Minimization of Age-of-Information in Remote Sensing with Energy Harvesting

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Abstract—In this paper, minimization of time-averaged age-of-information (AoI) in an energy harvesting (EH) source equipped remote sensing setting is considered. The EH source opportunistically samples one or multiple processes over discrete time instants, and sends the status updates to a sink node over a time-varying wireless link. At any discrete time instant, the EH node decides whether to probe the link quality using its stored energy, and further decides whether to sample a process and communicate the data based on the channel probe outcome. The trade-off is between the freshness of information available at the sink node and the available energy at the energy buffer of the source node. To this end, an infinite horizon Markov decision process theory is used to formulate the problem of minimization of time-averaged expected AoI for a single energy harvesting source node. The following two scenarios are considered: (i) single process with channel state information at transmitter (CSIT), (ii) multiple processes with CSIT. In each scenario, for probed channel state, the optimal source node sampling policy is shown to be a threshold policy involving the instantaneous age of the process(es), the available energy in the buffer and the instantaneous channel quality as the decision variables. Finally, numerical results are provided to demonstrate the policy structures and trade-offs.

Index Terms—Age-of-information, remote sensing, Markov decision process (MDP).

I. INTRODUCTION

In recent years, the need for combining the physical systems with the cyber-world has attracted significant research interest. These cyber-physical systems (CPS) are supported by ultra-low power, low latency IoT networks, and encompass a large number of applications such as vehicle tracking, environment monitoring, intelligent transportation, industrial process monitoring, smart home systems etc. Such systems often require deployment of sensor nodes to monitor a physical process and send the real-time status updates to a remote estimator over a wireless network. However, for such critical CPS applications, minimizing mean packet delay without accounting for delay jitter can often be detrimental for the system performance. Also, mean delay minimization does not guarantee delivery of the observation packets to the sink node in the same order in which they were generated, thereby often resulting in unnecessarily dedicating network resources towards delivering outdated observation packets despite the availability of a freshly generated observation packet in the system. Hence, it is necessary to take into account the freshness of information of the data packets, apart from the mean packet delay.

Figure 1: Pictorial representation of a remote sensing system where an EH source samples one of \(N\) number of processes at a time and sends the observation packet to a sink node.

Recently, a metric named Age of Information (AoI) has been proposed \([1]\) as a measure of the freshness of the information update. In this setting, a sensor monitoring a system generates time-stamped status update and sends it to the sink over a network. At time \(t\), if the latest monitoring information available to the sink node comes from a packet whose time-stamped generation instant was \(t'\), then the AoI at the sink node is computed as \((t - t')\). Thus, AoI has emerged as an alternative performance metric to mean delay \([2]\).

However, timely delivery of the status updates is often limited by energy and bandwidth constraints in the network. Recent efforts towards designing EH source nodes (e.g., source nodes equipped with solar panels) have opened a new research paradigm for IoT network operations. The energy generation process in such nodes are very uncertain and typically modeled as a stochastic process. The harvested energy is stored in an energy buffer as energy packets, and used for sensing and communication as and when needed. This EH capability significantly improves network lifetime and eliminates the need for frequent manual battery replacement, but poses a new challenge towards network operations due to uncertainty in the available energy at the source nodes at any given time.

Motivated by the above challenges, we consider the problem of minimizing the time-averaged expected AoI in a remote sensing setting, where a single EH source probes the channel state, samples one or multiple processes and sends the observation packets to the sink node over a fading channel. Energy generation process is modeled as a discrete-time i.i.d. process, and a finite energy buffer is considered. Two variants of the problem are considered: (i) single process with CSIT, (ii) multiple processes with CSIT. Channel state probing and process sampling for time-averaged expected AoI minimization problem is formulated as an MDP, and the threshold nature of the optimal policy is established analytically for each case. Numerical results validate the theoretical results and intuitions.

A. Related work

Initial efforts towards optimizing AoI mostly involved the analysis of various queuing models; e.g., \([1]\) for analysing a single source single server queueing system with FCFS service discipline. \([3]\) for LCFS service discipline for M/M/1 queue,
[4] and [5] for multi-source single sink system with M/M/1 queueing at each source, [6] for AoI performance analysis for multi-source single-sink infinite-buffer queueing system where unserved packets are substituted by available newer ones, etc.

