Regularized Harmonic Surface Deformation

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Abstract

Harmonic surface deformation is a well-known geometric modeling method that creates plausible deformations in an interactive manner. However, this method is susceptible to artifacts, in particular close to the deformation handles. These artifacts often correlate with strong gradients of the deformation energy. In this work, we propose a novel formulation of harmonic surface deformation, which incorporates a regularization of the deformation energy. To do so, we build on and extend a recently introduced generic linear regularization approach. It can be expressed as a change of norm for the linear optimization problem, i.e., the regularization is baked into the optimization. This minimizes the implementation complexity and has only a small impact on runtime. Our results show that a moderate use of regularization suppresses many deformation artifacts common to the well-known harmonic surface deformation method, without introducing new artifacts.

1 Introduction

Surface deformation is an important task in geometry processing. Deforming models involves interactive modeling sessions driven by a user, who deforms an object by manipulating a subset of the surface vertices. Linear deformation methods [BS08] have proven effective in this context as they often create plausible and realistic-looking deformations, while still allowing for interactive runtimes. Deformations are usually modeled as the minimizers of specific deformation energies that are defined locally at each point of the domain and measure a specific distortion property.

Harmonic surface deformation is a well-known linear gradient-domain method introduced by Zayer et al. [ZRKS05] that uses a differential surface representation to perform global mesh deformations. Deformation constraints are smoothly distributed over the entire mesh using harmonic functions and surface details are preserved in the reconstructed deformations. Furthermore, the method is parameter-free.

Linear methods such as harmonic deformations are prone to artifacts, as linear energy terms are inadequate to accurately model the non-linear physical forces and processes involved in a deformation. Common deformation artifacts include flipped triangles (in the case of planar surfaces), protruding triangles, degenerate elements, volume loss as well as local and global shape distortion. Many of those artifacts occur close to the deformation handles. Figure 1 shows an example.

A number of non-linear correction methods [Lip12, AL13, SKPSH13, KABL14] exists that suppress these artifacts in planar or volumetric settings. These methods are very powerful as they guarantee artifact-free deformations, but they typically have a big impact on the runtime. Most importantly, they are not applicable to deformations of surfaces. Martinez Esturo et al. [MRT14] introduce an alternative linear energy regularization method: a quadratic regularization term is proposed that is strongly coupled to the problem-specific deformation energy. For a number of problems, this regularization yields to artifact-free results, albeit it cannot be guaranteed. Technically, this energy regularization requires only minor modifications to the algorithm with little impact on the runtime. The amount of regularization can be adjusted using a single parameter.

In this work, we apply linear energy regularization to the harmonic surface deformation method of Zayer et al. [ZRKS05]. Hereby, we follow the general ideas of Martinez Esturo et al. [MRT14]. We demonstrate that energy regularization enhances harmonic deformation results and suppresses a variety of artifacts. Our main contributions are:

- We provide an energy-regularized formulation of harmonic surface deformation.
- We refine the discretization of the energy differential operator of Martinez Esturo et al. [MRT14] for better estimates in high curvature regions.
- We evaluate the effectiveness of our approach. In particular, we demonstrate that moderate use of energy regularization improves deformation results by resolving artifacts without introducing new ones.

This paper is structured as follows: we discuss related shape editing techniques and correction methods (Section 2) and review harmonic deformations as well as energy regularization (Section 3). Then we
introduce our approach to linear energy regularization for harmonic surface deformation (Section 4). We perform a qualitative analysis of our results (Section 5), followed by quantitative evaluation and discussion (Section 6). Lastly, we present our conclusions and outlook for future work (Section 7).

2 Related Work

The goal of interactive surface deformation is to create meaningful deformations while preserving surface properties such as local details and curvature. Linear deformation methods play a major role in this area, since they provide the interactivity and often produce plausible deformations. Most often, linear methods represent the surface using its differential properties [Sor06]. One can distinguish these methods with respect to their sensitivity regarding rotation and translation. Rotation sensitive methods such as Yu et al. [YZX+04] and Zayer et al. [ZRKS05] use the gradients of affine transformations to construct a deformation guidance field, and solve a Poisson problem for geometry reconstruction. Since translations introduce local changes to the tangent plane of the surface, these methods are not suitable for shape deformations that involve large translations. On the other hand, translation sensitive methods such as [SCL+04] can handle large translations but not rotations. We refer to the survey of Botsch and Sorkine [BS08] for a detailed review of linear deformation methods.

