Computational technique of thermal comparative examination of Cu and Au nanoparticles suspended in sodium alginate as Sutterby nanofluid via extending PTSC surface

Wasim Jamshed1, Rabia Safdar2, Zulfiqar Rehman3, Maha M. A. Lashin4, Mohamed Ehab5, Mohamed Moussa6 and Aysha Rehman7

Abstract
Current research underscores entropy investigation in an infiltrating mode of Sutterby nanofluid (SNF) stream past a dramatically expanding flat plate that highlights Parabolic Trough Solar Collector (PTSC). Satisfactory likeness factors are utilized to change halfway differential conditions (PDEs) to nonlinear conventional differential conditions (ODEs) along with relating limit requirements. A productive Keller-box system is locked in to achieve approximated arrangement of decreased conventional differential conditions. In the review, two sorts of nanofluids including Copper-sodium alginate (Cu-SA) and Gold-sodium alginate (Au-SA) are dissected. Results are graphically plotted as well as talked about in actual viewpoints. As indicated by key discoveries, an improvement in Brinkmann, as well as Reynolds number, brings about expanding the general framework entropy. Sutterby nanofluid boundary improves heat rate in PTSC. Additionally, Copper-sodium alginate nanofluid is detected as a superior thermal conductor than Gold-sodium alginate nanofluid. Further to that, the reported breakthroughs are beneficial to updating extremely bright lighting bulbs, heating and cooling machinery, fiber required to generate light, power production, numerous boilers, and other similar technologies.

Keywords
Parabolic trough solar collector, Sutterby-nanofluid, inclined MHD, variable thermal conductivity, entropy generation

Introduction
Solar energy the radiant light and heat from the Sun are utilized by diverse technology which includes sun strength to generate energy, sun thermal strength to combine sun water heating, and sun structures. There are levels of sun strength generation, Solar Photovoltaic and Solar Thermal. Solar Photovoltaic (or PV) generation that converts day-light into semi-conductors. In contrast, Solar Thermal is a generation that makes use of sun strength to heat or generate energy. Solar strength, radiation from the solar, can generate heat, cause chemical reactions, or generate energy. The quantity of sun strength within side the global is a whole lot more than the modern and anticipated worldwide strength needs. Properly covered, it’ll face up to an excellent deal of detrimental conditions. Solar strength
sorts are a photovoltaic device, a photovoltaic device, and a PV device or sun device, an electrical device designed to offer usable sun strength the usage of photovoltaic. Solar water heating structures, sun water heating structures use sun strength to heat water for home or industrial purposes. Solar water structures gather warmth from the solar thru sun collectors. Solar strength is a shape of sun strength production; that is frequently utilized in industrial structures. As maximum folks know, many strengths vegetation use non-renewable fossil fuels to boil water, and sun thermal generation and herbal air flow generation percentage the identical working device. Solar energy is derived from daylight. Whether you recognize it or not, the solar is already placing on our planet, presenting a whole lot wished strength to preserve the surroundings and populace growth. The quantity of daylight accomplishing the earth’s ecosystem is sufficient to strength all our needs, and so on. Solar strength use is photovoltaic (PV), road lights, sun water pumps, commercial strength projects, Net metering projects, and sun production generation. While agricultural specialists are debating the legitimacy of herbal and nearby meals production, grocery stores throughout the U.S. have delivered natural and nearby meals portions. Consumers are shopping for natural and shopping for locally, and the call for those merchandise has grown exponentially in latest years. The integration of herbal and nearby meals actions has caused quite a few lines conveying the working fluid. Rabl due to the development of the sun during the day, concentrating gatherers frequently require some type of sun oriented global positioning framework, and are in some cases alluded to “dynamic” authorities for this reason. Non-concentrating authorities are normally utilized in private, modern and business structures for space warming, while at the same time amassing authorities in concentrated sun based power plants produce power by warming a hotness move liquid to drive a turbine associated with an electrical generator. Sreekumar et al. Evacuated level plate sun based authorities require both a glass-metal envelope and an interior construction to help such plate against climatic tension. The safeguard must be divided or given reasonable openings to oblige such construction. Joining of all parts must be high vacuum-tight and just
materials with low fume strain can be utilized to forestall outgassing. Glass-metal seal innovation can be founded either on metallized glass, or vitrified metal and characterizes the kind of authority. Not the same as emptied tube authorities, they utilize non-evaporable getter (NEG) siphons to keep the inner tension stable through time. This getter siphon innovation enjoys the benefit of giving some recovery in-situ by openness to daylight. Emptied level plate sun oriented gatherers have been read up for sun powered cool and contrasted with reduced sunlight based concentrators. Illustrative Trough Solar Collector (PTSC) is one of the best current and an employable gadget that changes sun based radiation into a hotness or helpful energy. One more kind of CSP is the illustrative box sun oriented assortment (PTSC), which has four key parts: authority, beneficiary, heat move liquid (HTF), and motor hotness. Jamshed et al. top to bottom warm examination done on sustainable SE in explanatory box sun oriented savior with the assistance of Maxwell nanofluid was given by Shahzad et al. Mwesigye et al. introduced the working of PTSC. Sign was done about curiously mindful apportion connected with PTSC framework. Examination, Modeling, reproduction as well as investigation is introduced in Yılmaz and Mwesigye. Enhancement was finished by Derakhshan and Khorasvian about the capacity arrangement of fluid air energy with illustrative box sun based finder power plant. More Researches connected with PTSC can be seen.

Nanofluid is a liquid that contains nanometer-sized particles, called nanoparticles. These fluid colloidal suspensions are made up of nanoparticles in basic liquid. Nanoparticles used in Nano fluids are usually made of metals, oxides, caribides, or carbon nanotubes. Different standard fluids have been used as an active fluid to transfer heat to various processes. As an active liquid, water is widely used due to its high availability, but it is not considered an effective heat carrier due to its lack of thermal conductivity. Non-exchange fluids, such as engine oil, ethylene glycol, etc., are also used in a variety of applications, but high viscosity and toxic environment limit the use of these compounds in heat transfer processes. A mixture of ethylene glycol and water used worldwide as a car coolant, the addition of nanoparticle enhances the engine cooling rate. Electronic cooling of integrated circuit and microprocessor circuits has increased in recent years, high-performance computers with a maximum power of 100–300 w/cm². The heat transfer of natural convection of Newtonian Nano fluid on the outer surface of the laminar boundary is investigated in a systematic manner. It has been found that the convection heat transfer of natural convection is not only characterized by the active Nano fluid of thermal conductivity and that sensitivity to the viscosity model used appears undeniable and plays an important role in heat transfer behavior. The speculation of as far as possible layer has exhibited very significant and has given unbelievable power to the examination of liquid equipment since the turn of the century. One of the principle occupations of minor theory is the assessment of pulling bodies on the stream, for instance pulling a level plate in a zero position, this way and that, airfoil, plane body, or turbine edge. In the current survey, the hotness moves and stream of pseudo-plastic non-Newtonian Nano fluid over the entering surface was settled where there was imbuenment and ingestion. By imbuenment and non-mixture plate, non-Newtonian Nano fluid shows better hotness move execution appeared differently in relation to Newtonian Nano fluid. Nevertheless, altering the sort of nanoparticles fundamentally influences the hotness move process during maintenance.

