\( \mathcal{N} = 1 \) Super Feynman rules for any Superspin: Non-Canonical SUSY.

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ABSTRACT: Super Feynman rules for any superspin are given for massive \( \mathcal{N} = 1 \) supersymmetric theories, including momentum superspace on-shell legs. This is done by extending, from space to superspace, Weinberg’s perturbative approach to quantum field theory. Superfields work just as a device that allow to write superpoincaré-covariant superamplitudes for interacting theories, without relying neither in path integral nor canonical formulations. Explicit transformation laws for particle states under finite supersymmetric transformations are offered. \( C, P, T, \) and \( R \) transformations are also worked out. A key feature of this formalism is that it does not require the introduction of auxiliary fields, and when introduced their purpose is just to render supersymmetric invariant the time-ordered products in the Dyson-series. The formalism is tested for the cubic scalar superpotential. It is found that when a superparticle is its own anti-superparticle, the lowest order correction of time-ordered products, together with its covariant part, corresponds to the Wess-Zumino model potential.

KEYWORDS: Superpoincare, Super Feynman rules, On-shell lines.
1 Introduction.

From the inception of superspace by Salam and Strahdee[1], functional and path integral methods have been the preferred scheme to formulate field theory in superspace[2–4]. These formalisms allow us to write correlation functions that perturbatively give super Feynman rules with off-shell legs, making it unclear how to replace them by the corresponding momentum superspace on-shell legs. Perhaps, because realistic supersymmetric theories would never be symmetries of the S-matrix [5], this issue seems secondary. However, thinking of supersymmetry as a theoretical laboratory, the issue has its own importance. The purpose of this paper is to provide formulas for on-shell legs in order to construct superamplitudes $S_{\mathcal{N}\mathcal{M}}$ for scattering processes of massive superparticle states (or particle superstates), where $\mathcal{N}$ and $\mathcal{M}$ label Fock states, extended such that one superparticle carries momentum $\mathbf{p}$, spin-projection $\sigma$, and left or right fermionic four-spinors $s_+$ or $s_-$. These superamplitudes are constructed extending Weinberg’s approach[6, 7] from fields to superfields, that is from
(momentum and configuration) space to superspace. What is done here is to express
the potential appearing in the Dyson operator series
\[ S = T \exp \left[ i \int dt V(t) \right] \]  
(1.1)
as
\[ V(t) = \int d^3x d^4\vartheta V(x, \vartheta), \]  
(1.2)
where \( V(x, \vartheta) \) is a sum of free superfield products obtained as super momentum
Fourier transforms of creation-annihilation superparticle operators. These creation-
annihilation superparticle operators are used to write superparticle states that allow
us to write \( S_{N,M} \) in terms of super Feynman rules, after the appropriate Wick's
pairings.

This extension maintains all the properties of Weinberg’s approach, i.e. super
Feynman rules can be built for any superspin in a straightforward manner and one
can easily incorporate charge conjugation, parity, time reversal and \( \mathcal{R} \)-symmetries.
Furthermore, it also allows us to obtain economic and concise expressions.
A characteristic feature of supersymmetric theories[8] is that, when the La-
grangian does not contain auxiliary fields, the potential becomes not only a function
of the coupling constant \( g \), but also of its square \( g^2 \), relating one and the next order
in perturbation theory (otherwise ‘miraculous’ cancellations could not occur).
Thus it is difficult to see how a perturbative scheme can cope with this situation.
Considering \( V(x^\mu, \vartheta) \) as an invariant density under supersymmetry transformations:
\[ U(\xi)V(x^\mu, \vartheta)U^{-1}(\xi) = V(x^\mu + \vartheta^I \epsilon_{I57} \gamma^\mu \xi, \vartheta + \xi), \]  
(1.3)
is not sufficient to render supersymmetric invariant the time-ordered products ap-
pearing in (1.1), therefore we must introduce (local) non-covariant terms of higher
order in coupling constants. For this perturbative formalism this seems to be the
origin of auxiliary fields.

We adopt the notation and conventions of \([9, 10]\), except for right and left four-
spinors, which we write as \( 2\vartheta_\pm = (I \pm \gamma_5)\vartheta \) instead of \( \vartheta_{L,R} \). As for the methods
employed we use the standard techniques of the operators’ formalism and calculus in
superspace (see for example \([10, 11]\)). We present notation and all our conventions
in Appendix A. Also, we conjugate under the integrals of the fermionic variables and
explain this in Appendix B.

The article is structured as follows: In Section 2 unitary representations of the
superpoincare group are constructed. Section 3 deals with causal superfields, mean-
while Section 4 is devoted to time-ordered products and superpropagators. In Section
5 super Feynman rules are presented. Charge conjugation, parity, time-reversal and
\( \mathcal{R} \) transformation formulas are written in Section 6. The details of the cubic superpotential for a scalar superfield are worked out in Section 7. Finally our conclusions are presented in Section 8.

2 Creation-annihilation superparticle operators.

\( \mathcal{N} = 1 \) supersymmetric multiplets have four particle states with angular momentum \((j, j, j \pm \frac{1}{2})\). With this in mind, we embed these states into two superparticle states, one with \( s_+ \) and the other with \( s_- \), and their fermionic expansion coefficients represent the states of the supersymmetric multiplet. We show that superpoincare transformations are acting unitarily on these superstates, with the additional feature that finite supersymmetric transformations are also considered. To do so, instead of taking states with \( j + \frac{1}{2} \) and \( j - \frac{1}{2} \) angular momentum, we take these states to be in the tensorial representation \( j \otimes \frac{1}{2} \). That is, at the level of creation operators we start with

\[
a_+^\ast(p, \sigma), \quad a_-^\ast(p, \sigma), \quad l_a^\ast(p, \sigma), \quad a = \pm \frac{1}{2}, -\frac{1}{2}, \tag{2.1}
\]

that satisfy the following (non-zero) (anti)commutators

\[
[a_\pm(p, \sigma), a_{\pm}'(p', \sigma')] = \delta^3(p - p')\delta_{\sigma\sigma'},
\]

\[
\{l_a(p, \sigma), l_b^\ast(p', \sigma')\} = \delta^3(p - p')\delta_{ab}\delta_{\sigma\sigma'}, \tag{2.2}
\]

and under a Poincare transformation behave as

\[
U(\Lambda, x)a_\pm^\ast(p, \sigma)U(\Lambda, x)^{-1} = e^{-ip\cdot x}\sqrt{\frac{k_0}{p^0}}\sum_{\sigma'} U_{\sigma\sigma}^{(j)}[W(\Lambda, p)]a_\pm^\ast(p', \sigma'),
\]

\[
U(\Lambda, x)l_a^\ast(p, \sigma)U(\Lambda, x)^{-1} = e^{-ip\cdot x}\sqrt{\frac{k_0}{p^0}}\sum_{b, \sigma'} U_{ba}l_{\sigma}^{(j)}[W(\Lambda, p)]U_{\sigma\sigma}^{(j)}[W(\Lambda, p)]l_b^\ast(p', \sigma'), \tag{2.3}
\]

where \( U^{(j)} \) is the spin-\( j \) rotation matrix and \( W(\Lambda, p) \) is the so-called Wigner rotation:

\[
W(\Lambda, p) = L(\Lambda p)^{-1}\Lambda L(p), \quad p = L(p)k, \tag{2.4}
\]

with \( k = (0 \ 0 \ 0 \ m) \) as a standard vector and \( W(\Lambda, p) \) isomorphic to the rotation group. As a definition fermionic (bosonic) creation-annihilation particle operators

\footnote{Except for the case \( j = 0 \). We call superspin \( j \) to the set \( \{j, j, j \pm \frac{1}{2}\} \).

2 All states are constructed from \( a^\ast(\cdots)|\text{VAC}\rangle \), where \( |\text{VAC}\rangle \) is a superpoincare invariant vacuum. Here we denote the adjoint of an operator as \( * \). When the adjoint is accompanied by a transpose of some vector we denote it by \( \dagger \).

3 \{ \} is defined to be an anticommutation or commutation if \( \} \) is a commutation or an anticommutation, respectively.
remain fermionic (bosonic) with respect to supernumbers. A very important fact is that when a Lorentz transformation $R$ is an element of the rotation group, the following relation holds:

$$[D_\pm(R)]_{ab} = U_{ab}^{(\frac{\delta}{2})}(R),$$

where $D_\pm$ stands for the Weyl representations. We embed the operators $l^*_a$ in a four component vector

$$b(p, \sigma) \equiv D[L(p)]\left(\frac{l(p, \sigma)}{l(p, \sigma)}\right),$$

with $D[\Lambda] = D_+(\Lambda) \oplus D_-(\Lambda)$, the Dirac representation. In view of (2.3) and (2.5):

$$U(\Lambda, x)\bar{b}(p, \sigma)U(\Lambda, x)^{-1} = e^{-ip \cdot x} \sqrt{\epsilon_0 \over p^0} \left\{ \sum_{\sigma'\sigma} U_{\sigma'}^{(j)}(W(\Lambda, p))\bar{b}(p_\Lambda, \sigma')D_{\sigma}\right\},$$

(2.7)

where $\bar{b}$ is the Dirac adjoint $b^\dagger \beta$. The non-vanishing (anti)commutation relations of $(b, \bar{b})$ are

$$\{b_\alpha(p, \sigma), \bar{b}_\beta(p', \sigma')\} = [I + (-i\varphi) / m]_{\alpha\beta} \delta(p - p')\delta_{\sigma\sigma'}.$$ (2.8)

One can also show that

$$(-i\varphi) b(p, \sigma) = mb(p, \sigma),$$ (2.9)

which is a reminder that although we are using a four dimensional vector with $4(2j + 1)$ spin projections, only $2(2j + 1)$ of them are independent.

