METHOD FOR ASSESSING THE STRUCTURAL RELIABILITY OF NETWORKS WITH UNDETERMINED TOPOLOGY

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Abstract. This paper shows the relevance of the task of assessing the structural reliability of networks with undetermined topology. Proposed is a method for assessing the structural reliability of networks of undetermined topology based on taking into account the basic structural characteristics of the network (the number of nodes and branches, the degree of network connectivity, the maximum allowable rank of paths, and others). To obtain an estimate of the structural reliability for a network of any dimension and any topology, expressions are proposed in the scientific research to determine the number of paths of various ranks, which must be taken into account when calculating the structural reliability index by the upper and lower bounds method.

Keywords: network of undetermined topology, structural reliability, route rank, number of routes of a certain rank, upper and lower bounds

METODA OCENY STRUKTURALNEJ NIEZAWODNOŚCI SIECI O NIEOKREŚLONEJ TOPOLOGII

Słowa kluczowe: sieć o nieokreślonej topologii, niezawodność strukturalna, ranga ścieżek, liczba ścieżek o określonej randze, górne i dolne kresy

Introduction

The development strategy of modern communication networks is currently aimed at meeting the growing needs of users and ensuring the required quality of the services. In this regard, there is a tendency to a constant increase in the volume of transmitted data and the complexity of the network structure. The characteristic features of modern networks are the priority use of wireless access and the introduction of self-organization mechanisms into the network. Communication networks are increasingly becoming decentralized, wireless, do not have a constant structure, and the number of nodes and connections between nodes are random variables in time. Each node of such a network can forward data destined to other nodes. The data transfer route is determined dynamically, based on the connectivity of the network at a certain point in time [10]. Moreover, in networks with an undetermined topology, multi-path routing is often used, the aim of which is to provide the source node with the ability to select one of several possible routes to a specific destination node. This approach allows one to optimally use the capacity of the communication channel and increase the overall network bandwidth [8]. Additionally, network fault tolerance and transmission reliability are provided.

A number of scientific papers by A. Ye, Kucheryavy [10], A. V. Roslyakov [15], A. V. Prokopiev, Ye, A. Kucheryavy [9], A. Goldsmith [3], N. N. Egum and V. P. Shuvalov [5], A. V. Kharybin. The structural reliability of self-organized networks is the subject of a number of works by Rudenko and D. A. Migov [11]. In these works, the object of reliability assessment are networks of an initially given structure described by adjacency matrices and other network characteristics. The goal of the classical network analysis problem of a certain topology is to determine the structural reliability of a functioning network or the resulting design solution, presented in the form of some structure [6]. However, in the case where the scale of the network is known, however, the network topology has not yet been determined (at the design stage, for example) or is constantly changing (at the operation stage), for a given (known) set of nodes, the set of branches is unknown. In this case, the network is characterized only by the number of nodes and branches. Therefore, the existing methods for assessing structural reliability, focused on applications for networks with a previously known topology, in cases of networks with an undetermined topology, are of little use.

When considering problems related to the analysis of the structural reliability of communication networks, a random graph is usually used as a network model [1, 2, 11, 13]. Among these models classical are, first of all, the Erdős–Rényi (ER) model
The structure of the generalized network of the undetermined topology is described by a random ER-graph (model \( G(N, p) \), ER-network). The set of vertices \( N \) of the graph corresponds to points (nodes) of the network, \(|N| = N\). The set of edges \( L \) corresponds to the branches of the network – direct connections connecting pairs of nodes, \(|L| = L\). Considered an undirected connected network.

The connectivity between pairs of nodes is provided by routes in the form of chains of branches without cycles and loops. The degree of connectivity of a pair of nodes is determined by the number of routes (in the general case, dependent) connecting these nodes. The rank \( R \) of the route corresponds to the number of branches included in the route. In a network of undetermined topology, the set \( L \) is not defined, but its cardinality is known \( L \).

