Effective actions for compact objects in an effective field theory of gravity

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ABSTRACT: Using the effective field theory framework for extended objects and the coset construction, we build the leading order effective action for the most general compact object allowed in an effective theory of gravity as general relativity. By recognizing the symmetry breaking pattern of a charged spinning compact object, we derive all the covariant building blocks and constraints to build up the relevant invariant operators in the action to all orders. The breaking of symmetries implies the existence of Goldstone bosons that are used as building blocks through their derivatives. Due to the nature of gravity, which is a space-time symmetry, the inverse Higgs constraint is imposed to remove some of the bosons. The remaining building blocks made out of the Goldstone fields have a correspondence to the acceleration and the angular velocity degrees of freedom. By considering the allowed invariant operators, we derive the effective action for spinning extended objects used to obtain state-of-the-art results in the post-Newtonian expansion, as well as the correction to the point particle due to charge. Moreover, we build up the operators that take into account for the internal structure of the extended object, such as tidal effects, polarization and dissipation. The invariant operators in the action are accompanied by coefficients that encapsulates the properties of the compact object. We match the known coefficients of the effective action from the literature, and point out the unknown ones that are to be derived.
1 Introduction

The recent detection of gravitational waves (GWs) from various coalescing binaries [1–5] consisting of compact objects, such as black holes (BHs) and neutron stars (NSs), have opened up the possibility to test fundamental physics in the strong regime of gravity. With upcoming sensibility upgrades in current GW detectors, and future earth [6, 7] and space based detectors [8], the era of high precision gravity is arriving, and with it the potential of great discoveries. One of the key potentials is to probe the internal structure of the compact objects by matching the coefficients of the theory with GW observations [9]. Thus the need to develop theoretical models that describe the compact objects, taking into account for the different effects that can play a role, such as the spin, charge, and their internal structure.

Using the coset construction [10–13] in the effective field theory (EFT) framework for extended objects [14–16], we build the most general effective action allowed in an effective theory of gravity such as general relativity. In this theory, BHs are constrained by the no hair theorem, which states that a BH can be described by only three parameters, its mass, spin and charge, behaving effectively as a point particle [17, 18]. In this sense, we derive the leading order effective action for a charged spinning compact object. In the
EFT framework, we treat compact objects as point particles, with their additional effects and internal structure encoded as higher order corrections in the action, which are made out of the allowed invariant operators of the theory. These operators are accompanied by coefficients which encapsulate the properties and internal structure of the objects.

The coset construction is a very general technique from the EFT framework that can be used whenever there is a symmetry breaking. With this technique, it is possible to derive the covariant building blocks that transform correctly under the relevant symmetries, which then can be used to form invariant operators to build up an effective action. Any state other than vacuum will break some of the symmetries, and by correctly identifying the pattern of the symmetry breaking, we can derive its effective action. Within this approach, we can treat the coefficients that appears in front of the invariant operators as free parameters to be fixed by experiments or observations. In this sense, we are interested to know what was the full symmetry group, $G$, of the EFT, and what subgroup, $H$, was realized non-linearly, parametrized by the coset, $G/H$ [19].

An effective theory of gravity, such as general relativity, can be derived using the coset construction by weakly gauging the space-time symmetry group of gravity, the Poincaré group $ISO(3,1)$, and realizing translations non-linearly, which is parametrized by the coset $ISO(3,1)/SO(3,1)$ [16], with $SO(3,1)$ the Lorentz group. From this coset parametrization, it is possible to derive the widely known Einstein’s vierbein theory of curved space or tetrad formalism, which is a generalization of the theory of general relativity that is independent of a coordinate frame. Once the underlying theory of gravity is developed, one can proceed to identify the symmetries that an extended object breaks to derive its effective action. For instance, a point particle breaks spatial translations and boosts, while a spinning point particle breaks spatial translations and boosts, while a spinning point like object in curved space-time, implies the existence of a Nambu-Goldstone field, that corresponds to the angular velocity and acceleration of the object [16]. This yields a very natural construction for the action of spinning objects, and we show that it corresponds to the EFTs in [21, 22]. Although it has been pointed out that this is a theory for slowly spinning objects given that from construction the effective action corresponds to the low energy dynamics of the theory, it has been shown that we can consider the current observed compact objects through GWs as "slowly" spinning [9]. Thus, in this work we review and build on the EFT for spinning extended objects introduced in [16], and later developed in [23]. In [16], the building blocks of the theory are derived, and the lowest order action for spinning objects in curved space-time is built, while in [23], tidal effects and dissipation are considered to describe stellar objects in the Newtonian limit.

We derive in detail the building blocks of the effective theory and its constraints, which allows us to build up the tower of invariant operators and form the effective action to all orders. We show that the spin corrections considered in the effective theory for spinning extended objects in [22], which are used to obtain state of the art results needed for GW extraction, are encoded in the allowed boson couplings. We have extended the model by considering the electromagnetic charge $U(1)$ symmetry, as an internal symmetry in the coset parametrization, which allows us to derive the Einstein-Maxwell action. Then, by
identifying the symmetry breaking pattern, for which a point particle that is charged under a U(1) symmetry, corresponds to the eigenstate of the charge and does not break U(1), we derive the correction to the point particle.

On the internal structure of the objects, we take into account for size [14], and dissipative effects [15, 24]. The coefficients accompanying the operators of these effects, encode the microphysics of the object, and based on results in the literature, we can identify them without having to do the explicit computations. We consider tidal effects for spinning objects as in [21, 23, 25], by considering only rotationally invariant operators and associating departures from sphericity with higher order corrections. We take into account the size effects induced by charge as well, known as the polarizability. On the dissipative effects for spinning BHs, we consider the absorption of gravitational and electromagnetic waves [15], and the dissipation generated by the spin [23, 24, 26, 27]. For a NS, dissipation accounts for the energy loss due to the viscosity of the star. We match the known coefficients from the literature, and point out the unknown ones that are to be derived.

Our derived action has multiple applications in the description of the coalescence of binaries. The first application we have considered in [28], is on the post-Newtonian (PN) expansion in the non-relativistic regime of gravity [14, 29–31], which is a perturbative series in terms of the expansion parameter $v/c < 1$, with $v$ the relative velocity of the binary. The PN expansion can be used for extracting GWs, and in particular for performing numerical simulations in the late inspiral of the coalescence [9]. In the second application considered, we incorporate our action into the recently introduced EFT framework for the post-Minkowskian (PM) expansion [32–34], in which one expands over the gravitational constant, $G$, and which encodes the PN expansion. Finally, we are incorporating our derived effective action and its applications into the effective one body (EOB) formalism [35–39], cross checking known results, and implementing new ones. The EOB, by a combination of analytical and numerical results, can take into account for the full coalescence of a binary.

This work emphasizes on the foundations of the description of compact objects as an effective field theory, and provides a connection to the current used effective theories to obtain state-of-the art results required to extract high accuracy gravitational waveforms. Moreover, we shed light onto probing whether or not charged BHs can exists, and to describe neutron stars, such as pulsars and magnetars.

In section 2, we start with a very brief review of the basic ingredients of the coset construction to derive our effective action, and refer the reader to [16, 19] for a brief but more comprehensive review. In section 3, we start with a pedagogical introduction to the coset construction by deriving a classical theory of electromagnetism, as well as the effective action for charged spheres in flat space-time. This is done with particular emphasis to introduce the tools of the coset construction with a concrete and simple known example, compared to [16], where the examples of superfluids and membranes can complicate the understanding for non-experts on high energy physics. In section 4, we derive the Einstein-Maxwell action in the vierbein formalism, and then derive the covariant building blocks and constraints to build up the effective action of a charged spinning compact object in curved space-time. Then, we match the coefficients of the effective theory for compact objects from the literature. Finally in section 5, we conclude.
2 Basics of the coset construction

We start with the very basics of the coset construction to develop this paper. A brief but more comprehensive review can be found in [16, 19]. We use the notation as in [16] to consider the breaking of internal [10, 11] and space-time symmetries [12, 40] alike.

The coset construction is a very general technique from the EFT framework that can be used whenever there is a symmetry breaking. The breaking of some of the symmetries implies the existence of additional degrees of freedom, known as Nambu-Goldstone bosons or simply as Goldstone fields.\footnote{Goldstone theorem [20] implies the existence of a Goldstone field for each broken internal symmetry, but for the case in which space-time symmetries are broken, there can be a mismatch on the number of degrees of freedom and broken symmetries, for which additional constraints are needed. See the Inverse Higgs constraint below.} The coset construction is then used to derive building blocks for the Goldstone fields that transform correctly under the relevant symmetries, blocks that can be used to build up invariant operators to form an effective action. Any state other than vacuum breaks at least some of the symmetries, and by appropriately identifying the pattern of the symmetry breaking, we can use it as a guide to derive the effective action.

We can formulate an EFT using the symmetry breaking pattern as the only input, knowing the full symmetry group G that is broken, and the subgroup H that is non-linearly realized [19]. If the group is broken, $G \rightarrow H$, due to a spontaneous symmetry breaking, the coset recipe [16, 19] tells us that we can classify the generators into three categories:

\begin{equation}
\begin{align*}
P_a &= \text{generators of unbroken translations}, \\
T_A &= \text{generators of all other unbroken symmetries}, \\
X_\alpha &= \text{generators of broken symmetries},
\end{align*}
\end{equation}

where the broken generators, $X_\alpha$, and the unbroken ones, $T_A$, can be of space-time symmetries, as well as of internal ones. Whenever the set of generators for broken symmetries is non-zero, some Goldstone fields will arise. Thus, we must build an effective action for the Goldstone fields that is invariant under the whole symmetry group of consideration. The power of the coset construction, is that we can formulate an invariant EFT in which the broken symmetries and the unbroken translations are realized non-linearly on the Goldstone fields [16].

Following the coset recipe [16, 19], we do a local parametrization of the coset, $G/H_0$, with $H_0$, the subgroup of $H$ generated by the unbroken generators, $T$’s. The coset is parametrized as

\begin{equation}
g(x, \pi) = e^{iy^a(x)P_a}e^{i\pi^\alpha(x)X_\alpha},
\end{equation}

where the factor, $e^{iy^a(x)P_a}$, describes a translation from the origin of the coordinate system to the point, $x_a$, at which the Goldstone fields, $\pi^\alpha(x)$, are evaluated. This factor ensures that the $\pi$’s transform correctly under spatial translation. The group element $g$, which is
generated by the $X$’s and the $P$’s, is known as the coset parametrization. For the case of flat space-time, the translation is simply parametrized by, $e^{ix^aP_a}$, with $y(x) \equiv x$.

To derive the building blocks that depend on the Goldstone bosons and that have simple transformation rules, we first note that the Goldstone fields, when appearing in the Lagrangian, they are coupled through its derivatives. Then, we introduce the Maurer-Cartan form, $g^{-1}\partial_\mu g$, a very convenient quantity that is an element of the algebra of $G$, and that can be written as a linear combination of all the generators [16, 19],

$$\tag{2.3} g^{-1}\partial_\mu g = (e^a_\mu P_a + \nabla_\mu \pi^a X_\alpha + C^B_\mu T_B).$$

The coefficients $e^a_\mu$, $\nabla_\mu \pi^a$ and $C^B_\mu$, in general are non-linear functions of the Goldstones, and are basic ingredients of the effective theory, with $\nabla_\mu \pi^a = e^a_\mu \nabla_a \pi^a$ and $C^B_\mu = e^a_\mu C^B_a$. The explicit expression of the aforementioned ingredients can be obtained using the algebra of the group $G$.

Following the coset recipe [16, 19], we can use the coefficients of the unbroken symmetries, $C$’s, and its operators, $T$’s, to define the covariant derivative,

$$\nabla_a \equiv (e^{-1})^a_\mu (\partial_\mu + iC^B_\mu T_B). \tag{2.4}$$

This covariant derivative can be used to define higher covariant derivatives on the Goldstone fields, as well as on some of the building blocks and additional fields that transform linearly under the unbroken group. Then, by considering all the allowed contractions of the building blocks and their higher covariant derivatives, it is possible to build up an invariant effective action under the full symmetry group $G$.

