TeV NEUTRINOS IN A DENSE MEDIUM

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Abstract

The dispersion relation of energetic (few TeV) neutrinos traversing a medium is studied. We use the real time formalism of thermal field theory and we include the effects from the propagator of the W gauge boson. We consider then the MSW oscillations for cosmic neutrinos traversing the Earth, adopting for the neutrino parameters values suggested by the LSND results. It is found that the $\nu_\mu$ flux, for neutrinos passing through the center of the Earth, will appear reduced by 15\% for energies around 10 TeV.

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1 Introduction

The quest to learn whether neutrinos are massive has been long and arduous. In the recent years there are mounting astrophysical and laboratory data suggesting that neutrinos oscillate from one flavor to another, which can only happen if they have non-zero mass. The SuperKamiokande experiment on atmospheric neutrinos\cite{1}, solar neutrino experiments\cite{2} and LSND accelerator experiment\cite{3}, provide three distinct scales of neutrino mass-squared differences:

\[
\Delta m^2_{atm} \sim 5 \times 10^{-3} \text{eV}^2
\]

\[
\Delta m^2_{sun} \sim 10^{-5} \text{eV}^2
\]

\[
\Delta m^2_{LSND} \sim 10 \text{eV}^2
\]

To accommodate all data a mixing scheme with four massive neutrinos, three active (\(\nu_e, \nu_\mu, \nu_\tau\)) and one sterile (\(\nu_s\)) is required. The mass pattern contains two pairs, each consisting of two nearly degenerate states, separated by the ‘LSND gap’ of a few eV\cite{4}. A natural assignment would have (\(\nu_\mu, \nu_\tau\)) with a mass of few eV each and a mass-difference \(\Delta m^2_{atm}\) accounting for the atmospheric neutrino oscillations, while \(\nu_e\) and \(\nu_s\), much lighter and with a splitting \(\Delta m^2_{sun}\), provide the MSW solution for the solar neutrinos.

Few - eV mass neutrinos play a cosmological role as a hot dark matter (HDM) component in the popular cold+hot dark matter (CHDM) model\cite{5}, providing less structure on small scales. In addition, neutrinos with eV masses mimic extra radiation at the epoch of matter-radiation equality and affect the cosmic microwave background (CMB) anisotropy\cite{6}. Sterile background neutrinos may generate large neutrino asymmetries and influence the big bang nucleosynthesis (BBN)\cite{7}. Massive relic neutrinos participate in gravitational clustering around galaxies. Recent calculations\cite{8} indicate that gravitational clustering provides locally an increase in the neutrino density by a factor \(10^5\), over the uniform density of the big bang cosmology. It has been suggested\cite{9} that the highest energy cosmic rays result from the annihilation of extremely high energy cosmic neutrinos on the background of gravitationally clustered relic neutrinos. From all the above it is clear that the issue of neutrinos with a mass of few eV is an interesting one and de-
serves further study. We suggest in this paper another approach to explore this issue.

Solar neutrinos, in the MeV energy range, undergo resonant conversion with $\Delta m_{\text{sun}}^2 \sim 10^{-5} \text{ eV}^2$. We infer that the range $\Delta m_{\text{LSND}}^2 \sim 10 \text{ eV}^2$ can be studied more appropriately, via MSW oscillations, using neutrinos in the energy range of few TeV. The outer space provides powerful radiation sources, like Active Galactic Nuclei (AGN). AGN are the central regions of certain galaxies in which the emission of radiation can rival or even surpass the total power output of the entire galaxy by as much as a thousand fold. The source that powers AGN is believed to be gravity, i.e. matter accretion into a supermassive black hole located at the center of the galaxy. Accelerated protons may interact with matter or radiation in the AGN to produce pions whose decay products include neutrinos. It is anticipated that AGN could be the most luminous high energy neutrino sources in the universe and the diffuse isotropic neutrino flux from all AGN has been estimated. Also Gamma-Ray Burst (GRB) sources have been proposed as generators of high energy neutrinos.

