Bin Packing Under Multiple Objectives –
a Heuristic Approximation Approach

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Abstract—The article proposes a heuristic approximation approach to the bin packing problem under multiple objectives. In addition to the traditional objective of minimizing the number of bins, the heterogeneousness of the elements in each bin is minimized, leading to a biobjective formulation of the problem with a tradeoff between the number of bins and their heterogeneousness. An extension of the Best-Fit approximation algorithm is presented to solve the problem. Experimental investigations have been carried out on benchmark instances of different size, ranging from 100 to 1000 items. Encouraging results have been obtained, showing the applicability of the heuristic approach to the described problem.

I. INTRODUCTION

The term “bin packing” describes a class of well-known, classical problems with numerous applications in logistics, operations research and related disciplines. From single dimensional to multi-dimensional problems, various types can be identified in practice. Common to all is the overall task of packing a finite number of items into a minimum number of bins (knapsacks) subject to a set of practical constraints and requirements. These include given capacities of the bins, but also other considerations such as irregularly shaped bins, load balancing of the bins, etc.

Numerous approaches including exact, heuristic, and meta-heuristic algorithms have been proposed for the resolution of bin packing problems, and a rich literature on packing problems exists, with important classifications by Dyckhoff [1] and more recently Wäscher et al. [2]. While the majority of approaches is dedicated to single-objective models, only minimizing the number of bins used, the multi-objective nature of many of these problems becomes more and more obvious. Following early work of Wäscher [3], modern heuristics such as Particle Swarm Optimization have recently been applied to a multi-objective variant of the two-dimensional bin-packing problem [4]. For the here considered multi-objective bin-packing problem however, no corresponding studies have been carried out to our knowledge.

The remainder of the article is organized as follows. Section II describes the multi-objective bin packing problem and its underlying practical application. A heuristic approximation approach is presented in the following Section III. In brief, we propose an extension of the well-known best-fit heuristic, allowing the computation of a set of solutions that constitute an approximation to the set of efficient solutions. Experimental results of the approach to the problem are reported in Section IV, and conclusions are given in Section V.

II. PROBLEM DESCRIPTION

We consider a bin packing problem where a given number of \( n \) items has to be packed into \( n \) bins, each of capacity \( c \) [5]. Each item \( j \) is characterized by a weight \( w_j \) and an additional attribute \( a_j \). While the weights refer to the size of the items and therefore have to be taken into consideration when filling up a bin to at most its’ capacity \( c \), the attributes \( a_j \) describe properties of the items on a nominal scale. On the basis of this description, a comparison of two items \( i, j \) is possible such that they are either identical with respect to \( a_i \) and \( a_j \), \( a_i = a_j \) or not: \( a_i \neq a_j \). The goal of packing the items into bins can then be modeled as follows.

\[
\begin{align*}
\text{minimize} \quad & z_1 = \sum_{i=1}^{n} y_i \\
\text{minimize} \quad & z_2 = \frac{1}{z_1} \sum_{i=1}^{n} u_i \\
\text{s.t.} \quad & \sum_{j=1}^{n} w_j x_{ij} \leq c y_i & i \in N = \{1, \ldots, n\}, \\
& \sum_{i=1}^{n} x_{ij} = 1 & j \in N, \\
& y_i = 0 \text{ or } 1 & i \in N, \\
& x_{ij} = 0 \text{ or } 1 & i \in N, j \in N,
\end{align*}
\]

where

\[
\begin{align*}
y_i &= \begin{cases} 
1 & \text{if bin } i \text{ is used} \\
0 & \text{otherwise}
\end{cases} \\
x_{ij} &= \begin{cases} 
1 & \text{if item } j \text{ is assigned to bin } i \\
0 & \text{otherwise}
\end{cases} \\
u_i &= \text{counts the number of distinct attributes in bin } i
\end{align*}
\]

Expression (1) minimizes the number of bins. The second objective given in (2) minimizes the average heterogeneousness of the bins. To do this, the number of distinct attributes \( u_i \) is counted for each bin \( i \). Unused bins \( (y_i = 0) \) have a value of \( u_i = 0 \). Used bins \( (y_i = 1) \) have a possible minimum value of \( u_i = 1 \). This is the case when all items in the particular bin have the identical nominal attribute. The values of \( u_i \) are bounded by either the number of items assigned to a bin or the number of distinct attributes over all items \( i \)
Intuitively, the two objective functions are of conflicting nature. While a large number of bins allows the packing of bins which are each fully homogeneous, leading to a $z_2 = 1$, a solution being minimal for $z_1$ will require the packing of items $i, j$ of different $a_i$ and $a_j$ into the same bin. It can therefore be suspected that not a single solution $x$ exists in the set of feasible solutions $X$ that equally minimizes both objective functions $z_1$ and $z_2$. In brief, this leads to a vector optimization problem in which a solution $x \in X$ is evaluated with respect to a vector $Z(x) = (z_1(x), z_2(x))$. The resolution of the problem has consequently to be seen in the identification of all efficient outcomes or the Pareto set $P$, introduced in the following Definitions 2.1 and 2.2.

