Universal decay rule for reduced widths

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Emission processes including α-decay, heavy cluster decays, proton and di-proton emission are analyzed in terms of the well known factorisation between the penetrability and reduced width. By using a shifted harmonic oscillator plus Coulomb cluster-daughter interaction it is possible to derive a linear relation between the logarithm of the reduced width squared and the fragmentation potential, defined as the difference between the Coulomb barrier and Q-value. This relation is fulfilled with a good accuracy for transitions between ground states, as well as for most α-decays to low lying 2\textsuperscript{+} excited states. The well known Viola-Seaborg rule, connecting half lives with the Coulomb parameter and the product between fragment charge numbers, as well as the Blendowke scaling rule connecting the spectroscopic factor with the mass number of the emitted cluster, can be easily understood in terms of the fragmentation potential. It is shown that the recently evidenced two regions in the dependence of reduced proton half-lives versus the Coulomb parameter are directly connected with the corresponding regions of the fragmentation potential.

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1. INTRODUCTION

The family of emission processes triggered by the strong interaction contains various decays, namely particle (proton or neutron) emission, two-proton emission, Ω-decay, heavy cluster emission and binary or ternary fission. There are also other nuclear decay processes induced by electromagnetic (γ-decay) or weak forces (β-decays). The purpose of this work is to investigate only the first type of fragmentation, where the emitted fragments are left in ground or low-lying excited states. They are called cold emission processes and are presently among important tools to study nuclei far from the stability line. Nuclei close to the proton drip line are investigated through proton emission, while the neutron drip line region is probed by cold fission processes. On the other hand superheavy nuclei are exclusively detected by α-decays chains\textsuperscript{[1]}. Actually the first paper in theoretical nuclear physics applying quantum mechanics \textsuperscript{[2]} was devoted to the description of the α-decay in terms of the penetration of a preformed particle through the Coulomb barrier.

There are two goals of this paper. The first one is to explain the well known Viola-Seaborg rule \textsuperscript{[3]}, valid for all kinds of cold emission processes. It turns out that it is possible to give a simple interpretation of this rule in terms of two physical quantities, namely the Coulomb parameter, connected with the penetrability, and the fragmentation potential, connected with the reduced width. An universal linear dependence between the logarithm of the reduced width and fragmentation potential will be derived. It will be shown that this interpretation is valid not only for transitions between ground states, but also for transitions to excited states. On the other hand, the scaling dependence of spectroscopic factors in heavy cluster decays versus the mass numbers of the emitted cluster can be also understood in terms of the fragmentation potential.

2. EXPERIMENTAL DECAY RULES

Let us consider a binary emission process \( P \rightarrow D + C \) from a parent (\( P \)) to the daughter nucleus (\( D \)) and the lighter cluster (\( C \)), which can be in particular an α-particle or a proton. The total decay width is the sum of partial decay widths corresponding to different angular momenta, given by \textsuperscript{[4]}

\[
\Gamma_l = 2 \Gamma_l(E_l, r) \gamma_l^2(\beta, r),
\]

where it was introduced the standard penetrability and reduced width squared \textsuperscript{[5]}

\[
P_l(E_l, r) = \frac{\kappa_l r}{|H_l^{(+)}(\chi_l, \kappa_l r)|^2},
\]

\[
\gamma_l^2(\beta, r) = \frac{\hbar^2}{2 \mu r} s_l^{(\text{int})(\beta, r)}^2.
\]

