An effective associative memory for pattern recognition

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Abstract

Neuron models of associative memory provide a new and prospective technology for reliable date storage and patterns recognition. However, even when the patterns are uncorrelated, the efficiency of most known models of associative memory is low. We developed a new version of associative memory with record characteristics of its storage capacity and noise immunity, which, in addition, is effective when recognizing correlated patterns.

1 Introduction

Conventional neural networks can not efficiently recognize highly correlated patterns. Moreover, they have small memory capacity. Particularly, the Hopfield neural network \(^1\) can store only \(p_0 \approx N/2 \ln N\) randomized \(N\)-dimensional binary patterns. When the patterns are correlated this number falls abruptly. Few algorithms that can be used in this case (e.g., the projection matrix method \(^2\)) are rather cumbersome and do not allow us to introduce a simple training principle \(^3\). We offer a simple and effective algorithm that allows us to recognize a great number of highly correlated binary patterns even under heavy noise conditions. We use a parametrical neural network \(^4\)-\(^6\) that is a fully connected vector neural network similar to the Potts-glass network \(^7\) or optical network \(^8\),\(^9\). The point of the approach is given below.

Our goal is to recognize \(p\) distorted \(N\)-dimensional binary patterns \(\{Y^\mu\}, \mu \in 1, p\). The associative memory that meets this aim works in the following way. Each pattern \(Y^\mu\) from the space \(R^N\) is put in one-to-one correspondence with an image \(X^\mu\) from a space \(\mathbb{R}\) of greater dimensionality (Sect. \(3\)). Then, the set of images \(\{X^\mu\}\) is used to build a parametrical neural network (Sect. \(2\)). The recognition algorithm is as follows. An input binary vector \(Y \in R^N\) is replaced by corresponding image \(X \in \mathbb{R}\), which is recognized by the parametrical neural network. Then the result of recognition is mapped back into the original \(N\)-dimensional space. Thus, recognition of many correlated binary patterns is reduced to recognition of their images in the space \(\mathbb{R}\). Since parametrical neural network has an extremely large memory capacity (Sect. \(2\)) and the mapping process \(Y \rightarrow X\) allows us to eliminate the correlations between patterns almost completely (Sect. \(3\)), the problem is simplified significantly.

2 Parametrical neural network and their recognition efficiency

Let us described the parametrical neural network (PNN) \(^4\)-\(^6\). We consider fully connected neural network of \(n\) vector-neurons (spins). Each neuron is described by unit vectors \(\bar{x}_i = x_i \bar{e}_i\), where an amplitude \(x_i\) is equal \(\pm 1\) and \(\bar{e}_i\) is the \(l\)th unit vector-column of \(Q\)-dimensional space: \(i = 1,\ldots, n; 1 \leq l \leq Q\). The state of the whole network is determined by the set of vector-columns \(X = (\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n)\). According \(^5\), we define the Hamiltonian of the network the same as in the Hopfield model

\[
H = -\frac{1}{2} \sum_{i,j=1}^{n} \bar{x}_i^T \bar{T}_{ij} \bar{x}_j,
\]
where \( \vec{x}^+ \) is the \( Q \)-dimensional vector-row. Unlike conventional Hopfield model, where the interconnections are scalars, in Eq.(1) an interconnection \( \vec{T}_{ij} \) is a \((Q \times Q)\)-matrix constructed according to the generalized Hebb rule \[7\],

\[
\vec{T}_{ij} = (1 - \delta_{ij}) \sum_{\mu=1}^{p} \vec{x}_{i}^{\mu} \vec{x}_{j}^{\mu+},
\]
from \( p \) initial patterns:

\[
X^{\mu} = (\vec{x}_{1}^{\mu}, \vec{x}_{2}^{\mu}, ..., \vec{x}_{n}^{\mu}), \quad \mu = 1, 2, ..., p.
\]

It is convenient to interpret the network with Hamiltonian (1) as a system of interacting \( Q \)-dimensional spins and use the relevant terminology. In view of (1)-(3), the input signal arriving at the \( i \)th neuron (i.e., the local field that acts on \( i \)th spin) can be written as

\[
\vec{h}_{i} = \sum_{j=1}^{n} \vec{T}_{ij} \vec{x}_{j} = \sum_{i=1}^{Q} A_{i}^{(i)} \vec{e}_{l},
\]
where

\[
A_{i}^{(i)} = \sum_{j(\neq i)}^{n} \sum_{\mu=1}^{p} (\vec{e}_{i}^{\mu})(\vec{x}_{j}^{\mu} \vec{x}_{j}^{\mu+}), \quad l = 1, ..., Q.
\]

The behavior of the system is defined in the natural way: under action of the local field \( \vec{h}_{i} \) the \( i \)th spin gets orientation that is as close to the local field direction as possible. In other words, the state of the \( i \)th neuron in the next time step, \( t+1 \), is determined by the rule:

\[
\vec{x}_{i}(t+1) = \text{max} \vec{x}_{\text{max}}, \quad \vec{x}_{\text{max}} = \text{sgn} \left( A_{i}^{(i)} \right),
\]
where \( \text{max} \) denotes the greatest in modulus amplitude \( A_{i}^{(i)} \) in (5).

