A non-perturbative contribution to jet quenching

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Abstract. It has been argued by Caron-Huot that infrared contributions to the jet quenching parameter in hot QCD, denoted by $\hat{q}$, can be extracted from an analysis of a certain static-potential related observable within the dimensionally reduced effective field theory. Following this philosophy, the order of magnitude of a non-perturbative contribution to $\hat{q}$ from the colour-magnetic scale, $g^2T/\pi$, is estimated. The result is small; it is probably below the parametrically perturbative but in practice slowly convergent contributions from the colour-electric scale, whose all-orders resummation therefore remains an important challenge.

1 Introduction

One of the classic qualitative indications for the generation of a thermal medium in heavy ion collision experiments is the fact that high-$p_T$ jets get quenched [1,2]. In fact, not only do jets get quenched but they appear to do so more effectively than naive estimates suggest [3,4]. This motivates not only a complete leading-order weak-coupling analysis [5], but also developing methods to go beyond the leading order [6], taking into account large corrections from the infrared (IR) scales that are characteristic of thermal field theory [7,8]. (The weak-coupling regime has recently also been discussed in ref. [9].)

In a weakly coupled non-Abelian plasma, there are two momentum scales in the IR: the colour-magnetic scale, $g^2T/\pi$, and the colour-electric scale, $gT$ ($g^2 = 4\pi\alpha_s$ is the QCD gauge coupling). We refer to these through the gauge coupling, $g_E^2/\pi$, and the mass parameter, $m_E$, of the dimensionally reduced effective theory [10,11]. The contributions of the two scales have been disentangled for several thermodynamic observables, and a particularly rich source of information is the “spectrum” measured through screening masses [12]. For instance, the smallest screening mass (at temperatures below $\sim 10$ GeV), which is parametrically of the form $2m_E + \mathcal{O}(g_E^2/\pi)$, gets a numerically insignificant contribution from the colour-magnetic scale [13], whereas the Debye screening mass, which is parametrically of the form $m_E + \mathcal{O}(g_E^2/\pi)$, is in practice all but dominated by the magnetic scale [14]:

\[ M_D = m_E + \frac{g_E^2 N_c}{4\pi} \left( \ln \frac{m_E}{g_E^2} + 6.9 \right) + \mathcal{O}(g^3T) \]

\[ \simeq m_E + 1.65 g_E^2 + \mathcal{O}(g^3T). \]  

The purpose of the current study is to estimate whether one of these extreme scenarios could be relevant for $\hat{q}$.

2 Colour-electric contribution

The rate of jet broadening (transverse momentum diffusion) is often parametrized by a phenomenological coefficient, called the jet quenching parameter and denoted by $\hat{q}$ [15]. Although, strictly speaking, the definition of $\hat{q}$ is ambiguous even at leading order [16], it can be argued that the ambiguities can be hidden by considering a quantity called the transverse collision kernel, $C(q_\perp)$; then

\[ \hat{q} = \int_0^{q^*} \frac{d^2q_\perp}{(2\pi)^2} q_\perp^2 C(q_\perp). \]  

It is now the choice of the upper bound $q^*$ which reflects the ambiguities. The form of $C(q_\perp)$ for $q_\perp \sim \pi T$ was determined in ref. [5] and for $q_\perp \sim gT$ in ref. [6]; although the NLO contribution from $q_\perp \sim gT$ to $\hat{q}$ is parametrically suppressed by $\mathcal{O}(g)$, it is numerically large.

More precisely, denoting by $m_E = gT\sqrt{N_c/3 + N_f/6} + \mathcal{O}(g^3T)$ the Debye mass parameter and by $g_E^2 = g^2T + \mathcal{O}(g^4T)$ the coupling constant of the dimensionally reduced effective field theory, the next-to-leading order (NLO) expression for the IR part ($q_\perp \sim m_E$) of the collision kernel can be written as [6]

\[ C(q_\perp) = g_E^2 C_F \left\{ \frac{1}{q_\perp^2} - \frac{1}{q_\perp^2 + m_E^2} \right\} \]

