Semileptonic decays of heavy to light mesons from an MIT bag model.*

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Abstract
Using the (modified) MIT bag model we calculated formfactors and decay widths for pseudoscalar - pseudoscalar and pseudoscalar - vector semileptonic decays of heavy to light mesons. We discuss the physical phenomena which are important in these processes and their influence on the measurable quantities. Our results are consistent with the available experimental data. A comparison with the results of other models is also given.

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1 Introduction

There is a wide variety of interesting questions concerning semileptonic decays of heavy mesons into light ones. There was a lot of papers dealing with this subject and trying to resolve problems from different points of view (for example [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]), because the first principle approach fails for the obvious reason that nonperturbative effects of strong interactions affect the picture of the weak decay. Even the Heavy Quark Symmetry (HQS), such a powerful method in many other situations, can only relate the hadronic matrix elements of $D$ decays to the corresponding ones of $B$ decays [16]. For this reason pictures consistently describing possible semileptonic processes (within models) are still of importance.

An example of an interesting problem is to try to understand the suppression of the ratios of the decay widths between pseudoscalar — vector and pseudoscalar — pseudoscalar transitions. For example:

$$R_{DK\pi} = \frac{\Gamma(D \to K^+e^\nu_e)}{\Gamma(D \to Ke^\nu_e)}$$

is about 0.5 although in the case of $R_{BD\pi}$, when a heavy meson $B$ decays into a $D$ meson, experiments give us a number about six time larger. This fact is clearly connected with some nonperturbative phenomena in strong interactions.

Another interesting question concerns the relation between such processes as $D^0 \to K^+e^+\nu_e$ and $D_s \to \phi e^+\nu_e$. From the experimental results [17], taking the center value and forgetting about the large errors, it seems that both processes are very close in probability in spite of the fact that they differ by the spectator antiquark. This is in agreement with the naive $SU(3)$ flavour symmetry expectations, but we know that this symmetry is broken. Another unsolved problem is connected with the $q^2$ - dependence of the formfactors. Some models assume the pole dependence of the formfactors (which we also do), others try to check this dependence (for example [5, 7, 11, 15]). It is generally believed that in $D$ decays the pole form dependence is quite reasonable, but in the case of $B$ decays the question is still open [14, 15].

Finally, the values of the decay widths also are useful for the determination of the CKM matrix element $|V_{ub}|$ as well as are probes of our understanding of semileptonic processes.

In this paper we propose to apply the simplest version of the corrected MIT bag model ([18, 19]). This model was successfully applied to a calculation of the Isgur-Wise function [20] and now proves also promising in the case of the heavy to light transitions. The results agree well with the available data and the model also gives some predictions for other not yet measured processes.

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\(^1\) see Table III
2 Formfactors

In the case of pseudoscalar - pseudoscalar transitions the matrix elements of the hadronic currents can be described in terms of two invariant formfactors:

\[
\langle P(p')|V_\mu|H(p)\rangle = \frac{1}{2\sqrt{mm'}} \left( f_+(q^2)(p + p')_\mu + f_-(q^2)(p - p')_\mu \right),
\]

where \( P \) stands for a light pseudoscalar (\( \pi, K, \eta \)) and \( H \) for a heavy one (\( D, B, D_s, B_s \)), \( m', m \) are the masses of the \( P \) and \( H \) mesons respectively, \( p' \) and \( p \) are their four-momenta. In the case of the pseudoscalar - vector transitions we have four independent formfactors [21]:

\[
\langle V(\epsilon, p')|V_\mu|H(p)\rangle = \frac{i}{2\sqrt{mm'}} \frac{g(q^2)}{m + m'} \epsilon^{\mu\rho\sigma\tau} \epsilon_\nu(p + p')^\rho(p - p')^\tau,
\]

\[
\langle V(\epsilon, p')|A_\mu|H(p)\rangle = \frac{1}{2\sqrt{mm'}} \left[ (m + m') f(q^2) \epsilon_\mu + \frac{a_+(q^2)}{m + m'} (\epsilon \cdot p)(p + p')_\mu + \frac{a_-(q^2)}{m + m'} (\epsilon \cdot p)(p - p')_\mu \right],
\]

