The 3D Dust and Opacity Distribution of Protoplanets in Multifluid Global Simulations

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Abstract

The abundance and distribution of solids inside the Hill sphere are central to our understanding of the giant planet dichotomy. Here, we present a 3D characterization of the dust density, mass flux, and mean opacities in the envelope of subthermal and superthermal-mass planets. We simulate the dynamics of multiple dust species in a global protoplanetary disk model accounting for dust feedback. We find that the meridional flows do not effectively stir dust grains at scales of the Bondi sphere. Thus the dust settling driven by the stellar gravitational potential sets the latitudinal dust density gradient within the planet envelope. Not only does the planet’s potential enhance this gradient, but also the spiral wakes serve as another source of asymmetry. These asymmetries substantially alter the inferred mean Rosseland and Planck opacities. In cases with moderate-to-strong dust settling, the opacity gradient can range from a few percent to more than two orders of magnitude between the midplane and the polar regions of the Bondi sphere. Finally, we show that this strong latitudinal opacity gradient can introduce a transition between optically thick and thin regimes at the scales of the planet envelope. We suggest that this transition is likely to occur when the equilibrium scale height of hundred-micron-sized particles is smaller than the Hill radius of the forming planet. This work calls into question the adoption of a constant opacity derived from well-mixed distributions and demonstrates the need for global radiation hydrodynamics models of giant planet formation that account for dust dynamics.

Unified Astronomy Thesaurus concepts: Planet formation (1241); Planetary atmospheres (1244)

1. Introduction

The core accretion model for giant planet formation relies on the complex interplay of gas and solids in protoplanetary disks (PPDs; Pollack et al. 1996; Youdin & Kenyon 2013). Despite a mass fraction of only 1% relative to the gas, the solids control much of the planet formation process. Both planetesimals and pebbles impact the growth rate of the solid core and final metallicity (Ormel & Klahr 2010; Lambrechts & Johansen 2012; Alibert et al. 2018). Small grains dominate the envelope opacity (Pollack et al. 1994; Piso et al. 2015) and thus the gas cooling rate in the radiative zones (e.g., Podolak 2003; Hubickyj et al. 2005). In turn, the gas cooling rate controls the growth timescale of the planetary envelope throughout the onset of runaway growth, when the envelope starts to dominate the planet mass (Piso & Youdin 2014). Eventually the envelope separates from the disk, and hydrodynamic accretion is no longer limited by cooling but by the supply of gas that becomes depleted by gap opening and disk dispersal (Lissauer et al. 2009; Ginzburg & Chiang 2019). We emphasize that, in core accretion theory, envelope cooling, and therefore dust opacities, are essential for a planet’s transition to a gas giant.

In this work we present a 3D multispecies characterization of the dust density, dust mass flux, and mean opacities in the envelope of embedded protoplanets. The distribution of solids in the vicinity of a protoplanet is set by complex dynamical processes, including the gravity of the central star and of the planet, and drag forces with the disk and envelope gas, which are affected by flows triggered by the planet. Three-dimensional simulations have elucidated the hydrodynamic flows that couple the planet envelope and surrounding disk gas (Tanigawa et al. 2012). While local shearing-box simulations can more readily achieve high resolution within the Bondi sphere (e.g., Kuwahara & Kurokawa 2020), global simulations model planet–disk interactions more accurately, e.g., by capturing the full orbits in the horseshoe region. Global models have become increasingly sophisticated, moving from pure hydrodynamics (e.g., Wang et al. 2014; Fung et al. 2015) to radiation hydrodynamics (e.g., Ayliffe & Bate 2009; D’Angelo & Bodenheimer 2013; Lambrechts et al. 2016; Szulágyi et al. 2016; Kurokawa & Tanigawa 2018; Schulik et al. 2019) and magnetohydrodynamics (Gressel et al. 2013).

These simulations have altered the standard picture of 1D core accretion models, where low-mass gas envelopes grow hydrostatically around a core as they cool (Pollack et al. 1996; Piso & Youdin 2014). In particular, 3D simulations have shown that hydrodynamic flows penetrate within the Hill and Bondi radii, which limits the ability of this gas to cool (Ormel et al. 2015; Moldenhauer et al. 2021). However, the gas that is deeper in the envelope is more slowly recycled and can still cool, if at a reduced rate (Cimerman et al. 2017). Furthermore, radiation hydrodynamics calculations show that the strength of dust opacities has a significant effect on the nature of recycling flows and on envelope convection (Zhu et al. 2021). Thus a hydrodynamic determination of 3D distributions of dust opacities—an issue explored in this paper—is a crucial step to a fuller understanding of how 3D flows affect the accretion of planetary envelopes.

In this work, we move toward a self-consistent treatment of thermal physics at the planet–disk interface by exploring the

Note that we refer to “dust opacity” as the opacity of the gas that has small dust grains as its main source. Molecular opacity is usually negligible at the outer envelope for temperatures below 10^4 K (Freedman et al. 2008).
size-dependent particle distribution at scales of the Bondi sphere in global, 3D models. We use novel multifluid 3D simulations including dust feedback, to model the nonuniform dust distribution in the vicinity of the planet. We study the implications for envelope opacity and thus the thermal physics of the cooling and an accreting planet. Our simulations allow us to explore how the interplay between vertical settling and planet–disk interaction impacts the distribution of solids and mean opacities at the scale of the Bondi sphere of subthermal-mass planets. Thus, our simulations are complementary to previous studies that have focused on the large-scale meridional circulation of solids affected by gap-opening planets (Fouchet et al. 2007; Bi et al. 2021; Binkert et al. 2021; Szulágyi et al. 2022). In addition, these numerical simulations may inform updated boundary conditions for long-timescale, 1D core accretion models (e.g., Lee & Chiang 2015).

This work is organized as follows. Model setup and description of the 3D flow: In Section 2 we present the equations, numerical method, and disk model that we adopt in this work. In Section 3 we describe the gas and dust flows at the scale of the Bondi sphere, which are consistent with previous models that neglect focusing. Focusing on the dust density distribution, in Section 3.4 we investigate the deviations from the initial settling equilibrium introduced by the planet potential, highlighting the dust-to-gas ratio anisotropies due to settling induced latitudinal variations, and spiral density wave induced azimuthal ones.

Opacities calculation at the Bondi sphere and conclusions: In Section 4 we estimate how departures from spherical symmetry affect the Rosseland and Planck mean opacity at the Bondi radius. In Section 4.2 we show the existence of an anisotropic opacity distribution driven by the planet potential, highlighting the dust-to-gas ratio anisotropies due to settling induced latitudinal transition between optically thick and thin regimes at the Bondi sphere of a subthermal-mass planet in Section 4.3. We therefore identify the disk locations and conditions in which this transition may develop. Finally, in Section 5 we summarize the main conclusions and emphasize the need for multispecies self-consistent radiative transfer disk models for future work.

2. Numerical Method

The numerical simulations in this work are carried out using the multifluid code FARGO3D (Benítez-Llambay & Masset 2016; Benítez-Llambay et al. 2019). We first introduce the equations in Section 2.1, then continue with a summary of the key features of the numerical method in Section 2.2. We defer discussion of the post-processing analysis required to produce realistic opacity maps to Section 4.1.

2.1. Equations

Our multifluid hydrodynamic simulations solve the following set of equations. First, the continuity equations are

\[ \partial_t \rho_g + \nabla \cdot (\rho_g v_g) = 0, \]
\[ \partial_t \rho_d + \nabla \cdot (\rho_d v_d + J_d) = 0, \]

where \( \rho_g \), \( \rho_d \), \( v_g \), and \( v_d \) correspond to the gas and dust densities and velocities, respectively. The dust diffusion flux is given by

\[ J_d = -D(\rho_g + \rho_d) \nabla \left( \frac{\rho_d}{\rho_g + \rho_d} \right) \]

(2)

where \( D \) is the diffusion coefficient (Weber et al. 2019). The momentum equations for the gas and dust species are

\[ \partial_t v_g + v_g \cdot \nabla v_g = -\nabla P - \nabla \Phi + \frac{1}{\rho_g} \nabla \cdot \tau - F_g, \]
\[ \partial_t v_d + v_d \cdot \nabla v_d = -\nabla \Phi - F_d, \]

(3)

for \( j = 1, \ldots, N \). The gas pressure is defined as \( P = c_s^2 \rho_g \), where \( c_s \) is the sound speed. The terms \( F_g \) and \( F_d \) denote the accelerations due to the drag force between gas and dust species, respectively, and are defined as

\[ F_g = \frac{\Omega}{T_{sj}} \sum_{j=1}^{N} \rho_d T_{sj} (v_g - v_d), \]
\[ F_d = \frac{\Omega}{T_{sj}} (v_d - v_g), \]

(4)

where \( T_{sj} \) corresponds to the Stokes number of the \( j \)th dust species, and \( \Omega \) is the orbital frequency (e.g., Epstein 1924; Whipple 1972). In this work we assume a constant Stokes number and neglect the momentum and mass transfer between dust species.

