Exclusive $W^+ + \gamma$ production in proton-antiproton collisions II: results

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Abstract

We present results for total cross sections, single and double differential distributions and correlations between pairs of outgoing particles in the reactions $p + \bar{p} \rightarrow W^+ + \gamma$ and $p + \bar{p} \rightarrow W^+ + \gamma + {\rm jet}$ at $\sqrt{S} = 1.8$ TeV. Order $\alpha_S$ QCD corrections and leading logarithm photon bremsstrahlung contributions are included in the $\overline{MS}$ mass factorization scheme for three experimental scenarios: 1) 2-body inclusive production of $W^+$ and $\gamma$, 2) exclusive production of $W^+$, $\gamma$ and 1 jet and 3) exclusive production of $W^+$ and $\gamma$ with 0 jet. The latest CTEQ parton distribution functions, which fit the newly released HERA data, are used in our analysis. The dependence of our results on the mass factorization scale is used to place error bars on our predictions for the single differential distributions and correlations.
I INTRODUCTION.

In the first part of this study [1] we presented the general formalism for the computation of exclusive cross sections in $p + \bar{p} \rightarrow W^+ + \gamma$ and $p + \bar{p} \rightarrow W^+ + \gamma + \text{jet}$. We examined the order $\alpha_S$ and photon bremsstrahlung contributions to these cross sections in the $\overline{\text{MS}}$ mass factorization scheme. The analytical expressions obtained in that work were then combined with a Monte Carlo integration routine, namely Vegas [2], to produce numerical results for three different experimental scenarios: 1) 2-body inclusive production of $W^+$ and $\gamma$, 2) exclusive production of $W^+$, $\gamma$ and 1 jet and 3) exclusive production of $W^+$ and $\gamma$ with 0 jet. The resulting programs have been run for $\sqrt{S} = 1.8$ TeV in the proton-antiproton center of mass system (Fermilab Tevatron). We include 4 massless flavours $u, d, s$ and $c$ in our partonic hard scattering processes and set $\cos \theta_C = 0.95$ where $\theta_C$ is the Cabbibo angle. We use the one loop strong running coupling constant with $\Lambda_{QCD} = 0.139$ GeV. The $W$ boson is assumed to be an on shell physical particle with mass $M_W = 80.2$ GeV.

We use the latest available CTEQ parton distribution functions [3] in the $\overline{\text{MS}}$ mass factorization scheme (set CTEQ2M). These parton distribution functions are in agreement with the latest CCFR NLO analysis of the strange quark density and fit the newly released HERA data (for details see [3].)

The three experimental scenarios considered here are defined as follows (see also the discussion in [1]):

1) 2 BODY INCLUSIVE PRODUCTION OF $W^+$ AND $\gamma$.

In this scenario ("2 body inclusive scenario") the reaction $p + \bar{p} \rightarrow W^+ + \gamma + X$ with $X = 0$ or 1 jet is considered. In other words, one detects the outgoing $W^+$ boson and the photon but does not tag the outgoing jet. We therefore use the following kinematic restrictions on the outgoing particles, in the proton-antiproton center of mass frame:

$$|\cos \theta_\gamma|, |\cos \theta_W| < \cos(0.3 \text{ rad})$$
$$P_{t \gamma}, P_{t W} > 10.0 \text{ GeV}$$
$$R_{W, \gamma} > 0.2$$
$$(R_{\text{jet}, \gamma} < 0.2) \implies (s_{(\text{jet}, \gamma)} < 0.2)$$
$$(R_{\text{jet}, W} < 0.2) \implies (s_{(\text{jet}, W)} < 0.2), \quad (1.1)$$

where $\theta_i$ is the angle between the incoming proton axis and the axis of the outgoing particle $i$ and $P_{t i}$ is the transverse momentum of particle $i$. $R_{i,j}$ is the cone size between a pair of outgoing particles: $R_{i,j} = \sqrt{(\eta_{i,j})^2 + (\phi_{i,j})^2}$ with $\eta^*$ the pseudorapidity and $\phi$ the azimuthal angle; $s_{(\text{jet}, W)} = E_{\text{jet}}/E_W$ is the "shadowing ratio" between the untagged jet and the $W^+$ boson. The third condition in (1.1) removes events where the $W^+$ boson is too close to the photon. The last two conditions remove events where the jet which is too close to the $W^+$ boson or photon is at the same time of comparable energy so that it would "shadow" one of the two tagged particles making it undetectable. This might be the case for bremsstrahlung contributions with a small photon momentum fraction.

Several artificial parameters, namely $x_0, y_0$ and $v_0$ were introduced in [1] in order to control the numerical cancellation of singularities. Results for physical quantities like total cross sections and differential distributions and correlations do not depend on the choice of
these parameters, however, the requirement of stability and small numerical errors in the Monte Carlo runs lead us to the choice \( x_0 = (1 + \rho(s))/2, \) \( y_0 = v_0 = 0.05 \) for the partial cross sections in the \( q\bar{q} \) channel and \( y_0 = v_0 = 0.05 \) for the partial cross sections in the \( qg \) and \( g\bar{q} \) channels. For the cut parameters defined in section V of [1] we choose the following values: \( \Delta_x = 10^{-5} \) and \( \Delta_y = \Delta_v = 10^{-8} \). As discussed in the referred section these cuts introduce errors in the numerical results, however, we have checked that the above choice minimizes these errors while keeping the Monte Carlo program stable. Our tables of results for total cross sections include entries for the lowest order estimates of these errors and confirm that they are negligible. Thus the expressions for the errors are neglected in all the distributions and correlations.

