**ABSTRACT**

We examine the expected X-ray polarization properties of neutron-star X-ray sources of various types, e.g., accretion and rotation powered pulsars, magnetars, and low-mass X-ray binaries. We summarize the model calculations leading to these expected properties. We describe how a comparison of these with their observed properties, as inferred from GEMS data, will probe the essential dynamical, electromagnetic, plasma, and emission processes in neutron-star binaries.
discriminate between models of these processes, and constrain model parameters. An exciting goal is the first observational demonstration in this context of the existence of vacuum resonance, a fundamental quantum electrodynamical phenomenon first described in the 1930s.

1. Introduction

A major class of X-ray sources that the Gravity and Extreme Magnetism SMEX (GEMS) would be studying is neutron stars in various astrophysical situations, operating as accretion powered pulsars (APPs), rotation powered pulsars (RPPs), magnetars, accretion powered millisecond X-ray pulsars (AMXPs), and bright, non-pulsing low-mass X-ray binaries (LMXBs) \cite{Shapiro & Teukolsky 1983, Ghosh 2007}. APPs, AMXPs and LMXBs are binary systems where a neutron star accretes matter from its companion, and conversion of the gravitational binding energy of the accreted matter powers X-ray emission. RPPs are thought to generate X-rays by one or more of the mechanisms summarized in Sec.\ref{sec:3} the ultimate source of energy for this emission being that of the neutron star’s rotation. The majority of RPPs are single, relatively young neutron stars, but there is a population of RPPs in binaries, consisting mostly of old, recycled neutron stars with low-mass companions, and also a small number of young neutron stars with massive companions. Magnetars are believed to generate X-rays by conversion of the magnetic energy available from the superstrong magnetic fields of the neutron stars in them. These objects are usually thought to be single, although there have been occasional suggestions of magnetars in binaries. All of the above classes of neutron-star X-ray sources exhibit periodic pulses at the rotation period of the neutron star, except for LMXBs, for which these pulses are believed to be unobservable or absent for reasons summarized in Sec.\ref{sec:6} although the neutron stars in these sources are thought to be rotating fast as a result of the recycling process which they are undergoing. However, many LMXBs undergo X-ray bursts, and during the bursts from some of them, periodic oscillations \textit{have} been observed at high frequencies, which are believed to be the spin frequencies (or simple multiples/submultiples thereof) of the underlying neutron stars. A classic timing property exhibited by many LMXBs is quasiperiodic oscillations (QPO) at low, medium, and high frequencies, to which we return in Sec.\ref{sec:6.2} Correlating the above timing behaviors of the X-ray fluxes from these classes of neutron-star sources with those of their X-ray polarization properties (amount/degree of polarization and its position angle, or, alternatively, the Stokes parameters) will be a principal diagnostic available to GEMS for addressing the scientific questions listed in Sec.\ref{sec:7}.

The magnetic field of the neutron star is the most important parameter for categorizing
the X-ray polarization properties at the source (i.e., the environs of the neutron star), as expected, since the primary cause for the emission polarized X-rays from APPs, RPPs and magnetars is this magnetic field — directly or indirectly — as explained below. Among the above classes of neutron-star sources, APPs and RPPs are universally believed to have the “standard” surface field-strengths of $\sim 10^{12} \text{G}$ canonical for neutron stars, with a rough spread of about one order of magnitude around this value. Magnetars are thought to have superstrong fields $\sim 10^{14} - 10^{15} \text{G}$ by this standard, whence the name. By contrast, LMXBs and AMXPs are thought to have fields $\sim 10^8 - 10^9 \text{G}$, low by the above standards. Such low fields are believed to have been produced by a reduction of the standard neutron-star field strengths in the process of recycling which occurs during the long accretion phases in LMXBs (van den Heuvel 1992).

For APPs and magnetars, it is their magnetic fields that dominate both (a) the production processes of polarized X-rays at or near the neutron-star surface, and, (b) the subsequent propagation of these polarized X-rays through the magnetosphere surrounding the neutron star (see Secs 2 and 4). This happens because the magnetic field strongly influences both (a) the differential opacity between the ordinary (O) and extraordinary (E) polarization modes, and, (b) the scattering cross-sections (Meszaros & Nagel 1985a,b). For RPPs, the X-ray emission process in the magnetosphere is dominated by the local magnetic field, and so therefore is the polarization of the resultant radiation (Sec 3). For LMXBs and AMXPs, by contrast, the magnetic field is so low that its effects are thought to be negligible in the context of X-ray polarization, so that the latter is described in terms of non-magnetic Compton scattering (see Secs 5 and 6).

In addition to its magnetic field, the other essential property which characterizes a neutron star is its spin period, so that it is most useful to display neutron stars in a period - magnetic field ($P - B$) diagram, as shown in Fig 1. The different classes of neutron-star systems introduced above occupy characteristically different areas in this diagram, as shown. The dominant X-ray polarization-producing mechanisms are also characteristically different. For APPs and magnetars, it is the strong differential opacity between O and E modes, while for RPPs, it is the inherent strongly-polarized nature of the synchrotron/curvature radiation emitted from their magnetospheres, and in both cases the polarization degree can be very high, upto $\sim 80\%$. For LMXBs and AMXPs, it is the characteristically low polarization ($\sim 10\% - 15\%$) signature given to the radiation by non-magnetic Compton scattering in the accreting plasma around the neutron star.

