Design and Kinematics Analysis of Disc Cam with Translation Flat-face Follower

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Abstract: The profile equation of disc cam with translation flat-face follower is derived, the motion fidelity analysis is carried out, and the minimum size range of flat-face is deduced based on the rotation transformation tensor and meshing principle. The coordinate data of cam profile point is calculated by VC++, and the cam modeling and mechanism assembly are carried out with Creo software. The kinematics simulation analysis is completed, and the displacement, velocity and displacement of follower are obtained. The simulation analysis verifies the correctness of the design method, and the research has practical significance for the design of the disc cam with translation flat-face follower.

1. Introduction
Cam mechanism is the main driving component for mechanization and automation in engineering, and has been widely used in light industry, textile, food, medical, and other fields \cite{1}, while disc cam with translation flat-face follower is widely used in high-speed occasions because of significant advantages in bearing capacity, lubrication characteristics, working life, and high-speed performance \cite{2}. The design process of cam mechanism generally includes selecting the appropriate motion law according to the working conditions, determining the radius of the base circle, checking the pressure angle and radius of curvature, calculating the contour, and completing the CAD/CAM analysis and calculation. For the disc cam with translation flat-face follower, the size of the flat-face should be calculated to avoid the contact between the edge of the flat-face and the cam, which results in motion distortion, that is, motion fidelity analysis.

The common design methods of disc cam include drawing method and analytical method \cite{1}, and the solution of spatial cam mechanism mostly adopts the rotation transformation tensor and meshing principle. The drawing method is simple and low precision, which is difficult to meet the design requirements of high-precision cam. The analytical method \cite{3-5} has high precision in designing cam, the calculation formula of vector diameter length of contour points is deduced, which needs further processing in practical use. Using the rotation transformation tensor and meshing principle to solve the cam, the position information of the meshing point at any time can be accurately calculated, and the contour equations of various types of globoidal cam with cylindrical roller and tapered roller and cylindrical cam successfully solved \cite{6-9}.

At present, the research of disc cam with translation flat-face follower mainly focuses on the design of profile, the establishment of three-dimensional model, and motion analysis. Literature \cite{10} analyzes the parameter design method of the mechanism under the constraint of maximum pressure angle,
calculates the profile equation with MATLAB, and establishes a three-dimensional model with Pro/E. Literature [11-13] also uses different software tools such as VC++, Solidworks, UG and Adams to calculate the cam profile coordinates, establish three-dimensional models and carry out kinematic analysis. Literature [14] analyzes the factors affecting the motion fidelity of disk cam mechanism with oscillating flat-face push rod.

The simple and intuitive profile equation and motion fidelity calculation formula are the key to cam design. The Cartesian coordinate equation of profile of the disc cam with translation flat-face follower is solved by using the rotation transformation tensor and meshing principle, the minimum flat-face size of motion fidelity is determined, and the influence of inclination angle and eccentricity on the mechanism is analyzed. At the same time, with the help of the three-dimensional modeling and kinematics analysis function of Creo software, the virtual prototype of the mechanism is established, and kinematic analysis was carried out.

Fig.1 Coordinate system of disc cam with translation flat-face follower

2. Establishment of profile equation

2.1 Establishment of coordinate system

The coordinate system of the disc cam mechanism with translation flat-face follower is shown in Figure 1. Motion coordinate system O-xy of the translation flat-face follower is established, y-axis is the moving direction of the flat-face follower, and x-axis passes cam rotation center, the intersection of x-axis and y-axis is the coordinate origin O point. Rotation coordinate system O1-x1y1 of the disc cam is established, and cam rotation center is take the disc as the origin O1. The distance between the two origins is the eccentricity \(E\) of translation flat-face follower.

The base circle radius of the disc cam is \(R_b\), and the inclination angle between the flat-face and the moving direction of flat-face follower is \(\gamma\), the distance along the flat-face from the contact point \(M\) to the moving center of the flat-face follower is \(a\). the eccentricity of the translation flat-face follower is \(E\), when the translation flat-face follower meshes with the disc cam on the radius of base circle, the distance from the lowest point \(P\) of the straight moving rod to the x-axis is \(L\), \(L=(R_b-E\cos\gamma)/\sin\gamma\), and the disc cam turns counterclockwise at angular speed \(\omega\).

2.2 Establishment of profile equation

Disc cam rotation angle \(\theta\), Point \(M_f\) on the cam profile reaches point \(M\) and meshes with point \(M\) on the flat-face of follower, the distance from point \(M\) to the moving center of the flat-face follower is \(a\). According to the vector relationship and rotation transformation tensor:

\[
R_1 = R_s + R_E
\]

\[
R_2 = R_c + R_t
\]

\[
R_f = R_e^{(-\omega t)}
\]
The cam profile equation can be obtained as follows:

\[ R_j = e^{i(-j\theta)}(R_s + R_e - R_c) \]   \tag{4}

Where, \( R_s = [0 \ L + S \ 0] \), \( S \) is the displacement of the follower, \( R_c = [E \ 0 \ 0] \), \( R_e = [a * \sin \gamma \ a * \cos \gamma \ 0] \), \( e^{-j\theta} \) is the rotation factor, \( e^{-j\theta} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & -1 \end{bmatrix} \).

