Theoretical Modelling of Sound Radiation from Plate

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Abstract. Recently the development of aerospace, automotive and building industries demands the use of lightweight materials such as thin plates. However, the plates can possibly add to significant vibration and sound radiation, which eventually lead to increased noise in the community. So, in this study, the fundamental concept of sound pressure radiated from a simply-supported thin plate (SSP) was analyzed using the derivation of mathematical equations and numerical simulation of ANSYS®. The solution to mathematical equations of sound radiated from a SSP was visualized using MATLAB®. The responses of sound pressure level were measured at far field as well as near field in the frequency range of 0–200 Hz. Result shows that there are four resonance frequencies; 12 Hz, 60 Hz, 106 Hz and 158 Hz were identified which represented by the total number of the peaks in the frequency response function graph. The outcome also indicates that the mathematical derivation correlated well with the simulation model of ANSYS® in which the error found is less than 10%. It can be concluded that the obtained model is reliable and can be applied for further analysis such as to reduce noise emitted from a vibrating thin plate.

1. Introduction
Formerly about 4 decades ago, the vibrations of complex engineering systems was done using rough model with only a few degrees of freedom. However, since the advent of high-speed digital and interactive computers in the early 2000s had managed to produce an approximate solution that can display thousands of degrees of freedom efficiently through matrices. The concurrent development of the finite element method allows engineers to use the digital computer to perform a detailed vibration analysis of mechanical systems and complex structure [1,2]. According to Clough [3], the finite element method as known today was introduced by Turner, Clough, Martin and Topp through their research related to analysis of aircraft structures.

In engineering and construction industry, machines or equipment are certainly tool that used to help people to accomplish the objectives as planned by the company. Nevertheless, the use of process technology produces a huge problem to vibration and noise [4,5]. For example in the event of machinery and equipment are not treated, it can emit considerable sound power. In this case, workers who operating the equipment and machines as well as its surrounded people will be directly exposed to vibration and noise affection. For the employee, it can cause the loss of efficiency and discomfort.
Moreover, it also affects the public and the environment, and at the same time can bring harm to the machine itself.

Meanwhile, plate is one of the most extensively used structures in industrial due to its lightweight and economy advantages. Some of the example application of plates can be found in aircraft, automobile, machinery and modern houses. Despite this advantage, the utilization of plate has potential to contribute substantial vibration and sound radiation which leads to excessive noise. Putra and Thompson [6] in his study state the vibration of engineering structures, particularly those consisting of thin plate-like members can be a significant source of noise. This is why the study of sound radiation of a plate become of great interest to both researchers and practicing engineers since the noise produced from plate can be meaningfully reduced. In study by Snyder and Tanaka [7] stated that acoustic radiation from plates with various boundary conditions and loading features can be minimized by altering the relative amplitudes of the vibration modes. Similar study was carried out by Park et al. [8] who observed that the sound emitted from plate can be weaken by modifying the boundary support configurations.

Therefore, this paper strive to present details derivation of a mathematical model which allows the calculation of frequency response function and sound radiation of a simply-supported plate. The outcome of this study will provide a helpful reference to future researchers who endeavor to find an equation for sound radiation of simply-supported plate and use it for further analysis in sound reduction.

2. Mathematical Modelling
This section describes the derivation of a mathematical model which allows the calculation of sound radiation of a simply-supported plate. At first, the mathematical equation for dynamic response of a simply-supported plate is derived, followed by the sound radiation from a simply-supported plate.

