Determinations of quark mixing matrix elements $|V_{cd}|$ and $|V_{cs}|$ from leptonic and semileptonic $D$ Decays

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Abstract

With the recent measurements of purely leptonic $D^+_s$ decays and semileptonic $D$ decays in conjunction with decay constants $f_{D^+_s}$ and form factors $f^{(K)}_+$ (0) calculated in LQCD, we extract the magnitudes of $V_{cd}$ and $V_{cs}$ to be $|V_{cd}| = 0.218 \pm 0.005$ and $|V_{cs}| = 0.987 \pm 0.016$. Compared to those given in PDG2013, the precisions of these newly extracted $|V_{cd}|$ and $|V_{cs}|$ are improved by more than 2.0 and 1.5 factors, respectively. With the newly extracted $|V_{cd}|$ and $|V_{cs}|$ together with other CKM matrix elements given in PDG2013, we check the unitarity of the CKM matrix, which are $|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 0.997 \pm 0.002$, $|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1.027 \pm 0.032$ and $|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.023 \pm 0.032$.

1 Introduction

In the Standard Model (SM) of particle physics, the $D^+_s$ meson can decay into $\ell^+\nu_\ell$ (where $\ell = e, \mu, \text{or } \tau$) via annihilation mediated by a virtual $W^+$ boson. The decay rate depends upon the wave function overlap of the two quarks at the origin, which is parameterized by the $D^+_s$ decay constant, $f_{D^+_s}$. All of the strong interaction effects between the two initial-state quarks are absorbed into $f_{D^+_s}$. In the SM, the decay width of $D^+_s \rightarrow \ell^+\nu_\ell$ is given by

$$\Gamma(D^+_s \rightarrow \ell^+\nu_\ell) = \frac{G_F^2 f_{D^+_s}^2}{8\pi} |V_{cd(s)}|^2 m_\ell^2 m_{D^+_s} \left( 1 - \frac{m_\ell^2}{m_{D^+_s}^2} \right)^2,$$

where $G_F$ is the Fermi coupling constant, $V_{cd(s)}$ is the $c \rightarrow d(s)$ Cabibbo-Kobayashi-Maskawa (CKM) matrix element $[1]$, $m_\ell$ is the lepton mass, and $m_{D^+_s}$ is the $D^+_s$ meson mass.
Similarly, in the SM, neglecting the positron mass, the differential decay rate of \( D \to \pi(K)e^+\nu_e \) process is given by

\[
\frac{d\Gamma}{dq^2} = X \frac{G_F^2}{24\pi^3}|V_{cd(s)}|^2|\tilde{p}_{\pi(K)}|^3|f^\pi(K)(q^2)|^2,
\]

where \( \tilde{p}_{\pi(K)} \) is the three-momentum of the \( \pi \) (\( K \)) meson in the rest frame of the \( D \) meson, \( f^\pi(K)(q^2) \) represents the hadronic form factor of the hadronic weak current depending on the square of the four-momenta transfer \( q^2 \), and \( X \) is a factor due to isospin, which equals to 1 for \( D^0 \to \pi^-e^+\nu_e \), \( D^0 \to K^-e^+\nu_e \) and \( D^+ \to \bar{K}^0e^+\nu_e \), and equals to 1/2 for \( D^+ \to \pi^0e^+\nu_e \). The form factor \( f^\pi(K)(q^2) \) measures the probability to form the final state \( \pi \) (\( K \)) meson in this decay.

Recently, the branching fractions for leptonic \( D^+ \) and \( D^+_s \) decays were well measured at the \( e^+e^- \) experiments near threshold of the \( \bar{D}D \) production (CLEO-c and BESIII) and near 10.6 GeV (Belle and BaBar), and the decay constants \( f_{D^+} \) and \( f_{D^+_s} \) were calculated in LQCD at precisions of \( \sim 1.6\% \) and \( \sim 1.1\% \), respectively. With these measured branching fractions in conjunction with the \( f_{D^+_s} \) calculated in LQCD, the magnitudes of CKM quark mixing parameters \( V_{cd} \) and \( V_{cs} \) can be well extracted. In addition, the precisions of these measured branching fractions for \( D \to \pi e^+\nu_e \) and \( D \to K e^+\nu_e \) decays or measured products of \( |V_{cd(s)}| \) and \( f^\pi(K)(0) \) are at an accuracy level of about 1\%, while the LQCD calculations of these form factors \( f^\pi(K)(0) \) also reach to about 4.4\% and 2.5\%, respectively. With these measured products of \( |V_{cd(s)}| \) and \( f^\pi(K)(0) \) together with inputs of the form factors calculated in LQCD, the magnitudes of \( V_{cd} \) and \( V_{cs} \) can also be well extracted.

