Unified dark energy thermodynamics: varying $w$ and the $-1$-crossing

Emmanuel N Saridakis$^1$, Pedro F González-Díaz$^2$ and Carmen L Sigüenza$^3$

$^1$ Department of Physics, University of Athens, GR-15771 Athens, Greece  
$^2$ Colina de los Chopos, Instituto de Física Fundamental, Consejo Superior de Investigaciones Científicas, Serrano 121, 28006 Madrid, Spain  
$^3$ Estación Ecológica de Biocronología, Pedro de Alvarado 14, 06411 Medellín, Spain

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Abstract  
We investigate, in a unified and general way, the thermodynamic properties of dark energy with an arbitrary, varying equation-of-state parameter $w(a)$. We find that all quantities are well defined and regular for every $w(a)$, including the $-1$-crossing, with the temperature being negative in the phantom regime ($w(a) < -1$) and positive in the quintessence one ($w(a) > -1$). The density and entropy are always positive while the chemical potential can be arbitrary. At the $-1$-crossing, both temperature and chemical potential are zero. The temperature negativity can only be interpreted in the quantum framework. The regular behavior of all quantities at the $-1$-crossing leads to the conclusion that such a crossing does not correspond to a phase transition, but rather to a smooth crossover.

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1. Introduction

Recent cosmological observations support that the universe is experiencing an accelerated expansion, and that the transition to the accelerated phase has been realized in the recent cosmological past [1]. In order to explain this remarkable behavior, and despite the intuition that this can be achieved only through a fundamental theory of nature, physicists can still propose some paradigms for its description, such as theories of modified gravity [2] or ‘field’ models of dark energy. The field models that have been discussed widely in the literature consider a canonical scalar field (quintessence) [3], a phantom field, that is a scalar field with a negative sign of the kinetic term [4] or the combination of quintessence and phantom in a unified model named quintom [5]. The common feature of all the field models of dark energy is that the equation-of-state parameter $w$ of the dark energy ‘fluid’ is induced by the field evolution, and thus it acquires a dynamical nature ($w \rightarrow w(a)$ with $a$ being the scale factor).
On the other hand, there have been large efforts in order to study the thermodynamic properties of dark energy fluids with a constant $w = w_0 \neq -1$. For the case of quintessence scenarios, where $w = w_0 > -1$, one can use the results of conventional perfect fluid thermodynamics. However, in the case of phantom fields, where $w = w_0 < -1$, a thermodynamic investigation must be performed from first lines. Some authors assume a zero chemical potential $\mu$ and argue that the temperature of a phantom fluid must be negative, with the density and the entropy positive [6, 7], and the negative nature of the temperature is related to the quantum properties of phantom fields (although in some initial works on $\mu = 0$ the entropy was found to be negative and thus the phantom phase meaningless [8]). On the other hand, some authors consider the nonzero chemical potential case and argue that phantom fluids must have $\mu < 0$, with temperature, entropy and density being positive, with the phantom particles possessing a bosonic, massless nature [9, 10].

In our opinion, the central point of dark energy models is not so much that $w$ can take a constant value different from $-1$, but rather that $w(a)$ is varying (indeed since $w(a)$ is a ratio of two varying quantities, being a constant requires rather special constructions). In addition, the crossing of the phantom-divide $w = -1$ from above to below is not only possible but it could indeed be the basic requirement for a successful description of observations. Therefore, the question of investigating in a unified way the thermodynamics of dark energy fluids with an arbitrarily varying $w(a)$ becomes crucial.

However, a comment must be made here, concerning the applicability of thermodynamics to time-dependent systems such as the universe, where gravitational degrees of freedom are present. Although in principle statistical mechanics assumes equilibrium and ergodicity, which could be violated in time-dependent gravitational backgrounds, and despite the fact that a thermodynamic and statistical description of the dynamics of gravitational degrees of freedom does not fully exist yet, we believe that such an approach can be enlightening. This is similar to the thermodynamical investigation of the properties of black holes or de Sitter spaces, which is purely gravitational systems evolving in time. In addition, it is fairly known that black holes can be regarded as the time-reversed analog of the Big Bang universe, despite starting from static spacetime metrics. Having these points in mind, we proceed to the thermodynamic analysis of a general dark-energy fluid. The paper can be outlined as follows: in section 2, we present the thermodynamic properties of a general-$w(a)$ dark-energy fluid, while section 3 is dedicated to discussion, concluding remarks and a brief summary of the results.

