Quantum polariton trigger

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Abstract. We present a device model of the polariton logic element that is able to work both with classical and quantum signals. This device uses the nonlinear and quantum properties of exciton-polaritons. The mathematical description of our model is based on stochastic quantum equations with a complex diffusion in the Ito formalism with the corresponding quasi-probability in the quantum phase space. It is shown that the stationary states in the bistable mode can be controlled by both classical and quantum signals through the stochastic resonance within the region of quantum phase transition. Based on these properties, the model of the quantum logic elements has been developed.

1. Introduction
Exciton-polaritons are mixed states consisting of the photon and exciton parts [1]. Exciton-polariton designs are preferable over photonic structures because an exciton-polariton device combines optic and semiconductor physical properties; moreover, since a phonon energy is less than a polariton energy, the thermal noise is considerably smaller in comparison with optical systems [1]. The exciton-polariton system has cubic Kerr-like nonlinearities [1]; in the stationary condition, this system may show bistability, with several stable stationary solutions for one control parameter (for example, pump intensity) [2].

A bistable system may display the effect of stochastic resonance. If the external signal is powerful i.e. the signal amplitude is large enough to overcome the potential barrier (in the phase space), it can transfer the system from one stable state to another. Accordingly, a weak signal with the amplitude below the potential barrier cannot transfer the system from one mode to another. However, if there is noise in the system, at a certain signal frequency and noise power, a jump from one state to another becomes possible even for a weak signal – this effect is called the stochastic resonance (SR) [3].

The stochastic resonance is an interesting phenomenon enhancing the response of a nonlinear system due to additional noise perturbations. A lot of researches have reported about observations of the stochastic resonance in optical bistability states in laser systems [4], passive optical cavities [5], atomic systems [6], Shmitt-Trigger circuits [7] and confined 0D exciton-polaritons [8]. The paper [8]
investigates the impact of classical noise on the exciton-polariton bistable mode. However, the experimental research [9] shows what the bistable response of the 0D exciton-polariton structure has the narrow width of the bistable loops, known as the effect of constricted hysteresis loops. In the presence of quantum noise, the quantum stochastic resonance may be observed. The quantum stochastic resonance is an enhancement of a weak signal due to the quantum noise of the system [10]; the gain threshold determined by the quantum properties of the system is much lower than the classical threshold.

The first theoretical prediction and explanation of the bistable loops constriction is given in the article [11]. The constrictions exhibit the quantum influence on the bistability and prove the key role of quantum noise in the system. The effect of bistability makes the performance of classical polariton logical elements difficult.

The work [12] offers the NOT and control-AND logical element based on the bistability effect of polariton system. There are always noises in the system, including quantum ones; as a result, the hysteresis loops become constricted [13]. In this case, the curve of the first quantum moment respective to the photon number divides the semiclassical bistability loop into two parts [11, 13].

We suggest using this negative effect for our benefit. In this article, we describe the quantum polariton trigger based on the photon bistable system, switched by the quantum SR, with the external quantum noise as a switch. Our analysis is based on the results obtained in our work [11].

2. Methods

2.1. Model

The exciton polaritons are created in quantum wells (or quantum dots) embedded in the antinode microcavity region, see figure 1. In the microcavity, the photons continuously excite excitons in a quantum well. In the tight mode, when the Rabi frequency $\omega_k$ is greater than the system loss $\omega_k \gg \gamma_{ph,ex}$, a new quasiparticle, an exciton-polariton, appears due to the anti-crossing effect [1]. Since the system is not closed to maintain the stationary mode, an additional energy is pumped, for example, using a laser (the laser pumping).

We consider the system consisting of a 0D semiconductor pillar microcavity [14] with an embedded quantum well, see figure 1 excited by a coherent laser field $E = E_0 e^{i\omega t}$ with the frequency value close both to the photon and the exciton resonances of the structure. The coherent field $E$ is spatially homogeneous and intense.

A pillar microcavity is a one-dimensional structure with a quantum well embedded in the antinode region of the light wave. Due to the one-dimensionality, the light cone is rather narrow; therefore, one-dimensional excitons with the wave vector $k = 0$ are excited in the quantum well [1, 14].