\[ + g_E^4 C_F N_c \left\{ \frac{7}{32q_\perp^2} - \frac{m_E + 2(q_\perp - \frac{m_E^2}{q_\perp})}{4\pi(q_\perp^2 + m_E^2)^2} \right. \]

\[ \times \frac{m_E}{4\pi(q_\perp^2 + m_E^2)} \left[ \frac{3}{q_\perp^2 + m_E^2} - \frac{2}{q_\perp^2 + m_E^2} - \frac{1}{q_\perp^2} \right] \]

\[ - \frac{m_E}{2\pi q_\perp (q_\perp^2 + m_E^2)} \left( \frac{2}{q_\perp^2} \right) \]

\[ + \frac{m_E - \left( \frac{4}{q_\perp^2} + \frac{2m_E^2}{q_\perp^2} \right)}{8\pi q_\perp^4} \frac{\arctan \left( \frac{m_E}{2m_E} \right)}{\arctan \left( \frac{2m_E}{q_\perp^2} \right)} \]  

\[ + \mathcal{O}(g_E^6). \]
Here \( C_F = (N_c^2 - 1)/(2N_c) \) is the Casimir of the fundamental representation. Choosing \( m_e \ll q^* \ll \pi T \), the integral in eq. (2) yields
\[
\hat{q} = \frac{g_s^2 m_e^2 C_F}{2\pi} \ln \frac{q^*}{m_e} + \frac{g_s^2 m_e C_F N_c}{2\pi} \left\{ \frac{q^*}{16m_e} + \frac{3\pi^2 + 10 - 4 \ln 2}{16\pi} + \mathcal{O}\left(\frac{m_e}{q^*}\right) \right\} + \mathcal{O}(q_0^6). \tag{4}
\]
The terms involving \( q^* \) cancel against contributions from hard momenta, \( q_\perp \gg \pi T \) [5], and the \( q^* \)-independent middle term inside the curly brackets then represents the physical NLO contribution to the resonant term of eq. (2).

### 3 Colour-magnetic contribution

For small momenta, \( q_\perp \ll m_e \), the NLO part of eq. (3) behaves as
\[
C^{\text{NLO}}(q_\perp) = g_s^2 C_F N_c \left\{ \frac{7}{32q_\perp^2} - \frac{1}{48\pi m_e q_\perp^2} - \ldots \right\}. \tag{5}
\]
We note from eqs. (3), (5) that, as also suggested by the operator definition [6], the small-\( q_\perp \) limit \( (q_\perp \ll m_e) \) of \( C(q_\perp) \) goes over into minus the momentum-space static potential of three-dimensional pure Yang-Mills theory:
\[
C(q_\perp) = -\hat{V}(q_\perp), \tag{6}
\]
where \([17]\]
\[
\hat{V}(q_\perp) = -\frac{g_s^2 C_F}{q_\perp^4} - \frac{7g_s^4 C_F N_c}{32q_\perp^3} + \mathcal{O}\left(\frac{g_s^6}{q_\perp^2}\right). \tag{7}
\]
The Fourier transform to coordinate space, \( V(r) = \int \hat{f}(q_\perp) e^{i q_\perp \cdot r} - 1) \hat{V}(q_\perp) \), yields
\[
V(r) = \frac{g_s^2 C_F}{2\pi} \ln \frac{r}{r_*} + \frac{7g_s^4 C_F N_c}{64\pi} r + \mathcal{O}(g_s^6 r^2), \tag{8}
\]
where \( r_* \) is a regularization-dependent constant. However, at the next order the expansion breaks down: a direct momentum-space computation of \( \hat{V}(q_\perp) \) at 2-loop order produces in \( d = 3 - 2\epsilon \) dimensions the (gauge-independent) expression \([17]\]
\[
\hat{V}^{\text{NNLO}}(q_\perp) = \frac{g_s^6 C_F N_c^2}{(4\pi)^{1-2\epsilon} q_\perp^{4+\epsilon}} \left( \frac{1}{4\epsilon^2} + \frac{3}{4\epsilon} \right). \tag{9}
\]
A comparison with eq. (2) suggests the presence of a non-perturbative contribution to \( \hat{q} \) at \( \mathcal{O}(g_s^6) \), both because the integral is logarithmically divergent at the lower end, and because the coefficient in eq. (9) is IR divergent.

