Abstract

In this paper, the power flow in electrical systems is modelled in the time domain by using Geometric Algebra and the Hilbert Transform. The use of this mathematical framework unifies different approaches already raised so far to explain instantaneous power flows. Also, it overcomes some of the limitations shown by the existing methodologies under distorted supply. The proposed method can be applied for sinusoidal and non-sinusoidal regimes, non-linear loads, in single- and multi-phase systems, and it provides meaningful results with a compact formulation. Several examples have been included to prove the validity of the proposed theory.

Index terms— Time-domain power theories, geometric algebra, Hilbert transform.

1 Introduction

Pioneer power theories for analysing electrical systems were developed by Steinmetz, Kennelly and Heaviside, among others, by the end of the XIX century [1, 2, 3]. Nowadays, these theories are still a source of discussion and debate concerning their correctness and physical interpretation [4]. Some of them were formulated in the frequency domain, such as those proposed by Budeanu [5] and Czarnecki [6], while other ones where formulated in the time domain, like those presented by Fryze [7], Akagi [8] and Depenbrock [9]. More recently, Lev-Ari [10] and Salmerón [11] have made relevant contributions to the field by using the Hilbert transform and tensor algebra, respectively. All these theories are devoted to explain the power-transfer process between complex electrical systems and they establish mathematical concepts associated to fictitious powers (e.g. reactive power), which are of a great value from the engineering point of view. Unfortunately, none of the existing proposals can be used to separate current components in the time domain under any type of voltage distortion, asymmetry, non-linearity of the load or combinations thereof. Some of these limitations have been reported in the literature [4, 12]. In this work, a new proposal is presented to overcome these limitations by applying two powerful math tools: Geometric Algebra (GA) and the Hilbert Transform (HT). GA is the key tool to be used in this paper. Due to its versatility, GA has been identified as a unified language for physics and mathematics [13]. The application of GA would make it possible to separate current components that have a relevant engineering meaning for systems with any number of phases (including single-phase systems) [14, 15]. Current decomposition is addressed from both instantaneous and averaged
points of view, for systems with and without stored energy. Moreover, thanks to the use of the HT, the
proposed theory can be applied to single-phase systems seamlessly, which is a relevant advantage compared
to the existing theories. Mathematical demonstrations are omitted in this paper for the sake of conciseness.

2 Geometric Algebra and Power Theory

2.1 Hilbert Transform and Geometric Algebra Fundamentals

Consider a multi-phase system where \( u(t) \) is the voltage and \( i(t) \) is the current. Both signals are vectors of
multiple dimensions associated to a Hilbert space. Current and voltages are periodic and \( T \) is the period
value. The HT of \( u(t) \) is

\[
H[u(t)] = \frac{1}{\pi} \text{PV} \int_{-\infty}^{+\infty} \frac{u(\tau)}{t-\tau} d\tau
\]

where \( \text{PV} \) is the Cauchy principal value. Note that the Bracewell criteria \([16]\) is used for the sign of the
transformation. The HT is a crucial tool that will be applied later in the paper to calculate quadrature
signals and to define the impedance in the time domain \([10]\).

Consider now an orthonormal base \( \sigma = \{\sigma_1, \sigma_2, \ldots, \sigma_n\} \) defined for a vector space in \( \mathbb{R}^n \). Then, it is
possible to establish a new geometric vector space \( \mathbb{G}^n \). Under these assumptions, a vector can be represented
as:

\[
a = \sum_n a_n \sigma_n = a_1 \sigma_1 + \ldots + a_n \sigma_n
\]

In this new space, the geometric product between two vectors \( (a \text{ and } b) \) can be defined as:

\[
M = ab = a \cdot b + a \wedge b
\]

which can be seen as the sum of the traditional scalar or inner product plus the so-called wedge or Grassmann
product \([13]\). The latter fulfils the following property:

\[
a \wedge b = -b \wedge a
\]

This new geometrical entity is commonly known as bivector and it cannot be found in traditional linear
algebra \([13]\).

