Iterative Learning Control for Singular Network Control System with Time-Delay and Data Dropout

Zu-song Li, Fu-gui He and Shi-rong Li
School of Electronic and Information Engineering, West Anhui University
Lu’an, Anhui, 237012, China
Email: 67202167@qq.com

Abstract: Since the object becomes more and more complicated and the existed data dropout and network-introduced time-delay are random and varying in network control system (NCS), the traditional control method is more and more difficult to obtain a good control effect. In this paper, the iterative learning control (ILC) issue for a class of linear continuous singular network control system (SNCS) with time-delay and data dropout is investigated. According to the $\lambda$ norm and expectation algorithm, the convergence condition of the system is derived. At last, simulation example shows the effectiveness and superiority of the presented method.

1. INTRODUCTION
Iterative Learning Control (ILC), one of the important branches in the field of intelligent control with rigorous mathematical description [1], constructs the control rate in a different way on the basis of the previous control experience and the tracking errors so that the tracking tasks can be achieved by the object within a short time and some insurmountable difficulties in traditional control methods can also be overcome[2][3].

In addition, the research on network control system (NCS) has attracted much attention, it has made significant progress over the past few years. A central research area is to evaluate and compensate data dropout and time delays factors [4][5]. But in practice, many control objects present the singular system models, such as cold rolling mill, nonlinear loads power systems, electronic networks. Because the singular system has different characteristics (e.g. pulse entry) from the normal system, the conclusion which is suitable for the normal system can’t be directly applied to the singular system. Recently, the researches for singular network control system (SNCS) have arisen the attention of people[6].

With the Internet have been more and more applied in NCS, the data dropout and time-delay become random and varying. In this paper, we discuss the ILC applied in singular network control system (SNCS), which is still an open research area except for certain pioneer works that address singular system[7][8]. On the basis of previous researches, this article investigates the implementation of ILC for a class of SNCS with time-delay and data dropout. Based on the $\lambda$ norm and expectation algorithm, the convergence condition of the system is derived. The simulation example shows the effectiveness and superiority of the presented method.

The paper is organized as below. Section 2 describes the SNCS system and the effective ILC for the system. Section 3 analyzes the convergence condition of the ILC for the system. The simulation results are verified in section 4. Conclusions will be given in section 5.
2. PROBLEM FORMULATION

Definition 1: $E(x)$ denotes the expectation of the stochastic variable $x$, $P(x)$ stands for the occurrence probability of the event $x$ [5].

Definition 2: The $\lambda$ norm of vector function $h$ is defined as

$$\|h\|_\lambda = \sup_{t \in [0, T]} \{\exp(-\lambda t) \|h(t)\|\}, \lambda > 0,$$

where $\|\|_\lambda$ denotes a vector norm [9].

Lemma 1: If vector function $f$ and $h$ meet the formula:

$$\int_0^T f(\tau)d\tau h(t)\| \leq (1 - \exp(-\lambda t)) \|f(t)\|_\lambda,$$

then $h(t) = \int_0^T f(\tau)d\tau h(t)$.

Lemma 2: Suppose that $E \in C^{\infty\times\nu}, ind(E) = \nu$, and if $n$-dimensional matrix $X$ meets the following conditions: $E^\nu X E = E^\nu, X E X = X, E X = X E$, then $X$ is the Drazin inverse of $E$, defined as $E^D$ [10].

Assuming that the network only exists in the control layer, we consider a linear continuous singular system with time-delay given by the following equation:

$$\begin{align*}
\dot{x}_k(t) &= Ax_k(t) + Bu_k(t - \theta), \\
y_k(t) &= Cx_k(t)
\end{align*}$$

where $E \in R^{n \times \nu}, A \in R^{n \times n}, B \in R^{n \times m}, C \in R^{m \times n}$ are constant matrices and $E$ also is a singular matrix, $ind(E) = \nu$.

$x(t) \in R^n, u(t) \in R^m, y(t) \in R^q$ for all $t \in [0, T]$ are system states, inputs and outputs, respectively. $k$ and $\theta$ denote the iteration index and constant time-delay, respectively.