The outgoing spherical Coulomb-Hankel function \( H_l^{(+)}(\chi_l, \kappa_l r) \) depends upon two variables, namely the Coulomb (or twice the Sommerfeld) parameter

\[
\chi_l = \frac{Z_D Z_C e^2}{\hbar v_l},
\]

and the reduced channel radius \( \rho_l = \kappa_l r \). Here \( v_l = \hbar \kappa_l / \mu \) and \( \kappa_l = \sqrt{2 \mu E_l / \hbar} \) are the asymptotic relative velocity and momentum between the emitted fragments, respectively, in terms of the reduced mass of the daughter-cluster system \( \mu \). It is also defined the center of mass (cm) channel energy \( E_l = Q - E_l^{(ex)} \) of emitted fragments, in terms of the difference between the total energy (Q-value) and the excitation energy of the daughter nucleus \( E_l^{(ex)} \). The internal component \( s_l^{(\text{int})(\beta, r)} \) at a certain radius \( r \) inside the Coulomb barrier, is for deformed emitters a superposition of different Nilsson components.
multiplied by the propagator matrix \[4\], depending on deformation parameters \(\beta\), i.e.

\[
s^{(\text{int})}_l(\beta, r) = \sum_{l'} K_{ll'}(\beta, r) f^{(\text{int})}_{l'}(r) \tag{2.4}
\]

For spherical emitters with \(K_{ll'} = \delta_{ll'}\) it coincides with the wave function component \(f^{(\text{int})}_{l'}(r)\).

The half life is defined by the inverse of the total decay width, i.e.

\[
T = \frac{\hbar \ln 2}{\Gamma}. \tag{2.5}
\]

There were considered here for simplicity transitions between states with the same angular momentum \(l\), as for instance \(\alpha\)-decays or proton emission processes between ground states. Thus, the logarithm of the so-called reduced half-life, corrected by the exponential centrifugal factor squared \(C_l^2\), defined by the second line of this relation, i.e.

\[
\log_{10} T_{\text{red}} = \log_{10} \left( \frac{T}{C_l^2} \right) = \log_{10} \ln 2 - \log_{10} 2P_l - \log_{10} \gamma^2_l. \tag{2.8}
\]

The above relation is also called Geiger-Nuttall law, discovered in 1911 for \(\alpha\)-decays between ground states (where the angular momentum carried by the \(\alpha\)-particle is \(l = 0\)). The explanation of this law was given by G. Gamow in 1928 \[2\], in terms of the quantum-mechanical penetration of the Coulomb barrier, i.e. the first line of Eq. (2.2). It is characterized by the Coulomb parameter, which is proportional with the ratio \(Z_D/\sqrt{Q_\alpha}\).

The \(\alpha\)-decays between ground states are characterized by a remarkable regularity, especially for transitions between ground states of even-even nuclei. The fact that
$\alpha$-transitions along various isotopic chains lie on separate lines, as stated by the Viola-Seaborg rule [3], i.e.

$$\log_{10}T = \frac{a_1 Z_D + a_2}{\sqrt{Q_\alpha}} + b_1 Z_D + b_2,$$  \hspace{1em} (2.10)

is connected with different $\alpha$-particle reduced widths, multiplying the penetrability in (2.1). From Eq. (2.8) it becomes clear that the reduced width should depend upon the charge of the daughter nucleus. This feature is shown in figure 1. Still in doing systematics along neutron chains there are important deviations with respect to this rule, as for instance in $\alpha$-decay from odd mass nuclei, and this feature is strongly connected with nuclear structure details. Let us mention that different forms of the Viola-Seaborg relation were used in Refs. [4, 7].

The Viola-Seaborg rule can be generalized for heavy-cluster decays [8], as it is shown in figure 2. Here the angular momenta carried by emitted fragments are zero. In Ref. [9] it was proposed the following generalized Viola-Seaborg rule for the heavy cluster emission

$$\log_{10}T = a_1 \frac{Z_D Z_C}{\sqrt{Q_\alpha}} + a_2 Z_D Z_C + b_2 + c_2,$$  \hspace{1em} (2.11)

with the following set of parameters

$$a_1 = 1.517, \quad a_2 = 0.053, \quad b_2 = -92.911, \quad c_2 = 0 \quad (even), = 1.402 \quad (odd),$$

where $c_2$ is the blocking parameter for odd-mass nuclei. Thus, from Eq. (2.8) the reduced width in this case should depend upon the product between daugher and cluster charges.

