Numerical simulation on mixed magneto-convection in a tilted lid-driven box with sinusoidal heating

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Abstract. An unsteady combined convective flow and energy transfer in an inclined lid-driven square box with sinusoidal boundary temperatures at the vertical sidewalls in the presence of uniform magnetic field is numerically investigated. The horizontal sides of the box are thermally insulated. The governing equations are numerically solved by using finite difference method. The results are discussed for various groupings of the governing parameters, inclination angle, phase deviation, amplitude ratio, Richardson number and Hartmann number. The sinusoidal heating provides a higher energy transfer than a constant thermal distribution while the amplitude ratio can be raised to attain such heat distribution along any sidewall of the box.

Keywords: Mixed convection; MHD; Inclined box; Amplitude ratio; Phase deviation

1. Introduction

Mixed convection is a composite heat transfer mechanism that occurs due to the mutual action of the buoyancy effect and shear force [1-3]. The behavior of an electrically conducting liquid due to the Lorentz effect, which leads to magneto-hydro-dynamics (MHD), has also been widely deliberated [4-7]. Due to the extensive applications of MHD convection, many research works are in progress to investigate the liquid movement and heat transference of an industrial system under extensive magnetic effect. Oztop and Dagtekin [1] explored the effect of direction of moving walls on heat transfer and liquid flow in a two-sided wall-driven box. Ghasemi and Aminossadati [2] discovered the mixed convection effects in a square box. Sivakumar et al. [3] deliberated a work on mixed convective current in a wall-driven box with diverse sizes and locations of the heat source. Al-Najem et al. [4] calculated the effects on the buoyant convection in a slanted box in the existence of magnetic field. Chamkha [5] evaluated the mixed convective stream in a square box in the occurrence of the magnetic field and inner heat generation. Sarris et al. [6] calculated unsteady free convection in a box of an electrically conducting liquid under the impact of a magnetic field. An experimental analysis on convection of a liquefied metal in a closed box in the occurrence of an exterior magnetic field was considered by Xu et al. [7]. Kang and Hyun [8] explored buoyancy convection in a box with periodic magnetizing force and fixed fluid. Jalil and Tae’y [9] scrutinized the buoyant convection of molten sodium in a square box. Sivasankaran and Ho [10] examined the influence of variable properties of water in the presence of constant magnetic field on convective stream. Pirmohammadi and Ghassemi [11] scrutinized the natural convection in a tilted box in the occurrence of a magnetic field. Sathiyamoorthy and Chamkha [12] presented a study on buoyancy convection of liquid gallium in a box in the occurrence of inclined magnetic field. Sivasankaran et al. [13] inspected the Marangoni convection of chilly water in an open box under the impact of constant external magnetic field. Malleswaran et al. [14] established that the presence of the magnetic field declines the convection inside the box.

Recently, convection in closed regions with diverse thermal (boundary) settings has been focused extensively. Corcione [15] discussed the buoyant convective stream in a closed box cooled from above & heated from below for various thermal (boundary) conditions at the side walls. Saeid and Yaacob [16] calculated the buoyant-convection in a closed box with sinusoidally heated sidewall. Dalal and Das [17] deliberated the buoyant convection in a rectangle shaped box with spatially varying temperature on bottom wall. Deng and Chang [18] proposed a numerical examination on buoyant convection in a four-sided box with sinusoidal heat scatterings on sidewalls. Basak et al. [19] inspected the impact of non-uniform / uniform warming of bottom side on mixed convective stream in a top-driven box. Sivasankaran et al. [20] numerically examined the free convection of different nanofluids in a box with linearly changing wall temperature. Sivasankaran et al. [21, 22] discussed the mixed convective movement in a top-driven box with sinusoidal thermal sources on (vertical) walls in the absence/presence of a magnetic field. Bluwaneswari et al. [23] scrutinized the MHD convection in a box...
with sinusoidally varying heat distribution on both sidewalls. Sivasankaran and Pan [24] calculated the mixed convection in a porous box with sinusoidal thermal situation on (both) walls. Sivasankaran and Bhuvaneswari [25] numerically explored the natural convection in a porous box with non-uniform thermal heating on both vertical walls. They observed that averaged Nusselt number augments by non-uniform warming on both walls. Cheong et al. [26] discovered the influence of aspect ratio on buoyant convection in a tilted box with sinusoidal border condition at right sidewall. Sivasankaran and Pan [27] discovered from the study on convection of nanofluids in a box with sinusoidal thermal boundary conditions on side-walls that the heat transference rate rises with growing the values of amplitude ratio and nano-particles volume fraction.

