Research on the method of warship formation threat assessment based on structural contribution degree

Guibao Song, Zhenyu Liu, Tie Liu, Yugong Qiang

Department of Missile General and Engine, Naval Aviation University, Yantai, 264001, China
committee_abby@keoaeic.org.cn

Abstract. In this paper, a threat assessment method based on the structural contribution of equipment is proposed. The complex network theory, entropy weight method, grey correlation method and other assessment models are comprehensively used to deconstruct the structural contribution of equipment to combat tasks from the perspective of the contribution of each ship to the structure of the entire formation. The research results can help to determine the benefit value and priority of each ship attacking the enemy formation, guide our commanders and soldiers to destroy the air defense system of the enemy formation so as to reduce the function of the system, and provide reference for the target allocation of attacking the enemy formation.

1. Introduction

With the continuous development of the operational mode of modern naval warfare, the key to the outcome of battle no longer simply depends on the performance of a single weapon and equipment, but on the overall operational effectiveness formed by the coupling of multiple combat units. The structural contribution of equipment is the importance of equipment as nodes in the formation joint air defense combat system network, which is mainly reflected in the degree to which a node in the complex network affects the network structure and the integrity of the network function. Structural contribution is the embodiment of the functional status of equipment in the joint air defense system. Equipment with large structural contribution can often play a multi-functional role of crossing and connecting[1]. Considering the network topology characteristics of the system where the equipment is located, this paper proposes an evaluation model based on complex network theory to measure the contribution of the equipment structure.

2. Evaluation model research

2.1 Theoretical Basis

(1) Complex network theory

Complex networks are large-scale networks characterized by network topology and dynamic behavior. Network phenomena are described and studied by analyzing the relationship between network topology and system components[2]. Complex network modeling method based on the whole, the biggest characteristic is to comprehensively study the dynamic behavior of the system components and the interaction between components, stick to the decisive relationship between system structure and function, is to engrave the individual in the system, and can describe and analyze the individual and the whole system function because of the role relationship between the emergence of effect.

(2) Network description of the joint air defense combat system of naval formations
The network description of combat system is to use the network topology structure to reproduce the influence of equipment in the process of combat system from the perspective of graph theory, which mainly involves the type of node and edge, and the type of edge is determined by the type of node. Complex networks are generally divided into homogeneous networks and heterogeneous networks. The homogeneous network does not distinguish node types and homogenizes all nodes, so the connection relationship between nodes is also single. Heterogeneous networks are more common, and the types of functional edges are often distinguished by the different properties and functions of network nodes.

Obviously, the combat system formed by various types and various levels of equipment systems in the combat of the joint air defense system of naval formations is called the complex heterogeneous network of the joint air defense operation of naval formations. The equipment to be evaluated is the node of the network, and the influence relationship between each equipment is taken as the side for research.

2.2 Determine the evaluation index of structural contribution

The architecture is the way that the various elements in the weapon system cooperate and interact with each other. The architecture of weapons and equipment usually has certain basic characteristics, which are typically flexible, efficient and robust. In this paper, the structural contribution of warship formation is studied, and the evaluation index is determined based on the three typical characteristics.

Flexibility is a measure of the ability of the architecture of a weapon to adapt to combat missions and changing environments. The flexibility studied in this paper refers to the measurement of the ability of each ship's formation to adapt to the changing architecture with the changing combat environment in a complex and changeable combat environment, which is embodied in the coordination ability between each ship. When applied to air defense combat network, it represents the degree of communication between vessels as nodes and other nearby vessels. The higher the degree of communication, the more important the node is. The aggregation coefficient in the complex network theory can reflect the integrity of the network near the nodes, that is, it can effectively reflect the structural flexibility of each ship in the formation.

