Macroscopic Einstein - Maxwell equations for a system of interacting particles to second-order accuracy in the interaction constant.

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Abstract.

In this paper the macroscopic Einstein and Maxwell equations for system, in which the electromagnetic interactions are dominating (for instance, the cosmological plasma before the moment of recombination), are derived.

Ensemble averaging of the microscopic Einstein - Maxwell equations and the Liouville equations for the random functions leads to a closed system of macroscopic Einstein - Maxwell equations and kinetic equations for one-particle distribution functions. The macroscopic Einstein equations for a relativistic plasma differ from the classical Einstein equations in that their left-hand sides contain additional terms due to particle interaction. The terms are traceless tensors with zero divergence. An explicit covariant expression for these terms is given in the form of momentum-space integrals of expressions depending on one-particles distribution functions of the interacting particles of the medium.

The additional terms in the left-hand side of the macroscopic Einstein equations for a relativistic plasma has murch in common with the additional terms in the left-hand side of the macroscopic Einstein equations for a system of self - gravitating particles (refer to. [1], [2]).

The macroscopic Maxwell equations alsow differ from the classical macroscopic Maxwell equations in that their left-hand sides contain additional terms due to particle interaction as well the effects of general relativity.
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1. Introduction.

The idea of macroscopic gravity can be considered as an extension of Lorentz’ idea (refer to [3]), formulated first for electrodynamics, about the existence of two levels, microscopic and macroscopic, of understanding classical physical phenomena. Lorentz showed that Maxwell’s electrodynamics is a macroscopic theory of electromagnetism, and the Maxwell equations could be derived from a system of microscopic field equations called now the Maxwell - Lorentz ones, by infinitesimal space - time regions averaging (refer to [3], [4]).

As it is known the macroscopic Maxwell equation for continuous media can be also obtained from the microscopic Maxwell equations by ensemble averaging the latter (refer to [5]).

The Einstein equations, whose right-hand side contain the energy-momentum tensor of matter, are phenomenological equations. It is natural to suppose that the Einstein equations (or their generalizations) for continuous media can also be obtained from the microscopic Einstein equations, i.e., Einstein equations whose right-hand sides contain the sum of the energy - momentum tensors of individual particles. However, due to the nonlinearity of the left-hand side of Einstein equations, the averaging of the microscopic Einstein equations is more complicated than one of the microscopic Maxwell equations (refer to [6] - [10]).

In (refer to [1]) a method is developed for the ensemble averaging of the microscopic Einstein equations for interacting particles. (We use the ensemble averaging procedure introduced by Klimontovich (refer to [11, 12]) to derive the relativistic kinetic equation for a plasma. The same procedure was used by the present author in (refer to [13, 14]) to derive a relativistic kinetic equation for a system of gravitationally and electromagnetical interacting particles in General Relativity accurate to within the second order for the interaction constant.) This results to macroscopic Einstein equations for continuous media that are accurate to second-order terms in the interaction constant. The macroscopic Einstein equations for a system of interacting particles differ from the classical Einstein equations in that their left-hand sides contain additional components due to particle interaction. The components are expressed in terms of the two-particle correlation function of the particles.

In (refer to [2]) we got covariant expressions for additional components for the system of self-gravitating particles. The components are traceless tensors with zero divergence. The
expressions were obtained in the form of momentum-space integrals of expressions depend-
ing on one-particle distribution function of the gravitationally interacting particles of the
medium. The given expressions are proportional to the cube of the Einstein constant and
the square of the particle number density. The latter relationship implies that interaction
effects manifest themselves in systems of very high density (the Universe in the early stages
of its evolution, dense objects close to gravitational collapse, etc.)

The present paper is a direct continuation of earlier papers (refer to. [1], [2]), devoted to
the derivation of the macroscopic Einstein equations for a system of self-gravitating particles
to within terms of second order in the interaction constant.

The objective of this paper is to obtain the macroscopic Einstein equations for system,
in which electromagnetic interactions (for instance, cosmological plasma before a moment of
recombination,) are dominating.

The macroscopic Einstein equations for relativistic plasma differ from the classical Ein-
stein equations in that their left-hand side contains additional terms due to particle interac-
tion. The terms are traceless tensors with zero divergence. An explicit covariant expression
for these terms is given in the form of momentum-space integrals of expressions depending
on one-particles distribution functions of the interacting particles of the medium.

The additional terms in the left-hand side of the macroscopic Einstein equations for a
relativistic plasma has march in common with the additional terms in the left-hand side of
the macroscopic Einstein equations for a system of self - gravitating particles (refer to. [1],
[2]).

The macroscopic Maxwell equations also differ from the classical macroscopic Maxwell
equations for their left-hand sides contain additional terms due to particle interaction as well
the effects of general relativity.

2. Microscopic Einstein - Maxwell equations

The method of deriving the macroscopic Einstein equations is discussed in (refer to.[1]).
The notation we use here are the same that in (refer to.[1]).

Briefly, the method we used to obtain the macroscopic Einstein - Maxwell equations is
the following.

We start from the microscopic Einstein and Maxwell equations

\[ \tilde{G}^{ij} = \chi \tilde{T}^{ij}_{(m)} + \chi \tilde{T}^{ij}_{(el)}, \]  

\[ \nabla_k \tilde{F}^{ik} = -\frac{4\pi}{c} \tilde{J}^i. \]  

Here \( \tilde{G}^{ij} \) is the Einstein tensor in a Riemannian space with metric \( \tilde{g}_{ij} \), \( \chi = 8\pi k/c^4 \)
is Einstein’s constant (where \( k \) is the gravitational constant, \( c \) is the velocity of light),
\( T^{ij} \) is the microscopic energy-momentum tensor of particles, \( \tilde{F}^{ik} \) is the Maxwell’s tensor, \( \tilde{J}^i \) is the microscopic current vector of particles, \( T_{(el)}^{ij} \) is the energy-momentum tensor of electromagnetic field. Raising and lowering of indexes is accomplishment with the metric \( \tilde{g}_{ij} \), \( \tilde{\nabla}_k \) is a covariant derivative in a Riemannian space with metric \( \tilde{g}_{ij} \).

The tensor \( T_{(el)}^{ij} \) have the form

\[
T_{(el)}^{ij} = \frac{1}{4\pi} \left(-\tilde{F}^{ii}_{\ j} \tilde{F}_{ij} + \frac{1}{4} \tilde{g}^{ij} \tilde{F}_{lm} \tilde{F}^{lm}\right).
\]  

(3)

The tensors \( T^{ij}_{(m)} \) and \( \tilde{J}^i \) has the form

\[
T^{ij}_{(m)} = \sum_a m_a c^2 \int \frac{d^4p_a}{\sqrt{-g}} \tilde{u}_a^i \tilde{u}_a^j \tilde{N}_a(q^i, \tilde{p}),
\]

(4)

\[
\tilde{J}^i = \sum_b e_b c \int \frac{d^4\tilde{p}}{\sqrt{-g}} \tilde{u}_b^i \tilde{N}_b(q, \tilde{p}).
\]

(5)

Here \( e_a \) is the charge of particles of species "a", \( m_a \) is their mass, \( \tilde{g} \) is the determinant of \( \tilde{g}_{ij} \), \( \tilde{p}_i^j \) is the momentum of particles of species "a", \( \tilde{u}_a^i = \tilde{p}_a^i / \sqrt{\tilde{g}_{ij} \tilde{p}_a^i \tilde{p}_a^j} \).

\[ \frac{d^4\tilde{p}}{\sqrt{-g}} \]

- is the invariant volume element in momentum space [7].

\[ N_a(q^i, \tilde{p}_a) \] - is the Klimontovich random function [11]:

\[
\tilde{N}_a(q^i, \tilde{p}_j) = \sum_{i=1}^{n_a} \int d\tilde{s} \delta^4(q^i - q^i_{(l)}) \delta^4(\tilde{p}_j - \tilde{p}^{(l)}_j(\tilde{s})).
\]

(6)

Here \( n_a \) is the number of particles belonging to species "a", \( \tilde{s} \) is the canonical parameter along the particle trajectories: \( d\tilde{s} = \sqrt{\tilde{g}_{ij} dq^i dq^j} \); \( \tilde{q}^i_{(l)} \) and \( \tilde{p}^{(l)}_j \) are the coordinates and momentum of the l-th particle of species "a", which are found by solving the equations of motion:

\[
\frac{d\tilde{q}^i_{(l)}}{d\tilde{s}} = \tilde{p}^i_{(l)} / m_a c, \quad \frac{d\tilde{p}^{(l)}_j}{d\tilde{s}} = \frac{1}{m_a c} \tilde{\Gamma}_{j,ik} \tilde{p}^i_{(l)} \tilde{p}^k_{(l)} + \frac{e_a}{c} \tilde{F}_{ik} \tilde{p}^k_{(l)}.
\]

(7)

Here \( \tilde{\Gamma}_{j,ik} \) is the Christoffel symbol of the first kind given by the metric \( \tilde{g}_{ij} \).

In view of Eqs. (7) the random function (3) obeys the equation

\[
\tilde{p}^i \frac{\partial \tilde{N}_a}{\partial q^i} + \tilde{\Gamma}_{j,ik} \tilde{p}^j \tilde{p}^k \frac{\partial \tilde{N}_a}{\partial \tilde{p}_i} + \frac{e_a}{c} \tilde{F}_{ik} \tilde{p}^k \frac{\partial \tilde{N}_a}{\partial \tilde{p}_i} = 0.
\]

(8)

Next we write the metric \( \tilde{g}_{ij} \) as

\[ \tilde{g}_{ij} = g_{ij} + h_{ij}, \]

(9)
and $\tilde{F}_{ik}$ as
\begin{equation}
\tilde{F}_{ik} = F_{ik} + \omega_{ik},
\end{equation}
(10)

Here $g_{ij} = \langle \tilde{g}_{ij} \rangle$ is the ensemble average of the metric $\tilde{g}_{ij}$ \[1, 2\], $\langle \tilde{F}_{ik} \rangle$ is the ensemble average of $\tilde{F}_{ik}$. Note that $\langle h_{ij} \rangle \equiv 0$ and $\langle \omega_{ik} \rangle \equiv 0$.

Parallel with the momenta $\tilde{p}_{i(l)} = m_a c d q_{i(l)} / d\tilde{s}$ we use the momenta $p^i$ measured in the metric $g_{ij}$:
\begin{equation}
p_i = \alpha^{-1}(q, p) \tilde{p}_i,
\end{equation}
(11)

Here $s$ is the canonical parametr introduced by $g_{ij}$.

The transformation from $\tilde{p}_i$ to $p_i$ is given by
\begin{equation}
\tilde{p}_j = \tilde{g}_{jk} p^k = \alpha \tilde{g}_{jk} g^{ki} p_i.
\end{equation}
(12)

The Jacobian of transformation (12), is (see [14]):
\begin{equation}
\left| \frac{\partial \tilde{p}_i}{\partial p_j} \right| = \alpha^4 \frac{\tilde{g}}{g},
\end{equation}
(13)

where $g$ is the determinant of $g_{ij}$.

Now we introduce the function $N_a(q^i, \tilde{p}_j)$ defined in the eight - dimensional phase space with coordinates $(q, p)$ as
\begin{equation}
N_a(q, p) = \sum_{l=1}^{n_a} \int ds \delta^4(q^i - q^{i(l)}(s)) \delta^4(p_j - p_{j(l)}(s)),
\end{equation}
(14)

where $q^{i(l)}$, $p_{j(l)}$ are found by solving equations obtained from (7) with the transformation (12) taken into account ($p^i = g^{ij} p_j$).