The polariton Hamiltonian in terms of the second quantization [1]:

$$\hat{H}_{pol} = \hbar \omega_{ph} \hat{\phi}^* \hat{\phi} + \hbar \omega_\chi \hat{\chi}^* \hat{\chi} + \hbar \omega_k \left( \hat{\chi}^* \hat{\phi} + \hat{\phi}^* \hat{\chi} \right),$$

where $\hat{\phi} (\hat{\phi}^*)$ and $\hat{\chi} (\hat{\chi}^*)$ annihilates (creates) a photon and an exciton respectively, under the coherent field [1].
\[ \hat{H}_{\text{pump}} = i\hbar \left( F e^{i\omega_d t} \hat{\phi}_s + h.c. \right), \]  

where \( F \) is the electric field strength with the laser pump frequency \( \omega_d \).

**Figure 1.** A pillar microcavity design of the 0D polariton formed in the structure.

In the exciton gas, the collisions of excitons can be either elastic or not. These interactions between the particles of the exciton gas cause nonlinearity of the exciton mode. In turn, it affects polaritons: when generated at the laser pump frequency, they are scattered into the signal mode \((k = 0)\) and the idle mode emitted at an angle to the microcavity plane. Such a nonlinear parametric scattering occurs above a certain pump threshold and is called the optical parametric oscillator (OPO) mode. The nonlinear susceptibility of this parametric process is \( \chi^{(3)} \) [1, 2].

In the pillar microcavity, the photon mode interacts with the \( k=0 \) modes only, due to the waveguide effect; thus, we can write the following nonlinear Hamiltonian [1, 2]

\[ H_{NL} = h\omega \hat{\chi}^+ \hat{\chi}, \]  

where \( \chi = V_0 / 2 \) – is the nonlinear parameter (sometimes, called the Kerr nonlinearity); \( V_0 = \frac{6e^2 a_{\text{exc}}}{\varepsilon A} \) – is the Coulomb interaction coefficient that determines the potential of the exciton repulsing; \( a_{\text{exc}} \) – is the exciton radius; \( A \) – is the effective area of exciton interaction.

The total Hamiltonian of the system is

\[ H_{\text{total}} = H_{\text{pol}} + H_{\text{pump}} + H_{NL}. \]  

In the rotating wave approximation (RWA) the Hamiltonian (4) becomes
\[
\hat{H}_c = \hbar \Delta_{\text{ph}} \hat{\phi}^* \hat{\phi} - \hbar \Delta_{\text{ex}} \hat{\chi}^* \hat{\chi} + \hbar \omega_R \left( \hat{\chi}^* \hat{\phi} + \hat{\phi}^* \hat{\chi} \right) + \hbar \alpha \hat{\chi}^2 \hat{\chi}^2 + i \hbar \Gamma \left( \hat{\phi}^\dagger - \hat{\phi} \right). \tag{5}
\]

In the Hamiltonian (5), the following notation is used: \( \Delta_{\text{ph}} = (\Delta - \Omega) = \omega_{\text{ph}} - \omega_d \) — the detuning between the photon mode frequency and the pump frequency, \( \Delta_{\text{ex}} = (\Delta + \Omega) = \omega_{\text{ex}} - \omega_d \) — the detuning between the exciton mode frequency and the pump frequency, \( \Delta = (\omega_{\text{ph}} - \omega_{\text{ex}}) / 2 \) — a half of the exciton-photon detuning, \( \Omega = \omega_d - (\omega_{\text{ph}} + \omega_{\text{ex}}) / 2 \) — the detuning between the pump frequency and the average value of the frequencies of the exciton-photon system eigenstates (the laser detuning).

The exciton-polariton system is an open quantum system; thus, according to the fluctuation-dissipative theorem, noises occur there. Taking into account this fact, the Hamiltonian (1) should contain the terms \( \Gamma_{\text{ph}} \hat{\phi}^\dagger \hat{\phi} \) and \( \Gamma_{\text{ex}} \hat{\chi}^\dagger \hat{\chi} \), responsible for the dissipation of the exciton and resonator modes due to interaction with the corresponding reservoirs of external modes. When writing the equation for the density matrix \( \rho \), this approach allows describing the dissipative processes in the Markov terms. Basically, these terms can be omitted without consequences to further presentation. Then, the occurrence of relaxation terms in the master equation (6) can be explained by the fact that they are written in the generally accepted form of the Lindlab superoperators [15, 16].

