The $\lambda$ energy dependence deduced from Bose – Einstein correlations of $\pi\pi$ pairs produced in $pp$ collisions

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Abstract. The energy dependence of the chaoticity parameter $\lambda$, derived from Bose–Einstein correlations (BEC) of pion-pairs produced in $pp$ collisions, is investigated. Considered is the one dimension (1D) of the BEC analysis in terms of a Gaussian and/or exponential distributions. The experimental data are examined in terms of the relation between the pion sources and the BEC dimension $R$ which in turn are deduced from the charged outgoing particle multiplicity. This approach follows the general energy behavior as obtained from the 1D BEC analyzes of the $pp$ collision data. Prediction for the $\lambda$ dependence on energy is obtained over a multi-TeV energy range based on a model of independent pion sources. The decrease of the $\lambda$ value with energy expected within the framework of this approach is supported by the experimental findings.

1. Introduction
With the recent operation of the Large Hadron Collider (LHC) at CERN the opportunity to study the one dimension (1D) BEC of identical bosons at very high hadron–hadron collision energies has been utilized by the ALICE, ATLAS and CMS collaborations. In parallel the energy dependence of the BEC dimension $R$ has recently been investigated and found in proton-proton ($pp$) collisions to increase with $\log(\sqrt{s_{pp}})$ [1, 2, 3]. Here we report on a study of the chaoticity parameter $\lambda$ energy behavior which determines the strength of the BEC measured effect. To this end we utilize the relation between the BEC dimension $R$ and the number of the groups of pion sources which we stipulate to be proportional to the average charged particle multiplicity produced in the $pp$ reactions. One of the frequently used parametrization for the 1D BEC analysis of hadrons which emerge from a sphere volume, is that of Goldhaber [4], namely

$$C_{Gauss}(Q) = 1 + \lambda_{Gauss}e^{-Q^2R^2_{Gauss}},$$

which assumes a static source in the plane-wave approach of a spherical volume with a radial Gaussian distribution of the emitter. The second often used parametrization, which assumes a radial Lorentzian distribution of the source, is given by

$$C_{Expo}(Q) = 1 + \lambda_{Expo}e^{-QR_{Expo}},$$
where in both representations $Q^2 = -(p_1 - p_2)^2$ is the difference squared of the 4-momentum vectors of the two correlated identical bosons. The $\lambda$ factor can vary between 0 and 1. In the 1D analysis the relation between $R_{\text{Gauss}}$ and $R_{\text{Expo}}$ dimensions can be evaluated from the requirement that the first $Q$ moment in a given BEC distribution will be equal when treated by a fit to a Gaussian distribution or an Exponential one, namely

$$\frac{1}{S_2} \int_{Q_1}^{\infty} Q e^{-Q^2 R_{\text{Gauss}}} dQ = \frac{1}{S_1} \int_{Q_1}^{\infty} Q e^{-Q R_{\text{Expo}}} dQ, \quad \text{where } S_{j=1,2} = \int_{Q_1}^{\infty} e^{-(QR)^j} dQ. \quad (3)$$

This relation remains essentially the same as long as the upper integration is higher than 2 GeV. In particular for the value of $Q_1 = 0$ GeV one obtains the known relation

$$R_{\text{Gauss}} = R_{\text{Expo}}/\sqrt{\pi}. \quad (4)$$

A connection between $\lambda_{\text{Gauss}}$ and $\lambda_{\text{Expo}}$ can be estimated from the relations

$$\int_{Q_1}^{Q_2} \lambda_{\text{Expo}} e^{-QR_{\text{Expo}}} dQ \simeq \int_{Q_1}^{Q_2} \lambda_{\text{Expo}} e^{-QR_{\text{Gauss}}} \sqrt{\pi} dQ \simeq \int_{Q_1}^{Q_2} \lambda_{\text{Gauss}} e^{-Q^2 R_{\text{Gauss}}} dQ. \quad (5)$$

which are valid as long as the upper integration level is above 2 GeV. Where in the range between $Q_1$ and $Q_2$ the Gaussian and Exponential BEC parametrization fit equally well the measured $Q$ distribution. In setting $Q_1 = 0$ and $Q_2 = \infty$ GeV one finds that