2. General $w(a)$ thermodynamics

We consider dark-energy fluids, with energy density and pressure, respectively, $\rho(a)$ and $p(a)$, and with an arbitrary and general equation of state of the form

$$p(a) = w(a)\rho(a).$$

In these expressions, $a$ is the scale factor of a homogenous and isotropic Friedmann–Robertson–Walker (FRW) universe, with metric,

$$ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right),$$

where $k = 0, \pm 1$ is the curvature parameter and $t$ is the comoving time. Since we desire our analysis to be general, we do not consider specific cosmological models, with explicitly extracted Friedmann equations, but we just consider the resulting $w(a)$. Thus, the present investigation is valid for every FRW cosmological model. Finally, note that we
could equivalently express \( w(a) \) as \( w(t) \) or \( w(z) \) (with \( z \) being the redshift), since such a transformation is straightforward.

As usual, the equilibrium thermodynamic states of a relativistic perfect fluid are characterized by an energy–momentum tensor \( T^{\mu\nu} \),

\[
T^{\mu\nu} = (p + \rho)u^\mu u^\nu - pg^{\mu\nu},
\]

where \( u^\mu \) (\( \mu = \{0, 1, 2, 3\} \)) is the 4-velocity in the metric \( g^{\mu\nu} \). The particle and entropy currents, \( N^\mu \) and \( S^\mu \), respectively, are defined as

\[
N^\mu = nu^\mu, \quad S^\mu = su^\mu,
\]

with \( n \) and \( s \) being the densities of particle number and entropy. Denoting the covariant derivative by \( (\cdot; \cdot) \), the equations of motion are given by

\[
T^{\mu\nu};_\nu = 0, \quad N^\mu;_\mu = 0, \quad S^\mu;_\mu = 0,
\]

which in the case of the FRW geometry can be written as

\[
\dot{\rho} + 3[1 + w(a)]\rho \frac{\dot{a}}{a} = 0, \quad \dot{n} + 3n \frac{\dot{a}}{a} = 0, \quad \dot{s} + 3s \frac{\dot{a}}{a} = 0,
\]

with the dot denoting the comoving time derivative. In the case of a general \( w(a) \), the solution for \( \rho(a) \) reads

\[
\rho(a) = \rho_0 \exp \left\{ \int_{a_0}^{a} \frac{3[w(a) + 1]}{a} da \right\},
\]

\[
\Rightarrow \rho(a) = \rho_0 \left( \frac{a_0[a_0^{3[w(a)+1]}]}{a^{3[w(a)+1]}} \right) \exp \left[ -3 \int_{a_0}^{a} da \ w'(a) \ln a \right],
\]

where the prime denotes the \( a \)-derivative, and \( a_0 \) and \( w_0 \equiv w(a_0) \) are the present values of the scale factor and of the dark energy equation-of-state parameter, respectively. In this relation, we preferred to use \( w'(a) \), in order for the extraction of the \( w = \text{const} \) results (i.e., \( w'(a) \to 0 \)) to be straightforward. Similarly, the solutions for \( n(a) \) and \( s(a) \) are simply

\[
n(a) = n_0 \left( \frac{a_0}{a} \right)^3, \quad s(a) = s_0 \left( \frac{a_0}{a} \right)^3,
\]

with \( n_0 \) and \( s_0 \) being the values of the corresponding quantities at present (from now on, the index 0 stands for the present value of a quantity). Finally, we stress that \( w(a) \) is the value of the equation-of-state parameter when the scale factor is \( a \), while \( w_0 \) is its present value. Obviously, one can face all the evolution combinations, where \( w(a) \) can cross \(-1\) at a specific scale factor or cannot, and/or \( w_0 \) being \(-1\) or not, or the case \( w(a) = w_0 = \text{const} \) with the constant being not equal or equal to \(-1\) (the last case is the cosmological constant universe). Therefore, in the following one must examine the regularity of all expressions when \( w(a) \) and/or \( w_0 \) is equal to \(-1\).

