Effects of pairing correlations on the inverse level density parameter of hot rotating nuclei

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Abstract. Angular momentum dependence of the inverse level density parameter \( K \) in the excitation-energy region of \( \sim 30 \) – \( 40 \) MeV is studied within the finite-temperature Bardeen-Cooper-Schrieffer (FTBCS) theory and the FTBCS theory that includes the effect due to quasiparticle-number fluctuations (FTBCS1). The two theories take into account the noncollective rotation of the nucleus at nonzero values of \( z \)-projection \( M \) of the total angular momentum. The comparison between the results obtained within the FTBCS and FTBCS1 as well as the case without pairing correlations and the experimental data for two medium-mass even-even nuclei \(^{108}\)Cd and \(^{122}\)Te shows that by including the pairing corrections the FTBCS and FTBCS1 reproduces quite well all the experimental data, whereas the non-pairing case always overestimates the data. Due to the effect of quasiparticle-number fluctuations, the FTBCS1 gaps at different \( M \) values do not collapse at critical temperature \( T_C \) as in the FTBCS ones but monotonously decrease with increasing \( T \) and being finite even at high \( T \). As the result, the values of \( K \) obtained within the FTBCS1 are always closer to the experimental data than those obtained within the FTBCS.

1. Introduction

Pairing correlation is known to play a major role in strongly interacting many-body systems ranging from large ones such as superconductors to very small ones such as atomic nuclei [1]. In nuclear systems, pairing correlation has significant effects on various nuclear properties such as the binding and excitation energies, collective motions, rotations, level densities, etc. Dependence of pairing correlations on temperature and angular momentum has been widely studied within the finite-temperature Bardeen-Cooper-Schrieffer (FTBCS) theory taking into account the \( z \)-projection \( M \) of total angular momentum [2, 3]. It has been shown in Ref. [2] that the pairing gap obtained within the FTBCS theory decreases with increasing both temperature \( T \) and/or angular momentum \( M \) and vanishes at a given critical temperature \( T_C \) or angular momentum \( M_C \). However, by taking into account the quasiparticle-number fluctuations (QNF), which are neglected within the FTBCS, the pairing gap obtained within the so-called FTBCS1 theory, which is the FTBCS plus QNF, for light and medium mass nuclei does not collapse at
but monotonously decreases with increasing $T$ or $M$ and keeps finite even at high $T$ or $M$ [4, 5, 6]. The goal of present work is to apply the FTBCS and FTBCS1 theories to study the effect of pairing correlation on the inverse level density parameter of some medium-mass hot rotating nuclei. The latter is an important input in the calculations of cross sections of fission or fusion reactions as well as astrophysical reaction rate of the nucleosynthesis processes [7]. The results obtained are compared with the recent data extracted from the measured $\gamma$-ray multiplicity fold-gated $\alpha$-particle energy spectra of two medium-mass even-even nuclei $^{108}$Cd and $^{122}$Te at the excitation energy of $\sim 30 - 40$ MeV and angular momentum $\sim 0 - 20\hbar$ [8].

2. FORMALISM
2.1. Model Hamiltonian
In this work, we consider a system, which consists of $N$ particles (protons or neutrons) interacting via a constant pairing interaction $G$ and rotating about the symmetry axis (noncollective rotation) with a fixed projection $M$ of the total angular momentum along the $z$-axis. The Hamiltonian of this type has the form as [2, 5]

$$H = H_p - \lambda \hat{N} - \gamma \hat{M},$$

where $\lambda$ and $\gamma$ are the chemical potential and angular velocity, respectively and $H_p$ is the standard pairing Hamiltonian, which is given as

$$H_p = \sum_k \epsilon_k (a_k^\dagger a_k + a_{-k}^\dagger a_{-k}) - G \sum_{k,k'} a_k^\dagger a_{-k}^\dagger a_{-k'} a_{k'},$$

with $a_{\pm k}^\dagger (a_{\pm k})$ denoting the operator that creates (annihilates) a particle with angular momentum $k$, spin projection $\pm m_k$, and energy $\epsilon_k$. The subscripts $k$ here label the single-particle states $|k, m_k> in deformed basis, whereas $-k$ denotes the time-reversal ones $|k, -m_k>$ namely the states, which have the same spin projection with the states $k$ but having the opposite direction. In Eq. (1), the particle-number operator $\hat{N}$ and total angular momentum operator $\hat{M}$, which coincides with its $z$-projection, are given as

$$\hat{N} = \sum_k (a_k^\dagger a_k + a_{-k}^\dagger a_{-k}), \quad \hat{M} = \sum_k (a_k^\dagger a_k - a_{-k}^\dagger a_{-k}).$$

