Toward an understanding of the $R_{AA}$ and $v_2$ puzzle for heavy quarks

F. Scardina 1, 2, S. K. Das 1, 2, S. Plumari 1, 2, J. I. Bellone 1, 2, and V. Greco 1, 2

1 Department of Physics and Astronomy, University of Catania, Via S. Sofia 64, I-95125 Catania, Italy
2 Laboratori Nazionali del Sud, INFN-LNS, Via S. Sofia 62, I-95123 Catania, Italy

Abstract

One of the primary aims of the ongoing nuclear collisions at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) energies is to create a Quark Gluon Plasma (QGP). The heavy quarks constitutes a unique probe of the QGP properties. Both at RHIC and LHC energies a puzzling relation between the nuclear modification factor $R_{AA}(p_T)$ and the elliptic flow $v_2(p_T)$ related to heavy quark has been observed which challenged all the existing models. We discuss how the temperature dependence of the heavy quark drag coefficient can address for a large part of such a puzzle. We have considered four different models to evaluate the temperature dependence of drag and diffusion coefficients propagating through a quark gluon plasma (QGP). All the four different models are set to reproduce the same $R_{AA}(p_T)$ experimentally observed at RHIC energy. We have found that for the same $R_{AA}(p_T)$ one can generate 2 – 3 times more $v_2$ depending on the temperature dependence of the heavy quark drag coefficient.

1. Introduction

Heavy Quarks, charm and bottom, created in ultra-Relativistic Heavy Ion Collisions (uRHIC) represents ideal probes to study the Quark Gluon Plasma QGP [1, 2]. An essential feature in analyzing Heavy quarks motion in a QGP is that their mass is much larger than the typical momentum exchanged with the plasma particles entailing that many soft scatterings are necessary to change significantly the momentum and the trajectory of the heavy quarks. Therefore the propagation of heavy quarks has been usually treated as a Brownian motion that is described by means of the Fokker-Planck (FP) equation. In such an equation the interaction is encoded in the drag and diffusion coefficient. The two key observables related to HQ that have been measured in experiments are the nuclear suppression factor $R_{AA}$ and the elliptic flow $v_2$ [3, 4, 5, 6, 7]. Several theoretical efforts have been made using the Fokker Planck equation to reproduce the $R_{AA}$ and the $v_2$ experimentally observed [8, 9, 10, 11, 12, 13, 14, 15]. Another approach used to describe heavy quark propagation is the Boltzmann approach (BM) [16, 17, 18, 19, 20, 21]. Analyzing the $R_{AA}$ gives information on the average magnitude of the interaction. We point out that studying the relation between $R_{AA}$ and $v_2$ it is possible to deduce other informations on the interactions between HQ and bulk. All the approaches show some difficulties to describe simultaneously $R_{AA}$ and $v_2$. We have found that two ingredients assume a particular importance in reducing the differences between experimental data for $R_{AA}$ and $v_2$ and theoretical calculations: the temperature dependence of the interaction and the use of full Boltzmann collision integral to study the time evolution of HQ momentum. The proceeding is organized as follows. In section 2 we discuss the Fokker Planck approach which is used to describe the propagation of heavy quark through the QGP. In this section we present four different modelings to calculate the drag and diffusion coefficients. In section 3 we compare the result for $R_{AA}$ and $v_2$ obtained with FP approach with those obtained using the BM approach.
2. Fokker-Planck approach

The propagation of the Heavy quark is described by the Fokker Planck equations which is indicated in the following equation

$$\frac{df}{dt} = \frac{\partial}{\partial p_i} \left[ A_i(p)f + \frac{\partial}{\partial p_j} \left( B_j(p) \right) \right]$$

(1)

where $A_i$ and $B_{ij}$ are the drag and the diffusion coefficients. To study the propagation of HQ this equation is replaced by the relativistic Langevin equation, which is more suited for numerical simulations

$$dx_i = \frac{p_i}{E} dt,$$

$$dp_i = -A p_i dt + \left( \sqrt{2B_0} P_{ij}^1 + \sqrt{2B_1} P_{ij}^b \right) \frac{p_j}{\sqrt{2}} dt$$