Efficiency is a measure of the operational efficiency of the architecture of weapons and equipment. The efficiency studied in this paper refers to the measurement of the combat efficiency of each ship under the formation architecture when the formation is performing air defense combat tasks, which is specifically reflected in the transmission efficiency between each ship. When applied to air defense combat network, it represents the efficiency of each ship as node to transmit information to each other, and the shortest transmission route means that the node is more efficient. Degree centrality in complex network theory refers to the ratio between the vertex degree of a node and the maximum possible vertex degree of the node i, which reflects the local connectivity of the node. The greater degree centrality is, the easier and more efficient information transmission will be in the network. Therefore, degree centrality can reflect the high efficiency of the structure of each ship in the formation.

Robustness is a measure of the capability of each subsystem and its relationship to maintain the original operational effectiveness even after the failure. The robustness studied in this paper refers to the measurement of the combat effectiveness that the overall structure of the formation can maintain when the formation performs air defense tasks without a certain warship. The lower the combat effectiveness is maintained, the more important the missing warship is as a node. When applied to the air defense combat network, it means that the structure of the network can still maintain a certain structural integrity and the combat effectiveness will not be greatly affected if a certain ship is damaged. The toughness in the complex network theory can reflect the ability of the architecture to keep connected after the partial failure of each subsystem, that is, it can effectively reflect the structural robustness of each ship in the formation.

Therefore, based on the typical characteristics of weapon and equipment architecture, this paper selects three indicators, namely aggregation coefficient, degree centrality and degree of destruction resistance, to evaluate the flexibility, efficiency and robustness of nodes respectively.
(1) Flexibility index: aggregation coefficient
The definition of aggregation coefficient is as follows:
\[ O_i = \frac{2u}{k(k-1)} \]  
(1)
Among them,
\[ k \]: A node has \( k \) adjacent nodes;
\[ k(k-1) \]: Theoretically, the \( k \) nodes can be connected by \( k(k-1) \) edges at most;
\[ u \]: Actually these \( k \) nodes are connected by \( u \) edges.

(2) Efficiency index: degree centrality
Degree centrality is defined as follows:
\[ D_i = \frac{d_i}{N-1} \]  
(2)
Among them,
\[ d_i \]: Vertex degree of node \( i \);
\[ N \]: Total number of nodes in the network;
\[ N-1 \]: The number of maximum join edges that a node may have.

(3) Robustness index: toughness
The toughness is defined as follows:
\[ T_i = \frac{1+n}{m} \]  
(3)
Among them:
\[ n \]: The number of nodes with the largest branch of the network after the node is removed;
\[ m \]: The maximum number of branches of the network after node removal;
\[ l \]: Number of nodes removed.

2.3 Determine the evaluation model
The structural contribution of evaluation nodes is a multi-attribute evaluation problem that takes each node in the network as the evaluation object, takes the above three indexes as the evaluation attribute, and comprehensively evaluates under a certain weight. The core content of this research is to select the appropriate weighting and evaluation methods.

(1) Determine the weight analysis method
To determine the evaluation model, the weight of each index should be determined first. Entropy weighting method is a relatively objective weighting method, which is based on the amount of information contained in each index\[7\]. For evaluation indicators, if the entropy value of an indicator is higher, it means that the degree of variation of the indicator is smaller, the information provided is smaller, and the impact on the evaluation result is smaller, so the weight should be smaller. Conversely, the greater the weight. The entropy weight method is more objective to determine the index weight.

(2) Determine the evaluation method
The evaluation method is the key to the evaluation model. In this paper, three typical indexes are selected for the evaluation of structural contribution degree, but they are not complete, and the gray system is composed of small samples with incomplete information and not strong regularity, which can solve the problem of incomplete evaluation indexes in this paper. Therefore, this paper chooses grey relational method for evaluation.

