Polarization effects in electroweak vector boson productions

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Abstract

The electroweak production processes of vector bosons, $e^+e^- \to Z\gamma$, $\gamma e \to Ze$, and $\gamma e \to W\nu$ are considered simultaneously and in the Standard model the covariant polarization density matrices of vector bosons for these processes are obtained explicitly from one of the processes, $\gamma e \to Ze$. The effect of the photon polarization as well as the vector meson polarization is considered and some special cases are discussed.

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I. INTRODUCTION

The standard model (SM) provides an excellent description of physics below the electroweak scale but we also believe that new physics beyond the SM must exist at higher energy. Three processes, \( e^+e^- \rightarrow Z\gamma \), \( \gamma e \rightarrow Ze \), and \( \gamma e \rightarrow W\nu \) are the simplest ones to check the SM. The \( e^+e^- \rightarrow Z\gamma \) process can be investigated experimentally at relatively low energy scale. This process was also considered to search for the number of neutrino generations [1,2] since it is shown [2] that the major contribution to the process \( e^+e^- \rightarrow \nu\bar{\nu}\gamma \) at the low energy is the \( Z^0 \) exchange, i.e., near the \( Z^0 \) peak the \( e^+e^- \rightarrow Z^0\gamma \) and the subsequent decay of \( Z^0 \) into \( \nu\bar{\nu} \). The processes \( \gamma e \rightarrow Ze \) and \( \gamma e \rightarrow W\nu \) might be possible experimentally by using the backscattered laser beams at the future linear \( e^+e^- \) colliders.

These processes have been discussed in order to investigate the effect of possible deviations from the SM through the anomalous terms of vector meson couplings. The anomalous \( \gamma\gamma Z \) and \( \gamma ZZ \) vertices are considered in the process \( e^+e^- \rightarrow \gamma Z \) [3,4] and also in the process \( \gamma e \rightarrow Ze \) [5–7]. In the process \( \gamma e \rightarrow W\nu \), the anomalous \( \gamma WW \) terms are considered [8–11].

While the \( e^+e^- \rightarrow Z\gamma \) process can be described in the c.m. frame, the \( \gamma e \rightarrow Ze \) and \( \gamma e \rightarrow W\nu \) processes are not necessarily in the c.m. frame since the energy of the Compton backscattered photons is less than that of the initial electron beam. Usually the helicity formalism in the c.m. frame has been used.

The purpose of this paper is to consider the processes \( e^+e^- \rightarrow Z\gamma \), \( \gamma e \rightarrow Ze \), and \( \gamma e \rightarrow W\nu \) simultaneously in an arbitrary frame. The covariant density matrix formalism [12] is used to obtain the polarization of vector mesons. In the SM, it is shown that these three processes are interrelated and, therefore, starting from the process \( \gamma e \rightarrow Ze \), the result of other two processes can be obtained, even though the \( \gamma e \rightarrow Ze \) and \( \gamma e \rightarrow W\nu \) processes are described by different Feynman diagrams, i.e., in the \( \gamma e \rightarrow Ze \) there are no boson coupling terms like \( WW\gamma \) term in the \( \gamma e \rightarrow W\nu \). The process \( e^+e^- \rightarrow Z\gamma \) considered in the c.m. frame previously by us [13] can be reproduced from a general and much simpler form obtained from the \( \gamma e \rightarrow Ze \) process.
The general consideration of these processes in the SM can be useful in considering the new physics beyond the SM and it is also useful to discuss the contribution of the processes to the other final states like the $\gamma e \rightarrow \mu \nu \bar{\nu}$ even in the SM. If the anomalous vector coupling terms in these processes are considered beyond the SM, the situation is changed and the effect of additional terms can be checked through the polarization of vector bosons. The method discussed here can be used for the investigation of possible deviations from the SM.
II. TRANSITION AMPLITUDE OF VECTOR BOSON PRODUCTION

In the SM, the transition amplitude of the $\gamma e \rightarrow Ze$ process is described in the tree level by the transition amplitude

$$M_{\gamma e \rightarrow Ze} = \frac{e}{2} i g_Z \bar{u}(p_2, s_2) \left[ \frac{\not{k}_2 (\not{k}_1 + 2 \not{p}_1 \cdot \varepsilon_\gamma)}{k_1 \cdot p_1} - \frac{(2 \not{p}_2 \cdot \varepsilon_\gamma - \not{\gamma} \cdot \not{k}_1)}{k_1 \cdot p_2} \not{\gamma} \not{Z} \right] \times [\epsilon_L (1 - \gamma_5) + \epsilon_R (1 + \gamma_5)] u(p_1, s_1). \quad (1)$$

