Effects of raceway waviness on dynamic behaviors of deep groove ball bearing

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Abstract. Deep groove ball bearings (DGBB) are widely applied in the supporting structure of a variety of rotating machineries. A high rotating accuracy should be maintained for a more stable rotor system, which requires a carefully designed DGBB. For a designing purpose, dynamic behaviors of the DGBB under different structural parameters need to be investigated. Its dynamic responses are essentially excited by the contact forces induced by external loads. Besides the relative displacement between the inner- and the outer-raceway, and the clearance, ring waviness is the major factor that affects the values of the contact forces and the deformations of different rolling elements. Therefore, the control of raceway waviness amplitude and waviness order is of great significance for the design of a high-precision DGBB and moreover a rotor system with high rotating accuracy. In this work, the effects of different waviness order and waviness amplitude are investigated to study the dynamic behavior of a DGBB under different waviness parameter.

Keywords: Ring waviness, waviness order, waviness amplitude, deep groove ball bearing, dynamic behaviour.

1. Introduction

Deep groove ball bearings (DGBB) are widely applied in the supporting structure of a variety of rotating machineries. The dynamic behaviors of the DGBB under different structural parameters need to be investigated. Its dynamic responses are essentially excited by the contact forces induced by contact deformations of the rolling element [1]. Besides the relative displacement between the inner- and the outer-raceway, and the clearance, ring waviness is the major factor that affects the values of the contact deformations. Therefore, the control of raceway waviness amplitude and waviness amplitude is of great significance for the design of a high-precision DGBB.

Tallian [2] built a linear rolling bearing model to study the relationship between raceway waviness and vibration response. Yhland [3] and Harsha [4] considered the effect of waviness order through several experimental trials. Besides the waviness parameters, Wardle [5] also investigated the effect of ball number on waviness. Liu [6] took the waviness of inner-race rib into consideration.

In this work, the effects of different waviness order and waviness amplitude are induced to study the dynamic behavior of a DGBB under different waviness parameter.
2. Bearing dynamic model

2.1. Bearing model coupled with a Jeffcott rotor

The dynamic behavior of a DGBB was investigated in this contribution with the consideration of a coupled Jeffcott rotor, as shown in Fig. 1. The rotor was symmetrically supported by two DGBBs with the same type. Gyroscopic force and axial vibration were omitted and all the stiffness and damping coefficients are assumed to be linear in our model.

Both the horizontal and vertical responses \( [(x_e, y_e), (x_{ij}, y_{ij}), (x_{op}, y_{op})] \) of the eccentric mass \( m_e \), the inner-race \( m_{ij} \) and the outer-race \( m_{op} \) were studied under the Cartesian coordinate system. Besides, the vertical response \( y_r \) of a resonance changer \( m_r \) was also considered to keep the natural frequency of our model to be consistent with the one of our test bench. According to the above terminologies, a 7-DOF DGBB-rotor model was established.

\[
\begin{align*}
\dot{x}_e + \frac{k_e}{m_e} x_e - \frac{k_x}{m_e} x_{ij} &= F^x_e \\
\dot{y}_e + \frac{k_y}{m_e} y_e - \frac{k_y}{m_e} y_{ij} &= F^y_e \\
\dot{x}_{ij} + \frac{k_x}{m_{ij}} x_{ij} + \frac{k_x}{m_{ij}} x_e &= -F^x_{\text{contact}} \\
\dot{y}_{ij} + \frac{k_y}{m_{ij}} y_{ij} + \frac{k_y}{m_{ij}} y_e &= -F^y_{\text{contact}} + Q_r \\
\dot{x}_{op} + \frac{c_p}{m_{op}} \dot{x}_{op} + \frac{k_x}{m_{op}} x_{op} &= F^x_{\text{contact}} \\
\dot{y}_{op} + (c_p + c_r) \dot{y}_{op} - c_r \dot{y}_r + (k_p + k_r) y_{op} - k_r y_r &= F^y_{\text{contact}} + Q_r \\
\dot{y}_r - c_r \dot{y}_{op} + c_r \dot{y}_r - k_r y_{op} + k_r y_r &= 0
\end{align*}
\]

Synchronous vibration was excited by the centrifugal force \( F_e \) of the eccentric mass rotating at a speed of \( \omega_s \) and a radius of \( e_u \) around the shaft axis.

\[
\begin{align*}
F^x_e &= m_e \omega_s^2 e_u \cos(\omega t + \alpha) \\
F^y_e &= m_e \omega_s^2 e_u \sin(\omega t + \alpha)
\end{align*}
\]