In gauge symmetries, it is necessary to promote the partial derivative to a covariant one in the Maurer-Cartan form, $\partial_\mu \rightarrow D_\mu$ [16]. Consider the gauged generator, $E_I$, from a subgroup, $G' \subseteq G$, with corresponding gauge field, $w^I_\mu$. Thus, by replacing the partial derivative with a covariant one, we obtain the modified Maurer-Cartan form,

$$\tag{2.5} g^{-1}\partial_\mu g \rightarrow g^{-1}D_\mu g = g^{-1}(\partial_\mu + iw^I_\mu E_I)g.$$

This modification of the Maurer-Cartan form can also be written as a linear combination of the generators as in eq. (2.3), with a new building block made up of the gauge field, $w^I_\mu$, accompanying the gauged generator, $E_I$. Now the building blocks can also depend on the included gauge fields. The modified Maurer-Cartan form, $g^{-1}D_\mu g$, is invariant under local transformations, and its explicit components can be obtained using the commutation relations of the generators.

**Inverse Higgs constraint**

The Goldstone’s theorem [20], which states that a Goldstone mode exists for each broken generator, is only valid for internal symmetries. If space-time symmetries are spontaneously broken, there can be a mismatch in the number of broken generators and the number of bosons [41]. Nevertheless, we can preserve all the symmetries by imposing additional local constraints, which can be solved to write down some of the Goldstone’s modes.
in terms of others [19]. Using the inverse Higgs constraint [13], we can set to zero one or
more of the coset covariant derivatives, whenever \(X\) and \(X'\), are two multiplets of the
broken generator, such that the commutators of the unbroken translations, \(P\), and the broken
generator, \(X'\), yields a different broken generator, \([P,X'] \subset X\). If this is the case, we
can set some of the covariant derivatives of the Goldstones to zero. By imposing all possible
inverse Higgs constraints, one obtains the only relevant building blocks.

3 Electrodynamics of spheres

3.1 Classical electromagnetism

It is well known that a classical theory of electromagnetism obeys the symmetries of
special relativity determined by the Poincaré group, ISO(3,1), as well as the symmetries
of the U(1) charge symmetry group. Thus, the full group is, \(G = U(1) \times ISO(3,1)\), which
contains the generators for Lorentz transformations, \(J^{ab}\), and gauge field, \(\tilde{\omega}^{ab}_\mu\), the generators
of translations, \(P_a\), and gauge field, \(\tilde{e}^a_\mu\), and the generator of charge, \(Q\), and a gauge field
\(\tilde{A}_\mu\).\(^2\) Charge corresponds to a time invariant generator of the internal symmetry group,
U(1), while the symmetry of the Poincaré group is a space-time symmetry (See appendix
C for more details).

Therefore, we parametrize the coset, \(U(1) \times ISO(3,1) / U(1) \times SO(3,1)\), separating trans-
lations from the rest of the group. The coset parametrization in flat space-time is given by
the group element

\[
ge = e^{ix^a P_a} = e^{ix^a P_a} e^{ix^0 Q}, \quad (3.1)
\]

which contains all the unbroken translation, \(\bar{P}_a = \bar{P}_0 + P_a = P_a + Q\), with \(\bar{P}_0 = P_0 + Q\). Then, the Maurer-Cartan form reads

\[
g^{-1} D_\mu g = e^{-ix^a \bar{P}_a} \left( \partial_\mu + i \bar{A}_\mu Q + ie^a_\mu P_a + \frac{i}{2} \tilde{\omega}^{ab}_\mu J_{ab} \right) e^{ix^a \bar{P}_a} = \partial_\mu + i A_\mu Q + ie^a_\mu P_a + \frac{i}{2} \omega^{ab}_\mu J_{ab}, \quad (3.2)
\]

where we have used the commutation relation rules of the symmetries (See appendix C) to
obtain

\[
e^a_\mu = \bar{e}^a_\mu + \partial_\mu x^a + \tilde{\omega}^{ab}_\mu x_b, \quad (3.3)
\]

\[
A_\mu = \bar{A}_\mu + \partial_\mu \xi(x), \quad (3.4)
\]

\[
\omega^{ab}_\mu = \tilde{\omega}^{ab}_\mu. \quad (3.5)
\]

\(^2\)Note that we have defined the gauge fields, \(\tilde{o}'s\), compared to [16], in which the gauge fields are defined
as \(\tilde{o}'s\). We will reserve the tilde to refer to quantities in the comoving frame.
Before going any further, we consider the case of flat space-time. From a geometrical perspective, this limit implies that the curvature and the torsion tensor are equal to zero, which can be measured from the commutator of the two covariant derivatives, \([D_\mu, D_\nu]\). By considering the covariant derivative appearing in the Maurer-Cartan form, (3.2), we can cast the commutator as,

\[
[D_\mu, D_\nu] = i \hat{F}_{\mu\nu} Q + i T^a_{\mu\nu} P_a + \frac{i}{2} \check{R}_{\mu\nu}^{ab} J_{ab},
\]

(3.6)

Given that in flat space-time, \(\check{T}_{\mu\nu}^a = 0\) and \(\check{R}_{\mu\nu}^{ab} = 0\), it implies that

\[
\check{\omega}_{\mu}^{ab} = 0 \quad \text{and} \quad \check{e}_{\mu}^a = 0,
\]

(3.7)

for which eq. (3.3), takes the simple form

\[
e_{\mu}^a = \partial_{\mu} x^a = \delta_{\mu}^a.
\]

(3.8)

The field, \(e_{\mu}^a\), is known as the vierbein, which is used for deriving an invariant volume element \(d^4x \det e\), and which for the case of flat space-time is simply \(d^4x\). Given the coset recipe, we can use the coefficients of the unbroken U(1) generator in eq. (3.2), to define the gauge covariant derivative,

\[
\nabla_a \equiv \partial_a + i Q A_a,
\]

(3.9)

which is the usual covariant derivative in a classical field theory of electromagnetism. Having defined the vierbein and the gauge covariant derivative, we can proceed to build invariant actions [16].

The first building block can be extracted from the covariant commutator of the covariant derivatives, eq. (3.6), which in the flat space-time limit,

\[
g^{-1}[D_\mu, D_\nu]g = i(\partial_\mu A_\nu - \partial_\nu A_\mu)Q = i F_{\mu\nu} Q,
\]

(3.10)

with \(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\), the electromagnetic tensor. Thus, the first invariant term that can be built with our covariant building block, \(F_{\mu\nu}\), is,

\[
S = -\alpha \int d^4x F_{\mu\nu} F^{\mu\nu}.
\]

(3.11)
The coefficients of the theory can be treated as free parameters that are to be fixed by experiments or from the literature. Thus, to reproduce Maxwell’s theory, we find the coefficient, \( \alpha = (4\mu_0)^{-1} \), with \( \mu_0 \), the magnetic permeability of vacuum, such that we have the well known Maxwell action,

\[
S = -\frac{1}{4\mu_0} \int d^4x F_{ab} F^{ab},
\]

where we have used, \( F_{ab} = e^a_\mu e^b_\nu F^{\mu\nu} \). Eq. (3.12) lives in the bulk, while the action that describes the sphere, will live in the worldline.

### 3.2 Charged spheres

With the underlying theory of classical electromagnetism, we identify the symmetry pattern for a charged point particle, and derive the building blocks to construct an effective action. Then, by considering spherical objects at rest, and departures from sphericity, we add size effects to describe charged spheres.

#### Charged point particles

In general, a point particle breaks spatial translations, and boosts by choosing a preferred reference frame. Under a U(1) symmetry, the state of a charged point particle is an eigenstate of the charge, and does not break the U(1) symmetry. The symmetry breaking pattern reads

\[
\begin{align*}
\text{Unbroken generators} & = \left\{ \begin{array}{c}
P_0 = P_0 + Q \\
J_{ij}
\end{array} \right\} & \text{time translations, spatial rotations.} \\
\text{Broken generators} & = \left\{ \begin{array}{c}
P_i \\
J_{0i}
\end{array} \right\} & \text{spatial translations, boosts,}
\end{align*}
\]

where we have included the charge generator as a time invariant generator of translations [16]. Given this pattern, we can parametrize the coset as

\[
g = e^{ix^a(\lambda)P_a} e^{i\eta^i(\lambda)K_i} = e^{ix^a(\lambda)P_a} \tilde{g}.
\]

with \( \lambda \), the worldline parameter that traces out the trajectory of the particle, \( \eta^i \), the Goldstone mode, and \( \tilde{g} = e^{i\eta(\lambda)K_i} \).

The important building blocks are contained in the Maurer-Cartan form projected into the particle’s trajectory, \( \dot{x}^\mu g^{-1} D_\mu g \). It can be casted as a linear combination of the generators,

\[
\begin{align*}
\dot{x}^\mu g^{-1} D_\mu g & = i\dot{x}^\mu g^{-1} (\partial_\mu + i\dot{A}_\mu Q + i\dot{e}_\mu a P_a) g \\
& = i\dot{x}^\mu \tilde{g}^{-1} (\partial_\mu + iA_\mu Q + i\dot{e}_\mu a P_a) \tilde{g} \\
& = i\dot{x}^\mu (A_\nu \Lambda^\nu_{\mu} Q + e^b_\mu \Lambda^0_{a} P_0 + e^b_\mu \Lambda^i_{a} P_i) + i(\Lambda^{-1})^c_\mu \Lambda^\mu c K_i, \\
& = iE \left( P_0 + AQ + \nabla \pi^i P_i + \nabla \eta^i K_i \right),
\end{align*}
\]
where the dot means derivative with respect to the worldline parameter, \( \lambda \), and \( \Lambda^a{}_b(\eta) \equiv (e^{i\eta K})^a{}_b \), being the boost matrix of the Lorentz transformations, which is a function of the Goldstone field, \( \eta^i \). Thus, the covariant quantities are

\[
\begin{align*}
E &= \dot{x}^\mu e^b_\mu \Lambda^0_b, \\
A &= E^{-1}\dot{x}^\mu A^\nu_\mu, \\
\nabla \pi^i &= E^{-1}\dot{x}^\mu e^b_\mu \Lambda_{ib}^i, \\
\nabla \eta^i &= E^{-1}(\Lambda^{-1})^0_c \dot{\Lambda}^c_i. 
\end{align*}
\] (3.16)

These are some of the building blocks that we will use to build up an invariant action. Nevertheless, as pointed out before, given that we are working with space-time symmetries, there are some subtleties with the counting of Goldstone fields, for which the Inverse Higgs constraint can be placed.

In this case, the commutator between boosts and unbroken time translations gives broken spatial translations. Therefore, one can impose the inverse Higgs constraint and set to zero the covariant derivative, \( \nabla \pi^i = 0 \), such that we obtain the relation

\[
\nabla \pi^i = E^{-1}\dot{x}^\mu e^b_\mu \Lambda_{ib}^i = E^{-1}(\dot{x}^\mu e^0_\mu \Lambda_{ib}^i + \dot{x}^\mu e^j_\mu \Lambda_{ij}^i) = 0.
\] (3.17)

Solving for this constraint, we obtain the velocity (See eq. C.18)

\[
\beta^i \equiv \frac{\eta^i}{\eta} \tanh \eta = \frac{\dot{x}^j e^j_\mu}{\dot{x}^\mu e^0_\mu} = \tilde{u}^i,
\] (3.18)

where we have used the fact that, in flat space-time, \( e^a_\mu = \delta^a_\mu \), such that \( \tilde{u}^i = \partial_x x^i \), the four velocity measured in the proper frame. Using the inverse Higgs constraint we have expressed the \( \eta^i \)'s in terms of the \( \pi^i \)'s. This result can be interpreted as the Goldstones, \( \eta^i \)'s, or in terms of the, \( \beta^i \)'s, as parametrizing the boost necessary to get into the moving particle rest frame [16].

