The Promising Process to Distinguish Supersymmetric Models with Large tan$\beta$ from the Standard Model: $B \to X_s \mu^+ \mu^-$

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abstract

It is shown that in supersymmetric models (SUSYMs) the large supersymmetric contributions to $B \to X_s \mu^+ \mu^-$ come from the Feynman diagrams which consist of exchanging neutral Higgs bosons (NHBs) and the chargino-stop loop and are proportional to $m_\mu m_\mu \tan^3 \beta / m_h^2$ when $\tan \beta$ is large and the mass of the lightest neutral Higgs boson $m_h$ is not too large (say, less than 150 Gev). Numerical results show that the branching ratios of $B \to X_s \mu^+ \mu^-$ can be enhanced by more than 100% compared to the standard model (SM) and the backward-forward asymmetry of lepton is significantly different from that in SM when $\tan \beta \geq 30$.

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It is widely believed that supersymmetry (SUSY) is one of the most promising candidates for physics beyond SM since it offers a scheme to embed the SM in a more fundamental theory in which many theoretical problems such as gauge hierarchy, origin of mass and Yukawa couplings can be answered. One direct way to search for SUSY is to discover SUSY particles at colliders. But, unfortunately, so far no SUSY particles have been found. Another way is to search for its effect through indirect methods. In most of SUSYMs R parity is conserved so that SUSY contributions to an observable appear at the loop level. Therefore, it has been realized for a long time that rare processes can be used as a good probe for searches of SUSY, since in these processes the contributions of SUSY and SM arise at the same order in perturbation theory. The $B \to X_s l^+ l^-$ ($l=e, \mu, \tau$) process, one of rare processes, in SUSYMs has been extensively studied [1–5]. The effects of large tan$\beta$ have been noticed in recent papers [4, 5]. There is a stop-chargino

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loop diagram which gives a large contributions to $C_7$ when $\tan\beta$ is large \cite{7}. This leads to that in the minimal supergravity model (mSUGRA) there are regions in the parameter space where the branching ratio of $b \to s\tau^+\tau^-$ ($l=$ e, $\mu$) is enhanced by about 50% compared to the SM \cite{5}. However, the contributions from exchanging NHBs are ignored in these previous analyses. Recently, the contributions of NHBs in SUSYMs have been taken into account. Because the contributions to $b \to s\tau^+\tau^-$ coming from the chargino-stop loop diagram are proportional to $m_b m_{\tau} \tan^3\beta/m_h^2$ ($h = h^0, A^0$) when $\tan\beta$ is large, the branching ratio of $b \to s\tau^+\tau^-$ can be enhanced by about 200% compared to the SM \cite{7}.

