The Pomeron in QCD

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Giuliano PREPARATA
Dip. di Fisica, Univ. degli Studi, via Celoria 16, I-20133 Milano, Italy
and
INFN - Sezione di Milano via Celoria 16, I-20133 Milan, Italy

Philip G. RATCLIFFE
Dip. di Fisica, Univ. degli Studi, via Celoria 16, I-20133 Milano, Italy

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Abstract
In the framework of Anisotropic Chromodynamics, a non-perturbative realization of QCD, we develop the Low-Nussinov picture of the Pomeron. In this approach all the usual problems of low $p_T$ perturbative calculations (infrared divergence) are naturally absent. Thus, we are able to perform an ab initio calculation of the hadron-hadron total cross section. The result is a cross section of the same magnitude as indicated experimentally and approximately energy-independent (with a $\log^2 s$ growth). We further discuss the $p_T$ dependence of the hadron-hadron elastic-scattering cross section, which displays all the experimentally observed features.

1 Introduction

Ever since it was realized that hadrons are extended objects and that hadronic interactions at high energies have a finite (transverse) size, the natural description of high-energy scattering at small angles has always been in terms of diffraction. While the intuitive notion of diffraction is very easy to understand, the actual dynamical mechanisms that render a hadron a kind of “grey disk” at high energies have in the last thirty years been the subject of the most diverse speculation.

Under the hegemony of “Reggeology” during the sixties, when dynamics and (angular momentum) analyticity were thought to be tied together in a sort of one-to-one correspondence, it was only natural to look for the description/explanation
of the observed (approximate) constancy of hadronic cross sections in the high-energy asymptotic limit in terms of a Regge trajectory with intercept $\alpha_{P}(0) = 1$, the Pomeron \[^{[1]}\]. With the subsequent advent of higher energy accelerators (ISR, Fermilab, SpS at CERN, the Tevatron collider) it became ever clearer that the Pomeron singularity in the complex angular-momentum plane is quite different from that of the subleading Regge trajectories, whose connection with the low-lying meson spectrum completely corroborates the theoretical foundations of the Regge-pole approach to hadrodynamics. In particular, no meson has ever been found to be associated with the Pomeron trajectory. It thus became clear that the simple high-energy dynamics of Regge poles, relating a “tower” of $t$-channel exchanges to the characteristic power behaviour in $s$ (the CM energy squared) of the high-energy scattering amplitudes, is totally inadequate. But, if not a Regge pole, what then is the Pomeron?

![Figure 1: The Low-Nussinov picture of the Pomeron as a two-gluon exchange process (all permutations of the quark-gluon vertices should be summed).](image)

The answer that, in our opinion, came closer to the truth was provided in the middle of the seventies by the very interesting proposal of F. Low and S. Nussinov \[^{[2]}\]. According to these authors, if one considers the QCD Feynman diagrams of fig. 1, where the interaction is the simple perturbative one-gluon exchange between quarks (obviously modified by appropriate non-perturbative form factors) then the related hadronic amplitudes would exhibit the observed high-energy behaviour. In particular, the structure of the angular-momentum plane singularity associated with the “two-gluon exchange” diagrams of fig. 1 is certainly not that of a pole – the Pomeron trajectory – but rather that of a (possibly fixed) cut.

We should like to remark that the definite achievement of the Low-Nussinov proposal lies in its focussing attention on a simple dynamics of the hadronic fundamental degrees of freedom – quarks and gluons – showing that very interesting and sensible results can be obtained even in the regime where perturbative QCD (PQCD) is deemed to be completely inapplicable, owing to the outstanding problem of confinement. On the other hand, taken at face value, the diagrams of fig. 1
must be considered absolutely *unrealistic*, for their imaginary parts, which build up most of the high-energy amplitudes, arise from intermediate states in which there freely propagate coloured degrees of freedom – quarks and gluons – contrary to observations that attribute those imaginary parts (through the optical theorem) to rather complicated multihadron intermediate states.