On the other hand, a number of papers have considered AoI minimization problem under EH setting: [7] for derivation of minimal AoI for a single source having finite battery capacity, [8] for derivation of the minimal age policy for EH two hop network, [9] for average AoI expression for single source EH server, [10] for AoI minimization for wirelessly powered user, [11] for sampling, transmission scheduling and transmit power selection for single source single sink system over infinite time horizon where delay is dependent on packet transmit energy.

There have also been several other works on developing optimal scheduling policy for minimizing AoI for EH sensor networks [12]–[20]. For e.g., [12] has investigated optimal online policy for single sensor single sink system with a noiseless channel, for infinite, finite and unit size battery capacity; for the finite battery size, it has provided energy aware status update policy. The paper [13] has considered a multi-sensor single sink system with infinite battery size, and proposed a randomized myopic scheduling scheme. In [14], the optimal online status update policy to minimize the long run average AoI for EH source with updating erasures have been proposed. It has been shown that the best effort uniform updating policy is optimal when there is no feedback and best-effort uniform updating with retransmission (BUR) policy is optimal when feedback is available to the source. The authors of [15] examined the problem of minimizing AoI under a constraint on the count of status updates. The authors of [16] addressed AoI minimization problem for cognitive radio communication system with EH capability; they formulated optimal sensing and updating for perfect and imperfect sensing as a partially observable Markov decision process (POMDP). Information source diversity, i.e., multiple sources tracking the same process but with dissimilar energy cost, and sending status updates to the EH monitoring node with finite battery size, has been considered in [17] with an MDP formulation, but no structure was provided for the optimal policy. In [18], reinforcement learning has been used to minimize AoI for a single EH sensor with HARQ protocol, but no clear intuition on the policy structure was provided. The authors of [19] have developed a threshold policy for minimizing AoI for a single sensor single sink system with erasure channel and no channel feedback. For a system with Poisson energy arrival, unit battery size and error-free channel, it has shown that a threshold policy achieves average age lower than that of zero-wait policy; based on this, lower bound on average age for general battery size and erasure channel has been derived. In [20], the authors have proposed optimal sampling threshold policy using MDP formulation for a system consists of multiple sources RF powered by the destination.

B. Our contributions and organization

1) We formulate the problem of minimizing the time-averaged expected AoI in an EH remote sensing system with a single source monitoring one or multiple processes, as an MDP with two stage action model, which is different from standard MDP in the literature. Under the assumptions of i.i.d. time-varying channel with CSIT, channel state probing capability at the source, and finite battery size, we derive the optimal policy structures which turn out to be simple threshold policies. The source node, depending on the current age of a process, decides whether to probe the channel or not. Afterwards, based on the channel probe outcome, the source node decides whether to sample the process and send an observation packet, or to remain idle. Thus, the MDP involves taking action in two stages at each time instant.

2) We prove convergence of an analogue of value iteration for this two-stage MDP.

3) Numerical analysis shows that the threshold for multiple processes turns out to be a function of the relative age of the processes.

4) We also prove certain interesting properties of various cost functions and some properties of the thresholds as a function of energy and age of the process.

The rest of the paper is organized as follows. System model has been explained in Section II. Aoi minimization for the single source single process case is addressed in Section III. Aoi minimization policy for multiple process sensing is provided in Section IV. Numerical results are provided in Section V followed by the conclusions in Section VI. All proofs are provided in the appendices.

II. System model

We consider an EH source capable of sensing one out of N different processes at a time, and reporting the observation packet to a sink node over a fading channel; see Figure 1. Time is discretized with the discrete time index $t \in \{0, 1, 2, 3, \cdots\}$. At each time, the source node can decide whether to estimate the quality of the channel from the source to the sink, or not. If the source node decides to probe the channel state, it can further decide whether to sample a process and communicate the data packet to the sink, or not, depending on the instantaneous channel quality. The source has a finite energy buffer of size $B$ units, where $E_p$ unit of buffer energy is used to probe channel state information and $E_a$ unit of buffer energy is used in sensing and communication. The energy packet generation process in the energy buffer is assumed to be an i.i.d. process with known mean. In case the energy buffer in a source node is full, the newly generated energy packets will not be accommodated unless $E_p$ unit of energy packet is spent in probing. Let $A(t)$ denote the number of energy packet arrivals to the energy buffer at time $t$, and $E(t)$ denote the energy available to the source at time $t$, for all $t \geq 0$.

