Yang Mills Magneto-Fluid Unification

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We generalize the hybrid magneto-fluid model of a charged fluid interacting with an electromagnetic field to the dynamics of a relativistic hot fluid interacting with a non-Abelian field. The fluid itself is endowed with a non-Abelian charge and the consequences of this generalization are worked out. Applications of this formalism to the Quark Gluon Plasma are suggested.

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I. INTRODUCTION

Recent experiments at the relativistic heavy ion collider (RHIC) have shed light on the behavior of hot dense nuclear matter [1]. The conjecture, that quarks and gluons de-confine and become a plasma in extreme conditions [2], is close to being experimentally proved. However, it has been realized that the de-confined quark gluon matter that has been revealed at RHIC, far from being the weakly interacting collisionless plasma envisioned by theorists, is, in fact, behaving more like a quark gluon liquid, or a strongly interacting plasma [3]. The QGP liquid (or strongly interacting plasma) is dense, but seems to flow with very little viscosity. It flows so freely that it approximates an ideal, or perfect fluid, the kind governed by the standard laws of hydrodynamics. Thus, most of the phenomenological input for the explanation of the data at RHIC comes from adopting a hydrodynamic approach to the plasma and consequently, fluid dynamics is used to deduce its properties. For a proper fluid dynamical description of the quark gluon fluid the fact that quarks and gluons have a non-Abelian charge has to be taken into consideration. There have been theories focusing on various aspects of an ideal fluid in interaction with Yang-Mills fields called Yang Mills magneto-hydrodynamics or chromohydrodynamics [4, 5, 6, 8, 9, 10]. The relativistic heavy ion experiments coupled with recent studies by Jackiw and his collaborators on the "Clebsch representation" of Yang Mills fluid dynamics have lead to a resurgence of interest in these theories [11, 12]. In tune with this revival, we investigate the dynamics of a relativistic hot fluid with a non-abelian charge in terms of a model which unifies the Yang Mills field with the flow field strength tensor [13]. Apart from its possible phenomenological applications, the motivation for the following work is based on the aesthetic criterion of unifying the fluid field and the Yang Mills field into a Yang Mills "magneto-fluid" by a "gauge principle". The similarities between one gauge field theory - electromagnetism and fluid dynamics have been explored extensively and fluid flow has been shown to have a formal equivalence with a gauge theory [14]. It has also been shown that a fully antisymmetric flow tensor, resembling the electromagnetic field, can be constructed and the unification is achieved by defining an effective field strength tensor that combines appropriately weighted electromagnetic and flow fields [13]. Is a consistent and useful non-Abelian generalization of this genre of flow-field unification possible? This investigation constitutes the theme of this paper.

II. ABELIAN MAGNETO-FLUID UNIFICATION

First, let us recapitulate the salient features of Abelian(Maxwell) Magneto-fluid unification [13]. Although Maxwells electrodynamics provides equations of motion for the electric and magnetic fields, for describing their interaction with matter fields (charged particles), the Lorentz force law has to be independently postulated. In contrast, in a gravity coupled plasma, a natural consequence of general covariance is the conservation of energy and momentum, and the Lorentz force law for charged particles moving in a gravitational field [15] can be de-
derived from the field equations to be:
\[ \nabla \mathbf{U} = \frac{q}{m} \mathbf{U} \cdot \mathbf{F}, \]
(1)
where, \( \mathbf{U} \) is the tangent vector to a geodesic (the velocity vector). In the limit of a weak gravitational field (the flat space-time limit, \( \nabla \mathbf{U} \rightarrow 0 \)), the component form of Eq.(1) reads:
\[ U^\mu \partial_\mu U^\nu = \frac{q}{m} U^\mu F^\nu_{\mu}. \]
(2)
Because of the antisymmetry of \( F_{\mu\nu} \), contraction with \( U^\nu \) reduces the right hand side of Eq.(2) to zero. Thus, with no loss of generality, the coefficient of \( U^\nu \), on the left hand side of the equation, should also be anti-symmetrized.