Since there is no study on mixed magnetic-convection in a tilted wall-driven square box with sinusoidal heat distributions on sidewalls in the occurrence of heat generation, this work aims to explore such problem in the literature.

2. Mathematical model

The physical prototype is a tilted square box with angle $\gamma$, assessed from horizontal in the counter-clockwise way. The domain of length $L$ is filled with an electrically conducting liquid as displayed in Figure 1. The stream is presumed to be 2D, time-dependent, incompressible & laminar. The liquid properties are reserved as fixed except the density in buoyant term (Boussinesq’s approximation holds). The density differs linearly with temperature as $\rho = \rho_0[1-\beta(T-T_0)]$, where $\beta$ is thermal expansion coefficient & subscript 0 indicates reference state. The directions of $x$-axis and $y$-axis with velocities $(u_1, u_2)$ are indicated in figure with lower left corner of the box as origin.

The lid (box) moves with fixed velocity $U_0$. The vertical sides of the box have executed with varying sinusoidal thermal distributions. The gravitational acceleration performances downward. The uniform external magnetic field of fixed magnitude $B_0$ is involved horizontally. The viscous dissipation, induced magnetic field & Joule heating are neglected.

The mathematical model is derived as follows:

$$\frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} + u_2 \frac{\partial u_1}{\partial y} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} \right) + g\beta(\theta - \theta) \sin \gamma$$  

(3)

$$\frac{\partial u_2}{\partial t} + u_1 \frac{\partial u_2}{\partial x} + u_2 \frac{\partial u_2}{\partial y} = -\frac{1}{\rho_0} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2} \right) + g\beta(\theta - \theta) \cos \gamma - \frac{\sigma B_0^2 u_2}{\rho_0}$$  

(4)

$$\frac{\partial \theta}{\partial t} + u_1 \frac{\partial \theta}{\partial x} + u_2 \frac{\partial \theta}{\partial y} = \frac{k}{\rho_0 c_p} \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) + \frac{Q}{\rho c_p}(\theta - \theta_0)$$  

(5)

where $g$, $k$, $c_p$, $\rho$, $\gamma$, $\theta$, $\nu$ are the gravitational acceleration, thermal conductivity, specific heat, pressure, time, inclination angle, temperature, & kinematic viscosity, respectively.

The appropriate initial & boundary settings take the following form:

$$t = 0: \quad u_1 = 0, u_2 = 0, \quad \theta = 0, \quad 0 \leq y \leq L, \quad 0 \leq x \leq L$$
where the mean temperature of the sinusoidal thermal shapes on the right & left side-walls is $\theta_0$, the subscripts $r$ & $l$ denote right & left walls, respectively. The amplitude & phase of the (sinusoidal) thermal profile on right side are $A_r$ & $\phi$ and that of left side are $A_l$ & $\phi$.

The overhead modelled equations are suitably non-dimensionalized by presenting the following variables

\[
X = x/L, \quad Y = y/L, \quad U_1 = u_1/U_0, \quad U_2 = u_2/U_0, \quad \tau = tU_0/L, \quad T = (\theta - \theta_0)/(\Delta \theta), \quad \Psi = \psi/(U_0 L) \quad \text{&} \quad \zeta = v_0 L/U_0.
\]