The reliability index of all branches (the probabilistic structural reliability) \( \beta_{xy} \) is a branch in a cross section with the corresponding \( p_{xy} \) value – the probability of failure-free operation.

The set of dividing sections \( \gamma_{ij} \) is formed on the basis of a specific, proposed method using routes between \( i \) and \( j \), for each \( l \)-th section \( \gamma_{ij} \) out of \( \gamma_{ij} \), the section of \( \beta_{xy} \) is determined using the value \( p_{xy} \).

As follows from the presented expressions (3, 4), one of the basic structural characteristics used in this method is the number of routes of a certain rank. In a network with \( L \) branches, the number of routes of rank \( R \) (more precisely, the mathematical expectation of this number) can be obtained according to the following recursive expression proposed in \([6]\):

\[
M_{R,L} = M_{R,L+1} (1 - \frac{R}{L+1} )
\]

Since, to determine the number of routes of rank \( R \) in a network with \( L \) branches, one first needs to calculate the number of routes of rank \( R \) in a network with \( (L + 1) \) branches, recursive calculation begins with calculating the number of routes in a fully connected network with \( L_{\text{all}} = \frac{n(n-1)}{2} \) branches (\( n \) is the number of network nodes) \([6]\):

\[
M_{R,L_{\text{all}}} = \frac{n(n-1)!}{2(n-R-1)!}
\]

It is important to note that this method assumes uniformly random saturation of many nodes with branches and, thus, does not take into account the requirements for the formation of a fully connected network. As a result of this, it is possible to obtain the number of routes \( M_{L} < 1 \) for small \( L \) and large \( R \). This result is consistent with the probabilistic nature of the number of routes in ER-networks (as a mathematical expectation) and means that paths with given parameters will not be present in all implementations of a random graph.
The number of ranks of rank \( R \) per one connection is calculated as the ratio of the total number of \( M_R \) routes to the known number \( t \) of connections:

\[
m_{Rij} = \frac{M_R}{t}
\]  

(7)

The number of connections can be determined in accordance with expressions (8, 9) or it can be a given constant indicator for a particular network.

In an oriented network, the total number \( t_n \) of connections is determined on the basis of expression (8):

\[
t_n = n(n - 1).
\]  

(8)

If the network is non-oriented, then the total number of \( t_n \) connections is defined as:

\[
t_n = \frac{n(n-1)}{2}.
\]  

(9)

The above analytical method is not the only one route to obtain the structural characteristics of the network. The number of routes in ER networks of a relatively small dimension can be determined by the simulation of ER graphs.

2. Research results

In this research, using the proposed expressions (5–6), a series of experiments was carried out for ER networks of dimensions 20, 50, and 100 nodes with a different number of branches: from close to minimum \( L_{\text{min}} \), in which the network is connected (tree), to close to maximum based on expressions (5–6), in which the network is fully connected. To record large numbers, the decimal exponential notation of the form \( m \times 10^n \), standard for computer programs, is used.

The experimental results of determining the number of routes of different ranks by the analytical method are presented in tables. Table 1 shows the results of a study for an ER network with a dimension of 20 nodes with a different number of branches (20, 70, 120, and 170 branches).

Table 1. The number of routes of rank \( R \) in an ER network with \( N = 20 \) nodes and \( L \) branches

| \( L \) | \( R \) = 1 | \( R \) = 2 | \( R \) = 3 | \( R \) = 4 | \ldots | \( R \) = 19 |
|---|---|---|---|---|---|---|
| 20 | 20 | 36 | 59 | 86 | | 3.79E-07 |
| 70 | 70 | 460 | 2829 | 1.62E+04 | | 1.20E+09 |
| 120 | 120 | 1360 | 14511 | 1.45E+05 | | 1.11E+14 |
| 170 | 170 | 2376 | 41567 | 5.94E+05 | | 1.31E+17 |

The number of routes \( M_R \) of the rank \( R=1 \) corresponds to a given number of branches \( L \). The number of routes of a rank higher than 1 increases with the number of branches and the value of the route rank. In real networks, for the implementation of a single connection, routes of maximum ranks are practically not used for a reason of network performance. Constraints are usually fulfilled at the level of the third or fourth rank. According to this rule, the table shows the calculations of \( M_R \) for routes for rank \( R \leq 4 \).

