Quantum Character of Black Holes

Adam D. Helfer
Department of Mathematics, University of Missouri, Columbia, MO 65211

Abstract

Black holes are extreme manifestations of general relativity, so one might hope that exotic quantum effects would be amplified in their vicinities, perhaps providing clues to quantum gravity. The commonly accepted treatment of quantum corrections to the physics around the holes, however, has provided only limited encouragement of this hope. The predicted corrections have been minor (for macroscopic holes): weak fluxes of low-energy thermal radiation which hardly disturb the classical structures of the holes. Here, I argue that this accepted treatment must be substantially revised. I show that when interactions among fields are taken into account (they were largely neglected in the earlier work) the picture that is drawn is very different. Not only low-energy radiation but also ultra-energetic quanta are produced in the gravitationally collapsing region. The energies of these quanta grow exponentially quickly, so that by the time the hole can be said to have formed, they have passed the Planck scale, at which quantum gravity must become dominant. The vicinities of black holes are windows on quantum gravity.

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1 Introduction

Quantum theory and general relativity are at once practically compatible and theoretically irreconcilable. While there is a profound difficulty in melding the theories, it is not apparent at ordinary scales. It is expected to become pronounced rather in extreme circumstances, in the Planck regime.

The difficulty in reconciling the theories derives from a fundamental incompatibility in their hypotheses. According to quantum theory, the position and momentum of an object can never be known simultaneously with unlimited accuracy; but according to general relativity, we should always be able to imagine space–time populated with locally inertial observers who can make physical measurements to arbitrary accuracy and without disturbing the underlying geometry of space–time. For measurements at ordinary scales, this conflict is utterly insignificant. But for measurements at very fine scales we must use, according to quantum theory, particles with high momenta. And at some point — the Planck threshold — the momenta become so high that their gravitational effects interfere with the measurements being attempted. At such scales it becomes impossible to give operational significance to ordinary notions of space, time, momentum and energy [1]. The occurrence of Planckian or “trans-Planckian” (past the Planck scale) quantities in a conventional physical theory is a sign it has been applied beyond its realm of credibility.

I shall show here that, when space–time gravitationally collapses to form a black hole, quantum theory predicts the production of particles at, and the scattering of particles to, Planckian energies. This means the theory predicts its own inapplicability, and the onset of an essentially quantum-gravitational regime, in the vicinity of a black hole. The affected region of space–time extends outwards from the horizon a distance comparable to the size of the hole. These results do not allow
us to say precisely what the quantum-gravitational effects in the vicinities of black holes should be — for that, we would need a theory of quantum gravity. But we do have an unequivocal prediction that the quantum-gravitational effects should be significant in a substantial volume of space–time. Therefore observations of particles from the vicinities of black holes should be able to provide us with clues about quantum gravity.

The picture that is drawn here is very different from the accepted one (due to Hawking’s pioneering analysis [2, 3, 4]), in which black holes are essentially classical objects and quantum corrections to their structure are minor. The reason for this discrepancy is that concerns about the accepted theory, which had generally been dismissed as negligible, turn out to be very much on point and to alter the picture significantly. Trans-Planckian “virtual” effects (which had been considered an awkward peripheral feature of the conventional theory) and interactions between quantum fields (which had been largely neglected\(^1\)) combine to produce real Planckian effects.

A rather different argument, but reaching parallel conclusions, has been given elsewhere [6].

Here is the plan of the paper. Sections 2 and 3 review the relevant structure of a gravitationally collapsing space–time and of the Hawking process. They emphasize the origin of the Hawking quanta in ultra-high frequency vacuum fluctuations in the distant past; as the associated virtual quanta propagate through the gravitationally collapsing region they are distorted and red-shifted by an exponentially growing factor. Since they emerge with frequencies of the order of the characteristic Hawking frequency, the precursors’ frequencies grow exponentially.

Section 4 contains the computation of the effect of interactions, in the case of quantum electrodynamics. I show there that there is a non-trivial first-order amplitude to produce electron–positron pairs, at the exponentially growing energies associated with the Hawking quanta’s precursors. One can think of the process as being due to the quantum-electrodynamic dressing of the vacuum by virtual triples, each consisting of a photon and an electron–positron pair. For an ultra-energetic such triple, the photon may pass through the collapse region and be distorted to a Hawking quantum which can no longer recombine with the electron–positron pair to produce the proper dressing of the vacuum. The result is a real Hawking photon together with a real ultra-energetic electron–positron pair.

Section 5 discusses the consequences of the analysis. The exponential growth in the energy scales means that one very quickly passes the regime in which reliable theoretical computations of quantum-field-theoretic processes are possible, and in a few dozen e-foldings has arrived at the Planck regime, where the entire theory breaks down. Thus our main conclusion is that current theory is unable to make reliable predictions about quantum physics in the neighborhood of a black hole. Indeed, we suggest that observations of the vicinities of black holes may be able to provide us with evidence of the behavior of quantum field theories at ultra-high energies and the character of quantum gravity.

Conventions and terminology. We shall use natural units, so \(c, G\) and \(\hbar\) are unity. We shall need to discuss processes with energies approaching, but not at, the Planck energy; we shall refer to these as ultra-energetic. For simplicity, in most of the paper we consider only an isolated uncharged spherically symmetric black hole;\(^2\) its mass will be \(M\). Conventions for quantum fields are those of Schweber [7].

\(^1\)An important attempt to incorporate interactions is due to Gibbons and Perry [5]. The relation of their approach to the present work is discussed in the appendix.

