Estimation of time parameter proportionality ratios in large catchments: Case study of the Modder-Riet River Catchment, South Africa

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Abstract
Catchment response time parameters, for example, the time of concentration ($T_C$), lag time ($T_L$), and time to peak ($T_P$) are fundamental to event-based design flood estimation in ungauged catchments. Hydrological literature often fails to clearly differentiate between the seven different time parameter definitions interchangeably used in practice and consequently, proportionality ratios are commonly applied. This article aims to investigate and establish the suitability of the currently recommended time parameter definitions and proportionality ratios for small catchments in larger sub-catchments exceeding 50 km² by using an automated hyetograph–hydrograph analysis tool. The time parameter estimates proved to be highly variable, but the average time parameter proportionality ratios proved to be less significant and in all the sub-catchments, it was confirmed that the average proportionality ratios equal unity, that is, $T_C \approx T_L \approx T_P$. Hence, the currently recommended $T_C:T_L$ proportionality ratios of 1.417 and 1.667, are respectively not applicable at larger catchment levels. The establishment of unbiased empirical time parameter equations would remain an international challenge if the misinterpretation of the different time parameter definitions and use of incorrect time parameter proportionality ratios continue; hence, the results from this study, as well as others, should be considered for further investigation and/or implementation.

KEYWORDS
catchment response time, design flood estimation, lag time, time of concentration, time parameter proportionality ratios, time to peak

1 | INTRODUCTION
Understanding the nature of catchment response to rainfall input is at the core of applied hydrological applications, for example, design flood estimation, water resources management, and catchment parameter estimation (De Almeida, Almeida, Steffen, & Sobrinho, 2016; Gericke & Smithers, 2014; Michailidi, Antoniadis, Koukouvinos, Bacchi, & Efstratiadis, 2018). Catchment response reflects how a catchment converts rainfall into runoff, and it incorporates the influence of numerous catchment characteristics, for example,
catchment geomorphology, channel geomorphology, soils, land-use and vegetation, and developmental and climatological variables (Gericke, 2019; Royappen, Dye, Schulze, & Gush, 2002). Catchment response is normally studied using a comparative analysis of the temporal and spatial characteristics of a rainfall hyetograph and the resulting streamflow hydrograph (Dingman, 2002; Grimaldi, Petroselli, Tuaro, & Porfiri, 2012). Indices such as the peak discharge, runoff volume, baseflow index, recession constant, and response time could be obtained from rainfall hyetographs and streamflow hydrographs to provide first-order information to comprehend the rainfall–runoff relationship in a particular catchment (Dow, 2007; Elsenbeer & Vertessy, 2000; Sujono, Shikasho, & Hiramatsu, 2004).

Time variables describe the individual events defined on either a hyetograph or hydrograph, for example, peak rainfall intensity and/or discharge, the centroid of effective rainfall and/or direct runoff, end time of a rainfall event, and/or the inflection point on the recession limb of a hydrograph. As a result, time parameters are defined by the difference between two interrelated time variables (McCuen, 2009). The most commonly used time parameters are the time of concentration ($T_C$), lag time ($T_L$), and time to peak ($T_P$), all of which, are regarded as fundamental input to event-based methods when peak discharges and runoff volumes need to be estimated in ungauged catchments (Gericke & Smithers, 2014).

Traditionally, time parameters have numerous theoretical or computational definitions. Conceptually, $T_C$ is defined as the time required for runoff, due to effective rainfall, with a uniform spatial and temporal distribution over a catchment, to contribute to the peak discharge at the catchment outlet (McCuen, Wong, & Rawls, 1984; Seybert, 2006); whereas $T_L$ is regarded as a weighted $T_C$ value and is estimated by dividing a catchment into sub-areas with the total weighted travel times being estimated from the centroid of each sub-area to the catchment outlet (Woodward et al., 2010). However, the latter $T_L$ definition is not often used in flood hydrology, instead $T_L$ is directly expressed in terms of $T_C$ (Fang, Thompson, Cleveland, Pradhan, & Malla, 2008; Hood, Clausen, & Warner, 2007; Jena & Tiwari, 2006; McCuen, 2009). Theoretically, the $T_P$ is normally defined as the time from the beginning of effective rainfall (which coincide with the start of direct runoff) to the peak discharge in a single-peaked hydrograph (Kent et al., 1985; McCuen et al., 1984; Seybert, 2006).

