BINARY YORP EFFECT AND EVOLUTION OF BINARY ASTEROIDS

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ABSTRACT

The rotation states of kilometer-sized near-Earth asteroids are known to be affected by the Yarkevsky O’Keefe–Radzievskii–Paddack (YORP) effect. In a related effect, binary YORP (BYORP), the orbital properties of a binary asteroid evolve under a radiation effect mostly acting on a tidally locked secondary. The BYORP effect can alter the orbital elements over $\sim 10^4$–$10^5$ years for a $D_p = 2$ km primary with a $D_s = 0.4$ km secondary at 1 AU. It can either separate the binary components or cause them to collide. In this paper, we devise a simple approach to calculate the YORP effect on asteroids and the BYORP effect on binaries including $J_2$ effects due to primary oblateness and the Sun. We apply this to asteroids with known shapes as well as a set of randomly generated bodies with various degrees of smoothness. We find a strong correlation between the strengths of an asteroid’s YORP and BYORP effects. Therefore, statistical knowledge of one could be used to estimate the effect of the other. We show that the action of BYORP preferentially shrinks rather than expands the binary orbit and that YORP preferentially slows down asteroids. This conclusion holds for the two extremes of thermal conductivities studied in this work and the assumption that the asteroid reaches a stable point, but may break down for moderate thermal conductivity. The YORP and BYORP effects are shown to be smaller than could be naively expected due to near cancellation of the effects at small scales. Taking this near cancellation into account, a simple order-of-magnitude estimate of the YORP and BYORP effects as a function of the sizes and smoothness of the bodies is calculated. Finally, we provide a simple proof showing that there is no secular effect due to absorption of radiation in BYORP.

Key words: minor planets, asteroids: general – planets and satellites: dynamical evolution and stability

Online-only material: color figures

1. INTRODUCTION

When near-Earth asteroids (NEAs) orbit around the Sun, they are constantly subjected to the Sun’s radiation. In equilibrium, the total energy absorbed by the asteroid must be re-emitted. Yet, asymmetry in the asteroid’s geometry results in a residual force that tends to change its motion (Rubincam 2000). This residual force, called the Yarkevsky O’Keefe–Radzievskii–Paddack (YORP) effect, can significantly change the spin rate of kilometer-sized asteroids at a distance of 1 AU from the Sun after $10^3$–$10^4$ years. The YORP effect has been successfully measured for several NEAs (Taylor et al. 2007; Kaasalainen et al. 2008; Durech et al. 2008). The high abundance of fast rotators in the NEA family (Pravec et al. 2008) might be explained by the YORP effect. A rubble-pile NEA might undergo fission if the YORP effect accelerates it beyond its rotational breakup velocity (Vokrouhlický & Čapek 2002); this mechanism might explain the formation of binary NEAs.

The differential acceleration between the two components of the binary is mostly due to the acceleration of the secondary due to its larger surface-to-volume ratio. A coherent effect also requires at least one of the components to be tidally locked, which is more common for the secondary. The net radiation force acting on the secondary will produce a torque relative to the primary. This binary YORP (BYORP) effect, first suggested by Čuk & Burns (2005), evolves the orbit of the binary on fairly fast timescales ($\sim 10^5$ years for a $D_p = 2$ km primary with a $D_s = 0.4$ km at 1 AU separated by $a = 1.5D_p$). The spin rate and the obliquity of the asteroid evolve due to the YORP effect. Similarly, the semimajor axis of the binary and its inclination relative to the orbital plane around the Sun evolve due to the BYORP effect. In addition, the BYORP effect changes the eccentricity vector of the binary. We show that there are preferred end states for both effects. YORP tends to slow the spin rates, while BYORP tends to shrink the semimajor axis in the binary case.

In Section 2, we introduce our model method and assumptions. In Section 3, we show that neighboring areas on the surface of the asteroid tend to have opposing effects. We provide order-of-magnitude estimates of the YORP effect as a function of the sizes of the body and its smoothness. A method of precise calculation, as well as accounting for thermal lag due to finite thermal inertia, is shown in Section 4 and the results are discussed in Section 5. Analogous results for the BYORP effect are derived in Section 6.

2. STRUCTURE MODELS AND COORDINATE SYSTEMS

2.1. Modeling Method

We model the asteroids by means of tessellation, where the asteroid’s surface is described by a set of triangles. We neglect the effect of shadows cast by one facet over another and we assume that the emission from each point on the surface of the asteroid is isotropic. Since the orbital period of the asteroid around the Sun is much longer than its rotation time, it is assumed that there are no resonances between the asteroid’s orbit around the Sun and its revolution around itself. Constant density is assumed, although the possible effects of density non-uniformity will be briefly addressed.

2.2. Coordinate Systems

Two coordinate systems will be used throughout this paper.

1. An inertial frame with axes labeled $x$, $y$, $z$. The $z$-axis is perpendicular to the orbital plane of the asteroid around the Sun.astronomical Journal, 141:55 (10pp), 2011 February

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The x-axis is chosen to coincide with the projection of the asteroid’s spin vector on the xy plane. The origin is chosen to be the Sun. This system will be referred to as the inertial system.

2. The principal axes of the asteroid, labeled \(x', y', z'\), where \(z'\) is parallel to the spin vector. This system will be referred to as the asteroid system.

3. SCALING OF THE RADIATIVE TORQUE

In this section, we discuss the net acceleration and total torque that arises from the emission of radiation by a body of arbitrary shape. From symmetry, spherical bodies do not exhibit torque or acceleration, and we derive here the general scaling of the torque and acceleration as a function of the roughness of the body, or its deviations from sphericity. We treat only the effect that arises from the emission of radiation since Rubincam & Paddack (2010) have shown that there is zero secular change that arises from the emission of radiation by a body of arbitrary density, and \(v = \frac{4\pi cd}{3}\).

