Boundary value problems and the validity of the Post constraint in modern electromagnetism

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Abstract: When a (frequency–domain) boundary value problem involving a homogeneous linear material is solved to assess the validity of the Post constraint, a conflict arises between the fundamental differential equations of electromagnetism in the chosen material and a naive application of the usual boundary conditions. It is shown here that the conflict vanishes when the boundary conditions are properly derived from the fundamental equations, and the validity of the Post constraint in modern macroscopic electromagnetism is thereby reaffirmed.

Keywords: Boundary conditions; Electromagnetic theories; Linear materials; Magnetoelectric materials; Post constraint; Tellegen parameter

1 Introduction

The genesis of the Post constraint on the electromagnetic constitutive relations of linear mediums was described in detail quite recently [1]. This structural constraint was shown to arise from the following two considerations:

• Two of the four Maxwell postulates (containing the induction fields and the sources) should be independent of the other two Maxwell postulates (containing the primitive fields) at the macroscopic level, just as the two sets of postulates are mutually independent at the microscopic level.

• The constitutive functions must be characterized as piecewise uniform, being born of the spatial homogenization of microscopic entities. Therefore, if a certain constitutive function of a homogeneous piece of a medium cannot be recognized by proper electromagnetic experimentation, the assumption of a continuously nonhomogeneous analog of that constitutive function is untenable.

Available experimental evidence against the validity of the Post constraint for linear materials was shown to be incomplete and inconclusive, in addition to being based either on the physically inadmissible premise of purely instantaneous response and/or derived from a pre–modern version of electromagnetism [1].
Nevertheless, solutions of very simple (frequency-domain) boundary value problems can be invoked very easily to claim the invalidity of the Post constraint for linear materials. Indeed, when a boundary value problem involving a homogeneous linear material is formulated to assess the validity of the Post constraint, a conflict arises between the fundamental differential equations of electromagnetism in the chosen material and a naïve application of the usual boundary conditions. In this paper, that conflict is easily resolved—in favor of the Post constraint.

The organization of this paper is as follows: Section 2 contains a brief review of modern macroscopic electromagnetism, followed by a relevant presentation of linear constitutive relations in Section 3. The principal equations of a naïve formulation of boundary value problems are set up in Section 4, and the aforementioned conflict is presented and resolved in Section 5. The paper concludes with some remarks in Section 6.

2 Modern Macroscopic Electromagnetism

Let us begin with the fundamental equations of modern electromagnetism. The microscopic fields are just two: the electric field $\mathbf{\tilde{e}}(x, t)$ and the magnetic field $\mathbf{\tilde{b}}(x, t)$.

These two are accorded the status of primitive fields in modern electromagnetism, and their sources are the microscopic charge density $\tilde{c}(x, t)$ and the microscopic current density $\tilde{j}(x, t)$. Both fields and both sources appear in the microscopic Maxwell postulates:

\begin{align*}
\nabla \cdot \mathbf{\tilde{e}}(x, t) &= \varepsilon_0^{-1} \tilde{c}(x, t), \\
\nabla \times \mathbf{\tilde{b}}(x, t) - \varepsilon_0 \mu_0 \frac{\partial}{\partial t} \mathbf{\tilde{e}}(x, t) &= \mu_0 \tilde{j}(x, t), \\
\nabla \cdot \mathbf{\tilde{b}}(x, t) &= 0, \\
\nabla \times \mathbf{\tilde{e}}(x, t) + \frac{\partial}{\partial t} \mathbf{\tilde{b}}(x, t) &= 0.
\end{align*}

Spatial averaging of the microscopic primitive fields and source densities yields the macroscopic Maxwell postulates:

\begin{align*}
\nabla \cdot \mathbf{\tilde{E}}(x, t) &= \varepsilon_0^{-1} \tilde{\rho}(x, t), \\
\nabla \times \mathbf{\tilde{B}}(x, t) - \varepsilon_0 \mu_0 \frac{\partial}{\partial t} \mathbf{\tilde{E}}(x, t) &= \mu_0 \tilde{\mathbf{J}}(x, t), \\
\nabla \cdot \mathbf{\tilde{B}}(x, t) &= 0, \\
\nabla \times \mathbf{\tilde{E}}(x, t) + \frac{\partial}{\partial t} \mathbf{\tilde{B}}(x, t) &= 0.
\end{align*}

which involve the macroscopic primitive fields $\mathbf{\tilde{E}}(x, t)$ and $\mathbf{\tilde{B}}(x, t)$ as well as the macroscopic source densities $\tilde{\rho}(x, t)$ and $\tilde{\mathbf{J}}(x, t)$. Equations (5)–(8) are the fundamental (differential) equations of modern macroscopic electromagnetism. Let us note that