The Cartesian coordinate equation of the contour curve of disc cam with the translation flat-face follower can be obtained by substituting the above variables into equation (4), the equation is as follows:

\[
\begin{align*}
X &= (a * \sin \gamma - E) \cos \theta + (L + S + a * \cos \gamma) \sin \theta \\
y &= -(a * \sin \gamma - E) \sin \theta - (L + S + a * \cos \gamma) \cos \theta \\
L &= \frac{Rb - E \cos \gamma}{\sin \gamma} 
\end{align*}
\]  \tag{5}

When \( \gamma = 90^\circ \), the coordinate equation of disc cam profile curve is as follows:

\[
\begin{align*}
X &= (a - E) \cos \theta + (Rb + S) \sin \theta \\
y &= -(a - E) \sin \theta - (Rb + S) \cos \theta 
\end{align*}
\]  \tag{6}

2.3 Motion fidelity analysis

During the movement of disc cam, the condition that contact points on the edge of flat-face should be avoided when the disc cam meshes with flat-face, which can result in motion law distortion. That is, the flat-face should be large enough, the size of the flat-face is determined by the maximum and minimum values of parameter \( a \), and parameter \( a \) is also the key value for solving equation 5.

Meshing principle \[15\]:

\[ V_{12} \cdot n_1 = 0 \] \tag{7}

According to the equation (7), the relative sliding speed between the cam and the follower is perpendicular to the normal direction of the contact point, the normal direction of the contact point is perpendicular to the flat-face, therefore the normal direction is \( n_1 = [\cos \gamma \ -\sin \gamma \ 0] \).

The moving speed of point \( M \) on the cam is \( V_c = \omega \times R_c = -\omega [L + S + E * \cos \gamma \ E * \sin \gamma - a] \). The speed of point \( M \) on translation flat-face follower is \( V_2 = V_y [0 \ 1 \ 0] \).

Relative sliding speed is \( V_{12} = V_1 - V_2 \).

Substituting \( V12 \) and \( n1 \) into equation (7) to obtain:

\[ a = -(L + S) * \cos \gamma + E * \sin \gamma + \sin \gamma * V_c / \omega \] \tag{8}

Substituting \( L \) in to get:

\[ a = -Rb * \cos \gamma - S * \cos \gamma + E / \sin \gamma + \sin \gamma * V / \omega \] \tag{9}

According to formula (9), the \( a \) value corresponding to each rotation angle of cam can be calculated, and \( a_{\min} \) and \( a_{\max} \) can be got, that is, the minimum size of flat-face is determined.

When \( \gamma = 90^\circ \), \( a = E + V / \omega \)

The size of flat-face is \( B \geq B_+ - B_- \), and \( B_+ \geq a_{\max}, B_- \leq B_{\min} \).

2.4 Pressure angle analysis

The pressure angle refers to the acute angle between the force direction and the motion direction without considering the friction. As shown in Figure 1, the pressure angle is as follows:

\[ \alpha = 90^\circ - \gamma \] \tag{11}

The inclination angle is the only factor affecting the pressure angle.
2.5 Movement law

The modified sinusoidal acceleration motion law has the advantage of less impact and is suitable for medium speed occasions, its dimensionless time parameters are as follows:

Starting period of graduation period (0 < T < 1/8):

\[
S = \frac{1}{\pi+4} (\pi T - \frac{3}{4}\sin 4\pi T) \\
V = \frac{4\pi^2}{\pi+4} (1 - \cos 4\pi T) \\
A = \frac{4\pi^2}{\pi+4} \sin 4\pi T
\]

Intermediate stage: (1/8 < T < 7/8)

\[
S = \frac{1}{\pi+4} (2 + \pi T - \frac{9}{4}\sin \frac{\pi + 4\pi T}{3}) \\
V = \frac{\pi}{\pi+4} (1 - 3\cos \frac{\pi + 4\pi T}{3}) \\
A = \frac{4\pi^2}{\pi+4} \sin \frac{\pi + 4\pi T}{3}
\]

End of graduation period (7/8 < T < 1):

\[
S = \frac{1}{\pi+4} (4 + \pi T - \frac{1}{4}\sin 4\pi T) \\
V = \frac{\pi}{\pi+4} (1 - \cos 4\pi T) \\
A = \frac{4\pi^2}{\pi+4} \sin 4\pi T
\]

Its dimensionless displacement, velocity and acceleration motion curve is shown in Figure 2.