The Sutterby fluid model represents a highly purified polymer assembly and is one of the non-Newtonian-based liquids used to study the rheological properties of various materials. Revert the ODE framework using progress interactions. This program is mathematically approached with a unique shooting strategy. Boundaries of various limits related to velocity, temperature, and fixation are characterized. Speed, fixation, and temperature are determined by numbers. The results obtained show that the boundaries of the items slow down. Temperature and focus are improved by limitations. The study of the behavior of non-Newtonian nanofluids is mysterious and boring due to the detour relationship between endless horrors. This is because quite a few things that actually happen are not real and are not very similar. Misconstrued numbers are usually marked. Working on the line problem is very easy, but finding the answer to the sly problem is still a real challenge. However, the results obtained by mathematical methods give inconsistent emphasis when placed. In any case, it’s even harder to fully understand the roundabout problem. This adds to the complexity of the numbers, assuming that the wicked problem involves solidarity or has a large number of consensuses. While there are limits to mathematical and numerical methods for dealing with roundabout problems, they also have their advantages. Therefore, neither of these two strategies can be overlooked, but addressing sly problems through analysis is generally fun. In addition, punctured media are used to transfer and distribute energy in many modern structures such as heat pipes, powerful grid heat exchangers, electrical cooling, and synthetic reactors. A key factor in combining a fluid with a punctured area is convolution. This addresses an obstacle to the circulation of streams that are limited to near or neighborhood consistency.

Flow of fluid with inside the presence of appeal has wide collection of tasks in designing however scientific area. Hotness and mass transfer below appealing area has highly primary state of affairs in the ones areas. Attractive anxiety is finished to electrically doing beverages in MHD restriction layer streams. This area produces flows and makes a limiting Lorentz pressure on the fields. In distinct
liquid waft state of affairs impact of MHD is perceived, which have some tasks which includes strength generators, oil industry, MHD siphons, heat exchangers, appealing remedy targeted on, and so on. Due to its importance in magneto treatment, maximum malignant growths treatment, and a ton of revolutionary cycles, the restriction layer waft wherein the inclined appealing anxiety is done, can’t be disregarded. Hayat et al. explored appealing fields effects with tendency demeanor on heat transfer for low of Williamson liquid. Endalew and Nayak explored the effect of flimsy radiative waft on mass transfer with inclined MHD, implanted in a penetrable medium over adjusts plate. The MHD restriction layer move over a permeable extending surfaces have several programs under-way cycles, plasma studies, petrol ventures, MHD electricity generator, restriction layer oversee in stream-lined features, chilling of atomic reactors, valuable stone fiber assembling and paper fabricating. Numerous hypothetical and check examinations have analyzed diverse scientists. Animasaur researched MHD Casson liquid move with sights and non-instantly compound response. Megaheda focused on MHD Casson liquid over a porous extending sheet.

For fluid that are vital to grease hypothesis, the hotness created by inward grinding, and in this manner the comparing expansion in temperature, is reliable with the batch-eller and influences the consistency and warm conductivity of the liquid. Anyakoha and Meyers et al. Viscosity and warm conductivity are generally touchy to temperature increase. Expanded temperature prompts a development of the reach inside transport peculiarities in that the actual peculiarities of force lessen the thickness and the hotness move coefficient at the dividers is significantly more firmly impacted. Because of the above realities, the thickness and warm conductivity of a liquid are not consistent 100% of the time. Afify and Bazia embrace the Dybbs and Ling models of temperature-subordinate consistency and warm conductivity, individually, to impact the variable properties of nanofluids going through vertical plates on the regular convective material science. I checked. It was observed that an expansion in the variable consistency boundary prompted a diminishing in the Nusselt and Sherwood numbers, yet the contrary outcome was acquired for skin grinding. Jawaliet al. examined the joined impact of variable consistency and warm conductivity on the free convection of gooey liquids in vertical channels. He utilized Attia’s model for both temperature-subordinate thickness and warm conductivity. It has been observed that rising the variable consistency boundary increments liquid stream and hotness move, and expanding variable warm conductivity diminishes both hotness move and liquid stream. Bagai and Nishad utilize a mathematical technique (i.e. shooting strategy) to explore the impact of temperature-subordinate thickness on normal convective physical science streaming over an even plate implanted all through a permeable medium immersed with nanofluids. I did indeed. The thickness of a liquid is supposed to change dramatically with temperature. The warm conductivity is thought to be consistent and the radiation term is overlooked. It has been seen that hotness and mass exchange rates increment as the consistency boundary increments. contains new increases that consider regular nanofluids with warm and mass exchange under different states of being.

Entropy is a logical idea and quantifiable data often associated with states of disruption, disorder or vulnerability. Rudolf Clausius (1822–1888) is the author of the concept of entropy. Austrian physicist Ludwig Boltzmann described entropy as the fraction of the smallest conceivable number of frameworks or the atomic domains of indivi-dual 100% of the time. Afify and Bazia models of temperature-subordinate consistency and warm conductivity, individually, to impact the variable properties of nanofluids going through vertical plates on the regular convective material science. I checked. It was observed that an expansion in the variable consistency boundary prompted a diminishing in the Nusselt and Sherwood numbers, yet the contrary outcome was acquired for skin grinding. Jawaliet al. examined the joined impact of variable consistency and warm conductivity on the free convection of gooey liquids in vertical channels. He utilized Attia’s model for both temperature-subordinate thickness and warm conductivity. It has been observed that rising the variable consistency boundary increments liquid stream and hotness move, and expanding variable warm conductivity diminishes both hotness move and liquid stream. Bagai and Nishad utilize a mathematical technique (i.e. shooting strategy) to explore the impact of temperature-subordinate thickness on normal convective physical
instrument is dismissed, the plan is abbreviated, subsequently it would be not difficult to mathematically process. Nonetheless, the trouble in utilizing the model is that sure outcomes status is not the same as those acquired tentatively. For this model, the volume grouping of nanoparticles changes from 10% to 20%. Mathematical outcomes simply surmised the impacts in regards to Cu-SA and Au-SA nanofluids. Current examination intends to close the distinction by utilizing Keller-box based computational procedure. This depends on the impact of intense boundaries on liquid elements alongside Sutterby entropy inside the limit layer.

Flow model formulations

This segment will in general model the stream and warm aspects occupied with PTSC utilizing characterized nanofluids. Versatile level plate with non-ordinary extending speed is communicated as

\[ U_w(x, 0) = px, \]

In equation (1), \( p \) is signifying the underlying stretch rate. Temperature of the protection sheet is \( \Psi_w(x, 0) = \Psi_w + \rho^* x \) and for straightforwardness, it is expected that it is fixed at \( x = 0 \), where \( \rho^* \) is addressing temperature change rate, \( \Psi_w \) and \( \Psi_w \) separately represent the divider as well as natural temperature. Stream of nanofluid has elements like 2D, consistent, gooey in addition to incompressible. It is assumed that the levelness is tricky and the surface is exposed to a temperature inclination. The inside geometric PTSC is illustrated in Figure 1.