In the following sections, we summarize the current state of knowledge of the expected X-ray polarization properties of the above classes of neutron-star sources, and GEMS strategies for observing and utilizing them for diagnostics of emission geometry, magnetic field
structure, and fundamental quantum electrodynamic (QED) effects. In Sec.2, we consider APPs. RPPs are considered in Sec.3 and magnetars are dealt with in Sec.4. Low magnetic field systems are considered in later sections, Sec.5 dealing with AMXPs and Sec.6 describing LMXBs, including both burst oscillations and QPO sources. Finally, we summarize our conclusions in Sec.7 discussing the key scientific questions to be addressed by GEMS.

2. Accretion Powered Pulsars (APPs)

Plasma accreting onto the neutron star in an APP is channeled by the stellar magnetic field to the magnetic poles and so forms accretion columns above these poles (Lamb et al. 1973, Davidson & Ostriker 1973). X-rays generated at the bases of the accretion columns are transported through the highly magnetized plasma there, and the transport properties (e.g., the scattering and absorption cross-sections) depend strongly on the polarization of the X-rays, being very different for the ordinary (O) mode and the extraordinary (E) mode. When vacuum polarization effects (see below) are negligible, the O mode has its electric vector (which is the conventional definition of the direction of polarization) lying in the plane defined by the external (i.e., stellar) magnetic field $B$ and the wave vector $k$, while the E mode has its direction of polarization perpendicular to the $(k, B)$ plane. When vacuum polarization effects are significant, the mode characteristics are more complicated (Pavlov et al. 1980). This strong disparity between the transport of O and E modes leads to a strong polarization of the emergent radiation. A simple, rule-of-thumb argument widely used to illustrate the point is that, at photon energies $E = h\nu$ far below the cyclotron energy $E_c = \hbar (eB/m_e c) \approx 11.6B_{12}$ keV (here, $B_{12}$ is $B$ in units of $10^{12}$ G), the opacity $\kappa_E$ of the E mode is drastically reduced compared to that of the opacity $\kappa_O$ of the O mode, scaling roughly as $\kappa_E \sim (E/E_c)^{2}\kappa_O$ (Lai & Ho 2003). Consequently, the emergent radiation is dominated by the E mode, which comes from the deeper and hotter layers of plasma, and so is strongly polarized in the direction of the E mode. We shall see below that the actual situation is more complicated, but that the general thrust of the above argument is qualitatively correct.

Emission of X-ray pulses from APPs is conceptually visualized in terms of rotating X-ray beams which can be characterized as either pencil- or fan-shaped in two useful limits. In the former, the opacity along the accretion column is sufficiently low that the beam emerges preferentially along the column, in a pencil shape. In the latter, the opacity along the column is so high that the beam emerges preferentially through the sides of the column, in a fan shape (Lamb et al. 1973). Pulse profiles for these beam geometries are different (Meszaros & Nagel 1985b), and so are the polarization properties (Meszaros et al. 1988).
We follow here the description of these properties pioneered by Meszaros and co-authors on the basis of their extensive numerical treatment of radiative transfer through the strongly magnetized plasma in the accretion columns (Meszaros & Nagel 1985a).

In the Meszaros et al. approach, the pencil beam is modeled as emission from the face of a plasma slab lying on the stellar magnetic pole (the stellar magnetic field being perpendicular to the slab surface), while the fan beam is modeled as emission from the sides of a plasma cylinder sticking out of the stellar surface around the magnetic pole (the stellar magnetic field being along the cylinder axis). Transfer of radiation is handled in either geometry through the Feautrier formalism (Feautrier 1964), using scattering and absorption cross-sections for O and E modes, and a redistribution matrix that keeps track of the scattering of photons from one angle to another, from one energy to another, and from one polarization mode to the other (Meszaros & Nagel 1985a).

In a strong magnetic field, not only is the plasma birefringent, but so also is the vacuum, the latter being a fundamental quantum electrodynamical (QED) effect first pointed out long ago (Heisenberg & Euler 1936). These two effects generally work “against” each other in a sense, and so “compensate” each other at a point called vacuum resonance (Meszaros & Ventura 1979), where the degree of linear polarization vanishes. This vacuum resonance energy $E_v$ depends on both the magnetic field strength $B$ and the electron density $n_e$, and is given by $E_v \approx 13B_{12}^{-1}n_{e,22}^{1/2}$ keV (here, $n_{e,22}$ is $n_e$ in units of $10^{22}$ cm$^{-3}$, and $B_{12}$ as before). For photon energies below $E_v$, the normal modes of the plasma dominate in the radiation-transfer process, while those of the vacuum dominate at $E > E_v$. Cross-sections for scattering and absorption for both E and O modes show strong features both at the cyclotron energy $E_c$ and at the vacuum resonance energy $E_v$ (Meszaros & Nagel 1985a), so that the polarization properties of the emergent X-rays from APPs go through strong changes when the X-ray photon energy passes through either of these resonances, providing valuable diagnostics of magnetic fields and plasma conditions. For canonical field strengths of APPs (see above), it is clear that $E_c$ would be outside the energy range of GEMS operation, so that our focus here will necessarily be on the expected changes in the essential polarization parameters at $E_v$.

