The Impact of Government Subsidies on Single-Channel Recycling Based on Recycling Propaganda

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Abstract: The recycling of waste products is an important way to achieve global sustainable development. To analyze the impact of different objects of government subsidies on single-channel recycling based on recycling propaganda, four theory game models of single-channel recycling based on government subsidies and recycling propaganda are established. By comparing and analyzing the effects of different subsidies and propaganda strategies on the recycling of waste products in the four models, this article mainly draws the following conclusions: the government selecting different objects to subsidize has the same effect on the unit recycling price, quantity, and revenue of waste products; when the government subsidizes the processors, the consigned recycling price of waste products will increase, but when the government subsidizes recyclers, it will decrease; when the propagandist is determined, the optimal value of propaganda is related to the sensitivity of residents to the unit recycling price of waste products, the unit propaganda of waste products, and the expenses of propagating waste products.

Keywords: waste products; single-channel recycling; government subsidies; recycling propaganda

1. Introduction

At present, the world is facing serious environmental pollution and a resource recycling dilemma. Green and low-carbon recycling development is an important goal of economic development and global governance systems. The construction of resource recycling systems is an effective way to achieve low-carbon development [1]. However, due to the different resource endowments of such systems, there are significant differences in carbon dioxide emissions, the proportion of non-fossil energy consumption, and the amount of forest stock. Thus, in order to further reduce environmental pollution and resource use efficiency, the resource recycling system must be improved. This is an important way to achieve carbon neutrality and establish a green economic system. To accelerate the establishment of the recycling system, most governments encourage resource recycling through various subsidies [2–6]. For example, China encourages step-by-step recycling of electric vehicle batteries and has established waste sorting and recycling systems in major cities [7–9].

Due to the poor public awareness of environmental protection, the imperfect resource recycling system, and the conflict between economic development and environmental protection, all countries have many problems in the construction of recycling supply chain systems [10]. Among them, the importance of public awareness regarding the recycling of waste products is increasing, and it has gradually become an important driving force affecting resource recycling and green development. For example, the resolute attitude of the public has always been an important reason for the difficulty of water recycling in the United States. Strengthening recycling propaganda and raising public awareness are effective means of promoting the recycling of water resources in the United States [11].
resource-based cities to green cities is environmental protection propaganda [12]. Public awareness is also a prerequisite for the establishment of a rural waste management system. Effective waste classification propaganda is an important means of waste classification repurchase and classification management [13–15]. The dynamic dissemination of domestic waste classification information in relation to environmental protection will increase the willingness of waste classification and then determine the behavior of waste classification [16]. Recycling propaganda is also an important dimension in the construction of a recycling supply chain system and the development of a supply chain partnership [17].

However, recycling promotion requires a high cost, and companies, consumers, and recyclers are unwilling to pay it [18,19]. Therefore, the governments of some countries try to subsidize recycling propaganda and encourage recyclers to recycle and remanufacture products through subsidies. Scholars generally believe that government subsidies are an important means of promoting recycling and remanufacturing, especially in the initial stage of the development of remanufactured products [20–26]. Government subsidies are more effective than policies such as carbon taxes, cap-and-trade, take-back regulation, and carbon permit allocation [27–40]. In particular, government subsidies are conducive to improving the recycling performance and the level of recycling research of recycling companies [41,42].

Scholars have studied the impact of subsidies on the supply chain of different channels and the impact of different subsidy models on recycling channels [2–6]. However, scholars rarely study the impact of subsidy models based on recycling propaganda on recycling channels. This article analyzes the role of government subsidy strategies (subsidizing recyclers or processors) and propaganda methods in promoting the recycling of waste products and establishes a game model based on two subsidy strategies and two propaganda methods to determine the best subsidy and propaganda mode.

2. Model Formulation

2.1. Problem Statements

Recycling waste products requires the participation of both recyclers and processors. Recyclers profit from waste product recycling commissioned by processors, and processors directly dispose of waste products to obtain revenue. In single-channel recycling, the processors obtain the waste products from the recyclers and then recycle them. In order to further enhance recycling efficiency of waste products, the government chooses to subsidize recyclers or processors, and the recyclers or processors propagate them, as shown in Figure 1. Therefore, it is necessary to consider government subsidies for different objects and who will carry out the promotion. Different objects of subsidies and different propagandists can have different effects on the recycling of waste products.

