Numerical Modeling of Proppant Embedment in Viscoelastic Formations with the Fractional Maxwell Model

Xiang Ding,* Fan Zhang, Na Chen, and Yan Zhang

ABSTRACT: Hydraulic fracturing is often used to exploit unconventional hydrocarbons, and proppants are usually added during hydraulic fracturing to keep the fractures induced open. Nevertheless, time-dependent proppant embedment has often been neglected in previous studies. In this survey, the fractional Maxwell model is first proposed to describe the viscoelastic deformation of tight sandstones. Then, the fractional rheological model is incorporated into the finite element framework in ABAQUS to establish a numerical model to investigate the time-dependent embedment of proppants in viscoelastic formations. Parameter sensitivity studies are also performed to investigate the influences of the mechanical characteristics of proppants and formation on the embedment depth. Several factors that influence proppant embedment are also discussed.

INTRODUCTION

During hydraulic fracturing, fractures are initially created with fluids without solids and then followed with fluids with proppants.1−4 Proppants are transported by the fracturing fluid into the newly created fractures to sustain the fracture width after the pressure is removed and improve the efficiency of transportation of underground hydrocarbons to the wellbore.5 However, several mechanisms may result in the loss of fracture conductivity, such as fines migration,6 proppant diagenesis,7 proppant crushing,8−10 and proppant embedment,11 which is defined as proppant particles being embedded into the rock mass under pressure, causing a reduction in the fracture width and conductivity.12 Among these mechanisms, proppant embedment has been investigated via experiments,13−21 numerical simulation,22−34 and analytical modeling.35−46 However, as has been confirmed by a larger number of laboratory experiments conducted with proppants to reproduce the in situ fracturing process, laboratory observations greatly overestimate the conductivity of real wells.16,47 This great discrepancy may arise because in these studies, the underground rocks are often regarded as elastic/elastoplastic.16,48 However, increasing reservoir depth, high temperature, high pressure, and high stress may result in extreme geological conditions, which may transform the mechanical properties of reservoir rocks from elastic to viscoelastic or viscoplastic.49 Hard rocks can also exhibit time-dependent deformation50,51 under such extreme conditions. The viscoelastic/viscoplastic deformation of reservoir rocks may aggravate the proppant embedment and ultimately result in a significant reduction in fracture conductivity.48 According to Rybacki et al., at a depth of 2−4 km, a fracture may totally heal at a closure pressure of 30 MPa in 0.3−13.9 years by creep-induced proppant embedment. As the lifetime of a well typically lasts for decades, it is necessary to quantify the viscoelasticity-induced embedment of proppants in formations to make a rational estimation of the fracture width in production prediction.

In this study, the fractional Maxwell model (FMM) is first introduced to describe the creep deformation of sandstones. Then, the FMM is incorporated into ABAQUS to simulate time-dependent proppant embedment in a viscoelastic formation. The limitations of this study are also discussed.

MATERIALS AND METHODS

Fractional Modeling. Various rheological models have been suggested to express the creep deformation of sandstones,52−58 and most of these models are based on the series connection and/or parallel connection of basic mechanical elements—Hookean spring element, Newtonian dashpot, and Saint Venant element. In recent decades, applications of fractional calculus in modeling time-dependent deformation have increased rapidly.59−65 The successful application of fractional modeling in the description of various hereditary
phenomena may originate from its superiority to interpret the underlying physical essences better than traditional phenomenological models. Compared with traditional integer models, fractional models have shown the advantage of more concise equations, fewer parameters, and higher precision.\textsuperscript{62,63,66} In addition to these advantages mentioned above, it also should be noted that creep experiments always last shorter than engineering practices. Take the creep tests of sandstones as examples,\textsuperscript{54} the result of which can be used in well instability prevention, the experiment time of 72 h is much shorter than the life span of an oil/gas well, which may be decades. Fortunately, fractional modeling is a good tool to bridge the gap between creep deformation measured in the laboratory and creep deformation measured in real engineering projects, due to its ability to extrapolate the laboratory results to field applications with improved confidence. These advantages of fractional modeling guarantee the continued application in rheology.\textsuperscript{67}

**Constitutive Equations for the Fractional Maxwell Model.** The time-dependent deformation of tight sandstones was discovered and verified in a previous study.\textsuperscript{64} Here, the constitutive modeling process, which was not specified in detail in the previous study, is clarified step by step. The FMM\textsuperscript{61,68,69} resembles the traditional integer Maxwell model (IMM),\textsuperscript{70} and the only difference is that the Newtonian dashpot in the IMM is replaced by the Abel dashpot, as illustrated in Figure 1.