Definition 2.1 (Dominance): A vector $Z(x)$ is said to dominate $Z(x')$ iff $z_k(x) \leq z_k(x') \forall k = 1, \ldots, K \land \exists k \mid z_k(x) < z_k(x')$. We denote the dominance of $Z(x)$ over $Z(x')$ with $Z(x) \leq Z(x')$.

Definition 2.2 (Efficiency, Pareto-optimality): The vector $Z(x), x \in X$ is said to be efficient iff $\nexists x' \in X \mid Z(x') \leq Z(x)$. The corresponding alternative $x$ is called Pareto-optimal, the set of all Pareto-optimal alternatives Pareto-set $P$.

Numerous practical applications of the formal model exist. In many cases, a minimum number of bins should be used when packing a given set of items, however assuring a maximum possible homogeneousness of the items being packed together into a single bin. Applications include the storage of goods, the storage of music/video data on optical discs, etc.

III. A HEURISTIC APPROXIMATION APPROACH

As already mentioned in Section I, numerous algorithms have been proposed to solve the single-objective variant of the bin packing problem. One important heuristic is the Best-Fit algorithm, important for both for its’ time complexity as well as for its’ worst-case complexity [6]. Best-Fit subsequently assigns items to the feasible bin having the smallest residual capacity. If no such bin exists, the item is assigned to a previously unused (new) bin.

Unfortunately however, Best-Fit only takes into consideration the weights $w_j$ of the items and the residual capacities of the bins when selecting the ‘best-fitting’-bin. In order to address the problem described in Section II, a method of controlling the heterogeneousness of the bins needs to be included in the algorithm. Algorithm 1 describes such an attempt, allowing the successive computation of alternatives with different heterogeneousness levels and therefore providing an idea of how to compute an approximation to the vector optimization problem given in Section II.

The modified Best-Fit algorithm is based in principle on the conventional method. However, in order to control the heterogeneousness of the bins, an additional control parameter $u_{max}$ is used as described in step 8 of Algorithm 1. Starting with an initial value of $u_{max} = u = 1$, only Best-Fit-bins are allowed which are fully homogeneous. This means that an item may only be assigned to a bin containing other elements of identical attributes $a_j$. In this stage of the algorithm, solutions are computed that lead to the lowest possible value of $z_2 = 1$ as all bins contain homogeneous items.

Algorithm 1 Multi-objective Best-Fit algorithm

Require: $s, \varpi$

1: Compute the maximum possible heterogeneousness of a bin $\varpi$
2: Set $u = 1$
3: $\mathcal{P}^{approx} = \emptyset$
4: repeat
5: for $m = 1$ to $\varpi$ do
6: Construct a new solution $x$:
7: for all $n$ items do
8: Compute the maximally allowed heterogeneousness $u_{max}$ of the Best-Fit-bin:
9: $u_{max} = [u]$ with probability $1 - (u - [u])$ and
10: $u_{max} = [u]$ with probability $u - [u]$
11: Compute the Best-Fit-bin with respect to the randomly determined $u_{max}$
12: Assign $i$ to the Best-Fit-bin
13: end for
14: Update $\mathcal{P}^{approx}$ with $x$: Remove all elements in $\mathcal{P}^{approx}$ which are dominated by $Z(x)$;
15: Add $x$ to $\mathcal{P}^{approx}$ if $Z(x)$ is not dominated by any element in $\mathcal{P}^{approx}$
16: Return $\mathcal{P}^{approx}$

With increasing value of $u$, incremented by $s$ in step 14 of the algorithm, Best-Fit-bins become possible that have a higher heterogeneousness. This concept is randomized throughout the generation of the solutions, allowing a gradual transition from $u_{max} = u = 1$ to the maximum possible heterogeneousness $u_{max} = \varpi$. Due to the randomness in the algorithm, different runs lead to different outcomes. We therefore propose to compute a number of solutions with each setting of $u$, given as control parameter $\varpi$.

Throughout the algorithm, an archive $\mathcal{P}^{approx}$ of the best solutions is kept which is returned after the algorithm terminates. This archive represents an approximation to the true Pareto-set $P$.

IV. EXPERIMENTAL INVESTIGATION

A. Generation of test instances and experimental setup

In order to test the effectiveness of the multi-objective extension of the Best-Fit approximation algorithm, four multi-objective test instances have been computed with values of $n = 100, n = 200, n = 500,$ and $n = 1000$. The data has been derived taking $\frac{n}{5}$ bins, each of capacity $c = 1000$, and randomly splitting the capacity into five items $j, \ldots, j + 4$ such that the weights $w_j$ of the items add up to $c: \sum_{j=1}^{j+4} w_j = c$. This means that the so constructed instances have a solution for which the minimum number of bins is $\frac{n}{5}$ and therefore equal to the trivial lower bound $\left\lfloor \frac{\sum_{j=1}^{n} w_j}{c} \right\rfloor$.

