Explaining $b \to s\ell^+\ell^-$ and the Cabibbo Angle Anomaly with a Vector Triplet

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The statistically most significant hints for new physics in the flavour sector are the disagreement between $b \to s\ell^+\ell^-$ data and its Standard Model (SM) predictions as well as the tensions between different determinations of $V_{us}$ (from Kaon and tau decays vs super-allowed beta decays) known as the Cabibbo angle anomaly (CAA). We examine how these discrepancies can be reconciled within a simplified model with massive gauge bosons transforming in the adjoint representation of $SU(2)_L$, i.e. by adding $W'$ and $Z'$ bosons coupling to left-handed SM fermions. We find that the $W'$ boson can account for the CAA while the $Z'$ can explain the tensions in $b \to s\ell^+\ell^-$ data. Furthermore, since the $W'$ and $Z'$ couplings are related via $SU(2)_L$, gauge invariance, we observe interesting correlations between electroweak precision data, the CAA and $b \to s\ell^+\ell^-$ within our global fit. In fact, we find that our model can provide a consistent common explanation of both anomalies, giving an excellent fit to data, far superior to the one of the SM.

I. INTRODUCTION

In 2012, the LHC confirmed the Standard Model (SM) of particle physics by discovering the (Brout-Englert) Higgs boson [1, 2]. However, so far no particles beyond the ones of the SM have been observed in high energy searches. Therefore, great hopes of finding physics beyond the SM rest on the low energy precision frontier. Here, fortunately, flavour experiments have accumulated intriguing hints for new physics (NP) within the recent years. Among them, the statistically most significant ones are the deviations between the SM predictions and the measurements in many observables containing $b \to s\ell^+\ell^-$ transitions and hints for a (apparent) violation of 1st row CKM unitarity, known as the “Cabibbo Angle Anomaly” (CAA).

Concerning $b \to s\ell^+\ell^-$ transitions, the LHCb measurements [3, 4] of $R(K^{(*)}) = \frac{Br(B\to K^{(*)}\mu^+\mu^-)}{Br(B\to K^{(*)}\ell^+\ell^-)}$ indicate lepton flavour universality (LFU) violation with a combined significance of $\approx 4\sigma$ [5,13]. A consistent pattern of tensions in the angular distribution of the muonic channel $B \to K^\mu \mu^-$, most noticeably in the angular observable $P_{1\ell}$ [16,17], has also been recently confirmed by the LHCb [18,19]. Taking into account all available measurements of $b \to s\ell^+\ell^-$ observables, the most up-to-date global analyses find several NP scenarios to be preferred over the SM at the 5 – 6$\sigma$ level [12,13]. This therefore constitutes some of the most compelling evidence for NP in the LHC era, hinting at the violation of LFU with a NP structure mostly related to muons, while an effect related to electrons is possible but not mandatory.

The CAA, which is due to the disagreement between the CKM element $V_{us}$ extracted from Kaon and tau decays and the one determined from beta decays (using CKM unitarity), has a significance of $\approx 4\sigma$ [20,21]. Interestingly, this discrepancy can also be interpreted as a sign of LFU violation [22,23] where the sensitivity to NP in the determination via beta decays is enhanced by a factor $V_{ud}^2/V_{us}^2$ compared to the NP sensitivity of $V_{us}$ from Kaon or tau decays [23, 24].

It therefore seems plausible that a connection between the $b \to s\ell^+\ell^-$ anomalies and the CAA exists, and it is both interesting and important to explore which NP models can provide a common explanation. In order to account for the CAA, NP must in some way be related to the charged current, which can be achieved in the form of modified $W\ell\nu$ couplings and/or by effects in $u\ell\nu$ operators. Both of these possibilities can be realized with a $W'$ boson coupling to left-handed SM fermions; the first one via $W \to W'$ mixing, the second one through a tree-level contribution. Furthermore, due to $SU(2)_L$ gauge invariance, a left-handed $W'$ boson always comes together with a left-handed $Z'$ [29] which is a prime candidate for an explanation of the $b \to s\ell^+\ell^-$ anomalies [20,62], obviously opening up the possibility of a combined explanation in this setup.

A minimal dynamical model including a left-handed $W'$ and $Z'$ can be obtained by extending the SM with massive vector bosons transforming in the adjoint representation (or equivalently as a triplet) of $SU(2)_L$, and with zero hyper-charge [63,65]. This Lagrangian can be generated by various NP models, for instance composite Higgs and extra dimensional models [66,74] or mod-

1 Alternatively, it can be interpreted as a sign of (apparent) CKM unitarity violation [26,25]. However, a sizeable violation of CKM unitarity is in general difficult due to the strong bounds from flavor-changing neutral currents, such as Kaon mixing (see e.g. Ref. [26]) Furthermore, a right-handed $W$ coupling [27,28] can only partially account for it [21].

