Blagonravov Continuously Variable Transmission: computational model of load distribution over elements of control system

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Abstract. The paper presents a kinematic scheme of a differential planetary control gear designed to change the vibration amplitude of the intermediate links of the Blagonravov mechanical continuously variable transmission on the move and under load. A computational model of load distribution over the elements of the planetary gear is developed. The influence of the internal gear ratio and vibration amplitude on the load distribution is estimated. The established dependencies are recommended to be applied when choosing the drive motor power of the transmission control system.

1. Introduction

The transforming property of the Blagonravov burst mechanical continuously variable transmission [1,2,3] is formed, firstly, due to self-regulation (automatic change in the torsion shaft twist angle when the moment of resistance at the output changes) and, secondly, due to the forced change in the vibration amplitude of intermediate links by the control gear.

A large number of such mechanisms are known [4,5,6]. They are designed to change on the move and under load either the length of one of the links of the converting mechanism, or change the position of its pivot point. The mechanisms differ in design, influence over the overall transmission efficiency, and the amount of required control forces. As a rule, they are of the differential type – either screw, planetary, or combined. This makes it possible to reduce the work expended in the control compared to any conventional mechanisms in which a reactive torque acts on the control gear. For the automatic transmission control system [7], especially when choosing power actuators, this fact becomes critical. Therefore, in the considered Blagonravov transmission, a differential mechanism is used to change the vibration amplitude of the intermediate links.

The planetary control gear, in comparison with the screw gear, has such a disadvantage that with a constant amplitude the planetary gears rotate under load. This can cause additional power loss. However, due to the high friction in the screw mechanism, the work expended on friction can be greater than in the planetary gear when the amplitude changes [4]. The obtained test results of the experimental model of the Blagonravov transmission with a planetary-type control gear [8] showed acceptable values of idle losses in this gear, which amounted to 0.003% of the total power losses in the transmission.
At the design stage of the Blagonravov transmission, to calculate the strength of parts and to make a reasonable choice of the power of the drive motor that regulates the vibration amplitude, knowledge of the load on the control gear is required. The development of a computational model for the distribution of loads over its elements is the task of this paper. Its solution is one of the stages in the development of a methodology for designing a mechanical continuously variable transmission with internal force functions – the Blagonravov transmission.

2. Arrangement and Principle of Operation of Planetary Control Gear for Blagonravov Mechanical continuously Variable Transmission

To solve the problem posed here – to determine the distribution of loads over the elements of the vibration amplitude control gear – it is sufficient to consider the part of the kinematic transmission diagram [8] connecting the drive shaft with the crank shaft of the vibration converter. It is shown in fig. 1. a.

![Figure 1](image.png)

**Figure 1.** Planetary mechanism for changing the vibration amplitude of the rocker arms of the rectifiers of the Blagonravov transmission.

- a – kinematic scheme; b – drive shaft assembly with crank shaft; c – general view of the control gear of the experimental transmission model (without epicyclic gears 2 and 4).
- 1 – drive shaft; 2 – general carrier; 3 – rocker arm link; 4 – guide; 5 – crank shaft.

The drive shaft 1 has an enlarged section (Fig. 1. b.). Inside this section, parallel to its axis O–O, but at a distance a (axis O*–O*), a crank shaft 5 is installed, the radius r of which is equal to a. The crank shaft 5, with the help of the guide 4 which is fixed on it and which passes through the lateral split of the shaft 1, can be rotated 180° by means of the rocker arm link 3 (the length of the rocker arm is h). The eccentricity e of the vibration converter, i.e. the distance from the axis K–K of the crank shaft 5 to the axis O–O of the drive shaft 1, can vary from 0 to 2a. In this case, the vibration amplitude φ of the intermediate links – the rocker arms of the rectifiers (with harmonic vibration) will be equal to φ = e/l, where l is the length of the rocker arm.

The overall kinematic scheme of all the transmission is presented in detail in work [8]. Here we just state that on the axis K–K of the crank shaft 5, a cage is installed on the bearing, which, with the help of connecting rods pivotally connected to it, vibrates the rocker arms of five mechanical rectifiers – freewheel mechanisms. They are evenly spaced around the circumference and are equipped with torsion shafts connecting their driven parts with the peripheral gears of the compound. Its central gear is connected to the driven transmission shaft.

