Effect of gravitational settling on the collisions of small inertial particles with a sphere

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The rate at which small inertial particles collide with a moderate-Reynolds-number spherical body is found to be strongly affected when the formers are also settling under the effect of gravity. The sedimentation of small particles indeed changes the critical Stokes number above which collisions occur. This is explained by the presence of a shielding effect caused by the unstable manifolds of a stagnation-saddle point of an effective velocity field perceived by the small particles. It is also found that there exists a secondary critical Stokes number above which no collisions occur. This is due to the fact that large-Stokes number particles settle faster, making it more difficult for the larger one to catch them up. Still, in this regime, the flow disturbances create a complicated particle distribution in the wake of the collector, sometimes allowing for collisions from the back. This can lead to collision efficiencies higher than unity at large values of the Froude number.

I. INTRODUCTION

The capture of small suspended particles by a streamlined or bluff body is an important process in many natural systems. Besides wind pollinisation [1] or the collection of phytoplankton by passive suspension-feeding invertebrates [2], it plays a crucial role in planet formation [3], in particular when estimating the growth rate of planetesimals by sweep-up and accretion of small-size dust grains [4]. Several atmospheric processes also involve the capture of small particles by a larger drop or ice crystal, and estimating such phenomena is key in the parameterization of cloud-resolving meteorological models [5]. Collection rates are for instance needed when accounting for the growth of raindrops by accretion of cloud droplets [6], for the riming of supercooled droplets by ice crystals [7], and for the scavenging of aerosols during wet deposition [8]. Capturing particles in dirty gases is also an important industrial challenge. Most techniques rely on the collection of particles by water drops, such as in reverse jet scrubbers [9]. In all these applications, achieving precise estimates requires, on the one hand, elucidating mesoscopic fluid-dynamical effects that determine whether or not impaction occurs, and on the other hand, specifying the microphysical features and processes that affect the outcome of such collisions and a possible capture by the collector [10].

A large object moving across a fluid creates a flow that pushes fluid elements away from its surface. Small-size particles uniformly suspended in the fluid are then drifted aside, so that the rate at which they are collected is less than the ideal rate obtained by considering the volume swept by the large object. The fraction of particles that actually collide defines the collision efficiency, a quantity that enters most kinetic models. An efficiency larger than zero requires that particles detach from the fluid streamlines. This can be brought about by several effects, including Brownian motion, boundary interception, and inertia [11]. While the formers can be treated analytically by probabilistic and geometric arguments, inertial impaction is still mainly addressed with empirical approaches. By fitting numerical and experimental measurements at moderate values of the large-object Reynolds number, one obtains formulae expressing the collision efficiency as a function of the small-particle stopping time [12, 13] that are today at the basis of model parameterization.

Much work has been recently devoted to improving collision efficiencies and providing refined statistics that are of importance to the collisional processes. Many aspects have been covered, including the flow modifications due to the collected particles [14, 15], the statistics of impact velocities [16], fluctuations in the particle concentration [17], the effect of a large Reynolds number of the collector [18], the presence of turbulent fluctuations in the surrounding fluid [19, 20], the outcomes of elastic rebounds [21], and the fluid-structure interaction between the collector and the flow [22]. However an effect of particular importance to atmospheric applications has been neglected so far. Usually, while the collecting object (raindrop or ice crystal) falls through the fluid under the effect of gravity, the collected, small-size particles are themselves settling and decouple from the fluid. Gravitational settling is known to have drastic impacts on the dynamics and collisions between small inertial particles, in particular when the carrier flow is turbulent [23, 24]. Except in specific cases related to aerosol washout [25, 12, 25], not much is known on the way settling affects collision efficiencies due to inertial impaction.