Here $k_1, p_1, k_2,$ and $p_2$ are momenta of incoming photon, incoming electron, outgoing $Z^0$, and outgoing electron, respectively. Also $\varepsilon_\gamma^* \not{Z}$ and $\varepsilon_Z^* \not{Z}$ are the wave vectors of photon and $Z^0$, respectively, and $g_Z, \epsilon_L,$ and $\epsilon_R$ are defined as

$$g_Z = \left( \sqrt{2} G_F m_Z^2 \right)^{\frac{1}{2}}, \quad \epsilon_L = -\frac{1}{2} + \sin^2 \theta_W, \quad \epsilon_R = \sin^2 \theta_W. \quad (2)$$

The process $\gamma e \rightarrow W\nu$ is described by the transition amplitude

$$M_{\gamma e \rightarrow W\nu} = \frac{e}{2} i g_W \bar{u}(p_2, s_2) \left\{ \frac{\not{\gamma} \not{W}}{k_1 \cdot p_1} \frac{\not{k}_2 (\not{k}_1 + 2 \not{p}_1 \cdot \varepsilon_\gamma)}{k_1 \cdot p_1} \right\} (1 - \gamma_5) u(p_1, s_1), \quad (3)$$

where

$$g_W = \left( 2^{-\frac{1}{2}} G_F m_W^2 \right)^{\frac{1}{2}} = \frac{g_Z \cos \theta_W}{\sqrt{2}}. \quad (4)$$

The transition amplitudes for the $\gamma e \rightarrow Ze$ and $\gamma e \rightarrow W\nu$ look quite different due to their Feynman diagrams as shown as Figs. 1 and 2. But the term in the bracket of Eq. (3) can be converted into the form in the bracket of Eq. (1) by using the relation,

$$\left\{ \frac{\not{k}_2 (\not{k}_1 + 2 \not{p}_1 \cdot \varepsilon_\gamma)}{k_1 \cdot p_1} \right\} + \frac{2}{k_1 \cdot k_2} \left\{ (\varepsilon_\gamma \cdot \varepsilon_W^*) \not{k}_1 - (\varepsilon_W^* \cdot k_1) \not{\gamma} - (\varepsilon_\gamma \cdot k_2) \not{\gamma} \not{W} \right\} \not{\gamma} \not{W}$$

$$= -\frac{k_1 \cdot p_2}{k_1 \cdot k_2} \left\{ \frac{\not{k}_2 (\not{k}_1 + 2 \not{p}_1 \cdot \varepsilon_\gamma)}{k_1 \cdot p_1} \right\} - \frac{(2 \not{p}_2 \cdot \varepsilon_\gamma - \not{\gamma} \cdot \not{k}_1)}{k_1 \cdot p_2} \not{\gamma} \not{W} \not{W}. \quad (5)$$
Therefore, various results obtained in the $\gamma e \rightarrow Ze$ process can be used in the $\gamma e \rightarrow W\nu_e$ process after $\epsilon_R, m_Z$, and $\epsilon_L g_Z$ are replaced by 0, $m_W$, and $-(k_1 \cdot p_2 / k_1 \cdot k_2) g_W$, respectively.

Since the transition amplitude of the $e^+ e^- \rightarrow Z\gamma$ process is described by

$$M_{e^+ e^- \rightarrow Z\gamma} = \frac{e}{2} i g_Z \bar{v}(p_2, s_2) \left[ \frac{g_Z^* (k_1^* \gamma^* - 2p_1 \cdot \varepsilon^*_\gamma)}{k_1 \cdot p_1} + \frac{(2p_2 \cdot \varepsilon^*_\gamma - \frac{1}{2} k_1 \cdot p_2)}{k_1 \cdot p_2} \right] g_Z^*$$

where $p_1, p_2, k_1$, and $k_2$ are momenta of positron, electron, photon, and $Z^0$, respectively, one can see that it can be obtained from Eq. (1) just by replacing $k_1, p_2, \epsilon^*_\gamma$, and $\bar{v}(p_2, s_2)$ by $-k_1, -p_2, \varepsilon^*_\gamma$, and $\bar{v}(p_2, s_2)$, respectively. The differential cross sections as well as polarization vectors and tensors of vector bosons can be obtained from those obtained in the process $\gamma e \rightarrow Ze$. The transition amplitudes become simplified if the helicity formalism is used, but the covariant formalism is used here in order to treat the $e^+ e^- \rightarrow Z\gamma$, $\gamma e \rightarrow Ze$, and $\gamma e \rightarrow W\nu$ simultaneously.

### III. POLARIZATION EFFECTS

Once the transition amplitude producing the vector mesons is given, it is straightforward to obtain the polarization vector and tensor of the spin-1 particle according to the following method.

