Turbulent transport close to marginal instability: role of the source driving the system out of equilibrium

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Abstract. When modeling plasma turbulence, two different means of driving the system out of equilibrium are considered. On one hand, flux driven (FD) approaches are based on the idea that no scale separation can be assumed in a turbulent system. On the other hand, gradient driven (GD) approaches rely on the idea that the back-reaction of fluctuations on the mean profiles is not a critical ingredient for turbulence self-organization and saturation. We find that FD and GD systems strongly differ in regimes close to marginal stability. The characteristic non linear upshift is recovered in GD simulations but no comparable behavior is possible in FD case. Discrepancy between the various analysis in terms of diffusion and pinch velocities and between the models is also discussed.

1. Introduction
Modeling plasma turbulent transport remains a challenging task and many issues that drive fusion performance are unresolved. The complexity of the so-called anomalous transport, in the light of the recent results with tungsten walls [1], is the cause of major uncertainties and risk regarding the coming high performance experiences whether D-T experiments on JET or ITER $Q = 10$ goal [2]. State of the art simulations with gyro-kinetic codes require long computational time to yield realistic results to be compared in detail to a growing wealth of experimental measurements.

Among the issues that are presently addressed are those related to the way the system is driven out of equilibrium and the associated uncertainty regarding the predictive capability of these codes. Two means of addressing this issue are implemented in the codes.

On the one hand, some simulations rely on scale separation, i.e. the slow evolution of the mean profiles can be addressed on a different footing than the fluctuations. Such an assumption lead to developing a series of so-called local codes where the mean profiles are considered constant and set by the user. A recent focal point of this activity is evaluating the range of predicted fluxes and density fluctuation levels when the prescribed gradients are changed within the
experimental error bars. Together with an effort towards completeness of the turbulence models this opens the way to quite a broad range of tunable parameters. The effort of first principle modeling then leads to a large farming effort to explore the range of possible parameters and determine the best fit to experiments [3, 4]. A procedure that seems more in line with empirical modeling.

On the other hand, a series of experimental results have shown the limit of the local assumption [5–8]. In the framework of understanding and explaining such non-local aspect of plasma transport, global Flux-Driven models (i.e. with no scale separation assumption) have been developed during the last twenty years [9–14]. Non-local aspects have been identified, such as avalanche-like events [12, 15, 16] or jet-like patterns, named as ‘E × B staircases’ [17]. Large scale predator-prey cycles are also observed in flux-driven simulations [18]. As a consequence, the standard local diffusion process ansatz appeared to fall short in describing the large scale transport [12, 17, 19, 20].

Analyzing the difference between local and global, as well as between Gradient and Flux Driven approaches can lead to a better understanding of turbulent transport and help in improving the predictive capability of the modeling effort. It is interesting to note that the impact of different forcing on transport is addressed by a wider research community than fusion plasma physics [21].

In this paper, we present a quantitative comparison of the two forcing mechanisms and their impact on the regimes explored by the system [21]. We focus our work on the transition regime from stability to turbulence. As a versatile tool we use the TOKAM2D code, a minimum turbulent model initially developed to address SOL turbulence. In Section 2, we describe the equations solved by the TOKAM2D code [12] in the original flux driven version (FD). We then define the fixed gradient (GD) version and the general method applied to compare the two cases, Section 3. The results of the comparison are addressed in Section 4. In Section 5 are found the conclusion and a discussion of the ongoing work.

2. The TOKAM2D model

The TOKAM2D system only addresses particle transport with given constant temperatures so that the pressure nT is in fact proportional to the density. The model is of interchange governed turbulence [22]. It solves the density balance equation and the charge balance equation \( \nabla \cdot \vec{j} = 0 \), \( \vec{j} \) being the total current. The turbulent transport is described by the three fluid drifts, i.e. the electric drift \( v_{\text{elec}} \), the diamagnetic drift \( v_{\text{dia}} \) and the polarization drift \( v_{\text{pol}} \) [12, 22]. The model is considered in the flute approximation, with no variation along the magnetic field lines. This allows one to reduce the model to two dimensions, the radial direction \( x \), \( x = (r - a)/\rho_s \) where \( a \) is the plasma minor radius, \( r \) the radius of the considered magnetic surface and \( \rho_s \) the hybrid Larmor radius, and an angle that describes the poloidal angle on the magnetic surface \( y \), \( y = a \theta / \rho_s \) where \( \theta \) is the actual poloidal angle. The only terms that contribute when averaging over the parallel coordinate are the divergence of the parallel particle flux flowing to the limiters \( \nabla_{||} \vec{j} || \), governed by the sheath physics, and the divergence of the diamagnetic current \( \nabla_{\perp} j_{\text{dia}} \) that takes into account the curvature of the magnetic field. Their derivation is treated in detail in [12].