Let $\Delta u_k(t) = u_k(t) - u_k(t)$, $\Delta x_k(t) = x_k(t) - x_k(t)$, $e_k(t) = y_k(t) - y_k(t)$ where $u_k(t), x_k(t)$ and $y_k(t)$ is ideal control input, object state and output, respectively.

In order to make the problem be simply analyzed but without losing typicality, we make the following assumption for system (1).

Assumption 1: $(E, A)$ is regular.

Assumption 2: The desired trajectory $y_d(t)$ can be obtained within $t \in [0, T]$, there exists a unique input $u_d(t)$ such that

$$\begin{align*}
\dot{x}_d(t) &= Ax_d(t) + Bu_d(t - \theta), \\
y_d(t) &= Cx_d(t)
\end{align*}$$

Assumption 3: System initialization is $x_k(0) = x_k(0), k = 0,1,2\cdots$

From lemma 2, we have

$$\begin{align*}
E^D E \dot{x}_k(t) &= E^D Ax_k(t) + E^D Bu_k(t - \theta) \\
y_k(t) &= Cx_k(t)
\end{align*}$$

The formula (2) can be converted into

$$\begin{align*}
E^D E \dot{x}_k(t) &= E^D A(EE^D x_k(t)) + E^D B(EE^D u_k(t - \theta)) \\
y_k(t) &= CEE^D x_k(t)
\end{align*}$$

Setting $A^- = E^D A, B^- = E^D B, x_k(t) = EE^D x_k(t), u_k(t - \theta) = EE^D u_k(t - \theta)$, finally formula(1) can be converted into[8]
\[ x_k^* = A^* x_k^*(t) + B^* u_k^*(t - \theta) \]
\[ y_k^*(t) = C x_k^*(t) \] (4)

Open loop PID type iteration learning rate is defined as follows:
\[ u_{k+1}(t) = u_k(t) + \eta \Gamma e_k(t + \theta) + Q_o e_k(t + \theta) + \int_0^\tau e_k(\tau + \theta) d\tau \] (5)

Where \( \Gamma, Q_o, Q_i \) are constant gain matrices, \( \eta \in \{0,1\} \), where \( \eta = 1 \) denotes a normal transmission, and \( \eta = 0 \) denotes an occurrence of data dropout[11][12][13]. The probabilities of \( \eta \) is \( P(\eta = 1) = E(\eta) = \gamma \), while \( P(\eta = 0) = 1 - \gamma \) denotes the data dropout rate, which is a known constant.

3. CONVERGENCE ANALYSIS

Theorem 1: If there exists an iterative learning control system satisfied formula (1) and (5), and the inequality \( \|I - \gamma^\Gamma CB\| \leq \rho < 1 \) is established, then expectation of the iteration tracking error \( E[e_k(t)] \) converges to zero.

Proof: Setting \( \Delta u_k(t) = EE^D \Delta u_k(t) \), \( e_k(t) = EE^D e_k(t) \), then
\[ \Delta u_{k+1}(t) = \Delta u_k(t) - \eta \Gamma [e_k(t + \theta) + Q_o e_k(t + \theta) + \int_0^\tau e_k(\tau + \theta) d\tau] \]
\[ = \Delta u_k(t) - \eta \Gamma \left\{ C A^* \Delta x_k^*(t + \theta) + CB^* u_k^*(t) + Q_o C \Delta x_k^*(t + \theta) \right\} \]
\[ + Q_o \int_0^\tau C \Delta x_k^*(t + \theta) d\tau \] (6)

From definition 1 and formula (6), we have
\[ E[\Delta u_{k+1}(t)] = E[\Delta u_k(t)] - \gamma E \left\{ \Gamma [C A^* \Delta x_k^*(t + \theta) + CB^* u_k^*(t)] \right\} \]
\[ + Q_o C E \int_0^\tau C \Delta x_k^*(t + \theta) d\tau \] (7)

