Generalized supersymmetric cosmological term in
N=1 Supergravity

P. K. Concha$^{1,2,3}$, E. K. Rodríguez$^{1,2,3}$, P. Salgado$^1$

$^1$Departamento de Física, Universidad de Concepción,
Casilla 160-C, Concepción, Chile.
$^2$Dipartimento di Scienza Applicata e Tecnologia (DISAT),
Politecnico di Torino, Corso Duca degli Abruzzi 24,
I-10129 Torino, Italia.
$^3$Istituto Nazionale di Fisica Nucleare (INFN) Sezione di Torino,
Via Pietro Giuria 1, 10125 Torino, Italia.

E-mail: patrickconcha@udec.cl, everodriguez@udec.cl, pasalgad@udec.cl

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Abstract

An alternative way of introducing the supersymmetric cosmological term in a supergravity theory is presented. We show that the $AdS$-Lorentz superalgebra allows to construct a geometrical formulation of supergravity containing a generalized supersymmetric cosmological constant. The $N = 1, D = 4$ supergravity action is built only from the curvatures of the $AdS$-Lorentz superalgebra and corresponds to a MacDowell-Mansouri like action. The extension to a generalized $AdS$-Lorentz superalgebra is also analyzed.

1 Introduction

A good candidate to describe the dark energy corresponds to the cosmological constant \[1,2\]. It is well known that a cosmological term can be introduced in a $D = 4$ gravity theory using the Anti de Sitter ($AdS$) algebra. In particular the supersymmetric extension of gravity including a cosmological term can be obtained in a geometric formulation. In this framework, supergravity is built from the curvatures of the $osp(4|1)$ superalgebra and the action is known as the MacDowell-Mansouri action \[3\].

Recently it was presented in ref. \[4\] an alternative way of introducing the generalized cosmological constant term using the Maxwell algebra. It is usually accepted that the symmetries of Minkowski spacetime are described by the
Poincaré algebra. In refs. [5, 6] this spacetime was generalized extending its symmetries from the Poincaré to the Maxwell symmetries whose generators satisfy the following commutation relations

\[ [P_a, P_b] = \Lambda Z_{ab}, \quad Z_{ab} = -Z_{ba}, \quad (1) \]

\[ [J_{ab}, Z_{cd}] = \eta_{bc}Z_{ad} - \eta_{ac}Z_{bd} - \eta_{bd}Z_{ac} + \eta_{ad}Z_{bc}, \quad (2) \]

\[ [Z_{ab}, Z_{cd}] = 0, \quad [Z_{ab}, P_c] = 0. \quad (3) \]

Here \( Z_{ab} \) correspond to tensorial Abelian charges and the constant \( \Lambda \) can be related to the cosmological constant when \( [\Lambda] = M^2 \). If we put \( \Lambda = e \), where \( e \) is the electromagnetic coupling constant, we have the possible description of spacetime in presence of a constant electromagnetic background field.

The deformations of the Maxwell symmetries lead to the \( \mathfrak{so}(D-1,2) \oplus \mathfrak{so}(D-1,1) \) or \( \mathfrak{so}(D,1) \oplus \mathfrak{so}(D-1,1) \) algebra [7, 8]. In this case the \( Z_{ab} \) generators are non-abelian. If spacetime symmetries are considered local symmetries then it is possible to construct Chern-Simons gravity actions where dark energy could be interpreted as part of the metric of spacetime.

Subsequently it was shown in ref. [9] that the generalized cosmological constant term can also be included in a Born-Infeld like action built from the curvatures of the \( \text{AdS-Lorentz}^{(2)}(\text{AdS} - L_4) \) algebra. Alternatively the \( \text{AdS-Lorentz} \) algebra can be obtained as an abelian semigroup expansion (S-expansion) of the \( \text{AdS} \) algebra using \( S^{(2)}_M \) as the relevant semigroup [10].

The S-expansion procedure is based on combining the multiplication law of a semigroup \( S \) with the structure constants of a Lie algebra \( g \). The new Lie algebra obtained using this method is called the S-expanded algebra \( \mathfrak{S} = S \times g \). Diverse (super)gravity theories have been extensively studied using the S-expansion approach. In particular, interesting results have been obtained in refs. [12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. An alternative expansion method can be found in ref. [22].

In this paper we analyze the consequence of consider the supersymmetric extension of the \( \text{AdS-Lorentz} \) algebra in the construction of a supergravity theory. This superalgebra has the following anticommutation relation,

\[ \{Q_\alpha, Q_\beta\} = -\frac{1}{2} \left[ (\gamma^a C)_{\alpha\beta} Z_{ab} - 2 (\gamma^a C)_{\alpha\beta} P_a \right], \quad (4) \]

where \( Q_\alpha \) represents a 4-component Majorana spinor charge. Unlike the Maxwell superalgebra the new generators \( Z_{ab} \) are not abelian and behave as a Lorentz generator,

\[ [Z_{ab}, Z_{cd}] = \eta_{bc}Z_{ad} - \eta_{ac}Z_{bd} - \eta_{bd}Z_{ac} + \eta_{ad}Z_{bc}. \quad (5) \]

The presence of the \( Z_{ab} \) generators implies the introduction of a new bosonic "matter" field \( k_{ab} \) which modify the definition of the different curvatures allowing us to introduce a generalized supersymmetric cosmological constant term.

\[ ^1 \text{Also known as } \mathfrak{so}(D-1,1) \oplus \mathfrak{so}(D-1,2) \text{ algebra.} \]
It is the purpose of this work to construct a supergravity action which contains a generalized supersymmetric cosmological constant from the $AdS$-Lorentz superalgebra. To this aim, we apply the $S$-expansion method to the $osp(4|1)$ superalgebra and we build a MacDowell-Mansouri like action with the expanded 2-form curvatures. The result presented here corresponds to an alternative way of introducing the supersymmetric cosmological term and can be seen as the supersymmetric extension of refs. [4, 9]. We extend our result introducing the generalized minimal $AdS$-Lorentz superalgebra and we build the most general $D = 4, N = 1$ supergravity action involving a supersymmetric cosmological term.

This work is organized as follows: in section 2 we review the construction of the $AdS$-Lorentz superalgebra using the $S$-expansion procedure. Sections 3 and 4 contain our main results. In section 3, we present the $D = 4, N = 1$ supergravity action including a generalized supersymmetric cosmological constant. We show that this action corresponds to a MacDowell-Mansouri like action built from the curvatures of the $AdS$-Lorentz superalgebra. In section 4 we extend our results to the generalized minimal $AdS$-Lorentz superalgebra. Section 5 concludes the work with some comments.

2 $AdS$–Lorentz superalgebra and the abelian semigroup expansion procedure

The abelian semigroup expansion procedure ($S$-expansion) is a powerful tool in order to derive new Lie (super)algebras [11]. Furthermore, the $S$-expansion method has the advantage to provide with an invariant tensor for the $S$-expanded algebra $\mathfrak{g} = S \times \mathfrak{g}$ in terms of an invariant tensor for the original algebra $\mathfrak{g}$.

Following refs. [11, 18], it is possible to obtain the $AdS$–Lorentz superalgebra as an $S$-expansion of the $osp(4|1)$ superalgebra using $S^{(2)}_M$ as the abelian semigroup.

Before to apply the $S$-expansion method it is necessary to consider a decomposition of the original algebra $\mathfrak{g} = osp(4|1)$ in subspaces $V_p$.

$$\mathfrak{g} = osp(4|1) = so(3, 1) \oplus \frac{osp(4|1)}{sp(4)} \oplus \frac{sp(4)}{so(3, 1)} = V_0 \oplus V_1 \oplus V_2,$$

(6)

where $V_0$ is generated by the Lorentz generator $\tilde{J}_{ab}$, $V_1$ corresponds to the fermionic subspace generated by a 4-component Majorana spinor charge $\tilde{Q}_a$, and $V_2$ corresponds to the $AdS$ boost generated by $\tilde{P}_a$. The $osp(4|1)$ generators
satisfy the following (anti)commutation relations

\[
\begin{align*}
\left[ J_{ab}, J_{cd} \right] &= \eta_{bc}J_{ad} - \eta_{ac}J_{bd} - \eta_{ad}J_{bc} + \eta_{bd}J_{ac}, \\
\left[ J_{ab}, \tilde{P}_a \right] &= \eta_{bc}\tilde{P}_c - \eta_{ac}\tilde{P}_b, \\
\left[ \tilde{P}_a, \tilde{P}_b \right] &= \tilde{J}_{ab}, \\
\left[ \tilde{J}_{ab}, \tilde{Q}_{\alpha} \right] &= -\frac{1}{2} (\gamma_{ab}\tilde{Q})_\alpha, \\
\left[ \tilde{P}_a, \tilde{Q}_{\alpha} \right] &= -\frac{1}{2} (\gamma_{a\alpha}C)_{\alpha\beta} \tilde{J}_{ab} - 2 (\gamma^aC)_{\alpha\beta} \tilde{P}_a.
\end{align*}
\]

Here, \( \gamma_a \) are Dirac matrices and \( C \) stands for the charge conjugation matrix.

