Ammunition Consumption Prediction Method Based on Data Fusion

Xianming Shi, Rudong Zhao, Guangsheng Jiang, Kang Li and Yuan Li

ABSTRACT

Aiming at the limitation of traditional ammunition consumption in the experimental sample, and it is difficult to comprehensively consider the influence of many factors, a new ammunition consumption prediction method based on Bayes system fusion is proposed. This method analyzes and obtains the key influencing factors when extracting the factors affecting the consumption of new ammunition, and generates the system contribution model according to the weight. After the system samples of the ammunition consumption test samples under the influence of single factors are merged with the field test data, it is expected that the new ammunition consumption under the influence of complex factors will be obtained. Based on the data of the penetration effect of a new type of ammunition, two key factors of the incident angle and impact velocity of the projectile are obtained by multi-factor analysis method. Using this method to statistically infer the target to achieve the ammunition consumption under severe damage, verify the scientific and feasible method.

KEYWORDS
Bayesian fusion, System contribution degree, Degree of damage, New ammunition, Consumption prediction.

INTRODUCTION

Consumption forecast is the basic work of ammunition support decisions. Accurately predicting ammunition consumption is the key to the efficient support of ammunition. Estimated data on traditional ammunition consumption are often derived from performance tests under a single specific condition. In the process of estimating ammunition consumption, it is difficult to consider the influence of multiple factors, and the new ammunition is expensive, so it is hard to verify it through a large number of experiments. How to comprehensively consider the many factors affecting the ammunition consumption, and to achieve the most accurate estimation of ammunition consumption in the case of small sample size, is a hot issue in current research.

At present, many domestic experts predict the ammunition consumption from the macroscopic damage effect. Shi Quan[1] established an ammunition effectiveness model and a target damage simulation model based on target damage simulation. He also developed an ammunition consumption prediction
model and system simulation platform based on damage to enemy firepower. Zhi Yonglei\textsuperscript{(2)} proposed the calculation method of the average ammunition consumption required for the damage target of ship-to-air missiles. This method can calculate the average ammunition consumption required for a ship-to-air missile to damage a single target, an evacuated target, or a dense target.

The principle of statistics is widely used in the estimation of ammunition consumption. Hu Jiang\textsuperscript{(3)} studied the shooting characteristics of the ship-launched rockets and proposed the calculation model of the average ammunition consumption of the ship-launched rockets. It uses statistical analysis to establish a damage probability evaluation formula based on uniform distribution, and then determines the average ammunition consumption by the probability of damage. Song Xieen\textsuperscript{(4)} used the most favorable firepower distribution method based on single target and the minimum ammunition consumption method to establish a hybrid target optimal firepower allocation scheme and a minimum ammunition consumption calculation model. Zhang Tong\textsuperscript{(5)} used the statistical principle to establish a prediction model of ammunition consumption through the SVM, and carried out simulation experiments.

Judging from the results of the existing literature search, the methods of estimating ammunition consumption are mostly guided by combat missions, based on the degree of target damage. Most of them are based on the macroscopic damage effect, and there are few uncertain microscopic factors that influence the damage effect of ammunition characteristics. The existing statistical methods do not fully consider the reality that the price of new ammunition is high and it is difficult to carry out a large number of experiments. How to reasonably predict ammunition consumption under small sample conditions is an urgent problem to be solved. Based on this, this paper proposes a new ammunition consumption prediction method based on Bayesian system fusion. The method treats the microscopic factors with uncertainties in the operational effectiveness of ammunition as key information and integrates them into the expected stage of ammunition consumption. The fusion weight model based on the system contribution degree proposed in this paper can solve the problem that the combat effectiveness has a significant impact on the ammunition consumption. Firstly, the construction of the Bayesian system fusion model is introduced. Furthermore, the ammunition penetration effect system is established, and the application of this method is carried out based on the penetration effect. The influence of the uncertainty microscopic information such as the incident angle and impact velocity of the projectile on the ammunition consumption is analyzed. Finally, the application examples are used to infer the optimal ammunition consumption when the target is severely damaged, which proves that this method is feasible.

