The Fundamental Metallicity Relation Emerges from the Local Anti-correlation between Star Formation Rate and Gas-phase Metallicity that Exists in Disk Galaxies

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\textbf{Abstract}

The fundamental metallicity relation (FMR) states that galaxies of the same stellar mass but larger star formation rate (SFR) tend to have smaller gas-phase metallicity ($\langle Z_g \rangle$). It is thought to be fundamental because it naturally arises from the stochastic feeding of star formation from external metal-poor gas accretion, a process extremely elusive to observe but essential according to the cosmological simulations of galaxy formation. In this Letter, we show how the FMR emerges from the local anti-correlation between SFR surface density and $Z_g$ recently observed to exist in disk galaxies. We analytically derive the global FMR from the local law, and then show that both relations agree quantitatively when considering the star-forming galaxies of the Mapping Nearby Galaxies at APO (MaNGA) survey. Thus, understanding the FMR becomes equivalent to understanding the origin of the anti-correlation between SFR and metallicity followed by the set of star-forming regions of any typical galaxy. The correspondence between local and global laws is not specific to the FMR, so that a number of local relations should exist that are associated with known global relations.

\textit{Key words:} galaxies; abundances – galaxies: evolution – galaxies: formation – galaxies: fundamental parameters – galaxies: star formation – methods: analytical

1. Rationale

The scaling relations followed by galaxies play a central role in our understanding of galaxy formation. The nonlinear evolution of the density perturbations created during the big bang gives rise to galaxies in a sequence that can be followed only through numerical simulations. Many key physical processes (e.g., the feedback produced by stars and black holes) have to be taken into account through sub-grid recipes tuned to reproduce some of the scaling properties observed in galaxies (e.g., the distribution of stellar masses). Other independent scaling relations allow us to test the consistency of the resulting model galaxies (e.g., Vogelsberger et al. 2014; Schaye et al. 2015). Among them, the fundamental metallicity relation (FMR) stands out because it relates (and so constrains) the main ingredients involved in the star formation process, its fueling, and its self-regulation. Ellison et al. (2008), Mannucci et al. (2010), and Lara-López et al. (2010) found out that galaxies of the same stellar mass with higher star formation rate (SFR) show lower gas-phase metallicity. As it was already put forward by Ellison et al. (2008) and Mannucci et al. (2010), the existence of this relation suggests the feeding of star formation from external stochastic metal-poor gas accretion, a process that is extremely elusive to observe (e.g., Sánchez Almeida et al. 2014) but essential according to the numerical simulations of galaxy formation (e.g., Dekel et al. 2009). The accretion of metal-poor gas triggers star formation, which consumes the accreted gas, which eventually leads to reducing the SFR and increasing the gas-phase metallicity. For recent reviews on the FMR, see, e.g., Sánchez Almeida (2017), Sánchez et al. (2017), Cresci et al. (2018), or Maiolino & Mannucci (2019).

It has been recently found that star-forming disk galaxies are chemically inhomogeneous, with a decrease of metallicity spatially coinciding with enhanced SFR (Sánchez Almeida et al. 2018; Hwang et al. 2019; Sánchez-Menguiano et al. 2019). Notably, this is not only a property of some rare extremely metal-poor galaxies (e.g., Sánchez Almeida et al. 2013, 2015) but it happens to be characteristic of the full population of local star-forming galaxies with stellar mass $\log(M_*/M_\odot) \lesssim 10.5$, as portrayed by the Mapping Nearby Galaxies at APO (MaNGA) survey (Sánchez-Menguiano et al. 2019).

In this Letter, we show that the FMR emerges from the local anti-correlation between SFR surface density and metallicity. We derive the global FMR from the local law (Section 2), and then show that both relations agree observationally (Section 3). The agreement is qualitative when considering FMRs from assorted references, but it becomes quantitative when using the same MaNGA galaxies for both the local and the global law (Section 3). Thus, understanding the FMR becomes equivalent to understanding the origin of the anti-correlation between SFR and metallicity followed by the set of star-forming regions of low-mass galaxies (Section 4). The equivalence between local and global laws is to be expected also in other known global scaling relations (Section 4).