The subspace structure may be written as

\[
\begin{align*}
[V_0, V_0] &\subset V_0, \quad [V_1, V_1] \subset V_0 \oplus V_2, \\
[V_0, V_1] &\subset V_1, \quad [V_1, V_2] \subset V_1, \\
[V_0, V_2] &\subset V_2, \quad [V_2, V_2] \subset V_0.
\end{align*}
\]

Following the definitions of ref. \cite{1}, let \( S^{(2)}_M = \{ \lambda_0, \lambda_1, \lambda_2 \} \) be an abelian semigroup whose elements satisfy the multiplication law,

\[
\lambda_\alpha \lambda_\beta = \begin{cases} 
\lambda_{\alpha+\beta}, & \text{if } \alpha + \beta \leq 2 \\
\lambda_{\alpha+\beta-2}, & \text{if } \alpha + \beta > 2
\end{cases}
\]

Let us consider the subset decomposition \( S^{(2)}_M = S_0 \cup S_1 \cup S_2 \), with

\[
\begin{align*}
S_0 &= \{ \lambda_0, \lambda_2 \}, \\
S_1 &= \{ \lambda_1 \}, \\
S_2 &= \{ \lambda_2 \}.
\end{align*}
\]

One sees that this decomposition is said to be resonant since it satisfies the same structure as the subspaces \( V_\rho \) [compare with eqs \( \text{(12)} \)]

\[
\begin{align*}
S_0 \cdot S_0 &\subset S_0, \quad S_1 \cdot S_1 \subset S_0 \cap S_2, \\
S_0 \cdot S_1 &\subset S_1, \quad S_1 \cdot S_2 \subset S_1, \\
S_0 \cdot S_2 &\subset S_2, \quad S_2 \cdot S_2 \subset S_0.
\end{align*}
\]

Following theorem IV.2 of ref. \cite{1}, we can say that the superalgebra

\[
\mathfrak{G}_R = W_0 \oplus W_1 \oplus W_2,
\]

is a resonant subalgebra of \( S^{(2)}_M \times \mathfrak{g} \), where

\[
\begin{align*}
W_0 &= (S_0 \times V_0) = \{ \lambda_0, \lambda_2 \} \times \{ \tilde{J}_{ab} \} = \{ \lambda_0 \tilde{J}_{ab}, \lambda_2 \tilde{J}_{ab} \}, \\
W_1 &= (S_1 \times V_1) = \{ \lambda_1 \} \times \{ \tilde{Q}_{\alpha} \} = \{ \lambda_1 \tilde{Q}_{\alpha} \}, \\
W_2 &= (S_2 \times V_2) = \{ \lambda_2 \} \times \{ \tilde{P}_a \} = \{ \lambda_2 \tilde{P}_a \}.
\end{align*}
\]
Thus the new superalgebra obtained is generated by \( \{ J_{ab}, P_a, Z_{ab}, Q_\alpha \} \) where these new generators can be written as

\[
\begin{align*}
J_{ab} &= \lambda_0 \tilde{J}_{ab}, \\
Z_{ab} &= \lambda_2 \tilde{J}_{ab}, \\
P_a &= \lambda_2 \tilde{P}_a, \\
Q_\alpha &= \lambda_1 \tilde{Q}_\alpha.
\end{align*}
\]

The expanded generators satisfy the (anti)commutation relations

\[
\begin{align*}
\{ J_{ab}, J_{cd} \} &= \eta_{bc} J_{ad} - \eta_{ac} J_{bd} - \eta_{bd} J_{ac} + \eta_{ad} J_{bc}, \\
\{ J_{ab}, Z_{cd} \} &= \eta_{bc} Z_{ad} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac} + \eta_{ad} Z_{bc}, \\
\{ Z_{ab}, Z_{cd} \} &= \eta_{bc} Z_{ad} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac} + \eta_{ad} Z_{bc}, \\
\{ J_{ab}, P_c \} &= \eta_{bc} P_a - \eta_{ac} P_b, \\
\{ Z_{ab}, P_c \} &= \eta_{bc} P_a - \eta_{ac} P_b, \\
\{ J_{ab}, Q_\alpha \} &= -\frac{1}{2} \left( \gamma_{ab} Q \right)_\alpha, \\
\{ P_a, Q_\alpha \} &= -\frac{1}{2} \left( \gamma_a Q \right)_\alpha, \\
\{ Q_\alpha, Q_\beta \} &= -\frac{1}{2} \left[ \left( \gamma^{ab} C \right)_{\alpha \beta} Z_{ab} - 2 \left( \gamma^a C \right)_{\alpha \beta} P_a \right],
\end{align*}
\]

where we have used the multiplication law of the semigroup (13) and the commutation relations of the original superalgebra. The new superalgebra obtained after a resonant \( S_M^{(2)} \)-expansion of \( osp(4|1) \) corresponds to the \( AdS - Lorentz \) superalgebra \( sAdS - L_4 \) in four dimensions. The details of its construction can be found in ref. [18]. An extensive study of the relations between Lie algebras and the semigroup expansion method can be found in ref. [23].

One can see that the \( AdS - Lorentz \) superalgebra contains the \( AdS - L_4 \) algebra \( \{ J_{ab}, P_a, Z_{ab} \} \) as a subalgebra. The \( AdS - L_4 \) algebra and its generalization has been extensively studied in ref. [9]. In particular it was shown that this algebra allows to include a generalized cosmological constant in a Born-Infeld gravity action.

On the other hand it is well known that an In"{o}n"{u}-Wigner contraction of the \( AdS - Lorentz \) superalgebra leads to the Maxwell superalgebra. In fact, the rescaling

\[
\begin{align*}
Z_{ab} &\rightarrow \mu^2 Z_{ab}, \\
P_a &\rightarrow \mu P_a \\
\end{align*}
\]

provide the Maxwell superalgebra in the limit \( \mu \rightarrow \infty \).

\[ \text{Also known as Poincaré semi-simple extended algebra.} \]
3 Generalized supersymmetric cosmological term from \(AdS\)-Lorentz superalgebra

In ref. [3] it was introduced a geometric formulation of \(N = 1, D = 4\) supergravity using the \(osp(4|1)\) gauge fields. The resulting action is known as the Mac Dowell-Mansouri action whose geometrical interpretation can be found in ref. [26]. In a very similar way to ref. [20] in which a Mac Dowell-Mansouri like action was built for the minimal Maxwell superalgebra, we will construct an action for the \(AdS\)-Lorentz superalgebra using the useful properties of the \(S\)-expansion procedure.

We have shown in the previous section that the \(D = 4\) \(AdS\)-Lorentz superalgebra can be found as an \(S\)-expansion of the \(osp(4|1)\) superalgebra. Following the definitions of ref. [11], let \(S_M^{(2)} = \{\lambda_0, \lambda_1, \lambda_2\}\) be an abelian semigroup whose elements satisfy the multiplication law (13). After the extraction of a resonant subalgebra one finds the \(AdS\)-Lorentz superalgebra whose generators \(\{J_{ab}, P_a, Z_{ab}, Q_\alpha\}\) satisfy the commutations relations (22) − (29).