The building block, \( E = |E| = \sqrt{E^2} \), can be rewritten

\[
|E| = \sqrt{(E \nabla \pi^i)^2 + (E^2 - (E \nabla \pi^i)^2)}
\]

\[
= \sqrt{(E \nabla \pi^i)^2 - (\dot{x}^\mu e^a_\mu \Lambda^a_0 \dot{x}^\mu e^b_\mu \Lambda^b_0)},
\]

\[
= \sqrt{-\eta_{ab} e^a_\mu e^b_\mu \dot{x}^\mu \dot{x}^\mu} = \sqrt{-\eta_{ab} \dot{x}^a \dot{x}^b} = \frac{d\tau}{d\lambda},
\] (3.19)

where we have imposed the inverse Higgs constraint, and used the orthogonality property of the boost matrices, \( \Lambda^a_b \Lambda^b_a = \delta^a_c \), and \( e^a_\mu = \delta^a_\mu \). Therefore, we can rewrite the inverse Higgs constraint in a way that the physical interpretation is transparent [16]. This is done by expressing eq. (3.17), in terms of the velocity in the proper frame, \( \tilde{u}^a \), as
\[ \tilde{u}^a \Lambda_{a}^i = 0, \]  
\begin{equation} \tag{3.20} \end{equation}

and defining the set of Lorentz vectors, \( \hat{n}^a \),

\[ \hat{n}^a_{(0)} \equiv \tilde{u}^a = \Lambda^a_0(\eta), \quad \hat{n}^a_{(i)} \equiv \Lambda^a_i(\eta). \]  
\begin{equation} \tag{3.21} \end{equation}

This set of Lorentz vectors define an orthonormal basis in the particle’s comoving frame.

Then, we analyse the covariant derivative of the boost Goldstone, \( \eta^i \), in eq. (3.16), which can be rewritten as

\[ \nabla \eta^i = \hat{n}^a_{(i)} \partial_{\tau} \tilde{u}^a = \hat{n}^a_{b} \epsilon_{\mu}^b \hat{a}^\mu = \tilde{a}^i, \]  
\begin{equation} \tag{3.22} \end{equation}

with, \( \tilde{a}^i \), the acceleration of the particle in the comoving frame. Thus, the covariant derivatives, \( \nabla \eta^i \), corresponds to the component of the acceleration projected on the i-th vector defined by the set of Lorentz vectors, \( \hat{n}^a_{(b)} \), measured in the proper frame of the particle [16].

In the absence of external forces, \( \nabla \eta^i = 0 \). Nevertheless, this building block is needed to take into account for a full description of the system, i.e. when the charge from an external body is relevant, such that the acceleration is non-zero.

Thus, we are left with the building blocks,

\[ \tilde{A}_\nu = \frac{d\tau}{d\lambda}, \]  
\[ A = \tilde{u}^\mu \tilde{A}_\mu, \]  
\[ \nabla \eta^i = \tilde{a}^i, \]  
\begin{equation} \tag{3.23} \end{equation}

with \( \tilde{A}_\nu = A_\nu \Lambda^\nu_\mu \), which are to be used in order to build an invariant action. We can build an invariant term made up of the building block \( E \),

\[ S = -n_E \int d\lambda E = -n_E \int d\lambda \frac{d\tau}{d\lambda} = -m \int d\tau, \]  
\begin{equation} \tag{3.24} \end{equation}

and match the coefficient, \( n_E = m \), to the action of a free relativistic point particle with mass, \( m \). The last expression shows the action for a free relativistic point particle to all orders in the absence of external forces [16].

For the next covariant block, \( A \), we add the point particle correction due to charge as,

\[ S = \int d\lambda E(-mc^2 + n_A A) = \int d\tau(-mc^2 + q \tilde{u}^a \tilde{A}_a), \]  
\begin{equation} \tag{3.25} \end{equation}

with, \( q \), the net charge of the charged particle, where the coefficient, \( c_A \), has been matched from the classical theory of electrodynamics. Therefore, equation (3.25), is the action in the proper frame that describes a charged point particles in classical electromagnetism.
Size effects

In the context of the EFT for extended objects, the addition of size effects in the action of a point particle was introduced in [14], which can be done in a systematic fashion. Size corrections must respect the symmetries of the theory, Lorentz and gauge U(1) symmetry, and thus contained in an infinite series expansion of all possible invariant operators made up of our covariant building blocks. To build up invariant terms made out of $\tilde{F}_{ab}$, we first define its transformation properties under local Lorentz transformations. By considering the Lorentz parametrization of eq. (3.14), $g_L$, the electromagnetic tensor transforms as

$$\tilde{F} \equiv g_L^{-1} F,$$  \hspace{1cm} (3.26)

such that $F$ is transformed in a linear representation under a Lorentz transformation as expected. The explicit transformation reads

$$\tilde{F}_{ab} = (\Lambda^{-1})_a^c (\Lambda^{-1})_b^d F_{cd},$$  \hspace{1cm} (3.27)

Having the correct transformation rules, we can form rotationally invariant objects made out of $\tilde{F}_{ab}$, $\tilde{u}^a$, and $\tilde{a}^b$, to build up the invariant action. These corrections in the action reads,

$$S = \sum_n \int d\tau c_n \mathcal{O}_n(\tilde{F}_{ab}, \tilde{u}_c, \tilde{a}_d),$$  \hspace{1cm} (3.28)

with, $\mathcal{O}_n$, the invariant operators, $c_n$, their corresponding coefficients, and $n$, being chosen to the desired accuracy. The leading order, electric parity terms are [14]

$$S = \int d\tau \left( n_q \tilde{F}^{ba} \tilde{F}_c \tilde{u}_b \tilde{u}_c + n_{qa} \tilde{u}_a \tilde{a}_b \tilde{F}^{ab} + \ldots \right),$$  \hspace{1cm} (3.29)

where the ellipsis denotes higher order corrections. The first term correspond to the induced, electric parity dipolar moment, while the second term can be seen as a higher order correction to the latter due to the acceleration of the body.

The electric parity building block is identified, $\tilde{E}_a = \tilde{F}_a$. For the size effects in the gravitational case, which are taken into account with invariant combinations of the Riemann tensor defined in next section, the magnetic parity is subleading with respect to the electric one, therefore restricting our discussion to the electric parity terms. Nevertheless, a term with magnetic parity, $\tilde{B}_a = \frac{1}{2} \epsilon_{abcd} \tilde{F}^{bc} \tilde{a}^d$, can be build as well, $\propto \tilde{B}^a \tilde{B}_a$. The leading order corrections in (3.29), with their corresponding coefficients, $c_q$, and $c_{qa}$, encode the short distance information, or the size structure, which are responsible for the polarization of the object. The polarizability accounts for the deformation of the sphere in the presence of an external electromagnetic field. In general, the coefficients, $c_q$ and $c_{qa}$, are dependent on the radius of the sphere, therefore taking into account for the fact that a sphere is an extended object. The specific form of these coefficient for a charged sphere is beyond the scope of this work.
Dissipative effects

Dissipation in the EFT of extended objects was introduced in [15]. Dissipative effects of the sphere takes into account for its absorption of electromagnetic waves. These large number of degrees of freedom can be encoded in operators allowed by the symmetries of the object. For instance, for a rigid sphere, the allowed operators for rotations, SO(3), as well as parity eigenvalue, allows us to build the action [15]

\[ S = \int d\tau \tilde{P}^a(\tau)\tilde{u}^b\tilde{F}_{ab}, \]

(3.30)

with \( \tilde{P}(\tau) \), a composite dynamical operator corresponding to the electric parity, electromagnetic dipole moment. The specific form of the operator, \( \tilde{P}(\tau) \), and its coefficient, are discussed in detail in the next section.

3.3 The effective action

Finally, by considering, the electric parity only, we write down the leading order effective action of a charged sphere in a classical theory of electromagnetism,

\[ S_{\text{eff}} = \int d\tau \left( -mc^2 + q\tilde{u}^a \tilde{A}_a + c_q \tilde{E}^a \tilde{E}_a + c_{qa} \tilde{E}^a \tilde{a}_a + \tilde{P}^a \tilde{F}_{ab} \tilde{u}^b + \ldots \right) + S_0, \]

(3.31)

with

\[ S_0 = -\int d^4x \frac{1}{4\mu_0} \tilde{F}_{ab} \tilde{F}^{ab}. \]

(3.32)

We have developed a theory of classical electromagnetism and described a charged sphere as a charged point particle with the finite size structure encoded in higher order operators in the action. In this approach, the description of the sphere lives in the worldline, while eq. (3.32), lives in the bulk.

4 Compact objects in general relativity

Before constructing the effective action for a compact object, we review how a theory of gravity can be derived from the coset construction as in [16], where a frame independent generalization of general relativity, known as Einstein’s vierbein field theory, is derived. This is a theory that can naturally incorporate spinning objects. The difference of our construction compared to the one in [16], is the inclusion of the gauge symmetry group, \( U(1) \), which allows us to derive the Einstein-Maxwell action in the vierbein formalism, as well as the correction to the point particle due to charge. Once the underlying theory of gravity has been developed, then we derive the leading order action for a charged spinning compact object in the effective theory of general relativity (vierbein formalism).
4.1 Effective theory of gravity

There are two symmetries in gravity to consider: Diffeomorphisms invariance, and Poincaré symmetry, determined by the Poincaré group, ISO(3,1), which contains the generators for translations, \( P_a \), and Lorentz transformations, \( J_{ab} \), with their corresponding gauge fields, \( \hat{e}_\mu^a \) and \( \hat{\omega}_{\mu}^{ab} \). Both of the aforementioned symmetries of the system can be separated by considering the principal bundle, \( P(M,G) \), with base manifold, \( M \), and structure group, \( G \). In this way, we realize the matter fields as sections of their respective fiber bundle [16]. The coordinates, \( x^\mu \), describing the position on the considered manifold, \( M \), are not affected by the local Poincaré group, but it is transformed under diffeomorphisms. The local Poincaré transformations act along the fiber, while diffeomorphisms can be considered as relabeling the points on the manifold.

To incorporate electrodynamics in the theory of gravity, we add the U(1) symmetry of electromagnetism with its gauge field, \( \hat{A}_\mu \), and generator \( Q \), and proceed with the coset construction by gauging the Poincaré group and realizing translations non-linearly [40]. The coset then reads, \( \text{U}(1) \times \text{ISO}(3,1)/\text{U}(1) \times \text{SO}(3,1) \), with the coset parametrization

\[
g = e^{i y^a(x) \bar{P}_a},
\]

where \( \bar{P}_a = \bar{P}_0 + P_i = P_a + Q \).

The Maurer-Cartan form from the coset parametrization (4.1), is expressed as a linear combination of the generators of the theory,

\[
g^{-1} D_\mu g = \exp\left( -i y^a(x) \bar{P}_a \left( \partial_\mu + i \hat{A}_\mu Q + i \hat{e}_\mu^a P_a + \frac{i}{2} \hat{\omega}_{\mu}^{ab} J_{ab} \right) \right) e^{i y^a(x) \bar{P}_a}
\]

where we have used the commutation relation rules of the symmetries (See appendix C) to obtain

\[
\hat{e}_\mu^a = \hat{e}_\mu^a + \partial_\mu y^a + \hat{\omega}_{\mu b} y^b,
\]

\[
\hat{A}_\mu = \hat{A}_\mu + \partial_\mu \xi(y),
\]

\[
\hat{\omega}_{\mu}^{ab} = \hat{\omega}_{\mu}^{ab}.
\]

The field, \( \hat{e}_\mu^a \), is the vierbein, that appears in the tetrad (vierbein) formalism, which defines the metric as \( g_{\mu\nu} = \eta_{ab} e_\mu^a e_\nu^b \). It can be used to build up the invariant element, \( d^4x \det e \), as well as to change from orthogonal frame, i.e. \( A_\mu = e_\mu^b A_b \). The field, \( \omega_{\mu}^{ab} \), is known as the spin connection, and it is named given its transformation properties (See appendix C).

