Sensitivity of the baryon asymmetry produced by leptogenesis to low energy CP violation

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If the baryon asymmetry of the Universe is produced by leptogenesis, CP violation is required in the lepton sector. In the seesaw extension of the Standard Model with three hierarchical right-handed neutrinos, we show that the baryon asymmetry is insensitive to the PMNS phases: thermal leptogenesis can work for any value of the observable phases. This result was well-known when there are no flavour effects in leptogenesis; we show that it remains true when flavour effects are included.

Introduction: CP violation is required to produce the puzzling excess of matter (baryons) over anti-matter (anti-baryons) observed in the Universe. If this Baryon Asymmetry of the Universe (BAU) was made via leptogenesis, then CP violation in the lepton sector is needed. So any observation thereof, for instance in neutrino oscillations [2], then CP violation in the lepton sector is needed. It was shown in [3] that the BAU produced by thermal leptogenesis in the type 1 seesaw, without “flavour effects”, is insensitive to PMNS phases: thermal leptogenesis can vanish for arbitrary values of the PMNS phases. In fact, the “unflavoured” asymmetry is controlled by phases from the RH sector only, and it would vanish were this sector CP conserving. However, it was recently realised that lepton flavour matters in leptogenesis [4, 5, 6]: in the relevant temperature range \(10^9 \rightarrow 10^{12}\) GeV, the final baryon asymmetry depends separately on the lepton asymmetry in \(\tau \)s, and on the lepton asymmetry in muons and electrons. So in this paper, we revisit the question addressed in [3], but with the inclusion of flavour effects. Our analysis differs from recent discussions [7] (2RHN model), [8, 9] (CP as a symmetry of the N sector), [10] (sequential N dominance) in that we wish to do a bottom-up analysis of the three generation seesaw. Ideally, we wish to express the baryon asymmetry in terms of observables, such as the light neutrino masses and PMNS matrix, and free parameters. Then, by inspection, one could determine whether fixing the baryon asymmetry constrained the PMNS phases.

Notation and review: We consider a seesaw model [11], where three heavy (\(M \gtrsim 10^9\) GeV) majorana neutrinos \(N_I\) are added to the Standard Model. The Lagrangian at the \(N_I\) mass scale is

\[
\mathcal{L} = \overline{\nu}_R Y_{\alpha J} H \nu_{L}^T + \sum_{i} \lambda_{iJ} H_u \ell_i^T + \sum_{i} \frac{M_{JK}}{2} N_{iK} + h.c.
\]

where the flavour index order on the Yukawa matrices \(Y_{\alpha \lambda}, \lambda = \text{left-right}, \) and \(H_u = i\sigma_2 H_u^T\).

There are 6 phases among the 21 parameters of this Lagrangian. We can work in the mass eigenstate basis of the charged leptons and the \(N_I\), and write the neutrino Yukawa matrix as

\[
\lambda = V_L^T D_\lambda V_R \quad ,
\]

where \(D_\lambda\) is real and diagonal, and \(V_L, V_R\) are unitary matrices, each containing three phases. So at the high scale, one can distinguish CP violation in the left-handed doublet sector (phases that appear in \(V_L\)) and in the right-handed singlet sector (phases in \(V_R\)). Leptogenesis can work when there are phases in either or both sectors. At energies accessible to experiment, well below the \(N_I\) mass scale, the light (LH) neutrinos acquire an effective Majorana mass matrix [20]:

\[
[m] = \lambda M^{-1} \lambda^T v^2 = U D_m U^T
\]

where \(v = 174\) GeV is the Higgs vev, \(D_m\) is diagonal with real eigenvalues, and \(U\) is the PMNS matrix. There are nine parameters in \([m]\), which is “in principle” experimentally accessible. Two mass differences and two angles of \(U\) are measured, leaving the mass scale, one angle and three phases of \(U\) unknown.

From the above we can write

\[
D_m = U^T V_L^T D_\lambda V_R D_\lambda^{-1} V_R^T D_\lambda V_L^* U^T v^2
\]
so we see that the PMNS matrix will generically have phases if $V_L$ and/or $V_R$ are complex. Like leptogenesis, it receives contributions from CP violation in the LH and RH sectors. Thus it seems “probable”, or even “natural”, that there is some relation between the CP violation of leptogenesis and of the PMNS matrix. However, the notion of relation or dependence is nebulous [12], so we address the more clear and simple question of whether the baryon asymmetry is sensitive to PMNS phases. By this we mean: if the total baryon asymmetry is fixed, and we assume to know all the neutrino masses and mixing angles, can we predict ranges for the PMNS phases?

