The decays $B_c \to J/\psi + \ell \nu_{\ell}$ and $B_c \to J/\psi + \pi(K)$ in covariant confined quark model

A.Issadykov$^{1,2,3,*}$ and M.A.Ivanov$^{1,†}$

$^1$Joint Institute for Nuclear Research, Dubna, Russia
$^2$The Institute of Nuclear Physics, Ministry of Energy of the Republic of Kazakhstan, Almaty, Kazakhstan
$^3$Al-Farabi Kazakh National University, SRI for Mathematics and Mechanics, Almaty, Kazakhstan

Abstract

In the wake of the recent measurements of the decays $B_c \to J/\psi \pi(K)$ and $B_c \to J/\psi \ell \nu_{\ell}$ reported by the LHCb Collaboration we calculate the form factors for the $B_c \to J/\psi$ and $B_c \to \eta_c$ transitions in full kinematical region within covariant confined quark model. Then we use the calculated form factors to evaluate the partial decay widths of the above-mentioned semileptonic and nonleptonic decays of the $B_c$ meson. We find that the theoretical predictions on the ratios of $\mathcal{R}_{K^+/\pi^+}$ and $\mathcal{R}_{\pi^+/\mu^+\nu}$ are in good agreement with last LHCb-data. However, the prediction for the $\mathcal{R}_{J/\psi}$ is found to be underestimated.

PACS numbers: 13.20.He, 12.39.Ki

*Electronic address: issadykov@theor.jinr.ru
†Electronic address: ivanovm@theor.jinr.ru
I. INTRODUCTION

The first measurement that relates semileptonic and hadronic decay rates of the $B_c^+$ meson was performed by the LHCb Collaboration [1]. The measured value of the ratio of branching fractions,

$$R_{\pi^+/\mu^+\nu} = \frac{B(B_c^+ \to J/\psi\pi^+)}{B(B_c^+ \to J/\psi\mu^+\nu)} = 0.0469 \pm 0.0028({\text{stat}}) \pm 0.0046({\text{syst}}),$$

was found at the lower end of available theoretical predictions. Among them one can mention a nonrelativistic reduction of the Bethe-Salpeter equation [2–4], a light-front constituent quark model [5, 6], QCD sum rules [7], a relativistic quasipotential Schrödinger model [8], and a relativistic constituent quark model [9].

The decay $B_c^+ \to J/\psi K^+$ was observed for the first time by the LHCb Collaboration [10]. The ratio of the branching fractions were measured to be

$$R_{K^+/\pi^+} = \frac{B(B_c^+ \to J/\psi K^+)}{B(B_c^+ \to J/\psi\pi^+)} = \begin{cases} 0.069 \pm 0.019({\text{stat}}) \pm 0.005({\text{syst}}) [10] \\ 0.079 \pm 0.007({\text{stat}}) \pm 0.003({\text{syst}}) [11] \end{cases}$$

The theoretical predictions for this ratio given in Refs. [2, 3, 6, 8, 9, 12–14] lie in the range from 0.054 to 0.088.

Recently, LHCb collaboration reported about measurement of the ratio of semileptonic branching fractions $R_{J/\psi}$ [15]:

$$R_{J/\psi} = \frac{B(B_c^+ \to J/\psi\tau^+\nu_{\tau})}{B(B_c^+ \to J/\psi\mu^+\nu_{\mu})} = 0.71 \pm 0.17({\text{stat}}) \pm 0.18({\text{syst}}).$$

This result lies within 2 standard deviations above the predictions obtained in several theoretical models Refs. [6, 13, 16–19] based on the Standard Model. Note that the semileptonic $B_c$ decays provide an excellent laboratory to measure the CKM-matrix elements: $V_{cb}$, $V_{ub}$, $V_{cs}$ and $V_{cd}$. The theoretical description of semileptonic and nonleptonic decays is, however, nontrivial problem because one needs to know the transition form factors characterizing the strong transition of the $B_c$ to the charmonium.