\(^2\)In subsection 4.4 we shall explain that the results here hold too for holes with angular momentum and charge.
Figure 1: A diagram of a black-hole space–time, suppressing angular variables. (The left-hand edge is the spatial origin.) Time increases upwards, and lines at \(45^\circ\) represent the paths of radial light rays. The scale has been distorted so that the entire space–time, and some ideal points at infinity, can be represented. The region occupied by the collapsing matter is shaded. The event horizon \(\mathcal{H}\) is the dashed line; the black hole itself is the set of points at and above this. The dotted line represents a radial light ray beginning at a point on \(I^-\) (an ideal set in the past, coordinatized by the advanced time \(v\)), moving radially inwards and passing through the spatial origin (where, in the diagram, it appears to reflect from the left-hand edge), and escaping to a point on the ideal set \(I^+\) (coordinatized by \(u\)). The sets \(I^-\) and \(I^+\) are really unbounded, but, because of the distortion of scales appear in the diagram to be finite.

2 Black Holes

While the event horizon of a black hole is best known as a “point of no return,” it is a different property which is most important both in Hawking’s analysis and here. This is that light rays (and other fields) passing close to the event horizon are red-shifted, the shifts increasing exponentially as later and later rays are considered.

It is convenient to introduce a diagram in which the past and future limits of the light rays can be represented. In order to do this, we distort the scales so that these limits appear at finite positions on the diagram. It is possible to do this while retaining an accurate representation of causal relations.

This is done in Fig. 1, which suppresses the angular variables and thus represents the radial and temporal degrees of freedom. The spatial origin is at the left-hand edge, and radial light rays are at \(45^\circ\). The event horizon is the dashed line, and the points at and above this form the black hole itself.

The future limits of radial light rays are on the upper line at \(45^\circ\), coordinatized by the “retarded time” \(u\), and denoted collectively \(I^+\). (Similarly, the past limits are coordinatized by “advanced time” \(v\) and denoted \(I^-\).) One should think of the neighborhood of \(I^+\) as the locations of observers far from the collapsing object who look back into the space–time with \(u\) their appropriate measure of time. (The neighborhood of \(I^-\) is the set of locations of distant actors who might send signals into the space–time.) These external observers are, by definition, everywhere below the event
horizon. This means that later and later external observers appear, in Fig. 1, to crowd the upper-right corner. This is only the effect of the distortion of scales, however; the corresponding portion of space–time is in reality unbounded. Similarly, the set $I^+$ is really unbounded in the past (that is, towards the right-hand corner in Fig. 1), and the set $I^−$ is likewise really unbounded.

An observer at a retarded time $u$ near $I^+$, looking inwards, would see a radial light ray. Tracing this ray backwards in time, it would move inwards to the spatial origin, pass through this (appearing, in the diagram, to reflect from the left-hand edge), and emerge finally at an advanced time $v(u)$ on $I^−$ (dotted line in Fig. 1). The periods $dv$ and $du$ of a light ray at $I^−$ and $I^+$ are related by $dv = v'(u) du$, so $v'(u)$ is the red-shift suffered by the ray.

The red-shift factor $v'(u)$ plays a key role in quantum physics around black holes. It can be shown to have the asymptotic form
\[ v'(u) \simeq \exp \left( -\frac{u}{4M} \right) \quad \text{as} \quad u \to +\infty , \]
where $M$ is the mass of the hole [8, 9]. This exponential decay drives $v'(u)$ to zero very quickly. (For example, for a solar-mass object, the $\epsilon$-folding time is $\simeq 2 \times 10^{-5} \text{ s}$.) Although observers in the exterior of the black hole (by definition) never cross the event horizon and so never, strictly speaking, see the hole, the nominal time of formation of the hole may be taken as the point where $v'(u)$ has decreased to become practically indistinguishable from zero.

## 3 Fluctuations, Hawking Radiation and the Trans-Planckian Problem

Because the uncertainty principle forbids measuring all degrees of freedom of a quantum field simultaneously, one can never speak of a state of identically zero field. This means that even in a space–time unaffected by gravity the vacuum is populated by quantum fluctuations in the field, sometimes called zero-point fluctuations. While they do not contribute any real particles to the state, they can have significant indirect effects.

The lowest-order vacuum fluctuations in the absence of gravity are represented in Feynman-type diagrams by closed loops (Fig. 2). These loops suggest particles coming into existence and annihilating themselves. While this is not wholly correct — the use of the term “particle” in this context is oversimplistic, and the loops do not have any distinguished points at which one can say creation or annihilation occurs — it will not be necessary for us to refine this. In general, intermediate “particles” are called “virtual”. (While its literal significance will not be needed here, Fig. 2 corresponds to the effect of zero-point energy on the propagation of the field. This effect can be consistently subtracted when there is no explicit time-dependence, and in those cases diagrams like Fig. 2 can be neglected and are rarely drawn.)

In principle, according to quantum field theory, vacuum fluctuations occur for all field modes, including those of arbitrarily high frequencies, corresponding to virtual particles of arbitrarily high
Figure 3: One can think of propagation through the gravitationally collapsing space–time as opening some of the vacuum fluctuation loops, resulting in real particle production. Although, for fixed endpoints, one could draw vacuum fluctuation arcs occupying any portion of space–time, the dominant contributions to the Hawking process come from ones like that shown here. Ultra-high frequency fluctuations in the past propagate through the collapsing space–time, where they are both redshifted and distorted in a way which produces a low flux of real low-energy photons. (The red-shift and distortion are not represented in this diagram.) Thus while the outgoing arms of the diagram represent real particles, the portion deep within space–time has a more schematic significance.

energies. This would be a problem if the effects of arbitrarily high-energy fluctuations had always to be considered, because quantum field theory itself should break down at the Planck scale and quantum gravity should take over. However, in realistic theories, for any given process, there is an energy scale beyond which the fluctuations decouple and can be ignored. As long as this energy is below the Planck scale there is no trouble. We shall see, however, that Hawking’s model requires the use of virtual particles at arbitrarily high energies, and so its reliance on conventional quantum field theory is inappropriate.