An interesting feature can be seen by plotting in figure 3 the logarithm of the reduced half life (2.8) versus the Coulomb parameter for various proton emitters [10, 11]. In this case most of emitters have non-vanishing angular momentum. The data are roughly divided into two regions, corresponding to $Z < 68$ (open circles) and $Z > 68$ (dark circles). This pattern can be assimilated with a generalized Viola-Seaborg rule for the two groups of charge numbers.

The situation with binary cold fission is quite different. Here there is not a simple linear dependence between the half-life for a given isotopic partition and the Coulomb parameter, due to the fact that mass asymmetry changes during the scission process.

3. A SIMPLE MODEL FOR THE REDUCED WIDTH

The decay processes can be schematically described by the following cluster-daugher spherical potential

$$V(r) = \hbar \omega \left( \frac{\beta(r - r_0)^2}{2} + v_0 \right), \quad r \leq r_B$$

$$= \frac{Z_D Z_C e^2}{r} \equiv V_C(r), \quad r > r_B$$  \hspace{1em} (3.1)

where $r_0 = 1.2 A_0^{1/3}$ is the surface radius of the daughter nucleus. Indeed, microscopic calculations have shown that the preformation factor of the $\alpha$-particle has a Gaussian shape, centered on the nuclear surface [12]. Moreover, the spherical component of the preformation amplitude gives more than 90% contribution in the $\alpha$-decay width of deformed nuclei. Notice that the radial equation of the shifted harmonic oscillator (ho) potential is similar with the equation of the one-dimensional oscillator, but having approximate eigenvalues given by

$$E_{nl} = \hbar \omega \left( n + \frac{1}{2} \right) + \frac{\hbar^2 l(l + 1)}{2\mu r_0^2}.$$  \hspace{1em} (3.2)

By considering $Q$-value as the first eigenstate in the shifted ho well $Q - v_0 = \frac{1}{2} \hbar \omega$, together with the continuity condition at the top of the barrier $r_B$, one obtains the following relation

$$\frac{\hbar \omega \beta(r_B - r_0)^2}{2} = V_{frag}(r_B) + \frac{1}{2} \hbar \omega,$$  \hspace{1em} (3.3)

where it was introduced the so called fragmentation (or driving) potential, as the difference between the top of the Coulomb barrier and $Q$-value

$$V_{frag}(r_B) = V_C(r_B) - Q.$$  \hspace{1em} (3.4)

The second component in Eq. (2.8) contains the logarithm of the Coulomb-Hankel function inside the Coulomb barrier which, according to Eq. (2.6), is proportional with the Coulomb parameter $\chi$. The third part

\[\text{FIG. 3: Logarithm of reduced half lives (2.8) for proton emitters versus Coulomb parameter (2.9). Open circles denote emitters with } Z < 68, \text{ while dark circles emitters with } Z > 68.\]
contains the reduced width squared which, according to Eq. (2.2), is proportional with the modulus of the internal wave function squared. For a shifted ho well one has for the ground state

$$|f_0^{(int)}(r)|^2 = A_2^2 e^{-\beta(r-r_0)^2}.$$  \hspace{1cm} (3.5)

By using the notation $\gamma \equiv \gamma_0$ and Eq. (3.3) one obtains the following relation

$$\log_{10} \gamma^2(r_B) = -\frac{\log_{10}e^2}{\hbar \omega} V_{frag}(r_B) + \log_{10} \frac{\hbar^2 A_0^2}{2\epsilon \mu r_B}$$ \hspace{1cm} (3.6)

4. DECAY RULE FOR REDUCED WIDTHS

Most of experimental data refer to the $\alpha$-decay. Therefore there were analyzed reduced widths in $\alpha$-decays connecting ground states of even-even nuclei. In figure it is plotted the logarithm of the experimental reduced width squared, by using the above relation versus the fragmentation potential. The data are divided into five regions of even-even $\alpha$ emitters as follows

1) $Z < 82, 50 < N < 82$ Fig. (c), stars ;  \hspace{0.5cm} (4.1)
2) $Z < 82, 82 < N < 126$ Fig. (a), crosses ;  \hspace{0.5cm} (4.2)
3) $Z > 82, 82 < N < 126$ Fig. (b), circles ;  \hspace{0.5cm} (4.3)
4) $Z > 82, 126 < N < 152$ Fig. (c), squares ;  \hspace{0.5cm} (4.4)
5) $Z > 82, N > 152$ Fig. (d), triangles .