The gravitational potential \( \Phi \) includes the contributions from the central star and the planet and neglects the indirect term; thus,

\[ \Phi = -\frac{GM_s}{r} - \frac{GM_p}{\sqrt{r^2 - r_s^2}} \]

(5)

where \( r_s \) is a softening length used to avoid a divergence at the planet location, and \( M_p \) and \( r_p \) correspond to planet mass and radial vector position. The viscous stress tensor, \( \tau \), is given by

\[ \tau = \rho_g \nu \left( \nabla v + (\nabla v)^T - \frac{2}{3} (\nabla \cdot v) I \right) \]

(6)

with \( \nu \) the gas viscosity.

2.2. FARGO3D Numerical Simulations

We solve Equations (1) and (3) using an improved version of the code FARGO3D with a more efficient velocity update method, which also allows for effective parallelization of multiple fluids (Krapp & Benítez-Llambay 2020). We perform global 3D simulations on a spherical mesh centered at the star, with coordinates \((r, \phi, \theta)\). The numerical domain comprises only half of the disk in the vertical direction, with \( \theta_{\text{min}} = \pi/2 - 0.1 \) and \( r \in [0.4, 2.1] \). Results will also be described as a function of spherical coordinates centered at the planet defined as \((\hat{r}, \hat{\phi}, \hat{\theta})\), which is a coordinate transformation rather than the simulated domain.

We use a nonuniform static mesh in \( r \) and \( \theta \), defined to provide adequate resolution at the Hill sphere and close to the midplane where dust settles. The grid is obtained using the grid density function \( \psi(s) = a/s + \xi/((s - s_0)^2 + \xi^2) \) (Benítez-Llambay & Pessah 2018). For the radial coordinate \( s = r \), \( s_0 = \hat{r} \), \( a = 1 \), and \( \xi = 0.05 \), whereas for the vertical polar coordinate, we set \( s = \theta \), \( s_0 = \pi/2 \), \( a = 0 \), and \( \xi = 0.02 \). This
The symbols are: $\epsilon$ is the total dust-to-gas mass ratio, $\delta$ is the settling-diffusion equilibrium parameter, $\alpha$ is the effective viscosity parameter, $\Delta \phi$ is the extension of the azimuthal domain, and $N_r$, $N_\theta$, and $N_\phi$ are the number of cells in azimuth, radius, and latitude, respectively. The last column corresponds to the minimum and maximum Stokes numbers of the dust distribution.

configuration provides about 16–20 cells at the Bondi radius for the SUB-runs in the radial and vertical direction with $N_r \times N_\theta = 192 \times 36$.

Unlike $r$ and $\theta$, the grid is uniform in the azimuthal direction, which is required for this implementation of the FARGO scheme for orbital advection (Masset 2000). To achieve a resolution at the Bondi radius comparable to that of the radial and vertical direction, we set $N_\phi = 4800$. In all of the runs, the softening length $r_s$ is set to approximately two cells in the azimuthal direction. A numerical convergence study of our simulations is presented in the Appendix.

### 2.2.1. Boundary Conditions

At the inner and outer disk radius, we employ reflecting boundary conditions for $v_{gr}$ and $v_{dr}$. The gas density and azimuthal velocity are extrapolated to match the initial conditions, whereas for the dust, the radial derivatives of the density and dust azimuthal velocity are set to zero. We additionally include wave-damping buffer zones that restore the density to the initial value while preventing undesired reflections that may perturb the flow (de Val-Borro et al. 2006).

At the upper boundary, the gas density and azimuthal velocity are extrapolated to match the initial conditions, while for the dust, $\partial \phi_{v_{\phi}} = \partial \phi_{\rho_{\phi}} = 0$. A reflecting boundary is adopted for $v_{g\theta}$ and $v_{d\theta}$. Since we simulate only one disk hemisphere, we also adopt reflecting boundary conditions at the disk equator for $v_{g\phi}$ and $v_{d\phi}$; otherwise, the latitudinal derivative is set to zero. The initial conditions and parameters of all simulations are described in Section 2.3 and Table 1.

### 2.3. Disk Model

Our base disk model is adapted from Appendix A of Masset & Benítez-Llambay (2016). We adopt as a reference distance to the central star the planet semimajor axis, $r_p = r_0 = 1$ and typically quote time units in units of the inverse of the orbital frequency $\Omega_0 = \sqrt{GM_*/r_0^3}$. The gas sound speed and disk aspect ratio are defined as

$$c_s = c_s(\frac{r}{r_0})^{-\beta/2}, \quad h = c_s v_K^{-1},$$

where $v_K = \sqrt{GM_*/r}$ is the Keplerian velocity. We set $\beta = 1$ to simulate a disk with constant aspect ratio, that is a gas scale height $H = hr$. We fix the disk aspect ratio to $h = 0.035$ in all of our runs. The gas density is given by

$$\rho_g = \frac{\Sigma_0}{\sqrt{2\pi \rho_0}^3} \left(\frac{r}{r_0}\right)^{-\sigma-1} \sin(\theta)^{-(\beta-\sigma-1+1/h^2)}$$

(8)

with $\sigma = 1/2$ and the surface density $\Sigma_0 = 6.366197 \times 10^{-4} M_*/r_0^2$, which corresponds to $\Sigma_0 \approx 200 \text{ g cm}^{-2}$ at $r_0 = 5.2 \text{ au}$. The radial and vertical velocity are set to zero, whereas the azimuthal velocity is

$$v_{g\phi} = v_K \sqrt{1 - (\beta + \sigma + 1)h^2} = v_K \sqrt{1 - 2.5h^2}.$$  

(9)

The gas viscosity corresponds to $\nu = \alpha c_s^2/\Omega_0$, with $\Omega_K = v_K/r$ the orbital Keplerian frequency. The values of $\alpha$ adopted for each run are shown in Table 1.

The dust azimuthal velocity is initialized with a Keplerian rotation profile; although this neglects the size-dependent impact of gas drag, the velocities quickly re-adjust under the influence of the sub-Keplerian gas. The dust density of the $j$th species is given by

$$\rho_{dj} = \frac{\epsilon_j \Sigma_0}{\sqrt{2\pi h_{dj} r_0^3}^3} \left(\frac{r}{r_0}\right)^{-\sigma-1} \sin(\theta)^{-(\beta-\sigma-1+1/h^2)}$$

(10)

where

$$\epsilon_j = \epsilon \frac{T_4^{j-1} - T_4^{j-s}}{T_4^{j, \text{max}} - T_4^{j, \text{min}}},$$

(11)

where $\epsilon$ is the total dust-to-gas mass ratio in the simulated domain. We fix $s = 3.5$ and consider dust species with $T_{4, \text{max}} = 0.01$, which corresponds to nearly centimeter-sized particles at the Hill radius assuming $r_0 = 5.2 \text{ au}$ in the adopted disk model. The aspect ratio of the $j$th dust species, $h_{dj}$, is obtained from the settling and diffusion equilibrium of dust particles (e.g., Dubrulle et al. 1995).

$$h_{dj} \equiv h \sqrt{\frac{\delta}{\delta + T_{dj}}}$$

(12)

where $\delta$ is a free parameter that sets the scale height of the dust, $H_d \equiv h_{dj} r$, from the balance between settling (due to vertical stellar gravitational acceleration) and the diffusion flux defined