The total strain of the FMM is composed of the elastic strain represented by the Hooke spring and the viscous strain represented by the Abel dashpot. With the elastic constitutive equation of the Hooke spring and the fractional constitutive equation of the Abel dashpot, the constitutive relation of the FMM is expressed as

\[
\begin{align*}
\varepsilon &= \varepsilon_e + \varepsilon_v \\
\sigma &= \sigma_e = \varepsilon_e \\
\sigma_v &= E\varepsilon_v \\
\alpha \eta^\alpha \frac{d^\alpha \varepsilon_v}{dt^\alpha}
\end{align*}
\]  
\( (1) \)

in which \( \varepsilon \), \( \varepsilon_e \), and \( \varepsilon_v \) are the total strain, elastic strain, and viscous strain, respectively; \( \sigma \), \( \sigma_e \), and \( \sigma_v \) are the stresses corresponding to the total strain, elastic strain, and viscous strain, respectively; \( \alpha \) is the fractional order of the Abel dashpot; \( E \) is the elastic modulus of the spring; \( t \) is the time; \( \eta^\alpha \) is the viscosity of the dashpot; and the superscript \( \alpha \) is utilized to fulfill the principle of dimensional homogeneity.

After simplification, eq 1 can be rewritten as

\[
\frac{d^\alpha \sigma}{dt^\alpha} + \frac{E}{\eta^\alpha} \sigma = E \frac{d^\alpha \varepsilon}{dt^\alpha}
\]  
\( (2) \)

Equation 2 can be rewritten with the Laplace transform technique as

\[
s^\alpha \sigma(s) + \frac{E}{\eta^\alpha} \sigma(s) = E s^\alpha \varepsilon(s)
\]  
\( (3) \)

where \( s \), \( \sigma(s) \), and \( \varepsilon(s) \) are the complex variable, stress, and strain in the Laplace domain, respectively. The creep modulus \( J(s) \) in the Laplace domain is expressed with stress and strain as follows\textsuperscript{65}:

\[
J(s) = \frac{\varepsilon(s)}{s \sigma(s)}
\]  
\( (4) \)

So, eqs 3 and 4 give

\[
J(s) = \frac{\varepsilon(s)}{s \sigma(s)} = \frac{\alpha \varepsilon(s) + \frac{E}{\eta^\alpha} \sigma(s)}{E s^\alpha \sigma(s)} = \frac{1}{E s} + \frac{1}{\eta^\alpha s^{\alpha+1}}
\]  
\( (5) \)

Applying the inverse Laplace transform to eq 5 yields

\[
J(t) = \frac{1}{E} + \frac{1}{\eta^\alpha} \frac{t^\alpha}{\Gamma(1 + \alpha)}
\]  
\( (6) \)

The expression of \( J(t) \) in eq 7 is the creep modulus of the FMM in the time domain. If the fractional order in eq 6 is 1, then eq 6 reduces to

\[
J(t) = \frac{1}{E} + \frac{t}{\eta}
\]  
\( (7) \)

The Data Fitting with the Fractional Maxwell Model. Four tight sandstone samples are subjected to constant uniaxial pressures of 5, 10, 20, and 30 MPa, and the axial strains under each pressure are recorded as shown in Figure 2.
Under a constant loading stage, the creep strain $\varepsilon(t)$ can be expressed in terms of the creep modulus $J(t)$ and the constant load $\sigma_0$ as $\varepsilon(t) = J(t) \sigma_0$, and then the strains of the four tight sandstone samples under constant load, as shown in Figure 2, are fitted with the FMM and plotted in Figure 3.

NUMERICAL IMPLEMENTATION

Although the history of fractional calculus can be dated back to 1695, it seems novel to researchers who do not specialize in physics or mathematics. Although fractional calculus has successfully been applied in various disciplines, most of these studies focus on the constitutive modeling of anomalous phenomena that involve memory, heredity, path dependence, and global correlation in physics and mechanics, and the application of fractional calculus in engineering practice may be rare. One reason contributing to the imbalance between booming fundamentals research studies and depressed engineering applications may be the difficulties of the numerical implementation and integration of fractional calculus in common design and simulation software. Two of the most frequently used finite element software in simulations—ABAQUS and ANSYS—do not include any rheological model based on fractional calculus, which has been verified to be an excellent tool to depict time-dependent phenomena, as mentioned above. As one of the most popular finite element software, ABAQUS offers secondary development capabilities to build user-defined models into the simulation packages. ABAQUS provides users numerous subroutines, such as UMAT, UEL, DLOAD, FRIC, and CREEP, to tackle different requirements of engineering designs and simulation with the secondary development platform. The user subroutine CREEP allows the development of self-defined viscoelastic/viscoplastic models that have not been included in the material library via FORTRAN. The FMM discussed above, unsurprisingly, is not yet listed in the material library of ABAQUS. Here, in this section, the numerical implementation of the FMM in ABAQUS with the user subroutine CREEP is presented in detail. Specifically, in the definition of time-dependent material behavior with the user-defined subroutine CREEP, the “uniaxial” creep laws are expressed in a general time-dependent viscoelastic or viscoplastic material formulation. In the user-defined CREEP subroutine, different variables need to be defined according to different occasions. The CREEP subroutine only defines the time-dependent behaviors and the time-independent characteristics, such as elasticity and plasticity, are defined in the material defining module. The coupling of time-dependent and independent characteristics is carried out by ABAQUS. The elastic or plastic properties are defined as usual in the material library, but the models chosen in this step determine the meaning of variables defined in this subroutine. Taking the variable DECRA, which is most commonly used in the subroutine, as an example, it usually represents the equivalent creep strain rate increase. As an array, DECRA is composed of
five components, DECRA($i$), in which $i$ ranges from 1 to 5. In a subroutine, all or part of DECRA($i$) need to be defined depending on usage. Depending on the time-independent model selected in the material library, the component DECRA(1) may donate the equivalent (uniaxial) deviatoric/cohesion/compressive creep strain increase. More details about the variable definitions can be found in the ABAQUS manual.\textsuperscript{72} The main efforts made in developing the CREEP subroutine focus on defining the uniaxial equivalent creep strain rate and volumetric swelling strain rate (if needed) in terms of stresses and time.

