Factorization in Color-Suppressed B Meson Decays

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Abstract

We summarize the status of factorization hypothesis in the color-suppressed B meson decays: $B \to J/\psi K^{(*)}$. We present the general formalism for decay rates and polarization fractions with considering all possible non-factorizable contributions which also include the color-octet contributions, and a new factorization scheme comes appear when the universality is exist: $\chi_{F1} = \chi_{A1} = \chi_{A2} = \chi_{V} = \chi$. We consider various phenomenological models to compare their theoretical predictions with the recent CLEO II experimental measurements.
1 Introduction

One of the interests in $B \to J/\psi K^*$ decays is their role in CP violation measurements at asymmetric B-factories. The vector-vector decay $B^0 \to J/\psi K^{*0}$, with $K^{*0} \to K^0 \pi^0$, is a mixture of CP-even and CP-odd eigenstates since it can proceed via an S, P, D wave decay. If one CP eigenstate dominates or if the two CP eigenstates can be separated, this decay can be used to measure the angle $\beta$ of the unitarity triangle in a manner similar to which the CP-odd eigenstates $B^0 \to J/\psi K_S^0$ is used.

Measurements of the decay amplitudes of $B \to J/\psi K^{(*)}$ transitions also provide a test of the factorization hypothesis in decays with internal W-emission, so called Color-Suppressed decay modes. Several phenomenological models, based on the factorization hypothesis, predict the logitudinal polarization fraction in $B \to J/\psi K^*$, denoted $\Gamma_L/\Gamma$, and the ratio of vector to pseudoscalar meson production, $R_\psi \equiv B(B \to J/\psi K^*)/B(B \to J/\psi K)$ [1, 2, 3, 4, 5]. It has been noted [5, 6] that usual form factor models can not simultaneously explain the earlier experimental data for these two quantities. As shown in table 1, the high values of $\Gamma_L/\Gamma$ measured by ARGUS [7] and CLEO II [8], with low statistics, are not consistent with factorization and the measured value of $R$. The CDF collaboration has measured a lower value of $\Gamma_L/\Gamma$ [9].

Additional information about the validity of factorization can be obtained by a measurement of the decay amplitude phases, since any non-trivial phase differences indicate final state interactions and the breakdown of factorization [11]. In recent CLEO collaboration [12] presented a complete angular analysis and an update of the branching fractions for $B \to J/\psi K^{*0}$ using the full CLEO II data sample. They measured five quantities include $\Gamma_L/\Gamma = 0.52 \pm 0.07 \pm 0.04$, and $R_\psi = 1.45 \pm 0.20 \pm 0.17$. From the data of the relative phases $\phi(A_\perp), \phi(A_\parallel)$ with respect to $\phi(A_0)$, the amplitudes are relatively real, and there is no significant signature of the final state interaction.
2 General formalism for Decay rates and Polarization in Color-suppressed Decay Modes

Using the effective Hamiltonian that contains the short distance wilson coefficients $C_1$ and $C_2$, the decay amplitude for such processes is written as

$$A(B \rightarrow P(V)J/\psi) = \langle P(V)J/\psi|H_{\text{eff}}|B \rangle = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{cb} \left[ a_2(\mu) \langle P(V)J/\psi|O^{(1)}|B \rangle + 2C_1(\mu) \langle P(V)J/\psi|O^{(8)}|B \rangle \right]$$

(1)

where

$$O^{(1)} = \bar{c}_i \gamma_\mu (1 - \gamma_5) c^j \bar{s}_j \gamma_\mu (1 - \gamma_5) b^l,$$

$$O^{(8)} = \bar{c}_i \gamma_\mu (1 - \gamma_5) \frac{\lambda_{ij}^l}{2} c^j \bar{s}_k \gamma_\mu (1 - \gamma_5) \frac{\lambda_{kl}^l}{2} b^l$$

(2)

and

$$a_2 = \frac{C_1}{N_c} + C_2$$

(3)

$N_c$ is the number of colors and $\lambda^a$ is the Gell-Mann matrices.