Note that the functions $\tilde{N}_a$ and $N_a$ are related in the following manner:
\begin{equation}
\tilde{N}_a(q, \tilde{p}) = \frac{g}{g^\alpha} N_a(q, p).
\end{equation}
(15)

Equation for $N_a(q, p)$ can be obtained directly from (8) by replacing the variables (12) and (13):
\begin{equation}
p^i \frac{\partial N_a}{\partial q^i} + \Gamma_{j,ik} p^j p^k \frac{\partial N_a}{\partial p_i} + \frac{e_a}{c} F_{ik} p^k \frac{\partial N_a}{\partial p_j} =
\end{equation}
\begin{equation}
= \frac{\partial}{\partial p_i} \left[ \left( \Omega^{m}_{jk} \Delta_{m} p^j p^k - \frac{e_a}{c} \psi^l_{jk} \Delta_{l} p^k \right) N_a \right].
\end{equation}
(16)

Here
\begin{equation}
\Delta_{ki} = g_{ki} - u_{ki} u_i; \quad u^k = \frac{p^k}{\sqrt{p^l p_l}}; \quad \Omega^{m}_{kj} = \Gamma^{m}_{kj} - \Gamma^{m}_{kj} - \Omega^{m}_{dk}.
\end{equation}
(17)
- is the difference of the Christoffel symbols of second kind for the metrics $\tilde{g}_{ij}$ and $g_{ij}$, 

$$\psi_{\alpha}^{l} = \frac{1}{\alpha(q,p)} \tilde{F}_{l}^{k} - F_{l}^{k} = \frac{1}{\alpha(q,p)} \tilde{g}^{lm} \tilde{F}_{mk} - g^{lm} F_{mk}. \quad (18)$$

If in (11) and (12) we turn to the variables $p_i$ and $N_a$ we get 

$$\tilde{T}_{ij} = \sum_{a} m a^{2} \int \frac{d^{4} p_{a}}{\sqrt{-g}} \alpha(q,p) \sqrt{\frac{g}{\tilde{g}}} u_{a}^{i} u_{a}^{j} N_{a}(q,p_{a}), \quad (19)$$

$$\tilde{J}^{i} = \sum_{b} e_{b} \int \frac{d^{4} p_{b}}{\sqrt{-g}} \sqrt{\frac{g}{\tilde{g}}} u_{b}^{i} N_{b}(q,p_{b}). \quad (20)$$

where $d^{4} p/\sqrt{-g}$ is the invariant volume element in the unperturbed momentum space.

For subsequent calculation it is convenient to write the Einstein equations as 

$$R_{ij} + \nabla_m \Omega_{ij}^{m} - \nabla_j \Omega_{im}^{m} + \Omega_{mn}^{m} \Omega_{ij}^{n} - \Omega_{jn}^{m} \Omega_{im}^{m} =$$

$$= \chi \sum_{a} m a^{2} \int \frac{d^{4} p_{a}}{\sqrt{-g}} \sqrt{\frac{g}{\tilde{g}}} \left( g_{ik} g_{jm} - 1 \right) u_{a}^{k} u_{a}^{m} N_{a}(q,p_{a}) + \chi \tilde{T}^{(el)}_{ij}. \quad (21)$$

Here $R_{ij}$ is the Ricchi tensor of the Riemannian space with the metric $g_{ij}$, $\nabla_m$ is the covariant derivative in this space.

With the help (9) and (10) we can write the Maxwell equations (2) to within the first-order terms in $h_{ij}$: 

$$\nabla_{k} F_{ik}^{m} + \nabla_{k} \left( h_{m}^{i} F_{km}^{m} - h_{m}^{k} F_{im}^{k} \right) + \frac{1}{2} F_{ik} \nabla_{k} h + \nabla_{k} \omega^{ik} +$$

$$+ \nabla_{k} \left( h_{m}^{i} \omega^{km} - h_{m}^{k} \omega^{im} \right) + \frac{1}{2} \omega^{ik} \nabla_{k} h = -4 \pi \sum_{a} e_{a} \int \frac{d^{4} p_{a}}{\sqrt{-g}} u_{a}^{i} \left( 1 - \frac{1}{2} h \right) N_{a}(q,p_{a}). \quad (22)$$

In (21), (22) and below when raising and lowering the indexes we use the averaged metric $g_{ij}, \ h = h_{ij}^{1}.$

Let’s expand the Einstein equations (21) up to the second-order members in small $h_{ij}$ and $\omega_{ij}$:

$$R_{ij} + R^{(1)}_{ij} + R^{(2)}_{ij} + \cdots = \chi \sum_{a} m a^{2} \int \frac{d^{4} p_{a}}{\sqrt{-g}} \left( g_{ik} g_{jm} - 1 \right) u_{a}^{k} u_{a}^{m} N_{a}(q,p_{a}) +$$

$$+ \sum_{a} \chi m a^{2} \int \frac{d^{4} p_{a}}{\sqrt{-g}} R^{(1)}_{ijkm}(h) u_{a}^{k} u_{a}^{m} N_{a} + \cdots + \chi T^{(el)}_{ij} + \chi \left( T^{(el)}_{ij} \right)^{(1)} + \chi \left( T^{(el)}_{ij} \right)^{(2)}. \quad (23)$$
Here $R_{ij}^{(1)}$ is the sum of all terms of expansion $\tilde{R}_{ij}$ that are first-order in $h_{ij}$, $R_{ij}^{(2)}$ is the sum of all second-order terms in $h_{ij}$, etc.; $L_{ijkm}^{(1)}(h)$ is the sum of all first-order terms in $h_{ij}$ of expansion the expression

$$\alpha \sqrt{\frac{g}{g}} \left( \bar{g}_{ik} \bar{g}_{jm} - \frac{1}{2} \bar{g}_{ij} \bar{g}_{km} \right),$$

$T_{ij}^{(el)}$ is the energy-momentum tensor of averaged electromagnetic field $F_{ij}$, $(T_{ij}^{(el)})^{(1)}$ is the sum of all terms of expansion $\tilde{T}_{ij}^{(el)}$ that are first-order in $h_{ij}$ and $\omega_{ij}$, $(T_{ij}^{(el)})^{(2)}$ is the sum of all terms of expansion $\tilde{T}_{ij}^{(el)}$ that are second-order in $h_{ij}$ and $\omega_{ij}$.

This expressions has the forms:

$$R_{ij}^{(1)} = \nabla_m \Omega_{ij}^{(1)m} - \nabla_j \Omega_{im}^{(1)m},$$

$$R_{ij}^{(2)} = \nabla_m \Omega_{ij}^{(2)m} - \nabla_j \Omega_{im}^{(2)m} + \Omega_{im}^{(1)m} \Omega_{ij}^{(1)n} - \Omega_{jm}^{(1)m} \Omega_{il}^{(1)n},$$

$$\Omega_{ij}^{(1)m} = \frac{1}{2} g^{ml} (-\nabla_l h_{ij} + \nabla_i h_{lj} + \nabla_j h_{li}),$$

$$\Omega_{ij}^{(2)m} = \frac{1}{2} h^{ml} (-\nabla_l h_{ij} + \nabla_i h_{lj} + \nabla_j h_{li}) = -\frac{1}{2} h^{ml} \Omega^{(1)l} \Omega^{ij}.$$

$$L_{ijkm}^{(1)} = -\frac{1}{2} \left( h_{st} u^s u^t + h_{st} g^{st} \right) \left( g_{ik} g_{jm} - \frac{1}{2} g_{ij} g_{km} \right) +$$

$$+ h_{ik} g_{jm} + g_{ik} h_{jm} - \frac{1}{2} h_{ij} g_{km} - \frac{1}{2} g_{ij} g_{km},$$

$$T_{ij}^{(el)} = \frac{1}{4\pi} \left( -F_{il} F_{jm} g^{lm} + \frac{1}{4} g_{ij} F_{lm} F^{lm} \right),$$

$$\left( T_{ij}^{(el)} \right)^{(1)} = \frac{1}{4\pi} \left[ F_{il} F_{jm} h^{lm} + \frac{1}{4} h_{ij} F_{lm} F^{lm} - \frac{1}{2} g_{ij} F_{lm} F_{km} g^{lk} h^{mn} - \omega_{il} F_{jl} - \omega_{jl} F_{il} + \frac{1}{2} F^{lm} \omega_{lm} \right],$$

$$\left( T_{ij}^{(el)} \right)^{(2)} = \frac{1}{4\pi} \left[ - F_{il} F_{jm} g_{st} + \frac{1}{2} g_{ij} F_{lk} F_{m}^{k} g_{st} + \frac{1}{4} g_{ij} F_{lm} F_{st} - \frac{1}{2} F_{il}^{k} F_{km} F_{ij} g_{lt} g_{js} \right] h^{ls} h^{lm} +$$

$$+ \frac{1}{4\pi} \left[ F_{it} g_{jt} g_{sm} + F_{jt} g_{it} g_{sm} - g_{ij} F_{lm} g_{st} + \frac{1}{2} g_{it} g_{jm} F_{ls} \right] \omega^{ls} h^{lm} +$$

$$+ \frac{1}{4\pi} \left( - g_{it} g_{is} + \frac{1}{4} g_{ij} g_{st} \right) \omega^{ls}.\right.$$
We average (22) and (23) over the paths (Ref. [11]-[14]) and introduce the one-particle distribution function

\[ f_a(q, p) = \langle \int ds \delta^4(q^i - q^i_{a(l)}(s)) \delta^4(p_j - p^a_{j(l)}(s)) \rangle = \frac{1}{n_a} \langle N_a \rangle. \]  

As a result we have the averaged Einstein equations in the form

\[ R_{ij} + \Lambda_{ij} = \chi \left( T_{ij}^{(m)} - (1/2) T^{(m)} g_{ij} \right) + \chi \left( T_{ij}^{(el)} - \frac{1}{2} T^{(el)} g_{ij} \right) + \chi \left( T_{ij}^{(r)} - \frac{1}{2} T^{(r)} g_{ij} \right). \]  

Here

\[ T^{ij}_{(m)} = \sum_a n_a m_a c^2 \int \frac{d^4p_a}{\sqrt{-g}} u_a^i u_a^j f_a(q, p) \]  

is the macroscopic energy-momentum tensor of medium, \( T^{ij}_{(el)} \) is the energy-momentum tensor of macroscopic electromagnetic field (see. (29),

\[ T^{ij}_{(el)} = \frac{1}{4\pi} \langle -\omega^i l \omega^j l + \frac{1}{2} g^{ij} \omega_m \omega^m \rangle - \]  

is the macroscopic energy-momentum tensor of radiation in plasma,

\[ \Lambda_{ij} = \langle R^{(2)}_{ij} \rangle - \sum_a \chi m_a c^2 \int \frac{d^4p_a}{\sqrt{-g}} \langle N_a L^{(1)}_{ijkl} u^m_a u^k_a \rangle. \]  

When obtained (33) we taking into account that

\[ \langle R^{(1)}_{ij} \rangle = 0, \quad \langle \left( T^{(el)}_{ij} \right)^{(2)} \rangle = 0. \]  

Next we assum that

\[ F_k h^k_j \ll \omega_{ij} \quad \text{ets.} \]  

inside the correlation region. Consequently \( \langle \left( T^{(el)}_{ij} \right)^{(2)} \rangle \) is equal approximately to \( T^{(r)}_{ij} \).

