Deterministic generation of N00N states using quantum dots in a cavity

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Compared to classical light sources, quantum sources based on N00N states consisting of N photons achieve an N-times higher phase sensitivity, giving rise to super-resolution. N00N-state creation schemes based on linear optics and projective measurements only have a success probability p that decreases exponentially with N, e.g., p = 4.4 × 10^{-14} for N = 20. Feed-forward improves the scaling but N fluctuates nondeterministically in each attempt. Schemes based on parametric down-conversion suffer from low production efficiency and low fidelity. A recent scheme based on atoms in a cavity combines deterministic time evolution, local unitary operations, and projective measurements. Here we propose a novel scheme based on the off-resonant interaction of N photons with four semiconductor quantum dots (QDs) in a cavity to create N00N states deterministically with p = 1 and fidelity above 90% for N ≤ 60, without the need of any projective measurement or local unitary operation. Using our method we obtain maximum N-photon entanglement E_N = 1 for arbitrary N. Our method paves the way to the miniaturization of N00N-state sources to the nanoscale regime, with the possibility to integrate them on a computer chip based on semiconductor materials.

In quantum metrology, the creation of a photonic Greenberger-Horne-Zeilinger (GHZ) state also known as N00N state consisting of N photons, can be used to achieve ultrahigh phase sensitivity providing viable mechanisms to beat the shot-noise limit in optical interferometry and enabling super-resolution by beating the Rayleigh diffraction limit by a factor of N. Since the demonstration that two-photon N00N states exhibit super sensitivity and super resolution, impressive progress has been made to experimentally realize N = 3, N = 4, and N = 6 N00N states. Recently an N = 8 N00N polarization state with a fidelity of 70% has been created. It is well known that coherent Schrödinger cat states \(|\psi_{CSC}\rangle = (|+\rangle + |-\rangle)/\sqrt{2 + 2e^{-2|\alpha|^2}}\) can be produced by a combination of Mach-Zehnder interferometer, cross-Kerr interaction, and postselection which can also be used to create N00N states. Alternatively, N00N states can be produced by means of linear optics and postselection with a success rate that decays exponentially with N. Here we demonstrate that N photons in a cavity interacting off-resonantly with four QDs evolve deterministically from a non-entangled product state \(|\psi_0\rangle\) into a N00N state \(|N:\psi\rangle\) with probability p = 1 for arbitrary N and with fidelity above 90% for N ≤ 60. The time evolution is governed by an effective photon Hamiltonian which we derive using our many-photon entanglement formalism based on Schwinger angular momentum operators. The loading of the Fock state \(|N:\psi\rangle\) into the cavity can be done deterministically using simulated Raman adiabatic passage (STIRAP). We adapt the loading process to QDs.

Our goal is to create the N-photon N00N state

\[|N:\psi\rangle = \frac{1}{\sqrt{2}} \left(|N\rangle_+ |0\rangle_- + |0\rangle_+ |N\rangle_- \right) \tag{1} \]

in terms of the polarization degree of freedom, where + and − denote right and left circular polarization, resp. Operating with a phase shifter for the − mode of the form \(U_{PG} = \exp(i\delta a_+^\dagger a_-)\) on the N00N state, one obtains

\[U_{PG} |N:\psi\rangle = \frac{1}{\sqrt{2}} \left(|N\rangle_+ e^{iN\delta} |0\rangle_- + |0\rangle_+ e^{-iN\delta} |N\rangle_- \right) \tag{2} \]

which improves the phase sensitivity by a factor of N compared to a single-photon state

\[U_{PG} |1:\psi\rangle = \frac{1}{\sqrt{2}} \left(|1\rangle_+ e^{iN\delta} |0\rangle_- + |0\rangle_+ e^{-iN\delta} |1\rangle_- \right) \tag{3} \]

