The Electroweak Phase Transition in Minimal Supergravity Models

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ABSTRACT

We have explored the electroweak phase transition in minimal supergravity models by extending previous analysis of the one-loop Higgs potential to include finite temperature effects. Minimal supergravity is characterized by two higgs doublets at the electroweak scale, gauge coupling unification, and universal soft-SUSY breaking at the unification scale. We have searched for the allowed parameter space that avoids washout of baryon number via unsuppressed anomalous Electroweak sphaleron processes after the phase transition. This requirement imposes strong constraints on the Higgs sector. With respect to weak scale baryogenesis, we find that the generic MSSM is not phenomenologically acceptable, and show that the additional experimental and consistency constraints of minimal supergravity restricts the mass of the lightest CP-even Higgs even further to \(m_h \lesssim 32\text{ GeV} \) (at one loop), also in conflict with experiment. Thus, if supergravity is to allow for baryogenesis via any other mechanism above the weak scale, it must also provide for B-L production (or some other ‘accidentally’ conserved quantity) above the electroweak scale. Finally, we suggest that the no-scale flipped \(SU(5)\) supergravity model can naturally and economically provide a source of B-L violation and realistically account for the observed ratio \(n_B/n_\gamma \sim 10^{-10}\).
1. Introduction

It is now well known that finite temperature effects can considerably alter the vacuum symmetry of a gauge theory \[1\], and can lead to restoration or anti-restoration \[2,3\] of a particular global or gauge symmetry due to the interaction of the theory with a plasma. In addition, the non-trivial structure of the electroweak (EW) vacuum naturally leads to the possibility of unsuppressed baryon number violation at the weak scale via finite temperature non-perturbative sphaleron transitions \[4,5,6\]. It was subsequently suggested that these baryon number violating effects could actually lead to baryogenesis at the weak scale \(\mathcal{O}(10^2\text{ GeV})\), since the necessary conditions of (i) C and CP-violation, (ii) B-violation and (iii) thermal non-equilibrium could in principle be satisfied. Recently, several new mechanisms have been proposed which can apparently account for the observed ratio \(n_B/n_\gamma \sim 10^{-10}\). Some of these mechanisms are rather economical, and involve \textit{e.g.} two higgs doublets \[8,9,10,11\], supersymmetry \[12\], left-right models \[13\], new heavy Majorana neutrino decays \[14\], and a \(CP\)-violating neutrino mass matrix \[15\].

In the standard scenario for the electroweak phase transition involving the decay of the false vacuum, thermal non-equilibrium requires the transition to be first order. This crucial requirement has led to a recent reappraisal of the EW phase transition beyond the classic treatment of Dolan and Jackiw \[16\] and Weinberg \[2\]. Although there has been some recent controversy regarding the generic form of the standard model (SM) scalar potential, a consensus now seems to have been reached \[17\], and the higher order effects considered (to order \(\lambda^3\)) tame the infrared divergences and effectively rescale the cubic term in the scalar potential by a factor of \(\frac{2}{3}\). In addition, no linear terms are present.

Irrespective of the details of baryogenesis, by insisting that unsuppressed sphaleron transitions after the EW phase transition do not wash out the observed BAU (Baryon Asymmetry of the Universe), an upper bound to the SM Higgs mass ensues, since the quartic coupling is bounded from above. This ensures an adequate finite-temperature vacuum expectation value at the critical temperature, \(v(T_c)\) so that the sphaleron transition rates are sufficiently small. In other words, the sphaleron mass must be sufficiently large \(M_{sph} \sim v(T_c)\) so that the Boltzmann factor \(e^{(-M_{sph}/T)}\) sufficiently suppresses the transition rate. In the SM, this translates into the limit \(m_H \lesssim 45(37)\text{ GeV} \) which is in conflict with the LEP result of \(m_H \gtrsim 60\text{ GeV} \) \[18\]. In extensions of the SM, the Higgs sector is

\[1\] Hereafter, Higgs mass limits in parenthesis represent the \(\sqrt{2/3}\) reduction due to the higher order effects.
usually enlarged by including singlets or additional doublets which generically relax the SM bound quoted above ($m_H \leq 45$ GeV). For a general two Higgs-doublet scenario, in order to avoid washout of $B + L$, the upper Higgs limit can possibly be as large as 120(98) GeV \cite{11}, and by adding a singlet \cite{19}, the limit $m_H \lesssim 150(122)$ GeV results.