In considering observed rainfall and streamflow data to define and estimate time parameters, hydrological literature often fails to clearly differentiate between the different time parameter definitions used in practice. Typically, as shown in Figure 1, a simplified convolution process between a single rainfall hyetograph (small catchment areas) or a catchment rainfall hyetograph (derived from numerous rainfall stations in larger catchments)
catchments), and the resulting outflow hydrograph is used to define and estimate these time parameters. In essence, a synthetic transfer function is used to transform the effective runoff producing rainfall into direct runoff based on the principle of linear super-positioning, that is, multiplication, translation, and addition. As a result, the following seven different time parameter definitions are interchangeably used to obtain these time parameters from observed data sets (Gericke & Smithers, 2014; McCuen, 2009):

- $T_C$ (a): The time from the end of effective rainfall to the inflection point on the hydrograph recession limb; that is, the end of direct runoff;
- $T_C$ (b): The time from the centroid of effective rainfall to the peak discharge;
- $T_C$ (c): The time from the maximum rainfall intensity to the peak discharge;
- $T_C$ (d): The time from the start of direct runoff (rising limb of hydrograph) to the peak discharge;
- $T_L$ (a): The time from the centroid of effective rainfall to the peak discharge associated with direct runoff;
- $T_L$ (b): The time from the centroid of effective rainfall to the peak discharge associated with total runoff; and/or
- $T_L$ (c): The time from the centroid of effective rainfall to the centroid of direct runoff.

In considering the above-listed definitions, it is obvious that time intervals from various points during a rainfall event extracted from a hyetograph to various points on the resultant hydrograph, are often misinterpreted as $T_C$, and/or $T_L$. For example, $T_C$ definitions (a) and (b) were, respectively, used by Clark (1945) and Snyder (1938) to define $T_L$. However, in principle, all these definitions are reliant on the conceptual definition of $T_C$. Furthermore, the seven definitions shown in Figure 1 are based on time variables with an associated degree of uncertainty. The centroid values denote “average values,” which are regarded to be more stable time variables representative of the catchment response, especially in larger catchments where runoff volumes are central to the design (Gericke & Smithers, 2014; McCuen, 2009). In contrast to large catchments, the time variables associated with peak rainfall intensities and peak discharges are preferred to estimate the catchment response time in small catchments.

Alternative $T_L$ estimation techniques have been proposed in literature, due to the difficulty in estimating the centroid values of hyetographs and hydrographs. Hence, instead of using $T_L$ as an input for event-based design flood estimation methods, it is preferably used as input to compute $T_C$. In considering $T_L$ definition (c), $T_C$ and $T_L$ are related by $T_C = 1.417T_L$, while considering $T_L$ definitions (a) and (b), the proportionality ratio increases to 1.667 (McCuen, 2009). However, in contradiction to the above-mentioned proportionality ratios, Schultz (1964) demonstrated that $T_C \approx T_L$ in small catchments in Lesotho and South Africa. In addition, Gericke and Smithers (2017, 2018) also showed that $T_C \approx T_P$ at medium to large catchment scales in South Africa, but the relevance of the $T_L$ proportionality ratio ($x = 1.667$), that is, $T_L = 0.6T_C$, was not established.

The analysis of above-mentioned hyetograph–hydrograph relationships to obtain time variables and time parameters is often done manually, relying on visual examination and interpretation. As a result, considerable time is required to implement these analyses and in general, results could be regarded as inconsistent and subjective. In contrast, automated hydrograph analyses provide objective and consistent results (White & Sloto, 1990). Former automated tools for hydrograph analyses primarily focused on the selection of hydrograph characteristics and the incorporation of baseflow separation, recession analyses, and direct runoff estimation (Arnold, Allen, Muttitia, & Bernhardt, 1995; Chapman, 1999; Lim et al., 2005; Piggott, Moin, & Southam, 2005; Rutledge, 1998; Sloto & Crouse, 1996). However, the use of automated tools to extract and analyze rainfall hyetographs, is not common practice and most of the rainfall-based time variables are extracted manually. In essence, none of the automated tools developed include both rainfall hyetograph and streamflow hydrograph characteristics, while the relationship between rainfall-based and runoff-based time variables is not defined.

Therefore, in considering the inconsistent use of time parameter definitions and the inherent procedural limitations associated with the rainfall–runoff convolution process as discussed above, the overall purpose of this article is to investigate and establish the suitability of the currently recommended time parameter definitions and proportionality ratios for small catchments in larger sub-catchment areas (exceeding 50 km$^2$) of the Modder-Riet River Catchment (MRRC) in South Africa. The focus is on the development of an automated hyetograph–hydrograph analysis tool to estimate time parameters and average time parameter proportionality ratios at a catchment level.

