Goldilocks Supersymmetry: Simultaneous Solution to the Dark Matter and Flavor Problems of Supersymmetry

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Neutralino dark matter is well motivated, but also suffers from two shortcomings: it requires gravity-mediated supersymmetry breaking, which generically violates flavor constraints, and its thermal relic density \(\Omega\) is typically too large. We propose a simple solution to both problems: neutralinos freezeout with \(\Omega \sim 10^{-100}\), but then decay to \(\sim 1\) GeV gravitinos, which are simultaneously light enough to satisfy flavor constraints and heavy enough to be all of dark matter. This scenario is naturally realized in high-scale gauge-mediation models, ameliorates small scale structure problems, and implies that “cosmologically excluded” models may, in fact, be cosmologically preferred.

Supersymmetric extensions of the standard model of particle physics are among the prime candidates for new microphysics. Among their many virtues, supersymmetric models naturally predict new particles that are candidates for dark matter. The most well studied of these are thermal relic neutralinos \(\tilde{\chi}\), superpartners of the Higgs and electroweak gauge bosons. The thermal relic density of neutralinos is dependent on unknown supersymmetry parameters. However, order-of-magnitude estimates yield relic densities that are consistent with

\[
\Omega_{\text{DM}} h^2 = 0.1050^{+0.0041}_{-0.0040} \, (1\sigma) ,
\]

where \(\Omega_{\text{DM}}\) is the observed energy density of non-baryonic dark matter in units of the critical density, and \(h \simeq 0.73\) is the normalized Hubble parameter. This remarkable fact has not only motivated supersymmetry, but has also focused attention on “cosmologically preferred” models, in which the neutralino thermal relic density is exactly that required for dark matter. Such studies have implications for a large range of experiments, from direct and indirect dark matter searches to those at the Large Hadron Collider (LHC) at CERN.

The neutralino dark matter scenario is not without its blemishes, however. First, for the neutralino to be stable, it must be the lightest supersymmetric particle (LSP). In particular, it must be lighter than the gravitino. This requires gravity-mediated supersymmetry breaking models, in which low energy bounds on flavor and CP violation are generically violated by several orders of magnitude. Gauge-mediated supersymmetry breaking (GMSB) models\(^\text{\[8\]}\) elegantly avoid these constraints, but such models have gravitino LSPs and so are incompatible with neutralino dark matter.

Second, although general arguments imply that the neutralino thermal relic density is of the right order of magnitude, in concrete models, it is often too large: Neutralinos are Majorana fermions, and so annihilation to quarks and leptons is \(P\)-wave suppressed. In addition, gauge coupling unification and radiative electroweak symmetry breaking typically imply Bino-like neutralinos, which suppresses annihilation to gauge and Higgs bosons. These effects together enhance relic densities to values that may far exceed those given in Eq. (1).

These two shortcomings of neutralino dark matter are usually considered unrelated and addressed separately. One may, for example, consider gravity-mediated scenarios, such as minimal supergravity, where low energy constraints are satisfied by unification assumptions. One then further focuses on special regions of parameter space in which the neutralino relic density is reduced to acceptable levels through, for example, resonant annihilation\(^\text{\[4\]}\), stau co-annihilation\(^\text{\[3\]}\), or significant Higgsino mixing\(^\text{\[6\]}\). Alternatively, one may simply abandon the hope that the order-of-magnitude correctness of the neutralino thermal relic density is a significant lead in the hunt for dark matter and explore other mechanisms for dark matter production. For example, one may consider GMSB models with thermally produced gravitinos\(^\text{\[8\]}\). (Note, however, that recent Lyman-\(\alpha\) constraints requiring \(m_{\tilde{G}} \gtrsim 2\) keV\(^\text{\[8\]}\) imply that the gravitino thermal relic density \(\Omega_{\tilde{G}} h^2 \approx 1.2\, (m_{\tilde{G}}/\text{keV})\) must be significantly diluted through late entropy production\(^\text{\[9\]}\) for this possibility to be viable.) More recently, GMSB-like models with gravitino dark matter produced by late decaying gauge singlets have also been proposed\(^\text{\[10\]}\).