The diminished conditions from Bouslimi et al. were utilized for the progression of gooey Sutterby nanofluid by standard limit layer approximations including inclined MHD, thermal radiation, variable thermal conductivity, and inclined joule heating impacts which are outlined as follows:

\[ (L_1)_y + (L_2)_y = 0, \]

\[ L_1(L_1)_y + L_2(L_2)_y = \frac{\mu_{nf}}{\rho_{nf}} \left( L_1 c_{nf} \right)_y \left( 1 - \frac{n b^2}{2} (L_1)_y \right) \]

\[ - \frac{\sigma_{nf} B^2}{\rho_{nf}} \sin^2(\sigma) L_1, \]

\[ L_1 \Psi_y + L_2 \Psi_y = \frac{1}{\rho_{nf} (C_p)_nf} \left( \kappa_{nf}(\Psi) \Psi_y \right)_y - \frac{1}{\rho_{nf} (C_p)_nf} (q_y)_y + \frac{\mu_{nf}}{\rho_{nf} (C_p)_nf} (L_1)_y^2 + \frac{\sigma_{nf} B^2}{\rho_{nf} (C_p)_nf} \sin^2(\sigma) L_1 \]

the relevant boundary conditions are

\[ L_1(x, 0) = U_w + N_\Omega (L_1), L_2(x, 0) = V_\Omega, -\kappa_\Omega \left( \Psi_y \right)_y \]

\[ = h_\Omega (\Psi_w - \Psi) \]

\[ L_1 \to 0, \Psi \to \Psi_w as y \to \infty, \]

where the stream speed vector is \( \bar{L} = [L_1(x, y), L_2(x, y), 0] \). \( \Psi \) images a nanofluid temperature. The vulnerability of the broadening levelness is indicated as \( V_{\Omega} \). The slip length indicates by \( N_\Omega \). The attractive field strength indicates by \( \sigma \). The additional variables like strong warm conductance and hotness transport factor are represented by \( \kappa_\Omega \) and \( h_\Omega \) individually.

Table 1 summarizes the material characteristics of Sutterby nanofluid. The additional properties \( \rho_s \), \( (C_p)_s \), and \( \kappa_s \) are the nanoparticle density, actual heat capacitance, and thermal conductance, correspondingly. \( \kappa_{nf}(\Psi) \) represents the temperature dependent thermal conductivity and \( \epsilon \) is its contact. The physical characteristics of the standard liquid sodium alginat and diverse nanoparticle employed in the current research are given in Table 2 (see for details).

In the case of Sutterby’s non-Newtonian nanofluids, the radiation propagates only a small distance due to the fluid’s thickness. On account of this phenomenon, we use the Rosseland approximation in the equation (4) for radiation to obtain

\[ q_y = \frac{-4\sigma^*}{3k^2} \Psi^4_y, \]
Here \( \sigma^* \) is the number of Stefan-Boltzmann and \( k^* \) is the mean-absorption coefficient.

### Problem resolution

The boundary layer equations (2) to (6) have been transformed through a similarity process that renovates PDEs to ODEs. Using the \( \psi \) flow functions of the form is given as

\[
\frac{D_1}{\psi_y} = \psi_x = 0
\]

and similarity variables as \( \lambda = \frac{b}{V_f} y, \psi(x, y) = \sqrt{V_f b} x f(\lambda), \theta(\lambda) = \frac{\psi - \psi_\infty}{\psi_y - \psi_\infty} \)

into equations (2) to (6). We get

\[
\phi_1 = (1 - \phi)^2, \phi_2 = \left( 1 - \phi + \frac{\rho_f}{\rho} \right), \phi_3 = \left( 1 - \phi + \phi \frac{(\rho \mathcal{C}_p)_{nf}}{(\rho \mathcal{C}_p)_f} \right),
\]

\[
\phi_4 = \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)}, \phi_5 = \frac{3(\frac{\sigma}{\sigma_f} - 1)\phi}{(\frac{\sigma}{\sigma_f} + 2) - (\frac{\sigma}{\sigma_f} - 1)\phi}
\]

### Table 1. Thermophysical properties for Sutterby nanofluid.

| Features          | Nanofluid                      |
|-------------------|--------------------------------|
| Dynamical viscidness (\( \mu \)) | \( \mu_{nf} = \mu_f (1 - \phi)^{-2} \) |
| Density (\( \rho \))      | \( \rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s \) |
| Heat capacity (\( \rho \mathcal{C}_p \)) | \( (\rho \mathcal{C}_p)_{nf} = (1 - \phi)(\rho \mathcal{C}_p)_f + \phi(\rho \mathcal{C}_p)_s \) |
| Thermal conductivity (\( \kappa \)) | \( \frac{\kappa_{nf}}{\kappa_f} = \left( \frac{k_s + 2k_f}{k_s + 2k_f} + \phi(k_f - k_s) \right) \) |
| Electrical conductivity (\( \sigma \)) | \( \frac{\sigma_{nf}}{\sigma_f} = \left[ \frac{\sigma_f}{\sigma} + (\frac{\sigma_f}{\sigma} - 1)\phi \right] \) |
| Variable thermal conductivity (\( \kappa^* \)) | \( \kappa^*_\psi = k_{nf} \left[ 1 + \epsilon \frac{\psi - \psi_\infty}{\psi_y - \psi_\infty} \right] \)

### Table 2. Standard values of nanoliquid and solid-particles thermal characteristics at 293 K.

| Thermophysical | \( \rho \) (kg/m\(^3\)) | \( \mathcal{C}_p \) (J/kgK) | \( k \) (W/mK) | \( \sigma \) (S/m) |
|---------------|----------------------|-----------------|-----------------|---------------|
| Copper (Cu)   | 8933                 | 385.0           | 401.00          | 5.96 \times 10^7 |
| Sodium alginate (SA) | 989                  | 4175            | 0.6376          | 2.6 \times 10^{-4} |
| Gold (Au)    | 19,300               |                 | 318             | 4.1 \times 10^6  |

Herein \( \sigma^* \) is the number of Stefan-Boltzmann and \( k^* \) is the mean-absorption coefficient.

\[
f^{\prime \prime \prime \prime} + 2f^{\prime \prime \prime} - f^{\prime \prime} - \Phi_1 f^{\prime \prime} - 2f^{\prime \prime} f^{\prime \prime} f^{\prime} - \Phi_1 f^{\prime \prime} f^{\prime} = 0
\]

\[
\theta^*(1 + \epsilon \theta + P, N) \left[ f^{\prime \prime} f^{\prime \prime} f^{\prime} + \Phi_1 f^{\prime \prime} f^{\prime} + \Phi_2 f^{\prime \prime} f^{\prime} \right] = 0
\]

with

\[
f(0) = S, f^{\prime}(0) = 1 + \Lambda f^{\prime}(0), \theta^{\prime}(0) = -B_0 (1 - \theta(0)) \]

\[
f^\prime (\lambda) \rightarrow 0, \theta (\lambda) \rightarrow 0, \text{as} \lambda \rightarrow \infty.
\]
It is remarked that equation (8) is immediately verified. In overhead equations, ′ takes differentiation w.r.t \( \chi \). Deborah number and magnetic field are specified as \( G_\Omega = B^2 \rho \) and 

\[
M_\Omega = \frac{\sigma_f B^2}{C_p f}
\]

respectively. \( P_r = \frac{v_f}{\alpha_f} \) signifies the number of Prandtl. The diffusion parameter, mass-transport, and radiative flow parameters are specified as 

\[
S = -V_\Omega \sqrt{\frac{1}{v_f}}, \quad N_\Omega = \frac{16}{3} \frac{\sigma \Psi_x}{\kappa v_f (\rho C_p)_f}
\]

respectively. 

\[
\Lambda_\Omega = \sqrt{N_\Omega}, \quad E_\Omega = \frac{b}{(C_p)_f (V_w - V_x)}\n\]

is the Eckert number, and 

\[
B_\Omega = \frac{b}{\Omega} \sqrt{\frac{v_f}{b}}\n\]

symbols the Biot amount.

