Solution of internal ballistic problem for SRM with grain of complex shape during main firing phase

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Abstract. Solid rocket motor (SRM) internal ballistics problems are related to the problems with moving boundaries. The algorithm able to solve similar problems in axisymmetric formulation on Cartesian mesh with an arbitrary order of accuracy is considered in this paper. The base of this algorithm is the ghost point extrapolation using inverse Lax-Wendroff procedure. Level set method is used as an implicit representation of the domain boundary. As an example, the internal ballistics problem for SRM with umbrella type grain was solved during the main firing phase. In addition, flow parameters distribution in the combustion chamber was obtained for different time moments.

1. Introduction
Solid rocket motors (SRM) have a relative simplicity of design and reliability in compare with other types of rocket motors. Despite this fact, they have a number of disadvantages, such as thrust control. Due to certain difficulties in active control of the SRM thrust, the main interests of the designers lie in various ways of "passive" thrust control. These ways include the choice of propellant, as well as the form of charge. Depending on the purpose of the SRM, an acceptable curve of the thrust versus time is determined. Then the shape of propellant grain, which provides the required dependence of the burning surface on time, is designed. In recent years, numerical modeling of processes in combustion chambers of rocket engines, as one of the tools for designing new SRMs, has been widely used. The ability to model internal chamber processes of SRM over firing period without the cost of production or using an engine test stand provides the opportunity for testing SRMs varied in form and allows one to obtain, ultimately, the optimal design of the rocket engine. As a rule, modern SRMs have grains of complex shape, characterized by 3-D or axisymmetric geometry. For some grains, the flow of combustion products in the chamber free volume of SRM does not significantly change in space. In this case, numerical models taking into account the complex shape of the grain with either a zero-dimensional or quasi-one-dimensional formulation of the problem for describing the flow of combustion products are applicable [1, 2].

However, in some cases it is necessary to consider a three-dimensional (axisymmetric) flow in conjunction with a change of the grain surface geometry, which greatly complicates the numerical implementation. This class of problems relate to problems with moving boundaries. For the numerical solution of this type of problem, unstructured moving computational grids can be used. Despite the simplicity of discretization of equations on such grids, this approach has a number of drawbacks. Firstly, the construction of a ”qualitative” computing grid can occupy
a considerable part of the total time of the problem solution. Secondly, this approach requires permanent rebuilding of the computational grid, which can lead to loss of accuracy. Thirdly, with time, various topological changes in the surface can occur, which leads to a significant complication of the algorithm. Fourthly, such schemes have an order of approximation not higher than the second.

In this paper, an algorithm that makes it possible to solve such problems with moving boundaries on a fixed Cartesian grid with an arbitrary order of accuracy in space and time has been developed. The peculiarities of the proposed algorithm are the following: 1) boundaries cross the computational grid in an arbitrary way, which complicates the specification of boundary conditions; 2) the realization of a numerical scheme in the vicinity of the boundaries requires the determination of the parameter values at the "ghost" points; 3) it is necessary to track the moving surface.

For the geometric representation of the surface and tracking its evolution over time, the level set method [3] is used. To take into account the boundary conditions and find the values of the flow parameters at points outside the computational domain ("ghost" points), the inverse Lax-Wendroff method developed by Shu [4, 5] is used.

2. Description of the numerical scheme

Let us consider the unsteady equations of gas dynamics with two spatial coordinates for the vector-function of conservative variables \( U(x,y,t) \):

\[
\begin{align*}
U_t + F(U)_x + G(U)_y &= S(U),
U(x,y,0) &= U_0(x,y),
\end{align*}
\]

(1)

in a bounded region \( \Omega(t) \) with the relevant conditions specified on the boundary \( \Gamma(t) = \partial \Omega(t) \) at the time \( t \). The region \( \Omega(t) \) is covered by a homogeneous Cartesian grid with steps along the x and y coordinates, equal, respectively \( \Delta x = h_x \) and \( \Delta y = h_y \), but the boundary of the region does not coincide with any of the coordinate lines. Schematically region \( \Omega(t) \) and computational grid are shown in Figure 1. Then the semi-discrete approximation of equations (1) can be written in the form:

\[
\frac{d}{dt} U_{i,j}(t) = -\frac{1}{h_x} (F_{i+\frac{1}{2},j} - F_{i-\frac{1}{2},j}) - \frac{1}{h_y} (G_{i,j+\frac{1}{2}} - F_{i,j-\frac{1}{2}}) + S_{i,j},
\]

(2)

where \( F_{i+\frac{1}{2},j} \) and \( G_{i,j+\frac{1}{2}} \) are numerical fluxes.

To integrate the system of ordinary differential equations (2) with respect to time, we use the Runge-Kutta method. To obtain the solution of a high order of accuracy over space either
an ENO scheme of the third order of accuracy or a WENO scheme of the fifth order of accuracy are used. Those schemes have a stencil consisting of seven points, three of which can be ghost points that are outside the solution region.