The GEMS energy-band of $\sim 1 – 10$ keV defines an allowed region in the Magnetic Field - Luminosity ($B - L$) plane of APPs, so that, within this region, $E_v$ lies inside the GEMS band, and therefore valuable diagnostics at $E_v$ would be possible with GEMS. A

\[ \text{Actually, the directions of the dominant linear polarizations generated by plasma and vacuum birefringence are perpendicular to each other. At vacuum resonance, their strengths are equal, so that the resultant polarization is circular, not linear.} \]
rough sketch of this allowed region is shown in Fig.2, for canonical properties of accreting neutron stars and accretion geometry (Ghosh et al. 2013).

Model X-ray pulse profiles of APPs are shown in Figs.3 and 4, together with the (linear) X-ray polarization degree $P_L$ and the polarization angle $\chi$ as functions of the pulse phase $\phi$, as obtained from the numerical scheme of Meszaros et al., referred to henceforth as the $M$ scheme. For the particular neutron-star and accreting-plasma parameters used in the results displayed in Figs.3 and 4, the cyclotron resonance energy is $E_c \approx 38$ keV, and the vacuum resonance energy is $E_v \approx 22$ keV. (Note that we have shown here for illustration the original $M$ scheme results obtained for the parameters used by those authors. Recent estimates of these parameters for well-known, bright APPs indicate that $E_v$ is expected to be in the GEMS band for a substantial number of them (Ghosh et al. 2013).) We shall henceforth call libraries of such model profiles of total X-ray intensity ($I$), $P_L$, and $\chi$ as functions of essential system parameters (e.g., the inclination angle and the observer angle) as atlases. Comparison of such atlases with GEMS data on APPs will be a major diagnostic probe in our work, and we hereby name this probe pulse phase polarimetry, which will indeed be a most useful probe for X-ray pulsars of any kind, as we shall see in subsequent sections.

A key signature of the beam geometry (pencil or fan) is the phase correlation between the pulse profile (which we can also call the intensity profile or $I$-profile) and the $P_L$-profile, which can be seen clearly in Figs.3 and 4. For pencil beams, the maximum in $P_L$ is generally in phase with the pulse maximum for photon energies below $E_v$, but generally out of phase for photon energies above $E_v$. For fan beams, the situation is exactly the opposite. In a similar vein, the phase correlation between the pulse profile and the $\chi$-profile is also a good diagnostic. For pencil beams, $\chi$ generally goes from positive to negative values at the pulse maximum, and from negative to positive values at the pulse minimum. Again, for fan beams, the situation is exactly the opposite.

It follows from the above that a good pictorial representation of the $I - P_L$ diagnostic would be a direct $I - P_L$ plot, which can be obtained from the $I$- and $P_L$-profiles, treating the phase $\phi$ as a parameter. This $I - P_L$ plot would be a closed, ellipse-like, but more complicated curve, in analogy with the familiar Lissajous figure which would occur if the above profiles were sinusoids (Ghosh et al. 2013). The major axis of this figure would have a positive slope below $E_v$, but a negative slope above this energy for pencil beams, so that the figure would turn dramatically by about 90 degrees as the observation energy passes

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2We exclude here discontinuous jumps $\chi$ by 180 degrees, which occur when the line of sight points directly down at the magnetic pole at some pulse phase, and there is a discontinuous jump in the angle between sky-projected stellar magnetic field vector and the sky-projected rotation axis.
through $E_v$. For fan beams, the situation would be the opposite, and the turn would be in the opposite direction (Ghosh et al. 2013). Similar considerations apply to the $I - \chi$ plot.

The slab or cylinder shaped emission regions envisaged in the above M scheme are typically of radial extents $\sim 1 - 10^3$ m (Meszaros & Nagel 1985a,b), i.e., small compared to the neutron-star radius, and quite tiny compared to the typical extent of an APP magnetosphere ($\sim 10^6 - 10^7$ m). Polarized X-rays produced by these regions propagate subsequently through the magnetospheres of APPs, which lead to possible modifications of their properties. There are two major propagation effects whose polarization signatures on X-rays from APPs need to be quantified fully.

The first effect is the resonant Compton scattering of an outgoing photon generated by the M scheme from those points in the accretion column where the photon's frequency equals the local cyclotron frequency in the rest frame of the infalling plasma. In the rest frame of the neutron star, then, the corresponding condition in terms of the energies defined earlier can be written as

$$E = \frac{E_c}{\gamma(1 - \beta \mu)}.$$  \hspace{1cm} (1)

Here, $\mu \equiv \hat{k} \cdot \hat{B}$ is the cosine of the angle between the photon propagation ($\hat{k}$) and magnetic field ($\hat{B}$) directions, $\beta \equiv v/c$ is the dimensionless electron velocity, and $\gamma \equiv (1 - \beta^2)^{-1/2}$ is the Lorentz factor. There is a limiting surface of last scattering, or escape surface, outside which it is not possible to satisfy the condition given by Eq. (1). The location of this escape surface is given by the condition $(E_c/E)^2 + \mu^2 = 1$ (Fernández & Davis 2011). Its radius $r_{es}$ has been calculated for magnetars (see Sec. 4), and the values for APPs with characteristically lower magnetic fields (see Sec. 1) are correspondingly lower, being given for a purely dipolar neutron-star magnetic field roughly as

$$\frac{r_{es}}{R_{NS}} \approx 2.3 \left[ \left( \frac{E}{1\text{keV}} \right)^{-1} B_{12} (1 - \mu^2)^{-1/2} \right]^{1/3}.$$  \hspace{1cm} (2)