Here, \(\Phi\) is the solar radiation momentum flux given by

\[
\Phi = \frac{L_\odot}{4\pi d^2\sqrt{1 - e^2}}.
\]

(2)

Here, \(L_\odot\) is the solar luminosity, \(e\) is the eccentricity, \(d\) is the speed of light, \(d\) is the orbital semimajor axis, \(\rho\) is the density, and \(R\) is the length scale of the asteroid. The eccentricity dependence arises from averaging the torque over a heliocentric orbit.

We construct simple models to account for the asymmetry of the asteroid. We choose \(n\) points randomly distributed on the unit sphere and connect them to create a tessellation of small triangles that encloses the asteroid (based on the Quickhull algorithm, Barber et al. 1996, which produces 2 \(n\) triangular facets for a given \(n\)). This method of construction eliminates shadowing of one facet over another. For this body, we now calculate the radiation effects, and their scaling with \(n\) or with the deviation of the shape of the body from a sphere. In the estimates below, we assume \(n \gg 1\).

We define the deviation of the asteroid from a sphere with the same volume by comparing the normalized difference in their surface areas:

\[
d_s = \frac{S - 4\pi r^2}{4\pi r^2},
\]

(3)

where \(4\pi r^3/3\) is the volume of the asteroid and \(S\) is its surface area. This definition of spherical deviation is shown to be \(d_s \propto n^{-1}\). In order to simplify the derivation, we will assume that all of the facets are equilateral and that the center of mass (CM) is located at the origin. If we connect each one of the facets to the CM and thereby create a tetrahedron, the relation between \(\theta\), which is the vertex angle of a tetrahedron face, and \(\Omega \approx 2\pi/n\), the solid angle that the tetrahedron subtends, can be found by making use of L’Huillier’s theorem:

\[
\theta^2 = \frac{4\Omega}{\sqrt{3}}.
\]

(4)

The area of each facet is

\[
S_j = \frac{\sqrt{3}}{4} R^2 \sin^2 \left(\frac{\theta}{2}\right) \approx \frac{\sqrt{3}}{4} R^2 \left(\theta - \frac{\theta^3}{12}\right).
\]

(5)

where \(R\) is the radius of the sphere that covers the asteroid. The height of the tetrahedron is

\[
h \approx R \left(1 - \frac{\theta^2}{6}\right).
\]

(6)

The volume that is enclosed between the facet and the sphere is

\[
V \approx \frac{R^3\Omega - S \cdot h}{3} \approx \frac{\sqrt{3} R^3 \theta^4}{48} = \frac{R^3 \Omega^2}{3\sqrt{3}}.
\]

(7)

The total difference in the volume between the asteroid and the sphere is roughly \(2n \cdot V\):

\[
\frac{4\pi}{3} R^3 - \frac{4\pi}{3} r^3 \approx \frac{8\pi^2 R^3}{3\sqrt{3}n}.
\]

(8)

The ratio of the radii is

\[
\frac{r}{R} \approx 1 - \frac{2\pi}{3\sqrt{3}n}.
\]

(9)

By equating the volume of the asteroid with the volume of a sphere with radius \(r\), we have

\[
\frac{h}{3} \sum_{j=1}^n S_j = \frac{4\pi r^3}{3}
\]

(10)

and Equations (9) and (6) yield

\[
S = \sum_{j=1}^n S_j \approx 4\pi r^2 \left(1 + \frac{2\pi}{3\sqrt{3}n}\right).
\]

(11)

Therefore, we obtain

\[
d_s \propto \frac{1}{n}.
\]

(12)

The angle, \(\gamma\), between the normal of a non-equilateral facet and the vector joining the sphere’s center to the facet’s centroid, is of order \(\theta\), so

\[
\gamma \propto \frac{1}{\sqrt{n}}.
\]

(13)

The torque that is produced by a single facet is

\[
\tau_j \propto S_j \gamma_j \propto n^{-3}.
\]

(14)

Naively, we might expect that the total torque is a random walk summation over the individual facets, and therefore

\[
\tau_{\text{naive}} \approx \sqrt{n} \tau_j \propto \frac{1}{n}.
\]

(15)
However, the torques from neighboring facets are not independent. This dependence leads to a partial cancellation of the torque produced by neighboring facets that share a point. The torque is given by

$$\mathbf{r} \propto (\mathbf{a} + \mathbf{b} + \mathbf{c}) \times ((\mathbf{b} \times \mathbf{a} + \mathbf{b} + \mathbf{a} + \mathbf{c}) \cdot \cos \lambda), \quad (16)$$

where $\lambda$ is the angle between the normal to the facet and the Sun's flux and $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors from the origin to the vertices located on the surface of the sphere. Consider a displacement of a single vertex, either radially or tangentially on the surface of the sphere, by an amount that would create a change of order unity in the torque; then the difference between the initial torque and the torque after displacing $\mathbf{a}$ by an amount $d\mathbf{a}$ is

$$d\mathbf{a} \times ((\mathbf{b} \times \mathbf{a} + \mathbf{b} + \mathbf{a} + \mathbf{c}) \cdot \cos \lambda)$$

$$+ (a + b + c) \times ((b - c) \times d\mathbf{a}) \cdot \cos \lambda$$

$$+ (a + b + c) \times ((b \times a + c \times b + a + c) \cdot d(\cos \lambda)). \quad (17)$$

If we displace $\mathbf{a}$ by an angle $|d\mathbf{a}| \sim n^{-\frac{1}{2}}$, then the first two terms in Equation (17) scale as $O(n^{-\frac{1}{2}})$, while the third term scales as $O(n^{-\frac{1}{2}})$. Likewise, if we displace $\mathbf{a}$ radially by $|d\mathbf{a}| \sim n^{-1}$ the second term will scale as $O(n^{-\frac{1}{2}})$, while the first and third terms will scale as $O(n^{-2})$.

