MIMO Multiple Access Channel with an Arbitrarily Varying Eavesdropper

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Abstract—A two-transmitter Gaussian multiple access wiretap channel with multiple antennas at each of the nodes is investigated. The channel matrices at the legitimate terminals are fixed and revealed to all the terminals, whereas the channel matrix of the eavesdropper is arbitrarily varying and only known to the eavesdropper. The secrecy degrees of freedom (s.d.o.f.) region under a strong secrecy constraint is characterized. A transmission scheme that orthogonализes the transmit signals of the two users at the intended receiver and uses a single-user wiretap code is shown to be sufficient to achieve the s.d.o.f. region. The converse involves establishing an upper bound on a weighted-sum-rate expression. This is accomplished by using induction, where at each step one combines the secrecy and multiple-access constraints associated with an adversary eavesdropping a carefully selected group of sub-channels.

I. INTRODUCTION

Information theoretic security was first introduced by Shannon in [1], which studied the problem of transmitting confidential information in a communication system in the presence of an eavesdropper with unbounded computational power. Since then, an extensive body of work has been devoted to studying this problem for different network models by deriving fundamental transmission rate limits [2]–[4] and designing low-complexity schemes to approach these limits in practice [5], [6].

Secure communication using multiple antennas was extensively studied as well, see e.g., [7]–[15]. These works investigated efficient signaling mechanisms using the spatial degrees of freedom provided by multiple antennas to limit an eavesdropper’s ability to decode information. The underlying information theoretic problem, the multi-antenna wiretap channel, was studied and the associated secrecy capacity was identified. We note that these works assumed that the eavesdropper’s channel state information is available either completely or partially, although such an assumption may not be justified in practice.

As a more pessimistic but stronger assumption, references [16]–[18] study secrecy capacity when the eavesdropper channel is arbitrarily varying and its channel states are known to the eavesdropper only. Reference [17] studies the single-user Gaussian multi-input-multi-output (MIMO) wiretap channel and characterizes the secrecy degrees of freedom (s.d.o.f.). The same paper extended the single user analysis to the two user Gaussian MIMO multiple access (MIMO-MAC) channel. This was possible only when all the legitimate terminals have equal number of antennas, leaving the MIMO-MAC with arbitrary number of antennas at the terminals an open problem.

Our main contribution is to fully characterize the s.d.o.f. region of the two-transmitter MIMO MAC channel when the eavesdropper channel is arbitrarily varying. We show that the s.d.o.f. region can be achieved by a scheme that orthogonализes the transmit signals of the two users at the intended receiver. Moreover, it suffices to use a single-user wiretap channel code [17] and no coordination between the users is necessary except for synchronization and sharing the transmit dimensions. To establish the optimality of this scheme, our converse proof decomposes the MIMO MAC channel into a set of parallel and independent channels using the generalized singular value decomposition (GSVD). A set of eavesdroppers, each monitoring a subset of links, is selected using an induction procedure and the resulting secrecy constraints are combined to obtain an upper bound on a weighted sum-rate expression. The outer bound matches the achievable rate in terms of the s.d.o.f. region, thus settling the open problem raised in [17] for the case of two transmitters.

Interestingly, the s.d.o.f. region remains open for this model when the eavesdropper channel is perfectly known to all terminals. A significant body of literature already exists on this problem, see e.g., [19]–[22]. If the channel model has real inputs and outputs, Gaussian signaling is in general suboptimal and user cooperating strategies as well as signal alignment techniques are necessary [23]. In [24] it is established that s.d.o.f. of 1/2 is achievable using real interference alignment for almost all configurations of channel gains. If the channel model has complex inputs and outputs, it is shown in [25] Section 5.16 that in general s.d.o.f. of 1/2 is achievable using asymmetric Gaussian signaling. In contrast, the best known upper bound on the s.d.o.f. of individual rates is 2/3 for both cases, established in [25] Section 5.5.

The remainder of this paper is organized as follows. In Section II we describe the system model. The main result is stated as Theorem 1 in Section IV. The proof of the theorem is divided into two parts. First, we establish the result for the case of parallel channels in Section V. Subsequently, in Section VI we establish the result for the general case by decomposing the MIMO-MAC channel into a set of independent parallel

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channels. Such a reduction is used both in the proof of the converse as well as the coding scheme. Section VII concludes the paper.

We use the following notation throughout the paper: For a set \( A \), \( V_i \in A \) and \( V_A \) denote the set of random variables \( \{V_i,j \in A\} \) and \( \{V_j,j \in A\} \) respectively. \( \{\delta_n\} \) denotes a non-negative sequence of \( n \) that converges to 0 when \( n \) goes to \( \infty \). We use bold upper-case font for matrices and vectors and lower-case font for scalars. The distinction between matrices and vectors will be clear from the context. For a set \( A \), \([A]\) denotes its cardinality and a short hand notation \( \phi \) denotes the empty set.

II. SYSTEM MODEL

As shown in Figure 1, we consider a discrete-time channel model where two transmitters communicate with one receiver in the presence of an eavesdropper. We assume transmitter \( i \) has \( N_{T_i} \) antennas, \( i = 1, 2 \), the legitimate receiver has \( N_R \) antennas whereas the eavesdropper has \( N_E \) antennas. The channel model is given by

\[
\begin{align*}
Y(i) &= \sum_{k=1}^{2} H_k X_k(i) + Z(i) \quad (1) \\
\hat{Y}(i) &= \sum_{k=1}^{2} \hat{H}_k(i) X_k(i) \quad (2)
\end{align*}
\]

where \( i \in \{1, \ldots, n\} \) denotes the time-index, \( H_k, k = 1, 2 \), are channel matrices and \( Z \) is the additive Gaussian noise observed by the intended receiver, which is composed of independent rotationally invariant complex Gaussian random variables with zero mean and unit variance. The sequence of eavesdropper channel matrices \( \{\hat{H}_k(i), k = 1, 2\} \), is an arbitrary sequence of length \( n \) and only revealed to the eavesdropper. In contrast, \( H_k, k = 1, 2 \) are revealed to both the legitimate parties and the eavesdropper(s). We assume \( N_E \), the number of eavesdropper antennas, is known to the legitimate parties and the eavesdropper.

User \( k, k = 1, 2 \), wishes to transmit a confidential message \( W_k, k = 1, 2 \), to the receiver over \( n \) channel uses, while both messages, \( W_1 \) and \( W_2 \), must be kept confidential from the eavesdropper. We use \( \gamma \) to index a specific sequence of \( \{\hat{H}_k(i), k = 1, 2\} \) over \( n \) channel uses and use \( Y^{\gamma}_n \) to represent the corresponding channel outputs for \( \hat{Y}^n \). The strong secrecy constraint is [17]:

\[
\lim_{n \to \infty} I \left( W_1, W_2; \hat{Y}^n \right) = 0 \quad \forall \gamma \quad (3)
\]

where the convergence must be uniform over \( \gamma \). The average power constraints for the two users are given by

\[
\lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} |X_k(i)|^2 \leq \bar{P}_k, \quad k = 1, 2. \quad (4)
\]

The secrecy rate for user \( k, R_{s,k} \), is defined as

\[
R_{s,k} = \lim_{n \to \infty} \frac{1}{n} H(W_k), \quad k = 1, 2. \quad (5)
\]

such that \( W_k \) can be reliably decoded by the receiver, and \( 3 \) and \( 4 \) are satisfied.

