Large photon productions in a gravitational collapsing

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We study a possible gravitational vacuum-effect, in which vacuum-energy variation is due to variation of gravitational field, vacuum state gains gravitational energy and releases it by spontaneous photon emissions. Based on the path-integral representation, we present a general formulation of vacuum transition matrix and energy-momentum tensor of a quantum scalar field theory in curved spacetime. Using analytical continuation of dimensionality of the phase space, we calculate the difference of vacuum-energy densities in the presence and absence of gravitational field. Using the dynamical equation of gravitational collapse, we compute the rate of vacuum state gaining gravitational energy. Computing the transition amplitude from initial vacuum state to final vacuum state in gravitational collapsing process, we show the rate and spectrum of spontaneous photon emissions for releasing gravitational energy. The possible connection of our study to the genuine origin of gamma ray bursts is discussed. We compare our idea with the Schwinger idea for Sonoluminiscence and contrast our scenario with the Hawking effect.

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I. INTRODUCTION

The issue of quantum field theories of elementary particles in curved spacetime has played a tremendously important role in understanding quantum phenomenon of particle creations from the vacuum, when gravitational field is present. In quantum field theories, the vacuum state consists of a large number of virtual particles, that are quantum-field fluctuations whose energy-momentum are off-shell. The vacuum energy (zero-point energy) is attributed to the energy-momentum of virtual particles. These virtual particles interact with a external gravitational field via their energy-momentum. Such interaction results in vacuum-energy variations and particle creations from the vacuum.

The Hawking radiation[1, 2] is the phenomenon of particle creations occurring around black hole’s horizon. The quantum radiation of the Hawking type is rather general in curved spacetime[3]. Although, such quantum radiation is too small to be detected in the present Universe, its impact

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on theoretical understanding of particle creations due to a static gravitational field is indeed far reaching. On the other hand, it is important to study the phenomenon of particle creations in a non-static curved spacetime. In fact, the rate and spectrum of particle and antiparticle creations in extremely early Universe are possibly related to the CMB pattern and the large-scale structure of the present Universe, which are most exciting arena of theoretical and observational physics today. Beside, in a gravitational collapse process, violent variation of gravitational field possibly causes a large number of particle creations, which might be account for most energetical events of gamma ray bursts. This is the topic that we attempt to further discuss in this article.

The phenomenon of particle creations from the vacuum was first studied by Euler, Heisenberg and Schwinger in the quantum field theory of electromagnetic dynamics (QED). A strong external electric field, its strength is larger than \( m_e^2 c^3 / \hbar \), greatly reduces the energy-mass gap \( \sim 2m_e \) of charged virtual particles in the QED vacuum, so that virtual particles undergo a quantum tunneling process, leading to large pair-productions of electrons and positrons. The mechanism and phenomenon of electron and positron pair-productions have advocated a numerous studies both in experimental physics and theoretical physics, as well as in astrophysics. In addition, it is worthwhile to mention that an external magnetic field can induce vacuum decay leading to spontaneous photon emissions. The dynamics of this effect, which is completely different from the Schwinger one, is that the external magnetic field modifies the vacuum-energy spectrum so that the vacuum state gains magnetic energy and becomes unstable. This effect can be possibly tested in laboratories and account for the phenomenon of anomalous x-ray emission from pulsars.

Casimir first considered that boundary conditions (two conducting plates) modify the energy spectrum of the vacuum state and vacuum-energy variation is the Casimir energy \( \delta E_{\text{casimir}} \). An attractive force between two plates is observed as the Casimir effect. The reasons for the Casimir effect are that the vacuum state gains the Casimir energy and becomes energetically unstable, quantum-field fluctuations result in releasing the Casimir energy. Recently, there is much theoretical and experimental attention on the dynamical Casimir effect. In such an effect, boundary conditions are dynamically time-dependent, the Casimir energy is released by spontaneous photon emissions.

Sonoluminescence is another most interesting vacuum effect which shows a flash of spontaneous photon emissions, when gas bubbles in water collapse, driven by a sound-wave. The vacuum-energy variation is due to the variation of dielectric constant, rather than the modification of boundary conditions as in the Casimir effect. We will have a more detailed discussion in this
The mystery of energetic sources generating gamma ray bursts\cite{16, 20} is a prompt emission (seconds) of extremely huge energy output ($\sim 10^{54}$ergs for isotropic emission) from rather compact sources ($\sim 10^8$cm) at cosmological distance ($z \sim O(1)$). These have stimulated many studies in connection with electromagnetic properties of black holes\cite{8, 11, 18}. Various astrophysical scenarios are discussed in literatures\cite{19}. It is worthwhile to mention that via the Schwinger mechanism of electron-positron pair productions, the “dyadosphere”\cite{8, 20} of photons, electrons and positrons, is formed during the process of gravitational collapse of a massive star with electromagnetic structure.

In this article, we want to study a possible gravitational vacuum-effect, in which vacuum-energy variation is due to the variation of gravitational field, vacuum state gains gravitational energy and releases it by spontaneous photon emissions. In section (II), using path-integral representation, we present a general formulation of quantum scalar field theories in curved spacetime, where the vacuum states are defined, the transition matrix from vacuum to vacuum, vacuum-energy spectrum and vacuum energy-momentum tensor are obtained. In sections (III) and (IV), we specify two static observers respectively in the Schwarzschild and flat spacetime; we analyze the eigenvalues (spectrum) of the transition matrix and define the phase space of vacuum states; by using analytical continuation of dimensionality of the phase space, we obtain vacuum-energy density. In sections (V) and (VI), we calculate vacuum-energy density and discuss why the characteristic energy-scale should be the ultraviolet cutoff for considering the difference of vacuum-energy densities in the presence and absence of gravitational field. In section (VII), we define vacuum energy with respect to two static observers and compute the difference of vacuum energies in the presence and absence of gravitational field, which shows vacuum state gains gravitational energy. In section (VIII), we adopt a simple model describing the gravitational collapse of a massive shell that is infinitesimally thin, and using the dynamical equation of gravitational collapse, we compute the rate of vacuum state gaining gravitational energy. In section (IX), based on the transition amplitude from initial vacuum state to final vacuum state at each step of gravitational collapsing process, we compute the rate and spectrum of spontaneous photon emissions for releasing gravitational energy. In sections (X) and (XI), we compare our proposal for gamma ray bursts with the Schwinger proposal for Sonolumininescence and contrast our scenario with the Hawking effect. In the final section (XII), we make some remarks on this preliminary study and the possible connection of our proposal to the genuine origin of gamma ray bursts.
II. GENERAL FORMULATION.

We assume that the structure of spacetime is described by the pseudo-Riemannian metric $g_{\mu\nu}$ associated with the line element

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \quad \mu, \nu = 0, 1, 2, 3,$$

(1)

and the spacetime point is coordinated by $x = (x^0, x^i) = (t, \vec{x})$. The special geometrical symmetries of the spacetime $\mathcal{S}$ are described by using Killing vectors $\xi^\mu$, which are solutions of Killing’s equation

$$L_\xi g_{\mu\nu}(x) = 0, \quad \xi_{\mu;\nu} + \xi_{\nu;\mu} = 0,$$

(2)

where $L_\xi$ is the Lie derivative along the vector field $\xi^\mu$, orthogonal to the spacelike hypersurface $\Sigma_t \ (t = \text{constant})$ of the spacetime $\mathcal{S}$. A static observer $\mathcal{O}$ is at rest in this hypersurface $\Sigma_t$.

We consider that at the initial time ($t_{\text{in}} = -\delta t/2$), the spacetime is asymptotically flat, described by asymptotically flat geometry $\bar{g}_{\mu\nu}$ in Eq.(1); while at the final time ($t_{\text{out}} = +\delta t/2$), the spacetime is curved and stationary, described by a non-trivial geometry $g_{\mu\nu}$ in Eq.(1). The characteristic time-scale $\delta t$ of such variation of spacetime geometry is supposed to be much larger than the characteristic time-scale (e.g., $1/m_e$) of a quantum-field transition, i.e., $\delta t \gg 1$ in the unit of the quantum time-scale. We attempt to study quantum-field fluctuations interacting with the variation of spacetime geometry. A specific model for such variation of spacetime geometry and quantum-field transition will be presented in due course.

In order to clearly illustrate the physics content, we first consider a complex scalar field $\phi$ in curved spacetime. The simplest coordinate-invariant action is given by ($\hbar = c = G = k = 1$)

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[ g^{\mu\nu} \phi_{,\mu} \phi_{,\nu}^* + (m^2 + \xi \mathcal{R}) \phi \phi^* \right],$$

(3)

where $m$ is particle mass and $\mathcal{R}$ the Riemann scalar. The quantum scalar field $\phi$ can be in principle expressed in terms of a complete and orthogonal basis of quantum-field states $u_k(x)$:

$$\phi(x) = \sum_k \left( a_k u_k(x) + a_k^\dagger u_k^*(x) \right), \quad \left[ a_k, a_{k'}^\dagger \right] = \delta_{k,k'},$$

(4)

where $a_k^\dagger$ and $a_k$ are creation and annihilation operators of the $k$-th quantum-field state $u_k(x)$. This quantum field state obeys the following equation of motion,

$$(\Delta_x + m^2 + \xi \mathcal{R}) u_k(x) = 0,$$

(5)
and appropriate boundary conditions for selected values of \( k \). In Eq. (5), \( \Delta_x \) is the Laplacian operator in curved spacetime:

\[
\Delta_x = \frac{1}{\sqrt{-g}} \partial_{\mu} \left[ \sqrt{-g} g^{\mu\nu} \partial_{\nu} \right].
\] (6)

In Eq. (4), we assume that \( u_k(x) \) are positive energy \((\omega)\) states, satisfying

\[
\mathcal{L}_\xi u_k(x) = -i\omega u_k(x), \quad \omega > 0,
\] (7)

with respect to the timelike Killing vector field \( \xi^\mu \) [2] associated to the static observer \( O \) rest in hypersurface \( \Sigma_t (t = \text{constant}) \) of the spacetime \( S \).

At the initial time \( (t_{in} = -\delta t/2) \), the spacetime is approximately flat, quantum-field states \( u_k(x) \) [5] are asymptotically free states \( \bar{u}_k(x) \), obeying Eq. (5) for the asymptotically free geometry \( \bar{g}_{\mu\nu} \). Then, the quantum scalar field \( \phi(x) \) is an asymptotically free field in the hypersurface \( \Sigma_{-\delta t/2} \) of the asymptotically free spacetime \( \tilde{S} \):

\[
\phi_{in}(x) = \sum_k \left( \bar{a}_k \bar{u}_k(x) + \bar{a}_k^\dagger \bar{u}_k^*(x) \right), \quad \left[ \bar{a}_k, \bar{a}_{k'}^\dagger \right] = \delta_{k,k'}
\] (8)

where \( \bar{x} \in \Sigma_{-\delta t/2}, \bar{a}_k^\dagger \) and \( \bar{a}_k \) are creation and annihilation operators of the \( k \)-th asymptotically free quantum-field state \( \bar{u}_k(x) \). Corresponding Lie derivative along the Killing vector is \( \xi^\mu \) [2] is \( \partial_t \), positive energy states are \( \bar{u}_k(x) \), satisfying Eq. (7). Then we may construct the standard Minkowski space quantum vacuum state \( |\bar{0}, \text{in}\rangle \):

\[
\bar{a}_k |\bar{0}, \text{in}\rangle = 0, \quad \langle \bar{0}, \text{in}|\bar{a}_k^\dagger = 0.
\] (9)

\( |\bar{0}, \text{in}\rangle \) is an initial quantum vacuum state at the initial time \( (t_{in} = -\delta t/2) \) with respect to the static observer \( O \) rest in hypersurface \( \Sigma_{-\delta t/2} \) of the spacetime \( \tilde{S} \).