In this article, we extract \( |V_{cd}| \) and \( |V_{cs}| \) with these measured branching fractions and/or \( |V_{cd(s)}|f^\pi(K)(0) \) in conjunction with decay constants \( f_{D^+_s} \) and/or form factors \( f^\pi(K)(0) \) calculated in LQCD. In determinations of \( |V_{cd}| \) and \( |V_{cs}| \), we use \( G_F \), masses of \( D^+_s \) meson and leptons, and lifetimes of \( D^+_s \) meson given in PDG2013 [1].

## 2 Recent experimental measurements

### 2.1 Purely leptonic \( D^+ \) decays

In 2008, the CLEO-c Collaboration accumulated 460055 \( \pm 787 \) \( D^- \) tags by analyzing 818 \( \text{pb}^{-1} \) data taken at 3.773 GeV and selecting \( D^- \) mesons from 6 hadronic decay modes of the \( D^- \) meson. They observed 149.7 \( \pm 12.0 \) signal events for \( D^+ \to \mu^+\nu_\mu \) decays in the system recoiling against these \( D^- \) tags. They measured the decay branching fraction \( B(D^+ \to \mu^+\nu_\mu) = (3.82 \pm 0.32 \pm 0.09) \times 10^{-4} \) [2].

In 2014, the BESIII Collaboration measured the branching fraction for \( D^+ \to \mu^+\nu_\mu \) decays by analyzing 2.92 \( \text{fb}^{-1} \) data taken at 3.773 GeV. From 9 hadronic decay modes of
$D^-$ meson, the BESIII Collaboration accumulated 1703054 ± 3405 $D^-$ tags. In this $D^-$ tag sample they observed 409.0 ± 21.2 signal events for $D^+ \rightarrow \mu^+\nu_\mu$ decays and measured branching fraction $B(D^+ \rightarrow \mu^+\nu_\mu) = (3.71 \pm 0.19 \pm 0.06) \times 10^{-4}$ [3].

Averaging these two branching fractions, we obtain

$$B(D^+ \rightarrow \mu^+\nu_\mu) = (3.74 \pm 0.17) \times 10^{-4},$$

where the error is the combined statistical and systematic errors together.

### 2.2 Purely leptonic $D_s^+$ decays

In 2009, the CLEO-c Collaboration studied the $D_s^+ \rightarrow \ell^+\nu_\ell$ decays based on 600 pb$^{-1}$ data taken at 4.17 GeV. From this data sample, they tagged $D_s^-$ mesons from 9 hadronic decay modes. By examining distribution of missing mass-squared of the $D_s^-$ and $\gamma$ system they accumulated 43859±936 $D_s^+$ mesons; by analyzing distribution of missing mass-squared of the $D_s^+\gamma\mu^+$ system, they selected $D_s^+ \rightarrow \mu^+\nu_\mu$ decay events and measured the branching fraction $B(D_s^+ \rightarrow \mu^+\nu_\mu) = (0.565 \pm 0.045 \pm 0.017)\%$ [4]. Using similar method, the CLEO-c Collaboration also measured the branching fraction $B(D_s^+ \rightarrow \tau^+\nu_\tau) = (5.58 \pm 0.33 \pm 0.13)\%$, which is the average of three measured branching fractions obtained with $\tau^+ \rightarrow \pi^+\nu_\tau$ [1], $\tau^+ \rightarrow e^+\nu_\tau\bar{\nu}_\tau$ [5] and $\tau^+ \rightarrow \rho^+\bar{\nu}_\tau$ decays [6].

