Comparative analysis of unbalanced moment of reinforced concrete slab-column connections

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Abstract. The existing comparative analysis of the unbalanced moment mainly compares the differences between the calculated value and the experimental value of the unbalanced moment in different national codes and different calculation methods. There is less analysis on the change trend of the important parameters in the unbalanced moment calculation methods in national codes. A comparison of the provisions for unbalanced moment of reinforced concrete slab-column connections was studied based on Chinese code (GB 50010-2010), American code (ACI318-14) and European code (EN1992-04). The effects of concrete strength, longitudinal reinforcement ratio, the use of shear reinforcement and gravity-shear ratio on the unbalanced moment were studied through the calculation example and experimental results of 186 specimens. The results indicate that the calculation values of Chinese and American codes are conservative and dispersive, because the longitudinal reinforcement ratio and gravity-shear ratio on distribution coefficient of unbalanced moment were not considered, while the calculation results of European code are the closest to the test values.

1. INTRODUCTION
The unbalanced moment of reinforced concrete slab column connections under horizontal load is an important problem in the study of slab column connections. At present, many scholars have proposed a variety of calculation methods for unbalanced moment, such as pseudo beam method, yield line theory and eccentric shear stress, but a complete set of theory has not yet been formed. At the same time, the equivalent methods of unbalanced moment are different in different countries. In Chinese code (GB50010-2010)\cite{1} and American code (ACI318-14)\cite{2}, unbalanced moment is mainly equivalent to concentrated reaction, while in European Code (EN1992-04)\cite{3}, unbalanced bending moment is equivalent to amplification factor. Moreover, the calculation methods of codes in different countries are different, so the formulas for calculating the unbalanced moment derived from codes in different countries are also different. The punching failure mode and bending failure mode are considered for calculation value of unbalanced moment in Chinese code and American code. Tang Ming\cite{4} and Yi Wei Jian\cite{5} made a comparative analysis on the unbalanced moment of Chinese, American and European codes, and Tian \cite{6} made a comparative analysis on the unbalanced moment calculation results of American code and pseudo beam method. The existing comparative analysis of the unbalanced moment mainly compares the differences between the calculated and experimental values of the unbalanced
moment in the national codes, and there is less analysis on the change trend of the important parameters in the calculation methods of the unbalanced moment in the national codes. This paper compares and analyzes the relevant provisions on the unbalanced moment of reinforced concrete slab column connections in Chinese, American and European codes, and analyzes the influence of the concrete strength, longitudinal reinforcement ratio, punching shear reinforcement and gravity-shear ratio on the unbalanced moment in Chinese, American and European codes through calculation example and a large number of test data. The research content can provide reference and basis for the future design and research of reinforced concrete slab column joints.

2. CALCULATION METHOD OF UNBALANCED MOMENT IN NATIONAL CODES

2.1. Chinese code (GB50010-2010)

For slab column connections under vertical and horizontal loads, the eccentric shear stress model is adopted in appendix F of Chinese code, and the calculation method of equivalent concentrated reaction design value $F_{l,eq}$ is given:

$$F_{l,eq} = F_l + \frac{\alpha_0 M_{un} (h_1 + h_0)}{I_c}$$

$$I_c = \frac{1}{6} h_0 (h_1 + h_0)^3 (h_2 + h_0) \left( \frac{h_1 + h_0}{2} \right)^2$$

$$\alpha_0 = 1 - \frac{1}{1 + \frac{2}{3} \frac{h_1 + h_0}{h_0}}$$

Where, $F_l$ is the interlayer difference of the design value of the axial pressure on the column under the action of vertical load and horizontal load minus the design value of the load on the plate within the scope of punching failure vertebral body; $\alpha_0$ is the calculation coefficient of unbalanced moment distribution; $b_0$ and $h_c$ are the side lengths of columns perpendicular to and parallel to the action direction of unbalanced bending moment, respectively; $u_m$ is the critical interface perimeter, which is $h_0/2$ away from the column edge; $h_0$ is the effective height of the plate; $I_c$ is the similar polar moment of inertia of the critical section.