In order to avoid unnecessary repetition, this part of the kinematic diagram in Fig. 1. a is not shown, since it is not related to the problem solved in this paper. Counterweights are not shown either, with the help of which the center of mass of the parts fixed on the axis K–K of the crank shaft 5 is aligned with its axis O*–O*, and the center of mass of shaft 1 together with shaft 5 is aligned with the axis O–O.

The rotation of the crank shaft 5 is carried out by means of the rocker arm link 3 connecting the guide 4 with the hub of the sun gear z3 of the planetary gear set with the stationary epicycle z4. This planetary gear set has the carrier 2 connected also with the other planetary gear set, where the sun gear z1(z1 = z3) is fixed on the drive shaft 1, and the epicycle z2(z2 = z4), with the help of the drive of the
control gear, can be rotated at the angle $\phi$. Fig. 1. c shows the general view of the control gear of the assembled experimental transmission model. When the epicyclic gears of the planetary gears $z_2$ and $z_4$ are stationary, the sun gears of these rows $z_1$ and $z_3$ rotate at the same rotational speed, which coincides with the rotational speed $\omega_1$ of the drive shaft 1, and the general carrier 2 rotates with a frequency proportional to the characteristic of the planetary gears $k = z_2 / z_1 = z_4 / z_3$, where $z_2$, $z_3$ are the number of teeth of the epicyclic and sun gears, respectively. Let us write the equations of motion for each planetary gear set

$$\omega_{s1} - \omega_{s1} (1+k) + \omega_{s1} = 0,$$

$$\omega_{s2} - \omega_{s2} (1+k) + \omega_{s2} = 0,$$

where $\omega_s$, $\omega_e$, $\omega_i$ are rotational speeds of the sun gear, the carrier and the epicyclic gear of the planetary gear set, indexes 1 and 2 refer to the planetary gear set with a controlled and stationary epicycle, respectively. In accordance with the kinematic scheme, the rotational speed of the carrier makes $\omega_{s1} = \omega_{s2} = \omega_{s}$(the general carrier). With the stopped epicycles $\omega_{s1} = \omega_{s2} = 0$ from (1) and (2) we obtain $\omega_{s1} = \omega_{s2} = \omega_{s}$ and $\omega_{e} = \omega_{s} / (1+k)$.

When turning the controlled epicyclic gear, $\omega_{s1} \neq 0$ and $\omega_{s2} = 0$ we obtain $\omega_{s1} - \omega_{s2} = -k\omega_{s1}$. Moving on to the angles of rotation of the gears, we obtain

$$\Delta \phi = -k \cdot \phi,$$

where $\Delta \phi$ is relative reversal of the sun gears $z_1$ and $z_3$ of the planetary gears, $\phi$ is angle of rotation of the controlled epicyclic gear $z_2$.

With regard to the above mentioned, in accordance with the kinematic scheme (Fig. 1. a), the angle of rotation of the crank shaft 5 is proportional to $\Delta \phi$ and determines the vibration amplitude, and the torque $M_y$, loading the driven epicyclic gear $z_2$ to the approximation of the efficiency factor, will make as follows:

$$M_y = k \cdot M_{z3},$$

where $M_{z3}$ is the module of the torque on the sun gear $z_3$.

It should be noted that the effective torques on both solar and epicyclic gears of different planetary gears are equal in magnitude, but opposite in sign. If a torque on one sun gear is known, then a torque on any link of the gear is easily determined.

The torque on the drive shaft $M_d$ branches into two streams. One stream $M_c$ is transmitted through the sun gear $z_1$ of the left planetary gear set (see Fig. 1. a) of the control gear to the crank shaft 5, overcoming the torque on it, and the other one $M_y$ overcomes resistance on the part of the drive shaft (section N–O), loaded through the bearings of the crank shaft. Thus, at the point N they are summed

$$M_d = M_c + M_y.$$  
In expression (5), it is necessary to take into account the direction of action of the torques $M_c$ and $M_y$. With $M_c = -M_y$, the motor load torque $M_d = 0$. However, in this case the absolute values $M_c$ and $M_y$ can be significant. The magnitude of these loads does not affect the value of the motor load torque.