We consider here the fundamental problem of a large-size spherical object freely falling in an incompressible flow at rest with a small or moderate Reynolds number. On its way it collects small-size heavy inertial particles that themselves settle at lower terminal speeds. Using numerical simulations and phenomenological arguments we quantify the collision efficiency as a function of the three dimensionless parameters characterizing the problem, namely the large-particle Reynolds number $Re$, the small particle Stokes number $St$, and the Froude number $Fr$, that measures the importance of the involved hydrodynamical forces with respect to gravity. As already
known in the case without gravity, inertial impaction occurs only if the small-particle Stokes number is large enough. We find that gravity leads to a shielding effect that increases the corresponding critical Stokes number. Furthermore, in the presence of gravity, as already observed in [23], there exists a second critical Stokes number above which small particles fall so fast that they are never collected by the sphere. Concretely, this means that inertial impaction can only occur when the small-particle sizes belong to a specific window. We moreover remark that inertial impaction can only occur when the small-particle Stokes number is large enough. We find that gravity leads to a shielding effect and the observed spreading comes from variations in pressure, temperature, and drop’s shape, which tend such considerations to the case of a finite Reynolds number of the large sphere and find that our results stay there valid. Finally, section VII encompasses concluding remarks and perspectives.

II. MODEL AND PARAMETERS

We consider a large spherical particle with diameter d immersed in a three-dimensional incompressible fluid whose dynamics solves the Navier–Stokes equation. The flow is assumed at rest at infinity and the velocity field obeys a no-slip boundary condition at the surface of the sphere. The spherical particle represents a collector. It moves with a steady speed U obtained by balancing gravity, buoyancy and the drag exerted by the fluid. Without loss of generality, we work in the reference frame attached to the sphere and whose origin is at its center. The fluid flow is thus at rest at the particle surface, namely the velocity field satisfies \( \mathbf{u}(x, t) = 0 \) at \( |x| = d/2 \). It tends to \( \mathbf{U} \mathbf{e}_z \) when \( |x| \to \infty \).

Small heavy particles are suspended in the fluid. Their positions \( x_p \) and velocities \( v_p \) follow the dynamics

\[
\frac{dx_p}{dt} = v_p,
\]

\[
\frac{dv_p}{dt} = \beta \frac{Du}{dt} (x_p, t) - \frac{1}{\tau_p} |v_p - u(x_p, t)| + (1 - \beta) \mathbf{g},
\]

where \( \beta = 3 \rho_f / (2 \rho_p + \rho_f) \) is the added mass factor, \( \tau_p = a^2 / (3 \beta \nu) \) the particle response time, \( a \) designating its radius, \( \rho_p \) its mass density, \( \rho_f \) and \( \nu \) the fluid density and kinematic viscosity, respectively, and \( \mathbf{g} = -g \mathbf{e}_z \) is the acceleration of gravity. The particle sizes are assumed sufficiently small to consider only three forces in the right-hand side of (1), namely the added-mass, the viscous drag, and buoyancy, and to neglect the Basset–Boussinesq history term and Faxén’s finite-size corrections. In addition, particles are sufficiently diluted to neglect their possible feedback onto the fluid. We moreover suppose that they are uniformly distributed at \( z \to -\infty \) with a velocity equal to their terminal speed, namely \( v_p = (1 - \beta) \tau_p g + \mathbf{u} \mathbf{e}_z \) in the reference frame of the large spherical particles.

One usually writes the system in a non-dimensional form by expressing length scales in units of the particle diameter \( d \) and timescales in terms of the sweeping time \( d/U \). This leads to introduce three non-dimensional parameters:

- the Reynolds number \( Re = U d/\nu \), which characterizes the flow around the large sphere,
- the Froude number \( Fr = U^2 / (d g) \), which measures the importance of gravity,
- the Stokes number \( St = \tau_p U / d \), which quantifies the small-particles inertia.

The Stokes number, together with the added mass factor \( \beta \), are specified by the nature of the collected particles. We consider them as variable parameters, because in most applications, such particles are polydisperse and have a broad distribution of sizes and masses. The Stokes number measures the inertia of the small particles. When \( St = 0 \), they behave as tracers, exactly follow the fluid streamlines, and collide with the large object only by diffusion or interception. When \( St \to \infty \), the particles completely detach from the flow and collide with the collector from the moment that they are located on its path. In the presence of gravity, the small particles are themselves settling. The importance of gravity with respect to inertia is often measured in terms of the gravitational Stokes number \( Sv = (1 - \beta) \tau_p g / U = (1 - \beta) St / Fr \) (see, e.g., [23]), which corresponds in our case to the ratio between the terminal velocities of the small particles and that of the large sphere. Clearly, when \( Sv > 1 \), the inertial particles settle faster than the sphere and collisions occur in the wake.

