Fractal techniques as alternative of Fourier transform for fast search of periodic events in time series

A Prosvetov

Space Research Institute, Russian Academy of Science
Cleverdata

e-mail: prosvetov@gmail.com

Abstract. In our day Fourier analysis is commonly used in varies fields, that require information extraction form time series data. However, the Fourier technique has own pros and cons and we can found, that more contemporary approaches can give significant improvements in some cases. We take the example of astrophysical problem and try to resolve it with Fourier analysis in comparison with Fractal analysis. From our results one can find, that in cases of noise extraction Fractal techniques are more efficient and decrease the time of analysis of time series. The proposed approaches can be applied on other fields, to analyze the data sets with dramatically increased volume.

1. Introduction

In Astrophysics exist a problem of quasi-periodic oscillations in power spectra of X-ray light curves of black holes: the current physical models can’t gives a complete, comprehensive picture of their formation, while the QPO generation mechanism seems to remain completely unexplained. The constructed power spectra usually consist of one or more band-limited (“red”) noise components and, occasionally, quasi-periodic oscillation (QPO) peaks at low (0.1–1 Hz) and/or high (~10 Hz) frequencies (figure 1). Being the second-order statistical moments, the power spectra inevitably carry only limited information about the nonlinear variability mechanisms compared to the original light curves. Band-limited noise in the power spectrum is one of the signatures of not just a random but chaotic behavior of a physical system, i.e., the existence of fractal properties: this means a scale invariance, a self-similarity on various time scales. The chaotic variability, moderate on a time scale of tens of seconds, is seen to be repeated and amplified manifolds when passing to seconds and fractions of a second. To determine the degree of scale invariance of a light curve more rigorously (quantitatively), its fractal or Hausdor dimension can be calculated. In particular, the degree of nonlinearity of the equations describing the physical process can be judged from the fractal dimension. Also the Hausdor dimension of time series is connected to the noise power spectrum, and that fact gives a opportunity to extract information about noise spectrum without Fourier transform. The connection between fractal dimension of light curve of black holes and QPO is studied in our previous work [1]. The previous main result was that a correlation between the fractal dimension of the light curves and the frequency of the QPO peak has been revealed.
In current paper we propose to use fractal techniques for fast search of observations with QPO in big arrays of observational data.

2. Data and instruments

For our study we used history observations of black holes (GX 339-4, Cyg X-1, Swift J1729) obtained with the following X-ray observatories: Integral, Swift, RXTE. The data were processed with the NASA/HEASARC software package. For our analysis, we used data in a wide energy range, 2.9–36 keV. The size of the time bin Δt was always taken to be 0.01 s.

We used the following algorithm (usually called R/S analysis) to determine D. We divided the time series being investigated into intervals and calculated the following in each of them: (1) the range R equal to the difference between the maximum and minimum of the function of the accumulated deviation from the mean in the interval, (2) the accumulated standard (root mean square) deviation S, and (3) the number of points in the interval N. For all intervals with the same N the value R/S is averaged. For all intervals with different N, we constructed the dependence of ln(R/S) on ln(N/2) and fitted the derived points of the dependence by a straight line using the least-squares method. The slope of the straight line specifies the Hurst exponent H. The Hurst exponent allows the fractal dimension of the time series to be determined, which is $D = 2 - H$ and reflects the scale invariance of the time series [2].

Figure 1. Power spectrum of GX 339-4 from its PCA/RXTE observations on February 16, 2007, in the energy range 2.9–36.0 keV. The data were fitted by King’s model with a line that had a Lorentz profile. The power is given in percentage normalization after subtraction of the Poisson noise. Powerful “red” (band-limited) noise with the QPO peak at 0.32 Hz is seen.
For additional interpretation we also performed the following operations with the data, by analogy with the approach to analyzing time series called Max-Spectrum [3]. The distribution of extrema depending on the time scale in the signal is also analyzed in the mentioned Max-Spectrum approach. To completely smooth out the correlations in the signal without changing its amplitude distribution, a random sample is additionally constructed from the original sample, with the new signal being obtained from the randomly mixed old signal. In our case, the $\ln(R/S) - \ln(N/2)$ diagram of the new time series will differ from the original one. In the Max-Spectrum approach, $y = \alpha x + \text{const}$ for the mixed signal and $y = \alpha x + a \log_2(\theta) + \text{const}$ for the original signal, which allows the parameter $\theta$ to be determined. The original of this statistical signal from the Max-Spectrum analysis is called an extremal index and it has been studied in detail by Leadbetter et al. (1983). The informative interpretation of the extremal index is that $\theta^{-1}$ is equal to the mean size of the cluster of extremes in the time interval being investigated [4].

