Resummation of next-to-leading logarithms in top quark production

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Abstract

We discuss the resummation of next-to-leading logarithms (NLL) in heavy quark production near threshold. Results are presented for top quark production at the Fermilab Tevatron.

1 Introduction

There has been a lot of interest recently in the resummation of soft gluon radiation near threshold for heavy quark production in hadronic collisions [1-4]. This resummation is a direct consequence of the factorization of short-from long-distance effects in QCD scattering [5]. In the perturbative expansion for the heavy quark cross section one finds logarithmic terms that can be resummed to all orders of perturbation theory. These logarithms come from distributions that are singular for $z = 1$, where $z = Q^2/s$, with $Q^2$ and $s$ the invariant mass squared of the produced heavy quark pair and the partons in the incoming hadrons, respectively. One of the interesting features of singular distributions in QCD-induced cross sections is their sensitive dependence on the color exchange in the hard scattering. This feature as well as the presence of final state interactions are the main complications relative to the Drell-Yan process [6]. We will show that these effects contribute

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only at NLL. As we discuss below, resummation is conveniently carried out in terms of moments, and logarithmic dependence on the moment variable exponentiates. The moments have to be inverted to derive the physical cross section. Previous formalisms [1-4] resum only leading logs (and some NLL) and there has been a debate as to which subleading logs to keep. In our analysis (in moment space) we have for the first time resummed all the NLL so we have effectively pushed the debate in [1-3] to the next-to-next-to-leading logarithmic level. In Section 4 we invert moments using the method in Ref. 1.

2 Resummation for QCD hard scattering

For the hadronic process $h_1(p_1) + h_2(p_2) \rightarrow Q \bar{Q} + X$ the cross section $\sigma$, can be written as a convolution of parton distribution functions $\phi$, with the partonic hard scattering $\hat{\sigma}$,

$$\sigma_{h_1h_2\rightarrow QQ} = \sum_{f=q,\bar{q},g} \phi_{f/h_1} \otimes \hat{\sigma}_{ff\rightarrow QQ} \otimes \phi_{f/h_2}. \quad (2.1)$$

This convolution becomes a product in terms of moments,

$$\tilde{\sigma}_{h_1h_2\rightarrow Q\bar{Q}}(N) = \sum_{f=q,\bar{q},g} \tilde{\phi}_{f/h_1}(N) \hat{\sigma}_{ff\rightarrow Q\bar{Q}}(N) \tilde{\phi}_{f/h_2}(N), \quad (2.2)$$

where $\tilde{\sigma}(N) = \int_0^1 d\tau \, \tau^{N-1} \sigma(\tau)$, $\hat{\sigma}(N) = \int_0^1 dz \, z^{N-1} \hat{\sigma}(z)$, and $\tilde{\phi}(N) = \int_0^1 dx \, x^{N-1} \phi(x)$. Here $\tau = Q^2/S$ where $S$ is the invariant mass squared of the incoming hadrons. Singular distributions in $z$ give rise under moments to logarithmic $N$-dependence, as in

$$\int_0^1 dz \, z^{N-1} \left[ \frac{\ln^m(1-z)}{1-z} \right]_+ \propto \ln^{m+1} N. \quad (2.3)$$

Thus, the large-$N$ behavior is a diagnostic for singular distributions in $1 - z$. We can derive the resummation of soft gluon contributions by reexpressing moments of the cross section (with $h_1 = f$, $h_2 = \bar{f}$) in a form in which collinear gluons are factorized into alternate parton distributions $\psi$, and soft gluons into a function $S_{JI}$,

$$\sigma_{ff\rightarrow QQ} = \psi_{ff} \otimes h^*_I h_I \otimes S_{JI} \otimes \psi_{ff}. \quad (2.4)$$
This factorization takes into account the color exchange at the hard scatterings, $h_I$ and $h_J^*$ in the amplitude and its complex conjugate, respectively, where $I$ and $J$ are color tensor indices.

From Eqs. (2.1) and (2.4) we find $\hat{\sigma}(N) = [\psi(N)/\phi(N)]^2 h_J^* S_{JI}(N) h_I$. The ratio of the universal $\psi$ and $\phi$ has been analyzed in Drell-Yan production \cite{6}.