The second effect is that of the polarization freezing radius $r_{pl}$. As the X-ray photons propagate outward in the magnetosphere, normal modes do not mix significantly at first: this propagation is called adiabatic, as each mode follows its own evolution in the changing magnetic field without being affected by the other one. At a sufficiently large radius $r_{pl}$ and small magnetic field strength, however, this adiabaticity breaks down and the modes mix, thereby fixing the polarization degree and direction for all further outward propagation. $r_{pl}$ has been calculated for magnetars (see Sec. 4), where the entire magnetosphere is dominated by vacuum polarization effects (Fernández & Davis 2011). This can be seen by calculating the radius $r_v$ of the vacuum resonance point (see above), and remembering that vacuum
polarization effects dominate for \( r < r_v \), while plasma polarization effects dominate for \( r > r_v \). For magnetars, \( r_v \approx 3000R_{NS} \), where \( R_{NS} \) is the neutron-star radius (Fernández & Davis 2011). For APPs, on the other hand, the magnetosphere is almost entirely dominated by plasma polarization effects, since the vacuum resonance radius is given by (Ghosh et al. 2013)

\[
\frac{r_v}{R_{NS}} \approx 0.91 \left( \frac{E}{1\text{Kev}} \right)^{4/7} B_{12}^{20/49} R_6^{1/49} \left( \frac{M}{M_\odot} \right)^{19/49} L_{37}^{-10/49}.
\]

(3)

It is clear from Eq. (3) that, except at high photon energies, high magnetic fields, and very low luminosities, the entire magnetosphere is dominated by plasma polarization effects. Thus, a calculation of \( r_{pl} \) for APPs involves only the properties of plasma polarization modes, and so is entirely different from that for magnetars (Ghosh et al. 2013).

A third, interesting, fundamental effect is that of general relativity on photon propagation near the neutron-star surface, but it appears that its strength is not large enough for neutron-star radii given by modern equations of state (EOS) to warrant a detailed inclusion into a polarization computational scheme at this stage. Gravitational bending of photon trajectories can lead to interesting effects on the pulse profiles of APPs which were studied in the 1980s (Rieffert & Meszaros 1988; Meszaros & Rieffert 1988). Briefly, a fraction of the photons emitted near the stellar surface can be so bent as to propagate backward, so that this backward flux would partly (a) be blocked by the stellar surface, and partly (b) enhance the flux from the other magnetic pole (Rieffert & Meszaros 1988). This would occur for both pencil and fan beams in the M scheme introduced above. Consequent changes in the pulse profile, particularly in the degree of modulation, were studied by the above authors (Meszaros & Rieffert 1988). As expected, the strength of the GR effects increases with increasing compactness of the neutron star, i.e., decreasing values of the parameter \( \alpha \equiv R_{NS}/R_s \), where \( R_s = 2GM_{NS}/c^2 \) is the Schwarzschild radius. For \( M_{NS} = 1.4M_\odot \), \( R_s \approx 4.2 \text{ km} \). The above authors studied the parameter space \( 1.6 \leq \alpha \leq 4.0 \) (which correspond to \( R_{NS} \) between 6.7 km and 16.8 km) and showed that GR effects became important for \( \alpha \leq 2.0 \), which corresponds to \( R_{NS} \leq 8.4 \text{ km} \). Those neutron-star EOS which are currently considered viable give values of \( R_{NS} \) in a narrow range around \( \approx 12 \text{ km} \) (corresponding to \( \alpha \approx 3 \)), where GR effects would be minor, estimated to be \( \approx 10\% \) or less, for both pulse profiles and polarization properties (Meszaros et al. 1988). Accordingly, we shall not consider these effects further in this paper.
3. Rotation Powered Pulsars (RPPs)

Radio polarization measurements have been a valuable probe of the emission geometry in RPPs, and the only measurement (i.e., not an upper limit) of X-ray polarization from neutron stars so far has been from the well-known RPP, the Crab pulsar, and its nebula (Weisskopf et al. 1976). However, the emission sites and mechanisms for X-ray and radio emission are believed to be entirely different because the X-ray and radio pulse profiles are quite different in general. There are three main types of models for X-ray (and other high-energy) emission from RPPs, namely, (a) polar cap models (see Daugherty & Harding 1996 and references therein), where the emission occurs within a few stellar radii of the neutron-star surface, (b) the outer gap models (Cheng et al. 1986), where the emission occurs in the outer magnetosphere near the light cylinder, and, (c) striped wind models (Petri & Kirk 2005), where the emission occurs in the pulsar wind outside the light cylinder. In the slot gap model, a recently explored variation of polar cap-type models, the emission occurs at high altitudes along the last open magnetic field line, in the outer magnetosphere (Muslimov & Harding 2003).

