On correlators of Reggeon fields and operators of Wilson lines in high energy QCD

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Abstract

In this note we derive Dayson-Schwinger hierarchy of the equations for the correlators of reggeized gluon fields in the framework of Lipatov’s high energy QCD effective action formalism, [1–5]. The explicit perturbative expressions for the correlators till correlator of four-fields inclusively are obtained and different perturbative schemes for the solutions of the equation for the two-field correlator are discussed. A correspondence between the correlators of reggeized gluon fields and Wilson line operators of longitudinal gluon field is established with the help of [5] paper results and the connection between the JIMWLK-Balitsky formalism and Lipatov’s effective action approach is clarified. The discussion of the obtained results is also present.

1 Introduction

The action for interaction of reggeized gluons introduced in [1], see also [2–5], describes quasi-elastic amplitudes of high-energy scattering processes in the multi-Regge kinematics. The applications of this action for the description of high energy processes and calculation of sub-leading, unitarizing corrections to the amplitudes and production vertices can be found in [6]. The generalization of the formalism for the case of production amplitudes was considered in [4] as well, where the prescription of the calculation of S-matrix elements of the different processes was given following to the approach of [7]. This effective action formalism, based on the reggeized gluons as main degrees of freedom, can be considered as reformulation of the RFT (Regge Field Theory) calculus introduced in [8] for the case of high energy QCD degrees of freedom which are reggeized gluons, see [9]. It was underlined in [1] that the main purposes of the approach is the construction of the S-matrix unitarity in the direct and crossing channels of the scattering processes through the multi-reggeon dynamics described with the use of the vertices of multi-reggeon interactions, see similar approaches in [10–13].

An application of the shock wave and large $N_c$ approximations in the framework of the formalism, see [2][4], allowed to establish a connection between the Lipatov’s approach to the high energy processes and the CGC (Color Glass Condensate), BK and Balitsky-JIMWLK approaches to high energy scattering [12,15,17]. Besides these approximations, the another difference between the approaches is the degrees of freedom under consideration. In the effective action formalism these are reggeized gluons fields, see [1][9], whereas in the shock wave formalisms operates with the correlators of Wilson lines constructed from the gluon or quark fields. These degrees of freedom are different but are complimentary each to other in the sense that the Lipatov’s effective action can be considered as reformulation of the theory of interacting operators[4] of Wilson lines without any reggeons introduced,
see details and discussion in [5]. Therefore, the new formulation of the effective action approach proposed in [5] allows to establish the connection between the correlators of the reggeized gluon fields and correlators of Lipatov’s operators, i.e. it allows to establish the connection between two main but different degrees of freedom in high energy QCD beyond any approximations.

In order to reproduce the Balitsky-JINWLK hierarchy of the equations for the correlators of the Wilson lines operators we need, at least, to obtain the similar hierarchy of the equations for the correlators of the reggeized fields. In terms of BFKL physics, [9], it means that we have to derive in the formalism the BFKL like equations for the any correlators of interests. The straightforward way to achieve that is a derivation of the Dayson-Schwinger hierarchy of the equations for the correlators starting directly from the generating functional for the reggeized fields, this is done in the second Section of the paper. The perturbative expressions for the reggeon fields correlators till correlator of four-fields inclusively are written in Appendixes A-D, where we solved perturbatively the hierarchy of the correlator’s equations and write explicitly the form of the corrections to the BFKL like correlators of one, two, three and four reggeons. In Section 3 we discuss the leading order BK like equation for the correlator of two reggeon fields which form depends on the perturbative scheme used in the calculations. The connection between the correlators of Lipatov’s operators and reggeized fields correlators is established and discussed in Section 4, where also the precise form of the connection between the correlator of two Lipatov’s operators and correlator of two reggeon fields is written. The last Section 5 is the conclusion of the paper.

2 Lipatov’s effective action and correlators of reggeon fields

The Lipatov’s effective action for reggeized gluons $B_{\pm}$, formulated as RFT (Regge Field Theory), can be obtained by an integration out the gluon fields $v$ in the generating functional for the $S_{eff}[v, B]$:

$$e^{i\Gamma[B]} = \int Dv e^{iS_{eff}[v, B]}$$  \hspace{1cm} (1)

where

$$S_{eff} = -\int d^4x \left( \frac{1}{4} G_\mu^a G^{\mu}_a + tr \left[ (T_+(v_+) - B_+) j^+_\text{reg a} + (T_-(v_-) - B_-) j^-_\text{reg a} \right] \right),$$  \hspace{1cm} (2)

with

$$T_\pm(v_\pm) = \frac{1}{g} \partial_\pm O(v_\pm) = v_\pm O(v_\pm), \quad j^\pm_\text{reg a} = \frac{1}{C(R)} \partial^2 B^\pm_a,$$  \hspace{1cm} (3)

here $C(R)$ is eigenvalue of Casimir operator in the representation $R$, $tr(T^aT^b) = C(R) \delta^{ab}$ see [1][4].

The form of the Lipatov’s operator $O$ (and correspondingly $T$) depends on the particular process of interests, in the simplest case it has the form of the Wilson line (ordered exponential) for the longitudinal gluon fields in an arbitrary representation:

$$O(v_\pm) = Pe^{g \int_{-\infty}^{x_\pm} dx_\pm v_\pm(x^+, x^-, x_\perp)}, \quad v_\pm = iT^a v^a_\pm$$  \hspace{1cm} (4)

see details in [1][3]. A Hermitian form of the operators was also derived in [5], in this case the operator is represented by a combination of the different Wilson lines:

$$O(v_\pm) = \frac{1}{4} \left( Pe^{g \int_{-\infty}^{x_\pm} dx_\pm v_\pm(x^+, x^-, x_\perp)} - Pe^{g \int_{x_\perp}^{\infty} dx_\pm v_\pm(x^+, x^-, x_\perp)} - Pe^{-g \int_{-\infty}^{x_\perp} dx_\pm v_\pm(x^+, x^-, x_\perp)} + Pe^{-g \int_{x_\perp}^{\infty} dx_\pm v_\pm(x^+, x^-, x_\perp)} \right),$$  \hspace{1cm} (5)

see also [18].
The effective action $\Gamma$ of the interactions of reggeized gluons was calculated to one-loop precision in [3] for the case of the adjoint representation of gluon fields. This actions has the following form:\footnote{In order to make the notations shorter, we write + and − indexes of the vertices as upper and lower indexes in the expressions for the vertices, i.e. we write: $K^{+a_1 \ldots a_n}_{-b_1 \ldots b_m} = (K^{+ \ldots +}_{- \ldots -})^{a_1 \ldots a_n}_{b_1 \ldots b_m}$.}:

$$\Gamma = \sum_{n,m=1} \left( B^{a_1 \ldots a_n}_{+} \ldots B^{a_m}_{+} (K^{+ \ldots -}_{- b_1 \ldots b_m} B^{b_1}_{-} \ldots B^{b_m}_{-}) \right) = -B^{a}_{+} \partial_i B^{a}_{-} + B^{a}_{+} \left( K^{ab}_{xy} \right)_{-} B^{b}_{-} + \ldots , \tag{6}$$

where $B_{\pm}$ are the reggeized gluon fields and shorthand notations for the integration over the variables in the action were used. The effective vertices (kernels) $K$ in Eq. (6) represents the processes of multi-reggeon interaction in t-channel of the high energy scattering amplitude. For example, the vertex responsible for the gluon’s reggeization has the following form\footnote{In the language of high-energy perturbative QCD these vertices are BFKL-like kernels of the integro-differential equations for the objects of interests or, equivalently, they can be considered as analog of the parts of a Hamiltonian in Balitsky-JIMWLK approach.} to LO:

$$\delta Z^{x y} = -\delta(x^{+}) \delta(x^{-}) \delta^{a b} \frac{g^{2} N}{8 \pi} \partial_{x}^{2} \int_{\Lambda}^{\Lambda} \frac{dp_{-}}{p_{-}} \int \frac{d^{2} p_{\perp}}{(2\pi)^{2}} \int \frac{d^{2} k_{\perp}}{(2\pi)^{2}} \frac{k_{\perp}^{2}}{p_{\perp}^{2}} \left( p_{\perp} - k_{\perp} \right)^{2} e^{-i k_{i}(x_{i} - y_{i})}, \tag{7}$$

see details of the calculations in [3]. The rapidity interval $\eta$ in Eq. (7) is an analog of the ultraviolet cut-off in the relative longitudinal momenta. Physically it determines the value of the cluster of the particles in the Lipatov’s effective action approach, see [1]. In order to calculate the correlators of the reggeon $B_{\pm}$ fields, we will use the following generating functional for reggeon fields:

$$Z[J] = \int DB \exp \left( i \Gamma[B] - i \int d^{4}x J^{a}_{-} (x^{+}, x_{\perp}) B^{a}_{+}(x^{+}, x_{\perp}) - i \int d^{4}x J^{a}_{+} (x^{-}, x_{\perp}) B^{a}_{-}(x^{-}, x_{\perp}) \right), \tag{8}$$

see [5]. Correspondingly, the Schwinger-Dayson equations for the correlators we obtain now taking derivative of the field’s variation of $Z[J]$ in respect to the currents and taking them equal to zero at the end. Namely, to the first order we have:

$$\delta Z[J] = \int DB \delta B_{\pm} \left( \frac{\delta \Gamma[B]}{\delta B_{\pm}} - i \int d^{4}x J^{a}_{\mp} (x^{\mp}, x_{\perp}) \right) \exp \left( i \Gamma[B] - i \int d^{4}x J^{a}_{-} B^{a}_{+} - i \int d^{4}x J^{a}_{+} B^{a}_{-} \right) = 0. \tag{9}$$

Therefore, for the one-field correlator we obtain:

$$\frac{\delta \Gamma[B]}{\delta B_{\pm}} = 0 \tag{10}$$

that, using Eq. (9) expansion, provides:

$$\langle \tilde{B}^{a}_{\pm}(x^{\mp}, x_{\perp}, \eta) \rangle = \sum_{n=1} \left( \tilde{K}(\eta) \right)^{a a_{1} \ldots a_{n}}_{x x_{1} \ldots x_{n}} \langle B^{a}_{\pm_{1}} \ldots B^{a_{n}}_{\pm_{n}} \rangle, \tag{11}$$

see Appendix A. In the r.h.s. of the expression we introduce the shorthand notation $\tilde{K}$ for the modified vertices of Eq. (5) expansion, which are arising after including of any non-leading perturbative corrections in the definition of the r.h.s of the expression and accounts permutations of the reggeized fields after the variation with respect to the fields in Eq. (9), see Appendixes A-D. Taking the derivative from the Eq. (11) with respect to the $\eta$, we can also obtain the evolution equation for the reggeized fields:

$$\frac{\partial}{\partial \eta} B^{a}_{\pm}(x^{\mp}, x_{\perp}, \eta) = \frac{\partial}{\partial \eta} \left( \sum_{n=1} \left( \tilde{K}(\eta) \right)^{a a_{1} \ldots a_{n}}_{x x_{1} \ldots x_{n}} \langle B^{a}_{\pm_{1}} \ldots B^{a_{n}}_{\pm_{n}} \rangle \right). \tag{12}$$
The next derivative of Eq. (9) with respect to the currents provides the following equation:

\[
< \frac{\delta \Gamma[B]_{1}}{\delta B_{\pm}^{a_{1}}} B_{\pm}^{a_{2}} - i \delta^{a_{1}a_{2}} \delta_{\pm 1} \pm 2 \delta(x_{\perp 1} - x_{\perp 2}) > = 0 .
\]  

(13)

Considering, for example, the expression for the \( < B_{\pm} B_{\mp} > \) correlation function in adjoint representation, we have:

\[
\partial_{\perp}^{2} < B_{\pm}^{a_{1}} B_{\pm}^{a_{2}} > = - i \delta^{a_{1}a_{2}} \delta_{\pm 1} \pm 2 \delta(x_{\perp 1} - x_{\perp 2}) + (K_{x_{\perp}}^{\pm})^{a_{1}} < B_{\pm}^{b_{1}} B_{\pm}^{a_{2}} > + \cdots.
\]  

(14)

We see here that this equation indeed reproduces the equation for the propagator of the reggeized fields, see [3] and Eq. (B.3).

The general Schwinger-Dayson system of the equations for the field’s correlators, correspondingly, can be written as

\[
< \frac{\delta \Gamma[B]}{\delta B_{\pm}^{a_{1}}} B_{\pm}^{a_{1}} \cdots B_{\pm}^{a_{n}} >_{J} = - i \sum_{i=1}^{n} < B_{\pm}^{a_{1}} \cdots \delta^{a_{i}a_{i}} \delta_{\pm a_{i}} \delta(x_{\perp a} - x_{\perp a_{i}}) \cdots B_{\pm}^{a_{n}} >_{J} = 0 .
\]  

(15)

that, with the help of Eq. (13), provides

\[
< B_{\pm x}^{a} B_{\pm x_{1}}^{a_{1}} \cdots B_{\pm z_{m}}^{a_{m}} > = \sum_{n=1}^{N} \left( \hat{K} (\eta) \right)^{a_{b_{1}} \cdots b_{n}}_{x_{x_{1}} \cdots x_{n}} < B_{\pm x_{1}}^{b_{1}} \cdots B_{\pm x_{n}}^{b_{n}} B_{\pm x_{1}}^{a_{1}} \cdots B_{\pm z_{m}}^{a_{m}} > .
\]  

(16)

Examples of the equations for the two, three and four reggeon fields correlators are presented in the Appendixes. Now, taking derivatives of the l.h.s. and r.h.s. of the Eq. (16) with respect to the parameter \( \eta \) we obtain the same system of the equations as a system of coupled evolution equations for the correlators:

\[
\frac{\delta}{\delta \eta} < B_{\pm x}^{a} B_{\pm x_{1}}^{a_{1}} \cdots B_{\pm z_{m}}^{a_{m}} > = \frac{\delta}{\delta \eta} \sum_{n=1}^{N} \left( \hat{K} (\eta) \right)^{a_{b_{1}} \cdots b_{n}}_{x_{x_{1}} \cdots x_{n}} < B_{\pm x_{1}}^{b_{1}} \cdots B_{\pm x_{n}}^{b_{n}} B_{\pm x_{1}}^{a_{1}} \cdots B_{\pm z_{m}}^{a_{m}} > .
\]  

(17)

Formally, the form of Eq. (17) is similar to the Balitsky-JIMWLK hierarchy of the correlators of Wilson lines. In the paper [5] the connections between the correlators of the Reggeon fields and operators constructed from the Wilson lines was established, therefore further we will use the results of [5] in order to derive hierarchy of Lipatov’s operators in terms of reggeon correlators.