We define two types of creation superparticle (sparticle) operators:

$$a_\pm^*(p, s_\pm, \sigma) \equiv a_\pm^*(p, \sigma) \pm \sqrt{2m b(p, \sigma)} s_\pm \pm 2m \delta^2(s_\pm) a_\pm^*(p, \sigma),$$ (2.10)

with their corresponding annihilation sparticle operators

$$a_\mp(p, s_\mp, \sigma) \equiv (a_\pm^*(p, (\epsilon\gamma_5 b^* s_\pm^\dagger, \sigma)))^* = a_\pm(p, \sigma) \pm \sqrt{2m s_\mp^\dagger \epsilon\gamma_5 b(p, \sigma)} \mp 2m \delta^2(s_\mp)a_\mp(p, \sigma).$$ (2.11)

Creation-annihilation sparticle operators have the Poincare transformation property:

$$U(\Lambda, x)a_\pm^*(p, s_\pm, \sigma)U(\Lambda, x)^{-1} = e^{-ip \cdot x} \sqrt{\epsilon_0 \over p^0} \left\{ \sum_{\sigma'\sigma} U_{\sigma'}^{(j)}(W(\Lambda, p))a_\pm^*(p, D(\Lambda)s_\pm, \sigma')\right\},$$

$$U(\Lambda, x)a_\pm(p, s_\pm, \sigma)U(\Lambda, x)^{-1} = e^{ip \cdot x} \sqrt{\epsilon_0 \over p^0} \left\{ \sum_{\sigma'\sigma} U_{\sigma'}^{(j)*}(W(\Lambda, p))a_\pm(p, D(\Lambda)s_\pm, \sigma')\right\},$$

(2.12)
and the (non-zero) anti(commutation) relations:

\[
[a_\pm(p, s_\pm, \sigma), a^*_\pm(p', s'_\pm, \sigma')] = \delta^3(p' - p)\delta_{\sigma\sigma'}\exp\left[2s^\top \epsilon\gamma_5 (-i\not{p}) s'_\pm\right], \\
[a_\pm(p, s_\pm, \sigma), a^*_\pm(p', s'_\pm, \sigma')] = \pm 2m\delta^3(p' - p)\delta_{\sigma\sigma'} \delta^2 \left[(s' - s)_\pm\right].
\]

The (+) and (−) creation-annihilation sparticle operators are not independent, they are related by a Fourier transformation in fermionic variables. For creation type we have

\[
a^*_\pm(p, s_\pm, \sigma) = \mp (2m)^{-1} \int d^2 s'_\pm \exp\left[2s^\top \epsilon \gamma_5 (+i\not{p}) s'_\pm\right] a^*_\pm(p, s'_\pm, \sigma),
\]

meanwhile for the annihilation type

\[
a_\pm(p, s_\pm, \sigma) = \mp (2m)^{-1} \int d^2 s'_\pm \exp\left[-2s^\top \epsilon \gamma_5 (+i\not{p}) s'_\pm\right] a_\pm(p, s'_\pm, \sigma).
\]

Now we introduce the Majorana fermionic operators:

\[
U(\Lambda)Q_\alpha U^{-1}(\Lambda) = \sum_\beta D(\Lambda^{-1})_{\alpha\beta} Q_\beta,
\]

\[
\{Q_\alpha, \overline{Q}_\beta\} = (-2i)(\gamma^\mu)_{\alpha\beta} P_\mu, \quad [Q_\alpha, P^\mu] = 0,
\]

that are supersymmetry generators. We define a supersymmetric transformation through the exponential mapping

\[
U(\vartheta) = \exp\left[i\vartheta^\top \epsilon \gamma_5 \vartheta\right],
\]

where \(\vartheta\) is a fermionic four-spinor that parametrizes the transformation. The composition rule for the supersymmetric transformation is given by

\[
U(\vartheta')U(\vartheta) = \exp\left[i\vartheta'^\top \epsilon \gamma_5 \vartheta'\right] U(\vartheta + \vartheta').
\]

We take the action of a supersymmetric transformation on creation-annihilation sparticle operators as

\[
U(\vartheta) a^*_\pm(p, s_\pm, \sigma) U(\vartheta)^{-1} = \exp\left[\vartheta^\top \epsilon \gamma_5 (+i\not{p}) (2s + \vartheta)_\pm\right] a^*_\pm(p, (s + \vartheta)_\pm, \sigma),
\]

\[
U(\vartheta) a_\pm(p, s_\pm, \sigma) U(\vartheta)^{-1} = \exp\left[(2s + \vartheta)^\top \epsilon \gamma_5 (+i\not{p}) \vartheta\right] a_\pm(p, (s + \vartheta)_\pm, \sigma).
\]

This equation is consistent with the composition property (2.18), with (2.14), and (2.15). From here we can write the finite supersymmetric transformations in
In the rest frame \( L \) consistently. In other words, the sparticle state condition \( \vartheta \) is invariant under a superponcaire transformation. When that is, the (anti)commutator of creation-annihilation sparticle operators remains recovering the structure of laddering operators of the fermionic generators (with steps \( \pm \frac{1}{2} \) in angular momentum). Equations (2.12) and (2.19) show that under the superpoincare group \( U(\Lambda, x, \vartheta) \equiv U(\Lambda, x)U(\vartheta) \):

\[
U(\Lambda, x, \vartheta) \left[ a_+^\dagger(p, s, \sigma), a_+^\dagger(p', s', \sigma') \right] U(\Lambda, x, \vartheta)^{-1} = \left[ a_+^\dagger(p, s, \sigma), a_+^\dagger(p', s', \sigma') \right],
\]

that is, the (anti)commutator of creation-annihilation sparticle operators remains invariant under a superponcaire transformation. When \( \vartheta \) satisfies the Majorana condition \( \vartheta = \vartheta^* \), equation (2.23) allows us to write \( (U(\Lambda, x, \vartheta)^{-1})^* = U(\Lambda, x, \vartheta) \) consistently. In other words, the sparticle state

\[
|p, s, \sigma\rangle^\pm = a_\pm(p, s, \sigma) |\text{VAC}\rangle,
\]

We note that \( U(\vartheta) \bar{b}(p, \sigma)U(\vartheta)^{-1} \) is consistent with (2.9). Taking \( \vartheta \) infinitesimal, equation (2.20) give us the following (anti)commutation relations:

\[
i \left[ a_\pm(p, \sigma), Q_\alpha \right] = + (2m)^{+1/2} \left[ b_\pm(p, \sigma)\epsilon \gamma_5 \right]_\alpha ,
\]

\[
i \left[ a_\pm(p, \sigma), Q_\alpha \right] = - (2m)^{+1/2} \left[ b_\pm(p, \sigma)\epsilon \gamma_5 \right]_\alpha ,
\]

\[
i \left\{ b_\alpha(p, \sigma), Q_\delta \right\} = + (2m)^{+1/2} a_\pm(p, \sigma) \left[ (I + \gamma_5) (m - i\varphi) \right]_{\delta\alpha} ,
\]

\[
i \left\{ b_\alpha(p, \sigma), Q_\delta \right\} = - (2m)^{+1/2} a_\pm(p, \sigma) \left[ (I - \gamma_5) (m - i\varphi) \right]_{\delta\alpha} .
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U(\Lambda, x, \vartheta) \left[ a_\pm(p, s, \sigma), a_\pm(p', s', \sigma') \right] U(\Lambda, x, \vartheta)^{-1} = \left[ a_\pm(p, s, \sigma), a_\pm(p', s', \sigma') \right],
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\]

\[
i \left\{ b_\alpha(p, \sigma), Q_\delta \right\} = + (2m)^{+1/2} a_\pm(p, \sigma) \left[ (I + \gamma_5) (m - i\varphi) \right]_{\delta\alpha} ,
\]

\[
i \left\{ b_\alpha(p, \sigma), Q_\delta \right\} = - (2m)^{+1/2} a_\pm(p, \sigma) \left[ (I - \gamma_5) (m - i\varphi) \right]_{\delta\alpha} .
\]
transforms unitarily under the superpoincare group. Note also that
\[ U(\Lambda, x, \vartheta) \left[ a_{\pm}(p, s, \sigma), a_{\pm}(p', s', \sigma') \right] U(\Lambda, x, \vartheta)^{-1} = \left[ a_{\pm}(p, s, \sigma), a_{\pm}(p', s', \sigma') \right] . \] (2.25)

It is also possible to eliminate the quadratic phase factor appearing in (2.19) by defining
\[ a_{\pm}^*(p, s, \sigma) \equiv \exp \left[ s^T \gamma_5 (-i\varrho) s \right] a_{\pm}^*(p, s, \sigma) , \]
\[ a_{\pm}(p, s, \sigma) \equiv \left( a_{\pm}^*(p, \gamma_5 \beta s^*, \sigma) \right)^* , \] (2.26)
leading to
\[ U(\Lambda, x) a_{\pm}^*(p, s, \sigma) U(\Lambda, x)^{-1} = e^{-i\varrho x} \sqrt{\frac{k^0}{p^0}} \sum_{\sigma'} U^{(j)}_{\sigma'\sigma} [W(\Lambda, p)] a_{\pm}^*(p, D(\Lambda)s, \sigma') , \]
\[ U(\Lambda, x) a_{\pm}(p, s, \sigma) U(\Lambda, x)^{-1} = e^{+i\varrho x} \sqrt{\frac{k^0}{p^0}} \sum_{\sigma'} U^{(j)*}_{\sigma'\sigma} [W(\Lambda, p)] a_{\pm}(p, D(\Lambda)s, \sigma') , \]
\[ U(\vartheta) a_{\pm}^*(p, s, \sigma) U(\vartheta)^{-1} = \exp \left[ \vartheta^T \gamma_5 (+i\varrho) s \right] a_{\pm}^*(p, s + \vartheta, \sigma) , \]
\[ U(\vartheta) a_{\pm}(p, s, \sigma) U(\vartheta)^{-1} = \exp \left[ \vartheta^T \gamma_5 (-i\varrho) s \right] a_{\pm}(p, s + \vartheta, \sigma) . \] (2.27)