Assuming that the micropillar temperature of the GaAs-based systems used in the experiments is several Kelvin, the thermal noise contribution is rather insignificant in comparison with the polariton energy. In fact, the noise power is proportional to the reservoir thermal population, which decreases exponentially with the ratio \( \omega_{\text{ph,ex}} / k_B T \) increasing. Since \( \hbar \omega_{\text{ph,ex}} \gg k_B T \), the thermal noise contribution can be neglected. In this work, we also neglect the noises created by the devices themselves, for example, the pump intensity fluctuations [17]. Thus, the only noise source taken into account in the model is the quantum noise, which is expected to make a significant contribution to the polariton system (due to the nonlinearity of the exciton mode).

Therefore, neglecting the thermal noise, the kinetic equation for the density matrix in the Markov and Born approximations is [15, 16]

\[
\frac{\partial \rho}{\partial t} = \frac{1}{\hbar} \left[ \hat{H}_c, \rho \right] + \gamma_{\text{ph}} \left( 2 \hat{\phi} \rho \hat{\phi}^\dagger - \rho \hat{\phi}^\dagger \hat{\phi} - \hat{\phi} \hat{\phi}^\dagger \rho \right) + \gamma_{\text{ex}} \left( 2 \hat{\chi} \rho \hat{\chi}^\dagger - \rho \hat{\chi}^\dagger \hat{\chi} - \hat{\chi} \hat{\chi}^\dagger \rho \right),
\tag{6}
\]

where \( \gamma_{\text{ph}} \) and \( \gamma_{\text{ex}} \) are dissipations of the photon and exciton modes respectively.

Further, the following parameters of the system are used for calculations [18]: \( \hbar \omega_R = 0.001 \text{ meV} \), \( \hbar \omega_K = 2.5 \text{ meV} \), \( \gamma_{\text{ex}} = 0.01 \text{ ps}^{-1} \) and \( \gamma_{\text{ph}} = 0.1 \text{ ps}^{-1} \).

### 2.2 Governing equations

In works [12, 19] we conducted a quantum investigation (namely, quantum average and quantum statistics of excitons and photons in a pillar microcavity) for the Hamiltonian (1) using generalized P-representation [20] in the adiabatic limit [20].
We can write the SDEs for the Hamiltonian (1) in the Ito form corresponding to the Fokker-Planck equation (9) from article [11] according to the quantum phase space formalism [11, 21, 22]

\[
\frac{d\mathbf{x}}{dt} = -A\mathbf{x} + \mathbf{F}(\mathbf{x})\tilde{\eta}(t),
\]

(7)

where \( \mathbf{x} = \left( \phi, \phi^*, \chi, \chi^* \right)^T \), \( \mathbf{F} = \left( \mathbf{F}_p, \mathbf{F}_p^*, 0, 0 \right)^T \), and \( \tilde{\eta}(t) = \left( \xi_1, \xi_1^*, \xi_2, \xi_2^* \right)^T \).

Drift matrix is

\[
A[\mathbf{x}] = 
\begin{pmatrix}
(i\Delta_{ph} + \gamma_{ph}) & 0 & i\omega_R & 0 \\
0 & (-i\Delta_{ph} + \gamma_{ph}) & 0 & -i\omega_R \\
i\omega_R & 0 & (-i\Delta + \gamma_{\alpha}) + 2i\chi^*\chi & 0 \\
0 & -i\omega_R & 0 & (i\Delta + \gamma_{\alpha}) - 2i\chi^*\chi
\end{pmatrix}
\]

Diffusion matrix is

\[
D[\mathbf{x}] = 
\begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & -2i\alpha\chi^2 & 0 \\
0 & 0 & 0 & 2i\alpha\chi^2
\end{pmatrix}
\]

Here \( \phi \) and \( \chi \) are c-numbers (i.e. eigenvalues) of the operators \( \hat{\phi}, \hat{\chi} \) respectively; \( \xi(t) \) – the independent Gaussian stochastic function that meets the following correlations expressions: \( \langle \xi(t) \rangle = 0, \langle \xi^*(t) \rangle = 0 \), and \( \langle \xi(t) \xi^*(t') \rangle = \delta(t-t') \).

