Detecting spatial homogeneity in the World Trade Web with Detrended Fluctuation Analysis

Riccardo Chiarucci\textsuperscript{a,b}, Franco Ruzzenenti\textsuperscript{a,c,d,}, Maria I. Loffredo\textsuperscript{a,b}

\textsuperscript{a}CSC—Complex Systems Community, Siena
\textsuperscript{b}Department of Information Engineering and Mathematics, University of Siena, Italy
\textsuperscript{c}Department of Economics and Statistics, University of Siena, Italy
\textsuperscript{d}Department of Biotechnology, Chemistry and Pharmacy, University of Siena, Italy

Abstract

In a spatially embedded network, that is a network where nodes can be uniquely determined in a system of coordinates, links' weights might be affected by metric distances coupling every pair of nodes (dyads). In order to assess to what extent metric distances affect relationships (link's weights) in a spatially embedded network, we propose a methodology based on DFA (Detrended Fluctuation Analysis). DFA is a well developed methodology to evaluate autocorrelations and estimate long-range behaviour in time series. We argue it can be further extended to spatially ordered series in order to assess autocorrelations in values. A scaling exponent of 0.5 (uncorrelated data) would thereby signal a perfect homogeneous space embedding the network. We apply the proposed methodology to the World Trade Web (WTW) during the years 1949-2000 and we find, in some contrast with predictions of gravity models, a declining influence of distances on trading relationships.

Keywords: DFA analysis, distance puzzle, spatially embedded networks, spatial homogeneity, exponential random graphs

1. Introduction

In studying dynamical systems, homogeneity of space is a fundamental assumption of particle interactions. As is well known, invariance under translation or rotation or, alternatively, breaking of symmetries in presence of a
force field, play a crucial role in physical systems. In these cases distances certainly do count. Moreover the question of defining and assessing the role of distances in shaping social and economic complex systems - like communities, cities and whole economies - has always been addressed by social sciences, economics, human geography and more recently by network theory of spatial networks [1]. The most remarkable example of such system, where the distance factor has undergone a long-standing investigation, is given by the trading relationships between countries. So far as the early 1960s gravity models were proposed to explain bilateral trades flows [2]. Gravity models functionally resemble the two bodies’ interaction gravity formula, where the mass of the particle is replaced by the GDP (Gross Domestic Product) or by any other measure of the size of the economy. Gravity models proved to work pretty well in predicting the intensity of trading flows between any pair of countries, given the size of the two economies and the geographic distance. Nevertheless, they fail to predict the existence of the flow. In other words, they are not able to explain any zero-intensity flow between any pair of countries with a non-zero GDP [3]. Therefore, according to gravity models, any country should trade with any other, or, in a mathematical jargon, the network of the world-wide trading relationships should be a full graph. Conversely, we know that the World Trade Web (WTW) has a non-trivial topological structure, resembling those of many complex networks, in society and nature [4, 5, 6]. A second major shortcoming of gravity models concerns the consistency and robustness of the distance parameter estimations along time. Despite being very recent, the so called distance puzzle is a very debated riddle in the economic literature [7, 8]. According to standard gravity models, based on log-linear regressions, the parameter defining the weight of distances has always been considered constant, indicating that, contrary to expectations, transport costs, barrier removal and market integration - among the many factors favoring globalization - never affected the role of distances in shaping the WTW [9]. In this paper we propose to test the relevance of distances in the trading network by using a procedure based on the detrended fluctuation analysis. Even if this methodology is typically applied to analyze autocorrelation properties and long-range behavior of data which are temporally ordered, we propose to extend it in order to verify the homogeneity of the space in which a network is embedded and the role of the distances in it. This goal can be achieved only after a proper procedure of spatial reordering of the data - based on metric distances between vertices - has been implemented. Moreover the homogeneity of the space embed-
ding the network can be considered as the null hypothesis corresponding to absence of correlations, hypotheses that can in fact be tested by using the DFA. We apply the proposed procedure to the Trading Network during a temporal window of fifty years starting from the year 1949. We find - as a main outcome - a declining influence of distances on trading relationships. As a byproduct our results could in part explain the “distance puzzle” which is one of the drawback inside the gravity models.

