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GAMA/WiggleZ: The 1.4GHz radio luminosity functions of high- and low-excitation radio galaxies and their redshift evolution to z=0.75

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GAMA/WiggleZ: The 1.4 GHz radio luminosity functions of high- and low-excitation radio galaxies and their redshift evolution to z=0.75

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ABSTRACT

We present radio Active Galactic Nuclei (AGN) luminosity functions over the redshift range 0.005 < z < 0.75. The sample from which the luminosity functions are constructed is an optical spectroscopic survey of radio galaxies, identified from matched Faint Images of the Radio Sky at Twenty-cm survey (FIRST) sources and Sloan Digital Sky Survey (SDSS) images. The radio AGN are separated into Low Excitation Radio Galaxies (LERGs) and High Excitation Radio Galaxies (HERGs) using the optical spectra. We derive radio luminosity functions for LERGs and HERGs separately in the three redshift bins (0.005 < z < 0.3, 0.3 < z < 0.5 and 0.5 < z < 0.75). The radio luminosity functions can be well described by a double power-law. Assuming this double power-law shape the LERG population displays little or no evolution over this redshift range evolving as \( \sim (1+z)^{0.60^{+0.17}_{-0.17}} \) assuming pure density evolution or \( \sim (1+z)^{0.46^{+0.21}_{-0.21}} \) assuming pure luminosity evolution. In contrast, the HERG population evolves more rapidly, best fitted by \( \sim (1+z)^{2.93^{+0.36}_{-0.47}} \) assuming a double power-law shape and pure density evolution. If a pure luminosity model is assumed the best fitting HERG evolution is parameterised by \( \sim (1+z)^{7.41^{+1.76}_{-1.76}} \). The characteristic break in the radio luminosity function occurs at a significantly higher power \( (\gtrsim 1 \text{ dex}) \) for the HERG population in comparison to the LERGs. This is consistent with the two populations representing fundamentally different accretion modes.

Key words: galaxies: evolution, accretion – galaxies: active – radio continuum: galaxies
1 INTRODUCTION

The evolution of galaxies and the supermassive black holes at their centres appear to be closely connected. This connection is evident in the correlation between black hole mass and the stellar bulge mass (Magorrian et al. 1998) and black hole mass and stellar velocity dispersion (e.g. Gebhardt et al. 2000). One way this coupling may occur is via feedback processes from the Active Galactic Nuclei (AGN) depositing energy, released by the accretion of matter onto the black hole, into their host galaxy and its surrounding environment. For example, radio jets from AGN are commonly invoked as a feedback mechanism to inhibit gas cooling and suppress star-formation in massive galaxies and clusters of galaxies (e.g. Binney & Tabor 1995; Fabian et al. 2002; Best et al. 2006; Croton et al. 2006; Bower et al. 2006; McNamara & Nulsen 2007). Such a feedback cycle can simultaneously solve the cooling flow problem; the sharp bright-end turn down in the optical galaxy luminosity function and the fact that the most massive bulge dominated galaxies contain the oldest stellar populations (Croton et al. 2006).

The properties of the AGN population indicate that there are two fundamentally different accretion modes operating (Hardcastle et al. 2007; Best & Heckman 2012). In the classical ‘cold mode’, material is accreted onto the supermassive black hole via a small, geometrically thin, optically luminous accretion disc. This disc is the source of ionising photons producing both broad and narrow line emission in the optical spectrum of AGN and x-ray emission via the inverse-Compton process. In the unified AGN model (Antonucci 1993), the broad line region is obscured by a dusty torus when viewed from certain orientations resulting in the type I (not obscured) and type II (obscured) AGN classifications. As a result of the emission lines in their optical spectra, these ‘cold-mode’ AGN are also referred to as ‘quasar mode’ or ‘high-excitation’ AGN. However, there is a large population of AGN observable by their radio emission but without the bright high-ionisation emission lines in their optical spectra (e.g. Hine & Longair 1979; Laing et al. 1994; Jackson & Rawlings 1997; Best & Heckman 2012). These display no evidence for the presence of an accretion disc or dusty torus (e.g. Chiaberge et al. 2002; Whysson & Antonucci 2004) and are likely powered by radiatively inefficient accretion possibly from a hot gas halo (e.g. Hardcastle et al. 2007; Best & Heckman 2012). These ‘hot-mode’ AGN (also referred to as ‘radio mode’ or ‘low-excitation’ AGN) are generally more massive, have higher mass-to-light ratios, are redder, and are of earlier morphological type than strong-lined AGN (e.g. Kauffmann et al. 2003, 2008; Best & Heckman 2012). The ‘cold-mode’ AGN display a higher rate of interactions and peculiarities in their morphology (e.g. Smith & Heckman 1989).

Both AGN modes will inject energy into their environment and have the potential to influence star-formation and galaxy evolution. In the ‘hot mode’ the radio jets heat the surrounding hot gas atmosphere. This is the same gas that fuels the AGN allowing a self regulating feedback cycle with a balance established between heating and cooling (Croton et al. 2006; Best et al. 2006). In the ‘cold mode’ AGN are known to drive winds in their host galaxy (e.g. Ganguly & Brotherton 2008; Harrison et al. 2014; McElroy et al. 2015) and have been suggested as mechanisms to both enhance (Silk & Nusser 2010) and suppress (Shabala et al. 2011; Page et al. 2012; Davis et al. 2012) star formation.

In the local Universe the number of radio AGN with low-excitation optical spectra (Low Excitation Radio Galaxies, hereafter LERGs) outnumber the High-Excitation Radio Galaxies (hereafter, HERGs) at all but the highest radio luminosities (Best & Heckman 2012). The change in the dominant population occurs at $L_{1.4\text{GHz}} \sim 10^{26}$ W Hz$^{-1}$. The cosmic evolution of the space density of radio AGN is sensitive to radio luminosity. It is well established that the density of the most powerful radio AGN increases rapidly with increasing redshift out to $z \sim 2$ (Longair 1966; Doroshkevich et al. 1970; Willott et al. 2001; Sadler et al. 2007) – increasing by a factor of $\sim 1000$. At higher redshift the space density flattens and then decreases. The decrease occurs at higher redshift for more luminous sources (e.g. Peacock 1985; Dunlop & Peacock 1990; Cirasuolo et al. 2006). This fast evolving high luminosity population is mostly associated with Fanaroff-Riley II radio sources (Jackson & Wall 1999) and objects with strong optical emission lines (Willott et al. 2001; Best & Heckman 2012) i.e. HERGs. Conversely, the low luminosity radio sources are more commonly associated with Fanaroff-Riley I sources and objects which lack strong optical emission lines i.e. LERGs. The space density of these low luminosity radio galaxies shows little (a factor $\sim 2$) or no redshift evolution out to $z\sim 1$ (e.g. Clewley & Jarvis 2004; Sadler et al. 2007; Donoso et al. 2009; Smolčić et al. 2009; McAlpine et al. 2013).

Best & Heckman (2012) constructed the first radio galaxy luminosity function with the LERG and HERG contributions separated. They found, that while the LERGs outnumber the HERGs below $L_{1.4\text{GHz}} \sim 10^{26}$ W Hz$^{-1}$, examples of both classes are found at all radio luminosities. The HERG population shows evidence of evolution over the redshift range of their sample ($z < 0.3$) while there is no evidence for cosmic evolution of the LERG population. This implies the evolution of the radio luminosity functions depend on radio luminosity, and at least in part, be explained by a changing mix in the populations (Best & Heckman 2012). Best et al. (2014) using a composite of 8 published AGN samples (Wall & Peacock 1985; Gendre et al. 2010; Rigby et al. 2011; Lacy et al. 1999; Hill & Rawlings 2003; Waddington et al. 2001; Simpson et al. 2006) constructed a catalogue of 211 radio-loud AGN in the range $0.5 < z < 1.0$. Using this to compare with local samples (Best & Heckman 2012; Heckman & Best 2014) they made the first measurement of the evolution of the radio AGN population separated, spectroscopically, into LERGs (jet-mode in their nomenclature) and HERGs (dubbed radiative-mode). They found that the space density of the HERGs in their sample increased by around an order of magnitude between $z=0$ and $z=1$ in contrast, the LERG AGN density decreased with redshift at luminosities below $L_{1.4\text{GHz}} \sim 10^{26}$ W Hz$^{-1}$ and increased at higher luminosities.

