Generalized Fully Coherent Closed-Form Receiver Design for Joint Radar and Communication System

Muhammad Zubair, Sajid Ahmed*, Senior Member, IEEE, and Mohamed-Slim Alouini**, Fellow, IEEE

Abstract—Conventional radars repeat the transmission of the same waveform after a predefined interval of time called pulse-repetition-interval (PRI). This technique helps estimate the range and Doppler shift of the targets and suppress clutter. However, in dual-function radar communication (DFRC), a different symbol waveform is transmitted after each PRI. Depending on the number of targets, radar receiver output yields several peaks representing different targets’ ranges. Each peak comes with its side-lobes called range-side-lobes (RSL). In DFRC, due to different symbol waveform transmission, peaks and RSLs do not remain coherent, making Doppler shift estimation and clutter suppression challenging tasks. In most of the available literature, iterative receive filters have been designed for DFRC to minimize RSLs and achieve coherent output for different waveforms. However, the proposed receive filter does not guarantee coherent output for more than two-way. In contrast, we proposed two novel closed-form algorithms to design receive filters for DFRC that guarantee coherent output response for several waveforms and suppress RSLs. Simulation results demonstrate that the proposed receivers achieve full coherency, and the RSLs are significantly lower than the conventional method. Furthermore, the advantage of achieving coherent output response is shown in target detection and bit-error-rate improvement.

Index Terms—Joint radar communication, constrained optimization, closed-form receive filters.

I. INTRODUCTION

In wireless communication, the radio spectrum is the most precious and expensive resource. Radio spectrum extending from 1MHz to 300GHz has favorable propagation characteristics, therefore, this spectrum is being used for different purposes. The use of this band for different applications is summarized in Table I. There are currently 7.9 billion mobile subscriptions and it is predicted that the total mobile subscriptions by the end of 2026 will reach 8.8 billion. Similarly, the overall Internet-of-Things (IoT) connections have reached 12.6 billion and in 2026 they are expected to reach 26.9 billion. To accommodate such high demand for subscriptions within the available spectrum, the fifth-generation (5G) standard was introduced in 2020. The 5G will be much faster, reliable, more efficient, and will support data rates up to 35.46 Gbps [1]. It will use frequency range 1 (FR1), also known as a sub-6GHz band (2-7.125GHz), and frequency range 2 (FR2), known as mm-Wave radio band (24.25-52.6GHz), as shown in Table I. Future applications, such as holographic calls and flying networks, will require even more high data rates. In this perspective, quest for 6G and 7G has been already started, where 5G, satellite network, and space roaming will be integrated [1], [2], [3], [4]. Spacecraft, drones, and radars will be considered a simple IoT devices [5], [6], [7].

As cellular communication systems have evolved from 1G to 5G (please see Table I) and on the way to 6G, radar systems are also evolving at a much faster pace. Until the second world war, radar systems had a significant size, were bulky, power-hungry, and the cost was enormous; moreover, the use of radar was restricted to the military only. Now, the size and cost of radar have significantly reduced, and it is entering the consumer market, such as automotive industry, medical devices for non-invasive testing, gesture control smart devices, and wearable devices [8], [9], [10], [11].

Until now, radar and cellular communication systems have been developed and studied separately. Both systems use their frequency spectrum. In Table I, it is summarized how radar bands for different applications underutilized the limited radio spectrum. Now the evolution of applications requiring high data rate communication has completely occupied the limited radio spectrum and started to overlap with existing radar bands as shown in Table I. The scarcity of radio spectrum urged researchers to develop new solutions to overcome this problem. One of the possible solutions is the integration of both systems. As both technologies rely on the same electromagnetic radiation, the same equipment, spectrum, and signals can be used for both systems. Therefore, to efficiently use the available resources for both communication and radar, extensive research is being done to devise new techniques. The spectrum sharing and the use of identical waveforms for communications and radar will likely be the main feature of future communications systems in 5G and beyond.

In the literature, there are mainly four categories of joint radar and communication research, which are as follows:

1) Spectrum Sharing: In this technique, both communication and radar systems share the same spectrum when it is free from the other [14], [15], [16], [17], [18]. For strategic applications, use of this technology is not preferred.
2) Communication Centric Joint Communication Radar (JCR): In JCR, radar parameters are estimated from the transmission of the communication system [19], [20], [21], [22].

3) Radar Centric Joint Radar Communication (JRC): In JRC technique, communication is realized on a primary radar system. The pioneering work in this domain is presented in [23] and [24], where OFDM communication is employed on MIMO radar. In JRC framework, the waveform design for ultra-broad bandwidth THz systems is investigated in [25].

4) Dual Function Radar Communication (DFRC): In DFRC, radar and communication functions are implemented on a single hardware platform. Here, a single waveform serve the purpose of both data communication and radar parameter estimation. Based on the system model, DFRC can be further classified into two categories: i) Communication Centric DFRC [26], [27], [28], [29], and ii) Radar Centric DFRC [30], [31], [32], [33], [34], [35], [36], [37], [38], [39], [40], [41], [42], [43], [44], [45], [46]. In Communication Centric DFRC, the communication performance matrices are optimized under the radar constraint, and in radar Centric DFRC, performance matrices are optimized under the communication constraint.

Although all of the above techniques, i.e., spectrum sharing, JCR, and JRC, have their significance, DFRC has more applications than the other technologies due to its single hardware for dual functionality. Therefore, the focus of this paper will be DFRC, and a few challenges will be addressed.

The main challenge in DFRC is detecting and estimating target parameters in the presence of clutter (the transmitted signal echo from mountains, ground, and buildings). After demodulation, the received signal, a combination of the reflected signal from the target and clutter, is passed through the receive filter. The indices of peak values at the receiver
output represent the location of the target and clutter. Each peak comes with its side lobes, known as range-side-lobes (RSLs), which can mask weak targets.