In this letter we wish to show that in the Anisotropic Chromodynamics (ACD) realization of non-perturbative QCD one can develop the seminal idea of Low and Nussinov in a completely consistent way, without the grave difficulties we have just pointed out. Without entering into a detailed description of ACD and how it describes non-perturbatively QCD, space does not permit, we wish to remind the reader that the theoretical framework we shall adopt in this paper has been derived from a detailed analysis of the structure of the QCD vacuum [3]. From this analysis a picture emerges that, through a remarkable process of condensation of large chromomagnetic fields, explains the mysterious phenomenon of colour confinement. In addition to colour confinement, ACD has so far scored notable successes in the description of the meson \((q\bar{q})\) [4] and baryon \((qqq)\) spectra [5].

The basic picture that ACD paints of the hadronic world is easily outlined [6]: the only sector of the theory that must be analysed non-perturbatively regards the spectrum and wave functions (in terms of a minimal number of quarks, antiquarks and gluons) of colourless hadronic states. Such a Primitive World (PW) of hadronic states is (slightly) modified by the residual quark-gluon and gluon-gluon interaction, which is obtained by subtraction of the well-defined confining interaction terms. This perturbative strategy is expected to account for all three-meson and baryon-baryon-meson couplings, which give finite widths to the unstable members of the PW (which comprises zero-width hadronic states). Some preliminary satisfactory results have recently been achieved in this direction [7].

In this letter, on the other hand, we shall bring this perturbative strategy to bear upon the solution in QCD of the fundamental problem of the nature and structure of high-energy hadronic diffraction, i.e., the Pomeron. In the following section we outline the calculation leading to the total hadron-hadron cross section, which in section 3 is extended to the \(p_T\) dependence in the case of elastic scattering.

## 2 The Hadron-Hadron Total Cross-Section in ACD

As already stated, the Pomeron, being an effective exchange with zero quantum numbers, is suggestive of gluons, which do carry colour however. Thus, in order not to “uncover” the colour of the initially white states, one must necessarily think in terms of two-gluon exchange (as the leading mechanism), as suggested
in the Low-Nussinov approach \[2\].

In ACD the inherent infra-red problems are under control, owing to the fact that confinement is incorporated into the scheme from the outset and thus, for example, the gluon is not allowed to propagate over large distances, as in a standard perturbative QCD. Moreover, from an analysis of the experimental meson spectrum, which is well described in ACD \[4\], we also have access to completely determined wave-functions for the scattering hadronic states (at least in the meson sector).

![Diagram](attachment:image.png)

Figure 2: Meson scattering into fire-string final states, the leading contribution to the total cross-section at high energy.

The calculation we are thus led to contemplate is represented graphically in fig. 2. In this diagram, \(M_1, M_2\) represent the initial-state mesons and \(FS_{1,2}\) the final fire-string states of ACD, which then decay into the hadronic states detected in the laboratory \[8\]; \(g\) is just the ACD gluon exchange we are considering and the gluon-hadron blobs stand for the two possible gluon-quark vertices. In terms of hadronic strings, the picture one then has is an “entanglement” of the colliding hadrons due to the rearrangement of the colour charge, leading to two high-mass fire-strings, whose decays provide the final hadronic states in the total cross section.

Since we are not interested in the structure of the final state we may appeal to the property of completeness and sum over all possibilities and so avail ourselves of the result \[9\] that one may perform such a calculation simply by replacing the final states with on-shell quarks. Therefore, squaring the amplitude, we obtain the cross-section represented in fig. 3.

