An interpretable predictive modelling framework for the turning process by the use of a compensated fuzzy logic system

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ABSTRACT
This research presents a compensated fuzzy logic system that integrates an interval type-2 fuzzy logic system (IT2FLS) with the Gaussian mixture model (GMM) to model the turning process. First, an IT2FLS is elicited to model the turning process by mapping its input variables to the cutting force and the surface quality. Second, the GMM is incorporated in the IT2FLS structure to compensate for the error residuals. The idea of such an incorporation stems from the fact that the majority of the models are constructed based on the normality assumption of the error. The GMM is developed in a way that refines the extracted rules and considers stochastic unmodelled behaviours. Validated on real experiments, it has been demonstrated that the compensated fuzzy logic system has the ability to accurately predict the cutting force and the surface quality; deal with uncertainties; and provide users with comprehensive understanding of the turning process.

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1. Introduction
The manufacturing industry has witnessed a revolution upon a revolution. In the recent Fourth Industrial Revolution (4IR), predictive analytics has been widely utilized to make manufacturing systems predictable, flexible and also controllable (Lasi et al., 2014). Among the various types of the manufacturing processes, the turning process, as a machining one, can considerably benefit from the 4IR. In general, the turning process is one of the four well-known metal cutting processes that are commonly utilized to produce rotational and perhaps axisymmetric parts that usually include many features (e.g. grooves, threads and holes; Lin et al., 2001). In such a process, a single-point cutting tool or multi-point ones feed into a rotating workpiece to remove materials in the form of chips to produce the parts required (Lin et al., 2001).
Since the turning process is considered to be one of the key manufacturing processes, a considerable research work has, therefore, been conducted to understand and model such a process (Bhattacharya et al., 2009; Davim et al., 2008; Sharma et al., 2008). Some of the related books and scientific research papers have focused on understanding the influence of the various cutting variables (e.g. cutting (rotational) speed and feed rate) and tool geometry (e.g. back rake angle) on the machined specimen’s surface roughness, cutting force and tool wear (Bhattacharya et al., 2009; Davim et al., 2008; Sharma et al., 2008). Such research papers and books have presented a comprehensive understanding of the turning process and the parameters that affect it. However, there is a strong need to enhance such an understanding by developing and implementing various predictive modelling paradigms that have the ability to provide users with quantitative predictions of the effect of the machined parameters on the output parameters (Sharma et al., 2008). Therefore, modelling and predicting the cutting force of the turning process that is essential for determining the power required by the cutting machine, and surface roughness of the machined part have received a great deal of interest (S. Kumar et al., 2018). The various predictive modelling approaches that have hitherto been implemented can be either physical-driven models or data-based models (S. Kumar et al., 2018). Physical-based models have been implemented to understand the process behaviour and to develop the related mathematical equations that aim at finding the relationships between the various cutting input parameters and the process responses (e.g. cutting force and surface quality represented by surface roughness; Box & Draper, 1987). Due to the lack of the related physical-based equations that can be utilized to represent the turning process and with the huge advances in the recent computing power, data-driven paradigms have found their way into many academic and industrial applications to represent input/output relationships (S. Kumar et al., 2018). Multiple linear regression approaches have been established to represent and predict various process parameters (e.g. cutting force, tool wear, etc.) and the properties of a machined workpiece (e.g. surface roughness; Bhardwaj et al., 2014; Uthayakumar et al., 2012). For instance, a regression model was utilized to represent the relationship between the cutting force, as a dependent output, and the feed rate, cutting speed and depth of cut, as independent inputs (Uthayakumar et al., 2012). Such approaches cannot account for nonlinear relationships between inputs and outputs, and cannot consider the sophisticated interactions that usually exist among the process parameters (AlAlaween et al., 2016). Therefore, artificial neural networks (ANNs) have been extensively utilized in modelling various machining processes (Liu & Ko, 1990; Purushothaman & Srinivasa, 1994; Yao & Fang, 1993). For example, ANNs were implemented to estimate the performance of the different metal cutting processes, and to control the surface roughness characteristics (Grzesik & Brol, 2003; Purushothaman & Srinivasa, 1994). In addition, a multilayer neural network was utilized to predict the surface roughness of Ti-13Zr-13Nb alloy (Khanlou et al., 2016). In general, these modelling paradigms are considered to be powerful interpolators that can be used to represent complex input/output relationships in various processes, in particular, the cutting processes (Grzesik & Brol, 2003). Because these models cannot provide users with information about the process under investigation, they are commonly referred to as black-box (Bishop, 2006), fuzzy logic systems (FLSs) have, therefore, been implemented to map the process variables to the output parameters and the properties of a produced part (Kamatala et al., 1996; Kuo & Cohen, 1998). For instance, the neuro-fuzzy paradigm
was implemented to predict the surface roughness (Kamatala et al., 1996). It was demonstrated that such a model outperformed other modelling approaches (e.g. regression models and ANNs) in terms of the predictive performance (Kamatala et al., 1996). In addition to their ability to successfully represent complex relationships among the examined inputs and outputs, FLSs, as data-driven models, can provide users with a comprehensive understanding of the investigated process (AlAlaween et al., 2018). Likewise, FLSs, as it is well-known, can effectively deal with uncertainties that result from measurement uncertainties and from any uncontrollable variables, which may in some cases have conflicting influences on the outputs under investigation. For instance, various fuzzy logic based systems were developed to predict and optimize the cutting force and/or the surface roughness for different materials while using different sets of the process variables (Barzani et al., 2015; Marani et al., 2020, 2021). Likewise, the fuzzy logic system was utilized to predict the material removal rate and surface roughness during diamond surface grinding (Unune et al., 2018) However, the majority of the presented paradigms (i.e. both the physical- and data-driven ones) are constructed based on the normality assumption of the resulted errors (Yang et al., 2012). Such an assumption, which is, more often than not, invalid in reality, can result in performance deterioration, this being due to the unmodelled behaviour, stochastic or otherwise (Yang et al., 2012).

