One scheme of stabilization and sample-data control for T-S fuzzy time-delay systems

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Abstract. In this article, a refined Lyapunov functional is proposed to study the stabilization problem for Takagi-Sugeno (T-S) fuzzy systems with time delays. Considering both the time-varying delays of the system and the constant delays of the part of sample-data control loop, the refined Lyapunov function in this paper makes it more practical in actual sampling control pattern compared with some existing research. Some techniques of dealing the linear matrix inequalities (LMIs) are used to derive the less conservative criteria and an example is presented in the end to prove the availability of the work.

1. Introduction

It is widely acknowledged that T-S fuzzy system model is an efficient method to deal with complex nonlinear system since the model could be used to approximate practical nonlinear system. With the help of T-S fuzzy system model, considering each weight of linear system, a nonlinear systems could be translated into a sum of different linear systems so that the methods of studying linear systems are available for nonlinear system[1, 2, 3]. Thus, in recent decades, researchers have paid so much attention on analyzing nonlinear based on appropriate T-S fuzzy system.

Time delays inevitably occur and affect the system performance such as switched systems, T-S fuzzy systems, neural networks[4, 5, 6] and so on. It is so significant to analyze the stability and accomplishing the stabilization of time-delayed systems in order to improve system performance through diminishing the conservatism of systems. Therefore, in recent decades, researchers have paid so much attention on analyzing the stability and accomplishing the stabilization of time-delayed systems. The Lyapunov-Krasovskii functional (LKF), recognized as one of the most efficient tools for these questions, offers both the methods as well as the challenges for dealing with such problems. On one hand, like some precious papers[7, 8], the way to construct LKF with augmented vector is commonly accepted. Many innovative integral inequalities such as Jensen inequality, Wirtinger-based inequality, Bessel-Legendre inequality, free-matrix-based integral inequality and many new refined types of these integral inequalities such as shown in [9, 10, 11, 12] have been proposed to deal with the derivative of LKF. On the other hand, it is important to pick proper sample intervals to design controllers for data-sampling.
Motivated by the contents above, an attempt to construct the LKF has been made for the T-S fuzzy systems with time-varying delays in this paper. Consulting the methods of LMIs and some similar systems in previous articles, analysis of a T-S fuzzy system with less conservatism is revealed.

This paper consists of the following parts: Section 2 states the description and definition of the system as well as the lemmas used later. Then the achievements of this paper are proved in Theorem 1. A numerical example is provided to confirm the results at the end of this paper.

Throughout this paper, \( \mathbb{R}^n \) denotes the n-dimensional Euclidean space. \( P > 0 \) means that \( P \) is a real symmetric and positive definite matrix. The superscripts "-1" and "T" indicate the inverse and the transpose of a matrix respectively. \( \text{Sym}\{X\} = X + X^T \) and \( \text{diag}\{\ldots\} \) represents the block diagonal matrix.

2. Problem statement

Take the following T-S Fuzzy system model with time-varying delays into account

Plant Rule \( i(i=1,2,...,r) \): If \( \theta_j \) is \( M_{i1} \) and ... and \( \theta_p(t) \) is \( M_{ip} \), then

\[
\begin{aligned}
\dot{x}(t) &= A_i x(t) + A_{di} x(t - d(t)) + B_i u(t) \\
x(t) &= \gamma(t) \quad t \in [-\tau, 0]
\end{aligned}
\]  

(1)

Where \( x(t) \in \mathbb{R}^n \) represents the state vector. \( \gamma(t) \) is the initial condition. \( A_i \), \( A_{di} \) and \( B_j \in \mathbb{R}^{n \times n} \) are system matrices with compatible dimensions. The continuous system time delay function \( d(t) \) is defined as

\[
0 \leq d(t) \leq d, \quad d_{\min} \leq \dot{d}(t) \leq d_{\max}
\]  

(2)