2 | STUDY AREA

2.1 | Location and general characteristics

South Africa is located on the southern tip of Africa and is demarcated into 22 primary drainage regions, which
are further subdivided into 148 secondary drainage regions. The MRRC comprises of the C5 secondary drainage region located within primary drainage region C (Midgley, Pitman, & Middleton, 1994). The MRRC covers 34,795 km² and is located between 28°25' and 30°17' S and 23°49' and 27°00' E (DWAF, 1995). The Modder and Riet Rivers are the principal river reaches in the MRRC and discharge into the Orange-Vaal River drainage system (Midgley et al., 1994). The native vegetation consists of Grassland of the Interior Plateau, False Karoo, and Karoo. Agricultural land is the largest human-induced modification in the rural areas, while residential and suburban areas govern the urban areas (CSIR, 2001).

Practically, 99.1% of the MRRC comprises of rural areas, while 0.7% and 0.2% denote urban areas and water bodies, respectively (DWAF, 1995). The landscape is gentle with slopes between 2.4 and 5.5% (USGS, 2016), while runoff has a tendency to pool easily; hence, affecting the attenuation and translation of floods.

In the MRRC, the average Mean Annual Precipitation (MAP) is 424 mm, varying from 275 mm in the west to 685 mm in the east (Lynch, 2004). The rainfall is primarily classified as convective rainfall, which is regarded as highly variable in both time and space. The rainy season commences in early September and ends in mid-April with a dry winter.

2.2 Flow-gauging and rainfall monitoring network

There are 16 gauged sub-catchment areas ranging between 39 km² and 33,278 km² in the MRRC. The sub-catchments are regarded as “gauged”, since the Department of Water and Sanitation (DWS) flow-gauging stations are located at the outlet of each sub-catchment. The layout of each sub-catchment, the river network, and location of each individual flow-gauging station are shown in Figure 2.

There are 185 South African Weather Services (SAWS) daily rainfall stations located within the boundaries of the MRRC. However, currently, there are only 40 active SAWS rainfall stations available in the MRRC, while only 169 SAWS rainfall stations, as shown in Figure 2, proved to have adequate historical data both in terms of record length and data quality to conduct this study. It is apparent from the rainfall monitoring network in Figure 2 that it is more condensed in the mid-
eastern parts than in the north-western parts of the MRRC (Pietersen, 2016).

3 | METHODOLOGY

3.1 | Establishment of rainfall database

A daily rainfall database was established by evaluating, preparing, and extracting daily rainfall data from the SAWS and the Agricultural Research Council—Institute for Soil, Climate, and Water (ARC-ISCW) rainfall stations present in the MRRC. In total, 169 rainfall stations were used due to a lack of data from 16 stations within the MRRC.

The Daily Rainfall Extraction Utility (DREU; Lynch, 2004) was used for the extraction of the daily rainfall data series. Infilling of missing rainfall data to extend the rainfall data series was not considered. In cases where inactive SAWS rainfall stations lacked data, data from the ARC-ISCW database were combined with the SAWS database as far as possible to extend the rainfall data series. The ARC-ISCW rainfall stations used to extend the data series of inactive SAWS stations were in close proximity to the inactive stations.

The Geographical Information Systems (GIS) feature classes (shape files) containing the spatial features of the complete daily rainfall database were generated in the ArcGIS™ 10.7.1 environment. During the analyses, care was taken to ensure that all the stations within a sub-catchment contributed to the rainfall data series. In cases where missing rainfall data are present during the analyses, the Automated Toolkit developed (cf. Section 3.3), would caution the user about the presence of a negative Phi-index and that an alternative rainfall–runoff event needs to be selected.

3.1.1 | Synchronization of rainfall data

The degree of synchronization between the point rainfall data sets at each rainfall station was established by considering recorded rainfall with mutual time intervals. The rainfall data series at each rainfall station was firstly exported and converted to a Microsoft Excel file format (e.g., *.xlsx). Thereafter, the rainfall data files were imported to the Automated Toolkit (cf. Section 3.3). In essence, a number of logical and synchronization functions are available in the Visual Basic for Applications (imbedded in Microsoft Excel) environment to enable the automatic synchronization of daily rainfall data, for example, “INDEX” and “MATCH”. The use of the Automated Toolkit ensured that large data sets from numerous rainfall stations within a particular sub-catchment could be synchronized within minutes.

3.1.2 | Averaging of observed rainfall data

In the calculation of total quantities of rainfall over large areas, the frequency of storms and their contribution to single rainfall stations are unknown. Therefore, it is necessary to convert numerous observed point rainfall depths to provide an average rainfall depth over a certain area (Wilson, 1990).

