Deformed Hidden Conformal Groups for Rotating Black Holes

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Abstract. The general non-extremal Kerr black hole is holographically dual to a conformal field theory in two dimensions. It is known that two CFT duals or pictures, can describe the charged rotating black holes. They correspond respectively to the angular momentum $J$ and the electric charge $Q$ of the black hole. Moreover, these two pictures can be incorporated by a CFT dual or general picture, which is generated by an $SL(2,Z)$ modular group. To construct the general conformal structure, one can look at the charged scalar wave equation, in some appropriate values of frequency and charge. We consider the wave equation of a charged massless scalar field in the background of Kerr-Sen black hole and show, the wave equation can be reproduced by the Casimir operator of a local $SL(2,R) \times SL(2,R)$ hidden conformal symmetry, in the near region limit. We can find the exact agreement between macroscopic and microscopic physical quantities like entropy and absorption cross section of scalars for Kerr-Sen black hole. We then find an extension of vector fields that in turn yields an extended local family of $SL(2,R) \times SL(2,R)$ hidden conformal symmetries, parameterized by one parameter. For some special values of the parameter, we find a copy of $SL(2,R)$ hidden conformal algebra for the charged Gibbons-Maeda-Garfinkle-Horowitz-Strominger black hole in the strong deflection limit. This proceedings article is entirely based on the results in the published paper [1].

1. Introduction
The conjectured Kerr/CFT states that the physical properties of an extremal Kerr black hole are related to properties of a conformal field theory. In fact, the microscopic entropy and near-super radiant modes of four-dimensional extremal Kerr black hole can be derived by using the dual chiral conformal field theory associated with the diffeomorphisms of near horizon geometry of the Kerr black hole [2].

The Kerr/CFT correspondence has been studied extensively for different four and higher dimensional extremal rotating black holes which the dual chiral conformal field theory always contains a left-moving sector [3]-[13]. For all the extremal black holes, the near horizon geometry contains a copy of AdS space with isometries that could be extended to Virasoro algebra, hence it may explain the appearance of conformal structure.

We should note that the standard techniques of Kerr/CFT correspondence for extremal rotating black holes can not be applied directly to the non-extremal black holes. The reason is that no simple symmetry exists near the non-extremal black hole horizon that may point to conformal structure. Moreover for non-extremal black holes, the right-moving sector of dual
CFT turns on and there is no consistent boundary conditions that allow for both left and right-moving sectors in CFT.

However, as it is noted in [14], there is other conformal invariance, known as hidden conformal invariance, in the solution space of the wave equation in background of rotating non-extremal black holes. This means the existence of conformal invariance in a near horizon geometry is not a necessary condition, and the hidden conformal invariance is sufficient to have a dual CFT description. The idea of hidden conformal symmetry in the solution space of wave equation for a neutral scalar field in different rotating backgrounds was explored in detail in [15, 16].

A class of rotating black hole solutions is Kerr-Sen geometry. The solutions includes some non gravitational fields: an antisymmetric tensor field, a vector field and a dilaton. The solution is an exact solutions to the equations of motion of effective action of heterotic string theory in four dimensions. In [17], it was shown that for extremal Kerr-Sen black hole, the central charges of dual chiral CFT don’t get any contributions from the non-gravitational fields. Moreover, the central charges lead to the microscopic entropy of black hole that is in perfect agreement with Bekenstein-Hawking entropy. We consider the generic non-extremal Kerr/CFT correspondence for the class of Kerr-Sen black hole and show the existence of hidden conformal symmetry in the solution space of a charged scalar test field [1]. The U(1) gauge symmetry associated with the electric charge of scalar field enables us to find a class of conformal field theories; quite distinct from what has been found for a neutral scalar field in [16]. Although the Kerr-Sen solution contains a scalar dilatonic field, but the solution space of dilaton equation does not show any conformal symmetry. This in turn is in the same direction and in agreement with previously observation of no contribution of non-gravitational fields to the central charges of dual CFT in extremal case [17]. We also discuss the absorption of scalars in the near region of non-extremal black hole. Moreover we find an extended version of hidden conformal generators that involve one parameter for the class of Kerr-Sen solutions. These conformal generators provide a completely new set of conformal symmetry generators for the charged Gibbons-Maeda-Garfinkle-Horowitz-Strominger black hole.

