I. INTRODUCTION

With the remarkable development of the cooling technique for two-component atomic Fermi gases and magnetic-field Feshbach resonance [1], which can change both the strength and sign of the atomic interaction characterized by the scattering length \( a \) [2], experiments have explored the crossover region between the strongly attractive and repulsive interaction. For the spin mixture of the fermionic atoms, there would be dimers of fermionic atom pairs in different form dependent on the attractive or repulsive interaction between atoms. On the side of repulsive interaction (BEC side), there exists molecule which is a short-range fermionic atom pairs [3]. In several recent experiments [3, 4, 5], the evidence for Bose-Einstein condensates of diatomic molecules has been given. On the side of attractive interaction (BCS side), one expects that the dimer is in a state of long-range fermionic pairs analogously to the electronic Cooper pairs. The magnetic-field Feshbach resonance gives us an important opportunity to investigate the pairing phenomena in the ultracold Fermi gases especially the BCS-BEC crossover which has been discussed in a lot of theoretical investigations [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16] and experimental researches [17, 18, 19, 20, 21, 22, 23, 24, 25] such as condensate fraction [17, 18], collective excitations [21, 22, 23], pairing gap [24], and the recent measurement of heat capacity [25].

In the BCS-BEC crossover, a particular interesting problem is the properties of the ultracold gases at the location of the Feshbach resonance where the scattering length between atoms is divergent. At or extremely close to the location of the Feshbach resonance, the absolute value of the scattering length \( a \) can be comparable to or even much larger than the average distance \( l \) between particles. In this situation, the ultracold gas can be described in the unitarity limit [26, 27, 28, 29, 30], where there is a universal behavior for the system. The Feshbach resonant technique has given us the experimental means to investigate the ultracold gas with \( |a| >> l \). On resonance, the fraction of dimers in zero-momentum state is investigated experimentally in [17, 18] after the conversion of pairs of atoms into bound molecules. The experiment by MIT group [18] shows that there is a high fraction of dimers of fermionic atoms extremely close to the Feshbach resonance. On resonance, the frequency of the radial breathing mode is given in [21] by observing the collective oscillations of the system. In the present work, based on a mixture of the atomic and dimeric gases in the unitarity limit \( |a| >> l \), a very simple theory is developed to explain the recent experimental results [18] and [21] at the location of the Feshbach resonance.

II. CHEMICAL EQUILIBRIUM ON RESONANCE

In the present experiments on BCS-BEC crossover, an equal mixture of fermionic atoms in two different spin states are confined in a harmonic trap to realize atomic collisions. In thermal equilibrium, the system comprises fermionic atoms and dimers, and its dynamic equilibrium is characterized by the fact that the Gibbs free energy \( G \) of the system is a minimum. Assuming that \( \mu_{F↑} \) and \( \mu_{F↓} \) being the chemical potential of the Fermi gases and...
\( \mu_D \) being the chemical potential of the dimeric gas, the minimum of the Gibbs free energy means that

\[
2\mu_F^* = 2\mu_F = \mu_D. \tag{1}
\]

The chemical potential of the dimeric gas is \( \mu_D = \varepsilon_D + \mu_t \) with \( \varepsilon_D \) being the dimeric energy and \( \mu_t \) being the contribution to the chemical potential due to the thermal equilibrium of the dimeric gas.

Assuming that \( n_F \) is the total density distribution of the Fermi gases without the presence of dimers and \( x \) is the fraction of the dimers immersed in the ultracold Fermi gases, the density distribution of the dimeric gas \( n_D \) and Fermi gas \( n_{F^1} \) (and \( n_{F^2} \)) in a spin state are respectively given by \( n_D = x n_F/2 \) and \( n_{F^1} = n_{F^2} = (1 - x) n_F/2 \). For the unitarity limit \(|a| \gg \tilde{t}_1\), there is a universality hypothesis (see [24]) that although the scattering length \( a \) will play an important role, it will not appear in the final result of a physical quantity such as chemical potential because it can be regarded as infinity. In this case, the length scale \( \tilde{t}_1 \sim n_F^{-1/3} \) rather than \( a \) will appear in the final result of a physical quantity which means a universal behavior for a system. By using the universality hypothesis and local density approximation, at zero temperature, the chemical potential \( \mu_F^* \) (= \( \mu_{F^1} \)) of the Fermi gas is given by

\[
\mu_F^* = (1 + \beta_1) \frac{\hbar^2 (6\pi^2)^{2/3}}{2m} \left( \frac{1 - x}{} \right)^{2/3} + V_{\text{ext}}(r), \tag{2}
\]

where \( V_{\text{ext}}(r) \) is the external potential of the fermionic atoms. Without \( \beta_1 \), the above expression gives the chemical potential of an ideal Fermi gas in the local density approximation. The parameter \( \beta_1 \) shows the role of the extremely large scattering length \( a \) in the unitarity limit. The parameter \( \beta_1 \) was firstly measured in [1] with a careful experimental investigation of the strongly interacting degenerate two-component Fermi gases near the Feshbach resonance. \( \beta_1 \) has been also calculated with different theoretical methods [21, 22].