In this paper we present the radio luminosity function of LERGs and HERGs and their cosmic evolution to $z=0.75$, corresponding to the second half of the history of the Universe. These luminosity functions are based on a new sample of over five thousand radio galaxies ($0.005 < z < 0.75$) with confirmed spectroscopic redshifts and spectroscopic classifications. Throughout this paper we convert from observed
to physical units assuming a $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ and $H_0 = 70\,\text{km\,s}^{-1}\text{Mpc}^{-1}$ cosmology.

2 SAMPLE CONSTRUCTION

The underlying sample from which we constructed the luminosity distributions of various radio galaxy populations is the Large Area Radio Galaxy Evolution Spectroscopic Survey (LARGESS; Ching 2015). This sample was constructed by matching the Faint Images of the Radio Sky at Twenty-centimetre survey (FIRST; Becker et al. 1995) with the photometric catalogue of the Sloan Digital Sky Survey data release 6 (SDSS; Adelman-McCarthy et al. 2008). The catalogues were matched to all objects in the FIRST catalogue ($\lesssim 0.5\,\text{mJy}$) and an optical apparent magnitude limit of $i = 20.5$ (SDSS $i$-band model magnitudes). The matching procedure was designed to include both point and extended radio sources, including those with multiple components. This matching was performed over $\approx 900$ square degrees of sky in regions where a high completeness of optical spectroscopy could be obtained. A careful estimation of the matching reliability and completeness was performed by Ching (2015) using monte-carlo simulations of randomised catalogs. The overall reliability of the matching is estimated as $\approx 93.5$ per cent and the overall completeness is $\approx 95$ per cent. The incompleteness and inclusion of false matches will somewhat offset in the effects on the normalisation of the luminosity functions. We do not do any scaling of the luminosity functions to account for input catalog incompleteness but note that its effect on the normalisation could not be more than a few per cent.

A possible systematic could arise if the incompleteness or false matches are biased toward a particular source type or source properties. The most likely source of such a bias is the behaviour of the matching algorithm for sources with complex (non-point source) radio morphologies. Such sources are more common at low redshift and lower luminosity as a result of angular resolution effects and the presence of extended star-forming galaxies. There is no evidence for such a bias. Only $\approx 10$ per cent of the sample is matched to more than one FIRST component, which is consistent with other determinations from the literature (Ivezić et al. 2002). Of those $\approx 10$ per cent the fraction of matches to the random catalogs are almost identical for FIRST sources with two, three and four or more components.

While the high spatial resolution of the FIRST catalogue makes it preferable for matching to the optical photometry it will also resolve out extended radio emission resulting in lower flux density measurements. Since angular size decreases with redshift this flux loss will be more significant at low redshift and could mimic evolution in the radio luminosity function. To avoid this we replace the flux densities with those measured from the lower spatial resolution NRAO VLA Sky Survey (NVSS; Condon et al. 1998) catalogue. In the case where a single NVSS source coincides with more than one FIRST source the NVSS flux is split between objects with the same flux ratio as measured by FIRST (see Ching (2015) for details of the matching of FIRST and NVSS sources).

At the faintest flux densities the NVSS catalogue will suffer from significant incompleteness. To avoid working with a parent sample that suffers from such incompleteness we only include objects with flux density greater than 2.8 mJy in constructing our luminosity function. This is the same flux limit adopted by Sadler et al. (2007) when matching radio sources to the luminous red galaxies in the 2dF-SDSS LRG and QSO survey (2SLAQ; Croom et al. 2009). With this flux-density limit the source counts are still rising at the faintest flux densities; as demonstrated in Figure 1. This results in a sample of 10827 matched radio/optical sources.

The optical spectroscopy for the sample was gathered either from publicly available archival sources such as the SDSS (Adelman-McCarthy et al. 2008), 2SLAQ (Cannon et al. 2006; Croom et al. 2009) and the 2dF QSO Redshift survey (2QZ; Croom et al. 2004) or from a dedicated ‘spare-fibre’ campaign using the 2dF/AAOmega instrument on the Anglo Australian Telescope. This was done by ‘piggy-backing’ on the WiggleZ Dark Energy Survey (WiggleZ; Drinkwater et al. 2010) and GAMA (Baldry et al. 2010; Driver et al. 2011; Liske et al. 2015) large survey programs by using a small number of fibres on each survey field to target radio galaxies (see Ching (2015) for details). This has almost no impact on the large survey efficiency but over time results in a large number of ‘spare-fibre’ spectra being obtained. Of the 10827 radio galaxies satisfying our 1.4 GHz flux density criterion, the number with optical spectra is 7088. Of these 6215 are deemed to have reliable redshift determinations ($\approx 87$ per cent).

2.1 Optical spectroscopic classification

In addition to providing an accurate measure of the redshift the optical spectroscopy can be used to determine the physical origin of the radio emission i.e. star-formation or AGN. The optical spectroscopy can also be used to separate the AGN into LERGs and HERGs. Full details of the semi-automatic classification procedure is given in Ching (2015). In brief, FIRST galaxies whose radio emission is dominated
by star-formation are identified using a Baldwin, Philips and Terlevich (hereafter BPT; Baldwin et al. 1981) diagram. Only galaxies with radio luminosity $L_{1.4\text{GHz}} \leq 10^{24}$ W Hz$^{-1}$ and $z < 0.3$ are considered, since galaxies more powerful than this would require unrealistically high star-formation rates and so are assumed to have their radio emission generated by an AGN. The BPT diagnostic will not work in the case where the galaxy has a radio-quiet optical AGN and radio emission powered by star-formation (Best & Heckman 2012; Ching 2015). These cases were identified based on a comparison of the inferred star-formation rates from H$\alpha$ and radio luminosity. The remaining radio galaxies are classified as AGN and further subdivided using the equivalent width of the [OIII]5007 emission line, where galaxies are classified as HERGs if they have SNR([OIII]5007) > 3 and EW([OIII]5007) > 5 Å. See Ching (2015) for a detailed description of the methods used to make the line strength measurements. The 5 Å demarcation is the same as that used by Best & Heckman (2012).

The [OIII]5007 line is redshifted out of the WiggleZ and SDSS spectra at $z \sim 0.83$ and out of the GAMA spectra at $z \sim 0.76$. These also correspond, approximately, to the redshift where our optical k-corrections are expected to be reliable (Blanton & Roweis 2007). In addition, as a result of the magnitude limit of the optical spectroscopic follow up, above $z \sim 0.75$ the spectroscopic sample is dominated by broad line AGN and there are almost no narrow-line AGN or LERGs in the sample (see Figure 2). We therefore impose a redshift limit of $z = 0.75$ for construction of our luminosity functions. We also apply a lower redshift limit of $z = 0.005$ to remove Galactic objects. With these cuts the final number of objects, with reliable redshifts, is 5026. A summary of the number of spectra from each survey in this final 5026 objects is given in Table 1.

Not all objects can be classified using the semi-automated methods of Ching (2015). Out of the 5026 objects in our sample 284 (~ 6.7 per cent) were not classified by Ching (2015). A fraction of the spectra in the Ching (2015) sample had been inspected and classified visually, and in the cases where there is no automated classification but a visual one, we use this visual classification. This leaves 220 objects (corresponding to ~ 4.6 per cent) without a classification. Rather than carry these unclassified objects through the analysis we performed visual classification of these sources. For the most part this was straight-forward. The fraction of objects classified as each type visually are similar to the overall fractions but with a lower frequency of ‘star-forming’ objects and a higher fraction of HERGs. The fractions of objects of each type prior to visual classification are ~75.2, 14.3 and 10.5 per cent for the LERGs, HERGs and star-forming galaxies, respectively. The corresponding fractions in the visual classification are ~78.6, 20.9 and 0.5 per cent. The increase in the HERG fraction (including quasars) and the decrease in the star-forming fraction is expected since the redshift distribution of the unclassified objects is, not surprisingly, peaked at the high redshift end of the sample.