The Reynolds and the Froude numbers depend upon the considered application. Figure I sketches the typical ranges covered by these parameters in atmospheric physics, in oceanology, in planet formation and in industrial scrubbers. In the case of raindrops falling in the atmosphere, the velocity \( U \) is given by the large drop terminal speed, \( g \) is the near-Earth-surface acceleration of gravity and the observed spreading comes from variations in pressure, temperature, and drop’s shape, which tends to become more oblate when its diameter increases above 1 mm (see, e.g., [25]). Setting crystals define a broader range of parameters depending on meteorological conditions and how densely ice is packed. Organic matter in the ocean, such as phytoplankton, has a mass.
density very close to water and is thus settling at a rather low speed \( \beta \), whence small values of the parameters \( Re \) and \( Fr \). Understanding the rate at which the larger particles accrete smaller sediments is important to quantify the downward piggy-back transport \( 27 \), and thus the efficiency of the CO2 oceanic pump \( 29 \). For planet formation, gravity depends upon the distance \( r \) between the planetesimals and the star, which is expressed here in astronomical units (a.u.). The protoplanetary disk is made of gas whose thermodynamical properties depend on \( r \). The radial pressure of the disk maintains the gas at a sub-Keplerian orbiting speed. The planetesimals, whose sizes range from several meters to hundreds of kilometers, have a drag with the gas and slowly drift inward at velocities of the order of tens of meters per seconds \( 19 \). On their path, they collect additional dust to eventually reach the sizes of planetary embryos \( 30 \). Finally, in the case of industrial wet scrubbers, a jet of water droplets with sizes of the order of hundreds of microns is sent against a flow of polluted gas in order to collect suspended particulate matter. The slip velocity \( U \) of droplets depends upon the distance from the jet’s nozzle.

In all these applications, we aim at understanding the efficiency with which the large spherical collector accretes smaller particles. In the absence of fluid flow, the rate at which collisions occur is obtained by considering the number of small particles contained in the volume swept by the large sphere per unit time. This leads to the “ideal” collision rate \( Q_{\text{no fluid}} = (\pi d^2/4) U n \), where \( n \) designates the small-particles number density that is assumed uniform. In the presence of a fluid flow, small particles are possibly drifted away from the sphere, so that the actual collision rate \( Q_{\text{fluid}} \) differs from the above estimate. Such a discrepancy is measured in terms of the collision efficiency defined as \( \mathcal{E} = Q_{\text{fluid}}/Q_{\text{no fluid}} \). This quantity, which enters into all model parametrizations, is our main observable. We aim at understanding how it depends both on the features of the small accreted particles (\( \beta \) and \( St \)) and on the collector parameters (\( Re \) and \( Fr \)).

We focus here on moderate values of the falling-sphere Reynolds number \( Re \), so that the perturbed flow is steady and axisymmetric \( 31 \). This allows for a more systematic investigation as a function of the other parameters. We first study the case when the flow has no inertia (\( Re = 0 \)). The velocity field is given by the Stokes’ equation, for which an explicit analytical solution is known (see, e.g., \( 32 \)). By comparing to the results of direct numerical simulations, we find that, as expected, this simplified case reproduces all the qualitative features observed for \( Re \lesssim 15 \). Simulations are performed using the CimLib CFD finite-element code \( 33 \), which is able to solve the Navier–Stokes equations with an arbitrary immersed object using an adaptive meshing \( 34 \).

### III. Collision Efficiency

In this section, we neglect the fluid inertia and fix the Reynolds number to \( Re = 0 \). This allows for a general overview on how the collision efficiency \( \mathcal{E} \) depends on the two parameters \( St \) and \( Fr \), as shown in Fig. 2 for very heavy small-size particles (\( \beta = 0 \)). A first observation is that \( \mathcal{E} > 0 \) only for values of the Stokes and the Froude numbers in a set with a triangular shape. In other words, collisions occur only when \( Fr > Fr^\star \approx 11 \) and for a fixed Froude number above this value, the efficiency is non-zero only for Stokes numbers in the interval \( St^\star_1(Fr) < St < St^\star_2(Fr) \). When \( Fr \to \infty \), the lower critical Stokes number \( St^\star_1 \) approaches from above the value \( St_{\text{no grav}}^\star \approx 0.605 \) already identified in the absence of gravity \( 13, 21 \). This means that gravitational settling tends to increase

