Efficient Amplitude-Modulated Pulses for Triple- to Single-Quantum Coherence Conversion in MQMAS NMR

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Supporting Information

S1. Procedure for high-throughput computer-based optimisation of FAM-N pulses
S2. Comparison of FAM-N and other multiple-pulse FAM-II-like conversion schemes
S3. Pulses used for $^{87}$Rb MQMAS NMR spectra of RbNO$_3$
S4. Pulses used for $^{87}$Rb MQMAS NMR spectra of Rb$_2$SO$_4$
S5. References
S1. Procedure for computer-based optimisation of FAM-N pulses

High-throughput computer optimisation was performed using the SIMPSON density matrix simulation program,\textsuperscript{51} executed using MATLAB\textsuperscript{52} routines. Specified within the optimisation is the external magnetic field strength, the MAS rate, the inherent radiofrequency (rf) nutation rate and the quadrupolar parameters $C_Q$ and $\eta_Q$. The simulation determines the amount of central-transition single-quantum coherences generated from conversion of a unit amount of triple-quantum coherences of the same sign. This process is illustrated below using the example of the conversion of triple- to single-quantum coherences for a single $^{87}\text{Rb}$ ($I = 3/2$) species with $C_Q = 1.2$ MHz and $\eta_Q = 0$, at $B_0 = 14.1$ T, using rf pulses with $\omega_1/2\pi = 150$ kHz at an MAS rate, $\omega_R/2\pi$, of 12.5 kHz.

Step 1: The amount of central-transition single-quantum coherence generated from unit triple-quantum coherence of the same sign is monitored in a series of simulations as the duration of the rf pulse is varied. The point at which maximum conversion is obtained (for one pulse, p1) is highlighted by the red dotted line (Figure S1).

![Figure S1](image-url)

**Figure S1.** Plot of the amount of single-quantum coherences generated from unit triple-quantum coherences for a single $^{87}\text{Rb}$ ($I = 3/2$) species with $C_Q = 1.2$ MHz and $\eta_Q = 0$. The pulse duration resulting in the maximum conversion efficiency is highlighted by the red dotted line.
Step 2: When the pulse duration resulting in the maximum amount of single-quantum coherences has been determined, a new SIMPSON input file is written by the code, in which a second, oppositely-phased pulse (p2) is added, and the simulation is repeated. The point at which maximum conversion is now obtained (for two pulses) is highlighted by the red dotted line (Figure S2).

![Figure S2](image)

**Figure S2.** A second oppositely-phased pulse is added to the SIMPSON input file after p1. The duration of this pulse is varied and the amount of single-quantum coherences generated is plotted. The new point at which maximum conversion is obtained is highlighted by the red dotted line.

Step 3: The duration of p1 is then increased by one increment and the variation of p2 repeated. If a further increase is observed, this process is repeated, *i.e.*, more increments are added to p1). The point at which maximum conversion efficiency is obtained is again highlighted by the red dotted line, while the dark blue dotted line shows the duration of p1 for which maximum efficiency in the two pulse conversion is obtained (Figure S3).
Figure S3. The duration of p1 is incremented and variation of p2 repeated. If efficiency improves this process is repeated. The total pulse duration resulting in maximum efficiency is shown by the red dotted line, while the dark blue dotted lines shows the duration of p1 when maximum efficiency (of the two pulse sequence) is obtained.

Step 4: The program will continue to increase the length of p1, even if the amount of single-quantum coherences is decreasing, for a fixed number of times, determined by the parameter "countmax" specified in the MATLAB routine. If the maximum signal increases in any one incrementation of p2, countmax is reset to zero. Once countmax is reached, the incrementation of p1 is stopped, and the value of the maximum amount of single-quantum coherences compared to that obtained with the previous increment of p1.

Step 5: For the point where maximum efficiency has been obtained, a new pulse is added (again with opposite phase to the previous one) and steps 3 and 4 are repeated, with a variation in the duration of the N\textsuperscript{th} pulse for a number of increments of the (N – 1)\textsuperscript{th} pulse. (Figure S4).
Figure S4. A third (oppositely-phased) pulse is added and the process is repeated.