![Game diagram of single-channel recycling with government subsidies and propaganda.](image_url)
2.2. Notation Descriptions

Table 1 gives all the Notation and Definition, as shown in Table 1.

| Symbol | Definition |
|--------|------------|
| EN     | The model of propagating by processors, government-subsidized processors; |
| EO     | The model of propagating by processors, government-subsidized recyclers; |
| EP     | The model of propagating by recyclers, government-subsidized processors; |
| EQ     | The model of propagating by recyclers, government-subsidized recyclers; |
| VC     | Government-subsidized processors; |
| VR     | Government-subsidized recyclers; |
| A      | The profit obtained by processors from the disposal of the unit waste product; |
| v      | The subsidy amount of the unit waste product when the government determines the subsidy strategy; |
| P_{ij} | The unit recycling price of the waste products when the government determines strategy \( i, i \in \{N, VC, VR\}; \) |
| W_{ij} | The unit-consigned recycling price of waste products when the government determines strategy \( i, i \in \{N, VC, VR\}; \) |
| \tau_{ij} | The unit propaganda of recycling waste products when the government determines strategy \( i, i \in \{N, VC, VR\}; \) |
| k      | The residents’ sensitivity to expenses of propagating waste products; |
| k^2 \tau^2_{ij} | The expenses of propagating waste products in single-channel recycling when the government determines strategy \( i, i \in \{N, VC, VR\}; \) |
| q_{ij} | The recycling volume of waste products when the government determines strategy \( i, i \in \{N, VC, VR\}; \) |
| \pi_{id} | The processors’ profits when the government determines strategy \( i, i \in \{N, VC, VR\}; \) |
| \pi_{ij} | The recyclers’ profits when the government determines strategy \( i, i \in \{N, VC, VR\}; \) |
| Q      | The recycling number of waste products when the unit recycling price of waste products is 0; |
| \alpha | The residents’ sensitivity to the unit recycling price of waste products; |
| \beta  | The residents’ sensitivity to the unit propaganda of recycling waste products; |

2.3. Model Function

According to the model symbols and referring to the literature [6,16], the quantity of recycling in a single channel can be obtained as follows:

\[ q_{ij} = Q + \alpha p_{ij} + \beta \tau_{ij}, i \in \{N, VC, VR\} \]

2.4. The Sequence of Decision Making

Firstly, we can list the profit functions of recyclers and processors in different subsidy and propaganda modes and decide the unit-consigned recycling price of waste products based on profit functions. Secondly, based on the obtained unit-consigned recycling price of waste products, the unit recycling price of waste products can be determined. Finally, based on the above conclusions, the propaganda of waste products disseminated by recyclers or processors can be determined.

According to the reverse solution of the game theory, the propaganda of waste products disseminated by recyclers or processors in different models can be first determined when we make decisions. Secondly, the unit recycling price of waste products can be set based on propaganda. Finally, we can determine the unit-consigned recycling price of waste products.

3. Model Analysis

3.1. The Model Function of Recycling in a Single Channel

(1) EN:

\[ \pi_{VCd} = (A - w_{VC} + v)(Q + \alpha p_{VCj} + \beta \tau_{VCj}) - \frac{k}{2} \tau^2_{VCj} \] (1)

\[ \pi_{VCj} = (w_{VC} - p_{VCj})(Q + \alpha p_{VCj} + \beta \tau_{VCj}) \] (2)

(2) EO:

\[ \pi_{VRd} = (A - w_{VR})(Q + \alpha p_{VRj} + \beta \tau_{VRj}) - \frac{k}{2} \tau^2_{VRj} \] (3)
The above functions are the profit functions of recyclers and processors in single-channel recycling with different subsidy and publicity modes. By solving the above functions, the optimal solutions of the recycling number, the unit-consigned recycling price, and the unit recycling price of waste products in these four models can be obtained; the details are as follows:

\[ w^*_V = (A + v)\alpha - Q - \beta \tau_{VCj} \]
\[ w^*_V = (A - v)\alpha - Q - \beta \tau_{VRj} \]
\[ P^*_V = P^*_{VCEP} = \frac{(A + v)\alpha - 3Q - 3\beta \tau_{VCj}}{4\alpha} \]
\[ P^*_V = P^*_{VCEO} = \frac{(A + v)\alpha - 3Q - 3\beta \tau_{VRj}}{4\alpha} \]
\[ q^*_V = q^*_{VCEP} = \frac{(A + v)\alpha + Q + \beta \tau_{VCj}}{4} \]
\[ q^*_V = q^*_{VCEO} = \frac{(A + v)\alpha + Q + \beta \tau_{VRj}}{4} \]