In calculations it was used the value of the touching radius, i.e.

$$r_B = 1.2(A_D^{1/3} + A_C^{1/3})$$ \hspace{1cm} (4.2)

where the approximation $K_{\omega'} \approx \delta_{\omega'}$ in Eq. (2.3) is fulfilled with 90% accuracy for the most deformed nuclei. Notice that regions 1-4 contain rather long isotopic chains, while in the last region 5 one has not more than two isotopes/chain. This is the reason why, except for the last region 5, the reduced width decreases with respect to the fragmentation potential, according to the theoretical prediction given by (3.6). Notice that in the regions 1 and 4, above $^{100}$Sn and $^{208}$Pb double magic emitters, respectively, the $\alpha$ parameter of the $\alpha$-daughter potential is larger than for regions 2 and 3, corresponding to charge numbers around the double magic nucleus $^{208}$Pb. On the other hand, one obtains the largest amplitudes of the cluster wave function in the regions 2 and 3.

An interesting observation can be made from figure where it is plotted the difference $\log_{10} T - c_2V_{frag}(r_B) - c_3(r_B, A_0^2)$ versus the Coulomb parameter $\chi$, by using the same five symbols for the above described regions. Amazingly enough there were obtained three lines, corresponding to different amplitudes of the cluster wave function. The regions 1 and 4, corresponding to emitters above double magic nuclei $^{50}$Sn and $^{208}$Pb, respectively, have practically the same internal amplitudes $A_0$. The same is true for the regions 3 and 5.

The linear dependence of $\log_{10} \gamma^2$ versus the fragmentation potential (3.6) remains valid for any kind of cluster emission. This fact is nicely confirmed by heavy cluster emission processes in figure (a), where it is plotted the dependence between the corresponding experimental values for the same decays in figure. Here it is also plotted a similar dependence for $\alpha$-decays corresponding to the same heavy cluster emitters. The straight line is the linear fit for cluster emission processes, except $\alpha$-decays

$$\log_{10} \gamma^2 = -0.586(V_C - Q) + 15.399$$ \hspace{1cm} (4.3)

FIG. 4: The logarithm of the $\alpha$-decay reduced width squared versus the fragmentation potential (3.4) for regions of the nuclear chart described by (4.1).

In this way one obtains that indeed the logarithm of the half life is of the Viola-Seaborg type

$$\log_{10} T = c_1(r_B)\chi + c_2V_{frag}(r_B) + c_3(r_B, A_0^2)$$ \hspace{1cm} (3.7)

because the fragmentation potential contains the product $Z_D Z_C$. Its coefficient depends upon the touching radius, but this radius has a very small variation along various isotopic chains. Notice that the slope in Eq. (3.6) has a negative value and it is connected with the shape of the interaction potential (ho energy $\hbar \omega$), while the free term gives information about the amplitude of the cluster wave function.

Our calculation has shown that the linear relation (3.6) but with different coefficients, remains valid in the most general case of the double folding plus repulsive interaction between fragments, used in Refs. [13, 14].
FIG. 5: The difference \( \log_{10} T - c_2 V_{\text{frag}}(r_B) - c_3 \) versus the Coulomb parameter \( \chi \) for five different regions described by Eq. (4.1). The straight lines are the corresponding linear fits.

The above value of the slope \(-\frac{\log_{10} e^2}{\hbar \omega}\) in Eq. (3.6) leads to \( \hbar \omega \approx 1.5 \) MeV, with the same order of magnitude as in the \( \alpha \)-decay case. The relative large scattering of experimental data around the straight line in figure 6 can be explained by the simplicity of the used cluster-core potential (3.1).