(i) all four equations contain only two fields, both primitive, and

2 The lower-case letter signifies that a field or a source density is microscopic, while the tilde indicates dependence on time. Furthermore, $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m and $\mu_0 = 4\pi \times 10^{-7}$ H/m are the permittivity and the permeability of matter–free space in the absence of an external gravitational field (which condition is assumed here).
Indeed, modern electromagnetism may be called EB–electromagnetism to indicate the central role of $\vec{E}(x, t)$ and $\vec{B}(x, t)$.

Equations (5)–(8) are not, however, the textbook form of the Maxwell postulates. In order to obtain that familiar form, source densities are decomposed into free and bound components, and the bound components are then quantified through the polarization and the magnetization, both of which are in turn subsumed in the definitions of the electric induction $\vec{D}(x, t)$ and the magnetic induction $\vec{H}(x, t)$. Then, (5)–(8) metamorphose into the following familiar form:

$$\nabla \cdot \vec{D}(x, t) = \tilde{\rho}_{so}(x, t),$$
$$\nabla \times \vec{H}(x, t) - \frac{\partial}{\partial t} \vec{D}(x, t) = \tilde{J}_{so}(x, t),$$
$$\nabla \cdot \vec{B}(x, t) = 0,$$
$$\nabla \times \vec{E}(x, t) + \frac{\partial}{\partial t} \vec{B}(x, t) = 0.\quad (12)$$

Here, $\tilde{\rho}_{so}(x, t)$ and $\tilde{J}_{so}(x, t)$ represent free or externally impressed source densities. Let us note that $\vec{H}(x, t)$ and $\vec{D}(x, t)$ do not have microscopic counterparts and therefore are not considered fundamental in modern electromagnetism.

### 3 Linear Constitutive Relations

The most general linear constitutive relations may be written as [1]

$$\vec{D}(x, t) = \int \int \tilde{\epsilon}(x, t; x_h, t_h) \cdot \vec{E}(x - x_h, t - t_h) dx_h dt_h$$
$$+ \int \int \tilde{\alpha}(x, t; x_h, t_h) \cdot \vec{B}(x - x_h, t - t_h) dx_h dt_h$$
$$+ \int \int \tilde{\Phi}(x, t; x_h, t_h) \vec{B}(x - x_h, t - t_h) dx_h dt_h$$
$$- \int \int \tilde{\Phi}(x, t; x_h, t_h) \vec{E}(x - x_h, t - t_h) dx_h dt_h\quad (13)$$

and

$$\vec{H}(x, t) = \int \int \tilde{\beta}(x, t; x_h, t_h) \cdot \vec{E}(x - x_h, t - t_h) dx_h dt_h$$
$$+ \int \int \tilde{\nu}(x, t; x_h, t_h) \cdot \vec{B}(x - x_h, t - t_h) dx_h dt_h$$
$$- \int \int \tilde{\Phi}(x, t; x_h, t_h) \vec{E}(x - x_h, t - t_h) dx_h dt_h$$
$$- \int \int \tilde{\Phi}(x, t; x_h, t_h) \vec{E}(x - x_h, t - t_h) dx_h dt_h\quad (14)$$

wherein the integrals extend only over the causal values of $(x_h, t_h)$ in relation to $(x, t)$. Five constitutive functions are present in the two foregoing equations: $\tilde{\epsilon}$ is the permittivity tensor; $\tilde{\nu}$ is the permeability tensor; $\tilde{\alpha}$ and $\tilde{\beta}$ are the magnetoelectric tensors such that

$$\text{Trace} \left[ \tilde{\alpha}(x, t; x_h, t_h) - \tilde{\beta}(x, t; x_h, t_h) \right] = 0;\quad (15)$$

3
and $\tilde{\Phi}$ may be called the Tellegen parameter.

When (13) and (14) are substituted in (9)–(12) to retain only the primitive fields and the source densities, the resulting four equations contain $\tilde{\epsilon}$, $\tilde{\alpha}$, $\tilde{\beta}$, and $\tilde{\nu}$ in two ways:

(i) by themselves, and
(ii) through their space- and time-derivatives.