In this work, we consider the possibility that the two shortcomings described above are not separate issues, but are in fact pointing to a single resolution. We propose that neutralinos do, in fact, freezeout with very large densities. However, they then decay to gravitinos, which are light enough to accommodate the GMSB solution to the flavor and CP problems, but heavy enough to be all of dark matter. In analogy to Goldilocks planets, which have temperatures that lie within the narrow window required to support life, these supersymmetric models have gravitino masses in the narrow window required to satisfy both particle physics and cosmological constraints, and so we call this “Goldilocks Supersymmetry.”

The essential features of this scenario may be illustrated by simple scaling arguments. Consider models

\[
\Omega_{\text{DM}} h^2 = \frac{\sigma v}{16\pi^2 m_\chi} = \left( \frac{\sigma v}{16\pi^2 m_\chi} \right) \left( \frac{m_\chi}{M_{\text{pl}}} \right)^2 ,
\]

where \(M_{\text{pl}}\) is the Planck mass, \(\sigma v\) is the annihilation cross section times the thermal velocity, and \(m_\chi\) is the neutralino mass. The relic density is determined by the relative strengths of \(\sigma v\), \(m_\chi\), and \(M_{\text{pl}}\). For example, in minimal supergravity, \(\sigma v\) is suppressed by the strong coupling constant, \(\alpha_s\), while in GMSB, \(\sigma v\) is enhanced by the weak coupling constant, \(\alpha\). In both cases, \(m_\chi\) is given by the superpartner mass scale, \(M\). The predicted relic density is then

\[
\Omega_{\text{DM}} h^2 = \left( \frac{\sigma v}{16\pi^2 m_\chi} \right) \left( \frac{m_\chi}{M_{\text{pl}}} \right)^2 = \left( \frac{\sigma v}{16\pi^2 m_\chi} \right) \left( \frac{m_\chi}{M_{\text{pl}}} \right)^2 ,
\]

where \(H\) is the Hubble parameter. This result is consistent with the observed relic density, \(\Omega_{\text{DM}} h^2 = 0.1050^{+0.0041}_{-0.0040}\, (1\sigma)\), provided that \(\sigma v / M_{\text{pl}}^2 \approx 10^{-10}\).
in which there are two mass scales: the scale of the standard model superpartner masses $\tilde{m}$, and the gravitino mass $m_{\tilde{G}}$. The freezeout density of neutralinos is inversely proportional to the gravitino annihilation cross section, and so by dimensional analysis, $\Omega_\chi h^2 \sim \langle \sigma v \rangle^{-1} \sim \tilde{m}^{-2}$. The gravitino relic density is therefore $\Omega_{\tilde{G}} h^2 = (m_{\tilde{G}}/\tilde{m}) \Omega_\chi h^2 \sim m_{\tilde{G}} \tilde{m}$. At the same time, a natural solution to the supersymmetric flavor and CP problems requires $m_{\tilde{G}} \ll \tilde{m}$. We find, then, that we can always make $\Omega_{\tilde{G}}$ large enough to explain dark matter by raising $m_{\tilde{G}}$ and $\tilde{m}$ together with their ratio fixed. The essential question, then, is whether the scenario may be realized with $\tilde{m} \lesssim \text{TeV}$, as required for a natural solution to the gauge hierarchy problem, and whether it passes all other particle physics and astrophysical constraints.

To analyze this question concretely, we consider the example of minimal GMSB models [11]. Such models are specified by the 4+1 parameters $M_m, \Lambda, N_m, \tan \beta$, and $\text{sign}(\mu)$, where $M_m$ is the messenger mass, $\Lambda = F/M_m$, where $F$ is the supersymmetry breaking scale in the messenger sector, $N_m$ is the number of $SU(5)$ messenger pairs, $\tan \beta = \langle H_u^0 \rangle/\langle H_d^0 \rangle$, and $\mu$ is the supersymmetric Higgsino mass. In terms of these parameters, the gauge-mediated contributions to squark and slepton masses are

$$m_{\tilde{f}}^2 (M_m) = 2 N_m \Lambda^2 \sum_{i=1}^{3} C_i^f \left( \frac{g_i^2 (M_m)}{16 \pi^2} \right)^2,$$