At the final time \( (t_{out} = +\delta t/2) \), the spacetime \( S \) is curved, described by a stationary non-trivial geometry \( g_{\mu\nu} \). We assume that quantum-field states \( u_k(x) \) [5] are asymptotical states \( \tilde{u}_k(x) \), obeying Eq. (5) for the stationary geometry \( g_{\mu\nu} \). In the hypersurface \( \Sigma_{+\delta t/2} \) of the spacetime \( S \), the asymptotical quantum scalar field \( \phi_{out} \) is expressed in terms of \( \tilde{u}_k(x) \):

\[
\phi_{out}(x) = \sum_k \left( \tilde{a}_k \tilde{u}_k(x) + \tilde{a}_k^\dagger \tilde{u}_k^*(x) \right), \quad \left[ \tilde{a}_k, \tilde{a}_{k'}^\dagger \right] = \delta_{k,k'}
\] (10)

where \( \tilde{x} \in \Sigma_{+\delta t/2}, \tilde{a}_k^\dagger \) and \( \tilde{a}_k \) are creation and annihilation operators of the \( k \)-th quantum-field state \( \tilde{u}_k(x) \). Corresponding Lie derivative along the Killing vector is \( \xi^\mu \) [2], positive energy states are \( \tilde{u}_k(x) \) satisfying Eq. (7). Then we may construct the quantum vacuum state \( |\tilde{0}, \text{out}\rangle \):

\[
\tilde{a}_k |\tilde{0}, \text{out}\rangle = 0, \quad \langle \tilde{0}, \text{out}|\tilde{a}_k^\dagger = 0,
\] (11)
in curved spacetime. $|\tilde{0},\text{out}\rangle$ is an final quantum vacuum state at the final time ($t_{\text{out}} = +\delta t/2$) with respect to the same static observer $O$ rest in hypersurface $\Sigma_{+\delta t/2}$ of the spacetime $S$.

It is worthwhile to note that $\phi_{\text{out}}(x)(\{\tilde{u}_k(x)\})$ are not asymptotically free field(states), instead they are asymptotical field(states) in the presence of external and stationary gravitational field, so that the final vacuum state $|\tilde{0},\text{out}\rangle$ (11) is different from the initial vacuum state $|\bar{0},\text{in}\rangle$ (9). Such a difference is not only a unitary phase. The final vacuum state $|\tilde{0},\text{out}\rangle$ (11), may not necessarily be measured as devoid of particles, in contrast to the initial vacuum state $|\bar{0},\text{in}\rangle$ defined by Eq. (9) relating to the asymptotically free field $\phi_{\text{in}}(x)$. In fact, as will be shown, the final vacuum state $|\tilde{0},\text{out}\rangle$ is a quantum-field state of particle and antiparticle creations upon the initial vacuum state $|\bar{0},\text{in}\rangle$. This indicates gravitational field interacting with quantum-field fluctuations of positive and negative energy states of the initial vacuum $|\bar{0},\text{in}\rangle$, and the quantum scalar field evolves throughout intermediate quantum-field states $\phi(\vec{x}, t)$ (12) for $-\delta t/2 < t < +\delta t/2$. We speculate that this evolution is adiabatic for $\delta t$ being much larger than quantum-field transition time.

To deal with all possible intermediate states, represented by $\phi(x)$ or $u_k(x)$ in Eq. (11) for $-\delta t/2 < t < +\delta t/2$, we use path-integral representation to study the transition amplitude between the initial vacuum state and final vacuum state:

$$\langle \tilde{0}, \text{out} | \bar{0}, \text{in} \rangle = \int [D\phi D\phi^*] \exp(iS),$$

(12)

where

$$\int [D\phi D\phi^*] = \Pi_{-\delta t/2 < t < +\delta t/2} \Pi_{\vec{x} \in \Sigma_t} \int [d\phi(\vec{x}, t)\phi^*(\vec{x}, t)].$$

(13)

The intermediate quantum-field states contributions to the transition amplitude (12) can be formally path-integrated,

$$\langle \tilde{0}, \text{out} | \bar{0}, \text{in} \rangle = \det^{-1}(\mathcal{M}), \quad \mathcal{M} = \Delta_x + m^2 + \xi \mathcal{R}.$$

(14)

This result clearly depends on the initial vacuum state $\phi_{\text{in}}$ (8) and final vacuum state $\phi_{\text{out}}$ (10), which are not explicitly written. The effective action $S_{\text{eff}}$ is defined as

$$S_{\text{eff}} = -i \ln \langle \tilde{0}, \text{out} | \bar{0}, \text{in} \rangle,$$

(15)

which relates to the phase of the $S$-matrix transition from the initial vacuum state $|\bar{0},\text{in}\rangle$ to the final vacuum state $|\tilde{0},\text{out}\rangle$. The averaged energy-momentum tensor $\langle T_{\mu\nu} \rangle$ of the quantum-field vacuum is given by:

$$\langle T^{\mu\nu}(x) \rangle = \frac{2}{\sqrt{-g}} \frac{\delta S_{\text{eff}}}{\delta g_{\mu\nu}(x)}.$$

(16)
Main efforts are calculations of the effective action (15) and energy-momentum tensor (16) in this article.

In order to evaluate the path-integral (12) over all intermediate quantum-field states $\phi(x)$, it is convenient to introduce operators $\hat{X}_\mu$ and $\hat{K}_\mu$ defined on the states $|x\rangle$ and $|k\rangle$:

$$\hat{X}_\mu |x\rangle = x_\mu |x\rangle; \quad \hat{K}_\mu |k\rangle = k_\mu |k\rangle. \quad (17)$$

They enjoy the canonical communication,

$$[\hat{X}_\mu, \hat{K}_\nu] = -ig_{\mu\nu}. \quad (18)$$

The states $|x\rangle$ and $|k\rangle$ satisfy:

$$\langle x|x' \rangle = \delta(x - x'), \quad \int dx |x\rangle \langle x| = 1$$
$$\langle k|k' \rangle = 2\pi\delta(k - k'), \quad \int dk |k\rangle \langle k| = 1, \quad (19)$$

and intermediate quantum-field state $u_k(x)$ can be represented as

$$u_k(x) = \langle x|k \rangle. \quad (20)$$

Using these matrix notations, we write the operator $\mathcal{M} (\hat{X}, \hat{K})$ (14) as a hermitian matrix

$$\mathcal{M}^x_{k,k'} |x\rangle \langle x'| = \langle k| \mathcal{M}(\hat{X}, \hat{K}) |x\rangle \langle x'| \rangle \langle x'| k' \rangle$$

$$= u_k^*(x) \mathcal{M}(x, \hat{K}) u_{k'}(x') \delta(x - x'),$$

$$= u_k^*(x) \mathcal{M}(x, \hat{K}) u_{k'}(x') \delta_{kk'} \delta(x - x'), \quad (21)$$

where

$$\langle x| \mathcal{M}(\hat{X}, \hat{K}) |x' \rangle = \mathcal{M}(x, \hat{K}) \delta(x - x'), \quad (22)$$

is diagonal in the coordinate space $\{x\}$. In the representation of $\{u_k(x)\}$, the hermitian matrix $\mathcal{M}^{x,x'}_{k,k'}$ is also diagonal in the energy-momentum space $\{k\}$, since $\{u_k(x)\}$ are eigenstates of the operator $\mathcal{M}$ (5). By normalizing the quantum field $\phi$, we define the normalized diagonal element of the matrix (21) as

$$\lambda^2_k(x) \equiv \frac{1}{|u_k(x)|^2} u_k^*(x) \mathcal{M}(x, \hat{K}) u_k(x), \quad (23)$$

and formally compute the effective action $S_{\text{eff}}$ given in Eqs. (15,14):

$$iS_{\text{eff}} = -\ln \langle \bar{0}, \text{out} | \hat{0}, \text{in} \rangle iS_{\text{eff}}$$

$$= -\int \sqrt{-g} d^4x d^4k \ln \lambda^2_k(x) |_{\text{out}}$$

$$- \left( -\int \sqrt{-g} d^4x d^4k \ln \lambda^2_k(x) |_{\text{in}} \right), \quad (24)$$
where the \( \{ \lambda^2_k \}_\text{out} \) and \( \{ \lambda^2_k \}_\text{in} \) are the diagonal elements. The \( \{ \lambda^2_k \}_\text{out} \) is in terms of the final vacuum state and geometry \( g_{\mu\nu} \), whereas the \( \{ \lambda^2_k \}_\text{in} \) is in terms of the initial vacuum state \( \phi_{\text{in}} \) and geometry \( \bar{g}_{\mu\nu} \):

\[
\lambda^2_k|_{\text{out}} = \frac{1}{|\tilde{u}_k(x)|^2} \tilde{u}_k^*(x) \mathcal{M}(x, \tilde{K}) \tilde{u}_k(x), \\
\lambda^2_k|_{\text{in}} = \frac{1}{|\bar{u}_k(x)|^2} \bar{u}_k^*(x) \mathcal{M}(x, \bar{K}) \bar{u}_k(x).
\]

(25) (26)

The operator \( \lambda^2_k(x) \) and the number of quantum-field states \( \int \sqrt{-g}d^4xd^4k/(2\pi)^4 \) are invariant in arbitrary coordinate systems, later is the Liouville theorem for the phase-space invariance.

By using Eqs. (16) and (24), the variation \( \langle T_{\mu\nu}(x) \rangle_{\text{diff}} \) of averaged energy-momentum tensor in the time interval \( \delta t \) is given by the difference:

\[
\langle T_{\mu\nu}(x) \rangle_{\text{diff}} = \langle T_{\mu\nu}(x) \rangle_{\text{out}} - \langle T_{\mu\nu}(x) \rangle_{\text{in}},
\]

(27)

where \( \langle T_{\mu\nu}(x) \rangle_{\text{out}} \) is the averaged energy-momentum tensor computed by the variation of geometry \( g_{\mu\nu} \), corresponding to the final quantum vacuum state \( \phi_{\text{out}} \), whereas \( \langle T_{\mu\nu}(x) \rangle_{\text{in}} \) is the averaged energy-momentum tensors computed by the variation of geometry \( \bar{g}_{\mu\nu} \), corresponding to the initial quantum vacuum state \( \phi_{\text{in}} \).

In general, using the definition (24), we formally calculate the averaged energy-momentum tensor \( \langle T_{\mu\nu}(x) \rangle \) as,

\[
\langle T_{\mu\nu}(x) \rangle = \langle T_{\mu\nu}(x) \rangle^{(1)} + \langle T_{\mu\nu}(x) \rangle^{(2)},
\]

(28)

where

\[
\langle T_{\mu\nu}(x) \rangle^{(1)} = ig^{\mu\nu}(x) \int \frac{d^4k}{(2\pi)^4} \ln(\lambda^2_k(x))
\]

(29)

\[
\langle T_{\mu\nu}(x) \rangle^{(2)} = 2i \int \frac{d^4k}{(2\pi)^4} \frac{\delta \ln(\lambda^2_k(x))}{\delta g_{\mu\nu}(x)}.
\]

(30)

In these equations

\[
\frac{\delta \sqrt{-g}(x)}{\delta g_{\mu\nu}(y)} = \frac{1}{2} \sqrt{-g(x)}g^{\mu\nu}(x)\delta^4(x - y);
\]

\[
\int d^4x \sqrt{-g}\delta^4(x - y)f(x) = f(y),
\]

(31)

for an arbitrary function \( f(x) \). The calculations of energy-momentum tensor (29,30) are main tasks in the following two sections.
III. VACUUM ENERGY-SPECTRUM.

In order to illustrate physical idea in a mathematically tractable way, we model a collapsing massive star as an infinitesimally thin and spherical shell. This massive shell separates the spacetime into two regions: (i) internal region $\bar{S}$ described by the flat geometry $g_{\mu\nu} = (1, -1, -1, -1)$,

$$\begin{align*}
   ds^2 &= dt_0^2 - dr^2 - r^2 d\Omega, \quad r < R; \\
   \text{and (ii) the external spacetime } S \text{ described by the stationary Schwarzschild geometry,}
\end{align*}$$

$$\begin{align*}
   ds^2 &= g_{tt} dt^2 + g_{rr} dr^2 - r^2 d\Omega, \quad r > R \\
   g_{tt} &= -(g_{rr})^{-1} = g \equiv (1 - \frac{2M}{r}),
\end{align*}$$

where $r, \theta, \phi$ are spherical-polar coordinates, $d\Omega = d\theta^2 + \sin^2\theta d\phi^2$, $t$ and $t_0$ are the Schwarzschild-like coordinates in the external and internal spacetime respectively. $R$ indicates the radial position of the shell. $M$ is the total mass-energy of the shell.