### 3 Causal Superfields.

Now we are in a position to define causal quantum superfields out of momentum superspace Fourier transformations of the creation-annihilation sparticle operators. We choose supersymmetric transformations in configuration superspace that induce linear-homogeneous ones in the spacetime variable \( x^\mu \), they in turn generate symmetric covariant superderivatives[12]. It has to be noted that in this formalism these superderivatives arise directly from considering the most general superfield, without any other extra input. As in ordinary quantum field theory we introduce two kinds of superfields:
\[ \Xi_{\pm n}(x, \vartheta) \equiv \sum_\sigma \int d^8 p \ d^4 s \ a_{\pm}^*(p, s, \sigma) v_{\pm n}(x, \vartheta; p, s, \sigma) , \] (3.1)
\[ \Xi_{\pm n}(x, \vartheta) \equiv \sum_\sigma \int d^8 p \ d^4 s \ a_{\pm}(p, s, \sigma) u_{\pm n}(x, \vartheta; p, s, \sigma) , \] (3.2)
that give a total of four superfields. We demand for \( \Xi_{\pm n}^* \) the superpoincare transformation:
\[ U(\Lambda, a) \Xi_{\pm n}^*(x, \vartheta) U(\Lambda, a)^{-1} = \sum_{\pm m} [S(\Lambda^{-1})]_{\pm n, \pm m} \Xi_{\pm m}^* (\Lambda x + a, D(\Lambda)\vartheta) , \] (3.3)
\[ U(\xi) \Xi_{\pm n}^*(x, \vartheta) U(\xi)^{-1} = \Xi_{\pm n}^* (x^\mu + \vartheta^T \gamma_5 \gamma^\mu \xi, \vartheta + \xi) , \] (3.4)
where $S_{\pm n, \pm m}$ is a finite-dimensional Lorentz representation that in principle could be different for $\Xi^*_{+n}$ and $\Xi^*_{-n}$. With the help of (2.27), the general solution of (3.1), and including the requirements in (3.4), can be expressed as

$$
\Xi^*_\pm (x, \vartheta) = \sum_\sigma \int p^3 \, d^4 s \, e^{-ix \cdot p} e^{\vartheta \cdot \epsilon \gamma_5 (\pm ip)^s} A^*_\pm (p, s, \sigma) v_{\pm n} (p, (-ip)^r [s - \vartheta], \sigma) .
$$

(3.5)

The coefficients $v_{\pm n} (p, (-ip)^r [s - \vartheta], \sigma)$ are given in the rest frame:

$$
v_{\pm n} (p, (-ip)^r [s - \vartheta], \sigma) = \sqrt{\frac{p^0}{p^3}} \sum_{\pm m} [S(L(p))]_{\pm n, \pm m} v_{\pm n} (k, (-i\bar{k}) D[L(p)]^{-1} [s - \vartheta], \sigma) .
$$

(3.6)

Given a unitary representation for the superstate of superspin $j$, the coefficients in the rest frame are required to satisfy

$$
\sum_{\sigma'} v_{\pm n} (k, (-i\bar{k}) [s - \vartheta], \sigma') U^{(j)\sigma'}_{\sigma\sigma} (W) = \sum_{\pm m} [S(W)]_{\pm n, \pm m} v_{\pm n} (k, (-i\bar{k}) D [W^{-1}] [s - \vartheta], \sigma) ,
$$

(3.7)

with $W$ being a little group transformation of the form (2.4). Equations (3.6) and (3.7) have to be satisfied by the expansion coefficients of the $\vartheta - s$ variables independently, showing that the superfield (3.5) is a reducible realization of the superpoincare symmetry.

Consider the zero order fermionic expansion in $v_{\pm n}$ for the annihilation superfield:

$$
\chi^*_{\pm n} (x, \vartheta) \equiv -\frac{1}{m^2} \sum_\sigma \int d^3 p \, d^4 s \, e^{-ix \cdot p} e^{\vartheta \cdot \epsilon \gamma_5 (\pm ip)^s} A^*_\pm (p, s, \sigma) v_{\pm n} (p, \sigma) .
$$

(3.8)

Since we can generate terms of the form $[\bar{\psi} (\vartheta - s)]_\alpha$ by applying the superderivative defined as

$$
\mathcal{D} \equiv (\epsilon_5 \partial_\vartheta - \gamma^\mu \vartheta \frac{\partial}{\partial x^\mu}) ,
$$

(3.9)

we can reconstruct the reducible superfields $\Xi^*_\pm (x, \vartheta)$ from superfields of the form (3.8). We can also introduce a zero order creation superfield $\chi^*_{\pm n} (x, \vartheta)$:

$$
\chi_{\pm n} (x, \vartheta) \equiv -\frac{1}{m^2} \sum_\sigma \int d^3 p \, d^4 s \, e^{i x \cdot p} e^{\vartheta \cdot \epsilon \gamma_5 (\mp ip)^s} A_{\pm} (p, s, \sigma) u_{\pm n} (p, \sigma) .
$$

(3.10)

Given $n = (a, b)$, where $a = -A, -A + 1, \ldots, A - 1, A$ and $b = -B, -B + 1, \ldots, B - 1, B$, and $2A, 2B = 0, 1, 2, \ldots$, we enumerate irreducible finite representations of the Lorentz group by the $SU(2)$ pair of indices $(A, B)$. 

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Depending on whether we operate an even or odd number of times the $D$’s, we obtain all the possible superspins that an irreducible representation $S_{\pm m \pm n}$ can carry.

For the zero order and the first superderivative we have

\begin{align}
|\mathcal{A} - \mathcal{B}| &\leq j \leq |\mathcal{A} + \mathcal{B}|, \quad \text{zero order in } \mathcal{D}_\alpha; \\
|\mathcal{A} - \mathcal{B} + \frac{1}{2}| &\leq j \leq |\mathcal{A} + \mathcal{B} + \frac{1}{2}|, \quad \text{linear in } \mathcal{D}_\alpha. \quad (3.11) \quad (3.12)
\end{align}

These relations follow from (3.7) and the product rules of $(\mathcal{A}, \mathcal{B}) \otimes \left[\left(\frac{1}{2}, 0\right) \oplus (0, \frac{1}{2})\right]$. With the help of equation (2.26), we can integrate explicitly the superfields (3.8) and (3.10) in the fermionic variable $s$ to obtain:

\begin{align}
\chi^*_\pm n(x, \vartheta) &= \sum_\sigma \int d^3p \ e^{-ix^\pm p} a^*_\pm(p, \vartheta, \sigma)v_n(p, \sigma), \quad (3.13) \\
\chi_{\pm n}(x, \vartheta) &= \sum_\sigma \int d^3p \ e^{+ix^\pm p} a_{\pm}(p, \vartheta, \sigma)u_n(p, \sigma), \quad (3.14)
\end{align}

where $x^\mu_\pm = x^\mu - \vartheta^\Gamma \gamma^\mu \vartheta_\pm$. Note that in equations (3.13) and (3.14), we are dropping the sign $\pm$ in the Fourier coefficients $u_n$ and $v_n$ because the inequalities (3.11)-(3.12) allow us to consider $\pm$-superfields for one and the same representation.

From now on we will suppose that this is case. We can see that these zero order superfields are chiral:

\begin{align}
\mathcal{D}_+ \left(\frac{\chi^*_\pm n(x, \vartheta)}{\chi_{\pm n}(x, \vartheta)}\right) &= 0, \quad (3.15)
\end{align}

and also that

\begin{align}
\mathcal{D}_+^1 \mathcal{D}_- \left(\frac{\chi^*_\pm n(x, \vartheta)}{\chi_{\pm n}(x, \vartheta)}\right) &= \mp 4m \left(\frac{\chi^*_\pm n(x, \vartheta)}{\chi_{\pm n}(x, \vartheta)}\right). \quad (3.16)
\end{align}

The last set of equations are usually taken as the free equations of motion. For us they mean we can work with $\chi_{+n}(x, \vartheta)$ and $\chi_{-n}(x, \vartheta)$ without the need of introducing $\mathcal{D}_+^1 \mathcal{D}_-^1$, or just work with $(+)$ superfields $\chi_{+n}(x, \vartheta)$ and $\mathcal{D}_+^1 \mathcal{D}_-^1 \chi_{+n}(x, \vartheta)$ (similar remarks for $\chi^*_{\pm n}$). From the relation

\begin{align}
\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} &= + 2 (\gamma^\mu \epsilon \gamma_5)_{\alpha\beta} \partial_\mu, \quad (3.17)
\end{align}

and equation (3.28), $p_\pm$ products of $\mathcal{D}_{+\alpha}$ superderivatives together with $p_-$ products of $\mathcal{D}_{-\beta}$ superderivatives acting on $\chi_{\pm n}(x, \vartheta)$, are equivalent to $p_\pm$ products of $\mathcal{D}_{\pm \alpha}$ acting on $\chi_{\pm n}(x, \vartheta)$ plus sums of $p'_\pm < p_\pm$ products of $\mathcal{D}_{\pm \alpha}$ times ordinary derivatives $\partial_\mu$ acting on $\chi_{\pm n}(x, \vartheta)$. Also from (3.17), $\{\mathcal{D}_{\pm \alpha}, \mathcal{D}_{\pm \beta}\} = 0$, which means that non-zero products of superderivatives of the same sign end at the second order $\mathcal{D}_{\pm \alpha} \mathcal{D}_{\pm \beta}$, but $\mathcal{D}_{\pm \alpha} \mathcal{D}_{\pm \beta} = \frac{1}{2} (1 \pm \gamma_5)_{\alpha\beta} (\mathcal{D}_{\pm \alpha}^1 \mathcal{D}_{\pm \beta}^1)$, which due to (3.16), flips the signs of $\chi_{\pm n}(x, \vartheta)$ to $\chi_{\mp n}(x, \vartheta)$ (same remarks for $\chi^*_{\pm n}(x, \vartheta)$). Finally since derivatives of superfields can
be taken as superfields without derivatives, with complete generality we can consider superfields of the form\footnote{Expressions $(D\chi_n)_{\pm \alpha}$ and $(D\chi_n^*)_{\pm \alpha}$ are shorthand notations for $D_{\pm \alpha} \chi_n \pm \alpha$ and $D_{\pm \alpha} \chi_n^* \pm \alpha$, respectively.}

\begin{equation}
\chi_{\pm n}, \quad \chi_{\pm n}^*, \quad (D\chi_n)^{\pm \alpha}, \quad (D\chi_n^*)^{\pm \alpha}.
\end{equation}