Both the radial and tangential displacements of the vertex $\mathbf{a}$ cause a change in the torque of order unity ($O(n^{-\frac{1}{2}})$). However, the same vertex $\mathbf{a}$ is shared by several neighboring tetrahedrons. Summing the changes in the torque over all of the facets which share the vertex $\mathbf{a}$, we find that all contributions up to $O(n^{-\frac{1}{2}})$ cancel out and we are left with a contribution of $O(n^{-2})$. The effective contribution of each such group will therefore scale as $O(n^{-2})$. A random walk process will now give

$$\mathbf{r} \propto d\mathbf{a}^2. \quad (18)$$

Following Goldreich & Sari (2009, hereafter GS), we define $f_Y$ to be the ratio between the actual torque and the torque that would be exerted if all of the received radiation were emitted tangentially from the body's equator:

$$f_Y = \frac{2}{3} \pi R^3 \Phi f_Y, \quad (19)$$

where the $2/3$ arises from assuming isotropic emission. We therefore find

$$f_Y \sim d\mathbf{a}^2. \quad (20)$$

We have shown that the total YORP effect is comparable to the effect of a single facet. Therefore, the uncertainties in the asteroid's shape will induce errors in the torque estimate by an amount

$$\frac{\Delta \tau}{\tau} \approx \frac{\Delta R}{R(N)} \approx \frac{\Delta R}{d_3 R}, \quad (21)$$

where $\Delta R$ is the physical length scale of the modeling error and $R$ is the length scale of the asteroid.

4. DETAILED CALCULATION AND THERMAL LAG

In order to compute the total average torque, we first compute the average torque arising from each facet's emission independently. Let $\hat{n}$ be the unit vector pointing from the facet to the Sun, $d\mathbf{s}$ the vector perpendicular to the facet with a magnitude equal to its area, and $\mathbf{r}$ the vector pointing from the asteroid's CM to the centroid of the facet; then the torque of each facet due to Lambertian reflection while neglecting specular reflection is

$$\mathbf{r}_{\text{reflect}} = -\frac{2}{3} \frac{A \Phi d_3^2}{D^2} (\hat{n} \cdot d\mathbf{s}) (\mathbf{r} \times d\mathbf{s}), \quad (22)$$

where $D$ is the distance between the Sun and the asteroid and $A$ is the asteroid's albedo, which is assumed to be constant. Thermal lag due to a finite thermal conductivity, $\kappa$, might influence the obliquity change rate (it can also reverse sign in extreme cases) but it hardly affects the spin change rate (Čapek & Vokrouhlický 2004). The effect of thermal lag can be estimated by having some constant temperature per facet, $T_{eq}$, and a time-varying component, $\Delta T$, which lags in time relative to the solar insolation. A facet with a constant temperature produces a torque which is a constant in the asteroid frame. Averaging of this torque over the asteroid's revolution around itself leaves us only with the projection of the torque on the spin axis. Therefore, the contribution of the constant temperature to the obliquity term vanishes. The spin component has no preference regarding the phase at which the emission takes place, hence the spin component is unaffected by thermal conductivity.

As the thermal conductivity increases, the ratio $T_{eq}/\Delta T$ increases (Rubincam 1995). For a typical asteroid, if $\kappa \cdot s \gtrsim 6 \times 10^{-5} \text{Wm}^{-1}\text{s}^{-1}\text{K}^{-1}$ then the equilibrium temperature dominates over the time-varying component. Recent studies have shown that some asteroids could have high enough thermal conductivity (Opeil et al. 2010). For these asteroids, one can neglect the obliquity term due to thermal re-emission. In this paper, we explore two extremes which bound the possible behaviors: the first is the Rubincam approximation ($\kappa = 0$) and the second is the high thermal conductivity regime.

The torque due to thermal re-emission is

$$\mathbf{r}_{\text{emission}} = -\frac{2}{3} \frac{(1 - A) \Phi d_3^2}{D^2} (\hat{n} \cdot d\mathbf{s}) (\mathbf{r} \times d\mathbf{s}). \quad (23)$$

For the high thermal conductivity regime, we evaluate only the spin component of this torque.

The general case of non-zero obliquity requires averaging the torque over both (1) the orbit of the asteroid around the Sun and (2) a revolution of the asteroid around itself.

For a zero obliquity orbit, it is sufficient to average over either of these. Since we assumed that the spin period and the orbital period are non-commensurate, we can calculate these averages to arbitrary order. We find that it is more convenient to first average over the orbit around the Sun while holding each facet pointing in a fixed direction.

4.1. Heliocentric Orbit Average

The insolation, averaged over the orbit of the asteroid around the Sun, is given by (Ward 1974)

$$\langle I \rangle = \frac{1}{T} \int_0^T \frac{L}{4\pi c D^2} \hat{n} \cdot d\mathbf{s} \, dt = \frac{\Phi \sin(\theta)}{\pi}, \quad (24)$$

where $T$ is the heliocentric orbital period and $\theta$ is the angle between $d\mathbf{s}$ and the normal to the orbital plane.

4.2. Asteroid Spin Averaging

Since we are interested in calculating the torque in the inertial system (system 1), we need to transform the torque from the
asteroid system (system 3) to the inertial system. In the asteroid system, the torque per facet is

\[ \mathbf{r}'_j = \mathbf{r}'\text{reflection}_j + \mathbf{r}'\text{emission}_j = -\mathbf{r}' \times \frac{2\mathbf{S}_j}{3}(I)\mathbf{d}s' \]

\[ = -\frac{2\mathbf{S}_j}{3}(I)\left(\frac{\psi'}{\sin(\theta'_j/\cos(\theta'_j) \sin(\theta'_j))} \right) \sin(\theta'_j/\cos(\theta'_j) \sin(\theta'_j)) \sin(\theta'_j/\cos(\theta'_j) \sin(\theta'_j)). \]  

(25)

where \( \mathbf{S}_j \) is the facet’s area and \( \theta'_j, \phi'_j \) are the polar and azimuthal angles accordingly. The torque is then transformed into system 1 with the standard Euler angle rotation matrix:

\[ \mathbf{R}(\psi) = \begin{pmatrix} -\cos \epsilon & \sin \psi & -\epsilon \\ -\sin \epsilon & \cos \psi & 0 \\ \epsilon & 0 & \cos \epsilon \end{pmatrix}. \]  

(26)

where \( \psi \) is the rotation angle of the asteroid around itself and \( \epsilon \) is the obliquity. For \( \psi = 0 \) the asteroid system’s \( \mathbf{x}' \)-axis coincides with the \( \gamma \)-axis of the inertial system.