In this research paper, the main aim is to present an interpretable and more accurate predictive modelling framework to successfully model the turning process. For this purpose, a modelling structure called a compensated fuzzy logic system that integrates deterministic and stochastic modelling paradigms is proposed. First, an interval type-2 fuzzy logic system (IT2FLS) is implemented to (i) represent the turning process; (ii) predict the cutting force and surface roughness that is represented by the arithmetic mean value (Rₐ), it is worth emphasising at this stage that the cutting force plays an important role in determining the fate of some of the outputs of the turning process in terms of productivity and quality (i.e. it affects, for instance, the surface texture, which by itself determines the fate of the machined parts, and tool wear; AlAlaween et al., 2021); (iii) provide a simple understanding of such a process; and (iv) take into consideration the uncertainties, which commonly surround the turning process. Then, the resulted error residuals are characterised using the Gaussian mixture model (GMM), which compensates for such resulted errors, to take into consideration any unmodelled behaviour, stochastic or otherwise. The incorporation of the GMM in the modelling framework can improve the modelling performance of the IT2FLS, this being due to its ability to (i) compensate for the implicit normality assumption of the error residuals, and (ii) extract information that may perhaps be hidden in the form of error residuals. The rest of the paper is organized as follows: in Section 2, the set of experiments that were conducted using a lathe machine is briefly mentioned. The background of the FLS, in particular, the IT2FLS and the obtained results are summarized in Section 3. The GMM and its results are presented in Section 4. Finally, Section 5 summarizes the conclusions of the whole paper.

2. Experimental work

In the turning process, there are many parameters that have an influence on the cutting force and the quality of a machined specimen represented by the surface roughness measured via the Rₐ value. Since they are considered to be the most crucial parameters for
the turning process (Bhattacharya et al., 2009; Davim et al., 2008; Sharma et al., 2008), cutting (rotational) speed, depth of cut, feed rate and the use of lubricant were investigated in this research work, as listed in Table 1. The levels of these parameters were also defined in the same table. It is worth emphasizing at this stage that the levels of the four variables investigated in this research work were defined by conducting a set of trial experiments using the same material and cutting tool. Based on a full factorial design of experiments, a total of 54 experiments were conducted using different input settings. It is worth emphasising that each experiment was repeated three times. A lathe machine (Colchester Master 2500, UK) was used to machine a cylindrical AISI D2 steel specimen that has a diameter of 20 mm and length of 110 mm, as shown in Figure 1.

