Research on Polar-region Coarse Alignment Algorithm Based on Grid System

Ming Tian1*
1Frontier Interdisciplinary Academy, National University of Defense and Technology, Hunan, 410000, China
*Corresponding author’s e-mail: 2913463551@qq.com

Abstract. Due to the convergence of meridians in polar regions, the geography-based northern-azimuth mechanics arrangement has problems such as increase of error and computational overflow in polar navigation, and the traditional compass alignment scheme is unable to operate normally. In this paper, the problem in polar navigation is solved by using the mechanics arrangement scheme, and a simulated analysis has been carried out on the application of compass alignment in high latitudes through static data, which verifies that it is unable to operate normally at the pole. Then compass alignment has been replaced by anti-sloshing coarse alignment and a simulated verification has been conducted on its correctness. After that, the latitude increment method is used to convert the sports car data at low-latitude regions to high-latitude regions and carry out a semi-physical simulation, which further verifies the correctness of the grid inertial navigation and the coarse alignment method in anti-sloshing polar region.

1. Introduction

With the development of military science and technology and the variability of the international political, economic, and military situation, all military powers are demanding more rapid response capability and global combat capability of fighters. It is requiring to shorten the flight routes as much as possible based on a higher speed, which requires that the aircraft should have global flight capability and, if necessary, be able to fly through the polar regions. For civil airliners, polar routes can be straight flight, shorten the range, reduce passenger fatigue during long-range flight, increase passenger capacity, save fuel, and reduce costs [1-3]. In addition to the demand to shorten the range, the strategic significance and economic research value such as the oil and gas resources reserves in polar regions, strategic “observation and control” status, and channel control status are also potential powers for flying over the polar regions.

Since the geographical meridians converge to the pole at high latitudes, the traditional inertial navigation mechanics arrangement of northern-azimuth platform has the problems such as computational overflow and error amplification in the high-latitude regions. The wander-azimuth inertial navigation system can complete the calculation of attitude direction cosine matrix and position direction cosine matrix, but there are singular values in the extraction of heading and position information from the direction cosine matrix [4]. Grid navigation with the Greenwich meridian as the heading reference is an effective means to solve the problem that there is no direction guideline for polar navigation [5]. Meanwhile, compared with non-polar regions, the meridian convergence in polar regions may cause the invalidation of conventional navigation algorithm based on the geographic-north direction reference line [4-5]. And the error of the corresponding traditional compass
alignment algorithm increases, which cannot be used at the pole. Based on the mechanics arrangement of the grid coordinate system, this paper simulates and analyzes the application of traditional compass alignment in high latitudes, uses a new polar alignment method, and conducts a verification on it. Then, it is converted to the high latitude region by using the latitude increment method, and the semi-physical simulation is carried out, which further illustrates the adaptability of grid navigation and the anti-sloshing polar region’s coarse alignment method in the polar region.

2. Mechanics Arrangement of Grid Inertial Navigation

As shown in Figure 1, the center-of-gravity position of the carrier is set as the origin point \( O \), \( ox'y'z'_e \) is the earth coordinate system and \( ONED \) is the navigation coordinate system of the current position of the carrier. The plane passing through the origin point \( O \) and parallel to the prime meridian plane is set as the grid tangent plane, and the horizontal plane tangent to the reference ellipsoid at the origin point \( O \) is set as the horizontal tangent plane. Thus, there is an intersecting line between the grid tangent plane and the horizontal tangent plane, which is defined as the grid northern direction of the current position where the carrier is located. The ground direction of the grid coordinate system coincides with the that of the navigation coordinate system, and the z-axis of grid coordinate system appearing below represents the ground rotation axis of the grid coordinate system. The grid east is in the tangent plane and perpendicular to the grid north and the ground direction, and constitutes the right-handed rectangular coordinate system. This coordinate system is the grid inertial navigation coordinate system, which is recorded as \( O N^e E^e D^e \).

