Quantum information dynamics in a high-dimensional parity-time-symmetric system

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Non-Hermitian systems with parity-time (PT) symmetry give rise to exceptional points (EPs) with exceptional properties that arise due to the coalescence of eigenvectors. Such systems have been extensively explored in the classical domain, where second or higher order EPs have been proposed or realized. In contrast, quantum information studies of PT-symmetric systems have been confined to systems with a two-dimensional Hilbert space. Here by using a single-photon interferometry setup, we simulate quantum dynamics of a four-dimensional PT-symmetric system across a fourth-order exceptional point. By tracking the coherent, non-unitary evolution of the density matrix of the system in PT-symmetry unbroken and broken regions, we observe the entropy dynamics for both the entire system, and the gain and loss subsystems. Our setup is scalable to the higher-dimensional PT-symmetric systems, and our results point towards the rich dynamics and critical properties.

I. INTRODUCTION

A fundamental postulate of quantum theory is that the Hamiltonian of an isolated system is Hermitian. This Hermiticity ensures real eigenvalues and a coherent, unitary time evolution for the system. This conventional wisdom was upended two decades ago by Carl Bender and co-workers, who showed that a non-Hermitian Hamiltonian with parity-time (PT) symmetry can exhibit entirely real spectra [1–4]. Over time, it has become clear that non-Hermitian Hamiltonians with PT symmetry can provide an effective description for systems with balanced, spatially separated gain and loss [5]. This concept has been extensively, and fruitfully, explored in classical (wave) systems where the number of energy quanta is much larger than one [6–15]. A PT-symmetric system is described by an effective, non-Hermitian Hamiltonian \( H_{PT} \) that is invariant under the combined parity and time-reversal operation [16]. As the gain-loss strength is increased, the spectrum of \( H_{PT} \) changes from real into complex conjugate pairs, and the corresponding eigenvectors cease to be eigenvectors of the PT operator. This PT-symmetry-breaking transition occurs at an EP of order \( n \) (EP\(_n\)), where \( n \) eigenvalues, as well as their corresponding eigenvectors, coalesce [17–19]. The PT transition and the non-unitary time evolution generated by \( H_{PT} \) have been observed in classical systems with EP2 [9,14,20,26], EP3 [15], and higher order EPs [27,28].

Due to the quantum limit on noise in linear (gain) amplifiers [29], creating a photonic system with balanced gain and loss in the quantum domain is not possible [30]. However, the EP degeneracies also occur in dissipative systems with mode-selective losses. Such passive PT-symmetric systems have been realized in the quantum domain with lossy, single-photon [31–33], ultracold atoms [34], and a superconducting transmon [35]. These realizations are limited to effective two-dimensional Hamiltonians with second-order EPs, and their quantum information studies are confined to global properties [34]. Here we present experimental quantum simulation of entropy dynamics in a four-dimensional, passive PT-symmetric system with an EP4.

II. IMPLEMENTING PT-SYMMETRIC QUDIT WITH AN EP4

Let us consider an open, four-mode system described by a 4 \( \times \) 4 Hamiltonian

\[
H_{PT} = -JS_x + i\gamma S_z, \tag{1}
\]

where \( S_x \) and \( S_z \) are spin-3/2 representations of the SU(2) group. It can be written in the matrix form as

\[
H_{PT} = \frac{1}{2} \begin{pmatrix}
3i\gamma & -\sqrt{3}J & 0 & 0 \\
-\sqrt{3}J & i\gamma & -2J & 0 \\
0 & -2J & -i\gamma & -\sqrt{3}J \\
0 & 0 & -\sqrt{3}J & -3i\gamma
\end{pmatrix} \tag{2}
\]