2. FMR from the Local Anti-correlation between $\Sigma_{\text{SFR}}$ and $Z_g$

As discussed in Section 1, the galaxies forming stars have been shown to present an anti-correlation between the SFR surface density, $\Sigma_{\text{SFR}}$, and the gas-phase metallicity, $Z_g$, i.e.,

\begin{equation}
\Delta \log Z_g = m \Delta \log \Sigma_{\text{SFR}},
\end{equation}

with $\Delta X$ being the residual of parameter $X$ once large-scale variations have been removed or, equivalently, the difference between $X$ in two nearby regions of the same galaxy. Equation (1) has been shown to hold by Sánchez-Menguiano et al. (2019), where the slope $m$, which quantifies the strength and sign of the correlation, is found to vary systematically depending on the global properties of the galaxy. Considering...
two galaxies with the same global properties, and thus the same $m$, Equation (1) also describes the difference between two regions in the two galaxies, provided that the regions are at the same galactocentric distance. Then the relation (1) also describes the excess of $\log Z_{g}$ and $\log \Sigma_{\text{SFR}}$ in one particular region of a particular galaxy with respect to the average alike galaxy $^3$ at the position of the region, i.e.,

$$\Delta \log Z_{g} = \log Z_{g} - \log Z_{g0},$$
$$\Delta \log \Sigma_{\text{SFR}} = \log \Sigma_{\text{SFR}} - \log \Sigma_{\text{SFR0}},$$

(2)

where $Z_{g0}$ and $\Sigma_{\text{SFR0}}$ stand for the large-scale variation of the gas-phase metallicity and SFR, respectively, in the mean alike galaxy. Defining $\Delta Z_{g} = Z_{g} - Z_{g0}$ and $\Delta \Sigma_{\text{SFR}} = \Sigma_{\text{SFR}} - \Sigma_{\text{SFR0}}$, and assuming $|\Delta Z_{g}| \ll Z_{g}$ and $|\Delta \Sigma_{\text{SFR}}| \ll \Sigma_{\text{SFR}}$, then Equation (1) can be re-written as

$$\Delta Z_{g}/Z_{g0} \simeq m \Delta \Sigma_{\text{SFR}}/\Sigma_{\text{SFR0}}.$$  

(3)

Note that, except for $m$, all variables in Equations (1) and (3) depend on the position on the galaxy disk, defined by the Cartesian coordinates $x$ and $y$. Thus, the mean metallicity of the galaxy is defined as

$$\langle Z_{g} \rangle = \langle M_{g} \rangle^{-1} \int W Z_{g} \Sigma_{g} \, dx \, dy,$$
$$\langle M_{g} \rangle = \int W \Sigma_{g} \, dx \, dy.$$

(4)

with $\Sigma_{g}$ being the surface density of the gas, and $W$ being an arbitrary weight that goes to zero outside the galaxy so that the integral extends to all of the $x$-$y$ plane. The symbol $\langle M_{g} \rangle$ stands for the total gas mass of the galaxy only if $W$ equals 1 within the galaxy. (To facilitate distinguishing local from galaxy-integrated quantities, the latter are always represented as symbols within brackets.) The excess mean metallicity can be defined as

$$\Delta \langle Z_{g} \rangle = \langle Z_{g} \rangle - \langle Z_{g0} \rangle,$$

(5)

with $\langle Z_{g0} \rangle$ being the mean metallicity to be expected considering all of the alike galaxies. Equations (3)–(5) lead to

$$\Delta \langle Z_{g} \rangle \simeq m \langle M_{g} \rangle^{-1} \int W Z_{g0} \Delta \Sigma_{\text{SFR}} \Sigma_{\text{SFR0}} \, dx \, dy.$$  

(6)