The soft matrix $S_{JI}(Q/(N\mu))$ requires renormalization: $S_{JI}^{(0)} = (Z_S^I)_{JB} S_{BA} \times Z_{S, AI}$, where $Z_{S, AI}$ is a matrix of renormalization constants \cite{7}. Hence $S_{JI}$ satisfies the renormalization group equation

$$\mu \frac{dS_{JI}}{d\mu} = - (\Gamma_S^I)_{JB} S_{BI} - S_{JA}(\Gamma_S^A)_{AI}.$$ (2.5)

In a minimal subtraction scheme, and with $\epsilon = 4 - n$, the anomalous dimension matrix $\Gamma_S$ is

$$\Gamma_S(g) = -g \frac{\partial}{\partial g} \text{Res}_{\epsilon \to 0} Z_S(g, \epsilon).$$ (2.6)

At the level of leading logarithms of $N$ in $S_{JI}$, and therefore at NLL of $N$ in the cross section as a whole, we choose a color basis in which the anomalous dimension matrix is diagonal, with eigenvalues $\lambda_I$ for each basis color tensor labelled by $I$. Then, the solution to Eq. (2.5) is

$$\tilde{S}_{JI} \left( \frac{1}{N}, \alpha_s(Q^2) \right) = \tilde{S}_{JI} \left( 1, \alpha_s \left( \left[ \frac{Q^2}{N} \right] \right) \right) \times \exp \left[ - \int_{Q/N}^Q \frac{d\bar{\mu}}{\bar{\mu}} \left( \lambda_I(\alpha_s(\bar{\mu}^2)) + \lambda_J^*(\alpha_s(\bar{\mu}^2)) \right) \right].$$ (2.7)

Thus, we find for the partonic cross section \cite{8,9}

$$\hat{\sigma}_{f\bar{f} \to \bar{q}q}(N) = A'e^{E_{JI}(N, \theta, Q^2)} h_J^* \left( 1, \alpha_s(Q^2) \right) \tilde{S}_{JI} \left( 1, \alpha_s \left( \left[ \frac{Q^2}{N} \right] \right) \right) h_I \left( 1, \alpha_s(Q^2) \right)$$ (2.8)

where $A'$ is an overall constant and $\theta$ is the center-of-mass scattering angle. The exponent is

$$E_{JI}(N, \theta, Q^2) = E_{DY}(N, Q^2) - \int_0^1 dz \frac{z^{N-1} - 1}{1-z} \left[ g_3^{(I)} [\alpha_s((1-z)^2 Q^2), \theta] + g_3^{(J)^*} [\alpha_s((1-z)^2 Q^2), \theta] \right],$$ (2.9)
where $E_{DY}$ is the Drell-Yan exponent, and
\[
g_{3}^{(T)}[\alpha_{s}, \theta] = -\lambda_{I}[\alpha_{s}, \theta] + \frac{\alpha_{s}}{\pi} C_{F,A}. \tag{2.10}
\]

3 The $\Gamma_{S}$ matrices for $q\bar{q} \to Q\bar{Q}$ and $gg \to Q\bar{Q}$

We now give explicit results for the anomalous dimension matrices $\Gamma_{S}$. We begin with quark-antiquark annihilation, $q(p_{a}) + \bar{q}(p_{b}) \to \bar{Q}(p_{1}) + Q(p_{2})$. The explicit calculation is given in [9]. Here we give the results for the anomalous dimension matrix in a color tensor basis consisting of singlet and octet exchange in the $s$ channel,

\[
c_{1} = \delta_{ab} \delta_{12}, \quad c_{2} = -\frac{1}{2N} c_{1} + \frac{1}{2} \delta_{a2} \delta_{b1}. \tag{3.1}
\]