Conventional radars transmit the same waveform multiple times, for some interval called coherent-processing-interval (CPI), to detect weak targets in clutter. Therefore, each waveform yields a coherent response at the receiver output for clutter, helping to detect targets. In contrast, DFRC systems transmit different symbol waveforms in CPI, and each waveform does not yield a coherent output response for clutter. Consequently, detecting weak targets becomes challenging.

In [47], waveforms are designed to jointly optimize the radar and communication performance matrices. This work optimizes waveforms’ covariance matrices corresponding to different users to maximize the signal-to-interference-plus-noise ratio (SINR) and minimize the ripple in the main lobe of the beam pattern. The main drawback of the algorithm is that it requires the design of waveforms to realize the designed covariance matrices in the second stage [48], [49], [50], [51]. Therefore, the computational complexity of this technique can be very high. Similarly, the coherent output response of different waveforms and the transmission of random data for communication without affecting correlation are questionable.

The study in [34] proposes an iterative joint least-squares (JLS) algorithm to achieve coherent receiver output and minimize RSLs for different symbol waveforms. However, the designed receive filter does not guarantee a fully coherent response for more than two waveforms. One closed-form and two iterative algorithms proposed in [26] satisfy constant modulus and similarity constraints but do not guarantee a fully coherent receiver output nor suppress RSLs. In [27] integrated RSLs constraint is incorporated in the objective function to reduce RSLs but output coherency is not considered in the objective function. Similarly, to reduce RSLs, iterative receive filters are proposed in [52], [53], and [54] but here again coherent output response is not accounted for in the receive filter design. In another DFRC framework, frequency-diverse-array (FDA) is exploited, where frequency increment is applied across antenna array elements [41], [42], [43], [44], [45], [46]. In FDA, the antenna array beampattern becomes dependent on angle, time, and range [55], [56], [57]. Further developments on DFRC waveforms are presented in [35], [36], [37], [38], [39], [40], [28], [29], [30], [31], [32], [33], and [58].

As far as we know, no algorithm guarantees a fully coherent receiver output for different waveforms. Therefore, the target detection is poor in most of the proposed algorithms. In contrast, our proposed algorithm reduces RSLs and achieves a fully coherent output response for several waveforms.

Contributions: The main contribution of this paper are:

1) We transform the problem into a constrained optimization problem to guarantee fully coherent RSLs for different waveforms.

2) The reported result in [34] claims that the design of a fully coherent receiver is possible only for two waveforms. In contrast, we transformed the problem into a constrained optimization problem that guarantees a fully coherent receiver for any number of waveforms.

3) Two generalized closed-form receivers are designed based on linear- and circular-convolution models for multiple waveforms. The corresponding output response of both receivers is fully coherent, and the RSL levels are lower than the existing iterative method in [34].

Paper Organization: The rest of the paper is organized as follows: The problem formulation is discussed in Sec. II and proposed closed-form design of receive filter is described in Sec. III. The receive signal model of radar and communication is discussed in Sec. IV. Experimental results are discussed in Sec. V, and conclusions are drawn in Sec. VI.

Notations: Bold upper case letters, X represents matrices and lower case letters, x, denote vectors. Transpose and conjugate transposition of a matrix are denoted by (·)T and (·)H, respectively. The conjugate of a scalar is denoted by (·)*. Convolution operator is denoted by * and the close interval \{x : a ≤ x ≤ b\} is denoted as [a, b].

II. PROBLEM FORMULATION

Consider a joint radar-communication scenario shown in Fig. 1; a base-station (BS) intends to communicate with multiple mobile-users (MUs) and localize targets using the same transmitted waveforms. The transmitted signal propagates in different directions depending on the antenna’s beampattern. In the figure blue, yellow, and red waveforms represent the received signal by moving targets, MUs, and stationary objects, respectively. Multiple objects, such as moving vehicles, airplanes, ground, and buildings, receive the transmitted signal at different range bins. If the target’s radar-cross-section (RCS) is sufficiently large, part of the received signal energy reflects towards BS, where it can be used to localize that target. At the BS, a significant portion of the received signal energy is due to the ground and buildings. The received signal from such targets is called clutter, and it makes small target detection challenging. Since the Doppler shift due to the stationary object is zero and the moving is more significant than zero, we can easily exploit this fact to localize moving objects. Similarly, the BS’s transmitted signals are decoded using conventional techniques on the MU end. To isolate one user’s data from the other, techniques such as time-division-multiplexing and spreading codes can be used at the BS. MUs modulate their data symbols at a different carrier frequency to avoid interference with the reflected signals from the targets. The received signal is demodulated at the communication and radar units at the BS. The communication unit decodes received symbols transmitted from different users, while the radar unit localizes the targets. The main focus of this work is on the radar unit. It should be noted that the radar unit knows the transmitted symbols from the BS.

As mentioned earlier, a coherent-processing-interval (CPI) technique is used in conventional radar systems to estimate the target’s Doppler shift [59]. However, in DFRC, a non-CPI (NCPI) technique is required to transmit different symbols in each waveform. In NCPI, the receiver output response does not remain coherent for clutter, affecting the receiver’s ability to detect small targets. In the next section, we design receive filters for K waveforms and constrain them to yield coherent output responses to address this problem. For the transmission
Fig. 1. Joint Radar-Communication scenario: Transmitted signal from the base-station is used for radar (blue waveform) and communication (yellow waveform) functions. The reflected signal from the stationary objects is considered as a clutter signal.

Fig. 2. Embedding data bits in radar pulses. Here, $\Psi_k(t)$ is the $k$th waveform (symbol), $T$ is the pulse/symbol duration, $T_{\text{PRI}}$ is pulse repetition interval, and $\text{NCPI}$ is the non-coherent processing interval. Of $M$ symbols, the NCPI technique is illustrated in Fig. 2. The system can select a waveform from a set of $K$ waveforms to transmit a particular symbol. Each waveform/symbol carries $\log_2(K)$ bits.