We define the secrecy degrees of freedom as:

\[
\left\{ (d_1, d_2) : d_k = \limsup_{\bar{P}_1 = \bar{P}_2 = \bar{P} \to \infty} \frac{R_{s,k}}{\log_2 \bar{P}}, \quad k = 1, 2 \right\} \quad (6)
\]

III. MOTIVATION

Before stating the main result, we illustrate the main difficulty in characterizing the s.d.o.f. region through a simple example. As illustrated in Figure 2(a), in this example, each transmitter has 2 antennas and the intended receiver has 3 antennas, while the eavesdropper has only 1 antenna. Let \( x_1, x_2, x_3, x_4 \) denote the transmitted signals from the two users and \( y_1, y_2, y_3 \) denote the signals observed by the intended receiver. And the main channel is given by

\[
\begin{align*}
y_1 &= x_1 + z_1, \quad y_3 = x_4 + z_3 \quad (7) \\
y_2 &= x_2 + x_3 + z_2 \quad (8)
\end{align*}
\]

where \( z_i, i = 1, 2, 3 \) denote additive channel noise. As shown in [17], a secrecy degree of freedom min(\( N_{T_i}, N_R \)) = \( N_E = 1 \) is achievable for a user if the other user remains silent. Time sharing between these two users lead to the following achievable s.d.o.f. region:

\[
d_1 + d_2 \leq 1, \quad d_k \geq 0, \quad k = 1, 2 \quad (9)
\]

For the converse, we begin by considering a simple upper bound, which reduces each channel to a single-user MIMO wiretap channel. First, by revealing the signals transmitted by user 2 to the intended receiver and assuming that the eavesdropper monitors either \( x_1 \) or \( x_2 \) we have that \( d_1 \leq 1 \).
Similarly we argue that \( d_2 \leq 1 \). To obtain an upper bound on the sum-rate we let the two transmitters to cooperate and reduce the system to a \( 3 \times 3 \) MIMO link. The s.d.o.f. of this channel (17) yields \( d_1 + d_2 \leq 2 \). This outer bound, illustrated in Figure 2(b), does not match with the achievable region given by (9).

As we shall show in Theorem 1 (9) is indeed the largest possible s.d.o.f. region for Figure 2(a). In Section V-C, in generalizing this example we are required to express our result, we define

\[
\text{bound on } R \times \text{effective signal dimension at the receiver from three to one,}
\]

and the second eavesdropper implies the following upper bound on \( R_1 \):

\[
n(R_1 - \delta_n) \leq I \left( x_n^2; y_n^2 | y_n^1, x_n^{(3,4)} \right)
\]

and the second eavesdropper implies the following upper bound on \( R_2 \):

\[
n(R_2 - \delta_n) \leq I \left( y_n^1, x_n^{(3,4)}; y_n^2 \right)
\]

Their joint effect can be captured by adding (10) and (11), which lead to:

\[
n(R_1 + R_2 - 2\delta_n) \leq I \left( x_n^2, y_n^1, x_n^{(3,4)}; y_n^2 \right)
\]

Since there is only one term, which is \( y_n^2 \), at the right side of the mutual information \( I \left( x_n^2, y_n^1, x_n^{(3,4)}; y_n^2 \right) \), we observe the sum s.d.o.f. can not exceed 1, thereby justifying that (9) is indeed the largest possible s.d.o.f. region for Figure 2(a).

As captured by (10) and (11), a simultaneous selection of two different eavesdroppers for the two users reduces the effective signal dimension at the receiver from three to one, thus leading to a tighter converse. As we shall show later in Section V-C in generalizing this example we are required to systematically select a sequence of eavesdroppers using an induction procedure.

IV. MAIN RESULT

In this section, we state the main result of this work. To express our result, we define \( r_t \) as the rank of \( H_i \), \( t = 1, 2 \) and \( r_0 \) as the rank of \( [H_1 \mid H_2] \). We will refer to \( r_t \) as the number of transmit dimensions at user \( t = 1, 2 \) and \( r_0 \) as the number of dimensions at the receiver.

**Theorem 1**: The secrecy degrees of freedom region of the MIMO multiple access channel with arbitrarily varying eavesdropper channel is given by the convex hull of the following five points of \((d_1, d_2)\):

\[
\begin{align*}
  p_0 &= (0, 0) \\
p_1 &= (r_1 - N_E^+, 0) \\
p_2 &= (0, r_2 - N_E^+) \\
p_3 &= (r_1 - N_E^+, [r_0 - r_1 - N_E^+]^+) \\
p_4 &= ([r_0 - r_2 - N_E^+]^+, [r_2 - N_E^+]^+)
\end{align*}
\]

where we use \( [x]^+ \triangleq \max \{x, 0\} \).

Fig. 3 illustrates the structure of the s.d.o.f. region as a function of the number of eavesdropping antennas. In Fig. 3(a) we have \( N_E \geq \min\{r_0 - r_1, r_0 - r_2\} \). In the case the s.d.o.f. region is a polyomatroid (see e.g., [27] Definition 3.1) described by \( d_1 \leq r_1 - N_E \) and \( d_1 + d_2 \leq r_0 - 2N_E \). Fig. 3 (b) illustrates the shape of the s.d.o.f. region when \( \min \{r_0 - r_1, r_0 - r_2\} \leq N_E \leq \max \{r_0 - r_1, r_0 - r_2\} \). In Fig. 3(b), without loss of generality, we assume \( r_1 < r_2 \) and the s.d.o.f. region is bounded by the lines \( d_1 \geq 0 \), \( d_1 \leq r_1 - N_E \) and

\[
(r_1 + r_2 - r_0)d_1 + (r_1 - N_E)d_2 \\
\leq (r_1 - N_E) \times (r_2 - N_E).
\]

When \( \min \{r_1, r_2\} > N_E \geq \max \{r_0 - r_1, r_0 - r_2\} \), the s.d.o.f. region, as illustrated in Fig. 3(c) is bounded by \( d_1 \geq 0 \) and the line

\[
\frac{d_1}{r_1 - N_E} + \frac{d_2}{r_2 - N_E} \leq 1.
\]

The s.d.o.f. region in Theorem 1 allows the following simple interpretation: The region can be expressed as a convex hull of a set of rectangles shown by Figure 4 (illustrated for Figure 3 (a)). Each rectangle is parameterized by the dimensions of the subspace occupied by the transmission signals from the two users, denoted by \((t_1, t_2)\), where \( t_i \) indicates the dimension of user \( i \), \( i = 1, 2 \). Then in order for the signals from both transmitters to be received reliably
The parallel channel model is a special case of (1) with
\[
H_1 = \begin{bmatrix} I_{|A|} & I_{|B|} \\ O_{|C|} & I_{|C|} \end{bmatrix}, \quad H_2 = \begin{bmatrix} O_{|A|} & I_{|B|} \\ I_{|A|} & I_{|C|} \end{bmatrix},
\]
where \(I_{|A|}, I_{|B|}\) and \(I_{|C|}\) denote the identity matrices of size \(|A|, |B|\) and \(|C|\) respectively, and \(O_{|A|}\) and \(O_{|B|}\) denote the matrices, all of whose entries are zeros. Note that we do not make any assumption on the eavesdropper’s channel model (2).

