$B_s^0 - \bar{B}_s^0$ mixing and $b \to s$ transitions in isosinglet down quark model

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Abstract

The recent observation of the mass difference in $B_s$ system seems to be not in complete agreement with the corresponding standard model value. We consider the model with an extra vector like down quark to explain this discrepancy and obtain the constraints on the new physics parameters. Thereafter, we show that with these new constraints this model can successfully explain other observed deviations associated with $b \to s$ transitions, namely, $B_s \to \psi \phi$, $B \to K \pi$ and $B \to \phi K_s$.

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I. INTRODUCTION

The results of the currently running two asymmetric B factories confirmed the fact that the phenomenon of CP violation in the Standard Model (SM) is due to the complex phase in the CKM quark mixing matrix \([1]\). The observed data are almost in the line of the SM expectations and there is no clear indication of new physics so far. However, there are some interesting deviations from that of the SM expectations which could provide us an indirect signal of new physics. Here we are concentrating on few such deviations which are associated with the CP violation parameters of flavor changing neutral current (FCNC) mediated \(b \to s\) transitions. A partial list includes:

- The observed mass difference measured between heavy and light \(B_s\) mesons \([2]\) seems to be inconsistent with its SM value with a deviation of few sigma.
- The observed discrepancy between the measured \(S_{\phi K_s}\) and \(S_{\psi K_s}\) \([3]\) already gave an indication of the possible existence of NP in the \(B \to \phi K_s\) decay amplitude. Within the SM, these CP symmetries are expected to be same with a deviation of about 5% \([4]\).
- The recent observation of a very large \(S_{\psi \phi}\) by the CDF collaboration \([5]\) is in contrast to its expected SM value i.e., \(S_{\psi \phi} \approx 0\). This may be considered as a clear signal of new physics in the \(b \to s\) transitions.
- There appears to be some disagreement between the direct CP asymmetry parameters of \(B^- \to \pi^0 K^-\) and that of the \(\bar{B}^0 \to \pi^+ K^-\), \(\Delta A_{CP}(K\pi)\), which is the difference of these two parameters, is found to be around 15% \([3]\), whereas the SM expectation is vanishingly small. This constitutes what is called \(\pi K\) puzzle in the literature and is believed to be an indication of the existence of new physics.
- \(B_s \to \mu^+ \mu^-\) problem has been widely discussed in the literature. The SM value is quite small (we have only upper limit for the branching ratio) and it is very clean mode so if we have any smoking gun signal of new physics elsewhere in \(b \to s\) transitions it is quite likely that it could also be found in this mode. Therefore, \(B_s \to \mu^+ \mu^-\) is a golden mode to detect new physics.

In this paper, we would like to see the effect of the extra vector like down quark \([6]\) in explaining the above mentioned observed discrepancies. It is a simple model beyond the standard model with an enlarged matter sector due to an additional vector like down quark \(D_4\). Isosinglet quarks appear in many extensions of the SM like the low energy limit of
the $E_6$ GUT models \cite{7}. The mixing of this singlet type down quark with the three SM down type quarks provides a framework to study the deviations of the unitarity constraint of the $3 \times 3$ CKM matrix. To be more explicit, the presence of an additional down quark implies a $4 \times 4$ matrix $V_{i\alpha}$ ($i = u, c, t, 4, \alpha = d, s, b, b'$) would diagonalize the down quark mass matrix. Due to this, some new features appear in the low energy phenomenology. The charged currents are unchanged except that the $V_{CKM}$ is now the $3 \times 4$ upper sub-matrix of $V$. However, the distinctive feature of this model is that the FCNC interaction enters at tree level in the neutral current Lagrangian of the left handed down quarks as \cite{6}

$$L_Z = \frac{g}{2 \cos \theta_W} \left[ \bar{u}_L \gamma^\mu u_L - \bar{d}_L U_{\alpha\beta} \gamma^\mu d_L - 2 \sin^2 \theta_W J_{em}^\mu \right] Z_\mu \ , \quad (1)$$

with

$$U_{\alpha\beta} = \sum_{i=u, c, t} V_{i\alpha}^* V_{i\beta} = \delta_{\alpha\beta} - V^*_{4\alpha} V_{4\beta} \ , \quad (2)$$

where $U$ is the neutral current mixing matrix for the down sector, which is given above. As $V$ is not unitary, $U \neq 1$. In particular the non-diagonal elements do not vanish

$$U_{\alpha\beta} = -V^*_{4\alpha} V_{4\beta} \neq 0 \quad \text{for} \quad \alpha \neq \beta \ . \quad (3)$$

Since the various $U_{\alpha\beta}$ are non vanishing they would signal new physics and the presence of FCNC at the tree level which can substantially modify the predictions of SM for the FCNC processes. Of course, these low energy couplings are severely restricted by the low energy results available on different FCNC processes i.e., $Br(K_L \rightarrow \mu \bar{\mu})_{SD}$, $Br(K^+ \rightarrow \pi^+ \nu\bar{\nu})$, $\epsilon_K$, $\Delta M_K$, $\Delta M_{B_d}$, $\Delta M_{B_s}$, $Br(B \rightarrow X_{d,s} l^+l^-)$ etc \cite{8}. Nevertheless, it is well known that even fulfilling these strong constraints there could still be large effects on $B$ factory experiments on CP violation. The implications of the FCNC mediated $Z$ boson effect has been extensively studied in the context of $b$ physics \cite{9, 10, 11}.

