Constructive algebraic renormalization of the abelian Higgs-Kibble model

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Abstract

We propose an algorithm, based on Algebraic Renormalization, that allows the restoration of Slavnov-Taylor invariance at every order of perturbation expansion for an anomaly-free BRS invariant gauge theory. The counterterms are explicitly constructed in terms of a set of one-particle-irreducible Feynman amplitudes evaluated at zero momentum (and derivatives of them). The approach is here discussed in the case of the abelian Higgs-Kibble model, where the zero momentum limit can be safely performed. The normalization conditions are imposed by means of the Slavnov-Taylor invariants and are chosen in order to simplify the calculation of the counterterms. In particular within this model all counterterms involving BRS external sources (anti-fields) can be put to zero with the exception of the fermion sector.

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1 Introduction

Few gauge models of physical interest enjoy a symmetrical regularization of Feynman amplitudes (as QCD in dimensional regularization). In particular for the standard model the difficulty comes from the endemic presence of $\gamma_5$ and of the complete antisymmetric tensor. Thus, if the regularization breaks the desired symmetries, one has to recover the correct Green’s functions by finite renormalization in order to fulfill the Slavnov-Taylor Identities (STI). Algebraic Renormalization (AR) ([1], [2], [3], [4] and [5]) theory gives the conditions under which this strategy is possible: in particular there should be no anomalies in the STI. Thus in principle the renormalization program can be performed. However it is not an easy task beyond the one-loop approximation, since a high number of vertex functions at lower order must be evaluated for generic external momenta in order to restore the STI.

In this paper we propose a strategy for the evaluation of the counterterms, based on the zero-momentum subtraction [6]. The final result is an explicit solution of the STI where the counterterms are given in terms of a set of finite vertex functions and their derivatives evaluated at zero momentum. Our strategy is based on various results taken from BPHZL renormalization scheme and from the Algebraic Renormalization theory. We show that the zero momentum subtraction and a judicious use of the normalization conditions allows a practical evaluation of the counterterms by means of a relevant set of finite vertex functions. In particular the choice of the normalization conditions entails a diagonal block structure of the matrices that fix the counterterms.

As a starting point we assume that a consistent subtraction procedure allows the evaluation of the n-loops vertex functions $\Gamma^{(n)}$ when the correct vertex functional $I^{(j)}$ is given for any $j < n$. I.e. we assume that our procedure has successfully worked for the lower orders and we proceed to restore ST invariance on $\Gamma^{(n)}$. The n-order vertex functions are constructed by iterative use of subgraphs and counterterms according to the scheme of Bogoliubov[7]. The regularization can be any, provided it respects the Quantum Action Principle [8] (i.e. it is correct up to counterterms in the action). In order to make the discussion simpler we assume also that the regularization procedure respects some basic symmetries of the classical action, as Lorentz covariance, Faddeev-Popov (FP) charge conservations and any possible further symmetry (as charge conservation $C$). Then we expect that Slavnov-Taylor identities (STI) are broken

$$S(\Gamma)^{(n)} = \int d^4x \left[ \partial^\mu c_{\Gamma^{(n)}} + (\partial^\mu A_\mu + \frac{e\nu}{\alpha} \phi_2) I^{(n)}_{\bar{c}} \right] + (\Gamma, \Gamma)^{(n)} = \Delta^{(n)}$$

where the bracket is

$$(X, Y) = \frac{\delta X}{\delta J_1} \frac{\delta Y}{\delta \phi_1} + \frac{\delta X}{\delta J_2} \frac{\delta Y}{\delta \phi_2} - \frac{\delta X}{\delta \psi} \frac{\delta Y}{\delta \eta}$$

$J_1, J_1, \eta, \bar{\eta}$ are the sources coupled to the BRS variations, i.e. the anti-fields (see the eq. (93)). We use the convention that derivatives are always from left and any field with fermi character anti-commute. Although STI are broken, the QAP guarantees that at every order $\Delta^{(n)}$ is a local insertion (provided that STI are valid at the lower orders), has the correct invariance properties under exact symmetries (e.g. Lorentz, Faddeev-Popov (FP) charge, etc.) and it is consistent with the power counting. Thus we can expand $\Delta$ on a suitable basis

$$\Delta = \sum_i c_i M_i = \sum_i c_i \int d^4x f_i(\phi, \partial \phi)(x)$$
where $\mathcal{M}_i$ is any Lorentz scalar monomial $f_i$ in the fields and their derivatives, integrated over the Minkowski space. Residual symmetries restrict the basis of $\mathcal{M}_i$; for the present Higgs-Kibble model $C$ invariance constrains the possible breaking-terms. By construction the canonical dimension of $\mathcal{M}_i$ is less or equal five.

The renormalization of the model consists in finding the finite counterterms in the action that restore the validity of the STI and consequently the physical unitarity (Algebraic Renormalization). Let us denote by $\Gamma$ the vertex functions resulting from this procedure. The locality and covariance of $\Delta$ suggests to consider the Taylor expansion in momentum space. Let $t^\delta$ be the projector of the polynomials of degree $\delta$ (the Taylor expansion in the independent external momenta up to degree $\delta$) and $\delta_{pc}$ denotes the superficial degree of some given amplitude.

These facts suggest a strategy in the evaluation of the counterterms. The first step consists in the zero momentum subtraction compatible with the power counting

$$(1 - t^{\delta_{pc}})\Gamma.$$  \hspace{1cm} (4)

The above expression $t^\delta \Gamma$ is a short-hand notation of the following procedure: one considers first the relevant amplitude (the functional derivatives respect to fields are denoted by subscripts)

$$\Gamma_{\phi_1(p_1)\phi_2(p_2)\ldots\phi_m(p_m)} \left|_{p_m = \sum_{j=1}^{m-1} p_j} \right.$$  \hspace{1cm} (5)

then the Taylor expansion $t^\delta$ in the independent momenta up to degree $\delta$. Formally one has

$$t^\delta \Gamma = \sum_{m=1}^{\infty} \int \prod_{i=1}^{m} dp_i \phi_i(p_i) \delta^4(\sum_{j=1}^{m} p_j) t^\delta \Gamma_{\phi_1(p_1)\phi_2(p_2)\ldots\phi_m(p_m)} \left|_{p_m = \sum_{j=1}^{m-1} p_j} \right.$$  \hspace{1cm} (6)

Thus we consider the lowest $\delta_D$ such that

$$(1 - t^{\delta_D})S(\Gamma^{(n)}) = (1 - t^{\delta_D})\Delta^{(n)} = 0$$  \hspace{1cm} (7)

at every order in the perturbation expansion ( $\delta_D$ does not depend on $n$). Also the expression $t^{\delta_D} \Delta^{(n)}$ has to be intended in the above sense and $\delta_D = 5 - \sum_{i} d_{\phi_i}$ ( $d_{\phi}$ are the naïve dimensions of the fields, entering in the functional derivatives of $\Delta^{(n)}$). In the above equation the relevant term for a recursive construction of the invariant vertex functions is the linear operator on $\Gamma^{(n)}$

$$S_0(\Gamma^{(n)}) = \int d^4 x \left[ \partial^\mu c \Gamma_A^{(n)} + (\partial^\mu A_\mu + \frac{e}{\alpha} \phi_2) \Gamma_c^{(n)} \right] + \left( \Gamma^{(0)}, \Gamma^{(n)} \right) + \left( \Gamma^{(n)}, \Gamma^{(0)} \right)$$  \hspace{1cm} (8)

where $S_0$ is the linearized ST operator. We assume that zero momentum subtraction is possible and focus our attention on other effects of the subtraction. In general $S_0$ is not homogeneous in the dimensions of the fields (e.g. in presence of a spontaneous breaking of symmetry). As a consequence the action of $(1 - t^{\delta_D})$ on each single terms of $S_0(\Gamma^{(n)})$ induces some over-subtractions of $\Gamma^{(n)}$. These over-subtractions manifest theirselves as local breaking terms $\Delta$, as can be seen by re-shuffling the eq. (5) in the form

$$(1 - t^{\delta_D})S_0(\Gamma^{(n)}) = S_0((1 - t^{\delta_{pc}})\Gamma^{(n)}) + S_0(t^{\delta_{pc}} \Gamma^{(n)}) - t^{\delta_D} S_0(\Gamma^{(n)})$$  \hspace{1cm} (9)
The last terms show that the zero momentum subtraction does not give ST invariant vertex functions and that order-by-order we have to introduce counterterms in the action. Let us make explicit the STI. The recursive procedure gives

$$S_0((1 - t^{δ_{νσ}})\Gamma^{(n)}) + \sum_{j=1}^{n-1} \left( \Pi^{(j)}, \Pi^{(n-j)} \right) = \left[ t^{δ_{νσ}} S_0 - S_0 t^{δ_{νσ}} \right] \Gamma^{(n)} + t^{δ_{νσ}} \sum_{j=1}^{n-1} \left( \Pi^{(j)}, \Pi^{(n-j)} \right) \equiv \Psi^{(n)}. \quad (10)$$

The $\Pi$ terms are computed at the lower orders in the perturbative expansion. They are supposed to satisfy STI at every order less than $n$. In our strategy one of the criteria in the choice of the normalization conditions is the suppression of the above bilinear contributions.

If the model has no anomalies the problem is then to find the counterterms $\Xi^{(n)}$ which satisfy

$$S_0(\Xi^{(n)}) = -\Psi^{(n)} \quad (11)$$

or

$$S_0(\Xi^{(n)}) = - \left[ t^{δ_{νσ}} S_0 - S_0 t^{δ_{νσ}} \right] \Gamma^{(n)} - t^{δ_{νσ}} \sum_{j=1}^{n-1} \left( \Pi^{(j)}, \Pi^{(n-j)} \right). \quad (12)$$

Finally the correct vertex functions are

$$\Pi^{(n)} = (1 - t^{δ_{νσ}})\Gamma^{(n)} + \Xi^{(n)} \quad (13)$$

The zero momentum subtraction, as intermediate renormalization, has the advantage to reduce the renormalization in any subtraction procedure to a common ground: the algorithm is then the same and it consists in the evaluation of a set of finite amplitudes and their derivatives at zero momenta. Moreover, as we will discuss later, it suggests a natural choice of the normalization conditions. Finally in the zero momentum subtraction the contributions of the lower orders of perturbation to $\Psi$ is consistently reduced (eq. (10)).

