The mechanism of stability of fault system inducing roof water-inrush

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Abstract
This paper analyzes the strain stability during mining, which often causes a water inrush. Mining causes constant stress on the fault zone, which is a loading process on the system composed of fault material and surrounding medium. A cusp catastrophe model is presented and the necessary and sufficient conditions leading to fault systems are discussed. The fault zone is assumed to be planar and is a combination of two media: medium-1 is elastic-brittle or strain-hardening and medium-2 is strain-softening. The shear stress-strain constitutive model for the strain-softening medium is described by the Weibull’s distribution law. It was found that the instability of a fault system mainly relies on the ratio between the stiffness of medium-1 to the post-peak stiffness of the strain-softening medium, and the homogeneity index of strain-softening medium and the bifurcation point, \( k/C_{20} \), which is the turning point of the fault system from stability to potential instability. One can judge the occurrence of fault instability from this feature and regard the index \( D \) as a parameter, which reflects the precursory abnormality of a fault.

Keywords
Fault instability, Weibull’s distribution, catastrophe, stiffness ratio, critical displacement

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Introduction

Water inrush is experienced in underground mining at various locations around the world, causing death and serious injury to underground miners and damage to mine facilities (Zhang et al., 1997). Although the phenomenon of water inrush has been experienced in mines for decades (Axel et al., 1998; Bell et al., 1997; Bell and Jermy, 2002; Bu and Mao, 2009; Santos and Bieniawski, 1989; Shi and Singh, 2001; Tang et al., 2002; Wang and Park, 2003; Wu et al., 2004; Yang, 1995), the harsh reality of failing to predict many catastrophic inrushes around the world shows that, so far, our understanding about water inrushes under mining conditions is still limited. Particularly when induced by fault activation. Some preventive and monitoring approaches (Dou et al., 2012; Zhu et al., 2012), such as the effusion method, micro-gravity method, rheological method, rebound method, drilling-yield method, and micro-seismic method, have been applied in many underground openings to assist understanding the mechanisms of water inrush and predicting its occurrence. However, few approaches have been found to be particularly successful. The reasons for this may be due to two factors: (1) that the precise physical mechanism of water inrush is very complicated, therefore, it is difficult to establish a simple mechanical model; and (2) the data monitored in situ are not completely or appropriately utilized.

In this paper, the water inrush induced by the fault activation is studied. The fault zone is assumed to be planar and is the combination of two media: medium-1 is elastic-brittle or strain-hardening and medium-2 is strain-softening. The shear stress-strain constitutive model for the strain-softening medium is described by Weibull’s distribution law. A cusp catastrophe model will be presented for the system that is composed of the fault and potential damaged rock mass. The unstable conditions of such a system leading to a fault system will be suggested using the catastrophe theory.

Mechanical model

Figure 1 shows the model analyzed in this article. The fault zone with dip angle $\theta$ is a nonuniform intercalation, the potential damaged rock mass with weight $Wg$ ($g$ is gravity acceleration) is a rigid body, the potential water pressure is $P$ and the joint surface resistance is $Fk$. Under the action of the driving force caused by the weight of the rock mass and potential water pressure, the creep displacement is $u$ along the intercalation. Due to the different stress levels, material composition, textures, potential water pressure and structures.
at different segments of the intercalation can comprise many kinds of media with different mechanical properties, such as elastic-brittle, elastic-ductile (strain-hardening) and strain-softening, etc. To simplify the analysis, the intercalation is regarded as the combination of only two kinds of media with different mechanical properties, i.e., one (medium-1) possesses a complicated elastic-brittle (such as a strong interlocking rock block or so-called rock bridge) behavior and the other (medium-2) has a strain-softening property (such as fault gouge).

The shear stress-displacement relation, for the strong rock block with pre-existing multi-cracks or flaws, can generally be assumed to be linear prior to failure. At failure, the rock block is bisected and a sudden stress drop will take place. After failure of the rock block, the combined cracks could resemble a stepped or saw-toothed pattern, through on going discontinuity, resulting in a slow or sometimes quick stress increment (Lajtai, 1967; Gehle and Kutter, 2003) stress increment instead of a decrease with growing displacement. Finally, the stepped or saw toothed discontinuity is leveled out, which may lead to a rapid stress release with growing displacement. According to the above assumptions and direct shear tests on intermittent rock joints performed by (Gehle and Kutter, 2003), the constitutive model for medium -1 (Figure 2) can be written as

\[
\tau_e = \begin{cases} 
G_{e1} \frac{u}{h} & (u < u_b) \\
G_{e2} \left(\frac{u - u_b}{h}\right) + \tau_b & (u \geq u_b)
\end{cases}
\]

where \(u_b\) is the critical displacement corresponding to the failure of the rock block, \(\tau_b\) is the value of \(\tau_e\) at point \(u_b\), \(G_{e1}\) and \(G_{e2}\) are the shear modulus for \(u < u_b\) and \(u \geq u_b\), and \(h\) is the layer thickness of the intercalation.