Let us now refer to thermodynamics. Considering \( (n, T) \) as the 2D thermodynamical base, we use the identity (Gibbs law),

\[
T \left[ \frac{\partial w(a) \rho(a)}{\partial T} \right]_n = [w(a) + 1] \rho(a) - n(a) \left[ \frac{\partial \rho(a)}{\partial n} \right]_T,
\]

where the prime denotes the \( a \)-derivative, and \( a_0 \) and \( w_0 \equiv w(a_0) \) are the present values of the scale factor and of the dark energy equation-of-state parameter, respectively. In this relation, we preferred to use \( w'(a) \), in order for the extraction of the \( w = \text{const} \) results (i.e., \( w'(a) \to 0 \)) to be straightforward. Similarly, the solutions for \( n(a) \) and \( s(a) \) are simply

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with \( n_0 \) and \( s_0 \) being the values of the corresponding quantities at present (from now on, the index 0 stands for the present value of a quantity). Finally, we stress that \( w(a) \) is the value of the equation-of-state parameter when the scale factor is \( a \), while \( w_0 \) is its present value. Obviously, one can face all the evolution combinations, where \( w(a) \) can cross \(-1\) at a specific scale factor or cannot, and/or \( w_0 \) being \(-1\) or not, or the case \( w(a) = w_0 = \text{const} \) with the constant being not equal or equal to \(-1\) (the last case is the cosmological constant universe). Therefore, in the following one must examine the regularity of all expressions when \( w(a) \) and/or \( w_0 \) is equal to \(-1\).
On the other hand, we can express $\dot{\rho}$ as $\dot{\rho} = \dot{n} \left( \frac{\partial n}{\partial T} T + \frac{\partial T}{\partial n} n \right)$. Combining this relation with (10) and (5) we acquire $\left[\frac{3}{a} T(a) w(a) + T'(a)\right][w(a) + 1] = -3 \frac{\dot{T}}{a} \rho(a) w'(a)$ or using (8):

\[
\left[\frac{3}{a} T(a) w(a) + T'(a)\right][w(a) + 1] = T(a) w'(a). 
\]

Note that this general differential equation leads straightforwardly to the general physical implication that $T(a)$ must be zero when $w(a)$ crosses $-1$. Furthermore, note that in the case where $w(a) \equiv -1 = \text{const}$ at all times, i.e. of a pure cosmological constant, equation (11) is trivially satisfied, not allowing for the determination of $T(a)$, and this has led some authors to argue that the temperature of the vacuum state is ill-defined [7]. However, we see that considering a general $w(a)$, i.e. with $w'(a) \neq 0$, we are led to $T(a) = 0$ for the vacuum state, which is a self-consistent result.

If $w(a) \neq -1$, the general solution of (11) is

\[
T(a) = T_0 \left[ \frac{w(a) + 1}{w_0 + 1} \right] \left[ \frac{a_0^{w_0} - a^{w(a)}}{a_0^{w_0} - w(a)} \right] \left[ -3 \int_a^{a_0} \, da \, w'(a) \ln a \right],
\]

with $T_0$ being the present temperature value. Solution (12) holds for $w(a) < -1$ or $w(a) > -1$. For $w(a) = -1$, the only information that we can obtain straightaway from (11) is that $T(a)$ must be zero. However, this requirement is indeed satisfied by (12), and furthermore we can see that

\[
\lim_{w(a) \to -1^\pm} T(a) = 0.
\]

Thus, we can safely consider that (12) is the general solution for every $w(a)$, including the $-1$-crossing. We mention that if $w_0 \to -1$, then as required by (11) $T_0 \to 0$ too, and $T(a)$ remains regular.

Expression (12) has an interesting and general physical implication, namely that $T(a)$ and $w(a) + 1$ (and thus $T_0$ and $w_0 + 1$) have always the same sign. Therefore, in summary, the temperature is always negative for $w(a) < -1$, it is zero for the cosmological constant bound, and it is always positive for $w(a) > -1$. Later on, we will discuss these points in detail. Finally, note that we could denote the regular ratio $\frac{T_0}{w_0 + 1}$ by a positive constant $C_0$, but we prefer keeping it in order for the various quantities at present to be straightforwardly obtained.

A useful relation can be obtained using (8) and (12), namely

\[
\rho(a) = \rho_0 \left\{ \frac{T(a)}{T_0} \left[ \frac{w_0 + 1}{w(a) + 1} \right] \frac{a_0^{w_0}}{a} \right\}^3,
\]

which is regular at $w(a) = -1$ (and at $w_0 = -1$), as can be seen from (12). Equivalently, eliminating $a^3$ between (12) and (14) we acquire the generalized Stefan–Boltzmann law:

\[
\rho(T) = \rho_0 \left\{ \frac{T(a)}{T_0} \left[ \frac{w_0 + 1}{w(a) + 1} \right] \frac{a_0^{w_0}}{a_0^{w_0} - w(a)} \right\} \cdot \exp \left[ -3 \int_a^{a_0} \, da \, w'(a) \ln a \right].
\]

Note that in the case $w(a) = 0$, i.e when the dark energy fluid is a dust, one can only use relation (14), since the elimination of $a^3$ between (12) and (14) is not possible.