At the initial time $t_{in} = -\delta t/2$, the shell radius is $R$ and a static observer $O$ is located at $\bar{x}(R - 0^+, \Omega) \in \Sigma_{-\delta t/2}$ in the internal spacetime $\bar{S}$. His four velocity $u_\mu$ and Killing vector $\xi_\mu$ are given by,

$$u_\mu = (1, 0, 0, 0), \quad \xi_\mu = (1, 0, 0, 0).$$

The initial quantum field is $\phi_{in}$ and quantum vacuum state is $|\bar{0}, in\rangle$. After $\delta t$, the shell gravitationally collapses and its radial position $R$ moves inwards to $R - \delta R$. At the final time $t_{out} = +\delta t/2$, the same static observer $O$ turns out to be in the external spacetime $S$. His four velocity $u_\mu$ and Killing vector $\xi_\mu$ are then given by,

$$u_\mu = (g_{tt}(r)^{1/2}(0, 0, 0), \quad \xi_\mu = (g_{tt}(r), 0, 0, 0).$$

The final quantum field is $\phi_{out}$ and quantum vacuum state is $|\bar{0}, out\rangle$. In the following, we respectively compute the vacuum-energy spectrum of the initial and final vacuum states.

In the external spacetime, the Riemann scalar $\mathcal{R} = 0$ and the Laplacian operator $\hat{L}$ is given by:

$$\begin{align*}
   \Delta_x &= g^\mu \frac{\partial^2}{\partial t^2} + \frac{1}{r^2} \frac{\partial}{\partial r} r^2 g^{rr} \frac{\partial}{\partial r} + \hat{L}^2 \frac{1}{r^2} \\
   &= g_{tt} \frac{\partial^2}{\partial t^2} + g^{rr} \left( \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} \right) \\
   &\quad - \frac{2M}{r^2} \frac{\partial}{\partial r} + \hat{L}^2 \frac{1}{r^2},
\end{align*}$$

(36)
where $\hat{L}^2$ is the angular momentum operator, $g^{tt} = (g_{tt})^{-1}$ and $g^{rr} = (g_{rr})^{-1}$.

The appropriate basis of quantum field states is chosen as

$$\tilde{u}_k(x) = \langle t, r, \theta, \phi | \omega, k_r, l, m \rangle = R_{l\omega}(r) Y_{lm}(\theta, \phi) e^{-i\omega t},$$

where $Y_{lm}(\theta, \phi)$ is the standard spherical harmonic function: $\hat{L}^2 Y_{lm}(\theta, \phi) = l(l+1) Y_{lm}(\theta, \phi)$ and $k$ indicates a set of quantum numbers $(\omega, k_r, l, m)$. $\omega$ is the energy-spectrum and the radial momentum $k_r$ will be defined soon. From Eq. (35), the radial function $R_{l\omega}(r)$ obeys the following differential equation for $r > R > 2M$,

$$\left[ g^{tt} \omega^2 + g^{rr} \hat{k}_r^2 + \frac{2M}{r^2} \hat{k}_r - V_l \right] R_{l\omega}(r) = 0,$$

where the hermitian radial momentum operator,

$$\hat{k}_r = \frac{1}{ir} \frac{\partial}{\partial r} r, \quad \hat{k}_r^2 = -\left( \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} \right),$$

and potential

$$V_l = \frac{l(l+1)}{r^2} + \frac{2M}{r^3} + m^2.$$

Eq. (38) is exactly equivalent to the Regge and Wheeler equation. The radial function $R_{l\omega}(r)$ is an orthogonal and complete basis, asymptotically behaves as the Hankel function $h_{l\omega}(r)$ for $r \gg R > 2M$. The matrix operator $\mathcal{M}$ in Eq. (23) is given by,

$$\mathcal{M}(r, \hat{k}_r) = g^{tt} \frac{\partial^2}{\partial t^2} - g^{rr} \hat{k}_r^2 - i \frac{2M}{r^2} \hat{k}_r + V_i.$$

Eqs. (37-41) define a complex eigen-value problem to find the energy spectrum of the final vacuum state in an external gravitation field. The imaginary part of the operator $\mathcal{M}(r, \hat{k}_r)$ results in the quantum radiation of Hawking type in curved spacetime, as discussed in ref. [3].

The diagonal matrix $\lambda_k^2$ is given by:

$$\lambda_k^2(x) |_{out} = -g^{tt} \omega^2 - g^{rr} k_r^2 + V_l - i \frac{2M}{r^2} k_r,$$

where we define the values of “radial momentum” $k_r$ and $k_r^2$ of the quantum field state $\tilde{u}_k(x)$:

$$k_r \equiv \frac{\tilde{u}_k^* \hat{k}_r \tilde{u}_k}{|\tilde{u}_k|^2} = \frac{R_{l\omega}^* \hat{k}_r R_{l\omega}}{|R_{l\omega}|^2};$$

$$k_r^2 \equiv \frac{\tilde{u}_k^* k_r^2 \tilde{u}_k}{|\tilde{u}_k|^2} = \frac{R_{l\omega}^* k_r^2 R_{l\omega}}{|R_{l\omega}|^2}. \tag{44}$$

In the internal spacetime $\bar{S}$, the Laplacian operator is given by:

$$\Delta_x = \frac{\partial^2}{\partial t_0^2} - \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{\hat{L}^2}{r^2}. \tag{45}$$
The appropriate basis of quantum field states is chosen as
\[ \bar{u}_k(x) = \langle t_0, r, \theta, \phi | \omega_0, k_{r0}, l, m \rangle = \bar{R}_{l\omega_0}(r) Y_{lm}(\theta, \phi) e^{-i\omega_0 t_0}. \] (46)

The radial function \( \bar{R}_{l\omega_0}(r) \) obeys the following differential equation,
\[ \left[ \omega_0^2 - \hat{k}^2 - \frac{l(l+1)}{r^2} - m^2 \right] \bar{R}_{l\omega_0}(r) = 0. \] (47)

The radial function \( \bar{R}_{l\omega_0}(r) \) forms a orthogonal and complete basis. The matrix operator \( \mathcal{M} \) in Eq. (23) is given by,
\[ \mathcal{M}(r, \hat{k}_r) = \frac{\partial^2}{\partial t_0^2} + \frac{l(l+1)}{r^2} + m^2. \] (48)

The diagonal matrix \( \lambda^2_k \) is given by:
\[ \lambda^2_k(x)|_{in} = -\omega_0^2 + k_{r0}^2 + \frac{l(l+1)}{r^2} + m^2, \] (49)

where we define the values of “radial momentum” \( k_{r0} \) and \( k_{r0}^2 \) of the quantum field state \( \bar{u}_k(x) \):
\[ k_{r0} \equiv \frac{\bar{u}_k^* \hat{k}_r \bar{u}_k}{|\bar{u}_k|^2} = \frac{\bar{R}_{\omega_0 l}^* \hat{k}_r \bar{R}_{\omega_0 l}}{|\bar{R}_{\omega_0 l}|^2}; \] (50)
\[ k_{r0}^2 \equiv \frac{\bar{u}_k^* \hat{k}_{r0} \bar{u}_k}{|\bar{u}_k|^2} = \frac{\bar{R}_{\omega_0 l}^* \hat{k}_{r0}^2 \bar{R}_{\omega_0 l}}{|\bar{R}_{\omega_0 l}|^2}. \] (51)

For \( m = 0 \) in Eq. (refeq2f), \( \bar{R}_{\omega_0 l} = 2\omega_0 j_l(\omega_0 r) \), \( j_l(\omega_0 r) \) is the spherical Bessel function. As a particular case, we adopt the spherically symmetric solution \( l = 0 \) and \( m = 0 \) in the differential equation Eq. (17), such that the spherically symmetric solution \( \bar{R}_{0\omega_0 l}(r) \sim e^{ik_r r}/r \) is the eigenstate of the radial momentum operators \( \hat{k}_r \) and \( \hat{k}_{r0} \). The eigenvalues \( k_{r0} \) and \( k_{r0}^2 \) are related to radial momentum of the spherical quantum field \( l = 0 \). This indicates that the values of “radial momentum” \( k_r \) and \( k_{r0}^2 \) are consistent with the radial momentum of quantum field states.

**IV. VACUUM ENERGY DENSITY**

Armed with the energy-momentum tensor \( \langle T^{\mu\nu}(x) \rangle \) and the energy-spectrum \( \lambda^2_k(x)|_{out} \) in the external spacetime \( S \), we are ready to compute the energy-density,
\[ \langle T^t_t \rangle_{out} = g_{tt} \langle T^{tt} \rangle_{out}. \] (52)

Given the volume of spherical shell \( 4\pi r^2 dr dt \), we have the number of quantum field states in the energy-momentum phase-space,
\[ \int \frac{d^4k}{(2\pi)^4} = \int \frac{d^2k}{(2\pi)^2} \int \frac{d\omega d\mathbf{k}_r}{(2\pi)^2} = \frac{1}{4\pi r^2} \sum_{l,m} \int \frac{d\omega d\mathbf{k}_r}{(2\pi)^2}. \] (53)
where $k_r$ is the “radial momentum” defined in Eq. (43) and $\vec{k}_\perp$ are the transverse momenta, perpendicular to the radial direction.

Starting with the first part $\langle T^{\mu\nu}(x) \rangle^{(1)}$ of the energy-momentum tensor (29), we compute the vacuum energy-density (52) as:

$$\langle T_{tt} \rangle^{(1)} = \frac{i}{4\pi r^2} \sum_{l,m} \int \frac{d\omega dk_r}{(2\pi)^2} \ln(\lambda_k^2).$$  \hspace{1cm} (54)

Using the identity:

$$\ln \frac{a}{b} = \int_0^\infty \frac{ds}{s} \left( e^{is(b+i\epsilon)} - e^{is(a+i\epsilon)} \right),$$  \hspace{1cm} (55)

we are able to write the vacuum energy-density (54),

$$\langle T_{tt} \rangle^{(1)} = \frac{i}{4\pi r^2} \sum_{l,m} \int \frac{d\omega dk_r}{(2\pi)^2} \int_0^\infty \frac{ds}{s} e^{is(\lambda_k^2+i\epsilon)}$$

$$+ (\lambda_k^2 \to 1).$$  \hspace{1cm} (56)

The second term indicated by $(\lambda_k^2 \to 1)$ is the same as the first term with $\lambda_k^2 \to 1$, and this term is a constant. The logarithmic function in Eq. (54) is represented by an $s$-integration in Eq. (56) and infrared convergence at $s \to 0$ is insured by $i\epsilon$ prescription ($\epsilon \to 0$).

Introducing the variable $\xi = -i\omega$ (the Wick rotation) and the integral representation,

$$\int_{-\infty}^\infty \frac{d\xi}{2\pi} e^{-i\beta\xi^2} = \frac{1}{2\sqrt{i\pi\beta}},$$  \hspace{1cm} (57)

for $\text{Im}(\beta) < 0$, we express Eq. (56) as,

$$\langle T_{tt} \rangle^{(1)} = -\frac{i}{8\pi r^2} \sum_{l,m} \int \frac{dk_r}{(2\pi)^2} \frac{1}{\sqrt{-\pi g_{tt}}} \int_0^\infty \frac{ds}{s^{3/2}} e^{is(\lambda_k^2+i\epsilon)},$$

$$+ (\lambda_k^2 \to 1)$$  \hspace{1cm} (58)

where and henceforth, $\lambda_k^2$ is given by Eq. (42) without the $\omega^2$-term.

In order to compute the integration over “$s$” in Eq. (58), we introduce a complex variable $z = -1/2 + \delta$ ($|\delta| \to 0$), and use the following integral representation of the $\Gamma(z)$-function by an analytical continuation for $\text{Im}(\alpha) > 0$:

$$\int_0^\infty e^{ias} s^{-1} ds = (-i\alpha)^{-z} \Gamma(z).$$  \hspace{1cm} (59)

This analytical continuation is equivalent to analytical continuation of dimensionality of the momentum-space $\int \frac{d\omega}{(2\pi)}$ in Eq. (57). In the neighborhood of singularity, where $|\delta| \to 0$ and $z \to -1/2$ in Eq. (59), we have

$$\Gamma(z) = -2\sqrt{2\pi}, \quad \alpha^{-z} = \sqrt{g k_r^2 + V_l - \frac{2M}{r^2} k_r}. \hspace{1cm} (60)$$
Substituting Eqs. (59, 60) into Eq. (58), we obtain the vacuum energy-density:

$$\langle T^t_t \rangle^{(1)} = \frac{g^{1/2}}{4\pi r^2} \sum_{l,m} \int \frac{dk_r}{(2\pi)} \sqrt{gk_r^2 + V_l - \frac{2M}{r^2} k_r},$$  

(61)

where the “radial momentum” integration \(\int dk_r\) integrates from “0” to a ultra violate cutoff \(\Lambda\).