Time normalization in the code is in \( 1/\omega_i \), hence the inverse of the ion Larmor frequency. The angle coordinate is periodic so that the average of any particle flux on a given magnetic surface yields the radial component of the flux. In this model, field lines connect region with different properties so that parallel loss terms govern transport that tends to equilibrate these regions. Both parallel loss terms to end plates in a quasi flute approximation or parallel transport taking into account the ballooned cross field transport lead to an outward particle flux and a current exchange. The latter is particularly important since it bounds the inverse cascade that would otherwise take place.
2.1. flux driven version

In the flux driven approach we assume that the density and the potential profile can fluctuate not only at small but also at large scale, i.e. the fluctuations size can be comparable to the size of the box. Such an assumption implies that the driving of the system is a particle source that is localized radially. In this way the initial profile is null and it will build up through the source a density gradient that will drive transport in bursts in the box. The equations solved by the FD version of TOKAM2D are

\[ \frac{\partial}{\partial t} n + [\phi, n] = D_\perp \Delta_\perp n - \sigma_n \Gamma + S \] (1)

\[ \frac{\partial}{\partial t} W + [\phi, W] + g \frac{n}{n} \partial_\perp n = \nu_\perp \Delta_\perp W + \sigma_\phi J \] (2)

where:

\[ W = \nabla_\perp^2 \phi \] (3)

and where the parallel loss currents in the SOL case are defined as:

\[ \Gamma = n \exp(\Lambda - \phi) \] (4)

\[ J = \left(1 - \exp(\Lambda - \phi)\right) \] (5)

The particle balance equation Eq.(1) includes the leading order particle flux due to the electric drift velocity, which takes the form of the Poisson bracket between the normalized electric potential \( \phi \) and the density, together with a source term \( S \), the parallel particle sink \( \Gamma \), defined for the SOL case in (4), and a small transverse diffusion \( D_\perp \). It is to be noted that while the source term is localized radially, the loss terms is distributed in the simulation region. The particle flux that drives the turbulence therefore decays as the radial distance from the source increases. In these expressions, \( \Lambda \) is the plasma potential with respect to the reference potential of the wall, it includes the plasma floating potential and the biasing potential if applied. The control parameter of the parallel particle loss term \( \sigma_n \) is proportional to the normalized saturation current. The charge balance equation Eq.(2) takes the form of the evolution equation of the vorticity \( W \) defined in (3), with an electric drift convection, the Poisson bracket term between the electric potential and the vorticity, together with a diffusive damping of the convective motion with diffusion coefficient \( \nu_\perp \). The curvature charge separation is simplified to a constant interchange term \( g \). This \( g \)-term is the standard gravity term when addressing buoyancy effects in a neutral fluid. A similar term should be included in the density equation, however this term is a lower order term than the electric drift convection term. Consequently it is only retained in the charge conservation equation because the latter only includes lower order terms for the cross-field current, i.e. the curvature and polarization currents. The parallel loss current \( J \), defined for the SOL case by (5), closes the current loop with a restoring term that tends to compensate charge separation and is all the more efficient when the cross-section is large.

3. How to compare different forcing terms?

The definition of gradient driven forcing is often implicit and thus confusing. Generally, the gradient driven approach is a simplified version to model turbulent transport based on the assumption that the fluctuation scale is much smaller than the mean profile scale. This opens the way to scale separation, we can study only the fluctuating terms and consider a constant mean profile. This approach is quite handy when comparing numerical and experimental results. In this case the density profile measured in tokamaks can be supplied as input for the simulation. Furthermore, this approach reduces drastically the computational time: the system starts already from a steady state condition, nullifying the time interval needed to reach confinement in the FD simulations.
3.1. The local like approach
In the first and most conventional GD version the profile is defined by an input coefficient $-Ln$ that defines the ratio of the mean density and the gradient of the latter and such value is considered constant all over the simulation domain. Splitting the density and potential into the fluctuating part $\tilde{n} = n - <n>_y$ and $\tilde{\phi} = \phi - <\phi>_y$, and the mean part such that $\bar{\phi} = <\phi>_y$ and $\bar{n} = <n>_y$, we can then rewrite the equations of the TOKAM2D for the FD version, hence:

$$\partial_t \tilde{n} + \left[ \frac{\tilde{n}}{\bar{n}} \right] = D_{\perp} \Delta_{\perp} \bar{n} - \sigma_n \bar{\Gamma}$$
$$\partial_t \tilde{\phi} + \left[ \frac{\tilde{\phi}}{\bar{\phi}} \right] = D_{\perp} \Delta_{\perp} \bar{\phi} + \sigma_{\phi} \bar{\phi}$$