In this paper we use the form factors for the $B_c \to J/\psi$ and $B_c \to \eta_c$ transitions calculated in the full kinematical region within the covariant confined quark model [20, 21]. We evaluate the partial decay widths of the above-mentioned semileptonic and nonleptonic decays of the $B_c$ meson and compare our predictions for the ratios $R_{\pi^+/\mu^+\nu}$, $R_{K^+/\pi^+}$, $R_{J/\psi}$ with available experimental data given by Eqs. (1)-(3) and the results obtained in other approaches.
The paper is organized in the following manner. In Sec. II we define the hadronic matrix elements in terms of the invariant and helicity form factors. We calculate the form factors within the covariant confined quark model (CCQM) in the full kinematical region of the momentum transfer squared with all model parameters to be fixed.

In Sec. III we present our numerical results for the semileptonic and nonleptonic branching ratios of the $B_c$ meson calculated within our model and compare them with those calculated in other approaches. Finally, we briefly conclude in Sec. IV.

II. FORM FACTORS, HELICITY AMPLITUDES AND DECAY WIDTHS

The form factors of the transitions of the $B_c$ into charmonia $\eta_c$ and $J/\psi$ have been calculated in our recent papers [20, 21] in the framework of the covariant confined quark model. The accuracy of calculation was estimated of 10%.

The behavior of the calculated form factors in the kinematical region $0 \leq q^2 \leq q^2_{\text{max}}$ is shown in Fig. 1. Note that $q^2_{\text{max}} = (m_{B_c} - m_{\eta_c})^2 = 10.9 \text{ GeV}^2$ for the $\eta_c$ and $q^2_{\text{max}} = (m_{B_c} - m_{J/\psi})^2 = 10.1 \text{ GeV}^2$ for the $J/\psi$.

![Graph](image-url)

FIG. 1: The $F_+$ and $F_-$ form factors for the $B_c \rightarrow \eta_c$ transition (left panel) and $A_0, A_-, A_+$ and $V$ form factors for the $B_c \rightarrow J/\psi$ transition (right panel).

The results of our numerical calculations can be approximated with high accuracy by the parameterization

$$F(q^2) = \frac{F(0)}{1 - a \hat{s} + b \hat{s}^2}, \quad \hat{s} = \frac{q^2}{m^2_{B_c}}, \quad (4)$$
the relative error of the approximation is less than 1%. The values of $F(0)$, $a$, and $b$ are listed in Table I.

**TABLE I:** Parameters of the approximated form factors for $B_c \rightarrow J/\psi(\eta_c)$ transitions.

|     | $A_0$ | $A_+$ | $A_-$ | $V$ | $F_+$ | $F_-$ |
|-----|-------|-------|-------|-----|-------|-------|
| $F(0)$ | 1.65  | 0.55  | −0.87 | 0.78 | 0.75  | −0.40 |
| $a$  | 1.19  | 1.68  | 1.85  | 1.82 | 1.31  | 1.25  |
| $b$  | 0.17  | 0.70  | 0.91  | 0.87 | 0.33  | 0.25  |

The invariant form factors for the semileptonic $B_c$-decay into the hadron with spin $S = 0, 1$ are defined by

$$
\mathcal{M}^\mu_{S=0} = P^\mu F_+(q^2) + q^\mu F_-(q^2),
$$

(5)

$$
\mathcal{M}^\mu_{S=1} = \frac{c_\mu}{m_1 + m_2} \left\{ -g^\mu P q A_0(q^2) + P^\mu P^\nu A_+(q^2) + q^\mu P^\nu A_-(q^2) + i\epsilon^{\mu\nu\alpha\beta} P_\alpha q_\beta \right\},
$$

where $P = p_1 + p_2$, $q = p_1 - p_2$ and $\epsilon^{\mu\nu\alpha\beta} \equiv \epsilon^{\alpha\beta\mu\nu} P_\alpha q_\beta$. It is convenient to express all physical observables through the helicity form factors $H_m$. The helicity form factors $H_m$ can be expressed in terms of the invariant form factors in the following way [22]:

(a) Spin $S = 0$:

$$
H_t = \frac{1}{\sqrt{q^2}} \left\{ (m_1^2 - m_2^2) F_+ + q^2 F_- \right\}, \quad H_\pm = 0, \quad H_0 = \frac{2 m_1 |p_2|}{\sqrt{q^2}} F_+.
$$

(6)

