Declarative Stream Runtime Verification (hLola)

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Abstract. Stream Runtime Verification is a formal dynamic analysis technique that generalizes runtime verification algorithms from temporal logics like LTL to stream monitoring, allowing to compute richer verdicts than Booleans (including quantitative and arbitrary data). In this paper we study the problem of implementing an SRV engine that is truly extensible to arbitrary data theories, and we propose a solution as a Haskell embedded domain specific language.

In spite of the theoretical clean separation in SRV between temporal dependencies and data computations, previous engines include ad-hoc implementations of a few data types, requiring complex changes to incorporate new data theories.

We propose here an SRV language called hLOLA that borrows general Haskell types and embeds them transparently into an eDSL. This novel technique, which we call lift deep embedding, allows for example, the use of higher-order functions for static stream parameterization. We describe the Haskell implementation of hLOLA and illustrate simple extensions implemented using libraries, which require long and error-prone additions in other ad-hoc SRV formalisms.

1 Introduction

In this paper we study the problem of implementing a generic Stream Runtime Verification (SRV) engine, and show the advantages of using an embedded domain specific language (eDSL).

Runtime Verification (RV) is an area of formal methods for reactive systems, that analyses dynamically one trace of the system at a time. Compared to static techniques like model checking, RV sacrifices completeness to obtain an applicable and formal extension of testing and debugging. The main idea of RV is to generate monitors from formal specifications which then inspect a single trace of execution at a time. Early RV languages were based on logics like LTL or past LTL adapted for finite paths. Other approaches followed, based on regular expressions, timed regular expressions, rule based languages, or rewriting. These specification languages come from static verification with decidable decision problems to obtain algorithmic model checking. Therefore, the observations and verdicts are typically Boolean values.

Stream Runtime Verification starts from the observation that most monitoring algorithms for these logics can be generalized to the computation of
richer outcomes and also to allow richer observations, by generalizing the data types their operations. Languages for SRV, pioneered by LOLA \[12\], describe monitors declaratively via equations that relate streams of input and streams of output, offering a clean separation between the time dependencies and the concrete operations. The temporal part of the monitoring algorithm is a sequence of operations on abstract data that must be followed to compute the final verdict, where each individual operation is performed by a concrete data type implementation. Most SRV developments \[11,23,16\] focus on efficiently implementing the temporal engine, promising that the clean separation allows incorporating off-the-self implementations of data types. However, in practice these implementations fail to fulfill this promise because each new data-type requires modifying the parser of the input language, the internal representation, and the concrete evaluation function in the run-time system. Consequently, these tools only support a limited hard-wired collection of data types. In this paper, we give a general solution to this problem via an embedded DSL with Haskell as the host language.

The language hLOLA\[4\] and the engine that we describe in this paper implement a generic SRV monitoring algorithm that works for any data-type from the host language. We introduce a novel technique called lift deep embedding, which lifts a data-types from the host language into hLOLA and then use a single deep embedding. Our technique also allows us to exploit advanced features of the host language. For example, higher-order functions describe transformations that produce stream declarations from stream declarations, obtaining static parametrization for free. Libraries collect these transformers, which allows defining in a few lines new logics like LTL, MTL, etc. or quantitative semantics for these logics. Supporting these concepts in previous SRV languages requires to re-invent and implement ad-hoc features (like macro expansions or parametrization). Polymorphism is used both for genericity (to simplify the engine construction) and to allow describing generic stream specifications, which, again, is not allowed by previous SRV engines. Finally, we also exploit the features of Haskell to offer IO for every stream data-type for free.

Related work. SRV was pioneered by \[12\], which introduced the language LOLA for synchronous monitoring. LOLA uses stream equations with explicit shifts and introduces the idea of separating time dependencies and value computations, but it only supports Integers and Booleans. Copilot \[27\] is a Haskell implementation that offers a collection of building blocks to transform streams into other streams. Copilot generates C code suitable for embedded systems. LOLA 2.0 \[14\] extends LOLA with special constructs for run-time parametrization and real-time features \[16\] (see also \[15\]). TeSSLa \[11\] and Striver \[18\] are two SRV approaches to real-time event streams. All these languages still support only limited data-types, hard-wired in the parser, AST and run-time systems. The version of hLOLA that we describe here is only for synchronous systems (future work includes extending it to timed event streams), but hLOLA can be extended without complex refactorizations.

\[4\] available open source at http://github.com/imdea-software/hlola
Contributions. In summary, the contributions of the paper are:
1. An implementation of SRV as an eDSL that exploits advanced features of Haskell to build a generic engine and to improve the expressivity of specifications.
2. The concept of lift deep embedding where data-types are lifted from Haskell into hLOLA, and then deep embedded into Haskell transparently.
3. An implementation of many existing RV specification languages in hLOLA, which illustrates the simplicity of extending the language.
4. A brief empirical evaluation, which suggests that the hLOLA engine executes using only the theoretically predicted resources.

The rest of the paper is structured as follows. Section 2 presents the preliminaries. Section 3 describes the run-time system of hLOLA, including the engine and static parameterization. The library system of Haskell is used to extend hLOLA to support logics like LTL, MT-LTL and MTL, shown in Section 4. Section 5 describes how to leverage Haskell tools to assess the quality of hLOLA specifications, and a brief empirical evaluation. Finally, Section 6 concludes.

2 Preliminaries

We briefly introduce Stream Runtime Verification using LOLA (see [31] for a longer presentation) and then present the features of Haskell as a host language.

2.1 Stream Runtime Verification: Lola

Intuitively speaking, LOLA is a specification language and a monitoring algorithm for synchronous systems. LOLA programs describe monitors as stream transformers. Streams are finite sequences of values of a given type, for example, a Boolean stream is a sequence of Boolean values. The main idea of SRV is to cleanly separate the temporal dependencies from the data computation.

For the data, LOLA uses multi-sorted first order interpreted theories. A data theory is simply a finite collection of interpreted sorts and a finite collection of (interpreted) function symbols. Sorts are interpreted in the sense that each sort is associated with a domain, for example the domain of sort Bool is associated to the set of values \{true, false\}. Function symbols are interpreted meaning that every function symbol $f$ is both a constructor (used to build expressions of its return type, when given expressions of the type of its arguments), and a total function as its interpretation (to evaluate). For example, the theory of natural numbers uses two sorts (Nat and Bool) and contain constant function symbols 0, 1, 2, · · · of type Nat and $T$ and $F$ of type Bool, as well as functions $+$, $\cdot$, · · · $Nat \times Nat \rightarrow Nat$ (with their usual interpretations). Other function symbols in this theory are predicates $<$, $\leq$, · · · , that are symbols of type $Nat \times Nat \rightarrow Bool$. We assume that our theories include equality, and also that for every sort $T$ there is a ternary function if · then · else · of type $Bool \times T \times T \rightarrow T$. We use $e : T$ to represent that $e$ has sort $T$. 
Let $Z$ be a set of (sorted) atoms, that we call stream variables (for example $Z = \{ x : \text{Bool}, y : \text{Nat} \}$). To build stream declarations we use a special kind of expressions called offset expressions: $v[k, d]$ where $v : T \in Z$, $d$ is a constant of sort $T$ and $k$ is an integer number. For example, $x[-1, \text{false}]$ and $y[+3, 5]$ are offset expressions. The intended meaning of an offset expression $v[k, d]$ is to represent that the value of the offset expression at a point in time $n$ depends on the value of the stream $v$ at point $n + k$, while the second argument $d$ indicates the value used in case $n + k$ falls beyond the beginning or end of the stream. The set of stream expressions for stream variables from $Z$ (denoted $\text{Expr}(Z)$) is the smallest set containing $Z$, all offset expressions of variables from $Z$ and closed under constructor symbols of the data theory used. For example $(x[-1, \text{false}] \lor x)$ is a stream expression of sort $\text{Bool}$ and $(y + y[+3, 5] * 7)$ is a stream expression of sort $\text{Nat}$.