In order to write down an action for \(AdS\)-Lorentz superalgebra we start from the one-form connection

\[
A = A^A T_A = \frac{1}{2} \omega^{ab} J_{ab} + \frac{1}{l} e^a P_a + \frac{1}{2} k^{ab} Z_{ab} + \frac{1}{\sqrt{l}} \psi^\alpha Q_\alpha, \tag{31}
\]

where the one-form gauge fields are given in terms of the components of the \(osp(4|1)\) connection,

\[
\omega^{ab} = \lambda_0 \tilde{\omega}^{ab}, \\
e^a = \lambda_2 e^a, \\
k^{ab} = \lambda_2 \tilde{k}^{ab}, \\
\psi^\alpha = \lambda_1 \tilde{\psi}^\alpha.
\]

The associated two-form curvature \(F = dA + A \wedge A\) is given by

\[
F = F^A T_A = \frac{1}{2} R^{ab} J_{ab} + \frac{1}{l} R^a P_a + \frac{1}{2} F^{ab} Z_{ab} + \frac{1}{\sqrt{l}} \Psi^\alpha Q_\alpha, \tag{32}
\]

where

\[
R^{ab} = d\omega^{ab} + \omega^{a} e^b + \omega^{b} e^a - \frac{1}{2} \bar{\psi} \gamma^a \psi, \\
R^a = de^a + \omega^a e^b + k^a e^b - \frac{1}{2} \bar{\psi} \gamma^a \psi, \\
F^{ab} = dk^{ab} + \omega^a k^{cb} - \omega^b k^{ca} + k^a k^{cb} + \frac{1}{l^2} e^a e^b + \frac{1}{2l} \bar{\psi} \gamma^a \psi, \\
\Psi = d\psi + \frac{1}{4} \omega_{ab} \gamma^{ab} \psi + \frac{1}{2l} e^a \gamma_\alpha \psi + \frac{1}{4} k^{ab} \gamma^{ab} \psi.
\]

The one-forms \(\omega^{ab}, e^a, \psi\) and \(k^{ab}\) are the spin connection, the vielbein, the gravitino field and a bosonic "matter" field respectively. Here \(\psi\) corresponds to a
Majorana spinor which satisfy $\bar{\psi} = \psi C$, where $C$ is the charge conjugation matrix. Naturally when $F = 0$ the Maurer-Cartan equations for the $AdS$–Lorentz superalgebra are satisfied.

In order to interpret the gauge field as the vielbein, it is necessary to introduce a length scale $l$. In fact, if we choose the Lie algebra generators $T_A$ to be dimensionless then the 1-form connection fields $A = A^A_{\mu} T_A dx^\mu$ must also be dimensionless. Nevertheless, the vielbein $e^a = e^a_\mu dx^\mu$ must have dimensions of length if it is related to the spacetime metric $g_{\mu\nu}$ through $g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$. Thus the "true" gauge field must be of the form $e^a / l$. In the same way we must consider that $\psi / \sqrt{l}$ is the "true" gauge field of supersymmetry since the gravitino $\psi = \psi^\mu dx^\mu$ has dimensions of $(\text{length})^{1/2}$.

From the Bianchi identity $\nabla F = 0$, with $\nabla = d + [A, \cdot]$, it is possible to write down the Lorentz covariant exterior derivatives of the curvatures as

\begin{align}
DR^{ab} &= 0, \\
DR^a &= R^a_{\phantom{a}^b} e^b + R^a_{\phantom{a}^c} k^c + \bar{\psi}^{\alpha} \Psi^\alpha, \\
DF^{ab} &= R^a_{\phantom{a}^c} k^b - R^b_{\phantom{b}^c} k^a + F^a_{\phantom{a}^c} k^b - F^b_{\phantom{b}^c} k^a + \frac{1}{l^2} \left( R^a e^b - e^a R^b \right) \\
&\quad+ \frac{l}{2} \bar{\psi}^{\gamma} \gamma^{ab} \psi, \\
D\Psi &= \frac{1}{4} R_{\alpha\beta} \gamma^\alpha \gamma^\beta \psi + \frac{1}{4} F_{\alpha\beta} \gamma^\alpha \gamma^\beta \psi - \frac{1}{4} k_{\alpha\beta} \gamma^\alpha \gamma^\beta \Psi + \frac{1}{2l} R^a \gamma_a \psi \\
&\quad- \frac{1}{2l} e^a \gamma_a \Psi. 
\end{align}

The general form of the Mac Dowell-Mansouri action built with $osp(4|1)$ two-form curvature is given by

\begin{equation}
S = 2 \int (F \wedge F) = 2 \int F^A \wedge F^B \langle T_A T_B \rangle, 
\end{equation}

with the following choice of the invariant tensor

\begin{equation}
\langle T_A T_B \rangle = \begin{cases} 
\langle J_{ab} J_{cd} \rangle = \epsilon_{abcd} \\
\langle Q_{\alpha} Q_{\beta} \rangle = 2 \langle \gamma_5 \rangle_{\alpha\beta}. 
\end{cases}
\end{equation}

It is important to note that if $\langle T_A T_B \rangle$ is an invariant tensor for the $osp(4|1)$ superalgebra then the action corresponds to a topological invariant. The action can be seen as the supersymmetric generalization of the $D = 4$ Born-Infeld action in which the action is built from the $AdS$ two-form curvature using $\langle T_A T_B \rangle$ as an invariant tensor for the Lorentz group.

In order to build a Mac Dowell-Mansouri like action for the $AdS$–Lorentz superalgebra we will consider the $S$-expansion of $\langle T_A T_B \rangle$ and the 2-form curvature given by (32).

Thus, the action for $AdS$–Lorentz superalgebra can be written as

\begin{equation}
S = 2 \int F^A \wedge F^B \langle T_A T_B \rangle_{sAdS-L_4}, 
\end{equation}
where \( \langle T_A T_B \rangle_{s AdS-L_4} \) can be derived from the original components of the invariant tensor \( [58] \). Using Theorem VII.1 of ref. [11], it is possible to show that the non-vanishing components of \( \langle T_A T_B \rangle_{s AdS-L_4} \) are given by

\[
\begin{align*}
\langle J_{ab} J_{cd} \rangle_{s AdS-L_4} &= \alpha_0 \langle J_{ab} J_{cd} \rangle, \\
\langle J_{ab} Z_{cd} \rangle_{s AdS-L_4} &= \alpha_2 \langle J_{ab} J_{cd} \rangle, \\
\langle Z_{ab} Z_{cd} \rangle_{s AdS-L_4} &= \alpha_2 \langle J_{ab} J_{cd} \rangle, \\
\langle Q_\alpha Q_\beta \rangle_{s AdS-L_4} &= \alpha_2 \langle Q_\alpha Q_\beta \rangle,
\end{align*}
\]

where \( \alpha_0 \) and \( \alpha_2 \) are dimensionless arbitrary independent constants. This choice of the invariant tensor breaks the \( AdS \)-Lorentz supergroup to their Lorentz like subgroup.

Then considering the non-vanishing components of the invariant tensor \([40]-[43]\) and the 2-form curvature \([32]\), it is possible to write down an action as

\[
S = 2 \int \left( \frac{1}{4} \alpha_0 \epsilon_{abcd} R^{ab} R^{cd} + \frac{1}{2} \alpha_2 \epsilon_{abcd} R^{ab} F^{cd} + \frac{1}{4} \alpha_2 \epsilon_{abcd} F^{ab} F^{cd} + \frac{2}{l} \alpha_2 \bar{\psi} \gamma_5 \Psi \right).
\]

Explicitly, the action takes the form

\[
S = \int \left( \frac{\alpha_0}{2} \epsilon_{abcd} R^{ab} R^{cd} + \alpha_2 \epsilon_{abcd} \left( R^{ab} D_k^{cd} + R^{ab} k^c_k e^{ed} + \frac{1}{l^2} R^{ab} e^{c} e^{d} \\
+ \frac{1}{2l} D_k^{ab} \bar{\psi} \gamma^{cd} \psi + \frac{1}{2} D^{ab} D^c_d k^e_k + D^{ab} k^c_k + \frac{1}{l^2} D^{ab} e^{c} e^{d} \\
+ \frac{1}{2l} D^{ab} \bar{\psi} \gamma^{cd} \psi + \frac{1}{2l} \bar{\psi} \gamma_5 k^{a} k^{b} \gamma^{cd} \psi \right) + \alpha_2 \left( \frac{4}{l^2} \bar{\psi} \gamma_5 D \psi + \frac{4}{l^2} \bar{\psi} \gamma_5 D \psi \right) + \frac{2}{l} \bar{\psi} \gamma_5 k_{ab} \gamma^{ab} \psi + \frac{1}{l} \bar{\psi} \gamma_5 \bar{\psi} \gamma_5 \bar{\psi} \gamma_5 \psi + \frac{1}{l^2} \bar{\psi} \gamma_5 \bar{\psi} \gamma_5 \psi \right).
\]

The action can be written in a more compact way using the gamma matrix identity

\[
\gamma_{ab} \gamma_5 = -\frac{1}{2} \epsilon_{abcd} \gamma^{cd},
\]

and the gravitino Bianchi identity

\[
DD \psi = \frac{1}{4} R^{ab} \gamma_{ab} \psi.
\]

