Mathematical Model of Gunfire Location

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Abstract. This paper makes use of the feature that the supersonic projectile will generate shock signals when flying, a system composed of 5 acoustic sensor arrays is constructed to detect the orientation and distance of the shock wave, and to determine the location of the terrorists hiding. A new method to determine the location of terrorists is established in the model. This model does not require projectile coefficient calibration, which solves a big problem in the development of the system.

1. Introduction

When the supersonic projectile flies, shock signal will be generated, sensor array will be constructed to detect the location of the sound source, and then the shooting location of the terrorist can be obtained. It is an effective method. The traditional method of calculating gunshot source point is obtained by dB calculation,

\[ d_B = C \left[ T_F - \frac{C_2}{CM_P} \right]^4 \left( \frac{M_P^2 - 1}{M_P^3} \right) \]

In the formula,

- \( d_B \): Distance from sound source transmitting point to detection point;
- \( C \): the speed of sound
- \( M_P \): The Mach number of the projectile;
- \( C_1, C_2 \): Calibration coefficient of projectile
- \( T_F \): Time for the projectile head and tail wave to sweep the detection sensor

In calculation, \( C_1 \) and \( C_2 \) should be obtained through range calibration test, which is time-consuming, laborious and expensive. A new algorithm is developed in this paper, which can obtain the sound source position according to \( d_B \) calculation without passing the test.
2. Basic Principle [1] [2]

Figure 1. Schematic diagrams of conical waves

Figure 2. N wave schematic diagrams

While bullets at supersonic speeds through the atmospheric flight, the air disturbance and formation density variation of pressure wave, commonly known as the warhead wave. Each point on the ballistic warheads wave before and after the wave of the path is a vertex in the bullet in the head and tail cone. Wave front and wave after all present a cone (as shown in figure 1), the wave front half Angle of cone is associated with the flight Mach number of bullets.

When every point on the trajectory has a pressure wave whose direction of motion is at right angles to the wave front and propagating outward at the speed of sound, the pressure wave is similar to the letter "N" (as shown in Figure 2), so it is also called N wave. On the other hand, the projectile will produce a shock wave, which will propagate outward in the form of sound velocity C and spherical wave.

If the time difference between the arrival of bullet wave and the arrival of the shock wave from the muzzle of the projectile can be detected on the detector (sensor) array, then the hiding position of the sniper can be determined according to the mathematical model.

Figure 3. Schematic diagram of sniper position determination

Figure 3 depicts the relationship between the misalignment distance and other parameters obtained by the anti-terrorist vehicle during sniper fire. When the projectile is fired at car 2, the gun is aimed at the future point of car S, before the car enters this diagram. When the projectile is at P_B point, it is still outside the figure, and the n-wave front 4 is far away from propagation in the direction parallel to line d_b. When the projectile reaches the position 3', the n-wave front 4 'reaches the detector at the front of the vehicle 5', at which time the vehicle is at 2 '. The N-wave generated at the P_B point will meet the detector at the position 5 '. P_B point is called the bang point, which is an important reference point for
calculating the hiding places of terrorists.

Figure 4. Schematic diagram of sensor array installed in anti-terrorist vehicle

Sensor array installed vertically above the terror car (see figure 4), the sensor array is composed of five sensors (see figure 5)

3. Mathematical Models

3.1. Establishment of sensor detection array
From the perspective of spatial positioning, three measurement points can determine the spatial position. For convenience of modeling and practical, this subject adopts 5 sensor probe array, 4 sensors located in regular tetrahedron apex Angle, and 1 front position, located in the center of the regular tetrahedron as auxiliary positioning.

3.2. Establishment of sensor array coordinate system[3][4]

Figure 6. Sensor array coordinate system
Sensor array coordinate system as shown in figure 6, 0 to coordinate system origin, the origin in the sensor \( M_1, M_2, M_3, M_4 \) form an equilateral quadrilateral center, and in front of the equilateral quadrilateral \( X \) axis to increase a \( M_5 \) sensors, used to increase a set of information to accurately distinguish ballistic direction, wasn't illustrates the \( M_5 \) sensor here. Pointing to the direction of vehicle running, is the X-axis forward direction, \( M_2, M_3 \) is parallel to the Y-axis, the Y-axis forward direction is the left side of the X-axis, and \( 0M_4 \) upward is the positive direction of the z-axis.

