Stochastic Unit Commitment in Electricity-Gas Coupled Integrated Energy Systems based on Modified Progressive Hedging

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Abstract—The increasing number of gas-fired units has significantly intensified the coupling between power and gas networks. Traditionally, the nonlinearity and nonconvexity in gas flow equations, together with renewable-induced stochasticity, result in a computationally expensive model for unit commitment in electricity-gas coupled integrated energy systems (IES). To accelerate stochastic day-ahead scheduling, we applied and modified Progressive Hedging (PH), a heuristic approach that can be computed in parallel to yield scenario-independent unit commitment. By applying a termination and enumeration technique, the modified PH algorithm saves considerable computational time, especially when the unit production prices are similar for all generators, and when the scale of IES is large. Moreover, an adapted second-order cone relaxation (SOCR) is utilized to tackle the nonconvex gas flow equation. Case studies are performed on the IEEE 24-bus system/Belgium 20-node gas system and the IEEE 118-bus system/Belgium 20-node gas system. The computational efficiency when employing PH is 188 times that of commercial software, even outperforming Benders Decomposition. Meanwhile, the gap between the PH algorithm and the benchmark is less than 0.01% in both IES systems, which proves that the solution produced by PH reaches acceptable optimality in this stochastic UC problem.

Keywords—coupled electricity-gas network, integrated energy system, unit commitment, modified Progressive Hedging, second-order cone relaxation.

NOMENCLATURE

A. Indexes and Sets
- \( i \in \mathcal{G} \) Index of gas-fired units.
- \( i \in \mathcal{C} \) Index of coal-fired units.
- \( g \in \mathcal{GW} \) Index of gas wells.
- \( sc \in \mathcal{S}\mathcal{C} \) Index of typical scenarios.
- \( r \in \mathcal{L} \) Index of electric buses.
- \( n \in \mathcal{N} \) Index of gas nodes.
- \( s \in \mathcal{S} \) Index of gas storage.
- \( w \in \mathcal{W} \) Index of wind farms.
- \( t \in \mathcal{T} \) Hourly time periods for the following day.
- \((a,b) \in \mathcal{BR}\) Transmission lines from bus a to b.
- \((c,d) \in \mathcal{PL} \) Gas pipelines from gas node c to d.
- \((q,j) \in \mathcal{PC} \) Compressors; q and j denote the starting and terminal nodes of a compressor, respectively.

B. Parameters
- \( C_{iG}^{ci} \) Unit production cost of generators (MS/GW).
- \( C_{iG}^{cg} \) Unit cost of gas generation (MS/GW).
- \( p^{sc} \) Probability of occurrence for a typical wind output scenario \( sc \).
- \( C_{il}^{cp} \) Unit cost for non-served power at bus \( l \) (MS/GWh).
- \( C_{in}^{cg} \) Unit cost for non-served gas at node \( n \) (MS/MSm³).
- \( C_{is}^{cs} \) Unit cost for storage \( s \) (MS/MSm³).
- \( C_{iw}^{cw} \) Unit cost for wind curtailment at wind farm \( w \) (MS/GW).
- \( RU_{it}, RD_{it} \) Ramp up/down limits for generator \( i \) (GW).
- \( P_{f}, T_{f} \) Minimum and maximum power output of unit \( i \) (GW).
- \( P_{F_{ab}} \) Maximum power flow of branch \( ab \) (GW).
- \( l'_{i}, t' \) Power demand at bus \( l \) and time \( t \) (GW).
- \( X_{ab} \) Inductance of branch \((a,b)\) (p. u.).
- \( W_{w}, \overline{W}_{g} \) Minimum and maximum production for gas well \( g \) (MSm³).
- \( S_{w}, \overline{S}_{s} \) Minimum and maximum storage levels for storage \( s \) (MSm³).
- \( WR_{i}, IS_{s} \) Withdrawal and injection limits for storage \( s \) (MSm³/h).
- \( D_{cd} \) Diameter of pipeline \((c,d)\) (m).
- \( L_{cd} \) Length of pipeline \((c,d)\) (m).

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Due to the increasing demand for clean and efficient energy resources, electricity-gas coupled networks have garnered considerable interest in recent decades, as an example of integrated energy systems (IESs). Gas-fired units are pollutant-free in terms of sulfur or nitrogen oxides, and highly efficient in energy conversion compared to coal-fired units [1]. It has been predicted that power generated by gas-fired units will increase by 230% by 2030 [2][3], and natural gas consumption would take up 28% of all energy demand by then [4]. Moreover, the refinement of Power-to-Gas (PtG) techniques [5] has strengthened the interaction of power and gas systems. In addition to environmental benefits such as carbon consumption, PtG units enable large-scale energy storage, which would further improve the flexibility of IESs.

