It is shown how the time series obtained from searches for ultralight bosonic dark matter (DM), such as the axion, can be used to determine whether it is in a coherent or incoherent quantum state. The example is essentially trivial, but it is hoped that explicitly addressing it will provoke experimental exploration. In the standard coherent state, $O(1)$ oscillations in the number density occur over the coherence time, $\tau_c = \hbar/m^2v$, where $m$ is the particle mass and $v$ is the galactic virial velocity, leading to a reduction in the constraining power of experiments operating on timescales $T < \tau_c$, due to the unknown global phase. On the other hand, if the DM is incoherent then no such strong number oscillations occur since the ensemble average over particles in different streams gives an effective phase average. If an experiment detects a signal then the coherent or incoherent nature of DM can be determined by time series analysis over the coherence time. This finding is observationally relevant for DM masses, $10^{-17}$ eV $\lesssim m \lesssim 10^{-11}$ eV (corresponding to coherence times between a year and 100 s).

### 1. Introduction

The quantum nature of dark matter (DM) has important observational consequences. If the particle mass, $m$, is less than a few hundred eV, then the Pauli exclusion principle applied to galactic halos implies that the DM must necessarily be bosonic.$^{[1,2]}$ If the mass is less than around 1 eV, then the occupation number squeezed,$^{[11]}$ and highly homogeneous state that arises during Bose–Einstein condensation$^{[27]}$. The timescales are shown in Figure 1.

In order for coherence to be measurable, $\tau_c$ should be shorter than the experimental timescale. Assuming a reasonable campaign of $O(1)$ year implies that coherence can be measured only for $m \gtrsim 10^{-17}$ eV. We see that in this case, the relaxation and condensation times in the Milky Way are much longer than the age of the Universe, and thus are not dynamically relevant.$^{[28–30]}$

There are two quantum timescales to consider: the quantum break time, and the decoherence timescale. Ref. [31] computed the gravitational decoherence time, which becomes shorter than the
The local DM field can be modeled stochastically as:

\[ \phi(t) = \sqrt{2 \rho_{\text{loc}}/m} \sum_{i=0}^{\text{max}} \alpha_i \cos \left( \frac{1 + \nu_i^2}{2} \right) t + \delta_i \]  

(1)

where \( \delta_i \in U[-\pi, \pi] \) is a random phase and \( \alpha_i \) and \( \nu_i \) are random Rayleigh distributed variables, that is

\[ P(x) = 2xe^{-x^2} \, dx \]  

(2)

where \( x = \alpha/a_0 \) or \( \nu/v_0 \) and \( a_0, v_0 \) are the root-mean-square values. The distribution for \( \nu_i \) follows from the 3D Maxwell–Boltzmann velocity distribution assumed in the standard halo model (SHM) of DM, while the distribution for \( \alpha_i \) follows from considering a random walk under phase averaging.\(^{36,37}\) The normalization by \( \sqrt{2 \rho_{\text{loc}}/m} \) in Equation (1) ensures the correct average local DM density. The maximum number of draws from the distributions, \( i_{\text{max}} \), is a free parameter that should be large enough to well sample the distributions. The model Equation (1) models the field as a sum of coherent states of definite phase and energy, and corresponds to the coherent state \( |\phi\rangle \) under which the expectation value of the field operator obeys the classical equations of motion. Note that \( |\phi\rangle \) is not an eigenstate of the number operator, \( \hat{N} = \hat{a}^{\dagger} \hat{a} \).

The energy density of the field is

\[ \rho_\phi = \frac{1}{2} \phi^2 + \frac{1}{2} m^2 \phi^2 \]  

(3)

which can be interpreted in the particle picture as a number density, \( n_\phi = \rho_\phi/m \).

The coherent stochastic model, Equation (1), and its (statistical) agreement with MFT simulation,\(^{20}\) is illustrated in Figure 2. The MFT simulation solves the classical equation of motion of the scalar field in the WKB limit:

\[ \phi = \psi e^{imt} + \psi^* e^{-imt} \]  

(4)

Clearly visible in the figure is the coherence time, \( \tau_c \), over which the number density undergoes \( O(1) \) fluctuations. Note that the coherent stochastic model cannot predict the exact time series of the MFT simulation, only the coherence time and amplitude of the number density fluctuations. On the other hand, since the MFT simulations are performed under the WKB approximation, they do not include the rapid oscillations on time scale \( 1/m \) shown in the zoom in of the stochastic model in Figure 2b.