We denote by $p(t)$ the probability of packet transmission success from the source to the sink node at time $t$. In this paper, we consider fading channel where $p(t) \in \{p_1, p_2, \cdots, p_m\}$ is i.i.d. across $t$, with $P(p(t) = p_j) = q_j$ for all $j \in \{1, 2, \cdots, m\}$. The channel state corresponding to channel success probability $p_j$ is denoted by $C_j$, and the packet success probability corresponding to channel state $C_j$ is given by...
Let us also denote by \( r(t) \in \{0, 1\} \) the indicator that the packet transmission from the source to the sink at time \( t \) is successful. Hence, \( \mathbb{P}(r(t) = 1 | C(t) = C_j) = p_j \) for all \( j \in \{1, 2, \ldots, m\} \). It is assumed that the channel state \( C(t) \) is learnt perfectly via a channel probe.

At time \( t \), let \( b(t) \in \{0, 1\} \) denote the indicator of deciding to probe the channel, and \( a(t) \in \{0, 1, \ldots, N\} \) denote the identity of the process being sampled, with \( a(t) = 0 \) meaning that no process is sampled, and \( b(t) = 0 \) meaning that the channel is not probed. Also, \( b(t) = 0 \) implies \( a(t) = 0 \). The set of possible actions or decisions is denoted by \( A = \{\{0, 0\} \cup \{1 \times \{0, 1, 2, \ldots, N\}\}\} \), where a generic action at time \( t \) is denoted by \((b(t), a(t))\).

Let us denote by \( \tau_k(t) \equiv \sup\{0 \leq \tau < t : a(\tau) = k, r(\tau) = 1\} \) the last time instant before time \( t \), when process \( k \) was sampled and the observation packet was successfully delivered to the sink. The age of information (AoI) for the \( k \)-th process at time \( t \) is given by \( T_k(t) = (t - \tau_k(t)) \). However, if \( a(t) = k \) and \( r(t) = 1 \), then \( T_k(t) = 0 \) since the current observation of the \( k \)-th process is available to the sink node.

A generic scheduling policy is a collection of mappings \( \{\mu_t\}_{t \geq 0} \) from the available energy level, probed channel capability, and process sampling and data transmission history summarized in the AoI of various processes, to \( A \), which basically decides the decision rule at each time. Thus, the decision rule \( \mu_t \) at time \( t \) takes the current state \( s(t) \) as input and maps it to one decision in the action space \( A \). If \( \mu_t = \mu \) for all \( t \geq 0 \), the policy is called stationary, else non-stationary.

We seek to find a stationary scheduling policy \( \mu \) that minimizes the expected AoI, summed over nodes and averaged over time. In other words, we seek to solve the following mathematical problem:

\[
\min \mu \frac{1}{T} \sum_{t=0}^{T} \sum_{k=1}^{N} \mathbb{E}_\mu(T_k(t))
\]  

III. SINGLE SOURCE SENSING SINGLE PROCESS

In this section, we derive the optimal channel probing, source activation and data transmission policy for a single EH source sampling a single process \( (N = 1) \), which will provide insights to develop process sampling policy for \( N > 1 \).

Here, we formulate (1) as a long-run average cost MDP with state space \( S = \{0, 1, \ldots, B\} \times \mathbb{Z}_+ \) and an intermediate state space \( V = \{0, 1, \ldots, B\} \times \mathbb{Z}_+ \times \{C_1, C_2, \ldots, C_m\} \) where a generic state \( s = (E, T) \) means that the energy buffer has \( E \) energy packets, and the source was last activated \( T \) slots ago. A generic intermediate state \( v = (E, T, C) \) which additionally means that the current channel state \( C \), obtained via probing, has packet success probability \( p(C) \). The action space \( A = \{\{0, 0\} \cup \{1 \times \{0, 1\}\}\} \) with \( a(t), b(t) \in \{0, 1\} \). At each time, if the source node decides not to probe the channel state then it will not perform sampling, and thus \( b(t) = 0, a(t) = 0 \), and the expected single-stage AoI cost is \( c(s(t), b(t), a(t)) = T \). However, if the source node decides to probe the channel state, the expected single-stage AoI cost is \( c(v(t), b(t), a(t)) = T \) for \( a(t) = 0 \), and \( c(v(t), b(t), a(t)) = 1 \) for \( a(t) = 1 \), \( T(1 - p(C)) \), where the expectation is taken over packet success probability \( p(C) \).

We first formulate the average-cost MDP problem as an \( \alpha \)-discounted cost MDP problem with \( \alpha \in (0, 1) \), and derive the optimal policy, from which the solution of the average cost minimization problem can be obtained by taking \( \alpha \to 1 \).