We suppose that the baryon asymmetry is made via leptogenesis, in the decay of the lightest singlet $N_1$, with $M_1 \sim 10^{10}$ GeV. Flavour effects are relevant in this temperature range [4, 5, 6, 21]. $N_1$ decays to leptons $\ell_\alpha$, an amount $\epsilon_{\alpha\alpha}$ more than to anti-leptons $\bar{\ell}_\alpha$, and this lepton asymmetry is transformed to a baryon asymmetry by SM processes (sphalerons). We will further suppose that the partial decay rates of $N_1$ to each flavour are faster than the expansion rate of the Universe $H$. This implies that $N_1$ decays close to equilibrium, and there is a significant washout of the lepton asymmetry due to $N_1$ interactions (strong washout regime); we discuss later why this assumption does not affect our conclusions.

Flavour effects are relevant in leptogenesis [4, 5, 6] because the final asymmetry cares which leptons $\ell$ are distinguishable. $N_1$ interacts only via its Yukawa coupling, which controls its production and destruction. The washout of the asymmetry, by decays, inverse decays and scatterings of $N_1$, is therefore crucial for leptogenesis to work, because otherwise the opposite sign asymmetry generated at early times during $N_1$ production would cancel the asymmetry produced as they disappear. To obtain the washout rates (for instance, for $\ell + H_\alpha \to N_1$), one must know the initial state particles, that is, which leptons are distinguishable.

At $T \sim M_1$, when the asymmetry is generated, SM interactions can be categorised as much faster than $H$, of order $H$, or much slower. Interactions that are slower than $H$ can be neglected. $H^{-1}$ is the age of the Universe and the timescale of leptogenesis, so the faster interactions should be resummed—for instance into thermal masses. In the temperature range $10^9 \lesssim T \lesssim 10^{12}$ GeV, interactions of the $\tau$ Yukawa are faster than $H$, so the $\ell_\tau$ doublet is distinguishable (has a different “thermal mass”) from the other two lepton doublets. The decay of $N_1$ therefore produces asymmetries in $B/3 - L_\alpha$, and in $B/3 - L_\alpha$, where $\ell^c$ (“other”) is the projection in $\ell^c$ and $\ell^o$ space, of the direction into which $N_1$ decays [14]:

$$\epsilon_{\alpha\alpha} = (\lambda_{\alpha 11} + \lambda_{\alpha 1e})/\sqrt{|\lambda_{\alpha 11}|^2 + |\lambda_{\alpha 1e}|^2}.$$ 

Following [6], we approximate these asymmetries to evolve independently. In this case, the baryon to entropy ratio can be written as the sum over flavour of the flavoured CP asymmetries $\epsilon_{\alpha\alpha}$ times a flavour-dependent washout parameter $\eta_\alpha < 1$ which is obtained by solving the relevant flavoured Boltzmann equations [4, 5, 6]:

$$Y_B \simeq \frac{12}{3\pi^2} \frac{1}{3g_\ast} \left( \epsilon_{\tau\tau} \eta_\tau + \epsilon_{oo} \eta_o \right)$$  

(5)

where $g_\ast = 106.75$ counts entropy, and the $12/37$ is the fraction of a $B - L$ asymmetry which, in the presence of sphalerons, is stored in baryons.

In the limit of hierarchical RH neutrinos, the CP asymmetry in the decay $N_1 \to \ell_\alpha H$ can be written as

$$\epsilon_{\alpha\alpha} \simeq -\frac{3M_1}{16\pi v^2 |\lambda|_{11}} \text{Im} \left\{ |\lambda|_{\alpha 1} |m^\dagger \lambda|_{\alpha 1} \right\}$$  

(6)

where $m$ is defined in eqn (5).

In the case of “strong washout” for all flavours, which corresponds to $\Gamma(N_1 \to \ell_\alpha H_\alpha) > H$ for $\alpha = \tau, o$, the washout factor is approximately [4, 15]

$$\eta_\alpha \simeq 1.3 \left( \frac{m_s}{6A_{\alpha\alpha} \bar{m}_{\alpha\alpha}} \right)^{1.16} - \frac{m_s}{5A_{\alpha\alpha} \bar{m}_{\alpha\alpha}}$$  

(7)

where there is no sum on $\alpha$, $m_s \simeq 10^{-3}$ eV, and $A_{\tau\tau} \simeq A_{oo} \sim 2/3$ [6, 14, 22]. The (rescaled) $N_1$ decay rate is

$$\tilde{m} = \sum_{\alpha} \bar{m}_{\alpha\alpha} = \sum_{\alpha} |\lambda_{\alpha 1}|^2 M_1^{-1} o^2$$  

(8)