Let us mention that a relation expressing the spectroscopic factor (proportional with the reduced width) for cluster emission processes was derived in Ref. [15]

\[
S = S_\alpha^{(A_C-1)/3},
\]

where \( A_C \) is the mass of the emitted light cluster and \( S_\alpha \sim 10^{-2} \). As can be seen from figure 6 (b) between \( A_C \) and \( V_{\text{frag}} \) there exists a rather good linear dependence and therefore the above scaling law can be easily understood in terms of the fragmentation potential.

Concerning the reduced widths of proton emitters in Refs. [11, 16] it was pointed out the correlation between the reduced width and the quadrupole deformation. This fact can be seen in figure 7 (a), where the region with \( Z < 68 \) corresponds to \( \beta > 0.1 \) (open circles), while the other one with \( Z > 68 \) to \( \beta < 0.1 \) (dark circles). The two linear fits have obviously different slopes. This dependence is induced by the propagator matrix \( K_{\ell \ell'}(\beta, r) \) in Eq. (2.4).

Notice that the two dark circles with the smallest reduced widths correspond to the heaviest emitters with \( Z > 80 \). At the same time one sees from figure 7 (b) that the same data are clustered into two regions, which can be directly related with the fragmentation potential (3.4). Here the two linear fits in terms of the fragmentation potential, corresponding to the two regions of charge numbers, have roughly the same slopes, but different values in origin. Thus, the two different lines in figure 3 can be directly connected with similar lines in figure 7 (b). They correspond to different orders of magnitude of the fragmentation potential, giving different orders to wave functions and therefore to reduced widths.

A special case is given by the two-proton emission. This process was predicted log time ago [17], but only few such emitters were recently detected until now. Let us mention that the most recent general treatment of the two-proton emission process, assuming a three-body dynamics, is given in Refs. [18] (and the references therein).

The experimental half-lives versus Coulomb parameter are given in figure 8 (a) by triangles. The emitter charges are also pointed out. Here it is assumed a simplified version, where the light emitted cluster is supposed to be a di-proton with the charge \( Z_C = 2 \). In this case one can use the factorisation of the decay width (2.1). One sees that half lives (triangles) follow the general trend (dashed line) of the usual proton emitters, given in the same figure by the symbols in figure 3. In figure 8 (b) it is plotted the logarithm of the reduced width squared versus the fragmentation potential by triangles. One indeed observes that the slope of the fitting dashed line has a negative value \(-\frac{\log_{10} e^2}{\hbar \omega}\), but with a much larger value in comparison with proton emitters, given by the same symbols as in figure 7 (b). The three lines in this figure, corresponding to proton and two-proton radioac-
activity, respectively, are given by

\[ \log_{10} \gamma^2 = -0.283(V_C - Q) + 1.329, \quad Z < 68 \]
\[ \log_{10} \gamma^2 = -0.365(V_C - Q) + 3.440, \quad Z > 68 \]
\[ \log_{10} \gamma^2 = -2.075(V_C - Q) + 4.403. \quad (4.5) \]

The \( h \omega \) energy is \( h \omega \approx 1.5 \) MeV for proton emission (i.e. the same order as for heavy cluster radioactivity and \( \alpha \)-decay) and \( h \omega \approx 0.2 \) MeV for two-proton emission, by considering the di-proton approximation. This small \( h \omega \) energy is connected with the use of Eq. (4.2) in estimating the spatial extension of the di-proton system \( R_C \) and therefore the fragmentation potential. In reality the di-proton is not a bound system and it changes its size during the barrier penetration. Our microscopic estimate, by using a pairing residual interaction between emitted protons from \( ^{45}\text{Fe} \), evidenced that the size of the wave packet increases in the relative coordinate by 1 fm over a distance of 1 fm in the region of the nuclear surface. Actually the reduced width defined by (2.1) can be generalized by using the other extreme scenario, given by a sequential emission, where this relation is integrated over all possible energies of emitted particles \( [2] \).