Through the utilization of center-average defuzzifier, product inferences and singleton fuzzier, model (1) would be

\[
\dot{x}(t) = \sum_{i=1}^r \lambda_i(\theta(t))[A_i x(t) + A_{di} x(t - d(t)) + B_i u(t)]
\]  

(3)

Where \( \lambda_i(\theta(t)) \) is the normalized membership function and \( M_{ij}(\theta(t)) \) represents the grade of membership of \( \theta_j(t) \) in \( M_{ij} \). Assuming \( \omega(\theta(t)) > 0 \) for any \( t \geq 0 \) and \( \sum_{i=1}^r \omega_i(\theta(t)) > 0 \), one has

\[
\lambda_i(\theta(t)) = \frac{\alpha_i(\theta(t))}{\sum_{i=1}^p \alpha_i(\theta(t))} \geq 0, \quad \omega_i(\theta(t)) = \prod_{j=1}^p M_{ij}(\theta(t)), \quad \sum_{i=1}^r \lambda_i(\theta(t)) = 1, \quad \sum_{i=1}^r \dot{\lambda}_i(\theta(t)) = 0
\]  

(4)

Assume that the T-S fuzzy system is controlled by a sampling-based communication network and the control signal is generated through a zero-order holder (ZOH). The sampling time of the ZOH is defined as

\[
0 = t_0 < t_1 < ... < t_k < t_{k+1} < ... < +\infty, \quad k \geq 0
\]  

(5)

The interval between two sampling points \( [t_k, t_{k+1}) \) is defined as \( t_{k+1} - t_k = h_k, \quad h_k \in \Psi^k \) and the output of control signal \( u(t) \) with transmission delay \( \lambda \) is defined as

Plant Rule \( i(i=1,2,...,r) \): If \( \theta_j(t_k) \) is \( M_{i1} \) and ... and \( \theta_p(t_k) \) is \( M_{ip} \), then

...
\[ u(t) = \sum_{j=1}^{r} K_j x(t_k) + K_{\alpha} x(t_k - \lambda) \quad t \in \Psi^k \] (6)

Thus, the system function with the defuzzified output of the control signal is

\[ \dot{x}(t) = \sum_{i=1}^{r} \lambda_i(\Theta(t)) \sum_{j=1}^{r} \lambda_j(\Theta(t_k))[A_j x(t) + A_{\alpha} x(t - d(t)) B_j K_j x(t_k) + B_{\alpha} K_{\alpha} x(t_k - \lambda)] \quad t \in \Psi^k \] (7)

Before establishing the results of this paper, some lemmas would be applied for calculating through the derivation process.

**Lemma 1** [13] If a constant matrix \( \lambda \), a positive scalar \( \sigma \) and continuously differentiable function \( \epsilon : [0, \sigma] \rightarrow \mathbb{R}^n \) exist, the following inequality holds

\[ \sigma \int_0^\sigma \epsilon^T(s) \lambda \epsilon(s) ds \geq \left( \int_0^\sigma \epsilon(s) ds \right)^T \lambda \left( \int_0^\sigma \epsilon(s) ds \right) + 3 \zeta^T \lambda \zeta \] (8)

where \( \zeta = \int_0^\sigma \epsilon(s) ds - \frac{2}{\sigma} \int_0^\sigma d\theta \int_0^\sigma \epsilon(s) ds ds \).

**Lemma 2** [14] Let \( x \) be a continuous and differentiable function: \( [\alpha, \beta] \rightarrow \mathbb{R}^n \). For any constant matrix \( M > 0 \), the following inequality holds:

\[ -(\beta - \alpha) \int_\alpha^\beta x^T(s) M x(s) ds \leq -\delta_1^T M \delta_1 - 3\delta_2^T M \delta_3 - 5\delta_3^T M \delta_3 \] (9)

where \( \delta_1 = x(\beta) - x(\alpha) \), \( \delta_2 = x(\beta) + x(\alpha) - \frac{2}{\beta - \alpha} \int_\alpha^\beta x(s) ds \), \( \delta_3 = x(\beta) - x(\alpha) + \frac{6}{\beta - \alpha} \int_\alpha^\beta x(s) ds - \frac{12}{(\beta - \alpha)^2} \int_\alpha^\beta x(s) ds d\theta \).

