Isospin effects in a covariant transport approach to spallation reactions: Analysis of $p + Fe$ and $Pb$ reactions at 0.8, 1.2 and 1.6 GeV

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Abstract

We have investigated the influence of different non-linear relativistic mean field models ($NL$, $NL\rho$ and $NL\rho\delta$) on spallation neutrons for p+Fe and Pb reactions at 0.8, 1.2 and 1.6 GeV by means of a relativistic Boltzmann Uehling Uhlenbeck (RBUU) approach plus a statistical multifragmentation (SM) decay model. We find that the "evaporation shoulder", i.e. the neutron energy spectrum from 3 to 30 MeV, almost for any emission angle is quite sensitive to the isospin part of the mean field. For the more neutron-rich Pb-target the evaporation component can be directly related to the low density behavior on the symmetry energy in the thermal expansion phase of the excited compound system. It turns out that the spallation data for the reactions under study are shown to be more consistent with RBUU+SM employing the $NL\rho$ effective lagrangian.

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I. INTRODUCTION

Spallation reactions are very important for their wide applications in accelerator technology, production of energy, astrophysics and transmutation of nuclear waste [1, 2, 3, 4, 5, 6]. The theoretical description of the spallation process is essential in understanding better the physical mechanism of each of the mentioned cases.

Several dynamical models have been constructed for the theoretical description of spallation process [7, 8, 9, 10, 11]. All of them have the same basis, they describe the reaction as a cascade of nucleon-nucleon (NN) collisions, but employing different assumptions. The main difference concerns implementing mean field dynamics. One can distinguish the simplest models, which neglect features of the mean field dynamics and employ constant static potential, like a class of Intra-nuclear cascade (INC) models [7]. Other, more sophisticated approaches comprise changing field and real fluctuations obtained due to use of two and three body potentials e.g., quantum molecular dynamics (QMD) models [10]. Boltzmann-Uehling-Uhlenbeck (BUU) type models use both non-relativistic and relativistic mean field potentials [9, 12]. An important advantage of relativistic mean field models is the distinct separation of scalar and vector Lorentz-components of the nuclear mean field potential. The scalar part is associated with the in-medium dependence of the effective mass (Dirac mass), while the vector one changes the properties of the particle momenta inside the hadronic environment. This separation between scalar and vector self energies is very useful when extending mean field models to asymmetric nuclear matter, in which the role of isovector mesons can be clearly associated with the in-medium properties, symmetry potentials and effective masses of protons and neutrons.

In this work, we study the double differential cross sections (DDCS) of emitted neutrons in the reactions induced by 0.8, 1.2 and 1.6 GeV proton on targets Fe and Pb by using a covariant transport model of a relativistic BUU (RBUU) type [12]. The RBUU model is found to be a useful tool in describing the time evolution of the nucleus-nucleus reaction dynamics [13, 14, 15, 16]. As a genuine feature of transport theories it has two important ingredients: a relativistic mean field (RMF) based on quantum hadrodynamics [17] and isospin effects that account for the elastic and inelastic channels in resonance production and decay (cf. Refs. [13, 18] for different numerical realizations). In our approach, the non-linear (NL)-RMF mean-field in RBUU consists of isoscalar and isovector parts with different
Lorentz properties. The isoscalar part is characterized by an attractive scalar (\(\sigma\)-meson) and a repulsive vector field (\(\omega\)-meson), which is of importance in describing saturation properties of nuclear matter \[17\]. The isovector part of NL-RMF is also characterized by a competition between vector (\(\rho\)-meson) and scalar (\(\delta\)-meson) fields, which is responsible for the density dependence of the symmetry energy \[16,19,20,21\]. It should be noted that, the scalar nature of the \(\delta\)-meson leads even to a new interesting effect, important in the reaction dynamics, the splitting of the neutron and proton effective masses in isospin asymmetric systems. The numerical values of the field parameters, fixed to nuclear matter and symmetry energy at saturation, are chosen similar to the QMD parameters for a better comparison with the results presented in \[10,22,23\].

An important feature of our transport model is that isospin effects are also explicitly included in the collision integral, see later. The results presented here are based on the RBUU code presented in Refs. \[13,14,16,20,21\], where also the properties of the NL-RMF, cross sections and the collision integral are discussed. For a meaningful comparison with spallation neutron data we use the statistical multifragmentation (SM) model as an afterburner \[24\].

The paper is organized as follows. Section 2 defines the basic ingredients of the RBUU model. In Sec.3, we apply the RBUU+SM code systematically to the experimental neutron DDCS in the interactions of 0.8, 1.2 and 1.6 GeV proton on Fe and Pb at fixed angles of 0\(^\circ\), 10\(^\circ\), 25\(^\circ\), 40\(^\circ\), 55\(^\circ\), 85\(^\circ\), 100\(^\circ\), 115\(^\circ\), 130\(^\circ\), 145\(^\circ\) and 160\(^\circ\). We summarize and conclude this work in Sec. 4.