The insolation in system 1 is

\[ \Phi \sin(\theta_j) = \frac{\Phi \sin(\cos^{-1}(\mathbf{R} \cdot \mathbf{d}s' \cdot \hat{\mathbf{z}}))}{\sin(\theta'_j)} 

= \Phi[(\cos(\psi + \phi'_j) \sin(\theta'_j))^2 

+ \cos\epsilon \sin(\psi + \phi'_j) \sin(\theta'_j)^2 \frac{\psi}{\sin(\theta'_j)}]. \]  

(27)

The averaged torque in the inertial system can now be rewritten as

\[ \mathbf{r}_j = -\frac{\mathbf{S}_j \Phi}{3\pi^2} \int_0^{2\pi} \sin(\theta_j) (\mathbf{R} \cdot \mathbf{r}_j) d\psi. \]  

(28)

In order to calculate the torque, we define the following numerical functions:

\[ B(\theta', \epsilon) \equiv \frac{1}{2\pi^2} \int_0^{2\pi} [(\cos \psi \sin \theta')^2 

+ (\cos \theta' \sin \epsilon - \cos \epsilon \sin \psi \sin \theta')^2] d\psi. \]  

(29)

and

\[ G(\theta', \epsilon) \equiv \frac{1}{2\pi^2} \int_0^{2\pi} \sin \psi [(\cos \psi \sin \theta')^2 

+ (\cos \theta' \sin \epsilon - \cos \epsilon \sin \psi \sin \theta')^2] d\psi. \]  

(30)

Note that the physical interpretation of the function \( B \) is simply the average insolation a facet receives at a given angle \( \theta' \). This result is similar to that derived in Ward (1974) and Nesvorný & Vokrouhlický (2007). Figure 1 shows \( B \) and \( G \) as a function of \( \theta' \) for various values of \( \epsilon \).

Rewriting Equation (28) with Equations (29) and (30) and using Equations (26) and (25), we obtain by summing over all of the facets \( \mathbf{d}s_j \) (assuming \( n \) facets)

\[ \tau_j = -\sum_{j=1}^n \frac{2\mathbf{S}_j}{3} B(\theta'_j, \epsilon) \sin \epsilon \sin(\psi'_j \sin \phi'_j - \psi'_j \cos \phi'_j) \]

\[ + G(\theta'_j, \epsilon) \cos \epsilon \sin(\psi'_j \sin \phi'_j - \psi'_j \cos \phi'_j) \]  

(31)

We have so far treated the asteroid as having homogeneous density, so that the CM is at the geometric center. Inhomogeneous density would result in a displacement of the CM from the geometric center and can easily be treated by adding the displacement term, a vector pointing from the geometric CM toward the true CM, to \( \mathbf{r}' \) in Equation (25).

4.3. Spin Evolution

In the inertial system the unit spin vector is

\[ \hat{s} = \begin{pmatrix} \sin \epsilon \\ 0 \\ \cos \epsilon \end{pmatrix}. \]  

(32)
The rate of change of the spin vector’s magnitude is

\[
\dot{s} = \frac{\tau \cdot \dot{s}}{C},
\]

where \( C \) is the asteroid’s moment of inertia. Substituting Equation (31) into Equation (33), we obtain

\[
\langle \dot{s} \rangle = -\sum_{j=1}^{n} \frac{2S_j \Phi B(\theta'_j, \epsilon) \sin \theta'_j}{3C} (x'_j \sin \phi'_j - y'_j \cos \phi'_j). \tag{34}
\]

This result depends only on \( B \) since the torque along the \( \zeta' \)-axis is the only vector that is fixed in all coordinate systems. It has the simple physical interpretation as the average insolation time due to the torque that each facet contributes. Thermal conductivity has no effect on the spin evolution. An interesting result is that when the obliquity is approximately \( \epsilon \approx 55^\circ \), \( B \) is nearly a constant with respect to \( \theta' \), and as a result, \( \dot{s} \) tends to vanish due to the same argument that was used to show that \( \dot{s} \) vanishes due to absorption (see Rubincam & Paddack 2010). A clue as to why \( B \) is roughly \( \theta' \) independent for \( \epsilon \approx 55^\circ \) might be found in the fact that a related expression

\[
\int_0^{2\pi} (\cos \psi \sin \theta')^2 + (\cos \theta' \sin \epsilon - \cos \epsilon \sin \psi \sin \theta')^2 d\psi
\]

is independent of \( \theta' \) for \( \cos 2\epsilon = -1/3 \). This result coincides with the value calculated by Nesvorný & Vokrouhlický (2007). Note, however, the critical obliquity, \( \epsilon \approx 55^\circ \), is by no means a stable point. Even though \( \dot{s} = 0 \), the obliquity will evolve away from the critical obliquity (see Section 4.4). Our calculations were tested by computing the YORP effect on Kleopatra and 1998KY26, presented in Figure 2, and comparing it with the results of Čapek & Vokrouhlický (2004) in Figure 3 of their paper for 1998KY26. Our results are about 5% smaller than theirs.