The scattering length \( a_D \) (\( \approx 0.6a \)) of the dimers is comparable to the scattering length \( a \) between fermionic atoms, thus the dimeric gas will also show a universal behavior on resonance. In the unitarity limit, for the bosonic dimer gas, the scattering length \( a_D \) should not appear in the final result of a physical quantity too. According to the universality hypothesis, we assume here that the bosonic dimer gas shows an analogous universal behavior with the Fermi gas (see also [24]). Based on this assumption, one has the chemical potential for the dimeric gas at zero temperature

\[
\mu_D = (1 + \beta_2) \frac{\hbar^2 (6\pi^2)^{2/3}}{2 \times 2m} \left( \frac{x n_F}{2} \right)^{2/3} + 2V_{\text{ext}}(r) + \varepsilon_D. \tag{3}
\]

The parameter \( \beta_2 \) in \( \mu_D \) is also due to the large scattering length \( a_D \) between dimers and it can be calculated in the unitarity limit. Near the Feshbach resonance, the dimeric energy \( \varepsilon_D \sim \hbar^2/ma^2 \). Thus, on resonance, \( \varepsilon_D \) can be omitted in the above expression. From the dynamic equilibrium condition \( 2\mu_{F^*} = \mu_D \), one gets the following general equation to determine the fraction of dimers at zero temperature and Feshbach resonance:

\[
4\beta_r (1 - x)^{2/3} = x^{2/3}, \tag{4}
\]

where \( \beta_r = (1 + \beta_1)/(1 + \beta_2) \). We show the relation between the fraction of dimers \( x \) and different value of \( \beta_r \) in Fig. 1.

In the presence of the dimers, the chemical potential of the system is obviously lower than the case without the dimers. Thus, the mixture gas of the atoms and dimers is a stable state of the system. For \(|a| \gg \tilde{t}_1\), there are three important characteristics for the dimeric gas:

(i) The above result for the fraction of the dimeric gas is a general result once the system is in the unitarity limit.

(ii) There is a quite high fraction of the dimers as shown in Fig. 1. In the present experiments, the fraction of dimers is determined by the molecules in zero-momentum state after a fast magnetic field transfer (the magnetic field is swept below the Feshbach resonant magnetic field \( B_0 \) so that the scattering length between fermionic atoms becomes positive) to create bound molecules from the dimers. After the dimer-molecule conversion, in [13] the maximum fraction of the molecules in zero-momentum state is observed to be 80% which agrees with our theoretical result. From this experimental result, \( \beta_r \) is estimated to be 0.63.

(iii) The dimer comprising two fermionic atoms is quite stable. At zero temperature, the dimeric gas is immersed in the degenerate Fermi gases. Due to Pauli blocking comes from the degenerate Fermi gases, any dimer can not be dissociated into two fermionic atoms once the equilibrium is attained so that \( 2\mu_{F^*} = \mu_D \). In fact, the stability of the dimeric gas is consistent with the experiment [13] that extremely close to the Feshbach resonance there is no obvious decreasing of the molecules in zero-momentum state after the dimer-molecule conversion process even after 10 s hold time of the final magnetic field. Due to the stability and Pauli blocking, the dimer-molecule conversion will always convert the fermionic atom pairs in a same dimer into bound molecules. Thus at zero temperature, all the dimers will be converted into molecules with zero-momentum state during the dimer-molecule conversion process. This means that the fraction of dimers in thermal equilibrium investigated here can be used to explain the experimental result of the fraction of zero-momentum molecules after the dimer-molecule conversion.

In a recent experiment [21], the frequency of a radial breathing mode is observed extremely close to the Feshbach resonance and found to agree well with the theo-
the Fermi gases.

On resonance, the mixture of the fermionic atoms and dimers will play a role in the width of the gases confined in the harmonic trap. From the dynamic equilibrium $2\mu_{F} = \mu_{D}$, the width of the Fermi gas is equal to that of the dimeric gas. Assume that $R_{a-d}$ is the width of the mixture gases for the atoms and dimers, while $R_{a}$ is the width of the gases without considering the existence of the dimers. After a simple calculation, one has the ratio $R_{a-d}/R_{a} = (1 - x)^{1/6}$. The dashed line in Fig.1 shows $R_{a-d}/R_{a}$ as a function of $\beta r$. We see that the existence of dimers in dynamic equilibrium will decrease the width of the gases.

III. SUMMARY

In summary, the role of the dimers of fermionic atoms is discussed extremely close to the Feshbach resonance where both the fermionic atoms and dimers show a universal behavior based on the universality hypothesis. The dynamic equilibrium of the mixed atom-dimer gas is considered for the case of a harmonic trap through the analysis of the Gibbs free energy. Based on this simple theoretical model, it is found that the high fraction of the stable dimeric gas in the present work agrees well with the experiment [18] where there is a high fraction of molecules in zero-momentum state after the conversion of the dimers into molecules. Extremely close to the Feshbach resonance, the dimeric gas plays a dominant role for the frequency of the radial breathing mode observed in [21]. On resonance, the stability of the dimeric gas would be useful in the creation of molecular BEC on the BEC side and condensate of atomic Cooper pairs on the BCS side. Crossing the Feshbach resonant magnetic field $B_0$ slowly from above (or below) would contribute to the creation of molecular BEC (or condensate of atomic Cooper pairs) because there is a formation of a stable dimeric gas when $B_0$ is swept cross slowly. For example, in [5], the molecular BEC is created after the magnetic field is swept slowly from above to below $B_0$.

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