2) **Exclusive production of \( W^+, \gamma \) and 1 jet.**

In this scenario (“1 jet scenario”) one detects three outgoing particles, namely the \( W^+ \), \( \gamma \) and 1 jet. This scenario is thus defined by the following kinematic conditions:

\[
|\cos \theta_\gamma|, |\cos \theta_W|, |\cos \theta_{\text{jet}}| < \cos(0.3 \text{ rad})
\]

\[
Pt_\gamma, Pt_W, Pt_{\text{jet}} > 10.0 \text{ GeV}
\]

\[
R_{W,\gamma} > 0.2
\]

\[
R_{\text{jet},\gamma} > 0.2
\]

\[
R_{\text{jet},W} > 0.2.
\]

(1.2)

The above conditions will automatically remove bremsstrahlung contributions which tend to produce jets parallel to the outgoing photon. In the Monte Carlo runs we choose the values for the artificial parameters of this scenario in all channels as: \( x_0 = 1, \ y_0 = v_0 = 0 \). This choice and the above cuts guarantee that no unphysical dependence is introduced in the calculation of physical quantities related to the jet. Moreover, the cuts \( \Delta_x, \Delta_y, \Delta_v \) introduced in the first scenario are not necessary here and thus the associated errors are zero in this case.

3) **Exclusive production of \( W^+ \) and \( \gamma \) with 0 jet.**

In this scenario (“0 jet scenario”) we select events where the \( W^+ \) boson and the photon are detected and no outgoing jet is detected. This will include 2 to 3 body events where the outgoing jet is not detected because it has either a small angle with respect to the beam or a small transverse momentum or it is “shadowed” by the \( W^+ \) boson or the photon. The following kinematic conditions define this scenario:

\[
|\cos \theta_\gamma|, |\cos \theta_W| < \cos(0.3 \text{ rad})
\]

\[
Pt_\gamma, Pt_W > 10.0 \text{ GeV}
\]

\[
R_{W,\gamma} > 0.2
\]

\[
(R_{\text{jet},\gamma} < 0.2) \implies (s_{(\text{jet},\gamma)} < 0.2)
\]

\[
(R_{\text{jet},W} < 0.2) \implies (s_{(\text{jet},W)} < 0.2)
\]

\[
(\cos \theta_{\text{jet}} > \cos(0.3 \text{ rad})) \quad \text{or} \quad (Pt_{\text{jet}} < 10.0 \text{ GeV}).
\]

(1.3)

As already discussed in [1] the results for this scenario can be obtained by subtracting the results for the 1 jet scenario from the corresponding results for the 2 body inclusive scenario.
In section II we present results for total cross sections. Section III is devoted to the discussion of single differential distributions and correlations. In section IV we present and discuss results for double differential cross sections and correlations. In section V we end our study with some concluding remarks.
II TOTAL CROSS SECTIONS.

II.A Brief Review of Relevant Formulae.

We write the total hadronic cross section as:

\[
\sigma^H = \int_0^1 d\tau_1 \int_0^1 d\tau_2 \{f_{qg}(\tau_1)f_{qg}(\tau_2)\sigma_{q\bar{q}} + f_{qg}(\tau_1)f_{gq}(\tau_2)\sigma_{qg} + f_{gp}(\tau_1)f_{qg}(\tau_2)\sigma_{g\bar{q}} + (p \leftrightarrow \bar{p}, \tau_1 \leftrightarrow \tau_2, p_1 \leftrightarrow p_2, P_1 \leftrightarrow P_2)\}
\]

with

\[
\begin{align*}
\sigma_{q\bar{q}} &= \sigma_{q\bar{q}}^{\text{Born}} + \sigma_{q\bar{q}}^P (SV) + \sigma_{Ia} + \sigma_{Ib} + \sigma_{I\alpha} + \sigma_{q\bar{q}}^{\text{finite}} (\text{finite}) + \sigma_{q\bar{q}} (\text{Brems}) + \sigma_{q\bar{q}} (\text{error}) \\
\sigma_{qg} &= \sigma_{qg,\text{finite}} + \sigma_{qg,\text{coll}} + \sigma_{qg}^{\text{col}} + \sigma_{qg,\text{Brems}} + \sigma_{qg} (\text{Brems}) + \sigma_{qg} (\text{error}) \\
\sigma_{g\bar{q}} &= \sigma_{g\bar{q},\text{finite}} + \sigma_{g\bar{q},\text{coll}} + \sigma_{g\bar{q}}^{\text{coll}} + \sigma_{g\bar{q}} (\text{Brems}) + \sigma_{g\bar{q}} (\text{error})
\end{align*}
\]