The stochastic differential equations describe the quantum behavior of photons and excitons in the quantum phase space with multiplicative diffusion.

Note that these equations are not everywhere adequate for the system description, as Drummond wrote. In general case \( \chi^* \neq \chi \) and SDE (7) in the Ito form are wrong; in more details, it is discussed in work [20]. This fact is taken into account when developing the general P representation [20, 23] or positive P-function. We consider the region of parameters with the bistability present. It has been shown in [11] that this region is well described by the stochastic quantum equations and the results coincide with the calculations through quasiprobability [20, 23] (for example, P-function or Wigner function).

2.3. Quantum stochastic jumps

In case of bistability in polariton systems, quantum phase transitions are observed with characteristic quantum statistical features [11, 19] (for example, a pronounced peak of the second moment of the correlation function). It is proved, that the presence of quantum noise causes metastability of a homogeneous bistable solution. In this case, there are jumps [24] between the system states from one stationary solution to another (provided that such a state exists in the bistability region) with the same
order (pump) parameter. We have found the parameters of a system where both types of bifurcations are observed: a rigid type (a sharp transition to the opposite state, with the system remaining there), and a soft type (continuous jumps between states). In this case, there is a characteristic threshold of the semiclassical and quantum behavior of the system (without hysteresis). These areas are metastable, that is, there is a possibility of gigantic fluctuations that can push the system out of the attraction basin of stable points to another stationary point. This is similar to tunneling between potential wells of a bistable potential. The exact moment of a jump depends on the protogator parameter calculated in the analogue work [4] that plays the role of tunneling time between branches. Physically, the system must remain in this state longer than this period of time in order to detect a jump. In works [9, 19], this effect is observed as the hysteresis loop constrictions.

The SDEs (7) were numerically solved by the Ito integration method [20] using the Euler scheme [25] in MATLAB. To find the first moment of the photons number, we used numerical integration over the simulation time T,

\[ \phi^* \phi = \int_0^T \phi^*(t) \phi(t) dt , \]

where the angle brackets \(< >\) indicate the time averaging.

Figure 2a and figure 2c show the first moment of the photons number (4) (the result of the SDEs (7) numerical simulation) in comparison with the semiclassical solution (bistability loop) for different detuning parameters. The region of the pump scale, where the photons number in the quantum limit (the dashed curve) is very different from the semiclassical solution, can be very large, as shown in figure 2a; in this region, there are jumps in the photons number, see figure 2b, from the region in the lower branch of the bistability loop to the value in the upper branch of the bistability loop (from the region indicated by the double arrow in figure 2a). With the pump increasing, the curve corresponding to the average value of the photons number (8) approaches the upper branch of the bistability loop.

In the case shown in figure 2c, a sharp transition to the upper branch of the bistable curve is observed; in this case, a rigid bifurcation occurs and the system “gets stuck” in the upper state. The numerical simulation in figure 2d clearly demonstrates this behavior.

Up to the boundary of the quantum jump (the quantum threshold) of the average number of particles, the system is stable with some stationary noise. The quantum statistics is described by the Poisson distribution with the average value equal to the semiclassical average. In the region of the quantum threshold, the switching between branches is observed; we describe it in details below, using two Poisson distributions. Beyond the quantum threshold, the lower bistability curve is metastable, i.e. has a finite lifetime; in a while, the system goes to the upper branch of the bistability loop, which is described by the Poisson distribution.
Figure 2. (a, c) The blue line shows the bistability of the photons population, the green dashed line corresponds to the quantum averaged photons population, calculated by the computing simulation of the SDEs (7) for the quantum case – panel (a) with the detuning parameters are $\Delta = 3.1 \text{ ps}^{-1}$ and $\Omega = -4.85 \text{ ps}^{-1}$ and the quasiadiabatic case – panel (c) with the detuning parameters are $\Delta = 3.1 \text{ ps}^{-1}$ and $\Omega = -4.81 \text{ ps}^{-1}$; (b) The right panel shows the quantum photons population jumping between two stability states of classical bistability (the red double-arrow in the panel (a)) calculated by the computing simulation of the SDEs (7); (d) The right panel shows the simulated of the stochastic jumps from the lower branch to the upper branch in the bistability loop, in the region indicated by the red arrow in the panel (c).
2.4. Stochastic resonance