For a fixed irreducible representation of the Lorentz group, due to (3.11) and (3.12), chiral superfields and linear superderivatives of chiral superfields are incompatible. Now we introduce causal superfields

\begin{equation}
\Phi_{\pm n}(x, \vartheta) = (2\pi)^{-3/2} \sum_{\sigma} \int d^3 p \left\{ e^{+i(x_+ - p)} a_{\pm}^\sigma (p, \vartheta, \sigma) u_n(p, \sigma) + (-)^{2B} e^{-i(x_+ - p)} a_{\pm}^{\sigma*} (p, \vartheta, \sigma) v_n(p, \sigma) \right\},
\end{equation}

\begin{equation}
\Phi^*_{\pm n}(x, \vartheta) = (2\pi)^{-3/2} \sum_{\sigma} \int d^3 p \left\{ (-)^{2B} e^{+i(x_+ - p)} a_{\pm}^{\sigma} (p, \vartheta, \sigma) (v_n(p, \sigma))^* + e^{-i(x_+ - p)} a_{\pm}^{\sigma*} (p, \vartheta, \sigma) (u_n(p, \sigma))^* \right\},
\end{equation}

with $v_n(p, \sigma) = (-)^{i + \sigma} u_n(p, -\sigma)$. Note that they are related by

\begin{equation}
\Phi^*_{\pm n}(x, \vartheta) = (\Phi_{\pm n}(x, e\gamma_5 \beta \vartheta^*))^*.
\end{equation}

Consider now another superfield $\tilde{\Phi}_{\pm n}^*(x', \vartheta')$ for the same sparticle. Introducing

\begin{equation}
(x_{12}^\mu) = x_{1}^\mu - x_{2}^\mu + (\vartheta_{2} - \vartheta_{1})^T \epsilon \gamma_5 \gamma^\mu (\vartheta_{2} + \vartheta_{1}) = -(x_{21}^\mu),
\end{equation}

we can we write the (anti)commutator of $\Phi_{\pm n}(x, \vartheta_{1})$ and $\tilde{\Phi}_{\pm n}^*(x', \vartheta')$ as

\begin{equation}
\left[ \Phi_{\pm n}(x, \vartheta_{1}), \tilde{\Phi}_{\pm n}^*(x', \vartheta') \right]_\varepsilon = (2\pi)^{-3} \int d^3 p (2p^0)^{-1} \exp \left[ +ix_{12}^{\pm} \cdot p \right] P_{n,\tilde{n}}(p, p^0)
+ \varepsilon (-)^{2(B+\tilde{B})} (2\pi)^{-3} \int d^3 p (2p^0)^{-1} \exp \left[ -ix_{12}^{\pm} \cdot p \right] P_{n,\tilde{n}}(p, p^0),
\end{equation}

with $\varepsilon = -1$ for commutator and $\varepsilon = +1$ for anticommutator. $P_{n,\tilde{n}}(p, p^0)$ can be expressed as\cite{7}

\begin{equation}
P_{n,\tilde{n}}(p, p^0) = P_{n,\tilde{n}}(p) + p^0 Q_{n,\tilde{n}}(p),
\end{equation}

where $P_{n,\tilde{n}}(p)$ and $Q_{n,\tilde{n}}(p)$ polynomials in $p$ obtained from

\begin{equation}
(2p^0)^{-1} P_{n,\tilde{n}}(p, p^0) = \sum_{\sigma} u_n(p, \sigma) \tilde{u}_{\tilde{n}}^\sigma(p, \sigma) = \sum_{\sigma} v_n(p, \sigma) \tilde{v}_{\tilde{n}}^\sigma(p, \sigma).
\end{equation}
Weinberg has shown [7] that \( P_{n,n}(p,p^0) = (-)^{2(A+B)} P_{n,-}(p,-p^0) \), therefore at \((x_1 - x_2)^2 > 0\):

\[
\left[ \Phi_{\pm n}(x_1, \partial_1), \Phi^*_{\mp n}(x_2, \partial_2) \right]_\varepsilon = \left( 1 + \epsilon(-)^{2(A+B)} \right) P_{n,-}(i\partial_1) \Delta_+(x^\pm_{12}) ,
\]

with

\[
\Delta_+(x^\pm_{12}) = (2\pi)^{-3} \int d^3 p \, (2p^0)^{-1} \exp \left[ +ix^\pm \cdot p \right] .
\]

Equation (3.25) vanishes provided that \( \epsilon = -(-)^{2(A+B)} = -(-)^{2j} \). For linear superderivatives of chiral superfields, the vanishing of the expression

\[
\left[ (D\Phi_{n'}(x_1, \partial_1))_{\pm\alpha}, (D\Phi^*_{n'}(x_2, \partial_2))_{\mp\beta} \right]_{(-\varepsilon')}
\]

at spacelike separations gives \( \varepsilon' = -(-)^{2j} = -\varepsilon \), therefore making \( \Phi_{\pm n} \) and \( (D\Phi_{n'})_{\pm\alpha} \) incompatible. Since \( \Phi_{\pm n} \) goes in accordance with the spin statistics theorem, from now on we will leave out \( (D\Phi_{n'})_{\pm\alpha} \) from the discussion.

Causal superfields are also chiral:

\[
D_\mp \left( \frac{\Phi^*_{\pm n}(x, \partial)}{\Phi_{\pm n}(x, \partial)} \right) = 0 ,
\]

and satisfy

\[
D_\mp \left( \frac{\Phi^*_{\pm n}(x, \partial)}{\Phi_{\pm n}(x, \partial)} \right) = \mp 4m \left( \frac{\Phi^*_{\mp n}(x, \partial)}{\Phi_{\mp n}(x, \partial)} \right) .
\]

Expanding the superfields as

\[
\Phi_{\pm n}(x, \partial) = \phi_{\pm n}(x) \mp \sqrt{2} \partial_\alpha \gamma_5 \psi_n(x) \pm 2m \delta^2(\partial_{\pm}) \phi_{\mp n}(x) ,
\]

we have

\[
\phi_{\pm n}(x) = (2\pi)^{-3/2} \sum_\sigma \int d^3 p \left\{ e^{ix \cdot p} a_{\pm}(p, \sigma) u_n(p, \sigma) \right. \]

\[
+ \left. (-)^{2B} e^{-ix \cdot p} a^*_{\pm}(p, \sigma) v_n(p, \sigma) \right\} ,
\]

\[
\left[ \psi_n(x) \right]_\alpha = \sqrt{2m} (2\pi)^{-3/2} \sum_\sigma \int d^3 p \left\{ e^{ix \cdot p} [ b(p, \sigma) ]_\alpha u_n(p, \sigma) \right. \]

\[
\left. - (-)^{2B+2j} e^{-ix \cdot p} \gamma_5 \beta^c(p, \sigma) ]_\alpha v_n(p, \sigma) \right\} ,
\]

with \( \psi_n(x) \) satisfying Dirac's equation: \((\hat{\partial} + m) \psi_n(x) = 0\). Now it is clear that one the roles of the superfields \( \Phi_{-n} \) and \( \Phi_{+n} \) is to allow us to use \((0, \frac{1}{2}) \otimes (A, B)\) and \((\frac{1}{2}, 0) \otimes (A, B)\), respectively, for their linear term. These component fields satisfy Klein-Gordon equations, since \( \psi_n \) fields also satisfy the Dirac equation, the number of independent components of \( \phi_{+n} \) and \( \phi_{-n} \) are equal to number of independent components of \( \psi_n \). There could be more redundancy equations that the three fields will share.
4 Time-ordered products and superpropagators.

So far everything has gone as in ordinary quantum field theory, but things are different for superpropagators: time-ordered products in Dyson-series are not supersymmetric invariant and we need to correct them in order to write superpropagators properly. We start by writing the superpropagator that follows from Wick’s pairing rules:

\[-i\tilde{\Delta}_{n,\bar{n}}^{\pm}\left(x_1, \vartheta_1, x_2, \vartheta_2\right) = \omega(x_{12}^0)(2\pi)^{-2} P_{n,\bar{n}} \left(-i\frac{\partial}{\partial x_1}\right) \Delta_+ \left(x_{12}^+\right) + \omega(x_{12}^0)(2\pi)^{-2} P_{n,\bar{n}} \left(-i\frac{\partial}{\partial x_1}\right) \Delta_+ \left(-x_{12}^\pm\right), \tag{4.1}\]

where \( \omega(x_{12}^0) = \omega(x_{1}^0 - x_{2}^0) \) is the step function. To illustrate that this superpropagator is not supersymmetric invariant, we consider interactions restricted to superpotentials

\[V(x, \vartheta) = V_\pm(x, \vartheta) + V_\mp(x, \vartheta), \tag{4.2}\]

Its general component expansion can be expressed as

\[V_\pm(x, \vartheta) = C(x_\pm) + \sqrt{2} \vartheta^\dagger \gamma_5 \Omega(x_\pm) + \delta^2(\vartheta_\pm) F(x_\pm). \tag{4.4}\]

Further restricting it to scalar superfields, the superpropagator then becomes (dropping the \(-i\) factor for now)

\[\delta^2(\vartheta_+\mp) \delta^2(\vartheta_{2\pm}) \tilde{\Delta}_{n,\bar{n}}^{\pm}(x_1, \vartheta_1, x_2, \vartheta_2) = \delta^2(\vartheta_+\mp) \delta^2(\vartheta_{2\pm}) \left[1 + 2\vartheta^\dagger \gamma_5 (-\vartheta_1) \vartheta_{2\mp} - 4m^2 \delta^2(\vartheta_+\pm) \delta^2(\vartheta_{2\mp}) \right] \Delta_F(x_1 - x_2), \tag{4.5}\]

with

\[\Delta_F(x) = (2\pi)^{-2} \int dq \frac{\exp[iq \cdot x]}{m^2 + q^2 - i\varepsilon}. \tag{4.6}\]

Making use of \((\Box - m^2) \Delta_F(x) = -\delta^4(x)\) we write

\[\delta^2(\vartheta_+\pm) \delta^2(\vartheta_{2\pm}) \tilde{\Delta}_{n,\bar{n}}^{\pm}(x_1, \vartheta_1, x_2, \vartheta_2) = \delta^2(\vartheta_+\mp) \delta^2(\vartheta_{2\pm}) \Delta_F \left(x_{12}^\pm\right) - 4 \delta^4(\vartheta_+) \delta^4(\vartheta_2) \delta^4(x_1 - x_2). \tag{4.7}\]