3 THE RADIO LUMINOSITY FUNCTION

We first wish to construct the bivariate luminosity function for all galaxies in the optical-radio matched sample in multiple redshift bins. That is, we calculate the volume density of galaxies per interval of radio luminosity per interval of optical luminosity in each redshift bin. To do this we use the standard $1/V_{\text{max}}$ method (Schmidt 1968), where the number density in each bin is given by:

$$
\Phi(M_i, L_{1.4\text{GHz}}) \Delta M_i \Delta \log L_{1.4\text{GHz}} = \sum_{\text{gal}} \frac{w_{\text{gal}}}{V_{\text{max}}(\text{gal})}
$$

The sum on the right-hand side is over all galaxies in the bin. The weight, $w_{\text{gal}}$, given to each galaxy corrects for completeness in the spectroscopic follow up. For a complete survey $w_{\text{gal}} = 1$ and for incomplete samples $w_{\text{gal}}$ is given by the reciprocal of the completeness. The denominator, $V_{\text{max}}(\text{gal})$, is the maximum volume over which the galaxy could have been observed given the selection limits in both the radio and optical. We outline our methods for estimating $w_{\text{gal}}$ and $V_{\text{max}}(\text{gal})$ below.

3.1 Estimating the completeness

The overall spectroscopic completeness is given by the number of targets for which we obtained a spectrum of sufficient quality to make a reliable redshift measurement divided by the number of targets in the parent radio/optical matched sample. That is $0.6215/10827$ or ~57 per cent. This is mostly targeting incompleteness; the percentage of spectroscopically targeted objects which resulted in a good quality redshift classification, i.e. spectroscopic completeness, is ~90 percent. This spectroscopic incompleteness is not random. It will depend sensitively on the optical magnitude for at least two reasons. Firstly, the optical spectroscopy of the sample is constructed from spectroscopy from multiple surveys, which have different limiting magnitudes. In addition, the spectroscopic completeness within an individual survey generally decreases with optical magnitude as the noisier spectra obtained for fainter galaxies make it increasingly difficult to identify the redshift. Ching (2015) used repeat observations to demonstrate that there is little difference in the likelihood of obtaining a redshift for different spectral classes (with and without emission lines) once optical magnitude is taken into account. They did this by comparing the fraction of objects of different spectral classes that required more than one repeat observation to acquire a good redshift. The idea being that if for a particular source type it is easier to identify a redshift (at give apparent magnitude) than it should, on average, require less repeat observations. While they did find a slightly higher tendency for spectra without emission lines to require a repeat observation than those with emission lines, it was not statistically significant. One important caveat on this technique is that it can only be used in cases where a reliable redshift is eventually obtained by one of the observations.

The completeness will also be sensitive to colour, again for at least two distinct reasons. Firstly, while most of spectra were obtained from surveys that do not apply colour selection criteria (e.g. our dedicated follow-up, targets with SDSS spectra and the GAMA main survey), a fraction of the spectra come from surveys that do (e.g. the WiggleZ main
Table 1. A summary of the number of spectra from each survey in this final 5026 objects which are used in the measurement of the luminosity functions.

| Survey                      | z < 0.3 | 0.3 ≤ z < 0.5 | 0.5 ≤ z < 0.75 | Total  |
|-----------------------------|---------|---------------|----------------|--------|
| N(SDSS)                     | 1958    | 846           | 221            | 3025   |
| N(WiggleZ main)\(^a\)       | 26      | 137           | 151            | 314    |
| N(WiggleZ radio)\(^b\)      | 72      | 354           | 389            | 815    |
| N(WiggleZ other)\(^c\)      | 9       | 15            | 40             | 64     |
| N(GAMA)                     | 154     | 298           | 221            | 673    |
| N(2SLAQ LRGs) \(^d\)       | 0       | 32            | 95             | 127    |
| N(2SLAQ QSOs) \(^e\)       | 1       | 1             | 3              | 5      |
| N(2QZ/6QZ) \(^f\)          | 1       | 1             | 1              | 3      |
| Total                       | 2221    | 1684          | 1121           | 5026   |

\(^a\)WiggleZ main survey targets.

\(^b\)Spare fibre targets for this project.

\(^c\)Target for other spare fibre programs.

Figure 2. Left panel: The redshift distribution of all radio galaxies with reliable spectroscopic redshifts. The redshift distributions for star-forming galaxies, LERGs and HERGs are shown separately. Above \(z \sim 0.8\) the sample is dominated by broad-line AGN. Right panel: The \(n(z)\) of the sample used to construct luminosity functions in this paper after selecting objects with \(0.005 < z < 0.75\) and \(S_{1.4\,\text{GHz}} > 2.8\,\text{mJy}\).

survey, 2SLAQ and 2QZ). Another reason completeness will vary with colour, is that colour correlates with other spectral properties which make a redshift easier to identify. In particular, galaxies with bluer colour are more likely to have easy to identify emission lines. Although, as mentioned above, this appears to cause little bias.

There may be other properties which influence the spectroscopic completeness (e.g. changes in the observable spectral features in a fixed observing band with redshift) but it is likely that magnitude and colour account for the dominant dependencies. We therefore choose to construct our spectroscopic completeness in this plane, specifically we calculate our completeness (and hence our \(w_{\text{gal}}\)’s) in the \(i\)-magnitude versus \(g - i\) plane.

We calculate the completeness for each object (or position) in the \(i\)-magnitude versus \(g - i\) plane by averaging over an adaptively sized circle in this plane. The size of the circle is calculated by using the distance in the plane to the 50th nearest neighbour in the photometric input catalog. Using this adaptive bin size for estimating the completeness means in regions of the plane with few objects we still use a reasonable number of objects in estimating the completeness, while in high density regions we obtain a finer more local sampling of the completeness. The completeness in this plane is shown in Figure 3. Each galaxy is assigned a \(w_{\text{gal}}\) given by the reciprocal of the completeness. Using this nearest neighbour method for estimating the completeness even objects with large weights have had their completeness estimated using a reasonable number of objects. For example, for an object with a weight of 10 (completeness of 0.1) the estimate is based on five objects with good spectra and 50 objects in the input catalogue. We also checked that our luminosity functions are not largely affected by objects with the highest weights. Removing objects with weights greater than 10 makes no significant quantitative difference and no qualitative difference to our results.

3.2 Estimating \(V_{\text{max}}\)

To evaluate Equation 1, and correctly weight the contribution to the luminosity function of each galaxy in our sample, we need to estimate the volume within which each galaxy...
Figure 3. The completeness map in the $i$-magnitude versus $g-i$ colour plane, used to generate the weight ($w_{gal}$; shown in sidebar) assigned to each galaxy in constructing the luminosity function. Overlaid blue points are the photometric sample objects above our 2.8 mJy flux limit for which a reliable redshift has been obtained. The overlaid red points are objects for which no spectroscopic follow observation was made. The yellow points represent objects which were observed spectroscopically but a reliable redshift could not be determined. The structure in the completeness map, such as the low completeness for faint objects with $g-i$ colours of $\sim 1-2$ mag, is mostly the result of targeting completeness in the constituent surveys and demonstrates the importance of constructing our completeness estimates in this plane.

would satisfy the optical and 1.4 GHz flux density selection limits i.e. $V_{\text{max}}$. This volume will depend on the redshift of the source, its optical magnitude, 1.4 GHz flux density, and the spectral shape in both the optical and radio regimes. To do this, for each source, we first calculate the absolute $i$-band magnitude including a k-correction and evolutionary correction:

$$M_i = m_i - DM - K_i(z) - e(z)$$

where $DM$ is the distance modulus, $K_i(z)$ is the k-correction and $e(z)$ is the evolutionary correction (e-correction). The k-correction is calculated using version v4_2 of the kcorrect package (Blanton & Roweis 2007) using the 5 band (u,g,r,i,z) SDSS photometry. The e-correction accounts for the fading of stellar populations with time. For the LERGs we use the e-corrections for early type galaxies from Poggianti (1997).

The HERGs are generally bluer, more likely to be interacting and star-forming (or recently star-forming) systems which mitigates the effect on the overall optical luminosity of the ageing of the underlying older stellar population. Given this we set $e(z) = 0$ for the HERGs and discuss the effects this assumption has on the evolution of the luminosity function in Section 4. It is worth noting the magnitude of the e-correction is significant: increasing from $\sim 0.1$ mag ($i$-band) at $z=0.1$ to $\sim 0.8$ mag at $z=0.75$.

We then calculate the radio luminosity, also including a k-correction:

$$L_{1.4\text{GHz}} = 4\pi d_L^2 \frac{1}{(1+z)^{1+\alpha}} S_{1.4\text{GHz}}$$

here $d_L$ is the luminosity distance and $\alpha$ is the spectral index, defined as $S_{\nu} \propto \nu^\alpha$. We assume the canonical $\alpha = -0.7$ (Sadler et al. 2002; Condon et al. 2002). Within reasonable spectral
index limits this assumption has no significant effect on the derived luminosity functions.