If $\Psi_k(t)$ is the $k$th symbol waveform, $h_k(t)$ is the corresponding receive filter to be designed, and $e_k(t)$ is the desired output response, mathematically, the receive filter design problem can be expressed as

$$\Psi_k(t) \ast h_k(t) = e_k(t), \quad k = 1, 2, \ldots, K$$

subject to constraint

$$\Psi_1(t) \ast h_1(t) = \Psi_2(t) \ast h_2(t) = \cdots = \Psi_K(t) \ast h_K(t). \quad (1)$$

The solution of (1) can yield receive filters to produce coherent output response for non-coherent waveforms. In the following, the generation of waveforms and the solution of optimization problem setup in (1) will be discussed.

III. PROPOSED CLOSED-FORM RECEIVE FILTER DESIGN

An iterative algorithm is proposed in [34] to minimize RSLs and achieve coherent output response for different waveforms. The proposed algorithm suppresses RSLs, but a coherent output response for more than two waveforms is not guaranteed; therefore, Doppler shift measurement can be challenging. Similarly, the proposed algorithm in [60] uses a continuous-phase-modulation (CPM) framework [52], and based on match and mismatch filters proposes a cascaded filter design. Using these cascaded filters, one can enhance the coherency at the cost of higher RSLs that can reduce the target sensing capability of the radar system. The motivation
of our proposed work is to provide a generalized closed-form solution applicable for multiple waveforms to communicate any number of information bits without sacrificing the performance of a DFRC system.

To design our proposed receive filters for $K$ waveforms, consider $x_k(p)$ and $h_k(l)$ are $p$th and $l$th sampled values of $\Psi(t)$ and $h(t)$, respectively. Using sample values, the convolution in (1) in vector form can be written as

$$\Psi_k h_k = e_k,$$ \hspace{1cm} (2)

where

$$\Psi_k = \begin{bmatrix} x_k(0) & 0 & \cdots & 0 \\ x_k(1) & x_k(0) & \cdots & \vdots \\ \vdots & \vdots & \ddots & x_k(L - L_f - 1) \\ x_k(L - 1) & \vdots & \cdots & x_k(L - L_f) \\ 0 & x_k(L - 1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & x_k(L - 1) \end{bmatrix},$$

$$h_k = \begin{bmatrix} h_k(0) \\ h_k(1) \\ \vdots \\ h_k(L_f - 1) \end{bmatrix},$$

and

$$e_k = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}^T.$$

Moreover, $\Psi_k \in \mathcal{C}^{(L+L_f-1) \times L_f}$ while $L$ is the total number of samples corresponding to the waveform $\psi_k(t)$ and $L_f$ is the length of filter coefficients. Similarly, the constraints in (1) can also be written as

$$\Psi_1 h_1 = \Psi_2 h_2 = \cdots = \Psi_K h_K.$$ \hspace{1cm} (4)

To design receive filters for $K$ waveforms, the objective function in (1) can be written in vector-matrix form as

$$Xh = e,$$ \hspace{1cm} (5)

and

$$\tilde{X}h = 0,$$ \hspace{1cm} (6)

where

$$X = \begin{bmatrix} \Psi_1 & 0 & \cdots & 0 \\ 0 & \Psi_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \Psi_K \end{bmatrix},$$

$$\tilde{X} = \begin{bmatrix} -\Psi_2 & 0 & \cdots & 0 & 0 \\ 0 & -\Psi_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & -\Psi_K & 0 \end{bmatrix},$$

$h = \begin{bmatrix} h_k^T \\ h_2^T \\ \vdots \\ h_K^T \end{bmatrix}^T$, and

$e = \begin{bmatrix} e_1^T \\ e_2^T \\ \vdots \\ e_K^T \end{bmatrix}^T$,

where $X \in \mathcal{C}^{(K(L+L_f-1)) \times KL_f}$ and $\tilde{X} \in \mathcal{C}^{(K-1)(L+L_f-1) \times KL_f}$. Using (5) and (6), the constrained optimization problem to estimate $h$ can be defined as

$$\text{minimize } \|Xh - e\|^2$$

subject to $\tilde{X}h = 0$. \hspace{1cm} (7)

The above constrained optimization problem can be solved using the Lagrangian multiplier. Therefore, the cost-function to be minimised can be written as

$$J = \|Xh - e\|^2 + h^H \tilde{X}^H \lambda,$$ \hspace{1cm} (8)

where vector $\lambda$ is called Lagrangian multiplier. The minimization of (8) with respect to $h$ yields

$$h = (X^H X)^{-1} \left[ X^H e - \tilde{X}^H \lambda \right].$$ \hspace{1cm} (9)

Substituting (9) back into (8) yields

$$\lambda = D^{-1} \tilde{X} \left( X^H X \right)^{-1} X^H e,$$ \hspace{1cm} (10)

where $D = \left( \tilde{X} \left( X^H X \right)^{-1} \tilde{X}^H \right)$. Now substituting (10) into (9), we get

$$h = (X^H X)^{-1} X^H e - (X^H X)^{-1} \tilde{X}^H D^{-1} \tilde{X} \left( X^H X \right)^{-1} X^H e,$$

$$= (X^H X)^{-1} \left( I - \tilde{X}^H D^{-1} \tilde{X} \left( X^H X \right)^{-1} \right) X^H e.$$ \hspace{1cm} (11)

Similarly, if the symbol waveform is contaminated with noise, the constrained optimization problem can be formulated as

$$\text{minimize } \| (X + W)h - e \|^2$$

subject to $\tilde{X}h = 0$, \hspace{1cm} (12)

where $W$ is the matrix of white Gaussian noise samples each of variance $\sigma^2$. Solving (12), again using the Lagrangian multiplier, the minimum-mean-squares-error (MMSE) receive filter can be derived as

$$h = \left( X^H X + \sigma^2 I \right)^{-1} X^H e - \left( X^H X + \sigma^2 I \right)^{-1} \tilde{X}^H D^{-1} \tilde{X} \left( X^H X + \sigma^2 I \right)^{-1} X^H e,$$

$$= \left( X^H X + \sigma^2 I \right)^{-1} \left( I - \tilde{X}^H D^{-1} \tilde{X} \left( X^H X + \sigma^2 I \right)^{-1} \right) X^H e.$$ \hspace{1cm} (13)

Remark: Estimating vector $h$ in (13) requires the inversion of two matrices, $X^H X$ and $D = \tilde{X} \left( X^H X \right)^{-1} \tilde{X}^H$. Since $X \in \mathcal{C}^{(K(L+L_f-1)) \times KL_f}$, $(X^H X)$ will be non-singular iff $KL_f \leq K(L + L_f - 1)$, and this condition always holds. Similarly, $\tilde{X} \in \mathcal{C}^{(K-1)(L+L_f-1) \times KL_f}$, and for the matrix $D$ to be full rank, $(K-1)(L+L_f-1) \leq KL_f$, which simply impose the condition $L_f \geq (K-1)(L-1)$.