The cylindrical AISI D2 steel has a chemical composition of 1.5% Carbon, 0.3% Silicon, 12% Chromium, 0.8% Molybdenum and 0.9% Vanadium, as summarized in Table 1. Such a cylindrical specimen was placed with its longitudinal axis aligned with the feed direction. The cutting tool that was utilized to conduct all experiments is Carbide tipped insert coated with TiN. Once the cutting process was completed, the cutting force and the surface roughness represented by $R_a$ were measured. The cutting force and the surface roughness were measured to be used as target outputs to develop the proposed model. In this research work, the cutting force was investigated because it is considered to be one of the outputs that can be utilized to characterize the turning process in terms of tool wear and more importantly surface texture (Piotrowska et al., 2009). The $R_a$ value was measured using a Portable Stylus-Type Profilometer with LCD display (Senze Instruments, Italy), while the cutting force was measured using two Dial Gauges Type

| Inputs              | Inputs’ levels                  | Outputs                      |
|---------------------|--------------------------------|------------------------------|
| Cutting speed       | 175, 235 and 320 (rpm)         | Cutting force (KGF)          |
| Depth of cut        | 0.1, 0.15 and 0.2 (mm)         | Surface roughness* (µm)      |
| Feed rate           | 0.05, 0.1 and 0.16 (mm/rpm)    |                              |
| Use of lubricant    | Dry and Cutting fluid (Zinol, UAE) |                              |
| Chemical composition| 1.5% Carbon, 0.3% Silicon, 12% Chromium, 0.8% Molybdenum and 0.9% Vanadium | |

*Surface roughness was measured via the arithmetic mean value ($R_a$).

Figure 1. The lathe machine (Colchester Master 2500, UK).
Table 2. The correlation coefficients.

| Inputs      | Dry          | Cutting fluid          |
|-------------|--------------|------------------------|
| Cutting speed | -0.34        | 0.16                   |
| Depth of cut  | -0.38        | -0.02                  |
| Feed rate    | 0.16         | 0.13                   |
|              | 0.31         | 0.32                   |
|              | 0.42         | 0.46                   |

*Surface roughness was measured via the arithmetic mean value (Ra).

60/0.002 mm (TecQuipment, UK). It is worth mentioning that the two Dial Gauges were utilized to measure the tool/tool holder deflection that was proportional to the magnitude of the cutting force as explained in (H. Kumar & Kumar, 2011). The average values of the cutting force and surface roughness for the three replicates of each experiment were determined in this research work. The variability values for the cutting force and the surface roughness were in the range of 1.7 to 4.5 and in the range of 0.6 to 5.6, respectively.

In order to evaluate the strength of the linear relationships between the investigated quantitative inputs and outputs, the statistical linear correlation analysis was conducted between the examined process variables and the measured cutting force and the surface roughness represented by Ra. Reasonable correlation coefficient values among most of the investigated parameters can be noticed in Table 2. However, some of the examined variables have different correlation coefficient values between the experiments that were carried out without the use of lubricant/cutting fluid (i.e. dry process) and the ones that were conducted with the use of lubricant. For instance, the relationship between the cutting speed and the cutting force without the use of cutting fluid is relatively stronger when compared to the same relationship when the cutting fluid was used. By implementing the analysis of variance test, as a statistical test, it was noticeable that the use of lubricant, as a classical input parameter, has a significant influence on both the cutting force and the surface roughness represented by the Ra values (i.e. the P-values are less than 0.05).