Let us first focus briefly on the loading of the Fock state \(|N:\psi\rangle\) into the cavity. It has been shown in Ref. that N atoms can be used to create a N-photon Fock state inside the cavity by means of STIRAP. Instead of using atoms, we propose to use QDs with compressive strain in growth plane, such as standard GaAs QDs embedded in Al_{x}Ga_{1-x}As. In the case of compressive strain, the light hole states lie below the heavy hole states at an energy \(\Delta\), as shown in Fig. 1. The advantage of using QDs is that it is possible to select the three states connected by the red (blue) transitions for creating the right-(left-) circularly polarized \(|N:\psi\rangle\) state by means of a left-(right-)circularly polarized electric field of the pump laser \(E_{p,-} = \partial A_{p,-}/\partial t\) (\(E_{p,+} = \partial A_{p,+}/\partial t\)). The classical pump laser field is given by the vector potential

\[A_{p,\pm} = \frac{1}{(2\pi)^{3/2}} \int d^3k e_{k,\pm} \frac{1}{\sqrt{2\omega_{k,\pm}}} e^{i\mathbf{k}\cdot\mathbf{x}} a_{k,\pm} + \text{h.c.}, \tag{4} \]

where \(e_{k,\pm}\) are the unit polarization vectors, \(\mathbf{x}\) is the coordinate, and \(a_{k,\pm}\) is the photon annihilation operator. The Hamiltonian describing the STIRAP process in the rotating frame is given by

\[H_{\text{STIRAP}} = \delta \left(|-\frac{1}{2}\rangle_c |0\rangle_+ - \frac{1}{2} |\frac{1}{2}\rangle_c |0\rangle_+ + |\frac{1}{2}\rangle_c |0\rangle_+ + |\frac{1}{2}\rangle_c |0\rangle_+ \right) + \Omega_{p,-}(t) \left(|-\frac{1}{2}\rangle_c |0\rangle_+ + |\frac{1}{2}\rangle_c |0\rangle_+ - \frac{1}{2} |\frac{1}{2}\rangle_c |0\rangle_+ \right) \tag{5} \]

where \(\Omega_{p,-}(t)\) is the electric field strength, and \(\delta\) is the detuning with respect to the right-hand circular transition.
and the red transitions give rise to the $|0, N\rangle$ state, and the red transitions give rise to the $|N, 0\rangle$ state inside the cavity.

$$
\Omega_{p,\mp}(t)\left|\frac{1}{2}\right\rangle_c \langle 0 | - \frac{1}{2} \right|_v \langle 0 | + \left| - \frac{1}{2} \right\rangle_v \langle 0 | \langle 1 \right|_c \langle 0 | \right) + \Omega_{c,\mp}\left| - \frac{1}{2} \right\rangle_c \langle 0 | \langle - \frac{1}{2} \right|_v \langle 0 | + \left| - \frac{1}{2} \right\rangle_v \langle 0 | \langle 1 \right|_c \langle 0 | \right) \right.
$$

where $\Omega_{c,\mp} = 1/\sqrt{\Omega_{c,\mp}^2 + \Omega_{c,\pm}^2}$. The eigenspace of zero eigenvalue for a particular number of photons $n_{\pm}$ is $n_{\pm/2,v} - q$ in the cavity is spanned by

$$
|\psi_{0,q}\rangle = \frac{1}{Z_q} \sum_{j=0}^{N_q} \left[ -\Omega_{p,\mp}/\Omega_{c,\pm} \right]_{N_q} \left( |N_q - j, 0, j, j, q\rangle \right)
$$

where $Z_q$ is a normalization constant and we adopted the second quantization state $n_{\pm/2,v} + n_{\pm/2,v}$, $n_{\pm/2,c}$, and $n_{\pm/2,v}$ count the number of QDs being in the state $|\pm\frac{1}{2}\rangle_c$ and $|\pm\frac{3}{2}\rangle_v$, resp.

