Studying the effect of the bending magnet parameters on focusing of charged particles in beam transport systems via the matrix representation method

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Abstract. The focusing of charged particles beam passing through the transport system is affected by many parameters and depends on the elements that used in the transport beam systems. In this study, a permanent bending C-shape magnet is used to investigate the effect of the geometrical configuration parameters of bending magnet on the focusing of the passing charged particles beam. This investigation is implemented via the method of the matrix’s representation. The results showed that the edges of the bending magnet have opposite focus in the horizontal and vertical planes of beam motion. The deflecting angle effect appeared as a rotation of the phase space ellipse of the charged particle beam without focusing of this beam. The results also, showed that the focus of the charged particles beam is unlike in both horizontal and vertical planes of beam motion.

Keywords: beam transport systems, bending magnet, phase space ellipse.

1. Introduction

Beam transport system usually contains of focusing, deflection, acceleration and sometimes additional energy or momentum selection elements. Often these devices are magnetic elements, although at low energy beam, electrostatic elements are used. Quadrupoles and bending magnets with free drift spaces are the main elements of any beam transport system. The dipole magnet produce bending and dispersion effect, while quadrupoles produce focusing effects, the drift spaces is regions where there is no force acting on the particle motion. The performance of beams can be studied in two aspects either by tracing a large number of individual particles (single particle dynamics) or by studying the transfer properties of bundle (the collective effects caused by the electric and magnetic fields generated by the beam itself), in the present study the charged-particles treated as a bundle of particles, linear phase space [1].

Importance of studying the beam transport systems lies in determining the role of each element of the system in the focusing, accelerating and transporting beam particles from the source to the target with more control on particles beam properties.

Any charged particles beam can be represented in six-dimensional phase space (ellipsoid) that enclose all the particle in the beam, these particles expected to occupy the volume enclosed by the ellipsoid, and each point representing a possible ray. The total sum of all phase points, or the phase space volume, is generally referred to as the phase space occupied by the beam [2].
The projection of the ellipsoid in two dimensions is named phase ellipse, this phase space ellipse shape and orientation change as the beam moves along the beam line.

2. Bending Magnet

The main function of bending magnet, mostly named as mass analyzer, is bend the path of the particle on a curve with radius (R) which leads to clearing the natural particles from charged particles beam, then, focusing these particles into the central orbit of the magnet. In such this analyzer the ions are accelerated as focused beam with same kinetic energy. And then the ions are deflected according to their masses, the magnetic field bend the lighter ions more than the heavier ones which leads to separate different ions. In this study a C- shape magnet was used to transport ions generated by plasma source. Figure 1 illustrates the longitudinal section for the design of the magnet used in present study.

![Figure 1](image1.png)

Figure 1. Longitudinal section of bending magnet.

The bending magnet consist basically of two regions, the good field region in which the magnetic field stay particularly constant and the other region is the fringing field which the magnetic field decreases to approach zero in a transition region far from the edges of the magnet, extending from each side of the terminal cross section for a distance depends on the order of the width of the gap between the poles [3].

![Figure 2](image2.png)

Figure 2. Main parameters of bending magnet.

From configuration of bending magnet there are three angles play important role in the focusing of the charged particles beam passing through the magnet. Deflecting angle (φ), as indicates in Figure 2,
which is defined as the angle between the line connecting the center of particle orbit and points of entrance and exit of central trajectory [4] this angle cause a rotation of the beam, this angle in some magnets with range of (45°-90°) [5, 6], while in others (135°) or (270°) [4,7].

The entrance angle ($\beta_1$), as shown in Figure 2, is defined as the angle between the normal to magnet pole boundary and the beam center line in entrance space, and the exit angle ($\beta_2$) is the angle between the normal and the beam line on the exit side. 

The edge of deflecting magnet poles acts as focusing regions that are resulted by a non-homogenous magnetic field. These regions are called a fringing field. So at the entrance and exit angle are very important parameters for the focusing beam passing through the deflecting magnet.

The motion of beam through each region in beam transport line can be describe in term of R-matrix, the total R-matrix of the magnet region is:

$$R_{x,y}^{\text{(total)}} = R_{x,y}^{\text{(exit edge)}}, R_{x,y}^{\text{(inside the magnet)}}, R_{x,y}^{\text{(entrance edge)}}$$

In other form [8,9]

$$R_{x,y} = \begin{pmatrix}
\pm \frac{\tan(\beta_1 - \psi)}{R_o} & 0 \\
\cos k_{x,y}l & \frac{1}{k_{x,y}} \sin k_{x,y}l \\
-k_{x,y} \sin k_{x,y}l & \cos k_{x,y}l
\end{pmatrix}
\begin{pmatrix}
\pm \frac{1}{\tan(\beta_2 - \psi)} & 0 \\
\frac{1}{k_{x,y}} & \frac{1}{k_{x,y}} \sin k_{x,y}l \\
-k_{x,y} \sin k_{x,y}l & \cos k_{x,y}l
\end{pmatrix}
\begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix}
(1)

where $\psi$ is angle of fringing field related to nature of bending magnet manufacturing, $k_{x,y}$ is the wavenumber for the horizontal and vertical oscillations respectively, related to field index ($n$) which take value $0 < n < 1$ and $l$ is the path length inside the bending magnet.

