Steering line waves at a dual metasurface for optical applications

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Abstract

Line waves are defined as confined edge modes propagating at the interface of dual electromagnetic metasurfaces that preserve mirror reflection symmetries. Previous works have theoretically and practically explored these waves, showing that they occur at microwave regimes and terahertz ranges. It is also demonstrated that line waves can happen when there is a symmetric resistance discontinuity from negative to positive values and a uniform surface reactance. Line waves are of tunable mode confinement, direction-dependent polarizations, and singular field enhancement. This study presents a graphene patch design and demonstrates that the line waves associated with this structure can travel in the optical domain. Our design consists of a graphene metasurface on an epsilon near zero (ENZ) substrate. While our approach considers both dual reactive impedances and homogeneous reactive ones, we only concentrate on line wave utilization in a dual-impedance structure.
Introduction

Line waves (LW) are unique electromagnetic modes that are supported on the junctions of various metasurfaces, such as capacitive and inductive metasurfaces. These edge modes can efficiently propagate through arbitrary one-dimensional paths and are strongly constrained to the interface [1-8]. LWs were also discovered to be supported by non-Hermitian metasurface systems with parity-time symmetry, where the interface between two metasurfaces relies on the difference in resistance (gain and loss) rather than reactance (capacitance and inductance) [9,10]. Additionally, it was shown that LWs can be supported by parallel plate waveguides having electromagnetic (EM) duality [11]. These waves display the in-plane localization along the discontinuities in addition to the out-of-plane localization that is typical of conventional surface waves on reactive metasurfaces, which allows the energy to be transported along a 1-D track [12-16]. Since LWs have remarkable properties including singular field enhancement, a wide bandwidth, propagation-dependent polarization, and robustness to perturbations, they become promising for numerous applications in integrated photonics or optical sensing.

The physical mechanism related to the propagation of LWs is studied based on a dual impedance metasurface system at microwave bands in Bisharat et al work [1]. In their paper, two different types of tiny unit cells are presented to develop capacitive and inductive metasurfaces whose dispersions may be modified by changing just a few crucial geometrical factors without altering the lattice period. Xu et al investigate the intrinsic characteristics of LWs such as spin-momentum locking, confinement, and dispersion by proposing a compact-dual impedance metasurface [11]. Line wave propagation in non-Hermitian metasurfaces is studied by Moccia et al. both analytically (using the Sommerfeld-Maliuzhinets approach) and numerically (using finite-element simulations) [9]. The possibility of supporting LWs in the terahertz regime using gate-tunable graphene sheets is demonstrated as well [2].

One noteworthy aspect of the graphene-based line wave implementation is the ability to actively control the phase, intensity, and the guided wavelength of the mode by tuning surface
impedance values across the interface. However, line wave movements in the interface of complementary metallic metasurfaces lack this property. Compared to conventional edge modes, LWs benefit from singular field enhancement. This characteristic of LWs in which infinite energy is focused on a line is another important aspect of these modes [17-21]. Modulation and switching in nonlinear photonic devices are applications for this feature.

It is in this context that we propose a graphene patch design to present LWs in the optical domain. Our design is composed of a graphene metasurface on an epsilon near zero (ENZ) substrate. Both the dual reactive (inductive-capacitive) impedance and the impedance with a homogenous reactive component and a dual resistance (gain-loss) are satisfied by our approach. However, in our study, we exclusively use LWs making utilization of dual-impedance structures.

Results and discussion

Theory and method

The propagation of two surface modes with perpendicular polarizations can be attributed to homogeneous surface impedances as follows

\[
Z_{TM}^S = j \eta_0 / \zeta_{TM} \\
Z_{TE}^S = -j \eta_0 * \zeta_{TE}
\]

when \(\zeta_{TM} = \zeta_{TE}\), two inductive-capacitive surface impedances support the propagation of symmetric LWs moving in adverse directions, as depicted in Figure 1a. Indeed, as the line wave with spin-up can only go forward when the line wave with spin-down may only go backward, the spin-momentum locking property enforces the directionality of decoupled LWs [22-24]. Differently, surface impedances of non-complementary EM response support a quasi-line mode with a distribution similar to that of the edge modes.
Considering that graphene has the ability to support both TM and TE surface waves, we may use freestanding graphene to support LWs [25-27]. The dispersion relations of surface waves supported by a graphene sheet are determined by the following equations [28]