1) Optimality equation: Let \( J^*(E, T) \) be the optimal value function for state \( (E, T) \) in the discounted cost problem, and let \( W^*(E, T, C) \) be the cost-to-go from an intermediate state \( (E, T, C) \). The Bellman equations are given by:

\[
J^*(E \geq E_p + E_s, T) = \min \left\{ T + \alpha \mathbb{E}_A J^*(\min\{E + A, B\}, T + 1), \right. \\
V^*(E, T) \bigg\}
\]

\[
V^*(E, T) = \sum_{j=1}^{m} \mathbb{P}(C_j) W^*(E, T, C_j)
\]

\[
W^*(E, T, C) = \min \left\{ T(1 - p(C)) + \alpha_{E,A} J^*(E - E_p - E_s + A, 1) + \alpha(1 - p(C)) \mathbb{E}_A J^*(E - E_p - E_s + A, T + 1) \right\}
\]

\[
J^*(E < E_p + E_s, T) = T + \alpha \mathbb{E}_A J^*(\min\{E + A, B\}, T + 1)
\]

The first expression in the minimization in the R.H.S. of the first equation in (2) is the cost of not probing channel state \( b(t) = 0 \), which includes single-stage AoI cost \( T \) and an \( \alpha \) discounted future cost with a random next state \( \{\min\{E + A, B\}, T + 1\} \), averaged over the distribution of the number of energy packet generation \( A \). The quantity \( V^*(E, T) \) is the expected cost of probing the channel state, which explains the second equation in (2).

At an intermediate state \( (E, T, C) \), if \( a(t) = 0 \), a single stage AoI cost \( T \) is incurred and the next state becomes \( (E - E_p + A, T + 1) \); if \( a(t) = 1 \), the expected AoI cost is \( T(1 - p(C)) \) (expectation taken over the packet success probability \( p(C) \)), and the next random state becomes \( (E - E_p - E_s + A, 1) \) and \( (E - E_p - E_s + A, T + 1) \) if \( r(t) = 1 \) and \( r(t) = 0 \), respectively. The last equation in (2) follows similarly since \( b(t) = 0, a(t) = 0 \) is the only possible action when \( E < E_p + E_s \).

Substituting the value of \( V^*(E, T) \) in the first equation of (2), we obtain the following Bellman equations:

\[
J^*(E \geq E_p + E_s, T) = \min \left\{ T + \alpha \mathbb{E}_A J^*(\min\{E + A, B\}, T + 1), \right. \\
E \left( \min\{T + \alpha \mathbb{E}_A J^*(E - E_p + A, T + 1), \right. \\
T(1 - p(C)) + \alpha_{E,A} J^*(E - E_p - E_s + A, 1) + \alpha(1 - p(C)) \mathbb{E}_A J^*(E - E_p - E_s + A, T + 1) \bigg) \bigg\}
\]

\[
J^*(E < E_p + E_s, T) = T + \alpha \mathbb{E}_A J^*(\min\{E + A, B\}, T + 1)
\]

2) Policy structure:

Proposition 1. The value function \( J^{(k)}(s) \) converges to \( J^*(s) \) as \( k \) tends to \( \infty \).

Proof. See Appendix A \)

We provide the convergence proof of value iteration since we have a two-stage decision process as opposed to traditional MDP where a single action is taken.

Lemma 1. For \( N = 1 \), the value function \( J^*(E, T) \) is increasing in \( T \) and \( W^*(E, T, C) \) is decreasing in \( p(C) \).
**Theorem 1.** For $N = 1$, at any time, if the source decides to probe the channel, then the optimal sampling policy is a threshold policy on $p(C)$. For any $E \geq E_p + E_s$ and probing channel state, the optimal action is to sample the source node if and only if $p(C) \geq p_{th}(E,T)$ for a threshold function $p_{th}(E,T)$ of $E$ and $T$.

Proof. See Appendix B

**Conjecture 1.** For $N = 1$, the optimal probing policy for the $\alpha$-discounted AoI cost minimization problem is a threshold policy on $T$. For any $E \geq E_p + E_s$, the optimal action is to probe the channel state if and only if $T \geq T_{th}(E)$ of $E$.

**Theorem 2.** For $N = 1$, after probing the channel state, the optimal source activation policy for the $\alpha$-discounted cost problem is a threshold policy on $p(C)$. For any $E \geq E_p + E_s$ and probed channel state, the optimal action is to sample the process $\arg \max_{1 \leq k \leq N} T_k$ if and only if $p(C) \geq p_{th}(E,T)$ for a threshold function $p_{th}(E,T)$ of $E$, $T$.