The ill matrix condition can be removed using a square circular convolution matrix, $\Psi_{ck}$, instead of tall linear convolution matrix, $\Psi_k$, defined as

$$\Psi_{ck} = \begin{bmatrix} x_k(0) & 0 & \cdots & \cdots & x_k(1) \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ x_k(L-1) & x_k(0) & \cdots & x_k(L-1) \end{bmatrix},$$

where

$$\Psi_{ck} h_{ck} = e_{ck}.$$ \hspace{1cm} (14)

The corresponding desired output response of the designed receive filter, $h_{ck}$, can be written as
where $\Psi_{\cdot\cdot\cdot}$ is generated as $\mathcal{C}^{(L+L_f-1)\times(L+L_f-1)}$. Following similar strategy used to generate $X$ and $\hat{X}$ with $\Psi_{\cdot\cdot\cdot}$, we can generate $X_c$ and $\hat{X}_c$ with $\Psi_{\cdot\cdot\cdot}$ and can find corresponding $h_c$ by solving the following constrained optimization problem,

$$
\begin{align*}
\text{minimize} & \; \|X_c h_c - e_c\|^2 \\
\text{subject to} & \; \hat{X}_c h_c = 0,
\end{align*}
$$

(15)

where the dimension of $X_c \in \mathcal{C}^{(K(L+L_f-1))\times(K(L+L_f-1))}$ and $\hat{X}_c \in \mathcal{C}^{(K-1)(L+L_f-1)\times(K(L+L_f-1))}$, while the dimensions of $h_c$ and $e_c$ are adjusted accordingly. Solving (15) yields,

$$
\begin{align*}
h_c &= (X_c^H X_c)^{-1} X_c^H e_c \\
&= (X_c^H X_c)^{-1} (I - \hat{X}_c^H D_c^{-1} \hat{X}_c (X_c^H X_c)^{-1}) X_c^H e_c,
\end{align*}
$$

(16)

where $D_c = (\hat{X}_c (X_c^H X_c)^{-1} X_c^H)$.

We will assess the performance of the designed filters on differential phase-shift-keying (DPSK) and minimum-phase-shift keying (MSK) symbol waveforms. Due to continuous phase change, DPSK and MSK waveform gives narrow bandwidth and good autocorrelation compared to the other phase modulation schemes. The DPSK waveform can be generated in two steps. In the first step, a digital waveform using binary random sequence is generated as

$$
s_d(t) = \sum_{n=0}^{N-1} e^{j\pi(b(n)\frac{t}{T})} \left[ u(t - nt_c) - u(t - (n+1)t_c) \right],
$$

(17)

where $b(n) \in \{\pm 1\}$ generated from Gaussian distribution, $u(t)$ is the unit step function, and $t_c$ is chip interval. While in the second step, pulse shaping is applied across bit boundaries to convert discontinuous phase transition to continuous. Thus final DPSK waveform for baseband frequency $f_b = \frac{1}{2T_c}$ can be expressed as

$$
\Psi_{\text{DPSK}}(t) = s_d \left( t - \frac{t_c}{2} \right) \cos(2\pi f_b t) + js_d(t) \sin(2\pi f_b t),
$$

$$
= s_c(t) + js_c(t), \quad 0 \leq t \leq T
$$

(18)

where $s_c(t) = s_d \left( t - \frac{t_c}{2} \right) \cos(2\pi f_b t)$ and $s_s(t) = s_d(t) \sin(2\pi f_b t)$. If $t_c$ is a single chip interval, the $N$ chips symbol waveform duration $T = Nt_c$. By changing sequence of random binary numbers, $\{b(n)\}$, in (17), a unique digital waveform $s_d(t)$ can be generated. The corresponding modulated signal at frequency $f_c$ can be written as

$$
\Psi(t) = s_c(t) \cos(2\pi f_c t) - s_s(t) \sin(2\pi f_c t),
$$

$$
= \text{Re} \left( \Psi_{\text{DPSK}}(t)e^{j2\pi f_c t} \right).
$$

Similar to DPSK, minimum-shift-keying (MSK) can be generated as [34]

$$
\Psi_{\text{MSK}}(t) = e^{-j(\theta_0 + s_d(t)\pi f_b t)},
$$

(20)

where $\theta_0$ is the final phase value from previous chip interval. In the following section, we will discuss the receive signal model at the BS and MU.

### IV. RECEIVE SIGNAL MODEL

This work considers a broadcast system, where BS transmits symbols to communicate with the MUs and detect targets. At the BS, we have radar and communication receive units. The radar receive unit estimates the targets’ ranges and velocities, while the communication receive unit decodes the MUs transmitted symbols. Generally, different frequencies are transmitted for BS-to-MU and MU-to-BS communication to avoid interference between the uplink and downlink signals. Therefore, radar and communication receive units can be designed independently at the BS. In DFRC, the design of the radar receive unit is more challenging; therefore, in the following, more focus will be on it. We will also briefly discuss the receiver at the MU end.