**Lemma 3** [15] Let \( x \) be a differentiable function in \( [\alpha, \beta] \rightarrow \mathbb{R}^n \). For any matrix \( R > 0 \), vector \( \vartheta(t) \in \mathbb{R}^k \), matrices \( N_1, N_2, N_3 \in \mathbb{R}^{k \times n} \), the following inequalities hold

\[ \int_{\alpha_1}^{\beta_1} x^T(s) R_1 \dot{x}(s) ds \leq \vartheta^T(t)(\beta_1 - \alpha_1) N_1 R_1 N_1^T \vartheta(t) + \text{Sym}[\vartheta^T(t) N_1(x(\beta_1) - x(\alpha_1))] \] (10)

\[ \int_{\alpha_2}^{\beta_2} x^T(s) R_2 \dot{x}(s) ds \leq \vartheta^T(t)(\beta_2 - \alpha_2) N_2 R_2 N_2^T \vartheta(t) + \frac{1}{3} N_2 R_2 N_2^T \vartheta(t) + \text{Sym}[\vartheta^T(t) N_2(x(\beta_2) - x(\alpha_2))] \] (11)

3. **Main result**

The stability problem of T-S fuzzy system is studied in this section. For revealing the layout of this paper concisely, the following nomenclatures are used to simplify vector and matrix representations.
By adopting the two-sided looped-functional method, the criterion of Fuzzy system with time delays referred above is presented as the follow

Theorem 1 Given known gain matrices \(K_j, K_{dj}\), system (7) with a sampled-data fuzzy controller would be asymptotically stable if such symmetrical positive matrices \(P, Q_j, R_j, T, Z_j\), \(a = 1, 2, 3, 4\) and any other matrices \(U_j, U_2, N_{jk}, b = 1, 2, \ldots, 6\) and \(Y\) with appropriate dimension exist and satisfy the following LMIs.

\[
\begin{bmatrix}
\Omega_1 + h_j K_1 & h_j N_2 & h_j N_3 & h_j N_4 & h_j N_5 \\
* & -Z_1 & 0 & 0 & 0 \\
* & * & -3Z_2 & 0 & 0 \\
* & * & * & -Z_3 & 0 \\
* & * & * & * & -Z_4 \\
\end{bmatrix} < 0
\]

(12)

\[
\begin{bmatrix}
\Omega_1 + h_j K_2 & h_j N_1 & h_j N_6 & h_j N_5 \\
* & -Z_1 & 0 & 0 \\
* & * & -3Z_1 & 0 \\
* & * & * & -Z_3 \\
\end{bmatrix} < 0
\]

(13)

\[
\Omega_1 < 0
\]

(14)
\[\Omega_i = \Phi_i + F_i\]

\[\Phi_i = \text{Sym}\left\{\Pi_i^1 T\Pi_i^2\right\} + h_i^e T e_i - \frac{\pi^2}{4} \Pi_i T \Pi_i + 2\Pi_i T (U_i \Pi_i + U_i \Pi_i) + d^2 (e_i^T R_e e_i + e_i^T R_e e_i)\]

\[+ (1 - d(t)) Q_i^T (Q_i - Q_i) \Pi_i + \Pi_i^2 Q_i^T Q_i + \text{Sym}\{N_i \Pi_i\} + \text{Sym}\{N_i \Pi_i\}\]

\[+ \text{Sym}\{N_\pi \Pi_\pi\} + \text{Sym}\{N_\pi \Pi_\pi\} + \text{Sym}\{N_\pi \Pi_\pi\} + \Theta_1 + \Theta_2\]

\[\Theta_i = -\Pi_i^2 R_i \Pi_i - 3\Pi_i^2 R_i \Pi_i; \quad \Theta_1 = -\Pi_i^2 R_i \Pi_i - 3\Pi_i^2 R_i \Pi_i - 5\Pi_i^2 R_i \Pi_i\]