### A. Achievability

It suffices to establish the achievability of points \(p_3\) and \(p_4\) in (16) and (17) respectively. The rest of the region follows through time-sharing between these points. Note that for the proposed parallel channel model
\[
p_3 = \left( |A| + |B| - N_E \right)^+, \left( |C| - N_E \right)^+ \quad (26)
\]
\[
p_4 = \left( |A| - N_E \right)^+, \left( |B| + |C| - N_E \right)^+ \quad (27)
\]

To prove the achievability of \(p_3\) we restrict user 2 to transmit only on the last \(|C|\) components of in (24) and allow user 1 to transmit over all of the components of \(A \cup B\) in (22) and (23). Note that in this case, the signals of these two users do not interfere with each other at the intended receiver. From (17), user 1 can transmit \(W_1\) such that \(d_1 = (|A| + |B| - N_E)^+\) and
\[
\lim_{n \to \infty} I(W_1; \tilde{H}_1^n X_1^n) = 0
\]
and user 2 can transmit \(W_2\) such that \(d_2 = (|C| - N_E)^+\) and
\[
\lim_{n \to \infty} I(W_2; \tilde{H}_2^n X_2^n) = 0
\]
where we use \(\tilde{H}_k^n X_k^n\) to denote the sequence \(\{\tilde{H}_k(i) X_k(i), i = 1, \ldots, n\}\). Furthermore since \((W_1, X_1^n)\) is independent of \((W_2, X_2^n)\) we have that
\[
\lim_{n \to \infty} I(W_1; \tilde{H}_1^n X_1^n, \tilde{H}_2^n X_2^n) = 0 \quad (30)
\]
\[
\lim_{n \to \infty} I(W_2; \tilde{H}_1^n X_1^n, \tilde{H}_2^n X_2^n) = 0 \quad (31)
\]
which imply:
\[
I \left( W_1; \tilde{H}_1^n X_1^n, \tilde{H}_2^n X_2^n | W_2 \right) \leq I \left( W_1; W_2, \tilde{H}_1^n X_1^n, \tilde{H}_2^n X_2^n \right) \quad (32)
\]
\[
= I \left( W_1; \tilde{H}_1^n X_1^n, \tilde{H}_2^n X_2^n \right) + I \left( W_1; W_2 | \tilde{H}_1^n X_1^n, \tilde{H}_2^n X_2^n \right) \quad (33)
\]
\[
\leq I \left( W_1; \tilde{H}_1^n X_1^n, \tilde{H}_2^n X_2^n \right) + I \left( W_1, \tilde{H}_1^n X_1^n, W_2, \tilde{H}_2^n X_2^n \right) \quad (34)
\]
\[
= I \left( W_1; \tilde{H}_1^n X_1^n \right) \quad (35)
\]
where the last step follows from the fact that \((W_2, \tilde{H}_2^n X_2^n)\) is independent from \((W_1, \tilde{H}_1^n X_1^n)\). Therefore (28) implies
\[
\lim_{n \to \infty} I \left( W_1; \tilde{H}_1^n X_1^n, \tilde{H}_2^n X_2^n | W_2 \right) = 0. \quad (36)
\]
Adding (36) and (31), we obtain
\[
\lim_{n \to \infty} I \left( W_1, W_2; \mathbf{H}_1^n X_1^n, \mathbf{H}_2^n X_2^n \right) = 0
\] (37)
and the secrecy constraint (3) follows from the data-processing inequality. Also, since the convergence in \( n \) in (30) and (31) is uniform [17], the convergence in (37) and hence in (3) is uniform as well. Hence we have proved the point \( p_3 \) is achievable.

The achievability of \( p_3 \) is proved by repeating the argument above by exchanging user 1 with user 2.

Remark 1: As is evident from (37), the secrecy guarantee achieved by one user is not affected by the transmission strategy of the other user.

B. Converse : \( N_E \leq \min(\{|A|, |C|\}) \)

We need to show that the s.d.o.f. region is contained within
\[
d_1 \leq |A| + |B| - N_E
\] (38)
\[
d_2 \leq |C| + |B| - N_E
\] (39)
\[
d_1 + d_2 \leq |A| + |B| + |C| - 2N_E
\] (40)
Since (38) and (39) directly follow from the single user case in [17], we only need to show (40).

Let \( E_i \) be the set of links such that an eavesdropper is monitoring for \( W_k \), \( k = 1, 2 \). Let \( |E_1| = |E_2| = N_E \). \( A \supseteq E_1 \), \( C \supseteq E_2 \). We establish the following upper bound on the achievable rate pairs.

Lemma 1:
\[
n(R_{s,1} - \delta_n) \leq I \left( X_{1,A\setminus E_1}^n; Y_{A\setminus E_1}^n \right) + I \left( X_{1,B}^n; Y_{1,B}^n \right)
\] (41)
\[
n(R_{s,2} - \delta_n) \leq I \left( X_{2,C\setminus E_2}^n; Y_{C\setminus E_2}^n \right) + I \left( M; Y_{B}^n \right)
\] (42)
where \( M = \{ Y_{1,A\setminus E_1}, X_{2,B\setminus C}^n \} \).

Proof: The proof is provided in Appendix A.

The proof is completed upon adding (41) and (42) so that
\[
n(R_{s,1} + R_{s,2} - 2\delta_n) \leq (X_{1,A\setminus E_1}^n; Y_{A\setminus E_1}^n) + I \left( X_{2,C\setminus E_2}^n; Y_{C\setminus E_2}^n \right) + I \left( M, X_{1,B}^n; Y_{B}^n \right)
\] (43)
and using
\[
d \left( \frac{1}{n} I \left( X_{A\setminus E_1}^n; Y_{A\setminus E_1}^n \right) \right) \leq |A| - N_E \] (44)
\[
d \left( \frac{1}{n} I \left( X_{C\setminus E_2}^n; Y_{C\setminus E_2}^n \right) \right) \leq |C| - N_E \] (45)
\[
d \left( \frac{1}{n} I \left( M, X_{1,B}^n; Y_{B}^n \right) \right) \leq |B|
\] (46)
where \( d(x) \triangleq \lim_{P \to \infty} \frac{x(P)}{\log_2 P} \) characterizes the pre-log scaling of \( x \) with respect to \( P \).

C. Converse : \( N_E \geq \max(\{|A|, |C|\}) \)

Without loss of generality, we assume \( |C| \geq |A| \). Let \( E_i \) be the set of links such that an eavesdropper is monitoring for \( W_k \), \( k = 1, 2 \). Let \( |E_1| = |E_2| = N_E \), \( A \supseteq E_1 \), \( C \supseteq E_2 \). We establish the following upper bound on the achievable rate pairs.

Lemma 2: For any choice of \( F \subseteq B \) and \( G \subseteq B \) with appropriate cardinalities the rates \( R_{s,1} \) and \( R_{s,2} \) are upper bounded by
\[
n(R_{s,1} - \delta_n) \leq I \left( X_{1,F}; Y_{F}^n \right) + I \left( M, X_{1,B\setminus F}^n \right)
\] (48)
\[
n(R_{s,2} - \delta_n) \leq I \left( M, X_{1,B}^n; Y_G^n \right)
\] (49)
where \( M = \{ Y_{1,A\setminus E_1}, X_{2,B\setminus C}^n \} \).

