Physics Based on Physical Monism

Kim, Seong Dong
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Based on a physical monism, which holds that the matter and space are classified by not a difference of their kind but a difference of magnitude of their density, I derive the most fundamental equation of motion, which is capable of providing a deeper physical understanding than the known physics. For example, this equation answers to the substantive reason of movement, and Newton’s second law, which has been regarded as the definition of force, is derived in a substantive level from this equation. Further, the relativistic energy-mass formula is generalized to include the potential energy term, and the Lorentz force and Maxwell equations are newly derived.

I. INTRODUCTION

René Descartes requested a system of science that explains both mode and reason of phenomenon (i.e., how and why) [1], but it seems that he failed in explaining correctly either of them. Subsequently, Sir Isaac Newton made a coup in explaining the mode of phenomena, but even the Newtonian mechanics, which is the matrix of current physics, failed in explaining the reason of phenomena. As known in Newton’s own endeavor[2], the correct understanding of the reason of phenomena is indispensable for completing a natural philosophy with consistency. Nevertheless, a question about the reason of phenomena has been forgotten under the admirable success of Newtonian mechanics that has been revealed in description and prediction of phenomena.

This success of Newtonian physics results from adopting a quantitative description of phenomena, which can be improved more and more by comparing with experimental results. Here, the quantitative description in theories of physics substantially corresponds to mathematical abstraction, which is the major feature of modern physics. Nonetheless, if an essence of mathematical abstraction cannot be understood concretely, this abstraction leads us to understand a phenomenon as just the phenomenon. In other words, the mathematical abstraction is merely a quibble for evading the essence of phenomenon and hinders us from understanding the reason of phenomenon. In this sense, abstract concepts need to be re-interpreted using substantial concepts in order that we have the natural philosophy with consistency.

Meanwhile, some scientific philosophers have said that the reason of phenomenon cannot be explained[3][4]. Of course, if we have an interest in only describing an empirical phenomenon by using abstract concepts, it seems that their despair is unavoidable. But, I believe that their despair can be overcome. As will be shown later in this paper, careful considerations to an entity and a process of cognizing it enable us to understand the reason of phenomenon in a substantial level. These considerations will start from statements on the entity that is a metaphysical subject. Nevertheless, it seems that our starting statements, which will be given as postulates, can be highly justified in several philosophical viewpoints and are compatible with the known results of current physics. In addition to this point, given the successful results that will be shown in this paper, I think that questions related to phenomena, which have been the subject of science up to now, can be reduced to metaphysical questions as the subject of philosophy.

The construction of this paper is as follows. In Sec.II, I will introduce some postulates to provide that an object of physical inquiry (i.e., the matter and space), which will be called an ex-entity, has transcendental, objective, independent, conservative and singular characteristics. Here, it is worth noting that the objective characteristic of ex-entity is the main basis of all arguments that will be made in this paper. Thereafter, I will introduce additional postulates that confine possible existential-modes of the ex-entity; these postulates require that we describe the magnitude, position and change of ex-entity. In sequential consideration to the process of cognition, it will be explained that the physical world can be recognized by only the perception of dissimilarity. Next, we will discuss how to describe the magnitude, position and change of ex-entity. In this discussion, we will come to conclusions that the concept of density is required for describing the magnitude of ex-entity and the density can be written by a function of position and velocity.

In Sec.III.A, the mode, reason and magnitude of change will be discussed on the basis of the objectivity of ex-entity. In Sec.III.B, we will discuss methods of describing the magnitude, position and change of ex-entity in order to establish a precondition for describing objectively the physical world. In Sec.III.C, we will obtain the law of motion, which prescribes a relation between the motion of object and the external density of ex-entity, on the basis of the objectivity and conservativeness of ex-entity. Next, we will compare quantities of ex-entity that are contained in the stationary cube and the moving cube, and from this comparison, we will come to a conclusion that Lorentz factor, which is the keyword of special relativity, represents a change of density caused by the movement of object. In Sec.III.D, we will discuss the origin of relativity, which appears to be incompatible with the objectivity, on the basis of the objectivity of ex-entity. From this discussion, we will see that the theory of rela-
tivity can be explained from the objectivity of ex-entity and the afore-mentioned objective description; that is, we will verify that our conclusions related to the length contraction, the time dilation and the Lorentz transformation coincide with those of the theory of relativity. In Sec.III.E, the gravitational field will be considered in connection with the law of motion obtained in Sec.III.C., and next, we will discuss how to express mathematically the density distribution of ex-entity, which generates the gravitational field. Especially, the fact that the Lorentz factor represents the density of ex-entity will be importantly used in this discussion. In this section, the afore-mentioned reason of phenomenon will be answered quantitatively, and some issues related to the general theory of relativity will be examined further.

The aim of Sec.III.F is to expand the idea suggested in this paper. For this, we will discuss the electromagnetic and quantum mechanical issues; e.g., a stability of matter, a force and field, a relationship between electric charge, mass and quantity of ex-entity, a spin, a size of particle, the Lorentz force and the Maxwell equations. But, to tell the truth, I fail to develop completely and sufficiently my arguments related to these issues, because these issues are deeply connected with difficult problems that have not solved in even the present physics. For all that, I think that these issues merit reader’s sober reflection—particularly, the Lorentz force and Maxwell equations will be plausibly derived from results obtained on the basis of the objectivity and new acceptable assumptions such as a conservation of momentum, in Sec.III.F.6.

II. PHILOSOPHICAL CONSIDERATION

A. Entity

Let us define ‘entity’ as anything that can be said to exist and ‘cognition system’ as every mental process performed in the human head. Then, the entity can be classified into ’ex-entity’ and ’in-entity’ depending on whether it exists inside or outside the cognition system.

1. Ex-entity

According to this classification, the ex-entity corresponds to the thing-in-itself mentioned by Kant and is also the subject of physical science under the conviction of its objectivity. However, since the ex-entity has a transcendental characteristic as will be mentioned in the following postulate 1, it is the source of consumptive arguments in that it can be interpreted in various ways from philosophical viewpoints. In order to avert the consumptive argument and make the starting premises of this paper clear, I will introduce the following postulates concerning the ex-entity.

Postulate 1 : Ex-entity is a transcendental basis that makes cognition possible.
Postulate 2 : Ex-entity exists objectively.
Postulate 3 : Ex-entity never disappears.
Postulate 4 : Ex-entity is of only one kind.
Postulate 5 : Ex-entity exists spatially.
Postulate 6 : Ex-entity changes.

Postulate 1 provides the transcendental characteristic of ex-entity as a relationship between the ex-entity and the cognition. Postulate 2 represents that the ex-entity is an objective real existence that is independent of the cognition system; therefore, we can say that the ex-entity has objectivity and independence. Postulates 3 and 4 provide the conservative and singular characteristics of ex-entity that are important for the following physical consideration. In conclusion, from the postulates 1 to 4, the ex-entity has the transcendental, objective, independent, conservative and singular characteristics. Postulates 5 and 6 are statements on possible existential modes of ex-entity, as will be discussed in detail later.

2. In-entity

The in-entity is the entity that exists within the cognition system, constitutes the cognition system, and serves for cognitive processes. If we exclude egregious mysticism, it is obvious that the world outside the cognition system (i.e., the world of ex-entity or the physical world) can be perceived by means of only sensory perceptions on dissimilarities of ex-entity. (Hereinafter, we will refer to the ‘sensory perceptions’ and the ‘dissimilarities of ex-entity’ as ”perceptions” and ”dissimilarities”, respec-
tively, for brevity’s sake.) In other words, if there is no perceivable dissimilarity, the cognition system is isolated from the external world (i.e., the physical world). For this reason, it can be concluded that every cognitive process starts from the perception of dissimilarity, and most of abstract concepts, which do not represent directly the dissimilarity of ex-entity, are obtained by processing the perceived dissimilarity within the cognition system. For example, energy is not measured directly by a sense organ or measuring equipment; it is just a property of physical system that is mentioned as something maintained constantly when the perceived dissimilarities are processed in knowledge system of physics.

In the meantime, since the concept of energy is defined mathematically, it is clear in the mathematical structure of physics at least. But, the concept of energy is just a vague quibble in the substantial aspect; that is, it is unclear what is constant. Of course, such abstract concepts are manifestly useful to explain a manner of phenomena (i.e., how). Nevertheless, they are substantially useless to study a reason of phenomena (i.e., why). For example, energy is not measured directly by a sense organ or measuring equipment; it is just a property of physical system that is mentioned as something maintained constantly when the perceived dissimilarities are processed in knowledge system of physics.

The existence of an object can be recognized by perceiving its $\Delta m$. Specifically, if there is no perceivable $\Delta m$ between an object and its vicinity, the existence of object cannot be recognized. In addition, if no object can be recognized by perceiving its $\Delta m$, either one of its $\Delta p$ and $\Delta c$ cannot be perceived. It is necessary to remember this point to understand a concept of complementarity, which will be discussed later.

A degree of $\Delta m$ is typically described using a physical concept of quantity. The concept of quantity can be understood as a combination of the ex-entity for substance and the number for form. Also, the concept of quantity seems to be most intuitive and essential because it correlates directly the ‘ex-entity’ outside the cognition system with the concept of ‘be’, which is the most fundamental concept of the cognition system. Nevertheless, the concept of quantity has arbitrariness because it does not have a criterion for comparison; for example, a comparison of quantities contained in two boxes having different volumes is generally meaningless because of the volume difference. This arbitrariness in the concept of quantity can be overcome by using the concept of density that is defined as a quantity of ex-entity contained in unit volume. (Unless there is any room for confusion, the ‘density of ex-entity’ and the ‘quantity of ex-entity’ will now be referred to as ‘density’ and ‘quantity’, respectively, for convenience.)

In the meantime, given the relation of mass and acceleration in the Newtonian mechanics, the quantity or density of ex-entity cannot be directly determined by perceiving the $\Delta m$ of ex-entity. The density can be determined only through calculation using information about $\Delta p$ and $\Delta c$. This density determination process will be discussed in detail later.

2. Magnitude

By perceiving $\Delta p$, we can recognize that one object is not identical with the other. That is, unless $\Delta p$ between two different objects can be perceived, we cannot know that the objects are different from each other.

As is well known, a position and a degree of $\Delta p$ can be described by using the spatial coordinates and spatial length, respectively. Here, contrary to the density, the spatial length can be determined through an observation. For example, in the case of two meter-sticks, one’s length can be directly compared with the other’s length by observing scales graduated on them, and this comparison makes it possible to determine the spatial length. Of course, quantities and densities of meter-sticks are determined by means of not an observation but a calculation process based on knowledge of physics, e.g., the Newton’s second law.

In the meantime, as mentioned in the previous section, if there was no object whose existence can be recognized,
it would be impossible for us to perceive \( \Delta p \). In this sense, the perception of \( \Delta m \) is a precondition for perception of \( \Delta p \). On the contrary, if \( \Delta m \) of objects are described without specifying their positions, it is obscure which one of two objects is described. For this reason, \( \Delta m \) and \( \Delta p \) are complementary to each other.

Given this complementarity between \( \Delta m \) and \( \Delta p \), the magnitude of the \( \Delta m \) of ex-entity should always be described in connection with the position of ex-entity, for the clarity of representation. Accordingly, we will introduce a density distribution function, \( \rho = \rho(r) \), which expresses the density of ex-entity as a function of position and is calculated from the above-mentioned density determination process.

C. Matter and Space

According to the conventional physical viewpoint, it is understood that matter is intrinsically of a different kind from space, and the matter is subdivided into several fundamental particles according to physical properties such as mass, electric charge and spin. Furthermore, according to this viewpoint, the space is a vacuum state, and the matter is an existing object that is wandering in the space: it is generally taken for granted that there is a substantial boundary surface between the matter and the space to separate being and nothing.

But, from the singleness of postulate 4, the entity outside the cognition system (i.e., ex-entity) is of only one kind. Thus, we can say that the matter and space, which are definitely present outside the cognition system, are not intrinsically of different kinds from each other. Judging from the non-vanishment of minimum magnitude for distinction and the significance of perception of \( \Delta m \) explained in Sec.III.B, it can be explained that differentiation of the matter from the space is based on not the kind of ex-entity but the magnitude of density of ex-entity. That is, the matter corresponds to a local region of ex-entity where existence can be recognized by perceiving its \( \Delta m \), and the space corresponds to the other region of ex-entity where existence cannot be recognized - where density is less than the minimum magnitude for distinction in \( \Delta m \).

From this analysis, the space is a non-empty portion of ex-entity. Consequently, the length, which was introduced to express the degree of \( \Delta p \), can be understood as a physical magnitude that represents the quantity of ex-entity corresponding to \( \Delta p \): the length can be described in terms of the quantity of ex-entity. Specially, given that the quantity of ex-entity has essential objectivity, which is the origin of physical objectivity, the length must be described in terms of the quantity of ex-entity for its objective description. (The essential objectivity of the quantity of ex-entity will be discussed in connection with relativity in detail later.)

Although it was concluded that the space is not empty, this conclusion should be distinguished from any attempt for resurrecting the ether that was introduced to explain the propagation of light. According to the ether hypothesis, the ether was interpreted as a kind of matter that is different from conventional matters, such as apples and electrons, and has transparent and undetectable properties. Contrary to this, in our above conclusion, the matter and space are regarded as entity of the same kind. In this sense, our conclusion of space is definitely different from the ether hypothesis. As is well known, the ether hypothesis in which the ether is regarded as another matter is incompatible with the Michelson-Moley experiment and the special relativity, but as we shall see later, our conclusion of matter and space leads to results that are compatible with them. Furthermore, our conclusion will provide us with profound knowledge that has not been revealed in the special theory of relativity.

Similarly, the matter cannot be classified on the basis of the kind of ex-entity because of its singular characteristic; that is, the matter is also of only one kind. In addition, given the transcendental and independent characteristics of ex-entity, it can be concluded that the physical properties for classifying the matter do not exist objectively outside the cognition system. Rather, such physical properties for classifying the matter are merely abstract concepts that express phenomenal regularities detected by making observations of the physical world. This is because the ex-entity has only the a priori characteristics written in the above postulates, but the regularities are one of a posteriori characteristics that are learned only by experiences on ex-entity. For this reason, it can be concluded that all the a posteriori properties including the regularities originate from the a priori characteristics of ex-entity. Particularly, the regularities result from objectivity of ex-entity, as will be argued later.

III. PHYSICAL CONSIDERATION

The laws of physics are universal statements representing regularities that can be learned from observations of physical phenomena, and most of them are generally expressed by mathematical equations, each of which prescribes a quantitative relation between its left and right sides. In this aspect, physical regularity means quantitative regularity that is found in a relation between physical quantities. As we have discussed, the regularity itself should be interpreted as the result of a priori characteristics (esp., objectivity) of ex-entity. Furthermore, in the following section III.A, we will discuss the reason that the physical regularity can be expressed in a quantitative form.