![FIG. 1. Typical ranges of Reynolds and Froude numbers encountered in applications. Different examples illustrate typical sphere radii and settings.](image)

![FIG. 2. Collision efficiency as a function of the small-particles Stokes number \( St \) and of the Froude number \( Fr \) for the Stokes flow \( Re = 0 \) and for \( \beta = 0 \). The black dashed line \( Fr = St \) corresponds to a settling Reynolds number of unity. The white curve in the colour area delimits the parameter region where backward collisions occur. The bold black curve is the fit \( 2 \) to the critical line. The red curve corresponds to the bounds \( 43-45 \) obtained in the next section.](image)
its value. The upper critical Stokes number (above which no collisions occur) tends asymptotically to infinity as $St_2^* \propto Fr$ when $Fr \to \infty$. This critical Froude number is bounded from below by the line $Su = (1-\beta)St/ Fr = 1$. As stated above, when $Su > 1$, the small particles settles faster than the collector and collide with it from the back. Such settings could for instance be encountered when interested in the collection of small particles by a rising spherical bubble. Such settings are beyond the scope of this work.

As can be seen from Fig. 2 the critical Stokes numbers $St_1^*$ and $St_2^*$ actually define a single curve in the $(St, Fr)$ parameter space. An approximation is more conveniently found in the $(St, Su)$ space by looking for a fit of the form

$$St^*(Su) = \frac{St_{no\ grav}}{f(\beta_1 Su) + \beta_2 Su},$$

where we use the function $f(x) = (1 - 7x^{2/3}/3 - 5x/3)/(1 + 3x/2)$, and $St_{no\ grav}$ is the critical Stokes number below which no collisions occur in the absence of gravity. The constants $\beta_1, \beta_2$ are two adjustable parameters that possibly depend on the Reynolds number. The black curve shown in Fig. 2 was obtained by choosing $\beta_1 = 1.38$ and $\beta_2 = 0.014$. It gives a very good approximation of the critical curve. As we will see in Sec. 4, this form can also be used to fit the measurements made at finite values of the Reynolds number.

Another observation from Fig. 2 is the presence of a second maximum of the collision efficiency close to the boundary $St = St_2^*$. This elongated region leads to efficiencies higher than unity at large values of both $Fr$ and $St$. Such a surprising behavior comes from collisions by small particles from the back of the sphere. Such collisions are present in the hook-shaped parameter-space region delimited by a white curve in Fig. 2.

Figure 3 represents four horizontal cuts of the previous figure at different values of the Froude number. We separate there contributions from forward and backward collisions. For an infinite value of $Fr$, that is in the absence of gravity, all collisions occur on the head-on hemisphere of the large particle and the obtained efficiency is that known for Stokes flow around a sphere, which grows linearly from zero at $St = St_{no\ grav}^* \approx 0.605$ and approaches 1 when $St \to \infty$. For the smallest value of the Froude number ($Fr = 12$), which is right above the threshold $Fr^2 \approx 11$, one clearly observes that small particles collide only if their Stokes number in a narrow range (here $1.4 < St < 3.8$). Again, there are no backward collisions. They occur only at intermediate values of the Froude number. As can be seen from Fig. 3 for the curves associated to $Fr = 24, 48$, and 80, collisions that occur from the back lead to a non-trivial dependence of $E$ upon $St$ with several maxima. At the largest values of the Froude number, this can lead to collision efficiencies larger than unity. We will turn back to this behavior in the next two sections.