### 4 Transformation to configuration space

In view the delicate nature of the IR contribution, it is useful to re-express the small-\( q_\perp \) part of eq. (2) in another way. The idea is to make use of eq. (6), in combination with a non-perturbative understanding of \( V(r) \), in order to get a handle on IR physics.

To be concrete, let \( \hat{\theta}_\perp(q_\perp) \) be some cutoff function, where \( A \) is chosen to be formally in the range \( \frac{g_s^2}{\pi} \ll A \ll m_e \). Then we can rephrase the IR part of eq. (2), to be denoted by \( \hat{q}_\perp \), as
\[
\hat{q}_\perp = \int \frac{d^2q_\perp}{2\pi^2} q_\perp^2 C(q_\perp) \hat{\theta}_\perp(q_\perp) = \int d^2r \nabla^2 V(r) \, \theta_\perp(r), \tag{10}
\]
where \( \theta_\perp(r) = \theta(A - q_\perp) \), then \( \theta_\perp(r) = A J_1(Ar)/2\pi r \). This very choice is not particularly convenient, however, since the Bessel function \( J_1 \) is oscillatory. We find it more transparent to choose a Gaussian,
\[
\hat{\theta}_\perp(q_\perp) \equiv \exp(-\frac{q_\perp^2}{A^2}), \quad \theta_\perp(r) = \frac{A^2}{4\pi} \exp\left(-\frac{r^2 A^2}{4}\right). \tag{11}
\]
because both functions are elementary and positive.

Now, because of rotational symmetry and the form of eq. (8), the behaviour of \( \nabla^2 V(r) \) appearing in eq. (10) is
\[
\nabla^2 V(r) = g_s^2 C_F \delta^{(2)}(r) + \left[ V''(r) + \frac{V'(r)}{r} \right]_{r>0} \tag{12}
\]
where we have adopted the notation \( F(r) \equiv V'(r) \) and \( c(r) \equiv r^2 V''(r)/2 \) from ref. [18].

We subsequently need to subtract the known perturbative terms, eq. (8), from the IR contribution to \( \hat{q}_\perp \), given that they were already included in eqs. (3), (4). Thereby we obtain an expression for the remaining IR contribution, let us call it \( \delta \hat{q}_\perp \):
\[
\delta \hat{q}_\perp = 2\pi \int_0^{\infty} dr \left[ F(r) + \frac{2c(r)}{r^2} - \frac{7g_s^4 C_F N_c}{64\pi} \right] \theta_\perp(r) \tag{13}
\]
where we rescaled the integration variable as \( \tilde{r} = rA \) and defined
\[
\phi(r) = F(r) + \frac{2c(r)}{r^2} - \frac{7g_s^4 C_F N_c}{64\pi}. \tag{14}
\]
As eq. (13) shows, only the short-distance part of \( \phi(r) \), terms linear in \( r \) (modulo logarithms), contributes to \( \delta \hat{q}_\perp \); this corresponds to terms quadratic in \( r \) in \( V(r) \).

The nature of the short-distance behaviour of \( V(r) \) can be discussed within a framework similar to the Operator Product Expansion [19,20]. The lowest-dimensional con-
densate in three-dimensional pure SU(3) evaluates to
\[ \frac{1}{2g_0^4} \langle \text{Tr}[F_k F_k] \rangle = \frac{6g_0^4 C_F N_c^4}{(4\pi)^4} \times \left[ \left( \frac{43}{12} - \frac{157}{708}\pi^2 \right) \left( \ln \frac{\bar{\mu}}{2N_c g_0^2} - \frac{1}{3} \right) - 0.2 \pm 0.4^{\text{(MC)}} \pm 0.4^{\text{(NSPT)}} \right], \tag{15} \]

where the errors come from Monte Carlo simulations (MC) [21] and a scheme conversion (NSPT) [22], and the numerical values apply for \( N_c = 3 \). Just cancelling the scale dependence from eq. (15) one might expect a short-distance behaviour of the form
\[ V(r) \sim g_0^4 r^2 \ln \left( \frac{1}{g_0^2 r} \right). \tag{16} \]