According to the optimal solution obtained above, a comparative analysis of it can draw the following conclusions:

**Conclusion 1.** The influence of different propagandists on the optimal solution in single-channel recycling is as follows:

(i) \( w^*_V, p^*_V, q^*_V \) in the EN model are the same as those in the EP model;
(ii) \( w^*_V, p^*_V, q^*_V \) in the EN model are the same as those in the EP model.

Conclusion 1 shows that no matter who the propaganda party is, as long as the government subsidizes the same party, the optimal solutions of the unit-consigned recycling price, unit recycling price, and recycling quantity of waste products in single-channel recycling are consistent.

**Conclusion 2.** According to the above conclusions and the article [7], when propaganda is not considered, the government subsidies for different objects will not affect the unit recycling quantity and the recycling price of waste products. When we consider the propaganda, whether the government’s selection of different subsidy objects affects the quantity and unit recycling price depends on whether the propaganda is the same.

**Management Implication 1.** If the objects of government subsidies are the same, the different propagandists will not change the corresponding function of the optimal solution of the consigned recycling price, unit recycling price, and recycling quantity of waste products in single-channel recycling.

The obtained optimal solutions of the recycling quantity, unit-consigned recycling price, and unit recycling price of waste products in single-channel recycling are added into profit functions of recyclers and processors in different modes. We can obtain the following profit functions about propaganda:

\[ \pi^*_V = \frac{[\alpha + Q + \beta \tau_{VCj}]^2}{8\alpha} - \frac{k}{2} \tau_{VCj} \]
\[ \pi^*_V = \frac{[\alpha + Q + \beta \tau_{VRj}]^2}{16\alpha} \]
\[
\pi_{VRd}^* = \frac{[(A + v)\alpha + Q + \beta \tau_{VRj}]^2}{8\alpha} - \frac{k}{2} \tau_{VRj}^2
\]

(11)

\[
\pi_{VRj}^* = \frac{[(A + v)\alpha + Q + \beta \tau_{VRj}]^2}{16\alpha}
\]

(12)

\[
\pi_{VCd}^* = \frac{[(A + v)\alpha + Q + \beta \tau_{VCj}]^2}{8\alpha}
\]

(13)

\[
\pi_{VCj}^* = \frac{[(A + v)\alpha + Q + \beta \tau_{VCj}]^2}{16\alpha} - \frac{k}{2} \tau_{VCj}^2
\]

(14)

\[
\pi_{VRd}^* = \frac{[(A + v)\alpha + Q + \beta \tau_{VRj}]^2}{8\alpha}
\]

(15)

\[
\pi_{VRj}^* = \frac{[(A + v)\alpha + Q + \beta \tau_{VRj}]^2}{16\alpha} - \frac{k}{2} \tau_{VRj}^2
\]

(16)

When studying the impact of different propagandists on the recycling of waste products, it needs to be carried out with the following hypothesis:

**Hypothesis 1.** The value of propaganda should be greater than 0. Otherwise, it is meaningless to analyze the impact of different propagandists on the recycling of waste products. Therefore, \( \tau_j > 0 \).

**Lemma 1.** (i). When \( \beta^2 < 4\alpha \), Equations (9) and (11) are concave functions of \( \tau_{VCj} \) and \( \tau_{VRj} \); otherwise, when \( \beta^2 > 4\alpha \), Equations (9) and (11) are convex functions of \( \tau_{VCj} \) and \( \tau_{VRj} \). Equations (10) and (12) are increasing functions of \( \tau_{VCj} \) and \( \tau_{VRj} \). Meanwhile, according to Equations (9)–(12), the optimal solutions of \( \tau_{VCj}^* \), \( \tau_{VRj}^* \), \( \pi_{VCj}^* \), \( \pi_{VRj}^* \) can be obtained.

