Method for Calculating the Cutting Coefficient of Rocks

G D Pershin¹, L V Kosarev², N G Karaulov³

¹Mining Machines and Transportation Technological Complexes Department, Nosov Magnitogorsk State Technical University, Lenin Street, 38, Magnitogorsk city, Chelyabinsk Region, Russian Federation, 455000, Russia
²Mining Department, Technical Institute (branch) of North-Eastern Federal University named after M. K. Ammosov, st. Kravchenko 16, Neryungi city, 678960, Russia
³Mining Department, Nosov Magnitogorsk State Technical University, Lenin Street, 38, Magnitogorsk city, Chelyabinsk Region, Russian Federation, 455000, Russia

E-mail: n_karaulov@mail.ru

Abstract. Research objective is improvement of engineering methods for calculating the main indicators and parameters of the process of cutting rocks with a carbide tool. The sawing coefficient is defined as the ratio of the total forces that arise during the cutting of rocks with a carbide tool. Numerical estimation of the coefficient is conducted using two methods, both on the basis of experimental data from previous studies, and according to the developed analytical theory for calculating the system of contact forces, which characterize the resistance of the rock material to shear and compression deformations. Energy-power assessment of the process of separation of the surface layer of rock with a cutting tool is reduced to determination the system of contact forces, which on the one hand characterize the resistance of the rock material to compression and shear deformations (reactive forces), and on the other - acting from the cutter to the rock active forces. The conducted research makes it possible to calculate the effective power spent on the cutting process, as well as to determine the share of energy that goes to the destruction of rock, tools and heat generation as a result of mutual friction. These results will be in demand by production companies engaged in the extraction, processing of facing stone and also design of stone-cutting equipment.

1. Introduction
Facing and building stones are fundamental resources for economic, urban and social development. They are widely used in all construction projects that emphasize the expediency and economical use of stone [1, 2]. Despite constant competition with various artificial imitations appearing on the market the consumption of natural stone materials worldwide increases by 7 – 9% annually and is currently at the level of about 30 million m³ (650 million m²). This indicates the high competitiveness of natural stone. [3, 4, 5].

The presented research is aimed at improving engineering methods for calculating the main indicators and parameters of the process of cutting natural facing stone with a carbide tool, which is currently the most common rock-breaking working organ of many mining machines.
2. Theoretical substantiation

Energy-power assessment of the process of separation of the surface layer of rock with cutting tool is reduced to determining the system of contact forces, which on the one hand characterize the resistance of the rock material to compression and shear deformations (reactive forces), and on the other - acting from the cutter to the rock active forces. For practical purposes, we are interested in productive from the point of view of the effectiveness of the destruction of the rock material the constituent forces \( P_t, P_n \) acting in the direction of the cutting speed vector and perpendicular to it. The tangential force \( P_t \) cleaves (cuts) the rock with a certain specified chip thickness \( h \), and the normal to the coalface force \( P_n \) provides the thickness of the cut in accordance with the specified power or kinematic mode of operation of the working body of the mining machine.

Moving the tool along the coalface at a depth \( h \) from the upper free surface will cause a shift (chip) in the separated rock layer at a sliding angle \( \beta_0 \) to the direction of movement. If the cutter is loaded only by the force \( P_t \), then its further movement will occur along the shift line with access to the upper free surface. To maintain a constant chip thickness during the entire cutting process, it is necessary to apply a normal \( P_n \) force to the cutter. Based on the noted functional purpose of the forces \( P_t \) and \( P_n \), they have a relationship through the sliding angle \( \beta_0 \) (see Fig. 1):

\[
P_n = \frac{P_t}{\tan \beta_0}
\]

which is justified in [6] and developed [7].

The obtained ratio makes it possible to calculate the coefficient of rock sawing with a carbide tool:

\[
\mu_{pacn} = \frac{P_{pac}}{P_{n0n}} = \frac{P_t + P_{tr} + P_{iz}}{P_n + P_{ntr}}
\]

where \( P_{pac} \) – total tangential force (sawing force), H;
\( P_{n0n} \) – the total normal force (feed force), H;
\( P_{iz} \) – abrasive wear force of a carbide tool, H;
\( P_{tr} \) – respectively, the constituent forces of the friction in the tangential and normal directions, H.