2 A modification of the Fermi-constant also affects $V_{ud}$ from beta decays. However, we checked that such an effect is too tightly constrained from EW precision data to account for the CAA.
els based on $SU(2) \times SU(2)_L$ [75][79]. Because of the many possible UV completions, we find that in order to understand the effects of generic vector triplets, it is convenient to focus on a simplified model. To determine the viability of a heavy vector $SU(2)_L$ triplet, we will perform a global fit including the most relevant observables that are modified by tree level effects, i.e. $b \to s\ell^+\ell^-$ and $V_{us}$ as well as EW precision data, the observables testing LFU and LHC direct searches.

In the next section we will define our setup before we study in Sec. III the impact of the vector triplet on the various observables to be used in the global fit. In Sec. IV we present the results of the global fit and conclude in Sec. V.

II. SETUP

In our peruse of a common explanation of $b \to s\ell^+\ell^-$ data and the CAA, we supplement the SM by an $SU(2)_L$ triplet of heavy vector bosons $X_a^\mu$ with zero hypercharge [64][65]. Therefore, following the conventions of Ref. [64] we write $\mathcal{L} = \mathcal{L}_\text{SM} + \mathcal{L}_X^0 + \mathcal{L}_X^\text{int}$ with

$$\mathcal{L}_X^0 = -\frac{1}{2} [D_\mu X_a^\nu] [D^\mu X_a^\nu] + \frac{1}{2} |D_\mu X_a^\nu|^2 + \frac{\mu_0^2}{2} X_a^\mu X_a^\mu,$$

$$\mathcal{L}_X^\text{int} = -g_j X_a^\mu j^\alpha j^\beta \frac{\sigma^\alpha}{2} \ell_i - g_j X_a^\mu \gamma_5 \gamma_\mu \frac{\sigma^\alpha}{2} q_i - \left(i g_X^D X_a^\mu \phi^\dagger \sigma^\mu D_\mu \phi + h.c.\right) + g_X^D X_a^\mu X_a^\mu \phi^\dagger \phi,$$

where $D_\mu = \partial_\mu + ig_2 \sigma^\alpha W_\mu^{\alpha(0)}/2 + ig_1 Y B_\mu^{(0)}$, $\sigma^\alpha$ are the Pauli matrices and $W_\mu^{\alpha(0)}, B_\mu^{(0)}$ correspond, in the absence of $SU(2)_L$ breaking, to the SM gauge bosons. The first two terms in $\mathcal{L}_X^0$ generate the interactions of the new gauge bosons with the SM ones while the last term gives them masses even before EW symmetry breaking (EWSB). In $\mathcal{L}_X^\text{int}$ the terms proportional to $g_j^{(0)}$ parametrize the couplings of the new gauge bosons to left-handed quarks (leptons) and the term containing $g_X^D$ gives rise to a mass mixing between $X_a^\mu$ and the SM gauge bosons after EWSB. The last term in $\mathcal{L}_X^\text{int}$ creates interactions between $X_a^\mu$ and the SM Higgs, which gives an additional contribution to their mass after EWSB.

Consider the mass spectrum of the gauge bosons, the zero mass eigenstate is identified with the photon $\gamma$ and does not mix with $X_a^\mu$. Hence the SM relation between $g, g'$ and the measured $\alpha$ is not modified [65]. Thus, one can consider the mass matrices after EWSB in the basis $(Z^{(0)}, A_\mu)$. Taking $\langle \phi \rangle = (0, v/\sqrt{2})^T$, we have

$$M_0^2 = \begin{pmatrix} M_{Z^{(0)}}^2 - \frac{\epsilon v}{c_W} & M_{Z^{(0)}} M_{A_\mu} \sqrt{2} \\ M_{Z^{(0)}} M_{A_\mu} \sqrt{2} & M_{A_\mu}^2 \end{pmatrix},$$

where the superscript (0) refers to the SM fields in the absence of mixing, $M_{W^{(0)}} = g v/\sqrt{2}$, $M_{Z^{(0)}} = M_{W^{(0)}/c_W}$, while $M_{A_\mu} = \mu_0 + x^2 v^2$, $x = M_{W^{(0)}}(g_X^D v/2)$ and $c_W \equiv g/\sqrt{g_2^2 + g_1^2}$ is the cosine of the Weinberg angle. Provided that $|x| \ll M_X$ one can work in the approximation $M_{W^\prime} \approx M_Z \approx M_X$ while the $M_W$ and $M_Z$ masses are shifted by

$$\frac{M_W^2}{M_{W^{(0)}}^2} \approx \frac{M_Z^2}{M_{Z^{(0)}}^2} \approx \left( 1 - \frac{|g_X^D v|^2 x^2}{4 M_X^2} \right),$$

respecting the SM tree-level relation $M_{W^{(0)}} = c_W M_{Z^{(0)}}$. When mixing is present, the eigenvalues are linear combinations of $(Z^{(0)}, X^0)$ and $(W^{(0)}, X^0)$ which for $|x| \ll M_X$ yield the following mixing angles

$$\sin \alpha_{ZZ^\prime} \approx \frac{x}{M_X c_W}, \quad \sin \alpha_{WW^\prime} \approx \frac{x}{M_X}.$$