After eliminating pressure, vorticity - stream function form of the model (2)-(5) is acquired as

\[
\frac{\partial \zeta}{\partial \tau} + U_1 \frac{\partial \zeta}{\partial x} + U_2 \frac{\partial \zeta}{\partial y} = \frac{1}{Re} \left[ \frac{\partial^2 \zeta}{\partial x^2} + \frac{\partial^2 \zeta}{\partial y^2} \right] + \frac{Gr}{Re} \left( \frac{\partial T}{\partial x} \cos \gamma - \frac{\partial T}{\partial y} \sin \gamma \right) - Ha \frac{\partial U_2}{\partial x}
\]

\[
\zeta = -\nabla^2 \Psi
\]

\[
\frac{\partial T}{\partial \tau} + U_1 \frac{\partial T}{\partial x} + U_2 \frac{\partial T}{\partial y} = \frac{1}{Re Pr} \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] + \frac{Hg}{Re Pr} T
\]

\[
\zeta = \frac{\partial U_2}{\partial x} - \frac{\partial U_1}{\partial y}, \quad U_1 = \frac{\partial \Psi}{\partial y}, \quad U_2 = -\frac{\partial \Psi}{\partial x}
\]

where $Gr = g\beta\Delta \theta L^3/\nu^2$, Grashof number, $Ha = L B_0 \sqrt{\sigma_v/\mu}$, Hartmann number, $Hg = Q \ell c_\ell /k$, heat generation parameter, $Pr = \nu/\alpha$, Prandtl number, $\tilde{Ri} = Gr/Re^2$, the Richardson number & $Re = LU_0/\nu$, Reynolds number.

The dimensionless initial & boundary settings are derived as follows:

\[
\begin{align*}
\tau = 0: & \quad U_1 = 0, U_2 = 0, \quad T = 0, \quad Y = 0 \quad 0 \leq (X, Y) \leq 1 \\
\tau > 0: & \quad U_1 = 0, U_2 = 0, \quad T = 0, \quad Y = 0 \\
& \quad U_1 = 1, U_2 = 0, \quad T = 0, \quad Y = 1 \\
& \quad U_1 = 0, U_2 = 0, \quad T = \sin(2\pi Y), \quad X = 0 \\
& \quad U_1 = 0, U_2 = 0, \quad T = \varepsilon \sin(2\pi Y + \phi), \quad X = 1
\end{align*}
\]

where $\varepsilon = A_r/A_l$ is the amplitude ratio on right side to that on left side of sinusoidal thermal condition.

The local Nusselt number along the left & right sides are designed as

\[
Nu_l = \left. \frac{\partial T}{\partial x} \right|_{x=0} \quad \text{&} \quad Nu_r = \left. \frac{\partial T}{\partial x} \right|_{x=1}
\]

Since the liquid in the box gets warm at the heating portion, i.e., $Nu > 0$. Similarly, the liquid at the cooling portion tend to drop the heat and henceforth, $Nu < 0$. Hence, the total heat transfer rate through the box is attained by totaling the averaged Nusselt numbers along the heating portions of left & right side-walls. Thus, the averaged Nusselt number is measured as follows

\[
\overline{Nu} = \int_{\text{heating half}} Nu_l \, dY + \int_{\text{heating half}} Nu_r \, dY
\]
The present numerical solutions have been verified against the numerical results reported on convective movement in lid-driven box by Sharif [28] and Iwatsu et al. [29]. The averaged Nusselt numbers from present simulation and existing results have been evaluated to be: 

\[ \text{Nu} = \frac{hL}{k} \]

where \( h \) is the average Nusselt number, \( L \) is the characteristic length, and \( k \) is the thermal conductivity. The Nusselt number is defined as the ratio of the heat transfer rate to the rate of heat conduction through the wall.

3. Solution approach

The dimensionless governing models (7)-(9) with the boundary settings (11) are discretized by using the finite difference method. The resulting discretized equations are applied in the solution domain which involves a number of grid points with uniform grid system in both directions, respectively. In order to confirm the grid independence of the results, the grid sensitivity tests are executed. The numerical experiments are performed for the uniform grids from 41x41 to 161x161 with \( \text{Pr} = 0.054, \text{Re} = 1, \) and \( \text{Ha} = 0 \). It is witnessed from the grid independence checks that an 81x81 grid is adequate for the preferred precision of solutions. The time step is uniformly chosen as \( \Delta \tau = 10^{-4} \). An iterative process based on Gauss-Seidel approach is applied to solve the resulting sets of discretized algebraic equations. The convergence of the solutions is achieved by repeating the above process so that the relative error for each of the variables with tolerance \( 10^{-9} \). The present numerical solutions have been verified against the numerical results reported on convective movement in lid-driven box by Sharif [28] and Iwatsu et al. [29]. The averaged Nusselt numbers from present simulation and existing results

![Figure 2. Isotherms (up) & streamlines (down) for various Ha & inclinations with Ri=0.01 and Ri=100](image)
are compared in Table 1. The comparisons illustrate a good proximity in the predictions made between the present and existing solutions.