where \( V(\epsilon, p') \) describes the vector particle (\( \rho, K^*, \phi \)) with momentum \( p' \) and polarization \( \epsilon_\mu \). For vanishing electron mass the formfactors \( a_-, f_- \) do not contribute to the decay probabilities. The formfactors defined in (3), (4) are related to the other commonly used set (e.g. [12]) by the simple formulae: \( g = V, f = A_1, a_+ = -A_2 \). The strategy of our calculation is standard. At first we calculate the values of the formfactors in the point of maximum four-momentum transfer \( q_{\text{max}}^2 \) and then we assume the pole dependence of the formfactor:

\[
f(q^2) = \frac{f(0)}{1 - q^2/m'^2},
\]

where \( m'^* \) are: 2.01 GeV for \( c \to q \) (\( q = u, d \)) transitions, 2.11 GeV for \( c \to s \). For \( B \) decays, in the case of the \( a_+ \) and \( f \) formfactors we used the pseudovector pole 5.71 GeV [14] and for the \( g \) formfactor the vector pole 5.32 GeV [17].

3 Formfactors at \( q_{\text{max}}^2 \)

In the corrected version of the MIT bag model [18], [19] the light mesons are treated conventionally but the heavy ones are built like the hydrogen atom. The heavy quark defines the center of mass of the system and occupies the center

\(^2\)We neglect the different masses of poles in different formfactors in the case of \( D \) decay, because the dependence of the mass \( m'^* \) on the decay channel is known to have little effect on the results [4].
of the spherical bag. Considering the limit of the infinite mass of the heavy quark there is no difference between the pseudoscalar and the vector heavy mesons, because the colour magnetic interactions between the components of the particles are proportional to the inverse of the heavy quark mass. Similarly, there is no difference in structure between corresponding mesons with $c$ and $b$ quarks. E.g. $B$ and $D$ mesons posses the same structure. These two statements follow directly from the HQS. The situation is quite different for light mesons. In this case there is an important difference between pseudoscalar and vector particles due to the colour magnetic interaction of their components. In the language of the bag model this is expressed by the difference in the bag radius, namely about $3.4 GeV^{-1}$ for pseudoscalar mesons and about $4.4 GeV^{-1}$ for the vector mesons \cite{19}. There is also the difference between particles with the $u$ or $d$ and the $s$ quarks, e.g. between $K$ and $\rho$ mesons. The last difference is a consequence of the broken $SU(3)$ flavour symmetry, because the mass of the $u,d$ is approximately 0 and the mass of the $s$ quark (from fit from \cite{19}) is 0.273 GeV. As we shall show in the next section, the colour magnetic interaction can explain the suppression of the ratio (1). The direct influence of the spectator antiquark explains the similarity between the decay widths of $D \to K^+ e^+ \nu_e$ and $D_s \to \phi e^+ \nu_e$.

The matrix elements of the hadronic current in the bag model can be found in \cite{22}:

$$\langle X(p')|J_\mu|H(p)\rangle = \langle X(p')|\int_{Bag} d^3r e^{-ik \cdot r}\bar{\psi}_q(r,t)\gamma_\mu(1 - \gamma_5)\psi_Q(r,t)|H(p)|\rangle|_{t=0} \cdot \langle \bar{\psi}_{q'}|\bar{\psi}_{q'}\rangle|_{t=0},$$

where $\psi_q$ and $\psi_Q$ are spinors describing respectively the light quark and the heavy quark taking part in the weak decay. The bracket at the end of the formula is the overlap function of the spectator antiquark in the parent ($\bar{q}$) and the daughter ($\bar{q'}$) particle. This overlap corresponds to the Isgur-Wise function in the case of the heavy to heavy meson transitions. $k = p - p'$ is the momentum transfer in the process. The integral is over the whole space filled by the (anti)quark fields at the moment of the decay. In the case of the bag model this is the intersection of the bags of the parent and daughter particles. The moment of the decay is arbitrary and is here chosen to be zero. The integral must be performed in some well defined reference frame and in principle should give us the full dependence of the formfactors in terms of the momentum transfer $k$. Unfortunately, the prescription is not covariant because we do not know the boost operators acting on the quark fields. Such operators are necessary, because the spinors $\psi$ are known only in the reference frame in which the bag is at rest:

$$\psi^0(r,t) = \begin{bmatrix} iF(r)\chi \\ G(r)\sigma \cdot r\chi \end{bmatrix}.$$
Here the functions \( F \) and \( G \) for the light quark are proportional to the spherical Bessel functions \( j_0 \) and \( j_1 \); \( \chi \) is a Pauli spinor. Generally in the case of the decay it is not possible to find a frame where both bags are at rest. For this reason only the calculation for small values of \( k \) is reliable. Expanding the right hand side of equation (6) in powers of \( |k| \) and comparing with (2), (3), (4) one can find the formfactors in the point of maximum four - momentum transfer \( q_{\text{max}}^2 \), where \( k = 0 \). The details of these calculations can be found in papers [21, 23]. As the most useful frame of reference we consider the modified Breit frame [24]. As results we have:

\[
f_+(q_{\text{max}}^2) = \frac{m + m'}{2\sqrt{mm'}}[G_0 - \delta G_2],
\]

\[
g(q_{\text{max}}^2) = \frac{m + m'}{2\sqrt{mm'}}G_1,
\]

\[
f(q_{\text{max}}^2) = \frac{2\sqrt{mm'}}{m + m'}H_1,
\]

\[
a_+(q_{\text{max}}^2) = \frac{1}{1 - \delta^2}\sqrt{\frac{m'}{m}}[-H_0 - (1 + \frac{1}{2}\delta)H_1 - H_2],
\]

where \( \delta = (m - m')/(m + m') \) and \( G_0, G_2, G_1, H_0, H_1, H_2 \) are the Sachs formfactors defined in [21]. The first two formfactors describe pseudoscalar - pseudoscalar transitions the next four correspond to pseudoscalar - vector transitions. These formfactors are expressed by the bag integrals:

\[
G_0 = \hat{N}\hat{N}',
\]

\[
G_2 = -\beta B r \hat{d}\hat{N}',
\]

\[
G_1 = (\hat{g}_A + \beta B r \hat{\mu})\hat{N}',
\]

\[
H_0 = -\beta B r \hat{d}\hat{N}',
\]

\[
H_1 = \hat{g}_A\hat{N}',
\]

\[
H_2 = (-\frac{1}{2}\hat{g}_A - \beta B r \hat{\mu} - \frac{2}{15}\hat{g}^2 \hat{B})\hat{N}',
\]

where:

\[
\hat{N} = \int d^3r[F_q^* F_Q + G_q^* G_Q],
\]

\[
\hat{g}_A = \int d^3r[F_q^* F_Q - \frac{1}{3}G_q^* G_Q],
\]

\[
\hat{\mu} = -\frac{1}{3}(m + m') \int d^3r[F_q^* G_Q + G_q^* F_Q],
\]

\[
\hat{d} = \frac{1}{3}(m + m') \int d^3r[F_q^* G_Q - G_q^* F_Q],
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a_+(q_{\text{max}}^2) = \frac{1}{1 - \delta^2}\sqrt{\frac{m'}{m}}[-H_0 - (1 + \frac{1}{2}\delta)H_1 - H_2],
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\]

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H_1 = \hat{g}_A\hat{N}',
\]

\[
H_2 = (-\frac{1}{2}\hat{g}_A - \beta B r \hat{\mu} - \frac{2}{15}\hat{g}^2 \hat{B})\hat{N}',
\]

where:

\[
\hat{N} = \int d^3r[F_q^* F_Q + G_q^* G_Q],
\]

\[
\hat{g}_A = \int d^3r[F_q^* F_Q - \frac{1}{3}G_q^* G_Q],
\]

\[
\hat{\mu} = -\frac{1}{3}(m + m') \int d^3r[F_q^* G_Q + G_q^* F_Q],
\]

\[
\hat{d} = \frac{1}{3}(m + m') \int d^3r[F_q^* G_Q - G_q^* F_Q],
\]
\[ \hat{B} = (m + m')^2 \int d^3 r r^2 G^*_q G_Q, \]

\[ \hat{N}' = \int d^3 r [F^*_q F_q + G^*_q G_q]. \]