Following the coset recipe, we can introduce the covariant derivative for matter fields by using the coefficients from the unbroken Lorentz generators. The covariant derivative reads

\[
\nabla_\mu^a = (e^{-1})_a^\mu \left( \partial_\mu + \frac{i}{2} \hat{\omega}_{\mu bc} J_{bc} \right),
\]
where the upper index, $g$, denotes gravity. In a similar manner we can define the covariant derivative for charged fields. Using the coefficients and generators from the U(1) symmetry, the covariant derivative for charged fields reads

$$\nabla_a^g = (e^{-1})_a^\mu (\partial_\mu + i A_\mu Q).$$

(4.7)

The only required ingredients to describe the non-linear realizations of translations and the local transformations of the Poincaré and U(1) group, are the covariant derivatives and the vierbein [16]. Having defined these elements, we identify the building blocks of the bulk.

The curvature invariants can be obtained from the covariant version of the commutator of the covariant derivative that appear in the Maurer-Cartan form,

$$g^{-1}[D_\mu, D_\nu]g = i F_{\mu\nu} Q + i T_{\mu\nu}^a P_a + \frac{i}{2} R_{\mu\nu}^{ab} J_{ab},$$

(4.8)

with $T_{\mu\nu}^a = \tilde{T}_{\mu\nu}^a + R_{\mu\nu}^{ab} y_b$, and $R_{\mu\nu}^{ab} = \tilde{R}_{\mu\nu}^{ab}$, the covariant torsion and Riemann tensor respectively. The covariant quantities have been defined in this way, from, $[D_\mu, D_\nu] = i \tilde{F}_{\mu\nu} Q + i \tilde{T}_{\mu\nu}^a P_a + \frac{i}{2} \tilde{R}_{\mu\nu}^{ab} J_{ab}$, in eq. (3.6), such that by construction, $T_{\mu}^a$ and $R_{\mu\nu}^{ab}$ transforms independently under the local transformations [16].

We are interested in a gravitational theory as general relativity, where the torsion tensor is zero. Solving for the vanishing torsion tensor, we obtain an equation for the spin connection in terms of the vierbein [16],

$$\omega^{ab}(e) = \frac{1}{2} \left\{ e^{\nu a} (\partial_\mu e_\nu^b - \partial_\nu e_\mu^b) + e_{\mu c} e^{\nu a} e^{\lambda b} \partial_\lambda e_\nu^c - (a \leftrightarrow b) \right\}. $$

(4.9)

From the expression for the torsion in eq. (4.8), one can read the Christoffel symbols, $T_{\mu\nu}^a = \Gamma_{\mu\nu}^a - \Gamma_{\nu\mu}^a$. Therefore, the Christoffel symbol reads

$$\Gamma_{\mu\nu}^a = \partial_\mu e_\nu^a - e_{\nu b} \omega^{ab}_\mu.$$

(4.10)

After imposing the condition that the torsion tensor vanishes, eq. (4.8) reads

$$g^{-1}[D_\mu, D_\nu]g = i F_{\mu\nu} Q + \frac{i}{2} R_{\mu\nu}^{ab} J_{ab},$$

(4.11)

Therefore, we can use the tensors, $F_{\mu\nu}$ and $R_{\mu\nu}^{ab}$, to build up a Lagrangian that matches to the one of the theory of general relativity, as well as the theory of electrodynamics in curved space-time. Such action reads:

$$S = \int \det e \, d^4 x \left\{ - \frac{1}{4 \mu_0} F_{ab} F^{ab} + \frac{1}{16 \pi G} R + \ldots \right\},$$

(4.12)
with, \( R = R^{ab}_{\ ab} = e^a_{\ mu} e^b_{\ nu} R^ab_{\ mu\ nu} \), the Ricci scalar, which is the low energy term of the theory of gravity. The second term is the Einstein-Hilbert action, with \( G \), the gravitational constant, while the first term is the Maxwell’s action in curved space-time, with \( \mu_0 \), the magnetic permeability of vacuum. The coefficients of eq. (4.12), have been matched from the known theory, which allows us to obtain the Einstein-Maxwell action in the vierbein formalism. The ellipsis stands for higher order terms from a more fundamental theory of gravity.

One can easily obtain the well known gravitational action by changing to the space-time indices:

\[ S = \int \sqrt{-g} \, d^4x \frac{1}{16\pi G} R, \quad (4.13) \]

with \( R = R^{\mu\nu}_{\ \mu\nu} = e^a_{\ mu} e^b_{\ nu} R^ab_{\ mu\ nu} \), and where we have used \( g_{\mu\nu} = e^a_{\ mu} e^b_{\ nu} \eta_{ab} \). Furthermore, one can also recover the Christoffel symbol, eq. (4.10), in terms of the metric via, \( \Gamma^a_{\ \mu\nu} = e^a_{\ \mu} \Gamma^a_{\ \mu\nu} \).

### 4.2 Charged spinning compact objects

The description of extended objects in the EFT framework was first introduced in [14, 15] for the non-spinning case. Then, spinning extended objects was introduce in [21], and later in [42], theories whose constructions differ. Finally, the effective theory for spinning extended objects derived using the coset construction was introduced in [16]. Although in [15], BHs electrodynamics is introduced, it was until [43] that charge was considered in the EFT for extended objects to obtain the dynamics. In the following, we will use the coset construction and extend the work on spinning extended objects in [16], to include the U(1) symmetry in order to describe charged spinning compact objects.

#### Charged spinning particles

An invariant action for an extended object can be constructed by identifying the symmetry breaking pattern that such object generates. In order to describe a charged spinning extended object, we need to consider the full group before being broken, \( G = U(1) \times \text{ISO}(3,1) \times S \), with \( S \subseteq \text{SO}(3) \), the internal symmetry of a spinning extended object which characterizes the low energy dynamics [16]. We characterize the state of a charged extended object as the one of a charged point particle, such that its state is an eigenstate of the charge, which does not break the U(1) symmetry. When considering spin, one can choose a coset parametrization such that a spinning extended object breaks the full Poincaré group: spatial translations, boosts and rotations [16].

In the comoving frame, the group \( G \) is broken into a linear combination of internal rotations, \( S_{ij} \), and spatial rotations, \( J_{ij} \), such that the symmetry breaking pattern for a charged spinning extended object reads,
Unbroken generators = \{
\bar{P}_0 = P_0 + Q \quad \text{time translations},
\bar{J}_{ij} \quad \text{internal and space - time rotations}.
\}

Broken generators = \{
P_i \quad \text{spatial translations},
J_{ab} \quad \text{boosts and rotations},
\}

(4.14)

with, \bar{J}_{ij}, the sum of the internal and space-time rotations \cite{16}. We consider a spherical extended object at rest, for which \( S_{ij} \) are the generators of the internal \( SO(3) \) group, such that \( \bar{J}_{ij} = S_{ij} + J_{ij} \). Translations are non-linearly realized and the local Poincaré and U(1) transformations are considered to take place along the fiber.

Given that Lorentz transformations can be parametrized as the matrix product of a boost and a rotation, the coset parametrization reads

\[
g = e^{iy_a \bar{P}_a} e^{i\alpha_{ab} J_{ab}} = e^{i\eta_i J_{0i}} e^{i\xi_{ij} J_{ij}} = e^{i\eta_a \bar{P}_a} \bar{g},
\]

(4.15)

which implies a correspondence between the Goldstone fields, \( \alpha_{ab} \) and \( \eta_i, \xi_{ij} \) \cite{16}. By parametrizing the coset with the generators of the broken spatial rotations, the Maurer-Cartan form can be computed without the need to specify the explicit unbroken generators of rotations, \( \bar{J}_{ij} \).

The relevant degrees of freedom can be identified from the projected Maurer-Cartan form to the worldline of the object,

\[
\dot{x}^\mu \bar{g}^{-1} D_\mu g = \dot{x}^\mu \bar{g}^{-1} (\partial_\mu + i \bar{A}_\mu Q + i e_\mu^a \bar{P}_a + i \omega_{\mu}^{ab} J_{ab}) \bar{g}
= \dot{x}^\mu \bar{g}^{-1} (\partial_\mu + i A_\mu Q + i e_\mu^a \bar{P}_a + i \omega_{\mu}^{ab} J_{ab}) \bar{g}
= iE(P_0 + AQ + \nabla \pi^i P_i + \frac{1}{2} \nabla \alpha_{cd} J^{cd}).
\]

(4.16)

The building blocks of the low energy dynamics are,

\[
E = \dot{x}^\mu e_\mu^a \Lambda_a^0
\]

(4.17)

\[
A = E^{-1} \dot{x}^\mu A_\mu \Lambda^\nu
\]

(4.18)

\[
\nabla \pi^i = E^{-1} \dot{x}^\mu e_\mu^a \Lambda_a^i
\]

(4.19)

\[
\nabla \alpha^{ab} = E^{-1} \left( \Lambda_c^{a} \Lambda^{cb} + \dot{x}^{\nu} \omega_{\nu}^{cd} \Lambda_c^{a} \Lambda_d^{b} \right)
\]

(4.20)

with the \( \Lambda \)'s, the Lorentz transformations parametrized by \( \alpha \), or equivalently by \( \eta \) and \( \xi \).

As pointed out above, the parametrization (4.15), implies that no connection proportional to \( J \) appears, which make the building blocks independent of the residual symmetry group. The residual symmetry, \( SO(3) \), requires all spatial indices to be contracted in an \( SO(3) \) invariant manner \cite{16}.
In the breaking of space-time symmetries, one can impose the inverse higgs constraint to remove some of the Goldstones \[13\]. Given that the commutator between the unbroken time translations and boosts gives broken spatial translations, \([K_i, P_0] = iP_i\), we impose the constraint and set to zero the covariant derivative of the Goldstone,

\[
\nabla \pi^i = E^{-1}(\dot{x}^\nu e^a_\nu \Lambda^a_i) = E^{-1}(\dot{x}^\nu e_\nu^0 \Lambda^0_i + \dot{x}^\nu e^i_\nu \Lambda^i_j) = 0,
\]

and solve it, to express the boost as a function of velocities \[16\],

\[
\beta_i \equiv \frac{\eta_i}{\eta} \tanh \eta = \frac{\dot{x}^\nu e^i_\nu}{\dot{x}^\nu e^0_\nu}
\]

where, \(\eta\), is identified as the rapidity (See appendix C). To obtain the physical interpretation of last equation, we rewrite the building block, \(E\), such that \[16\]

\[
E = \sqrt{(E \nabla \pi^i)^2 - (\dot{x}^\nu e^a_\nu \Lambda^a_i \dot{x}^\mu e^b_\mu \Lambda^b_c)}
\]

\[
= \sqrt{-\eta_{ab} e^a_\nu e^b_\mu \dot{x}^\nu \dot{x}^\mu} = \sqrt{-g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu} = \frac{d\tau}{d\lambda},
\]

with, \(\lambda\), the worldline parameter that traces out the trajectory of the particle. To obtain last equation, we have imposed the inverse Higgs constraint, and the property of the boost matrices, \(\Lambda^b_a \Lambda^a_c = \delta^b_c\).

