A “head/tail” plasmon model with a Hubble law velocity profile

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\textbf{ABSTRACT}

We present a model of a hypersonic, collimated, “single pulse” outflow, produced by an event with an ejection velocity that first grows, reaches a peak, and then decreases again to zero velocity in a finite time (simultaneously, the ejection density can have an arbitrary time-variability). We obtain a flow with a leading “head” and a trailing “tail” that for times greater than the width of the pulse develops a linear, “Hubble law” velocity vs. position. We present an analytic model for a simple pulse with a parabolic ejection velocity vs. time and time-independent mass-loss rate, and compare it to an axisymmetric gasdynamic simulation with parameters appropriate for fast knots in planetary nebulae. This “head/tail plasmon” flow might be applicable to other high-velocity clumps with “Hubble law” tails.

\textbf{Key words:} hydrodynamics – shock waves – stars: winds, outflows – ISM: jet and outflows – ISM: Herbig-Haro objects – ISM: planetary nebulae

1 INTRODUCTION

A pattern that is sometimes seen in collimated stellar outflows is a high-velocity, compact “clump”, joined to the outflow source by fainter emission with a linear ramp of increasing velocity as a function of distance from the source. This results in striking “position-velocity” (PV) diagrams (obtained, e.g., from long-slit, high resolution spectra or from millimetre interferometric “position-velocity cubes”) with a linear ramp ending in a bright, high-density condensation.

Alcolea et al. (2001) proposed that clumps with “Hubble law tails” (observed in the CO emission of a collimated, protoplanetary nebula outflow) are produced in “explosive events” (i.e., with a duration much shorter than the evolutionary time of the outflow). A “velocity sorting” mechanism (with higher velocity material racing ahead of slower ejecta) would then produce the observed linear velocity vs. position structure of the tails.

The most dramatic example of “Hubble law tail clumps” is of course found in the molecular fingers pointing away from the Orion BN-KL region (see, e.g., Allen & Burton 1993; Zapata et al. 2011; Bally et al. 2017). The \approx 100 fingers all show CO emission with linearly increasing radial velocities away from the outflow centre, and terminate in compact clumps (observed in H\textsubscript{2} and in optical atomic/ionic lines).

Dennis et al. (2008) presented numerical simulations of variable jets and of outflows composed of discrete “clumps”, and conclude that the clump-like outflows produce a compact “head” (i.e., the clump), followed by a tail of decreasing velocity material. They favour this “clump scenario” for explaining the observed “Hubble law” PV diagrams of clumps in planetary nebulae (PNe). However, even though they obtain trails of decreasing velocity material (between the clumps and the outflow source), these trails do not show either the length or the very dramatic linear velocity vs. position signatures of the observed clumps.

In the present paper we explore a scenario similar to the one of Dennis et al. (2008), but instead of imagining a “clump” ejected from the source (with a well defined ejection velocity), we propose a “single pulse”-type ejection velocity (and density) variability. Basically, during a finite time the source ejects material first at increasing velocities, then reaching a maximum ejection velocity, and finally decreasing down to zero. In principle, within this “ejection episode”, the density of the ejected material could also vary in an arbitrary way.

In sections 2-5 we present a simple analytic model of the resulting “head/tail plasmon” flow, calculate its time-evolution and obtain predicted PV diagrams. We also com-
The position $x_j$ of the fluid parcels (if they had not merged) is given by the free-streaming relation:

$$x_j = (t - \tau')u_0(\tau'),$$

where $t$ is the “evolutionary time” (different from the ejection time $\tau'$, satisfying the condition $t \geq \tau'$). The upper limit $\tau$ of the integrals is given by the free-streaming flow condition:

$$x_{cm} = (t - \tau)u_0(\tau),$$

for the ejected fluid parcels currently (i.e., at time $t$) entering the working surface.

Now, combining equations (13), and considering a uniform environment (with $\rho_a = \text{const}$), we obtain:

$$\frac{\rho_a x_{cm}^2}{2} + x_{cm} \int_{-\tau_0}^{\tau} \rho_0 u_0 d\tau' - \frac{1}{u_0(\tau)} \int_{-\tau_0}^{\tau} \rho_0 u_0^2 d\tau' =$$

$$\tau \int_{-\tau_0}^{\tau} \rho_0 u_0^2 d\tau' - \int_{-\tau_0}^{\tau} \tau' \rho_0 u_0^2 d\tau', \quad (4)$$

which, once the appropriate integrals over $\tau'$ have been carried out, is a quadratic equation which gives us $x_{cm}(\tau)$. If we want to know the position of the working surface as a function of the evolutionary time $t$, we can calculate $t$ as a function of $\tau$ and $x_{cm}(\tau)$ from equation (6).