Proof: The proof is provided in Appendix B.

For the remainder of the proof we assume without loss of generality that \( B = \{1, \ldots, |B|\} \). We fix \( G = \{1, \ldots, |G|\} \) while choosing \( |G| \) different sets of \( |F| \) elements: \( F_1, \ldots, F_{|G|} \), the sets \( V_0, V_1, \ldots \) and a sequence of \( c_i \) in the following recursive manner.

Definition 1: Let \( V_0 = G \), \( c_0 = 1 \). For \( i \geq 1 \) recursively construct \( F_i \) as follows.

1) Case I: \( |V_{i-1}| \geq |F| \)
Let \( F_i = \{ V_{i-1}(1), \ldots, V_{i-1}(|F|) \} \), where \( V_{i-1}(k) \) denotes the \( k \)th smallest element in \( V_{i-1} \). Let \( V_i = V_{i-1} \setminus F_i \) and \( c_i = c_{i-1} + 1 \). This case is illustrated in Figure 6(a) for \( i = 1 \).

2) Case II: \( |V_{i-1}| < |F| \)
Let \( F_i = V_{i-1} \cup H_i \), and \( V_i = G \setminus H_i \), and \( c_i = c_{i-1} + 1 \), where \( H_i = \{1, 2, \ldots, |F| - |V_{i-1}|\} \). This case is illustrated in Figure 6(b) for \( i = 4 \).

To interpret the above construction, we note that the set \( G \) is a row-vector with \( |G| \) elements and let \( G^c \) be obtained by concatenating \( |F| \) identical copies of the \( G \) vector i.e.,
\[
G^c = [G | G | \ldots G]^{\otimes |F|}
\] (50)
As shown in Figure 6 by our construction, the vector $F_i$ spans the first $|F|$ elements of $G^i$, the vector $F_2$ spans the next $|F|$ elements of $G^i$ etc. The constant $c_i$ denotes the index number of copies of the $G$ vector necessary to cover $F_i$.

When $i = |G|$ the row-vector $F_i$ terminates exactly at the end of the last $G$ vector in $G^i$. Hence,

$$c_{|G|} = |F|, \quad \forall |G| = \phi.$$ (51)

By going through the above recursive procedure and invoking Lemma 2 repeatedly, each time by setting $F$ in (48) and $V_i$ to be $F_i$, we establish the following upper bound on the rate region.

**Lemma 3:** For each $i = 0, 1, \ldots, |G|$ and the set of channels $F_1, F_2, \ldots, F_{|G|}$ defined in Def. 1 the rate pair $(R_{s,1}, R_{s,2})$ satisfies the following upper bound

$$i \cdot n(R_{s,1} - \delta_n) + c_i \cdot n(R_{s,2} - \delta_n) \leq \sum_{j=1}^{i} I(M, X_{1:B}^n; Y_{F_j}^n) + I(M, X_{1:B}^n; Y_{V_i}^n).$$ (52)

Before providing a proof, we note that (47) follows from (52) as described below. Evaluating (52) with $i = |G|$, using (51) and letting $R_{s,i} = R_{s,1} - \delta_n$,

$$n|G| R_{s,1} + n|F| R_{s,2} \leq \sum_{j=1}^{|G|} I(M, X_{1:B}^n; Y_{F_j}^n) = \sum_{j=1}^{|G|} \{h(Y^n_{F_j}) - h(Y^n_{F_j} | M, X_{1:B}^n)\} = n \{ |G| \cdot |F| \cdot \log_2 P + \Theta(1) \},$$ (53)

where the last step uses the fact that

$$h(Y^n_{F_j}) \leq \sum_{k \in F_j} h(Y^n_{k}) \leq n \{ |F| \log_2 P + O(1) \},$$ (54)

and

$$h(Y^n_{F_j} | M, X_{1:B}^n) = h(Y^n_{F_j} | X_{1:F_j}, X_{2:F_j}) = n \cdot O(1).$$ (55)

Dividing each side of (55) by $\log_2 P$ and taking the limit $P \to \infty$ yields (47).

**Proof of Lemma 2** We use induction over the variable $i$ to establish (52). For $i = 0$, note that $c_0 = 0$ and $V_i = G$ and hence (52) is simply (51). This completes the proof for the base case.

For the induction step, we assume that (52) holds for some $t = i$, we need to show that (52) also holds for $t = i + 1$, i.e.,

$$(i + 1) \cdot n(R_{s,1} - \delta_n) + c_{i+1} \cdot n(R_{s,2} - \delta_n) \leq \sum_{j=1}^{i+1} I(M, X_{1:B}^n; Y_{F_j}^n) + I(M, X_{1:B}^n; Y_{V_{i+1}}^n).$$ (58)

holds. For our proof we separately consider the cases when $|F| \leq |V_i|$ and when $|V_i| < |F|$ holds.

When $|F| \leq |V_i|$, from Definition 1

$$F_{i+1} \subseteq V_i, \quad V_{i+1} = V_i \setminus F_{i+1}, \quad c_{i+1} = c_i$$ (59)

holds. Then (58) follows by combining (52) with (48) as we show below. Note that

$$I(M, X_{1:B}^n; Y_{V_{i+1}}^n) = I(M, X_{1:B}^n; Y_{V_i}^n | Y_{V_i}^n)$$

$$+ I(M, X_{1:B}^n; Y_{V_{i+1}}^n | Y_{V_i}^n)$$

$$\leq I(M, X_{1:B}^n; Y_{V_i}^n | Y_{V_i}^n)$$

$$+ I(M, X_{1:B}^n; Y_{V_{i+1}}^n | Y_{V_i}^n)$$

$$\leq I(M, X_{1:B}^n; X_{1:B}^n | Y_{V_i}^n)$$

$$+ I(M, X_{1:B}^n; Y_{V_{i+1}}^n | Y_{V_i}^n)$$

$$= I(M, X_{1:B}^n; Y_{F_{i+1}}^n) + I(M, X_{1:B}^n; Y_{V_{i+1}}^n)$$ (60)

where (60) follows from the chain rule of the mutual information and the definition of $V_{i+1}$ in (59), while (62) follows from the Markov condition

$$(X_{1:B}^n | Y_{V_i}^n) \leftrightarrow (X_{1:B}^n; Y_{V_i}^n) \leftrightarrow (M, Y_{F_{i+1}}^n, X_{1:B}^n)$$ (61)

and the fact that $M = (X_{1:B}^n, Y_{V_i}^n)$ already includes $X_{1:B}^n | Y_{V_i}^n$, (63) follows from the fact that $V_i \subseteq G$, while (64) follows from the fact that $\{B \setminus G\} \cup \{G \setminus F_i\} = \{B \setminus F_i\}$.

