Bending analysis of sandwich plate by using WRBF

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Abstract. In the present paper, an accurate approach is made for the bending analysis of sandwich plate by using Wendland-C2 radial basis function (WRBF) based meshfree method. Governing differential equations (GDEs) are derived based on the Energy principle. WRBF based meshfree method is implemented for discretizing the GDEs. Convergence and validation of the developed MATLAB code is obtained. Accuracy of the present work is determined by comparing the obtained results with available published results.

Article I. Keywords: Sandwich plate, bending, WRBF, meshfree

1. Introduction

Sandwich structures are made of the outer facings which are strong and hard thus providing strength, while the soft core placed between the outer facingare light in weight. Owing to these adjacent stiffer composite layers, the soft core is deformed in shear thus sandwich composite materials are highly useful in shock absorption and noise reduction. Stiffness-to-weight ratio, fatigue strength, high specific modulus and high specific strength over traditional materials are some of their characteristics which lead to their increasing demand in various fields such as mechanical, transportation, aeronautical and many more.

Sandwich plates bearing transverse load show greater shear deformation effects. Various researchers have focused upon analysis of sandwich composite plates over recent couple of years. Mantari et al., 2012 [1] determined a solution for laminated composite and sandwich plates using new TSDT. Ferreira, 2004 [2] studied bending of layerwise modeled laminated and sandwich plates using poly-harmonic thin plate radial basis function. Fazzolari and Carrera, 2013 [3] used hierarchical trigonometric Ritz formulation for vibration analysis of sandwich plates in thermal environment. Ćetković and Vuksanović, 2009 [4] used layerwise displacement model for bending, free vibration and buckling analysis of sandwich composite plates.
2. Mathematical formulation

The mathematical formulation of the rectangular sandwich plates subjected to transverse loading is presented in Figure 1. The displacement field at any point in the sandwich plate is expressed as Kumar et al. [5]:

\[ U(x, y, z) = u_0(x, y) - \frac{\partial w(x, y)}{\partial x} + \left( -\frac{9}{10} \frac{z}{h} - \frac{27}{40h} \right) \psi_y(x, y) \]

\[ V(x, y, z) = v_0(x, y) - \frac{\partial w(x, y)}{\partial y} + \left( -\frac{9}{10} \frac{z}{h} - \frac{27}{40h} \right) \psi_x(x, y) \]

\[ W(x, y, z) = w_0(x, y) \]

\[ U, V \text{ and } W \] are the in-plane and transverse displacements of the plate at any point \((x, y, z)\) in \(x, y\) and \(z\) directions, respectively. \(u_0, v_0\) and \(w_0\) are the displacements at mid plane of the plate at any point \((x, y)\) in \(x, y\) and \(z\) directions, respectively. The functions \(\psi_x\) and \(\psi_y\) are the higher order rotations of the normal to the mid plane due to shear deformation about \(y\) and \(x\) axes, respectively.

The governing differential equations of plate are obtained using energy principle and expressed as[6]:

\[ \delta u_0 \frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \]

\[ \delta v_0 \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} = 0 \]

\[ \delta w_0 \frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} - q_x = 0 \]

\[ \delta \psi_x \frac{\partial M_{xx}'}{\partial x} + \frac{\partial M_{xx}'}{\partial y} - Q_x' = 0 \]
\[ \delta \psi_j = \frac{\partial M_{xx}'}{\partial x} + \frac{\partial M_{yy}'}{\partial y} - Q'_j = 0 \]  \hspace{1cm} (6)

The boundary conditions for simply supported edges are:

\[ \begin{align*}
  x &= 0, a : \nu = 0; \psi_x = 0; w = 0; M_x = 0; N_x = 0 \\
  y &= 0, b : u = 0; \psi_x = 0; w = 0; M_y = 0; N_y = 0
\end{align*} \hspace{1cm} (7)\]

\[ \begin{align*}
  x &= 0, a : \nu = 0; \psi_x = 0; w = 0; M_x = 0; N_x = 0 \\
  y &= 0, b : u = 0; \psi_x = 0; w = 0; M_y = 0; N_y = 0
\end{align*} \hspace{1cm} (8)\]