Step 6: This procedure is repeated until the maximum amount of single-quantum coherence achieved with $N$ pulses is less than that achieved with $N - 1$ pulses. (Figures S5 and S6).

Figure S5. Additional pulses are added and the process is repeated.
Figure S6. This procedure is repeated until the maximum amount of single-quantum coherence achieved with \( N \) pulses is less than that achieved with \( N - 1 \) pulses.

Step 7: At this point the program terminates. The composite pulse that produced the maximum amount of single-quantum coherence is saved as a text file and as a MATLAB file. If desired, the program can also include the FAM-N pulse directly into a Bruker Topspin 3 pulse sequence for use experimentally. Examples of the MATLAB input and output (text) files for this calculation can be found in the Supporting Information.

The composite pulse obtained in this way is termed here FAM-N. In the example described here, the FAM-N pulse obtained consists of 6 oppositely-phased pulses, with a total duration of 5.63 \( \mu \text{s} \), with components of 1.77, 1.26, 0.81, 0.81, 0.70 and 0.44 \( \mu \text{s} \), respectively, as shown in Figure S7.
The optimisation procedure described here does not take into account the effects of varying the lengths of the pulses before the \((N - 1)^{th}\) pulse on the overall efficiency of the FAM-N pulse. However, as shown in Table S1, in simulations where the lengths of the first, second, third and fourth pulses of a composite FAM-N pulse, comprising 6 oppositely-phased pulses, were incremented after generation of the optimum pulse following Steps 1-7 above, negligible improvement (a maximum of 0.39%) in the conversion efficiency was observed. Therefore, owing to the additional time cost and very limited benefits of such an additional variation of the pulses, the optimisations reported here omit this process. It should also be noted that the pulse length increments considered here are smaller than the pulse digitisation possible for most spectrometers – for example, the Bruker Avance III spectrometer used in this work has a digitisation limit of 75 ns (twice the value of the pulse length increment during FAM-N optimisation), meaning that any small theoretical gains in signal intensity during the optimisation are likely to be lost during the experimental execution of the pulse.

**Figure S7.** Final FAM-N pulse produced.
Table S1. The changes to the pulse lengths and signal enhancement (relative to an unmodified pulse) of a FAM-N pulse (N = 6) optimised (for $^{87}\text{Rb}$ with $C_Q = 1.2$ MHz, $\omega_1/2\pi = 150$ kHz) using the method outlined in the text, when pulses before the $(N - 1)^{th}$ pulse were reoptimised.

| Pulse varied | Change to pulse length / ns (points) | Signal enhancement |
|--------------|-------------------------------------|--------------------|
| 1            | 37 (1)                              | 0.15%              |
| 2            | 74 (2)                              | 0.25%              |
| 3            | 74 (2)                              | 0.39%              |
| 4            | 37 (1)                              | 0.07%              |
S2. Comparison of FAM-N and other multiple-pulse FAM-II-like conversion schemes

Two previous attempts have been made in the literature to improve the efficiency of pulses based on FAM-II.\textsuperscript{53,54} In the approach of Goldbourt \textit{et al.},\textsuperscript{53} the length of the initial pulse was varied until a maximum in conversion efficiency was reached, at which point a second pulse was added with opposite phase. The length of the second pulse was incremented until a maximum in conversion efficiency was reached, and then a third pulse of opposing phase was added. This approach was continued with up to four pulses of opposing phase reported in the original work and with an enhancement factors of ~3 (200\% more signal) reported, relative to the most efficient single-pulse conversion. In the approach of Morais \textit{et al.},\textsuperscript{54} it was noted that, for spin I = 3/2, the optimum length of the first pulse corresponded to an inherent flip angle of 90°, \textit{i.e.}, the point of the echo/anti-echo crossing, and this value was used as a constraint. While the precise details of the remainder of the optimisation procedure are somewhat unclear from the original work, composite pulses were generated, comprised of up to six oppositely-phased pulses, and yielding signal enhancement factors slightly higher than those obtained by Goldbourt \textit{et al.}.\textsuperscript{3}

One possible reason for the improved efficiency of the optimised pulses of Morais \textit{et al.}, relative to those of Goldbourt \textit{et al.}, is the removal of the constraint that the phase must be inverted whenever an efficiency maximum is obtained. Morais \textit{et al.} demonstrated that, by increasing the length of the preceding pulse beyond an efficiency maximum, greater overall efficiency could be achieved by the following pulse. The pulses developed in this way appear to be essentially invariant with both the rf nutation frequency and the magnitude of the quadrupolar coupling (see, for example, Table 1 of the original work\textsuperscript{54}), suggesting that they may have essentially universal applicability (at least under the conditions explored). However, the optimisation procedure of Morais \textit{et al.} still appears to have been constrained so that each pulse is shorter than the preceding pulse.