2.2 Geometric Power in Time Domain

For a \( n \)-phase electrical system, a vector containing the instantaneous voltages can be defined as follows:

\[
\vec{u}(t) = [u_1(t), u_2(t), \ldots, u_n(t)]
\]

where each voltage is referred to a virtual star point that might not be the same as the neutral conductor.
Also, a vector that includes the line currents is defined:

\[
\vec{i}(t) = [i_1(t), i_2(t), \ldots, i_n(t)]
\]

In order to simplify the notation, \( u_k(t) = u_k \) and \( i_k(t) = i_k \). The expression for the instantaneous power
consumed by the circuit is widely known, and it can be calculated as:

\[
p(t) = u_1 i_1 + u_2 i_2 + \ldots + u_n i_n
\]

which is the inner product between \( \vec{u}(t) \) and \( \vec{i}(t) \).

By using the HT, \( \vec{u}(t) \) and \( \vec{i}(t) \) can be represented in the geometric domain as the geometric vectors \( \vec{u} \)
and \( \vec{i} \), respectively:

\[
\vec{u} = u_1 \sigma_1 + \mathcal{H}[u_1] \sigma_2 + \cdots + u_n \sigma_2n - 1 + \mathcal{H}[u_n] \sigma_{2n}
\]

\[
\vec{i} = i_1 \sigma_1 + \mathcal{H}[i_1] \sigma_2 + \cdots + i_n \sigma_{2n - 1} + \mathcal{H}[i_n] \sigma_{2n}
\]
The use of the HT in (8) becomes essential to overcome the shortcomings of existing power theories and it is one of the main contributions of this work. Note that HT is only required for time average single-phase calculations and it can be omitted in the case of systems with more than one phase.

The norm of the geometric voltage and current vectors is:

$$\|u\| = \sqrt{uu} = \sqrt{\sum_{k=1}^{n} u_k^2 + \mathcal{H}^2[u_k]}$$

$$\|i\| = \sqrt{ii} = \sqrt{i \cdot i} = \sqrt{\sum_{k=1}^{n} i_k^2 + \mathcal{H}^2[i_k]}$$

The instantaneous geometric power is defined as the product of voltage and current vectors, thus

$$M = ui = u \cdot i + u \wedge i$$

The inspection of (10) reveals the potential of GA applied to electrical circuits. This expression encompasses the existing knowledge developed by different authors and by using different techniques (complex numbers, matrices, vector calculus, quaternions, tensors, etc.), but in a very compact way.

It is relevant to highlight that (10) consists of two terms that have a different nature, i.e., a scalar and a bivector term. This mathematical entity is a multivector and it can be written as

$$M = M_p + M_q$$

where

$$M_p = u \cdot i = \sum_{k=1}^{n} u_k i_k + \mathcal{H}[u_k] \mathcal{H}[i_k] = u_1 i_1 + \mathcal{H}[u_1] \mathcal{H}[i_1] + \ldots + u_n i_n + \mathcal{H}[u_n] \mathcal{H}[i_n]$$

$$M_q = u \wedge i = \left| \begin{array}{cccc} \sigma_1 & \sigma_2 & \ldots & \sigma_{2n-1} & \sigma_{2n} \\ u_1 & \mathcal{H}[u_1] & \ldots & u_n & \mathcal{H}[u_n] \\ i_1 & \mathcal{H}[i_1] & \ldots & i_n & \mathcal{H}[i_n] \end{array} \right|$$

(12)

$$M_p$$ is the scalar part and includes the instantaneous active power $$p(t)$$. It will be referred to as the parallel geometric power. $$M_q$$ is the bivector part and will be named quadrature geometric power. It comprises the well-known instantaneous reactive power in the IRP theory and its further refinements [17, 18, 19, 11]. In (12), the determinant can be calculated by using Leibniz or Laplace formula.

The instantaneous geometric power $$M$$ can also be written in terms of a commutative and anti-commutative part:

$$M_p = \frac{1}{2} (ui + iu) = \frac{1}{2} (M + M^\dagger)$$

$$M_q = \frac{1}{2} (ui - iu) = \frac{1}{2} (M - M^\dagger)$$

where $$M^\dagger$$ is the reverse of the instantaneous geometric power. It is worth noting that no matrices nor tensors are used in the definitions of powers presented in (10)–(14). This leads to a very compact formulation and simplifies mathematical expressions.

2.3 Current Decomposition

If $$i(t)$$ is the instantaneous current demanded by a load, it can be separated into meaningful engineering components by operating (10). Indeed, left multiplying by the inverse of the voltage vector leads to:

$$u^{-1}M = u^{-1}ui = i$$

(15)
since $u^{-1}u = 1$.