\[F_i = \text{Sym}\left\{(x_i + k_2 + k_3 + k_4) \times (-e_i + A_i e_i + A_i^d e_i + B_i K_i e_i + B_i K_i e_i)\right\}\]

\[\Omega_2 = 2\Pi_i^2 (U_i \Pi_i + U_i \Pi_i) + \text{Sym}\{\Pi_i^2 (U_i \Pi_i)\} + \Pi_i^2 Y \Pi_i + h_i^e T \Sigma e_i + h_i^e T \Sigma e_i + N_i Z_i^{-1} N_i^T\]

\[+ \frac{1}{3} N_i Z_i^{-1} N_i^T + N_i Z_i^{-1} N_i^T\]

\[\Pi_1 = \text{col}\{e_1, e_2, e_3, d(t)e_13, (d - d(t))e_1, (d - d(t))e_1\}\]

\[\Pi_2 = \text{col}\{e_1, (d - d(t))e_13, (d - d(t))e_1, (1 - d(t))e_13 - e_1, (1 - d(t))e_13 - e_1\}\]

\[\Pi_3 = \text{col}\{e_1, e_3\}\]

\[\Pi_4 = \text{col}\{e_1, e_3\}\]

\[\Pi_5 = e_i - e_i\]

\[\Pi_6 = e_i - e_i, e_i - e_i, e_i - e_i, e_i - e_i\]

\[\Pi_7 = \text{col}\{e_1, e_i, e_i, e_i, e_i, e_i, e_i, e_i, e_i, e_i\}\]

\[\Pi_8 = e_i - e_i, e_i - e_i, e_i - e_i, e_i - e_i\]

\[\Pi_9 = e_i - e_i, e_i - e_i, e_i - e_i, e_i - e_i\]

\[\Pi_{10} = e_i - e_i, e_i - e_i, e_i - e_i, e_i - e_i\]

\[\Pi_{11} = e_i - e_i, e_i - e_i, e_i - e_i, e_i - e_i\]

\[\Pi_{12} = e_i - e_i, e_i - e_i, e_i - e_i, e_i - e_i\]

\[\Pi_{13} = e_i - e_i, e_i - e_i, e_i - e_i, e_i - e_i\]

\[\Pi_{14} = e_i - e_i, e_i - e_i, e_i - e_i, e_i - e_i\]

\[\Pi_{15} = e_i - e_i, e_i - e_i, e_i - e_i, e_i - e_i\]

\[\Pi_{16} = e_i - e_i, e_i - e_i, e_i - e_i, e_i - e_i\]

\[\Pi_{17} = e_i - e_i, e_i - e_i, e_i - e_i, e_i - e_i\]

\[\Pi_{18} = e_i - e_i, e_i - e_i, e_i - e_i, e_i - e_i\]

\[\Pi_{19} = e_i - e_i, e_i - e_i, e_i - e_i, e_i - e_i\]

\[\Pi_{20} = e_i - e_i, e_i - e_i, e_i - e_i, e_i - e_i\]

\[\Pi_{21} = e_i - e_i, e_i - e_i, e_i - e_i, e_i - e_i\]

\[\Pi_{22} = e_i - e_i, e_i - e_i, e_i - e_i, e_i - e_i\]

\[\Pi_{23} = e_i - e_i, e_i - e_i, e_i - e_i, e_i - e_i\]

\[\Pi_{24} = e_i - e_i, e_i - e_i, e_i - e_i, e_i - e_i\]

\[\Pi_{25} = e_i - e_i, e_i - e_i, e_i - e_i, e_i - e_i\]

\[\Pi_{26} = e_i - e_i, e_i - e_i, e_i - e_i, e_i - e_i\]

\[\text{Proof. In this work, the Lyapunov functional for the system is designed as follows}\]

\[V(t) = \sum_{m=1}^{4} V_m(t) + \sum_{n=1}^{4} W_n(t)\]