The last integral describes the overlap of the spectator antiquarks in the process. \( F_q \) and \( G_q \) are the upper and the lower component of spinor \( \psi \) describing the light quark produced in the process. \( F_Q, G_Q \) describe the heavy quark. \( \beta \) is the so called retardation factor and comes from the relativistic corrections to the transformation of time:

\[ \beta_{Br} = 1 - \frac{E_q + E_Q}{m + m'}, \]

where \( E_q \) and \( E_Q \) are the quark eigenenergies in the rest frames of the bags. Unfortunately it is impossible to calculate the integrals, because the wavefunction \( \psi_Q \) of the heavy quark is not known within the bag model. Nevertheless, we can still obtain some interesting results. First of all let us take the infinite mass limit in the bag integrals. Using the fact that the lower component of the spinor \( \psi_Q \) must vanish and parameter \( \beta_{Br} \) is very small (because \( E_Q \approx m_Q \approx m >> m' \)) one finds that the Sachs formfactors \( G_2, H_0 \) are negligible and \( G_1 = H_1 = -2H_2 \). In terms of the invariant formfactors this means that:

\[ f_+(q^2_{\text{max}}) = \frac{m + m'}{2\sqrt{mm'}} \hat{N} \]

for pseudoscalar - pseudoscalar transitions and

\[ -a_+(q^2_{\text{max}}) = g(q^2_{\text{max}}) = \frac{m + m'}{2\sqrt{mm'}} \hat{N}, \]

\[ f(q^2_{\text{max}}) = \frac{2\sqrt{mm'}}{m + m'} \hat{N}, \]

for pseudoscalar - vector transitions. Here \( \hat{N} \) is a product of two overlaps: first \( \hat{N} \) between the heavy quark and the produced light quark and second \( \hat{N}' \) between the spectators of the process:

\[ \hat{N} = \hat{N}' = \langle \psi_q | \psi_Q \rangle \cdot \langle \psi_{\bar{q}} | \psi_{\bar{q}} \rangle. \]

Thus the calculation of all the formfactors is reduced to the calculation of the quantity \( \hat{N} \). First of all it is straightforward to see that in the case of the heavy to heavy meson transition one can reproduce the HQS prediction connected with the Isgur-Wise function. In this case both overlaps are equal to unity. This is true because for the decay in the limit \( k = 0 \) a heavy quark at rest is replaced by another heavy quark at rest so that \( \hat{N} = 1 \). Also \( \hat{N}' = 1 \), because the wavefunction of the light antiquark in the meson does not depend on the heavy quark flavour. In the case of the heavy to light transition, the overlap of the spectator antiquark can be calculated easily, but the second overlap requires more attention. For calculating this overlap let us assume that the decay process proceeds in two steps:
rapid transition between the heavy and the light quark

transition of the light quark into its final state

The first step is very difficult to calculate. We know neither the heavy quark wavefunction, nor the light quark, besides the fact that both wavefunctions are similar. We propose to describe this transition by the unknown dimensionless function $\alpha(\mu a)$ of the product of some parameter $\mu$ (dimension of energy) and the radius $a$ of the region previously occupied by the heavy quark. The radius $a$ should scales like $1/m_Q$. The parameter $\mu$ can be constructed only from quantities describing the light quark just after the transition such as momentum, energy or mass. The second step gives us more information. Although we do not know the wavefunction of the heavy quark (or of the light quark just after transition) we can use the fact that it occupies the center of the bag. The density of the heavy quark behaves like a delta function so its wavefunction is the "square root" of delta. Let us use for the estimation of the overlap (18) the Gaussian representation of the delta function. Thus the heavy quark wavefunction is represented by the formula:

$$F_Q(r) = Q_a(x, y, z) = \frac{1}{(a\sqrt{\pi})^{3/2}} e^{-r^2/2a^2}. \quad (29)$$

The overlap of $Q_a$ and an arbitrary wavefunction $h(r)$ is:

$$\int d^3r Q_a(r) h(r) \approx 2^{3/2} \pi^{3/4} a^{3/2} h(0). \quad (30)$$

Parameter $a$ has an obvious interpretation as the radius of the region occupied by the heavy quark. In the limit $a \to 0$ ($a$ scales like $1/m_Q$) $Q_a$ is proportional to a delta function, but integral (30) vanishes. It may seem that there is no interesting information here, but if we want to calculate the overlap (18), we have to take into account both steps of the decay. After incorporating (30) in (18) we find:

$$\hat{N} = \alpha(\mu a) 2^{3/2} \pi^{3/4} a^{3/2} F_q^*(0), \quad (31)$$

where $F_q$ is the wavefunction of the produced light quark in its final state in the light meson. Taking the limit $a \to 0$ and simultaneously requiring that $\hat{N}$ should be different from zero, we have to assume that:

$$\alpha(\mu a) = \frac{k}{(\mu a)^{3/2}}, \quad (32)$$

where $k$ is some constant. Finally:

$$N' = 2^{3/2} \pi^{3/4} k \frac{F_q^*(0)}{\mu^{3/2}} (\bar{\psi}_q \psi_\bar{q}), \quad (33)$$