2. Method

2.1. Applying DFA to time series

DFA has been extensively used to estimate long-range power-law correlation exponents in noisy signals \[10, 11, 12\]. More recently, DFA has been further improved in order to cope with series characterized by high non-linearity and periodicity \[16\]. To illustrate the DFA method, we consider a noisy time series, \(u(i), (i = 1, \ldots, N)\). We integrate the time series \(u(i)\) and we get the profile

\[ y(j) = \sum_{i=1}^{j} (u(i) - \langle u \rangle) \]

where

\[ \langle u \rangle = \frac{1}{N} \sum_{i=1}^{N} u(i) \]

We divide the profile into \(N_s = N/s\) boxes of equal size \(s\). Since the length of the series is generally not an integer multiple of time scale \(s\), a small portion of data at the end of the profile will not be included in any interval. In order not to overlook this small part we repeat the entire procedure starting from the end of the series, obtaining \(2N_s\) segments. Then in each box, we fit the integrated time series by using a polynomial function, \(y_{\text{fit}}(i)\), which is called the local trend. For \(order - l\) DFA polynomial functions of order \(l\) should be applied for the fitting. We calculate the local trend for each interval of width \(s\) via a linear fit of data. Indicating with \(y_{\nu\text{fit}}(i)\) the fit in the \(\nu\)-th box we define the detrended profile as

\[ Y_s(i) = y(i) - y_{\nu\text{fit}}(i) \]

where

\[(\nu - 1)s < i < \nu s\]
For each of $2N_s$ intervals we calculate the mean square deviation from the local trend

$$F^2_s(\nu) = \frac{1}{s} \sum_{i=1}^{s} Y^2_s[(\nu - 1) + i]$$  \hspace{1cm} (5)

Finally, we calculate the mean on all segments to obtain the fluctuation function

$$F(s) = \sqrt{\frac{1}{2N_s} \sum_{\nu=1}^{2N_s} F^2_s(\nu)}$$  \hspace{1cm} (6)

The above computation is repeated for box sizes $s$ (different scales) to provide a relationship between $F(s)$ and $s$. A power-law relation between $F(s)$ and the box size $s$ indicates the presence of scaling: $F(s) \propto s^\alpha$. As an example, Figure 2.1 shows the behavior of the fluctuation function for the WTW (2000), anticipating the use of the corresponding time series as obtained through the procedure explained in the next section. The parameter $\alpha$, called the scaling exponent or correlation exponent (which can be directly related to the Hurst Exponent $H$ if the series is stationary) gives a measure of the correlation properties of the signal: if $\alpha = 0.5$, there is no correlation and the signal is an uncorrelated signal (white noise); if $\alpha < 0.5$, the signal is anticorrelated; if $\alpha > 0.5$, there are positive correlations in the signal [13, 14].
2.2. Encoding spatial series

So far, DFA has been applied mainly to time series, with some remarkable exceptions. Peng et al. [15], applied DFA to evaluate long-range power-law correlations for DNA sequences containing noncoding regions. Can we apply DFA to assess autocorrelations in signals which are spatially embedded? By doing that, we aim at assessing the homogeneity of space for the signal generating system, in the same way as a 0.5 exponent would indicate a purely uncorrelated time series. Indeed, in unidimensional spaces, like the DNA chain, the analogy with the time series is straightforward and the application of DFA thereof. What if the space dimension is greater than one? In what follows, we will explain how we generated a signal spatially encoded from an embedded network. In a network embedded in a two-dimensional space, nodes can be univocally defined by a couple of coordinates (Figure 2). For every vertex, it is possible to array the outgoing links according to the distance of the receiving node. In Figure 2 we selected node C as the starting node and then we order the outgoing links according the distance of the incoming vertex. The order of the links are explicited in Figure 3.

Figure 2: Network embedded in a metric space
It is possible to obtain a series of \( n - 1 \) weights (strength of links) for each node. That is to say: we can array the entries of the rows of the weighted matrix according to the distance from the row’s first entry. We are therefore applying a central symmetry scheme, curiously, like in gravity models. Once engulfed all the nodes, we moved to another node and started again encoding from the first neighbour onward (Figure 3). At first, we order the list on \( n \) sequences according to columns’ order of the entries in the matrix of the weighted directed network \((a \rightarrow b \rightarrow c \rightarrow \ldots \rightarrow f)\) in Figure 2. While encoding the signal according to nodes’ distances, it is possible to overrate the role of space versus the role of topology. Strongly connected nodes (hubs) tend to connect mutually regardless of the distance (c and f in the example). Similarly, countries with high GDP values tend to trade more than countries with a low GDP \([5]\). It was recently proved that, for the case of the WTW, the topology and the symmetry structure of the binary network are actually more important in defining trading relationship than distances \([17]\). In the weighted structure of the WTW too topology plays a fundamental role, both on a local and global scale \([18]\). Therefore, in order to discount topological
effects, we deflate the trading values of the network (volumes), with deflators generated by a Null Model based on the Exponential Random Graph methodology. We calculate the expected value of trades over a graph ensemble of randomized networks that preserved the observed strength sequence (see Appendix) and we divide the observed values by the randomly generated values:

\[ w_{ij}^{\text{deflated}} = \frac{w_{ij}}{\langle w_{ij} \rangle} \]  \hspace{1cm} (7)

Figure 5 reports results for both the deflated and non-deflated series in the WTW.