A coherent state virialized model like the one expressed in Equation (1) is explicitly assumed in many UBDM direct detection data analyses, including refs. [37, 38], and the model is implemented in the AxiScan data analysis software.\(^{36,39}\)

\section*{3. Incoherent Particle Model}

Ref. [40] approximate quantum evolution by a sum of coherent MFT simulations with classically distinct phases. It is found during gravitational collapse and virialization the phases become incoherent in different DM “streams.” I approximate this incoherent model by taking a sum of independent copies of the coherent stochastic model above. First, we write the WKB envelope function, \( \psi \), in the stochastic model

\[ \psi = \sum_{k} \psi_k e^{ik \nu_k t} \]  

(5)

Labeling each stream by the index \( k \) defines

\[ \psi_k = \sqrt{2 \rho_{\text{loc}}/m} \sum_{i=0}^{\text{max}} \alpha_k \exp \left[ i \left( \frac{1}{2} m \nu_k^2 t + \delta_k + \epsilon_k \right) \right] \]  

(6)

where \( \alpha_k, \nu_k, \) and \( \delta_k \) are the same random draws as for the classical stochastic model, but we add a new phase \( \epsilon_k \) to every term in the sum. The local number density is given by the ensemble average over streams

\[ n = \frac{1}{2} m \langle |\psi_k|^2 \rangle_k \]  

(7)

The number density is shown Figure 2d. Since the phases are completely uncorrelated for a given energy, the stream ensemble
average leads to a number density with no coherent time variation, consistent with the results of ref. [41]. Averaging the number density oscillations requires observing many particles from different streams. In the example in Figure 2d, I found that of order 100 draws are required to visibly reduce the oscillations; the precise number necessary will in reality depend on the experimental precision required.

The incoherent and coherent models of DM give consistent predictions when appropriately averaged (see e.g., ref. [42]). In the case of a DM halo simulated using classical fields, the ensemble average in the particle case is recovered from either the angle average of the classical field (as expressed by the radial DM density profile $\rho(r)$, which has no significant time dependence over the coherence time $t_{\text{coh}}$), or by the coherence time average.

4. Measuring the Quantum State of DM

4.1. Preliminaries

In an experiment, the DM is converted into some observable signal, with rate $\Gamma_{\phi}$, leading to a power

$$P = VT_\phi E_\phi n_\phi$$  \hspace{1cm} (8)

where $V$ is the effective volume and $E_\phi$ is the particle energy (see below). The specific details of the conversion from DM to signal are not important here, but for example this can occur by inducing electric fields,[43] magnetization,[44] effective currents,[45] or atomic energy level shifts,[46] for scalar, axion, or dark photon DM, depending on details of the DM particle physics model and the specific experiment (see e.g., refs. [47, 48]). A reference scale for the power in the case of the QCD axion in a resonant cavity haloscope at around 1 GHz is $10^{-22}$ W. The power, $P$, has a characteristic linewidth, $\Delta \omega/\omega$. In the coherent model, the linewidth can be derived from the time series power spectrum (i.e. Fourier transform) of the coherent classical field. In the incoherent case the linewidth arises from the energy spread, via $E_i = \hbar \omega_i \approx mc^2 + \frac{1}{2}mv_i^2$ (restoring $\hbar$ and $c$ units for clarity), with $v_i$ the velocity of an individual particle. In both cases, the linewidth is determined by the Maxwell–Boltzmann distribution of velocities in the SHM, and is of order $\Delta \omega/\omega \approx (v/c)^2 \approx 10^{-6}$. Linewidth models used, for example, in an axion haloscope analysis such as described in refs. [49, 50] are therefore not sensitive to the model of the quantum state of the field presented here. Haloscopes such as ADMX[51] operating at sensitivity to the QCD axion with $m_a \approx 3 \mu$eV typically scan a single frequency for $T = 100$ s. Operating only in the frequency domain and averaging over many coherence times, the resulting power is insensitive to the DM quantum state.

In many cases the experimental signal of ultralight DM is derived assuming a coherently oscillating classical source,[43,45,52–55] however the presence of the signal does not necessarily depend on the coherence of the field. Indeed, in the case of axion DM, it has been shown in detail that the rate $\Gamma_{\phi}$ agrees between calculations using classical MFT (i.e., axion electrodynamics) and tree level quantum field theory.[56]
4.2. The Crux of the Argument

If the DM is in a classical coherent state, then the models outlined above and shown in Figure 2 predict that the measured power undergoes $O(1)$ oscillation over the time period $\tau_c$. In the incoherent quantum model, the possibilities are more complex since the state is a superposition of classical states. Measurement projects the quantum state onto the pointer state (in the case of the cat: alive or dead). Measurement can occur in the DM detector, but also occurs due to entanglement with the environment, that is, decoherence, which occurs for a cat state. The pointer states are those states that remain as pure as possible under decoherence. Ref. [31] showed that DM can be decohered due to gravity in the environment on a short timescale.