In fact one can see that

\[
\frac{1}{2} \epsilon_{abcd} R^{ab} \bar{\psi} \gamma^{cd} \psi + 4D \bar{\psi} \gamma_5 D \psi = d \left( 4D \bar{\psi} \gamma_5 \psi \right),
\]

\[
\frac{1}{2} \epsilon_{abcd} D_k^{ab} \bar{\psi} \gamma^{cd} \psi + 2D \bar{\psi} \gamma_5 k^{a} k^{b} \gamma^{cd} \psi = d \left( \bar{\psi} k^{a} k^{b} \gamma_5 \gamma_5 \psi \right).
\]
Furthermore it is possible to show that
\[
\bar{\psi} e^a \gamma_a \gamma_5 \tilde{e}^b \gamma_5 \gamma_b \psi = \frac{1}{2} e^a e^b \bar{\psi} \gamma^{cd} \psi \varepsilon_{abcd},
\]
\[
\frac{1}{4} \bar{\psi} k_{ab} \gamma_a \gamma_5 k_{cd} \gamma_5 \gamma_b \gamma_5 \gamma_c \gamma_d \psi = -\frac{1}{2} k^a_f k^b_f \bar{\psi} \gamma_5 \gamma_c \gamma_d \psi \varepsilon_{abcd},
\]
\[
\bar{\psi} e^a \gamma_a \gamma_5 \tilde{e}^b \gamma_5 \gamma_b \psi = \varepsilon_{abcd} k_{ab} e^c \bar{\psi} \gamma^d \psi,
\]
where we have used the following identities
\[
\gamma_a \gamma_b = \gamma_{ab} + \eta_{ab},
\]
\[
\gamma^{ab} \gamma^{cd} = \varepsilon^{abcd} \gamma_5 - 4 \delta_5^{[a} \delta_5^{b]} - 2 \delta_{cd},
\]
\[
\gamma^a \gamma^b = -2 \gamma^a \delta^b - \varepsilon^{abcd} \gamma_5 \gamma_d,
\]
and the fact that \(\gamma_5 \gamma_a\) is an antisymmetric matrix. Thus the Mac Dowell-Mansouri like action for the \(AdS\)–Lorentz superalgebra takes the form
\[
S = \int \frac{\alpha_0}{2} \varepsilon_{abcd} R^{ab} R^{cd} + \frac{\alpha_2}{l^2} \varepsilon_{abcd} R^{ab} e^c e^d + 4 \bar{\psi} e^a \gamma_5 D \psi
\]
\[
+ \alpha_2 \varepsilon_{abcd} \left( R^{ab} D k_{cd} + R^{ab} k_{e}^{c} k_{cd} + \frac{1}{2} D k^{ab} D k^{cd} + D k^{ab} k_{e}^{c} k^{cd} + \frac{1}{2} k^{a}_f k^{b}_f k^{c}_g k^{d}_g \right)
\]
\[
+ \frac{1}{l^2} k^{ab} e^c \bar{\psi} \gamma_d \psi + \frac{1}{2l^4} e^a e^b e^c e^d \right) + \alpha_2 \left( 4 D \bar{\psi} \gamma_5 \psi + \bar{\psi} k^{ab} \gamma_5 \gamma_5 \psi \right). \tag{48}
\]
This action has been intentionally separated in five pieces where the first term is proportional to \(\alpha_0\) and corresponds to the Gauss Bonnet term. The second term contains the Einstein-Hilbert term plus the Rarita-Schwinger (RS) Lagrangian describing pure supergravity. The third piece corresponds to a Gauss Bonnet like term containing the new super \(AdS\)–Lorentz fields. This piece does not contribute to the dynamics and can be written as a boundary term. The fourth term corresponds to a generalized supersymmetric cosmological term which contains the usual supersymmetric cosmological constant plus three additional terms depending on \(k^{ab}\). The last piece is a boundary term.

One can see that the Mac Dowell-Mansouri like action built using the useful definitions of the \(S\)-expansion procedure describes a pure supergravity theory with a generalized supersymmetric cosmological term.

From \(48\) we can see that the bosonic part of the action corresponds to the one found for \(AdS\)–Lorentz algebra in ref. \(9\). Besides the action contains the generalized cosmological term introduced in ref. \(3\) for the Maxwell algebra.

One can note that if we omit the boundary terms in \(48\), the action can be written as
\[
S = \int \frac{\alpha_2}{l^2} \varepsilon_{abcd} R^{ab} e^c e^d + 4 \bar{\psi} e^a \gamma_5 D \psi + \alpha_2 \varepsilon_{abcd} \left( \frac{1}{l^2} D k^{ab} e^c e^d + \frac{1}{l^2} k^{a}_f k^{b}_f e^c e^d \right)
\]
\[
+ \frac{1}{l^4} e^a e^b e^c e^d \right) + \alpha_2 \left( 4 D \bar{\psi} \gamma_5 \psi + \bar{\psi} k^{ab} \gamma_5 \gamma_5 \psi \right). \tag{49}
\]
or equivalently

\[
S = \int \frac{\alpha^2}{l^2} \left( \epsilon_{abcd} R^{ab} e^c e^d + 4 \tilde{\psi} e^a \gamma_5 D \psi \right) + \alpha_2 \epsilon_{abcd} \left( \frac{2}{l^2} k^{ab} \tilde{T} e^d + \frac{1}{l^2} k^a k^b f^c e^d + \frac{1}{l^3} e^a k^b \tilde{\psi} e^c \gamma_5 \psi + \frac{1}{2l^4} e^a e^b e^c e^d \right),
\]

(50)

where we have used

\[
\epsilon_{abcd} D k^{ab} e^c e^d = 2 \epsilon_{abcd} k^{ab} T e^d + \frac{1}{l^2} \epsilon_{abcd} k^{ab} e^c e^d,
\]

\[
\tilde{T}^a = D e^a - \frac{1}{2} \tilde{\psi} \gamma^a \psi = T^a - \frac{1}{2} \tilde{\psi} \gamma^a \psi.
\]

Interestingly if we consider \( k^{ab} = 0 \) in our action we obtain the usual Mac Dowell-Mansouri action for the \( Osp(4|1) \) supergroup.

In order to obtain the field equations let us compute the variation of the Lagrangian with respect to the different super \( AdS-Lorentz \) fields. The variation of the Lagrangian with respect to the spin connection \( \omega^{ab} \), modulo boundary terms, is given by

\[
\delta \omega L = \frac{\alpha^2}{l^2} \epsilon_{abcd} \left( 2 \delta \omega^a D e^d + 2 \delta \omega^a k^b e^c e^d \right) + \frac{\alpha_2}{l^2} \tilde{\psi} e^a \gamma_5 \delta \omega^{cd} \gamma_{cd} \psi
\]

\[
= 2 \alpha_2 \epsilon_{abcd} \delta \omega^{ab} \left( T^c + k^c e^f - \frac{1}{2} \tilde{\psi} \gamma^c \psi \right) e^d
\]

\[
= 2 \alpha_2 \epsilon_{abcd} \delta \omega^{ab} R e^d.
\]

(51)

Here we see that \( \delta \omega L = 0 \) leads to the following field equation for the \( AdS-Lorentz \) supertorsion

\[
\epsilon_{abcd} R^{a} e^{d} = 0.
\]

(52)

On the other hand, the variation of the Lagrangian with respect to the vielbein \( e^a \) is given by

\[
\delta e L = \frac{\alpha^2}{l^2} \epsilon_{abcd} \left( 2 R^{ab} e^c + 2 D k^{ab} e^c + 2 k^a k^b e^c + \frac{2}{l} \tilde{\psi} \gamma^{ab} \psi e^c + \frac{2}{l^2} e^a e^b e^c \right) \delta e^d
\]

\[
+ \frac{\alpha_2}{l^2} \left( 4 \tilde{\psi} \gamma_5 \gamma_5 D \psi + \tilde{\psi} \gamma_5 k^{ab} \gamma_5 \gamma_5 k^{ab} \psi \right) \delta e^d.
\]

\[
= 2 \alpha_2 \epsilon_{abcd} \left( R^{ab} e^c + F^{ab} e^c \right) \delta e^d + \frac{\alpha_2}{l^2} \left( 4 \tilde{\psi} \gamma_5 \gamma_5 \Psi \right) \delta e^d,
\]

(53)

where we have used the \( AdS-Lorentz \) 2-form curvatures \( \Omega_2 \) and the fact that

\[
\epsilon_{abcd} \tilde{\psi} \gamma^{ab} \psi e^c = 2 \tilde{\psi} \gamma_5 \gamma_5 \gamma_5 \gamma_5 \psi,
\]

\[
\epsilon_{abcd} k^{ab} e^c \tilde{\psi} \gamma_5 \gamma_5 \psi = \tilde{\psi} e^a \gamma_5 \gamma_5 \gamma_5 \gamma_5 \psi.
\]

Then the field equation is obtained imposing \( \delta e L = 0 \)

\[
2 \epsilon_{abcd} \left( R^{ab} + F^{ab} \right) e^c + 4 \tilde{\psi} \gamma_5 \gamma_5 \Psi = 0.
\]

(54)
One can see that the rescaling
\[ k_{ab} \rightarrow \mu^2 k_{ab}, \quad e^a \rightarrow \mu e^a \quad \text{and} \quad \psi \rightarrow \sqrt{\mu} \psi \]
and dividing (53) by \( \mu^2 \) provide us with the usual field equation for supergravity in the limit \( \mu \rightarrow 0 \),
\[ \epsilon_{abcd} R^{c}{}_{e} + 4 \bar{\psi} \gamma_{d} \gamma_{5} D \psi = 0, \quad (55) \]
where \( D \) corresponds to the Lorentz covariant exterior derivative.