2.2 Solution for a parabolic $u_0(\tau)$ pulse with constant mass loss rate

Let us now consider an ejection velocity pulse with $u_0(\tau) = 0$ for $|\tau| > \tau_0$ and:

$$u_0(\tau) = v_0 \left[ 1 - \left( \frac{\tau}{\tau_0} \right)^2 \right]; \quad \text{for } |\tau| < \tau_0,$$

a parabola that goes to zero at $\tau = \pm \tau_0$ and has a peak velocity $v_0$ at $\tau = 0$. For the ejection density $\rho_0(\tau)$, we assume that it is proportional to the inverse of the ejection velocity, so that the mass loss rate (per unit area)

$$\dot{m} = \rho_0(\tau) u_0(\tau),$$

is time-independent. However, an arbitrary ejection density variability could be considered within our analytic framework.

With $u_0(\tau)$ and $\rho_0(\tau)$ given by equations (5) we compute the integrals in equation (4), obtaining:

$$\sigma \left( \frac{x_{cm}}{v_0 \tau_0} \right)^2 + f \frac{\tau}{\tau_0} \frac{x_{cm}}{v_0 \tau_0} = g \left( \frac{\tau}{\tau_0} \right), \quad (7)$$

with

$$\sigma \equiv \frac{\rho_a v_0}{2 \dot{m}}, \quad (8)$$

$$f(\eta) \equiv \frac{(2\eta - 1)(\eta + 1)}{3(\eta - 1)}; \quad g(\eta) = \frac{(3 - \eta)(\eta + 1)^2}{12}. \quad (9)$$

3 THE “FREE PLASMON”, $\sigma = 0$ CASE

In the $\sigma \to 0$ limit (see equation (5) of a very low density environment, equation (7) has the solution:

$$\frac{x_{cm}}{v_0 \tau_0} = \frac{g(\tau/\tau_0)}{f(\tau/\tau_0)}, \quad (10)$$

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4 THE $\sigma > 0$ CASE

For $\sigma > 0$, equation (7) can be inverted to obtain:

$$\frac{x_{cm}}{v_0\tau_0} = \frac{1}{2\sigma} \left[ -f \left( \frac{\tau}{\tau_0} \right) + \sqrt{f^2 \left( \frac{\tau}{\tau_0} \right) + 4\sigma g \left( \frac{\tau}{\tau_0} \right)} \right].$$

The centre of mass positions and velocities as a function of $t$ obtained for different $\sigma$ values are shown in Figure 2.

For the $\sigma = 0.1$ case (see Figure 2), $x_{cm}$ and $v_{cm}$ initially follow the $\sigma = 0$ solution (see equation (10)) and start deviating for $t > 0$, when the plasmon head begins to brake in an appreciable way. The $\sigma = 1$ and $10$ solutions show substantial braking for all $t$.

The $\sigma > 0$ solutions show plasmon heads that first accelerate, then reach a maximum velocity, and subsequently brake for increasing times $t$. For $\sigma \ll 1$, the plasmon head first reaches a velocity similar to the asymptotic velocity $v_a$ of the “free plasmon” (see equation (14)) and then slowly slow down for increasing times. For $\sigma > 1$, the velocity of the plasmon head does not reach values $\sim v_a$.

We should note that in the $\sigma > 0$ solutions, all of the mass ejected in the pulse eventually ends up in the plasmon head.

5 POSITION-VELLOCITY DIAGRAMS

Evidently, the “head/tail plasmon” model is attractive for trying to explain fast moving clumps which have a tail of decreasing velocity emission towards the outflow source. When observed with spatially resolved spectroscopy or with interferometric millimeter observations these clumps show position-velocity (PV) diagrams with a high velocity, compact emission at a given position, and a ramp of emission with increasing radial velocities from the source out to the clump.

In Figure 3 we show the positions and velocities of the head at different times, and the velocity of the material in the “tail” of the plasmon. This velocity is directly obtained from the free-streaming relation:

$$u(x,t) = u_0(\tau) = \frac{x}{t - \tau},$$

where $u_0(\tau)$ is given by equation (5). This can be easily done in a parametric way by varying $\tau$ (at a fixed evolutionary time $t$), using the first equality to calculate the velocity $u(x,t)$ and then the second equality for obtaining the corresponding position $x$ along the tail of the plasmon.

Figure 3 shows the PV diagrams obtained for different values of $t$ for four models with $\sigma = 0, 0.1, 1$ and 2. The $\sigma = 0$ model has a PV diagram that becomes more extended along the outflow axis with time, with a plasmon head that shows a decreasing acceleration for increasing times.