![Figure 2](image1.png)

3. Establishment of cam model and kinematics simulation

3.1 Cam contour point calculation and output

The calculation of cam contour point coordinates is the key to cam modeling. Coordinate calculation system of cam contour is programmed using VC++, and cam contour coordinate point data is calculated with equations (5) and equations (8), and is outputted in IBL file format, and the format of IBL file is shown in Figure 3.

![Figure 3](image2.png)
3.2 Motion fidelity analysis

Parameter $a$ is calculated according to equation (9), and it determines the minimum flat-face size without motion distortion. The change trend of $a$ is shown in Figure 4 and Figure 5 when the cam parameters are shown in Table 1. As shown in the two figures the contact point moves in the position direction of the X axis in the first half of the push, and it returns after rest when it reaches the maximum value, then it moves in the negative direction of the X axis in the second half of the push, and it returns.

As can be seen in Figure 4, the maximum value and minimum value of $a$ are the same corresponding to different tilt angles, but the starting value and change process are changed, it means the same size of flat-face follower can be used when tilt angles changes. It also can be determined that when the size of the flat-face is bigger than 60mm in the positive direction of X axis and is smaller than 40mm in the negative direction of X axis, the flat-face will always contact with the cam to ensure the motion law.

Figure 5 shows the effect of the eccentricity $E$ on the parameter $a$ when the inclination angle is maintained at 90°, it can be concluded that eccentricity $E$ has no effect on the size of flat-face follower, but it changes the eccentricity of the flat-face follower.

3.3 Establishment of cam model

Creo is a 3D software with excellent performance, which has the functions of 3D model and kinematics simulation. The cam contour curve is generated by importing contour point coordinate data in the format of IBL file, and the cam model is generated by stretching function, and the model of translation flat-face follower is established, and the simulation model is completed according to the motion relationship and position relationship at last.

In order to carry out kinematic simulation analysis of the mechanism, the following kinematic relationships and constraints are added to the cam mechanism:

1. Establish the cam connection relationship between the cam and the flat-face follower;
2. Establish the slider relationship between the flat-face follower and the earth;
3. Establish the pin connection relationship between camshaft and earth;
4. Add servo motor for the pin connection relationship between camshaft and earth, and set the servo motor speed as a constant, $\omega=360°/s$.

The established kinematics simulation analysis model is shown in Figure 6, and the parameters of cam are shown in Table 1.
3.4 Kinematic simulation

Kinematic simulation is completed, and the displacement, velocity and acceleration of the translation flat-face follower are obtained and are shown in Figure 7-9 respectively, and the curves of them are consistent with the standard motion rule in Figure 2.

From Figure 7 it can be seen that the cam stroke is 120mm, which is consistent with the design value. The dimensionless velocity equation of the cam is \( V = \frac{ds}{dt} = \frac{\theta f}{h} \cdot \frac{v}{\omega \theta} \). \( Vm \) is the maximum velocity factor of modified sinusoidal acceleration motion law, \( Vm=1.76 \), \( v_{max} \) can be calculated when \( Vm \) and \( \omega \theta \) were substituted, \( \theta f=316.8 \text{mm/s} \), which corresponds to the maximum speed in Figure 8, dimensionless acceleration equation of cam is \( A = \frac{d^2 s}{dt^2} = \frac{\theta f^2}{h} \cdot \frac{a}{\omega \theta^2} \). \( Am \) is maximum acceleration factor of modified sinusoidal acceleration motion law, \( Am=5.53 \), \( a_{max} \) can be calculated when \( Am \) and \( \omega \theta \) were substituted, \( a_{max}=2986 \text{mm/S}^2 \), which is consistent with the maximum acceleration value in Figure 9.

| parameter          | attribute | parameter          | attribute               |
|--------------------|-----------|--------------------|-------------------------|
| Base circle radius | 30mm      | Motion law         | Modified sinusoidal     |
|                   |           |                    | acceleration motion law |
| Inclination angle | 80°       | Cam stroke angle   | 120°                    |
| Eccentricity E    | 10mm      | Stroke h           | 60mm                    |

4. Conclusion

The Cartesian coordinate equation of the cam profile and the formula of the size of the flat-face are deduced by using rotation transformation tensor and meshing principle. The analysis proves that the tilt
angle and eccentricity of the flat-face have no influence on the size of the flat-face. The tilt angle of the flat-face only affects the beginning point of contact and the eccentricity affects the eccentricity of the flat-face to translation acting guide. The size of flat-face is determined by motion law and cam parameters. Coordinate data of contact point of the cam are generated by programming with VC++, and the three-dimensional model of the disc cam and flat-face follower are established, then virtual prototype model are established with Creo. Kinematics simulation is completed, and displacement, speed and acceleration of transition flat-face follower are obtained, which verifies the correctness of formula and model.

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