Let us now return to thermodynamics, using the general identity $Ts = \rho + p - \mu n$, with $\mu$ being the chemical potential of the dark energy fluid, considered nonzero in general. This relation can be used to determine $\mu$. In particular, it can be written as

\[
\mu(a) = \frac{[w(a) + 1]\rho(a)}{n(a)} - \frac{T(a)s(a)}{n(a)},
\]
and using (8) and (12) it leads to

\[ m u(a) = \mu_0 \left[ \frac{w(a) + 1}{w_0 + 1} \right] \left[ a_0^3 w_0 \right] a^3 \exp \left[-3 \int_a^{a_0} da' w'(a) \ln a' \right]. \] (17)

where we have defined

\[ \mu_0 = \rho_0 (w_0 + 1) - T_0 s_0, \] (18)

the chemical potential value at present. Expressions (17) and (18) are regular at \( w_0 = -1 \) (since in this case \( T_0 = 0 \), too), and furthermore at \( w(a) = -1 \) (17) leads to \( \mu(a) = 0 \).

Note, however, that in general the sign of \( \mu_0 \), and thus of \( \mu(a) \), can be arbitrary. Therefore, we conclude that the chemical potential for the cosmological constant bound is zero, but it is arbitrary at the two sides of that bound, depending on the specific cosmological model. Later on, we will discuss these points in detail. Finally, (16) with the use of (14) leads to the simple relation

\[ \mu(T) = \mu_0 \left[ \frac{T(a)}{T_0} \right]. \] (19)

Relation (16), with \( s(a) = S(a)/V(a) \), can be used to calculate the total entropy \( S \) of the universe with physical volume \( V(a) = a^3 \). Indeed, using (9), (15) and (19), we acquire

\[ S(T) = s_0 V(a) \left\{ \frac{T(a)}{T_0} \right\}^{\frac{w_0 + 1}{w_0 + 1}} a_0^{\frac{w_0 + 1}{w_0}} \exp \left[\frac{3}{w_0} \int_a^{a_0} da' w'(a) \ln a' \right]. \] (20)

which is regular and nonzero at \( w(a) = -1 \). It can easily be seen, using (14), that

\[ S(a) = s_0 V(a) \left\{ \frac{a_0}{a} \right\}^3 = s_0 V_0 = s(a) V(a), \] (21)

with \( V_0 = a_0^3 \) being the current physical volume of the universe. Expression (21) is just the statement of the conservation of entropy in the whole universe, and provides a self-consistency test for our calculations. Finally, note that in the case \( w(a) = 0 \), i.e. when the dark energy fluid is a dust, relations (20) and (15) lead to a trivial result, and one can use only relation (21).

In the aforementioned investigation, we desired to remain general, and we did not use any ansatz for \( w(a) \). However, phenomenologically, one can consider various \( w(z) \)-parametrizations [11] at will, in order to extract quantitative predictions for the quantities at hand. Finally, taking the limit \( w'(a) \to 0 \) (or equivalently just setting \( w'(a) = 0 \) since the terms that need caution have been written as separated pre-factors), we extract the corresponding relations for the \( w(a) = w_0 = \text{const} \) case:

\[ \begin{align*}
\rho(a) &= \rho_0 \left( \frac{a_0}{a} \right)^{3[w_0+1]} \\
T(a) &= T_0 \left( \frac{a_0}{a} \right)^{3w_0} \\
\mu(a) &= \mu_0 \left( \frac{a_0}{a} \right)^{3w_0} \\
\rho(T) &= \rho_0 \left( \frac{T}{T_0} \right)^{\frac{w_0+1}{w_0}} \\
\mu(T) &= \mu_0 \left[ \frac{T}{T_0} \right] \\
S(T) &= s_0 V(a) \left[ \frac{T}{T_0} \right]^{\frac{1}{w_0}}. \end{align*} \] (22)
We mention that these relations are also valid for the simple cosmological constant \( w_0 = -1 \) and \( T_0 = \mu_0 = 0 \), giving \( \rho = \rho_0 = \text{const} \), \( T = \mu = 0 \) and \( S = S_0 V_0 = s(a) V(a) = \text{const} \).

3. Discussion

Let us discuss the physical context of the obtained results. First, we see that all quantities are regular and well defined, for all values of \( w(a) \). In addition, the density and entropy are always positive, consistently with the basic requirements of classical and quantum field theory. This behavior clarifies some ambiguities about the phantom nature, since one does not need integrability conditions [7] or special constructions [8–10] in order to make this ‘phase’ possible (which remarkably might indeed be the current phase of the universe).

