\textbf{\textit{\Upsilon \bar{B}B} COUPLINGS, SLOPE OF THE ISGUR-WISE FUNCTION
AND IMPROVED ESTIMATE OF $V_{cb}$}

S. Narison

Theoretical Physics Division, CERN
CH - 1211 Geneva 23
and
Laboratoire de Physique Mathématique
Université de Montpellier II
Place Eugène Bataillon
34095 - Montpellier Cedex 05

\textbf{ABSTRACT}

We estimate the sum of the $\Upsilon \bar{B}B$ couplings using QCD Spectral Sum Rules (QSSR). Our result implies the phenomenological bound $\xi'(vv' = 1) \geq -1.04$ for the slope of the Isgur-Wise function. An analytic estimate of the (physical) slope to two loops within QSSR leads to the accurate value $\xi'(vv' = 1) \simeq -(1.00 \pm 0.02)$ due to the (almost) complete cancellations between the perturbative and non-perturbative corrections at the stability points. Then, we deduce, from the present data, the improved estimate $|V_{cb}| \simeq (1.48 \text{ps}/\tau_B)^{1/2} (37.3 \pm 1.2 \pm 1.4) \times 10^{-3}$ where the first error comes from the data analysis and the second one from the different model parametrizations of the Isgur-Wise function.
1 Introduction

With the advent of the heavy quark symmetry [1], there has been considerable interest and progress in the understanding of the semileptonic form factors of the transition of a heavy quark into another heavy quark, as in this infinite mass limit all semileptonic form factors reduce to the single Isgur-Wise function [2]. The question is whether this result in the heavy quark infinite mass limit can be applied to the physical B and D mesons. Progress has been made for including the $1/M_Q$ corrections. At the non-recoil point, the $1/M_Q$ terms cancel in the form factors [3], while for the leptonic decay constant, one expects a large $1/M_Q$ correction as indicated by lattice [3] and QCD Spectral Sum Rules (QSSR) estimates. QCD vertex sum rules have also been applied for evaluating the semi-leptonic form factors at $q^2 = 0$ [3–8] and the Isgur-Wise function at $q_{\text{max}}^2$ [8–10]. Although the QSSR results are quite impressive and the authors [8–10] expect that working with the ratio of the vertex over the two-point pseudoscalar sum rules eliminates different systematic uncertainties, one should not forget that, in this derivation, certain assumptions on the choice of the QSSR scales and QCD continuum thresholds, which introduce uncertainties, have to be done. Most of these choices are based on the model discussed in Ref. [11]. More rigorous and less model dependent is the bound obtained recently by [12], where one exploits the analyticity and the unitarity of the $b$-number form factor in the limit $M_c = M_b$ (see also [13], [14]). The bound on the slope of the form factor in [12] depends on the residues and on the positions of Υ poles. The most rigorous bound comes from the normalization condition $F(0) = 1$ of the $b$-number form factor leading to

$$F'(v,v' = 1) \geq -6,$$

while the inclusion of $\Upsilon \bar{B}B$ couplings, assumed to be of order one each, leads to phenomenological bound [12]:

$$F'(v,v' = 1) \geq -1.5,$$

which is 4 times much stronger than the previous one.

The purpose of this note is to present an estimate of the $\Upsilon \bar{B}B$ couplings which play an important role in the phenomenological derivation of the previous bounds [12], [13]. We shall also estimate directly this slope from analytic expressions to two-loops in the QSSR approach. Then, we shall deduce an estimate of the CKM mixing angle $V_{cb}$.

As the $\bar{B}B$ pairs are below the $\Upsilon(1S,2S,3S)$ states, there are no available data for estimating such couplings. The only experimental available information concerns the coupling of the $\Upsilon(4S)$ which lies above the $\bar{B}B$ threshold. From the data of its leptonic and total widths, one can deduce [12]:

$$|\eta_{\Upsilon_4}| \equiv \frac{g_{\Upsilon_4 \bar{B}B}}{2\gamma_{\Upsilon_4}} \leq 0.75 \pm 0.15,$$  \hspace{1cm} (2)