5The use of \(\delta^2(\vartheta_+) \tilde{W}_+(x, \vartheta)\) or \(\delta^2(\vartheta_-) \tilde{W}_-(x, \vartheta)\) in the first term of the superpotential is merely conventional since we can always make the redefinition \(\tilde{W}_\pm(x, \vartheta) = \tilde{W}_\pm^*(x, \vartheta)\).
The term \( + 4i \delta^4(\theta_1) \delta^4(\theta_2) \delta^4(x_1 - x_2) \) is Lorentz but not supersymmetric invariant. Since this expression is local in superspace, the non-covariant part of the superpropagator induces non-covariant terms in the interactions. In order to gain some insight on their form, we recall that although the step function is translational and Lorentz invariant (except at spacelike separations where to achieve Lorentz invariance commutators must vanish), it is not supersymmetric invariant. \( \omega \) would be supersymmetric invariant if it was evaluated at \( x_{12}^{\pm 0} \) or even at \( x_{12}^0 - \theta_1^T \epsilon \gamma_5 \gamma_0 \theta_2 \). Keeping in mind that the \( \Delta_+ \) functions in (4.1) are evaluated at \( x_{12}^{\pm 0} \) we write

\[
\omega \left( x_{12}^0 \right) = \omega \left( x_{12}^{\pm 0} \right) + \varsigma_\pm (z_1, z_2), \quad z = (x, \vartheta),
\]

(4.8)

with \( \varsigma_\pm (z_1, z_2) \) given by the negative of the next to zero order fermionic expansion coefficients in \( \omega \left( x_{12}^{\pm 0} \right) \). The second order of the unitary operator in expansion (1.1) is given by

\[
U^{(2)} = (-i)^2 \int d^8 z_1 d^8 z_2 \omega \left( x_{12}^0 \right) V(z_1) V(z_2), \quad (4.9)
\]

and for superpotentials can be written as

\[
U^{(2)} = (-i)^2 \int d^8 z_1 d^8 z_2 \left[ \omega \left( x_{12}^0 \right) V_\pm(z_1) V_\mp^*(z_2) + \omega \left( x_{12}^0 \right) V_\mp^*(z_1) V_\pm(z_2) + \ldots \right]
\]

\[
= U^{(2)}_1 + U^{(2)}_{n,1} + \ldots , \quad (4.10)
\]

with the superpoincare covariant term

\[
U^{(2)}_1 = (-i)^2 \int d^8 z_1 d^8 z_2 \left( \omega \left( x_{12}^{\pm 0} \right) V_\pm(z_1) V_\mp^*(z_2) + \omega \left( x_{21}^{\mp 0} \right) V_\mp^*(z_2) V_\pm(z_1) \right),
\]

(4.11)

and the non-covariant term

\[
U^{(2)}_{n,1} = (-i)^2 \int d^8 z_1 d^8 z_2 \left( \varsigma_\pm (z_1, z_2) V_\pm(z_1) V_\mp^*(z_2) + \varsigma_\mp (z_2, z_1) V_\mp^*(z_2) V_\pm(z_1) \right).
\]

(4.12)

Due to the fermionic delta functions in the superpotentials, we can evaluate invariant step functions at \( x_{12}^0 - 2 \theta_1^T \epsilon \gamma_5 \gamma_0 \theta_2 \), allowing us to write the non-covariant part of the step functions as

\[
\varsigma_\pm (z_1, z_2) = 2 \theta_1^T \epsilon \gamma_5 \gamma_0 \theta_2 \delta \left( x_{12}^0 \right) - 4 \delta^2(\theta_1 \pm) \delta^2(\theta_2 \mp) \frac{\partial}{\partial x_{12}^0} \delta \left( x_{12}^0 \right) . \quad (4.13)
\]

We can see from this that the other terms (expressed by \ldots in (4.10)) do not need to be corrected. Noting that \( \varsigma_\pm (z_1, z_2) = - \varsigma_\mp (z_2, z_1) \) we write

\[
U^{(2)}_{n,1} = (-i)^2 \int d^8 z_1 d^8 z_2 \varsigma_\pm (z_1, z_2) \left[ V_\pm(z_1), V_\mp^*(z_2) \right]. \quad (4.14)
\]
Using equation (4.4), we can integrate the fermionic variables to obtain

\[ U_{n,1}^{(2)} = 4 \int d^4x_1 d^4x_2 \left( i \delta(x_{12}^0) \sum_\alpha \{ [\Omega(x_1)]_{\alpha+}, [\Omega^*(x_2)]_{\alpha+} \} ight. \\
\left. + \delta(x_{12}^0) \frac{\partial}{\partial x_{10}} [C(x_1), C^*(x_2)] \right). \tag{4.15} \]

Any (anti)commutator will generate products of fields multiplied by \( \Delta(x) = \Delta_+(x) - \Delta_+(-x) \) functions and derivatives. Due to delta functions in time and \((\Box \Delta(x) = m^2 \Delta(x))\), the only surviving terms in the anticommutator(commutator) of equation (4.15) are the ones in which an odd(even) number of time derivatives act on \( \Delta(x) \), generating four dimensional delta functions \( \delta(x_{12}^0) \frac{\partial}{\partial x_{10}} \Delta(x_1 - x_2) = -i \delta^4(x_1 - x_2) \). This lets us write

\[ U_{n,1}^{(2)} = i \int d^4x_1 d^4x_2 \delta^4(x_1 - x_2) F(x_1), \tag{4.16} \]

with \( F(x_1) \) given explicitly by the term of the integral factor of \( d^4x_1 \) in (4.15). Therefore by making the substitution

\[ \mathcal{V}(x, \vartheta) \rightarrow \mathcal{V}(x, \vartheta) + \delta^4(\vartheta) F(x) \tag{4.17} \]

to cancel the non-covariant term (4.16), we are introducing higher order terms in coupling constants to the potential. Replacing (4.1) by

\[ -i \Delta_{n,\tilde{n}}^{\pm\mp}(x_1, \vartheta_1; x_2, \vartheta_2) = \omega(x_{12}^0) P_{n,\tilde{n}} \left( -i \frac{\partial}{\partial x_1} \right) \Delta_+(x_{12}^\pm) \\
+ \omega(-x_{12}^0) P_{n,\tilde{n}} \left( -i \frac{\partial}{\partial x_1} \right) \Delta_+(-x_{12}^{\pm}), \tag{4.18} \]

is effectively the same as replacing (1.1) by its supersymmetrized version

\[ \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int d^8 z_1 d^8 z_2 \ldots d^8 z_n \mathcal{T}_s \left[ \mathcal{V}(z_1) \mathcal{V}(z_2) \ldots \mathcal{V}(z_n) \right], \tag{4.19} \]

where \( \mathcal{T}_s \) is the time ordered product evaluated with the supersymmetric invariant step functions \( \omega(x_{12}^\pm) \).

Before presenting the final form of the superpropagator, we move the on-shell propagators \( P_{n,\tilde{n}}(p, p^0) \) in (4.18) to the left of \( \omega \). To do so, we extend (3.23) to the off-shell momentum case and then rewrite it as the sum of a Lorentz covariant part

\[ P_{n,\tilde{n}}^{(\text{off})}(\Lambda q) = \sum_{m,\tilde{m}} S_{n,m}(\Lambda) S^*_n_{\tilde{m}}(\Lambda) P_{m,\tilde{m}}^{(\text{off})}(q) \tag{4.20} \]
plus a Lorentz non-covariant term originated at \((x_1 = x_2)\). Following \([7]\), we drop this last term (and the label 'off') ending up with

\[ -i \Delta_{n,\tilde{n}}^{\pm\mp}(x_1, \vartheta_1, x_2, \vartheta_2) = (-i) P_{n,\tilde{n}} \left( -i \frac{\partial}{\partial x_1} \right) \Delta_F(x_{12}^\pm). \]  

(4.21)

From this we can obtain the superpropagator that arises from the pairing of \(\Phi_{\pm n}\) with \(\Phi^{*}_{\pm \tilde{n}}\):

\[ -i \Delta_{n,\tilde{n}}^{\pm\mp}(x_1, \vartheta_1, x_2, \vartheta_2) = \pm 2m(-i) \delta^2(\vartheta_1 - \vartheta_2) \pm P_{n,\tilde{n}} \left( -i \frac{\partial}{\partial x_1} \right) \Delta_F(x_{12}^\pm). \]  

(4.22)

We refer the reader to \([7]\) for the explicit formulas of the covariant off-shell propagator \(P_{n,\tilde{n}}\).

By restricting us to perturbation theory, and in analogy to the case of the Lorentz non-covariant terms, when defining supersymmetrized Dyson operators, we are also dropping supersymmetric non-covariant terms, i.e. we take equation (4.19) to be the definition of what we mean by a supersymmetric theory, and in doing so, we recognize that in passing from (1.1) to (4.19), we are introducing interactions from possible non trivial dynamics.

5 Super Feynman Rules.

Having all the ingredients, now we can state super Feynman rules. Since we are not including superderivatives, these rules can be written in similar manner to ordinary Feynman rules, the extra ingredient is that we have to add \((\pm)\) signs for every vertex formed by the superfields \(\Phi_{n+}\) and \(\Phi_{n-}\). For a theory written as sum of superfield polynomials \(\mathcal{H}_\ell\), of degree \(N_\ell\), the potential is:

\[ V(x, \vartheta) = \sum_{\ell} g_\ell \mathcal{H}_\ell(x, \vartheta). \]  

(5.1)

Now, the super Feynman rules are:

6(a) Include a factor of \(-i g_\ell\) for every vertex.