In the minimal supersymmetric model (MSSM), the Higgs sector contains two complex Higgs doublets. After spontaneous EW symmetry breaking, the physical Higgs are the $h, H$ (CP-even), $A$ (CP-odd) neutral fields, and the charged $H^\pm$ Higgs (see Ref. \cite{20} for a complete discussion). Now that one-loop corrections to the Higgs masses are regarded as essential in certain regions of parameter space, the Higgs masses depend explicitly on the 21 parameters of the MSSM. Thus, any constraints to $m_h$ (the lightest CP-even Higgs) at the EW phase transition also depends explicitly on these many parameters. In order to simplify matters, the allowed parameter space of a SUSY model which employs a single SUSY breaking parameter has been explored in Ref. \cite{21}. A more general case has been considered in Ref. \cite{22}, however the conclusions drawn in Refs. \cite{21,22} are not necessarily in agreement regarding the upper limit to $m_h$. Nonetheless, an apparent upper limit to $m_h$ does depend strongly on $\tan \beta$ and $m_t$. Overall, it appears that $m_h \lesssim 65(53)$ GeV, corresponding to $m_t = 200$ GeV \cite{21}. Given the experimental model independent LEP limit of $m_h > 43$ GeV \cite{18}, it might appear that this simplified SUSY model is still barely viable, and would favor a very heavy top quark with a relatively light squark spectrum. We argue however that the resulting Higgs spectrum is very SM-like, and leads to the much more restrictive limit $m_h \gtrsim 60$ GeV. It is unlikely that the MSSM is involved in weak scale baryogenesis.

The consideration of two Higgs doublets at the EW scale has further motivation in the context of unified models. From the perspective of SUSY unification, the restriction to two Higgs doublets has been made quite explicit \cite{23}. By including additional doublets in the theory, the gauge couplings either fail to unify, or results in a unification scale $M_U$ that leads to unacceptably fast proton decay \cite{23}. In this letter we show that if one considers the EW phase transition in completely realistic supergravity models, the constraints of unification combined with the experimental and consistency constraints restricts $m_h \lesssim 32(26)$ GeV in order to avoid a washout of $B + L$ after the EW phase transition. This is even more restrictive than the MSSM. Therefore, unless there is an additional source of $B - L$ production above the weak scale, (or possibly some other ‘accidentally’ conserved quantity \cite{24}), baryon number would not survive, and would be completely washed out in this class of supergravity unified models.
We therefore present a natural solution to this problem by considering the flipped $SU(5)$ supergravity model [25]. The model also possesses two light higgs doublets, so $B + L$ could also in principle (and most likely will) be washed out. However, in this scenario out of equilibrium heavy Majorana neutrino decay provides a natural source of $\Delta L \neq 0$. As $B + L$ is washed out, the sphaleron transitions effectively process this $L$-number into a net $B$-number (and ultimately a BAU), since $B - L$ is conserved during the transitions. The model therefore naturally connects a massive neutrino sector (relevant to the MSW solution to the solar neutrino problem [26]) to the issue of baryogenesis and provides a comprehensive picture of solar neutrino physics, dark matter, and baryogenesis.