The evolution of the network is a sequence of changes of neurons states according to (6) with energy (1) going down. Sooner or later the network finds itself at a fixed point.

Let us see how efficiently PNN recognizes noisy patterns. Let the distorted \( m \)th pattern \( X^{m} \) come to the system input, i.e. the neurons are in the initial states described as \( \vec{x}_{i} = \hat{a}_{i} \hat{b}_{i} \vec{x}_{i}^{m} \). Here \( \hat{a}_{i} \) is the multiplicative noise operator, which changes the sign of the amplitude \( x_{i}^{m} \) of the vector-neuron \( \vec{x}_{i}^{m} = x_{i}^{m} \vec{e}_{i}^{m} \) with probability \( a \) and keeps it the same with probability \( 1 - a \); the operator \( \hat{b}_{i} \) changes the basis vector \( \vec{e}_{i}^{m} \in \{\vec{e}_{j}^{m}\}^{Q}_{1} \) by any other from \( \{\vec{e}_{j}^{m}\}^{Q}_{1} \) with probability \( b \) and retains it unchanged with probability \( 1 - b \). In other words, \( a \) is the probability of an error in the sign of the neuron ("+" in place of "-") and \( b \) is the probability of an error in the vector state of the neuron. The network recognizes the reference pattern \( X^{m} \) correctly, if the output of the \( i \)th neuron defined by Eq.(6) is equal to \( \vec{x}_{i}^{m} \), that is \( \vec{x}_{i} = \vec{x}_{i}^{m} \). Otherwise, PNN fails to recognize the pattern \( X^{m} \). According to the Chebyshev-Chernov method \[10\] (for such problems it is described comprehensively in \[4\]-\[6\], \[9\]) the probability of recognition failure is

\[
P_{\text{err}} \leq n \exp \left[ -\frac{nQ^{2}}{2p} (1 - 2a)^{2}(1 - b)^{2} \right].
\]

The inequality sets the upper limit for the probability of recognition failure for PNN. The memory capacity of PNN (i.e., the greatest number of patterns that can be recognized) is found from (7):

\[
p_c = nQ^{2}(1 - 2a)^{2}(1 - b)^{2}.
\]

Comparison of (8) with similar expressions for the Potts-glass neural network \[7\] and the optical network \[8, 9\] shows that the memory capacity of PNN is approximately twice as large as the memories of both aforementioned models. That means that under other conditions being equal its recognition power is 20-30\% higher.

It is seen from (7) that with growing \( Q \) the noise immunity of PNN increases exponentially. The memory capacity also grows: it is \( Q^{2} \) times as large as that of the Hopfield network. For example, if \( Q = 32 \), the 180-neuron PNN can recognize 360 patterns \((p/n = 2)\) with 85\% noise-distorted components (Fig.1). With smaller distortions, \( b = 0.65 \), the same network can recognize as many as 1800 patterns
\( (p/n = 10), \) etc. Let us note, that some time ago the restoration of 30% noisy pattern for \( p/n = 0.1 \) was a demonstration of the best recognition ability of the Hopfield model [11].

Of course, with regard to calculations, PNN is more complex than the Hopfield model. On the other hand the computer code can be done rather economical with the aid of extracting bits only. It is not necessary to keep a great number of \( (Q \times Q) \)-matrices \( \hat{T}_{ij} \) (2) in your computer memory. Because of the complex structure of neurons, PNN works \( Q \)-times slower than the Hopfield model, but it makes possible to store \( Q^2 \)-times greater number of patterns. In addition, the Potts-glass network operates \( Q \)-times slower than PNN.

Fig.2 shows the recognition reliability \( \bar{P} = 1 - P_{err} \) as a function of the noise intensity \( b \) when the number of patterns is twice the number of neurons \( (p = 2n) \) for \( Q = 8, 16, 32 \). We see that when the noise intensity is less than a threshold value

\[
b_c = 1 - \frac{2}{Q} \frac{\sqrt{p}}{n},
\]

the network demonstrates reliable recognition of noisy patterns. We would like to use these outstanding properties of PNN for recognition of correlated binary patterns. The point of our approach is given in next Sections.