Now one can calculate the quantities, for which the unknown factor in (33) cancels. Moreover, making simple assumptions about the parameter $\mu$, one can find the values of the decay widths.
4 Numerical results

The bag model has some parameters that were adopted from paper [19] in which they had been fitted to the spectroscopy of the light and heavy particles. The wavefunctions of the light quarks are normalized to unity inside the bag. For the light mesons we used radii calculated in [19]; in the case of the heavy mesons we take the infinite mass limit and we find $R_{D(B)} = 3.9 \, GeV$ and $R_{D_s(B_s)} = 4.0 \, GeV$. For the decay width calculation we used the pole form ansatz for the form factor dependence on $q^2$ (see equation (5)).

Our knowledge of the formfactors is up to the unknown factor $k\mu^{-3/2}$, but we have still got the possibility to calculate quantities, for which this factor cancels. These quantities are: the polarization factors $\Gamma_L/\Gamma_T$ and the $R_{HV}^{LP}$ parameters (cf. (1)). The second parameter is calculable only for transitions for which parameter $\mu$ can cancel. This parameter depends not only on the created light quark but also can be indirectly influenced by the spectator of the process as we shall show in a moment. Using formulae (25) - (27), (33) and the assumption of pole dependence (5), without any fitting, we obtain the results written in the Tables I, II.

The results from Tables I, II agree well with the available data and are

Table I: Parameters $R_{HV}^{LP}$ (def. (1)).

| Decay | our work | QCD sum rules | lattice | chiral model [10] | quark model [1, 2] | Exp. [25] |
|-------|---------|---------------|---------|------------------|------------------|---------|
| $D \rightarrow K^*$ | 1.03 | $0.86 \pm 0.06$ [26] | $1.4 \pm 0.3$ [12] | 1.31 | 0.89 | 1.15 $\pm 0.17$ |
| $D \rightarrow \rho$ | 0.92 | $1.31 \pm 0.11$ [11] | $1.86 \pm 0.56$ [7] | 1.4 | 0.91 | - |
| $D_s \rightarrow K^*$ | 0.98 | - | - | 1.25 | - | - |
| $D_s \rightarrow \phi$ | 1.07 | - | $1.49 \pm 0.19$ [7] | - | - | - |
| $B \rightarrow \rho$ | 0.78 | $0.06 \pm 0.02$ [11] | $0.125 \pm 0.08$ [15] | 0.36 | 1.34 | - |
| $B_s \rightarrow K^*$ | 0.43 | - | - | 0.28 | - | - |

Table II: Polarization parameters $\Gamma_L/\Gamma_T$.

| Decay | our work | QCD sum rules | lattice | chiral model [10] | quark Model [1, 2] | Exp. [25] |
|-------|---------|---------------|---------|------------------|------------------|---------|
| $D \rightarrow K^*$ | 1.03 | $0.86 \pm 0.06$ [26] | $1.4 \pm 0.3$ [12] | 1.31 | 0.89 | 1.15 $\pm 0.17$ |
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| $D_s \rightarrow \phi$ | 1.07 | - | $1.49 \pm 0.19$ [7] | - | - | - |
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| $B_s \rightarrow K^*$ | 0.43 | - | - | 0.28 | - | - |
comparable with the calculations from other models. We predict the suppression of the ratios $R_{D^0}^{D\rho}$ and $R_{D^0}^{D_s\pi}$, but not of $R_{B_s}^{B\rho}$. This tendency is qualitatively confirmed by QCD sum rules calculations \cite{11} and partially agrees with chiral model results. We also predict a large longitudinal polarization in the decays $B \to \rho$ and $B_s \to K^*$, although not as large as according to QCD sum rules and chiral model calculations. The lattice results have large errors and it is difficult to draw from them definite conclusions.