**BAYESIAN SYSTEM FUSION MODEL**

**CALCULATION OF PRIOR INFORMATION CREDIBILITY**

Because the ammunition consumption under the influence of single factor is deviated from the performance data of range test, this paper uses the prior information credibility to measure the difference between them. The prior information credibility reflects the degree of consistency between the prior information and the parameter distribution to be estimated\textsuperscript{(6)}. The credibility of Bayesian inference results is often judged by the credibility of prior information.
The higher the credibility is, the more credible the inference results are. The credibility is expressed as $P(H_i/A)$. The higher the $P(H_i/A)$, the higher the credibility of the prior information, the more accurate the Bayesian inference results. According to the Bayesian formula, $P(H_i/A)$ is:

$$
P(H_i/A) = \frac{P(A|H_i)P(H_i)}{P(A|H_i)P(H_i) + P(A|H_0)P(H_0)} = \frac{P(A|H_0)P(H_0)}{P(A|H_0)P(H_0)P(H_0)[1-P(H_0)]}
$$

(1)

**Osterial Distribution Processing of Prior Information**

Assume that the ammunition performance parameter $L_1, L_2, \cdots, L_n$ is a discrete random variable. Under the premise that the variable is the above single factor, the prior information sample is the ammunition consumption under the influence of single factor, expressed as $Z_i^b : (Z_i^b, Z_i^{d_1}, \cdots, Z_i^{d_k})$. In order to meet the needs of the research, this paper will continuously process the ammunition performance parameters and fit the scattered points of the discrete distribution into the distribution curve. The distribution of $Z_i^b, Z_i^{d_1}, \cdots, Z_i^{d_k}$ can be obtained by statistical analysis of the target data $Z_i^k, Z_i^{d_1}, \cdots, Z_i^{d_k}$ under a certain damage level. This paper takes the normal distribution as an example to explore the system fusion method. Assume that $Z_i^b, Z_i^{d_1}, \cdots, Z_i^{d_k}$ obey the normal distribution of parameters $\mu_i^b, \sigma_i^b, \mu_i^{d_1}, \sigma_i^{d_1}, \cdots, \mu_i^{d_k}, \sigma_i^{d_k}$ respectively, denoted as $Z_i^b \sim N(\mu_i^b, \sigma_i^b)$ among $i = 1, 2, \cdots, n$. The priori probability density of ammunition consumption $Z_i^b$ is:

$$
f_i^b(z_i^b) = \frac{1}{\sqrt{2\pi}\sigma_i^b} e^{-\frac{(z_i^b-\mu_i^b)^2}{2\sigma_i^b}}
$$

(2)

The priori probability function of ammunition consumption $Z_i^b$ is:

$$
F_i^b(z_i^b) = \frac{1}{\sqrt{2\pi}\sigma_i^b} \int_{-\infty}^{z_i^b} \frac{1}{\sqrt{2\pi}\sigma_i^b} e^{-\frac{(t-\mu_i^b)^2}{2\sigma_i^b}} dt
$$

(3)

In the field test, the actual ammunition consumption data is $Z_i = (z_i, z_i^{d_1}, \cdots, z_i^{d_k})$. According to the field test probability density $f(z_i/\mu_i^b, \sigma_i^b)$ and the prior probability density $f_i^b(z_i^b)$, the Bayesian formula can be used to calculate the post-test probability density.

$$
f_i^b(\mu_i^b, \sigma_i^b/|z_i) = \frac{f_i^b(z_i^b) \cdot f(z_i/\mu_i^b, \sigma_i^b)}{m(z_i^b)}
$$