We now have to specify the form of the quark-quark, quark-antiquark potential. The ACD picture of the hadronic wave-function leads to a system of \(q\bar{q}\) confined inside a needle-like domain (in which the colour field is also confined) of length corresponding to the effective gluon mass of the theory \[3\]. And the potential takes the form

\[
V(r) = \mu^2 r e^{-m_g r}, \quad (1)
\]
where the string tension $\mu$ and the gluon mass $m_g$ are fixed from the meson-spectrum analysis \cite{4}:

$$\mu \simeq 0.48 \text{ GeV},$$

$$m_g \simeq 0.42 \text{ GeV}.$$  \hspace{1cm} (2)

Note that there is also an overall sign that is positive (attractive) for $q\bar{q}$ and negative (repulsive) for $qq$ and $\bar{q}\bar{q}$.

Transforming into momentum space, we obtain an effective potential for such a gluon exchange of the form

$$V(Q) = 8\pi \mu^2 \left( \frac{3m_g^2 - Q^2}{(m_g^2 + Q^2)^3} \right).$$  \hspace{1cm} (3)

For the meson wave-functions we use the following infinite-momentum frame (IMF) variables:

$$\vec{p}_1 = x\vec{p} + \vec{k}_T,$$

$$\vec{p}_2 = (1 - x)\vec{p} - \vec{k}_T,$$

where one has the usual restrictions $0 \leq x \leq 1$, $|k_T| \ll p$. In this frame, with the quarks approximately on-shell, it is easy to show that energy conservation demands that the longitudinal momentum fraction carried by the gluon be $\approx O(k_T^2/p^2)$ (i.e., vanishingly small in the IMF) and so may be neglected. With this approximation the calculation simplifies considerably, as all longitudinal dynamics drops out.

Thus, we obtain for the total cross-section

$$\sigma_{\pi\pi}^{\text{total}} = \frac{k_{\text{tot}}}{(2\pi)^6} \int d^2k_T \ d^2k_T' \frac{1}{2} d^2q_T \ |V(q_T)|^2$$

$$\times \left| \phi_T(k_T - \frac{1}{2}q_T) - \phi_T(k_T + \frac{1}{2}q_T) \right|^2 \left| \phi_T(k'_T - \frac{1}{2}q_T) - \phi_T(k'_T + \frac{1}{2}q_T) \right|^2,$$  \hspace{1cm} (5)
where \( f_{\text{col}} = 2/9 \) is the colour factor from tracing the vertex colour matrices.

We parametrize the transverse part of the wave-functions with

\[
\phi_T(k_T) = N \exp \left( -\frac{k_T^2}{2\kappa^2} \right),
\]

(6)

where the normalisation fixes \( N \) and the transverse-momentum cut-off parameter \( \kappa \), as obtained from the meson spectrum analysis \([4]\), is \( \kappa \approx 0.15 \text{ GeV} \). So the \( k_T \) and \( k'_T \) integrals may be performed analytically and we arrive at

\[
\sigma_{\pi\pi}^{\text{tot}} = \frac{f_{\text{col}}}{2(2\pi)^2} \int d^2q_T \left| V(q_T^2) \right|^2 4 \left( 1 - e^{-q_T^2/4\kappa^2} \right)^2,
\]

(7)

This final integral may be performed numerically to give an answer of the form

\[
\sigma_{\pi\pi}^{\text{tot}} = 32\pi f_{\text{col}} I(\xi) \left( \frac{\mu}{m_g} \right)^4 \frac{1}{m_g^2},
\]

(8)

with

\[
I(\xi) = \int_0^\infty dy \left( 1 - e^{-\xi y} \right)^2 \left[ \frac{(3 - y)}{(1 + y)^3} \right]^2,
\]

(9)

where \( \xi = m_g^2/4\kappa^2 \) and \( y = q_T^2/m_g^2 \). Thus, we finally obtain

\[
\sigma_{\pi\pi}^{\text{tot}} \simeq 0.09\pi f_{\text{col}} \left( \frac{\mu}{\kappa} \right)^4 \frac{1}{m_g^2} \simeq 15 \text{ nb}.
\]

(10)

This figure is to be compared with the experimentally accessible

\[
\sigma_{pp}^{\text{tot}} \simeq 39 \text{ nb} \quad \text{and} \quad \sigma_{\pi p}^{\text{tot}} \simeq 24 \text{ nb}.
\]