\[V_1(t) = \eta_i^2(t)^T P \eta_i(t)\]

\[V_2(t) = \int_{t_d(t)}^{t_d(t)} \eta_2^T(s) Q_2 \eta_2(s) ds + \int_{t_d(t)}^{t_d(t)} \eta_2^T(s) Q_2 \eta_2(s) ds\]

\[V_3(t) = d \int_{t_d(t)}^{t_d(t)} \int_{t_d(t)}^{t_d(t)} T(s) R_i x(s) ds d \theta + d \int_{t_d(t)}^{t_d(t)} \int_{t_d(t)}^{t_d(t)} T(s) R_i x(s) ds d \theta\]

\[V_4(t) = h \int_{t_d(t)}^{t_d(t)} T(s) R_i x(s) ds d \theta\]

\[V_5(t) = 2 \int_{t_d(t)}^{t_d(t)} (U_i \zeta_i(t) + U_2 \zeta_2(t))\]

\[W_2(t) = (t_{k+1} - t) (t_{k+1} - t) \eta_i^2(t) Y \eta_i(t)\]

\[W_3(t) = (t_{k+1} - t)(t_{k+1} - t) \int_{t_k}^{t_{k+1}} T(s) Z_i \dot{x}(s) ds - (t - t_k) (t_{k+1} - t_k) \int_{t_k}^{t_{k+1}} T(s) Z_i \dot{x}(s) ds\]

\[W_4(t) = (t_{k+1} - t)(t_{k+1} - t_k) \int_{t_k}^{t_{k+1}} T(s) Z_i \dot{x}(s) ds - (t - t_k)(t_{k+1} - t_k) \int_{t_k}^{t_{k+1}} T(s) Z_i \dot{x}(s) ds\]

The derivative of \( V(t) \) is as

\[\dot{V}(t) = \sum_{m=1}^{4} \dot{V}_m(t) + \sum_{n=1}^{4} \dot{W}_n(t)\]

