Evaluation of Normal Meshing Profile Deviation based on 3D Measurement

Baoya Zhao¹,²*, Zhaoyao Shi¹ and Bo Yu¹

¹ Beijing Engineering Research Center of Precision Measurement Technology and Instruments, Beijing University of Technology
² School of Mechatronics Engineering, North China Institute of Aerospace Engineering
*Corresponding author’s e-mail: baoya_yg@126.com

Abstract. The feature lines on involute surface include profile, helix, normal meshing profile and contact line. In helical gear transmission, the deviation of normal meshing tooth shape can reflect the transmission quality of gear better. In this paper, a method is proposed to obtain the tooth flank normal deviation from the 3D measuring point cloud data of gear, and to obtain the normal meshing profile deviation curve from it, and to evaluate it.

1. Introduction

Gear is the most widely used key basic component[1], which plays an important role in industry, aerospace, national defense and other fields. Gear accuracy evaluation is an indispensable link in gear manufacturing process[2]. When evaluating gears, the characteristic lines are usually measured on several tooth flanks[3].The parameter curve with specific meaning on the tooth flank is called characteristic line. Although there are many kinds of tooth flank parameter curves, limited by the working mechanism of the workpiece, design principle, processing method, use state, curve measurability and many other factors, the selection of feature line is ultimately attributed to the error of four kinds of feature lines on the gear tooth flank, which are helix, involute, normal meshing profile and contact line[4].

Helix and involute are commonly used tooth flank feature lines. Only a few error curves are used as the evaluation basis for the above deviations in ISO1328 series precision standard documents. For the profile deviation and helix deviation, only a few teeth are measured, and only one curve is measured in the middle of the tooth surface for each tooth. This kind of geometric error based on the local (feature points or feature lines) of the gear tooth flank can only reflect part of the information of the complex shape of the tooth flank, and the measurement data obtained as the evaluation basis is only a very small sample of all the geometric information of the gear to be measured. For a large number of data obtained from gear measurement, using small samples to evaluate the overall performance will inevitably lead to obvious differences between the evaluation results and the actual performance of gears. In helical gear transmission, the normal meshing tooth profile is the actual working line on the tooth flank, which can better reflect the performance of the gear, so it has special significance. But in practice, the normal meshing tooth profile and contact line are seldom used[5], mainly because the latter two characteristic lines are difficult to be measured and evaluated by traditional measurement methods[6].
In this paper, a method of extraction and evaluation of normal meshing profile error is proposed for normal meshing profile of four kinds of feature lines. The tooth flank deviation in the normal direction is extracted from the tooth flank measuring point, and then the normal meshing tooth profile deviation curve on the tooth flank is extracted and evaluated in the meshing plane coordinate system.

2. Equation of normal meshing profile

According to the geometric construction method of involute helicoid, when the tangent plane is rolling along the base cylinder, the motion track of the oblique line AA inclined to the axis at $\beta_b$ angle is the involute helicoid (Figure. 1) [7]. For parallel axis helical cylindrical gear transmission, there are many contact points at every moment of rotation, which form a contact line, and there are also many parallel meshing lines, which form a meshing plane. For helical gear transmission, there is only one contact point at each moment of rotation, so there is only one meshing line, which is perpendicular to the generating line. See line in Figure.1. The meshing and merging of the tooth flank is not along the involute of the end face, but along the normal meshing profile.

The normal meshing tooth profile is the locus of the instantaneous contact point on the tooth flank when two tooth flanks mesh in the helical gear transmission. For hobbing, shaving, honing and grinding with worm wheel, the flank of helical gear can be regarded as the enveloping of normal meshing profile when it moves along the helix of flank. For this kind of gear, measuring the normal meshing profile is better than measuring the involute of the end face.

If there is no error between the two meshing gears, their transmission will be ideal and there will be no transmission error. When there is an error in the gear, the ideal transmission will be destroyed and the transmission error will be produced. It is obvious that the fundamental effect on the transmission error is the error of tooth flank contact, that is, the error of "normal meshing profile", which will be brought into the transmission by 1:1 along the direction of meshing line. Therefore, the normal profile error can reflect the accuracy and smoothness of the transmission motion of the gear better than the profile error defined on the end face.

The equation of normal meshing profile in gear coordinate system. The involute helicoid equation of Figure. 2 is known.
The un
unit normal vector on the tooth flank is

\[
\begin{aligned}
\mathbf{n}_x &= \frac{1}{\sqrt{1 + \tan^2 \beta_b}} \sin(\varphi + \theta(z)) \\
\mathbf{n}_y &= \tan \beta_b \cos(\varphi + \theta(z)) \\
\mathbf{n}_z &= -\cot \beta_b (\theta(z) - \eta_b)
\end{aligned}
\]

Let two coordinate systems \(O_{xyz}\) and \(O_{x'y'z'}\), as shown in Fig. 3. \(O_{xyz}\) coincides with the gear coordinate system in Figure 1. As the gears rotate together, \(O_{x'y'z'}\) is fixed, and its \(x_0\) is the radial direction passing through the tangent point of the two indexing cylinder. The \(z_0\) and \(z\) axes coincide with the gear circumference. When the tooth flank of the gear and the coordinate system \(O_{xyz}\) rotate around the \(z\)-axis, corresponding to each angle \(\psi\), there is a cross-section tooth profile at one end and an instantaneous contact point. The track of the contact point in the coordinate system \(O_{x'y'z'}\) is the normal meshing profile.