Therefore, with eq. (4.23), we can rewrite the constraint, eq. (4.21), in a way that makes manifest its physical interpretation \[16\]. For rotations, \(\Lambda^0_a(\xi) = \delta^0_a\) and \(\Lambda^i_j(\xi) = R^i_j(\xi)\), with \(R(\xi)\) an \(SO(3)\) matrix, such that the constraint (4.21), now reads

\[
\tilde{u}^a \Lambda^a_i(\eta) R^j_i(\xi) = 0,
\]

with, \(\tilde{u}^a \equiv e_\mu^a \partial_\tau x^\mu\), the velocity measured in the local inertial frame defined by the vierbein. Given that the matrix \(R^j_i(\xi)\) is invertible, we obtain

\[
\tilde{u}^a \Lambda^a_i(\eta) = 0.
\]

These quantities have a clear geometrical interpretation: the set of local Lorentz vectors,

\[
\hat{n}^a(0) \equiv \tilde{u}^a = \Lambda^a_0(\eta), \quad \hat{n}^a(i) \equiv \Lambda^a_i(\eta),
\]

define an orthonormal local basis with respect to the local flat metric, \(\eta_{ab}\), in the frame that is moving with the particle \[16\]. Orthogonality is obtained from the boost matrices, \(\Lambda^b_a \Lambda^a_c = \delta^b_c\). Therefore, the set \(\hat{n}^a(\eta)\) defines the local inertial frame on the particle worldline. One can also define the orthonormal basis in terms of the space-time vectors, \(\hat{m}^a(\eta) \equiv e_\mu^a \hat{n}^\mu(\eta)\), with respect to the full metric, \(g_{\mu\nu}\).

Moreover, an additional set of orthonormal vectors is obtained \[16\],

\[
\hat{m}^b(\alpha) = \Lambda^b_a(\eta) R^c_a(\xi),
\]
with the zeroth vector, \( \hat{m}_b^{(0)} = \hat{n}_b^{(0)} = \tilde{u}_b \), the velocity of the object in the proper frame. The rest of the vectors differs by a rotation, \( \mathcal{R}(\xi) \), compared to (4.26), from which one can observe that the set of vectors, \( \hat{m}_b^{(a(i))} \), contain information about the rotation, parametrized by the degrees of freedom of \( \xi \).

With these definitions, the covariant derivatives, \( \nabla^0 \alpha^i \), can be expressed as

\[
\nabla^0 \alpha^i = \mathcal{R}^i_j(\xi)\Lambda^j_\alpha(\eta)(\partial_\tau \tilde{u}^a + \tilde{u}^\mu \omega_\mu^a \tilde{u}^\xi) = \Lambda^i_\alpha a = \tilde{a}^i, \tag{4.28}
\]

which is a rotated version of the acceleration, projected into the orthonormal basis defined by the \( \hat{n} \)'s basis in the proper frame of the particle. In the absence of external forces, this building block is zero. Nevertheless, it must be considered to build up invariant operators when an external force, such as the one from an external gravitating object, or from the charge of another object, is strong enough to make this building block relevant.

The rest of the covariant derivatives for the Goldstone, reads

\[
\nabla^{ij} = \Lambda^i_k(\eta^{kl}\partial_\tau + \partial_\tau x^\mu \omega_\mu^{kl})\Lambda^j_l = \tilde{\Omega}^{ij}, \tag{4.29}
\]

which is the relativistic angular velocity of the object in the proper frame. From the covariant quantity, \( \tilde{\Omega}^{ij} \), it is possible to define the angular velocity vector with the epsilon tensor, which reads

\[
\tilde{\Omega}_i = -\frac{1}{2} \epsilon_{ijk} \Lambda^j_\alpha(\eta^{ab}\partial_\tau + \partial_\tau x^\mu \omega_\mu^{ab})\Lambda^k_b. \tag{4.30}
\]

Thus, we are left with the building blocks

\[
E = d\tau/d\lambda, \tag{4.31}
\]
\[
A = \tilde{u}^\mu \tilde{A}_\mu, \tag{4.32}
\]
\[
\nabla^0 \alpha^i = \tilde{a}^i, \tag{4.33}
\]
\[
\nabla^{ij} = \tilde{\Omega}^{ij}, \tag{4.34}
\]

with \( \tilde{A}_\mu = A_\mu \Lambda^\nu_\mu \). We consider the blocks \( A \) and \( E \), to derive the effective action of a charged point particle in curved space-time. The action takes the form

\[
S = \int d\lambda E(-n_E + n_A A) = \int d\tau(-mc^2 + q\tilde{u}^\alpha \tilde{A}_\alpha), \tag{4.35}
\]

where we have matched the coefficient, \( n_E = mc^2 \), from the action of a point particle [14] with mass, \( m \), and the coefficient, \( n_A = q \), from the action of a charged point particle [14, 43] with net charge, \( q \).

Now we include the building block, \( \nabla^{ab} \alpha \), neglecting charge for simplicity. The leading order action reads [16],

\[
S = \int d\lambda E(-mc^2 + n_\alpha \nabla^{ij} \nabla^{ij} + \ldots) = \int d\lambda E(-mc^2 + n_\Omega \tilde{\Omega}^{ij} \tilde{\Omega}^{ij}), \tag{4.36}
\]
where a term linear in $\tilde{\Omega}$, has been discarded by time reversal symmetry, and we have considered spherical objects at rest. The ellipses denotes higher order corrections made out of $\nabla\alpha^{ab}$. In the usual mechanics of rotational dynamics, to characterize a rigid sphere only two parameters are needed, which is the mass, $m$, and moment of inertia, $I$. By comparing our action to the one of a relativistic spinning point particle \cite{21, 44}, we match the coefficient, $n_\Omega = I/4$, to obtain the relativistic action for spinning particles in curved space-time \cite{16, 23},

$$S = \int d\tau \left\{ -mc^2 + \frac{I}{4}\tilde{\Omega}_{ab}\tilde{\Omega}^{ab} + \ldots \right\},$$  

(4.37)

where we have defined, $\tilde{\Omega}_{0i} = 0$, as in \cite{23}. A higher order correction made out of the Goldstone building block,

$$S = \int d\tau c_{\alpha, a} \nabla\alpha^{ac} \nabla\alpha^{eb} \tilde{u}^a \tilde{u}^b \tilde{u}^2 + \ldots.$$  

(4.38)

We can connect our action to the one used to obtain the PN expansion for spinning objects \cite{22, 45}, with the introduction of the relativistic spin degree of freedom \cite{21, 22, 38}

$$S_{ab} = 2\frac{\partial L}{\partial \tilde{\Omega}_{ab}},$$  

(4.39)

which is the conjugate variable of the angular velocity, with associated spin tensor $\tilde{S}_{ab} = \epsilon^{ab}_{\ c}\tilde{S}^c$. Then, by considering the action that describes a spinning extended object in the proper frame,

$$S = \int d\tau \left\{ -mc^2 + \frac{I}{4}\tilde{\Omega}_{ab}\tilde{\Omega}^{ab} + c_{\Omega, a}\tilde{\Omega}_{ab}\tilde{u}^a \tilde{u}^0 + \ldots \right\},$$  

(4.40)

we Legendre transform it to write it down in terms of the spin $\tilde{S}_{ab}$, and transform it to the lab frame, with

$$\Omega^{ab} = \Lambda^a_c \Lambda^b_d \tilde{\Omega}^{cd} = \Lambda^a_c \Lambda^b_d \left[ \Lambda^e_c (\eta^{ef} \partial_r + \partial_r x^\mu \omega^e_\mu) \Lambda^d_f \right]$$  

(4.41)

$$= -\Lambda^a_c \partial_r \Lambda^{bc} + \partial_r x^\mu \omega^a_\mu,$$  

(4.42)

and $S^{ab} = \Lambda^a_c \Lambda^b_d \tilde{S}^{cd}$. In the lab frame, in which the PN expansion is computed, we set $u^a = v^a$ with $v^0 = 1$, and $\lambda = t$, such that we obtain

$$L = -mc^2 + \frac{1}{2}S_{ab}\Omega^{ab} + c_{\Omega, a} S_{ab} \frac{a^a v^b}{v^2} - \frac{1}{4I}S_{ab}S^{ab},$$  

(4.43)

with the expected acceleration correction in \cite{22} when setting $c_{\Omega, a} = I$.

Since $\tilde{u}^a \Lambda^i_a = 0$, the tensors, $\nabla\alpha^{ab}$ and $\tilde{S}_{ab}$, are orthogonal to the four velocity. Therefore, we can obtain a constraint on the angular velocity.
\[ \tilde{u}_a \nabla^{ab} = \tilde{u}^\mu \nabla_\mu \tilde{u}^b + \tilde{u}_a \tilde{\Omega}^{ab} = 0, \quad (4.44) \]

as well as on the spin,

\[ \tilde{u}_a \tilde{S}^{ab} = \sqrt{\tilde{u}_a \tilde{S}^0_0} + \tilde{u}_a \tilde{S}^a = 0. \quad (4.45) \]

The latter, known as the spin supplementary condition, is equivalent to the relativistic Price-Newton-Wigner spin supplementary condition \cite{22, 46}, while the former is the constraint on the angular velocity derived in \cite{23}.

With all of our derived invariant operators, we write down the effective action of a charged spinning point particle in the particle’s rest frame \cite{46},

\[
S = \int d\tau \left\{ -mc^2 + q\tilde{u}^a \tilde{A}_b + \frac{I}{4} \tilde{\Omega}_{ab} \tilde{\Omega}^{ab} + I \tilde{\Omega}_{ab} \tilde{a}^b \tilde{u}^a \tilde{u}_0 + \ldots \right\}. \quad (4.46)
\]

**Size effects**

Size effects in the EFT for extended objects was introduced in \cite{14}, which can be systematically taken into account by building invariant operators made up of the Weyl curvature tensor, \( W_{abcd} \). The Weyl tensor is obtained from the Riemann tensor, \( R_{abcd} \), by subtracting out various traces. Defining the transformation \cite{16}

\[
\tilde{R} \equiv g_L^{-1} R, \quad (4.47)
\]

with, \( g_L \), the Lorentz part of the parametrization in eq. (4.15), the Riemann tensor transforms linearly under Lorentz transformations as expected. The explicit transformation reads

\[
\tilde{R}_{abcd} = (\Lambda^{-1})_a^e (\Lambda^{-1})_b^f (\Lambda^{-1})_c^g (\Lambda^{-1})_d^h R_{efgh}, \quad (4.48)
\]

with, \( \tilde{R}_{abcd} \), the Riemann tensor in the local rest frame of the object. Having the correct transformations, we define the Weyl tensor as usual,

\[
W_{abcd} = R_{abcd} + \frac{1}{2} (R_{adgb} - R_{acgb} + R_{bcgd} - R_{bdgc}) + \frac{1}{6} R (g_{ac} g_{bd} - g_{ad} g_{bc}), \quad (4.49)
\]

which have the physical content \cite{14}. The Weyl tensor measures the curvature of the spacetime and contains the tidal force exerted on an extended particle that is moving along the worldline, taking into account for how the shape of the body is distorted. It transforms in the same way as eq. (4.48).

Furthermore, we can also use the electromagnetic tensor to build invariant operators that take into account for the polarizability of the object. Following the above discussion, we define its transformation rule,

\[
\tilde{F} \equiv g_L^{-1} F. \quad (4.50)
\]
Having defined the correct transformation rules for the Riemman and the electromagnetic tensor, we now form rotationally invariant objects.