The inverse of a vector in the geometric domain is:

$$u^{-1} = \frac{u}{\|u\|^2}$$

(16)

which can be used in (15) to find the current decomposition. By replacing (11) in (15):

$$i = u^{-1}M = \frac{u}{\|u\|^2}M = \frac{u}{\|u\|^2}(M_p + M_q) = \frac{u}{\|u\|^2}M_p + \frac{u}{\|u\|^2}M_q = i_p + i_q$$

(17)

The term $i_p$ is commonly known as instantaneous geometric parallel current, while the term $i_q$ is the instantaneous geometric quadrature current and is orthogonal to $i_p$. It should be noted that $i_q$ is not a reactive current since it is not related to any physical phenomenon such as energy oscillations in reactive elements like inductors or capacitors [20]. In fact, it includes the effects generated by asymmetries of voltage sources and load unbalance.

The Fryze current can also be defined by using GA:

$$i_F = \frac{\bar{M}_p}{\|\bar{u}\|^2}u$$

(18)

where $\bar{M}_p$ is the mean value of the geometric parallel power and $\|\bar{u}\|$ is the RMS value of the geometric voltage. It can be readily demonstrated that $\bar{M}_p = 2P$, where $P$ is the active power. Moreover, the reactive current defined by Budeanu and supported by Willems [21], Lev-Ari [10] and Jeltsema [22] (among others) is:

$$i_B = \frac{\bar{M}_q}{\|\bar{u}\|^2}\mathcal{H}[u]$$

(19)

where $\bar{M}_q$ is the mean value of the quadrature geometric power. Similarly, $\bar{M}_q = 2Q$, where $Q$ is the reactive power defined by Budeanu. Therefore, the current expression can be fully decomposed as follows:

$$i = i_p + i_q = i_F + i_f + i_B + i_b$$

(20)

where $i_f$ is the Fryze complementary current required to conform the parallel current. Similarly, $i_b$ is the Budeanu complementary current required to conform the quadrature current. The asymmetry or unbalance current is included in $i_f$ and $i_b$.

The use of GA allows a natural decomposition of currents without requiring additional tools such as Clarke or Park transformations. The proposed methodology can be seamlessly applied to any distorted system since no constraints have been imposed to the waveforms of voltages and currents in this regard. This theory can be applied to any circuit with an arbitrary number of phases, including single-phase systems. This is not possible so far with other theories such as the IRP and it is a relevant feature of this proposal.

The transformation of a given current in a $n$-phase system from the geometric to the time domain can be written as:

$$i(t) = \sum_{k=1}^{n}[\bar{u}]_{2k-1}$$

(21)

where $[\bar{u}]_k$ refers to the $k$-th component of the geometric vector of the current $\bar{u}$.

### 2.4 Properties of $M$ and $i$

The instantaneous geometric power $M$ and the instantaneous geometric current $i$ fulfil a number of mathematical properties that reinforce the geometric interpretation of the proposed theory. These are explained below.
2.4.1 Orthogonality of the components of $M$

The parallel geometric power $M_p$ is a scalar number, while the quadrature geometric power $M_q$ is a bivector. Therefore, their internal product is zero and this implies orthogonality:

$$M_p \cdot M_q = 0$$  \hspace{1cm} (22)

In addition, the norm of the instantaneous geometric power is:

$$\|M\| = \sqrt{\langle MM^\dagger \rangle_0} = \|u\|\|i\|$$  \hspace{1cm} (23)

which can further be developed, yielding:

$$\|M\|^2 = \left\langle (M_p + M_q) (M_p + M_q)^\dagger \right\rangle_0 = \|M_p\|^2 + \|M_q\|^2$$

2.4.2 Orthogonality of the components of $i$

As shown in (17), the instantaneous geometric current $i$ can be decomposed into two vectors, $i_p$ and $i_q$. Based on (22), these vectors are orthogonal:

$$i_p \cdot i_q = u \frac{u}{\|u\|^2} M_p \cdot \frac{u}{\|u\|^2} M_q = \frac{1}{\|u\|^2} M_p \cdot M_q = 0$$  \hspace{1cm} (24)

Also, it can be demonstrated that all the terms in (20) are also orthogonal to each other.