### 4.4. Obliquity Evolution

The obliquity change rate can be derived as follows. From Equation (32), we have

\[
\cos \epsilon = \frac{s \cdot \hat{z}}{s}.
\]

Differentiating with respect to time, we obtain

\[
-\dot{\epsilon} \sin \epsilon = \frac{\dot{s} \cdot \hat{z}}{s} - \frac{(s \cdot \hat{z}) \dot{s}}{s^2},
\]

and rearranging yields

\[
\dot{\epsilon} = \frac{\tau \cos \epsilon - \hat{z}}{C s \sin \epsilon} = \frac{\tau_x \cos \epsilon - \tau_z \sin \epsilon}{C s}.
\]

Substituting in Equation (23), we get

\[
\langle \dot{\epsilon} \rangle = -\sum_{j=1}^{n} \frac{2S_j \Phi \Theta(\theta'_j, \epsilon) \cos \theta'_j}{3Cs} \left( x'_j \sin \phi'_j - y'_j \cos \phi'_j \right) \Theta,
\]

where \( \Theta = 1 \) for the Rubincam approximation and \( \Theta = A \) for the high thermal conductivity regime. Just as the function \( B \) governs the spin’s evolution, \( \Theta \) governs the obliquity’s evolution.

### 4.5. Comparison of Qualitative with Precise Results

In order to check our scaling results of Section 3, 2500 random bodies were constructed in the manner described in Section 3. For each body, we computed \( f_Y \) and \( d_s \). Figure 3 shows \( f_Y \) for the randomly constructed bodies as a function of spherical deviation, \( d_s \), along with the best fit for Equation (20). In addition, \( f_Y \) for 18 real asteroids (their shape was taken from ECHO JPL) was computed using our algorithm and plotted together with the two measured YORP for comparison. For most of the asteroids, we can see a good correlation between our qualitative results and our precise calculations, although our random asteroids tend to have a higher \( f_Y \). The ratio between our calculated results and the measured YORP for 1620 Geographos and 2000 PH5 are 2.18 and 5.22, respectively. Taking the mean of the real asteroids, we find that \( f_Y \approx 2.8 \times 10^{-3} \). Re-examining the order-of-magnitude approach that was taken by GS, we can see that their choice of \( f_Y (6 \times 10^{-3}) \) is somewhat low, but still within an acceptable range.
having a spin change rate that is positive for \( \epsilon > 0 \) because every asteroid, regardless of its shape, has at least two equilibrium points with regard to the change in the obliquity since for every asteroid, regardless of its shape, has at least two equilibrium points and a spin change rate that is positive for \( \epsilon > 0 \). A Type I asteroid will evolve toward \( \epsilon = \pi / 2 \) where its spin change rate is negative. Note that once the spin rate reaches zero, the asteroid does not simply change its obliquity from \( \epsilon \) to \( \pi - \epsilon \) since that would also involve inverting the \( z' \)-axis. Rather, the obliquity remains fixed while \( G \) changes its sign. This causes the equilibrium point to lose its stability and results in the asteroid evolving to its other equilibrium point where the spin will once again slow down.

A viable explanation for this phenomena can be found by inspecting the stability at \( \epsilon = 0 \):

\[
\frac{d\dot{\epsilon}}{d\epsilon}_{\epsilon=0} = \sum_{j=1}^{n} \frac{\Phi}{3\pi C_{s}} \cos^{2}\theta_{j}'(S_{j}x_{j}'\sin\phi_{j}' - S_{j}y_{j}'\cos\phi_{j}').
\]

(40)

The spin change rate with zero obliquity is

\[
\dot{s}(\epsilon = 0) = -\sum_{n=1}^{N-1} (x_{n+1} - x_{n}) \sin^{2}\theta_{n}',
\]

(42)

and for the obliquity stability

\[
\frac{d\dot{\epsilon}}{d\epsilon}_{\epsilon=0} = \sum_{n=1}^{N-1} (x_{n+1} - x_{n}) \cos \theta_{n}',
\]

(43)

where \( \theta' \) is a randomly drawn vector of length \( N - 1 \) ranging from \( 0 \) to \( \pi \) and sorted in an increasing manner. Since in asteroids we have

\[
\sum_{j=1}^{n} S_{j} \sin \theta_{j}'(x_{j}' \sin \phi_{j}' - y_{j}' \cos \phi_{j}') = 0,
\]

(44)

we set \( x_{N} - x_{N-1} = \sum_{n=1}^{N-2} (x_{n+1} - x_{n}) \sin \theta_{n}' / \sin \theta_{N-1}' \).

The sign of \( \dot{\epsilon} / d\epsilon_{\epsilon=0} \cdot \dot{s}(\epsilon = 0) \) for our simple model has a likelihood of 85% being positive, while the likelihood of \( \dot{\epsilon} / d\epsilon_{\epsilon=0} \cdot \dot{s}(\epsilon = 0) \) for our randomly constructed asteroids being positive is 88% (it is more than 78% because it accounts for cases with more than only two stable obliquity points). The strong tendency of asteroids to spin down is merely a result of the correlations between neighboring facets. Asteroids with moderate thermal conductivity might exhibit different obliquity behavior which is beyond the scope of this work (Čapek & Vokrouhlický 2004).

When the spin of the asteroid is sufficiently low, the asteroid might fall into a chaotic tumbling rotation state and it is not clear if the asteroid can recover principal axis rotation...
(Vokrouhlicky et al. 2007). If the asteroid is a rubble pile, the spin-up might eventually break up the asteroid due to centrifugal force. However, rubble-pile asteroid deformations are not well understood (Holsapple 2010; Scheeres 2009; Walsh et al. 2008). Inspection of the shape of binary 1999KW4 (Ostro et al. 2006) suggests that break up is possible.