3.3. Model parameters[5]

Five sensors are used to form a test array. The following physical quantities and computations should be referred in the mathematical model establishment.

![Figure 7. Calculation model of sniper position determination](image)

Note: It is assumed in Figure 7 that the \( M_2 \) sensor first receives the projectile shock signal, and so on.

1. The propagation distance \( d_i \) \((i = 1, 2, 3, 4)\) from the shock wave generated by \( P_B \) at the projectile's bang point to the sensor \( M_i \).

2. Projectile N wave front propagation inverse direction vector \( n_i \).

3. The time difference \( t_i \) \((i = 1, 2, 3, 4)\) between the pressure wave cone and each sensor in the sensor array.

4. The velocity \( C \) of the sound wave.

5. The projectile velocity is \( V_P \) near the sensor. The vehicle's speed is \( V_M \).

6. The Mach Angle of the projectile is \( \mu \) \((\mu = \arcsin C/V_P)\).

7. The pressure wave is equivalent to the sensor's propagation velocity \( V_K \).

8. Vehicle body jolt causes rotation, sensor array along the movement direction (X axis direction) around the rolling Angle for \( \varphi \), along the Y axis rotation Angle for \( \gamma \), along the Z axis rotation Angle for \( \omega \).

9. After the projectile comes out of the muzzle, the time difference between the time when the N
wave first meets a sensor (for example $M_2$) and the time when the detonation wave meets $M_2$ is $\Delta t_i$.

(10) The atmospheric pressure value of the vehicle position is $P_0$.

(11) The sniper's azimuth is $\alpha$. The altitude angle of the sniper is $\beta$.

(12) Distance between muzzle and sensor array is $|SM_2|$ (the distance between the muzzle and the car).

3.4. Calculation Model

This is depicted in Figure 8 that $\vec{n}_K$ can be solved by the time difference between the projectile N wave arrive before $M_1$, $M_2$, $M_3$ and $M_4$ each sensor. The projection length of vector $\overrightarrow{M_iM_2}$ on $\vec{n}_K$ of N wave propagation direction should be: $\overrightarrow{M_iM_2} \cdot \vec{n}_K$ (i=1, 3, 4)

The propagation velocity of N wave along $\vec{n}_K$ is C, cost the time $t_1$, $t_3$, $t_4$, so $\overrightarrow{M_iM_2} \cdot \vec{n}_K = c t_i$ (i=1,3,4)

According to these three equations, a system of linear equations with three variables can be formed.

It can solve to be $\vec{n}_K \rightarrow (n_{Kx}, n_{Ky}, n_{Kz})$.

When the first sensor is $M_1$, $\vec{n}_K = \begin{bmatrix} \sqrt{2} & \sqrt{2} & 0 \\ -\sqrt{6} & \sqrt{6} & 0 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} t_2 \\ t_3 \\ t_4 \end{bmatrix} = \begin{bmatrix} \sqrt{2}t_2 + \sqrt{2}t_3 \\ \sqrt{6}t_3 - \sqrt{6}t_2 \\ t_2 + t_3 - 3t_4 \end{bmatrix}; \quad (1)$

When the first sensor is $M_2$ [1], $\vec{n}_K = \begin{bmatrix} -2\sqrt{2} & \sqrt{2} & 0 \\ 0 & \sqrt{6} & 0 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} t_1 \\ t_3 \\ t_4 \end{bmatrix} = \begin{bmatrix} \sqrt{2}t_3 - 2\sqrt{2}t_1 \\ \sqrt{6}t_3 \\ t_1 + t_3 - 3t_4 \end{bmatrix}; \quad (2)$
When the first sensor is $M_3$, $n_k = \begin{bmatrix} -2\sqrt{2} & \sqrt{2} & 0 \\ 0 & -\sqrt{6} & 0 \\ 1 & 1 & -3 \end{bmatrix}$, $t = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix}$; (3)