2. The EW Phase Transition and Weak Scale $\Delta B \neq 0$

Issues related to both weak scale $\Delta B \neq 0$ and the EW phase transition have been discussed recently and extensively in the literature [17,19]. We confine ourselves here to a brief discussion of the issues relevant to our calculations and conclusions. At finite temperature, the SM scalar Higgs potential takes the following form at one-loop [19]:

$$V_T(\phi, T) = D(T^2 - T_c^2)\phi^2 - ET\phi^3 + \frac{1}{4}\lambda_T\phi^4$$

(2.1)

where $\phi$ is the real, neutral component of the scalar Higgs field and $T_c, E, D, \lambda_T$ are calculable parameters that depend on the matter content (see Refs. [19], the first Ref. of [17], or Ref. [22] for the details). Finally, $T_c$ is the critical temperature; to a good approximation, it can be obtained from:

$$\frac{\partial^2 V_T(\phi, T_c)}{\partial^2 \phi} |_{\phi=0} \simeq 0$$

(2.2)

The finite temperature vacuum expectation value $v(T)$ is obtained from the usual $\frac{\partial V_T}{\partial \phi} = 0$ condition, and at the critical temperature $T_c$,

$$v(T_c) = \frac{3ET_c}{\lambda_T}.$$  

(2.3)

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2 This condition effectively determines the spinodal point, which is different than the condition $V_T(T_c, 0) = V_T(T_c, v_c)$. The difference is expected to be small for the range of Higgs masses we consider here [27].
We thus see the crucial role of the cubic term in obtaining a non-zero \( v(T_c) \), necessary for the first-order phase transition where the true/false vacuum is separated by a barrier.

Due to the non-trivial EW vacuum and the chiral nature of the theory, unsuppressed, topology changing \( \Delta(B + L) \neq 0 \) transitions become possible, particularly at high temperatures, where the probability to go over the classical barrier is enhanced enormously. The essential requirement for ensuring that the baryon number is not washed out at the weak scale is that the sphaleron transition rate \( \Gamma_{\text{sph}} < H \sim e^{-40} T \), where \( H \) is the Hubble expansion parameter. Since \( \Gamma_{\text{sph}} \sim T e^{-M_{\text{sph}}/T} \), this naive analysis implies \( M_{\text{sph}}/T_c \gtrsim 40 \). A more detailed calculation \cite{28} shows that baryon number is safe from ‘washout’ provided that

\[
\frac{M_{\text{sph}}(T_c)}{T_c} \geq 45 \to \frac{v(T_c)}{T_c} \equiv R_c \gtrsim 1.3.
\]

The last inequality is obtained from the specific SM sphaleron solution \cite{4,5}. From Eqn. (2.3) one can easily see that as \( \lambda_T \) grows, \( R_c \) is obviously diminished, thus the zero temperature Higgs mass \( m_H \) has a natural upper limit. Fig.1 shows the ratio \( R_c \), for \( m_t = 115 \) (solid line), and 150 GeV (dashed line). One can observe an overall asymptotic decrease in \( R_c \) with increasing \( m_H \), and by requiring \( R_c \gtrsim 1.3 \), the SM limit \( m_H \lesssim 45 \text{ GeV} \) is evident. Also shown in Fig. 1 is the effect of higher order terms (dotted line) which serve to reduce this limit to \( m_H \lesssim 37 \text{ GeV} \). As we have discussed, the generic constraint (2.4) has been imposed on extensions of the standard model and constrains the masses in these extended Higgs sectors. We now consider the EW constraints on the Higgs sector in minimal supergravity models.

3. Minimal Supergravity

Minimal supergravity models can be regarded as \( SU(3) \times SU(2) \times U(1) \) models with the minimal three generations and two Higgs doublets of matter representations at the EW scale (along with superpartners), and are assumed to unify into a larger gauge group (\( SU(5), SO(10) \) or \( E6 \)) at a unification mass of \( M_U \approx 10^{16} \text{ GeV} \). The five dimensional parameter space of this model can be described in terms of three universal soft-supersymmetry breaking parameters at \( M_U \): \( m_{1/2}, m_0, A \); the top-quark mass \( m_t \), and finally the ratio of Higgs vacuum expectation values \( \tan \beta = v_2/v_1 \). We re-define two of the independent soft-SUSY breaking parameters as \( \xi_0 = m_0/m_{1/2}, \xi_A = A/m_{1/2} \). The
sign of the superpotential Higgs mixing term $\mu$ is also undetermined. The low energy physical masses are then determined from a detailed RG-analysis of the gauge and Yukawa couplings, the scalar masses and the trilinear $A$-terms that all evolve separately from $M_U$ down to $M_Z$. Thus, all of the low-energy parameters are correlated to GUT-scale parameters (along with $m_t, \tan \beta$), and the 21 parameters of a generic, global SUSY analysis are dramatically reduced to five.