(11)

Note that if one assumes the factorization implicit in our calculation then these numbers are perfectly compatible, since one then expects

\[
\sigma_{pp}^{\text{tot}} \cdot \sigma_{\pi\pi}^{\text{tot}} = \left( \sigma_{\pi p}^{\text{tot}} \right)^2.
\]

(12)

So far, we have considered the final-state fire-strings as having an energy-independent transverse radius. However, owing to the logarithmic growth in multiplicity \([8]\) and the consequent logarithmic growth of the transverse momentum “kicks” in the decay process, there will be a logarithmic dependence on energy of the transverse size of the fire-string states. Such a growth spoils the naive approximation \([9]\) necessary to arrive at the sum of quark states and a residual \( \log s \) dependence is introduced into the amplitude. Unfortunately we are not at present able to perform the calculations necessary to obtain a precise parametrisation of this growth, although we note that the implied \( \log^2 s \) growth of the total cross section represents well the available experimental data.
3 The Elastic Scattering Cross-Section

We can take this calculation a step further: by using the optical theorem, the diagram of fig. 3 is just the imaginary part of the forward elastic $\pi\pi$ cross-section. If the outgoing pion pair are now assigned a different momentum, $p'$ say, then writing $p' = xp + p_T$ we see that for $p_T$ small $x \approx 1$ and the only alteration is that the two exchanged gluons now have different transverse momenta. The calculation is then a straightforward modification of the previous and one easily arrives at

$$\text{Im } A_{\pi\pi}^{el}(p_T) = \frac{f_{\pi\pi}}{2(2\pi)^2} \int \text{d}^2q_T \ V(q_T - \frac{1}{2}k_T) V^\dagger(q_T + \frac{1}{2}k_T) \times 4 \left[e^{-\frac{(4k_T)^2}{4\kappa^2}} - e^{-q_T^2/4\kappa^2}\right]^2,$$  \hspace{1cm} (13)

In fig. 4 we display the $\pi\pi$ differential elastic cross-section obtained in our calculation as a function of $p_T^2$. Several features are noteworthy. First and foremost, the striking dip at around 1 GeV, which is well-established in the case of high-energy $pp$ scattering at a similar value of $p_T$. Less obvious, but equally established experimentally, is the break in the slope parameter in the region of $p_T^2 \sim 0.2$ GeV$^2$, above this value and below the dip a parameterization of the form $\exp(-bp_T^2)$ gives $b \sim 10$ GeV$^{-2}$. Asymptotically, for large $p_T$, the behaviour takes on the form of a power law $\sim p_T^{-8}$.

The transparency of the calculations, performed in the infinite-momentum frame, make it rather easy to elucidate the origin of these effects. The finite range of the potential, due to confinement, implies the existence of zeroes in the scattering amplitude starting at a value of $p_T$ of the order of $\pi/r_{\text{conf}}$, where the confinement radius $r_{\text{conf}} \sim (0.4 \text{ GeV})^{-1}$ as discussed above. The higher zeroes are, of course, to a large extent smeared out in the integral, a full treatment including an Eikonal re-summation may eventually also lead to some smearing of the primary zero. The form of the final integrand in $q_T$, see eq. 13, then explains the lower $p_T$ behaviour: for very low $p_T$ the behaviour is dominated by the first exponential in eq. 13 and thus by the width of the $p_T$ distribution (obtained from the meson spectrum analysis), this is then overtaken by the second term, which leads to a partial cancellation that is, however, never clearly evident due to the very steep fall-off of the amplitude in this region and only the slight break in slope parameter remains. The large-$p_T$ behaviour is dictated entirely by the form of the potential used and is seen to be approximately of the form $p_T^{-8}$.

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Figure 4: Our results for the $\pi\pi$ differential elastic cross-section as a function of $p_T^2$.

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