The mass eigenstates $Z^{(0)}$ can then be expressed as

$$\left(\begin{array}{c} Z' \\ Z \end{array}\right) = \left(\begin{array}{cc} X^3 \cos \alpha_{ZZ^\prime} - Z^{(0)} \sin \alpha_{ZZ^\prime} \\ X^3 \sin \alpha_{ZZ^\prime} + Z^{(0)} \cos \alpha_{ZZ^\prime} \end{array}\right),$$

and similarly for the charged gauge bosons $W$ and $W'$. After $SU(2)_L$ symmetry breaking the $W'$ and $Z'$ couplings to quarks differ by a CKM rotation. Working in the down-quark basis we have

$$L_\mu^\text{QW'} = \frac{g_3^d}{2} (\bar{d}_f \gamma_\mu P_L d_i) Z'_\mu - \frac{g_3^d}{2} (\bar{u}_f \gamma_\mu P_L u_i) Z'_\mu,$$

with $g_3^d = g_1^d$, $g_3^d = V_{jk} g_3^d V_{ik}$, and $g_3^d = V_{jk} g_3^d$. In our phenomenological analysis we will assume that the $Z', W'$ couplings to quarks respect an (approximate) $U(3)^3$ flavour symmetry [55][92]. This means that to a good approximation

$$g_{12}^{ud} \approx g_{12}^{ud} \approx g^d, \quad g_{12}^{ud} = V_{us} g^d, \quad g_{23}^{ud} = O(V_{cb}).$$

Notice that in this setup the $Z'$ coupling $g_{12}^d$ is of third order in the Wolfenstein parameter [35][61], i.e. $O(10^{-3})$ so that with this ansatz for the $Z'$ couplings dangerously large effects in $K - \bar{K}$ and/or $D - \bar{D}$ mixing are avoided.

\footnote{Note that this differs from “standard” minimal flavour violation [32][35] (MFV) which is based on $U(3)^3$ [92], however, $U(3)^3$ is anyway strongly broken to $U(2)^3$ by the large third-generation Yukawa couplings.}
Since the coupling $g_{34}^g$ does not affect the observables that we consider (except for a small effect in LHC searches) we disregard it from here on after.

We can neglect the corrections to the $W'$ and $Z'$ couplings to fermions originating from gauge boson mixing as this leads to dim-8 operators. However, the couplings of $W$ and $Z$ to leptons are modified as

$$L_{W,Z} = \frac{g_{34}}{2\sqrt{2}} \left[ \bar{\ell}_i \gamma^\mu (\Delta_{ji} P_L - 2 s_W^2 \delta_{ji}) \ell_j Z_\mu \right. 
- \sqrt{2} \Delta_{ji} (\bar{\nu}_j \gamma^\mu P_L \nu_i) W_\mu - \Delta_{ji} (\bar{\nu}_j \gamma^\mu P_L \nu_i) Z_\mu \left. \right],$$

with $\Delta_{ji} = \delta_{ji} + x \frac{g_{34}^g}{M_X}$. 

III. OBSERVABLES

In this section we collect the relevant observables and show how they are affected by our NP contributions. Here, we will only give the explicit formulas for the direct $Z'$ and $W'$ contributions. One can easily recover the mixing induced effects by replacing the SM $W$ and $Z$ couplings by their modified versions given in Eq. (8).

A. Lepton Flavour Violation

If $g_{34}^g$ has off-diagonal elements, flavour violating decays of charged leptons are generated. Here the most stringent bounds come from radiative lepton decays $\ell \rightarrow \ell' \gamma$, decays to three charged leptons (like $\mu \rightarrow 3e$) and $\mu \rightarrow e$ conversion in nuclei.

The loop effects giving rise to $\ell \rightarrow \ell' \gamma$ can be calculated in the unitary gauge since a finite result is obtained in our simplified model setup, which includes unavoidable Goldstone effects present in a UV complete model. Using the expressions given in Ref. [97] we obtain

$$\text{Br}[\ell_i \rightarrow \ell_j \gamma] = \frac{m_{\ell_i}^3}{4\pi \Gamma_{\ell_i}} (|c_{ji}^g|^2 + |c_{ij}^g|^2),$$

with

$$c_{ji}^g \approx \frac{cm_{\ell_i} g_{34}^g g_{ji}^g}{16\pi^2} \frac{1}{M_X^2}. \tag{10}$$

The current experimental limits for lepton flavour violation processes are [98][100] and yield the 90% CL bounds

$$\text{Br} [\mu \rightarrow e \gamma] \lesssim 4.2 \times 10^{-13}, \quad |g_{ek} g_{34}^g| \lesssim 0.06, \quad \text{Br} [\tau \rightarrow \mu \gamma] \lesssim 4.4 \times 10^{-8}, \quad |g_{ek} g_{34}^g| \lesssim 112, \quad \text{Br} [\tau \rightarrow e \gamma] \lesssim 3.3 \times 10^{-8}, \quad |g_{ek} g_{34}^g| \lesssim 96,$$

where we used $M_X = 10$ TeV as a reference point and the sum over $k$ is implied.