**II. $B_s - \bar{B}_s$ MIXING**

We will first concentrate on the mass difference between the neutral $B_s$ meson mass eigenstates ($\Delta M_s$) that characterizes the $B_s - \bar{B}_s$ mixing phenomena. In the SM, $B_s - \bar{B}_s$ mixing occurs at the one-loop level by flavor-changing weak interaction box diagrams and hence is very sensitive to new physics effects.
In the SM, the effective Hamiltonian describing the $\Delta B = 2$ transition, induced by the box diagram, is given by [12]

$$H_{\text{eff}} = \frac{G_F^2}{16\pi^2} \lambda_t^2 M_W^2 S_0(x_t) \eta_t (\bar{s}b)_{V-A} (\bar{s}b)_{V-A}$$

where $\lambda_t = V_{tb} V_{ts}^*$, $\eta_t$ is the QCD correction factor and $S_0(x_t)$ is the loop function

$$S_0(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1-x_t)^2} - \frac{3\log x_t x_t^3}{2(1-x_t)^3}$$

with $x_t = m_t^2/M_W^2$. Thus, the $B_s - \bar{B}_s$ mixing amplitude in the SM can be written as

$$M_{12}^{\text{SM}} = \frac{1}{2M_{B_s}} (\bar{B}_s | H_{\text{eff}} | B_s) = \frac{G_F^2}{12\pi^2} M_W^2 \lambda_t^2 \eta_t B_{s} f_{B_s}^2 M_{B_s} S_0(x_t)$$

where the vacuum insertion method has been used to evaluate the matrix element

$$\langle \bar{B}_s | (\bar{s}b)_{V-A} (\bar{s}b)_{V-A} | B_s \rangle = \frac{8}{3} B_s f_{B_s}^2 M_{B_s}^2$$

The corresponding mass difference is related to the mixing amplitude through $\Delta M_s = 2 |M_{12}|$.

Recently, Lenz and Nierste [13] updated the theoretical estimation of the $B_s$ mass difference in the SM, with the value $(\Delta M_{B_s})^{\text{SM}} = (19.30 \pm 6.68) \text{ ps}^{-1}$ (for Set-I parameters) and $(\Delta M_{B_s})^{\text{SM}} = (20.31 \pm 3.25) \text{ ps}^{-1}$ (Set-II).

The CDF [2] and DØ [14] collaborations have also recently reported new results for the $B_s - \bar{B}_s$ mass difference

$$\Delta M_{B_s} = (17.77 \pm 0.10 \pm 0.07) \text{ ps}^{-1} \quad (\text{CDF})$$

$$17 \text{ ps}^{-1} < \Delta M_{B_s} < 21 \text{ ps}^{-1} \quad 90\% \text{ C.L. (DØ)} .$$

Although the experimental results appear to be consistent with the standard model prediction, but they do not completely exclude the possible new physics effects in $\Delta B = 2$ transitions. In the literature, there have already been many discussions both in model independent [15, 16, 17] and model dependent way [18] regarding the implications of these new measurements. In this work we would like to see the effect of the extended isosiglet down quark model on the mass difference of $B_s$ system and its possible implications for the other $b \rightarrow s$ transition processes.

In the model with an extra vector like down quark there will be two additional contributions to the $B_s - \bar{B}_s$ mixing amplitude. The first one is induced by tree level FCNC
mediated $Z$ boson, with two non-standard (flavor-changing) $Z - b - s$ coupling as shown in Figure-1(a) and the second contribution contains one non-standard $Z - b - s$ coupling and one SM loop-induced $Z - b - s$ coupling as depicted in Figure-1(b). With these new contributions the mass difference between $B_s^H$ and $B_s^L$ deviates significantly from its SM value.

To evaluate these two additional contributions, one can write from Eq. (1) the effective FCNC mediated Lagrangian for $Zbs$ interaction as

$$
\mathcal{L}_{FCNC}^Z = -\frac{g}{2\cos\theta_W} U_{sb} \bar{s}_L \gamma^\mu b_L Z_\mu .
$$

(9)

This gives the effective Hamiltonian induced by tree level FCNC mediated $Z$ boson (Figure-1(a)) as

$$
\mathcal{H}_{eff}^Z = \frac{G_F}{\sqrt{2}} U_{sb}^2 \eta_Z (\bar{s}_L \gamma^\mu b_L) (\bar{s}_L \gamma b_L),
$$

(10)

where $\eta_Z = (\alpha_s(m_Z))^6/23$ is the QCD correction factor. Using the matrix elements as defined in Eq. (7) we obtain

$$
M_{12}^Z = \frac{G_F}{3\sqrt{2}} U_{sb}^2 \eta_Z B_s f_{B_s}^2 M_{B_s} .
$$

(11)

The effective Hamiltonian induced by the SM penguin at one vertex and $Z$ mediated FCNC coupling on the other (Figure-1(b)) is given as

$$
\mathcal{H}_{eff}^{SM+Z} = \frac{G_F^2}{4\pi^2} \lambda_t \eta_{Zt} M_W^2 U_{sb} C_0(x_t) (\bar{s}b)_{V-A} (\bar{s}b)_{V-A}
$$