#### A. Receive Signal Model for Radar

Consider a DFRC system shown in Fig. 1 transmitting $M$ waveforms separated by a pulse-repetition-interval (PRI) $T_{\text{PRI}}$ and $P$ scatterers located at different ranges. Some scatterers are stationary, known as clutter, and others are moving targets. If a scatterer $p$ is located at an initial range $R_{0p}$ from the BS, the echo will be received after time $t_p = 2R_{0p}c$. Moreover, if target is moving with a velocity $v_p$, its range at the start of the $m$th pulse will be equal to $R_{0p} + mv_p T_{\text{PRI}}$ depending on the target’s velocity. After the transmission of the $m$th transmitted symbol waveform, the complex received signal can be written as

$$
r_m(t) = \sum_{p=1}^{P} \beta_p \Psi_m \left( t - 2 \frac{(R_{0p} - mv_p T_{\text{PRI}})}{c} \right) + \tilde{w}(t),
$$

$$
m = 0, \ldots, M - 1
$$

(21)

where $\Psi_m(t)$ is the $m$th symbol waveform drawn from DPSK or MSK alphabets defined in (18) and (20), $\tilde{w}(t)$ is the white Gaussian noise of variance $\sigma^2$ and $\beta_p$ is the reflection coefficient of $p$th scatterer. Using (19) in (21), the receive signal can be written as

$$
r_m(t) = \sum_{p=1}^{P} \beta_p \left[ s_m^c(t - \tau_p) \cos(2\pi f_c t + \phi_m(t, R_{0p})) - s_m^s(t - \tau_p) \sin(2\pi f_c t + \phi_m(t, R_{0p})) \right] + \tilde{w}(t),
$$

(22)

where $\tau_p \leq t \leq \tau_p + T$, $s_m^c(t)$ and $s_m^s(t)$ are waveforms used to form the $m$th transmitted waveform, $\phi_m(t, R_{0p}) = 2\pi f_{dp}mT_{\text{PRI}} - \frac{4\pi R_{0p}}{\lambda}$ while $f_{dp} = \frac{2v_p c}{\lambda}$ is the Doppler shift due to the $p$th target. Demodulating (22), noise free in-phase, $I(t)$, and quadrature-phase, $Q(t)$, components of the baseband received signal can be written as

$$
I(t) = \sum_{p=1}^{P} \beta_p \left[ s_m^c(t - \tau_p) \sin \left( 2\pi f_{dp}mT_{\text{PRI}} - \frac{4\pi R_{0p}}{\lambda} \right) \right. \left. + s_m^s(t - \tau_p) \cos \left( 2\pi f_{dp}mT_{\text{PRI}} - \frac{4\pi R_{0p}}{\lambda} \right) \right],
$$

$$
Q(t) = \sum_{p=1}^{P} \beta_p \left[ s_m^c(t - \tau_p) \cos \left( 2\pi f_{dp}mT_{\text{PRI}} - \frac{4\pi R_{0p}}{\lambda} \right) \right. \left. - s_m^s(t - \tau_p) \sin \left( 2\pi f_{dp}mT_{\text{PRI}} - \frac{4\pi R_{0p}}{\lambda} \right) \right].
$$
Using in- and quadrature-phase components in the above expression, the complex baseband received signal can be written as

\[ y_m(t) = I(t) + jQ(t) + w(t), \]

\[ = \sum_{p=1}^{P} \beta_p \Psi_m(t - \tau_p) e^{j(2\pi f_{dp}mT_{PRI} - \frac{4\pi R_{0p}}{c})} + w(t), \]

(23)

where \( w(t) \) is the complex white Gaussian noise of variance \( \sigma_w^2 \). After demodulation, \( y_m(t) \) is sampled for further processing. Therefore, after sampling, the receive signal in discrete form can be written as

\[ z_m(n) = \sum_{p=1}^{P} \beta_p e^{-j\frac{4\pi R_{0p}}{c}} \Psi_m(n - \tilde{n}_p) e^{j2\pi f_{dp}mT_{PRI}} + w(n), \]

(24)

where \( \tilde{n}_p \) is the time delay index of the \( p \)th scatterer. It can be noted that terms \( e^{-j\frac{4\pi R_{0p}}{c}} \) and \( e^{j2\pi f_{dp}mT_{PRI}} \) do not change with respect to sample number \( n \). In radar signal processing, the received signal is convolved with the filter designed for the transmitted waveform to find the range of the target. The index of the peak value in the output convolution signal depends on the target’s range. The convolution of \( z_m(n) \) with the proposed receive filter, \( h_m(n) \), designed for \( m \)th symbol waveform can be written as

\[ \tilde{z}_m(n) = z_m(n) * h_m(n), \]

\[ = \sum_{p=1}^{P} \beta_p e^{j2\pi f_{dp}mT_{PRI}} e^{-j\frac{4\pi R_{0p}}{c}} \Psi_m(n - \tilde{n}_p) * h_m(n) + w(n) * h_m(n), \]

\[ = \sum_{p=1}^{P} \beta_p e^{j2\pi f_{dp}mT_{PRI}} e^{-j\frac{4\pi R_{0p}}{c}} \Psi_m(n - \tilde{n}_p - l) h_m(l) + w(n), \]

(25)

where \( \tilde{w}(n) = w(n) * h_m(n) \). According to the filter design problem formulated in (7), the maximum value of \( \tilde{z}_m(n) \) will occur at \( n = \tilde{n}_p \) for \( p = 1, 2, \ldots, P \), and at this value

\[ \sum_l \Psi_m(n - \tilde{n}_p - l) h_m(l) \approx 1. \]

Therefore, at \( n = n \), we can write

\[ \tilde{z}_m(\tilde{n}_p) = \sum_{p=1}^{P} \beta_p e^{j2\pi f_{dp}mT_{PRI}} e^{-j\frac{4\pi R_{0p}}{c}} + \tilde{w}(n). \]

(26)

It can be noted in (26), for target \( p \), the value of convolution at \( n = \tilde{n} \) depends on \( m \) and \( R_{0p} \). However, during the NCPI’s short duration, the target movement can be considered negligible, and we can say that during NCPI, the value of convolution at \( n = \tilde{n} \) depends only on \( m \). Therefore, by collecting \( \tilde{z}_m(\tilde{n}_p) \) samples from all consecutive waveforms

\footnote{Here for simplicity, we have assumed that for a target at zero delay, the peak value of the convolution occur at \( n = 0 \).}

in the NCPI and applying the FFT, we can easily estimate the Doppler shift due to the \( p \)th target. The following subsection describes the range and Doppler shift estimation algorithm in detail.