After applying the non-dimensional transformations equation (16) on reduction drag force \( C_f \), Nusselt amount \( (Nu_e) \), and entropy generation \( (N_G) \) the subsequent equations are obtained

\[
C_f Re_x = \left[ \frac{1}{\phi_f} \left( f^* + \frac{n}{3} G_\Omega M_\Omega f^* \right) - N_\Omega \right] Re_x \frac{v_x}{v_f} = -\frac{k_w}{k_f} (1 + N_\Omega) \theta'(0), \quad \text{(14)}
\]

\[
N_G = R_\beta \left[ \frac{\phi_f}{\phi_m} \left( f^{*2} + \phi_f M_\Omega \sin^2(\sigma) f^{*2} \right) \right], \quad \text{(15)}
\]

Where \( Re_x = \frac{U_w x}{v_f} \) is the local-Reynolds amount.

**Numerical process: Keller box method**

The Keller-box strategy (KBM)\(^8^1\) is utilized to decide the mathematical answer for displayed conditions. On account of its quick combination, it is preferred over numerous different methodologies. The Keller box approach is naturally steady and united to the subsequent request. It finishes the Von Neumann steadiness assessment, which lays out the standard for mathematical arrangement union as far as certifiable PDE arrangements while tending to consistency and mathematical arrangement soundness. The understanding of KBM, which is one of the fundamental methodologies for getting estimated answers for limit three-layer issues, yields a restricted arrangement of articulations (10–11) dependent upon requirements (12). KBM offers a wide assortment of utilizations in laminar limit layer streams and delivers more effective outcomes than different methodologies. In Figure 2, the Keller box approach incorporates the accompanying advances:

The preliminary stage needs to shift all the ODEs (14) to (16) into first order ODEs, that is

\[
q_1 = f', \quad q_2 = c_1', \quad q_3 = \theta', \quad \text{(16)}
\]

\[
q_1^2 + 2\phi\phi_2 \left[ f q_2 - q_1^2 \right] - \frac{n}{2} G_\Omega q_2^2 q_2' - 2\phi M_\Omega \sin^2(\sigma)q_1 = 0, \quad \text{(19)}
\]
\[ q_3 = \left(1 + \epsilon \frac{1}{\phi_3} P_f N_{\Omega} \right) \]
\[ + \frac{\phi_3}{\phi_4} \left[ f q_3 - q_1 \theta + \epsilon q_2^2 + \frac{E_{\Omega} q_2}{\phi_3} \right] = 0. \] (20)

\[ f(0) = S, \quad q_1(0) = 1 = \Lambda_{\Omega} q_2(0), \quad q_3(0) \]
\[ = -B_{\Omega}(1 - \theta(0)), \quad q_1(\infty) \to 0, \quad \theta(\infty) \to 0. \] (21)

**Authentication of code**

The rightness of the mathematical method was estimated by correlation the results on the heat transmission rate from the present methodology against this outcomes got underway.\(^82,^83\) Table 3 sums up the correlation of simultaneous current assessment through the past works.

**Analysis of results**

Our examination is based on the mathematical outcomes provided by the system introduced in the previous segment. This segment makes sense of the impacts of a few expected factors, like \( G_{\Omega}, M_{\Omega}, \phi, \epsilon, \Lambda_{\Omega}, N_{\Omega}, E_{\Omega}, B_{\Omega}, S, \sigma, R_{\Omega}, \) and \( B_f \). The actual way of behaving of various boundaries, like stream speed, temperature, and entropy, is addressed in Figures 3 to 17, contingent upon the aforementioned factors. The discoveries concern non-Newtonian Cu-SA and Au-SA Sutterby nanofluids. The actual qualities for the skin contact coefficient and temperature change are displayed in Table 4. The accompanying qualities have been doled out to the potential variables: \( G_{\Omega} = 0.1, \quad M_{\Omega} = 0.6, \quad \sigma = \pi / 2, \phi = 0.18, \quad \epsilon = 0.1, \Lambda_{\Omega} = 0.3, \quad P_f = 6.5, \quad N_{\Omega} = 0.3, \quad B_{\Omega} = 0.1, \quad E_{\Omega} = 0.1, S = 0.1, \quad R_{\Omega} = 5, \) and \( B_f = 5 \).

Figures 3 and 4 show how nanoparticle fixation \( \phi \) influences fluid movement and temperature dispersion. The speed drops as \( \phi \) boundary increments, decreasing the fluid motion’s limit layer thickness. The thickness of the liquid increments as the centralization of nanomolecules increments, and thus, the movement limit layer develops more slender. This is on the grounds that the volume part of nanoparticles initiates an expansion in liquid temperature. Because of the expanded warm conductance, a pattern for a bringing down speed limit layer can be taken note. Nonetheless, when the amount of nanomolecules develops, the thermal conductance of nanofluids lifts, and this influences nanofluid temperatures. Table 4 shows the component \( \phi \) related quickness and warm changes close to the limit. As indicated by Figure 5, the entropy of the framework increments as the boundary \( \phi \) increments. Following the evaluation of the qualities alluded to in Table 4 for boundary \( \phi \), the similar extent of hotness move rate is improved. Figures 6 and 7 show the effect of the \( G_{\Omega} \) boundary on the stream and temperature profiles, separately. \( G_{\Omega} = 0.1, 0.3, 0.5 \) was utilized in the estimations. Sutterby nanofluid in light of sodium alginate that is non-Newtonian. The speed profile diminishes as \( G_{\Omega} \) increments, owing for the most part to an abatement in stimulus limit layer thickening. How much protection from which the liquid is oppressed lessens its speed. An expansion in the flexibility stress component can be displayed to fortify the warm limit layer. At the point when the driving force limit layers of the nanofluid Cu-SA and Au-SA are analyzed in Figure 6, the previous is more apparent than the last option. Nusselt’s number for Cu-SA and Au-SA drops in this situation. At the point when the degrees of \( G_{\Omega} \) increment the entropy of the framework expands (Figure 8). Whenever the \( G_{\Omega} \) values in Table 4 are expanded, the similar extent of hotness move rate increments. The inclination angle \( \sigma \) sway on speed, temperature, and entropy fields are shown in Figures 9 to 11, individually. A decrease in tendency point with respect to attractive field gives fantastic protection from the stream, diminishing the speed (Figure 9), and expanding the temperature (Figure 10). The impact of \( \sigma \) on entropy is depicted in Figure 11 a positive tendency in regards to attractive field works on the entropy of the framework. Truly, an increase in \( \sigma \) of the attractive field fosters a power, to be specific Lorentz force, which intensifies energy scattering on the grounds that the effect on the framework’s entropy is articulated. Individual variation in thermal conductivity \( \epsilon \) appears to be dominated in both thermal and entropy aspects when compared to the thermal ability of nano and hybrid combinations. The influence of the variable thermal conductivity parameter \( \epsilon \) on the thermal state and entropy generation is depicted in both Figures 12 and 13. Despite the fact that the changing parameter tends to raise the thermal and entropy ranges, the narrower thermal layers and closer entropy fluctuation likely to demonstrate the nominal impact of \( \epsilon \). Both of these parametric behaviors are Cu-SA nanofluid underplays the Au-SA nano nanofluid. Figure 14 depicts the temperature bend for different upsides of the dimensionless radiation boundary \( N_{\Omega} \). We see that rising the radiation boundary \( N_{\Omega} \) causes an expansion in the mean stream temperature profile. As the hotness created \( N_{\Omega} \) develops, the temperature profiles of nanofluids improve genuinely. Notwithstanding expanding the thermal radiation boundary, the radiative transition