The Kennicutt–Schmidt relation shows that the observed SFR surface density scales gas mass surface density through a law assumed to have the form of

$$\Sigma_{\text{SFR}} = A \Sigma_{g}^{N},$$

(7)

where $A$ and $N$ are numerical constants, with $N \simeq 1$ (e.g., Kennicutt 1998; Bigiel et al. 2008). Keeping only first-order terms in both $N - 1$ and $\Delta \Sigma_{\text{SFR}}$, Equation (7) becomes

$$\frac{\Sigma_{g}}{\Sigma_{\text{SFR0}}} \simeq A^{N-2} \left[1 + (1 - N) \ln \Sigma_{\text{SFR0}} + \frac{\Delta \Sigma_{\text{SFR}}}{\Sigma_{\text{SFR0}}} \right].$$  

(8)

Introducing Equation (8) into Equation (6), and retaining only first-order terms,

$$\Delta \langle Z_{g} \rangle \simeq m \langle \text{SFR} \rangle^{-1} \int W Z_{g0} \Delta \Sigma_{\text{SFR}} \, dx \, dy,$$

(9)

with the integrated SFR defined as

$$\langle \text{SFR} \rangle = \int W \Sigma_{\text{SFR}} \, dx \, dy.$$  

(10)

Equation (9) can be re-written as

$$\Delta \langle Z_{g} \rangle \frac{\langle \text{SFR} \rangle}{m} \simeq \langle Z_{g0} \rangle \left[\langle \text{SFR} \rangle - \langle \text{SFR0} \rangle \right] + \int W \langle Z_{g0} \rangle \Delta \Sigma_{\text{SFR}} \, dx \, dy.$$  

(11)

The second term in the right-hand side of Equation (11) is negligible if the local excess of SFR ($\Delta \Sigma_{\text{SFR}}$) is uncorrelated with the large-scale variations of metallicity ($Z_{g0} - \langle Z_{g0} \rangle$). In principle, local and large-scale variations are expected to be independent from each other, allowing the condition to be met. However, for the moment, we neglect this term just as an ansatz to be justified later on in Section 3 using real galaxies. Thus, one can rewrite Equation (11) as

$$\Delta \langle Z_{g} \rangle / \langle Z_{g0} \rangle \simeq m \Delta \text{log} \langle \text{SFR} \rangle.$$

(12)

or, neglecting second-order terms,

$$\Delta \text{log} \langle Z_{g} \rangle \simeq m \Delta \text{log} \langle \text{SFR} \rangle.$$  

(13)

Equation (13) shows the global FMR, which emerges from the local anti-correlation given in Equation (1) with exactly the same slope.

### 3. Comparison with Observations

Sánchez-Menguiano et al. (2019) measured the slope $m$ in a set of 736 nearby spiral galaxies from the fifteenth data release (DR15) Sloan Digital Sky Survey (SDSS) IV MaNGA survey (Bundy et al. 2015). Galaxies were selected the full MaNGA parent sample to be both star-forming and large enough to allow measuring local variations of SFR and $Z_{g}$. The value of $m$ was inferred from a linear fit to the scatter plot $\Delta Z_{g}$ versus $\Delta \Sigma_{\text{SFR}}$ in each individual galaxy computed after removing the large-scale trends. $Z_{g}$ and SFR were deduced using the strong line ratio O3N2 and the H$\alpha$ flux, respectively (for details, see Sánchez-Menguiano et al. 2019). The slope $m$ was found to depend on $M_{*}$, going from negative at low mass (log[M$_{*}$/M$_{\odot}$] < -11.5) to slightly positive at the high-mass end of the mass distribution (log[M$_{*}$/M$_{\odot}$] > 11). The thick red line in Figure 1 shows the resulting mean $m$ for a given $M_{*}$.