2.2. FTBCS1 theory
The FTBCS1 equations at finite temperature and angular momentum are derived in the same way as the conventional FTBCS ones, namely based on the variational procedure to minimize the expectation value of the pairing Hamiltonian (1) in the grand-canonical ensembles. The only difference between the FTBCS and FTBCS1 is that the FTBCS1 equations contain the corrections due to the quasiparticle-number fluctuations (QNF) on the pairing gap, average particle number $N$, and angular momentum $M$, whereas the QNF are zero within the FTBCS. The derivation of the FTBCS1 equations was mentioned in details in Refs. [5, 6]. The final FTBCS1 equations for the pairing gap, particle number and total angular momentum are then given as

$$\Delta_k = \Delta + \delta \Delta_k,$$

$$N = 2 \sum_k \left[ (1 - n_k^+ - n_k^-) v_k^2 + \frac{1}{2} (n_k^+ + n_k^-) \right],$$

$$M = \sum_k m_k (n_k^+ - n_k^-),$$

with $\Delta_k$ and $\delta \Delta_k$ being the pairing gap at $k$ and its fluctuations, respectively.
\[ \Delta = G \sum_k u_k v_k (1 - n_k^+ - n_k^-); \quad \delta \Delta_k = G \frac{\delta N_k^2}{1 - n_k^+ - n_k^-} u_k v_k, \]  
\[ u_k^2 = \frac{1}{2} \left( 1 + \frac{\epsilon_k - \lambda}{E_k} \right); \quad v_k^2 = 1 - u_k^2, \]  
\[ E_k = \sqrt{(\epsilon_k - \lambda)^2 + \Delta_k^2}, \]  
\[ n_k^\pm = \frac{1}{1 + e^{\beta(E_k \mp \gamma m_k)}}; \quad \beta = 1/T, \]  
with the quasiparticle-number fluctuations \( \delta N_k^2 \) at nonzero angular momentum being defined as 
\[ \delta N_k^2 = (\delta N_k^+)^2 + (\delta N_k^-)^2 = n_k^+(1 - n_k^+) + n_k^-(1 - n_k^-). \]  
The conventional FTBCS equations are obtained from the FTBCS1 by setting \( \delta N_k^2 = 0 \). The FTBCS1 (FTBCS) total (internal) energy and entropy are calculated as 
\[ E(T, M) = \langle H \rangle, \]  
\[ S(T, M) = -\sum_k [n_k^+ \ln n_k^+ + (1 - n_k^+)[\ln(1 - n_k^+) + n_k^- \ln n_k^- + (1 - n_k^-)\ln(1 - n_k^-)]. \]  

2.3. Inverse level density parameter 

The level density parameter \( a \) as a function of angular momentum \( M \) and temperature \( T \) is given as \[ a(T, M) = \frac{U(T, M)}{T^2}, \]  
where \( U(T,M) \) is the corrected total excitation energy of the system 
\[ U(T, M) = E^*(T, M) - E_{\text{pair}} - E_{\text{rot}}, \]  
with \( E_{\text{pair}} \) and \( E_{\text{rot}} \) being the corrections due to the pairing and rotational vibration of the total excitation energy, respectively \[ E_{\text{pair}} = 12/\sqrt{A}, \quad E_{\text{rot}} = \frac{M(M + 1)}{5mR^2} \left( 1 + \sqrt{\frac{5}{15m}\beta_2} \right) \]  
In Eq. (11), \( \beta_2 \) is the quadrupole deformation parameter (\( \beta_2 = 0 \) for a spherical nucleus) and \( R = 1.2A^{1/3} \) (\( A \) is mass number) is the nuclear radius. The total excitation energy \( E^*(T, M) \) is calculated based on the FTBCS (FTBCS1) total energy as 
\[ E^*(T, M) = E(T, M) - E(0, 0). \]  
The inverse level density parameter \( K \) is obtained as the mass number \( A \) divided by the level density parameter \( a \) 
\[ K(T, M) = \frac{A}{a(T, M)}. \]
3. NUMERICAL RESULTS AND DISCUSSION

3.1. Ingredients of numerical calculations

The numerical calculations are carried out for two medium-mass even-even nuclei, namely $^{108}$Cd and $^{122}$Te. The single-particle spectra are obtained within the axially deformed Woods-Saxon (WS) potential including the spin-orbit and Coulomb interactions. The parameters of the WS potential are taken from Ref. [11], in which the quadrupole deformation parameters $\beta_2$ are fixed and equal to 0.135 (prolate shape) for $^{108}$Cd and -0.139 (oblate shape) for $^{122}$Te, respectively. The pairing interaction parameter $G$ is adjusted so that the pairing gap at $T = 0$ and $M = 0$ obtained within the FTBCS (FTBCS1) fits the experimental odd-even mass difference [12]. In order to compare the results of $K$ obtained within the FTBCS (FTBCS1) with the experimental values reported in Ref. [7], we perform the calculations at the same angular momenta and in the same interval of excitation energy $(30 \text{ – } 40)$MeV as those of the measured data.