In turn, the expression (2) allows us to calculate the coefficient of surface destruction of rock \( \mu_p^n \), with a cutting tools:

\[
\mu_p^n = \tan \beta_0 = \mu_{pac} - \mu_{tr} - \mu_{tr}^n
\]

where \( \mu_{tr} \) – total coefficient of friction of the cutter on the rock;
\( \mu_p^n \) – coefficient of destruction (abrasive wear) of the tool.
Calculation of coefficients of sawing surface destruction of rock with a carbide tool using formulas (2) and (3) is possible, if the numerical values of the forces of $P_{pac}$ and $P_{pod}$ are known, which can be obtained in two ways: by analytical formulas; by empirical dependencies.

In the future, we will use the experimental data obtained and presented in [8,9].

The following empirical dependences were obtained by statistical processing for an unblunted cutter:

- sawing force (kg): 
  \[ P_{pac} = K_p h^k n^h \]  
  (4)

- feed force (kg): 
  \[ P_{pod} = K_n h^{0.33} \]  
  (5)

where 
  \[ K_p = 1.1 e^{0.00074 \gamma_g \sigma_{csh}}; \]
  \[ K_n = 0.9 e^{0.00077 \gamma_g \sigma_{csh}}; \]
  \[ \gamma_g - \text{the volumetric weight of the rock, t/m}^3; \]
  \[ \sigma_{csh} - \text{the tensile strength of rocks for uniaxial compression, kg/cm}^2. \]

As shown by experimental data of work [8], the feed force (5) is practically not affected by the type of contact between the tool and the rock, exponents for parameters $e$ and $h$ are constant values.

The system of forces acting on the rock from the cutting tool can be reduced to the resultant force \( R \) (see Fig. 1), called the cutting force:

\[ R = \sqrt{P_{pac}^2 + P_{pod}^2 + P_B^2}, \]  
(6)

where $P_{pac}$, $P_{pod}$ and $P_B$ are the constituent forces. The $P_B$ force occurs as a side force in a semi-free cutting scheme.

The above researches determine the first methodological approach to calculating of therequired energy-power parameters and characteristics of the rock cutting process with a cutting tools. The second method of calculation follows from the expression (1), (2) and assumes finding the angle of sliding (shear, chip) of the rock material, along which there is a deformation and separation of the element surface volumes of chips. A complete and accurate analytical solution to the problem of surface shear for an elastic-fragile body, the model of which can be attributed to the majority of rocks, has not been obtained at present. In this regard we will use experimental data to calculate the sliding angle in accordance with the described method [6].

In works [8,10,11] experimental data of multi-dimensional research of the process of directed destruction of natural stone of various strengths with a cutting tool are presented. As a result by processing experimental data a geometric equation of the breakaway line was obtained, as a projection of the breakaway plane on the vertical direction, in the form of a second-order curve:

\[ y = ax^2 + bx, \]  
(7)

where - \( a \) and \( b \) are the proportionality coefficients.

For felsitic tuff \( \sigma_{csh} = 35 \text{ MPa} \) by free cutting with a cutter, the width of which was 5 mm, and the front angle was $10^o$, the above equation (7) was written as follows:

\[ h = 0.0285 c^2 - 0.586 c, \]  
(8)

where \( c \) – the length of the breakaway line, mm.

In figure 2, equation (8) is presented in graphical form, which visually shows how the length of the chip element changes from the depth of cut \( h \).
Figure 2. Graphs of the dependence of the cutting depth and sliding angle on the chip length.

The given curve has two characteristic sections. In the first section \((h = 0÷0,5\text{mm})\) where the rock is destroyed by crumbling, compression and fine crushing, the coalface surface is straight. At cutting depths \(h > 0,5\text{ mm}\), the contact height of the front face of the cutter with the rock and the shear force increase, which leads to the formation of a fracture crack. With further movement of the cutter, the crack sprouts and bends as a result, a large volume of the chip element is separated.

Geometric equation \((7)\) allows us to find the angle \(\beta_0\) by differentiating it in the form of the following dependence:

\[
tg \beta_0 = \frac{dy}{dx} = 2ax - b, \tag{9}
\]

or for a specific example according to the expression \((9)\)

\[
tg \beta = \frac{dh}{dc} = 2 * 0,0285c - 0.586. \tag{10}
\]

An analysis of the development of the theory of rock cutting with a carbide tool is presented in the works \([12,13,14,15]\), where a wide variation of this characteristic in the range from \(18^\circ\) to \(45^\circ\) is noted in relation to the value of the shear angle (chipping) \(\beta_0\), without specifying the dependencies for which it is calculated.