where $C_i^f = \frac{5}{3} Y^2$, with hypercharge $Y = Q - T_3$, and $C_i^f = 0$ for gauge singlets, $\frac{1}{2}$ for $SU(2)_L$ doublets, and $\frac{2}{3}$ for $SU(3)_C$ triplets. The gaugino masses are

$$M_i (M_m) = N_m \Lambda C_i \left( \frac{g_i^2 (M_m)}{16 \pi^2} \right),$$

where $i = 1, 2, 3$ for the $U(1)_Y, SU(2)_L,$ and $SU(3)_C$ groups, $c_1 = \frac{5}{3},$ and $c_2 = c_3 = 1$. As indicated, these masses are generated at the energy scale $M_m$. We determine physical masses through renormalization group evolution to the weak scale and radiative electroweak symmetry breaking with SoftSUSY 2.0 [12].

In addition to the gauge-mediated masses, there are gravity-mediated contributions. These generate the gravitino mass $m_{\tilde{G}} = \sqrt{3} M_*$, where $F_0$ is the total supersymmetry breaking scale and $M_* \approx 2.4 \times 10^{18}$ GeV is the reduced Planck mass. Because $F_0$ receives contributions from all supersymmetry breaking $F$-terms, $F_0 \geq F$. For direct gauge mediation, $F_0 \sim F$, but this is model-dependent. Here, we assume $F_0 = F$, and so

$$m_{\tilde{G}} = \frac{F}{\sqrt{3} M_*} = \frac{M_m A}{\sqrt{3} M_*}.$$

Our results are not changed significantly for $F_0 > F$.

Gravity-mediation also generates flavor- and CP-violating squark and slepton mass parameters $(m_{ij}^{AB})_{AB}$, where $i, j = 1, 2, 3$ label generation, $A, B = L, R$ label chirality, and $f = l, u, d$. The chirality-preserving parameters are naturally $\sim m_{\tilde{G}}$: for concreteness, we assume $|m_{ij}^{AB}| \sim |m_{ij}^{A'B'}| \sim m_{\tilde{G}}$. The chirality-violating masses require the breaking of electroweak gauge symmetry (and possibly horizontal symmetries); we assume $|m_{ij}^{A'B'}| \ll m_{\tilde{G}}$, where the $m_{ij}^{A'B'}$ are Yukawa couplings. Finally, we assume $O(1)$ CP-violating phases for both the gravity- and gauge-mediated masses, as detailed below.

Given these assumptions, the most stringent constraints are the flavor-changing observables $\Delta m_K$ and $\epsilon_K$, and the CP-violating, but flavor-preserving, electron and neutron electric dipole moments (EDMs) [13, 14, 15]:

$$\Delta m_K^{\text{SUSY}} < 3.5 \times 10^{-12} \text{ MeV}$$

$$\epsilon_K^{\text{SUSY}} < 2.3 \times 10^{-3}$$

$$d_e < 1.6 \times 10^{-27} \text{ e cm}$$

$$d_n < 2.9 \times 10^{-26} \text{ e cm}$$.

In the mass insertion approximation, these constrain $(\delta_{ij}^{\text{AB}})_{AB} \approx (m_{ij}^{AB})_{AB}/\tilde{m}_j$, where $\tilde{m}_j$ is an average $\tilde{m}$ mass. The leading constraints are from $\Delta m_K$ on $\text{Re} [\delta_{12}^{dL} L L (\delta_{ij}^{dL})_{RR}], \text{Im} [\delta_{12}^{dL} L L (\delta_{ij}^{dL})_{RR}],$ and from the EDMs on the gauge-mediated masses.

The supersymmetric contributions to the kaon observables are $\Delta m_K^{\text{SUSY}} = \text{Re}(M)$ and $\epsilon_K^{\text{SUSY}} = \text{Im}(M)/(\sqrt{3} \Delta m_K^{\text{SUSY}})$, with $M$ as given in Ref. [16]. For concreteness, we choose the $\delta$ phases to maximize the supersymmetric contribution for each kaon observable. The constraints from $\Delta m_K$ and $\epsilon_K$ are therefore not simultaneously applicable, but the most stringent constraint smoothly interpolates between these as the phase varies. For the EDMs, we first use micrOMEGAs 1.3.7 [17] to determine the supersymmetric contribution to $\alpha_{\mu}$, the anomalous magnetic moment of the muon. The EDMs are then,