**Table 3. Comparing values of heat transmission rate \(-\theta'(0)\)**

| \( P_f \) | Makinde and Aziz\(^82\) | Abu-Hamdeh et al.\(^83\) | Present |
|---------|--------------------------|--------------------------|--------|
| 0.72    | 0.1691                   | 0.1690                   | 0.1690 |
| 1.00    | 0.4539                   | 0.4537                   | 0.4537 |
| 3.00    | 0.9114                   | 0.9113                   | 0.9113 |
| 7.00    | 1.8954                   | 1.8958                   | 1.8958 |
drives the nano polymeric wave, which adds nuclear power to the interaction. The line layer is supported due to this glow. The discoveries of changes in temperature dissemi-
nations under fluctuated amounts of radiative boundary $N\beta$ are displayed in Figure 14. It was found that when $N\Omega$ develops, so does the mean stream energy profile. Despite the fact that the warm radiation boundary is expanded, the radiative transition drives the nano-polymeric wave, adding nuclear power to the cycle. The limit layer thickens as the temperature increases. The impact of entropy creation
Figure 5. Entropy variation on various $\phi$.

Figure 6. Velocity variation on various $G_\Omega$. 
Figure 7. Temperature variation on various $G_{\Omega}$.

Table 4. Calculation of $\overline{C_{r}Re_{\omega}^{\frac{1}{2}}}$ and $\overline{Nu_{r}Re_{\omega}^{\frac{1}{2}}}$ for $Pr = 6.5$.

| $G_{\Omega}$ | $\phi$ | $\epsilon$ | $B_{x}$ | $S_{x}$ | $\overline{C_{r}Re_{\omega}^{\frac{1}{2}}}$ Cu-SA | $\overline{C_{r}Re_{\omega}^{\frac{1}{2}}}$ Au-SA | $\overline{Nu_{r}Re_{\omega}^{\frac{1}{2}}}$ Cu-SA | $\overline{Nu_{r}Re_{\omega}^{\frac{1}{2}}}$ Au-SA |
|-------------|--------|------------|--------|--------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
| 0.1         | $\pi/2$| 0.18       | 0.3    | 0.1    | 0.1                                           | 2.3112                                        | 2.0587                                        | 1.1324                                        | 1.0825                                        |
| 0.3         |        |            |        |        | 2.2965                                        | 2.0360                                        | 1.1124                                        | 1.0684                                        |
| 0.5         |        |            |        |        | 2.2754                                        | 2.0184                                        | 1.0964                                        | 1.0492                                        |
| $\pi/6$     |        |            |        |        | 2.2713                                        | 2.0125                                        | 1.1710                                        | 1.1399                                        |
| $\pi/4$     |        |            |        |        | 2.2940                                        | 2.0323                                        | 1.1541                                        | 1.1083                                        |
| $\pi/2$     |        |            |        |        | 2.3112                                        | 2.0587                                        | 1.1324                                        | 1.0825                                        |
| $\pi/2$     | 0.1    |            |        |        | 2.2539                                        | 2.0092                                        | 1.0821                                        | 1.0339                                        |
| $\pi/2$     | 0.15   |            |        |        | 2.2854                                        | 2.0459                                        | 1.1136                                        | 1.0626                                        |
| $\pi/2$     | 0.18   |            |        |        | 2.3112                                        | 2.0587                                        | 1.1324                                        | 1.0825                                        |
| $\pi/2$     | 0.1    |            |        |        | 2.3644                                        | 2.1136                                        | 1.1838                                        | 1.1255                                        |
| $\pi/2$     | 0.2    |            |        |        | 2.3421                                        | 2.0859                                        | 1.1519                                        | 1.1034                                        |
| $\pi/2$     | 0.3    |            |        |        | 2.3112                                        | 2.0587                                        | 1.1324                                        | 1.0825                                        |
| $\pi/2$     | 0.1    |            |        |        | 2.3421                                        | 2.0859                                        | 1.1519                                        | 1.1034                                        |
| $\pi/2$     | 0.4    |            |        |        | 2.3112                                        | 2.0587                                        | 1.1324                                        | 1.0825                                        |
| $\pi/2$     | 0.7    |            |        |        | 2.3112                                        | 2.0587                                        | 1.1324                                        | 1.0825                                        |
| $\pi/2$     | 0.1    |            |        |        | 2.3421                                        | 2.0859                                        | 1.1519                                        | 1.1034                                        |
| $\pi/2$     | 0.5    |            |        |        | 2.3112                                        | 2.0587                                        | 1.1324                                        | 1.0825                                        |
| $\pi/2$     | 0.7    |            |        |        | 2.3112                                        | 2.0587                                        | 1.1324                                        | 1.0825                                        |
| $\pi/2$     | 0.1    |            |        |        | 2.3112                                        | 2.0587                                        | 1.1324                                        | 1.0825                                        |
| $\pi/2$     | 0.3    |            |        |        | 2.3112                                        | 2.0587                                        | 1.1324                                        | 1.0825                                        |
| $\pi/2$     | 0.6    |            |        |        | 2.3112                                        | 2.0587                                        | 1.1324                                        | 1.0825                                        |
Figure 8. Entropy variation on various $G_\Omega$.

Figure 9. Velocity variation on various $\varpi$. 
Figure 10. Temperature variation on various $\omega$.

Figure 11. Entropy variation on various $\omega$. 

$\omega = \pi/6, \pi/4, \pi/2$
Figure 12. Temperature variations on various $\varepsilon$.

Figure 13. Entropy variations on various $\varepsilon$. 
on the nanofluid movement of the radiation boundary $N_\Omega$ is portrayed in Figure 15 for the two kinds of Cu-SA and Au-SA nanoparticles. It is examined that rising measures of $N_\Omega$ cause a nonstop expansion in the entropy circulation. The radiation boundary firmly affected the entropy rate in the extended permeable gadget, as found in Figure 13. The reaction in Reynolds number $R_\Omega$ on the entropy creation profiles in the extended limit layer region is portrayed in Figure 16. It tends to be shown that rising the Reynolds number $R_\Omega$ expands the entropy impact. More prominent upsides of Reynolds number $R_\Omega$ cause retrogression of frictional powers and will more often than not not gasify the entropy profiles. Figure 17 shows the vacillation in the entropy age rate against the upsides of the Brinkman number $B_r$, demonstrating an expansion in entropy age by expanding the Brinkman number $B_r$. Due to the real world and such way of behaving, the Brinkman number $B_r$ researches the gooey impact of the liquid. Subsequently, enormous Brinkman numbers $B_r$ exhibit prevailing impacts of liquid rubbing, which is a huge wellspring of entropy development. The graphical way of behaving of $N_G$ versus important elements (Figures 16 and 17) shows that it is more delicate to variety at the surface than away from it.

Cu and Au nanomolecules of steady size when contrasted with Au-SA nanofluid, Cu-SA nanoliquid has a more noteworthy heat bandwidth. Cu works on warm conduction by expanding the liquid’s thermal conductance since it is a superior mode for heat move in a nanofluid than Au. This conduct is recommended for frameworks where heat transmission is basic. This is exhibited by the general $\text{Nu}_e$, inferred for different actual boundaries. Table 4 delineates the repercussions for the perusers.

