AN EFFICIENT POWER REDUCTION TECHNIQUE IN ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING SYSTEM USING TONE RESERVATION

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ABSTRACT
In a basic communication system, the data’s are modulated onto a single carrier frequency. The available bandwidth is then totally occupied by each symbol. This kind of system can lead to Inter-Symbol-Interference (ISI) in case of frequency selective fading. The basic idea of Orthogonal Frequency Division Multiplexing (OFDM) is to divide the available spectrum into several orthogonal sub channels so that each narrowband sub channel experiences almost flat fading. In OFDM system, orthogonally placed sub-carriers are used to carry the data from the transmitter end to the receiver end. However, the major challenging issue is to reduce PAPR and Bit Error Rate (BER). The reduction algorithm for OFDM signals is presented in this project using convex optimization technique along with Tone Reservation (TR). Tone reservation uses other unused or reserved tones to design a peak-cancelling signal that lowers the PAR of a transmit OFDM block. Similarly it lowers the bit error rate. In contrast to previous methods, the tone reservation with convex optimization implies a very quick convergence of minimum-PAR and bit error rate solution at a lower computational cost. Also the effects of oversampling at \( l = 1, 2, 4, 8 \) on the performance of proposed OFDM system is discussed.

Keywords: Orthogonal Frequency Division Multiplexing (OFDM), Convex Optimization, Peak to Average Power Ratio (PAPR), Tone Reservation (TR), Complementary Cumulative Distribution Function (CCDF), Bit Error Rate (BER), Signal to Noise Ratio (SNR)

1. INTRODUCTION

1.1. Multi-Carrier Modulation
With the ever growing demand of this generation, need for high speed communication has become an utmost priority. Various Multi-Carrier Modulation (MCM) techniques (Krongold and Jones, 2004) have been evolved in order to meet these demands. Orthogonal Frequency Multiplexing (OFDM) is a Frequency Division Multiplexing (FDM) scheme utilized as a digital MCM method. A large number of closely spaced orthogonal sub-carriers (Bauml et al., 1997) are used to carry data. The data is divided into several parallel streams of channels, one for each sub-carrier. Each sub-carrier is modulated with a conventional modulation scheme.

In spite of the attractive attributes such as high spectral efficiency, crosstalk elimination due to orthogonal sub-carriers which avoids inter-carrier guard band, robustness against frequency selective fading and these advantages are outweighed by the high Peak-to-Average Power Ratio (PAPR) of OFDM systems that results from the summation of multiple carriers with random phases.

1.2. OFDM Spectrum
OFDM is a special form of MCM, which is particularly suited for transmission over a dispersive channel. Here the different carriers are orthogonal to each other and they are totally independent of one another. This is achieved by placing the carrier exactly at the nulls in the modulation spectra of each other.

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The basic principle of OFDM is to split a high-rate data stream into a number of lower rate streams that are transmitted simultaneously over a number of subcarriers. Because the symbol duration increases for lower rate parallel subcarriers, the relative amount of dispersion in time caused by multipath delay spread is decreased. Figure 1 represents the spectrum of a single OFDM subchannel and OFDM spectrum.

1.3. Problem Formulation

High PAPR has two undesirable effects when these signals are passed through nonlinear devices such as High Power Amplifier (PA): First, an increase of the equivalent noise in the signal band and second, an increase of the out-of-band signal power. The PA will be no longer in the linear range of operation. A simple approach is to sufficiently back off the PA, so that it operates in the linear region, which is far below the saturation point. This results in the significant amount of energy loss, generation of high temperatures due to the large power supply.

While this approach was acceptable for 2G systems, it is inappropriate for 3G and 4G systems, where power efficiency is a critical issue. So reducing PAPR (Cimini and Sollenberger, 2000) and Bit Error Rate (BER) is a major concern.

1.4. System Model

Figure 2 shows the generation of OFDM signal. The signal to be transmitted is in serial form and it is
1.5. OFDM System Parameters

1.5.1. OFDM Symbol

Let \( c \in \mathbb{C}^N \) (Aggarwal and Meng, 2006) be the frequency domain OFDM symbol and \( \{c(i), i = 1, \ldots, N\} \) be the symbol value carried in the \( i \)-th sub-carrier. Then, the time domain OFDM symbol, \( x \in \mathbb{C}^N \) corresponding to \( c \) with 1 times over-sampling is expressed as Equation 1 and 2:

\[
x(k) = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} c(i) e^{j\frac{2\pi k i}{N}}
\]

\[
c(i) = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} x(k) e^{-j\frac{2\pi k i}{N}}
\]

where, \( i = 1, \ldots, N \).