The efficiency curves as a function of the Stokes number can be fitted. We use the approximation

$$E(St; Fr) = E_{front}(St; Fr) + E_{rear}(St; Fr)$$

with

$$E_{front} \approx \alpha_1 \frac{(St_1^* - St - 3/2)^{2/3}(St - St_1^*)}{St_2^*^{1/3} + (St - St_1^*)(2 St_2^* - St_1^*)},$$

$$E_{rear} \approx \alpha_2 \frac{St_1^* 0.88}{40 + 15 St - 13.8 St_2^*}$$

for $St_1^* < St < St_2^*$, and where by convention $E_{rear}$ vanishes when the right-hand side is negative. The constants are adjusted to $\alpha_1 = 0.95$ and $\alpha_2 = 0.04$ for $Re = 0$ but might depend on the Reynolds number. The results of such fits are shown in Fig. 4. While the approximation of front collisions is given by a smooth rational function, the complexity of backward collisions is reduced to a simple piecewise linear function.
We next turn to investigate the influence on the collision efficiency of the parameter $\beta$, which controls both the added-mass force and buoyancy effects. Figure 5 shows $\mathcal{E}$ as a function of $St$ for $Fr = 24$ fixed and various values of $\beta$ that approximately correspond to mass density ratios $\rho_p/\rho_t \approx 1000, 300, 100, 30,$ and $10$. At the smallest values of $\beta$, corresponding for instance to water droplets in the air, one observes a very tiny difference with the case $\beta = 0$. Collision efficiency is slightly depleted by less than $10\%$. For $St$ and $Fr$ fixed, the effect of buoyancy is to decrease the settling velocity of small particles. In principle, this should thus increase their relative velocity with the collector and lead to higher collision rates. The observed depletion must hence be due to added-mass effects. This force is proportional to the fluid acceleration at the particle position. Upstream the collector, the fluid is decelerated in the direction $z$ and accelerated in the transverse directions. The particles are thus pushed aside, decreasing their probability to collide with the sphere.

At larger values of $\beta$, of relevance for instance when considering slowly sinking organic matter in the ocean, the interplay between buoyancy and added mass becomes much less trivial. One observes for the largest values of $\beta$ of Fig. 5 that the collision efficiency even largely exceeds unity, meaning that the flow perturbations are able to make far-side particles converge toward the sphere’s trajectory. As we will see in the next section, this somewhat surprising findings originate from a non-trivial dynamics of the small particles in the vicinity of the collector.

IV. SHIELDING AND CRITICAL STOKES NUMBERS

We have seen in Figs. 2 and 4 that, for a given value of the Froude number, the collision efficiency vanishes outside the interval bounded by two critical Stokes numbers. We have moreover observed that the lowest critical Stokes number, $St_1^\star$ increases when gravity effects increase (i.e. when $Fr$ decreases), while at the same time, the upper critical value $St_2^\star$ decreases. Let us first focus on the case $\beta = 0$. The effect of gravity is then equivalent to considering that the small particles are in an effective fluid flow $\mathbf{u} = \mathbf{u} + \tau_p \mathbf{g}$ given by the sum of the fluid velocity and their settling speed. When the gravitational Stokes number $Sv$ is less than unity, $\mathbf{u}$ has two stagnation points, at the front and at the rear of the sphere. The left-hand panel of Fig. 6 shows the streamlines of the effective flow $\mathbf{u}$ represented in the plane of symmetry ($\rho, z$), where $\rho^2 = x^2 + y^2$. The two stagnation points sit on the $z$-axis of symmetry and are saddle. The unstable manifolds of the upstream point are heteroclinic orbits that delimit a recirculation zone around the particle. Such separatrices act as a shield around the large sphere. Small particles approaching the collector are pushed away from it, as for instance illustrated by the blue trajectories on the left half-plane. For the larger Stokes number whose trajectories are shown on the right half-plane, the effective vortices of this recirculation zone are even able to entrain particles that have sufficiently decelerated to project them toward the back of the sphere.
cause \((1 - St/Fr)^2 - 1 < 0\). Only one of them satisfies \(z* < -d/2\) and reads \(d/(2z*) = -2\sin(\varphi/3)\) with \(\sin \varphi = 1 - St/Fr\) and \(0 < \varphi < \pi/2\). One clearly observes that if \(Fr \rightarrow \infty\), then \(\varphi \rightarrow \pi/2\) and \(d/(2z*) \rightarrow -1\), so that the stagnation point is at the sphere’s surface in the absence of gravity. If \(Fr \rightarrow St\), then \(\varphi \rightarrow 0\) and \(d/(2z*) \rightarrow 0\), so that the stagnation point goes to \(-\infty\).