We describe now the minimal configuration we found that is able to create a N00N state of arbitrary photon number $N$. Without loss of generality, we choose a left-circularly polarized pump field, resulting in an $|N, 0\rangle$ Fock state, which can then be transferred to an adjacent cavity containing four QDs that will transform the $|N, 0\rangle$ Fock state to a $|N, 0\rangle$ N00N state (see Fig. 3). We choose the four QDs to have tensile strain, such that the heavy-hole states lie below the light-hole states, such as already demonstrated for GaAsN QDs embedded in InP. We choose the four QDs to have tensile strain, such that the heavy-hole states lie below the light-hole states, such as already demonstrated for GaAsN QDs embedded in InP. We choose the four QDs to have tensile strain, such that the heavy-hole states lie below the light-hole states, such as already demonstrated for GaAsN QDs embedded in InP. We choose the four QDs to have tensile strain, such that the heavy-hole states lie below the light-hole states, such as already demonstrated for GaAsN QDs embedded in InP. We choose the four QDs to have tensile strain, such that the heavy-hole states lie below the light-hole states, such as already demonstrated for GaAsN QDs embedded in InP. We choose the four QDs to have tensile strain, such that the heavy-hole states lie below the light-hole states, such as already demonstrated for GaAsN QDs embedded in InP. We choose the four QDs to have tensile strain, such that the heavy-hole states lie below the light-hole states, such as already demonstrated for GaAsN QDs embedded in InP. We choose the four QDs to have tensile strain, such that the heavy-hole states lie below the light-hole states, such as already demonstrated for GaAsN QDs embedded in InP. We choose the four QDs to have tensile strain, such that the heavy-hole states lie below the light-hole states, such as already demonstrated for GaAsN QDs embedded in InP. We choose the four QDs to have tensile strain, such that the heavy-hole states lie below the light-hole states, such as already demonstrated for GaAsN QDs embedded in InP. We choose the four QDs to have tensile strain, such that the heavy-hole states lie below the light-hole states, such as already demonstrated for GaAsN QDs embedded in InP. We choose the four QDs to have tensile strain, such that the heavy-hole states lie below the light-hole states, such as already demonstrated for GaAsN QDs embedded in InP. We choose the four QDs to have tensile strain, such that the heavy-hole states lie below the light-hole states, such as already demonstrated for GaAsN QDs embedded in InP. We choose the four QDs to have tensile strain, such that the heavy-hole states lie below the light-hole states, such as already demonstrated for GaAsN QDs embedded in InP. We choose the four QDs to have tensile strain, such that the heavy-hole states lie below the light-hole states, such as already demonstrated for GaAsN QDs embedded in InP. We choose the four QDs to have tensile strain, such that the heavy-hole states lie below the light-hole states, such as already demonstrated for GaAsN QDs embedded in InP. We choose the four QDs to have tensile strain, such that the heavy-hole states lie below the light-hole states, such as already demonstrated for GaAsN QDs embedded in InP. We choose the four QDs to have tensile strain, such that the heavy-hole states lie below the light-hole states, such as already demonstrated for GaAsN QDs embedded in InP. We choose the four QDs to have tensile strain, such that the heavy-hole states lie below the light-hole states, such as already demonstrated for GaAsN QDs embedded in InP. We choose the four QDs to have tensile strain, such that the heavy-hole states lie below the light-hole states, such as already demonstrated for GaAsN QDs embedded in InP. We choose the four QDs to have tensile strain, such that the heavy-hole states lie below the light-hole states, such as already demonstrated for GaAsN QDs embedded in InP. We choose the four QDs to have tensile strain, such that the heavy-hole states lie below the light-hole states, such as already demonstrated for GaAsN QDs embedded in InP. We choose the four QDs to have tensile strain, such that the heavy-hole states lie below the light-hole states, such as already demonstrated for GaAsN QDs embedded in InP.
the linear J\(_B\)n\(\) arrive at the effective \(N\)\(\) distribution comes from 4th-order terms, corresponding to vanish as well. This means the first nonvanishing con- momentum operators \(J\) all the odd powers in \(P\)\(\) the photons by means of \(g\)\(\) and \(B\)\(\) to denote the Zeeman splittings of the conduction, heavy-hole valence, and light-hole valence band states. We choose the special symmetry \(g_{hh1} = g_{hh2}; g_{hh3} = g_{hh4}\), \(B_{1e} = -B_{2e}; B_{1h} = -B_{2h}, B_{3e} = B_{3e}, B_{3hh} = -B_{4hh}, \) and \(B_{3h} = -B_{4h}.\) This combination ensures that all the linear \(J\_x\) terms vanish, so that there is no \(J\_x\) term that could lift the degeneracy between the \(|M = J\) and \(|M = -J\) states (see below).