and

\[
\begin{align*}
\sigma_{Ia} &= \sigma_{Ia,1} + \sigma_{Ia,2} + \sigma_{Ia,3} \\
\sigma_{Ib} &= \sigma_{Ib,1} + \sigma_{Ib,2} + \sigma_{Ib,3} \\
\sigma_{q\bar{q}}^{\text{(finite)}} &= \sigma_{q\bar{q}} f_{1,1,a} + \sigma_{q\bar{q}} f_{1,2,a} + \sigma_{q\bar{q}} f_{1,3,a} + \sigma_{q\bar{q}} f_{1,1,b} + \sigma_{q\bar{q}} f_{1,2,b} + \sigma_{q\bar{q}} f_{1,3,b} \\
&+ \sigma_{q\bar{q}} f_{2,1,a} + \sigma_{q\bar{q}} f_{2,2,a} + \sigma_{q\bar{q}} f_{2,3,a} + \sigma_{q\bar{q}} f_{2,1,b} + \sigma_{q\bar{q}} f_{2,2,b} + \sigma_{q\bar{q}} f_{2,3,b} + \sigma_{q\bar{q}} f_{3} \\
\sigma_{qg,\text{finite}} &= \sigma_{qg,\text{finite}} f_{1,1,a} + \sigma_{qg,\text{finite}} f_{1,2,a} + \sigma_{qg,\text{finite}} f_{1,3,a} + \sigma_{qg,\text{finite}} f_{1,1,b} + \sigma_{qg,\text{finite}} f_{1,2,b} + \sigma_{qg,\text{finite}} f_{1,3,b} \\
\sigma_{qg}^{\text{finite}} &= \sigma_{qg}^{\text{finite}} f_{1,1,a} + \sigma_{qg}^{\text{finite}} f_{1,2,a} + \sigma_{qg}^{\text{finite}} f_{1,3,a} + \sigma_{qg}^{\text{finite}} f_{1,1,b} + \sigma_{qg}^{\text{finite}} f_{1,2,b} + \sigma_{qg}^{\text{finite}} f_{1,3,b} \\
\sigma_{Ig,\text{finite}} &= \sigma_{Ig,\text{finite}} f_{1,1,a} + \sigma_{Ig,\text{finite}} f_{1,2,a} + \sigma_{Ig,\text{finite}} f_{1,3,a} + \sigma_{Ig,\text{finite}} f_{1,1,b} + \sigma_{Ig,\text{finite}} f_{1,2,b} + \sigma_{Ig,\text{finite}} f_{1,3,b} \\
\sigma_{g\bar{q}}^{\text{finite}} &= \sigma_{g\bar{q}}^{\text{finite}} f_{1,1,a} + \sigma_{g\bar{q}}^{\text{finite}} f_{1,2,a} + \sigma_{g\bar{q}}^{\text{finite}} f_{1,3,a} + \sigma_{g\bar{q}}^{\text{finite}} f_{1,1,b} + \sigma_{g\bar{q}}^{\text{finite}} f_{1,2,b} + \sigma_{g\bar{q}}^{\text{finite}} f_{1,3,b} \\
\end{align*}
\]

Each term in (2.2) and (2.3) has been explained in section V of [1]. Note that the replacements in parenthesis in (2.1) apply to all explicit as well as implicit variables in the above expressions. The last replacement in the parenthesis in (2.1) will act on physical variables that go into experimental cut functions and histograms (see section IV.D of [1].)

For the bremsstrahlung contributions we use here the so called leading logarithm approximation, which has been previously used by other authors [1],[3],[4]. In this approximation we write the photon fragmentation functions as:

\[
\begin{align*}
f_{\gamma q}(x,M) &= \frac{\alpha}{2\pi} \ln \left( \frac{M^2}{\Lambda_{QCD}^2} \right) \left[ \frac{e_{q\bar{q}}^2 (2.21 - 1.28 x + 1.29 x^2) x^{-0.951}}{1 - 1.63 \ln (1 - x) + 0.002 (1 - x)^{2.0} x^{-2.54}} \right] \\
f_{\gamma g}(x,M) &= \frac{\alpha}{2\pi} \ln \left( \frac{M^2}{\Lambda_{QCD}^2} \right) 0.0243 (1 - x)^{1.03} x^{-1.97},
\end{align*}
\]

where \(e_{q\bar{q}}\) denotes the charge of the outgoing quark (antiquark) \(q\) (\(\bar{q}\)) in units of \(e\), \(M\) is the mass factorization scale and the running electromagnetic fine structure constant is \(\alpha = e^2 (\mu^2) / 4\pi\).
II.B  Total cross section for the 2 body inclusive reaction \( p + \bar{p} \rightarrow W^+ + \gamma + X \).

