Modeling of scattering intensity of spheroid particles with a Gaussian beam

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Abstract. Based on the Generalized Lorenz Mie Theory (GLMT), the scattering intensity of spheroid particles is investigated within the Gaussian beam. The spheroid model is applied to represent the real non-spherical particles, and the scattering intensity of particles is deduced according to the GLMT. On the other hand, the sphere of the same volume for the spheroid is used for calculating the scattering intensity of the spheroid within the Gaussian beam. For a comparison, the scattering intensity of the spheroid with the plane wave is also calculated. Simulation data results indicate that fairly reasonable results of the scattering intensity for the spheroids can be obtained with this model, and it can provide a reliable and efficient approach to research the scattering intensity of the non-spherical particles by the Gaussian beam.

1. Introduction
The light scattering of particles is very important in the engineering, Chemical, medicine and other regions. Since the theory of light scattering has been established, some researches have studied the electromagnetic light scattering of particles for the plane wave case [1]. And some common theories and methods have been utilized to analyze this problem. When the plane light wave is incident the particle, the classical Mie theory, the discrete dipole approximation (DDA), the T matrix method, and the finite difference time domain (FDTD) method can be used for sphere and non-sphere particles [2-4]. Nevertheless, none of those methods can be applied to analyze and calculate the scattering of particles for the non-plane waves, i.e., the Gaussian beam incidence.

In this paper, we study the scattering intensity of particles based on the Gaussian beam incidence. Since the computation of light scattering by the non-spherical particles is very important in many measurement areas, great progress has been made in the study of light scattering for the non-spherical particles [5]. Actually, the non-spherical calculations and measurements show significant differences from the sphere particles. Here, we choose the spheroid model as the non-spherical particles to study the scattering intensity of non-spherical particles within the Gaussian beam incidence.

2. Theory and method
With the development of the laser technique, the laser are usually used for the measuring the light scattering of particles. Especially, many optical particle sizing techniques rely on the interaction between the particles and the laser sources. So this research on the light scattering by the particles and the interaction between the particles and the incident non-plane beam for the case in which the particles are illuminated by the laser beam has been on growing up [6].
It is well known that the Generalized Lorenz Mie Theory (GLMT) proposed by Gouesbet et al. is a generalization of the Lorenz Mie theory for the Gaussian beam [7]. The GLMT can describe the properties of incident Gaussian beam and the scattered field of the homogeneous spherical particles as well as the interaction between the spheres and the Gaussian beam. Actually, the GLMT is a direct extension of the plane wave Mie theory to the case of Gaussian beam. What is most important is that the GLMT is a rigorous theory that depends on the Maxwell electromagnetic scattering equations [6].

In the GLMT framework, the incident Gaussian beam field including the electric and the magnetic fields can be described by the Bromwich Scalar Potentials (BSP) in the spherical coordinate system (r, θ, φ) [6]. The field components are then found to be:

\[
E_i^r = \frac{\partial^2 U_{TM}}{\partial r^2} + k^2 U_{TM}
\]

\[
E_i^\theta = \frac{1}{r} \frac{\partial^2 U_{TM}}{\partial \theta \partial r} - \frac{i \omega \mu}{r} \frac{\partial U_{TE}}{\partial \phi}
\]

\[
E_i^\phi = \frac{1}{r \sin \theta} \frac{\partial^2 U_{TM}}{\partial \phi \partial r} + \frac{i \omega \mu}{r} \frac{\partial U_{TE}}{\partial \theta}
\]

\[
H_i^r = \frac{\partial^2 U_{TE}}{\partial r^2} + k^2 U_{TE}
\]

\[
H_i^\theta = \frac{i \omega \varepsilon}{r \sin \theta} \frac{\partial U_{TM}}{\partial \phi} + \frac{1}{r \sin \theta} \frac{\partial^2 U_{TE}}{\partial \phi \partial \theta}
\]

\[
H_i^\phi = \frac{1}{r \sin \theta} \frac{\partial^2 U_{TE}}{\partial \theta \partial r} - \frac{i \omega \varepsilon}{r} \frac{\partial U_{TM}}{\partial \phi}
\]

(1)

where \(U_{TM}\) and \(U_{TE}\) are the transverse magnetic (TM) and transverse electric (TE) BSP, respectively, \(i = \sqrt{-1}\), \(k\) is the wave number, \(\omega\) is the angular frequency of the electromagnetic wave, \(\mu\) and \(\varepsilon\) are the permeability and the permittivity of the medium, respectively.