Under constant stress $\sigma_0$, creep strain $\varepsilon(t)$ is the product of creep compliance $J(t)$ and constant stress $\sigma_0$

$$\varepsilon(t) = J(t)\sigma_0$$ \hfill (8)

Replacing the creep compliance $J(t)$ in eq 8 with the expression indicated in eq 6, the uniaxial equivalent creep strain rate of the FMM is obtained by taking the derivative of eq 8 with respect to time

$$\dot{\varepsilon}(t) = \dot{J}(t)\sigma_0 = \frac{\alpha\sigma_0}{\eta^\alpha} \frac{t^{\alpha-1}}{\Gamma(1 + \alpha)}$$ \hfill (9)

To accommodate different solution schemes, the expressions of the equivalent creep strain increase are defined under both explicit integration and implicit integration conditions. In the CREEP subroutine, the variable LEXIMP is introduced to indicate these two situations. More specifically, if LEXIMP is set to be 0, then the explicit integration scheme is used and the equivalent deviatoric creep strain rate needs to be defined in advance; otherwise, the implicit integration scheme is adopted and then another new variable—the derivative of the equivalent deviatoric creep strain increase with respect to the equivalent stress $\partial \varepsilon / \partial q$—requires a definition. Under the circumstance of implicit integration, the variable $\partial \varepsilon / \partial q$ for the FMM is expressed as

$$\frac{\partial \varepsilon}{\partial q} = \frac{\alpha t}{\eta^\alpha} \frac{t^{\alpha-1}}{\Gamma(1 + \alpha)}$$ \hfill (10)

The most challenging part of developing a user-defined subroutine has been finished, and now it is time to validate the CREEP subroutine of the FMM incorporated into ABAQUS; thus, a numerical simulation of the creep experiment of a tight sandstone sample H20-6 is conducted with the newly developed CREEP subroutine of the FMM. As a comparison, the IMM is also compiled and built into ABAQUS with the CREEP subroutine. The parameters used in the simulation are retrieved from the fitting parameters listed in Figure 3a.

Three loading steps, namely, the initial, elastic, and viscous steps, are designated to reproduce the sequential loading procedure of a real creep experiment. The Initial step adopts a default setting. Detailed information on the loading procedure of the simulation of the creep experiment is listed in Table 1.

The rock volume is meshed along the vertical and circumferential directions into 40 segments, as shown in Figure 4a. The pressure with a magnitude of 5 MPa is applied on the top of the rock volume, and a fixed boundary condition is adopted at the bottom, as shown in Figure 4b. Figure 4c shows the axial displacement distribution at the end of the elastic step.

The axial displacements of the top surface during the viscous step, simulated by both the FMM and the IMM, are plotted together with the experimental data, as shown in Figure 5. The axial displacements simulated with the CREEP subroutine of the FMM nearly overlap with the experimental results. However, the displacement results determined with the IMM simulation are not similar to the experimental results. Consequently, to simulate the creep deformation of sand-

---

### Table 1. Loading Procedures of the Simulated Creep Experiment

| Step   | Time/s | Amplitude | Initial Step/s | Minimum Step/s | Maximum Step/s |
|--------|--------|-----------|----------------|----------------|----------------|
| Initial| 0      | 0         |                |                |                |
| Elastic| 7199   | 1         | $10^{-2}$      | $10^{-7}$      | 0.05           |
| Viscous|        |           | $10^{-3}$      | 50             |                |

---

**Figure 4.** (a) Mesh scheme, (b) loading and boundary conditions, and (c) contour plot of axial displacement.

**Figure 5.** Comparison of the numerical simulation results and the experimental data; reprinted with permission from Ding et al.\textsuperscript{74} Copyright 2020 Elsevier.
stones, the CREEP subroutine of the FMM may be superior to that of the IMM.

■ RESULTS

With the validated CREEP subroutine of the fractional nonlinear creep model, the time-dependent proppant embed-

Table 2. Loading Procedures of the Simulated Creep Experiment

| step  | time      | amplitude | initial step/s | minimum step/s | maximum step |
|-------|-----------|-----------|----------------|----------------|--------------|
| initial | 0 | 0 | 10^-2 | 10^-7 | 0.05 s |
| elastic | 1 s | 1 | 10^-7 | 10^-4 | 30 days |
| viscous | 360 days | 1 | 10^-4 | 30 days |

Figure 6. (a) Mesh of the proppant, (b) mesh of the formation, (c) contact pattern, and (d) boundary conditions of the numerical model.