3. Solution methodology:

Radial basis function based meshless formulation works on the principle of interpolation of scattered data over entire domain. For present analysis the WRBF function used is as follows:

\[ g = (1 - c r)^{4}(4cr + 1) \]  \hspace{1cm} (9)

Where,

\[ r = \|X - X_c\| = \sqrt{(x - x_c)^2 + (y - y_c)^2} \]  \hspace{1cm} (10)

and ‘c’ is shape parameter. In present analysis

\[ c = \frac{1.01N^{1/25}N}{\sqrt{m}} \]  \hspace{1cm} (11)

is taken after validation and convergence study and the value of ‘m’ taken here is 2.6 (Kumar et al. [5]). The field variables (displacements) in terms of radial basis function are expressed as:

\[ \begin{align*}
  u &= \sum_{j=1}^{K} a_{j} g \|X - X_c\|, v = \sum_{j=1}^{K} a_{j} g \|X - X_c\|, w = \sum_{j=1}^{K} a_{j} g \|X - X_c\|, \psi_x = \sum_{j=1}^{K} a_{j} g \|X - X_c\|, \psi_y = \sum_{j=1}^{K} a_{j} g \|X - X_c\|
\end{align*} \hspace{1cm} (12)\]

Where, \( g \|X - X_c\| \) is radial basis function, \( \alpha_{j} \) is unknown coefficient, \( \|X - X_c\| \) is the radial distance between two nodes.

The static problem in terms of WRBF can be expressed as:

\[ \begin{bmatrix} L \\ B \end{bmatrix}_{N \times N} \begin{bmatrix} \alpha^u \\ \alpha^w \end{bmatrix}_{N \times 1} = \begin{bmatrix} F \end{bmatrix}_{N \times 1} \]  \hspace{1cm} (13)\]

Where,

\[ \begin{align*}
  L &= \left[ Lg g \|X - X_c\| \right]_{N \times N}, B = \left[ Lg g \|X - X_c\| c \right]_{N \times N}
\end{align*} \hspace{1cm} (14)\]

From equation (7.1), the value of unknown coefficient \( \alpha^u \) is obtained.

4. Results and discussion

A simply supported square sandwich plate is considered under uniform load. The span to thickness ratio \( (a/h) \) is taken as 10. The sandwich plate is composed of two outer layers (skins) of thickness \( h_1 = h_3 = 0.1h \) and one inner layer (core) of thickness \( h_2 = 0.8h \). The skin orthotropic properties are obtained by multiplying the core orthotropic properties with an integer \( R \), and they are given below:
And the skin properties are obtained by,

$$\mathbf{\bar{G}}_{\text{skin}} = R \mathbf{\bar{G}}_{\text{core}}$$  \hspace{1cm} (16)

The normalized stresses and transverse displacements of simply supported three layered sandwich plates subjected to uniformly distributed load $q_0$ is obtained for different values of $R$. The results are presented in non-dimensional form as:

$$\bar{w} = \frac{0.999781 \times w(a/2,a/2,0)}{q_0 h}, \quad \bar{\sigma}_{xx} = \frac{\sigma_{xx}(a/2,a/2,h/2)}{q_0}, \quad \bar{\sigma}_{yy} = \frac{\sigma_{yy}(a/2,a/2,h/2)}{q_0}, \quad \bar{\sigma}_{xy} = \frac{\sigma_{xy}(a/2,a/2,h/2)}{q_0}$$

In order to demonstrate the accuracy and applicability of present solution methodology and results, a WRBF based meshless code in MATLAB is developed for the bending analysis. Several examples have been analyzed and the computed results are compared with the published results.

Figure 2 is the convergence study of normalized central deflection of square simply supported sandwich plate. Span to thickness ratio is taken as 10.