When the first sensor is $M_4$, $n_k = \begin{bmatrix} -2\sqrt{2} & \sqrt{2} & \sqrt{2} \\ 0 & -\sqrt{6} & \sqrt{6} \\ 1 & 1 & 1 \end{bmatrix}$, $t = \begin{bmatrix} \sqrt{2}t_2 + \sqrt{2}t_3 - 2\sqrt{2}t_1 \\ 6t_3 - \sqrt{6}t_2 \\ t_1 + t_2 + t_3 \end{bmatrix}$ (4).

When calculate out $n_k$, the shooting point $S(S_x, S_y, S_z)$ can be directly calculated coordinate, according to the coordinates of Point S, the following equation is satisfied:

$$\frac{\overrightarrow{SM_i}}{C} \cdot \frac{\overrightarrow{n_k}}{C} = \Delta t_i \quad (i=1,3,4)$$

The first term on the left of the previous equation represents the time taken for the muzzle blast wave to travel at the speed of sound from point S to (i =1, 3, 4). The second term represents the time for $n_k$ to reach $M_i$ along the propagation direction of N wave generated by the muzzle, the difference between the two terms is just $\Delta t_i \quad (i=1,3,4)$. By solving the three dimensional quadratic equations, the coordinates of Point S can be obtained. There are exactly two solutions to the equations, but the correct solution can be judged by the vehicle motion direction and the first encounter sensor.

So once we figure out the coordinates of point S, we can figure out

$$\alpha = \arcsin\left(\frac{S_y}{\sqrt{x^2 + y^2}}\right)$$

Azimuth Angle:

$$\beta = \arctg\left(\frac{S_z}{\sqrt{x^2 + y^2}}\right)$$

Pitching Angle:

3.5 Projectile velocity estimation

$$V_p = \frac{\overrightarrow{SM_2} \cdot C}{\overrightarrow{SM_2} \cdot \Delta t \cdot C}$$

According to the projectile velocity and shooting distance, as well as the computer pre-loaded with the current use of several firearms shooting table, can find out snipers use firearms caliber, to provide reference for counterterrorism personnel.

3.6 Distance between muzzle and sensor array

$$\left|\overrightarrow{SM_2}\right| \left|\overrightarrow{SM_2}\right| \approx OS = \sqrt{S_x^2 + S_y^2 + S_z^2}$$

3.7 Error Correction

(1) Error correction caused by temperature influence of sound velocity $C$

$$C = \sqrt{402.94t_h}$$
\[ t_h = t_c - \frac{6.328h}{1000} \]
\[ t_x = \frac{273 + T}{1 - \frac{3\cdot a \cdot H_u}{80000P_o}} \]

(11) \hspace{1cm} (12)

Where, \( P_o \) is the gas flame, \( \alpha \) is the saturation air pressure, \( H_u \) is the height of the car, \( T \) is the temperature, \( t_h \) is the height of the car, and \( t_x \) is the converted virtual temperature.

At close range for shooting, presumably the vehicle with a sniper in the same height, the error due to this is very small, so \( t_h = t_x \).

(2) Error caused by vehicle movement

The movement of the vehicle body will cause turbulence, resulting in changes of \( \varphi \), \( \gamma \) and \( \omega \) angles, which will cause the sensor array to swing and lead to the error of receiving signals, which can be eliminated by installing the indicator image stabilization platform, the accuracy can be satisfied if the angle is stable in the range of \( 1^\circ \).

4. conclusions

The application shows that the model is correct and practical. It solves the shortcoming of using other models that the projectile coefficient must be acquired by firing live ammunition in the range to calculate the hiding position of terrorists. The model can be further extended to other missile sight deviation testing systems, and has a good application prospect.

References

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