The effect of change in entropy on primordial nucleosynthesis

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Abstract. One of the most important features of the Universe in studies of evolution of the early Universe is conservation of entropy of particles being in thermal contact with each other in comoving volume. Destruction of any particles (for example, annihilation of electrons and positrons) or thermalization processes lead to transfer or sharing of entropy. In this paper hypothetical process of occurrence of thermal contact between photons and axions in the early Universe is considered to examine the effect of the process on yields of D, ⁴He and ⁷Li in Primordial Nucleosynthesis. It is found that change in photon entropy caused by thermalization of photons and axions lead to stronger overlapping between the intervals of acceptable values of η for ⁴He and ⁷Li. At the same time interval of acceptable values of η for D/H is shifted in opposite direction relatively to ⁴He.

1. Introduction

The result of Primordial Nucleosynthesis (the abundances of light nuclei) depends significantly on the total energy density of all components of primordial matter filling the Universe at that cosmological epoch (times of order [1 ÷ 1000] seconds after the Big bang), as well as on the temperature, which may be different for various components of the matter (see about Primordial Nucleosynthesis in detail in [1, 2, 3, 4, 5]). The temperature defines rates of nuclear reactions proceeding in the plasma of the Universe and the energy density defines the rate of expansion of the Universe. At the epoch of Primordial Nucleosynthesis the ultrarelativistic matter of the Universe consists of two components: electromagnetic (the particles taking part in the processes of electromagnetic interaction) and neutrino component (neutrino and antineutrino particles, taking part only in the processes of weak interaction in addition to gravitational interaction). At that time the rate of the electromagnetic processes exceed significantly the Universe expansion rate [1, 2, 3] and this ensures thermal equilibrium between distinct particles of the electromagnetic component. This allows one to employ the equilibrium thermodynamics for description of these particles. At the same time the rate of the weak interactions is insufficient for keeping neutrino in thermal equilibrium with the rest matter. Accordingly, the law of entropy conservation in comoving volume in the expanding Universe can be applied to each of the two non-interacting components of matter independently. Therefore at the epoch of electron-positron annihilation (when concentration of electrons falls roughly by nine orders of magnitude) entropy of annihilating electron-positron pairs is almost completely transferred into entropy of photons resulting in increase of photon temperature relative to neutrino temperature by approximately 40 percents. This difference affects significantly results of Primordial Nucleosynthesis and this
Figure 1. Reduced entropy of electromagnetic component of cosmic plasma in the cases of \( \rho_{\text{rad}} = \rho_{\gamma} + \rho_e + \rho_\nu \) (\( S_0 = 11/4 \)) and \( \rho_{\text{rad}} = \rho_{\gamma} + \rho_e + \rho_\nu + \rho_a \) (\( S_0 = 11/6 \)) (see the text for description of the variables).

suggests the importance of studying of possible processes, which could lead to additional change in entropy of components of cosmic plasma. In this work various models of change in entropy and their effect on final abundances of light elements in Primordial Nucleosynthesis are considered.

2. Entropy of the matter in the early Universe

The entropy of matter plays crucial role in thermodynamics of the early Universe because its conservation defines the dependence of scale factor of the Universe on the temperature of photons, as well as relation between the photon and the neutrino temperatures. Reduced entropy of electromagnetic component of cosmic plasma \( s/T^3 \) divided by reduced entropy of photons \( 4\pi^2/45 \) [1]:

\[
S(x) \equiv 1 + \frac{45}{2\pi^2} \int_0^\infty y^2 \, dy \left( \frac{\sqrt{y^2 + x^2}}{3\sqrt{y^2 + x^2}} + \frac{y^2}{3\sqrt{y^2 + x^2}} \right) \exp \sqrt{y^2 + x^2 + 1},
\]

where \( x = m_e/kT \). The meaning of this function is that it shows how many times entropy of all particles interacting with photons greater than photon entropy at a given temperature. The law of entropy conservation written in terms of the reduced entropy

\[
(a(T_1)T_1)^{\frac{3}{T}}S(x_1) = (a(T_2)T_2)^{\frac{3}{T}}S(x_2)
\]
Figure 2. Abundances of nuclei of light elements produced in Primordial Nucleosynthesis. Solid lines correspond the classical case, where $\rho_{\text{rad}} = \rho_\gamma + \rho_e + \rho_\nu$ after neutrino decoupling, while dashed lines correspond the case, when after neutrino decoupling thermal contact between photons and axions occurs and in this case $\rho_{\text{rad}} = \rho_\gamma + \rho_e + \rho_\nu + \rho_a$. The calculations are performed using updated our numerical code [8].