where $dx_i$ and $dp_i$ are the coordinates and momenta changes in each time step $dt$; $A$ is the drag force and $B_0$ and $B_1$ are respectively the longitudinal and the transverse diffusion coefficients; $p_j$ is the stochastic Gaussian distributed variable; $P_{ij}^1 = \delta_{ij} - p_i p_j/p^2$ and $P_{ij}^b = p_i p_j/p^2$ are the transverse and longitudinal projector operators respectively. We employ the common assumption, $B_0 = B_1 = D$ \cite{9, 10, 12, 13}. To solve the Langevin equation a background medium which describes the evolution of the bulk is necessary. In our work the evolution of the bulk is provided by a 3+1D relativistic transport code tuned at fixed $\eta/s$ \cite{22, 23} which is able to reproduce the same results of hydrodynamical simulations. The transport code provides at each time step the density profile and the temperature profile of the bulk which are necessary to calculate the drag coefficients. We have performed simulations of $Au + Au$ collisions at $\sqrt{s} = 200$ AGeV for the minimum bias. In both cases the initial conditions in coordinate space are given by the Glauber model, while in the momentum space a Boltzmann-Juttner distribution function up to a transverse momentum $p_T = 2$ GeV has been considered. At larger momenta mini-jet distributions, as calculated within pQCD at NLO order, have been employed \cite{24}. The HQ distribution in momentum space is in accordance with the one in proton-proton that have been taken from \cite{25}. We have used four different modelings to calculate the drag coefficient entailing different temperature dependence of the interaction . The diffusion coefficient is instead calculated in accordance with the Einstein relation $D = \frac{\pi \alpha \lambda^2}{2}$. Our purpose is to investigate the relation between $R_{AA}$ and $v_2$ for different temperature dependence of the energy loss. In the first modeling we have evaluated the drag coefficient from (pQCD) and we have considered elastic interaction among HQ and the bulk (light quarks and gluons). The scattering matrix related to these processes $M_{HQB}$, $M_{HQG}$ and $M_{HQG}$ in leading order are the well known Combridge matrix. The infrared singularity in the $t$-channel is regularized introducing a Debye screening mass $m_D = \sqrt{4\pi\alpha_s} T$ with a running coupling \cite{26}. In another modeling we have evaluated the drag force from the gauge/string duality \cite{27} through the following equation

$$\Gamma_{conf} = C \frac{T^2_{QCD}}{M_c}$$

(3)

where $C = \frac{\pi^2}{2} = 2.1 \pm 0.5$ \cite{14}. We have also considered another modeling in which the drag coefficient is evaluated considering a bulk consisting of particles with a $T$-dependent quasi-particle
masses, \( m_q = 1/3 2g^2 T^2, m_e = 3/4 g^2 T^2 \). This model is able to reproduce the thermodynamics of lattice QCD \([28]\) (see also \([29], [30], [31]\)) by fitting the coupling \( g(T) \). Such a fit leads to the following coupling \([28]\).

\[
g^2(T) = \frac{48\pi^2}{[\langle 11N_c^2 - 2N_f \rangle \ln(\frac{T_c}{T} - \frac{T}{T_c})]}
\]

where \( \lambda = 2.6 \) and \( T/T_c = 0.57 \).

Finally we have considered a model in which the light quarks and gluons are massless but the coupling is from the QPM as indicated in Eq. 4. This last case is indicated in the figure as \((\alpha_{QPM}(T), m_q = m_e = 0) \) and has to be considered as an expedient to have a drag which decreases with the temperature.

For all the 4 cases considered the interaction has been rescaled to reproduce the \( R_{AA} \) observed in experiments. We have simulated HQ propagation with the Langevin dynamics for the four different models presented above. The Langevin equation gives as output the momentum distributions of HQ at the quark-Hadron transition temperature \( T_c \). The momenta distributions are convoluted with the Peterson fragmentation functions of the heavy quark indicated in Eq. 5 in order to get the momentum distribution of D and B mesons.

\[
f(z) \propto \frac{1}{[z[1 - \frac{1}{z} - \frac{c}{1-\frac{z}{T_c}}]]}
\]

where \( c_e = 0.04 \) for charm quarks and \( c_b = 0.005 \) for bottom quark. In figure 3, the nuclear modification factor \( R_{AA} \) of the D and B mesons is shown as a function of \( p_T \) for RHIC (200AGeV). Instead in figure 4 the elliptic flow \( (v_2 = \langle (p_T^2 - \langle p_T^2 \rangle)/(p_T^2 + \langle p_T^2 \rangle) \rangle \) at the same energy as a function of \( p_T \) is depicted. We observe that the larger is the interaction in the region of low temperature the larger is the elliptic flow. The same conclusions has been discussed also in the light flavor sector as shown in Refs. \([32], [33]\). The reason of such a strong dependence of the elliptic flow on the temperature dependence of the drag coefficient is due to the fact that the elliptic flow is generated in the final stage of the evolution of the fireball when the temperature is lower.