Proof. See Appendix E

The first expression in the minimization in the R.H.S. of the first equation in (4) is the cost of not probing channel state ($b(t) = 0$), which includes single-stage AoI cost $\sum_{i=1}^{N} T_i$ and an $\alpha$ discounted future cost with a random next state $(\min \{E + A, B\}, T_1 + 1, T_2 + 1, \cdots, T_N + 1)$, averaged over the distribution of the number of energy packet generation $A$. The quantity $V^*(E, T_1, T_2, \cdots, T_N)$ is the optimal expected cost of probing the channel state, which explains the second equation in (4). At an intermediate state $(E, T_1, T_2, \cdots, T_N, C)$, if $a(t) = 0$, a single stage AoI cost $\sum_{i=1}^{N} T_i$ is incurred and the next state becomes $(E - E_p + A, T_1 + 1, T_2 + 1, \cdots, T_N + 1)$; if $a(t) = k$, the expected AoI cost is $\sum_{i \neq k} T_i + T_k(1 - p(C))$ (expectation taken over the packet success probability $p(C)$), and the next random state becomes $(E - E_p + E_s + A, T_1 + 1, T_2 + 1, \cdots, T_N + 1)$ and $(E - E_p + E_s + A, T_1 + 1, T_2 + 1, \cdots, T_N + 1)$ if $r(t) = 1$ and $r(t) = 0$, respectively. The last equation in (4) follows similarly since $b(t) = 0, a(t) = 0$ is the only possible action when $E < E_p + E_s$.

Substituting the value of $V^*(E, T_1, T_2, \cdots, T_N)$ in the first equation of (4), we obtain the Bellman equations (5):

**Lemma 2.** For $N > 1$, the value function $J^*(E, T_1, T_2, \cdots, T_N)$ is increasing in each of $T_1, T_2, \cdots, T_N$ and $W^*(E, T_1, T_2, \cdots, T_N, C)$ is decreasing in $p(C)$.

Proof. See Appendix D

Let us define $T = [T_1, T_2, \cdots, T_N]$ and $T_{-k} = [T_1, T_2, \cdots, T_{k-1}, T_{k+1}, \cdots, T_N]$.

**Conjecture 2.** For $N > 1$, the optimal probing policy for the $\alpha$-discounted AoI cost minimization problem is a threshold policy on $\arg \max_{1 \leq k \leq N} T_k$. For any $E \geq E_p + E_s$, the optimal action is to probe the channel state if and only if $\arg \max_{1 \leq k \leq N} T_k \geq T_{th}(E, T_{-k})$ for a threshold function $T_{th}(E, T_{-k})$ of $E$ and $T_{-k}$.

**Theorem 2.** For $N > 1$, after probing the channel state, the optimal source activation policy for the $\alpha$-discounted cost problem is a threshold policy on $p(C)$. For any $E \geq E_p + E_s$ and probed channel state, the optimal action is to sample the process $\arg \max_{1 \leq k \leq N} T_k$ if and only if $p(C) \geq p_{th}(E,T)$ for a threshold function $p_{th}(E,T)$ of $E$, $T$.

Proof. See Appendix E

The policy structure upholds the two intuitions, (i) for a given $E$ and $(T_1, T_2, \cdots, T_N)$, the source decides to probe...
Given the channel state if highest AoI is greater than some threshold value, (ii) for a given $E$ and $(T_1, T_2, \ldots, T_N)$ and probed channel state, if the channel condition is better than a threshold, then the optimal action is to sample the process with highest AoI and send the observation to the sink node. We will later numerically demonstrate some intuitive properties of $T_{th}(E, T, \lambda)$ as a function of $E$, $T$, and $\lambda$ and $p_{th}(E, T, \lambda)$ as a function of $E$, $(T_1, T_2, \ldots, T_N)$ and $\lambda$ in Section V.

