Flavor structure with multi moduli in 5D SUGRA

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Abstract. We investigate 5-dimensional supergravity on $S^1/Z_2$ with a physical $Z_2$-odd vector multiplet, which yields an additional modulus other than the radion. We find additional terms in the 4-dimensional effective theory that are peculiar to the multi moduli case. Such terms can make the soft masses are non-tachyonic and almost flavor-universal at tree-level, in contrast to the single modulus case. This provides a new possibility to solve the SUSY flavor problem.

Keywords: 5-dimensional supergravity, supersymmetry breaking

PACS: 04.50.-h,12.60.Jv

INTRODUCTION

One of the simplest setup for the extra-dimensional model with supersymmetry (SUSY) is five-dimensional (5D) supergravity compactified on $S^1/Z_2$, which is extensively studied in a large number of papers. Most of such works assume or consider a situation that the radius of the extra dimension $r$ is determined by the vacuum expectation value (VEV) of a single chiral multiplet $T$ (the radion multiplet), that is, $\pi r = \text{Re} \langle T \rangle$. In general, however, there are also cases where $r$ is determined by VEVs of more than one chiral multiplets, such as

$$\pi r = \mathcal{F}(\text{Re} \langle T^1 \rangle, \text{Re} \langle T^2 \rangle, \cdots),$$

where $\mathcal{F}$ is some function, and $T^i$ ($i = 1, 2, \cdots, n$) are chiral multiplets which we call the moduli in this talk.

The single modulus case ($n = 1$) corresponds to the case that there are no physical 5D vector multiplets whose scalar components have zero-modes. The radion mode comes from the zero-mode of the extra-dimensional component of the fünfbein $e^i_4$. When some scalar components of physical 5D vector multiplets have zero-modes, the radion mixes with them to form chiral multiplets in the four-dimensional (4D) effective theory. Thus $n$ moduli consist of zero-modes for $e^1_4$ and $(n - 1)$ scalar components of the 5D vector multiplets. In this case the orbifold radius $r$ is given by a combination of VEVs of the $n$ moduli as shown (1).

In the multi moduli case ($n \geq 2$), the low-energy physics can be changed from that in the single modulus case due to the existence of the physical 5D vector multiplets. In this talk we investigate 4D effective theory of 5D supergravity in the multi moduli case and see the difference from that in the single modulus case. Especially we focus on the flavor structure of the soft SUSY breaking masses [1].

SETUP

The off-shell formulation is useful to describe the 5D supergravity action. Here we adopt the conformal supergravity formulation developed by Ref. [2]. In this formulation, the $(n - 1)$ physical 5D vector multiplets are expressed as $n$ off-shell vector multiplets by adding unphysical degrees of freedom. One combination of the $n$ vector components is identified with the graviphoton, which belongs to the supergravity multiplet in the on-shell description.

As the simplest case, we consider the two moduli case. Namely we introduce two vector multiplets $\gamma^1 = (V^1, \Sigma^1)$, $\gamma^2 = (V^2, \Sigma^2)$ which include the moduli as zero-modes. We also introduce a hypermultiplet $(X, X^c)$ which is relevant to the SUSY breaking besides the matter hypermultiplets $(Q_i, Q_i^c)$. The index $i$ run over quarks, leptons and Higgses. We use $N = 1$ superfield notation to express each 5D multiplet. $V^1$, $V^2$ are $N = 1$ vector multiplets while the others are chiral multiplets. Among them, $\Sigma^1$, $\Sigma^2$, $X$ and $Q_i$ are even under the $Z_2$-parity around the orbifold boundaries, while the others are $Z_2$-odd. Thus the former have zero-modes $T^1$, $T^2$, $X_0$ and $Q_{0i}$, respectively [3].

In 5D supergravity, every mass scale in the bulk Lagrangian is introduced as a gauge coupling constant (in the unit of the 5D Planck mass) for some vector multiplet

1 Talks given by Y.S. at PASCOS’08 (Perimeter Institute, Canada, June 2-6, 2008) and at SUSY’08 (Seoul, Korea, June 16-21, 2008).