2.1. Simply-Supported Plate (SSP)
The equation of motion of a simply supported plate is written as:

\[ EI \left( \frac{\partial^4 \omega}{\partial x^4} + 2 \frac{\partial^4 \omega}{\partial x^2 \partial y^2} + \frac{\partial^4 \omega}{\partial y^4} \right) + \rho h \frac{\partial^2 \omega}{\partial t^2} = -F(x, y, t) \]  

(1)

where \( E \) is the Young’s modulus, \( I \) is the area moment of inertia, \( \rho \) is the density of plate and \( h \) is the thickness of plate. Eq. (2) defined the area moment of inertia for plate, where \( \nu \) is the Poisson’s ratio.

\[ I = \frac{h^3}{12(1-\nu^2)} \]  

(2)

The solution of transverse modal displacement for a plate is given by Eq. (3) which is the summation of all individual modal amplitude responses multiplied by their mode shapes at that point.

\[ \omega(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \Psi_{mn}(x, y) e^{i\omega_{mn}t} \]  

(3)

where \( W_{mn} \) is the modal amplitude, \( \Psi_{mn}(x, y) \) is the mode shape of plate, and \( m \) and \( n \) are modal integers.

The general mode shape of a simply-supported plate can be calculated with:

\[ \Psi_{mn}(x, y) = 2 \sin \left( \frac{m\pi x}{a} \right) \sin \left( \frac{n\pi y}{b} \right) \]  

(4)

where \( a \) and \( b \) are the length and width of a plate, respectively.
The natural frequencies of a simply-supported plate can be calculated from:

$$\omega_n = \sqrt{\frac{EI}{\rho h}} \left[ \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \right]$$  \hspace{1cm} (5)

By neglecting the exponential time varying term, an expression of the total response if simply-supported plate incorporating the viscous damping $\zeta$ is given in Eq. (6) as derived in our previous study [9,10].

$$\omega(x,y,t) = \frac{F}{\rho hab} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \psi_{mn}(x,y)\psi_{mn}(x_0,y_0)$$ \hspace{1cm} (6)

2.2. Sound Radiation by a Simply-Supported Plate

The simplest approach to calculate the sound field radiated by a vibrating surface that is surrounded by a rigid infinite panel is the evaluation of the Rayleigh Integral, which given as follows [11]:

$$p(x_0,y_0,t_0) = \frac{j\omega_p}{2\pi r} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \psi_{mn}(x_0,y_0)$$

\hspace{1cm} \times \exp(i\omega t) \exp(-jkR) \frac{dS}{2\pi r} \hspace{1cm} (7)

where $\omega(x,y,z)$ is the component of the complex velocity normal to the surface, $\rho_v$ is the density of the acoustic medium, $\omega$ is the frequency in rad/s, $r$ is the distance from the observation point $(x_0,y_0,z_0)$ to the coordinate origin and $R^2 = (x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2$.

The classical assumption made in order to evaluate the far-field pressure is that the value of $R$ is approximately by [11]:

$$R \approx r - x \sin \theta \cos \phi - y \sin \theta \sin \phi$$ \hspace{1cm} (8)

where $x$ and $y$ define the coordinate position on the plate and $(r, \theta, \phi)$ are the coordinate of the field point. This assumption is valid for provided $R \geq a,b$.

A particular form of out-of-plane vibration for a simply-supported rectangular plate with the above assumptions, leads to an analytically tractable form of equation which is given by:

$$\omega(x,y,z) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin \left( \frac{m\pi x}{a} \right) \sin \left( \frac{n\pi y}{b} \right)$$ \hspace{1cm} (9)

By substituting Eq. (8) into Eq. (9), the sound pressure radiated by a simply-supported plate in an infinite baffle then can be written as:

$$p(r,\theta,\phi) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{j\omega_p}{2\pi r} \psi_{mn}(x_0,y_0)$$

\hspace{1cm} \times \exp(i\omega t) \exp(-jkR) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \exp \left[ j \left( \frac{\alpha x}{a} + \frac{\beta y}{b} \right) \right] dx dy \hspace{1cm} (10)

where $\alpha = k \sin \theta \cos \phi$ and $\beta = k \sin \theta \sin \phi$. 


