Evaluation of the curvature ductility ratio of a circular cross-section of concrete reinforced with GFRP bars

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Abstract. The present study deals with the use of fiberglass reinforced polymer bars (GFRP) as a replacement for the common steel of a reinforced concrete circular pile, in order to avoid the corrosion of durability of reinforcing bars and thus improve them. The comparative analysis was carried out between a pile reinforced with GFRP and another with steel, where the ductility was evaluated by obtaining moment-curvature diagram. As a result, said idealized moment-curvature diagrams and ductility indices are presented, concluding the ductility of the section reinforced with GFRP in 20% more than that of steel.

1. Introduction
Fiberglass composite bars have emerged in recent years as a possible and efficient alternative to replace steel in reinforced concrete elements due to the problem corrosion [1]. Some of the advantages of this material are the high resistance-to-weight ratio, high corrosion resistance and low maintenance cost [2]. These bars are made up of fiberglass, which in combination with a polymer improves stress resistance and can be used as structural reinforcement [3].

A methodology for the construction of the interaction diagram for a composite section with GFRP bars subjected to eccentric axial force was proposed [4]. An experimental research on moment-resisting frame reinforced with GFRP bars and steel bars subjected to lateral cyclic loads and axial loads was conducted [5]; it was observed that the GFRP-reinforced section developed greater elastic deformations than steel creep deformations. Finally, a series of tests on columns reinforced with fiberglass bars and carbon bars subjected to axial load was carried out and it was developed an analytical method for determining the moment-curvature diagram and the lateral displacement [6].

In this investigation, a comparative analysis of the moment-curvature diagrams of a circular cross-section of concrete reinforced with steel bars and GFRP bars subjected to axial load and lateral force was carried out. In addition, the diagrams were idealized into trilinear curves in order to assess the index of ductility curvature for both types of reinforcement through a program code made in MATLAB [7].

2. Method

2.1. Procedure Diagram Moment-Curvature
The fiber method was used for the analysis of a circular section of reinforced concrete. This analysis was carried out computationally as follows:

i. Define the mechanical properties of materials, dimensions, and distribution of rebar in the section. Divide the section in 1 mm thick fibers.

ii. Determine the design resistance compression of the section $\phi P_n$. It will depend of the reinforcement material (steel or GFRP). The section is subjected to a constant axial load $P$ equal to 15, 30, 45 and 60% of $\phi P_n$. 

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iii. Assume an initial value of strain to compression of the top fiber (\(\varepsilon_{c1}\)) and a value of the distance to the neutral axis of that fiber (\(c_1\)).

iv. In each fiber, calculate the core area (\(A_{core}\)) and the area of the cover (\(A_{cover}\)) and the deformation of the fiber (\(\varepsilon_{ci}\)) with the \(c_i\) assumed. Calculate the stress of the concrete (\(f_c\)) unconfined for the cover according to [8] and for confined core according to [9].

v. Calculate the resulting force as the product of the stress by the fiber area, both in the core and cover area. Then, sum of all the resulting forces in each fiber to determine the total force of the concrete (Fc); also, multiply the resulting forces by the distance from the centroid and sum the moments in each fiber to determine the resulting moment relative to the neutral axis (Mc).

vi. Calculate the contribution on each axis of rebar by calculating the reinforcement area (\(A_{s(j)}\)), the deformation (\(\varepsilon_{s(j)}\)), stress (\(f_{s(j)}\)) and force (\(F_{s(j)}\)) from its respective elasticity module (\(E_s o E_f\)). Finally, sum the forces and moments relative to the neutral axis to determine the resulting force (Fs) and the resulting moment relative to the neutral axis (Ms).

vii. \(P_1\) is obtained from the sum of the resultant forces Fc and Fs. In turn, the sum of the resultant moments Mc and Ms is \(M_1\).

viii. If the value of \(P_1\) is verified to be equal to \(P\), then the assumed value \(c_1\) is correct, otherwise iterations of step 5 to 8 are performed with different values of "c" until you find the right one.

ix. Once the value of \(c\) is set, the curvature \(\Psi\) is determined as \(\varepsilon_{c1}/c\) and with its respective \(M_1\), the ordered pair is formed (\(\Psi, M_1\)) and a point of the curve is obtained.

x. This process is repeated for a range of \(\varepsilon_{c1}\) values. In this case, values from 0.0005 to 0.008 were chosen with increments of 0.00025 in order to build the diagram until the higher fiber reaches the last deformation of the unconfined concrete.

It is important to consider the degradation of the concrete, that is, if the upper fiber has a deformation equal to or greater than the maximum deformation of concrete \(\varepsilon_{cu}\) equal to 0.003, its contribution is disregarded. In addition, the contribution of GFRP bars in compression is not considered.

2.2. Idealization del diagram moment-curvature

According to [10], the curvature moment diagram can be idealized as a trilinear diagram (figure 1).