2 The standard model gauge multiplets are contained in 5D vector multiplets whose vector components are $Z_2$-even. For simplicity, we omit them in this talk because they are irrelevant to the tree-level soft masses.
whose lowest component is $Z_2$-even. Thus we assume that $Q_0$ and $X_i$ are charged for $Y$ with the gauge couplings $c_i$ and $c_X$ in order to introduce the bulk mass parameters for the hypermultiplets.\(^3\) The localization of the wave functions for $Q_0$ and $X_0$ in the bulk is controlled by these bulk mass parameters $c_i$ and $c_X$ and the hierarchical structure of the Yukawa couplings can be realized just in the similar way to Ref. \([4]\). Besides these gauging, 5D supergravity is characterized by a cubic polynomial called the norm function:

$$\mathcal{N}(Y) \equiv C_{IJK} Y^I Y^J Y^K.$$  \hfill (2)

A real constant tensor $C_{IJK}$ is completely symmetric for the indices $I, J, K$.

**4D effective theory**

The multi moduli case has not been studied very much just because of a technical reason. In that case the derivation of the 4D effective theory becomes much more complicated than that in the single modulus case. In our previous work \([3]\), we developed a systematic method to derive the 4D effective theory for general 5D supergravity model, which is based on an $N = 1$ superspace description of 5D conformal supergravity \([3]\) and developed in subsequent works \([6]\). The procedure is as follows. We start from the $N = 1$ off-shell description of 5D action. After some gauge transformation, we drop kinetic terms for $Z_2$-odd multiplets which are negligible at low energy. Then these multiplets play a role of Lagrange multiplier and their equations of motion extract zero-modes from the $Z_2$-even multiplets. After these steps, we obtain the following Kähler potential in the 4D effective theory.

$$\Omega \equiv -3e^{-K_{\text{eff}}/3} = \mathcal{N}^{1/3} \left\{ -\frac{3}{4} \sum_i Z_i \left| Q_0 \right|^2 + \sum_i \Omega_X \left| Q_0 \right|^2 \left| X_0 \right|^2 \right\} + \mathcal{O} \left( \left| Q_0 \right|^4, \left| X_0 \right|^4 \right),$$  \hfill (3)

where

$$Z_i = \frac{1 - e^{-2c_i \Re T^1}}{c_i \Re T^1}.$$  \hfill (4)

are coefficients of $\left| Q_0 \right|^2$ in the effective Kähler potential, i.e.,

$$K_{\text{eff}} = -3 \mathcal{N}^{1/3} \left( \sum_i Z_i \left| Q_0 \right|^2 + \ldots \right).$$

The arguments of $\mathcal{N}$ and $\Omega_X$ are $(\Re T^1, \Re T^2)$. The essential difference between the single and multi moduli cases appears in $\Omega_X$.

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**SOFT SUSY BREAKING MASSES**

In this talk, we assume that $\langle X_0 \rangle \ll 1$ in the unit of the 5D Planck mass and the F-term of $X_0$ is the dominant source of SUSY breaking. In Ref. \([1]\), we provide a specific example that realizes such a situation explicitly. Under this assumption, the soft masses are expressed as

$$m_{\text{soft}}^2 \simeq -\left| F_+^i \right|^2 \frac{\tilde{\Omega}_X}{Z_i}.$$  \hfill (5)

Let us first review the single modulus case. In this case, we obtain

$$\Omega_X = \frac{\left( \Re T^1 \right)^3}{3(c_i + c_X)},$$  \hfill (6)

We can see from \((6)\) that the soft masses are tachyonic irrespective of the values of the bulk mass parameters $c_i$ and $c_X$. Thus we have to choose $c_i$ and $c_X$ such that $e^{-2c_i \Re T^1} \gg 1$ and $e^{-2c_X \Re T^1} \ll 1$ so that the tree-level contribution \((5)\) is exponentially suppressed and the quantum effects dominate. Such quantum effects can save the tachyonic masses at tree level. From the 5D point of view, the condition $e^{-2c_i \Re T^1} \gg 1$ and $e^{-2c_X \Re T^1} \ll 1$ means that the wave functions for $Q_0$ and $X_0$ are localized around the opposite boundaries of $S^1/Z_2$. Namely the SUSY breaking sector is geometrically sequestered from our visible sector. This is the situation usually considered in most of the works on 5D supergravity models.