3. Interval type-2 fuzzy logic systems

3.1. Model development

With the huge advances in recent computing power, the development of computational intelligence has positive effects on several areas including, but not limited to, healthcare and manufacturing. Utilizing computer systems has also changed how researchers think in industry and academia. The observed/collection data are, therefore, used to develop and construct data-driven modelling approaches that mimic the human way of thinking. Such modelling paradigms can complement or replace the so-called physical-based models, in particular, for those processes where such models do not exist or they can be too complex to derive. Therefore, a plethora of data-driven paradigms (e.g. regression models and ANNs) have hitherto been developed and implemented in various research areas, such as healthcare, manufacturing and marine technology (AlAlaween et al., 2017; Shahani et al., 2009). Despite their powerful algorithms, some of the presented models (e.g. regression paradigms) are incapable of representing complex highly nonlinear
relationships. Likewise, some of these models, in particular ANNs, are referred to as black-box ones, this being due to the low interpretability of such models (AlAlaween et al., 2018). Therefore, the fuzzy logic system (FLS) has hitherto been utilized in various applications to develop an interpretable model that can efficiently take into consideration uncertainties (i.e. measurement uncertainties and uncertainties that may result from any uncontrollable and difficult to consider variables; AlAlaween et al., 2018).

In general, FLSs can usually be described by fuzzy sets. In general, type-1 and type-2 are the common types of these fuzzy sets. The system that its antecedents and consequent of the rules are characterised by the former fuzzy sets is referred to as the type-1 fuzzy logic system (T1FLS). Whereas a type-2 fuzzy logic system (T2FLS) is the one that at least one of its antecedents and consequent of the rules is characterised by the latter fuzzy sets whose membership functions are fuzzy (Karnik & Mendel, 2001). The T2FLS, as it is known, is able to handle uncertainties more effectively and efficiently compared to the T1FLS. However, as it is well known, such a system is considered to be computationally expensive. Thus, an interval type-2 fuzzy logic system (IT2FLS) was presented and thenceforth it has been implemented and utilized (Tan & Chua, 2007). The corresponding fuzzy set for the IT2FLS can be written as presented in Equation (1; Tan & Chua, 2007):

$$
\% A = \frac{1}{\mu \left( x \right) \int_{\mu \left( x \right) \leq 0.1}^{1} \int_{J_x} \frac{1}{x, u} \right)
$$

where $x$ and $X$ represent the primary variable and its measurement domain, respectively. The parameters $J_x$ and $u$ stand for the primary membership degree and the secondary variable; where $u$ belongs to $J_x$ at each $x$ belongs to $X$, respectively.

The IT2FLS framework is shown in Figure 2. As shown in such a figure, the first step is to fuzzify the crisp inputs ($x_1, x_2 \ldots x_n$) into type-2 fuzzy sets ($\tilde{A}_j$); where $\tilde{A}_j$ represents the $i$th fuzzy set for the $j$th parameter. In the fuzzification step, the membership functions both the upper and lower $\left[ \mu_{\tilde{A}_j}, \bar{\mu}_{\tilde{A}_j} \right]$ are determined. The smoothness and continuity of the Gaussian function commonly allow the fuzzy system to be implemented in the form

![Figure 2. The framework of IT2FLS.](https://via.placeholder.com/150)
of a universal approximator. Therefore, such a function is a common choice for the membership function. The Gaussian membership function can simply be written as represented in Equation (2) (Tan & Chua, 2007).

\[ \mu_j^i(x_j) = \exp \left[ -\frac{1}{2} \left( \frac{x_j - m_j^i}{\sigma_j^i} \right)^2 \right], \quad m_j^i \in [m_{j1}^i, m_{j2}^i] \]  

(2)

where \( m_j^i \) and \( \sigma_j^i \) represent the mean and the standard deviation values of the \( i \)th set for the \( j \)th variable, respectively. The subscript number is used to distinguish the lower mean (\( m_{j1}^i \)) from the upper one (\( m_{j2}^i \)). The area shaded between the lower and the upper membership functions is usually known as the footprint of uncertainty.