Several consistency and phenomenological constraints restrict the range of the model parameters, such as the requirement of radiative EW symmetry breaking, a potential bounded from below, $m_{\tilde{q}}, m_{\tilde{l}} > 0$ ($\tilde{q}, \tilde{l}$ correspond to the squark (slepton) fields) and the CDF, LEP experimental constraints to $m_{\tilde{g}, \tilde{\chi}}, m_{\tilde{\chi}^+, \tilde{\chi}^0}, m_{h, A}$. For a thorough discussion of these relevant details, see Ref. [29]. After all of these constraints have been imposed, one is left with a bounded region in the $m_t, \tan \beta$ parameter space for a given $\xi_0, \xi_A, m_{1/2}$. If one makes the choice of a specific model with an underlying gauge group, the Yukawa relations at $M_U$ will further constrain the allowed points. In order to make our analysis here as general as possible, we initially do not specify the unifying gauge group, however in our conclusions, we address the consequences with respect to the specific $SU(5)$ supergravity model.

Regarding the breaking of the EW symmetry, spontaneously broken supergravity models achieve this goal by inducing radiative corrections to the parameters in the higgs potential. This has the effect of dynamically generating vacuum expectation values for the neutral components of the two higgs doublets. At zero temperature, the tree-level potential involving the neutral Higgs fields is [29]:

$$V_0 = (\mu^2 + m_{H_1}^2)h_1^2 + (\mu^2 + m_{H_2}^2)h_2^2 + 2B\mu h_1 h_2 + \frac{(g_2^2 + g'^2)}{8}(h_1^2 - h_2^2)^2$$

where $g' = \sqrt{\frac{3}{5}} g_1$ and $g_2$ are the $U_Y(1)$ and $SU_L(2)$ gauge couplings; $h_1 = \phi_3, h_2 = \phi_7$ are the real, neutral components of the $H_1, H_2$ complex Higgs doublet fields respectively (and contain the eight real degrees of freedom $\phi_i = 1, ..., 8$), $\mu$ is the Higgs mixing term in the superpotential, $B$ is a soft-SUSY breaking parameter and finally we require $B\mu < 0$ [3]. Furthermore, $M_W^2 = \frac{g_2^2}{2} v^2, M_Z^2 = \frac{(g_2^2 + g'^2)}{2} v^2$, and $m_t = \frac{1}{\sqrt{2}} \lambda_t v \sin \beta, m_b = \frac{1}{\sqrt{2}} \lambda_b v \cos \beta$.

The normalization conditions for $H_1, H_2$ are chosen so that the minimum of $V$ is located at $h_1 = v_1, h_2 = v_2$; this is in contrast to the potential in [30]. We have compensated for a $\frac{1}{\sqrt{2}}$ discrepancy accordingly in Eqn. (2.4).
where $\lambda_{t,b}$ are the usual top and bottom quark Yukawa couplings, and $v^2 = v_1^2 + v_2^2$ (where $v = 246$ GeV).

In the one-loop approximation $V = V_1 = V_0 + \Delta V$, where \[ \Delta V = \frac{1}{64 \pi^2} \text{STr} \mathcal{M}^4 \left( \ln \frac{\mathcal{M}^2}{Q^2} - \frac{3}{2} \right) \] (3.2)
in the $\overline{MS}$ scheme, and the supertrace is defined as $\text{STr} f(\mathcal{M}) = \sum_j (-1)^{2j}(2j + 1) \text{Tr} f(\mathcal{M}_j)$. $\mathcal{M}_j$ are the higgs-field dependent spin $j = 0, 1/2, 1$ mass matrices, and $Q$ is the renormalization scale. We obtain the one-loop corrected higgs boson masses numerically from the standard mass matrix:

\[
M_{ij}^2 = \frac{1}{2} \left( \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} \right)_{\langle \phi_3 \rangle = v_1, \langle \phi_7 \rangle = v_2}
\] (3.3)