Now we turn to the computations of the second part \(\langle T^\mu_\nu(x)\rangle^{(2)}\) of the energy-momentum tensor (30). Using Eqs. (30, 52) and exchanging momentum-integration and metric-variation, we write

$$\langle T^t_t \rangle^{(2)} = 2g_{tt} \frac{\delta}{\delta g_{tt}(x)} \left[ i \int \frac{d^4k}{(2\pi)^4} \ln(\lambda_k^2(x)) \right],$$  

(62)

where the bracket \([\cdots]\) was computed, as shown in Eq. (58). In addition, we have \(\delta(g^{tt}g_{tt}) = 0\), \(g^{tt} \delta g_{tt} = -g_{tt} \delta g^{tt}\) and

$$g_{tt} \frac{\delta}{\delta g_{tt}} \frac{1}{\sqrt{g^{tt}}} = -g^{tt} \frac{\delta}{\delta g^{tt}} \frac{1}{\sqrt{g^{tt}}} = \frac{1}{2} \frac{1}{\sqrt{g^{tt}}}.$$  

(63)

Using Eq. (58) and relationship (63), we take metric-derivative in Eq. (62) and obtain the result:

$$\langle T^t_t \rangle^{(2)} = \langle T^t_t \rangle^{(1)}.\tag{64}$$

Thus the vacuum-energy density in the external spacetime \(S\) is

$$\langle T^t_t \rangle_{\text{out}} = \langle T^t_t \rangle^{(1)} + \langle T^t_t \rangle^{(2)} = 2 \frac{g^{1/2}}{4\pi r^2} \sum_{l,m} \int \frac{dk_r}{(2\pi)} \sqrt{gk_r^2 + V_l - \frac{2M}{r^2} k_r}.\tag{65}$$

This is the vacuum energy-density corresponding to the final vacuum state \(\phi_{\text{out}}\).

In the internal flat spacetime \(\bar{S}\), the vacuum-energy density (65) is reduced to,

$$\langle T^t_t \rangle_{\text{in}} = 2 \frac{1}{4\pi r^2} \sum_{l,m} \int \frac{dk_{r0}}{(2\pi)} \sqrt{k_{r0}^2 - \frac{l(l+1)}{r^2} + m^2}.\tag{66}$$

This is the vacuum energy-density corresponding to the final vacuum state \(\phi_{\text{in}}\). We adopt the transverse momenta \(\vec{k}_\perp\) to replace the quantum numbers \((l, m)\) of angular momenta:

$$|\vec{k}_\perp|^2 \simeq \frac{l(l+1)}{r^2}, \quad \int d^2k_\perp \frac{(2\pi)^2}{(2\pi)^2} = \frac{1}{4\pi r^2} \sum_{l,m}.$$  

(67)

Eq. (66) can be written as

$$\langle T^t_t \rangle_{\text{in}} = \int \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + m^2},\tag{68}$$

where \(k^2 = k_{r0}^2 + |\vec{k}_\perp|^2\), \(d^3k = dk_{r0}d^2k_\perp\) and integration range \([-\Lambda, \Lambda]^3\). Eq. (68) is consistent with the vacuum-energy density for two components of quantum scalar field.
Comparing the vacuum-energy density $\langle T_i^i \rangle_{\text{out}}$ in the Schwarzschild geometry with the vacuum-energy density $\langle T_i^i \rangle_{\text{in}}$ in the flat geometry, we find the vacuum-energy of the quantum-field fluctuations (virtual particles) of the vacuum couples to the gravitational field in the following aspects: (i) an addition term $2M/r^3$ in the potential $V_l$ and an imaginary term $i2Mk_r/r^2$; (ii) the “radial momentum” $k_r$ and $k_r^2$ are modified from Eqs.(43,44) to Eqs.(50,51); (iii) modification $gk_r^2 \to k_r^2$. This shows the radial modes described by $k_r$ directly interact with gravitational field. While, the transverse momenta $\vec{k}_\perp$ in Eq.(65) are the same as its counterpart in Eq.(66), showing the transverse modes described by $\vec{k}_\perp$ do not directly interact with gravitational field. In following sections, we attempt to calculate the difference of vacuum-energy densities,

$$\langle T_i^i(x) \rangle_{\text{diff}} = \langle T_i^i(x) \rangle_{\text{out}} - \langle T_i^i(x) \rangle_{\text{in}}, \quad (69)$$

where $\langle T_i^i(x) \rangle_{\text{out}}$ is in terms of the final quantum vacuum state $\phi_{\text{out}}$ and geometry $g_{\mu\nu}$; whereas $\langle T_i^i(x) \rangle_{\text{in}}$ is in terms of the final quantum vacuum state $\phi_{\text{in}}$ and geometry $\bar{g}_{\mu\nu}$.

V. THREE ENERGY SCALES $\Lambda, m$ AND $r^{-1}$

The vacuum-energy densities Eqs.(65) and (66) depend on the three very different energy scales: $\Lambda, m, r^{-1}$ ( $\Lambda \gg m \gg r^{-1}$). The scale $M$ is considered the same as the scale $r^{-1}$. One would expect that final results of vacuum-energy densities $\langle T_i^i \rangle_{\text{out}}$ and $\langle T_i^i \rangle_{\text{in}}$ should be in terms of finite terms: $r^{-4}, mr^{-3}, m^2r^{-2}$ and $m^4$, since all “divergent terms” containing $\Lambda$ should be removed away. In what follows, we argue that $\Lambda$ is a physical cutoff of its own right for calculating the difference (69) of vacuum energies in the presence and absence of gravitational field; “divergent terms” containing $\Lambda$ should not be simply removed away.

We know that a quantum field theory in the flat spacetime, although its characteristic energy-scale is the measured mass $m$ of particles, has involved high-energy modes in the computations of virtual particle contributions (loops in Feynman diagrams) to quantum corrections. The ultraviolet divergent terms arise from the contributions of these high-energy virtual particles. A ultraviolet cutoff $\Lambda$ or another method is introduced to regularize these ultraviolet divergent terms, which are then consistently removed away by renormalizing quantum fields and parameters of theory. Only relevant contributions of virtual particles, whose wave-lengths are the order of $m^{-1}$, are taken into account. This is due to the fact that high-energy virtual particles characterized by the cutoff $\Lambda$ must not contribute to a physical process characterized by the energy scale $m$, that is much smaller than $\Lambda$. In a sensible quantum field theory, theoretical results corresponding to experimental low-
energy measurements, must be independent of the ultraviolet cutoff. The ultraviolet divergent terms can be consistently removed, if and only if the number of renormalized fields and parameters (for instance, coupling and mass) is exactly equal to the number of different types of ultraviolet divergent terms. This is normally guaranteed by internal symmetries of a renormalizable quantum field theory.

The vacuum energy of a quantum field theory is a “divergent constant”, since it is resulted from virtual particles whose energy range from 0 to the cutoff \( \Lambda \). In the flat spacetime, only the energy-difference of quantum states can be measured, as a consequence the vacuum energy can be discarded, by normal ordering of quantum fields. However, in studying the Casimir effect, we need to calculate the difference of vacuum energies in presence and absence of two conducting plates, since the presence of two conducting plates modifies vacuum state and its energy. This difference of vacuum energies turns out to be a measurable Casimir effect. The resultant Casimir energy is characterized by the size of the distance \( L \) between two conducting plates. This is because:

- (i) those virtual particles of wavelengths comparable with \( L \) strongly impacted by two plates and their contributions to vacuum energy are modified, giving rise to the Casimir effect;
- (ii) whereas those virtual particles of wavelength incomparable (either much smaller or greater than) with \( L \) are not much affected by two plates, so that their contributions to vacuum energy are same, and as a result these virtual particles have no contributions to the Casimir effect.

The Hawking effect is certainly due to virtual particles in the vacuum interacting with an external static gravitational field near the horizon of a black hole. The reasons that this effect is characterized by the energy scale \( T \sim 1/M \), which is the gravitational potential at the size of a black hole, are the following:

- (1) the dynamics of such an effect is virtual particles quantum-mechanically tunneling through the gravitational potential near the horizon of a black hole;
- (2) those virtual particles, whose energy is comparable with the gravitational potential around the horizon, have a large probability of undergoing the tunneling process, since there are more crossing energy-levels and virtual particles at these energy-levels can tunnel out of the gravitational potential into infinity;
- (3) those virtual particles, whose energy is much smaller than the gravitational potential, have a small probability of undergoing the tunneling process, since there are not crossing
energy-levels and virtual particles at these energy-levels cannot tunnel out of the gravitational potential into infinity;

- (4) those virtual particles, whose energy is much larger than the gravitational potential, are not disturbed by the gravitational potential and remain as virtual particles in the vacuum.

Analogously, the quantum radiation in curved spacetime discussed in ref.[3] has the characteristic energy scale \( \sim \frac{1}{r} \), where is the gravitational potential that virtual particles tunnel through.

We turn back to our case: we study a quantum field theory in an external gravitational field of a collapsing massive shell and compute the difference (69) of vacuum energies in the presence and absence of gravitational field. Gravitational field is a classical field of long wavelengths \( O(r) \). In curved spacetime, the vacuum energy cannot be simply be discarded, since gravitational field couples to the vacuum energy. As we can see the vacuum to vacuum transition matrix Eq.(42), the term

\[- g^{tt} \omega^2 - g^{rr} k_r^2 \]

shows that gravitational field couples to the energy-momentum \((\omega, k_r)\) of virtual particles in the vacuum. High-energy modes of virtual particles have much stronger interactions with gravitational field than low-energy modes of virtual particles do. The cutoff \( \Lambda \) should be a real physical cutoff, determined by an energy scale, where the difference (69) of vacuum energies in the presence and absence of gravitational field vanishes. We speculate that the cutoff \( \Lambda \) seems to be the Planck energy scale \( \Lambda_p \) for the reasons: (i) virtual particles of high-energy up the Planck energy scale interact with gravitational field and (ii) we do not see any intermediate energy scale, where the difference (69) of vacuum energies in the presence and absence of gravitational field vanishes. However, we leave the ultraviolet cutoff \( \Lambda \) as a parameter determined by the phenomenon of gamma ray bursts.

We will also have a similar discussion on this point, in comparison with the ultraviolet cutoff \( K \) introduced by Schwinger for the phenomenon of Sonoluminescence in section (X).

To end this section, we note two points: (i) the difference (69) of vacuum-energy densities \( \langle T_t^t \rangle_{\text{out}} \) and \( \langle T_t^t \rangle_{\text{in}} \) receives dominate contributions from high-energy modes of virtual particles, in particular, those modes at the ultra violate cutoff \( \Lambda \); (ii) the back action of vacuum-energy variation to gravitational field is not considered.
VI. COMPUTATIONS OF VACUUM ENERGY DENSITIES

We are interested in computing vacuum-energy densities \(\langle T_t^t\rangle\) in the presence of gravitation field and \(\langle T_t^t\rangle\) in the absence of gravitation field. Our aim is to find the difference of these vacuum-energies. Therefore the modes of virtual particles that do not interact with the gravitational field should be discarded. As we can see from the potential term \(V_l\) in Eq.(65), the radial modes described by \(k_r\) and the transverse modes by angular quantum numbers \((l, m)\) or transverse momenta \(\vec{k}_\perp\) play very different roles in contributing to the vacuum-energy density. The radial modes of virtual particles of high energy up to the cutoff scale \(\Lambda\), directly interacting with gravitational field, are most relevant, since they make dominate contributions to the difference of vacuum energies. While, the transverse modes do not directly interact with gravitational field, since the term \(k^2_\perp = l(l+1)/r^2\) in Eq.(65) is the same as that in Eq.(66). Thus, we separate the S-wave \((l = 0)\) contributions from non S-wave \((l \neq 0)\) contributions to energy-momentum tensors in the following computations.