This gives the general relation for the dependence of the scale factor of the Universe on the temperature of photons in terms of the reduced entropy:

$$a(T) = S(T)^{-1/3} T_0 \frac{T_0}{T},$$

(3)

where $T_0$ is the current temperature of photons. Relation between photon and neutrino temperatures in these terms takes the form
\[ T_\nu = S(T \to \infty)^{-1/3} S(T)^{1/3} T = S_0^{-1/3} S(T)^{1/3} T, \]  

(4)

where \( S_0 \) is the value of function (1) at temperatures at which the neutrino and photon temperatures are equal. At \( T \to \infty \) function (1) tends to limit \( S_0 = 11/4 \).

3. Possible scenarios of change in entropy

The classical scenario, when after neutrino decoupling \( \rho_{rad} = \rho_\gamma + \rho_e + \rho_\nu \) leads to

\[ \rho_{rad} = \rho_\gamma + \rho_e + \rho_\nu = \rho_\gamma \left[ 1 + \frac{\rho_e}{\rho_\gamma} + N_{\text{eff}} \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} S^{4/3}(x) \right], \]  

(5)

where \( 4/11 \) is \( S_0^{-1} \) in this scenario.

Alternative scenarios come when entropy exchange between electromagnetic component and additional possible components is allowed. In this case the total initial budget of entropy of electromagnetic component is not entirely kept in photons, but some its part is transferred into these additional components. The part of the entropy which is withdrawn from photons is defined by the number of relativistic degrees of freedom of particles which shares entropy with photons. This also changes the value of the normalization \( S_0 \). One of the possible candidates to come into thermal contact with photons are axions.

Axions are broad class of particles appearing in various theories related to CP-violation in the Standard Model, Supersymmetric theories and theories with extra dimensions, including string theory. Moreover, axions appear to be an exceptionally good candidate to be Dark Matter due to their physical features. Thereby, axions are widely employed when numerous alternative cosmological scenarios are considered (see [6] for detailed review on axions in cosmology).

In the case of the reaching thermal contact between photons and axions [7] during radiation dominated era at least one more relativistic degree of freedom is added to particles being in thermal equilibrium with photons at any cosmological epoch after neutrino decoupling. This modifies the normalization value of the function \( S(x) \) so that it takes value \( S_0 = 11/4 \cdot 2/3 = 11/6 \), where \( 2/3 \) is the ratio of relativistic degrees of freedom of electromagnetic component before and after photons and axions come into thermal equilibrium. In this scenario a new term – the axion energy density \( \rho_a \) – enters the total energy density of ultrarelativistic matter:

\[ \rho_{rad} = \rho_\gamma + \rho_e + \rho_\nu + \rho_a = \rho_\gamma \left[ 1 + S^{4/3}_a(x) + N_{\text{eff}}^\text{st} \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} S^{4/3}_\gamma(x) + \frac{\rho_e}{\rho_\gamma} \right], \]  

(6)

where \( N_{\text{eff}}^\text{st} \) is the value \( N_{\text{eff}} = 3.046 \) in standard scenario.

4. Effect on Primordial Nucleosynthesis

We used the above two models (classical and one taken from [7]) of change in entropy of the electromagnetic component of the cosmic plasma to obtain the dependence of light nuclei abundances on value of baryon-to-photon ratio \( \eta \) and results of calculations are presented on Figure 2. It is seen in this figure, that decrease in entropy of photons resulted in shifts of values of \( \eta \), which correspond observed light nuclei abundances. In this model the intervals of acceptable values of \( \eta \) for \( ^4\text{He} \) and \( ^7\text{Li} \) overlaps stronger in comparison with the standard scenario. At the same time interval of acceptable values of \( \eta \) for \( \text{D}/\text{H} \) is shifted in opposite direction relatively to \( ^4\text{He} \).

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