Initially, all of the parameters in $V$ need to be specified at zero temperature. For a given point in the five-dimensional parameter space $(m_t, \tan \beta, \xi_0, \xi_A, m_{1/2})$ we numerically solve for $\mu$ and $B$ from the minimization conditions for the scalar potential; for a given $\tan \beta$ and $M_Z$ at zero temperature (i.e., $v_1(T = 0)$ and $v_2(T = 0)$) we find the values of $\mu$ and $B$ which solve the following conditions:

\[
\left( \frac{\partial V}{\partial h_{1,2}} \right)_{\langle h_1 \rangle = v_1, \langle h_2 \rangle = v_2} = 0
\] (3.4)

where $V$ is to zero-temperature scalar Higgs potential. At finite temperature, each fermion/boson of species $i$ adds the following standard term to the effective potential:

\[
\Delta V_T = \eta_i \left( \frac{T^4}{2\pi^2} \right) F_\pm(y_i), \quad y_i = m_i(h_1, h_2)/T,
\] (3.5)

and $\eta_i$ is the multiplicity for each boson/fermion. In our calculation, we have included the following fields in the supertrace appearing in Eqn. (3.2): the third generation quarks and squarks $t, b(12), \tilde{t}_{1,2}(6), \tilde{b}_{1,2}(6)$, the gauge bosons $W(6), Z(3)$, and the Higgs bosons $h, H, A, H^\pm(1)$ (the numbers in parenthesis specify the multiplicity $\eta_i$). The functions
where \( F_-(bosons), F_+(fermions) \) are given by the following standard high (\( y_i < 1 \)) and low (\( y_i > 1 \)) temperature expansions:

\[
F^h_-(y_i) \simeq -\frac{1}{45}\pi^4 + \frac{1}{12}\pi^2 y_i^2 - \frac{1}{6}\pi y_i^3 - \frac{1}{32}y_i^4\log(y_i^2/c_b)
\]

\[
F^h_+(y_i) \simeq -\frac{7}{8}\left(\frac{1}{45}\right)\pi^4 + \frac{1}{24}\pi^2 y_i^2 + \frac{1}{32}y_i^4\log(y_i^2/c_f)
\]

and \( \log(c_b) = \frac{3}{2} + 2\log(4\pi) - 2\gamma_E \simeq 5.41, \log(c_f) = \frac{3}{2} + 2\log(\pi) - 2\gamma_E \simeq 2.64 (\gamma_E \) is the standard Euler-Mascheroni constant). Notice the \( y_i^3 \) infrared divergent bosonic contribution in Eqn. (3.6). For the low temperature expansion,

\[
F^l_\pm(y_i) \simeq -\frac{1}{2}\pi e^{-y_i}(1 + \frac{15}{8y_i}).
\]

We have used these expansions, as well as the tenth-order polynomial expansions for \( F_\pm(y_i) \) given in ref [9] which is valid for (1 < \( y_i < 3 \)); the results agree to within 10%.

In order to find \( T_c \), we employ \( V_T = V_1 + \Delta V_T \) in eqn.(3.3) and require

\[
\det\left(\frac{\partial^2 V_T}{\partial h_i \partial h_j}\right)_{(h_1) = 0,(h_2) = 0} \simeq 0
\]

for \( i, j = 1, 2 \), in analogy to Eqn. (2.2). Having determined \( T_c, V_T(T_c) \) is then minimized with respect to \( h_1, h_2 \), using the following conditions in order to determine \( v_1(T_c), v_2(T_c) \):

\[
\left.\frac{\partial V_T}{\partial h_1}\right|_{(h_1) = v_1(T_c), (h_2) = v_2(T_c)} \left.\frac{\partial V_T}{\partial h_2}\right|_{(h_1) = v_1(T_c), (h_2) = v_2(T_c)} \simeq 0
\]