Polar-cap models assume that particles begin accelerating near the neutron-star surface, and emit high-energy radiation at radii comparable to $R_{NS}$ through curvature radiation and/or inverse Compton-induced pair cascade in the strong magnetic fields there. The slot-gap model is somewhat similar, except that the curvature, synchrotron, and inverse Compton components of the radiation originate in the outer magnetosphere. Outer-gap models assume that the acceleration occurs in vacuum gaps that form between the null-charge surfaces and the light cylinder in the outer magnetosphere, and that the observed high-energy radiation is curvature radiation and photon-photon pair-production induced cascades from these sites. Striped-wind models envisage the pulsed emission as originating in the pulsar wind beyond the light cylinder, due to toroidal magnetic fields of alternating sign in this wind, which can form current sheets for pulsars with high inclination, and the reconnection of these magnetic fields can convert magnetic energy into particle energy, leading to sharply (double) peaked pulse profiles if the dissipation region is small enough. Both slot-gap and outer-gap models have caustics in their emission pattern, i.e., photons emitted over a broad range of altitudes tend to pile up at roughly the same phase, due to an almost complete cancellation between (a) the phase delay in photon emission with increasing altitude along a trailing field line, and, (b) special-relativity effects like the aberration of photon-emission directions and the time-of-flight effects caused by the finite speed of light. In the two-pole-caustic (TPC) configuration, strong caustics form on field lines trailing both polar caps, leading to light curves with widely-separated sharp double peaks.

In the light of extensive studies of high-energy emission from RPPs with Fermi over the
last several years, it is now clear that observations strongly disfavor polar-cap type models where this emission occurs near the neutron-star surface, and strongly support those types of models in which this emission takes place in the outer magnetosphere or beyond the light cylinder (Harding 2010 and references therein). Accordingly, we shall consider only the latter types here for X-ray polarization studies, giving an occasional reference to results for polar-cap type models to show the difference. Expected pulse-phase variation profiles of the X-ray polarization have to be calculated numerically for these models, since simple, empirical models like the rotating vector model, so useful for lower-altitude radio emission (see, e.g., Ghosh 2007 and references therein), are no longer relevant here. Results from such calculations for the slot-gap and outer-gap models are shown in Fig.[5] with that for the polar-cap model also shown for reference. The latter shows an S-shaped swing in the polarization angle, reminiscent of the rotating vector model, but irrelevant here for the above reasons and also in strong disagreement with optical polarization measurements of the Crab pulsar. The polarization signatures of the TPC/slot-gap and outer-gap models are very different indeed. In particular, for the TPC/slot-gap models, the polarization angle shows very rapid sweeps through the main peak and the interpulse, and a strong depolarization associated with each peak. These features are actually observed in the optical polarization profile of the Crab pulsar (Smith et al. 1988).

As the different outer-magnetosphere models produce high energy RPP pulse profiles which are rather similar, it would be difficult to discriminate between them on this count. However, since different models of this type yield quite different polarization profiles at these energies because of their different magnetic field structures in the outer magnetosphere, pulse-phase polarimetry in the X-rays would serve as an excellent discriminator between models, and this would be a major direction of RPP study with GEMS. To this end, we shall have atlases of the profiles of $I$, $P_L$, and $\chi$ corresponding to the above models, each such atlas detailing the profiles for a particular model for a grid of values for the essential parameters, e.g., the inclination angle and the observer angle. The idea is exactly analogous to what we described in Sec.[2] for APPs, and examples of such RPP atlases are shown in Figs.[6] and [7].

4. Magnetars

Magnetars are believed to generate X-rays by conversion of the magnetic energy stored in the twisted magnetospheres of neutron stars with superstrong magnetic fields (see Sec.[1]). This energy release near the stellar surface is thought to produce “hot spots”, which emit thermal X-rays through the very thin (typically 1 – 10 cm thick) neutron-star atmosphere. The polarization properties of this primary radiation are determined by the interplay be-
tween the radii of the photospheres of O and E modes on the one hand, and the vacuum resonance radius \( r_v \) on the other (see Lai & Ho 2003 and references therein). At magnetar field strengths, the vacuum resonance lies between the O- and E-mode photospheres (the latter always lying deeper because of the lower opacity of the E-mode), and the emergent radiation from the atmosphere is dominated by the E mode at all photon energies of interest for GEMS observations. Expected pulse-phase variations in the polarization of this radiation, as given by model calculations (Lai & Ho 2003), are shown in Fig. 8.

Propagation of the above polarized X-rays through magnetar magnetospheres has two major effects on the polarization, which have been already introduced in Sec. 2, viz., resonant Compton scattering and polarization freezing. The first is described by the limiting surface of last scattering or the *escape* surface, outside which it is not possible to satisfy the condition given by Eq. (1). The location of this escape surface is given by the condition \( (E_c/E)^2 + \mu^2 = 1 \) and its radius \( r_{es} \) has been calculated (Fernández & Davis 2011) for the standard twisted-dipole magnetospheric structure (Thompson et al. 2002) of magnetars to be

\[
\frac{r_{es}}{R_{NS}} \approx 12 \left( \frac{E}{1\text{keV}} \right)^{-1} B_{14} \xi (1 - \mu^2)^{-1/2} \right]^{1/2.88}.
\]

Here, \( B_{14} \) is the polar magnetic field of the magnetar in units of \( 10^{14} \) G, \( \xi \) is a geometrical factor describing the angular dependence of the magnetic field, and the specific result given by Eq. (1) is for a twist parameter of unity (Fernández & Davis 2011).