Substituting (64) into the last term in (52) we get

$$i \cdot n(R_{s,1} - \delta_n) + c_i \cdot n(R_{s,2} - \delta_n) \leq \sum_{j=1}^{i} I(M, X_{1:B}^n; Y_{F_j}^n) + I(M, X_{1:B}^n; Y_{V_i}^n)$$

$$\leq \sum_{j=1}^{i} I(M, X_{1:B}^n; Y_{F_j}^n) + I(M, X_{1:B}^n; Y_{V_{i+1}}^n)$$

$$+ I(M, X_{1:B}^n; Y_{V_{i+1}}^n)$$ (66)

Finally combining (66) with (48) and using $c_{i+1} = c_i$ (c.f. (59)) we have

$$(i + 1) \cdot n(R_{s,1} - \delta_n) + c_{i+1} \cdot n(R_{s,2} - \delta_n) \leq \sum_{j=1}^{i} I(M, X_{1:B}^n; Y_{F_j}^n) + I(M, X_{1:B}^n; Y_{V_{i+1}}^n)$$

$$+ I(M, X_{1:B}^n; Y_{V_{i+1}}^n)$$

$$+ I(X_{1:F_{i+1}}; Y_{F_{i+1}}^n | M, X_{1:B}^n)$$ (67)

$$= \sum_{j=1}^{i+1} I(M, X_{1:B}^n; Y_{F_j}^n) + I(M, X_{1:B}^n; Y_{V_{i+1}}^n)$$ (68)

as required.
holds. From (49) and (58) we have that

\[ i \cdot n(R_{s,1} - \delta_n) + (c_i + 1) \cdot n(R_{s,2} - \delta_n) \]

\[ = \sum_{j=1}^{i} I(M, X_{1,B}^{n}; Y_{F_j}^{n}) + I(M, X_{1,B}^{n}; Y_{V_i}^{n}) \]

\[ + I(M, X_{1,B}^{n}; Y_{G}^{n}) \]

\[ = \sum_{j=1}^{i} I(M, X_{1,B}^{n}; Y_{F_j}^{n}) + I(M, X_{1,B}^{n}; Y_{V_i}^{n}) \]

\[ + I(M, X_{1,B}^{n}; Y_{H_{i+1}}^{n} \mid Y_{G}^{n} \mid Y_{H_{i+1}}^{n}) \]

\[ \leq I(M, X_{1,B}^{n}; Y_{V_i}^{n}) \quad \text{(70)} \]

As we will show subsequently,

\[ I(M, X_{1,B}^{n}; Y_{V_i}^{n}) + I(M, X_{1,B}^{n}; Y_{H_{i+1}}^{n} \mid Y_{G}^{n} \mid Y_{H_{i+1}}^{n}) \]

\[ \leq I(M, X_{1,B}^{n}; Y_{V_i}^{n}) \quad \text{(70)} \]

\[ \text{(71)} \]

Combining (48), (71) and (72) and using \( c_{i+1} = c_i + 1 \) we get that

\[ (i+1) \cdot n(R_{s,1} - \delta_n) + c_{i+1} \cdot n(R_{s,2} - \delta_n) \]

\[ \leq \sum_{j=1}^{i} I(M, X_{1,B}^{n}; Y_{F_j}^{n}) + I(M, X_{1,B}^{n}; Y_{H_{i+1}}^{n} \mid Y_{G}^{n} \mid Y_{H_{i+1}}^{n}) \]

\[ \leq I(M, X_{1,B}^{n}; Y_{V_i}^{n}) \quad \text{(71)} \]

(72)

Thus establishing (72).

This completes the proof.

D. Converse: \( \min(|A|, |C|) \leq N_E \leq \max(|A|, |C|) \)

We assume without loss of generality that \( |C| \geq |A| \) and and before let \( E_k \) be the set of links such that an eavesdropper is

\[ \text{monitoring for message } W_k. \]

\[ \text{Since } |E_1| = |E_2| = N_E \text{ and } |A| \leq N_E \leq |C| \text{ holds, we select the sets such that the relations } \]

\[ A \subseteq E_1 \subseteq A \cup B \text{ and } C \geq E_2 \text{ are both satisfied. Define } F = B \setminus E_1 \text{ and note that } |F| = |A| + |B| - N_E. \]

Theorem 1 reduces to the following region:

\[ 0 \leq d_1 \leq |F| \]

\[ 0 \leq d_2 \leq |B| + |C| - N_E \]

\[ |B|d_1 + |F|d_2 \leq (|B| + |C| - N_E) \times |F| \]

Since (83) and (84) directly follow from the single user case (77), we only need to establish (85). As in earlier cases we begin by establishing the following bounds on the rate pair \((R_{s,1}, R_{s,2})\):

\[ n(R_{s,1} - \delta_n) \leq I(X_{1,A}^{n}; Y_{F}^{n} \mid M, X_{1,B}^{n} \setminus F) \]

\[ n(R_{s,2} - \delta_n) \leq I(M; Y_{B}^{n}) + I \left( X_{2,C}^{n}; Y_{C}^{n} \right) \]

\[ \text{(86)} \]

\[ \text{(87)} \]

where \( M = (X_{1,B}^{n} \cup X_{1,A}^{n}) \).

Proof: The proof for (86) is identical to (48) in Lemma 2 since the proof does not depend on the choice of \( E_2 \). The proof for (87) is identical to (42) in Lemma 1.

To establish (83)–(85), note that by defining

\[ R_{s,2}' = R_{s,2} - \frac{1}{n} I \left( X_{2,C}^{n}; Y_{C}^{n} \right) \]

\[ \text{(88)} \]

we have from (77) that

\[ n(R_{s,2}' - \delta_n) \leq I(M; Y_{B}^{n}) \]

(89)

and the bounds on \( R_{s,1} \) and \( R_{s,2}' \) in (86) and (89) are identical to the bounds (48) and (49) in Lemma 2 with \( G = B \). Applying Lemma 3 to \( R_{s,1} \) and \( R_{s,2}' \) for each \( i = 0, 1, \ldots, |G| \), it follows that

\[ i \cdot n(R_{s,1} - \delta_n) + c_i \cdot n(R_{s,2}' - \delta_n) \]

\[ \leq \sum_{j=1}^{i} I(M, X_{1,B}^{n}; Y_{F_j}^{n}) + I(M, Y_{V_j}^{n}) \]

\[ \text{(90)} \]

where the sets \( V_i, F_i \) and the sequence \( c_i \) are as in Definition 1. Substituting (89) into (90) and evaluating the bound for \( i = |B| \)
we have that
\[
|\mathcal{B}| n(R_s, 1 – \delta_n) + |\mathcal{F}| n(R_s, 2 – \delta_n) \leq |\mathcal{F}| I(\mathcal{X}_2^n, \mathcal{C}_1; \mathcal{Y}_2^n, \mathcal{C}_2)
\]
\[
+ \sum_{j=1}^{[\mathcal{B}]} I(M, \mathcal{X}_1^n, \mathcal{C}_1; \mathcal{Y}_j^n, \mathcal{C}_j).
\] (91)

Finally substituting
\[
d\left(\frac{1}{n} I(M, X^n, C_1, C_2; Y^n, C_2)\right) \leq |\mathcal{C}| – N_E
\] (92)
\[
d\left(\frac{1}{n} I(M, X^n, C_1; Y^n, C_1)\right) \leq |\mathcal{F}|,
\] (93)
in (91) we obtain (85).