\[
\frac{1}{2} \langle \frac{1}{2} | \psi_i \rangle + g_{hh1} \langle \frac{1}{2} | \psi_i \rangle + g_{hh2} \langle \frac{1}{2} | \psi_i \rangle + \frac{1}{2} \langle \frac{1}{2} | \psi_i \rangle \otimes |N - 1\rangle \langle N| + \sum_{i=3} \langle \psi_i | \frac{1}{2} \rangle \langle \frac{1}{2} | \psi_i \rangle + \frac{1}{2} \langle \frac{1}{2} | \psi_i \rangle \langle \frac{1}{2} | \psi_i \rangle \otimes |N - 1\rangle \langle N| + \text{h.c.,}
\]

where \(g_{hh1} = \sqrt{N}A(x_i)P_{hh1}; \) \(g_{hh2} = \sqrt{N}A(x_i)P_{hh2}\) are the heavy-hole and light-hole constants for the coupling between the \(N\) cavity photons and QD \(i.\) \(A(x_i)\) denotes the photon field amplitude at the position \(x_i\) of QD \(i,\) and \(P_{hh1} \) and \(P_{hh2}\) are the Kane interband matrix elements between the conduction band and the heavy- and light-hole band states, resp. Since in general the Zeeman splittings differ among the bands due to the variation in effective mass, we use \(B_e, B_{hh}\), and \(B_h\) to denote the Zeeman splittings of the conduction, heavy-hole valence, and light-hole valence band states.

We choose the special symmetry \(g_{hh1} = g_{hh2}; g_{hh3} = g_{hh4}\), \(B_{1e} = -B_{2e}; B_{1h} = -B_{2h}, B_{3e} = B_{3e}, B_{3hh} = -B_{4hh}, \) and \(B_{3h} = -B_{4h}.\) This combination ensures that all the linear \(J_x\) terms vanish, so that there is no \(J_x\) term that could lift the degeneracy between the \(|M = J\) and \(|M = -J\) states (see below).

\[
\text{FIG. 4: 4th-order diagrams in } V. \text{ The left term corresponds to two serially excited virtual excitons, the right term corresponds to an excited virtual biexciton.}
\]

We first switch to second-quantization operators for the photons by means of \(a_{\pm} = \sum_{n=0}^{\infty} \sqrt{\frac{1}{2n+1}} \langle n_{\pm} - 1 | n_{\pm} \rangle\) and \(a_{\pm}^\dagger = \sum_{n=0}^{\infty} \sqrt{\frac{1}{2n+1}} \langle n_{\pm} | n_{\pm} + 1 \rangle.\) Then we transform to the Schwinger representation using the angular momentum operators \(J_z = \frac{i}{2} (a_{+}^a a_{-} - a_{+}^a a_{-}), J_x = a_{+}^a a_{-}, J_+ = a_{+}^a a_{-}, J_- = a_{+}^a a_{-},\) where \(J_x = J_y = iJ_y.\) Taking advantage of the off-resonant interaction, we make the approximation that the excited states are not populated. Consequently, all the odd powers in \(V\) are approximately zero in the perturbation series. Because of the special symmetry of the magnetic fields, all the 2nd-order terms in \(V,\) which correspond to the excitation of virtual single excitons, vanish as well. This means the first nonvanishing con- contribution comes from 4th-order terms, corresponding to the serial excitation of two non-interacting virtual excitons (denoted by subscript \(x\)) and the excitation of virtual biexcitons (denoted by subscript \(XX\)), as depicted in Fig. 4. After long but straightforward calculations we arrive at the effective \(N\)-photon Hamiltonian

\[
H_N = A_N J_z^2 + D_N (J_x^2 - J_y^2) + B_N J_x,
\]

where \(n = n_+ + n_- = a_{+}^a a_{+} + a_{+}^a a_{-}, A_N = (A_X + A_{XX}), B_N = (B_{XX} + B_{XX}^{(n)}) n, D_N = (D_X + D_{XX}).\) Note that the linear \(J_x\) term comes from a 4th-order term of the form \(i[J_y, J_z].\) \(A_N, D_N,\) and \(B_N\) correspond to the longitudinal anisotropy, the transverse anisotropy and the effective magnetic field for the angular momentum \(J,\) resp.