All the various methods proposed for the averaging of point rainfall depths over an area were considered in this study. However, Gericke and Du Plessis (2011) confirmed that there is a high degree of association ($r^2$ values >0.9) between the various averaging methods when applied to the MRRC, with percentage differences <17%. The latter results actually confirmed the even spatial distribution of the rainfall stations and the relatively flat topography of the MRRC. Based on these findings and in conjunction with the large amount of data and computations required, the Thiessen polygon method was selected as the most suitable method to use.

The weighting procedure as applicable to the Thiessen polygon method [Equation (1)] defines the zone of influence of each rainfall station by drawing lines between pairs of stations, bisecting the lines with perpendiculars. The total area enclosed within the polygon formed by these intersecting perpendiculars has rainfall of the same amount as the enclosed rainfall station (Wilson, 1990).

$$
\bar{P} = \sum \frac{A_S P_i}{A_T}
$$

where $\bar{P}$ is the average rainfall depth (mm), $A_S$ is the area of the polygon surrounding a particular rainfall station (km$^2$), $A_T$ is the total catchment area (km$^2$), and $P_i$ is the point rainfall depth at a particular rainfall station (mm).

In essence, the Thiessen polygon method was used in each sub-catchment to convert the individual point rainfall hyetographs into an average catchment rainfall hyetograph using the Create Thiessen Polygons tool in the Proximity toolset contained in the Analysis Tools toolbox of ArcGIS™. The boundary of the resultant Thiessen polygons was selected in each case by the applicable sub-catchment boundary (polygon feature class). Thereafter, the areas of the polygons surrounding the stations within each sub-catchment were exported and converted to a Thiessen weight using the total sub-catchment area. The Thiessen weights were then utilized to approximate each rainfall station’s contribution to the daily point rainfall within each sub-catchment.
3.2 Establishment of streamflow database

A streamflow database was established by evaluating, preparing, and extracting primary flow data from the DWS flow database for the 16 continuous flow-gauging stations present in the MRRC. The average data record length of all the flow-gauging stations is 46 years (Gericke & Smithers, 2018). The screening criteria used to select the stations for the analyses include the following.

1. Stations common to previous flood studies: Sixteen continuous flow-gauging stations used by Gericke and Smithers (2018) in the MRRC were considered.

2. Record length: Only streamflow records longer than 10 years were considered; as a result, one of the 16 flow-gauging stations did not meet the criteria. However, this flow-gauging station met the criteria as stipulated in (1.) and (3.); hence, it was included in the analysis. This also ensured that a consistent approach is followed when the event-specific and average time parameter proportionality ratios are estimated at a sub-catchment level.

3. Catchment area: In addition to above-listed criteria, the catchment areas of the selected flow-gauging stations should cover the range of sub-catchment areas present in the MRRC.

The next stage involved the identification and extraction of complete flood hydrographs from the primary flow data sets. The Flood Hydrograph Extraction Software (EXHYD) developed by Görgens et al. (2007) was used to assist in identifying and extracting the complete flood hydrographs. Complete flood hydrographs were extracted using the selection criteria as proposed by Gericke and Smithers (2017, 2018). In applying the latter selection criteria, a total of 1,134 complete flood hydrographs or runoff events were extracted from the primary flow data sets.

3.3 Development of automated toolkit

The Automated Toolkit consists of a collection of functions required to estimate the temporal characteristics from rainfall and streamflow records, including (i) baseflow separation, (ii) time variable identification and estimation, (iii) time parameter estimation, and (iv) the estimation of time parameter proportionality ratios. Typically, the following modules are available in the Automated Toolkit: (i) general catchment information, (ii) processing of observed daily rainfall data, (iii) extracted streamflow data, (iv) analysis and plotting of hyetograph–hydrograph relationships, and (v) exporting of individual hyetograph–hydrograph pairs and summary of results.

The function for baseflow separation is based on the Hydrograph Analysis Tool (HAT) developed by Gericke (2016), while the remaining functions are proposed as a mechanism to extract compounded catchment hyetographs from multiple rainfall stations with mutual or synchronized events of recorded rainfall. Since the EXHYD software developed by Görgens et al. (2007) was used to assist in identifying and extracting the complete flood hydrographs (cf. Section 3.2), this function was not included in the toolkit. The Automated Toolkit attempts to mimic the typical convolution procedure practitioners would follow to visually inspect and interpret hyetograph–hydrograph data sets. Rainfall and streamflow data are exported to corresponding modules in the toolkit, followed by the working processes and analyses as summarized in Figure 3.