At intermediate values of the Froude number, the shield is thus at a finite distance from the spherical collector. This stagnation point, associated with \(v_p = 0\), defines a fixed point of the position-velocity particle dynamics. A linear stability analysis restricted to the plane \(\rho = 0\), \(v_p = 0\) gives the two eigenvalues

\[
\lambda_{\pm} = \frac{1}{2}\tau_p \left( \pm \sqrt{1 + 4\tau_p \partial_z u_z(z*)} - 1 \right).
\]

As \(\partial_z u_z(z*) < 0\), this fixed point is always stable on the \(\rho = 0\) manifold. The two eigenvalues are real negative if \(\partial_z u_z(z*) > -1/(4\tau_p)\). In this case, all trajectories located upstream on the axis of symmetry converge to the stagnation point located at \((0, z*)\). This is illustrated by the trajectory associated to \(St = 1\) on the right-hand panel of Fig. 3. A necessary condition for collisions to occur is that the particles located on the axis of symmetry reach the right-hand side of this fixed point (cases \(St = 6\) and \(St = 12\) in the right panel of Fig. 3). This is just necessary, but not sufficient, as illustrated by the \(St = 6\) case, in which the spiralling trajectory is not broad enough to hit the sphere. Collisions require that

\[
\partial_z u_z(z*) = \frac{3U}{d} \left[ d/(2z*) \right]^4 - \frac{d}{[d/(2z*)]^2} < -1/(4\tau_p),
\]

so that \(St > 1/3\) and

\[
\frac{1}{2} \left( 1 - \sqrt{1 - \frac{1}{3St}} \right) < \left[ \frac{d}{2z*} \right]^2 < \frac{1}{2} \left( 1 + \sqrt{1 - \frac{1}{3St}} \right).
\]

These two conditions lead to

\[
Fr > \frac{St}{1 - \sin \left[ 3 \arcsin \left( \sqrt{\frac{1}{2} \left( 1 - \sqrt{1 - 1/(3St)} \right)} \right) \right]},
\]

\[
Fr < \frac{St}{1 - \sin \left[ 3 \arcsin \left( \sqrt{\frac{1}{2} \left( 1 + \sqrt{1 - 1/(3St)} \right)} \right) \right]}.
\]

The two corresponding curves are represented in Fig. 4. They indeed give bounds on the lower and upper critical Stokes numbers but can unfortunately not be used as fits. As they just correspond to a necessary condition, they clearly stand below the approximation previously proposed. The discrepancy is larger for the upper critical Stokes number for which the stagnation point is located far upstream the collector. It is in that case clear that penetrating the shield is markedly not sufficient to warrant collisions. Still, the arguments leading to these two branches explain why there exist two critical Stokes numbers and show that both stem from similar mechanisms. Below \(St^*\), as well as above \(St^*\), the unstable manifold of the upstream saddle stagnation point act as a shield that prevent particles from penetrating too far in the recirculation zone and thus from hitting the collector.

It is clear from the considerations drawn above in the case \(\beta = 0\), that the near-sphere stagnation points and the associated shield strongly determines possible collisions of small-size particles with the collector. We now examine the effects of a finite added-mass parameter. This time, the particles are as if suspended in an effective fluid flow, which incorporates the added-mass term, namely \(\vec{u} = \vec{u} + (1 - \beta) \tau_p \vec{g} + \vec{\tau}_p (D\vec{u}/Dt)\) and thus adds up a compressible component. Figure 7 shows the streamlines of this effective flow for the same parameters as in Fig. 4 but with this time \(\beta = 0.05\) (left panel) and \(\beta = 0.15\) (right panel), thus stressing the modifications of the physical-space diagrams due to a finite added mass. Differences occur at a qualitative level. First of all, the two centers that were sitting on the side of the sphere and defining the effective recirculation flow are now sources. Moreover, the fore-aft symmetry is broken. For the smallest value of the added mass parameter, the two upstream and downstream saddle fixed points are still present, but they are no more connected by any heteroclinic orbit. For the larger value of \(\beta\), a bifurcation has occurred and the upstream saddle point has now become a combination of two saddles and a sink located on the axis of symmetry \(\rho = 0\). In both cases, particle trajectories experience a compression in the transverse direction and are pushed inward before approaching the sphere, explaining the efficiencies above unity that are observed in previous section. Additionally, one observes for the larger value of \(\beta\) that the sink present in the effective flow gives rise to a stable fixed point for the particles dynamics. Some particles are expected to get trapped there.

