Real space Dynamical Super Cell Approximation for interacting disordered systems.

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Effective medium super-cell approximation method which is introduced for disordered systems is extended to a general case of interacting disordered systems. We found that the dynamical cluster approximation (DCA) and also the non local coherent potential approximation (NLCPA) are two simple case of this technique. Whole equations of this formalism derived by using the effective medium theory in real space.

Theoretical understanding of strongly correlated systems such as high temperature superconductors, heavy fermions and magnetism require appropriate techniques to obtain their physical properties. Recently, by the many body theory techniques, it has been shown that both alloys and strongly correlated systems in the infinite limit dimensions are mapped to a single site which embedded in an effective medium. This can be described by restriction on locality of self energy, that is, \( \Sigma(i,j;\omega_n) = \Sigma(\omega_n)\delta_{ij} \). This single site technique for strongly correlated systems and alloys systems called, Dynamical Mean Field Approximation (DMFA) and Dynamical Cluster Approximation (DCA) respectively. The single site nature of the infinite dimensional limit is imply that the inter sites correlations and also inter-site multiple scattering are negligible. But in the ordinary dimensions such as one, two and three, both inter sites correlation and inter-site multiple scattering have a significant contribution on the self energy, therefore not only the self energy is not local but also it is very sensitive with respect to the dimension. Recently by coarse graining of the self energy in k-space, we obtain a closed set of equations to do calculations in a general EMSCA. Also whole relevant DCA equations are derived by the EMSCA method.

We start our investigation on a general tight binding model for an interacting alloy system, which is given by,

\[
H = -\sum_{ij\sigma} t_{ij}^{\sigma\sigma'} c_{i\sigma}^\dagger c_{j\sigma'} + \sum_{\sigma} \left( \varepsilon_i - \mu \right) n_{i\sigma} + \sum_{ij\sigma} U_{ij}^{\sigma\sigma'} \hat{n}_{i\sigma} \hat{n}_{j\sigma'},
\]

where \( c_{i\sigma}^\dagger (c_{i\sigma}) \) is the creation (annihilation) operator of an electron with spin \( \sigma \) on lattice site \( i \) and \( \hat{n}_{i\sigma} = c_{i\sigma}^\dagger c_{i\sigma} \) is the number operator. \( t_{ij}^{\sigma\sigma'} \) are the random hopping integrals between \( i \) and \( j \) lattice sites with spin \( \sigma \) and \( \sigma' \) respectively. \( \mu \) is the chemical potential and \( \varepsilon_i \) is the random on-site energy, where takes \(-\delta/2\) with probability \( 1-c \) for the host sites and \( \delta/2 \) with probability \( c \) for impurity sites. \( U_{ij}^{\sigma\sigma'} \) is a positive or negative interaction potential between electrons on the lattice site \( i \) and \( j \).

The equation of motion for electrons corresponding to the above Hamiltonian, Eq. (1) is given by,

\[
\sum_{l\sigma''} \left( \left( \frac{\partial}{\partial \tau} - \varepsilon_i + \mu \right) \delta_{il} \delta_{\sigma\sigma''} - U_{ij}^{\sigma\sigma''} \right) G^{\sigma''\sigma}(l\tau,j\tau') + \sum_{l\sigma'} U_{ij}^{\sigma\sigma'} G_2^{\sigma''\sigma'}(l\tau,l\tau',j\tau') = \delta(\tau - \tau') \delta_{ij} \delta_{\sigma\sigma'}
\]

where \( \sigma \) and \( \sigma' \) are imaginary time, \( G^{\sigma\sigma'}(i\tau,j\tau') \) is the random interacting single particle Green function and \( G_2^{\sigma\sigma'}(i\tau,j\tau') \) is the two particle Green function. The random hopping integrals, \( t_{ij}^{\sigma\sigma'} \), can be defined in terms of clean system hoppings, \( t_{0ij}^{\sigma\sigma'} \), and the hopping integral deviations, \( \delta t_{ij}^{\sigma\sigma'} \), in a such way that the hopping

