Improving Graduation Rate Estimates Using Regularly Updated Markov Chains

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ABSTRACT

American universities use a rolling six-year graduation rate (SYGR) to calculate statistics regarding their students’ final educational outcomes (graduate or not graduate). Meanwhile application of absorbing Markov chains (AMC) is commonly used to estimate graduation rates in research settings. In both cases a frequentest approach is used by counting the number of students who finished their program within six years for the standard SYGR method; in the case of Markov chains a frequentest approach is used to compute the associated transition matrix. Both approaches have significant limitations related to sensitivity when applied to small sample sizes or sub-populations at a university. In this paper, we use sensitivity analysis to compare the performance of the standard SYGR method, and absorbing Markov chains. We also propose and evaluate the use of a regularly updating Markov chain in which the transition matrix is updated year-to-year. Results indicate that the regularly updating Markov chain approach reduces the estimation variation by 50%, especially for population with small sample sizes.

Keywords Graduation rate estimation, Absorbing Markov chain, Higher education

1 Introduction

American universities commonly use a standard 6-year graduation rate (SYGR) calculation to report their students’ outcomes. Based on federal regulations, a program’s graduation rate is defined as the percentage of first-time-in-college (FTIC) students who complete the program within 150% of the standard enrollment time to degree [8]. For example, for a four-year program, students who earn degrees within 6 years are considered as graduates. The SYGR method has some disadvantages. For example the method only considers FTIC students, which excludes transfer students who make-up up to 38% at many public universities [23]. Also students who complete their program in more than 6 years, common to student who enroll part-time, are reported as not graduating in this method.

Based on the definition of the SYGR a operational discussion of calculating the SYGR is useful in understanding its features and limitations. Consider the case of $N_y^s$ FTIC students starting at a university degree-program in year $y$. After 6 full years assume that of the original $N_y^s$ students $N_y^g$ are observed to graduate. Accordingly, the SYGR for year $y$ is calculated and reported as

$$G_y^r = 100 \cdot \frac{N_y^g}{N_y^s}. \quad (1)$$

Immediately, the first issue with this approach is that the reporting of the graduation rate for a group of students occurs 6-years after their initial matriculation in year $y$. As such, there is an underlying assumption that students entering the university in year $y + 6$ and later will bear out similar results; the reported statistic is arguably a stale statistic. Moreover, the accuracy of using the standard SYGR calculation to estimate graduation and retention rates is a direct
function of the data available; small data sets produce sensitive estimations. That is to say graduation rate estimations may vary significantly from the true value. Another common approach to estimating graduation rates is to build a Markov Chain based on historical data. One advantage of this method over the standard SYGR is that the Markov Chain method can be adapted to capture and represent student progress at a university throughout the same six-year period. In other words, the method models some temporal aspects of student progress which SYGR does not model. However, as it will be demonstrated in this paper, the accuracy of estimating graduation rates using Markov chains is quite sensitive to data availability as well. This disadvantage makes the use of absorbing Markov chains unreliable for many instances in the context of education when sample sizes are small.

At issue in this paper is how small universities estimate their graduation rates. Or even in the case of larger universities, how they go about estimating their graduation rates for degree programs with lower enrollments (e.g. Physics, Mathematics) or for sub-populations with low representation (e.g. Women in certain STEM degree programs). As an example of a small sub-population, consider the case of female students majoring in physics. At the University of Central Florida – one of the top 5 largest universities in America for the last 5 years – only 3 female students have been observed to both start and graduate from the Physics department at UCF between the years of 2008 and 2016. The low number is a reflection of multiple factors. First, representation of female students in physics is low; as reported by, females students only made up 21% of all physics students across the United States in 2017. More practically however, when calculating the SYGR a sizeable fraction of students are missed because their academic careers will start or end outside the time period for which data is available. For example, during the 8-year time period of data, the number of female students that are observed to declare themselves as physics majors is 79. Over the 8 years of available data, the SYGR can only be calculated for 3 of the years. So even when generously summing and averaging over the 3 available years, the reported graduation rate for women in physics would be 18% (3 of 17) — the reliance of such a metric is questionable, and more so any implications that might be drawn from it.