The anomalous dimension matrix (shifted by $C_{F}\alpha_{s}/\pi$ as in Eq. (2.10)) in this color basis and in an axial gauge $A^{0} = 0$ is [9]

\[
\Gamma_{S,11} = -\frac{\alpha_{s}}{\pi} C_{F}(L_{\beta} + 1 + \pi i), \\
\Gamma_{S,21} = \frac{2\alpha_{s}}{\pi} \ln \left( \frac{u_{1}}{t_{1}} \right), \quad \Gamma_{S,12} = \frac{C_{F}}{2C_{A}} \Gamma_{S,21}, \\
\Gamma_{S,22} = \frac{\alpha_{s}}{\pi} \left\{ C_{F} \left[ 4 \ln \left( \frac{u_{1}}{t_{1}} \right) - L_{\beta} - 1 - \pi i \right] \\
+ \frac{C_{A}}{2} \left[ - \ln \left( \frac{u_{1}^{2} m_{s}^{2}}{t_{1}^{4}} \right) + L_{\beta} + \pi i \right] \right\}, \tag{3.2}
\]

where $s = (p_{a} + p_{b})^{2}$, $t_{1} = (p_{a} - p_{1})^{2} - m^{2}$, $u_{1} = (p_{b} - p_{1})^{2} - m^{2}$, and $m$ is the heavy quark mass. Also

\[
L_{\beta} = \frac{1 - 2m^{2}/s}{\beta} \left( \ln \frac{1 - \beta}{1 + \beta} + \pi i \right), \tag{3.3}
\]

where $\beta = \sqrt{1 - 4m^{2}/s}$. $\Gamma_{S}$ is diagonalized in this singlet-octet basis for arbitrary $\beta$ when the scattering angle is $\theta = 90^{\circ}$ (where $u_{1} = t_{1}$). It is also diagonalized at $\beta = 0$.

Next we discuss gluon fusion, $g(p_{a}) + g(p_{b}) \to \bar{Q}(p_{1}) + Q(p_{2})$. We choose the following color basis:

\[
c_{1} = \delta^{ab} \delta_{21}, \quad c_{2} = d^{abc} T^{c}_{21}, \quad c_{3} = if^{abc} T^{c}_{21}. \tag{3.4}
\]
Then in the $A^0 = 0$ gauge the anomalous dimension matrix (shifted by $C_A \alpha_s / \pi$ as in Eq. (2.10)) is 

$$
\begin{align*}
\Gamma_{S,11} &= \frac{\alpha_s}{\pi} \left[ -C_F (L_\beta + 1) - C_A \pi i \right], \\
\Gamma_{S,21} &= 0, \quad \Gamma_{S,12} = 0, \\
\Gamma_{S,31} &= 2 \frac{\alpha_s}{\pi} \ln \left( \frac{u_1}{t_1} \right), \\
\Gamma_{S,22} &= \frac{\alpha_s}{\pi} \left\{ -C_F (L_\beta + 1) \right. \\
&\quad \left. + \frac{C_A}{2} \left[ \ln \left( \frac{t_1 u_1}{m^2} \right) + L_\beta - \pi i \right] \right\}, \\
\Gamma_{S,32} &= \frac{N^2 - 4}{4N} \Gamma_{S,31}, \quad \Gamma_{S,13} = \frac{1}{2} \Gamma_{S,31}, \\
\Gamma_{S,23} &= \frac{C_A}{4} \Gamma_{S,31}, \quad \Gamma_{S,33} = \Gamma_{S,22}. 
\end{align*}
$$

(3.5)

We note that the matrix is diagonalized at $\theta = 90^\circ$ and also at $\beta = 0$.

We have checked that the one-loop expansion of our results for both channels are consistent with [10].

4 Results for top quark production at the Fermilab Tevatron

We are now interested in the magnitude of the NLL terms, particularly the $g_3$ contribution, relative to the LL results of [1]. Therefore we use a similar cutoff scheme, modifying the definitions of the exponent in Eq. (2.9) to work directly in momentum space. This method uses the correspondence between logarithms of the moment variable $N$ and logarithmic terms in the momentum space variable $1 - z$. We work at $\theta = 90^\circ$ to avoid the diagonalization of the anomalous dimension matrices.