in the computational basis \( \{|1\rangle, |2\rangle, |3\rangle, |4\rangle\} \), and represents a PT-symmetric qudit with \( d = 4 \). The Hamiltonian \( H_{PT} \) commutes with the antilinear PT operator where the parity operator is \( P = \text{antidiag}(1,1,1,1) \) and a time-reversal operator is given by complex conjugation, \( T = * \). It follows from Eq. (2) that the first two computational modes represent the “gain sector” and the last two represent the “loss sector” in the system. The four equally spaced eigenvalues of \( H_{PT} \)
are given by \( \lambda_k = \{-3/2, -1/2, +1/2, +3/2\} \sqrt{J^2 - \gamma^2} \) \((k = 1, 2, 3, 4)\), which give rise to an EP4 at the \( PT \)-
breaking threshold \( \gamma = J \). The advantage of choosing
Hamiltonian \((1)\) is that it can be easily generalized to an
arbitrary dimensional system where it still remains
analytically solvable and has an EP with the order equal
to the system dimension \([15, 41, 42]\). Since \( H_{P\,T} \) has a
single energy gap \( \Delta = \sqrt{J^2 - \gamma^2} \), it follows that the \( PT \)
symmetric qudit has a sinusoidal dynamics in the \( PT \)-
symmetry unbroken region \((\gamma < J)\), and a monotonic,
exponential growth behavior in the \( PT \)-broken region
\((\gamma < J)\).

The coherent, non-unitary time evolution operator for
the system is given by \( U(t) = \exp(-iH_{P\,T}t) \) where we
have set \( h = 1 \). For \( \gamma = 0 \), the system is Hermitian
and the fermionic nature of spin-3/2 representation is
manifest in the anti-periodicity of \( U \), i.e., \( U(T) = -I_4 \)
where \( T(0) = 2\pi/J \) for \( \gamma = 0 \). In this case, the mode-
occupations \( P_{\gamma}(t) = |\langle k|\psi(t)\rangle|^2 \) of the four modes obey
a shifted mirror symmetry with \( P_{\gamma}(t) = P_{\gamma-k}(t+T/2) \),
which indicates a perfect state transfer occurring from
mode \( k \) to mode \((5-k)\) at \( T/2 \). Here \( |\psi(t)\rangle = U(t)|\psi(0)\rangle \)
is the time-evolved state. For \( \gamma < J \), the system is in the
\( PT \)-symmetry unbroken region, the dynamical evolu-
tion is anti-periodical with period \( T(\gamma) = 2\pi/\Delta \). At
the EP4 \((\gamma = J)\), \( U(t) \) ceases to be periodic and
has an operator norm that grows as \( t^6 \), reflecting the fourth
order of the EP. In the \( PT \)-symmetry broken region, the
mode occupations grow exponentially with time. How-
ever, the quantum information metrics, such as the von
Neumann entropy, are defined with respect to the in-
stantaneously normalized state (indicating post-selection
that eliminates the quantum jumps \([38, 40, 42]\)). Therefore,
the EP in \( PT \)-symmetry, for the EP4 region, these quan-
tities reach a steady-state value. These results are appli-
cable to all finite-dimensional representation of the \( SU(2) \)
group.

The four-dimensional Hamiltonian \( H_{P\,T} \) is particularly
interesting because it can be viewed as a system of two
interacting, non-Hermitian qubits. This mapping is pro-
vided by the identities \( 2S_x = \sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y + \sqrt{3} \sigma_z \otimes \sigma_z \),
\( 2S_z = \sigma_z \otimes I_2 + I_2 \otimes \sigma_z / 2 \), and \( P = \sigma_y \otimes \sigma_y \), where \( \sigma_k \)
\((k = x, y, z)\) are the standard Pauli matrices. Using this
insight, we investigate the quantum information dynam-
ics in the gain and loss subsystems of the \( PT \)-symmetric
qudit.

We encode the four modes of the qudit in the spa-
tial and polarization degrees of freedom of a single pho-
ton, and label them as \(|1\rangle = |UH\rangle, |2\rangle = |UV\rangle, |3\rangle = |DV\rangle \),
\(|4\rangle = |DH\rangle \). Here \(|{\{H, V}\rangle}\) are the horizontal
and vertical polarizations, and \(|{\{U, D}\rangle}\) denote the up-
ner and lower paths, which undergo gain and loss respectively
(Fig. 1(a)). As illustrated in Fig. 1(b), pairs of single photons are generated via type-I spontaneous
parametric down conversion (SPDC) using a non-linear
\( \beta \)-Barium-Borate (BBO) crystal. One photon serves as a
trigger and the other signal photon is prepared in an arbi-
trary qudit state using a polarizing beam splitter (PBS),
wave plates with certain setting angles and a beam dis-
placer (BD).