Regarding coupled IES systems, reliability and security are two of the most crucial topics. Generally, sources of uncertainty in IESs include renewable generation fluctuation [6], load variation [7], N-1 contingencies [8] and malicious attacks [9]. Gas systems are generally considered an ideal choice for accommodating renewable energy, such as wind and solar energy. The pipelines serve as flexible storage that draws in/out gases by self-adjusting nodal pressures, outperforming power reserves in terms of cost and stability. However, eco-friendly renewables introduce operational uncertainties that would jeopardize the coupled systems. [10] introduced the concept of distributed slack nodes to analyze gas flow when wind power is included. [11] established an EGTran model that aims to solve for the wind-penetrated stochastic unit commitment (UC). However, the UCs vary among different scenarios, and the gas dynamics are not considered, leading to a less instructive conclusion for operation in the following day.

When addressing renewable-induced uncertainties, two branches emerge: robust optimization (RO) and stochastic optimization (SO). RO recasts a deterministic model over an uncertainty set with upper and lower bounds, followed by identifying and optimizing the worst-case scenarios [12]. Although fewer data are required, RO is still limited due to its overconservativeness. On the other hand, SO resolves a problem by selecting typical scenarios and optimizing the resulting joint model, thus making it popular in research on UC. [11][13][14]. One of the problems in SO is the explosion of scenarios, which significantly complicates the model. Stochastic models are normally accompanied by algorithms for acceleration. One of the most famous algorithms is the Benders Decomposition [15], the acceleration performance of which has been confirmed.

However, these algorithms are not specifically designed for stochastic scenarios. Scenario-independent variables (SIVs) are still correlated among scenarios, which make the computation unable to be performed in parallel. In light of this, Progressive Hedging (PH) is introduced as a scenario-based decomposition method. First proposed by [16], PH is known as a distributed technique that keeps solving each scenario individually. Convergence of this article is also strictly proven [17]. PH largely resembles the Alternating Direction Method of Multipliers (ADMM) as a heuristic and distributed algorithm [18], but relaxes non-anticipativity constraints instead of linking constraints, therefore not requiring auxiliary functions or matrix inversion [19]. Moreover, PH is inherently supportive of parallel computing, which may further facilitate the optimization due to the uniformity of the decomposed scenarios [17]. The PH algorithm has been applied to the stochastic UC of power systems [20][21], and the optimality gap has been

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**Variables**

- $\rho$ Density of natural gas (kg/m³).
- $F$ Friction coefficient.
- $R$ Specific gas constant (m³·bar K⁻¹·kg⁻¹).
- $T$ Temperature of the gas system (K).
- $Z$ Average compressibility factor of gas.
- $GTP_i$ Gas-to-power conversion efficiency for gas turbine $i$ (MSm³/GWh).
- $\Pi_{a,b}$ Minimum and maximum pressure for gas node $n$ (bar).
- $CM_{ij}$ Compression factor, for compressor located between gas nodes $q$ and $j$.

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**I. INTRODUCTION**

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confirmed to tighten toward zero with a larger number of iterations [22][23]. However, the exploration of this algorithm in the operation of IESs has been rare. The introduction of PH can be meaningful, especially in a natural gas system where each scenario contains hundreds of nonlinear and nonconvex gas flow equations.

To model an electricity-gas coupled IES, the steady-state formulation has been widely adopted due to its simplicity [24][25]. Nevertheless, in reality, pipelines are equipped with the flexibility to adjust their gas storage level by changing the nodal pressures. Hence, the dynamics of pipelines remain a widely investigated topic, especially for short-term operation where pipeline storage makes a difference in economic dispatch. Liu et al. first established a gas transmission model that describes a slow transient through partial differential equations and algebraic equations [26]. Correa-Posada et al. further simplified the dynamic model into a mixed-integer linear programming (MILP) problem. They formed a linear lineup equation and gas flow equation by considering the gas travel velocity and compressibility [27]. Although less accurate, the linear lineup model is still favorable in short-term operation, as it recovers most of the dynamics.

To address the nonconvex gas flow constraints, various piecewise linearization techniques (summarized in [28]) have been proposed. These algorithms can mostly recover a bilinear equation, yet a large number of segments are required to guarantee the exactness. This would introduce hundreds of binary variables, and is therefore inefficient in computation. Second-order cone relaxation (SOCR), as an alternative in addressing gas flow, has been extensively investigated in the past 10 years. It does not require additional binary variables, which significantly speeds up the calculation. Cone reformulation techniques including Taylor expansion-based linearization [29], sequential-cones [30] and convex envelopes [31], can further reduce the error to negligible levels. The post-extraction issues generated from SOCR are addressed in [32], [33] and [34].