We form all leading order invariant operators that contribute to the dynamics, by combining all of our covariant quantities: \( \tilde{W}^{abcd} \), \( \tilde{F}^{ab} \), \( \nabla^{ab} \), and \( \tilde{u}^a \), in all possible ways allowed by the symmetries. In particular, for the electromagnetic and Weyl tensor, the building blocks are the electric like parity tensors, \( \tilde{E}_a = \tilde{F}_{ab}\tilde{u}^b \), and \( \tilde{E}_{ab} = \tilde{W}_{abcd}\tilde{u}^c\tilde{u}^d \), respectively [15]. By considering the electromagnetic dipolar and gravitational quadrupolar moments, we build the following leading order relevant operators for finite-size effects:

\[
\mathcal{O}(\tilde{u}^a, \tilde{\Omega}^a, \tilde{E}^{ab}, \tilde{E}^a) = \begin{cases} 
\tilde{E}^{ab}\tilde{E}_{ab} & \text{Gravity}, \\
\tilde{\Omega}^a\tilde{\Omega}^b\tilde{E}_{ab} & \text{Spin – gravity}, \\
\tilde{E}^a\tilde{E}_a & \text{Electromagnetic}, \\
\tilde{\Omega}^a\tilde{\Omega}_a & \text{Spin – electro}.
\end{cases} \tag{4.51}
\]

One can consider the magnetic parity operators as well, \( \tilde{B}^{ab} = (1/2)\epsilon_{cdea}\tilde{W}^{cde}f_b\tilde{u}^c\tilde{u}^d \) and \( \tilde{B}_a = \epsilon_{abcd}\tilde{F}^{bc}\tilde{u}^d [15] \), for the gravitational and electromagnetic case respectively, which are subleading with respect to the electric parity terms (at least for the gravitational case), therefore restricting our discussion to the electric parity action. The leading order magnetic parity operators can be build in analog to (4.51) i.e. \( B^{ab}\tilde{B}_{ab}, B^a\tilde{B}_a, \text{etc}. \)

Higher order terms to the ones shown in eqs. (4.51), can be built from the derived building blocks and the covariant derivatives, eqs. (4.6) and (4.7), i.e. \( \tilde{\nabla}^a\tilde{E}_{ab} \) and \( \tilde{\nabla}^a\tilde{E}_a \). Furthermore, it is worth noting that size effects can be seen as encoded in a composite operator \( \tilde{Q}^{ab} \), which we comment below.

**Dissipative effects**

Dissipation, due to the internal structure of an extended object, was introduced in EFT description in [15], where the existence of gapless modes that are localized on the worldline of the particle take into account for the energy and momentum loss from the interaction with external sources. Dissipative effects for slowly spinning objects have been considered in [23, 26], and for maximally spinning in [24].

These large number of degrees of freedom can be encoded in operators allowed by the symmetries of the object. For a compact object, the allowed operators due to its symmetries gives rise to the invariant operators [15, 24]:

\[
\text{Dissipative operators} = \begin{cases} 
\tilde{P}^a(\tau)\tilde{E}_a & \text{Electro}, \\
\tilde{D}^{ab}(\tau)\tilde{E}_{ab} & \text{Gravity},
\end{cases} \tag{4.52}
\]

with \( \tilde{P}(\tau) \) and \( \tilde{D}(\tau) \), composite operators corresponding to the electric parity of the electromagnetic dipole and the gravitational quadrupole moment respectively, encoding the dissipative degrees of freedom.

For a non-spinning BH, dissipation takes into account for the absorption of electromagnetic and gravitational waves, while for a non-spinning NS, dissipative effects take into
account for the energy loss during the interaction with an external source given the internal viscosity of its equation of state of matter. On spinning objects, the spin has a time dependence between the object and its environment which generates dissipative effects. The operators in eq. (4.52), take into account for the spin dissipative degrees of freedom as well.

The coefficients encoding the internal structure are encoded in the dynamical moments, $\tilde{P}$ and $\tilde{D}$, which are dependent on the internal degrees of freedom of the compact object in an unspecified way, but which explicit form is not necessary to obtain the dynamics [15, 24]. The dynamics of the system containing the dissipative degrees of freedom can be obtained using the in-in closed time path [47], a formalism that allows us to treat dissipative effects in a time asymmetric approach [15].

The expectation values of these operators, $\langle \tilde{P}^a(\tau) \rangle$, $\langle \tilde{D}^{ab}(\tau) \rangle$, are defined through the in-in path integral, which is the expectation value in the initial state of the internal degrees of freedom, and which in general is a function of the building blocks, $\tilde{E}^a$ and $\tilde{E}^{ab}$, respectively [24]. In the gravitational case, by considering the linear response in a weak external field, the in-in formalism implies the form of the expectation value [24]

$$\langle \tilde{D}^{ab}(\tau) \rangle = \int d\tau' G_{R}^{ab,cd}(\tau - \tau') E_{cd}(\tau') + O(E^2),$$

where the expectation values of the retarded Green’s function,

$$G_{R}^{ab,cd}(\tau - \tau') = i\theta(\tau - \tau') \langle [\tilde{D}^{ab}(\tau), \tilde{D}^{cd}(\tau')] \rangle,$$

are obtained at the initial state of the interaction where the external field is zero.

By considering low frequencies, from which we assume that the degrees of freedom from the operator, $\tilde{D}^{ab}$, are near equilibrium, the time ordered two point correlation function imply that the Fourier transform, $\tilde{G}_R$, must be an odd, analytic function of the frequency, $\omega > 0$ [15]. Therefore, the retarded correlation function, $\tilde{G}_R$, reads

$$\tilde{G}_R^{ab,cd}(\omega) \simeq ic_g \omega \left( \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{cb} - \frac{2}{3} \delta^{ab}\delta^{cd} \right),$$

with the coefficient for dissipative effects, $c_g \geq 0$. Note that, in contrast to [15], we have absorbed the $1/2$ factor appearing in front of (4.55) into the dissipative coefficient.

By considering the response of the interaction to be nearly instantaneous, the operator due to gravitational dissipative effects takes the form [15, 24]

$$\langle \tilde{D}^{ab}(\tau) \rangle \simeq ic_g \frac{d}{d\tau} \tilde{E}^{ab} + \ldots$$

In general, one can use the dynamical composite operator, $\tilde{Q}^{ab}(\tau)$, to account for the tidal response function, for which the above reasoning leads to [15, 24]

$$\langle \tilde{Q}^{ab}(\tau) \rangle \simeq n_g \tilde{E}^{ab} + ic_g \frac{d}{d\tau} \tilde{E}^{ab} + n_g' \frac{d^2}{d\tau^2} \tilde{E}^{ab} + \ldots$$
The first term in last equation is the static quadrupolar tidal effect considered above, while the third term is a dynamical tidal effect in the quasi-static limit. The second term, which is the imaginary part of the response function, takes into account for the dissipative degrees of freedom. In our work we consider the static tidal response only, and use the operator, $\tilde{D}_{ab}$, to contain only the dissipative degrees of freedom. For NSs, dynamical oscillations in the quasi-static limit have been taken into account in [9]. Nevertheless, dynamical oscillations should be considered in a different fashion, as in [38], which is beyond our scope.

On the electromagnetic side, an analog procedure can be taken, for which retarded correlation function, $\tilde{G}_R$, reads [15]

$$\tilde{G}_R^{ab}(\omega) \simeq ic_q\delta^{ab}\omega,$$  \hspace{1cm} (4.58)

with the coefficient, $c_q \geq 0$. Therefore, the operator for electromagnetic dissipative effects reads

$$\langle \tilde{P}_a(\tau) \rangle \simeq ic_q \frac{d}{d\tau} \tilde{E}_a + \ldots$$  \hspace{1cm} (4.59)

4.3 Effective action for compact objects

Gathering all our results, we construct the most general, leading order and electric like parity, effective action for a compact object in the theory of general relativity,

$$S_{\text{eff}} = \int d\tau \left\{-mc^2 + q\tilde{u}^a \tilde{A}_a + \frac{I}{4} \tilde{\Omega}_{ab} \tilde{\Omega}^{ab} + I\tilde{\Omega}_{ab} \tilde{u}^a \tilde{u}_b \frac{\tilde{\Omega}_{0}}{\tilde{u}^2} \right. \right.$$  \hspace{1cm} (4.60)

$$\left. + n_q \tilde{\Omega}^a \tilde{E}_a + n_g \tilde{\Omega}^a \tilde{\Omega}^b \tilde{E}_{ab} + n_q \tilde{E}_a \tilde{E}_a + \ldots \right\} + S_0,$$

with the electric parity tensor, $\tilde{E}_{ab} = \tilde{W}_{abcd} \tilde{u}^c \tilde{u}^d$, and $\tilde{E}_a = \tilde{F}_{ab} \tilde{u}^b$, that corresponds to the gravitational quadrupole and electromagnetic dipole moment respectively. The interaction action,

$$S_0 = \int \det e \ d^4x \left\{ -\frac{1}{4\mu_0} F_{ab} F^{ab} + \frac{1}{16\pi G} R + \ldots \right\},$$  \hspace{1cm} (4.61)

is the Einstein-Maxwell action. The action describing the charged spinning compact object lives in the worldline, while $S_0$ lives in the bulk.

The coefficients of the effective theory

The coefficients of the effective theory encode the microphysics of the compact objects, which are determined through a matching procedure to the full known theory, and ultimately from GW observations. We identify them from the results in literature, without the need to do the explicit calculations here. We have already pointed out the coefficients appearing in the action describing a charged spinning point particle. The coefficient of the
point particle term \( [14] \), \( n_E = mc^2 \), the coefficients in the spin corrections, \( n_\Omega = I/4 \) and \( n_{\Omega,a} = I \) \([21, 22]\), and the coefficient from the correction due to electromagnetic charge, \( n_A = q \) \([14, 43]\). Now, we point out the rest of the coefficients due to the internal structure of the compact object.

We start with the coefficient due to static tidal effects, \( n_g \), which is a coefficient that depends on the internal structure of the star through a parameter known as the Love number. Any stellar object that can be described by an equation of state of matter, can be described approximately in terms of its Love numbers. In the case of BHs, it has been found that their Love numbers vanishes \([48]\), which also occurs for the case of spinning BHs \([27, 49]\), therefore setting \( n_g = 0 \) and \( n_{g,\Omega} = 0 \). Furthermore, it has been shown that, for both non-spinning \([50]\) and spinning BHs \([49]\), the coefficients due to the polarizability vanishes as well, \( n_q = 0 \) and \( n_{q,\Omega} = 0 \).

For a NS, the coefficients are different from zero. The leading order quadrupolar Love coefficient reads \([51]\)

\[
n_g = \frac{2\ell^5 k_2}{3G},
\]

with \( k_2 \), the quadrupolar dimensionless Love number, and \( \ell \) the radius of the star.\(^3\) The Love numbers are dimensionless parameters that measure the rigidity and tidal deformability of the compact object, and varies given different equations of state of matter \([52]\). This number is related to other parameters of the star from the relativistic I-Love-Q relations \([52, 53]\), which relates the moment of inertia, \( I \), the tidal deformability parameter, or Love number, \( k_2 \), and the quadrupole moment. These relations, which are empirically found to hold for a wide range of equations of state of matter, are only approximate relations and are not exact first principles relations.

On the coefficient due to the coupling of spin-gravity size effects for spinning NSs, in the slow rotation limit, the relativistic coefficient, \( n_{g,\Omega} \), is obtained through the Love-Q part of the I-Love-Q relations \([53]\). The latter coefficient in the Newtonian limit is the same as the one for static tides, \( n_{g,\Omega} = n_g \) \([53]\). For the charge-spin size coupling, which take into account polarization effects, given that charge in compact objects has been mostly neglected, the rest of the coefficients, \( n_{q,\Omega} \) and \( n_q \), are unknown, and are to be derived by analytical and numerical means.

Moving on into dissipative effects, although their coefficients are not explicitly shown in the action, they are encoded in the operators, \( \tilde{\mathcal{P}} \) and \( \tilde{\mathcal{D}} \), as shown above. We point out the known coefficients for dissipative effects in BH interactions from the existing literature. The operator \( \tilde{\mathcal{D}}^{ab} \), for non-spinning BHs, contains the coefficient, \( c_g \), which encodes the capacity of the BH to absorb GWs, and which can be read off from the response function derived in \([15]\),

\[
c_g = \frac{16 G^5 M^6}{90 c^{13}} = \frac{\ell^6 s^6}{360 G c},
\]

\(^3\)We have chosen to denote the radius of the object with \( \ell \) as in \([9, 23]\), rather than with \( R \) as commonly used, given that we use \( R \) for the Ricci scalar.
with \( M \) the mass of the BH, and \( \ell_s \) its radius. The coefficient in eq. (4.63), includes the factor of \( 1/2 \) mentioned in eq. (4.55). In the same way, we can obtain the coefficient for electromagnetic dissipative effects from the composite operator, \( \tilde{\mathcal{P}}(\tau) \) [15],

\[
c_q = \frac{2\pi \ell_s^4}{3\mu_0 c},
\]

which encodes the capacity of the BH to absorb electromagnetic waves.