II. DESCRIPTION OF THE RBUU MODEL

In this section, an outline of the RBUU approach is given, which is described in detail in Refs. \[13,14,16,20,21\]. The RBUU equation has the form

\[
\left[ p^{*\mu} \partial_\mu + (p_x^{*} F^{\mu\nu} + m^{*} \partial^{\mu} m^{*}) \partial^{\nu} \right] f(x,p^*) = \mathcal{I}_{\text{coll}}.
\]

Eq. 1 describes the evolution of the single particle distribution function \(f(x,p^*)\) under the influence of a mean-field, which enters via effective masses \(m^*\), effective momenta \(p^{*\mu}\) and the field tensor \(F^{\mu\nu} = \partial^{\mu} \Sigma^{\nu} - \partial^{\nu} \Sigma^{\mu}\), and 2-body collisions \(I_{\text{coll}}\). \(\Sigma_s\) and \(\Sigma^\mu\) are the scalar and vector self energies, respectively.
In actual simulations the test particle (relativistic Landau Vlasov (RLV)) method \cite{23,26} is used for the numerical treatment of the Vlasov part, the lhs of Eq. (1). In this work we use a covariant Gaussian (in coordinate and momentum space) shape for the test particles. In \cite{25} it was shown that the use of a Gaussian shape for the test particles is appropriate to produce smooth fields and it is possible to determine local quantities, such as densities, currents, etc, without introducing additional grids.

With the introduced Gaussians the one-body phase space distribution function \( f(x, p^*) \) is simulated in the following way

\[
f(x, p^*) = \frac{1}{N_{\text{test}} A} \sum_{i=1}^{A \cdot N_{\text{test}}} \int_{-\infty}^{\infty} d\tau g(x - x_i(\tau)) g(p^* - p_i^*(\tau))
\]  

(2)

where \( A \) is the number of nucleons and \( N_{\text{test}} \) is the number of test particles per nucleon. The four dimensional Gaussian weights of the test particles take the form

\[
g(x - x_i(\tau)) = \alpha_s \exp \left( R^\mu_i(x) R_i^\mu(x)/\sigma_s^2 \right) \delta \left[ (x_\mu - x_i(\tau)_\mu) u^\mu_i(\tau) \right]
\]

(3)

and

\[
g(p^* - p_i^*(\tau)) = \alpha_p \exp \left( (p^* - p_i^*(\tau))^2/\sigma_p^2 \right) \delta \left[ p_i^* \cdot p_i^* - m_i^* \right]
\]

(4)

with \( \alpha_s = (\sqrt{\pi} \sigma_s)^{-3} \) and \( \alpha_p = (\sqrt{\pi} \sigma_p)^{-3} \). The widths \( \sigma_s \) and \( \sigma_p \) are kept constant, \( \sigma_s \) is fixed normalizing the space Gaussian to unity and \( \sigma_p \) is correlated making use of the uncertainty relation to \( \sigma_s \cdot \sigma_p = \hbar/2 \). In Eq. (3),

\[
R_i^\mu(x) = (x^\mu - x_i^\mu(\tau)) - (x_\nu - x_i(\tau)_\nu) u^\mu_i(\tau) u^\nu_i(\tau)
\]

(5)

is the projection of the distance \( x - x_i(\tau) \) on the hyperplane perpendicular to the velocity \( u_i(\tau) \). \( \tau \) refers to the eigentime of the particle.

A non-linear QHD model is adopted for the relativistic mean field potential, with isoscalar, scalar \( \sigma \) - and vector \( \omega \)-meson fields and with inclusion of the isovector channel through the exchange of the virtual charged \( \delta \)- and, vector \( \rho \)-mesons \cite{16,19,20,21}. In this model, in the mean field approximation, the self energies \( \Sigma_s \) and \( \Sigma^\mu \) are proportional to the expectation values of the isoscalar and isovector fields with coupling constants \( g_\sigma, g_\omega, g_\rho \) and \( g_\delta \). The scalar and vector components are given by

\[
\Sigma^\mu_i = g_\omega \omega^\mu_i(x) \pm g_\rho b^\mu_i(x)
\]

(6)

\[
\Sigma_{si} = g_\sigma \sigma_i(x) \pm g_\delta \delta_i(x)
\]

(7)
with the upper (lower) sign corresponds to the proton $p$ (neutron $n$). The self energies characterize the in-medium properties of the nucleons inside the hadronic environment in terms of kinetic momenta and effective masses

$$p_i^{\mu} = p_i^{\mu} - \Sigma_i^{\mu}$$  \hspace{1cm} (8)

$$m_i^* = M - \Sigma_{si}$$  \hspace{1cm} (9)

which is different for protons and neutrons due to the appearance of the isovector $\rho$ and $\delta$ mesons.

In the local density approximation, the scalar and vector meson fields, determined by the scalar density $\rho_s$ and the baryonic current $J_\mu$, respectively, result from the solution of the corresponding equations

$$m_\sigma^2 \sigma(x) + a\sigma^2(x) + b\sigma^3(x) = g_\sigma \rho_s(x) = g_\sigma \int d^4 p^* \frac{m^*(x)}{E^*(x)} f(x, p^*)$$  \hspace{1cm} (10)

$$\omega_\mu(x) = \frac{g_\omega}{m_\omega^2} J_\mu(x) = \frac{g_\omega}{m_\omega^2} \int d^4 p^* p_\mu^* f(x, p^*)$$  \hspace{1cm} (11)

$$b_\mu(x) = \frac{g_\rho}{4m_\rho^2} J_{3\mu}(x) = \frac{g_\rho}{4m_\rho^2} \int d^4 p^* p_\mu^* f_3(x, p^*)$$  \hspace{1cm} (12)

$$\delta_\mu(x) = \frac{g_\delta}{4m_\delta^2} \rho_{s3}(x) = \frac{g_\delta}{4m_\delta^2} \int d^4 p^* \frac{m^*(x)}{E^*(x)} f_3(x, p^*)$$  \hspace{1cm} (13)

The isospin vector phase-space and scalar densities are given by $f_3 = f_p - f_n$ and $\rho_{s3} = \rho_{sp} - \rho_{sn}$, respectively.

