Natural convection in a three-dimensional cavity filled with nanofluid and partially heated: Effect of the heating section dimension

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Abstract. This study focuses on heat transfer by natural convection in a three-dimensional cavity filled with nanoparticles and partially heated from the side with a uniform temperature. The opposite wall of the cavity is maintained in a cold temperature. The effect of nanofluid type on thermal phenomena within the cavity was analyzed for different sizes of the heating section, using the control volume method. The governing parameters are: the Rayleigh number \(10^3 \leq \text{Ra} \leq 10^5\), the volume fraction \(0 \leq \Phi \leq 0.1\), the heating section size \(0.5 \leq \epsilon \leq 1\), and the nanofluid type. The results represent a great interest in terms of the flow and heat transfer through the cavity depending on the chosen parameters sets.

Key words. Natural convection, Nanofluids, Tridimensional cavity.

1. Introduction

The natural convection has been particularly and considerably studied these past years because of its many practical applications in relation with the energy management, air conditioning, cooling of the electronic components etc. In order to improve the heat transfer by convection in these applications, the researchers started looking deeper into the structure of the matter at the molecular level which has given birth to the development of nanofluids: a colloidal solution composed of particles of nanometric size in suspension in a conventional fluid. The term nanofluid was proposed for the first time in 1995 by Choi [1] and opened the door to a new technology which is characterized by higher thermal properties related. In fact, this new concept has been proposed as a technique to improve the performance of the thermal transmission of conventional fluids. Thus, the interest presented by the nanofluids has given rise to numerous studies [2-4], which have focused on the determination of their thermophysical properties such as the thermal conductivity, specific heat and the dynamic viscosity and their impact on the flow and heat transfer. However, it should be noted that most of the available studies have considered the two-dimensional case when a three-dimensional approach presents more flow visualization opportunities and better approach of the heat transfer.

2. Problem Formulation

A schematic of the physical problem and coordinates are shown in figure 1. A part of the left vertical wall of the cavity is maintained at a hot temperature \(T_h\) while the opposite one is maintained at a cold temperature \(T_c\). The other walls are considered adiabatic. The governing parameters are: the volume fraction \(\Phi\) \((0 \leq \Phi \leq 0.1)\), the Rayleigh number \(\text{Ra} \) \(10^3 \leq \text{Ra} \leq 10^5\) and the heating section size \(0.5 \leq \epsilon \leq 1\).

The considered nanofluids are: water + \(\text{Al}_2\text{O}_3\); Water + Cu and Water + \(\text{TiO}_2\).
Figure 1. Studied configuration and coordinates.

The governing equations are discretized by finite volume method, adopting the Boussinesq approximation and neglecting the viscous dissipation. The physical parameters are given by the following equations:

- The density:
  \[ \rho_{nf} = (1 - \phi) \rho_f + \phi \rho_{np} \]  
  \[ (1) \]

- The heat capacitance of the nanofluid:
  \[ (\rho C_p)_{nf} = (1 - \phi) (\rho C_p)_f + \phi (\rho C_p)_{np} \]
  \[ (2) \]

- Thermal expansion coefficient:
  \[ \beta_{nf} = (1 - \phi) \beta_f + \phi \beta_{np} \]
  \[ (3) \]

- The dynamic viscosity of the nanofluid:
  It's given by Brinkman [5] who has extended the Einstein formula to cover a wide range of volumetric concentrations.
  \[ \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}} \]
  \[ (4) \]

- Thermal diffusivity of nanofluids:
  \[ \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}} \]
  \[ (5) \]

- The thermal conductivity of nanofluids:
  It is given by Maxwell [6] as follows:
  \[ \frac{k_{nf}}{k_f} = \frac{k_{np} + 2k_f - \left[ 2(k_f - k_{np})\phi \right]}{k_{np} + 2k_f + \left[ \phi(k_f - k_{np}) \right]} \]
  \[ (6) \]
\( k_{nf}, k_f \) and \( k_{np} \) are respectively the thermal conductivities of the nanofluid, the basic fluid and solid nanoparticles.