We then evaluate $S_i$ and $m_i$ using Equations 2 and 3 at a series of redshifts ($\Delta z = 0.005$) and find the minimum and maximum redshifts where the source satisfies both the optical and radio selection criteria as well as the limits of the redshift range being analysed (i.e. the appropriate redshift bin boundaries). We can then calculate the integrated co-moving volume over which the source could have been observed i.e. $V_{\text{max}}$.

### 3.3 The bivariate luminosity function

In Figure 4 we have plotted the bivariate luminosity function for all radio galaxies (including HERGs, LERGs and star-forming) constructed using Equation 1 in three redshift bins. The left panel is the luminosity function from our ‘local’ redshift bin: $0.005 < z < 0.3$. It can be seen that the number density of radio galaxies rises steeply with decreasing radio luminosity. There is a significant contribution to the radio galaxy density from objects at faint optical magnitudes and low radio luminosities. There is a large variance between the bivariate luminosity function bins for these objects. Much of this contribution comes from local radio galaxies where the radio emission arises from star-formation rather than from an AGN (see right-hand column of Figure 6). The noise is due to the small volume (and hence absolute numbers) in which these objects satisfy the selection limits. The middle and right panels show the bivariate luminosity function for redshift bins $0.3 < z < 0.5$ and $0.5 < z < 0.75$, respectively. As a consequence of the flux density limits in both the optical and radio, the luminosity function is restricted to progressively brighter (in both optical luminosity and radio luminosity) galaxies in the higher redshift bins. In all redshift bins the space density increases with decreasing radio luminosity. Later, when we consider redshift evolution in the radio luminosity function we will restrict our analysis to a subset of galaxies with $M_i < -23$ (corresponding to the faintest galaxies we can detect at $z=0.75$). This limit is over-plotted as a vertical black line on Figure 4.

### 3.4 The local radio luminosity function

In order to produce a radio luminosity function we marginalize the bivariate luminosity function over optical luminosity. That is, we calculate the space density of radio galaxies, per radio luminosity bin, integrated over all observed optical luminosities. In Figure 5 we produce the local radio galaxy luminosity function. The radio luminosity function for all radio galaxies is plotted as open diamonds and the luminosity functions separated for AGN and star forming galaxies are plotted as open triangles and open squares, respectively. Below a radio luminosity of $L_{1.4\text{GHz}}\sim10^{23}\text{W Hz}^{-1}$ the star-forming galaxies dominate the space density of sources, while above this luminosity the AGN population dominates. These luminosity functions are also tabulated in Table 2.

Also in Figure 5 we overplot as blue lines the local radio luminosity function of 6dFGS galaxies from Mauch & Sadler (2007) for all radio galaxies (solid line), AGN (dotted line) and star-forming galaxies (dashed line). Our luminosity functions and those of Mauch & Sadler (2007) are in good agreement and the local radio luminosity function of Mauch & Sadler (2007) is known to be in good agreement with determinations measured by other authors (Machalski & Godlowski 2000; Sadler et al. 2002; Mao et al. 2012; Best & Heckman 2012). It should be noted that the comparison of these luminosity functions is not expected to be exact. It is clear from the bivariate luminosity functions in Figure 4 that the space density of radio galaxies measured will depend on the range of optical luminosities sampled. The fainter our optical limits the more galaxies at a given radio luminosity we will find. Strictly then, to compare radio luminosity functions we should integrate over the same optical magnitude range to ensure consistency. For local luminosity functions this effect should be small, since at the lowest redshifts all surveys will probe down to very faint optical magnitudes (albeit noisily because of the reducing volume) and the $1/V_{\text{max}}$ correction will return approximately the same luminosity function. However, when comparing luminosity functions at different redshifts it is important to use the same range of optical luminosities, as demonstrated in Figures 4 and 6.

### 3.5 Separating the luminosity function of radio AGN by accretion mode

As described in Section 2.1 the AGN in our sample have been classified as either LERGs or HERGs based on their optical spectral properties. In Figure 6 we show the bivariate luminosity function for LERGs (left column), HERGs (middle column) and star forming radio galaxies (right column) in three redshift bins (top to bottom). The star forming radio galaxies are shown only for the low redshift bin, since there are no such objects in the catalogue with higher redshifts.

In Figure 7 and tabulated in Table 2) we show the radio luminosity functions for LERGs and HERGs separately, for AGN in our ‘local’ $0.005 < z < 0.3$ redshift bin. We have summed over optical luminosities down to the limit of our sample ($m_i < 20.5$). The local LERG and HERG luminosity functions of Best & Heckman (2012, parameters taken from Heckman & Best 2014) are also shown (solid lines). The LERG luminosity functions are similar in shape and normalisation. The HERG luminosity functions, however, are significantly different. The HERG luminosity function presented here has a higher space density, especially at low radio luminosities. As noted earlier, we don’t expect radio luminosity functions to agree unless they sample the same range of optical luminosities, however, when measured locally this effect should be small and cannot explain the discrepancy.

In the case of the HERGs there are also differences in the classification techniques. Best & Heckman (2012) were deliberately strict in removing star-forming radio galaxies. This conservative approach is well justified since at the faint end of the radio luminosity function the star-forming radio galaxies can dominate in number density over the AGN by approximately an order of magnitude (see Figure 5). In this case mis-classifying a small fraction of star-forming galaxies as AGN can have a considerable effect on the measured space density of AGN.

Based on comparison of common objects, some radio galaxies classified as star-forming radio galaxies by Best & Heckman (2012) are expected to be classified as HERGs in this sample (Ching 2015). Best & Heckman (2012) use a combination of three tests to remove radio galaxies where
the radio emission is suspected to arise from star-formation: a standard BPT emission line diagnostic; a ratio of radio-to-emission-line luminosity; and a method using the strength of the 4000Å break (measured via the D4000 index) and the ratio of radio luminosity to stellar mass. While Ching (2015) used methods similar to the first two — they do not apply the third. The reason for this is two-fold. Firstly, some of the spectra (such as those derived from the WiggleZ survey) have low continuum signal-to-noise and poor apparent magnitude selection limits correspond to progressively brighter radio and optical luminosity limits with increasing redshift. The absolute magnitude limit applied when analysing the redshift evolution of the radio luminosity function is shown as the vertical line. This corresponds to the absolute magnitude limit at the high-redshift edge of the highest redshift bin not inclusive of an e-correction.

The local \((0.005 < z < 0.3)\) 1.4 GHz radio luminosity function from this sample separated into radio AGN and star-forming galaxies. The radio AGN are further separated into LERGs and HERGs.