The coherent and incoherent models differ in their prediction for the time series of the measurement power if the pointer state is not the classical coherent state (the relevance of pointer states for UBDM was first discussed in refs. [32, 41, 57]). In this case, the incoherent state can be measured as such, with no time variation of the measurement power over the coherence time. Therefore experiments operating at low particle mass that can resolve the time dependence of the signal on the order of $\tau_c$ can distinguish between the two models and pointer states. The fact that the quantum versus classical description of the DM field $\phi$ is encoded in the time evolution of the number density was noted already in ref. [56].

Electromagnetism has the pointer state as the classical classical field, as do Bose–Einstein condensates with certain types of interactions, but not others: coherent states can be considered the natural pointer states for harmonic oscillators coupled linearly to an environment. On the other hand, for interactions such as those of UBDM, both in experiments and gravitationally, it is not a priori obvious what the pointer state should be. Measurement can determine what the DM quantum state is by time series analysis over the classical coherence time.

4.3. Example

It is helpful to give a concrete example of the proposed measurement, for which we use the CASPEr experiment,[44,59] and specifically the analysis model described in ref. [37]. CASPEr-gradient proposes to measure the spin-precession of hyperpolarized liquid Xenon nuclei induced by axion-like DM in an external magnetic field, $B_0$. The axion field, $\phi$, acts like an effective magnetic field, $B_{\text{eff}} = g \nabla \phi / \gamma$, where $g$ is the axion-nucleon coupling constant, and $\gamma$ is the gyromagnetic ratio (a particle description exists just as for nuclear magnetic resonance). The effective magnetic field would induce anomalous magnetization in the Xe sample, which is resonantly enhanced when the axion frequency, $\omega$, is equal to the Larmor frequency due to the external field, $\mu_B B_0$. The experiment aims to measure the induced magnetization using a magnetometer. In the absence of a detection, an upper limit can be set on $g$ at a given value of the resonant frequency, which is scanned by varying $B_0$.

The relevant timescales are: field oscillation time, $1/m$, coherence time, $\tau_c$, and integration time, $T$. I consider a reference integration time of 100 s. If a detection is made, then an experimental campaign can make $N_{\text{tot}}$ measurements in some reasonable time scale, for example 1 year, in order to determine the quantum state of the field. In order for the quantum state of the field to be measurable, we require $T < \tau_c < N_{\text{tot}} T$.

We begin with the case of the coherent state. In order for the MFT model to be valid the integration time should satisfy $T > 1/m$ to average over many oscillations of field, which for our reference parameters corresponds to $m \gtrsim 10^{-37}$ eV. With $T < \tau_c$, then the stochastic nature of the field over the coherence time becomes important. In particular, ref. [37] argued that a Bayesian model should marginalize over the unknown phase, leading to a degradation in the limits that can be set on the unknown coupling constant $g$. A deterministic prediction in the coherent model is only made when averaging over a time scale longer than $\tau_c$. The coherent state leads to fluctuations in the signal power for repeated measurements in a campaign if the campaign time $N_{\text{tot}} T \gtrsim \tau_c$.

In the incoherent model of the axion field $T$ should be long enough to measure many different axion events. If the events sample many streams with different phases, then the prediction for $n$ has very small fluctuations, as shown in Figure 2. In this model, all $N_{\text{tot}}$ repeated measurements should measure the same signal strength. The event of a detection in the incoherent quantum model, the induced magnetization at frequency $\omega$ will have no time dependence, and in particular no time variation between successive measurements even if $N_{\text{tot}} T \gtrsim \tau_c$.

Since one does not know, a priori, whether the DM is in a coherent or incoherent state, or what the UBDM pointer state is, it is therefore necessary in the event of any detection to make multiple measurements for an extended time period $N_{\text{tot}} T > \tau_c$ in order to break the degeneracy between the unknown phase in the coherent state model, and the inferred value of $g$. Measurement in the time domain is sensitive to the phase coherence of the UBDM.

5. Discussion

The first question one may now ask is: to what significance can such a difference in signals between the coherent and incoherent quantum states be detected? The measurement of the quantum state is a post discovery measurement, so I assume that a signal has already been detected at high signal-to-noise (SNR) somewhere in the parameter space $(m, g)$ of particle mass and coupling constant. Intuitively, in any case where a single measurement with integration time $\Delta t < \tau_c$ corresponds to a detection, then quasi-periodic time variation present in the coherent model on time scale $\tau_c = N \Delta t$ should be distinguishable from no time variation with high confidence.