Figure 2. Constitutive curves of two media along the fault zone.
A simplified constitutive model of medium $-2$ can generally be expressed as Weibull’s distribution law and is adopted for the strain-softening media (Qian et al., 2006; Qin et al., 2001, 2006, 2010a, 2010b).

\[
\tau_s = G_s \frac{u}{h} \exp \left[ -\left( \frac{u}{u_0} \right)^m \right]
\] (2)

where $G_s$ is the initial shear modulus, $u_0$ is a measurement of average strength and $m$ is the shape parameter. Since $m$ is a measurement of the local strength variability, it can be considered as a homogeneity index (Tang et al., 1993). The larger the index $m$, the more homogeneous is the rock.

**Cusp catastrophe model**

The overall potential energy for the system illustrated in Figure 1 equals the sum of the strain energy and driving potential energy, i.e.

\[
V = \int_0^u \frac{G_s u}{h} l_1 \exp \left[ -\left( \frac{u}{u_0} \right)^m \right] du + \int_0^u \frac{G_{e1} u}{h} l_2 du - (P + Wg) u \sin \theta \quad (u < u_b)
\]

\[
+ \int_{u_b}^u \left[ \tau_b + \frac{G_{e2} (u - u_b)}{h} \right] l_2 du - (P + Wg - F_k) u \quad (u \geq u_b)
\]

(3)

where $l_e$ and $l_s$ are the length of the fault zone for media 1 and 2, respectively, and $l_e + l_s = l$; $u$ can be regarded as the state variable in the cusp catastrophe analysis.

Let $V' = 0$ and the equilibrium surface equation is expressed as

\[
V' = \begin{cases} 
\frac{G_s u}{h} l_1 \exp \left[ -\left( \frac{u}{u_0} \right)^m \right] + \frac{G_{e1} u}{h} l_2 - (P + Wg - F_k) \sin \theta & (u < u_b) \\
\frac{G_s u}{h} l_1 \exp \left[ -\left( \frac{u}{u_0} \right)^m \right] + \frac{G_{e2} u}{h} l_2 + \left( \tau_b - \frac{G_{e2} u_h}{h} \right) l_2 - (P + Wg - F_k) \sin \theta & (u \geq u_b)
\end{cases}
\]

(4)

Equation (4) is the equilibrium condition of forces. By the smoothness property of the equilibrium surface, the cusp can be solved using the condition $V'' = 0$, i.e.

\[
u = u_1 = \left( \frac{m + 1}{m} \right)^{\frac{1}{n}} u_0
\] (5)
Equation (5) shows that the displacement value at the cusp is exactly the displacement value at the turning point of the constitutive curve of medium 2.

By applying the Taylor series expansion with respect to \( u_1 \) for equation (4), discarding all the terms but the first three because the third order term is the minimum one away from zero, and substituting equation (5) into equation (4), one has a standard form of cusp catastrophe, which is

\[
x^3 + ax + b = 0
\]  

(6)

where

\[
x = \frac{u - u_1}{u_1}
\]  

(7)

\[
a = \frac{6}{(m + 1)^2} (k - 1)
\]  

(8)

\[
b = \frac{6}{m(m + 1)^2} (mk + 1 - \xi)
\]  

(9)

\[
k = \begin{cases} 
\frac{G_{e1}l_e}{mG_{ls}} \exp\left(\frac{m + 1}{m}\right) & (u < u_b) \\
\frac{G_{e2}l_e}{mG_{ls}} \exp\left(\frac{m + 1}{m}\right) & (u \geq u_b)
\end{cases}
\]  

(10)

\[
\xi = \begin{cases} 
\frac{(P + Wg - F_k)h}{G_{sls}u_1} \exp\left(\frac{m + 1}{m}\right) & (u_1 < u_b) \\
\frac{(P + Wg - F_k) - (\tau_b - \frac{G_e u_b}{h})l_h}{G_{sls}u_1} \exp\left(\frac{m + 1}{m}\right) & (u_1 \geq u_b)
\end{cases}
\]  

(11)

where \( k \) is the ratio of the stiffness of medium 1 to the stiffness at the turning point of the constitutive curve of medium-2. \( \xi \) is relative to the weight of the rock mass, geometric size of the system, and mechanical parameters of media (referred to as the geometric-mechanical parameter).