A. Change

Considering the postulate 1, knowledge of the world outside the cognition system can be obtained from the experience of ex-entity. Here, given the postulates 2 and
6, the change of ex-entity is an objective actuality and enables us to experience the outer world. In the following subsections, we will discuss the implications of the aforementioned postulates in the mode, reason and magnitude of change. Here, the following conclusions will be obtained from explanations on the basis of the dissimilarity of ex-entity that can be solely objectified; hence, they are statements having cognition-independence. Therefore, the following conclusions should be distinguished from both the phenomenal explanation of Descartes\(^3\) and the personified and teleological explanation such as the least action principle\(^4\).

1. **Mode of Change**

A mode of change of ex-entity is restricted by the postulates 3 and 4-conservative and singular characteristics. That is, since the ex-entity never disappears and is of only one kind, it is hard to escape a conclusion that the change of ex-entity is only the change of density distribution. Especially, if we exclude mysterious answers, this conclusion is inevitable. In this respect, we can conclude that all phenomenal concepts, such as movement of particle, wave and collision, are derived from processing of the perceived change of density distribution: they are just derivative concepts. In conclusion, "The change of ex-entity can only be achieved by the change of density distribution." In this case, the change of ex-entity can be described in two ways - a position-based description and a density-based description; the former is the way of representing a change of density at a fixed position, and the latter is the way of representing a change of position having a fixed density. By comparison with the descriptive ways of wave, the position-based description has affinity with a method of expressing a change of amplitude of wave at a fixed position, and the density-based description has affinity with a method of expressing a change of position having fixed amplitude. Of course, in both the descriptions, time is inevitably used as a parameter to describe the change. (We will discuss the essence of time in Sec.III.B.3-4.)

Notwithstanding, contrary to the vibration of string, the density of ex-entity cannot be measured directly as discussed above. Hence, a position-based description cannot be directly used to describe the change of density distribution. In contrast to this, since we can find an object with a fixed density in a restricted scope\(^5\), the density-based description can be used to describe the change of object: that is, we can describe effectively the movement of object in the way of density-based description. For this reason, the density-based description will be mainly adopted for the following discussions related to the description of change, and we will use some terms for the density-based description, such as matter, movement, velocity and acceleration, if required.

However, it is worth noting that the density-based description can be performed only on a perceivable object (e.g., matter and light): it cannot provide us with any information on regions outside the perceivable object. As a result, the density-based description can be used to describe the motion of object but not to obtain the density distribution of the entire space. In order to obtain the density distribution of space, we need to establish a quantitative relation between the density distribution and the movement of object, as will be concretely discussed later.

2. **Reason of Change**

If we want to make physics objective, it is obvious that the reason of change should be examined in connection with the ex-entity and its three dissimilarities whose objectivity can be assured. Also, explanations based on mysticism must be excluded in this examination. These requirements related to the reason of change lead to the conclusion that the dissimilarity of ex-entity causes a new change (i.e., acceleration) of ex-entity. To put it more concretely, it seems reasonable to conclude that the uniform motion of the universe with a uniform density distribution can make no new change in the moving state of an object that is co-moving with the universe. We can therefore conclude that the non-uniform distribution of ex-entity is the unique and general state of dissimilarities of ex-entity that can change the moving state of the object. Accordingly, the reason of acceleration can be concretely expressed as follows:

\[ \text{A change in the moving state of an object results from non-uniformity of density distribution at a position where the object } \]

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\(^3\) Descartes wanted to explain the movement of matter on the basis of phenomena such as collision and vortex. Although such phenomenal concepts have empirical intuitiveness, they should be analyzed, in a substantial level, based on the dissimilarity of ex-entity because phenomena are substantially only the derivative results of dissimilarities of ex-entity as mentioned above.

\(^4\) The least action principle demands that a physical object should search for a course in which the abstract quantity of action is minimized. But, it is obvious that the ex-entity cannot do such physical thought.

\(^5\) For example, if a change in \(\Delta m\) cannot be detected from a perceivable object such as matter and light, the density of object can be interpreted as being constant at least within the limit of minimum magnitude for distinction. Nevertheless, the above-mentioned perceivability of object enables us to measure the positional change (i.e., movement) of object. The density-based description is therefore possible within this restriction.
is located.

This conclusion explains the reason of phenomena requested by Descartes. In particular, since only the entity and its dissimilarities can be objectified as mentioned repeatedly above, only the above conclusion is an explanation having cognition-independence. Moreover, the above conclusion is the most fundamental explanation on the origins of all physical changes in that it is the universal statement without any confinement; it implies that the physical regularity (i.e., the law of physics) is singular in that the above conclusion restricts the reason of change to only one.

Besides, according to the above conclusion, the concept of action-at-a-distance must be excluded from physical considerations. In spite of this exclusion of action-at-a-distance, in order to explain interactions between distant objects, it is necessary to introduce the concept of field, as is well known. But, the field should be also related to the density distribution on the same ground. For example, in the case of a gravitational force between the earth and the sun, it can be understood that one object (e.g., the earth) is affected by the density distribution of the sun. This is similar to the concept of scalar potential except for the ontological reality of density; however, it is obvious that the density distribution function is not identical with the gravitational field in that the density distribution function is a scalar field but the acceleration is a vector quantity. A relation between the field and the density distribution function will be minutely discussed later. Additionally, given the sun’s strong stability, it is highly probable that the density distribution function of the sun has a particular mathematical structure to maintain the sun as it is. This subject will be also discussed later.

In addition to the new change of moving state (i.e., the accelerative motion), the matter may move with a constant velocity (i.e., uniform motion). From the above conclusion, we can see that if the cause of change disappears (i.e., if a density distribution become uniform), the moving state of an object is not changed. In fact, this is identical with Newton’s first/second laws. Nonetheless, both the above conclusion and the Newton’s laws are explaining a condition for uniform motion, but not the essence of motion$^6$: that is, we don’t know yet why movement occurs. We will make an answer to the essence of motion on the basis of density and its conservativeness.

3. Magnitude of Change

If we accept the afore-mentioned reason of acceleration, it is obvious that the resultant acceleration of object is only dependent on the magnitude of density distribution in the neighborhood of the object. Here, since the density distribution function $\rho(r)$, which was introduced in Sec.II.B.3, expresses the distributional magnitude of density, the acceleration of object should be represented in connection with the density distribution function. Similarly, given that the uniform motion of matter is also dependent on the density of space where the matter is located, we can say that the velocity of matter is dependent on the density distribution function. Beyond this qualitative dependence, quantitative relations between the acceleration/velocity and the density distribution function will be discussed in detail later. At all events, the magnitude of change can be concluded as follows:

[ Magnitude of Change ] - Physical magnitudes related to the moving state of object, e.g., velocity and acceleration, are dependent on the density distribution function.

In the meantime, given that the acceleration is dependent on the density distribution function as stated in the above conclusion, we can see that the density distribution function of space can be determined in connection with the acceleration of phenomenon that occurs at the very space. It is worth noting that this conclusion enables us to justify the process of density determination, which will be discussed later. In addition, since the density distribution function, which determine the magnitude of change, can be objectified and quantified, the above conclusion explains not only the origins of physical quantitative regularities but also the reason of every regularity related to the magnitude of change, on the objective ground. Given that these quantitative regularities are expressed by equations defining the quantitative relations between physical magnitudes, not only descriptions of respective physical magnitudes but also establishments of relations between them should be objectified so that we can express the quantitative regularity correctly.

B. Objective Description of Physical Magnitude

1. Reference Magnitude and Ratio

Every physical magnitude is represented as a ratio to a predetermined reference magnitude. Concretely, a magnitude of comparative object (hereinafter, a comparative magnitude) in a certain physical substance is represented as a ratio to a magnitude of predefined reference object (hereinafter, a reference magnitude) in the same physical substance. As a result, the physical substance of comparative magnitude is expressed by the reference

$^{6}$ Note that the essence of motion means not a reason of acceleration but that of movement itself.
magnitude, and a numerical relation between the reference and comparative magnitudes is expressed by a ratio that is a dimensionless number. Here, the reference magnitude is not measured or calculated but merely defined as a unit value, and the ratio is empirically determined by means of a measurement. Hence, in order to describe the physical magnitude objectively, both the definition of reference magnitude and the determination of ratio must be executed through objective methods.

Although a physical measurement has uncertainty that depends on the minimum magnitude for distinction, its method is reliably objective. That is, if two physicists measure ratios by using the same method of measurement, there is no denying the objectivity of measured ratios. Of course, a wrong measurement gives rise to a wrong result, but this is irrelevant to the topic of objectivity under discussion.

Given that the reference magnitude is just defined as the unity, even if two reference magnitudes defined independently by two physicists are expressed by the same number (i.e., unity), they may be substantially different from each other. For example, both one meter and one inch are identically expressed by the number of one, but there is a manifest spatial difference between them. The units of 'meter' and 'inch' are used to differentiate such substantial difference. In conclusion, one reference magnitude defined by one physicist is objective for the physicist's own sake but not for the others, and two reference magnitudes defined independently by two physicists cannot be objectified until a substantial difference between them is revealed (i.e., converted) quantitatively.

In the meantime, given the objectivity of ex-entity, it is obvious that an object for defining the reference magnitude can be freely selected, and that reference magnitudes can be objectively converted to each other. Nevertheless, it is necessary to take facility of conversion between reference magnitudes into consideration, because the conversion is actually one of complex procedures for determining a ratio between magnitudes. In particular, the dissimilarities of ex-entity are the most fundamental components that represent the possible existential mode of ex-entity as discussed above, thus we will discuss how to define the reference magnitudes of dissimilarities with regard to the facility of conversion, in the following section.

2. Reference Magnitudes of Density and Length

As discussed in Sec.II.B.2-3, the density and length are physical quantities that represent \( \Delta m \) and \( \Delta \rho \), respectively. Given that an object for defining the reference magnitude can be freely selected as mentioned in the previous section, space can be selected as a reference object for defining reference magnitudes of density and length. Especially, this selection of space is justified from the fact that the space is a portion of ex-entity with objectivity as discussed in Sec.II.C. (Hereinafter, we will refer to the reference magnitude of density as 'reference density' or 'unit density' and the reference magnitude of length as 'reference length' or 'unit length', for brevity's sake.) For example, an observer A can select an arbitrary position \( r_A \) in space to define his reference density \( \rho_A^0 \), [i.e., \( \rho_A^0 \equiv \rho(r_A) \)], and select a distance between two arbitrary positions \( r_1 \) and \( r_2 \) to define his reference length \( L_A^0 \) (i.e., \( L_A^0 \equiv r_1 - r_2 \)).

This reference density \( \rho_A^0 \) serves as a standard for describing the density distribution function \( \rho(r) \), which was introduced in Sec.II.B.3. For this description, let us introduce the distribution factor \( \phi(r) \) that represents a ratio of a density at a position \( r \) to the reference density \( \rho_A^0 \) - a spatial variation of density. Then, the density distribution function described by the observer A can be given by

\[
\rho(r) = \rho_A^0 \phi(r) = \rho(r_A)\phi(r). \tag{1}
\]

From the definition of density, \( \rho(r) \) of Eq. (1) represents a quantity of ex-entity contained in the unit volume at \( r \). This meaning of \( \rho(r) \) should be remembered, because it is related to the problem of singularity as will be discussed in the Sec.III.F.4.

In the meantime, similar to the case of the observer A, another observer B can select other position \( r_B \) to define his reference density \( \rho_B^0 \), [i.e., \( \rho_B^0 \equiv \rho(r_B) \)]. But, as mentioned above, the reference magnitudes are defined as merely unity and may make a substantial difference. That is, even if \( \rho_A^0 \) and \( \rho_B^0 \) are expressed by the same value of unity, they may different from each other, because of the positional difference between \( r_A \) and \( r_B \).

The difference between \( \rho_A^0 \) and \( \rho_B^0 \) can be written by

\[
\rho_0^B = \rho_A^0 \phi(r_B). \tag{2}
\]

Nevertheless, it is not until the distribution factor (eventually, the density distribution function) is determined that we can know the substantial difference.

As discussed in Sec.II.C, the length, which was introduced to represent a degree of \( \Delta \rho \), expresses the quantity of ex-entity corresponding to \( \Delta \rho \). In order to express a relation between length and its corresponding quantity, let us define length quantity \( Q_L \) as the quantity of ex-entity corresponding to a length \( L \). Then, from the definition of density, the quantitative relation between \( L \) and \( Q_L \) is given by

\[
L = Q_L / A\rho, \tag{3}
\]

where \( \rho \) denotes a density of space where the length \( L \) is measured, and the term \( A \) denotes a unit area perpendicular to the direction of \( L \) so that a volume equation of \( V = AL \) is satisfied. As a result, we can conclude that the relation between \( L \) and \( Q_L \) is also dependent on the density. Hence, the reference volumes and the reference lengths that are respectively defined by the observers A and B may make a substantial difference depending on the densities of spaces where they are defined. Similar to the above argument on density, substantial differences
in volume and length cannot be revealed quantitatively until a function of density is known quantitatively.

3. Change and Time

As is well known, time is used as a standard for describing the magnitude of change (hereinafter, 'a reference magnitude of change'). It seems that this usage of time as the reference magnitude of change results from a peculiar property of time that can be said as uniform passage. In that case, is time an ontological real existence having the property of uniform passage? Or, do physical phenomena keep in step with the passage of time?

Time cannot be understood as a real existence having ontological objectivity, because the ex-entity and its dissimilarities are the only ontologically objective things as discussed above. Especially, if we exclude the personification of the physical world, it is obviously impossible that physical phenomena keep voluntarily in step with the passage of time, even though such personified interpretation is greatly useful for physical descriptions. Then, what is the uniform passage of time? It seems that this is related to the regularity of magnitudes of changes that can be perceived from various phenomena.

To put it more concretely, let us suppose that change-magnitudes of phenomena P1, P2 and P3 remains the constant ratios of l:m:n. This relation holds approximately true for the most cases comprising movements of pendulum, sun, moon, earth, and light. Here, the change-magnitude of one of them can be selected as a reference to describe those of the others. For example, the change-magnitudes of P2 and P3 can be expressed in constant ratios to P1 (i.e., m/l and n/l). It seems that our clocks have been fabricated on the basis of this relation among change-magnitudes of phenomena. Of course, this constant ratio relation does not hold true for movements of free-falling apple and accelerating car. However, magnitudes of such accelerative movements are also not perfectly random in that they have quantitative regularities that can be described by the change-magnitude of P1.