In view of the above, this article applies and modifies the PH algorithm to solve stochastic UC problems in IESs, with a dynamic model and an iteration-free SOCR modeling the gas flow. The contribution of this article is therefore two-fold:
1) Propose a stochastic UC model for electricity-gas coupled IESs, where SOCR-based dynamics of the natural gas system are considered. To the best of our knowledge, this topic has not been explored due to problems in computation efficiency. Additionally, we use an iteration-free SOCR technique to model the gas flow, in which a linear constraint is proposed to compensate for the inexactness in relaxation.
2) Accelerate solving the stochastic UC problem based on a modified PH algorithm, which has outperformed commercial software and even the Benders Decomposition. In cases where the network is large, or the unit production prices are disparate for either generator, traditional Progressive Hedging (TPH) is sufficient. However, when the scale of the IES increases, or when unit power production prices are similar, we develop modified Progressive Hedging (MPH) to further reduce the computational burden, in which the proposed termination criteria control the number of iterations required for convergence.

The remainder of this paper is structured as follows: Section II formulates the stochastic UC model, with the generation of wind output scenarios. Section III illustrates the TPH/MPH algorithm and SOCR method for improving the performance of optimization. Section IV tests the novel algorithms on medium and large-scale IES systems to evaluate the algorithm’s performance, and compares them with a benchmark method and Benders Decomposition. Section V concludes the article.

II. MODEL FORMULATION

A. Objective Function

The stochastic UC model for IESs, adapted from [27], considers multiple scenarios of wind power. The objective is to minimize the total expected cost incurred within the coupled system, over the time horizon $t \in T$ and scenarios with superscript $sc \in SC$

$$\min \sum_{t = 1}^{T} \left( \sum_{s \in S_t} \left[ \sum_{g \in G_t} C_{pg}^g \cdot p_{g,t}^s + \sum_{s \in S_t} C_{st} \cdot s_{t}^s \right] + \sum_{l \in L_t} \sum_{s \in S_t} C_{l} \cdot l_{t}^s \right)$$

where $\sum_{l \in L_t} \sum_{s \in S_t} C_{l} \cdot l_{t}^s$ calculates the generation cost of all gas- and coal-fired generators, and $\sum_{g \in G_t} C_{pg}^g \cdot p_{g,t}^s + \sum_{s \in S_t} C_{st} \cdot s_{t}^s$ considers the additional costs within the natural gas system, respectively gas production from wells and withdrawal from storage. Finally, non-served power for power and gas, together with curtailment of wind power, are penalized.

B. Power System Constraints

The output power from generators is constrained by unit commitment, unit capacity limits (2) and up/down ramping limits (3). Note that the unit commitment $c_{i,t}^g$ is the only scenario-independent variables.

$$c_{i,t}^g \cdot P_{i,t}^g \leq p_{d,t}^s \leq c_{i,t}^g \cdot \overline{P}_{i,t}^g, \forall i \in G \cup C$$

$$-R_D \leq p_{d,t}^s - p_{d,t}^{s_{t-1}} \leq R_U, \forall i \in G \cup C$$

For power transmission, a lossless DC model is employed for simplicity in (4). Constraint (5) restricts the power flow of each transmission line by its capacity. Constraint (6) outlines the nodal balance at each bus, which involves power generation, transmission, consumption and VOLL.

$$p_{f_{a,b}}^t = \left( \theta_{a,t} - \theta_{b,t} \right) / X_{a,b}, \forall (a,b) \in BR$$

$$-P_F \leq p_{f_{a,b}}^t \leq P_F, \forall (a,b) \in BR$$

$$p_{d,t}^s + \sum_{b \in B} p_{f_{a,b}}^t = L_{i,t}^s, \forall i \in BR$$

C. Natural Gas System Constraints

A typical natural gas system consists of gas wells, storage, compressors and pipelines. Constraint (7) describes the production limit of natural gas wells. Constraints for storage...
include storage capacities (8), injection/withdrawal upper limits (9), and the variation in storage levels with time (10). For compressors, Constraint (11) describes the nodal pressure requirements between the starting and ending gas nodes. For simplicity, we assume that compressors do not consume energy.