In 2013, the Belle Collaboration measured the branching fractions for leptonic $D_s^+$ decays. They selected leptonic $D_s^+$ decays from the $e^+e^- \rightarrow c\bar{c}$ continuum production, in which the $D_{\text{tag}}K_{\text{frag}}X_{\text{frag}}D_s^{\pm}\gamma$ is produced from the quark fragmentation, where $D_s^{\pm} \rightarrow \gamma D_s^+$, $K_{\text{frag}}$ is either $K^+$ or $K_0^0$, and $X_{\text{frag}}$ indicates several pions or photons. By reconstructing the recoil mass of the $D_{\text{tag}}K_{\text{frag}}X_{\text{frag}}\gamma$, they observed clear $D_s^+$ signal in the system recoiling against the $D_{\text{tag}}K_{\text{frag}}X_{\text{frag}}\gamma$. By fitting the recoil mass spectra of $D_{\text{tag}}K_{\text{frag}}X_{\text{frag}}\gamma$, they accumulated 94360 ± 1310 ± 1450 inclusive $D_s^+$ mesons. To search for $D_s^+ \rightarrow \mu^+\nu_\mu$ decays, they examined the missing mass-squared $M_{\text{miss}}^2(D_{\text{tag}}K_{\text{frag}}X_{\text{frag}}\gamma\mu)$ distribution of the $D_{\text{tag}}K_{\text{frag}}X_{\text{frag}}\gamma\mu$ system. Fitting the $M_{\text{miss}}^2(D_{\text{tag}}K_{\text{frag}}X_{\text{frag}}\gamma\mu)$ distribution yields 492 ± 26 signal events for $D_s^+ \rightarrow \mu^+\nu_\mu$ decays. With these numbers of events, the Belle Collaboration measured the decay branching fraction $B(D_s^+ \rightarrow \mu^+\nu) = (0.531 \pm 0.028 \pm 0.020)\%$ [7]. In addition, the Belle Collaboration observed 2217 ± 83 signal events for $D_s^+ \rightarrow \tau^+\nu_\tau$ decays with $\tau^+ \rightarrow e^+\nu_\tau\bar{\nu}_\tau$, $\tau^+ \rightarrow \mu^+\nu_\mu\bar{\nu}_\mu$ and $\tau^+ \rightarrow \pi^+\bar{\nu}_\tau$ decays, and measured the decay branching fraction $B(D_s^+ \rightarrow \tau^+\nu_\tau) = (5.70 \pm 0.21^{+0.31}_{-0.30})\%$ [7].

In 2010, using the similar technique as the one used by the Belle Collaboration, the BaBar Collaboration made measurements of the branching fractions for leptonic $D_s^+$ decays. By analyzing 521 fb$^{-1}$ data taken at 10.6 GeV, the BaBar Collaboration measured the decay branching fractions $B(D_s^+ \rightarrow \mu^+\nu_\mu) = (0.602 \pm 0.038 \pm 0.034)\%$ and $B(D_s^+ \rightarrow \tau^+\nu_\tau) = (5.00 \pm 0.35 \pm 0.49)\%$ [8].
Combining these branching fractions measured by the CLEO-c, Belle and BaBar Collaborations, we obtain

$$B(D^+_s \rightarrow \mu^+ \nu_\mu) = (0.556 \pm 0.025)\% \quad (4)$$

and

$$B(D^+_s \rightarrow \tau^+ \nu_\tau) = (5.54 \pm 0.24)\%, \quad (5)$$

where the errors are the combined statistical and systematic errors together.

### 2.3 Semileptonic $D$ decays

In 2008, the CLEO-c Collaboration studied the semileptonic decays of $D^0 \rightarrow \pi^- e^+ \nu_e$, $D^0 \rightarrow K^- e^+ \nu_e$, $D^+ \rightarrow \pi^0 e^+ \nu_e$ and $D^+ \rightarrow K^0 e^+ \nu_e$ by analyzing 818 pb$^{-1}$ data taken at 3.773 GeV. They extracted the products $f_\pi^+(0)|V_{cd}| = 0.150 \pm 0.004 \pm 0.001$ and $f_K^+(0)|V_{cs}| = 0.719 \pm 0.006 \pm 0.005$ by fitting their measured partial decay rates with form factor parameterized with three parameter series expansion [9].