3. Inverse Lax-Wendroff procedure for fixed boundary

We write down the boundary conditions for the burning propellant surface, which will be assumed to be stationary. Those boundary conditions have the form of equations (3)-(5), where \( \rho, p, u_n, u_\tau \) are the density, the pressure, the normal and tangential components of velocity at the boundary, respectively, and \( m_t, H_0, \nu \) are constants characterizing the propellant.

\[
\rho u_n = m_t p^\nu, \tag{3}
\]

\[
\frac{k}{k-1} \frac{p}{\rho} + \frac{u_n^2 + u_\tau^2}{2} = H_0, \tag{4}
\]

\[
u \tau = 0. \tag{5}
\]

As noted in [4], to obtain values at ghost points it is more convenient to use the system of equations of gas dynamics written in a non-divergent form for primitive variables

\[
W_t + A(W) W_x + B(W) W_y = 0,
\]

where \( W = (\rho, u, v, p)^T \), \( A(W) = \begin{pmatrix} u & \rho & 0 & 0 \\ 0 & u & 0 & 1/\rho \\ 0 & 0 & u & 0 \\ 0 & \rho c^2 & 0 & u \end{pmatrix} \) and \( c \) is the sound speed.

The system of equations of gas dynamics written in this form has the following left eigenvectors for the matrix \( A(W) \):

\[
L(W) = \begin{pmatrix} 0 & -1/2c & 0 & 1/2 \rho c \\ -1/2 \rho & 0 & 1/2 & 1/2 \rho c \\ 1/2 \rho & 0 & 1/2 & -1/2 \rho c \\ 0 & 1/2c & 0 & -1/2 \rho c \end{pmatrix}
\]

When applying the inverse Lax-Wendroff procedure [4], the fourth extrapolation equation should be added to the three boundary conditions, which is written taking into account the zero components of the fourth eigenvector as:

\[
l_{4,2} u_n + l_{4,2} p = V_4. \tag{6}
\]

To solve the system of equations (3)-(5), (6) we rewrite it in another form:

\[
u_n = a + bp, \tag{7}
\]

\[
\rho = \frac{m_t}{u_n} p^\nu, \tag{8}
\]

\[
u_\tau = 0, \tag{9}
\]

\[
\frac{b^2}{2} p^2 + bcp^{2-\nu} + abp + acp^{1-\nu} + \frac{a^2}{2} - H_0 = 0, \tag{10}
\]
where \( a = \frac{V_4}{l_{4,2}} \), \( b = \frac{1}{l_{4,2}} \), \( c = \frac{k}{k - 1} m_t \).

Thus, it is required to solve numerically a single nonlinear equation (10) with respect to pressure, and the remaining parameters are found from the corresponding equations (7) - (9). To solve Eq. (10), one can use the Newton method.

In order to obtain the first derivatives with respect to space, it is necessary to take the time derivatives of the boundary conditions (3)-(5), convert these derivatives to spatial ones using gas dynamic equations and add one extrapolation equation. First order scheme is used for calculations. Thus, there is no need in calculating first derivatives and it is omitted in this paper.

4. The level set method for representing the region boundary
As can be seen from the description of the inverse Lax-Wendroff procedure algorithm for determining the parameter values at ghost points, it is necessary to know a nearest point on the boundary and the normal to it. In the case of moving boundaries, it is also necessary to know the algorithm for changing the position of the boundary with time. Relatively simple regions, for the boundaries of which there exist analytic expressions, are considered in [4, 5]. However, such an analytical description of moving boundaries of burning propellant for the problems of SRM internal ballistics is possible only in rare cases. Thus, to keep track the burning surface, a numerical technique is needed. In this paper we used the level set method, which seems to be the most effective way to determine the position of a moving surface on a Cartesian grid. The level set method is widely used for various problems with interface boundaries and the necessary information can be found in the book [3].

5. Algorithm for calculation of flow parameters for SRM internal ballistics problems with account for propellant burn-out
The main firing phase of SRM is characterized by relatively small changes in parameters over time as compared with their change in space, which suggests that the combustion of propellant occurs in a quasi-stationary mode, and the pressure field has time to adjust to the change in the burning surface. In addition, the characteristic times for the flow of combustion products in a rocket engine and the motion of a burning surface differ by approximately ten times. Furthermore, the characteristic times for the flow of combustion products in SRM and for the motion of a burning surface differ by approximately \( 10^6 \) times.

The following algorithm is proposed:

1) The propellant surface is assumed to be fixed at each time step. The system of gas dynamics equations is solved on a fixed Cartesian grid according to the scheme (2) until a stationary solution is obtained. Values in ghost points are given by the inverse Lax-Wendroff method; 2) For each point near the burning surface, the speed of surface motion is calculated to integrate the level function in time [3]; 3) The new position of the burning propellant surface is determined; 4) The results are written to a file. This procedure is repeated until all the propellant is burnt out.