Figure 1 also includes the slopes worked out by Sánchez Almeida et al. (2018; blue stars with error bars) for the local anti-correlation found in a number of dwarf galaxies of the local universe, which agree quite well with the estimates by Sánchez-Menguiano et al. Slopes inferred from other assorted FMRs in the literature are also included in Figure 1 (references are given in the inset). The slopes corresponding to the FMRs were computed from the published equation $Z_{g} = Z_{g0}(\text{SFR}, M_{*})$ as

$$m = \frac{\partial \log Z_{g}}{\partial \log \text{SFR}} \bigg|_{M_{*}}.$$  

(14)
As one can see from Figure 1, the $m$ values inferred from local and global variations agree qualitatively; the slopes are negative, and there is a tendency for $|m|$ to decrease as $M_*$ increases (see, e.g., the values from Salim et al. 2014). Qualitative differences are also evident, though. We think that these differences are mostly due to systematic errors in the various calibrations used to estimate $Z_g$ and SFR (e.g., Kewley & Ellison 2008; Kennicutt & Evans 2012), and also due to the actual set of galaxies employed to infer $m$ (e.g., Salim et al. 2014). This impression is corroborated by the exercise described in the next paragraph, where local and global laws obtained using the same galaxies and identical $Z_g$ and $\Sigma_{\text{SFR}}$ are shown to coincide.

The equivalence between the local properties and the integrated quantities is a mathematical identity; therefore, it holds provided that certain conditions are met. The main ones are: (1) perturbations are small so that second- and higher-order terms can be neglected, and (2) large-scale variations of metallicity and small-scale variations of SFR are uncorrelated.

In order to check whether these two conditions are usually met, we have computed $\Delta \Sigma_{\text{SFR}}$ and $\Delta (Z_g)$ for the same 736 MaNGA galaxies from which Sánchez-Menguiano et al. (2019) worked out the local anti-correlation between $\Delta \Sigma_{\text{SFR}}$ and $\Delta Z_g$ shown in Figure 1. We aim at comparing the slope inferred from integrated quantities, without approximations (Equations 4, 10, and 13), with those obtained from the spatially resolved quantities (Equation 1). Mean metallicities are derived in four different ways to test the dependence of the conclusions on the way the spatial average $(Z_g)$ is computed. The result for weighting with $\Sigma_{\text{SFR}}$ is shown in Figure 2 (according to Equation 7), $\Sigma_{\text{SFR}}$ is a proxy for $\Sigma_g$ when $N = 1$. We employ Equation 4, replacing $\Sigma_g$ with $\Sigma_{\text{SFR}}$, and setting $W$ to a constant $\neq 0$ only in the star-forming regions. MaNGA galaxies follow the global FMR as evidenced in the left panel of Figure 2, where the mass-metallicity relation is color-coded with SFR. It is also clear from the right panel of Figure 2, which shows the scatter plot $(Z_g)$ versus SFR for all galaxies within a narrow range of stellar masses $(9.2 \leq \log (M_*/M_\odot) \leq 9.6)$. $Z_g$ and $\Sigma_{\text{SFR}}$ were computed with exactly the same recipes employed by Sánchez-Menguiano et al. (2019), and the actual galaxies and the star-forming regions within them were also the same. A linear fit to scatter plots like the one in the right panel of Figure 2 provides $m$ at a particular mass range. Repeating the exercise for different mass bins renders the dependence of $m$ on $M_*$ represented in Figure 3.

Equation 7 shows the result based on weighting with $\Sigma_{\text{SFR}}$ (the red symbols), with $\Sigma_{\text{SFR}}$ (the orange symbols), without weighting (the blue symbols), and using the metallicity at the effective radius (the green symbols). The three first averages correspond to weighting with $\Sigma_{\text{SFR}}$ if the Kennicutt–Schmidt relation holds (Equation 7) and $N = 1$, 1.4, and 0, respectively. $N = 1.4$ corresponds to the canonical value in the calibration by Kennicutt (1998).

The three main conclusions ensuing from Figure 3 are as follows. (1) There is a very good agreement between the slope derived from local and global laws, which reinforces the approximations leading to the analytical derivation in Section 2. (2) The way in which the average is computed does not seem to be very important. Four different ways of averaging yield similar results. (3) The number of galaxies used to construct Figure 3 is large enough for the conclusions to be representative of local galaxies (as portrayed by MaNGA).