(b) Spin $S = 1$:

$$
H_t = \frac{1}{m_1 + m_2} \frac{m_1 |p_2|}{\sqrt{q^2}} \left\{ (m_1^2 - m_2^2) (A_+ - A_0) + q^2 A_- \right\},
$$

$$
H_\pm = \frac{1}{m_1 + m_2} \left\{ -(m_1^2 - m_2^2) A_0 \pm 2 m_1 |p_2| V \right\},
$$

(7)

$$
H_0 = \frac{1}{m_1 + m_2} \frac{1}{2 m_2 \sqrt{q^2}} \left\{ -(m_1^2 - m_2^2) (m_2^2 - m_2^2 - q^2) A_0 + 4 m_1^2 |p_2|^2 A_+ \right\}.
$$

where $|p_2| = \lambda^{1/2}(m_1^2, m_2^2, q^2)/(2 m_1)$ is the momentum of the outgoing particles in the $B_c$ rest frame.

The semileptonic decays $B_c \rightarrow M_{c\bar{c}} + \ell\nu_\ell$ are described by the tree diagram shown at the left panel in Fig. [2]. Note that $M_{c\bar{c}}$ denote both the $\eta_c$ and $J/\psi$ states. The decay widths
are written down as

$$\Gamma(B^+_c \to M_{c\bar{c}} \ell \bar{\nu}_\ell) = \frac{G_F^2}{(2\pi)^3} |V_{cb}|^2 \int_{m_t^2}^{q_{\text{max}}^2} dq^2 \frac{(q^2 - m_\ell^2)^2}{12 m_t^2 q^2} |P_2|$$

$$\times \left\{ \left(1 + \frac{m_t^2}{2 q^2} \right) \sum_{i=\pm,0} \left( H_{i}^{B_c \to M_{c\bar{c}}}(q^2) \right)^2 + \frac{3 m_t^2}{2 q^2} \left( H_{i}^{B_c \to M_{c\bar{c}}}(q^2) \right)^2 \right\}. \tag{8}$$

The effective Hamiltonian which is needed to describe the nonleptonic decays $B_c \to M_{c\bar{c}} + \pi(K)$ is given by

$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{cb} V_{uq}^\dagger \left( C_1 (\bar{q}b)_{V-A}(\bar{c}u)_{V-A} + C_2 (\bar{c}b)_{V-A}(\bar{q}u)_{V-A} \right)$$

where the subscript $V-A$ refers to the usual left–chiral current $O^\mu = \gamma^\mu(1-\gamma^5)$ and $q = d, s$. The Feynman diagram describing such decays is shown at the right panel in Fig. 2. The nonleptonic $B_c$-decay widths in terms of the helicity amplitudes are given by

$$\Gamma(B^+_c \to P^+ M_{c\bar{c}}) = \frac{G_F^2}{16\pi m_t^2} \left| V_{cb} V_{uq}^\dagger a_1 f_{P M} \right|^2 \left( H_{i}^{B_c \to M_{c\bar{c}}}(m_{P M}^2) \right)^2,$$

$$(P^+ = \pi^+, K^+, \text{ and } q = d, s, \text{ respectively}),$$

$$\Gamma(B^+_c \to V^+ M_{c\bar{c}}) = \frac{G_F^2}{16\pi m_t^2} \left| V_{cb} V_{uq}^\dagger a_1 f_{V M} \right|^2 \sum_{i=0,\pm} \left( H_{i}^{B_c \to M_{c\bar{c}}}(m_{V M}^2) \right)^2,$$

$$(V^+ = \rho^+, K^{*+}, \text{ and } q = d, s, \text{ respectively}),$$

where $a_1 = C_2 + \xi C_1$ with $\xi = 1/N_c$. The leptonic decay constants are also calculated $f_{P(V)}$ in the framework of the CCQM.
TABLE II: Branching ratios (in %) of semileptonic $B_c$ decays into ground state charmonium states.