3 Instantaneous Geometric Impedance

The instantaneous relationship between the voltage and current for each phase is defined for the first time in this paper as instantaneous geometric impedance. For each phase $k$, the current and voltage can be represented in rectangular and polar form as:

$$[u]_k = u_k \sigma_{2k-1} + \mathcal{H} [u_k] \sigma_{2k} = U e^{j(\omega k t)} \sigma_{(2k-1)(2k)} \sigma_{2k-1}$$
$$[i]_k = i_k \sigma_{2k-1} + \mathcal{H} [i_k] \sigma_{2k} = I e^{j(\omega k t)} \sigma_{(2k-1)(2k)} \sigma_{2k-1}$$  \hspace{1cm} (25)

and therefore:

$$Z_k = [u]_k [i]_k^{-1} = U e^{j\phi_k} \sigma_{(2k-1)(2k)} \sigma_{2k-1} I^{-1} e^{j\phi_k} \sigma_{(2k-1)(2k)} \sigma_{2k-1}$$

$$= U \frac{e^{j(\phi_k - \phi_k')}}{\sigma_{(2k-1)(2k)}} = Z_k \angle \phi_k = R_k + X_k \sigma_{(2k-1)(2k)}$$  \hspace{1cm} (26)

In the above expression, $Z_k$ is the instantaneous impedance module, $\phi_k$ is the instantaneous phase, $R_k$ represents the instantaneous resistance and $X_k$ is the instantaneous reactance.

4 Example

In the following sections, a number of classic examples solved in the time domain with the new GAPoT theory are presented. These examples are both single and three-phase circuits.

4.1 Single-phase circuits

Figure 1 shows two simple but widely studied circuits in the literature by different authors. They are of interest since traditional theory is not able to distinguish them in terms of complex apparent power. Consider a non-sinusoidal voltage supply $u(t) = 100\sqrt{2} (\sin t + \sin 3t)$. The frequency domain resolution shows that the reactive power (in the Budeanu sense) for each harmonic is equal and of opposite sign, so the total average reactive power is 0 for both circuits. Unfortunately, the use of complex algebra does not allow finding the
interaction between voltage and current harmonics of different frequency and that causes that the circuit a) can be completely compensated by passive elements, not so b) that needs an active compensator to achieve unity power factor.

This problem has already been solved by GAPoT in the geometric frequency domain in [23]. It proves how it is possible to identify differences at power level through the use of GA. Therefore, it is possible to distinguish between the two circuits by using the geometric apparent power. The two circuits will be solved in the time domain using GAPoT as formulated in this paper. By applying (8), the geometric voltage and current in the time domain are:

\[ u = u(t) \sigma_1 + \mathcal{H}[u(t)] \sigma_2 \]
\[ i = i(t) \sigma_1 + \mathcal{H}[i(t)] \sigma_2 \]  

(27)

The analytical resolution of the current yields \( i_a = 50\sqrt{2} (\sin t + \cos t + \sin 3t - \cos 3t) \) for circuit a), and \( i_b = \sqrt{2} (10 \sin t + 30 \cos t + 90 \sin 3t - 30 \cos 3t) \) for circuit b), so

\[ u = 100\sqrt{2} (\sin t + \sin 3t) \sigma_1 + 100\sqrt{2} (\cos t + \cos 3t) \sigma_2 \]
\[ i_a = 50\sqrt{2} (\sin t + \cos t + \sin 3t - \cos 3t) \sigma_1 + 50\sqrt{2} (\cos t - \sin t + \cos 3t + \sin 3t) \sigma_2 \]
\[ i_b = \sqrt{2} (10 \sin t + 30 \cos t + 90 \sin 3t - 30 \cos 3t) \sigma_1 + \sqrt{2} (10 \cos t - 30 \sin t + 90 \cos 3t + 30 \sin 3t) \sigma_2 \]

It is therefore possible to calculate the geometric apparent power according to (10)

\[ M_a = M_p^a + M_q^a = 20,000 \left(1 + \sin 2t + \cos 2t\right) \]
\[ M_b = M_p^b + M_q^b = 20,000 + 12,000 \sin 2t + 20,000 \cos 2t - 16,000 \sin 2t \sigma_{12} \]

The active power is the same in both cases, \( P = M_p / 2 = 10,000 \) as explained in Section 2.3. However, the quadrature geometric power is different, since \( M_q^a = 0 \), while \( M_q^b \neq 0 \). The above implies that circuit b) must be necessarily compensated by active elements to achieve a unit power factor. Note that in both circuits the reactive power (in the Budeanu sense) is \( Q = M_q / 2 = 0 \), so it is not feasible to know the eventual passive compensation in the time domain. However, by using GAPoT in the frequency domain, it is possible to disaggregate the reactive power for each harmonic [23].