2.4.1. Quantum stochastic resonance. Let us consider solutions of the SDEs (7) under the quantum threshold (the green curve in figure 2a, c) in the bistable region with the initial conditions that satisfy the stationary mode in the lower branch of the bistability loop; as known, the system will remain in a stable state infinitely long (as long as this mode is maintained). However, if we modulate the pumping with a signal whose amplitude is much smaller than the pumping itself, but sufficient to overcome the quantum threshold, the system will jump to the upper branch of the bistability loop under the influence of the quantum noise, if the pump frequency is less than the reciprocal of the lower metastable state lifetime. It should be noted, that this noise is a manifestation of the nonlinear and quantum features of the system and cannot be eliminated in principle. Schematically, the process is shown in figure 4a.

We add the external coherent resonant signal to the SDEs (7)
\[
\frac{dx}{dt} = -Ax + F + F_s \cdot D(x) \eta(t),
\]
where \( F_s = (F_s^+, 0, 0)^T \) is the signal amplitude.

Figure 3a shows the signal to noise ratio (SNR) for the quasiadiabatic and quantum limit (figure 2a and figure 2c) cases. In our case, the SNR is the ratio between the output signal and the quantum noise. We can see that for some signal pumps the SNR increases, which demonstrates the stochastic resonance effect. In this region, the external signal pump adds to the background pump and reaches the quantum threshold (see figure 2a and 2c). In the quantum limit, we can see a peak of the SNR vs. some values of the signal intensity, where quantum jumps occur.

**Figure 3.** (a) Signal to noise ratio vs. dimensionless pump in the quasiadiabatic and quantum limit corresponding to figure 2a and figure 2c respectively; (b) The uncertainty region for coherent state \( \alpha \) (the blue circle) and the squeezed state.
In the quasiadiabatic limit, we can see a sharp jump of the SNR value, while the SNR remains mostly the same when the signal value increases. This demonstrates the quantum stochastic resonance due to a rigid bifurcation, an asymmetric potential wells in the quantum phase space. The pulsation of the permanent pump causes the quantum stochastic resonance and the switching in bistability.

2.4.2. External quantum noise. In the second case, we apply squeezed light [26] onto the system, see figure 4b. A source of non-classical squeezed light, for example, may be a source of parametric down-conversion [27] (second order nonlinearity $\chi^{(2)}$). Squeezed light is a light field with the noise below the vacuum level.

Consider the electric field $E$, written through the creation and annihilation operators [28]

$$E = \frac{1}{2} \zeta \left( \hat{a} + \hat{a}^\dagger \right), \quad (10)$$

where we introduced the notation $\zeta = 2^{3/2} \frac{h \omega_p}{\sqrt{2e_0 j'}}$.

Figure 3b shows the quadratures (the real and imaginary parts) of the electric field, the dispersion of the electric field of coherent and squeezed light.

The dispersion of squeezed light quadratures creates an ellipsoid body of uncertainty (see figure 3b), while the vacuum or coherent state creates a sphere of uncertainty.
Mathematically, the squeeze is described by the following statement [26, 28]

\[ S(r, \theta) = \exp \left( \frac{1}{2} \xi^2 \delta^2 - \frac{1}{2} \xi \delta^2 \right), \]  

(11)

where \( \xi = re^{i\theta} \) – is the complex squeeze parameter.