When the punching failure of the joint occurs, in equation (1), $F_{l,eq}$ reach the punching $F_l$ of the connection, and the first item on the right is represented by $V_g$, then unbalanced bending moment of slab column connection can be deduced from equation (1):

$$M_{un} = \frac{(F_l - V_g) I_c}{u_m h_0 \alpha_0 (h_1 + h_0) / 2}$$

For slab column connections without shear reinforcement:

$$F_l = 0.7 \beta_h f_t \rho u_m h_0$$

$$\eta = \min \left\{ \eta_1 = 0.4 + \frac{1.2}{\beta_1}, \eta_2 = 0.5 + \frac{\alpha f_t}{4 \rho u_m} \right\}$$

For slab column connections with shear reinforcement:

$$F_l = 0.5 f_t \rho u_m h_0 + 0.8 (f_{ys} A_{sva} + f_{ys} A_{sba} \sin \alpha)$$

Where, $M_{un}$ is the unbalanced moment when punching failure occurs; $F_l$ is the punching shear capacity of slab column connections; $V_g$ is the punching force produced by the gravity load on the critical section of the floor; $f_t$ is the design value of concrete tensile strength; $\beta_h$ is the height coefficient of plate section; $\eta_1$ is the influence coefficient of local load or concentrated reaction area shape; $\eta_2$ is the influence coefficient of the ratio of the section perimeter to the effective height of the plate section; $\beta_1$ is the ratio of the long side to the short side of the load area; $f_t$ is the design value of stirrup tensile strength; $A_{sva}$ and $A_{sba}$ are the total area of the intersection of stirrup and flexural reinforcement with the inclined section of $45^\circ$ punching failure cone, respectively; $\alpha$ is the angle between the bent steel bar and the bottom of the slab.
When bending failure occurs, the unbalanced moment of concrete slab column connections is reduced:

\[
M_{f,u} = \frac{M_{f,1} + M_{f,2}}{1 - \alpha_1} \quad (8)
\]

\[
M_{f,1} = f_y,1 A_{s,1} h_0 \left( 1 - 0.5 \frac{f_y,1 \rho_1}{\alpha_1 f_c} \right) \quad (9)
\]

\[
M_{f,2} = f_y,2 A_{s,2} h_0 \left( 1 - 0.5 \frac{f_y,2 \rho_2}{\alpha_2 f_c} \right) \quad (10)
\]

Where, \( M_{f,u} \) is the unbalanced moment of the connections under bending failure; \( M_{f,1} \) and \( M_{f,2} \) are the positive and negative bending capacity in the width range of \( b_c + 3h \); \( f_y,1 \), \( A_{s,1} \) and \( \rho_1 \) is the yield strength, cross-sectional area and reinforcement ratio of bottom reinforcement in \( b_c + 3h \) slab width, \( f_y,2 \), \( A_{s,2} \) and \( \rho_2 \) are the indexes of the steel bars in the corresponding range of the top of the slab; \( \alpha_1 \) is the figure coefficient of equivalent rectangular stress of concrete under compression, which is taken according to the code [1].

2.2. American code (ACI318-14)

For slab column connections under vertical and horizontal loads, the American code adopts the eccentric shear stress model to give the calculation formula of the maximum shear stress on the critical section:

\[
v_u = \frac{V_u}{b_d d} \left[ 1 + \frac{\gamma_v M_{u}(c + d)/2}{J_c} \right] \quad (11)
\]

\[
J_c = \frac{d(c_1 + d) c_2 + 2d(c_1 + d) c_2 + c_1 + d c_2}{6} \quad (12)
\]

\[
\gamma_v = 1 - \frac{2}{3} \left( \frac{c_1 + d}{3 c_1 + d} \right)^2 \quad (13)
\]

Where, \( V_u \) and \( M_u \) are the vertical load and unbalanced bending moment of the node respectively; \( \gamma_v \) is the calculation coefficient of unbalanced bending moment distribution; \( b_0 \) and Chinese specification \( u_m \); \( c_1 \) and \( c_2 \) are in line with Chinese specifications \( h_c \) and \( b_c \); \( J_c \) is the polar-like moment of inertia of the critical section.

When the punching failure occurs, the maximum shear stress \( v_u \) of the critical section in equation (11) reaches the nominal shear stress \( v_n \), then the unbalanced moment of slab column connection can be deduced from equation (11):

\[
M_{f,u} = \frac{(v_n b_d - V_u) J_c}{b_d d (c_1 + d)/2} \quad (14)
\]

For slab column connections without shear reinforcement, \( v_n \) is the minimum of the following three formulas:

\[
v_n = v_c = 0.083(2 + \frac{A_{s}}{b_0}) \lambda \sqrt{f_c} \quad (15)
\]

\[
v_n = v_c = 0.083(2 + \frac{A_{s}}{b_0}) \lambda \sqrt{f_c} \quad (16)
\]

\[
v_n = v_c = 0.333 \lambda \sqrt{f_c} \quad (17)
\]