6. BYORP

The BYORP effect is very similar to the YORP effect so the understanding of YORP extends into BYORP. We will assume that the secondary is synchronously locked with the primary, and that the secondary’s spin vector is aligned with the orbital angular momentum. This assumption is reasonable due to the strong tidal interactions between the binary components. The primary is assumed not to be locked with the secondary (i.e., not a double synchronous state), and thus the evolution of the binary orbit does not depend on the orientation of the primary. In the following discussion, the origin of the asteroid system (system 3) will be at the CM of the secondary asteroid with its smallest principal axis, labeled the $x'$-axis, pointing away from the primary. The inertial system’s (system 1) origin will now be located at the CM of the primary, the $z$-axis aligned with the orbital angular momentum, and the $x$-axis pointing toward the periapsis of the binary’s orbit. For the special case where the orbit is circular, we define the $x$-axis to point along the line of the ascending node. The angle $\psi$ is defined relative to the line of nodes where our plane of reference is the orbit of the primary around the Sun. In order to be consistent with our previous definitions, the rotation angle $\psi$ is now defined as $\psi = \lambda + w$, where $\lambda$ is the mean anomaly and $w$ is the argument of the periapsis. In addition, the role of the inclination of the binary orbit relative to the primary’s orbit around the Sun in BYORP takes the role of the obliquity in YORP, $\epsilon \leftrightarrow i$.

In YORP, the main interest is to find the evolution of the obliquity and spin. In BYORP, we are interested in the evolution of the binary orbit’s inclination relative to the orbit of the primary around the Sun, the evolution of the binary semimajor axis, and the evolution of the binary’s eccentricity. Since the secondary is assumed to be locked, changes in the orbital spin vector correspond to a change in the semimajor axis, while the change in the inclination corresponds to the change in obliquity in YORP. There is no analog for the eccentricity change in YORP.

Our derivation of BYORP neglects the distance of the secondary’s facets from its CM and we assume that all of the facets are located at the CM of the secondary. This is justified since in typical cases the error in the torque due to errors in the facets’ locations will be at least an order of magnitude less than the actual torque.

Just as in YORP, we will inspect the same two extremes of very high and very low thermal conductivity. In the following section, the eccentricity of the orbit of the secondary around the primary will be denoted $\epsilon$, to distinguish it from the eccentricity of the binary orbit around the Sun, $e$.

Absorption of radiation leads to no secular changes in the semimajor axis, eccentricity, and inclination of the binary. This can be shown using a similar argument as in Rubincam & Paddack (2010). Fixing the binary’s orbital elements, the torque at a given point along the heliocentric orbit cancels the torque arising at a point with the true anomaly of the heliocentric orbit changed by 180°, since the radiation force changes sign. This is true for any point along the orbit, so the net effect of absorption vanishes.

6.1. Semimajor Axis Evolution

The change in the energy of a binary orbit. To zero order in the eccentricity the change is

$$\frac{1}{a} \dot{a} = \frac{2}{na} F_t,$$  

(45)

where $a$ is the binary’s semimajor axis, $n$ is the mean orbital angular velocity, and $F_t$ is the transverse orbital force per unit mass arising due to BYORP, positive in the direction of movement. In order to compute $\langle \dot{a} \rangle$ we define two rotation matrices. The first, $\mathbf{R}_t$, describes a rotation around the $z$-axis in the inertial frame by an angle $f + w$, where $f$ is the true anomaly. The second rotation matrix, $\mathbf{R}_s$, describes a rotation around the $z$-axis by an angle $\psi$. With our choice of coordinates

$$F_t = -\sum_{j=1}^{n} \frac{2 S_j}{3 M_s} \cdot \langle I \rangle (\mathbf{R}_t \cdot \mathbf{d}'_j) \cdot (\mathbf{R}_t \cdot \mathbf{\hat{y}}'),$$

(46)

where $M_s$ is the secondary’s mass. By first averaging over the heliocentric orbit and then averaging over the binary orbit, we obtain

$$\langle F_t \rangle = -\sum_{j=1}^{n} \frac{2 S_j}{3 M_s} B(\theta_j', f) \Phi \sin \theta_j' \sin \phi_j'. $$

(47)

This result is intuitive since the expression merely sums the transverse force from each facet. Equation (47) resembles the YORP spin change rate in Equation (34). Just as the spin change rate for YORP was unaffected by thermal lag, the semimajor axis change rate is also unaffected by thermal lag. When $i \approx 55^\circ$, $\langle \dot{a} \rangle$ vanishes for the same reasons that $\dot{\epsilon}$ vanishes. We define $f_{BY}$, similar to $f_Y$, to be the ratio between the BYORP actual effect and the effect it would have on the semimajor change rate if all of the received radiation had been emitted tangentially from the secondary. Therefore, the torque can be rewritten as

$$\tau = \frac{2}{3} \pi a R^2 f_{BY} \Phi.$$  

(48)

In the derivation in Section 3, the angle between the force and the lever that produced the torque scaled as $O(d_s^2)$. However, in BYORP there is no correlation between the force and the lever so we would expect

$$f_{BY} \sim d_s \sim f_Y^2.$$  

(49)

Fitting the ratio between the coefficients with $f_{BY} = c \cdot f_Y^2$, we find that the ratio for our randomly constructed bodies satisfies

$$f_{BY} = 0.43 f_Y^2.$$  

(50)

The 2σ dispersion around the best-fit number has an upper limit of $f_{BY} = 1.85 f_Y^2$ and a lower limit of $f_{BY} = 0.1 f_Y^2$. Figure 4 shows the ratio for our random asteroids as well as for our real asteroid models. Taking the mean of the real asteroid’s $f_{BY}$ yields

$$f_{BY} \approx 0.01.$$  

(51)

The uncertainties in the asteroid’s shape will induce errors in the torque estimate by an amount

$$\frac{\Delta \tau}{\tau} \approx \frac{2 \Delta R}{3 (R_e/N)} \approx \frac{2 \Delta R}{3 d_s R_e},$$  

(52)
where $\Delta R$ is the physical length scale of the modeling error, $R_s$ is the length scale of the secondary, and the factor $2/3$ arises from the relation of YORP to BYORP strength.