However, according to ref. [23] the shape at \( g_0^2 r \ll 1 \) is really more complicated, \( \sim g_0^4 r^2 \ln[\ln(1/g_0^2 r)/\ln(1/g_0^2 r)] \), where \( g_0^2 N_c \ln(1/g_0^2 r) \) represents the difference of octet and singlet potentials. Note that such a tail does not contribute in eq. (13) for \( \Lambda \gg g_0^2/\pi \). Unfortunately the analysis is not valid for the range \( g_0^2 r \sim 1 \) that is relevant for us here, so we resort to modelling in the following.

5 Modelling of lattice data

In fig. 1, numerical data for \( r_0^2 \phi(r) \) from ref. [18] is shown. The parameter \( r_0 \) is defined from
\[ r_0^2 F(r_0) = 1.65 \tag{17} \]
for any given lattice spacing [24]. In the continuum limit [18],
\[ g_0^2 r_0 \approx 2.2, \tag{18} \]
and we have inserted this estimate in order to combine the last term of eq. (14) with lattice data. (Obviously, it would be nice to add data at shorter distances, but this task is non-trivial because strong cutoff effects appear and a careful extrapolation to the continuum limit is needed.)

Now, motivated by the naive eq. (16) and the definition in eq. (14), we describe the data through the function
\[ r_0^2 \phi(r) = \frac{a r}{r_0} \ln \frac{r_0}{r} + \frac{b r^2}{r_0^2} + c r^2, \quad r \lesssim 2r_0. \tag{19} \]

The data (at \( \beta = 20 \)) are well modelled \((\chi^2/\text{d.o.f.} = 0.18)\) with \( a = 0.72(2), b = 0.55(1), c = 0.18(1) \). We remark that around \( r/r_0 \approx 2 \) this function is close to the asymptotic value given by the non-perturbative string tension minus the perturbative subtraction, \( \{(0.553(1))^2 - 7/16\pi\} g_0^2 r_0^3 \approx 0.81 \) [25]. In the following we assume that eq. (19) should reflect the magnitude of the true function for \( g_0^2 r \sim 1 \).

Carrying out the integral in eq. (13), we get
\[ \delta \tilde{q}_\perp = \frac{1}{r_0^2} \left[ a \left( \ln \frac{\Lambda_0}{2} + \frac{\gamma_E}{2} \right) + b + o \left( \frac{1}{\Lambda_0^0} \right) \right]. \tag{20} \]

6 Phenomenological interpretation

Omitting from eq. (20) terms vanishing for \( Ar_0 \gg 1 \); replacing the cutoff inside the logarithm with the scale \( \sim m_\perp \) at which other physics sets in; inserting \( r_0 \) from eq. (18); and estimating \( m_\perp^2 / g_0^2 \sim 1 \) (cf. fig. 1 of ref. [13]); we get the order-of-magnitude estimate
\[ \delta \tilde{q}_\perp \approx \frac{g_0^2}{2.2^3} \left[ 0.72 \left( \ln \frac{2.2}{2} + \frac{\gamma_E}{2} \right) + 0.55 \right] \approx 0.08 g_0^6. \tag{21} \]

This can be compared with the middle term from the curly brackets in eq. (4),
\[ \delta \tilde{q}_\perp \approx \frac{g_0^2 m_\perp C_F N_c}{32\pi^2} (3\pi^2 + 10 - 4 \ln 2) \approx 0.47 g_0^4 m_\perp. \tag{22} \]

The non-perturbative contribution, eq. (21), is clearly below the NLO perturbative contribution from the colour-electric scale, eq. (22).

In conclusion, the contribution to \( \tilde{q} \) from the colour-magnetic scale, \( q_\perp \sim g_0^2/\pi \), may well be of modest magnitude. Thus the phenomenological motivation for its theoretically consistent determination may be feasible; rather, it should probably be measured as a part of the total IR contribution from both the colour-electric and colour-magnetic scales, e.g. along the lines suggested in ref. [6].

Fig. 1. The function \( \phi \) defined by eq. (14), in units of the parameter \( r_0 \) defined by eq. (17). The lattice data is from tables 5 and 7 of ref. [18].