The wave vector $\varepsilon^\mu(p, \lambda)$ of a massive spin-1 particle is expressed as

$$\varepsilon^\mu(p, \lambda) = (1 - |\lambda|) n_3^\mu + \frac{1}{\sqrt{2}} |\lambda| (\lambda n_1^\mu - i n_2^\mu),$$

where $p$ and $\lambda$ are its four-momentum and spin components, ±1 or 0, respectively, and the $n_i^\mu$ ($i = 1, 2, 3$) are the Lorentz boosts of an arbitrary orthogonal basis ($\hat{\theta}, \hat{\phi}, \hat{s}$) in three dimensional space. Together with $n_0^\mu = p^\mu / m$, the $n_i^\mu$ form a tetrad. The wave vector $\varepsilon^\mu(p, \lambda)$ enables one to construct the spin-1 projection operator

$$\varepsilon^\mu(p, \lambda) \varepsilon^{*\mu}(p, \lambda') = \frac{1}{3} \left[ I^{\mu\nu} - \frac{3}{2m} i \varepsilon^{\mu\rho\sigma} p_\rho n_\nu(S^\sigma) - \frac{3}{2} n_i^\mu n_j^\nu(S^{ij}) \right]_{\lambda' \lambda},$$

where $I^{\mu\nu}$ is the identity tensor.
where $I^{\mu\nu} = -g^{\mu\nu} + p^{\mu}p^{\nu}/m^2$, the $S^i$ is the standard spin-1 angular momentum matrix, and traceless, symmetric matrix $S^{ij}$ is

$$(S^{ij})_{\lambda\lambda'} = (S^i S^j + S^j S^i - \frac{4}{3} \delta^{ij} I)_{\lambda\lambda'}. \tag{9}$$

When a spin-1 particle is produced, the general form of the transition amplitude reads

$$\mathcal{M} = T_\mu \varepsilon^{*\mu}(p, \lambda), \tag{10}$$

and the physical properties of the outgoing spin-1 particle are determined by a vector field $\phi^\mu$ defined as

$$\phi^\mu = \sum_\lambda T_\alpha \varepsilon^{*\alpha}(p, \lambda) \varepsilon^\mu(p, \lambda)$$

$$= \sum_\lambda \mathcal{M}(\lambda) \varepsilon^\mu(p, \lambda). \tag{11}$$

The covariant density matrix is then obtained as

$$\rho^{\mu\nu} = -\frac{<\phi^\mu \phi^{*\nu}>}{g_{\alpha\beta} <\phi^\alpha \phi^{*\beta}>}$$

$$= \sum_\lambda \sum_{\lambda'} \varepsilon^\mu(p, \lambda) \rho_{\lambda\lambda'} \varepsilon^{*\nu}(p, \lambda'), \tag{12}$$

where the bracket $< >$ denotes the ensemble averaged value, and $\rho_{\lambda\lambda'}$ is another type of the polarization density matrix obtained by folding $\rho^{\mu\nu}$ with the projection operator $\varepsilon^{*\mu}(p, \lambda) \varepsilon_\nu(p, \lambda')$ as

$$\rho_{\lambda\lambda'} = \frac{\mathcal{M}(\lambda) \mathcal{M}(\lambda')}{Tr(|\mathcal{M}|^2)}$$

$$= \sum_\mu \sum_\nu \varepsilon^{*\mu}_\mu(p, \lambda) \rho^{\mu\nu} \varepsilon_\nu(p, \lambda'). \tag{13}$$

The density matrix $\rho^{\mu\nu}$ and $\rho_{\lambda\lambda'}$ can be decomposed into three parts;

$$\rho^{\mu\nu} = \frac{1}{3} I^{\mu\nu} - \frac{i}{2m} \varepsilon^{\mu\nu\lambda\tau} p_\lambda P_\tau - \frac{1}{2} Q^{\mu\nu}, \tag{14}$$

$$\rho_{\lambda\lambda'} = \frac{1}{3} \delta_{\lambda\lambda'} - \frac{1}{2} P_\mu n^{\mu}_\lambda (S^i)_\lambda\lambda' + \frac{1}{4} Q_{\lambda\lambda'} n^{\mu}_\lambda n^{\nu}_\lambda (S^{ij})_{\lambda\lambda'}, \tag{15}$$

with the polarization variables $P^\mu$ and $Q^{\mu\nu}$ called the covariant vector and tensor polarization, respectively. Conversely, for a given density matrix $\rho^{\mu\nu}$, the $P^\mu$ and $Q^{\mu\nu}$ are given by
\[ P^\mu = \frac{i}{m} \epsilon^{\mu \alpha \beta \gamma} p_\alpha p_\beta p_\gamma, \]

\[ Q^{\mu \nu} = \frac{2}{3} T^{\mu \nu} - (\rho^{\mu \nu} + \rho'^{\mu \nu}). \quad (16) \]