1.6. PAPR

Let \( c^0 \) and \( x^0 \) represents the original frequency domain and time domain OFDM symbols respectively. Also, let \( c \) and \( x \) denote the processed frequency domain and time domain OFDM symbols. Then the PAPR of the original time domain symbol is defined as Equation 3:

\[
PAPR = \frac{\max_{k=1}^{N} |x^0(k)|^2}{\frac{1}{N} \sum_{k=1}^{N} |x^0(k)|^2}
\]

1.7. Existing Technique

1.7.1. Tone Reservation

Multicarrier modulation has become a key communication systems technology; for example, Discrete Multi Tone modulation (DMT) is used in the Asymmetric Digital Subscriber Line (ADSL) standard and coded OFDM schemes are used for wireless LAN (802.11a and Hiperlan2) and terrestrial digital television and audio broadcasts in Europe. Multicarrier communication systems have distinct advantages over single-carrier systems, but suffer from a serious drawback: the approximately Gaussian-distributed output samples cause a high Peak-To-Average Power Ratio (PAR). The analog hardware at the transmitter requires an expensive High-Power Amplifier (HPA) to avoid clipping and/or soft thresholding that causes nonlinear output. The power consumption of a HPA depends largely on its peak power output rather than the average output power and thus handling occasional large peaks leads to low power efficiency common to all OFDM systems is a large Peak-To-Average-Power Ratio (PAR), which can lead to low power efficiency and nonlinear distortion at the transmit power amplifier. Tone reservation (Mobasher and Khandani, 2006) uses other unused or reserved tones to design a peak-cancelling signal that lowers the PAR of a transmit OFDM block. In contrast to previous methods, the new active-set method proposed here converges very quickly toward a minimum-PAR solution at a lower computational cost.

1.8. Differential Scaling Technique

The probability distribution of amplitudes of the OFDM signal follows Rayleigh distribution and thus the probability of high peaks is very less. An upper threshold above which the signal amplitudes do not contribute much to the signal is determined as follows. Using simulations, we have determined BER for the modified signals along with PAPR. We select the clipping threshold at which the BER is degraded from \( 1.5 \times 10^{-3} \) to \( 3.5 \times 10^{-3} \) at SNR of 10dB and the amplitudes above this clipping threshold are clipped. Instead of clipping the signal further to reduce the PAPR, we consider a reversible process-Differential Scaling which would reduce the PAPR but not deteriorate the BER. Since different ranges of amplitudes of the signal are scaled in a different manner, it is called Differential Scaling. We have considered three types of scaling as described below.

1.9. Scale Up

In this method, we scale up the lower amplitudes of the signal by a factor of \( \beta \). This leads to increase the average value without affecting the peak values. Therefore, the resulting PAPR reduces. The PAPR reduction function can be defined as:
where, $x_p$ is the amplitude peak value occurring in an OFDM symbol block, $\alpha$ is the factor deciding the clipping threshold in terms of percentage of the peak value and $\beta$ is the scaling factor for the range $(0, A)$ whose value is greater than one. The values of the parameters used are mentioned at the end of this section.

1.10. Scale Down

In this method, we scale down the higher amplitudes of the signal by a factor of $\gamma$. This leads to decrease the peak value. Although the average value would also fall down, the resulting PAPR reduces. Because the reduction in peak power is greater than the reduction in the average power. The PAPR reduction function can be defined as:

$$h(x) = \alpha x_p, \quad \text{if} \quad x > \alpha x_p$$

$$= \beta x, \quad \text{if} \quad B \leq x \leq \alpha x_p$$

$$= x, \quad \text{if} \quad A > B$$

where, $x_p$ is the amplitude peak value occurring in an OFDM symbol block, $\alpha$ is the factor deciding the clipping threshold in terms of percentage of the peak value and $\gamma$ is the scaling factor for the range $(\beta, \alpha x_p)$ whose value is less than one. The values of the parameters used are mentioned at the end of this section.

1.11. Scale Up and Down

In this method, we combine both the above-mentioned approaches i.e. up-scaling and down-scaling. This method exploits the advantages of both the methods. Hence, a PAPR can be reduced considerably. The PAPR reduction function can be defined as:

$$h(x) = \alpha x_p, \quad \text{if} \quad x > \alpha x_p$$

$$= \beta x, \quad \text{if} \quad x < A$$

$$= x, \quad \text{if} \quad A \leq x \leq B$$

where, $x_p$ is the amplitude peak value occurring in an OFDM symbol block, $\alpha$ is the factor deciding the clipping threshold in terms of percentage of the peak value. $\beta$ is the scaling factor for the range $(0, A)$ and $\gamma$ is the scaling factor for the range $(B, \alpha x_p)$.