3 Leading order solutions for two-fields correlator

The BFKL like equation for the two-fields correlator, Eq. (B.3), can be obtained with the help of Appendixes A-D results. Accounting the leading perturbative contributions in the equations only, the form of the equations is depending on the perturbative scheme of the calculations chosen. Namely, in the RFT scheme the bare propagator of the scheme is given by Eq. (B.4) and instead of an analog of JIMWLK-Balitsky equation

\[
\partial_{\perp}^{2} < B_{\pm}^{a} B_{\mp}^{a_{1}} > = - i \delta^{a_{1}a_{2}} + (K_{a_{2}}^{a_{1}})_{\perp} < B_{\pm}^{a_{2}} B_{\mp}^{a_{1}} > + 2 (K_{a_{1}a_{2}}^{a_{1}})_{\perp} < B_{\pm}^{a_{3}} B_{\mp}^{a_{2}} B_{\mp}^{a_{1}} >
\]  

(18)

the following expression will be obtained

\[
\partial_{\perp}^{2} < B_{\pm}^{a} B_{\mp}^{a_{1}} > = - i \delta^{a_{1}a_{2}} + (K_{a_{2}}^{a_{1}})_{\perp} < B_{\pm}^{a_{2}} B_{\mp}^{a_{1}} > - 2 i (K_{a_{1}a_{2}}^{a_{1}})_{\perp} < B_{\pm}^{a_{3}} B_{\mp}^{a_{2}} > G_{0}^{a_{3}a_{1}} - 2 i (K_{a_{1}a_{2}}^{a_{1}})_{\perp} G_{0}^{a_{3}a_{2}} + < B_{\pm}^{a_{1}} B_{\mp}^{a_{1}} >
\]  

(19)

Further we will use \( \delta^{ab} \) note for the notation of 4-delta function plus color indexes included, denoting full coordinate and color indexes dependence in the delta functions only where it will be needed.
after insertion of Eq. (D.3) in Eq. (B.3). Similarly to routines of the BFKL calculus, we assume that the vertex $K^{++}_a$ is local in rapidity space. Therefore, the tadpole contributions in Eq. (19) can be represented with the help of redefined effective vertex in the equation:

$$\left(\hat{K}_b^{b_1}\right)_-^{++} = \left(K_{b_1 b_1}^{b_2 b_3}\right)_-^{++} G_0^{b_2 b_1} = \left(K_{b_1 b_1}^{b_2 b_1}\right)_-^{++} G_0^{b_2 b_1} + ,$$

where another bare $\hat{G}_0^+$ propagator is introduced:

$$\delta^{ab} \delta^4(x - y) \partial^2_{x} \hat{G}_0^+(y, z) = \delta^{ac} \delta^4(x - z) .$$

Now we can rewrite Eq. (19) as

$$\left(\partial^2_{x} \delta^{ab} - \left(K_a^{b}\right)_-^{++} + 2 t \left(\hat{K}_a^{b}\right)_-^{++}\right) < B^b \ B^d > = - i \delta^{ad} - 2 t \left(K^{a_1 a_2}_a\right)_-^{++} < B^a_{a_1} B^{a_2} > G^{a_1 a_1}_0 ,$$

which is analog of the equation for the NLO reggeized gluon propagator in the proposed RFT scheme. Taking into account the locality of the $K$ vertex in the rapidity space, in the last r.h.s. of the Eq. (22) equation the $< B^a_{a_1} B^{a_2} > \rightarrow - i G^{a_1 a_2}_0$ replacement can be performed. Therefore, we obtain:

$$\left(\partial^2_{x} \delta^{ab} - \left(K_a^{b}\right)_-^{++} + 2 t \left(\hat{K}_a^{b}\right)_-^{++}\right) < B^b \ B^d > = - i \delta^{ad} - 2 \left(\hat{K}_a^{b}\right)_-^{++} G^{b_0 a_1}_0 .$$

In terms of Eq. (B.5) expansion, to the leading perturbative order, we correspondingly have:

$$< B^a_+ B^{-a}_- >_1 = - 4 G^{ab}_0 + \left(\hat{K}_a^{b}\right)_-^{++} G^{b_1 a_1}_0 ,$$

that allows to rewrite Eq. (B.5) as

$$< B^a_+ B^b_- > = - i G^{ab}_0 + - i G^{ac}_0 \left(\delta^{cb} - 4 t \left(\hat{K}_c^{c}\right)_-^{++} G^{c_1 a_1}_0 \right) .$$

We see, that this correction to the propagator can be considered as consequence of the redefinition of the intercept of the reggeon fields propagator, i.e. with the given precision we can rewrite Eq. (23) as

$$\left(\partial^2_{x} \delta^{ab} - \left(K_a^{b}\right)_-^{++} + 4 t \left(\hat{K}_a^{b}\right)_-^{++}\right) < B^b_+ B^d_- > = - i \delta^{ad}$$

that corresponds to the redefinition of the intercept of the reggeized gluons propagator in the coordinate space as

$$K^+ \rightarrow K^+ - 4 t \left(\hat{K}\right)_+^+, \quad (27)$$

see [3].

The equation for the two-fields correlator can be written differently if the usual perturbative scheme based initially on the Eq. (21) bare propagator instead propagator of Eq. (B.4) will be used. In this case, we can assume to the leading order:

$$< B^a_+ B^{a_1}_+ B^a_+ B^{a_2}_- B^{-a_1}_- > = 2 \ < B^a_+ B^{a_2}_- B^a_+ > < B^a_+ B^{a_1}_- > .$$

Inserting that factorized expression back in the Eq. (11), we will obtain the expression for the two fields correlator to leading order precision in this perturbative scheme:

$$\hat{\partial}^2_x \ < B^a_+ B^{a_2}_+ > = - i \delta^{a a_1} + \left(K^{a_2}_a\right)_-^{++} < B^a_+ B^{a_2}_- B^{a_1}_+ > ,$$

\[6\]

The $K^{++}_a$ vertex is known from the BFKL physics but did not calculated in the framework of [2, 3] formalism yet.
see Eq. \([33]\), that justifies Eq. \([28]\) assumption. Therefore, to leading order, we have for Eq. \([33]\):

\[
\partial^2 \phi - B^a_+ B^a_- = -i \delta^a v + (K^{a_2})_+ - B^a_+ B^a_- + 4 (K^{a_2 a_1})_+ - B^a_+ B^a_- < B^a_+ B^a_- > .
\]

(30)

In this case, in the tadpole-like term we can not replace the full correlator on the bare one similarly to done above and we will obtain:

\[
< B^a_+ B^a_- > = -i \tilde{G}^{a_1}_+ + \tilde{G}^{a_2}_- (K^{b_1}_b)_+ < B^b_+ B^a_- > + 4 \tilde{G}^{a_2}_- (K^{b_2 b_1}_b)_+ < B^b_+ B^b_- > < B^b_+ B^a_- > .
\]

(31)

This equation we can consider as analog of BK equation written for the correlator of the reggeized gluon fields.