Among the problems of this approach is that the vertex functions and their derivatives with respect external momenta must have regular behavior at zero momenta. In the presence of massless and massive fields, this requirement implies the introduction of infra-red cut-offs and the Taylor operator $t^{δ_{νσ}}$ has to be modified (see [9]); however this possibility will not be explored in the present work.

There is a fairly large amount of freedom in the choice of the counterterms $\Xi$ (eq. (13)). This is due to the presence of a certain number of ST invariant terms explicitly given in the Appendix (C). This freedom will be exploited in order to obtain the most efficient strategy in the evaluation of $\Xi$ and in order to reduce the contribution to $\Psi$ (eq. (12)) due to the lower perturbative terms. Any choice of $\Xi$ fixes automatically the normalization conditions.

The use of ST invariants and the normalization conditions is organized by introducing a hierarchy for the counterterms (choice of a basis of non-invariant counterterms). They will be grouped into disjoint sets: the $S_0$ of two different sets have no common elements. Subsequently the elements of a single set can be organized with a nesting structure. By following this hierarchy decomposition, in the present model it is
possible to avoid all counterterms involving the external sources $J$, tadpoles and out-of-diagonal bilinear expressions. As a consequence the mass counterterms turn out to be zero. The ghost equation, which guarantees the nilpotency of the ST operator, plays also an important rôle in the control of some of the counterterms.

By construction the functional $\Psi$ contains only finite vertex functions, i.e. at every order of the perturbative expansion $n$ it can be evaluated independently from the regularization procedure (once $\Pi^{(j<n)}$ is correctly constructed). The counterterm functional $\Xi$ is determined by eq. (11). In general there are more equations than unknowns (over-determined problem). However the system of equations has a solution since there are consistency conditions \[ S_0(\Psi^{(n)}) = 0. \] Most of them are consequence of the nilpotency of the ST operator $S_0(\Psi^{(n)}) = 0$. (14)

The evaluation of $\Xi$ can be performed either by imposing the consistency conditions on $\Psi^{(n)}$ or by a choice of the linearly independent equations. It should be remarked that the expression of $\Xi^{(n)}$ in terms of $\Psi^{(n)}$ is a simple linear relation independent from the order of the perturbation expansion.

The really hard work is the evaluation of $\Psi^{(n)}$. It consists in the computation of vertex functions and of some of their derivatives at zero momenta. The number of graphs turns out to be very large (especially for amplitudes involving scalars). For this reasons it is important to find possible relations among the amplitudes, e.g. Callan-Symanzik Equation (CSE), and to use automatic calculus to generate and evaluate the graphs. Particularly interesting is the CSE (see for example [13], [14], [12] and [15]). The consistency conditions imposed by the CSE on the breaking terms $\Psi^{(n)}$ allows the evaluation of some amplitudes in terms of simpler vertex functions. Moreover some amplitude can be obtained as the result of mass insertions on vertex functions with less external legs. The automatic calculus is particularly useful since the external momenta are zero.

It is important to reduce the contributions to $\Psi^{(n)}$ of the lower terms in the perturbation expansion. Eq. (10) allows the direct control of the consequences of any particular choice for the basis of the non-invariant counterterms, i.e. of the choice of the normalization conditions. This point of view is at variance with the on-shell conditions, which cannot dispose this particular problem. For instance it is clear that by dropping external sources counterterms one can eliminate most of the terms coming from the lower order in the perturbation expansion.

The physical amplitudes necessitate the study of the zeros of the two-point-functions. Then the free parameters of the action have to be tuned in order to obtain the physical masses and the correct coupling constants.

The present paper is devoted to the $U(1)$ abelian Higgs-Kibble model ([1], [11]) for reasons of simplicity. The model has the advantage of admitting dimensional regularization (if there is no fermion sector). It is non trivial, since the presence of $\gamma_5$ requires the full generality of the Algebraic Renormalization. Moreover the model has no anomalies: the Adler-Bardeen-Jackiw anomaly is zero due to C-conjugation.

In the following to make the formalism simpler and more direct we use to give a compact notation for the breaking-terms $\Psi^{(n)}$ and its coefficients:

$$\Psi^{(n)} = \sum_i \psi^{(n)}_{M_i} M_i \tag{15}$$

and in the same way we will denote the counterterms $\Xi^{(n)}$ by

$$\Xi^{(n)} = \sum_k \xi^{(n)}_{P_k} P_k \tag{16}$$
where $P_i$ is a single monomial with dimension less or equal to four and null Faddeev-Popov charge. We may omit also the sign of integral $\int d^4x$, when not necessary, e.g. $\xi^{(n)}_\phi \equiv \xi^{(n)}_\phi \int d^4x \phi^2$

Eq. (13) can be looked from the point of view of a different renormalization scheme. Let $\Gamma^{(n)}$ be the result of any (non-symmetric) renormalization. One needs to introduce a set of counterterms $\Gamma^{(n)}_{CT}$ order-by-order:

$$\Gamma^{(n)} + \Gamma^{(n)}_{CT}$$

By comparing with our procedure we have

$$\Gamma^{(n)}_{CT} = -\xi^{(n)} + \sum_j v_j I_j.$$  

The first term is just a Taylor expansion of the action-like amplitudes. The second term is evaluated in terms of finite amplitudes and of some of their derivatives all at momentum zero. The later computation can be easily performed by automatic calculus. The last term contains the ST invariants and accounts for the differences between the normalization conditions in the two schemes.

Section 2 is devoted to the separation of the counterterms into sectors. By a judicious choice of the normalization conditions we can drop the tadpole and most of the external source counterterms. Only in the fermion sector the external source terms are modified by the renormalization procedure. Moreover we can identify a bosonic, a kinetic-gauge sector and a fermionic sector.

Section 3 contains a study of the breaking term functional $\Psi$. In particular the ST linearized operator $S_0$ of eq. 8, which enters in expression for $\Psi$, is modified in order to keep track of the ghost equation.

Section 4 provides the complete list of the counterterms in terms of finite amplitudes. The solution contains the contribution of the lower terms of the perturbative expansion. Moreover some consistency conditions are shown to be present among the finite amplitudes.

Technical detail are in the Appendices. In Appendix A we give the essential elements of the BRS transformations and of the model. In Appendix B we list all possible counterterms and their ST transforms. In Appendix C we discuss the important issue of the linearly independent ST invariants. Finally Appendix D contains all the relevant functional derivatives of the breaking term $\Psi$. The expansion of the functional $\Psi$ in terms of Lorentz invariant amplitudes allows the evaluation of the solutions given in Section 4.

### 2 Hierarchy of counterterms and breaking terms

The complexity of the problem is somehow distributed on two different steps. The evaluation of the breaking-terms functional $\Psi$ is probably the most complex part. Once $\Psi$ is given, one has to evaluate the counterterms $\Xi$ by eq. (14). The present section is devoted to this last problem. In order to reduce the problem of managing the complete set of STI simultaneously, we introduce a hierarchy for the counterterms $\Xi$ and breaking terms $\Psi$. This problem has been already discussed in previous works (see 8 and 9) on algebraic renormalization.

$S_0$ is a mapping of $\mathcal{V}_\Xi$ on $\mathcal{V}_\Psi$:

$$S_0 : \mathcal{V}_\Xi \to \mathcal{V}_\Psi$$
where the vector spaces are given by the relevant monomials

\[ V_\Xi \equiv \left\{ \sum_k x_k P_k | x_k \in \mathcal{C}, \dim(P_k) \leq 4, \text{FP charge}(P_k) = 0 \right\} \]  
(20)

and

\[ V_\Psi \equiv \left\{ \sum_i x_i \mathcal{M}_i | x_i \in \mathcal{C}, \dim(M_i) \leq 5, \text{FP charge}(\mathcal{M}_i) = 1 \right\} . \]  
(21)

The set of all action-like functionals \( \{ I_i \} \) which are invariant under ST transformations form the kernel of \( S_0 \)

\[ \ker(S_0) = \left\{ \sum_i v_i I_i | v_i \in \mathcal{C}, \dim(I_i) \leq 4, \text{FP charge}(I_i) = 0 \right\} . \]  
(22)

Some of the ST invariants are genuine BRS invariants. The trivial ST invariants are given by all elements which are \( S_0 \)-variation of local functionals of dimension \( \leq 3 \) and FP charge = -1. The subspace \( \ker(S_0) \) induces an equivalence relation among the counterterms. The freedom of the choice of the representative of the equivalence classes will be used as one of the tools to organize the counterterms in a hierarchy, according to a strategy aiming to reduce the complexity of AR. This choice amounts to fix the normalization conditions; in fact in this way we select a basis on which we write the counterterm functional \( \Xi \). Therefore all monomials outside the basis do not appear as counterterms. It should be mentioned here that the sub-space \( \ker(S_0) \) is further restricted by the condition imposed by the ghost equation of motion. The necessity to impose this condition as a first step comes from the fact that the ghost equation of motion is the statement of the nilpotency of \( S_0 \).

The image of \( V_\Xi \) is a proper subspace of \( V_\Psi \)

\( S_0(V_\Xi) \subset V_\Psi . \)  
(23)

By construction

\[ \Psi \in S_0(V_\Xi) \]  
(24)
since there are no anomalies. It is convenient to use a basis

\[ \mathcal{M}_i e_{ik} = S_0(P_k) \]  
(25)

where \( k \) labels the chosen representatives of the equivalence classes in \( V_\Xi \). Finally we have

\[ \Xi = \sum_k \xi_k P_k \]  
(26)

where \( \xi_k \) are determined from

\[ \Psi = \sum_k \mathcal{M}_i e_{ik} \xi_k \]  
(27)
\[ \psi_i = \sum_k e_{ik} \xi_k \] (28)

In general the number of \( \psi_i \) is higher than the number of the unknowns \( \xi_k \). The solution exists since the theory is assumed to satisfy STI (no anomalies). Most of the consistency conditions can be derived from the nilpotency of \( S_0 \)

\[ S_0(\Psi) = 0. \] (29)

It should be noticed that \( e_{ki} \) is a matrix fixed by the model and by the choice of the basis \( \{ P_k \} \). It can be evaluated solely by using the BRS transformations given in Appendix B. In particular it does not depend on the order of the perturbation expansion.

The choice of the representatives and of the linearly independent equations in (11) is performed according to the following strategy, which aims to reduce the complexity of AR. First, we look for a block or triangular structure of the matrix \( e_{ki} \) (hierarchy). Second, we reduce the number of terms coming from the lower perturbation expansion (see eq. (11)). Third, the choice of the linearly independent equations is done by preferring the breaking terms with lower number of external legs and higher derivatives in the external momenta. In this way the number of graphs is reduced at the cost of some derivatives on external momenta. This strategy might look unnecessary in the present simple model. However it will be useful in a more complicated situation as, e.g., in the Standard Model.