3.4 Analyses of hyetographs

To analyze rainfall hyetographs, the associated runoff events (as discussed in Section 3.2) need to be identified first. Consequently, a Visual Basic search algorithm was used to identify the causal rainfall event in a window spanning \( n \) days before the start of the identified runoff event to the time of the last streamflow recording, where \( n \) is a user-defined parameter. For example, if \( n \) is set as 12 days, all rainfall records located in the window 12 days before the start of the runoff event to the last streamflow recording will be identified. The rainfall event starts at the first zero rainfall record in the search window and ends at the last zero recording. Subsequently, after the averaging of observed rainfall data per rainfall station and the synchronization of mutual time interval rainfall–runoff events, the daily spatial distribution of any rainfall event could be estimated using Equation (2):

\[
S_d = \left( \frac{\sum A_T TW_i}{A_T} \right) \times 100
\]

where \( S_d \) is the daily spatial distribution (%), \( A_T \) is the total catchment area (km\(^2\)), and \( TW_i \) is the Thiessen weight of each rainfall station that contributed to the daily rainfall.

During a rainfall event, not all the rainfall contributes to direct runoff. Initial abstractions, for example, evaporation, transpiration, depression, detention, infiltration, and interception by vegetation, reduce the effective runoff producing rainfall that a catchment receives. The Phi-
index method [Equation (3)] was used to yield an effective rainfall hyetograph.

\[ I = \frac{P_T - Q_D}{t} \]  

(3)

where \( I \) is the Phi-index (mm/hr), \( P_T \) is the total rainfall (mm), \( Q_D \) is the direct runoff, which equals the effective rainfall (mm), and \( t \) is the time period during which effective rainfall occurred (hr).

Hence, Equation (3) enabled the plotting of possible hyetograph–hydrograph combinations to ultimately translate the effective runoff producing rainfall into direct runoff using a simplified convolution process as shown in Figure 4. The selection of an appropriate hyetograph–hydrograph event is characterized by the effective rainfall being equal to the direct runoff (as obtained from the baseflow separation applied to the hydrographs in Section 3.5). In cases where the effective rainfall and direct runoff volumes are not in equilibrium, an alternative rainfall period is selected and the process is repeated until equilibrium is reached. In each case, the event spatial distribution [Equation (4)] is also automatically estimated for each rainfall period.

\[ S_e = \left[ \frac{\sum_{i=0}^{r-1} P_i}{\sum P_i} \times S_{di} \right] \times 100 \]  

(4)

where \( S_e \) is the event spatial distribution (%), \( i \) is the number of frequency, \( P_i \) is the weighted daily rainfall (mm), \( \sum P_i \) is the cumulative frequency of weighted daily rainfall (mm), \( r \) is the range of frequency, and \( S_{di} \) is the daily spatial distribution (%).

The application of Equation (4) and matching of rainfall–runoff events with corresponding effective rainfall and direct runoff volumes are discussed in the next section. However, it is important to note that the identification and estimation of time variables, for example, start of effective rainfall \( (t_{\text{er0}}) \), centroid of effective rainfall \( (t_{\text{erc}}) \), end of effective rainfall \( (t_{\text{ere}}) \), and time of maximum rainfall \( (t_{\text{rmax}}) \) for each rainfall–runoff event, are already possible at this stage.

3.5 Analysis of hydrographs

The convolution process required to assess the time parameters, for example, \( T_C \), \( T_L \), and \( T_P \), was based on
the temporal relationship between an average compounded catchment rainfall hyetograph and a corresponding hydrograph in each sub-catchment. Conceptually, the proposed procedure is based on the definition that the volume of effective rainfall equals the volume of direct runoff when a hydrograph is separated into direct runoff and baseflow. The separation point on the hydrograph is also regarded as the start of direct runoff which coincides with the start of effective rainfall.

A number of methods, for example, graphical, recursive digital filter algorithms, and recession analysis have been proposed to separate direct runoff and baseflow (Arnold et al., 1995; Lyne & Hollick, 1979; Nathan & McMahon, 1990; Smakhtin, 2001). Smakhtin and Watkins (1997) adopted the methodology as proposed by Nathan and McMahon (1990) with some modifications in a national-scale study in South Africa, while Hughes, Hannart, and Watkins (2003) and Gericke and Smithers (2017) also adopted the latter methodology in pilot-scale studies in South Africa.

Hence, based on these local studies, as well as the need for consistency and reproducibility, Equation (5) was used in this study. Figure 4 (cf. Section 3.4) is also illustrative of a typical baseflow separation.

$$Q_{Dx_i} = \alpha Q_{Dx(i-1)} + \beta (1 + \alpha) (Q_{Tx_i} - Q_{Tx(i-1)})$$  \hspace{1cm} (5)

where $Q_{Dx_i}$ is filtered direct runoff at time step $i$, which is subject to $Q_{Dx_i} \geq 0$ for time $i$ (m$^3$/s), $\alpha$, $\beta$ are filter parameters (0 < $\alpha$ < 1; 0 < $\beta$ < 0.5), and $Q_{Tx_i}$ is the total streamflow (i.e., direct runoff plus baseflow) at time step $i$ (m$^3$/s).

