Overview of Calculation Methods of Structural Time-Dependent Reliability

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Abstract. Under the action of natural erosion, the strength, durability and other safety performance of structures and elements gradually decrease with time, which has a great impact. To solve the above problem, a series of time-dependent reliability analysis methods were proposed. Based on different structural performance functions, this paper analyzes and discusses different time-dependent reliability theories, including outcrossing-based reliability method, Monte Carlo simulation method, extremum method and other new methods proposed in recent years, which provides reference for later research.

Keywords. Structural reliability; Time-dependent reliability; Calculation method; Performance function of structure.

1. Introduction

The traditional reliability problems are mainly load-based stationary binomial stochastic process model [1] and the specific calculation methods include Monte Carlo method, first-order second-moment theory [2], high-order moment method and so on. Traditional methods usually assume that the structural resistance and loading effect are fixed, and they are random variables at a certain time. However, because of an oversized variety of uncertainties within the actual project, the reliability of the structure is constantly changing during the service period, resulting in that the traditional theories cannot meet the requirements of the particular project. Thus, factors such as time and resistance degradation should be considered in the design [3]. To solve the influence of uncertain factors on the safety performance of structures or elements, researchers have proposed a series of time-dependent reliability analysis methods. However, there has not been an accurate and unified theoretical method, nor a lot of practice.

Starting from different structural performance functions, this paper analyzes and discusses different time-dependent reliability theories, including outcrossing-based reliability method, Monte Carlo simulation method, extremum method and other new methods proposed in recent years, so as to provide reference for later research.

2. Performance Function of Structure

As shown in figure 1, the performance functions of structure in service include monotonic and non-monotonic variations, which are applicable to different specific situations.
Figure 1. Structural response-time curves.

(1) Monotonic variation is principally applicable to the case of structural aging and cumulative damage, which is generally described by random variable model and random process model [4]. The structural performance function can be expressed as a random process $g(x, t)$, where $x$ is the vector of structural input random variables, and $t$ is time. The instantaneous failure probability of structures is defined as:

$$P_{f,i}(t) = P\{g(x, t) < 0\}$$

When $t$ is assumed to be a fixed value, the formula (1) is expressed as the failure probability of time-invariant reliability. Therefore, the traditional time-invariant reliability analysis method can be used to replace the time-dependent reliability at the endpoint of the time interval, such as Monte Carlo simulation method [5] and first-order second-moment method [6-7].

(2) Non-monotonic variation is mainly applicable to wind load and seismic load. Compared with monotonic variation mode, the performance function is more changeable and the calculation is more difficult. The mature theories are outcrossing-based reliability method [8], extremum method [9] at present and some novel methods, which are shown in the detailed analysis below.

3. Theories of Time-Dependent Reliability

There are two main ways to analyze the time-dependent reliability at present: (1) The service time of the structure is divided into several discrete time periods, and the problem is transformed into time-invariant reliability problem respectively [10]. (2) Based on the stochastic process theory, the resistance and load of the structure are treated as stochastic processes [11]. Researchers have applied and further analyzed the above theories in different fields for different research objects.

3.1. Outcrossing-Based Reliability Method

Outcrossing-based reliability method [8] is characterized by whether the dynamic response (e.g., displacement and velocity) exceeds the safety limit or not. The structure fails when the boundary is first exceeded, which is generally applicable to special stochastic processes such as Gaussian processes. The commonly used analysis methods are Poisson model method, modified Poisson model method, Volterra series method [12], stochastic averaging method [13] and integral equation method.

On the basis of this method, Andrieu-Renaud et al. [14-15] proposed and improved the PH2 method for establishing the reliability numerical equation in the discrete time period, which reduced the influence of time step on the results. Zheng et al. [16] considered the memory of the first crossing time and proposed a calculation method for brittle structures based on integral equation method. Li et al. [17-19] gave the expressions of structural crossing probability and failure probability under different loads, and proposed a modified method for calculating the time-dependent reliability of structures according to the correlation of crossing occurrence. Considering the randomness of resistance degradation process, Zhang et al. [20] gave the calculation formula of time-varying reliability of structure by combining point estimation method. Considering discrete load process based on the boundary method of penetration
threshold, an outcrossing-based reliability assessment method was recommended by Wang et al. [21-22] in order that the load model was not restricted to Poisson process.

However, the outcrossing-based reliability method is only applicable to some specific stochastic processes such as Gaussian processes, which requires complex multidimensional numerical integration and requires more efficient and wider application methods.

3.2. Monte Carlo Simulation Method

Monte Carlo simulation method is one of the most effective methods to evaluate time-dependent reliability, especially for high-dimensional problems [5] [23]. This method obtains the performance function or structural response value of the structure through random sampling test, and calculates the time-dependent reliability of the structure according to the number of tests for structural failure and the total number of sampling tests.