Two \( A, B \) subspaces of \( \mathcal{V}_\Xi \) are disjoint if

\[ S_0(A) \cap S_0(B) = \{0\} \] (30)

practically this means that the ST transforms of \( A, B \) do not shear any monomial \( \mathcal{M}_i \). \( A \) includes \( B \) if

\[ S_0(B) \subset S_0(A) \] (31)

These definitions are the guide for the hierarchy structure of the counterterms. If they can be grouped into disjoint sets then we have a block diagonalization of \( e_{ki} \). If we get an including structure then the matrix is triangular. In both cases the task is consistently reduced. Moreover we can use the ST invariants in order to improve the structure of the matrices \( e_{ki} \) by choosing appropriate normalization conditions. This is performed by exploiting the invariance of eq. (11) under the transformation

\[ \Xi \to \Xi + \sum_j v_j \mathcal{I}_j. \] (32)

The coefficients \( v_j \) will be determined by excluding some monomials \( P_k \) from the basis for \( \Xi \).

### 2.1 Ghost equation and invariant counterterms

The proof of physical unitarity relay on the property of \( S \) of being nilpotent. In the present on-shell formalism the ghost equation guarantees the above requirement

\[ \alpha \Box c + ev \Gamma J_2 = \Gamma \bar{c} \] (33)
This requirement excludes a mass-term in $\Gamma^{(0)}$ of the form
\[
M^2 \left( \frac{A^2}{2} + \bar{c}c - \frac{1}{2\alpha} (\phi_1^2 + \phi_2^2) \right). \tag{34}
\]
The present approach is equivalent to the Nakanishi-Lautrup formulation of the gauge fixing. The ghost equation must be valid after the renormalization procedure. For $n > 1$ we have
\[
ev J_{2c}^{(n)} = J_{2c}^{(n)}, \quad \ev J_{2c\phi_1}^{(n)} = J_{2c\phi_1}^{(n)}, \quad \ev J_{2c\phi_2}^{(n)} = J_{2c\phi_2}^{(n)}, \quad \ev J_{2cA_\mu}^{(n)} = J_{2cA_\mu}^{(n)}. \tag{35}
\]
These equations fix the counterterms
\[
\xi_{\bar{c}c}, \xi_{\bar{c}c\phi_1}, \xi_{\bar{c}c\phi_2}, \xi_{\bar{c}cA_\mu} \tag{36}
\]
since they are related to superficially finite vertex functions. The remaining counterterms
\[
\xi_{\bar{c}c}, \xi_{\bar{c}c\phi_1} \tag{37}
\]
are related to counterterms involving external sources
\[
\xi_{J_{2c}}, \xi_{J_{2c\phi_1}}. \tag{38}
\]
Appendix C lists the linearly independent ST invariants with charge conjugation +1. Any linear combination of ST invariants
\[
\Xi \rightarrow \Xi + \sum_{j=1, \ldots, 11} v_j I_j. \tag{39}
\]
can be added to the vertex functional. A straightforward analysis shows that the ghost equation is preserved provided
\[
v_7 = v_8 = 0 \tag{40}
\]
and moreover that, under such circumstances, the monomial $\int d^4x \bar{c}c$ is absent in the rest of the ST invariants in eq. (39).

For further use we notice that the rest of the constants $\{v_j\}$ can be determined by fixing the coefficients of the following nine monomials
\[
\phi_1, \phi_2^2, \phi_1 A^2, \phi_1 F_{\mu\nu}, i \bar{c} \gamma_5 \psi_1 \phi_2, i \bar{c} \gamma_5 \psi_2, J_{2c}, J_{2c\phi_1}, J_{1c\phi_2} \tag{41}
\]
as can be seen from the matrix given in Appendix C.\footnote{The Nakanishi-Lautrup formulation requires a Lagrange multiplier $b$ coupled to the gauge fixing function $F(A, \phi)$ (see the eq. (11)) and whose BRS transformation is simply given by $s b = \bar{c}, s \bar{c} = 0$. This provides an off-shell nilpotent BRS transformations avoiding the constraints (33) in order to guarantee the nilpotency of $S_0$.}
2.2 Sector 0

The counterterms containing external sources $J_i, \eta, \bar{\eta}$ are the right group to start with.

\[ \xi_{J_i c}, \xi_{J_2 c \phi_1}, \xi_{J_1 c \phi_2}, \xi_{\bar{\eta} c \psi}, \xi_{\bar{\psi} c \eta}. \]  

(42)

Their ST transforms (see Appendix B) contain the equations of motion and therefore one expects that they belong to the subspace that includes (in the sense of eq. (31)) most of the subspaces of counterterms. Moreover in the recursive equation (30) the counterterms which contain external sources are present in almost every terms. Thus it is advantageous to set all possible loop corrections to the BRS external sources to zero by using the freedom in the choice of the coefficients \{v_j\} in eq. (32). By using the ST invariants $I_9^{\Gamma - 11}$ given in Appendix C we impose the normalization conditions ($n > 0$)

\[ I_{\Gamma}(n)_{J_2 c} = 0 \]
\[ I_{\Gamma}(n)_{J_2 c \phi_1} = 0 \]
\[ I_{\Gamma}(n)_{J_1 c \phi_2} = 0. \]

(43)

As a consequence of this choice eq. (35) now fixes the counterterms in eq. (37), by using the relation

\[ I_{\Gamma} = (1 - t^{\phi c}) \Gamma + \Xi \]  

(44)

one gets ($n > 0$)

\[ \xi_{J_2 c}^{(n)} = 0 \]
\[ 8 \xi_{c c \phi_1}^{(n)} = 0 \]
\[ 8 \xi_{c c \phi_2}^{(n)} = 0 \]
\[ 8 \xi_{c A_2}^{(n)} = 0. \]

(45)

Since the ghost equation fixes all counterterms involving the ghost field, we drop the analysis of the ghost sector. The ghost part of $\Xi$ is

\[ \Xi_{\text{GHOST}} = \int d^4x \left[ \xi_{c c c c} c \Box c + \xi_{c c \phi_1} c \phi_1^2 + \xi_{c c \phi_2} c \phi_2^2 + \xi_{c A_2} c A_2^2 \right]. \]

(46)

2.3 Sector I

The next sector is selected by the condition

\[ N_{\phi} \leq 4, N_A = N_{\psi} = N_{\bar{\psi}} = 0 \]

(47)

where $N_{\phi}, N_A, N_{\psi}$ and $N_{\bar{\psi}}$ respectively count the number of $\phi, A, \psi, \bar{\psi}$. The coefficients of of the monomial of this sector are

- mass terms (3): $\xi_{\phi_1}, \xi_{\phi_2 \phi_2}$

The number in brackets counts the number of counterterms of the corresponding sub-sector
• trilinear self-interacting terms (2): $\xi_{\phi_1 \phi_2 \phi_2}$, $\xi_{\phi_1 \phi_1 \phi_1}$
• quadrilinear interacting terms (3): $\xi_{\phi_1 \phi_2 \phi_2 \phi_2}$, $\xi_{\phi_1 \phi_1 \phi_1 \phi_1}$, $\xi_{\phi_2 \phi_2 \phi_2}$

The sector can be further decomposed into two sub-sectors with $N_{\phi} \leq 2$ and $N_{\phi} > 2$. These two sub-sectors turn out to be disjoint if we put to zero the coefficient $\xi_{\phi_1 \phi_2 \phi_2}$ (see Appendix B). This can be achieved by the ST invariant $I_2$. The contribution from the lower order of perturbation are reduced if we put equal zero the coefficient $\xi_{\phi_1}$ of the tadpole. The ST invariant necessary to impose these conditions is $I_1$. Finally six coefficients have to be evaluated. A direct inspection of the ST transforms of the corresponding monomial shows that the breaking terms to be evaluated are six out of the following set

$$\psi_{\xi_{\phi_1 \phi_2 \phi_2}} = \{\psi_{\xi_{\phi_1 \phi_2 \phi_2}}, \psi_{\xi_{\phi_1 \phi_1 \phi_1}}, \psi_{\xi_{\phi_2 \phi_2 \phi_2}}\}.$$ (48)

With the above conventions it is straightforward, with the help of the BRS transformations in Appendix B, to construct the reduced matrix in eq. (28)

$$\psi_{\xi_{\phi_1 \phi_2 \phi_2}} = \begin{pmatrix} \xi_{\phi_1 \phi_2}^2 & \xi_{\phi_1 \phi_2}^2 & \xi_{\phi_1 \phi_2}^2 & \xi_{\phi_1 \phi_2}^2 & \xi_{\phi_1 \phi_2}^2 & \xi_{\phi_1 \phi_2}^2 \\ \psi_{\xi_{\phi_1 \phi_2 \phi_2}} & 0 & -2ev & 0 & 0 & 0 \\ \psi_{\xi_{\phi_1 \phi_2 \phi_1}} & 2e & -2e & 0 & 0 & 0 \\ \psi_{\xi_{\phi_1 \phi_2 \phi_1}} & 0 & 3e & -2ev & 0 & 0 \\ \psi_{\xi_{\phi_1 \phi_2 \phi_2}} & 0 & 0 & 2e & 0 & -4ev \\ \psi_{\xi_{\phi_1 \phi_2 \phi_2}} & 0 & 0 & 0 & 2e & 0 & -4ev \end{pmatrix}.$$ (49)

### 2.4 Sector II

This sector deals with the kinetic terms of the scalar fields and the corresponding terms coming form the covariant derivatives, that is the interaction terms of the scalar fields and the gauge fields. This sector also deals with the mass of the gauge bosons. This sector is selected by the condition $N_{\phi} \leq 2, N_{\phi} = 0, N_{\bar{\psi}} = 0, N_{\bar{\psi}} = 2$. $\xi^{II}$ are

• mass term for gauge field (1): $\xi_{A_\mu}^2$
• kinetic terms for scalar fields (2): $\xi_{\partial^\mu \phi_1 \partial^\mu \phi_1}, \xi_{\partial^\mu \phi_2 \partial^\mu \phi_2}$
• mixing terms between scalar field and gauge field (1): $\xi_{\partial^\mu A_\mu \phi_2}$
• coupling scalar-gauge fields (2): $\xi_{A_\mu \partial^\mu \phi_2 \phi_2}, \xi_{A_\mu \partial^\mu \phi_2 \phi_1}$
• trilinear term (1): $\xi_{A_\mu^2 \phi_1}$
• quadrilinear terms (2): $\xi_{A_\mu^2 \phi_1^2}, \xi_{A_\mu^2 \phi_2^2}$