We have scanned over the \( \xi_0, \xi_A, m_{1/2}, \tan \beta, m_t \) parameter space in an effort to find points for which \( R_c > 1.3 \). Points in the parameter space which violate this condition are regarded as baryonically ‘unstable’ at the weak scale. There are several approximations involved in the calculation which introduce uncertainties in the whole procedure at the anticipated 10 – 20% level, such as the uncertainty in \( T_c \), the use of the SM sphaleron mass [9], and the neglect of two-loop (and higher) order EW and QCD effects. Nonetheless, in the following section we quote specific numerical results, which we therefore consider to be \( \gtrsim 80\% \) accurate.
4. Results

In order to gain confidence with our procedure as well as to make contact with earlier results, we repeated the analysis of Ref. [21] in the case of a simplified MSSM model, where only degenerate $\tilde{t}_L, \tilde{t}_R$ contributions along with a single soft-SUSY breaking term was included. We find numerical agreement of $T_c, m_h^{max}$ to within 10% in a point-by-point comparison; the difference is expected to lie in the different approximations for $\Delta V_T$ that was used, as well as the numerical methods employed. In all points considered, the largest (and acceptable) values of $R_c$ required very large values of $m_3$ ($m_3^2 = \mu B$ for the potential we consider). For example, for $\tan \beta = 1.52, m_t = 115, m_3 \approx 1000$ GeV. This corresponds to $m_h \sim 48$ GeV, $m_A \sim 1475$ GeV, and a very SM-like Higgs spectrum, since the $hZZ (\sim \sin(\alpha - \beta)), hbb (\sim -\sin \alpha/\cos \beta)$ couplings $\rightarrow 1$ as $m_A \rightarrow \infty$. The preference for this limit was in fact pointed out in Ref. [9] for the tree-level situation. Due to these SM-like couplings, we expect the much more restrictive SM Higgs experimental limit to apply here. The reason is the following: for the set of allowed points in the simplified MSSM model considered in [21], we find the value of $\sin(\alpha - \beta) > 0.99$ with the coupling $hbb > 1$. In this case, $h$ production is not suppressed compared to the SM, and the experimentally preferred 2-jet signal from the overwhelmingly dominant $h \rightarrow bb$ will be comparable to the SM (see ref [32] for a more thorough discussion of these details). Therefore, the present SM analysis should be applicable to the $h$ Higgs, and the experimental limit $m_h \gtrsim 60$ GeV results. It is therefore unlikely that $B + L$ would survive in this MSSM model. Therefore, the likelihood that the MSSM alone is involved in weak-scale baryogenesis is rather remote.

The question we consider next is whether or not these conclusions hold for realistic minimal supergravity models as well.

For the realistic supergravity case, the parameters are obviously more constrained. In the case of $m_3$, once the initial values for $\tan \beta, \xi_0, \xi_A, m_{1/2}, m_t$ are given, $\mu, B$ (and thus $m_3$) are determined. As a result, $m_3$ is not a free parameter. We find that the allowed region of $\tan \beta$ for which $v(T_c)/T_c \geq 1.3$ is even more restricted than the generic MSSM limit $\tan \beta \lesssim 1.7$. Fig. 2 shows a scatter plot of $R_c$ versus $m_h$ for 100 distinct minimal supergravity models. We have fixed $\xi_0 = 1, \xi_A = 0, \tan \beta = 1.2; m_{1/2}, m_t$ are allowed to vary over their allowed values. For $\tan \beta = 1.2$, the tree-level perturbative unitarity constraint $\lambda_t \lesssim 5$ at all scales restricts $m_t \lesssim 148$ GeV [33]. We find that increasing $\tan \beta, \xi_0$ drives $R_c$ to smaller values quite rapidly. Also shown in Fig.2 is a set of points

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4 For the small $\tan \beta$ case considered ($\tan \beta \lesssim 2$), this approximation is perfectly adequate.
(circles with crosses) which correspond to $m_t = 95\text{ GeV}$. One can see from the figure that near $m_h^{\text{max}} \simeq 32\text{ GeV}$, increasing $m_t$ lowers the value for $R_c$, however this shift can be compensated by decreasing $\xi_0$. This behavior qualitatively reproduces the result obtained in Ref. [21], where increasing values of $m_t$ correspond to a smaller SUSY breaking scale. For all points considered, $m_h$ grows with increasing $m_{1/2}$, but never exceeds $m_h \simeq 32\text{ GeV}$.

