A Curvature Principle for the interaction between universes

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Abstract

We propose a Curvature Principle to describe the dynamics of interacting universes in a multi-universe scenario and show, in the context of a simplified model, how interaction drives the cosmological constant of one of the universes toward a vanishingly small value. We also conjecture on how the proposed Curvature Principle suggests a solution for the entropy paradox of a universe where the cosmological constant vanishes.

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1 Introduction

The fundamental underlying principle of Einstein’s theory of general relativity is the connection between curvature and matter-energy. This relationship, as established by Einstein’s field equations, is consistent with all experimental evidence to considerable accuracy (see e.g. [1, 2] for reviews), however, there are a number of reasons, theoretical and experimental, to question the general theory of relativity as the ultimate description of gravity.

From the theoretical side, difficulties arise from the strong gravitational field regime, associated to the existence of spacetime singularities, and the cosmological constant problem. Quantization of gravity is likely to bring relevant insights to overcome these problems, however, despite the success of quantum gauge field theories in describing the electromagnetic, weak, and strong interactions, the recipes they suggest to describe gravity at the quantum level are not sufficient to achieve a fully consistent formulation. At a more fundamental level, one can say that, the two cornerstones of modern physics, quantum mechanics and general relativity, are not compatible with each other.

On the experimental front, recent cosmological observations leads one to conclude that the standard Big Bang scenario of the origin and evolution of the universe requires the introduction of “invisible” fields, as most of the energy content of the Universe seems to be composed of presently unknown components, dark matter and dark energy, which permeate much, if not all spacetime. Nevertheless, general relativity allows for quite detailed predictions, for instance, of nucleosynthesis yields and the properties of the Microwave Background Radiation, and hence one can use the theory to establish the specific properties of the missing links. In fact, it is widely believed that one has to admit new fundamental scalar fields to achieve a fully consistent picture of universe’s evolution. Indeed, scalar fields are required to obtain a successful period of inflation (see e.g. [3] for a review), to account for the late accelerated expansion of universe, either through, for instance, a quintessence scalar field (see e.g. [4] for a review) or via the Chaplygin gas model [5], and in the case of some candidates for dark matter, either self-interacting [6] or not [7].

Furthermore, given that Einstein’s theory does not provide the most general way to establish the spacetime metric, it is natural to consider additional fields, especially scalar fields. Of particular relevance, are the scalar-tensor theories of gravity as they mimic a great number of unification models. The graviton-dilaton system in string/M-theory can, for instance, can be seen as an specific scalar-tensor theory of gravity. For an updated discussion of the implications for these theories of the latest high-resolution measurements of the PPN parameters $\beta$ and $\gamma$, see e.g. [8] and references therein.

However, likewise general relativity, none of its extensions seem to warrant a fully consistent description of our universe given the huge discrepancy between the observed value of the cosmological constant and the one arising from the Standard Model. Many solutions have been proposed to tackle this major difficulty (see e.g. [9]) and it has been remarked that it should admit a solution along the lines of the strong CP problem have [10], which might be implemented in the context of a S-modular invariant $N = 1$ supergravity quantum cosmological model in a closed homogeneous and isotropic spacetime [11]. Nevertheless, none of the mechanisms pro-
posed to solve the problem are quite consistent (see e.g. Refs. [12] for recent reviews). Actually, even in the context of string theory, the most studied quantum gravity approach, no satisfactory solution has ever been advanced [13], even though more recently, it has been argued that a solution arises if the “landscape” of vacua of the theory is interpreted as a multi-universe (see e.g. [14] and references therein). In this approach, each vacuum configuration in the multitude of about $10^{500}$ vacua of the theory [15] is regarded as a distinct universe, from which follows that some criteria is required for the selection of the suitable choice for the vacuum of our universe. Anthropic arguments [16] and quantum cosmological considerations [17] have been suggested for this vacuum selection, and hence, as a meta-theory of initial conditions. These proposals are a relevant contribution to a better understanding of the problem, although may not be the last word as it should be kept in mind that a non-perturbative formulation of string theory is largely unknown [18].

In this work we propose a new mechanism for achieving a vacuum with a vanishingly small cosmological constant. It is based on the assumption that, likewise the dynamics of matter in the physical spacetime, vacua dynamics and evolution should emerge from a Curvature Principle that sets the way how different components of a multi-universe interact. Actually, the interaction between different universes has already been suggested as a possible way to obtain a vanishing cosmological constant [19]. In quantum cosmology, in some attempts to solve the cosmological constant problem, a “third quantization” has been suggested where universes could be created and destroyed through quantum transitions [20]. Our approach follows the same logics, but it assumes that the relevant quantities to consider are the curvature invariants of each universe of the multi-universe network. It is suggested that these invariants evolve in a “meta cosmological time” scale, so to relax the curvature of one universe and place it into another. This is the core of the proposed curvature principle.