On the dissipative effects of rotating BHs, the coefficients can be obtained from the response function derived in [27] using the Teukolsky equation, for both non-spinning and spinning case. We start by reading off the coefficient for a non-spinning BH from the response function in [27], which in their notation, reads

\[
\frac{1}{2} N_2 \ell_s^5 \mathcal{F}^{Sch}_{2m_a} = i \frac{\ell_s^5 M}{90c^3} \omega + \mathcal{O}(\omega^3) \approx i \frac{\ell_s^5}{300c^6} \omega,
\]

with \( m_a \), the azimuthal number, \( \ell_s \), the Schwarzschild radius, and where we have used the leading order mode, \( l = 2 \), of the angular momentum number, such that \( N_2 = 1/3 \).

To obtain eq. (4.65) as eq. (4.63), we have substituted the mass of the BH in terms of its radius, \( M = \ell_s c^2/2G \), and considered the extra factor of \( 1/2 \), as for eq. (4.63).

In the spinning case, the response function reads [27]

\[
\mathcal{F}^{I,Kerr}_{2m_a} = -i \frac{iam_a}{30c(\ell_+ - \ell_-)} + i \frac{\ell_+ M}{15c^3 (\ell_+ - \ell_-)} \omega + \mathcal{O}(\omega^3)
\]

\[
\approx -i \frac{Im_a}{30c Mc(\ell_+ - \ell_-)} \Omega + i \frac{\ell_+ M}{15c^3 (\ell_+ - \ell_-)} \omega,
\]

where \( a = J/Mc = I\Omega/Mc \), with \( J = I\Omega \), the scalar value of the angular momentum, and \( \ell_+ \) and \( \ell_- \), the outer and inner radius of the Kerr BH. The moment of inertia of a BH is \( I = 4GM^2/c^4 \). Therefore, the normalized response function, reads

\[
\frac{1}{2} N_2 \ell_s^5 \mathcal{F}^{I,Kerr}_{2m_a} \approx -i \frac{Im_a \ell_s^5}{180 Mc(\ell_+ - \ell_-)} \Omega + i \frac{\ell_s^5 M}{90c^6 (\ell_+ - \ell_-)} \omega.
\]

We have obtained a response function for the dissipative effects of the form, \( \langle \tilde{\mathcal{D}}_{ab}(\tau, \Omega) \rangle \propto i(c_g d\Omega + c_g d/d\tau) \tilde{E}_{ab} \). We can identify that, for a rotating extended object, tidal dissipation arises due to two separate contributions. The first term on the right hand side of eq. (4.67), for which is non-zero even in the case of \( \omega = 0 \), arises given that the spin of the body has a time dependence between the object and its tidal environment. This can be seen, from the object perspective in its proper frame, as the external environment rotating with the frequency of the spin of the object.

---

4 To make the reading of the coefficients accessible, we have written the response function as in [27], and then converted it to our notation.
Furthermore, now we have an expression which is explicit on the azimuthal numbers, $m_a$. For the dominant perturbation mode, $l = 2$, then, $m_a = [-2, -1, 0, 1, 2]$. Nevertheless, we can not identify a specific value of $m_a$ to be dominant. Therefore, it is necessary to sum over all possible values of $m_a$. This can be done by considering the electric tidal moment, $\tilde{E}_{l m_a}$, for which leading order is $\tilde{E}_{2m_a}$. Then, we one can proceed to identify the different elements of $\tilde{E}_{2m_a}$, given the possible values of $m_a$, to then couple each response function with its electric tidal moment. We would couple schematically, $\tilde{Q}_{2m_a} \propto \tilde{D}_{2-2} \tilde{E}_{2-2} + \tilde{D}_{2-1} \tilde{E}_{2-1} + \tilde{D}_{20} \tilde{E}_{20} + \tilde{D}_{21} \tilde{E}_{21} + \tilde{D}_{22} \tilde{E}_{22}$. Such procedure for maximally spinning BHs has been performed in [24], where the causal response function is matched.

The coefficients for NSs from dissipative effects are still unknown, and must be determined from hydrodynamical simulations, and ultimately from observations. Nevertheless, it is worth commenting that for a stellar object that is described by an equation of state of matter, there exists the weak friction model [54]. In the latter, the coefficient, $c_g = \Theta n_g$ [54], with $\Theta$, being the time lag, which accounts for the tidal bulge formed in the stellar object during the interaction with another gravitational object. It is unknown whether or not this model applies for NSs, but perhaps such coefficient can allows us to get an insight into the dissipative description of NSs.

5 Discussion

In this work we have reviewed and extended the model for spinning extended objects introduced in [16], which is derived using the coset construction [11, 40], a very powerful method that allows us to construct an effective theory from the symmetry breaking pattern as the only input. In this approach, a spinning extended object whose ground state breaks space-time symmetries, is coupled to a gravitational theory formulated as a gauge theory with local Poincaré symmetry and translations being non-linearly realized. We have included the internal structure [14, 15, 23] and electromagnetic charge [15, 43], such that we describe charged spinning extended objects, the most general extended object allowed in a theory of gravity such as general relativity.

We have derived the covariant building blocks of the effective theory, to build up invariant operators to form an action. We built our underlying theory and matched the coefficients to the full known theory, to obtain the Einstein-Maxwell action in the vierbein formalism. Then, by recognizing the symmetry breaking pattern of a charged spinning extended object, we have built the leading order invariant operators that are allowed by the symmetries, to describe it as a worldline point particle with its properties and internal structure encoded in higher order corrections in the action. Such corrections take into account for the basic necessary ingredients to completely describe an extended object in an effective theory of gravity as general relativity. By matching the coefficients of the effective action from the literature, we have described charged spinning compact objects, such as BHs and NSs.

Although this effective theory for spinning extended objects [16] by construction is a low energy description of the dynamics, compact objects which are described classically, fit into the description of "slowly" spinning [9]. We have shown the equivalence of our
effective theory to the ones currently used to obtain state of the art perturbative results of the binary dynamics [21, 22], with the advantage that the covariant building blocks to construct the tower of invariant operators to all orders have been derived. Therefore, our work complements the aforementioned theories for spinning extended objects, and lays on the foundations for a full description of the possible compact objects that can exist.

The most direct application of our derived action is on the PN expansion [28], where we have shown that our theory reproduces the well known results for spinning [22] and charged [43] extended objects. Moreover, novel results in the PN expansion have been derived on the internal structure of charged spinning compact objects [28]. More applications of the derived effective action are coming in the next series of papers, in which we approach the PM expansion, and introduce the new perturbative results into the EOB framework.

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A Conventions

We differentiate between space-time and local Lorentz indices as in [16]:

- $\mu, \nu, \sigma, \rho...$ denote space-time indices.
- $a, b, c, d...$ denote Lorentz indices.
- $i, j, k, l...$ denote spatial components of the Lorentz indices.

We denote the time of occurrence and the location in space of an event with the four component vector, $x^a = (x^0, x^1, x^2, x^3) = (t, \vec{x})$, and define the flat space-time interval, $ds$, between two events, $x^a$ and $x^a + dx^a$, by the relation

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2,$$

which we write using the notation

$$ds^2 = \eta_{ab} dx^a dx^b; \quad \eta_{ab} = \text{diag}(-1, +1, +1, +1). \quad (A.2)$$

B Worldline point particle dynamics

Our most important consideration when modeling compact objects with EFTs is that they can be treated as point particles, with the properties and internal structure encoded
in higher order corrections. To describe the dynamics of point particles, it is necessary to specify a continuous sequence of events in the space-time by giving the coordinates, $x^a(\lambda)$, of the events along a parametrized curve, defined in terms of a suitable parameter, $\lambda$. There is one curve among all possible curves in the space-time, which describes the trajectory of a material particle moving along some specified path, known as the worldline. We can consider it as a curve in the space-time, with $\lambda = ct$, acting as a parameter so that $x^a = (ct, \vec{x}(\tau))$. The existence of a maximum velocity $|\vec{u}| < c$, which requires the curve to be time-like everywhere, $ds < 0$, allows us to have a direct physical interpretation for the arc length along a curve.

Consider a clock attached to the particle frame, or the proper frame $\tilde{F}$, which is moving relative to some other inertial frame $F$ on an arbitrary trajectory. As measured in the frame $F$, during a time interval between $t$ and $t + dt$, the clock moves through a distance $|\vec{d}x|$. The proper frame $\tilde{F}$, which is moving with same velocity as the clock, will have $d\vec{x} = 0$. If the clock indicates a lapse of time, $dt \equiv d\tau$, the invariance of the space-time interval, eq. (A.1), implies that

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2 = ds^2 = -c^2 d\tau^2.$$  

(B.1)

Thus, we obtain the lapse of time in a moving clock,

$$d\tau = dt \sqrt{1 - \frac{v^2}{c^2}},$$  

(B.2)

along the trajectory of the clock. The total time that has elapsed in a moving clock between two events, known as the proper time $\tau$, is

$$\tau = \int d\tau = \int_{t_1}^{t_2} dt \sqrt{1 - \frac{v^2}{c^2}}.$$  

(B.3)

To build the action of a free point particle, we must construct a Lagrangian from the trajectory, $x^a(\tau)$, of the particle, which should be invariant under Lorentz transformations. The only possible term is proportional to the integral of $d\tau$, yielding the action

$$S = -\alpha \int d\tau = -\alpha \int \sqrt{1 - \frac{v^2}{c^2}} dt,$$  

(B.4)

where $\alpha$ is a dimensionful constant. To recover the action of a free point particle from non-relativistic mechanics, we take the limit $c \to \infty$, for which the Lagrangian yields $L = \alpha v^2/2c^2$. By comparing our point particle Lagrangian with the non-relativistic one, $L = (1/2)mv^2$, with $m$ the mass of the particle, we find that, $\alpha = mc^2$. Thus, the action for a relativistic point particle is

$$S = -mc^2 \int d\tau,$$  

(B.5)

which corresponds to the arc length of two connecting points in the space-time.
C  Symmetries in classical field theory

We consider symmetries that can be labelled by a continuous parameter, \( \theta \). Working with the Lie algebras of a group \( G \), we write a group element as a matrix exponential

\[
U = e^{i\theta u_T},
\]

where the generators \( T_u \), with \( u = 1, \ldots, n \), form a basis of the Lie algebra of \( G \). The \( T \)'s generators are hermitian if \( U \) is unitary. For each group generator, a corresponding field arises, which in this case is the field, \( \theta \).

The properties of a group \( G \), are encoded in its group multiplication law

\[
[T_u, T_v] = T_u T_v - T_v T_u = i c_{uvw} T_w,
\]

where, \( c_{uvw} \), are the structure constant coefficients. The last expression defines the Lie algebra of the group \( G \). The Lie bracket is a measure of the non-commutativity between two generators.

In the local framework of field theory, it is also possible to consider continuous symmetries that have a position dependent parameter, \( \theta = \theta(x) \). The space-time dependent symmetry transformation rules are called local or gauge symmetries. For global symmetries, \( \theta \) do not depends on the position. There is also a distinction between internal symmetries and space-time symmetries, on whether they act or not on space-time position. An example of an internal symmetry, where \( x \) is unchanged, is

\[
\phi^u(x) \rightarrow U^{-1} \phi^u(x) U = \mathcal{U}^u_v \phi^v(x),
\]

while an example for a space-time symmetry is the transformation

\[
\phi^u(x) \rightarrow V^{-1} \phi^u(x) V = \mathcal{V}^u_v \phi^v(x'),
\]

with \( x'^u = \mathcal{V}^u_v x^v \). Both internal and space-time symmetries can arise in global or gauged varieties.