For the decay width calculations we need the knowledge of the $\mu$ parameter. The only thing we know is that it should be described by some observables connected with the light quark just after the transition. We choose this parameter with two assumption: to get as good agreement with the data as we can and, if possible, to remove the constant $k$ (it means to put $k = 1$). Fortunately we find such a quantity: $\mu = \bar{p} + m_q$, where $\bar{p}$ is the average momentum of the produced light quark just after the transition and $m_q$ is its mass. Within the bag model the momentum $\bar{p}$ is calculable as follows. In the point of zero momentum transfer it has to be equal to the momentum of the heavy quark before decay. On the other hand in the reference frame chosen, the momentum of the heavy quark is equal to the momentum of the spectator antiquark $\sqrt{<\vec{p}^2>}$.

Table III: Decay widths in units $|V_{cq}|^2 10^{11}s^{-1}$ (upper entries) and the corresponding branching ratios (lower entries) calculated assuming $|V_{cd}| = 0.221$, $|V_{cs}| = 0.974$ and life times from \cite{17}.

| Decay           | our work | QCD sum rules | lattice | chiral model [10] | quark model [1, 2] | Exp. Br. [17] |
|-----------------|----------|---------------|---------|------------------|--------------------|---------------|
| $D^0 \to K^-$   | 0.77     | 2.7 ± 0.6%    | 0.54 ± 0.3 ± 0.14 | 0.69*            | 0.83              | (3.31 ± 0.29)% |
|                 | 3.24%    |               |         |                  |                    |               |
| $D^\circ \to K^*$ | 0.43     | 0.39 ± 0.15   | 0.64 ± 0.28 | 0.37             | 0.95              | (1.7 ± 0.6)%  |
|                 | 1.71%    |               |         |                  |                    |               |
| $D^\circ \to \pi^-$ | 1.9      | 0.8 ± 0.17    | 1.01 ± 0.61 ± 0.2 | 1.8              | 1.41              | (0.39 ± 0.12)% |
|                 | 0.39%    |               |         |                  |                    |               |
| $D^\circ \to \rho^-$ | 1.14     | 0.24 ± 0.07   | 1.2 ± 0.61 ± 0.2 | 0.92             | 1.38              | -             |
|                 | 0.23%    |               |         |                  |                    |               |
| $D_s^+ \to \Phi^0$ | 0.38     |               |         |                  |                    |               |
|                 | 1.6%     |               |         |                  |                    | (1.6 ± 0.7)%   |
| $D_s^+ \to K^0$  | 1.26     |               |         |                  |                    |               |
|                 | 0.28%    |               |         |                  |                    |               |
| $D_s^+ \to K^{*0}$ | 0.9      |               |         |                  |                    |               |
|                 | 0.2%     |               |         |                  |                    |               |

\textsuperscript{a} We used $|V_{cs}| = 0.975$ and $|V_{cd}| = 0.222$ for undoing $\Gamma$ dependence of CKM matrix elements. \textsuperscript{b} This is $D^- \to K^0$ channel. \textsuperscript{c} This is $D^+ \to K^{*0}$. \textsuperscript{d} To get decay width we used $\tau_{D^+} = 1.07 ps$.
in the heavy meson. In this way, besides the overlap of the spectator antiquark \( \bar{N}' \) \(^{22}\), we have also the additional influence in \(^{23}\) of the spectator through the \( \mu \) parameter. This phenomenon causes the decay \( D_s \to \phi \) to be similar to the decay \( D \to K^* \). With the same \( \mu \) for both processes, \( D_s \to \phi \) decays would be about 1.7 times more probable than the \( D \to K^* \) decays. This is also the case for other "twin" decays as: \( D^0 \to \rho^- \) vs. \( D_s^+ \to K^{*0} \) or \( B^0 \to \pi^+ \) vs. \( B_s^0 \to K^+ \). The decay widths are collected in Tables III, IV.

The agreement with the available data is quite impressive. In the case of \( D \) decays our decay widths are almost the same as, those in the chiral model and agree with the quark model in size (except for the \( D \to K^* \) decay). The QCD sum rules give smaller widths for \( D \to \pi \) and \( D \to \rho \) decays. On the other hand in the case of \( B \) decays our results are rather closer to the QCD sum rules predictions (for \( B \to \pi \) almost within errors \(^{11}\)) and do not agree with the chiral model calculations which give much larger widths. One can also compare

Table IV: Decay widths in units \( |V_{ub}|10^{13}s^{-1} \) and corresponding branching ratio calculated assuming \( \tau_B = 1.49ps \) \(^{27}\) and \( |V_{ub}| = 0.0043 \) \(^{28}\).