The goal of this paper is to more accurately assess retention and graduation rates of the university as a whole, as well as for specific target sub-populations. This includes particular majors, under-represented populations, or transfer students. Prior efforts have dealt with the issues of decreasing data availability according to specification. In particular, Hierarchical Linear models (HLM) have tackled the problem and sought to overcome data availability by understanding particular effects layered on top of main effects.

The rest of the paper is organized as follows: In Section II, we show that how the accuracy of six-year graduation rate and absorbing Markov chain is a function of data availability. In Section III we explain our regularly updating absorbing Markov chain approach. Results analysis is presented in Section IV. Finally, Sections V and VI correspond to discussion and conclusion respectively.

2 Estimating graduation rate

In this section, we discuss the standard SYGR method and elaborate on the sensitivity of this method. Similarly, we introduce and compare usage of absorbing Markov chain to small populations size when estimating graduation rates.

2.1 Standard 6-year graduation rate

Suppose we are interested in estimating a population’s six-year graduation rate, θ, given some observed data, D. Since only two final 6-year outcomes are possible, that is graduate or not graduate (according to Federal guidelines), each student’s outcome can be modeled as a Bernoulli trial. With this assertion, the number of students who graduate follows the Binomial distribution with parameter θ. Therefore, the probability of k students graduating out of n, given θ (the probability of graduating in six years for each student), is:

\[ P(D|θ) = \binom{n}{k} θ^k (1 - θ)^{n-k} \]

The standard six-year graduation rate method corresponds to estimating the graduation rate by maximizing the likelihood function in Equation 2. Based on the Maximum Likelihood Estimation (MLE), the graduation rate estimation follows the frequentist approach whereby, \( \hat{θ} = k/n \) is an unbiased estimator for graduation rate.

1The notion of a true value graduation rate appears odd in practice, however, here we refer to true value in the statistical sense as it relates to parameter estimation.
In order to demonstrate the sensitivity of the MLE approach, we use data collected from University of Central Florida (UCF). Based on historic data, the six-year graduation rate at UCF for FTIC students starting in 2008 is 71%; 29% of students graduate in more than six years or halted enrollment. Assuming 71% to be the true value parameterizing a binomial distribution representing the number of students graduating within six years (i.e. $\theta = 0.71$), we randomly simulate 10,000 outcomes of different sample sizes, $n$ with the resulting SYGR calculated using Equation 1. Each sample represents an incoming Freshman class. The corresponding probability density function (pdf) of the 6-year graduation rate for each incoming class, representing an estimate, is shown in Figure 1. As shown in the figure, the graduation rate estimates for populations with small sample sizes can vary significantly from the actual value (see the case for $n = 50$). Also, sample variation for populations with small sample sizes ($n = 50$) compared to the larger sample sizes ($n = 5000$) are quite high (6.4% versus 0.6%). In fact, when $n = 50$ and $n = 500$, the probability that the SYGR is reported to be lower than 66% or higher than 76% is 0.53 and 0.03, respectively; these differences are considered significant in the context of college rankings and when being evaluated by government or accreditation boards.

2.2 Absorbing Markov Chain

Different approaches are used to evaluate students performance and persistence in higher education systems, among which Machine learning algorithms and stochastic models are the most common [25, 19, 10, 7]. Markov model have been used in many educational studies to analyze students’ academic progress and academic behavioral [14, 13, 20, 15, 2, 5]. For example Nicholas [14], analyzed students progress and performance such as expected time to graduation and graduation rate for doctoral and master degrees candidates in an Australian higher education institution. The proposed model includes two absorbing states, withdrawal from program and thesis accepted and students move between the transient states at the end of each year based on their performance. Depending on academic policies, different measures might be used for defining different states. For example in the research conducted by Bairagi et al [3], states are defined based on the semester examination in which the students are going to appear. To move between states, students need to pass certain examinations.