The equation of motion for the test particles are obtained by substituting Eq. (2) into
Eq. (11) and putting the collision term equal to zero, such that
\[
\frac{d}{d\tau} x_i^\mu(\tau) = u_i^\mu
\]
\[
\frac{d}{d\tau} u_i^\mu = \frac{1}{m^*(x_i)} \sum_{j=1}^{2N_{\text{test}}} \frac{2}{\sigma_s^2} \left[ g_{2\omega}^2 m_{\omega}^2 u_{i\nu}^\mu \left( R_j^\mu(x_i) u_j^\nu - R_j^\nu(x_i) u_j^\mu \right) 
- g_{\sigma} \frac{\partial \sigma(x_i)}{\partial \rho_s} \left( R_j^\mu(x_i) - u_i^\mu u_j^\nu R_{j\nu}(x_i) \right) \right] \frac{\alpha_s \exp(R_j^2(x_i)/\sigma_s^2)}{N_{\text{test}}}
\]
\[
\pm \frac{1}{m^*(x_i)} \sum_{j=1}^{2N_{\text{test}}} \frac{2}{\sigma_s^2} \left[ g_{2\rho}^2 4m_{\rho}^2 u_{i\nu}^\mu \left( R_j^\mu(x_i) u_j^\nu - R_j^\nu(x_i) u_j^\mu \right) 
- \frac{g_2^3}{4m_{\rho}^2} \left( R_j^\nu(x_i) - u_i^\mu u_j^\nu R_{j\nu}(x_i) \right) \right] \frac{\alpha_s \exp(R_j^2(x_i)/\sigma_s^2)}{N_{\text{test}}}
\]
\[
\pm \frac{1}{m^*(x_i)} \sum_{j=Z-N_{\text{test}}}^{2N_{\text{test}}} \frac{2}{\sigma_s^2} \left[ g_{2\delta}^2 4m_{\delta}^2 u_{i\nu}^\mu \left( R_j^\mu(x_i) u_j^\nu - R_j^\nu(x_i) u_j^\mu \right) 
- \frac{g_2^3}{4m_{\delta}^2} \left( R_j^\nu(x_i) - u_i^\mu u_j^\nu R_{j\nu}(x_i) \right) \right] \frac{\alpha_s \exp(R_j^2(x_i)/\sigma_s^2)}{N_{\text{test}}}
\]
\[\alpha_s \exp(R_j^2(x_i)/\sigma_s^2) \quad . \tag{15}\]

Eq. (15) is the Vlasov term of the transport equation in terms of the test particle representation of the phase-space distribution function. It describes the propagation of the test particles under the influence of the nuclear mean field, which enters here via the coupling functions \(f_n = \frac{g^2}{m_n^2}\) \(n = \sigma, \omega\) and \(n = \rho, \delta\), for the isoscalar and isovector parts of the nuclear potential, respectively). For charged baryons the Coulomb force represents an additional term in the equation of motion.

In the RBUU code, not only protons and neutrons are propagating separately according to their hadronic fields and Coulomb interaction but also \(\Delta^{0,\pm,++}\)-resonances. \(N^*\)-resonances are not fully accounted for, which may set the upper limit of the incident energy of the code to 1.5 GeV for the nucleon induced reactions. The self energies of the resonances are built as [27, 28, 29]

\[
\Sigma_i(\Delta^-) = \Sigma_i(n)
\]
\[
\Sigma_i(\Delta^0) = \frac{2}{3} \Sigma_i(n) + \frac{1}{3} \Sigma_i(p)
\]
\[
\Sigma_i(\Delta^+) = \frac{1}{3} \Sigma_i(n) + \frac{2}{3} \Sigma_i(p)
\]
\[
\Sigma_i(\Delta^{++}) = \Sigma_i(p) \quad , \tag{16}
\]

where \(i = \text{scalar, vector}\).
Differently from $\Delta$ and nucleons, pions are propagated as free particles with respect to the nuclear mean field, but experience the Coulomb potential generated by all the other charged particles.

The collision integral, i.e., the r.h.s of Eq. (11) is modeled by a parallel ensemble Monte Carlo algorithm [13, 14, 16, 28]. A geometrical interpretation of the cross section is used, so that in each ensemble two test-particles are allowed to collide if their relative distance $d$ fulfills the relation:

$$d \leq \sqrt{\frac{\sigma_{\text{tot}}}{\pi N_{\text{test}}}}.$$  \hspace{1cm} (17)

The collision is accepted if the final state Pauli blocking is not violated.

The total cross section $\sigma_{\text{tot}}$ depends on the center of mass energy and on the species of the incoming particles. The distance $d$ is calculated using the covariant calculation proposed by Kodama et al. [30].