| Decay        | our work | QCD sum rules | lattice \(^{12}\) | chiral model \(^{10}\) | quark model \(^{10}\) | Exp.  |
|--------------|----------|---------------|-------------------|-------------------|--------------------|-------|
| \( B^0 \to \pi^+ \) | 0.74     | 0.51 ± 0.11   | 0.9 ± 0.6         | 5.43c             | 0.74               | < 0.014% \( @90\% \) CL \(^{29}\) |
| \( B^0 \to \rho^+ \) | 2.18     | 1.2 ± 0.4     | 1.4 ± 1.2         | 3.4               | 2.6                | (0.103 ± 0.036 ± 0.025)% \(^{30}\) |
| \( B_s^0 \to K^+ \) | 0.89     | -             | -                 | 5.43              | -                  | -     |
| \( B_s^0 \to K^{*+} \) | 1.43     | 0.04%         | -                 | 3.4               | 0.087%             | -     |

\(^a\) This is upper limit on \( Br(B \to \pi l \nu) \). \(^b\) This is \( Br(B \to \rho^0 l^- \bar{\nu}_l) \). \(^c\) This is \( B^0 \to \pi^- \) channel.

Table V: Formfactors \( f_{+}(0) \) for pseudoscalar - pseudoscalar transitions at \( q^2 = 0 \).

| \( f_{+}(0) \) | our work | QCD sum rules | lattice | chiral model \(^{10}\) | quark model \(^{10}\) | Exp.  |
|--------------|----------|---------------|---------|-------------------|--------------------|-------|
| \( f_{+}^{D,K} \) | 0.71     | \( 0.6^{+0.15}_{-0.1} \) \(^{26}\) | \( 0.6 \pm 0.15 \pm 0.07 \) \(^{12}\) | 0.67              | 0.76               | 0.76 ± 0.02 \(^{13}\) \(^{31}\) |
| \( f_{+}^{B,K} \) | 0.8      | 0.5 ± 0.1     | 0.58 ± 0.09 \(^{21}\) | 0.79              | 0.69               | 0.8^{+0.24}_{-0.14} \(^{17}\) |
| \( f_{+}^{D,K} \) | 0.74     | -             | \( 0.84 \pm 0.08 \pm 0.18 \) \(^{12}\) | 0.78              | 0.64               | -     |
| \( f_{+}^{B,K} \) | 0.33     | 0.26 ± 0.02   | 0.3 ± 0.14 ± 0.05 \(^{12}\) | 0.89              | 0.33               | -     |
| \( f_{+}^{B,K} \) | 0.36     | -             | -                   | 0.89              | -                  | -     |
the values of the formfactors at $q^2 = 0$. These numbers are collected in Tables V, VI.

In the case of $f_+$ formfactors we agree with the QCD sum rules calculations, except for the $D \to \pi$ transition where our result is larger. We also have good agreement with the quark model for all calculated decays and with the chiral model predictions for $D$ decays. For $B$ decays the chiral model gives larger values than any other model considered here. In the case of the formfactors for the pseudoscalar - vector transitions, the situation is more complicated. Our results for the $f$ formfactors roughly agree with other models and only in $B$ decays the QCD sum rules calculations give a little bit larger numbers. In the