By mapping the \( PT \)-symmetric Hamiltonian \( H_{P\,T} \) into
a passive \( PT \)-symmetric one with mode-selective losses
\( H_{L} = H_{P\,T} - 3i\gamma L_3^2/2 \), we implement the \( 4 \times 4 \) lossy,
time-evolution operator

\[
U_L(t) = \exp(-iH_L t)
\]

via a lossy linear optical circuit, which is related to
\( U(t) \) through \( U(t) = U_L(t) \exp(3\gamma t/2) \) \([11]\). The evolu-
tion operator \( U_L(t) \) is realized by BDs, half-wave plates
(HWP), and sandwich-type QWP-HWP-QWP setups,
where QWP is an abbreviation for quarter-wave plate.

We experimentally measure and then obtain scaled
mode occupations \( P_k(t) \) by projecting the time-evolved state
\(|\psi(t)\rangle\) onto \(|k\rangle\). The initial state is chosen to be
\(|\psi(0)\rangle = (|1\rangle + |2\rangle + |3\rangle + |4\rangle)/2 \). The projective
measurement and the quantum state tomography on the
qudit state are realized by BDs, wave plates and a PBS.
followed by avalanche photodiodes (APDs). Only coincidences between the heralded and trigger photons are registered. The perfect state transfer for $\gamma = 0$ is confirmed by the transfer of occupation from the first mode to the fourth mode (Fig. 2(a)). In the $\mathcal{PT}$-unbroken phase with a finite $\gamma = 0.2J$, there is no perfect state transfer at time $T(\gamma)/2$ due to the non-unitary dynamics (Fig. 2(b)). The measured occupations are, however, periodic in time with a period $T(\gamma)$. At the EP4 with $\gamma = J$, the scaled mode occupation $P_k(t)$ grows algebraically with time as $t^6$ (Fig. 2(c)). Such a scaling is dictated by the order of the EP. At $\gamma = J$, the Hamiltonian obeys $H_{\mathcal{PT}}(\gamma = J) = 0$ and the power-series expansion of $U(t)$ terminates at the third order, giving rise to $t^6$ dependence for the occupation numbers. By projecting the time-evolved state onto $|k\rangle$, we can obtain the occupation at the EP4 and its power-law behaviour (Fig. 2(c)). In the $\mathcal{PT}$-broken phase, the scaled mode occupation grows exponentially with time as expected (Fig. 2(d)). We note that while the simulation time-range is limited to two periods for $\gamma < J$, we restrict to $0 < t \leq 4.5$ due to the rapid growth of the scaled mode occupation at the EP4 and in the broken $\mathcal{PT}$ region.

When the $\mathcal{PT}$-symmetric Hamiltonian is perturbed from the EP4 by a small detuning $\delta$, the resulting complex eigenvalues in the vicinity of EP$n$ are given by a Puiseux series in $\delta^{1/n}$, indicating enhanced classical sensitivity proportional to the order of the EP \textsuperscript{[13, 43]}. In addition to the behavior of the mode occupations at the EP, this serves as a complementary check of the order of the EP. To that end, we experimentally measure the complex eigenvalues of the perturbed Hamiltonian $H_\delta = H_{\mathcal{PT}}(\gamma = J) + iJ\delta|1\rangle\langle 1|$. Figure 3 shows that the real and imaginary parts of the eigenvalues of $H_\delta$ indeed scale as $\delta^{1/4}$, consistent with the EP4 that occurs at $\gamma = J$.