| Mode                  | This work | [3] | [4] | [5] | [7] | [8] | [9] | [16] | [18] | [19] | [22] | [26] |
|-----------------------|-----------|-----|-----|-----|-----|-----|-----|------|------|------|------|------|
| $B_c^+ \rightarrow \eta_c\mu^+\bar{\nu}_\mu$ | 0.95 ± 0.19 | 0.76 | 0.15 | 0.59 | 0.75 | 0.42 | 0.81 | 0.55 | $4.5^{+1.66}_{-1.24}$ | 0.44 | 0.98 | 0.52 |
| $B_c^+ \rightarrow \eta_c\tau^+\bar{\nu}_\tau$ | 0.24 ± 0.05 | 0.20 | 0.23 | 0.22 | 2.8$^{+1.01}_{-0.73}$ | 0.14 | 0.27 |
| $B_c^+ \rightarrow J/\psi\mu^+\bar{\nu}_\mu$ | 1.67 ± 0.33 | 2.01 | 1.47 | 1.20 | 1.9 | 1.23 | 2.07 | 1.73 | $5.7^{+1.1}_{-0.92}$ | 1.01 | 2.30 | 1.47 |
| $B_c^+ \rightarrow J/\psi\tau^+\bar{\nu}_\tau$ | 0.40 ± 0.08 | 0.34 | 0.48 | 0.49 | 1.7$^{+0.51}_{-0.33}$ | 0.29 | 0.59 |

III. NUMERICAL RESULTS

First, we show up the input parameters used in calculations. The central values of the relevant CKM-matrix elements $|V_{cb}| = 0.0405$, $|V_{ud}| = 0.974$ and $|V_{us}| = 0.225$ are taken from the PDG [23]. The values of leptonic decay constants were calculated in our previous papers and are given in Eq. (10) (all in MeV).

$$
\begin{align*}
\frac{f_\pi}{f_K} &= 130.3 & \frac{f_\rho}{f_{K^*}} &= 221.0 & 226.8 \\
\end{align*}
$$

We will use the numerical values of the Wilson coefficients from [24] obtained at the scale $\mu = 4$ GeV at leading order with $\Lambda_{MS}^{(5)} = 225$ MeV. One has $C_2 = 1.141$ and $C_2 = -0.310$ that gives $a_1 = C_2 + \xi C_1 = 1.038$. Note that this value has been also used in the paper [25]. It differs from the most old papers where the color-suppressed factor $\xi$ was set to zero.

The results of theoretical predictions of the branching ratios of the semileptonic $B_c$ decays and ratio $R_{J/\psi}$ in comparison with LHCb data [15] are shown in Table II and in Fig 3, where $R_{\eta_c} = \frac{B(B_c^+\rightarrow\eta_c\mu^+\bar{\nu}_\mu)}{B(B_c^+\rightarrow\eta_c\tau^+\bar{\nu}_\tau)}$ and $R_{J/\psi} = \frac{B(B_c^+\rightarrow J/\psi\mu^+\bar{\nu}_\mu)}{B(B_c^+\rightarrow J/\psi\tau^+\bar{\nu}_\tau)}$. The experimental errors and some theoretical uncertainties are taken in quadrature. We estimate the uncertainties of our calculation of the branching fractions as about 20% because they are proportional to form factors squared.

The results of theoretical predictions for the nonleptonic decay widths of the $B_c$ meson...
FIG. 3: Theoretical predictions and LHCb data [15] for \( R_{J/\psi} \) are shown in Table III in units of \( a_1^2 \cdot 10^{-15} \) GeV obtained in our approach, whereas the absolute values of the branching fractions in % are displayed in Table IV.

The values of the ratios branching fractions obtained by LHCb and calculated in several theoretical approaches are given in Table V.

IV. SUMMARY AND DISCUSSION

We have calculated the semileptonic and nonleptonic decays of the \( B_c \) meson within CCQM. We have found that the ratios of the branching fractions \( R_{\pi^+/\mu^+\nu} \) and \( R_{K^+/\pi^+} \) are in good agreement with the LHCb data and other theoretical approaches. At the same time the theoretical predictions for the ratio \( R_{J/\psi} \) are more than 2 \( \sigma \) less than the experimental data. This may indicate on the possibility of New physics effects in this decay. The possible influence of such effects or some physical observables in the decays mediated \( b \to c l \nu \)
TABLE III: Exclusive nonleptonic decay widths of the $B_c$ meson in units of $a_1^2 \cdot 10^{-15}$ GeV for general value of the Wilson coefficient $a_1$. 