The equations solved by the code for the fluctuating density and potential will be respectively equal to eq.1-eq.7 and eq.2-eq.7:

$$\partial_t \tilde{n} + \left[ \frac{\tilde{n}}{\bar{n}} \right] - \left[ \frac{\tilde{\phi}}{\bar{\phi}} \right] = D_{\perp} \Delta_{\perp} \bar{n} - \sigma_n \bar{n} + \sigma_n \bar{\Gamma}$$
$$\partial_t \tilde{\phi} + \left[ \frac{\tilde{\phi}}{\bar{\phi}} \right] = \nu_{\perp} \Delta_{\perp} \bar{\phi} + \sigma_{\phi} \bar{\phi}.$$}

We then further assume $\Phi > y$ to be constant in the radial direction and equal to $\Phi = \Lambda$, so that $\bar{\Gamma} = \bar{n}$, and $<n>_y$ to be constant in time with e-folding length $1/Ln$ in the radial direction. A more accurate presentation would introduce two evolution time scales and assume that the mean density evolves slowly compared to the fluctuations. Implicitly, one then assumes that the two transport terms $\left[ \tilde{\phi}/\bar{\phi} \right]$ and $\left[ \tilde{\phi}/\bar{\phi} \right]$ are small (they contribute on the long time scale evolution of the mean values). Consistently, one can consider to remove them in the GD evolution equations for the fluctuations, however, at the cost of losing a symmetry in the fluctuation evolution equations.

$$\partial_t \tilde{n} + \left[ \frac{\tilde{n}}{\bar{n}} \right] = D_{\perp} \Delta_{\perp} \bar{n} - \sigma_n \bar{n} + \sigma_n \bar{\Gamma}$$
$$\partial_t \tilde{\phi} + \left[ \frac{\tilde{\phi}}{\bar{\phi}} \right] = \nu_{\perp} \Delta_{\perp} \bar{\phi} + \sigma_{\phi} \bar{\phi}.$$}

In the latter set of equations, the field $\tilde{n}$ stands in fact for $\bar{n}/\bar{n}$ hence the relative fluctuations.

3.2. A general comparison method
In order to define an appropriate method to compare the two approaches we decide to follow the same process applied for the comparison between experimental and numerical results. Namely, one takes the steady state density profile from the experiment as input for the simulation and then compare the computed and experimental flux. For our purpose, the flux driven simulation is compared to the fixed gradient one along the same line. We run long a FD simulation in order to define a steady state condition, then we calculate the mean density profile averaged in time and poloidal direction to smooth out the fluctuations. Such a profile $\bar{n}(r)$ or its gradient, normally defined as $1/Ln = -\partial_r \bar{n}/\bar{n}$, is then inserted as parameter for GD simulation, in which the source is set at zero.

The sketch shown in fig.(1) summarizes this comparison approach.

4. Different transport regimes at large scale
In order to decide which parameters to set for the comparison, we analyze the dispersion relation of the interchange instability. We want to be able to bring the system from a stable condition to a fully turbulent regime.
Figure 1. Sketch of the procedure used to compare the two different mechanisms that are used to drive the system out of equilibrium

Assuming that the fluctuations are small for both the density and potential field, we can then investigate the stability of the chosen reference state $\bar{n}$ and $\bar{\phi}$ when considering the linearized set of equations (10, 11).

$$\tilde{n}_k = \hat{n}_{kx,ky} \exp(i(k_x x + k_y y)) \exp(\Omega_k t) + \text{cc}$$  \hspace{1cm} (12)

$$\tilde{\phi}_k = \hat{\phi}_{kx,ky} \exp(i(k_x x + k_y y)) \exp(\Omega_k t) + \text{cc}$$ \hspace{1cm} (13)

where ‘cc’ means ‘complex conjugates’ and $\gamma_k$ represents the real part of $\Omega_k$. The perturbation will be excited if $\gamma_k$ is positive, i.e. the system is unstable. Substituting eq.12 and eq.13 in eq.10 and eq.11 defined for the GD approach, we recover the following dispersion relation:

$$\left(\Omega_k + Dk^2 + \sigma_n\right) \hat{n} + \left(\frac{i k_y}{L_n} - \sigma_n\right) \hat{\phi} = 0$$ \hspace{1cm} (14)

$$\left(-i g k_y^2 \frac{k_y}{k^2}\right) \hat{n} + \left(\Omega_k + \nu k^2 + \frac{\sigma_\phi}{k^2}\right) \hat{\phi} = 0$$ \hspace{1cm} (15)