Cosmic high energy muon neutrinos ($E_\nu > 1 \text{ TeV}$) can be observed by neutrino telescopes by detecting the long range muons produced in charged current muon neutrino-nucleon interactions. The effective detector volume is enhanced in proportion to the range of the produced muon (typically several kilometers). To reduce background, at the detection site one looks for upward moving muons induced by neutrinos traversing the Earth. We study then neutrino matter oscillations inside the Earth ($\nu_\mu \leftrightarrow \nu_\tau$) for neutrino energies above 1 TeV and with mass-squared difference and vacuum mixing angle for the neutrinos as suggested by the LSND results.

The usual MSW analysis has previously been applied to low energy neutrinos, where the Fermi model is an adequate description. Since we are interested in energetic neutrinos ($E_\nu > 1 \text{ TeV}$), the gauge-boson propagator effects might be important. In the next section we study the dispersion relation for energetic neutrinos in a medium, using the real-time formalism of finite-temperature field theory. In section 3 we apply our results to neutrino matter oscillations inside the Earth and present our conclusions.
2 Dispersion relation for neutrinos

At finite density and temperature the properties of particles deviate from their vacuum values. This applies in particular to the dispersion relation which governs their plane-wave propagation. For neutrinos traversing a "flavor birefringent" medium, the effective mixing angle can become large, inducing a complete reversal of the flavor content of the neutrino state (MSW effect\[14\]). The real-time formalism of thermal field theory\[15\] is well suited to analyze such a problem. The real-time propagator consists of a sum of two parts - one corresponding to the propagator at zero temperature and the other corresponding to a temperature dependent part. The fermion propagator is given by

$$S_f(p) = (\not{p} + m)[\frac{1}{p^2 - m^2 + i\varepsilon} + i\Gamma(p)]$$

$$\Gamma(p) = 2\pi \delta(p^2 - m^2) \theta(p_o) \eta_f(|p_o|)$$

$$\eta_f(|p_o|) = \frac{1}{e^{\beta(|p_o| - \mu)} + 1}$$

where $\eta_f$ is the occupation number for the background fermions at temperature $T = \frac{1}{\beta}$ and chemical potential $\mu$. While the zero temperature part of the propagator can be thought of in the conventional manner as representing the exchange of a virtual particle, the temperature dependent part, see eq.(3), represents an on-shell contribution. In a plasma of particles, there is a distribution of real particles and these real particles participate in the scattering process in addition to the exchange of virtual particles.

Neutrinos propagating in a medium acquire an effective mass. Denoting by $\Sigma$ the modified self-energy, the propagation of a neutrino in a medium is governed by the Dirac equation

$$[\not{k} - \Sigma(k)]\psi = 0$$

At one-loop level $\Sigma$ has the general form\[16\]

$$\Sigma = m - (\alpha \not{k} + b\not{\nu})\frac{1}{2}(1 - \gamma_5)$$

where $m$ is vacuum neutrino mass, $k$ is the neutrino four-momentum and $u$ is the four-velocity of the medium. The coefficients $\alpha$, $b$ are functions of the
scalar quantities $k^2$, $u^2 = 1$ and $k \cdot u = \omega$, with $\omega$ the energy of the neutrino in the rest frame of the medium. The dispersion relation is given by

$$det(k - \Sigma) = 0$$

and to lowest order in small quantities provides

$$\omega^2 - |\vec{k}|^2 = m^2 - 2b\omega$$

Equivalently the refraction index $n$ ($|\vec{k}| = n\omega$) becomes

$$n = 1 - \frac{m^2}{2\omega^2} + \frac{b}{\omega}$$

Neutrinos traversing the Earth interact with electrons via neutral current and charged current interactions. While all active neutrinos interact via the neutral current, only the $\nu_e$ participate in the charged current interaction. The relevant one loop contribution to the self-energy of the neutrino is given by (see Fig. 1)

$$i\Sigma = -\frac{ig}{2\sqrt{2}} \int \frac{d^4p}{(2\pi)^4} \gamma^\mu (1 - \gamma_5) i(\not{p} + m)i\Gamma(p)D_{\mu\nu}(k - p)$$

where $g$ is the SU(2) coupling constant and $D_{\mu\nu}$ is the propagator for the W gauge boson. In the unitary gauge we have

$$D_{\mu\nu}(k - p) = \frac{-i}{(k - p)^2 - M_W^2}[g_{\mu\nu} - \frac{(k - p)_\mu(k - p)_\nu}{M_W^2}]$$

