Numerical Solution of Unsteady $\text{Al}_2\text{O}_3$-Water Nanofluid Flow Past A Magnetic Porous Sliced Sphere

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Abstract. This study aims to investigate the MHD stagnation point flow of $\text{Al}_2\text{O}_3$-Water nanofluid with mixed convection effect pass a sliced magnetic porous sphere. The unsteady $\text{Al}_2\text{O}_3$-Water nanofluid mixed convection boundary layer flows the stagnation point is an interesting study. This implementation of this concept can use in the problems of engineering and industrial fields such as propulsion ships system, oil drilling, chemical piping, and nuclear reactor coolant. In this paper, we consider the effect of magnetic, porosity, and slicing angle parameters towards the velocity and temperature profiles. We construct a mathematical model of MHD flow in $\text{Al}_2\text{O}_3$-Water nanofluid to obtain dimensional governing equations. Then, the equations converted into non-dimensional equations by using non-dimensional variables and unsimilarity equations using stream function. The solution of the model solved by numerically using the Keller-Box scheme. The velocity and temperature profiles are determined for various non-dimensional parameters such as magnetic parameter, sliced angle, and porosity parameter. We obtain that the velocity of $\text{Al}_2\text{O}_3$-Water nanofluid decreases but the temperature decreases when magnetic parameters increases. For the porosity parameter increases then the velocity of the fluid decreases but the temperature of the fluid increases. For the sliced angle parameter increases then the velocity of the fluid increases but temperature of the fluid decrease. For the volume fraction parameter increases then the fluid velocity decreases but the fluid temperature increases.

1. Introduction
Magnetic fields affect many natural currents as well as human intervention. Magnetic fields are routinely used in industry to heat, pump, stir and lift molten metal. Various magnetic fields include terrestrial magnetic fields that are influenced by fluid motion in the earth’s core, solar magnetic fields that produce sunspots and solar flares, and galactic magnetic fields which are thought to influence star formation from interstellar clouds. The study of this flow is called magnetohydrodynamics (MHD). Formally, MHD is concerned with the mutual interactions of fluid flow and magnetic fields. The fluid is conducting electricity and non-magnetic, some examples are liquid metal, ionized gas (plasma) and strong electrolytes (Davidson, 2001).

Magnetohydrodynamics are widely applied in the engineering and industrial fields. Some technological developments related to MHD are MHD power generators and accelerators, nuclear reactor coolers and crystal growth. Therefore, magnetohydrodynamics are very interesting and important to continue to be developed and studied in a variety of types and influences.
recent years there have been many studies on various types of fluids which flow through various different geometric forms. Rahma et al (2017) investigated MHD passing objects with porous spherical geometry. Khalimah (2015) examined the flow of viscous fluid that passes through the elliptic cylinder. Widodo et al (2016) investigated viscous fluid that passes through a porous cylinder. Abu (2018) examined the nanofluid passing through the porous magnetic sphere, and Wijaya (2016) examined the viscous fluid passing through sliced sphere.

In this research investigate the nanofluid through a porous sliced magnetic sphere under the influence of mixed convection. Nanofluid is a basic fluid mixed with nanoparticles. The nanoparticle used is $\text{Al}_2\text{O}_3$. The nanofluid flow is then given a mixed convection that is a combination of natural convection and forced convection. Nanofluid flow in this study is considered to flow from the bottom up and pass through a sliced porous magnetic ball.

The slicing in this research is limited to sliced sphere with a certain pair of symmetrical angles ($\theta_s$). Fluid flow through the sliced sphere will form a boundary layer. Furthermore, this boundary layer can be expressed in a certain equation. The equation is then transformed into a form of flow function and unsimilarity equation. Then the transformation results are numerically solved using the Keller-box method. Simulation results are obtained with several variations of magnetic parameters, porosity parameters, slices angle parameters, and volume fraction parameters. From the numerical simulation results, we obtain the velocity and temperature profile of fluid $\text{Al}_2\text{O}_3$ - nanofluid fluid past a magnetic porous sliced sphere.

2. Numerical Method

A nanofluid with water as base fluid and $\text{Al}_2\text{O}_3$ as nano particle considered when flow past a magnetic porous sliced sphere, and we assume that ambient temperature is $T_\infty$ and upstream velocity is $U_\infty$. Further a mixed convection is assumed to be applied on the system. The physical model and coordinat system of this problem is shown in Figure 1.

We develop the governing equations of this problem from continuity equation, momentum equations, and energy equation, as follow.