Applying the Bogoliubov transformation to the operators of creation and annihilation, we obtain [28]

\[ \hat{a} \rightarrow S^+(r, \theta) \hat{a} S(r, \theta) = \cosh(r) \hat{a} - e^{i\alpha} \sinh(r) \hat{a}^*, \]  

(12)

\[ \hat{a}^* \rightarrow S^+(r, \theta) \hat{a}^* S(r, \theta) = \cosh(r) \hat{a}^* - e^{-i\alpha} \sinh(r) \hat{a}. \]  

(13)

Thus, the quadratures variances

\[ \Delta X^2 = \frac{1}{4} \left( e^{-2\theta} \cos \left( \frac{1}{2} \theta \right)^2 + e^{2\theta} \sin \left( \frac{1}{2} \theta \right)^2 \right), \]  

(14)

\[ \Delta Y^2 = \frac{1}{4} \left( e^{2\theta} \cos \left( \frac{1}{2} \theta \right)^2 + e^{-2\theta} \sin \left( \frac{1}{2} \theta \right)^2 \right). \]  

(15)

For \( \theta = 0 \), the quadratures variances are

\[ \langle \Delta X^2 \rangle = e^{-\gamma/2} / 4, \]  

(16)

\[ \langle \Delta Y^2 \rangle = e^{\gamma/2} / 4. \]  

(17)

On the other hand, it is possible to model the electric field with quantum fluctuations as a complex signal with complex diffusion [27] having a certain quasi-probability distribution (Wigner function, Husimi function or others) in a certain quantum phase space [27].

\[ F_{\text{total}} = F_p + F_s = \bar{F}_p + D_r \xi(t), \]  

(18)

where the noise power \( D \) is related to the squeeze parameter given by

\[ D_r = \langle \Delta X^2 \rangle, \]  

(19)

\[ D_i = \langle \Delta Y^2 \rangle. \]  

(20)

The diffusion in the SDEs (7) can be rewritten as

\[ \mathbf{D}'(x) = \mathbf{D}(x) + D_n(x), \]  

(21)
where \( \mathbf{D}_n = (D_n, D_n^*, 0, 0)^T \).

The squeezed signal (with a certain intensity \( I_s^0 \)) is fed to the input from a quantum parametric oscillator. The signal amplitude in the polariton system depends on the signal intensity at the microcavity input, as follows [1]:

\[
F_s = \frac{\gamma_c I_s^0}{2\hbar\omega_s},
\]

where \( \omega_s \) – is the laser frequency of the signal itself.

Note that the signal intensity is below the quantum threshold, i.e. for a coherent signal, no jump from the lower stable state to the upper state is observed; however, if we feed a squeezed signal to the input, then, at a certain squeeze, a transition to the upper state occurs (figure 5a). This is caused by the quantum noise of the squeezed signal that shifts the quantum threshold, in this case, reducing its value. It should be noted, that the switching effect is also observed in case of a squeezed vacuum (when \( I_s = 0 \)), see figure 5a (the red dash-dotted curve). Also, such a switch is observed if we fix the squeeze amplitude while varying the squeeze phase, see figure 5b.

![Figure 5](image)

**Figure 5.** (a) The quantum phase transition caused by an external squeezed signal; (b) The quantum phase transition caused by a squeezed vacuum (i.e. \( I_s = 0 \)) for \( r = 0.37 \).

3. Results and Discussion

Based on the principles described in sections 2.4.1 and 2.4.2, we have developed the following model of quantum logic elements.

The principle of operation of the polariton quantum logic NOT-switch:

The operating mode of the device is selected under the quantum threshold in the bistable quasiadiabatic limit in the lower branch of the bistability loop. A quantum squeezed signal is fed to the input, the control parameter is the complex squeeze value, see figure 5. Let the logic “1” be a phase
less than $\pi/4$ for a squeezed vacuum ($I_s = 0$), and the logic “0” be a phase less than $\pi/4$ or no signal, see figure 5b. For operation, the interrupt principle is selected, i.e. the system continuously sends a logic “0”, while the incoming squeezed pulses modulate the signal. If you select an operating point in the upper branch of the bistability loop, the logic NOT can be organized in the same way.

The principle of operation of the polariton quantum logic control-AND-switch.

1st scheme.