In contrast, $\tilde{\Phi}$ does not occur by itself, but only in terms of derivatives [1]. The elimination of this anomalous situation leads to the Post constraint

$$\tilde{\Phi}(x, t; x_h, t_h) \equiv 0.$$  

(16)

Arguments in favor of and against the Post constraint were cataloged some years ago [3], with the opposing arguments based on the so-called EH electromagnetism wherein $\tilde{\mathbf{H}}(x, t)$ is regarded as the primitive magnetic field and $\tilde{\mathbf{B}}(x, t)$ as the induction magnetic field. The EH–electromagnetism is a pre–modern formulation that is still widely used in frequency–domain research. Opposing arguments of a similar nature have also been made under the rubric of the heterodox EDBH–electromagnetism [4], wherein $\tilde{\mathbf{D}}(x, t)$ and $\tilde{\mathbf{H}}(x, t)$ are also supposed to have microscopic counterparts and are therefore also considered primitive.

4 Boundary Value Problems

Constitutive functions are macroscopic entities arising from the homogenization of assemblies of microscopic bound source densities, with matter–free space serving as the reference medium [5]. In any small enough portion of matter that is homogenizable, the constitutive functions are uniform. When such a portion will be interrogated for characterization, it will have to be embedded in matter–free space. Typically, macroscopically homogeneous matter is characterized in the frequency domain. Hence, it is sensible to investigate if the Tellegen parameter can be determined by such a measurement.

Without loss of generality, let us consider therefore that all space is divided into two regions, $V_+$ and $V_-$, separated by a boundary $S$. The region $V_+$ is not filled with matter, whereas the region $V_-$ is filled with a spatially homogeneous, temporally invariant and spatially local matter characterized by the constitutive relations

$$\begin{align*}
\mathbf{D}(x, \omega) &= \varepsilon(\omega) \cdot \mathbf{E}(x, \omega) + \alpha(\omega) \cdot \mathbf{B}(x, \omega) + \Phi(\omega) \mathbf{B}(x, \omega), \\
\mathbf{H}(x, \omega) &= \mu(\omega) \cdot \mathbf{E}(x, \omega) + \nu(\omega) \cdot \mathbf{B}(x, \omega) - \Phi(\omega) \mathbf{E}(x, \omega),
\end{align*}$$  

(17)

where $\omega$ is the angular frequency, and $\mathbf{D}(x, \omega)$ is the temporal Fourier transform of $\tilde{\mathbf{D}}(x, t)$, etc.

The frequency–domain differential equations

$$\begin{align*}
\nabla \cdot \mathbf{B}(x, \omega) &= 0, \\
\nabla \times \mathbf{E}(x, \omega) - i\omega \mathbf{B}(x, \omega) &= 0,
\end{align*}$$  

(18)

$$x \in V_+ \cup V_-,$$
are applicable in both $V_+$ and $V_-$, with $i = \sqrt{-1}$.

The remaining two Maxwell postulates in matter–free space may be written as

\[
\begin{align*}
\epsilon_0 \nabla \cdot \mathbf{E}(x, \omega) &= \rho_{so}(x, \omega) \\
\mu_0^{-1} \nabla \times \mathbf{H}(x, \omega) + i \omega \epsilon_0 \mathbf{E}(x, \omega) &= \mathbf{J}_{so}(x, \omega)
\end{align*}
\]

in terms of only the macroscopic primitive fields, with sources that are sufficiently removed from the boundary $S$ [6]. The fields $\mathbf{E}(x, \omega)$ and $\mathbf{B}(x, \omega)$ in $V_+$ can be represented using standard techniques [2, 7], and the representations of $\mathbf{D}(x, \omega) = \epsilon_0 \mathbf{E}(x, \omega)$ and $\mathbf{H}(x, \omega) = \mu_0^{-1} \mathbf{B}(x, \omega)$ in $V_+$ then follow.