We have seen that the collisions between the small
particles and the collector are largely determined by the near-sphere dynamics. This process is strongly influenced by the shield-like structure and the resulting recirculation that appear in the local topology of the effective flow. Such effects give rise to a variety of behaviors of the particles trajectories, including quasi-rebounds, head-on collisions, deflections, trapping and backward collisions.

V. CAUSTICS AND BACKWARD COLLISIONS

Another way to interpret the various behaviors observed in the previous section consists in drawing, instead of individual trajectories, the steady-state density of particles with given characteristics. Figure 8 gives such an overview for a fixed value of the Froude number, here Fr = 24, and different Stokes numbers representative of the observed regime. In the case $A$, the Stokes number is below the lower critical value $St_{\text{c}}$. The shield effect prevents small particles from attaining the collector. They slightly concentrate along an envelope that surrounds the sphere and create a void that extends far away in the wake. The case $B$ is representative of what is happening above the lower critical Stokes number. A fraction of the particles collides with the sphere, while another is deflected and passes around the sphere. A void is again created in the wake, but it gets refilled rather rapidly under the influence of the converging streamlines of the fluid flow. Still, particles have a rather large inertia that make them overshoot this tendency, leading to an over-concentration downstream, along the axis of symmetry. This is a manifestation of the formation of caustics, and thus of the presence of regions where particles overlap in space with different velocities. These behaviours are still present in the case $C$ where, in addition, some particles experience backward collisions. It is clear from the density profile that such particles are first strongly decelerated when penetrating the shielded area and are then entrained by the recirculating effective flow before being pushed back and collected on the tail of the sphere. This process creates an intricate pattern of caustics in the vicinity of the collector. Finally, case $D$ is above the upper critical Stokes number $St_{\text{c}}$. The shield effect again prevents particles from touching the collector and yields the creation around it of a void with a tear-drop shape. This process occurs after several dynamical rebounds of the particles, suggesting that they oscillate along the unstable manifold of the upstream stagnation point. This is evidenced by the presence of several concentric layers in the concentration profile.

To complete the picture, we next turn to quantify further backward collisions by measuring their spatial spread on the collector. Figure 9 represents the distribution of the angle $\theta$ made by the particles position with $-e_z$ at the instant when they impact the sphere. In case $B$, for which there are no collisions from the back, the distribution is peaked over small values of $\theta$, and most collisions occur on the collector’s head. In case $C$, backward collisions are dominant. There is no preferential alignment of the head-on collisions but backward collisions concentrate in several strips on the downstream hemisphere, with a maximum sitting on the collector’s tail. Such strong dependences of the angular distribution of impacts on the small-particle features can have important consequences on the microphysics of accretion and on the shape evolution of the collector.

VI. FINITE REYNOLDS NUMBERS

To substantiate the global picture drawn from previous sections, we here investigate the effect of a finite Reynolds number of the collector. At moderate values of $Re$ (less than $\approx 15$), while the fluid velocity field remains axisymmetric, the “fore-aft” symmetry observed for Stokes flow $(z,t) \mapsto (-z,-t)$ is broken. This clearly implies that the two saddle fixed points of the effective
FIG. 10. Physical-space phase diagram for $S_v = 1/2$ (to be compared to the left-hand panel of Fig. 6). The gray streamlines show the trajectories of tracers in the effective flow $\tilde{u}$ for various Reynolds numbers, as labeled. The bold orange lines are the stable and unstable manifolds associated to the two saddle stagnation points located upstream and downstream the spherical collector.

The velocity field $\tilde{u}$ introduced in Sec. IV are no more symmetric but could also implicate that they are no more connected by the heteroclinic trajectories at the origin of the shield effect. The results of numerical simulations of the Navier–Stokes equation have been used to reconstruct the physical-space streamlines of the effective velocity field. The results presented in Fig. 10 in the case $S_v = 1/2$ show that finite, moderate values of $Re$ do not alter the topology of the effective flow. The heteroclinic orbits are preserved and still bound a well-defined recirculation zone around the collector. The shape of this region varies when increasing the Reynolds number and progressively approach an egg-like shape.