From experimental points of view, the observation of $B \to X_s l^+ l^-$ ($l=$ e, $\mu$) is easier accessible than that of $B \to X_s \tau^+ \tau^-$. The inclusive decay $B \to X_s \gamma$ has been observed by CLEO. Meantime, experiments at $e^+e^-$ and hadron colliders are closing in on the observation of $B \to K^* l^+ l^-$ ($l=$ e, $\mu$) \cite{8}. The B factories presently under construction will collect some $10^7$–$10^8$ B mesons per year which can be used to obtain good precision on low branching fraction modes. Therefore, it is meaningful to pay attention to the process $B \to X_s l^+ l^-$ for ($l=$ e, $\mu$). As pointed above, the contributions of NHBs are proportional to the mass of a lepton and $\tan^3\beta$. For $B \to X_s e^+ e^-$, the contributions can be safely neglected due to the smallness of $m_e$, no matter how large $\tan\beta$ is (of course, in the theoretically allowed range, say, in SUSYGUT, $\tan\beta \leq 50$). However, for $b \to s\mu^+\mu^-$, $m_{\mu} \tan\beta$ can be as large as $m_\tau$ as long as $\tan\beta \geq 17$. Thus one can expect that for $b \to s\mu^+\mu^-$, in addition to the enhancement coming from the possible change of the sign of $C_7$ (the value of $C_7$ is fixed by the measurement of $b \to s\gamma$ with a branching ratio of $(2.32 \pm 0.57 \pm 0.35) \times 10^{-4}$ and 95% C.L. bounds of $1 \times 10^{-4} < Br(B \to X_s\gamma) < 4.2 \times 10^{-4}$ \cite{9}), a even more significant enhancement coming from exchanging NHBs arises in SUSYMs with large $\tan\beta$. In the letter we calculate the invariant mass distribution and backward-forward asymmetry of dilepton angular distribution for $B \to X_s \mu^+ \mu^-$ in SUSYMs. Our results show that the branching ratio of $B \to X_s \mu^+ \mu^-$ is enhanced by at least 100% compared to the SM and the back-forward asymmetry is more sensitive to $\tan\beta$ than the invariant mass distribution when $\tan\beta$ is larger than 30 and masses of Higgs bosons, stops and charginos are in the reasonable range (i.e., all constraints from phenomenology are satisfied). Note that the invariant mass distribution of $B \to X_s \mu^+ \mu^-$ in a two Higgs doublet model with large $\tan\beta$ is not enhanced compared to the SM. Therefore, the rare process $B \to X_s \mu^+ \mu^-$ provides a good opportunity to distinguish SUSYMs with large $\tan\beta$ from the SM and it is possible that the first distinct signals of SUSY could come from deviations from the SM in the inclusive decay $B \to X_s \mu^+ \mu^-$. Inclusive decay rates of $B \to X_s \mu^+ \mu^-$ can be calculated in the $1/m_Q$ expansion and it has been shown that the leading order term turns to be the decay of a free $b$ quark and corrections stem from the $1/m_Q^2$ order and are small about a few percent \cite{10}. Therefore in what follows we limit our analyses to the leading order term, i.e. the decay $b \to s\mu^+\mu^-$. There are several classes of new contributions in SUSYMs and the dominant ones are arising
from chargino(\tilde{\chi})-uptype squark loop and charged Higgs boson (\(H^\pm\))-uptype quark loop. Because of small generation mixing of squarks coming from phenomenological constraints on \(K^0 - \bar{K}^0\) and \(D^0 - \bar{D}^0\), the contributions from gluino-downtype squark loop and neutralino-downtype squark loop are much smaller than the dominant ones and are neglected in the following.

The effective Hamiltonian relevant to the \(b \to s\mu^+\mu^-\) process is

\[
H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left( \sum_{i=1}^{10} C_i(\mu) O_i(\mu) + \sum_{i=1}^{10} C_Q_i(\mu) Q_i(\mu) \right)
\]

(1)

where \(O_i\) (i=1, 2, ..., 10) are given in Ref. [11], and \(Q_i\)'s come from exchanging neutral Higgs bosons and have been given in Ref. [12]. The coefficients \(C_i(m_w)\) in SUSYMs have been calculated [1] [3]. We calculate the coefficients \(C_Q_i(m_W)\) in SUSYMs and the results are:

\[
C_{Q1}(m_W) = \frac{m_b m_\mu}{4m_\rho^2 \sin^2 \theta_W} \operatorname{tg}^2 (\beta) \left\{ (\sin^2 \alpha + h \cos^2 \alpha) \left[ \frac{1}{x_{Wt}} (f_1(x_{Ht}) - f_1(x_{Wt})) + \sqrt{2} \sum_{i=1}^{m_\chi} \frac{m_i}{m_W} \frac{U_{iv}}{\cos \beta} (V_{i1} f_1(x_{Ht}) + V_{i2} f_1(x_{tH})) \right] \right. \\
+ \left. (1 + \frac{2m_W}{m_W}) f_2(x_{Ht}, x_{Wt}) \right\} - \frac{m_h^2}{m_W^2} f_2(x_{Ht}, x_{Wt}) \\
+ 2 \sum_{i'=1}^{2} \left\{ B_1(i, i') \Gamma_1(i, i') + A_1(i, i') \Gamma_2(i, i') \right\} \\
C_{Q2}(m_W) = -\frac{m_b m_\mu}{4m_{A_0}^2 \sin^2 \theta_W} \operatorname{tg}^2 (\beta) \left\{ \frac{1}{x_{Wt}} (f_1(x_{Ht}) - f_1(x_{Wt}))) + 2f_2(x_{Ht}, x_{Wt}) \right. \\
+ \left. \sqrt{2} \sum_{i=1}^{m_\chi} \frac{m_i}{m_W} \frac{U_{iv}}{\cos \beta} (V_{i1} f_1(x_{Ht}) + V_{i2} f_1(x_{tH})) \right\} \\
+ 2 \sum_{i'=1}^{2} (U_{i'2} V_{i1} \Gamma_1(i, i') + U_{i'1}^* V_{i2} \Gamma_2(i, i')) \right\}
\]

(2)

where the definitions of the functions \(f_i, \Gamma_i\) (i=1, 2), \(A_1, B_1, \Lambda\) and the meaning of the matrices \(U, V, T\) have been given in [7] and we have omitted less important terms because they are numerically negligible compared to those given in eq. (2) when \(\tan \beta \geq 20\).