Taking into account the (25) - (28) we can write \( \Lambda_{ij} \) in the form

\[ \Lambda_{ij} = \nabla_k \varphi_{ij}^k + \mu_{ij}, \]  

where

\[ \varphi_{ij}^k = -\frac{1}{2} \left( \delta^k_n \delta^s_j - \delta^k_j \delta^s_n \right) P^n_{is}, \]  

\[ P^n_{is} = \langle h^{n}_{i} \Omega^{(1)l}_{is} \rangle, \]  

\[ \mu_{ij} = \left( \delta^k_n \delta^s_j - \delta^k_j \delta^s_n \right) Q^n_{kis} + \lambda_{ij}, \]  

\[ Q^n_{kis} = \langle \Omega^{(1)l}_{kis} \rangle, \]  

\[ \lambda_{ij} = \]
\[\lambda_{ij} = -\sum_a \chi m_a c^2 \int \frac{d^4p}{\sqrt{(-g)}} \left\{ -\frac{1}{2} u_i u_j u^k u^m - \frac{1}{4} g_{ij} u^k u^m - \frac{1}{2} u_i u_j g^{km} + \right. \\
+ \frac{1}{4} g_{ij} g^{km} + u_i u^k \delta_j^m + u_j u^k \delta_i^m - \frac{1}{2} \delta_i^k \delta_j^m \right\} <N_a h_{km}>. \tag{43}\]

In (43) we reject the indices "a" on momentums and velocities of particles of species "a".

The macroscopic Maxwell equations, obtained from (22) after averaging, have the form

\[\nabla_k F^{ik} + \nabla_k \varphi^{ik} + \mu^i = -\frac{4\pi}{c} J^i, \tag{44}\]

where

\[\varphi^{ik} = \langle h^i_m \omega^{km} \rangle - \langle h^k_m \omega^{im} \rangle, \tag{45}\]

\[\mu^i = \frac{1}{2} \langle \omega^{ik} \nabla_k h \rangle + \lambda^i, \tag{46}\]

\[\lambda^i = -2\pi \sum_b e_b \int \frac{d^4p}{\sqrt{-g}} u^i_b (N_b h), \tag{47}\]

\[J^i = \sum_b e_b c n_b \int \frac{d^4p}{\sqrt{-g}} u^i_b f_b - \tag{48}\]

is the macroscopic current vector.

To simplify still further, we only have to calculate \(h_{ij}\) and \(\omega_{ij}\) inside the region determined by the correlations radius and corresponding correlation time. Note, That distant collisions provide the main contribution to calculated macroscopic quantities. To consider this contribution it is enough to find \(h_{ij}\) and \(\omega_{ij}\) from the Einstein-Maxwell equations linearised with respect to the metric \(g_{ij}\) and the macroscopic electromagnetic field \(F_{ij}\).

We assume the average gravitational field generated by the particles to be constant within the correlation region. In this case we can interpret \(g_{ij}\) within the correlation region as the Minkowski metric. We assume also, that the influence of macroscopic electromagntical field on the microscopic field in correlation region is small (see (37)).

As a result we have linearized Einstein and Maxwell equations with respect the Minkowski metric \(g_{ij}\). By employing the gauge \(\nabla_k \gamma^{ik} = 0\), where \(\gamma_{ij} = h_{ij} - (1/2)h g_{ij}\), we get the following form of the linearized Einstein and Maxwell equations

\[\Box \gamma^{ij} = -\sum_b 2\chi m_b c^2 \int d^4p_b \Phi_b(q,p'_b) u^i_b u'^i_b, \tag{49}\]

\[\nabla_k \omega^{ik} = -4\pi \sum_b e_b \int d^4p_b \Phi_b(q,p'_b) u^i_b, \tag{50}\]

Here \(\Box = g^{ij} \nabla_i \nabla_j\), \(\Phi_b = N_b - n_b f_b\).

Thus subsequent calculatios do not have a covariant form, but they are all done for the purpose of determining the components of the tensors \(\varphi^{k}_{ij}, \mu_{ij}, \varphi_{ij}, \mu_i\) and \(T_{ij}^{(r)}\) at some
(arbitrary) point \((q)\) in the locally Lorentzian frame. In this reference frame the interval 
\(ds^2 \) has the form
\[
ds^2 = d\eta^2 - (dq^1)^2 - (dq^2)^2 - (dq^3)^2. \tag{51}
\]
The final result must be written in covariant form.

The expressions for \(h_{ij} \) and \(\Omega_{ijk} \) we get from the Einstein equations \((49)\) linearized with
respect to the Minkowski metric (which we still denote by \(g_{ij} \)) were found in (\(\text{refer to.}[1]\)):
\[
h_{ij}(\eta, q) = \sum_b \int d^4p' \int d^3q' \int d^3k \int_{-\infty}^{\eta} d\eta' e^{-ik(q-q')} \times
h_{ij}^{(b)}(\eta, \eta', p', k) \Phi_b(\eta', q', p'),
\tag{52}
\]
\[
\Omega_{ijk}(\eta, q) = \sum_b \int d^4p' \int d^3q' \int d^3k \int_{-\infty}^{\eta} d\eta' e^{-ik(q-q')} \times
\Omega_{ijk}^{(b)}(\eta, \eta', p', k) \Phi_b(\eta', q', p').
\tag{53}
\]
where \((q = (q^1, q^2, q^3)\) is the three-dimensional radius vector in the given reference frame, and \(k = (k_1, k_2, k_3)\),
\[\Phi_b = N_b - n_b f_b,\]
\[
h_{ij}^{(b)}(\eta, \eta', p', k) = -\frac{i\chi mc^2}{(2\pi)^3k} (u_i' u_j' - \frac{1}{2} g_{ij}) \{e^{ik(\eta'-\eta)} - e^{-ik(\eta'-\eta)}\},
\tag{54}
\]
\[
\Omega_{ijk}^{(b)}(\eta, \eta', p', k) = \frac{\chi mc^2}{2(2\pi)^3k} \left\{[u'_j u'_k - \frac{1}{2} g_{jk})k_i^+ - (u'_j u'^i - \frac{1}{2} \delta_j^i)k_k^+ -
-(u'_k u'^i - \frac{1}{2} \delta_k^i)k_j^+] e^{ik(\eta'-\eta)} - [(u'_j u'_k - \frac{1}{2} g_{jk})k_i^- - (u'_j u'^i - \frac{1}{2} \delta_j^i)k_k^- -
-(u'_k u'^i - \frac{1}{2} \delta_k^i)k_j^-] e^{-ik(\eta'-\eta)}\right\}.
\tag{55}
\]
In \((54)\) and \((55)\) the following vectors were introduced:
\[k_i^+ = (k, k), \quad k_i^- = (-k, k),\]
where
\[k = \sqrt{(k_1)^2 + (k_2)^2 + (k_3)^2} = |k|\]
Obviously, \(k_i^- (k) = -k_i^+ (-k)\).

To obtain the additional terms \(\nabla_k \varphi^i \) and \(\mu^i \) in macroscopic Maxwell equations we have to calculate the \(h = h^i_l \) and \(\nabla_k h\):
\[ h(\eta, \mathbf{q}) = \sum_b \int d^4 p' \int d^3 q' \int d^3 k \int_{-\infty}^\eta d\eta' e^{-ik(q-q')} \]
\[ h^{(b)}(\eta, \eta', p', k) \Phi_b(\eta', \mathbf{q}', p'), \] (56)

where
\[ h^{(b)}(\eta, \eta', p', k) = \frac{i \chi m_b c^2}{(2\pi)^3 k} \left\{ e^{ik(\eta'-\eta)} - e^{-ik(\eta'-\eta)} \right\}. \] (57)

\[ \nabla_k h(\eta, \mathbf{q}) = \sum_b \int d^4 p' \int d^3 q' \int d^3 k \int_{-\infty}^\eta d\eta' e^{-ik(q-q')} \]
\[ h^{(b)}_{ik}(\eta, \eta', p', k) \Phi_b(\eta', \mathbf{q}', p'), \] (58)

where
\[ h^{(b)}_{ik}(\eta, \eta', p', k) = \frac{\chi m_b c^2}{(2\pi)^3 k} \left\{ k^+_{ik} e^{ik(\eta'-\eta)} - k^-_{ik} e^{-ik(\eta'-\eta)} \right\}. \] (59)

Let's write the solution of (50) in the form
\[ \omega_{ik} = \partial_i A_k - \partial_k A_i, \]

where [11]:
\[ A_i(\eta, \mathbf{q}) = \sum_b \int d^4 p' \int d^3 q' \int d^3 k \int_{-\infty}^\eta d\eta' e^{-ik(q-q')} \]
\[ A^{(b)}_i(\eta, \eta', p', k) \Phi_b(\eta', \mathbf{q}', p'). \] (60)

Here
\[ A^{(b)}_i(\eta, \eta', p', k) = \frac{i c_b}{(2\pi)^2 k} \bar{u}_i' \left\{ e^{ik(\eta'-\eta)} - e^{-ik(\eta'-\eta)} \right\}. \]

The \( \omega_{ik} \) have the form:
\[ \omega_{ik}(\eta, \mathbf{q}) = \sum_b \int d^4 p' \int d^3 q' \int d^3 k \int_{-\infty}^\eta d\eta' e^{-ik(q-q')} \times \]
\[ \times \omega^{(b)}_{ik}(\eta, \eta', p', k) \Phi_b(\eta', \mathbf{q}', p'), \] (62)

where
\[ \omega^{(b)}_{ik}(\eta, \eta', p', k) = \frac{e_b}{(2\pi)^2 k} \left\{ (k_i^+ u_k' - k_k^+ u_i') e^{ik(\eta'-\eta)} - (k_i^- u_k' - k_k^- u_i') e^{-ik(\eta'-\eta)} \right\}. \] (63)