Taking the \( \lambda \) norm \( \|\cdot\| \) on both sides of (7), then
\[ \|E[\Delta u_{k+1}(t)]\| \leq \rho \|E[\Delta u_k(t)]\| + \|\gamma \Gamma (CA^* + Q_o C) E[\Delta x_k^*(t + \theta)]\| \]
\[ + \|Q_o C \| E \int_0^\tau \|C \Delta x_k^*(t + \theta)\| d\tau \] (8)

Besides \( \|E[\Delta x_k^*(t + \theta)]\| \leq \frac{b \|E[\Delta u_k(t)]\|}{I - a \lambda^{-1}} \) (9)

where \( a = \|A\|, b = \|B\| \), \( \lambda^{-1} = 1/\gamma \).

From lem1, so the formula (8) can be converted into
\[
\|E[\Delta u_{k+1}(t)]\|_k \leq \rho \|E[\Delta u_k(t)]\|_k + \left( \|PA(CA^* + Q_0 C)\| + \|PQ_1 C \| \circ (\lambda^{-1}) \right)
\]
\[
\cdot \left( \|E[\Delta x_k(t + \theta)]\|_k \right) 
\]
\[
\leq \rho \|E[\Delta u_k(t)]\|_k + \kappa \|E[\Delta x_k(t + \theta)]\|_k 
\]
\]

where \( \kappa = \|PA(CA^* + Q_0 C)\| + \|PQ_1 C \| \circ (\lambda^{-1}) \).

Combined the formula (9) and (10), we can get
\[
\|E[\Delta u_{k+1}(t)]\|_k \leq \varepsilon \|E[\Delta u_k(t)]\|_k ,
\]
where \( \varepsilon = \rho + \kappa b \circ (\lambda^{-1})/(I - a \circ (\lambda^{-1})) \). From theorem 1, \( \rho < 1 \), it is possible to choose \( \lambda \) sufficiently large so that \( \varepsilon < 1 \), so \( \|E[\Delta u_k(t)]\|_k \rightarrow 0 \) as \( i \rightarrow \infty \), from (9), so \( \|E[\Delta x_k(t)]\|_k \rightarrow 0 \) as \( i \rightarrow \infty \), then
\[
\|E[e_k(t)]\|_k = \|E[C\Delta x_k(t)]\|_k = \|E[CE^D \Delta x_k(t)]\|_k = \|E[C\Delta x_k(t)]\|_k \rightarrow 0 \) as \( i \rightarrow \infty \). Hence \( E[e_k(t)] \rightarrow 0 \) as \( i \rightarrow \infty \).

4. NUMERICAL EXAMPLES

Considering the following linear singular system with time-delay
\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x(t) \\
x(t-\theta)
\end{bmatrix}
= 
\begin{bmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
-1 & 0 & 0 & 1 \\
0 & 1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
x(t) \\
x(t-\theta)
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]
where \( x_i(0) = 0 \), \( y_d(t) = 3 \sin(t) \), \( t \in [0,7] \). Sample time is 10ms, Two probabilities of the data dropout and time-delay \( \gamma = 0.9 \), \( \gamma = 0.6 \) and \( \theta = 0.005 \), \( \theta = 0.02 \) are considered. The learning gain is \( \Gamma = 0.5 \). The study shows that the data is suitable for the convergence condition for the theorem. The tracking performances of the system are given in the following figures, where the ordinate denotes the root mean square error of each iteration.
As can be seen from the figure that it is effective for using ILC algorithm to control the SNCS, meanwhile, with the growth of time-delay and data dropout rate, the system can still work stably, and the control performance is satisfactory.

5. CONCLUSION
In this paper, the ILC issue for a class of SNCS with time-delay and data dropout is investigated. The convergence condition for the ILC is derived based on the $\lambda$-norm and expectation algorithm. Simulation example shows the effectiveness and superiority of the presented method. In future work, nonlinear MIMO singular system which is more close to the actual industry system will become the key point.

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