3. Boltzmann approach

The Boltzmann equation for the HQ distribution function is indicated here

\[
p^\mu \partial_p \chi_{Q}(x, p) = C[\chi_{Q}(x, p)]
\]

where \( C[\chi_{Q}(x, p)] \) is the relativistic Boltzmann-like collision integral which is solved by means of a stochastic algorithm. In such an algorithm whether a collision happen or not is sampled stochastically comparing the collision probability \( P_{rr} = v_{rel}C_{x+Q+Q+Q} \cdot \Delta t/\Delta x \) with a random number extracted between 0 and 1. We use the Boltzmann equation to describe the propagation of the heavy quark and the evolution of the bulk.

The comparison between LV and BM approach has been studied in these references \([18], [19]\) where it is shown that for charm quark FP deviates significantly from the BM and such a deviation significantly depends on the the values of the Debye screening mass, whereas for bottom quarks the FP is a very good approximation. We considered in references \([18], [19]\) three values of \( m_D: 0.4 \text{ GeV}, 0.83 \text{ GeV} \) and 1.6 GeV. Here we have not considered a fixed value of the Debye screening mass but a value which depends on the temperature according to \( m_D = g_0 \). In figures 2 and 3 the comparison for the \( R_{AA} \) and \( v_2 \) at RHIC between the BM (orange lines) and the FP (black lines) are shown. We found that using the BM for the same values of the \( R_{AA} \) we get larger values for \( v_2 \) with respect to those obtained using the FP.

With the Boltzmann approach using the \((\alpha_{QPM}(T), m_q = m_e = 0) \) we get a value of the elliptic flow even larger with respect to the experimental data, however this case represent an extreme and not realistic case. Our results show a non-negligible impact of the approximation in the collision integral involved in the Fokker Plack equation on the relation between \( R_{AA} \) and \( v_2 \). We summarize the results introducing a new plot in Fig. 4 in which \( R_{AA} \) vs \( v_2 \) at a given momentum \( p_T = 1.3 \text{ GeV} \) is shown. This figures clearly shows how the building up of the \( v_2 \) can differ.
Figure 4: $R_{AA}$ vs $v_2$ obtained with the FP for the four different T-dependences of the drag coefficient with the experimental data at RHIC energy at $p_T = 1.3$ GeV. The open symbols indicate the results obtained using the BM approach up to a factor 3 for the same $R_{AA}$ depending on the temperature dependence of the interaction and on the approach, BM or FP, used to describe the propagation of the heavy quark in the QGP.

4. Conclusions

We have found that different temperature dependences of the interaction can lead to different in the elliptic flow $v_2$ by 2-3 times even if leading to the same $R_{AA}$. Our studies suggest that the correct temperature dependence of the drag coefficient may not be larger power of $T$ (as in pQCD or AdS/CFT) rather a lower power of $T$ or may be constant in $T$. Moreover we have studied the difference in the building up of the elliptic flow between Fokker-Planck and Boltzmann approach for a fixed $R_{AA}$. We observe that the Boltzmann approach generates a larger elliptic flow than the Fokker-Planck [18][19].

