Optimization on Preventive Maintenance and Spare Unit Number for Multi-unit Parallel System

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Abstract. Preventive maintenance (PM) for multi-unit parallel system with limited production resource is a crucial issue. To find a solution for the issue, this study focused on a multi-unit parallel system with one repairman and some spare units, and developed an optimization model for the system. In the model, PM policy and cold spare unit number were jointly considered. After optimization, an optimal PM policy with PM interval of running and spare units, PM number and cold spare unit number were obtained. Then, a numerical example was analyzed to illustrate the proposed model, and some results were attained based on the sensitive analysis.

Notations

| Symbol | Description |
|--------|-------------|
| $C_f$  | cost of minimal failure |
| $C_p$  | cost of PM |
| $C_r$  | cost of replacement |
| $C_d$  | loss rate of down time |
| $T$    | PM interval of operating-unit |
| $T_{st}$ | PM interval of spare unit |
| $\lceil\cdot\rceil$ | integral operation |
| $\lfloor\cdot\rfloor$ | rounding operation |
| $g(t)$ | the system long-running cost rate |
| $h(t)$ | failure intensive function |
| $h(u)$ | factor of $M$ |
| $n$    | times of PM for spare units in every PM |
| $n_d$  | interval of operating unit |
| $T_p$  | PM time |
| $M$    | operating unit number |
| $M_s$  | spare unit number |
| $N_p$  | PM times of spare unit |
| $N$    | PM times of operating unit |

$$H(t) = \int_{0}^{t} h(u) du$$

1. Introduction

A joint consideration of preventive maintenance (PM) for a multi-unit parallel production system with limited production resource is a crucial issue, which widely exists in railway, civil aviation and production fields. The configuration of production resource, such as spare unit, repairman or maintenance team, is a main part in production process. A rich configuration can cause waste of resource, while a poor one may result in outage. Therefore, how to configure spare unit for the multi-unit production system with one repairman based on a minimal running cost rate is an exceedingly valuable research issue. To resolve this issue, maintenance, operating unit number, repairmen and production should be jointly considered in the resource configuration and maintenance decision-making[1-3].

Research on resource configuration, Ni et al, [4] pointed out that production is often interrupted by pre-scheduled PM without considering the throughput target. PM tasks are not cost-effectively conducted, and investigated the extra hidden opportunities for PMs during production time without...
violating the system throughput requirement. Liao et al.[5] oriented a mono-machine-based optimization model of production scheduling and PM under group production is proposed. Xiao et al.[6] proposed a joint optimization model to minimize the total cost including production cost, PM cost, minimal repair cost for unexpected failures and tardiness cost. Hidayanto et al.[7] take the salt crusher machine as object, proposed four PM plans based on the reliability analysis, compared the cost, reliability and production of these PM plans, and obtained optimal PM plans in the cost, reliability and production. Some research regarded that grouping replacement or maintenance for all units of multi-unit parallel production system can reduce setup cost[8]. Pargar et al.[9] developed an integrated optimization method to schedule PM and renewal projects by grouping them and simultaneously finding the optimal balance between them. Do et al.[10] proposed a dynamic maintenance grouping approach for multi-unit system, and the approach can provide a minimum number of repairman to ensure the maintenance plan. Bai et al.[11] introduced grouping maintenance strategy for optimizing the compound maintenance intervals of multi-component system, and proposed the optimization steps and methods. However, grouping maintenance policy can result in outage to production system, and the outage is not agreed in some fields. Zhang et al.[12] developed a PM policy which jointly considered the PM policy and the operating unit number for a multi-unit parallel production system. Whereas the number of a parallel running unit is a fixed value for most multi-unit parallel system, and usually configured by engineering experience. Therefore, it is necessary to optimize the number of cold spare unit under a fixed running unit system.