Let $I = \{ s_1, \ldots, s_m \}$ be a set of stream variables (called input stream variables) and $O = \{ t_1, \ldots, t_n \}$ (called output stream variables), a LOLA specification is given by a collection of expressions $e_i \in \text{Expr}(I \cup O)$, one per output variable. The intention of a defining equation is that the output stream for stream $t_i$ must satisfy its defining expression $e_i$. We sometimes write $t_i = e_i(s_1, \ldots, s_m, t_1, \ldots, t_n)$ to emphasize that the defining equation can depend on all input and output streams (including $t_i$ itself).

Example 1. Let $s : \text{Bool}$ be an input stream variable and $\text{once}_s : \text{Bool}$ be an output stream variable, with defining equation $\text{once}_s = (\text{once}_s[-1, \text{false}] \lor s)$. When expressed as a LOLA program, this would be written:

```lol
input bool s
output bool once_s = once_s[-1|false] || s
```

This example corresponds to the LTL specification $\Diamond s$. A modified specification that counts how many times $s$ holds in the past is:

```lol
output int c_once_s = c_once_s[-1|0] + int(s)
```

The semantics of LOLA specifications is defined in terms of evaluation models, which describe the relation between input streams and output streams. Recall that a stream is a finite sequence of values of the domain of its sort. A valuation of a specification is a stream for each of its variables, all of the same length $N$. Given a stream $\sigma_i$ for each input stream variable $s_i$ and a stream $\tau_i$ for each output stream variable $t_i$, we can define the evaluation $[e]$ of every expression $e \in \text{Expr}(I \cup O)$ as the stream of the same length $N$:

1. $[c](j) = c$ for constants, and $[s_j] = \sigma_j$ and $[t_i] = \tau_i$ for stream variables;
2. $[f(e_1, \ldots, e_n)] = f([e_1], \ldots, [e_n])$;
3. $[v[k, d]](j) = v[j + k]$ if $0 \leq j + k < N$, and $[v[k, d]](j) = d$ otherwise.

Note that every expression is mapped into a stream of values of length $N$. We say that a valuation is an evaluation model, if $[t_i] = [e_i]$ for each output variable $t_i$, that is, if every output stream satisfies its defining equation. However, not every specification is well-defined in the sense that there is a unique output for every input. In [1231] it is shown that if the specification has no circularities in its
dependency graph, then for every input there is a single output. The dependency graph contains one node for every stream variable and an edge from \( t_i \) to \( v \) to indicate the shift \( k \) for every offset \( v[k,d] \) in the defining expression of \( t_i \). The shift indicates the value of \( t_i \) at a given time may depend on the value of \( v \) shifted \( k \) positions. Absence of zero-weight cycles in the dependency graph guarantees well-definedness of the specification.

Another important notion is that of **efficient monitorability**, which states that the specification contains only negative cycles. Efficiently monitorable specifications can be monitored in a forward manner with constant memory (independently of the length of the trace). Finally, the dependency graph can also be used to decide when a value for stream \( v \) is no longer needed (known as the back-off reference of \( v \), given by the most negative edge incoming \( v \)) and when a stream \( u \) is guaranteed to be resolved (known as the latency of \( u \), given by the largest weight of an outgoing path from \( u \)). Efficiently monitorable specifications have bounded latency for every output stream \( u \), which indicates that the value of \( u \) at \( n \) is totally determined at time \( n + \text{lat}(u) \). See [31] for longer formal definitions.

All these notions are the core of evaluation engines for LOLA, provided that data theories can be incorporated and parsed in the syntax, represented in the AST and evaluated at run-time.

### 2.2 Haskell as a host language for an eDSL

An *embedded Domain Specific Language* (eDSL) is a DSL that can be used as a language by itself, and also as a library in its host programming language. An eDSL inherits the language constructs of its host language and adds domain-specific primitives. In this paper we show how we implemented hLola as an eDSL in Haskell. In particular, we exploit *metaprogramming* from Haskell to obtain static parameterization, which provides a way to programatically create specifications from other specifications. This is used to extend hLola to support many temporal logics proposed in RV.

Other SRV implementations, in their attempt to offer expressive data theories in a standalone tool, require a long and costly implementation of features that are readily available in higher-order expressive languages like Haskell. Using an eDSL, we can effectively focus our development efforts on the temporal aspects of LOLA. We describe in the next section the novel technique of *lift deep embedding* which allows to automatically lift Haskell data-types to LOLA, with a single deep embedding for all lifted data-types. This technique fulfills the promise of a clean separation of time and data, and eases the extensibility to new data theories, while keeping the amount of code at a minimum. Additionally, using eDSLs brings benefits beyond data theories, including leveraging Haskell’s parsing, compiling, type-checking, and modularity.

The drawback is that specifications have to be compiled with a Haskell compiler, but once a specification is compiled, the resulting binary is agnostic of the fact that an eDSL was used. Therefore, any target platform supported by Haskell can be used as a target of hLola. Moreover, improvements in the Haskell com-
pilr and runtime systems will be enjoyed seamlessly, and new features ready to be used in hLOLA.

Haskell \cite{haskell} is a pure, statically typed functional programming language, that has been reported to be an excellent host language for eDSLs \cite{edsl}. Functions are values, and function application is written simply as a blank space without parentheses, which helps eDSLs look cleaner. Haskell also allows custom polymorphic datatypes—which eases the definition of new data theories—and generic programming—which is the crucial feature that we use to abstract away the types of the streams effectively allowing the expression of generic specifications. Our engine implementation also uses generic programming to incorporate new types without modification. New data-types developed by the active Haskell community can be incorporated off-the-shelf into hLOLA.

Haskell is declarative and statically typed, just like LOLA. In LOLA, functions are functions in the mathematical sense, this is, they do not have side effects. LOLA does not make assumptions about when these functions will be called, and guarantees that a function yields the same result when applied to the same arguments twice. This is aligned with the Haskell purity of functions.

