The proton’s gluon structure

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The proton’s gluon structure function at small $x$ is larger than nowadays is commonly believed.

1. Introduction

We do not understand perturbative QCD at small $x$. In particular, as I will explain, we do not understand how to apply DGLAP evolution there. However, if we combine it with Regge theory and use an important message from the HERA data for the charm structure function $F_2^c(x, Q^2)$, it is possible reliably to extract the gluon structure function $g(x, Q^2)$ at small $x$. It turns out to be larger than nowadays is commonly believed. This is seen in figure 1. The most recent CTEQ and MRST structure functions agree well with each other and with those extracted by the two HERA experiments because they all use similar procedures; however, Donnachie and I believe that the old MRSG structure function is nearer the truth.

2. Regge theory – the two pomerons

When one tries to fit data, it is usually sensible to start with the simplest assumptions and then refine them later. In its simplest form, Regge theory leads to fixed powers of $x$ at small $x$, and it turns out that two terms are enough:

$$F_2(x, Q^2) \sim f_0(Q^2)x^{-\epsilon_0} + f_1(Q^2)x^{-\epsilon_1}$$

(1)

The second term corresponds to soft-pomeron exchange, with $\epsilon_1 \approx 0.08$ determined from soft reactions. The data need a term that rises more rapidly at small $x$; one needs $\epsilon_0 \approx 0.4$. By fitting the data at each $Q^2$, Donnachie and I found that a successful and economical parametrisation of the coefficient functions is provided by

$$f_0(Q^2) = A_0(Q^2)^{1+\epsilon_0}/(1 + Q^2/Q_0^2)^{1+\epsilon_0/2}$$

$$f_1(Q^2) = A_1(Q^2)^{1+\epsilon_1}/(1 + Q^2/Q_1^2)^{1+\epsilon_1}$$

(2)

with $Q_0 \approx 3$ GeV and $Q_1 \approx 0.8$ GeV. To make the fit, we used real-photon data and DIS data with $x \leq 0.001$, so that $Q^2$ ranges from 0.045 to 35 GeV$^2$. If we then simply multiply the resulting form (1) by $(1 - x)^7$, as is suggested by the dimensional counting rules, it agrees quite well with the HERA data even beyond $x = 0.1$ and up to $Q^2 = 5000$ GeV$^2$. This is shown in figure 1. Note that this factor $(1 - x)^7$ should not be taken too seriously; it is much too simple.

Data for the charm structure function $F_2^c(x, Q^2)$ have the remarkable property that, at all available $Q^2$, they fit to just the single hard-pomeron power of $x$. Further, to an excellent approximation the coupling of the hard pomeron appears to be flavour blind:

$$F_2^c(x, Q^2) = f_c(Q^2)x^{-\epsilon_0}$$

(3)

with

$$f_c(Q^2) = 0.4 f_0(Q^2)$$

(4)

So if we define a charm-production cross section

$$\sigma_c(W) = \frac{4\pi^2\alpha_{EM}}{Q^2} |F_2^c(x, Q^2)|^2 \bigg|_{x = Q^2/(W^2 + Q^2)}$$

(5)

it behaves as $W^{-\epsilon_0}$ at all $Q^2$, even down to $Q^2 = 0$: see figure 3. Perturbative QCD directly relates $F_2^c(x, Q^2)$ to the gluon structure function, so that at small $x$ it too must be dominated by hard-pomeron exchange alone, even at quite small values of $Q^2$. This what causes the rapid rise at small $x$ of the DL curve in figure 1.
Figure 1. Gluon structure functions from http://cpt19.dur.ac.uk/hepdata/pdf3.html and reference 1.

Figure 2. Regge fit to ZEUS and H1 data for $F_2(x, Q^2)$ for $Q^2$ between 0.045 and 5000 GeV$^2$. The parameters were fixed using only data for $x < 0.001$ and therefore $Q^2 \leq 35$ GeV$^2$.

Figure 3. Data for the electroproduction of charm at various $Q^2$, with $W^{0.88}$ and pQCD fits (upper and lower curves, respectively).

Figure 4. NLO evolution of the hard-pomeron coefficient function (solid curve) with the phenomenological fit (broken curve).
3. DGLAP evolution

The singlet DGLAP equation introduces the two-component quantity

$$u(x, t) = \left( x \sum f(q + \bar{q}) \right)$$

where $t = \log(Q^2/\Lambda^2)$. If we Mellin transform with respect to $x$, the equation becomes very simple:

$$\frac{\partial}{\partial t} u(N, Q^2) = P(N, \alpha_s(Q^2)) u(N, Q^2)$$

(7)

The usual approach is to expand the splitting matrix $P$ in powers of $\alpha_s$. However, this is mathematically illegal when $N$ is small. Compare

$$\sqrt{N^2 + \alpha_s - N = \alpha_s/2N - \alpha_s^2/8N^3 + \ldots}$$

(8)

Here, each term in the expansion is singular at $N = 0$ but the function itself is regular there: the expansion is illegal for $N^2 \leq \alpha_s$. Similarly, it is likely that whenever expanding $P(N, \alpha_s(Q^2))$ makes it large, and therefore makes $u(N, Q^2)$ vary rapidly with $Q^2$, the expansion is dangerous.