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**Table 1**

List of Numerical Simulations

| Run         | $\epsilon$ | $\delta$ | $\alpha$ | $M_r/M_*$ | $\Delta \phi$ | $N_r \times N_\theta \times N_\phi$ | $N_{\text{dust}}$ | $T_{s, \text{min}} - T_{s, \text{max}}$ |
|-------------|------------|----------|-----------|-----------|--------------|----------------------------------|-----------------|-------------------------------|
| SUB-d4      | 0.01       | $5 \times 10^{-4}$ | $2 \times 10^{-4}$ | $2.6 \times 10^{-5}$ | $2\pi$ | $4800 \times 192 \times 36$ | 11   | $2.3 \times 10^{-6}$–$10^{-2}$ |
| SUB-d5      | 0.01       | $5 \times 10^{-5}$ | $2 \times 10^{-5}$ | $2.6 \times 10^{-5}$ | $2\pi$ | $4800 \times 192 \times 36$ | 11   | $2.3 \times 10^{-6}$–$10^{-2}$ |
| SUB-d6      | 0.01       | $5 \times 10^{-6}$ | $2 \times 10^{-6}$ | $2.6 \times 10^{-5}$ | $2\pi$ | $4800 \times 192 \times 36$ | 11   | $2.3 \times 10^{-6}$–$10^{-2}$ |
| SUB-d5-half-sig0 | 0.01 | $5 \times 10^{-5}$ | $2 \times 10^{-5}$ | $2.6 \times 10^{-5}$ | $\pi$ | $2400 \times 192 \times 36$ | 11   | $2.3 \times 10^{-6}$–$10^{-2}$ |
| SUB-d5-e1    | 0.1        | $5 \times 10^{-5}$ | $2 \times 10^{-5}$ | $2.6 \times 10^{-5}$ | $\pi$ | $4800 \times 192 \times 36$ | 11   | $2.3 \times 10^{-6}$–$10^{-2}$ |
| SUB-d5-half  | 0.01       | $5 \times 10^{-5}$ | $2 \times 10^{-5}$ | $2.6 \times 10^{-5}$ | $\pi$ | $2400 \times 192 \times 36$ | 11   | $6.3 \times 10^{-6}$–$10^{-2}$ |
| SUB-d5-half-cnv | 0.01 | $5 \times 10^{-5}$ | $2 \times 10^{-5}$ | $2.6 \times 10^{-5}$ | $\pi$ | $4800 \times 384 \times 72$ | 5    | $6.3 \times 10^{-6}$–$10^{-2}$ |
| SUB-d5-half-adia | 0.01 | $5 \times 10^{-5}$ | $2 \times 10^{-5}$ | $2.6 \times 10^{-5}$ | $\pi$ | $4800 \times 384 \times 72$ | 5    | $6.3 \times 10^{-6}$–$10^{-2}$ |
| SUP-nd       | ...        | ...      | $2 \times 10^{-5}$ | $7.8 \times 10^{-5}$ | $2\pi$ | $4800 \times 192 \times 36$ | NO  | ...                         |
| SUB-nd       | ...        | ...      | $2 \times 10^{-5}$ | $7.8 \times 10^{-5}$ | $2\pi$ | $4800 \times 192 \times 36$ | NO  | ...                         |
in Equation (2). The diffusion coefficient $D = \delta c_s^2 / \Omega$; the value of $\delta$ is shown (see Table 1) for each run.

Note that in absence of a self-consistent mechanism that sustains particles above the midplane, the choice of $\delta$ is unconstrained. However, in viscous disk models meant to approximate isotropic turbulent diffusion, it is usually assumed that $\delta$ is of the order of the effective viscosity parameter $\alpha$. Moreover, since the diffusion coefficient satisfies $D \sim \nu$ for $T_s \ll 1$, we safely neglect the size dependency of the diffusion coefficient (e.g., Youdin & Lithwick 2007). Therefore, the adopted initial settling equilibrium may slightly deviate from that obtained with self-consistent stirring at scales $z \simeq H$ (see, e.g., Fromang & Nelson 2009). The dust distributions are fixed for each run and do not account for coagulation, collisions, or evaporation of larger solids, which would also alter the opacity and size distributions.

2.4. Planet Model

Most of our analysis focuses on the gas and dust dynamics at the Hill and Bondi spheres, with radius $r_1 = r_p(M_p/3M_0)^{1/3}$ and $r_B = G M_p/c_s^2$, respectively. We refer to the gas on these scales as part of the planet’s envelope, regardless of whether the gas is bound to the planet or cycling through. Because our simulations are locally isothermal, these definitions are fixed throughout each run. We consider only two planet masses (see Table 1), and denote our simulations with the prefix $\text{SUB}$ (subthermal) when $M_p/M_0 = r_0/H \simeq 0.6$, where $M_0 = \dot{M}_\text{in}/c_s$ is the disk thermal mass. Equivalently, the prefix $\text{SUP}$ is used accordingly for runs with $M_p/M_0 = r_0/H \simeq 1.8$ (superthermal). The planet is implemented as a potential only, meaning that the initial conditions within the Hill sphere are identical to that in the disk. We do not consider any accretion sink for the planet; that is, we do not remove mass and momentum inside the smoothing length. The resolution of our 3D global simulations is adequate to capture the dynamics at the interface between the planet envelope and the protoplanetary disk. This interface is traditionally defined as $r_{\text{envelope}} = \min(r_0, r_1)$, although it is a reference location, not a sharp barrier.

3. Gas and Dust flow

Our simulations demonstrate that at scales of the envelope, the dust flow, and thus dust-to-gas density ratio, is inherently 3D and decouples from the gas dynamics, meaning that dust is not well described by a well-mixed fluid with a fixed $c_s$. The stellar vertical gravity, spiral wakes, and the planet potential drive strong variations in the dust-to-gas density ratio at scales of the Bondi sphere. The meridional circulation within the planet envelope is too weak to lift dust grains above the equilibrium scale height set by the balance of vertical gravity and diffusion. The dust distribution deviates from the settling equilibrium only where the spiral wakes intersect the Bondi sphere. We furthermore find that the multispecies dust feedback has minimal impact on the meridional flow patterns and overall gas mass flux cross the Hill surface for a total dust-to-gas mass ratio $\epsilon \leq 0.1$.

We focus on the outcome from runs $\text{SUB-d4, SUB-d5, and SUB-d6}$. These simulations have a planet mass of $M_p = 2.6 \times 10^{-5} M_\odot \simeq 0.6 M_\oplus$ and only differ by the strength of diffusion and therefore the dust scale height relative to the Hill radius. For comparison, we also include the run $\text{SUP-d5}$ with a planet mass of $M_p = 7.8 \times 10^{-5} M_\odot \simeq 1.8 M_\oplus$. Since our simulations feature near-thermal-mass planets, we focus our discussion on calculations at the Hill hemisphere. Note that for all of the $\text{SUB}$-runs, $r_1 \simeq r_B$.

3.1. Overview of 3D Morphology

We begin with a qualitative summary of the 3D morphology that characterizes our runs. In Figure 1 we show a snapshot of the run $\text{SUB-d5}$ at $t = 20 \times 2\pi \Omega_0^{-1}$ that includes the total dust density (top panel) and the vertical gas velocity (bottom panel). Qualitatively similar flow patterns are obtained at the scales of the Hill radius and at the horseshoe region in all of our runs. Dust grains excited by the planet’s spiral wakes are lofted above their initial scale height $H_0$. The lifted stream of dust seems to have a vertical turn-over of about $z \simeq H_0$, where $v_{z \text{gr}}$ seems to change sign. The zoomed-in plots correspond to the Bondi semisphere, and show that the perturbation introduced by the spiral wakes inside the planet envelope disturbs the dust flow down to planetary smoothing length.

In the bottom panel of Figure 1, we show the gas vertical velocity for run $\text{SUB-d5}$. While the gas vertical velocity is subsonic everywhere in the disk, the Mach number, $v_{z \text{gas}}/c_s$, approaches unity inside the Bondi sphere, at the polar region near the smoothing length of the planet potential. This strong inflow is also seen in the dust species, with a vertical velocity that slightly deviates from that of the gas, consistent with the moderate-to-strong coupling. Mach numbers in regions with uplift ($v_{z \text{gr}} > 0$) can reach values of $v_{z \text{gr}}/c_s \sim 0.05$ near the horseshoe region, whereas $v_{z \text{gr}}/c_s \sim 0.2$ inside the Bondi sphere.

Comparing the zoomed-in top and bottom panel of Figure 1 reveals that regions with gas $v_{z \text{gr}} > 0$ are traced by an enhancement of the dust density. Dust grains are lifted at these locations, generating the anisotropic dust–gas mixture, which deviates not only from the commonly assumed well-mixed composition, but also that of an equilibrium settled disk model (see, e.g., Chachan et al. 2021).

3.2. Meridional Flow Pattern

Recycling of material between the disk and the envelope may be crucial for understanding planet growth, and in particular may slow atmospheric cooling and contraction (e.g., Ormel et al. 2015). Even with our moderate resolution inside the Hill sphere, we still observe this characteristic meridional flow pattern. Note that the in the Appendix we directly compare our meridional flow pattern with previous works, and evaluate the numerical convergence.

In this section we focus on runs $\text{SUB-d5 and SUP-d5}$ to compare flow patterns with fixed $\delta$ across the transition from sub- to superthermal masses, where we might expect rapid gap opening to commence.

In Figure 2 we show the azimuthal average of the gas and dust density (only $T_s = 0.01$, as it is the most decoupled species in our runs). On top of the mean density, we plot the azimuthal average of the meridional velocity field, $\bar{v}_{mer} = (\bar{v}_r, \bar{v}_z)$, where the $\bar{v}_r$ and $\bar{v}_z$ correspond to the radial and vertical velocities obtained in a cylindrical coordinate system centered on the planet.