Figure 7. (a) Vertical displacement at the end of 360 days, (b) displacement of the formation, and (c) local amplification of the displacement of the contact area (deformation scale factor 2).

Figure 8. (a) Deformed view of the formation at Step 198, (b) deformed view of the formation at Step 271, and (c) deformed view of the formation at Step 52, deformation scale factor ×1.
proppant and the formation are presented in Figure 6a,b. The proppant is meshed into 2248 R3D3 elements, and the formation is meshed into 124,511 C3D10 elements. It should be noted that both the proppant and the formation adopted a single bias mesh scheme, which defined a nonuniform distribution of elements along the selected edges by defining the ratio of the sizes of the coarsest and finest elements along the edges to ensure that the contact areas of the proppant and the formation had the finest mesh, as shown in Figure 6c. The boundary conditions are illustrated in Figure 6d. The bottom of the formation is fixed, and two vertical surfaces are imposed with symmetrical boundary conditions. The contact pattern between the proppant and the formation is surface-to-surface contact. The arc surface of the proppant is specified as the master surface, while the top face of the formation is specified as the slave surface. The vertex of the proppant is specified as the reference point to constrain the rigid body and apply the concentrated force.

To focus on the influences of the time-dependent deformation of the formation on the proppant embedment, the proppant is modeled as a discrete grid body, and no deformation occurs during the simulation. The formation is modeled as a viscoelastic or viscoelastic-plastic body, depending on whether the stress exceeds the yield stress. The rheological properties of the sandstone formation are depending on whether the stress exceeds the yield stress. From Figure 8, the proppant embedment depth is found to increase with time due to the time-dependent deformation of the reservoir rock. The embedment depth is $U_e = 5.69 \times 10^{-2}$ at the end of elastic deformation and is $U = 11.47 \times 10^{-2}$ at the end of 360 days. The time-dependent embedment depth $U_e = U - U_e = 0.1147 - 0.0569 = 5.78 \times 10^{-2}$ is larger than the instantaneous embedment depth $U_e$. As the production of oil/gas wells lasts for decades, during such a long time, the time-dependent embedment depth may become larger than the instantaneous embedment depth. The proppant embedment depth due to the time-dependent deformation of reservoir rocks cannot be ignored, especially in the long-term productivity prediction of hydrocarbon reservoirs.

Parameter sensitivity studies are also conducted to investigate the influence of the viscoelastic characteristics on the embedment depth. Different orders of the FMM represent different creep tendencies of the material studied. Fractional order $\alpha = 0$ indicates a linear elastic material, but with increasing $\alpha$, the material becomes more prone to creep. As expressed in eqs 6 and 7, $\alpha = 1$ indicates the traditional IMM, for which the creep strain increases linearly with time.

With the data listed in Table 3, the different components of the embedment depth are plotted in Figure 8. The fractional order $\alpha$ is an index of the viscous properties of the reservoir rock and does not affect the instantaneous embedment depth, as shown by the blue line in Figure 9a. The time-dependent embedment depth increases with the fractional order, as shown by the green line in Figure 9a, and a larger fractional order means a large creep strain rate. It can also be seen from Figure 9b that the ratio of time-dependent embedment to instantaneous embedment also increases with the fractional order $\alpha$.

### DISCUSSIONS

By retrospecting the steps of the current study, the time-dependent deformation of reservoir rocks has been taken into consideration to investigate its influences on proppant embedment, which has been ignored in most previous studies, though the time-dependent proppant embedment may significantly reduce the fracture width and long-term conductivity.\(^{28,36}\) In addition, a more advanced and accurate fractional rheological model is introduced to depict the time-dependent deformation of reservoir rocks. The superiority of the fractional Maxwell model has been fully discussed by comparing it with other traditional integer-order rheological models in our previous study.\(^{64}\) Moreover, the numerical implementation of fractional models in general purpose finite element software greatly facilitates their applications in various simulations.\(^{74}\) In other words, without the successful combination of fractional rheological models with the finite element platform, the more advanced fractional rheological models may be limited only to theoretical analyses.

As this paper does not serve as a comprehensive review of proppant embedment, many factors, including fracture closure stress, rock type, proppant distribution, proppant type, proppant size, fluid deterioration of rocks, and stress cycling as shown in Figure 10, which may influence proppant embedment, are not fully introduced in this study, and we
focus on only the proppant embedment resulting from the time-dependent deformation of reservoir rocks.

Additionally, the closure pressure exerted on the proppant is assumed to remain constant during the whole production process, which is a simplification of the real downhole condition. Actually, during the production process, the underground pore pressure gradually decreases, which thus increases the effective closure pressure and accelerates the proppant embedment. Moreover, the cyclic loading of proppants with periodic production and shut-ins also increases the embedment and decreases the fracture width. In this study, the most conservative estimation of proppant embedment is made because the variation in closure pressure is ignored to emphasize that the proppant embedment originates from the time-dependent deformation of reservoir rocks.