Here we consider all possible non-factorizable contributions in Eq.(1) and parameterize them as the following:

$$\langle P(V)J/\psi|O^{(1)}|B \rangle = \langle J/\psi|\bar{c}c|B \rangle \langle P(V)|\bar{b}s|V \rangle$$

$$\langle P(V)J/\psi|O^{(8)}|B \rangle = 2m_\psi f_\psi (\epsilon \cdot p_B) F_1^{(1,8)}(q^2)$$

(9)
\[
\langle V J/\psi | O^{(1,8)} | B \rangle_{NF} = -m_\psi f_\psi \left[ (m_B + m_V) (\epsilon \cdot \eta^\ast) A_1^{(1,8)NF}(q^2) - \frac{2 i}{m_B + m_V} (\epsilon \cdot p_B) (\eta^\ast \cdot p_B) A_2^{(1,8)NF}(q^2) \right] \\
- \frac{2 i}{m_B + m_V} \epsilon_{\mu\nu\rho\sigma} \epsilon^\mu \eta^{\nu\rho} p_B^\nu p_V^\rho V^{(1,8)NF}(q^2)
\]

\[
(10)
\]

where \( q = [p_B - p_{P(V)}]_\mu = (p_\psi)_\mu \). The polarization vectors \( \epsilon^\mu \) and \( \eta^\mu \) correspond to the two vector mesons \( J/\psi \) and \( V \), respectively.

Substituting (4-10) into the decay amplitude (1), we can calculate decay rates for the processes \( B \rightarrow P(V) J/\psi \) and polarization for the \( B \rightarrow V J/\psi \) process.

The decay widths for each process are presented below:

\[
\Gamma(B \rightarrow P J/\psi) = \frac{G_F^2 m_B^5}{32\pi} |V_{cb}|^2 |V_{cs}|^2 a_2^2 \left( \frac{f_\psi}{m_B} \right)^2 k^3(t^2) \left| F_1(m_\psi^2) \right|^2 \left| 1 + \frac{2C_1}{a_2} \chi_{F1} \right|^2,
\]

\[
\Gamma(B \rightarrow V J/\psi) = \frac{G_F^2 m_B^5}{32\pi} |V_{cb}|^2 |V_{cs}|^2 a_2^2 \left( \frac{f_\psi}{m_B} \right)^2 \left| A_1(m_\psi^2) \right|^2 k(t^2) t^2 (1 + r)^2 \sum_{\lambda\lambda} H_{\lambda\lambda},
\]

where

\[
H_L = H_{00} = \left[ a \left( 1 + 2 \frac{C_1}{a_2} \chi_{A1} \right) - bx \left( 1 + 2 \frac{C_1}{a_2} \chi_{A2} \right) \right]^2,
\]

\[
H_T = H_{++} + H_{--} = 2 \left[ \left( 1 + 2 \frac{C_1}{a_2} \chi_{A1} \right)^2 + c^2 g^2 \left( 1 + 2 \frac{C_1}{a_2} \chi_V \right)^2 \right],
\]

\[
\chi_{F1} = \left( F_1^{(8)NF}(m_\psi^2) + \frac{a_2}{2C_1} F_1^{(1)NF}(m_\psi^2) / F_1(m_\psi^2) \right) / A_1(m_\psi^2),
\]

\[
\chi_{A1} = \left( A_1^{(8)NF}(m_\psi^2) + \frac{a_2}{2C_1} A_1^{(1)NF}(m_\psi^2) / A_1(m_\psi^2) \right) / A_2(m_\psi^2),
\]

\[
\chi_{A2} = \left( A_2^{(8)NF}(m_\psi^2) + \frac{a_2}{2C_1} A_2^{(1)NF}(m_\psi^2) / A_2(m_\psi^2) \right) / V(m_\psi^2),
\]

\[
\chi_V = \left( V^{(8)NF}(m_\psi^2) + \frac{a_2}{2C_1} V^{(1)NF}(m_\psi^2) / V(m_\psi^2) \right) / V(m_\psi^2),
\]

Here subscripts \( L \) and \( T \) in (13) and (14) stand for longitudinal and transverse, and 00, ++ and −− represent the vector meson helicities. In (1) - (13) we have introduce the following dimensionless parameters:

\[
r = \frac{m_{P(V)}}{m_B}, \quad t = \frac{m_\psi}{m_B},
\]

\[
k(t^2) = \sqrt{(1 - r^2 - t^2)^2 - 4t^2r^2},
\]

(19)
\[ a = \frac{1 - r^2 - t^2}{2rt}, \quad b = \frac{k^2(t^2)}{2rt(1+r)^2}, \quad c = \frac{k(t^2)}{(1+r)^2}. \]  

(21)

The numerical values of parameters \( a, b, \) and \( c \) for the processes \( B \to J/\psi K(K^*) \) are given as

\[ a = 3.165, \quad b = 1.308, \quad c = 0.436. \]  

(22)

Furthermore \( x, y, \) and \( z \) represent the following ratios,

\[ x = \frac{A_2^{BK^*}(m_{\psi}^2)}{A_1^{BK^*}(m_{\psi}^2)}, \quad y = \frac{V^{BK^*}(m_{\psi}^2)}{A_1^{BK^*}(m_{\psi}^2)}, \quad z = \frac{F_1^{BK}(m_{\psi}^2)}{A_1^{BK^*}(m_{\psi}^2)}. \]  