2. A Scalar Field in the Background of Kerr-Sen Black Hole

The Kerr-Sen [18] is an exact classical four dimensional black hole solution in the low energy heterotic string field theory which the line element is given by

\[ ds^2 = -\left(1-\frac{2Mr}{\rho^2}\right)dt^2 + \rho^2\left(\frac{dr^2}{\Delta} + d\theta^2\right) - \frac{4Mr a}{\rho^2} \sin^2 \theta dt d\phi + \left\{r(r+2b)+a^2 + \frac{2Mra^2 \sin^2 \theta}{\rho^2}\right\} \sin^2 \theta d\phi^2, \tag{1} \]

where \( \rho^2 = r(r + 2b) + a^2 \cos^2 \theta \), \( \Delta = r(r + 2b) - 2Mr + a^2 \), and \( b = Q^2/2M \). The dilaton of the theory is \( \Phi = -\frac{1}{2} \ln \frac{r(r+2b)+a^2 \cos^2 \theta}{r^2+a^2 \cos^2 \theta} \). The non-zero components of the gauge field are \( A_t = -\frac{rQ}{\rho^2}, \ A_\phi = \frac{rQ \sin^2 \theta}{\rho^2} \), and the antisymmetric tensor field is given by \( B_{t\phi} = \frac{bra \sin^2 \theta}{\rho^2} \).

The event horizon of black hole is located at \( r_+ = M - b + \sqrt{(M-b)^2 - a^2} \), and the Hawking temperature is \( T_H = \frac{4\pi M}{(2M^2 - Q^2 + \sqrt{(2M^2 - Q^2)^2 - 4J^2})} \) and the angular velocity of horizon is \( \Omega_H = \frac{J}{M(2M^2 - Q^2 + \sqrt{(2M^2 - Q^2)^2 - 4J^2})} \). For \( b = 0 \), we note that all non-gravitational fields vanish and the metric (1) changes simply to the metric of Kerr black hole. The Kerr-Sen black hole (1) approaches to the metric of charged Gibbons-Maeda-Garfinkle-Horowitz-Strominger black hole in the strong deflection limit [21] where the rotational parameter \( a \rightarrow 0 \).

As in [1], we consider a massless scalar field \( \Phi \) with charge \( e \) as a probe in background (1). The minimally coupled Klein-Gordon equation for the massless scalar field \( \Phi \) is

\[ (\nabla_\mu - ieA_\mu) (\nabla^\mu - ieA^\mu) \Phi = 0. \tag{2} \]
After separating the coordinates as $\Phi (r, t, \theta, \phi) = \exp (im\phi - i\omega t) S(\theta) R(r)$, we find two separated differential equations [1]

$$\partial_r (\Delta \partial_r R(r)) + \left(\frac{\alpha r - ma}{\Delta} + \omega^2 \Delta + 2\delta r - \sigma\right) R(r) = 0,$$

(3)

and

$$\frac{1}{\sin \theta} \partial_\theta ((\sin \theta) \partial_\theta S(\theta)) + \left(\sigma - \frac{m^2}{\sin^2 \theta} + \omega^2 a^2 \sin^2 \theta\right) S(\theta) = 0,$$

(4)

where $\alpha = 2M\omega - eQ$, $\delta = \alpha \omega$ and $\sigma$ is the separation constant. We rewrite the radial equation (3) as

$$\partial_r (\Delta \partial_r R(r)) + \left(\frac{(2M\omega r_+ - eQr_+ - ma)}{(r - r_+)(r_+ - r_-)} - \frac{(2M\omega r_+ - eQr_- - ma)}{(r - r_-)(r_+ - r_-)} + f(r)\right) R(r) = \sigma R(r),$$

(5)

where $f(r) = (\Delta + 4M(M + r))\omega^2 - (2M + r) 2eQ\omega + e^2 Q^2$, and $r_- = M - b - \sqrt{(M - b)^2 - a^2}$ is the inner horizon of black hole. We can simplify the equation (5) and ignore the function $f(r)$ by considering a low frequency scalar field $\omega << 1/M$ in the near region geometry where $r << 1/\omega$ and with assumption that electric charge of scalar field satisfies $eQ << 1$.