The bilinear out-of-diagonal counterterm can be put to zero

$$t^2 \Gamma_{A_\mu \phi_2}(0) = \Xi_{A_\mu \phi_2} = 0.$$ (50)
by using the ST invariant $I_3$. Finally one has to evaluate eight coefficients

$$\xi^{II} = \left\{ \xi_{A_2^2}, \xi_{\bar{\theta}_1\partial_\mu \phi_1}, \xi_{\bar{\theta}_2\partial_\mu \phi_2}, \xi_{A_\mu \phi_1 \phi_2}, \xi_{A_\mu \phi_1}, \xi_{A_2^2 \phi_1^2}, \xi_{A_2^2 \phi_2^2} \right\}$$

in terms of the following breaking terms

$$\psi_{II}^{\dagger} = \left\{ \psi_{c\phi_2}, \psi_{c\phi_1}, \psi_{c\phi_1 \phi_2}, \psi_{c\phi_2 \phi_1}, \psi_{c\phi_1}, \psi_{c\phi_2}, \psi_{c\phi_1 \phi_2}, \psi_{c\phi_2 \phi_1} \right\}$$

The transformation matrix $e_{ik}$ (eq. (28)) is

$$
\begin{pmatrix}
\xi_{A_2^2} & \xi_{\bar{\theta}_1 \partial_\mu \phi_1} & \xi_{\bar{\theta}_2 \partial_\mu \phi_2} & \xi_{A_\mu \phi_1 \phi_2} & \xi_{A_\mu \phi_1} & \xi_{A_2^2 \phi_1^2} & \xi_{A_2^2 \phi_2^2} \\
\psi_{c\phi_2} & 0 & 0 & 2e & 0 & 0 & 0 \\
\psi_{c\phi_1} & -2e & 0 & 1 & 0 & 0 & 0 \\
\psi_{c\phi_1 \phi_2} & 0 & 0 & 0 & 1 & 1 & 0 \\
\psi_{c\phi_2 \phi_1} & 0 & 0 & 0 & 0 & 0 & 0 \\
\psi_{c\phi_2 \phi_1} & 2e & 0 & 0 & 0 & 0 & 0 \\
\psi_{c\phi_1} & 0 & 0 & 0 & e & 2 & 0 \\
\psi_{c\phi_1} & 0 & 0 & 0 & 0 & 4 & 0 \\
\psi_{c\phi_2} & 0 & 0 & 0 & 0 & 0 & e \\
\psi_{c\phi_1} & 0 & 0 & 0 & 0 & 0 & 0 \\
\psi_{c\phi_2} & 0 & 0 & 0 & 0 & 0 & 2e \\
\psi_{c\phi_1} & 0 & 0 & 0 & 0 & 0 & -2e \\
\psi_{c\phi_2} & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

2.5 Sector III

In the present model the kinetic terms for the gauge fields are trivial because of the abelianity of the gauge group. The eigenvalues of the counting operators are given by:

$$N_\phi = N_\psi = 0, (N_A + N_\bar{\phi}) = 4$$

The sector contains

- kinetic terms for gauge fields (2): $\xi_{\bar{\theta}_1 \partial_\mu \partial_\nu A_\mu}$, $\xi_{\bar{\theta}_2 \partial_\mu \partial_\nu A_\mu}$
- interacting terms (1): $\xi_{A_\mu}$
The corresponding breaking-terms are given by:

\[ \psi_{IV}^I = \left\{ \psi_c \partial_\mu A^\mu, \psi_c \partial_\mu A_\mu A^\mu, \psi_c \partial_\mu A_\mu A^\mu A_\nu A^\nu \right\} \]  \hfill (55)

The ST invariant \( I_4 \) can be used in order to put equal zero the counterterm corresponding to the transverse part

\[ \xi F^2_{\mu \nu} = 0. \]  \hfill (56)

By looking at eq. (57) we have the following relations

\[ \begin{align*}
\psi_c \partial_\mu A^\mu &= -2 \xi \partial_\mu A^\mu \\
\psi_c \partial_\mu A_\mu A^\mu &= 4 \xi A_\mu - \xi \bar{c} c A^2_\mu \\
\psi_c \partial_\mu A_\mu A^\mu A_\nu A^\nu &= 8 \xi A_\mu 
\end{align*} \]  \hfill (57)

where the coefficient \( \xi \bar{c} c A^2_\mu \) is known from the ghost equation (45).

### 2.6 Sector IV

This sector contains the Green’s functions with fermion fields, and it can be further divided into the sector of mass terms of fermion fields and their coupling with the scalar fields and the sector of the kinetic terms and the interaction with the gauge fields. The present sector is completely decoupled for the previous sectors, it is specified by the following eigenvalues:

\[ N_{\phi} \leq 1, N_{\bar{c} c} = 2, N_A + N_{\psi} \leq 1 \]  \hfill (58)

and the counterterms are

- mass term (1): \( \xi \bar{c} c \psi \)
- Yukawa term (2): \( \xi \bar{c} c \psi \phi_1, \xi \bar{c} c \psi \gamma_5 \psi \phi_2 \)
- kinetic term and interaction with the gauge field (2): \( \xi \bar{c} c \psi \phi_1, \xi \bar{c} c \psi \phi_2 \)

The breaking-terms are given by:

\[ \psi_{IV}^I = \left\{ \psi_c \bar{\psi}_5 \gamma_5 \psi, \psi_c \bar{\psi}_5 \gamma_5 \psi, \psi_c \bar{\psi}_5 \gamma_5 \psi, \psi_c \bar{\psi}_5 \gamma_5 \psi, \psi_c \bar{\psi}_5 \gamma_5 \psi, \psi_c \bar{\psi}_5 \gamma_5 \psi \right\} \]  \hfill (59)

There are two invariants (\( I_5 \) and \( I_6 \)), pertinent to this sector. They are used to impose the following normalization conditions

\[ \begin{align*}
\xi \bar{c} c \phi &= 0 \\
\xi \bar{c} c \psi \phi_2 &= 0.
\end{align*} \]  \hfill (60)
The matrix $e_{ik}$ which express the functional $\psi^{(IV)}$ in terms of $\xi^{(IV)}$ is given by

$$
\begin{pmatrix}
\xi_{\bar{\psi}\psi} & \xi_{\bar{\psi}\phi_1} & \xi_{\bar{\psi}\phi_2} \\
\psi_{\bar{c}\psi\gamma_5\psi} & -1 & 0 & 0 \\
\psi_{\bar{c}\phi_1\bar{\psi}\gamma_5\psi} & 0 & -1 & 0 \\
\psi_{\bar{c}\phi_2\bar{\psi}\psi} & 0 & 1 & 0 \\
\psi_{\bar{c}\bar{c}\gamma_5\psi} & 0 & 0 & -\frac{1}{2} \\
\psi_{\bar{c}\bar{c}\gamma_5\gamma_5\psi} & 0 & 0 & -\frac{1}{2}
\end{pmatrix}
$$

(61)

2.7 Summary of the normalization conditions

For $n > 0$ we have imposed the normalization conditions

$$
\xi^{(n)}_{\phi_1} = 0 \quad \xi^{(n)}_{\phi_2 \phi_1} = 0 \quad \xi^{(n)}_{\partial A_{\mu} \phi_2} = 0 \quad \xi^{(n)}_{\bar{c} c_{\mu} \phi_2} = 0 \quad \xi^{(n)}_{\bar{c} c_{\gamma_5} \phi_2} = 0 \quad \xi^{(n)}_{\bar{c} \gamma_5 \phi_2} = 0 \\
\xi^{(n)}_{J_2 c} = 0 \quad \xi^{(n)}_{\bar{c} c_{\phi_1} \phi_2} = 0 \quad \xi^{(n)}_{\bar{c} c_{\phi_2} \phi_1} = 0.
$$

(62)

The evaluation of physical S-matrix elements requires the evaluations of the eigenvalues and of the eigenvectors of the two-points vertex functions. The physical amplitudes are then obtained from the connected and truncated Feynman amplitudes evaluated on the physical states obtained from the diagonalization procedure (LSZ reduction formalism). Thus on-shell normalization can be by-passed. The coupling constants and masses in $\Gamma^{(0)}$ are dummy parameters which can be obtained from a sufficient number of physical processes.

3 ST breaking terms

In the strategy outlined before the counterterm functional $\Xi$ is obtained by solving a set of linear equations (28). The restoration of ST invariance consists in the evaluation of a certain number of (finite) vertex functions (the functional $\Psi$). This fact puts in clear evidence that it is the finite part of the perturbative expansion that fixes the counterterms in the action.

In this section we discuss some aspects of this procedure. The first step consists in the evaluation of the functional derivatives of $\Psi$. It is of some help to remember that, in absence of anomalies, $\Psi$ is the image through $S_0$ of non-invariant counterterms ($\Xi$). Therefore it has FP-charge equal $+1$, $C = 0$ and dimension less or equal five. The next step is to find the coefficients $\psi_i$ in the expansion in terms of Lorentz scalar monomials

$$
\Psi = \sum_i \psi_i M_i
$$

(63)

Let us write explicitly, for $n > 0$, the operator $S_0$, where we impose the ghost equation of motion given in eq. (33) i.e.

$$
ge v J_2^{(n)} = 1^{(n)} \quad \text{for } n > 0
$$

(64)
\[ S_0(\Gamma^{(n)}) = \int d^4x \left\{ \partial_\mu c \Gamma_{A_\mu}^{(n)} - ec\phi_2 \Gamma_{\phi_1}^{(n)} + ec(\phi_1 + v)\Gamma_{\phi_2}^{(n)} + i\frac{e}{2} \bar{c}\psi\gamma_5\Gamma_{\psi}^{(n)} + i\frac{e}{2} \bar{c}\gamma_5\psi\Gamma_{\psi}^{(n)} + \Gamma_{\phi_1}^{(0)} \Gamma_{J_1}^{(n)} + \left[ \frac{e}{\alpha} \Gamma_{\phi_2}^{(0)} + ec(\phi_2) \right] \Gamma_{J_2}^{(n)} - \Gamma_{\psi}^{(0)} \Gamma_{\phi_1}^{(n)} + \Gamma_{\psi}^{(0)} \Gamma_{\phi_2}^{(n)} \right\} \]  

(65)

By imposing the condition (64) the breaking term \( \Psi \) changes. We denote this change with the notation \( \Psi \rightarrow \hat{\Psi} \).