Our results for the gas meridional velocity resemble the characteristic flow described in both local (e.g., Béthune & Rafikov 2019) and global (e.g., Fung et al. 2019) isothermal simulations, where inflow occurs mainly at the polar regions,
Figure 1. Top panel: dust-to-gas density ratio for the run SUB-d5 for the full domain, with a zoom-in at the Bondi sphere. The dust perturbation driven by the spiral wakes extends well within the Bondi sphere, as described in Section 4.2. Bottom panel: vertical gas velocity normalized by the sound speed for the run SUB-d5. The horseshoe region is characterized by meridional gas outflow where $v_{g z} \sim 0.05c_s$.

Figure 2. Azimuthally averaged (along $\tilde{\phi}$) meridional flow pattern for the runs SUB-d5 and SUP-d5. The left panels correspond to the gas flow, whereas the right panels show the results for dust with Stokes number $T_s = 0.01$ (the most decoupled species in this work). Gas density is normalized by the initial density, whereas dust density is normalized by gas density.
while outflows are prominent at the midplane. We find that at the disk midplane, both inflow and outflow occur (see Section 3.3), and therefore the azimuthal average provides an incomplete description of the flow at $\theta = \pi/2$.

The gas density for the subthermal-mass planet remains nearly symmetric, while the superthermal-mass case shows hints of rotational flattening. However, these density structures are close enough to the smoothing length that they are resolution dependent (see the Appendix).

Overall, the dust meridional flow follows the gas since there is a moderate-to-strong coupling. However, regions with strong gas vertical infall are depleted of dust (see Section 3.3, Figure 3). Therefore the transport of solids toward the planetary core is dominated by midplane flows.

For the lower-mass case, the average dust layer thickness seen in Figure 2 is similar to the equilibrium $H_{\text{dust}}$ far from the planet. The higher-mass planet stirs dust a bit more, unsurprisingly. The higher-mass case also shows a dust gap at $0.25 < r/r_{\text{H}} < 0.5$, but this small-scale feature could be resolution dependent, and needs further study to be confirmed.

### 3.3. Mass Flux at the Hill Radius

Our simulations do not permit accretion onto the planet since we adopt a softened gravitational potential with no envelope self-gravity, nor removal of mass and angular momentum. Thus, we cannot explicitly measure envelope or core growth. Instead we calculate the gas mass flux at the Hill semisurface to characterize the balance between inflow and outflow and assess the steady-state nature of our simulations. We compare our results with previous simulations to determine if the dust back reaction onto the gas significantly affects the gas mass flux.

The mass flux is estimated from the output density and velocity fields and integrated at the Hill semisurface as

$$M = \int_0^{2\pi} \int_{r_{\text{H}}}^{r_{\text{H}}/2} \rho(\mathbf{v} \cdot \mathbf{\hat{n}}) r_{\text{H}}^2 \sin(\theta) d\theta \, d\phi.$$  \hspace{1cm} (13)

The values obtained for the net flux $M_{\text{net}}$ as well as inflow ($\mathbf{v} \cdot \mathbf{\hat{n}} > 0$), $M_{\text{in}}$, and outflow ($\mathbf{v} \cdot \mathbf{\hat{n}} < 0$), $M_{\text{out}}$, are shown in Table 2. All of the values are normalized to $10^{-5} M_{\text{jup}} \, \text{yr}^{-1}$ and obtained after time-averaging between the $t = 2 \times 2\pi \Omega_{\text{H}}^{-1}$ and the final integration time of each run.

In Figure 3 we illustrate the mass flux through the Hill sphere for both the gas and the dust at $r_{\text{H}}$. We also show in Figure 4 the time evolution of $M_{\text{in}}$, $M_{\text{out}}$, and $M_{\text{net}}$ for several runs. For the gas, we find that $M_{\text{in}}$ and $M_{\text{out}}$ are typically $\approx 7 \times 10^{-5} M_{\text{jup}} \, \text{yr}^{-1}$ and both fluxes balance within a few percent, which is highlighted by the substantially smaller net flux, typically $\approx 10^{-6} M_{\text{jup}} \, \text{yr}^{-1}$. The minimal net mass flux is consistent with the observed quasi-steady state and near hydrostatic balance within the Bondi sphere (see the Appendix).

Note that for the superthermal-mass case, the inflow/outflow values increase to $\approx 2 \times 10^{-4} M_{\text{jup}} \, \text{yr}^{-1}$. As shown in Table 2, this is an approximately threefold increase between the mass fluxes for run SUP-d5 compared to SUB-d5, which correlates linearly with the planet mass.

Similar inflow and outflow distribution at $r = r_{\text{H}}$ has been reported by Cimerman et al. (2017), although their simulations were local and included radiative transfer. The measured fluxes are also in reasonable agreement with those from the previous

### Table 2

Simulations Diagnostics at the Hill Radius

| Run   | $(e_{\text{mid}}/e_{\text{zero}})_{\text{p}}$ | $R_{5.2}$ | $W_{5.2}$ | $R_{30}$ | $W_{30}$ | $M_{\text{net}}^\text{5.2}$ | $M_{\text{net}}^\text{30}$ | $M_{\text{in}}^\text{5.2}$ | $M_{\text{in}}^\text{30}$ | $M_{\text{out}}^\text{5.2}$ | $M_{\text{out}}^\text{30}$ |
|-------|---------------------------------|----------|-----------|----------|----------|-------------------------|-------------------------|------------------------|------------------------|------------------------|------------------------|
| SUB-d5 | 96.2                            | 2.7      | 0.93      | 287.0    | 0.18     | $-0.061$                | $6.982$                 | $7.043$                | $0.023$                 | $0.141$                | $0.117$                |
| SUP-d5 | 49.6                            | 2.3      | 0.82      | 87.8     | 0.15     | $0.926$                 | $19.735$               | $18.810$               | $0.007$                 | $0.231$                | $0.224$                |
| SUB-d4 | 8.9                             | 1.4      | 0.96      | 9.3      | 0.93     | $-0.054$                | $6.744$                 | $6.798$                | $0.023$                 | $0.117$                | $0.094$                |
| SUB-d6 | 534.9                           | 13.7     | 0.81      | 937.7    | 0.06     | $-0.071$                | $7.001$                 | $7.072$                | $0.024$                 | $0.156$                | $0.132$                |
| SUB-d5-mSig0 | 166.5                           | 12.4     | 0.88      | 2595.6   | 0.04     | $-0.011$                | $0.706$                 | $0.717$                | $0.009$                 | $0.017$                | $0.007$                |
| SUB-d5-e1  | 53.5                            | 2.3      | 0.94      | 169.9    | 0.21     | $-0.230$                | $6.978$                 | $7.208$                | $0.060$                 | $1.507$                | $1.447$                |
| SUB-d5-half | 60.1                            | 2.4      | 0.93      | 147.9    | 0.31     | $-0.058$                | $6.803$                 | $6.861$                | $0.020$                 | $0.121$                | $0.101$                |
| SUB-d5-half-conv | 170.6                           | 4.4      | 0.78      | 564.7    | 0.13     | $-0.321$                | $7.020$                 | $7.340$                | $0.036$                 | $0.146$                | $0.110$                |
| SUB-d5-half-adia | 36.9                            | 2.8      | 0.92      | 63.4     | 0.38     | $-0.020$                | $5.760$                 | $5.780$                | $0.022$                 | $0.136$                | $0.114$                |

Note. Simulation diagnostics are obtained at 20 and 40 planet orbits for subthermal (SUB–) and superthermal (SUP–) mass planets, respectively. $R$ is the midplane-to-pole opacity ratio, while $W$ represents the ratio of the shell-averaged opacity to that for a well-mixed distribution (defined in Section 4.1). Note that the subscripts 5.2 and 3 refer to the radial distance in astronomical unit (the locations of Jupiter and Neptune in our solar system, respectively) chosen to evaluate $R$ and $W$. The estimated mass fluxes $M_{\text{in}}$ and $M_{\text{out}}$ correspond to the time-averaged values (from the first two orbits until the final integration time) across the Hill radius. The mass flux values are normalized to $10^{-5} M_{\text{jup}} \, \text{yr}^{-1}$.
works of Bate et al. (2003) and Ayliffe & Bate (2009) for the isothermal runs with an accreting planet with mass $M_p = 10 M_\oplus$, which is the closest case in units of the thermal mass to our work. Higher resolution may narrow the balance to $\lesssim 1\%$ between $M_{\text{in}}$ and $M_{\text{out}}$ as reported for subthermal-mass planet simulations (Fung et al. 2015, 2019).