(12)

where $\eta_{Zt}$ is the QCD correction factor and

$$
C_0(x_t) = \frac{x_t}{8} \left( \frac{x_t - 6}{x_t - 1} + \frac{3x_t + 2}{(x_t - 1)^2} \log x_t \right).
$$

(13)

This gives

$$
M_{12}^{SM+Z} = \frac{G_F^2}{3\pi^2} \lambda_t U_{sb} \eta_{Zt} M_W^2 C_0(x_t) B_s f_{B_s}^2 M_{B_s} .
$$

(14)
Thus, the mass difference $\Delta M_s$ in this model can be given as

$$\Delta M_s = 2 |M_{12}^{SM} + M_{12}^Z + M_{12}^{SM+Z}| = \Delta M_s^{SM} \left| 1 + a \left( \frac{U_{sb}}{\lambda_t} \right) + b \left( \frac{U_{sb}}{\lambda_t} \right)^2 \right|$$

(15)

with

$$a = 4 \frac{C_0(x_t)}{S_0(x_t)}, \quad b = \frac{2\sqrt{2}\pi^2}{G_FM_W^2 S_0(x_t)},$$

(16)

where we have assumed $\eta_t \approx \eta_Z \approx \eta_{Zt}$. The coupling $U_{sb}$ characterizing the $Z - b - s$ strength is in general complex and can be parameterized as $U_{sb} = |U_{sb}| e^{i\phi_s}$, where $\phi_s$ is the new weak phase. The constraints on these parameters can be obtained using the recent measurement on $\Delta M_s$.

Since $V_{tb}V_{ts}^* = -|V_{tb}V_{ts}| e^{i\beta_s}$, we parametrize

$$\frac{U_{sb}}{V_{tb}V_{ts}} = -\left| \frac{U_{sb}}{V_{tb}V_{ts}} \right| e^{i(\phi_s - \beta_s)} \equiv -x e^{i(\phi_s - \beta_s)}. \quad (17)$$

For numerical evaluation we use the CKM elements as $|V_{tb}| = 0.999176^{+0.000031}_{-0.000044}$, $|V_{ts}| = 0.03972^{+0.00015}_{-0.000077}$, $\beta_s = -1.1^\circ$, the masses of $W$ boson and $t$ quark as $M_W = 80.4$ GeV, $m_t = 168$ GeV. For $\Delta M_s$, we use the CDF result [2] $\Delta M_s = 17.77 \pm 0.12$ ps$^{-1}$ and for $\Delta M_s^{SM} = 19.30 \pm 6.68$ ps$^{-1}$ [13], which yields $\Delta M_s/\Delta M_s^{SM} = 0.92 \pm 0.32$. Varying $(\Delta M_s/\Delta M_s^{SM})$ within its $1 - \sigma$ range the allowed parameter space in the $\phi_s - |U_{sb}|$ plane is shown in Figure-2. From the figure it can be seen that for higher value of $|U_{sb}|$ the phase $\phi_s$ is very tightly constrained. However, for $|U_{sb}| \leq 0.0015$ there is no constraint on the new weak phase $\phi_s$ i.e., the whole range $0 - 2\pi$ is allowed. The constraint on $|U_{sb}|$ obtained from $B \to X_s l^+ l^-$, i.e., $|U_{sb}| \leq 0.002$, [8] is consistent with the constraint obtained from $B_s - \bar{B}_s$ mixing. We now use the allowed values of $|U_{sb}|$ (i.e., we use $|U_{sb}| \leq 0.002$ so that constraints coming from both the observables will be satisfied) and $\phi_s$ to study some anomalies associated with $b \to s$ transitions. In particular, we would like to see whether the constraints obtained above in the extended isosinglet down quark model, consistent with $B_s - \bar{B}_s$ mixing, can also explain the discrepancies in the modes $B_s \to \psi\phi$, $B_s \to \mu^+ \mu^-$, $B \to \pi K$ and $B \to \phi K_s$.

III. MIXING INDUCED CP ASYMMETRY IN $B_s \to J/\psi\phi$ ($S_{\psi\phi}$)

We now consider the effect of the isosinglet down quark on the mixing induced CP asymmetry in $B_s \to J/\psi\phi$ mode. Recently a very largish CP asymmetry has been measured by
FIG. 2: The 1 − σ allowed range of (ΔM_s/ΔM_s^{SM}) in the φ_s − |U_{sb}| plane.

the CDF collaboration [5] in the tagged analysis of B_s → J/ψφ with value S_{ψφ} ∈ [0.23, 0.97].

Within the SM this asymmetry is expected to be vanishingly small, which comes basically from B_s − B_s mixing phase. Since this mode receives dominant contribution from b → cūs tree level transition, the NP contribution to its decay amplitude is naively expected to be negligible. Therefore, the observed large CP asymmetry is believed to be originating from the new CP violating phase in B_s − B_s mixing.