The application of gray correlation method to evaluate the importance index of nodes is to determine the theoretical optimal node index, then compare each node index with the optimal node.
(3) Model construction

By multiplying the index weight obtained by entropy weight method and the correlation coefficient obtained by grey correlation method, the weighted correlation degree, namely the structural contribution degree of the node index studied, is obtained. This method is used for evaluation, and the specific steps are as follows:

Step 1: Determine the initial evaluation matrix. The evaluation matrix is composed of evaluation nodes and evaluation indexes. Given the above three evaluation indexes, and assuming that there are N evaluation nodes to be evaluated, the matrix can be expressed as $R = \left[ r_{ij} \right]_{3 \times n}$, which can be obtained after standardized treatment:

$$
R' = (r'_{ij})_{3 \times n}
$$

The standardized treatment can remove the differences of dimension and quantity of each element and facilitate unified calculation. The standardized calculation formula is as follows:

$$
r_{ij}' = \frac{r_{ij} - min_{i}}{max_{i} - min_{i}}
$$

Among them:

- The optimal value in different nodes under the i-th index;
- The worst value of the different nodes under the j-th index.

Step 2: Calculate the index entropy value and entropy weight. Firstly, under the j-th evaluation index, the proportion of the index value of each node in $\chi_i$ was determined, and the formula was as follows:

$$
\chi_{ij} = \frac{r_{ij}'}{\sum_{i}^n r_{ij}'}
$$

According to the definition, the entropy value is calculated as follows:

$$
E_{ij} = \frac{\sum_{i}^n \chi_{ij} \ln \chi_{ij}}{\ln n}
$$

Since entropy is inversely proportional to the amount of information, and the amount of information is proportional to the ability of indicators to influence the evaluation results, the entropy weight calculation formula can be expressed as:

$$
\omega_j = \frac{1 - E_{ij}}{\sum_{i}^3 (1 - E_{ij})}
$$

Step 3: Determine the reference sequence and construct the absolute difference matrix

The reference sequence $\partial_0$ is composed of the maximum node evaluation value under each index. Since three evaluation indexes are selected, the reference number is listed as follows:

$$
\partial_0 = \{ \partial_0 (1), \partial_0 (2), \partial_0 (3) \}
$$

Among them:
\( \mathcal{G}_0(j) \): the optimal value of each node's evaluation value under the j-th evaluation index (since the four indexes are all profitability indexes, the optimal value is the maximum value) the comparison sequence is the row vector composed of the evaluation value of each node under the j-th evaluation index:

\[
\varepsilon_j(i) = \{r_{1j}', r_{2j}', \ldots, r_{nj}'\}
\]

(10)

Construct the absolute difference matrix:

\[
\psi = (\psi_{ji})_{3n \times n} = \begin{bmatrix}
\psi_{11} & \psi_{12} & \cdots & \psi_{1n} \\
\psi_{21} & \psi_{22} & \cdots & \psi_{2n} \\
\psi_{31} & \psi_{32} & \cdots & \psi_{3n}
\end{bmatrix}
\]

(11)

Among them,

\[
\psi_{ji} = |\varepsilon_j(i) - \mathcal{G}_0(j)|: The absolute value of the difference between the evaluated value and the optimal value at the JTH node of the ith evaluation index.
\]

Step 4: Solve the correlation coefficient

The formula for calculating the correlation coefficient can be expressed as follows:

\[
I_j = \frac{\min_{i,j} \psi_{ij} + f_{ib} \max_{i,j} \psi_{ij}}{\psi_{ij} + f_{ib} \max_{i,j} \psi_{ij}}
\]

(12)

Among them:

\( I_j \) is the correlation coefficient, which indicates the correlation degree between the evaluation value of the j-th node and the optimal value of the index under the i-th index.

\( f_{ib} \): Resolution coefficient, 0.5 under normal conditions;

\( \min_{i,j} \psi_{ij} \): Second-order minimum difference;

\( \max_{i,j} \psi_{ij} \): Maximum difference of level 2.

The weighted average value of the correlation coefficient was calculated to obtain the grey relational degree \( \rho_i \) of node j, namely the comprehensive correlation degree between the j-th node and the optimal value of each index, and the formula was as follows:

\[
\rho_i = \sum_{j=1}^{3} I_j \cdot \omega_j
\]

(13)

Obviously, the closer it is to 1, the closer the j-th node is to the optimal index, and the greater the structural contribution of this node is, so it is recorded as the structural contribution of equipment j.