In this section, we compute the S-wave \((l = 0)\) contributions \(\langle T_t^t\rangle_{l=0}\) and \(\langle T_t^t\rangle_{l=0}\) to energy-momentum tensors in and out, respectively. The calculations of non S-wave \((l \neq 0)\) contributions \(\langle T_t^t\rangle_{l \neq 0}\) in and out will be presented in appendix A.

We first compute vacuum-energy density \(\langle T_t^t\rangle_{\text{in}}\) in the absence of gravitational field. Separating the S-wave contributions \(\langle T_t^t\rangle_{l=0}\) from non S-wave contributions \(\langle T_t^t\rangle_{l \neq 0}\), we write Eq.(66) as,

\[
\langle T_t^t\rangle_{\text{in}} = \langle T_t^t\rangle_{l=0} + \langle T_t^t\rangle_{l \neq 0},
\]

where

\[
\langle T_t^t\rangle_{l=0} = 2 \frac{1}{4\pi r^2} \int \frac{dk_r}{(2\pi)} \sqrt{k^2_r + m^2},
\]

\[
= \frac{\Gamma(-\frac{\epsilon}{2})}{4\pi^2 r^2} \left(m^2\right)^{(1+\epsilon)}.
\]

In Eq.(73), we use the dimensional regularization of \(\int dk_r \to \int d^{(1+\epsilon)}k_r\) in Eq.(72) and analytic continuation representation:

\[
\int d^{(1+\epsilon)}k_r \sqrt{k^2_r + A^2} = \Gamma(-\frac{\epsilon}{2})[A^2]^{(1+\epsilon)},
\]

where \(\epsilon \to 0\) and \(\Gamma(-\epsilon/2) \sim -2/\epsilon\). The divergent term \(2/\epsilon\) represents the ultraviolet cutoff \(\Lambda\) of “radial momentum” \(k_r\).

We turn to compute the vacuum-energy density \(\langle T_t^t\rangle_{\text{out}}\) in the presence of gravitational field. Separating the S-wave contributions from non S-wave contributions in Eq.(65), we have,

\[
\langle T_t^t\rangle_{\text{out}} = \langle T_t^t\rangle_{l=0} + \langle T_t^t\rangle_{l \neq 0},
\]
where
\[ \langle T^t_t \rangle_{l=0}^{\text{in}} = 2 \frac{g^{1/2}}{4\pi r^2} \int \frac{dk_r}{(2\pi)} \sqrt{g k_r^2 + V_{l=0} - \frac{2M}{r^2} k_r}, \]  
(76)

where \( V_{l=0} \) is \( V_l(r) \) for \( l = 0 \). Changing the integrating variable in Eq. (76):
\[ k_r \rightarrow k'_r = g^{1/2} k_r, \quad k'_r \rightarrow \bar{k}_r = k'_r - i\frac{M}{r^2}, \]  
(77)

we write Eqs. (76) as,
\[ \langle T^t_t \rangle_{l=0}^{\text{out}} = 2 \frac{g^{1/2}}{4\pi r^2} \int \frac{d\bar{k}_r}{(2\pi)} \sqrt{\bar{k}_r^2 + m^2 + Q^2}, \]  
(78)

\[ = \frac{\Gamma(-\epsilon)}{4\pi^2 r^2} \left( m^2 + Q^2 \right)^{(1+\epsilon)} , \]  
(79)

where
\[ Q^2 = \frac{2M}{r^3} + \frac{M^2}{gr^4} . \]  
(80)

In Eq. (79), we use the formula (74) to compute the \( \bar{k}_r \)-integrations in \( \langle T^t_t \rangle_{l=0}^{\text{out}} \) (78), analogously to the calculations of \( \langle T^t_t \rangle_{l=0}^{\text{in}} \) (73).

Using \( \langle T^t_t \rangle_{l=0}^{\text{in}} \) (73) and \( \langle T^t_t \rangle_{l=0}^{\text{out}} \) (79) for \( \epsilon \rightarrow 0 \) up to \( O(2/\epsilon) \), we find that the term \( m^2 \) is canceled in the difference \( \langle T^t_t \rangle_{l=0}^{\text{in}} \) (69). To compute the \( k_{r,0} \)- and \( \bar{k}_r \)-integrations in \( \langle T^t_t \rangle_{l=0}^{\text{in}} \) (72) and \( \langle T^t_t \rangle_{l=0}^{\text{out}} \) (78), we use the formula,
\[ \int_0^\Lambda dx \sqrt{ax^2 + b^2} = \frac{\Lambda}{2} \sqrt{a\Lambda^2 + b^2} \]  
(81)

\[ + \frac{b^2}{2a^{1/2}} \ln \left( \frac{\Lambda a^{1/2} + \sqrt{a\Lambda^2 + b^2}}{b} \right) . \]

For \( \Lambda \gg m \gg r^{-1} \), to the leading term \( (O(\Lambda^2)) \) we approximately have,
\[ \langle T^t_t \rangle_{l=0}^{\text{in}} \simeq \frac{1}{4\pi r^2} \left( \frac{\Lambda^2}{2\pi} \right) ; \]  
(82)

\[ \langle T^t_t \rangle_{l=0}^{\text{out}} \simeq \frac{g}{4\pi r^2} \left( \frac{\Lambda^2}{2\pi} \right) . \]  
(83)

The vacuum-energy densities Eq. (82) and Eq. (83) are mainly contributed from virtual particles at the ultraviolet cutoff scale \( \Lambda \).

Taking into account the non S-wave contributions obtained in appendix A, we obtain the total vacuum-energy densities \( \langle T^t_t \rangle_{\text{in}} \) (74) and \( \langle T^t_t \rangle_{\text{out}} \) (75),
\[ \langle T^t_t \rangle_{\text{in}} \simeq \frac{1}{4\pi r^2} \left( \frac{\Lambda^2}{2\pi} \right) ; \]  
(84)

\[ \langle T^t_t \rangle_{\text{out}} \simeq \frac{g}{4\pi r^2} \left( \frac{\Lambda^2}{2\pi} \right) , \]  
(85)
up to the leading order $O(\Lambda^2)$.

We rewrite vacuum-energy densities Eq. (84) and Eq. (85) as

$$\langle T_{tt}^i \rangle_{\text{in}} \simeq \frac{\Lambda}{4\pi r^2 dr} \frac{5}{6} \left( \Lambda dr \right);$$  \hspace{1cm} (86)

$$\langle T_{tt}^i \rangle_{\text{out}} \simeq \frac{g^{1/2} \Lambda}{4\pi g^{-1/2} r^2 dr} \frac{5}{6} \left( \frac{g^{1/2} \Lambda g^{-1/2} dr}{2\pi} \right).$$  \hspace{1cm} (87)

Eq. (86) indicates that in the shell-volume $4\pi r^2 dr$, the number of quantum-field states is $(\Lambda dr/2\pi)$ and these states carry the energy $\Lambda$. Eq. (87) shows that the number of quantum-field states $(\Lambda dr/2\pi)$ is invariant, the energy of these states receives the gravitational red-shift $g^{1/2}(r)$ and the shell-volume $4\pi r^2 dr$ receives the gravitational factor $g^{-1/2}(r)$.

**VII. VACUUM-ENERGY DIFFERENCE AND ITS VARIATION**

We have obtained the vacuum-energy densities $\langle T_{tt}^i \rangle_{\text{in}}$ (84) and $\langle T_{tt}^i \rangle_{\text{out}}$ (85), and in this section we compute vacuum energies in the presence and absence of gravitational field. With respect to a static observer in spacetime $\mathbf{2}$, whose killing vector and four velocity are $\xi_\mu$ and $u_\mu$, we define the vacuum energy as

$$\mathcal{E}(x) \equiv u^\mu \langle T_{\mu\nu} \rangle u^\nu d\Sigma = n^\mu \langle T_{\mu\nu} \rangle d\Sigma^\nu,$$  \hspace{1cm} (88)

and

$$\mathcal{E}_{\text{total}} \equiv \int_{\Sigma_t} u^\mu \langle T_{\mu\nu} \rangle u^\nu d\Sigma = \int_{\Sigma_t} n^\mu \langle T_{\mu\nu} \rangle d\Sigma^\nu,$$  \hspace{1cm} (89)

where $\Sigma_t$ is the hypersurface described by the equation $t = \text{const}$, $d\Sigma^\nu$ its hypersurface element vector,

$$d\Sigma^\nu = n^\nu d\Sigma, \quad n^\nu \equiv \frac{\xi^\nu}{\sqrt{|\xi^\alpha \xi_\alpha|}} = \frac{\xi^\nu}{\sqrt{g_{tt}}},$$  \hspace{1cm} (90)

and the hypersurface element $d\Sigma = \sqrt{h}d^3x$, $h_{ij}$ is the spatial metric.

At the initial time ($t_{\text{in}} = -\delta t/2$), with respect to the static observer $O$ (34) located in the internal flat spacetime $\bar{\mathcal{S}}$, the vacuum energy is given by

$$\mathcal{E}_{\text{in}}(r) = 4\pi r^2 dr \langle T_{tt}^i \rangle_{\text{in}} \simeq \frac{5}{6} \left( \frac{\Lambda^2 dr}{2\pi} \right),$$  \hspace{1cm} (91)

where the vacuum-energy density $\langle T_{tt}^i \rangle_{\text{in}}$ is approximately given by Eq. (84). While, at the final time ($t_{\text{out}} = +\delta t/2$), with respect to the same static observer $O$ (35) located in the external spacetime
\( S \), the vacuum energy is given by

\[
E_{\text{out}}(r) = \sqrt{\frac{4\pi r^2}{1 - g_{rr}}} \langle T^t_{\text{t}} \rangle_{\text{out}},
\]

where the vacuum-energy density \( \langle T^t_{\text{t}} \rangle_{\text{out}} \) is approximately given by Eq. (85). Note that Eq. (92) is the same as Eq. (6.184) in the book by Birrell and Davies [2].

Using Eq. (91) for \( E_{\text{in}}(r) \) and Eq. (93) for \( E_{\text{out}}(r) \), we approximately obtain the vacuum energy \( E_{\text{out}}(r) \) in the presence of gravitational field, in terms of the vacuum energy \( E_{\text{in}}(r) \) in the absence of gravitational field,

\[
E_{\text{out}}(r) = g^{1/2}(r)E_{\text{in}}(r),
\]

which are the same as the results obtained by using the Heisenberg uncertainty relationship in ref. [4]. We find that the vacuum energy \( E_{\text{out}}(r) \) in the presence of gravitational field is gravitationally red-shifted from the vacuum energy \( E_{\text{in}}(r) \) in the absence of gravitational field.

Corresponding to the difference \( \langle T^t_{\text{t}}(x) \rangle_{\text{diff}} \), the difference between the vacuum energy \( (93) \) and vacuum energy \( (91) \) is

\[
\delta E(r) \equiv E_{\text{out}}(r) - E_{\text{in}}(r) = (g^{1/2}(r) - 1)E_{\text{in}}(r) < 0,
\]

which shows the vacuum energy gets smaller, implying that the vacuum state gains gravitational energy when gravitational field is turned on in the time interval \( \delta t = t_{\text{out}} - t_{\text{in}} \). For \( r \gg 2M \), we approximately obtain the difference \( (95) \) of vacuum energies,

\[
\delta E(r) = E_{\text{out}}(r) - E_{\text{in}}(r) \simeq -\frac{M E_{\text{in}}(r)}{r}.
\]

This indicates an interacting energy due to the vacuum energy \( E_{\text{in}}(r) \) coupling to the negative gravitational potential \(-M/r\).

For the static observer \( O \) located at \( r \) in the flat spacetime \( S \), absolute value of the vacuum energy \( E_{\text{in}}(r) \) \( (91) \) is not measurable. Analogously, the static observer \( O \) located at \( r \) in the curved spacetime \( S \) is not able to measure the absolute value of the vacuum energy \( E_{\text{out}}(r) \) \( (93) \). The question is then how to show the difference \( (95) \) of vacuum energies in the presence and absence of a static gravitational field on the Earth. In the short letter [4], author suggested to measure the Casimir effect at different altitudes above the Earth so as to reveal the difference of vacuum energies due to gravitational field acting on the vacuum. Although the gravitational field is static, it varies.
in the radial position, the static observer can possibly detect the variation of such difference, by measuring the Casimir effect at different altitude \( r_1 \) and \( r_2 \),

\[
\delta \mathcal{E}(r_2)|_{\text{Casimir}} - \delta \mathcal{E}(r_1)|_{\text{Casimir}},
\]

where \( \delta \mathcal{E}(r)_{\text{Casimir}}(r_2) \) is the difference of vacuum energies in the presence and absence of two conducting plates. Such a gravitational effect on the vacuum energy seems too small to be seen.