The RBUU code describes the propagation and mutual interactions of nucleons, Delta resonances and also $\pi$-mesons. The following hadronic reactions are treated explicitly:

- $NN \rightarrow NN$ (elastic scattering)
- $\Delta\Delta \rightarrow \Delta\Delta$ (elastic scattering)
- $NN \leftrightarrow N\Delta$
- $N\pi \leftrightarrow \Delta$

The different $NN \leftrightarrow N\Delta$ and $NN \leftrightarrow \Delta\Delta$ isospin channels are treated according to [31], in which the cross sections are evaluated on the basis of the One Boson Exchange model and then fits to the analytical calculations are given. For the elastic cross sections and the angular distributions, the free parametrization according to Cugnon et al. [33] is used. Besides production and absorption in $NN$-channels, resonance states can be populated also through $\pi N$ collisions. It should be mentioned that, in contrast to other transport approaches [7, 9, 10], the RBUU code accounts for both the relativistic kinematics and the isospin effects in resonance production and decay, due to the Lorentz structure of the isovector channel.

The mean field is determined by four coupling constants, $f_i = g_i^2/m_i^2$, $i = \sigma, \omega, \rho, \delta$ and the two parameters of the non-linear self-interactions of the $\sigma$-meson field $a, b$. In this
\[ f_i (fm^{-2}) = \frac{g_i^2}{m_i^2} \text{ for } i = \sigma, \omega, \rho, \delta, \quad B = \frac{a}{g_\omega^2} \quad \text{and} \quad C = \frac{b}{g_\delta^2} \]

for the different \( NL - RMF \) models.

\begin{table}
\begin{tabular}{|c|c|c|c|c|c|}
\hline
 & \( f_\sigma (fm^{-2}) \) & \( f_\omega (fm^{-2}) \) & \( f_\rho (fm^{-2}) \) & \( f_\delta (fm^{-2}) \) & \( B (fm^{-1}) \) & \( C \) \\
\hline
\( NL \) & 9.3 & 3.6 & 0.0 & 0.0 & 0.015 & -0.004 \\
\hline
\( NL_\rho \) & 9.3 & 3.6 & 1.22 & 0.0 & 0.015 & -0.004 \\
\hline
\( NL_\rho \delta \) & 9.3 & 3.6 & 3.4 & 2.4 & 0.015 & -0.004 \\
\hline
\end{tabular}
\end{table}

TABLE I: Coupling constants \( f_i = \frac{g_i^2}{m_i^2} \) for \( i = \sigma, \omega, \rho, \delta \), \( B = \frac{a}{g_\omega^2} \) and \( C = \frac{b}{g_\delta^2} \) for the different \( NL - RMF \) models.

work, for the isoscalar part the parameter set NL of Ref. [32] has been used. For symmetric matter it gives reasonable values for the incompressibility \( K = 200 \text{ MeV} \) with a nucleon effective mass of \( m^* = 0.83M \) at saturation density \( \rho_0 = 0.148 \text{ fm}^{-3} \). In order to unravel the role played by the different density behavior of the symmetry energy, we will discuss the results obtained with the two parameter sets \( NL_\rho \) and \( NL_\rho \delta \) in the isovector channel. In \( NL_\rho \) and \( NL_\rho \delta \) models the symmetry energy parameter is fitted at saturation to the value of 30.7 MeV. The values of all the coupling constants are given in Table 1. In Fig. [1] we plot the density dependence of the symmetry energy for the \( NL, NL_\rho \) and \( NL_\rho \delta \) models.

We note two features that will be relevant for spallation reactions: i) The symmetry energy is in general larger when isovector bosons are inserted (in the NL model only the kinetic Fermi contribution is present); ii) The contribution of the \( \delta \)-meson has different effects on the symmetry term, a decrease below saturation and an increase above \( \rho_0 \) \[16, 19, 20\].

It should be noted that the \( NL, NL_\rho \) and \( NL_\rho \delta \) parameters have been already used for pion/kaon production and isospin tracer calculations in relativistic heavy ion collisions with an overall agreement with the data \[13, 14, 28, 29\].

Event Characterization

Due to the stochastic nature of the RBUU simulations the transport code is used as an event generator. In order to reduce statistical fluctuations 500 events are analyzed for each initial condition (target, beam energy and centrality).

An important issue for the description of spallation data within the RBUU model is an appropriate initialization of the ground state nuclei. We will follow here in particular the
initialization of Ref. [31], which aims to construct the static properties of the ground nuclear state such as binding energy and root mean square radius. Moreover the evolution of these quantities should be stable enough, and the behavior of the density and momentum distributions should be approximately correct. To get the initial nuclear phase space distribution, we first distribute the $N_{test} \times A$ test-particles according to a Saxon-Wood shape with neutron (proton) radius $R_{n,p}(A)$ and diffuseness parameter $a_{n,p}(A)$ depending on the target mass $A$. The parameterizations of $R_{n,p}(A)$ and $a_{n,p}(A)$ as a function of mass $A$ for stable nuclei have been deduced from the adjustment of the Saxon-Wood function on the density matter distributions calculated with the model of Ref. [13]. The Saxon-Wood tail is cut off at $R_{max} = R_{n,p} + 2a_{n,p}$. The next step is to determine the local density of all nucleons generated by all the other nucleons. The initial momenta of $N_{test} \times A$ test-particles are randomly chosen between zero and the local Fermi momentum: $p_F^{max} = \hbar c(3\pi^2 \rho_{Sax})$, with $\rho_{Sax}$ being the corresponding Saxon-Wood-neutron or proton density. Finally, an initial ground state configuration is accepted when (a) the sum of the total energy is equal to $E_B/A \pm 0.5\text{MeV}/A$, where $E_B/A$ is the ground state energy of a nucleus of mass $A$ and charge $Z$ given by the liquid drop model and (b) the binding energy and root mean square radius keep smooth variation with time and without spurious particle emission.