In this sense, the uniformity of passage means just the constancy in ratios of change-magnitudes (hereinafter, constancy in change ratio). Here, it is obvious that the constancy in change ratio results from the regularity of change-magnitude mentioned in Sec.III.A.3. As discussed there, the regularity of change-magnitude results from the fact that every change-magnitude depends on the density of ex-entity that can be objectified. Consequently, we can say that the afore-mentioned peculiar property of time (i.e., the constancy in change ratio or the uniformity of passage) results from the objectivity of ex-entity and its dissimilarities. As a result, the magnitude of time should be also expressed in connection with the density of ex-entity.

4. Reference Magnitude of Change

Given the freedom of selecting a reference, an arbitrary phenomenon can be selected as a reference for defining the reference magnitude of change (hereinafter, reference phenomenon). Furthermore, as discussed in Sec.III.A.1, the change of ex-entity can be only accomplished by the change of density distribution, and this change of density distribution can be validly expressed by the density-based description that represents a change of position having a fixed density. Considering these conclusions, it is clear that the reference magnitude of change can be defined by an advancing length of reference phenomenon. (Here, the advancing length of reference phenomenon means a magnitude in positional change of point having a fixed density. For brevity’s sake, we will now refer to the advancing length of reference phenomenon as a reference advancing length.) In conclusion, the reference magnitude of change can be represented by using a spatial length.

Furthermore, since the spatial length represents the corresponding quantity of ex-entity as mentioned above, the reference magnitude of change can be represented by using a quantity of ex-entity. Similar to the Eq. (3), the reference advancing length \( L_T \) can therefore be expressed in terms of a corresponding quantity of ex-entity \( Q_T \) (hereinafter, time quantity) as follows:

\[
L_T = Q_T / A \rho
\]  

where \( \rho \) denotes the density of space where the reference phenomenon takes place, and the term A denotes the unit area of reference phenomenon perpendicular to the advancing direction of reference phenomenon.

The reference advancing length \( L_T \) defined by this way can be used for two purposes – a common reference for describing the change-magnitude of every phenomenon in a system and a specific reference for describing that of individual phenomenon. The \( L_T \) as the common reference serves as a parameter for describing diverse phenomena and, eventually, corresponds to the parametric time that is generally used for our physical descriptions. That is, an observer can determine the magnitude of temporal passage (i.e., a temporal length) in his system by measuring the \( L_T \). On the contrary, the \( L_T \) as the specific reference is used to describe speeds of individual phenomena, as will be discussed in the next paragraph. Of course, the speed can be also described by the parametric time, as usual; that is, the parametric time can take the place of \( L_T \) that is used as the specific reference. In fact, this conclusion is natural in that the parametric time – the reference magnitude of change – can be expressed by a spatial length.

Let us discuss further a description of speed using the reference advancing length \( L_T \). Since the change is literally not static unlike the length or the density, a phenomenon having a fixed change-magnitude cannot be used as an objective reference for describing diverse
phenomena any more. A change-magnitude of each phenomenon should therefore be expressed by a ratio to that of a covariant reference phenomenon as follows:

$$\frac{[\text{advancing length of comparative phenomenon}]}{[\text{advancing length of reference phenomenon, } L_T]}.$$  

Given that a standard for comparison (i.e., $L_T$) is clearly specified in the above definition, the magnitude of change described in this way is objective. In addition, given that the reference advancing length serves as the parametric time, it is obvious that the change-magnitude expressed by this way is equivalent to the aforementioned speed of each phenomenon. Particularly, we can say that the speed is substantially a dimensionless magnitude, because both the numerator and denominator of the above expression have dimensions of spatial length. In addition, we can say that the reference magnitude, which is the basis of objectification, is already implied into the definition of speed: the speed itself is a physical concept having its reference magnitude. In this sense, the length and the density are distinguished from the speed, because they are objectified only when their reference magnitudes are specified.

Meanwhile, the Lorentz factor is expressed by a ratio of speeds of object and light (i.e., $v/c$), but in this ratio term, the dimension of time in the numerator and denominator are canceled each other. We can therefore say that the Lorentz factor is actually a function depending on a ratio of an advancing length of object to that of light. Furthermore, if the advancing length of light satisfies some requisites for the reference advancing length, we can say that this ratio term is also equivalent to the afore-defined speed. Of course, given the freedom of selecting reference, it is natural that the advancing length of light can be used as the reference advancing length, and moreover, the light can be preferred as the reference phenomenon, for its peculiarity. In the following Sec.III.C.2, we will concretely discuss this peculiarity of light in connection with features of space and light: for instance, the facts that 1) the space is the common ground where every phenomenon occurs, and 2) the speed of light is entirely dependent on the density of space.

Let us discuss the density dependence of temporal length. Similar to the case of spatial length, since $L_T$ is dependent on the density as written in Eq. (4), $L_T$ corresponding to the same $Q_T$ is also changed with density. That is, Eq. (4) implies the relativistic conclusion that the time is not a physical quantity regardless of a state of system. In the Sec.III.D.3, we will see that the famous relativistic time dilation can be explained from this density dependence of temporal length. Meanwhile, the spatial and temporal lengths can be mathematically expressed by a speed of object and a position in a gravitational field, as shown respectively in the Lorentz transformation and the general relativity. We can therefore say that the spatial and temporal lengths have physical regularities. Here, considering the origin of quantitative regularity discussed in the Sec.III.A.3, we can conclude that these regularities of spatial and temporal lengths result from the objectivity of ex-entity (especially, its density). The equations (3) and (4) are the quantitative explanation that reconfirms this conclusion.

Finally, as we have seen, a phenomenon having a fixed change-magnitude cannot be selected as the reference phenomenon: the reference magnitude of change itself – time – changes ceaselessly unlike the reference magnitude of length. For this reason, if $L_T$ is used as the common reference for expressing other change phenomena (i.e., as time), we need to define additionally its reference magnitude (i.e., a unit time) for describing the magnitude of $L_T$ objectively. Of course, the unit time must be also defined in connection with the quantity of ex-entity. For example, the unit time can be defined as a time needed for the reference phenomenon to advance the reference length. In this case, even if the density of space varies, we can see that the unit time and the unit length of system are changed at the same rate regardless of the density of space. In addition, given the objectivity of ex-entity and the quantitative regularity derived from it, it is obvious that the times needed for same phenomena (i.e., phenomena that are subject to the same mechanism) to advance the same quantity of ex-entity are equal to each other. I believe that these two conclusions, which are originated from the objectivity of ex-entity, enable us to explain the first postulate of relativity, i.e., the constancy of light speed. That is, the constancy of light speed is just other statement that expresses the above conclusions based on the objectivity.

C. Density Determination I

From the preceding arguments, the density is the physical magnitude that represents the substance of ex-entity (i.e., the existence or the $\Delta m$), and the spatial coordinates and the velocity are magnitudes for representing the existential modes of ex-entity (i.e., the spatial dissimilarity and the change). And, from the postulate 2, 5 and 6, these three kinds of dissimilarities can be only objectified. We can therefore conclude that every difference in a physical substance (or content) must be capable of being completely expressed by a density function of ex-entity, and this density function must be capable of being described by using the spatial coordinates and velocity as variables. For these reasons, in addition to the distribution factor that describes a change of density caused by a positional difference, we need to introduce a kinetic
factor, which is a function of velocity (i.e., that of change magnitude), to describe a change of density caused by a movement.

1. The Law of Motion

From the discussion in the Sec.III.A.1, a non-uniform density distribution causes a change of movement state of object. In this section, we will concretely discuss a quantitative relation between the distribution factor and the kinetic factor on the basis of objectivity of ex-entity. As written in Eq. (1), in this case, the observer A will describe a density distribution of space as follows:

$$\rho(\mathbf{r}) = \rho_A^0 \phi(\mathbf{r}) = \rho(\mathbf{r}_A) \phi(\mathbf{r}).$$

Here, $\rho_A^0$ – the reference density for the observer A – denotes the density of ex-entity at the reference position $\mathbf{r}_A$, which is selected by the observer A, and is just defined as unity in value. Contrary to this, the distribution factor $\phi(\mathbf{r})$ is an unknown function because $\Delta m$ cannot be measured directly as mentioned above. Meanwhile, since the reference density is dependent on a position selected by an observer, the reference density $\rho_A^0$ may vary with the reference position $\mathbf{r}_A$. Hence, let us assume for convenience that the reference position $\mathbf{r}_A$ is fixed with respect to the observer A. Then, $\rho_A^0$ is a time-independent constant.

Now, let us denote the density, position and velocity of test object, which are described by the observer A, as $\rho_A^0$, $\mathbf{r}_A^0$ and $\mathbf{v}_A^0$, respectively. Here, a ratio of $\rho_A^0$ to $\rho_b^0$ may vary with $\mathbf{r}_p^0$ and $\mathbf{v}_p^0$, but at this stage, we cannot know the ratio owing to the impossibility of measuring the $\Delta m$. We can therefore express $\rho_A^0$ by using an unknown ratio $X$ to $\rho_A^0$; that is, $\rho_A^0 = \rho_A^0 X (\mathbf{r}_p^0, \mathbf{v}_p^0)$. And, the position $\mathbf{r}_p^0$ varies with the movement of test object, thus it can be described by using time as a parameter: $\mathbf{r}_p^0 = \mathbf{r}_p^0(t)$. Since the velocity $\mathbf{v}_p^0$ denotes the velocity of test object at $\mathbf{r}_p^0$, it is dependent on $\mathbf{r}_p^0$: $\mathbf{v}_p^0 = \mathbf{v}_p^0(\mathbf{r}_p^0)$. As a result, the density of test object described by the observer A-$\rho_p^0$ - can be written by

$$\rho_p^0(t) = \rho_A^0 X \{\mathbf{r}_p^0(t), \mathbf{v}_p^0(\mathbf{r}_p^0(t))\}.$$  

(5)

In the meantime, the test object can be described in the same manner by another observer B who is co-moving with the test object. That is, let us assume that a reference density for B (i.e., $\rho_B^0$) is defined by the density of ex-entity at the position $\mathbf{r}_B$, which is fixed with respect to the observer B. Then, similar to the observer A, the observer B will describe the density of test object as follows:

$$\rho_p^B(t) = \rho_A^0 Y \{\mathbf{r}_p^B(t), \mathbf{v}_p^B(\mathbf{r}_p^B(t))\},$$

(6)

where $\rho_p^B$ and $\mathbf{v}_p^B$ denote the density, position and velocity of test object, which are described by the observer B, and $Y$ denotes an unknown ratio of $\rho_B^0$ to $\rho_A^0$ for the observer B. Here, similar to $\rho_A^0$, $\rho_B^0(t)$ is a density at the position that is fixed with respect to the co-moving observer B, thus it is also a time-independent constant to the observer B: $\rho_B^0(t)$ is a constant that is unity in value. In addition, since the observer B is co-moving with the test object, we have

$$\mathbf{r}_p^B(t) = \mathbf{r}_p^B(0) = \mathbf{r}_p^A(t) - \mathbf{r}_B^0(t) \quad \text{(const.)},$$

(7a)

$$\mathbf{v}_p^B(t) = \mathbf{v}_p^B(0) = \mathbf{v}_A^0 \mathbf{v}_B^0 \mathbf{v}_p^0(\mathbf{r}_p^0(t)) - \mathbf{v}_B^0(\mathbf{r}_B^0(t)) = 0 \quad \text{(const.)},$$

(7b)

where $\mathbf{r}_B^0$ and $\mathbf{v}_B^0$ denote the position and velocity of the observer B, which are described by the observer A.

Similar to the conventional physical consideration, let us assume for convenience that the movement of test object is a complete kinematical phenomenon that is not accompanied by a thermal or internal process. Then, although the ratio $Y$ is still an unknown number for B, the ratio $Y$ is independent of the movement of test object, contrary to the ratio $X$ for the observer A; that is, $Y$ is constant regardless of the movement of test object. As a result, $\rho_B^0(t)$ - the density of test object written by the co-moving observer B - is always a time-independent constant. That is,

$$\rho_p^B(t) = \rho_A^0 Y \quad \text{(const.)}.$$  

(8)

But, considering that the reference position $\mathbf{r}_B$ where $\rho_B^0(t)$ is defined is moving with respect to the fixed observer A, the reference density $\rho_A^0(t)$ is not constant for the observer A. As discussed in Eq. (2), this variation of $\rho_A^0(t)$ can be revealed when $\rho_B^0(t)$ is described on the basis of $\rho_A^0$ – the reference density for A. That is, a substantial magnitude of $\rho_B^0(t)$ can be given by the product of the reference density for A and a magnitude of distribution factor at $\mathbf{r}_B^0(t)$. We can express $\rho_B^0(t)$ of Eq. (8) in terms of $\rho_A^0(t)$ as follows:

$$\rho_B^0(t) = \rho_A^0 Y \quad \text{(const.)},$$

(9)

where $\rho_A^0(t)$ denotes the magnitude of $\rho_A^0(t)$, at t=t, that is converted in terms of $\rho_A^0$; that is, $\rho_A^0(t)$ is the magnitude of $\rho_B^0(t)$ that is described by A. Using Eq. (9), we can express $\rho_B^0(t)$ of Eq. (8) in terms of $\rho_A^0(t)$.

That is, substituting $\rho_A^0(t)$ of Eq. (9) into $\rho_B^0(t)$ of Eq. (8), we have

$$\rho_B^0(t) = \rho_A^0 \phi(\mathbf{r}_B^0(t)) Y,$$

(10)

where $\rho_A^0(t)$ denotes the magnitude of $\rho_B^0(t)$ that is converted in terms of $\rho_A^0$; that is, $\rho_B^0(t)$ is the magnitude of $\rho_B^0(t)$ that is described by A.