A non trivial solution is possible when $\Omega_k$ satisfies the second order equation:

$$\Omega_k^2 + B_k \Omega_k + C_k = 0$$ \hspace{1cm} (16)

where $B_k = (D+\nu)k^2+\sigma_n+\sigma_\phi/k^2$ and $C_k = (Dk^2+\sigma_n)(\nu k^2+\sigma_\phi/k^2)-(g k_y^2)/(L_n k^2)-i \sigma_n g k_y/k^2$. Finally if we consider no dissipation or loss term ($D=0$, $\nu=0$, $\sigma_n=0$, $\sigma_\phi=0$), hence $B_k = 0$ and $C_k = (g k_y^2)/(L_n k^2)$, one can state that the growth rate of the unstable modes is proportional to $\sqrt{g/L_n}$. Specifically, decreasing $g$ or increasing $L_n$, we reduce the linear growth rate and consequently the characteristic time of the turbulence.

Table 1. parameter used in the following simulation both for flux and gradient driven case

|   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|
| $D$ | $\nu$ | $\sigma_n$ | $\sigma_\phi$ | $N_x$ | $N_y$ | $L_x$ | $L_y$ | $t_{\text{diag}}$ |
| $10^{-2}$ | $10^{-2}$ | $6.1 \times 10^{-5}$ | $6.1 \times 10^{-5}$ | 128 | 128 | 128 | 128 | 32 |

The parameters used for the following simulations are specified in table 1. For this set of parameters the maximum of the growth rate is at $k y_{\text{max}} = 8/128$ and $k x_{\text{max}} = 0$ and the critical
value corresponds to \((g/Ln)_{crit}\) equal to 0.4 \(10^{-5}\). If the ratio of the two quantities is smaller than \((g/Ln)_{crit}\), the growth rate is negative for all modes and the system is linearly stable. Conversely, if we set the system just above the critical value, only a few modes are linearly unstable, namely around \(k y_{max}\) and \(k x_{max}\).

For the flux driven simulation, we cannot control the two terms, the gradient being in fact self consistently defined. For the GD case, both parameters can be set by the user and therefore we can study how and when the system departs from the linear one. We start analyzing the FD case in order to define which appropriate value or range of values of \(1/Ln\) has to be used for the GD simulations.

A scan on the g-term is performed to analyze the response of the FD system close to criticality. The number of points of the simulation grid, \(N_x\) and \(N_y\), have also been varied in order to test the sensitivity of the code to the grid-size. The increase of \(N_x\) and \(N_y\) only impacts the high frequency part of the spectrum. Since the transport analysis is based on time and space averages, the simulation output is readily observed not to exhibit relevant differences when increasing the grid-size.

4.1. FD CASE: self-organization close to marginal stability

First, we define an effective diffusion \(D_{eff}\) in order to quantify the turbulent transport [10,23]. The effective diffusivity is used to quantify the macroscopic transport.

\[
< \frac{\Gamma_{turb}}{\bar{n}}>_{t,y} = -D_{eff} < \frac{\partial \bar{n}}{\bar{n}} >_{t,y}
\]  

A coarse graining approach with flux surface average and long time averaging is required to determine \(D_{eff}\) with a smooth behavior as shown in ref. [19]. Should one require that large scale transport follows a Fick’s law, namely that \(D_{eff}\) is a scalar with a single value would require further coarse graining in the radial direction [19]. Another possible transport law can be considered by combining a diffusion \(D_V\) and a pinch velocity \(V\).

\[
< \frac{\Gamma_{turb}}{\bar{n}}>_{t,y} = -D_V < \frac{\partial \bar{n}}{\bar{n}} >_{t,y} + V
\]  

Compared to the expression of \(D_{eff}\) which is simply defined as the ratio < \(\Gamma > L n / < \bar{n} >\), the latter transport law is based on a linear approximation of the data such that the slope is governed by \(D_V\) and the value at \(1/Ln \rightarrow 0\) determines \(V\). It is possible to generalize the linear fitting process by identifying within the data several regions where the linear fit holds, hence introducing a non-linear dependence of the transport law on the gradient. From fig.(2) one defines three different regions:

- a flat top region, region I, localized immediately after the source, in this region the flux is maximum and nearly constant,
- region II that connects the first and third region and where the particle flux decreases,
- region III, where the density gradient is positive and turbulence stabilized.