All terms in expressions (2.2) and (2.3) will produce nonvanishing contributions in this scenario. In table 1 we show the results for the \( q\bar{q} \) channel cross sections in pb at the three scales \( r = 0.5, 1.0 \) and 2.0, where \( r = M/M_W = \mu/M_W \) and \( M, \mu \) are the mass factorization and renormalization scales, respectively. The second entry of table 1 shows the value used for \( \alpha_S \) at these scales. In all our runs we use \( \alpha = 1/137.036 \) for the fine structure constant and \( G_F = 1.16639 \cdot 10^{-5} \text{GeV}^{-2} \) for the Fermi coupling constant.

The corresponding results for the \( qg \) channel contributions to the 2-body inclusive hadronic reaction \( p + \bar{p} \rightarrow W^+ + \gamma + X \) are summarized in table 2. Finally, the corresponding results for the \( g\bar{q} \) channel are summarized in table 3.

If we compare these results with the ones previously reported for the single particle inclusive total cross section in [7] we note a considerable reduction of the total cross section in the present calculation. The smaller values can be attributed to larger cut in the photon angle and the effect of experimental cuts on the \( W^+ \) boson which was treated previously in an inclusive fashion.

We also note that in the approximation we are using here the bremsstrahlung contributions are negligible in the \( q\bar{q} \) channel and that their contribution in the other channels amounts to not more than 2% of the total cross section. This would make big deviations from the leading logarithm bremsstrahlung approximation easy to study.

The next to last rows in tables 1, 2 and 3 show that the errors introduced by the \( \Delta_x, \Delta_y \) and \( \Delta_v \) cuts (see section V of [1]) are negligible and thus our prescription for splitting the \( x, y \) and \( v \) integrals when adding up the histograms is consistent.

II.C  Total cross sections of \( p + \bar{p} \rightarrow W^+ + \gamma + jet \) and \( p + \bar{p} \rightarrow W^+ + \gamma \).

With the choice of parameters \( x_0, y_0 \) and \( \nu_0 \) explained in section I the only terms in (2.2) contributing to the reaction \( p + \bar{p} \rightarrow W^+ + \gamma + jet \) are:

\[
\begin{align*}
\sigma_{q\bar{q}} &= \sigma_{f,1,1,a} + \sigma_{f,1,1,b} \\
\sigma_{qq} &= \sigma_{qg,f,1}^I + \sigma_{qg,f,1}^{II} + \tilde{\sigma}_{qg}^P \\
\sigma_{g\bar{q}} &= \sigma_{gq,f,1}^I + \sigma_{gq,f,1}^{II} + \tilde{\sigma}_{gq}^P.
\end{align*}
\]

The results for the contributions of all these terms to the hadronic cross section are shown in table 1 for the three scales \( r = 0.5, 1.0 \) and 2.0. At these scales we thus obtain for the total cross sections of the 1 jet reaction values of 2.15, 1.79 and 1.51 pb respectively. As in the 2 body inclusive case the variation is large in the 1 jet scenario so we need to include higher order QCD corrections to make more accurate predictions. Unfortunately these higher order corrections are not available.
Table 1: Results for the partial hadronic cross sections (in pb) for the 2-body inclusive reaction \( p + \bar{p} \rightarrow W^+ + \gamma + X \) in the \( q\bar{q} \) channel at \( \sqrt{S} = 1.8 \) TeV.
|   | $r$ | 0.50 | 1.0 | 2.0 |
|---|---|---|---|---|
| $\alpha_S$ |   | 0.145 | 0.129 | 0.116 |
| $\sigma_{qq,f,1}$ |   | 0.38 | 0.31 | 0.26 |
| $\sigma_{qq,f,2}$ |   | 0.78 | 0.66 | 0.56 |
| $\sigma_{qq,f,3}$ |   | -0.80 | -0.67 | -0.57 |
| $\sigma_{gq,finite}$ |   | 0.38 | 0.31 | 0.26 |
| $\sigma_{gq}$ |   | -0.22 | -0.25 | -0.26 |
| $\sigma_{gq}$ |   | 0.12 | 0.10 | 0.09 |
| $\sigma_{gq}$ |   | 0.15 | 0.13 | 0.11 |
| $\sigma_{gq}$ |   | -0.08 | -0.07 | -0.06 |
| $\sigma_{gq,finite}$ |   | 0.19 | 0.16 | 0.13 |
| $\sigma_{gq,finite}$ |   | -0.04 | -0.04 | -0.04 |
| $\sigma_{gq}$ |   | 0.15 | 0.12 | 0.10 |
| $\sigma_{gq}(Brems)$ |   | 0.02 | 0.02 | 0.02 |
| $\sigma_{gq}(error)$ |   | $O(10^{-9})$ | $O(10^{-9})$ | $O(10^{-9})$ |
| $\sigma_{gq}$ |   | 0.48 | 0.33 | 0.21 |

Table 2: Results for the partial hadronic cross sections (in pb) for the 2-body inclusive reaction $p + \bar{p} \rightarrow W^+ + \gamma + X$ in the $qg$ channel at $\sqrt{S} = 1.8$ TeV.