and each part in \( V(t) \) is therefore calculated as
\[ V_i(t) = \xi^T(t) \text{Sym} \left\{ \Pi_i^T \Pi_i \right\} \xi(t) \]
\[ V_4(t) = \xi^T(t) \left( \left( 1 - \dot{d}(t) \right) \Pi_4^T(Q_i - Q_j) \Pi_4 + \Pi_4^T Q_i \Pi_4 + \Pi_4^T Q_j \Pi_4 \right) \xi(t) \]
\[ V_j(t) = \xi^T(t) \left( d_x^T e_j^T R_i e_j + d_y^T e_j^T R_i e_j \right) \xi(t) - d \int_{t-d}^{t} \dot{x}^T(s) R_j x(s) ds - \int_{t-d}^{t} x^T(s) R_j x(s) ds \]
\[ V_4(t) = \xi^T(t) \left( b^T e_j^T T e_j - \frac{\pi^2}{4} \Pi_4^T T \Pi_4 \right) \xi(t) \]
\[ W_i(t) = \xi^T(t) \left( t_{k+i} - t \right) 2 \Pi_i^T \left( U_{i1} + U_{i2} \right) + \text{Sym} \left\{ \Pi_i^T U_{i1} \right\} \right) + \left( t_{k+i} - t \right) 2 \Pi_i^T \left( U_{i1} + U_{i2} \right) + \text{Sym} \left\{ \Pi_i^T U_{i1} \right\} \right) \xi(t) \]
\[ W_j(t) = \xi^T(t) \left( t_{k+j} - t \right) 2 \Pi_j^T \left( U_{j1} + U_{j2} \right) + \text{Sym} \left\{ \Pi_j^T U_{j1} \right\} \right) \xi(t) \]
\[ W_k(t) = \xi^T(t) \left( t_{k+i} - t \right) e_j^T Z_i e_j + \left( t_{k+j} - t \right) e_j^T Z_j e_j \right) \xi(t) - \left( t_{k+i} - t \right) \int_{t-d}^{t} \dot{x}^T(s) Z_i \dot{x}(s) ds \]
\[ W_k(t) = \xi^T(t) \left( t_{k+i} - t \right) e_j^T Z_i e_j + \left( t_{k+j} - t \right) e_j^T Z_j e_j \right) \xi(t) - \left( t_{k+i} - t \right) \int_{t-d}^{t} \dot{x}^T(s) Z_i \dot{x}(s) ds \]
\[ \text{With lemma 1, one has} \]
\[ -d \int_{t-d}^{t} \dot{x}^T(s) R_i x(s) ds \leq -\Pi_i^T R_i \Pi_i - 3 \Pi_i^T R_i \Pi_i \]
\[ \text{Meanwhile, with lemma 2, one has} \]
\[ -d \int_{t-d}^{t} \dot{x}^T(s) R_j x(s) ds \leq -\Pi_j^T R_j \Pi_j - 3 \Pi_j^T R_j \Pi_j - 5 \Pi_j^T R_j \Pi_j \]
\[ \text{Additionally, by adopting lemma 3, the other integrals in} \ W_3(t) \text{ and } W_4(t) \text{ could be estimated as} \]
\[ -\int_{t-d}^{t} \dot{x}^T(s) Z_i \dot{x}(s) ds \leq \xi^T(t) \left( t_{k+i} - t \right) \left\{ N_z \Pi_z + \frac{1}{3} N_z^2 \Pi_z^2 \right\} \xi(t) + \text{Sym} \left\{ N_z \Pi_z + N_z^2 \Pi_z^2 \right\} \]
\[ -\int_{t-d}^{t} \dot{x}^T(s) Z_j \dot{x}(s) ds \leq \xi^T(t) \left( t_{k+j} - t \right) \left\{ N_z \Pi_z + \frac{1}{3} N_z^2 \Pi_z^2 \right\} \xi(t) + \text{Sym} \left\{ N_z \Pi_z + N_z^2 \Pi_z^2 \right\} \]
\[ -\int_{t-d}^{t} \dot{x}^T(s) Z_k \dot{x}(s) ds \leq \xi^T(t) \left( t_{k+i} - t \right) \left\{ N_z \Pi_z + \frac{1}{3} N_z^2 \Pi_z^2 \right\} \xi(t) + \text{Sym} \left\{ N_z \Pi_z + N_z^2 \Pi_z^2 \right\} \]
\[ -\int_{t-d}^{t} \dot{x}^T(s) Z_k \dot{x}(s) ds \leq \xi^T(t) \left( t_{k+j} - t \right) \left\{ N_z \Pi_z + \frac{1}{3} N_z^2 \Pi_z^2 \right\} \xi(t) + \text{Sym} \left\{ N_z \Pi_z + N_z^2 \Pi_z^2 \right\} \]
\[ \text{Based on the system equation, the following equality holds with matrices } \kappa_1, \kappa_2, \kappa_3, \kappa_4 \text{ with} \]
\[ 2 \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i(\theta(t)) \lambda_j(\theta(t)) \xi^T(t) (\kappa_1 + \kappa_2 + \kappa_3 + \kappa_4) \times (-e_0 + A e_1 + A e_2 + B \kappa_1 e_4 + B \kappa_2 e_5) \xi(t) = 0 \]
\[ \text{Combining (15)-(22), } V(t) \text{ could be presented as a simple form as} \]
\[ \bar{V}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i(\theta(t)) \lambda_j(\theta(t)) \xi^T(t) \bar{O} \xi(t) \]
Where \( \Omega = \Omega_1 + (t_{k+1} - t)\Omega_2 + (t - t_k)\Omega_3 \).