Thus, the angle of rotation of the crank shaft 5 determines the magnitude of the vibration amplitude $\phi$, and the torque on this shaft determines the loading of the control gear. Determining the distribution of these values is the task of developing a computational model.
3. Calculation and Computational Model of Loading of Control Gear Elements

From the side of the swirling torsion shafts during transmission operation [1,3], the average per cycle (one revolution of the drive shaft) force $P$ is applied to the axis $K–K$ of the crank shaft, which is always directed towards the axis $O–O$ of the drive shaft. In stop mode (speed ratio $i = \omega_2 / \omega_1 = 0$, where $\omega_2$ is the driven shaft rotational speed), this force is directed along the eccentricity $e$. The motor load torque $M_d = 0$. In any other transmission modes, it is deflected through the angle $\alpha$ as shown in Fig. 2. a (view of the drive shaft along arrow A, see Fig. 1. a).

The scheme uses the following designations: $\beta$ is half the angle of rotation of the crank shaft (a positive change in the angle is taken clockwise and coincides with the direction of rotation of the input shaft); $H$ is the lever of force $P$ action relative to the axis $O–O$. When $\beta = 0$, the axis $K–K$ of the crank shaft coincides with the axis $O–O$ of the drive shaft, the eccentricity $e = 0$; when $\beta = \pi / 2$, the axis $K–K$ is located at the maximum distance from the axis $O–O$, with the eccentricity $e = 2r$. Taking into account that $H = e \cdot \sin(\alpha)$, and $e = 2r \cdot \sin(\beta)$, the torque $M_d$ is determined by the following formula

$$M_d = 2P \cdot r \cdot \sin(\beta) \cdot \sin(\alpha). \quad (6)$$

![Figure 2](image1.png)

**Figure 2.** Calculation scheme of loading the elements of the Blagonravov transmission planetary control gear.

- a – scheme of the application of force to the crank shaft from the side of the torsion bars (view along arrow A, see Fig. 1.a); b – crank shaft loading scheme; c – scheme of loading of the drive shaft part.

As the crank shaft 5 (see Fig. 1. a) is fixedly connected (through the spline connection) with the guide 4, its rotation is equal to the angle of rotation of the guide. In turn, the guide is connected via a rocker arm link to the sun gear $z_3$ of the eccentricity changing planetary mechanism (Fig. 2. b). The variable length of the guide $b$ turning the crank shaft is determined depending on the angle of rotation of the rocker arm according to the cosine theorem

$$b = \left( a^2 + h^2 - 2ah \cos(\gamma) \right)^{0.5}, \quad (7)$$

where $\gamma$ is the angular position of the rocker arm relative to the vertical axis crossing the axis $O–O$ and $O^*–O^*$. The negative value $\gamma$ means counterclockwise, the positive value means clockwise (Fig. 2. b.).

The angle $\gamma$ is related to the relative rotation $\Delta \varphi$ (3) of the sun gears $z_1$ and $z_3$ of the planetary gears by the following ratio

$$\gamma = \gamma_0 + \Delta \varphi \quad (8)$$
where \( \gamma_0 \) is the initial angular position of the rocker arm, which is determined by the assembly ‘guide – crank shaft’.

The initial position of the rocker arm is taken with the guide being located horizontally to the left
\[
\gamma_0 = -a \cos\left(\frac{a}{h}\right)
\]
with the alignment of the axes \( K-K \) of the crank shaft and \( O-O \) of the drive shaft.

Thus, depending on \( \Delta \varphi \), the crank shaft rotation angle \( 2\beta \) can be determined, taking into consideration (7) and (8), from the geometrical relationship (see Fig. 2. b)
\[
\sin\left(\frac{\gamma}{2}\right) \sin\left(\frac{\beta}{2}\right) = -\frac{\alpha}{b}
\]
by the following formula
\[
2\beta = \frac{0.5 \pi - a \sin\left(h \sin(\gamma) + \Delta \varphi\right)}{b}
\]  
edenoted as (9).

Force \( P \) is transmitted to the drive shaft, as mentioned above, through the crank shaft and its bearings. In Fig. 2. b, the crank shaft loading scheme is shown. In this case, in the stationary mode, the sum of the torques about the axis \( O*O* \) is equal to zero. The equilibrium equation is as follows
\[
Pr \cos(\beta - a) = Fb,
\]
where \( F \) is the force acting perpendicular to the guide at the point \( G \) of the rocker arm link.

This force is determined through the torque \( M_c \) by the following formula
\[
F = \frac{M_c}{h \cos(e)},
\]
where \( e \) is angle determined from geometric constructions (see Fig. 2. b) by the following formula
\[
e = \pi / 2 - \gamma - 2\beta.
\]

Then the value of the torque transmitted to the sun gear \( z_3 \), is determined as follows
\[
M_c = M_3 = \frac{Prh \cos(\beta - a) \sin(\gamma + 2\beta)}{b}
\]  
edenoted as (12).