This integral equation has been evaluated by [12] who gives the solution:

\[ p(r, \theta, \phi) = \frac{j \omega p_a W_{na}}{2\pi} \exp(j \omega t) \exp(-j kR) \cdot \frac{ab}{mn\pi^2} \times \left[ (-1)^n \exp(-j \alpha) - 1 \right] \times \left[ (-1)^n \exp(-j \beta) - 1 \right] \]  \hspace{1cm} (11)

3. Result and Discussion
A sound radiation from SSP in an infinite baffle subjected to a harmonic load \( F(x_i, y_i, z_i) \) is shown in Figure 1. The point load, \( F \) (1N) is applied at the center of plate \((0,0,0)\) which has a uniform thickness, \( h \); width, \( a \); and length, \( b \). In this analysis, the sound pressure \( p(x_\circ, y_\circ, z_\circ) \) radiated from vibrating plate was determined by using finite element analysis (FEA) and mathematical model. The FEA was carried out using ANSYS® and mathematical model was plotted in MATLAB®. The frequency range was set between 0 to 200 Hz with approximately step size of 2 Hz increments. The responses of sound pressure level were measured at far field coordinates \((0,0,0.8)\) and near field \((0,0,0.4)\). The parameters of SSP and air used in this analysis are shown in Tables 1 and 2, respectively.

![Figure 1. Starting coordinate and size of plate](image)

**Table 1. Parameters of SSP structure**

| Parameters | Description         | Value       | Units |
|------------|---------------------|-------------|-------|
| \( A \)    | Length              | 0.2         | m     |
| \( B \)    | Width               | 0.2         | m     |
| \( H \)    | Thickness           | 0.0001      | m     |
| \( I \)    | Area moment of inertia | \( 9.2 \times 10^4 \) | m²    |
| \( E \)    | Young’s modulus     | 2.1 \times 10^2 | GPa   |
| \( \nu \)  | Poisson ratio       | 0.3         | -     |
| \( \rho \) | Density             | 7.85 \times 10^3 | kg/m³ |
| \( \zeta \) | Damping ratio       | 1           | %     |
### Table 2. Parameters of air medium

| Parameters   | Description            | Value  | Units   |
|--------------|------------------------|--------|---------|
| $\rho_a$     | Density                | 1.21   | kg/m$^3$|
| $C$          | Speed of sound         | 95450.0| m/s     |
| $R_a$        | Radius of infinite acoustic elements | 1.0 | m       |

Figures 2 and 3 show the comparison graph of sound pressure level radiated from SSP by using ANSYS® and mathematical model of MATLAB® in which the pressure measured at the respective far field and near field. Obviously, it can be seen that the pressure level increases when the sound was measured at near field [11]. Both of the graph show almost identical pattern which indicate that the mathematical model derived in Eq. (11) is in a good agreement with FEA result of ANSYS®.

![Figure 2](image1.png)

**Figure 2.** Graph comparison between ANSYS® and MATLAB® on point (0,0,0.8)

![Figure 3](image2.png)

**Figure 3.** Graph comparison between ANSYS® and MATLAB® on point (0,0,0.4)
There are four resonance frequencies were identified in the frequency range of 0–200 Hz which represented by the total number of the peaks shown in Figures 2 and 3. Table 3 tabulates the value of the natural frequencies of sound radiation from SSP which determined directly from frequencies where the peaks are occurred. These natural frequencies are found similar for both sound pressure measurement at far field (0,0,0.8) and near field (0,0,0.4). The errors between the result obtained from ANSYS® and MATLAB® are found below 10% which again demonstrate the reliability of mathematical model derived in the study.

Table 3. Errors of sound radiation from SSP between theory and FEM at (0,0,0.8) and (0,0,0.4)

| Mode n<sup>th</sup> | ANSYS® | MATLAB® | Error (%) |
|---------------------|---------|----------|-----------|
| 1                   | 12      | 13.06    | 8.83      |
| 2                   | 60      | 61.3     | 2.17      |
| 3                   | 106     | 109.5    | 3.30      |
| 4                   | 158     | 159.8    | 1.14      |

4. Conclusion
A mathematical model was developed to enable the prediction of sound pressure level radiated from a simply-supported plate. Later, a comparison with finite element analysis of ANSYS® is performed. The outcome displays errors difference between FEA and mathematical model are less than 10% which can be negligible.

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