Contact forces \( F_{\text{contact}} \) between the rolling elements and the raceway were calculated according to the Hertzian contact theory as
The angular position of the \( j \)th \((j = 1, \ldots, n_b)\) ball is expressed as

\[
\phi_j = \frac{2\pi (j - 1)}{n_b} + \omega t + \phi_0
\]  

(4)

In which \( n_b \) represents the ball number. \( \phi_0 \) represents the initial position of the bearing retainer. The cage speed is defined as

\[
\omega_c = \frac{\omega_s}{2} \left( 1 - \frac{d_b}{D_p} \right)
\]  

(5)

In which \( \omega_s \) is the shaft speed. The diameters of the ball and the pitch circle are \( d_b \) and \( D_p \).

The deformation between the inner-raceway and the ball, and the counterpart between the outer-raceway and the ball, both constitute the total deformation of a DGBB as

\[
\delta_j = (x_{ij} - x_{op}) \cos \phi_j + (y_{ij} - y_{op}) \sin \phi_j - c + w
\]  

(6)

In Eq. 6, \( c \) is the working clearance, and its value is determined as to be 25 \( \mu m \) according to the model number of the DGBB simulated in our numerical studies.

State variable \( \gamma_j \) defines the compression status between the raceways and the balls.

\[
\gamma_j = \begin{cases} 
1 & \delta_j > 0 \\
0 & \text{otherwise}
\end{cases}
\]  

(7)

2.2. Ring waviness modeling

Ring waviness changes the contact angle and curvature radius of a rolling bearing as the profile of a bearing deviates from its theoretical one shown in Fig. 2. Direct result is that the dynamic response together with its frequency constitution changes accordingly.
Figure 2. Schematic diagram of bearing ring waviness

In our work, the ring waviness $w$ in Eq. 6 is modeled as the summation of both the inner- and outer-raceway waviness

$$w = w_{\text{out}} + w_{\text{in}}, \quad (8)$$

In which the outer-raceway waviness is modeled as

$$w_{\text{out}} = \sum_{m=1}^{n} A_{m}^{\text{out}} \sin \left\{ m \left[ \frac{2\pi(j-1)}{n_b} \right] + \beta_{m}^{\text{in}} \right\} \quad (9)$$

And the inner-raceway waviness is modeled as

$$w_{\text{in}} = \sum_{l=1}^{h} A_{l}^{\text{in}} \sin \left\{ l \left[ \frac{2\pi(j-1)}{n_b} \right] + \beta_{l}^{\text{in}} \right\} \quad (10)$$

In the expressions in Eq. 8 and Eq. 9, $m$ and $l$ are respectively the order of the outer- and inner-race waviness, while $A_{m}^{\text{out}}$ and $A_{l}^{\text{in}}$ are the waviness amplitude of a certain waviness order. $\beta_{m}^{\text{out}}$ and $\beta_{l}^{\text{in}}$ are the waviness initial angle of a certain waviness order.

3. Numerical studies of ring waviness

In this section, numerical studies are conducted on the established rotor-DGBB system to investigate the effects of waviness amplitude and waviness order on the dynamic behavior of the ball bearing. The type of the bearing and some of the main simulation parameters are listed in Table 1. The fourth order Runge-Kutta method is applied to solve the system equations, whose initial responses are $(x_0, y_0) = (x_0, y_0) = 0$. The rotational speed of the inner-race and the shaft is 2880 r/min ($f_s = 48$ Hz) and the radial load $Q_r$ applied on the shaft is 5 kg. According to the bearing parameters listed in Table
1, pass frequency of the outer-race and the inner-race are respectively \( f_o = 146.5 \text{ Hz} \) and \( f_i = 237.5 \text{ Hz} \), and the cage frequency is 18.3 Hz.

### Table 1. Bearing type and simulation parameters

| Parameter                        | Value                  |
|----------------------------------|------------------------|
| Bearing model                    | NSK 6204               |
| Number of balls \( n_b \)        | 8                      |
| Outer ring/Inner ring/Ball diameter [mm] | 42.45/26.56/7.94    |
| Radial clearance \( c \) [mm]     | 0.025                  |
| Eccentric mass \( m_e \) [kg]     | 0.01                   |
| Outer/Inner ring waviness order \( n [h=n] \) | 5, 8, 13, 16          |
| Waviness amplitude [\( \mu \)m]  | 0.4, 1, 4, 7, 10       |
| Contact stiffness \( k \) [N/m]   | 1.9 \times 10^{10}    |
| System damping \( c_r/c_p \) [N/m \( \cdot \) s^{-1}] | 2000 |