Fig. 2. c shows the scheme of loading of a part of the drive shaft in section \( N-O \). Taking into account the fact that the lever of the action of the force \( P \) relative to the axis \( O-O \) of the drive shaft
\[
h = a \sin\left(\frac{\pi}{2} - (\beta + a)\right),
\]
the torque value \( M_c \) is determined by the following formula
\[
M_c = \frac{Pa \sin\left(\pi / 2 - (\beta + a)\right) + Fa \cos(\pi / 2 - 2\beta)}{2b}.
\]  
edenoted as (13).

Using expressions (11) and (12), equation (13) after transformations is as follows
\[
M_v = \frac{Pa \left(2b \cos(\beta + a) + r \sin(\beta + a) + r \sin(3\beta - a)\right)}{2b}.
\]  
edenoted as (14).

The direction and value of the torques \( M_c \) and \( M_v \) are determined by the ratio of the angles \( \beta, \alpha \), dimensions \( a, r, h \), as well as the gear ratio \( i \). The authors of the article «The Blagonravov Continuously Variable Transmission: Computational Model of Oscillation Generator Loading» have calculated the force \( P \), acting on the axis \( K-K \) of the crank shaft, with the help of the following expression
\[
P = \frac{c_T \varphi_{r\tau_r}^n}{l \left[2.138 - \left(0.019 + 1.864i_T - 0.597i_T^2\right)^{0.5}\right]},
\]  
edenoted as (15).

and they also have calculated its direction as follows
\[
\alpha = \arcsin\left[i_T \left[2.138 - \left(0.019 + 1.864i_T - 0.597i_T^2\right)^{1/2}\right]\right]
\]  
edenoted as (16).

where \( i_T \) is the internal gear ratio [1], characterizing the property of self-regulation of transmission and changing from 0 to 1, \( \varphi_{r\tau_r} \) is average angle of swirling the torsion shaft [1], \( c_T \) is the torsion shaft rigidity, \( n \) is a number of transmission torsion shafts. Expressions (12) and (14) are the computational
model of the distribution of loads over the elements of the control gear of the vibration amplitude of Blagonravov transmission.

As an illustration, Fig. 3. a shows the dependences calculated in accordance with (12) and (14) – $M_c$ (dashed line) and $M_v$ (full line) on the internal gear ratio $i_r$ in the values relative to $c_r\phi_0$. The direction of the torque $M_c$ is negative, of the torque $M_v$ - positive. The verification of the correctness of these dependencies is carried out by the comparison calculated according to (5) – dark points and according to (6) – full line, of the torque $M_d$ (Fig. 3. b) on the drive shaft. The results are identical. The calculations have been performed taking into account (15) and (16) at $a = r = 0,01$ m, $h = 0,04$ m, $n = 1$, $\phi_0 = 0,1$, 0,2 and 0,299 rad.

![Figure 3](image)

Figure 3. Load of the elements of the control gear depending on the internal gear ratio: 1 – $\phi_0 = 0,1$; 2 - $\phi_0 = 0,2$; 3 - $\phi_0 = 0,299$ rad.

a is the torque $M_c$ on the sun gear of the planetary control gear and the torque $M_v$ on the part of the drive shaft (section N–O) of the transmission

b is the torque $M_d$ on the drive shaft (section O–N).

Using dependences (12) and (4), it is possible to determine the load on the drive shaft of the control mechanism (epicyclic gear $z_2$), and by setting the maximum allowable time for changing the vibration amplitude, it is possible to determine the power required for control.

4. Conclusions

The dependences of the distribution of torques over the elements of the differential planetary gear for controlling the Blagonravov mechanical continuously variable transmission are established. The transmitted torque is influenced by the set vibration amplitude of the intermediate links and the internal gear ratio characterizing the transmitted load. In the stop mode of operation, the values of the torques on the controlled sun gear of the planetary mechanism and the section of the drive shaft loaded through the bearings of the crank shaft, are equal in magnitude, but opposite in direction. The magnitude of the internal moments on the elements of the control gear can significantly exceed the torque loading the transmission drive shaft.

The obtained dependences are recommended to be used in strength calculations of the planetary gear parts and the choice of the power of the drive motor of the control system.

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