### Table 1. Comparison of averaged Nusselt number for lid-driven cavity.

| Gr  | Re = 400 |          |          | Re = 1000 |          |          |
|-----|----------|----------|----------|-----------|----------|----------|
|     | Present  | Iwatsu et al. [29] | Sharif [28] | Present  | Iwatsu et al. [29] | Sharif [28] |
| 10^3 | 4.08     | 3.84     | 4.05     | 6.48     | 6.33     | 6.55     |
| 10^4 | 3.84     | 3.62     | 3.82     | 6.47     | 6.29     | 6.50     |
| 10^5 | 1.10     | 1.22     | 1.17     | 1.66     | 1.77     | 1.81     |

### 4. Results

During the study, the Prandtl number (Pr = 0.054) and Grashof number (Gr = 10^3) are kept constant. The Richardson number is taken as 0.01 ≤ Ri ≤ 100 by varying the Reynolds number from 10 to 1000. The Hartmann number values are 0, 25 and 100. The phase deviation choose as ϕ = 0, π/4, π/2, 3π/4 and π. The amplitude ratio selects as ε = 0, 0.5 & 1. The inclination angle selects as 0°, 30°, 45°, 60° and 90°. The heat generation parameter takes the values -10, -5, 0, 5, 10.

Figure 2 shows the effect of tilting angle, Hartmann number & Ri number on stream and thermal fields. It is evidently seen from the figures that a single rotating eddy occupies the box in the forced convection regime whereas multi-cellular stream exists in the buoyant convection regime. When rising the strength of the magnetic field (Ha=25), the flow speed is reduced due to Lorenz force acting on the fluid particle and the size of the eddy shrinks. Further, strengthening the magnetic field (Ha=100), the eddy shrinks and exist only in the upper portion of the box in the forced convective regime. It is also observed that the tilting angle did not produce much effect on flow and thermal fields in the forced convective regime for all values of Ha. However, the tilting angle shows its impact on stream and thermal fields in the free convective regime. Multi-cell pattern exists for non-tilting box and a single eddy occupies in the case of tilting box in the absence of magnetic field. The multi-cell flow occupies the box for all tilting and non-tilting cases in the occurrence of magnetic field. The corresponding isotherms clearly shows the convection type of thermal transmission inside the box.

**Figure 3.** Isotherms (left) & streamlines (right) for various ϕ & γ with Ha=25, ε=1, Ri=100.
When raising the value of ε, it further enhanced at ε = 0, the thermal distribution in the right wall is very weak and conduction type heat transfer started on right wall and it further enhanced at ε = 1. As far as flow, the similar trend as discussed in Figure 2 is observed here.

The impact of heat generation/absorption of stream and thermal field is portrayed in the Figure 5 for various tilting angle and Ri. It is evidently seen from the figures that a single eddy exists in the heat field. When altering the phase deviation, the thermal distribution near the right wall is altered much because the phase deviation makes influence on the right wall temperature. This also makes much impact on stream pattern and flow field drastically changed by various multi-cellular pattern. Figure 4 displayed the streamlines & isotherms for diverse amplitude ratios with φ = 0, γ = 45, and Ha=25. When ε = 0, the thermal distribution is very weak in the right wall and conduction type heat transfer is involved here. When raising the value of ε = 0.5, the convective heat transport started on right wall and it further enhanced at ε = 1. As far as flow, the similar trend as discussed in Figure 2 is observed here.