| form factors | our work | QCD sum rules | lattice | chiral model [10] | quark model [1, 2] | Exp. [10, 32, 33] |
|-------------|---------|---------------|---------|------------------|------------------|------------------|
| $D \to K^*$ |         |               |         |                  |                  |                  |
| $f(0)$      | 0.55    | 0.5 ± 0.15 [26] | 0.64 ± 0.16 [12] | 0.48 | 0.88 | 0.48 ± 0.05 |
| $-a_+ (0)$  | 0.63    | 0.6 ± 0.15 | 0.4 ± 0.28 ± 0.04 | 0.27 | 1.15 | 0.27 ± 0.11 |
| $g(0)$      | 0.63    | 1.1 ± 0.25 | 0.86 ± 0.24 | 0.95 | 1.23 | 0.95 ± 0.2 |
| $D \to \rho$ |         |               |         |                  |                  |                  |
| $f(0)$      | 0.69    | 0.5 ± 0.2 [11] | 0.45 ± 0.04 [7] | 0.55 | 0.78 | - |
| $-a_+ (0)$  | 0.83    | 0.4 ± 0.1 | 0.02 ± 0.26 | 0.28 | 0.92 | - |
| $g(0)$      | 0.83    | 1.0 ± 0.2 | 0.78 ± 0.12 | 1.01 | 1.23 | - |
| $D_s \to K^*$ |         |               |         |                  |                  |                  |
| $f(0)$      | 0.61    | -             | -       | 0.52 | 0.717 | - |
| $-a_+ (0)$  | 0.71    | -             | -       | 0.3 | 0.853 | - |
| $g(0)$      | 0.71    | -             | -       | 1.08 | 1.250 | - |
| $D_s \to \phi$ |         |               |         |                  |                  |                  |
| $f(0)$      | 0.53    | -             | 0.52 ± 0.03 [7] | - | 0.820 | - |
| $-a_+ (0)$  | 0.59    | -             | 0.17 ± 0.17 | - | 1.076 | - |
| $g(0)$      | 0.59    | -             | 0.86 ± 0.1 | - | 1.319 | - |
| $B \to \rho$ |         |               |         |                  |                  |                  |
| $f(0)$      | 0.27    | 0.5 ± 0.1 [11] | 0.38 ± 0.04 [15] | 0.22 ± 0.05 [12] | 0.21 | 0.28 | - |
| $-a_+ (0)$  | 0.61    | 0.4 ± 0.2 | 0.49 ± 0.21 ± 0.05 | 0.2 | 0.28 | - |
| $g(0)$      | 0.46    | 0.6 ± 0.2 | 0.37 ± 0.11 | 1.04 | 0.33 | - |
| $B_s \to K^*$ |         |               |         |                  |                  |                  |
| $f(0)$      | 0.25    | -             | -       | 0.2 | 0.328 | - |
| $-a_+ (0)$  | 0.5     | -             | -       | 0.21 | 0.331 | - |
| $g(0)$      | 0.38    | -             | -       | 1.08 | 0.369 | - |
case of the \( a_+ \) and \( g \) formfactors we find that in the infinite mass limit they are equal at the point of maximum four-momentum transfer. From this fact and from the pole dominance assumption it follows that they are also similar at zero four-momentum transfer. This is not the case for other models (except of the quark model). If the experimental results and other model calculations do not change, this would mean that finite heavy quark mass corrections are quite large for the \( a_+ \) and \( g \) formfactors. Moreover, comparing the numbers one can risk the conclusion that mass corrections decrease the values of \( -a_+ \) and increase the values of the \( g \). For \( B \) decays corrections should be smaller. From comparison with the available data it follows that the corrections to the decay widths are little affected by these changes. One may hope that this may be also true for other not yet measured processes.

We also checked our predictions for another choice of the parameter \( \mu \) putting it equal to the energy of the light quark \( \mu = \sqrt{\bar{p}^2 + m_q^2} \). We find \( k = 0.66 \) from fitting to the \( D \to K \) decay. The only difference in the results for this new choice of \( \mu \), as compared to the previous one, was the suppression of \( Q \to q \) transitions (\( Q = b, c \) and \( q = u, d \)) relative to \( Q \to s \) transitions. The formfactors describing \( Q \to q \) transitions are smaller by a factor \( k = 0.66 \). It means that our prediction for \( D \to K^* \) and \( D_s \to \phi \) do not change. For the \( D \to \pi \) decay we found \( f_+(0) = 0.53 \) and for the \( B \to \pi \) decay \( f_+(0) = 0.22 \). These predictions are quite similar to the QCD sum rules results \([11], [15]\), however, our first choice agrees better with the \( D \to \pi \) decay data.

In conclusion, we performed an analysis of the semileptonic decays of heavy to light mesons in the framework of the corrected MIT bag model. We explain the suppression of pseudoscalar–vector transition \( D \to K^* \) relative to the pseudoscalar–pseudoscalar transition \( D \to K \) as an effect of the colour magnetic interaction between the produced light quark and the antiquark spectator. We can also identify the source of the restored \( SU(3) \) flavour symmetry in the case of the ”twin” decays e.g. \( D_s \to \phi \) and \( D \to K^* \) as an additional influence of the spectator antiquark in the decay ”two step” process and of momentum conservation. Additionally, we calculated the decay widths of several semileptonic processes for heavy–light transitions. Agreement with the experimental data, whenever available is good.

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