At this time, there is an angle between the geographically real north direction and the grid north, which is defined as the grid angle. The positive direction rotates anticlockwise around the z-axis of grid coordinate system. The grid angle in Figure 2 is defined as the positive direction [6].

![Figure 1. Grid Coordinate System](image)

According to Figure 1, the cosine matrix relation between the grid coordinate system and the navigation coordinate system is the rotation of the angle around the z-axis of grid coordinate system. Thus, the cosine matrix relation of the earth coordinate system, the grid coordinate system and the navigation coordinate system can be expressed as the following equations:
The cosine matrix relation between the earth coordinate system and the grid coordinate system can be obtained by multiplying the two, which is as

\[
\begin{align*}
C_n^G &= \begin{bmatrix}
\cos \sigma & \sin \sigma & 0 \\
-sin \sigma & \cos \sigma & 0 \\
0 & 0 & 1
\end{bmatrix} \\
C_e^n &= \begin{bmatrix}
-sin L \cos \lambda & -sin L \sin \lambda & \cos L \\
-sin \lambda & \cos \lambda & 0 \\
-cos L \cos \lambda & -cos L \sin \lambda & -sin L
\end{bmatrix}
\end{align*}
\]  

The cosine matrix relation between the earth coordinate system and the grid coordinate system can be obtained by multiplying the two, which is as

\[C_e^G = C_e^n C_n^G \]  

It can be known from the definition of the grid coordinate system that the north direction of the grid coordinate system is always perpendicular to the y-axis of the earth coordinate system. Thus, it can be expressed as below

\[
\begin{align*}
(N^G)_x &= [\cos \sigma \sin \sigma 0]^T \\
(Y^G)_x &= [-\sin L \sin \lambda \cos \lambda - \cos L \sin \lambda]^T
\end{align*}
\]  

It is obtained as follows from the above two equations:

\[\tan \sigma = \frac{\sin \lambda \sin L}{\cos \lambda} \]  

According to the general equation of navigation solution in the form of first-order differential equation, the mechanical arrangement of grid inertial navigation system can be expressed as follows:

\[
\begin{align*}
\dot{C}_b^G &= \dot{C}_b^b (\omega_{ib}^b \times) \\
\dot{v}^G &= f^G - (2\omega_{ic}^G + \omega_{ic}^G \times v^G + g^G \\
\dot{C}_e^G &= -\omega_{ec}^G \times C_e^G
\end{align*}
\]  

3. Application of Traditional Compasses Alignment at High Latitudes

In order to verify whether the traditional compass alignment is applicable at high latitudes, the grid inertial mechanics arrangement based on compass alignment is adopted to carry out a simulated analysis on static data. The simulation parameters are as follows: the total time of simulation data is 6h, the compass alignment time is 1h, longitude is set as east longitude of 112°, the latitude is taken as north latitude of 80°, 82°, 84°, 86°, 88° and 90° respectively, and the height is 0m. The gyro bias is \( \varepsilon_\omega = \varepsilon_\omega = 0 \) 1/h, and the error of accelerometer is \( \varepsilon_{\varphi} = \varepsilon_{\varphi} = \varepsilon_{\psi} = 50 \mu g \). The simulation results are as follows:
According to figure 2 and figure 3, as the latitude increases, the errors of the pitch angle and the roll angle increase continuously. When the latitude increases to 90°, the pole point, the outputs of pitch angle and the roll angle are almost zero, which verifies that the compass alignment can not apply at the pole. The reason is that there is an azimuth misalignment angle when carrying out compass alignment, and the platform rotates relatively to the local horizontal plane to generate a compass item, which is as follows:

$$\cos \gamma_0x_z = \omega_\phi \omega_{ie} \cos L$$

The compass alignment is to use the $\gamma_0x_z$ caused by the compass item and the loop feedback method to control the rotation of the platform around the azimuth axis to gradually reduce to the limit value [6-7]. As the latitude increases, the compass item becomes smaller. When it reaches the polar region, the compass item tends to be zero, causing the error to become larger. When it reaches the pole, the compass item is zero, so the compass alignment is no longer applicable.