$$\lambda_{\text{Gauss}} = 2\lambda_{\text{Expo}}/\pi. \quad (6)$$

It has generally been found that at low $Q$ values, e.g. $Q_{\text{low}} \leq 0.1$ GeV, the Gaussian parametrization fails to yield a satisfactory fit to the measured BEC distribution and the Exponential one is describing it much better [13]. Therefore it is not surprising that the measured ratios $\lambda_{\text{Expo}}/\lambda_{\text{Gauss}}$ deduced from table 1 are significantly different than the one given by equation (6) of $\pi/2$ and are lying in the neighborhood of 2. A relation between the pairs ($R_{\text{Gauss}}, \lambda_{\text{Gauss}}$) and ($R_{\text{Expo}}, \lambda_{\text{Expo}}$) obtained in 1D BEC fits to the $Q$ distribution are analyzed in details in [16] where the obtained $\lambda_{\text{Expo}}/\lambda_{\text{Gauss}}$ ratios are very similar to the measured ones listed in table 1.

2. The BEC parameters dependence on energy

It has been shown that in the 1D BEC there exists a relation between $R_{n=1}$ and $R_n$ where $n$ is the number of the independent similar emitting pion group sources (see e.g. [17]), namely

$$R_n = n(\lambda_n / \lambda_{n=1}) R_{n=1}. \quad (7)$$

which relates the one group sources dimension $R_1$ at low energies with its value $R_n$ for $n$ identical group sources. The $\lambda_{n=1}$ and $\lambda_n$ parameters are the chaoticity values respectively for $n = 1$ and $n$ group sources. Inasmuch that one aims for the energy behaviour of $\lambda$ rather than on its absolute value sufficient is to determine the energy dependence of the ratio $\lambda = \lambda_n/\lambda_{n=1}$. Recently the BEC one dimension $R$ dependence on energy was observed in $pp$ collisions to increase from low to high energies and was fitted [1] to yield ($s$ is in $\text{TeV}^2$ units and $s_0 = 1 \text{ TeV}^2$)

$$R(s) = \{(1.64 \pm 0.11) + (0.07 \pm 0.01) \ln \varepsilon\} R_{n=1}, \quad \varepsilon \equiv s/s_0. \quad (8)$$

Thus the behavior of $\lambda(s)$ in terms of relations (7) and (8) requires a solution of the equation

$$\{(1.64 \pm 0.11) + (0.07 \pm 0.01) \ln \varepsilon\} R_{n=1} = n(s) \lambda R_{n=1}, \quad (9)$$
where $n(s)$ is equal to the number of group of identical sources as a function of energy. As it has already been seen that $R$ increases with the average charged multiplicity, $\langle N_{ch}\rangle$, of the colliding hadrons and since we aim only to estimate the $\lambda$ dependence on energy it is sufficient to require that the number of sources is proportional to the average charged multiplicity.

A compilation of $\lambda_{Gauss}$ and $\lambda_{Expo}$ obtained from the 1D BEC of pion-pairs produced in $pp$ collisions are given in table 1 and shown in figure 1 as a function of energy for the $\lambda_{Gauss}$. The data are seen to scatter somewhat in the energy region of 20 to 60 GeV which may well be attributed to problems concerning the reference samples and different analysis criteria. Despite of this discrepancy, a general decrease with energy of the $\lambda_{Gauss}$ values is apparent.