Linear techniques often cannot guarantee that the used deformations are smooth everywhere [JBPS11]. This leads to deformation artifacts such as flipped triangles, protruding elements and volume loss due to rotations. Artifacts can be avoided by improving the smoothness of the transformation interpolation field. However, bi- or tri-harmonic weights create additional local extrema in the interpolation field that lead to unintuitive deformations results. Jacobson et al. [JWS12] tackle this problem by forcing a desired topology for the interpolation field. This requires solving a non-linear conic problem.

Several correction methods have been explored as another means to reducing artifacts in various geometry processing tasks. Lipman [Lip12] presents a generic tool for constructing orientation preserving (i.e., no triangle flips allowed) triangle mesh mappings, while limiting worst-case conformal distortion. This method has non-interactive run times and is only defined for planar meshes. Schüller et al. [SKPSH13] propose a specialized optimization based on a barrier energy function to repress zero-area elements and flipped triangles at interactive rates. Their iterative scheme solves for a injective mapping to a new mesh configuration. They guarantee inversion-free mappings of planar triangular and volumetric tetrahedral meshes. Aigerman and Lipman [AL13] extend Lipman to volumetric meshes. Their algorithm takes a deformation created by common deformation techniques and returns a similar deformation that is injective and minimizes the distortion of the mesh volumetric elements. The method is not interactive. Most recently, Kovalsky et al. [KABL14] present a method based on linear matrix inequalities for restricting the range of singular values. It enables, e.g., bounded distortion mappings of planar or volumetric domains, but is also computationally to expensive for interactive applications.

These correction methods guarantee that deformations are inversion-free, and in some cases even protrusion-free. However, they all require solving non-linear systems, which results in loss of interac-
tivity for moderate to large meshes. In contrast, we follow the recent linear approach to regularization by Martinez Esturo et al. [MRT14], which has no significant impact on runtime. While we cannot guarantee artifacts-free deformations, our method successfully suppresses usual deformation artifacts. Furthermore, the correction methods mentioned above are not applicable to surface meshes embedded in $\mathbb{R}^3$. In contrast, our method is well-defined for surface meshes.

3 Background

In this Section, we continue to review the formal details required in our work. We consider triangulated surface meshes $M = (T, V, E)$ defined by sets of vertices $i \in V$, oriented edges $E \subset V^2$, and triangles $T \subset V^3$. Coordinates of vertices $i \in V$ are denoted by $x_i \in \mathbb{R}^3$. A missing subscript either indicates a vector of stacked coefficients, e.g., $x \in \mathbb{R}^{|V|}$, the vector of stacked vertex coordinates $x_i$, or a matrix of component-wise coefficients, e.g., $X \in \mathbb{R}^{|V| \times 3}$. For a triangle $t \in T$, $X_t \in \mathbb{R}^3$ denotes the column-wise concatenation of the coefficients of its vertices. Using the notations

$$
\|y\|_N^2 = y^T N y \quad \text{and} \quad \|Y\|_N^2 = \text{Tr}(Y^T N Y),
$$

we denote (squared) vector and matrix norms that are induced by symmetric and positive definite matrices $N$. ($\text{Tr}(\cdot)$ denotes the trace of a matrix.)

For the piecewise linear functions on $M$ a discrete gradient operator $G \in \mathbb{R}^{|T| \times |V|}$ can be assembled from local per-triangle gradient operators $G_t$: for triangles $t = (i, j, k) \in T$ with normalized normals $n_t$, the local gradient operators are given by

$$
G_t = \begin{bmatrix}
(x_j - x_i)^T \\
(x_k - x_i)^T \\
n_t^T
\end{bmatrix}^{-1}
\begin{bmatrix}
-1 & 1 & 0 \\
-1 & 0 & 1 \\
0 & 0 & 0
\end{bmatrix},
$$

see, e.g., [BS08]. Given a scalar function on $M$ defined by the vertex-based coefficients $u \in \mathbb{R}^{|V|}$, $G u$ is the vector of stacked and constant per-triangle gradients. Note that gradients computed by $G$ are defined in a common coordinate system.