### Table 1. Thermophysical properties of water and nanoparticles

|          | \( \rho (\text{kg.m}^{-3}) \) | \( \beta (\text{K}^{-1}) \) | \( k (\text{W.m}^{-1}.\text{K}^{-1}) \) | \( C_p (\text{J.kg}^{-1}.\text{K}^{-1}) \) |
|----------|-------------------------------|----------------------------|---------------------------------|---------------------------------|
| Pure     | 997.1                         | 21x10^{-5}                | 0.613                           | 4179                            |
| Water    |                               |                           |                                 |                                 |
| AL_2O_3  | 3970                          | 0.85x10^{-5}             | 40                              | 765                             |
| Cu       | 8933                          | 1.67x10^{-5}             | 400                             | 385                             |
| TiO_2    | 4250                          | 0.90x10^{-5}             | 8.9538                          | 686.2                           |

Hence, the obtained dimensionless governing equations are:

\[
\begin{align*}
\frac{\partial U}{\partial X} + \frac{\partial U}{\partial Y} + \frac{\partial U}{\partial Z} &= 0 \\
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} + W \frac{\partial U}{\partial Z} &= -\frac{\partial P}{\partial X} + \frac{\mu_d}{\alpha_r \rho_d} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial Z^2} \right) \\
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} + W \frac{\partial V}{\partial Z} &= -\frac{\partial P}{\partial Y} + \left( \frac{P_r \beta_r}{\rho_n \beta_n} + \frac{\mu_d}{\alpha_i \rho_i} \right) \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} + \frac{\partial^2 V}{\partial Z^2} \right) \\
U \frac{\partial W}{\partial X} + V \frac{\partial W}{\partial Y} + W \frac{\partial W}{\partial Z} &= -\frac{\partial P}{\partial Z} + \left( \frac{\mu_d}{\alpha_i \rho_i} \right) \left( \frac{\partial^2 W}{\partial X^2} + \frac{\partial^2 W}{\partial Y^2} + \frac{\partial^2 W}{\partial Z^2} \right) \\
U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} + W \frac{\partial \theta}{\partial Z} &= \frac{\alpha_d}{\alpha_f} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} + \frac{\partial^2 \theta}{\partial Z^2} \right)
\end{align*}
\]

The adopted thermal boundary conditions are:
\[
\theta = -0.5 \quad \text{for} \quad X = 1 \quad \text{on the cold wall}
\]
\[
\theta = 0.5 \quad \text{for} \quad X = 0 \quad \text{on the heating section}
\]
\[
\frac{\partial \theta}{\partial n} = 0 \quad \text{On the adiabatic walls}
\]

Where \( n \) is the normal to the considered wall.

The adopted hydrodynamic boundary conditions are:

\[
U = V = W = 0 \quad \text{for all walls.}
\]

The local Nusselt number is defined as:

\[
Nu_l (y,z) = \frac{k_{nf} \nabla \theta \cdot \hat{e}_x}{k_f}
\]

The average Nusselt number, \( Nu_a \), is defined as the integral of the temperature flux through the vertical cold wall and formulated as:

\[
Nu_a = \frac{k_{nf}}{k_f} \int_S \nabla \theta \cdot \hat{e}_s \, dydz
\]
3. Numerical method
The Navier-Stokes and energy equations governing the problem are discretized by the finite volume method, using the Boussinesq approximation and neglecting the viscous dissipation. To overcome the difficulty associated with the determination of the pressure, the SIMPLEC algorithm is used to solve the momentum equations coupled with the continuity one. The Alternating Direction Implicit scheme (ADI) is then adopted to solve the algebraic discretized equations. The obtained numerical code was validated by comparing its results with those of Ravnik et al. [2]. The results show that the differences in terms of Nusselt do not exceed 0.5% for $Ra = 10^{3}$ and 2% to $Ra = 10^{5}$. Figure 2 shows also a good agreement between the two works in terms of the velocity components V(X) and V(Y) for $Ra = 10^{3}$ and Pr = 6.2.