Since \( U^\mu \), the four-velocity, obeys \( U^\mu U_\mu = -1 \) (implying \( U^\mu \partial_\mu U^\nu = 0 \)), Eq.(2) may be written as
\[ U^\mu \partial_\mu U^\nu - U^\nu \partial^\mu U_\mu = \frac{q}{m} U^\mu F^\nu_{\mu}. \]
(3)

With the definition
\[ P_{\mu\nu} = \partial_\mu U_\nu - \partial_\nu U_\mu. \]
(4)
we can write Eq.(3) as
\[ U^\mu (\frac{m}{q} P_{\mu\nu} + F_{\mu\nu}) = 0. \]
(5)

For deriving equations of motion for point particles, usually a limiting procedure is invoked \[12]. For the motion of fluids, however, a small volume element of the fluid is the limiting element and the statistical properties of the fluid come into play. Recently \[13\] suggested that the \( S_{\mu\nu} = \partial_\mu f U_\nu - \partial_\nu f U_\mu \) must replace the particle \( P_{\mu\nu} \) for a new and natural "minimal coupling" to describe a fluid interacting with the Maxwell fields, where \( f \) represents a temperature dependent statistical attribute of the fluid, and is related to the enthalpy \( h \), number density \( n \) and mass \( m \) of the fluid by the relation \( h = mn f(T) \). In terms of \( S_{\mu\nu} \), the "fluid Lorentz equation" derived in \[13\] is
\[ T \partial^\sigma \sigma = q (F_{\mu\nu} + \frac{m}{q} S_{\mu\nu}) U_\mu \]
(6)
where, \( \sigma \) is an entropy density. The limiting procedure to the point particle case, then, is simply equivalent to \( f U^\mu \rightarrow U^\mu, S_{\mu\nu} \rightarrow P_{\mu\nu} \), as \( f \rightarrow 1 \) (\( T \rightarrow 0 \)).

The curvature \( F_{\mu\nu} \) corresponding to the connection \( A_\mu \) is obtained from the commutator of two covariant derivatives \( D_\mu = \partial_\mu - iq A_\mu \). In a similar vein, we can define a "unified" connection \( Q^\mu = A^\mu + \frac{m}{q} f U^\mu \), which corresponds to a minimally coupled hot magnetofluid. The new Abelian covariant derivative for the unified field
\[ D_\mu = \partial_\mu - iq Q_\mu = \partial_\mu - iq A_\mu - im f U_\mu. \]
(7)
leads to the unified curvature
\[ [D_\mu, D_\nu] = -iq F_{\mu\nu} - im S_{\mu\nu}, \]
(8)
where \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) and \( S_{\mu\nu} = \partial_\mu f U_\nu - \partial_\nu f U_\mu \).

This new, temperature dependent minimal coupling procedure has many interesting consequences which are explored in \[13\], \[14\].

### III. THE YANG MILLS MAGNETO-FLUID TENSOR

Now we return to the main theme of this work-the dynamics of a non-Abelian fluid. The non-Abelian gauge field is represented by
\[ F_{\mu\nu}^a = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - ig [A_\mu, A_\nu]^a. \]
(9)
where, \([A_\mu, A_\nu]^a = iC_{bc}^a A^b_\mu A^c_\nu, \) \( C_{bc} \) are the structure constants of the gauge group and \( g \) is the gauge "charge".

Since \( F \) now has a gauge index, the right hand side of the Abelian equations of motion suggests a generalization of the fluid flow vector to include a gauge, or a non-Abelian index. The RHS of the equations \[3\] can then be written as : \((q/m) U_{\mu} F^a_{\mu\nu}^a\). Correspondingly, the LHS, \( U_{\mu} \partial_\mu U_\nu \) requires a non-Abelian generalization. This mandates giving the flow field \( U_\mu \), a non-Abelian index, and we are led to a generalization of the Abelian flow tensor \( S_{\mu\nu} \), to \( S^a_{\mu\nu} \).