To evaluate the $\lambda_{Gauss}$ dependence on energy we follow the formalism outlined in [18, 19] where the hadron-hadron collisions are contributed by a “hard” and a “soft” component. In this approach the charged multiplicity of $pp$ collisions per rapidity $\eta$ is equal to $dN_{ch}/d\eta = N_{pp}$. Since however it has been found that $dN_{ch}/d\eta$ is independent of $\eta$ it follows that $N_{pp}$ is proportional to the total charge multiplicity of the $pp$ collision. Inserting the $N_{pp}$ reported expression

$$ N_{pp}(s_{pp}) = 3.43 \varepsilon^{0.11} , $$

and $n(s_{pp}) = N_{pp}(s_{pp})$ into equation (9) one obtains to a good approximation the $\lambda$ behaviour (not absolute values) as a function of energy

$$ \lambda_{Gauss}(s_{pp}) \simeq \{0.288 + 0.013 \ln \varepsilon\} \varepsilon^{-0.11} , $$

which is plotted in figure 1 normalized to the measured value at 200 GeV. As seen equation (11) describes the experimental energy dependence of $\lambda_{Gauss}$ quite reasonable and predicts a decrease of the chaoticity parameter with the increase of the collision energy. Note that at $\sqrt{s_{pp}} = 14$ TeV, the highest planned LHC energy, the 1D $\lambda_{Gauss}$ should reach the $0.20 \pm 0.04$ value. Due to the slow decrease of the chaoticity parameter at high energies the expected $\lambda$ value and its errors at the current LHC energy of 13 TeV should be similar to that predicted at 14 TeV.

### Table 1. A compilation of $\lambda$ values obtained from the 1D BEC of pion-pairs produced in $pp$ collisions. The statistical and systematic errors are added in quadrature in the table.

| Experiment/Collaboration | $\sqrt{s_{pp}}$ [GeV] | $\lambda_{Gauss}$ | $\lambda_{Expo}$ | References |
|--------------------------|------------------------|-------------------|------------------|------------|
| BNL-E766                 | 7.21                   | 0.466 ± 0.015     | —                | [5]        |
| NA23                     | 26.0                   | 0.32 ± 0.08       | —                | [6]        |
| ABCDHW                   | 31.0                   | 0.35 ± 0.04       | —                | [7]        |
| ABCDHW                   | 44.0                   | 0.42 ± 0.04       | —                | [7]        |
| AFS                      | 58.1                   | 0.34 ± 0.04       | —                | [8]        |
| ABCDHW                   | 62.0                   | 0.42 ± 0.08       | —                | [7]        |
| AFS                      | 63.0                   | 0.40 ± 0.03       | 0.77 ± 0.07      | [9]        |
| SFM                      | 63.0                   | 0.42 ± 0.04       | —                | [10]       |
| STAR                     | 200                    | 0.353 ± 0.003     | —                | [11]       |
| ALICE                    | 900                    | 0.31 ± 0.03       | 0.55 ± 0.05      | [12]       |
| ATLAS                    | 900                    | 0.34 ± 0.03       | 0.74 ± 0.11      | [13]       |
| CMS                      | 900                    | 0.315 ± 0.014     | 0.63 ± 0.03      | [14, 15]  |
| CMS                      | 2360                   | 0.32 ± 0.01       | 0.66 ± 0.09      | [14]       |
| ATLAS                    | 7000                   | 0.25 ± 0.02       | 0.53 ± 0.05      | [13]       |
| CMS                      | 7000                   | —                  | 0.62 ± 0.04      | [15]       |
Figure 1. The 1D BEC $\lambda_{Gauss}(\sqrt{s_{pp}})$ as deduced from the measured $\pi \pi$ produced in $pp$ collisions. The $\lambda_{Gauss}$ values at $\sqrt{s_{pp}}$ close to the 63 GeV and at $\sqrt{s_{pp}} = 900, 7000$ GeV are averages over experiments at $\sqrt{s_{pp}} = 62 - 63, 900$ and 7000 GeV respectively. The expected $\lambda_{Gauss}(\sqrt{s_{pp}})$ derived in this work is shown by the continuous line, normalized to the measured $\lambda_{Gauss}$ at 200 GeV where the dotted lines represent its $\pm 1$ s.d. limits.

3. Summary
The 1D BEC measured values of $\lambda_{Gauss}$ are seen to decrease with energy. The approach of relating the $R$ and multiplicity increase with $pp$ collisions energy with the number of sources results in a decrease of the 1D $\lambda_{Gauss}$ similar to the one obtained experimentally. Prediction for the chaoticity dependence on energy is obtained over a multi-TeV energy range.

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