Magnetic field orientation effect on specific heat in a p-wave superconductor

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The effect of magnetic field on specific heat of a p-wave spin triplet superconductor has been analyzed. To describe superconductivity we used a single band model with various realizations of p-wave order parameter and analyzed the effect of a Doppler shift in different temperatures.

1 Introduction

The discovery of superconductivity is strontium ruthenate [1] and subsequent proposal of its spin triplet odd parity symmetry [2] has triggered a lot of experimental and theoretical activities [3]. One of the issues of great importance is the identification of the pairing symmetry of this superconductor. The expected sizable spin-orbit coupling and the temperature independent Knight shift measured with magnetic field in the basal a-b plane [4] suggest the realization of the state

\[ \Delta(k) = \Delta_x \sin k_x + \Delta_y \sin k_y; \]

Fig. 1 (a) Fermi surface of the gamma band Sr2RuO4. (b) Pairing potential for three different p-wave solutions \( \Delta(k) = \Delta_x \sin k_x + \Delta_y \sin k_y \): '1' complex \( (\Delta_y = i\Delta_x) \), '2' real \( (\Delta_y = \Delta_x) \), and '3' dipole \( (\Delta_x = 0, \Delta_y = 0) \).

System parameters in the electron nearest neighbour hopping unit \( t \): next nearest neighbour hopping \( t'/t = 0.45 \), nearest neighbour attraction \( U/t = -0.446 \), chemical potential \( \mu = 1.5966 \).

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Fig. 2  Eigenvalues at the Fermi surface $E_k(E_f)$ (measured from the Fermi surface) for complex, real and dipole solutions, ((a), (b) and (c) as in Fig. 1) ‘1’, ‘2’ and ‘3’, respectively) calculated in the temperature $T/t = 0.006$. Dashed lines correspond to lines of zeros or minima in eigenvalues at the Fermi surface.

\[
d(\mathbf{k}) = \Delta (\sin k_x + i \sin k_y) \hat{e}_z,\]

(1)

which is chiral and has d-vector pointing in z-direction.

The chiral state (Eq. (1)) has been deduced [5, 6] from specific heat measurements on Sr$_2$RuO$_4$ in the presence of the magnetic field. To shed some light on the issue we shall study the magnetic field effect on the specific heat of a superconductor with different order parameters. To describe the various experiments on strontium ruthenate one needs a 3-dimensional, 3-band model [7]. In this work we shall study a fairly simpler model consisting of a single band of a two-dimensional character [8, 9, 10] where the magnetic field is applied at an angle $\phi$ with respect to the $x$-axis, and study its effect on the specific heat. The spectrum we start with corresponds to the $\gamma$ band in Sr$_2$RuO$_4$ [7]. However, we neglect the other two bands and limit our consideration to two dimensions. Our approach of introducing the magnetic field is similar to that of Tanaka et al. [11], who studied the influence of an in-plane magnetic field on the thermal conductivity of the same material with the spectrum approximated by the $\alpha$ and $\beta$ bands.

The study here allows a better understanding of the influence of a magnetic field and of temperature on the specific heat of superconductors with different order parameters.

### 2 The model

The system is described by negative $U$ Hubbard model [8] with the Hamiltonian

\[
H = \sum_{i,j,\sigma} (t_{ij} - \mu \delta_{ij}) c_i^{\sigma\dagger} c_j^{\sigma} + \frac{1}{2} \sum_{i,j,\sigma,\sigma'} U_{ij}^{\sigma\sigma'} n_{i\sigma} n_{j\sigma'},
\]

(2)

where $i, j$ label sites of a square lattice, $t_{ij} = -t$ is the hopping integral between nearest-neighbour sites, $\sigma$ is the electron spin and $\mu$ is the chemical potential. $U_{ij}^{\sigma\sigma'} < 0$ describes attraction between electrons with spins $\sigma$ and $\sigma'$ occupying sites $i$ and $j$, respectively. We have assumed the hopping parameter $t$ to be our energy unit. In Fig. (1a) we show the Fermi circle of the $\gamma$ band of interest.
The mean-field type decoupling of the interaction, together with Bogolubov-Valatin quasi-particle transformation have been used to derive the superconducting equations

$$\sum_{j,\sigma'} \left( E^\nu - H(ij); \Delta^{\sigma\sigma'}(ij); E^\nu + H(ij) \right) \left( \begin{array}{c} u^\nu_{i\sigma'} \\ v^\nu_{j\sigma'} \end{array} \right) = 0,$$

(3)

where $H(ij)$ is the normal spin independent part of the Hamiltonian, and the $\Delta^{\sigma\sigma'}(ij)$ is self consistently given in terms of the pairing amplitude, or order parameter, $\chi^{\sigma\sigma'}(ij)$,

$$\Delta^{\sigma\sigma'}(ij) = U^{\sigma\sigma'}(ij) \chi^{\sigma\sigma'}(ij).$$

(4)

defined by the usual relation

$$\chi^{\sigma\sigma'}(ij) = \sum_{\nu} u^\nu_{i\sigma} v^{\nu\ast}_{j\sigma'} (1 - 2 f(E^\nu)),$$

(5)

where $\nu$ enumerates the solutions of Eq. 3.