Following the Abelian route, an explicit form of \( S^a_{\mu\nu} \) is given by evaluating the non-Abelian curvature for the generalized non-Abelian covariant derivative,
\[ D_n = \partial_\mu - ig[A_\mu, \cdot] - im[f U_\mu, \cdot]. \]
(10)
The corresponding curvature
\[ [D_\mu, D_\nu]^a = -ig F_{\mu\nu}^a - im S_{\mu\nu}^a \]
(11)
coupled with Eq.9, defines the YM generalization of \( S_{\mu\nu} \)
\[ S_{\mu\nu}^a = \partial_\mu f U^a_\nu - \partial_\nu f U^a_\mu - im f^2 U_{\mu} U_{\nu}^a \]
\[- ig f [A_\mu, U_\nu]^a - ig f [U_\mu, A_\nu]^a. \]
(12)
written, succinctly, as
\[ S_{\mu\nu}^a = D^\rho (f U^\rho_{\nu}) - D^\rho (f U^\rho_{\mu}) - im f^2 [U^\rho_{\mu}, U^\rho_{\nu}] \]
(13)
where \( D_\rho = \partial_\rho - ig[A_\rho, \cdot] \) is the ordinary non-Abelian gauge covariant derivative. Notice that \( S^a_{\mu\nu} \) encompasses the pure flow-field as well as the interaction.

In the presence of a matter gauge current, the Yang Mills Field evolves as
\[ D_\mu F^a_{\mu\nu} = -J^a_{\nu}. \]
(14)
The L.H.S of Eq.(14) is easily related to the energy momentum tensor of the gauge field
\[ \Theta_{\mu\nu} = F_{\alpha a}^\mu F^{a\sigma}_{\nu} - \frac{1}{4} \eta_{\mu\nu} F^a_{\alpha \rho} F^{a\rho}_{\sigma} \]
(15)
through the Bianchi identity
\[ D^\rho F^{\rho\sigma} + D^\sigma F^{\rho\nu} + D^\nu F^{\rho\sigma} = 0, \]
(16)
leading to
\[ \partial^\nu \Theta_{\nu\mu} = -F_{\nu\mu} D^\rho F^{\rho\nu} = F^a_{\mu\nu} J^a_{\nu}. \]
(17)
The evolution of Yang Mills potential $A_{\mu} = A_{\mu}^a T_a$ is dictated by the matter current $J_\mu$, in addition to the inherent non-linearities in field equations.

In the conventional particle description, the current $J_\mu$ is constructed from the wave function, which itself evolves covariantly in the gauge field. Ideally, for strongly interacting matter such as a quark gluon plasma, such a current has to be constructed from a collective many-body wave function. This procedure is very cumbersome and sometimes not very illuminating. Thus, a fluid description, in terms of flow and thermodynamic variables, is useful, because it captures a complicated dynamics in terms of a few collective variables. We are "viewing" a strongly interacting many particle system through a set of representative "flow" fields. We consider not just one but several flow fields (labeled by a species index $s$), a set of representative "flow" fields. We consider not just one but several flow fields (labeled by a species index $s$) denoting the particles (quarks, anti-quarks, gluons) interacting with the non-Abelian gauge field. Each species can in principle have different charges, densities, temperatures etc. Each species is taken to be a perfect fluid and denoting the particles (quarks, anti-quarks, gluons) interacting with the non-Abelian gauge field. Ideally, for strongly nonlinearities in field equations.

\[ \Theta^{\mu \nu} = 0, \]

justifying the expression for the current. For the rest of the paper, we shall drop the species index unless it is essential for clarity.

From the continuity equation, generalized to the non-Abelian case,
\[ \mathcal{D}_\mu \Gamma^{\mu a} = 0. \]

or
\[ \partial_\mu (n_R U^{\mu a}) = -g n_R C^{abc} A_\mu U^{\nu \mu}. \]

and from the the definition of $T^{\mu \nu}$ (for a perfect fluid), we have
\[ \partial_\mu T^{\mu \nu} = \partial^\nu p + m n_R (f n_R U^{\mu a} U^{\nu}_a) \]
\[ = \partial^\nu p + m n_R (f n_R U^{\mu a} \partial_\mu (f U^{\nu}_a) - g n_R m f U^{\nu}_a A_\mu U^{\nu \mu} C^{abc}) \]
\[ = \partial^\nu p + m n_R (D_{\nu} U^{\mu a}) = 0, \]

where $T^{\mu \nu}$ is the total fluid tensor. Combining it with \[ \Theta^{\mu \nu} = 0, \]

we arrive at the expected conservation law for the total energy momentum tensor (matter plus field)
\[ \partial^\mu (\Theta^{\mu \nu} + \mathcal{T}^{\mu \nu}) = 0, \]

justifying the expression for the current. For the rest of the paper, we shall drop the species index unless it is essential for clarity.