Now parameterizing the new physics contribution to the B_s − B_s mixing amplitude as

\[ M_{12} = M_{12}^{SM} + M_{12}^{Z} + M_{12}^{SM+Z} = M_{12}^{SM} C_{B_s} e^{2iθ_s}, \]  

one can obtain

\[ S_{ψφ} = -\eta_{ψφ} \sin(2|β_s| + 2θ_s), \]  

where β_s is the phase of V_{ts} = −|V_{ts}|e^{−iβ_s} and η_{ψφ} is the CP parity of the ψφ final state. Taking η_{ψφ} = +1 and β_s ≈ −1.1° we obtain the mixing induced CP asymmetry as

\[ S_{ψφ} = \sin(2|β_s| - 2θ_s). \]  

Now substituting the expressions for M_{12}^{SM}, M_{12}^{Z} and M_{12}^{SM+Z} from Eqs. (6), (11) and (14) in Eq. (18), we obtain the new CP-odd phase of B_s − B_s mixing as

\[ 2θ_s = \arctan \left( \frac{-a \ x \sin(φ_s + |β_s|) + b \ x^2 \sin(2φ_s + 2|β_s|)}{1 - a \ x \cos(φ_s + |β_s|) + b \ x^2 \cos(2φ_s + 2|β_s|)} \right), \]  

where a, b and x are defined in Eqs. (16) and (17) respectively. In Figure-3 we show the variation of S_{ψφ} (20) with the new weak phase φ_s for two representative values of |U_{sb}|. From
FIG. 3: Variation of $S_{\psi\phi}$ with the new weak phase $\phi_s$ where the solid and dotted lines are for $|U_{sb}| = 0.002$ and $0.0015$ respectively. The horizontal line represents the lower limit of the experimental value.

The figure it can be seen that the observed largish $S_{\psi\phi}$ can be explained in the model with an extra vector-like down quark for $|U_{sb}| \geq 0.0015$.

IV. $B_s \rightarrow \mu^+\mu^-$

Now let us consider the FCNC mediated leptonic transition $B_s \rightarrow \mu^+\mu^-$. This decay mode has attracted a lot of attention recently since it is very sensitive to the structure of the SM and potential source of new physics beyond the SM. Furthermore, this process is very clean and the only nonperturbative quantity involved is the decay constant of $B_s$ meson which can be reliably calculated by the well known non-perturbative methods such as QCD sum rules, lattice gauge theory etc. Therefore, it provides a good hunting ground to look for for new physics. The recent updated branching ratio $\text{Br}(B_s \rightarrow \mu^+\mu^-) = (3.35 \pm 0.32) \times 10^{-9}$ in the SM [16] is well below the present experimental upper limit [3]

$$\text{Br}(B_s \rightarrow \mu^+\mu^-) < 4.7 \times 10^{-8}.$$  \hspace{1cm} (22)

This decay has been analyzed in many beyond the SM scenarios in a number of papers [20]. Let us start by recalling the result for $B_s \rightarrow \mu^+\mu^-$ in standard model. The effective
Hamiltonian describing this process is

\[ H_{\text{eff}} = \frac{G_F \alpha}{\sqrt{2} \pi} V_{ts} V_{ts}^* \left[ C_9 (\bar{s} \gamma_\mu P_L b)(\bar{\mu} \gamma^\mu \mu) + C_{10} (\bar{s} \gamma_\mu P_L b)(\bar{\mu} \gamma^\mu \gamma_5 \mu) 
- \frac{2C_7 m_b}{q^2} (\bar{s} i\sigma_{\mu\nu} q^\nu P_R b)(\bar{\mu} \gamma^\mu \mu) \right], \]

(23)

where \( P_{L,R} = \frac{1}{2} (1 \mp \gamma_5) \) and \( q \) is the momentum transfer. \( C_i \)'s are the Wilson coefficients evaluated at the \( b \) quark mass scale in NLL order with values \cite{21}

\[ C_7 = -0.308, \; C_9 = 4.154, \; C_{10} = -4.261. \]

(24)

To evaluate the transition amplitude one can generally adopt the vacuum insertion method, where the form factors of the various currents are defined as follows

\[ \langle 0 | \bar{s} \gamma_\mu \gamma_5 b | B_s^0 \rangle = i f_{B_s} p_B^\mu, \quad \langle 0 | \bar{s} \gamma_5 b | B_s^0 \rangle = i f_{B_s} m_{B_s}, \quad \langle 0 | \bar{s} \sigma_{\mu\nu} P_R b | B_s^0 \rangle = 0. \]

(25)

Since \( p_B^\mu = p_+^\mu + p_-^\mu \), the contribution from \( C_9 \) term in Eq. \( (23) \) will vanish upon contraction with the lepton bilinear, \( C_7 \) will also give zero by \( (25) \) and the remaining \( C_{10} \) term will get a factor of \( 2m_\mu \).

Thus the transition amplitude for the process is given as

\[ \mathcal{M}(B_s \to \mu^+ \mu^-) = i \frac{G_F \alpha}{\sqrt{2} \pi} V_{ts} V_{ts}^* f_{B_s} C_{10} m_\mu (\bar{\mu} \gamma_5 \mu), \]

(26)

and the corresponding branching ratio is given as

\[ Br(B_s \to \mu^+ \mu^-) = \frac{G_F^2 \tau_{B_s}}{16 \pi^3} \alpha^2 f_{B_s}^2 m_{B_s}^2 m_\mu^2 |V_{tb} V_{ts}^*|^2 C_{10}^2 \sqrt{1 - \frac{4m_\mu^2}{m_{B_s}^2}}. \]

(27)

Helicity suppression is reflected by the presence of \( m_\mu^2 \) in \( (27) \) which gives a very small branching ratio of \( (3.35 \pm 0.32) \times 10^{-9} \) for \( \mu^+ \mu^- \) \cite{16}.