From eq. (2), we see that, for large \(\tan \beta \geq 20\), the coefficients \(C_{Q_i}\) are proportional to \(m_b m_\mu / m_h \tan^3 \beta (h = h^0, A^0)\). One factor of \(\tan \beta\) comes from the chargino-up-type squark loop and \(\tan^2 \beta\) from exchanging the neutral Higgs bosons (note that in the large \(\tan \beta\) approximation \(\cos^{-1} \beta \approx \tan \beta\)). Therefore, \(C_{Q_i}\) can compete with \(C_i\) and even overwhelm \(C_i\) as long as \(\tan \beta\) is large enough. We remark that the chirality structure of the \(Q_i\) (i=1, 2) operators allows a large \(\tan \beta\) enhancement for the \(C_{Q_i}\) (i=1, 2) coefficients, as happened for the magnetic moment operator \(O_7\), and there is no such a large \(\tan \beta\) enhancement for the \(C_i\) (i=8, 9) coefficients due to the different chirality structure of the \(O_i\) (i=8, 9) operators. Incorporating the QCD corrections to the coefficients \(C_i\) and \(C_{Q_i}\) in the standard way, we calculate these coefficients at \(\mu=m_b\).
The differential branching ratio and the forward-backward asymmetry of the dimuon angular distribution for $B \to X_s \mu^+ \mu^-$ can be obtained from [7] with substituting $m_\mu$ for $m_\tau$. The numerical results of the invariant mass distribution and backward-forward asymmetry are shown in Fig. 1 (a) for the mSUGRA model and Fig. 1 (b) for the MSSM with a typical choice of masses of sparticles and Higgs bosons respectively. The mSUGRA parameters $(m_0, m_{1/2}, A)=(190, 190, 380)$ Gev, Higgs mass mixing parameter $\mu < 0$ and tan$\beta=30$ have been chosen in Fig. 1 (a). In the computations of sparticle mass spectra and mixings we neglect the Yukawa couplings of the first two generations. The chosen values of masses of relevant sparticles and Higgs bosons in MSSM are given in the Figure Captions. The constraint from the LEP and $b \to s\gamma$ has been imposed in our numerical calculations. One can see from the Fig. 1 that a large enhancement of the differential branching ratio $d\Gamma/ds$ shows up and the enhancement can reach 100% compared to SM when tan$\beta=30$. The backward-forward is significantly different from that in SM. The predictions without including the contributions of exchanging NHBs are also shown in Fig. 1 in order to compare. It is evident from the figure that the contributions of exchanging NHBs to the differential branching ratio are the same order of magnitude as supersymmetric contributions without including exchanging NHBs in the low s region ($s \leq 0.4$) and larger than those in the high s region ($s > 0.4$). There are regions in the parameter space where the contributions of NHBs alone make a large enhancement of the differential branching ratio. For example, for a set of values of parameters ($\theta_{\tilde{t}}=-20^\circ, m_{\chi_2}=220$ Gev, $m_{\chi_1}=100$ Gev, $m_{\tilde{\tau}}=430$ Gev, $m_{\tilde{t}_1}=250$ Gev, $m_{\tilde{t}_2}=500$ Gev, $m_{A_0}=80$ Gev, $m_{\tilde{\mu}_L}=160$ Gev, and tan$\beta=30$) the enhancement of $d\Gamma/ds$ coming from NHBs is about 80% compared to SM.

We would like to make some remarks:

(i) The large enhancement of the invariant mass distribution of $B \to X_s \mu^+ \mu^-$ compared to the SM is of a common feature of SUSYMs with large tan$\beta$ in some region of the parameter space. The Fig. 2 shows the results for tan$\beta=30$. For larger tan$\beta$, for example, tan$\beta=45$, the enhancement can reach 200%. The enhancement exists as long as the mass splitting of stops is large enough (say, $\geq 100$ Gev). The condition is necessary because if all the squark masses are degenerate ($m_{\tilde{t}_1} = m_{\tilde{t}_2} = \tilde{m}$), the large contributions arising from the chargino-squark loop exactly cancel due to the GIM mechanism [8]. In order to illustrate this point we show the $C_{Q_i}$ as a function of the stop mixing angle $\theta_{\tilde{t}}$ under the above condition in Fig. 2. As can be seen from the Figure, $C_{Q_i}$ is large enough to make an large enhancement of $d\Gamma/ds$ in a wide range of $\theta_{\tilde{t}}$ (about from $-\frac{2\pi}{3}$ to $-\frac{\pi}{8}$).