(ii). Equations (13) and (15) are increasing functions of \( \tau_{VCj} \) and \( \tau_{VRj} \). When \( \beta^2 < 8\alpha \), Equations (14) and (16) are concave functions of \( \tau_{VCj} \) and \( \tau_{VRj} \); otherwise, when \( \beta^2 > 8\alpha \), Equations (14) and (16) are convex functions of \( \tau_{VCj} \) and \( \tau_{VRj} \). According to Equations (13)–(16), the optimal solutions of \( \tau_{VCj}^* \), \( \tau_{VRj}^* \), \( \pi_{VCj}^* \), \( \pi_{VRj}^* \) can be obtained.

See Appendix A for the proof of Lemma 1. According to Lemma 1, we can obtain the optimal solutions of the value of propaganda in different models, as shown in Conclusion 3.

**Conclusion 3.** The optimal solutions of propaganda in different models are shown in Table 2:

| EN | When \( \beta^2 < 4\alpha \), \( \pi_{VCd} \rightarrow \max \), then \( \tau_{VCj}^* = \frac{\beta[(A + v)\alpha + Q]}{4\alpha - \beta^2} \) |
|----|----------------------------------------------------------------------------------------------------------------------------------|
|    | When \( \beta^2 > 4\alpha \), \( \pi_{VCd} \rightarrow \max \), then \( \tau_{VCj}^* = 0 \) or \( \tau_{VCj}^* \rightarrow \max \) |
|    | When \( \pi_{VCd} \rightarrow \max \), then \( \tau_{VCj}^* \rightarrow \max \) |
| EO | When \( \beta^2 < 4\alpha \), \( \pi_{VRd} \rightarrow \max \), then \( \tau_{VRj}^* = \frac{\beta[(A + v)\alpha + Q]}{4\alpha - \beta^2} \) |
|    | When \( \beta^2 > 4\alpha \), \( \pi_{VRd} \rightarrow \max \), then \( \tau_{VRj}^* = 0 \) or \( \tau_{VRj}^* \rightarrow \max \) |
|    | When \( \pi_{VRd} \rightarrow \max \), then \( \tau_{VRj}^* \rightarrow \max \) |
| EP | When \( \beta^2 < 8\alpha \), \( \pi_{VCj} \rightarrow \max \), then \( \tau_{VCj}^* = \frac{\beta[(A + v)\alpha + Q]}{8\alpha - \beta^2} \) |
|    | When \( \beta^2 > 8\alpha \), \( \pi_{VCj} \rightarrow \max \), then \( \tau_{VCj}^* = 0 \) or \( \tau_{VCj}^* \rightarrow \max \) |
|    | When \( \pi_{VCd} \rightarrow \max \), then \( \tau_{VCj}^* \rightarrow \max \) |
| EQ | When \( \beta^2 < 8\alpha \), \( \pi_{VRj} \rightarrow \max \), then \( \tau_{VRj}^* = \frac{\beta[(A + v)\alpha + Q]}{8\alpha - \beta^2} \) |
|    | When \( \beta^2 > 8\alpha \), \( \pi_{VRj} \rightarrow \max \), then \( \tau_{VRj}^* = 0 \) or \( \tau_{VRj}^* \rightarrow \max \) |
3.2. Analysis of Recycling Model in Single Channel
3.2.1. Impact of Government Subsidies on Waste Product Recycling

When the government selects different subsidy objects, the influence of residents’ green consciousness on recycling channels for waste products can be seen in Conclusion 4.

**Conclusion 4.** If the government subsidizes the processors, the impact of residents’ green consciousness on the choice of recycling channels for waste products is as follows:

When \( Q < (A + v)\alpha - \beta \tau_{VC_j} \), the best decision is for the recyclers to recycle the waste products; when \( Q = (A + v)\alpha - \beta \tau_{VC_j} \), recyclers or processors have the same effect on recycling waste products; when \( Q > (A + v)\alpha - \beta \tau_{VC_j} \), the best decision is for the processors to directly recycle the waste products. In short, if the residents’ green consciousness is relatively low, the processors decide to obtain the waste products from the recyclers and then recycle them; if the residents have a high green consciousness, the processors will directly recycle the waste products.