\[
W_s \leq pg_{sc} \leq W_c, \forall g \in GW
\]  
(7)

\[
S_{sc} \leq sl_{sc} \leq \bar{S}_{sc}, \forall s \in S
\]  
(8)

\[
sto_{sc} \leq WR, \forall s \in S
\]  
(9)

\[
sl_{sc} = \bar{sl}_{sc} + sto_{sc}, \forall s \in S
\]  
(10)

\[
\pi^s_{sc} \leq \pi^s_{sc} \leq CM, \forall (q, j) \in PC
\]  
(11)

For pipelines, we apply a quasi-dynamic model following [27]. Constraint (12) denotes the equation of mass within a pipeline, and (13) denotes its variation with time. The average gas flow, defined as the mean of pipeline, and (13) denotes its variation with time. The average gas flow, defined as the mean of pipeline, and (13) denotes its variation with time. The average gas flow, defined as the mean of pipeline, and (13) denotes its variation with time. The average gas flow, defined as the mean of pipeline, and (13) denotes its variation with time.

\[
\sum_{d \in D_n c n s n i} d c d \sum_{n c d} \Delta t_{c d} = \sum_{t \in T} g f_{d c d} \sum_{n c d} \Delta t_{c d} \sum_{n c d} \Delta t_{c d} \sum_{n c d} \Delta t_{c d} \sum_{n c d} \Delta t_{c d}
\]  
(12)

\[
\sum_{d \in D_n c n s n i} d c d \sum_{n c d} \Delta t_{c d} = \sum_{t \in T} g f_{d c d} \sum_{n c d} \Delta t_{c d} \sum_{n c d} \Delta t_{c d} \sum_{n c d} \Delta t_{c d} \sum_{n c d} \Delta t_{c d}
\]  
(13)

\[
\sum_{d \in D_n c n s n i} d c d \sum_{n c d} \Delta t_{c d} = \sum_{t \in T} g f_{d c d} \sum_{n c d} \Delta t_{c d} \sum_{n c d} \Delta t_{c d} \sum_{n c d} \Delta t_{c d} \sum_{n c d} \Delta t_{c d}
\]  
(14)

\[
\sum_{d \in D_n c n s n i} d c d \sum_{n c d} \Delta t_{c d} = \sum_{t \in T} g f_{d c d} \sum_{n c d} \Delta t_{c d} \sum_{n c d} \Delta t_{c d} \sum_{n c d} \Delta t_{c d} \sum_{n c d} \Delta t_{c d}
\]  
(15)

As shown in (16), each node in the gas system is subject to upper/lower pressure boundaries. Constraint (17) states the pressure difference by the Weymouth equation (15).

\[
\Pi^s_{sc} \leq \Pi^s_{sc} \leq \Pi^s_{sc}, \forall n \in N
\]  
(16)

\[
\sum_{d \in D_n c n s n i} d c d \sum_{n c d} \Delta t_{c d} = \sum_{t \in T} g f_{d c d} \sum_{n c d} \Delta t_{c d} \sum_{n c d} \Delta t_{c d} \sum_{n c d} \Delta t_{c d} \sum_{n c d} \Delta t_{c d}
\]  
(17)

\[
\sum_{d \in D_n c n s n i} d c d \sum_{n c d} \Delta t_{c d} = \sum_{t \in T} g f_{d c d} \sum_{n c d} \Delta t_{c d} \sum_{n c d} \Delta t_{c d} \sum_{n c d} \Delta t_{c d} \sum_{n c d} \Delta t_{c d}
\]  
(18)

D. Model of Wind Power Generation

Generating characteristic output curves of wind turbines is the key to construct accurate stochastic scenarios. Herein the real-time windspeed \( v(t) \) is viewed as the summation of both the forecasted windspeed \( v(f) \) and the prediction error \( v(e) \).

\[
v(t) = v(f)(t) + v(e)(t)
\]  
(19)

Considering the difficulty in predicting windspeed from only historical records, \( v(f)(t) \) is obtained by importing a 24-hour windspeed curve, from the National Data Buoy Center (NDBC) dataset [35]. \( v(f)(t) \) is generated through the auto-regressive moving average (ARMA) series. Here a simplified ARMA(1,1) model, in which the current error is only correlated with error in the past hour with additional noise, is chosen in accordance with references [6] and [36] as follows:

\[
v(t) = \alpha v(f)(t-1) + \beta \xi(t-1) + \xi(t)
\]  
(20)

where \( \xi(t) \) is a random variable following a Gaussian distribution, with a mean of 0 and a standard deviation of \( \sigma \). Random scenarios can therefore be generated from (20).

Subsequently, k-means clustering is applied to extract typical scenarios of windspeed. k-means will inevitably lead to deviations in the distribution of windspeed, but the computational burden can be greatly relieved. Otherwise the optimality gap would hardly converge even for decomposition algorithms. The reduced scenarios are finally mapped to the power output through a typical power curve [37].