The RBUU calculation is carried out up to a time scale referred to as the transition time $t_{tr}$, when an excited compound system is formed, the "prefragment" source that will decay via statistical emissions. At $t_{tr}$, the position of each nucleon is used to calculate the distribution of mass and charge. The minimum spanning tree method [35] is employed, a prefragment is formed if the centroid distances are lower than $R_{clus}$. In this paper, $R_{clus}$ is fixed at 4 fm. The prefragment thus identified is then Lorentz boosted into his rest frame to evaluate the excitation energies. The excitation energy ($\epsilon_{exc}$) is calculated as the difference between the binding energy of the hot prefragment and the binding energy of this prefragment in the ground state, evaluated as indicated before. When the prefragment is in the excited state, the SM model [23] is employed as an afterburner, which is shown to be well suited for the description of the slow evaporated neutrons [11, 23, 34].

In order to determine $t_{tr}$, it must be verified, whether information obtained from the transport code are sensitive to the time duration of the first stage of the reaction. For this purpose, the time variation of the average values (over 100 events) of three physical quantities: excitation energy per nucleon $<\epsilon_{exc}>$, angular momentum $<L>$ and mass
number of the excited prefragments \( < A_{\text{exc}} > \) after the first RBUU stage, have been analyzed. As an example, Fig. 2 shows the time evolution of these quantities for \( p + Pb \) collisions at 1.2 GeV, obtained with a \( NL\rho \) interaction. As one can see, large prefragments are produced at earlier times \( 5 < t_{\text{tr}} < 10 \, fm/c \) with a maximal value of \( < \epsilon_{\text{exc}} > \, MeV/A \) and relatively large \( < L > \). Then, both quantities \( < \epsilon_{\text{exc}} > \) and \( < L > \) drop very rapidly in the time range 10-35 fm/c, in correspondence of a fast, pre-equilibrium, nucleon emission as shown by the decrease of \( < A_{\text{exc}} > \) in the bottom panel. The dropping of \( < \epsilon_{\text{exc}} > \), \( < L > \) and \( < A_{\text{exc}} > \) stops at around 40 fm/c and the average values remain stable. At later times, \( t > 80 \, fm/c \), spurious increase (decrease) of \( < \epsilon_{\text{exc}} > \) (\( < L > \) and \( < A_{\text{exc}} > \)) is observed, which is unphysical and arises due to spurious numerical fluctuations. This may indicate that the first dynamical stage of the reaction evolution should be terminated at about 80 fm/c and a statistical decay procedure could be inserted to get the final reaction products.

Let us now check the dependence on the \( t_{\text{tr}} \) choice of the final results. In Fig. 3 we show the neutron double differential cross section for the reaction \( p + Pb \) at 1.2 GeV. The line histograms denote the RBUU+SM calculations using \( NL\rho \) parameterizations with three different transition times, 60 fm/c (thin lines), 75 fm/c (thick lines), and 100 fm/c (dot-dashed lines). As one can see, calculations with different transition times yield the same results in the low energy neutron region \( \approx E_n \leq 30 \, MeV \). This may indicate that the decay processes of the excited prefragments described by RBUU+SM are equivalent in a \( t_{\text{tr}} \) choice from 60 to 100 fm/c. This is important for the study of the symmetry term dependence of this component of the neutron spectrum presented in the next section.

On the other hand, in the "cascade region" (see later), \( 30 < E_n < 60 \, MeV \), a systematic difference between the calculations is seen as the angle increases. This difference decreases within the time interval from 75 to 100 fm/c. We observe that the neutron data, in this energy range, are better reproduced by the RBUU+SM at \( t_{\text{tr}} = 75 \, fm/c \). We, thus, use this value for all systems in the present study, which is also supported by Fig. 2.

It should be pointed out here that in the SM model, standard symmetry coefficients \( \gamma_{\text{sym}} = 23 \) to 25 MeV of the fragment binding energy are used to describe light and heavy fragments produced in proton/nucleus-nucleus collisions [24]. On the other hand, the symmetry energies within the different parameterizations used in the transport model may differ from those used in the statistical approach for densities beyond saturation. Therefore, it is worthwhile to examine the consequences of changing \( \gamma_{\text{sym}} \) in SM calculations, as has been
done in different studies that focused on isotopic distributions [36]-[39]. Fig. 4 displays the evaporation neutron spectra obtained using RBUU (with NL)+SM calculations for p+Pb reactions at 0.8 GeV, and the symmetry term $\gamma_{\text{sym}}$ was varied between 5 to 25 MeV. As one can see, with $\gamma_{\text{sym}} = 5$ MeV, the SM processes causes a slight increase of low energy neutrons at $E_n < 10$ MeV. For larger values of $\gamma_{\text{sym}}$, however, the evaporation neutron spectra are not much affected by the symmetry coefficient difference in the SM code. We note that at variance with the Heavy Ion Collisions studies of Refs.[36-38] in our case we do not have large compression-expansion effects. The final equilibrated source is then at densities close to the saturation value. This has been checked analyzing the time evolution of the root mean square radius of the composite system in the (p+Pb) case at 0.8 GeV. After the thermal expansion the system shows a rather good stability with smooth monopole oscillations, see also Ref. [40]. This may imply that the equilibrating source is formed at about normal nuclear density and it is not necessary to use much reduced symmetry coefficients. Thus, in what follows the value of $\gamma_{\text{sym}} = 25$ MeV will be kept fixed in the SM code, which is supported by the experimental analysis of $\gamma_{\text{sym}}$ with the SM model in $p,d$ and $\alpha$ induced reactions at relativistic energies up to 15 GeV [39].