The input fuzzy sets are then mapped to the output fuzzy sets by combining the defined rules, such a process is called the inference process. Commonly, the rules can be extracted from an available data set (i.e. the experimental data) or can be noticed and provided by experts. These rules are usually expressed in the form of a set of IF-THEN rules, as follows:

**Rule:** IF \( x_1 \) is \( \tilde{A}_1^i \) and \( x_n \) is \( \tilde{A}_n^i \), THEN \( y \) is \( \tilde{B}^j \).

where \( \tilde{A}_j^i \) and \( \tilde{B}^j \) stand for the membership functions of \( j \)th antecedent of the \( i \)th rule and the consequent of the same rule, respectively. It is worth emphasising that the rules for the T1FLS and the T2FLS have a similar form, the only difference is related to the membership functions nature. In the presented research work, the Mamdani fuzzy system is utilized. In such a system, \( \tilde{B}^j \) is represented by a membership function. Membership functions that are generally expressed by words (e.g. low and medium) are usually described by fuzzy sets, which commonly represent subjective information provided about an examined process. The output of the inference process is the type-2 output fuzzy sets that are then reduced into type-1 ones. In such a step, the lower and upper limits are usually determined using Karnik-Mendel (KM) algorithm (Karnik & Mendel, 2001). It is worth emphasizing that the type reduction step incurs most of the computational effort needed. Finally, a defuzzification process is utilized to calculate a crisp output by simply calculating the average value (Tan & Chua, 2007).

### 3.2. Results and discussion

In this research work, the IT2FLS was implemented to predict the cutting force and the quality of a machined specimen represented by the surface roughness measured via the \( R_a \) value. The IT2FLS was employed in this research work because it can, in general, (i) represent highly nonlinear input/output relationships; (ii) handle the uncertainties that may surround the process effectively; and (iii) provide users with a simple and linguistic understanding of the process examined in the form of If/Then rules. In order to develop such a model and due to the limited amount of data points, the data collected were classified into two sets only: training set that contains 38 experiments and testing set that includes 16 experiments. The training data set is usually utilized to extract rules and, thus, it allows the paradigm to learn the input/output relationships, whereas the testing one is utilized to examine the FLS generalization capabilities. It is worth mentioning at this stage that different division methods have hitherto been investigated in the related literature (e.g. the 10-fold cross-validation technique and random division method). In
In this research paper, it was found that dividing the data randomly into training and testing sets was the best and simple method. To successfully model the turning process, the nature of the examined input parameters should be understood (i.e. discrete or continuous). In this research work, all the input parameters were considered as continuous ones except the use of lubricant parameter, which was considered as a crisp one. For a specific number of rules, and by using the Gaussian membership function, the IT2FLS parameters, such as mean and standard deviation, were initialized by utilizing the interval type-2 fuzzy clustering algorithm (Rubio & Castillo, 2013). Then, they were optimized by implementing the steepest descent algorithm that is commonly integrated with the back-propagation network (Karnik & Mendel, 2001). The best number of rules was the one that resulted in the minimum difference between the target and predicted values. Such a difference was estimated by the root mean square error (RMSE).

The performance of the IT2FLS for the cutting force is presented in Figure 3, with a RMSE (training, testing) = [1.197, 1.230]. Obviously, one can notice that the RMSE value for the testing data set is slightly greater than the one for the training data set (approximately 3% higher). This can indicate that an overtraining problem has occurred.

![Figure 3](image-url)
However, it does not seem to be the case in this model, where this difference is associated to the cutting force values in both the training and the testing data sets. To elucidate further, three data points out of 16 in the testing data set have values that are greater than 20KGF, thus, the error values are quite large, but less than 10% of the target value, these points can significantly affect the RMSE value in the testing set. This can be demonstrated by estimating the coefficient of determination, \( R^2 \) (training, testing) \( = [0.888, 0.883] \). Furthermore, it is noticeable that the majority of the predicted values fit properly in a 90% confidence interval, as shown in Figure 3.

Out of a total of 6, two rules, as examples, are represented in Figure 4, where the footprint of uncertainty is represented by the shaded area, and the corresponding linguistic forms of such rules can be as follows:

**Rule 1:** *IF* no cutting fluid is used and cutting speed is medium and feed rate is small and depth of cut is small, *THEN* the cutting force is medium.

**Rule 2:** *IF* cutting fluid is used and cutting speed is high and feed rate is high and depth of cut is medium, *THEN* the cutting force is high.