In this paper the real space effective medium super-cell approximation (EMSCA) is generalized and extended to treat a random interacting system. We show that neglecting of the interaction between electrons in different super-cells, multiple scattering by sites in different super-cells and also deviations of the hopping integrals, \( \delta t_{ij}^{\sigma\sigma'} \), naturally leads to the super-cell periodicity of the self energy, \( \Sigma(i,j;\omega_n) \), with respect to the super-cell translation vectors, \( r_{ij} \). This provide us the DCA coarse graining of the self energy in k-space. We are obtain a closed set of equations to do calculations in a general EMSCA. Also whole relevant DCA equations are derived by the EMSCA method.
The imaginary time Fourier transform of Eq.4 leads to, 

\[ \bar{G}(\tau'; \kappa') = \frac{1}{N} \sum_{i,j} \langle \psi_i | \hat{S}^{\dagger} \hat{a}_i \hat{a}_{\kappa}^\dagger | \psi_j \rangle \delta(\tau - \tau') \delta_{ij} \delta_{\sigma \sigma'} \]  

where the self energy, \( \Sigma_{\sigma \sigma'}(i \tau', l \tau') \), is defined by, 

\[ \bar{G}^{\sigma \sigma'}(l \tau', i \tau') = \frac{1}{N} \sum_{l' \tau'} \langle \psi_l | \hat{S}^{\dagger} \hat{a}_{l'} \hat{a}_{\sigma'}^\dagger | \psi_i \rangle \delta(\tau - \tau') \delta_{ll'} \delta_{\sigma \sigma'} \]  

(4)

So, the real space correspondence of Eq.7 can be written as, 

\[ G^{\sigma \sigma}(i \tau; l \tau; i \omega_n) = G^{\sigma \sigma}(i \tau; l \tau; i \omega_n) + \sum_{l' \tau'} G^{\sigma \sigma}(i \tau; l' \tau; i \tau') G^{\sigma \sigma}(l' \tau; j \tau; i \omega_n) \]  

(15)

where the real space Green function matrix, \( G^{\sigma \sigma}(i \tau; l \tau; i \omega_n) \), is, 

\[ G^{\sigma \sigma}(i \tau; l \tau; i \omega_n) = \frac{1}{N} \sum_{\kappa} e^{-i \kappa \cdot r_{ij}} G^{\sigma \sigma}(i \tau; l \tau; i \omega_n) \]  

(16)

and the average single particle Green function is defined by, 

\[ G_{\sigma \sigma}(i \tau; l \tau; i \omega_n) = \frac{1}{N} \sum_{\kappa} e^{-i \kappa \cdot r_{ij}} G_{\sigma \sigma}(i \tau; l \tau; i \omega_n) \]  

(17)

Eq.14 can not be solved exactly. We extend the effective medium super-cell approximation (EMSCA) which is recently introduced by us for a disordered system, to the case of interacting disordered systems. The EMSCA for such system based on three assumptions: first, neglecting interaction between electrons on different super-cells, 

\[ U_{\tau \gamma} = 0, \quad \text{if } i \text{ and } j \text{ are same super-cell,} \]  

(18)

second, neglecting hopping integral deviations, \( \delta t_{\tau \gamma} \), when \( i \) and \( j \) are in the different super-cells, 

\[ \delta t_{\tau \gamma} = 0, \quad \text{if } i \text{ and } j \text{ are same super-cell.} \]  

(19)

and finally, neglecting multiple impurity scattering and also correlations between different super-cells. These conditions imply that no correlation between super-cells, hence we have, 

\[ \Sigma_{\gamma \gamma}(i \tau; l \tau; i \omega_n) = \Sigma_{\gamma \gamma}(i \tau; l \tau; i \omega_n) + \sum_{l' \tau'} \Sigma_{\gamma \gamma}(i \tau; l' \tau; i \omega_n) \Sigma_{\gamma \gamma}(l' \tau; j \tau; i \omega_n) \]  

