Spin-Flavour Oscillations and Neutrinos from SN1987A

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Abstract

The neutrino signal from SN1987A is analysed with respect to spin-flavour oscillations between electron antineutrinos, $\bar{\nu}_e$, and muon neutrinos, $\nu_\mu$, by means of a maximum likelihood analysis.

Following Jegerlehner et al. best fit values for the total energy released in neutrinos, $E_t$, and the temperature of the electron antineutrino, $T_{\bar{\nu}_e}$, for a range of mixing parameters and progenitor models are calculated. In particular the dependence of the inferred quantities on the metallicity of the supernova is investigated and the uncertainties involved in using the neutrino signal to determine the neutrino magnetic moment are pointed out.

14.60.St, 97.60.Bw
I. INTRODUCTION

At the end of their lives massive stars explode in a type II supernova event releasing almost their entire binding energies in form of neutrinos of all flavours. Depending on the model for the progenitor, the equation of state and the treatment of the neutrino transport the theoretical prediction for the total energy emitted in neutrinos is

\[ E_t \approx 2 - 4 \times 10^{53} \text{erg} \]  

(1)

and for the mean neutrino temperature is

\[ T_\nu \approx \begin{cases} 
3 - 4 \text{ MeV for } \nu_e, \\
5 - 6 \text{ MeV for } \bar{\nu}_e, \\
7 - 9 \text{ MeV for all other species.}
\end{cases} \]  

(2)

[1,2] The inferred values from the observed neutrino signal from the explosion of SN1987A in the Large Magellanic Cloud (LMC) are \( E_t \approx 2 - 3 \times 10^{53} \text{erg} \) and \( T_{\bar{\nu}_e} \approx 3 \text{ MeV} \) [3–5]. While the energy, \( E_t \), is in good agreement with supernova theory, the temperature of the electron anti-neutrinos lies below the predicted range. Neutrino oscillations could significantly alter the energy spectra as observed by neutrino detectors on Earth and have implications for the interpretation of the observed neutrino signal. In fact an admixture of the hotter \( \nu_\mu \)-spectrum through oscillations would shift the inferred value for \( T_{\bar{\nu}_e} \) towards lower values thus making it even more difficult to reconcile theory with observation.

This analysis follows closely the method by Jegerlehner et al. [6], who investigated the implications of flavour oscillations (\( \bar{\nu}_e \leftrightarrow \bar{\nu}_\mu \)) on the interpretation of the supernova neutrino signal using a maximum likelihood method. The impact of the MSW-effect on the neutrino spectra has also been studied in Ref. [7–9]. For a ‘normal’ mass hierarchy (i.e. \( m_{\nu_e} < m_{\nu_\mu} \)) flavour oscillations between antineutrinos are effective only for a very limited choice of mixing parameters. However spin-flavour (SF) oscillations, which arise from the interaction of the (hypothetical) magnetic moment of the neutrino with an external magnetic field, could have a serious effect for a wide range of reasonable parameters.
In this paper I will investigate the implications of SF-conversion between right-handed $\bar{\nu}_e$s and left-handed $\nu_\mu$s (or $\nu_\tau$s) on the interpretation of the neutrino signal from supernova SN1987A. The distortion of the electron anti-neutrino spectrum is of special interest since almost all the neutrinos detected from SN1987A were $\bar{\nu}_e$s. I will also consider the possibility of deriving an upper bound on the neutrino magnetic moment from the observed neutrino spectra.

The conversion probabilities (i.e. the probability that a neutrino emitted from the neutrinosphere has been converted into a different species by the time it leaves the supernova) are taken from [10], which were calculated on the basis of the precollapse model of Woosley and Weaver (1995) [11] for a progenitor mass of 15 and 25 solar masses and solar ($Z_\odot$) as well as zero ($Z_0$) metallicity. The mass-squared difference $\Delta m^2 = m^2_{\nu_\mu} - m^2_{\bar{\nu}_e}$ and the vacuum mixing angle $\theta_v$ are assumed to take the values suggested by the MSW-solution to the solar neutrino problem.