The considered methods for determining and calculating the rock sawing coefficient are based on experimental data obtained in the laboratory on special bench installations. When considering the scheme of these forces, simplifications are allowed:

- the shearing chip element has the form of a parallelogram and is separated along straight sliding lines at an angle \(\beta_0\) to the direction of relative motion;
- chip formation is a cyclical process.
- resistance to chipping during cyclic destruction is not constant.

With the assumptions made, the shear force is written:

\[
P_{cd} = \tau_{cd}f_{cd} = \tau_{cd} \frac{h_n}{\sin \beta_0}, \tag{11}
\]

where \(f_{cd}\) – the area of the shear, \(\text{m}^2\);
\(\tau_{cd}\) – the tensile strength of the rocks to the shear, \(\text{Pa}\)

The value of the \(P_{cd}\) force makes it possible to find in the conditions of free cutting the tangential force of rock destruction with a cutting tool that's the force \(P_t\), which does not include energy losses due to mutual friction of the tool against the rock and abrasive wear of the tool (see Fig. 1):

\[
P_t = \frac{P_{cd}}{\cos \beta_0} \tag{12}
\]

The force \(P_t\) corresponds to the normal force \(P_n\) (see Fig. 1), which is calculated by the formula

\[
P_n = \frac{P_t}{tg \beta_0} = \frac{P_{cd}}{\sin \beta_0} \tag{13}
\]

The friction forces on the front and back faces of the cutter are calculated using the formulas:
where $N_{n,g}$ is the normal pressure force of the front face of the tool on the rock, H;  
$\mu_{n,g}$ and $\mu_{z,g}$ - accordingly the friction coefficients on the front and back faces of the cutter; 
$\gamma$ – front angle of the cutter.

Taking into account the unproductive but unavoidable forces (14) and (15), we find the sum of the projections of forces acting on both the front and back surfaces on the x and y axes:

$$\sum P_{xi} = P_{aac} = \frac{p_{ca}}{c_s \cos \beta_0} (1 + \mu_{n,r} \sin \gamma \cos \gamma + \mu_{z,r} \cos^2 \gamma)$$  \hspace{1cm} (16)

$$\sum P_{yi} = P_{noc} = \frac{p_{ca}}{c_s \cos \beta_0} (c_t \gamma_0 + \mu_{n,r} \cos^2 \gamma)$$  \hspace{1cm} (17)

By the ratio of forces $P_{aac}/P_{noc}$ we find the sawing coefficient:

$$\mu_{pac} = \frac{1 + \mu_{n,r} \sin \gamma \cos \gamma + \mu_{z,r} \cos^2 \gamma}{c_t \gamma_0 + \mu_{n,r} \cos^2 \gamma}$$  \hspace{1cm} (18)

3. Results and discussion

The analytical expression (18) is functionally related to the value of the sliding angle $\beta_0$, which in turn depends on the physical and mechanical properties of the rock and the parameter $h$ according to formulas (8) and (9).

A feature of the implementation of friction forces along the front face of the cutter is the impact on it of crushed rock (sludge), squeezed out of the contact zone along the surface. In this case, the friction is carried out by the loose abrasive and the coefficient of friction has a lower value in comparison with the friction on the fixed abrasive. In further calculations we will take $\mu_{n,z} \approx 0.5 \mu_{z,g}$.

The numerical calculations $\mu_{pac}$ by expression (19) are shown in Fig. 4 as graphical dependences on the value of the sliding angle $\beta_0$, which is functionally related to the cutting depth $h$, according to the geometric relations (10) and (11).

Figure 3. Dependence of the sawing coefficient on the size and energy coefficient of efficiency of the sawing process on the cutting depth $h$.

The graph in Fig. 3 shows an increase $Q_{E_{n,g}}$ with an increase in the cutting depth $h$, while the share of productive energy costs is from 65 to 80% of the total.
The integrating indicator of energy consumption is the capacity, which is determined by the action of the forces of $P_{pac}$, $P_{rod}$ and $P_e$. The numerical values of the capacity from the forces $P_{rod}$ and $P_e$ are usually small, so the effective capacity is practically determined from the force $P_{pac}$ using the formula:

$$N_s = P_{pac}V,$$

where $V$ is the linear velocity of movement of the cutter along coalface, m/s.