The polarization effect of the initial photon beam as in the $\gamma e$ reactions can be obtained easily in the c.m. frame using the Coulomb gauge. But, sometimes it is convenient to choose the polarization vector of the photon in a covariant form as

\[ \varepsilon^\mu(\lambda) = -\frac{1}{\sqrt{2}} (\lambda n_1^\mu + i n_2^\mu). \quad (17) \]

where $\lambda$ is ±1 for right/left handed circular polarization and $n_1^\mu, n_2^\mu$ are different from the $n_i^\mu$ which are defined in massive spin-1 particle case in Eq. (7). A natural cartesian basis for a polarization vector $\varepsilon^\mu(\lambda)$ with a momentum $k_1$ can be given in terms of two arbitrary four momenta $p_1$ and $p_2$ satisfying the constraints $p_1^2 = p_2^2 = 0$. For simplicity, we use $p_1$ and $p_2$ for the momenta of the incident electron and final fermion in the $\gamma e$ reaction. Then we can choose the basis consisting of two orthonomal four-vector $n_1$ and $n_2$ such that

\[ n_1^\mu = N\{(k_1 \cdot p_2)n_1^\mu - (k_1 \cdot p_1)p_2^\mu\}, \quad (18) \]

\[ n_2^\mu = N\epsilon^{\mu \alpha \beta \gamma} k_{1 \alpha} p_{1 \beta} p_{2 \gamma}, \quad (19) \]

where $N$ is a normalization factor given by

\[ N = [2(k_1 \cdot p_1)(k_1 \cdot p_2)(p_1 \cdot p_2)]^{-\frac{1}{2}}. \quad (20) \]

In the covariant density matrix formalism, the photon polarization operator $\varepsilon^\mu(\lambda)\varepsilon^{\nu}(\lambda')$ for an incident photon beam is replaced by its photon covariant density matrix

\[ \rho^{\mu \nu} = \frac{1}{2}[\{n_1^\mu n_1^\nu + n_2^\mu n_2^\nu\} + \xi_1(n_1^\mu n_2^\nu + n_2^\mu n_1^\nu)] - \frac{1}{2}[\{n_1^\mu n_2^\nu - n_2^\mu n_1^\nu\} + \xi_2(n_1^\mu n_2^\nu - n_2^\mu n_1^\nu) + \xi_3(n_1^\mu n_1^\nu - n_2^\mu n_2^\nu)], \quad (21) \]

where the $\xi_i$ are Stokes parameters of the photon beam.

In the process $\gamma e \rightarrow Z e$, the wave vector $\phi^\mu$ of the $Z^0$ vector boson can be described in the SM as
\[
\phi^\mu_Z = \frac{e}{2} g_z \bar{u}(p_2, s_2) \left[ \frac{\gamma_\alpha (k_1 \not \cdot 2 - 2 p_1 \cdot \varepsilon)}{k_1 \cdot p_1} - \frac{(2 p_2 \cdot \varepsilon - \not \cdot k_1) \gamma_\alpha}{k_1 \cdot p_2} \right] \\
\times [\epsilon_L (1 - \gamma_5) + \epsilon_R (1 + \gamma_5)] u(p_1, s_1) \sum_\lambda \epsilon^*_Z(k_2, \lambda) \varepsilon^\mu_Z(k_2, \lambda),
\]

(22)

where \(\varepsilon^\mu\) implies the wave vector of the photon, i.e., \(\varepsilon^\mu_\gamma\).

Then the density matrix \(\rho^{\mu\nu}\) of \(Z^0\) can be obtained from Eqs. (12) and (22) explicitly. In particular, one obtains the denominator of Eq. (12) as

\[
-\rho^{\mu\nu} < \phi^\mu_Z \phi^\nu_Z > = \frac{e^2 g_Z^2}{2 s^2 u^2} A_0,
\]

(23)

\[
A_0 = ([|\epsilon_R|^2 + |\epsilon_L|^2]) [\varepsilon \cdot \varepsilon^* (s^2 + u^2) s u - 4 m_Z^2 \varepsilon \cdot A \varepsilon^* \cdot A] \\
+ 2i(|\epsilon_R|^2 - |\epsilon_L|^2) s(s - u)(t + m_Z^2) < \varepsilon \varepsilon^* k_1 p_2 >,
\]

(24)

where \(s, t,\) and \(u\) are the usual Mandelstam variables, and \(A^\mu\) and \(< \varepsilon \varepsilon^* k_1 p_2 >\) are defined as

\[
A^\mu = u p_1^\mu + s p_2^\mu,
\]

\[
< \varepsilon \varepsilon^* k_1 p_2 > = \epsilon_{\mu \lambda \tau} \varepsilon^\mu \varepsilon^* \lambda k_1^\tau p_2^\tau.
\]

(25)