For slab column connections without shear reinforcement:

\[
v_n = 0.5 v_c + \frac{A_{s} f_c}{b_0 s} \left( \leq 0.5 \lambda \sqrt{f_c} \right) \quad (18)
\]

Where, \( M_{f,u} \) is the of unbalanced moment when punching failure occurs; \( f_c \) is the compressive strength of the cylinder; \( \lambda \) is the reduction factor of mechanical properties of lightweight concrete, 1.0 for ordinary concrete; \( A_{s} \) is the area of shear reinforcement in one perimeter with the same geometry as the column; \( s \) is the spacing of shear reinforcement in the direction perpendicular to the cylinder.
When bending failure occurs, the calculation formula of American code is the same as that of (7), (8) and (9), and American code specifies that $\alpha_1$ is 0.85.

2.3. European Code (EN 1992-04)

For slab column connections under both vertical and horizontal loads, the European code adopts the method based on uniform distribution of shear stress on both sides of the central axis, and gives the maximum shear stress generated by increasing the coefficient:

$$\beta = 1 + K \frac{M_{ud}}{V_{ud} W_f}$$

$$W_f = \frac{c_1^2}{2} + c_1 c_2 + 4c_2 d + 16d^2 + 2\pi c_1$$

Where, $V_{Ed}$ and $M_{Ed}$ are the vertical loads and unbalanced bending moments of the nodes; $u_1$ is the critical section perimeter, and the distance from the column edge is $2d$. $K$ is the coefficient related to the ratio of column dimensions $c_1$ and $c_2$, and its value is a function of the bending moment, bending and torsion ratio of non-uniform shear force transfer. For square columns, 0.6 is taken. $W_f$ is a function of the critical section perimeter $U_1$, calculated according to equation (21).

For slab column connections without shear reinforcement, when the maximum shear stress $v_{Ed}$ of critical section reaches the stress $v_{Rd,c}$, the unbalanced moment of slab column connections is deduced from equation (19):

$$M_{Ed} = (v_{Ed} - \frac{V_{ud}}{u_1}) \frac{1}{K} W_f d$$

$$v_{Ed} = 0.18 k(100\rho_f f_{yk})^{1/3}$$

$$k = 1 + \frac{\sqrt{200}}{d} \leq 2$$

$$\rho_f = \sqrt{\rho_{lz} \cdot \rho_{ly}} \leq 0.02$$

Where: $M_{Ed}$ is the unbalanced bending moment of the joint; $V_{Ed}$ and $V_g$ have the same meaning; $v_{Rd,c}$ are the shear stress of critical section without punching reinforcement; $c_1$, $c_2$, and $d$ have the same meaning as American code; $\rho_{lz}$, $\rho_{ly}$ is the reinforcement ratio within the range of the sum of the effective height of the slab and the size of the column head three times from the column edge along the Z and Y directions of the slab.

For slab column connections with shear reinforcement, when maximum shear stress $v_{Ed}$ of critical section reaches the stress $v_{Rd,cs}$, the unbalanced moment of slab column connections is deduced from equation (19):

$$M_{Ed} = (v_{Rd,cs} - \frac{V_{ud}}{u_1}) \frac{1}{K} W_f d$$

$$v_{Rd,cs} = 0.75 v_{Rd,c} + 1.5 \frac{dA_{sw} f_{ywd,ef} \sin \alpha}{S_{yd}}$$

$$f_{ywd,ef} = 250 + 0.25d \leq f_{ywd}$$

Where, $v_{Rd,cs}$ is the critical section shear stress with stirrups or bent bars; $A_{sw}$ and $S_{yd}$ are the same as American code; $f_{ywd,ef}$ and $f_{ywd}$ are the effective design strength and yield strength design values of reinforcement respectively.

3. CONVERSION OF MATERIAL STRENGTH INDEX IN NATIONAL CODES

Due to the different value methods of material strength index in Chinese, American and European codes, it is necessary to establish the conversion relationship between the material strength index in different countries in the comparative analysis of unbalanced bending moment.
The standard value of cube compressive strength $f_{cu,k}$ with a side length of 150 mm is adopted in Chinese code, and the cylinder compressive strength $f'_c$ with a diameter of 150 mm and a height of 300 mm is adopted in American code. The conversion formula between the standard value of cube compressive strength in Chinese code and that in Chinese code:

$$f_{cu,k} = 1.25 \times \frac{1 - 1.645\delta_{cu}}{1 - 1.28\delta_{cu}} f'_c$$

The characteristic value of the compressive strength of a cylinder with a diameter of 150 mm and a height of 300 mm is adopted in the European code, and the conversion relationship between it and the standard value of the compressive strength of concrete cube in the Chinese code is as follows:

$$f_{cu,k} = 1.226 f'_c$$

The specifications of different countries are different for the reinforcement strength index. Chinese code and European code use the "strength standard value" divided by the "sectional coefficient". For the provisions of the sectional coefficient, the Chinese standard is 1.1, the European standard is 1.15, and the American standard directly uses the standard value.

