from \( SF_4 \) by duplicating of each vertices \( x_i \), \( i=1,2,3,4 \) by new vertices \( y_i \), and double duplication of each vertices \( x_i \), \( i=1,2,3,4 \) by new vertices \( z_i \), \( i=1,2,3,4 \) also graph obtained from \( SF_4 \) by subdividing the edges \( x_i v_i \) and \( x_i v_{i+1}(mod \ n) \) for \( i=1,2,3,4 \) by a vertex \( a_i^1 \) and \( a_i^2 \), subdividing the edges \( y_i v_i \) and \( y_i v_{i+1}(mod \ n) \) for \( i=1,2,3,4 \) by a vertex \( b_i^1 \) and \( b_i^2 \) and subdividing the edges \( z_i v_i \) and \( z_i v_{i+1}(mod \ n) \) for \( i=1,2,3,4 \) by a vertex \( c_i^1 \) and \( c_i^2 \), respectively for \( i=1,2,3,4 \) and it is a Edge product cordial labeling is shown in the Fig 1.
Presenting the letter codes:

(a) The spoke edges 4, subdivision edges, the rim edges 4 and the LWE and RWE between \(v_3\) to \(v_4\) and to \(v_1\) of the inner, middle and outer subdivided petals are assigned the number 0.

(b) The LWE and RWE of the two whorls (Inner, middle and outer) also subdivision edges from \(v_1\) to \(v_3\) and the LWE and RWE of the outer whorls between \(v_3\) to \(v_4\) and \(v_4\) to \(v_1\) are assigned the number 1.

From (3) and (4), \(|v_f(0) - v_f(1)| = 1 \leq 1\) is satisfied.

Therefore it is a Edge product cordial labeling.

The Pictorial representation as similar to Figure 1.

(v) Coding: (word wise)

| Secret | bag | in | your | hand |
|--------|-----|----|------|------|
| \(-6(L(1,0)/3)(R(2,1)/2)2(L(2,1)/2)R(3,1)\) \(L(2,1)/2R(4,1)5(L(2,1)/3)R(2,1)/2\) \(2(L(1,1)/2)R(4,1)7(L(1,1)/3)R(2,1)/2\) | \(-L(1,1)/3R(3,1)L(3,1)R(4,1)3(L(1,1)/2)R(4,1)\) | \(-4(L(1,1)/2)R(4,1)L(2,1)/3R(2,1)/2\) | \(-6(L(1,1)/3)6(R(1,1)/2)2L(1,1)/3R(1,1)/2L(2,1)/2\) | \(-2(R(2,1)/3)5(L(1,1)/3)R(1,1)/2\) |
| \(-2(L(2,1)/3)R(3,1)L(4,1)R(3,1)L(1,1)/3R(1,1)/2\) | | | \(-2(L(2,1)/3)R(4,1)\) |

(vi) Presenting the letter codes:

There are 4 petals in the sunflower taken. Divide the letters in the message into sets of 4 letters. The left part of these letters are written horizontally first, followed by the right part separated by a ‘∗’.

The Horizontal string is written and the string is given below.

(vii) Horizontal string:

\((L(1,0)/3)2(L(2,1)/2)(L(2,1)/2)(5(L(2,1)/3) ∗ (R(2,1)/2)R(3,1)R(4,1)\) \((R(2,1)/2) ∗ 2L(1,1)/27L(1,1)/3L(1,1)/3L(3,1) ∗ R(4,1)R(2,1)/2R(3,1)\) \(R(4,1) ∗ 3(L(1,1)/2)4L(1,1)/2L(2,1)/3L(2,1)/3 ∗ R(4,1)R(4,1)(R(2,1)/2)\) \(6(R(1,1)/2) ∗ 2L(1,1)/2)/2L(2,1)/2)5L(1,1)/34L(1,1)/3 ∗ R(1,1)2(R(2,1)/3)\)
\((R(1,1)/2)R(3,1) * L(4,1)L(1,1)/3(2L(2,1)/3) * R(3,1)(R(1,1)R(4,1))\)

(viii) Matrix coding:

\[
\begin{pmatrix}
((1,0)/3)2((2,1)/2) & (2,1)/2(5((2,1)/3) & ((2,1)/2)(3,1) \\
(4,1)(2,1)/2 & 2((1,1)/2)7((1,1)/3) & (1,1)/3)(3,1) \\
(4,1)(2,1)/2 & (3,1)(4,1) & 3((1,1)/2)4((1,1)/2) \\
(2,1)/3((2,1)/3) & (4,1)(4,1) & ((2,1)/2)6((1,1)/2) \\
2((1,1)/3)/2((2,1)/2) & 5((1,1)/3)4((1,1)/3) & (1,1)/2(2,1)/3) \\
((1,1)/2)(3,1) & (4,1)(1,1)/3 & 2((2,1)/3)(3,1) \\
((1,1)/4,1) & (0,0)(0,0) & (0,0)(0,0)
\end{pmatrix}
\]

4. Algorithm
- **Step 1**: Sunflower graph has to be taken using clue.
- **Step 2**: Apply product cordial labeling on the graph.
- **Step 3**: Split the alphabets using PPST.
- **Step 4**: Method of coding for each letter is stated.
- **Step 5**: The message to be coded is written.
- **Step 6**: Coding word wise.
- **Step 7**: Representing codes into Horizontal string.
- **Step 8**: Representing string into Matrix form.

The sender has to forward to the receiver the following:
- Clue to guess the graph
- Clue to guess the numbering of alphabets without explanation.
- Letter codes along a matrix

5. Conclusion and future work:
In this paper we have settled the graph \(SF_4D_2\) and \(SF_6D_1\) is an Edge product cordial labeling. Similar problem can be discussed for other graph families. We illustrated how the text message is converted to matrix code. This research meant for communicating any message personal, official, governmental or pertaining to military services with a high level secrecy, intricacy and a sense of sufficiency which are the most important factors for
coding. In this research article we have shown how the text message is transformed into coded message using sunflower graphs. In our future work we would like to apply vertex product cordial labeling for sunflower graphs and develop coding techniques for them.

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