2 The Model

Let us consider universes whose spaces that are globally hyperbolic and satisfy the weak and strong energy conditions. Furthermore, we assume, for simplicity, that the topology of the components of the multi-universe is trivial and that the overall geometrical characterization of each universe, labeled with the index, $i$, is fully specified by the curvature invariant $I_i = R^i_{\mu\nu\lambda\sigma} R^{\mu\nu\lambda\sigma}$, where $R_{\mu\nu\lambda\sigma}$ is the Riemann tensor of each universe. This invariant stands out in comparison to other known curvature invariants as it is not a total derivative in 4 dimensions, as is the case of Euler densities, and it is sensitive to the presence of singularities and of a non-vanishing the vacuum energy. For sure, the dynamics of each universe is described by its Einstein equation, but its vacuum also depends on the interaction with another universes through the Curvature Principle that is suggested as follows. Since we are concerned with the vacuum of each universe, which is supposed to be homogeneous, isotropic and Lorentz invariant\(^1\), then:

\(^1\)The connection between the cosmological constant and Lorentz invariance has been discussed in different contexts in Refs. [21, 22].
\[
R^i_{\mu\nu\lambda\sigma} = k_i [g^i_{\mu\lambda} g^i_{\nu\sigma} - g^i_{\mu\sigma} g^i_{\nu\lambda}],
\]
(1)

for a constant \( k_i \), which correspond to de Sitter (dS), anti-de Sitter (AdS) or Minkowski spaces whether \( k_i < 0, k_i > 0 \) or \( k_i = 0 \). From the vacuum Einstein equation with a cosmological constant, \( \Lambda_i \), it follows that \( \Lambda_i = 3k_i(1 - N/2) \), where \( N \) is the number of spacetime dimensions. Moreover, it is clear that the curvature invariant is proportional to the square of the cosmological constant. For sure, the chosen curvature invariant is unsuitable to distinguish between AdS and dS spaces; however, this is not of particular relevance for our discussion as we will be primarily concerned with dS spaces. Notice that dS and AdS spaces are related by analytic continuation and that invariance under complex transformations has been proposed a possible way to solve the cosmological constant problem [23].

Let us now propose a scheme for the evolution of the curvature invariants in a “meta cosmic time”, \( T \), a time that is related to the dynamics of interacting universes. The relation between this time and the usual cosmic time will be discussed in a while. Clearly, one must endow the vacuum of each universe with a dynamics. For simplicity, let us consider only two universes and assume that their evolution is determined by the “Lagrangian” function:

\[
L = \frac{1}{2} \left( \frac{dI_1}{dT} \right)^2 + \frac{1}{2} \left( \frac{dI_2}{dT} \right)^2 - V(I_1, I_2),
\]
(2)

where \( V(I_1, I_2) \) is a “potential” function. Of course, the construction of the potential function is at the very heart of the proposed mechanism. Clearly, what is needed are well defined minima for the curvature invariants and an interaction term. A fairly generic possibility is the following:

\[
V(I_1, I_2) = \alpha_1 I_1 + \beta_1 (I_1 - I_1^{(0)})^2 + \alpha_2 I_2 + \beta_2 (I_2 - I_2^{(0)})^2 - \gamma I_1 I_2,
\]
(3)

where \( I_1^{(0)} \) and \( I_2^{(0)} \) correspond to the minimal values of the curvature invariants for universes 1 and 2, respectively. All coefficients of the potential are positive and \( V(I_1, I_2) \geq 0 \). One can easily see that if \( \alpha_1 = \alpha_2 = 0 \), then \( I_1^{(0)} = I_2^{(0)} = 0 \). A more interesting possibility arises when, say \( \alpha_2 = 0 \), but \( \alpha_1 \neq 0 \) as in this case \( I_1 = I_1^{(0)} = 0 \), however \( I_2 = I_2^{(0)} \neq 0 \), that is to say that the interaction between the two universes drives the curvature invariant of universe 1 toward a vanishing cosmological constant, while toward a non-vanishing value for the universe 2. Notice that the condition of minima requires that \( 4\beta_1\beta_2 > \gamma^2 \).

It is easy to see that from the “integral of motion”

\[
E = H = \frac{1}{2} \left( \frac{dI_1}{dT} \right)^2 + \frac{1}{2} \left( \frac{dI_2}{dT} \right)^2 + V(I_1, I_2),
\]
(4)

that \( E = 0 \) and that one can obtain a suitable Lyapunov function, \( Ly = -H \), from which one can show that the minimum for the case where \( \alpha_2 = 0 \), but \( \alpha_1 \neq 0 \) are attractors of the autonomous dynamical system associated to the motion of \( I_1 \) and \( I_2 \).