Another feature that improves syntax readability is Haskell type classes, which allows overloading methods. We can redefine functions that are typically native in other languages, such as Boolean operators (\(\lor\)) and (\(\land\)), and the arithmetic operators (\(+\)), (\(-\)) and (\(*\)), as well as define and use custom infix operators. Haskell has let-bindings, comprehensions, anonymous functions, higher-order, and partial function evaluation, which eases writing specifications. Finally, hLOLA uses Haskell’s module system to allows modular specifications and to build language extensions.

3 Implementation

3.1 Language design

We model input and output stream variables using:
- **Input Stream** declarations, modeling LOLA’s input variables simply as a name. During evaluation the engine can look up a name in the environment and fetch the corresponding value at any required time instant.
- **Output Stream** declarations, which model output streams in LOLA. The value of these streams will be computed based on Stream declarations from some specific data theory. These declarations bind the name of the stream with its **Expression**, which represents the defining expression of the LOLA output stream definition.

Revisiting the LOLA specification in Ex. 1 in hLOLA, \(s\) will be an Input Stream declaration and \(once\_s\) an Output Stream declaration.

We seek to handle many data theories and incorporate new ones transparently, so we abstract away concrete types in the eDSL. For example, we want to use the data theory of *Boolean* without adding the constructors that a usual deep embedding would require. One alternative to obtain this generality is to use
modular data-types, as presented in [33]. However, this approach requires the final user to implement every data theory element as a new Haskell data-type, which is against our goal of using data-types completely off-the-shelf. Another option is to revisit the very essence of functional programming. Every expression in a functional language—as well as in mathematics—is built from two basic constructions: values and function applications. Hence, we base our eDSL in these two constructions, plus two additional stream access primitives to implement offset expressions. The resulting data-type resembles Free Applicative Functors [7], with the limitation that some Haskell data-types cannot be handled. In particular, we cannot handle data-types that are not instances of the Typeable type class, due to our use of the Dynamic package. We define expressions in Haskell as a parametric data-type Expression with a polymorphic argument domain. An e :: Expression domain represents an expression e over the domain domain. For example, to use the Integers domain, we can use Expression Int, automatically instantiated inside Haskell.

Here we present in more detail the Expression construction in Haskell, which again illustrates the clean separation between constructions for data theories and time constructions. The first two constructors (Leaf and App) are the data constructions of the language, and they let us handle elements in the data theories. These constructions are aligned with the notions of Free Applicative Functor mentioned above, and they enable us to encode data theoretic-domain expressions seamlessly. The other two constructors (Now and (s@)) represent the offset expressions:

- Leaf :: a → Expression a: contains an element of the theory;
- App :: Expression (b → a) → Expression b → Expression a: represents the application of a function Expression to a value Expression;
- Now :: Declaration a → Expression a, which represents the value of a stream in the current instant; and
- (s@) :: Declaration a → (Int, a) → Expression a (read as at), represents the value of a stream at an instant in the future or in the past, specifying the default value to use if the access falls off the trace. Notice that the expression Now s is equivalent to s s@ (0, ⊥).

We have omitted the type constraint in the constructors: every type involved in the construction of an Expression has to be an instance of the Typeable type class. These constructions allow us to lift operations from domain values to Expressions directly. For example, we can create an Expression that represents the sum of two Expression Int without defining a dedicated type of Expression. Using this approach, we avoid to define a constructor for all elements in the data theory, making every extension transparent.

Example 2. The LOLA specification from Ex. 1 is in hLOLA:

\[
\begin{align*}
\text{s :: Stream Bool} & & \text{once_s :: Stream Bool} \\
\text{s = Input "s"} & & \text{once_s = Output ("once_s", App (App (Leaf (∨)) (once_s s@ (−1, False))) (Now s))}
\end{align*}
\]
The expression of $\text{once}_s$ takes the application of the (data theory) function ($\lor$) to the value of $\text{once}_s$ at $-1$, using $\text{False}$ as the default value, and applying the result to the current value of $s$. This style of writing declarations is cumbersome and error prone. Instead, we can use function overloading and infix operators definition to make the hLOLA Output Stream declaration look almost like a LOLA expression:

$$\text{once}_s :: \text{Stream} \ \text{Bool}$$
$$\text{once}_s = "\text{once}_s" =: \text{once}_s \oplus (-1, \text{False}) \lor \text{Now} \ s$$

To express the property $\text{once}_r$ for a different Boolean stream $r$, we would require to replicate the definition above, which leads to code duplication. We show in Section 3.4 how to avoid this duplication using static parameterization.

### 3.2 Static analysis

Not every grammatically correct LOLA specification is valid. Some errors like syntactic errors, missing references and type checking can be performed by the Haskell compiler. In the case of LOLA, we also need to check that the specification is well-formed by examining the dependency graph of the specification. To do so, hLOLA performs the following pre-processing stages. First, we convert every Expression $a$ to an Expression of Dynamic so we can mix Expressions of different types in the same specification. One drawback of this approach is that the Haskell typesystem can no longer track the original type of an expression. However, we keep the information on how to parse the input streams and how to show values of output streams based on the stream names, safely casting from and into Dynamic, and avoiding type mismatches when converting from dynamically-typed objects. We make the following claim (whose proof is beyond the scope of this paper):

**Lemma 1.** Every conversion from a Dynamic object back into a Haskell value is of the correct type.

After the conversion, we obtain the dependency graph by traversing the stream definitions in a specification recursively, which is analyzed to guarantee that there are no closed walks of weight zero. During this stage, the tool also calculates the minimum back reference in the dependency graph, which we call $\text{minBackRef}$, and is an upper bound on the number of clock instants after which it is guaranteed that a value will not be accessed any longer. The tool also calculates the maximum latency in the dependency graph, called $\text{maxLatency}$, that indicates the furthest in the future we may need to look to resolve the streams at the current instant. The dependency graph of the specification in Ex. 1 is shown on the right. The $\text{minBackRef}$ is $-1$, because $\text{once}_s$ depends on the previous value of itself, and the $\text{maxLatency}$ is 0 because there are no references to future values of streams, which means that at every instant, the engine will keep the values of the streams at that instant and the instant before it.
3.3 Runtime System

Execution data structures. We describe now some internal data-types used in the implementation of the execution engine. An *Instant* is a map that binds the name of a stream to an *Expression*. Given a specification with *m* streams $s_1, \ldots, s_m$, an instant can be interpreted as a vector of size *m*. A *Sequence* is an ordered collection of *Instants*, where one of those *Instants* is said to be “in focus”. The *Instants* in the past of the one in focus are stored in the *Sequence* in an array of size $(\text{maxLatency} - \text{minBackRef})$. On the other hand, the *Instants* in the future of the one in focus are stored as a list. Even though this list can be (implicitly) as long as the full trace, the elements in the list will not actually exist until they are needed, due to the lazy evaluation of Haskell. We can think of a *Sequence* as a matrix of expressions, where each column is an *Instant* vector, and one of them is in focus. A specification with *m* streams over a trace of *n* instants is then an $n \times m$ matrix.