Results obtained for deflection of simply supported sandwich plate (a/h=10 and R1=R2=5, 10 and 15, thickness of skin (inner and outer) is h1=h3=h/10 and the thickness of core is 8 h/10) under uniformly distributed load. WRBF based results here show a good convergence of less than 0.7% at 19×19 nodes. Considering this, a 19×19 node is used throughout the study.
Table 1 Comparison study of normalized maximum deflection and stress in sandwich plate (a/h=10, R=15).

| Method                     | \( \bar{w} \) | \( \sigma_{xx}^1 \) | \( \sigma_{xx}^2 \) | \( \sigma_{yy}^1 \) | \( \sigma_{yy}^2 \) | \( \sigma_{yy}^3 \) | \( \sigma_{zz}^1 \) |
|----------------------------|--------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 3D-elasticity [7]          | 121.720      | 66.787         | 48.299         | 3.238          | 46.424         | 34.955         | 2.494          | 3.964          |
| Pandya and Kant [8]        | 110.430      | 66.620         | 51.970         | 3.465          | 44.920         | 35.410         | 2.361          | 3.035          |
| Pandya and Kant [8]        | 90.850       | 70.040         | 56.030         | 3.753          | 41.390         | 33.110         | 2.208          | 3.091          |
| Ferreira et al. [9]        | 114.644      | 66.920         | 50.323         | 3.355          | 45.623         | 35.170         | 2.345          | 3.021          |
| Xiang et al. [10]         | 113.088      | 66.539         | 50.043         | 3.336          | 45.293         | 34.903         | 2.326          | 3.254          |
| Present (15 x 15)         | 113.676      | 66.773         | 50.223         | 3.348          | 45.453         | 35.036         | 2.336          | 3.564          |
| Present (17x17)           | 113.916      | 66.786         | 50.214         | 3.348          | 45.485         | 35.058         | 2.337          | 3.659          |
| Present (19 x 19)         | 114.076      | 66.853         | 50.272         | 3.352          | 45.520         | 35.087         | 2.339          | 3.733          |

Table 1 is a benchmark problem, which deals with the bending analysis of the simply supported, orthotropic sandwich plates, was presented by Srinivas and Rao [7] and is used here to validate the accuracy and convergence rate of WRBF based meshfree method. The present results are accurate and converge rapidly, and the convergence rate of WRBF based meshfree method is very fast. The present result shows good agreement to 3D-elasticity (Srinivas and Rao [7]) and other published results.

Table 2 is the effect of modular ratio of individual layers on transverse deflection and stresses of simply supported square sandwich plate. The span to thickness is 10. It can be observed that the value of R1 and R2 is low the deflection is high and by increase the value the deflection also decreases. Figure 3 and Figure 4 are effect of the stresses according to thickness.

Table 2. Effect of modular ratio of individual layers on transverse deflection and stresses of simply supported square sandwich plate (a/h=10, a=b)

| Effects of modular ratio | \( \bar{w} \) | \( \sigma_{xx}^1 \) | \( \sigma_{xx}^2 \) | \( \sigma_{yy}^1 \) | \( \sigma_{yy}^2 \) | \( \sigma_{yy}^3 \) | \( \sigma_{zz}^1 \) |
|-------------------------|--------------|----------------|----------------|----------------|----------------|----------------|----------------|
| R1=5, R2=5              | 256.24       | 60.23          | 46.89          | 9.38           | 38.37          | 30.17          | 6.03           | 4.60           |
| R1=10, R2=10            | 154.00       | 65.30          | 49.91          | 4.99           | 43.15          | 33.57          | 3.36           | 4.10           |
| R1=15, R2=15            | 114.08       | 66.85          | 50.27          | 3.35           | 45.52          | 35.09          | 2.34           | 3.73           |
| R1=5, R2=10             | 200.83       | 36.67          | 26.39          | 5.28           | 23.66          | 17.30          | 3.46           | 4.35           |
| R1=5, R2=15             | 176.55       | 26.94          | 17.98          | 3.60           | 17.43          | 11.87          | 2.37           | 4.07           |
| R1=10, R2=15            | 133.29       | 47.38          | 34.24          | 3.42           | 31.68          | 23.46          | 2.35           | 3.90           |
Figure 3  Effects of modular ratio on span to thickness ratio of $\sigma_{xy}$. (a/h=10, a=b)

Figure 4  Effects of modular ratio on span to thickness ratio of $\sigma_{xz}$. (a/h=10, a=b)

5. Conclusion

Bending response of sandwich plate was presented using WRBF based meshfree method. It was observed that the present results were accurate and converge rapidly, and the convergence rate of WRBF based meshfree method is very fast. The present result was good agreement with the published results. So, it can be concluded that the present WRBF is acceptable for the bending analysis of sandwich plate.

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