In $V_-$, the remaining two Maxwell postulates are expressed as follows:

\[
\begin{align*}
\nabla \cdot \mathbf{D}(x, \omega) &= 0 \\
\nabla \times \mathbf{H}(x, \omega) + i \omega \mathbf{D}(x, \omega) &= 0
\end{align*}
\]

Substituting (17) therein, we obtain

\[
\begin{align*}
\nabla \cdot \left[ \epsilon(\omega) \mathbf{E}(x, \omega) + \alpha(\omega) \mathbf{B}(x, \omega) \right] \\
+ \Phi(\omega) \nabla \cdot \mathbf{B}(x, \omega) &= 0, \quad x \in V_-
\end{align*}
\]

and

\[
\begin{align*}
\nabla \times \left[ \beta(\omega) \mathbf{E}(x, \omega) \right] + i \omega \epsilon(\omega) \mathbf{E}(x, \omega) \\
+ \nabla \times \left[ \mu(\omega) \mathbf{B}(x, \omega) \right] + i \omega \alpha(\omega) \mathbf{B}(x, \omega) \\
- \Phi(\omega) \left[ \nabla \times \mathbf{E}(x, \omega) - i \omega \mathbf{B}(x, \omega) \right] &= 0, \quad x \in V_-
\end{align*}
\]

These equations simplify to

\[
\begin{align*}
\nabla \cdot \left[ \epsilon(\omega) \mathbf{E}(x, \omega) + \alpha(\omega) \mathbf{B}(x, \omega) \right] &= 0, \quad x \in V_-
\end{align*}
\]

and

\[
\begin{align*}
\nabla \times \left[ \beta(\omega) \mathbf{E}(x, \omega) \right] + i \omega \epsilon(\omega) \mathbf{E}(x, \omega) \\
+ \nabla \times \left[ \mu(\omega) \mathbf{B}(x, \omega) \right] + i \omega \alpha(\omega) \mathbf{B}(x, \omega) &= 0, \quad x \in V_-
\end{align*}
\]

by virtue of (18). For many classes of materials and shapes of $S$, $\mathbf{E}(x, \omega)$ and $\mathbf{B}(x, \omega)$ in $V_-$ can also be adequately represented [8, 9]; and thereafter so can be $\mathbf{D}(x, \omega)$ and $\mathbf{H}(x, \omega)$ in $V_-$.

In order to solve the boundary value problem, the boundary conditions

\[
\begin{align*}
\mathbf{B}^{norm}(x+, \omega) &= \mathbf{B}^{norm}(x-, \omega) \\
\mathbf{D}^{norm}(x+, \omega) &= \mathbf{D}^{norm}(x-, \omega) \\
\mathbf{E}^{tan}(x+, \omega) &= \mathbf{E}^{tan}(x-, \omega) \\
\mathbf{H}^{tan}(x+, \omega) &= \mathbf{H}^{tan}(x-, \omega)
\end{align*}
\]

\[x \in S, \]
have to be imposed on the boundary $S$. Here, $\mathbf{B}^{\text{norm}}(x, \omega)$ indicate the normal components of $\mathbf{B}(x, \omega)$ on either side of $S$, whereas $\mathbf{E}^{\text{tan}}(x, \omega)$ denote the tangential components of $\mathbf{E}(x, \omega)$ similarly, etc. Some resulting set of equations can then be solved to determine the scattering of an incident field by the material contained in $V_-$.

Much effort is not required to solve the simplest boundary value problems. Relevant to the Post constraint, reference is made to two papers wherein the boundary $S$ is a specularly smooth plane of infinite extent [10, 11]. More complicated boundaries have also been tackled [9, 12, 13]. The inescapable conclusion from examining the results of boundary value problems is that the fields scattered in $V_+$ by the material contained in $V_-$ are affected by the Tellegen parameter (if any). Yet that conclusion is naïve and incorrect, as we see next.

## 5 The Conflict and Its Resolution

We have two very sharply contrasting Statements emanating from the foregoing frequency–domain exercise:

A. The Tellegen parameter $\Psi$ vanishes from the fundamental equations (18), (23) and (24) for the material of which the chosen scatterer is made.

B. The fields scattered by the chosen scatterer contain a signature of the Tellegen parameter (if any).

In other words, the Tellegen parameter is a *ghost*: it does not have a direct existence in the fundamental differential equations, but its presence may be indirectly gleaned from a scattering measurement.