(4)
FUSION WEIGHT MODEL BASED ON SYSTEM CONTRIBUTION

System contribution[9] refers to the influence of weapons and equipment on the comprehensive operational capability of the combat system. By analyzing the effect of ammunition efficiency on consumption, the ammunition damage effectiveness system is established, and the system contribution is used as the weight of Bayesian fusion. We use the intuitionistic fuzzy membership function[10] method to solve the weights. \( \Omega = \{ L_1, L_2, \ldots, L_n \} \) indicates the ammunition information set; \( \gamma_i^m(m = L_1, L_2, \ldots, L_n) \) indicates how much experts \( P_d(d = 1, 2, \ldots, m) \) believe that the effectiveness of the ammunition \( L_1, L_2, \ldots, L_n \) are severely harmful to the target; \( \eta_i^m (m = L_1, L_2, \ldots, L_n) \) indicates how much experts \( P_d(d = 1, 2, \ldots, m) \) believe that the effectiveness of the ammunition \( L_1, L_2, \ldots, L_n \) are not severely harmful to the target. The intuitionistic fuzzy evaluation information of expert \( P_d(d = 1, 2, \ldots, m) \) on the effectiveness \( L_1, L_2, \ldots, L_n \) of ammunition is expressed as:

\[
\Phi_i^d = \{ L_i, (\gamma_i^d, \eta_i^d) \}
\]  

(5)

Among: \( 0 \leq \gamma_i^d \leq 1, 0 \leq \eta_i^d \leq 1 \), and \( 0 \leq \gamma_i^d + \eta_i^d \leq 1 \).

The intuition indicator indicates the degree of hesitation that the expert believes that the ammunition effectiveness \( L_1, L_2, \ldots, L_n \) is vested in the evaluation target. We use half the degree of hesitation to correct the weight. The intuition indicator expression is:

\[
\tau_i^d = 1 - \gamma_i^d - \eta_i^d
\]  

(6)

Membership is expressed as:

\[
\varphi_i^d = \frac{1}{n} \sum_{j=1}^{n} \gamma_i^j - \frac{1}{n} \sum_{j=1}^{n} \eta_i^j + \frac{1}{2} \left( 1 - \frac{1}{n} \sum_{j=1}^{n} \gamma_i^j - \frac{1}{n} \sum_{j=1}^{n} \eta_i^j \right)
\]  

(7)

Normalization processing:

\[
\varphi_{i}^d = \frac{\varphi_i^d}{\sum_{i=1}^{n} \varphi_i^d}
\]  

(8)

Bayesian fusion formula is rewritten as:

\[
f(\mu, \sigma^2/z) = \sum_{i=1}^{n} \varphi_i f_i(z^i) \cdot f(z_i/\mu, \sigma^2)
\]  

(9)

Among \( \sum_{i=1}^{n} \varphi_i = 1 \).

\[
m(z) = \sum_{i=1}^{n} f_i(z^i) \cdot f(z_i/\mu, \sigma^2) \cdot \mu, d \sigma^2 = \sum_{i=1}^{n} m(z_i) \text{ is a fixed value and is not affected by the parameters.}
\]
BAYESIAN STATISTICAL INFERENCE OUTPUT

On the basis of a variety of ammunition information $\Omega = \{L_1, L_2, \ldots, L_n\}$, the weight is determined by the contribution degree of the system, and the weighted prior probability density $f(z)$ is obtained.

$$f(z) = \sum_{i=1}^{n} \varphi_i \cdot f_i(z_i) \quad (10)$$

Combined with formula (4)(9)(10), the formula for calculating the posterior density after fusion is:

$$f(\mu, \sigma^2 / z) = \frac{\sum_{i=1}^{n} \varphi_i \cdot m(z_i) \cdot f_i(\mu, \sigma_i^2 / z_i)}{m(z)}$$

$$= \sum_{i=1}^{n} \varphi_i \cdot m(z_i) \cdot f_i(\mu, \sigma_i^2 / z_i) \quad (11)$$

Equation (11) shows that the posterior density after fusion is weighted by the density of $Z^h, Z^{h'}, \ldots, Z^s$. According to the prior probability density $f_i(z_i)$, the posterior density obtained by combining the field performance data $Z^h : (Z^h, Z^{h'}, \ldots, Z^s)$.

In order to simplify the calculation, the $f(z_i / \mu_i, \sigma_i^2) / m(z)$ in the formula (4) can be replaced by the likelihood function when the operation is performed. Because the difference between the likelihood probability and the posterior probability is only a constant coefficient $\lambda / m(z)$.