Initial sequence. Given a specification and a list of values, we first create a *Sequence* with an empty past and the focus on the first instant. If we inspect the *Sequence* at the beginning of the execution, we will find that, for every input stream $s_i$ and instant $n$, the value $(s_i, n)$ in the *Sequence* matrix is a *Leaf* containing the value read for the stream of $s_i$ at time instant $n$. Similarly, the value of every output stream $t_j$ and instant $n$ is the defining *Expression* for $t_j$ in the specification, waiting to be specialized and evaluated. The goal of the engine is to compute a *Leaf* expression (this is, a ground value) at every position in the matrix, particularly for output streams.

System execution. Starting from the initial state, the engine will solve every output stream at the instant in focus, and then move the focus one step forward. This algorithm guarantees that all elements in the past of the focus are leaves. The figure on the right illustrates the *Sequence* of an execution at time instant 3, where some of the output expressions $e_{1,3}, \ldots, e_{m,n}$ can be leaves too. At the end of the execution, the focus will be on the last column of the matrix, and all its elements will be of type *Leaf*.

The output will be calculated and output incrementally while new data is retrieved for the input streams. The engine will block when it needs the value of an input stream that has not been generated yet. These characteristics of the Haskell runtime system allows the monitor to run online processing events from the system under analysis on the fly, or offline over dumped traces.

Example 3. Consider the specification from Ex. 1 and suppose that the first three elements read for input stream $s$ are *False*, *True* and *False*. Then, the initial state of our system will be a *Sequence* containing these values as leaves for $s$. 
and for once\_s its defining expression once\_s \oplus (-1, False) \lor \textbf{Now } s. After the first iteration, the first instant of once\_s is resolved as follows. Since once\_s does not exist at instant \(-1\), the default value False is used to compute the disjunction with the current value of s, resulting in False. Therefore once\_s at position 1 is replaced with \textbf{Leaf} False. In the next iteration, with the focus in 2 the expression \(e\) is evaluated using the previous value of once\_s, which is still False, and calculate its disjunction with the value of s at instant 2, which is True, and therefore the evaluation becomes True. Since True is the absorbing element of the disjunction, the value of once\_s will be True from instant 3 onwards, as shown on the right.

3.4 Static parameterization

Haskell’s ability for metaprogramming can be used to programmatically create specifications, providing support for template definitions, a feature called static parameterization. Static parameterization is useful to reuse repetitive specifications, making them more readable and easier to maintain. Section 4 show how this feature is used to concisely implement several monitoring languages.

Consider again the specification of Ex. 1. Instead of defining an output stream once\_s specifically for s, we aim to write a general stream once that receives a Boolean stream as an argument.

Example 4. The definition of once in hLOLA using static parameterization is:

\[
\begin{align*}
\text{once} & : \text{Stream Bool} \to \text{Stream Bool} \\
\text{once } s & = "\text{once}" \left< s \right> \equiv \text{once } s \oplus (-1, False) \lor \textbf{Now } s
\end{align*}
\]

Note that we simply abstracted away the occurrences of s. To avoid name clashes among different instantiations of once, we concatenate the string "once" with the name of the argument stream s, by using the operator \(<\). Assume that s\(_0\) and s\(_1\) are two Stream Bool. The following specification checks whether either of s\(_0\) or s\(_1\) has been True in the past:

\[
\begin{align*}
\text{anyEverTrue} & : \text{Stream Bool} \\
\text{anyEverTrue} & = "\text{anyEverTrue}" \equiv \textbf{Now } (\text{once } s_0) \lor \textbf{Now } (\text{once } s_1)
\end{align*}
\]

3.5 Input/Output

Another feature that simplifies the engine is Haskell’s IO facilities to automatically parse and generate outputs for arbitrary types.

In the current implementation, events can be read in either JSON or CSV format. For CSV, the first line must be a header with the names of the input streams, which defines the order of the values in the following rows. The following lines must have the values as strings separated by commas. The types of the
input streams must be instances of the `Read` class, meaning that a value of the corresponding type can be constructed from a `String`. The output streams have to be instances of the `Show` class, which means that we can get a `String` from a value of the corresponding type.

If we use JSON as the Input/Output format, each line must be a string representation of a JSON object with one field per input stream. The types of the input streams have to be instances of the `FromJSON` class, meaning that a value of the corresponding type can be constructed from a serialized JSON `Object`. Output streams must be instances of the `ToJSON` class, which means that we can get a JSON `Object` from a value of the corresponding type.

Haskell allows defining custom data-types via the `data` statement. Once defined, these types can be used just like any other type in Haskell. Most of the time, we can use Haskell’s `deriving` mechanism to make our custom types instances of the corresponding classes, if needed. Section 4 contains examples of custom data-types for input values.

4 Extensible libraries in HLola

One of the benefits of an eDSL is to use the library system of the host language. The Haskell module system allows including third parties libraries, as well as developing new libraries. HLOLA ships with some predefined theories and stream-specific libraries. The following subsections show an overview of the stream-specific libraries.

4.1 Past-LTL

The operators of Past-LTL [5] can be described using the LOLA specification language, and the properties in Past-LTL are very efficiently monitorable. We have shown the definition of the Past-LTL operator $\diamond$ in Example 4. We now show the implementations of $\diamond$ (“since”) and $\otimes$ (“yesterday”), as can be found in `Lib.LTL`, with types:

\[
\text{since} :: \text{Stream Bool} \rightarrow \text{Stream Bool} \rightarrow \text{Stream Bool}
\]
\[
\text{yesterday} :: \text{Stream Bool} \rightarrow \text{Stream Bool}
\]

\[
\text{since } p \ q = \begin{array}{c}
\text{name} = \text{"since" } \triangleleft p \triangleleft q \\
\text{body} = \text{Now } q \lor \\
(\text{Now } p \land p \text{\ 'since' } q :@ (-1, \text{False}))
\end{array}
\]
\[
\text{in name } =: \text{body}
\]
\[
\text{yesterday } p = \begin{array}{c}
\text{name} = \text{"yesterday" } \triangleleft p \\
\text{body} = p :@ (-1, \text{False})
\end{array}
\]
\[
\text{in name } =: \text{body}
\]

Given two Boolean streams $p$ and $q$, the Boolean stream $p \text{ 'since' } q$ is True if $q$ has ever been True, and $p$ has been True since the last time $q$ became True.