where

$$\Gamma(\Upsilon(4S) \rightarrow e^+e^-) = \frac{1}{3}\pi\alpha^2 \left( \frac{1}{3} \right)^2 \frac{M_{\Upsilon_4}}{\gamma_{\Upsilon_4}^2},$$  \hspace{1cm} (3)

and

$$\Gamma(\Upsilon(4S) \rightarrow \bar{B}B) = M_{\Upsilon_4} \left( \frac{g_{\Upsilon_4 \bar{B}B}}{48\pi} \right) \left( 1 - \frac{4M_{\bar{B}}^2}{M_{\Upsilon_4}^2} \right)^{3/2},$$  \hspace{1cm} (4)

with the normalization:

$$< 0 | -\bar{b}\gamma\mu b | \Upsilon_i > = e^\mu \frac{M_{\Upsilon_i}^2}{2\gamma_{\Upsilon_i}},$$  \hspace{1cm} (5)
where we have included in the definition of the current the negative sign related to the \( b \)-quark charge. In principle, the couplings of the three lightest states are unconstrained. However, using the location of the three \( \Upsilon \) poles and the fact that the \( b \)-number form factor of the \( B \)-meson is \( 1 \) at \( q^2 = 0 \), Ref. [12] obtains the following bounds:

\[
|\eta_{\Upsilon 1}| \leq 9.9 \times 10^3, \quad |\eta_{\Upsilon 2}| \leq 17.1 \times 10^3, \quad |\eta_{\Upsilon 3}| \leq 8.1 \times 10^3,
\]  \( 6 \)

which, as already pointed by the authors, are quite weak compared to the one for \( \eta_4 \) obtained previously from the data.

## 2 QSSR estimate of the \( \Upsilon \bar{B}B \) coupling

In so doing, we consider the vertex correlator:

\[
V^\mu(q^2, p^2, p'^2) = (i)^2 \int d^4x \ d^4y e^{i(p_x - p'_x y)} < 0|TJ_5(x)J^\mu(0)J_5(y)|0 > \\
\equiv (p + p')^\mu V_+(q^2, p^2, p'^2),
\]  \( 7 \)

built with the local quark currents:

\[
J_5(x) = (m_d + M_b) : \bar{d}(i\gamma_5)b : , \quad J^\mu = - : \bar{b}\gamma^\mu b : .
\]  \( 8 \)

\( V_+ \) obeys the double dispersion relation:

\[
V_+(q^2, p^2, p'^2) = -\frac{1}{\pi^2} \int_{M_b^2}^{\infty} \frac{dt}{t - p^2} \int_{M_b^2}^{\infty} \frac{dt'}{t' - p'^2} \text{Im}V_+(q^2, t, t') + ..., \]  \( 9 \)

where \( ... \) means polynomial subtraction constants and \( q^2 \equiv (p - p')^2 \leq 0 \). For the estimate of the \( \sum \eta_{\Upsilon i} \) couplings, we evaluate the correlator at \( q^2 = 0 \) similarly to the estimate of the form factors of the \( B \) semi-leptonic decays using vertex sum rules. We shall limit ourselves to the lowest order contribution in \( \alpha_s \) but include the non-perturbative condensate contributions in the OPE. This approximation has also given a quite good prediction in different estimates of the \( B \) and \( D \) decay form factors. So we, a priori, expect that a similar feature will hold in our analysis. The QCD expression of the three-point function has been evaluated in the literature [1], including higher dimension condensates. Our case corresponds to putting \( M_b = M_c \) in this paper. Therefore, the perturbative contribution reads, to leading order in \( \alpha_s \):

\[
\text{Im}V_+^\text{pert}(q^2, t, t') \simeq -\frac{3}{4} q^2 \frac{(M_b^4 - tt')}{((t + t' - q^2)^2 - 4tt')^{3/2}},
\]  \( 10 \)

which shows that at \( q^2 = 0 \), the lowest order perturbative contribution to the spectral function vanishes. We have also checked this result from a direct evaluation of the triangle perturbative diagram. Therefore, in this case, the leading contribution comes from the light quark condensate and reads:

\[
V_+(0, p^2, p'^2) = M_b^3 < \bar{d}d > \frac{1}{(p^2 - M_b^2)(p'^2 - M_b^2)},
\]  \( 11 \)
which is not the case of the other B decay form factors studied previously at $q^2 = 0$. So, from this particular feature, we (intuitively) expect that $V_+(0)$ is much smaller than the previous form factors. We parametrize the phenomenological side of the vertex by introducing the B and $\Upsilon$ couplings via:

$$<0|J_{5}|B> = \sqrt{2} f_B M_B^2,$$

$$<0|J_{\mu}|\Upsilon_i> = \epsilon_{\mu} \frac{M_B^2}{2 \gamma_{\Upsilon_i}},$$

and we insert the intermediate states in (7). Using the definition of the $\Upsilon BB$ coupling in (2), we have at $q^2 = 0$:

$$V_+(0, p^2, p'^2) \simeq -\frac{2 f_B^3 M_B^4}{(p^2 - M_B^2)(p'^2 - M_B^2)} \sum_i \eta_{\Upsilon_i}.$$

If one uses the quark hadron (semi)local duality picture by simply equating the phenomenological and QCD sides of the vertex, one obtains the sum rule:

$$\sum_i \eta_{\Upsilon_i} \simeq \left(\frac{M_b}{M_B}\right)^3 <\bar{d}d>.$$

Using for $f_B$ the local duality constraint \cite{13}:

$$2 f_B^2 M_B \simeq \frac{1}{\pi^2} \left(\frac{E_c^B}{M_B}\right)^3 \left(\frac{M_b}{M_B}\right)^3,$$

where $E_c^B \simeq 1.3$ GeV \cite{13} is the B continuum energy, one obtains:

$$\sum_i \eta_{\Upsilon_i} \simeq \frac{\pi^2}{(E_c^B)^3} <\bar{d}d> \simeq -0.07,$$

if one uses $<\bar{d}d> (E_c^B) \simeq -(250$ MeV$)^3$. The previous constraint indicates that $\eta_{\Upsilon_i}$ remains constant for $M_b \to \infty$, in agreement with the expectation from the “naive $M_b$ counting rule”:

$$g_{\Upsilon BB} \to M_b^{1/2}, \quad \gamma_{\Upsilon} \to M_b^{1/2}. \quad (17)$$

This feature increases our confidence on the physical meaning of the constraint in (14). Moreover, the previous constraint also indicates that the sum of couplings in (16) is almost independent of the b-quark mass as we shall check later on in the case of the J/ψ.

One can improve further the previous constraint by working with the Laplace (Borel) sum rules and by including the contribution of the quark-gluon mixed condensate $g <\bar{d}G_{\mu\nu}^a G^a_{\mu\nu} d> \equiv M_0^2 <\bar{d}d>$. In this way, the relativistic Laplace operator leads to the change:

$$\frac{1}{p^2 - M_P^2} \to \tau e^{-\tau M_P^2}, \quad \frac{1}{p'^2 - M_P^2} \to \tau' e^{-\tau' M_P^2},$$

where $\tau$ (resp. $\tau'$) are the sum rule variables associated to $p^2$ (resp. $p'^2$). For convenience and because of the symmetry of the vertex, we shall take the natural choice $\tau = \tau'$ in our
analysis. However, this choice does not have any noticeable effect in our conclusion. We use the following values of the QCD parameters [5],[16]:

\[ f_B \simeq (1.59 \pm 0.26)f_\pi \]
\[ M_b(p^2 = M_b^2) \simeq (4.59 \pm 0.05) \text{GeV} \]
\[ < \bar{d}d > = -((189 \pm 7) \text{MeV})^3 \left(-\log(\tau^{-1/2}\Lambda)\right)^{-2/\beta_1} \]
\[ M_0^2 \simeq (0.80 \pm 0.10) \text{GeV}^2, \quad (19) \]

where \(-\beta_1 = \frac{1}{2}(11 - \frac{2n}{3})\) for SU(\(n\))\(_f\) and \(\Lambda = 260 \pm 50 \) MeV. We give the result for \(\sum \eta_\Upsilon_i\) in Fig. 1 versus the sum rule variable \(\tau\). Due to the absence of the perturbative contribution in our leading order analysis, the result is insensitive to the continuum contribution. The stability in \(\tau\) is reached at 0.2 GeV\(^{-2}\), a value that is quite similar to the one where \(f_B\) is optimal [5]. At the stability point, one obtains:

\[ \sum_i \eta_\Upsilon_i \simeq -0.224, \quad (20) \]

where the error is negligible (0.004), if one uses the correlated values of \(f_B\) and \(M_b\). We estimate the maximal error by taking the uncorrelated values of the previous two parameters. Then, we obtain, at the order we are working, the estimate:

\[ \sum_i \eta_\Upsilon_i \simeq -(0.224 \pm 0.064). \quad (21) \]