3.1 Harmonic Guidance for Surface Deformation

Zayer et al. [ZRKS05] propose a variant of gradient-domain deformations in which local deformation constraints are propagated using harmonic functions. Deformed surfaces are reconstructed from manipulated surface gradients by minimizing the global deformation energy

$$
E(x) = \sum_{t \in T} A_t \|G_t X_t - Z_t\|_F^2
$$

subject to suitable boundary constraints. Here, $A_t$ denotes the area of triangle $t$, $\|M\|_F^2 = \text{Tr}(M^T M)$ is the (squared) Frobenius norm of $M$, and $Z_t \in \mathbb{R}^{|V|}$.
R^{3×3} are prescribed component-wise guidance gradients that are constant per triangle. Dense guidance gradients are computed from user-specified transformations associated to a set of handle regions. In [ZRKS05], harmonic functions h(x) given by solutions of the Poisson equation \( L \mathbf{h} = \mathbf{0} \) are used for the global propagation of the sparse set of given transformations. Here, \( L = G^T A G \) is a discretization of the Laplace-Beltrami operator [BS08], in which \( A \) is a diagonal matrix of replicated triangle areas. Please see Zayer et al. [ZRKS05] for further details on the quaternion-based propagation of transformations using harmonic functions. As minimizers of (3) are characterized by the same differential operator, in practice a single factorization of \( L \) can be used to perform both transformation propagation and energy minimization.

**Drawbacks.** Linear deformation methods are susceptible to various artifacts. This is because the physical energies involved in deformations are nonlinear by nature, and are only approximated by linear methods [BS08]. Specifically, harmonic surface deformation is susceptible various deformation artifacts, which are in particular related to the size of the handle regions. Small deformation handles are likely to cause protruding and intruding triangles. Large deformation handles cause local shape distortion on the boundary between the constrained and free mesh areas. Other artifacts include volume loss close to the boundary edges. We continue to show that the concept of energy regularization is applicable to harmonic surface deformation.

### 3.2 Linear Energy Regularization

Martinez Esturo et al. [MRT14] propose a generic linear energy regularization scheme that suppresses geometric artifacts in a number of different applications. It is applicable to regularize problem-specific squared energies of the general form

\[
E_P(\mathbf{u}) = ||\mathbf{E} \mathbf{u} - \mathbf{c}||_{A}^2
\]

over \( d \)-dimensional piecewise linear functions defined by vertex-based coefficients \( \mathbf{u} \in \mathbb{R}^{d|V|} \). Here, \( \mathbf{E} \in \mathbb{R}^{n|T|×d|V|} \) is a problem-specific linear energy operator that maps unknown functions \( \mathbf{u} \) to triangle-constant local energies of dimension \( n \). \( \mathbf{c} \) are problem-specific energy constants, and \( A \) is a diagonal matrix of replicated triangle areas that performs domain-wide integration of triangle-constant quantities. Energies \( E_P \) are regularized by introducing a regularization term

\[
E_R(\mathbf{u}) = ||\mathbf{D} (\mathbf{E} \mathbf{u} - \mathbf{c})||_{B}^2
\]

that measures squared variations of local energies. For piecewise constant local energies, pointwise energy variations are estimated by the sparse differential operator \( \mathbf{D} \). For each pair of neighboring triangles, it can be discretized along all internal edges \( e \in \mathcal{E} \), from the set of non-boundary edges \( \mathcal{E}_i \subset \mathcal{E} \): let \( l(e) \) and \( r(e) \) denote the left and right triangle at \( e \), respectively. Then, for scalar local energies \( n = 1 \), the nonzero coefficients of \( \mathbf{D} \) are given by \( D_{et} = \begin{cases} 1 & \text{if } l(e) = t \\ -1 & \text{if } r(e) = t' \end{cases} \) for all internal edges \( e \in \mathcal{E}_i \) and triangles \( t \in \mathcal{T} \). For vector-valued local energies \( n > 1 \), the differential operator is given by a component-wise replication, which can be expressed as \( \mathbf{D} \otimes \mathbf{I}_n \) using the Kronecker product \( \otimes \) and the \( n \times n \) identity matrix \( \mathbf{I}_n \). The constant estimates of pointwise local energy variations are integrated using the diagonal matrix \( \mathbf{B} \) of replicated internal edge lengths.

The total regularized energy is given by a weighted combination of both terms

\[
E_{\beta}(\mathbf{u}) = (1 - \beta) E_P(\mathbf{u}) + \beta E_R(\mathbf{u}) = ||\mathbf{E} \mathbf{u} - \mathbf{c}||_{W_\beta}^2
\]

that can be expressed compactly using the \( \beta \)-weighted norm

\[
W_\beta = (1 - \beta) A + \beta D^T B D.
\]

The amount of regularization is steered by \( \beta \in [0, 1] \). Note that this formulation of energy regularization is also valid for energies in the components of the unknown functions \( \mathbf{u} \), which are then given by \( E_{\beta}(\mathbf{U}) = ||\mathbf{E} \mathbf{U} - \mathbf{C}||_{W_\beta}^2 \). Please see Martinez Esturo et al. [MRT14] for further details and applications on this energy regularization scheme.