One can simply import `Lib.LTL` and then then define streams like `property = yesterday $\diamond p \text{ 'since' } q$`. An example of a Past-LTL property for a sender/receiver model “abp4” from [5]:

\[
\square (\text{snd.state } = \text{waitForAck} \rightarrow \diamond \square \text{snd.state } \neq \text{waitForAck})
\]
Using hLOLA, we can define a type to represent the possible states of the sender, deriving a `FromJSON` instance to use it as the type of an input stream with

```haskell
data SndState = Get | Send | WaitForAck deriving (Generic, Read, FromJSON, Eq)
```

Using this data-type as the type of the input stream `senderState`, we can define the property as a Boolean output stream:

```haskell
sndState :: Stream SndState
sndNotWaiting :: Stream Bool

sndState = Input "senderState"
sndNotWaiting = "senderNotWaiting" ==: Now sndState /== Leaf WaitForAck

property :: Stream Bool
property = let
  sndWaitingAck = Now senderState === Leaf WaitForAck
  startedWaiting = yesterday $ historically $ sndNotWaiting
in "property" ==: sndWaitingAck 'implies' Now startedWaiting
```

### 4.2 Utils

Often the value of a stream `s` at a certain point is a combination of the previous value `s` and the current value of another stream. For example, this is the pattern of the definitions many Past-LTL operators like $\Diamond$ and $\Box$, where the combination operations are $\lor$ and $\land$ respectively, and the default elements are the combiner neutral element. This behaviour is captured by folds like Haskell’s `foldl`, so hLOLA incorporates a parametric stream `hFoldl` as follows:

```haskell
hFoldl :: (Streamable a, Streamable b) =>
Ident -> (b -> a -> b) -> b -> Stream a -> Stream b

hFoldl name combiner d dec = let
  acc = hFoldl name combiner neutral dec @ (-1, d)
in name <$> dec ==: combiner ($) acc (*) Now dec
```

This way we redefine the Past-LTL operators $\Box$ and $\Diamond$ as follows:

```haskell
historically :: Stream Bool -> Stream Bool
once :: Stream Bool -> Stream Bool
historically = hFoldl "historically" (\) True
once = hFoldl "once" (\) False
```

The same pattern can be used to implement quantitative semantics, like counting how many times a Boolean stream has been false:

```haskell
nViolations :: Stream Bool -> Stream Int
nViolations = hFoldl "nViolations" (\n b -> n + if b then 0 else 1) 0
```

### 4.3 MTL

Metric Temporal Logic \[22\] is an extension of LTL with time constraints, that give upper and lower bounds on the temporal intervals. In \[29\], Reinbacher et al.
introduce Mission-Time LTL, a projection of LTL for systems which are bounded to a certain mission time. They propose a translation of each LTL operator to its corresponding MTL operator, using $[0, \text{mission}_t]$ as the temporal interval, where $\text{mission}_t$ represents how long the mission took. The ability of $\text{hLola}$ to monitor MLTL can be used to monitor Mission-time LTL through this translation.

We show the implementation of $U_{[a,b]}$ and $\Diamond_{[a,b]}$ (here the definitions of the name $\text{name}$ associated with the stream have been omitted for clarity).

\[
\text{untilMTL} :: (\text{Int}, \text{Int}) \rightarrow \text{Stream Bool} \rightarrow \text{Stream Bool} \rightarrow \text{Stream Bool} \\
\text{untilMTL} (a, b) \ p \ q = \text{name} == \text{until'} a \ b \ p \ q \\
\text{until'} a \ b \ p \ q = \begin{cases} 
\text{a} \equiv \text{b} = \text{q} & \Rightarrow (\text{b}, \text{False}) \\
\text{otherwise} = q \ ? (\text{a}, \text{False}) \lor (p \ ? (\text{a}, \text{True}) \land \text{until'} (a+1) \ b \ p \ q) 
\end{cases}
\]

\[
\text{eventuallyMTL} :: (\text{Int}, \text{Int}) \rightarrow \text{Stream Bool} \rightarrow \text{Stream Bool} \\
\text{eventuallyMTL} (a, b) \ p = \text{name} == \text{foldl} (\lor) (\text{Leaf False}) [p \ ? (i, \text{False}) | i \leftarrow [a \ldots b]]
\]

The stream $\text{untilMTL}$ is parameterized by two integers, which are the boundaries of the interval, and two Boolean streams to model the formula $p U_{[a,b]} q$. We use metaprogramming to unfold the recursive definition of the temporal operator, which can be expanded at compile time. This expansion is shown in the dependency graph of a specification that uses $\text{untilMTL}$, for example $\text{property} = \text{untilMTL} (-1,1) p q$, which checks that a stream $p$ is True until $q$ is True in the interval $(-1,1)$, which is shown on the right. Appendix A contains an example on MTL formula $\Box (\text{alarm} \rightarrow (\Diamond [0,10] \text{allclear} \lor \Diamond [10,10] \text{shutdown}))$ from [6] implemented in $\text{hLola}$.

5 Implementation and Empirical evaluation

The implementation of $\text{hLola}$ requires no code for the parser and type checker, since it reuses those from Haskell. The table below shows the number of lines for the full $\text{hLola}$ implementation. In summary, the core has 597 lines, the utils 314 for a total of 911 lines. This compares to the tens of thousands of lines of a parser or run-time system of a typical stand-alone tool.

| Engine and language | Syntax | Libraries | Theories |
|---------------------|--------|-----------|---------|
| Files: ./ | LoC | Files: Syntax/LoC | Files: Lib/LoC | Files: Theories/LoC |
| DecDyn.hs | 101 | | LTL.hs | 38 |
| Engine.hs | 204 | Booleans3.hs | 18 | MITL.hs | 35 |
| Focus.hs | 65 | HLPrelude.js | 3 | Pinescript.js | 49 |
| InFromFile.hs | 47 | Num.js | 26 | Utils.js | 15 |
| Lola.js | 85 | Ord.js | 18 | | |
| Total | 502 | Total | 102 | Total | 137 |

Well-formedness check

| Files: ./ | LoC |
|-----------|-----|
| StaticAnalysis.js | 95 |

| Total | 95 |
5.1 Haskell tools

The use of Haskell as a host language allows us to use existing tools to improve HLOLA specifications, such as LiquidHaskell and QuickCheck. Liquid Haskell enriches the type system with refinement types that allow to define a specification with more precision. For example, we can prevent a specification that adds the last $n$ elements to be used with negative numbers:

$$\begin{align*}
    \{- \text{sumLast :: Stream Int} \to \text{Nat} \to \text{Stream Int} \- \}
    
nsum :: \text{Stream Int} \to \text{Int} \to \text{Stream Int} \\
    nsum s n = "n\_sum" \lhd s \lhd n \equiv: nsum s n \oplus (\neg 1, 0) + \text{Now } s - s \oplus (\neg n, 0)
\end{align*}$$

Then, we can use it to define a stream $s$ that computes the sum of the last 5 values on stream $r$ as $s = nsum r 5$ were $r$ is a Stream Int. Running LiquidHaskell with --no-termination allows the recursive definition of $n$ over this specification, which yields no error, but running LiquidHaskell on $s' = nsum r (\neg 1)$ produces a typing error.

QuickCheck is a tool to perform random testing Haskell programs, which can now be used over specifications. For example, we can assess that the first index at which a Boolean stream $p$ is False is exactly one instant less than the last index at which $p$ is True, increasing our confidence on the implementation of the Past-LTL $\Box$ operator. See Appendix C for more details.

5.2 Empirical evaluation

We have preformed an empirical evaluation to empirically assess whether that the engine behaves as expected in terms of memory usage. The hardware platform is a MacBook Pro with macOS Mojave Version 10.14.6, with an Intel Core i5 at 2.5 GHz and 8 GB of RAM.

