Relativistic corrections to the nuclear Schiff moment

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Parity and time invariance violating ($P, T$-odd) atomic electric dipole moments (EDM) are induced by interaction between atomic electrons and nuclear $P, T$-odd moments which are produced by $P, T$-odd nuclear forces. The nuclear EDM is screened by atomic electrons. The EDM of a non-relativistic atom with closed electron subshells is induced by the nuclear Schiff moment. For heavy relativistic atoms EDM is induced by the nuclear local dipole moments which differ by 10-50\% from the Schiff moments calculated previously. We calculate the local dipole moments for $^{199}$Hg and $^{205}$Tl where the most accurate atomic \cite{1} and molecular \cite{2} EDM measurements have been performed.

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I. INTRODUCTION

The best limits on Parity($P$) and Time($T$) invariance violating nuclear forces are obtained from the measurement of electric dipole moment (EDM) of $^{199}$Hg atom \cite{1}. The mechanism of this EDM generation is the following. $P, T$-odd nuclear forces produce $P, T$-odd nuclear moments. In turn, these moments can induce electric dipole moments in atoms through the mixing of electron wave functions of opposite parity. The EDM of a point-like nucleus according to the Schiff theorem \cite{3, 4, 5} is completely screened by atomic electrons. However, due to finite nuclear size, some dipole component of the electrostatic potential still exists. It is generated by the Schiff moment \cite{6}, the higher moment in the dipole density distribution.

The electrostatic potential produced by the Schiff moment is usually presented in the form \cite{6}

$$\phi(R) = 4\pi S \cdot \nabla \delta(R),$$

where $S$ is the Schiff moment. Presence of the gradient of $\delta(R)$ means that the matrix element of the potential given by Eq. (1) between $s$- and $p$-atomic states is proportional to $\langle r^{k+1} \rangle - \frac{k+4}{3} \langle r \rangle \langle r^{k+1} \rangle$. However, at high $Z$ the relativistic wave functions of atomic electrons vary significantly ($\sim Z^2 \alpha^2$; for $^{199}$Hg $Z^2 \alpha^2 = 0.34$) inside the nucleus. This produce a corrections from higher moments of the dipole density distribution leading to a local dipole moment \cite{7}

$$L = e \sum_{k=1}^{\infty} \frac{b_k}{b_1} \frac{1}{(k+1)(k+4)} \left[ \langle r^{k+1} \rangle - \frac{k+4}{3} \langle r \rangle \langle r^{k+1} \rangle \right] = LI/I,$$

(2)

The summation is carried over odd powers of $k$, $k = 1, 3, 5, ...$. The first term ($k = 1$) in Eq. (2) is just the Schiff moment and the first nonzero correction ($k = 3$) to it comes from $b_3/b_1$. The factors $b_3/b_1$ being both of the order of $Z^2 \alpha^2$ differ for $p_{1/2}$ and $p_{3/2}$ atomic states \cite{7}

$$\frac{b_3}{b_1} = \frac{3}{5} \frac{Z^2 \alpha^2}{R_N^2},$$

(3)

for $s - p_{1/2}$ matrix element, and

$$\frac{b_3}{b_1} = \frac{9}{20} \frac{Z^2 \alpha^2}{R_N^2},$$

(4)

for $s - p_{3/2}$ matrix element. Here $R_N$ is the charge radius of uniformly charged nucleus.

When doing atomic EDM calculations one should use a corrected $P, T$-odd interaction Hamiltonian $W_{PT}$ for relativistic atomic electrons containing the local dipole moment \cite{6}:

$$W_{PT}/(-e) = \phi_L(R) = 4\pi L \cdot \nabla \delta(R),$$

(5)

To use this formula one must assume that the local dipole moment is placed in the centre of a finite-size nucleus.