(b) For every internal line running from a \((\pm)\)-vertex at \((x_1, \vartheta_1)\) to a \((\mp)\)-vertex \((x_2, \vartheta_2)\), include a superpropagator:

\[ (-i) P_{n,\tilde{n}} \left( -i \frac{\partial}{\partial x_1} \right) \Delta_F(x_{12}^\pm). \]  

(5.2)

(c) For every internal line running from a \((\pm)\)-vertex at \((x_1, \vartheta_1)\) to a \((\mp)\)-vertex \((x_2, \vartheta_2)\) include a superpropagator:

\[ \pm 2m(-i) \delta^2(\vartheta_1 - \vartheta_2) \pm P_{n,\tilde{n}} \left( -i \frac{\partial}{\partial x_1} \right) \Delta_F(x_{12}^\pm). \]  

(5.3)

\(^6\)We are following very close the form presented in \([6]\).
(d) For every external line corresponding to a sparticle of superspin \( j \), superspin z-projection \( \sigma \) and supermomentum \((p, s)\), include:

\[(\mp)\)-sparticle created at vertex \((\pm)\):
\[
\mp 2m (2\pi)^{-3/2} e^{-ixp/\epsilon} e^{i(\vartheta - 2s)T} e_{\gamma\delta}^{\epsilon} (\vartheta \pm) u_n^*(p, \sigma).
\]  
\(5.4\)

\[(\pm)\)-sparticle created at vertex \((\pm)\):
\[
\mp 2m (2\pi)^{-3/2} e^{-ixp/\epsilon} \delta^2 \left[ (\vartheta - s) \pm \right] u_n(p, \sigma).
\]  
\(5.5\)

\[(\mp)\)-sparticle destroyed at vertex \((\pm)\):
\[
(2\pi)^{-3/2} e^{i\xi p/\epsilon} e^{i(\vartheta - 2s)T} e_{\gamma\delta}^{\epsilon} (\vartheta \pm) n(p, \sigma).
\]  
\(5.6\)

\[(\pm)\)-sparticle destroyed at vertex \((\pm)\):
\[
\pm 2m (2\pi)^{-3/2} e^{-ixp/\epsilon} \delta^2 \left[ (s - \vartheta) \pm \right] n(p, \sigma).
\]  
\(5.7\)

\[(\mp)\)-antisparticle created at vertex \((\pm)\):
\[
(2\pi)^{-3/2} e^{-ixp/\epsilon} e^{i(\vartheta - 2s)T} e_{\gamma\delta}^{\epsilon} (\vartheta \pm) v_n(p, \sigma).
\]  
\(5.8\)

\[(\pm)\)-antisparticle created at vertex \((\pm)\):
\[
\mp 2m (2\pi)^{-3/2} e^{-ixp/\epsilon} \delta^2 \left[ (\vartheta - s) \pm \right] v_n(p, \sigma).
\]  
\(5.9\)

\[(\mp)\)-antisparticle destroyed at vertex \((\pm)\):
\[
(2\pi)^{-3/2} e^{i\xi p/\epsilon} e^{i(\vartheta - 2s)T} e_{\gamma\delta}^{\epsilon} (\vartheta \pm) n^*(p, \sigma).
\]  
\(5.10\)

\[(\pm)\)-antisparticle destroyed at vertex \((\pm)\):
\[
\pm 2m (2\pi)^{-3/2} e^{i\xi p/\epsilon} \delta^2 \left[ (s - \vartheta) \pm \right] v^*_n(p, \sigma).
\]  
\(5.11\)

(e) Integrate all superspacetime vertex indices \((x, \theta)\), etc., and sum all discrete indices \(n, n'\), etc. (that come from Lorentz tensor products of the superfields in \(H_\ell\)).

(d) Supply minus signs that arise in theories with fermionic superfields.

To derive the wave superfunctions \((5.4)-(5.11)\), we have taken (anti)commutators of superfields and creation-annihilation (anti)sparticle operators. For external legs we can use any combination of + or − signs, since they are related by \((2.14)\) and \((2.15)\).

These rules work for general supersymmetric potentials, including Kähler type potentials.

6 \(C, P, T\) and \(R\) symmetries.

To explore the \(C, P, T\) and \(R\) transformation properties of the superfields, we have to turn-on the full notation of the \((A, B)\)–superfields: \(\Phi_{\pm n} \to \Phi_{\pm ab}^{AB}\). The transfor-
mation of annihilation and creation (anti)sparticle operators goes as follows

\[
C a_{\pm}(p, s_{\pm}, \sigma) C^{-1} = \zeta^+_* a^*_+ (p, \zeta_{s_{\pm}}, \sigma),
\]
\[
C a^c_{\pm}(p, s_{\pm}, \sigma) C^{-1} = \zeta^c_* a^*_c (p, \zeta_{s_{\pm}}, \sigma),
\]
\[
P a_{\pm}(p, s_{\pm}, \sigma) P^{-1} = \eta^+_* \eta a^*_+ (-p, \eta_{s_{\pm}} (\beta s_{\pm}), \sigma),
\]
\[
P a^c_{\pm}(p, s_{\pm}, \sigma) P^{-1} = \eta^c_* \eta c a^*_c (-p, \eta_{s_{\pm}} (\beta s_{\pm}), \sigma),
\]
\[
T a_{\pm}(p, s_{\pm}, \sigma) T^{-1} = \zeta^+_* \zeta (-)^{-\sigma} a_{\pm} (-p, \zeta_{s_{\pm}}, \sigma),
\]
\[
T a^c_{\pm}(p, s_{\pm}, \sigma) T^{-1} = \zeta^c_* \zeta (-)^{-\sigma} a^c_{\pm} (-p, \zeta_{s_{\pm}}, \sigma),
\]  

(6.1)

where some of the phases are restricted to

\[
\zeta_+ = \zeta^*, \quad \eta_+ = -\eta^*, \quad \zeta_+ = -\zeta^*, \quad \zeta^c_+ = \zeta^{c*}, \quad \eta^c_+ = -\eta^{c*}, \quad \zeta^c_+ = -\zeta^{c*}.
\]  

(6.2)

The numbers that have \( \pm \) signs have to be the same for all sparticles, this in order to guarantee supersymmetric covariance (this is due to the fact that they appear in the algebra of the transformations with fermionic generators). These relations can be obtained by starting with component transformations, then require invariance under (2.13), and consistency with (2.14). We should mention that to obtain appropriate relations for time reversal we have defined \( Ts = i s^* T \) for any fermionic number, in particular this guarantees that \( Ts s' = (s s')^* T \) for any pair of fermionic numbers. In order to perform superfield transformations we use [7]

\[
\begin{align*}
(u_{ab}^{AB}(p, +\sigma))^* &= (-)^{-a-b-j} v_{-a-b}^{BA}(p, -\sigma), \\
(v_{ab}^{AB}(p, +\sigma))^* &= (-)^{a-b-j} v_{b-a}^{BA}(p, +\sigma), \\
(u_{ab}^{AB}(p, \sigma))^* &= (-)^{a+b+\alpha A+B-j} u_{-a-b}^{AB}(-p, -\sigma), \\
(v_{ab}^{AB}(p, \sigma))^* &= (-)^{a+b+\alpha A+B-j} v_{b-a}^{BA}(-p, -\sigma), \\
u_{ab}^{AB}(-p, \sigma) &= (-)^{A+B-j} u_{ba}^{BA}(p, \sigma), \\
v_{ab}^{AB}(-p, \sigma) &= (-)^{A+B-j} v_{ba}^{BA}(-p, \sigma),
\end{align*}
\]

(6.3)

and the properties of the exponential factor in (3.31)

\[
\begin{align*}
ix_\pm \cdot (\Lambda_\rho p) &= i (\Lambda_\rho x) \cdot p - (\varepsilon_\rho \beta \bar{\theta})^T \epsilon_5 (\bar{\psi} + i \bar{\psi}) (\varepsilon_\rho \beta \bar{\theta})\pm, \\
ix_\pm \cdot p &= - (ix \cdot p - (\varepsilon_\rho \epsilon_5 \beta \bar{\theta}^*)^T \epsilon_5 (\bar{\psi} + i \bar{\psi}) (\varepsilon_\rho \epsilon_5 \beta \bar{\theta})\pm)^*, \\
(i x_\pm \cdot (\Lambda_\rho p))^* &= i (\Lambda_\rho x) \cdot p - (\varepsilon_\rho \epsilon \bar{\theta}^*)^T \epsilon_5 \epsilon_5 (\bar{\psi} + i \bar{\psi}) (\varepsilon_\rho \epsilon \bar{\theta})\pm,
\end{align*}
\]  

(6.4)

with \( (\varepsilon_\rho)^2 = (\varepsilon_\rho c)^2 = -1 \) and \( \Lambda_\rho = -\Lambda_\rho = diag(1 1 1 -1) \). For a superfield transforming onto another superfield, we must impose

\[
\eta_+ = \eta^c_+ = \varepsilon_\rho, \quad \zeta_+ = \zeta^c_+ = \varepsilon_\tau, \quad \zeta_+ = \zeta^c_+ = \varepsilon_\epsilon
\]

(6.5)
and

\[ \eta^e = \eta(-)^{2j}, \quad \zeta^e = \zeta, \quad \zeta = \zeta^e, \quad (6.6) \]

giving

\[
C \Phi_{\pm,ab}^A(x, \vartheta) C^{-1} = \zeta (-)^{2A-a-b-j} \Phi_{\pm,-b,-a}^B(x, \varepsilon_c \vartheta), \\
P \Phi_{\pm,ab}^A(x, \vartheta) P^{-1} = \eta (-)^{A+B-j} \Phi_{\pm,ba}^A(\Lambda_\rho x, \varepsilon_\rho \beta \vartheta), \\
T \Phi_{\pm,ab}^A(x, \vartheta) T^{-1} = \zeta (-)^{a+b+\sigma + A+B-j} \Phi_{\pm,-a,-b}^A(\Lambda_\tau x, \varepsilon_\tau \epsilon \vartheta^*). \quad (6.7)
\]

The combined CPT transformation becomes

\[
(CPT) \Phi_{\pm,ab}^A(x, \vartheta) (CPT)^{-1} = \zeta \eta \zeta (-)^{2A} \Phi_{\mp,ab}^A(-x, \varepsilon_c \varepsilon_\tau \beta \epsilon \vartheta^*). \quad (6.8)
\]

This last equation implies

\[
(CPT) \mathcal{V}(x, \vartheta) (CPT)^{-1} = \mathcal{V}(-x, \varepsilon_c \varepsilon_\tau \beta \epsilon \vartheta^*). \quad (6.9)
\]

Note that when applying T to \( \mathcal{V}(x, \vartheta) \) we pass through \( d^4x d^4\vartheta \), and because \( \varepsilon_c \varepsilon_\tau \varepsilon_\tau \) is just a sign, we can write \( T d^4\vartheta = (d^4\vartheta)^* T = d^4(\varepsilon_c \varepsilon_\tau \varepsilon_\tau \beta \epsilon \vartheta^*) T \), giving a proof of CPT invariance for massive supersymmetric theories.