The first two Stream declarations calculate if a Boolean stream $q$ is True in the last $n$ instants, and if $p$ is True from then up to the present. This specifications is an interval version of $S$ of Past-LTL ($pS_{[-n,0]}q$). We specify this property
in two different ways. In the first Stream declaration, we programatically create \( n \) streams with the following meaning: since \( n \) \( p \) \( q \) indicates that either \( q \) is True now, or \( p \) is True now and since \( (n - 1) \) \( p \) \( q \) was True at the previous position (and since \( 0 \) \( p \) \( q \) = Now \( q \) as the base case). In the second Stream declaration, we define a single stream which unfolds \( n \) times the definition of the \( p \mathcal{S}_{[-n,0]} q \) operator, creating a single streams that accesses the input streams \( p \) and \( q \) with an offset of up to \( n \). This is done using a fold of \( \land \) over the offset \( p @ (i, True) \). See Appendix D for details.

We also run a quantitative version of \( \mathcal{S} \), which counts the number of times that \( q \) was True since \( p \) was True, which due to the flexibility of hLOLA only requires switching False and True by 0 and 1, and \( \land \) and \( \lor \) by \( \ast \) and \( + \) respectively. In the first experiment, we run a specification that calculates the qualitative and quantitative semantics of \( \mathcal{S} \) using both Stream declarations over traces of varying length, as shown on the diagram below. Both curves are roughly constant, indicating that the memory is independent on the trace length, and the monitor runs in constant space, as theoretically predicted. In the second experiment, we use the width of the interval to assess how the number of streams affect the memory usage, showing has a linear impact only, as shown in Fig. 1(a). In the third experiment, we show how the width of the interval to increase minBackRef using the second technique, making the memory consumption again grow roughly linearly, as shown in Fig. 1(b). We can also see that the memory consumption is unaffected by the fact that we are working with quantitative data-types or restricting ourselves to Boolean values.

6 Conclusion and future work

We have presented hLOLA, an engine for SRV implemented as a Haskell eDSL. We have used a lift deep embedding to fulfill the promise of using data-types off-the-self, in this case from the host language. The use of Haskell enables metaprogramming, obtaining features like static parametrization for free, which allows us to easily implement many languages common in runtime verification. The resulting system is very concise, yet efficient. While the current version only supports efficiently monitorable specifications, we are working on extending hLOLA to arbitrary specifications both for online and offline monitoring. We are also extending hLOLA to support real-time event streams.

From the point of view of Haskell, future work includes to use LiquidHaskell more aggressively to prove properties of specifications and memory bounds. We will also use QuickCheck to generate test traces from specifications and study how to use model-based testing to improve the test suites obtained.
References

1. E. Asarin, P. Caspi, and O. Maler. Timed regular expressions. J. ACM, 49(2):172–206, 2002.
2. H. Barringer, A. Goldberg, K. Havelund, and K. Sen. Rule-based runtime verification. In Proc. of VMCAI’04, LNCS 2937, pages 44–57. Springer, 2004.
3. E. Bartocci and Y. Falcone, editors. Lectures on Runtime Verification - Introductory and Advanced Topics, volume 10457 of LNCS. Springer, 2018.
4. A. Bauer, M. Leucker, and C. Schallhart. Runtime verification for LTL and TLTL. ACM T. Softw. Eng. Meth., 20(4):14, 2011.
5. M. Benedetti and A. Cimatti. Bounded model checking for past LTL. In Proc. of TACAS’03, volume 2619 of LNCS, pages 18–33. Springer, 2003.
6. F. Boniol, P.-E. Hladik, C. Pagetti, F. Aspro, and V. Jégou. A framework for distributing real-time functions. In Proc. of FORMATS’08, volume 5215 of LNCS, pages 155–169. Springer, 2008.
7. P. Capriotti and A. Kaposi. Free applicative functors. volume 153 of EPTCS, pages 2–30, 2014.
8. K. Claessen and J. Hughes. QuickCheck: A lightweight tool for random testing of Haskell programs. In Proc. of (ICFP’00), pages 268–279. ACM, 2000.
9. E. M. Clarke and E. A. Emerson. Design and synthesis of synchronization skeletons using branching time temporal logic. In Proc. of Workshop on Logic of Programs, pages 52–71, 1981.
10. E. M. Clarke, O. Grunberg, and D. A. Peled. Model checking. MIT Press, 1999.
11. L. Convent, S. Hungerecker, M. Leucker, T. Scheffel, M. Schmitz, and D. Thoma. TeSSLa: Temporal stream-based specification language. In Proc. of SBMF’18, volume 11254 of LNCS. Springer, 2018.
12. B. D’Angelo, S. Sankaranarayanan, C. Sánchez, W. Robinson, B. Finkbeiner, H. B. Sipma, S. Mehrotra, and Z. Manna. LOLA: runtime monitoring of synchronous systems. In Proc. of TIME’05, pages 166–174. IEEE, 2005.
13. C. Eisner, D. Fisman, J. Havlicek, Y. Lustig, A. McIsaac, and D. V. Campenhout. Reasoning with temporal logic on truncated paths. In Proc. of CAV’03, volume 2725 of LNCS 2725, pages 27–39. Springer, 2003.
14. P. Faymonville, B. Finkbeiner, S. Schirmer, and H. Torfah. A stream-based specification language for network monitoring. In Proc. of RV’16, volume 10012 of LNCS, pages 152–168. Springer, 2016.
15. P. Faymonville, B. Finkbeiner, M. Schledzewski, M. Schwenger, M. Stenger, L. Tentrup, and T. Hazem. StreamLAB: Stream-based monitoring of cyber-physical systems. In Proc. of CAV’19, volume 11561 of LNCS, pages 421–431. Springer, 2019.
16. P. Faymonville, B. Finkbeiner, M. Schwenger, and H. Torfah. Real-time stream-based monitoring. CoRR, abs/1711.03829, 2017.
17. A. Gill. Domain-specific languages and code synthesis using Haskell. CACM, 57:42–49, 06 2014.
18. F. Gorostiaga and C. Sánchez. Striver: Stream runtime verification for real-time event-streams. In Proc. of RV’18, volume 11237 of LNCS, pages 282–298. Springer, 2018.
19. K. Havelund and A. Goldberg. Verify your runs. In Proc. of VSTTE’05, LNCS 4171, pages 374–383. Springer, 2005.
20. K. Havelund and G. Roqu. Synthesizing monitors for safety properties. In Proc. of TACAS’02, LNCS 2280, pages 342–356. Springer, 2002.
21. P. Hudak. Building domain-specific embedded languages. *ACM Comput. Surv.*, 28(4es), Dec. 1996.
22. R. Koymans. Specifying real-time properties with metric temporal logic. *Real-time Systems*, 2(4):255–299, 1990.
23. M. Leucker, C. Sánchez, T. Scheffel, M. Schmitz, and A. Schramm. TeSSLa: Runtime verification of non-synchronized real-time streams. In *Proc. SAC’18*, pages 1925–1933. ACM, 2018.
24. M. Leucker and C. Schallhart. A brief account of runtime verification. *J. Logic Algebr. Progr.*, 78(5):293–303, 2009.
25. Z. Manna and A. Pnueli. *Temporal Verification of Reactive Systems: Safety*. Springer, New York, 1995.
26. S. Marlow. Haskell language report, 2010.
27. L. Pike, A. Goodloe, R. Morisset, and S. Niller. Copilot: A hard real-time runtime monitor. In *Proc. of RV’10*, LNCS 6418. Springer, 2010.
28. J.-P. Queille and J. Sifakis. Specification and verification of concurrent systems in cesar. In *Proc. International Symposium on Programming*, volume 137 of *LNCS*, pages 337–351. Springer, 1982.
29. T. Reinbacher, K. Rozier, and J. Schumann. Temporal-logic based runtime observer pairs for system health management of real-time systems. 04 2014.
30. G. Roșu and K. Havelund. Rewriting-based techniques for runtime verification. *Automated Software Engineering*, 12(2):151–197, 2005.
31. C. Sánchez. Online and offline stream runtime verification of synchronous systems. In *Proc. of RV’18*, volume 11237 of *LNCS*, pages 138–163. Springer, 2018.
32. K. Sen and G. Roșu. Generating optimal monitors for extended regular expressions. *ENTCS*, 89(2):226–245, 2003.
33. W. Swierstra. Data types à la carte. *J. Funct. Program.*, 18(4):423–436, 2008.
34. N. Vazou, E. L. Seidel, and R. Jhala. Liquidhaskell: experience with refinement types in the real world. In *Proc. of Haskell’14*, pages 39–51. ACM, 2014.
A MTL Example