The likelihood function is expressed as:

$$L(\mu, \sigma^2) = \prod_{i=1}^{n} f(z_i / \mu_i, \sigma_i^2) \quad (12)$$

Then the weighted posterior probability density is expressed as:

$$f(c, \lambda / \theta) = \frac{\sum_{i=1}^{n} \varphi_i \cdot f(\theta^i; c_i, \lambda_i) \cdot L(\theta; c, \lambda) \cdot f(\theta; c, \lambda) \cdot f(c, \lambda) \cdot dc \cdot d\lambda}{\int_{\theta} f(\theta; c, \lambda) \cdot f(c, \lambda) \cdot d\theta} \quad (13)$$

APPLICATION EXAMPLE

Assume that the armor target achieves severe damage, that is, the ratio of armor penetration depth to armor thickness is 0.6 to 0.8. Only the distribution of the incident angle $X$ of the projectile is considered, and other factors are constant. The ammunition consumption at this time is shown in Table I. Other factors remain the same, as shown in Table II for the study of ammunition consumption at the impact velocity $Y$. The amount of ammunition consumed in the field test is shown in Table III. The above distributions are subject to a normal distribution, and the next step is to resolve reasonable ammunition consumption.


| Serial number | Ammunition Quantity(pcs) | Serial number | Ammunition Quantity(pcs) |
|---------------|--------------------------|---------------|--------------------------|
| 1             | 3                        | 11            | 4                        |
| 2             | 4                        | 12            | 2                        |
| 3             | 5                        | 13            | 3                        |
| 4             | 2                        | 14            | 4                        |
| 5             | 1                        | 15            | 1                        |
| 6             | 2                        | 16            | 5                        |
| 7             | 4                        | 17            | 4                        |
| 8             | 2                        | 18            | 3                        |
| 9             | 3                        | 19            | 3                        |
| 10            | 2                        | 20            | 2                        |

The calculated mean is 2.95, the variance is 1.42, and the probability density function is:

\[
 f(z) = \frac{1}{\sqrt{2.84\pi}} e^{-\frac{(z-2.95)^2}{2.84}}
\]  

\[ (14) \]

| Serial number | Ammunition Quantity(pcs) | Serial number | Ammunition Quantity(pcs) |
|---------------|--------------------------|---------------|--------------------------|
| 1             | 2                        | 11            | 5                        |
| 2             | 2                        | 12            | 5                        |
| 3             | 4                        | 13            | 3                        |
| 4             | 4                        | 14            | 4                        |
| 5             | 3                        | 15            | 1                        |
| 6             | 2                        | 16            | 2                        |
| 7             | 5                        | 17            | 4                        |
| 8             | 4                        | 18            | 1                        |
| 9             | 3                        | 19            | 3                        |
| 10            | 2                        | 20            | 4                        |

By calculation, the mean value is 3.15 and the variance is 1.61. The probability density function is:

\[
 f(z) = \frac{1}{\sqrt{3.22\pi}} e^{-\frac{(z-3.15)^2}{3.22}}
\]  

\[ (15) \]

| Serial number | Ammunition Quantity(pcs) | Serial number | Ammunition Quantity(pcs) |
|---------------|--------------------------|---------------|--------------------------|
| 1             | 0.5                      | 11            | 1                        |
| 2             | 1                        | 12            | 0.8                      |
| 3             | 1                        | 13            | 1                        |
| 4             | 0                        | 14            | 1                        |
| 5             | 0.5                      | 15            | 1                        |
| 6             | 2                        | 16            | 0.6                      |
| 7             | 1                        | 17            | 1                        |
| 8             | 0.9                      | 18            | 2                        |
| 9             | 2                        | 19            | 0.8                      |
| 10            | 0.5                      | 20            | 1                        |
Zero number of ammunition in the field test indicates that the equipment is naturally damaged. The average value of the field test is 0.98, the variance is 0.25, and the probability density is:

\[
f(z_i; \mu, \sigma^2) = \frac{1}{\sqrt{0.5\pi}} e^{-\frac{(z_i-\mu)^2}{0.5\sigma^2}}
\]  

We invited 10 experts in the same field to score the incident angle X and impact velocity Y of the projectile. The scores are shown in Table IV and Table V. According to the formula (12) (13) and the normalization process, the system contribution degree of the two systems can be determined and used as the weight. The system contribution degree of the two are \(\phi_x = 0.53\) and \(\phi_y = 0.47\).