V. Numerical results

A. Single source, single process ($N = 1$)

We consider five channel states $(m = 5)$ with channel state occurrence probabilities $q = [0.2, 0.2, 0.2, 0.2, 0.2]$ and the corresponding packet success probabilities $p = [0.9, 0.7, 0.5, 0.3, 0.1]$. Energy arrival process is i.i.d. Bernoulli($\lambda$) with energy buffer size $B = 12$, $E_p = 1$ unit, $E_s = 1$ unit, and the discount factor $\alpha = 0.99$. Numerical exploration revealed that, there exists a threshold policy on $T$ in decision making for channel state probing; see Figure 2(a). It is observed that this $T_{th}(E)$ decreases with $E$ since higher available energy in the energy buffer allows the EH node to probe the channel state more aggressively. Similar reasoning explains the observation that $T_{th}(E)$ decreases with $\lambda$. For probed channel state, Figure 2(b) shows the variation of $p_{th}(E, T)$ with $E, T, \lambda$. It is observed that $p_{th}(E, T)$ decreases with $E$, since the EH node tries to sample the process more aggressively if more energy is available in the buffer. Similarly, higher value of $T$ results in aggressive sampling, and hence $p_{th}(E, T)$ decreases with $T$. By similar arguments as before, we can explain the observation that this $p_{th}(E, T)$ decreases with $\lambda$.

B. Single source, multiple processes ($N > 1$)

We choose $N = 3, \alpha = 0.99, B = 12, E_p = 1, E_s = 1$ and the same channel model and parameters as in Section V-A. Figure 3(a) shows the variation on the threshold on $T_1$ for given $T_2, T_3$, for channel state probing. It is observed that $T_{th}(E, T_2, T_3)$ decreases with $E$ and $\lambda$. Extensive numerical work also demonstrated that this threshold decreases with each of $T_2, T_3$. For probed channel state, Figure 3(b) shows that $p_{th}(E, T_1, T_2, T_3)$ decreases with $E$ and $\lambda$. Further numerical analysis also demonstrated that this threshold decreases with each of $T_1, T_2, T_3$.

\begin{align*}
J^*(E \geq E_p + E_s, T_1, T_2, \ldots, T_N) &= \min \left\{ \sum_{i=1}^{N} T_i + \alpha E_T J^*(\min\{E + A, B\}, T_1 + 1, T_2 + 1, \ldots, T_N + 1), \right. \\
& \left. \mathbb{E}_C \left( \min \left\{ \sum_{i=1}^{N} T_i + \alpha E_T J^*(E - E_p + A, T_1 + 1, T_2 + 1, \ldots, T_N + 1), \right. \\
& \left. \min_{1 \leq k \leq N} \left\{ \sum_{i \neq k} T_i + T_k (1 - p(C)) + \alpha p(C) E_T J^*(E - E_p - E_s + A, T_1 + 1, T_2 + 1, \ldots, T_N + 1) \right\} \right) \right\} \\
J^*(E < E_p + E_s, T_1, T_2, \ldots, T_N) &= \sum_{i=1}^{N} T_i + \alpha E_T J^*(\min\{E + A, B\}, T_1 + 1, T_2 + 1, \ldots, T_N + 1)
\end{align*}

Figure 2: For $N = 1$, (a) Variation of $T_{th}(E)$ with $E, \lambda$ and (b) Variation of $p_{th}(E, T)$ with $E, T, \lambda$.

Figure 3: For $N = 3$, (a) Variation of $T_{th}(E, T_2, T_3)$ with $E, T_2, T_3, \lambda$ and (b) Variation of $p_{th}(E, T_1, T_2, T_3)$ with $E, T_1, T_2, T_3, \lambda$.

VI. Conclusions

In this paper, we derived the optimal policy structures for minimizing the time-averaged expected AoI under an energy-harvesting source. We considered single and multiple
processes, i.i.d. time varying channels, and channel probing capability at the source. The optimal source sampling policy turned out to be a threshold policy. Numerical results illustrated the policy structures and trade-offs. We will extend this work for unknown energy generation rate in our future research endeavours.

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APPENDIX A

PROOF OF PROPOSITION [1]

We prove max_{s \in S} |J^{(k)}(s) - J^*(s)| \uparrow k as k \uparrow \infty.

Let us define the error e_k = \max_{s \in S}|J^{(k)}(s) - J^*(s)| and J^{(0)}(s) as initial estimate for J^*(s).