Tables 2 and 3 show the results of similar calculations for an ER network with \( N = 50 \) and \( N = 100 \) nodes.

Table 2. The number of routes of rank \( R \) in an ER network with \( N = 50 \) nodes and \( L \) branches

| \( L \) | \( R \) = 1 | \( R \) = 2 | \( R \) = 3 | \( R \) = 4 | \ldots | \( R \) = 49 |
|---|---|---|---|---|---|---|
| 50 | 50 | 96 | 177 | 314 | | 5.87E-23 |
| 100 | 100 | 3518 | 40285 | 450384 | | 7.29E+32 |
| 150 | 150 | 11841 | 249371 | 5134767 | | 4.04E+46 |
| 200 | 200 | 25067 | 768725 | 2.31E+07 | | 7.68E+54 |

The above tables can be used in practical calculations of the reliability of networks of undetermined topology. Using the obtained values of the number of routes \( M_R \), given in Table 1, will provide an example of calculating indicator of the structural reliability for a non-oriented communication network with the number of nodes \( N = 20 \) and the number of branches \( L = 20 \).

Based on expressions (7) and (9), we determined the average number of routes of each rank \( (R = 1, ..., 4) \) per one bond \((i,j)\):

\[
m_{ij} = \frac{2 \cdot 20}{(20 \cdot (20 - 1))} = 0.105,
\]

\[
m_{ij} = 0.189,
\]

\[
m_{ij} = 0.311,
\]

\[
m_{ij} = 0.453.
\]  

(10)

In real systems, devices with a low reliability value are not used. According to statistics, the probability of uptime is usually in the range of 0.96–0.99. Basing on this, in this example, we take the probability of failure-free operation of the network branches \( p = 0.98 \).

The upper boundary of the structural reliability \( P_{\text{UBSRij}} \) is determined by expression (4). Transforming it, according to the source data, obtains the following expression:

\[
P_{\text{UBSRij}} = 1 - ((1-p)^{m_{ij}} \cdot (1-p^8)^{m_{ij}} \cdot (1-p^{10})^{m_{ij}} \cdot (1-p^{12})^{m_{ij}}).
\]  

(11)

In order to be able to record the set of cross sections in disjunctive normal form, the obtained values of the number of routes \( m_{ij} \) of each rank \((R = 1, ..., 4)\) per one connection \((i,j)\), obtains:

\[
P_{\text{UBSRij}} = 1 - ((1-p^{0.98})^{0.105} \cdot (1-p^{0.98})^{0.189} \cdot (1-p^{0.98})^{0.311} \cdot (1-p^{0.98})^{0.453}) = 0.99999639.
\]

To obtain the value of the lower boundary of the structural reliability \( P_{\text{LSRij}} \), based on the set of routes \( M_R \), one should obtain the set of dividing cross sections \( \gamma_{ij} \). In order to be able to record the set of cross sections in disjunctive normal form, the obtained in (10) of the number of routes of each rank per one bond to the nearest integer should be rounded off. The result values are:

\[
m_{ij} = 1, \ m_{ij} = 1, \ m_{ij} = 1, \ m_{ij} = 1.
\]

Represented a set of paths \( M_R \) in disjunctive normal form:

\[
M_{ij} = k_1 + k_2k_1 + k_3k_2k_1 + k_4k_3k_2k_1.
\]  

(12)