3 Mapping algorithm

Let \( Y = (y_1, y_2, ..., y_N) \) be an \( N \)-dimensional binary vector, \( y_i = \{ \pm 1 \} \). We divide it mentally into \( n \) fragments of \( k + 1 \) elements each, \( N = n(k + 1) \). With each fragment we associate an integer number \( \pm l \) according the rule: the first element of the fragment defines the sign of the number, and the other \( k \) elements determine the absolute value of \( l \):

\[
l = 1 + \sum_{i=1}^{k} (y_i + 1) \cdot 2^{i-2}; \quad 1 \leq l \leq 2^k.
\]

Now we associate each fragment with a vector \( \vec{x} = \pm \vec{e}_l \), where \( \vec{e}_l \) is the \( l \)th unit vector of the real space \( \mathbb{R}^Q \) and \( Q = 2^k \). We see that any binary vector \( Y \in \mathbb{R}^N \) one-to-one corresponds to a set of \( n \) \( Q \)-dimensional unit vectors, \( X = (\vec{x}_1, \vec{x}_2, ..., \vec{x}_n) \), which we call the internal image of the binary vector \( Y \). (In the next Section we use the internal images \( X \) for PNN construction.) The number \( k \) is called a mapping parameter.

For example, the binary vector \( Y = (-1, 1, -1, -1, 1, -1, 1) \) can be split into two fragments of four elements: \((-11-11)\) and \((1-1-1-1)\); the mapping parameter \( k \) is equal 3, \( k = 3 \). The first fragment \(("-2" \) in our notification) corresponds to the vector \(-\vec{e}_2 \) from the space of dimensionality \( Q = 2^3 = 8 \), and the second fragment \(("+5" \) in our notification) corresponds to the vector \(+\vec{e}_5 \in \mathbb{R}^8 \). The relevant mapping can be written as \( Y \rightarrow X = (-\vec{e}_2, +\vec{e}_5) \).

It is important that the mapping is biunique, i.e., the binary vector \( Y \) can be restored uniquely from its internal image \( X \). It is even more important that the mapping eliminates correlations between the original binary vectors. For example, suppose we have two 75% overlapping binary vectors

\[
Y_1 = (1, -1, -1, -1, -1, -1, 1)
\]

\[
Y_2 = (1, -1, -1, -1, 1, 1, -1, -1, 1)
\]

Let us divide each vector into four fragments of two elements. In other words, we map these vectors with the mapping parameter \( k = 1 \). As a result we obtain two internal images \( X_1 = (+\vec{e}_1, -\vec{e}_1, -\vec{e}_1, -\vec{e}_2) \) and \( X_2 = (+\vec{e}_1, -\vec{e}_2, +\vec{e}_1, -\vec{e}_2) \) with \( \vec{e}_i \in \mathbb{R}^2 \). The overlapping of these images is 50%. If the mapping parameter \( k = 3 \) is used, the relevant images \( X_1 = (+\vec{e}_1, -\vec{e}_5) \) and \( X_2 = (+\vec{e}_5, +\vec{e}_5) \) with \( \vec{e}_i \in \mathbb{R}^8 \) do not overlap at all.

4 Recognition of the correlated binary patterns

In this Section we describe the work of our model as a whole, i.e. the mapping of original binary patterns into internal images and recognition of these images with the aid of PNN. For a given mapping parameter \( k \) we apply the procedure from Sect. 3 to a set of binary patterns \( \{Y^\mu\} \in \mathbb{R}^N, \mu \in \mathbb{T, p} \). As a result we
obtain a set of internal images \( \{X^\mu\} \in \mathbb{R} \), allowing us to build PNN with \( n = N/(k + 1) \) Q-dimensional vector-neurons, where \( Q = 2^k \). Note, the dimension \( Q \) of vector-neurons increases exponentially with \( k \) increasing, and this improves the properties of PNN.