An equation: Combining equations (5), (6), and (7), we obtain $Y_B \propto \epsilon_{\tau\tau}/\tilde{m}_{\tau\tau} + \epsilon_{oo}/\tilde{m}_{oo}$, where ($\alpha$ not summed)

$$\epsilon_{\alpha\alpha}/\tilde{m}_{\alpha\alpha} = \frac{3M_1}{16\pi v^2 \tilde{m}} \sum_{\beta} \text{Im} \left\{ \lambda_{\alpha 1} m_{\alpha 1} \lambda_{\beta \beta} \right\} |\lambda_{\beta 1}| / |\lambda_{\alpha 1}|$$  

(9)

and the Yukawa couplings of $N_1$ have been written as a phase factor times a magnitude: $\lambda_{\alpha 1} \lambda_{\alpha 1} = \lambda_{\alpha 1}^*$. So the baryon asymmetry can be approximated as

$$Y_B \simeq Y_B^{bd} \left( \frac{\text{Im} \{ \lambda_{\nu 1} m_{\nu 1} \lambda_{\nu 1} \}}{m_{\nu 1}} + \frac{\text{Im} \{ \lambda_{\tau 1} m_{\tau 1} \lambda_{\tau 1} \}}{m_{\tau 1}} \right) \frac{1}{A_{\tau\tau}}$$  

(10)

The prefactor of the parentheses $Y_B^{bd} = \frac{12}{3\pi^2} \frac{M_1 m_{\nu 1}}{16\pi v^2 g_\ast}$ is the upper bound on the baryon asymmetry; that would be obtained in the strong washout case by neglecting flavour effects. Recall that this equation is only valid in strong washout for all flavours.

This equation reproduces the observation [6], that:

(i) for equal asymmetries and equal decay rates of all distinguishable flavours, flavour effects increase the upper bound on the baryon asymmetry by $\sum_{\alpha} A_{\alpha\alpha}^{-1} \sim 3$.

(ii) More interestingly, having stronger washout in one flavour, can increase the baryon asymmetry [via the term in brackets]. So models in which the Yukawa coupling
\( \lambda_{11} \) is significantly different from \( \lambda_{\mu 1}, \lambda_{\tau 1} \), can have an enhanced baryon asymmetry (with cooperation from the phases).

Finally, this equation is attractive steps towards writing the baryon asymmetry as a real function of real parameters \( \chi_{bl} \), depending on \( M_1 \) and \( \hat{m_1} \), times a phase factor \( \Theta \). In this case, the phase factor is a sum of three terms, depending on the phases of the \( N_1 \) Yukawa couplings, light neutrino mass matrix elements normalised by the heaviest mass, and a (real) ratio of Yukawas.

**CP violation:** In this section, we would like to use eqn (10) to show that the baryon asymmetry is insensitive to the PMNS phases. The parameters of the lepton sector can be divided into “measurables”, which are the neutrino and charged lepton masses, and the three angles and three phases of the PMNS matrix \( U \). The remaining 9 parameters are unmeasurable. We want to show that for any value of the PMNS phases, there is at least one point in the parameter space of the unmeasurables where a large enough baryon asymmetry is obtained. The approximations leading to eqn (10) are only valid in a subset of the unmeasurable parameter space, but if we can find points in this subspace, we are done. We first show analytically that such points exist, then we do a parameter space scan to confirm that leptogenesis can work for any value of the PMNS phases.

If the phases of the \( \lambda_{11} \) were independent of the PMNS phases, and a big enough \( Y_B \) could be obtained for some value of the PMNS phases, then our claim is true by inspection: for any other values, the phases of the \( \lambda_{11} \) can be freely varied without affecting the “measurables”. Then \( \lambda_{11} \) of parameter space where the phases of the PMNS phases could be chosen to reproduce the same value of the PMNS phases. However, there is in general some relation between the phases of \( m \) and those of \( \lambda_{11} \), so we proceed by looking for an area of parameter space where the phases of the \( \lambda_{11} \) can be freely varied without affecting the “measurables”. Then we check that a large enough baryon asymmetry can be obtained.

Such an area of parameter space can be found using the \( R \) matrix parametrisation of Casas-Ibarra \[17\], where the complex orthogonal matrix \( R \) is defined such that \[18\]. Taking a simple \( R \) of the form

\[
R = \begin{bmatrix}
\cos \phi & 0 & -\sin \phi \\
0 & 1 & 0 \\
\sin \phi & 0 & \cos \phi 
\end{bmatrix}
\]

and parametrising \( U = VP \), where \( V \) is a CKM-like unitary matrix with one “Dirac” phase \( e^{-i \delta} \) appearing with \( \sin \theta_{13} \), and \( P = \text{diag}\{e^{i \phi_1/2}, e^{i \phi_2/2}, 1\} \), gives