For any state s, we can establish relation between the error at time k+1 and the error at time k in the following way:

\[
J^{(k+1)}(E \geq E_p + E_s, T) = \min \left\{ T + \alpha \mathbb{E}_A J^{(k)}(\min\{E + A, B\}, T), T(1 - p(C)) + \alpha p(C) \mathbb{E}_A J^{(k)}(E - E_p - E_s + A, 1) + \alpha(1 - p(C)) \mathbb{E}_A J^{(k)}(E - E_p - E_s - A, T + 1) \right\}
\]

We assume there exists an optimal value function J^*(E, T) for the discounted cost problem and substituting J^{(k)}(E, T) by J^{(k)}(E, T) \leq J^*(E, T) + e_k in [6] we get following equation:

\[
J^{(k+1)}(E \geq E_p + E_s, T) \leq \min \left\{ T + \alpha \mathbb{E}_A J^*(\min\{E + A, B\}, T + 1) + e_k, T(1 - p(C)) + \alpha p(C) \mathbb{E}_A J^*(E - E_p - E_s + A, 1) + \alpha(1 - p(C)) \mathbb{E}_A J^*(E - E_p - E_s - A, T + 1) + e_k \right\}
\]

(6)

Similarly, substituting J^{(k)}(E, T) by J^{(k)}(E, T) \geq J^*(E, T) - e_k in [6] we get following equation:

\[
J^{(k+1)}(E \geq E_p + E_s, T) \geq \min \left\{ T + \alpha \mathbb{E}_A J^*(\min\{E + A, B\}, T) - e_k, T(1 - p(C)) + \alpha p(C) \mathbb{E}_A J^*(E - E_p - E_s + A, 1) - e_k + \alpha(1 - p(C)) \mathbb{E}_A J^*(E - E_p - E_s - A, T + 1) - e_k \right\}
\]

(7)

Combining the results obtained from equations (6) and (7), we get:

\[
J^*(E \geq E_p + E_s, T) - \alpha e_k \leq J^{(k+1)}(E \geq E_p + E_s, T) \leq J^*(E \geq E_p + E_s, T) + \alpha e_k
\]

(8)

\[
|J^{(k+1)}(E \geq E_p + E_s, T) - J^*(E \geq E_p + E_s, T)| \leq \alpha e_k
\]

(9)

\[
\max_{s \in S} |J^{(k+1)}(s) - J^*(s)| \leq \alpha e_k
\]

(10)

\[
e_k \leq \alpha e_k
\]

(11)

\[
e_{k+1} \leq \alpha e_k
\]

(12)

Thus, equation (12) gives the relation between the error at time k+1 to the error at time k. By backward substitution we get,

\[
\max_{s \in S} |J^{(k)}(s) - J^*(s)| \leq \alpha^k \max_{s \in S} |J^{(0)}(s) - J^*(s)|
\]

(13)

From equation (13), as k \uparrow \infty, the error reduces to zero. Hence, J^{(k)}(s) converges to J^*(s).

APPENDIX B

PROOF OF LEMMA [1]

We prove this result by value iteration:

\[
J^{(k+1)}(E \geq E_p + E_s, T) = \min \left\{ T + \alpha \mathbb{E}_A J^{(k)}(\min\{E + A, B\}, T + 1), T(1 - p(C)) + \alpha p(C) \mathbb{E}_A J^{(k)}(E - E_p - E_s + A, 1) + \alpha(1 - p(C)) \mathbb{E}_A J^{(k)}(E - E_p - E_s + A, T + 1) \right\}
\]

(14)

Let us start with J^{(0)}(s) = 0 for all s \in S. Clearly, J^{(1)}(E \geq E_p + E_s, T) = \min\{T, \mathbb{E}_C(\min\{T, T(1 - p(C))\})\} = \min\{T, \mathbb{E}_C(T(1 - p(C)))\} and J^{(1)}(E < E_p + E_s, T) = T.

Hence, for any given E, the value function J^{(1)}(E, T) is an increasing function of T and decreasing function of p(C).

As induction hypothesis, we assume that J^{(k)}(E, T) is also increasing function of T.

Now,

\[
J^{(k+1)}(E \geq E_p + E_s, T) = \min \left\{ T + \alpha \mathbb{E}_A J^{(k)}(\min\{E + A, B\}, T + 1), T(1 - p(C)) + \alpha p(C) \mathbb{E}_A J^{(k)}(E - E_p - E_s + A, 1) + \alpha(1 - p(C)) \mathbb{E}_A J^{(k)}(E - E_p - E_s + A, T + 1) \right\}
\]

(15)

We need to show that J^{(k+1)}(E \geq E_p + E_s, T) is also increasing in T. The first term inside the minimization operation in [15] is increasing in T, by the induction hypothesis and from the fact that expectation is a linear operation. On the other hand, the second term has linear expectation over channel state and another minimization operator. Also, the first and second terms inside the second minimization operation in [15] are increasing in T by the induction hypothesis and the linearity of expectation operation. Thus, J^{(k+1)}(E \geq E_p + E_s, T) is also increasing in T. By similar arguments, we can claim that J^{(k+1)}(E < E_p + E_s, T) is increasing in T. Now, since J^{(k)}(\cdot) \uparrow J^*(\cdot) as k \uparrow \infty by proof of Proposition 1, J^*(E, T) is also increasing in T. Hence, the lemma is proved.