If substitute the (52), (53), (56), (58), (62) to (41), (42), (43), (44) — (47) we get the following expressions for \( P^n_{is}, Q^n_{is}, \lambda_{ij}, \langle h_i^m \omega_{km} \rangle, \langle \omega^{ik} \nabla_k h \rangle, \lambda_i \):
\[ P^n_{is} = \sum_{bc} \int d^4 p' \int d^4 p'' \int d^3 q' \int d^3 q'' \int_{-\infty}^\eta d\eta' \int_{-\infty}^\eta d\eta'' \int d^3 k' \int d^3 k'' \times \]
\[ e^{-ik'(q-q')}e^{-ik''(q-q'')}h_i^{\text{b}}(\eta, \eta', p', k')\Omega_i^{\text{c}}(\eta, \eta'', p'', k'')n_b n_c g_{bc}(x', x''), \quad (64) \]

\[ Q_{kis}^{\text{d}} = \sum_{bc} \int d^4 p' \int d^4 p'' \int d^3 q' \int d^3 q'' \int_{-\infty}^{\eta} d\eta' \int_{-\infty}^{\eta''} d\eta'' \int d^3 k' \int d^3 k'' \times \]

\[ e^{-ik'(q-q')}e^{-ik''(q-q'')}\Omega_{k_{li}}^{\text{c}}(\eta, \eta', p', k')\Omega_{k_{li}}^{\text{c}}(\eta, \eta'', p'', k'')n_b n_c g_{bc}(x', x''), \quad (65) \]

\[ \lambda_{ij} = -\sum_{bc} \chi_m c^2 \int d^4 p' \int d^4 p'' \int d^3 q' \int d^3 q'' \int_{-\infty}^{\eta} d\eta' \int_{-\infty}^{\eta''} d\eta'' \int d^3 k' \int d^3 k'' \times \]

\[ e^{-ik'(q-q')}e^{-ik''(q-q'')}h_i^{\text{b}}(\eta, \eta', p', k')n_b n_c g_{bc}(x', x''), \quad (66) \]

\[ \langle h_{\text{m}}^{\text{k}} \rangle = \sum_{bc} \int d^4 p' \int d^4 p'' \int d^3 q' \int d^3 q'' \int_{-\infty}^{\eta} d\eta' \int_{-\infty}^{\eta''} d\eta'' \int d^3 k' \int d^3 k'' \]

\[ e^{-ik'(q-q')}e^{-ik''(q-q'')}h_{\text{m}}^{\text{k}}(\eta, \eta', p', k')\omega_{(c)}^{\text{k}}(\eta, \eta'', p'', k'')n_b n_c g_{bc}(x', x''), \quad (67) \]

\[ \langle \omega^{ik} \nabla_k h \rangle = \sum_{bc} \int d^4 p' \int d^4 p'' \int d^3 q' \int d^3 q'' \int_{-\infty}^{\eta} d\eta' \int_{-\infty}^{\eta''} d\eta'' \int d^3 k' \int d^3 k'' \]

\[ e^{-ik'(q-q')}e^{-ik''(q-q'')}h_{\text{b}}^{\text{k}}(\eta, \eta', p', k')\omega_{(c)}^{\text{k}}(\eta, \eta'', p'', k'')n_b n_c g_{bc}(x', x''), \quad (68) \]

\[ \lambda^i = -\sum_{bc} 2\pi c e \int d^4 p' \int d^4 p'' \int d^3 q' \int d^3 q'' \int_{-\infty}^{\eta} d\eta' \int_{-\infty}^{\eta''} d\eta'' \int d^3 k' e^{-ik'(q-q')}u_{ii}^{\text{m}} \]

\[ h^{\text{b}}(\eta, \eta', p', k')n_b n_c g_{bc}(x'; \eta, \eta', p''), \quad (69) \]

In this expressions unprimed, primed and double-primed quantities refer to particles belonging to species "a", "b" and "c" respectively.

In (64) - (68) we introduced the two-particle correlation function \( g_{ab}(x', x'') \) (see Ref. [1], [2], [11], [12]):

\[ f_{ab}(x, x') = f_a(x) f_b(x') + g_{ab}(x, x'). \]

Here \( f_a(x) \), \( f_{ab}(x, x') \), \( f_{abc}(x, x', x'') \) are the one-particle, two-particle and tree-particle distribution functions respectively:

\[ \langle \int ds \delta(x - x_a(s)) \rangle = f_a(x), \]

\[ \langle \int ds \delta(x - x_a(s)) \int ds' \delta(x' - x_b(s')) \rangle = f_{ab}(x, x'), \]

\[ \langle \int ds \delta(x - x_a(s)) \int ds' \delta(x' - x_b(s')) \int ds'' \delta(x'' - x_c(s'')) \rangle = f_{abc}(x, x', x''), \]

where

\[ \delta(x - x_a(s)) = \delta^4(q^i - q^i_a(s))\delta^4(p^j - p^j_a(s)). \]
We denote the set of all variables \((\eta, \mathbf{q}, p_i)\) by \(x\), the set \((\eta', \mathbf{q}', p_i')\) by \(x'\), while the momenta \(p_{(b)}\) are denoted by \(p'\), and the \(p_{(c)}'\) by \(p''\).

For the moments of random functions we have the formulas (Ref.[11], [12]):

\[
\langle N_a(x) \rangle = n_ah_a(x),
\]

\[
\langle N_a(x) N_b(x') \rangle = (n_an_b - n_a\delta_{ab})f_{ab}(x, x') + \]

\[
+ n_a\delta_{ab}f_a(x) \int ds'd\delta(x' - x_a(s'/x)),
\]

\[
\langle N_a(x) N_b(x') N_c(x'') \rangle = (n_an_bn_c - n_an_b\delta_{ac} - n_an_b\delta_{bc} - n_an_c\delta_{ab} + 2n_a\delta_{ab}\delta_{bc})f_{abc}(x, x', x'') +
\]

\[
+ (n_an_c - n_a\delta_{ac})\delta_{ab}f_{ac}(x, x', x'') \int ds'd\delta(x' - x_a(s'/x)) +
\]

\[
+ (n_an_b - n_a\delta_{ab})\delta_{ac}f_{ab}(x, x', x'') \int ds''\delta(x'' - x_a(s''/x)) +
\]

\[
+ (n_an_b - n_a\delta_{ab})\delta_{bc}f_{ab}(x, x', x'') \int ds''\delta(x' - x_b(s''/x')) +
\]

\[
+ n_a\delta_{bc}\delta_{ab}f_a(x) \int ds'd\delta(x' - x_a(s'/x)) \int ds''\delta(x'' - x_a(s''/x)).
\]

Here \(x_a(s/x)\) stands for the particle path through point \(x\) of the phase space. Bearing in mind that \(\Phi_a = N_a - n_ah_a\) and that \(f_a\) is not a random function, we can easily obtain expressions for the averages

\[
\langle N_a(x)\Phi_b(x') \rangle, \quad \langle N_a(x)\Phi_b(x')\Phi_c(x'') \rangle.
\]

In deriving (64) — (69) we assumed that \(n_a \gg 1\) and that \(x'' = x_b(s''/x')\), i.e. point \(x''\) is not on the path of particles of species ”b” passing through the point \(x'\) of the phase space.

In work [2] two-particles correlation functions \(g_{ab}(x', x'')\) are found for the system gravitationally interacting particles. The two-particles correlation functions the for system of electromagnetically interacting particles were found by the author in Ref. [13], when getting the relativistic kinetic equation for the plasma (Eq. (18) from Ref. [13]).

In our case we should find two-particles correlation function \(g_{ab}(x, x')\) caused by electromagnetic and gravitationally interactions simultaneously.

To obtain correlation function inside the correlation region we assume that \(h_{ik} \ll 1\), \(\omega_{ik} \ll 1\). Therefore \(\Omega^{k}_{ij} \simeq \Omega^{(1)}_{ij}, \, \psi_{ij}^l \simeq \omega_{ik}^l\).

After substituting (53), (52) into (14), multiplying (16) by \(\Phi_b(x')\) and averaging we get

\[
p_i \frac{\partial}{\partial q^i} \langle N_a(x)\Phi_b(x') \rangle + \Gamma_{ijk}p_jp_k^l \frac{\partial}{\partial p_i} \langle N_a(x)\Phi_b(x') \rangle + \frac{e_a}{c} F_{jk}p_k^l \frac{\partial}{\partial p_j} \langle N_a(x)\Phi_b(x') \rangle =
\]
In view of Eqs. (72), (73) the two partial correlation function $g_{ab}(x, x')$ obeys the equation

$$\frac{p^i}{\partial q^i} g_{ab}(x, x') + \Gamma_{ijk} p^j \frac{\partial}{\partial p_j} g_{ab}(x, x') + \frac{e_a}{c} \omega_{jk} p^b \frac{\partial}{\partial p_j} g_{ab}(x, x') =$$

$$= \frac{\partial}{\partial p_i} \left\{ \int d^4 p'' \int d^3 q'' \int d^3 k \int_{-\infty}^{\eta''} d\eta'' \exp[-i k (q - q'')] \times \right.$$}

$$\times \left[ \Omega^{(c)}_{lm} (\eta, \eta'', p''_l, k) p^l p^m \Delta_{ji} - \frac{e_a}{c} \omega^{(c)}_{jk} (\eta, \eta'', p'', k) p^k \right] \times$$

$$\times f_a(x) f_b(x') \int ds'' \delta(x'' - x_b(s''/x')) \right\}. \quad (75)$$

In this equation for $g_{ab}(x, x')$ we should put $\Gamma_{ijk} = 0$, $F_{jk} = 0$, since we assume that within the correlation region the metric $g_{ij}$ are constant and that the influence of macroscopic electromagnetic field is small.

So we have the first order linear equation (75), whose right-hand side contains the sum of two terms. The first term caused by gravitational interactions, the second one caused by electromagnetic interaction.

Consequently, we can write the solution of Eqs. (75) in the form

$$g_{ab}(x, x') = g^{(gr)}_{ab}(x, x') + g^{(el)}_{ab}(x, x'). \quad (76)$$

Here $g^{(gr)}_{ab}(x, x')$ caused by gravitational interaction, and $g^{(el)}_{ab}(x, x')$ caused by electromagnetic interaction.

The equation for $g^{(gr)}_{ab}(x, x')$ coincides with one in Ref. [1]. In [1] we got the $g^{(gr)}_{ab}(x, x')$ in the form (see Eq. (46) in Ref. [1]):

$$g^{(gr)}_{ab}(x, x') = \int d^3 k \int_{-\infty}^{\eta} \frac{d\tau}{p^0} \left[ \frac{\partial}{\partial p_i} (p^j p^m \Delta_{ji} f_a(x)) \right] \tau \int_{-\infty}^{\tau} \frac{d\tau'}{u^{\prime 0}} f_b(x') \times$$

$$\times \Omega^{(b)}_{lm} (\tau, \tau', p', k) \exp[-i k (q - q')] + \frac{i}{c} (kv)(\eta - \tau) + \frac{i}{c} (kv')(\tau' - \eta') +$$

$$+ \int d^3 k \int_{-\infty}^{\eta'} \frac{d\tau'}{p^{\prime 0}} \left[ \frac{\partial}{\partial p_i} (p^{\prime j} p^{\prime m} \Delta_{ji} f_b(x')) \right] \tau' \int_{-\infty}^{\tau'} \frac{d\tau}{u} f_a(x) \times$$
\[ \times \Omega_{lm}^{(a)}(\tau', \tau, p, k) \exp[-i k (q - q') + \frac{i}{c} (kv')(\eta' - \tau') + \frac{i}{c} (kv)(\tau - \eta)], \]  

(77)

where \( v = cu_a/u_a^0 \), \( v' = cu'_b/u'_b^0 \), \( u_a = (u^1, u^2, u^3) \), \( u'_b = (u'^1, u'^2, u'^3) \). Here the subscript \( \tau \) indicates that after calculating the derivatives with respect \( p \) we must replace the arguments \( \eta \), and \( q \) by \( \tau \) and \( q + \frac{\eta}{c}(\tau - \eta) \), respectively. The subscript \( \tau' \) indicates that after calculating the derivatives with respect \( p' \) we must replace the arguments \( \eta' \), and \( q' \) by \( \tau' \) and \( q' + \frac{\eta}{c}(\tau' - \eta') \), respectively.

The equation for \( g^{(el)}_{ab}(x, x') \) coincide with one in Ref.[13]. In Ref. [13] we got the solution for \( g^{(el)}_{ab}(x, x') \) (See (18) from Ref. [13]).