Recently, the BESIII Collaboration reported their new preliminary results of $D^0 \rightarrow \pi^- e^+ \nu_e$ and $D^0 \rightarrow K^- e^+ \nu_e$ decays obtained by analyzing 2.92 fb$^{-1}$ data taken at 3.773 GeV. They obtained $f_\pi^+(0)|V_{cd}| = 0.1420 \pm 0.0024 \pm 0.0010$ and $f_K^+(0)|V_{cs}| = 0.7196 \pm 0.0035 \pm 0.0041$ by fitting differential decay rates with the three parameter series expansion [10].

In 2007, the BaBar Collaboration measured the form factors $f_\pi^+(q^2)$ by analyzing 75 fb$^{-1}$ data collected at 10.6 GeV and determined $f_\pi^+(0) = 0.727 \pm 0.007 \pm 0.005 \pm 0.007$ [11]. Multiplying this form factor by $|V_{cs}| = 0.9729 \pm 0.0003$ used in their paper, we obtain the product $f_\pi^+(0)|V_{cd}| = 0.707 \pm 0.007 \pm 0.005 \pm 0.007$. Using the same technique, the BaBar Collaboration also studied the $D^0 \rightarrow \pi^- e^+ \nu_e$ decay by analyzing 347.2 fb$^{-1}$ data collected at $\Upsilon(4S)$ and reported preliminary results at ICHEP2014. They measured $f_\pi^+(0)|V_{cd}| = 0.1374 \pm 0.0038 \pm 0.0022 \pm 0.0009$ [12].

Combining these $f_\pi^+(K)|V_{cd(s)}|$ measured at the CLEO-c, BESIII and BaBar experiments, we obtain

$$f_\pi^+(0)|V_{cd}| = 0.143 \pm 0.002 \quad (6)$$

and

$$f_K^+(0)|V_{cs}| = 0.718 \pm 0.004, \quad (7)$$

where the errors are the combined statistical and systematic errors together.

### 3 Determinations of $|V_{cd}|$

Before 2012, the CKM matrix element $|V_{cd}|$ was usually determined with the $\nu \bar{\nu}$ interaction or the semileptonic decay of $D \rightarrow \pi e^+ \nu_e$. Actually, using the measured branching fraction for $D^+ \rightarrow \mu^+ \nu_\mu$ decays in conjunction with the LQCD calculation on $D^+$
meson decay constant, the magnitude of $V_{cd}$ can also be extracted via the Eq. (1). At Charm2012, the BESIII Collaboration reported preliminary result on the determination of $|V_{cd}|$ based on their measured branching fraction for $D^+ \rightarrow \mu^+\nu_\mu$ decay, which is $|V_{cd}| = 0.2218 \pm 0.0062 \pm 0.0047$ \cite{13}. Recently, the Flavor Lattice Averaging Group (FLAG) made an average of several values of the $f_{D^+}$ calculated in LQCD. The averaged $D^+$ decay constant calculated in LQCD is $f_{D^+} = (209.2 \pm 3.3)\text{ MeV}$ \cite{14}. Inserting the averaged branching fraction for $D^+ \rightarrow \mu^+\nu_\mu$ decays as given in Eq. (3) and this averaged $f_{D^+}$ into Eq. (1) yields

$$|V_{cd}|_{D^+\rightarrow\mu^+\nu_\mu} = 0.219 \pm 0.005 \pm 0.004,$$

where the first uncertainty is from the measured branching fractions and the second mainly from the uncertainties of $f_{D^+}$ and the lifetime of $D^+$ meson.

Dividing the averaged $f_\pi(0)|V_{cd}|$ from semileptonic $D \rightarrow \pi\ell^+\nu_\ell$ decays by the form factor $f_\pi(0) = 0.666 \pm 0.029$ calculated in LQCD \cite{15} yields

$$|V_{cd}|_{D\rightarrow\pi\ell^+\nu_\ell} = 0.215 \pm 0.003 \pm 0.009,$$

where the first uncertainty is from the measured $f_\pi(0)|V_{cd}|$, and the second uncertainty is from $f_\pi(0)$.