| \(\log_{10}P_{1.4}\) (W Hz\(^{-1}\)) | All galaxies | SF galaxies | Radio AGN | LERGs | HERGs |
|----------------|-------------|-------------|-----------|-------|-------|
| \(N\) | \(\log(\Phi)\) (mag\(^{-1}\) Mpc\(^{-3}\)) | \(N\) | \(\log(\Phi)\) (mag\(^{-1}\) Mpc\(^{-3}\)) | \(N\) | \(\log(\Phi)\) (mag\(^{-1}\) Mpc\(^{-3}\)) | \(N\) | \(\log(\Phi)\) (mag\(^{-1}\) Mpc\(^{-3}\)) |
| 21.00 | 1 | \(-3.58^{+0.30}_{-0.30}\) | 1 | \(-3.58^{+0.30}_{-0.30}\) | 2 | \(-4.00^{+0.13}_{-0.13}\) | 1 | \(-4.75^{+0.30}_{-0.30}\) |
| 21.40 | 14 | \(-3.28^{+0.10}_{-0.10}\) | 14 | \(-3.28^{+0.10}_{-0.10}\) | 12 | \(-4.17^{+0.09}_{-0.09}\) | 22 | \(-4.17^{+0.10}_{-0.10}\) |
| 21.80 | 67 | \(-3.15^{+0.08}_{-0.08}\) | 67 | \(-3.21^{+0.08}_{-0.08}\) | 64 | \(-4.56^{+0.05}_{-0.05}\) | 46 | \(-4.72^{+0.06}_{-0.06}\) |
| 22.20 | 156 | \(-3.55^{+0.04}_{-0.04}\) | 128 | \(-3.64^{+0.04}_{-0.04}\) | 124 | \(-4.88^{+0.07}_{-0.07}\) | 22 | \(-5.06^{+0.10}_{-0.10}\) |
| 22.60 | 208 | \(-4.03^{+0.03}_{-0.03}\) | 144 | \(-4.18^{+0.03}_{-0.03}\) | 164 | \(-5.04^{+0.07}_{-0.07}\) | 234 | \(-5.09^{+0.09}_{-0.09}\) |
| 23.00 | 255 | \(-4.51^{+0.03}_{-0.03}\) | 190 | \(-4.84^{+0.03}_{-0.03}\) | 234 | \(-5.09^{+0.09}_{-0.09}\) | 39 | \(-5.84^{+0.11}_{-0.11}\) |
| 23.40 | 314 | \(-4.95^{+0.03}_{-0.03}\) | 41 | \(-5.79^{+0.08}_{-0.08}\) | 273 | \(-5.04^{+0.07}_{-0.07}\) | 234 | \(-5.09^{+0.09}_{-0.09}\) |
| 23.80 | 440 | \(-5.26^{+0.03}_{-0.03}\) | 21 | \(-6.54^{+0.13}_{-0.13}\) | 419 | \(-5.25^{+0.07}_{-0.07}\) | 368 | \(-5.34^{+0.10}_{-0.10}\) |
| 24.20 | 349 | \(-5.46^{+0.03}_{-0.03}\) | 2 | \(-7.63^{+0.53}_{-0.53}\) | 347 | \(-5.47^{+0.07}_{-0.07}\) | 319 | \(-5.52^{+0.09}_{-0.09}\) |
| 24.60 | 242 | \(-5.65^{+0.03}_{-0.03}\) | 242 | \(-5.65^{+0.03}_{-0.03}\) | 217 | \(-5.71^{+0.07}_{-0.07}\) | 25 | \(-6.52^{+0.09}_{-0.09}\) |
| 25.00 | 112 | \(-6.03^{+0.03}_{-0.03}\) | 112 | \(-6.03^{+0.03}_{-0.03}\) | 101 | \(-6.07^{+0.07}_{-0.07}\) | 11 | \(-7.04^{+0.11}_{-0.11}\) |
| 25.40 | 27 | \(-6.57^{+0.08}_{-0.08}\) | 27 | \(-6.75^{+0.08}_{-0.08}\) | 20 | \(-6.75^{+0.11}_{-0.11}\) | 7 | \(-6.98^{+0.11}_{-0.11}\) |
| 25.80 | 17 | \(-6.83^{+0.13}_{-0.13}\) | 17 | \(-6.83^{+0.13}_{-0.13}\) | 7 | \(-7.26^{+0.11}_{-0.11}\) | 10 | \(-7.03^{+0.11}_{-0.11}\) |
| 26.20 | 8 | \(-7.07^{+0.13}_{-0.13}\) | 8 | \(-7.07^{+0.13}_{-0.13}\) | 3 | \(-7.42^{+0.13}_{-0.13}\) | 5 | \(-7.33^{+0.13}_{-0.13}\) |

Figure 4. The bivariate luminosity distribution. The space density represented by a colour-scale in the optical magnitude-radio luminosity plane. The panels are for the different redshift bins: left: \(0.005 < z < 0.3\); middle: \(0.30 < z < 0.50\); and right: \(0.5 < z < 0.75\). The flux-density and apparent magnitude selection limits correspond to progressively brighter radio and optical luminosity limits with increasing redshift. The absolute magnitude limit applied when analysing the redshift evolution of the radio luminosity function is shown as the vertical line. This corresponds to the absolute magnitude limit at the high-redshift edge of the highest redshift bin not inclusive of an e-correction.

Table 2. The local \((0.005 < z < 0.3)\) 1.4 GHz radio luminosity function from this sample separated into radio AGN and star-forming galaxies. The radio AGN are further separated into LERGs and HERGs.
In Figure 8 we demonstrate that the differences between our local HERG luminosity function and that of Best & Heckman (2012) can be explained by differences in the classifications in the star-forming objects (at low radio powers) and the inclusion of broad-line objects (at \( L_{1.4\, \text{GHz}} \sim 10^{24} \text{ W Hz}^{-1} \)). In the top panel of Figure 8 we show the local luminosity function of the star-forming galaxies (dotted lines) and HERGs (dashed lines) for our sample (blue) and that of Best & Heckman (2012) (red). The solid lines show the sum of the HERGs and star-forming luminosity functions. The luminosity functions of the sum are well matched at the faint-end between the two samples. The difference being some objects classified as HERGs by Ching (2015) were removed as star-forming galaxies by Best & Heckman (2012).

There is, however, a difference at \( L_{1.4\, \text{GHz}} \sim 10^{24} \text{ W Hz}^{-1} \) with the number density of objects in our sample being higher by a factor of \( \sim 2 \). In the bottom panel we show the same luminosity functions but with the broad-line objects removed from our HERG luminosity function. The two determinations are in much better agreement and the remaining difference is not statistically significant.

The determination of the true HERG luminosity function at low radio power is a fundamentally difficult problem stemming from the fact that the number density of star-forming radio galaxies is so high in this regime. A faultless classification would likely require high spatial resolution radio continuum imaging. However, it is not critical in the context of this paper i.e. measuring the redshift evolution of the luminosity function. This is because the radio luminosity limits in our higher redshift bins (set by our flux limit) are outside the regime where star-formation powered radio galaxies are important. For example, even in our intermediate redshift bin we are restricted to radio luminosities \( L_{1.4\, \text{GHz}} \gtrsim 10^{24} \text{ W Hz}^{-1} \). Above these radio luminosities the number density of HERGs quickly rises above the star-forming galaxies, regardless of the classification method used.

3.6 Fitting radio luminosity functions

Several analytical forms have been proposed as good representations of galaxy luminosity functions. The radio luminosity function is often fitted with a double power-law (e.g. Dunlop & Peacock 1990; Brown et al. 2001; Mauch & Sadler 2007) of the form:

\[
\Phi(L) = \Phi^* \left( \frac{L}{L^*} \right)^{-\alpha} \exp\left( -\frac{1}{2\sigma^2} \log^2(1 + L/L^*) \right).
\]

(Saunders et al. 1990) proposed a function which behaves as a power-law at low radio luminosity and log-normal at high radio luminosities:

\[
\Phi(L) = \Phi^* \left( \frac{L}{L^*} \right)^{-\alpha} \exp\left( -\frac{\log(1 + L/L^*)}{\sigma^2} \right).
\]

which has been commonly used to parameterize the radio galaxy luminosity function (e.g. Saunders et al. 1990; Sadler et al. 2002; Smolčić et al. 2009). Optical galaxy luminosity functions are usually represented by a power-law with an exponential cut-off at the bright-end:

\[
\Phi(L) = \Phi^* \left( \frac{L}{L^*} \right)^{-\alpha} \exp(-L/L^*)
\]

i.e. the Schechter function (Schechter 1976). This function has the advantage of having one less parameter than the other two. We tested each of these functional forms in fitting the radio luminosity functions in this paper, including fitting for redshift evolution.

In the case of the HERGs all three functional forms perform comparably with, for example, the difference in Akaike Information Criterion (AIC: Akaike 1974) in every case being less than 3. An issue which arises in fitting the HERG population is the small number of objects at the bright-end where the number density decreases rapidly. In the case of the double-power law fit (equation 4) this manifests itself as a poor constraint on the bright-end slope \( \beta \). The sharp drop off in density at high radio luminosities and poor statistics means that a good fit can be obtained for arbitrarily large-negative \( \beta \) (i.e. as \( \beta \to -\infty \) the power-law slope becomes vertical). Similarly, for the fitting-function of equation 5 the \( \sigma \) parameter is poorly constrained. The Schechter function fits the data well at the bright-end, although it is not significantly preferred to the other models overall.

For the LERGs both equation 4 and 5 allow good analytical representations of the data, with little difference in the maximum likelihoods. On the other hand, the Schechter (1976) function is entirely inappropriate (\( \Delta \text{AIC} \gtrsim 20 \) in comparison to the other models when fitting for redshift evolution). For our purposes, the most important check is that measuring the rate of redshift evolution (i.e. the \( K_z \) and \( K_B \) parameters in equations 7 and 9) is robust. This is true, since the change in the best-fitting value of these parameters between the models is small in comparison to their uncertainties.

In this paper we have chosen to use the double power-law representation of equation 4 when fitting the radio luminosity functions. Because of the poor constraints at the...
bright-end of the HERGs we use a logarithmic prior on the $\beta$ parameter. Nevertheless, for the HERGs this parameter essentially only gives an upper limit on the bright-end slope.