To focus attention on the influence of the time-dependent deformation of reservoir rocks on proppant embedment, only the viscoelastic characteristics of the reservoir rocks are considered, and the proppant is modeled with a discrete analytical rigid body. However, the proppant cannot be infinitely rigid, and the high-pressure and high-temperature downhole conditions and the underground fluid serve as catalysts for the change in proppants from a rigid body to a viscoelastic material. The introduction of the viscoelastic...
properties of proppants may increase the predicted embedment of proppants.

■ CONCLUSIONS

With experiment-calibrated parameters, an FMM is utilized to describe the viscoelastic deformation of tight sandstones and is built into finite element software to simulate the time-dependent embedment of proppants into a fracture surface. The main conclusions of this study are as follows.

1. Tight sandstones creep under constant stresses, and the FMM is an excellent tool to describe these time-dependent deformations.
2. The flexibility of the finite element software ABAQUS enables researchers to develop their own models, which greatly facilitate the numerical simulations of various engineering problems.
3. The time-dependent embedment of proppants relies on the viscoelastic properties of the reservoir rocks, and in 1 year, the time-dependent embedment of proppants can be one to several times the instantaneous embedment depth according to different viscoelasticities of reservoir rocks; thus, it is of great significance to consider the time-dependent embedment of proppants in long-term production prediction.

■ AUTHOR INFORMATION

Corresponding Author
Xiang Ding – School of Civil Engineering, Architecture and Environment, Hubei University of Technology, Wuhan 430068, China; Sino-French Joint Research Collaboration for Geomechanics and Concrete Materials, Hubei University of Technology, Wuhan 430068, China; orcid.org/0000-0002-9940-9257; Email: dingxiang@hbut.edu.cn

Authors
Fan Zhang – School of Civil Engineering, Architecture and Environment, Hubei University of Technology, Wuhan 430068, China; Sino-French Joint Research Collaboration for Geomechanics and Concrete Materials, Hubei University of Technology, Wuhan 430068, China
Na Chen – School of Civil Engineering, Architecture and Environment, Hubei University of Technology, Wuhan 430068, China
Yan Zhang – School of Petroleum Engineering, Yangtze University, Wuhan 430100, China

Complete contact information is available at: https://pubs.acs.org/10.1021/acsomega.1c02407

Author Contributions
The manuscript was written through the contributions of all authors. All authors have given approval to the final version of the manuscript. Dr. X.D. proposed, designed, and conducted the study and prepared the first draft of this article. Dr. F.Z. oversaw the work and provided technical input. Dr. N.C. reviewed the manuscript and provided suggested changes. Dr. Y.Z. provided the final editing of the document.

Notes
The authors declare no competing financial interest.

■ ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (Grant Nos. 51979100 and 52009038) and middle-aged Talents Project of Department of Education of Hubei Province (Q20201407). The authors thank Prof. Shiyuan Li in China University of Petroleum-Beijing and Daobing Wang in Beijing Institute of Petrochemical Technology for useful discussions.

■ REFERENCES

(1) Yew, C. H.; Weng, X. In Mechanics of Hydraulic Fracturing, 2nd ed.; Hammon, K., Ed.; Gulf Professional Publishing: Oxford, 2015; p 49.
(2) Al Mteiri, S.; Suboyn, A.; Rahman, M. M.; Haroun, M. Hydraulic Fracture Propagation and Analysis in Heterogeneous Middle Eastern Tight Gas Reservoirs: Influence of Natural Fractures and Well Placement. ACS Omega 2021, 6, 799–815.
(3) Yang, L.; Shi, F.; Sun, X.; Wang, S.; Jiang, Q.; Lu, H. Experimental Investigation of the Pressure Decay Characteristics of Oil Reservoirs after Fracturing Operations. ACS Omega 2020, 5, 26441–26453.
(4) Wang, F.; Chen, Q.; Lu, Y.; Zhang, S. Fracturing-Fluid Flowback Simulation with Consideration of Proppant Transport in Hydraulically Fractured Shale Wells. ACS Omega 2020, 5, 9491–9502.
(5) Zoback, M. D.; Kohli, A. H. Unconventional Reservoir Geomechanics; Cambridge University Press: Cambridge, 2019; p 254.
(6) Pope, C.; Peters, B.; Benton, T.; Palisch, T. In Haynesville Shale - One Operator’s Approach to Well Completions in this Evolving Play, SPE Annual Technical Conference and Exhibition, New Orleans, Louisiana, 2009.
(7) LaFollette, R. F.; Carman, P. S. In Proppant Diagnostics: Results So Far, SPE Unconventional Gas Conference, Pittsburgh, Pennsylvania, 2010.
(8) Terracina, J. M.; Turner, J. M.; Collins, D. H.; Spillars, S. In Proppant Selection and Its Effect on the Results of Fracturing Treatments Performed in Shale Formations, SPE Annual Technical Conference and Exhibition, 2010.
(9) Reinicke, A.; Rybacki, E.; Stanchits, S.; Huenges, E.; Dresen, G. Hydraulic fracturing stimulation techniques and formation damage mechanisms—Implications from laboratory testing of tight sandstone–proppant systems. Geochemistry 2010, 70, 107–117.
(10) Han, J.; Wang, J. Y. In Fracture Conductivity Decrease Due to Proppant Deformation and Crushing, a Parametrical Study, SPE Eastern Regional Meeting, Charleston, WV, 2014.
(11) Huitj, J.; McGlothlin, B., Jr. et al. The Propping of Fractures in Formations Susceptible to Propping-Sand Embedment. In Drilling and Production Practice; OnePetro: New York, 1958.
(12) Tang, Y.; Ranjith, P. An experimental and analytical study of the effects of shear displacement, fluid type, joint roughness, shear strength, friction angle and dilation angle on proppant embedment development in tight gas sandstone reservoirs. Int. J. Rock Mech. Min. Sci. 2018, 107, 94–109.
(13) Volk, L. J.; Raible, C. J.; Carroll, H. B.; Spears, J. S. In Embedment of High Strength Proppant Into Low-Permeability Reservoir Rock, SPE/DOE Low Permeability Gas Reservoirs Symposium, Denver, Colorado, 1981.
(14) Penny, G. In An Evaluation of the Effects of Environmental Conditions and Fracturing Fluids Upon the Long-Term Conductivity of Proppants, SPE Annual Technical Conference and Exhibition, Dallas, Texas, 1987.
(15) Lacy, L. L.; Rickards, A. R.; Ali, S. A. In Embedment and Fracture Conductivity in Soft Formations Associated with HEC, Borate and Water-Based Fracture Designs, SPE Annual Technical Conference and Exhibition, 1997.
(16) Lacy, L.; Rickards, A.; Bilden, D. Fracture Width and Embedment Testing in Soft Reservoir Sandstone. SPE Drill. Completion 1998, 13, 25–29.
(17) Wen, Q.; Zhang, S.; Wang, L.; Liu, Y.; Li, X. The effect of proppant embedment upon the long-term conductivity of fractures. J. Pet. Sci. Eng. 2007, 55, 221–227.
(18) Alamahi, B.; Sundberg, M. In Proppant Embedment and Conductivity of Hydraulic Fractures in Shales, 46th US Rock Mechanics/Geomechanics Symposium, Chicago, Illinois, 2012.