In the following section I will briefly review the theory of SF oscillations. In section III I will discuss the various progenitor models with their associated conversion probabilities and in section IV I will present the numerical results which will be followed by the discussion.

II. SPIN-FLAVOUR OSCILLATIONS

Assuming that the neutrino is a Majorana particle with a non-vanishing transition magnetic moment, an anti-neutrino of one flavour can be resonantly converted into a neutrino of another flavour on traversing a magnetic field $B$ and vice versa [12–14]. These spin-flavour oscillations are governed by the evolution equation

$$i \frac{d}{dt} \begin{pmatrix} \bar{\nu}_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} 0 & \mu_\nu B \\ \mu_\nu B & \Delta H \end{pmatrix} \begin{pmatrix} \bar{\nu}_e \\ \nu_\mu \end{pmatrix},$$

(3)

where $\mu_\nu$ denotes the neutrino transition magnetic moment and $B$ the component of the magnetic field which is transverse to the direction of propagation of the neutrinos. The diagonal component of the Hamiltonian is given by
\[ \Delta H = \frac{\Delta m^2}{2E_\nu} \cos 2\theta_\nu - \Delta V \]  

(4)

with \( \Delta V = \sqrt{2}G_F \rho/m_N(1 - 2Y_e) \), where \( \rho \) denotes the density, \( G_F \) the Fermi coupling constant, \( m_N \) the nucleon mass, \( E_\nu \) the neutrino energy and \( Y_e \) the electron fraction. Resonance occurs when \( \Delta H = 0 \) and the conversion is adiabatic if

\[ \gamma \equiv \frac{\mu_B}{\sqrt{|\frac{d\Delta H}{dv}|}} \geq 1. \]  

(5)

If the neutrino is a Dirac particle, the effective potential is proportional to \((1 - 3Y_e)\), which is zero in the strongly neutronized core. However the oscillations are highly non-adiabatic in the high density core region for all but unrealistically high magnetic fields so that a fine tuning would be required for any oscillations to have a noticeable effect. Moreover SF-oscillations for Dirac neutrinos are strongly constrained by observations \[3,4,15,17\] and subsequently it will be assumed here that the neutrino is a Majorana particle.

It has been pointed out by Athar et al. \[7\] that in the isotopically neutral (i.e. number of protons equals number of neutrons) region above the iron core and below the hydrogen envelope the effective matter potential is suppressed by 3-4 orders of magnitude. Recently Totani and Sato \[10\] noted that the potential is even further suppressed in the O+C and He layers in the absence of heavy nuclei which, with their excess of neutrons, are mostly responsible for the deviation from isotopic neutrality and it turns out that the conversion probability is very sensitive to the metallicity of the supernova progenitor.

According to the electroweak standard model the neutrino does not possess a magnetic moment and an indication of a non-vanishing magnetic moment would be a sign for physics beyond the standard model and have profound implications for particle physics.

The current limit on the magnetic moment of the electron anti-neutrino from laboratory experiments is \( \mu_{\bar{\nu}_e} \leq 2.4 \times 10^{-10}\mu_B \) \[15,19\] for the Majorana neutrino. Astrophysical constraints on the magnetic moment from the SN1987A neutrino signal have been derived by Nötzold \[13\] and others \[16,17,20\] excluding values for \( \mu_\nu \) of greater than a few times \( 10^{-12}\mu_B \). For a review see \[21\]. In the following analysis a transition magnetic moment of
around $\mu_\nu \approx 10^{-12}\mu_B$ will be considered.