### TABLE IV. EXPERTS SCORE THE ANGLE OF INCIDENCE OF THE PROJECTILE.

| Serial number | \(\gamma_{iX}\) | membership degree | \(\eta_{iX}\) | membership degree |
|---------------|-----------------|------------------|-------------|-----------------|
| 1             | 1               | 0.6              | \(\eta_1^X\) | 0.3             |
| 2             | 2               | 0.4              | \(\eta_1^X\) | 0.5             |
| 3             | 3               | 0.8              | \(\eta_1^X\) | 0.2             |
| 4             | 4               | 0.5              | \(\eta_1^X\) | 0.5             |
| 5             | 5               | 0.7              | \(\eta_1^X\) | 0.2             |
| 6             | 6               | 0.6              | \(\eta_1^X\) | 0.3             |
| 7             | 7               | 0.6              | \(\eta_1^X\) | 0.2             |
| 8             | 8               | 0.4              | \(\eta_1^X\) | 0.5             |
| 9             | 9               | 0.6              | \(\eta_1^X\) | 0.3             |
| 10            | 10              | 0.8              | \(\eta_1^X\) | 0.2             |

### TABLE V. EXPERTS SCORE THE BALLISTIC END SPEED.

| Serial number | \(\gamma_{iY}\) | membership degree | \(\eta_{iY}\) | membership degree |
|---------------|-----------------|------------------|-------------|-----------------|
| 1             | 1               | 0.5              | \(\eta_1^Y\) | 0.4             |
| 2             | 2               | 0.7              | \(\eta_1^Y\) | 0.3             |
| 3             | 3               | 0.6              | \(\eta_1^Y\) | 0.2             |
| 4             | 4               | 0.4              | \(\eta_1^Y\) | 0.5             |
| 5             | 5               | 0.3              | \(\eta_1^Y\) | 0.6             |
| 6             | 6               | 0.7              | \(\eta_1^Y\) | 02              |
| 7             | 7               | 0.6              | \(\eta_1^Y\) | 0.3             |
| 8             | 8               | 0.6              | \(\eta_1^Y\) | 0.2             |
| 9             | 9               | 0.4              | \(\eta_1^Y\) | 0.5             |
| 10            | 10              | 0.7              | \(\eta_1^Y\) | 0.1             |

The weighted prior probability density is:

\[
f(z) = \sum_{i=1}^{10} \phi_i f(z) = 0.53 \times \frac{1}{\sqrt{2.84\pi}} e^{-\frac{(z-2.95)^2}{2.84}} + 0.47 \times \frac{1}{\sqrt{3.22\pi}} e^{-\frac{(z-3.15)^2}{3.22}}
\]  

According to the Bayesian formula, the fusion posterior probability density can be obtained as:
\[ f(\mu, \sigma^2|z) = f(z) \frac{1}{\sqrt{0.5\pi}} e^{-\frac{(z-\mu)^2}{0.5\sigma^2}} \]  

(18)

We can get \( \mu = 2.983 \), and a confidence level of \( \mu \) with a confidence interval of 0.95 is:

\[ (\bar{X} - \frac{1}{\sqrt{20}} z_{0.025}, \bar{X} + \frac{1}{\sqrt{20}} z_{0.025}) = (2.54, 3.42) \]  

(19)

Because 2.983 \( \in (2.54, 3.42) \), we can accept this result. The amount of ammunition consumed when the armored target reaches severe damage can be determined to be 2.983. The ammunition consumption calculated in this example combines the historical information and field data of the incident angle and impact velocity of the projectile. This method can achieve more accurate ammunition consumption prediction under small sample conditions, overcome the impact of insufficient ammunition efficiency test, and effectively verify that the ammunition consumption prediction method based on Bayesian system fusion is correct and feasible.