Acknowledgements

F. Scardina, S. K. Das, S. Plumari and V. Greco acknowledge the support by the ERC StG under the QG-PDyn Grant n. 259684

References

[1] Svetitsky B 1988 Phys. Rev. D 37, 2484
[2] R. Rapp and H van Hees, R. C. Hwa, X. N. Wang (Ed.) Quark Gluon Plasma 4, 2010, World Scientific, 111
[3] A. Adare et al. (PHENIX Collaboration), Phys. Rev. Lett. 98, 172301 (2007).
[4] B. I. Abeleb (STAR Collaboration), Phys. Rev. Lett. 98, 192301 (2007).
[5] A. Adare et al. (PHENIX Collaboration) A. Adare et al. (PHENIX Collaboration), Phys. Rev. C 84, 044905 (2011)
[6] B. Abelev et al.(ALICE Collaboration) JHEP 1209 (2012) 112
[7] B. Abelev et al. (ALICE Collaboration) Phys. Rev. Lett. 111,(2013) 102301.
[8] Mustafa M G, Pal D and Srivastava D K 1998 Phys. Rev. C 57, 889
[9] Hees H van, Greco V and Rapp R 2006 Phys. Rev. C 73, 034913; Hees H van, Mannearelli M, Greco V and Rapp R 2008 Phys. Rev. Lett. 100 192301
[10] Das S K, Alam J, Mohanty 2009 Phys. Rev. C 80 054916; 2010 Phys. Rev. C 82 014908; Majumdar S, Bhattacharyya T, Alam J, Das S K 2012 Phys. Rev. C 84 044901
[11] Alberico W M et al. 2011 Eur. Phys. J. C 71 1666; Alberico W M et al. 2013 Eur. Phys. J. C 73 2481
[12] S. Cao, G. Y. Qin and S. A. Bass, Phys. Rev. C 88 (2013) 4, 044907
[13] M. He, R. J. Fries and R. Rapp, Phys. Rev. Lett. 110, 112301 (2013)
[14] S. K. Das and A. Davody, Phys. Rev. C 89, 054912 (2014)
[15] S. K. Das, F. Scardina, S. Plumari and V. Greco, Phys. Rev. Lett. B747 (2015) 260-264
[16] Gossiaux P B, Aichelin J 2008 Phys. Rev. C 78 014904
[17] Uphoff J, Fochler O, Xu Z and Greiner C 2011 Phys. Rev. C 84 024908
[18] S. K. Das, F. Scardina, S. Plumari and V. Greco,Phys. Rev. C 90 044901 (2014)
[19] F. Scardina,S. K. Das, S. Plumari and V. Greco, J.Phys.Conf.Ser. 535 (2014) 012019; F. Scardina,S. K. Das, S. Plumari and V. Greco, J.Phys.Conf.Ser. 636 (2015) 012017
[20] Younas M, Coleman-Smith C E,Bass S A and Srivastava D K, Phys.Rev. C 91 (2015) 2, 024912
[21] T. Song, H. Berrehrah, D. Cabrera, J. M. Torres-Rincon, L. Torres-Rincon, W. Cassing and E. Bratkovskaya, arXiv:1503.03039 [nucl-th].
[22] G. Ferrini, M. Colonna, Di Toro M and Greco V, 2009 Phys. Lett. B670, 325; V. Greco, M. Colonna, Di Toro M and Ferrini G, 2009 Progr. Part. Nucl. Phys. 62, 562
[23] S. Plumari, A. Puglisi, F. Scardina and V. Greco, Phys. Rev. C 86 (2012) 054902
[24] V. Greco, C. M. Ko and P. Levai, Phys. Rev. Lett. 90 (2003) 202302; V. Greco, C. M. Ko and P. Levai, Phys. Rev. C 68 (2003) 034904
[25] M. Cacciari, P. Nason and R. Vogt, Phys. Rev. Lett. 95 (2005) 122001; M. Cacciari, S. Frixione, N. Housse, M. D. Mangano, P. Nason and G. Ridolfi, JHEP 1210 (2012) 137
[26] O. Kazcmarek and F. Zantow, Phys. Rev. D, 71, 114510(2005)
[27] J. M. Maitalacena, Adv. Theor. Math. Phys. 2, 231 (1998)
[28] S. Plumari, W. M. Alberico, V. Greco and C. Ratti, Phys. Rev. D, 84, 094004 (2011)
[29] S. K. Das, V. Chandra, J. Alam, J. Phys. G 41 015102 (2014)
[30] H. Berrehrah, E. Bratkovskaya, W. Cassing, P.B. Gossiaux, J. Aichelin, and M. Bleicher, Phys.Rev. C 89, 054901 (2014),
[31] H. Berrehrah, P.B. Gossiaux, W. Cassing, J. Aichelin.E. Bratkovskaya, Phys.Rev. C 90, 051901 (2014)
[32] F. Scardina, Di Toro and V. Greco, Phys. Rev. C 82 (2010) 054901.
[33] J. Liao and E. Shuryak, Phys. Rev. Lett. 202302 (2009)
[34] A. Lang et al., Jour. of Comp. Phys. 106, 391 (1993)
[35] F. Scardina, D. Perricone, S. Plumari, M. Ruggieri and V. Greco Phys. Rev. C 90 (2014) 054904
[36] Xu Z, Greiner C 2005 Phys. Rev. C 71, 064901