III. SUPERNOVA MODELS AND CONVERSION PROBABILITIES

The value of the effective matter potential $\Delta V$ was calculated from the composition and density profiles of the precollapse model by Woosley and Weaver (1995) [11], which includes about 200 isotopes up to $^{71}$Ge with solar abundances. The magnitude of $\Delta V$ as function of radius is shown in Fig. 1.(a) and (b) for a progenitor mass of 15 $M_\odot$ and 25 $M_\odot$ respectively. It is about $10^{-5}$ eV in the centre and quickly decreases by about 10 orders of magnitude as the density decreases and $Y_e$ approaches 0.5. Then at around 0.1 $R_\odot$ the effective potential falls discontinuously by several orders of magnitude and continues to decrease until it changes sign at roughly the solar radius, where the He layer ends and the hydrogen envelope starts. In this region $Y_e$ is very close to 0.5 (isotopically neutral) and the matter potential is suppressed by several orders of magnitude. In this region the deviation from $Y_e = 0.5$ is mainly due to isotopes like Ne, Mg, Al, S and Ar. Totani and Sato [10] have reduced the abundance of these isotopes over solar abundances thus suppressing the effective matter potential even further. These models, subsequently called low metallicity ($Z_0$) models, are shown in Fig. 1. (c) and (d). The investigation of low metallicity progenitors seems especially important given the low metallicity encountered in the LMC of around $Z \approx Z_\odot/4$ and various other observations that have led to the belief that the progenitor of SN1987A was a low metallicity blue super-giant (BSG). For a study of the progenitor of SN1987A see e.g. Langer et al. 1989 [22]. It can be seen from Fig.1 that in the $Z_0$ - models the isotopically neutral region is broader and the effective potential in this region is suppressed by another 1-2 magnitudes over the solar metallicity models.

For the purpose of calculating the conversion probabilities it can be assumed that the composition of the envelope remains unchanged during infall since the collapse is homologous and the shock wave cannot reach the isotopically neutral region during the neutrino burst (1-10 s). Resonance occurs in the isotopically neutral region for values of $10^{-10} \ eV^2/\text{MeV} < \frac{\Delta m^2}{E_\nu} <$
There is no direct information on the magnitude of magnetic fields inside a collapsing star, but a rough estimate may be obtained from the observed field on the surface of white dwarfs [23] suggesting magnetic fields of up to \( B_0 \approx 10^{10} \) Gauss at the surface of the iron core. Assuming that the radial dependence of the magnetic field above the core follows a dipole field, \( B \propto r^{-3} \), which would give a field in the isotopically neutral region of about \( 10^4 \) G. Higher estimates may be obtained by equating the magnetic field energy to the thermal energy of the gas (see [10]). In following analysis I will assume a dipole field with field strengths at the surface of the iron core of \( B_0 \approx 10^9 \) G and \( B_0 \approx 10^{10} \) G.

Also I assumed mixing parameters which are favoured by the MSW-interpretation of the solar neutrino problem: the large angle solution with \( \Delta m^2 \approx 10^{-6} \text{eV}^2 \), \( \sin^2 2\theta_v \approx 0.7 \), and the small angle solution with \( \Delta m^2 \approx 6 \times 10^{-6} \text{eV}^2 \) and \( \sin^2 2\theta_v \approx 10^{-2} \).

The conversion probabilities corresponding to the models discussed above have been calculated by Totani and Sato [10] and some typical cases are shown in Fig. 2 for \( \Delta m^2 = 10^{-6} \text{eV}^2 \) and \( \sin^2 2\theta_v \approx 10^{-2} \). For other mixing angles or neutrino masses the energy can simply be rescaled with \( \left( \frac{\text{eV}}{\text{MeV}} \frac{E_{\nu}}{\Delta m^2} \cos 2\theta_v \right) \).