Figure 5 portrayed the thermal and stream fields for several values of phase deviation and tilting angle. When altering the phase deviation, the thermal distribution near the right wall is altered much because the phase deviation makes influence on the right wall temperature. This also makes much impact on stream pattern and flow field drastically changed by various multi-cellular pattern. Figure 4 displayed the streamlines & isotherms for diverse amplitude ratios with φ = 0, γ = 45, and Ha=25. When ε = 0, the thermal distribution is very weak in the right wall and conduction type heat transfer is involved here. When raising the value of ε = 0.5, the convective heat transport started on right wall and it further enhanced at ε = 1. As far as flow, the similar trend as discussed in Figure 2 is observed here. The impact of heat generation/absorption of stream and thermal field is portrayed in the Figure 5 for various tilting angle and Ri. It is evidently seen from the figures that a single eddy exists in the heat field. When altering the phase deviation, the thermal distribution near the right wall is altered much because the phase deviation makes influence on the right wall temperature. This also makes much impact on stream pattern and flow field drastically changed by various multi-cellular pattern. Figure 4 displayed the streamlines & isotherms for diverse amplitude ratios with φ = 0, γ = 45, and Ha=25. When ε = 0, the thermal distribution is very weak in the right wall and conduction type heat transfer is involved here. When raising the value of ε = 0.5, the convective heat transport started on right wall and it further enhanced at ε = 1. As far as flow, the similar trend as discussed in Figure 2 is observed here. The impact of heat generation/absorption of stream and thermal field is portrayed in the Figure 5 for various tilting angle and Ri. It is evidently seen from the figures that a single eddy exists in the heat field.
absorption case while dual eddies occupy in the heat generation case when Ri=0.01. The isotherms indicate the convection mode in all cases. The flow field is strongly affected by heat generation parameter in the natural convective regime. The isotherms show the horizontal stratification in the heat generation case at Ri=100.

**Figure 6.** Mean Nusselt number for various inclination angle & Ri with $\varepsilon=1$ and $\phi=0$.

In order to explore the mean energy transport across the domain, the averaged Nusselt number is exposed against the tilting angle, and Ri for various values of Ri & Ha in Figure 6. It is visibly
demonstrated that the averaged Nusselt number declines with tilting angle in the occurrence of magnetic field. But, \( \overline{Nu} \) decreases with tilting angle in the forced convection regime and it enhances with tilting angle in the free convection regime in the nonexistence of magnetic field. It is detected that \( \overline{Nu} \) enhances on reducing the values of \( Ri \). The \( \overline{Nu} \) declined on rising the values of \( Ha \). The impact of sinusoidal heating on averaged Nusselt number is discovered in Figure 7 for diverse values of inclinations of the box and Richardson numbers with \( Ha=25 \). The averaged thermal transport enhances first on growing the phase deviation upto \( \phi = \pi / 2 \) and then it declines on rising the phase deviation. The averaged Nusselt number rises when growing the values of amplitude ratio for all values of \( Ri \) and \( Ha \).

Figure 7. Averaged Nusselt numbers for various \( \varepsilon \) with \( Ri=0.01 \) (a, c & e) and \( Ri=100 \) (b, d & f)
The influence of heat generation and absorption on the thermal and flow fields for several values of tilting angle, Ha and Ri is shown in Figure 8. The averaged heat transport performs nonlinearly with heat generation parameter. The averaged energy transport augments in the absence of heat generation or heat absorption in the forced convection regime. But, averaged energy transport augments due to heat generation in the free convection regime. At Ri=0.01, the averaged Nusselt number gets its lowest values due to heat generation.

Figure 8. Averaged Nusselt numbers versus Hg for different with Ri = 0.01 (a) & Ri = 100 (b)

5. Conclusions

The problem of mixed magnetic-convection in a square titled driven box with sinusoidal thermal settings on walls in the occurrence of heat generation has been numerically investigated. The results of the present mathematical study lead to the subsequent deductions. Multiple flow patterns are observed due to non-uniform thermal conditions. The averaged heat transfer enhances on reducing the values of Ri and Ha. The averaged Nusselt number augments when growing the values of amplitude ratio for all values of Ri and Ha. The averaged Nusselt number boosts in the nonexistence of heat generation or heat absorption in forced convective regime while it enhances due to heat generation in the free convection regime.

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