(20)

This means that the self energies in each super-cell are independent of other super-cells and they are periodic with respect to the super-cell translation vectors, \( \mathbf{r}_c \), 

\[ \Sigma_{\gamma \gamma}(i \tau; l \tau; i \omega_n) = \Sigma_{\gamma \gamma}(i \tau; l \tau; i \omega_n) \]  

(21)
where $I$ and $J$ refer to sites in a same super-cell. Also Eq. 18 leads to the super-cell periodicity for the interaction potential matrix, $U_{sc}(I, J)$, where

$$U_{sc}(I, J) = \begin{pmatrix} U_{sc}^{++}(I, J) & U_{sc}^{+\downarrow}(I, J) \\ U_{sc}^{\downarrow+}(I, J) & U_{sc}^{\downarrow\downarrow}(I, J) \end{pmatrix}, \quad (22)$$

which is,

$$U_{sc}(r_{IJ} + r_{Nc}) = U_{sc}(r_{IJ}). \quad (23)$$

The Fourier transformation of Eqs. 21 and 23 imply that,

$$e^{-i\mathbf{k} \cdot \mathbf{r}_{Nc}} = 1. \quad (24)$$

Thus the wave vectors of the self energy, $\Sigma(\mathbf{k}; \omega_n)$, and also of the interaction potential, $U(\mathbf{k})$ are restricts to the $\mathbf{K}_n$ which are given by,

$$\mathbf{K}_n = \sum_{i=1}^{3} \frac{l_i}{N_{ci}} \mathbf{b}_i, \quad (25)$$

where $\mathbf{b}_i$ are the reciprocal lattice primitive vectors, $N_{ci}$ are the number of the lattice sites in a super-cell at $\mathbf{a}_i$ direction ($\mathbf{a}_i$ are the lattice primitive vectors) and $l_i$ is a integer number. Therefore by inserting Eqs. 24 and 20 in to Eq. 3 we found that,

$$\Sigma_{sc}(\mathbf{K}_n; \omega_n) = \frac{1}{N_c} \sum_{I,J} e^{i \mathbf{K}_n \cdot r_{IJ}} \Sigma_{sc}(I, J; \omega_n). \quad (26)$$

Also by applying the EMSCA conditions, Eqs. 18 and 24 to the following exact relation,

$$U(\mathbf{k}) = \frac{1}{N} \sum_{i,j} U(i, j) e^{i \mathbf{k} \cdot \mathbf{r}_{ij}}, \quad (27)$$

we found a similar coarse graining for the interaction potential,

$$U(\mathbf{K}_n) = \frac{1}{N_c} \sum_{I,J} U_{sc}(I, J) e^{i \mathbf{K}_n \cdot r_{IJ}}, \quad (28)$$

The inverse Fourier transformation of the self energy, $\Sigma(\mathbf{K}_n; \omega_n)$, to the real super-cell is,

$$\Sigma_{sc}(I, J; \omega_n) = \frac{1}{N_c} \sum_{\mathbf{K}_n} e^{-i \mathbf{K}_n \cdot r_{IJ}} \Sigma(\mathbf{K}_n; \omega_n). \quad (29)$$

and also for $U(\mathbf{K}_n)$ is,

$$U_{sc}(I, J) = \frac{1}{N_c} \sum_{\mathbf{K}_n} U(\mathbf{K}_n) e^{-i \mathbf{K}_n \cdot r_{IJ}}, \quad (30)$$

where the orthogonality condition in a super-cell is given by,

$$\frac{1}{N_c} \sum_{\mathbf{K}_n} e^{-i \mathbf{K}_n \cdot r_{IJ}} = \delta_{IJ}. \quad (31)$$