### 4 Enhancing Harmonic Surface Regularization

We continue to show that the concept of energy regularization is applicable to harmonic surface deformation in a straightforward way. For this, we rewrite the component-wise deformation energy (3) to the equivalent formulation

\[
E(\mathbf{X}) = ||\mathbf{G} \mathbf{X} - \mathbf{Z}||_{A}^2
\]

using the global gradient operator \( \mathbf{G} \), the matrix \( \mathbf{Z} \) of all stacked prescribed gradients, and diagonal matrix \( \mathbf{A} \) of replicated triangle areas. Note that the local energies correspond to the to the summed terms of (9). Comparing our problem-specific energy (4) and the regularizable generic energy (4), we obtain the correspondences that the generic energy operator \( \mathbf{E} \) is given by the gradient operator \( \mathbf{G} \), the constant energy term \( \mathbf{C} \) is given by the gradient field \( \mathbf{Z} \), and the dimension of the local energies is \( n = 3 \). Hence, the energy-regularized version of the deformation energy (8) is given by

\[
E_{\beta}(\mathbf{X}) = ||\mathbf{G} \mathbf{X} - \mathbf{Z}||_{W_\beta}^2.
\]
by simply applying the weighted norm $W_\beta$ for energy integration and smoothness estimation. Technically, to apply regularization, this substitution of $A$ for $W_\beta$ allows for a straightforward implementation. Specifically, the remainder of the original harmonic surface deformation approach is unchanged, in particular the harmonic function-based transformation propagation for the guidance gradients $Z$.

Our experiments demonstrate that this simple energy modification suppresses a variety of deformation artifacts of the original energy formulation (see Section 5). Still, the original discretization of the energy differential operator $D$ is defined independently of the local surface curvature, which leads to poor estimates of energy variation in high curvature regions. We continue to provide a refined differential operator discretization that is based on surface curvature and yields better estimates of energy variation.

Curvature-based Energy Differential Operator. Martinez Esturo et al. [MRT14] estimate the curvature-based energy differential operator $D$ is defined independently of the local surface curvature, which leads to poor estimates of energy variation in high curvature regions. The operators $G$, $D^R$, $A$, and $B$ as well as $L$ are assembled once when the surface mesh is loaded. For each deformation of the model the guidance field $Z$ is computed and the corresponding normal equations

$$G^t W_\beta G X = G^t W_\beta Z$$

of (9) are solved for the coordinates $X$ of the deformed mesh. The norm $W_\beta$ is assembled for a given $\beta$ value using the refined differential operator $D^R$. After elimination of positional hard boundary constraints, the system (11) is symmetric positive definite and it is solved using a Cholesky factorization with fill-in reducing reordering [GJ10]. Similar to [ZRKS05], this factorization can be reused for different guidance fields $Z$ as long as the handle configuration and $\beta$ values are unchanged.

5 Results

We evaluate our approach qualitatively on a number of different models. The models presented have $3 \sim 36,6k$ vertices. Deformations are performed using single and multiple handles, varying sizes of the handle regions and the regularization weight $\beta$.

Energy Visualization. We use color to visualize the squared magnitude of local triangle-constant energies $||E_l||_F^2$. Energies are linearly mapped to the color space (shown in Figure 1) by setting the maximum of the color interval to correspond to the 95th percentile of the energy values. The top 5% of the energy values is clipped to allow visualization of the variation of lower energies with higher contrast.