|   | $r$ | 0.50 | 1.0 | 2.0 |
|---|---|---|---|---|
| $\alpha_S$ |   | 0.145 | 0.129 | 0.116 |
| $\sigma_{gq,f,1}$ |   | 0.14 | 0.12 | 0.10 |
| $\sigma_{gq,f,2}$ |   | 0.60 | 0.50 | 0.42 |
| $\sigma_{gq,f,3}$ |   | -0.61 | -0.51 | -0.43 |
| $\sigma_{gq,finite}$ |   | 0.13 | 0.11 | 0.09 |
| $\sigma_{gq}$ |   | -0.18 | -0.20 | -0.20 |
| $\sigma_{gq}$ |   | 0.38 | 0.31 | 0.26 |
| $\sigma_{gq}$ |   | 0.48 | 0.40 | 0.34 |
| $\sigma_{gq}$ |   | -0.27 | -0.23 | -0.20 |
| $\sigma_{gq,finite}$ |   | 0.59 | 0.49 | 0.41 |
| $\sigma_{gq,finite}$ |   | -0.13 | -0.13 | -0.13 |
| $\sigma_{gq}$ |   | 0.11 | 0.09 | 0.07 |
| $\sigma_{gq}(Brems)$ |   | 0.08 | 0.08 | 0.07 |
| $\sigma_{gq}(error)$ |   | $O(10^{-8})$ | $O(10^{-8})$ | $O(10^{-8})$ |
| $\sigma_{gq}$ |   | 0.61 | 0.43 | 0.31 |

Table 3: Results for the partial hadronic cross sections (in pb) for the 2-body inclusive reaction $p + \bar{p} \rightarrow W^+ + \gamma + X$ in the $g\bar{q}$ channel at $\sqrt{S} = 1.8$ TeV.
| $r$  | 0.50 | 1.0  | 2.0  |
|------|------|------|------|
| $\alpha_S$ | 0.145 | 0.129 | 0.116 |
| $\sigma_{f,1.1,a}$ | 0.54  | 0.46  | 0.39  |
| $\sigma_{f,1.1,b}$ | 0.52  | 0.44  | 0.38  |
| $\sigma_{qg}$ | 1.05  | 0.89  | 0.77  |
| $\sigma_{qg,f,1}$ | 0.26  | 0.21  | 0.17  |
| $\sigma_{qg,f,1,1}$ | 0.14  | 0.11  | 0.09  |
| $\sigma_{qg}$ | 0.12  | 0.10  | 0.08  |
| $\sigma_{qg}$ | 0.52  | 0.42  | 0.35  |
| $\sigma_{qg,f,1}$ | 0.07  | 0.06  | 0.05  |
| $\sigma_{qg,f,1,1}$ | 0.41  | 0.33  | 0.27  |
| $\sigma_{qg}$ | 0.10  | 0.08  | 0.06  |
| $\sigma_{qg}$ | 0.58  | 0.47  | 0.39  |
| $\sigma_{total}$ | 2.15  | 1.79  | 1.51  |

Table 4: Results for the partial and total hadronic cross sections (in pb) for the reaction $p + \bar{p} \rightarrow W^+ + \gamma + jet$ at $\sqrt{S} = 1.8$ TeV.

Subtracting the above numbers from the ones for the 2 body inclusive reaction $p + \bar{p} \rightarrow W^+ + \gamma + X$ we obtain for the 0 jet reaction $p + \bar{p} \rightarrow W^+ + \gamma$ the values of 2.55, 2.50 and 2.53 pb at the scales $r = 0.5, 1.0$ and 2.0 respectively. In this scenario we do not need to include order $\alpha_S^2$ QCD corrections to make our predictions more reliable, since they are already very stable.
III SINGLE DIFFERENTIAL CROSS SECTIONS AND CORRELATIONS.

We now turn to the analysis of the single differential distributions and correlations. Our results are shown in figures 1 to 25. The error bars represent the theoretical uncertainty associated with the dependence of our results on the mass factorization and renormalization scales. They have been obtained evaluating the distributions at the two scales $r = 0.5$ and 2.0. Note that the central values and the upper and lower limits still contain a numerical error introduced via the Monte Carlo. This error is negligible in our 1 jet results but it is larger in our 2 body inclusive and 0 jet results. Regions with big error bars in our plots may be interpreted as regions where perturbation theory at order $\alpha_S$ is not reliable and thus higher order QCD corrections would be needed to make more accurate theoretical predictions.

In general, the single differential distributions and correlations have very little dependence on the scale $r$ in the scenario where the $W^+$ and photon are produced with 0 outgoing jet. If we include contributions from 1 jet processes we increase the statistics by more than 60%, but a nonnegligible theoretical uncertainty is introduced by the scale dependence of the 1 jet processes. To make theoretical predictions for single distributions and correlations more reliable in the 2 body inclusive scenario, we would thus need to include the order $\alpha_S^2$ corrections.

Figures 1 to 6 show results for single differential distributions for the $W^+$ boson and figures 7 to 11 show the corresponding photon distributions. These plots demonstrate that the $W$ boson is mainly concentrated in the regions of small energy and transverse momentum in all three scenarios. In the 1 jet scenario $E_W$ peaks at around 85 GeV (figure 1) while $P_{tW}$ peaks at around 30 GeV (figure 2). In the other two scenarios the single differential distributions of $E_W$ and $P_{tW}$ are monotonically decreasing functions except for the physical and experimental lower cuts.