1) The Delay and Doppler Estimation Algorithm: At the BS, the proposed algorithm transmits \( M \) waveforms during the NCPI interval and uses \( K \) filters to receive the echoes of these waveforms. The received signal after demodulation passes through all \( K \) filters. If a waveform, \( \psi_k(t) \) is convolved with \( h_k(t) \), our objective function guarantees a coherent output response for all waveforms. After transmitting the \( k \)th waveform, the algorithm collects samples from the \( k \)th filter output that yields peak values at different indices depending on the ranges of different targets. The peak values represent the presence of targets at different ranges that can be estimated using the indices of peak values. After finding each target’s range, the algorithm estimates each target’s velocity. In (25), we have observed that the term \( e^{j2\pi f_{dp}mT_{PRI}} \) changes with \( m \), but not with \( n \). Therefore, after the transmission of the first waveform, the algorithm finds the indices of all peak values. Let us assume the index of the first peak is denoted by \( \tilde{n}_1 \). After the transmission of each of the \( M \) waveforms, the algorithm collects \( \tilde{n}_1 \)th sample from the filter corresponding to the transmitted waveforms. Finally, by applying FFT on these \( M \) samples, our proposed algorithm estimate the Doppler shift of the target present at the \( \tilde{n}_1 \) index location. Similarly, the proposed algorithm estimates the Doppler shift of each target present at different index locations.

At some range given by \( \tilde{n}_p \), along with moving target, we may have stationary or slow-moving objects such as buildings, ground, mountains, and trees, called clutter. The received signal from the clutter can be high (due to huge radar-cross-section) and mask the received signal from the moving target. Since, the Doppler shift of clutter is close to zero, we can easily minimize the contribution of clutter in the overall received signal by applying DC and moving-target-indicator (MTI) filtering.

It should be noted that MUs communicate with the BS at a different frequency; they do not interfere with the signals received after the reflection from the target and clutter. For performance analysis of the system, the clutter-to-noise ratio (CNR) for \( P \) stationary or slowly moving targets can be defined as

\[ \text{CNR} = \frac{\sum_{p=1}^{P} | \beta_p |^2}{\sigma_w^2}. \]

(27)

In the above discussion, we considered a broadcast system. The BS can also transmit data for different users, where the combination of different symbol waveforms for different users can be transmitted. In such a system, we have to mitigate the interference effects at the MU end. We leave it for future work.

B. Receive Signal Model for Communication

The communication link between the BS and MU is very straightforward. In this paper, as mentioned earlier, we have only considered the transmission of broadcast symbols from
the BS. Therefore, the received signal by MU $q$, in response to $m$th transmitted waveform, can be written as

$$y_{cm}(t) = d_{bq} \Psi_m(t) + v(t),$$

where $d_{bq}$ is the flat faded channel between the BS and MU $q$ and it can be easily estimated using the conventional techniques [61]. The received signal, $y_{cm}(t)$, is passed through all filters designed for $K$ symbol waveforms and output response of the filters is sampled. When the received signal passes through the filter designed for the transmitted waveform, it yields a maximum peak value. Therefore, the filter that yields maximum output peak defines the transmitted symbol waveform. The sampled output response of the filter designed for $m$th symbol waveform, after passing it through the filter designed for $m$th transmitted waveform can be written as

$$z_{cq}(n) = d_{bq} h_{m}(n) * \Psi_m(n) + v(n).$$

If the BS transmitted signal contain a $\Psi_m(t)$ symbol waveform for user $q$, along with the other symbol waveform, the received signal in (29) can be written as

$$z_{cq}(n) = d_{bq} + d_{bq} Q \sum_{i=1}^{Q} h_{m}(l) \Psi_i(n-l) + v(n).$$

The second term in (30) is the interference, which can degrade the detection performance of the receiver. Therefore, the interference should be minimized. The SINR at the MU $q$ can be written as

$$\text{SINR} = \frac{|d_{pq}|^2}{d_{bq}^2 \sum_{i=1}^{Q} h_{m}(l) \Psi_i(n-l) + \sigma_v^2}. $$

To minimize the interference, different MU’s symbol waveforms can be multiplied by the different spreading codes known to the corresponding user before the transmission at the BS. We can also exploit other innovative techniques. Since this work focuses on the radar receiver design, we leave the design of the MU receiver for our future research.
V. SIMULATION RESULTS

In this section, several simulations are performed to assess the performance of our designed closed-form receive filters. For all simulations, the number of symbol waveforms $K = 4$ and are DPSK. To generate each waveform, the number of chips $N = 30$, and the duration of each chip $t_c = 1\text{ms}$. Therefore, the total duration of each symbol waveform is $T = 30\text{ms}$. The baseband frequency of pulse shaping sines and cosines functions is $f_b = 500\text{Hz}$. The sampling frequency at the receiver is $F_s = 3\text{kHz}$, which yield $L = 90$ samples corresponding to each waveform. As derived in Sec. III the length of the filter $L_f \geq (K-1)(L-1) = 267$, we set it to $L_f = 450$. Therefore, the length of receiver output response will be $L + L_f - 1 = 539$ and we can detect targets up to range bins 270.

In the first simulation, we compare the RSLs of our proposed receive filter with the iterative JLS receive filter [34]. The simulation results are shown in Fig. 3. Fig. 3a shows that the output response of iterative JLS receive filters is not coherent for any waveform. Therefore, this receiver can mask weak targets in the presence of clutter, and the estimation of Doppler can be a challenging task. Similarly, Fig. 3b, and 3c shows the output response of our proposed receive filters designed using linear and circulant convolution model. Both figures show that the output response of the designed filters is fully coherent. The RSLs of the filter designed using the linear-convolution model is comparatively lower than the iterative JLS receiver filter. While the RSLs of the designed filters using the circulant convolution model are remarkably lower than both of the above techniques. It is important to note that when cascaded filter design [60] improve coherency, the RSLs become approximately $-15\text{dB}$. In contrast, our proposed filters designed using linear and circular convolution models achieve full coherency, but the RSLs do not exceed $-40\text{dB}$.