III. RESULTS AND DISCUSSIONS

In this section we present and discuss the predictions of the RBUU+SM model using different NL parameter sets along with the measurements [41, 42] of energy-angle DDCS of neutrons induced by 0.8 GeV, 1.2 GeV and 1.6 GeV protons on Fe and Pb.

The experimental energy spectra (see Figs. 5 and 6 and similar ones) show at 0° two prominent peaks. These peaks are less pronounced at 10° and are insignificant at 25° and larger. The (quasi-elastic) peak, characterized by a narrow peak at a kinetic energy near that of the beam energy, is due to a single $(p,n)$ elastic scattering in the forward direction. The (quasi-inelastic) peak, centered around 873 $MeV$ and 760 $MeV$ at 0° and 10°, respectively, is about 400 $MeV$ wide and is thought to be due to $\Delta$-resonance excitation. In addition to these two peaks, it seems that two components exist for all of the spectra: one is a "shoulder" below $E_n \approx 20$ (30) $MeV$ for Fe (Pb), the other is a wide peak extending up to a few hundred MeV. The low energy neutrons mostly come from evaporation of the excited target residues formed through the equilibration process. The other component becomes less pronounced
with increasing angles. This component arises from "cascade" processes involving several
$NN$-collisions. Below we are going to investigate the effect of the $NL$, $NL\rho$ and $NL\rho\delta$
models on the three components of the neutron spectra by employing RBUU+SM code.
We have performed 500 simulations at various impact parameters, from 0 to 4(7.5) fm for
Fe(Pb), respectively. In order to have sufficient statistics, calculations were done for angular
bins of $\pm 3.5$ at $0^\circ$ and $\pm 5$ for larger angles.

Before confronting RBUU+SM with experimental data, it is worthwhile to investigate
the sensitivity of the calculations to the number of test particles ($N_{\text{test}}$). This has been done
(not shown here) for $p+Fe$ reactions. We used $NL\rho$ parameter set and the calculations were
performed with 40 and 100 test particles per nucleon. It was found that the overall behavior
of the neutron DDCS are nearly the same for both calculations at all angles. Since the
computational time of the collision integral scales as ($N_{\text{test}} \times A$) (parallel ensemble method),
a value of $N_{\text{test}} = 40$ test particles per nucleon has been chosen.

All the results are presented and compared to data in the Figs. 5-10. Before passing
to a more detailed analysis we like to note that all the components of the neutron spectra
are rather well reproduced by this relativistic transport approach. We remark that no free
parameters are used while genuine relativistic effects are present via the full covariance
of the model and the natural presence of Lorentz forces due to the coupling to vector fields.

Let us first focus on the role of symmetry energy on the neutron spectra. This is illustrated
in Figs. 5-8 by comparing the RBUU calculations without (Figs. 5 and 6) and with
(Figs. 7 and 8) evaporation using $NL$ (dot-dashed lines) and $NL\rho$ (thin lines) models. The
two models differ only in the isospin part, with $\rho$-coupling giving an almost linear density
dependence of the symmetry energy with value $\approx 30.7$ MeV at saturation (see Fig. 1).

Figs. 5 and 6 show that the results of both calculations, $NL$ and $NL\rho$, are close together
and can describe the high energy part of the neutron spectra (above $E_n \approx 20$ and 30 MeV
for $p+Fe$ and Pb, respectively). It is interesting to see that without statistical decay the
"low energy shoulder" is absolutely missed. For the more neutron-rich Pb target some
difference originating from the $\rho$-meson becomes more pronounced in the lower part of the
neutron energy spectrum (below 30 MeV), especially with increasing emission angle. Such
behaviour can be related to the difference in the symmetry energy: neutrons experience a
stronger repulsive field going from $NL$ to $NL\rho$, at any density (see Fig. 1) [16, 20].

In fact the largest isospin effects are observed just on the low energy evaporation compo-
nent which appears when the afterburner statistical decay is included. In Figs. 7 and 8 we compare the full RBUU+SM calculations (with evaporation) along with the experimental neutron spectra. As one can see, the NL results, with no (potential) symmetry energy, show a broad maximum at $E_n \approx 3 - 50$ MeV in all angular intervals for all studied collisions. On the other hand, the NLρ calculations reduce the neutron yield in this energy region and more closely reproduce the experimental data, especially as the incident energy increases. The importance of the isovector interaction is shown to increase when both the incident energy and the isospin asymmetry of the target system increase. It should be pointed out here that, the effect of the symmetry potential on the three components of the neutron spectra has already been discussed by a non-relativistic QMD approach for p+Pb at 1.2 and 1.6 GeV [23]. It was shown that of all the three components of the neutron spectra, the evaporation part is the most sensitive to the symmetry potential.

Next we compare, in Figs. 9 and 10, the RBUU+SM results using NLρ without (solid lines) and with (dot-dashed lines) the δ-field. In the case of p+Fe interactions, there is almost no significance differences between NLρ and NLρδ results. However, for p+Pb interactions, with increasing neutron to proton asymmetry, NLρδ generates an excess of slow neutrons ($E_n \approx 3 - 50$ MeV) compared to NLρ.