The models with higher $\sigma$ values have PV diagrams that have lower peak velocities as a function of $t$. Regardless of the value of $\sigma$, the “head/tail” plasmons develop an almost linear velocity vs. distance “Hubble law” velocity profile at evolutionary times $t \gg \tau_0$. This result follows from equation (10), which in the $t \gg \tau \sim \tau_0$ limit gives $u(x,t) \approx x/t$ (i.e., at a given time $t$ we have a “Hubble law” of slope $1/t$).
The velocity along the outflow axis vs. distance from the outflow source at different evolution times. The plots are labeled with the value of $\sigma$ of the model (from $\sigma = 0$ at the top to $\sigma = 2$ on the bottom). The $\sigma = 0$ frame (top graph) shows the velocity along the tail as a function of $x$ for times $t/\tau_0 = 0$ (shortest curve), 1, 3, 5 and 7 (spatially more extended curve). The $\sigma = 0.1$ frame shows the velocity vs. position at times $t/\tau_0 = 0, 2, 4, 6$ and 8. The $\sigma = 1$ and 2 frames (two bottom graphs) show the velocity vs. position at times $t/\tau_0 = 0, 5, 10, 15$ and 20. The open circles located at the end of each curve show the position and velocity of the head of the plasmon.

### 6 NUMERICAL SIMULATION

We have computed an axisymmetric gasdynamic simulation of a “head/tail plasmon” with parameters for a high-velocity clump in a PN (see, e.g., Alcolea et al. 2001) using the Walicxe-2D code (Esquivel et al. 2009). We use a setup with an adaptive mesh with 5 refinement levels giving a maximum resolution of 14.64 AU in a computational domain of 15000 $\times$ 3750 AU. We used a reflective boundary condition on the symmetry axis and free outflow for all of the other boundaries.

The ejection velocity pulse is imposed at $x = 0$, with a radius $r_j = 10^{16}$ cm, a time half-width $\tau_0 = 50$ yr and a peak velocity $v_0 = 200$ km s$^{-1}$ (see equation (5)). The total mass of the pulse is $M_p = 10^{-4}$ $M_\odot$. For calculating the ejection density, we impose a constant mass loss rate per unit area $\dot{m} = M_p/(2\pi r_j \tau_0) = 2.0 \times 10^{-13}$ g cm$^{-2}$ s$^{-1}$, and calculate the density as:

$$\rho_0(\tau) = \frac{\dot{m}}{\text{max}[\rho_0(\tau), v_{\text{min}}]}.$$

with $v_{\text{min}} = 1$ km s$^{-1}$ ($v_{\text{min}}$ is introduced in order to avoid the divergence of the density for $v_0 \to 0$). Initially, the computational domain is filled with a uniform environment of numerical density $n_a = 1963.3$ cm$^{-3}$, which, combined with the properties of the pulse, gives $\sigma = 0.1$ (see equation (5)).

Both the environment and the ejected material have an initial temperature of $10^4$ K, and have singly ionized H.

In the simulation, a minimum temperature of $10^4$ K is imposed in all cells at all times (also assuming that H is always fully ionized), and the parametrized cooling function of Biro & Raga (1994) is used for $T > 10^4$ K. This setup is meant to approximate the behaviour of the gas within a photoionized region. Throughout our simulation, the bow shock has a shock velocity $\sim 100$ km s$^{-1}$, which together with the pre-shock ambient density ($n_a \approx 2000$ cm$^{-3}$, see above) gives a cooling distance $d_c \sim 1$ AU to $10^4$ K (from the plane-parallell shock models of Hartigan et al. 1987), which is unresolved in our simulation. The slower “jet shock” develops velocities as low as $\sim 20$ km s$^{-1}$, and does not have substantial cooling in this regime.

From this simulation, we have calculated predicted H$\alpha$ maps and PV diagrams. These are obtained by computing the H$\alpha$ emission coefficient (using the interpolation of Aller 1994), and integrating it through lines of sight.

Figure 4 shows the H$\alpha$ emission maps obtained for evolutionary times $t/\tau_0 = 4, 6$ and 8 (top, middle and bottom panels, respectively), assuming a $\phi = 30^\circ$ angle between the outflow axis and the plane of the sky. From this figure we see that the H$\alpha$ emission has two components: the plasmon head and the tail. This latter component is brightest close to the outflow source. The bow shock at the head of the plasmon is rather broad, which is a result of the fact that the Mach number of the flow is not so high (going down to $\sim 10$ towards the end of the simulation).