To sum up, based on all the content above, inequality (23) can be transformed into another form

\[
\dot{\tilde{W}}(t) \leq \sum_{i=1}^{r} \sum_{j=1}^{r} \tilde{\beta}_i(\theta(t))\tilde{\beta}_j(\theta(t_k))\xi^T(t) \left( \frac{t-t_k}{h_k} (\Omega_1 + h_k\Omega_2) + \frac{t_{k+1} - t}{h_k} (\Omega_1 + h_k\Omega_2) \right) \xi(t) \tag{24}
\]

Based on convex combination technique, \( \tilde{\Omega} \leq 0 \) holds if \( h_k \in (0, h] \) exits while \( \Omega + h\Omega_3 \leq 0 \) and \( \Omega_1 + h\Omega_2 \leq 0 \) respectively. By applying Schur complement, inequality (24) would be guaranteed by LMIs (12)-(14) and thus \( \dot{V}(t) \) is negative. Therefore, the fuzzy system (7) is stable and the theorem above is proved.

**Remark 1** In this article, compared with some other existing papers, both the system delays \( d(t) \) and the control loop delays \( \hat{\lambda} \) are taken into account since they inevitably occur in practical system. In addition, the design of \( V_m(t) \) and \( W_i(t) \) is aimed at research the relationships between \( x(t), x(t_k), x(t_{k+1}), x(t - d(t)), x(t_k - d(t)) \) and \( x(t_{k+1} - d(t)) \) which would help reduce the system conservativeness.

**Remark 2** In this paper, the sampling interval \( h_k = t_{k+1} - t_k \) is not required that to be fixed since in practical system many inevitable factors will affect the system. On the other hand, the mismatched membership functions are shared for the system since the available time-stamped packet to derive the premises in the system and the controller are asynchronous.

### 4. Controller design

**Theorem 2** The system (7) would be asymptotically stable if such symmetrical positive matrices \( P, Q_1, Q_2, R_1, R_2, T, Z_a, a = 1, 2, 3, 4 \) and any other matrices \( \tilde{U}_1, \tilde{U}_2, \tilde{Y}, \tilde{N}_b, b = 1, 2, \ldots, 6 \) with appropriate dimension exist and satisfy the following LMIs.

\[
\begin{bmatrix}
\tilde{\Omega}_1 + h\tilde{\Omega}_2 & h_4\tilde{N}_3 & h_5\tilde{N}_4 & h_6\tilde{N}_6 \\
* & -\tilde{Z}_2 & 0 & 0 \\
* & * & -3\tilde{Z}_2 & 0 \\
* & * & * & -\tilde{Z}_4
\end{bmatrix} < 0 \tag{25}
\]

\[
\begin{bmatrix}
\tilde{\Omega}_1 + h\tilde{\Omega}_3 & h_1\tilde{N}_1 & h_2\tilde{N}_2 & h_3\tilde{N}_3 \\
* & -\tilde{Z}_1 & 0 & 0 \\
* & * & -3\tilde{Z}_1 & 0 \\
* & * & * & -\tilde{Z}_2
\end{bmatrix} < 0 \tag{26}
\]

\[
\tilde{\Omega}_4 < 0 \tag{27}
\]

where
\[ \Omega_i = \Phi_i + \dot{F}_i \]

\[ \Phi_i = \text{Sym}\left\{ \Pi_i^T \tilde{P} \Pi_i \right\} + h^2 e_i^T \tilde{F} e_{i2} - \frac{\pi^2}{4} \Pi_i^T \tilde{F} \Pi_i + 2 \Pi_i^T (\tilde{U}_i \Pi_{10} + \tilde{U}_j \Pi_{11}) + d^2 (e_i^T \tilde{R} e_i + e_j^T \tilde{R} e_j) \]

\[ + (1 - d(t)) \Pi_i^T (\tilde{Q}_i - \tilde{Q}_j) \Pi_i - \Pi_i^T \tilde{Q}_i \Pi_i + \Pi_i^T \tilde{Q}_j \Pi_i + \text{Sym}\left\{ \tilde{N}_i \Pi_{21} \right\} + \text{Sym}\left\{ \tilde{N}_i \Pi_{22} \right\} \]