An improvement of this result needs an evaluation of the radiative correction to the vertex function. However, we do not expect that the higher order terms will modify the present leading order results by more than a factor 2-3, if no marginal terms break the conventional OPE or/and if there are no anomalous couplings that drastically modify the parametrization of the spectral function in (13).

The result from the simple local duality relation in (16) corresponds to the case where \(\tau \to 0\). The difference between the local duality and Laplace sum rules results can indicate the possible large role of the continuum for \(\tau \to 0\) which is negligible in the Laplace sum rule analysis.

We test the quality of our estimate by applying the method in the J/\(\psi\) channel where the couplings of the 3S and 4S states are also bounded experimentally to be:

\[ \eta_{\psi_3} \leq 0.47, \quad \eta_{\psi_4} \leq 0.17. \quad (22) \]

As we have discussed in the case of the decay constant \([13],[14]\), the simple duality constraint in (15) reproduces correctly the ratio \(f_B/f_D\), though its prediction for the absolute value is inaccurate. Then, we also expect that the ratio of the coupling \(\eta_i\) is well reproduced by (16). It gives:

\[ \sum_i \eta_{\psi_i} \simeq -\left(\frac{E_{\psi_3}^{D_c}}{E_{\psi_4}^{D}}\right)^3 \sum_i \eta_\Upsilon_i \simeq 0.40, \quad (23) \]

where we have used \(E_{\psi_3}^{D_c} \simeq 1.08 \text{GeV} \quad ([3],[13]). \) A direct Laplace (Borel) sum rule estimate analogous to the one used to get (21) gives an optimization scale \(\tau \simeq 0.8 \text{GeV}^{-2}\), very similar to the one for \(f_D \quad [3]. \) Using \(f_D \simeq (1.31 \pm 0.12)f_\pi\) and \(M_c(p^2 = M_c^2) \simeq (1.45 \pm 0.05)\text{GeV} \quad [3],\) the sum rule gives the estimate:

\[ \sum_i \eta_{\psi_i} \simeq 0.34 \pm 0.02. \quad (24) \]
This result compares quite well with the previous leading order estimate from local duality sum rule in (23) and with the experimental bounds in (22). It also indicates that the sum of couplings is almost independent of the heavy quark mass, which is manifest in the local duality constraints. This test increases further our confidence on the numbers obtained in (21).

Moreover, our result applies if the quark structure of the vertex remains valid for this particular process but cannot be used if the $\bar{B}B, \bar{B}^*B^*$ pairs are molecules formed by Van Der Vaals like forces. In this case a vertex sum rule approach with four-quark currents similar to the one done for the $K\bar{K}$ molecule [17] becomes more adequate.

### 3 Bound on the slope of the Isgur-Wise function

We have estimated the sum of the $\Upsilon \bar{B}B$ couplings using a conventional vertex sum rules analysis within the quark structure of the $\bar{B}B$ meson pairs. Despite the leading order approximation that we have used for deriving the values of the couplings, we expect that our results are valid within a factor of 2-3, which is a conservative estimate of the radiative corrections not included here. A comparison of the result with the experimental bound in (2) indicates that the bound for the $\Upsilon(4S)$ coupling is satisfied by the sum of the couplings of the different $\Upsilon$ states. In order to see the effects of our results for the bound on the slope of the Isgur-Wise function as derived in [12], we shall consider the following different scenarios on the strength of each coupling given the constraint in (19) for the sum: The first scenario, which seems to be the most plausible phenomenologically, due to the experimental suppression of the electronic widths of the 2S and 3S states, is the one where the coupling of the 1S state is much larger than the previous ones (vector meson dominance) (first row in the tables). The second and third scenarios (second and third rows) are the ones where the absolute values of the couplings are equal, with also the possibility to have a cancellation between the couplings of the 2S and 3S. The fourth possibility (fourth row) is the one where the coupling of the 3S almost saturates the experimental bound for the 4S given in (1). The one of the 2S is assumed to be about the meson mass squared ratio $(M_3/M_2)^2$ times the one of the 2S.