Three body decays to charged leptons are already mediated at tree-level but are phase space suppressed. In our model we find that

$$\text{Br}(\mu \rightarrow 3e) = \frac{m_{\mu}^5}{3072 \pi^3 M_X^4} \frac{|g_{ek} g_{34}^g|^2}{16}, \quad \text{Br}(\tau \rightarrow e \mu \mu) = \frac{m_{\tau}^5}{1536 \pi^3 M_X^4} \frac{|g_{ek} g_{34}^g|^2}{16}, \tag{12}$$

where we neglected contributions involving two flavour changing couplings. Together with the experimental results [101] this yields the following 90% CL bounds (for $M_X = 10$ TeV)

$$\text{Br}(\mu \rightarrow eee) \lesssim 1.0 \times 10^{-12}, \quad |g_{ek} g_{34}^g| \lesssim 0.008, \quad \text{Br}(\tau \rightarrow \mu \mu \mu) \lesssim 1.2 \times 10^{-8}, \quad |g_{ek} g_{34}^g| \lesssim 3.4, \quad \text{Br}(\tau \rightarrow eee) \lesssim 1.4 \times 10^{-8}, \quad |g_{ek} g_{34}^g| \lesssim 3.7, \quad \text{Br}(\tau \rightarrow e \mu \mu) \lesssim 1.6 \times 10^{-8}, \quad |g_{ek} g_{34}^g| \lesssim 4.0, \quad \text{Br}(\tau \rightarrow \mu \mu e) \lesssim 1.1 \times 10^{-8}, \quad |g_{ek} g_{34}^g| \lesssim 3.3.$$

Finally, following the conventions of Refs. [102][103] we have that for $\mu \rightarrow e$ conversion in nuclei

$$\Gamma^{\text{N}}_{\mu \rightarrow e} = \frac{m_{\mu}^5 |g_{ek} g_{34}^g|^2}{16 M_X^3} \left| (V_{N}^{(n)} - V_{N}^{(p)})^2 \right|,$$

which has to be normalized to the capture rate $\Gamma_{\text{capture}}^N$ for gold [104]

$$V_{Au}^{(n)} - V_{Au}^{(p)} = -0.0486, \quad \Gamma_{\text{capture}}^N = 8.7 \times 10^{-15} \text{ MeV}.$$

The current 90% CL experimental limits are [83]

$$\text{Br}^{\text{Au}}_{\mu \rightarrow e} \lesssim 7.0 \times 10^{-13}, \quad |g_{ek} g_{34}^g| \lesssim 5.8 \times 10^{-8}, \tag{16}$$

again for $M_X = 10$ TeV.

Given these strong constrains on flavor changing $W'$ and $Z'$ couplings to leptons, we will in the following assume $g_{34}^g$ to be diagonal. This turns out to be a very good approximation, not only due to these stringent bounds, but also because flavour changing effects do not interfere with the SM contribution, such that they are suppressed by $1/M_X^2$.

B. Electroweak Precision Observables

The quantities $G_F$, $\alpha_{\text{em}}$ and $M_Z$ have been measured with the highest accuracy among the EW observables. Therefore, they are commonly taken as Lagrangian parameters (fixed to their experimental values) and used to calculate all other EW observables within the SM. Beyond the SM, this method can still be used, but the relations between the Lagrangian parameters and the measurements are changed. In particular, in our model the
TABLE I. Electroweak observables [106, 107] used in our fit which are calculated (as a function of \(M_Z^2\), \(\alpha\) and \(G_F^\ell\)) by HEPfit [105].

- Fermi-constant \(G_F = 1.16637(1) \times 10^{-5}\)GeV\(^{-2}\), measured from muon decays, is then given in terms of the one in the Lagrangian as

\[
G_F = G_F^L + \frac{g_1^\ell g_2^\ell}{4\sqrt{2}M_Z^2}, \quad (17)
\]

- likewise the measured \(M_Z\) mass is given by

\[
M_Z^L = \left(M_Z^0\right)\left(1 - \frac{1}{\sqrt{2}G_F^L + 4M_Z^2}\right), \quad (18)
\]

where \(G_F^L = 1/(\sqrt{2}\alpha)\) and \(M_Z^L = M_Z^0\) is the \(Z\) mass within the SM. In addition, the gauge bosons mixing induces corrections to \(W\) and \(Z\) couplings to fermions. The modified couplings affect \(W\) and \(Z\) decays and the corresponding list of observables given in Table I. For the numerical analysis, we implemented them in HEPfit [105].