See Appendix A for the proof of Conclusion 4. According to Conclusion 4, the impact of residents’ green consciousness on waste product recycling when the government subsidizes recyclers can be similarly obtained, as shown in the following corollary:

**Corollary 1.** If the government subsidizes the recyclers, the impact of residents’ green consciousness on the choice of recycling channels for waste products is as follows:

When \( Q < (A - v)\alpha - \beta \tau_{VR_j} \), the best decision is for the recyclers to recycle the waste products; when \( Q = (A - v)\alpha - \beta \tau_{VR_j} \), recyclers or processors have the same effect on recycling waste products; when \( Q > (A - v)\alpha - \beta \tau_{VR_j} \), the best decision is for the processors to directly recycle the waste products.

Conclusion 4 and Corollary 1 show that residents’ green consciousness can influence the choice of recycling channels. The crucial content of this article is the impact of government subsidies and propaganda on waste product recycling. Therefore, this article is based on the following hypothesis:

**Hypothesis 2.** In single-channel recycling, if the government subsidizes processors, then \( Q < (A + v)\alpha - \beta \tau_{VC_j} \); if the government subsidizes recyclers, then \( Q < (A - v)\alpha - \beta \tau_{VR_j} \).

3.2.2. Impact of Propaganda on the Recycling of Waste Products

By comparing and analyzing the optimal solutions of propaganda in the four different models, the following conclusions can be drawn:

**Conclusion 5.** According to Conclusion 2, the quantity and unit recycling price of waste products will not be affected by the different objects of government subsidies when the processors conduct propaganda; similarly, when propagated by recyclers, the government selects different subsidy objects can obtain same optimal solutions of the unit recycling price and recycling quantity of waste products.

**Conclusion 6.** When propagated by processors, the optimal solution of propaganda when processors obtain the maximum benefit is independent of the objects of government subsidies, and the optimal solution is related to \( \beta^2 - 4\alpha \); when \( \beta^2 - 4\alpha < 0 \), the optimal solution of propaganda in two models of propagating by processors is \( \frac{\beta(A + v)\alpha + Q}{4\alpha - \beta^2} \); the optimal solution of propaganda when recyclers obtain the maximum benefit is also independent of the objects of government subsidies, and it is the maximum value of \( \tau \). In the same way, when propagated by recyclers, the optimal solution of propaganda when recyclers obtain the maximum benefit is independent of the objects of government subsidies, and the optimal solution is related to \( \beta^2 - 8\alpha \); when \( \beta^2 - 8\alpha < 0 \), the optimal solution of propaganda in two models of propagating by recyclers is \( \frac{\beta(A + v)\alpha + Q}{8\alpha - \beta^2} \); the optimal solution of propaganda when processors obtain the maximum benefit is also independent of the objects of government subsidies, and it is the maximum value of \( \tau \).
**Conclusion 7.** In single-channel recycling, the optimal solutions of propaganda when processors or recyclers obtain the maximum benefit in different subsidies and propaganda models are not consistent. Moreover, according to Conclusion 6, when the processors propagate, the optimal solution of propaganda is the value corresponding to the maximum profit obtained by the processors. Similarly, when the recyclers propagate, the optimal solution of propaganda is the value corresponding to the maximum profit obtained by the recyclers.

3.2.3. Model Analysis

From the above analysis of the impact of different government subsidy objects and propagandists on the recycling of waste products, we can obtain the impact of different subsidies and propaganda models on the unit-consigned recycling price, unit recycling price, and recycling quantity of waste products in single-channel recycling. The details can be seen in the following conclusions:

**Conclusion 8.** The influence of government subsidies and propaganda on the unit-consigned recycling price of waste products is as follows:

(i) \( \frac{\partial w^*_V}{\partial v} > 0, \frac{\partial w^*_R}{\partial v} < 0; \)
(ii) \( \frac{\partial w^*_V}{\partial \tau} < 0, \frac{\partial w^*_R}{\partial \tau} < 0. \)