With the preceding notation the result (18) from Ref. [13] takes the form:

\[ g^{(el)}_{ab}(x, x') = -\frac{e_b e_c}{2\pi^2 c} \int \frac{d^3 k}{k} \int_{-\infty}^{\eta'} d\tau \int_{-\infty}^{\tau'} d\tau' \frac{u^k}{u^{00}} f_c(x') \frac{\partial f_b(x')}{\partial p'_{ik}} \int_{-\infty}^{\tau'} d\tau'' \times \]

\[ \times \left\{ \frac{u'_k}{u'^{00}} \frac{\partial}{\partial q'^{rk}} [\exp(-i k (q' - q'') \sin(k(\tau' - \tau'')))] - \right. \]

\[ \left. - \frac{u'^{00}}{u'^{00} q'^{rk}} [\exp(-i k (q' - q'') \sin(k(\tau' - \tau'')))] \right\} \times \]

\[ \times \exp[i \frac{\tau'}{c} (kv')(\eta' - \tau') + \frac{i}{c} (kv'')(\tau'' - \eta'')] - \]

\[ - \frac{e_b e_c}{2\pi^2 c} \int \frac{d^3 k}{k} \int_{-\infty}^{\eta''} d\tau'' \int_{-\infty}^{\tau''} d\tau' \times \]

\[ \times \left\{ \frac{u'_k}{u'^{00}} \frac{\partial}{\partial q'^{rk}} [\exp(-i k (q'' - q') \sin(k(\tau'' - \tau')))] - \right. \]

\[ \left. - \frac{u'^{00}}{u'^{00} q'^{rk}} [\exp(-i k (q'' - q') \sin(k(\tau'' - \tau')))] \right\} \times \]

\[ \times \exp[i \frac{\tau'}{c} (kv')(\eta' - \tau') + \frac{i}{c} (kv'')(\tau'' - \eta'')]. \]  

(78)

Here \( q'^{rk} = (\tau, q'^r) \).

After performing a differentiation with respect to \( q'^r \) in (78) we get a following expression for correlation function:
\[-u'_i u''_k \{ (k^+_k e^{ik(\tau' - \tau''}) - k^-_k e^{-ik(\tau' - \tau'')}) \exp[i(\mathbf{k}v')(\eta'' - \tau'')] + \frac{i}{c}(\mathbf{k}v')(\tau' - \eta') \} \]  

(79)

It is evident, that the electromagnetic interactions in plasma are dominating. Consequently

\[ g_{ab}^{(el)}(x, x') \ll g_{ab}^{(et)}(x, x'). \]

That is why one can put the \( g_{ab}^{(el)}(x, x') \) instead of \( g_{ab}(x, x') \) in (64) - (69).

If we now substitute (73) into (62) - (63) and integrate with respect \( \mathbf{q}', \mathbf{q}'', \mathbf{k}', \mathbf{k}'' \) we get the following expressions:

\[
P_{is}^n = \sum_{bc} \left( -\frac{e_b e_c n_b n_c (2\pi)^6}{4\pi^2 c} \right) \int d^4 p' \int d^4 p'' \int_{-\infty}^{\eta'} d\eta' \int_{-\infty}^{\eta''} d\eta'' \int \frac{d^3 k}{k} \times \]

\[
\times \left\{ \int_{-\infty}^{\eta'} \frac{d\tau'}{u_{00}} \int_{-\infty}^{\tau'} \frac{d\tau''}{u_{00}} f_\mathbf{c}(x'') \frac{\partial f_\mathbf{b}(x')}{\partial p_j'} [(u' u'') \delta_k^k - u'' u''] \times \right. \]

\[
\times \left( -k^-_k e^{-ik(\tau' - \tau''}) + k^+_k e^{ik(\tau'' - \tau')} \right) \exp[i(\mathbf{k}v'')(\eta'' - \tau'')] + \frac{i}{c}(\mathbf{k}v')(\tau' - \eta')] \right\} \times \]

\[
\times h^n_{t}(\eta, \eta', p', -\mathbf{k}) \Omega^{(c)}_{is}(\eta, \eta'', p'', \mathbf{k}), \quad (80) \]

\[
Q_{kis}^n = \sum_{bc} \left( -\frac{e_b e_c n_b n_c (2\pi)^6}{4\pi^2 c} \right) \int d^4 p' \int d^4 p'' \int_{-\infty}^{\eta'} d\eta' \int_{-\infty}^{\eta''} d\eta'' \int \frac{d^3 k}{k} \times \]

\[
\times \left\{ \int_{-\infty}^{\eta'} \frac{d\tau'}{u_{00}} \int_{-\infty}^{\tau'} \frac{d\tau''}{u_{00}} f_\mathbf{c}(x'') \frac{\partial f_\mathbf{b}(x')} {\partial p_j'} [(u' u'') \delta_k^k - u'' u''] \times \right. \]

\[
\times \left( -k^-_k e^{-ik(\tau' - \tau'')}) + k^+_k e^{ik(\tau'' - \tau'')} \right) \exp[i(\mathbf{k}v'')(\eta'' - \tau'')] + \frac{i}{c}(\mathbf{k}v')(\tau' - \eta')] \right\} \times \]

\[
\times \left. \Omega^n_{k}(\eta, \eta', p', -\mathbf{k}) \Omega^{(c)}_{is}(\eta, \eta'', p'', \mathbf{k}), \quad (81) \right. \]

\[
\lambda_{ij} = \sum_{bc} \chi(2\pi) e_b e_c n_b n_c c \int d^4 p' \int d^4 p'' \left[ -\frac{1}{2} u'' u'' u''' u''' - \frac{1}{4} g_{ij} u'' u''' u''' \right. \right]
\]

\[
- \frac{1}{2} u'' u'' g_{km} + \frac{1}{4} g_{ij} g_{km} + u'' u'' \delta_j^k + u'' u'' \delta_i^m \right] \times \]

15
\begin{align}
&\times \left( \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{d^3k}{k} \int_{-\infty}^{\infty} d\eta h^{(b)}_{km}(\eta, \eta', p', -k) \left\{ \int_{-\infty}^{\eta'} \frac{d\tau'}{u^{\tau'}} \int_{-\infty}^{\tau'} \frac{d\tau''}{u^{\tau''}} \int_{-\infty}^{\eta''} d\eta f_c(x'') \frac{\partial f_b(x')}{\partial p'_l} \times \\
&\quad \times ((u'u'')\delta_r - u''_ru'^l) \left( k^+_l e^{ik(\tau''-\tau')} - k^-_l e^{ik(\tau''-\tau')} \right) \times \\
&\quad \times \exp\left[ \frac{i}{c} (kv'')(\eta - \tau'') + \frac{i}{c} (kv')(\tau' - \eta') \right] + \\
&\quad + \int_{-\infty}^{\eta''} \frac{d\tau''}{u^{\tau''}} \int_{-\infty}^{\tau''} \frac{d\tau'}{u^{\tau'}} \int_{-\infty}^{\eta'} \frac{d\eta}{u^{\eta'}} f_c(x') \frac{\partial f_c(x'')}{\partial p''_l} \times ((u'u'')\delta_r - u''_ru'^l) \times \\
&\quad \times \left( k^+_l e^{ik(\tau''-\tau')} - k^-_l e^{ik(\tau''-\tau')} \right) \exp\left[ \frac{i}{c} (kv'')(\eta - \tau'') + \frac{i}{c} (kv')(\tau' - \eta') \right] \right\} \right) \\
&\quad \times (h_{km}^{\omega, km}) = \sum_{bc} \left( -\frac{e_b e_c n_b n_c (2\pi)^6}{4\pi^2 c} \right) \left( -\frac{i\chi m_b c^2}{(2\pi)^3 k} \right) \left( \frac{e_c}{4\pi^2 k} \right) \int d^4p' \int d^4p'' \times \\
&\quad \times \left( \int_{-\infty}^{\eta''} d\eta' \int_{-\infty}^{\eta'} d\eta'' \int_{-\infty}^{\eta''} d\eta' \int_{-\infty}^{\eta''} \frac{d^3k}{k} \left\{ \int_{-\infty}^{\eta'} \frac{d\tau'}{u^{\tau'}} \int_{-\infty}^{\tau'} \frac{d\tau''}{u^{\tau''}} \int_{-\infty}^{\eta''} d\eta f_c(x'') \frac{\partial f_b(x')}{\partial p'_l} \times \\
&\quad \times (u^n u'_m - \frac{1}{2}\delta^i_{mn}) \left( e^{ik(\eta'-\eta)} - e^{-ik(\eta'-\eta)} \right) \left[ u^{nm} \left( k^+_m e^{ik(\eta'-\eta)} - k^-_m e^{-ik(\eta'-\eta)} \right) \right. \\
&\quad \left. - u^{nk} \left( k^+_n e^{ik(\eta'-\eta)} - k^-_n e^{-ik(\eta'-\eta)} \right) \right] \right) \\
&\quad \times (u^n u'_m - \frac{1}{2}\delta^i_{mn}) \left( e^{ik(\eta'-\eta)} - e^{-ik(\eta'-\eta)} \right) \left[ u^{nm} \left( k^+_m e^{ik(\eta'-\eta)} - k^-_m e^{-ik(\eta'-\eta)} \right) \right. \\
&\quad \left. - u^{nk} \left( k^+_n e^{ik(\eta'-\eta)} - k^-_n e^{-ik(\eta'-\eta)} \right) \right],
\end{align}

(82)
\[
\lambda^i = \sum_{bc} i\chi e_b e_c^2 m_b m_c n_b n_c \frac{\int d^4 p' \int d^4 p'' \int_0^n d\eta' \int_0^n d\eta'' \int_0^n \frac{d^3 k}{k^2}}{2\pi} \times \\
\times u'^i \left( e^{ik(\eta'-\eta)} - e^{-ik(\eta'-\eta)} \right) \left\{ \int_{-\infty}^\eta \frac{d\tau'}{u^{00}} \int_{-\infty}^{\tau''} \frac{d\tau''}{u^{00}} f_c(x'') \frac{\partial f_b(x')}{\partial p'_{\eta}} \left( (u' u'') \delta^s_i - k_\alpha e^{-ik(\tau''-\tau')} \right) + \int_{-\infty}^\eta \frac{d\tau''}{u^{00}} \int_{-\infty}^{\tau''} \frac{d\tau'}{u^{00}} f_b(x') \frac{\partial f_c(x'')}{\partial p'_{\eta}} \left( (u' u'') \delta^s_i - u'_i u''^s \right) \left( k_\alpha e^{-ik(\tau''-\tau')} - k_\alpha e^{ik(\tau''-\tau')} \right) \right\}.
\]

To simplify (80) — (83) still further, we proceed as follows. We assume that the distribution function changes little inside the correlation region, so that in calculating the integrals in (80) — (83) we can ignore, in first approximation, the temporal coordinate dependence on \( f \). We substitute the explicit expressions for \( h_{ij}^{(b)} \) and \( \Omega_{ij}^{(b)} \) (Eqs. (74) and (53)) into (80) - (82) and evaluate the integrals with respect to \( \tau', \tau'', \eta', \eta'' \) and \( k \). Then the expression for \( P_{is}^n \) becomes

\[
P_{is}^n = \sum_{bc} \frac{\chi^2 e_b e_c m_b m_c n_b n_c c^3}{8(\pi)^2} \int d^4 p' \int d^4 p'' (u''^m u^i - \frac{1}{2} \delta^m_i) \left\{ \left( u'_i u''^j - \frac{1}{2} \delta^m_i \right) f_c(x') \frac{\partial f_b(x'')}{\partial p'_{\eta}} K_{fm}^{(1)}(u', u'') + \delta^m_j f_b(x') \frac{\partial f_c(x'')}{\partial p'_{\eta}} K_{fm}^{(2)}(u', u'') \right\}.
\]