4. Anti-sloshing Coarse Alignment of Polar Navigation
Since the traditional compass alignment method is invalid at high latitudes, it is replaced by a new polar-region coarse alignment method based on grid coordinate system, namely anti-sloshing coarse alignment method.

The initial direction cosine matrix between the earth-centered-earth-fixed coordinate system e and the grid coordinate system G are determined through the initial latitude and longitude;
\[ C^G_e = C^a_e C^G_n = \begin{bmatrix} -\cos \sigma \sin \lambda + \sin \sigma \cos \lambda \sin L & \cos \sigma \cos \lambda + \sin \sigma \sin \lambda \sin L & -\sin \sigma \cos L \\ -\sin \sigma \sin \lambda - \cos \sigma \cos \lambda \sin L & \sin \sigma \cos \lambda - \cos \sigma \sin \lambda \sin L & \cos \sigma \cos L \\ \cos L \cos \lambda & \cos L \cos \lambda & \sin L \end{bmatrix} \] (7)

The angle between the skyward component of the grid coordinate system and the skyward component of the geographic coordinate system can be determined by the following equations.

\[ \sin \sigma = \sin \lambda \sin L / \sqrt{1 - \cos^2 L \sin^2 \lambda} \]
\[ \cos \sigma = \cos \lambda / \sqrt{1 - \cos^2 L \sin^2 \lambda} \] (8)

The time interval \( t \) is selected, and the direction cosine matrix \( C^e_i \) between the inertial coordinate system \( i \) and the earth-centered-earth-fixed coordinate system \( e \) is determined.

\[ \Delta t = t - t_0 \]
\[ C^e_i = \begin{bmatrix} \cos \omega_n \Delta t & \sin \omega_n \Delta t & 0 \\ -\sin \omega_n \Delta t & \cos \omega_n \Delta t & 0 \\ 0 & 0 & 1 \end{bmatrix} \] (9)

The direction cosine matrix \( C^b_i \) between the carrier coordinate system \( b \) and the carrier coordinate system \( b_0 \) which is fixed to the inertial coordinate system at the initial time is determined by the output of the gyroscope in the strapdown inertial navigation system.

\[ C^b_i = C^b_i [\omega_{b_0} \times] = C^b_i [\omega_{b_0} \times] \] (10)

The direction cosine matrix \( C^b_i \) of the carrier coordinate system \( b_0 \) fixed to the inertial coordinate system at the initial alignment time to the inertial coordinate system \( i \) is determined. In the geographic coordinate system and the inertial coordinate system, the gravitational acceleration can be expressed respectively as:

\[ g^e = [0 \ 0 \ -g]^T \]
\[ g^i = C^e_i C^n_g = \begin{bmatrix} -g \cos L \cos(\lambda + \omega_n \Delta t) \\ -g \cos L \sin(\lambda + \omega_n \Delta t) \\ -g \sin L \end{bmatrix} \] (11)

In order to suppress interference errors, an integral operation is introduced.

\[ V'(t_k) = \int_{t_{k-1}}^{t_k} -g^i dt = \begin{bmatrix} (g \cos L[\sin(\lambda + \omega_n \Delta t_k) - \sin \lambda]) / \omega_n \\ (g \cos L[\cos \lambda - \cos(\lambda + \omega_n \Delta t_k)]) / \omega_n \\ g \sin L \Delta t_k \end{bmatrix} \] (12)

Since the specific force at stationary state is composed of gravitational acceleration, disturbing acceleration and accelerometer offset, the specific force and its integral operation can be expressed respectively as:
Ignoring the smaller quantity after the integration of disturbing acceleration and accelerometer offset, the above equations are simplified as:

\[ V^{b_i}(t_k) = C^{bh_i} \int_{t_0}^{t_k} -g^i dt + \int_{t_0}^{t_k} C^{bh_i} (a^i + \nabla b) dt \]

(13)

The time \( t_0 \) and time \( t_k \) are respectively selected and substituted in the above equation to obtain the direction cosine matrix \( C^{bh_i} \) of the carrier coordinate system \( b_i \) which is fixed to the inertial coordinate system at the initial alignment time to the inertial coordinate system \( i \).