| Mode                        | This work | [2] | [3] | [4] | [5] | [8] | [9] | [16] | [27] |
|-----------------------------|-----------|-----|-----|-----|-----|-----|-----|------|------|
| $B_c^+ \to \eta_c\pi^+$    | 2.28 ± 0.46 | 2.07 | 1.59 | 0.28 | 1.47 | 0.93 | 2.11 | 1.18 | 0.10 | 1.49 |
| $B_c^+ \to \eta_c\rho^+$   | 3.15 ± 0.63 | 5.48 | 3.74 | 0.75 | 3.35 | 2.3  | 5.10 | 2.89 | ±0.51| 3.93 |
| $B_c^+ \to \eta_cK^+$      | 0.17 ± 0.03 | 0.16 | 0.12 | 0.023| 0.15 | 0.07 | 0.166| 0.092| ±0.0078| 0.12 |
| $B_c^+ \to \eta_cK^{*+}$   | 0.19 ± 0.04 | 0.09 | 0.20 | 0.04 | 0.24 | 0.12 | 0.276| 0.17 | ±0.02| 0.20 |
| $B_c^+ \to J/\psi\pi^+$    | 1.22 ± 0.24 | 1.97 | 1.22 | 1.48 | 0.82 | 0.67 | 1.93 | 1.24 | ±0.11| 1.01 |
| $B_c^+ \to J/\psi\rho^+$   | 2.03 ± 0.41 | 5.95 | 3.48 | 4.14 | 2.32 | 1.8  | 5.49 | 3.59 | ±0.64| 3.25 |
| $B_c^+ \to J/\psiK^+$      | 0.09 ± 0.02 | 0.15 | 0.09 | 0.08 | 0.08 | 0.05 | 0.15 | 0.095| ±0.008| 0.08 |
| $B_c^+ \to J/\psiK^{*+}$   | 0.13 ± 0.03 | 0.32 | 0.20 | 0.23 | 0.18 | 0.11 | 0.31 | 0.226| ±0.03| 0.17 |

TABLE IV: Branching ratios (in %) of exclusive nonleptonic $B_c$ decays with the choice of Wilson coefficient $a_1 = 1.038$ within covariant confined quark model.

| $B_c^+ \to \eta_c\pi^+$ | $B_c^+ \to \eta_c\rho^+$ | $B_c^+ \to \eta_cK^+$ | $B_c^+ \to \eta_cK^{*+}$ |
|--------------------------|--------------------------|-----------------------|---------------------------|
| 0.189 ± 0.037            | 0.518 ± 0.104            | 0.015 ± 0.003         | 0.029 ± 0.006             |
| $B_c^+ \to J/\psi\pi^+$  | $B_c^+ \to J/\psi\rho^+$ | $B_c^+ \to J/\psiK^+$  | $B_c^+ \to J/\psiK^{*+}$  |
| 0.101 ± 0.02             | 0.334 ± 0.067            | 0.008 ± 0.002         | 0.019 ± 0.004             |

transitions have been discussed in [17, 18, 28].

V. ACKNOWLEDGMENTS

The work has been carried out under financial support of the Program of the Ministry of Education and Science of the Republic of Kazakhstan IRN number AP05132978. Author
TABLE V: The ratios of branching fractions.

| Ref.     | $\mathcal{R}_{\pi^+}/\mu^+\nu$ | $\mathcal{R}_{\kappa^+}/\pi^+$ | $\mathcal{R}_{\eta_c}$ | $\mathcal{R}_{J/\psi}$ |
|----------|-------------------------------|---------------------------------|-----------------|-----------------|
| LHCb [1] | 0.0469 ± 0.0054               |                                 |                 |                 |
| LHCb [10]| 0.069 ± 0.019                 |                                 |                 |                 |
| LHCb [11]| 0.079 ± 0.0076                |                                 |                 |                 |
| LHCb [15]|                                 |                                 | 0.71 ± 0.25     |                 |
| This work| 0.0605 ± 0.012                | 0.076 ± 0.015                   | 0.26 ± 0.05     | 0.24 ± 0.05    |

[3] A. Abd El-Hady, J. H. Munoz and J. P. Vary, Phys. Rev. D 62 (2000) 014019 [hep-ph/9909406].
[4] P. Colangelo and F. De Fazio, Phys. Rev. D 61 (2000) 034012 [hep-ph/9909423].
[5] A. Y. Anisimov, I. M. Narodetsky, C. Semay and B. Silvestre-Brac, Phys. Lett. B 452,
FIG. 4: Theoretical predictions and LHCb data [1] for $R_{\pi^+}/\mu^+$.  