\[ + \text{Sym}\left\{ \tilde{N}_i \Pi_{13} \right\} + \text{Sym}\left\{ \tilde{N}_i \Pi_{24} \right\} + \text{Sym}\left\{ \tilde{N}_i \Pi_{25} \right\} + \Theta_3 + \Theta_4 \]

\[ \Theta_3 = -\Pi_i^T \tilde{R} \Pi_{16} - 3 \Pi_i^T \tilde{R} \Pi_{17} - 2 \Pi_i^T \tilde{R} \Pi_{18} - 5 \Pi_i^T \tilde{R} \Pi_{19} - 5 \Pi_i^T \tilde{R} \Pi_{20} \]

\[ \dot{F}_i = \text{Sym}\left\{ (k_1 + k_2 + k_3 + k_4) \times (-e_3 + A_i e_i + A_d e_d + B_i \tilde{e}_j e_j + \tilde{B}_i e_j e_j) \right\} \]

After that, the gain matrices \( K_j \) and \( K_{dj} \) of the controller for system (7) would be like \( K_j = \tilde{K}_j G^{-1} \) and \( K_{dj} = \tilde{K}_{dj} G^{-1} \).

Proof. It is not difficult to prove the new stability criterion on the basis of Theorem 1 if (11)-(13) pre-multiply and post-multiply by \( \text{diag}\left\{ G, G, G \cdots, G, G \right\}^{18} \) and \( \text{diag}\left\{ G, G, G \cdots, G, G \right\}^{T} \). The matrices should also be changed through pre-multiplying and post-multiplying by the diagonal matrix made of matrix \( G \) according to their own dimension.

For example, \( \tilde{P} = \text{diag}\left\{ G, G, G \cdots, G, G \right\} \gamma \text{diag}\left\{ G, G, G \cdots, G, G \right\}^{T} \), the details are here omitted.

5. Numerical example
In this part, one numerical example is given to prove the advantage of the work.

Take the nonlinear time-delay system of stirred tank reactor as well as the same parameters referred in [16] as the research system. Picking different values of \( \lambda_1 = 0.01, \lambda_2 = 0.22 \) and \( \lambda_3 = 0.49 \), the system matrices \( A_i, A_{dj} \) and \( B_i \) are defined as

\[
A_1 = \begin{bmatrix} -1.4274 & 0.0757 \\ -1.4189 & -0.9442 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -2.0508 & 0.3958 \\ -6.4066 & 1.6268 \end{bmatrix}, \quad A_3 = \begin{bmatrix} -4.5279 & 0.3167 \\ -26.2228 & 0.9387 \end{bmatrix}
\]

\[
A_{dj} = A_{d1} = A_{d3} = [0.25, 0; 0, 0.25], \quad B_1 = B_2 = B_3 = [0, 0.3]^T
\]

Table 1. MVSI of different methods for example.

| MVSI   | Corollary 4 [17] | Corollary 3 [17] | Corollary 2 [17] | [16] | Example 1 |
|--------|-----------------|-----------------|-----------------|------|-----------|
| \( h = h_k \) | 0.3410 | 0.6953 | 0.8338 | 0.9046 | 0.9327 |
Figure 1. state response with initial condition.

Based on the parameters above, under the initial condition $x(0)=[0.8, 0.4]^T$, the maximum value of sampling interval (MVSI) is as $h=h_k=0.9327$. Table 1 shows the MVSI of different methods and Figure 1 illustrates the state responses. The corresponding controller gain matrices are as follow

$$K_1 = \begin{bmatrix} 19.6433 & -3.7063 \\ 66.4223 & -10.4159 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 26.5227 & -3.7261 \\ 8.0531 & -1.1077 \end{bmatrix}, \quad K_3 = \begin{bmatrix} 13.3484 & -1.7913 \end{bmatrix}$$

5. Conclusion

A further investigation of the stability for the T-S fuzzy system under sample-data control has been figured out in this paper. With different methods of solving inequalities and mismatched membership function introduced, results of less conservative stability criteria have been achieved. At the end of the paper, one example, compared with results from other papers, is represented in order to show the advanced availability of this work.

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