As discussed in Section 3.4, the volumes of effective rainfall and direct runoff need to be in equilibrium when a causal rainfall event of appropriate duration prior to the resulting runoff event is selected. This was done by matching the direct runoff depth ($Q_{D}$) with the effective rainfall depth ($P_{E}$) in Equation (6).

$$P_{E} = \frac{\sum (Q_{Dx_i} + Q_{Dx(i-1)}) \times \Delta T_{xi}}{1000A_{T}S_{e}}$$  \hspace{1cm} (6)

where $P_{E}$ is the effective rainfall (mm), $A_{T}$ is the total catchment area (km$^2$), $Q_{Dx_i}$ is the filtered direct runoff at time step $i$, which is subject to $Q_{Dx_i} \geq 0$ for time $i$ (m$^3$/s), $S_{e}$ is the event spatial distribution (%), and $\Delta T_{xi}$ is the absolute change in time at time step $i$ (s).

As a result, time variables, for example, start of total runoff ($t_{q0}$), time of peak discharge ($t_{qpk}$), centroid of direct runoff ($t_{qc}$), and time of the inflection point on the recession limb ($t_{ip}$) could be identified and estimated for each rainfall–runoff event at a sub-catchment level.

### 3.6 Estimation of time parameters and proportionality ratios

Table 1 provides a summary of the different Time Parameter (TP) equations and Time Parameter Proportionality Ratio (TPPR) estimation procedures included in the Automated Toolkit.
runoff events could also not be analyzed due to the difficulty experienced to identify the inflection point on the recession limb of hydrographs or due to multi-peaked hydrographs. In essence, only 35% of the extracted runoff events could be analyzed, that is, the 394 rainfall–runoff events.

4.2 | Estimation of time parameters

In considering the analyses of the 394 hyetograph–hydrograph events, it was quite evident that the seven different time parameter definitions contribute to the time parameter variability, which is also influenced by the event spatial distribution ($S_e$), the variation in peak discharge ($Q_p$) and the distance ($L$) between the rainfall station (where the maximum rainfall depth was recorded) and the sub-catchment outlet. In general, the largest $Q_p$ and direct runoff ($Q_d$) values are associated with the likelihood of the entire sub-catchment receiving rainfall of a high intensity for the critical storm duration, which in principal, represents the conceptual $T_C$. Shorter response times, that is, lower $T_C$, $T_L$, and $T_P$ values could be expected to occur when the effective rainfall does not cover the entire catchment, especially when a rainfall event is centered near the outlet of a sub-catchment.

In considering the average time parameters illustrated in Figures 5–7 for each sub-catchment, it is evident that these average time parameters are very similar, that is, $T_C \approx T_L \approx T_P$.

Figure 5 shows a clear association between average time parameters and the average $Q_p$ values associated with all the rainfall events at a sub-catchment level in the MRRC. However, in considering Figures 6 and 7, there is no apparent association evident between the average time parameters and the average distance ($L$) of all rainfall events from the catchment outlet and the average spatial distribution ($S_e$) thereof. The insignificance of $T_C$ definition (d), as shown in Figure 1, when compared to the other $T_C$ definitions, are also evident from Figures 5–7. The latter difference could be ascribed to the fact that $T_C$ definition (d) is also used to define the time to peak for any specific rainfall event, and/or could also be ascribed to the runoff events being wrongfully regarded as baseflow instead of being part of the rising limb of the hydrograph, that is, direct runoff.

4.2.1 | Influence of rainfall event locality on time parameters

An example of the locality analysis results, that is, the establishment of a relationship between the distance of
rainfall events from the catchment outlet and the different time parameters in sub-catchment C5H035 \((A = 17,359 \text{ km}^2)\), is listed in Table 2 and illustrated in Figure 8, respectively.

Generally, the results in Table 2 and Figure 8 demonstrate that the \(Q_P\) values are influenced by the different time parameter definitions used to define the time parameter values, and the distance between the rainfall station that received the maximum rainfall depth and the sub-catchment outlet. In considering all the sub-catchments, an increase in the distance of a rainfall event from the sub-catchment outlet was generally associated with an increase in the time parameter values, while rainfall events which occurred close to the sub-catchment outlets, are more susceptible to shorter response times. However, in some cases, due to low rainfall intensities resulting in lower \(Q_P\) values, the time parameter values are higher and the distance from the sub-catchment outlet has no apparent effect on the response time.