Figure 1 shows the comparison of $|V_{cd}|$ determined from purely leptonic $D^+$ decay and semileptonic $D$ decay. Averaging the determined $|V_{cd}|_{D^+\rightarrow\mu^+\nu_\mu}$ and $|V_{cd}|_{D\rightarrow\pi\ell^+\nu_\ell}$ yields

$$|V_{cd}| = 0.218 \pm 0.005.$$

Figure 2 shows the comparison of the newly determined $|V_{cd}|$ and the one given in PDG2013 \cite{1}.

## 4 Determinations of $|V_{cs}|$

Using the measured decay branching fractions for $D_{s^+}^+ \rightarrow \ell^+\nu_\ell$ together with the $D_{s^+}^+$ meson decay constant calculated in LQCD, the magnitude of $V_{cs}$ can be extracted via Eq. (1). We herein use the value of $f_{D_{s^+}^+} = (248.6 \pm 2.7)\text{ MeV}$, which is the FLAG average of several decay constants calculated in LQCD \cite{14}, to extract $|V_{cs}|$. Inserting the averaged branching fractions for $D_{s^+}^+ \rightarrow \ell^+\nu_\ell$ decays and the $f_{D_{s^+}^+}$ into Eq. (1) yields

$$|V_{cs}|_{D_{s^+}^+\rightarrow\ell^+\nu_\ell} = 1.001 \pm 0.022 \pm 0.013,$$

and

$$|V_{cs}|_{D_{s^+}^+\rightarrow\tau^+\nu_\tau} = 1.011 \pm 0.022 \pm 0.013,$$

where the first uncertainties are from the measured branching fractions, and the second uncertainties are mainly from $f_{D_{s^+}^+}$ and the lifetime of $D_{s^+}^+$ meson. Combining the above two values, we obtain

$$|V_{cs}|_{D_{s^+}^+\rightarrow\ell^+\nu_\ell} = 1.006 \pm 0.016 \pm 0.013,$$

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### Figure 1: Comparison of $|V_{cd}|$ determined from leptonic $D^+$ and semileptonic $D$ decays.

| Experiment       | Value               |
|------------------|---------------------|
| CLEO-c (2008)    | $0.222 \pm 0.009 \pm 0.004$ |
| BESIII (2014)    | $0.219 \pm 0.006 \pm 0.004$ |
| Average          | $0.219 \pm 0.005 \pm 0.004$ |

| Experiment       | Value               |
|------------------|---------------------|
| CLEO-c (2009)    | $0.225 \pm 0.006 \pm 0.010$ |
| BESIII (2014) Preliminary | $0.213 \pm 0.004 \pm 0.009$ |
| BaBar (2014) Preliminary | $0.206 \pm 0.007 \pm 0.009$ |
| Average          | $0.215 \pm 0.003 \pm 0.009$ |

### Figure 2: Comparison of the newly determined $|V_{cd}|$ from both the leptonic $D^+$ and semileptonic $D$ decays and the one given in PDG2013.

| Experiment       | Value               |
|------------------|---------------------|
| PDG2013          | $0.230 \pm 0.011$   |
| This work        | $0.218 \pm 0.005$   |
where the first uncertainty is from the measured branching fractions, the second uncertainty is mainly from $f_{D_s^+}$ and the lifetime of $D_s^+$ meson.

Dividing the averaged $f^K_+(0)|V_{cs}|$ from semileptonic $D \to Ke^+\nu_e$ decays by the form factor $f^K_+(0) = 0.747 \pm 0.019$ calculated in LQCD \cite{16} yields

$$|V_{cs}|_{D \to Ke^+\nu_e} = 0.961 \pm 0.005 \pm 0.024,$$

where the first uncertainty is from the measured $f^K_+(0)|V_{cs}|$ and the second uncertainty is from $f^K_+(0)$.