### III. OBSERVING INFORMATION DYNAMICS

A crucial aspect of dynamics of a high-dimensional $\mathcal{PT}$-symmetric system is the flow of information among its different parts, and the information retrieval phenomena between the whole system and its environment. To that end, we consider the qudit entropy

$$S(t) = -\text{Tr} \left[ \hat{\rho}(t) \log_2 \hat{\rho}(t) \right],$$

where $\hat{\rho}(t) = \rho(t)/\text{Tr}[\rho(t)]$ is the instantaneously normalized density matrix and $\rho(t) = U(t)\rho(0)U^\dagger(t)$ is the time-evolved density matrix of the system with a time-dependent trace. The gain- and loss-sector entropies are $S_{\text{Gain}}(t)$ and $S_{\text{Loss}}(t)$, respectively. These are obtained from the gain- and loss-sector reduced density matrices $\rho_{\text{Gain}}(t) = \text{Tr}_{3,4}[\rho(t)]$ and $\rho_{\text{Loss}}(t) = \text{Tr}_{1,2}[\rho(t)]$, respectively.

A full knowledge of the time-dependent density matrix through the quantum-state tomography allows us to experimentally explore the information flow. We focus on the quantum dynamics with the fully symmetric initial state $|\psi(0)\rangle$ (Figs. 4(a)-(c)) and a mixed initial state $\rho(0) = 0.925|1\rangle\langle 1| + 0.025|2\rangle\langle 2| + |3\rangle\langle 3| + |4\rangle\langle 4|$ (Figs. 4(d)-(f)) in the $\mathcal{PT}$-symmetry broken region. Since the qudit undergoes a coherent, non-unitary evolution for any gain-loss strength $\gamma$, a pure state remains a pure state and the entropy of the entire system $S(t)$ remains constant with time (Fig. 4(a)). For a mixed...
initial state, the entropy is constant only in the Hermitian limit, $\gamma = 0$. In the $\mathcal{PT}$-symmetry unbroken region, the entropy $S(t)$ shows periodic oscillations. This demonstrates an exchange of quantum information between the $\mathcal{PT}$-symmetric qudit and its environment, and the oscillations observed here may be interpreted as an evidence of information backflow from the environment and a signature of non-Markovianity in the $\mathcal{PT}$-unbroken phase [44].

At the EP4 or in the $\mathcal{PT}$-symmetry broken region, due to the diverging occupation, the normalized density matrix $\hat{\rho}(t)$ approaches a pure state, and the total system entropy, therefore, approaches zero [44] [44] [45]. In all cases, the experimental simulation results agree well with the theoretical prediction. Importantly, this observed behavior of entropy does not depend on the details of the system, which signifies its universality. In this case, information flows unidirectionally and the dynamics is asymptotically Markovian [44].

In a sharp contrast to the results for the entire system, the behavior of subsystem entropies for pure and mixed initial states is qualitatively similar. In either case, the gain-sector entropy $S_{\text{Gain}}(t)$ and the loss-sector entropy $S_{\text{Loss}}(t)$ oscillate in the $\mathcal{PT}$-symmetry unbroken region including the Hermitian limit. On the other hand, they reach nonzero steady-state values at EP4 and in the broken $\mathcal{PT}$-symmetry region. It is worth its while to point out that although the gain and loss entropies show qualitatively similar behavior, the trajectories traced out by the instantaneously normalized, reduced density matrices $\hat{\rho}_{\text{Gain}}(t)$ and $\hat{\rho}_{\text{Loss}}(t)$ in the Bloch ball are distinctly different (Fig. 5). The trajectory of the gain-sector density matrix is weighted towards the northern hemisphere, representing the largest amplifying mode, whereas the loss-sector density matrix trajectory is less heavily weighted. These differences lead to the slightly different behaviors of $S_{\text{Gain}}$ and $S_{\text{Loss}}$.

In this paper, we realize a four-level system dynamics under a non-Hermitian Hamiltonian in either $\mathcal{PT}$-symmetric unbroken, broken or at the exceptional point with single photons and a cascaded interferometric setup. We realize $4 \times 4$ non-unitary evolution operations with six BDs and use another one for state preparation. Two different measurements—projective measurement and the quantum state tomography of a four-level system—are carried out at the output. In contrast, the setup in [44] is much simpler; a two-level system dynamics under a non-Hermitian Hamiltonian is realized with two BDs, and only a single-qubit state tomography is carried out to reconstruct the final state. Our experimental method to
implement a non-unitary, loss time evolution operator is scalable, and therefore can be used to simulate higher-dimensional $\mathcal{PT}$-symmetric systems in the future.