3.2. Inverse level density parameter

Shown in Fig. 1 are the neutron and proton pairing gaps $\Delta_N$ [Fig. 1 (a)] and $\Delta_Z$ [Fig. 1 (b)], excitation energy $E^*$ [Fig. 1 (c)], and inverse level density parameter $K$ [Fig. 1 (d)] obtained within the FTBCS and FTBCS1 calculations for prolately deformed $^{108}$Cd nucleus. In Fig. 1 (d), the filled circles with error bars denote the experimental data of $K$, whereas the triangles depict the results obtained within the case of zero pairing ($\Delta = 0$). Similar results obtained for an oblately deformed $^{122}$Te nucleus are displayed in Figs. 2 (a) – (d).

**Figure 1.** (a) - (c): neutron and proton pairing gaps $\Delta_N$ (a) and $\Delta_Z$ (b) and total excitation energy $E^*$ (c) as functions of temperature $T$ obtained within the FTBCS (thin lines) and FTBCS1 (thick lines) at different values of total angular momentum $M$ for $^{108}$Cd nucleus. (d): inverse level density parameter $K$ as functions of $M$ obtained within the FTBCS (open circles), FTBCS1 (squares), and the case without pairing correlation (triangles) versus the experimental data (full circles with error bars) taken from Ref. [8].
In Figs. 1 (a)−(b) and Fig. 2 (a), one can see that the FTBCS and FTBCS1 gaps both decrease with increasing either temperature $T$ or total angular momentum $M$. However, the FTBCS1 gaps at different $M$ do not collapse at critical temperature $T_C$ as in the case of the FTBCS ones. In stead of that the FTBCS1 gaps monotonously decrease with increasing $T$ and being finite even at $T$ as high as 2 MeV. This feature is well-known due to the quasiparticle-number fluctuations within the FTBCS1 [4, 5]. Due to the shell structure in an oblate deformed $^{122}$Te nucleus, the proton gap obtained within the FTBCS1 is nearly similar as that of the FTBCS one as seen in Fig. 2 (b). As a result, the excitation energy $E^*$ obtained within the FTBCS1 is lower than that obtained within the FTBCS only in the low and intermediate regions of temperature and total angular momentum [See e.g., the thin and thick lines in the Figs. 1 (c) and 2 (c)], whereas in the high $T$ and $M$ regions both FTBCS and FTBCS1 predict nearly the same excitation energies. Consequently, the corresponding inverse level density parameter $K$ obtained within the FTBCS1 in the region of intermediate $M \sim 10\hbar - 18\hbar$ for both nuclei [See Fig. 1 and 2 (d)] is closer to the experimental data than that obtained within the FTBCS. It is also clear to see from Figs. 1 (d) and 2 (d) that the $K$ values obtained within the zero-pairing case always deviate significantly from the measured data in both nuclei. This results show that pairing correlation has significant effects on the inverse level density parameter of light and medium-mass nuclei at low and intermediate temperature and angular momentum. In addition, when the total angular momentum $M$ is high enough (but not too high), one can see clearly the reentrance effect [5, 6, 13] of the pairing gap of $^{108}$Cd nucleus [See e.g., red dash dotted lines of Fig. 1 (a) for the neutron gap or blue dotted lines in Fig. 1 (b) for the proton gap], in which the pairing gap at low $T$ increases to reach a maximum and decreases with further increasing of $T$, whereas this effect is not seen in $^{122}$Te nucleus due to the shell structure. However, this pairing reentrance does not affect the $K$ value considered here as it appears only in the low temperature region, whereas the experimental data are measured in the intermediate region of temperature.

Figure 2. The same as in Fig. 1 but for $^{122}$Te nucleus
and/or excitation energy $\sim 30 - 40$ MeV.

4. CONCLUSIONS

Present work studies the temperature and angular momentum dependence of the inverse level density parameter within the conventional FTBCS theory and the FTBCS theory taking into account the quasiparticle-number fluctuations (FTBCS1) as well as the case without pairing. The numerical calculations are carried out for two medium-mass even-even $^{108}$Cd and $^{122}$Te nuclei with the use of single-particle spectra obtained within the axially deformed Woods-Saxon potential at the fixed values of the quadrupole deformation parameters. The results obtained show that due to the effects of quasiparticle-number fluctuations, the pairing gaps obtained within the FTBCS1 for the neutrons and protons in $^{108}$Cd and for the neutrons in $^{122}$Te at the given total angular momenta $M$ do not collapse at critical temperature as those obtained within the FTBCS but monotonously decrease with increasing $T$ and keep finite even at high $T$. The FTBCS1 and FTBCS gaps for the protons in $^{122}$Te are nearly the same due to the shell structure of this nucleus. This feature of the FTBCS1 pairing gaps leads to the lowering of the FTBCS1 excitation energies around the regions of intermediate temperature and angular momentum as compared with the FTBCS ones. As a result, the inverse level density parameters obtained within the FTBCS1 are always closer to the experimental data than that of the FTBCS, whereas the results obtained within the zero-pairing case are quite far from the data. Based on these results, we conclude that pairing correlations have significant effects on the inverse level density parameters of hot rotating nuclei.

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