A New Equation of State for Dark Energy Model

Lei Feng1,3 and Tan Lu2,3
1Department of Physics, Nanjing University, Nanjing 210093, China
2Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008, China
3Joint Center for Particle, Nuclear Physics and Cosmology, Nanjing University – Purple Mountain Observatory, Nanjing 210093, China

A new parameterization for the dark energy equation of state (EoS) is proposed and some of its cosmological consequences are also investigated. This new parameterization is the modification of Efstathiou’ dark energy EoS parameterization. w(z) is a well behaved function for z \( \gg 1 \) and has same behavior in z at low redshifts with Efstathiou’ parameterization. In this parameterization there are two free parameter \( w_0 \) and \( w_a \). We discuss the constraints on this model’s parameters from current observational data. The best fit values of the cosmological parameters with 1\( \sigma \) confidence-level regions are: \( \Omega_m = 0.2735^{+0.0071}_{-0.0063} \), \( w_0 = -1.0537^{+0.1432}_{-0.1511} \) and \( w_a = 0.2738^{+0.8018}_{-0.8288} \).

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I. INTRODUCTION

In recent years, the discovery of accelerating expansion of the universe is an amazing development. It was firstly discovered by observing type Ia supernova (SNe Ia) [1, 2], which can be used as standard candles [3, 4]. The cosmic microwave background (CMB) measurements from Wilkinson Microwave Anisotropy Probe (WMAP) [5] and the large scale structure survey by Sloan Digital Sky Survey (SDSS) [6, 7] confirm this accelerating expansion universe model. There are two kinds of ideas, i.e. the existence of the dark energy or modifications of the gravity theory, to explain this concept. The first scheme is most popularly discussed, and many models have been proposed, such as the holographic dark energy models [8] and the Chaplygin Gas [9]. In addition there are also many modified gravity models, such as the brane world [10] and \( f(R) \) [11] and so on.

The dark energy equation of state (EoS) which is the ratio of pressure to energy density, is a prefect quantity to study the behavior of dark energy. If EoS is a constant \(-1\), dark energy is the \( \Lambda \) Cold Dark Matter Model (LCDM), and maybe dark energy is vacuum energy. Otherwise the dark energy would be dynamical scalar field, such as the Quintessence [12], the quintom [13].

There are several way to explore the behavior of dark energy EoS. The most popular way is to build a functional form for EoS in terms of some free parameters. Lots of EoS parameterizations have already been discussed in the literature (such as [14–24] and Refs. therein). Another way is picking a simple local basis representation for \( w(z) \) (bins, wavelets), and estimate the associated coefficients [25–27]. In addition, there are also some nonparametric way [28, 29].

In this paper, we consider a new parameterization for the dark energy EoS. In [24], the author developed a new dark energy EoS parameterization: \( w(z) = w_0 + w_a \ln(1 + z) \). It is obvious that when \( z \to \infty \), \( w(z) \) has poor behavior and becomes infinite. This dark energy EoS parameterization can only describe the behavior of dark energy when \( z \) is not very large. To avoid this problem, we consider a new dark energy EoS parameterization, which is the modification of Efstathiou’ parameterization: \( w(z) = w_0 + w_a \ln(1 + \frac{z}{z_0}) \). The value of \( w(z) \) is \( w_0 \) at present and \( w(z) \) becomes to \((w_0 + w_a \ln z)2\) when \( z \to \infty \).

In this model there are three parameters in all, which is \( w_0, w_a \) and \( \Omega_m \). As shown in [30], this EoS will get to a nonphysical value in the far future time when redshift \( z \) approaches \(-1\), namely, \( w(z) \) will grow rapidly and diverge.

In this paper, we perform a global data fitting analysis on this new dark energy EoS parameterizations, and present constraints on the model parameters from the current observational data, including the seven-year WMAP data, Baryon Acoustic Oscillations (BAO) data, Observational Hubble data and SN Union2 sample. Since dark energy parameters are tightly correlated to some other cosmological parameters, such as the matter density parameter \( \Omega_m \) and the Hubble constant \( H_0 \), it is necessary to consider a global fit procedure in the investigation of the dynamical dark energy. The paper is organized as follows: In section II, we review the new dark energy EoS parameterization. In section III, we describe Current Observational Data we used. In section IV, we perform the cosmic observation constraint, the results are also presented. The last section is the conclusion.