However, in a gravitational collapsing process approaching to the horizon of a black hole, with respect to a static observer located at \( R(t) \), where is the collapsing shell and gravitational field strongly and rapidly varies, the difference \( \delta \mathcal{E}(r) \) could be very large and the vacuum state gains a large amount of gravitational energy.

\section*{VIII. GRAVITATIONAL COLLAPSING}

In order to be able to analytically study the vacuum-energy gain and in particular vacuum decay leading to photon productions in the following sections, we simplify the dynamical process of a gravitational collapse and adopt a model with the following approximations:

- gravitational collapsing of massive and spherical shell that is infinitesimally thin;
- exactly spherical symmetry in gravitational collapsing;
- stationary Schwarzschild geometry in the external spacetime \( S \) outside the massive shell;
- flat geometry in the internal spacetime \( \bar{S} \) inside the massive shell.

This spherical shell has a total mass-energy \( M \) and rest mass \( M_0 \). In this model, the solution to the Einstein equations was studied in refs.\cite{21} and such a gravitational collapse process can be described by the equation,

\[
\frac{\delta R}{\delta t} = \frac{g(R) \sqrt{h^2(R) - g(R)}}{h(R)},
\]

\[
h(R) = \Gamma - \frac{2M}{4R \Gamma},
\]

where \( V(t) \equiv \frac{\delta R}{\delta t} \), in the unit of \( c \), is the collapsing velocity in the opposite radial direction, and the collapsing parameter \( \Gamma \equiv M/M_0 \). The solution of this equation \( R = R(t) \) is the radial location
of the collapsing shell at the moment \( t \). At the moment \( t_0 \) when the collapsing velocity \( V(t_0) = 0 \), the shell, that is at rest and starts to collapse, is located at the radial position:

\[
R_0(t_0) = \frac{1}{4} \frac{2M}{\Gamma(1 - \Gamma)}. \tag{99}
\]

We chose \( M = 10M_\odot \) and \( \Gamma = 0.0257 \), Eq. (99) gives \( R_0(t_0) = 10(2M) \). In Fig. (11), we plot the collapsing velocity \( V(t) \) as a function of \( \bar{R} \equiv R/2M \), shell’s radius in unit of \( 2M \). We find that the collapsing process \( R(t) \) for \( R_0(t_0) \geq R(t) > 2M \) undergoes almost in the speed of light, however, it becomes very slow when it approaches the horizon: \( R(t) \rightarrow 2M \). The collapsing process takes about 0.004 seconds.

When such collapsing shell \( R(t) \) sweeps inwards \( \delta R \) in the time interval \( \delta t \), with respect to the static observer located at \( R(t) \), the vacuum state changes from \( |\bar{0}, \text{in}\rangle \) (9) to \( |\tilde{0}, \text{out}\rangle \) (11). The amplitude of vacuum state transition is given by Eq. (24), corresponding the variations of energy-momentum densities are given by Eqs. (27, 69). The vacuum-energy variation is given by Eq. (95),

\[
\delta E(R) = \left( \frac{g^{1/2}(R) - 1}{2\pi} \right) \frac{\Lambda^2}{2\pi} \delta R,
\]

indicating that vacuum state \( |\tilde{0}, \text{out}\rangle \) (11) gains gravitational energy with respect to \( |\bar{0}, \text{in}\rangle \) (9). By using the collapsing equation (98), we obtain the rate of vacuum-energy variation per unit of time:

\[
\frac{\delta E(R)}{\delta t} = \left( \frac{5}{6} \right) \frac{\Lambda^2}{2\pi} \left( 1 - g^{1/2}(R) \right) \frac{\delta R}{\delta t}, \tag{101}
\]

where and henceforth we take the absolute value of the collapsing velocity \( V = \delta R/\delta t \). This is the rate of the vacuum states gaining gravitational energy in gravitational collapsing process.

In order to see the numbers of this rate in Eq. (101), we take the ultraviolet cutoff \( \Lambda = \Lambda_p \) and convert the natural unit \( (\Lambda_p = 1) \) into: \( \Lambda_p^2 = \frac{\Lambda}{t_p} = 1.95 \cdot 10^{16} \text{ ergs}/5.4 \cdot 10^{-44} \text{ sec} = 3.6 \cdot 10^{59} \text{ ergs/sec} \),

\[
\frac{\delta E(R)}{\delta t} = 4.78 \cdot 10^{58} (1 - g^{1/2}(R)) \frac{\delta R}{\delta t} \left( \frac{\text{ergs}}{\text{sec}} \right). \tag{102}
\]

It is worthwhile to point out that the number \( 3.6 \cdot 10^{59} \text{ ergs/sec} \) in the rate of the vacuum-energy gain is completely determined by natural constants, independently of any free parameters.

Eqs. (102) completely determines the rate of vacuum-energy gain \( \delta E(R)/\delta t \) in the spherical shell \( 4\pi R^2 \delta R \) that the collapsing shell sweeps inwards in the time interval \( \delta t \). We plot the rate of vacuum-energy variation (gain) \( \delta E(R)/\delta t \) in Fig. (2). The result shows that the rate \( \delta E(R)/\delta t \) rapidly increases to \( 10^{57} \text{ erg/sec} \), as the radius \( R(t) \) of the collapsing shell moves, almost in the
speed of light, inwards to the horizon. Whereas, in the vicinity of the horizon, the collapsing process becomes slow and the rate $\delta \mathcal{E}(R)/\delta t$ decreases and goes to zero.

The total amount of vacuum-energy gain from gravitational field at the end of gravitational collapse is given by integrating Eq.(100):

$$\mathcal{E}_{\text{total}} = \int_R \delta \mathcal{E}(R) = \left( \frac{5}{6} \right) \frac{\Lambda_p^2}{2\pi} \int_{R_o}^{2M} (g^{1/2}(R) - 1) \delta R$$

$$= \left( \frac{5}{6} \right) \frac{2M}{2\pi} \int_{10}^{1} (g^{1/2}(\bar{R}) - 1) \delta \bar{R} \simeq 0.1M$$

where $R_o = 10(2M)$. The maximum variation of gravitational energy is $M/2$ in the collapse process, which can be derived from differentiating the gravitational potential $-M/r$ from $r \sim \infty$ to $r = 2M$.

Due to this vacuum-energy gain $\delta \mathcal{E}(R)$ (100), vacuum states become energetically unstable, have to spontaneously undergo a quantum transition to lower energy states via quantum-field fluctuations, which leads to particle productions. As a consequence, the vacuum-energy $\delta \mathcal{E}(R)$ (100) gained from gravitational field must be released and deposited in the region from $r = 2M$ to $r = R_o$.

IX. ENERGY RELEASE AND PHOTON PRODUCTIONS

Which process of quantum transition releases this vacuum-energy gain $\delta \mathcal{E}(R)$ (100)? One of possibilities is spontaneous photon emission, analogously to the spontaneous photon emission taking place in the atomic physics. In ref.[4], we mentioned the possibility of such a spontaneous photon emission can be induced by the four-photon interacting vertex in the Quantum Electromagnetic Dynamics (QED). Yet, we have not been able to calculate the rate of such spontaneous photon-emission, since we are studying quantum scalar fields, instead of the QED, in curved spacetime. In this section, we try to compute the quantum transition amplitude between the initial and final vacuum states, corresponding to before and after gravitational field is turned on, at each step of gravitational collapsing process. This quantum transition amplitude is related to the probability of spontaneous particle (“photon”) productions.

First, we define the invariant scalar product of initial vacuum state $\phi_{\text{in}}(x)$ and final vacuum state $\phi_{\text{out}}(x)$ (see Eq.(3.28) in the book by Birrell and Davies[2]):

$$\langle \phi_{\text{out}}, \phi_{\text{in}} \rangle = -i \int_{\Sigma} \phi_{\text{out}}(x) \bar{\partial}_\mu \phi_{\text{in}}^*(x) d\Sigma^\mu$$

$$\phi_{\text{out}}(x) \bar{\partial}_\mu \phi_{\text{in}}^*(x) = \phi_{\text{out}}(x) \partial_\mu \phi_{\text{in}}^*(x) - \partial_\mu [\phi_{\text{out}}(x)] \phi_{\text{in}}^*(x)$$

(104)
where $d\Sigma^\mu = n^\mu d\Sigma$, with a future-directed unit orthogonal to the spacelike hypersurface $\Sigma$ and $d\Sigma$ is the volume element in $\Sigma$. Since the value of $(\phi_{\text{out}}, \phi_{\text{in}})$ is independent of $\Sigma$, we rewrite Eq. (104) as,

$$(\phi_{\text{out}}, \phi_{\text{in}}) = -i \int_{\Sigma_t} d\Sigma^{t} \phi_{\text{out}}(x) \overline{\partial_t \phi_{\text{in}}}(x)$$ (105)

where $\Sigma_t$ is the spacelike hypersurface for $t =$constance and its element $d\Sigma^{t} = n^t d\Sigma$ is given in Eq. (90).

With respect to the rest observer $O$ located at the radial position $r = R(t)$ and at the moment $t$, where and when the collapsing shell $R(t)$ sweeps inwards, the gravitational field is turned on for the time interval $\delta t$. This time scale $\delta t$ is determined by the gravitational collapsing process $\delta R$. In this time interval $\delta t$, the vacuum state changes from the initial vacuum state $\phi_{\text{in}}(x)$:

$$\phi_{\text{in}}(x) = \begin{cases} \frac{1}{\sqrt{2\omega_0}} e^{-i\omega_0 t_0} \tilde{R}_{\omega l_0}(r) Y_{l_0 m_0}(\Omega), & r < R + \frac{\delta R}{2} \\ \frac{1}{\sqrt{2\omega}} e^{-i\omega t} \tilde{R}_{\omega l}(r) Y_{l m}(\Omega), & r > R + \frac{\delta R}{2} \end{cases}$$ (106)

to the final vacuum state $\phi_{\text{out}}(x)$:

$$\phi_{\text{out}}(x) = \begin{cases} \frac{1}{\sqrt{2\omega_0}} e^{-i\omega_0 t_0} \tilde{R}_{\omega l_0}(r) Y_{l_0 m_0}(\Omega), & r < R - \frac{\delta R}{2} \\ \frac{1}{\sqrt{2\omega}} e^{-i\omega t} \tilde{R}_{\omega l}(r) Y_{l m}(\Omega), & r > R - \frac{\delta R}{2} \end{cases}$$ (107)

We assume that the time scale of quantum-field transition from the initial vacuum state $\phi_{\text{in}}(x)$ to the final $\phi_{\text{out}}(x)$ induced by the variation of gravitational field is very much shorter than the gravitational collapsing time-scale $\delta t$, so that the gravitational field is adiabatically turned on at $r = R(t)$, the initial vacuum state $\phi_{\text{in}}(x)$ and final vacuum state $\phi_{\text{out}}(x)$ are considered as their asymptotic eigenstates $(\omega_0, l_0, m_0)$ and $(\omega, l, m)$ respectively.