When the quantum fields propagate through a time-dependent region of space–time, they are distorted and the balance of the vacuum fluctuations is upset. This is what, in Hawking’s analysis, gives rise to thermal radiation. We can represent this by drawing Feynman-type diagrams on top of space–time diagrams, as in Fig. 3. In these diagrams, one can think of the effect of the passage through space–time as opening some of the vacuum fluctuation loops. The results are real particles emitted, and those which escape to \( \mathcal{I}^+ \) are the Hawking radiation.

Although virtual particles can propagate throughout space–time, only certain vacuum fluctuations propagated in particular ways will be distorted in the correct manner to give rise to significant real particle production. The most important contributions to the Hawking process come from modes like those shown in Fig. 3. These originate as vacuum fluctuations in the past, propagate inwards, pass through the origin and then move outwards to \( \mathcal{I}^+ \) just before the event horizon. Hawking’s prediction of thermal radiation, although not originally expressed diagrammatically, is equivalent to the evaluation of diagrams like Fig. 3, and Hawking’s remarkable insight was (again, recast in diagrammatic form) that the main contributions would come from the modes shown there.
The red-shift of the fluctuations as they propagate through space–time and reify is crucial. The predicted frequencies of the Hawking particles are \( \sim \frac{\omega_H}{v'}(u) \simeq \omega_H \exp \left( \frac{u}{4M} \right) \). So the precursors — which should be thought of as the portion of Fig. 3 around and including the semicircular portion at the lower right — have frequencies which grow exponentially and rapidly reach the Planck scale. Then quantum-gravitational effects must take over.

This is the “trans-Planckian problem” \[10, 4\]. The prediction of thermal radiation relies on assuming that conventional physics applies to vacuum fluctuations at exponentially increasing energy scales. The scales rapidly pass the Planck threshold, at which quantum-gravitational effects must supplant conventional physics. The conventional analysis \[2, 3\] ignores this, and uses vacuum fluctuations at the exponentially increasing, trans-Planckian, scales. For us, however, the ultra-energetic portion of Fig. 3 will be of central significance.\(^3\)

### 4 Effects of Interactions

Hawking’s analysis rested on several assumptions, one of which was that interactions between quantum fields could be neglected. To my knowledge, there has been no attempt to trace through the essential logic of the analysis while taking interactions into account. And yet this is surely worthwhile, for, while interacting quantum field theories are admittedly difficult, they are well understood (at least perturbatively).

We shall consider for definiteness quantum electrodynamics, the theory of photons, electrons and positrons, but it will be apparent that the main ideas are more general. Because the coupling is weak, in most familiar situations, it is legitimate to treat this theory as a perturbation of a “bare” theory of noninteracting photons and charged particles. The interaction is determined by the vertex in Fig. 4, which may represent scattering of a charged particle by a photon, or a pair of charged particles converting to or from a photon.

We shall see that the perturbation theory in the presence of gravitational collapse is qualitatively different from that in Minkowski space, and that this difference becomes apparent at first order (in

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\(^3\)There is one approach to the trans-Planckian problem which has been much pursued in recent years and is worth mentioning here. Unruh \[11\], Corley and Jacobson \[12\] and others (see ref. \[4\] for more references and discussion) aim to resolve the problem by substituting non-standard rules for the propagations of the fields. Their goal is to “insulate” Hawking’s result from trans-Planckian physics, and this is done by radically altering the actual mechanism that is used to get that result. At the moment, work on these approaches is ad-hoc and preliminary, and the extent to which they can be said to eliminate the trans-Planckian problem is not entirely clear. If, however, one of these non-standard approaches were to prove correct (that is, could be developed into a theory which turned out to be the way the world works), then the present analysis, which relies heavily on the original Hawking model, would have to be reconsidered.
the electric charge). At this order, we might expect to draw diagrams like Fig. 5, which (intuitively, at least) would represent the production of a pair of charged particles, together with a Hawking-type photon, by a vacuum fluctuation. A diagram like this would vanish in Minkowski space, by conservation of energy–momentum, but in a gravitationally collapsing space–time some of the energy–momentum may be exchanged with the incipient black hole. We shall see that this diagram has a non-zero value, and in fact represents the pair production of ultra-energetic charged particles.

We shall now make this precise. However, we shall do so in terms of conventional operator-theoretic methods, rather than Feynman-type diagrams. (There is no standard definition of such diagrams in curved space–time, and making one and carefully connecting it with conventional field theory would be lengthy; also the operator-theoretic treatment is better suited to some aspects of the discussion.)

It should be emphasized that our goal here is to clearly establish the existence of real ultra-energetic effects. It is not to make quantitatively accurate estimates of them in very general circumstances, or even to discuss the variety of different possible effects. While such estimates and discussion are of interest in themselves, they are not of primary interest in the present work. This is because, once we know that real ultra-energetic effects exist, the exponentially increasing energies quickly move the effects beyond those which can be reliably computed. The main point of showing that the effects are real is to show that the theory itself breaks down.