(23)

The longitudinal polarization fraction \( \Gamma_L/\Gamma \) and the ratio \( R_\psi \) are defined:

\[ \Gamma_L \equiv \frac{\Gamma(B \to J/\psi K^*)_L}{\Gamma(B \to J/\psi K^*)} = \frac{H_L}{H_L + H_T}, \]  

(24)

\[ R_\psi \equiv \frac{\Gamma(B \to J/\psi K^*)}{\Gamma(B \to J/\psi K)} = 1.08 \frac{(H_L + H_T)}{z^2 \left| 1 + 2 \frac{a_2}{a_2} \chi \right|^2} . \]  

(25)

And the parity-odd (P-wave) transverse polarization measured in the transversity basis \([12, 13]\) is given:

\[ |P_\perp|^2 = \frac{|A_\perp|^2}{|A_\parallel|^2 + |A_\perp|^2} = 2c_2^2 y^2 \left( 1 + 2 \frac{a_2}{a_2} \chi \right)^2 \]  

(26)

When \( \chi_{F1} = \chi_{A1} = \chi_{A2} = \chi_V = \chi \), a new factorization scheme comes appear. In this case, the nonfactorizable terms only affect the coefficient \( a_2 \) as below:

\[ a_2 \quad \rightarrow \quad a_2^{\text{eff}} = a_2 \left( 1 + 2 \frac{C_1}{a_2} \chi \right) \]

\[ = a_2 + 2C_1 \chi \]

\[ = \left( C_2 + \frac{1}{N_c} C_1 \right) + 2C_1 \chi \]

\[ = C_2 + \xi C_1, \quad \xi = \frac{1}{N_c} + 2 \chi. \]  

(27)

However the predictions of \( \Gamma_L/\Gamma, R_\psi, \) and \( |P_\perp|^2 \) in the normal factorization method \([1]\) remain intact since all nonfactorizable terms are cancelled out in Eq. (24 - 26).

### 3 Phenomenological Model

Let us see if the experimental measurements can be explained within the context of the factorization approach. To proceed, we consider several phenomenological models of form factors:
1. The Bauer-Stech-Wirbel model (called BSW I here) \cite{1} in which $B \rightarrow K(K^*)$ form factors are first evaluated at $q^2 = 0$ and then extrapolated to finite $q^2$ using a monopole type $q^2$-dependence for all form factors $F_1, A_1, A_2,$ and $V$.

2. The modified BSW model (called BSW II here) \cite{2}, takes the values of the form factors at $q^2 = 0$ as in BSW I but uses a monopole form factor for $A_1$ and a dipole form factor for $F_1, A_2,$ and $V$.

3. The non-relativistic quark model by Isgur et al (ISGW) \cite{3} with exponential $q^2$ dependence for all form factors.

4. The model of Casalbuoni et al and Deandrea et al (CDDFGN) \cite{4} in which the normalization at $q^2 = 0$ is obtained in a model that combines heavy quark symmetry with chiral symmetry for light vector degrees of freedom and also introduces light vector degrees of freedom. Here all form factors are extrapolated with monopole behavior.

Several authors have derived the $B \rightarrow K(K^*)$ form factors from experimentally measured $D \rightarrow K(K^*)$ form factors at $q^2 = 0$ using the Isgur-Wise scaling laws based on the SU(2) heavy quark symmetry \cite{17}, which are allowed to relate $B$ and $D$ form factors at $q^2$ near $q^2_{max}$.

1. The $B \rightarrow K(K^*)$ form factors are calculated in Ref.\cite{15} by assuming a constant for $A_1$ and $A_2$, a monopole type form factor for $F_1$, and dipole type for $V$.

2. An ansatz proposed in Ref.\cite{5}, which relies on “soft” Isgur- Wise scaling laws and a monopole type for $A_1$ and a dipole type for $A_2, V, F_1$.

3. For Ref.\cite{16}, they are computed by advocating a monopole extrapolation for $F_1, A_0, A_1$, a dipole behavior for $A_2, V$, and an approximately constant for $F_0$.

Table 1 summarizes the predictions of $\Gamma_L/\Gamma, R_\psi$ and $|P_|^2$ in above-mentioned various form factor models within the factorization approach by assuming the absence of inelastic final-state interactions. It appears that Keum’s and CT’s predictions are most close to the data.

