Enhanced Josephson tunneling between high-temperature superconductors through a normal pseudogap underdoped cuprate with a finite-energy cooperon

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Abstract – The Josephson coupling between optimally cuprate superconductors separated by a spacer with a finite-energy cooperon excitation, which contributes to the Josephson coupling strength, is examined. For an underdoped-cuprate barrier in its normal state, the YRZ model gives a good description of the temperature-dependent enhanced Josephson coupling. A detailed examination of the origin of the enhancement shows a significant contribution from the cooperon excitation which is comparable to that from nodal quasiparticles.

We are motivated by the recent ARPES [8–11] (angle-resolved photoemission spectroscopy) and other experiments [12–20], which support rather a 2-gap scenario with an insulating pseudogap for underdoped cuprates. This leads us to examine the superconducting penetration at $T>T_c$ using the phenomenological 2-gap YRZ theory put forward by Yang, Rice and Zhang for the pseudogap phase. The YRZ model has had considerable success in describing many anomalous properties of the pseudogap phase. For a recent review see Rice, Yang and Zhang [21]. As discussed below, this model has a low-energy cooperon pole at temperatures $T>T_c$ which enhances the penetration.

An analysis of the single-particle propagator in a 2-dimensional array of 2-leg Hubbard ladders was an important input in the formulation of the YRZ ansatz. A recent extension by Konik, Rice and Tsvelik to an array of 4-leg Hubbard ladders exhibited 4 bands crossing the Fermi surface. They showed that at low doping there is an insulating gap with a cooperon resonance in the outer-band pair which coexists with metallic behavior in the inner-band pair. The virtual exchange of the cooperon resonance with support on the outer bands introduces a pairing mechanism in the inner-band Fermi surface [6].

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A decade ago Nozieres and Pistolesi [22] studied the transition for fermions with an attractive interaction between a superconducting and a semiconducting state within BCS mean-field theory as the one-particle band gap is increased. They found that at a critical value of the band gap, which is simply related to the superconducting gap of the starting metallic state, the pairing amplitude and phase stiffness dropped continuously to zero while the single-particle gap remained finite. Recently Rice et al. [21] showed that the approach to this quantum critical point (QCP) from the semiconducting state was characterized by a softening of the cooperon mode, dropping to zero energy at the QCP [21]. Support for the existence of cooperons in strongly underdoped cuprates also comes from exact diagonalization results of the strong coupling t-J models on small clusters. When extrapolated to an infinite lattice, these show a low-lying cooperon resonance in the antinodal regions of the Brillouin zone [6].

We use the phenomenological YRZ theory to describe the pseudogap phase with input parameters chosen to have the same values as in the original work [8] to model the experiments by Bozovic et al. on the enhanced superconducting penetration [2] into underdoped LSCO cuprates at temperatures $T > T_c$. In the YRZ model, the coherent part of the Green’s function is given by

$$G_{YRZ}(k, \omega) = \frac{g_t}{\omega - \epsilon_k - \Sigma_R(k, \omega)}. \quad (1)$$

The self-energy is given by $\Sigma_R(k, \omega) = [\Delta_R^0(k)]^2/\omega + \epsilon(k)]$ with $\epsilon_k = -2t(\cos k_x + \cos k_y) - 2t''(\cos 2k_x + \cos 2k_y) - \mu_p$, and the RVB gap $\Delta_R^0(k) = \Delta_0(x) (\cos k_x - \cos k_y)$. $g_t = 2x/(1+x)$ is the doping-dependent renormalization factor [23]. $t$, $t'$, $t''$ are the renormalized hopping integrals, and $\mu_p$ is an effective chemical potential determined by the Luttinger sum rule according to which the area enclosed by the four Fermi pockets equals the doped hole density [8,24]. The modified pairing form for the self-energy leads to two quasiparticle bands with energy dispersion and the corresponding spectral weight,

$$E_{k}^\pm = (\epsilon_k - \epsilon_k)/2 \pm \sqrt{(\epsilon_k + \epsilon_k)^2 + 4 \Sigma_R^2(k)}, \quad \Sigma_R^2 = [1 + (\epsilon_k + \epsilon_k)/(2E_k - \epsilon_k + \epsilon_k)]/2. \quad (2)$$