The second effect, *viz.*, polarization freezing, is described in terms of a polarization freezing radius \( r_{pl} \), the physics of which has been introduced in Sec. 2. Magnetar magnetospheres are completely dominated by vacuum polarization effects, as seen by calculating the vacuum resonance point radius (see Sec. 2) \( r_v \approx 3000R_{NS} \) (Fernández & Davis 2011). Using the properties of the vacuum modes, \( r_{pl} \) for magnetars has been calculated as

\[
\frac{r_{pl}}{R_{NS}} \approx 146 \left( \frac{E}{1\text{keV}} \right)^{-1} B_{14} R_6 \xi^2 \zeta^{-1} (1 - \mu^2)^{-1/2} \right]^{1/4.76}.
\]

Here \( \zeta \) is a dimensionless function of order unity involved in calculating the scale length on which \( B \) changes in the magnetosphere.

The above separation by roughly one order of magnitude between \( R_{NS}, r_{es}, \) and \( r_{pl} \) for magnetars makes the calculation of propagation effects relatively straightforward for them, since resonant scattering is basically decoupled from subsequent polarization freezing (Fernández & Davis 2011), and both are, in a sense, decoupled from the processes in the neutron star’s atmosphere. Such calculations have been done for the propagation of an
assumed completely polarized radiation from the neutron-star atmosphere through a model magnetar magnetosphere, and the results for the final emergent radiation are summarized in Fig.9. The propagation effects tend to reduce the amount of polarization, but polarization degrees are still relatively high (∼40–80%) at lower (∼2–4 keV) energies, and particularly so around the pulse minima. The calculated pulse-phase variation profiles of the X-ray polarization of magnetars are generally similar to those of pencil-beam APPs (described by the slab model in the M scheme: see Sec.2), as expected, since hot-spot emission from neutron-star surfaces in magnetars would have a pencil-beam character.

5. Accretion Powered Millisecond X-ray Pulsars (AMXPs)

In this and the next section, we consider systems containing accreting neutron stars with low magnetic fields ∼10⁸ – 10⁹ G, where the magnetic field is assumed to have a negligible effect on radiation transport, and X-ray polarization properties of such systems are described in terms of Compton scattering in a plasma configuration with a geometry appropriate for the system. We consider AMXPs first, of which there are about thirteen currently known. These systems are believed to have been produced by the recycling of neutron stars in low-mass X-ray binaries (LMXBs: see next section), where accretion had both (a) spun up the neutron stars to millisecond periods and (b) reduced their magnetic fields to the above levels.

The emission sites of AMXPs are modeled as “hot spots” on the neutron star surface, in analogy with APPs. Thermal radiation from these hot spots undergoes Compton scattering in the plasma behind the accretion shock, which is modeled as a plane-parallel slab of optical depth of order unity (τ ∼ 1). The basic polarization signatures of Compton scattering are well-known (Chandrasekhar 1960; Loskutov & Sobolev 1982; Nagirner & Poutanen 1993).

For an optically thick (τ >> 1) slab the direction of the linear polarization is preferentially in the plane of the slab, and for the classic problem of a semi-infinite slab in the Thompson scattering limit, the maximum degree of polarization possible is the Chandrasekhar limit ≈12% (Chandrasekhar 1960). Detailed calculations for τ = 1 have been performed, which yield typical polarization degrees ∼10% – 15% (Poutanen & Svensson 1996).

Numerical calculations of the expected polarization properties of Compton-scattered radiation emitted by such hot spots on neutron stars have been done, including special relativistic boosting and aberration, and general relativistic light bending in Schwarzschild space-time. For slowly-rotating neutron stars, the polarization vector lies in the plane defined
by the normal to the hot spot and line of sight, as expected, since these are the basic directions in a non-magnetic system. In fact, the polarization angle can be described by a rotating-vector type model (see Sec.3) which includes the special and general relativistic effects (Viironen & Poutanen 2004). Model pulse-phase profiles of $I$, $P_L$, and $\chi$ from such calculations are shown in Fig.10. The idea in this case is also to have an atlas of these profiles for a grid of values of the essential parameters like spot colatitude (referred to the rotation axis of the neutron star) and observer angle, to be compared with GEMS data on AMXPs. Such pulse phase polarimetry will be an excellent diagnostic probe of the emission geometry in these systems.

6. Low-Mass X-ray Binaries (LMXBs)

Bright, accretion-powered galactic-bulge LMXBs are among the brightest X-ray binaries known, and are widely believed to be the nurseries producing AMXPs by recycling. Their persistent X-ray emission does not show periodicity at any possible rotation period relevant for neutron stars, although they are thought to harbor fast-rotating neutron stars with millisecond periods. Various explanations have been offered for this, e.g., (a) that the emitted pulses are “washed out” by the dense plasma surrounding the neutron star, or (b) that their rotation and magnetic axes have become aligned by accretion torques, so that they actually do not pulse. Thus, the kind of pulse phase polarimetry we have discussed so far in this paper is not possible for them.