Similar to [3], Conclusion 8 shows that, when the government chooses different subsidy objects, with the increase in the value of propaganda, the unit-consigned recycling price will decrease. If government subsidizes recyclers, recyclers will lower the unit-consigned recycling price of waste products to gain more subsidies from the government. Recyclers should consider the sum of government subsidies and the unit-consigned recycling price of waste products, and appropriately consider raising the unit-consigned recycling price of waste products to reduce propaganda. If the government subsidizes processors, processors will raise the unit-consigned recycling price of waste products to gain more subsidies. Processors should consider the difference between government subsidies and the unit-consigned recycling price of waste products. In general, government subsidies can fully compensate for the losses caused by increasing the unit-consigned recovery price of waste products. Moreover, processors should properly consider lowering the unit-consigned recycling price of waste products and promote propaganda to motivate waste product recycling and increase their profits.

**Management Inspiration 2:** Recyclers and processors can choose to raise or lower the unit-consigned recycling price of waste products to obtain more government subsidies and expectations of propaganda. That is, no matter which party is the propagandist, if the government subsidizes the recyclers, the recyclers will lower the unit-consigned recycling price of waste products to gain more subsidies from the government. Recyclers should consider the sum of government subsidies and the unit-consigned recycling price of waste products, and appropriately consider raising the unit-consigned recycling price of waste products to reduce propaganda. If the government subsidizes processors, processors will raise the unit-consigned recycling price of waste products to gain more subsidies and lower the unit-consigned recycling price of waste products to promote propaganda.

**Conclusion 9.** The influence of government subsidies and propaganda on the unit price of waste product recycling is as follows:

(i) \( \frac{\partial \rho^*_V}{\partial \tau} > 0, \frac{\partial \rho^*_R}{\partial \tau} > 0; \)
(ii) \( \frac{\partial \rho^*_V}{\partial v} < 0, \frac{\partial \rho^*_R}{\partial v} < 0. \)

Different from [6], Conclusion 9 shows that no matter whom the government subsidizes, with the increase in propaganda, the unit price of waste products will decrease. If government subsidizes the recyclers, the recyclers will raise the unit recycling price of waste products to obtain more government subsidies. Meanwhile, to obtain more profits, the recyclers should consider the difference between the government subsidies and the
unit price of waste products. Moreover, the recyclers should properly consider lowering the unit-consigned recycling price of waste products to promote propaganda. If the government subsidizes the processors, the processors will raise the unit-consigned recycling price, which indirectly encourages the recyclers to raise the recycling price to obtain more government subsidies and reduce propaganda.

Management Inspiration 3: Recyclers and processors can raise or lower the unit recycling price of waste products to obtain more government subsidies and expectations of propaganda. That is, no matter which party is the propagandist, if the government subsidizes the recyclers, the recyclers will raise the unit recycling price to obtain more subsidies and lower the unit recycling price to reduce propaganda. If the government subsidizes the processors, the processors will raise the unit-consigned recycling price to obtain more subsidies and promote propaganda.

Conclusion 10. The influence of government subsidies and propaganda on the recycling number of waste products is as follows:

(i) \( \frac{\partial q^*_{VCj}}{\partial v} > 0, \frac{\partial q^*_{VRj}}{\partial v} > 0; \)
(ii) \( \frac{\partial q^*_{VCj}}{\partial \tau_{VCj}} > 0, \frac{\partial q^*_{VRj}}{\partial \tau_{VRj}} > 0. \)

Conclusion 10 shows that no matter whom the government subsidizes, the recycling number of waste products will increase with the increase in propaganda. If government subsidizes recyclers, recyclers will raise the unit price of waste product recycling and increase propaganda to increase the recycling number in order to obtain more government subsidies. If the government subsidizes processors, processors will increase the consigned recycling price and increase propaganda, as in the case of recyclers, to increase the recycling quantity in order to obtain more government subsidies.

Management Inspiration 4: Both recyclers and processors can obtain more government subsidies by increasing the recycling quantity of waste products. That is, no matter which party propagates and which party is subsidized by the government, the recyclers and processors will increase the propaganda and the recycling quantity of waste products.