However, the row’s order of the adjacency matrix we have so far chosen for arraying the \( n \) lists of signals must be considered arbitrary. We asked ourselves whether this arbitrary choice is fatal to DFA by operating a randomization, for both the volumes’ series and the deflated series, over the row’s order (preserving the distance ordering criteria for every row). We compared those randomizations with randomizations over all the values (rows and columns). It is noteworthy that, whereas the \( \alpha \) value for the fully randomized series, as expected, floats around 0.5, with a standard deviation of 0.01, the \( \alpha \) value of the series randomized over the rows is positive (with a standard deviation of 0.01), indicating that the degree of freedom in selecting the order of nodes still preserves autocorrelation in the overall signal (Figure 4). Nevertheless, it is still possible that preserving also the spatial order of the rows, that is, arraying the starting points of each signal series according to the distance from a first node (pivot), will affect the autocorrelation of the global signal. This may intuitively depend on the fact that neighbors are more likely to share neighbors than distant nodes. Neighboring nodes will thus have similar series’ order, whereas for a random order of nodes, the series’ order will be generated by a larger number of transpositions. For a more formal description see [20, 21]. We ultimately tested this hypothesis by arraying the nodes according to the distance from a pivot vertex and obtained a higher autocorrelation (Figure 6). We applied DFA to spatially encoded series generated by the Gleditsch database with the aim of assessing homogeneity of

\[ \text{The Weighted Directed Configuration Model (WDCM) underestimates the reciprocal weighted structure of the WTW and a better null model would probably be the one that preserves the reciprocated strength together with the in- and out- strength. However, this latter model has been recently formalized but not implemented [19]. This is subject for a new research and enhancement of the herein proposed method.} \]
space in the WTW and its evolution in time\cite{22}. Figure 5 displays the $\alpha$ value of deflated and non-deflated trade volumes, with the series composing the signal not ordered according to the distance from one starting vertex\cite{3}.

As expected, we observe an autocorrelation in space indicating that space matters in shaping trading relationships. The non-deflated signal seems to exhibit a stationary trend, hinting that the role of distance has not changed throughout time. However, if we look at the deflated signal we can clearly spot three main periods: a growing trend that goes from the post-war to the 1960s, a descending trend from the early 1980s onward and a stationary period in between. We recently investigated the development of the spatial filling - the degree of stretchiness of the WTW - and obtained similar results: the WTW undergone to a shrinking phase up to the mid 1960s and an expanding phase from the mid 1970s, though followed by a new stationary phase in the late 1990s\cite{23}.

Finally, we ordered the rows according to the spatial distance from a starting node set as pivot. As expected, the $\alpha$ exponent of the spatial series that preserved both, the order of neighbors for every node and the global order of nodes, is greater than the $\alpha$ exponent that preserves just the former one, indicating a higher autocorrelation in the

\footnote{Missing points refer to missing values due to computational problems. Nevertheless, the trend is still easily inferred and the time sample significant.}
Figure 5: Evolution of the $\alpha$ exponent for the WTW, deflated and non-deflated signal (Figure 6). Furthermore, even in this case, the $\alpha$ exponent seems not to exhibit any stationarity. The same three phases, though smoothed, are still present. Our results are consistent with results from recent analysis based on improved methodologies of gravity models that show that, after the late 1970s, distances became less constraining in the WTW, hinting declining costs of transports worldwide [24, 25, 26, 27]. We performed the DFA on a different set of data with the aim of extending the analysis beyond the year 2000. We used the BACI data set developed by CEPII and we obtained a declining trend that further confirms previous results, suggesting that transport costs kept decreasing after the year 2000. However, albeit we were able to resolve the distance puzzle, we attained a new, intriguing puzzle that we could call, mimicking the former one, mass puzzle. BACI data set provides trade flows in both mass unit and monetary unit (volumes). Surprisingly, the $\alpha$ exponent of the signal concerning trades in mass unit is lower than the signal expressing trades in monetary unit. This result overturns our expectations that trades in mass should be more bond to distances than trades in money. We conjectured that bulk and heavy commodities might be hauled with more efficient transport modes, like sea shipping, and thereby could be

[http://www.cepii.fr](http://www.cepii.fr)
less affected by the distance travelled per unit of mass. Nevertheless, this is just a conjecture and more investigation is needed.