We shall therefore choose to examine a particular situation in which the effects are revealed with a minimum of computational and interpretational technicalities. This will be the main point of subsection 4.2 below. First, however, we must understand what the physical vacuum is in the interacting case.
4.1 Dressing the Vacuum

It will be crucial to understand how the vacuum is altered when the interaction is “switched on.” The vacuum in question is the in-vacuum, defined as the state of lowest energy in the distant past. In this regime, we shall assume that space–time, at least exterior to the collapsing body, is to good approximation stationary (in fact, in the case we are considering, spherical symmetry and vacuum exterior force it to be exactly Schwarzschild in this region). Then we may write the Hamiltonian in the past as \( H_p = H_b + H_{int} \), where \( H_b \) is the “bare,” noninteracting, term and \( H_{int} \) is the interaction. According to standard perturbation theory, the bare in-vacuum \( |0_b\rangle \) is perturbed to first order to a “dressed” vacuum

\[
|0_d\rangle = \left(1 - H_b^{-1}H_{int}\right)|0_b\rangle.
\]

(2)

The interaction Hamiltonian is \( H_{int} = -e\int_\Sigma \hat{\psi}^\gamma a\Phi\psi \), where the integration is over the initial surface in question, and \( \Phi \) and \( \psi \) are the electromagnetic and charged-particle field operators.

Each of these operators is a sum of (bare) creation and annihilation terms. Combinations involving annihilation terms will not contribute to the perturbation of the vacuum, but those involving creation terms from \( \hat{\psi} \), \( \Phi \) and \( \psi \) will. This is one way of understanding how the interaction populates the vacuum with virtual quanta.

4.2 Charged-Pair Production

While, as emphasized above, our computation will be in operator-theoretic terms and will not rely on diagrams, the diagrams do provide a useful intuitive picture, and we shall begin by explaining the connection between the quantity to be computed and them.

We shall compute an amplitude which can be thought of as corresponding to the diagram in Fig. 5. Our point of view is that this diagram is a modification of the conventional Hawking process, where one branch of the electromagnetic line has been converted to an electron–positron pair. Thus the diagram represents the destruction of a Hawking electromagnetic quantum (relative to the in-vacuum — recall that the in-vacuum is the state which has Hawking quanta in the out-region) and the creation of a charged-particle pair. We may write the amplitude as

\[
\langle 0_d|\hat{\psi}_+^+(k')\Phi^-_F(k)\psi^+(k'')|0_d\rangle,
\]

(3)

where \( \hat{\psi}_+^+, \Phi^-_F, \psi^+ \) are annihilation and creation operators in the distant future, and \( k', k, k'' \) are mode labels.

In order to simplify the analysis, we shall choose to examine the amplitude \( \langle 3 \rangle \) only for certain modes. Most importantly, the charged-particle modes will be restricted so that to zeroth order they correspond to wavepackets propagating everywhere throughout the vacuum region. This means that to zeroth order the charged-particle modes are unaffected by the dynamic, collapse, portion of the space–time. (In particular, we do not then have to consider Hawking-production of charged particles, for Hawking-produced particles appear to emanate from the collapsing region.) It will also be convenient to consider only modes corresponding to propagations everywhere far from the strong-field region. This is not really essential, but allows us to use a conventional momentum-space representation of the field operators.

With these restrictions on the modes to be examined, the amplitude \( \langle 3 \rangle \) vanishes to zeroth order, because the charged-particle field \( \psi^+(k'') \) annihilates the quantum state in this order. We shall see, however, that there are non-trivial first-order effects.

There are two sorts of first-order contributions to the amplitude \( \langle 3 \rangle \): those where the first-order correction is to one of the operators; and those where it is to the state vector. As it turns out, those due to corrections to the operators do not produce very significant effects.
(The reason for this is essentially the same as in Minkowski space. The corrections due to the
perturbations of the operators are
\[
\langle b | (\tilde{v}^+_l \Phi^{b} - \Delta \psi^+_l + \tilde{v}^+_b \Delta \Phi^{b}_{l} \psi^+_b + \Delta \tilde{v}^+_l \Phi^{b}_{l} \psi^+_b ) | b \rangle ,
\]
where in each case the subscript “b” denotes the bare term and the prefix \( \Delta \) the first-order correction. Of these, the latter two vanish since the charged-particle annihilation operator is applied to the vacuum; we are left with \( \langle b | \tilde{v}^+_l \Phi^{b} \Delta \psi^+_l | b \rangle \). This leads to an interaction integral of the form
\[
\int \tilde{v}^a A_a u d\tau ,
\]
where \( \tilde{v} \), \( A_a \) and \( u \) are the mode functions for the fields, determined from their data in the future
and propagated through space–time by the free field equations, and the integration extends over the
space–time volume in question. If the mode functions correspond to particle wavepackets sufficiently
well-localized that the volume in which there is a significant interaction is small compared to the
space–time curvature, then in this volume the interaction must conserve energy–momentum. In
this interaction volume, both the electromagnetic and electron–positron mode functions can be
thought of as on-shell modes in Minkowski space. The electron–positron mode functions will
 correspond to a superposition of terms, each with timelike future-pointing energy–momentum.
The electromagnetic mode functions, on the other hand, will be a superposition of terms each with
null energy–momentum. It will therefore be impossible for energy–momentum to be conserved: the
interaction integral will vanish due to destructive interference.)

The interesting first-order contributions to the quantity \( \Phi \) come from the perturbation of the vacuum. In principle, there are two of these, one from the perturbation of the bra and the other from the perturbation of the ket, but the first of these vanishes as in it annihilation operators are
applied directly to the bare vacuum. We have, therefore,
\[
\langle d | \tilde{v}^+_f \Phi^- \psi^+_f | d \rangle = -\langle b | \tilde{v}^+_f \Phi^- \psi^+_f H^{-1}_b H_{int} | b \rangle ,
\]
to the approximation required, where the field operators may be taken to be bare.

We should, strictly speaking, do the details of the computation with wave-packets to represent
the mode functions determining the fields. However, as is conventional, we shall work directly with
the momentum-space representations of the fields, and understand that the computation really only has meaning when averaged over appropriate wave-packets (corresponding to the assumptions
we have made about the modes we consider).