The direction cosine matrix \( C^{G_i} \) of the carrier coordinate system \( b \) to the grid coordinate system \( G \) is determined through the results of above steps.

\[ C^{G_i} = C^{G_e} C^{e_i} C^{h_i} C^{h_i} \]

(16)

The initial alignment attitude matrix \( C^{G_i} \) is obtained, and the initial attitude angle obtained after the initial coarse alignment.

The simulation verification is carried out as below, and the simulation parameters are set as follows: initial position is \([112\degree E, 90\degree N, 0]\), initial velocity is \([0, 0, 0]\), simulation time is 6h, alignment time is 600s, gyro zero offset is \( \varepsilon_x = \varepsilon_y = \varepsilon_z = 0.1 \degree / h \), and the error of accelerometer is \( \nabla x = \nabla y = \nabla z = 50 \mu g \). The results are as follows:

![Figure 6. Pitch Angle Output at the Pole](image6)

![Figure 7. Roll Angle Output at the Pole](image7)

According to figure 6 and figure 7, the anti-sloshing coarse alignment method works normally at the pole point compared to the compass alignment. In the 6-hours simulation time, the highest pitch angle is 0.05\degree, and the highest roll angle is 0.042\degree. Therefore, the anti-sloshing coarse alignment is an
effective method used for polar coarse alignment.

5. Experimental Data Simulation of Latitude Increment Method

In the high-latitude polar regions, using pure mathematical methods to verify the feasibility of navigation algorithm arrangement of grid coordinate system cannot fully explain the problem and lacks persuasiveness [9]. Thus, the latitude increment method is used to convert the experimental data sports cars at low latitudes to high latitudes and to carry out the semi-physical simulation.

Firstly, the vertical line of the meridian plane of the current carrier is taken as the rotation axis (the direction of the rotation axis is parallel to the lower east of the carrier’s local horizontal geographic coordinate system), and the carrier rotates $\Delta L$ around the rotation axis in the ascent direction of the latitude, thereby obtaining a new grid coordinate system which is defined as $G'$. The grid coordinate system $G$ at the original position rotates around the vertical line of the carrier’s radial plane and forms a new grid coordinate system $G''$ after rotating $\Delta L$ around the rotation axis in the ascent direction of the latitude, which is as shown in figure 8[10].

![Figure 8. Schematic Diagram of Latitude Increment Method under Earth Coordinate System](image)

The $G$ system differs from the $G'$ system by a z-axis rotation, but the relation between the $G'$ system and other coordinate systems is enhanced, which is shown in the following equations:

$$
\begin{align*}
C^G_{G'} &= C^G_n \\
C^b_{G'} &= C^b_n \\
C^b_{G} &= C^b_G 
\end{align*}
$$

According to the design idea of the ideal trajectory in figure 8, the angular rate output increment $\int_{t_i}^{t_{i+\Delta t}} \omega^b_{b'}$ of the gyro is decomposed as:

$$
\int_{t_i}^{t_{i+\Delta t}} \omega^b_{b'} = C^b_{G'} \int_{t_i}^{t_{i+\Delta t}} \omega^G_{G'} + \int_{t_i}^{t_{i+\Delta t}} \omega^G_{b'} \\
= C^b_{G'} \int_{t_i}^{t_{i+\Delta t}} \omega^G_{G'} + C^b_G \int_{t_i}^{t_{i+\Delta t}} \omega^G_{G'} + \int_{t_i}^{t_{i+\Delta t}} \omega^b_{G'} \\
= C^b_{G'} \int_{t_i}^{t_{i+\Delta t}} \omega^G_{G'} + C^b_G \int_{t_i}^{t_{i+\Delta t}} \omega^G_{G'} + \int_{t_i}^{t_{i+\Delta t}} \omega^b_{G'}
$$

Before and after the conversion, the attitude-direction cosine matrix between $G'$ system to $b'$ system and $G$ system to $b$ system is consistent.
\[ \int_{t}^{t+\Delta t} \omega_{G\text{e}G}^{G'} = \int_{t}^{t+\Delta t} \phi_{G\text{e}G}^{G'} \]  

(19)

Then the unknown quantities are \( \int_{t}^{t+\Delta t} \omega_{G\text{e}G}^{G'} \) and \( \int_{t}^{t+\Delta t} \omega_{G\text{e}G}^{G'} \), which are solved separately as below.