In order to make all these scaling techniques realizable, a marker needs to be used. The marker is basically a small set of signal values that needs to be transmitted along with the information signal. Its job is to keep track of values which have been scaled at the transmitter. The same values would be reversibly scaled at the receiver. The marker may be accommodated like the pilot carriers or sent on another frequency orthogonal to the carriers. The optimum values of simulation parameters for which PAPR is minimum is documented in the following table:

| Clipping threshold ($\alpha$) | 0.47 |
| Scale down factor ($\gamma$) | 0.8  |
| Lower limit for scale down (B) | 1.2  |
| Scale up factor ($\beta$) | 2.0  |
| Upper limit for scale up (A) | 0.5  |

1.12. Proposed Technique

1.12.1. Tone Reservation and Convex Optimization

In this study, we reformulate the TR method as a convex optimization problem in which our goal is to maximize the Signal-to-Distortion Ratio (SDR) of the OFDM system instead of minimizing the PAPR. We assume that the power amplifier of the transmitter has fixed level hard clipping and our objective is to reduce the peak of signal magnitudes to become smaller than this fixed level. The performance of the proposed algorithm compared with that of up-down scaling, the proposed algorithm reveals that the SDR is a better criterion than the PAPR for TR approach. Interestingly, the algorithm in results in smaller PAPR, however optimizing SDR criterion results in an enhanced Bit-Error-Rate (BER) performance for the same transmit power. In contrast to some existing methods such as our proposed method does not take advantage of the frequency selectivity of the multi-path fading channel. However, those techniques (such as pre-coding) could be easily combined with our approach in order to further improve the error rate.

1.13. Convex Optimization

It refers to minimization of a convex objective function subject to convex constraints. A local optimum in convex problem is also a global optimum. Convex Optimization is used widely in design and analysis of communication systems.

Condition for convexity:

- $f(ax + by) \leq a f(x) + b f(y)$
- General form of optimization problem
- Minimize $f(x)$ (objective function)
- Subject to $g(x) \leq b$ (constraint function)
Fig. 3. Flowchart for PAPR reduction Technique
where, $x$ is the optimization variable and the vector $x^*$ is called optimal solution if it has the smallest objective value among all the vectors that satisfy the constraints. The techniques used for the reduction of Squared Crest Factor was shown in flowchart (Fig. 3) below.
1.14. Simulation Results

Simulation and analysis of the proposed method is carried out for 128 sub-carrier OFDM system with QPSK modulation scheme.

The data phase representation of the transmitted signal was shown in Fig. 4. In the Scatter plot diagram (Fig. 5) the transmitted data is displayed as collection of points in which the horizontal axis represents In-phase points and the vertical axis represents Quadrature phase points. Figure 6 represents the OFDM signal that is transmitted. The BER vs. SNR plot and CCDF Vs. Squared Crest Factor plot (Cimini and Sollenberger, 2000; Jiang et al., 2007) for our proposed technique were shown in Fig.7 and 8. The above table (Table 1) deals with the BER performance of various sampling rate.

![OFDM signal](image1)

**Fig. 6.** Transmitted OFDM signal

![Bit error probability curve](image2)

**Fig. 7.** The BER Vs SNR Tone Reservation and Differential scaling technique
Table 1. BER Performance for various sampling rate and no of sub carriers for SNR of 9.8dB

| Sampling Rate | Number of subcarriers (N) | Bit Error Rate (BER) |
|---------------|---------------------------|----------------------|
| L = 2         | 128                       | 0.104                |
|               | 256                       | 0.110                |
| L = 4         | 128                       | 0.088                |
|               | 256                       | 0.094                |
| L = 8         | 128                       | 0.079                |
|               | 256                       | 0.086                |

2. CONCLUSION

The proposed Tone reservation technique modifies the filter response and hence produces better reduction in PAPR after applying convex optimization the PAPR is reduced from 11dB to 4.36dB. Better BER performance is achieved when compared with differential scaling. Simulation results clearly define that this method offers less distortion in the processed OFDM symbols with better out-of-band radiation. The algorithm was defined for QPSK but it equally applies for other modulation techniques as well. The future works will the implementation of customized convex optimization.

3. ACKNOWLEDGMENT

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