\*\*\*\*\*

\*8 For an imperfect kinematical process, it seems that a careful consideration is needed. But we will not discuss this subject here.
Now, let us discuss a relation between the unknown ratios \( X \) and \( Y \). For this, let us assume that at \( t=0 \), both the test object and the observer B are at rest relative to the observer A. Then, from Eq. (5), the initial density of test object described by A is given by

\[
\rho_p^A(0) = \rho_0^A X(\mathbf{r}_p^A(0), 0).
\]  

(11)

The initial density of test object can be similarly described by the observer B. But, for the objective comparison, we need to convert the initial density of test object described by B in terms of \( \rho_0^A \). That is, from Eq. (10), we have

\[
\rho_p^B(0) = \rho_0^A \phi(\mathbf{r}_B^A(0)) Y. \tag{12}
\]

Given the objectivity of ex-entity density, Eqs. (11) and (12) must be equal to each other, because they express the identical substance (i.e., the initial density of test object) using the common reference density (i.e., \( \rho_0^A \)). Consequently, we have

\[
X(\mathbf{r}_p^A(0), 0) = \phi(\mathbf{r}_B^A(0)) Y. \tag{13}
\]

In the meantime, since the ratio X of Eq. (5) is not known, all the observer A can say at \( t=t \) is only the fact that the test object whose initial density factor was \( X(\mathbf{r}_p^A(0), 0) \) is moving with a velocity \( \mathbf{v}_p^A[\mathbf{r}_p^A(t)] \) at a position \( \mathbf{r}_p^A(t) \) at \( t=0 \). This statement of A can be mathematically written using the kinetic factor as follows:

\[
\rho_p^A(t) = \rho_0^A X[\mathbf{r}_p^A(0), 0] \gamma\{\mathbf{v}_p^A[\mathbf{r}_p^A(t)]\}. \tag{14}
\]

Similarly, since Eqs. (10) and (14) also express the density of same test object at the same time using the common reference density (i.e., \( \rho_0^A \)), they must be equal to each other due to the objectivity of ex-entity density. That is, we have

\[
\phi[\mathbf{r}_B^A(t)] Y = X[\mathbf{r}_p^A(0), 0] \gamma\{\mathbf{v}_p^A[\mathbf{r}_p^A(t)]\}. \tag{15}
\]

Substituting Eqs. (7) and (13) into Eq. (15), we have

\[
\frac{\gamma\{\mathbf{v}_B^A[\mathbf{r}_B^A(t)]\}}{\phi[\mathbf{r}_B^A(t)]} = \frac{1}{\phi[\mathbf{r}_B^A(0)]}. \tag{16}
\]

The above equation has only the position and velocity of the observer B that are described by the observer A as variables. Hence, let us remove superscriptions and subscripts from the above equation, for convenience. In addition, although the left side of the above equation is a function of time that expresses the ratio of the kinetic factor to the distribution factor, it is a time-independent constant because its right side is constant. Hence, if we define the left side of the above equation as a ratio function \( M \), the above equation can be written in the simple form as follows:

\[
M[\mathbf{r}(t), \mathbf{v}(t)] = \frac{\gamma[\mathbf{v}(t)]}{\phi[\mathbf{r}(t)]} = \frac{1}{\phi[\mathbf{r}(0)]} \quad (\text{const.}) \tag{17}
\]

This equation is a universal statement obtained from the objectivity of ex-entity density and prescribes the mode of movement that should be obeyed by the test object. In this sense, I will designate Eq. (17) as the law of motion of ex-entity hereinafter. Here, since the ratio function \( M \) included in the law of motion is the time-independent constant as mentioned above, it can be understood as the constant of motion that plays an important role in a physical analysis. We will see in Sec.III.E3 that the energy, which is the famous constant of motion, is closely related to the ratio function \( M \).

Meanwhile, if the reference positions \( \mathbf{r}_A \) and \( \mathbf{r}_B \) coincide with each other at \( t=0 \), the ratio function \( M \) becomes unity by Eq. (17); that is, the kinetic factor is always equal to the distribution factor. Here, it can be comprehended that the kinetic factor represents the internal density of test object, while the distribution factor represents the density of space that causes the movement of test object. In this sense, we can conclude that a test object moves such that its internal density is equal to the density of external space. In order to make a profound comprehension of this conclusion, we will now discuss a relation between speed and density and possible modes of change of density.

2. Determination of Kinetic Factor

In order to establish a quantitative relation between the kinetic factor and the velocity, let us make a comparison of quantities of ex-entity contained in two virtual cubes having the same volume. First, let us assume that one cube is at rest relative to an observer A and the other cube is moving with velocity \( \mathbf{v} \) relative to the observer A. (See FIGs. 1 and 2.) In addition, we will assume that each of the cubes has a fixed volume regardless of its movement. Of course, this assumption is incompatible with the theory of relativity and the Michelson-Moley experiment. But this assumption is just suggested to objectively compare quantities of ex-entity contained in the virtual cubes. That is, in the following Sec.III.D, we will come to the relativity-compatible conclusion that the volume of a real cube must be contracted in its moving direction.

Before making the comparison of quantities in earnest, let us discuss how to determine a quantity of ex-entity contained in the virtual cube. The quantity of ex-entity in the cube can be determined by using the relation between the density and the advancing length of reference phenomenon, as written in Eq. (4). A matter may however move with various speeds depending on its physical circumstance. Hence, if the physical circumstance is not specified, it is hard to select a movement of matter as the reference phenomenon for describing the changes of ex-entity. Contrary to the matter, the speed of light is dependent only on the properties of space (e.g., permittivity and permeability), thus the light can be desirably
selected as the reference phenomenon. Especially, given that the space is the common ground where every physical phenomenon occurs, the speed of light, which depends only on the properties of space, is enough to be the reference magnitude of change for describing change magnitudes of various phenomena. For this reason, we will use the speed of light as the reference magnitude of change to determine a quantity of ex-entity contained in the virtual cube.

From Eq. (4), the advancing length of reference phenomenon represents the corresponding quantity of ex-entity. Hence, by measuring a time taken for the light to travel from one surface of cube to the opposite surface thereof, we can compare the quantities of ex-entity contained in the stationary and moving cubes. Here, note that the compared volumes of cubes should be equal in two cases to make the comparison exact.

First, let us consider the case of rest cube. For convenience, let us assume that the advancing direction of measurement light is perpendicular to the moving direction of moving cube. That is, we can select the regular tetragon defined by four points A, B, C and D as a starting surface from which the measurement light starts, as shown in FIG. 1. Since the cube under consideration is at rest, the time \( t_s \), which is taken for the light to travel from the points A, B, C and D to the points E, F, G and H, is given by

\[
 t_s = \frac{l_0}{c},
\]

where \( l_0 \) denotes the length of each side of the cube and \( c \) denotes the speed of light.

Now, let us consider the case of moving cube. Just as the case of rest cube, let us select the regular tetragon defined by points A, B, C and D as the starting surface. But, since this cube is moving with a velocity \( v \) relative to the observer A in the direction parallel to the starting surface, the observer A will observe that the light beams, which start from the points A, B, C and D, arrive at points \( E' \), \( F' \), \( G' \) and \( H' \), which are shifted from the original arrival points E, F, G and H, respectively. See FIG. 2. In this case, the time \( t_m \) measured in the moving cube can be calculated in the same way as in the conventional relativistic argument on the time dilation. That is, by the Pythagorean theorem, \( t_m \) is given by

\[
 t_m = \frac{l_0}{c} \frac{1}{\sqrt{1 - v^2/c^2}}
 = t_s \frac{1}{\sqrt{1 - v^2/c^2}}.
\]

Here, note that the stationary and moving cubes have not only the same area of starting surface but also the same volume. As mentioned above, \( t_s \) and \( t_m \) represent the quantities of ex-entity contained in the stationary and moving cubes, respectively. We can therefore conclude that, from a comparison between Eqs. (18) and (19), the density of cube moving with the velocity \( v \) is equal to the density of cube at rest.
product of the density of stationary cube and the Lorentz factor as follows:

$$\rho' = \rho \sqrt{\frac{1}{1 - v^2/c^2}}$$  \hspace{1cm} (20)$$

where $\rho'$ and $\rho$ denote the densities of stationary and moving cubes, respectively. From this result, we can say that the Lorentz factor, which is a keyword of special relativity, is the kinetic factor that expresses the change of density induced by a movement of object. That is,

$$\gamma(v) = \frac{1}{\sqrt{1 - v^2/c^2}}.$$  \hspace{1cm} (21)$$

Meanwhile, in the above thought experiment, the virtual cube was introduced to mark merely the boundary of fixed volume. In this sense, the ex-entity quantity calculated in the above argument corresponds to the ex-entity quantity contained in the local space that is defined by the virtual cubes. That is, the quantity considered in the above argument is the quantity of space confined by the cube rather than the cube’s own quantity. Nevertheless, considering Eq. (17), we can say that an object’s own density does also increase in the ratio of Eq. (21) with the object’s velocity.

**D. Relativistic Consideration**

1. Relativity

Let us apply the same thought experiment to the case of a new observer who is co-moving with the moving cube. Then, the new observer will come to the same conclusion that the previous observer $A$ has obtained, similar to the special relativity. That is, the new observer will conclude that the moving cube, which was at rest relative to $A$, has an increased density more than the rest cube that was moving relative to $A$. Nonetheless, since such relative description between two observers seems to be contradictory to the fact that the density is an objective magnitude, we should explain the reason why such relative description is possible. For this, it is necessary to discriminate between an essential objectivity and a descriptive objectivity.

Given the objectivity of ex-entity, the quantity of ex-entity is essentially objective; therefore, a relative description of ex-entity quantity is meaningless and is not allowed to assure the physical regularity. Contrary to this, it can be stated that the length and time have only descriptive objectivity because they are merely magnitudes that can be objectively described using the quantity of ex-entity, as shown in Eqs. (3) and (4). Of course, from the postulates 5 and 6, it is obvious that $\Delta p$ and $\Delta c$ are objective actualities. But, their magnitudes (i.e., the spatial and temporal lengths) are expressed merely in ratios to the defined reference magnitudes, and each of the defined reference magnitudes can be objectified only when it is expressed using the quantity of ex-entity with the essential objectivity. As a result, every physical magnitude including the spatial and temporal lengths can be objectified only when it is expressed on the basis of the quantity of ex-entity.

In this sense, we can conclude that the relativity related to the spatial and temporal lengths is obtained as the result of descriptive objectivity. In order to justify this conclusion, we will verify, in the following Sec.III.D.4, that comparisons of the spatial and temporal lengths corresponding to the same quantity of ex-entity lead to the Lorentz transformation, which implies the relativity in the spatial and temporal lengths. For all that, in order to prevent any misunderstanding about the relativity, it is necessary to remember that the objectivity in the descriptive objectivity can be achieved on the basis of the quantity of ex-entity with the essential objectivity. Given that the density of object is dependent on the volume of object, we can see that the afore-mentioned relative description of density results from the relativity of spatial length.

2. Mode of Density Change

In view of the definition of density, the change of density can be accomplished through two different ways – a change of quantity contained in a fixed volume and a change of volume occupied by a fixed quantity. Nevertheless, as mentioned above, since the quantity of ex-entity has the essential objectivity, the statements on quantity cannot be relative to each other. Hence, the change of density induced by a movement of object cannot be accomplished by a change of ex-entity quantity. Furthermore, the change of density without any change of volume is incompatible with the Michelson-Moley experiment that excluded the theory of ether from physics. In conclusion, the change of density induced by a movement of object is accomplished by means of not the change in the ex-entity quantity of object but just the change in the volume of object. In this sense, the kinetic factor represents not the change of quantity of ex-entity but the changes of density and volume, which are caused by the movement of object. This conclusion enables us to explain the relativistic contraction of length, as will be discussed concretely in the following section.

In this sense, we can say that the acceleration of object is a compression process with invariance of quantity in that it is accomplished by a contraction of its volume without any change of quantity. The kinetic energy that increases with acceleration is therefore related not to an increase in the quantity of ex-entity but to a compression in the volume of object. For the same reason, the deceleration of object is an expansion process with invariance of quantity in that it is accomplished by an expansion of compressed volume without any change of quantity. But, a volume expanding in the deceleration process will compress repeatedly a corresponding quantity of space,
because the space is not empty. In addition, given the conservative characteristic of ex-entity, an expansion of volume in the deceleration will cause a continuous propagation of compressed density through the space. In this connection, we can interpret the movement of matter as a wavelike propagation of density of ex-entity and may intuitively explain the conservative characteristics of kinetic energy and momentum. (We will discuss quantitatively the conservative characteristics of kinetic energy and momentum later.)

In the meantime, from the relativistic mass formula \( m(v) = m_0 \gamma(v) \), an acceleration in the special relativity is understood as a non-invariant process accompanied with an increase of mass. In this sense, if we regard mass as the quantity of ex-entity, the above interpretation is incompatible with the relativistic understanding. The mass should therefore be distinguished from the quantity of ex-entity in spite of similarity in meaning between them. The reason of this discrepancy between the mass and quantity of ex-entity will be explained on the ground of representativeness of mass in the Sec.III.F.3.

Despite this discrepancy, considering the invariance of quantity of ex-entity in the acceleration and deceleration, it is obvious that the afore-mentioned interpretation of kinetic energy is compatible with the definition of energy — the capacity of a physical system to do work. Owing to this compatibility, we can interpret the relativistic formula such as \( E = mc^2 \), which has been experimentally verified, in the same way as the conventional viewpoints of current physics. In this respect, we can say, without any violation of known empirical results, that the relativistic mass formula represents not the increase of rest mass but the compression of rest volume.

On the other hand, given that the general relativity, it seems that the distribution factor does not cause contradictory statements of two observers. In this sense, the distribution factor, which is related to the position-dependent change of density, can be understood to represent an actual change in the quantity of ex-entity. Even so, the quantity of ex-entity corresponding to the reference magnitude may be changed with the density of ex-entity, and this density dependence of reference magnitude can be objectified only when the reference magnitudes are described on the basis of the same quantity of ex-entity, as discussed above. The dependence of reference magnitudes related to the distribution factor will be again discussed in the relation to the general relativity, in the Sec.III.E.4.

In addition, from the preceding considerations, we can conclude that, even in space with uniform density, the movement of object is the only method that can change the density of object without any change in quantity of ex-entity. That is, only the movement of object can satisfy the relation between internal density (i.e., \( \gamma \)) and external density (i.e., \( \phi \)), which is required by Eq. (17), without any change in quantity of ex-entity. This conclusion is an explanation for the substantial reason of movement, which was asked above.

3. Density dependence of behavior of meter sticks and clocks

As we have seen thus far, the relative description of quantity is not permitted for the essential objectivity of quantity. The density dependence of behavior of meter sticks can therefore be objectified when it is described by a quantitative relation between lengths corresponding to the same quantity of ex-entity. To describe this quantitative relation, let us refer to the lengths of objects with densities of \( \rho \) and \( \rho' \), which correspond to the same quantity of ex-entity, as \( L \) and \( L' \) respectively. Then, from Eq. (3), the quantitative relation between \( L \) and \( L' \) is given by

\[
L' = \frac{\rho}{\rho'} L. \tag{22}
\]

To verify the special relativistic results, let us assume that a difference between \( \rho \) and \( \rho' \) results from the relative motion of objects. For example, if \( \rho' \) denotes the density of an object moving with velocity \( v \) and \( \rho \) denotes that of a stationary object, a relation between \( \rho \) and \( \rho' \) is written by the above Eq. (20). For convenience, let us assume that both the objects have the same shape and volume when both of them are at rest, and that \( L \) and \( L' \) denote the lengths of stationary and moving objects, respectively, in the direction of motion. Here, as is well known, the length perpendicular to the direction of motion is independent of the motion of object\(^2\). Hence, from Eqs. (20) and (22), a relation between \( L \) and \( L' \) is given by

\[
L' = \frac{L}{\gamma(v)}. \tag{23}
\]

This equation shows that the length of moving object becomes shorter than that of stationary object in the direction of motion: as a result, this is equivalent to the special relativistic conclusion of length contraction.