The Monte Carlo simulation method does not need to consider the dimension of random variables and omits the complex mathematical deduction process. However, the sampling calculation is huge, and sufficient precision sampling and tests are required to support its accuracy. Researchers have developed different selection and variance reduction techniques for these shortcomings, and combined them with other methods to improve the efficiency of Monte Carlo method [24-28]. Luo et al. [29] simplified the calculation process by Monte Carlo sampling method combined with series system model, and gave the time-dependent reliability calculation method of corroded elements. An improved response surface method combined with Monte Carlo sampling method was put forward by Zhang [30], which can effectively improve calculation accuracy of the reliability and reduce the design sample points. Considering the statistical uncertainty, Zhang et al. [31] theorized to combine Bootstrap method with weighted analysis of probability density to calculate the reliability of foundation pit deformation by Monte Carlo method. Ji et al. [32] improved the WUS (Weighted Uniform Simulation) method by combining Monte Carlo method to calculate the failure probability of slope system more efficiently.

3.3. Extremum Method

The extremum method [9] focuses on the global maximum and minimum of structural response. When the extreme value exceeds the safety range, it is the structural failure, which is a typical method to transform the time-dependent problem into the time-invariant problem. In recent years, researchers have proposed different methods: Gong et al. [33-34] put forward the concept of time synthesis of load to calculate cumulative failure probability. Li et al. [35] developed the concept of equivalent extreme events and analyzed the system reliability of dynamic and static problems. NERS (Nested Extreme Response Surface) was proposed by Wang et al. [36] based on Kriging model and nested extreme response surface was established. Considering the time-dependent limit state equation involving stochastic processes, Hu et al. [37] estimated the corresponding distribution of the extreme values of stochastic processes by sampling and substituted the extreme values for stochastic processes. Huang [4] theorized that the extremum method combined with AK-SS method was used to analyze the time-dependent reliability of the structure, and the Kriging model of the time-varying function was established by the mixed EGO algorithm to improve the accuracy of the extreme response surface and calculate the failure probability.

3.4. Reliability Theory Based on Probability Density Evolution

In order to solve the problem that the outcrossing-based reliability method is only applicable to certain random processes, researchers have proposed the reliability theory based on the evolution of probability density, which has become one of the most popular theories in recent years. This method constructs a virtual random process, and takes the extreme value of the required solution as the cross-sectional random variable of the random process, so the obtained structural dynamic reliability has high accuracy [38-39].

Based on Lagrange description of dynamic system, Li et al. [40-42] incorporated deterministic system and stochastic system into a unified framework, gave a generalized density evolution equation
with universal significance and developed different point selection methods [43-44]. Fan [45] introduced the Dirac sequence solution of the generalized probability density evolution equation without the first-order derivative of the target quantity with respect to time, which simplified the calculation process. Zhou [1] put forward a hybrid method using the Dirac sequence to. Wang et al. [46] used covariance matrix to obtain joint probability density function based on KDE (Kernel Density Estimation) method to calculate reliability and prove its accuracy. The reliability theory based on probability density evolution has been widely concerned because of its wide application and high precision.

However, there are still some limitations, which are mainly reflected in the strong experience and subjectivity in tangential radius, the number of selected points and the selection of the median value of the Dirac sequence solution, and needs further improvement. Further improvement is a problem that researchers need to pay attention to in the future.

3.5. New Theories of Time-Dependent Reliability
In addition to the above classical time-dependent reliability theories, researchers have also proposed many new reliability theories in recent years, introduced different models or theories, and applied them to different fields: Mourelatos et al. [47] theorized the limit state equation of explicit time-dependent function including random variables and random processes, and calculated the time-dependent conditional probability by combining FORM and CLS methods, which was suitable for solving low dimensional failure probability problems. Wang et al. [48] introduced a non-probabilistic time-dependent reliability estimation model and a sequential multidisciplinary optimization, combined with SMO-NTRA (sequential multidisciplinary optimization and non-probabilistic time-dependent reliability assessment) method. Gong et al. [49] improved the Poisson outcrossing rate method of the first-order reliability, to analyze the time-dependent reliability more easily. TSM method (two-step method) was developed by Shi et al. [50], which simplified analysis of time-dependent reliability and some deterministic optimization.

4. Conclusion
In this paper, monotonic and non-monotonic structural functions are introduced, and different time-dependent reliability theories are analyzed and discussed, including several classical methods such as outcrossing-based reliability method, Monte Carlo simulation method, extremum method, as well as novel methods proposed in recent years such as reliability theory based on probability density evolution. Basic problems in engineering can be solved using those several theories at present, but the limitations are also obvious, such as the low efficiency of Monte Carlo simulation method, the subjectivity of probability density evolution method, and the special limitation of outcrossing-based reliability method. Thus, it is necessary to apply a wider and more convenient theoretical method as a support, and it is also a problem that researchers need to consider in the future.

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