The variation of the Lagrangian with respect to the new \( AdS-Lorentz \) field \( k_{ab} \), modulo boundary terms, gives
\[
\delta_{k} \mathcal{L} = \frac{\alpha_{2}}{l^{2}} \epsilon_{abcd} \left( 2 \delta k_{ab} D e^{c} e^{d} + 2 \delta k_{a}{}^{f} k_{b}{}^{e} e^{c} e^{d} + \frac{1}{l^{2}} \delta k_{ab} \bar{\psi} \gamma_{d} \gamma_{5} \psi e^{c} \right)
\]
\[ = 2 \frac{\alpha_{2}}{l^{2}} \epsilon_{abcd} \delta k_{ab} \left( T^{e} + k_{j}^{e} e^{f} - \frac{1}{2} \bar{\psi} \gamma_{e} \psi \right) e^{d} \]
\[ = 2 \frac{\alpha_{2}}{l^{2}} \epsilon_{abcd} \delta k_{ab} R^{c} e^{d}, \quad (56) \]
where we have used the gamma matrix identities
\[ \gamma_{ab} \gamma_{5} = - \frac{1}{2} \epsilon_{abcd} \gamma^{cd}, \]
\[ \gamma^{c} \gamma_{a} = - 2 \gamma^{[a} \delta_{b]} - \epsilon^{abcd} \gamma_{d}. \]

Here we see that \( \delta_{k} \mathcal{L} = 0 \) leads to the same field equation than \( \delta_{\omega} \mathcal{L} = 0 \)
\[ \epsilon_{abcd} R^{a} e^{d} = 0. \quad (57) \]

Let us consider the variation of the Lagrangian with respect to the gravitino field \( \psi \), modulo boundary terms
\[
\delta_{\psi} \mathcal{L} = \frac{\alpha_{2}}{l^{2}} \left( 4 \delta \bar{\psi} e^{a} \gamma_{a} \gamma_{5} D \psi + 4 D \bar{\psi} e^{a} \gamma_{a} \gamma_{5} \delta \psi - 4 \bar{\psi} D e^{a} \gamma_{a} \gamma_{5} \delta \psi \right.
\]
\[ + 2 \delta \bar{\psi} e^{a} \gamma_{a} \gamma_{5} k^{bc} \gamma_{bc} \psi + \frac{\alpha_{2}}{l^{2}} \epsilon_{abcd} \left( \frac{2}{l} e^{a} e^{b} \delta \bar{\psi} \gamma^{cd} \psi \right) \]
\[ = \frac{\alpha_{2}}{l^{2}} \left( 4 \delta \bar{\psi} e^{a} \gamma_{a} \gamma_{5} D \psi + 4 \bar{\psi} \gamma_{a} \gamma_{5} D e^{a} \gamma_{a} \gamma_{5} \psi \right.
\]
\[ + 2 \delta \bar{\psi} e^{a} \gamma_{a} \gamma_{5} k^{bc} \gamma_{bc} \psi + \frac{4 \alpha_{2}}{l^{2}} \delta \bar{\psi} e^{a} \gamma_{a} \gamma_{5} \gamma_{b} e^{b} \gamma_{b} \psi \]
\[ = \frac{\alpha_{2}}{l^{2}} \delta \bar{\psi} \left( 8 e^{a} \gamma_{a} \gamma_{5} D \psi + 4 \gamma_{a} \gamma_{5} \psi D e^{a} + 2 e^{a} \gamma_{a} \gamma_{5} k^{bc} \gamma_{bc} \psi + \frac{4}{l} e^{a} \gamma_{a} \gamma_{5} e^{b} \gamma_{b} \psi \right) \]
\[ = \frac{\alpha_{2}}{l^{2}} \delta \bar{\psi} \left( 8 e^{a} \gamma_{a} \gamma_{5} \Psi + 4 \gamma_{a} \gamma_{5} \psi D e^{a} \right). \quad (58) \]

Then, using the definition of the supertorsion
\[ R^{a} = D e^{a} + k_{a}^{b} e^{b} - \frac{1}{2} \bar{\psi} \gamma^{a} \psi, \]
and the Fierz identity
\[ \gamma_a \psi \bar{\psi} \gamma^a \psi = 0, \]
we find the following field equation,
\[ 8 e^a \gamma_a \gamma_5 \Psi + 4 \gamma_a \gamma_5 \psi \left( R^a - k_{[a} e_{b]} \right) = 0. \]  
(59)
We can see that the introduction of a generalized supersymmetric cosmological constant leads to field equations very similar to those of \( \mathfrak{osp}(4|1) \) supergravity. The differences appear in the definition of the two-form curvatures due to the presence of the new matter field \( k_{ab} \).

On the other hand, although the Lagrangian is built from the \( \text{AdS}-\text{Lorentz} \) superalgebra it is not invariant under gauge transformations. In fact, the Lagrangian does not correspond to a Yang-Mills Lagrangian, nor a topological invariant.

As we can see the variation of the action (48) under gauge supersymmetry can be obtained using \( \delta R = [\epsilon, R] \),
\[ \delta_{\text{susy}} S = -\frac{4\alpha^2}{l^2} \int R^a \bar{\Psi} \gamma_a \gamma_5 \epsilon. \]
(60)
Thus in order to have gauge supersymmetry invariance it is necessary to impose the supertorsion constraint
\[ R^a = 0. \]
(61)
However this leads to express the spin connection \( \omega_{ab} \) in terms of the others fields \( \{e^a, k_{ab}, \psi\} \).

Nevertheless, it is possible to have supersymmetry invariance in the first formalism adding an extra piece to the gauge transformation \( \delta \omega_{ab} \) such that the variation of the action can be written as
\[ \delta S = -\frac{4\alpha^2}{l^2} \int R^a \left[ \bar{\Psi} \gamma_a \gamma_5 \epsilon - \frac{1}{2} e_{abcd} \delta_{\text{extra}} \omega_{cd} \right], \]
(62)
where the supersymmetry invariance is fulfilled when
\[ \delta_{\text{extra}} \omega_{ab} = 2 e_{abcd} \left( \bar{\Psi} \gamma_c \gamma_d \gamma_5 \epsilon + \bar{\Psi} \gamma_5 \gamma_5 \epsilon - \bar{\Psi} \gamma_5 \gamma_5 \epsilon \right) e^c, \]
(63)
with \( \bar{\Psi} = \bar{\Psi}_{ab} e^a e^b \).

Thus the action (48) in the first order formalism is invariant under the following supersymmetry transformations
\[ \delta \omega_{ab} = 2 e_{abcd} \left( \bar{\Psi} \gamma_c \gamma_d \gamma_5 \epsilon + \bar{\Psi} \gamma_5 \gamma_5 \epsilon - \bar{\Psi} \gamma_5 \gamma_5 \epsilon \right) e^c, \]
(64)
\[ \delta k_{ab} = -\frac{1}{l} \bar{\epsilon} \gamma_{ab} \psi, \]
(65)
\[ \delta e^a = \bar{\epsilon} \gamma^a \psi, \]
(66)
\[ \delta \psi = d \epsilon + \frac{1}{4} \omega_{ab} \gamma_{ab} \epsilon + \frac{1}{4} k_{ab} \gamma_{ab} \epsilon + \frac{1}{2 l} e^a \gamma_a \epsilon. \]
(67)
4 The Generalized minimal $AdS$–Lorentz superalgebra

In this section, we show that a particular choice of an abelian semigroup $S$ leads to a new Lie superalgebra. For this purpose we will consider the $\mathfrak{osp}(4|1)$ superalgebra as a starting point.

Let us consider a decomposition of the original superalgebra $g = \mathfrak{osp}(4|1)$ as

\[ g = \mathfrak{osp}(4|1) = \mathfrak{so}(3,1) \oplus \frac{\mathfrak{osp}(4|1)}{\mathfrak{sp}(4)} \oplus \mathfrak{sp}(4) \oplus \mathfrak{so}(3,1) \]

where $V_0$, $V_1$ and $V_2$ satisfy $[\mathfrak{osp}(4|1)]$ and correspond to the Lorentz subspace, the fermionic subspace and the $AdS$-boost respectively.