![Figure 2. Comparison of the velocity V(X) and V(Y) between our results and those of Ravnik et al. [2]](image)

To check the effect of the grid size, preliminary tests have been conducted for different sets of the governing parameters. The presented results (Table 2) are relative to the nanofluid water-Al$_2$O$_3$ ($\Phi=0.1$) and $Ra=10^{5}$. Hence, the non-uniform staggered grid of 41x41x41 nodes was estimated to be appropriate for the present study since it permits a good compromise between the computational cost (a significant reduction of the execution time) and the accuracy of the obtained results.

| Grid       | 31x31x31 | 41x41x41 | 51x51x51 | 61x61x61 |
|------------|----------|----------|----------|----------|
| $Nu_a$     | 1.357282 | 1.353103 | 1.350895 | 1.349503 |

4. Results
A numerical study was conducted to investigate the effects of heating condition on natural convection in an enclosure filled with a nanofluid. The Prandtl number of water which is the base fluid is equal to 6.2, and the nanoparticle volume fraction is varying between 0 and 10%. Three types of nanoparticles are considered in the present work: Al$_2$O$_3$, Cu and TiO$_2$. Their properties are listed in Table 1. The range of Rayleigh number is $10^3 \leq Ra \leq 10^5$ and the heating section size is $(0.5 \leq \varepsilon \leq 1)$.
In this section, we present the isotherms obtained for different values of the governing parameters and also the speed variation and the resulting Nusselt number.
Figure 3 presents the temperature distribution within the three dimensional cavity for Al$_2$O$_3$, $Ra = 10^5$, $\Phi = 0.04$ and $\varepsilon=0.75$. The figure shows that the fluid is moving from the heating wall to the cold one, in such a way that the heat transfer rate is permanently maintained within the cavity.
Based on different preliminary tests; the plan Z = 0.5 was chosen for the following presentations. In fact, this plan, perpendicular to the vertical wall containing the heating section, is characterized by high activity and presented adequately the fluid motion and the heat transfer within the enclosure. In order to study the effect of the heating section dimension \( \varepsilon \), the hydrodynamic and thermal fields in the cavity are shown in figure 4 for Al\(_2\)O\(_3\), Ra = 10\(^5\), \( \Phi = 0.04 \) and three dimensions \( \varepsilon = 0.5 \), \( \varepsilon = 0.75 \) and \( \varepsilon = 1 \). Hence, figures 4a and 4b present respectively the streamlines and the isotherms in the plan (Z=0.5) for the three dimensions. For all the considered cases, the flow consists mainly of two counter-rotating cells and secondary flows appearing for \( \varepsilon = 0.75 \) and in the perpendicular direction. The resulting flow is symmetrical to the mid plan in the case of \( \varepsilon = 1 \). The isotherms show an increase of the heat transfer with increasing \( \varepsilon \).
Figure 4. the streamlines (left) and isotherms (right) at $Z=0.5$ for $Al_2O_3, Ra = 10^5$, $\Phi=0.04$ and different $\varepsilon$.

Figure 5 illustrates the temperature contours in the central plan $Z=0.5$ for $Ra=10^3, 10^5$, $\Phi$ varying between 0 and 0.04 and $0.5 \leq \varepsilon \leq 1$. The figures show that for all the considered nanofluids, the temperature distribution has the same behavior in the central part of the enclosure. However, differences are observed near the walls due to the increasing size of the heating section. In fact, increasing $\varepsilon$ causes a better heat transfer from the left wall of the cavity to the cold one as shown in the corresponding isotherms. This effect is observed for all the considered nanofluids. The differences between the pure water and the nanofluid are shown by the variation of the volume fraction between 0 and 0.04. The figures corresponding to $Ra=10^3$, show a stratification of the isotherms from the hot wall to the cold one. This is due to the dominance of conduction in this case. The differences due to the increase of $\Phi$ are very small in the considered range and disappear totally in the center of the cavity. This trend is encountered for all the considered nanofluids. The same remarks can be said when increasing $Ra$ to $10^5$. However, a distortion of the isotherms can be seen in this case, attesting of a better heat transfer by convection through the cavity.
| \( \varepsilon \) | 
|---|---|---|
| 0.75 | 1 | 0.5 |