This is the main result of this paper. In order to satisfy the condition \(D_N \ll A_N\) and stay in the regime of perturbation theory, we must be able to choose \(P_{hh1}\) independent of \(P_{hh3}\). This can be achieved by placing the QDs 1 and 2 in a position in the cavity that has a different electromagnetic field amplitude than for the position of the QDs 3 and 4, as shown in Fig. 5.

\[
|N : 0\rangle = \frac{1}{\sqrt{2}} (|J, M = +J\rangle + |J, M = -J\rangle).
\]

In order to maximize entanglement, we get rid of the \(B_NJ_x\) term by choosing \(B_3\) such that the numerator vanishes. From the resulting 3rd-order polynomial equation, we choose the real root for \(B_3.\)

We discuss now our method to create the \(|N : 0\rangle\) \(\text{N00N state. After loading the cavity containing our four QDs with the Fock state } |N_+ , 0\rangle, \) we let the system of \(N\) photons evolve in time according to the Hamiltonian \(H_N\) given in Eq. (7). The time evolution is best described in terms of the total angular momentum states of the \(N\) photons. Using the standard Schwinger representation \(|J, M\rangle = 1/\sqrt{\langle J + M | J - M \rangle} (a_{+}^a)^{J+M} (a_{-}^a)^{-M} | 0\rangle\) with \(J = (N_+ + N_-)/2, M = (N_+ - N_-)/2\) the time evolution corresponds to a Rabi oscillation between the states \(|J, M = J\rangle = |N_+ , 0\rangle\) and \(|J, M = -J\rangle = |0, N_-\rangle\) in good approximation if \(D_N \ll A_N.\) At half the Rabi oscillation period \(T_N/2\), we obtain our desired \(\text{N00N state}\)

\[
|N : 0\rangle = \frac{1}{\sqrt{2}} (|J, M = +J\rangle + |J, M = -J\rangle).
\]

This form is shown in Figs. 1 and 2. In order to satisfy the condition \(D_N \ll A_N\) and stay in the regime of perturbation theory, we must be able to choose \(P_{hh1}\) independent of \(P_{hh3}\). This can be achieved by placing the QDs 1 and 2 in a position in the cavity that has a different electromagnetic field amplitude than for the position of the QDs 3 and 4, as shown in Fig. 5.

It has been shown in Refs. 1, 4, 9 that an effective Hamiltonian of the form \(H_{\text{Rydberg}} = \eta (J_x^2 + J_y^2)\) can be used to produce \(\text{N00N states of atomic states of Rydberg atoms. However, the time evolutions of } |M = J\rangle\) governed by \(H_N\) and \(H_{\text{Rydberg}}\) are completely different.
While $H_{\text{Rydberg}}$ leads the atomic state through all the $|M\rangle$ states with $M = -J, \ldots, J$ and therefore requires very high phase accuracy and timing control to create the N00N state, our effective Hamiltonian $H_N$ mixes in the regime $D_N \ll A_N$ only the states $|M = J\rangle$ and $|M = -J\rangle$, which makes our method much more robust against phase and timing errors.

We present now our numerical results. Keeping in mind that photons can leak out of a cavity on timescales of $\tau_{\text{md}} = 0.4 \mu s$ for microdisk cavities\cite{26} with quality factor $Q = 10^8$ and $\tau_{\text{ms}} = 33 \text{ ms}$ for glass microsphere cavities\cite{27} with $Q = 8 \times 10^{10}$, we try to minimize the time it takes to create a N00N state. Therefore we relax the condition $D_N \ll A_N$ to $D_N \lesssim A_N$. Consequently, we need to introduce the fidelity $F_N = |\langle N : 0 | \psi(t = T_N/2) \rangle|^2$ to describe the deviation of our result from the ideal N00N state. In Fig. 6 we plot the amplitudes $c_M$ of the N-photon states $|\psi(t = T_N/2)\rangle$ for $N = 20, 40, 60$, corresponding to a total angular momentum of $J = 10, 20, 30$, resp. For the calculations we use values that enhance the fidelity of the output states, while being realistic. The values common for all number of photons are $A(x_1)P_{hh1} = 40 \mu eV, B_{e1} = -100 \mu eV, B_{e3} = 500 \mu eV, B_{hh1} = -50 \mu eV, B_{hh3} = 600 \mu eV, B_{hh5} = 600 \mu eV, \Delta = 10 \text{ meV}, \delta_2 = 1000 \mu eV$. Additionally for $N = 20, 40, 60$ we choose $A(x_3)P_{hh3} = 2.5, 3.0$ and 3.2 $\mu eV$, respectively.