**Final remarks**

Computational examinations of limit layer flow for Cu and Ag sodium alginate-based nanofluids are performed through a permeable growing surface in PTSC using Sutterby model, which is a basic model to mimic the viscoelastic shear diminishing elements of non-Newtonian nanofluids. Examination was made under the successful presence of inclined MHD, viscous dissipation, variable thermal conductivity, and inclined joule heating along warm radiative stream with the guide of Keller-box procedure. The outline of acquired perceptions is as per the following: Speed is diminished with expanding impacts of non-Newtonian shear diminishing Sutterby boundary $G_\Omega$ as well as the volumetric size of nanoparticles $\phi$. Temperature is upgraded with $G_\Omega, M_\Omega, \phi, E_\Omega, B_\Omega$, and $N_\Omega$. Sutterby nano-fluid boundary improves heat rate in PTSC. Additionally, Copper-sodium alginate nanofluid is detected as a superior thermal conductor than Gold-sodium alginate nanofluid. Entropy is raised with $G_\Omega, M_\Omega, \phi, E_\Omega, B_\Omega, N_\Omega, B_r$, and $R_\Omega$ however a decrease is gotten with $\Lambda_\beta$, which works on the productivity of PTSC too.
Figure 15. Entropy variations on various $N_\Omega$.

Figure 16. Entropy variations on various $R_\Omega$. 
Future scope

Aftereffects of the examination can be a reference for future explores in which warm execution of PTSC should be possible by different types of non-Newtonian nanoliquids (e.g. Casson, second grade, Carreau, Maxwell, micropolar nanofluids, and so on). Moreover, conditions can be summed up to incorporate impacts in regards to thickness, porosity in view of temperature as well as multi-faceted slip magneto-transition. The Keller box technique could be applied to a variety of physical and technical challenges in the future.84–94

Acknowledgements

This research was funded by the Princess Nourah bint Abdulrahman University Researchers Supporting Project number (PNURSP2022R152), Princess Nourah bint Abdulrahman University, Riyadh, Saudi Arabia.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This research was funded by the Princess Nourah bint Abdulrahman University Researchers Supporting Project number (PNURSP2022R152), Princess Nourah bint Abdulrahman University, Riyadh, Saudi Arabia.