$$\beta_{TM} = \frac{\omega}{c} \sqrt{1 - (2/\sigma_g \eta_0)^2} \quad (3)$$

$$\beta_{TE} = \frac{\omega}{c} \sqrt{1 - (\sigma_g \eta_0/2)^2} \quad (4)$$

where $\sigma_g$ is the surface conductivity of graphene that can be derived using Kubo formula as [29]

$$\sigma_g(\omega, E_F, \tau, T) = \frac{-je^2}{\pi \hbar (\omega - j\tau^{-1})} \left\{ E_f + 2k_BT \ln \left[ \exp \left( \frac{-E_F}{k_BT} \right) + 1 \right] \right\}$$

$$+ \frac{e^2}{4\hbar} \left( \frac{1}{2} + \frac{1}{\pi} \text{arctan} \left[ \frac{\omega\hbar - 2E_F}{2k_BT} \right] \right) + \frac{j}{2\pi} \ln \left[ \frac{(\omega\hbar + 2E_F)^2}{(\omega\hbar - 2E_f)^2 + (2k_BT)^2} \right] \quad (5)$$

where $E_f$ is the Fermi energy, $\hbar$ is the Plank constant, $\omega$ is the angular frequency, $e$ is the electron charge, $T=300K$ is the temperature, and $\tau = 10^{-13}s$ is the relaxation time.

Figure 2 shows graphene conductivity versus frequency at different $E_f$ values. The real value of $\sigma_g$ is positive at all frequencies and for all values of $E_f$, as shown in Figure 2a. According to the equation 2, for real values of $\sigma_g$, $\beta_{TM}$ and $\beta_{TE}$ are purely imaginary. This corresponds to the propagation of evanescent surface modes. In general, real values of $\sigma_g$ exhibit their effect on the dissipation loss of waves supported by the graphene sheet. The imaginary value of $\sigma_g$ can change sign by altering $E_f$ from negative to positive or vice versa, as shown in Figure 2b. Therefore, the graphene structure can support the TE or TM surface wave depending on the $E_f$ value. To exemplify, consider the propagation of surface plasmons in graphene at a frequency of 50 THz. For $E_f = 0.4$, the graphene surface conductivity is $\sigma_g = 3.05 \times 10^{-5} - j1.34 \times 10^{-6}$. This is equivalent to TM surface wave propagation in graphene. The normalized propagation constant in this case is obtained $Re[\beta/k_0] = 37.667$, which demonstrates that the wave is strongly bound to the structure and that the
guided wavelength for TM surface wave propagation is much smaller than the wavelength of free space, i.e. $\lambda_{\text{guided-TM}} \ll \lambda_0$. For $E_f = 0.1$, the graphene surface conductivity is $\sigma_g = 4.02 \times 10^{-5} + j3.38 \times 10^{-6}$. This is equivalent to TE surface wave propagation with normalized propagation constant $Re[\beta/k_0] = 1$, so the TE mode propagation in graphene is very similar to TEM mode propagation in free space, i.e., $\lambda_{\text{guided-TM}} \approx \lambda_0$, and the wave can easily radiate into the free space. The equality of the phase velocity of surface waves with perpendicular polarization is a requirement for the excitation of a line wave in a structure. However, freestanding graphene cannot meet this criterion, making it incapable of guiding LWs [2].

**Proposed structure**

On the other hand, graphene-based metasurfaces with ordinary substrates require higher values of chemical potential to extend the operating range of LWs. To address this issue, we propose a graphene patch metasurface exhibiting dual capacitive and/or inductive EM response, placed on an ENZ material, as shown in Figure 3a. The metasurface consists of periodically formed structures with subwavelength graphene patches. Hence, the entire array can be regarded as a homogeneous surface impedance $Z_s$ [30,31]

$$Z_s = Z_{s1} + Z_{s2} = \frac{D}{(D-g)\sigma} - j \frac{2\omega\varepsilon_0\varepsilon_r}{D \ln(csc(\pi g/2D))} \frac{\pi}{g}$$ (6)