These plots demonstrate that the magnitude of the conversion probability as well as the dependence on energy are very different for the solar and low metallicity models. For solar metallicity the conversion probability is generally very small for all but the most extreme values of \( B \) or very high values of \( \Delta m^2 \), which we will not consider here. For a field strength of \( B_0 \approx 10^9 \) G the conversion probability is negligible and not shown in Fig. 2 and even for \( B_0 \approx 10^9 \) G the conversion probability is hardly significant. As expected the efficiency of the conversion improves with increasing \( \mu B \) (and thus increasing \( \gamma \)). The conversion probability decreases with increasing \( E_\nu \) as seen from the dependence of \( \gamma \) on \( \frac{\Delta m^2}{E_\nu} \). However for progenitors of low metallicity the conversion probability increases with energy and remains constant at high energies due to the precession of the magnetic moment in the broad conversion region. The conversion probability is periodic in \( \mu B \) since the
precession length is dependent on \( \mu B \), which determines the phase at the outer edge of the isotopically neutral region. The conversion probability for a low metallicity, 25 M\(_\odot\) model is not shown here. Like for the 15M\(_\odot\) model conversion is effective for a field of \( B_0 = 10^{10} \) G but it contains a number of complicated features due to precession in the inner part of the isotopically neutral region.

Also it is found that in contrast to the metallicity the dependence on the progenitor mass is comparatively weak [10].

**IV. STATISTICAL ANALYSIS**

In total 19 neutrinos from SN1987A have been detected: Kamiokande recorded 11 and IMB eight events [3,4]. A maximum likelihood analysis is frequently used as an effective and unbiased method to interpret this sparse data [3,24]. I will be using the likelihood function suggested by Jegerlehner et al. [3]. If the expected spectrum of detected energies is \( n(E) \), the total number of events \( N_{\text{obs}} \) and the observed energies \( E_i \), the likelihood function \( L \) is given by

\[
L \propto \exp \left( \int_0^\infty n(E)dE \right)^{N_{\text{obs}}} \prod_{i=1}^{N_{\text{obs}}} n(E_i).
\]  

(6)

For a joint analysis of the signal from both detectors, the likelihood function is the product of the individual \( L \)'s. The expected spectrum of detected energies is given by

\[
n(E') = \int_0^\infty dE'P(E, E')\eta(E')n_p(E'),
\]  

(7)

where \( P \) denotes the probability of detecting an energy \( E \) if the real energy is \( E' \), which is assumed to be Gaussian with an energy-dependent dispersion:

\[
P(E, E') = \left( \frac{2\pi E_s E'}{\text{MeV}^2} \right)^{-1/2} \exp \left( -\frac{(E - E')^2}{2E_s E'} \right).
\]  

(8)

where \( E_s = 0.75 \) MeV for Kamiokande and \( E_s = 1.35 \) MeV for IMB. \( \eta \) represents the efficiency curve for the respective detector. Analytic fits to these curves [4] for Kamiokande are...
\[ \eta(E) = 0.93 - \exp[-(E/9.0 \text{ MeV})^{2.5}] \] (9)

for \( E \geq 7.0 \text{ MeV} \) and \( \eta = 0 \) for \( E \leq 7.0 \text{ MeV} \). For the IMB detector

\[ \eta(E) = 0.3975 \frac{E}{10 \text{ MeV}} - 0.02625 \left( \frac{E}{10 \text{ MeV}} \right)^2 - 0.59 \] (10)

for \( 1.9 < E/10 \text{ MeV} < 7.6 \), \( \eta = 0.915 \) for \( E/10 \text{ MeV} > 7.6 \) and \( \eta = 0 \) for \( E/10 \text{ MeV} < 1.9 \).

Lastly

\[ n_p(E) = \frac{N_p}{4\pi D^2} \sigma(E + Q) F_{\bar{\nu}_e}(E + Q), \] (11)

where \( Q \) is the mass difference between neutron and proton, \( Q = 1.29 \text{ MeV} \), \( \sigma = 2.295 \times 10^{-44} \text{ cm}^2 \) is the cross-section for the superallowed reaction \( \bar{\nu}_e(p, n)e^+ \), \( F_{\bar{\nu}_e} \) is the electron antineutrino flux at the detector, \( D = 50 \text{ kpc} \) is the distance to the supernova and \( N_p \) is the number of target protons in the detector: \( N_p = 1.43 \times 10^{32} \) for Kamiokande and \( N_p = 4.55 \times 10^{32} \) for IMB.