In the linearized form \( (S_0) \) one of the factors in each monomial contains the vertex function at zero loop \( \Gamma^{(0)} \). All these facts have some interesting consequences

1. The functional derivatives of \( \Psi \) relevant for the evaluation of the counterterm can be read directly from the BRS transforms of all action-like terms (see Appendix B).

2. Let \( \delta \) be the total dimension of the fields we use for the functional derivative of \( \Psi \). Then the order of the Taylor operator \( \delta_D \) is (see eq. (7))

\[ \delta_D = 5 - \delta \]  

(66)

3. Let us consider a generic term of \( S_0 \) for instance \( \Gamma_{J_1}^{(0)} \Gamma_{\phi_1}^{(n)} \) or \( \Gamma_{J_1}^{(n)} \Gamma_{\phi_1}^{(0)} \). If \( \Gamma^{(0)} \) does not contain any dimensional parameter, then

\[ t^\delta_D \Gamma_{J_1}^{(0)} \Gamma_{\phi_1}^{(n)} = \Gamma_{J_1}^{(0)} t^{\delta_D} \Gamma_{\phi_1}^{(n)}. \]  

(67)

In the above equation we use a rather short-hand writing and to be more explicit we give an example: by taking the functional derivative of the \( \Gamma_{J_1}^{(0)} \Gamma_{\phi_1}^{(n)} \) term with respect to \( c\phi_2 \), the \( \delta_D \) is equal to 3 and we get

\[ t^3 \Gamma_{J_1}^{(0)} c\phi_2 \Gamma_{\phi_1}^{(n)} = -ct^2 \Gamma_{\phi_1}^{(n)}. \]  

(68)

where \( \delta_{pc} = 3 \) is the superficial degree of divergence of \( \Gamma_{\phi_1}^{(n)} \).

4. The above point implies that \( \Psi \) (eq. (10)) gets contributions only from those terms of \( \Gamma^{(0)} \) which carry a dimensioned parameter \( (v \) and masses).

The functional derivatives of \( \Psi \) are performed in Appendix B. It should be noticed that, due to our choice of normalization conditions, few other counterterms turn out to be zero at every order.

\[ \int d^4x \phi_1^2 \quad \int d^4x \phi_2^2 \quad \int d^4x (\partial_\mu \phi_2)^2 \quad \int d^4x A_\mu^2. \]  

(69)

This is due to the combined effects of our choice of normalization conditions and of the zero momentum subtraction procedure. Moreover the contribution to STI from the lower order amplitudes appear only in few functional derivatives of \( \Psi \). One can describe this fact by saying that the set of STI becomes almost linear in \( \Gamma \).
4 Solution for counterterms

The relations obtained in Appendix D can be expanded in terms of Lorentz invariant amplitudes. Thus one can express the invariant amplitude for Ξ in terms of the invariant amplitude for Ψ. This amounts to solve the linear algebra problem given in equation (28) where the matrices are given in (49), (53), (49), (57) and (61). We remind our notations where the small letters ξ, γ and ψ denote the coefficients of the Lorentz invariant monomials respectively of of Ξ (counterterms), Γ and Ψ (breaking terms) indicated by the subscript. The order of perturbation theory is not shown and it is understood to be n, unless explicitly exhibited.

The ghost equation (33) fixes the kinetic counterterms of the ghost (see eq. (45))

\[ ξ_{\text{c@c}} = ev_{\gamma_1J_2c} \]  

(70)

4.1 Counterterms of sector I

In this sector we have the same number of equations and unknowns. The solution is (including the normalization conditions)

\[
\begin{align*}
ξ_{φ_1} &= 0 \quad ξ_{φ_2} = 0 \quad ξ_{φ_1^2} = 0 \quad ξ_{φ_2^2} = 0 \quad ξ_{φ_1^2φ_1} = 0 \quad ξ_{φ_2^2φ_1} = 0 \\
ξ_{φ_1^2} &= \frac{1}{3e} \left\{ -m_1^2γ_{J_2cφ_2} + 4ev^2γ_{φ_1^2φ_1} - m_1^2vγ_{J_1cφ_1} - \frac{3}{2}m_1^2γ_{J_1cφ_2φ_1} \right\} \\
ξ_{φ_2^2φ_1} &= \frac{1}{2e} \left\{ -2λνγ_{J_2cφ_1} - vλγ_{J_1cφ_2φ_1} + 4evγ_{φ_1^2φ_1} - m_1^2γ_{J_1cφ_2^2} \right\} \\
ξ_{φ_2^2φ_1} &= \frac{1}{4e} \left\{ -2vλγ_{J_2cφ_1} - 4vλγ_{J_1cφ_2φ_1} - m_1^2γ_{J_1cφ_2φ_1} + 2evγ_{φ_2^2φ_1}^3 \\
&\quad - 2λνγ_{J_2cφ_1} + 4evγ_{φ_2^2φ_1} - m_1^2γ_{J_1cφ_2^2} + 3\sum_{j=1}^{n-1} γ_{J_1cφ_2φ_1} ξ_{φ_1^2}^{(n-j)} \right\}
\end{align*}
\]

(71)

4.2 Counterterms of sector II

In this sector the problem is over-determined. We use the first six and the last two rows of the matrix (53) . The solution is (including the normalization conditions)

\[
\begin{align*}
ξ_{A^2} &= 0 \quad ξ_{A^νφ_1} = 0 \quad ξ_{φ_1 φ_2} = 0 \\
ξ_{A^νφ_1 φ_2} &= -m_1^2γ_{J_1cφ_2} - 2evγ_{φ_1 φ_2} - m_1^2vγ_{J_2c} - 2λνγ_{J_2c} \\
ξ_{A^νφ_1 φ_2} &= m_1^2γ_{J_1cφ_2} - m_1^2γ_{J_1cφ_2} - 2λνγ_{J_2c} \\
ξ_{φ_1 φ_1} &= \frac{1}{2e} \left\{ m_1^2γ_{J_1cφ_2} + evγ_{φ_1 φ_2} - m_1^2γ_{φ_1 φ_2} + m_1^2γ_{J_1cφ_2} \right\} \\
ξ_{A^φ_1} &= -\frac{1}{2}m_1^2γ_{J_1cφ_2} + \frac{1}{2}m_1^2evγ_{J_1cφ_2} + (ev)^2γ_{φ_1 φ_2} + \frac{1}{2}m_1^2evγ_{J_2c} \\
ξ_{A^φ_1} &= \frac{1}{2e} ξ_{A^2 φ_1}
\end{align*}
\]
The rest of the equations provided by the matrix provides consistency conditions. However not all of them are linear independent, in fact one can easily check that the linear combination implied by

\[-2\psi_c A^2 \phi_2 + e\psi_v A^2 \phi_2 - e v J_1 c \phi_2 - \gamma J_1 c A^\mu - \gamma J_1 c A^\mu = 0\]  

(73)

1. \(e v[\gamma \partial_{\nu, J_1 c \phi_2} - \gamma J_1 c \phi_2] + e v J_1 c A^\mu + \gamma J_1 c A^\mu = 0\)  

(74)

2. \(6\lambda v(\gamma J_1 c c A^\mu - \gamma J_1 c A^\mu) - m_1^2 \gamma J_1 c A^\mu - m_1^2 \gamma J_1 c A^\mu + 2m_1^2 \gamma J_1 c A^\mu - m_1^2 \gamma J_1 c c A^\mu - 4\lambda v J_1 c \phi_2 + 2 v J_1 c \phi_2 A^\mu + 2 v J_1 c \phi_2 A^\mu = 0\)  

(75)

3. \(+ \lambda v^2 e^2 J_1 c \phi_2 X + e^3 v \gamma \partial_{\nu, J_1 c \phi_2} - \gamma J_1 c \phi_2 - 2 e^2 J_1 c \phi_2 A^\mu + e^2 \gamma J_1 c \phi_2 A^\mu + 2 e v J_1 c \phi_2 A^\mu - 2 v J_1 c \phi_2 A^\mu + 2 v J_1 c \phi_2 A^\mu = 0\)  

(76)

4. \(+ 3 e^3 v \gamma J_1 c \phi_2 X + e^3 v \gamma \partial_{\nu, J_1 c \phi_2} - \gamma J_1 c \phi_2 - 3\lambda v J_1 c \phi_2 - \gamma J_1 c \phi_2 - e v J_1 c \phi_2 A^\mu + \gamma J_1 c \phi_2 A^\mu = 0\)  

(77)

### 4.3 Counterterms of sector III

The counterterms of the sector III, together with the normalization condition, are

\[\xi_{F^2_{\mu \nu}} = 0\]

\[\xi_{\partial_{\mu, A^\nu} \partial_{\mu, A^\nu}} = \frac{1}{2} \{ e v A_{\mu, \partial_{\mu, A^\nu} - e v J_1 c \phi_2} \}

(78)

\[\xi_{A^2} = \frac{1}{4} \{ e v J_1 c A^2 + e^2 v \gamma \partial_{\nu, J_1 c A^\mu} + \gamma J_1 c A^\mu \} \]

16
In this sector there is one consistency condition

\[
\gamma^\phi_2 A_\mu \partial^\mu A^2 + e\gamma J_1 c \partial^\mu A^\mu + \gamma J_2 c A^2 + \sum_{j=1}^{n-1} \gamma^{(j)}_1 \partial^\mu A_\mu \xi^{(n-j)}_1 A^2 \\
= \gamma^\phi_2 A_\mu A^2 + e\gamma J_1 c A^\mu + \sum_{j=1}^{n-1} \gamma^{(j)}_1 \partial^\mu A_\mu \xi^{(n-j)}_1 A^2.
\]

(79)

It is remarkable that contribution from the lower order terms appear only in three counterterms (see the eqs. (71), (72) and (73)).