These qualitative similarities are confirmed when measuring collision efficiencies for finite values of the collector Reynolds number. Figure 11 represents $E$ as a function of $St$ for the fixed representative value $Fr = 24$. Two effects are visible when increasing the Reynolds number. First, the contribution of forward collisions becomes larger. This is due to sharper variations of the fluid velocity at $z < 0$ and thus the observed shift of the upstream stagnation point toward the collector. The second observation is that the backward collisions occurring close to the maximum of $E$ has a non-monotonic behavior as a function of the Reynolds number, with a maximum around $Re \approx 5$. This comes from a balance between particles attaining the sphere with a larger speed, and thus less likely to get captured in the effective recirculation zone, and the increased extension of this region allowing more particles to being pushed back toward the collector. The efficiencies shown in Fig. 11 are fairly well fitted by the approximation (3) introduced for $Re = 0$ with fitting parameters $\alpha_1$ and $\alpha_2$ adjusted as a function of $Re$, as reported in Tab. I.

| $Re$ | $\alpha_1$ | $\alpha_2$ | $\beta_1$ | $\beta_2$ |
|------|-------------|-------------|-----------|-----------|
| 0    | 0.95        | 0.04        | 1.38      | 0.014     |
| 1    | 1.10        | 0.067       | 1.22      | 0.0090    |
| 5    | 1.25        | 0.078       | 1.18      | 0.0059    |
| 15   | 1.32        | 0.070       | 1.15      | 0.0042    |

TABLE I. Values of the fitting parameters $\alpha_1$ and $\alpha_2$ used for the approximation (3) of the efficiency in Fig. 11 together with the parameters $\beta_1$ and $\beta_2$ used for the approximation (2) of the critical lines of Fig. 12 for the various Reynolds numbers considered.

To conclude this section, we represent in Fig. 12 the phase diagram in the $(St, Fr)$ parameter space that separates regions with no collisions from those where $E > 0$. Numerical results are shown as symbols, while the solid lines are approximations obtained from the formula (2) with parameters $\beta_1$ and $\beta_2$ fitted to the data. The reported measurements clearly confirm that increasing the Reynolds number systematically broadens the parameter range over which collisions occur.

FIG. 11. Collision efficiency at $Fr = 24$, as a function of the small particles Stokes number and different values of the collector’s Reynolds number, as labeled. The symbols reports the results of numerical simulation, while the solid lines are fits of the form (3) with parameters given in Tab. I.

FIG. 12. Critical lines in the parameter space $(St, Fr)$ for various values of the Reynolds number, as labelled. The symbols are the results of numerical simulations, the solid lines are fit of the form (2) with parameters given in Tab. I.
VII. CONCLUDING REMARKS

We reported in this paper new findings on the effects of small-particle gravitational settling on their accretion by a large spherical collector. Our study shows that such a sedimentation plays a crucial role in determining collision efficiencies and outcomes of accretions. The more noticeable conclusions include the presence of a secondary critical Stokes number above which no collisions occur and the occurrence of backward collisions where accreted particles are swept around the spherical collector before falling on its tail. We provided a physical interpretation of these phenomena in terms of an effective velocity field in which the particles are suspended, that is responsible for the creation of a shield and of an effective recirculation zone around the collector.

The effects that we describe should clearly be taken into consideration in order to improve the parametrization of models used in atmospheric physics and in astrophysics. Neglecting the gravitational settling of the small collected particles lead to drastically misestimating the growth rates of raindrops by collection or the wet deposition rates of heavy aerosols. Additionally, the occurrence of backward collisions can have an important impact on the shape of ice crystals during their growth by riming or of planetesimal in the early Solar system when they accrete dust particles by filtering. We provide in this paper fitting formulae both for the critical Stokes numbers and for the efficiencies that can be of interest to improve parametrizations.

We finally presented some results on the influence of the small-particle added-mass forces. Surprisingly, we found that such effects can be much more significant than those of a finite Reynolds number of the collector. This is particularly relevant at small values of the Froude number, or when the mass density ratio between the fluid and particles cannot be neglected. Added-mass forces could hence be a key ingredient when estimating the collection rate of sinking organic matter in the oceans. The revealed importance of such effects suggests that added-mass forces are clearly needing more attention and require further studies.

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