Here we have introduced the notation \( K_{fm}^{(1)}(u', u'') \) and \( K_{fm}^{(2)}(u', u'') \) for tensors that in locally Lorentzian reference frame, in which \( g_{ij} = \eta_{ij} \) is the Minkowski tensor, have the following form:

\[
K_{fm}^{(1)}(u', u'') = \frac{i}{u'^0 u''^0} \int d^3 k \int_{-\infty}^\eta d\eta' \int_{-\infty}^\eta d\eta'' \int_{-\infty}^{\tau''} d\tau'' \int_{-\infty}^{\tau'} d\tau' (e^{ik(\eta'-\eta)} - e^{-ik(\eta'-\eta)}) \left( k_\alpha e^{ik(\eta'-\eta)} - k_\alpha e^{-ik(\eta'-\eta)} \right) \left( k_\alpha e^{ik(\tau''-\tau')} - k_\alpha e^{-ik(\tau''-\tau')} \right) \times \\
\times \exp \left\{ \frac{i}{c} (k \cdot v')(\eta'' - \tau'') + \frac{i}{c} (k \cdot v') (\tau' - \eta') \right\},
\]

\[
K_{fm}^{(2)}(u', u'') = \frac{i}{u'^0 u''^0} \int d^3 k \int_{-\infty}^\eta d\eta' \int_{-\infty}^\eta d\eta'' \int_{-\infty}^{\tau''} d\tau'' \int_{-\infty}^{\tau'} d\tau' (e^{ik(\eta'-\eta)} - e^{-ik(\eta'-\eta)}) \left( k_\alpha e^{ik(\eta'-\eta)} - k_\alpha e^{-ik(\eta'-\eta)} \right) \left( k_\alpha e^{ik(\tau''-\tau')} - k_\alpha e^{-ik(\tau''-\tau')} \right) \times \\
\times \exp \left\{ \frac{i}{c} (k \cdot v')(\eta'' - \tau'') + \frac{i}{c} (k \cdot v') (\tau' - \eta') \right\}.
\]
\[ \times \exp[\frac{i}{c}(kv''(\eta'' - \tau'') + \frac{i}{c}(kv'(\tau' - \eta'))], \]

After carrying out the integrals with respect to \( \tau', \tau'', \eta', \) and \( \eta'' \) these expressions take the form:

\[
K_{fm}^{(1)}(u', u'') = \frac{2\pi c^5}{u^0 u^{0'}} \int \frac{d^3k}{k^2} \delta(kv'' - kv') \left\{ \frac{k_f^+ k_m^+}{(kc - kv'')(kc + kv'')} + \frac{k_f^- k_m^+}{(kc - kv'')(kc + kv'')} \right\} = K_{fm}(u', u''),
\]

\[
K_{fm}^{(2)}(u', u'') = -K_{fm}^{(1)}(u', u'') = -K_{fm}(u', u'').
\]

The above equalities hold only in a locally Lorentzian reference frame. To obtain covariant expressions for the tensors \( K_{fm}^{(1)}(u', u'') \) and \( K_{fm}^{(2)}(u', u'') \), we take into account the following fact. The quantities \( K_{fm}^{(1)}(u', u'') \) and \( K_{fm}^{(2)}(u', u'') \) appeared in (86) after the correlation function \( g_{ab}(x', x'') \) was substituted to (84) and result was integrated with respect to \( q', q'', k' \) and \( k'' \). But the expression (79) for two-particle correlation function is a sum of two terms, which differ in that primed quantities referring to particles of species \( "a" \) are replaced by double-primed quantities referring to particles of species \( "b" \), and vice versa. It is after these terms were integrated with respect to \( q', q'', k' \) and \( k'' \) that \( K_{fm}^{(1)}(u', u'') \) and \( K_{fm}^{(2)}(u', u'') \) appeared in (86). Obviously, the both must be calculated in the same reference frame, for which it is convenient to take the center-of-mass reference frame, in which

\[ v' = v, \quad v'' = -v, \quad u^{0'} = u'^{0'} = 1/\sqrt{1 - v^2/c^2} = u^0 \]

In this reference frame

\[ K_{00} = K_{0\alpha} = 0, \quad K_{\alpha\beta} = \frac{2\pi^2 c}{v u_0^2 k_{min}} \left( \delta_{\alpha\beta} - \frac{v_{\alpha} v_{\beta}}{v^2} \right) \]

(89)

Here \( v = \sqrt{v_1^2 + v_2^2 + v_3^2} \), where \( v_{\alpha} = v^\alpha = u^\alpha/u^0 \) are spatial components of the vector \( v \).

A covariant generalization of (89) has the form

\[
K_{ij}(u', u'') = \frac{4\pi^2}{k_{min}^2 [(u'u'')^2 - 1]^{3/2}} \left\{ -[(u'u'')^2 - 1]g_{ij} - u_i'u'_j - u_i' u_j'' + (u'u'')(u_i'u_j' + u_j'u_i') \right\}
\]

(90)

The expressions for \( K_{fm}^{(1)}(u', u'') \) and \( K_{fm}^{(2)}(u', u'') \) diverge as \( k \to 0 \), i.e., for large impact parameters. The reason is that we integrate over an infinite region, while actually we should integrate only over the correlation region, where the metric is assumed to vary only weakly.
This difficulty is resolved, as well as in the case of kinetic equation deriving, by introducing a cutoff procedure in the divergent integral

\[ \int_0^\infty \frac{dk}{k^3} \]

We set the lower integration limit to \( k_{\text{min}} = 1/r_{\text{max}} \), rather than zero, where \( r_{\text{max}} \) is the size of the correlation region (the correlation radius). Then the above integral assumes the value \( 1/2k_{\text{min}}^2 = (1/2)r_{\text{max}}^2 \).

As the experience of deriving the relativistic kinetic equation (refer to. [13], [14], [16], [20]) shows, more thorough investigations suggest that the integrals become convergent as \( r \to \infty \), with the contribution from the region where \( r > r_{\text{max}} \) being infinitesimal. In Ref. [14],[16] there are estimates for \( r_{\text{max}} \) in the case of the metric \( g_{ij} \) is the metric of isotropic cosmological model and in the case of gravitational interaction of particles.

In the case of electromagnetical interaction of particles the parameter \( k_{\text{min}} \) is equal to \( \frac{1}{r_D} \), where \( r_D \) a radius of Debit, since electromagnetic interactions in the plasma are shielded under \( r > r_D \).

The tensor (90) possesses the following properties:

\[ K_{ij}(u', u'') = K_{ij}(u'', u'); K_{ij}u'^i = K_{ij}u''^i = 0; K_{ij} = K_{ji}. \] (91)

Because of this the expression for \( P^n_{is} \) simplifies considerably. The macroscopic Einstein equations incorporate not \( P^n_{is} \), but the tensor \( \varphi^k_{ij} = -(1/2)(\delta^k_i \delta^j_s - \delta^k_j \delta^s_i)P^n_{is} \). The expression for this tensor can be written as follows:

\[
\varphi^k_{ij} = -(1/2)(\delta^k_i \delta^j_s - \delta^k_j \delta^s_i)P^n_{is} = \sum_{bc} \frac{\chi^2 e_b e_c m_b m_c n_b n_c c^3}{16\pi^2} \int \frac{d^4p'}{\sqrt{(-g)}} \int \frac{d^4p''}{\sqrt{(-g)}} \left[ \frac{1}{2} g^{ik} u''^i u''^k + u'^k (u'^i u''^i + \delta^i_k u''^i) \right] (u'^j u''^j) K_{fr}(u', u'') \left( f_c(x') \frac{\partial f_b(x')}{\partial p_r'} - f_b(x') \frac{\partial f_c(x'')}{\partial p_r''} \right) \]

(92)

Note that

\[ g^{ij} \varphi^k_{ij} = 0, \quad \varphi^i_{ij} = 0, \quad \varphi^k_{ij} = \varphi^k_{ji}. \] (93)

Reasoning along similar lines, we can simplify the expression for the tensor \( \mu_{ij}, \varphi_{ij}, \mu_i \), which assumes the following form:

\[
\mu_{ij} = \sum_{bc} \frac{\chi^2 e_b e_c m_b m_c n_b n_c c^3}{16\pi^3} \int \frac{d^4p'}{\sqrt{(-g)}} \int \frac{d^4p''}{\sqrt{(-g)}} \left[ (z^2 + \frac{1}{2})(u''^i u''^j) + u'^i u'_j g^{rr} + (z^2 - \frac{1}{2})g_{ij} g^{rr} - 2z(u''^i u''^j + u''^i u'_j) g^{rr} - (z^2 - \frac{1}{2}) (\delta^i_k \delta^j_s + \delta^j_k \delta^s_i) \right] \times
\]

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Lorentzian reference frame have the form

\[ J_{\alpha\beta} = \sum_{\delta, \gamma} \frac{\chi_{\delta c} e^2 m c n c}{2\pi} \int \frac{d^4 p}{\sqrt{(-g)}} \int \frac{d^4 p''}{\sqrt{(-g)}} (u'u'') K_{\delta i}(u', u'') \times \]

\[ \left( (u'u'')(u''g'\delta - u''k'\delta) - (u''g'\delta - u''k'\delta) \right) \left( f_c(x'')(\partial f_b(x') / \partial p'_i) - f_b(x') \partial f_c(x'') / \partial p'_i \right) \times \]

\[ ((u'u'')\delta^i - u'_i u''_i) \times \]...

Here \( z = (u'u'') \).

In (94), (96) we have introduced the notation \( J_{\alpha\beta}(u', u'') \) for tensor that in locally Lorentzian reference frame have the form

\[ J_{\alpha\beta}(u', u'') = \frac{1}{u'^0 u''^0} \int \frac{d^3 k}{k^3} \int_{-\infty}^{\infty} dy' \int_{-\infty}^{\infty} dy'' \int_{-\infty}^{\infty} d\tau' \int_{-\infty}^{\infty} d\tau'' \left( k_i^+ e^{-ik(\eta' - \eta)} - k_i^- e^{-ik(\eta'' - \eta)} - k_m^+ e^{-ik(\eta' - \eta)} - k_m^- e^{-ik(\eta'' - \eta)} \right) \]

\[ -k_i^+ e^{-ik(\eta'' - \eta)}(k_m^+ e^{ik(\eta'' - \eta)} - k_m^- e^{ik(\eta'' - \eta)}) \]

\[ -k_n^+ e^{-ik(\tau'' - \tau)} e^{ik(\nu'' - \nu)} \exp \left( \frac{i}{\hbar} (k\nu'')(\eta'' - \eta'') + \frac{i}{\hbar} (k\nu')(\tau' - \eta') \right) \]

After evaluating the integrals with respect \( \eta', \eta'', \tau' \) and \( \tau'' \), we get

\[ J_{\alpha\beta}(u', u'') = \frac{c^4}{u'^0 u''^0} \int \frac{d^3 k}{k^3} \frac{V_p.}{(k\nu'' - k\nu')} \left\{ \frac{k_i^+ k_m^+ k_n^+}{(k\nu'' - k\nu')^3} + \right. \]

\[ \frac{k_i^+ k_m^+ k_n^- + k_i^- k_m^- k_n^+ + k_i^- k_m^+ k_n^-}{(k\nu'' - k\nu')^2(k\nu'')} + \frac{k_i^+ k_m^- k_n^- + k_i^- k_m^- k_n^+ + k_i^- k_m^- k_n^-}{(k\nu'' - k\nu')^2(k\nu'')} + \frac{k_i^- k_m^- k_n^-}{(k\nu'' - k\nu')^3} \]...