We also calculate the PV diagrams for evolutionary times $t/\tau_0 = 4, 6$ and 8, and a $\phi = 30^\circ$ angle between the outflow axis and the plane of the sky (see Figure 5). For the PV diagrams, we have assumed that we have a spectrograph slit with a full width of 100 AU, centred on the outflow axis. The resulting PV diagrams show a clear “Hubble law” ramp.

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of increasing radial velocities vs. distance from the source, ending in a broad emission line region corresponding to the head of the plasmon.

In Figure 5, we also plot the (appropriately projected) velocity vs. position obtained from the analytical model (see section 5 and Figure 3). The "Hubble law" feature of the tail agrees very well with the results obtained from the numerical simulation. Also, the analytic position of the plasmon head falls in the middle of the spatially quite extended emission predicted from the numerical simulation (this spatial extent being partly the result of the projection of the wide bow shock onto the plane of the sky).

Figure 5. Position-velocity (PV) diagrams obtained from the simulation for evolutionary times 0.4, 6 and 8 (top, middle and bottom panels, respectively), assuming a $\phi = 30^\circ$ angle between the outflow axis and the plane of the sky. The PV diagrams are normalized to the peak emission of the top frame, and are shown with the logarithmic colour scale given by the bar to the right of the top frame. The (appropriately projected) velocity vs. position obtained from the analytical solution (for the corresponding evolutionary times) is shown with the dashed white curves.

7 CONCLUSIONS

We present a model for a hypersonic "single pulse jet", produced by a collimated outflow event with an ejection velocity history with a single peak, and wings of decreasing velocity (at earlier and later times). An arbitrary form for a simultaneous ejection density variability is also possible.

Such an ejection results in the formation of a "head" associated with a working surface travelling through the surrounding environment, and a "tail" of slower material (formed by the decaying velocity tail of the outflow event) which rapidly develops a linear, "Hubble law" kinematical signature. We call this flow configuration a "head/tail plasmon".

We study the simple case of a parabolic ejection velocity pulse (which could be viewed as a second order Taylor series of the peak of an arbitrary ejection pulse), with a time-independent mass loss rate (i.e., the ejection density is proportional to the inverse of the ejection velocity). With a "centre of mass formalism", we obtain the motion of the head of the "head/tail plasmon" (see section 2).

In the limit of a very low density environment (see section 3) the head of the plasmon reaches a constant velocity, and the material in the tail at all times retains a substantial fraction (asymptotically approaching $1/4$) of the total mass of the ejection event. For denser environments (see section 4), the plasmon slows down, and the head ends up incorporating most of the mass of the ejection pulse. For all flow parameters, the predicted PV diagrams rapidly develop a "Hubble law" kinematical signature (see section 5).

Finally, we compute an axisymmetric gasdynamic simulation with parameters appropriate for a high velocity clump in a PN (see section 6). We compute Hα emission maps and PV diagrams showing the observational characteristics this flow. The predicted PV diagrams obtained from the simulation agree very well with the analytic model (see Figure 5).

This paper represents a first exploration of a different kind of jet or plasmon flow. A detailed application of this model to different objects will be necessary to show what improvements are found with respect to previous models, such as the ones of Dennis et al. (2008) for knots in PNe, or the ones of Rivera et al. (2019a, b) for the Orion BN-KL fingers.

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Data availability: The lead author may be contacted for access to the results of the simulations.

REFERENCES

Alcolea, J., Bujarrabal, V., Sánchez Contreras, C. et al. 2001, A&A, 373, 932
Allen, D. A., Burton, M. G. 1993, Nature 363, 54A
Aller, L. H. 1994, Physics of Thermal Gaseous Nebulae. Reidel, Dordrecht
Bally, J., Ginsburg, A., Arce, H. 2017, ApJ, 837, 60
Biro, S., Raga, A. C. 1994, ApJ, 434, 221
Cantó, J., Raga, A. C., D’Alessio, P. 2000, MNRAS, 313, 656
Dennis, T. J., Cunningham, A. J., Frank, A. et al. 2008, ApJ, 679, 1327
De Young, D. S., Axford, W. I., 1967, Nature, 216, 129
Esquivel, A., Raga, A. C., Cantó, J., Rodríguez-González, A. 2009, A&A, 507, 855E
Hartigan, P., Raymond, J. C., Hartmann, L. W. 1987, ApJ, 316, 323
Rivera-Ortíz, P. R., Rodríguez-González, A. Hernández-Martínez, L., Cantó, J. 2019a, ApJ, 874, 38R
Rivera-Ortíz, P. R., Rodríguez-González, A. Hernández-Martínez, L., Cantó, J. 2019b, ApJ, 885, 104R
Zapata, L., Loinard, L., Schmid-Burgk, J., et al. 2011, ApJ, 726L, 12Z

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