Gourdin et al \cite{19} also have suggested that the ratio $R_{\eta_c} = B(B \rightarrow \eta_cK^*)/B(B \rightarrow \eta_cK)$ would provide a good test of the factorization hypothesis in Class II decays. Using data of
Particle Data Group [20] of $\mathcal{B}(B^+ \rightarrow K^+ J/\psi) = (1.02 \pm 0.14)\%$, we expect $\mathcal{B}(B^+ \rightarrow K^+ \eta_c) = (1.14 \pm 0.31) \times 10^{-3}$, which could be within reach of near future experimental data accumulation. Other ratios of decay rates in modes with charmonium mesons may also be used to test for the violation of factorization [15, 18].

References

[1] Wirbel M., Stech B., and Bauer M., Z.Phys. C 29 (1985) 637; Z.Phys. C 34 (1987) 103.

[2] Neubert M., Rieckert V., Xu Q.P., and Stech B. in Heavy Flavours, edited by A. J. Buras and H. Lindner, World Scientific, Singapore (1992).

[3] Isgur N., Scora D., Grinstein B., and Wise M.B., Phys. Rev D 39 (1989) 799; Isgur N. and Scora D., Phys. Rev. D 40 (1989) 1491.

[4] Casalbuoni R., Deandrea A., Bartolomeo N.Di., Feruglio F., Gatto R., and Nardulli G., Phys. Lett. B 292 (1992) 371; 299 (1993) 139; Deandrea A., Bartolomeo N. Di., Gatto R. and Nardulli G., Phys. Lett. B 318 (1993) 549.

[5] Aleksan R., Yauoanc A. Le., Oliver L., Pène O., and Raynal J.C., Phys. Rev. D 51 (1995) 6235.

[6] Gourdin M., Kamal A. N., and Pham X. Y., Phys. Rev. Lett. 73 (1994) 3355.

[7] Albrecht H. et al, Phys. Lett B 340 (1994) 217.

[8] Alam M.S. et al, Phys. Rev. D 50 (1994) 43.

[9] Abe F. et al, Phys. Rev. Lett. 75 (1995) 3068.

[10] Abe F. et al, Phys. Rev. Lett. 76 (1996) 2015.

[11] Kroner J.G. and Goldstein G. R., Phys. Lett. B 89 (1979)105.

[12] Jessop C. P. et al, CLEO collaboration, Preprint CLNS 96/1455, CLEO 96-24.
[13] Dighe A.S., Dunietz I., Lipkin H.J. and Rosner J.L., Phys. Lett. B 369 (1996) 144.

[14] Jaus W., Phys. Rev. D41 (1990) 3394; Jaus W. and Wyler D., Phys. Rev. D41 (1990) 3405.

[15] Keum Y.Y., Proceedings of the APCTP-ICTP Joint International Conference ’97, Seoul, Korea; Preprint APCTP-97-23.

[16] Cheng H.-Y. and Tseng B., Phys. Rev. D51 (1995) 6259.

[17] Isgur N. and Wise M.B., Phys. Rev. D42 (1990) 2388.

[18] Gourdin M., Keum Y.Y., and Pham X.Y., Phys. Rev. D52 (1995) 1597; Keum Y.Y., Ph.D. thesis, Université Pierre et Marie Curie (Pais VI) (1996), unpublished.

[19] Gourdin M., Keum Y.Y., and Pham X.Y., Phys. Rev. D51 (1995) 3510.

[20] Particle Data Group, Phys. Rev. D54 (1996) 1.
Table 1: Experimental data and theoretical predictions for $\Gamma_L/\Gamma$, $R_\psi$, and $|P_\perp|^2$.

| - | $\Gamma_L/\Gamma$ | $R_\psi$ | $|P_\perp|^2$ |
|---|---|---|---|
| ARGUS [7] | 0.97 ± 0.16 ± 0.15 | - | - |
| CLEO II(95) [8] | 0.80 ± 0.08 ± 0.05 | 1.71 ± 0.34 | - |
| CDF [9, 10] | 0.65 ± 0.10 ± 0.04 | 1.32 ± 0.23 ± 0.16 | - |
| CLEO II(96) [12] | 0.52 ± 0.07 ± 0.04 | 1.45 ± 0.20 ± 0.17 | 0.16 ± 0.08 ± 0.04 |
| BSW I [1] | 0.57 | 4.23 | 0.09 |
| BSW II [2] | 0.36 | 1.61 | 0.24 |
| ISGW [3] | 0.07 | 1.72 | 0.52 |
| CDDFGN [4] | 0.36 | 1.50 | 0.30 |
| JW [14] | 0.44 | 2.44 | |
| Orsay [5] | 0.45 | 2.15 | 0.25 |
| Keum [15] | 0.59 ± 0.07 | 1.74 ± 0.38 | 0.14 ± 0.05 |
| CT [16] | 0.56 | 1.84 | 0.16 |