4. Numerical Analysis

In order to further analyze the influence of different objects of government subsidies and different propagandists on the recycling of waste products, this article takes Bora 2002 cars with 1.8 L emissions produced by FAW-Volkswagen as an example of the data analysis. By investigating enterprises, we set \( A = 4000, \alpha = 3, \beta = 1, k = 0.5 \), and, in this case, \( \beta^2 - 4k\alpha < 0 \) and \( \beta^2 - 8k\alpha < 0 \). The specific analysis is as follows:

4.1. The Impact of Q on \( \tau_{ij} \) in Single-Channel Recycling

As can be seen from Figure 2, the value of propaganda corresponding to the promotion carried out by the processors is higher than that corresponding to the promotion carried out by the recyclers. The value of propaganda has nothing to do with the different objects of government subsidies. No matter which party conducts propaganda, the value of the propaganda increases with the increase in recycling quantity. Therefore, recyclers and processors can increase the recycling quantity of waste products by increasing propaganda. In addition, from the steepness of the two curves in the figure, it can be seen that when the recycling quantity of waste products increases to a certain extent, the value of propaganda disseminated by the processors is higher than that disseminated by the recyclers. From the linear relationship between Q and \( \tau_{ij} \) in the figure and the optimal solution of \( w_i \) above, we can further analyze the impact of Q on \( w_i \) in single-channel recycling. The specific analysis is as follows.
4.2. The Impact of $Q$ on $w_i$ in Single-Channel Recycling

As can be seen from Figure 3, the unit-consigned recycling price of waste products is the maximum when the recyclers conduct propaganda and the government subsidizes the processors. It is the second largest when the processors conduct propaganda and the government subsidizes the processors. When the recyclers conduct propaganda and the government subsidizes the recyclers, it is the third largest. When the processors conduct propaganda and the government subsidizes the recyclers, the unit-consigned recycling price is the lowest.

Figure 2. The impact of $Q$ on $\tau_{ij}$.

Figure 3. The impact of $Q$ on $w_i$. 
According to the case of the Bora 2002 cars, when the recyclers conduct propaganda and the government subsidizes the processors, based on residents’ green awareness, if the recycling quantity of waste products is less than 12,000, the processors will recycle the waste products from the recyclers; if the recycling quantity of waste products exceeds 12,000, the unit-consigned recycling price is lower than 0, and the processors will directly recycle the waste products. Similarly, we can determine the boundary value of the unit-consigned recycling price in other models of different recycling channels. In short, the boundary value of the unit-consigned recycling price is different in different recycling models; that is, the unit-consigned recycling price is the largest when propagation is carried out by the recyclers and the government subsidizes the processors, and it is the second largest when processors conduct propaganda and the government subsidizes processors. When recyclers conduct propaganda and the government subsidizes recyclers, the unit-consigned recycling price is the third largest; when propagation is carried out by recyclers and government subsidizes the recyclers, the unit-consigned recycling price is the lowest.

4.3. The Effect of Government Subsidies and Propaganda on Profits in Single-Channel Recycling

Figure 4 demonstrates the influence of propaganda and government subsidies on the profit of recyclers in single-channel recycling. Figure 5 demonstrates the influence of propaganda and government subsidies on the profit of processors in single-channel recycling. Figures 4 and 5 show that both government subsidies and propaganda can promote the benefits of processors and recyclers, and the superposition works better than when these two aspects are used alone. When the propagandist is determined, the different objects of government subsidies will not have an impact on the benefits of the processors and recyclers. When the processors propagate, the processors’ benefit is lower than that when the recyclers propagate; the recyclers’ benefit is higher than that when the recyclers propagate. From the above analysis, Corollary 2 can be drawn.

**Corollary 2.** The influence of government subsidies and publicity on profits of recyclers and processors in single-channel recycling:

(i) \( \frac{\partial \pi_{VCd}}{\partial v} > 0, \frac{\partial \pi_{VCj}}{\partial v} > 0, \frac{\partial \pi_{VCd}}{\partial \tau} > 0, \frac{\partial \pi_{VCj}}{\partial \tau} > 0; \)

(ii) \( \frac{\partial \pi_{VRd}}{\partial v} > 0, \frac{\partial \pi_{VRj}}{\partial v} > 0, \frac{\partial \pi_{VRd}}{\partial \tau} > 0, \frac{\partial \pi_{VRj}}{\partial \tau} > 0. \)
5. Conclusions