The likelihood function is a function of the free parameters of the model \( x_i \) and the data \( y_i \). The best-fit values \( \bar{x}_i \) are the parameters which maximize the likelihood function

\[ \mathcal{L}(\bar{x}, y) = \max \mathcal{L}(x, y) \] (12)

Under the assumption that \( \mathcal{L} \) is Gaussian, the confidence region around the best fit values \( \bar{x}_i \) can be estimated by

\[ \ln \mathcal{L}(\bar{x}, y) - \ln \mathcal{L}(x, y) \leq \frac{1}{2} \chi, \] (13)

where \( \chi \) is 2.3, 4.61 and 6.17 for the 68.3\%, 90\% and 95.4\% confidence levels respectively.

In the presence of SF-oscillations \( (\bar{\nu}_e \leftrightarrow \nu_\mu) \) the primary electron anti-neutrino spectrum becomes distorted due to the admixture of the \( \nu_\mu \) spectrum, so that the flux at some distance from the source is related to the primary spectra \( F_\nu^0 \) by

\[ F_{\bar{\nu}_e} = (1 - p) F_{\bar{\nu}_e}^0 + p F_{\nu_\mu}^0, \] (14)
where the conversion probability $p$ is a function of the neutrino energy $E$ and the mixing parameters. The time-integrated primary spectra are approximated by a Maxwell-Boltzmann distribution

$$F^0_\nu(E) = CE^2e^{-E/T_\nu}$$

with the parameters $C$ and $T_\nu$. Although there is little reason to believe that the actual spectra really follow a Maxwell-Boltzmann distribution, it allows to fit the first and second moments, $<E>$ and $<E^2>$, of the spectra and seems reasonable in the light of an exponential cooling model.

Equipartition of energy between all (anti-) neutrino species is assumed, but different neutrino ‘temperatures’ for the $\bar{\nu}_e$s and $\nu_\mu$s, $T_{\bar{\nu}_e}$ and $T_{\nu_\mu}$, are allowed. Due to their lower interaction cross-section the temperature of the $\nu_\mu$s is higher and I define

$$T_{\nu_\mu} \equiv \alpha T_{\bar{\nu}_e}$$

with $\alpha$ predicted to lie between 1.5 and 2.0. The total energy $E_t$ is six times the energy emitted in $\bar{\nu}_e$s which is found to be

$$E_t = 36CT^4.$$ \hspace{1cm} (17)

As in Ref. [6] the detector background is expected to have a negligible influence on the results and is ignored here.

V. NUMERICAL RESULTS

Using the method described above the best-fit values for $T_{\bar{\nu}_e}$ and $E_t$ were first found in the absence of oscillations. The Kamiokande data yielded a value for $T_{\bar{\nu}_e}$ of 2.6 MeV with a total energy of $E_t = 4.8 \times 10^{53}$ erg while the IMB data gave $T_{\bar{\nu}_e} = 3.6$ MeV and $E_t = 4.7 \times 10^{53}$ erg. These results are in good agreement with the values found by Jegerlehner et al. [6] of $T_{\bar{\nu}_e} = 2.5$ MeV and $E_t = 4.9 \times 10^{53}$ erg for Kamiokande and $T_{\bar{\nu}_e} = 3.7$ MeV and $E_t = 5.4 \times 10^{53}$ erg for IMB and thus independently confirm their results.
Subsequently I calculated the best-fit parameters for a variety of models. Best fit values for $E_t$ and $T_{\bar{\nu}_e}$ together with errorbars indicating the 90% confidence levels are shown in Fig. 3 and 4 for the Kamiokande and the IMB data respectively. For the $Z_\odot$ - models the mixing is too small to have a significant effect and the results are virtually indistinguishable from the ones obtained in the absence of oscillations and not shown in Fig. 3 and 4. However the strong conversion in the $Z_0$- models shifts the inferred $T_{\bar{\nu}_e}$ to significantly lower values. Also a higher relative temperature of the muon neutrinos (i.e. a higher $\alpha$) lowers the inferred neutrino temperature. For the Kamiokande data $T_{\bar{\nu}_e}$ is lowered to 1.9 MeV and 1.5 MeV for $\alpha = 1.5$ and 2.0 respectively. Similarly for the IMB set the inferred value for $T_{\bar{\nu}_e}$ is 2.5 MeV and 1.8 MeV.