We introduce superconductivity by adding a pairing potential with d-wave symmetry. The magnitude of the pairing potential is chosen to reproduce mean-field values for $T_c$ in the Bozovic experiments, i.e., $T_c = 25 \text{K}$ (doping $x = 0.1$) in the underdoped junction and $T_c = 45 \text{K}$ (doping $x' = 0.15$) in the optimally doped leads. The tunneling setup is represented by a set of layers, typically 10 for each of the leads and 10 for the junction, as shown in fig. 1. Next we obtain a set of coupled Bogoliubov-de Gennes (BdG) equations for the layers,

$$\sum_j H_{ij}^{\alpha} \begin{bmatrix} u_{n_k}(j) \\ v_{n_k}(j) \end{bmatrix} = \epsilon_{n_k}^{\alpha} \begin{bmatrix} u_{n_k}(i) \\ v_{n_k}(i) \end{bmatrix}, \quad \text{with $i,j$ the layer indices, $k$ the in-plane momentum, $E_{n_k}^{\alpha}$ the YRZ quasiparticle dispersion given by eq. (2) which depends on the doping value of the layer $i$, $\Delta_{n_k}(k)$ is the superconducting gap of the layer $i$ with the d-wave factor $d(k) = \cos k_x - \cos k_y$, $E_{n_k}^{\alpha}$ and $[u_{n_k}(k), v_{n_k}(k)]^T$ are the eigenvalues and the corresponding orthonormal eigenvectors of the BdG equation.}

The Nambu-Green’s function for this multi-layer system can be constructed by the wave functions as [26]

$$G_{ij}^{\alpha}(k, i\omega_n) = \sum_{\alpha, \nu} Z_{k}^{\alpha, \nu} \begin{bmatrix} w_{n_k}(i) \\ v_{n_k}(i) \end{bmatrix} \begin{bmatrix} u_{n_k}(j) \\ v_{n_k}(j) \end{bmatrix}. \quad (5)$$

Choosing a $d$-wave attractive interaction $V_{ij}(k_\parallel, q_{\parallel}) = V_{d}(k_\parallel) d(q_{\parallel})$ with $V_d < 0$, then the self-consistent superconducting-gap equation is given by

$$\Delta_i = |V_i| \text{Im} \sum_{k_j} \int \frac{d\omega}{2\pi} d(k_j) f(\beta \omega) [F_{k_j}^{\alpha}(k_\parallel, \omega) - F_{k_j}^{\alpha}(k_\parallel, \omega)], \quad (6)$$

with $\beta = 1/(k_B T)$ the reciprocal temperature, $f(x) = 1/(e^x + 1)$ the Fermi function, $F_{k_j}^{\alpha}$ the retarded/advanced anomalous Green’s function. According to eq. (5), the gap equation (6) can be simplified as

$$\Delta_i = \frac{|V_i|}{2} \sum_{k_j} \sum_{n=\pm} d(k_j) \langle \beta E_{n_k}^{\alpha} w_{n_k}(i) | v_{n_k}(i) \rangle^*. \quad (7)$$

The Josephson current between nearest-neighbor layers is given by

$$I_{ij} = (2e/h) \sum_{k_j} \text{Re} \langle \epsilon_{k_j}^{\alpha} c_{k_j,i}^{\dagger} c_{k_j,j}^{\dagger} + \epsilon_{k_j}^{\alpha} c_{k_j,i}^{\dagger} c_{k_j,j} \rangle. \quad (8)$$

Fig. 1: (Colour on-line) Schematic illustration of the setup. The leads are two optimally doped LSCO materials, while the center is an underdoped LSCO with doping $x = 0.1$. The interface is parallel with the CuO$_2$ plane.

where the BdG Hamiltonian $H_{ij}^{\alpha}$ is given by

$$H_{ij}^{\alpha} = \begin{bmatrix} E_{n_k}^{\alpha}(k_\parallel) \delta_{ij} - t_{\perp,i,j}(k_\parallel) & \Delta_{n_k}(k_\parallel) \delta_{ij} \\ \Delta_{n_k}(k_\parallel) \delta_{ij} & -E_{n_k}^{\alpha}(k_\parallel) \delta_{ij} + t_{\perp,i,j}(k_\parallel) \end{bmatrix} \quad (4)$$
This can be evaluated as [27]

\[
I_{i,j} = -\frac{4e}{\hbar} \sum_{k_i, k_j} t_{k_i, k_j} \int \frac{d\omega}{2\pi} f(\beta\omega) \text{Im} \left[ G^{\alpha}_{ji}(k_{||}, \omega) - G^{\alpha}_{ij}(k_{||}, \omega) \right]_{11},
\]

(9)

\( I_{i,j} \) satisfies the continuity equation [28,29] between the leads, i.e., independently of the layer index. Figure 2 gives the result of the critical Josephson current as a function of temperature obtained from the YRZ model. We choose the thickness of underdoped LCO \( d \sim 10 \) layers, the perpendicular hopping \( t_c = 0.05t_0 \) with \( t_0 \sim 300 \text{meV} \) [8,20], the doping \( x = 0.1 \), the magnitude of the d-wave attractive interaction \( V = -0.54t_0 \), which gives a mean-field critical temperature of 25 K as in the Bozovic experiments [2]. We find a decay length of the Josephson current around \( \lambda \sim 10 \) (unit cell) at a temperature \( T = 30 \text{K} \) which is higher than the critical temperature of the underdoped spacer \( T_c \approx 25 \text{K} \). This long range penetration at temperatures above the superconducting transition of the underdoped spacer is due to an enhanced proximity effect. In contrast, for an insulating spacer without cooperon states (induced by attractive interaction), a single layer is enough to kill the Josephson coupling in our formalism. These results agree with the experiments [2,30] on the stoichiometric LCO quite well.