Now by inserting Eq. 26 and 20 in to Eq. 17 the super-cell average Green function, $\bar{G}_{sc}(I, J; \omega_n)$, is given by,

$$\bar{G}_{sc}(I, J; \omega_n) = \frac{1}{N_c} \sum_{\mathbf{K}_n} e^{i \mathbf{K}_n \cdot r_{IJ}} \bar{G}(\mathbf{K}_n; \omega_n) \quad (32)$$

where

$$\bar{G}(\mathbf{K}_n; \omega_n) = \frac{N}{N_c} \sum_{\mathbf{K}_n'} \left( \bar{G}^{0}(\mathbf{K}_n + \mathbf{k}; \omega_n) - \Sigma(\mathbf{K}_n; \omega_n) \right)^{-1}. \quad (33)$$

In order to obtain $\bar{G}_{sc}(I, J; \omega_n)$ from Eq. 32 we should have $\Sigma_{sc}(I, J; \omega_n)$, thus we need to have another equations to complete the self consistency loop. These equations are obtained by applying the EMSCA to the system partition function, as follow. The partition function of the system with the Hamiltonian Eq. 11 is given by,

$$Z = \langle Tr e^{-\beta H} \rangle_r, \quad (34)$$

where $\langle \rangle_r$ denotes the configurational average over random energies, $\varepsilon_i$. The partition function, Eq. 33 can be rewrite as

$$Z = \langle \int D\bar{\Psi} D\bar{\Psi} e^{-S} \rangle_r, \quad (35)$$

where the action $S$ is,

$$S = \sum_{ij} \int_{\tau_0}^{\beta} d\tau \bar{\psi}_{i\sigma}(\delta_{ij} \delta_{\sigma\sigma'} (\frac{\partial}{\partial \tau} - \mu + t_{ij}^{\sigma\sigma'}) \bar{\psi}_{j\sigma'} (\tau)) + S_{r-1} \quad (36)$$

and $S_{r-1}$ is,

$$S_{r-1} = \sum_{ij} \int_{\tau_0}^{\beta} d\tau \bar{\psi}_{i\sigma}(\delta_{ij} \delta_{\sigma\sigma'} (\frac{\partial}{\partial \tau} - \mu + t_{ij}^{\sigma\sigma'}) \bar{\psi}_{j\sigma'} (\tau))$$

$$+ \sum_{ij} \int_{\tau_0}^{\beta} d\tau \bar{\psi}_{i\sigma}(\tau) \bar{\psi}_{j\sigma'} (\tau) \delta_{\sigma\sigma'}$$

$$+ \sum_{ij} \int_{\tau_0}^{\beta} d\tau \bar{\psi}_{i\sigma}(\tau) \delta_{\sigma\sigma'} \bar{\psi}_{j\sigma'} (\tau), \quad (37)$$

in which $D\bar{\Psi} = \Pi_i d\bar{\psi}_{i\sigma} d\bar{\psi}_{i\sigma}$ and $D\bar{\Psi} = \Pi_i d\bar{\psi}_{i\sigma} d\bar{\psi}_{i\sigma}$, where $d\bar{\psi}_{i\sigma} = \lim_{M \to \infty} \Pi_{m=1}^{M} d\bar{\psi}_{i\sigma}(\tau_m)$, $d\bar{\psi}_{i\sigma} = \lim_{M \to \infty} \Pi_{m=1}^{M} d\bar{\psi}_{i\sigma}(\tau_m)$. The Eq. 37 can be written as,

$$S = \int_{\tau_0}^{\beta} d\tau d\tau' \sum_{ij} \bar{\psi}_{i\sigma}(\tau) (G^{0-1})_{ij\sigma\sigma'} \bar{\psi}_{j\sigma'} (\tau') + S_{r-1}, \quad (39)$$

where,

$$\frac{1}{\beta} \sum_{\omega_n} \left( (\delta_{ij} \delta_{\sigma\sigma'} (\omega_n - \mu + t_{ij}^{\sigma\sigma'}) \bar{\psi}_{j\sigma'} (\tau)) = \int_{\tau_0}^{\beta} d\tau' \times$$