The \( \mathcal{R} \) transformations on annihilation-creation (anti)particle operators are

\[
U(\theta_\mathcal{R}) a_{\pm}(p, s_\pm, \sigma) U(\theta_\mathcal{R})^{-1} = e^{-i(q \cdot \vartheta_\mathcal{R})} a_{\pm}(p, e^{i(q \cdot \vartheta_\mathcal{R})} s_\pm, \sigma), \\
U(\theta_\mathcal{R}) a_{\pm}^*(p, s_\pm, \sigma) U(\theta_\mathcal{R})^{-1} = e^{-i(q \cdot \vartheta_\mathcal{R})} a_{\pm}^*(p, e^{i(q \cdot \vartheta_\mathcal{R})} s_\pm, \sigma), \quad (6.10)
\]

where \( q_0 \) is the same for all superparticle species. With the help of

\[
x_{\pm} \cdot p = x \cdot p - (e^{i(q \cdot \vartheta_\mathcal{R})})^\dagger \epsilon_\gamma \sigma_\beta (e^{i(q \cdot \vartheta_\mathcal{R})})^\dagger \epsilon_\gamma \sigma_\beta, \quad (6.11)
\]

we can write

\[
U(\theta_\mathcal{R}) \Phi_{\pm,ab}^A(x, \vartheta) U(\theta_\mathcal{R})^{-1} = \exp [-i(q \cdot \vartheta_\mathcal{R})] \Phi_{\pm,ab}^A(x, \mathcal{R} \vartheta), \quad (6.12)
\]

with

\[
\mathcal{R}_{\alpha \beta} = \begin{pmatrix} \exp [-i \theta_\mathcal{R} q_0] & 0 \\ 0 & \exp [+i \theta_\mathcal{R} q_0] \end{pmatrix}_{\alpha \beta}. \quad (6.13)
\]

In defining \( \mathcal{R} \)-symmetries we allow \( U(\theta_\mathcal{R}) \) to be a discrete or continuous symmetry, restricting \( \{\theta_\mathcal{R}, q, q_0\} \) to take values in a discrete set in the former case.
7 Scalar superpotentials.

In this section we restrict ourselves to a theory of a sparticle with zero superspin whose interactions are constructed with cubic polynomials of the scalar superfield. We calculate the lowest order correction to time-ordered products and construct a superamplitude for a sparticle-antisparticle collision.

The parity and $\mathcal{R}$ transformations appearing in equations (6.7) and (6.12) become

\[
\begin{align*}
P \Phi_\pm (x, \vartheta) P^{-1} &= \eta \Phi_\pm (\Lambda_p x, \varepsilon_p \beta \vartheta) , \\
P \Phi^*_\pm (x, \vartheta) P^{-1} &= \eta^* \Phi^*_\pm (\Lambda_p x, \varepsilon_p \beta \vartheta) , \\
U (\theta_\mathcal{R}) \Phi_\pm (x, \vartheta) U (\theta_\mathcal{R})^{-1} &= \exp [-i(q \mp q_0) \theta_\mathcal{R}] \Phi_\pm (x, \mathcal{R} \vartheta) , \\
U (\theta_\mathcal{R}) \Phi^*_\pm (x, \vartheta) U (\theta_\mathcal{R})^{-1} &= \exp [+i(q \pm q_0) \theta_\mathcal{R}] \Phi^*_\pm (x, \mathcal{R} \vartheta) .
\end{align*}
\]

(7.1)

For a sparticle that is its own antiparticle, the first equation in (6.7) implies

\[
\Phi_\pm (x, \vartheta) = \Phi^*_\pm (x, \vartheta) ,
\]

(7.2)

with $\eta = \eta^*$. For the cubic superpotential, we have the following stock of possibilities to form interactions:

\[
\Phi_\pm \Phi_\pm \Phi_\pm , \quad \Phi_\pm \Phi_\pm \Phi^*_\pm , \quad \Phi_\pm \Phi^*_\pm \Phi_\pm , \quad \Phi^*_\pm \Phi^*_\pm \Phi^*_\pm .
\]

(7.3)

Under $\mathcal{R}$ transformations, together with $\delta^2 (\mathcal{R}^{-1} \vartheta_\pm) = \exp [\pm 2i q_0] \delta^2 (\vartheta_\pm)$, these terms generate the following phases in the superpotential:

\[
-3q \pm q_0 , \quad -q \pm q_0 , \quad +q \pm q_0 , \quad 3q \pm q_0 .
\]

(7.4)

Therefore for $\mathcal{R}$-symmetric cubic superpotentials, only one term (of the four possible) survives. For a sparticle that is its own antiparticle, due to (7.2) the four possibilities shrink to one.

Now consider a superpotential for a sparticle with different antiparticle \(^7\)

\[
\begin{align*}
\mathcal{W}_+ (x, \vartheta) &= \frac{g_+}{3!} \left( \Phi_+ (x, \vartheta) \right)^3 + \frac{g^*_-}{3!} \left( \Phi^*_+ (x, \vartheta) \right)^3 , \\
\mathcal{W}^*_+ (x, \vartheta) &= \frac{g^*_+}{3!} \left( \Phi^*_+ (x, \vartheta) \right)^3 + \frac{g_-}{3!} \left( \Phi_+ (x, \vartheta) \right)^3 .
\end{align*}
\]

(7.5)

When either $g_+$ or $g_-$ is zero, if $\mathcal{R}$-charges are properly chosen, we obtain $\mathcal{R}$-invariant superpotentials.

\(^7\)The name 'complex' superfield for such superfield is not appropriate since superfields are always chiral.
From (4.4) and (3.30) we can see that
\[
C(x) = \frac{g_+}{3!} (\phi_+)^3 + \frac{g_-}{3!} (\phi_-)^3
\]
\[
\Omega(x) = \frac{g_+}{2} (\phi_+)^2 \psi + \frac{g_-}{2} (\phi_-)^2 [-\epsilon \gamma_5 \beta \psi^*]
\]
\[
F(x) = g_+ \left( -\phi_+ \psi^\dagger \psi_+ + m (\phi_+)^2 \phi_- \right) + g_- \left( -\phi_- \psi^\dagger \psi_- + m (\phi_-)^2 \phi_+ \right).
\]
(7.6)

For this superpotential, the two lowest order correction terms in (4.15) are \(^8\)
\[
\delta \sum \alpha \left\{ [\Omega(x_1)]_{\alpha \pm}, [\Omega^*(x_2)]_{\alpha \pm} \right\} = -2\delta \left( x_0^0 \right) \frac{\partial}{\partial x_0} \sum [C(x_1), C^*(x_2)]
\]
\[
= \frac{1}{2} \left[ \delta^4 (x_1 - x_2) \right] F(x_2),
\]
(7.7)
where \( F(x_2) \) is the function appearing in (4.16) given by
\[
F(x_2) = |g_+|^2 (\phi_+ (x_2))^2 (\phi_+^* (x_2))^2 + |g_-|^2 (\phi_- (x_2))^2 (\phi_-^* (x_2))^2.
\]
(7.8)

The covariant spacetime potential
\[
-iV(x) = F(x) - F(x)^*
\]
(7.9)
aquires the form
\[
-iV(x) = g_+ \left( -\phi_+ \psi^\dagger \psi_+ + m (\phi_+)^2 \phi_- \right) + g_- \left( -\phi_- \psi^\dagger \psi_- + m (\phi_-)^2 \phi_+ \right)
\]
\[
+ g_+ \left( -\phi_+ \psi^\dagger \psi_-^* + m (\phi_+^*)^2 \phi_+^* \right) + g_- \left( -\phi_- \psi^\dagger \psi_-^* + m (\phi_-^*)^2 \phi_-^* \right).
\]
(7.10)

Finally, after integrating the fermionic variables in (4.17), the resulting corrected spacetime potential is
\[
-H_{\text{int}}(x) = -F(x) - V(x)
\]
\[
= -ig_+ \left( -\phi_+ \psi^\dagger \psi_+ + m (\phi_+)^2 \phi_- \right) - ig_- \left( -\phi_- \psi^\dagger \psi_- + m (\phi_-)^2 \phi_+ \right)
\]
\[
- ig_+ \left( -\phi_+ \psi^\dagger \psi_-^* + m (\phi_+^*)^2 \phi_+^* \right) - ig_- \left( -\phi_- \psi^\dagger \psi_-^* + m (\phi_-^*)^2 \phi_-^* \right)
\]
\[
- \left( |g_+|^2 (\phi_+^*)^2 + |g_-|^2 (\phi_-^*)^2 \right).
\]
(7.11)

For the case when a particle is its own antiparticle, the component fields satisfy
\[
\phi = \phi_+ = \phi_-^*, \quad \epsilon \gamma_5 \beta \psi = -\psi^*.
\]
(7.12)
The most general (corrected) spacetime cubic potential for this case is
\[
-\mathcal{H}_\text{int}'(x) = -ig (\phi \bar{\psi} \psi + m (\phi)^2 \phi^*) + ig^* (\phi^* \bar{\psi} \psi + m (\phi^*)^2 \phi) - |g|^2 (\phi)^2 (\phi^*)^2.
\]  
(7.13)

Making \( ig = \sqrt{2} \lambda e^{i\alpha} \) and \( \sqrt{2} \phi = e^{-i\alpha} (A + iB) \), this last equation can be written as
\[
-\mathcal{H}_\text{int}'(x) = -\lambda A (\bar{\psi} \psi) - i\lambda B (\bar{\psi} \gamma_5 \psi) - m\lambda A (A^2 + B^2) - \frac{\lambda^2}{2} (A^2 + B^2)^2
\]  
which is the interaction Lagrangian of the Wess-Zumino model[8]. Thus equation (7.11) generalizes to the case where a sparticle is different from its antisparticle and where possibly parity and \( \mathcal{R} \) symmetries are not conserved.

We now are ready to compute a superamplitude of a sparticle-antisparticle process for either \( g_+ \) or \( g_- \) zero in (7.5).