Now let us analyze the decay modes \( B_s \to \mu^+ \mu^- \) in the model with the \( Z \) mediated FCNC occurring at the tree level. The effective Hamiltonian for \( B_s \to \mu^+ \mu^- \) is given as

\[ H_{\text{eff}} = \frac{G_F}{\sqrt{2}} U_{sb} \left[ \bar{s} \gamma^\mu (1 - \gamma_5) b ] \left[ \bar{\mu} (C_V^\mu \gamma_\mu - C_A^\mu \gamma_\mu \gamma_5 \mu) \right], \]

(28)

where \( C_V^\mu \) and \( C_A^\mu \) are the vector and axial vector \( Z \mu^+ \mu^- \) couplings, which are given as

\[ C_V^\mu = -\frac{1}{2} + 2 \sin^2 \theta_W, \quad C_A^\mu = -\frac{1}{2}. \]

(29)
Since, the structure of the effective Hamiltonian (28) in this model is the same form as that of the SM, like $\sim (V - A)(V - A)$ form, therefore its effect on the various decay observables can be encoded by replacing the SM Wilson coefficients $C_9$ and $C_{10}$ by

$$C_9^{\text{eff}} = C_9 + \frac{2\pi U_{sb} C_V^\mu}{\alpha V_{tb} V_{ts}^*}, \quad C_{10}^{\text{eff}} = C_{10} - \frac{2\pi U_{sb} C_A^\mu}{\alpha V_{tb} V_{ts}^*}. \quad (30)$$

Thus, one can obtain the branching ratio including the NP contributions by substituting $C_{10}^{\text{eff}}$ from (30) in (27). Now varying the the value $|U_{sb}|$ between 0 and $1.5 \times 10^{-3}$ and the phase $\phi_s$ between $(0 - 360)^{\circ}$ the branching ratio for $B_s \to \mu^+\mu^-$ is shown in Figure-4. From the figure one can conclude that the branching ratio of $B_s \to \mu^+\mu^-$ in this model can be significantly enhanced from its SM value. Observation of this mode in the upcoming experiments will provide additional constraints on the new physics parameters.

FIG. 4: The allowed range of the branching ratio for $B_s \to \mu^+\mu^-$ process in the $B_r - |U_{sb}|$ plane. The horizontal line represents the experimental upper limit.

V. $\Delta A_{CP}(K\pi)$ PUZZLE

The $\Delta A_{CP}(K\pi)$ puzzle refers to the difference in direct CP asymmetries in $B^- \to \pi^0 K^-$ and $\bar{B}^0 \to \pi^+ K^-$ modes. These two modes receive similar dominating contributions from tree and QCD penguin diagrams and hence one would naively expect that these two channels will have the same direct CP asymmetries i.e., $A_{\pi^0 K^-} = A_{\pi^+ K^-}$. In the QCD factorization approach, the difference between these asymmetries is found to be $^{22}$

$$\Delta A_{CP} = A_{K^+\pi^-} - A_{K^-\pi^0} = (2.5 \pm 1.5)\% \quad (31)$$
whereas the corresponding experimental value is

\[ \Delta A_{CP} = (14.8 \pm 2.8)\% , \tag{32} \]

which yields nearly 4\( \sigma \) deviation.

In the SM, the relevant effective Hamiltonian describing the decay modes \( B^- \to \pi^0 K^- \) and \( \bar{B}^0 \to \pi^+ K^- \) is given by

\[
\mathcal{H}_{\text{eff}}^{\text{SM}} = \frac{G_F}{\sqrt{2}} \left[ V_{ub} V_{us}^* (C_1 O_1 + C_2 O_2) - V_{tb} V_{ts}^* \sum_{i=3}^{10} C_i O_i \right], \tag{33}
\]

where \( C_i \)'s are the Wilson coefficients evaluated at the \( b \) quark mass scale and \( O_i \)'s are the four-quark current operators.

Thus, one can obtain the transition amplitudes in the QCD factorization approach as \[23\], where the CKM unitarity \( \lambda_u + \lambda_c + \lambda_t = 0 \) has been used

\[
\sqrt{2} A(B^- \to \pi^0 K^-) = \lambda_u (A_{\pi K} \alpha_1 + \beta_2 + A_{K\pi} \alpha_2) + \sum_{q=u, c} \lambda_q \left( A_{\pi K} (\alpha_4^q + \alpha_{4,EW}^q + \beta_3^q + \beta_{3,EW}^q) + \frac{3}{2} A_{K\pi} \alpha_{3,EW}^q \right), \tag{34}
\]

and

\[
A(B^0 \to \pi^+ K^-) = \lambda_u (A_{\pi K} \alpha_1) + \sum_{q=u, c} \lambda_q A_{\pi K} \left( \alpha_4^q + \alpha_{4,EW}^q + \beta_3^q - \frac{1}{2} \beta_{3,EW}^q \right), \tag{35}
\]

where

\[
A_{\pi K} = i \frac{G_F}{\sqrt{2}} M_B^2 F_{B^0 \to \pi}^0 \alpha K \quad \text{and} \quad A_{K\pi} = i \frac{G_F}{\sqrt{2}} M_B^2 F_{B^0 \to K}^0 \alpha \pi . \tag{36}
\]

The parameters \( \alpha_i \)'s and \( \beta_i \)'s are related to the Wilson coefficients \( C_i \)'s and the corresponding expressions can be found in \[23\].