The cooperon excitation in underdoped LSCO at temperatures \( T > T_c \) with total momentum \( \mathbf{q}_z \), appears as the pole at an energy \( \Omega_{\mathbf{q}_z} \) in the pair propagator which is defined in real space as

\[
G_2(d, \tau) = \sum_{k_i, k_j} d(k_{||})d(k'_{||}) \times \langle \mathcal{T} [c_{k_i \uparrow}(0,0)c_{-k_i \downarrow}(0,0)c_{k_j \downarrow}^{\dagger}(d, \tau)c_{-k_j \uparrow}^{\dagger}(d, \tau)] \rangle.
\]

(10)

It's Fourier transform in momentum space can be estimated by summing over the particle ladder diagrams [32,33]. See fig. 3, which gives the result

\[
G_2(0, i\omega_n) \approx \frac{\sum_{k_{||}, \alpha} (Z_{k_{||}})_{\alpha}^2 X_{k_{||}}(i\omega_n) d^2(k_{||})}{1 + V \sum_{k_{||}, \alpha} d^2(k_{||}) Z_{k_{||}} X_{k_{||}}(i\omega_n)}
\]

(11)

when the total momentum \( \mathbf{q}_z = 0 \) and the perpendicular hopping \( t_c \) is much smaller than the in-plane hopping \( t_0 \). Here the factor \( X_{k_{||}}(i\omega_n) = \text{tanh}(\beta E_{k_{||}}^\alpha/2)/[2E_{k_{||}}^\alpha + i\omega_n - i\omega_n] \) with \( E_{k_{||}}^\alpha \) and \( Z_{k_{||}} \) denotes the dispersion and spectral weight of YRZ dispersion given in eq. (2). \( \eta = 0.01t_0 \) is a small imaginary part, representing the decay of the cooperon state [6]. The zero-momentum cooperon energy \( \Omega_0 \) is determined by the real part of the pole of pair propagator given by eq. (11), i.e.,

\[
\text{Re} \left[ 1 + V \sum_{k_{||}, \alpha} d^2(k_{||}) Z_{k_{||}} X_{k_{||}}^\alpha(\Omega_0) \right] = 0.
\]

(12)

The insert in fig. 2 shows the decrease (in electron notation) of \( \Omega_0 \) from zero as \( T \) increases above \( T_c \), in line with the behavior seen in numerical simulations [7] of a related model.

The contribution of the cooperon excitation can be deduced from the distribution of the Josephson current in momentum space of the underdoped sample (doping \( x \approx 0.1 \)). In the antinodal region of the Brillouin zone where cooperon states play an important role [6], the Josephson current is enhanced by cooperon states. On the other hand, the Josephson current can also be contributed from the quasiparticle excitation around the Fermi pockets.
The analogy between the 2-dimensional and finite-ladder systems has led to proposals according to which a cooperatoron resonance appears as the antinodal energy gap opens up. The aim of this paper is to test this proposal by examining the Josephson coupling between optimally doped (or overdoped) cuprate superconductors separated by a barrier of an underdoped-cuprate in its pseudogap state with a finite-energy cooperatoron excitation which contributes to the Josephson coupling strength. Here the calculations based on the YRZ model give a good description of the temperature-dependent enhanced Josephson tunneling. A detailed examination of the origin of the enhancement shows that the contribution from the cooperatoron excitation is significant and is comparable with that from the nodal quasiparticles.

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Fig. 4: (Colour on-line) Left panel: the contribution to the Josephson current from nodal and antinodal region, respectively. Right panel: the distribution of the Josephson current in the Brillouin zone. The parameters are same as the ones in fig. 2. The region between the two white lines is the nodal region, while the others are the antinodal region. Although the Josephson current is strongest at the “banana tips” of Fermi pockets, the total contribution from the nodal region is comparable with the one from the broadly spreading current around the antinodal region.

The phenomenological YRZ theory for the pseudogap phase at underdoping interprets the antinodal energy gap as a precursor to the Mott insulator at zero doping.
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