One can then identify the incremental transport coefficients, in region I from \(x_1 = 15\) to \(x_2 = 30\), width \(d_1 = 15\), \(D_I \approx 0.076\) and is associated to an outward pinch velocity of \(V_I \approx 0.008\), while in region II from \(x_2\) to \(x_3 = 80\), width \(d_{II} = 50\), \(D_{II} \approx 0.44\) and the outward pinch velocity is \(V_{II} \approx 0.001\). Here the radial units are units of \(\rho_s\), the pinch velocity in units of \(c_s\), and correspond therefore to Mach numbers, and the diffusion coefficients are normalized to the Bohm diffusion coefficient, i.e. \(\rho_s c_s\). For this data, the value of g-term is equal to \(5.7 \ 10^{-4}\), a standard value for TOKAM2D simulations determined for Tore Supra parameters, and the value of \(1/Ln\)
Figure 2. (a) Time and y-averaged density gradient $1/Ln$. (b) Time and y-averaged turbulent flux divided by the Time and y-averaged density $\Gamma/\bar{n}$, (c) $\Gamma/\bar{n}$ as a function of $1/Ln$. The turbulent flux $\Gamma$ is normalized by $N_x/(2\pi)$.

varies from $-1.15 \times 10^{-2}$ to $2.9 \times 10^{-2}$ as indicated on fig.(2). The ratio $D/(dV)$ allows one to determine the dominant transport mechanism (since both act in the same direction). Hence, in region I, $D_\varphi/(d\varphi I) \approx 0.625$ while in region II $D_\varphi/(d\varphi I) \approx 8.8$: in first region the transport is governed by the convection while in the second one the diffusion dominates the transport. On fig.(3) a scan of the $g$-term allows one to see its impact on the radial transport coarse grained in time and on a flux surface. For all values of $g$, the plot of $<\Gamma>/\langle n \rangle$ versus $g/Ln$ is characterized by a sub-threshold transport governed by turbulent spreading into the stable region at weak density gradient, a sharp rise of transport in the vicinity of the threshold (indicative of profile stiffness at lowest values of $g$) followed by a less stiff regime. One can also observe a specific aspect of the effect of the $g$-term on the transport properties. In particular the stiffness of the turbulent response seems to decrease as $g$ is increased (hence at smaller $\rho_*$). For $g$ equal or smaller than $2 \times 10^{-4}$, the flux and the density gradient only explore the vicinity of the threshold. Conversely, for the larger values of $g$, the flux dependence on $1/Ln$ recalls more conventional diffusion-convection regimes.

Let us analyze the transport regime for $g = 2 \times 10^{-4}$, hence close to criticality where one can expect that self-organization dominates the transport response. Furthermore this regime is likely more relevant for ITER size devices. The characteristic time of turbulence is proportional to $(g/Ln - g/Ln_*)^{1/2}$, where $g/Ln_*$ is the threshold value. Decreasing $g$, one thus expects a slower self-organization time of the turbulent patterns. This is readily seen on fig.(4) where is shown the contour plot of the evolution of the radial profile of flux surface averaged electric potential $\Phi_{zf}(r,t)$. One can observe that $\Phi_{zf}$ tends to develop long lasting structures, akin to the $E \times B$ staircases. The shearing effect associated to the zonal flow structure then acts as micro barriers, dump the turbulence locally. Such structures seem to disappear for larger values of $g$. It appears that these micro transport barriers are mostly localized in the region I. One can notice that this region tends to shrink radially as $g$ is increased.

4.2. Dimits shift region in gradient driven simulations

In the GD case we assume that the gradient is constant in the all the domain. Unlike the flux driven simulations that are characterized by a curve in the $(1/Ln, <\Gamma>/\langle n \rangle)$ plane, the gradient driven simulation yield a single point. In order to compare with the FD case we must therefore perform a scan of Ln for different values of $g$-term. This provides a means to compare the response of the system to the way it is driven out of equilibrium.
Let us first compare the case with $g = 2.2 \times 10^{-4}$ and we scan $1/Ln$ from $L^* = 0.02$ (corresponding to the critical value) to a maximum set at 0.05. In fig.(5), the two curves of $<\Gamma>/ <n>$ in terms of the driving term $g/Ln$ are displayed. One finds an unexpected feature of the local (fixed gradient) approach. Just above the threshold, in the region referred to as the Dimits region [24], the system is linearly unstable but the flux is not governed by the conventional Fick’s law since it first decays as the drive is increased. The system only recovers the standard behavior with increasing $1/Ln$ when the latter exceeds $2.5 \times 10^{-4}$. This new onset value plays the role of a secondary non-linear threshold.

The decrease of the flux can be explained by coupling the interchange instability and the Kelvin Helmholtz (KH) instability. The interchange instability linearly grows with $g/Ln$ and is maximum for small values of $kx$. It thus grows into a nonlinear state of streamer like modes. These become KH unstable which leads to the build-up of Zonal flows. Such a mechanism leads to a predator-prey behavior. Acting as weak transport barriers, these flows reduce the perturbation amplitude and stabilize the non-linear mode. Further increasing $1/Ln$ also increases the number of linearly unstable modes until the interchange instability becomes dominant and the system is fully turbulent.