In the meantime, we can say that the change of density related to the kinetic factor has an anisotropic property in that the length changes only in the direction of motion. Contrary to this, if we consider a small region of space, it seems that such anisotropy of density is not found in connection with the distribution factor. We can therefore expect that the volume of object changes isotropically with the position of object, unlike the case of kinetic factor. But, as far as I know, there is no experiment for verifying a relation between distribution factor and volume and in fact, I am not certain whether such experiment has a physical meaning and whether it is possible.

Now, let us consider the time dilation that is another famous result of special relativity. The temporal length can be determined by measuring the advancing length of reference phenomenon, as discussed in Sec.III.B.4. In addition, similar to the case of spatial length, the density dependence of behavior of clocks can be also objectified by making a comparison between temporal lengths corresponding to the same quantity of ex-entity. For this
comparison, at first, let us assume that two reference phenomena occur in regions with densities of $\rho$ and $\rho'$ and that their advancing lengths that correspond to the same quantity of ex-entity are denoted by $L_T$ and $L'_T$, respectively. Then, from Eq. (4), a relation between $L_T$ and $L'_T$ can be written by

$$L'_T = \frac{\rho'}{\rho} L_T. \quad (24)$$

Like the case of length, let us assume that $\rho'$ denotes the density of object moving with velocity $v$ and $\rho$ denotes the density of stationary object$^{11}$. Then, since $L_T$ and $L'_T$ denote the temporal lengths of stationary and moving clocks respectively, by substituting Eq. (20) into Eq. (24), we have

$$L'_T = \frac{L_T}{\gamma(v)}. \quad (25)$$

Here, since $L_T$ and $L'_T$ are the reference advancing lengths, they correspond to the numbers of ticks of clocks that are initialized to zero; that is, $L_T$ and $L'_T$ represent frequencies of stationary and moving clocks, respectively. Hence, Eq. (25) also coincides with the special relativistic conclusion that a moving clock runs more slowly than a stationary clock.

As a result, the conclusions of the present paper on behavior of meter sticks and clocks coincide with those of special relativity. In this sense, it is clear that the Michelson-Moley experiment, which excluded the concept of ether from physics, can be also explained based on the above conclusions. Nevertheless, given that the present conclusions were obtained from the attempt of describing the quantity of non-empty space objectively, we can say that the Michelson-Moley experiment is compatible with the idea of non-empty space, unlike usual interpretations. Of course, if we interpreted the space as a kind of matter like in the ether theory, the compatibility would be impossible as in the past 1900 or thereabouts. But, if we interpret the space and matter as regions of ex-entity that are classified according to density and correlate the movement of matter with the density of space based on objectivity and conservativeness of ex-entity, this compatibility between the Michelson-Moley experiment and the idea of non-empty space is possible, as discussed above.

In conclusion, the objectivity of ex-entity (especially, in quantity), which can be found from only the non-empty space, is the root of all physical regularities, and moreover it is paradoxically the ground on which the special relativity, which denied the necessity of non-empty space, is proved valid. In particular, it is obvious that the aforementioned descriptive objectivity in length and time is one of such physical regularities and that the invariance of space-time interval – the keyword of relativistic consideration – is another way of representing the objectivity of quantity of ex-entity.

4. Coordinate Transformation

Now, we will briefly discuss relations between the coordinates of an event in primed and unprimed coordinate systems, where the primed coordinate system is moving with a velocity $v$ relative to the stationary unprimed coordinate system. To avoid any unnecessary complication, we assume for convenience that the primed and unprimed coordinate systems have their axes parallel, that the $x$ and $x'$ axes coincide, that the origins $O$ and $O'$ coincide at $t = t' = 0$, and that the direction of motion is parallel to the $x$ and $x'$ axes as shown in Fig. 3.

Consider an event E which occurs at a point $(x, y, z)$ at a time $t$ in the unprimed coordinate system. Let the primed coordinates of this event be $(x', y', z', t')$. According to the well-known Galilean transformation, the primed coordinates are given by

$$x' = x - vt, \quad y' = y, \quad z' = z, \quad t' = t$$

But, since the coordinates are ratios to the corresponding reference length, they are in inverse proportion to their reference lengths. That is, as is well-known, the Galilean transformation is not correct. Given this point and the relation of Eq. (23), we can see that the primed coordinate $x'$ is equal to the multiplication of the Galilean coordinate (i.e., $x-vt$) and the kinetic factor (i.e., $\gamma$), as follows:

$$x' = \gamma(v) (x - vt), \quad (26a)$$

Unlike the primed coordinate $x'$ in the direction of motion, the primed coordinates $y'$ and $z'$, which are perpendicular to the direction of motion, are equal to those of unprimed system because kinetic factors in these directions are the unity. That is, we have

$$y' = y, \quad z' = z. \quad (26b)$$

---

$^{11}$This assumption is justified by equivalence between densities of object and space, which is written by Eq. (17) and has been mentioned in the last paragraph of Sec.III.C.2.
Finally, the primed time coordinate $t'$ can be easily obtained by conventional methods that use Eq. (26a) in order to derive the Lorentz transformation, as follows:

$$t' = \gamma(v) \left( t - \frac{vx}{c^2} \right). \quad (27)$$

E. Density Determination II

1. Equation of Motion

From the law of motion written by Eq. (17), the ratio function $M$, which is a function of position and velocity, is a time-independent constant. Thus, we have

$$\frac{d}{dt} M(\mathbf{r}(t), \mathbf{v}(t)) = 0. \quad (28)$$

Using the chain rule, this equation can be rewritten by

$$\sum_{i=1,2,3} \left( \frac{\partial M}{\partial x_i} \frac{dx_i}{dt} + \frac{\partial M}{\partial v_i} \frac{dv_i}{dt} \right) = 0. \quad (29)$$

To simplify this equation, let us define gradient operators related to position and velocity as follows:

$$\nabla \equiv \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k},$$

$$\nabla_v \equiv \frac{\partial}{\partial v_x} \hat{i} + \frac{\partial}{\partial v_y} \hat{j} + \frac{\partial}{\partial v_z} \hat{k}.$$ Using these operators, Eq. (29) is given by

$$\mathbf{v} \cdot \nabla M + \mathbf{a} \cdot \nabla_v M = 0. \quad (30)$$

Substituting Eq. (17) into Eq. (30), we have

$$\mathbf{a} \cdot \nabla_v \gamma = \gamma^3 \frac{\mathbf{v}}{c^2}. \quad (31)$$

From Eq. (21), the term $\nabla_v \gamma$ of Eq. (31) is expressed by

$$\nabla_v \gamma = \gamma^3 \frac{\mathbf{v}}{c^2}. \quad (32)$$

Substituting this result into Eq. (31) and then using Eq. (17) to eliminate $\gamma$ in the resultant expression, we can obtain the equation of motion that expresses the quantitative relation between $\mathbf{a}$ and $\nabla \phi$, as follows:

$$\mathbf{v} \cdot \left( \mathbf{a} - \frac{c^2}{M^2} \frac{1}{\phi^3} \nabla \phi \right) = 0. \quad (33)$$

It is worth noting that this equation is expressed by a power per unit mass. In addition, as mentioned in Sec.III.A, this equation shows that a non-uniform distribution of density can lead to an accelerative motion of object and the magnitude of acceleration is dependent on the function of density distribution (i.e., the distribution factor). Now, if we know the distribution factor, we can describe the motion of object using the above Eq. (33). But the distribution factor should be determined empirically, because it cannot be directly measured as mentioned repeatedly above. The next section is related to a process of determining the distribution factor.

2. Determination of Distribution Factor

At first, let us consider a gravitational field that is one of the most important fields in the history of physics. As is known, the gravitational field is written by

$$\mathbf{g} = -\frac{Gm}{r^2} \hat{r}, \quad (34)$$

where $G$ denotes the gravitational constant and $m$ does the mass of source object that generates the gravitational field. Meanwhile, given a vector equation $\mathbf{A} \cdot \mathbf{B} = \mathbf{A} \cdot \mathbf{C}$, two vectors $\mathbf{B}$ and $\mathbf{C}$ should generally satisfy other vector equation $\mathbf{A} \times \mathbf{B} = \mathbf{A} \times \mathbf{C}$ so that $\mathbf{B}$ is equal to $\mathbf{C}$. We cannot therefore remove the term $'v'$ from Eq. (33) freely. But, in the above Eq. (33), if $\mathbf{a}$ is parallel with $\nabla \phi$, the equation

$$a = \frac{c^2}{M^2} \frac{1}{\phi^3} \nabla \phi. \quad (35)$$

holds as the solution of Eq. (33). To satisfy this requirement, let us assume that $\nabla \phi$ is radial (i.e., $\phi$ is isotropic). It is empirically obvious that this assumption is approximately valid for many physical situations. Thus, substituting Eq. (34) into Eq. (35), we have

$$\frac{c^2}{M^2} \frac{1}{\phi^3} \frac{d\phi}{dr} = -\frac{Gm}{r^2}. \quad (36)$$

Solving this differential equation with respect to $\phi$ and $r$, we have

$$\phi(r) = \left[ \frac{1}{\phi_0} - \frac{2GmM^2}{c^2} \left( \frac{1}{r} - \frac{1}{r_0} \right) \right]^{-1/2}. \quad (37)$$

For convenience, let us select the infinity for the reference position $r_0$ where the reference density is defined. Then, since the reference density is unity as mentioned above, Eq. (37) can be written in the simple form as follows:

$$\phi(r) = \left[ 1 - \frac{2GmM^2}{r r_0 c^2} \right]^{-1/2}. \quad (38)$$

Given that the value of $M$ is dependent on how to select the reference position as mentioned above, we can properly select the reference position such that $M$ becomes the unity. For $M=1$ like this, the distribution factor of Eq. (38) seems to be related to the Schwarzschild solution of the general relativity. It is worth noting that both the Eq. (38) and the Schwarzschild solution are related to a static field having a spherical symmetry. In this respect, if the kinetic factor is a keyword of special relativity, we can say that the distribution factor of Eq. (38) for $M=1$ is a keyword for general relativity. In the following sections, we will examine the meaning of the constant $M$ and then discuss behaviors of meter sticks and clocks under the gravitational field.
3. Meaning of the Constant of Motion

Substituting Eqs. (21) and (38) into Eq. (17) for $M$, we have

$$M(r, v) = \left[1 - \frac{v^2}{c^2} + \frac{2Gm}{rc^2}\right]^{-1/2}. \quad (39)$$

As in the relativistic analysis of the famous formula $E = mc^2$, expanding the right side of Eq. (39) and then multiplying the resultant expansion by $m_0c^2$, we have

$$Mm_0c^2 = m_0c^2 + \frac{m_0v^2}{2} - \frac{Gm_0m}{r} + O(v^4, \frac{1}{r^2}). \quad (40)$$

The first term of the right side of this equation corresponds to the energy of rest mass, and the second and third terms do the kinetic and potential energies of the classical mechanics, respectively. The remaining terms of right side can be interpreted as the relativistic kinetic and potential energies. In conclusion, we can say that the constant of motion $M$ represents (total mechanical energy)/$m_0c^2$, which is expressed as a function of position and velocity.

4. Distribution Factor dependence of behavior of meter sticks and clocks

As have been discussed in Sec.III.D.3, the density dependence of behavior of meter sticks and clocks can be objectified by the comparisons of spatial and temporal lengths corresponding to the same quantity of ex-entity. In that section, we have also obtained Eqs. (22) and (24) for such objective comparisons of spatial lengths and of temporal lengths, and the density dependence of behavior of meter sticks and clocks has been discussed in connection with the kinetic factor. But, unlike the kinetic factor, the change of distribution factor represents not a change of volume occupied by the same quantity of ex-entity but an actual variation of quantity of ex-entity contained in the unit volume, as mentioned in Sec.III.D.2. In this sense, it is expected that a change of density, which is caused from a positional change, does not result in an anisotropic change of length (e.g., the length contraction in the moving direction induced by a movement); that is, a change of length related to the distribution factor seems to be isotropic. Given this isotropic property, if the distribution factor of space is expressed by Eq. (38), a relation between two lengths $L$ and $L'$ can be expressed by

$$L' = L\left(1 - \frac{2Gm}{rc^2}\right)^{1/6}, \quad (41)$$

where $L$ and $L'$ denote spatial lengths that are defined at the reference position $r_0$ and an arbitrary position $r$, respectively, and correspond to the same quantity of ex-entity. But if ever there was unknown anisotropy, a relation between lengths in the anisotropic direction may be written by

$$L' = L\sqrt{1 - \frac{2Gm}{rc^2}}. \quad (42)$$

At all event, given that a change of distribution factor represents an actual variation in the quantity of ex-entity, we can say that $L$ and $L'$ in Eq. (41) or Eq. (42) represent not real lengths but just the magnitudes of length corresponding the same quantity of ex-entity (hereinafter, length-values). On the contrary, a variation of length-value related to the kinetic factor is necessarily accompanied by an actual change of length due to the invariance of quantity of ex-entity, as we have seen above.

In the meantime, we can identically apply the argument made in Sec.III.D.3 to a distribution factor dependence of temporal length. That is, if the distribution factor of space is given by Eq. (38) and two clocks are disposed at the reference position $r_0$ and an arbitrary position $r$, respectively, a relation between temporal lengths, which are expressed by the two clocks, can be obtained from Eqs. (24) and (38) as follows:

$$L'_T = L_T\sqrt{1 - \frac{2Gm}{rc^2}}, \quad (43)$$

where $L_T$ denotes a temporal length of one clock at $r_0$ and $L'_T$ denotes that of the other clock at $r$. Here, as mentioned above, a frequency of light corresponds to a temporal length of clock. Thus, the distribution factor dependence of temporal length expressed by Eq. (43) can be quantitatively verified by measuring how a frequency of light, which is generated at $r_0$, changes at $r$ or vice versa, as suggested in the theory of general relativity\textsuperscript{12}. As a result, the above Eq. (43) coincides with the gravitational time dilation predicted by the theory of general relativity, and therefore, we can say that our argument is compatible with the theory of general relativity at least within the scope that has been discussed so far\textsuperscript{12}.