$$d_e = \frac{\alpha_{\mu} \alpha_{\tau}}{\alpha_{\tau}} \tan \theta_{\text{CP}}$$

$$d_n = \frac{\alpha_{\mu} \alpha_{\tau}}{\alpha_{\tau}} \tan \theta_{\text{CP}}$$,

where $d_e$ and $d_n$ are determined from $d_e$ with $\alpha \rightarrow \alpha_s, M_1 \rightarrow M_3, m_1 \rightarrow m_{2,3,6}$, and the introduction of appropriate color factors [18]. We set $\tan \theta_{\text{CP}} = 1$ in the EDMs. Note that the EDMs may be suppressed, depending, for example, on the origin of the $\mu$ and $B$ parameters.

The resulting constraints are given in Fig. 1. The observables $\Delta m_K$ and $\epsilon_K$ require $m_{\tilde{G}} \lesssim 30 \text{ GeV}$ (500 GeV) for neutralino mass $m_\chi \sim 100 \text{ GeV}$ (1 TeV). In contrast, the EDMs are insensitive to $m_{\tilde{G}}$, since they do not rely on gravity-mediated contributions. They are found to require $m_\chi \gtrsim 1 \text{ TeV}$, in agreement with earlier work [18]. These results are, of course, subject to the assumptions we have made. However, they imply that in any model in which gravity-mediated contributions are at their natural scale and all mass parameters have $O(1)$ phases, the standard model superpartners must be heavy, and the LSP is the gravitino, not the neutralino.

For $N_m = 1$, the lightest standard model superpartner is the lightest neutralino $\chi$ throughout parameter space. In Fig. 1, we also show the freezeout density $\Omega_\chi h^2$, that
is the relic density if neutralinos were stable, determined using micrOMEGAs [17]. These results illustrate the difficulties for neutralino dark matter. At the weak scale, typically $\mu, M_2 > M_1$, and $\chi$ is Bino-like. Its annihilation is therefore suppressed for the reasons noted above. For $m_\chi = 100$ GeV, $\Omega_\chi h^2 \sim 1$ is already far too large, and for the heavier superpartner masses favored by the EDM constraints, it grows to values of $\sim 10 - 100$.

In the scenario proposed here, however, neutralinos are not stable, but decay to gravitinos. The resulting gravitino relic density is given in Fig. 2. In the dark green shaded region, $\Omega_\chi h^2$ is in the range required to account for all of non-baryonic dark matter. We see that parts of this shaded region are consistent with low energy flavor and CP constraints. In this scenario, very large neutralino freezeout densities are a virtue, not a problem, as they allow light gravitinos to have the required relic density, despite the significant dilution factor $m_\tilde{G}/m_\chi$. In this simple example of minimal GMSB, the Goldilocks window, in which both relic density and low energy constraints are satisfied, has $m_\tilde{G} \sim 1 - 10$ GeV.

So far, we have considered constraints from particle physics and $\Omega_{DM}$. We now turn to astrophysical constraints. In the preferred band, the gravitino is light and dominantly couples through its Goldstino components. The neutralino decay widths are $\Gamma(\chi \rightarrow \gamma G) = (\cos^2 \theta_W/48\pi)(m_\chi^2/m_\tilde{G}^2 M_2^2)$ and $\Gamma(\chi \rightarrow Z G) = (\sin^2 \theta_W/48\pi)(m_\chi^2/m_\tilde{G}^2 M_2^2)[1 - (m_2^2/m_\chi^2)]^4$. As shown in Fig. 3, these imply lifetimes $\tau \gtrsim 0.01$ s in the preferred band. Such late decays are constrained by entropy production, $\mu$ distortions of the cosmic microwave background, Big Bang nucleosynthesis (BBN) [19], and small scale structure [20]. We find that the last two are most stringent, and so focus on them here.