Such upshift is not present in the flux driven simulation. We thus find a different behavior of the turbulent flux predicted by the two ways the system is driven out of equilibrium in the vicinity of the threshold, the so-called Dimits shift region. The systems approach to the stable region is markedly different. For the FD case the immediate positive response of the flux for $1/Ln - 1/Ln^* > 0$ shows that the Dimits shift cannot be recovered. We can state that the observed non linear upshift is characteristic of local simulations. In the FD case we cannot constrain the gradient profile and the system strongly differs an ideal predator-prey behavior although some features of the latter reduced model still hold.
Figure 5. $\Gamma$ as a function of $1/Ln$ for $g = 2 \times 10^{-4}$ in FD and GD approaches. The values of $\Gamma$ are divided by $N_x/(\bar{n}2\pi)$ in the FD approach while these values are divided by $N_x/(2\pi)$ in the GD case.

Figure 6. $\Gamma/\bar{n}$ as a function of $g/Ln$, for $g = [2 \times 10^{-4}, 10^{-3}]$ in the GD case. The values of $\Gamma$ are divided by $N_x/(2\pi)$.

On the one hand, one can notice on fig.(5) that in region I of flux driven simulation the effective diffusivity defined as $-\langle \Gamma_I >_{y,t,dx} / < Ln_I >_{y,t,x}$, as previously the subscript $I$ stands for region I, is comparable to the effective diffusivity of the local case for $Ln$ equal to $< Ln_I >_{y,t,d1}$. On the other hand, the flux calculated in GD simulations is far more sensitive to small variations of $1/Ln$ than in FD ones. From fig.(6) we can see that the effective diffusivity grows with the curvature term, in agreement with the FD case, but, in the local case, the vicinity of the threshold is still subject to a Dimits shift like effect, although its impact is less stringent on large scale transport as $g$ is increased. When we increase the $g$-term we do not recover a strong variation of the flux as in the case presented in fig.(5), but we can see still identify a region with a sudden variation on the incremental diffusivity.

5. Discussion and conclusion

We have analyzed in this paper the impact of various forcing on turbulent transport. Particularly we have focused our attention on understanding and quantifying how two different means of forcing the system out of equilibrium modify transport properties in the vicinity of marginal criticality. A characteristic aspect of the Gradient Driven(GD) case is the presence of a non linear upshift of the turbulence threshold. A comparable aspect cannot be found for the Flux Driven (FD) case. Such results can be explained by the fact that in the Gradient Driven case it is possible to constrain the transport in regimes that are not explored self consistently by the system in the flux driven case. In strongly turbulent, hence well above the Dimits shift [24] for GD and at large interchange drive, it is possible to find comparable values of the effective diffusivity for the two different cases. This is all the more striking that the patterns that develop close to the source in the FD case are reminiscent of staircases reported in FD gyro-kinetic simulations [17]. The latter are long living structures that behave like transport barriers. The observation of similar effective diffusivity in a region with self-regulation of transport as in the local GD case is thus rather surprising. However, when considering linear approximations of the particle flux in terms of the density gradient length -transport modeled in terms of a diffusion
coefficient complete by a pinch term- a clear departure is found as reported previously [19]. Incremental transport analysis further underlines the latter feature.

These results are being further investigated, in particular regarding the interplay between the various regions and patterns that develop in the radial direction. Key issues are: a ρ∗ analysis in terms of a scan of the box size, a more detailed analysis of the source region and its neighborhood to understand its specific effect and the analysis of the particular role of the stable region via spreading or coupling of the larger modes to stable regions.

All these issues are relevant when analyzing the gyro-kinetic simulations which is the ultimate goal of this work. In particular, quantifying the difference between flux and gradient driven gyrokinetic simulations [25, 26] is needed. Various source term models have been implemented in TOKAM2D that allow one to drive the FD systems with prescribed profiles that depart in particular from a constant gradient length. As with Krook source terms, the effect of a local restoring force as well as larger scale patterns of the restoring forces must be investigated and their effect quantified. The case of prescribed thermal bath boundary conditions completed by a Krook like restoring force which is the standard drive in gyrokinetic codes, is an example of such a way of driving the system out of equilibrium based on two characteristic scales in time and space. Such a source term is now implemented in the GYSELA code allowing to step the present analysis to the case of gyrokinetic turbulence.