In the second simulation, we compare the target detection performance of the proposed receivers with the iterative JLS receiver proposed in [34]. We consider a scenario of $P = 6$ targets, in which four targets are moving towards the BS.
Fig. 5. Detection of six targets at CNR = 70dB and SNR 10dB. (a) Iterative receiver proposed in [34], targets cannot be identified, (b) Linear-convolution based receiver, all targets can be easily identified, (c) Linear-convolution based receiver, all targets can be easily identified.

and two are moving away. The normalized Doppler frequency of the first three targets moving towards the BS is 0.3Hz, and the fourth target is 0.2Hz. Similarly, the normalized Doppler frequencies of the targets moving away from the BS are −0.25 and −0.3. We divide the overall range of the DFRC system into 450 range bins and place the targets around the central range bins. The first three targets having a Doppler shift of 0.3Hz are placed in range bins 225, 228, and 221, while the fourth one is placed in the range bin 245. The other two targets with Doppler frequencies of −0.25Hz and −0.3Hz are placed in range bins 235 and 215, respectively. To detect these targets, we transmit $M = 50$ symbol waveforms in each NCPI and randomly select each waveform from the set of $K = 4$ alphabet waveforms. The received signal is modelled using (24) by incorporating the above defined parameter values. For clutter, we choose five objects having Doppler frequency range between −0.1Hz and 0.1Hz. The radar receiver unit knows the transmitted symbol waveforms; therefore, the received samples are collected only from the filter designed for the transmitted waveform. Following this, samples are collected after the transmission of 50 symbol waveforms, and a matrix of dimension $520 \times 50$ is formed. Depending on the number of targets, each matrix column shows peaks representing the presence of targets. Since, in practice, the duration of each symbol waveform is kept low, the target’s location does not change during several transmitted waveforms. Therefore, the peak corresponding to a particular target appears at an identical location in each matrix column. However, identical samples phase can change from one column to the next. Finally, by applying fast-Fourier-transform (FFT) to these samples, the target’s velocity can be easily estimated. The same procedure is repeated for each peak to estimate the velocity of the target present at the corresponding range. We minimize the contribution of stationary and slowly moving targets in the identical index samples corresponding to a particular peak by MTI and DC filtering. For simulation two scenarios are considered. In the first scenario the signal-to-noise ratio (SNR) is 10dB and CNR is 50dB. Fig. 4 shows the corresponding detection performance of all receivers. Note that only 100 central range bins are shown in the simulation results for better visualization. Fig. 4a shows that with the JLS iterative receive filter, the three closely located targets in range bins 225, 228, and 221 each of Doppler shift 0.3Hz are poorly resolvable. The other three widely
Fig. 6. Probability-of-detection comparison of proposed schemes with the JLS iterative receiver proposed in [34].

separated targets are resolvable, but due to the non-coherence response of the receive filter, targets are masked by nearby clutters. In contrast, Fig. 4b shows the detection performance of our proposed linear-convolution-based receiver. As can be seen in the figure, the closely located targets are resolved efficiently due to the fully coherent response of our proposed receiver. Similarly, the performance of our proposed circular-convolution-based receiver is shown in Fig. 4c, which is even better than the linear-convolution-based receiver (please compare the amplitudes of peaks). Similarly, in the second scenario SNR = 10 dB and CNR = 70 dB. Fig. 5 shows the corresponding detection performance of all receivers. It can be seen in Fig. 5a, the JLS iterative receiver completely fails to detect the targets in the presence of nearby strong clutters. On the other hand, Fig. 5b shows the linear-convolution-based receivers have successfully resolved and unmasked the targets buried in heavy clutter. Similarly, Fig. 5c shows that the circular-convolution-based receivers, as in the first scenario, have significantly better performance compared to JLS and linear-convolution-based receivers.

In the third simulation, we compare the probability-of-detection (PD) using different designed receivers, which is defined as

$$PD = \frac{d}{P},$$

where $d$ is the number of targets detected correctly, and $P$ is the total number of targets. For this simulation, six targets are located at different locations as specified in the previous simulation. The first six peaks are selected from the range-Doppler grid and compared with the ground truth. Fig. 6 compares the PD of the proposed receivers with the JLS iterative receiver. It can be seen in Fig. 6 that the performance of proposed linear- and circular-convolution-based receivers is much better than the JLS iterative receiver. The PD is more than 0.9 using linear- and circular-convolution-based receivers at 5.0 dB and 3.5 dB, respectively. In contrast, the iterative JLS algorithm’s PD reaches 0.9 at 11 dB. Therefore, our proposed receivers yield a considerable gain in SNR.

Finally, we simulate symbol-error rate (SER) at the MU. For simplicity, we consider a frequency-flat channel and a BS broadcast same symbol waveforms for all users. The received signal at the MU is convolved with $K$ receive filters. The receive filter that yields the maximum peak value, the symbol waveform corresponding to this filter, defines the detected symbol. Fig. 7 compares the SER rate of different receivers. The figure shows that the proposed receivers designed using linear- and circular-convolution perform better than the iterative JLS receiver. Furthermore, at low values of SNR ($\leq 5$ dB), the SER of the linear-convolution-based receiver is better than the circular-convolution-based receiver. While, at high values of SNR ($>5$ dB), the SER of the circular-convolution-based receiver start performing better than the linear-convolution-based receiver. This is because a circular-convolution-based receiver performs better at high SNRs. We can easily extend the model for a frequency selective channel and multiple user data transmission from the BS.