3. Results
The form of typical light curve is represented on figure 2 and is highly jagged on a scale of 1 s (top) and on scales of 0.1 s (middle) and 0.01 s (bottom). There is a self-similarity of the light curve on various scales suggesting that the light curve can possess the property of fractality. The comparison of Hurst exponent for two observations is present on the figure 3. One of the observation has red noise without registered QPO, the other has QPO in power spectrum.

Figure 2. PCA/RXTE light curve from GX 339-4 in the energy range 2.9–36 keV. The typical measurement errors are shown on the right. The parts of the light curves separated by the vertical line on the upper panels are given on the lower panels with a better angular resolution.
One can find, the slope change on time scale corresponded to QPO frequency. The slope change can be understood the following way:

1) for time scales larger than period of QPO oscillations and Fourier analysis and R/S analysis can find only red noise in time series;

2) for time scale smaller than period of QPO oscillations Fourier analysis can observe only red noise again, but R/S analysis is highly dependent on peaks presence in time series. Due to periodicity of peaks, the majority of intervals for R/S calculations on the selected time scale have QPO oscillation peaks inside and that fact makes the difference on the step of averaging.

3) the lower is time scale (after QPO period), the higher is the number of intervals for R/S calculation, and lower is R/S value for intervals with QPO peaks and the mean log(R/S) is proportional to log(N/2) where N is number of points on each time interval.

Thus, to find the presence of QPO in time series we don’t need to perform Fourier analysis. Instead we can calculate several points in log(R/S) – log(N/2) diagram on time scale lower then period of QPO and get the value of Hust exponent using least-squares method.

QPO can naturally appear in the framework of the short noise model under the assumption that individual shots are not completely independent [5]. Under this assumption we prepared a modeled series of light curves with Poisson distributed peaks, but the form of peaks were a hight peak with

\[ \text{Figure 3. } \ln(R/S) - \ln(N/2) \text{ diagrams for GX 339-4: at the top with QPOs (D = 1.02 for the scale } \ln(N/2) < 6); \text{ at the bottom without QPOs (D = 1.45). Along the axes: R is the range equal to the difference between the maximum and minimum of the function of the accumulated deviation from the mean in the interval; S is the accumulated standard deviation from the mean; and N is the number of points in the interval.} \]
exponential decay on several next time bins. The overlapping of peaks gives the power spectrum
similar to the spectrum of black holes ([5]), but the fractal dimension of obtained time series was
strongly connected with “dead time” interval between peaks. The founded connection requires
additional studies.

The main advantage in using fractal techniques in order to search QPO in array of time series
is requirements of minimal observation length: based on our observations, fractal technique can get
the periodic structure in data on the time scale of several QPO periods, while for statistically good
power spectrum one require several time series with the same length. This efficiency allow to change
the observational program in the process of observation, if the object in field of view shows
the symptoms of changed fractal dimension and consequently has the QPO in power spectrum.

4. Conclusions
The following can be concluded from our analysis:
1) the R/S analysis can be applied for studying QPO in Black Holes;
2) the average case performance of R/S algorithm is O(n), while performance of Fourier transform
is O(n log(n)), that lead to the decreasing of required time especially for big data arrays.
3) every point, calculated by R/S algorithm, is calculated independently that fact give potential
for parallel computing;
4) the proposed approach can be used to correct observational program in the process of observation
to obtain more data from interesting objects with QPO;
5) the same approach can be used in other fields of time series analysis to find structured parts
in big data arrays especially in cases when the data consist of colored noise.

References
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