REMARKS ON CP ASYMMETRIES IN $D^0/\bar{D}^0 \to K_S\pi^+\pi^-$

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Abstract

We consider the interference of resonant amplitudes leading to the final state $K_S\pi^+\pi^-$ in $D^0/\bar{D}^0$ decays. Each of these amplitudes consists of both Cabibbo allowed and doubly Cabibbo suppressed transitions. The role of strong phase arising out of Breit-Wigner resonant propagators is emphasised. Invoking the $\Delta S = \Delta Q$ rule, $K_S$ in the final state is identified as the mass eigenstate of superposed weak eigenstates $K^0$ and $\bar{K}^0$. A nonzero CP asymmetry appears to be possible in three body decays.

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As immense activities are on at charm/B factories searching for CP violating effects, the study of weak dynamics of charm sector acquires renewed interest. The CP asymmetry occurs if there exist nonvanishing weak and strong phases provided by a pair of amplitudes which are distinct by both phases with respect to one another, through interference \[1\]. For instance, in the two body $D^0$ decays into $K^\pm\pi^\mp$, the strong phase arises due to strong interaction effects such as rescattering and final state interaction. But, such a strong phase is small, for example it is about $13^\circ$ in the model of Buccella \textit{et al} \[2\], thus resulting in a CP asymmetry of $O(10^{-3})$ \[3,4\].

Alternatively, one can look at three body modes. The three body final state is reached by more than one way, namely, nonresonant and resonant modes. There are many resonant modes: That is, $D^0 \rightarrow M_1 R \rightarrow M_1[M_2M_3]_R$, where $R$ stands for resonance and $M$’s for mesons. Given two resonant modes that lead to the same final state, there arise distinct strong phase between them due to the Breit-Wigner (BW) resonant propagator and the angular momentum quantum numbers of the resonance and of its decay products \[5\]. Thus, a strong phase is nontrivial.

Besides, the correlation between the net strangeness (produced)\footnote{In the strange decays, this is change in strangeness.} and the net charge (involved), namely, the $\Delta S = \Delta Q$ rule. Unlike in strange decays where violation strangeness-charge symmetry means the mixing of $K_S$ and $K_L$ \[6\], in charm decays into strange final state, this rule qualifies $K_S$ is the mixed state of the weak eigen states $K^0$ and $\bar{K}^0$. This rule implies the coherent superposition of CA and DCS transitions.

In this letter, we look at the significance of strong phase that arises due to BW propagators of intermediate resonances which lead to a common final state in $D^0$ three body decays, as an extension of the Atwood and Soni’s proposal for $B$ decays \[3\] to charm sector and the importance of invoking the $\Delta S = \Delta Q$ rule thereof. For $D^0 \rightarrow K_S\pi^+\pi^-$, we consider the resonant modes of $K^*(890)\pm, \rho^0(770)$ and $f_0(990)$:
\[ D^0 \to K^+\pi^- \to \bar{K}^0\pi^+\pi^- \quad (1) \]
\[ D^0 \to \bar{K}^0\rho^0 \to \bar{K}^0\pi^+\pi^- \quad (2) \]
\[ D^0 \to \bar{K}^0f_0 \to \bar{K}^0\pi^+\pi^- \quad (3) \]

These resonances show up in the Dalitz plot of BABAR data [7].

Let us write the amplitude for the resonant decay \( D^0 \to K_S\pi^+\pi^- \) mediated by a resonance \( i \) as

\[ M_i = A_i \Pi_i B_i \quad (4) \]

where \( A_i \) is the weak part of the amplitude containing the CKM phase, \( \Pi_i = [s - m_i^2 + i\Gamma_i m_i]^{-1} \) the BW propagator that provides the strong phase and the strong coupling \( B_i = (16\pi m_i^3\Gamma_i/\lambda^{1/2}(m_i^2, m_1^2, m_2^2))^{1/2} \). In order to extract the strong phase, we rewrite \( \Pi_i \) as

\[ \Pi_i = \bar{\Pi}_i e^{i\delta_i} \quad (5) \]

The strong phase \( \delta \) is given by the width and mass of the resonance.

We take into account four amplitudes due to them: both the CA and DCS transitions of each. The amplitude for \( D^0 \to K_S\pi^+\pi^- \) is then expressed after factoring out the strong and weak phases as

\[ M = e^{i\gamma_1} \left[ e^{i\delta_i} M_i^C + e^{i\delta_j} M_j^C \right] + e^{i\gamma_2} \left[ e^{i\delta_i} M_i^S + e^{i\delta_j} M_j^S \right] \quad (6) \]
\[ \bar{M} = e^{-i\gamma_1} \left[ e^{i\delta_i} \bar{M}_i^C + e^{i\delta_j} \bar{M}_j^C \right] + e^{-i\gamma_2} \left[ e^{i\delta_i} \bar{M}_i^S + e^{i\delta_j} \bar{M}_j^S \right] \quad (7) \]

where \( \gamma_{1,2} \) stands for the CA and DCS weak phase, \( \delta_{i,j} \) the strong phase and the superscript \( C \) and \( S \) for CA and DCS transitions. The amplitude \(|M|\) is resultant of coherent superposition of the four amplitudes [8]. As notation, \( K^* \) is identified with \( i \) and \( \rho^0 \) and \( f_0 \) with \( j \).