2. Maintenance and assumption
In this section, a multi-unit parallel production system with \( M \) operating units, \( M_s \) spare units and one repairman will be considered. In this system, the operating and spare units arise out of the same type. In accordance with the schedule plan, the sequentially PM activities of operating units are performed at \( kT (k=1,2,\ldots,N-1) \) and a replacement is carried out at \( NT \). As for the spare units, keeping the same running time with operating unit is hard to achieve, so if the accumulated operating time of spare unit \( T_{st} \) is within \( [\alpha_1T, \alpha_2T] \) (\( \alpha_1<1, \alpha_2>1 \)), it will be preventively maintained. In order to ensure the normal implementation of the schedule plan, one of the spare units performs operating when one of the operating units executes PM activity. The repairman can maintain only one unit at the same time and just for PM activity.
Take \( M=6, N=4 \) and \( M_s=2 \) as an example to introduce the operation and maintenance process. The parameter \( n \) means the times of PM for spare units in every PM interval of operating units. As for this system, let the parameter \( n=1,1\frac{1}{2},1\frac{1}{4},2 \) and 3. Then, the process diagram respectively showed as subplot (a) to (e) of Fig.1.

Figure 1. The operating process of system.

To convenience for modeling, the following assumptions are proposed firstly, 1) All failures can be discovered and repaired immediately, and the time for repair and replacement are ignored, while the time of PM is considered.

| subplot (a) | n=1 |
| subplot (b) | n=1\frac{1}{4} |
| subplot (c) | n=1\frac{1}{2} |
| subplot (d) | n=2 |
| subplot (e) | n=3 |

- Operating time
- Spare unit operating time
- PM time
- Replacement time
2) The hazard rate function of a new unit is \( h(t) \), which is monotonically increasing with respect to \( t \) and each PM can make it return to 0. PM can change the hazard rate of unit following \( h_i(t)=a_ih(t) \) \((i=1,2,...,N)\) before the \( i \)th PM, where \( 1=a_i<a_2<...<a_N \). The unit can be restored to “as good as new” state after repair.

3) The cost of minimal failure \( C_f \) is independent of the time of the failures and the total sever of the system. The cost of PM \( C_p \) is irrelevant to the total operational time of the system. The loss rate of outage time of spare unit is \( C_d \). After the \( N \)-1th PM, the operating unit is replaced and the cost is a constant \( C_r \), where \( C_r>C_f \).

4) The PM activity of all units can be finished in the PM time. PM interval of each operating unit is \( T \) while that of the spare unit is \( T_{st} \) which is within \([a_1T, a_2T]\), where \( a_1 \leq 1 \), \( a_2 \geq 1 \).

3. Modeling and optimization

In this section, the long-term running cost rate and optimization model of system will be illustrated in detail.

3.1 Modeling

As for the multi-unit production system, a renewal cycle is defined as the time from when the system starts running until all units finish the first replacement, or the interval between two successive replacements of all units which consist of \( M \) operating units and \( M_s \) spare units.

In the PM model, three actions are comprehensively considered: minor repairs, PMs and replacements. Minor repairs can restore the minor failures and the replace cycle is renewal process. Let \( Y \) denote the length of a replace cycle, \( C \) means the total cost of renewal cycle and \( g(T, n, T_{st}, N, M) \) denote the long-term running cost rate of system within the interval \((0, t)\). Thus, \( (Y, C) \) constitutes a renewal reward process. In accordance with renewal theory, the function of total \( s \)-expected long-term running cost rate of renewal cycle can be described as the Eq.(1).

\[
g(T, n, T_{st}, N, M) = \frac{E[C]}{E[Y]} = \frac{C_f \left( M \sum_{i=1}^{N} H_i(T) + M_s \sum_{i=1}^{N_s} H_i(T_{st}) + \left\{ Nn - M_s \left[ Nn/M_s \right] \right\} T_{st} \right)}{C_p (M(N-1) + N_p) + C_r (M + M_s) + C_d (M_s N(T + T_p) - MNT_p - N_p T_p)}
\]

\[
N_p = \begin{cases} 
0 & \text{if } Nn \leq M_s \\
\left\lfloor \frac{Nn}{M_s} \right\rfloor & \text{other cases}
\end{cases}
\]

\[
T_{st} \geq MT_p
\]

\[
T_{st} = \left( M + \left\lfloor \frac{Np}{N-1} \right\rfloor - 1 \right) T_p
\]

\[
a_1 T_{st} \leq T \leq a_2 T_{st} \Rightarrow \frac{T_{st}}{a_2} \leq T \leq \frac{T_{st}}{a_1}
\]