The ghostly nature of the Tellegen parameter is a consequence of the boundary conditions (25)$_2$ and (25)$_4$. Even more specifically, it arises from the representations of $\mathbf{D}(x, \omega)$ and $\mathbf{H}(x, \omega)$ in $V_-$. It is instructive to decompose the macroscopic induction fields as [14] $\mathbf{D}(x, \omega) = \mathbf{D}_{\text{actual}}(x, \omega) + \mathbf{D}_{\text{excess}}(x, \omega) \quad \mathbf{H}(x, \omega) = \mathbf{H}_{\text{actual}}(x, \omega) + \mathbf{H}_{\text{excess}}(x, \omega) \quad x \in V_-$, (26)

where $\mathbf{D}_{\text{actual}}(x, \omega) = \varepsilon(\omega) \cdot \mathbf{E}(x, \omega) + \mu(\omega) \cdot \mathbf{B}(x, \omega) \quad \mathbf{H}_{\text{actual}}(x, \omega) = \mu(\omega) \cdot \mathbf{E}(x, \omega) + \varepsilon(\omega) \cdot \mathbf{B}(x, \omega) \quad x \in V_-$, (27)

are retained in (23) and (24). On the other hand, $\mathbf{D}_{\text{excess}}(x, \omega) = \Phi(\omega) \mathbf{B}(x, \omega) \quad \mathbf{H}_{\text{excess}}(x, \omega) = -\Phi(\omega) \mathbf{E}(x, \omega) \quad x \in V_-$, (28)

are filtered out of (23) and (24) by (18) but do affect the boundary conditions (25)$_2$ and (25)$_4$. 


The fundamental differential equations in $V_-$ can now be written as follows:

$$
\begin{align*}
\nabla \cdot B(x, \omega) &= 0 \\
\nabla \times E(x, \omega) - i\omega B(x, \omega) &= 0 \\
\nabla \cdot D_{\text{actual}}(x, \omega) &= 0 \\
\nabla \times H_{\text{actual}}(x, \omega) + i\omega D_{\text{actual}}(x, \omega) &= 0
\end{align*}
$$

\begin{equation}
\text{, } x \in V_-. \quad (29)
\end{equation}

Boundary conditions in electromagnetics emerge from the fundamental equations [15]. Therefore, consistently with (29), the correct boundary conditions on $S$ are

$$
\begin{align*}
B_{\text{norm}}(x^+, \omega) &= B_{\text{norm}}(x^-, \omega) \\
D_{\text{norm}}(x^+, \omega) &= D_{\text{norm}}(x^-, \omega) \\
E_{\text{tan}}(x^+, \omega) &= E_{\text{tan}}(x^-, \omega) \\
H_{\text{tan}}(x^+, \omega) &= H_{\text{tan}}(x^-, \omega)
\end{align*}
$$

\begin{equation}
\text{, } x \in S, \quad (30)
\end{equation}

instead of (25). Thus the correct formulation of the boundary value problem involves (30)$_2$ and (30)$_4$ instead of (25)$_2$ and (25)$_4$.

To sum up, the conflict between Statements A and B arises from a naïve and incorrect formulation of the boundary value problem. The correct formulation does not contain $D_{\text{excess}}(x, \omega)$ and $H_{\text{excess}}(x, \omega)$ in $V_-$ as well as in the boundary conditions.

6 Concluding Remarks

Any field that cannot survive in the fundamental differential equations is superfluous. Neither $H_{\text{excess}}(x, \omega)$ nor $D_{\text{excess}}(x, \omega)$ survives, and may therefore be discarded \textit{ab initio}. The Post constraint thus removes the nonuniqueness inherent in (17), not to mention in (13) and (14), which can appear in two of the four Maxwell postulates in relation to the other two postulates. No wonder, de Lange and Raab [16, 17] could recently complete a major exercise — whereby a multipole formulation of linear materials that was initially noncompliant with the Post constraint was made compliant.

In addition, the Post constraint also removes two anomalies: the first is that of a constitutive function not appearing by itself but only through its derivatives [1]; the second is that of the Tellegen “medium” which is isotropic (i.e., with direction–independent properties) but wherein propagation characteristics in antiparallel directions are different.

A simple exercise shows that isolated magnetic monopoles can negate the validity of the Post constraint [18, 19], but the prospects of observing such a magnetic monopole are rather remote [20, 21]. Furthermore, although the electromagnetic characterization of matter–free space, even in the context of general relativity, is compliant with the Post constraint [22], the axion concept renders that constraint invalid [4]. No axions have yet been detected however [23]. Finally, available data on magnetoelectric materials seems to negate the Post constraint [24, 25, 26], but that data is faulty [1] as it is based on the neglect of causality [27] and a false manipulation of the Onsager principle [28]. Needless to add, if either an isolated magnetic monopole or an axion is ever discovered, or if a magnetoelectric material is properly characterized to have the
electromagnetic properties claimed for it by virtue of misapplications of various principles, the Post constraint would be invalidated and the basics of EB–electromagnetism would have to thought anew.

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