4.4 Counterterms of the fermion sector

The analysis of the breaking terms in terms of the Lorentz invariant amplitude performed in the Appendix reveals that the fermion source counterterms are non vanishing. The counterterms of this sector are

\[
\xi^{\tilde{2}}_2 (\bar{\psi} \gamma_5 \psi + c\bar{\psi} \gamma_5 \eta) = -\frac{1}{2G} \left\{ 2ev\gamma^\phi_2 \bar{\psi} \psi - 2G\gamma^\phi_2 \bar{\psi} \psi \\
+ ev\gamma^\phi_2 \bar{\psi} \gamma_5 \psi + 2G\gamma^\phi_2 \bar{\psi} \gamma_5 \psi - m^2 \gamma^\phi_2 \bar{\psi} \gamma_5 \psi \\
- \sum_{j=1,n-1} 2\xi^{(n-j)}_2 \bar{\psi} \left[ (n-j) \gamma^\phi_2 \bar{\psi} \gamma_5 \eta + \gamma^\phi_2 \bar{\psi} \gamma_5 \eta + \xi^{(n-j)}_1 \bar{\psi} \gamma_5 \eta \right] \right\}
\]

(80)

The other counterterms are

\[
\xi^{\tilde{1}}_2 \bar{\psi} \psi = \frac{1}{2e} \left\{ 2ev\gamma^\phi_2 \bar{\psi} \psi - 2G\gamma^\phi_2 \bar{\psi} \psi \\
- ev\gamma^\phi_2 \bar{\psi} \gamma_5 \psi + 2G\gamma^\phi_2 \bar{\psi} \gamma_5 \psi - m^2 \gamma^\phi_2 \bar{\psi} \gamma_5 \psi \\
- \sum_{j=1,n-1} 2e\xi^{(n-j)}_2 \bar{\psi} \left[ (n-j) \gamma^\phi_2 \bar{\psi} \gamma_5 \eta + \gamma^\phi_2 \bar{\psi} \gamma_5 \eta + \xi^{(n-j)}_2 \bar{\psi} \gamma_5 \eta \right] \right\}
\]

(81)

\[
\xi^{\tilde{1}}_2 \bar{\psi} \psi = -\frac{1}{e} \left[ G\xi^{\tilde{1}}_2 (\bar{\psi} \gamma_5 \psi + c\bar{\psi} \gamma_5 \eta) - \sum_{j=1,n-1} \xi^{(n-j)}_2 \bar{\psi} \gamma_5 \eta \right]
\]

(82)

and

\[
\xi^{\tilde{1}}_1 \bar{\psi} \gamma_\mu \partial^\mu \psi = \frac{2}{e} \left\{ -\frac{1}{2} \xi^{\tilde{1}}_2 (\bar{\psi} \gamma_5 \psi + c\bar{\psi} \gamma_5 \eta) \\
+ G\gamma^\phi_2 \bar{\psi} \gamma_5 \partial^\mu \psi + G\gamma^\phi_2 \bar{\psi} \gamma_5 \partial^\mu \psi - ev\gamma^\phi_2 \bar{\psi} \gamma_\mu \gamma_5 \partial^\mu \psi \\
- \sum_{j=1,n-1} \xi^{(n-j)}_2 \bar{\psi} \left[ (n-j) \gamma^\phi_2 \bar{\psi} \gamma_5 \eta + \gamma^\phi_2 \bar{\psi} \gamma_5 \eta + \xi^{(n-j)}_2 \bar{\psi} \gamma_5 \eta \right] \right\}
\]

(83)
Since we require hermiticity and charge conjugation invariance, there is one consistency condition left, given by the equation
\begin{equation}
0 = -e v \gamma_{\phi_2} \gamma^\mu A_\mu \psi \overline{\psi} + G v \gamma_{c \bar{e} \bar{\gamma}^5} \gamma_5 \psi A_\mu - G v \gamma_{c \bar{e} \bar{\gamma}^5} \gamma^\mu \eta A_\mu + 
\sum_{j=1,n-1} \left( \epsilon(j) \gamma^{n-j} A_\mu - \eta(j) \gamma^{n-j} A_\mu \right) (84)
\end{equation}

5 Conclusions

The absence of anomalies in the Higgs-Kibble model allows the explicit construction of counterterms which re-establish the Slavnov-Taylor invariance of the model. Therefore any regularization procedure which preserves the Lorentz covariance and the relevant discrete symmetries can be corrected by finite counterterms. In the present work we give explicitly the counterterms in terms of a set finite vertex functions. Our strategy relies on two essential ingredients. One is the possibility to perform subtraction at zero momentum. The second consists in the use of the normalization conditions which simplify the construction of explicit solutions. Quite a few counterterms turn out to be zero and moreover the contribution of the lower terms in the perturbative expansion is highly reduced. Although the solution look cumbersome we believe that it makes possible the automatic evaluation of the counterterms.

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A Classical action and BRS

A.1 Feynman rules

The Lagrangian density is
\begin{equation}
\mathcal{L} = -\frac{1}{4} F_{\mu \nu}^2 - \frac{\alpha}{2} (\partial A)^2 + |D_\mu \phi|^2 - \lambda (|\phi|^2 - \frac{v^2}{2})^2
\end{equation}
\begin{equation}
+ \overline{\psi} D \psi + \frac{G}{\sqrt{2}} \overline{\psi}(1 - \gamma_5) \psi \phi + \frac{G}{\sqrt{2}} \overline{\psi}(1 + \gamma_5) \psi \phi^* (85)
\end{equation}

where
\begin{align*}
D_\mu &= \partial_\mu - i e A_\mu \\
\mathcal{D} &= \partial_\mu - i e \gamma_5 A_\mu (86)
\end{align*}
BRS transformations

\[ \delta A_\mu = \partial_\mu c \]
\[ \delta \phi = iec\phi \]
\[ \delta \phi^* = -iec\phi^* \]
\[ \delta \psi = -ie 5\psi \gamma c \]
\[ \delta \bar{\psi} = ie 2\Delta \bar{\psi} \gamma c \]

(87)

Now we consider the spontaneous symmetry breaking

\[ \phi = \phi_1 + v + i\phi_2 \frac{\sqrt{2}} { \sqrt{2} } \]

(88)

The bilinear parts give an out-of-diagonal term

\[ ev\phi_2 \partial A \]

(89)

thus we need a gauge fixing ('t Hooft)

\[ -\frac{\alpha}{2} \left( \partial A + \frac{ev}{\alpha} \phi_2 \right)^2 \]

(90)

Thus we complete the BRS

\[ \delta \phi_1 = -ec\phi_2 \]
\[ \delta \phi_2 = ec(\phi_1 + v) \]
\[ \delta \bar{c} = \bar{F} = \partial A + \frac{ev}{\alpha} \phi_2 \]

(91)

Then the gauge fixing term is

\[ \Gamma^{(0)}_{GF} = \int d^4x \left[ -\frac{\alpha}{2} \bar{F}^2 + \alpha \bar{c} \delta \bar{F} \right] \]
\[ = \int d^4x \left[ -\frac{\alpha}{2} \bar{F}^2 + \alpha \bar{c} \bar{c} + (ev)^2 \bar{c} c + e^2 v \bar{c} \phi_1 \right] \]

(92)

and the zero-loop action is

\[ \Gamma^{(0)} = \int d^4x \left[ -\frac{1}{4} F^2_{\mu\nu} + \frac{e^2 v^2}{2} A^2_\mu \right] \]
\[ -\frac{\alpha}{2} \bar{A}^2 + \alpha \bar{c} \bar{c} + (ev)^2 \bar{c} c + e^2 \bar{c} \phi_1 \]
\[ + \frac{1}{2} (\partial_\mu \phi_1^2 + \partial_\mu \phi_2^2) - \lambda v^2 \phi_1^2 - \frac{(ev)^2}{2\alpha} \phi_2^2 \]
\[ + eA_\mu(\phi_2 \partial^\mu \phi_1 - \partial^\mu \phi_2 \phi_1) + e^2 v \phi_1 A^2 + \frac{e^2}{2} (\phi_1^2 + \phi_2^2) A^2 \]
\[-\lambda \nu \phi_1 (\phi_1^2 + \phi_2^2) - \frac{\lambda}{4} (\phi_1^2 + \phi_2^2)^2 \]
\[+ \bar{\psi} i \partial \psi + G \bar{\psi} \gamma_5 \psi A^\mu \]
\[+ G \bar{\psi} \psi \phi_1 - i G \bar{\gamma}_5 \psi \phi_2 \]
\[+ J_1 [-e c \phi_2 + J_2 e c (\phi_1 + v) + i e \bar{\gamma}_5 \psi c + i e \bar{\gamma}_5 \psi \eta] \]

The action is C invariant if the fields \(\phi_2, A_\mu, i \bar{\psi} \gamma_5 \psi, c, \bar{c}, J_2\) are C odd and the fields \(\phi_1, J_1, \bar{\psi} \psi\) are C even. This invariance can be extended to \(\eta, \bar{\eta}\) by requiring
\[C \psi C^{-1} = B \bar{\psi}^T, \quad C \eta C^{-1} = B \bar{\eta}^T \quad B^\dagger \gamma_\mu B = -\gamma_\mu^T \]
\[B^2 = 1 \quad B^* = B \quad B^T = -B \quad B^\dagger = B^{-1} \] (94)

Moreover we impose hermiticity for the low momentum expansion of the vertex amplitude \(\Gamma\) by requiring
\[c^\dagger = c \quad \bar{c}^\dagger = -\bar{c} \quad \bar{\eta}^\dagger = \gamma_0 \eta \] (95)

### B  ST transformation of counterterms

The ST of the counterterms.