As we notice in equation (5), the electric charge of scalar field contributes by extra terms to the radial equation, compared to the radial wave equation of neutral scalar field [16]. Applying the $U(1)$ gauge transformation $\Phi \rightarrow \exp (ie\lambda)\Phi$ associated with the electric charge of scalar field enable us to write the phase of scalar field $\exp (im\phi)$ as $\exp (im\phi^0)$. To do this, we consider the pair of linearly independent coordinates, $\phi^0 = \alpha \phi + (1 - \alpha) \lambda$, and $\phi^\beta = \beta \phi + (1 - \beta) \lambda$ such that $\exp (im\phi + i\epsilon \lambda) = \exp \left( im_\alpha \phi^0 + im_\beta \phi^\beta\right)$ where $m = a m_\alpha + \beta m_\beta$ and $\epsilon = (1 - \alpha) m_\alpha + (1 - \beta) m_\beta$, as in [1]. The above transformations give us some benefits such as getting the proper temperature and entropy from the conformal calculations [22]. We consider only the case where $m_\alpha = 0$, which means there is no momentum mode along $\phi^0$. We show the coordinate $\phi^\beta$ with $\phi$, for simplicity, in what follows. The other case with $m_\beta = 0$ gives similar results. We consider the radial equation (5) for low frequency massless charged (small probe) scalar field in near region Kerr-Sen can be rewritten finally as

$$\frac{\partial_r (\Delta \partial_r R(r))}{R(r)} + \left(\frac{(2Mr_+ \omega - (Qr_+ - (1 - \beta) a \beta) m_\beta)}{(r - r_+)(r_+ - r_-)} - \frac{(2Mr_+ \omega - (Qr_- - (1 - \beta) a \beta) m_\beta)}{(r - r_-)(r_+ - r_-)}\right) = \sigma.$$

(6)

3. Hidden Conformal Symmetry for the Charged Scalar Field

As in [1], we define the conformal coordinates $\omega^+ , \omega^-$ and $y$, in terms of coordinates $t, r$ and $\phi$ by

$$\omega^+ = \sqrt{\frac{r - r_+}{r - r_-}} \exp(2\pi T_R \phi + 2n_R t),$$

(7)

$$\omega^- = \sqrt{\frac{r - r_+}{r - r_-}} \exp(2\pi T_L \phi + 2n_L t),$$

(8)

$$y = \sqrt{\frac{r_+ - r_-}{r - r_-}} \exp(\pi(T_R + T_L) \phi + (n_R + n_L) t),$$

(9)
where the right and left temperatures \( T_R \) and \( T_L \) are given by \( T_R = \frac{r_+ - r}{4\pi a_{\beta}} \) and \( T_L = \frac{r_+ + r}{4\pi a_{\beta}} \), and \( n_R = -\frac{(r_+ - r_-)(1-\beta)Q}{8\pi a_{\beta}} \), \( n_L = -\frac{(2\alpha_\beta + (r_+ + r_-)(1-\beta)Q)}{8\pi a_{\beta}} \). We also define the right and left moving vector fields as

\[
H_1 = i\partial_+, \quad H_0 = i(\omega^+ \partial_+ + \frac{1}{2} y \partial_y), \quad H_{-1} = i((\omega^+)^2 \partial_+ + \omega^+ y \partial_y - y^2 \partial_-),
\]

and

\[
\bar{H}_1 = i\partial_-, \quad \bar{H}_0 = i(\omega^- \partial_- + \frac{1}{2} y \partial_y), \quad \bar{H}_{-1} = i((\omega^-)^2 \partial_- + \omega^- y \partial_y - y^2 \partial_+),
\]

respectively. The vector fields (10) satisfy the \( SL(2, R) \) algebra \( [H_0, H_{\pm 1}] = \mp i H_{\pm 1}, [H_{-1}, H_1] = -2i H_0 \), and similarly for \( \bar{H}_1, \bar{H}_0 \) and \( \bar{H}_{-1} \). The vectors \( \partial_+, \partial_- \), and \( \partial_y \) in terms of coordinates \( t, r \) and \( \phi \), can be written as

\[
\partial_+ = e^{-(2\pi T_R \phi + 2n a_{\beta} t)} \left( \Delta^{1/2} \partial_r + Z_{\phi+} \partial_\phi - Z_{t+} \partial_t \right),
\]