Water + Al\(_2\)O\(_3\) for Ra=10\(^3\) | Water + Cu for Ra=10\(^3\) | Water + TiO\(_2\) for Ra=10\(^3\) |

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As the hot and cold walls of the cavity are positioned in the X direction, the main vortex induced by the natural convection, is evidenced in the Z plan. Hence, the plan Z = 0.5 was chosen to present the profiles of horizontal velocity $V(X)$ and vertical one $V(Y)$. The comparison of the profiles corresponding to the three types of nanofluids is shown in Figure 6, for $Ra = 10^3$ and $10^5$ and the volume fraction varying in the range (0, 0.05, 0.1).

We can notice from the figures that the nanofluid moves slower than pure water. It is also observed that the reduction of velocity of the nanofluid with the volume fraction is more accentuated for small values of Rayleigh number.

The difference of speed between the three types of nanofluids decreases with increasing Rayleigh number. Therefore the nanoparticles speeds approach the speed of pure water with increasing Ra.

When comparing the speed profiles between the different nanofluids only small differences are observed. However, the nanofluid Cu reaches higher speeds than Al$_2$O$_3$ and TiO$_2$. 

\[
Ra = 10^3 \quad \text{Ra} = 10^5
\]
Figure 6. Profiles velocity V (X) and V (Y) of the nanofluids in the central plane (Z = 0.5H) for different values of the volume fractions of nanoparticle Φ (Φ=0, 0,05 et 0,1), Ra=10\(^3\), Ra=10\(^5\) and ε =0,75.

The Tables 3 and figure 7 show the variation of the average Nusselt number (\(Nu_a\)) with the Rayleigh number and the heating section size, for various nanofluid type. For all considered nanofluids, the average Nusselt number increases with the Rayleigh number and the size of the heating portion ε. This increase is due to the improvement of the effective thermal conductivity of the nanofluid, when Ra increases. In addition, the heat transfer rate is maximum for copper (Cu), which has the largest thermal conductivity compared to TiO\(_2\) and Al\(_2\)O\(_3\).

Table 3. the variation of the Nusselt number as a function of Ra, ε and volume fraction.

| \(Nu_a\) | ε =1 | ε =0.75 | ε =0.5 |
|-------|------|---------|--------|
| \(Al_2O_3\) | \(Φ =0.00\) | \(Φ =0.02\) | \(Φ =0.04\) | \(Φ =0.00\) | \(Φ =0.02\) | \(Φ =0.04\) | \(Φ =0.00\) | \(Φ =0.02\) | \(Φ =0.04\) |
| \(10^3\) | 1.080170 | 1.078004 | 1.177496 | 1.032905 | 1.078004 | 1.126788 | 0.8485183 | 0.8878419 | 0.9300382 |
| \(10^4\) | 2.121895 | 2.040174 | 2.164778 | 2.020745 | 2.040174 | 2.057273 | 1.544651 | 1.563698 | 1.581932 |
| \(10^5\) | 4.671789 | 4.422078 | 4.799010 | 4.352990 | 4.422078 | 4.486072 | 3.022230 | 3.078850 | 3.133210 |
5. Conclusion

The study of natural convection in a partially heated three-dimensional cavity, filled with nanofluid was performed numerically. The results show the effect of the governing parameters, namely the Rayleigh number \(10^3 \leq \text{Ra} \leq 10^5\), the heating section size \(\varepsilon \leq 1\), the volume fraction \(\Phi\) varying between 0 and 0.1 and nanofluid type \(\text{Al}_2\text{O}_3\), Cu and \(\text{TiO}_2\).

It should be noted as well as the increase of the heating section size \(\varepsilon\) and Ra results in increased the amount of heat removed by the same nanofluid. Similarly, increasing the volume fraction causes an intensification of the flow and an increase in heat exchange.
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