II. NEW PARAMETERIZATION

Let us start by presenting some of the most investigated EoS parameterizations(see also [22] for other parameterizations):

\[
\begin{align*}
\begin{aligned}
\text{(redshift)} & : \ w(z) = w_0 + w_a z \\
\text{(scale factor)} & : \ w(z) = w_0 + w_a z/(1 + z) \\
\text{(logarithmic)} & : \ w(z) = w_0 + w_a \ln(1 + z)
\end{aligned}
\end{align*}
\]

where \( w_0 \) is the current value of the EoS, and \( w_a \) indicate the revolution of the dark energy EoS. The value of these parameter is determined by the observational data. When \( w_a = 0 \) and \( w_0 = -1 \), the dark energy model becomes to LCDM model.

The first parameterization represents a good fit for low redshifts, but has serious problems to explain high-z observations since it blows up as \( \exp(3w_a z) \) when \( z > 1 \) and \( w_a > 0 \). For
example, it can not explain the estimated ages of high-$z$ objects [31]. The second one solves this problem, since $w(z)$ is a well behaved function for $z \gg 1$ and recovers the linear behavior in $z$ at low redshifts. The latter was introduced by Efstathiou [20]. It was built empirically to adjust some quintessence models at $z \leq 4$. When $z$ approaches infinity, $w(z)$ has poor behavior and becomes infinite and this is unnatural. Similar to the second model, let us consider the following EoS parameterization

$$w(z) = w_0 + w_a \ln(1 + \frac{z}{1 + z}).$$  (1)

where the value of $w(z)$ is $w_0$ at present and $w(z) \rightarrow (w_0 + w_a \times \ln 2)$ when $z \rightarrow \infty$.

If there is no interaction between dark energy and other component of the universe, one can show from the energy conservation law $\dot{\rho} = -3a(\rho + p)/a$ that the dark energy density evolves as

$$\rho_{\text{de}}(z) = \rho_c(1 - \Omega_m) \exp\left(3 \int_0^z [1 + w(z')] \frac{dz'}{1 + z'} \right).$$  (2)

where $\rho_c$ is the critical density, it is defined by the following equation

$$\rho_c \equiv \frac{3H_0^2}{8\pi G_N}.$$  (3)

In this new parameterization, it is hard to write out the analysis formula of this quantity. It is calculated through numerical method.

In order to study the evolution of cosmological perturbations, we use the public Parameterized Post-Friedmann (PPF) package developed by Wayne Hu (see e.g., [32] for detailed discussions on PPF method) to calculate the CMB anisotropy spectrum. In Figure 1, we plot the anisotropy spectrum for different choices of $w_0$ and $w_a$. We observe some obvious differences with respect to the LCDM case on larger scales (mulpole number $l < 10$). And for the model $w_0 = -0.7$ and $w_a = -0.2$, there is a slit deviation on the peaks of anisotropy spectrum.

III. CURRENT OBSERVATIONAL DATA

In order to test this new model, we use the most recent observational data currently available. In this section, we describe how we use these data.

A. Type Ia Supernovae constraints

We use the 557 SNe Ia Union2 dataset [33]. Following [34, 35], one can obtain the corresponding constraints by fitting the distance modulus $\mu(z)$ as

$$\mu_{\text{obs}}(z) = 5 \log_{10}[D_L(z)] + \mu_0.$$  (4)

where $\mu_0 \equiv 42.38 - 5 \log_{10} h$, and $h$ is the Hubble constant $H_0$ in units of $100$ km s$^{-1}$ Mpc$^{-1}$. In flat universe, the Hubble-free luminosity distance $D_L = H_0 d_L$ is

$$D_L(z) = (1 + z) \int_0^z \frac{dz'}{E(z')}.$$  (5)

where $E(z) \equiv H(z)/H_0$.

For the SN Ia dataset, the best fit values of the parameters can be determined by a likelihood analysis, based on the calculation of

$$\chi^2_{\text{SN}} = \sum_i \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{obs}}(z_i)]^2}{\sigma^2(z_i)}.$$  (6)