$$\int_{\tau_0}^{\beta} d\tau' (G^{0-1})_{ij\sigma\sigma'} \bar{\psi}_{j\sigma'} (\tau'), \quad (40)$$
and the clean non-interacting Green function matrix, \( \mathbf{G}^0(\omega_n) \), is defined by,
\[
(\mathbf{G}^0)^{-1}_{ij\sigma\sigma'}(\tau - \tau') = \frac{1}{\beta} \sum_{\omega_n} \left( \delta_{ij} \delta_{\sigma\sigma'} (\omega_n - \mu) + \epsilon_{ij} \delta_{\sigma\sigma'} \right) e^{i\omega_n (\tau - \tau')}
\]
By hint of Eq.35 the clean Green function, \( \mathbf{G}^0 \), can be express in terms of the average Green function, \( \bar{\mathbf{G}} \), and self energy, \( \Sigma \), as,
\[
\mathbf{G}^0 = \mathbf{G}^{-1} + \Sigma.
\]
By inserting Eq.42 in to Eq.39 we have,
\[
S = \int d\tau d\tau' \sum_{ij\sigma\sigma'} \bar{\psi}_{i\sigma}(\tau) (\mathbf{G}^{-1})_{ij\sigma\sigma'} \bar{\psi}_{j\sigma'}(\tau') + S_{r-i-s},
\]
where,
\[
S_{r-i-s} = \sum_{ij\sigma\sigma'} \int d\tau d\tau' \bar{\psi}_{i\sigma}(\tau) \bar{\psi}_{j\sigma'}(\tau) \left( \mathbf{G}^{-1} \right)_{ij\sigma\sigma'} \bar{\psi}_{j\sigma'}(\tau')
\]
Now by applying the EMSCA, where is taking average over all super-cells except one super-cell, which is denoted by \( I \), Eq.43 converts to,
\[
Z_{EMSCA} = Z_{sc} \times \prod_{\delta \neq \{ I \}, \sigma} (d\bar{\psi}_{i\sigma} d\psi_{i\sigma}) \delta^{sc} \bar{\psi}_{i\sigma}(\tau) \left( \mathbf{G}^{-1} \right)_{ij\sigma\sigma'} \bar{\psi}_{j\sigma'}(\tau') e^{-S_{r-i-s}}
\]
where the super-cell partition function, \( Z_{sc} \), is given by,
\[
Z_{sc} = \left( \prod_{\delta = 1}^{Nc} d\bar{\psi}_{i\sigma} d\psi_{i\sigma} \right) e^{-S_{sc}}
\]
and the super-cell action in the effective medium, \( S_{sc} \), is,
\[
S_{sc} = \sum_{ij\sigma\sigma'} \int d\tau d\tau' \bar{\psi}_{i\sigma}(\tau) \bar{\psi}_{j\sigma'}(\tau) \left( \mathbf{G}^{-1} \right)_{ij\sigma\sigma'} \bar{\psi}_{j\sigma'}(\tau') - \sum_{ij\sigma\sigma'} \int d\tau d\tau' \bar{\psi}_{i\sigma}(\tau) \left( \mathbf{G}^{-1} \right)_{ij\sigma\sigma'} \bar{\psi}_{j\sigma'}(\tau) + \sum_{ij\sigma\sigma'} \int d\tau d\tau' \bar{\psi}_{i\sigma}(\tau) \left( \delta_{ij} \delta_{\sigma\sigma'} + \delta t_{ij} \delta_{\sigma\sigma'} \right) \bar{\psi}_{j\sigma'}(\tau)
\]
in which the super-cell cavity Green function matrix, \( \mathbf{G} \), is defined by,
\[
\mathbf{G} = \mathbf{G}^{-1} - \bar{\Sigma}.
\]
The matrix element of Eq.48 is given by the following Dynos like equation for the super-cell sites,
\[
\bar{G}_{sc} (I, J; \omega_n) = \mathbf{G}(I, J; \omega_n) + \sum_{L, L'} \mathbf{G}(I, L; \omega_n) \mathbf{G}_{sc}(L, J; \omega_n) \mathbf{G}(L', J; \omega_n).
\]