We shall therefore take the mode labels \( k', k, k'' \) to refer to three-momenta in the out-region.
(We will supplement these with polarizations shortly.) Since to zeroth order the spinor field modes
of interest for us are unaffected by the space–time curvature, it is not necessary to distinguish, to
this order, the momentum-space decomposition of the spinor fields in the future from that in the
past. However, for the electromagnetic field this distinction is essential. We shall put
\[
\Phi^+_f (k) = \int \left[ \alpha (k, l) \Phi^+_p (l) + \beta (k, l) \Phi^-_p (l) \right] d^3 l ,
\]
where \( \alpha (k, l), \beta (k, l) \) are called the Bogoliubov coefficients of the (zeroth order) scattering.

We may now straightforwardly compute the amplitude \( \Phi \):
\[
\langle d | \tilde{v}^+_f (k') \Phi^- (k) \psi^+_f (k'') | d \rangle
= - \int d^3 l \langle b | \tilde{v}^+_f (k') \beta (k, l) \Phi^+_p (l) \psi^+ (k'') H^{-1}_b H_{int} | b \rangle
\]
\[ - \int d^3 l \left[ E(k') + \|l\| + E(k'') \right]^{-1} \times (\langle 0_b | \tilde{\psi}^+(k') \beta(k,l) \Phi^+(l) \psi^+(k'') H_{\text{int}} | 0_b \rangle ) , \]  

where \( E(k') = \sqrt{\|k'\|^2 + m^2} \) is the energy of a particle of mass \( m \) and momentum \( k' \). We have \( H_{\text{int}} = -e \int_\Sigma \tilde{\psi}^\gamma \Phi \psi \), and when we compute the expectation \( \langle 0_b | \tilde{\psi}^+(k') \Phi^+(l) \psi^+(k'') H_{\text{int}} | 0_b \rangle \), the annihilation operators applied to the fields in \( H_{\text{int}} \) simply produce the corresponding mode functions. With standard continuum normalizations for these (the operator \( \tilde{\psi}^+(k') \) corresponding to the mode \((2\pi)^{-3/2} (m/E(k'))^{1/2} e^{-ik'\cdot x} v(k')\), the operator \( \Phi^+(k) \) to \((2\pi)^{-3/2} (2k)^{-1/2} e^{-ik\cdot x} \epsilon(k) \) and \( \psi^+(k'') \) to \((2\pi)^{-3/2} (m/E(k''))^{1/2} e^{-ik''\cdot x} \tilde{\psi}(k'')\), where we have written the polarizations \( v(k'), \epsilon(k), \tilde{\psi}(k'') \) explicitly) we find, after a short calculation,

\[ \langle 0_a | \tilde{\psi}^+(k') \Phi^-(k) \psi^+(k'') | 0_a \rangle = e^{m\pi^{-3/2}} 4 \left[ E(k') + \|k' + k''\| + E(k'') \right]^{-1/2} \times \tilde{\psi}(k'')^\dagger \beta_{ab}(k,-k'-k'') \epsilon^a(k') v(k') , \]

where we have written the index structure on the Bogoliubov coefficient and the photon polarization explicitly.

### 4.3 Interpretation

The interpretation of the evaluated amplitude turns on the behavior of the Bogoliubov coefficient \( \beta(k,l) \). This coefficient was evaluated by Hawking; indeed, its evaluation was the main goal of his work, for the probability of producing Hawking photons in mode \( k \) is \( \sim d^3 k \int |\beta(k,l)|^2 d^3 l \).

Hawking found that the quantities were controlled by the characteristic frequency scale \( \omega_H = (8\pi M)^{-1} \) (where \( M \) is the mass of the collapsing object). Associated with this is a characteristic period \( \omega_H^{-1} \). In any epoch (interval of late time a few characteristic periods in duration), the main contributions to \( \beta(k,l) \) come from wavevectors \( k \) of magnitude \( \sim \omega_H \) but with \( l \) the corresponding, blue-shifted, precursor. Thus the magnitude of \( l \) is \( \sim \omega_H \exp + u/(4M) \) and it is directed inwards.

Here, we have \( l = -k' - k'' \) with \( k', k'' \) the momenta of the charged particles. Thus we have ultra-high momenta \( k' + k'' \) directed outward. This is our amplitude to produce ultra-energetic pairs. The corresponding probability, being proportional to \( |\beta(k,l)|^2 \) (modulo polarization effects), can be thought of as representing the conversion of a fraction of Hawking quanta into these pairs.

It is natural to ask what the implications of this result are for late times \( u \). However, once the momenta \( k', k'' \) have become ultra-relativistic (that is, \( \|k'\|, \|k''\| \gg m \)), first-order perturbation theory is no longer valid; one must take higher-order corrections into account. It is beyond the range of our present computational abilities to make accurate evaluations of the amplitudes when \( u \) increases very much. Indeed, the point of view we shall advocate is that the computation here shows that non-trivial ultra-energetic effects are possible, but it is unrealistic at present to expect to be able to make quantitative theoretic predictions of them. Rather, it may be that we shall learn experimentally the physics of ultra-energetic quantum fields by observing signals from the vicinities of black holes.

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4. In fact, Hawking used a decomposition in spherical harmonics rather than the momentum-space one here. However, it is easy to interconvert between the two.

5. There are essentially two competing effects as \( u \) increases. On the one hand, the blue-shifting of the modes goes on in a region of space–time which is itself being exponentially compressed; this tends to decrease the amplitudes exponentially. On the other, as the energies increase one has more possible processes contributing to a given amplitude; estimates of this effect are speculative.
4.4 Discussion

There is no wholly satisfactory brief account of the physics underlying this ultra-energetic pair production, because of the interpretational difficulties present in any interacting quantum field theory. However, the effect is important enough that some discussion is certainly in order.