To study the influence of different government subsidy objects and propagandists on the recycling of waste products in a single channel, this article establishes four game models of recycling in a single channel. Firstly, it analyzes how government subsidies affect waste product recycling and then analyzes how propaganda affects it. This study mainly draws the following conclusions:

(1) In the case of single-channel recycling, the optimal solutions of the unit-consigned recycling price, unit recycling price, and recycling quantity of waste products obtained when the government subsidizes the same object will not be affected by different propaganda parties. If the propagandist is certain, the unit recycling price and recycling quantity of waste products will not be affected by the different objects of government subsidies.

(2) In the case of single-channel recycling, if the propagandist is certain, the optimal solution of propaganda when processors and recyclers obtain the maximum benefits is independent of the objects of government subsidies, and it is related to residents’ sensitivity to the unit recycling price, propaganda, and the expenses of propagating waste products. For example, when propagated by recyclers, if \( \beta^2 - 8k\alpha < 0 \), the optimal solution of propaganda in two models is \( \beta(A + \pi_i) \); when propagated by processors, if \( \beta^2 - 4k\alpha < 0 \), the optimal solution of propaganda in two models is \( \beta(A + \pi_i) \).

From these conclusions, the following implications can be drawn:

(1) If the objects of government subsidies are the same, the different propagandists will not change the corresponding function of the optimal solution of the consigned recycling price, unit recycling price, and recycling quantity of waste products in single-channel recycling.

(2) Recyclers and processors can raise or lower the unit-consigned recycling price of waste products to obtain more government subsidies and expectations of propaganda. Recyclers and processors can raise or lower the unit recycling price of waste products to obtain more government subsidies and expectations of propaganda.
Both recyclers and processors can obtain more government subsidies by increasing the recycling quantity of waste products. No matter which party propagates and which party is subsidized by the government, the recyclers and processors will increase the propaganda and the recycling quantity of waste products.

The following research can be conducted on the basis of this study: (1) analysis of the impact of subsidizing traditional propaganda channels and online propaganda channels on recycling channels; (2) analysis of the impact of subsidy recycling propaganda on dual-channel recycling; and (3) analysis of the impact of consumers’ willingness to recycle on recycling subsidies. This article only uses the game model to analyze the impact of subsidies on single-channel recycling based on recycling propaganda. In the future, real data should be further used for supplementary analysis, combined with the Delphi method, case studies, and other research methods to achieve further improvements.

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**Appendix A. Proof of Lemma and Propositions**

**Proof of Lemma 1.** For Equation (9), calculate the first-order and second-order partial derivatives of $\tau_{VCj}$:

$$\frac{\partial \pi_{VCj}}{\partial \tau_{VCj}} = \frac{\beta [(A + v)\alpha + Q + \beta \tau_{VCj}]}{4\alpha} - k\tau_{VCj}$$

$$\frac{\partial^2 \pi_{VCj}}{\partial \tau_{VCj}^2} = \frac{\beta^2}{4\alpha} - k$$

when $\frac{\partial^2 \pi_{VCj}}{\partial \tau_{VCj}^2} = \frac{\beta^2}{4\alpha} - k$, namely, $\beta^2 < 4k\alpha$, Equation (9) is a concave function. Otherwise, it is a convex function.

The first-order partial derivatives of Equation (10) concerning $\tau_{VCj}$:

$$\frac{\partial \pi_{VCj}}{\partial \tau_{VCj}} = \frac{\beta [(A + v)\alpha + Q + \beta \tau_{VCj}]}{8\alpha} > 0$$

Thus, Equation (10) is an increasing function concerning $\tau_{VCj}$.

The proof of Conclusion 2 is completed.

Similarly, Conclusion 3 could be proven.

**Proof of Conclusion 4.** According to the optimal solutions of the above different models, $w_{VC}^* > 0$ indicates that the unit-consigned recycling price of waste products is greater than 0, which means the processors are willing to obtain the waste products from the recyclers to recycle them. $w_{VC}^* < 0$ indicates that the unit-consigned recycling price is less than 0; that is, the processors are not willing to obtain waste products from recyclers to recycle them.

The proof of Conclusion 4 is complete.
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