4 Correlators of Wilson lines operators

In this Section we consider the generating functional for the Lipatov’s operators introduced above in Eq. \([3]\):

\[
Z[J] = \frac{1}{Z_I} \int Dv \exp \left( i S^0[v] \right) + \frac{i}{2 C(R)} \int d^4 x \ T_+ \partial_+ \ T_- + \frac{i}{2 C(R)} \int d^4 x \ J_-(x^-, x_\perp) T_+ + \frac{i}{2 C(R)} \int d^4 x \ J_+(x^+, x_\perp) T_- .
\]

(32)

Taking, for example, two derivatives of log \(Z[J]\) with respect to the currents we obtain:

\[
C(R)^2 \left( -2 g \right)^2 \left( \delta^2 \frac{\delta}{\delta J^{a_1}_+ \delta J^{a_2}_-} \log Z[J] \right) \bigg|_{J=0} = \frac{1}{T^{a_1}_1} (O_1(v_\perp))_{x_\perp = -\infty} - O_1(v_\perp))_{x_\perp = -\infty} \otimes T^{a_2}_2 (O_2(v_\perp))_{x_\perp = \infty} - O_2(v_\perp))_{x_\perp = -\infty} > = \frac{1}{T^{a_1}_1} \hat{O}_1(v_\perp) \otimes \left( T^{a_2}_2 \hat{O}_2(v_\perp) \right) > ,
\]

(33)

that in the case of Eq. \([11]\) representation, for example, provides us with the correlators of usual Wilson lines:

\[
\hat{O}(v_\perp) = W_\pm = P e^{g \int_{-\infty}^\infty dx^\pm v_\perp(x^\pm, x^\perp)} - 1 , \quad v_\perp = g T^a A^{a_1}_\perp ,
\]

(34)

whereas in the case of Eq. \([33]\) representation of the operators the correlators of the following operators arise:

\[
\hat{O}(v_\perp) = \frac{1}{2} \left( P e^{g \int_{-\infty}^\infty dx^\pm v_\perp(x^\pm, x^\perp)} - \bar{P} e^{-g \int_{-\infty}^\infty dx^\pm v_\perp(x^\pm, x^\perp)} \right) ,
\]

(35)

these operators we also can call as Lipatov’s operators. In accordance with \([33]\), we can rewrite the generating functional Eq. \([32]\) with the help of new degrees of freedom, which we identify as the reggeized gluons fields:

\[
Z[J] = \frac{1}{Z_I} \int Dv \exp \left( i S^0[v] \right) + \frac{i}{2 C(R)} \int d^4 x \left( \partial^2_+ T_+ + J_+ \right) \left( \partial^2_+ T_+ + J_- \right) - \frac{1}{2 C(R)} \int d^4 x \ J_-(x^+, x_\perp) \left( \partial^2_+ \right)^{-1} J_-(x^-, x_\perp) = \frac{1}{Z_I} \int Dv DB \exp \left( i S^0[v] \right) - \frac{2 i}{C(R)} \int d^4 x B_+(x^+, x_\perp) \partial_+ B_-(x^-, x_\perp) + \frac{i}{C(R)} \int d^4 x T_+(x^+, x_\perp) \partial_+ B_-(x^-, x_\perp) + \frac{i}{C(R)} \int d^4 x T_-(x^+, x_\perp) \partial_+ B_+(x^-, x_\perp) - \frac{2 i}{C(R)} \int d^4 x J_+(x^+, x_\perp) \partial_+ J_-(x^-, x_\perp) + \frac{i}{C(R)} \int d^4 x J_-(x^+, x_\perp) \partial_+ J_+(x^-, x_\perp) + + \frac{i}{C(R)} \int d^4 x J_-(x^+, x_\perp) B_+(x^+, x_\perp)
\]

(36)
and which also can be written as
\[ Z[J] = \frac{1}{Z} \int Dv DB \exp \left( i S_{eff}[v, B] - \frac{i}{2C(R)} \int d^4x \left( J_+ (x^+, x_-) \left( \partial_+^2 \right)^{-1} J_- (x^-, x_-) + \frac{i}{C(R)} \int d^4x \left( J_+ (x^+, x_-) B_- (x^-, x_-) + \frac{i}{C(R)} \int d^4x \left( J_- (x^-, x_-) B_+ (x^+, x_-) \right) \right) \right. \]

see Eq. (2) definition. Now we can define an arbitrary correlator of the \( \hat{O} \) operators as
\[ \langle \left( T^{a_1} \hat{O}_1 (v_+) \right) \otimes \left( T^{a_2} \hat{O}_2 (v_+) \right) \otimes \cdots \otimes \left( T^{a_n} \hat{O}_n (v_+) \right) \rangle = C(R)^n \left( -2 g \right)^n \left( \frac{\delta}{\delta J_{a_1}^+ \cdots \delta J_{a_n}^+} \log Z[J] \right) \]

The general expression for the r.h.s of Eq. (38) is cumbersome\(^7\), therefore we write the general expression in the following form:
\[ \langle \left( T^{a_1} \hat{O}_1 (v_+) \right) \otimes \left( T^{a_2} \hat{O}_2 (v_+) \right) \otimes \cdots \otimes \left( T^{a_n} \hat{O}_n (v_+) \right) \rangle = C(R)^n g^n \int dx_{1+} \cdots \int dx_{n+} \left( \sum_{j=0}^{\infty} C_{b_1 \cdots b_{n-2j}}^{a_1 \cdots a_n} \left( \partial_-^{2j} \right)^n (2i)^{-n-j} B_{b_1 \cdots b_{n-2j}}^b \right) , \]

where \( J = \frac{n}{2} \) or \( J = \frac{n-1}{2} \) for the even or odd \( n \) correspondingly and coefficients \( C_{b_1 \cdots b_{n-2j}}^{a_1 \cdots a_n} \) is a product of all possible \( \delta_{a_i b_j} \) delta functions which account all color indexes permutations in the expression. Taking derivative of this expression in respect to the rapidity dependence of the reggeon fields correlators, see Eq. (17), we obtain the evolution equation for the correlators of \( \hat{O} \) operators as well:
\[ \frac{\partial}{\partial \eta} \langle \left( T^{a_1} \hat{O}_1 (v_+) \right) \otimes \left( T^{a_2} \hat{O}_2 (v_+) \right) \otimes \cdots \otimes \left( T^{a_n} \hat{O}_n (v_+) \right) \rangle = C(R)^n g^n \int dx_{1+} \cdots \int dx_{n+} \left( \sum_{j=0}^{\infty} C_{b_1 \cdots b_{n-2j}}^{a_1 \cdots a_n} \left( \partial_-^{2j} \right)^n (2i)^{-n-j} \frac{\partial}{\partial \eta} B_{b_1 \cdots b_{n-2j}}^b \right) \]

which determines the evolution of the Wilson line like operators of interests in terms of the effective vertices of reggeon fields interactions, see Eq. (11), Eq. (12)-Eq. (17). We conclude also, that due the structure of the reggeon fields in the r.h.s of the expressions, see details in [5], the LO contribution to the correlators of the \( \hat{O} \) operators is pure transverse in the quasi-multi-Regge kinematics for which the Lipatov’s effective action is defined.