Substituting equation (8) into equation (9), the bifurcation set (cusp) (Thom, 1972) can be expressed as

\[
D = 4a^3 + 27b^2 = 4b^3(k - 1)^3 + 27\left(\frac{\beta}{m}\right)^2 (1 + mk - \xi)^2 = 0
\]  

(12)

where \( \beta = 6/(m + 1)^2 \).
Criterion analysis

The bifurcation set defines the thresholds where sudden changes can take place. As long as the state of the system remains outside the bifurcation set \((D > 0)\), the behavior varies smoothly and continuously as a function of the control parameters. Even on entering the bifurcation set \((D < 0)\) no abrupt change is observed. When the control point passes all the way through the bifurcation set \((D = 0)\), however, a catastrophe is inevitable. Thus, equation (12) is the sufficient and necessary mechanical criteria for the fault system instability (which often leads to water inrush). In the following analysis, \(D\) is referred to as the catastrophic characteristic index. Obviously, only when \(k \leq 1\), the condition of equation (12) may be satisfied. Thus, the necessary condition of instability is shown in formula (13),

\[
k \leq 1
\]

From equations (9), (11) and (12), one can derive that \(b\) depends on the relative magnitude of the resisting and driving forces at the turning point of the strain-stress curve of medium-2. It is clear from equation (12) that \(b > 0\), \(b = 0\) and \(b < 0\) corresponds to the sliding acceleration of the rock mass: \(b < 0\) (primary creep), \(b = 0\) (secondary creep) and \(b > 0\) (tertiary creep), respectively.

Equation (9) \((b < 0)\) can be used to determine the critical displacement value at the catastrophic points corresponding to failure of medium-1 as follows:

\[
u_t = u_1 \left[ 1 - \frac{\sqrt{2}}{m+1} (1-k)^{\frac{1}{2}} \right] = \left[ 1 - \frac{\sqrt{2}}{m+1} (1-k)^{\frac{1}{2}} \right] \left( \frac{m+1}{m} \right)^{\frac{m}{2}} u_0
\]

It is known from equation (13), Figures 3 and 4 that \(-k\) decreases with an increase of \(m\) for the fixed values of \(G_{v/e} / G_{e/s}\), thus demonstrating that a more homogenous rock is more

![Figure 3. The relationship between \(m\), \(k\) and \(u_t\).](image-url)
prone to fault. There are two limited state models in equation (8), which are shown as follows.

**Model 1**

When \( u_b < u_1 \) (Figure 2), one has \( G_e = G_{e2} \). Substituting equation (11) into equation (12), leads to:

\[
(P + W_g - F_K) \sin \theta = \left( \tau_b - \frac{G_{e1} u_e}{h} \right) t_e + \frac{G_{s1} u_1}{\exp \left( \frac{m+1}{m} \right)} \left[ 1 + m k + \frac{2\sqrt{2}}{3} \frac{m}{m+1} (1-k)^3 \right] (b < 0)
\]  

(15)

**Model 2**

When \( u_b \geq u_1 \) (Figure 5), one has \( G_e = G_{e1} \). Substituting equation (11) into equation (12), leads to:

\[
(P + W_g - F_K) \sin \theta = \frac{G_{s1} u_1}{\exp \left( \frac{m+1}{m} \right)} \left[ 1 + m k + \frac{2\sqrt{2}}{3} \frac{m}{m+1} (1-k)^3 \right] (b < 0)
\]  

(16)

**Discussions**

It is known from equations (14) and (15) or (16) that the steady state of a fault system is relative to the weight of the rock mass, geometric size of the system, stiffness ratio, and mechanical parameters of media (referred to as the geometric-mechanical parameter). Considering the extreme situation that \( l_s \to 0 \), the instability of a fault system cannot happen, if \( k > 1 \), which means that the fault zone does not exist. When \( l_e \to 0 \), the stiffness ratio \( k \) is nearly zero, which means that the fault zone is completely through, and the fault system is prone to instability under mining conditions.

![Figure 4. The relationship between \( m \) of \( G_s \), \( k \) and \( u_t \).](image-url)
Equation (15) or (16) indicates that the fault system is prone to instability with an increase of dip angle $\theta$, which is consistent with the experimental results (Bu and Mao, 2009).

**Conclusions**

We studied a cusp catastrophe model based on the catastrophe theory and discuss the necessary and sufficient conditions leading to fault instability. It is assumed that the sliding surface of the fault zone is planar and is a combination of two media: one is elastic-brittle and the other is strain-softening. The shear stress-strain constitutive model for the strain-softening medium is described by Weibull’s distribution law. The conclusions obtained can be summarized as: (1) The instability of a fault system relies mainly on the ratio of the stiffness of medium-1 to the post-peak stiffness of the strain-softening medium, and the homogeneity index of strain-softening the medium; (2) One can judge the occurrence of fault instability from this feature and regard the index $D$ as a parameter reflecting the precursory abnormality of a fault; and (3) It is found that the bifurcation point, $k \leq 1$, is the turning point of the fault system from stable to potentially instable.

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