![Figure 1. Lowest order superdiagram for a sparticle-antisparticle collision.](image)

To lowest order, there is only one superdiagram for a sparticle-antisparticle collision (Figure 1). For the external legs we choose left or right fermionic four-spinors as follows:
\[
1 \rightarrow \pm, \quad 1^c \rightarrow \mp, \quad 2 \rightarrow \mp, \quad 2^c \rightarrow \pm.
\]  
(7.15)

The upper (lower) signs correspond to the case \( g_- = 0 \) (\( g_+ = 0 \)). After integrating out configuration superspacetime variables we are left with:
\[
S_{g_\mp} \left( \mathbf{p}_1, s_1 \mp, s_1^c \mp; \mathbf{p}_2, s_2 \mp; \mathbf{p}_2^c, s_2^c \mp \right) = (-4i)|g_\mp|^2 f \left( \mathbf{p}_1, \mathbf{p}_1^c, \mathbf{p}_2, \mathbf{p}_2^c \right) \times \frac{(p_1^c - p_2)^2}{m^2 + (p_1^c - p_2)^2} \times \exp \left\{ -2i \left( \bar{s}_2^c s_2^c - \bar{s}_1^c s_1^c \right) \epsilon_{\gamma_5} \frac{(p_1^c - p_2)^2}{(p_1^c - p_2)^2} \left( p_2 s_2 - \bar{s}_2^c s_1^c \right) \mp \right\},
\]  
(7.16)
where
\[
f(p_1, p'_1, p_2, p'_2) = (2\pi)^{-2} \left[ 16 (p_1)^0 (p'_1)^0 (p_2)^0 (p'_2)^0 \right]^{-1/2} \delta^4 (p_1 + p'_1 - p'_2 - p_2).
\]

(7.17)

To calculate the particle-antiparticle scattering-amplitude for particles that are created by the \( a^*_+ (p) \) and \( a^*_c (p) \), we take \( s_{i\pm} = s_{c1} = s_{c2} = s_{c2\pm} = 0 \) and the exponential factor in (7.16) vanishes. Then, since
\[
\left( \frac{p_1^c - p_2}{m^2 + (p_1^c - p_2)} \right)^2 = 1 - \frac{m^2}{m^2 + (p_1^c - p_2)^2},
\]
the zero component of the superamplitude is giving us the sum of two Feynman diagrams. These diagrams correspond to the interaction terms (present in (7.11)):
\[
(\mp im) g_\mp (\phi_\mp)^2 \phi_\pm + h.c + |g_\mp|^2 (\phi_\mp^*)^2.
\]

(7.19)

The particle-antiparticle scattering with three particles and three antiparticles gives us a total of \( 3^4 \) initial-final state combinations \(^{10}\). Therefore equation (7.16) represents a very economical expression for the set of all processes of these particles at order \( |g_\mp|^2 \).

8 Conclusions and outlook.

In this paper we obtain perturbative scattering superamplitudes as super Feynman diagrams for sparticles and antisparticles that carry any superspin. We accomplish this by introducing interactions out of superfields \( \Phi_{+n}, \Phi_{-n} \), and their adjoints, in any representation \( (A, B) \) of the Lorentz group. These superfields possess component fields \( \phi_{+n} \), \( \phi_{-n} \) in the representation \( (A, B) \) and \( \psi_n \) in the representation \( [(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})] \otimes (A, B) \).

It is striking that for scalar superfields, as we know from canonical and path integral formulations, the lowest order correction to time ordered-products seems to be necessary and sufficient to guarantee supersymmetric invariance at all orders, suggesting that perturbatively some sort of domino effect mechanism is occurring: lowest order corrections introduced at first order in Dyson series are canceling non-covariant terms in second order, these corrections then generate second order terms that seem to be canceling the non-covariant terms arising at third order, and so on. Since fermionic expansion coefficients of superamplitudes are picking up external lines, to any order in coupling constants, these coefficients are giving the sum of all possible diagrams originated at that order.

\(^9 a^*_+ \text{ and } a^*_c \text{ for } g_- = 0 \text{ and } a^*_c \text{ and } a^*_+ \text{ for } g_+ = 0.\)

\(^{10}\) Some of them are zero, for example all odd fermionic expansions in (7.16).
Pertubatively, most broken supersymmetric theories preserve the particle number of exact supersymmetric theories. Thus, the formalism presented in this work, can in principle be extended to compute superamplitudes in phases of the theory where non-degeneracy of the supermultiplet masses is unimportant. This can be done by extending the super Feynman rules to include symmetry breaking terms that originate as local couplings constants in the fermionic variables.

The case involving massless sparticles will be addressed in a forthcoming paper.

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A Notation and conventions.

We use repeatedly identities of Dirac matrices and fermionic four-spinor variables. Since these relations are standard, we limit ourselves to present the notation and conventions employed in the paper. We represent Dirac and Lorentz indices by $\alpha, \alpha', \beta, \beta'$, etc., and $\mu, \nu, \mu', \nu'$, etc., respectively. We take the Lorentz metric as $\eta_{\mu\nu} = \text{diag}(1 1 1 -1)$. The Dirac Representation $D(\Lambda)$ is generated by

$$D[\Lambda] = \exp \left[ -i \frac{1}{4} w_{\mu\nu} J^{\mu\nu} \right], \quad J^{\mu\nu} = \frac{i}{4} [\gamma^\mu, \gamma^\nu],$$

(A.1)

where the anticommutator of $\gamma$-matrices is taken positive: $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$. We stick to the representation

$$\gamma^0 = -i \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} = -i\beta, \quad \gamma_i = -i \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}. \quad \text{(A.2)}$$

Also we use

$$\gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \epsilon = \begin{pmatrix} e & 0 \\ 0 & e \end{pmatrix}, \quad e = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

(A.3)

that together with $\beta$ satisfy

$$\beta \gamma^\mu = -\gamma^{\mu T} \beta, \quad \epsilon \gamma_5 \gamma^\mu = -\gamma^{\mu T} \epsilon \gamma_5.$$ \quad \text{(A.4)}

For the standard transformation $p = L(p)k$, we take $k = (0 \ 0 \ 0 \ m)$ as standard vector.
For any four-spinor $v$, its right projection is written as $2v_+ = (I + \gamma_5)v$ and its left projection as $2v_- = (I - \gamma_5)v$. Useful identities for fermionic four-spinors are

\[
\begin{align*}
(s_\pm)(s_\pm)^T &= \frac{1}{2} \left[ \epsilon (I \pm \gamma_5) \right] \delta^2(s_\pm), \\
(s_\pm)(\epsilon \gamma_5 s_\pm)^T &= \frac{1}{2} \left( s^T \epsilon \gamma_5 \gamma_\mu s_\pm \right) \left[ (I \pm \gamma_5) \right] \gamma^\mu, \\
s^T \epsilon \gamma_5 \gamma_\mu s_\pm &= -s^T \epsilon \gamma_5 \gamma_\mu s_\mp, \\
(s^T \epsilon \gamma_5 \gamma_\mu s_\pm)^* &= (\epsilon \gamma_5 \beta s^*)^T \epsilon \gamma_5 \gamma_\mu (\epsilon \gamma_5 \beta s^*)_\pm,
\end{align*}
\]  

(A.5)

where $\delta^2(s)$ is defined by:

\[
\delta^2(s) \equiv \frac{1}{2} s^T \epsilon s, \quad [\delta^2(s)]^* = -\delta^2(s^*). \tag{A.6}
\]

A four-spinor satisfies the Majorana condition if

\[
s = \epsilon \gamma_5 \beta s^*. \tag{A.7}
\]

## B Fermionic Integrals.

Given a set of fermionic variables $\zeta_1 \ldots \zeta_N$, the Berizinian integral is defined to act from the left

\[
\int d\zeta_{N'} \ldots d\zeta_2 d\zeta_1 \{ \zeta_1 \zeta_2 \ldots \zeta_{N'} A \} = A, \quad N' \leq N. \tag{B.1}
\]

The lowest dimension (non-trivial) integral with this set of fermionic variables is the line integral:

\[
\sum_{ij}^n \int d\zeta_i^T \zeta_j C_{ij} = \text{Tr} C = \sum_{ij}^n \int d(D\zeta)^T_j (D\zeta)_j C_{ij}, \tag{B.2}
\]

where $D_{ij}$ is an invertible bosonic matrix, since $\text{Tr} C = \text{Tr} D^{-1} C D$ we have $d(D\zeta)^T = d\zeta^T D^{-1}$. This holds for any surface Berezinian integral:

\[
d(D\zeta)_1 d(D\zeta)_2 \ldots d(D\zeta)_{N'} = \left[ (D^{-1})^T d\zeta_1 \right]_1 \left[ (D^{-1})^T d\zeta_2 \right]_2 \ldots \left[ (D^{-1})^T d\zeta \right]_{N'}. \tag{B.3}
\]

The right side of the complex conjugate of (B.1) is $A^*$. If we allow conjugation to enter in the integral as $(\zeta_1 \zeta_2 \ldots \zeta_N)^*$, the net effect in the integral is

\[
\left( \int d\zeta_{N'} \ldots d\zeta_2 d\zeta_1 \{ \zeta_1 \zeta_2 \ldots \zeta_{N'} A \} \right)^* = \int (d\zeta_{N'} \ldots d\zeta_2 d\zeta_1)^* (\zeta_1 \zeta_2 \ldots \zeta_{N'})^* A^*. \tag{B.4}
\]

For fermionic four-spinors, two dimensional and four dimensional fermionic differentials are defined by

\[
d^2 s_\pm \equiv -\frac{1}{2} ds_\pm^T \epsilon ds_\pm, \quad d^4 s \equiv d^2 s_+ d^2 s_- . \tag{B.5}
\]
They give
\[
\int d^2 s \pm \delta^2 (s_\pm) = \int d^4 s \delta^4 (s) = 1 , \tag{B.6}
\]
where \( \delta^4 (s) = \delta^2 (s_+) \delta^2 (s_-) \). Under conjugation
\[
(d^2 s_\pm)^* = -d^2 s_\mp^* \quad (d^4 s)^* = d^4 s^* \ . \tag{B.7}
\]
From (B.3) we have
\[
d^4 s^* = d^4 (\epsilon s)^* = d^4 (\gamma_5 s)^* = d^4 (\beta s)^* = d^4 (\epsilon \gamma \beta s^* ) \ . \tag{B.8}
\]
For an arbitrary operator density \( \mathcal{K} (s) \) that appears as
\[
\int d^4 s \mathcal{K} (s) , \tag{B.9}
\]
due (B.4) and (B.8), hermiticity and Lorentz invariance in the higher order fermionic expansion \( s \) of \( \mathcal{K} (s) \) can be chosen as the requirement that
\[
\mathcal{K} (s) = [\mathcal{K} (\epsilon \gamma_5 \beta s^*)]^* . \tag{B.10}
\]
If \( s \) satisfies the Majorana condition (A.7), then equation (B.10) becomes \( \mathcal{K} (s) = [\mathcal{K} (s)]^* \). We also define fermionic derivatives to act from the left.

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