Figure 3 reveals that the dust shows a similar flow pattern to the gas, albeit with significantly smaller fluxes. While a reduction is expected simply due to lower dust-to-gas density ratios, $M_d \propto \epsilon M_g$. At latitudes near the midplane, the total dust mass flux is roughly $\sim 10$ times smaller than the gas mass flux, for $\epsilon = 0.01$. At the poles, $M_d < \epsilon M_g$, with inflow reduced to $M_d \ll 10^{-3} M_{\text{pp}}$ yr$^{-1}$. This trend is expected due to dust settling, and is enhanced by the fact that the dust mass distribution is tilted toward the largest Stokes numbers, which settle efficiently.

In addition to spatial variation in the mass flux ratios, the integrated dust mass flux is in disagreement with that expected for a well-mixed distribution, i.e., 1% of the gas (see Table 2). Moreover, the net dust mass flux is always toward the planet (inflow) even though the net gas mass flux is predominately an outflow. The difference between gas and dust mass fluxes can be attributed to the fact that the dust mass flux is dominated by the species with the largest size ($\sim 1$ cm) and therefore more decoupled. Thus, dust recycling may be less efficient.

To better understand the impact of dust feedback on the gas dynamics, Figure 4 we also include a run with no dust, SUB-dnod, and a run with a total dust-to-gas mass ratio of 10%, instead of the fiducial 1%. We find that while feedback does modify the gas velocity locally, especially at the midplane where the dust-to-gas ratio approaches unity, overall the gas mass flux, mean meridional circulation, and envelope hydrostatic equilibrium seems to be agnostic to the presence of the dust at scales of $r \sim r_H$ when $\epsilon = 0.01$.

The simulation with a larger dust-to-gas mass ratio (10%) shows a stronger gas inflow/outflow imbalance, with a mean mass flux $\sim 4$ times larger than the SUB-d5 case. Although the net dust mass flux is not significantly larger than the runs with $\epsilon = 0.01$, a larger dispersion is shown in the left panel of Figure 4 (in comparison with the 1% standard case), which suggests that longer integration times may be needed to reach equilibrium. Future numerical simulations with higher resolution at the scales on which circumplanetary disks form ($r \lesssim r_H$) will better determine if dust feedback remains negligible in the inner regions of the planet envelope, where larger concentrations of dust are expected.

3.4. Dust Density at the Hill Surface

The distribution of small solids plays a major role in shaping the opacity of the planet envelope. However, it is usually assumed that grains are all well-mixed with the gas. We, therefore, assess the validity of such approximation by describing the dust distribution obtained from our numerical simulations at the Hill hemisphere. We find that the planet’s potential steepens the latitudinal gradient in dust-to-gas ratio that is driven by the balance between settling and diffusion. Moreover, the planet interaction with the dusty-fluids induces an anisotropic density distribution along the longitudinal coordinate, $\phi$.

In Figure 5 we show the total dust-to-gas density ratio at the Hill radius. Each panel has the label of the run as shown in Table 1. As expected from dust settling, the largest values of dust-to-gas mass ratio are obtained at the midplane. Comparing the dust-to-gas mass ratios of runs SUB-d5, SUB-d4, and SUB-d6, it is clear that changing the diffusion parameter, which changes $H_d$ (see Equation (12)), primarily modulates the dust density gradient along the latitudinal direction of the Hill radius.

Figure 5 also shows that the anisotropic dust density distribution develops at latitudes of $\phi \sim 120^\circ$ and $\phi \sim 300^\circ$ for subthermal-mass planets. These dominant features are ubiquitous in all runs and seem to be driven at the location of the spiral wakes excited by the planet. The one at $\phi \sim 120^\circ$ corresponds to the inner wake, whereas the one at

![Figure 4](image_url)
being lifted from the midplane well above the dust scale height. Dust density increases along these wakes, where dust is being lifted from the midplane well above the dust scale height. However, the effective ∊ ≤ 0.01 in these wakes, and therefore dust feedback is negligible.

To facilitate a quantitative comparison between the results obtained for different δ, we show in the top panel of Figure 6 the azimuthal average of the dust-to-gas density ratio at the Hill radius. Moreover, we include the dust-to-gas mass ratio at t = 0 (dashed lines) to demonstrate that the planet potential plays a minor role in shaping the dust’s latitudinal distribution at the Hill radius.

The latitudinal deviation introduced by the planet is highlighted by the comparing the mean pro- 

cation of spherically symmetric, well- 

cedust from the standard simpli- 

cations of superthermal-mass planets (therefore stronger velocity perturbations), these features are extended and shifted by a full 60°. Dust density increases along these wakes, where dust is being lifted from the midplane well above the dust scale height. However, the effective ∊ ≤ 0.01 in these wakes, and therefore dust feedback is negligible.

To facilitate a quantitative comparison between the results obtained for different δ, we show in the top panel of Figure 6 the azimuthal average of the dust-to-gas density ratio at the Hill radius. Moreover, we include the dust-to-gas mass ratio at t = 0 (dashed lines) to demonstrate that the planet potential plays a minor role in shaping the dust’s latitudinal distribution at the Hill radius.

The latitudinal deviation introduced by the planet is highlighted by the comparing the mean pro-

cation of spherically symmetric, well-

cedust from the standard simplification of spherically symmetric, well-
mixed dust. We now translate our more realistic dust distributions into opacities, to better infer its impact on radiative transfer in the envelope, and thus planetary cooling rates. We focus our calculations on the values of the opacity at the Hill radius, and examine the contributions of vertical settling and planet–disk interactions on the opacity. We show that the planet envelope can have a latitudinal transition between optically thin and optically thick regimes, at certain locations in the disk, primarily driven by dust settling.

4. Mean Opacity

We have shown in Section 3 that the coupled gas and multispecies dust dynamics generates substantial departures from the standard simplification of spherically symmetric, well-mixed dust. We now translate our more realistic dust distributions into opacities, to better infer its impact on radiative transfer in the envelope, and thus planetary cooling rates. We focus our calculations on the values of the opacity at the Hill radius, and examine the contributions of vertical settling and planet–disk interactions on the opacity. We show that the planet envelope can have a latitudinal transition between optically thin and optically thick regimes, at certain locations in the disk, primarily driven by dust settling.

4.1. Methods: Opacity Calculations

The mean opacities per gram of dust are obtained using the publicly available DSHARP opacity module; the details of the assumed composition, and absorption and scattering opacities, are described in Birnstiel et al. (2018). To calculate the opacity per gram of total material, we multiply the opacities per gram of dust by the total dust-to-gas mass ratio. We denote the Rossetti and Planck mean opacity (per gram of material) as δR and δP, respectively. We calculate both opacities using the dust density from the numerical simulations from Section 3 (based on the disk model described in Section 2.3).

We will compare the values of δR and δP with equivalent opacities obtained assuming a well-mixed dust distribution. We denote these opacities as δR,mix(T) and δP,mix(T). Both are obtained assuming that each grid cell has a dust density equal to that of Equation (11) times the gas density. Therefore, gradients in δR,mix(T) and δP,mix(T) are only sensitive to gradients in the gas density, and not the dust density. Thus, these opacities are expected to be nearly constant in space and time since the gas density is in quasi-hydrostatic equilibrium at the Bondi radius (see Figure 12), and remains symmetric at the Bondi radius.

Since our simulations do not include radiative transfer and planet luminosity, we proceed by assuming a temperature, T, given by the condition T^4 = T^4_{disk} + L/(16πσSB^2), where L corresponds to the accretion luminosity of the planet, L = GM_p^2/(R_{planet}t_{double}). We assume a mass-doubling time t_{double} = 10^5 yr and the planet radius, R_{planet}^4 is obtained assuming a mean density of ρ_{planet} = 3 g cm^-3. This approximation for the temperature is valid in optically thin regimes, which may not apply inside the Bondi sphere in our simulations. However, we are focusing on the outer regions where δR ≈ ρ_{planet} in the cases discussed (e.g., see Figure 6).
of the envelope ($r \sim r_H$), where $T \sim T_{\text{disk}}$. Thus the choice of specific doubling time and corresponding planetary luminosity also has a minimal impact on the scales on which we focus.

The opacities per gram of dust (and therefore the Rosseland and Planck mean opacities) depend on the total monochromatic opacity, $\kappa_{\nu}^{\text{tot}}$. To calculate $\kappa_{\nu}^{\text{tot}}$ using the DSHARP module, we first interpolate the dust density from our simulations to a discretized size distribution with 200 bins that span from $a_{\text{min}} = 10^{-5}$ cm to $a_{\text{max}} = 10^{2}$ cm. This interpolation facilitates the calculations since it arranges the simulated dust distribution to match the size-domain of the opacity in the DSHARP module.