Now consider the equation for the correlator of two Wilson line operators. Writing explicitly Eq. (39) for \( n = 2 \) we obtain:
\[ \langle tr \left( T^a \hat{O}_1 (v_+) \right) \otimes tr \left( T^b \hat{O}_1 (v_-) \right) \rangle = 4C(R)^2 g^2 \int dx^+ \int dy^- \left( -\frac{i}{2} \hat{G}_0^{a b} + \langle B_0^a x B_0^b y \rangle \right) . \]

Multiplying Eq. (11) on \( T^a \) and \( T^b \) matrices and using completeness identities for the matrices in the fundamental representation, we can rewrite Eq. (11) in the large \( N_c \) limit as
\[ \langle \hat{O}_{ij} (v_+ \otimes \hat{O}_{kl} (v_-) \rangle = -g^2 \int dx^+ \int dy^- \left( i \delta_{ij} \delta_{jk} \hat{G}_0^\perp + 4 \langle (B_+ x)_{ij} (B_- y)_{kl} \rangle \right) . \]

\(^7\)The structure of the answer is easy reconstructed if we put attention that a relative dimension of the \( \partial_-^{2j} \) operator in terms of \( B \) or \( \hat{O} \) operators is 2. Therefore, requesting that the dimension of the r.h.s and l.h.s of the Eq. (38) will be equal we arrive to the following expression:
\[ \langle \hat{O}_1 \cdots \hat{O}_n \rangle \propto \sum_{j=0}^{\infty} \partial_-^{2j} B_1 \cdots B_{n-2j} \rangle , \]

where the sum over \( j \) is going till \( n/2 \) or \( n-1)/2 \) in the case of even or odd \( n \) correspondingly.
We obtain, that the expression is transverse to the leading order and in the shock wave approximation, where \( B_\pm = \delta(x^\pm) \beta_\pm(x_\perp) \) is assumed, it can be reduced to the well known JIMWLK-Balitsky equation for the correlators of the Wilson lines, see [12,16,17], with the use of the correspondingly simplified vertices in large \( N_c \) limit in Eq. (15.3), see [19,21]. Namely, in shock wave approximation the correlator of reggeon fields can be simply expressed through the correlator of Lipatov’s operator in Eq. (41) and inserting obtained expression back in Eq. (31) we reproduce the Balitsky-JIMWLK like equation for the correlators in large \( N_c \) limit.

Nevertheless, in general, \( B_\pm = B_\pm(x^\pm,x_\perp) + D(x^\pm,x^\mp,x_\perp) \) with \( D \) as some corrections to the mean value of reggeized field \( B \), see [5], and therefore the Eq. (41) can not be precisely transverse in the full kinematical region of interest. Additionally, due the complex structure of the all-order equation for the two-field correlator, see Appendix B, the dependence on the \( x^- \) and \( y^+ \) variables in Eq. (41) can arises also through the family of the non-local vertexes \( K \) in Eq. (B.3) beyond shock wave and large \( N_c \) approximations.

5 Conclusion

In this paper we investigated two interrelated tasks: construction of a hierarchy of correlators of reggeized gluon fields in the formalism of Lipatov’s effective action and connection of these correlators to the correlators of operators of Wilson lines built from the longitudinal gluon fields. The later objects are related to the correlators of Wilson lines which are described by Balitsky-JIMWLK hierarchy of equations obtained with the use of shock wave and large \( N_c \) approximations in high energy scattering, [12,16,17].

Formulated as RFT, the Lipatov’s approach to high energy scattering allows to derive the equations for the all types of correlators of the fields of reggeized gluons. Formally, in the language of BFKL physics, each correlator represents some bound state of the reggeons, i.e. Eq. (B.3) correlator, for example, represents a bound state of two reggeized gluon fields that corresponds to the propagator of reggeized gluons in regular BFKL calculus. Consequently, Eq. (D.4) correlator, is a bound state of four reggeized fields and represents BFKL Pomeron like bound state which is after a suitable projection of color indexexes will represent the BFKL Pomeron. Other correlators, considered in the paper, are not widely used in the high energy scattering approaches but they are important from the point of view of an accounting of the non-leading unitarity corrections to the scattering amplitudes, see [1,10–13].

The subsequent solution of the RFT equations of the hierarchy, see Appendixes A-D, with all vertices included, allows to determine the corrections to the leading poles of the correlators in the momentum space in the way different from the QCD perturbative scheme. The possible interconnection between the calculations of the kernels of interest in BFKL calculus, see [19,21], and RFT calculus it is a interesting problem which we we plan investigate in the future. We also note, that usual form of BFKL like evolution equations, see [11,9,21], arises naturally in the RFT as a consequence of the dependence of the effective vertices of the theory on the ultraviolet cut-off of the longitudinal momenta in the expressions, see Eq. (17).

We note also, that the system of equations for the correlators Eq. (16)-Eq. (17) determines any correlator of interest in terms of bare RFT propagator Eq. (B.4) and vertices of the effective action only, which is regular property of Dayson-Schwinger hierarhy derived with the help of particular perturbative scheme. The vertices in the Lipatov’s action are responsible for the account of the unitarity corrections to the amplitude due the general non-conservation of the number of reggeized gluons in the scattering amplitude, see also [10,13]. In this case, quantum corrections to the amplitudes are given in the framework of RFT, whereas initially the expressions for the vertices are determined by perturbative QCD only. Therefore, another important task for the future research is a comparison of

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8We plan to rederive this equation in some separate publication, see also [14].

9We note, that all these equations are integral ones and can be reduced to the form of evolution equations with respect to rapidity variable after the differentiation of the expression with respect to \( \eta \), see Eq. (12) and Eq. (17).
the perturbative corrections to the vertices arising from the QCD calculations, i.e., in BFKL formalism, with the perturbative corrections to the same vertices determined by the Dayson-Schwinger hierarchy of the equations derived in the RFT calculus formalism. Additionally, it will be interesting to investigate the connection between correlators of two and four reggeized gluons. It is known that the Hamiltonian systems for the both bound states are integrable, see \[1, 21, 22\]. In the RFT formalism these states are not independent and it is possible that some interconnection between the states on the language of integrable systems exists, see \[23\] for some discussion concerning this point.

Basing on results of \[5\], we rewrite the Lipatov’s effective action in the form of another effective action without reggeized gluons present. The later action describes an Wilson lines operators (Lipatov’s operators) built from the longitudinal gluons interacting in two-dimensional transverse plane with the help of corresponding two-dimensional propagator. An averaging of this interaction term over the gluon fields leads to the precise Lipatov’s action form, in which, nevertheless, the form of the corresponding Lipatov’s operators can be different, see Eq. (4)-Eq. (5) and Eq. (34)-Eq. (36) in the paper and remarks in \[5\]. The important advantage of Eq. (32) new action is that it allows to connect the reggeized gluons correlators with the correlators of Wilson lines operators built from gluons beyond any simplifying approximations, see Eq. (39)-Eq. (40). It is interesting to note also, that requesting Hermicity of the Lipatov’s operators, the form of the Lipatov’s action will be different from the standard one to the non-leading orders, see Eq. (5). That, in turn, affects on the form of non-leading corrections for the vertices in Eq. (6) that means the different expressions in the r.h.s. of Eq. (39)-Eq. (40) which correspond to the different combinations of the Wilson lines in the correlators in the l.h.s. of the equations. Namely, the correlators for the regular Wilson lines Eq. (34) will be different from the correlators of the Eq. (35) Lipatov’s operators and this difference is introduced by the different non-leading corrections to the effective vertices appearing in the correlators of the reggeon in the r.h.s. of the equations.

Taking large \(N_c\) and shock-wave approximations in Eq. (39) or Eq. (40), the Balitsky-JIMWLK like hierarchy of equations can be derived inside the framework of the formalism, see Eq. (11)-Eq. (12) and \[14\]. We also obtained, that the knowledge of the correlators of the reggeized gluon fields, i.e., vertices of Eq. (6) action, will allow to calculate any sub-leading correction in the corresponding Balitsky-JIMWLK hierarchy taking large \(N_c\) limit in the final expressions for the vertices, see \[19\]. Thereby, the connection between the different high energy QCD approaches is clarified that can help in further investigations of the subject.