III. Algorithms: TPH/MPH and SOCR

A. Traditional/Modified Progressive Hedging

As stated in Section I, a modified Progressive Hedging approach is applied to address the stochastic nature of wind turbines. For the sake of simplicity, the stochastic model is expressed in a compact form as follows:

\[
\begin{align*}
\min \quad & \lambda^l c + \sum_{n c d} \lambda^l \lambda^u x_{w_n}
\quad \text{subject to} \quad (c, x_{w_n}) \in Q_{w_n} \quad \text{where } c \text{ denotes UC variables, the vectorized SIVs. All other variables are categorized by scenario, and are subsequently vectorized into } x_{w_n}. \quad \text{Q}_{w_n} \text{ denotes the feasible region for each scenario, which is the feasible set defined by (2)-(18). } \lambda^l \text{ and } \lambda^u \text{ are linear coefficients of the corresponding variables.}
\end{align*}
\]  
(21)

Algorithm 1: TPH/MPH

1. Parallel For each scenario
2. \( c^0_{w_n}, x^0_{w_n} = \arg \min_{c, x_{w_n}} \lambda^l c + \lambda^u x_{w_n} \)
3. subject to \( (c, x_{w_n}) \in Q_{w_n} \)
4. End Parallel
5. \( \tau^0 = \sum_{w_n} \rho_{w_n} c^0_{w_n} \)
6. \( \text{iter} \leftarrow 0, \rho^0_{w_n} \leftarrow \kappa (c^0_{w_n} \tau^0) \)
7. \( \text{Do} \)
8. \( \text{iter} \leftarrow \text{iter} + 1 \)
9. Parallel For each scenario
10. \( c^0_{w_n}, x^0_{w_n} = \arg \min_{c, x_{w_n}} \lambda^l c + \lambda^u x_{w_n} + \rho^0_{w_n} \tau^0 \xi + \xi \frac{1}{2} \kappa (c^0_{w_n} \tau^0) \)
11. subject to \( (c, x_{w_n}) \in Q_{w_n} \)
12. \( \rho^0_{w_n} \leftarrow \rho^0_{w_n} + \kappa (c^0_{w_n} \tau^0) \)
13. End Parallel
14. \( \tau^w = \sum_{w_n} \rho_{w_n} c^w_{w_n}, \text{IndNo of non-binary elements in } \tau^w \)
15. Until \( \text{IndNo } \leq (c=0 \text{ for TPH, 1 or 2 for MPH}) \)
16. If Apply MPH:
17. Locate all the inconsistent UCs \( c_n \). Denote other UCs as \( c_n \).
18. Parallel For case from 1 to 2^{IndNo}
19. Select a distinct combination of \( c^w_{w_n} \).
20. \( \min_{c_{w_n}} \text{OBJ}_{w_n} = \sum_{w_n} \lambda^l c_{w_n} + \lambda^u x_{w_n} + \rho^0_{w_n} \tau^0 \xi + \xi \frac{1}{2} \kappa (c^0_{w_n} \tau^0) \)
21. subject to \( (c_{w_n}, c^w_{w_n}, x^w_{w_n}) \in Q_{w_n}, \forall c_{w_n} \in SC \)
22. End Parallel
23. Select the \( c^w_{w_n} \) with minimum \( \text{OBJ}_{w_n} \)
24. End If
PH aims to solve each scenario independently, by means of penalizing the differences in each SIV in the optimal solution of each scenario. The common flowchart is outlined in [17] (corresponding to lines 1-15 of Algorithm 1), with $s$ as SIVs.

The penalty function $\rho_s^{\omega_{\gamma}}$ is adjusted iteratively with respect to the current inter-scenario differences, and is added to the objective function until the differences shrink considerably. The penalty stems heuristically from two sources: inter-scenario differences, and a shift in the average UC (respectively corresponding to the terms $\rho_s^{\omega_{\gamma}}$ and $\frac{\kappa}{2}\|c - \bar{c}\|_2^2$). The penalty coefficient $\kappa$ is chosen proportional to its objective coefficients as discussed in [17].

The traditional PH algorithm is mostly effective in decreasing the $Ind$, or the number of inconsistent UCs, to a low level. In a medium-scale network where the unit production prices are disparate for different generators, the inconsistency levels easily converge to zero. However, after repeated tests, we discovered that $Ind$ may fail to converge to zero when the network is large, or when power generation prices are similar. Instead this indicator remains at 1 or 2 for the next 10+ iterations unless the penalty parameters are carefully chosen for a specific network. Hence, we modified the PH algorithm, by terminating the iteration whenever $Ind$ drops below 1 or 2. From then on, all possibilities in inconsistent UCs are enumerated, while the total cost is recomputed with predetermined binaries. The UC set with the lowest objective is selected as the final solution (lines 16-24 in Algorithm 1). Compared with TPH, the modified algorithm can effectively shorten the computational time. Only $2^{Ind} \leq 4$ iterations are required to compare the enumerated UCs.