With these initial state $\phi_{\text{in}}(x)$ and final states $\phi_{\text{out}}(x)$, Eq. (105) gives the vacuum to vacuum transition amplitudes, when the gravitational field is turned on at $r = R(t)$. These vacuum to vacuum transition amplitudes are just the Bogolubov coefficients:

$$\alpha_{ij} = (\phi_{\text{out}}, \phi_{\text{in}}); \quad \beta_{ij} = - (\phi_{\text{out}}^*, \phi_{\text{in}}),$$ (108)

where $|\beta_{ij}|^2$ describes the probability of particle productions. Using $\phi_{\text{in}}$ (106) and $\phi_{\text{out}}$ (107), we compute the transition amplitude $\beta_{ij} = (\phi_{\text{out}}, \phi_{\text{in}}^*)$

$$(\phi_{\text{out}}, \phi_{\text{in}}^*) = \int_{[0,R-\delta R/2]} d\Sigma^{t} (\omega_0 - \omega_0) \phi_{\text{out}} \phi_{\text{in}} + \int_{[R-\delta R/2,R+\delta R/2]} d\Sigma^{t} (\omega - \omega_0) \phi_{\text{out}} \phi_{\text{in}}$$
+ \int_{[R+\delta R/2, \infty]} d\Sigma^t (\omega - \omega) \phi_{\text{out}} \phi_{\text{in}} \\
= \int_{[R-\delta R/2, R+\delta R/2]} d\Sigma^t (\omega - \omega_0) \phi_{\text{out}} \phi_{\text{in}}, \quad (109)

where \([A, B]\) indicates the integration zone of \(\Sigma^t\) in the radial direction, and in the last line of equation

\[
\phi_{\text{out}} \phi_{\text{in}} = \frac{1}{\sqrt{2\omega}} e^{-i\omega t} R_{\omega l}(r) Y_{lm}(\Omega) \\
\quad \cdot \frac{1}{\sqrt{2\omega_0}} e^{-i\omega_0 t} \tilde{R}_{\omega_0 l}(r) Y_{l_0 m_0}(\Omega), \quad (110)
\]

where \(\omega = g^{1/2}(r)\omega_0\),

Summing over initial states, we obtain the probability of particle productions in final states within the energy interval \((\omega, \omega + d\omega)\),

\[
\frac{dN}{d\omega} = \sum_{\omega_0} |\beta_{ij}|^2 \\
= \int_{R-\delta R/2}^{R+\delta R/2} r^2 dr \frac{(1 - g^{1/2}(r))^2}{4g^{1/2}(r)} |R_{\omega l}(r)|^2 \\
\simeq \frac{(1 - g^{1/2}(R))^2}{4g^{1/2}(R)} \delta R R^2 |R_{\omega l}(R)|^2, \quad (111)
\]

where \(l = l_0\) and \(m = m_0\). To derive Eq. \((111)\) we use the orthogonality and closure relations of eigenfunctions \(R_{\omega_0 l_0}(r) Y_{l_0 m_0}(\Omega)\):

1. \(\int d\Omega Y_{lm}^*(\Omega) Y_{l_0 m_0}(\Omega) = \delta_{ll_0} \delta_{mm_0};\)
2. \(\sum_{\omega_0 l_0 m_0} (\tilde{R}_{\omega_0 l_0}(r) Y_{l_0 m_0}(\Omega))^* \tilde{R}_{\omega_0 l_0}(r') Y_{l_0 m_0}(\Omega') = \frac{1}{r^2} \delta(r - r') \delta^2(\Omega - \Omega');\)
3. \(\int_{\Sigma_t} d\Sigma^t \frac{1}{r^2} \delta(r - r') \delta^2(\Omega - \Omega') f(r, \Omega) = f(r', \Omega').\)

Using the rate \(\delta R/\delta t\) given by gravitational collapsing equation \((98)\), we obtain the rate of the particle productions,

\[
\frac{dN}{dt d\omega} \simeq \frac{(1 - g^{1/2}(R))^2}{4g^{1/2}(R)} \frac{\delta R}{\delta t} R^2 |R_{\omega l}(R)|^2. \quad (112)
\]

This equation gives the rate and spectrum of particle productions, corresponding to the vacuum-energy variation \(\delta \mathcal{E}(R)/\delta t \) \((101)\) in the simplest model of gravitational collapsing shell, as described in the beginning of section \((VIII)\).
In order to have an idea of the number of particle creations in a second, we approximately use the continuity of functions $R_{\omega l}(R)$ and $R_{\omega l}(R)$, at $r = R$

$$R_{\omega l}(R) \simeq \bar{R}_{\omega l}(R), \quad (113)$$

where is infinitesimally thin shell in gravitational collapsing. As a result, we have

$$\frac{dN}{dt}d\omega \simeq \frac{(1 - g^{1/2}(R))^2}{4g^{1/2}(R)} \frac{\delta R}{\delta t} R^2 |\bar{R}_{\omega l}(R)|^2, \quad (114)$$

where $\bar{R}_{\omega l} = 2\omega_0 j_l(\omega_0 R)$. Using the relation $\omega = g^{1/2}(R)\omega_0$ and integrating $\omega_0$ over $[0, \Lambda]$ in Eq. (114) for the S-wave ($l = 0$), we obtain,

$$\frac{dN}{dt} \simeq \frac{\Lambda}{2} (1 - g^{1/2}(R))^2 \frac{\delta R}{\delta t}, \quad \text{(115)}$$

where we take $\Lambda = \Lambda_p$ again. We plot the rate of particle creations $dN/dt$ in terms of $R = R/2M$ in Fig. (3), which shows that the rate $\delta N/\delta t$ rapidly increases to $10^{43}$/sec, as the radius $R(t)$ of the collapsing shell moves, almost in the speed of light, inwards to the horizon. Whereas, in the vicinity of the horizon, the collapsing process becomes slow and the rate $\delta N/\delta t$ decreases and goes to zero. The total number of particles created in the collapse process is given by integrating Eq. (115),

$$N = \frac{\Lambda_p}{2}(2M) \int_{1}^{10} (1 - g^{1/2}(\bar{R}))^2 \delta \bar{R}, \quad \text{(116)}$$

where $\Lambda_p = \Lambda_p \mu M_{\odot} = 6.69 \cdot 10^{38}$.

As shown in Figs. (2) and (3), the rate of vacuum-energy gain, and the rate of particle creations are very large, as the collapsing process approaching to the formation of black hole’s horizon $R = 2M$. The total energy output (103) and number (116) of particles created are enormous. These qualitatively agree to the characteristics of energetic sources for gamma ray bursts. It is indeed interesting that the Planck scale $\Lambda_p$ as the ultraviolet cutoff in our proposal neutrally gives rise to the characteristics of gamma ray bursts, instead of depending on an arbitrary energy scale.

The energy of photons spontaneously emitted can be larger than the energy threshold $2m_e$, so that electron and positron pairs are produced. These pairs, on the other hand, annihilate into two photons. As a consequence, a dense and energetic plasma of photons, electron and position pairs, called “dyadosphere” or “fireball” in literatures, could be formed. The energy and particle-number densities of this “dyadosphere” can be respectively obtained by Eq. (102) and Eq. (115), see Figs. (2, 3). The total energy and particle-number of “dyadosphere” are given by Eq. (108) and Eq. (116).
X. COMPARISON WITH SONOLUMINIESCENCE

Sonoluminescence is the phenomenon of the intense flashes of light emission by the pulsations of a gas bubble driven by sound-wave in fluid. Such experiments deal with the pulsations of bubbles of air in water, driven by a sound wave of frequency of 20-30 KHz. During the expanding phase, the bubble radius reaches maximum of order $R \sim 4.5\mu m$, followed by a rapid collapse down to a minimum radius of order $R \sim 0.5\mu m$. The photons are emitted, having a “quasi-thermal” spectrum with a “temperature” of several tens of thousands of degrees Kelvin. There are about $10^6$ photons emitted per flash, and the time-averaged total power emitted is between 30 and 100 mW. The photons appear to be emitted a very tiny spatio-temporal region: Estimated flash widths vary from less than 35 ps to more than 380ps depending on the gas in the bubble.

The fundamental mechanism of such photon emissions in this phenomenon is still very controversial. We do not want to enter these controversial discussions in this article. In this section, we attempt to briefly discuss the Schwinger proposal in the connection with our study of large photon productions in a gravitational collapse. Schwinger considered this phenomenon of photon emissions as the Casimir energy (vacuum energy) $E$ releasing, due to the variation of the Casimir energy when a very rapid collapse of dielectric material into a vacuum takes place,

$$E = - \int \frac{d\vec{r}d\vec{k}}{(2\pi)^3} \frac{1}{2} |\vec{k}| \left( 1 - \frac{1}{\sqrt{\epsilon(\vec{r})}} \right),$$

where $\epsilon(\vec{r})$ is dielectric constant and $|\vec{k}|$ the vacuum energy of zero-point fluctuation mode. The total excess energy is

$$|E| = \frac{1}{12\pi} R^3 K^4 \left( 1 - \frac{1}{\sqrt{\epsilon}} \right),$$

for a slow varying dielectric constant $\epsilon$, where $R$ is bubble’s radius and $K$ is a cut-off wavenumber. The dielectric constant $\epsilon \to 1$, with respect to the high-energy modes above the cutoff $K$. If this Casimir energy releasing is completely in form of photon emissions, one identifies the average number of photon emissions as,

$$N = \int \frac{d\vec{r}d\vec{k}}{(2\pi)^3} \frac{1}{2} (\sqrt{\epsilon} - 1),$$

$$= \frac{1}{9\pi} R^3 K^3 \left( \sqrt{\epsilon} - 1 \right).$$

The cut-off wavenumber $K \sim 10^5\text{cm}^{-1}$ within the ultraviolet region, the energy-budget and the number of photon emissions agree with the experiments, although the spectrum of Sonoluminiescence does not extend to the ultraviolet region. However, Schwinger neither explicitly
worked out the mechanism of photon productions nor computed the rate of photon productions in the dynamical circumstance that the vacuum-energy variation is very rapid, for very rapid collapse of bubble. On the basis of this proposal, there are many further studies in the literatures, concerning on mechanism of photon productions and other relevant aspects relating the Schwinger proposal to the experiment of Sonoluminiscence. We want compare our proposal presented for gamma ray bursts in this article with the Schwinger proposal for the phenomenon of Sonoluminiscence. Regarding the variation of the vacuum energy, we find that vacuum-energy variation (95) due to turning on gravitational field is similar to vacuum energy variation (117) due to changing the dielectric constant. The variation of vacuum energies in both equations is negative ($\epsilon > 1$), implying the vacuum state gains energy in both cases. In Eq. (95), the vacuum state gains the gravitational energy. While, in Eq. (117), the vacuum state gains the sound-wave energy. In both cases, the variation of vacuum energy is very rapid, because collapsing processes driven by either gravitational field or sound-wave are very rapid. The collapsing velocity $\dot{R} \simeq c$ in the gravitational collapsing case and $\dot{R} \simeq 4\text{March}$ in the Sonoluminiscence case. The cutoff wavenumber $K$ is a real physical cutoff of its own right that Eqs. (118, 119) make physical scenes up to this cutoff. Analogously, The scale $\Lambda$ is a real physical cutoff of its own right that Eqs. (94, 95) take into account the total variation of vacuum energy, attributed to turning on an external gravitational field. It is not an artificial cutoff introduced for regulating calculations of divergent terms, and then removed in renormalizable theories. Although Schwinger did not explicitly work out the mechanism of photon productions in the phenomenon of Sonoluminiscence, the basic idea for vacuum-energy variation and photon productions is similar to that we propose in ref.4 and this article: the vacuum state gains (sound-wave/gravitational) energy and becomes unstable and must decay to the lowest energy state, releasing the (sound-wave/gravitational) energy it gains.

XI. CONTRASTION WITH THE HAWKING EFFECT

It is important to differentiate the Hawking radiation from the effect discussed in this article. It is clear that both effects are attributed to an external gravitational field interacting with virtual particles in the vacuum. However, they are very different, not only in the phenomena of their appearances, but also dynamics of their origins.

First, we see the aspect of phenomenon. The Hawking radiation is black-body radiation from a thermal bath of the temperature $1/8\pi M$ determined by the scale $M$. Particle creations leading to the Hawking radiation do not depend on gravitational collapse processes, or in the other words,
the Hawking radiation can be created by an external static gravitational field. While, particle creations discussed in this article do not have a black-body spectrum and the energy-scale of particle creations processes is a ultraviolet cutoff $\Lambda$. Such particle creations strongly depend on the gravitational collapsing processes, or in the other words, particle creations discussed in this article cannot occur in an external static gravitational field.

Second, we discuss the aspect of dynamics. There are many elegant scenarios interpreting the origin of the Hawking radiation around the horizon of a black hole. In the recent article [3], based on the context of quantum field theories for particle and antiparticle creations in an external gravitational field, author presented a scenario for understanding the origin of quantum radiation of the Hawking type when gravitational field is present. Since the general formulation of quantum scalar-field theory discussed in sections III and IV is similar to that in ref. [3], we adopt our scenario to contrast two different dynamical origins of the Hawking radiation and particle creations discussed in this article.