4. Discussion and Conclusions

Provided the existence of a local correlation between $\Sigma_{\text{SFR}}$ and $Z_g$, the FMR is a must—Equation (1) $\Rightarrow$ Equation (13). Does the inverse hold true, i.e., does the existence of the FMR imply the existence of a local anti-correlation? The answer is yes. If a galaxy does not have a local correlation, it cannot show a correlation in integrated quantities (i.e., if $m = 0$ in Equation (1), $m$ cannot be $\neq 0$ in Equation (13)). Thus, understanding the FMR becomes equivalent to understanding the origin of the correlation between SFR and metallicity followed by the set of star-forming regions of any typical galaxy.

A potential flaw of the above argument is the breakdown of the approximations used to derive Equation (13) from Equation (1), namely, the small variation of the physical parameters, and the lack of correlation between large-scale metallicity variations and small-scale SFR variations. The validity of these approximations was tested using a set of 736 star-forming galaxies in MaNGA, which give the same slope $m$ from local variations and from spatially integrated quantities (Figure 3). We note that this set is representative of the local star-forming galaxies with masses in the range $8.7 \leq \log (M_*/M_\odot) \leq 11.2$. Still, this may break down when considering other more extreme galaxies observed at higher spatial resolution.

5 As discussed by, e.g., Sánchez et al. (2017, 2019), the use of finite mass bins may induce a spurious correlation between $(Z_g)$ and SFR. However, this artificial correlation is always positive because both $(Z_g)$ and SFR tend to increase with increasing $M_*$. Thus, $m \leq 0$ cannot result from such a bias. As a sanity check, we verified that dividing by half the mass bin does not produce a significant variation in Figure 3.
The exercise with MaNGA galaxies also allows us to conclude that the actual weight employed to compute the integrated quantities is not fundamental. On the other hand, the methods used to estimate SFR and $Z_g$ from emission line fluxes, and the set of galaxies included in the analysis, do influence the actual value inferred for $m$ (Figure 1).

Our derivation in Section 2 is quite general, and shows that any local relation between two variables gives rise to a global relation. This is also true for the gas mass, for which an alternative FMR is known to exist. Galaxies with larger gas mass (both atomic and molecular) have smaller $Z_g$ (e.g., Bothwell et al. 2016a, 2016b). The fact that this global law has been observed implies that there should be a local anti-correlation between $\Sigma_g$ and $Z_g$. Indeed, Barrera-Ballesteros et al. (2018) found a local anti-correlation between the gas fraction inferred from dust extinction and $Z_g$. Similarly, Lian et al. (2015) analyzed a set of local analogs to Ly break galaxies, finding that objects with younger stellar populations have lower metallicity at a fixed mass. This would imply the existence of a local relation between the age of the stellar population and metallicity of the gas-forming stars. These specific predictions of our equations can be tested observationally.

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Facility: MaNGA (Bundy et al. 2015).
Software: astropy (Astropy Collaboration et al. 2013).

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Figure 2. Left panel: FMR for the 736 MaNGA galaxies used by Sánchez-Menguiano et al. (2019), i.e., $\langle Z_g \rangle$ vs. $M_*$ color-coded with SFR. Given $M_*$, galaxies of lower $\langle Z_g \rangle$ tend to have higher SFR. Right panel: scatter plot $\langle Z_g \rangle$ vs. SFR for the galaxies in the mass bin $9.2 \leq \log(M_*/M_\odot) \leq 9.6$, which is marked with dashed lines in the left panel. The slope of the linear fit shown in red renders $m$.

Figure 3. Comparison of the slope $m$ inferred from local and global FMR relations derived from exactly the same data set. The solid line represents slopes obtained from spatially resolved MaNGA galaxies (Sánchez-Menguiano et al. 2019). The symbols represent slopes obtained from integrated values of metallicity and SFR using exactly the same galaxies. Different colors represent different ways of computing the average metallicity (see the text for details). The error bars correspond to the uncertainty associated with the determination of the slope (vertical), and the rms variation of the stellar masses of the galaxies in the mass bin used to infer $m$ (horizontal).
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