To account for this discrepancy here we consider the effect of the extra isosinglet down quark. As discussed earlier, in this model the \( Z \) mediated FCNC interaction is introduced at the tree level as shown in Eq. \[9\]. Because of the new interactions the effective Hamiltonian describing \( b \to s\bar{s}s\bar{s} \) process receives the additional contribution given as \[10\],

\[
\mathcal{H}_{\text{eff}}^Z = - \frac{G_F}{\sqrt{2}} [ \tilde{C}_3 O_3 + \tilde{C}_7 O_7 + \tilde{C}_9 O_9 ], \tag{37}
\]

where the four-quark operators \( O_3 \), \( O_7 \) and \( O_9 \) have the same structure as the SM QCD and electroweak penguin operators and the new Wilson coefficients \( \tilde{C}_i \)'s at the \( M_Z \) scale are
given by

\[ \tilde{C}_3(M_Z) = \frac{1}{6} U_{sb}, \]
\[ \tilde{C}_7(M_Z) = \frac{2}{3} U_{sb} \sin^2 \theta_W, \]
\[ \tilde{C}_9(M_Z) = -\frac{2}{3} U_{sb} (1 - \sin^2 \theta_W). \]  

(38)

These new Wilson coefficients will be evolved from the \( M_Z \) scale to the \( m_b \) scale using renormalization group equation given in [24], as

\[ \vec{C}(m_b) = U_5(m_b, M_W, \alpha) \vec{C}(M_W), \]  

(39)

where \( \vec{C} \) is the 10 \times 1 column vector of the Wilson coefficients and \( U_5 \) is the five flavor 10 \times 10 evolution matrix. The explicit forms of \( \vec{C}(M_W) \) and \( U_5(m_b, M_W, \alpha) \) are given in [24] as described earlier. Because of the RG evolution these three Wilson coefficients generate new set of Wilson coefficients \( \tilde{C}_i (i = 3, \cdots, 10) \) at the low energy regime (i.e., at the \( m_b \) scale) as presented in Table-1, where we have used \( \sin^2 \theta_W = 0.231 \).

| \( \tilde{C}_3 \) | \( \tilde{C}_4 \) | \( \tilde{C}_5 \) | \( \tilde{C}_6 \) | \( \tilde{C}_7 \) | \( \tilde{C}_8 \) | \( \tilde{C}_9 \) | \( \tilde{C}_{10} \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.19U_{sb}      | -0.066U_{sb}    | 0.009U_{sb}     | -0.031U_{sb}    | 0.145U_{sb}     | 0.053U_{sb}     | -0.566U_{sb}    | 0.127U_{sb}     |

TABLE I: Values of the new Wilson coefficients at the \( m_b \) scale.

As discussed earlier, due to the presence of the additional isosinglet down quark the unitarity condition becomes \( \lambda_u + \lambda_c + \lambda_t = U_{sb} \). Thus, replacing \( \lambda_t = U_{sb} - (\lambda_u + \lambda_c) \), one can write the transition amplitudes including the new contributions as

\[
\sqrt{2} A(B^- \to \pi^0 K^-) = \lambda_u (A_{\pi K}(\alpha_1 + \beta_2) + A_{\bar{K}\pi} \alpha_2) \\
+ \sum_{q=u, c} \lambda_q (A_{\pi K}(\alpha_1^q + \alpha_4^q + \alpha_5^q + \beta_3 + \beta_3^{q,EW}) + \frac{3}{2} A_{\bar{K}\pi} \alpha_3^{q,EW}) \\
- U_{sb} (A_{\pi K}(\Delta \alpha_4 + \Delta \alpha_4^{EW} + \Delta \beta_3 + \Delta \beta_3^{EW}) + \frac{3}{2} A_{\bar{K}\pi} \Delta \alpha_3^{EW}) \]  

(40)

and

\[
A(\bar{B}^0 \to \pi^+ K^-) = \lambda_u (A_{\pi K} \alpha_1) + \sum_{q=u, c} \lambda_q A_{\pi K} (\alpha_1^q + \alpha_1^{q,EW} + \beta_3^q - \frac{1}{2} \beta_3^{q,EW}) \\
- U_{sb} A_{\pi K} (\Delta \alpha_4 + \Delta \alpha_4^{EW} + \Delta \beta_3 - \frac{1}{2} \Delta \beta_3^{EW}), \]  

(41)
where $\Delta \alpha_i$’s and $\Delta \beta_i$’s are related to the modified Wilson coefficients $\Delta C_i = \tilde{C}_i(m_b) + C_i^0(m_b)$, where $C_i^0(m_b)$’s are the values of the Wilson coefficients at the $m_b$ scale due to $t$ quark exchange.