F. Further Consideration

1. Distribution Factor and Stability of Matter

As discussed in Sec.II.C, the matter is merely a local part whose existence can be perceived amid the universal distribution of object. Nevertheless, the mechanical description based on the matter has been successful as

\textsuperscript{12} The general theory of relativity have quantitatively predicted several amazing phenomena, such as precession of mercury, deflection of light, and radar echo delay, and these predictions have been verified experimentally so far. But, since these predictions are based on tensor analysis, which is difficult for me, I have not dealt with these phenomena quantitatively. I hope that these general relativistic subjects will be further studied by readers.
shown in the classical mechanics. Owing to this success of mechanical description, we may say that the matter is a representative that symbolizes the universal distribution of object. But, given that other parts of object whose existence cannot be perceived may still exist outside the matter, it is necessary to explain reasons that the mechanical description can be successful and that the matter can be regarded as the representative of universally distributed object.

Furthermore, the law of motion written by Eq. (17) governs motions of all points of universally distributed object, because the law of motion is the universal statement that can be applied for every case. We should therefore describe motions of not only the local part, which can be observed as the matter, but also each and every part of object, in order to give a complete account of phenomena. In this sense, the mechanical description based on the matter is not complete, though useful and effective.

Nevertheless, since we should consider motions of infinite points for this universal description, we are inevitably confronted with complexity and difficulty in the mathematical description of physical world. In addition, the universal distributions of objects result in a superposition or an entanglement of objects over the whole universe, which causes a difficulty in distinguishing a test object from a source object. In some respect, this difficulty of distinction casts doubt on whether the matter is proper as the representative of universally distributed object.

But, the matters such as electrons and protons are remarkably physically stable, as is well known. I believe that this stability of matter is a clue to the solution of afore-mentioned problems. That is, owing to the strong stability of matter, we can conjecture that the distribution factor of object has a special mathematical structure that ensures the stability of matter, and that such mathematical structure of distribution factor is preserved anyhow. In this case, the matter can be justified as the representative of universally distributed object and the complexity and difficulty in mathematical description can be alleviated by the mechanical description based on the matter. It is also possible to distinguish a test object from a source object, because the density distributions of test and source objects will be preserved independently. In analogy, even though two water waves generated at different positions are superposed at many positions, they can be independently described by different wave equations, because they propagate independently.

In the meantime, given that the distribution factor of source object written by Eq. (38) is not homogeneous, we can see that respective parts of test object will be differently accelerated with each other depending on their positions. This position-dependent acceleration of test object inevitably leads to deformation of distribution factor of test object. But, from the above conjecture, we can expect that a distribution factor of object will be restored to preserve the stability of object. I believe that this restoring process can be correlated with electromagnetic or quantum mechanic phenomena, such as the radiation caused by the transition of electron. Of course, to justify this belief, we should further study for answering to remaining issues, such as the reason that the distribution factor of matter has the form of Eq. (38) and the mathematical particularity of distribution factor. It seems that the remaining issues are closely related to the quantum mechanics.

2. Force and Field

The concept of force in Newtonian mechanics is definitely useful for the first step of analyzing unknown phenomena, but it is just a concept based on the representativeness of matter. Though the representativeness of matter can be successfully used for alleviating the difficulties in physical description, the success of mechanical description based on the representativeness of matter cannot justify the groundless belief that all physical substances of object are contained in the local region referred to as the matter. In this sense, we cannot say that the force, which describes only a motion of matter, is always useful for an investigation of the physical world. In fact, if we can fully know density functions of objects and calculate acceleration at every point of test object from the density functions, there is no need to depend on the representativeness of matter anymore. That is, further complete information on the physical world can be obtained from not the concept of Newtonian force but the acceleration at every point of object.

Contrary to the force, the physical field such as the gravitational field and the electric field provides us with physical information, which is related to the acceleration, at every point of the whole universe, as is well known. That is, the acceleration at every point may be obtained from the physical field. But, the physical properties of field, such as magnitude and direction, are determined only by the source object (especially, its density distribution), while the force or the acceleration is an interaction between the test object and the field. In this sense, we cannot identify the field with the real acceleration at every point of test object. To calculate the real acceleration of test object, we should know not only the density distribution of source object, from which the field is generated, but also a physical property of test object. Of course, the property of test object that affects the acceleration should be naturally related to the density of test object. In conclusion, we should know the acceleration at every point of test object in order to have a better understanding of the physical world, and we should consider both the densities of test and source objects to calculate this acceleration.

In the meantime, given the Galileo’s famous experiment in Pisa, it appears that the magnitude of acceleration of test object is independent of the absolute values of density of test object. Contrary to this, as we shall see in following section, the direction of acceleration is
dependent on the density directions of test and source objects. Here, the density direction is defined as a parameter for indicating whether the density of object is larger or lesser than unity, and can be understood as the sign of electric charge as will be argued later.

3. Charge, Ex-entity and Mass

In the above Sec.III.E.2, we have seen that the gravitational field can be generated from the density distribution of ex-entity written in Eq. (38). As is well known, there are electromagnetic, weak, and strong interactions besides the gravitational interaction in nature. Considering the singleness of ex-entity, we cannot however introduce extra distribution factors for each of the remaining fields. That is, if we can know the density function of source object completely and correctly, the remaining fields as well as the gravitational field should be obtained from the density function of source object.¹³

At first, let us glance over the electric force. As is well known, the electric force is analogous to the gravitational force in that the intensity of force varies inversely as the square of distance, but they are different from each other in that the direction of electric force is dependent on the sign of electric charge. In this sense, it is necessary to examine a relationship between the density of ex-entity and the sign of electric charge before having a concrete discussion on the electric field itself.

For this examination, to begin with, let us consider how to express the afore-mentioned density direction of object. Once, let us re-write the distribution factor of source object, written in Eq. (38), in a general form as follows:

\[ \phi_s(r) = \left[ 1 + \frac{2q_s k_s}{r^2} \right]^{-1}. \]  

(44)

where \( k_s \) is a parameter that characterizes the distribution factor of source object. Considering the above Eq. (35), if \( q_s \) is the sign of electric charge, the characteristic parameter \( k_s \) will be \( 1/4\pi\epsilon_0 \) for the electric force: \( k_s \) is a positive constant. Hence, the value of \( \phi_s \) is 1 or less at every point for positive \( q_s \) and is 1 or more in the most region (i.e., \( 2k/c^2 \leq r \leq \infty \)) for negative \( q_s \). As a result, the parameter \( q_s \) included in Eq. (44) represents the density direction of source object, which was defined in the previous section.

Next, let us verify that the parameter \( q \) can be understood as the sign of electric charge. For this, substituting Eq. (44) into Eq. (35), we can express a field \( \mathbf{E} \) generated from the distribution factor of Eq. (44) as follows:

\[ \mathbf{E} = q_s \frac{k_s}{r^2} \hat{r}. \] 

(45)

At this time, since the Eq. (45) is obtained only from the distribution factor of source object, it corresponds to not a measurable acceleration field but just some physical field (in fact, the electrostatic field). Of course, this is identical to the case of gravitational interaction. But, for the gravitational interaction, an acceleration of test object is independent of its density or its density direction as known empirically. The gravitational field can therefore be expressed as the acceleration field of test object, for convenience. Contrary to this, for the electric interaction under consideration, the acceleration acting on the test object is dependent not only on the density of source object but also on that of test object. (Strictly speaking, it depends on the density directions of objects—the signs of electric charges.) In this respect, we need to consider the density distribution of test object besides that of source object written in Eq. (44). For this, let us write the distribution factor of test object in the same form as that of source object, as follows:

\[ \phi_t(r) = \left[ 1 + \frac{2q_t k_t}{(r - r_t)^2} \right]^{-1}. \]

where \( q_t, k_t \) and \( r_t \) denote the density direction, the characteristic parameter and the position of distribution center respectively of test object. In addition, let us assume¹⁴ that the acceleration of test object in the field of Eq. (45) can be written by the product of \( q_t \) and \( \mathbf{E} \), as follows:

\[ \mathbf{a} = q_t q_s \frac{k_s}{r^2} \hat{r}. \] 

(46)

In this case, the \( q \)-values of source and test objects determine the direction of acceleration, because \( k_s \) is positive. That is, the acceleration of test object is repulsive in the case of the same \( q \)-values and is attractive in the case of different \( q \)-values. This feature of direction of acceleration coincides with that of the known electric force. We can therefore conjecture that the \( q \)-value denotes the sign of electric charge.

From this result, we can say that for the electric force, the \( q \) is a sign of electric charge and the characteristic parameter \( k \) is \( 1/4\pi\epsilon_0 \), while for the gravitational force, the characteristic parameter \( k \) is \(-Gm/q\) regardless of \( q \). But, given that extra distribution factors cannot be introduced for several interactions as stated above, we can conclude that at least one of the gravitational and electric interactions is a secondary effect. I do not know what the

¹³ I will discuss the electromagnetic field later in this paper, but not the strong and weak fields beyond my ability. In addition, not only the density function of source object but also that of test object may be needed to describe some interaction.

¹⁴ In fact, this assumption is introduced because of its likelihood; I cannot concretely explain its ground. Further discussion is needed.
primary one is. But given that the gravitational interaction dominates only the electrically neutral world and is manifestly weaker than the electric interaction, it seems that the gravitational interaction is the secondary one. Of course, all attempts at explaining the gravitational interaction based on the electric interaction have failed up to now. It is however likely that there is no attempt based on the idea presented in this paper. Hence, further studies are needed to verify this assumption.

As for another issue, let us discuss a relation between the quantity of ex-entity and the mass. From the definition of density, we should consider the volume of object in order to calculate a total quantity of ex-entity of object. That is, given the universal distribution of object, the total quantity of object can be obtained by integrating the distribution factor over the whole space. But, since the distribution factor $\phi$ is close to unity in the most regions regardless of q-value, such integral over the whole space does not converge. This approximate unity in the value of distribution factor may however be interpreted in connection with the fact that all existing objects are superposed and distributed over the whole space. In this sense, let us introduce a proper distribution factor of object that is defined as a function of subtracting unity, which can be apprehended as a consequence of superposition of distributions of other objects, from the distribution factor of object, as follows:

$$\phi_0(r) = \phi(r) - 1 = \left[1 \pm \frac{2k}{rc^2} \right]^{-1} - 1. \quad (47)$$

Someone may say that Eq. (47), which is approximately zero in the most regions, is relevant to the concept of mass, because the mass is a concept based on the belief that the space is a vacuum. But, even if the unity problem is taken into consideration, an integral of Eq. (47) over the whole universe does not converge as before.

In fact, the total ex-entity quantity of object cannot be equal to the mass of object, because the mass is just one of representative magnitudes characterizing the object. Concretely, the mass is defined as a ratio to acceleration in the Newtonian definition of force, and the force is one of concepts based on the representativeness of matter as mentioned above. In addition, the mass is based on the groundless beliefs that it is contained in the local region referred to as the matter and the space is a vacuum. In this sense, we can say that the mass is just a representative magnitude, as mentioned above. And, the mean value is no more equal to the total than the mass, which is a representative magnitude, is equal to the total ex-entity quantity of object. This situation is identical to the amount of electric charge that is expressed by the unit of Coulomb (C). As a result, even if we can calculate the total ex-entity quantity of object, the result may be not equal to the mass or the charge amount in general. Considering this point, the afore-mentioned q-value - density direction - is not the charge amount of object but just the density direction of object. This should be remembered to avoid unnecessary confusion in the following sec. III.F.6.

4. Singularity

For $q=-1$, Eq. (44) is singular at $r = 2k/c^2$ that corresponds to the Schwarzschild radius. This singularity may be analyzed in connection with the definition of density -- the quantity of ex-entity contained in the unit volume. To put it concretely, the singularity of density seems to be related to the fact that information on volume vanishes in a zero-dimensional point.

From the above discussion on the kinetic factor, the density of object becomes infinity as the speed of object reaches that of light. But, as mentioned above, the increase of kinetic factor is the process of compression with the invariance of quantity. Hence, even if the density of ex-entity is infinite at an arbitrary point, the quantity of ex-entity cannot be infinite at the point. In particular, given that the infinite quantity may literally fill the whole universe with infinite quantity, it is obvious that the quantity of point is physically impossible to be infinite. The infinity of density related to the distribution factor is equivalent to this, because the distribution factor should be determined from the kinetic factor as mentioned in sec. III.E.

This difference between density and quantity is caused by the fact that, from the definition of density, the density is calculated based on the unit volume. That is, whenever a finite quantity contained in a non-zero volume is compressed into a zero-dimensional point, the resultant density becomes infinity regardless of initial volume. In this sense, if we wish to calculate an ex-entity quantity of point having a specific density objectively, we should consider the unit volume, which is used as a reference for calculating the density.

In the meantime, I thought that the Archimedes’ idea could be used for a calculation of ex-entity quantity, but I found recently that my calculation was based on a fatal mistake. The subject on singularity will therefore not be discussed any more in this paper. Nevertheless, it is worth noting that the singularity of Eq. (47) is distinguished from singularities in conventional field physics. That is, the singularity in the field physics is generally related to the potential, but the potential is just an abstract concept that is generated from the formalistic approach based on the mathematics. Hence, this formalistic approach hinders us from understanding physical meanings of potential or singularity. Contrary to this, the singularity of Eq. (47) is related to the function of ex-entity density, and as mentioned above, the ex-entity density is the concept that can be understood intuitively and has volume-dependence. In this sense, if we take the unit volume into consideration, the singularity of density may be possibly understood in a substantial level.
5. Spin and Size of Particle

Up to now, we have discussed the situation in which the density distribution of source object is static. But, given that an object is distributed over the whole space, it is clear that the density distribution of object can be changed in various ways. In this section, we will discuss the rotation of object - the revolution of object on its own center-, and in the next section, we will consider the influences of the movement of test object caused by the movement of source object. But, in this paper, we do not consider the orbital motion of object in which the object revolves around some point or axis that is deviated from its own center.

As is well known, Stern-Gerlach’s experiment has shown the fact that the electron has a spin angular momentum, but the known spin angular momentum of electron is incompatible with the maximum size of electron that is calculated from scattering experiments. In addition, if we regard the electron as a point particle, due to the degree of freedom, we cannot explain the reason that the electron has a finite spin angular momentum. Owing to these contradictions, the modern physics explains that the electron is not a classical particle but a quantum mechanical particle. Frankly speaking, I am ignorant of Dirac’s relativistic quantum mechanics that is known as providing the substantial explanation for the spin of electron. Owing to my ignorance, I suspect that the modern physical explanation based on the quantum mechanics means not a solution of above contradictions but only a success of mathematical description of phenomena.