\( \Delta L \) is known, thus, the direction cosine matrix between the earth coordinate system and the navigation coordinate system in the new position can be known, which is shown as below:

\[
C_{e}^{G'} = \begin{bmatrix}
-\sin(L+\Delta L)\cos \lambda -\sin(L+\Delta L)\sin \lambda & \cos(L+\Delta L) \\
-\sin \lambda & \cos \lambda & 0 \\
-\cos(L+\Delta L)\cos \lambda -\cos(L+\Delta L)\sin \lambda & -\sin(L+\Delta L) & 0
\end{bmatrix}
\]  

(20)

Thus, \( \int_{t}^{t+\Delta t} \omega_{G\text{e}G}^{G'} \) can be decomposed in \( G' \) system, which is shown in the following equation:

\[
\int_{t}^{t+\Delta t} \omega_{G\text{e}G}^{G'} = C_{e}^{G'} \int_{t}^{t+\Delta t} \omega_{G\text{e}G}^{G'} = C_{n}^{G'} C_{e}^{n} \int_{t}^{t+\Delta t} \omega_{G\text{e}G}^{G'} 
\]  

(21)

\( \int_{t}^{t+\Delta t} \omega_{G\text{e}G}^{G'} \) is decomposed as:

\[
\int_{t}^{t+\Delta t} \omega_{G\text{e}G}^{G'} = C_{G}^{G'} (\int_{t}^{t+\Delta t} \omega_{G\text{e}G}^{G'} + \int_{t}^{t+\Delta t} \omega_{G\text{e}G}^{G'})
\]  

(22)

The grid angle \( \sigma' \) of \( G' \) system in the new position is defined as:

\[
\tan \sigma' = \sin(L+\Delta L)\sin \lambda / \cos \lambda
\]  

(23)

Thus, the direction cosine matrix between \( n' \) system and \( G' \) system is:

\[
C_{n}^{G'} = \begin{bmatrix}
\cos \sigma' & \sin \sigma' & 0 \\
-\sin \sigma' & \cos \sigma' & 0 \\
0 & 0 & 1
\end{bmatrix}
\]  

(24)

The conversion relation between \( G' \) system and \( G' \) system is:

\[
C_{G}^{G'} = C_{n}^{G'} C_{G}^{n} = C_{G}^{G'} C_{G}^{n}
\]  

(25)

The definition of \( \int_{t}^{t+\Delta t} \omega_{G\text{e}G}^{G'} \) can be written as the following equation:

\[
\int_{t}^{t+\Delta t} \omega_{G\text{e}G}^{G'} = \int_{t}^{t+\Delta t} \begin{bmatrix}
\frac{v_{x}^{G'} - v_{y}^{G'}}{R_{x}^{\tau'}} - \frac{v_{y}^{G'} - v_{x}^{G'}}{R_{y}^{\tau'}} \\
\frac{v_{y}^{G'}}{\tau_{f}^{R}} - \frac{v_{x}^{G'}}{\tau_{f}^{R}} \\
\frac{k_{f}^{\tau} v_{x}^{G'}}{R_{x}^{\tau'}} - \frac{k_{f}^{\tau} v_{y}^{G'}}{R_{y}^{\tau'}}
\end{bmatrix}
\]  

(26)

The carrier’s velocity in \( G' \) system is:

\[
v_{G}' = C_{n}^{G'} C_{G}^{n} v_{G}
\]  

(27)