Let $S_M^{(4)} = \{\lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ be the abelian semigroup whose elements satisfy the following multiplication law

\[ \lambda_\alpha \lambda_\beta = \begin{cases} 
\lambda_{\alpha+\beta}, & \text{if } \alpha + \beta \leq 4 \\
\lambda_{\alpha+\beta-4}, & \text{if } \alpha + \beta > 4 
\end{cases} \]  

Let us consider the decomposition $S = S_0 \cup S_1 \cup S_2$, with

\[ S_0 = \{\lambda_0, \lambda_2, \lambda_4\}, \]
\[ S_1 = \{\lambda_1, \lambda_3\}, \]
\[ S_2 = \{\lambda_2, \lambda_4\}. \]

One can see that this decomposition satisfies the same structure as the subspaces $V_p$ then we said that the decomposition is resonant [compare with eqs $[\mathfrak{osp}(4|1)]$]

\[ S_0 \cdot S_0 \subset S_0, \quad S_1 \cdot S_1 \subset S_0 \cap S_2, \]
\[ S_0 \cdot S_1 \subset S_1, \quad S_1 \cdot S_2 \subset S_1, \]
\[ S_0 \cdot S_2 \subset S_2, \quad S_2 \cdot S_2 \subset S_0. \]

Following theorem IV.2 of ref. [11], we say that the superalgebra

\[ \mathfrak{g}_R = W_0 \oplus W_1 \oplus W_2 \]

is a resonant subalgebra of $S_M^{(4)} \times g$, where

\[ W_0 = (S_0 \times V_0) = \{\lambda_0, \lambda_2, \lambda_4\} \times \{\tilde{J}_{ab}\} = \{\lambda_0 \tilde{J}_{ab}, \lambda_2 \tilde{J}_{ab}, \lambda_4 \tilde{J}_{ab}\}, \]
\[ W_1 = (S_1 \times V_1) = \{\lambda_1, \lambda_3\} \times \{\tilde{Q}_\alpha\} = \{\lambda_1 \tilde{Q}_\alpha, \lambda_3 \tilde{Q}_\alpha\}, \]
\[ W_2 = (S_2 \times V_2) = \{\lambda_2, \lambda_4\} \times \{\tilde{P}_a\} = \{\lambda_2 \tilde{P}_a, \lambda_4 \tilde{P}_a\}. \]

Then the new superalgebra is generated by \{\$J_{ab}, P_a, \tilde{Z}_a, \tilde{Z}_{ab}, Z_{ab}, Q_\alpha, \Sigma_\alpha\}$ with

\[ J_{ab} = \lambda_0 \tilde{J}_{ab}, \quad P_a = \lambda_2 \tilde{P}_a, \]
\[ \tilde{Z}_{ab} = \lambda_2 \tilde{J}_{ab}, \quad \tilde{Z}_a = \lambda_4 \tilde{P}_a, \]
\[ Z_{ab} = \lambda_4 \tilde{J}_{ab}, \quad Q_\alpha = \lambda_1 \tilde{Q}_\alpha, \]
\[ \Sigma_\alpha = \lambda_3 \tilde{Q}_\alpha. \]
where $\tilde{J}_{ab}$, $\tilde{P}_a$ and $\tilde{Q}_\alpha$ are the $\mathfrak{osp}(4|1)$ generators. The new generators satisfy the commutation relations

$$[J_{ab}, J_{cd}] = \eta_{bc} J_{ad} - \eta_{ac} J_{bd} - \eta_{bd} J_{ac} + \eta_{ad} J_{bc},$$

$$[Z_{ab}, Z_{cd}] = \eta_{bc} Z_{ad} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac} + \eta_{ad} Z_{bc},$$

$$[J_{ab}, Z_{cd}] = \eta_{bc} Z_{ad} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac} + \eta_{ad} Z_{bc},$$

$$[\tilde{J}_{ab}, \tilde{Z}_{cd}] = \eta_{bc} \tilde{J}_{ad} - \eta_{ac} \tilde{J}_{bd} - \eta_{bd} \tilde{J}_{ac} + \eta_{ad} \tilde{J}_{bc},$$

$$[\tilde{Z}_{ab}, \tilde{Z}_{cd}] = \eta_{bc} \tilde{Z}_{ad} - \eta_{ac} \tilde{Z}_{bd} - \eta_{bd} \tilde{Z}_{ac} + \eta_{ad} \tilde{Z}_{bc},$$

$$[J_{ab}, P_c] = \eta_{bc} P_a - \eta_{ac} P_b, \quad [Z_{ab}, P_c] = \eta_{bc} P_a - \eta_{ac} P_b,$$

$$[\tilde{J}_{ab}, \tilde{Z}_c] = \eta_{bc} \tilde{J}_a - \eta_{ac} \tilde{J}_b, \quad [\tilde{Z}_{ab}, \tilde{Z}_c] = \eta_{bc} \tilde{Z}_a - \eta_{ac} \tilde{Z}_b,$$

$$[P_a, P_b] = Z_{ab}, \quad [\tilde{Z}_a, P_b] = \tilde{Z}_{ab}, \quad [\tilde{Z}_a, \tilde{Z}_b] = Z_{ab},$$

$$[J_{ab}, Q_\alpha] = -\frac{1}{2} (\gamma_{ab} Q)_\alpha, \quad [P_a, Q_\alpha] = -\frac{1}{2} (\gamma_\alpha Q)_\alpha,$$

$$[\tilde{Z}_{ab}, Q_\alpha] = -\frac{1}{2} (\gamma_{ab} Q)_\alpha, \quad [\tilde{Z}_a, Q_\alpha] = -\frac{1}{2} (\gamma_\alpha Q)_\alpha,$$

$$[Z_{ab}, Q_\alpha] = -\frac{1}{2} (\gamma_{ab} Q)_\alpha, \quad [P_a, \Sigma_\alpha] = -\frac{1}{2} (\gamma_\alpha Q)_\alpha,$$

$$[J_{ab}, \Sigma_\alpha] = -\frac{1}{2} (\gamma_{ab} Q)_\alpha, \quad [\tilde{Z}_a, \Sigma_\alpha] = -\frac{1}{2} (\gamma_\alpha Q)_\alpha,$$

$$[\tilde{Z}_{ab}, \Sigma_\alpha] = -\frac{1}{2} (\gamma_{ab} Q)_\alpha, \quad [Z_{ab}, \Sigma_\alpha] = -\frac{1}{2} (\gamma_{ab} Q)_\alpha,$$

$$\{Q_\alpha, Q_\beta\} = -\frac{1}{2} \left[ (\gamma_{ab} C)_{\alpha\beta} \tilde{Z}_{ab} - 2 (\gamma^a C)_{\alpha\beta} P_a \right],$$

$$\{Q_\alpha, \Sigma_\beta\} = -\frac{1}{2} \left[ (\gamma_{ab} C)_{\alpha\beta} Z_{ab} - 2 (\gamma^a C)_{\alpha\beta} \tilde{Z}_a \right],$$

$$\{\Sigma_\alpha, \Sigma_\beta\} = -\frac{1}{2} \left[ (\gamma_{ab} C)_{\alpha\beta} \tilde{Z}_{ab} - 2 (\gamma^a C)_{\alpha\beta} P_a \right],$$

where we have used the commutation relations of the original superalgebra and the multiplication law of the semigroup \cite{69}. The new superalgebra obtained after a resonant $S$-expansion of $\mathfrak{osp}(4|1)$ superalgebra corresponds to a generalized minimal $AdS$-Lorentz superalgebra in $D = 4$.

One can see that a new Majorana spinor charge $\Sigma$ has been introduced as a direct consequence of the $S$-expansion procedure. The introduction of a second spinorial generator can be found in refs. \cite{24,25} in the supergravity and superstrings context respectively.

Let us note that a generalized $AdS$–Lorentz algebra $\{J_{ab}, P_a, \tilde{Z}_a, \tilde{Z}_{ab}, Z_{ab} \}$ forms a bosonic subalgebra of the new superalgebra and looks very similar to
the $AdS - \mathcal{L}_6$ algebra introduced in ref. [9]. In fact one could identify $\tilde{Z}_{ab}$, $Z_{ab}$ and $\tilde{Z}_a$ with $Z^{(1)}_{ab}$, $Z^{(2)}_{ab}$ and $Z_a$ of $AdS - \mathcal{L}_6$ respectively. Nevertheless, the commutation relations (87) are subtly different of those of the $AdS - \mathcal{L}_6$ algebra. On the other hand the usual $AdS - \mathcal{L}_4$ algebra = \{ $J_{ab}$, $P_a$, $Z_{ab}$ \} is also a subalgebra.