At all event, if the electron is distributed universally as mentioned above, the contradiction of classical models related to the spin of electron can be solved at least qualitatively. Firstly, it is obvious that the contradiction related to the degree of freedom disappears, because the universally distributed electron has the degree of freedom four and over. Of course, given the universal distribution of electron and the relativistic limit of speed, it is obvious that the electron is not a rigid body whose all parts revolve with the same angular velocity. Accordingly, the degree of freedom of electron becomes infinity actually. Nonetheless, we can conjecture that the electron has a finite degree of freedom in the macroscopic aspect, because the afore-mentioned constraint on the density distribution, which is required for the stability of matter, serves to reduce the degree of freedom of electron. I believe that the constraint is connected with the quantum numbers of particle including the spin angular momentum. As a result, a remaining contradiction is the inconsistency between the spin angular momentum of electron and the maximum size of electron.

To solve this inconsistency, it is necessary to understand that the size of particle is a concept based on the classical viewpoint of matter that classifies matter and space according to the substantial difference in kinds. According to the classical viewpoint of matter, there is a substantial boundary surface between the matter and the space to separate being and nothing, as mentioned above, and all physical contents related to the matter are contained in the internal region of the boundary surface. Here, the size of particle is a concept based on the belief that there is the substantial boundary surface, and it means generally a radius of the internal region of boundary surface. But, it seems that the classical viewpoint of matter is only originated from usual experience, because there is no proof that can justify the classical viewpoint of matter. That is, there is no evidence that the substantial boundary surface exists between being and nothing. In this sense, we need not to have a deep attachment to the concept of particle size. (I believe that the duality of matter and wave, which is the origin of quantum mechanics, is closely related to nonexistence of substantial boundary surface, though it will be not discussed here.)

If we exclude the belief in the substantial boundary surface, we can also solve the inconsistency between the spin angular momentum of electron and the maximum size of electron. That is, it can be understood that the measured spin angular momentum of electron results from not a rotation of local region but a revolution of every part of universally distributed electron. In this case, we can avoid the contradictory conclusion that the localized electron should be rotated with the angular speed, which exceeds the velocity of light, to satisfy the measured spin angular momentum of electron. Meanwhile, given that the size of electron is determined from the calculation based on scattering experiments, it can be concluded that the size of electron corresponds to not a size of imaginary ball that contains all physical contents of electron but just an impact parameter that is dependent on the kinetic energy of electron. This is because the scattering experiment does not prove that all physical contents related to the electron are confined in the internal region of measured impact parameter. The idea that the localized electron ball exists is just a groundless belief based on the classical viewpoint of matter, as explained above.

In the meantime, if someone particularly wishes to define the size of particle, the size of region having a peculiar density seems to be a good criterion for defining the size of particle. For example, we can define the size of electron as that of region with a zero density: from Eq. (44), we can say that the electron is a point particle of which radius is zero. Similarly, the size of proton can be defined as that of region with an infinite density; in this case, the maximum radius of proton can be given by an electrical Schwarzschild radius defined as follows:

$$r_{proton} = \frac{2e}{4\pi\epsilon_0 c^2 m_p} \approx 3.1 \times 10^{-18} m.$$  (48)

For all that, it is obvious from above arguments that these localized regions do not contain all contents of electron or proton.
6. Electromagnetism

It is obvious that a movement of test object caused by a non-static source should be different from that caused by a static source. In this section, we will discuss quantitatively a movement of test object under a moving source with a non-uniform density distribution. For such discussion, we need to express respective densities of objects by using their distribution and kinetic factors and to obtain an equation connecting quantitatively them, similar to the argument of Sec. III.C.1.

At first, let us suppose that, at an arbitrary position \( \mathbf{r} \), test and source objects move with velocities of \( \mathbf{v} \) and \( \mathbf{u} \) respectively relative to a fixed observer. Then, this observer will express the densities of test and source objects, \( \rho_1 \) and \( \rho_2 \), in terms of the products of the reference density, respective distribution factors and respective kinetic factors, as follows:

\[
\rho_1 = \rho_0 \phi_1(\mathbf{r}) \gamma_1(\mathbf{v}(\mathbf{r})), \quad (49a) \\
\rho_2 = \rho_0 \phi_2(\mathbf{r}) \gamma_2(\mathbf{u}(\mathbf{r})). \quad (49b)
\]

Here, it should be remembered that test and source objects are distributed over the whole universe as stated above, and that \( \mathbf{v} \) and \( \mathbf{u} \) denote respective velocities of one-points, which are positioned at \( \mathbf{r} \) of test and source objects. Meanwhile, considering this continuum-like property of objects and the aforementioned stability of matter, we can assume that, in most classical phenomena, a velocity of one-point is nearly equal to those of neighboring points. Hence, we will assume that

\[
\nabla \gamma_1(\mathbf{v}) \sim 0, \quad \nabla \mathbf{v} \sim 0, \quad \nabla \cdot \mathbf{v} \sim 0, \quad \nabla \times \mathbf{v} \sim 0, \quad (50a) \\
\nabla \gamma_2(\mathbf{u}) \sim 0, \quad \nabla \mathbf{u} \sim 0, \quad \nabla \cdot \mathbf{u} \sim 0, \quad \nabla \times \mathbf{u} \sim 0. \quad (50b)
\]

(Though it is not discussed in this article, if we want to expand our argument toward the region of quantum mechanics, it seems that the above neglecting needs to be reconsidered.)

In this case, by generalizing the argument of Sec. III.C.1, we can obtain a generalized equation of motion from Eqs. (49a) and (49b), as follows:

\[
M(\mathbf{r}(t), \mathbf{v}(t), \mathbf{u}(t)) = \frac{\gamma_1(\mathbf{v}(t))}{\phi_2(\mathbf{r}(t)) \gamma_2(\mathbf{u}(t))} = \frac{1}{\phi_2(\mathbf{r}(0)) \gamma_2(\mathbf{u}(0))} \quad (const). \quad (51)
\]

where \( M \) denotes the constant of motion depending of \( \mathbf{r} \), \( \mathbf{v} \) and \( \mathbf{u} \). But, contrary to the case of Sec. III.E.1, a time derivative of Eq. (51) includes an additional terms, which hinders from writing an acceleration of test object to a form of explicit function. Owing to such difficulty, I will adopt other approaching method in the following argument.\(^{15}\)

\[\text{Specifically, I will assume that a total acceleration of test object } \mathbf{a}_t \text{ is equal to the sum of a distribution acceleration } \mathbf{a}_d \text{, which results from the non-uniformity of density distribution of source, and a kinetic acceleration } \mathbf{a}_k, \text{ which results from the movement of source. That is,}
\]

\[
\mathbf{a}_t = \mathbf{a}_d + \mathbf{a}_k. \quad (52)
\]

In addition, I will assume that an acceleration effect related to the movement of source can be completely expressed by the kinetic acceleration \( \mathbf{a}_k \); that is, the distribution acceleration \( \mathbf{a}_d \) is independent of the movement of source object. (Strictly speaking, there is no sufficient ground for these assumptions, and therefore, my research related to the electromagnetism is incomplete yet. For all that, considering the following successful results obtained from these assumptions, I think that they disclose one aspect of a perfect theory that may be obtained from a thoughtful consideration: that is, it seems that they can be regarded as at least one of considerable theoretical models.) Meanwhile, as a matter of convenience, we leave the retarded time out of consideration in the following discussion.

Now, let us calculate the distribution acceleration \( \mathbf{a}_d \). If the distribution acceleration \( \mathbf{a}_d \) is independent of the movement of source as assumed above, it may be similar in form to Eq. (35), which expresses the acceleration of test object under the case of static source, as follows:

\[
\frac{c^2}{M(\phi_2 \gamma_2)^2} \nabla(\phi_2 \gamma_2),
\]

where \( M \) is the constant of motion given by Eq. (51). However, as explained in Sec. III.E.3, this physical quantity corresponds to not a real acceleration of test object, which can be measured directly, but only a field generated by the source object. As explained in Sec. III.6.3) and 4), for the electromagnetic phenomena, the real distribution acceleration of one point, positioned at \( \mathbf{r} \) of test object can be given by the product of the density direction (i.e., the sign of electric charge) of test object \( q_1 \) and the field generated by the source object, which is given by the above equation, as follows:

\[
\mathbf{a}_d = q_1 \frac{c^2}{M(\phi_2 \gamma_2)^2} \nabla(\phi_2 \gamma_2). \quad (53)
\]

Now, let us calculate the kinetic acceleration \( \mathbf{a}_k \). For this, let us define the momentum factor as the product of the distribution factor, the kinetic factor and the velocity at each points of object. According to this definition, the momentum factors of test and source objects \( \mathbf{p}_1 \) and \( \mathbf{p}_2 \) can be written by

\[
\mathbf{p}_1 = \phi_1(\mathbf{r}) \gamma_1(\mathbf{v}) \mathbf{v} \equiv \psi_1(\mathbf{r}, \mathbf{v}) \mathbf{v}, \quad (54a) \\
\mathbf{p}_2 = \phi_2(\mathbf{r}) \gamma_2(\mathbf{u}) \mathbf{u} \equiv \psi_2(\mathbf{r}, \mathbf{u}) \mathbf{u}. \quad (54b)
\]

where \( \psi_1 \) and \( \psi_2 \) denote the ratios of densities of test and source objects, respectively, to the reference density.

\(^{15}\) The properness of the following assumptions should be verified from further research for overcoming this difficulty.
\[ \rho_0 \] and can be defined as the product of distribution and kinetic factors at \( r \) as expressed in the above equations.

In the meantime, to know an effect on the movement of test object caused by the moving source, we need to obtain the equation connecting quantitatively the two momentum factors, as mentioned above. However, such equation is unknown for the present, because the momentum factor was not derived but introduced. That is, it is unclear whether the well-known conservation law of momentum can be applied for the situation under discussion. Therefore, for a generalized discussion, let us assume that the time derivative of the algebraic sum of momentum factors is equal to an unknown vector \( \mathbf{X} \). Here, for the electromagnetic phenomena, we should take the density directions of objects into consideration, as explained above. Therefore, this assumption may be expressed by an equation with a parameter \( Q \) that depends on the density directions of source and test objects, as follows:

\[
\frac{d}{dt}(\mathbf{p}_1 + Q\mathbf{p}_2) = \mathbf{X}. \tag{55}
\]

We will discuss a relation between the parameter \( Q \) and the density direction \( q_1 \) below.

Let us calculate \( \frac{d\mathbf{p}_1}{dt} \) and \( \frac{d\mathbf{p}_2}{dt} \), respectively. Given the necessity of density-based description discussed in section III.A.1, if we want to express a movement of test object, it is necessary to describe first a movement of one point of test object with a specific constant density. For this, let us select the specific point of test object whose the magnitude of distribution factor is \( \phi_{10} \) and calculate Eq. (55) along a trajectory of this point. Then, from Eq. (54a), the time derivative of momentum factor of selected point is given by

\[
\frac{d\mathbf{p}_1}{dt} = \phi_{10} \left[ \frac{d\gamma_1(\mathbf{v})}{dt} \mathbf{v} + a_1 \gamma_1(\mathbf{v}) \right], \tag{56}
\]

where \( a \) denotes the time derivative of velocity of selected point of test object. Since the kinetic factor is the function of velocity, which is given by Eq. (21), the \( d\gamma_1/dt \) in Eq. (56) can be given by

\[
\frac{d\gamma_1(\mathbf{v})}{dt} = a \cdot \nabla_\mathbf{v} \gamma_1(\mathbf{v}). \tag{57}
\]

where \( \nabla_\mathbf{v} \gamma_1(\mathbf{v}) \) denotes the velocity gradient of kinetic factor of selected point of test particle, which is equal to Eq. (32). Therefore, Eq. (56) can be written by

\[
\frac{d\mathbf{p}_1}{dt} = \phi_{10} \gamma_1 \left[ \left( \frac{\gamma_1^2}{c^2} \right) (a \cdot \mathbf{v}) \mathbf{v} + a \right]. \tag{58}
\]

Since the \( (a \cdot \mathbf{v}) \mathbf{v} \) equals to \( \mathbf{v} \times (\mathbf{v} \times a) + v^2 a \) by the BAC-CAB rule, Eq. (58) can be written by

\[
\frac{d\mathbf{p}_1}{dt} = \phi_{10} \gamma_1 \left[ a \left( 1 + \frac{v^2}{c^2} \gamma_1^2 \right) + \frac{\gamma_1^2}{c^2} \mathbf{v} \times (\mathbf{v} \times a) \right]. \tag{59}
\]

And since \( (1 + v^2 \gamma_1^2/c^2) \) equals to \( \gamma_1^2 \) by Eq. (21), Eq. (59) can therefore be written by

\[
\frac{d\mathbf{p}_1}{dt} = \phi_{10} \frac{\gamma_1^2}{c^2} \left[ a + \frac{1}{c^2} \mathbf{v} \times (\mathbf{v} \times a) \right]. \tag{60}
\]

Now, let us calculate the term of \( d\mathbf{p}_2/dt \) in Eq. (55). Given that an action-at-a-distance is impossible, to describe the movement of selected point of test object, we should calculate the time derivative of momentum factor of source object along the trajectory of selected point of test object; that is, the term of \( d\mathbf{p}_2/dt \) should be calculated as the convective derivative, as follows:

\[
\frac{d\mathbf{p}_2}{dt} = \frac{\partial \mathbf{p}_2}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{p}_2, \tag{61}
\]

where \( \mathbf{v} \) in Eq. (61) denotes the velocity of selected point of test object. Here, using the known vector identity \( (\mathbf{v} \cdot \nabla) \mathbf{p}_2 = \nabla(\mathbf{v} \cdot \mathbf{p}_2) - (\mathbf{v} \times (\nabla \times \mathbf{p}_2)) - (\mathbf{p}_2 \cdot \nabla) \mathbf{v} \), Eq. (61) can be written by

\[
\frac{d\mathbf{p}_2}{dt} = \frac{\partial \mathbf{p}_2}{\partial t} + \nabla(\mathbf{v} \cdot \mathbf{p}_2) - (\mathbf{v} \times (\nabla \times \mathbf{p}_2)) - \mathbf{p}_2 \times (\nabla \times \mathbf{v}) - (\mathbf{p}_2 \cdot \nabla) \mathbf{v}. \tag{62}
\]

Here, the last two terms of right side of Eq. (62) can be neglected by Eq. (50a). Hence, we have

\[
\frac{d\mathbf{p}_2}{dt} = \frac{\partial \mathbf{p}_2}{\partial t} + \nabla(\mathbf{v} \cdot \mathbf{p}_2) - \mathbf{v} \times (\nabla \times \mathbf{p}_2). \tag{63}
\]