where $D = \lambda_0/12$ is the period, $g = \lambda_0/50$ is the gap spacing between adjacent graphene patches, $\lambda_0 = 7.0588 \, \mu m$ is the operating wavelength, and $\varepsilon_r$ is the dielectric constant of the substrate. The EM response of the graphene patch can be regarded as a series R-L-C circuit where the first section of impedance, $Z_{s1}$, represents the series R-L resulting from the graphene surface impedance and the geometric parameters. The second part, $Z_{s2}$, corresponds to the capacitive response of the graphene impedance surface and it is related to the patch geometry and permittivity of the background medium. Here, we use an aluminum-
doped zinc oxide (AZO) as the ENZ substrate. The complex permittivity of the AZO can be described using the Drude oscillator model [32-34]

\[ \varepsilon = \varepsilon_r + j\varepsilon_i = \varepsilon_b - \frac{\omega_p^2}{\omega^2 + \gamma^2} + j\frac{\omega_p^2\gamma}{(\omega^2 + \gamma^2)\omega} \]  

(7)

where \(\varepsilon_b = 3.8\) is the high-frequency permittivity, \(\omega_p = 0.5 \times 10^{15}\) is the plasma which is proportional to the free carrier concentration, and \(\gamma = 0.25 \times 10^{13}\) is the Drude damping rate. The ENZ frequency at which \(\varepsilon_r\) reaches zero is tuned in the infrared regime, as shown in Figure 3b. Note that the graphene patch is of dual EM response, in contrast to the subwavelength metallic patch/grid metasurface, which exhibits a dominant reactive response. This dual characteristic of the graphene patch allows switching the EM response of the structure by varying \(E_f\) at a constant frequency.

Figure 3c shows the surface impedance of graphene patches at \(\nu = 42.5\) THz, and \(\varepsilon = 0.3 + j0.033\). The EM response of graphene patches with the ENZ substrate changes from a capacitive impedance at low values of \(E_f\) to an inductive one at high values of \(E_f\). It is to be noted that establishing the equality of \(\zeta_{TM} = \zeta_{TE}\) is only possible in a narrowband, yet the quasi-line mode appears at other wavelengths, asserting that regarding the quasi-line mode, the waveguide has a remarkable bandwidth. Figure 3d plots the confinement factor of surface waves supported by the graphene patches at room temperature. For the previously mentioned waveguide parameters, a TE mode confinement factor of \(1 \leq Re[\beta/k_0] \leq 11.16\) is obtained in the range of \(0.017 \leq E_f \leq 0.095\). In the meantime, as \(E_f\) becomes greater a TM mode appears with the confinement factor of \(1.01 \leq Re[\beta/k_0] \leq 64.3\) for \(0.096 \leq E_f \leq 0.8\).

Bear in mind that in practice, attaining the parity-time symmetry condition that requires changing the sign of resistance is rather challenging task. Alternatively, owing to gapless energy spectra of electrons and holes in an optically pumped graphene, the negative resistance merely takes place at terahertz regime [35,36]. Our proposed structure paves the way for establishing the gain-loss effect and realizing optical amplification. This allows implementa-
tion of LWs using both duality and parity-time symmetries. Furthermore, we propose the use of a multi-layer graphene patch so as to reduce the effect of dissipation losses [37].

Figure 4a shows the resistive part of the proposed impedance surface in different scenarios. The real part of the impedance is purely positive at $\varepsilon = 0.3 + j0.033$ while a resistance discontinuity from a positive resistance (loss) to a negative one (gain) appears at $\varepsilon = 0.05 + j0.036$. In addition, much reduced dissipation losses have been attained regarding the use of a multilayer graphene structure. Figure 4b depicts $\zeta$ values versus $E_f$. The maximum values of $\zeta$ are obtained as $\zeta_{TM} \leq 7.94$, and $\zeta_{TE} \leq 11.7$. In comparison to the case when a typical substrate is employed, the obtained values for $\zeta$ are significantly higher and their limit is significantly greater. This leads to the enhancement of LW’s operation range by controlling $\zeta$ values.