Absorbing Markov Chain (AMC) are a type of Markov model that has two classes of states: transient states and absorbing states. In our case, the number of states for each AMC is finite. When the system goes to one of the absorbing states it cannot exit. Each AMC like any other Markov models has a transition matrix with $P_{ij}$ representing probability of going from state $i$ to state $j$ [20]. The canonical form of an absorbing Markov chain with $r$ absorbing states and $t$ transient states is shown in Equation 3. In this equation, $R$ is a $t \times r$ matrix that shows transition probabilities from the transient states to the absorbing states, $Q$ is a $t \times t$ matrix that represents transitions probabilities between the transient states, $I$ is a $r \times r$ identity matrix, and $O$ is a $r \times t$ zero matrix [6]. While matrix $P$ shows one-step transition probabilities, matrix $P^n$ represent $n$-step probabilities of transitions between states. In other words, $P^n_{ij}$

\footnote{Similar sensitivity results are expected at universities, there are no unique factors regarding UCF in this analysis}
In order to use an absorbing Markov chain to estimate graduation rates, we consider students’ academic level (Freshmen, Sophomore, Junior, Senior) as transient states, and students’ final educational outcomes (graduate or halt) as absorbing states. All student start from a dummy state (the start state) and then based on their academic level are assigned to other states. After this initial assignment, students then move between transient states based on their academic performance and finally are absorbed into one of the absorbing states. For our purposes a student is defined to halt their education if they do not enroll for three consecutive semesters. Transition flows and the corresponding probabilities for students who start the education in Fall 2008 at UCF are shown in Figure 2. Each student state is updated at the end of academic year. For example at the end of a year, 10% of sophomore student remain sophomore, 75% and 8% of them move to junior and senior levels respectively, and finally 7% will halt their education. In order to find the percentage of students who are graduated within six years, we need to calculate $P^7$ (considering start state) and observe the entry that shows transition probability from state start to state graduate. For the Markov chain illustrated in Figure 2, the mentioned probability is 69%, which is close to the six-year graduation rate for the same set of students (71%).

To test the sensitivity of using an AMC in estimating graduation rates 10000 sub-populations with different sample sizes are generated based on UCF transition matrix parameters. The academic trajectory of the students is sampled directly from the Markov model. Examples of generated students’ academic trajectories are provided in Table 1.

The probability distribution function (PDF) of graduation rate estimations for the sub-populations with different sample sizes are shown in Figure 3. As the figure shows, the estimation variations for sub-populations with small sample sizes are high. Also Figure 4 compares 5%-95% inter-quartile that is a measure of performance (in term of estimation variation) for both absorbing Markov chain and 6-years graduation rate method. Based on results so far, we see that both SYGR and AMC produce estimates of the graduation rate with high variance when sub-populations have small sample sizes.

The difference comes from the non-zero probability for students to remain in the same states for multiple years, e.g Freshman-Sophomore-Sophomore-Sophomore-Sophomore-Junior-Senior.
In this paper we use regularly updating absorbing Markov chain transition matrix to cope with this challenge and provide sensitivity analysis to demonstrate the benefit of this methodology in graduation rate estimation accuracy. The detail of the propose methodology is explained in the next section.

3 Methodology

In this paper, we apply a regularly updating absorbing Markov chain approach to update the transition probabilities between states year-by-year during six years. In our study, transient states are defined as student academic levels and absorbing states are graduate or halt. In this approach, we assume all states are initially empty and new students join the degree program in a constant rate which is equal to the initial number of students enrolled. For example if 50 students initially enroll in a program, total number of enrolled students at the beginning of the second year is assumed to be \(50 + 50 \times \) (Freshman retention rate).