### 3.7 Fitting the local LERG and HERG populations

In Figure 9 we show fits of equation 4 to the local LERG and HERG populations. We add an upper limit point to include the information from a lack of a detection at high radio luminosities. The value of the upper limit is set such that we can be 68 per cent confident the ‘true’ volume density is lower, under the assumption of a Poisson distribution (Gehrels 1986). The luminosity distributions can be well represented by this function and we summarise the best fitting parameter values and their uncertainty in Table 3.

### 3.8 Redshift evolution

To examine the redshift evolution of the LERG and HERG population we construct radio luminosity functions for each in multiple redshift bins. We present the luminosity functions in the same redshift bins as those used for the bivariate luminosity functions shown in Figure 4. That is: $0.005 < z < 0.30; 0.30 < z < 0.50; \text{and } 0.5 < z < 0.75$. The luminosity functions for the LERGs and the HERGs are tabulated in Tables 5 and 6. We restrict our analysis to $M_i < -23$; corresponding to the faintest galaxies we can detect at $z = 0.75$. They are plotted in Figure 10. The LERG luminosity functions, shown in the left-hand panels of Figure 6.
The radio luminosity function

Figure 7. The radio luminosity function of AGN in our 0.005 < z < 0.3 redshift bin separated into LERGs (red filled circles) and HERGs (blue filled squares) summed over optical magnitudes down to our limit of i < 20.5. Over-plotted are the luminosity functions of local LERGs (red solid line) and HERGs (blue solid line) from Heckman & Best (2014).

Figure 8. Top panel: the radio luminosity function of star forming radio galaxies (blue dotted line), HERGs (blue dashed line) and their sum (blue solid line) in our sample and those from Best & Heckman (2012) (red dotted, red dashed and red solid lines, respectively). The bottom panel is the same as the top panel but with the broad line AGN removed from our HERG sample.

The other common parameterization is pure density evolution, such that:

\[ \Phi(z) = (1 + z)^{\beta \Phi(0)} \]

(9)

giving:

\[ \Phi(L,z) = C(1 + z)^{\beta \Phi(0)} \frac{L^*}{L^* + (L/L^*)^{\alpha}} \]

(10)

There is a degeneracy between luminosity and density evolution, especially when the bright population is not tightly constrained (Le Floc’h et al. 2005; Smolčić et al. 2009). We therefore do not fit jointly for luminosity and density evolution but restrict ourselves to the cases above.

We fitted, using a Markov Chain Monte Carlo (MCMC) method, both the LERGs (left column of Figure 10) and HERGs (right column of Figure 10) with a pure density evolution model (top row of Figure 10) and a pure luminosity evolution model (second row of Figure 10). The evolution in the LERGs can be well represented by either model with the pure luminosity evolution marginally preferred with difference in AIC of ~3.5 (equivalent to a maximum likelihood ratio of ~6). In the case of the HERGs there is a similar preference for the pure density evolution model with a difference in AIC ~3.1 equivalent to a maximum likelihood ratio of ~4.5. The best fitting parameters and their uncertainties are summarised in Table 4. Parameterized in this way, the HERGs evolve slowly with redshift as ~ (1 + z)^{0.46+0.22} assuming pure density evolution or ~ (1 + z)^{3.29+0.24} assuming pure luminosity evolution. Under both assumptions this is consistent with no-evolution within ~2σ. The HERGs evolve faster than the LERGs with ~ (1 + z)^{3.41+0.15} in the pure density evolution case. If a pure luminosity evolution model is used the parameterised redshift dependence is very rapid ~ (1 + z)^{4.9+0.7}.

4 DISCUSSION

It is well established that there is luminosity dependent evolution in the overall radio AGN population, in the sense that
Figure 9. The radio luminosity function of AGN in our $0.005 < z < 0.3$ redshift bin separated into LERGs (red diamonds and solid line) and HERGs (blue triangles and solid line). Left panel: integrated over $i < 20.5$ and right panel: galaxies with $M_i < -23$ only. Over-plotted are double power law fits to the data (solid lines).

Figure 10. The radio luminosity function for LERGs (left column) and HERGs (right column) separated into three redshift bins: $0.005 < z < 0.30$ (blue); $0.30 < z < 0.50$ (green); $0.5 < z < 0.75$ (red). The top row shows the fit to the data (solid lines) assuming pure density evolution and the bottom row show the fit to the data assuming pure luminosity evolution.
In this case a mild decrease with increasing redshift is expected to the stellar mass function of massive quiescent galaxies. The expectation is that the LERGs will evolve in a similar manner (e.g. Longair 1966; Doroshkevich et al. 1970; Willott et al. 2001; Sadler et al. 2007; Smolčić et al. 2009). This differential evolution can be explained by a two population scenario where the LERGs dominate the space density at all but the highest radio luminosity and evolve slowly with redshift, while the HERGs that dominate at the highest radio luminosities evolve more rapidly (Smolčić et al. 2009; Best & Heckman 2012; Best et al. 2014).

The expectation is that the LERGs are hosted by quiescent galaxies and powered by Bondi & Hoyle (1944) accretion from their hot-gas atmospheres (e.g. Hardcastle et al. 2007; Best & Heckman 2012). In this case, a first-order prediction is that the LERGs will evolve in a similar manner to the stellar mass function of massive quiescent galaxies. In this case a mild decrease with increasing redshift is expected over the redshift range considered here (i.e. $z < 0.75$) evolving down in space density as $\sim (1 + z)^{0.11}$ (Best et al. 2014). In reality the evolution will be complicated by dependence on quantities like the halo hot gas fraction and the cooling function (e.g. Croton et al. 2006) and the average density of the medium into which the radio jets are expanding (Best et al. 2014). Croton et al. (2006) predict an almost flat black hole accretion rate density over this redshift range (see Figure 13). In any case, the expectation is we should observe little evolution in our LERG luminosity function out to $z \approx 0.75$, and this is the case with our best-fitting pure density evolution model evolving as $\sim (1 + z)^{0.01\pm0.11}$ consistent with zero evolution. It should be noted that the evolution measured from our luminosity functions is only for the optically brightest galaxies ($M_R < -23.0$), and that our application of an e-correction means we are including the same stellar mass hosts at all redshifts. If an e-correction is not applied our luminosity functions would display a significant positive redshift evolution of the space density of $\sim (1 + z)^{0.81\pm0.15}$ (assuming pure density evolution).

In this dual accretion mode picture the HERGs are some subset of the optical quasars and Seyfert galaxies. Croton et al. (2009) measured the evolution of the QSO luminosity function in the interval $0.4 < z < 2.6$ using the 2SLAQ sample. At the lowest redshifts ($z \lesssim 1$) the number density evolves rapidly in a manner that depends on luminosity — in the sense that the brightest QSOs evolve faster. At the bright-end ($M_R \sim -24$) the space density increases between their lowest redshift bins ($0.4 < z < 0.68$ and $0.68 < z < 1.06$) by a factor of $\sim 3$ which is similar to the change expected from our best fitting pure density model which has space density evolving as $\sim (1 + z)^{2.90}$. We did not apply an e-correction for the HERGs when constructing the bivariate luminosity functions. Since the HERGs often have ongoing star-formation and it is plausible that the AGN activity and star-formation histories are related — it is inappropriate to represent them as having a stellar population which fades with time as it ages. Generally the recent star-formation will contribute much of the optical light, nevertheless there will be some fading of the underlying older stellar population. Applying an e-correction will reduce the magnitude of the evolution in the luminosity function since it shifts higher redshift objects fainter and out of the selection limits. To investigate the maximum difference an e-correction could make to the evolution of the HERG luminosity function we applied the same e-correction for the HERGs as was applied to the LERGs (which will overestimate the magnitude of the correction) and fitted for the evolution. In the pure density evolution case we find evolution in the space density of $\sim (1 + z)^{1.14\pm0.11}$. If a pure luminosity evolution model is used the redshift dependence is $\sim (1 + z)^{5.70\pm2.05}$.