**Eigen-mod analysis**

To investigate the effect of dissipation losses on propagation characteristics of line mode, we consider complex impedance surfaces of the form $Z_{TM} = R_{TM} + \delta \times j_0/\zeta$, $Z_{TE} = R_{TE} - \delta \times j_0 \times \zeta$ where $R_{TM} = 10$, $R_{TE} = 50$ and $\delta$ is a real parameter which spans $0 \leq \delta \leq 1$. The figure of merit representing the confinement-to-attenuation ratio of the line mode is shown in Figure 4a at $\nu = 42.5$ THz. The effective propagation length increases equivalently with increase of $\zeta$ while there is an evanescent wave along the propagation direction for $\zeta \leq 1.3$, which means that $Re[\beta] \leq Im[\beta]$. On the other hand, $\zeta < 1$ corresponds to the cut-off LWs that is in accordance with the observation. Note that $Im(Z_s)$ is a decisive factor in the propagation properties of the line mode so that $Re[\beta]/Im[\beta]$ reaches its maximum value when $\delta \to 0$.

Figure 4b shows the propagation property of a quasi-line mode. The mode has a confinement factor of $1.05 \leq Re[\beta]/k_0 \leq 9.49$ with $0.2 \leq \zeta \leq 50$. In case of the quasi line mode, a fragment of EM energy is confined to the supporting metasurfaces, so this mode bears a remarkable resemblance to well-known edge modes. Furthermore, unlike the conventional line
mode, there is a quasi-line mode with $\zeta_{TM_{or}TE} < 1$ as long as $\zeta_{TM} \times \zeta_{TE} > 1$. Be mentioned that akin to gradient metasurfaces, graphene patches provide us with an opportunity to exercise greater control over the phase, intensity, and propagation constant of EM fields. Strictly speaking, this allows for changing in the phase of E-fields of the line wave versus $\zeta$ variations, as shown in Figure 4c. This qualifies designing a straight phase shifter without there being any complexity. Figure 4d shows the E-field distribution of the line wave at different $\zeta$ values. Obviously, the line wave possesses higher field intensity at lower $\zeta$ values. Besides, the guided wavelength of LWs increases with increase of $\zeta$. As a result, the line wave with lower $\zeta$ is tightly confined to the interface line. This tunable feature of the line wave makes it attractive for radiation applications.

**Reconfigurable circuits**

It is possible to develop reconfigurable circuits, as shown in Figure 5a. Consequently, the guiding and transforming of the line mode can be adjusted to create flexible circuits by coding the impedance surfaces. The magic-T design for a line wave is implemented, as shown in Figure 5(b). This characteristic relies on modifying the impedance surfaces using adjustable pads over the structure. Figure 5(c) demonstrates a demultiplexer using a linear waveguide [38]. To accomplish this, we can control LWs via impedance surface coding and steering them toward the desired output. At $\nu = 42.5$ THz, the graphene patch design’s impedance changes to pure ohmic resistance for $E_f = 0.088$. This makes it possible to change the $E_f$ in various locations along the graphene patch to produce a not gate, as shown in Figure 5(d) [39].

**Quantum Spain Hall Effect of Light**

It has been shown that propagating light carries momentum and angular momentum (AM), serving as dynamic features of EM waves and quantum particles [40,41]. There exist two types of AM of light: the spin AM which is associated with, loosely speaking, the elliptical
polarization of EM waves, and the orbital AM which stems from the circulation of canonical momentum that is observed in vortex beams. In general, the spin AM which is perceived to be purely intrinsic can be classified as: the well-known longitudinal spin AM, which is parallel to momentum, the transverse (out-of-plane) spin AM which is orthogonal to momentum, and the non-orthogonal/transverse one carried by LWs (Figure 6a)[2,42-46]. Since the LW is a hybrid mode of two EM surface waves with orthogonal polarizations, its spin orientation is inherently different from that of evanescent surface waves spin AM, and its spin has both electric and magnetic origins (that is, $S = S^e + S^m$).