VI. GENERAL MIMO-MAC

The result for the general MIMO case follows by a transformation that reduces the model to the case of parallel independent channels in the previous section while preserving the secrecy degrees of freedom region. As we discuss next, this transformation involves the generalized singular value decomposition (GSVD) and a channel enhancement argument. For an analogous application of GSVD to broadcast channels see e.g., [18], [29], [30].

A. GSVD Transformation

Theorem 2: [28] Given a pair of matrices \( H_1 \) and \( H_2 \) such that the rank of \( H_i \) is \( r_i \), \( i = 1, 2 \), and the rank of \( [H_1 \mid H_2] \) is \( r_0 \), there exists unitary matrices \( U_1, U_2, W, Q \) and nonsingular upper triangular matrix \( R \) such that for \( s = r_1 + r_2 - r_0, \tilde{r}_1 = r_1 - s, \tilde{r}_2 = r_2 - s, \)
\[
U_1^H H_1^H Q = \Sigma_1(N_{r_1}, r_0) [W^H R(r_0, r_0), O]_{r_0 \times N_r}(104)
\]
\[
U_2^H H_2^H Q = \Sigma_2(N_{r_2}, r_0) [W^H R(r_0, r_0), O]_{r_0 \times N_r}(105)
\]
\[
\Sigma_1 = \begin{bmatrix}
1_{(r_1, \tilde{r}_1)} & S_1(s \times s) & O_{1((N_{r_1} - \tilde{r}_1) \times s)}
\end{bmatrix}
\]
(96)
\[
\Sigma_2 = \begin{bmatrix}
O_{2((N_{r_2} - \tilde{r}_2) \times \tilde{r}_1)} & S_2(s \times s)
\end{bmatrix}
\]
(97)
where \( I_1, i = 1, 2 \) are \( \tilde{r}_i \times \tilde{r}_i \) identity matrices, \( O_{1, i = 1, 2} \) are zero matrices, and \( S_i, i = 1, 2 \) are \( s \times s \) diagonal matrices with positive real elements on the diagonal line that satisfy \( S_1^2 + S_2^2 = I_s \), and \( \tilde{r}_1 + s + \tilde{r}_2 = r_0 \). For clarity, the dimension of each matrix is shown in the parenthesis in the subscript. \( I_1 \) has the same number of columns as \( O_2, I_2 \) has the same number of columns \( O_1 \). However, \( O_{1, i = 1, 2} \) are not necessarily square matrices and can be empty, i.e., having zero number of rows.

For convenience in notation we define \( A = W^H R \) and observe that \( A \) is a square and non-singular matrix. Then from Theorem 2 we have:
\[
Q^H H_i U_t = \begin{bmatrix}
A^H & 0
\end{bmatrix} \Sigma_i^H, t = 1, 2.
\] (98)
Without loss of generality, we can cancel \( Q \) and \( U_t \) and rewrite (1) as:
\[
Y = A_{r_0 \times r_0}^H (\Sigma_1^H X_1 + \Sigma_2^H X_2) + Z.
\] (100)

B. Converse

For establishing the converse, we further enhance the channel model in (100) to the following
\[
Y = \Sigma_1^H X_1 + \Sigma_2^H X_2 + \sigma + Z'
\] (101)
where \( \sigma \leq 1 \) is any sufficiently small constant such that, \( \sigma^2 \) times the maximal eigenvalue of \( A_{r_0 \times r_0}^H A_{r_0 \times r_0} \), is smaller than 1 and \( Z' \) is a circularly symmetric unit-variance Gaussian noise vector.

To establish (101), note that we can express
\[
Z = \sigma \cdot A^H Z' + Z''
\] (102)
where \( Z'' \) is a Gaussian random vector, independent of \( Z' \) and with a covariance matrix
\[
I_{r_0 \times r_0} - \sigma^2 A_{r_0 \times r_0} A_{r_0 \times r_0}^H
\] (103)
which is guaranteed to be positive semi-definite by our choice of \( \sigma \). Upon substituting (102) into (100), we have
\[
Y = A_{r_0 \times r_0}^H (\Sigma_1^H X_1 + \Sigma_2^H X_2 + \sigma + Z') + Z''.
\] (104)
We consider an enhanced receiver that is revealed \( Z'' \). Clearly this additional knowledge can only increase the rate and serves as an upper bound. It is also clear that since \( Z'' \) is independent of \( (X_1, X_2, Z') \), it suffices to use this information to cancel \( Z'' \) in (104) and then discard it. Furthermore since the matrix \( A \) is invertible, upon canceling it, we obtain (101).

We further enhance the receiver by replacing \( \Sigma_1^H \) and \( \Sigma_2^H \) with \( \tilde{\Sigma}_1^H \) and \( \tilde{\Sigma}_2^H \) so that the model reduces to
\[
Y = \tilde{\Sigma}_1^H X_1 + \tilde{\Sigma}_2^H X_2 + \sigma + Z'
\] (105)
where
\[
\tilde{\Sigma}_1^H = \begin{bmatrix}
I_{r_1 \times \tilde{r}_1} & 0_{1((r_2 \times (N_{r_1} - \tilde{r}_1)))}
\end{bmatrix}_{r_0 \times N_{r_1}}
\] (106)
noise which can be simulated at the receiver by adding additional noise vector. Since
than times the minimum eigenvalue of all the singular values of scaling down the transmit powers on each of the sub-channels can express \( \sigma \) except that the noise variance is reduced by a factor of \( A \) from the model (100) to V-C and V-D we establish the converse in Theorem 1. Since a fixed scaling in the noise power does not affect the secrecy degrees of freedom, the coding schemes described in section V-A achieves the lower bound in Theorem 1.

\[
\Sigma_2^H = \begin{bmatrix}
0_2((r_1 \times (N_{r_2} - r_2)) & I_{2(s \times s)} \\ I_{2(r_2 \times r_2)} & I_{r_0 \times N_{r_2}}
\end{bmatrix}
\]  

(107)

are obtained by replacing each diagonal \( S_i \) by the identity matrix. The model (105) can only have a higher capacity, since each diagonal entry in \( S_i \) is between \((0, 1)\). We observe that in the resulting channel model is identical to (22)-(24)

\[
|A| = r_0 - r_2
\]

(108)

\[
|B| = s = r_1 + r_2 - r_0
\]

(109)

\[
|C| = r_0 - r_1
\]

(110)

except that the noise variance is reduced by a factor of \( \sigma^2 \). Since a fixed scaling in the noise power does not affect the secure-degrees of freedom, an outer bound on the s.d.o.f. for the parallel channel model (22)-(24) with \( A, B \) and \( C \) defined via (105) continues to be an outer bound on the s.d.o.f. region for the general MIMO-MAC channel.

Substituting (108), (110) in the upper bounds in section V-B V-C and V-D we establish the converse in Theorem 1.

C. Achievability

To establish the achievability for the general MIMO case we further use a suitable degradation mechanism to reduce the model (106) to

\[
Y = \Sigma_1^H X_1 + \Sigma_2^H X_2 + \sigma Z''
\]

(111)

where \( \sigma \geq 1 \) is any sufficiently large constant such that, \( \sigma^2 \) times the minimum eigenvalue of \( A^{H}_{r_0 \times r_0} A_{r_0 \times r_0} \), is greater than 1 and \( Z'' \) is a circularly symmetric unit-variance Gaussian noise vector. Since \( A \) is non-singular we are guaranteed that all the singular values of \( A \) are non-zero and hence a \( \sigma < \infty \) exists.