6. Physical statement of the problem
As an example, the calculations were carried out in the region depicted in Figure 2. The region is the motor housing in which the propellant grain and the nozzle block are located. The geometric parameters are as follows. The grain length is \( L = 2.5 \) m, the initial radius of channel is \( R_0 = 0.2 \) m, the final radius of channel is \( R_k = 0.9 \) m, the length of umbrella is \( H = 0.523 \) m, the width of umbrella is \( h = 0.2 \) m, the distance from left edge of the grain to the begin of umbrella is \( l = 0.585 \) m, the throat radius is \( r_{cr} = 0.1 \) m. The angle of inclination of the ”umbrella” \( \alpha \) is variable.
In solving this problem, the following assumptions were used: flow field is axisymmetric, combustion products are inviscid and ideal gas, propellant is homogeneous and the burning rate depends on the pressure according to the law: \( \rho u = m_t p^\nu \).

7. Mathematical statement of the problem

The Euler equations written in the form (11)–(14) and the ideal gas law (15) are used to describe internal chamber processes for an axisymmetric region.

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial z} + \frac{\partial \rho v}{\partial r} &= -\frac{1}{r} \rho v, \\
\frac{\partial \rho u}{\partial t} + \frac{\partial (\rho u^2 + p)}{\partial z} + \frac{\partial \rho vu}{\partial r} &= -\frac{1}{r} \rho vu, \\
\frac{\partial \rho v}{\partial t} + \frac{\partial \rho uv}{\partial z} + \frac{\partial (\rho v^2 + p)}{\partial r} &= -\frac{1}{r} \rho v^2, \\
\frac{\partial \rho E}{\partial t} + \frac{\partial \rho u H}{\partial z} + \frac{\partial \rho v H}{\partial r} &= -\frac{1}{r} \rho v H, \\
p &= \rho RT
\end{align*}
\]

The system of equations (11) - (14) is supplemented by initial and boundary conditions. There are four types of boundaries for the problem. The impermeability condition \( u_n = 0 \) is specified at solid walls. Condition \( \frac{\partial}{\partial n} = 0 \) is specified at the axis of symmetry. Eqs (3)-(5) are specified at the inflow boundary. Constant pressure \( p = p^* \) is specified at the outflow boundary when the flow speed is subsonic.

Initial conditions are written as: \( p = p_0, \rho = \rho_0, u = 0, v = 0 \).

8. Values of parameters and constants

Numerical results were obtained with the following values of parameters and constants:

\( \alpha = 45^\circ, 60^\circ; \ m_t = 5.34 \cdot 10^{-3} \frac{kg}{m^2 s \cdot Pa^\nu}; \ H_0 = 10251150 \frac{J}{kg}; \nu = 0.5; \ \rho^T = 1700 \frac{kg}{m^3}; \)\n
\( p_0 = p^* = 101325 \ Pa; \ \rho_0 = 1.3 \frac{kg}{m^3}; \ h_x = h_y = 0.2 \ m. \)

9. Numerical results

A scheme of the first order of accuracy with respect to space and time was used for solving the equations of gas dynamics. A scheme of the fifth-order accuracy in space and the third-order accuracy with respect to time was used to determine the level function. In Figures 3 and 4, one can see the evolution of the combustion surface over time for angles \( \alpha = 45^\circ \) and \( \alpha = 60^\circ \), respectively. The time interval through which isosurfaces are represented is equal to \( \Delta t = 5 \ s. \)
It can be seen from the figures that for a given type of charge, combustion occurs in practically parallel layers, that is, the burning velocity along the surface varies insignificantly.

Figure 3. The shape of the propellant grain surface as a function of time, $\alpha = 45^\circ$.

Figure 4. The shape of the propellant grain surface as a function of time, $\alpha = 60^\circ$.

Figure 5 shows the dependencies of pressure averaged over the combustion chamber volume on time. This figure shows that the pressure in the combustion chamber increases to the point of time $t \approx 20$ s that corresponds to the complete combustion of propellant near the left wall of the housing. Then the combustion area begins to decrease, which leads to a drop in pressure. At the time $t \approx 70$ s, the propellant is completely burned out and the gas inflow becomes zero, after which the pressure falls monotonically.

Figure 5. The mean chamber pressure as a function of time, a) $\alpha = 45^\circ$, b) $\alpha = 60^\circ$.

Figures 6, 7 show the distribution of pressure in the combustion chamber for the times $t = 5$ s and $t = 40$ s, respectively. It can be seen from the figures that until the fuel burned out near the left wall, there is a pressure drop along the channel of about five atmospheres. For a period of time $40s < t < 70s$, the pressure in the combustion chamber can be considered constant.
10. Conclusion
In this paper, an algorithm was developed to solve the internal ballistics problem for solid-rocket motors with the umbrella type grain during main firing phase. The moving surface of the burning propellant was tracked using a level set method, the essence of which lies in the implicit representation of the burning surface as a zero level of some function. To obtain parameter values at ghost points the inverse Lax-Wendroff method, which provides the necessary order of the equations approximation, was used.

The coefficients of extrapolation polynomials that allow one to determine the values of the gas parameters at the gas-inflow boundary taking into account the non-linearity of the boundary conditions are obtained. In the case of the motion of the burning surface, a number of simplifying assumptions have been made, which make it possible to reduce the calculation time. As an example the dependencies of the pressure on time and the distributions of the flow parameters at different moments of time in the combustion chamber were obtained for a grain with an "umbrella" oriented at different angles of inclination to the channel generatrix.

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