The research carried out in this work makes it possible to divide the input energy (20) into productive and unproductive costs according to the values of the coefficients $\mu_p$, $\mu_r$ and $\mu_{np}$.

In this case the energy coefficient of efficiency of the cutting process is characterized by the expression:

$$\mathcal{E}_{K,N,D} = \frac{\mu_p}{\mu_{pac}} = \frac{\mu_p}{\mu_p + \mu_r + \mu_{np}}.$$  

(20)

For $\mu_p = \mu_r \rightarrow 0$ we have

$$\mu_{pac} = \mu_p + \mu_r = tg\beta_0 + tg\rho_n,$$

so reducing the friction coefficients on the front and back faces of the cutter up to zero values increases the energy coefficient of efficiency of the sawing process:

$$\mathcal{E}_{K,N,D} = \frac{\mu_p}{\mu_p + \mu_r} = \frac{tg\beta_0}{tg\beta_0 + tg\rho_n},$$

(22)

where $tg\rho_n = \frac{p_n}{p_{pac}}$

4. Conclusions

The conducted research makes it possible to calculate the effective power spent on the cutting process, as well as to determine the share of energy that goes to the destruction of rock, tools and heat generation as a result of mutual friction.

At the same time it was found that with increasing cutting depth, the share of productive energy costs increases. A fairly good correlation of the experimental theory allows us to use the developed methodology with high confidence for analytical calculation of energy-power indicators of the rock cutting process.

5. References

[1] Pershin G D, Karaulov N G, Ulyakov M S 2015 Selection of high-strength dimension stone cutting method, considering natural jointing Journal of Mining Science 1 pp 111-121
[2] Genkel A V, Grishin I A, Burmistrov K V, Velikanov V S 2015 Improving the efficiency of crushed stone production and methods of applying crushing screenings Mining 6(124) P 64
[3] Ahmed Barakat, Mohamed El Baghdadi, Jamila Rais A 2015 GIS-Based Inventory of Ornamental Stone and Aggregate Operations in the Beni-Mellal Region (Morocco) Arabian Journal for Science and Engineering Vol 40 Issue 7 pp 2021–2031
[4] Gavrishev S E, Burmistrov K V, Zalyadnov V Yu, Mikhailova G V 2018 Substantiation of exposure flowsheets and mining direction in reconstruction of building stone quarries Gornyi Zhurnal 1 pp 27-32
[5] Cheban A Yu, Khrunina N P, Yakimenko D V 2018 Improving the complex of mining transport equipment for working together with layer-by-layer milling machines Bulletin of Magnitogorsk state technical University named after G. I. Nosov Vol 16 3 p 40-45
[6] Protasov Yu I 2002 Destruction of rocks (Moscow: MGGU) p 453
[7] Pershin G D, Pshenichnaya E G, Yelzov A N 2015 Energy indicators of the process of surface destruction of rocks with cutting tool Mining, processing and application of natural stone: Collection of scientific papers-(Magnitogorsk: MGTU) pp 188-203
[8] Ter-Azavre I A 1959 Process dynamics of stone cutting (Yerevan: Armugpedgiz) p 105
[9] Polskoy A V, Sekretoy V M 2011 Effective spatial shape of teeth of the working tool impact boring machine Scientific Bulletin of the Moscow state mining University 2 pp 50-62
[10] Yasar S, Yilmaz A O 2018 Drag pick cutting tests: A comparison between experimental and theoretical results *Journal of Rock Mechanics and Geotechnical Engineering* Vol 10 Issue 5 pp 893-906

[11] Pershin G D, Karaulov N G, Kosarev L V 2008 Productivity and energy of the process of cutting natural stone of medium strength by working bodies equipped with carbide cutters In the collection: Extraction, processing and application of natural stone. Edited by Pershin G D Magnitogorsk pp 201-211

[12] Alimov O D, Dvornikov L T 1976 *Drilling machines* (Moscow: Engineering) p 295

[13] Liu W, Zhu X, & Jing J 2018 The analysis of ductile-brittle failure mode transition in rock cutting *Journal of Petroleum Science and Engineering* 163 pp 311–319

[14] Vtorushin E V, Dorovsky V N 2019 Application of non–stationary non–Euclidean model of inelastic deformations to rock cutting *Journal of Petroleum Science and Engineering* pp 508-517

[15] Jiajun W, Deyong Z, Renqing H 2014 Experimental study on force of PDC cutter breaking rock *Procedia Eng* 73 pp 258–263