The corresponding projection of the velocity is as following equation:
\[ v_{n}^{n'} = \frac{R'_{N} \cos(L + \Delta L)}{R_{N} \cos L} v_{n}^{n} \]

\[ v_{e}^{n'} = \frac{R'_{M} v_{e}^{n}}{R_{M}} \]  

(28)

Other parameters are defined as below:

\[ \frac{1}{\tau_{f}'} = \frac{\cos \sigma' \sin \sigma' - \cos \sigma' \sin \sigma'}{R_{M} + h} - \frac{\cos \sigma' \sin \sigma'}{R_{N} + h} \]

\[ \frac{1}{R_{x}'} = \frac{\sin \sigma' \sin \sigma' + \cos \sigma' \cos \sigma'}{R_{M} + h} + \frac{\sin \sigma' \sin \sigma'}{R_{N} + h} \]

\[ \frac{1}{R_{y}'} = \frac{\cos \sigma' \cos \sigma' + \sin \sigma' \sin \sigma'}{R_{M} + h} + \frac{\cos \sigma' \cos \sigma'}{R_{N} + h} \]

\[ \kappa' = \frac{\cos(L + \Delta L) \sin \sigma'}{\sin(L + \Delta L)} \]  

(29)

The equation is the transfer angular rate increment of system-to-system in the new position, which can be understood as the difference between the change rate of the grid angles of the two. The change rate of the grid angles of the two can be written as follows:

\[ \dot{\sigma} = -\frac{\dot{\lambda}}{\sin L} - \frac{\cos \sigma}{\sin L} (\cos L \sin \sigma L - \cos^{2} L \cos \sigma \dot{\lambda}) \]

\[ \dot{\sigma}' = -\frac{\dot{\lambda}'}{\sin L'} - \frac{\cos \sigma'}{\sin L'} (\cos L' \sin \sigma' L' - \cos^{2} L' \cos \sigma' \dot{\lambda}') \]  

(30)

In the equations, \( L' = L + \Delta L \), other variables are defined as follows:

\[ \dot{L}' = \frac{v_{n}}{R_{M}} \], \( \dot{\lambda}' = \dot{\lambda} = \frac{v_{e}}{R_{N} \cos L} \)  

(31)

The integral increment of the specific force is decomposed as shown in the following equation:

\[ \int_{l_{i}}^{l_{i} + \Delta t} \int_{l_{b}}^{l_{b}'} f_{lb}^{b'} = C_{G}^{G} \int_{l_{i}}^{l_{i} + \Delta t} \int_{l_{b}}^{l_{b}'} f_{lb}^{G}^{e} \]

\[ = C_{G}^{G} \int_{l_{i}}^{l_{i} + \Delta t} ((2 \omega_{e}^{G} + \omega_{e}^{G'}^{G}) \times v^{G'} - g^{G'} + v^{G'}) \]  

(32)

According to the above algorithm arrangement, the latitude increment method is to obtain the data of semi-physical simulation in polar regions through the data of low latitudes. The semi-physical simulation experiment of the feasibility of latitude increment method is verified as below.

The initial latitude of the original position is 28.222° N, the initial longitude is 112.9916° E, and the initial height is 0 m. The north and east initial velocity of the grid coordinate system is set as 0 m/s, the trajectory is static and rocking, the sampling frequency is 200 Hz, and the variation of latitude is 60°. Without considering the error, the navigation solution results under the original position and the initial position after data conversion are shown in below figures.
According to figure 9 - figure 10, at low latitudes, the positioning error at low latitudes is 0.19nm. At high latitudes, the positioning error at high latitudes is 0.15nm. After the data is converted to high latitudes, the accuracy of positioning error is almost.

According to figure 11 - figure 14, the magnitude of the northward velocity and the eastward velocity error are the same. Based on the above-mentioned, the latitude increment method can effectively convert the experimental data at low latitudes to high latitudes for semi-physical simulation, which provides an idea to verify the polar navigation and effectively solves the problems in polar navigation verification.
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