Photon production by a quark-gluon plasma

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In this talk, I review the status of the calculation of the photon production rate in a hot quark-gluon plasma. Particular emphasis is given to the discussion of the various length scales of the problem.

1. Introduction - Model

Photon or dilepton production by a hot plasma is expected to reflect rather cleanly the state of the system. Indeed, in the collision of two heavy nuclei, the typical size of the system is much smaller than the mean free path of particles that feel only the electromagnetic interactions, which enables photons and leptons to escape freely. However, it has been realized recently that those rates are non-perturbative quantities. In other words, even for an idealized situation where the QCD coupling constant is small, the rate at leading order receives contributions from an infinite set of diagrams.

To keep the model as simple as possible, I assume in this talk that the temperature is much larger than all the quark masses ($T \gg m_q$), that the strong coupling constant at this temperature scale is extremely small ($g \ll 1$), and that the system is in both thermal and chemical equilibrium. Under these conditions, thermal field theory seems to be the tool of choice to calculate the photon production rate, which is obtained by calculating the imaginary part of the retarded photon polarization tensor [1]:

$$\frac{dN}{dt dV d\omega d^3q} \propto \Im \Pi^\mu_{\text{ret}}(\omega, q) .$$ (1)

In turn, the discontinuity of the retarded self-energy can be obtained directly by using the cutting-rules of the retarded/advanced formalism [2]. This formula automatically sums over all the possible processes and interferences thereof at a given order in $g$ (and takes care of the statistical weights).

2. Lowest order

At lowest non-trivial order in the loop expansion for $\Im \Pi^\mu_{\text{ret}}$, the relevant processes for the production of hard photons are the ones depicted on the following figure:

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*Work done in collaboration with P. Aurenche and H. Zaraket.*
Ignoring prefactors, their contribution to $\text{Im} \Pi_{\mu \nu}^{\text{ret}}$ behave like

$$\text{Im} \Pi_{\mu \nu}^{\text{ret}}(\omega, q) \propto e^2 g^2 T^2 \ln \left( \frac{\omega T}{m_{\text{th}}^2} \right),$$

(2)

where $m_{\text{th}} \sim gT$ is a thermal mass of order $gT$. Such a regulator is needed here, because the production rate of massless photons exhibit a logarithmic collinear singularity if evaluated with massless quarks and gluons. The resummation of hard thermal loops naturally provides this thermal mass.

3. Bremsstrahlung-like processes

One can note that a process like bremsstrahlung never appears in the lowest order diagram. It shows up only in the next order (i.e. at two-loop in the effective theory resumming the HTLs), together with a process that differs from bremsstrahlung by crossing symmetry:

The contribution of these processes to $\text{Im} \Pi_{\mu \nu}^{\text{ret}}$ can be written in the following form:

$$\text{Im} \Pi_{\mu \nu}^{\text{ret}}(\omega, q) \propto e^2 g^4 T^2 m_{\text{th}}^2 \left[ \frac{T}{\omega} \oplus T \omega \right],$$

(3)

where the first term dominates the low energy part of the spectrum, while the second term dominates its high energy part. Naively, one would expect those processes to come with four powers of $g$. However, this is altered by very strong collinear singularities, that bring the factor $T^2/m_{\text{th}}^2 \sim g^{-2}$ after regularization by a thermal mass. Therefore, it turns out that these contributions are as large as the 1-loop ones (or even larger for soft photons). The natural question one should ask after observing this collinear enhancement at two loops is whether it happens also in higher order diagrams, and whether the perturbative expansion can be kept under control.

4. Photon formation time and other scales

The collinear singularities in photon production diagrams are controlled by the virtuality of the quarks. It is particularly instructive to calculate the virtuality of an off-shell quark of momentum $R \equiv P + Q$ splitting into a photon of momentum $Q$ (and invariant mass $Q^2$) and an on-shell quark of momentum $P$:

$$R^2 - M^2 \approx \frac{\omega}{p} \left[ p_\perp^2 + M_{\text{eff}}^2 \right] \text{ with } M_{\text{eff}}^2 \equiv M^2 + \frac{Q^2}{\omega^2 p_0 r_0}.$$  

(4)