Symmetries of special relativity

The full symmetry of special relativity is determined by the Poincaré symmetry. Its Lie group, known as the Poincaré group, \( G = \text{ISO}(3,1) \), is the group of Minkowski space-time isometries that includes all translations and Lorentz transformations.

The Lorentz group

The Lorentz group, \( \text{SO}(3,1) \), is the group of linear coordinate transformations

\[
x^a \rightarrow x'^a = \Lambda^a_b x^b,
\]

that leave invariant the quantity

\[
\eta_{ab} x^a x^b = -(ct)^2 + x_1^2 + x_2^2 + x_3^2,
\]
with \( \det \Lambda = 1 \). In order for eq. (C.6) to be invariant, \( \Lambda \) must satisfy

\[
\eta_{ab} x'^a x'^b = \eta_{ab} (\Lambda^c_d x^c) (\Lambda^b_d x^d) = \eta_{cd} x'^c x'^d,
\]

which implies the transformation of the metric as

\[
\eta_{cd} = \eta_{ab} \Lambda^a_c \Lambda^b_d.
\]

Consider an infinitesimal Lorentz transformation, with the Lorentz generators \( J_{ab} \), and its corresponding field, \( \alpha_{ab} \). We can expand

\[
\Lambda_{ab} = (e^{i \alpha^{cd} J_{cd}})^a_b = (e^\alpha)^a_b \approx \delta^a_b + \alpha^a_b.
\]

From equation (C.8) we find

\[
\alpha_{ab} = -\alpha_{ba},
\]

which is an antisymmetric 4x4 matrix with six components that are independent. Thus, the six independent parameters of the Lorentz group from the antisymmetric matrix, \( \alpha_{ab} \), corresponds to six generators which are also antisymmetric \( J^{ab} = -J^{ba} \).

Under Lorentz transformations, a scalar field is invariant,

\[
\phi'(x') = \phi(x).
\]

A covariant vector field, \( V^a \), transforms in a representation of the Lorentz group,

\[
V^a \rightarrow (e^{i \frac{1}{2} \eta_{cd} J_{cd}})^a_b V^b.
\]

If we consider an infinitesimal transformation, the variation of \( V^a \) reads,

\[
\delta V^a = \frac{i}{2} \alpha_{cd} (J^{cd})^a_b V^b,
\]

which is an irreducible representation.

The explicit form of the matrix \( (J^{ab})^c_d \), reads

\[
(J^{ab})^c_d = -i (\delta^c_d \eta_{a0} - \delta^c_b \eta_{ad}).
\]

Using the form of the generator in eq. (C.14), we can compute the commutator

\[
[J_{ab}, J_{cd}] = i (\eta_{ac} J_{bd} - \eta_{bc} J_{ad} + \eta_{bd} J_{ac} - \eta_{ad} J_{bc}),
\]

to find the Lie algebra. The components of \( J^{ab} \) can be rearranged into two spatial vectors

\[
J^i = \frac{1}{2} \epsilon^{ijk} J_{jk}, \quad K^i = J^{0i},
\]

with, \( J^{ij} \) and \( K^i \), the generators of rotations and boosts, respectively.

The Lorentz group has six parameters: Three rotations in three 2D planes that can be formed with the \((x, y, z)\) coordinates that leave \( ct \) invariant, which is the SO(3) rotation...
group, and three boost transformations in the \((ct, x)\), \((ct, y)\) and \((ct, z)\) planes that leave invariant \(- (ct)^2 + x^2\), \(- (ct)^2 + y^2\) and \(- (ct)^2 + z^2\), respectively. We parametrize the Lorentz matrix as

\[
\Lambda_{0}^{0} = \gamma, \quad \Lambda_{i}^{0} = \gamma \beta_{i}, \quad \Lambda^{i}_{j} = \delta^{i}_{j} + (\gamma - 1) \frac{\beta^{i}\beta_{j}}{\beta^{2}},
\]

(C.17)

with \(\gamma = (1 - v^2/c^2)^{-1/2}\), the Lorentz factor, and \(\beta^{i}\), the velocity

\[
\beta^{i} = \frac{v^{i}}{v^{0}} \tanh \eta,
\]

(C.18)

where \(\eta\) is the rapidity, defined as the hyperbolic angle that differentiates two inertial frames of reference that are moving relative to each other.

Therefore, the four vectors, \(V^a\) and \(V_a\), transforms under the Lorentz group as

\[
V^a(x) \rightarrow V'^a(x') = \Lambda^a_b V^b(x), \quad V_a(x) \rightarrow V'_a(x') = \Lambda_a^b V_b(x),
\]

(C.19)

with \(\Lambda^a_b = \eta^{ac} \eta^{bd} \Lambda^c_d\). The vectors are related via \(V_a = \eta_{ab} V^b\). A tensor, \(T^{ab}\), transforms as

\[
T^{ab}(x) \rightarrow T'^{ab}(x') = \Lambda^c_a \Lambda^d_b T^{cd}(x).
\]

(C.20)

In general, any tensor with arbitrary upper and lower indices transforms with a \(\Lambda^a_b\) matrix for each upper index, and with \(\Lambda^b_a\) for each lower one. We denote Lorentz transformations simply as, \(V^a = \Lambda^a_b V^b\) and \(T^{ab} = \Lambda_a^c \Lambda^b_d T^{cd}\).

**The Poincaré group**

To complete the Poincaré group, in addition to Lorentz invariance, we also require invariance under space-time translations. We can write a general element of the group of translations in the following form,

\[
U = e^{i z^a P_a},
\]

(C.21)

where \(z^a\) are the components of the translation,

\[
x^a \rightarrow x^a + z^a,
\]

(C.22)

and \(P^a\) their generators. Lorentz transformations plus translations form the Poincaré group, ISO(3,1). The Poincaré group algebra reads

\[
[P_a, P_b] = 0 \quad \text{(C.23)}
\]

\[
[P_a, J_{bc}] = i(\eta_{ac} P_b - \eta_{ab} P_c) \quad \text{(C.24)}
\]

\[
[J_{ab}, J_{cd}] = i(\eta_{ac} J_{bd} - \eta_{bc} J_{ad} + \eta_{bd} J_{ac} - \eta_{ad} J_{bc}). \quad \text{(C.25)}
\]
Gauge symmetry of classical electromagnetism

The gauge symmetry of classical electromagnetism is invariance under the U(1) gauge transformation. This is an internal symmetry for which the charge generator, $Q$, corresponds to a time invariant generator, with its corresponding gauge field, $A_\mu(x)$, which is the electromagnetic or photon gauge field. The local gauge symmetry is parametrized by the parameter, $\theta = \theta(x)$, with the group element

$$U(x) = e^{i\theta(x)}.$$  \hfill (C.26)

The gauge field, $A_\mu$, transforms under the U(1) symmetry as

$$A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu \theta(x).$$  \hfill (C.27)

Therefore, under Lorentz and U(1) transformations, the gauge field transforms as

$$A_\mu(x) \rightarrow \Lambda^\nu_\mu A_\nu(x) + \partial_\mu \theta(x, \Lambda).$$  \hfill (C.28)

The commutations relations of the charge generator, $Q$, with the generators of the Poincaré group, are constrained by the Coleman-Mandula theorem [55]. This theorem constrains the kinds of continuous space-time symmetries that can be present in an interacting relativistic field theory, and states that the most general possible transformation can be parametrized by

$$U = \exp \left\{ i \left( z^a P_a + i \sigma^a O_a + \frac{i}{2} \omega^{ab} J_{ab} \right) \right\},$$  \hfill (C.29)

with $P_a$, the generators of translations, $J_{ab}$, of Lorentz transformations, and $O_a$, the rest of the generators. The generators, $O_a$, must be from internal symmetries, and although they can fail to commute with themselves, $[O_a, O_b] \neq 0$, they must always commute with the space-time symmetry generators, $[P_a, O_b] = 0$ and $[J_{ab}, O_c] = 0$. The charge operator for the U(1) symmetry of electromagnetism commutes with itself, thus obtaining the commutation relations: $[P_a, Q] = 0$, $[J_{ab}, Q] = 0$ and $[Q, Q] = 0$.

### Transformation properties of gauge fields

The transformation properties of the gauge fields $\tilde{e}_\mu^a$, $\tilde{A}_\mu$ and $\tilde{\omega}_{\mu}^{ab}$, introduced in eq. (4.2), under local translations, $e^{iz^a P_a}$, local Lorentz transformations, $e^{\frac{i}{2} \omega_{\alpha \beta} J_{\alpha \beta}}$ and the local U(1) transformation, $e^{i\theta}$, read
\[ U = e^{i\theta} : \begin{cases} \tilde{A}_{\mu} & \rightarrow \tilde{A}_{\mu} - \partial_{\mu} \theta, \\
\tilde{e}_{a \mu} & \rightarrow \tilde{e}_{a \mu}, \\
\tilde{\omega}^{ab}_{\mu} & \rightarrow \tilde{\omega}^{ab}_{\mu}. \end{cases} \]

\[ U = e^{icP} : \begin{cases} \tilde{A}_{\mu} & \rightarrow \tilde{A}_{\mu}, \\
\tilde{e}_{a \mu} & \rightarrow \tilde{e}_{a \mu} - \tilde{\omega}^{ab}_{\mu} \xi^{\mu} b - \partial_{\mu} \xi^{a}, \\
\tilde{\omega}^{ab}_{\mu} & \rightarrow \tilde{\omega}^{ab}_{\mu}. \end{cases} \]

\[ U = e^{i\alpha J} : \begin{cases} \tilde{A}_{\mu} & \rightarrow \Lambda^{\nu \mu} \tilde{A}_{\nu}, \\
\tilde{e}_{a \mu} & \rightarrow \Lambda^{a \nu} \tilde{e}_{\nu \mu} + \alpha^{a \nu} \xi^{\mu}, \\
\tilde{\omega}^{ab}_{\mu} & \rightarrow \Lambda^{a \nu} \Lambda^{b \nu} \tilde{\omega}^{c d}_{\mu} + \Lambda^{a \nu} \partial_{\mu} (\Lambda^{-1})^{c b} = \tilde{\omega}^{ab}_{\mu} + \tilde{\omega}^{ac}_{\mu} \alpha^{c b} + \tilde{\omega}^{cb}_{\mu} \alpha^{a c} - \partial_{\mu} \alpha^{a b}. \end{cases} \] (C.30)

The gauge field, \( \tilde{e} \), transforms inhomogeneously under local translations. Under Lorentz transformations, \( \tilde{e} \) and \( \tilde{A} \), transforms linearly, while \( \tilde{\omega}^{ab}_{\mu} \) transforms as a connection. Under the U(1) transformation, only the gauge field, \( \tilde{A} \), is transformed.

Finally, the photon field, the vierbein and spin connection, under diffeomorphisms transforms as

\[ \begin{align*}
A_{\mu}(x) & \xrightarrow{\text{diffeo}} A_{\nu}(x) \partial_{\nu} \xi^{\mu} - \xi^{\mu}(x) \partial_{\nu} A_{\mu}(x), \\
\tilde{e}_{a \mu}(x) & \xrightarrow{\text{diffeo}} \tilde{e}_{a \nu}(x) \partial_{\nu} \xi^{\mu} - \xi^{\mu}(x) \partial_{\nu} \tilde{e}_{a \mu}(x), \\
\tilde{\omega}^{ab}_{\mu}(x) & \xrightarrow{\text{diffeo}} \tilde{\omega}^{ab}_{\nu}(x) \partial_{\nu} \xi^{\mu} - \xi^{\mu}(x) \partial_{\nu} \tilde{\omega}^{ab}_{\mu}(x),
\end{align*} \] (C.31)

which all of them transforms in the same way.

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