Continuity Equation:

$$\frac{\partial (\bar{r}\bar{u})}{\partial x} + \frac{\partial (\bar{r}\bar{v})}{\partial y} = 0$$

(1)
where $g$ is gravity, $g$ is defined as $g = g \tan \left(\frac{x \cos \theta}{\cos \theta} \right)$ and $g_y = g \tan \left(\frac{y}{\cos \theta} \right)$. $\rho$ is nanofluid density, $\mu$ is viscosity. Also magnetic parameter $M$, mixed convection parameter $\lambda$, Grashof number $Gr$, Prandtl number $Pr$, and porosity parameter $\phi$ are dimensionless parameter that defined as:

$$M = \frac{a \sigma B_0^2}{\rho U_\infty^2}, \lambda = \frac{Gr}{Re^2}, Gr = \frac{g \beta (T_w - T_\infty)}{v_{nf}},$$

$$Pr = \frac{v_{nf}}{\alpha_{nf}}, \phi = \frac{\alpha \mu_{nf}}{\rho_{nf} U_\infty K}$$

Nondimensional equations obtained by subtituting the non-dimensional variables into equations 1-4 as follows:
a. Continuity Equation

\[ \frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial y} + \frac{\partial (rw)}{\partial z} = 0 \]  

(5)

b. Momentum Equations in \( x \) and \( y \) respectively

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \frac{\partial p}{\partial x} + \frac{1}{Re} \frac{v_n f \partial^2 u}{\partial x^2} + \frac{v_n f \partial^2 u}{\partial y^2} + \frac{v_n f \partial^2 u}{\partial z^2} + Mu + \phi u + \lambda T \tan (\frac{x \cos x}{\cos \theta_s}) \]

\[ \frac{1}{Re} \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = - \frac{\partial p}{\partial y} + \frac{1}{Re} \frac{v_n f \partial^2 v}{\partial x^2} + \frac{1}{Re} \frac{v_n f \partial^2 v}{\partial y^2} + \frac{1}{Re} \frac{v_n f \partial^2 v}{\partial z^2} + M + \phi \frac{v}{\cos \theta} - \lambda T \frac{1}{Re} \frac{v_n f \partial^2 \theta_s}{\cos \theta_s} \]

c. Energy equation

\[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \frac{1}{Pr} \frac{1}{\alpha_f} \frac{\partial^2 T}{\partial x^2} + \frac{1}{Pr} \frac{1}{\alpha_f} \frac{\partial^2 T}{\partial y^2} + \frac{1}{Pr} \frac{1}{\alpha_f} \frac{\partial^2 T}{\partial z^2} \]

(6)

Boundary layer theory states that the kinematic viscosity value close to zero (\( v \to 0 \)) in the boundary region, resulting in \( \frac{1}{Re} = 0 \) in the boundary layer region, from that thing is obtained

Momentum equations in \( x \)

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \frac{\partial p}{\partial x} + \frac{v_n f \partial^2 u}{\partial y^2} + Mu + \phi u + \lambda T \tan (\frac{x \cos x}{\cos \theta_s}) \]

(7)

\[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \frac{1}{Pr} \frac{1}{\alpha_f} \frac{\partial^2 T}{\partial x^2} + \frac{1}{Pr} \frac{1}{\alpha_f} \frac{\partial^2 T}{\partial y^2} + \frac{1}{Pr} \frac{1}{\alpha_f} \frac{\partial^2 T}{\partial z^2} \]

(8)

subject to boundary conditions

\( t < 0, u = v = 0, T = 0 \) for all \( x, y \)

\( t > 0, u = v = 0, T = 1 \) for \( y = 0 \)

Further we transform those equations into stream function, we therefore introduce stream function \( \psi \) defined as [5]:

\[ u = \frac{1}{r} \frac{\partial \psi}{\partial y}, \quad v = -\frac{1}{r} \frac{\partial \psi}{\partial x} \]

(9)

By substituting both \( u \) and \( v \) into 7 and 8, we obtain

\[ \frac{1}{r} \frac{\partial^2 \psi}{\partial \theta \partial \theta} + \frac{1}{r^2} \frac{\partial \psi}{\partial \theta} \frac{\partial^2 \psi}{\partial \theta \partial \theta} - \frac{1}{r^2} \frac{\partial \psi}{\partial \theta} \frac{\partial^2 \psi}{\partial \theta \partial \theta} = \frac{1}{r^2} \frac{\partial \psi}{\partial \theta} \frac{\partial^2 \psi}{\partial \theta \partial \theta} + (M + \phi) \left( \frac{1}{r^2} \frac{\partial \psi}{\partial \theta} - u_e \right) + \lambda T \tan (\frac{x \cos x}{\cos \theta_s}) \]

(10)

\[ \frac{\partial T}{\partial t} + \frac{1}{r} \frac{\partial \psi}{\partial \theta} \frac{\partial T}{\partial \theta} - \frac{1}{r} \frac{\partial \psi}{\partial \theta} \frac{\partial T}{\partial \theta} = \frac{\alpha_n f}{\alpha_f} \frac{1}{Pr} \frac{\partial^2 T}{\partial \theta^2} \]

(11)
with respect to the following boundary conditions.