Examples of the response surfaces for the cutting force using two parameters at a time are shown in Figure 5. It is noticeable that the cutting force is a non-linear function of the cutting speed, depth of cut and feed rate. It is also noticeable that at a low level of depth of cut (less than 0.15 \( mm \)) and high cutting speed the cutting force is low, whereas the increase of the depth of cut increases the cutting force. Furthermore, when the depth of cut is in the range of 0.15 \( mm \) to 0.2 \( mm \), the cutting force is reaching a saturation level when a low level of feed rate (less than 0.08 \( mm/rpm \)) is used. In addition, a high level of cutting force can be noticed when both the levels of feed rate and depth of cut are high.

In a similar way, the IT2FLS was also implemented to predict the quality of a machined part that is represented by the surface roughness measured via the \( R_a \) value. By using eight rules, the IT2FLS performance for the \( R_a \) values is presented in

![Figure 4. The rule base of IT2FLS for the cutting force.](image-url)
Figure 5. Examples of the response surfaces for the cutting force.

Figure 6. The IT2FLS for the $R_a$ value ($\mu m$): (a) Training, (b) Testing (with a 90% confidence interval).

Figure 6, with RMSE (training, testing) = [1.223, 1.241] and $R^2$ (training, testing) = [0.856, 0.852]. One can notice that the performance measures for the surface roughness are low when compared to the performance measures for the cutting force, this being due to the high uncertainties in measuring the surface roughness.
The IT2FLS requires more computational efforts (i.e. computationally expensive) when compared to T1FLS, such a fact raises the question of whether such computational efforts have resulted in a superior paradigm for the turning process. Thus, the T1FLS was utilized to predict the cutting force and the surface roughness represented by the $R_a$ of the machined specimen. The performance measures represented by the $R^2$ and the RMSE values are summarized in Table 3. In such a table, one can notice that for both the cutting force and the surface roughness, the predictive modelling performance of the IT2FLS presented also in Table 3 is superior to that of the T1FLS; this being due to the fact that the IT2FLS can systematically deal with uncertainties more effectively compared to T1FLS. It is also worth noting that the predictive modelling performance for the surface roughness measured via the $R_a$ value is worse than the one for the cutting force for both T1FLS and IT2FLS.

### 4. Gaussian mixture model

#### 4.1. Model development

Most of the modelling paradigms, including a T1FLS and a T2FLS, are based on the assumption that the set of error residuals is distributed normally (Yang et al., 2012). On real-world applications with noisy measurable or non-measurable factors, this assumption, in fact, may not always be valid and, as a result, may lead to a loss of valuable information and to a paradigm with sub-optimal parameters (AlAlaween et al., 2018). Therefore, various modelling algorithms have hitherto been presented and implemented to extract such valuable information and improve the modelling results by characterizing the error residuals (Yang et al., 2012). For instance, the stochastic Gaussian mixture model (GMM) that can usually be presented as a linear combination of a number of the Gaussian components, has been implemented to refine the predictive performance of a model by providing a deeper interpretation of the density function. In the fuzzy logic systems, all of the presented algorithms including the GMM algorithm will, however, change the predicted output values without changing the extracted rules (i.e. the
consequents of the extracted rules). Consequently, these rules can no longer represent the process under investigation (AlAlaween et al., 2018). In this research work, the GMM is, therefore, incorporated in the fuzzy logic system, by such an incorporation the informative extracted rules are refined. The GMM was implemented because of its ability to represent the probability density function with a rational accuracy using the best number of Gaussian elements. The incorporation of the IT2FLS and the GMM is schematically represented in Figure 7.

As shown in Figure 7, the first step in such an incorporation is to select the set of the input variables that can be utilized to characterize the error. The optimal parameters of the GMM, namely; mean, covariance and mixing coefficient for each Gaussian element, are then determined by optimizing the well-known log-likelihood function. The set of optimal parameters can be written as shown in Equation (3; Bishop, 2006):

\[
\phi(z_{ij}) = \frac{\pi_j N(x^e_i | \mu^e_j, \Sigma^e_j)}{\sum_{j=1}^J \pi_j N(x^e_i | \mu^e_j, \Sigma^e_j)}, \quad \forall j
\]