![Figure 1. Idealized curve][10]

The first line of the idealized curve [0;\(\Psi Y\)] has a slope equal to effective lateral stiffness and represents the area where the section has elastic behavior. The second line [\(\Psi Y, \Psi d\)] represents the post-yield of positive slope and part of the yield point (\(\Psi y, M_y\)) until the start of the degradation zone (\(\Psi d, M_d\)) and represents the plastic behavior zone of the section. Finally, in the third line of negative slope, the value of which is estimated between the starting point of the degradation to the point where it is intercepted with the horizontal line at 60% of the yield moment [10]. It is important that the areas below the idealized curve and the actual curve approximately equal in order to have about the same energy dissipation.

3. Results
The analysis of a circular section reinforced with steel bars and the same with GFRP bars was performed. This section has 40 cm diameter, spirals and rebar are 3/8" and 5/8" in diameter, respectively. It is considered a 7.5 cm of cover. The compression resistance of the concrete is 350 kg/cm² and the yield strength of the steel and the ultimate strength of the GFRP are 4200 kg/cm² and 10193 kg/cm², respectively. The elasticity modulus of steel and GFRP are 2000000 kg/cm² and 713634 kg/cm², respectively; and the yield strain of the steel and ultimate strain of the GFRP is 0.0021 and 0.015. It is considered an environmental reduction factor CN equal to 0.8 set out in the [3].

Figure 2. Moment-Curvature diagram for circular section with steel

Figure 3. Moment-Curvature Diagram for circular section with GFRP

In figure 2 and 3, it is observed that, for both types of reinforcements, the axial load is higher, the maximum moment of the section increases, while, its respective curvature decreases for both types of reinforcements.

Figure 4. Idealized diagram Moment-Curvature for steel reinforcement subjected to an axial load of 15% of φPn.

Figure 5. Idealized diagram Moment-Curvature for GFRP reinforcement subjected to an axial load of 15% of φPn.
Figure 6. Idealized diagram Moment-Curvature for steel reinforcement subjected to an axial load of 30% of $\phi P_n$.

Figure 7. Idealized diagram Moment-Curvature for GFRP reinforcement subjected to an axial load of 30% of $\phi P_n$.

Figure 8. Idealized diagram Moment-Curvature for steel reinforcement subjected to an axial load of 45% of $\phi P_n$.

Figure 9. Idealized diagram Moment-Curvature for GFRP reinforcement subjected to an axial load of 45% of $\phi P_n$.

Figure 10. Idealized diagram Moment-Curvature for steel reinforcement subjected to an axial load of 60% of $\phi P_n$.

Figure 11. Idealized diagram Moment-Curvature for GFRP reinforcement subjected to an axial load of 60% of $\phi P_n$. 
The Moment-Curvature diagrams were idealized and a ductility curvature of 5.00 for steel (figure 4) and 6.67 for GFRP (figure 5) was obtained for 15% $\phi P_n$. Meanwhile, for 30% $\phi P_n$, a ductility curvature of 4.88 for steel (figure 6) and 6.38 for GFRP (figure 7) was obtained. In addition, a ductility curvature is obtained of 3.98 for steel (figure 8) and of 5.50 (figure 9) for 45% $\phi P_n$. Finally, for the section loaded with 60%$P_n$, a ductility curvature of 3.40 for steel (figure 10) and 4.68 for GFRP (figure 11). In addition, the slope of the second line of the idealized Moment-Curvature diagram for the steel reinforced section represents the bending stiffness in the plastic range, the which is on average 17.2% the slope of the first line, that is, the effective bending stiffness in the elastic range of the section in each case. On the other hand, for the GFRP-reinforced section, the slope in the plastic range of the second line of the idealized Moment-Curvature diagram is 20.3%, 17.3%, 10.0% and 9.4% the slope of the first one, respectively.

4. Conclusions
The analysis of a circular cross-section of concrete reinforced with steel bars and GFRP bars was carried out and the following conclusions were drawn [11]:

- The maximum resistance moment increases, while, their respective curvature decreases in both types of rebar.
- Although GFRP-bars has greater curvature compared to steel-bars, the maximum resistance moments associated with such curvatures are less than 22% on average.
- The curvature ductility ratio obtained for the GFR-rebar were 26% higher on average compared to the steel-rebar.
- The slope of the second branch, which represents the bending stiffness in the plastic range, is on average 15.5% of the slope of the first branch, which represents the effective bending stiffness for cases studied with steel rebar.
- The bending stiffness in the plastic range for GFRP-reinforced section for 15, 30, 45 and 60% of maximum design compression resistance is 20.3%, 17.3%, 10.0% and 9.4% of the elastic effective bending stiffness, respectively. Therefore, it is concluded that for lower axial load, this section can develop greater tension input of GFRP bars and the stiffness to effective bending increases.

5. References
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