3. Example analysis

According to the target assessment method of ships in the formation studied in this paper, the following operational scenarios are set:

The naval fleet shall, in carrying out the combat mission against the enemy naval fleet, assess the threat of the enemy naval vessels before formulating the combat plan. According to relevant intelligence, the enemy's driving and guarding formation is composed of three ships, which are now subdivided into three operational subsystems, namely, early warning system, command and control system and interception system. The three operational subsystems of the three ships can cooperate with each other, and the cooperative relationship among them is shown in Table 3-1. (CAC stands for Command and control system; EWS stands for Early warning system; IS stands for Intercepting system)
Table 1 Connection relation of each system of the fleet

| The node input | Node in the output | Interactive node |
|----------------|-------------------|------------------|
| EWS 1          | None              | CAC1, CAC2, CAC3 |
| CAC1           | EWS 1, EWS 3     | IS 1             |
| intercept1     | CAC1, CAC2, CAC3 | None             |
| warning2       | None              | CAC2, CAC3       |
| CAC2           | EWS 1, EWS 2, EWS 3 | IS 1, IS 2     |
| intercept2     | CAC2              | None             |
| warning3       | None              | CAC1, CAC2, CAC3 |
| CAC3           | EWS 1, EWS 2, EWS 3 | IS 1, IS 3     |
| intercept3     | CAC3              | None             |

According to Table 3-1, the network model of the combat system composed of nine combat subsystems was built (see Figure 3-1). The evaluation and calculation method above was used to calculate the structural contribution of each subsystem, and the sequence was carried out on this basis.

Figure 1 Network model of ship's systems

3.1 Calculate index weight

Step 1: Calculate the evaluation value of the structural contribution evaluation index
(1) Aggregation coefficient
According to the ship network model, k and u of different nodes can be obtained. Then, by applying formula (1), aggregation coefficients of different indicators can be calculated, as shown in Table 3-2 below:

Table 2 Aggregation coefficient evaluation values of each node

|          | EWS1 | CAC1 | IS1 | EWS2 | CAC2 | IS2 | EWS3 | CAC3 | IS3 |
|----------|------|------|-----|------|------|-----|------|------|-----|
| k        | 4    | 3    | 3   | 3    | 6    | 1   | 6    | 1    | 6   |
| u        | 2    | 1    | 1   | 2    | 6    | 0   | 5    | 6    | 0   |
| 0_k      | 0.5  | 0.333| 0.333| 0.667| 0.714| 0   | 0.6  | 0.4  | 0   |

(2) Degree centrality
According to the ship network model, d_i and N of different nodes can be obtained. Then, by applying formula (2), the degree centrality of different indexes can be calculated, as shown in Table 3-3 below:

Table 3 Evaluation values of degree centrality of each node

|          | EWS1 | CAC1 | IS1 | EWS2 | CAC2 | IS2 | EWS3 | CAC3 | IS3 |
|----------|------|------|-----|------|------|-----|------|------|-----|
| d_i      | 8    | 7    | 8   | 6    | 7    | 8   | 6    | 8    | 7   |
| N        | 29   | 34   | 33  | 34   | 15   | 39  | 26   | 27   | 36  |
(3) Toughness after modification

According to the ship network model, \( n \) and \( M \) of different nodes can be obtained, and then formula (3) is applied to calculate the toughness of different indexes after transformation, as shown in Table 3-4 below:

| \( \text{EWS} \) | \( \text{CAC} \) | \( \text{IS} \) | \( \text{EWS2} \) | \( \text{CAC2} \) | \( \text{IS2} \) | \( \text{EWS3} \) | \( \text{CAC3} \) | \( \text{IS3} \) |
|---|---|---|---|---|---|---|---|---|
| \( n \) | 8 | 7 | 8 | 6 | 7 | 8 | 6 | 7 |
| \( m \) | 29 | 34 | 33 | 34 | 15 | 39 | 26 | 27 | 36 |
| \( T_{ij} \) | 0.402 | 0.274 | 0.255 | 0.21 | 0.5 | 0.156 | 0.383 | 0.479 | 0.184 |