In analogy with the Abelian case \[ U^{\mu a} S^{\mu a} = \mathcal{D}^\mu U^{\mu a} \]

or
\[ \partial^\nu p - N n_R N^\nu f = g n_R [F_{\mu a} \partial^\mu + m g S^{\mu a}] U^{\mu a} \]

representing the matter and gauge field (including their interaction). Then, equation (30) becomes

\[ \partial^\nu p - N n_R N^\nu f = g n_R M_{\mu a} U^{\mu a} \]

Defining the entropy $\sigma$ from Eqn. \[ \sigma = \ln \left( \frac{p}{K_3} \right) \]

in analogy with \[ \sigma = \ln \left( \frac{p}{K_2} \right) \]

Eqs.30-32 may be combined to yield
\[ T \partial^\nu \sigma = g M^{\mu a} U^{\mu a} \]

For a homentropic fluid (a relevant limit for the QGP), the equation of motion becomes even simpler:
\[ g M^{\mu a} U^{\mu a} = \left[ F_{\mu a} + \frac{m g}{S^{\mu a}} \right] U^{\mu a} = 0 \]
We have just shown the existence of a unified "minimally" coupled potential for hot non-Abelian fluids

\[ Q_a^\mu = A_a^\mu + \frac{m}{g} f U^\mu, \tag{35} \]

with its corresponding field tensor

\[ M_{a\mu} = \partial_\mu Q_a^\nu - \partial_\nu Q_a^\mu + g c_a^{bc} Q_b^\mu Q_c^\nu. \tag{36} \]

Through \( M_{a\mu} \) and \( Q_a^\mu \), we have put the non-Abelian flow field and the non-Abelian gauge field on the same footing. The unification opens up an opportunity to apply the powerful machinery of gauge theories to the unified gauge-flow field; this formulation complements the previous work on the subject \[ \text{[12]}. \]

The equation of motion given by \[ \text{(34)} \] is the analogue of the two equations that Jackiw \[ \text{[12]} \] finds for the continuum version of Wong’s equations. However, unlike Jackiw’s generalization, which seems to couple Yang-Mills fields through the connection \( A^\mu a \). This natural non-linearity provides a mechanism to bypass the theorem forbidding the existence of soliton solutions in a plasma \[ \text{[7]}. \]

IV. TOPOLOGICAL INVARIANTS OF THE YANG-MILLS "MAGNETO-FUID".

In order to explore any interesting consequences of this formalism for Yang-Mills fluids, let us return first to the Abelian formalism and results on helicity conservation.

The spatial components of the equation of motion in the Abelian case may be spelled out as:

\[ U_0 (\frac{m}{q} S_0^0 + F_0^0) + U_j (\frac{m}{q} S_0^{ji} + F_0^{ji}) = 0 \quad \text{(37)} \]

Since \( F_0^0 \) is just the electric field, let us call the combined factor \( (\frac{m}{q} S_0^0 + F_0^0) \), the fluid-generalized electric field \( \hat{E} \). Let us make the same prescription for the magnetic components. Then, since \( U_0 \) is just the relativistic factor, \( \gamma \), and \( U_i = \gamma \hat{U}_i \), \text{eq. (37)} corresponds to:

\[ \gamma \hat{E} + \gamma \hat{U} \times \hat{B} = 0. \quad \text{(38)} \]

An immediate consequence is the condition

\[ \hat{E} \cdot \hat{B} = 0. \quad \text{(39)} \]

Since the product of \( M_{\mu\nu} = \frac{m}{q} S_{\mu\nu} + F_{\mu\nu} \) with its dual \( M_{\mu\nu} = \frac{i}{4} \epsilon_{\mu\nu\lambda\rho} M^{\lambda\rho} \), is proportional to \( \hat{E} \cdot \hat{B} \), \text{Eq. (39)} demands

\[ \frac{1}{2} M_{\mu\nu} M_{\mu\nu} = 0. \quad \text{(40)} \]