We show now an example of MTL in practice.

Example 5. Consider the following specification from [6] that uses MTL to establish deadlines between environment event the corresponding system responses $$\Box (\text{alarm} \rightarrow (\Diamond_{[0,10]} \text{allclear} \lor \Diamond_{[10,10]} \text{shutdown}))$$, that is, every alarm is followed by a shutdown event in 10 time units unless all clear is sounded first. We begin by defining a datatype for the input events and then define a Boolean output stream that captures whether the property holds:

```haskell
data Event = Alarm | AllClear | Shutdown deriving (Generic, Read, FromJSON, Eq)

event :: Stream Event

event = Input "event"

property :: Stream Bool

property = let
    nowIs x = Now event === Leaf x
    allClear = "allClear" =: nowIs AllClear
    willClear = eventuallyMTL (0, 10) allClear
    shutdown = "shutdown" =: nowIs Shutdown
    willShutdown = eventuallyMTL (10, 10) shutdown
    body = nowIs Alarm 'implies' (Now willClear \lor Now willShutdown)

    in "property" =: body
```

B Pinescript

TradingView is an online charting platform for stock exchange, which offers a series-oriented language to create customized studies and signals and run them in the company servers. This language implements some of the most common indicators in the stock exchange field as built in functions so they can be used in the users’ scripts. Some of the most widely used kind of indicators are the Moving Averages, which include the Standard Moving Average (SMA), the Weighted Moving Average (WMA) and the Exponential Moving Average (EMA), whose definitions can be found at [https://www.tradingview.com/wiki/Moving_Average](https://www.tradingview.com/wiki/Moving_Average).

We show how these indicators can be implemented in hLOLA. We first define two auxiliary streams that will help us in the definition of the indicators: `sumLast`, which calculates the sum of the last \( n \) elements in a stream \( s \); and `instantN`, which returns the current instant:

```haskell
sumLast :: (Num a, Streamable a) \rightarrow Stream a \rightarrow Int \rightarrow Stream a
sumLast strm n = let
    name = "sumLast" \&<\ strm \&<\ n
    in name =: sumLast strm n :@ (-1, 0) + Now strm - strm :@ (-n, 0)

instantN :: Stream Int
instantN = "N" =: (+1) (\$) instantN :@ (-1, 0)
```
We are now ready to show the implementation of the Simple Moving Average (SMA) and the Exponential Moving Average (EMA) of a stream:

```haskell
sma :: (Fractional a, Streamable a) ⇒ Stream a → Int → Stream a
sma strm n = let
    numerator = Now $ sumLast strm n
    slots = fromIntegral $ min $ Now instantN $ leaf n
  in "sma" $ strm $ n $ numerator / slots

ema :: (Fractional a, Streamable a) ⇒ Stream a → Int → Stream a
ema strm n = let
    emaprev = ema strm n :@ (-1, 0)
    slots = fromIntegral $ min $ Now instantN $ leaf n
    multiplier = 2 / slots
  in "ema" $ strm $ n $ (Now strm - emaprev) * multiplier + emaprev
```

Based on these signals, we can define the well-known indicator Moving Average Convergence Divergence (MACD) as is defined at https://www.tradingview.com/wiki/MACD_(Moving_Average_Convergence/Divergence) in the following way:

```haskell
macd :: (Fractional a, Streamable a) ⇒ Stream a → Stream a
macd strm = "macd" $ strm $ Now (ema strm 12) - Now (ema strm 26)

macd_signal :: (Fractional a, Streamable a) ⇒ Stream a → Stream a
macd_signal strm = "macd_signal" $ strm $ Now (ema (macd strm) 9)
```

We can also detect when a signal crosses over another one:

```haskell
crossover :: (Num a, Ord a, Streamable a) ⇒ Stream a → Stream a → Stream Bool
crossover d0 d1 = let
    name = "crossover" $ d0 $ d1
  in name $: Now d1 $: Now d0 $:@ (-1, 0) $: d1 $:@ (-1, 0)
```

The users of TradingView can share their scripts and stock buy/sell strategies with the community, either for free or by selling them. We can use hLola and the Pinescript library to define the trading strategy presented in https://www.tradingview.com/script/DushajXt-MACD-Strategy/:

```haskell
buy_size = 10
sell_size = 10

close = Input "close" :: Stream Double
high = Input "high" :: Stream Double
ema18 = ema close 18

signalLine = macd_signal close
macLine = macd close
macd_sell_sig = crossover signalLine macdLine
macd_buy_sig = crossover macdLine signalLine

buy = "buy" $: Now ema18 < Now high $: Now macd_buy_sig
sell = let
    readytosell = ¬ (macd_buy_sig $:@ (-1, False)) $: positionsize $:@ (-1, 0) > 0
```
in "since" =: readytosell ∧ Now macd_sell_sig

positionsize :: Stream Double

positionsize = "positionsize" =: positionsize :@ (-1,0) + 
(if Now buy then buy_size else 0) –
(if Now sell then sell_size else 0)

This strategy suggests that whenever the stream buy becomes true, buy_size units of stock must be bought, and when the stream sell becomes true, sell_size units of stock must be sold.

C  Haskell tools

The use of Haskell as a host language allows us to use existing tools to improve HLola specifications.