One way of understanding the physics is to realize that “dressing” a state does not commute with evolution through a time-dependent potential (here, the gravitational collapse).

The in-vacuum, as we have seen, can be thought of as a bare vacuum “dressed” by the contributions of interactions. This dressing was determined by the Hamiltonian $H_p$ in the distant past, where the state was specified, by the requirement that the state indeed be the one of lowest energy. By contrast, consider a physical out-state with a thermal distribution of photons. This state is, to zeroth order, the one described by Hawking. However, if we wish to dress this state and retain its physical interpretation, we must do so on the basis of the Hamiltonian $H_f$ in the distant future. (The dressed thermal state could be defined by requiring that the KMS condition with respect to $H_f$ hold.) This Hamiltonian differs substantially from $H_p$, in that the electromagnetic potential operators of the two are related by a Bogoliubov transformation.

The bare in-state dressed by $H_p$ will not evolve to the bare out-state dressed by $H_f$. To see this, let us consider the situation in more detail.

The in-vacuum was modified by the addition of triples of (bare) quanta. Each of these triples consists of an electromagnetic quantum and a pair of charged quanta. While the spatial momenta of each triple sum to zero, contributions with arbitrarily high individual momenta are present. The field modes describing these triples, after passage through the collapsing region, are distorted. We here neglect the distortion of the charged-particle modes (we are interested only in charged particles which are everywhere far from the collapse region). But the photon modes are very severely distorted. Each photon mode which passes through the collapse region is exponentially red-shifted; also the photon field modes undergo a squeezing transformation (mixing positive and negative frequencies) which alters the particle numbers.

This means that the field modes in the future cannot recombine undistorted to dress the bare out-state as would be prescribed by $H_f$. The out-state will thus have an ultra-high frequency content which differs from that of the bare out-state dressed by $H_f$: there will be ultra-energetic excitations. And these distortions will be due to missing inward-directed photons, leaving outward-directed charged-particle pairs.

The discussion just given points up a subtlety in dealing with interacting field theories — one must be careful to distinguish between bare and dressed quantities, and in general one must be careful to justify the physical interpretations of the quantities. In our case, we have worked out the amplitude $\langle 3 \rangle$, which in principle exists in full quantum electrodynamics, to lowest non-trivial order. We are thus justified in identifying it as a physically significant quantity. It is true that its simple interpretation in terms of particles is only valid insofar as we may identify the operators $\tilde{\psi}_T^+\Phi_T^-\psi_T^+$ as annihilation and creation operators for particles, and this becomes a non-trivial issue for very high energies. However, the problem of defining the particle-content is not really relevant here. What matters is that the amplitude unambiguously represents ultra-energetic excitations of the charged fields.

In this connection, it is worth pointing out that the amplitude $\langle 9 \rangle$ we have computed would vanish in a stationary space–time (assuming the positive-/negative-frequency decomposition of fields is with respect to Killing vector defining the stationarity), for then the Bogoliubov coefficient $\beta$ would be zero. Thus if one wishes to view the amplitude as a certain three-point function, the difference of this function between the case of gravitational collapse and the stationary case is a clear signal of the effect of gravitational collapse on the physics.
Finally, we point out that similar results will hold for holes with angular momentum and charge. We really only used the Schwarzschild character of the hole in two places: when we assumed the Bogoliubov coefficients were those of a Schwarzschild hole; and in estimating the magnitude of the momentum $l$ as $\sim \omega_H \exp + u/(4M)$. However, it is well understood what these quantities are in the more general, Kerr–Newman, case. The Bogoliubov coefficients were discussed by Hawking, and the momentum increases as $\sim \omega_H \exp + \kappa u$, where $\kappa$ is the surface gravity of the hole.

4.5 Other Processes

We chose to compute the amplitude for pair production because it was simple both technically and conceptually. Technically, it was a first-order process in which only one term contributed significantly. Conceptually, it had a clear interpretation, as the first-order contribution to an expectation-value which should exist in full, non-linear quantum electrodynamics.

Many other processes could be investigated, and similar conclusions reached. However, the results are harder computationally and are also subject to further interpretational complications (largely associated with renormalization). It should be clear, however, that not only production of ultra-energetic pairs, but also the scattering of particles initially present to ultra-high energies is possible. (Some processes would simply be by virtual photon exchange with the ultra-energetic pairs described here.) There are also corrections to vacuum polarization effects, where a virtual photon can pass through the collapsing region.

We shall not attempt to discuss any of these here. The main point of the present paper is that the exponential increase of the energies quickly carries us beyond the point where we can make reliable quantum-field-theoretic computations.

5 Consequences and Predictions

We have seen that when interactions are taken into account, quantum field theory in curved space–time leads to a very different picture of a black hole than the one drawn by Hawking. Instead of an essentially classical object accompanied by the emission of low-energy ($\sim \omega_H$) thermal quanta, we have a prediction of emissions at exponentially increasing energy scales ($\sim \omega_H \exp + u/(4M)$), where the e-folding time ($4M \simeq (M/M_\odot) \times 2.0 \times 10^{-5}$ s, with $M_\odot$ the mass of the Sun) is fairly short for known black-hole candidates.

This has consequences for the general theoretical picture (the link between black holes and thermodynamics) and possibly for experiment as well. Too, we must divide the discussion into the cis-Planckian regime, where the energies involved are below the Planck scale, and the Planckian regime.