(19) Zhang, J.; Ouyang, L.; Zhu, D.; Hill, A. Experimental and numerical studies of reduced fracture conductivity due to proppant embedment in the shale reservoir. J. Pet. Sci. Eng. 2015, 130, 37–45.

(20) Tang, Y.; Ranjith, P.; Wu, B. Experimental study of effects of shearing on proppant embedment behaviour of tight gas sandstone reservoirs. J. Pet. Sci. Eng. 2019, 172, 228–246.

(21) Alagöz, E.; Wang, H.; Russell, R.; Sharma, M. In New Experimental Methods to Study Proppant Embedment in Shales, 54th U.S. Rock Mechanics/Geomechanics Symposium, Golden, Colorado, 2020.

(22) Deng, S.; Li, H.; Ma, G.; Huang, H.; Li, X. Simulation of shale–proppant interaction in hydraulic fracturing by the discrete element method. Int. J. Rock Mech. Min. Sci. 2014, 70, 219–228.

(23) Cui, A.; Glover, K.; Wust, R. et al. In Elastic and Plastic Mechanical Properties of Liquids-Rich Unconventional Shales and Their Implications for Hydraulic Fracturing and Proppant Embdement: A Case Study of the Nordegg Member in Alberta, Canada, 48th US Rock Mechanics/Geomechanics Symposium, Minneapolis, Minnesota, 2014.

(24) Mueller, M.; Amro, M. In Indentation Hardness for Improved Proppant Embedment Prediction in Shale Formations, SPE European Formation Damage Conference and Exhibition, Budapest, Hungary, 2015.

(25) Fan, M.; Han, Y.; McClure, J.; Chen, C. In Hydraulic Fracture Conductivity as a Function of Proppant Concentration Under Various Effective Stresses: From Partial Monolayer to Multilayer Proppants, Proceedings of the 5th Unconventional Resources Technology Conference, Austin, Texas, 2017.

(26) Zhang, F.; Zhu, H.; Zhou, H.; Guo, J.; Huang, B. Discrete-Element-Method/Computational-Fluid-Dynamics Coupling Simulation of Proppant Embedment and Fracture Conductivity After Hydraulic Fracturing. SPE J. 2017, 22, 632–644.

(27) Zheng, X.; Chen, M.; Hou, B.; Ye, Z.; Wang, W.; Yin, C.; Chen, X. Effect of proppant distribution pattern on fracture conductivity and permeability in channel fracturing. J. Pet. Sci. Eng. 2017, 149, 98–106.

(28) Li, S.; Yang, Y.; Zhang, G. In Numerical Research of Creep Behavior of Shale Gas Reservoir Rock and Effects on Hydraulic Fracturing, 51st US Rock Mechanics/Geomechanics Symposium, San Francisco, California, 2017.

(29) Mirani, A.; Marongiu-Porcu, M.; Wang, H.; En kababian, P. Production Pressure Drawdown Management for Fractured Horizon-For Wells in Shale Gas Formations. SPE Reservoir Eval. Eng. 2018, 21, 550–565.