To conclude, we developed an approach to the calculation of the correlators of reggeized gluons based on the Dayson-Schwinger derivation of the hierarchy of the correlators in QFT. The equations obtained for the different correlators in the proposed RFT allows to investigate unitarity connections to the in the amplitudes in BFKL physics from the some new angle of view and hopefully will be useful for the calculation and verification of high order unitarity corrections in the high energy QCD. The relation of these correlators to the correlators of the Wilson lines operators built from the longitudinal gluons is also clarified and results of this relation, we hope, will help to understand general landscape of the high energy QCD approaches.

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Appendix A: Correlators of single reggeon field

We consider Eq. (10) and Eq. (11) together and obtain the following equations for the quantum $B^\pm_+$ reggeon field:

$$\partial^2_\perp < B^a_+ > = (K_{a_1})^+_+ < B^{a_1}_+ > + (K^{a_1}_a)_-^+ < B^{a_1}_+ > + (K^{a_1}_{a_2})_+^- < B^{a_1}_a B^{a_2}_+ > +$$

$$+ 2 (K^{a_1}_{a_2})_+^- < B^{a_1}_+ B^{a_2}_+ > + 3 (K^{a_1}_{a_2 a_3})^-_+ < B^{a_1}_+ B^{a_2}_a B^{a_3}_+ > +$$

$$+ 3 (K^{a_1}_{a_2 a_3})_+^- < B^{a_1}_+ B^{a_2}_a B^{a_3}_+ > + 4 (K^{a_1}_{a_2 a_3})_+^- < B^{a_1}_+ B^{a_2}_a B^{a_3}_+ > + \cdots$$

(A.1)

and

$$\partial^2_\perp < B^a_+ > = (K^{a_1}_a)^+_+ < B^{a_1}_+ > + (K_{a_1})^-_- < B^{a_1}_+ > + (K^{a_1}_{a_2 a_3})_+^- < B^{a_1}_+ B^{a_2}_+ B^{a_3}_+ > +$$

$$+ 2 (K^{a_1}_{a_2})_+^- < B^{a_1}_+ B^{a_2}_+ > + 3 (K^{a_1}_{a_2 a_3})^-_+ < B^{a_1}_+ B^{a_2}_a B^{a_3}_+ > +$$

$$+ 3 (K^{a_1}_{a_2 a_3})_+^- < B^{a_1}_+ B^{a_2}_a B^{a_3}_+ > + 4 (K^{a_1}_{a_2 a_3})_+^- < B^{a_1}_+ B^{a_2}_a B^{a_3}_+ > + \cdots$$

(A.2)

Now, using Eq. (13.3)-Eq. (13.5) expressions, we will get for the leading contributions to the quantum reggeon fields:

$$< B^a_+ > = -2 i G_0^{a_1} (K^{a_2 a_3})_+^- G_0^{a_2 a_3} + 2 G_0^{a a_1} (K^{a_2 a_3})_+^- < B^{a_2}_a B^{a_3}_+ >_1 =$$

$$= -2 i G_0^{a_1} (K^{a_2 a_3})_+^- G_0^{a_2 a_3} + 2 G_0^{a a_1} (K^{a_2 a_3})_+^- < B^{a_2}_a B^{a_3}_+ >_1$$

(A.3)

and

$$< B^a_+ > = -2 i G_0^{a_1} (K^{a_2 a_3})_+^- G_0^{a_2 a_3} + 2 G_0^{a a_1} (K^{a_2 a_3})_+^- < B^{a_2}_a B^{a_3}_+ >_1 =$$

$$= -2 i G_0^{a_1} (K^{a_2 a_3})_+^- G_0^{a_2 a_3} + 2 G_0^{a a_1} (K^{a_2 a_3})_+^- < B^{a_2}_a B^{a_3}_+ >_1$$

(A.4)

These expressions contain tadpole contributions built from the reggeon propagator, which is not enhanced due to the presence of the rapidity ordering of the reggeon fields in the propagator, see [1, 2]. Additionally, to leading order, these effective vertices are proportional to $f_{abc}$ anti-symmetrical structure, therefore further we take:

$$< B^a_+ > \approx 0$$

(A.5)

and

$$< B^a_+ > \approx 0,$$

(A.6)

that corresponds to the signature conservation law as well. We also note, that the coefficient 2 in the front of Eq. (A.3)-Eq. (A.4) arises due the fact we do not symmetrize the expressions in respect to $a_1$ and $a_2$ indexes. We will keep these form of the shorthand notations further in all the places where it will not lead to any confusion and will write the precise symmetric expressions in respect to the indexes where it will be important.
Appendix B: Correlators of two reggeon fields

The $<B^a_+ B^{a_1}_+>$ correlator of the reggeon fields is corresponding to the propagator of the reggeized gluons, whereas $<B^a_+ B^{a_1}_->$ and $<B^a_- B^{a_1}_->$ correlators are suppressed perturbatively in comparison to the first one and represent some kind of the ”mass” terms of the reggeon fields. Therefore, in the expression for the last two propagators only the first terms will be presented. We have for the correlators of these fields:

$$\partial^2_+ <B^a_+ B^{a_1}_+> = (K^a_{a_2})^+ - <B^{a_2}_+ B^{a_1}_+> + 2 (K^{a_2 a_1})^{++} <B^{a_2}_+ B^{a_1}_-> + \cdots$$ (B.1)

and correspondingly

$$\partial^2_+ <B^a_- B^{a_1}_-> = (K^a_{a_2})^- - <B^{a_2}_- B^{a_1}_-> + 2 (K^{a_2 a_1})^{--} <B^{a_2}_- B^{a_1}_+> + \cdots$$ (B.2)

For the correlator of $\pm$ reggeon fields we obtain:

$$\partial^2_+ <B^a_+ B^{a_1}_-> = -\delta^{a_1 a} + (K^a_{a_2})^+ - <B^{a_2}_+ B^{a_1}_-> + 2 (K^{a_2 a_1})^{++} <B^{a_2}_+ B^{a_1}_-> +$$

$$+ 3 (K^{a_2 a_1 a_2})^{--} <B^{a_2}_+ B^{a_1}_-> + <(K^{a_2 a_3 a_4})^{++} <B^{a_2}_+ B^{a_3}_+ B^{a_4}_- B^{a_1}_-> +$$

$$+ 2 (K^{a_3 a_4 a_2})^{--} <B^{a_3}_+ B^{a_2}_- B^{a_1}_-> + 3 (K^{a_4 a_2 a_3})^{--} <B^{a_4}_+ B^{a_3}_- B^{a_2}_- B^{a_1}_+> +$$

$$+ 4 (K^{a_2 a_3 a_4 a_2})^{--} <B^{a_2}_+ B^{a_3}_- B^{a_2}_- B^{a_1}_-> + \cdots$$ (B.3)

Now we can introduce the ”bare” propagator of the reggeon fields as

$$\left(\delta^{a b} \partial^2_+ - (K^a_b)^+ \right) G^{bc}_0 = \delta^{a c}$$ (B.4)

and obtain the following expression for the correlator:

$$<B^a_+ B^{a_1}_-> = <B^a_+ B^{a_1}_-> + <B^a_+ B^{a_1}_- >_1 = -i G^{ab}_0 + <B^a_+ B^{a_1}_- >_1 = -i \delta^{ab} G^{ab}_0 + <B^a_+ B^{a_1}_- >_1$$ (B.5)

with

$$<B^a_+ B^{a_1}_- >_1 \sim \delta^{ab}$$ (B.6)

as well. Therefore, we have for the leading contributions to the r.h.s. of Eq. [B.1] and Eq. [B.2]:

$$<B^a_+ B^{a_1}_-> = -2 i G^{a_2 a_1}_0 + (K^{a_1 a_2})^{++} G^{a_2 b}_0 + 2 G^{a_2 a_1}_0 + (K^{a_1 a_2})^{++} <B^{a_2}_+ B^{a_1}_- >_1$$ (B.7)

and

$$<B^a_+ B^{a_1}_-> = -2 i G^{a_2 a_1}_0 + (K^{a_1 a_2})^{--} G^{b_2 a}_0 + 2 G^{a_2 a_1}_0 + (K^{a_1 a_2})^{--} <B^{a_2}_+ B^{a_1}_- >_1 .$$ (B.8)