Thus, including the new contributions one can symbolically represent these amplitudes as

$$Amp = \lambda_u A_u + \lambda_c A_c - U_{sb} A_{new}. \quad (42)$$

$\lambda$’s and $U_{bs}$ contain the weak phase information and $A_i$’s are associated with the strong phases. Thus one can explicitly separate the strong and weak phases and write the amplitudes as

$$Amp = \lambda_c A_c \left[ 1 + r a e^{i(\delta_1 - \gamma)} - r' b e^{i(\delta_2 + \phi_s)} \right], \quad (43)$$

where $a = |\lambda_u/\lambda_c|$, $b = |U_{sb}/\lambda_c|$, $-\gamma$ is the weak phase of $V_{ub}$ and $\phi_s$ is the weak phase of $U_{sb}$. $r = |A_u/A_c|$, $r' = |A_{new}/A_c|$, and $\delta_1$ ($\delta_2$) is the relative strong phases between $A_u$ and $A_c$ ($A_{new}$ and $A_c$). Thus from the above amplitudes one can obtain the direct CP asymmetry parameter as

$$A_{CP} = \frac{2 \left[ r a \sin \delta_1 \sin \gamma + r' b \sin \delta_2 \sin \phi_s + r r' a b \sin(\delta_2 - \delta_1) \sin(\gamma + \phi_s) \right]}{\left[ R + 2 (r a \cos \delta_1 \cos \gamma - 2 r' b \cos \phi_s \cos \delta_2 - 2 r r' a b \cos(\gamma + \phi_s) \cos(\delta_2 - \delta_1)) \right]} \quad (44)$$

where $R = 1 + (r a)^2 + (r' b)^2$.

For numerical evaluation, we use input parameters as given in the S4 scenario of QCD factorization approach. For the CKM matrix elements we use the values from $[25]$, extracted from direct measurements and $\gamma = (67^{+32}_{-25})^\circ \quad [19]$. The particle masses are taken from $[25]$. We vary the $|U_{bs}|$ in the range $0 \leq |U_{sb}| \leq 0.002$ and the corresponding phase between $30^\circ \leq \phi_s \leq 150^\circ$ and the allowed region in $\Delta A_{CP}$ and $|U_{sb}|$ plane is shown in the Fig.-5. From the figure it can be seen that the observed $\Delta A_{CP}$ can be accommodated in the VLDQ model.

VI. $S_{\phi K_s}$

Next we consider the decay mode $\bar{B}^0 \rightarrow \phi K^0$. In the SM, it proceeds through the quark level transition $b \rightarrow s\bar{s}s$ and hence the mixing induced CP asymmetry in this mode ($S_{\phi K}$) is
FIG. 5: The allowed range of the CP asymmetry difference ($\Delta A_{CP}$) in the ($\Delta A_{CP} - |U_{sb}|$) plane as shown by the red region. The 30% error bars are due to hadronic uncertainties and shown by green bands. The horizontal lines correspond to the experimentally allowed $1 - \sigma$ range.

expected to give the same value as that of the $B \to J/\psi K_s$ with an uncertainty of around 5%. However, the present world average of this parameter is $S_{\phi K} = 0.44^{+0.17}_{-0.18}$, which has nearly $2.4\sigma$ deviation from the corresponding $S_{\psi K_s}$, with $S_{\phi K_s} < S_{\psi K_s}$. We would like to see whether the model with an extra vector like down quark can account for this discrepancy.

In this model one can write the amplitude for this process, analogous to $B \to \pi K$ processes, as

$$A(\bar{B}^0 \to \bar{K}^0 \phi) = A_{\phi K} \left[ \sum_{q=u,c} \lambda_q \left( \alpha_q^a + \alpha_q^d + \beta_q^a - \frac{1}{2} (\alpha_q^{a,EW} + \alpha_q^{d,EW} + \beta_q^{a,EW}) \right) - U_{sb} \left( \Delta \alpha_3 + \Delta \alpha_4 + \Delta \beta_3 - \frac{1}{2} (\Delta \alpha_{3,EW} + \Delta \alpha_{4,EW} + \Delta \beta_{3,EW}) \right) \right],$$

with $A_{\phi K} = -2 m_\phi (\epsilon_\phi \cdot p_B) F_{B^+ K} (0) f_\phi$, which again can be expressed as

$$A(\bar{B}^0 \to \bar{K}^0 \phi) = \lambda_u A'_u + \lambda_c A'_c - U_{sb} A'_{new} = \lambda_c A'_c [1 + r_1 e^{i(\delta - \gamma)} - r'_1 b e^{i\phi_s} e^{i\delta'}],$$

where

$$r_1 = |A'_u/A'_c|, \quad \delta = Arg(A'_u/A'_c) \quad r'_1 = |A'_{new}/A'_c|, \quad \delta' = Arg(A'_{new}/A'_c).$$

Thus one can obtain the expression for mixing induced CP asymmetry parameter as

$$S_{\phi K} = \frac{X}{\mathcal{R}' + 2r_1 a \cos \delta \cos \gamma - 2r'_1 b \cos \delta' \cos \phi_s - 2r_1 r'_1 a b \cos(\delta - \delta') \cos(\gamma + \phi_s)},$$