The upper and lower bounds on the slope of the $b$-number form factor are given in these tables [13]. One can notice that increasing the sum of couplings by a factor 3 only affects the bounds by 15%. From the tables, one can deduce the conservative phenomenological bounds for the slope of the $b$-number form factor:

$$-(0.88 \sim 1.34) \leq F'(vv' = 1) \leq (0.08 \sim 0.52).$$

The lower bound is comparable with and even slightly stronger than the conservative bound in (1) given by Ref. [14], while the upper bound is weaker than the Bjorken bound of $-1/4$ [15]. Using its relation with the slope of the IW function $\xi'$ [20]:

$$\xi'(1) \simeq F'(1) - \frac{16}{75} \log \alpha_s(M_b),$$

we can deduce the conservative lower bound:

$$\xi'(vv' = 1) \geq -1.04.$$
Table 1: Upper and lower bounds for the slope of the $b$-number form factor for various phenomenological values of the couplings $\eta_i$ with $\sum \eta_i = -0.224$.

| $\eta_1$ | $\eta_2$ | $\eta_3$ | $F'(1)_{\text{lower}}$ | $F'(1)_{\text{upper}}$ |
|----------|----------|----------|------------------------|------------------------|
| -0.224   | 0.000    | 0.000    | -0.967                 | 0.447                  |
| -0.224   | -0.224   | 0.224    | -0.882                 | 0.516                  |
| -0.080   | -0.080   | -0.080   | -0.950                 | 0.465                  |
| -1.760   | +0.890   | 0.720    | -1.183                 | 0.235                  |

Table 2: The same as in Table 1 but $\sum \eta_i = -3 \times 0.224$.

| $\eta_1$ | $\eta_2$ | $\eta_3$ | $F'(1)_{\text{lower}}$ | $F'(1)_{\text{upper}}$ |
|----------|----------|----------|------------------------|------------------------|
| -0.672   | 0.000    | 0.000    | -1.118                 | 0.306                  |
| -0.672   | -0.672   | 0.672    | -1.175                 | 0.250                  |
| -0.224   | -0.224   | -0.224   | -1.048                 | 0.373                  |
| -2.170   | 0.780    | 0.720    | -1.344                 | 0.081                  |

The previous results question the accuracy of the experimental domain [21]:

$$-2.3 \leq \xi'(vv' = 1) \leq -1.17,$$

obtained after extrapolating the data until the non-recoil point. Our results also indicate that the smallness of the sum of the $\Upsilon \bar{B}B$ couplings, and presumably, of each individual coupling derived in this paper, raises again some doubts on the accuracy of existing models and methods used for determining the mixing angle $V_{cb}$.

4 QSSR estimate of the slope of the IW function

In view of the former result, let us estimate $\xi'(1)$ \textit{analytically}, using QSSR in the Heavy Quark Effective Theory (HQET). In so doing, we work as [8]-[10], with the ratio of vertex over two-point function sum rules to two loops within the continuum model of [11]. Using the expressions in [8], [10], we can deduce the compact analytical expression of the \textit{physical} Isgur-Wise function:

$$\xi_{\text{phys}}(y \equiv vv') \simeq \left(\frac{2}{1 + y}\right)^2 \left(1 + \frac{\alpha_s}{\pi} f(y)\right)$$

$$-<\bar{q}q> \tau^3 \left(\frac{8\pi^2}{3I_0}\right) \left\{1 + \frac{\alpha_s}{\pi} 2.38\right\} \left(1 - \left(\frac{2}{1 + y}\right)^2\right) + \frac{\alpha_s}{\pi} g(y)$$

$$+ <\alpha_s G^2 > \tau^4 \left(\frac{8\pi^2}{3I_0}\right) \left(\frac{1}{192\pi}\right) (y^2 - 1)$$

$$+ <\bar{q}q> \tau^5 \left(\frac{M^2}{4}\right) \left(\frac{8\pi^2}{3I_0}\right) \left\{\frac{2y + 1}{3} - \left(\frac{2}{1 + y}\right)^2\right\},$$