To be more concrete I consider a toy model. I take a time series of the number density, which is proportional to the measured power, over $T \approx 2 \tau_c$ in $N = 10$ bins for each model. I assume that the coherent model is the true model, and that SNR $\approx 5$ for the average number density. I then draw Monte Carlo data points and compute $\Delta \chi^2$ between the coherent model and the incoherent model with Gaussian errors on the measurement, leading to $\Delta \chi^2 \sim \mathcal{O}(100)$. This is not a rigorous statistical test, since I do not marginalize the unknowns in the halo model, and do not consider the dependence on the unknown number of streams in the quantum model.

One cannot say from such an exercise what experiments can perform such a measurement, since we do not know a priori what
the true value of \((m, g)\) is, and thus how long the measurement \(\Delta t\) should be in order to achieve \(\text{SNR} = 5\). However such a measurement distinguishing the two models should be possible up to the projected sensitivity contours for experiments, assuming that the projections are constructed with integration time \(\Delta t < \tau_c\) to fixed SNR on \(g\) at each frequency, and that the Bayesian model for the reduced sensitivity in the coherent model is used following ref. [37]. As already indicated, for this to be possible, the coherence time should be neither too short nor too long, and I take a ballpark mass range of \(10^{-17} \text{ eV} \leq m \leq 10^{-11} \text{ eV}\) (corresponding to coherence times between a year and \(100\) s), which can be explored by experiments including CASPER,[44,60] DMRadio,[61] and AION.[62] The shape of the reach in the \((m, g)\) plane depends on the scan strategy, as is well known.

Next, one might wonder what the astrophysical and cosmological differences between the coherent and quantum models are. As indicated already in ref. [41], the quantum model appears to lessen density fluctuations, while increasing the effective Jeans scale. The Jeans scale drives cosmological constraints on UBDM coming from, for example, weak lensing[63] and the Lyman-alpha forest,[64] leading to the constraint that the UBDM mass should exceed \(2 \times 10^{-20} \text{ eV}\) if it is to compose all of the DM. This bound would be stronger in the quantum model than in the coherent model if the time scale of the quantum break occurs early enough to influence the cosmological matter power spectrum. This appears unlikely, since the power spectrum is not strongly influenced by the inner structure of halos, which is where the quantum break occurs on a few dynamical times in ref. [41].

On the other hand, UBDM is also constrained by density fluctuations inside halos, for example heating of the Milky Way disk[65] and star clusters in dwarf galaxies,[25,66] leading to the constraint that the UBDM mass should exceed \(3 \times 10^{-19} \text{ eV}\) if it is to compose all of the DM. In the quantum model, the break time on a few dynamical times should drastically reduce such density fluctuations inside halos, and lessen the effect of these constraints. A full exploration of such effects will require 3D and cosmological simulations beyond the spherical collapse case studied in ref. [41].

### 6. Conclusions

The possible quantum state of UBDM has provoked debate and discussion for decades. Any departure from standard MFT may change the astrophysical and cosmological phenomenology of UBDM, in particular that based on fluctuations on the coherence scale,[12,17,49,50] and affects the inference of constraints derived from experiment.[37] Cosmological simulations of UBDM beyond MFT are extremely challenging, and have only recently started to be developed.[33,40,41] Even so, current understanding of UBDM does not specify the pointer state. Thus, I have considered whether and how the quantum state of UBDM can be measured in the laboratory.

If an experiment can be carried multiple times over the coherence time then in a coherent state model the observed signal should fluctuate, while it will not in an incoherent model if the pointer state is not the classical coherent state. Such a measurement is feasible for \(\tau_c\) less than around one year, or \(m \geq 10^{-17} \text{ eV}\). To observe the time variation the signal should be strong enough to measure on integration times shorter than \(\tau_c\), which for a given experiment limits the range of possible UBDM coupling constants where coherence can be measured. Taking a benchmark value of \(100\) s of measurement time suggests an upper limit of where this measurement can be performed of around \(m \approx 10^{-11} \text{ eV}\), although this is by no means definitive and further exploration on a case-by-case basis of experiments is necessary. This simple example demonstrates how the issue of the DM field quantum state can be resolved experimentally if ultralight DM is detected in the laboratory. I hope this stimulates further exploration of the possible phenomenological consequences of the DM quantum state.

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### Conflict of Interest

The author declares no conflict of interest.

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