In this work, we carried out a procedure conceptually similar to that of Morais \textit{et al.}, but with no constraint on the relative lengths or efficiencies of the individual pulses, and
with the pulse durations simply optimised to obtain the overall maximum efficiency. For $^{87}\text{Rb}$ (9.4 T, $C_Q = 1$ MHz, $\omega_R/2\pi = 15$ kHz MAS, $\omega_1/2\pi = 100$ kHz), Table S2 shows the durations of the individual oppositely-phased pulses obtained by optimisation of the pulse lengths using the methods of Goldbourt et al., Morais et al. and this work. The optimisations using the Goldbourt method were carried out for this work, whereas there was insufficient information in the work of Morais et al. to generate a comparable composite pulse optimised for the specific parameters considered here, and the pulse optimised for $\nu_Q/\nu_{rf} = 5$ in the original work was used. It can be seen that both the composite pulses of Goldbourt et al. and Morais et al. are shorter than the FAM-N pulse. In addition, it is clear that FAM-N does not represent merely an extension to either of the earlier pulse schemes, as the lengths of the first four individual pulses of the FAM-N scheme do not resemble those of either of the other two schemes. Table S2 also shows the overall efficiency of the three different pulses, and it can be seen that efficiency of the FAM-N pulse optimised as described in this work is $\sim$33% greater than that of the pulse optimised using the method of Goldbourt et al. and $\sim$5% greater than that of the pulse optimised by Morais et al. (see Table 1 of the original work$^{54}$ for further details). However, all composite pulses were between 60 and 120% more efficient than single-pulse conversion.

Table S2. The durations and efficiencies (relative to FAM-N) of the conversion pulses described in the text, when applied to $^{87}\text{Rb}$ at 9.4 T with $C_Q = 1$ MHz, $\omega_R/2\pi = 15$ kHz MAS and $\omega_1/2\pi = 100$ kHz.

| Method   | 1   | 2   | 3   | 4   | 5   | 6   | Total | Relative efficiency |
|----------|-----|-----|-----|-----|-----|-----|-------|---------------------|
| Single pulse | 2.14 |     |     |     |     |     | 2.14  | 0.460               |
| Goldbourt$^{53}$ | 2.14 | 1.65 |     |     |     |     | 3.79  | 0.744               |
| Morais$^{54}$ | 2.52 | 1.75 | 0.97 | 0.50 |     |     | 5.75  | 0.953               |
| FAM-N    | 2.39 | 1.78 | 1.06 | 0.89 | 1.00 | 0.56 | 7.67  | 1.000               |
**S3. Pulses used for $^{87}$Rb MQMAS NMR spectra of RbNO$_3$**

$^{87}$Rb two-dimensional triple-quantum MAS NMR spectra of RbNO$_3$ were acquired using the phase-modulated split-$t_1$ shifted-echo pulse sequence shown in Figure 1 of the main text, with $\omega_R/2\pi = 12.5$ kHz. In each case, two-dimensional spectra were acquired by averaging 192 transients with a recycle interval of 0.25 s, for 280 increments of 142.2 µs. In all cases, the excitation of triple-quantum coherences was carried out using a pulse duration of 4.25 µs, with $\omega_1/2\pi = 114$ kHz. All conversion pulses used $\omega_1/2\pi = 114$ kHz, with the exception of the central-transition selective pulse used in SPAM where $\omega_1/2\pi = 8$ kHz. The conversion of triple- to single-quantum coherences was performed using one of the following pulses:

(a) single pulse  \hspace{1cm} p1 = 1.2 µs
(b) SPAM  \hspace{1cm} p1 = 1.5 µs, p2 = 13.6 µs
(c) FAM-I  \hspace{1cm} The block $[p1, \tau_1, p1, \tau_1]$ was repeated 3 times with $p1 = \tau_1 = 0.8$ µs.
(d) FAM-II  \hspace{1cm} p1 = 1.7 µs, p2 = 0.8 µs
(e) DFS  \hspace{1cm} A frequency-swept pulse (over the range 175 to 850 kHz) was used for 20 µs, (0.25 of $\tau_R$) with increments of 75 ns.
(f) FAM-N  \hspace{1cm} Generated using $C_Q = 1.9$ MHz, $\eta_Q = 0$

11 oppositely-phased pulses for a total duration of 7.85 µs, with the individual components of 1.66, 1.22, 0.98, 0.78, 0.59, 0.59, 0.63, 0.59, 0.34, 0.29 and 0.19 µs. In this case experimental reoptimisation was also carried out, resulting in an optimum conversion when $\omega_1/2\pi = 108$ kHz.
S4. Pulses used for $^{87}$Rb MQMAS NMR spectra of Rb$_2$SO$_4$

$^{87}$Rb two-dimensional triple-quantum MAS NMR spectra of Rb$_2$SO$_4$ were acquired using the phase-modulated split-$t_1$ shifted-echo pulse sequence shown in Figure 1 of the main text, with $\omega_r/2\pi = 12.5$ kHz. In each case, two-dimensional spectra were acquired by averaging 192 transients with a recycle interval of 0.25 s, for 160 increments of 71.1 $\mu$s. In all cases, the excitation of triple-quantum coherences was carried out using a pulse duration of 4.75 $\mu$s, with $\omega_1/2\pi = 123$ kHz. All conversion pulses used with $\omega_1/2\pi = 123$ kHz, with the exception of the central-transition selective pulse used in SPAM where $\omega_1/2\pi = 13$ kHz. The conversion of triple- to single-quantum coherences was performed using one of the following pulses:

(a) single pulse \hspace{1cm} p1 = 1.5 $\mu$s  
(b) FAM-N \hspace{1cm} Generated using $C_Q = 2.52$ MHz, $\eta_Q = 1.0$  
\hspace{1cm} 14 oppositely-phased pulses for a total duration of 8.58 $\mu$s, with the \hspace{1cm} individual components of 2.03, 1.26, 0.63, 0.54, 0.50, 0.45, 0.45, 0.45, \hspace{1cm} 0.45, 0.41, 0.36, 0.41, 0.41, 0.23 $\mu$s.  
(b) FAM-N \hspace{1cm} Generated using $C_Q = 5.30$ MHz, $\eta_Q = 0.11$  
\hspace{1cm} 26 oppositely-phased pulses for a total duration of 11.02 $\mu$s, with the \hspace{1cm} individual components of 1.40, 0.54, 0.41, 0.45, 0.40, 0.40, 0.36, 0.40, \hspace{1cm} 0.32, 0.77, 0.72, 0.27, 0.32, 0.32, 0.32, 0.32, 0.23, 0.32, 0.36, 0.27, 0.32, 0.41, \hspace{1cm} 0.32, 0.41, 0.50, 0.18 $\mu$s.
S5. References

S1. Bak, M.; Rasmussen, J.; Nielsen, N. SIMPSON: A General Simulation Program for Solid-State NMR Spectroscopy *J. Magn. Reson.* **2000**, *147*, 296-330.

S2. MATLAB Release 2011b, *The MathWorks, Inc., Natick, Massachusetts, United States.*

S3. Goldbourt, A.; Madhu, P. K.; Vega S. Enhanced Conversion of Triple to Single-Quantum Coherence in the Triple-Quantum MAS NMR Spectroscopy of Spin-5/2 Nuclei *Chem. Phys. Lett.* **2000**, *320*, 448-456.

S4. Morais, C. M.; Lopes, M.; Fernandez, C.; Rocha, J. Assessing the Potential of Fast Amplitude Modulation Pulses for Improving Triple-Quantum Magic Angle Spinning NMR Spectra of Half-Integer Quadrupolar Nuclei *Magn. Reson. Chem.* **2003**, *41*, 679-688.