In further analysis we use so-called biased patterns \( Y^\mu \) whose components \( y^\mu_i \) are either +1 or -1 with probabilities \((1 + \alpha)/2\) and \((1 - \alpha)/2\) respectively \((-1 \leq \alpha \leq 1)\). The patterns will have a mean activation of \( Y^\mu = \alpha \) and a mean correlation \( \langle Y^\mu, Y^{\mu'} \rangle = \alpha^2 \), \( \mu \neq \mu' \). Our aim is to recognize the \( \text{mth} \) noise-distorted binary pattern \( Y^m = (s_1y^m_1, s_2y^m_2, ..., s_Ny^m_N) \), where a random quantity \( s_i \) changes the variable \( y^m_i \) with probability \( s \), and the probability for \( y^m_i \) to remain the same is \( 1 - s \). In other words, \( s \) is the distortion level of the \( \text{mth} \) binary pattern. Mapped into space \( \mathbb{R} \), this binary vector turns into the \( \text{mth} \) noise-distorted internal image \( X^m \), which has to be recognized by PNN. Expressing multiplicative noises \( a \) and \( b \) as functions of \( s \) and substituting the result in (7), we find that the probability of misrecognition of the internal image \( X^m \) is

\[
P_{\text{err}} = n \left( \text{ch}(\nu \alpha^3) - \alpha \text{sh}(\nu \alpha^3) \right) \cdot \exp \left[ -\frac{\nu}{2\beta} (1 + \beta^2 \alpha^3) \right],
\]

where

\[
\nu = n(1 - 2s)^2(1 - s)^k, \quad \beta = p[A/(1 - s)]^k, \quad A = (1 + \alpha^2)[1 + \alpha^2(1 - 2s)]/4.
\]

Expression (9) describes the Hopfield model when \( k = 0 \). In this case, even without a bias \((\alpha = 0)\) the memory capacity does not exceed a small value \( p_0 \approx N/2 \ln N \). However, even if small correlations \((\alpha > N^{-1/3})\) are present, the number of recognizable patterns is \((N\alpha^3)\)-times reduced. In other words, the network almost fails to work as associative memory. The situation changes noticeably when parameter \( k \) grows: the memory capacity increases and the influence of correlations decreases. In particular, when \( k \) grows, \( \nu \alpha^3 < 1 \), we estimate memory capacity from (9) as

\[
p = p_0 \frac{(1 - s)^2/A}{k + 1}.
\]

Let us return to the example from [11] with \( \alpha = 0 \). When \( k = 5 \), in the framework of our approach one can use 5-times greater number of randomized binary patterns, and when \( k = 10 \), the number of the patterns is 80-times greater.

When the degree of correlations is high \((\nu \alpha^3 > 1)\), the memory capacity is somewhat smaller:

\[
p = \alpha^{-3}(1 - s)/A^{k}.
\]

Nevertheless, in either case increasing \( k \) gives us the exponential growth of the number of recognizable patterns and a rise of recognition reliability.

Fig.3 shows the growth of the number of recognizable patterns when \( k \) increases for distortions \( s \) of 10\% to 50\% \((\alpha = 0)\).

Fig.4 demonstrates how the destructive effect of correlations diminishes when \( k \) increases: the recognition power of the network is zero when \( k \) is small; however, when the mapping parameter \( k \) reaches a certain critical value, the misrecognition probability falls sharply (the curves are drawn for \( \alpha = 0, 0.2, 0.5, 0.6, p/N = 2 \) and \( s = 30\% \)). Our computer simulations confirm these results.

5 Concluding remarks

Our algorithm using the large memory capacity of PNN and its high noise immunity, allows us to recognize many correlated patterns reliably. The method also works when all patterns have the same fragment or fragments. For instance, for \( p/N = 1 \) and \( k = 10 \), this kind of neural network permits reliable recognition of patterns with 40\% coincidence of binary components.

When initially there is a set of correlated vector-neuron patterns which must be stored and recognized, the following algorithm can be used to suppress correlations. At first, the patterns with vector coordinates of original dimensionality \( q_1 \) are transformed into binary patterns. Then they are mapped into patterns with vector neurons of dimensionality \( q_2 < q_1 \). A PNN is built using patterns with \( q_2 \)-dimensional vector neurons. This double mapping eliminates correlations and provides a reliable pattern recognition.

In conclusion we would like to point out that a reliable recognition of correlated patterns requires a random numbering of their coordinates (naturally, in the same fashion for all the patterns). Then the
constituent vectors do not have large identical fragments. In this case a decorrelation of the patterns can be done with a relatively small mapping parameter $k$ and, therefore, takes less computation time.

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Fig.1. Process of recognition of letter "A" by PNN at $p/n = 2, Q = 32, b = 0.85$. Noise-distorted pixels are marked in gray.

Fig.2. Recognition reliability of PNN $\bar{P} = 1 - P_{err}$ as a function of noise intensity $b$.

Fig.3. Growth of memory capacity with the mapping parameter $k$ ($s = 0.1 - 0.5$).

Fig.4. Fall of the misrecognition probability with the mapping parameter $k$. 