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References
[1] M. Beurskens, L. Frassinetti, C. Challis, C. Giroud, S. Saarelma, B. Alper, C. Angioni, P. Bilkova, C. Bourdelle, S. Brezinsek, P. Buratti, G. Calabro, T. Eich, J. Flanagan, E. Giovannozzi, M. Groth, J. Hobirk, E. Joffrin, M. Leyland, P. Lomas, E. de la Luna, M. Kempenaars, G. Maddison, C. Maggi, P. Mautic, M. Maslov, G. Matthes, M.-L. Mayoral, R. Neu, I. Nunes, T. Osborne, F. Rimini, R. Scannell, E. Solano, P. Snyder, I. Voitsekhovitch, P. de Vries, and J.-E. Contributors, “Global and pedestal confinement in jet with a be/w metallic wall,” Nuclear Fusion, vol. 54, no. 4, p. 043001, 2014.

[2] M. Shimada, D. Campbell, V. Mukhovatov, M. Fujinawa, N. Kirneva, K. Lackner, M. Nagami, V. Pustovitov, N. Uckan, J. Wesley, N. Asakura, A. Costley, A. Donn, E. Doyle, A. Fasoli, C. Gormezano, Y. Gribov, O. Gruber, T. Hender, W. Houlberg, S. Ide, Y. Kamada, A. Leonard, B. Lipschultz, A. Loarte, K. Miyamoto, V. Mukhovatov, T. Osborne, A. Polevoi, and A. Sips, “Chapter 1: Overview and summary,” Nuclear Fusion, vol. 47, no. 6, p. S1, 2007.

[3] N. T. Howard, A. E. White, M. Greenwald, M. L. Reinke, J. Walk, C. Holland, J. Candy, and T. Grler, “Investigation of the transport shortfall in alcator c-mod l-mode plasmas,” Physics of Plasmas (1994-present), vol. 20, no. 3, pp. --, 2013.

[4] D. Told, F. Jenko, T. Grler, F. J. Casson, E. Fable, and A. U. Team, “Characterizing turbulent transport in asdex upgrade l-mode plasmas via nonlinear gyrokinetic simulations,” Physics of Plasmas (1994-present), vol. 20, no. 12, pp. --, 2013.

[5] TFR Group, “Pellet injection experiments on the tfr tokamak,” Nuclear Fusion, vol. 27, no. 12, p. 1975, 1987.

[6] K. W. Gentle, W. L. Rowan, R. V. Bravenc, G. Cima, T. P. Crowley, H. Gasquet, G. A. Hallock, J. Heard, A. Ouroua, P. E. Phillips, D. W. Ross, P. M. Schoch, and C. Watts, “Strong nonlocal effects in a tokamak perturbative transport experiment,” Phys. Rev. Lett., vol. 74, pp. 3620–3623, May 1995.

[7] X. L. Zou, A. Géraud, P. Gomez, M. Mattioli, J. L. Sgui, F. Clairet, C. D. Micheli, P. Devynck, T. D. de Wit, M. Erba, C. Fenzi, X. Garbet, C. Gil, P. Hennetquin, F. Imbeaux, E. Joffrin, G. Leclert, A. L. Pequet, Y. Peysson, R. Sabot, and T. F. Seak, “Edge cooling experiments and non-local transport phenomena in tore supra,” Plasma Physics and Controlled Fusion, vol. 42, no. 10, p. 1067, 2000.

[8] S. Inagaki, T. Tokuzawa, N. Tamura, S.-I. Itoh, T. Kobayashi, K. Ida, T. Shimozuma, S. Kubo, K. Tanaka, T. Ido, A. Shimizu, H. Tsuchiya, N. Kasuya, Y. Nagayama, K. Kawahata, S. Sudo, H. Yamada, A. Fujisawa,
K. Itoh, and the LHD Experiment Group, “How is turbulence intensity determined by macroscopic variables in a toroidal plasma?,” *Nuclear Fusion*, vol. 53, no. 11, p. 113006, 2013.

[9] P. H. Diamond and T. S. Hahm, “On the dynamics of turbulent transport near marginal stability,” *Physics of Plasmas (1994-present)*, vol. 2, no. 10, pp. 3640–3649, 1995.

[10] B. A. Carreras, C. Hidalgo, E. Sánchez, M. A. Pedrosa, R. Balbín, I. García-Cortés, B. van Milligen, D. E. Newman, and V. E. Lynch, “Fluctuation-induced flux at the plasma edge in toroidal devices,” *Physics of Plasmas (1994-present)*, vol. 3, no. 7, pp. 2664–2672, 1996.

[11] X. Garbet and R. E. Waltz, “Heat flux driven ion turbulence,” *Physics of Plasmas (1994-present)*, vol. 5, no. 8, pp. 2836–2845, 1998.