Small Deformation Handles. For small handle regions, harmonic surface deformation tends to create artifacts such as protruding triangles and local surface intersections close to the handles. We show two examples of these artifacts: the Hand deformation (Figure 3) is created by fixing the base of the model, and a single vertex on each finger acts as the deformation handle. All handles are rotated inwards,
Dozen vertices. In the Cactus 2
nitude, occur if handle regions include up to several
facts to protruding triangles, albeit of smaller mag-

Figure 4: Cow. The handle on face is fixed, and the
horns are rotated upwards. Regularization helps pre-
sure the volume near the base of the horns.
resulting in protruding triangles and local surface in-
tersections near the deformation handles. Regulariza-
tion (β > 0) suppresses these artifacts. Similar arti-
facts to protruding triangles, albeit of smaller mag-
nitude, occur if handle regions include up to several
dozen vertices. In the Cactus 2 example shown in
Figure 3 single vertices on the base and the top are
fixed, and a single vertex on the side of the cactus
is used as a deformation handle. The deformation
suffers from large protruding triangles. Introducing
regularization corrects this artifact. For both defor-
mations, even low amounts of regularization result in
a smoother energy distribution, affecting more trian-
gles in the mesh. This way the optimization favors
more global changes to the mesh instead of local con-
centrations of energies, which result in the observed
local artifacts.

Large Deformation Handles. Harmonic surface
deformation is commonly used with large deformation
handles, as careful design of these constraints can lead
to pleasing deformations. However, the boundary of
large handle regions is also susceptible to artifacts.
The Cactus 1 model in Figure 5 is based on a bench-
mark deformation from [BS08]. The base of the model
is fixed, and the top is rotated and translated. Us-
ing the original method by Zayer et al. [ZRKS05], the
boundary between the constrained and free mesh re-
gions exhibits strong changes of the directions of mesh
normals, distorting the local shape. Our regulariza-
tion reduces distortions of local geometry close to this
boundary, creating a smooth transition between the
constrained and free mesh regions.

Strong Rotations. Strong rotations, especially on
elongated limbs, are a challenge for a number of defor-
mation methods [KS12]. Using harmonic surface de-
formation, strong rotations usually cause loss of vol-
ume. This effect is illustrated in Figures 4 and 2. The
foot of the Horse is rotated. In the resulting deforma-
tion, most of the lower leg suffers from volume loss,
creating a “candy wrapper”-like artifact. Low values
of regularization are effective at preserving some of
the limb’s volume to create more plausible results. In
the Cow deformation, the horns are rotated upwards
while the front of the face is fixed. This deformation
is even more challenging, as the region deformed is
small and the mesh geometry is rather coarse. Regu-
larization helps to restore some of the lost volume.

6 Evaluation and Discussion

In this section, we perform a quantitative evaluation of
our approach. Deformations are created for β ∈ [0, 1],
using a higher sampling rate for lower β values, for
which we usually observe the strongest changes of the
defformation. Please, see the accompanying video for
the deformations at these β values.

Maximal Isometric and Conformal Errors. Two error measures proved to be most useful for
evaluating deformation quality (see, e.g., [LZXR08]):
the local per triangle isometric error εiso is given by
the sum of squared deviations from I of the singular
values of the deformation gradient. The local per
triangle conformal error εconf is computed by half of
the squared sum of the pairwise differences be-
tween the singular values of the deformation gradient.
The maximal isometric (conformal) error, max(εiso)
(max(εconf)), is given by the maximal value of the
isometric (conformal) error over all triangles. Both
of these error measures indicate strong distortions of
the mesh geometry. They are also loosely related to
artifacts such as protruding or intruding elements, de-
gerate triangles, and surface self-intersections. We
also examined the total (integrated and normalized)
isometric and conformal errors as possible indicators
for deformation quality evaluation, but these proved
ineffective.

Figure 6 shows the behavior of the maximal isometric
and conformal errors for different values of β. In al-
most all tested deformations, regularization strongly
decreases the maximal isometric and conformal er-
ors. The behavior of these error measurements is
different for the Cactus 1 example of Figure 5, as
they slightly increase with regularization. The reason
is that deformations defined using large handle re-
gions usually don’t result in artifacts associated with
formation quality does not derogate for higher introduced for moderate ally, we note that no new local maximal errors are large local isometric and conformal errors. Addition- 
ally, we note that no new local maximal errors are introduced for moderate $\beta$ values, meaning that de- 
formation quality does not derogate for higher $\beta$ val- 
ues. We also confirm this behavior in all other tested 
examples, as the total deformation energy of the reg- 
ularized deformation defined in (3) stays within the 
same order of magnitude as the original deformation.