Since we only calculate the opacity for specific shells inside the Hill sphere, the conversion from Stokes number to particle size is done assuming a mean density, $\rho_{\text{g}} = \langle \rho_{\text{g}} \rangle_{\tilde{\theta}, 0}$, and mean sound speed $c_{s} = \langle c_{s} \rangle_{\tilde{\theta}, 0}$. This approximation is justified because the gas density is nearly constant at a fixed radius given the local hydrostatic equilibrium inside the Hill sphere, as we show in the Appendix.

Thus, for the $j$th dust species, the grain size is

$$a_j \equiv T_y \sqrt{8/\pi} \rho_{\text{g}}/\rho_{\text{solid}} \tilde{c}_s/\Omega(r_p)$$

(14)

where $\rho_{\text{solid}} = 1.6686$ g cm$^{-3}$, and $\Omega(r_p)$ is the orbital frequency at the planet radius. The dust density is set to zero for those bins with maximum (minimum) size obtained from Equation (14) that are smaller (larger) than the limits set by $a_{\text{max}}$ ($a_{\text{min}}$). The Stokes numbers included in our simulations correspond to sizes in the range of $1 \mu\text{m} \lesssim a_i \lesssim 1$ cm at $r_0 = 5.2$ au at the Bondi radius. With the new interpolated distribution, we estimate the total extinction and absorption coefficients as follows:

$$\kappa_{\nu}^{\text{tot}} = \sum_{j=1}^{N=200} \rho_{\text{d}}(a_j)\kappa_{\nu}(a_j).$$

(15)

$\kappa_{\nu}(a)$ is obtained from the DSHARP module, and it corresponds to $\kappa_{\nu}^{\text{abs}}$ and $\kappa_{\nu}^{\text{em}}$ for the Planck and Rosseland mean opacity, respectively (see, e.g., Birnstiel et al. 2018).

### 4.2. Rosseland and Planck Mean Opacity

In this section, we present the calculations of the Rosseland and Planck mean opacity (per gram of total material) at the surface of the Hill sphere. Based on the outcome of our numerical simulations, we show that both opacities, $\kappa_{\nu}(T)$ and $\kappa_{\nu}(T)$, deviate from $\kappa_{\nu}^{\text{abs}}(T)$ and $\kappa_{\nu}^{\text{em}}(T)$. The latter correspond to opacities of a well-mixed grain distribution, which has a fixed dust–gas density ratio.

In Figure 7 we show the Rosseland mean opacity at the Hill radius for the runs SUB–d5, SUP–d5, SUB–d4, and SUB–d6. Overall, the distribution of $\kappa_{\nu}$ is traced by the total dust-to-gas mass ratio shown in Figure 5. In all cases, the mean opacity decreases from the midplane to the polar regions of the Hill sphere. At the locations of the spiral wakes, where dust grains are significantly elevated compared to the dust scale height $H_d$, expected from settling-diffusion equilibrium, the Rosseland mean opacity also increases, creating anisotropy in the azimuthal direction as well.

Interestingly, the opacity distribution is similar for sub- and superthermal-mass planets (comparing $2.6 \times 10^{-5}M_\oplus$ to $7.8 \times 10^{-5}M_\oplus$), as can be seen when comparing the top-left and top-right panels of Figure 7. Note however, that our results are obtained during the initial phase of gap formation for the run SUB–d5, and therefore we anticipate that gap formation may affect the opacity distribution at the Hill sphere on longer timescales, because gaps may preferentially filter certain grain sizes (Weber et al. 2018).

Figure 7 also demonstrates that the opacity strongly depends on settling:$^5$ Smaller values of $\delta$ generate more deviation of $\kappa_{\nu}$ from $\kappa_{\nu}^{\text{mix}}$. We recall here that the opacity per gram of material is the product between opacity per gram of dust and total dust-to-gas mass ratio. The opacity per gram-of dust (whose bulk value is dominated by the smaller grains) is nearly constant at latitudes $\tilde{\theta} \lesssim 60^\circ$; however, strong settling produces larger latitudinal gradients in the dust-to-gas mass ratio. Thus, the dust-to-gas density ratio plays a crucial role in shaping the opacity gradient.

To better compare the effect of dust settling on the opacity distribution, we show in Figure 8 the azimuthal average of the Rosseland mean opacity at the Hill sphere. All values were obtained assuming parameters from the disk model described in Section 2.3 with the planet’s orbit at $r = 5.2$ au. We also indicate the Planck mean opacity with dashed lines. The results shown in the top panel indicate that deviations between $\kappa_{R}$ and $\kappa_{\nu}^{\text{mix}}$ are minor for $\delta = 5 \times 10^{-4}$, while reaching a factor of a few for $\delta = 5 \times 10^{-5}$. When $\delta = 10^{-6}$, we find an order-o-
The magnitude discrepancy between the midplane and polar regions.

The latitudinal opacity gradients are stronger for lower disk surface densities, such as those expected in the outer disk. Therefore, the deviations from a well-mixed distribution are even more pronounced for a planet forming beyond 10 au. To assess this case, we consider a planet forming at a location with surface density \( \Sigma_0 = 20 \) g cm\(^{-2}\). This is shown in the middle panel of Figure 8. Since dust settling is the major driver of the opacity gradient, and the dust scale height for a given particle size depends on the gas density and local sound speed, lower-density regions favor the presence of smaller grains close to the midplane.

Figure 6 shows that the mean dust density profile at \( r_B \) deviates little from the initial settling equilibrium due to perturbations from the planet. To quantify how this profile impacts opacity, in Figure 9 we show the ratio between the azimuthal average at the pole and midplane: \( \langle \kappa_{R,5.2} \rangle_{\text{mid}} / \langle \kappa_{R,5.2} \rangle_{\text{pole}} = R_{5.2} \). To quantify the impact of the planet compared to dust settling alone, values are shown for \( t = 0 \) and after the envelope has reached steady state. \( R \) is evaluated for a shell at the Bondi semisphere as well as shells at \( r = 0.5r_B \) and \( r = 1.75r_B \), all assuming a planet at \( r = 5.2 \) au from the central star. In all cases displayed in Figure 9, the ratio \( R_{5.2} \) is well described by the initial settling equilibrium.

We include the values of \( R_{5.2} \) for all runs in Table 2; only a subset are shown in Figure 9. Generically, a larger opacity gradient results from increasing the ratio \( \langle \kappa_{\text{mid}} / \kappa_{\text{zero}} \rangle_{\phi} \), i.e., for stronger settling. A comparison between both quantities indicates that there is not a simple linear scaling between the opacity gradient and the density gradient, necessitating the full calculation of the opacity, not simply the dust-to-gas ratio, to assess the impact on planetary thermodynamics.

Furthermore, larger values of \( R \) are expected in outer regions of PPDs, where dust scale heights are reduced. For
instance, if we scale our numerical simulation using the disk model in Section 2.3 to \( r_p = n_0 = 30 \) au, so that \( \Sigma_0 \approx 6 \) g cm\(^{-2}\) and \( T \approx 27 \) K, the midplane-to-pole ratio \( R \) significantly increases. This estimation is shown in Table 2 and denoted by \( R_{30} \). In all of our runs, we find that \( R_{30} \gg R_{5.2} \).

As shown in Figure 5, the dust distribution is asymmetric both latitudinally and azimuthally. To better understand the effect of nonuniform dust on the escaping radiation, we compute shell averages of \( 1/\kappa_R \), which is proportional to the optically thick radiation flux. To obtain this single average and compare with previous work, including 1D models, we take the integral over a solid angle of the inverse of both \( \kappa_{R_{\text{mix}}} \) and \( \kappa_R \). We define the ratio between the two integrals, \( W \):

\[
W = \frac{2\pi}{\kappa_{R_{\text{mix}}}} \left( \int_0^{2\pi} \int_0^{\pi/2} \frac{1}{\kappa_R} \sin(\theta) d\phi d\theta \right)^{-1}. \tag{16}
\]

As we have done with \( \mathcal{R} \), we estimate \( W \) at \( r = 5.2 \) au and \( r = 30 \) au and show the results in Table 2. At 5.2 au, we found that the integrated opacity is lower by a few percent down to 20%, for the case of the run SUB-d6. Much smaller values are obtained for \( W \) at \( r = 30 \) au, consistent with the enhanced settling of dust. Overall, the integrated inverse opacity decreases with respect to the well-mixed case. Moreover, \( W \) decreases with increasing numerical resolution; therefore, the obtained values may be taken as an upper bound (see the Appendix for more details).

Moreover, we expect that the results obtained at \( r = 5.2 \) au are likely conservative in that they underestimate the impact of the flow perturbations introduced by the planet potential at the scales of the Bondi sphere; the deviations from a well-mixed distribution increase with increasing numerical resolution (see for example values of runs SUB-d5 and SUB-d5-half-cnv and discussion in the Appendix).