Then the differential cross section of the \(\gamma e \rightarrow Z e\) can be obtained from \(A_0\) as

\[
\frac{d\sigma}{dt}_{\gamma e \rightarrow Z e} = \frac{e^2 g_Z^2}{64 \pi s^4 u^2} A_0,
\]

\[
= \frac{e^2 g_Z^2}{64 \pi s^4 u^2} \left\{ ([|\epsilon_R|^2 + |\epsilon_L|^2]) [\varepsilon \cdot \varepsilon^* (s^2 + u^2) s u - 4 m_Z^2 \varepsilon \cdot A \varepsilon^* \cdot A] \\
+ 2i(|\epsilon_R|^2 - |\epsilon_L|^2) s(s - u)(t + m_Z^2) < \varepsilon \varepsilon^* k_1 p_2 > \right\}
\]

(26)

If the incident electron beam is polarized, then \(|\epsilon_R|^2\) and \(|\epsilon_L|^2\) in Eq. (24) should be replaced by \((1 + s_1 \cdot \hat{p}_1)|\epsilon_R|^2/2\) and \((1 - s_1 \cdot \hat{p}_1)|\epsilon_L|^2/2\), respectively. Also, Eq. (24) can be used in the process \(e^+ e^- \rightarrow Z \gamma\), if \(\varepsilon^\mu, k_1\) and \(p_2\) are replaced by \(\varepsilon^* \mu, -k_1\) and \(-p_2\) so that \(s\) and \(t\) are interchanged, and the explicit form in the c.m. frame was considered in Ref. [13].

It is found that in the SM Eqs. (23) and (24) can be used for the \(\gamma e \rightarrow W \nu\) process after \(\epsilon_R\) is neglected and also \(m_Z\) and \(g_Z\) are replaced by \(m_W\) and \(g_W\). Therefore, one can obtain the differential cross section immediately as following,
\[
\frac{d\sigma}{dt}_{\gamma e \rightarrow W \nu} = \frac{e^2 g_W^2}{64\pi s^4(s+u)^2} \left[ \varepsilon \cdot \varepsilon^*(s^2 + u^2) su - 4m_W^2 \varepsilon \cdot A \varepsilon^* \cdot A - 2is(s-u)(t+m_W^2) < \varepsilon \varepsilon^* k_1 p_2 > \right]. \tag{27}
\]

In particular, if the c.m. frame of $\gamma e$ and the Coulomb gauge are considered [13], one obtains

\[
\varepsilon \cdot \varepsilon^* = -1,
\]
\[
\varepsilon \cdot A \varepsilon^* \cdot A = \frac{1}{2}stu(1 + \xi_3 \cos 2\phi - \xi_1 \sin 2\phi),
\]
\[
< \varepsilon \varepsilon^* k_1 p_2 > = -i \frac{u}{2} \xi_2. \tag{28}
\]

where the azimuthal angle $\phi$ which specifies the linear polarization of the photon through the Stokes parameters $\xi_i (i = 1, 3)$ can be chosen to be 0 without the loss of generality [13, 14].

The differential cross section becomes

\[
\frac{d\sigma}{dt}_{\gamma e \rightarrow W \nu} = -\frac{e^2 g_W^2 u}{64\pi s^3(s+u)^2} \left[ (s^2 + u^2) + 2m^2t(1 + \xi_3 \cos 2\phi - \xi_1 \sin 2\phi), +\xi_2(s-u)(t+m^2) \right]. \tag{29}
\]

The result of Eq. (27) is of a covariant form and it can be obtained by using the covariant photon polarization, Eq. (21), but then one obtain the $\phi = 0$ case. It is noted that, if the incoming photon beam is unpolarized, the $\xi_i$ become zero and the result is the same as that in Ref. [8]. Also the relation in the square bracket of Eq. (29) is the same as that of Eq. (26) in the process $\gamma e \rightarrow Ze$ except $\epsilon_R = 0$, and it differs from Eq. (4.3) of Ref. [5] which must be corrected. On the other hand, one can obtain the same result of differential cross section for the process $\gamma e \rightarrow Ze$ as that in Ref. [4] in the center of mass frame of $\gamma$ and $e$ from Eqs. (26) and (28) when the incident photon beam is polarized.

From the explicit form of $\rho^{\mu\nu}$, one obtains explicit values of polarization vector $P^\mu$ and polarization tensor $Q^{\mu\nu}$ in the process $\gamma e \rightarrow Ze$ as

\[
P^\mu = \frac{2}{m_Z} \left[ i(\epsilon_R^2 + |\epsilon_L|^2) R^\mu_1 + (|\epsilon_R|^2 - |\epsilon_L|^2) R^\mu_2 \right] / A_0,
\]
\[
Q^{\mu\nu} = -\frac{1}{3} F^{\mu\nu} - 2 \left[ (|\epsilon_R|^2 + |\epsilon_L|^2) R_{1}^{\mu\nu} + i(|\epsilon_R|^2 - |\epsilon_L|^2) R_{2}^{\mu\nu} \right] / A_0, \tag{30}
\]