Additional adaptations to the PH algorithm are as follows:

1) In the objective function (1), no coefficients are assigned to the unit commitments $c_{i,s}$, which poses challenges to determining the penalty function. Regarding this, the objective function is reformulated by expressing the power production $pd_{i,s}$ in terms of $c_{i,s}$, $P_s$ and an artificial variable $pd_{i,s}^r$:

$$pd_{i,s} \rightarrow c_{i,s} \cdot P_s + pd_{i,s}^r$$ (22)

Each $c_{i,s}$ is thereby related to a unit cost of $c_{i,s}^\omega \cdot P_s$.

2) For continuous variables, PH typically converts the objective function from linear to quadratic as a result of the Euclidean norm $\frac{\kappa}{2}\|c - \bar{c}\|_2^2$. For this model, however, the objective function can retain an MILP format after the relaxation penalty. This is achieved by substituting $c^2$ with $c$, which are equivalent terms for both binary values 0 and 1, within the aforementioned norm. The retained MILP format greatly contributes to a higher calculation efficiency.

B. Second Order Cone Relaxation

To establish a fast and mostly accurate depiction of Constraint (15), an iteration-free second-order cone relaxation is applied. It largely outperforms piecewise linearization in terms of time consumption, due to the exclusion of integer variables. See Algorithm 2 for the procedures (For simplicity, we denote $\text{CONT} = \frac{0.617D^3_i}{L_i \text{FRTZ} \rho^2}$).

Similar to previous research on electricity-gas coupled systems [38], the first step is to determine the direction of gas flow $gf$ in (15), to remove the absolute sign. The modified equation is then transformed into a convex second-order inequality (line 5 or 9 in Algorithm 2) and a concave inequality. While the former inequality is easily solvable, the latter requires advanced techniques to prevent $\|\pi_{i,s}^r - \pi_{i,s}^\omega\|$ from expansion.

Algorithm 2: Second-Order Cone Relaxation

1. Determine the direction of gas flow ($gf > 0$ or $gf < 0$).
   2. For each scenario
      3. For each pipeline
         4. If $gf > 0$
            5. Add $\|gf_s^r\|_2 \leq \text{CONT} \cdot \pi_{i,s}^r$ into the constraint
            6. Add $gf_s^r \geq \text{CONT} \cdot (\pi_{i,s}^r - \pi_{i,s}^\omega)$ into the constraint
            7. Add $\gamma(\pi_{i,s}^r - \pi_{i,s}^\omega)$ into the objective function
         8. Else ($gf \leq 0$)
            9. Add $\|gf_s^r\|_2 \leq \text{CONT} \cdot \pi_{i,s}^r$ into the constraint
           10. Add $gf_s^r \leq \text{CONT} \cdot (\pi_{i,s}^r - \pi_{i,s}^\omega)$ into the constraint
           11. Add $\gamma(\pi_{i,s}^r - \pi_{i,s}^\omega)$ into the objective function
      12. End If
   13. End For
   14. End For

To boost the computational efficiency, we intentionally discarded iterative constraints to tighten the relaxation. Instead, a penalty function in the form of $\gamma(\pi_{i,s}^r - \pi_{i,s}^\omega)$ is added to the objective (line 7/11 in Algorithm 2). In addition, to compensate for over-relaxation from only adopting a rudimentary penalization approach, a set of linear constraints

$$gf \geq (\leq) \text{CONT} \cdot (\pi_{i,s}^r - \pi_{i,s}^\omega)$$ (23)

is imposed (line 6/10). (23) are weaker inequalities of the concave constraint; therefore, the convexity can be restored.

IV. CASE STUDY

In this section, the IEEE 24-bus power system/Belgium 20-node gas system is used to test the validity of the PH algorithm. We further applied PH to the IEEE 118-bus power system/Belgium 20-node gas system to study its scalability in IES.

The calculation is performed on a Win10 environment with 8 Intel® Core i7-6700 CPUs, which can simultaneously provide 4 workers for parallel computing. The optimization model is formulated with MATLAB R2016a and Yalmip R20200116 [39], and solved by CPLEX V12.10.0.