\[
\partial_y = e^{-(\pi (T_L + T_R) \phi + (n R + n L) t)} (Z_{r y} \partial_r + Z_{y y} \partial_y - Z_{t y} \partial_t),
\]

\[
\partial_- = e^{-2\pi T_L \phi + 2n a_{\beta} t} \left( \Delta^{1/2} \partial_r + Z_{\phi-} \partial_\phi - Z_{t-} \partial_t \right),
\]

where \( Z_{ij} \) are given explicitly in [1]. The quadratic Casimir operators of \( SL(2, R)_R \) and \( SL(2, R)_L \) with generators \( H_{\pm 1}, H_0 \) and \( H_{\pm 1}, H_0 \) respectively, are equal \( H^2 = -H_{0}^2 + \frac{1}{2}(H_1 H_{-1} + H_{-1} H_1) \). The quadratic Casimir operator is

\[
H^2 = \frac{1}{4}(y^2 \partial^2_y - 2y \partial_y) + y^2 \partial_+ \partial_-
\]

It is straightforward to show that the Casimir operator (15) reduces simply to the radial equation (6) where we consider the separation constant \( \sigma = l(l+1) \)

\[
H^2 R(r) = l(l+1) R(r),
\]

which in turn signals the existence of \( SL(2, R)_L \times SL(2, R)_R \) hidden conformal symmetry. We should emphasise that only a local \( SL(2, R)_L \times SL(2, R)_R \) hidden conformal symmetry for the solution space of massless charged scalar field in near region of Kerr-Sen geometry that is generated by the vector fields (10),(11). The reason is these vectors are not periodic under \( \phi \sim \phi + 2\pi \) identification (the periodicity of \( \phi \) comes simply from the periodicity of gauge transformation parameter \( \lambda \) and original black hole coordinate), so they can’t be defined globally [1]. We may conclude the existence of local \( SL(2, R)_L \times SL(2, R)_R \) hidden conformal symmetry suggests that we assume the dynamics of the near region can be described by a dual CFT. To verify this assumption, we try to find the microscopic entropy of the dual CFT which according to the Cardy formula, is given by

\[
S_{CFT} = \frac{\pi^2}{3} (c_L T_L + c_R T_R).
\]

The central charges of dual CFT for extremal Kerr-Sen black holes were obtained in [17] based on analysis of the asymptotic symmetry group. For the case of non-extremal black hole, we assume that the conformal symmetry connects smoothly to that of the extremal case; so we consider the central charges given by [1]

\[
c_R = c_L = 12 \beta J,
\]
where the factor $\beta$ appears due to the presence of electrically charged scalar probe in the background of charged black hole. In the absence of electric charge for the test field, $\beta$ is equal to one, and (18) reduces to $12J$ which is the same central charge for the extremal case. The central charges (18) make the microscopic entropy of CFT (17), that is given by $S_{CFT} = 2\pi M r_+$, which is in complete agreement with the macroscopic Bekenstein-Hawking entropy of Kerr-Sen spacetime [1]. The macroscopic Bekenstein-Hawking entropy of Kerr-Sen black hole is given by [17, 23]

$$S = \pi \left(2M^2 - Q^2 + \sqrt{(2M^2 - Q^2)^2 - 4J^2}\right),$$

which is equal to $S_{CFT}$ upon substitution $r_+ = M - b + \sqrt{(M - b)^2 - a^2}$, $J = aM$, and $b = Q^2/2M$.

4. Deformed Hidden Conformal Symmetry with Deformation Parameter $\kappa$

The Kerr-Sen black hole has two horizons $r_+$ and $r_-$, where the scalar wave equation (5) have poles in both horizons. As in [1], we deform the wave equation (5) near the inner horizon $r_-$. The reason for deformation is that for the non-extremal Kerr-Sen black hole where $r \gg r_-$, the linear and quadratic terms in frequency which are coming from the expansion near the inner horizon can be dropped. So, we consider the deformation of radial equation (6) by deformation parameter $\kappa$ as

$$\left[\partial_r(\Delta \partial_r) + \frac{(2Mr_+\omega - a_1m\beta)^2}{(r - r_+)(r_+ - r_-)} - \frac{(2Mr_-\omega - a_2m\beta)^2}{(r - r_-)(r_+ - r_-)}\right] R(r) = l(l + 1) R(r),$$