### 6.2. Inclination Evolution

#### 6.2.1. Zero Order

The evolution of the inclination can be calculated from (see, e.g., Burns 1976)

$$\frac{d\theta_i}{dt} = \left[d\mu^{-1}(1 - \bar{e}^2)\right]^{1/2} F_N \cos(f + w)/(1 + \bar{e} \cos f), \quad (53)$$

where $F_N$ is the normal component of the BYORP force per unit mass, relative to the binary orbit:

$$F_N = -\sum_{j=1}^{n} \frac{2S_j}{3M_{\mu}} \cdot \{I(I_{\lambda} \cdot \hat{d}s_j') \cdot (\hat{R}_f \cdot \hat{d}z'). (54)$$

Due to the quadrupole moment of the primary, characterized by its $J_2$, the angular momentum of the secondary will be coupled to the angular momentum of the primary (Cuč & Nesvorný 2010). This interaction causes the orbit of the secondary to rapidly precess, typically at timescales of a few months. This coupling requires us to modify the inclination’s evolution rate by a factor $\beta$, the ratio of the secondary’s orbital angular momentum relative to the total angular momentum (which may be dominated by the primary’s spin). In order to take into account the rapid precession, an averaging of the argument of the periastris is required. No averaging of the line of ascending node is necessary since it does not enter our equations.

The Sun will also cause the binary orbit to precess. However, since the timescale for this interaction is of the order of hundreds of years, the angular momentum of the secondary’s orbit will stay aligned with the primary’s angular momentum. This precession will introduce no additional change in our equations since the heliocentric line of nodes does not enter our equations.

The average inclination change rate, up to zero order with respect to the eccentricity is

$$\left\langle \frac{d\theta_i}{dt} \right\rangle = -\sum_{j=1}^{n} \frac{2S_j G(\theta_j', i) \Phi \cos \theta_j' \sin \phi_j', (55)}{3M_{\mu} a} \Theta \beta \cos \theta_j' \sin \phi_j,'$$

where we added the $\Theta$ term just as in the case for the obliquity evolution. This result resembles Equation (39).

#### 6.2.2. First Order

In order to simplify our calculations we expand $f$ to first order in the eccentricity, $f \approx \lambda + 2\bar{e}\sin\lambda$. To $O(\bar{e})$ order in eccentricity, the contribution is

$$\left\langle \frac{d\theta_i}{dt} \right\rangle = \sum_{j=1}^{n} \frac{S_j \bar{e} \Phi}{3M_{\mu} a} \cos \theta_j' [3B(\theta_j', i) \cos w - \Theta H(\theta_j', i) \times \cos(2\phi_j' + w)], \quad (56)$$

where the function $H$ is defined to be

$$H(\theta, i) \equiv \frac{1}{2\bar{e}^2} \int_{0}^{2\pi} \{\cos 2\psi(\cos \psi \sin \theta)^2$$

$$+(\cos \theta \sin \psi - \cos \psi \sin \theta \sin \theta)^2\} \frac{d\psi}{\bar{e}}. \quad (57)$$

Averaging over the precession of the argument of the periastris cancels the first-order correction due to eccentricity.

The reasoning presented in Section 5 leads us to expect a correlation between the change in the inclination and the change in the semimajor axis.

### 6.3. Eccentricity Evolution

Unlike the change in the semimajor axis and inclination, which have their analogies in YORP, there is no analogy for the eccentricity evolution. The eccentricity vector is defined as

$$\bar{e} = \frac{v \times h}{\mu} - \hat{r}, \quad (58)$$

where $h$ is the binary’s orbital angular momentum per unit mass, $\mu \equiv GM_{\mu}$, and $v$ is the orbital velocity of the secondary. Since in an unperturbed orbit this vector is conserved, its time derivative is

$$\bar{e} = \frac{F_{BYORP} \times h}{\mu} + \frac{v \times \tau_{BYORP}}{\mu}, \quad (59)$$

where $F_{BYORP}$ is the force that arises from BYORP and $\tau_{BYORP}$ is the torque that is produced by BYORP. The radius vector $r'$ in the asteroid system is

$$r' = \begin{pmatrix} a(1-\bar{e}^2) \\ 1+\bar{e}\cos f \\ 0 \\ 0 \end{pmatrix}. \quad (60)$$

The time derivative of the eccentricity vector can be written as

$$\bar{e} = -\sum_{j=1}^{n} \frac{2\Phi S_j}{3\mu} \{I(I_{\lambda} \cdot \hat{d}s_j') \times h$$

$$+ \frac{d((R_{\mu}r') \times (R_{\mu} \hat{d}s'))}{dt}\}. \quad (61)$$
Averaging this result over the binary’s period, expanding up to $O(\epsilon')$ and taking the projection on the secondary’s orbital plane yields

$$
\langle \dot{e}_x \rangle = \sum_{j=1}^{n} \frac{S_j \Phi \sin \theta_j'}{6M_j na} \left]\left(2\Theta G(\theta_j', i)[\cos(2\phi_j' + w) - 3 \cos w] + \epsilon[2B(\theta_j', i) \sin \phi_j' - \Theta H(\theta_j', i)(\sin(\phi_j' + 2w) + 3 \sin(3\phi_j' + 2w))]\right\right),
$$

$$
\langle \dot{e}_y \rangle = \sum_{j=1}^{n} \frac{S_j \Phi \sin \theta_j'}{6M_j na} \left]\left(2\Theta G(\theta_j', i)[\sin(2\phi_j' + w) - 3 \sin w] + \epsilon[-4B(\theta_j', i) \cos \phi_j' + \Theta H(\theta_j', i)(3 \cos(3\phi_j' + 2w) + \cos(\phi_j' + 2w))]\right\right).
$$

(62)

6.3.1. Eccentricity’s Magnitude Evolution

The change in the magnitude of the eccentricity is the projection of $\dot{e}$ on the x-axis. To zero order in eccentricity this change is

$$
\langle \dot{e} \rangle = \sum_{j=1}^{n} \frac{S_j G(\theta_j', i) \Phi \sin \theta_j'}{3M_j na} \frac{1}{\epsilon}[\cos(2\phi_j' + w) - 3 \cos w].
$$

(63)

This contribution is zero for $i = 0, \pi/2$.