(29)
with:
\[ f(y) = \gamma(y) I_x(\tau) + (y - 1) \left( \frac{16}{9} \log 2 - \frac{49}{54} \right) - (1 - y)^2 \left( \frac{8}{15} \log 2 - \frac{197}{600} \right) \]
\[ g(y) = \gamma(y) (\text{Ei}(-\omega_c \tau) - \gamma_E) + (y - 1) \left( \frac{16}{9} \log 2 - \frac{56}{27} \right) - (1 - y)^2 \left( \frac{8}{15} \log 2 - \frac{112}{225} \right) \]

where:
\[ \gamma(y) = 4 \left( \frac{\log \left( y + \sqrt{y^2 - 1} \right)}{\sqrt{y^2 - 1}} - 1 \right) \], \( \gamma(1) = 0 \), \( \gamma'(1) = 8/9 \), \( \gamma_E = 0.5772 \), \( \text{Ei}(-x) = -\int_x^\infty \frac{dx}{x} e^{-x} \)

and:
\[ I_x(\tau) \equiv \int_0^{\omega_c} d\omega \frac{\omega^2}{1 + \frac{\omega}{M_Q}} \log(\omega \tau) e^{-\omega \tau} / \left( I_0 \equiv \int_0^{\omega_c} d\omega \frac{\omega^2}{1 + \frac{\omega}{M_Q}} e^{-\omega \tau} \right) \]

We shall use in our numerical analysis the value: \( \omega_c \simeq (3.0 \pm 0.5) \) GeV with a generous error compared to the true error of 0.1 [1]. We leave \( \tau \) as a free parameter, which we shall fix from a variational method. The slope \( \xi'(1) \) is the value of the first derivative of the IW function with respect to \( y \) at \( y = 1 \), which we can deduce **analytically** from the previous sum-rule expression of \( \xi_{\text{phys}} \). Then, its expression reads:
\[ \xi'(y = 1) \equiv -1 + \delta_{\text{pert}} + \delta_{NP} \]

where:
\[ \delta_{\text{pert}} \simeq -\frac{\alpha_s}{\pi} \left( I_x \gamma'(1) + \frac{16}{9} \log 2 - \frac{49}{54} \right) \]
\[ \delta_{NP} \simeq -\frac{8 \pi^2}{3 I_0} \tau^3 < \bar{q}q > \left( 1 + 1.05 \frac{\alpha_s}{\pi} - \frac{5}{12} M_0^2 \tau^2 \right) \]
\[ + \frac{8 \pi^2}{3 I_0} \tau^4 < \alpha_s G^2 > \]

We optimize this previous sum rule for \( \xi' \) using a variational method. The stability of the result is reached for \( \tau^{-1} \simeq 1.7 \) GeV (which is about the characteristic scale of the \( b \) into \( c \) transition), while the result is very stable (less than 2% change) for a larger range of \( \tau^{-1} \) between 1.25 and 2.5 GeV. An analogous stability is also obtained by moving \( \omega_c \) in the range between 2.7 GeV (starting of \( \tau \) stability) to 3.5 GeV. We also notice that there is an almost complete cancellation between the perturbative radiative and non-perturbative corrections where each strength does not exceed 4%. This feature leads to the accurate estimate:
\[ \xi'(1) \simeq -(1.00 \pm 0.02) \]

We can multiply the previous error by a factor two in order to add a conservative systematic error inherent in the method and in the continuum model.