To establish (111), let \( Z' \) be a Gaussian noise vector with covariance

\[
\sigma^2 A_{r_0 \times r_0}^{H} A_{r_0 \times r_0} - I_{r_0 \times r_0}
\]

(112)

independent of \( Z \) and consider a degraded version of (100)

\[
Y_1 = A_{r_0 \times r_0} \left( \Sigma_1^H X_1 + \Sigma_2^H X_2 \right) + Z + Z'
\]

(113)

which can be simulated at the receiver by adding additional noise \( Z' \) to its output. Since \( Z + Z' \sim \mathcal{CN}(0, \sigma^2 A H A) \), we can express \( Z + Z' = \sigma A^{H} Z'' \). Substituting into (113) and canceling the non-singular matrix \( A \), we arrive at (111).

Let \( \bar{s} > 0 \) denote the minimum element on the diagonals of \( S_1 \) and \( S_2 \) in (96) and (97) respectively. By appropriately scaling down the transmit powers on each of the sub-channels we can further reduce (104) to

\[
Y = \bar{\Sigma}_{k}^H X_1 + \bar{\Sigma}_{k}^H X_2 + \frac{\sigma}{\bar{s}} Z''
\]

(114)

where \( \bar{\Sigma}_{k} \) are defined in (96) and (97) respectively. The model (114) is identical to the parallel channel model (22)-(24) with the size of sets \( A, B \) and \( C \) in (108), (110) and with a noise power that is larger by a factor of \( \sigma^2/\bar{s}^2 \). Since a constant factor in the noise power does not affect the secrecy degrees of freedom, the coding schemes described in section V-A achieves the lower bound in Theorem 1.

VII. Conclusion

In this work we have studied the two-transmitter Gaussian complex MIMO-MAC wiretap channel where the eavesdropper channel is arbitrarily varying and its state is known to the eavesdropper only, and the main channel is static and its state is known to all nodes. We have completely characterized the s.d.o.f. region for this channel for all possible antenna configurations. We have proved that this s.d.o.f. region can be achieved by a scheme that orthogonalizes the transmit signals of the two users at the intended receiver, in which each user achieves secrecy guarantee independently without cooperation from the other user. The converse was proved by carefully changing the set of signals available to the eavesdropper through an induction procedure in order to obtain an upper bound on a weighted-sum-rate expression.

As suggested by this work, the optimal strategy for a communication network where the eavesdropper channel is arbitrarily varying can potentially be very different from the case where the eavesdropper channel is fixed and its state is known to all terminals. This is also observed for example in the MIMO broadcast channel [3] and the two-way channel [31], [32]. Characterizing secure transmission limits for a broader class of communication models with this assumption is hence important and is left as future work.

APPENDIX A

PROOF OF LEMMA 1

For \( R_{s,1} \), from Fano’s inequality, we have

\[
n(R_{s,1} - \delta_n) \leq I(W_1; Y_{A\cup B\cup C}^n) - I(W_1; Y_{E_1}^n)
\]

(115)

\[
\leq I(W_1; Y_{A\cup B\cup C})|Y_{E_1}^n
\]

(116)

\[
\leq I(W_1; Y_{A\cup B\cup C}, X_{2, B\cup C}^n|Y_{E_1}^n
\]

(117)

\[
= I(W_1^n; A_{R_{s,1}}, X_{2, B\cup C}^n|Y_{E_1}^n
\]

(118)

where the last step (118) relies on the fact that the additive noise at each receiver end of each sub-channel in Figure 5 is independent from each other and hence

\[
Y_{C}^n \rightarrow X_{n}^{2, C} \rightarrow (W_1, Y_{A\cup B}, Y_{E_1}^n, X_{n}^{2, B})
\]

holds. Since \( (X_{n}^{2, C}, X_{n}^{2, B}) \) is independent from \( W_1 \), and \( E_1 \subseteq A \), (119) can be written as:

\[
I(W_1; Y_{A\cup B}^n, Y_{E_1}^n, X_{2, B\cup C}^n)
\]

(119)

\[
= I(W_1; Y_{A\cup B}^n, X_{2, B\cup C}^n + I(W_1; Y_{B}^n, Y_{A}, X_{2, B\cup C}^n)
\]

(120)

where the last step (120) follows from the fact \( E_1 \subseteq A \) and hence \( A = (A \cup E_1) \cup E_1 \). We separately bound each of the two terms above.

\[
I(W_1; Y_{A\cup B}^n, X_{2, B\cup C}^n)
\]

(121)

\[
\leq I(W_1, Y_{E_1}^n, X_{2, B\cup C}^n, Y_{A\cup B}^n)
\]

(122)

\[
= I(W_1, X_{n}^{2, A\cup B}, Y_{A\cup B}^n)
\]

(123)
where the last step follows from the Markov chain relation $Y_{1,A,E}^n \leftrightarrow X_{A,E}^n \leftrightarrow (W_1, Y_{A,E}^n, X_{2,B,uc}^n)$. We upper bound the second term in (120) as follows

$$I \left( W_1; Y_B^n | Y_A^n, X_{2,B,uc}^n \right) \leq I \left( X_{2,B,uc}^n; Y_B^n | Y_A^n, X_{2,B,uc}^n \right)$$

(124)

$$= I \left( X_{2,B,uc}^n; Y_B^n | Y_A^n, X_{2,B,uc}^n \right) + I \left( Y_{A,E}^n; Y_B^n | Y_A^n, X_{2,B,uc}^n, X_{1,E}^n \right)$$

(125)

$$= I \left( X_{2,B,uc}^n; Y_B^n | Y_A^n, X_{2,B,uc}^n \right)$$

(126)

where we use the Markov relation $W_1 \leftrightarrow X_{1,A,E}^n \leftrightarrow (Y_{A,E}^n, X_{2,B,uc}^n)$ in step (124) and (126) follows from the fact Markov relation $Y_B^n \leftrightarrow (X_{2,B,uc}^n, Y_A^n) \leftrightarrow (X_{2,B,uc}^n, X_{1,E}^n)$. Note that (41) follows upon substituting (123) and (126) into (120).