Since the left hand side of this equation is a boundary term, we can assume the existence of a \( K^\mu \), which for electrodynamics, is conserved

\[ \partial_\mu K^\mu = 0. \quad \text{(41)} \]

It is well known that \( K^\mu \) gives rise to helicity conservation and \( K^\mu \) is identified with the fluid field equivalent of the Abelian Chern-Simons vector \( A_\mu F^{\mu\nu} \) (see \[ \text{[12]} \]). The quantity \( \frac{1}{2} M_{\mu\nu} M_{\mu\nu} \) represents the topological winding number (charge) of solutions to the fluid field equations of motion and the fact that it is zero supports the widely held view that an ordinary electron positron plasma, for example, does not support stable, self confining knot like solutions. This is upheld by a virial theorem due to Shafranov which states that a static configuration of a plasma in isolation is dissipative. Recently in \[ \text{[17]} \] it has been proposed that this no go theorem is circumvented by introducing non linear interactions. We shall now show that the generalization to the YM plasma overrides this limitation and can support stable knot like solutions.

For the non-Abelian case, the spatial part of equation \[ \text{(34)} \] is some what more complicated:

\[ U_0 (\frac{m}{q} S_0^0 + F_0^0) + U_j (\frac{m}{q} S_0^{ji} + F_0^{ji}) = 0 \quad \text{(42)} \]

To manipulate the Eq. \[ \text{(42)} \], we have to face the question of factoring the non-Abelian four-velocity \( U_\mu^a \). At this stage there is no immediate compulsion for a factoring out of the generators (charges) of the gauge group. Instead, one may assume that \( U_\mu^a U_\nu^b = tr_{group} U_\mu U_\nu = -N \) \( (N \) being the dimension of the gauge group) implies the existence of a full non-Abelian flow and that the velocity four-vector is normalized in each flow. In terms of the fluid-generalized electric and magnetic fields, equation \[ \text{(42)} \] becomes:

\[ \sum_a \gamma \hat{E}_a + \gamma \hat{U}^a \times \hat{B}_a = 0 \quad \text{(43)} \]

Because of the trace over the group indices, Eq. \[ \text{(42)} \] implies that unlike in the Abelian case, the product \( M_{\mu\nu} M_{a\mu} \neq 0 \). Consequently, the non Abelian generalization of the topological charge is not necessarily zero opening up the possibility of non trivial topological structures being supported by the non-Abelian fluid plasma.

However, we may define a generalized Chern-Simons vector, \( C^\mu \) as

\[ \partial_\mu C^\mu = \frac{1}{2} M_{\rho\mu}^a M_{a\lambda\rho}. \quad \text{(44)} \]

In terms of the connection \( Q^a_\mu \), the generalized Chern-Simons vector \( C_\mu \) is given by:

\[ C^\mu = Q^a_\mu [M_{a\mu\nu} - \frac{g}{6} \epsilon^{\mu\nu\lambda\rho} C_{a\lambda\rho} Q_{a\lambda\rho}]. \quad \text{(45)} \]

To analyze this topological term, we look at the gauge properties of this potential \( Q^a_\mu \). Under a gauge transformation \( \Omega_\mu \),

\[ A'_\mu = \Omega_\mu A_\mu \Omega^{-1} - i \frac{g}{\Omega} (\partial_\mu \Omega_\nu) \Omega^{-1}. \quad \text{(46)} \]
it is not difficult to see that:
\[ \mathbf{Q}'_\mu = \mathbf{Q}_\mu \mathbf{Q}^{-1} - \frac{i}{g} (\mathbf{Q}_\mu \mathbf{Q}^{-1}) \mathbf{Q}_\mu \mathbf{Q}^{-1}. \tag{47} \]

This means that the non-Abelian fluid velocity vector, \( U^\mu \), transforms covariantly
\[ U'_\mu = \mathbf{Q}_\mu U^\mu \mathbf{Q}^{-1} \tag{48} \]
under a gauge transformation and \( Q^\alpha \) can hence be strictly identified with a non-Abelian gauge connection. An immediate consequence for our analysis is that the \( C_\mu \) that we have written above will indeed give us a generalized Chern-Simons invariant associated with \( Q^\alpha \). In addition, the Yang-Mills connection \( A^\alpha \), will provide us with the standard Chern-Simons invariant.