In the multi moduli case, the situation is quite different. The expression of $\tilde{\Omega}_X$ becomes more complicated.

$$\tilde{\Omega}_X = \frac{\mathcal{N}_1 \mathcal{N}_1 - 2 \mathcal{M}_1^2}{\mathcal{N}_1 \mathcal{N}_1 - 2 \mathcal{M}_1^2} \left( \frac{1 - e^{-2c_i \Re T^1}}{(1 - e^{-2c_X \Re T^1})}(1 - e^{-2c_X \Re T^1}) \right),$$  \hfill (7)

where $\mathcal{N}_1(X) \equiv \partial \mathcal{N} / \partial X$ and $\mathcal{M}_1(X) \equiv \partial^2 \mathcal{N} / (\partial X)^2$. The first term has a similar structure to the single modulus case, but now it can be negative if $3 \mathcal{N}_1 \mathcal{N}_1 - 2 \mathcal{M}_1^2 < 0$.\(^4\) Even if it is positive, $\tilde{\Omega}_X$ becomes negative when the second term dominates over the first term. Especially when $e^{-2c_i \Re T^1} \gg 1$ and $e^{-2c_X \Re T^1} \ll 1$, the second term of \((7)\) dominates and furthermore the $c_i$-dependence is cancelled in \((5)\).

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\(^3\) Of course we can also gauge the hypermultiplets by $Y^2$.

\(^4\) Note that we have to choose the norm function such that $\mathcal{N}_1 > 0$ in order to obtain the correct signs of kinetic terms for the scalar components of the 5D vector multiplets.
Namely we obtain non-tachyonic and flavor universal soft masses. As mentioned above, this situation corresponds to the geometrical separation of the SUSY breaking source $F^X$ from our visible world.

In contrast to the single modulus case, the tree-level contribution to the soft masses (5) remains finite even when $Q_0^i$ and $X_0$ are localized around the opposite boundaries. This fact indicates the existence of some heavy modes that couple to both $Q_0^i$ and $X_0$. After integrating them out, additional contribution to the contact terms $|Q_0^i|^2|X_0|^2$ in the Kähler potential appears. This corresponds to the difference between (6) and (7). Such heavy modes are the Kaluza-Klein modes for the $Z_2$-odd vector multiplets $(V_1, V_2)$. Notice that one combination of $V_1$ and $V_2$ is identified with the graviphoton multiplet, which does not contribute to the contact terms $|Q_0^i|^2|X_0|^2$. Thus the additional contribution mentioned above exists only in the multi moduli case.

**SUMMARY**

We study 4D effective theory of 5D supergravity on $S^1/Z_2$ in the case that the orbifold radius is determined by VEVs of more than one chiral multiplets. In such a multi moduli case, the flavor structure of the soft SUSY breaking parameters is quite different from the single modulus case that is usually considered. The additional terms appear in the effective Kähler potential and it can make the soft masses non-tachyonic and almost flavor-universal in contrast to the single modulus case. This fact provides a new possibility to solve the SUSY flavor problem.

In this talk, we do not discuss the other soft SUSY breaking parameters, such as the A-terms and the gaugino masses. In order to study them, we need to consider a specific model that stabilizes the moduli and breaks SUSY. We provide such a model in Ref. [1] and discuss the possibility to solve the SUSY flavor problem. Detailed phenomenological analysis based on this possibility is in progress.

**ACKNOWLEDGMENTS**

This work was supported in part by the Japan Society for the Promotion of Science for Young Scientists No.182496 (H.A.), and by the Special Postdoctoral Researchers Program at RIKEN (Y.S.).

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