B. Baryon Acoustic Oscillation constraints

The BAO data come from SDSS DR7 [36]. The datapoints we use are

$$\frac{r_s(z_d)}{D_v(0.275)} = 0.1390 \pm 0.0037.$$  (7)

and

$$\frac{D_v(0.35)}{D_v(0.2)} = 1.736 \pm 0.065.$$  (8)

where the spherical average gives us the following effective distance measure [37],

$$D_T(z) = \left(\int_0^\infty \frac{dx}{H(x)} 2 \frac{z}{H(z)} \right)^{1/3}.$$  (9)

and $r_s(z_d)$ is the comoving sound horizon at the baryon drag epoch. Also, $z_d$ can be obtained by using a fitting formula [38],

$$z_d = \frac{1291(\Omega_m h^2)^{0.251}}{1 + 0.659(\Omega_m h^2)^{0.828}}[1 + b_1(\Omega_m h^2)^{0.251}],$$  (10)

with

$$b_1 = 0.313(\Omega_m h^2)^{-0.419}[1 + 0.607(\Omega_m h^2)^{0.674}],$$  (11)

$$b_2 = 0.238(\Omega_m h^2)^{0.233}.$$  (12)
The function \( r_s(z) \) is the comoving sound horizon size

\[
r_s(z) = \frac{c}{\sqrt{3}} \int_{0}^{1/(1+z)} \frac{da}{a^2 H(a) \sqrt{1 + (3\Omega_b/4\Omega_r)a}}.
\]

where \( \Omega_r \) is 2.469 \times 10^{-3} h^{-2} \) for \( T_{\text{CMB}} = 2.725 K \). So the \( \chi^2 \) for the BAO data is given by

\[
\chi^2_{\text{BAO}} = \left( \frac{r_s(z_d)/D_V(z = 0.275) - 0.1390}{0.0037} \right)^2 + \left( \frac{D_V(z = 0.35)/D_V(z = 0.2) - 1.736}{0.065} \right)^2.
\]

C. Cosmic Microwave Background constraints

The CMB shift parameter \( R \) is provided by [39]

\[
R(z_{\text{rec}}) = \frac{\sqrt{\Omega_m H_0^2}}{\sqrt{4\pi}} \sin[\sqrt{4\pi}] \int_{z_{\text{rec}}}^{0} \frac{dz'}{H(z')},
\]

where \( \sin(x) \) for \( \Omega_k > 0 \), \( x \) for \( \Omega_k = 0 \), and \( \sin(x) \) for \( \Omega_k < 0 \), respectively. Here, the redshift \( z_{\text{rec}} \) (the decoupling epoch of photons) is obtained by using the fitting function [40]

\[
z_{\text{rec}} = 1048 \left[ 1 + 0.00124(\Omega_b h^2)^{0.738} \right] \left[ 1 + g_1(\Omega_m h^2)^{0.72} \right],
\]

where

\[
g_1 = 0.0783(\Omega_b h^2)^{-0.238} \left( 1 + 39.5(\Omega_b h^2)^{0.763} \right)^{-1},
\]

\[
g_2 = 0.560 \left( 1 + 21.1(\Omega_b h^2)^{1.81} \right)^{-1}.
\]

In addition, the acoustic scale is related to the distance ratio and is expressed as

\[
l_A = \frac{\pi}{r_s(z_{\text{rec}})} \sin[\sqrt{4\pi}] \int_{0}^{z_{\text{rec}}} \frac{dz'}{H(z')}. \tag{16}
\]

Following Ref. [41], the \( \chi^2 \) for the CMB data is

\[
\chi^2_{\text{CMB}} = (x_i^{\text{th}} - x_i^{\text{obs}})(C^{-1})_{ij}(x_j^{\text{th}} - x_j^{\text{obs}}), \tag{17}
\]

where \( x_i = (l_i, R, z_{\text{rec}}) \) is a vector and \((C^{-1})_{ij}\) is the inverse covariance matrix. The seven-year WMAP observations [41] give the maximum likelihood values: \( l_A(z_{\text{rec}}) = 302.09 \), \( R(z_{\text{rec}}) = 1.725 \) and \( z_{\text{rec}} = 1091.3 \). In Ref. [41], the inverse covariance matrix is also given as follows

\[
(C^{-1}) = \begin{pmatrix}
2.305 & 29.698 & -1.333 \\
29.698 & 6825.270 & -113.180 \\
-1.333 & -113.180 & 3.414
\end{pmatrix}. \tag{18}
\]

D. Observational Hubble data (OHD)

The Hubble parameter can be written as the following form:

\[
H(z) = -\frac{1}{1+z} \frac{dz}{dt}.
\]

So, through measuring \( dt/dz \), we can obtain \( H(z) \). In [42] and [43, 44], the author discuss the possibility of using absolute ages of galaxies to calculate the value of \( dt/dz \). In [44], the galaxy spectral data come from the Gemini Deep Deep Survey [45] and archival data [46–51]. Detailed calculations of \( dt/dz \) can be found in [44], and we do not discuss them here. Currently, we have a set of 12 values of the Hubble parameter versus redshift in total (see Table 2 of [52]). The data calculated by this way are less sensitive to systematic errors which is a great advantage [53].