4.2.2 | Influence of the spatial rainfall distribution on time parameters

An example of the spatial distribution analysis results for the different time parameters in sub-catchment C5H035 \((A = 17,359 \text{ km}^2)\) is listed in Table 3. It is evident from Table 3 that the largest \(Q_P\) and time parameter values are associated with the likelihood of the entire catchment receiving rainfall for the critical storm duration.

Lower time parameter values could be expected when effective rainfall of high intensity does not cover...
the entire catchment, that is, low $S_e$ values, especially when a storm is centered near the outlet of a sub-catchment. However, in some cases, low rainfall intensities and associated lower peak discharges are ascribed to larger time parameters values, that is, longer response times due to rainfall events having a low spatial distribution more distant from the sub-catchment outlet.
Figure 9 is illustrative of the influence which the different time parameter definitions and the spatial distribution of a rainfall event have on the peak discharge values. Hence, based on the results contained in Tables 2 and 3, and Figures 8 and 9, it is evident that time parameters are influenced by the distance of a rainfall event from the catchment outlet, as well as the spatial distribution of such an event. However, a clear relationship could not be...
found across all the sub-catchments, due to the high variability of the $Q_P$, $L$, and $S_e$ values.

4.3 Estimation of time parameter proportionality ratios

In considering the $T_C$, $T_L$, and $T_P$ pair values obtained from the 394 hyetograph–hydrograph events, a relatively low variability is evident between the different time parameter proportionality ratios (TPPR 1 to TPPR 8; cf. Table 1) at a sub-catchment level. In general, where $T_L$ is defined as the time from the centroid of effective rainfall to the peak discharge [$T_L$ definitions (a/b)], $T_C$ and $T_L$ are related on average by $T_C = 1.003T_L$ (TPPR 1 to TPPR 3, as illustrated in Figure 10). In using $T_L$ defined as the time from the centroid of effective rainfall to the centroid of direct runoff [$T_L$ definition (c)], the average proportionality ratio reduced to 0.992 (TPPR 5 to TPPR 7, as illustrated in Figure 10). The average time parameter proportionality ratios, in the MRRC, presented in Figure 10 also showed no clear association with the average $Q_P$, $L$, and $S_e$ values, thus these results are regarded as similar to those contained in Tables 2 and 3, and Figures 8 and 9, respectively. Hence, the specific results pertained to the influence of rainfall event locality and the spatial rainfall distribution thereof on time parameter proportionality ratios are not included again.

However, it is quite evident that $T_C$ and $T_L$ are related by $T_C \approx T_L$, where TPPR 1–3 $\approx$ TPPR 5–7, irrespective of the spatial distribution ($S_e$) of a rainfall event, as well as the distance ($L$) thereof from the sub-catchment outlet. Furthermore, the average time parameter proportionality ratios obtained highlighted the insignificance of TPPR 4 and TPPR 8, respectively. This is due to the fact that $T_C$ definition (d) is also one of the definitions used to quantify $T_P$ and in general, the average values of $T_C$ definition (d) were $\pm$ 21 times smaller compared to the other average $T_C$ definition values. In considering the average time parameter proportionality ratios illustrated in Figure 10 for each sub-catchment, the average time parameter proportionality ratios also confirm that $T_C \approx T_L \approx T_P$ at a medium to large catchment level.

5 DISCUSSION

As highlighted in the Introduction, hydrological literature often failed to differentiate between the seven different time parameter definitions used in practice to estimate time parameters from observed rainfall–runoff data sets. The latter misinterpretation almost exists as long as the conceptual definition of catchment response itself, for example, Snyder (1938) and Clark (1945) wrongfully used the same definition to estimate $T_C$ and $T_L$, respectively. The situation is further exacerbated by the high degree of uncertainty associated with the rainfall–runoff time variables used to estimate time parameters.

Due to the latter uncertainty and difficulty in estimating some of these rainfall–runoff time variables, for example, centroid values, the concept of time proportionality ratios was introduced. As a result, $T_L$ is commonly used as
input to compute $T_C$ and not as the main time parameter when event-based design floods are estimated. Typically, time parameter proportionality ratios of $T_C = 1.417 T_L$ and/or $T_C = 1.667 T_L$, are recommended. However, according to some hydrological literature, for example, Schultz (1964) and Gericke and Smithers (2017, 2018), the latter proportionality ratios are regarded as being limited to small catchments; hence, the need for this study to investigate and establish the time parameter proportionality ratios in larger catchments exceeding 50 km².