Figures 3, 4 and 6 show clearly that there are dips in the polar angle and pseudorapidity distributions of the $W$ boson at around $\theta_W = \pi/2$ in the 0 jet and 2 body inclusive scenarios, with smooth dips at around the same point in the 1 jet scenario. This is very different to what we observe for the photon polar angle and rapidity distributions, which present dips at around $\theta_W = 2\pi/3$ in the 0 jet scenario only (figures 9, 10 and 11).

The 0 jet scenario would so far be the best for studying deviations from the Standard Model by looking at photon distributions since on the one hand the theoretical uncertainties related to dependences on the scale $r$ are small and on the other hand the 2 to 3 body contributions are suppressed so that the partonic zeros still show up as wide dips in the angular and rapidity distributions of the photon (see figures 9, 10 and 11). The $\theta_{\gamma}$ distribution maintains its features when 1 jet events are added, as seen in figure 9, however, the dip in rapidity is smeared out, as seen in figure 11. If photon bremsstrahlung contributions are not well accounted for by the leading logarithm approximation, these may disturb the dips in the photon angle and the photon rapidity and all other photon single and double differential distributions and correlations in the 0 jet scenario. Therefore, to isolate this effect it may be important to look also at physical quantities in the 1 jet scenario, where there is no contribution from photon bremsstrahlung at present order in perturbation theory.

Figures 12 to 15 show single differential distributions of the outgoing jet in the 1 jet scenario. We see that the jet is concentrated in the low energy, low $P_{t}$ and central rapidity
regions. We also note that the scale dependence increases in these regions. The plot in figure 14 shows a quite uniform polar angle distribution for the jet.

Figures 16 to 25 summarize our results for single correlations between outgoing pairs of particles. The same comments about the scale dependences in the different scenarios observed in figures 1 to 15 are valid here.

The $R_{W,\gamma}$ cone size correlation in figure 16 presents a sharp peak followed by a sharp dip that falls below zero near $R_{W,\gamma} = \pi$ in the 0 jet and 2 body inclusive scenarios. A similar feature was observed by Mangano et al. in $R_{q,q}$ for heavy quark pair correlations with fixed quark rapidity. The corresponding correlation in the 1 jet process presents no anomalous behavior, suggesting that the observed dips in the other two scenarios are the effect of the subtraction pieces introduced when the factorization theorem is implemented to cancel collinear singularities associated with untagged jets produced parallel to the beams.

The $R_{W,jet}$ cone size correlation in figure 21 has a very symmetric shape with a peak centered at $\pi$. On the other hand, $R_{jet,\gamma}$ in figure 21 is much smoother between 0.2 and $\pi$, falling rapidly to zero outside this range. This means that the jet and the photon are approximately uncorrelated in cone size, however the jet and the $W$ boson are highly correlated. Figure 25 confirms that the latter pair of particles are mainly concentrated on a plane that contains the beams, i.e. the distribution of the azimuthal angle difference $\phi_{W,jet}$ peaks at $\pi$. However, $\phi_{jet,\gamma}$ is approximately flat over the whole range. The observed bump near $\phi_{jet,\gamma} = R_{jet,\gamma} = 0.2$ is caused by the required experimental cut on the cone size.

Pair mass correlations are of interest when studying deviations from the Standard Model, as pointed out in [6]. The pair mass correlations $M_{W,j} = \sqrt{(Q_1 + p_j)^2}$ with $j = \gamma, jet$ in figures 17 and 22 both peak at around 100 GeV, however the correlation $M_{jet,\gamma}$ has a plateau between 20 and 40 GeV slowly falling to zero above 40 GeV.

Figures 18 and 23 show the distributions of the angles $\alpha_{i,j}$ between pairs of particles $i, j$. As expected $\alpha_{W,\gamma}$ peaks near $\alpha = \pi$ in the 0 jet scenario, since here the main contributions come from back to back 2 to 2 body partonic reactions. The smearing is introduced when boosting the partonic system into the hadronic system. This is not the case for the corresponding correlation in the 1 jet scenario, which shows that the $W$ boson and the photon are uncorrelated in the angle difference. The angular correlations between the jet and the $W$ or the photon are also quite smooth.

Since transverse momenta are preserved under Lorentz boosts along the beamline, the distribution of the azimuthal angle difference $\phi_{W,\gamma}$ only makes sense in the 1 jet scenario, which is shown in figure 19. It also peaks at around $\phi_{W,\gamma} = \pi$, meaning that the $W$ and the photon are mainly concentrated on the plane that contains the beams. This, along with the information contained in figure 25, shows that the beams and the three particles produced in the 1 jet scenario are approximately coplanar.