Up to $O(\epsilon)$ the change is

$$
\langle \dot{e} \rangle = \sum_{j=1}^{n} \frac{S_j \Phi \sin \theta_j'}{6M_j na} \epsilon \left\{2B(\theta_j', i) \sin \phi_j' - \Theta H(\theta_j', i) \right\} \left\{\sin(\phi_j' + 2w) + 3 \sin(3\phi_j' + 2w)\right\}.
$$

(64)

Averaging over the precession of the argument of the periapsis retains only the term

$$
\langle \dot{e} \rangle = \sum_{j=1}^{n} \frac{S_j \Phi}{3M_j na} \epsilon B(\theta_j', i) \sin \theta_j' \sin \phi_j'.
$$

(65)

Comparing Equations (45) and (47) to Equation (65) we find, similarly to GS, that the eccentricity evolves as

$$
\dot{e} \propto a^{-\frac{3}{2}}.
$$

(66)

We recover the result of McMahon & Scheeres (2010), that circular orbits remain circular. An interesting result is that both the semimajor axis evolution and eccentricity evolution are independent of the asteroid’s albedo and thermal conductivity, in the two extreme regimes of high and low thermal conductivities.

6.4. Results

Our calculations were tested on the binary asteroid 1999KW4 which is the best modeled binary. Figure 5 shows our calculated change in the semimajor axis and in the inclination of the binary. Our calculations show that the secondary is currently drifting away from the primary at a rate of about $\dot{a} = 7 \text{ cm yr}^{-1}$, which is in agreement to within 5% with the result of McMahon & Scheeres (2010).

Due to the similarity between YORP and BYORP, the statistics found in Section 5 are similar for BYORP. We performed calculations on the 2500 randomly constructed asteroids. The stability of $di/dt$ with respect to a change in the inclination determines the stability of the rest of the equilibrium points. The correlation that was found between the sign of $\dot{s}(\epsilon = 0)$ and the stability at $\epsilon = 0$ also holds for the sign of $\dot{a}(i = 0)$ and the stability at $i = 0$. For randomly shaped asteroids, there is an equal likelihood for $i = 0$ to be a stable point or an unstable point and 73% were either Type I or Type II (with equal likelihood). Our calculations also show that 2 out of the 18 real asteroids are not Type I or II. Of the remaining 16 asteroids, 6 were Type I and 10 were Type II, which is consistent with our random asteroids.

Unlike YORP, the timescale for reaching the stable point can be longer than the lifetime of the system, so not every binary will reach it. In order for the system to reach its stable point, the difference between the starting inclination and the inclination at the stable point needs to be $\Delta i \approx \beta^{-1}$.

7. CONCLUSIONS

In this paper, we have developed an intuitive and simple analytical model that predicts the behavior of the YORP and BYORP effects. Our model was computed on randomly constructed asteroids and on shape models of real asteroids. The calculations of our simple model show that the YORP effect is dominated by the inner correlations between facets. These correlations explain the magnitude of the effect and its preference to spin down the asteroid. We have also shown that the dimensionless parameter of $d_i$ governs the strength of the effect (but not the sign).

Kilometer-sized or smaller NEAs undergo a relative fast spin up or spin down due to YORP. The YORP effect tends, in most cases, to change the initial obliquity of the asteroid to either $\epsilon = 0$ or $\epsilon = \pi/2$. For both extreme regimes of thermal conductivity, asteroids will most likely spin down. The rapid spin up of asteroids will eventually cause the asteroid to break up and possibly form a binary. The binary will evolve under BYORP and tidal forces; the former will tend to orient the binary into an $i = 0$ or $i = \pi/2$ orbit, if the system is
relatively near to the stable point. After the binary orbit reaches its stable inclination, the binary will most likely migrate inward until it is stopped by tidal forces (if it survives the migration). A simple relation between the magnitude of the YORP and BYORP effects is calculated, thus knowledge of the former can help us estimate the strength of the latter.

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REFERENCES

Barber, C. B., Dobkin, D. P., & Huhdanpaa, H. 1996, ACM Trans. Math. Softw., 22, 469
Burns, J. A. 1976, Am. J. Phys., 44, 944
Čapek, D., & Vokrouhlický, D. 2004, Icarus, 172, 526
Čuk, M., & Burns, J. A. 2005, Icarus, 176, 418
Čuk, M., & Nesvorný, D. 2010, Icarus, 207, 732
Durech, J., et al. 2008, A&A, 489, L25
Goldreich, P., & Sari, R. 2009, ApJ, 691, 54
Holsapple, K. A. 2010, Icarus, 205, 430
Kaasalainen, M., Durech, J., Warner, B. D., Krugly, Y. N., & Gaftonyuk, N. M. 2007, Nature, 446, 420
McMahon, J., & Scheeres, D. 2010, Icarus, 209, 494
Nesvorný, D., & Vokrouhlický, D. 2007, AJ, 134, 1750
Opeil, C. P., Consolmagno, G. J., & Britt, D. T. 2010, Icarus, 208, 449
Ostro, S. J., et al. 2006, Science, 314, 1276
Pravec, P., et al. 2008, Icarus, 197, 497
Rubincam, D. P. 1995, J. Geophys. Res., 100, 1585
Rubincam, D. P. 2000, Icarus, 148, 2
Rubincam, D. P., & Paddack, S. J. 2010, Icarus, 209, 863
Scheeres, D. J. 2009, Planet. Space Sci., 57, 154
Taylor, P. A., et al. 2007, Science, 316, 274
Vokrouhlický, D., Breiter, S., Nesvorný, D., & Bottke, W. F. 2007, Icarus, 191, 636
Vokrouhlický, D., & Čapek, D. 2002, Icarus, 159, 449
Walsh, K. J., Richardson, D. C., & Michel, P. 2008, Nature, 454, 188
Ward, W. R. 1974, J. Geophys. Res., 79, 3375