where $a_1 = Qr_+(1 - \beta) + a\beta$ and $a_2 = Qr_-(1 - \beta) + a\beta$. The deformation parameter $\kappa$ and $r - r_-$ should satisfy $\kappa M^2 a_2 m\beta \omega << 2\sqrt{(M - b)^2 - a^2}(r - r_-)$ as well as $\kappa^2 M^4 \omega^2 << 2\sqrt{(M - b)^2 - a^2}(r - r_-)$ to drop the linear and quadratic terms in frequency from the expansion near the inner horizon pole while we still keep the near region geometry and low frequency scalar field as an electrically charged probe [1]. We try now to find a new set of vector fields that generate $SL(2, R)$ algebra. Moreover, we require that the quadratic Casimir operator of the algebra represents the deformed radial equation (20). We consider the set of vector fields $L_{\pm}$ and $L_0$ given by

$$L_{\pm} = e^{\pm \rho t \pm \sigma \phi} \left[ \mp \sqrt{\Delta} \partial_r + \frac{C_2 - \delta r}{\sqrt{\Delta}} \partial_\phi + \frac{C_1 - \gamma r}{\sqrt{\Delta}} \partial_t \right],$$

$$L_0 = \gamma \partial_t + \delta \partial_\phi,$$

which satisfy $[L_+, L_-] = 2L_0, [L_{\pm}, L_0] = \pm L_0$. Moreover

$$L_0^2 - \frac{1}{2} (L_+ L_- + L_- L_+) = \partial_r(\Delta \partial_r) + \frac{(2Mr_+\omega - a_1m\beta)^2}{(r - r_+)(r_+ - r_-)} - \frac{(2Mr_-\omega - a_2m\beta)^2}{(r - r_-)(r_+ - r_-)}.$$

Using (23), we can find two sets of solutions for the constants $\rho, \sigma, \delta, \gamma, C_1$ and $C_2$ that appear in (21) and (22).

In the first class of solutions, that we call $a$-solutions [1], the generators of $SL(2, R)$ are

$$L_{\pm a} = e^{\pm \rho t \pm \sigma \phi} \left[ \mp \sqrt{\Delta} \partial_r + \left( Q (1 - \beta) \frac{r_+(r_- + \kappa r_+ - r_+(1 + \kappa))}{r_+ - r_-} \right) \frac{r_- - r_+}{2\pi \Omega H} \partial_\phi \right.$$

$$- \frac{\beta}{2\pi T_H} \frac{r - (M - b)}{2\pi \Omega H \beta (T_L + T_R)} \frac{r_+ - r_-}{2\pi T_H} \left. \partial_t \right].$$
and

\[ L_{0a} = \left( \frac{1}{2\pi T_H} - \frac{1}{2\pi \Omega_H \beta (T_L + T_R)} \right) \partial_t + \left( \frac{Q (1 - \beta) (1 + \kappa)}{8\pi MT_H} + \frac{\Omega_H \beta}{2\pi T_H} \right) \partial_\phi, \] (25)

where

\[ \rho_1 \equiv \frac{b}{(1 - \kappa)} + \frac{Q (1 - \beta)}{2\alpha \beta (1 - \kappa)} (Mr_+ (1 + \kappa) - \kappa r_+ (r_+ + r_-)). \]

For the second class of solutions, that we call \( b \)-solutions \([1]\), we find

\[ L_{zb} = e^{\pm i \rho_1 \pi (\frac{b}{\kappa} + 2\pi T_L) \phi} \left[ \mp \sqrt{\Delta} \partial_r + \left( 2Mr_+ \Omega_H \beta + \frac{Q (1 - \beta) (r_+ (1 + \kappa) - \kappa r_+ (r_+ + r_-))}{r_+ - r_-} \right) \frac{\partial_\phi}{\sqrt{\Delta}} \right. \]

\[ + \left. \left( 2Mr_+ + \frac{(r - r_+)}{2\pi \beta \Omega_H (T_L + T_R)} \right) \frac{\partial_t}{\sqrt{\Delta}} \right], \] (26)

and

\[ L_{0b} = \left( \frac{-1}{2\pi \beta \Omega_H (T_L + T_R)} \right) \partial_t + \left( \frac{Q (1 - \beta) (\kappa - 1)}{8\pi T_H M} \right) \partial_\phi, \] (27)

where

\[ \rho_2 = \frac{Q (1 - \beta) (r_+ (r_+ - \kappa r_+) + Mr_+ (\kappa - 1))}{2Mr_+ a \beta (\kappa - 1)} + 2\pi \Omega_H (T_R + T_L). \]