\footnote{This collinear factor decreases very fast if the photon has a non-zero invariant mass. When the invariant mass is maximal for a given energy, this factor is of order 1.}
This virtuality can be used to write down the expression of the more intuitive “photon formation time” $t_F$. Using the uncertainty principle, we have indeed:

$$t_F^{-1} \sim \Delta E \sim \frac{R^2 - M^2}{2r_0} \sim \frac{\omega}{p_0 r_0} \left[ p_\perp^2 + M_{\text{eff}}^2 \right] \sim \frac{\omega M_{\text{eff}}^2}{p_0 r_0}$$

We therefore arrive at an important observation: the collinear enhancement is due to very small quark virtualities, which corresponds to very long photon formation times. One can easily check that the photon formation time increases if the photon becomes soft, or if the invariant mass of the photon becomes very small.

There are a priori three other length scales that may also play a role in this problem:

- The mean free path of the quarks in the plasma $\lambda_{\text{mfp}} \sim (g^2 T \ln(1/g))^{-1}$.
- The range of the electric interactions $\lambda_{\text{el}} \sim (g T)^{-1}$.
- The range of magnetic interactions $\lambda_{\text{mag}} \sim (g^2 T)^{-1}$.

In the limit where $g \ll 1$, those scales satisfy $\lambda_{\text{el}} \ll \lambda_{\text{mfp}} \ll \lambda_{\text{mag}}$.

From there, one can check that the condition for having higher order diagrams at the same order in $g$ as the bremsstrahlung is $\lambda_{\text{mfp}} \leq t_F$, and that the relevant topologies correspond to multiple scatterings of the quark in the medium [6,8]. In other words, if producing the photon lasts more than the typical time between two scatterings of the quarks, then multiple scattering diagrams are also important. This effect has already been studied in a slightly different context, where a very fast fermion is going through some cold medium. In that case, it is known as the “Landau-Pomeranchuk-Migdal” effect. This effect reduces the rate of radiative energy loss in the low energy end of the spectrum. For photon production by a plasma, a preliminary (but incomplete) study indicates that the LPM effect reduces the photon rate both in the low end and in the high end of the spectrum [8].

5. Nature of the relevant diagrams

The last question we have to address is the nature of the multiple scattering topologies that are dominant, and the short answer is that it depends on the range of the interactions.

If the relevant interactions are short ranged, like the Debye-screened electric interactions for which $\lambda_{\text{el}} \ll \lambda_{\text{mfp}}$, then one can check that only independent scatterings can occur, as illustrated on the following figure:

In that case, only ladder topologies are important for the calculation of the photon self-energy (together with the appropriate modification of the quark propagators, in order to preserve gauge invariance). Indeed, configurations like the diagram on the right in the previous figure require an interaction range at least comparable to the mean free path.
The situation can however become much more complicated if the interactions are long ranged, like for static magnetic interactions, since for them $\lambda_{mfp} \ll \lambda_{mag}$. In that case, the above argument does not apply to restrict the set of relevant topologies: successive scatterings are not independent from one another and any topology can a priori contribute, unless some unexpected cancellations occur. Those cancellations occur when the process under study can only happen with hard scatterings, which means that the relevant mean free path is not $\sim (g^2 T \ln(1/g))^{-1}$ (this is the average distance between two soft scatterings) but rather $(g^4 T)^{-1}$ (which is the average distance between two hard scatterings). When the relevant mean free path if larger than all the interaction scales, then only ladder diagrams contribute.

Even if no extra cancellation occurs, a naive power counting seems to indicate that sensitivity to the magnetic mass is suppressed by $1/\ln(1/g)$ compared to the sensitivity on the mean free path. Therefore, even in the worst scenario, one could calculate the photon production rate at leading logarithmic accuracy by only summing ladder diagrams. If no cancellation occurs, then terms beyond the leading logarithm are truly non-perturbative.

6. Conclusions

The problem of collinear singularities in the photon rate by a plasma is closely related to the interplay between the various distance scales in the problem. Higher order topologies become important if the photon formation time is larger that the quark mean free path, and the nature and complexity of those topologies is controlled by the range of the interactions compared to the mean free path. Two questions can probably be addressed analytically: the production of massive hard photons is dominated by a single scattering if the invariant mass of the lepton pair is large enough and one can probably perform the resummation of ladder topologies, which can at least give the leading log rate.

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