C. \(V_{us}\) and the CAA

As outlined in the introduction, the Lagrangian parameter \(V_{us}^L\) of the (unitary) CKM can be determined from kaon, tau or beta decays, in particular super-allowed beta decays. Concerning the latter one, the master formula is [108]

\[
|V_{ud}|^2 = \frac{2984.432(3)s}{F_L(1 + \Delta_R)}, \quad (19)
\]

with \(F_L = 3072.07(63)\) and the two different sets of radiative corrections

\[
\Delta_R|_{SGPR} = 0.02467(22) \quad (109), \quad (20)
\]

\[
\Delta_R|_{CMS} = 0.02426(32) \quad (110), \quad (21)
\]

lead to

\[
V_{us}\big|_{SGPR} = 0.22782(62), \quad V_{us}\big|_{CMS} = 0.22699(78), \quad (22)
\]

where we used unitarity with \(|V_{ub}| = 0.003683\) from [111, 112], even though the precise value of \(|V_{ub}|\) is unimportant here. This has to be compared to the average of the PDG value [107] for \(V_{us}\) from Kaon and the HFLAV value [101] from inclusive tau decays: \(V_{us} = 0.2243 \pm 0.0005\) and \(V_{us} = 0.2195 \pm 0.0019\), to get

\[
V_{us}\big|_{K^{+}\tau} = 0.2240 \pm 0.0005. \quad (23)
\]

Comparing \(V_{us}\big|_{K^{+}\tau}\) with \(V_{us}\) we notice a \(\approx 3 - 5\sigma\) discrepancy which is the origin of the CAA. This situation is illustrated in Fig. 1 where the different determinations of \(V_{us}\) are compared.

Turning to NP corrections to these determinations, we have for \(V_{us}\) from semileptonic Kaon decays with muons

\[
|V_{us}^\beta| = |V_{us}^\beta|_3 \left(1 - \frac{(g^\ell - g_1^\ell)g_2^\ell M_W}{g_2^\ell}ight), \quad (24)
\]

and the determination of \(V_{us}/V_{ud}\) from \(B_K^{\tau \to \mu}\) is not modified. The same is true for its determination from \(\tau \to K\nu\tau\to\pi\nu\). For \(V_{us}\) there is, in addition to the direct modification of the transition \(d \to w\mu\) an indirect one from the modification of \(G_F\). Therefore, the element of the unitary CKM matrix in the Lagrangian

\[\text{FIG. 1. Comparison between the different determinations of } V_{us} \text{ resulting in the CAA.}\]
V^L_{us} is given in terms of the one extracted from experiment with the SM V^β_{us} as

\[ V^L_{us} \approx V^β_{us} \left( 1 + \frac{|V^L_{ud}|^2 |g_1|^2 (g^u g^d - g^u g^d) M^2_W}{|V^L_{us}|^2 g_2^2 M^2_X} \right), \quad (25) \]

Note the important enhancement of \(|V^L_{ud}|^2 / |V^L_{us}|^2 \approx 20\) [29]. This enhancement is not present in the modifications to the \(V_{us}\) determination from Kaon and tau decays. Therefore, the difference between Eq. (22) and Eq. (24) amounts to

\[ (85 \pm 17) \times 10^{-4} \approx \frac{g_1^2 (g_{12}^d - g^u g^d) M^2_W}{g_2^2 M^2_X} \quad \text{(SGPR)}, \quad (26) \]
\[ (66 \pm 20) \times 10^{-4} \approx \frac{g_1^2 (g_{12}^d - g^u g^d) M^2_W}{g_2^2 M^2_X} \quad \text{(CMS)}, \quad (27) \]

by naively averaging the errors. In the phenomenological section, we will consider the SGPR determination due to its smaller error. Nonetheless, we checked that choosing the CMS determination instead has only a marginal impact on the global fit.

D. LFU Violation

In order to assess directly the modifications with respect to the SM we define the ratios

\[ R(X) = \text{Br}[X]/\text{Br}[X]_{SM}, \quad (28) \]

such that in the limit without NP they are unity. These ratios are modified as

\[ R[\tau \rightarrow \mu \nu] = \frac{1 + \frac{g_{13}^2 (g_{12}^d - g_{12}^d)}{g_2^2} M^2_W}{M^2_W}, \quad (29) \]
\[ R[\tau \rightarrow e \nu] = \frac{1 + \frac{g_{11}^2 (g_{12}^d - g_{12}^d)}{g_2^2} M^2_W}{M^2_W}, \quad (30) \]
\[ R[\tau \rightarrow \mu \nu] = \frac{1 + \frac{g_{12}^d (g_{13}^d - g_{13}^d)}{g_2^2} M^2_W}{M^2_W}, \quad (31) \]
\[ R[\tau \rightarrow \pi \nu] = \frac{1 + \frac{g_{11}^2 (g_{12}^d - g_{12}^d)}{g_2^2} M^2_W}{M^2_W}, \quad (32) \]
\[ R[\tau \rightarrow \pi \nu] = \frac{1 + \frac{g_{12}^d (g_{13}^d - g_{13}^d)}{g_2^2} M^2_W}{M^2_W}. \quad (33) \]

by the tree-level \(W\) effects. The corresponding experimental values are given in Table II with the correlations given in Ref. [101].