For $R_{a_2}$, from Fano’s inequality and the secrecy constraint, we have:

$$n(R_{a_2} - \delta_{a}) \leq I \left( W_2; Y_{A\cup B\cup C}^n | X_{2,E}^n \right) - I \left( W_2; X_{2,E}^n \right)$$

(128)

$$\leq I \left( W_2; Y_{A\cup B\cup C}^n | X_{2,E}^n \right)$$

(129)

$$= I \left( W_2; Y_{B\cup C}^n | Y_A^n, X_{2,E}^n \right)$$

(130)

$$= I \left( W_2; Y_{B\cup C}^n | Y_A^n, Y_{C\cup E}^n \right)$$

(131)

$$= I \left( W_2; Y_{B\cup C}^n | Y_{C\cup E}^n \right)$$

(132)

where (130) follows from the fact that $Y_A^n$ is independent of $(W_2, X_{2,B,uc}^n)$ and (131) follows from the fact that $Y_{C\cup E}^n \rightarrow X_{2,E}^n \rightarrow (Y_{B\cup C}^n, W_2, Y_A^n)$ holds. We separately bound each term in (132).

$$I \left( W_2; Y_{B\cup C}^n | Y_A^n, Y_{C\cup E}^n \right) \leq I \left( W_2, Y_A^n, X_{2,E}^n, Y_{C\cup E}^n \right)$$

(133)

$$\leq I \left( X_{2,C\cup E}^n, Y_{A^n}, X_{2,E}^n, Y_{C\cup E}^n \right)$$

(134)

$$= I \left( X_{2,C\cup E}^n; Y_{A^n}, X_{2,E}^n, Y_{C\cup E}^n \right)$$

(135)

where the justification for establishing (135) is identical to (123) and hence omitted. We finally bound the second term in (132).

$$I \left( W_2; Y_B^n | Y_A^n, X_{2,E}^n, Y_{C\cup E}^n \right)$$

(136)

$$\leq I \left( X_{2,B,uc}^n; Y_B^n | Y_A^n, X_{2,E}^n, Y_{C\cup E}^n \right)$$

(137)

$$\leq I \left( Y_A^n, X_{2,B,uc}^n, X_{2,E}^n, Y_{C\cup E}^n; Y_B^n \right)$$

(138)

$$= I \left( Y_A^n, X_{2,B,uc}^n, Y_{C\cup E}^n; Y_B^n \right) + I \left( Y_{C\cup E}^n, Y_B^n | Y_A^n, X_{2,B,uc}^n, X_{2,E}^n \right)$$

(139)

$$= I \left( Y_A^n, X_{2,B,uc}^n; Y_B^n \right)$$

(140)

where the justification for arriving at (140) is similar to (126) and hence omitted. Substituting (135) and (140) into (132) we establish (42).

**APPENDIX B**

**Proof of Lemma 2**

Assume the eavesdropper monitors $Y_A^n$ and $X_{1,E\setminus A}^n$ for $W_1$. Then for $R_{a_1}$, from Fano’s inequality, we have:

$$n(R_{a_1} - \delta_{a}) \leq I \left( W_1; Y_{A\cup B\cup C}^n \right) - I \left( W_1; X_{1,E\setminus A}^n \right)$$

(141)

$$\leq I \left( W_1; Y_{A\cup B\cup C}^n | Y_{1,E\setminus A}^n \right)$$

(142)

$$= I \left( W_1; Y_{B\cup C}^n | Y_{1,E\setminus A}^n \right)$$

(143)

$$= I \left( W_1; Y_{B\cup C}^n | Y_{1,E\setminus A}^n \right)$$

(144)

$$= I \left( W_1; Y_{B\cup C}^n | Y_{1,E\setminus A}^n \right)$$

(145)

$$= I \left( W_1; Y_{B\cup C}^n | Y_{1,E\setminus A}^n \right)$$

(146)

$$= I \left( W_1; Y_{B\cup C}^n | Y_{1,E\setminus A}^n \right)$$

(147)

where (135) follows from the fact that $X_{2,B,uc}^n$ is independent of $(W_1, Y_{A\cup B\cup C}^n, X_{1,E\setminus A}^n)$. While (146) follows from the fact that since the noise across the channels is independent the Markov condition

$$(Y_{1,E\setminus A}^n, Y_{B\cup C}^n) \leftrightarrow (X_{1,E\setminus A}^n, X_{2,B,uc}^n) \leftrightarrow (W_1, Y_{B\cup C}^n, Y_{A^n})$$

holds and furthermore we have defined $F = B\setminus E$. Since the channel noise is independent of the message, $W_1 \leftrightarrow X_{1,A\cup B}^n \leftrightarrow (Y_{F\setminus A\cup B}^n, X_{1,B\setminus F}^n, X_{2,B,uc}^n)$ holds. Hence

$$I \left( W_1; Y_{F\setminus A\cup B}^n | Y_{A^n}, X_{1,B\setminus F}^n, X_{2,B,uc}^n \right)$$

(148)

$$\leq I \left( X_{1,A\cup B}^n; Y_{F\setminus A\cup B}^n | Y_{A^n}, X_{1,B\setminus F}^n, X_{2,B,uc}^n \right)$$

(149)

$$= I \left( X_{1,A\cup B}^n; Y_{F\setminus A\cup B}^n | Y_{A^n}, X_{1,B\setminus F}^n, X_{2,B,uc}^n \right)$$

(150)

where the last step uses the fact that the second term in (150) involves conditioning on $(X_{1,F}^n, X_{2,F}^n)$ and hence is zero. This establishes (48).

For $R_{a_2}$, we assume the eavesdropper is monitoring $X_{2,E\setminus C}^n, Y_{2,C\setminus E}^n$ for $W_2$. Using Fano’s inequality and the secrecy constraint, we have:

$$n(R_{a_2} - \delta_{a}) \leq I \left( W_2; Y_{A\cup B\cup C}^n \right) - I \left( W_2; X_{2,E}^n \right)$$

(152)

$$\leq I \left( W_2; Y_{A\cup B\cup C}^n | X_{2,E}^n \right)$$

(153)

$$\leq I \left( W_2; Y_{A\cup B\cup C}^n | X_{1,E\setminus A}^n | X_{2,E}^n \right)$$

(154)

$$= I \left( W_2; Y_{A\cup B\cup C}^n | X_{2,E}^n, Y_{1,E\setminus A}^n | X_{1,E\setminus A}^n \right)$$

(155)

$$\leq I \left( X_{2,B,uc}^n; Y_{B\cup C}^n | X_{2,E}^n, Y_{1,E\setminus A}^n \right)$$

(156)

$$= I \left( X_{2,B,uc}^n; Y_{B\cup C}^n | X_{2,E}^n, Y_{1,E\setminus A}^n \right)$$

(157)

$$= I \left( X_{2,B,uc}^n; Y_{B\cup C}^n | X_{2,E}^n, Y_{1,E\setminus A}^n \right)$$

(158)

$$= I \left( X_{2,B,uc}^n; Y_{B\cup C}^n | X_{2,E}^n, Y_{1,E\setminus A}^n \right)$$

(159)

$$\leq I \left( X_{2,B,uc}^n; Y_{B\cup C}^n | X_{2,E}^n, Y_{1,E\setminus A}^n \right)$$

(160)
where \( B \cup C \) follows from the fact that \( (X^n_{1|E_1 G^n}; Y^n_{G^n}) \) are the transmitted signals from user 1 and independent of \((W_2, X^n_{2|E_2})\) and \( B \cap C \) follows from the fact that \( C \subseteq E_2 \subseteq B \cup C \) and \( G = B \cap E_2 \) and hence \( E_2 \cap G = B \cup C \) holds. Eq. (159) follows from the fact that since the noise on each channel is Markov, we have \( Y^n_{E_2} \leftrightarrow (X^n_{2|E_2}, X^n_{1|E_1 E_2|B^n}) \leftrightarrow (Y^n_{A|G^n}, X^n_{B \cup C^n}) \) and hence the second term in (155) is zero.

Hence we have proved Lemma 2.

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