A clear difference between the luminosity functions of the LERGs and the HERGs is the radio power of the turnover. The bright-end turn down in the HERG luminosity function is $\sim 1$ dex brighter than that of the LERGs (see e.g. Figure 10 or Table 4). The bright-end cut-off in the i-band optical luminosity function of our HERGs and LERGs is approximately the same (see abscissa values of Figure 6); implying, to first order, a similar cut-off in the black hole mass function of the two samples. The origin of the high power cut off in the radio luminosity function is likely related to the cut-off in the black-hole mass function modulated in some way by the accretion rate and jet-production efficiency. The Eddington scaled accretion rate in the HERGs is expected to be 1 to 2 dex higher than in LERGs (Best & Heckman 2012; Mingo et al. 2014; Turner & Shabala 2015; Fernandes et al. 2015). This difference should be offset to some extent by the higher jet production efficiency in the LERGs (i.e. the ratio of mechanical power to total power; Turner & Shabala 2015). In combination the expected mechanical power in units of the Eddington luminosity should be $\sim 1$ dex lower for the LERGs than the HERGs (Turner & Shabala 2015). This is consistent with the differences in the characteristic radio power in the HERG and LERG luminosity functions, and is supportive of a picture in which these two populations are powered by different accretion modes.

### Table 4. Best fitted double power-law parameters for the redshift evolution of the LERGs and HERGs.

|         | LERGs |         | HERGs |
|---------|-------|---------|-------|
| $\log(C)$ | $-6.05^{+0.07}_{-0.06}$ | $-6.06^{+0.06}_{-0.05}$ | $-8.04^{+0.17}_{-0.16}$ | $-7.59^{+0.12}_{-0.12}$ |
| $\log(P_C)$ | $25.17^{+0.06}_{-0.05}$ | $25.12^{+0.06}_{-0.05}$ | $26.96^{+0.24}_{-0.24}$ | $25.66^{+0.24}_{-0.24}$ |
| $\alpha$ | $-0.31^{+0.01}_{-0.01}$ | $-0.32^{+0.01}_{-0.01}$ | $-0.32^{+0.01}_{-0.01}$ | $0.34^{+0.04}_{-0.04}$ |
| $\beta$ | $-1.88^{+0.10}_{-0.01}$ | $-1.92^{+0.12}_{-0.03}$ | $-1.75^{+0.29}_{-0.29}$ | $-2.17^{+0.49}_{-0.49}$ |
| $K_{DL}$ | $0.06^{+0.18}_{-0.18}$ | $0.46^{+0.24}_{-0.24}$ | $2.97^{+0.46}_{-0.47}$ | $7.41^{+0.86}_{-0.86}$ |

The 1.4 GHz radio luminosity function $\propto (1 + z)^{1.14\pm0.11}$. If a pure luminosity evolution model is used the redshift dependence is $\propto (1 + z)^{5.70\pm2.05}$.

4.1 Comparison with Best et al. 2014

The only previous work on the evolution of the radio luminosity function separated by ‘accretion mode’ using optical spectroscopy was made by Best et al. (2014) using a composite sample constructed from eight different radio surveys with a range of flux density limits. This sample contains 211 radio galaxies with redshifts in the range $0.5 < z < 1.0$. As their local comparison sample they use the radio luminosity functions of Best & Heckman (2012) but with the HERGs replaced above $L_{1.4GHz} = 10^{26}$ W Hz$^{-1}$ with the steep spectrum radio luminosity function from the ConFIG sample (Gendre et al. 2010; Heckman & Best 2014).
Table 5. Luminosity function of LERGs in 3 redshift bins (0.005 < z < 0.30, 0.30 < z < 0.50, 0.50 < z < 0.75). Only galaxies with optical I band absolute magnitude $M_i < -23$ are included.

| log$_{10}P_{1.4}$ (W Hz$^{-1}$) | 0.005 < z < 0.30 | 0.30 < z < 0.50 | 0.50 < z < 0.75 |
|-------------------------------|------------------|----------------|-----------------|
|                               | N                | log(Φ)         | N               | log(Φ)          | N               | log(Φ)          |
|                               | (mag$^{-1}$ Mpc$^{-3}$) |                   | (mag$^{-1}$ Mpc$^{-3}$) |                   | (mag$^{-1}$ Mpc$^{-3}$) |                   |
| 21.80                         | 1                | $-4.78^{+0.34}_{-0.36}$ | 22.20           | 4                | $-5.13^{+0.09}_{-0.10}$ |
| 22.60                         | 15               | $-5.16^{+0.13}_{-0.30}$ | 23.00           | 34               | $-5.42^{+0.07}_{-0.08}$ |
| 23.40                         | 96               | $-5.49^{+0.04}_{-0.05}$ | 23.80           | 216              | $-5.61^{+0.03}_{-0.06}$ |
| 24.20                         | 193              | $-5.77^{+0.06}_{-0.08}$ | 25.00           | 17               | $-6.84^{+0.13}_{-0.08}$ |
| 25.00                         | 7                | $-7.23^{+0.13}_{-0.16}$ | 26.20           | 1                | $-8.07^{+0.30}_{-3.00}$ |
| 26.60                         | 1                | $-8.36^{+0.30}_{-0.30}$ | 27.00           | 1                | $-8.60^{+0.30}_{-0.30}$ |
| 27.40                         | 1                | $-8.35^{+0.30}_{-0.30}$ |                |                  |                  |

Table 6. Luminosity function of HERGs in 3 redshift bins (0.005 < z < 0.30, 0.30 < z < 0.50, 0.50 < z < 0.75). Only galaxies with optical I band absolute magnitude $M_i < -23$ are included.

| log$_{10}P_{1.4}$ (W Hz$^{-1}$) | 0.005 < z < 0.30 | 0.30 < z < 0.50 | 0.50 < z < 0.75 |
|-------------------------------|------------------|----------------|-----------------|
|                               | N                | log(Φ)         | N               | log(Φ)          | N               | log(Φ)          |
|                               | (mag$^{-1}$ Mpc$^{-3}$) |                   | (mag$^{-1}$ Mpc$^{-3}$) |                   | (mag$^{-1}$ Mpc$^{-3}$) |                   |
| 22.20                         | 1                | $-5.34^{+0.30}_{-0.30}$ | 22.60           | 1                | $-6.53^{+0.30}_{-0.30}$ |
| 23.00                         | 2                | $-6.54^{+0.30}_{-0.30}$ | 23.40           | 7                | $-6.60^{+0.30}_{-0.30}$ |
| 23.80                         | 18               | $-6.60^{+0.30}_{-0.30}$ | 24.20           | 8                | $-7.04^{+0.30}_{-0.30}$ |
| 24.60                         | 12               | $-6.89^{+0.30}_{-0.30}$ | 25.00           | 5                | $-7.41^{+0.30}_{-0.30}$ |
| 25.40                         | 2                | $-7.48^{+0.30}_{-0.30}$ | 26.20           | 1                | $-8.07^{+0.30}_{-0.30}$ |
| 26.80                         | 4                | $-7.36^{+0.30}_{-0.30}$ | 27.00           | 3                | $-7.89^{+0.30}_{-0.30}$ |
| 27.80                         | 1                | $-8.35^{+0.30}_{-0.30}$ |                |                  |                  |

4.1.1 Comparison of High Excitation Radio Galaxies

The HERG radio luminosity function of Best et al. (2014) exhibits an increase in space density between the local measurement at z < 0.3 and their high-redshift measurement at 0.5 < z < 1.0 of a constant factor of ~ 7. This increase is independent of radio luminosity, and well described by pure density evolution. Our HERG radio luminosity functions are measured in three redshift bins with 0.3, 0.3 < z < 0.5, and 0.5 < z < 0.75. In the best fitting pure density evolution model the space density increases as $(1 + z)^{2.93^{+1.10}_{-0.45}}$ which, if extrapolated to the upper redshift bin of Best et al. (2014) corresponds to slightly slower evolution in the space density of a factor of ~ 4–5. This is illustrated in Figure 11 where we compare the best-fitting HERG radio luminosity functions of Best et al. (2014) with our HERG radio luminosity function data. This comparison is not quantitatively precise since the classification scheme, and redshift ranges are not identical. The high redshift bin of Best et al. (2014) is 0.5 < z < 1.0 (red line) whereas our highest redshift bin is 0.5 < z < 0.75. Furthermore, we only included the brightest optical galaxies ($M_i < -23.0$) when fitting for the redshift evolution. If the faint-optical population evolves more rapidly than this would increase the redshift evolution of the space density.