VI. CONCLUSION

In this paper, to reduce range side-lobes and achieve full coherency, two novel closed-form algorithms based on linear- and circular-convolution are proposed. The computational complexity of the proposed algorithms is lower than the proposed algorithms available in the literature and achieves full coherence for any number of waveforms. As far as we know no one has achieved full coherence for more than two waveforms. Simulation results demonstrate the superiority of our proposed algorithms over the others for target detection and symbol-error-rate (SER). Our future work will focus on handling data reception from multiple mobile-users at the base-station’s radar-unit; and interference suppression at the mobile-user and the base-station’s communication unit.

REFERENCES

[1] S. Dang, O. Amin, B. Shihada, and M.-S. Alouini, “What should 6G be?” Nature Electron., vol. 3, no. 1, pp. 20–29, Jan. 2020.
[2] B. Zong, C. Fan, X. Wang, X. Duan, B. Wang, and J. Wang, “6G technologies: Key drivers, core requirements, system architectures, and enabling technologies,” IEEE Veh. Technol. Mag., vol. 14, no. 3, pp. 18–27, Sep. 2019.
[49] T. Bouchoucha, S. Ahmed, T. Al-Naffouri, and M.-S. Alouini, “DFT-based closed-form covariance matrix and direct waveforms design for MIMO radar to achieve desired beampatterns,” IEEE Trans. Signal Process., vol. 65, no. 8, pp. 2104–2113, Apr. 2017.

[50] S. Jardak, S. Ahmed, and M.-S. Alouini, “Generation of correlated finite alphabet waveforms using Gaussian random variables,” IEEE Trans. Signal Process., vol. 62, no. 17, pp. 4587–4596, Sep. 2014.

[51] S. Ahmed and M.-S. Alouini, “MIMO radar transmit beampattern design without synthesising the covariance matrix,” IEEE Trans. Signal Process., vol. 62, no. 9, pp. 2278–2289, May 2014.

[52] S. D. Blunt, M. Cook, J. Jakabosky, J. D. Graaf, and E. Perrins, “Polyphase-coded FM (PCFM) radar waveforms, Part I: Implementation,” IEEE Trans. Aerosp. Electron. Syst., vol. 50, no. 3, pp. 2218–2229, Jul. 2014.

[53] S. D. Blunt, J. Jakabosky, M. Cook, J. Stiles, S. Seguin, and E. L. Mokole, “Polyphase-coded FM (PCFM) radar waveforms, Part II: Optimization,” IEEE Trans. Aerosp. Electron. Syst., vol. 50, no. 3, pp. 2230–2241, Jul. 2014.

[54] S. D. Blunt and E. L. Mokole, “Overview of radar waveform diversity,” IEEE Aerosp. Electron. Syst. Mag., vol. 31, no. 11, pp. 2–42, Nov. 2016.

[55] P. Antonik, M. C. Wicks, H. D. Griffiths, and C. J. Baker, “Range-dependent beamforming using element level waveform diversity,” in Proc. Int. Waveform Diversity Design Conf., Jan. 2006, pp. 1–6.

[56] P. Antonik, “An investigation of a frequency diverse array,” Ph.D. thesis, Dept. Electron. Elect. Eng., Univ. College London, London, U.K., Apr. 2009.

[57] M. Zubair, S. Ahmed, and M.-S. Alouini, “Frequency diverse array radar: New results and discrete Fourier transform based beampattern,” IEEE Trans. Signal Process., vol. 68, pp. 2670–2681, 2020.

[58] P. M. McCormick, S. D. Blunt, and J. G. Metcalf, “Simultaneous radar and communications emissions from a common aperture, part I: Theory,” in Proc. IEEE Radar Conf. (RadarConf), May 2017, pp. 1685–1690.

[59] M. A. Richards, Fundamentals of Radar Signal Processing, New York, NY, USA: McGraw-Hill, 2014.

[60] C. Sahin, J. G. Metcalf, and S. D. Blunt, “Filter design to address range sidelobe modulation in transmit-encoded radar-embedded communications,” in Proc. IEEE Radar Conf. (RadarConf), May 2017, pp. 1509–1514.

[61] M. Zubair, S. Ahmed, and M.-S. Alouini, “Frequency diverse array radar: New results and discrete Fourier transform based beampattern,” IEEE Trans. Signal Process., vol. 68, pp. 2670–2681, 2020.

Muhammad Zubair received the B.S. degree in electrical engineering from COMSATS University Islamabad, Lahore, Punjab, Pakistan, in 2014, and the M.S. degree in electrical engineering from Information Technology University (ITU), Lahore, in 2018. From 2017 to 2019, he worked as a Research Associate with the Radar Signal Processing and Design (RSPD) Laboratory, ITU. He is currently working as a Signal Processing Engineer at U-blox. His current research interests include radar and wireless communications signal processing.

Sajid Ahmed (Senior Member, IEEE) received the M.Sc. degree in communication engineering from the University of Manchester Institute of Science and Technology, U.K., in 2002, and the Ph.D. degree in digital signal processing from King’s College London and Cardiff University, U.K., in 2005. He was a Researcher at Queen’s University Belfast and the University of Edinburgh, and a Research Scientist at the King Abdullah University of Science and Technology (KAUST), Thuwal, Saudi Arabia. He worked as a Faculty Member at Information Technology University, Lahore, Pakistan. He is currently working with the KAUST. His current research interests include linear and non-linear optimization techniques, low complexity parameter estimation for communication and radar systems, and waveforms design for joint communication and radar systems.

Mohamed-Slim Alouini (Fellow, IEEE) was born in Tunis, Tunisia. He received the Ph.D. degree in electrical engineering from the California Institute of Technology (Caltech), Pasadena, CA, USA, in 1998. He worked as a Faculty Member of the University of Minnesota, Minneapolis, MN, USA, then with Texas A&M University at Qatar, Education City, Doha, Qatar, before joining the King Abdullah University of Science and Technology (KAUST), Thuwal, Makkah, Saudi Arabia, as a Professor of electrical engineering in 2009. His current research interests include the modeling, design, and performance analysis of wireless communication systems.