5.1 The Cis-Planckian Regime

While the computations above have been done only for one quantum-electrodynamic process and only to first order, it should be clear that the underlying principle is more general: the passage of virtual quanta through the collapsing region distorts them significantly and so upsets the balance of virtual processes, resulting in real effects which become ultra-energetic. The energy scale for these increases as $\sim \omega_H \exp + u/(4M)$.

Typically, these effects at a given point are due to contributions from the blue-shifted precursors of Hawking photons. Of all the photons that might be visible at a point, Hawking photons should come from (roughly) that fraction of the sky occupied by the incipient black hole. This fraction will be appreciable for observers within a few Schwarzschild radii of the hole, and will fall off as the
square of the distance from the hole at greater distances. Thus we may expect significant effects in a volume of space whose radius is a few Schwarzschild radii, and possibly measurable effects some distance further out.

One would like to know the rate of production of ultra-energetic particles. Unfortunately, the exponential increase of the energies makes reliable computations of this first difficult, and then impossible. The first-order quantum-electrodynamic computation done above would be expected to be valid for energies $\lesssim 1$ MeV (beyond which higher-order effects, due to further real and virtual electron–positron pairs, would have to be considered). Above about 100 MeV one would have to consider meson production and associated strong-force physics; above about 1 GeV there would be baryon production; above about 100 GeV, electroweak mixing. And much beyond this, we are in a regime in which we have few experimental data and theory is very much a matter of speculation.

It should be pointed out that there are essentially two competing effects determining how significant a particular ultra-energetic amplitude is. On one hand, the exponential compression of the modes takes place in a region of space–time which is itself being exponentially compressed (its extent in advanced time is of the order of the blue-shifted period), and this tends to decrease the amplitude. On the other, as the energies increase one has the potential of more processes contributing to a given amplitude, as well (perhaps) as the break-down of perturbation theory itself. We have no good ways of estimating these latter effects, and thus we have no good way of estimating how much radiation is produced. We cannot even say whether the total production would tend to increase or decrease with $u$. We do know, however, that the characteristic energy scale increases exponentially.

While this difficulty in making quantitative predictions is a very real one, we also have the exciting prospect of the neighborhood of a black hole being an experimental laboratory in which energies well beyond those terrestrially accessible will appear. And while the quantitative estimates of the rate of production are at present unavailable, it is worth noting that there would be one general characteristic of the effects described here which would distinguish them from other high-energy astrophysical effects in the neighborhood of the hole. The energies of the effects here would be set essentially by the red-shift factor $v'(u)$, and not the distance from the hole (except for additional red-shift effects very near the hole). (The probabilities of the effects would fall with the distance, as noted above, but the energies would not.) By contrast, most highly energetic astrophysical effects near a black hole are driven by loss of potential energy, and so the energies involved vary inversely with the distance from the hole.

As noted above, the exponential increase in the energy scales means that this regime of cis-Planckian physics is run through in short order, typically a few dozen e-foldings. One might think that this means that only if we catch a black hole on the verge of formation is this cis-Planckian regime accessible. This is not the case, however. What matters is the increasing blue-shift of the frequencies, and such an increase can occur in other ways. For example, it may occur for an observer moving relative to an established black hole, if as time passes in the observer’s frame there is a direction in which there are null geodesics passing closer and closer to the hole. It may also occur if matter accretes to an existing black hole [9].

It is worth noting that besides providing a window on physics at very high energies, the effects here would provide some constraints on very low energies. This is because they rely, like the original Hawking computation, on the assumption that there is an effectively massless field (the photon, in the work above). Here “effectively” massless means much lower in mass than the scale $\omega_H$. Since the photon’s mass is presently believed constrained to be $< 6 \times 10^{-17}$ eV [13] and $\omega_H \approx 5.3 \times 10^{-12}(M_\odot/M)$ eV (where $M_\odot$ is the mass of the Sun), a non-zero photon mass could cause a difference in the production of ultra-energetic effects by black holes of masses $\gtrsim 10^6 M_\odot$. Finally, in this connection we note that neutrinos of masses below $\omega_H$ could also contribute to
ultra-energetic effects in manners similar to that of the photon; current experiments only constrain
the mass of the lightest neutrino to be $\lesssim 10^{-2}$ eV.

5.2 The Planck Regime

The most interesting feature of the analysis here is that it unequivocally demonstrates the inade-
quacy of conventional quantum theory in curved space–time to describe physics in the neighborhood
of a black hole. That conventional theory would predict the existence of effects at energies increas-
ing exponentially beyond the Planck scale, and at that point the neglect of quantum-gravitational
effects is illegitimate. Without a theory of quantum gravity, we cannot say how this conventional
model breaks down — whether it is conventional quantum theory which becomes inadequate, or
the treatment of space–time by a classical model, or both — but we do know that one of these
elements must be altered.

In particular, we cannot say whether an established black hole (one for which the energy scale
in question has entered the Planck regime) will emit quanta at all, in any frequency range. We
cannot even say whether the quantum structure of the hole and its neighborhood will be effectively
stationary or not. Because of the appearance of the Planck scale, the problem is wholly dependent
on quantum-gravitational physics, not presently understood.

The simplest hypothesis would be that, whatever the details of the quantum-gravitational struc-
ture are, they result in the emission of particles at energies somewhat below the Planck energy.
(Of course, without a theory of quantum gravity, we could say nothing about the production rates
for different species.) If this is correct, and the cut-off is independent of position, then, as in the
cis-Planckian case, the limiting energy would be essentially independent of distance from the hole
(apart from possible red-shift effects for particles very near the hole), although the probability of
emission would fall with distance from the hole. Again, this would be a signature likely to distin-
guish these effects from other ultra-energetic astrophysical effects near the hole. And again, with
our present ignorance of the physics of the extreme energy scales involved, we can only speculate
about the sorts of particles that might be produced. It is possible that this mechanism is responsible
for the production of ultra-high energy cosmic rays.