The resummed partonic cross section is 

$$
\sigma_{ab}(s, m^2) = - \int_{s_{cut}}^{s - 2ms^{1/2}} ds_4 \ f_{ab} \left( \frac{s_4}{2m^2} \right) \left( \frac{d\sigma_{ab}^{(0)}(s, s_4, m^2)}{ds_4} \right),
$$

(4.1)

where $ab = q\bar{q}$ or $gg$ and $s_4 = 2m^2 (1 - z)$ at threshold. $d\sigma_{ab}^{(0)}/ds_4$ is the differential of the Born cross section [1]. After replacing $zN^{-1} - 1$ in Eq.
Fig. 1. The resummed top quark production cross section at $\theta = 90^\circ$ in the $\overline{\text{MS}}$ scheme. We show the resummed results without the $g_3$ terms (lower solid line) and the resummed results with all the NLL terms ($Q = m$, upper solid line; $Q = 2m$, lower dashed line; $Q = m/2$, upper dashed line).
by \(-1\) and introducing \(\omega = 1 - z\), the exponential function for the \(q\bar{q}\) channel is \([1]\)

\[
f_{q\bar{q}} \left( \frac{s_4}{2m^2} \right) = \exp[ E_{q\bar{q}}^{DY} + E_{q\bar{q}}(\lambda_{\text{octet}})],
\]

where \(E_{q\bar{q}}^{DY}\) is the Drell-Yan part. The color-dependent \(g_3\) contribution in Eq. (2.9) leads to

\[
E_{q\bar{q}}(\lambda_i) = - \int_{\omega_0}^{1} \frac{d\omega}{\omega} \left\{ \lambda_i \left[ \alpha_s \left( \frac{\omega^2 Q^2}{\Lambda^2} \right), \theta = 90^\circ \right] + \lambda_i^* \left[ \alpha_s \left( \frac{\omega^2 Q^2}{\Lambda^2} \right), \theta = 90^\circ \right] \right\}
\]

(4.3)

where \(i\) denotes singlet or octet. Since our calculation is not done in moment space, the \(\omega\) integral is cut off at \(\omega_0 = s_4/2m^2\). Because the running coupling constant diverges when \(\omega^2 Q^2/\Lambda^2 \sim 1\), the minimum cutoff in eq. (4.3) is \(s_{\text{cut}} = s_{4,\text{min}} \sim 2m^2\Lambda/Q\), where \(\Lambda\) is the QCD scale parameter. In general we choose a larger value for the cutoff consistent with the sum of the first few terms in the perturbative expansion; here we cut off the \(z\)-integration at \(z = 1 - 10\Lambda/Q\).

The treatment of the gluon-gluon channel in the \(\overline{\text{MS}}\) scheme is very similar but now we have three distinct color structures. However, only two of them are independent. We define \(f_{gg}\) for each eigenvalue so that \(f_{gg,i} = \exp[ E_{gg}^{DY} + E_{gg}(\lambda_i)]\), in complete analogy with the relations for the \(q\bar{q}\) channel. For details see Ref. 11.

In Fig. 1 we show results for the resummed top quark cross section at the Fermilab Tevatron with \(\sqrt{S} = 1.8\) TeV (with and without the NLL \(g_3\) terms) versus the top quark mass. We used the MRSD-‘ scheme with factorization mass at \(m = 175\) GeV/\(c^2\) and \(Q = m\) the total \(t\bar{t}\) resummed NLL \(\overline{\text{MS}}\) cross section at \(\theta = 90^\circ\) is 5.3 pb. Without the NLL \(g_3\) contribution the cross section is 3.7 pb. Thus the NLL \(g_3\) terms enhance the total \(\overline{\text{MS}}\) cross section considerably.

5 Conclusions

We have given explicit results for the resummation of next-to-leading logarithms in top quark production. We have shown that the NLL terms are numerically significant. Our methods have been applied to \(b\)-quark production at HERA-B, with similar conclusions \([1]\), and they can also be applied
to jet production [13]. We have recently completed the evaluation of the anomalous dimension matrices for $q\bar{q} \to gg$, $qg \to qg$, and the more complicated case of $gg \to gg$, all relevant in jet production [13].

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Questions

L. Trentadue, Parma:
You have shown some numerical outputs of your calculation. You have used the method of Laenen, Smith, and van Neerven to cut-off the $z \to 1$ limit. Could not the results be affected by this by hand limiting of the final phase space?

N. Kidonakis:
We were interested in the relative size of the NLL. Of course the result depends somewhat on the procedure used and the value of the cutoff. However, we calculated the cross section with different cutoffs and found that the NLL contribution is always significant, and that is our main conclusion.