The operating mode of the device is selected in the region of the quantum threshold $I_{qth}$, in the quasi-adiabatic limit, see figure 2c. A weak coherent light beam (a classical signal) with the intensity much lower than the background pump, $I_s << I_p$, is fed to the device input. In this case, $I_s + I_p > I_{qth}$, therefore, for such a device to operate, the quasi-adiabatic mode with a narrow or closed Liouvillian gap is chosen [17], where the parameters of laser and exciton-photon detuning should be selected [11]. On arriving at the device, if the pump speed is less than the relaxation time of the upper metastable region (or the signal is constant), the signal modulates quantum jumps within the system based on the quantum stochastic resonance; if the relaxation time is longer, the system remains in the upper branch of the bistability loop.

2nd scheme.

The operating mode of the device is selected under the quantum threshold $I_{qth}$ or near the quantum threshold $I_s \approx I_{qth}$. A weak coherent light beam (a classical signal) with the intensity much lower than the background pump $I_s << I_p$ with the squeeze amplitude $r$ is fed to the device input. In this case, $I_s + I_p < I_{qth}$, the input signal is so weak that the system operates in the mode of the quantum stochastic resonance with the coherent signal. However, when the signal is squeezed, the quantum threshold is shifted and, at a certain value of the squeeze ratio despite the same value of the input signal, see figure 5a, the bistability loop switches to the upper branch. When the squeeze ratio becomes less than the threshold value, the system returns to the quantum stochastic resonance mode, or remains in the lower branch of the bistability loop, regardless of the waveform (the signal is lower than the threshold).

Based on the 2nd scheme, it is possible to operate a quantum polariton logic element working both with classical and quantum signals. Note that an operation mode should be chosen so that the thermal fluctuations do not interfere with the device work. With this purpose in mind, a pump slightly below the quantum threshold should be selected, so that only an external quantum signal is able to shift this threshold sufficiently.

The operating mode of the device is selected under the quantum threshold $I_s < I_{qth}$. The first input is a classical signal, the second control input is a quantum channel – a quantum squeezed signal (for example, a squeezed vacuum). For the signal at logic “0” $I_s < I_{sth}$ for any noise in the quantum channel, the system cannot overcome the potential barrier (i.e., the noise is not sufficient to overcome the threshold) and remains at logic “0”. For the signal at logic “1” $I_s > I_{sth}$, if the squeeze is less than the threshold $r < r_{th}$, the control channel is at logic "0", then the system remains in the stationary mode at logic "0". If the signal and squeeze ratio are sufficiently large, $I_s > I_{sth}$ and $r > r_{th}$, i.e. logic "1" is fed both to the input and the quantum channel, the system switches to logic "1", the stationary
state in the upper branch of the bistability loop. Although, this is a rigid type of bifurcation, if the input or control channel returns to state "0", the system also returns to its previous state (see table 1).

Table 1. Truth table of the control-AND logic element is.

| Input Signal | Quantum Noise, Control Channel | Output |
|--------------|--------------------------------|--------|
| 0            | 0                              | 0      |
| 1            | 0                              | 0      |
| 0            | 1                              | 0      |
| 1            | 1                              | 1      |

4. Conclusions

In this work, we studied the impact of a quantum signal on a bistable exciton-polariton system. The system of SDEs (7) was solved using numerical methods. Jumps in the photons number between the bistability loop branches caused by the quantum noise were obtained. The impact of quantum noise on bistability had been also studied in our previous works [9, 13].

The quantum stochastic resonance was discovered in a bistable polariton system with quantum noise. The principle of quantum polariton logic elements based on the quantum stochastic resonance and the stochastic resonance under the external quantum noise was presented. The advantage of this approach is the explicit consideration of noise in the system, as well as the ability to combine quantum and classical signals in the device.

In work, the models of quantum logic elements based on an exciton-polariton nonlinear system controlled by both classical and quantum external signals were developed. Those devices are stable to external noise and combine work within the frames of classical and quantum optics. They can be useful in quantum cryptography for data preprocessing.

Acknowledgements

The research was supported by the Ministry of Science and Higher Education of the Russian Federation under Agreement No. 075-15-2019-1838. The work was also supported by the RFBR within the framework of the scientific project No. 20-02-00515.

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