An important physical consequence is that the temperature of a dark energy fluid with \( w(a) < -1 \) is negative, independently of the value of the chemical potential which can be arbitrary. This is in contrast with the works that required necessarily a negative \( \mu \) and a positive \( T \) for the constant-\( w \) phantom fluids [9, 10]. The misleading point in these considerations was that since \( w(a) \equiv w_0 = \text{const} \), the authors instead of equation (11) used \( 3T(a)w(a) + aT'(a) = 0 \), which leads to qualitative different results. The reason for this behavior is that we face a singular perturbation problem [12], i.e. the results for \( w'(a) \to 0 \) are more general and do not coincide in the whole variable-range with those for \( w(a) \equiv w_0 = \text{const} \). Furthermore, note that if one considers \( w(a) \equiv w_0 = \text{const} \), then he cannot deduce any result at all for the temperature sign, since this will coincide with the sign of \( T_0 \), which is arbitrary in principle. Only allowing for a \(-1\)-crossing (i.e., for a general \( w(a) \)) we can safely conclude that \( T(a) \) and \( w(a) + 1 \) (and thus \( T_0 \) and \( w_0 + 1 \)) have the same sign, as can be seen from (12).

Another significant physical implication is that the vacuum, either as a permanent state \( (w(a) \equiv w_0 = -1 \text{ at all times}) \) or as an instantaneous state at the time of the crossing of a varying-\( w(a) \), has the zero temperature and zero chemical potential. This was expected, since we can consider it to correspond to the zero mode of the various involved scalar fields. In contrast, a vacuum with nonzero \( T \) and \( \mu \) would require a non-trivial explanation. We mention that starting with \( w(a) \equiv w_0 = -1 \), one cannot find a relation for the temperature (and thus for the chemical potential, too), and therefore he concludes that the temperature and chemical potential for this state are ambiguous or ill-defined [7], or he is led to wrong results [9, 10]. The correct approach is to start with a general \( w(a) \) and then examine the limit \( w(a) \to -1 \).

The regular behavior of all quantities at the phantom-divide crossing leads to an additional interesting result. In particular, we conclude that such a crossing does not correspond to a phase transition, but rather to a crossover. An additional argument for this statement arises from the regular behavior of the ‘specific heat’, defined as \( \propto \frac{\rho}{T} \) or \( \propto T \frac{\partial S}{\partial T} \), at the crossing. The \(-1\)-crossing, dynamically is just a pass to the super-accelerated evolution, while thermodynamically is just a smooth pass from \( T > 0 \) to \( T < 0 \), without a phase transition. This behavior was observationally required, since if a radical cosmological phase transition had taken place in the recent cosmological past, it would have left observable imprints.

The temperature negativity can only be interpreted in the quantum framework. Although the discussion about the construction of quantum field theory of phantoms is still open in the literature (see, for example, [13] about causality and stability problems and the possible spontaneous breakdown of the vacuum into phantoms and conventional particles), there have been serious attempts at overcoming these difficulties and constructing a phantom theory consistent with the basic requirements of quantum field theory [14]. Thus, as was analyzed...
in detail in [6], in such a quantum consideration of the phantom fluid all novel phenomena (stimulated absorption of phantom energy, generalized Wien and Planck radiation laws) can be naturally positioned.

One of these novel phenomena, which can have interesting cosmological implications, is the accretion of dark energy onto black holes. In particular, in [15] it was shown that the solution for a stationary, spherically symmetric, accretion of a cosmological perfect fluid onto a Schwarzschild black hole with mass $M$, is given by $\dot{M} = 4\pi A M^2 (\rho + p)$, with $A$ being a positive constant, which in the case of a general $w(a)$ reads

$$\dot{M} = 4\pi A M^2[1 + w(a)]\rho(a).$$

Although a specific solution of this equation requires a constant $w(a)$ (see [6]), in general we observe that $\dot{M} > 0$ in the quintessence regime, $\dot{M} < 0$ in the phantom one, while $\dot{M} = 0$ at the $-1$-crossing, consistently with the generalized second law of thermodynamics (the Hawking radiation is neglected in this consideration). Relation (23) is independent of the phantom chemical potential (contrary to the requirement of [16]) and the change in the sign of $\dot{M}$ is consistent with the change in the $T$-sign [6]. Although smooth at the $-1$-crossing, such a change in the variation rate of a black hole mass, and thus of the total mass in black holes in the universe, could leave observable imprints in the celestial orbits.

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