\[
\begin{align*}
\mu^e_j &= \frac{\sum_{i=1}^I \phi(z_{ij}) x^e_i}{\sum_{i=1}^I \phi(z_{ij})} \\
\Sigma^e_j &= \frac{\sum_{i=1}^I \phi(z_{ij}) (x^e_i - \mu^e_j) (x^e_i - \mu^e_j)^T}{\sum_{i=1}^I \phi(z_{ij})} \\
\pi_j &= \frac{\sum_{i=1}^I \phi(z_{ij})}{I}
\end{align*}
\]

where \(x^e_i\) is the \(i\)th data vector that need to be included in the errors characterization GMM. The parameters \(\mu^e_j\), \(\pi_j\) and \(\Sigma^e_j\) stand for the mean, the mixing coefficient and the covariance of the \(j\)th Gaussian element, respectively. The parameter \(\phi(z_{ij})\) represents

![Figure 7. The incorporation of the fuzzy logic system and the modified GMM algorithm.](image-url)
the probability of the $i$th data vector belongs to the $j$th Gaussian element, thus, $z_{ij}$, as a $J$-dimensional binary parameter, is assigned a value of one when the $i$th vector belongs to the $j$th component where the other elements are assigned zero values. Since defining the set of optimal parameters analytically is not an easy task, the Expectation Maximization (EM) algorithm is usually implemented (Bishop, 2006). First, the parameters $\mu_j^i$, $\Sigma_j^i$, $\pi_i$ are carefully initialized using a clustering algorithm. In this research work, the K-means algorithm was utilized to initialized such parameters. Such a step is followed by determining the value of $\phi(z_{ij})$, such a step is called the E-step. Second, in the M-step, the estimated value of $\phi(z_{ij})$ is used to update the parameters, which are used to re-estimate the $\phi(z_{ij})$ value. The described procedure is continued until the EM algorithm converges or the pre-defined number of iterations is reached (Bishop, 2006). Commonly, the optimal number of the Gaussian elements is unknown, therefore, in this research paper, the performance measure that is employed to select such a number is the Bayesian information criterion (BIC; Rogers & Girolami, 2016).

The conditional error mean and the standard deviation can be estimated using numerical methods (Bishop, 2006). The optimal conditional mean value, as an indication of the exist bias, is then added up to the consequent mean of the related rule, by this the GMM algorithm can compensate for the bias. Finally, this step is, then, followed by the defuzzification process to determine the crisp output. Such steps are summarized in the flowchart presented in Figure 8.

### 4.2. Results and discussion

The main effects of the investigated process variables were considered by developing and optimizing the IT2FLS. Thus, only two variables out of a total of four were utilized to implement the GMM algorithm to compensate for the bias that could result from the error normality assumption. Different combinations of input variables were tested, the combination that was finally chosen was the one that resulted in the maximum error compensation (i.e. the minimum RMSE value). For the cutting force, the feed rate and the depth of cut were employed, in addition to the vector that consists of the errors resulted from the IT2FLS, to construct the GMM. The training data set was utilized to train such a model, whereas the testing one was kept hidden during the training process. By using six Gaussian elements, the predictive performance for the cutting force for the training and testing data sets is shown in Figure 9. The corresponding $R^2$ [training, testing] value is [0.949, 0.960].

An overall improvement of approximately 7% in $R^2$ value proves the ability of the GMM to detect possible unmodelled stochastic or deterministic behaviour. Figure 10 shows the two rules presented in Section 3 above after bias compensation (i.e. incorporating the GMM in the IT2FLS structure). It is noticeable that the antecedents of the two rules presented in Figure 10 and the ones presented in Figure 4 are the same, the difference is only in the consequents. To illustrate, the consequents of the first and second rules were amended by approximately 0.9KFG to the right and 0.85KFG to the left, respectively. It is worth emphasising that such changes in the consequent mean values had no effect on the linguistic forms of these rules.
Similarly, the GMM algorithm presented in this work to refine the FLS rules was also implemented to refine the IT2FLS that was developed to model the surface roughness, which is represented by the $R_a$ value of the produced product. Using seven Gaussian components, the performance measure values for the surface roughness are listed in Table 3. It is obvious that an improvement value of approximately 8% in the $R^2$ value was gained. It is also evident that the predictive performance of the compensated IT2FLS that was developed for the cutting force is better than the one of the compensated IT2FLS that was constructed for the surface roughness represented by the $R_a$ value, whereas the improvement value for the surface roughness is slightly large when compared to the improvement value for the cutting force.