**CONCLUSIONS**

The new ammunition consumption prediction method based on Bayesian system fusion proposed in this paper can provide reference for statistical analysis of data under small sample conditions. The fusion model based on system contribution solves the problem of the influence of combat effectiveness on ammunition consumption. Since multivariate is difficult to control, only the effect of the penetration effect on the target damage is studied. The assessment object is the degree of damage to the armor target, and the damage effect is evaluated by the depth of penetration. By using the Bayesian method to combine the uncertainty factors such as the incident angle and impact velocity of the projectile during the penetration process, the multi-source information fusion posterior density and Bayesian inference results based on the system contribution are obtained. Thus we can determine the amount of ammunition consumed when the target reaches a critical damage. The application example demonstrates that this method is operational and provides the basic algorithmic support for the ammunition consumption prediction system. In the next study, ammunition information will be fully considered and its uncertainties will be fully integrated. The purpose is to predict the ammunition consumption of the target under different damage levels, and Bayesian inference results are more accurate, so as to achieve the purpose of accurate guarantee.

**ACKNOWLEDGMENTS**

Corresponding author: Rudong Zhao

This work was supported by the Army Scientific Research (No. KYSZJWJK1744, 012016012600B11403).
REFERENCES

1. G.Y. Wang, Q. Shi, Z.F. You. Study on the Simulation Method of Target Damage of Ground Artillery Fire Strike Cluster Equipment[J]. Acta Armamentari SINICA, 2016, 37(1):36-43.
2. Y.L. Zhi et al. Evaluation of the required bomb consumption for air missile damage targets[J]. Command Control & Simulation, 2016, 38(5):82-84.
3. J. Hu, Y. Dai, J.D. Huang. A Calculation Model of Average Ammunition Consumption of Shipborne Rocket Projectiles[J]. Ordnance Industry Automation, 2012, 31(3):13-14.
4. X.E. Song, W.D. Song, C.W. Zhao, D.F. Zhai. A Method for Solving Hybrid Target Fire Distribution and Ammunition Consumption[J]. Journal of Ballistics, 2014, 26(3):37-40.
5. W. Zhang, L. Liu, H.J. Xu. Predictive Modeling Based on SVR Ammunition Consumption Quantity[J]. Fire Control & Command Control, 2010, 35(12):8-10.
6. Q. Liu, X.Y. Wu. The Consistency and Credibility Principle of Pretest Distribution Selection in Bayes Method Application[J]. System Engineering and Electronics, 2010, 32(11):2356-2359.
7. X.M. Luo, Y.L. Zhu, W. He. A Method for Evaluating the Contribution Degree of Weapon Equipment System Based on Complex Network[J]. Fire Control & Command Control, 2017, 42(2):83-87.
8. G.T. Men. Application of Intuitionistic Fuzzy Reasoning for Armed Police Equipment Support Command Decision[J]. Journal of Armed Police University, 2016, 32(2):41-45.
9. J. Feng, Z.Q. Pan, Q. Sun. Reliability information fusion method for small subcomplex systems and its application[M]. Beijing: Science Press, 2015.
10. J.P. Li, J. Cheng, W. Xiong, et al. The Mathematical Study on the Damage Effectiveness of Armor-piercing Projectiles to Homogeneous Armored Deck[J]. Journal of Projectiles, Rockets and Guidance, 2015, 35(2):76-79.
11. Z.B. Zhou, H.T. Li, X.M. Liu, et al. Bayes Information Fusion Method for Reliability Modeling and Evaluation of Aerospace Long Life Products[J]. Systems Engineering Theory & Practice, 2012, 32(11): 2517-2522.
12. T.X. Xu, Y. Liu, J.Z. Zhao, et al. Fusion method of maintainability prior information[J]. System Engineering and Electronics, 2014, 36(9):1887-1892.