We have solved the Bogolubov-deGennes equations for the temperature-dependent superconducting order parameter and the results for 3 different order parameters are shown in Fig. (1b). Note the difference between curve ‘2’ and curves ‘1’, ‘3’. As is plotted here, the amplitudes of these differences do not mimic the average value of pairing potential which would not differer so much. For better clarity we show the angular dependence of eigenvalues in Fig. 2. Straight lines correspond to nodal lines or minima of eigenvalues on the Fermi surface. Note that only the complex solution has a 4-fold symmetry (Fig. 2a), while the dipole and real solutions have a 2-fold symmetry (Fig. 2b-c). Moreover the real solution is the most anisotropic with respect to the angle and possesses three nodal lines. On the other hand the complex solution has no line of nodes and the dipole has one.

The low-temperature specific heat $C$ across the superconducting state has been calculated using the formula [12]

$$C = -2 k_B \beta^2 \frac{1}{N} \sum_k E_k \frac{\partial f(E_k)}{\partial \beta}$$

(6)

where $\beta$ denotes the inverted temperature $1/(k_B T)$ and $f(E_k)$ is the Fermi function defined for the eigenvalue $E_k$. 

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The specific-heat results for the three studied states are shown in Fig. 3. One can see that the lines are close to each other except for a very small temperature region where the complex solution behaves exponentially as in a typical s-wave superconducting state.

3 The effect of magnetic field.

In the presence of a weak in-plane magnetic field the normal electrons close to the sample surface or vortices experience Doppler shift \[\frac{\hbar}{2e}\]. To take the field into consideration we assume that the penetration depth \(\lambda\) is fairly larger than the coherence length \(\xi\) and shift the wave vectors \(k_x\) and \(k_y\) according to the formulae \[\frac{\hbar}{2e}\] where the angle \(\theta\) is measured between the \(x\)-axis and the \(B\)-field. The field \(H_0 = \phi_0/(\pi^2\xi\lambda)\), where \(\phi_0 = h/2e\) is the superconducting flux quantum. Using the relations (Eq. 7) and previously calculated \(\Delta(k)\) (Fig. 1b) we recalculated the specific heat in the presence of the magnetic field. The corresponding results for different temperatures are presented in Fig. 4. Note that the curves 1 and 2 for the temperature \(T/t = 0.006\) (Fig. 4a) can be clearly related to the 4-fold symmetry, while in the lower temperature \(T/t = 0.002\) (Fig. 4b) curves 1 and 3 seem to follow this symmetry. The other curves 3 for \(T/t = 0.006\) and 2 for \(T/t = 0.002\) show a 2-fold symmetry. Comparing Figs. 2 and 4 one can conclude that minima in the eigenvalues play a similar role as lines of nodes in these curves and the temperature range is very important in such studies. Clearly the specific heat in a lower temperature is more sensitive to the type of anisotropy in the eigenvalues and distinguishes easier between lines of nodes and the simple minima in the eigenvalue spectra. For higher temperature the oscillations \(\Delta(k)\) have a fairly small amplitudes, as evident from Fig. 1b. The change of the character of the specific heat oscillations is due to the combined effect of (i) smaller \(\Delta(k)\) amplitudes and (ii) smearing of the Fermi distribution function as seen from Eq. 6. Thus the minima and nodal lines are reflected in the angular dependence of the specific heat (caused by the magnetic-field orientation with respect to the crystal lattice) in a similar way.

Fig. 4 Specific heat over temperature \(\frac{C}{T}\) versus an orientation angle \(\phi\) for the applied Field \(H/\pi\xi H_0 = 0.005\) at the temperature \(T/t = 0.006\) (a) and 0.002 (b). Note that the different colours and numbers '1', '2' and '3' correspond to the three solutions: complex, real and dipole as in Figs. 2 and 3.
4 Summary and Discussion

We have calculated the specific heat of the 2-dimensional one-band p-wave superconductor subjected to an external magnetic field of different orientation in the lattice plane. This is a commonly used method to identify the superconducting gap symmetry and nodal states. The experimental studies of the gap symmetry in different superconductors by measuring the angle dependence of the specific heat, have been recently reviewed [18].

Our results show that in different ranges of temperature below the superconducting critical temperature $T_c$, the response could be different. As a matter of fact, the complex solution, showing 4-fold symmetry is reflected in the specific-heat results of the same symmetry, but the two other states can show 2-fold or 4-fold symmetry, depending on temperature. In relation to Sr$_2$RuO$_4$, where the 4-fold symmetry has been also found [5, 6], we conclude that the complex state can be a proper solution for that case. Note that this kind of solution can be generalized to build a 3-dimensional and 3-orbital solution of a more realistic model for superconductivity in Sr$_2$RuO$_4$, including the $k_z$-dependence of the superconducting gap $\Delta(k)$ [12, 19]. That model possesses the horizontal nodal lines in all three bands which is also consistent with other experiments on the specific heat at zero field [20] with quadratic dependence of $C$ as a function of temperature. It has to be remarked that chiral f-wave state have also been proposed to describe some experiments on strontium ruthenate [21, 22].

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