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where $R_1^\mu, R_2^\mu, R_1^{\mu\nu}$ and $R_2^{\mu\nu}$, are defined as

$$R_1^\mu = -s <\varepsilon\bar{\varepsilon}k_1p_2> \left[ 2m_Z^2(tk_1^\mu - sp_1^\mu + up_2^\mu) + (s^2 + u^2)k_2^\mu \right],$$

$$R_2^\mu = m_Z^2 \left[ su(\varepsilon\bar{\varepsilon}^* \cdot A + \varepsilon^*\varepsilon^* \cdot A) - su(s - u)\varepsilon \cdot \varepsilon^* K^\mu \right.$$ 

$$-2(p_1^\mu + p_2^\mu)\varepsilon \cdot A \varepsilon^* - A \varepsilon \cdot A - 2k_1^\mu(u^2 \varepsilon \cdot p_1 \varepsilon^* \cdot p_1 - s^2 \varepsilon \cdot p_2 \varepsilon^* \cdot p_2) \right],$$

$$R_1^{\mu\nu} = su\varepsilon \cdot \varepsilon^* \left[ suI^{\mu\nu} - 2m^2 K^\mu K^\nu \right] + [(su\varepsilon - \varepsilon_1 B^{\alpha\mu})(su\varepsilon^* - \varepsilon_1 B^{\alpha\nu}) + (\mu \leftrightarrow \nu)],$$

$$R_2^{\mu\nu} = s <\varepsilon\bar{\varepsilon}k_1p_2> \left\{ (u - s)(t + m_Z^2)\frac{k_2^\mu k_2^\nu}{m_Z^2} + 2 \left[ m_Z^2 k_1^\mu(p_1 + p_2)^\nu - k_2^\mu A^\nu + (\mu \leftrightarrow \nu) \right] \right\},$$

where $K^\mu$, and $B^{\alpha\mu}$, and $<\mu\varepsilon\varepsilon^*k_1>$ are defined as

$$K^\mu = k_1^\mu - \frac{k_1 \cdot k_2 k_2^\mu}{m_Z^2} = k_1^\mu - \frac{(s + u)}{2m_Z^2}k_2^\mu,$$

$$B^{\alpha\mu} = up_1^\alpha(2p_2 + k_2)^\mu + sp_2^\alpha(2p_1 - k_2)^\mu = (p_1 + p_2)^\mu A^\alpha + k_1^\mu(\varepsilon_1 p_1 - \varepsilon_1 p_2),$$

$$<\mu\varepsilon\varepsilon^*k_1^\mu> = \varepsilon^\mu_{\alpha\beta} \varepsilon_\nu \varepsilon_\alpha k_{1\beta}.$$

The corresponding relations of Eq. (31) in the process $e^+e^- \rightarrow Z\gamma$ was given in Ref. [13], where the c.m. frame of $e^+$ and $e^-$ and Coulomb gauge for the photon beams are chosen, and they can be obtained from Eqs. (30)- (31) as a special case. Moreover, the above results can be used to obtain for the polarization vector $P_W^\mu$ and polarization tensor $Q_W^{\mu\nu}$ of the $W$ boson in the process $\gamma e \rightarrow W\nu$, by putting $\varepsilon_R = 0$ and by replacing $m_Z$ by $m_W$ in Eqs. (24) and (31).

As we can see in Eq. (21), $\xi_1$ and $\xi_3$ terms are symmetric in the indices $\mu$ and $\nu$, and the $\xi_2$ term is antisymmetric in those. Therefore, $R_1^\mu$ and $R_2^{\mu\nu}$ in Eq. (31) depend on the degree of circular polarization of the incident photon beams, $\xi_2$, while $R_1^\mu$ and $R_2^{\mu\nu}$ can be obtained in terms of the linearly polarized parameters $\xi_1$ and $\xi_3$. Explicitly the influence of the photon polarization in the reactions can be seen through $A_0, R_1^\mu$, and $R_i^{\mu\nu}$ ($i = 1, 2$) as following,

$$A_0 = su \left\{ -(|\varepsilon_R|^2 + |\varepsilon_L|^2) \left[ 2tm_Z^2(1 + \xi_3) + s^2 + u^2 \right] + \xi_2(|\varepsilon_R|^2 - |\varepsilon_L|^2)(s - u)(t + m_Z^2) \right\},$$

$$R_1^\mu = \frac{i}{2} su\xi_2 \left[ 2m_Z^2(tk_1^\mu - sp_1^\mu + up_2^\mu) + (s^2 + u^2)k_2^\mu \right],$$