E. \(b \rightarrow s\ell^+\ell^-\)

For \(b \rightarrow s\ell^+\ell^-\) our \(Z\)’s contribution is purely left-handed and given by

\[ C^{ij}_{9} = -C^{22}_{9} = \frac{-\pi^2}{e^2} \frac{g_{23}^d g_{3}^{j}}{\sqrt{2} G_F M^2_X V_{ts} V_{ts}^*}, \quad (34) \]

where \(C^{ij}_{9(10)}\) and \(C^{22}_{9(10)}\) correspond to \(C^{NP}_{9(10)e}\) and \(C^{NP}_{9(10)\mu}\) in the language of Refs. [5, 12]. For the analysis of all available \(b \rightarrow s\ell^+\ell^-\) data we use the method and program of Ref. [123] with the data set given in Ref. [122]. Since our (two dimensional) scenario with \(C^{22}_{9} = -C^{22}_{10}, C^{9}_{9} = -C^{9}_{10}\), with a pull of 5.6 \(\sigma\) with respect to the SM, was not explicitly given in Refs. [5, 12, 124] we show the corresponding preferred regions in Fig. 2.

Gauge boson mixing induces extra LFU effects

\[ C^{ij}_{9} = -C^{22}_{9} = \frac{-\pi^2}{e^2} \frac{g_{23}^d g_{3}^{j}}{\sqrt{2} G_F M^2_X V_{ts} V_{ts}^*}, \quad (35) \]

Since \(C^{ij}_{9} \approx 0\), the 3-dimensional scenario

\[ (C_{9}^{22} = -C_{10}^{22}, C_{9}^{11} = -C_{10}^{11}, C_{9}^{U}) = (36) \]

is the most general scenario for the simplified model that we can explore. A global \(b \rightarrow s\ell^+\ell^-\) fit to this structure yields a pull of 6.2 \(\sigma\), with a best fit point and 68% CL intervals of \([-1.13, -0.78, -0.82]\) and \([-1.13, -0.96, [-0.99, -0.55, [-1.04, -0.59]]\), respectively.
FIG. 2. Preferred regions (68%, 95%, 99% CL) of the two dimensional ($C_{11} = -C_{11}^{10}$, $C_{22} = -C_{22}^{10}$) scenario (blue), and the three dimensional scenario which includes $C_{10}^{10}$ in addition (green).

F. $B_s - \bar{B}_s$ Mixing

The most important constraint on $Z^\prime - b - s$ couplings, i.e. $g_{23}^b$ in Eq. (6), comes from $B_s - \bar{B}_s$ mixing where the contribution to the Hamiltonian $H_{\text{eff}} = C_1 O_1$ with $O_1 = s\gamma^\mu P_L b \times s\gamma^\mu P_L b$ is given by

$$C_1 = \frac{1}{2M_X^2} \left( \frac{g_{23}^b}{2} \right)^2 \left( 1 + \frac{\alpha_s}{4\pi} \frac{11}{3} \right),$$

including the NLO matching corrections of Ref. [125].

Note that the mixing induced effect generating $s - b - Z$ couplings can be neglected as it is a dim-8 effect. Employing the 2-loop RGE [126, 127], this leads to an effect, normalized to the SM one, of a

$$\left( \frac{g_{23}^b}{0.26 \ 10^{16} \text{TeV}} \right)^2 = 0.110 \pm 0.090$$

with the bag factor of Ref. [128] and the global fit to NP in $\delta F = 2$ observables of Ref. [129].

G. LHC bounds

In our phenomenological analysis we will consider very heavy $Z^\prime$ and $W^\prime$ bosons, which cannot be produced on-shell at the LHC. In this case bounds from the tails of di-jet and di-lepton distributions apply. This allows us to put bounds on the $Z^\prime$ couplings directly from 4-fermion operators which have the same scaling in coupling vs mass than flavour bounds and can thus be directly compared. For 2-quark-2-lepton operators the bounds related to muons and electrons are [130]

$$- \frac{4\pi}{(20\text{TeV})^2} \leq \frac{g_{22}^q g_{11}^{\ell}}{4M_X^2} \leq \frac{4\pi}{(30\text{TeV})^2}.$$  \hspace{1cm} (35)

$$- \frac{4\pi}{(24\text{TeV})^2} \leq \frac{g_{11}^q g_{11}^{\ell}}{4M_X^2} \leq \frac{4\pi}{(37\text{TeV})^2}.$$  \hspace{1cm} (36)