Liquid Haskell  

Liquid Haskell [34] enriches the type system with refinement types that allow to define a specification with more precision. For example, we can prevent a specification that adds the last \( n \) elements to be used with negative numbers:

\[
\{ \text{sumLast :: Stream Int} \rightarrow \text{Nat} \rightarrow \text{Stream Int} \}\]  

\[\text{nsum :: Stream Int} \rightarrow \text{Int} \rightarrow \text{Stream Int}\]  

\[\text{nsum s n} = \text{"s_sum"} \odot s \odot n =: \text{nsum s n} :@ (-1,0) + \text{Now s} – s :@ (-n,0)\]

Then, we can use it to define a stream \( s \) that computes the sum of the last 5 values on stream \( r \) as \( s = \text{nsum} r 5 \) were \( r \) is an Stream Int:

\[
r :: \text{Stream Int} \quad s :: \text{Stream Int}
\]

\[
r = \text{Input } "r" \quad s = \text{sumLast} r 5
\]

Running LiquidHaskell with \(--no-termination\) allows the recursive definition of \( n \) over this specification, which yields no error. But running LiquidHaskell on \( s' = \text{nsum} r (-1) \) produces a typing error. On the contrary, if we try to define a stream \( s' \) with a negative argument:

\[
s' :: \text{Stream Int} \quad s' = \text{sumLast} r (-1)
\]

LiquidHaskell returns the following error:

\[
| s' = \text{sumLast} r (-1)
\]

Inferred type

\[\text{VV : \{}v : \text{GHC.Types.Int} \mid v == (-1)\}\]

not a subtype of Required type

\[\text{VV : \{}\text{VV : \text{GHC.Types.Int} \mid VV >= 0}\}\]
QuickCheck

QuickCheck \[\textit{QuickCheck}\] is a tool to test Haskell programs, which can now be used over specifications. For example, we can assess that the first index at which a Boolean stream \(p\) is \textit{False} is exactly one instant less than the last index at which \(\square p\) is \textit{True}:

\[
\text{main} = \text{quickCheck \textit{historically\_is\_correct}}
\]

\[
\text{historically\_is\_correct :: [Bool] \rightarrow Bool}
\]

\[
\text{historically\_is\_correct \ bs} = \text{let}
\]

\[
\text{thetrace} = \text{map (\text{unpack} \circ \text{encode} \circ \text{singleton} \ "p") bs}
\]

\[
\text{thejsons} = \text{lines \$ \text{runSpec\_JSON \ False \ myspec \$ unlines \ thetrace}
\]

\[
\text{themaps} = \text{map (\text{fromJust} \circ \text{decode} \circ \text{pack}) thejsons}
\]

\[
\text{mMinNotP} = \text{findIndex \ (\lambda m \rightarrow m ! "p") themaps}
\]

\[
\text{mMaxHistP} = \text{findLastIndex \ id \$ \text{map (\lambda m \rightarrow \"historically<\p\"\) themaps
\]

\[
\text{minNotP} = \text{fromMaybe (length themaps) mMinNotP}
\]

\[
\text{maxHistP} = \text{fromMaybe (1) mMaxHistP}
\]

\[
in \text{minNotP} - 1 \equiv \maxHistP
\]

\[
\text{where}
\]

\[
\text{findLastIndex \ f \ l} = \text{findIndex \ f \ (\text{reverse} \ l)} \gg \lambda a \rightarrow \text{Just (length l} - a - 1)
\]

\[
\text{myspec :: Specification}
\]

\[
\text{myspec = [\text{out \$ historically \ p, out \ p]}
\]

\[
\text{p :: Stream \ Bool}
\]

\[
\text{p = Input \ "p"}
\]

The fact that arbitrary generated tests pass increases our confidence on the correctness of the implementation of the Past-LTL \(\square\) operator.

\[\]

D Empirical Evaluation

We report on a preliminary empirical evaluation of the maximum memory consumption of the execution of \textit{hLola}. to show empirically that the engine behaves as expected. The hardware platform is MacBook Pro with macOS Mojave Version 10.14.6, with an Intel Core i5 at 2.5 GHz and 8 GB of RAM.

The first two \textit{Stream} declarations calculate if a \textit{Boolean} stream \(q\) is \textit{True} in the last \(n\) instants, and if \(p\) is \textit{True} from then up to the present. This specifications is an interval version of \(S\) of Past-LTL \(p S_{[-n,0]} q\). We specify this property in two different ways.

In the first \textit{Stream} declaration, we programatically create \(n\) streams with the following meaning: since \(n \ p \ q\) indicates that either \(q\) is \textit{True} now, or \(p\) is \textit{True} now and since \((n - 1) \ p \ q\) was \textit{True} at the previous:

\[
\text{since :: Int \rightarrow Stream \ Bool \rightarrow Stream \ Bool \rightarrow Stream \ Bool}
\]

\[
\text{since \ n \ p \ q} = \text{name} =: \text{Now} \ q
\]

\[
\text{otherwise} = \text{name} =: \text{Now} \ q \lor \text{(Now} \ p \land \text{since \((n-1) \ p \ q \sqsupset \ (-1, False))}
\]

\[
\text{where name} = \text{"since"} \lhd 0 \sqsubset p \sqsubset q
\]
In the second Stream declaration, we define a single stream which unfolds \( n \) times the definition of the \( p S_{-n,0} q \) operator, creating a single streams that accesses the input streams \( p \) and \( q \) with an offset of up to \( n \):

\[
\text{since} :: \text{Int} \to \text{Stream Bool} \to \text{Stream Bool} \to \text{Stream Bool}
\]

\[
\text{since } n \ p \ q = "\text{since}" \triangleleft n \triangleleft p \triangleleft q =: \text{sinceexpr } n \ p \ q
\]

where

\[
\text{sinceexpr } 0 \ q = \text{Now} \ q
\]

\[
\text{sinceexpr } n \ p \ q =
\text{sinceexpr } (n-1) \ p \ q \triangledown
(q \triangleleft (-n, \text{False}) \land
(foldl \land \text{Leaf True}) (\text{map } \lambda i \to p \triangleleft (i, \text{True}))
[(1-n) \ldots 0]))
\]

We also run a quantitative version of \( S \), which counts the number of times that \( q \) was \( True \) since \( p \) was \( True \), which due to the flexibility of hLOLA only
requires switching \textit{False} and \textit{True} by 0 and 1, and \(\land\) and \(\lor\) by \(\ast\) and \(+\) respectively.

In the first experiment, we run a specification that calculates the qualitative and quantitative semantics of \(S\) using both \textit{Stream} declarations over traces of varying length (plot (a) of Fig. 2). Both curves are roughly constant, suggesting that the memory required is independent on the length of the trace in all cases, as theoretically predicted.

In the second experiment, we show how the width of the interval affects the number of streams needed to monitor the qualitative and quantitative semantics of the operator using the first technique, which has an impact on the memory consumption, which increases roughly linearly (plot (b) of Fig. 2).

In the third experiment, we show how the width of the interval affects the \textit{minBackRef} needed to monitor the qualitative and quantitative semantics of the operator using the second technique, making the memory consumption grow roughly linearly (plot (c) of Fig. 2). All these results confirm the theoretical predictions, in spite of the fact of using a garbage collected programming language.

We can also see that the memory consumption is unaffected by the fact that we are working with quantitative data-types or restricting ourselves to \textit{Boolean} values.