Further, for expression (12), it obtains the dual Boolean function in conjunctive normal form:

\[
\gamma_{ij} = k_3k_2k_1k_4 + k_1k_2k_1k_4 + k_1k_2k_4 + k_1k_2k_4k_3 + k_1k_2k_3k_2 + \ldots + k_1k_3k_4k_3.
\]  

(13)

Performing the conversion of conjunctive normal form (13) to disjunctive normal form obtains a set of cross sections (14):

\[
\gamma_{ij} = k_3k_2k_1k_4 + k_1k_2k_1k_4 + k_1k_2k_4 + k_1k_2k_4k_3 + k_1k_2k_3k_2 + \ldots + k_1k_3k_4k_3.
\]  

(14)

According to (4), we it obtain the expression for calculating \( P_{\text{LSRij}} \):

\[
P_{\text{LSRij}} = (1 - (1-p_1) \cdot (1-p_2) \cdot (1-p_3) \cdot (1-p_4)) \cdot \ldots 
\]

\[
(1 - (1-p_1) \cdot (1-p_2) \cdot (1-p_3) \cdot (1-p_4)) \cdot \ldots
\]

\[
(1 - (1-p_1) \cdot (1-p_2) \cdot (1-p_3) \cdot (1-p_4))
\]  

(15)
Transforming (15) according to the initial data we obtain the following expression:

$$P_{LRBJ} = (1 - (1 - p)^4)^q,$$

where $q$ defined as:

$$q = \prod_{R=1}^{4} R^{m_{RJ}}.$$

Using the given values of the number of routes $m_{RJ}$ of each rank ($R = 1$, ..., 4) per one connection and the probabilities $p_{RJ}$ of the network branches to fail we obtain:

$$q = \prod_{R=1}^{4} R^{m_{RJ}} = 1^1 \cdot 2^1 \cdot 3^1 \cdot 4^1 = 24.$$

$$P_{LRBJ} = (1 - (1 - 0.98)^4)^{24} = 0.99999616.$$

Based on expression (2) for the values accepted for this example, $k_U = 0.55$ and $k_L = 0.45$, the value of the indicator $P_{ISRJ}$ for communication $(i-j)$ is determined:

$$P_{ISRJ} = P_{LRBJ} \cdot 0.55 + P_{LRBJ} \cdot 0.45 = 0.9999639 \cdot 0.55 + 0.99999616 \cdot 0.45 = 0.99999629.$$

Performing calculations for all connections $(i-j)$ makes it possible to determine the value of the indicator $P_{ISRJ}$ of the structural reliability of the entire network (based on expression (1), taking into account the values of weighting coefficients $w_{ij}$).

3. Conclusion

The paper shows the relevance of the task of assessing the structural reliability of networks of undetermined topology. A method for obtaining such an estimate based on the basic structural characteristics of the network – the number of nodes and branches, the maximum allowable rank of routes, and others – is presented. Presented is also a method for determining the number of routes of each rank per one connection. Calculations were carried out for ER networks of dimensions of 20, 50 and 100 nodes with different number of branches: from the close to the minimum $L_{re}$, at which the network is connected (tree), to close to the maximum $L_{all}$ (fully connected network).

The proposed method for assessing the structural reliability of networks with an undetermined topology is based on the formation of indicator $P_{ISRJ}$ of the structural reliability, which is determined using the lower and upper boundaries of structural reliability. Expressions are presented for determining the number of routes of different ranks that can be used to service applications that enter the network of the undetermined topology. Based on the upper and lower boundaries of the structural reliability of individual links, presented is an approach that allows one to obtain a weighted average estimate of the structural reliability of the entire network of the undetermined topology.

An example of the implementation of the method for determining the structural reliability indicator of a network of undetermined topology is performed.

Further development of this work is the solution of issues related to the development of approaches to determining the values of the probabilities of failure-free operation of network branches, as well as the values of weighting coefficients for determining the upper and lower boundaries of the structural reliability of both individual connections and the entire network of the undetermined topology.

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