Space of Regularized Deformations. We ob- 
serve that the initial introduction of regularization 
($\beta > 0$) has strong effects on deformations suf- 
ferring from strong artifacts for $\beta = 0$. After 
this initial reaction interval, regularization creates 
gradual changes to the mesh geometry, indicated both by our deformation results (e.g., in Figures 2 
and 3) and by the behavior of the error measures 
$\max(e_{i}^{iso})$, $\max(e_{i}^{conf})$ in Figure 6. As the regularization energy becomes more dominant with increasing $\beta$ values, $\max(e_{i}^{iso})$, $\max(e_{i}^{conf})$ also slightly increase. 
For very high values of $\beta > 0.9$, the regularized energy formulation strongly deviates from the original problem, which can create new artifacts. In our ex- 
erience, choosing a fixed value of $\beta \approx 0.2$ usually suppresses artifacts effectively without negatively af- 
flecting the mesh geometry. This means that a con- 
stant regularization can simply be added to existing implementations without exposing users to a new pa- 
parameter. In addition, although the linear system (11) has to be refactored when $\beta$ changes, examining dif- 
ferent $\beta$ values can usually be done at interactive rates due to the high performance of sparse linear solvers. 
Hence, the space of regularized deformations can also 
be explored interactively.

Curvature-based Differential Operator. Figure 7 demonstrates the benefits of using our curvature-based energy differential operator on the Accordion mesh with highly curved edges. The method by Zayer et al. [ZRKS05] suffers from lo- 
tal shape distortion near the deformation handle. The differential operator by Martinez Esturo et al. [MRT14] estimates the energy variation between neighboring triangles ineffectively, resulting in global shape distortions. Our curvature-based operator esti- 
mates the energy differential more effectively, result- 
ing in a deformation which has the same geometry 
as the solution by Zayer et al. [ZRKS05], but sup- 
presses the local artifact. The deviation of Martinez Esturo et al. [MRT14] from our results increases with 
higher regularization weight. For smooth meshes, us- 
ing our curvature-based differential operator has neg-
ligible effects, as vertex coordinates were only affected marginally.

Performance. Regularization affects the runtime in two ways: it reduces the sparsity of the linear system (11) being solved and it requires a one-time operation to setup of the curvature-based differential operator $D^R$. Our measurements on an Intel Core i7 2.2GHz system indicates that the effect of regular- 
ization on interactive runtime is insignificant for all tested meshes: For example, factorization time of the linear system for the Cactus model ($|V| = 10k$) is $\approx 0.04$ seconds for both $\beta = 0$ and $\beta > 0$, its solution time is $\approx 0.003$ seconds. Similarly, factorization time for the Hand model ($|V| = 36.6k$) is $\approx 1.13$ seconds for $\beta = 0$ and $\approx 1.14$ seconds for $\beta > 0$ with a so- 
olution time of $\approx 0.1$ seconds. Hence, changing the regularization weight to examine different regulariza-
tion weight can be done interactively.

7 Conclusions

In this work, we have provided an energy regular-
ized formulation of harmonic surface deformation. 
Our approach expands the capabilities of the orig- 
inal method, allowing the creation of artifact-free de- 
f ormations for a wider range of deformation con- 
straints and handle configurations. Our formulation 
of a curvature-based differential energy operator im- 
proves the estimation of energy differentials in high-
curvature mesh regions. This reduces geometric dis-
Original model  

Zayer et al. [ZRKS05]  

Martinez Esturo et al. [MRT14]  

Ours  

\[ \beta = 0.4 \]

Figure 7: **Accordion**. The base is fixed and two vertices on the top row are rotated. Harmonic surface regularization [ZRKS05] creates an artifact near the deformation handles. Martinez Esturo et al. [MRT14] suppress the artifact, but distorts the global mesh shape. Our curvature-based differential operator creates a smooth, artifact-free deformation without distorting the mesh geometry.

Tortions introduced by the original energy differential operator around these regions. The evaluation of our results demonstrates that even low regularization weights can effectively suppress many deformation artifacts without negatively affecting the performance of the original method. In addition, no new artifacts are created.

**Future work.** An interesting direction for future work is the application of energy regularization to other (non-linear) surface deformation approaches, e.g., to the work of Jacobson et al. [JBK+12], who observe similar artifacts. In addition, since artifacts are usually localized close to handle regions, a local energy smoothness formulation could achieve better control on deformation artifacts.

**Acknowledgment.** The Horse, Hand, and Cow models are provided by the AIM @ Shape project. The Cactus model courtesy of Botsch and Sorkine [BS08].

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Figure 6: Graphs of the maximal isometric and conformal errors over the regularization weight $\beta$. Regularization typically decreases these errors drastically already for small values of $\beta$. Note the logarithmic scale on the $y$-axes.

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