Now, using Eq. (B.3), Eq. (B.3) reads as:

$$<B^a_+ B^{a_1}_- >_1 = 2 G^{a b}_0 + (K^{b_1 b_1}_0) \cdot \cdot \cdot <B^{b_1}_+ B^{a_1}_- > + C^{a b}_0 + (K^{b_1 b_2}_0) \cdot \cdot \cdot <B^{b_1}_+ B^{b_2}_- B^{a_1}_- > +$$

$$+ 2 C^{a b +} (K^{b_1 b_2}_0)^+ \cdot \cdot \cdot <B^{b_1}_+ B^{b_2}_- B^{a_1}_- > + 3 C^{a b +} (K^{b_1 b_1}_0) \cdot \cdot \cdot <B^{b_1}_+ B^{b_2}_- B^{a_1}_- > +$$

$$+ 2 C^{a b +} (K^{b_1 b_2}_0)^+ \cdot \cdot \cdot <B^{b_1}_+ B^{b_2}_- B^{a_1}_- > .$$ (B.9)
Inserting LO values of Eq. (B.7), Eq. (C.7)-Eq. (C.8) and Eq. (D.3) expressions back into the Eq. (B.3), we obtain the following LO answer for the two reggeon fields correlator:

\[ \langle B^a_+ B^{a_1}_- \rangle_1 = -4 \left( G^{ab}_{0-} + (K_{k_{b1}})_{-} G^{b_{1b_2}}_{0-} + (K^{b_2b_3})_{-} G^{b_{b_4}a_1}_0 + - \right) \]

\[ -4 G^{ab}_{0-} + \left( K^{b_{1b_2}}_{k_{b1}} \right)_{-} G^{b_{1b_2}}_{0-} + G^{b_{b_2}b_3}_{0-} + \left( K^{b_{b_3b_4}}_{b_{b_2}} \right)_{-} G^{b_{b_4}a_1}_0 + - \]

\[ -6 G^{ab}_{0-} + \left( K^{b_{1b_2}}_{b_{b_2}} \right)_{-} G^{b_{b_3}b_4}_{0-} + G^{b_{b_4}b_5}_{0-} + \left( K^{b_{b_6b_7}}_{b_{b_5}} \right)_{-} G^{b_{b_7}a_1}_0 + - \]

\[ -4 G^{ab}_{0-} + \left( K^{b_{1b_2}}_{b_{b_2}} \right)_{-} G^{b_{b_3}b_4}_{0-} + G^{b_{b_4}b_5}_{0-} + \left( K^{b_{b_6b_7}}_{b_{b_5}} \right)_{-} G^{b_{b_7}a_1}_0 + - \]

\[ -18 G^{ab}_{0-} + \left( K^{b_{1b_2}}_{b_{b_2}} \right)_{-} G^{b_{b_3}b_4}_{0-} + G^{b_{b_4}b_5}_{0-} + \left( K^{b_{b_6b_7}}_{b_{b_5}} \right)_{-} G^{b_{b_7}a_1}_0 + - \]

\[ -4 G^{ab}_{0-} + \left( K^{b_{1b_2}}_{b_{b_2}} \right)_{-} G^{b_{b_3}b_4}_{0-} + G^{b_{b_4}b_5}_{0-} + \left( K^{b_{b_6b_7}}_{b_{b_5}} \right)_{-} G^{b_{b_7}a_1}_0 + . \]

(B.10)

Comparing the different contributions in the r.h.s. of the expression we see that the main contribution to the correlator comes from the last term in r.h.s. of Eq. (B.10).
Appendix C: Correlators of three reggeon fields

There are the following correlators of the three reggeon fields which we have to calculate, the first one is the following.

\[
\partial_+^2 \langle B^a_+ B^{a_1}_+ B^{a_2}_- \rangle = -i \delta^{aa_2} \langle B^{a_1}_+ \rangle + (K_{a_3}^{a_1})^+ \langle B^{a_3}_+ B^{a_1}_+ B^{a_2}_- \rangle + 2 (K_{a_3 a_4})^- \langle B^{a_3}_+ B^{a_1}_- B^{a_2}_+ \rangle + 2 (K_{a_4 a_1})^+ \langle B^{a_4}_- B^{a_3}_+ B^{a_2}_+ \rangle + 3 (K_{a_4 a_3 a_1})^+ \langle B^{a_4}_- B^{a_3}_- B^{a_1}_+ B^{a_2}_+ \rangle .
\]

(C.1)

The second one can be obtained from the first one by replace \( + \) on \( - \) in the expression:

\[
\partial_+^2 \langle B^a_+ B^{a_1}_+ B^{a_2}_- \rangle = -2i \delta^{aa_2} \langle B^{a_1}_+ \rangle + (K_{a_3}^{a_1})^+ \langle B^{a_3}_+ B^{a_1}_- B^{a_2}_+ \rangle + 2 (K_{a_3 a_4})^- \langle B^{a_3}_+ B^{a_1}_- B^{a_2}_+ \rangle + 2 (K_{a_4 a_1})^+ \langle B^{a_4}_- B^{a_3}_+ B^{a_1}_- B^{a_2}_+ \rangle + 3 (K_{a_4 a_3 a_1})^+ \langle B^{a_4}_- B^{a_3}_- B^{a_1}_- B^{a_2}_+ \rangle .
\]

(C.2)

and the third one:

\[
\partial_+^2 \langle B^a_+ B^{a_1}_- B^{a_2}_- \rangle = (K_{a_3}^{a_1})^+ \langle B^{a_3}_- B^{a_1}_- B^{a_2}_- \rangle + 2 (K_{a_3 a_4})^- \langle B^{a_3}_- B^{a_1}_- B^{a_2}_- \rangle + (K_{a_3 a_4}^{a_1})^+ \langle B^{a_3}_- B^{a_1}_+ B^{a_2}_- \rangle + 3 (K_{a_4 a_3 a_1})^+ \langle B^{a_1}_- B^{a_4}_+ B^{a_3}_- B^{a_2}_- \rangle .
\]

(C.3)

This system of equations can be solved perturbatively, using results of the Appendix A and Appendix D, here we will use the symmetric expression for Eq. (D.3):

\[
\langle B^a_+ B^{a_1}_+ B^{a_2}_- \rangle = -G_{0 a_3}^{a_1} - G_{0 a_2}^{a_1} - G_{0 a_3}^{a_2} - G_{0 a_3}^{a_1} + - i G_{0 a_3} + \langle B^{a_1}_+ B^{a_2}_- \rangle + i G_{0 a_2}^{a_1} + \langle B^{a_2}_- B^{a_1}_- \rangle .
\]

(C.4)

We obtain for Eq. (C.3):

\[
\langle B^a_+ B^{a_1}_- B^{a_2}_- \rangle = 2 G_{0 b_2}^{a a_2} + (K_{b_2 b_3 b_4}^{b_1})^+ - 6 G_{0 a_2}^{a b_2} + (K_{b_2 b_3 b_4}^{b_1})^+ + (G_{0 a_3}^{b a_2} + G_{0 a_1}^{b a_2} + i G_{0 a_2}^{b a_1} + \langle B^{b_3}_- B^{a_1}_- \rangle ) ,
\]