Of course, the solutions of the equations of motion, \( I_1 = I_1(T) \) and \( I_2 = I_2(T) \), correspond to extrema of the “action” that can be constructed from the “Lagrangian” function, Eq. (3).
However, this is not sufficient to fix the values of the curvature invariants. This is done thanks to a suitable potential. Our choice seems plausible, but it is clearly an ad hoc one. At the present stage of our knowledge on can only conjecture whether the suggested Curvature Principle can be accommodated within the framework of a fundamental quantum gravity proposal.

Let us now discuss the typical time of change of the curvature invariants. Even though the cosmological constant problem is an ubiquitous problem it arises more acutely during the cosmological phase transitions when the relevant effective potential changes from a situation where the order parameter vanishes to a situation where it is non-vanishing, generating in the process, a large cosmological constant. Hence, the typical time scale of change of the curvature invariants must be of order of the characteristic time of change of the cosmological phase transitions order parameter, that is to say that it is typically a microscopic time interval. Furthermore, given that one aims to set the overall geometrical features of each universe via the change of the curvature invariants, then it must not differ significantly of the Hubble characteristic time of each universe at the transition, that is:

$$T_i \equiv \left( \frac{1}{I_i} \frac{dI_i}{dT} \right)^{-1} \lesssim H_i^{-1}. \quad (5)$$

That is to say that while a phase transition takes place, interaction between different universes change so to cancel the curvature invariant associated with the vacuum of one of the universes. It is conceivable that the vanishing of the cosmological constant of a given universe after multiple phase transitions might require considering and modeling the interaction among various “nearby” universes.

A general point that one can make from the proposed mechanism is that according to Eq. (5), it is likely that the observed accelerated expansion of our universe is not due to some residual cosmological constant. Even though cosmological data do not exclude this possibility, supernovae data, baryon acoustic oscillations, microwave background radiation shift parameter and topological considerations are consistent with alternative sources for the accelerated expansion rather than the cosmological constant [24].

3 Discussion and Outlook

The cosmological constant problem challenges our knowledge about the vacuum of the theories that we regard as fundamental. It has also been shown to resist all attempts of a solution that rely on a single universe framework. Given, that a multi-universe complex has been recently discussed, most particularly, in the framework of the vacua landscape of string theory, it is natural to ask whether these universes might interact. On the other hand, it is clear that a vacua theory, i.e. the non-perturbative formulation of the fundamental quantum gravity theory, is needed to fully understand the cosmological constant problem and, it is then just logical that an important ingredient of this formulation involves the interaction between different universes.

In this work we have proposed a scheme involving the interaction of different universes
through their curvature invariants. The interaction is such that at vacuum it can drive one of the invariants to vanish. The main ingredient of the proposal is the interaction between different universes. This is the main difference from other schemes that constrain curvature invariants and metric related functions. Indeed, in the unimodular gravity proposal, for instance, the determinantal of the metric is non-dynamical and the cosmological constant is shown to be an integration constant [25, 26, 27]; in the limiting curvature proposal, the value of the curvature invariants are bound from above so to avoid singularities [28, 29]. Another, curvature-type principle arises in the context of the field theory of closed strings, where minimal area metrics are proposed to solve the problem of generating all Riemann surfaces [30]. We suggest that this interaction can be modeled via the curvature invariant of each universe depicted by the square of the Riemann tensor, which is sensitive the vacuum state and is determined in each universe by the Einstein equation. If a Curvature Principle like the one suggested here could bring some insight on the vacua properties, it would be a transcendental vindication of Einstein’s genius. For sure, if this type of principle can arise in the the context of some fundamental quantum gravity theory, it would be an important validation. On the other hand, it is conceivable that a theory of initial conditions and interactions between different universes lie beyond the realm of the fundamental theory and, if so, the cosmological constant might be the only guidance available to unravel the ultimate nature of our world.

Before drawing this work to an end, let us point out that a possible implication of the proposed Curvature Principle concerns the entropy paradox of our universe. Indeed, the fact that our universe seems to have emerged from a singular state suggests that its initial entropy is much larger than the one that can be accounted at the present. Penrose had suggested that the problem could be understood through the assignment of entropy to the gravitational field through a curvature invariant, the square of the Weyl tensor [31]. We propose, instead, that one should consider the curvature invariant \( I \) we have been discussing. In fact, from another universe point of view, our universe can be regarded as a Schwarzschild black hole with its mass all concentrated in some point and, hence \( I = 48 M^2 r^{-6} \), where \( r \) is the horizon’s radius and \( M \) its mass. We have used units where \( G = \hbar = c = 1 \). Therefore, if the entropy scales with the volume, then \( S \sim r^3 \sim I^{-1/2} \); if the entropy scales according to the holographic principle, suitable for AdS spaces [32, 33], then \( S \sim r^2 \sim I^{-1/3} \). In either case, one finds that \( S \to 0 \) in the early universe and, \( S \to \infty \) when \( \Lambda \to 0 \). As discussed, the latter corresponds to the universe at late time, which is consistent with the generalized second principle of thermodynamics for our universe.

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