FIG. 5: Theoretical predictions and LHCb data [11] for $R_{K^+/\pi^+}$.  

[1] A. Y. Anisimov, P. Y. Kulikov, I. M. Narodetsky and K. A. Ter-Martirosian, Phys. Atom. Nucl. 62, 1739 (1999) [Yad. Fiz. 62, 1868 (1999)] arXiv:hep-ph/9809249.  
[6] H. W. Ke, T. Liu and X. Q. Li, Phys. Rev. D 89 (2014) no.1, 017501 arXiv:1307.5925 [hep-ph].
[7] V. V. Kiselev, hep-ph/0211021.
[8] D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D 68 (2003) 094020 [hep-ph/0306306].
[9] M. A. Ivanov, J. G. Körner and P. Santorelli, Phys. Rev. D 73 (2006) 054024 [hep-ph/0602050].
[10] R. Aaij et al. [LHCb Collaboration], JHEP 1309 (2013) 075 [arXiv:1306.6723 [hep-ex]].
[11] R. Aaij et al. [LHCb Collaboration], JHEP 1609 (2016) 153 [arXiv:1607.06823 [hep-ex]].
[12] I. P. Gouz, V. V. Kiselev, A. K. Likhoded, V. I. Romanovsky and O. P. Yushchenko, Phys. Atom. Nucl. 67 (2004) 1559 [Yad. Fiz. 67 (2004) 1581]
[13] S. Naimuddin, S. Kar, M. Priyadarshini, N. Barik and P. C. Dash, Phys. Rev. D 86 (2012) 094028.
[14] C. F. Qiao, P. Sun, D. Yang and R. L. Zhu, Phys. Rev. D 89 (2014) no.3, 034008 [arXiv:1209.5859 [hep-ph]].
[15] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 120 (2018) no.12, 121801 [arXiv:1711.05623 [hep-ex]].
[16] C. Chang, H. F. Fu, G. L. Wang and J. M. Zhang, Sci. China Phys. Mech. Astron. 58 (2015) no.7, 071001 [arXiv:1411.3428 [hep-ph]].
[17] R. Dutta and A. Bhol, Phys. Rev. D 96 (2017) no.7, 076001 [arXiv:1701.08598 [hep-ph]].
[18] Z. Rui, H. Li, G. x. Wang and Y. Xiao, Eur. Phys. J. C 76 (2016) no.10, 564 [arXiv:1602.08918 [hep-ph]].
[19] W. F. Wang, Y. Y. Fan and Z. J. Xiao, Chin. Phys. C 37 (2013) 093102 [arXiv:1212.5903 [hep-ph]].
[20] S. Dubnicka, A. Z. Dubnickova, A. Issadykov, M. A. Ivanov and A. Liptaj, Phys. Rev. D 96 (2017) no.7, 076017 [arXiv:1708.09607 [hep-ph]].
[21] A. Issadykov, M. A. Ivanov and G. Nurbakova, EPJ Web Conf. 158 (2017) 03002.
[22] M. A. Ivanov, J. G. Körner and P. Santorelli, Phys. Rev. D 63 (2001) 074010 [hep-ph/0007169].
[23] C. Patrignani et al. [Particle Data Group], Chin. Phys. C 40 (2016) no.10, 100001.
[24] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996) [hep-ph/9512380].
[25] N. Kitazawa, K. s. Masukawa and Y. Sakai, arXiv:1802.05417 [hep-ph].
[26] M. A. Nobes and R. M. Woloshyn, J. Phys. G 26, 1079 (2000) arXiv:hep-ph/0005056.
[27] J. F. Liu and K. T. Chao, Phys. Rev. D 56 (1997) 4133.
[28] C. T. Tran, M. A. Ivanov, J. G. Körner and P. Santorelli, arXiv:1801.06927 [hep-ph].