Figure 8. Flowchart of the implementation of the GMM algorithm.
For comparison purposes, the GMM algorithm was utilized to amend the rules that were extracted by the T1FLSs that were developed for both the cutting force and the surface roughness, leading to significant improvement values. The performance measures represented by RMSE and $R^2$ are listed in Table 3. However, it is worth noting at this stage that the model integrating the IT2FLS with the GMM algorithm outperforms the model integrating the T1FLS with the GMM algorithm, with significant improvement values of approximately 13% and 12% in $R^2$ for the cutting force and the surface roughness, respectively. Such expected results indicate that the IT2FLS can deal with uncertainties better than the T1FLS and can, consequently, lead to a better predictive performance. Likewise, the traditional GMM presented in (Yang et al., 2012), which is usually utilized to refine the data points instead of rules similar to the other error characterization paradigms, was used to refine the IT2FLS, it was demonstrated that the algorithm presented in this research paper, where the extracted rules are refined, is superior to the traditional one, with improvement values of approximately 4% and 5% in the $R^2$ value for the cutting force and the surface roughness, respectively. In addition, the transparency of the IT2FLS was kept and maintained during the proposed error characterization approach. In addition, two proposed algorithms, namely,
a fuzzy logic based on sub-clustering approach (Rodić et al., 2021) and Taguchi artificial neural network hybrid with genetic algorithm (Nukman et al., 2013) were employed in this research work for comparison purposes. The results obtained are summarized in Table 3. It is noticeable that the results of the Fuzzy logic based on sub-clustering approach are close to the ones of the T1FLS. Furthermore, the results of the Taguchi artificial neural network hybrid with genetic algorithm are not as expected, this being due to the fact that the artificial neural network, in general, cannot deal with uncertainties.

In summary, the presented modelling framework, which integrates the IT2FLS and the GMM algorithm, has superior predictive performance when compared to the well-known IT2FLS and T1FLS as presented in Table 3. In addition, the proposed model and the well-known IT2FLS and T1FLS are considered to be transparent models that can provide users with a simple understanding of the process under examination and that can deal with uncertainties intrinsically. However, the proposed modelling framework requires more computational efforts (i.e. computationally expensive) when compared to IT2FLS and T1FLS. Such computational efforts have resulted in a superior predictive performance for the cutting force and the surface roughness. After being successfully trained, the presented model, as a data driven model, can also be used to predict the cutting force and the surface roughness for new materials and perhaps new variables. In addition, the proposed framework represents a promising development not only in the manufacturing industry but also in other industries, where one needs to develop a predictive model that can (i) accurately predict the properties of a produced product in a way that can guarantee and perhaps optimize the quality attributes that can be critical, (ii) provide and maintain a simple and comprehensive linguistic understating of the process under investigation, and (iii) characterize the error residuals to compensate for the normality assumption (i.e. the error is normally distributed) and, consequently, model the stochastic and deterministic behaviours.
5. Conclusions

The main aim of the presented research work was to develop an interpretable and more accurate predictive structure for the turning process. The developed structure incorporated a Gaussian mixture model (GMM) in the structure of an interval type-2 fuzzy logic system (IT2FLS). The IT2FLS was implemented first to represent the turning process by mapping the process parameters to the cutting force and the surface quality of a machined specimen. The IT2FLS was able to successfully predict the cutting force and the surface roughness and to deal with any uncertainties. In addition, it provided informative rules that can be easily understood and utilized to control the turning process. Since, more accurate predictions of the examined attributes are, more often than not, desired, the GMM was then utilized to characterise the resulted error residuals by refining the extracted informative rules to compensate for any potential biases and to, consequently, improve the predictive performance. An overall improvement of 7% was gained by incorporating the GMM in the IT2FLS structure.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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