For established black holes, besides the possibility of direct observation of particle emission,
there are other potentially significant astrophysical consequences. Whatever hypotheses on the
rate of particle production are made, the rate of mass loss for the hole is likely to be different
from that predicted by Hawking, leading to a different black-hole lifetime. These differences could
have significance for cosmological models, since density fluctuations in the early Universe may have
produced black holes. Up to now, the main constraints on the number of these have been due to
the cosmological implications of Hawking radiation. A difference in the predicted radiation would
alter these constraints.

5.3 Theoretical Implications

The results here, besides showing the necessity of developing some aspects of a theory of quantum
gravity for the treatment of quantum fields near black holes, imply that we must reconsider the
picture that has been broadly accepted of quantum theory and black-hole thermodynamics.

Let us recall that, before the theory of black-hole radiance, a strong formal parallel had been
noted between the theory of black holes and thermodynamics. The zeroth law (the existence of
a well-defined temperature) was the fact that the surface gravity of a stationary black hole was
constant over the horizon; the first law was conservation of energy; and the second was that the
area of a black hole could only increase. Thus it seemed that one should interpret the surface
gravity and area of a black hole as a sort of temperature and entropy, respectively.
The relation remained only formal, however, until an explicit link with ordinary thermodynamics could be found. Such a link was first proposed by Bekenstein [14], based on the notion of information loss. This argument was however difficult to make completely quantitatively precise. Then Hawking predicted that black holes actually radiate thermally with a well-defined temperature $\omega_H$ (which turns out to be the surface gravity over $2\pi$). This seemed to explain one half of the puzzle, that of giving a thermodynamic meaning to the black-hole temperature. It was also true that from that result and the thermodynamic relation $dQ = TdS$ a definite value for the black-hole entropy could be inferred, but the problem of providing a convincing independent thermodynamic interpretation for black-hole entropy remained open. And despite much ingenious work (e.g., attempts to count the number of black-hole states quantum-gravitationally, thought-experiments based on having ordinary thermal systems interact with black holes), the problem is still open [4].

The results here call into question the prediction of black-hole radiation, and with that the explanation of black-hole temperature. They do not, as emphasized above, mean that black holes do not radiate, but they do make the previous analysis untenable. Their real lesson is that the regime in question cannot be understood at all without quantum gravity.

5.4 Conclusions

We may thus summarize our picture as follows. The black hole has an essentially quantum character, with significant ultra-energetic quantum effects in a region extending on the order of a Schwarzschild radius beyond the hole. As the hole forms, the energy scales in question increase exponentially, making the neighborhood of the incipient hole a potential laboratory for studying quantum fields at all energy scales. (Such increasing energy scales may also be obtained for established holes, for observers moving relative to them or when matter accretes onto them.) After a finite number of e-foldings, however, the energy scales have reached the Planck regime, and we are at the limit of known physics. At this point, the neighborhood of the hole has an essentially quantum-gravitational structure. We may hope that studies of black holes may bring experimental evidence for the character of this structure.

Appendix: Previous Work on Interactions

The main argument of this paper has been that the effects of interactions among quantum fields substantially alter their physics in a gravitationally collapsing space–time. This differs from most previous thinking; I shall here briefly discuss the relation of the ideas here to previous work on interactions.

Probably the bulk of the work that has been done has grown out of attempts to model Hawking-radiating mini black holes whose temperatures are high enough to make them astrophysically interesting. However, to my knowledge, all such work is based on the assumption that the black hole acts as a black (or brown) body of a given size and temperature. Thus this approach assumes that a Hawking-type result applies even for interacting fields; it does not attempt to justify this result.

There has also been the suggestion, not fully developed, that general principles of quantum field theory and invariance should force a black hole to radiate thermally even when quantum-theoretic nonlinearities are present [15]. The main support for this notion comes from the Bisognano–Wichmann Theorem, which can be interpreted as stating that a uniformly accelerating observer in Minkowski space would perceive the $n$–point functions of the vacuum of even an interacting field theory to be the same as those of a thermal state at the corresponding Unruh temperature. However, it has so far not been possible to develop a theorem like this in the gravitational-collapse
case. (The Bisognano–Wichmann Theorem is very much a result of special-relativistic field theory, and in particular depends crucially on the existence of a semi-bounded self-adjoint Hamiltonian, a hypothesis which is invalid in the gravitational-collapse region [16].)

Perhaps the best attempt to confront the problem of interactions has been due to Gibbons and Perry [5]. These authors outlined an argument that the Feynman propagators for the theory would have a periodicity in imaginary time which would enforce thermality of the state even in the case of interactions. However, there are some subtleties which make it difficult to completely prosecute the argument. The details of the development of the theory by Feynman diagrams, and especially the question of the choice of dressing of the in- and out-states, were not spelled out. Also, while one of the authors’ main points was the necessity of showing that an initially vacuum state would dynamically equilibrate to a thermal one, in fact it is unrealistically hard to follow this dynamical process, and so they based the details of their analysis on the use of the Hartle–Hawking propagator [17]. This is a somewhat formal object usually interpreted as representing a black hole assumed in equilibrium with a thermal bath of radiation. It is an object which by construction has no explicit information about a collapse phase; it was derived, and is most often used, under the assumption that one is dealing with late-time physics for which neglect of the collapse phase is legitimate. Thus the Gibbons–Perry analysis would not uncover the sorts of effects investigated here, which turn on the propagation of virtual quanta from before the time of collapse through the collapse region.

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