From Ref. [131] we find the following bound on operators involving tau leptons

$$- 10.5 \frac{M_X^2}{(10\text{TeV})^2} < g_{33}^q g_{33}^{\ell} < 0$$

Additionally, from 2-jet events we find from Ref. [132] the following approximate bound

$$|g_{22}^q|^2 \lesssim 15 \frac{M_X^2}{(10\text{TeV})^2}.$$  \hspace{1cm} (37)

H. Parity Violation

Atomic parity violation in atoms, in particular Cesium, and parity violation in electron proton scattering place limits on electron-quark interactions. Here the APV experiment [133, 134] and the QWEAK collaboration [135] report

$$-2 (2C_{1u} + C_{1d}) = 0.0719 \pm 0.0045,$$

$$-2 (188C_{1u} + 211C_{1d}) = -72.62 \pm 0.43$$

respectively, with

$$C_{1d} = 0.3419 + \frac{\sqrt{2}}{G_F} \frac{g^q g_{11}^\ell}{16M_X^2},$$

$$C_{1u} = -0.1887 - \frac{\sqrt{2}}{G_F} \frac{g^q g_{11}^\ell}{16M_X^2}.$$  \hspace{1cm} (39)

Note that our NP contribution to $C_{1d}$ and $C_{1u}$ are of equal strength but have opposite sign. This nearly avoids the APV bound and significantly weakens the QWEAK one.

IV. PHENOMENOLOGICAL ANALYSIS

Let us now combine the observables discussed in the previous sections by performing a global fit. For this purpose we implemented in HEPfit [105] all the observables testing LFU (see Table I) and the CAA (encoded

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6 Since Ref. [132] did not distinguish between charged and neutral current contributions we estimated this bound from matching their EFT with our $Z^\prime$ only. From Fig. 5 we expect our results to be unchanged by small modifications of the bound in Eq. (35).
FIG. 3. Global fit in the $g_{11} - g^9$, $g_{22} - g^9$ and $g_{33} - g^9$ planes. Even though we included the LHC measurements into our global fit, we display them as well as hatched regions to show their constraining power and to verify treating as a hard cut would not change our results.

FIG. 4. Global fit in the $g_{11} - g_{22}$ plane. One can see that the blue region from EW+LFU data overlaps with the yellow one from $b \to s\ell^+\ell^-$ data and $B_s - \bar{B}_s$ mixing at the 95% CL. Note that the overlap between the EW+LFU region (which only mildly depends on $g_{X}^{0\beta}$) and the one from $b \to s\ell^+\ell^-$ is smaller when mixing is included. However, this does not mean that the agreement with data is reduced. It is rather due to the fact that the three dimensional scenario (including mixing) agrees better with data and its best fit point is further away from the SM hypothesis.

In this subset we also included the LHC bounds for which we assumed a Gaussian distribution. However, we checked that in case a hard cut is implemented (which we will show in addition in the figures) the results only change marginally.

As discussed in Sec. II we require that the $Z'$, $W'$ couplings to quarks respect an (approximate) $U(2)$ flavor symmetry such that potentially dangerous effects in $K^0 - \bar{K}^0$ or $D^0 - \bar{D}^0$ mixing are suppressed. Furthermore, we assume that the $Z'$, $W'$ mass is above the LHC production threshold such that the previously discussed bounds apply. For concreteness, we fix the common $Z'$, $W'$ mass $M_X$ to 10 TeV. In addition, we assume the couplings to leptons to be flavour diagonal, due to the stringent bounds from LFV observables (see Sec. IIIA) and taking into account that such contributions do not interfere with the SM ones for the observables considered here. Therefore, the free parameters in our fit are $g_{11}^l$, $g_{22}^l$, $g_{33}^l$, $g^9$, $g_X$ and $g_{d3}^2$ for which we used a generously large prior of $[-10, 10]$. 

in the measurement of $V_{us}$ and performed a global fit together with the standard EW observables shown in Table I calculated by HEPfit. In the following, we will refer to this set of observables as “EW+LFU”. Furthermore, we translated the output for $b \to s\ell^+\ell^-$ data obtained with the code of Ref. [123] into a likelihood profile and included this, as well as the bound from $B_s - \bar{B}_s$ mixing, into HEPfit. With this setup we can now perform a Bayesian statistical analysis whose Markov Chain Monte Carlo (MCMC) determination of posteriors is powered by the Bayesian Analysis Toolkit (BAT) [136].