\( t < 0 : \psi = \frac{\partial \psi}{\partial y} = T \) for all \( x, y \)

\( t \geq 0 : \psi = \frac{\partial \psi}{\partial y} = 0, T = 1 \) for \( y = 0 \)

\( \frac{\partial \psi}{\partial y} = u_e(x) r(x), T = 0 \) for \( y = 0 \)

Then, we again introduce some unsimilarity equations for small time \( t \leq t^* \) to easily solve it:

\[
\psi = t^2 u_e(x) r(x) f(x, \eta, t), \quad T = s(x, \eta, t) \quad \eta = \frac{y}{t^2} \tag{12}
\]

and for large time \( t > t^* : \)

\[
\psi = t^4 u_e(x) r(x) F(x, Y, t), \quad T = S(x, Y, t) \quad Y = y \tag{13}
\]

By substituting the unsimilarity equations into equation 10 and 11, we obtain equations for lower stagnation point \( x = 0 \) on term of small time:

\[
-\eta \frac{\partial^2 f}{2 \partial \eta^2} + t \frac{\partial^2 f}{\partial \eta \partial t} + \frac{3t}{2 \cos \theta_s} \left( \frac{\partial f}{\partial \eta} \right)^2 - \frac{3t}{2 \cos \theta_s} \frac{\partial^2 f}{\partial \eta^2} = \frac{3t}{2 \cos \theta_s} + \frac{v_{nf}}{v_f} \frac{\partial^3 f}{\partial \eta^3} \tag{14}
\]

\[
\frac{\alpha_{nf}}{\alpha_f} \frac{\partial^2 s}{\partial \eta^2} + \frac{Pr \eta}{2} \frac{\partial s}{\partial \eta} + \frac{3}{2 \cos \theta_s} Pr f \frac{\partial s}{\partial \eta} = Pr \frac{\partial S}{\partial t} \tag{15}
\]

with respect to the following boundary equation

\( t < 0 : f' = s = 0 \) for all \( x, \eta \)

\( t \geq 0 : f = f' = 0, s = 1 \) for \( \eta = 0 \)

\( f' = 1, s = 0 \) for \( \eta \to \infty \)

For large time we obtain

\[
\frac{v_{nf}}{v_f} \frac{\partial^3 F}{\partial Y^3} + \frac{3}{2 \cos \theta_s} \left[ 1 - \left( \frac{\partial F}{\partial Y} \right)^2 + F \frac{\partial^2 F}{\partial Y^2} \right] = \frac{\partial^2 F}{\partial t \partial Y} - (M + \phi) \left( \frac{\partial F}{\partial Y} - 1 \right) - \frac{2}{3} \lambda S \tag{16}
\]

\[
\frac{\alpha_{nf}}{\alpha_f} \frac{\partial^2 S}{\partial \eta^2} + Pr \frac{3}{2 \cos \theta_s} F \frac{\partial s}{\partial \eta} \frac{\partial S}{\partial t} = Pr \frac{\partial S}{\partial t} \tag{17}
\]

with respect to the following boundary conditions as below

\( F = \frac{\partial F}{\partial Y} = 0, S = 1 \) for \( Y = 0 \)

\( \frac{\partial F}{\partial Y} = 1, S = 0 \) for \( Y \to \infty \)

Further, we denote that \( \frac{\partial f}{\partial \eta} = f' \) and \( \frac{\partial s}{\partial \eta} = s' \), then equations 14 and 15 can be written as:

\[
\frac{v_{nf}}{v_f} f'' + \frac{3}{2 \cos \theta_s} t \left[ 1 - f' + f f'' \right] + \frac{\eta}{2} f'' + t(M + \phi)(f' - 1) + \frac{2}{3} t \lambda s = \frac{t f'}{\partial t} \tag{18}
\]

\[
\frac{\alpha_{nf}}{\alpha_f} s'' + Pr \frac{s'}{2} + \frac{3}{2} Pr f s' = Pr \frac{\partial s}{\partial t} \tag{19}
\]

According to [6], the density, viscosity, speisic heat, and thermal conductivity of nano fluid respectively defined as follows