Then we have the asymmetry as

\[ a_{CP} = \frac{2W \sin \Delta \sin \phi}{1 + X + 4Y \cos \Delta + 2Z \cos \phi + 2W \cos \Delta \cos \phi} \quad (8) \]

where \( \phi = |\gamma_1 - \gamma_2| \) is the weak phase and \( \Delta = |\delta_i - \delta_j| \) the strong phase and
\[ W = R_3 + R_1R_2, \quad X = R_1^2 + R_2^2 + R_3^2, \quad Y = R_1 + R_2R_3, \quad Z = R_2 + R_1R_3 \]  

(9) 

with \( R_1, R_2 \) and \( R_3 \) respectively the ratio of \( M_j^C, M_j^S \) and \( M_j^S \) with respect to \( M_j^C \):

\[
\begin{align*}
R_1 &= \frac{f_\rho \langle \bar{K}^0|(\bar{s}c)_{V-A}|D^0\rangle}{f_\pi \langle K^*|(\bar{s}c)_{V-A}|D^0\rangle} \frac{\hat{\Pi}_\mu B_\rho}{\Pi_{K^*} B_{K^*}}, \\
R_2 &= \frac{f_{K^*} \langle \pi|(\bar{d}c)_{V-A}|D^0\rangle}{f_\pi \langle K^*|(\bar{s}c)_{V-A}|D^0\rangle} \frac{\hat{\Pi}_\mu B_\rho}{\Pi_{K^*} B_{K^*}}, \\
R_3 &= \frac{f_{K^0} \langle \rho|(\bar{d}c)_{V-A}|D^0\rangle}{f_\pi \langle K^*|(\bar{s}c)_{V-A}|D^0\rangle} \frac{\hat{\Pi}_\mu B_\rho}{\Pi_{K^*} B_{K^*}}
\end{align*}
\]

(10) 

where in \( \text{(10)} \) factorisation approximation is applied for the weak amplitude. Similarly, \( R_i \)'s are for \( f_0 \). We note caution that not much is known learily about \( f_0 \). However, we have treated this on par with \( \rho^0 \). The width of these resonances are 50, 150 and 50-100 MeV respectively of \( K^*(890), \rho^0(770) \) and \( f_0(980) \). In this note, for calculational purpose, the width of \( f_0 \) is chosen as 75 MeV.

The strong phase \( \Delta \) is determined as a function of \( s \). As an order of magnitude, the weak phase \( \phi \) is chosen as \( 0.4 \times 10^{-3} \) as \( \phi \sim \arg(V_{cd}V_{us}^*/V_{cs}V_{ud}) \). The ratios of the weak matrix elements in \( \text{(10)} \) are assumed to be \( O(1) \) as they may turn out to be so in the \( SU(3) \) limit. The CP asymmetry is shown in Fig. \( \text{(1)} \) as a function of \( s \). The strong phase \( \Delta \) is about \( 57^{\circ} \) for \( K^* \) and \( \rho^0 \) and about \( 64^{\circ} \) for \( K^* \) and \( f_0 \) at \( s = m_{K^*}^2 \). At \( s = m_{K^*}^2 \), the amplitudes are in phase with respect to one another, yielding constructive interference of each pair leading to the same final state. The minima of \( a_{CP} \) for \( K^* - \rho \) and \( K^* - f_0 \) occurs respectively at \( s = 0.75 \) and \( 0.85 \) GeV\(^2\) respectively, whereas the maxima are respectively at \( m_\rho^2 \) and \( m_{f_0}^2 \).

\( \Delta S = \Delta Q \) rule: At quark level, \( D^0 \to K_S\pi^+\pi^- \) proceeds via \( c \to s\bar{d}u(q\bar{q}) \), internal \( W \)-emission, and \( c \to d\bar{s}u(q\bar{q}) \), external \( W \)-emission, corresponding to the CA and DCS transitions respectively. While the net charge \( \Delta Q \) is zero in both, the strangeness production \( \Delta S \) is \(-1 \) and \(+1 \) respectively. Thus, for \( D^0 \to K_S\pi^+\pi^- \), the \( \Delta S = \Delta Q \) is satisfied by \( \text{(8)} \) if and only if the mass eigenstate \( K_S \) in the final state is the superposition of the weak eigenstates \( K^0 \) and \( \bar{K}^0 \). In other words, in order to reach this final state, the coherent superposition of the CA and DCS transitions is the basic requirement.

In the case of \( D^\pm \to K_S\pi^\pm\pi^0 \), the intermediate resonances play similar role in giving rise to a strong phase. As noted in \( \text{(4)} \), the mixing of final state neutral \( K \) mesons brings
in an additional phase factor of $3.3 \times 10^{-3}$. The role of this extra contribution to the CP asymmetry would eventually be significant.

To conclude, we considered the effects arising out of the BW propagators. These effects are large and thus lead to a possible large CP asymmetry. Application of the strangeness-charge symmetry rule reveals the structure of $K_S$ in the final state and is expected to shed more light on the possible new physics which would be responsible for a reasonable CP violating effects. A precise determination of CP asymmetry depends on the contribution of angular momenta of the resonances and of their decay products.

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FIG. 1. CP asymmetry Vs. $s$: $\rho^0$ and $K^*$ (solid line), $f_0$ and $K^*$ (dashed line). The CP asymmetry exhibits minimum, sign change and maximum as a function $s$. The weak phase $\phi = 0.4 \times 10^{-3}$. 
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