We can use these data to constrain different kinds of dark energy models and modified gravity models by minimizing the quantity

\[
\chi^2_{\text{OHD}} = \sum_{j=1}^{12} \frac{(H(z_j) - H_{\text{obs}}(z_j))^2}{\sigma_j^2}, \tag{20}
\]

This test has already been used to constrain several cosmological models [54–67].

IV. RESULTS

In our analysis, we perform a global fitting to determine the cosmological parameters using the MCMC method. In our calculations, we have taken the total likelihood function \( L \propto e^{-\chi^2/2} \) to be the products of the separate likelihoods of SNe Ia, BAO, CMB and OHD. Then we get \( \chi^2 \) as

\[
\chi^2 = \chi^2_{SN} + \chi^2_{\text{BAO}} + \chi^2_{\text{CMB}} + \chi^2_{\text{OHD}}. \tag{21}
\]

The results on the best fit values of the cosmological parameters with 1σ confidence-level regions are: \( \Omega_m = 0.2735 \pm 0.0071 \), \( w_0 = -1.0537 \pm 0.132 \) and \( w_a = 0.2738 \pm 0.0018 \). The nuisance parameters \( H_0 \) used in the analysis is actually not model parameters with significant meanings, so we do not list it.

In Figures 2, we also show the parametric spaces \( \Omega_m - w_0 \) and \( \Omega_m - w_a \) that arise from the joint analysis described above. We note that the result is consistent with the LCDM model [13], whose \( \chi^2 \) of the EOS \( w(z) \) along with \( z \). The following is some discussion of these findings:

1. The best-fit results are: \( \Omega_m = 0.2735 \), \( w_0 = -1.0537 \) and \( w_a = 0.2738 \). Note that here the results are maximum likelihood values. The value of \( \chi^2 \) is -1.0537 at present and \( w(z) \) equals to \(-0.8639 \) when \( z \to \infty \).

2. We find that the best-fit dark energy model is a quintom model [13], whose \( w(z) \) crosses the cosmological constant boundary \( w = -1 \) during the evolution. And the redshift is 0.2766 when \( w(z) \) crosses the cosmological constant boundary \( w = -1 \).
FIG. 2: Marginalized probability contours at 1σ and 2σ CL in the \(\Omega_m - w_0\) and \(w_0 - w_a\) planes for the model considered in this manuscript. The result is consistent with the LCDM model in the 1σ CL.

2. With the current observational data, the variance of \(w_0\) and \(w_a\) we get are still large: the 1σ constraints on \(w_0\) and \(w_a\) are \(w_0 = -1.0537^{+0.1432}_{-0.1511}\) and \(w_a = 0.2738^{+0.8018}_{-0.8288}\). This result implies that though the dynamical dark energy models are mildly favored, the current data cannot distinguish different dark energy models decisively. With the fitting results we obtained, we can reconstruct the evolution of the EOS of dark energy, \(w(z)\) which is shown in Fig. 3. From the figure, we can directly see that although the quintom model is more favored, LCDM, however, still cannot be excluded.

3. From Fig. 3 one can see that the allowed value of \(w(z)\) is in the band \([-1.7, -0.4]\), which is relatively narrow.

4. The best fit value and 1σ confidence-level regions of \(\Omega_m\) is \(0.2735^{+0.0171}_{-0.0163}\), which is also consistent with the constraint on \(\Omega_m\) in the LCDM model and the CPL model.

FIG. 3: The evolution of \(w(z)\) along with \(z\) for the model considered in this manuscript. The result is consistent with the cosmological constant in the 1σ CL.

V. CONCLUSION

In this paper, we develop a new parameterization which is the modification of Efstathiou’ parameterization. In this new parameterization, there are three free parameters: \(w_0\), \(w_a\) and \(\Omega_m\). Then we carry out the global fitting on these model using the current data: SNe Ia, BAO, CMB and OHD. From the analysis, the best fit values of the cosmological parameters with 1σ confidence-level regions are: \(\Omega_m = 0.2735^{+0.0171}_{-0.0163}\), \(w_0 = -1.0537^{+0.1432}_{-0.1511}\) and \(w_a = 0.2738^{+0.8018}_{-0.8288}\). From the analysis, we can directly see that the quintom model is more favored, but this result is also consistent with the LCDM model in the 1σ CL.

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