The important point is that the minimal coupling procedure introduced in \( \mathbf{13} \) and generalized here to the non-Abelian case, allows us to link these two Chern-Simons terms through the fluid velocity vector. Unlike the Abelian case (where the divergence of the generalized helicity four vector for the combined system was forced to be zero), we have two topological quantities in the non-Abelian case: one coming from the combined fluid+YM case and one from the YM case with the fluid velocity vector tying the two together.

To understand the topological implications of our result, consider the transformation of the quantity
\[ \frac{1}{8\pi^2} \int C_\mu d^2x \text{ under gauge transformations. For this we use the wedge product notation } A \wedge F = \epsilon^{ijk} A_i F_{jk} d^3x \text{ to write} \]
\[ I = \frac{1}{8\pi^2} \int C_\mu d^3x = \frac{1}{8\pi^2} \int \text{Tr}(Q \wedge dQ - \frac{2}{3} Q \wedge Q \wedge Q) \]
\[ = \frac{1}{8\pi^2} \int \text{Tr}(Q \wedge M + \frac{1}{3} Q \wedge Q \wedge Q) \tag{49} \]

Using the transformation properties established for \( Q \) we get the gauge transformed \( I_g \) as \( I_g = \frac{1}{8\pi^2} \int \text{Tr}(Q \wedge M_g + \frac{3}{2} Q' \wedge Q' \wedge Q') \) where \( M_g = \mathbf{Q}_g M \mathbf{Q}^{-1} \).

Thus we have
\[ I_g - I = \frac{1}{24\pi^2} \int \text{Tr}[d\mathbf{Q}_g \mathbf{Q}^{-1} \wedge d\mathbf{Q}_g \mathbf{Q}^{-1} \wedge d\mathbf{Q}_g \mathbf{Q}^{-1}] \tag{50} \]
which is the second Chern Class which describes the winding number of the manifold. This implies that the integral of the invariant \( \text{tr} M^{\mu \nu} M^{\mu \nu} \) will lead us to just the one appropriate Pontryagin invariant for the Yang-Mills gauge group that is used for the dynamics. From the minimal coupling prescription we have given above, we can write:

\[ \int_M \text{tr}(M^{\mu \nu} M^{\mu \nu}) = \int_M \text{tr}(F_{\mu \nu} F^{\mu \nu}) + \frac{2m}{g} \int_M \text{tr}(S_{\mu \nu} F^{\mu \nu}) + \frac{m^2}{g^2} \int_M \text{tr}(S_{\mu \nu} S^{\mu \nu}) \tag{51} \]

where \( M, F, S \) are the duals of \( M, F, S \) respectively and the integral is over spacetime. Thus in the non-Abelian magneto fluid we may associate this non zero Pontryagin index with a non-Abelian magneto fluid helicity implying the existence of stable self confining non dissipative solutions. In fact, the non triviality of the Hopf invariant ensures that flux lines can be knotted and solitonic configurations are inevitable. These can have a number of consequences, such as the existence of glueballs, as knotted solitons having the non-Abelian helicity as a topological quantum number, which may survive in the quark gluon plasma in the interior of a heavy ion collision or in the early universe.

V. CONCLUSION.

We have given the foundations of a consistent theory of non-Abelian fluid field system, in which the flow field and the gauge field are "unified" in a single minimally coupled gauge flow field. We have shown that this gives rise to a quantity which is the fluid field generalization of the non-Abelian Chern Simons term, and shown that knotted fluid-field non-Abelian solitons may exist. We can, by using standard techniques in pure Yang Mills theories, find explicit forms of these and look for phenomenological signatures in the context of QGP. The formalism is simple and unified and should lead to new and interesting phenomena such as non-Abelian Alfvén waves and other non-Abelian counterparts of magnetohydrodynamics which may lead to new signals for collective flow in the QGP. Such studies are under investigation.
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