[12] Y. Sarazin and P. Ghendrih, “Intermittent particle transport in two-dimensional edge turbulence,” *Physics of Plasmas (1994-present)*, vol. 5, no. 12, pp. 4214–4228, 1998.

[13] P. Ghendrih, Y. Sarazin, G. Attuel, S. Benkadda, P. Beyer, G. Falchetto, C. Figarella, X. Garbet, V. Grandgirard, and M. Ottaviani, “Theoretical analysis of the influence of external biasing on long range turbulent transport in the scrape-off layer,” *Nuclear Fusion*, vol. 43, no. 10, p. 1013, 2003.

[14] S. Y. V. Grandgirard, E. Florence, X. Garbet, P. Ghendrih, F. Bertrand, and G. Depret, “Kinetic features of interchange turbulence,” *Plasma Physics and Controlled Fusion*, vol. 47, no. 10, p. 1817, 2005.

[15] Y. Sarazin, V. Grandgirard, J. Abiteboul, S. Allfrey, X. Garbet, P. Ghendrih, G. Latu, A. Strugarek, and G. Dif-Pradalier, “Large scale dynamics in flux driven gyrokinetic turbulence,” *Nuclear Fusion*, vol. 50, no. 5, p. 054004, 2010.

[16] S. Jolliet and Y. Idomura, “Plasma size scaling of avalanche-like heat transport in tokamaks,” *Nuclear Fusion*, vol. 52, no. 2, p. 023026, 2012.

[17] G. Dif-Pradalier, P. H. Diamond, V. Grandgirard, Y. Sarazin, J. Abiteboul, X. Garbet, P. Ghendrih, A. Strugarek, and C. S. Chang, “On the validity of the local diffusive paradigm in turbulent plasma transport,” *Phys. Rev. E*, vol. 82, p. 025401, Aug 2010.

[18] G. Darmet, P. Ghendrih, Y. Sarazin, X. Garbet, V. Grandgirard, “Intermittency in flux driven kinetic simulations of trapped ion turbulence,” *Communications in Nonlinear Science and Numerical Simulation*, vol. 13, no. 1, pp. 53 – 58, 2008. Vlasovia 2006: The Second International Workshop on the Theory and Applications of the Vlasov Equation.

[19] P. Ghendrih, C. Norscini, F. Hasenbeck, G. Dif-Pradalier, J. Abiteboul, T. Cartier-Michaud, X. Garbet, V. Grandgirard, Y. Marandet, Y. Sarazin, P. Tamm, and D. Zarzoso, “Thermodynamical and microscopic properties of turbulent transport in the edge plasma,” *Journal of Physics: Conference Series*, vol. 401, no. 1, p. 012007, 2012.

[20] V. Naulin, “Turbulent transport and the plasma edge,” *Journal of Nuclear Materials*, vol. 363-365, pp. 24 – 31, 2007. Plasma-Surface Interactions-17.

[21] B. Saint-Michel, B. Dubrulle, L. Marié, F. Ravelet, and F. Daviaud, “Evidence for forcing-dependent steady states in a turbulent swirling flow,” *Phys. Rev. Lett.*, vol. 111, p. 234502, Dec 2013.

[22] X. Garbet, L. Laurent, J.-P. Roubin, and A. Samain, “A model for the turbulence in the scrape-off layer of tokamaks,” *Nuclear Fusion*, vol. 31, no. 5, p. 967, 1991.

[23] C. C. Petty and T. C. Luce, “Scaling of heat transport with collisionality,” *Physics of Plasmas (1994-present)*, vol. 6, no. 3, pp. 999–921, 1999.

[24] A. M. Dimits, G. Bateman, M. A. Beer, B. I. Cohen, W. Dorland, G. W. Hammett, C. Kim, J. E. Kinsey, M. Kotschenreuther, A. H. Kritz, L. L. Lao, J. Mandrekas, W. M. Nevins, S. E. Parker, A. J. Redd, D. E. Shumaker, R. Sydora, and J. Weiland, “Comparisons and physics basis of tokamak transport models and turbulence simulations,” *Physics of Plasmas (1994-present)*, vol. 7, no. 3, pp. 969–983, 2000.

[25] T. Görler, X. Lapillonne, S. Brunner, T. Dannert, F. Jenko, S. K. Aghdam, P. Marcus, B. F. McMillan, F. Merz, O. Sauter, D. Told, and L. Villard, “Flux- and gradient-driven global gyrokinetic simulation of tokamak turbulencea),” *Physics of Plasmas (1994-present)*, vol. 18, no. 5, pp. –, 2011.

[26] M. Nakata and Y. Idomura, “Plasma size and collisionality scaling of ion-temperature-gradient-driven turbulence,” *Nuclear Fusion*, vol. 53, no. 11, p. 113039, 2013.