\[
\begin{align*}
\frac{\lambda_{11}}{\sqrt{M_{1m_1}}} &= U_{11} \sqrt{m_1 m_3} \cos \phi + U_{13} \sin \phi \simeq \frac{\sin \phi}{\sqrt{2}} \\
\frac{\lambda_{m1}}{\sqrt{M_{1m_3}}} &= U_{m1} \sqrt{m_1 m_3} \cos \phi + U_{m3} \sin \phi \simeq \frac{\sin \phi}{\sqrt{2}} \\
\frac{\lambda_{\tau 1}}{\sqrt{M_{1m_3}}} &= U_{c1} \sqrt{m_1 m_3} \cos \phi + U_{c3} \sin \phi
\end{align*}
\]

where \( \lambda_{11} \) is significantly different from \( \lambda_{\mu 1}, \lambda_{\tau 1} \), can have an enhanced baryon asymmetry (with cooperation from the phases).

Finally, we check that a large enough baryon asymmetry can be obtained in this area of parameter space. The parentheses of eqn (10) can be written explicitly as

\[
\text{Im} \left\{ \sin^2 \phi^* \left( m_{\tau \tau} + m_{\mu \mu} + 2m_{\mu \tau} \right) \right\} \frac{1}{m_{\text{atm}}} \tag{15}
\]

Writing \( \phi^* = \rho - i \omega \), the final baryon asymmetry can be estimated from eqn (10) as

\[
\frac{Y_B}{10^{-10}} \simeq -\left( \frac{M_1}{10^{11} \text{GeV}} \right) \frac{\sin \rho \cos \rho \sinh \omega \cosh \omega}{(\sin^2 \rho \cos^2 \omega + \cos^2 \rho \sinh^2 \omega)^2} \tag{16}
\]

which can equal the observed \( 8.7^{+0.3}_{-0.4} \times 10^{-11} \) for \( M_1 \sim \text{few } 10^{10} \text{ GeV} \), and judicious choices of \( \rho \) and \( \omega \).

A similar argument can be made if the light neutrino mass spectrum is inverse hierarchical.

The scatter plots of figure 1 show that a large enough baryon asymmetry can be obtained for any value of the PMNS phases.

![FIG. 1: A random selection of points where the baryon asymmetry is large enough, for some choice of the unmeasurable parameters of the seesaw. The light neutrino masses are taken non-degenerate, and the Majorana phase of the smallest one can be neglected. The “Dirac” phase \( \delta \) is defined such that \( U_{c3} = \sin \theta_{13} e^{-i \delta} \) and \( \beta \) is the majorana phase of \( m_2 = |m_2| e^{2i \beta} \). The baryon asymmetry arises in the decay of \( N_1 \) of mass \( M_1 = 10^{10} \text{ GeV} \).](image)
ues. To mimic the possibility that $\beta$ and $\delta$ could be determined $\pm 15^\circ$, $\beta-\delta$ space is divided into 50 squares. In each square, the programme randomly generates values for: $\beta, \delta, 0.001 < \theta_{13} < 0.2$, the smallest neutrino mass $< \sqrt{\sum m^2_{sol}}/10$, and the three complex angles of the $R$ matrix. It estimates the baryon asymmetry from the analytic approximations of \cite{6}, and puts a cross if it is big enough. The programme is a proto-Monte-Carlo-Markov-Chain, preferring to explore parameter space where the baryon asymmetry is large enough.

Parametrising with the $R$ matrix imposes a particular measure (prior) on parameter space. This could mean we only explore a class of models. This is ok because the aim is only to show that, for any PMNS phases, a large enough asymmetry can be found.

Discussion: The relevant question, in discussing the “relation” between CP violation in the PMNS matrix and in leptogenesis, is whether the baryon asymmetry is sensitive to the PMNS phases. The answer was “no” for unflavoured leptogenesis in the Standard Model seesaw\cite{3}. This was not surprising; the seesaw contains more phases than the PMNS matrix, and many unmeasurable real parameters which can be adjusted to obtain a big enough asymmetry. In this paper, we argue that the answer does not change with the inclusion of flavour effects in leptogenesis: for any value of the PMNS phases, it is possible to find a point in the space of unmeasurable seesaw parameters, such that leptogenesis works. This “flavoured” asymmetry can be written as a function of PMNS phases, and unmeasurable as entered the unflavoured calculation. These can still be adjusted to get a big enough asymmetry. In view of this discouraging conclusion, it is maybe worth to emphasize that CP violation from both the left-handed and right-handed neutrino sectors, contributes both to the PMNS matrix and the baryon asymmetry. Moreover, the answer to this question in an MSSUGRA framework, with additional information from lepton flavour violating observables\cite{14}, is still work in progress.

In the demonstration that the baryon asymmetry (produced via thermal leptogenesis) is insensitive to PMNS phases, we found an interesting approximation for the “phase of leptogenesis” (see eqn (10)), when all lepton flavours are in strong washout.

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