A. IEEE 24-bus power system/20-node gas system

In this medium-scale IES, there are 3 gas-fired generators, U1, U4 and U5, connected to the gas system. In addition, there
are 5 coal-fired generators (U2, U3, U6, U7 and U8), and 2 wind turbines (U9 and U10). The network parameters are available from [27].

The power and gas loads are considered deterministic. In contrast, the wind power is stochastic. A total of 3000 random windspeed error curves are generated from the ARMA(1,1) series, and reduced to 15 through k-means clustering. After adding error terms to the windspeed baseline, we yield the output curve through power-windspeed relationships.

Two sets of unit production prices are presented to test the performance of the TPH/MPH algorithm:

**Case 1**: Production costs for gas-fired generators are set much lower than those for coal-fired generators. Therefore, aside from renewable energies, gas-fired generators U1, U4 and U5 are prioritized units for power generation.

**Case 2**: Production costs for gas-fired generators are set similar to those for coal-fired generators. The output of each generator is thus more susceptible to the wind and load profiles.

### A1. Numerical Results and Analysis

Before the stochastic operation of electricity-gas coupled systems is investigated, the optimal result of a separate scenario with higher wind penetration is studied. Fig. 1 depicts the hourly dispatch of three generator types for both cases. In both cases, the gas- and coal-fired outputs fluctuate accordingly with the total demand not met by wind power. However, because of the lower production prices for gas-fired generators in Case 1, an average of 0.128 GW more power is produced from gas. This also leads to gas-fired generators more frequently operating at their full capacity, especially in hours 17 to 23 when the load demand is high.

To illustrate the necessity of considering stochasticity and prove the effectiveness of the proposed stochastic UC model, the UCs determined via the proposed model are compared with those obtained in each scenario separately. Most UCs among different scenarios are the same, with minor exceptions. Table 1 presents two sets of UCs in Case 2 where differences occur.

The first example features the occasional start-up of generator U3 during periods 3-7 and 9-14 in Scenario 1. In the separately optimized model, the high wind power combined with the low power loads forces U3 to be shut down during these periods. However, in hours 3, 7, 9, 10 and 12, the generator needs to start up as a compromise for the other scenarios, where wind power is insufficient. The second set of UCs in Scenario 8 demonstrates the opposite case: U1 needs to be shut down to reduce costs for the other scenarios.

### A2. Performance of the TPH/MPH algorithm

The TPH/MPH algorithm unifies the total inconsistent unit commitments with high efficiency. Fig. 5 plots the inconsistency level \( \sum_{k} \left\| w_k - z_k \right\|_2 \) with respect to PH iterations for both cases (\( \kappa \) set to 1).

For Case 1, the error converges quickly to zero in 6 iterations. For Case 2, the problem becomes more intricate. Within the first 4 iterations, the inconsistency level converges considerably due to the oriented penalty terms. Iterations 5-9 are a trial-and-error period with a fluctuating inconsistency level. Finally, the error is stabilized (see the zoomed-in figure), where Ind remains at 1 for a long period before converging to 0. For TPH, 42 iterations are required before convergence, which is not illustrated in Fig. 2. However, for the proposed MPH, the iterations can be effectively terminated when Ind reaches 1 (i.e. iteration 10). The algorithm then searches for the outlier among all UCs, and performs enumeration to determine the best UC.

### Table 2 Comparison of different cases and methods in the IEEE 24-bus power system/Belgium 20-node gas system.

| Case | Method | Computation Time (s) | Expected Cost (M$) | Coal Production Cost (M$) | Gas Production Cost (M$) | Non-served Power (GW, Scenario-Averaged) |
|------|--------|----------------------|--------------------|--------------------------|--------------------------|------------------------------------------|
| Case 1 | Method 1 (benchmark) | 4035 | 14.7299 | 4.7023 | 6.9010 | 0 |
| | Method 2 (Deterministic UC) | 18.74 | 164.90 | 6.7181 | 4.7940 | 0.303 |
| | Method 3 (Benders) | 641.27 | 14.7299 | 4.6604 | 6.9376 | 0 |
| | Method 4 (TPH, 1/2/4 workers) | 65.17/46.24/36.11 | 14.7354 | 4.7192 | 6.8906 | 0 |
| Case 2 | Method 1 (benchmark) | 10933 | 13.9762 | 5.2747 | 5.6602 | 0 |
| | Method 2 (Deterministic UC) | 21.63 | 165.91 | 5.1201 | 5.6977 | 0.307 |
| | Method 3 (Benders) | 407 | 13.9736 | 5.2003 | 5.7358 | 0 |
| | Method 4 (MPH, 1/2/4 workers) | 114.3/70.72/58.03 | 13.9754 | 5.0411 | 5.8752 | 0 |
for this outlier. The total computational time is equivalent to only 12 iterations in terms of time consumption.