In the above formula, \( N_p \) is the number of PM for spare unit and \( T_{st} \) is the interval between the adjacent PMs for spare unit. In addition, \( n \) is the times of PM for standby units in every PM interval, and the relationship among \( n \), \( M_s \), \( N \) and \( M \) is shown in Table 1.

**Table 1.** The relationship among \( n \), \( M_s \), \( N \) and \( M \).

| \( M_s \) | \( n \) |
|---|---|
| 1 | 1/k | 1 | nd1 | nd2 | ... | ndi | M |
| 2 | 1/k | M/(M-k) | nd1 | nd2 | ... | ndi | M |
| 3 | 1/k | M/(M-k) | nd1 | 2M/(M-k) | nd2 | ... | ndi | M |
| ... | 1/k | M/(M-k) | nd1 | 2M/(M-k) | nd2 | ... | (ms-1)M/(M-k) | ndi | M |
| ms | 1/k | M/(M-k) | nd1 | 2M/(M-k) | nd2 | ... | (ms-1)M/(M-k) | ndi | M |
| (k=1,..,N) | (k=1,..,M-1) | (k=1,..,Mnd1) | ... | (k=1,..,Mndi-1) |
3.2 Optimization
To obtain the optimal PM policy and spare unit number, the system long-term running cost rate is to be minimized. To find the optimal results, the following theorems are introduced first.

Theorem 1: Under a given $M_s$ and $N$. If the hazard rate function $h(t)$ is a monotonous increasing function of $t$, the optimal $T^*$ and $T_{st}^*$ will be obtained.

To jointly optimize PM and the number of spare unit, we need to minimize the long-term running cost rate $g(T, n, T_{st}, N, M_s)$. Therefore, let $\frac{dg(T, n, T_{st}, N, M_s)}{dT} = 0$ and get Eq.(6).

$$
\sum_{i=1}^{n} (T + T_p)h_i(T) - H_i(T) = 
C_f \left( M_s \sum_{l=1}^{N_s/M_s} H_l(T_{st}) + \left[ Nn - M_s[Mn/M_s] + H_{[Nn/M_s]}(T_{st}) \right] 
+ C_p (M(N-1) + N_p) + C_c (M + M_s) - C_d (M_s N T_p + M N T_p + N_p T_p) \right)
$$

In accordance with the assumption (2) and the given $M_s$ and $N$, the above equation exists unique solution. Thus, the optimal $T'$ could be obtained in accordance with the theorem 1 in reference[13]. According to Eq.4 and Table 1, the relationship between $T_{st}$ and $T$ and $T_{st}$ can be got because $n$ can be obtained when $M_s$, $M$ and $N$ are given. Thus, the optimal $T^*$ and $T_{st}^*$ can be attained because the $h(t)$ is a monotonous increasing function of $t$. The detailed proof is negligible.

Theorem 2: Under a given $M_s$ and $T$. If the hazard rate function $h(t)$ is a monotonous increasing function of $t$, the optimal $N^*$ will be obtained.

According to the theorem 2 in the reference[13], under a given $M_s$ and $T$, the optimal $N^*$ can be obtained since $h(t)$ is a monotonous increasing function in $t$. The detailed proof is negligible.

Theorem 3: Under a given $N$ and $T$. If the hazard rate function $h(t)$ is a monotonous increasing function of $t$, the optimal $M_s^*$ will be obtained.

The proof is the same as theorem 2 in the reference [13]. The detailed proof is negligible.