ORCID iD

Wasim Jamshed https://orcid.org/0000-0001-9438-6132

References

1. Kannan N and Vakeesan D. Solar energy for future world: a review. Renew Sustain Energ Rev 2016; 62: 1092–1105.
2. Fu R, Feldman DJ and Margolis RM. US solar photovoltaic system cost benchmark: Q1 (No. NREL/TP-6A20-72399). National Renewable Energy Lab. (NREL), Golden, CO, USA, 2018.
3. International Energy Agency. Archived from the original (PDF) on 13 January 2012, 2011.
4. Zarza E, Rojas ME, González L, Caballero JM and Rueda F. INDITEP: the first pre-commercial DSG solar power plant. Sol Energy 2006; 80(10): 1270–1276.
5. Chan HY, Riffat SB and Zhu J. Review of passive solar heating and cooling technologies. Renew Sustain Energ Rev 2010; 14(2): 781–789.
6. Schulze TF and Schmidt TW. Photochemical transformation: the current state and prospects for its use in converting solar energy. Energy Environ Sci 2015; 8(1): 103–125.
7. Bahnemann D. Photocatalytic water treatment: solar energy applications. Sol Energy 2004; 77(5): 445–459.
8. Meinel AB and Meinel MP. Applied solar energy: an introduction. NASA STI/Recon technical report A, 77, 1977, p.33445.
9. Ohunakin OS, Adaramola MS, Oyewola OM and Fagbenle RO. Solar energy applications and development in Nigeria: drivers and barriers. Renew Sustain Energ Rev 2014; 32: 294–301.
10. Reddy RG. Molten salts: thermal energy storage and heat transfer media. *J Phase Equilibria Diffus* 2011; 32: 269–270.
11. Norton B. *Harnessing solar heat*. Dordrecht: Springer Science + Business Media, 2013.
12. Rabl A. *Active solar collectors and their applications*. New York: Oxford University Press, 1983.
13. Sreekumar S, Joseph A, Sujith Kumar CS and Thomas S. Investigation on influence of antimony tin oxide/silver nanofluid on direct absorption parabolic solar collector. *J Clean Prod* 2020; 249: 588–601.
14. Mosk RW, Henshall P, Arya F, Shire GS, Hyde T and Eames A. *The SRB solar thermal panel*. *Europhys News* 2013; 44(3): 16–18.
15. DIN CERTCO. Register-Nr. 011-7S1890 F, https://www.dincertco.tuv.com/registrations/60081291 (2019).
16. Evacuable flat panel solar collector, https://patentimages.storage.googleapis.com/c1/93/5b/7266f66b605c84/US20070039611A1.pdf (2004).
17. Vacuum solar thermal panel with a vacuum-tight glass-metal sealing, https://patentimages.storage.googleapis.com/2d/1c/0d/5e6d8a9c030717/WO2010003653A3.pdf (2009).
18. Buonomano A, Calise F, d’Accadia MD, et al. Experimental analysis and dynamic simulation of a novel high-temperature solar cooling system. *Energy Convers Manag* 2016; 109: 19–39.
19. Jamshed W, Nasir NAAM, Isa SSPM, et al. Thermal growth in solar water pump using Prandtl-Eyring hybrid nanofluid: a solar energy application. *Sci Rep* 2021; 11(1): 18704.
20. Shahzad F, Jamshed W, Sathyanarayanan SUD, Aissa A, Madheshwaran P and Mourad A. Thermal analysis on Darcy-Forchheimer swirling Casson hybrid nanofluid flow inside parallel plates in parabolic trough solar collector: an application to solar aircraft. *Int J Energy Res* 2021; 45: 20812–20834.
21. Mwesigye A, Yilmaz IH and Meyer JP. Numerical analysis of the thermal and thermodynamic performance of a parabolic trough solar collector using SWCNTs-Therminol® VP-1 nanofluid. *Renew Energy* 2018; 119: 844–862.
22. Yilmaz IH and Mwesigye A. Modeling, simulation and performance analysis of parabolic trough solar collectors: a comprehensive review. *Appl Energy* 2018; 225: 135–174.
23. Derakhshan S and Khosravian M. Exergy optimization of a novel combination of a liquid air energy storage system and a parabolic trough solar collector power plant. *J Energy Resour Technol* 2019; 141: 081901.
24. Jamshed W, Eid MR, Azeany Mohd Nasir NA, et al. Thermal examination of renewable solar energy in parabolic trough solar collector utilizing Maxwell nanofluid: a noble case study. *Case Stud Therm Eng* 2021; 27: 101258.
25. Parvin S, Isa SSPM, Jamshed W, Ibrahim RW and Nisar KS. Numerical treatment of 2D-magneto double-diffusive convection flow of a Maxwell nanofluid: heat transport case study. *Case Stud Therm Eng* 2021; 28: 101383.
26. Sajid T, Jamshed W, Shahzad F, et al. Study on heat transfer aspects of solar aircraft wings for the case of Reiner-Philipppoff hybrid nanofluid past a parabolic trough: Keller box method. *Phys Scr* 2021; 96: 095220.
27. Pozhar LA. Structure and dynamics of nanofluids: theory and simulations to calculate viscosity. *Phys Rev E* 2000; 61(2): 1432–1446.
28. Raffat M, Hamidi AA and Shariati Niaer M. Application of nanofluids in computer cooling systems (heat transfer performance of nanofluids). *Appl Therm Eng* 2012; 45–46: 9–14.
29. Polidori G, Fohanno S and Nguyen CT. A note on heat transfer modelling of Newtonian nanofluids in laminar free convection. *Int J Therm Sci* 2007; 46(8): 739–744.
30. Huminic G and Huminic A. Application of nanofluids in heat exchangers: a review. *Renew Sustain Energ Rev* 2012; 16(8): 5625–5638.
31. Schlichting H and Gersten K. *Boundary-layer theory*. Berlin: Springer Science & Business Media, 2003.
32. Akram J, Akbar NS and Tripathi D. Blood-based graphene oxide nanofluid flow through capillary in the presence of electromagnetic fields: a Sutterby fluid model. *Microvasc Res* 2020; 132: 104062.
33. Hayat T, Ahmad S, Ijaz Khan M and Alsaeedi A. Modeling chemically reactive flow of Sutterby fluid nanofluid by a rotating disk in presence of heat generation/absorption. *Commun Theor Phys* 2018; 69(5): 569.
34. Khan WA, Ali M, Waqas M, Shahzad M, Sultan F and Irfan M. Importance of convective heat transfer in flow of non-Newtonian nanofluid featuring Brownian and thermophoretic diffusions. *Int J Numer Methods Heat Fluid Flow* 2019; 29: 4624–4641.
35. Crochet MJ and Walters K. Numerical methods in non-Newtonian fluid mechanics. *Annu Rev Fluid Mech* 1983; 15: 241–260.
36. Agrawal P, Dadheech PK, Jat RN, Bohra M, Nisar KS and Khan I. Lie similarity analysis of MHD flow past a stretching surface embedded in porous medium along with imposed heat source/sink and variable viscosity. *J Mater Res Technol* 2020; 9(5): 10045–10053.
37. Jat RN, Agrawal P and Dadheech K. MHD boundary layer flow and heat transfer of Caisson fluid over a moving porous plate with viscous dissipation and thermal radiation effects. *J R Acad Phys Sci* 2017; 16(3–4): 211–232.
38. Ibrahim W and Negera M. The investigation of MHD Williamson nanofluid over stretching cylinder with the effect of activation energy. *Adv Math Phys* 2020; 2020: 1–16.
39. Saranya S and Al-Mdallal QM. Non-Newtonian ferrofluid flow over an unsteady contracting cylinder under the influence of aligned magnetic field. *Case Stud Therm Eng* 2020; 21: 100679.
40. Hayat T, Bibi S, Rafiq M, Alsaeedi A and Abbasi FM. Effect of an inclined magnetic field on peristaltic flow of Williamson fluid in an inclined channel with convective conditions. *J Magn Magn Mater* 2016; 401: 733–745.
41. Endalew MF and Taye K. Thermal radiation and inclined magnetic field effects on MHD flow past a linearly accelerated inclined plate in a porous medium with variable temperature. *Heat Transf Asian Res* 2019; 48: 42–61.
42. Animasaun IL. Effects of thermophoresis, variable viscosity and thermal conductivity on free convective heat and mass transfer of non-Darcian MHD dissipative Casson fluid flow.
with suction and nth order of chemical reaction. *J Nigerian Math Soc* 2015; 34: 11–31.
44. Megahed AM. MHD viscous Casson fluid flow and heat transfer with second-order slip velocity and thermal slip over a permeable stretching sheet in the presence of internal heat generation/absorption and thermal radiation. *Eur Phys J Plus* 2015; 130: 81.
45. Batchelor GK. *An introduction to fluid mechanics*. London: Cambridge University Press, 1987.
46. Umavathi JC, Chamkha AJ and Mohiuddin S. Combined effect of variable viscosity and thermal conductivity on free convection flow of a viscous fluid in a vertical channel. *Int J Numer Methods Heat Fluid Flow* 2016; 26: 18–39.
47. Anyakoha MW. *New school physics*. 3rd ed. Enugu: Africana First Publisher PLC, 2010. pp.36–51.
48. Myers TG, Charpin JP and Tshehla MS. The flow of a variable viscosity fluid between parallel plates with shear heating. *Appl Math Model* 2006; 30: 799–815.
49. Afify AA and Bazid MA. Effect of variable fluid properties on the natural convection boundary layer flow of nanofluid past a vertical plate. *J Comput Theor Nanosci* 2014; 11: 210–218.
50. Dybb A and Ling JX. Force convection over a flat plate submerged in a porous medium: variable viscosity case. In: *SME, paper 87-WA/HF-23*, ASME winter annual meeting, Boston, MA, 1987.
51. Chiam TC. Heat transfer in a fluid with variable thermal conductivity over a linearly stretching sheet. *Acta Mech* 1998; 129: 63–72.
52. Attia HA. Unsteady hydromagnetic channel flow of dusty fluid with temperature dependent viscosity and thermal conductivity. *Heat Mass Transf* 2006; 42: 779–787.
53. Bagai S and Nishad MA. Effect of temperature dependent viscosity on natural convective boundary layer flow over a horizontal plate embedded in a nanofluid saturated porous medium. In: *5th international conference on porous media and their applications in science, engineering and industry*, 2014.
54. Song YQ, Ali Khan S, Imran M, et al. Applications of modified Darcy law and nonlinear thermal radiation in bioconvection flow of micropolar nanofluid over an off centered rotating disk. *Alex Eng J* 2021; 60(5): 4607–4618.
55. Waqas H, Farooq U, Khan SA, Alshehri HM and Goodarzi M. Numerical analysis of dual variable of conductivity in bioconvection flow of Carreau–Yasuda nanofluid containing gyrotactic motile microorganisms over a porous medium. *J Therm Anal Calorim* 2021; 145(4): 2033–2044.
56. Ahmad F, Waqas H, Ayed H, et al. Numerical treatment with Lobatto-IIIa scheme magneto-thermo-natural convection flow of Casson nanofluid (MoS2–Cu/SA) configured by a stretching cylinder in porous medium with multiple slips. *Case Stud Therm Eng* 2021; 26: 101132.
57. Khan SA, Waqas H, Naqvi SMRS, Alghamdi M and Al-Mdallal Q. Cattaneo-Christov double diffusions theories with bio-convection in nanofluid flow to enhance the efficiency of nanoparticles diffusion. *Case Stud Therm Eng* 2021; 26: 101017.
58. Waqas H, Khan SA, Farooq U, Khan I, Alotaibi H and Khan A. Melting phenomenon of non-linear radiative generalized second grade nanoliquid. *Case Stud Therm Eng* 2021; 26: 101011.
59. Muhammad T, Waqas H, Khan SA, Ellahi R and Sait SM. Significance of nonlinear thermal radiation in 3D Eyring–Powell nanofluid flow with Arrhenius activation energy. *J Therm Anal Calorim* 2021; 143(2): 929–944.
60. Dawar A, Shah Z, Tassaddiq A, Kumam P, Islam S and Khan W. A convective flow of Williamson nanofluid through cone and wedge with non-isothermal and non-isosolutal conditions: a revised Buongiorno model. *Case Stud Therm Eng* 2021; 24: 100869.
61. Rashid U, Iqbal A, Liang H, Khan W and Ashraf MW. Dynamics of water conveying zinc oxide through divergent-convergent channels with the effect of nanoparticles shape when Joule dissipation are significant. *PLOS One* 2021; 16(1): e0245208.
62. Rasheed HU, Khan W, Khan I, Alshammari N and Hamadneh N. Numerical computation of 3D Brownian motion of thin film nanofluid flow of convective heat transfer over a stretchable rotating surface. *Sci Rep* 2022; 12(1): 1–14.
63. Rasheed HU, Islam S, Khan W and Abbas T. Numerical modeling of unsteady MHD flow of Casson fluid in a vertical surface with chemical reaction and Hall current. *Adv Mech Eng* 2022; 14: 1687813221085429.
64. Usman AH, Shah Z, Kumam P, Khan W and Humphries UW. Nanomechanical concepts in magnetically guided systems to investigate the magnetic dipole effect on ferromagnetic flow past a vertical cone surface. *Coatings* 2021; 11(9): 1129.
65. Rasheed HU, Islam S, Khan Z, et al. Thermal radiation effects on unsteady stagnation point nanofluid flow in view of convective boundary conditions. *Math Probl Eng* 2021; 2021: 1–13.
66. Bein B. Entropy. *Best Pract Res Clin Anaesthesiol* 2006; 20(1): 101–109.
67. Gray RM. *Entropy and information theory*. New York: Springer Science & Business Media, 2011.
68. Sciacovelli A, Verda V and Sciubba E. Entropy generation analysis as a design tool—a review. *Renew Sustain Energ Rev* 2015; 43: 1167–1181.
69. Manjunath K and Kaushik SC. Second law thermodynamic study of heat exchangers: a review. *Renew Sustain Energ Rev* 2014; 40: 348–374.
70. Torabi M, Karimi N, Peterson GP and Yee S. Challenges and progress on the modelling of entropy generation in porous media: a review. *Int J Heat Mass Transf* 2017; 114: 31–46.
71. Mahian O, Kianifar A, Kleinstreuer C, et al. A review of entropy generation in nanofluid flow. *Int J Heat Mass Transf* 2015; 81: 514–532.
72. Zahid UM, Akbar Y and Abbasi FM. Entropy generation analysis for peristaltically driven flow of hybrid nanofluid. *Chin J Phys* 2020; 67: 330–348.
73. Jamshed W, Nisar KS, Ibrahim RW, Mukhtar T, Vijayakumar V and Ahmad F. Computational frame work of Cattaneo-Christov heat flux effects on engine oil based Williamson hybrid nanofluids: a thermal case study. *Case Stud Therm Eng* 2021; 26: 101179.
74. Bouslimi J, Alkathiri AA, Alharbi AN, Jamshed W, Eid MR and Bouazizi ML. Dynamics of convective slippery constraints on hybrid radiative Sutterby nanofluid flow by Galerkin finite element simulation. *Nanotechnol Rev* 2022; 11: 1219–1236.
75. Jamshed W. Thermal augmentation in solar aircraft using tangent hyperbolic hybrid nanofluid: a solar energy application. *Energy Environ.* Epub ahead of print 11 August 2021. DOI: 10.1177/0958305X211036671.