6.1. Burst Oscillations

However, many LMXBs undergo X-ray bursts, and coherent pulsations at millisecond periods have been detected during these bursts in about twenty of them so far. Identifying this periodicity with that of the neutron star’s rotation, we can again use the above ideas of pulse phase polarimetry as a diagnostic probe of the emission geometry in these systems. These bursts are thought to be due to the ignition of thermonuclear reactions in the accreted material upon reaching a critical temperature and density, deep in the atmosphere of the neutron star where the optical depth is extremely large, so that the semi-infinite slab limit discussed in Sec.5 applies. Furthermore, at sub-Eddington luminosities, the burst process is not expected to “lift” the atmosphere to any significant extent, so that the static slab limit still applies, and straightforward calculations can and have been done with the aid of the Chandrasekhar-Sobolev formalism mentioned above.
Results of such calculations (Viironen & Poutanen 2004) are shown in Fig. 11, again detailing the model pulse-phase profiles of $I$, $P_L$, and $\chi$. The procedure here is again to have an atlas of these profiles for a grid of values of the essential parameters like the colatitude of the of the spot where the burst occurs, and the observer angle, to be compared with GEMS data for diagnostics of the burst emission region.

6.2. Quasiperiodic Oscillations (QPOs)

A major property of bright LMXBs in the time or frequency domain is the quasiperiodic oscillation (QPO) exhibited by them. QPOs appear as relatively wide, but clearly discernible, peaks in the power spectra of LMXBs, as opposed to the extremely sharp peaks corresponding to periodic pulses that appear in the power spectra of APPs. QPOs occur at both low frequencies ($\nu_{QPO} \sim 6 - 50$ Hz) and high “kilohertz” frequencies ($\nu_{QPO} \sim 400 - 1000$ Hz), and they have also been detected recently at intermediate “hectohertz” frequencies (van der Klis 2006). The low-frequency (LF) QPOs are particularly strong, and so were the first to be discovered (van der Klis et al. 1985). The study of QPOs and their relations with LMXB spectral states has become a most valuable diagnostic tool for probing the dynamics of inner accretion disks and the interactions between neutron stars and accretion disks. Studies of the polarization properties of QPO sources have a great potential for adding a new dimension to this probe.

Instead of the pulse phase polarimetry methods available for periodic sources, the appropriate diagnostic approach for QPO sources would be through studies of the cross-correlations between intensity and polarization parameters, and the associated cross-spectral analysis, which, in a sense, are the natural extensions of pulse-phase polarimetry to quasiperiodic sources. Schemes for such analysis are being constructed (Ghosh & Swank 2013). The analog of the atlases for periodic sources referred to in the earlier sections of this paper would, in this context, be the atlases of cross-spectra for various QPO models, for a grid of values of the essential parameters of each model. Comparison of such atlases with the observed cross-spectra obtained from GEMS data would discriminate between models and constrain model parameters. As LF QPOs are particularly strong, these would be the ideal first testing grounds for this scheme.
7. Discussion

We have summarized here the principal polarization properties of the known classes of neutron-star X-ray sources, as expected from model calculational schemes summarized in the earlier sections. Comparison of the atlases of these expected polarization properties with GEMS data will address some key scientific questions on X-ray emission from neutron stars, which we now summarize.

For APPs, the most immediate returns will be on the pencil/fan beam issue. Whereas beam-shape diagnostics from pulse-profile studies have been indirect and sometimes ambiguous, pulse phase polarimetry provides a diagnostic, viz., the phase-relation between the intensity and the polarization degree or angle, which is simple, qualitatively different for pencil and fan beams, and requires only a coarse binning in pulse phase to achieve clear results. However, the most fundamental discovery for APPs would be the discovery of the signature of vacuum resonance. As explained in Sec.2, the above phase-relation for a given beam-shape changes sign at $E_v$. When $E_v$ lies in the GEMS band, even a very coarse energy-binning will be able to find this signature.

For RPPs, the major scientific issue to be addressed is the nature and site of the X-ray emission, as described by the various outer-magnetosphere models summarized in Sec.3. Together with the discrimination between these models through their polarization signature, it may also be possible to obtain valuable diagnostics of the magnetic field geometry, e.g., whether the field is more similar to a retarded vacuum dipole (Deutsch field) configuration, or to a force-free configuration.

For magnetars, the key issue is again the vacuum resonance point, if it is indeed within the GEMS band for the observed magnetars. In contrast to the phase-relation which changes sign at $E_v$ for APPs, it does not do so for the primary radiation emitted from the neutron-star atmosphere in magnetars (Lai & Ho 2003). While the impact of propagation effects on this diagnostic feature need to be understood fully, all indications at the present time are that they do not change it. This is thus a valuable signature of the superstrong field regime of magnetars, whose demonstration would impact the fundamental physics (QED) of vacuum resonance.

For AMXPs, the key subject amenable to polarizaton studies is the character of non-magnetic Compton scattering in neutron-star systems, for comparison with that of similar scattering from accretion disks is black-hole systems, particularly at those radii in the latter systems where GR effects are mild. This bears on the important issue of the possible impact of the remnant magnetic field in AMXPs on the scattering and polarization signature. This magnetic field is still invoked for envisaging magnetic chanelling and polar hot-spots of X-
ray emission, and indeed one of the model parameters obtained by comparing the atlases with GEMS data would be the angle between the magnetic and rotation axes. Any residual signature of this magnetic field on X-ray polarization is therefore an intriguing possibility.