3. Conclusions

We proposed a method based on DFA to assess the homogeneity of space for spatially embedded networks and tested this method on the World Trade Eeb. We showed that encoding the signal by preserving both the distance from every node and the distance of every node from a pivotal node, increases the autocorrelation in the series. Furthermore, we showed that washing out the topological effects from the signal, by means of a deflator generated with an ERG (Exponential Random Graphs) null model, the autocorrelation further, is increasing or decreasing in agreement with results obtained by other measures of the spatial embeddedness of the network. Finally, we showed that the degree of autocorrelation decreased after the mid 1970s, indicating decreasing cost of distances, as expected by the distance puzzle. We obtained this result without adding any additional variable to the model, merely relying on data of trade flows and their topology.
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Appendix A. Null Models

The method used to carry on the analysis of the World Trade Web implements a recently proposed procedure [28], developed inside the exponential random graph theoretical framework [29]. The method is composed by two main steps: the first one is the maximization of the Shannon entropy over a previously chosen set of graphs, \( \mathcal{G} \)

\[
S = - \sum_{G \in \mathcal{G}} P(G) \ln P(G) \tag{A.1}
\]

under a number of imposed constraints [28, 30], generically indicated as

\[
\sum_{G \in \mathcal{G}} P(G) = 1, \quad \sum_{G \in \mathcal{G}} P(G) \pi_a(G) = \langle \pi_a \rangle, \ \forall \ a \tag{A.2}
\]
(note the generality of the formalism, above: $G$ can be a directed, undirected, binary or weighted network). We can immediately choose the set $G$ as the grandcanonical ensemble of BDNs, i.e. the collection of networks with the same number of nodes of the observed one (say $N$) and a number of links, $L$, varying from zero to the maximum (i.e. $N(N-1)$). This prescription leads to the exponential distribution over the previously chosen ensemble

$$P(G|\tilde{\theta}) = \frac{e^{-H(G, \tilde{\theta})}}{Z(\tilde{\theta})}, \quad (A.3)$$

whose coefficients are functions of the hamiltonian, $H(G, \tilde{\theta}) = \sum_a \theta_a \pi_a(G)$, which is the linear combination of the chosen constraints. The normalization constant, $Z(\tilde{\theta}) \equiv \sum_{G \in G} e^{-H(G, \tilde{\theta})}$, is the partition function [28, 29, 30]. The second step prescribes how to numerically evaluate the unknown Lagrange multipliers $\theta_a$. Let us consider the log-likelihood function $\ln L(\tilde{\theta}) = \ln P(G|\tilde{\theta})$ and maximize it with respect to the unknown parameters. In other words, we have to find the value $\tilde{\theta}^*$ of the multipliers satisfying the system

$$\left.\frac{\partial \ln L(\tilde{\theta})}{\partial \theta_a}\right|_{\tilde{\theta}^*} = 0, \quad \forall a, \quad (A.4)$$

or, that is the same,

$$\pi_a(G) = \langle \pi_a(\tilde{\theta}^*) \rangle = \langle \pi_a \rangle^*, \quad \forall a \quad (A.5)$$

i.e. a list of equations imposing the value of the expected parameters to be equal to the observed one [28]. Note that the term “expected”, here, refers to the weighted average taken on the grandcanonical ensemble, the weights being the probability coefficients defined above. So, once the unknown parameters have been found, it is possible to evaluate the expected value of any other topological quantity of interest, $X$:

$$\langle X \rangle^* = \sum_{G \in G} X(G) P(G|\tilde{\theta}^*). \quad (A.6)$$

Because of the difficulty to analytically calculate the expected value of the quantities commonly used in complex networks theory, it is often necessary to rest upon the linear approximation method: $\langle X \rangle^* \approx X(\langle G \rangle^*)$, where $\langle G \rangle^*$ indicates the expected adjacency matrix, whose elements are $\langle a_{ij} \rangle^* \equiv p_{ij}^*$. 

12
This is a very general prescription, valid for binary, weighted, undirected or directed networks: since the WTW has been considered in its binary, directed representation, the generic adjacency matrix $G$ will be indicated, from now on, with the usual letter $A$. For the weighted directed version of the WTW, a very useful null model is the *weighted directed configuration model* (WDCM), that imposes the *in* and *out* strength sequence for every node. The Hamiltonian will thus be:

$$H_{WDCM} = \sum_i (\alpha_i s_i^{in} + \beta_i s_i^{out}) = \sum_{i \neq j} (\alpha_j + \beta_i) w_{ij}$$  \hspace{1cm} (A.7)

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