4. Numerical case study

4.1 Numerical example
To illustrate the proposed model, a Weibull distributed example will be provided to demonstrate the character of the model in this paper. Hence, that the lifetime of each unit obeys Weibull distribution and the hazard rate function of new unit as below.

$$
h(t) = 0.042 \left( \frac{t}{50} \right)^{1.1} 
$$

Now, further assume that $M=12$, $T_{st}=4.5$, $\alpha_1=4i/(3i+1)$, $C_f=2.5$, $C_p=2$, $C_c=3$, $C_d=0.01$, $\alpha_1=0.5$ and $\alpha_2=1.5$. Then, let the initial values of $M_s$ and $N$ be 1. The step size of iteration for $M_s$ and $N$ respectively are $\delta M_s$ and $\delta N$, and the value of $\delta M_s$ and $\delta N$ are 1. An optimal algorithm shown as Fig.2 is used to compute the optimal results.

![Figure 2. The optimal algorithm.](image-url)

With the above algorithm, the optimal results of $N$, $M_s$, $n$, $T$ and $T_{st}$ are obtained. The optimal results presented in Table2.
Table 2. Optimal results.

| N  | 2   | 3   | 4   | 5   | 6   | 7   |
|----|-----|-----|-----|-----|-----|-----|
| M  | 1   | 2   | 2   | 2   | 2   | 2   |
| n  | 1   | 1.5 | 1.5 | 1.3333 | 1.2 | 1.2 |
| T  | 54  | 49.5| 49.5| 49.5| 49.5| 49.5|
| Tₚ | 54  | 36  | 36  | 40.5| 45  | 45  |
| g  | 0.1046| 0.1036| 0.1029| 0.10281| 0.10284| 0.1031|

From the above, the optimal \( N^* = 5, M^*_s = 2, T^*_s = 49.5 \) and \( T^*_{s_t} = M^* \times T_p / n^* = 40.5 \) can be obtained, and the minimal long-term running cost rate of system is 0.10281. That is to say, one repairman can serve a parallel system with 12 operating units and 2 spare units, and the system could be taken preventive maintenance each 5 PM intervals and taken replacement action at the end of the last interval. In addition, the PM interval of operating units and spare units are 49.5 and 40.5 respectively.

4.2 Sensitive analysis
Due to the parameters \( T_p \) and \( M \) are crucial for the proposed model. And thus, the influence of \( T_p \) and \( M \) will be analyzed in this section.
Fig. 3 shows that the optimal value of $T_p$ is existent to minimize the long-term running cost rate. The optimal value of $T_p$ is 3.4 and $g_{min}=0.1026$. It means that the enterprises could decrease the cost by keeping the PM time at optimal value. Fig. 4 indicates that the value of $N$ is decreasing and the value of $M_s$ is increasing with the increase of $T_p$. In addition, the change of $N$ is clearer than $M_s$. Fig. 5 displays that the PM interval is increasing in the interval $[3, 3.8]$ and $[4, 5]$, the turning point is the same as the curve of $M_s$ in Fig. 4.

Fig. 6 exhibits that the long-term running cost rate has a minimum value 0.10027 when the value of $M$ is 9. That is to say, the enterprises can adjust the parameters according to the change of the number of operating units to keep the minimum long-term running cost rate. Fig. 7 shows that the value of $N$ is decreasing with the increase of $T_p$ while the value of $M_s$ is increasing. The $N$ has more significant effect than $M_s$. Fig. 8 indicates that the value of $T$ has a trend of increase with the growth of $M$. In addition, the turning point is the same as the curve of $M_s$ in Fig. 7.

5. Conclusion

In this paper, the joint optimization for multi-unit parallel system with one repairman and several spare units is studied. According to the renewal theory and renewal reward process, the function of long-term running cost rate for the proposed model is established. The optimization objective is minimizing the long-term running cost rate. A numerical example is used to prove the existence and uniqueness of the optimal solution. Afterwards, the sensitive analysis of two significant parameters is given to demonstrate the influence on the optimal solution. The results of sensitive analysis show that the PM time and the number of operating units both have the optimal solution that minimizes the long-term running cost rate of system. These results exhibit that the enterprises could decrease the long-term running cost rate by adjusting the value of two parameters.

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