Usually the energies involved are small compared to $M_W$ and the momentum dependence in the W propagator is ignored. Since we are interested
in energetic neutrinos ($E_\nu > 1$ TeV) we expand the propagator in powers of $M_W^{-2}$. We obtain

$$D_{\mu\nu}(k-p) \simeq i\left[\frac{g_{\mu\nu}}{M_W^2} + \frac{(k-p)^2 g_{\mu\nu} - (k-p)_\mu(k-p)_\nu}{M_W^4}\right]$$  \hspace{1cm} (12)

It is the above expression we have employed in our calculations. Notice also that from the electron propagator we kept only the term emanating from the medium. The first term in eq.(2), electron propagator in the vacuum, provides the wave-function renormalization of the neutrino in the vacuum (of no relevance to us).

Working within the Fermi model approximation, i.e. keeping only the first term from the $W$ propagator, eq.(12), we obtain for the corresponding self-energy $\Sigma_o$ (evaluated in the rest frame of the medium)

$$i\Sigma_o = iG_F\sqrt{2} \gamma^0 \frac{1}{2}(1-\gamma_5) 2 \int \frac{d^3p}{(2\pi)^3} \eta_f$$  \hspace{1cm} (13)

Since the number density of the background electrons is given by

$$N_e = 2 \int \frac{d^3p}{(2\pi)^3} \eta_f$$  \hspace{1cm} (14)

we arrive at the following expression for $\Sigma_o$, valid in an arbitrary frame

$$i\Sigma_o = iG_F\sqrt{2} N_e \frac{1}{2}(1-\gamma_5)$$  \hspace{1cm} (15)

We read off then that

$$b_o = -\sqrt{2}G_F N_e$$  \hspace{1cm} (16)

in agreement with previous calculations [16-21].

Including the second term from eq.(12), i.e. the correction from the finite mass $W$ propagator, we find for the corresponding self-energy $\Sigma_1$ (evaluated also in the rest frame of the medium)

$$i\Sigma_1 = -i\frac{g^2}{M_W^4} E_\nu \gamma^0 \frac{1}{2}(1-\gamma_5) \int \frac{d^3p}{(2\pi)^3} \eta_f(\varepsilon)(\varepsilon + \frac{\vec{p}^2}{3\varepsilon}) + \mathcal{K} \text{ terms}$$  \hspace{1cm} (17)

By $\mathcal{K}$ terms we denote all these terms proportional to $\mathcal{K}$, responsible for the $\alpha$ coefficient (see eqn.(6)). Since we are interested in finding the $b$ coefficient
we ignore these terms. $\varepsilon$ and $\vec{p}$ are the energy and the momentum of the background fermions. We consider two distinct cases: relativistic electrons (R), where $\vec{p}^2 \simeq \varepsilon^2$, and non-relativistic electrons (NR), where $\vec{p}^2 \ll \varepsilon^2$. Then we have

$$\varepsilon + \frac{\vec{p}^2}{3\varepsilon} \simeq c\varepsilon$$

with $c = \frac{4}{3}(1)$ for the R(NR) case. For $\Sigma_1$ in an arbitrary frame we obtain

$$i\Sigma_1 = -\frac{ig^2}{2M_W^4} E_\nu N_e c < \varepsilon > \frac{1}{2}(1 - \gamma_5) + \text{f\ terms}$$

with $< \varepsilon >$ the average energy of the electrons in the medium. We extract that

$$b_1 = 2\sqrt{2}cG_F N_e \frac{E_\nu < \varepsilon >}{M_W^2}$$

Altogether

$$b = b_o + b_1 = -\sqrt{2}G_F N_e [1 - 2c \frac{E_\nu < \varepsilon >}{M_W^2}]$$

With $E_\nu$ in the TeV range and electrons in a hot plasma the second term in the square bracket could be a significant correction. In our case the electrons inside the Earth can be considered as a Fermi gas and the maximum momentum $p_F$ is fixed by the local number density. For the actual Earth densities we find low values for $p_F$. Therefore we obtain

$$b = -\sqrt{2}G_F N_e [1 - 2 \frac{E_\nu m_e}{M_W^2}]$$

We deduce that the correction is not significant in the Earth and in our calculations we use the standard MSW formalism.