It is interesting to observe that an İnönü-Wigner contraction of the new superalgebra leads to a generalization of the minimal Maxwell superalgebra introduced in ref. [27]. After the rescaling

$$
\tilde{Z}_{ab} \rightarrow \mu^2 \tilde{Z}_{ab}, \quad Z_{ab} \rightarrow \mu^4 Z_{ab}, \quad P_a \rightarrow \mu^2 P_a,
$$

$$
\tilde{Z}_a \rightarrow \mu^4 \tilde{Z}_a, \quad Q_\alpha \rightarrow \mu Q_\alpha \quad \text{and} \quad \Sigma \rightarrow \mu^3 \Sigma,
$$

the limit $\mu \rightarrow \infty$ provides with a generalized minimal Maxwell superalgebra $s\mathcal{M}_4$ in $D = 4$. An extensive study of the minimal Maxwell superalgebra and its generalization has been done using expansion method in refs. [28, 29]. On the other hand it was shown in refs. [30, 20] that $D = 4$, $N = 1$ pure supergravity Lagrangian can be obtained as a quadratic expression in the curvatures associated with the minimal Maxwell superalgebra.

Analogously we can show that the generalized minimal $AdS$-Lorentz superalgebra found here can be used in order to build the most general supergravity action involving a generalized supersymmetric cosmological term.

As in the previous section, we start from the one-form gauge connection,

$$
A = \frac{1}{2} \omega^{ab} J_{ab} + \frac{1}{l} e^a P_a + \frac{1}{2} k^{ab} \tilde{Z}_{ab} + \frac{1}{2} k^{a} Z_a + \frac{1}{l} \tilde{h}^a \tilde{Z}_a + \frac{1}{\sqrt{l}} \psi_\alpha Q_\alpha + \frac{1}{\sqrt{l}} \xi_\alpha \Sigma_\alpha, \quad (96)
$$

where the one-form gauge fields are related to the $osp(4 | 1)$ gauge fields $\left( \tilde{\omega}^{ab}, \tilde{e}^a, \tilde{\psi}_\alpha \right)$ as

$$
\omega^{ab} = \lambda_0 \tilde{\omega}^{ab}, \quad e^a = \lambda_2 \tilde{e}^a,
$$

$$
\tilde{k}^{ab} = \lambda_2 \tilde{\omega}^{ab}, \quad \psi_\alpha = \lambda_1 \tilde{\psi}_\alpha,
$$

$$
k^{ab} = \lambda_4 \tilde{\omega}^{ab}, \quad \xi_\alpha = \lambda_3 \tilde{\psi}_\alpha,
$$

$$
\tilde{h}^a = \lambda_4 e^a.
$$

Then the corresponding two-form curvature $F = dA + A \wedge A$ is given by

$$
F = \frac{1}{2} R^{ab} J_{ab} + \frac{1}{l} R^a P_a + \frac{1}{2} \tilde{R}^{ab} \tilde{Z}_{ab} + \frac{1}{2} F^{ab} Z_{ab} + \frac{1}{l} \tilde{H}^a \tilde{Z}_a + \frac{1}{\sqrt{l}} \Psi_\alpha Q_\alpha + \frac{1}{\sqrt{l}} \Xi_\alpha \Sigma_\alpha, \quad (97)
$$

15
where

\[ R^{ab} = d\omega^{ab} + \omega^{c}_{\alpha} \omega^{ab}, \]
\[ R^{a} = de^{a} + \omega^{a}_{\alpha} e^{b} + k^{a}_{\alpha} e^{b} + \tilde{k}^{a}_{\alpha} \tilde{e}^{b} - \frac{1}{2} \tilde{\psi} \gamma^{a} \psi - \frac{1}{2} \tilde{\xi} \gamma^{a} \xi, \]
\[ \tilde{H}^{a} = d\tilde{h}^{a} + \omega^{a}_{\alpha} \tilde{h}^{b} + k^{a}_{\alpha} \tilde{h}^{b} + \tilde{k}^{a}_{\alpha} \tilde{h}^{b} - \tilde{\psi} \gamma^{a} \xi, \]
\[ \tilde{F}^{ab} = dk^{ab} + \omega^{a}_{\alpha} \tilde{k}^{b} - \omega^{b}_{\alpha} \tilde{k}^{a} + k^{a}_{\alpha} \tilde{k}^{b} - \tilde{k}^{a}_{\alpha} \tilde{k}^{b} + \frac{2}{l^{2}} e^{a}_{\alpha} \tilde{h}^{b} + \frac{1}{2l^{2}} \tilde{\gamma}^{ab} \psi + \frac{1}{2l^{2}} \tilde{\xi} \gamma^{ab} \xi, \]
\[ F^{ab} = dk^{ab} + \omega^{a}_{\alpha} \tilde{k}^{b} - \omega^{b}_{\alpha} \tilde{k}^{a} + k^{a}_{\alpha} \tilde{k}^{b} + k^{b}_{\alpha} \tilde{k}^{a} + \frac{1}{l^{2}} e^{a}_{\alpha} e^{b} + \frac{1}{l^{2}} \tilde{\gamma}^{ab} \psi + \frac{1}{l^{2}} \tilde{\xi} \gamma^{ab} \psi, \]
\[ \Psi = d\psi + \frac{1}{4} \omega_{\alpha\beta} \gamma^{ab} \psi + \frac{1}{4} \tilde{k}_{\alpha\beta} \gamma^{ab} \psi + \frac{1}{4} \kappa_{\alpha\beta} \gamma^{ab} \xi + \frac{1}{2l^{2}} \tilde{e}_{a} \gamma^{a} \psi + \frac{1}{2l^{2}} \tilde{h}_{a} \gamma^{a} \psi, \]
\[ \Xi = d\xi + \frac{1}{4} \omega_{\alpha\beta} \gamma^{ab} \xi + \frac{1}{4} \kappa_{\alpha\beta} \gamma^{ab} \xi + \frac{1}{4} \tilde{k}_{\alpha\beta} \gamma^{ab} \psi + \frac{1}{2l^{2}} \tilde{e}_{a} \gamma^{a} \psi + \frac{1}{2l^{2}} \tilde{h}_{a} \gamma^{a} \xi. \]

Here the new Majorana field \( \xi \) is associated to the fermionic generator \( \Sigma \), while the one-forms \( \tilde{h}^{a}, \tilde{k}^{ab} \) and \( k^{ab} \) are the matter fields associated with the bosonic generators \( Z_{a}, Z_{ab} \) and \( Z_{ab} \) respectively.

Using the two-form curvature \( F \) it is possible to write the action for the generalized minimal \( AdS \)-Lorentz superalgebra as

\[ S = 2 \int \langle F \wedge F \rangle = 2 \int F^{A} \wedge F^{B} \langle T_{A} T_{B} \rangle_{S}, \]

where \( \langle T_{A} T_{B} \rangle_{S} \) corresponds to an \( S \)-expanded invariant tensor which is obtained from the original components of the invariant tensor \( (98) \). Using Theorem VII.1 of ref. \[11\], it is possible to show that the non-vanishing components of \( \langle T_{A} T_{B} \rangle_{S} \) are given by

\[ \langle J_{ab} J_{cd} \rangle_{S} = \alpha_{0} \langle J_{ab} J_{cd} \rangle, \]
\[ \langle J_{ab} \tilde{Z}_{cd} \rangle_{S} = \alpha_{2} \langle J_{ab} J_{cd} \rangle, \]
\[ \langle \tilde{Z}_{ab} Z_{cd} \rangle_{S} = \alpha_{2} \langle J_{ab} J_{cd} \rangle, \]
\[ \langle \tilde{Z}_{ab} \tilde{Z}_{cd} \rangle_{S} = \alpha_{4} \langle J_{ab} J_{cd} \rangle, \]
\[ \langle Q_{a} Q_{b} \rangle_{S} = \alpha_{2} \langle Q_{a} Q_{b} \rangle, \]
\[ \langle \Sigma_{a} \Sigma_{b} \rangle_{S} = \alpha_{2} \langle Q_{a} Q_{b} \rangle, \]

where \( \alpha_{0}, \alpha_{2} \) and \( \alpha_{4} \) are dimensionless arbitrary independent constants and

\[ \langle J_{ab} J_{cd} \rangle = \epsilon_{abcd}, \]
\[ \langle Q_{a} Q_{b} \rangle = 2 \langle \gamma_{b} \rangle_{a \beta}. \]