Inserting Eqs. (60) and (63) into Eq. (55), the acceleration of selected point of test object can be written by

\[
a = -\frac{Q}{\phi_{10} \gamma_1^3} \left[ \frac{\partial \mathbf{p}_2}{\partial t} - (\mathbf{v} \times (\nabla \times \mathbf{p}_2)) \right] + \mathbf{O}, \tag{64a}
\]

where

\[
\mathbf{O} = \left( \frac{1}{\phi_{10} \gamma_1^3} \right) \left[ \mathbf{X} - Q(\mathbf{v} \cdot \mathbf{p}_2) \right] - \frac{1}{c^3} \mathbf{v} \times (\mathbf{v} \times \mathbf{a}). \tag{64b}
\]

Next, substituting the constant of motion \( M \) for \( \gamma_1 \) of Eq. (64a) using Eq. (51), we have

\[
a = -\frac{Q}{M^3 \phi_{10} \psi_2^2} \left[ \frac{\partial \mathbf{p}_2}{\partial t} - (\mathbf{v} \times (\nabla \times \mathbf{p}_2)) \right] + \mathbf{O}. \tag{65}
\]

Here, since the variable \( a \) in Eq. (65) is the acceleration of selected point of test object caused by the movement of source object, it corresponds to the afore-defined kinetic acceleration \( a_k \). Therefore, inserting Eqs. (53) and (65) into Eq. (52), the total acceleration of selected point of test object \( a_k \) which is caused by the moving source object with a non-uniform density distribution, can be written by
\[
a_t = \frac{q_1 c^2}{M^2} \left\{ \nabla \psi_2 - \frac{Q}{q_1 M \phi_{10} c^2} \left[ \frac{\partial p_2}{\partial t} - \mathbf{v} \times (\nabla \times p_2) \right] \right\} + O. \tag{66}
\]

Though similar, this equation is not identical with the known Lorentz force formula, because of the non-constant factor \(1/\psi_2^2\). But, we can overcome this difference using the following identities.

\[
\begin{align*}
\frac{1}{\psi_3^2} \nabla \psi &= -\frac{1}{2} \nabla \frac{1}{\psi_2^2}, \\
\frac{1}{\psi_3^2} \nabla \times (\psi \mathbf{u}) &= -\frac{1}{2} \nabla \times \left( \frac{\mathbf{u}}{\psi_2^2} \right) + \frac{3}{2 \psi_2^2} \nabla \times \mathbf{u}, \\
\frac{1}{\psi_3^2} \frac{\partial}{\partial t} (\psi \mathbf{u}) &= -\frac{1}{2} \frac{\partial}{\partial t} \left( \frac{\mathbf{u}}{\psi_2^2} \right) + \frac{3}{2 \psi_2^2} \frac{\partial \mathbf{u}}{\partial t}, \\
\frac{1}{\psi_3^2} \frac{\partial \psi}{\partial t} &= -\frac{1}{2} \frac{\partial \psi}{\partial t} \left( \frac{1}{\psi_2^2} \right), \\
\frac{1}{\psi_3^2} \nabla \cdot (\psi \mathbf{u}) &= -\frac{1}{2} \nabla \cdot \left( \frac{\mathbf{u}}{\psi_2^2} \right) + \frac{3}{2 \psi_2^2} \nabla \cdot \mathbf{u}. \tag{67a-c}
\end{align*}
\]

That is, using Eqs. (67a)-(67c), Eq. (66) can be re-written by

\[
a_t = q_1 \frac{c^2}{M^2} \left\{ -\nabla \left( \frac{1}{\psi_2^2} \right) + \frac{Q}{q_1 M \phi_{10} c^2} \left[ \frac{\partial \mathbf{u}}{\partial t} - \mathbf{v} \times (\nabla \times \mathbf{u}) \right] \right\} + O', \tag{68}
\]

where

\[
O' = O - \frac{3Q}{2M^3 \phi_{10}} \left( \frac{1}{\psi_2^2} \right) \left[ \frac{\partial \mathbf{u}}{\partial t} - \mathbf{v} \times (\nabla \times \mathbf{u}) \right]. \tag{68a}
\]

At this time, the known Lorentz force formula can be obtained completely from the terms in the brackets of Eq. (68), as will be shown below. In this sense, if the classical electromagnetic theory is exact, all of additional terms written in Eq. (68a) or the sum of them should be vanished. Particularly, we can neglect the last term of right side of Eq. (68a) in most classical cases, because of Eq. (50b). But, if not, it is expected that we can measure physical effects caused by the terms written in Eq. (68a), that is, discrepancies from the known electromagnetic theory.

Now, let us derive the Lorentz force formula and the Maxwell Equations. For this, let us assume that the term \(O'\) of Eq. (68a) can be neglected and that \(Q/q_1\), which depends on the signs of electric charges of objects, is equal to -1, as a matter of convenience. (Further study is needed to verify the properness of these assumptions.) In addition, let us define a scalar function and a vector function as follows:

\[
\begin{align*}
\Phi &= \frac{c^2}{2 \psi_2^2}, \tag{69a} \\
\mathbf{A} &= \frac{\mathbf{u}}{2 \psi_2^2} = \Phi \frac{\mathbf{u}}{c^2}. \tag{69b}
\end{align*}
\]

Then, the equation (68) can be simply expressed by

\[
a_t = q_1 \frac{c^2}{M^2} \left\{ -\nabla \Phi + \frac{1}{M \phi_{10}} \left[ \frac{\partial \mathbf{A}}{\partial t} - \mathbf{v} \times (\nabla \times \mathbf{A}) \right] \right\}. \tag{70}
\]

Eq. (70) is identical with the Lorentz force except the additional factors \(M^2\) and \(M^3 \phi_{10}\), which denote respectively the electric and magnetic susceptibilities as explained below. In this sense, the scalar and vector functions of Eqs. (69a) and (69b) can be understood as the scalar and vector potentials, respectively, in the classical electromagnetic theory, and the electric field \(\mathbf{E}\) and the magnetic field \(\mathbf{B}\) can be defined as follows:

\[
\begin{align*}
\mathbf{E} &= \frac{1}{M^2} \left[ -\nabla \Phi - \frac{1}{M \phi_{10}} \frac{\partial \mathbf{A}}{\partial t} \right], \tag{71} \\
\mathbf{B} &= \frac{1}{M^2} \left[ \frac{1}{M \phi_{10}} \nabla \times \mathbf{A} \right]. \tag{72}
\end{align*}
\]

Then, the divergences and curls of \(\mathbf{E}\) and \(\mathbf{B}\)-fields are
respectively given by
\[
\nabla \cdot \mathbf{E} = \frac{1}{M^2} \left[ -\nabla^2 \Phi - \frac{1}{M \phi_{10}} \frac{\partial (\nabla \cdot \mathbf{A})}{\partial t} \right], \tag{73}
\]
\[
\nabla \cdot \mathbf{B} = 0, \tag{74}
\]
\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{75}
\]
\[
\nabla \times \mathbf{B} = \frac{1}{M^3 \phi_{10}} \left[ \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} \right]. \tag{76}
\]

It is interesting that the Lorentz condition can be obtained from the continuity equation. Using Eqs. (71) and (80), as a result, Eq. (79) can be re-written by
\[
\frac{\partial \psi_2}{\partial t} + \nabla \cdot \mathbf{p}_2 = 0. \tag{77}
\]
Multiplying Eq. (77) by \(c^2 \psi_2^{-3}\) and using the above identities (67d) and (67e), we have
\[
\left[ \frac{\partial}{\partial t} \left( \frac{c^2}{2 \psi^2} \right) + \nabla \cdot \left( \frac{u c^2}{2 \psi^2} \right) \right] - \frac{3 c^2}{2 \psi^2} \nabla \cdot \mathbf{u} = 0. \tag{78}
\]
Using the scalar potential \(\Phi\) of Eq. (69a) and the vector potential \(\mathbf{A}\) of Eq. (69b), Eq. (78) can be re-written by
\[
\left[ \frac{\partial \Phi}{\partial t} + c^2 (\nabla \cdot \mathbf{A}) \right] - 3 \Phi (\nabla \cdot \mathbf{u}) = 0. \tag{79}
\]
Owing to Eq. (53a), we can also neglect the last term of Eq. (79). As a result, Eq. (79) can be re-written by
\[
\frac{\partial \Phi}{\partial t} + c^2 (\nabla \cdot \mathbf{A}) = 0. \tag{80}
\]
This equation coincides with the "Lorentz condition". It is interesting that the Lorentz condition can be obtained from the continuity equation. Using Eqs. (71) and (80), the above Eqs. (73) and (76) can be written respectively by
\[
\nabla \cdot (M^2 \mathbf{E}) = \frac{1}{M \phi_{10} c^2} \frac{\partial^2 \Phi}{\partial t^2} - \nabla^2 \Phi, \tag{81}
\]
\[
\nabla \times (M^3 \phi_{10} \mathbf{B}) = \frac{1}{c^2} \frac{\partial (M^2 \mathbf{E})}{\partial t} + \left[ \frac{1}{M \phi_{10} c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} - \nabla^2 \mathbf{A} \right]. \tag{82}
\]

If the scalar and the vector potentials satisfy the following wave equations,
\[
\frac{1}{M \phi_{10} c^2} \frac{\partial^2 \Phi}{\partial t^2} - \nabla^2 \Phi = \frac{\rho}{\epsilon_0}, \tag{83}
\]
\[
\frac{1}{M \phi_{10} c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} - \nabla^2 \mathbf{A} = \frac{j}{\epsilon_0 c^2}. \tag{84}
\]

the above equations (81) and (82) can be written in the simple form as follows:
\[
\nabla \cdot (M^2 \epsilon_0 \mathbf{E}) = \rho, \tag{85}
\]
\[
\nabla \times (M^3 \phi_{10} \epsilon_0 c^2 \mathbf{B}) = \frac{\partial}{\partial t} (M^2 \epsilon_0 \mathbf{E}) + \mathbf{j}. \tag{86}
\]
(The properness of above wave equations is also needed to study furthermore.) In addition, if we introduce the famous Maxwell relation \(c^2 = 1/\epsilon_0 \mu_0\) and define the electric displacement \(\mathbf{D}\) and the magnetic intensity \(\mathbf{H}\) as follows:
\[
\mathbf{D} = M^2 \epsilon_0 \mathbf{E} = (1 + \chi_e) \epsilon_0 \mathbf{E}, \tag{87}
\]
\[
\mathbf{H} = \frac{M^3 \phi_{10}}{\mu_0} \mathbf{B} = \frac{\mathbf{B}}{(1 + \chi_m) \mu_0}, \tag{88}
\]
the above equations (85) and (86) can be written by
\[
\nabla \cdot \mathbf{D} = \rho, \tag{89}
\]
\[
\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{j}. \tag{90}
\]
Equations (74), (75), (89) and (90) are identical with the known Maxwell equations: Maxwell equations are derived.

In addition, comparing the above Eqs. (87) and (88) with the known formulae between \(\mathbf{E}, \mathbf{D}, \mathbf{B}\) and \(\mathbf{H}\), we have
\[
M^2 = 1 + \chi_e = k_e, \tag{91}
\]
\[
M^3 \phi_{10} = \frac{1}{1 + \chi_m} = k_m. \tag{92}
\]
where \(\chi_e\) and \(\chi_m\) denote the electric and magnetic susceptibilities respectively, and \(k_e\) and \(k_m\) denote the dielectric constant and the relative permeability respectively. Given that the constant of motion \(M\) means (total mechanical energy)/\(\mu_0 c^2\) as discussed in the sections III.E.2-3, we can correlate \(\chi_e\) and \(k_e\) with the total mechanical energy by using Eq. (91). Though not discussed here, this issue is also needed to study furthermore.

Furthermore, from Eqs. (91) and (92), we can have
\[
M \phi_{10} c^2 = \frac{1}{\epsilon \mu} \equiv c_m^2, \tag{93}
\]
where \(c_m^2 = \frac{c^2}{(1 + \chi_e)(1 + \chi_m)}\),
\[
1 - \frac{\partial^2 \Phi}{c_m^2 \partial t^2} - \nabla^2 \Phi = \frac{\rho}{\epsilon_0}, \tag{95}
\]
\[
1 - \frac{\partial^2 \mathbf{A}}{c_m^2 \partial t^2} - \nabla^2 \mathbf{A} = \frac{j}{\epsilon_0 c^2}. \tag{96}
\]
Finally, from the above Eqs. (54b) and (69a), the scalar potential $\Phi$ can be expressed by using Eqs. (19) and (42), as follows:

$$\Phi = \frac{c^2}{2} \left( 1 - \frac{u^2}{c^2} \right) \left( 1 + \frac{2q k_s}{rc^2} \right).$$

(97)

Here, for $u/c \ll 1$, Eq. (97) can be approximately written by

$$\Phi \approx \frac{c^2}{2} + \frac{q_k k_s}{r}.$$  

(98)

Since the term $c^2/2$ of Eq. (98) is constant, it plays no role in the electromagnetic laws written as the differential form. In this sense, we may say that the classical Coulomb potential is approximation of Eq. (97) for the case of slow source object, because $k_s$ is $1/4\pi \varepsilon_0$ as mentioned above. Nevertheless, the above equation (97) has some difference with the known relativistic vector potential. As a result, the vector potential $A$ has also some difference with the known relativistic vector potential, because the vector potential $A$ is defined by the scalar potential $\Phi$ as seen in the Eq. (69b). I hope that further studies will be pursued for overcoming this difference and for completing the afore-mentioned some assumptions.

7. Quantum Mechanics

From the previous discussions, matter and space is classified based on the difference in the density of ex-entity, and the movement of universally distributed object can be understood as the phenomenon of wave-like propagation thereof. In this sense, it seems that wave and particle are not contradictory concepts but only concepts that fall under different categories. That is, the wave corresponds to the phenomenal concept that represents the change mode of ex-entity or the physical phenomena, and the particle does the concept on state that represents the quantitative state of ex-entity or the distributional state of ex-entity density that has strong stability. Hence, the duality of matter and wave, which was the starting point of quantum mechanics, is not contradictory.

In addition, the function of probability density in the Schrödinger equation is very similar to the afore-discussed function of ex-entity density in that they contain all physical information of system. In this sense, it is obvious that the function of probability density should be understood in connection with the function of ex-entity density, even though we cannot say that two density functions are identical with each other. This subject is also needed to study furthermore. Additionally, in this section III.F., we have mentioned several subjects required for further study, but it seems that most of these subjects are closely related to the quantum mechanics.

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