APPENDIX C

PROOF OF THEOREM [1]

From (3), it is obvious that for probed channel state the optimal decision for E \geq E_p + E_s is to sample the source if and only if the cost of sampling is lower than the cost of not sampling the source, i.e., T + \alpha \mathbb{E}_A J^*(E - E_p + A, T + 1) \geq T(1 - p(C)) + \alpha \mathbb{E}_A J^*(E - E_p - E_s + A, T + 1) - \alpha p(C) \mathbb{E}_A J^*(E - E_p - E_s + A, T + 1) - \alpha(1 - p(C)) \mathbb{E}_A J^*(E - E_p - E_s + A, T + 1).

Now, by Lemma 1, \mathbb{E}_A J^*(E - E_p - E_s + A, T + 1) - \mathbb{E}_A J^*(E - E_p - E_s + A, T + 1) is non-negative. Thus the R.H.S. decreases with p(C), whereas the L.H.S. is independent of p(C). Hence, for probed channel state the optimal action is to sample if and only if p(C) \geq p_{th}(E, T) for some suitable threshold function p_{th}(E, T).
APPENDIX D
PROOF OF LEMMA [2]

The proof is similar to the proof of Lemma [1] and it follows from the convergence of value iteration as given below:

\[
J^{(k+1)}(E \geq E_p + E_s, T_1, T_2, \ldots, T_N) = \min \left\{ \sum_{i=1}^{N} T_i + \alpha E_A J^{(k)}(min(E + A, B), T_1 + 1, T_2 + 1, \ldots, T_N + 1), E_C \left( \min \left\{ \sum_{i \neq k} T_i + T_k (1 - p(C)) + \alpha p(C) E_A J^{(k)}(E - E_p - E_s + A, T_1 + 1, T_2 + 1, \ldots, T_N + 1) \right\} \right) \right\}
\]

Now, by Lemma [2],

\[
E_A J^{(k)}(E - E_p - E_s + A, T_1 + 1, T_2 + 1, \ldots, T_N + 1) = \min \left\{ \sum_{i=1}^{N} T_i + \alpha E_A J^{(k)}(min(E + A, B), T_1 + 1, T_2 + 1, \ldots, T_N + 1) \right\}
\]

APPENDIX E
PROOF OF THEOREM [2]

It is obvious that \(J^*(\cdot)\) is invariant to any permutation of \((T_1, T_2, \ldots, T_N)\). Hence, by Lemma [2],

\[
\arg \min_{1 \leq k \leq N} \left( \sum_{i \neq k} T_i + T_k (1 - p(C)) + \alpha p(C) E_A J^{(k)}(E - E_p - E_s + A, T_1 + 1, T_2 + 1, \ldots, T_N + 1) \right) = \arg \max_{1 \leq k \leq N} \min \left( T_k \right)
\]

\(J^*(\cdot)\) is increasing in each of \((T_1, T_2, \ldots, T_N)\). Therefore, it follows that \(J^{(k+1)}(E \geq E_p + E_s, T_1, T_2, \ldots, T_N)\) is also increasing in each of \(T_1, T_2, \ldots, T_N\). The first term is the minimization operation in (17) is increasing in each of \(T_1, T_2, \ldots, T_N\), utilizing the induction hypothesis and linearity property of expectation operation. On the other hand, the second term has expectation over channel state and another minimization operator. Also, the fist and second term of second minimization operator is increasing in each of \(T_1, T_2, \ldots, T_N\) by using induction hypothesis and the linearity of expectation operation. Thus, \(J^{(k+1)}(E \geq E_p + E_s, T_1, T_2, \ldots, T_N)\) is also increasing in each of \(T_1, T_2, \ldots, T_N\). Similarly, we can assert that \(J^{(k+1)}(E < E_p + E_s, T_1, T_2, \ldots, T_N)\) is increasing in each of \(T_1, T_2, \ldots, T_N\). Now, since \(J^{(k)}(\cdot) \uparrow J^*(\cdot)\) as \(k \uparrow \infty\), \(J^*(E, T_1, T_2, \ldots, T_N)\) is also increasing in each of \(T_1, T_2, \ldots, T_N\). Hence, the lemma is proved.