$$R_2^\mu = sum_2^2 \left\{ (s - u)K^\mu - (1 + \xi_3) \left[ A^\mu + t(p_1 + p_2)^\mu \right] + 2\xi_1 <\mu k_1 p_1 p_2> \right\},$$
\[ R_1^{\mu \nu} = \frac{S_{\mu}}{t} \left\{ -t (s u I^{\mu \nu} - 2 m_Z^2 K^\mu K^\nu) ight. \\
\left. - (1 - \xi_3) \left[ 2 k_1^\mu k_1^\nu + s u t g^\mu t - t k_1^\mu (u p_1^\nu - s p_2^\nu) - t k_1^\nu (u p_1^\mu - s p_2^\mu) + A^\mu A^\nu \right] + (1 + \xi_3) [A^\mu + t(p_1 + p_2)^\mu] [A^\nu + t(p_1 + p_2)^\nu] \right\} \\
- 2 \xi_1 \left[ \langle k p 1 p 2 \rangle (A^\nu + t(p_1 + p_2)^\nu) + (\mu \leftrightarrow \nu) \right], \}
\]
\[
R_2^{\mu \nu} = - \frac{i}{2} s u \xi_2 \left\{ (u - s) (t + m_Z^2) \frac{k_2^\mu k_2^\nu}{m_Z^2} + 2 \left[ m_Z^2 k_1^\mu (p_1 + p_2)^\nu - k_2^\mu A^\nu + (\mu \leftrightarrow \nu) \right] \right\}. \quad (33)
\]

**IV. DECAY OF VECTOR BOSONS**

Once the vector bosons like \( Z^0 \)'s or \( W \)'s are produced, they have the polarization vector \( P^\mu \) and polarization tensor \( Q^{\mu \nu} \) and they decay into lighter particles immediately. The density matrix for the vector mesons can be used in this case. Since the \( Z^0 \) decay has been considered previously \[13,15\], consider here the case that a \( W \) decays into a lepton and a neutrino.

In the SM, the transition amplitude for the decay \( W \to l \bar{\nu}_l \) becomes

\[
\mathcal{M} = g_W \varepsilon_W^\mu \bar{u}(p_1') \gamma_\mu (1 - \gamma_5) u(p_2'), \quad (34)
\]

where \( p_1' \) and \( p_2' \) are the momenta of a lepton and a neutrino, respectively, and \( \varepsilon_W^\mu \) is the Proca vector of the \( W \) before its decay. The full amplitude for the production of \( W \) followed by its subsequent decay can be obtained if \( \varepsilon_W^\mu \) in Eq. (34) is replaced by \( \phi_W^\mu \) in Eq. (11) and multiplied by the \( W \) propagator in the Wigner-Breit form.

If the outgoing lepton is unpolarized, the absolute square of the amplitude, after replacing \( \varepsilon_W^\mu \varepsilon_W^\nu \) by \( \rho^{\mu \nu} \), becomes

\[
\sum_f |\mathcal{M}|^2 = \frac{4 g_W^2}{m_W m_{\nu}} \rho^{\mu \nu} \left[ p_{1 \mu} p_{2 \nu} + p_{1 \mu} p_{2 \nu} - p_{1 \nu} \cdot p_{2 \mu} + i \epsilon_{\mu \nu \lambda \tau} p_{1 \lambda} p_{2 \tau} \right]. \quad (35)
\]

From the explicit expression of \( \rho^{\mu \nu} \) in terms of \( P^\mu \) and \( Q^{\mu \nu} \) as in Eq. (14), one obtains the angular distribution for the outgoing lepton as

\[
\frac{d\Gamma}{d\Omega} = \frac{m_W g_W^2}{48 \pi^2} \left[ 1 + \frac{3}{m_W} P \cdot p_1' + \frac{3}{m_W^2} Q^{\mu \nu} p_{1 \mu} p_{1 \nu} \right]. \quad (36)
\]
Since the effect of photon polarization is contained in $P^\mu$ and $Q^{\mu\nu}$, the asymmetry due to the photon polarization can be obtained from Eq. (36).

If $\rho^{\mu\nu}$ is replaced by Eq. (12), one obtains

$$\sum |M|^2 = \frac{4g_W^2}{m_W m_\nu} \sum \sum \rho_{\lambda\lambda'} [p'_1 \cdot p'_2 \delta_{\lambda\lambda'} - 2\varepsilon(\lambda) \cdot p'_1 \varepsilon^*(\lambda') \cdot p'_1 + i < \varepsilon(\lambda)\varepsilon^*(\lambda') p'_1 p'_2 > ]$$  \hspace{1cm} (37)