#### 3.1. Effects of waviness amplitude

In this section, the waviness order is set as \( n=h=10 \) in order to investigate the effect of the waviness amplitude on bearing responses. Waviness amplitude of the outer-race and the inner-race are respectively 0.4 \( \mu \)m, 1 \( \mu \)m, 4 \( \mu \)m, 7 \( \mu \)m and 10 \( \mu \)m. The acceleration RMS value of the bearing with the above waviness amplitude is shown in Fig. 3. Results show that the RMS value of the DGBB increases significantly with the increment of the waviness amplitude. Fig 4 shows the amplitude spectrum of different waviness amplitude. It can be seen that the characteristic frequencies in the amplitude spectrum mainly reside on the position \( pf_i \pm qf_s \) Hz and \( pf_o \pm qf_s \) Hz, in which \( p \) and \( q \) are positive integers.

Furthermore, as the waviness amplitude increases, amplitudes of the characteristic frequencies turn to be prominent at the frequency range around 500 Hz. The modulation the raceway pass frequencies \( f_i/f_o \) by the shaft frequency \( f_s \) is demodulated by applying the Hilbert transform to obtain the envelope spectrum, as shown in Fig. 5. The cutoff frequencies are set according to the amplitude spectrum, while the major modulations are found at \( f_i/f_o \) and \( 2f_i/2f_o \). In general, the modulation effect of the inner-race is larger than the counterpart of the outer-race. Besides, modulation mainly happens at \( f_i, 2f_i, 3f_i, f_o \) and \( 2f_o \) for smaller waviness amplitude. As for larger waviness amplitude, modulations mainly focus at \( f_i \) and \( f_o \).

![Figure 3. RMS value of the acceleration data versus waviness amplitude](image-url)
3.2. Effects of waviness order

In this section, the waviness amplitude is fixed as $A^m_{av} = A^n_{av} = 4 \mu m$ for the investigation of the effect of the waviness order on bearing responses. Waviness order of the outer-race and the inner-race are respectively 5, 8, 13, 16. The acceleration RMS value of the bearing with the above waviness order is shown in Fig. 6. Results show that the RMS value of the DGBB increases significantly with the increasement of the waviness order. Fig 7 shows the amplitude spectrum of different waviness order. It can be seen that the characteristic frequencies in the amplitude spectrum mainly reside on the position $pf_i \pm qf_s \ Hz$ and $pf_o \pm qf_s \ Hz$, in which $p$ and $q$ are positive integers. Furthermore, lower waviness
order seems to make the amplitudes of the characteristic frequencies being more prominent at the frequency range around 500 Hz. The modulation the raceway pass frequencies $f_i/f_o$ by the shaft frequency $f_s$ is demodulated by applying the Hilbert transform to obtain the envelope spectrum, as shown in Fig. 8. The cutoff frequencies are set according to the amplitude spectrum, while the major modulations are found at $f_i/f_o$ and $2f_i/2f_o$. In general, the modulation effect of the inner-race is larger than the counterpart of the outer-race at higher waviness order, and for lower waviness order, modulation effect of the outer-race frequency seems to be larger. Besides, modulation mainly happens at $f_i$, $2f_i$, $f_o$ and $2f_o$ for higher waviness order. As for lower waviness order, modulations mainly focus at $f_i$ and $f_o$.

![Figure 6. RMS value of the acceleration data versus waviness order](image)

![Figure 7. Acceleration spectrum of the DGBB with different waviness order](image)
4. Conclusion

In this paper, the dynamic model of a DGBB is established considering the supporting Jeffcott rotor. The outer-raceway waviness is modeled as a summation of a series of harmonic functions with angular frequencies being the integral multiples of the cage frequency $\omega_c$, while the angular frequencies in the harmonic model of the inner-raceway waviness are the integral multiples of $\omega_c-\omega_s$.

Simulations of different waviness amplitude and waviness order are conducted to study their influences on acceleration responses. Results show that both the waviness amplitude and the waviness order increase the RMS values of the acceleration data. Moreover, major characteristic frequencies shown in the amplitude spectrum are the integer multiples of outer-race or inner-race pass frequencies modulated by shaft frequency. Higher waviness amplitude seems to lead a higher energy of the characteristic frequencies. As for the modulation level, envelope spectrum show that strong modulations take place at the inner-race pass frequency, while modulations turn to be mild for outer-race pass frequency. Furthermore, higher-order raceway pass frequencies modulate the responses for higher waviness order.

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