In this method, given the additional observations for new students during the six years horizon, the transition probabilities between every two consecutive states is learned and updated year-by-year. That implies more learning happens at
Table 2: Number of students observed in each state for different years with regularly updating Markov chain method

|     | 1  | 2  | 3  | 4  | 5  | 6  |
|-----|----|----|----|----|----|----|
| Fr  | 47 | 93 | 140| 186| 232| 270|
| So  | 40 | 80 | 120| 158| 190| 194|
| Ju  | 36 | 72 | 106| 134| 140| 141|
| Sc  | 19 | 36 | 48 | 51 | 52 | 52 |
| H   | 13 | 25 | 38 | 51 | 94 | 95 |

earlier states (e.g. Freshman and Sophomore) where more observations are received by the model. Table 2 illustrates an example of sample sizes (number of students) for observed in each state for in different years. As the table shows, sample size for Freshman and Sophomore states is larger when compared to the Junior and Senior states from first year to the sixth year. The increase in student samples for these earlier states helps to reduce uncertainty.

4 Results

In this section, we apply regularly updating absorbing Markov chain method to compute the graduation rates for sub-populations with different sample sizes. Probability density functions of the first to sixth estimations for sub-population with N=50 are shown in Figure 5. Each successive estimation is based on 1, 2, 3, ..., 6 incoming classes of students used to create the transition matrix for the AMC. As we see in the figure, our first estimation has a large variance as a result of the small number of student at the initial stage; this is equivalent to the standard AMC discussed in Section 2.2. For the second estimation, given that 50 new students are added to the previous pool of students, the corresponding PDF is narrowed compared to the first estimation, and so on. Finally, the sixth estimation which uses the transition matrices of five previous years, provides the most accurate measure. Estimation variation for each of the six years are provided in Table 3. As we observe in the table, the sampled standard deviation for the six-years graduation rate method (6.4%) is cut by more than 50% compared to sixth year estimation using the regularly updating absorbing Markov chain method (3.0%).

Figure 6 compares 5%-95% inter-quartiles of the six estimations obtained by regularly updating absorbing Markov chain alongside with the six-year graduation approach for different numbers of students added per year. As it is illustrated in the figure, for a fixed number of students added per year, the gap between 5%-95% inter-quartiles are reduced from the first to the sixth estimation. Also by increasing the number of students added per year, the estimations for the transition probabilities become increasingly accurate, along with the final graduation estimate.
Table 3: Standard deviation of estimated graduation rate from year 1 to 6 for sub-population with $N=50$

| Estimation number | Estimation standard deviation |
|-------------------|-----------------------------|
| 1                 | 6.5%                        |
| 2                 | 4.6%                        |
| 3                 | 3.8%                        |
| 4                 | 3.4%                        |
| 5                 | 3.1%                        |
| 6                 | 3.0%                        |

Figure 6: 5%-95% inter-quartiles for graduation rate of sub-populations with different size obtained by regularly updating absorbing Markov chain and 6 years graduation rate method

5 Conclusion

In this paper, we propose using a regularly updating absorbing Markov chain method as an alternative to the six-year graduation rate method for computing students’ graduation rate when sample size is small. In the propose approach, the transition matrix is updated year by year based on existing and joining pool of students and their academic performances. The transition states of the Markov Chain are defined as students academic level and the absorbing states are graduation and halt. Our sensitivity analysis shows that the estimated graduation rates obtained by the regularly updating absorbing Markov chain model gives a more robust measure of graduation rate even for small data sets. For sub-population with $N=50$, our proposed approach reduces estimation variation by more than 50% compared to the six-year graduation rate method.

While the regularly updating Markov chain approach requires inclusion of student data not in the same year as the initial entering class, we find this approach more appropriate than the standard SYGR. As mentioned previously, the SYGR is arguably a stale statistic. Assuming that graduation rates remain static through multi-year periods, then our proposed method is an improvement as it can capture changes in graduation rates should there be significant shifts in the degree program.

Also it has shown in the Discussion section that in reality, students’ academic level advancement does not follow a Markov chain behavior which causes absorbing Markov chain underestimates true value of graduation rate. In future work we will address this issue and propose more advanced modeling for students’ academic level trajectories to reduce the bias. Moreover, previous research has shown that there is a meaningful relationship between students educational behavioral patterns (e.g. learning trait patterns and enrollment status patterns) and their academic performances [11][12][4]. Therefore, as a promising avenue for future studies, our developed methodology can be applied for computing retention and graduation rate for students with different educational behavioral patterns.
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