The momentum and orbital AM densities are associated with scalar characteristics of EM waves, while the spin AM density is linked to the polarization of fields. The time-averaged momentum density of a monochromatic EM field in Gaussian units can be defined as [45]

$$ P = gkRe(E^* \times H) $$

(8)

where k is the wave number and $g = \frac{1}{8\pi}$. The momentum density can be split into two parts, $P = P_o + P_s$, describing spin and orbital as

$$ P_o = g/2[Im(E^* (\nabla)E + H^*(\nabla)H)] $$

(9)

$$ P_s = \frac{1}{2} \nabla \times s $$

(10)

Here we use the notation $X_i(Y)Z_i = \sum_i X_i Y Z_i$. Then the spin AM density can be deduced as

$$ s = \frac{g}{2} Im(E^* \times E + H^* \times H) $$

(11)

The transverse spin AM in evanescent waves was described by Bliokh and Nori for the first time [47,48]. Figure 6b plots the spin AM density of a surface wave propagating along the x-axis and decaying exponentially in the y-direction. The evanescent nature of the wave
results in an imaginary longitudinal component producing a rotation of EM fields in the x-z
plane with a purely transverse spin AM that has an electric origin (that is, \( S^e, S^m = 0 \)). This spin AM is momentum-dependent and helicity-independent [49].

Thanks to reconfigurability of the waveguide, it is viable to alter the impedances orientation. Hence, spin AM of different orientations can be generated. Here, we propose a hybrid spin AM consisting of both transverse AM and the spin AM of LWs. Figure 6c shows a capacitive-inductive-capacitive impedance platform which can support a coupled line mode. Also there is a co-propagating surface wave guided by the inductive part. Similarly, guided coupled line modes based on non-Hermitian, and complementary metasurfaces have been proposed and studied [5,6,10]. Figure 6d plots the spin AM density of coupled line modes at different \( \zeta \) values. Given that \( \zeta_{TM} \) and \( \zeta_{TE} \) are identical, there are both transverse and the line wave spin AM while different values of \( \zeta \) result in almost a pure transverse spin AM. Additionally, surface impedances with complex values carry longitudinal spin AM. This is analogues to the spin AM of circularly polarized plane waves in free space.

The hall effect and quantum hall effect occur in the existence of a magnetic field breaking the time-reversal symmetry of a system. The hall effect creates an electric field which is perpendicular to both current flow and the applied magnetic field. Alternatively, the spin hall effect of light (SHEL) is the photonic counterpart of the quantum spin hall effect characterising a process in which the intrinsic spin of electrons is uniquely defined by the direction of momentum [50-53]. To show the SHEL, we use a spin source consisting of two orthogonal dipoles with a specific phase relation (i.e., \( \pi/2 \) phase difference). The point source placed at a subwavelength distance above the structure carries longitudinal spin angular momentum. The opposite helicities of the incident wave are restricted by the waveguide to generate unidirectional waves propagating in adverse directions, as shown in Figure 7a [54]. This non-reciprocity in the configuration of one-way modes together with magneto-optical scattering or absorption brings about efficient optical diodes [55].

Figure 7b plots the normalised propagation constant of coupled line modes versus \( \zeta \) varia-
tions in which the normalised propagation constant of $1.11 \leq \text{Re}[\beta/k_0] \leq 2.24$ is obtained for $2 \leq \zeta \leq 10$. Compared with the line mode, coupled line modes are rather loosely bound to interface lines. Additionally, in case of coupled line modes, full wave simulation suggests that there is no propagating mode for $\zeta \leq 2$.

On the one hand, the coupling of LWs may be controlled by the separation of beams $d$. On the other hand, it is possible to control the field intensity of branches using the variation of $\zeta$ values that in turn results in a coupled-quasi line mode, as shown in Figure 7c. This feature offers a new degree of freedom in that the phase velocity of LWs for $d > \lambda$ can be accordingly tuned by $\zeta$. In general, this can be generalized to multi-beam line waves enabling the implementation of feeding networks with a desired phase shifting function in branches [56].

Recently, Vernier effect has been used in many types of optical fiber sensors for measurements of different parameters, for instance, refractive index [57-59], gas pressure [60], temperature [61-64], strain [65], and so on. The Vernier-effect which significantly improves the sensitivity and resolution of fiber sensors was firstly introduced by Vernier Calliper to enhance the accuracy of length and air pressure measurement. As a rule, a Vernier effect-based sensor consists of two cascaded or parallel interferometers, one as a sensing interferometer and the other as a reference interferometer. In this effect, the overlapping of two out of phase signals leads to envelope modulation at the output spectrum. This effect can be applied to different kinds of two beams interferometric structures such as Mach-Zehnder interferometers (MZIs), Michelson interferometers, and Sagnac interferometers [66-69]. Here we propose a line-wave sensor which is based on the Vernier-effect, as shown in Figure 7d. In order to be able to control the phase and amplitude of signals, the sensor is composed of two cascaded MZIs with different $\zeta$ values. Unlike traditional Vernier-effect structures in which it is necessary to change the length of the Vernier scale to achieve higher resolution and sensitivity, here this desired feature can be achieved simply by changing the values of $\zeta$. 