### 3 Results and Conclusions

Focussing our attention into $\nu_\mu < \rightarrow > \nu_e$ oscillations, any neutrino state can be written as

$$|\nu(t) > = \alpha_e(t)|\nu_e > + \alpha_\mu(t)|\nu_\mu >$$

and the time evolution is given by

$$i\frac{d}{dt} \begin{pmatrix} \alpha_e \\ \alpha_\mu \end{pmatrix} = A \begin{pmatrix} \alpha_e \\ \alpha_\mu \end{pmatrix}$$

with $A$ the PMNS matrix.
where
\[ A = \frac{1}{4p} \begin{pmatrix} -\Delta \cos 2\theta_o + 2p\gamma & \Delta \sin 2\theta_o \\ \Delta \sin 2\theta_o & \Delta \cos 2\theta_o - 2p\gamma \end{pmatrix} \] (25)

The neutrino momentum is denoted by \( p \), \( \theta_o \) is the vacuum mixing angle and \( \Delta \) is the mass-squared difference. Matter effects are determined by \( \gamma \) \[ \gamma = \sqrt{2} G_F N_e \] (26)

with \( N_e \) the density of electrons in the medium. For the neutrino parameters we have chosen values implied by the LSND results
\[ \Delta = 10 \text{ eV}^2, \quad \sin^2 2\theta_o = 0.07 \] (27)

The relative population of cosmic neutrinos at the source is 2 \( \nu_\mu : 1 \nu_e \). The smallness of the vacuum mixing angle suggests that vacuum oscillations (during the journey from the cosmic source to the Earth) do not alter the relative proportion. However during the passage through the Earth matter oscillations might change significantly the relative abundance of \( \nu_\mu \) and \( \nu_e \). The density of the Earth is accurately described by the two density model, where the core and the mantle each have a separate and constant density \[22\]. Defining by \( \phi \) the angle between the neutrino direction and the tangent to the surface of the Earth, we considered neutrinos incident at \( \phi = \frac{\pi}{2} \) (i.e. passing through the center of the Earth). In the absence of any oscillations, the muon neutrino flux exiting the Earth is identical to the initial incident muon neutrino flux. In the presence of oscillations, the muon neutrino flux to be detected, is equal to the initial incident muon neutrino flux times the factor \( P(\nu_\mu \rightarrow \nu_\mu) + \frac{1}{2} P(\nu_e \rightarrow \nu_\mu) \). Fig. 2 presents this factor as a function of the energy. At around 10 TeV and for \( \phi = \frac{\pi}{2} \), a 15% reduction in the number of muon neutrinos is observed. Notice that the observed dip corresponds to an MSW resonance. However at resonance the oscillation length is given by \[22\]
\[ L_R = \frac{\pi}{\gamma \tan 2\theta_o} \] (28)

For small vacuum mixing angle \( \theta_o \), \( L_R \) becomes large and exceeds the radius of the Earth. For this reason the MSW dip is not fully developed.

Neutrino telescopes will provide a new observation window on our cosmos and will help us probe the deepest reaches of distant astrophysical objects. The large energies involved might also allow us to enlarge our knowledge of particle physics. Determining the direction of the incident neutrino, we infer
the neutrino path through the Earth. Therefore the neutrino telescopes are suited for the study of neutrino oscillations in the GeV-TeV energy range, and for oscillation lengths of the order of the size of the Earth. Furthermore by comparing the event rates of neutrinos scratching the Earth, to neutrinos traversing the Earth, we obtain access to the neutrino oscillation probabilities. In our work we studied the oscillations among $\nu_\mu$ and $\nu_e$. Another possibility is oscillations among $\nu_\mu$ and $\nu_\tau$ and the appearance of $\nu_\tau$ at TeV energies [23, 24]. This interesting case is under study.

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Fig. 2