In small catchments, a uniform response to rainfall is assumed, while the spatially non-uniform antecedent soil moisture conditions within the catchment as a consequence of both the spatially non-uniform rainfall and the heterogeneous nature of soils and land cover in the catchment, are ignored. In this study (at a larger sub-catchment level ≥ 50 km²), the nature and spatial distribution of land cover which could affect the volume, peak, and temporal distribution of runoff, are acknowledged and ultimately reflected in the observed runoff data sets. In other words, despite uncertainties and errors in measurement, the observed rainfall and runoff events as analyzed in this study, are the best indication of the catchment response time and runoff generation. As a result, and bearing in mind the main study objective, the impact of the various catchment characteristics (e.g., geomorphology, soils, and land cover) might have on runoff generation. As a result, and bearing in mind the main study objective, the impact of the various catchment characteristics (e.g., geomorphology, soils, and land cover) might have on runoff generation. As a result, and bearing in mind the main study objective, the impact of the various catchment characteristics (e.g., geomorphology, soils, and land cover) might have on runoff generation.

All the above problems related to time parameter definitions and proportionality ratios have or could have an impact on empirical time parameter equations. As highlighted by Gericke and Smithers (2014), almost 95% of all the equations developed internationally are empirically based and conceptually, these equations are based on either a 1.417 or 1.667 proportionality ratio. For example, the well-known Soil Conservation Services (SCS) method (Reich, 1962) is based on a 1.417 proportionality ratio, while Snyder’s method (Snyder, 1938) is based on a 1.667 proportionality ratio. In this article, where $T_L$ is defined as the time from the centroid of effective rainfall to the peak discharge, $T_C$ and $T_L$ are on average related by $T_C = 1.003 T_L$ and where $T_L$ is defined as the time from the centroid of effective rainfall to the centroid of direct runoff, the average proportionality ratio reduces to 0.992. Hence, in all the sub-catchments under consideration, the average proportionality ratios equal unity, that is, $T_C \approx T_L \approx T_P$.

Consequently, if any of the above empirical equations should be applied for example in the study area, the results...
would be questionable due to the following “errors”: (i) the use of incorrect time parameter proportionality ratios, and (ii) application beyond the original developmental region(s) and areal range of the empirical equation without the use of any local correction factors. The “errors” listed in (i) and (ii) are widespread throughout many parts of the world. Although, it could be argued that most of the empirical equations developed internationally are applicable to and calibrated for small catchments; hence, the current time parameter proportionality ratios of 1.417 and/or 1.667 might apply. However, “error” (ii) would still be a concern, while both “errors” (i) and (ii) would be applicable to the empirical equations developed for larger catchments of up to 5,000 km², for example, Johnstone and Cross (1949), Pullen (1969), Mimikou (1984), Watt and Chow (1985), and Sabol (2008).

It is thus clearly evident that the establishment of unbiased empirical time parameter equations would remain an international challenge if the misinterpretation of the different time parameter definitions and use of incorrect time parameter proportionality ratios continue; hence, the results from this study, as well as others, should be considered for further investigation and/or implementation.

6 | CONCLUSIONS

The estimation of time parameters and time parameter proportionality ratios in large sub-catchments of the MRRC was the objective of this article. The major findings in the study area are as follows:

1. Time parameter estimates based on the seven different theoretical time parameter definitions proved to be highly variable due to the spatial and temporal distribution of rainfall events, the distance of the rainfall events from the catchment outlet, and the variation in peak discharges.

2. In this article, where \( T_L \) is defined as the time from the centroid of effective rainfall to the peak discharge, \( T_C \) and \( T_L \) are on average related by \( T_C = 1.003T_L \) and where \( T_L \) is defined as the time from the centroid of effective rainfall to the centroid of direct runoff, the average time parameter proportionality ratio reduces to 0.992.

3. The average time parameter proportionality ratios equaled unity and are in agreement with the preliminary findings of Gericke and Smithers (2017, 2018), that is, \( T_C \approx T_L \approx T_p \). In other words, it highlighted that the time parameter proportionality ratios currently proposed for small catchments, that is, \( T_C = 1.417T_L \) and \( T_C = 1.667T_L \), are not applicable at larger catchment levels.

It is envisaged that the implementation and expansion of both the identified research values and adopted methodology to other catchments in South Africa and internationally, will ultimately contribute towards improved empirical time parameter equations. Consequently, the improved time parameter estimations will also result in improved design flood estimations which are essential in the planning, design, and operation of hydraulic structures to ultimately limit the risk of failure and to safeguard human life.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

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