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3- Conclusion

In conclusion, we propose a graphene patch with dual capacitive-inductive characteristic for the implementation of LWs in the optical wavelength region. Placing this structure on an ENZ substrate enables the implementation of LWs with small $E_f$ values. We also demonstrate that by altering the epsilon value of the substrate, the structure can be transformed into an active medium, which provides the realization of a line wave with a gain/loss effect. The positive and negative resistance values of the proposed structure can be controlled by varying $E_f$ and the number of layers in the multilayer graphene structure, making it possible to control the transition properties of LWs. We also show how $\zeta$ modifications could alter the linear wave’s intensity, phase, and confinement. The possibility of reconfigurable circuit development is further explored. This property allows the development of controllable logic gates. Additionally, we suggest a hybrid AM spin that employs a linear waveguide where the type of carried spin AM can be adjusted. We show that the intensity and phase of the fields in the coupled LWs can be adjusted in each branch, which leads to the coupled quasi-line mode. Moreover, the Vernier effect based on the line wave platform is examined, which offers high sensitivity as a desirable feature for sensors.

4- Numerical analyses

The eigen mode analysis in our work relies on the finite-element based commercial software Comsol-Multiphysics. To calculate dispersion relations, we employ the ‘Mode Analysis’ study which is available in the RF Module, and consider a 2-D computational domain. Additionally, to obtain transmission data, we perform full wave simulations using Driven-mode setup in Ansys Hfss, which is also a finite-element method based commercial software. This time we regard a 3-D computational domain surrounded by a radiation box of height $\lambda/2$ in the vacuum region. We model the graphene patch with inductive and capacitive EM responses as two surface impedance boundaries with values which correspond to $Z_{TM}$ and $Z_{TE}$ across the interface line.
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Figure 1: Unique characteristics of the line wave.
(a) Unidirectional propagation of the line wave along a bend path exhibiting that the propagation of pseudo-spin states is tied to wave momentum. (b) Singularity of the line wave along the interface line of dual impedance surfaces.
Figure 2: Graphene surface conductivity. (a) Real and (b) imaginary parts of the graphene conductivity versus frequency and $E_f$ variations at room temperature.

Figure 3: Characteristics and details of the proposed dual impedance surface. (a) Schematic of the proposed dual metasurface on an ENZ substrate and associated circuit model. (b) Real and imaginary parts of aluminium zinc oxide permittivity as a function of frequency. (c) The imaginary part of graphene patch impedance as a function of chemical potential at $\nu = 42.5$ THz. (d) The associated normalized propagation constant of surface waves supported by the structure.
Figure 4: Impedance variation to satisfy the non-Hermitian line wave symmetry requirement. (a) The real part of the surface impedance of graphene patch at different scenarios. (b) $\zeta$ values of surface waves versus $E_f$.

Figure 5: Eigen-mode analysis. (a) Dispersion characteristics of the line mode and (b) the quasi-line mode at different $\zeta$ values (c) phase shift of the line wave that is corresponds to surface impedance variations across the interface line (d) the possibility of controlling the intensity and guided wavelength of LWs using $\zeta$ variation.
Figure 6: Illustration of possible applications of the line wave.
(a) An electrically reconfigurable optical circuit (b) a possible implementation of optical circuits such as a magic-T coupler, and (c) a demultiplexer, and (d) a not gate

Figure 7: Angular momentum of EM waves.
(a) The line wave carries non-transverse spin AM (b) the evanescent wave is characterized by purely transverse spin AM (c) schematic of coupled line waves platform (d) coupled line modes carry hybrid spin AM.
Figure 8: Potential applications of coupled line waves.
(a) One-way states excited by a chiral source which is located at the center of the structure
(b) normalized propagation constant of coupled line waves in case of lossless impedance surfaces (c) E-field distribution of at the coupled-quasi line mode over a cross-sectional area of the waveguide (d) a full wave simulation of line wave based Vernier effect sensor consisting of two cascaded MZIs with different $\zeta$ values.