For LMXB burst oscillations, a key question is the site of thermonuclear ignition and X-ray bursts on the neutron-star surface. Recent observations suggest various possible sites between the rotational equator and pole for the onset of ignition. Polarization diagnostics would give us the angular position of this site in relation to the rotation axis, thus clarifying how an X-ray burst develops as the burning front spreads on the rotating star. Once again, the role of the residual magnetic field in LMXBs remains an interesting issue here.

Finally, a key target of polarization studies in LMXBs with QPOs would be to discriminate between various QPO models, particularly those aspects of the models which deal with the interaction of the accretion disks in these systems with the remnant magnetic field in neutron stars in LMXBs. The role of rotating clumps or blobs of plasma in quasiperiodic X-ray emission has been studied intensively in this context, and polarization diagnostics will add a new dimension to these studies.

8. **Epilogue**

GEMS was proposed in response to a NASA SMEX announcement of opportunity in December 2008. It was selected for phase A development in 2009 and down-selected for phase B in 2010. A technically successful Preliminary Design Review was held in Feb 2012. NASA Science Mission Directorate (SMD) indicated their intention to non-confirm (or cancel) in May 2012; the SMD decision was based on concerns that the eventual cost would be too high.
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Fig. 1.— Period - Magnetic Field ($P - B$) diagram of neutron stars, showing regions containing accretion powered pulsars (APPs, examples denoted by asterisks), rotation powered pulsars (RPPs, examples denoted by dots, those in binaries encircled), and magnetars. Recycled RPPs include both millisecond radio pulsars (MSPs) and accreting millisecond X-ray pulsars (AMXPs). Several well-known binary systems and single recycled pulsars are labeled. After Ghosh 2007.
Fig. 2.— Allowed region, between the two solid lines, in the $B - L$ plane, where the vacuum resonance energy $E_v$ falls in the GEMS X-ray band (see text). Only the schematic form is shown, with a rough scaling given by standard accretion theory. The upper line corresponds to the lower end of the GEMS band, and the lower line to the upper end.
Fig. 3.— Atlas of pulse profiles (top panel), polarization amounts (middle panel), and position angles (bottom panel) for APPs with pencil beams. In each panel, the five columns correspond to different geometries, labeled by the value of $i_1/i_2$, where $i_1 =$ observer angle referred to the rotation axis, $i_2 =$ inclination angle between magnetic and rotation axes, both in degrees. The eight curves in each panel and column correspond to different photon energies, labels 1 through 8 referring to energies of 1.6, 3.8, 9.0, 18.4, 29.1, 38.4, 51.7 and 84.7 keV, respectively. From Meszaros et al. 1988.
Fig. 4.— Same as Fig. 3 but for fan beams. From Meszaros et al. 1988.
Fig. 5.— Pulse profiles, polarization amounts, and position angles for the Crab pulsar, showing optical observations from Slowikowska et al. (2009) compared with polar cap, slot gap/two-pole caustic, and outer gap models (see text) from Dyks et al. (2004) and the striped wind model from Petri & Kirk (2005). The latter model is calculated for two Lorentz factors, 20 and 50, shown as solid and dashed lines, respectively. α = inclination angle between rotation and magnetic axes, ζ = observer angle with respect to the rotation axis. From Slowikowska et al. (2009).
Fig. 6.— Atlas of pulse profiles (black), polarization amounts (blue), and position angles (red) for RPPs with a vacuum (Deutsch) magnetic field configuration in the two-pole caustic (TPC) model (see text). Angles $\alpha$ and $\zeta$ as defined in caption of Fig. From Harding 2012.
Fig. 7.— Same as Fig. 6 but for the outer gap (OG) model (see text). From Harding 2012.
Fig. 8.— Pulse profiles and polarization amounts for magnetar emission from neutron-star atmospheres. Line style coded by photon energy: 1keV (dashed), 2 keV (long-dashed), 5 keV (solid). $\gamma$ = observer angle referred to the rotation axis, $\beta$ = inclination angle between rotation and magnetic axes. From Lai and Ho 2003.
Fig. 9.— Pulse profiles, polarization amounts and position angles for the final magnetar emission after including propagation effects in the magnetosphere. Results shown for two energy bands: 2-4 keV and 4-12 keV. Line style/color coded by geometry, i.e., the combination (inclination angle, observer angle), as follows. (45,70): solid/black, (70, 45): dotted/red), (60, 70) : dashed/blue, and (90, 90) : dot-dashed/green. All angles in degrees. From Fernandez and Davis 2011.
Fig. 10.— Pulse profiles, polarization amounts and position angles for AMXPs. $\theta =$ inclination angle between rotation and magnetic axes, $i =$ observer angle referred to the rotation axis. From Poutanen 2010.
Fig. 11.— Pulse profiles, polarization amounts and position angles for burst oscillations from LMXBs. $\theta$ = colatitude of the ignition site and $i$ = observer angle, both referred to the rotation axis. $i$ is taken as 60 degrees in all cases shown. From Poutanen 2010.