This value differs from the previous result in [1] deduced from a numerical polynomial two-parameter fit of the Isgur-Wise function, where the errors given there are quite doubtful. Our result satisfies the bound derived previously and the one of [12].
5 Improved estimate of $V_{cb}$

As an application of our result, let us estimate the mixing angle $V_{cb}$ by using the ARGUS \[21\] and CLEO \[22\] data on $\xi(y) \times V_{cb}$. We shall use in our numerical analysis the following parametrizations:

\[
\begin{align*}
\xi(y) &\simeq 1 + \xi'(y - 1) \\
\xi(y) &\simeq \exp \{\xi'(y - 1)\} \\
\xi(y) &\simeq \{(1 + y)/2\}^{2\xi'} \\
\xi(y) &\simeq \left(\frac{2}{y + 1}\right) \exp \left\{(2\xi' + 1) \frac{y - 1}{y + 1}\right\}. \quad (36)
\end{align*}
\]

The two former are in line of the Taylor expansion used for $y$ around 1. The third is the pole parametrization and the fourth is based on overlap integrals of meson wave functions in a harmonic oscillator model. We shall normalize our result with the world average $\tau_B \simeq (1.48 \pm 0.10)$ ps of the $B$-lifetime given in \[23\]:

\[
V_{cb} \simeq \left(\frac{1.48\text{ps}}{\tau_B}\right)^{1/2} \tilde{V}_{cb}. \quad (37)
\]

Given the previous value of the slope in (35), we determine $V_{cb}$ from each data point and then, we make a weighted average of the different results. From $B^0 \rightarrow D^{*-}l\bar{\nu}$, we obtain in units of $10^3$:

\[
|\tilde{V}_{cb}| \simeq 36.9 \pm 3.3 \quad \text{ARGUS 91} \\
\simeq 36.0 \pm 1.5 \quad \text{CLEO 93}, \quad (38)
\]

while from $B^- \rightarrow D^{*0}l\bar{\nu}$, we obtain in units of $10^3$:

\[
|\tilde{V}_{cb}| \simeq 40.6 \pm 2.3 \quad \text{ARGUS 92}. \quad (39)
\]

Our results are the average of the ones from the previous alternative parametrizations of $\xi$ in (36). The last two parametrizations give almost the same results of $V_{cb}$. The errors given there are the largest ones from each parametrization and are only due to the data. The choice of the parametrizations induces an extra error of 1.4. Theoretical errors induced by the ones of the slope are negligible. We take the average of previous results in (38) and (39). Then, we obtain the final best estimate:

\[
|\tilde{V}_{cb}| \simeq (37.3 \pm 1.2 \pm 1.4) \times 10^{-3}, \quad (40)
\]

where we the first error comes from the data, while the second one is due to the different choices of parametrizations used in the literature. We consider this result as a noticeable improvement over the existing estimate of $V_{cb}$ (see e.g. \[23\], \[24\]) thanks to a better control of the value of the slope from the QSSR estimate in (35). This result also agrees with the lesser accurate estimate (after rescaling the lifetime used in \[1\]) from the $B$ into $D, D^*$ semi-leptonic decays within a QSSR estimate of the form factors at zero momentum, where finite corrections due to the $c$ and $b$ quark masses have been taken into account.

Moreover, a model-independent result from the phenomenological de Rafael-Taron-like bound in (27) gives:

\[
|\tilde{V}_{cb}| \leq 38.9 \times 10^{-3}, \quad (41)
\]

which is enough strong for eliminating some results given in the literature.
6 Conclusions

We have estimated in Eq. (21) the sum of the $\Upsilon BB$ couplings using vertex sum rules. Using this information into the analysis of [12], we have derived in Eq. (27) a phenomenological bound on the slope of the IW function. Finally, we have reestimated in Eq. (35) this slope analytically from QSSR. From the previous results, we have deduced the value of $V_{cb}$ in (40) and the bound in (41). The accuracy of this value of $V_{cb}$ is mainly due to the good control of the slope both from the sum rules and from the de Rafael-Taron-like bound.

Acknowledgements

I wish to thank Josep Taron and Eduardo de Rafael for numerous discussions and for reading the manuscript. I have also enjoyed useful conversations with Ahmed Ali, Hans Gunter Dosch and Thomas Mannel.

Figure captions

$\tau$-dependence of the $\Upsilon BB$ coupling in the Laplace sum rule analysis.

References

[1] For reviews see e.g. H. Georgi, Proceedings of TASI-91 (World Scientific, Singapore, 1991), edited by R.K. Ellis et al.; N. Isgur and M. Wise, Proceedings of Heavy Flavours (World Scientific, Singapore, 1992), edited by A. Buras and M. Lindner; T. Mannel, Talk given at the 5th International Symposium on Heavy Flavours, Montreal, Canada, 6-10th June 1993, CERN preprint TH-7052/93 (1993).