4. CALCULATION AND COMPARATIVE ANALYSIS

4.1. The analysis model
The thickness of slab column connection is 150 mm, the thickness of protective layer is 15 mm, the side length of square column section is 350 mm, and the strength grade of concrete cube is C30. The longitudinal reinforcement of the slab adopts double-layer bidirectional configuration, which is C12@110 within $b_C + 3h$ and C12@150 outside $b_C + 3h$, and the standard value of reinforcement strength is 400 MPa. The vertical load of slab column connection is 131 kN. When shear reinforcement is calculated, the stirrup is A8@60(110), and the standard value of reinforcement strength is 300 MPa. Fig. 1 is details of node construction.

![Fig.1 Node details](a) stirrup reinforcement drawing  (b) A-A cross-section

4.2. Influence of concrete strength
Figure 2 is a curve of the relationship between unbalanced moment and concrete strength calculated by national codes. It can be seen from the figure that the unbalanced moment calculated by American code is higher than that calculated by Chinese code, because the ultimate shear stress of concrete is considered to be 0.7 times larger than the design value of axial tensile strength of concrete adopted by Chinese code when calculating $v_c$, so the unbalanced moment calculated by American code is larger than that calculated by Chinese code when the section size is the same. The calculated value of American code tends to be flat with the increase of concrete strength. The reason is that the failure mode of connections changes from punching failure to bending failure with the increase of concrete strength. Unbalanced moment of connection is mainly borne by the negative and positive bending within the range of $b_C + 3h$ with the column as the center. Therefore, when the concrete strength increases to a certain value, improving the concrete strength has little influence on the calculated value of American code. The calculated value of unbalanced moment of European code is between that of Chinese and American.
codes, which is related to the cube root of concrete compressive strength. Therefore, with the increase of concrete strength, the slope of calculated value is the smallest. It can be seen from Fig.2 that when the concrete strength is the same, the calculated value of American code is greater than that of Chinese code, which means that the definition of material strength in Chinese code is relatively conservative.

![Fig.2 Relationship between unbalanced moment and concrete strength](image)

4.3. Influence of longitudinal reinforcement ratio

Fig.3 shows the curve of the relationship between the unbalanced moment and the reinforcement ratio of longitudinal bars calculated by the codes of various countries. It can be seen from the figure that the calculated value of unbalanced moment in Chinese and American codes increases at first and then remains unchanged, because when the reinforcement ratio is small, the ratio of longitudinal reinforcement is increased. The calculation results of Chinese and American codes show that the joint has bending failure. At this time, the calculated value of unbalanced moment in Chinese and American codes is mainly determined by $M_{fu}$ related to the ratio of longitudinal reinforcement. When the longitudinal reinforcement ratio increases to a certain value, the calculation results of the two codes show that the punching failure and the unbalanced bending moment are determined by $M_{vu}$, but the $M_{vu}$ calculation values of the Chinese and American codes are not affected by the longitudinal reinforcement ratio. Therefore, when the reinforcement ratio is low, increasing the reinforcement ratio has a greater impact on the calculation values of the two codes. The increase of reinforcement ratio has no effect on the calculated value. However, when the reinforcement ratio is greater than 2%, it is still taken as 2%. Therefore, when the reinforcement ratio is greater than 2%, the calculated value of unbalanced moment of Eurocode does not change.

![Fig.3 Relationship between unbalanced moment and longitudinal reinforcement ratio](image)
4.4. Influence of shear reinforcement

Fig.4 shows the curve of unbalanced moment of each code before and after the configuration of shear reinforcement. It can be seen from the figure that the calculation results of each code change after the same stirrup is configured, but the influence of each code on the calculation value of unbalanced moment is different before and after the stirrup is configured. After the same stirrup is configured, the increase of unbalanced moment in Eurocode is much greater than that in American code and Chinese code. The reason is that the reduction factor of concrete and the stirrup cross-section area selected by the codes are different. European Code reduces the concrete strength by 25%, Chinese code reduces the concrete tensile strength by 29%, and American code reduces the concrete strength by 50%. The stirrup area of Eurocode is 2.025 times and 1.5 times of that of Chinese and American codes respectively.