Fig. 2 The convergence of inconsistency levels.

Despite the heuristics applied, Progressive Hedging is still highly competent in obtaining the optimal cost. To illustrate, the following methods are proposed and applied to the model:

**Method 1**: Stochastic UC solved by CPLEX as a benchmark.

**Method 2**: Deterministic UC. A deterministic UC with one scenario is performed, and then the UC result is applied in multi-scenario dispatch to test the effectiveness.

**Method 3**: Stochastic UC solved by Benders Decomposition.

**Method 4**: Stochastic UC solved by TPH/MPH, with parallel computing enabled in MATLAB.

It’s noteworthy that we also used piecewise linearization to compare with the proposed iteration-free SOCR for natural gas pipeline modeling. However, it cannot be solved within acceptable time because too many binaries are introduced.

For Case 1, TPH is applied due to the disparate unit pricing, while in Case 2, MPH is applied. The program respectively accesses 1, 2 and 4 workers in the environment, to test the effectiveness of parallel computing in the algorithm.

The time consumption (including the model formulation stage) and expected cost for each method are summarized in Table 2. For the benchmark (CPLEX), difficulties in locating feasible solutions and decreasing the gap led to a total computational time of 4035 s/10933 s. Compared with that of Case 1, the optimization of Case 2 is twice as time-consuming, due to the proximity in unit prices which complicates the dispatch. It can be seen that deterministic UC, despite exhibiting a faster computation speed, is not an ideal alternative to stochastic UC, since a total non-served power of 0.3 GW appears in the final solution.

For the TPH/MPH algorithm, the computational burden is considerably reduced to ~65 s and ~114 s, respectively. The algorithm’s support for parallel computing further boosts the computational efficiency. For example, 4-workers in Case 2 shortened the computational time by 49.2% compared to 2-worker servers. The computational speed employing PH even outperforms the Benders Decomposition by at most 17 times.

In Case 1, the UC solutions of TPH and the benchmark method only differ in 1 generation unit. Therefore, the expected cost, as well as the gas-fired and coal-fired power production costs, only differ in 0.01 M$. Interestingly, in Case 2, although the UC solutions differ in 11 generation units, there is little difference in total cost (13.9762 vs 13.9754 M$). The shift in UCs can be explained by similar power production prices, where many solutions are considered optimal with a threshold of 10⁻⁴.

**B. IEEE 118-bus power system/20-node gas system**

This large-scale IES is utilized to test the scalability of the PH algorithm. The power system is composed of 36 coal-fired generators, 12 gas-fired generators and 6 wind turbines [39]. The gas system is not expanded correspondingly, since the Belgium 20-node gas system is already one of the largest systems in the real world, and investigated most frequently [27][38].

The performance of different methods is presented in Table 3. The PH method can still efficiently obtain the optimal solution. The early termination and enumeration method in MPH are especially useful in such large systems, even when the power production prices are already dissimilar. In this case, MPH significantly reduces the number of iterations needed from 110 to 60. As a result, only 270 s is required to compute the stochastic UC with 4 parallel workers. The total cost only differs from the benchmark by 0.0065%, which again proves the optimality of solution produced by PH.

Benders Decomposition, on the other hand, is inefficient in handling this problem. It is even slower than the benchmark method, which suggests that its efficiency might be heavily subject to the scale and parameters of the network.

| Method            | Computational Time (s) | Expected Cost (M$) |
|-------------------|------------------------|--------------------|
| benchmark         | 1227.0                 | 1.5425             |
| Benders           | 1962.5                 | 1.5426             |
| TPH               | 802.8/544.7/514.8      | 1.5426             |
| MPH               | 406.3/285.5/270.0      | 1.5426             |

**V. Conclusion**

In this article, we proposed a stochastic UC model for electric-gas coupled IESs. Moreover, we applied and modified the PH algorithm to accelerate the optimization of the stochastic UC. It was proven by case studies that, for a medium-scale electric-gas coupled IES, TPH easily converges when generators have disparate unit production prices, while for similar unit pricing, MPH is required for convergence via early termination and UC enumeration. For large-scale IES, MPH is inherently useful for saving computational time, regardless of generation prices. Combined with an iteration-free SOCR method to restore the convexity of the IES, the inconsistency level among scenarios converges quickly, and the computational speed of the proposed algorithm is at most 188 times faster than that of commercial software. The acceleration is attributable to the uniform Decomposition of the stochastic model, and the algorithm’s support for parallel computing. Meanwhile, the gap between PH algorithm and the benchmark
We will further explore this topic in the future.

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