Changing the value of $\xi_A$ has little effect on the result. For example, for $\xi_{0,A} = 1, 0$, $m_{1/2} = 70\text{ GeV}, m_t = 115\text{ GeV}$, $R_c \simeq 1.7$. When $\xi_A$ is varied from $0 \rightarrow 1$, $R_c$ varies from $1.7 \rightarrow 1.8$. For the $\mu < 0$ possibility, we find that there exists a lower bound for $m_h$ which exceeds $32\text{ GeV}$. Therefore, any possibility for $R_c \gtrsim 1.3$ is immediately ruled out in the $\mu < 0$ case. Higher order effects are expected to reduce the upper limit to $m_h \lesssim 26\text{ GeV}$. Given the fact that the recent LEP experiments restrict $m_h > 43\text{ GeV}$, it appears that a washout of $B + L$ is inevitable at the weak scale. Overall, we find that $R_c > 1.3$ is only possible for $\tan \beta \lesssim 1.3$, and for $\xi_0 \lesssim 5$. However, even for this region, $m_h$ is too small, and is experimentally excluded. Therefore, it is expected that $B + L$ is also washed out in realistic minimal supergravity models.

5. The Flipped $SU(5)$ Supergravity Scenario

Recently, one of us (D.V.N.) along with J. Ellis and K. Olive proposed a natural baryogenesis mechanism that exists in the flipped supergravity model [25], and utilizes the Fukugita-Yanagida scenario where heavy Majorana neutrino decay generates a net $L$-number which gets processed into $B$-number at the weak scale [14]. The crucial requirement is the existence of $\nu^c_i$ which naturally appear in the flipped case, but is ad-hoc in the minimal $SU(5)$ model.

The generation of a BAU in the flipped model is intimately connected to the seesaw mechanism for generating neutrino masses (see Ref. [25] for details), and in the flipped model considered here, the light neutrino masses are found to correspond roughly to the following hierarchy: $m_{\nu_\tau} \sim 10\text{ eV}, m_{\nu_\mu} \sim 10^{-3}\text{ eV}, m_{\nu_e} \sim 10^{-7}\text{ eV}$. Coupled with an anticipated $\sin^2 2\theta_{e\mu}$ mixing between $\nu_e, \nu_\mu$ of the order of $10^{-2}$, this implies that the model can provide for both an excellent hot dark matter candidate (the $\nu_\tau$, with $\Omega_{\nu_\tau} \simeq 0.3$), and an acceptable MSW solution to the solar neutrino problem with $\Delta m^2 \sim 10^{-6}$ [34].

For our purposes here regarding the BAU in supergravity models, it was shown that this same neutrino mass matrix can lead to an acceptable and natural value of $n_B/n_\gamma \sim 10^{-10}$ through out of equilibrium decays of the $\nu^c$. The final result for $n_B/n_\gamma$ can be expressed in
terms of the dilution factor $\Delta$, the unknown CP-violating phase factor $\delta$, the superheavy $\nu^c_i$ neutrino masses, the primordial microwave background fluctuations $\delta \rho / \rho \simeq 5 \times 10^{-6}$, and the top quark Yukawa coupling $|25|$

\[
\frac{n_B}{n_\gamma} \simeq \frac{9}{80 \pi} |\lambda_{233}|^2 \left( \frac{m_{\nu^c}}{m_{\nu^c}} \right) \sqrt{\frac{\delta \rho}{\rho} \frac{\delta}{\Delta}},
\] (5.1)

by making natural choices for parameters in Eqn. (5.1) (see $[25]$ again for the relevant details), one finds that

\[
\frac{n_B}{n_\gamma} \simeq 2 \times 10^{-6} \frac{\delta}{\Delta}.
\] (5.2)

Thus, with a very natural choice of $\delta$, the scheme is completely consistent with the observed value $n_B/n_\gamma \simeq 3 \times 10^{-10}$ (where $\Delta \sim 10^{-3}$). Although $B + L$ is most likely washed out at the weak scale, prior to equilibrium heavy neutrino decay allows the BAU to survive below the weak scale.