The second part of the right hand side of Eq.65 is the super-cell excluded effective medium partition function which is easily integrable, due to bilinearity of the Grassmann variables. But the first part is the partition function of the super-cell where is embedded in an effective medium environment, is not integrable directly due to four point Grassmann variable in its integrand. Note that for the case of only disordered systems, Eq.47 leads to,
\[
\mathbf{G}_{imp}^{-1}(\sigma \rightarrow \sigma') = \mathbf{G}(\omega_n)^{-1}(\sigma \rightarrow \sigma') - \delta(\sigma \rightarrow \sigma') \bar{\epsilon}_{sc}
\]
where \( \bar{\epsilon}_{sc} \) is the super-cell impurity matrix. The imaginary time Fourier transform of Eq.50 imply that,
\[
\mathbf{G}_{imp}(\omega_n)^{-1} = \mathbf{G}(\omega_n)^{-1} - \bar{\epsilon}_{sc}
\]
which are derived previously by us. Therefore, for a disordered system, Eqs.51 and 52 are construct a close set of equations that should be solved self consistently.
In the general case, where is included both interaction and randomness, to calculate the super-cell partition function, \( Z_{sc} \), it is possible to use the Hirsch-Hubbard-Stratonovich transformation (HHST) to decouple the interaction term and map it to an auxiliary Ising filed. Although our method is general with respect to the interaction potential and randomness, but for simplicity and also to see the DCA derivation we concentrate our discussion on a repulsive on-site potential, \( U_{ij}^{imp} = U_{ij} \delta_{\sigma\sigma} \), and \( \delta t_{ij}^{imp} = 0 \). The HHST procedure is as following, dividing the imaginary time interval, \( [0, \beta] \), into \( M \) subintervals, \( \Delta \tau = \beta/M \), hence the imaginary time at the slice is given by \( \tau_l = l \beta/M \). Therefore the discretizing of imaginary times leads to \( \int_0^\beta d\tau = \sum_{l=0}^{M} \Delta \tau \), thus,
\[
Z_{sc} = \left( \prod_{\sigma} \prod_{l=1}^{Nc} \left( d\bar{\psi}_{i\sigma} d\psi_{i\sigma} \sum_{s_l = \pm 1} \right) e^{-S_{sc}^{l}} \right)_{r-sc},
\]
where the super-cell action \( S_{sc}^{l} \) is,
\[
S_{sc}^{l} = (\Delta \tau)^2 \sum_{l \delta} \bar{\psi}_{i\sigma}(\tau_l) \times
\]
\[
\left( \mathbf{G}^{-1} \right)_{I,J} + \delta_{JJ'} \left( \frac{\lambda \sigma_s l_I}{(\Delta \tau)^2} - \frac{\varepsilon_I}{\Delta \tau} + \frac{U}{2 \Delta \tau} \right) \delta_{l_I' + 1} + \delta_{II'} \left( \frac{\lambda \sigma_s l_I}{(\Delta \tau)^2} - \frac{\varepsilon_I}{\Delta \tau} + \frac{U}{2 \Delta \tau} \right) \delta_{l_I' + 1} \right) \psi_{J\sigma}(\tau_{I'}). 
\]