The aim of the present work is to calculate the local dipole moment $L$ for two most important nuclei $^{199}$Hg (with valence neutron) and $^{205}$Tl (with valence proton).
II. OUTLINES OF THE THEORY

In calculations we used a finite range P- and T-violating nucleon-nucleon interaction of the form
\[
W(r_a - r_b) = -\frac{g_s}{8\pi m_p} \left[ (g_0 \tau_a \cdot \tau_b + g_2 (\tau_a \cdot r_b - 3 r_a \cdot r_b^2)) \right]
\]
\[
(\sigma_a - \sigma_b) + g_1 (\tau_a \sigma_a - \tau_b \sigma_b) \cdot \nabla_a \frac{e^{-m_r r_{ab}}}{r_{ab}}, \tag{6}
\]
where \(m_p\) is the proton mass and \(r_{ab} = |r_a - r_b|\).

For a nucleus having an odd proton there are two kinds of contributions to the local dipole moment even in the absence of a strong residual interaction between the odd proton and the nucleons in the core. One contribution comes from the odd proton. Another contribution comes from the core nucleons. We start from the odd proton contribution. The weak interaction, Eq. (4), generates a weak correction to the nuclear mean field which, in turn, produces a correction \(\delta \psi_\nu(r)\) to the single particle wave function \(\psi_\nu(r)\) of the odd proton. With this correction the expectation value of the local dipole moment is
\[
L_{sp} = \langle \delta \psi_\nu | \vec{L}_z | \psi_\nu \rangle + \langle \psi_\nu | \vec{L}_z | \delta \psi_\nu \rangle. \tag{7}
\]
Note that this single particle contribution is absent for neutron odd nuclei.

Another type of contribution to the local dipole moment comes from off diagonal matrix elements of the two body weak interaction.
\[
L_{core} = \sum_{\nu_1 \nu_2} \langle \nu_1 | W | \nu_2 \rangle \frac{n_{\nu_1} - n_{\nu_2}}{\epsilon_{\nu_1} - \epsilon_{\nu_2}} \langle \nu_2 | \vec{L}_z | \nu_1 \rangle, \tag{8}
\]
where \(n_\nu\) are the occupation numbers and \(\epsilon_\nu\) are the single particle energies. Here the operator \(\vec{L}_z\) acts on the core protons. The magnitudes of \(S_{sp}\) and \(S_{core}\) are comparable for proton odd nuclei. This is in contrast to the P odd and T even nuclear anapole moment where the core contribution is smaller by a factor \(1/A^{1/3}\).

When the strong residual interaction leading to core polarization is taken into account the contributions to the local dipole moment can be written as a sum of three terms 
\[
L = \langle \delta \psi_\nu | \vec{L}_z | \psi_\nu \rangle + \langle \psi_\nu | \vec{L}_z | \delta \psi_\nu \rangle + \langle \nu | \delta L | \nu \rangle. \tag{9}
\]
The first two terms are those where the weak correction enters via the odd particle wave function. \(\vec{L}\) satisfies the equation
\[
\langle \nu' \vec{L} | \nu \rangle = \langle \nu' | \vec{L} | \nu \rangle + \sum_{\nu_1 \nu_2} \langle \nu' \nu_1 | F | \nu_2 \nu \rangle \frac{n_{\nu_1} - n_{\nu_2}}{\epsilon_{\nu_1} - \epsilon_{\nu_2}} \langle \nu_2 | \vec{L} | \nu_1 \rangle, \tag{10}
\]
Here only the residual strong interaction \(F\) enters in the equation. The effects of the weak interaction are entirely in the wave functions of the odd proton. Eq. (10) describes the well known effect of renormalization of nuclear moments due to coupling with particle-hole states in the core. The third term in (9) satisfies the equation
\[
\langle \nu' | \delta L | \nu \rangle = \langle \nu' | \delta L_0 | \nu \rangle + \sum_{\nu_1 \nu_2} \langle \nu' \nu_1 | F | \nu_2 \nu \rangle \frac{n_{\nu_1} - n_{\nu_2}}{\epsilon_{\nu_1} - \epsilon_{\nu_2}} \langle \nu_2 | \delta L | \nu_1 \rangle, \tag{11}
\]
that looks similar to (10). However, the inhomogenous term \(\delta S_0\) is completely different, namely
\[
\langle \nu' | \delta L_0 | \nu \rangle = \sum_{\nu_1 \nu_2} \langle \nu' \nu_1 | W | \nu_2 \nu \rangle \frac{n_{\nu_1} - n_{\nu_2}}{\epsilon_{\nu_1} - \epsilon_{\nu_2}} \langle \nu_2 | \vec{L} | \nu_1 \rangle + \\
\sum_{\nu_1 \nu_2} \langle \nu' \nu_1 | F | \nu_2 \nu \rangle \frac{n_{\nu_1} - n_{\nu_2}}{\epsilon_{\nu_1} - \epsilon_{\nu_2}} \langle (\delta \psi_{\nu_2} | \vec{L} | \nu_1) + \langle \nu_2 | \delta \psi_{\nu_1} \rangle \rangle.
\tag{12}
\]
TABLE I: Schiff moment $S$ and the ratio of relativistic correction $L'$ to $S$ ($L = S + L'$) for proton odd $^{205}$Tl and neutron odd $^{199}$Hg nuclei. $S_0$ and $L'_0$ are the bare values, without strong residual nuclear forces. $S_{tot}$ and $L'_{tot}$ are the total results with full account of core polarization effects. The values of $L'/S$ are given for $s - p_{1/2}$ transition. For $s - p_{3/2}$ transition they differ by the factor 3/4. To obtain values of $L$ and $S$ one should sum up contributions of three interaction constants $g_0$, $g_1$ and $g_2$.