(ii) The QCD corrections to coefficients $C_i$ and $C_{Q_i}$ are incorporated in the leading logarithmic approximation in our numerical computations. The one-loop mixing of $Q_i$ with $O_7$ has been analyzed [12] and leads to about 5% correction to $C_7(m_b)$ when tan$\beta=30$. A next-to-leading order(NLO) analysis without including $Q_i$ for $B \to X_s l^+ l^-$ has been performed, where it is stressed that a scheme independent result can only be obtained by including the LO and NLO.
corrections to $C_{8}^{eff}$ while retaining only the LO corrections in the remaining Wilson coefficient $C_{8}$. Because we did not include the NLO corrections the theoretical uncertainty due to the renormalization scale $\mu$ dependence is about $\pm 20\%$ as $\mu$ is varied in the range $1/2 \, m_b \leq \mu \leq 2 \, m_b$. We expected that a full NLO analysis including $Q_i$ for $B \rightarrow X_s l^+ l^-$ will appear in the near future. As pointed in the Ref. citebsg5, there is a SUSY high scale uncertainty and it is possible that the scale $\mu$ dependence is large enough to effectively encompass the uncertainty.

(iii) The following values of parameters have been used in the numerical calculations: $m_t = 175 \text{Gev}$, $m_c/m_b = 0.3$, $\eta = \alpha_s(m_b)/\alpha_s(m_w) = 0.548$. We have estimated the uncertainties from the parameters and results are that the $m_t$ dependence is weak and the uncertainties are about ten percent. The error from neglecting the strange quark mass $m_s$ is of order $m_s^2/m_b^2$ and consequently is very small.

In summary, we have investigated the differential branching ratio and backward-forward asymmetry of lepton for $B \rightarrow X_s \mu^+ \mu^-$ in SUSYMs with large $\tan \beta$. There is a 100% enhancement of the differential branching ratio compared to SM if $\tan \beta \geq 30$ and the masses of Higgs bosons, squarks and charginos are in the reasonable range. Because there is almost no enhancement till $\tan \beta \approx 50$ in a two Higgs doublet, one can make the conclusion that the first distinct signals of SUSY could come from the observation of $B \rightarrow X_s \mu^+ \mu^-$. This research was supported in part by the National Natural Science Foundation of China and partly supported by Center of Chinese Advanced Science and Technology (CCAST).

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**Figure Captions**

Fig. 1 $d\Gamma/ds$ and $A(s)$ for the case $\mu < 0$ and $\tan\beta=30$, a) $m_{\frac{1}{2}} = m_0 = 190$ GeV, $A = 380$ GeV in the mSUGRA and b) $\theta_t=-40^\circ$, $m_{\tilde{t}_2}=200$ GeV, $m_{\tilde{\chi}_1}=90$ GeV, $m_{\tilde{q}}=350$ GeV, $m_{\tilde{t}_1}=220$ GeV, $m_{\tilde{t}_2}=450$ GeV, $m_{A^0}=80$ GeV, $m_{\tilde{\nu}}=160$ GeV in the MSSM. The solid, dashed and dotted lines represent the predictions of the SUSYMs, the SUSYMs without including contributions of NHBs and SM respectively.

Fig. 2 $C_{Q1}$ and $C_{Q2}$ varying with the mass splitting of stop, the mass splitting of chargino and the stop mixing angle $\theta_t$ for $\mu < 0$, $\tan\beta=30$, $m_{\tilde{t}_1}=150$ GeV, $m_{\tilde{\chi}_1}=90$ GeV and $m_{A^0} = 80$ GeV; the characters following the line style indicate the mass splittings: the first character means the mass splitting of stop (h represents 300 GeV and l 100 GeV), and the second character means that of chargino (h represents 410 GeV, m 210 GeV, and l 110 GeV).
Fig. 1: $b \rightarrow s \mu^+ \mu^-$

(a) $\tan \beta = 30, m_0 = m_{1/2} = 190 \text{ GeV}, A = 380 \text{ GeV}$

(b) $m_{\chi_1} = 90 \text{ GeV}, m_A = 80 \text{ GeV}, m_{t_1} = 220 \text{ GeV}$
Fig. 2