4.1.2 Comparison of Low Excitation Radio Galaxies

For the LERGs, Best et al. (2014) do find luminosity dependent evolution in the radio luminosity function. At low radio luminosities ($L_{408M} = 10^{23}$ W Hz$^{-1}$) they find the space density is nearly constant or increases slowly out to z ~ 0.5–0.7 and then begins to decrease out to z = 1. At high radio luminosity they measure a more rapid increase in space density of LERGs with redshift, rising by factor of ~ 10 for the high-
As the solid lines. The three redshift intervals for our data are: 0.005 < \( z \) < 0.30 (blue); 0.30 < \( z \) < 0.50 (green); 0.5 < \( z \) < 0.75 (red). Whilst Best et al. (2014) give their radio luminosity function parameters for 0.005 < \( z \) < 0.30 (blue line) and 0.5 < \( z \) < 1.0 (red line).

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Figure 11. A comparison of the radio luminosity functions for the HERGs measured in this work (data points) and the radio luminosity functions of the HERGs from Best et al. (2014) shown as the solid lines. The three redshift intervals for our data are: 0.005 < \( z \) < 0.30 (blue); 0.30 < \( z \) < 0.50 (green); 0.5 < \( z \) < 0.75 (red). Whilst Best et al. (2014) give their radio luminosity function parameters for 0.005 < \( z \) < 0.30 (blue line) and 0.5 < \( z \) < 1.0 (red line).

Figure 12. A comparison of the radio luminosity functions for the LERGs measured in this work (data points) and the radio luminosity functions of the ‘jet mode’ AGN from Best et al. (2014) shown as the solid lines. The three redshift intervals for our data are: 0.005 < \( z \) < 0.30 (blue); 0.30 < \( z \) < 0.50 (green); 0.5 < \( z \) < 0.75 (red). Whilst Best et al. (2014) give their radio luminosity function parameters for the LERGs at 0.005 < \( z \) < 0.30 (blue line), 0.5 < \( z \) < 0.7 (red line) and 0.7 < \( z \) < 1.0 (black line).

4.2 Radio mode feedback

There is substantial evidence that radio jets from LERGs are important in regulating star-formation in massive galaxies and clusters of galaxies by injecting energy into the hot gas atmosphere and inhibiting gas cooling and star formation. This radio-mode AGN feedback can simultaneously explain the ‘cooling flow problem’, the exponential cut-off in the bright end of the optical galaxy luminosity function and the old stellar populations of the most massive bulges (e.g. Croton et al. 2006; Bower et al. 2006). Estimates of the energy associated with bubbles and cavities in the intergalactic medium surrounding elliptical galaxies and galaxy clusters can be used to estimate the mechanical power associated with the radio jets producing the cavities. The empirical correlation of these energies with monochromatic radio luminosity can be used to transform between the two (Dunn et al. 2005; Rafferty et al. 2006; Birzan et al. 2008; Cavagnolo et al. 2010). Although, these relations have large intrinsic scatter of \( \sim 0.7 \) dex (Cavagnolo et al. 2010). Using such relations the monochromatic radio luminosity function can be transformed into a mechanical power density function, and integrated to calculate the total mechanical power (per unit volume) available for radio mode feedback (e.g. Best et al. 2014).
That is, we calculate:

\[ \int \phi(P_m) P_m d(0.4 \log_{10} P_m) = \frac{2.5}{\ln(10)} \int \phi(P_m) dP_m \]

(11)

where \( P_m \) is the mechanical power which we calculate from the 1.4 GHz luminosity using equation (1) of Cavagnolo et al. (2010). The factor of \( 2.5/\ln(10) \) comes about since our luminosity function is in units of \( \text{mag}^{-1} \) rather than units of the natural logarithm.

In Figure 13 we show this integral as a function of redshift using our pure density evolution fits to the LERGs and HERGs. We follow Smolčić et al. (2009) and integrate above a mechanical power equivalent to \( L_{1.4 \text{GHz}} = 10^{25} \text{W Hz}^{-1} \). The shaded regions illustrate the uncertainties from the conversion of 1.4 GHz radio luminosity to mechanical power using the uncertainties quoted in Cavagnolo et al. (2010) equation (1) (large shaded regions), and from the uncertainties in our parameter values from fitting the radio luminosity function; constructed by sampling the posterior distribution of the parameters obtained from the MCMC fitting (smaller dark shaded regions).

There is little evolution with redshift in the volume density of mechanical power from the LERGs (left panel of Figure 13), consistent with the prediction from the cosmological model of Croton et al. (2006); shown as the dashed line. It should be noted these mechanical powers only include emission from massive galaxies since our radio luminosity functions are restricted to \( M_i < -23 \), including fainter optical galaxies will increase the normalisation further. Also, in the left panel of Figure 13 we show the mechanical power calculated from fits to the radio luminosity function of low luminosity VLA-COSMOS AGN (Smolčić et al. 2009). The low luminosity selection means the radio luminosity function should be dominated by LERGs although it will still contain a contribution from the HERGs. The normalisation of our estimate of the total mechanical power is a factor of ~4 lower than that measured by Smolčić et al. (2009). This difference can be attributed to our restriction to only the very brightest optical galaxies (\( M_i < -23 \)) causing the normalisation of our luminosity functions to be lower. The difference in normalisation of the luminosity functions accounts for all of this factor of 4 (c.f. our Figure 10 with Smolčić et al. (2009) Figure 3). We also show the evolution of the mechanical power calculated for the HERGs (right panel). The normalisation of this is approximately an order of magnitude smaller then that of the LERGs at \( z = 0 \) but becomes comparable at \( z = 1 \). Since the measurement of Smolčić et al. (2009) includes both HERGs and LERGs, this evolution in the HERGs can account for their factor of ~2 evolution in the total mechanical power out to \( z = 1 \).

5 SUMMARY

We have constructed radio luminosity functions out to a redshift of \( z = 0.75 \) using a new sample of 5026 radio galaxies with 1.4 GHz flux density: \( S_{1.4} > 2.8 \text{ mJy and optical magnitude: } m_i < 20.5 \text{ mag. These radio galaxies have confirmed spectroscopic redshifts. The optical spectra are also used to classify the radio galaxies as star-forming or AGN. The AGN are further subdivided into High Excitation Radio Galaxies (HERGs) or Low Excitation Radio Galaxies (LERGS). Using a subset of the brightest optical galaxies with \( M_i < -23 \) we characterise the evolution in the radio luminosity function of these objects. We find:

- The space density of the LERGs exhibits little evolution with redshift. The LERGs evolve as \( \sim (1 + z)^{0.06^{+0.14}_{-0.18}} \) under the assumption of pure density evolution or \( \sim (1 + z)^{0.06^{+0.14}_{-0.18}} \) under the assumption of pure luminosity evolution. Both are consistent with zero evolution at less than 2\( \sigma \). Since the LERGs dominate the number density of radio AGN at low radio luminosity this result is consistent with the mild evolution seen in the total radio luminosity function at low radio power.
- The HERGs evolve more rapidly best fitted by \( \sim (1 + z)^{2.91^{+3.25}_{-2.96}} \) assuming pure density evolution or \( \sim (1 + z)^{2.41^{+3.25}_{-1.73}} \) under the pure luminosity evolution assumption. Since the HERGs dominate the space density at only the highest luminosities, this is consistent with the more rapid evolution of bright radio galaxies observed in the overall radio AGN population.
- The bright-end turn down in the radio luminosity function occurs at a significantly higher power (\( \gtrsim 1 \text{ dex} \)) for the HERG population than the LERG population. This is consistent with the two populations representing fundamentally different accretion modes.
- Converting the LERG luminosity function to a mechanical power density function using empirical relations and integrating, results in a total mechanical power per unit volume available for radio mode feedback that remains roughly constant out to \( z = 0.75 \). This is consistent with cosmological models.

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Figure 13. The total mechanical power per unit volume as function of redshift estimated from the pure density evolution fits to our radio luminosity functions. The conversion to mechanical power from 1.4 GHz luminosity uses the relation of Cavagnolo et al. (2010). The shaded regions represent the uncertainty from the radio luminosity function fits (smaller dark shaded regions) and the uncertainty in the Cavagnolo et al. (2010) relation (the larger shaded regions). The left panel is for the LERGs and the right panel is for the HERGs. Also over-plotted on the LERGs are the prediction from the cosmological model of Croton et al. (2006) and the measurement from Smolčić et al. (2009). The difference in normalisation between our measurement and that of Smolčić et al. (2009) can be entirely attributed to our measurement only including the contribution from the brightest optical galaxies (see text for details).

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