Figure 3 shows the comparison of $|V_{cs}|$ determined from purely leptonic $D_s^+\to \ell^+\nu_\ell$ decays and semileptonic $D \to Ke^+\nu_e$ decays. Averaging the determined $|V_{cs}|_{D_s^+\to \ell^+\nu_\ell}$ and $|V_{cs}|_{D \to Ke^+\nu_e}$ yields

$$|V_{cs}| = 0.987 \pm 0.016.$$ (15)

Figure 4 shows the comparison of the newly determined $|V_{cs}|$ and the one given in PDG2013 \cite{1}. 5 Unitarity checks

Using the newly extracted $|V_{ud}| = 0.218 \pm 0.005$, the PDG values $|V_{ud}| = 0.97425 \pm 0.00022$ and $|V_{td}| = (8.4 \pm 0.6) \times 10^{-3}$ \cite{1}, the first column unitarity of CKM matrix is checked, which is

$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 0.997 \pm 0.002.$$ (16)

Using the newly extracted $|V_{cs}| = 0.987 \pm 0.016$, the PDG values $|V_{us}| = 0.2252 \pm 0.0009$ and $|V_{ts}| = (42.9 \pm 2.6) \times 10^{-3}$ \cite{1}, we find

$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1.027 \pm 0.032$$ (17)

for the second column of the CKM matrix. Using these newly extracted $|V_{cd}|$ and $|V_{cs}|$, and the PDG value $|V_{cb}| = (40.9 \pm 1.1) \times 10^{-3}$ \cite{1}, we find

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.023 \pm 0.032$$ (18)

for the second row of the CKM matrix. The unitarity check results for the first column, second column and second row of the CKM matrix are shown in Fig. 5 together with the unitarity checks given in PDG2013 \cite{1}. The newly determined $|V_{cd}|$ and $|V_{cs}|$ give more stringent checks of the CKM matrix unitarity compared to those in PDG2013.

6 Summary

Combining the precise measurements of leptonic $D_{(s)}^+ \to \mu^+\nu_\mu$ decays and semileptonic $D \to \pi(K)e^+\nu_e$ decays at the CLEO-c, Belle, BaBar and BESIII together with the improved
| Experiment          | Value               |
|---------------------|---------------------|
| CLEO-c (2009)       | $1.009 \pm 0.040 \pm 0.020$ |
| Belle (2013)        | $0.978 \pm 0.026 \pm 0.022$ |
| BaBar (2010)        | $1.041 \pm 0.033 \pm 0.032$ |
| Average             | $1.001 \pm 0.022 \pm 0.013$ |
| CLEO-c (2009)       | $1.015 \pm 0.030 \pm 0.018$ |
| Belle (2013)        | $1.025 \pm 0.019 \pm 0.031$ |
| BaBar (2010)        | $0.960 \pm 0.034 \pm 0.049$ |
| Average             | $1.011 \pm 0.022 \pm 0.013$ |
| CLEO-c (2009)       | $0.963 \pm 0.010 \pm 0.024$ |
| BESIII (2014) Preliminary | $0.963 \pm 0.007 \pm 0.024$ |
| BaBar (2007)        | $0.946 \pm 0.015 \pm 0.024$ |
| Average             | $0.961 \pm 0.005 \pm 0.024$ |

Figure 3: Comparison of $|V_{cs}|$ determined from leptonic $D_s^+$ decays and semileptonic $D \to Ke^+\nu_e$ decays.
Figure 4: Comparison of the newly determined $|V_{cs}|$ from both the leptonic $D_s^+$ and semileptonic $D$ decays and the one given in PDG2013.

| $|V_{cs}|$ | PDG2013 | This work |
|---------|---------|---------|
|         | $1.006 \pm 0.023$ | $0.987 \pm 0.016$ |

Figure 5: Unitarity checks for the first column, second column and second row of the CKM matrix.

| Expression | Value |
|-----------|-------|
| $|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2$ | $0.997 \pm 0.002$ |
| $|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2$ | $1.027 \pm 0.032$ |
| $|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2$ | $1.023 \pm 0.032$ |
$D_{(s)}$ decay constants and semileptonic $D$ decay form factors calculated in LQCD, we extract the magnitudes of $V_{cd}$ and $V_{cs}$ to be $|V_{cd}| = 0.218 \pm 0.005$ and $|V_{cs}| = 0.987 \pm 0.016$, which improve the precisions of those values given in PDG2013 by more than 2.0 and 1.5 factors, respectively. These improved determinations of $|V_{cd}|$ and $|V_{cs}|$ give more stringent unitarity checks of the CKM matrix compared to those given in PDG2013.

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