When the beam is incident upon a spherical particle with radius \(r\) and complex refractive index \(M\), the scattered field components in the far field approximation are given as [6]:

\[
E_s^r = H_s^r = 0
\]

\[
E_s^\theta = \frac{i E_0}{kr} \exp(-ikr) S_2
\]

\[
= \frac{i E_0}{kr} \exp(-ikr) \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \frac{2n+1}{n(n+1)} \left[ a_n g_{n,TM}^m \tau_n^{\|}(\cos \theta) + ib_n g_{n,TE}^m \pi_n^{\|}(\cos \theta) \right] \exp(im \phi)
\]

\[
E_s^\phi = -\frac{E_0}{kr} \exp(-ikr) S_1
\]

\[
= -\frac{E_0}{kr} \exp(-ikr) \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \frac{2n+1}{n(n+1)} \left[ ma_n g_{n,TM}^m \tau_n^{\|}(\cos \theta) + ib_n g_{n,TE}^m \pi_n^{\|}(\cos \theta) \right] \exp(im \phi)
\]

\[
H_s^\theta = -\frac{H_0}{E_0} E_s^\phi
\]

\[
H_s^\phi = \frac{H_0}{E_0} E_s^\theta
\]

(2)

where \(\tau_n\) and \(\pi_n\) are the generalized Legendre functions, \(a_n\) and \(b_n\) are the scattering coefficient of Mie theory, \(S_1\) and \(S_2\) are the scattering amplitudes.

The far field scattering intensity can be expressed:
\[ I = \lim_{r \to \infty} \frac{\beta^2 E_0}{4\pi^2 r^2} \left( |S_1|^2 + |S_2|^2 \right) \]  

(3)

In order to study the scattering intensity of spheroid particles based on the GLMT, it is still practical to use the sphere model to approximate the scattering properties of spheroid particles. A spheroid particle may be represented by a sphere of the same volume. This method could be used to readily calculate the scattering intensity of spheroids.

3. Numerical calculations

In this section, we will discuss the scattering properties of spheroids with different sizes and ratio aspects in the case of the incident Gaussian beam.

Fig. 1 shows the scattering intensity of spheroid particles. In this figure, the incident wavelength is 0.5\(\mu\)m, the beam waist \(w_0=1\mu\)m, and the relative complex reflective indices of particles is 1.33. We can see that the scattering of spheroid is different from the sphere, and the difference is more obvious with the increasing aspect of spheroid.

Fig. 2 shows the scattering intensity of spheroid particles with different sizes. In this figure, the incident wavelength is 0.5\(\mu\)m, the beam waist \(w_0=1\mu\)m, and the relative complex reflective indices of particles is 1.33. The aspect of spheroid is invariant and the size of minor axis of the spheroid is increasing. We can see that the scattering of spheroid is also different from the sphere.

4. Conclusions

In this paper, the scattering of spheroid particle with in the Gaussian beam based on the GLMT is studied. In the framework of GLMT, the scattering intensity of particles is calculated. In order to research the scattering of spheroid particle with more efficiency, the a sphere of the same volume with the spheroid is used to calculate the scattering intensity of spheroid particles within the Gaussian beam, and then the scattering intensity of spheroid particles is also calculated within the plane wave incidence. In the simulations, the results show that the scattering of spheroid is different from the sphere, and the difference is more obviator with the increasing aspect of spheroid.
Fig. 2 Scattering intensity of spheroid with different sizes

(a) \( a=0.06 \mu m, \text{aspect}=0.2 \)

(b) \( a=2.7 \mu m, \text{aspect}=0.2 \)

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References

[1] James A. Lock and Gérard Gouesbet, Rigorous justification of the localized approximation to the beam-shape coefficients in generalized Lorenz–Mie theory. I. On-axis beams, Journal of the Optical Society of America A Vol. 11, Issue 9, pp. 2503-2515 (1994)

[2] Wenbo Sun, Gorden Videen, Qiang Fu, Yongxiang Hu. Scattered-field FDTD and PSTD algorithms with CPML absorbing boundary conditions for light scattering by aerosols. Journal of Quantitative Spectroscopy and Radiative Transfer, Volume 131, December 2013, Pages 166-174

[3] Cui, Z.W. Han, Y.P.; Wang, J.J.; Zhao, W.J. Scattering of Gaussian beam by arbitrarily shaped inhomogeneous particles, Journal of Quantitative Spectroscopy and Radiative Transfer, v 113, n 6, p 480-488, 2012

[4] Zhou. B., S.S.Li, and K.Stamnes (2003), Geometrical-optics code for computing the optical properties of large dielectric spheres, Applied Optics 42 (21), 4295-4306

[5] Haihua Wang, Xianming Sun, Huayong Zhang. Scattering by a spheroidal particle illuminated with a couple of on-axis Gaussian beams, Optics & Laser Technology, Volume 44, Issue 5, July 2012, Pages 1290-1293

[6] G. Gouesbet, J.J. Wang, Y.P. Han, Transformations of spherical beam shape coefficients in generalized Lorenz–Mie theories through rotations of coordinate systems: I. General formulation. Optics Communications, Volume 283, Issue 17, 1 September 2010, Pages 3218-3225

[7] G. Gouesbet, F. Xu, Y.P. Han, Expanded description of electromagnetic arbitrary shaped beams in spheroidal coordinates, for use in light scattering theories: A review. Journal of Quantitative Spectroscopy and Radiative Transfer, Volume 112, Issue 14, September 2011, Pages 2249-2267.