The scalar boson sectors.

\[S_0[\int d^4 x \phi_1] = -e \int d^4 x (c \phi_2) \]
\[S_0[\int d^4 x \phi_2] = -2e \int d^4 x (c \phi_1 \phi_2) \]
\[S_0[\int d^4 x \phi_3] = 2e \int d^4 x (c \phi_2 (\phi_1 + v)) \]
\[S_0[\int d^4 x \phi_4] = -3e \int d^4 x (c \phi_2^2 \phi_2) \]
\[S_0[\int d^4 x \phi_5] = e \int d^4 x [2c \phi_2 (\phi_1 + v) \phi_1 - \phi_2^3] \]
\[S_0[\int d^4 x \phi_6] = -4e \int d^4 x (c \phi_3^2 \phi_2) \]
\[S_0[\int d^4 x \phi_7] = 4e \int d^4 x (c \phi_3^2 (\phi_1 + v)) \]
\[S_0[\int d^4 x \phi_8] = e \int d^4 x (2c \phi_2 \phi_1^2 (\phi_1 + v) - 2c \phi_2 \phi_2 \phi_1) \] (96)

The kinetic boson sector

\[S_0[\int d^4 x A^2] = -2 \int d^4 x (c \partial_\mu A^\mu) \]
The ghost sector

\[ S_0 \int d^4 x \psi \gamma^0 \gamma^5 \gamma^\mu \nu \psi = \int d^4 x \left[ \psi \gamma^0 \gamma^5 \partial^\mu \psi + \gamma^\mu \gamma^5 \psi \right] \]

The ghost sector

\[ S_0 \int d^4 x \bar{c} c = \int d^4 x F c \]
\[ S_0 \int d^4 x \bar{c} c = \int d^4 x F \square c \]
\[ S_0 \int d^4 x \bar{c} \phi_1 = \int d^4 x F c \phi_1 \]
\[ S_0 \int d^4 x \bar{c} \phi_2 = \int d^4 x F c \phi_2 \]  
\[ S_0 [ \int d^4 x \bar{c} \phi_1 ] = \int d^4 x F c \phi_1 \]
\[ S_0 [ \int d^4 x \bar{c} \phi_2 ] = \int d^4 x F c \phi_2 \]  
\[ S_0 [ \int d^4 x \bar{c} \phi_3 ] = \int d^4 x F c \phi_3 \]  
\[ S_0 [ \int d^4 x \bar{c} \phi_4 ] = \int d^4 x F c \phi_4 \]  
\[ S_0 [ \int d^4 x \bar{c} \phi_5 ] = \int d^4 x F c \phi_5 \]  
\[ S_0 [ \int d^4 x \bar{c} \phi_6 ] = \int d^4 x F c \phi_6 \]  
\[ S_0 [ \int d^4 x \bar{c} \phi_7 ] = \int d^4 x F c \phi_7 \]

Fermion sources sector

\[ S_0 [ \int d^4 x \frac{i}{2} (\bar{\eta} \gamma_5 \psi c + c \bar{\psi} \gamma_5 \eta)] = \epsilon \left[ \frac{1}{2} \partial \mu (\bar{\psi} \gamma_\mu \gamma_5 \psi) \right] + i G v \bar{\psi} \gamma_5 \psi + i G \bar{\psi} \gamma_5 \psi + G \bar{\psi} \phi_1 + G \bar{\psi} \phi_2 \]

\[ \int d^4 x \frac{i}{2} (\bar{\eta} \gamma_5 \psi c + c \bar{\psi} \gamma_5 \eta)] = \epsilon \left[ \frac{1}{2} \partial \mu (\bar{\psi} \gamma_\mu \gamma_5 \psi) \right] + i G v \bar{\psi} \gamma_5 \psi + i G \bar{\psi} \gamma_5 \psi + G \bar{\psi} \phi_1 + G \bar{\psi} \phi_2 \]

\[ \text{C ST invariants} \]

We have two classes of ST invariants: the BRS invariants where the sources do not intervene

\[ I_1 = \int d^4 x (\phi_1^2 + \phi_2^2 + 2v \phi_1) \]
\[ I_2 = \int d^4 x (\phi_1^4 + \phi_2^4 + 2\phi_1^2 \phi_2^2 + 4v \phi_1^2 + 4v \phi_2^2 + 4v^2) \]
\[ I_3 = \int d^4 x |D_\mu \phi|^2 \]
\[ I_4 = \int d^4 x (F_{\mu\nu})^2 \]
\[ I_5 = \int d^4 x \bar{\psi} i \gamma_\mu D^\mu \psi \]
\[ I_6 = \int d^4 x \bar{\psi} (\phi_1 + v) - i \gamma_5 \phi_2) \psi \]
\[ I_7 \equiv I_7 = \int d^4 x \left( \frac{1}{2} \mathcal{F}^2 + \bar{c} \delta_{\text{BRS}} \mathcal{F} \right) \]
\[ I_8 \equiv I_8 = \int d^4 x (\frac{1}{2} A^2 + \phi + \frac{v^2}{\alpha_1}) \]

and ST invariants with external sources:

\[ I_9 = \int d^4 x [A^\mu \Gamma^{(0)}_{A^\mu} + e \Gamma^{(0)} + \alpha (\mathcal{F} \partial^\mu A_\mu - \bar{c} \square c)] \]
\[ I_{10} = S_0 (\int d^4 x J_1) = \int d^4 x \Gamma^{(0)}_{\phi_1} \]
\[ I_{11} = S_0 (\int d^4 x J_1 \phi_1) = \int d^4 x (\phi_1 \Gamma^{(0)}_{\phi_1} + e J_1 \phi_2) \]
There are other invariants which are linearly dependent from the previous ones.

\[ \mathcal{I}_{12} = \int d^4x \left[ \phi_2 \Gamma_{\phi_2}^{(0)} - e J_2 c(\phi_1 + v) + ev(\mathcal{F} \phi_2 - e \bar{c}c(\phi_1 + v)) \right] \]

\[ = -\lambda v^2 \mathcal{I}_1 - \lambda \mathcal{I}_2 + 2 \mathcal{I}_3 + G \mathcal{I}_6 - ev \mathcal{I}_{10} - \mathcal{I}_{11} \]

\[ \mathcal{I}_{13} = \int d^4x \left( \Gamma_{\psi}^{(0)} \psi - \frac{1}{2} \bar{c}c \gamma^5 \psi \right) = -\mathcal{I}_5 - G \mathcal{I}_6 \]

\[ \mathcal{I}_{14} = \int d^4x \left( \bar{\psi} \Gamma_{\psi}^{(0)} + \frac{i}{2} \bar{c}c \gamma^5 \eta \right) = -\mathcal{I}_{13} \]  

The coefficients of the invariant counterterms can be fixed by choosing the normalization conditions on some monomials. The following matrix provides an example of the linear dependence of the ST invariants from a set of monomials (for comparison an extra row is added involving the fermi external source)

\[
\begin{pmatrix}
\phi_1 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_9 & v_{10} & v_{11} \\
2v & 0 & 0 & 0 & 0 & 0 & -2\lambda v^2 & 0 \\
\phi_2 & 0 & 4v & 0 & 0 & 0 & 0 & -\lambda & -\lambda v \\
A^2 & 0 & 0 & e^2v & 0 & 0 & 2e^2v & e^2 & e^2v \\
P_{\mu \nu} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\bar{\psi} \gamma^\mu \gamma^\nu \psi A^\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
i \bar{\psi} \gamma^\mu \gamma^\nu \phi_2 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
J_2 c & 0 & 0 & 0 & 0 & 0 & 0 & ev & e \\
J_2 c \phi_1 & 0 & 0 & 0 & 0 & 0 & e & 0 & e \\
J_1 c \phi_2 & 0 & 0 & 0 & 0 & 0 & -e & 0 & e \\
i \bar{\psi} \gamma^\mu \psi c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{e}{2} \\
\end{pmatrix}
\]

D Functional derivatives of \( \Psi \)

1. We project out the lowest powers of the momenta with \((1 - t^3)\) and moreover we use the normalization conditions

\[ \tilde{\Psi}_{c \phi_2} = -ev \Xi_{c \phi_2}^{(n)} \]

\[ = \left( \frac{ev}{\alpha} - m_{2 \phi_2}^2 (t^3 - t^1) \Gamma_{\phi_2}^{(n)} + t^3 \sum_{j=1}^{n-j} \Pi_{c \phi_2}^{(j)} \Pi_{\phi_2}^{(n-j)} \right) = t^3 \sum_{j=1}^{n-j} \Pi_{c \phi_2}^{(j)} \Pi_{\phi_2}^{(n-j)} \]

\[ = 0 \]

In the Taylor expansion denoted by \( t^3 \), the odd-number derivative of the vertex functions at zero momentum are zero, by Lorentz covariance. Moreover the constant term is zero due to the normalization condition \( \Pi_{c \phi_2}^{(0)}(0) = 0 \) thus only \( \xi_{\phi_2}^{(n)} \) can be non-zero. However we get

\[ - ev \Xi_{c \phi_2}^{(n)} = \sum_{j=1}^{n-j} \left( (t^2 - t^1) \Pi_{c \phi_2}^{(j)} \Pi_{\phi_2}^{(n-j)}(0) + \Pi_{c \phi_2}^{(j)}(0)(t^2 - t^1) \Pi_{\phi_2}^{(n-j)} \right) = 0 \]
Thus finally \( \Xi_{\phi_2} = 0 \) is zero.

2. Take the relevant functional derivatives of eq. (65). By using the normalization conditions and \( \Xi_{\phi_2} = 0 \) one gets

\[
\hat{\Psi}^{(n)}_{c\phi_2(p)\phi_1(q)} = -[i(p + q)\mu\Xi_{A^\mu A_\nu A_\mu A_\nu}^{(n)}] - c\Xi_{\phi_1,\phi_1}^{(n)} + \nu\int d^4x \hat{\Psi}_{c\phi_2}^{(n)}(x) = -m_1^2(t^2 - t^1)\Gamma_{cJ_1(p)\phi_2}^{(n)}(q) + ev(t^2 - t^1)\Gamma_{cJ_2(p)\phi_1}^{(n)}(q) - 2\lambda v(t^2 - t^1)\Gamma_{cJ_2(p+q)}^{(n)}(q)
\]

(107)

The lower order contributions are zero by the normalization conditions. Since \( \Psi^{(n)}_{c\phi_2\phi_1} \) contains only terms quadratic in the momenta, then there is no counterterm as \( \int d^4x \hat{\Psi}_{c\phi_2}^{(n)} \). It should be reminded that we have already chosen \( \Xi_{\phi_2\phi_1} = 0 \).

3. 

\[
\hat{\Psi}^{(n)}_{c\phi_2\phi_1} = -[e\Xi_{\phi_1}^{(n)} + ev\Xi_{\phi_2\phi_1}^{(n)}] = -2m_1^2(0)\Gamma_{cJ_1(p)\phi_2}^{(n)}(0)
\]

(108)

4. 

\[
\hat{\Psi}^{(n)}_{c\phi_2 \phi_1} = -[e\Xi_{\phi_1}^{(n)} + 3ev\Xi_{\phi_2\phi_1}^{(n)}] = -18\lambda ev\Gamma_{cJ_1(p)\phi_2}^{(n)}(0) - 3m_1^2(0)\Gamma_{cJ_1(p)\phi_2}^{(n)}(0) + ev\Gamma_{cJ_2(p)\phi_1}^{(n)}(0) - 6\lambda ev\Gamma_{cJ_2(p)\phi_1}^{(n)}(0)
\]

(109)

5. 

\[
\hat{\Psi}^{(n)}_{c\phi_2} = 0
\]

(110)

6. 

\[
\hat{\Psi}^{(n)}_{c\phi_2 \phi_1} = -[3e\Xi_{\phi_2\phi_1}^{(n)} + e\Xi_{\phi_1}^{(n)}] = -6\lambda ev\Gamma_{cJ_1(p)\phi_2}^{(n)}(0) + ev\Gamma_{cJ_1(p)\phi_2}^{(n)}(0) - m_1^2(0)\Gamma_{cJ_1(p)\phi_2}^{(n)}(0) - 6\lambda ev\Gamma_{cJ_2(p)\phi_1}^{(n)}(0)
\]

(111)

7. 

\[
\hat{\Psi}^{(n)}_{cA_\mu} = -ip\mu\Xi_{A^\mu A_\nu}^{(n)} = ev(t^3 - t^2)\Gamma_{cJ_1(p)\phi_2}^{(n)}(0) + ev\Gamma_{cJ_2(p)\phi_1}^{(n)}(0) - iev\Gamma_{cJ_2(p)\phi_1}^{(n)}(0) - ev\Gamma_{cJ_2(p)\phi_1}^{(n)}(0)
\]

(112)

Then there is no contribution to \( A^2 \) and to the transverse part of \( A_\mu \), i.e. \( \int d^4x E^2 \). Only to \( (\partial_\mu A^\mu)^2 \).