As discussed in the ref. [3], a gravitational field polarizes the vacuum: quantum field fluctuations (creation and annihilation) of virtual particles and antiparticles (positive and negative energy states) in the vacuum are “aligned” by an external gravitational field. By this gravitational polarization effect, the vacuum gains gravitational energy. This energy-gain reduces the energy-mass gap that is a barrier, preventing virtual particles in the vacuum from tunneling and creating particles. As a result, the probability of such quantum tunneling is increase. This causes the vacuum decay and creations of particles and antiparticles. This quantum tunneling effect is quantitatively described by the imaginary term, $i2M/r^2$ in Eq. (12) and imaginary effective action. This is rather analogous to the QED vacuum in the presence of an external electric field. The quantum-filed fluctuations of charged virtual particles are polarized by the electric field. Particles and antiparticles are created by the Schwinger mechanism [4], when electric field strength is strong enough to overcome energy-mass gap $\sim 2m_e$. The thermal nature of quantum radiation of the Hawking type is due to the CTP invariance in the processes of particle creations and annihilations. The temperature of thermal radiation is determined by the vacuum-energy gain when gravitational field polarizes the vacuum. The reason why the temperature (or vacuum-energy gain) is the order of $M^{-1}$ has been discussed in section V.

Instead, as discussed in previous sections, the dynamics of particle creations we discussed in this article is very similar to the dynamics of mechanism that Schwinger discussed for Sonoluminescence. It is also rather similar to the dynamics of photon-creations of the dynamical Casimir effect [13]. For the reasons that gravitational field interacting with virtual particles in the vacuum
and the variation of gravitational field in the collapse process, the vacuum gains gravitational energy and the large variation of vacuum energy occurs in a very short time and small space. Such a large vacuum-energy gain makes the vacuum state to be energetically unstable. Via quantum-field fluctuations, unstable vacuum state has to transit to lower energetical vacuum-state. Such vacuum to vacuum transition releases the gravitational energy, that the vacuum state gains, by spontaneous photons emissions. On the basis of its dynamical origin, this process of photon productions clearly does not take place in an external static gravitational field, very differently from the thermal radiation of the Hawking effect. It can be seen from computations in previous sections that this process is mainly contributed by the variation of terms \(-g^{tt}\omega^2 - g^{rr}k_r^2\) in Eq.(42), during a gravitational collapse. This contrasts with the imaginary term \(i2M/r^2\) in Eq.(42), describing quantum tunneling process for the Hawking effect in a static gravitational field. The energy scales of two processes are very different, which have been discussed in section (V).

XII. SOME REMARKS

The research of our proposal is at a preliminary step. We adopt the action (3) for scalar fields, rather than the vectorial field of electrodynamics dynamics (QED) in curved spacetime. The notion of photon productions in the title and text of the present article should be replaced by “photon” productions. In appendix A, we make an approximation in computing non S-wave contributions. We would like to consider the results (84,85) as the S-wave contribution only. In addition, we adopt an approximate model (infinitesimally thin shell) for describing the process of gravitational collapse. Learning a controversy[15] on the ultraviolet cutoff introduced in the Schwinger proposal for Sonoluminiscence, we need to further strengthen our discussions and arguments on the ultraviolet cutoff \(\Lambda\) and its value in our proposal. As discussed in section (X), Schwinger introduced the ultraviolet cutoff \(K\) to agree with the energy budget and particle number of Sonoluminiscence, although the spectrum of Sonoluminiscence does not extend to this ultraviolet region. Analogously, the ultraviolet cutoff \(\Lambda\) at the Planck scale is introduced in our proposal to be consistent with the energy budget and particle number of gamma ray bursts in sections (VIII,IX), although the spectrum of gamma ray bursts is \(O(\text{MeV})\). From the spectrum of particle creations Eqs.(112) or (114), we find that particle creations are dominated in the low-energy region, since the spectrum is approximately related to the function \(\sin^2(\omega R)/(\omega R)^2\) for a given value of collapsing radius \(R\). We speculate that high-energy particles should lead to multiparticle productions, and total number of particles produced is much larger than \(N\). We are still far from a complete understanding...
of our proposal for gamma ray bursts. Nevertheless, it is highly deserved to study the proposal presented in this article in connection with the origin of gamma ray bursts.

It is a good analogy to compare our proposal for the origin of gamma ray bursts with Schwinger proposal for the origin of Sonoluminiscence. Further experimental and theoretical studies on the Schwinger proposal for Sonoluminiscence will definitely help us to have a better understanding of the origin of gamma ray bursts in our proposal. Beside, experimental and theoretical studies on photon productions in the dynamical Casimir effect are essential for us to further understand the mechanism of photon productions in our proposal.

In literatures[23], we find that the Schwinger idea for Sonoluminiscence has been applied for explaining the origin of gamma ray bursts, on the basis of the variation of dielectric constant during a gravitational collapse. These scenarios seem interesting, in particular, in explaining the total energy budget of gamma ray bursts. Analogously, conducting electron gas is used as boundary conditions for computing the Casimir energy to discuss possible huge output of cosmic energy accounting for Quasars[24].

In future work, we expect to be able to study the quantum field theory of electrodynamics dynamics (QED) in curved spacetime and use more precise model describing the process of gravitational collapse, as well as elaborate calculations of vacuum-energy density and rate of gravitational energy releasing by spontaneous photon productions.

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XIV. APPENDIX A

In this appendix, we calculate the non S-wave \((l \neq 0)\) contributions \(\langle T^l_{\in} \rangle_{l \neq 0}\) in Eq.(71) and \(\langle T^l_{\in} \rangle_{l \neq 0}\) in Eq.(75).

In Eq.(71), the non S-wave \((l \neq 0)\) contributions \(\langle T^l_{\in} \rangle_{l \neq 0}\) in is given by,

\[
\langle T^l_{\in} \rangle_{l \neq 0} = \frac{1}{4\pi r^2} \sum_{l=1}^{\infty} (2l + 1) \int \frac{d\kappa_{l0}}{(2\pi)} \cdot \sqrt{\frac{k^2_{l0} + \frac{l(l+1)}{r^2} + m^2}{r^2}},
\]

(120)
\[ \Gamma(-\frac{1}{2}) = \frac{4\pi^{\frac{1}{2}}}{\sqrt{r^2}} \sum_{l=1}^{\infty} (2l + 1) \left[ \frac{l(l + 1)}{r^2} + m^2 \right]^{(1+\epsilon)}, \quad (121) \]

where we use the formula (74) in Eq. (121). Using Eqs. (74) and (77), we compute \( \langle T^t_{l\neq 0} \rangle_{\text{out}} \) in Eq. (121),

\[
\langle T^t_{l\neq 0} \rangle_{\text{out}} = \frac{2}{4\pi r^2} \sum_{l=1}^{\infty} (2l + 1) \int \frac{dk_r}{(2\pi)} \sqrt{gk_r^2 + V_{l\neq 0} - \frac{2M}{r^2} k_r}, \quad (122)
\]

\[
= \frac{\Gamma(-\frac{1}{2})}{4\pi^2 r^2} \left( \frac{l(l + 1)}{r^2} + m^2 + Q^2 \right)^{1+\epsilon}, \quad (123)
\]

Based on \( \langle T^t_{l\neq 0} \rangle_{\text{in}} \) (121) and \( \langle T^t_{l\neq 0} \rangle_{\text{out}} \) (123) for \( \epsilon \to 0 \) up to \( O(2/\epsilon) \), we find that the terms \( m^2 \) and \( l(l + 1)/r^2 \) are canceled in the difference \( \langle T^t_{l\neq 0} \rangle_{\text{diff}} \) Eq. (69).

In the following calculations, we only keep up to the terms that are \( O(2/\epsilon) \) in the limit of \( \epsilon \to 0 \).

Thus, considering the difference \( \langle T^t_{l\neq 0} \rangle_{\text{diff}} \) Eq. (69), we write Eq. (121) and Eq. (122) as,

\[
\langle T^t_{l\neq 0} \rangle_{\text{in}} = \frac{1}{4\pi r^2} \sum_{l=1}^{\infty} (2l + 1) \int \frac{dk_r}{(2\pi)} \sqrt{k_r^2}, \quad (124)
\]

\[
\langle T^t_{l\neq 0} \rangle_{\text{out}} = \frac{2}{4\pi r^2} \int \frac{dk_r}{(2\pi)} \sum_{l=1}^{\infty} (2l + 1) \sqrt{k_r^2 + Q^2}. \quad (125)
\]

The summation over “l” in Eqs. (124) and (125) is given by

\[
\sum_{l=1}^{\infty} (2l + 1) = 2\zeta(-1) + \zeta(0), \quad (126)
\]

where \( \zeta(n) \) is the Riemann zeta function

\[
\zeta(n) = \sum_{l=1}^{\infty} \frac{1}{l^n}, \quad n = 0, 1, 2, 3, \ldots \quad (127)
\]

Resulted from summing over “l” in Eqs. (124) and (125), \( \zeta(-1) \) and \( \zeta(0) \) are divergent. As has been discussed, the transverse momenta \( \vec{k}_\perp \) of transverse modes do not directly couple to the gravitational field, we discard these divergent terms, by assuming these divergent terms are independent of gravitational field \( g(r) \). To eliminate these divergent terms, we use the reflection formula of analytic continued \( \Gamma \)- and \( \zeta \)-functions for complex variable \( z = -n + \delta \) and \( \delta \to 0 \),

\[
\Gamma(-\frac{1}{2})\zeta(-z) = \pi^{-z-1/2}\Gamma(-\frac{z+1}{2})\zeta(z+1). \quad (128)
\]

As results, We obtain

\[
\langle T^t_{l\neq 0} \rangle_{\text{in}} = -\left( \frac{1}{6} \right) \frac{2}{4\pi r^2} \int \frac{dk_r}{(2\pi)} \sqrt{k_r^2},
\]

\[
\langle T^t_{l\neq 0} \rangle_{\text{out}} = \frac{2}{4\pi r^2} \int \frac{dk_r}{(2\pi)} \sqrt{k_r^2 + Q^2}. \quad (125)
\]
\[ \simeq - \left( \frac{1}{6} \right) \frac{1}{4\pi r^2} \left( \frac{\Lambda^2}{2\pi} \right); \]  
(129)

\[ \langle T^L \rangle_{\text{out}}^{l \neq 0} = - \left( \frac{1}{6} \right) \frac{2}{4\pi r^2} \int \frac{dk_r}{(2\pi)} \sqrt{k_r^2 + Q^2}, \]

\[ \simeq - \left( \frac{1}{6} \right) \frac{g}{4\pi r^2} \left( \frac{\Lambda^2}{2\pi} \right). \]  
(130)

In the second lines of these equations (129) and (130), we only keep the leading order \( O(\Lambda^2) \), which is in accordance with the limit of \( \epsilon \to 0 \).

It should be pointed that the definitions of \( k_{r0} \) (50) and \( k_{r0}^2 \) (51) depend on the angular quantum number “l” and this implies that \( \epsilon \) in Eqs.(121) is \( l \)-dependent. The exchanging the order of \( k_{r0} \)-integration and \( l \)-summation in Eqs.(120-121) is not exact for a finite \( \epsilon \), and the same problem for Eqs.(122-123). On the other hand, the \( l \)-dependence in Eq.(120) is dominated by the degeneracy term \( 2l + 1 \) and angular momentum term \( l(l + 1) \). Compared with these dominate terms, the \( l \)-dependence in \( k_{r0} \) and \( k_{r0}^2 \) is very weak in terms of the radial wave-function \( j_l(\omega_0) \), whose value is limited for \( l \to \infty \). Thus, we neglect the \( l \)-dependence of \( (k_{r0}, k_{r0}^2) \) and \( (k_r, k_r^2) \) in computations. We consider this exchanging to be a good approximation for \( \epsilon \to 0 \). The S-wave results (82,83) are free from these approximations.

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FIG. 1: The velocity of gravitational collapsing shell in the unit of $c$ as a function of radius $R$ in the unit of $2M$.

FIG. 2: The rate of vacuum-energy gain (ergs/sec) as a function of radius $R$ in the unit of $2M$. 
FIG. 3: The rate of particle creations (1/sec) as a function of radius $R$ in the unit of $2M$. 