$$k_x = \frac{(1-n)^{1/2}}{R_o}, k_y = \frac{(n)^{1/2}}{R_o}$$

$$0 < n < 1$$

$$l = \varphi R_o$$

(3)

The sign (+) for horizontal plane and (−) for vertical plane

The linear phase space matrix ($\sigma$) is

$$\sigma = \begin{pmatrix}
\sigma_{11} & \sigma_{12} \\
\sigma_{12} & \sigma_{22}
\end{pmatrix}
(4)

The relation between $\sigma$-matrices in two positions of the beam line (between $\sigma_{x,y}$ (in) and $\sigma_{x,y}$ (out) ), is giving by the following inverting equation

$$\sigma_{x,y}(\text{out}) = R_{x,y} \sigma_{x,y}(\text{in}) R_{x,y}^T$$

(5)

For phase space ellipse:

$$x_{\text{max}} = \sqrt{\sigma_{11}}$$ the maximum spatial extent of ellipse which is called by beam envelope.

$$x'_{\text{max}} = \sqrt{\sigma_{22}}$$ the maximum angular divergence of the beam within the phase ellipse.

3. Results and Discussion

Present design of beam transport system was include the regions; extraction region (plasma source), first drift space region, bending magnet, and then second drift space region. The angles of bending magnet were changed as, the deflecting angle ($\varphi$) values in the range (30°-90°) while the entrance and exit angles ($\beta_1$) was take values in the range (5°-20°).

The effect of deflection angle ($\varphi$) on the phase space ellipse of charged particles beam, at constant values of entrance and exit angles of bending magnet, shows in figures (3 and 4), which indicates that the increasing of ($\varphi$) leads to more rotation for the charged particles beam in both horizontal and vertical planes. Also, the slope of phase space ellipse became negative beyond ($\varphi = 60^0$), which means converge (focusing) of charged particles beam. One could be note that the area of the phase space ellipse remained constant for all values of ($\varphi$).
Figure 3. Phase space ellipse of horizontal plane for different deflecting angles.

Figure 4. Phase space ellipse of vertical plane for different deflecting angles.

The effect of entrance edge angle ($\beta_1$) on the phase space ellipse of charged particles beam for horizontal and vertical planes shows in Figures 5 and 6, at constant values of deflection and exit angles of bending magnet, which indicates that the entrance edge angle ($\beta_1$) of the bending magnet acted as a thin lens, as the distance does not increase through it, but changes the focus of the charged particles beam and that increasing the entrance edge angle increases the focus of the charged particle beam. The entrance edge angle ($\beta_1$) acts as a diverge thin lens in the horizontal plane, while in the vertical plane acts as a converging thin lens. Also, the area of the phase space ellipse remained constant for all values of ($\beta_1$).
As entrance angle, the effect of exit edge angle ($\beta_2$) on the phase space ellipse of charged particles beam for horizontal and vertical planes shows in Figures 7 and 8, at constant values of entrance and deflection angles of the bending magnet, indicates that the entrance edge angle ($\beta_1$) of the bending magnet acts as a converging thin lens in both horizontal and vertical planes. The increasing of ($\beta_2$) cause an increase in the converging of charged particle beam.
It is useful to study the envelope of charged particles beam passing through the transport system by plotting the maximum displacement of particles at every region along transport system. Beam envelop is indicate the focusing regions along the beam path.

From the results of phase space ellipse for horizontal and vertical planes, for different configurations for transport system, shown in Figures (3-8), the best configuration of bending magnet that gives best focusing for beam transport passing through it, found to be $\beta_1 = \beta_2 = 20^\circ$ and $\varphi = 90^\circ$.

Figures 9 and 10 show the beam envelope along beam transport system best configuration that obtained in present work for horizontal and vertical planes. These figure show that the best values of
\( \beta_1, \beta_2 \) and \( \varphi \) gives minimum values of distance (focusing) in both horizontal and vertical planes at same time.

![Graph showing beam profile in horizontal plane](image)

**Figure 9.** Beam profile in horizontal plane at \( \beta_1 = \beta_2 = 20^\circ \) and \( \varphi = 90^\circ \)

![Graph showing beam profile in vertical plane](image)

**Figure 10.** Beam profile in vertical plane \( \beta_1 = \beta_2 = 20^\circ \) and \( \varphi = 90^\circ \).

4. **Conclusions**

Focusing of the charged particles beam vary with bending magnet geometrical parameters. The edges of bending magnet act as thin lens with opposite focusing in horizontal and vertical planes of beam motion. The deflecting angle of the bending magnet leads to clear rotation for charged particles beam without focusing of charged particles beam. The focusing of the charged particles beam is unlike in both horizontal and vertical planes of beam motion. Best focusing of beam obtained at \( \beta_1 = \beta_2 = 20^\circ \) and \( \varphi = 90^\circ \).
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