In particular, when the $W$ decays in its rest frame, the $n_i^\mu(i = 1, 2, 3)$ becomes $\hat{\theta}, \hat{\phi}$, and $\hat{s}$ which can be chosen as unit vectors along the $x$, $y$, and $z$ direction, respectively. Using the explicit form of $\varepsilon^\mu(\lambda)$ in Eq. (37) given by Eq. (7), one obtains the angular distribution of the outgoing lepton in terms of density matrix elements $\rho_{\lambda\lambda'}$ as

$$I(\theta, \phi) = \frac{3}{16\pi} \left[ 1 + \cos^2 \theta + \rho_{00}(1 - 3 \cos^2 \theta) - 2 \cos \theta (\rho_{11} - \rho_{-1-1}) + 2 \sin^2 \theta \ \text{Re}\{\rho_{1-1}e^{2i\phi}\} \\
- 2\sqrt{2} \sin \theta \ \text{Re}\{(\rho_{10} + \rho_{0-1})e^{i\phi}\} + \sqrt{2} \sin 2\theta \ \text{Re}\{(\rho_{10} - \rho_{0-1})e^{i\phi}\} \right] \hspace{1cm} (38)$$

The density matrix elements can be obtained experimentally by investigating the angular dependence of the outgoing lepton beam. On the other hand, $\rho_{\lambda\lambda'}$ can be obtained from Eqs. (13) and (15) and, in particular, in the $W$ rest frame it becomes

$$\rho_{\lambda\lambda'} = \left[ \frac{1}{3}I + \frac{1}{2}P^i S^i + \frac{1}{4}Q^{ij} S^{ij} \right]_{\lambda\lambda'} \hspace{1cm} (39)$$

Therefore, it depends on the polarization vector $\vec{P}$ and polarization tensor $Q^{ij}$ again, and explicit values $\rho_{\lambda\lambda'}$ in Eq. (37) depends on the axis one chooses,

\begin{align*}
\rho_{00} &= \frac{1}{3} - \frac{1}{2}Q^{33}, \\
\rho_{11} - \rho_{-1-1} &= P^3, \\
\rho_{10} + \rho_{0-1} &= \frac{1}{\sqrt{2}}(P^1 - iP^2), \\
\rho_{10} - \rho_{0-1} &= \frac{1}{2\sqrt{2}}(Q^{13} - iQ^{23}), \\
\rho_{1-1} &= \frac{1}{4}(Q^{11} - Q^{22} - iQ^{12}),
\end{align*}  \hspace{1cm} (40)

where explicit forms of $\vec{P}$ and $Q^{ij}$ in the $W$ rest frame are given as following
\[ P^i = -\frac{2m_W[(s + t)k_1^i + (t + m_W^2)p_1^i]}{2m_W^2t + s^2 + u^2} \]
\[ Q^{ij} = -\frac{1}{3}\delta^{ij} + \frac{2m_W^2(k_1^i k_1^j + 2p_1^i p_1^j) + (t + m_W^2)(p_1^i k_1^j + p_1^j k_1^i)}{2m_W^2t + s^2 + u^2}. \] (41)

Usually, the \( x-z \) axis is chosen in the production plane.

V. CONCLUSION AND DISCUSSION

The polarization vector \( P^\mu \) and polarization tensor \( Q^{\mu\nu} \) in the \( \gamma e \to Ze \) process are obtained explicitly by means of the covariant density matrix formalism. Using the crossing symmetry, these results can be used to obtain the polarization of \( Z^0 \) beam produced by the \( e^+e^- \to Z\gamma \) process. It is shown that in the SM, the polarization of the \( W \) boson polarization in the \( \gamma e \to W\nu \) process can be obtained from that of \( Z^0 \) in the \( \gamma e \to Ze \) process just by putting \( \epsilon_R = 0 \) and by replacing \( m_Z \) and \( g_Z \) by \( m_W \) and \( -k_1 \cdot p_2 / k_1 \cdot k_2 g_W \), respectively.

Since the results are of the covariant form, they can be used in any frame, e.g., in the c.m. frame or in the rest frame of the vector bosons. The usual results discussed in the c.m. frame using the helicity formalism can be reproduced from the covariant results.

Some possible deviations from the SM have been considered by several authors through the anomalous gauge-boson-couplings, in the CP conserving cases \[3,8,10,11\] as well as CP violating cases \[4,7\], by introducing additional terms. Such additional terms change \( A_0, P^\mu, \) and \( Q^{\mu\nu} \). Therefore, the universality considered in the SM would not be held in general, and the underlying models for such changes can be investigated through the polarization effects of vector bosons as well as those of fermion and photon involved in the reactions.

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FIGURE CAPTIONS

Fig. 1 Feynman diagrams for the process $\gamma e \rightarrow Z^0 e$ in the SM (a) and (b), and the anomalous contribution (c).

Fig. 2 Feynman diagrams for the process $\gamma e \rightarrow W \nu$ in the SM (a) and (b), and the anomalous contribution (c).

Fig. 3 Feynman diagrams for the process $e^+ e^- \rightarrow Z^0 \gamma$ in the SM (a) and (b), and the anomalous contribution (c).
Fig. 1

Fig. 2

Fig. 3