To proceed with current decomposition, the expressions (17) to (20) shall be used. The complete current decomposition for each circuit is shown in table 1. It has been taken into account that the inverse of the voltage vector is
It can be remarked that current decomposition also leads to results that clearly highlights the differences between the two circuits. Although the total current module is the same for both, it is clear that their waveforms differ from each other. The parallel current $i_p$ is slightly lower in circuit b), while the quadrature current $i_q$ is zero in circuit a). Both circuits exhibit equal Fryze current $i_F$ because their active power $P$ is the same. This implies that an active compensator should be able to compensate it efficiently until a unity power factor is achieved. In both circuits the current $i_B$ is zero, which confirms that it is not possible to compensate them by means of passive elements in the time domain.

By applying (21) it is possible to retrieve the source currents in the time domain. Last column in table 1 shows the module of this current. Figure 2 shows the current decomposition for both circuits. Finally, the instantaneous impedance of the circuit can be found by (26)

$$Z = u^{-1} = u \frac{i}{\|i\|^2} = \frac{M}{\|i\|^2}$$

so, for circuits a) and b), it follows that

$$Z_a = 1 + \frac{\cos 2t}{1 + \sin 2t} \quad Z_b = 1 + \frac{\cos 2t}{\frac{5 \cos 2t}{5 + 3 \sin 2t} - \frac{4 \sin 2t}{5 + 3 \sin 2t}}$$

Again, the two circuits behave differently because their instantaneous impedances are distinct. While circuit a) does not have instantaneous reactance, circuit b) does. The resistive part also differs.
Figure 2: Current in time domain for circuits in figure 1.
4.2 Three-phase circuits

The circuit in figure 3 has been used by several authors to undermine the IRP theory [12, 4]. This theory does not make use of averaged quantities and therefore obtains results that can be considered uncommon or unnatural. This does not preclude some recognition of the merit of instantaneous current compensation. Through GAPoT it is also possible to obtain the same results as the IRP theory just by excluding the Hilbert transform from (8). For a three-phase system the voltage and current would be

\[
\begin{align*}
  u &= u_R \sigma_1 + u_S \sigma_2 + u_T \sigma_3 \\
  i &= i_R \sigma_1 + i_S \sigma_2 + i_T \sigma_3
\end{align*}
\]

so that, by applying again (10), a similar procedure would be performed as in the previous single-phase example. However, if the complete transformation is used and the Hilbert transform is included, the new results are in line with what is expected from an electrical point of view. In the circuit of figure 3, the supply voltage is balanced and sinusoidal with a value of

\[
\begin{align*}
  u_R(t) &= \sqrt{2}U \cos \omega t, \\
  u_S(t) &= \sqrt{2}U \cos (\omega t - 120), \\
  u_T(t) &= \sqrt{2}U \cos (\omega t + 120)
\end{align*}
\]

so that there is only current for the R phase, since the system is unbalanced

\[
i_R(t) = \sqrt{2}GU \cos \omega t
\]

Based on this information, the instantaneous geometrical vector of voltage and current can be derived as

\[
\begin{align*}
  u &= \sqrt{2}U [\cos \omega t \sigma_1 - \sin \omega t \sigma_2 + \cos (\omega t - 120) \sigma_4 - \sin (\omega t - 120) \sigma_5 + \cos (\omega t + 120) \sigma_6] \\
  i &= \sqrt{2}GU [\cos \omega t \sigma_1 - \sin \omega t \sigma_2]
\end{align*}
\]

which leads to the apparent geometric power

\[
M = 2GU^2 [1 - \cos \omega t \cos (\omega t - 120) \sigma_{13} + \cos \omega t \sin (\omega t + 120) \sigma_{14} - \cos \omega t \cos (\omega t + 120) \sigma_{15} \\
+ \cos \omega t \sin (\omega t + 120) \sigma_{16} - \sin \omega t \cos (\omega t - 120) \sigma_{23} + \sin \omega t \cos (\omega t - 120) \sigma_{24} \\
- \sin \omega t \cos (\omega t + 120) \sigma_{25} + \sin \omega t \sin (\omega t + 120) \sigma_{26}]
\]