Standard BBN agrees reasonably well with observations. This agreement constrains electromagnetic (EM) and hadronic energy release in late decays, which may be parameterized by $\xi_i \equiv e_i B_i Y_\chi$, where $i = EM, had$, $e_i$ is the EM/hadronic energy released in each neutralino decay, $B_i$ is the branching fraction into EM/hadronic components, and $Y_\chi \equiv n_\chi/n_\gamma^{BG}$, where $n_\gamma^{BG} = 2\zeta(3)T_3^3/\pi^2$. We have determined the $\xi_i$ following the prescription of Refs. [21] and compared them to the constraints given in

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FIG. 1: Neutralino thermal relic density $\Omega_\chi h^2$ in the $(m_\tilde{G}, \Lambda)$ plane, for $N_m = 1$, $\tan \beta = 10$, $\mu > 0$ and $m_t = 175$ GeV. The right-hand axis gives the neutralino mass $m_\chi \approx 1.3 \times 10^{-3}\Lambda$. Regions to the right of the $\epsilon_k$ and $\Delta m_K$ contours and below the $d_e$ and $d_\nu$ contours are disfavored. The neutralino is the LSP in the shaded region.

FIG. 2: Contours of $\Omega_\chi h^2$ in the $(m_\tilde{G}, \Lambda)$ plane. The thick contour is the 2$\sigma$ allowed region. Low-energy constraints and fixed GMSB parameters are as in Fig. 1.

FIG. 3: Contours of $\lambda_{FS}$ (solid) and lifetime $\tau(\chi \rightarrow \tilde{G})$ (dotted) in the $(m_\tilde{G}, \Lambda)$ plane, for $N_m = 1$, $\tan \beta = 10$, $\mu > 0$, and top quark mass $m_t = 175$ GeV. In the light yellow (medium blue) shaded region, hadronic (electromagnetic) showers from $\chi$ decays produce discrepancies with BBN observations. The band with the correct $\Omega_\chi h^2$ is as in Fig. 2 and the neutralino LSP region and fixed GMSB parameters are as in Fig. 1.
The BBN constraints are shown in Fig. 3 and are stringent — in this scenario, neutralinos are long-lived and greatly overproduced, resulting in large energy release. In the region of parameter space with \( 0.097 < \Omega_G h^2 < 0.113 \), the EM (hadronic) constraint requires lifetimes \( \tau \lesssim 10^5 \text{ s} \) (0.1 s) and \( m_\chi \gtrsim 200 \text{ GeV} \) (1 TeV).

Dark matter produced in late decays also may suppress structure on small scales \( \tau \). The free-streaming scale \( \lambda_{FS} = \int_{\tau}^{t_{end}} [v(t)/a(t)] dt \) is well approximated by

\[
\lambda_{FS} \simeq 1.0 \text{ Mpc} \left[ \frac{u_\tau^2}{10^6 \text{ s}} \right]^{1/2} \left[ 1 - 0.07 \ln \left( \frac{u_\tau^2}{10^6 \text{ s}} \right) \right]
\]

in the present context, where \( u_\tau \equiv |\vec{p}_\chi|/m_\chi \) at decay time \( \tau \), and we have neglected the effect of \( m_Z \) on kinematics and other small effects. Values of \( \lambda_{FS} \) are given in Fig. 3; they are essentially independent of \( \lambda \) and \( \tau \), respectively.

Current constraints \( \lambda_{FS} \lesssim 0.2 \text{ Mpc} \), but values near this bound may be preferred by observations. Remarkably, constraints from small scale structure are satisfied in the region of parameter space allowed by BBN, flavor and CP bounds, but just barely — Goldbergs suppressed sym-}

metry therefore predicts “warm” dark matter and may explain the suppression of power on scales \( \sim 0.1 \text{ Mpc} \).

In summary, we have proposed a simple model in which the flavor and overdensity problems of neutralino dark matter are simultaneously solved. In the specific framework considered here, the preferred model is high-scale GMSB, with \( m_\chi \sim 1 \text{ GeV}, \sqrt{F} \sim 10^9 \text{ GeV}, \Omega_\chi \sim 100, \) and \( m_\chi \sim 2 \text{ TeV} \). This last mass scale is unnaturally high, but is dictated by EDM constraints, irrespective of cosmology. More generally, this scenario de-emphasizes “cosmologically preferred” models with \( \Omega_\chi \sim 0.1 \), and implies that models typically considered excluded by neutralino overclosure may, in fact, be viable and preferred.

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