|        | $^{205}$Tl |        | $^{199}$Hg |
|--------|-----------|--------|-----------|
| $g_0$  | $-0.075$  | $0.014$ | $-0.18$   | $-0.085$  | $-0.1$   | $-0.006$ | $-0.05$ |
| $g_1$  | $-0.028$  | $-0.082$ | $-0.18$   | $-0.085$  | $-0.1$   | $-0.036$ | $-0.15$ |
| $g_2$  | $0.237$   | $-0.08$ | $-0.007$  | $-0.51$   | $0.17$   | $-0.1$   | $0.019$ | $-0.08$ |

The first term on the rhs of Eq. (12) is the core contribution $\mathbf{2}$, where instead of the bare local dipole moment operator $\mathbf{1}$ enters the renormalized operator $\mathbf{1}$. The second and the third terms correspond to additional contributions where the weak interaction enters via corrections to the intermediate single particle states $|\nu_1\rangle$ and $|\nu_2\rangle$. Equations (9-11) describe all of the core polarization effects.

Results of the calculations of the Schiff moment and the first relativistic correction for the proton odd nucleus $^{205}$Tl and the neutron odd nucleus $^{199}$Hg are shown in Table I. We see that usually the difference between the local dipole moment (which is actually measured in atomic EDM experiments) and the Schiff moment is not very large, about 10-20%. However, the correction $L'$ may reduce the result two times if the value of the Schiff moment is anomalously small (see the $g_2$ contribution for $^{205}$Tl).

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[1] M.V. Romalis, W.C. Griffith, J.P. Jacobs, and E.N. Fortson, Phys. Rev. Lett. 86, 2505 (2001).
[2] D. Cho, K. Sangster, E.A. Hinds, Phys. Rev. Lett. 63, 2559 (1989); Phys. Rev. A 44, 2783 (1991).
[3] E.M. Purcell and N.F. Ramsey, Phys. Rev. 78, 807 (1950).
[4] L.I. Schiff, Phys. Rev. 132, 2194 (1963).
[5] P.G.H. Sandars, Phys. Rev. Lett. 19, 1396 (1967).
[6] O.P. Sushkov, V.V. Flambaum and I.B. Khriplovich, Zh. Eksp. Teor. Fiz., 87, 1521 (1984) [Sov. Journ. JETP, 60, 873 (1984)]. V.V. Flambaum, I.B. Khriplovich, and O.P. Sushkov, Nucl. Phys. A449, 750 (1986).
[7] V.V. Flambaum and J.S.M. Ginges, Phys. Rev. A 65, 032113 (2002).
[8] V.F. Dmitriev and R.A. Sen’kov, Yad.Fiz. 66 1988 (2003), (Phys.Atom.Nucl. 66, 1940 (2003) ), nucl-th/0304048
[9] V.F. Dmitriev, R.A. Sen’kov and N. Auerbach, nucl-th/0408065