Then considering the two-form curvature \( (97) \) and the non-vanishing components of the invariant tensor \( (99) - (103) \) we found that the action can be
written as a MacDowell-Mansouri like action,

$$
S = 2 \int \left( \frac{\alpha_0}{4} \epsilon_{abcd} R^{ab} R^{cd} + \frac{\alpha_2}{2} \epsilon_{abcd} R^{ab} F^{cd} + \frac{\alpha_2}{2} \epsilon_{abcd} F^{ab} F^{cd} + \frac{\alpha_4}{4} \epsilon_{abcd} F^{ab} F^{cd} \right) + \frac{\alpha_4}{4} \epsilon_{abcd} \tilde{F}^{ab} F^{cd} + \frac{\alpha_2}{2} \epsilon_{abcd} F^{ab} F^{cd} + 2 \frac{l}{l} \alpha_2 \tilde{\psi} \gamma_5 \psi + 2 \frac{l}{l} \alpha_2 \tilde{\bar{\psi}} \gamma_5 \bar{\psi} + 4 \frac{l}{l} \alpha_4 \tilde{\bar{\psi}} \gamma_5 \bar{\psi} \right).
$$

(104)

Since we are interested in obtaining the Einstein-Hilbert and the Rarita-Schwinger like Lagrangian with a generalized supersymmetric cosmological term, we shall consider only the piece proportional to $\alpha_4$. Using the useful gamma matrix identities and the Bianchi identities ($dF + [A, F] = 0$) it is possible to write explicitly the $\alpha_4$-term as

$$
S = \alpha_4 \int \epsilon_{abcd} \left( R^{ab} K^{cd} + \frac{1}{2} \tilde{K}^{ab} \tilde{K}^{cd} + \frac{1}{2} K^{ab} K^{cd} \right) + \frac{1}{l^2} \left( \epsilon_{abcd} R^{ab} e^c e^d + 4 \tilde{\psi} e^a \gamma_5 D \psi + 4 \tilde{\psi} e^a \gamma_5 D \xi \right) + \frac{1}{l^2} \left( \epsilon_{abcd} R^{ab} \tilde{\bar{\psi}} \tilde{\bar{\psi}} e^d + 4 \tilde{\psi} \gamma_5 \gamma_a \gamma_5 D \psi + 4 \tilde{\psi} \gamma_5 \gamma_a \gamma_5 D \xi \right) + \frac{1}{l^2} \left( 2 \epsilon_{abcd} \tilde{\bar{\psi}} \tilde{\psi} e^d + \epsilon_{abcd} \tilde{\bar{\psi}} \tilde{\psi} e^d \right) + \frac{1}{l^2} \left( 2 \epsilon_{abcd} \tilde{\bar{\psi}} \tilde{\psi} e^d \right) + \frac{1}{l^2} \left( \tilde{\bar{\psi}} \psi \gamma^a \gamma_5 D \psi + \tilde{\bar{\psi}} \psi \gamma^a \gamma_5 D \xi \right) + \frac{1}{l^2} \left( \tilde{\bar{\psi}} \psi \gamma^a \gamma_5 D \psi + \tilde{\bar{\psi}} \psi \gamma^a \gamma_5 D \xi \right) + \frac{1}{l^2} \left( \tilde{\bar{\psi}} \psi \gamma^a \gamma_5 D \psi + \tilde{\bar{\psi}} \psi \gamma^a \gamma_5 D \xi \right)
$$

(105)

where we have defined

$$
\tilde{K}^{ab} = D \tilde{k}^{ab} + k^a_{\ c} \tilde{k}^{cb} + k^b_{\ c} \tilde{k}^{ac},
$$

$$
K^{ab} = D k^{ab} + k^a_{\ c} k^{cb} + k^b_{\ c} k^{ac}.
$$

Here we can see that the first piece corresponds to an Euler invariant term which can be seen as a Gauss-Bonnet like term and can be written as a boundary contribution. The second piece contains the Einstein-Hilbert term $\epsilon_{abcd} R^{ab} e^c e^d$ and the Rarita-Schwinger like Lagrangian. The novelty consists on the contribution of the new spinor field $\xi$ which is related to the Majorana spinor charge $\Sigma$. The fourth term corresponds to a generalized supersymmetric cosmological term built from the new $AdS$-Lorentz fields. The last piece is a boundary term and does not contribute to the dynamics.

A significant difference with the previous case (see eq. (103)) is the presence of the matter field $\tilde{h}^a$ which is related to the new generator $\tilde{Z}_a$. In particular, if we consider $\tilde{h}^a = 0$ and omit boundary contributions, the term proportional...
to $\alpha_4$ can be written as

$$S = \alpha_4 \int \frac{1}{l^2} \left( \epsilon_{abcd} R^{ab} e^c e^d + 4 \bar{\psi} e^a \gamma_a \gamma_5 \nabla \psi + 4 \bar{\xi} e^a \gamma_a \gamma_5 \nabla \xi \right)$$

$$+ \frac{1}{l^2} \left( \epsilon_{abcd} \mathcal{K}^{ab} e^c e^d + 4 e^a \gamma_a \nabla \bar{\psi} + 2 \bar{\psi} \gamma^a \phi^c \right),$$

(106)

with

$$\nabla \psi = D \psi + \frac{1}{4} k_{ab} \gamma^{ab} \psi + \frac{1}{4} \bar{\ell}_{ab} \gamma^{ab} \xi,$$

$$\nabla \xi = D \xi + \frac{1}{4} k_{ab} \gamma^{ab} \xi + \frac{1}{4} \bar{\ell}_{ab} \gamma^{ab} \psi.$$

The action (106) corresponds to a four-dimensional supergravity action with a generalized supersymmetric cosmological term.

It is interesting to observe that an Inönü-Wigner contraction of the action (106) leads us to the $D = 4$ pure supergravity action. In fact after the rescaling

$$\omega_{ab} \to \omega_{ab}, \quad \bar{k}_{ab} \to \mu^2 \bar{k}_{ab}, \quad k_{ab} \to \mu^4 k_{ab},$$

$$e_a \to \mu^2 e_a, \quad \psi \to \mu \psi \quad \text{and} \quad \xi \to \mu^3 \xi,$$

and dividing the action by $\mu^4$, the $D = 4$ pure supergravity action is retrieved by taking the limit $\mu \to \infty$,

$$S = \alpha_4 \int \frac{1}{l^2} \left( \epsilon_{abcd} R^{ab} e^c e^d + 4 \bar{\psi} e^a \gamma_a \gamma_5 D \psi \right).$$

(107)

It was shown in refs. [20, 30] that the $N = 1$, $D = 4$ pure supergravity can be derived from the minimal Maxwell superalgebra $sM_4$. This result is not a surprise since the Inönü-Wigner contraction of the generalized minimal $AdS$–Lorentz superalgebra corresponds to the minimal Maxwell superalgebra $sM_4$.

Let us notice that the procedure considered here could be extended to bigger $AdS$–Lorentz superalgebras whose Inönü-Wigner contractions lead to the Maxwell superalgebras type defined in [29]. These Maxwell superalgebras correspond to the supersymmetric extension of the Maxwell algebras type introduced in refs. [19, 21].

5 Comments and possible developments

In the present work we have presented an alternative way to introduce the supersymmetric cosmological constant to supergravity. Based on the $AdS$–Lorentz superalgebra we have built the minimal $D = 4$ supergravity action which includes the generalized supersymmetric cosmological constant term. For this purpose we have considered the semigroup expansion method to the $osp(4|1)$ superalgebra allowing us to construct a MacDowell-Mansouri like action. The geometric formulation of the supergravity theory found here corresponds to a supersymmetric generalization of the results of refs. [4, 9].
We have also presented the most generalized supergravity action that contains a cosmological constant. For this aim we have introduced the generalized minimal $AdS$-Lorentz superalgebra using a bigger semigroup. This new superalgebra requires the introduction of a second Majorana spinorial generator $\Sigma$ leading to additional degrees of freedom. In particular we have shown that the Inönü-Wigner contraction of the generalized $AdS$-Lorentz superalgebra leads to the minimal Maxwell superalgebra.

Our results provide one more example of the advantage of the semigroup expansion method in the geometrical formulation of a supergravity theory. The approach presented here could be useful in order to analyze a possible extension to higher dimensions. Nevertheless, it seems that in odd dimensions the Chern-Simons (CS) theory is the appropriate formalism in order to construct a supergravity action. For instance, for $D = 3$ interesting CS (super)gravity theories are obtained using the $AdS$-Lorentz (super)symmetries [10, 18, 31].

On the other hand it would be interesting to study the extended supergravity in the geometrical formulation. A future work could be analyze the $N$-extended $AdS$-Lorentz superalgebra introduced in ref. [32] and the construction of $N$-extended supergravities. It seems that the semigroup expansion procedure used here could have an important role in the construction of matter-supergravity theories.

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