The items of each instance have been randomly assigned nominal attributes from a set of five different attributes. Each
attribute has been selected with equal probability of 0.2.

We tested the randomized Best-Fit algorithm from Section III on the proposed benchmark instances with control parameters \( s = 0.1 \) and \( \bar{m} = 100 \). This means that the probability of selecting a Best-Fit-bin of maximum heterogeneity \( u_{\text{max}} = \lceil u \rceil \) over a Best-Fit-bin of maximum heterogeneity \( u_{\text{max}} = \lfloor u \rfloor \) increases in each step 14 of Algorithm 1 with 10%. The inner loop of the algorithm computes \( \bar{m} = 100 \) solutions with each setting of the parameters.

In addition to the multi-objective Best-Fit algorithm, a Random-Fit algorithm has been tested for comparison reasons. This algorithm randomly selects a bin that allows the assignment of the currently considered item with respect to the chosen maximum heterogeneity \( u_{\text{max}} \). Apart from that aspect, the algorithm is implemented identically to the pseudo-code given in Algorithm 1.

Three different input sequences of the items have been tested:

- Sorting the items in decreasing order of \( w_j \)
- Sorting the items in increasing order of \( w_j \)
- Arranging the items in random order

B. Results

The experimental investigations revealed that only few efficient outcomes exist for the instances. Instead of plotting the outcomes in figures, we chose to give the data of all found best vectors \( Z(x) = (z_1(x), z_2(x)) \). The following Table I shows the results for the smallest instance with \( n = 100 \). It can be seen, that both Best-Fit and Random-Fit perform comparably good given a decreasing or random order of the items.

| Item order | Best-Fit | Random-Fit |
|------------|----------|------------|
| Decreasing \( w_j \) | (22, 1,000) | (22, 1,000) |
| Increasing \( w_j \) | (25, 1,000) | (25, 1,000) |
| Random | (22, 1,000) | (22, 1,000) |

Similar results have been obtained for the instance with \( n = 200 \) as shown in Table II. Again, Best-Fit and Random-Fit lead to the best results given an order of the items with decreasing \( w_j \). While both identify the vector \((43, 1,000)\), they are incomparable with respect to the other best found outcomes \((42, 1.214)\) and \((41, 1.902)\).

For the next instance with \( n = 500 \), given in Table III, Random-Fit appears to lead to superior results, however with a very small distance to Best-Fit. While the assignment of items in increasing order of \( w_j \) is clearly inferior, the random ordering of items appears to become less and less favorable in comparison to the decreasing order.

The results of the largest instance, shown in Table IV, confirm that the ordering of the items becomes more influential with increasing size of the instance. The best results have been obtained assigning the items in decreasing order of \( w_j \) while the random ordering turned out to be comparably weak. When comparing Best-Fit and Random-Fit, Best-Fit appears to lead to more efficient outcomes, but still Random-Fit is able to identify solutions that have not been found by the Best-Fit algorithm.

Common to the results of all investigated instances is that the approximation algorithms have not been able to identify the minimal solution for \( z_1 \). The best solutions with respect to \( z_1 \) are still one bin larger than the minimum possible value.

V. Conclusions

The article presented a study on the multi-objective bin packing problem. We considered the objective of minimizing the number of bins as well as the objective of minimizing the average heterogeneousness of the bins, based on nominal
attributes of the items. The two conflicting objectives led to the formulation of the problems as a vector optimization problem.

A modified Best-Fit approximation algorithm has been presented to compute an approximation of the set of efficient solutions. The procedure allows the controlled consideration of the heterogeneousness of the bins, integrating the parameter in the selection process of the bins in a randomized fashion.

Experimental investigations have been carried out on a set of benchmark instances, and comparison results have been obtained from a Random-Fit heuristic. The results are encouraging, as very close approximations to the set of efficient outcomes have been identified. It has become clear, that with growing size of the instances, measured by the number of items \( n \), the processing order of the items plays an increasingly important role. For small instances, a random order of the items turns out to be feasible. For large instances however, an order of decreasing \( w_j \) is necessary to obtain good results.

In conclusion, the presented multi-objective Best-Fit algorithm led to satisfying solutions. The running times of the approach remained on an Intel Pentium IV 1.8 GHz processor within a few seconds for each test run. We conclude that the algorithm may also be beneficial when computing a first, qualitatively good approximation of the Pareto-set which is then used in a more complex improvement (meta-)heuristic.

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