6. Conclusions

The recent developments in baryon number violation at the EW scale may lead the way to a completely new understanding of the value $n_B/n_\gamma \simeq 10^{-10}$ at a regime possibly within reach of future experimental probes. Given the fact that $n_B/n_\gamma \neq 0$, any mechanism which hopes to explain this number with or without utilizing the non-trivial, baryon-changing vacuum of the EW sector must confront the ‘washout problem’, where baryon number can be enormously reduced after the EW phase transition if the higgs mass(es) are too heavy, and ‘initial conditions’ at the weak scale after the phase transition dictate that $B - L = 0$. The constraints of gauge coupling unification and proton decay imply that no more than two doublets can be considered in this type of analysis. We have demonstrated here that supergravity models with the minimal two Higgs doublet structure and SM particle content (along with superpartners) at the weak scale generically lead to a washout of $B + L$ below the EW phase transition, since $m_h \lesssim 32$ GeV, in obvious contradiction with experiment. Our results here are completely numerical, and a general, analytical treatment of the two doublet scalar Higgs potential at one-loop (and higher) perhaps deserves further study. However, until further progress is made to reduce the uncertainties inherent in the calculation, this may be premature.
We therefore believe that we have explicitly confirmed the suspicion that additional out of equilibrium $\Delta(B - L) \neq 0$ mechanisms (or some other ‘accidentally’ conserved quantity) are necessary in order to generate a BAU in the context of supergravity unified models above the weak scale. Although the MSSM does contain additional sources of CP-violation in the gaugino/higgsino sector, we argue that supersymmetric baryogenesis at the weak scale is no longer possible since the scenario is most likely already experimentally excluded by SM Higgs searches. We therefore believe that the mechanism for the BAU resides elsewhere. In the case of the minimal $SU(5)$ supergravity model where $\Delta(B - L) = 0$, higher dimensional, non-renormalizable interactions which lead $R$-parity breaking may be required \cite{35,36}; this may arise from as yet unknown ‘Planck slop’ effects. As a predictive and economical alternative, we discussed the flipped supergravity scenario where the observed $n_B/n_\gamma \simeq 3 \times 10^{-10}$ can be naturally accounted for. Baryogenesis originates in the neutrino sector of the model via out of equilibrium heavy Majorana neutrino decay. In addition, the model is found to be quite consistent with the MSW solution to the solar neutrino problem, and satisfactorily addresses several dark matter issues.

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Figure Captions

Fig. 1. Fig. 1 shows the ratio $R_c \equiv v(T_c)/T_c$ as a function of the SM Higgs mass $m_H$ for $m_t = 115$ GeV (solid line), $m_t = 150$ GeV (dashed line), and the result for the $\frac{2}{3}$ reduction of the cubic term for $m_t = 115$ GeV (dotted line). The critical Higgs mass $m_H \lesssim 45$ GeV can be seen by imposing $R_c \gtrsim 1.3$ on the solid line.

Fig. 2. Fig. 2 shows a scatter plot of $R_c \equiv v_c(T_c)/T_c$ versus the lightest CP-even $h$ Higgs for 100 distinct minimal supergravity models for $\xi_0 = 1, \xi_A = 0$ and $\tan \beta = 1.2$. The values for $m_{1/2}, m_t$ are allowed to vary continuously; $m_t \lesssim 148$ GeV due to tree-level perturbative unitarity constraints. The circles with crosses correspond to the case where $m_t = 95$ GeV. Increasing $\tan \beta, \xi_0$ causes a systematic downward shift in the scatter plot. For every point shown, the corresponding value for the lightest Higgs is $m_h \lesssim 32$ GeV, and is therefore experimentally excluded.