As we notice, the generators (24), (25), (26) and (27) of \( SL(2, R) \times SL(2, R) \) reduce exactly to the generators of generalized hidden conformal symmetry of Kerr black hole \([24, 25]\), in the limit where \( \beta = 1 \) and \( b = 0 \). The left and right temperatures are given by \( T_L = T_R \frac{1 + \kappa}{1 - \kappa} \) and \( T_R = \frac{r_+ - r_0}{4\pi \beta a} \) respectively \([1]\). This means the right temperature of generalized hidden CFT doesn’t get any contribution from the deformation parameter \( \kappa \) and so is the same as the right temperature of hidden CFT while the left temperature is affected by the deformation parameter \( \kappa \). Demanding the agreement of microscopic entropy of CFT given by (17) to the Bekenstein-Hawking entropy of Kerr-Sen black hole (19) requires the central charges are given by \([1]\)

\[ c_L = c_R = \frac{6 (1 - \kappa) a \beta M r_+}{\sqrt{(M - b)^2 - a^2}}, \] (28)

These central charges reduce to central charges of generalized hidden CFT of Kerr black hole where \( \beta = 1 \) and \( b = 0 \). The charged Gibbons-Maeda-Garfinkle-Horowitz-Strominger black hole is a special case of Kerr-Sen black hole when the rotational parameter is zero. In this limit, the only acceptable values of \( \kappa \) are \( \pm 1 \). The generators of \( SL(2, R) \) for \( b \)-solutions (26), (27) with \( \kappa = -1 \) reduce to \([1]\)

\[ L_{zb} = e^{\pm i \rho_1 \pi (\frac{b}{\kappa} + 2\pi T_L) \phi} \left[ \mp \sqrt{\Delta} \partial_r - \frac{2Q (1 - \beta) (M - b - r)}{\sqrt{\Delta}} \partial_\phi + \frac{4M (r - M + b)}{\sqrt{\Delta}} \partial_t \right], \] (29)

\[ L_{0b} = -4M \partial_t - 2Q (1 - \beta) \partial_\phi. \] (30)

So, these are the generators of \( SL(2, R) \) for the Gibbons-Maeda-Garfinkle-Horowitz-Strominger black hole. The generators (24) and (25) of \( SL(2, R) \) for \( a \)-solutions with \( \kappa = 1 \) give the same copy of generators as in (29) and (30) with renaming the generators by \( L_+ \rightarrow -L_+, L_0 \rightarrow -L_0 \) \([1]\). We also note that generators (29) and (30) in the special case of \( Q = 0 \) reduce to the generators of \( SL(2, R)_{Sch} \) for Schwarzschild black hole \([26]\).
5. Conclusions
In this proceedings article, that is based entirely on [1], we consider the concept of Kerr/CFT correspondence for the non-extremal Kerr-Sen black hole in heterotic string theory. We discuss the hidden conformal symmetry as well as a family of generalized hidden conformal symmetry for the Kerr-Sen black hole. The latter is realized by a family of generators that is parameterized by one parameter $\kappa$. In the special limit of the family parameter, the generalized hidden conformal symmetry reduces to a single copy of symmetry for the charged Gibbons-Maeda-Garfinkle-Horowitz-Strominger black hole. The crucial point in this paper is that the charged Klein-Gordon test field in the background of Kerr-Sen reveals the (generalized) hidden conformal symmetry. The equation of motion for the dilaton in Kerr-Sen geometry is different from the one for the Klein-Gordon test field and this renders the possibility of writing the equation in terms of Casimir invariants (15) of $SL(2,R)_{\mathcal{R}}$ and $SU(2,R)_{\mathcal{R}}$. This observation is somehow in agreement with the fact that the non-gravitational fields don’t contribute to the central charge of conformal field theory [17]. It would be also an interesting result to derive the central charges of generalized hidden conformal symmetry from some extension of asymptotic symmetry group.

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