The symbol \( V_p. \) indicates that the integral is calculated as a principal value.

Just as in the previous case, we specify (97) in the center-of-mass reference frame, where

\[ v' = v, \quad v'' = -v, \quad u'^0 = u''^0 = 1/\sqrt{1 - v^2/c^2} = u^0 \]

In this reference frame the components of \( J_{\alpha\beta}(u', u'') \) have the following form (the spatial indexes of three-dimensional velocity \( v^\alpha \) are lowered by using the tree-dimensional Kronecker symbol \( \delta_{\alpha\beta} \))

\[ J_{000} = -\alpha(v) \frac{v^2}{c^2}, \quad J_{00\alpha} = -\alpha(v) \frac{v_\alpha}{c}, \quad J_{0\alpha\beta} = -\alpha(v) \delta_{\alpha\beta} + \beta(v) \left( \delta_{\alpha\beta} - \frac{v_\alpha v_\beta}{v^2} \right) \]
\[ J_{\alpha\beta\gamma} = -\frac{c^2}{v^2} \alpha(v) \left[ \delta_{\alpha\beta} \frac{v_{\gamma}^2}{c} + \delta_{\alpha\gamma} \frac{v_{\beta}^2}{c} + \delta_{\beta\gamma} \frac{v_{\alpha}^2}{c} - 2 \frac{v_{\alpha} v_{\beta} v_{\gamma}}{c v^2} \right] + \]
\[ + \frac{c^2}{v^2} \beta(v) \left[ \left( \delta_{\alpha\beta} - \frac{v_{\alpha} v_{\beta}}{v^2} \right) \frac{v_{\gamma}^2}{c} + \left( \delta_{\alpha\gamma} - \frac{v_{\alpha} v_{\gamma}}{v^2} \right) \frac{v_{\beta}^2}{c} + \left( \delta_{\beta\gamma} - \frac{v_{\beta} v_{\gamma}}{v^2} \right) \frac{v_{\alpha}^2}{c} \right]. \] \hspace{1cm} (99)

The function \( \alpha \) and \( \beta \) in (98) and (99) depend on the velocity \( v = \sqrt{v_1^2 + v_2^2 + v_3^2} \) only and have the explicit form
\[ \alpha = \frac{\pi c^3}{u_0 v^3 k_{\text{min}}} \left[ \frac{2}{c} \left( 1 + \frac{v}{c}^2 \right)^2 + \ln \left( \frac{1 - \frac{v}{c}}{1 + \frac{v}{c}} \right) \right], \] \hspace{1cm} (100)
\[ \beta = \frac{\pi c^3}{2 u_0^3 v^3 k_{\text{min}}} \left[ \frac{2}{c} \left( 3 - 2 \frac{v^2}{c^2} + 3 \frac{v^4}{c^4} \right) \frac{1}{(1 - \frac{v^2}{c^2})^2} + 3 \left( 1 + \frac{v^2}{c^2} \right)^2 \ln \left( \frac{1 - \frac{v}{c}}{1 + \frac{v}{c}} \right) \right]. \] \hspace{1cm} (101)

Here we introduced the following notation for the integral
\[ \frac{1}{k_{\text{min}}} = \int_{k_{\text{min}}}^{\infty} \frac{dk}{k^2}. \]

We set the lower integration limit to \( k_{\text{min}} = 1/r_D \).

A covariant generalization of this results, which were obtained in the locally Lorentzian center-of-mass reference frame, to arbitrary reference frames has the form
\[ J_{ijk}(u', u'') = A \left[ (g_{ij} u_k' + g_{ik} u_j' + g_{jk} u_i') - z (g_{ij} u_k'' + g_{ik} u_j'' + g_{jk} u_i'') \right] - \]
\[ - (u_i' u_j'' u_k' + u_i'' u_j' u_k' + u_i'' u_j' u_k') + 3 z u_i'' u_j'' u_k' \] + \[ + C \left[ u_i' u_j'' u_k' - z (u_i' u_j'' u_k' + u_i'' u_j' u_k') + u_i'' u_j' u_k' \right] \] + \[ + z^2 (u_i' u_j'' u_k' + u_i'' u_j' u_k') - z^3 u_i'' u_j'' u_k' \], \hspace{1cm} (102)

where \( z = (u' u'') = (u'' u') \),
\[ A = -\frac{2 \pi \sqrt{2}}{k_{\text{min}}} \left[ \frac{(z - 2)}{(z - 1)^2 (z + 1)^{1/2}} + \frac{(2z - 1)}{(z + 1)(z - 1)^{5/2}} \ln \left( z + \sqrt{z^2 - 1} \right) \right], \] \hspace{1cm} (103)
\[ C = -\frac{2 \pi \sqrt{2}}{k_{\text{min}}} \left[ \frac{(z - 6)}{(z - 1)^3 (z + 1)^{3/2}} + \frac{(6z - 1)}{(z + 1)^2 (z - 1)^{7/2}} \ln \left( z + \sqrt{z^2 - 1} \right) \right]. \] \hspace{1cm} (104)

The tensor \( J_{ijk}(u', u'') \) satisfies the identity
\[ J_{ijk}(u', u'') u^{ik} = 0. \] \hspace{1cm} (105)

Note that the tensor \( \mu_{ij} \) is traceless:
\[ g^{ij} \mu_{ij} = 0. \] \hspace{1cm} (106)
Let us now simplify the tensor $T_{ij}^{(r)}$ (see. (35)). Substitution of (52) to (35) yields the expression for $T_{ij}^{(r)}$. In view (70) - (73) we have (if $n_a \gg 1$):

$$T_{ij}^{(r)} = \frac{1}{4\pi} \left( -g_{ij}g_{js} + \frac{1}{4} g_{ij}g_{ts} \right) \sum_{bc} \int d^4p' \int d^4p'' \int d^3q' \int d^3q'' \times$$

$$\times \int_{-\infty}^{\eta} d\eta' \int_{-\infty}^{\eta} d\eta'' \int d^3k' \int d^3k'' e^{-ik'(q-q')} e^{-ik''(q-q'')} \times$$

$$\times \omega^{l(b)}(\eta, \eta', p', k') \omega^{c(s)}(\eta, \eta'', p'', k'') n_b n_c g_{bc}(x', x''),$$

(107)

Here we can not neglect by $g_{bc}^{(gr)}(x', x'')$ in expression (76) for $g_{bc}(x', x'')$.

That is why

$$T_{ij}^{(r)} = \tau_{ij}^{(r)} + \tau_{ij}^{(gr)}.$$  

(108)

One can get $\tau_{ij}^{(r)}$ if replace $g_{bc}(x', x'')$ by $g_{bc}^{(el)}(x', x'')$ (see (79)) in expression (107) and replace $g_{bc}(x', x'')$ by $g_{bc}^{(gr)}(x', x'')$ (see (74)) to obtain the expression for $\tau_{ij}^{(gr)}$.

Substitution (77) and (79) to (107) yields the following expressions for $\tau_{ij}^{(r)}$ and $\tau_{ij}^{(gr)}$:

$$\tau_{ij}^{(r)} = \sum_{bc} \frac{e^2}{2(2\pi)^4} \frac{m_b n_c}{16\pi^2} \int \frac{d^4p'}{\sqrt{(-g)}} \int \frac{d^4p''}{\sqrt{(-g)}} \left[ -2z\delta_i^j \delta_j^f + zg_{ij}g^{rf} + 

+ (\delta_i^j u'' + \delta_j^i u'') u'' - g^{rf}(u'^i u''_j + u''_i u') \right] \times$$

$$\times (z\delta_i^j - u''_i u'^j) J^{(el)}_{fs}(u', u'') f_c(x'') \frac{\partial f_b(x')}{\partial p'_n}.$$  

(109)

$$\tau_{ij}^{(gr)} = \sum_{bc} \frac{e^2}{16\pi^2} \frac{m_b m_c n_b n_c e^3}{16\pi^2} \int \frac{d^4p'}{\sqrt{(-g)}} \int \frac{d^4p''}{\sqrt{(-g)}} \left[ 2z\delta_i^j \delta_j^q - zg_{ij}g^{pq} - 

- (\delta_i^j u'' + \delta_j^i u'') u'^q + g^{pq}(u'^i u''_j + u''_i u') J^{(gr)}_{pqf}(u', u'') f_c(x'') \times$$

$$\times \frac{\partial}{\partial p'_n} \left\{ f_b(x') \left[ \left( z^2 - \frac{1}{2} \right) \delta_i^j + \left( z^2 + \frac{1}{2} \right) u'_i u'^j - 2zu''_i u'^j \right] \right\}. $$  

(110)

Note that the tensors $\tau_{ij}^{(gr)}$ and $\tau_{ij}^{(r)}$ are traceless:

$$g^{ij} \tau_{ij}^{(gr)} = 0, \quad g^{ij} \tau_{ij}^{(r)} = 0.$$  

(111)

In (109) and (110) the tensors $J^{(el)}_{pq}(u', u'')$ and $J^{(gr)}_{pq}(u', u'')$ have the form (102), where $A$ and $B$ have the forms (103) and (104) respectively. But in the expression for $J^{(el)}_{pq}(u', u'')$ we must put $k_{min} = 1/r_D$, where $r_D$ is the radius of Debit, since the electromagnetic interaction in plasma are shielded under $r > r_D$. In the expression for $J^{(gr)}_{pq}(u', u'')$ we must put $k_{min} = 1/r_g$, where $r_g$ is the radius of correlation for gravitational interaction. As the experience of deriving the relativistic kinetic equation (refer to. [13], [14], [16], [20]) shows, more thorough investigations suggest than the integrals become convergent as $r \to \infty$, with
the contribution from the region where $r > r_g$ being infinitesimal. In Ref. [14],[16] there are estimates for $r_g$ in the case where the average metric $g_{ij}$ is the metric of isotropic cosmological model and in the case of gravitational interaction of particles.

4. Macroscopic system of Einstein and Maxwell equations for relativistic plasma

As a result were obtained the macroscopic Einstein and Maxwell equations in relativistic plasma. They have the forms:

$$G_{ij} + \nabla_k \varphi_{ij}^k + \mu_{ij} - \chi \tau_{ij}^{(gr)} = \chi T_{ij}, \quad (112)$$

$$\nabla_k F^{ik} + \nabla_k \varphi_{ik} + \mu^i = -\frac{4\pi}{c} J^i. \quad (113)$$

Here $G_{ij}$ is the Einstein’s tensor of the Riemannian space with macroscopic metric $g_{ij}$, $F^{ik}$ is the macroscopic tensor of electromagnetic field (Maxwell’s tensor), $T_{ij}$ is macroscopic energy-momentum tensor. The last is the sum of macroscopic energy-momentum tensor of medium $T_{ij}^{(m)}$, energy momentum tensor of macroscopic electromagnetical field $T_{ij}^{(el)}$ and macroscopic energy-momentum tensor $\tau_{ij}^{(r)}$ of electromagnetical radiation in plasma. (In cosmological plasma in the last case one should say about the energy - momentum tensor of relict radiation.)