(C.5)

that to leading order gives

\[
\langle B^a_+ B^{a_1}_- B^{a_2}_- \rangle = -6 G_{0 a_2}^{a b_2} + (K_{b_2 b_3 b_4}^{b_1})^+ + (G_{0 b_2}^{a a_3} + G_{0 b_2}^{a a_3} + i G_{0 b_2}^{a b_2} + \langle B^{a_1}_+ B^{a_2}_+ \rangle ) .
\]

(C.6)

Correspondingly, for Eq. (C.1) we have to leading order:

\[
\langle B^a_+ B^{a_1}_+ B^{a_2}_- \rangle = -i G_{0 a_2}^{a_1} + \langle B^{a_1}_+ \rangle + 2 G_{0 b_2}^{a b_2} + (K_{b_2 b_1}^{b_1})^+ - 4 G_{0 b_2}^{a b_2} + (K_{b_2 b_1}^{b_1})^+ + 2 i G_{0 b_2}^{a b_2} + (K_{b_2 b_1}^{b_1})^+ + 2 G_{0 a_2}^{a b_2} + (K_{b_2 b_1}^{b_1})^+ + 2 i G_{0 a_2}^{a b_2} + (K_{b_2 b_1}^{b_1})^+ - i G_{0 a_2}^{a b_2} + (K_{b_2 b_1}^{b_1})^+ + \langle B^{a_1}_+ B^{a_2}_+ \rangle - i G_{0 a_2}^{a b_2} + (K_{b_2 b_1}^{b_1})^+ + \langle B^{a_1}_+ B^{a_2}_+ \rangle .
\]

(C.7)

we note, that we wrote Eq. (C.7) in the mostly symmetrical way, whereas, for example, there is no difference between two last terms in the expression. Correspondingly, the answer for Eq. (C.2) can be
obtained from Eq. (C.7) by replace of the + and − signs in Eq. (C.7):

\[
\langle -B_a^1 B_a^1 B_a^2 \rangle = -2 \bar{G}_0^a + \bar{G}_0^b + K_{b_1}^{b_1 b_2} \bar{G}_0^1 - \langle -B_a^1 B_a^1 B_a^2 \rangle =
\]

\[
= -4 G_0^- + G_0^b + K_{b_1}^{b_1 b_2} - 2 \bar{G}_0^- + 2 G_0^b - K_{b_2}^{b_2 b_1} \bar{G}_0^1 + \langle -B_a^1 B_a^1 B_a^2 \rangle =
\]

\[
- 2 \bar{G}_0^- + (K_{b_1}^{b_1 b_2})^{b_1 b_2} + G_0^b + G_0^a + 2 \bar{G}_0^- + 2 G_0^b + (K_{b_2}^{b_2 b_1})^{b_2 b_1} \bar{G}_0^1 + \langle -B_a^1 B_a^1 B_a^2 \rangle =
\]

\[
- 2 \bar{G}_0^- + (K_{b_1}^{b_1 b_2})^{b_1 b_2} + G_0^b + G_0^a + \langle -B_a^1 B_a^1 B_a^2 \rangle =
\]

\[
(C.8)
\]

that can be verified by the direct calculation of Eq. (C.2).
Appendix D: Correlators of four reggeon fields

We limit the chain of the equations by the 4-reggeon correlators, therefore the equations for these correlators are simple. We have for the symmetrical correlator of four reggeon fields:

$$\partial_+^2 <B^a_+B^a_+B^a_+B^a_+> = -2i\delta^{aa_3} <B^a_+B^a_+> + (K^{a_4})^+ <B^{a_4}B^{a_1}B^{a_2}B^{a_3}> + 2(K_{aa_4})_+ <B^{a_4}B^{a_1}B^{a_2}B^{a_3}>, \quad (D.1)$$

where correspondingly:

$$\partial_+^2 <B^a_-B^a_-B^a_-B^a_-> = -i\delta^{aa_3} <B^a_-B^a_-> + (K^{a_4})^+ <B^{a_4}B^{a_1}B^{a_2}B^{a_3}> + 2(K_{aa_4})_+ <B^{a_4}B^{a_1}B^{a_2}B^{a_3}>, \quad (D.2)$$

Solving these equations perturbatively and using Eq. (B.5)-Eq. (B.7) we obtain:

$$<B^a_+B^a_+B^a_+B^a_+> = -2iG_{0-}^{aa_3} + <B^a_+B^a_+> = -2G_{0-}^{aa_3} + G_{0-}^{a_1a_2} - 2iG_{0-}^{aa_3} <B^{a_1}B^{a_2}> , \quad (D.3)$$

and

$$<B^a_-B^a_-B^a_-B^a_-> = -6G_{0-}^{aa_3} + G_{0-}^{a_1b} + (K^{bb_1})^{++} G_{0-}^{b_1a_2} + , \quad (D.4)$$

We note, that the leading contribution of Eq. (D.1) correlator is to $g^2$ order, i.e. a leading order of the $K^{++}$ vertex is $g^0$ at least. Corresponding $<B_-B_-B_+B_->$ and $<B_+B_+B_-B_->$ correlators can be obtained from the Eq. (D.3)-Eq. (D.3) expressions by the change + to − and vice versa in the effective vertices in the expressions.

For the other correlators we correspondingly have:

$$\partial_+^2 <B^a_+B^a_+B^a_+B^a_+> = -3i\delta^{aa_1} <B^a_+B^a_+> + (K^{a_4})^+ <B^{a_4}B^{a_1}B^{a_2}B^{a_3}> + 2(K_{aa_4})_+ <B^{a_4}B^{a_1}B^{a_2}B^{a_3}>, \quad (D.5)$$

and

$$\partial_+^2 <B^a_-B^a_-B^a_-B^a_-> = (K^{a_4})^+ <B^{a_4}B^{a_1}B^{a_2}B^{a_3}> + 2(K_{aa_4})_+ <B^{a_4}B^{a_1}B^{a_2}B^{a_3}>, \quad (D.6)$$

Solving the system perturbatively, we obtain:

$$<B^a_-B^a_-B^a_-B^a_-> = 2G_{0-}^{ab} + (K^{bb_1})^{++} <B^{b_1}B^{a_1}B^{a_2}B^{a_3}>, \quad (D.7)$$

that provides for the first correlator:

$$<B^a_+B^a_+B^a_+B^a_+> = -6\left[ (\delta - 4G^+_0K_-G^+_0K^{++})_a^b \right]^{-1} G^{b_1a_1}_0 + G^{a_2b_2}_0 + (K^{b_2b_3})^{++} G^{b_3a_3}_0 + , \quad (D.8)$$

or to leading approximation

$$<B^a_+B^a_+B^a_+B^a_+> = -6G_{0-}^{aa_1} + G_{0-}^{a_2b_2} + (K^{b_2b_3})^{++} G_{0-}^{b_3a_3} + , \quad (D.9)$$

that precisely reproduce Eq. (D.4) expression. Hence we obtain for Eq. (D.7):

$$<B^a_-B^a_-B^a_-B^a_-> = -12G_{0-}^{ab} + (K^{bb_1})^{++} G_{0-}^{b_1a_1} + G_{0-}^{a_2b_2} + (K^{b_2b_3})^{++} G_{0-}^{b_3a_3} + . \quad (D.10)$$
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