My own belief is that $P(N, \alpha_s(Q^2))$ has no singularities in the complex-$N$ plane, or at least no relevant singularities. My reason is that solving \[6\] would cause a singularity of $P(N, \alpha_s(Q^2))$ to induce an essential singularity in $u(N, Q^2)$ (that is, a nasty one). The variable $N$ is closely related to the orbital angular momentum $l$, and I was brought up to believe that matrix elements such as $u(N, Q^2)$ do not have essential singularities in the complex $l$-plane. This point of view contrasts with that of those who believe that the value of $\epsilon_0$ is associated with a singularity of $P(N, \alpha_s(Q^2))$ and may even be calculated, perhaps by refining the BFKL approach. I think that very probably $\epsilon_0$ is a nonperturbative quantity that therefore cannot be calculated.

A fixed-power behaviour $x^{-\epsilon_0}$ of $F_2(x, Q^2)$, such as in \[6\], corresponds to an $N$-plane pole:

$$u(N, Q^2) \sim \frac{f(Q^2)}{N - \epsilon_0} f(Q^2) = \left( \frac{f_0(Q^2)}{f_1(Q^2)} \right)$$

(9)

If we insert this into the DGLAP equation (8), and equate the residue of the pole on each side of the equation, we find

$$\frac{\partial}{\partial t} f(Q^2) = P(N = \epsilon_0, \alpha_s(Q^2)) f(Q^2)$$

(10)

$\epsilon_0$ is far enough from 0 for the expansion of $P(N = \epsilon_0, \alpha_s(Q^2))$ to be reasonably safe. So we may easily use the DGLAP equation to calculate the evolution of the hard-pomeron component of $F_2(x, Q^2)$. But this is not the case for the soft-pomeron component, because $\epsilon_1 \approx 0.08$ is too close to 0.

According to figure \[4\], the various gluon structure functions come together at $x \approx 0.01$. It is reasonable to assume that for values of $x$ larger than this the evolution of the two elements of $u(x, Q^2)$ does not use values of $N$ close to 0 and therefore the conventional analysis is correct. So we can start at some not-too-large value of $Q^2$, 20 GeV$^2$ say. We determine the value of $f_0(Q^2)$ there from the phenomenological fit \[3\] and $f_g(Q^2)$ from the MRST gluon structure function $xg(x, Q^2)$, which for $x$ greater than about 0.01 fits very well to $x^{-\epsilon_0}(1 - x)^5$. We choose $\Lambda_{\text{NLO}}$ such that $\alpha_s(M_Z^2) = 0.116$ and use \[11\] to calculate the NLO evolution of $f_0(Q^2)$ and $f_g(Q^2)$ in both directions. The result for $f_0(Q^2)$ is the continuous curve in figure \[5\]. The dashed curve is the phenomenological form \[2\]. Provided we adjust $\Lambda$ so that still $\alpha_s(M_Z^2) = 0.116$, LO evolution gives almost identical results; this is shown in figure \[4\].

The agreement between the pQCD calculation and the phenomenological curve is a success not only for the concept of the hard pomeron, but also for pQCD itself. The evolution is from a single value of $Q^2$, not the customary global fit \[4\], and it introduces far fewer parameters.

Notice that, as $Q^2$ increases the large-$x$ behaviour of $xg(x, Q^2)$ becomes steadily steeper than $(1 - x)^5$, and so the largest value of $x$ for which $x^{-\epsilon_0}$ is a good approximation to the structure function steadily decreases. Figure \[2\] shows an estimate of this.

We may use the gluon structure function to calculate the charm structure function $F_2^c(x, Q^2)$. The result, using just LO photon-gluon fusion with a charm-quark mass $m_c = 1.3$ GeV, is the solid curves in figure \[3\]. This is an important check on the consistency of the approach. As is seen in figure \[5\], a steep gluon distribution is needed to fit the data at small $Q^2$. 
In conclusion, the conventional approach to evolution needs modifying at small $x$. It can be corrected if we combine it with Regge theory, but only partly — we can only treat the hard-pomeron part. The resulting gluon distribution is larger at small $x$ than has so far been supposed and gives a good description of charm production. I should add that we want good data for the longitudinal structure function, because this gives the most direct window on the gluon distribution.

Figure 7. Charm structure functions at $Q^2 = 1.8$ GeV$^2$, with ZEUS data.

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Figure 5. LO and NLO evolution of the hard-pomeron and gluon coefficient functions

Figure 6. Gluon structure function at $Q^2 = 20$ and 200 GeV$^2$