On the basis of obtaining the evaluation values of the three structural contribution indicators, standardized treatment was carried out according to formula (5). The results after treatment are shown in Table 3-5:

| node | Aggregation coefficient | Degree of centrality | Toughness after modification |
|---|---|---|---|
| EWS1 | 0.5 | 0.5 | 0.804 |
| CAC1 | 0.333 | 0.5 | 0.548 |
| IS1 | 0.333 | 0.5 | 0.51 |
| EWS2 | 0.667 | 0.333 | 0.42 |
| CAC2 | 0.714 | 1 | 1 |
| IS2 | 0 | 0.167 | 0.312 |
| EWS3 | 0.6 | 0.5 | 0.766 |
| CAC3 | 0.4 | 0.833 | 0.958 |
| IS3 | 1 | 0.333 | 0.368 |

Step 2: Calculate entropy and entropy weight

On the basis of obtaining the evaluation value of the indicators, the entropy weight method is adopted to calculate the entropy value and entropy weight of the three evaluation indexes according to formulas (6), (7) and (8). The results are shown in Table 3-6:

| \( X_{ij} \) | \( E_{n,j} \) | \( \omega_j \) |
|---|---|---|
| 0.256 | 0.317 | 0.28 |
| 0.336 | 0.423 | 0.40 |
| 0.408 | 0.551 | 0.32 |

3. 2 Calculate the structural contribution value

In this paper, we study the three assessment evaluation index of the largest value of 1, according to the formula (10), (11) calculate the absolute difference value matrix \( \Psi \), the results are as follows:

\[
\Psi = \begin{pmatrix}
0.5 & 0.5 & 0.196 \\
0.667 & 0.5 & 0.452 \\
0.667 & 0.5 & 0.491 \\
0.333 & 0.667 & 0.58 \\
0.286 & 0 & 0 \\
1 & 0.833 & 0.687 \\
0.4 & 0.5 & 0.234 \\
0.6 & 0.167 & 0.042 \\
0 & 0.667 & 0.632
\end{pmatrix}
\]
The correlation coefficient of each indicator is calculated according to formula (10). The calculation results are shown in Table 3-7:

|       | Aggregation coefficient | Degree of centricity | Invulnerability degrees |
|-------|-------------------------|----------------------|-------------------------|
| EWS1  | 0.46                    | 0.46                 | 0.69                    |
| CAC1  | 0.39                    | 0.46                 | 0.49                    |
| intercept1 | 0.39                   | 0.46                 | 0.47                    |
| EWS2  | 0.56                    | 0.39                 | 0.43                    |
| CAC2  | 0.6                     | 1                    | 1                       |
| intercept2 | 0.3                    | 0.34                 | 0.38                    |
| EWS3  | 0.52                    | 0.46                 | 0.65                    |
| CAC3  | 0.42                    | 0.72                 | 0.91                    |
| intercept3 | 1                      | 0.39                 | 0.40                    |

On the basis of the correlation coefficient of each index and its weight, the structural contribution value of each node is calculated according to formula (13). The results are shown in Table 3-8:

|       | EWS1 | CAC1 | IS1 | EWS2 | CAC2 | IS2 | EWS3 | CAC3 | IS3 |
|-------|------|------|-----|------|------|-----|------|------|-----|
| Contribution to the structure | 0.53 | 0.45 | 0.44 | 0.45 | 0.89 | 0.34 | 0.54 | 0.70 | 0.56 |

4. Conclusion
In this paper, the target fleet formation is taken as the main body of research and a threat assessment method based on structural contribution degree is proposed. In the evaluation, an evaluation model is established from the perspective of structural contribution, and an example is given to verify the feasibility of the evaluation method studied.

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