For $N = 20, 40, 60$ we obtain fidelities of $F_{20} = 96\%, F_{40} = 92\%$, $F_{60} = 90\%$ and half Rabi oscillation periods of $T_{20}/2 = 23 \text{ ns}, T_{40}/2 = 0.67 \mu s, T_{60}/2 = 98 \mu s$. Comparing to the lifetimes of the photons inside the cavities, we see that a N00N state with $N = 20$ photons can be already produced inside a microdisk cavity due to $T_{20}/2 \ll \tau_{\text{md}}$, and N00N states with $N = 40$ and $N = 60$ can be produced inside a glass microsphere due to $T_{40}/2 \ll \tau_{\text{ms}}$ and $T_{60}/2 \ll \tau_{\text{ms}}$.

In order to calculate the many-photon entanglement, we use our many-particle expression for the concurrence\cite{11}

$$C_N = \sqrt{4\text{det} \langle a_\eta^\dagger a_\lambda \rangle} / N = \sqrt{1 - J^2}, \quad (9)$$

where $\tilde{J} = 2 |\langle J \rangle|/N$, $N = N_+ + N_-$, and $\kappa, \lambda = \pm$. The entanglement of formation can then be calculated using Wootter’s formula\cite{12} $E_N = -\xi \log_2 \xi - (1 - \xi) \log_2 (1 - \xi)$, where $\xi = (1 + \sqrt{1 - C_N^2})/2$. We obtain $C_N = 1$ and $E_N = 1$ for arbitrary $N$, i.e. all the N-photon states $|\psi(t = T/2)\rangle$ are maximally entangled. The reason is that the effective Hamiltonian $H_N$ in Eq. (7) is quadratic in $J$, because we made $B_N = 0$.

Using $N$th-order perturbation theory, it is possible to approximate the Rabi oscillation energy by $2D_N^{N/2} (A_N N^\eta)^{1-N/2}$, where $1/2 < \eta < 1$. Thus, in order to keep the fidelity high with increasing $N$, we need $D_N/A_N = \lambda N^\eta$ with $0 < \lambda \ll 1$ by choosing the appropriate locations in the cavity for the QDs. This means $T_N/2$ increases exponentially as $1/(4D_N \lambda N^{\eta/2-1})$. This increase can be partially compensated by increasing $D_N$, which can be achieved by making the light-QD interaction larger and adding more QDs.

In conclusion, we have shown that N00N states with $N = 20, 40$, and 60 photons are already within experimental reach. The next challenge is to find a scheme based on light-QD interaction that scales more favorably for the Rabi oscillation period. Nevertheless, we developed here a scheme that provides the possibility for a 10-fold improvement over current state-of-the-art. Our proposal paves the way to deterministic nanoscale N00N-state sources on a chip based on semiconductor structures.

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I. SUPPLEMENTAL INFORMATION

We give here the coefficients of the Hamiltonian $H_N$.

$$A_X = \{64\tau_{h,h}^2\tau_{h,3}\delta^2(B_{e,3}^2 - B_{h,h}^2 - \delta^2)(B_{e,3}^4 \Delta + \Delta(B_{h,h}^2
-\delta^2)^2 - 2B_{e,3}^4(B_{h,h}^2\Delta - 2B_{h,h}B_{h,3}\delta + \Delta \delta^2))\}

/((B_{e,3}^2 - \Delta^2)(B_{e,3} + B_{h,h} - \delta)^2(B_{e,3} - B_{h,h}
+\delta)^2(-B_{e,3} + B_{h,h} + \delta)^2(B_{e,3} + B_{h,h} + \delta)^2))$$

$$D_X = \{128B_{h,h}B_{h,h}g_{h,h}^2g_{h,3}^2g_{h,3}3\delta_3(-B_{e,3}^4 + (B_{h,h}^2 - \delta^2)^2)\}

/((B_{h,h}^2 - \Delta^2)(B_{e,3} + B_{h,h} - \delta)^2(B_{e,3} - B_{h,h}
+\delta)^2(-B_{e,3} + B_{h,h} + \delta)^2(B_{e,3} + B_{h,h} + \delta)^2))$$

$$B_{XX} = -\{4B_{e,3}^4\tau_{h,h}(B_{e,3}^2 - 2B_{h,h}^3 + B_{h,h}^2\delta_3 - \delta^3
+B_{e,3}^2(2B_{h,h} + 3\delta_3) + B_{e,3}(-5B_{h,h}^2 + \delta_3^2))\}