Figure 20 shows the distributions of the pseudorapidity difference between the $W$ and the photon in the three scenarios. In all cases the events are concentrated along the negative region, which means that the photon is produced with an angle with respect to the incoming antiproton beam that is usually smaller than the corresponding angle of the $W$ boson. We note the presence of two dips in the 0 jet scenario: one around $\eta_{W,\gamma}^* = -1$ and the other around $\eta_{W,\gamma}^* = 1.3$. These dips are smeared into plateaus in the inclusive scenario by the effect of 1 jet processes. In figure 24 we see a very different behavior for the pseudorapidity differences of jet and $W$ or photon: $\eta_{W,jet}$ is symmetric around 0 while $\eta_{jet,\gamma}^2$ receives bigger contributions on the negative axis but both correlations peak at 0.
IV DOUBLE DIFFERENTIAL CROSS SECTIONS AND CORRELATIONS.

Double differential cross sections and correlations help us visualize the qualitative features of the spatial distributions of particles hitting an experimental detector. We present in figures 26.a to 45 results obtained by averaging Monte Carlo runs at $r = 0.5$ and $r = 2.0$. Figures 26.a to 27.c correspond to double differential distributions of the $W$ while figures 28.a to 29.c show the corresponding double differential distributions of the photon. Figures 30.a to 34.c contain the plots of the correlations between the $W$ boson and the photon and figures 35 to 45 the various correlations between the jet and either the $W$ or the photon.

First we look at the double differential distributions of $P_t$ vs $\theta$ for the $W$ boson (figures 26.a, b and c). We conclude that the dip in $\theta_W$, already observed in the single differential distribution (figure 3), increases in relative depth as $P_t_W$ decreases. In the 1 jet scenario the double differential distribution clearly peaks along the line $P_t_W = 30$ GeV for all values of $\theta_W$ (figure 26.b). In the other two scenarios the peaks move down to small values of $P_t_W$, but in figures 26.a and 26.c they are not clearly seen due to the low density of bins that have been used on each axis; these peaks show up in the single differential distributions in figure 2, however, where the density of bins is four times bigger. Similar conclusions are obtained when looking at $P_t$ vs pseudorapidity for the $W$ in figures 27.a, b, and c.

The corresponding double differential distributions for the photon have been plotted in figures 28.a to 29.c. The effect of the 1 jet processes on the dips in angle and rapidity which reflect the partonic zeros is clearly visible in these figures. We note that in the 0 jet scenario the dips increase their relative depth as $P_t_\gamma$ decreases. This suggests that the effects of deviations from the Standard Model on the single differential distributions in the polar angle and rapidity of photon would be enhanced if $P_t_\gamma$ is constrained to take values below certain bound, say 25 GeV.

From the results in the previous subsection we already know that the events are mainly concentrated in the regions of small $E_W$ and small $E_\gamma$. Moreover, figures 30.a, b and c show that $E_W$ and $E_\gamma$ are not very correlated and that the bulk of the events is always in the small $E_W(\gamma)$ region, regardless of the value of $E_\gamma(W)$. The peak in the double differential cross section is steepest in the 0 jet scenario, however (see figure 30.c).

Since $P_t$ is conserved under Lorentz boosts along the beams, the $W$ and the photon are exactly correlated in transverse momentum space in the 0 jet scenario (i.e. $P_t_W = P_t_\gamma$), so this correlation only makes sense in the 1 jet scenario, where it gets smeared and it peaks around the line $P_t_W = 30$GeV regardless of the value of $P_t_\gamma$ (see figure 31).

The pseudorapidity correlation between $W$ and $\gamma$ clearly shows the effect of the partonic zero as we can appreciate in figure 32.c in the 0 jet scenario. The double differential cross section shows a valley in the negative $\eta_\gamma$ region along the whole range of $\eta^*_W$. Inclusion of 1 jet processes (see figure 32.b) would push the valley up producing a plateau instead, as seen in figure 32.a for the 2 body inclusive scenario and smearing out the dip in the single distribution, as already noted in figure 11.

Due to the exact correlation between $P_t_W$ and $P_t_\gamma$ in the 0 jet scenario, the correlation between $P_t_W$ and $\eta_\gamma$ in the 0 jet scenario (figure 33.c) looks similar to the double differential distribution of $P_t_\gamma$ vs $\eta_\gamma$ (figure 29.c), i.e., it shows an increasing effect of the partonic zero as $P_t_W$ decreases so that the photon rapidity dip becomes relatively deeper at small $P_t_W$. 1 jet processes present no dip in this double correlation, as seen in figure 33.b, and their effect
when looking at the 2 body inclusive process in figure 33.a is to smear out the dip, which reappears below $P_{tW} = 30$ GeV.

The pseudorapidity of the $W$ shows dips in all three scenarios (figure 6) and these dips are enhanced if $P_{t\gamma}$ is constrained to be smaller than 30 GeV, as inferred from figures 34.a, b and c. We contrast this with the results for the rapidity of the photon, which only shows a clear dip in the 0 jet scenario, even when looking at regions of low $P_{t\gamma}$ (figures 29.a, b and c).