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where \( R' = 1 + (r_1a)^2 + (r'_1b)^2 \) and

\[
X = \sin 2\beta + 2r_1 a \cos \delta \sin(2\beta + \gamma) - 2r'_1 b \cos \delta' \sin(2\beta - \phi_s) + (r_1a)^2 \sin(2\beta + 2\gamma) \\
+ (r'_1b)^2 \sin(2\beta - 2\phi_s) - 2r_1r'_1ab \cos(\delta - \delta') \sin(2\beta + \gamma - \phi_s). \tag{49}
\]

For numerical evaluation we use the input parameters as given in S4 scenario of QCD factorization. Using the CKM elements, as discussed earlier, along with \( \beta = (21.1 \pm 0.9)^\circ \), the variation of \( S_{\phi K} \) with \( \phi_s \) for different values of \( |U_{sb}| \) is shown in Figure-6. From the figure it can be seen that the experimental value of \( S_{\phi K} \) can be accommodated in this model.

![Figure 6](image)

**FIG. 6:** The variation of \( S_{\phi K} \) (in S4 scenario) with the new weak phase \( \phi_s \), where the dot-dashed, short-dashed and solid curves are for \( |U_{sb}| = 0.001, 0.0015 \) and 0.002. The horizontal band corresponds to experimental allowed 1\( \sigma \) range.

**VII. SUMMARY AND CONCLUSION**

Recent result of \( B_s - \bar{B}_s \) mixing has created a lot of attention in B decays and furthermore it is also claimed in the literature that it could be the first evidence of physics beyond the SM in the b-sector. Of course, there are many candidate beyond the SM scenarios which can explain such a discrepancy but here we will employ the model with an extended isosinglet down quark to study the same and explore whether other seemingly problematic deviations in the \( b \to s \) sector, as indicated by the data at present, can also be explained simultaneously.
A minimal extension of the SM with only addition of an extra isosinglet down quark in a vector like representation of the SM gauge group that induces FCNC couplings in the $Z$ boson couplings. These models naturally arise for instance as the low energy limit of an $E_6$ grand unified theory. From the phenomenological point of view models with isosinglet quarks provide the simplest self-consistent framework to study deviations of $3 \times 3$ unitarity of the CKM matrix as well as flavor changing neutral currents at the tree level.

As stated earlier, we impose the extended isosinglet down quark model to explain the deviation of $B_s - \bar{B}_s$ mixing from that of the SM expectation and obtained the constraints on the parameters of the new physics model and checked whether these severely constrained parameters still can explain other $b \rightarrow s$ processes, which appear to be not in agreement with the SM expectations.

Recently, CDF observed that the mixing induced parameter $(S_{\psi \phi})$ for the decay mode $B_s \rightarrow \psi \phi$ appears to be not in agreement with the SM expectation. In the SM, the value of $B_s \rightarrow \psi \phi$ is vanishingly small but the experiment has found a rather large value which might be an indication of new physics. We applied the constraints of the new physics model, obtained from the $B_s - \bar{B}_s$ mixing, to see whether one can explain the same. It can be seen from the figure-3 that one can explain the discrepancy in the NP model under consideration.

Next we consider the decay mode $B_s \rightarrow \mu^+ \mu^-$, which is believed to be a very clean mode and only the upper limit ($< 4.7 \times 10^{-8}$) on its branching ratio has been obtained so far which is much larger than the SM value. We used the constraints of the isosinglet down quark model and see that (figure-4) a huge enhancement can be possible due to its effect and can reach the upper limit obtained by the experiment.

Thereafter, we consider the $\pi K$ puzzle, which is basically the difference of direct CP asymmetry parameters, represented by $\Delta A_{CP}(K\pi)$, of the modes $B^- \rightarrow \pi^0 K^-$ and $\bar{B}^0 \rightarrow \pi^+ K^-$. In the SM value of $\Delta A_{CP}(K\pi)$ is expected to be close to zero whereas the experimental value is found to be around 15%. Invoking the new physics constraints, obtained before, we have shown that the observed asymmetry can be obtained in this scenario.

Finally, we consider the long standing problem of $S_{\phi K_s}$ corresponding to the decay mode $B \rightarrow \phi K_s$, which has about 2.5 sigma deviation from that of the $S_{\psi K_s}$. This large deviation is believed to be the due to beyond the SM physics. We employed the NP model under consideration and found that it can easily explain such a discrepancy (figure-6).

To conclude, in this paper we employed the model with an extended isosinglet down
quark to constrain the parameters of the model using the $B_s - \bar{B}_s$ mixing result. Thereafter, we checked whether deviations in other $b \to s$ modes, namely, $B_s \to \psi \phi$, $B_s \to \mu^+ \mu^-$, $B \to \pi K$ and $B \to \phi K_s$ can also be understood in this model, and found that the new physics parameters allowed by $B_s - \bar{B}_s$ mixing result can explain these discrepancies successfully. With more data in the future we will have a better understanding of these problems and possibly we shall be able to ascertain the nature of the new physics or else rule out some of the existing beyond the SM scenarios, which appear to be allowed at present.

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