(30) Liu, Y.; Leung, J. Y.; Chalaturnyk, R. Geomechanical Simulation of Partially Propped Fracture Closure and Its Implication for Water Flowback and Gas Production. SPE Reservoir Eval. Eng. 2018, 21, 273–290.

(31) Zhu, S.; Elsworth, D.; Wang, J.; Gan, Q.; Liu, S. Hydraulic fracturing for improved nutrient delivery in microbially-enhanced coalbed-methane (MECBM) production. J. Nat. Gas Sci. Eng. 2018, 60, 294–311.

(32) Wang, J.; Elsworth, D.; Denison, M. K. Propagation, proppant transport and the evolution of transport properties of hydraulic fractures. J. Fluid Mech. 2018, 855, 503–534.

(33) Shi, F.; Wang, X.; Liu, C.; Liu, H.; Wu, H. An XFEM-based numerical model to calculate conductivity of propped fracture considering proppant transport, embedment and crushing. J. Pet. Sci. Eng. 2018, 167, 615–626.

(34) Liu, Y.; Wang, J.; Gao, J.; Zhu, H.; Zeng, J. Numerical Modeling of the Conductivity of the Particle Monolayer with Reduced Size. Geoﬂuids 2018, 2018, 1–10.

(35) Gao, Y.; Lv, Y.; Wang, M.; Li, K. In New Mathematical Models for Calculating the Proppant Embedment and Fracture Conductivity, SPE Annual Technical Conference and Exhibition, San Antonio, Texas, 2012.

(36) Guo, J.; Liu, Y. In Modeling of Proppant Embedment: Elastic Deformation and Creep Deformation, SPE International Production and Operations Conference & Exhibition, Doha, Qatar, 2012.

(37) Khanna, A.; Koutousov, A.; Sobey, J.; Weller, P. Conductivity of narrow fractures filled with a proppant monolayer. J. Pet. Sci. Eng. 2012, 100, 9–13.

(38) Neto, L. B.; Koutousov, A. Residual opening of hydraulic fractures filled with compressible proppant. Int. J. Rock Mech. Min. Sci. 2013, 61, 223–230.

(39) Li, K.; Gao, Y.; Lyu, Y.; Wang, M. New Mathematical Models for Calculating Proppant Embedment and Fracture Conductivity. SPE J. 2015, 20, 496–507.

(40) Yan, X.; Huang, Z.; Yao, J.; Song, W.; Li, Y.; Gong, L. Theoretical analysis of fracture conductivity created by the channel fracturing technique. J. Nat. Gas Sci. Eng. 2016, 31, 320–330.

(41) Chen, D.; Ye, Z.; Pan, Z.; Zhou, Y.; Zhang, J. A permeability model for the hydraulic fracture filled with proppant packs under combined effect of compaction and embedment. J. Pet. Sci. Eng. 2017, 149, 428–435.

(42) Chen, M.; Zhang, S.; Liu, M.; Ma, X.; Zou, Y.; Zhou, T.; Li, N.; Li, S. Calculation method of proppant embedment depth in hydraulic fracturing. Pet. Explor. Dev. 2018, 45, 159–166.

(43) Huang, J.; Safari, R.; Perez, O.; Fragachan, F. E. In Reservoir Deposition-Induced Proppant Embedment and Dynamic Fracture Closure, SPE Middle East Oil and Gas Show and Conference, Manama, Bahrain, 2019.

(44) Jia, L.; Li, K.; Zhou, J.; Yan, Z.; Wan, F.; Kaita, M. A mathematical model for calculating rod-shaped proppant conductivity under the combined effect of compaction and embedment. J. Pet. Sci. Eng. 2019, 180, 11–21.

(45) Luo, Z.; Zhang, N.; Zhao, L.; Liu, F.; Liu, P.; Li, N. Modeling of pressure dissolution, proppant embedment, and the impact on long-term conductivity of propped fractures. J. Pet. Sci. Eng. 2020, 186, No. 106693.

(46) Liu, H.; Bedrikovetsky, P.; Yuan, Z.; Liu, J.; Liu, Y. An optimized model of calculating optimal packing ratio for graded proppant placement with consideration of proppant embedment and deformation. J. Pet. Sci. Eng. 2021, 196, No. 107703.

(47) Bandara, K.; Ranjith, P.; Rathnaweera, T. Improved understanding of proppant embedment behavior under reservoir conditions: A review study. Powder Technol. 2019, 352, 170–192.

(48) Liu, Y.; Guo, J.; Jia, X.; Duan, Y.; Ma, J.; Hu, J. Long Term Conductivity of Narrow Fractures Filled with a Proppant Monolayer. J. Pet. Explor. Dev. 2017, 5, 326–249.

(49) Yao, J.; HUANG, Z.; LIU, W.; ZHANG, Y.; ZENG, Q.; YAN, X. Key mechanical problems in the development of deep oil and gas reservoirs. Sci. China: Phys., Mech. Astron. 2018, 1527–1547.

(50) Malan, D. F. Time-dependent Behaviour of Deep Level Tabular Excavations in Hard Rock. Rock Mech. Rock Eng. 1999, 32, 123–155.