The shell-integrated quantity \( W \) may prove useful for benchmarking 1D evolutionary calculations against 3D multifluid simulations with radiative transfer where the opacity is updated based on the dust dynamics. For example, thermal gradients induced by opacity anisotropy are not included in this analysis and are expected to affect the dust distributions. We caution that a single value cannot likely capture the full impact of the observed asymmetry. We stress that the integrated inverse of the opacity by definition favors areas with lower opacity values and therefore may underestimate the impact of the optically thick (midplane) regions on cooling. As we discuss in Section 4.3, shell-averaged opacities may not be adequate to compute planetary cooling, since optically thin and thick regions coexist for a single planet. Such gradients cannot trivially be cast into a 1D envelope model.

4.2.1. The Adiabatic Case

So far we have considered only simulations with a locally isothermal equation of state. However, simulations with an adiabatic equation of state show different gas meridional flow (Kurokawa & Tanigava 2018; Fung et al. 2019) inside the Bondi sphere. To investigate the consistency of our findings with an adiabatic equation of state, we expand our parameter exploration to include a simulation where the pressure and internal energy \( \gamma \), satisfy \( P = (\gamma - 1)e \). We assume a ratio of the specific heats \( \gamma = 1.4 \), consistent with an ideal diatomic gas.

In the bottom panel of Figure 8, we show the azimuthal average of the Rosseland and Planck mean opacity for the run SUB-half-adia (adiabatic) and SUB-d5. At the scales of the Bondi sphere of the subthermal-mass planet, there are only minor differences between the mean opacity gradients. While we do observe circulation patterns, these do not appear to substantially change the dust distribution as a function of height. Since the latitudinal gradient is primarily driven by the initial settling equilibrium, the deviations in the opacity compared to the well-mixed case persist.

We emphasize that because we adopt an initial condition for this run that is in equilibrium for an isothermal equation of state, longer integration times may show different results. Nevertheless, we expect irradiated disks in the regime of interest to be closer to the isothermal regime than adiabatic (due to efficient cooling), even if the gas within the planetary envelope undergoes adiabatic perturbations. A full radiative transfer treatment will be necessary to explore more realistic planetary cooling, and is the subject of future work.

4.3. Optically Thin and Thick Regimes

The results described in Section 4.2 suggest that the latitudinal opacity gradient is driven by dust settling, and modestly steepened by the planet potential. Therefore, to first order, we can determine conditions that lead to this opacity gradient assuming axisymmetric disk models, with different size distributions and dust scale heights. Moreover, to better

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6 Since we consider an adiabatic equation of state, we explicitly integrate the internal energy. The initial condition remains as described in Section 2.3, although we set \( \beta_{(\gamma)} = \beta(\gamma) = (\gamma - 1) \). The additional boundary condition is set to \( \beta_{(\gamma)} = \beta = 0 \).
estimate the impact of this opacity gradient on the planet’s radiative processes, we calculate the photon mean free path defined as $\lambda_{\text{mfp}} \equiv 1/(\delta \rho_{\text{g}})$. We adopt a cylindrical coordinate system $R, Z$, and assume a settling-diffusion equilibrium dust distribution where the density associated with a given size $a_k$ is obtained as

$$\rho_{\text{d}, k} = \rho_{\text{g}} \frac{\epsilon(a_k)}{h_{\text{d}, k}} \exp\left(-\frac{T_{\text{d}, k}}{\delta} \left[e^{\frac{a_k}{3H}} - 1\right]\right),$$

$$\rho_{\text{g}} = \frac{\Sigma_0}{\sqrt{2\pi} H} \frac{\rho_{\text{d}, k}}{R_{\text{au}}^{-3}} e^{-\frac{a_k}{3H}},$$

(17)

where $R_{\text{au}}$ corresponds to the distance to the central star in astronomical unit. The gas scale height is denoted as $H = h(R) R$ with $h(R) = h_0 R_{\text{au}}^f$, and the Stokes number is defined as $T_{\text{d}, k} = \pi a_k \rho_{\text{sol}} / (4 \Sigma_0 R_{\text{au}}^{-3})$ (see, e.g., Dipierro et al. 2018). The values of $\epsilon(a_k)$ and $h_{\text{d}, k}$ are obtained replacing from Equations (11) and (12), respectively. We focus on scales of the Bondi radius of subthermal-mass planets in disks with moderate-to-strong dust settling.

We adopt a criterion for transition between optically thick and thin regimes when $\lambda_{\text{mfp}} = H$. Note that this is consistent with a criterion given by $\lambda_{\text{mfp}} = r_B$ (e.g., Rafikov 2006) for planets with nearly one thermal mass.

In Figure 10 we show $\lambda_{\text{mfp}}$ for different disk models, with variable gas surface density, diffusion parameters, and dust properties, assuming the opacity gradient is set solely by dust settling. The top-left panel corresponds to a case similar to SUB-d5-sigma0, while the others were obtained for a minimum mass solar nebula (MMSN) disk model (Hayashi 1981). Note that the MMSN models are flared, rather than having a constant aspect ratio. For all disk models, the well-mixed distribution of solids has a transition between optically thin and optically thick regimes that is mainly a function of the radial distance to the central star for $Z \lesssim H$.

On the other hand, when accounting for settling, we find a shallower transition between the two regimes as a function of radius. Therefore the envelopes of subthermal-mass planets may be optically thick near the midplane and optically thin near the polar regions. This transition is confirmed for our run SUB-d5-sigma0 (see Figure 11) where the planet is assumed to be at $r = 5.2$ au from the central star.

As can be appreciated from Figure 10, the transition near $\lambda_{\text{mfp}} \sim H$ is more likely to occur at intermediate regions of PPDs, whereas inner regions are optically thick and outer

Figure 10. Photon mean free path, $\lambda_{\text{mfp}} \equiv 1/(\delta \rho_{\text{g}})$ for different disk models with initial dust settling profiles and size distributions. The minimum particle size is fixed to $1 \mu m$, and the maximum particle size is assumed to be $a_{\text{max}} = 1$ cm unless specified. Except for the top-left panel, where the gas density and scale height were adopted from the disk model used for the run SUB-d5-sigma0, the rest of the results are obtained from an MMSN disk model with $h_0 = 0.033, f = 0.25$, and $\sigma = 3/2$ (see Equation (17)). Our estimations suggest that planet envelopes may have a transition between optically thin and optically thick regimes at the Bondi radius in outer regions of the disk. The radial location and planet mass where the transition occurs are a function of the adopted dust size distribution, the gas surface density, disk temperature, and flaring index.
regions are optically thin. At a distance from the star $r \lesssim 10$ au, such a transition would likely occur in nearly laminar and/or gas depleted disks, as the required settling-diffusion equilibrium parameter should be $\delta < 10^{-5}$.

We emphasize that the estimates of $\lambda_{\text{mfp}}$ strongly depend on the size distribution and the adopted disk model. In particular, the gas surface density and the diffusion parameter will set the equilibrium scale height of dust grains. In addition, the maximum grain size and the slope of the particle-size distribution also impact the mean free path transition as they set the mass load toward the micron-sized grains for a fixed dust-to-gas ratio.

The results obtained in Figures 10 and 11 suggest that some planets may occupy a regime not captured by 1D models that are either purely thick ($\lambda_{\text{mfp}} \ll r_B$) or thin regimes ($\lambda_{\text{mfp}} \gg r_B$), as studied in Rafikov (2006). Future 1D models may benefit from seeking an approximation that captures the transition of $\lambda_{\text{mfp}}$ reported in this work.

Similar axisymmetric dust opacity calculations have been used to study thermal relaxation in PPDs including the exchange of energy between gas and dust via collisions (Malýgin et al. 2017; Barranco et al. 2018; Bae et al. 2021), and self-consistent coagulation models (Sengupta et al. 2019; Chachan et al. 2021). Ultimately, whether the observed latitudinal gradient plays a significant role in the thermodynamic evolution of planets requires properly coupling the planet radiation, dust distribution, and disk hydrodynamics.

5. Conclusions

In this work, we have performed 3D multifluid simulations of nearly thermal-mass planets embedded in a protoplanetary disk. Our simulations, applied to a standard disk model with a planet at 5.2(30) au, allowed us to characterize a dust distribution with particle sizes spanning from $\sim 1 \mu m$ ($\lesssim 0.1 \mu m$) to $\sim 1$ cm ($\lesssim 0.05$ cm) at the Bondi sphere of an 8.7 $M_\odot$ planet and at the Hill sphere of a 26 $M_\odot$ planet. While we have included dust feedback in all of our runs, a comparison with gas-only simulations confirmed that the overall planet atmospheric structure is insensitive to the dust drag force, at least for dust-to-gas mass ratios of $\lesssim 0.1$.