An interesting observation concerns the amplitude \( A_0 \) in Eq. (3.6), given by the value of fitting lines in origin. It has similar values for both two-proton and proton emitters with \( Z > 68 \).

Now let us analyze \( \alpha \)-decay processes to excited low-lying \( 2^+ \) states. There were considered more than 70 decays of even-even rotational, vibrational and transitional nuclei \( [13, 14] \). The hindrance factor (HF) is defined as the ratio of reduced widths squared connecting the ground states and ground to excited states with the angular momentum \( l = 2 \), i.e.

\[ HF = \frac{\gamma_2^0}{\gamma_2^2}. \quad (4.6) \]

Thus, by using (3.6), the logarithm of the HF becomes proportional with the excitation energy of the daughter nucleus

\[ \log_{10} HF = \frac{\log_{10} e^{-2E_{(ex)}^{(2)}}}{h/\omega} + \log_{10} \frac{A_0^2}{A_2^2}. \quad (4.7) \]

It is worth mentioning that this relation is equivalent with the Boltzman distribution for the reduced width to the excited state \( \gamma_2 \). In Refs. \( [20, 21] \) such a dependence was postulated in order to describe HF’s.

In figure 9 it is plotted the logarithm of the HF versus the excitation energy for rotational nuclei with \( E_{(ex)}^{(2)} < 0.1 \) MeV, by using the same notations given by Eq. (4.1). As a rule the HF’s have small values and therefore the wave functions have similar amplitudes \( A_0 \approx A_2 \). Notice that the region 4 in figure 9 (b), with \( Z > 82, 126 < \( Z < 68 \).
FIG. 9: The logarithm of the hindrance factor versus the excitation energy of the daughter nucleus for rotational nuclei. The symbols and numbers correspond to the regions given by Eq. (4.1).

N < 126, contains most of rotational emitters (33 out of 41). Moreover, our analysis has shown that here the HF has an almost constant value along various isotopic chains, due to the fact that the energy range is very short (about 100 keV).

In figure 10 it is given the same quantity, but for transitional and vibrational nuclei, with $E^{(ex)} > 0.1$ MeV. The situation looks here to be different and more complex, with respect to rotational nuclei. The best example is given by the same region 4 in figure 10 (c), where the slope has a positive value, as predicted by Eq. (4.7). Notice that this region contains almost half of the analyzed vibrational emitters (14 out of 31).

5. CONCLUSIONS

Cold emission processes in terms of the well known factorisation of the decay width between the penetrability and reduced width squared, were analyzed. Based on a simple model of the two-body dynamics, namely a shifted harmonic oscillator potential surrounded by the Coulomb interaction, it was derived an universal analytical relation expressing the logarithm of the reduced width squared as a linear function in terms of the fragmentation potential, defined as the difference between the Coulomb barrier and the Q-value. Notice that the slope has the same order of magnitude, corresponding to an ho energy $\hbar \omega \approx 1.5$ MeV, for all decay processes, except the di-proton emission, where the factorisation (2.1) is not anymore valid.

This rule is a consequence of the fact that the logarithm of the wave function squared is proportional with the difference between the height of the Coulomb potential at a given radius and the energy of the system. It is fulfilled with a reasonable accuracy by experimental data, describing transitions between ground states as well as for $\alpha$-transitions to excited states in the most relevant region with $Z > 82$, $126 < N < 152$. As a particular case, the two regions in the dependence of proton emitters half-lives, corrected by the centrifugal barrier, versus the Coulomb parameter are directly related with the corresponding regions of the fragmentation potential.

Thus, the clustering character of the wave function inside the Coulomb barrier (i.e. the fragments are already preformed here) is evidenced by this linear rule in terms of the fragmentation potential. On the other hand the well known scaling relation, connecting the spectroscopic factor with the mass of the emitted cluster, can be nicely explained in terms of the fragmentation potential.

6. ACKNOWLEDGMENTS

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