(51) Rybacki, E.; Herrmann, J.; Wirth, R.; Drensen, G. Creep of Posidonia Shale at Elevated Pressure and Temperature. Rock Mech. Rock Eng. 2017, 50, 3121–3140.

(52) Jiang, Q.; Qi, Y.; Wang, Z.; Zhou, C. An extended Nishihara model for the description of three stages of sandstone creep. Geophys. J. Int. 2013, 193, 841–854.

(53) Zhang, Y.; Ya Xu, W.; Jian Gu, J.; Wang, W. Triaxial creep tests of weak sandstone from fracture zone of high dam foundation. J. Cent. South Univ. 2013, 20, 2528–2536.

(54) Cao, Y.; Deng, J.; Yu, B.; Tan, Q.; Ma, C. Analysis of sandstone creep and wellbore instability prevention. J. Nat. Gas Sci. Eng. 2014, 19, 237–243.

(55) Yang, S.-Q.; Jing, H.-W.; Cheng, L. Influences of pore pressure on short-term and creep mechanical behavior of red sandstone. Eng. Geol. 2014, 179, 10–23.

(56) Heap, M. J.; Brantut, N.; Baud, P.; Meredith, P. G. Time-dependent compaction band formation in sandstone. J. Geophys. Res.: Solid Earth 2015, 120, 4808–4830.
(57) Zhang, Y.; Shao, J.; Xu, W.; Jia, Y.; Zhao, H. Creep behaviour and permeability evolution of cataclastic sandstone in triaxial rheological tests. *Eur. J. Environ. Civ. Eng.* 2015, 19, 496−519.
(58) Wang, X.; Yin, Y.; Wang, J.; Lian, B.; Qiu, H.; Gu, T. A nonstationary parameter model for the sandstone creep tests. *Landslides* 2018, 15, 1377−1389.
(59) Friedrich, C. Relaxation and retardation functions of the Maxwell model with fractional derivatives. *Rheol. Acta* 1991, 30, 151−158.
(60) Podlubny, I. *Fractional Differential Equations: An Introduction to Fractional Derivatives, Fractional Differential Equations, to Methods of their Solution and Some of their Applications;* Academic Press: San Diego, California, 1999.
(61) Mainardi, F. In *Fractional Calculus and Waves in Linear Viscoelasticity: An Introduction to Mathematical Models;* Lydon, K., Ed.; World Scientific: London, 2010; pp 57−74.
(62) Zhou, H.; Wang, C.; Han, B.; Duan, Z. A creep constitutive model for salt rock based on fractional derivatives. *Int. J. Rock Mech. Min. Sci.* 2011, 48, 116−121.
(63) Mainardi, F. An historical perspective on fractional calculus in linear viscoelasticity. *Fract. Calc. Appl. Anal.* 2012, 15, 712−717.
(64) Ding, X.; Zhang, G.; Zhao, B.; Wang, Y. Unexpected viscoelastic deformation of tight sandstone: Insights and predictions from the fractional Maxwell model. *Sci. Rep.* 2017, 7, No. 11336.
(65) Diethelm, K. In *The Analysis of Fractional Differential Equations;* Morel, J., Ed.; Springer: Berlin, 2010; pp 1−6.
(66) Lai, J.; Mao, S.; Qiu, J.; Fan, H.; Zhang, Q.; Hu, Z.; Chen, J. Investigation progresses and Applications of Fractional Derivative Model in Geotechnical Engineering. *Math. Probl. Eng.* 2016, 2016, 1−15.
(67) Welch, S. W.; Rorrer, R. A.; Duren, R. G. Application of Time-Based Fractional Calculus Methods to Viscoelastic Creep and Stress Relaxation of Materials. *Mech. Time-Depend. Mater.* 1999, 3, 279−303.
(68) Kang, J.; Zhou, F.; Liu, C.; Liu, Y. A fractional non-linear creep model for coal considering damage effect and experimental validation. *Int. J. Non-Linear Mech.* 2015, 76, 20−28.
(69) Carpinteri, A.; Mainardi, F. In *Fractals and Fractional Calculus in Continuum Mechanics;* Kaliszky, S., Ed.; Springer: Vienna, 1997; pp 291−348.
(70) Pipkin, A. C. In *Lectures on Viscoelasticity Theory,* 2nd ed.; John, F., Ed.; Springer-Verlag: New York, 1986; pp 7−11.
(71) Ding, X.; Zhang, F.; Zhang, G. Modelling of time-dependent proppant embedment and its influence on tight gas production. *J. Nat. Gas Sci. Eng.* 2020, 82, No. 103519.
(72) ABAQUS. *Abaqus User Subroutines Reference Manual;* Dassault Systs Simulia Corp.: Providence, RI, 2014.
(73) ANSYS. *Software and User Manual;* ANSYS, Inc.: Canonsburg, PA, 2011.
(74) Ding, X.; Zhang, F.; Zhang, G.; Yang, L.; Shao, J. Modeling of hydraulic fracturing in viscoelastic formations with the fractional Maxwell model. *Comput. Geotech.* 2020, 126, No. 103723.