We found that the dust equilibrium scale height plays a primordial role in setting the much stronger latitudinal gradient of the (azimuthally averaged) opacities. The planet’s gravitational force only modestly steepens the latitudinal gradient at the scales of the Bondi and Hill semispheres.

Our numerical simulations revealed the presence of a persistent anisotropy in the dust opacities on the relevant scales of $r_B$ and $r_H$. The anisotropy is established after two planetary orbits and remains in a steady state up to 80 orbits. While smaller grains have even longer settling times, they are more well-mixed and do not significantly participate in the anisotropy. We find that spiral wakes from the planet generically introduce azimuthal dust asymmetries near the midplane.

Our results indicate that planetary envelopes may have a latitudinal transition between optically thin and optically thick regimes, which may limit the applicability of 1D models where such a transition cannot be captured. This transition is likely to occur when the scale height of grains that set the peak wavelength of a blackbody radiation is smaller than the planet Bondi radius. At temperatures range between 10 K and 100 K, the peak wavelength corresponds to grains with sizes smaller than 100 $\mu m$. Hence we expect that disks with lower surface densities and/or weaker gas stirring will show this transition. We found that for standard MMSN surface density and settling parameters $\delta \lesssim 5 \times 10^{-4}$, the transition between optically thick and thin regimes occurs for nearly thermal-mass planets at $r \gtrsim 10$ au.

A latitudinal transition in the mean free path differs from the traditional, symmetric case where envelopes are either optically thick or thin (see, e.g., Rafikov 2006; Lee & Chiang 2015). Moreover, the 3D dust dynamics produces an opacity that can deviate from opacity calculations based on the results from Bell & Lin (1994) and Semenov et al. (2003). The integrated opacity, and consequently the cooling time, is reduced in comparison with that obtained from a well-mixed size distribution (see, for example, cases with $W_{30}$, which could favor the faster growth of massive planets (Hubickyj et al. 2005; Movshovitz et al. 2010). Note that our opacity calculations for a planet at $r = 30$ au show even larger opacity gradients, and smaller integrated opacities, implying a greater impact on planets forming in the outer regions of PPDs.

To better understand the scope of our results, future simulations should include turbulence and/or winds as a mechanism of stirring dust, rather than a standard diffusion flux. We, however, stress that the assumed dust equilibrium scale heights are in reasonable agreement with regimes dominated by Magnetorotational instability turbulence (Flock et al. 2017) and MHD winds at $r \gtrsim 30$ au (e.g., Riols & Lesur 2018). Moderate-to-strong settling of dust grains in the outer regions of PPD has been also inferred from observations of HL Tau (Pinte et al. 2016). Note that this is not necessarily the case for self-consistent stirring driven by turbulence as a byproduct of the vertical shear instability (see, e.g., Stoll & Kley 2016; Picogna et al. 2018; Flock et al. 2017).

Ultimately, 3D multifluid simulations coupled with self-consistent radiative transfer are required to fully address the scope of our results. In addition to impacts on cooling and accretion rates, we anticipate that our results may also have implications for thermal physics of the heating torque (Benítez-Llambay et al. 2015), the evolution of eccentricity and inclination of hot protoplanets (e.g., Chrenko et al. 2017;...
Eklund & Masset 2017), and the excitation of buoyancy resonances (e.g., Zhu et al. 2012; Lubow & Zhu 2014; McNally et al. 2020; Bae et al. 2021).

6. Software and Third-party Data Repository Citations

All of the data was generated with a modified version of the open-source software FARGO3D available at https://bitbucket.org/fargo3d/public.git. The data underlying this article will be shared on reasonable request to the corresponding author.

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Appendix

Convergence with Numerical Resolution

To validate our results, we perform a resolution study varying the number of grid cells across the Bondi radius and the extent of the azimuthal domain. We also benchmark our results against both isothermal and adiabatic simulations from the literature with planet mass \( M_p \simeq 0.6 M_\oplus \). Referencing Table 1, we utilize runs SUB-d5, SUB-d5-half, SUB-d5-half-cnv, and SUB-d5-half-adia.

We focus our literature comparisons on Fung et al. (2019), who performed 3D global simulations to study the formation of circumplanetary disks, employing an isothermal and an adiabatic equation of state. They found that envelopes of subthermal-mass planets are nearly in hydrostatic equilibrium with little rotational support at the scales of the Bondi radius. Neglecting the stellar potential, the hydrostatic equilibrium around the planet gives a density profile that follows as

\[
\rho_\phi = \rho_0 \exp \left( \frac{r_B}{\sqrt{r^2 + r_s^2}} \right),
\]

for the isothermal case, whereas for the adiabatic equation of state (assuming an isentropic process), the density profile is

\[
\rho_\phi = \rho_0 \left( 1 - \left( \frac{1}{\gamma} - 1 \right) \frac{r_B}{\sqrt{r^2 + r_s^2}} \right)^{-\frac{1}{\gamma - 1}},
\]

where \( \gamma = 1.4 \) is the adiabatic index, and \( r_s \) is the smoothing length (for the runs SUB-d5-half-cnv and SUB-d5-half-adia, \( r_s \) is about 6% of the Bondi radius).

In the left panel of Figure 12, we compare the radial profile for the runs SUB-d5, SUB-d5-half-cnv, and SUB-d5-half-adia with the solutions given in Equations (A1) and (A2). In all cases, the numerical solutions are in good agreement with the hydrostatic equilibrium solution at \( r/r_B > 0.2 \). The differences between the density profiles at \( r/r_B < 0.2 \) are driven by the choice of smoothing length, which is set to 2 cells in azimuth, 2.3 cells in radius, and 3 cells in latitude.

In the middle panels of Figure 12, we show the radial profile of the azimuthal velocity in a cylindrical coordinate system centered on the planet. The profiles were calculated in the midplane after taking the azimuthal average on a cylindrical mesh centered on the planet. Pressure support dominates from the outermost regions of the Bondi sphere down to \( r \sim r_B/2 \). These results are in agreement with previous work showing increasing rotational support closer to the planet, where circumplanetary disks can form (Fung et al. 2019). Our resolution is insufficient to identify any Keplerian rotation below \( r \sim r_B/2 \).

In the rightmost panel of Figure 12, we show the azimuthal average of the total dust-to-gas density ratio at the Bondi sphere for the runs SUB-d5 (full azimuthal domain), SUB-d5-half,
and SUB-half-cnv (half azimuthal domain). The mean dust-to-gas density ratio is in good agreement for the runs SUB-d5 and SUB-d5-half, indicating that the choice of a half-domain in azimuth may not affect the overall dust density and therefore opacity distribution at the scales of the Bondi sphere.

At scales of \( r_B \), the runs have not entirely converged in the polar regions. We found that the dust-to-gas density ratio is more depleted at regions \( \tilde{\theta} \sim 0 \) for the case SUB-half-cnv. This has an impact on the opacity gradient as mentioned in Section 4.2. More precisely, we found that the integrated inverse opacity is \( \sim 7\% \) smaller than the well-mixed case for the run SUB-d5, whereas the largest resolution gives a discrepancy of \( \sim 22\% \). Thus we expect that our results are conservative, in that we have likely underestimated the magnitude of the opacity gradient.

Finally, in Figure 13, we show the azimuthal average of the density along with the meridional velocity field for the same runs displayed in Figure 12. Our results are in good agreement with those described in Fung et al. (2019). Note that the detailed behavior of the flow at \( r < r_H/2 \) may not be accurate given the simulations typically have \( \lesssim 8 \) cells at that radius, except for runs SUB-half-cnv and SUB-half-adia, where we have \( \lesssim 15 \) cells at half of the Bondi radius. Our comparison suggest that a minimum of 30 cells/\( r_B \) is required to characterize the meridional circulation pattern and dust density distribution below half of the Bondi radius. Moreover, for the explored parameters and timescales (less than 100 orbits), we found that simulations with reduced azimuthal domain may help to expedite the calculations without significantly affecting the dynamics near the planet.

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Figure 13. Azimuthal average of the gas density and meridional velocity field for the runs SUB, SUB-half-cnv, and SUB-adia. The left and center panels correspond to the isothermal case but with different resolutions, whereas the results from the adiabatic are displayed in the right panel. The meridional flows show a recycling pattern with polar inflow and midplane outflow. While these simulations include dust, the dust feedback seems negligible, and therefore these results may be compared with those from Fung et al. (2019). The important discrepancies between the left and center panels highlight the inadequate resolution of our fiducial run below \( r = r_H/2 \).