If the ansatz of eq. (19) were correct, the coefficient \( a/r_0^3 \), which comes together with a dependence on the cutoff \( \Lambda \), should be an analytically computable function, cancelling against an NNLO contribution from the colour-electric scale. Within the model the coefficient \( b/r_0^3 \) represents a “genuine” colour-magnetic contribution although, because of the logarithm, this notion is somewhat ambiguous.
Note added

Recently a paper appeared [26] in which, as acknowledged there, some basic ideas of the current study, communicated to one of the authors several years ago, were revealed. Unfortunately the practical implementation, citing e.g. a peculiar temperature dependence of \( \delta \hat{q}_A \), appears to suffer from misunderstandings.

References

1. K. Adcox et al. [PHENIX Collaboration], Suppression of hadrons with large transverse momentum in central Au+Au collisions at \( \sqrt{s_{NN}} = 130 \) GeV, Phys. Rev. Lett. 88 (2002) 022301 [nucl-ex/0109003].
2. C. Adler et al. [STAR Collaboration], Centrality dependence of high \( p_T \) hadron suppression in Au+Au collisions at \( \sqrt{s_{NN}} = 130 \) GeV, Phys. Rev. Lett. 89 (2002) 202301 [nucl-ex/0206011].
3. J. Casalderrey-Solana and C.A. Salgado, Introductory lectures on jet quenching in heavy ion collisions, Acta Phys. Polon. B 38 (2007) 3731 [0712.3443].
4. A. Majumder and M. van Leeuwen, The Theory and Phenomenology of Perturbative QCD Based Jet Quenching, Prog. Part. Nucl. Phys. A 66 (2011) 41 [1002.2206].
5. P.B. Arnold and W. Xiao, High-energy jet quenching in weakly-coupled quark-gluon plasmas, Phys. Rev. D 78 (2008) 125008 [0810.1026].
6. S. Caron-Huot, \( O(g) \) plasma effects in jet quenching, Phys. Rev. D 79 (2009) 065039 [0811.1603].
7. A.D. Linde, Infrared Problem in Thermodynamics of the Yang-Mills Gas, Phys. Lett. B 96 (1980) 289.
8. D.J. Gross, R.D. Pisarski and L.G. Yaffe, QCD and Instantons at Finite Temperature, Rev. Mod. Phys. 53 (1981) 43.
9. F. D’Eramo, C. Lee, M. Lekaveckas, H. Liu and K. Rajagopal, Momentum Broadening in Weakly Coupled Quark-Gluon Plasma, AIP Conf. Proc. 1441 (2012) 895 [1110.5363].
10. P. Ginsparg, First and second order phase transitions in gauge theories at finite temperature, Nucl. Phys. B 170 (1980) 388.
11. T. Appelquist and R.D. Pisarski, High-temperature Yang-Mills theories and three-dimensional Quantum Chromodynamics, Phys. Rev. D 23 (1981) 2305.
12. P.B. Arnold and L.G. Yaffe, The Non-Abelian Debye screening length beyond leading order, Phys. Rev. D 52 (1995) 7208 [hep-ph/9508280].
13. M. Laine and M. Vepsäläinen, On the smallest screening masses in hot QCD, JHEP 09 (2009) 023 [0906.4450].
14. M. Laine and O. Philipsen, The non-perturbative QCD Debye mass from a Wilson line operator, Phys. Lett. B 459 (1999) 259 [hep-lat/9905004].
15. R. Baier, D. Schiff and B.G. Zakharov, Energy loss in perturbative QCD, Ann. Rev. Nucl. Part. Sci. 50 (2000) 37 [hep-ph/0002198].
16. R. Baier, Y.L. Dokshitzer, A.H. Mueller, S. Peigné and D. Schiff, Radiative energy loss and \( p_T \) broadening of high-energy partons in nuclei, Nucl. Phys. B 484 (1997) 265 [hep-ph/9608322].
17. Y. Schröder, The static potential in QCD, DESY-THESIS-1999-021: The Static potential in QCD to two loops, Phys. Lett. B 447 (1999) 321 [hep-ph/9812205].