8. 

\[
\hat{\Psi}^{(n)}_{cA_\mu(p)\phi_1(q)} = -[i(p + q)\mu\Xi_{A^\mu A_\nu A_\mu A_\nu}^{(n)}] = -m_1^2(t^2 - t^1)\Gamma_{cJ_1(p)\phi_2}^{(n)}(q) + ev(t^2 - t^1)\Gamma_{cJ_2(p)\phi_1}^{(n)}(q)
\]

(113)

The last term is zero because of covariance.
9.

\[
\begin{align*}
\hat{\psi}^{(n)}_{cA_\nu(p_1)\phi_1(q_1)\phi_1(q_2)} &= - \left[ i(p + q_1 + q_2)\mu\Xi^{(n)}_{\nu A_\nu(p_1)\phi_1(q_1)\phi_1(q_2)} + e\Xi^{(n)}_{\phi_2 A_\nu(p_1)\phi_1(q_1)} + e \Xi^{(n)}_{\phi_2 A_\nu(p_1)\phi_1(q_2)} \right] \\
&= -m_0^2(t^1 \Gamma^{(n)}_{cJ_1 A_\nu(p_1)\phi_1(q_1)} + t^1 \Gamma^{(n)}_{cJ_1 A_\nu(p_1)\phi_1(q_2)}) - 6\lambda v(t^1 - t^0) \Gamma^{(n)}_{cJ_1 (q_1 + q_2) A_\nu(p_1)} \\
&+ ev(t^1 - t^0) \Gamma^{(n)}_{\phi_2 A_\nu(p_1)\phi_1(q_1)\phi_1(q_2)} - ive\nu^\nu \Gamma^{(n)}_{cJ_2 (p_1)\phi_1(q_1)\phi_1(q_2)} + \sum_{j=1}^{n-1} \left( \Pi^{(n-j)}_{\phi_1 A_\mu(0)} t^1 \Pi^{(j)}_{cJ_1 A_\nu} \right)
\end{align*}
\]

(114)

Notice that the breaking-term \( \Gamma^{(0)}_{\phi_2 A_\nu(0)} \) is zero and therefore it has been omitted.

10.

\[
\hat{\psi}^{(n)}_{cA_\nu(p_1)\phi_2(q_1)\phi_2(q_2)} = - \left[ i(p + q_1 + q_2)\mu\Xi^{(n)}_{\nu A_\nu(p_1)\phi_2(q_1)\phi_2(q_2)} - e\Xi^{(n)}_{\phi_1 A_\nu(p_1)\phi_2(q_1)} - e \Xi^{(n)}_{\phi_1 A_\nu(p_1)\phi_2(q_2)} \right] \\
= -2\lambda v(t^1 - t^0) \Gamma^{(n)}_{cJ_1 (q_1 + q_2) A_\nu(p_1)} + ev(t^1 - t^0) \Gamma^{(n)}_{\phi_2 A_\nu(p_1)\phi_2(q_1)\phi_2(q_2)} + ip\nu^\nu \Gamma^{(n)}_{cJ_2 \phi_2(0)}
\]

(115)

11.

\[
\hat{\psi}^{(n)}_{cA_\nu(p_1)A_\nu(p_2)\phi_2(q)} = - \left[ -e\Xi^{(n)}_{\phi_1 A_\nu(p_1)A_\nu(p_2)} + ev\Xi^{(n)}_{\phi_2 A_\nu A_\nu \phi_2(q)} \right] \\
= ev(t^1 - t^0) \Gamma^{(n)}_{\phi_2 A_\nu(p_1)A_\nu(p_2)\phi_2(q)} + 2ev^2(t^1 - t^0)g^\nu\nu \Gamma^{(n)}_{cJ_1 (p_1 + p_2)\phi_2(q)}\Gamma^{(n)}_{cJ_1 (p_1 + p_2)\phi_2(q)} + ive\nu^\nu \Gamma^{(n)}_{cJ_2 \phi_2(0)}
\]

(116)

\( \xi \)From the Lorentz structure we see that all terms are zero and therefore

\[
\hat{\psi}^{(n)}_{cA_\nu(p_1)A_\nu(p_2)\phi_2(q)} = e\Xi^{(n)}_{\phi_1 A_\nu(p_1)A_\nu(p_2)} - ev\Xi^{(n)}_{\phi_2 A_\nu A_\nu \phi_2(q)} = 0
\]

(117)

12.

\[
\hat{\psi}^{(n)}_{cA_\nu A_\nu \phi_2} = - \left[ -e\Xi^{(n)}_{\phi_1 A_\nu A_\nu \phi_1} + ev\Xi^{(n)}_{\phi_2 A_\nu A_\nu \phi_1} \right] \\
= ev\Gamma^{(n)}_{\phi_2 A_\nu A_\nu \phi_2(0)} - m_0^2\Gamma^{(n)}_{cJ_1 A_\nu \phi_2(0)} + 2ev^2g^\nu\nu \Gamma^{(n)}_{cJ_1 \phi_2 \phi_2(0)}
\]

(118)

\[
-2\lambda v\Gamma^{(n)}_{cJ_2 A_\nu(0)} + \sum_{j=1}^{n-1} \left( \Pi^{(j)}_{cJ_1 \phi_2 \phi_1} \Pi^{(n-j)}_{\phi_1 A_\nu(0)} \right)
\]

(118)
14. \[
\hat{\Psi}^{(n)}_{c\phi A_\mu A_\nu} = -\left[i (p_1 + p_2 + p_3) \mu \xi^{(n)}_{A_\mu A_\nu A_\sigma} \right] \\
= eu t^{(n)}_{\phi_2 A_\mu A_\nu A_\sigma} + 2eu^2 g^{\rho\sigma} (t^1 - t^0) \Gamma_{cJ_1 (p_2 + p_3) A_\nu}^{(n)} \\
+ 2eu^2 g^{\rho\sigma} (t^1 - t^0) \Gamma_{cJ_1 (p_1 + p_3) A_\nu}^{(n)} \\
+ i evp_1 c^{(n)}_{cJ_2 (p_1) A_\mu A_\nu} (0) + i evp_2 c^{(n)}_{cJ_2 (p_2) A_\mu A_\nu} (0) \\
+ \sum_{j=1}^{n-1} \left( t^{(j)}_{cJ_1 A_\nu} \right)^{(n-j)} \phi_1 A_\mu A_\nu (0) + \sum_{j=1}^{n-1} \left( t^{(j)}_{cJ_1 A_\nu} \right)^{(n-j)} \phi_1 A_\mu A_\nu (0) \\
\]

(119)

15. \[
\hat{\Psi}^{(n)}_{c\psi (p_1) \psi (p_2) \phi_1 (q)} = -\left[i \frac{e}{2} \gamma_5 \xi^{(n)}_{\psi \psi (p_2) \phi_1 (q)} + i \frac{e}{2} \xi^{(n)}_{\psi \psi (p_1) \phi_1 (q)} \gamma_5 - e \xi^{(n)}_{\psi \psi (p_1) \phi_2 (q)} - G e \xi^{(n)}_{\psi \psi (p_2) \phi_1 (q)} - G e \xi^{(n)}_{\psi \psi (p_2) \phi_1 (q)} \right] \\
= -ev (t^1 - t^0) \Gamma_{\phi_2 \psi (p_2) \psi (p_1) \phi_1 (q)}^{(n)} \\
- G e (t^1 - t^0) \Gamma_{\phi_2 \psi (p_2) \psi (p_1) \phi_1 (q)}^{(n)} \\
- t^0 \sum_{j=1}^{n-1} \left( \Gamma_{\psi (p_2) \phi_1 (q)}^{(n-j)} \phi_1 (q) + \Gamma_{\psi (p_1) \phi_1 (q)}^{(n-j)} \phi_1 (q) \right) \\
- t^0 \sum_{j=1}^{n-1} \left( \Gamma_{\psi (p_2) \phi_1 (q)}^{(n-j)} \phi_1 (q) + \Gamma_{\psi (p_1) \phi_1 (q)}^{(n-j)} \phi_1 (q) \right) \\
\]

(120)

16. \[
\hat{\Psi}^{(n)}_{c\psi (p_1) \psi (p_2) \phi_2 (q)} = -\left[i \xi^{(n)}_{\psi (p_2) \phi_1 (q)} - i G e \xi^{(n)}_{\psi (p_1) \phi_2 (q)} - G e \xi^{(n)}_{\psi (p_2) \phi_1 (q)} \right] \\
= -ev \phi_2 \psi (p_2) \phi_1 (q) \\
- G e \phi_2 \psi (p_2) \phi_1 (q) \\
- t^0 \sum_{j=1}^{n-1} \left( \Gamma_{\psi (p_2) \phi_1 (q)}^{(n-j)} \phi_1 (q) + \Gamma_{\psi (p_1) \phi_1 (q)}^{(n-j)} \phi_1 (q) \right) \\
\]

(122)

\( \hat{\Psi}_{c\bar{\bar{\psi}} \gamma_\mu \gamma_5 \psi A^\mu} = 0 \). In fact the only possible counterterm is \( \bar{\bar{\psi}} \gamma_\mu \gamma_5 \psi A^\mu \) and this is excluded by the normalization conditions.
The above analysis shows that at every order the following counterterms are absent to all orders.

\[ \int d^4x\phi_1^2 \int d^4x\phi_2^2 \int d^4x(\partial_\mu\phi_2)^2 \int d^4x A_\mu^2. \]

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