The overlap between the values deduced from the two data sets is poor and I refrain from a joint analysis of the data.

VI. CONCLUSION

In agreement with other authors it is found that in the absence of oscillations the results for the total energy released in neutrinos are in excellent concordance with supernova theory (1), whereas the temperature $T_{\bar{\nu}_e}$ lies slightly below the predicted range (2). The lower end of the predicted range lies within the 90% confidence region.

As expected any admixture of the hotter $\nu_\mu$ spectrum through SF-oscillations aggravates the discrepancy between the observed and predicted neutrino spectra. It is seen that a bound on the neutrino transition magnetic moment $\mu_\nu$ which is deduced from the observation of SN neutrinos is strongly dependent on the metallicity of the progenitor. It is found that a low metallicity can substantially increase the sensitivity to $\mu$ and has to be taken into account when deducing bounds on $\mu_\nu$ from supernova neutrinos. Therefore bounds on $\mu_\nu$ derived from the neutrino signal from SN1987A may be less stringent than earlier thought [7]. For solar metallicities the data from SN1987A is consistent with a magnetic moment of $\mu_\nu \approx 10^{-12} \mu_B$ for any realistic magnetic field inside the supernova and mixing parameters
chosen from the MSW solution to the solar neutrino puzzle. However lower metallicities as found in the LMC make a magnetic moment as high as $10^{-12} \mu_B$ seem less likely unless the magnetic field is lower than argued here.

Thus under the assumption that the progenitor of SN1987A in fact had a metallicity as low as $Z_0$ and a magnetic field as high as $B_0 \approx 10^{10}$ G the data seems to restrict the magnetic moment to $\mu_\nu < 10^{-13} \mu_B$.

The aim of this article is to point out the dependence of the inferred value for the neutrino magnetic moment on the metallicity of the progenitor thus highlighting the uncertainties involved in interpreting the data. I should emphasize again that the poor statistics of the data as well as the many unknowns of the source do not allow a reliable determination of neutrino properties such as the transition magnetic moment. However there is hope that with the advent of a new generation of neutrino telescopes as Superkamiokande, the Large Volume Detector (LVD), the Sudbury Neutrino Observatory (SNO) and others a future supernova explosions in our neighbourhood will provide us with richer data allowing better estimates of neutrino properties and variables of the supernova explosion.

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FIGURES

FIG. 1. Magnitude of the effective potential $\Delta V$ as a function of radius for several progenitor masses and metallicities. $\Delta V$ is measured in eV and the radius is measured in units of solar radii. (a) $15 \, M_\odot$, $Z_\odot$, (b) $15 \, M_\odot$, $Z_0$, (c) $25 \, M_\odot$, $Z_\odot$, (d) $25 \, M_\odot$, $Z_0$

FIG. 2. Conversion probabilities as function of neutrino energy $E_\nu$, solid line: $Z_\odot$, $15 \, M_\odot$, $B_0 = 10^{10}$ G, dotted line: $Z_\odot$, $25 \, M_\odot$, $B_0 = 10^{10}$ G, dashed line: $Z_0$, $15 \, M_\odot$, $B_0 = 10^{10}$ G, dot-dashed line: $Z_0$, $25 \, M_\odot$, $B_0 = 10^9$ G

FIG. 3. Best fit values for $E_t$ and $T_{\bar{\nu}_e}$ for the Kamiokande data. The errorbars correspond to 90% confidence level.

FIG. 4. The same as Fig. 3 but for the IMB data.
Kamiokande

$\alpha = 2.0$

$\alpha = 1.5$

no mixing