**IV. DISCUSSION**

In this section we briefly present the analytical derivation for the entropy of the $\mathcal{PT}$-symmetric system. If we start with a pure state, it remains pure under the coherent, non-unitary evolution that is generated by a $\mathcal{PT}$-symmetric, non-Hermitian Hamiltonian. Therefore, the entropy of such a state continues to remain zero. If the initial state is mixed, i.e., $\rho(0) = \sum_i \alpha_i |\psi_i\rangle \langle \psi_i|$, we can express the orthonormal vectors $|\psi_i\rangle = \sum_k \beta_{ik} |\zeta_k\rangle$ in terms of the non-orthogonal right eigenvectors $|\zeta_k\rangle$ of $H_{\mathcal{PT}}$. The initial state, thus, can be rewritten as

$$\rho(0) = \sum_{k,i} \alpha_i \beta_{ik} \beta_{ij}^* |\zeta_k\rangle \langle \zeta_j|.$$  

The final state is then given by

$$\rho(t) = \sum_{k,i} \alpha_i |\beta_{ik}|^2 |\zeta_k\rangle \langle \zeta_k| + \sum_{k\neq j,i} \alpha_i \beta_{ik} \beta_{ij}^* \kappa_{kl} e^{-i(\lambda_k - \lambda_j)t} |\zeta_k\rangle \langle \zeta_j|.$$  

We further express the right eigenvectors of $H_{\mathcal{PT}}$ in terms of the orthonormal eigenvectors of the instantaneous density matrix $\rho(t)$ as $|\zeta_k\rangle = \sum_i \kappa_{kl} |\phi_i\rangle$. It allows us to obtain the time-dependent occupation eigenvalues $p_k(t) = \langle \phi_i | \rho(t) | \phi_i \rangle$ as

$$p_l = \sum_{k,i,l} \alpha_i |\beta_{ik}|^2 |\zeta_l\rangle \langle \zeta_l| + \sum_{k\neq j,i,l} \alpha_i \beta_{ik} \beta_{jl}^* \kappa_{kl} \kappa_{jl}^* e^{-i(\lambda_k - \lambda_j)t}.$$  

In the Hermitian limit, the eigenvectors of $H_{\mathcal{PT}}$ are orthonormal, and the time evolution acts as the rotation of coordinates. Therefore the eigenstates $|\phi_i\rangle$ are unchanged and the entropy remains a constant of motion. In the non-Hermitian case, $\{|\zeta_k\rangle\}$ are not orthonormal, and the time-dependent entropy is then given by

$$S(t) = -\sum_t \tilde{p}_l \log_2 \tilde{p}_l,$$

where the fractional occupations are given by $\tilde{p}_l(t) = p_l(t)/\sum_k p_k(t)$. The entropy of time-evolved state oscillates periodically in $\mathcal{PT}$-symmetric unbroken region. At the EP4, $p_l(t)$ grow algebraically with time as $t^\delta$. By writing $p_l = \lambda_l t^\delta + \mu_l$ where $\lambda_l$ and $\mu_l$ are constant, it is straightforward to see that the entropy approaches a steady-state value polynomially with time. In contrast, in the $\mathcal{PT}$-broken region, $p_l(t)$ grow exponentially with time, leading to a steady-state value for the entropy that is approaches in an exponential manner.
V. SUMMARY

Higher-dimensional PT systems, which can be treated as composites of two or more minimal, non-Hermitian, quantum systems, provide a starting point for interacting quantum models with PT-symmetry and EP degeneracies. In this work, we experimentally simulate and observe the quantum information dynamics in a four-dimensional system with EP4. We show that the subsystem-entropy behavior for gain or loss subsystems can be either qualitatively different from or similar to the dynamics for the total entropy of the four-dimensional system. Our work is the first experimental demonstration of critical phenomena in four-dimensional PT-symmetric quantum dynamics, and shows the versatility of the single-photon interferometric network platform for simulating interacting, non-Hermitian, quantum systems.

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