The calculated value of unbalanced moment in Eurocode increases significantly with the increase of concrete strength, while the slope of the calculated value in Chinese and American code decreases with the increase of concrete strength. The calculated value of unbalanced moment in American code remains unchanged with the increase of concrete strength. The reason is that punching shear failure and bending failure are considered in Chinese and American codes, and stirrup has no effect on the calculated value of unbalanced moment when bending failure occurs. Therefore, the increase of concrete strength after stirrup configuration has no obvious effect on the unbalanced moment in Chinese and American codes.

5. COMPARATIVE ANALYSIS OF TEST DATA

In this paper, 186 groups of test data of slab column joints without shear reinforcement under the combined action of vertical and horizontal load[4,6-16]. Fig 5 shows the scatter comparison between the test value and the calculated value of unbalanced moment bearing capacity in national codes. The solid line represents 1 times of the test value, and the dotted line represents 0.75 times and 1.25 times of the test value respectively.

It can be obtained from a series of scatter comparison charts in Fig.5:

1) The unbalanced moment calculated by national codes is different from the test value. The unbalanced moment calculated by European codes is ideal, with an average value of 1.02 and a coefficient of variation of 0.29. The average values calculated by American codes and Chinese codes are 1.55 and 1.43, with coefficient of variation of 0.58 and 0.49, respectively.

2) The calculated values in European codes mainly focus on 0.75-1.25 times of test values, and 15% of them are outside this range, while 40% in American codes are outside the range of 0.75-1.25 times of test values, 45% in Chinese codes are outside the range of 0.75-1.25 times of test values, and the calculated results in Chinese and American codes are mainly above 1.25 times of test values.
Fig. 5 Comparison of test value and calculation value of unbalanced moment bearing capacity

Fig. 6~8 shows the classification of calculation results according to the influencing factors of unbalanced moment. The main influencing factors are concrete strength, longitudinal reinforcement ratio and gravity-shear ratio.

It can be seen from Fig. 6 that the calculation results of each code have no significant change with the concrete cube strength, and the calculation data group of American code is higher than that of Chinese code as a whole, because the definition of concrete strength in Chinese and American codes is different. It can be seen from Fig. 7 that when the longitudinal reinforcement ratio is greater than 0.8%, the ratio between the test value and the calculated value in Chinese and American codes has a slight downward trend.

Fig. 8 shows the scatter diagram of gravity-shear ratio and the calculated value of each code. It can be seen from the figure that the ratio of the test value to the calculated value of the Chinese and American codes decreases significantly with the increase of the gravity-shear ratio, while the variation law of the gravity-shear ratio from the calculated value of the European code is relatively unclear and the dispersion is low, because the influence of the longitudinal reinforcement ratio is not considered in the Chinese and American codes, Durrani[17] have proved the proportion of the moment of plate bending
resistance in the unbalanced moment is different under different weight shear ratio. However, in the eccentric shear stress model adopted by Chinese and American codes, the influence of weight shear ratio is not considered, so the calculation data points of Chinese and American codes are relatively discrete and have a significant downward trend.

6. CONCLUSION

(1) Due to the eccentric shear stress model adopted in Chinese and American codes, the calculated value of unbalanced moment is related to punching shear failure and bending failure modes of slab column connections. Therefore, when the concrete strength, effective height of slab and other parameters increase, the calculated value gradually tends to change unobvious with the increase of parameters, The calculated value of Eurocode increases with the increase of parameters.

(2) Compared with the Chinese and American codes, the European Code considering the influence of longitudinal reinforcement ratio is the closest to the test value and has the highest accuracy.

(3) Because the influence of longitudinal reinforcement ratio and gravity-shear ratio on unbalanced moment distribution is not considered in both Chinese and American codes, the calculated values in both codes are conservative and have high dispersion. Therefore, the influence of longitudinal reinforcement ratio and gravity-shear ratio should be considered when the unbalanced moment bearing capacity of slab column joints is calculated according to the Chinese code.

(4) In this paper, the variation trend of the parameters in the calculation methods of unbalanced moment bearing capacity in Chinese code, American code and European code is compared and analyzed from the calculation examples and test data, which makes up for the lack of existing research. This paper mainly compares the calculation methods of unbalanced moment in existing codes, and can be compared with other existing calculation methods.

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