76. Jamshed W and Aziz A. Entropy analysis of TiO$_2$-Cu/EG Casson hybrid nanofluid via Cattaneo-Christov heat flux model. *Appl Nanosci* 2018; 8: 1–14.

77. Alwawi FA, Alkasasbeh HT, Rashad AM and Idris R. MHD natural convection of sodium alginate Casson nanofluid over a solid sphere. *Results Phys* 2020; 16: 102818.

78. Ouni M, Ladhar LM, Omri M, Jamshed W and Eid MR. Solar water-pump thermal analysis utilizing copper–gold/engine oil hybrid nanofluid flowing in parabolic trough solar collector: thermal case study. *Case Stud Therm Eng* 2022; 30: 101756.

79. Brewster MQ. *Thermal radiative transfer and features*. New York: John Wiley and Sons, 1992.

80. Jamshed W, Şirin C, Selimefendigil F, Shamshuddin M, Altowairqi Y and Eid MR. Thermal characterization of coolant Maxwell type nanofluid flowing in parabolic trough solar collector (PTSC) used inside solar powered ship application. *Coatings* 2021; 11(12): 1552.

81. Keller HB. A new difference scheme for parabolic problems. In: Hubbard B (ed.) *Numerical solutions of partial differential equations*. New York: Academic Press, 1971, vol. 2, pp.327–350.

82. Makinde OD and Aziz A. Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition. *Int J Therm Sci* 2011; 50: 1326–1332.

83. Abu-Hamdeh NH, Aljinaidi AA, Eltaher MA, et al. Implicit finite difference simulation of Prandtl-Eyring nanofluid over a flat plate with variable thermal conductivity: a Tiwari and das model. *Mathematics* 2021; 9(24): 3153.

84. Chu YM, Khan U, Zaib A, Shah S and Marin M. Numerical and computer simulations of cross-flow in the streamwise direction through a moving surface comprising the significant impacts of viscous dissipation and magnetic fields: stability analysis and dual solutions. *Math Prob Eng* 2020; 2020: 1–11.

85. Qiang X, Mahboob A and Chu YM. Numerical approximation of fractional-order Volterra integrodifferential equation. *J Funct Spaces* 2020; 2020: 1–12.

86. Nazeer M, Hussain F, Khan MI, El-Zahar ER, Chu YM and Malik MY. Theoretical study of MHD electro-osmotically flow of third-grade fluid in micro channel. *Appl Math Comput* 2022; 420: 126868.

87. Chu YM, Shankaralingappa BM, Gireesha BJ, Alzahrani F, Khan MI and Khan SU. Combined impact of Cattaneo-Christov double diffusion and radiative heat flux on bio-convective flow of Maxwell liquid configured by a stretched nano-material surface. *Appl Math Comput* 2022; 419: 126883.

88. Zhao T, Khan MI and Chu Y. Artificial neural networking (ANN) analysis for heat and entropy generation in flow of non-Newtonian fluid between two rotating disks. *Math Methods Appl Sci.* Epub ahead of print 7 April 2021. DOI: 10.1002/mma.7310.

89. Abd El, Salam MA, Ramadan MA, Nassar MA, Agarwal P and Chu YM. Matrix computational collocation approach based on rational Chebyshev functions for nonlinear differential equations. *Adv Differ Equ* 2021; 2021: 1–17.

90. Chu YM, Nazir U, Sohail M, Selim MM and Lee JR. Enhancement in thermal energy and solute particles using hybrid nanoparticles by engaging activation energy and chemical reaction over a parabolic surface via finite element approach. *Fractal Fract* 2021; 5(3): 119.

91. Asjad M, Zahid M, Chu YM and Baleanu D. Prabhakar fractional derivative and its applications in the transport phenomena containing nanoparticles. *Therm Sci* 2021; 25(2): 411–416.

92. Hussain SM and Jamshed W. A comparative entropy based analysis of tangent hyperbolic hybrid nanofluid flow: implementing finite difference method. *Int Commun Heat Mass Transf* 2021; 129: 105671.

93. Jamshed W, Eid MR, Hussain SM, et al. Physical specifications of MHD mixed convective of Ostwald-de Waele nanofluids in a vented-cavity with inner elliptic cylinder. *Int Commun Heat Mass Transf* 2022; 134: 106038.

94. Jamshed W. Finite element method in thermal characterization and streamline flow analysis of electromagnetic silver-magnesium oxide nanofluid inside grooved enclosure. *Int Commun Heat Mass Transf* 2022; 130: 105795.