**Density** : \( \rho_{nf} = (1 - \chi) \rho_f + \chi \rho_s \)

**Viscosity** : \( \mu_{nf} = \frac{\mu_f}{(1 - \chi)^{2.5}} \)

**Specific heat** : \( (\rho C_p)_{nf} = (1 - \chi) (\rho C_p)_f + \chi (\rho C_p)_s \)

**Thermal conductivity** :

\[
k_{nf} \frac{k_f}{k_f} = \frac{(k_s + 2k_f) - 2\chi (k_f - k_s)}{(k_s + 2k_f) + \chi (k_f - k_s)}
\]
Therefore, the ratio of $v_{nf}$ with $v_f$ and $\alpha_{nf}$ with $v_f$ can be written as

$$\frac{v_{nf}}{v_f} = \frac{1}{(1 - \chi)^{2.5} \left( (1 - \chi) + \chi \left( \frac{\rho_s}{\rho_f} \right) \right)}$$

$$\frac{\alpha_{nf}}{\alpha_f} = \frac{(k_s + 2k_f) - 2\chi(k_f - k_s)}{(k_s + 2k_f) + \chi(k_f - k_s)} \frac{1}{(1 - \chi) + \chi \left( \frac{\rho_s}{\rho_f} \right) \left( \frac{\rho_s}{\rho_f} \right)}$$

Furthermore, the Equations (18) and (19) solved numerically by using Keller-Box method. First, (18) and (19) reduced to first order ordinary differential equations. Then we do discretization using finite difference method to discretize the first order equations. Further, by using the Newton’s Method the resulting algebraic equations linearized and written in matrix-vector form. The last step is solving the linear system by using the technique of tridiagonal block elimination [3]

3. Result and Discussion

In this paper, the $Al_2O_3$-Water nanofluid flow past a sliced magnetic porous sphere under the influence of mixed convection problem is solved numerically by using Keller-Box Scheme. The aims of this paper is to investigate the velocity and temperature of $Al_2O_3$-Water nanofluid flow with the variation of magnetic parameter ($M$), angle of sliced ($\theta_s$), porosity ($\phi$), and volume fraction parameter ($\chi$).

The simulation results of the velocity and temperature of the unsteady $Al_2O_3$-Water nanofluid flow at various value of magnetic and porosity parameter are illustrated in Figures 2 and 3. These results have been obtained at fixed values of $\lambda = 2$, $Pr = 0.7$, and $\chi = 0.1$.

![Figure 2: (a) Velocity, and (b) Temperature with various magnetic parameter](image)

The variation of magnetic parameter that use in this research is $M=1.3$ (iron), 1.8 (Cobalt), 2 (Steel) and 2.3 (Zinc). Figure (2a) shows that the increases of the velocity is at $f' = 0$ to $f' = 1$. The velocity of unsteady $Al_2O_3$-Water nanofluid decreases when magnetic parameter increases. This happens because by the Lorentz force ($M$) is proportional to magnetic field $B_0$. Figure (2b) shows that the decreases of the temperature is at $s = 1$ to $s = 0$. Furthermore, the temperature of unsteady $Al_2O_3$-Water nanofluid increases when the magnetic parameter increases. Figure (3a) shows that the velocity decreases when the parameter porosity increases. This is because the porosity parameter is proportional to the dynamic viscosity of nanofluid and
Figure 3: (a) Velocity, and (b) Temperature with various porosity parameter inversely proportional to the density of nanofluid. Figure (3b) shows that the temperature increases when the porosity parameter increases. In addition, with a greater parameter value of porosity, it can be seen that the fluid temperature is lower.

Figure 4: (a) Velocity, and (b) Temperature with various slicing angle parameter

From Figure (4a) the velocity of the fluid increases with increasing slice angle. This is because the surface of the sliced sphere gets wider, resulting in a decrease in fraction between the fluid and the surface so that the velocity increases, Figure (4b) shows that the fluid temperature decreases with increasing slicing angle. This occurs because the greater of $\theta_s$ can cause the surface to be wider which impacts in the distribution of heat of the sphere faster than the distribution of heat of the fluid so that the temperature becomes decreases.

From Figure 5(a) the velocity of the fluid decreases with increasing porosity. This happens because the increased volume fraction makes more and more particles inside fluid, thus creating greater friction between particles in the liquid. Figure 5(b) shows that the fluid temperature increases when the volume fraction increases.
Figure 5: (a) Velocity, and (b) Temperature with various volume fraction parameter

4. Conclusion
The study of Numerical Solution of Unsteady Al₂O₃-Water Nanofluid Flow Past A Magnetic Porous Sliced Sphere was already studied. We conclude that when magnetic parameter increases then fluid velocity decreases but fluid temperature increases. For the porosity parameter increases then the velocity of the fluid decreases but the temperature of the fluid increases. For the sliced angle parameter increases then the velocity of the fluid increases but temperature of the fluid decrease. For the volume fraction parameter increases then the fluid velocity decreases but the fluid temperature increases.

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