The origin of the different behaviour between the results in the slow neutron energy region ($\leq 50$ MeV) is clearly due to the excitation energy and isospin content of the hot prefragments. The properties of these prefragments at the final stage of the interactions (at 75 fm/c) are affected by the number of pre-equilibrium neutrons ($N_{em}$) emitted at the earlier times: less $N_{em}$ leads to larger excitation energy and larger N/Z of the prefragment. The NLρ calculations of the p+Pb interactions at 1.2 GeV show, in Fig. 2, that nearly 20 nucleons are emitted at the earlier times (between 20 and 40 fm/c). In Fig. 11 we examine the distributions of emitted neutrons $N_{em}$ before 75 fm/c by employing RBUU with NL, NLρ and NLρδ cases for the p+Pb reactions at 0.8 GeV. One observes that for $N_{em} \geq 8$, less pre-equilibrium neutrons are emitted with $NL < NLρδ < NLρ$. Consequently, the excitation energy and isospin asymmetry of the prefragments should appear larger for NL calculations with respect to NLρ (and $> NLρδ$) as observed. The sequence ($NL < NLρδ < NLρ$) agrees with the sequence of the three forms of the symmetry term at the densities below saturation (see Fig. 1), which confirms that $N_{em}$ is generated at earlier times where an expansion mode, due to thermal pressure, is likely occurred.
The sensitivity of the evaporation component of the neutron spectrum to the low density behavior of the symmetry term can be then used to disentangle among different effective fields and to get an independent information on the symmetry energy below saturation, see some general discussions in the reviews [16, 43]. The best agreement with the data in the more n-rich Pb case is obtained within the $NL\rho$ choice, which corresponds to a rather stiff increase of the symmetry energy below saturation, roughly proportional to the total density. It is also interesting to notice (see Figs. 8 and 10) that the $NL$ and $NL\rho\delta$ give similar extra yields with respect to the $NL\rho$ case, which implies that the excitation energies and N/Z of the Pb hot prefragments are similar. This can be also explained by considering the weaker symmetry term at densities below saturation (see Fig. 1).

Finally, let us identify the effects of the mean field on the high energy part of the neutron spectra ($E_n > 40$ MeV) for the reactions under study. This is illustrated in Figs. 5 and 6 and Fig. 12 which show a close up of this region at very forward angles ($\theta \leq 10^\circ$). Two different scenarios are explored: RBUU calculations with $NL\rho$ (thin lines) are contrasted by RBUU with deactivated mean field (CASCADE) (thick lines). We observe that at $10^\circ < \theta \leq 55^\circ$ the $NL\rho$ calculations are practically identical to CASCADE calculations as the incident energy increases, showing the mean field effects to become negligible. At $55^\circ < \theta \leq 160^\circ$ the $NL\rho$ calculations give lower values compared to CASCADE calculations, in good agreement with the data. However, the $NL\rho$ calculations underestimate the high energy neutrons (70-110 MeV) at $\theta = 160^\circ$ for p+Fe at 1.6 GeV and p+Pb at 1.2 GeV, contrary to CASCADE calculations. Note that, both calculations resemble each other at $\theta = 160^\circ$ for p+Pb at 1.6 GeV. Thus, it seems that the high energy neutron spectra evaluated by RBUU (with and without $NL\rho$) have different beam energy dependence compared to the experimental data. It should be pointed out that our results in the backward direction are nevertheless better with our present RBUU code than with the non-relativistic INC [8], QMD [22] and BUU [44] calculations.

Fig. 12 shows how the quasi elastic and quasi inelastic peaks are changed due to the mean field. In the CASCADE calculations, the maximum of the calculated quasi-elastic peak is shifted toward a higher energy in comparison to the data. The shape and yield of the peaks are in better agreement with the CASCADE calculations at 1600 MeV (and at $10^\circ$) than at 800 and 1200 MeV. On the other hand, as the mean field calculations are taken into account in RBUU, the maximum outgoing neutron energy is strongly attenuated at incident energy.
of 1200 MeV and 1600 MeV. As for the quasi-inelastic peak, i.e., the peak located at the beam energy minus \( \approx 300 \, \text{MeV} \), the evolution of the shape with the incident energy follows more or less the data for both calculations. We notice that the \( NL\rho \) calculations have a tendency to become smaller compared to the CASCADE ones at lower incident energy and need to be improved.

In [45] the quasi-elastic and inelastic peaks at 1.2 GeV were underestimated by the ultrarelativistic QMD (UrQMD) calculations, which was attributed to the free angular distributions of the scattered NN-elastic and inelastic collisions adopted in the UrQMD model, and it was suggested to introduce medium modified angular distributions (MMAD) into the UrQMD model for reducing the deviation. We then further test the influence of the MMAD on the reactions under study. We introduce the MMAD adopted in [45] in the collision term of RBUU (with \( NL\rho \)) code. The dot-dashed lines in Fig[12] illustrate these calculations for p+Pb at 800 MeV. As one can see, the introduction of MMAD in RBUU improves the intensity of both the quasi-elastic and inelastic peaks. For the location, however, the maximum of the calculated quasi-elastic peak is 50 MeV downward shift in comparison to the data. At higher incident energy (not shown here), this shift is increased even further to 100 and 150 MeV for the reactions under study at 1200 MeV and 1600 MeV, respectively. We attribute this failure to the linear energy dependence of the Schrodinger equivalent optical potential of the NL-RMF models at high energies [46]. Indeed, Dirac phenomenology on elastic proton-nucleus scattering predicts a nonlinear energy dependence of Schrodinger equivalent optical potential for incident energy above 400 MeV, which leads to a softening of the optical potential and, consequently, may improve the dynamics of NN-elastic collisions at high energies.