The Einstein equations of the gravitational field for continuum media, obtained here, differ from the classical Einstein equations by the presence of additional terms $\nabla_k \varphi_{ij}^k$, $\mu_{ij}$ and $-\chi \tau_{ij}^{(gr)}$ in the left-hand side. It caused by particle interaction. The forms of this tensors are (92), (94) and (110). The third term, $(-\chi \tau_{ij}^{(gr)})$ is the addition to macroscopic energy - momentum tensor of electromagnetic radiation, caused by gravitational interaction which multiplying on $\chi$ and moving from the right-hand side of macroscopic equations to the left-hand side.

The macroscopic Maxwell equations differ from the classical Maxwell equations by the presence of additional terms $\nabla_k \varphi_{ij}^k + \mu^i$. The additional terms in Maxwell equations caused by particle interaction and by effects of general relativity.

The tensors $\nabla_k \varphi_{ij}^k$, $\mu_{ij}$, $\tau_{ij}^{(gr)}$, $\nabla_k \varphi_{ki}$ and $\mu^i$ are expressed in (92) — (96) and (110) in terms of one-particle distribution function $f_b$ specified in the eight-dimensional phase space in which all four components of momentum are independent. The transition to the seven-dimensional distribution function $F_a(q^i, p_\alpha)$ is made according to the formula

$$n_a f_a(q^i, p_j) = F_a(q^i, p_\alpha) \delta(\sqrt{g^{lm} p_lp_m - m_a c}). \quad (114)$$

Here the function $F_a$ depends on the spatial components of momentum only. Greek indexes are used to denote spartial components.
By integrating (92) — (96) and (110) with respect to \( p'_0 \) and \( p''_0 \) we can write down the tensors \( \nabla_k \varphi_{ij}^k \), \( \mu_{ij} \), \( \tau_{ij}^{(gr)} \), \( \nabla_k \varphi^{ki} \) and \( \mu^i \) as

\[
\varphi_{ij}^k = \sum_{bc} \frac{\chi^2 \epsilon_b \epsilon_c m_b^2 m_c^2 c^5}{16(\pi)^2} \int \frac{d^3p'}{p'^0 \sqrt{(-g)}} \int \frac{d^3p''}{p''^0 \sqrt{(-g)}} \left[ \frac{1}{2} g^{fk} u''_{i} u''_{j} + u'^{k}(u'^{n}u'^{m}) \delta_{ij}^k \right] K_{f\alpha}(u', u'') \frac{\partial F_c(x'')}{\partial p'^{\alpha}} - F_b(x') \frac{\partial F_c(x'')}{\partial p''^\alpha} \right)
\]

\[
(115)
\]

\[
\mu_{ij} = \sum_{bc} \frac{\chi^2 \epsilon_b \epsilon_c m_b^2 m_c^2 c^5}{16\pi^2} \int \frac{d^3p'}{p'^0 \sqrt{(-g)}} \int \frac{d^3p''}{p''^0 \sqrt{(-g)}} \left[ (u'^{n}u'^{m}) \right] K_{f\alpha}(u', u'') \times (z\delta^{m}_{\alpha} - u''_{\alpha} u'^{m}) J_{rqm}(u', u'') F_c(x'') \frac{\partial F_b(x')}{\partial p'^{\alpha}} - F_b(x') \frac{\partial F_c(x'')}{\partial p''^\alpha}
\]

\[
(116)
\]

\[
\varphi^{ik} = \sum_{bc} \frac{\chi^2 \epsilon_b \epsilon_c m_b m_c c^3}{2\pi} \int \frac{d^3p'}{p'^0 \sqrt{(-g)}} \int \frac{d^3p''}{p''^0 \sqrt{(-g)}} (u'^{n}u'^{m}) K_{f\alpha}(u', u'') \times \left[ (u'^{n} u'^{m}) (u'^{k} g^{kj} - u'^{m} g^{kj}) - (u'^{n} g^{kj} - u'^{m} g^{kj}) \right] \left( F_c(x'') \frac{\partial F_b(x')}{\partial p'^{\alpha}} - F_b(x') \frac{\partial F_c(x'')}{\partial p''^\alpha} \right)
\]

\[
(117)
\]

\[
\mu^i = \sum_{bc} \frac{\chi^2 \epsilon_b \epsilon_c m_b m_c c^3}{4\pi} \int \frac{d^3p'}{p'^0 \sqrt{(-g)}} \int \frac{d^3p''}{p''^0 \sqrt{(-g)}} \left[ (u'^{n} u'^{m}) (u'^{k} J_{i\alpha\beta}(u', u'') F_c(x'') \frac{\partial F_b(x')}{\partial p'^{\alpha}} - F_b(x') \frac{\partial F_c(x'')}{\partial p''^\alpha} \right)
\]

\[
(118)
\]

\[
\tau^{(gr)}_{ij} = \sum_{bc} \frac{\chi^2 \epsilon_b \epsilon_c m_b m_c c^5}{16\pi^2} \int \frac{d^3p'}{p'^0 \sqrt{(-g)}} \int \frac{d^3p''}{p''^0 \sqrt{(-g)}} \left[ 2z\delta^{m}_{\alpha} \delta^{n}_{\alpha} \right] - z g_{ij} g^{pq} - (\delta^{m}_{\alpha} u'^{n} u'^{p} + \delta^{p}_{\alpha} u'^{n} u'^{m}) J_{pqf}(u', u'') F_c(x'') \times \delta^{f}_{\alpha} \left\{ F_b(x')[\left( z^2 - \frac{1}{2} \right) \delta^{f}_{\alpha} + \left( z^2 + \frac{1}{2} \right) u'^{n} u'^{f} - 2z u''_{n} u'^{f} \right] \right\}
\]

\[
(119)
\]

Here

\[
\frac{d^3p'}{p'^0 \sqrt{(-g)}} \quad \text{and} \quad \frac{d^3p''}{p''^0 \sqrt{(-g)}}
\]

are the invariant volume elements in tree-dimensional momentum space of particles species "b" and "c" respectively.

The greek index \( \alpha \) in (117) - (118) takes the values 1, 2 and 3 only (the spartial index). The derivative with respect to \( p'_n \) in (113) should by calculated as all four components
of momentum are independent. The dependence of \( p'_0 \) on \( p'_\alpha \) is taken into account after differentiation with respect \( p'_\alpha \) is completed only.

In (118) and (119) the tensors \( J^{(el)}_{rpq}(u', u'') \) and \( J^{(gr)}_{rpq}(u', u'') \) have the form (102), where \( A \) and \( B \) have the forms (103) and (104) respectively. But in expression for \( J^{(el)}_{rpq}(u', u'') \) we must put \( k_{min} = 1/r_D \), where \( r_D \) is the radius of Debit, since the electromagnetic interaction in plasma are shielded under \( r > r_D \). In the expression for \( J^{(gr)}_{rpq}(u', u'') \) we must put \( k_{min} = 1/r_g \), where \( r_g \) is the radius of correlation for gravitational interaction. As the experience of deriving the relativistic kinetic equation (refer to. [13], [14], [16], [20]) shows, more thorough investigations suggest that the integrals become convergent as \( r \to \infty \), with the contribution from the region where \( r > r_g \) being infinitesimal. In Ref. [14],[16] there are estimates for \( r_g \) in the case where the average metric \( g_{ij} \) is the metric of isotropic cosmological model and in the case of gravitational interaction of particles.

The tensors \( \varphi^{k}_{ij} \), \( \mu_{ij} \), \( \tau^{(gr)}_{ij} \) \( \mu^i \) must obey the additional conditions

\[
 g^{ij} \nabla_l \left( \nabla_k \varphi^{k}_{ij} + \mu_{ij} - \chi \tau^{(gr)}_{ij} \right) = 0, \tag{120}
\]

\[
 \nabla_i \mu^i = 0, \tag{121}
\]

since the divergences of \( G_{ij} \), \( T_{ij} \), \( \nabla_k F^{ik} \), \( \nabla_k \varphi^{ik} \), \( J^i \) vanish.

Equations (120), (121) impose some restrictions on the parameters \( r_D \) and \( r_g \) dependence on the coordinates and the relative velocity of particles. The latter can be expressed via of \( z = (u'u'') \).

The macroscopic energy-momentum tensor \( T^{(m)}_{ij} \) of medium and the current vector \( J^i \) can be also written in terms of one-particle distribution function as follows:

\[
 T^{(m)}_{ij} = \sum_a c \int \frac{d^3p}{p^0 \sqrt{\lvert -g \rvert}} p_i p_j F_a(p), \tag{122}
\]

\[
 J^i = \sum_a e_a c \int \frac{d^3p}{p^0 \sqrt{\lvert -g \rvert}} p^i F_a(p). \tag{123}
\]

The system of equations (112), (113) must by augmented by the kinetic equation for \( F_b \) in relativistic plasma. In the case when the electromagntical interaction of particles are dominating, the equation for \( F_a \) was derived in Refs. [11], [13].

The covariant form of this kinetic equations is

\[
 u^i \frac{\partial f_a}{\partial q^i} + \Gamma_{j,ik} \frac{\partial f_a}{\partial p_i} + \frac{e_a}{c} < F_{ik} > u^k \frac{\partial f_a}{\partial p_i} = \]

\[
 = \sum_b \frac{\partial}{\partial p_i} \int \frac{d^4p'}{\sqrt{\lvert -g \rvert}} E_{ij}(p, p') \left( \frac{\partial f_a}{\partial p_b} f'_b - \frac{\partial f'_b}{\partial p_b} f_a \right), \tag{124}
\]
where

\[
E_{ij}(p, p') = \frac{2\pi e^2 e^2_b L n_b [(u, u')^2 - 1]^{-3/2}(u', u)^2 \times \\
\times \{ - g_{ij}[(u, u')^2 - 1] - u_i u_j - u'_i u'_j + (u, u')(u_i u'_j + u'_i u_j) \}
\]

(125)

with \((u, u') = u'_i u^i\). Primed and nonprimed quantities refer to particles belonging to species \(a\), and \(b\) respectively and \(L\) is the Coulomb logarithm [15]

\[
L = \int_{k_{\text{min}}}^{k_{\infty}} \frac{dk}{k}
\]

(126)

5. Conclusion

The macroscopic equations of the gravitational field in relativistic plasma differ from the classical Einstein equations by the presence of additional terms

\[
Z_{ij} = \nabla_k \phi_k^{ij} + \mu_{ij} - \chi^{(gr)}_{ij}
\]
on the left-hand side due to partial interaction.

These terms are proportional to the square of Einstein constant and to the square of particle number density.

The macroscopic equations of the electromagnetical field in relativistic plasma differ from classical Maxwell equations by the presence of additional terms

\[
Z^i = \nabla_k \phi_k^i + \mu^i
\]
on the left-hand side due to particle interaction and due to effects of general relativity.

This terms are proportional to the first power of Einstein constant and to the square of the particle number density.

Hence these terms can play an important role in continuous media of very high density only. Such density are possible in the early stages of the evolution of the Universe and inside objects hear gravitational collapse. Therefore, it is natural to look for applications of the derived equations primarily in the theory of early stages of the Universe evolution and in gravitational collapse theory.
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