In the above expression, the parallel power is constant with value \(M_p = 2GU^2\). This is consistent with expectations, since we have a pure resistive circuit whose active power is \(P = GU^2 = M_p/2\). The rest of the bivector elements have to do with the unbalance of the load. Note that the terms \(\sigma_{12}, \sigma_{34}\) and \(\sigma_{56}\) do not exist, so it follows that there is no reactive power in the Budeanu sense, as would be expected in the absence of inductive or capacitive elements.
Once the geometric power has been found, the currents can be calculated according to (17)-(20). Considering that $\|u\|^2 = 6U^2$, then

\[
i_p = \frac{u}{\|u\|^2} M_p = \frac{G}{3}u = \sqrt{2} \frac{GU}{3} \left[ \cos \omega t \sigma_1 - \sin \omega t \sigma_2 + \cos (\omega t - 120) \sigma_3 - \sin (\omega t + 120) \sigma_4 + \cos (\omega t + 120) \sigma_5 - \sin (\omega t + 120) \sigma_6 \right]
\]

\[
i_q = \frac{u}{\|u\|^2} M_q = i - i_p = \sqrt{2} \frac{GU}{3} \left[ 2 \cos \omega t \sigma_1 - 2 \sin \omega t \sigma_2 - \cos (\omega t - 120) \sigma_3 + \sin (\omega t + 120) \sigma_4 - \cos (\omega t + 120) \sigma_5 + \sin (\omega t + 120) \sigma_6 \right]
\]

Furthermore, for this example, the Fryze current matches the parallel current, that is, $i_p = i_f$, and therefore, $i_f = 0$. There is also no reactive Budeanu current since $\bar{M}_q = 0$, so $i_B = 0$. Therefore, the current $i_b = i_q$, so that it contains all the asymmetry components, that is, the zero sequence current $i_0$ and the inverse sequence current $i_\perp$.

\[
i_q = i_b = \sqrt{2} \frac{GU}{3} \left[ \cos \omega t \sigma_1 - \sin \omega t \sigma_2 + \cos \omega t \sigma_3 - \sin \omega t \sigma_4 + \cos \omega t \sigma_5 - \sin \omega t \sigma_6 \right]
\]

\[
+ \sqrt{2} \frac{GU}{3} \left[ \cos \omega t \sigma_1 - \sin \omega t \sigma_2 + \cos (\omega t + 120) \sigma_3 - \sin (\omega t + 120) \sigma_4 + \cos (\omega t + 120) \sigma_5 - \sin (\omega t + 120) \sigma_6 \right]
\]

In figure 4, the decomposition of currents for this problem is depicted assuming that $U = 230$, $\omega = 1$ and $G = 1$ ohm. Note that the time domain current is recovered by applying (21) to the geometric vector

\[
i_p(t) = \frac{\sqrt{2}GU}{3} \begin{bmatrix} i_{R_p}(t) \\ i_{S_p}(t) \\ i_{T_p}(t) \end{bmatrix} = \begin{bmatrix} \cos \omega t \\ \cos (\omega t - 120) \\ \cos (\omega t + 120) \end{bmatrix}
\]
\[ i_q(t) = \begin{bmatrix} i_{R_q}(t) \\ i_{S_q}(t) \\ i_{T_q}(t) \end{bmatrix} = \frac{\sqrt{2}GU}{3} \begin{bmatrix} 2 \cos \omega t \\ - \cos (\omega t - 120) \\ - \cos (\omega t + 120) \end{bmatrix} = \frac{\sqrt{2}GU}{3} \begin{bmatrix} \cos \omega t \\ \cos \omega t \\ \cos \omega t \end{bmatrix} + \frac{\sqrt{2}GU}{3} \begin{bmatrix} \cos \omega t \\ \cos (\omega t + 120) \\ \cos (\omega t - 120) \end{bmatrix} \]

5 Conclusion

In this work, the power flow in electrical circuits has been redefined by using GA and HT. It has been shown that the use of these tools greatly simplify mathematical expressions, leading to a very compact formulation. The proposed formulation can be applied to single- and multi-phase systems, and it provides a robust engineering interpretation that is in good agreement with other power theories. Current decomposition can be easily carried out and the results match with those previously obtained in the literature. GAPoT can apply instantaneous and averaged time based approaches, obtaining results according to engineering and physical expectations. Further research will be focus on the application of this new theory to calculate power flows in distorted electrical networks with non-linear loads.

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