As discussed in Sec. II we require that the $Z'$, $W'$ couplings to quarks respect an (approximate) $U(2)$ flavor symmetry such that potentially dangerous effects in $K^0 - \bar{K}^0$ or $D^0 - \bar{D}^0$ mixing are suppressed. Furthermore, we assume that the $Z'$, $W'$ mass is above the LHC production threshold such that the previously discussed bounds apply. For concreteness, we fix the common $Z'$, $W'$ mass $M_X$ to 10 TeV. In addition, we assume the couplings to leptons to be flavour diagonal, due to the stringent bounds from LFV observables (see Sec. IIIA) and taking into account that such contributions do not interfere with the SM ones for the observables considered here. Therefore, the free parameters in our fit are $g_{11}^l$, $g_{22}^l$, $g_{33}^l$, $g^9$, $g_X$ and $g_{d3}^2$ for which we used a generously large prior of $[-10, 10]$. 

In this subset we also included the LHC bounds for which we assumed a Gaussian distribution. However, we checked that in case a hard cut is implemented (which we will show in addition in the figures) the results only change marginally.
Let us start with the combined fit EW+LFU where $g_{23}^d$ does not enter. For $M_X = 10$ TeV, we find $g_X = -0.352 \pm 0.381$, $g_{11} = 1.571 \pm 0.860$, $g_{22} = 1.283 \pm 1.437$, $g_{13} = 3.234 \pm 2.321$, $g^b = -2.006 \pm 0.785$ and a mirror solution obtained by switching simultaneously the sign of all couplings. The corresponding projections in the $g_{11}$, $g^b$, $g_{22}$ and $g_{13}$ planes are shown in blue in Fig. 3 and the $g_{11}$-$g_{22}$ plane in Fig. 4. Here one can see that the SM point lies outside the 95% CL regions, indicating a significantly better NP fit compared to the SM hypothesis. Furthermore, the bounds from LHC searches and parity violation experiments (hatched regions) are respected by the preferred regions.

Next we include the effect of $g_{23}^d$ which enters $b \to s\ell^+\ell^-$ transitions and $B_s - \bar{B}_s$ mixing. Here we combined both classes of observables in the $g_{11}$-$g_{22}$ plane by marginalizing over $g_{23}^d$ and $g_X$, resulting in the green region in Fig. 3. Interestingly one can see that this region overlaps significantly with the one favoured by EW+LFU data. Therefore, we can combine all data, $b \to s$ transitions and EW+LFU observables, in one fit, resulting in the red regions of Figs. 3 and 4. The corresponding best fit point are: $g_X = -0.684 \pm 0.253$, $g_{11} = 1.762 \pm 0.450$, $g_{22} = 3.791 \pm 0.613$, $g_{13} = 5.559 \pm 1.769$, $g^b = -1.221 \pm 0.526$ and we find a NP IC value of $\approx 113$ compared to $\approx 167$ within the SM. This clearly shows that our dynamical model describes data significantly better than the SM hypothesis. In particular, we find that our global fit improves the agreement with $b \to s\ell^+\ell^-$ data by $\approx 5\sigma$ compared to the SM, and that the CAA is alleviated by more than $2\sigma$ at our best fit point, further improving the posterior of $V^L_{ub}$ from $\beta$-decays has a large uncertainty such that at 68% CL all determinations of $V^L_{ub}$ can be brought into agreement.

V. CONCLUSIONS

In this article we studied a simplified model with massive vector bosons transforming as a $SU(2)_L$ triplet in the context of the CAA anomaly and the hints for NP in $b \to s\ell^+\ell^-$ data. Within our setup, these anomalies clearly cannot be addressed without affecting other observables, in particular EW precision data, ratios testing LFU, LHC bounds, parity violation experiment and $B_s - \bar{B}_s$ mixing. Therefore, assuming a $U(2)^3$ flavour symmetry in the quark sector, we performed a combined fit to five free parameters finding that the global EW+LFU fit is significantly improved. Furthermore, the preferred region of this fit overlaps with the one favoured by $b \to s\ell^+\ell^-$ data and LHC bounds as well as $B_s - \bar{B}_s$ mixing is respected. In particular, the $b \to s\ell^+\ell^-$ fit is improved by $5 - 6\sigma$ with respect to the SM while at the same time the CAA is reduced by more than $2\sigma$. This shows that our model describes data significantly better than the SM hypothesis, testified by an IC value of $\approx 113$ compared to the SM one of $\approx 167$.

Looking towards the future, our model can be test by improved measurements of LFU ratios (like $\pi \to \mu\nu/\pi \to e\nu$ at PEN [137] or $\tau \to \mu\nu\tau/\tau \to e\nu$ at BELLE II [138]), by additional data and modes for $b \to s\ell^+\ell^-$ transitions to be obtained by BELLE II [138], the LHC [139], by LHC searches with increased luminosity [140] and by $Z$-pole measurements at future colliders such as CLIC [141], ILC [142] or FCCee [143, 144]. This, together with the accurate description of current data by our dynamical model clearly motivates the construction of UV complete realizations as a very promising direction for future research.

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