The $P_{t\gamma}$ versus $P_{t\gamma}$ and $P_{tW}$ are quite different: in the $P_{t\gamma}$ versus $P_{t\gamma}$ space the events are mainly concentrated along the small $P_{t\gamma}$ and small $P_{t\gamma}$ regions (see figure 35) while in the $P_{tW}$ versus $P_{t\gamma}$ space they are concentrated along the $P_{tW} = P_{t\gamma}$ line (see figure 36). Something similar happens in the case of $\eta_{\gamma}$ correlations with $P_{t\gamma}$ and $P_{tW}$: in the $P_{t\gamma}$ versus $\eta_{\gamma}$ space the events are concentrated mainly along the small $P_{t\gamma}$ regions (see figure 42) while in the $P_{tW}$ versus $\eta_{\gamma}$ space they peak along the $P_{tW} = 30$ GeV line (see figures 41 and 2).

Although the single rapidity distribution of the photon does not present a dip in the 1 jet scenario (figure 11), the dip in $\eta_{\gamma}$ reappears if we fix the rapidity of the jet, as can be appreciated in figure 37. The dip moves up in $\eta_{\gamma}$ as $\eta_{\gamma}$ is increased and it is steeper in the regions of negative $\eta_{\gamma}$. This figure also shows that the double differential cross section peaks along the line $\eta_{\gamma} = \eta_{\gamma}$.

Figures 43 to 45 show the correlations between cone sizes $R_{i,j}$, $R_{i,j,\gamma}$ versus $R_{W,jet}$ (figure 43) shows a symmetric shape around the plane $R_{W,jet} = \pi$ at which the correlation peaks with a dip around $R_{i,j,\gamma} = 1$; these features still show up in the single distributions in figure 21. The correlation of $R_{W,jet}$ with $R_{W,\gamma}$ (figure 45) peaks around $R_{W,jet} = R_{W,\gamma} = \pi$ and also has a symmetric shape with respect to the plane $R_{W,jet} = \pi$, confirming again the observations regarding the plot of $R_{W,jet}$ in figure 21. Although $R_{i,j,\gamma}$ versus $R_{W,jet}$ (figure 44) shows no simple symmetry it also peaks near the planes $R_{i,j,\gamma} = \pi$ and $R_{W,jet} = \pi$. In general the $R_{i,j}$ cone sizes peak around $R_{i,j} = \pi$ and fall off quickly above $R_{i,j} = 4$, and, in particular, $R_{W,jet}$ is symmetric around $\pi$ even when plotted against other variables.
V CONCLUDING REMARKS.

We have completed an analysis of the exclusive reactions $p\bar{p} \rightarrow W^+\gamma$ and $p\bar{p} \rightarrow W^+\gamma + jet$ by generalizing the methodology used by other authors in the context of heavy quark and $Z$ pair production [8],[9], which consistently includes all divergent regions of phase space in the framework of the factorization theorem and dimensional regularization. We have taken into account order $\alpha_S$ QCD corrections and leading logarithm bremsstrahlung contributions and used the latest CTEQ set of parton distribution functions, which fit the newly released HERA data. We have treated the $W$ boson as a real particle with a mass of 80.2 GeV.

All our analytical results were presented in our paper [1] and have been used in a Fortran code with Monte Carlo integration techniques. Histograms of single and double differential cross sections and correlations for all outgoing particles ($W$ boson, photon and, when applicable, jet) have been obtained in each of three experimental scenarios, namely 2 body inclusive, 1 jet and 0 jet scenarios.

Our results for total, single and double differential cross sections and correlations for the production of $W^+$ and photon accompanied by 0 jet show the smallest theoretical uncertainty under variations of the mass factorization and renormalization scales when compared with the predictions in the 2 body inclusive and 1 jet scenarios. This means that accurate predictions are available in the 0 jet scenario for quantities related to the $W$ boson and the photon without including order $\alpha_S^2$ corrections, however, they would have to be included for better accuracy in the other two scenarios.

Previous work on the reaction $p\bar{p} \rightarrow W^+\gamma + X$ has been devoted to the study of single photon distributions, photon-$W$ boson pair mass and photon-charged lepton pseudorapidity correlations. The complete set of distributions and correlations including the $W$ boson and the jet therefore complements the studies of the Electroweak sector of the Standard Model (i.e. the magnetic moment of the $W$) and provides further checks for the QCD sector and the photon bremsstrahlung process. We note here that our results for the 0 jet and 2 body inclusive reactions include leading logarithm photon fragmentation functions, so deviations of the observed experimental data from theoretical predictions could in part be accounted for by errors introduced by this approximation. To discriminate the effect of poorly known photon bremsstrahlung contributions from the effects of deviations from the Standard Model, the analysis of distributions and correlations in the 1 jet scenario, which, at the present order in perturbation theory are free of photon bremsstrahlung, would provide a valuable tool. With regard to total cross sections the two reactions $p\bar{p} \rightarrow W^+\gamma$ and $p\bar{p} \rightarrow W^+\gamma + jet$ are of similar importance, but, as we pointed out before, the latter reaction really requires even higher order QCD corrections to provide more accurate predictions.

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