/((B_{e,3}^2 - B_{h,h}^3)(B_{e,3} - \delta_3)(B_{e,3}^2 - B_{h,h}^2 - 2B_{e,3}\delta_3
+\delta_3^2)(B_{e,3}^2 - B_{h,h}^2 - 2B_{e,3}\delta_3 + \delta_3^2))$$

$$B_{XX}^{(n)} = \{4B_{e,3}^4\tau_{h,h}(B_{e,3}^2 - 2B_{h,h}^3 + B_{h,h}^2\delta_3 - \delta^3
+B_{e,3}^2(2B_{h,h} + 3\delta_3) + B_{e,3}(-5B_{h,h}^2 + \delta_3^2))\}

/((B_{e,3}^2 - B_{h,h}^3)(B_{e,3} - \delta_3)(B_{e,3}^2 + (B_{h,h}^2 - \delta^2)^2
-2B_{e,3}^2(B_{h,h}^2 + \delta_3^2))$$

Interestingly, $A_{XX}$ is a function of $D_{XX}$, which is given by

$$D_{XX} = (2\tau_{h,h}^2(7B_{e,3}^2 + B_{e,3}^2(2B_{h,h}^2 + 5\delta_3) + 8B_{h,h}^2\delta_3^2(B_{h,h}^2 - \delta_3^2)^4 - B_{e,3}^2(15B_{h,h}^2 + 37\delta_3^2)

-B_{e,3}^2(10B_{h,h}^2 - 147B_{h,h}^2\delta_3 + 8B_{h,h}^2\delta_3^2 + 19\delta_3^3)

-2B_{e,3}^2(5B_{h,h}^2 + 62B_{h,h}^2\delta_3^2 - 39\delta_3^3) - B_{e,3}(B_{h,h}^2 - \delta_3^2)^3(2B_{h,h}^2 + 25B_{h,h}^4\delta_3 - 2B_{h,h}^2\delta_3^2 - 10B_{h,h}^2\delta_3^3 + \delta_3^5)

+2B_{e,3}^2(10B_{h,h}^2 - 223B_{h,h}^2\delta_3 + 8B_{h,h}^2\delta_3^2 + 98B_{h,h}^2\delta_3^2 + 6B_{h,h}^2\delta_3^4 + 13\delta_3^6)

+B_{e,3}^2(50B_{h,h}^2 + 490B_{h,h}^4\delta_3^2 - 202B_{h,h}^2\delta_3^4 - 82\delta_3^6)

-2B_{e,3}^2(10B_{h,h}^2 - 203B_{h,h}^2\delta_3^2 + 38B_{h,h}^2\delta_3^4 + 167B_{h,h}^2\delta_3^6 + 4B_{h,h}^2\delta_3^8 + 7\delta_3^10)

+B_{e,3}^2(-45B_{h,h}^2 - 452B_{h,h}^6\delta_3^2 + 64B_{h,h}^6\delta_3^4 + 332B_{h,h}^6\delta_3^6 + 43\delta_3^8)

+B_{e,3}^2(10B_{h,h}^2 - 87B_{h,h}^2\delta_3 - 167B_{h,h}^2\delta_3^2 - 268B_{h,h}^6\delta_3^4 + 4B_{h,h}^2\delta_3^6 + 350B_{h,h}^4\delta_3^8 + 4B_{h,h}^2\delta_3^6 + 2B_{h,h}^2\delta_3^8 + \delta_3^10)

+B_{e,3}^2(13B_{h,h}^2 + 115B_{h,h}^2\delta_3^2 + 226B_{h,h}^2\delta_3^4 - 346B_{h,h}^2\delta_3^6 + B_{h,h}^2\delta_3^8 - 9\delta_3^10))\}

/((B_{e,3}^2 - B_{h,h}^2)(B_{e,3} - \delta_3)(B_{e,3} - B_{h,h} - \delta_3)^3(B_{e,3} + B_{h,h} - \delta_3)^3(B_{e,3} - B_{h,h} + \delta_3)^3(B_{e,3} + B_{h,h} + \delta_3)^3))$$