IV. SUMMARY AND CONCLUSIONS

We have studied proton-induced Fe and Pb reactions at 0.8, 1.2 and 1.6 GeV by a hybrid model, using the RBUU transport approach for the first pre-equilibrium stage and the SM decay model for the second slow stage of the reaction. The NL-RMF model of the present approach consists of isoscalar (\( \sigma - \omega \) mesons) and isovector (\( \rho - \delta \) mesons) parts with different Lorentz covariant properties. The impact of different NL-RMF models (\( NL, \, NL\rho \) and \( NL\rho\delta \)) on the neutron spectra of the studied reactions has been investigated and the
following conclusions can be drawn:

- Some effects of the mean fields are found on the high energy part of the neutron spectra, when switching off the mean field and turning to the CASCADE mode within the RBUU calculations.

- The low energy neutron spectrum ($E_n \approx 3 - 50 \text{ MeV}$) is found to be quite sensitive to the details of the relativistic structure of the isovector part of the mean field: the inclusion of the $\rho$-field improves the comparison between theory and experiment to large extent. The effect is related to the properties (rate and isospin content) of the fast nucleon emission during the thermal expansion of the system and so it is sensitive to the low density behaviour of the symmetry term.

- The competition between a scalar attractive ($\delta$) and a vector repulsive ($\rho$) fields affects low energy neutrons ($E_n \leq 30 \text{ MeV}$) only for the more neutron-rich Pb target.

- The introduction of medium modified angular distribution in RBUU (with $NL\rho$) improves the intensity of p+Fe and Pb reactions at 0.8 GeV.

Thus, the present comparisons suggest a better agreement with $NL\rho$ model, however, several other aspects of the mean field dynamics of the RBUU model need further work. For example, the nucleon self-energies of the current version of the model have no momentum dependencies, which might improve the extremity of the calculated spectra at $0^\circ$ and $160^\circ$. This work is in progress.

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FIG. 1: (color online) Density dependence of the symmetry energy for the NL-RMF models.
FIG. 2: (color online) Calculated average excitation energy ($<\epsilon_{exc}>$), angular momentum ($<L>$) and mass number of prefragments ($<A_{exc}>$) as a function of transition time ($t_{tr}$) after the RBUU (with $NL\rho$) initiated by a proton on Pb at 1.2 GeV.
FIG. 3: (color online) Neutron energy-angle double differential cross sections for p+Pb at 1.2 GeV calculated by RBUU(with NLρ)+SM with different transition times ($t_{tr}$). Data (solid circles with error bars) are from [41]. For clarity, only the histograms and the data for the smallest angles are given in absolute value. The other ones have been multiplied by $10^{-1}$, $10^{-2}$, ... for other angles in increasing order.
FIG. 4: (color online) Evaporation neutron double differential cross sections for p+Pb reactions at 0.8 GeV calculated by RBUU(with NL)+SM. The lines are the results obtained by changing the symmetry coefficients ($\gamma_{\text{sym}}$) in the SM code.
FIG. 5: (color online) Neutron energy-angle double differential cross sections for p+Fe interactions at 0.8 GeV (left panel), 1.2 GeV (middle panel) and 1.6 GeV (right panel) as compared to RBUU calculations (without evaporation). The dashed and thin lines denote calculations with \( N_L \) and \( N_L\rho \), respectively. The thick lines show calculations without mean field. The data [41] (solid circles with error bars) and the histograms for the smallest angles are given in absolute value. The other ones have been multiplied by \( 10^{-1} \), \( 10^{-2} \), \ldots \) for other angles in increasing order.
FIG. 6: (color online) Same as Fig. 5, but for p+Pb reactions. Solid triangles with error bars (left panel) are data taken from [42].
FIG. 7: (color online) Same as Fig. 5, but the solid and dashed lines denote the RBUU+SM with $NL_\rho$ and $NL$, respectively.
FIG. 8: (color online) Same as Fig.7, but for p+Pb reactions. Solid triangles with error bars (left panel) are data taken from [42].
FIG. 9: (color online) Same as Fig.5, but the dashed and solid lines denote the RBUU+SM results with $NL\rho\delta$ and $NL\rho$, respectively.
FIG. 10: (color online) Same as Fig. 9, but for p+Pb reactions. Solid triangles with error bars (left panel) are data taken from [42].
FIG. 11: (color online) Normalized distribution of pre-equilibrium neutrons for p+Pb reactions at 0.8 GeV. The lines represent the RBUU (without SM) with different NL-RMF models.
FIG. 12: (color online) Neutron energy spectrum at $0^\circ$ and $10^\circ$ from 0.8 GeV (left panel), 1.2 GeV (middle panel) and 1.6 GeV (right panel) proton interactions with Fe (top panels) and Pb (bottom panels) targets. The thin lines denote RBUU calculations with $NL\rho$, while the thick lines are those without mean field. The dashed lines denote RBUU (with $NL\rho$) calculations that include the medium modified angular distributions of Ref. [45].