(54)

We define the cluster impurity Green function \( \mathbf{G}_{sc}^{imp} \) as

\[
\left( \mathbf{G}_{sc}^{imp-1} \right)_{I,J} = \left( \mathbf{G}^{-1} \right)_{I,J} + \delta_{JJ'} \left( \frac{\lambda \sigma_s l_I}{(\Delta \tau)^2} - \frac{\varepsilon_I}{\Delta \tau} + \frac{U}{2 \Delta \tau} \right) \delta_{l_I' + 1}. 
\]

(55)

The matrix form of Eq.55 can be written as,

\[
\left( \mathbf{G}_{sc}^{imp-1} \right)_{I,J} = \mathbf{T}(\varepsilon^\prime - 1),
\]

(56)

where, \( \mathbf{T}_{I,J} = \delta_{JJ'} \left( \frac{\lambda \sigma_s l_I}{(\Delta \tau)^2} \right) \) and matrix elements of \( \mathbf{V}_\sigma \) are,

\[
\mathbf{V}_{I\sigma} = \frac{\lambda \sigma_s l_I}{(\Delta \tau)^2} + \frac{U}{2 \Delta \tau}.
\]

(57)

Eqs. 56 and 55 for the case of just interacting systems are derived by the DCA method. Details of QMC solving of Eqs.55 can be find in elsewhere. The average of impurity Green function, \( \mathbf{G}_{sc}^{imp} \), over Ising fields and also impurities configurations is the super-cell effective medium Green function

\[
\left( \mathbf{G}_{sc}^{imp}(I;\tau_I,\tau_{I'}) \right) = \bar{\mathbf{G}}_{sc}(I;\tau_I,\tau_{I'}). 
\]

(58)

The Fourier transform of Eq.58 is give by,

\[
\mathbf{G}_{sc}(K_n;\omega_n) = \frac{1}{N_c} \times \sum_{l_I} \sum_{l_I'} e^{iK_n l_I l_I'} e^{i\omega_n(\tau_I - \tau_{I'})} \mathbf{G}_{sc}(I;\tau_I,\tau_{I'}). 
\]

(59)

Eqs. 56, 58 and 59 construct a close set of equations that should be solved self consistently.

The algorithm of numerical proses is as following,

1- A guess for the initial cluster self energy, \( \Sigma(K_n;\omega) \), usually zero.

2- From Eq.55 calculate the cluster Green function, \( \mathbf{G}(K_n;\omega) \).

3- Calculate the cavity Green function \( \mathbf{G}(K_n;\omega) \) from Fourier transform of Eq.59 \( \bar{\mathbf{G}}^{-1}(K_n;\omega_n) = \mathbf{G}^{-1}(K_n;\omega_n) + \Sigma(K_n;\omega_n) \).

4- Calculate the Fourier transform of the cavity Green function,

\[
\mathbf{G}(I;\tau_I - \tau_{I'}) = \frac{1}{\beta} \sum_n \sum_{K_n} \mathbf{G}(K_n;\omega_n) e^{i\omega_n} e^{-iK_n l_I l_I'} 
\]

(60)

5- Calculate the new cluster Green function \( \bar{\mathbf{G}}(I;\tau_I - \tau_{I'}) \) from Eq.59.

6- Calculate the inverse Fourier transform of \( \bar{\mathbf{G}}(I;\tau_I - \tau_{I'}) \).

7- Calculate the new self energy \( \Sigma(K_n;\omega_n) \) from

\[
\Sigma(K_n;\omega_n) = \mathbf{G}^{-1}(K_n;\omega_n) - \mathbf{G}^{-1}(K_n;\omega_n).
\]

(61)

8- Go to step 2 and repeat whole proses until convergence.

In conclusion we are extended the EMSCA to the case of an interacting disordered system. Similar to the EMSCA we showed that the periodicity of self energy with respect to super-cell translation vector leads to the coarse graining of self energies and hence the average Green function in k-space. Then by applying the effective medium theory on the system partition function, we find two equations where are relates the super-cell impurity Green function, \( \mathbf{G}_{sc}^{imp}(I;\tau_{I,n}) \), and super cell average Green function, \( \mathbf{G}_{sc}(I;\tau_{I,n}) \), to the super-cell cavity Green function, \( \mathbf{G}(I;\tau_{I,n}) \). This completes the whole formalism of a new real space method for disordered interaction systems. In especial case of interacting system our formalism leads to real space derivation of DCA while for a disordered system is converts to the EMSCA.

Now we are established that the DCA and also the NL-CPA are two especial case of the, real space dynamical effective medium super-cell approximation (DEMSCA).

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