![Figure 3 coordinate systems \(O_{xyz}\) and \(O_{x'y'z'}\)](image)

![Figure 4 local coordinate system on tooth flank](image)

The equation of involute helicoid and the normal vector of tooth flank are transformed into a fixed coordinate system. According to the principle of gear meshing, the relationship between the involute helicoid and the normal vector of tooth flank under meshing condition can be obtained

\[
\theta(z) = \tan^2 \beta_b (\varphi - \tan \alpha) + \eta_b
\]

By substituting (3) into (1), the equation of normal meshing profile in gear coordinate system can be obtained

\[
\begin{aligned}
\Phi &= \varphi + (\varphi - \tan \alpha) \cdot \tan^2 \beta_b + \eta_b \\
x_{\text{nom}, n} &= \eta_b \cdot \cos \Phi + \varphi \sin \Phi \\
y_{\text{nom}, n} &= \eta_b \cdot \sin \Phi - \varphi \cos \Phi \\
z_{\text{nom}, n} &= \eta_b \cdot \tan \beta_b (\varphi - \tan \alpha)
\end{aligned}
\]

According to \((x_{\text{meas}}, y_{\text{meas}}, z_{\text{meas}}), \left| \mathbf{d}_{\text{nom}} \right|\) can be expressed as a function

\[
F(x_{\text{meas}}, y_{\text{meas}}, z_{\text{meas}}, \varphi, \theta, \eta_b, \beta_b)
\]

In order to characterize the surface morphology, as shown in Figure 4, the normal deviation of the tooth flank is described by a pair of local coordinates in the contact surface and the corresponding normal distance \((y_n, z_n, d_{\text{nom}}(y_n, z_n))\). The \(y_n\) axis and \(z_n\) axis are along the profile and helix
respectively. The Cartesian coordinate system for locating any point \((y_n, z_n)\) on the tooth flank, with the origin \(y_n = 0, z_n = 0\), is located at the axial and radial center of the selected analysis area of axial dimension \(b\) and radial dimension \(h\).

The relationship between the local coordinate system \((y_n, z_n)\) on the tooth flank and the gear coordinate system is shown in equation (6)

\[
\begin{align*}
y_n &= r_n (\phi - \varphi_b + \eta_b) \\
z_n &= -r_n \cot \beta_b (\theta(z) - \eta_b) + b / 2
\end{align*}
\]

Combining (4) and (6), the equation of normal meshing profile on tooth flank in local coordinate system is obtained

\[
z_n = -\tan \beta_b \cdot y_n + C
\]  

(7)

\(-\tan \beta_b\) indicates that the angle between the equation of normal tooth profile in local coordinate system and the coordinate axis \(z_n\) is \(\beta_b\), and intercept \(C\) indicates different positions along the direction of helix, thus the normal tooth profile of normal tooth flank deviation \(\delta_{norm}\) in local can be characterized

\[
\begin{align*}
|\delta_{norm}| &= f(y_n, z_n) \\
z_n &= -\tan \beta_b \cdot y_n + C
\end{align*}
\]

(8)

That is, the intersection of normal tooth flank deviation and \(y_n = -\tan \beta_b \cdot z_n + C\) is the normal meshing tooth profile error.

3. Extraction and evaluation of normal meshing profile

In the local coordinate system \((y_n, z_n, \delta_{norm}(y_n, z_n))\), the normal meshing tooth profile error curve at any position on the tooth flank can be obtained by intercepting the corresponding feature plane at different positions of the tooth flank error \(\delta_{norm}(y_n, z_n)\). The extraction method is shown in Figure 5. At this time, the 2D curve is obtained, so the information obtained from the 3D measurement and error characterization is converted to the 2D plane for evaluation. The evaluation method in ISO1328 is still applicable[8,9]. Figure 6 shows a two-dimensional method for evaluating normal meshing tooth profile.

![Figure 5 Schematic diagram of normal meshing profile deviation extraction](image-url)
4. Data processing examples
In order to verify the correctness of the above model and method, a test gear (basic parameters in Table 1 is measured by optical non-contact measurement method, and the tooth flank data is modeled according to the model proposed by the above theory. The normal meshing profile feature line on the tooth flank is extracted and evaluated.

Table 1. Measuring test gear basic parameters

| No. | Name                        | Symbol (unit) | Value |
|-----|-----------------------------|---------------|-------|
| 1   | Number of teeth             | z             | 28    |
| 2   | Normal modulus              | m_n (mm)      | 3     |
| 3   | Normal pressure angle       | α_n (°)       | 27.5  |
| 4   | Helix angle                 | β (°)         | 2     |
| 5   | Tooth width                 | B (mm)        | 10    |
| 6   | Coefficient of displacement | x             | 0     |

5. Conclusion
Normal meshing profile is an important feature line of tooth flank, and the deviation of normal meshing profile can well reflect the performance of gear in use. In this paper, a method of extraction and evaluation of normal meshing tooth profile is proposed, which can extract and evaluate the normal meshing tooth profile deviation curve along the normal direction of meshing in the meshing plane coordinate system.

Figure 7 Extracting the test gear normal meshing profile deviation
Figure 8 normal meshing profile deviations

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