[2] N. Isgur and M.B. Wise, Phys. Lett. B332 (1989) 113.

[3] M. Luke, Phys. Lett. B252 (1990) 447.

[4] C. Sachrajda, Talk given at the EPS conference, July 1993, Marseille and references quoted therein; L. Maiani, Helv. Phys. Acta 64 (1991) 853; G. Martinelli, Talk given at the Third $\tau$Cf Workshop, 1-6th June 1993, Marbella, Spain; S. Sharpe, Nucl. Phys. (Proc. Suppl) 17 (1990) 146; G. Alexandrou et al., Phys. Lett. B256 (1991) 60.

[5] S. Narison, Phys. Lett. B198 (1987) 104; Lecture Notes in Physics, Vol. 26, QCD Spectral Sum Rules (World Scientific, Singapore, 1989) and references therein; Z. Phys. C55 (1992) 55; Phys. Lett. B308 (1993) 365 and Talk given at the Third $\tau$Cf Workshop, 1-6 June 1993, Marbella, Spain, CERN preprint TH-7042/93 (1993) and references therein.
[6] A. Ovchinnikov and V.A. Slobodenyuk, *Z. Phys.* **C44** (1989) 433. V.N. Baier and A.G. Grozin, *Z. Phys.* **C47** (1990) 669.

[7] S. Narison, *Phys. Lett.* **B283** (1992) 384.

[8] P. Ball, V.M. Braun and H.G. Dosch, *Phys. Rev.* **D44** (1991) 3567; E. Bagan et al, *Phys. Lett.* **B278** (1992) 457.

[9] E. Bagan, P. Ball and P. Gosdzinsky, *Phys. Lett.* **B301** (1993) 101; E. Bagan and P. Gosdzinsky, *Phys. Lett.* **B305** (1993) 157.

[10] M. Neubert, *Phys. Rev.* **D45** (1992) 2451, **D46** (1992) 1076 and **D47** (1993) 4063.

[11] B. Blok and M.A. Shifman, *Phys. Rev.* **D47** (1993) 2949.

[12] E. de Rafael and J. Taron, Marseille preprint CPT-93/P.2908 (1993).

[13] A.F Falk, M. Luke and M.B. Wise, *Phys. Lett.* **B299** (1993) 123; B. Grinstein and P.F. Mende, *Phys. Lett.* **B299** (1993) 127; C.E. Carlson, J. Milana, N. Isgur, T. Mannel and W. Roberts, *Phys. Lett.* **B299** (1993) 133; C.A. Dominguez, J.G. Korner and D. Pirjol, *Phys. Lett.* **B301** (1993) 257; P. Ball, H.G. Dosch and M.A. Shifman, *Phys. Rev.* **D47** (1993) 4077.

[14] E. de Rafael and J. Taron, *Phys. Lett.* **B292** (1992) 215.

[15] S. Narison and K. Zalewski, CERN preprint TH-7058/93 (1993) (Phys. Lett. B in press); S. Narison, CERN preprint TH-7094/93 (1993) (Phys. Lett.B in press).

[16] S. Narison, *Phys. Lett.* **B216** (1989) 191.

[17] S. Narison, *Phys. Lett.* **B175** (1986) 88.

[18] J. Taron (private communication).

[19] J.D. Bjorken, SLAC-PUB- 5278 (1990).

[20] A.F. Falk, H. Georgi and B. Grinstein, *Nucl. Phys.* **B343** (1990) 1.

[21] H. Albrecht et al. (ARGUS), *Phys. Lett.* **B275** (1992) 195; *Z. Phys.* **C57** (1993) 533.

[22] G. Crawford et al. (CLEO), CLEO CONF 93-30 (1993), *Talk presented at the XVI Int. Symp. on Lepton-Photon Interactions, Ithaca, New York 1993*

[23] For a review, see e.g: M. Danilov, *Talk given at the EPS conference, July 1993, Marseille*.

[24] For a review, see e.g: A. Ali, CERN preprint TH-7123/93 *Talk given at the Abdus Salam fest, Trieste 1993*.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9403253v1