Dynamic Analysis of the influence of the Rotor Centroid Offset of Gyrowheel

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Abstract. Gyrowheel is a new electromechanical servo device which can realize the integration of spacecraft attitude control and attitude measurement. Due to the existence of machining and assembly errors, there must be a deviation between the rotor centroid of the gyrowheel and the support center, that is, the centroid offset. Firstly, to solve this problem, the dynamic modeling of rotor center of mass offset is carried out from the perspective of vector mechanics by using D’Alembert principle. Secondly, the established dynamic model is simplified, and the influence of rotor center of mass offset on the micro vibration of gyrowheel is analyzed. Finally, the correctness of this method is verified by simulation.

1. Introduction
With the rapid development of spacecraft technology, Gyrowheel has great advantages in mass, power consumption, size and functional integration density, so its application in space is becoming more and more urgent[1]. At present, the domestic research on gyrowheel mostly focuses on the theoretical level. Although some research institutions have completed the development of engineering prototype, there is no on orbit application case[2].

The main reason is that there are a lot of non ideal factors in the processing and assembly process of gyrowheel, such as mismatch angle of torque converter, non coplanar and non vertical orthogonal support, non coincidence of rotor center of mass and orthogonal support center[3], etc. In view of these non ideal factors, some scholars at home and abroad have deduced the influence of the non ideal factors of the DTG[4], such as the internal and external torsion bars are not orthogonal, not vertical and the rotor mass is not balanced. Staley studies the characteristic polynomials of the equation of rotor system with internal and external torsion bar supporting scheme by the dynamic equation of DTG[5]. Since the gyrowheel works in a large number of non tilting states in the torque output process, its nonlinear characteristics are particularly critical.

Based on the above analysis, this paper first establishes the dynamic model of the gyro flywheel when the orthogonal support center and the center of mass are not coplanar. Second, it is properly simplified under engineering constraints, and the analytical formula of the disturbance torque caused by the rotor centroid offset characteristic is given, and the theoretical analysis is carried out by using the expressions. Finally, the effectiveness of this study is verified by simulation.
2. Dynamic modeling of the rotor centroid offset

Gyrowheel rotor is the main component to realize momentum exchange. In the process of machining and assembly, it is affected by process technology. Due to technical limitations, it is difficult for the rotor centroid to completely coincide with the support center, that is, the rotor centroid is offset as shown in Fig.1.

This error source will inevitably lead to changes in the dynamic characteristics of the gyrowheel. Therefore, in this section, the mathematical model of the gyrowheel in the presence of centroid offset is established from the perspective of vector mechanics by using the D’Alembert principle, which is used to provide a theoretical basis for the analysis of the dynamic characteristics of the gyrowheel under this nonideal factor. The support points $O_r$, $O_g$, $O_m$ of each body coincide with the centroid. And the radius vector $\mathbf{r}_{mc}$, $\mathbf{r}_{gm}$, $\mathbf{r}_{rg}$ of each body coincide with the centroid. It is assumed that the vector $\mathbf{r}_c$ can be expressed as $\mathbf{r}_c^{(r)} = \begin{bmatrix} R_{rx}^{(c)} & R_{ny}^{(c)} & R_{rz}^{(c)} \end{bmatrix}^T$ and $\mathbf{r}_c^{(c)} = \begin{bmatrix} R_{rx}^{(c)} & R_{ry}^{(c)} & R_{rz}^{(c)} \end{bmatrix}^T$ in the rotor coordinate system and shell system, and according to the coordinate transformation relationship, the following equation can be obtained.

$$\mathbf{r}_c^{(c)} = \mathbf{C}_{mc}^{-1} \cdot \mathbf{C}_{gm}^{-1} \cdot \mathbf{C}_{rg}^{-1} \cdot \mathbf{r}_c^{(r)}$$

Using the above description, in order to derive the mathematical description of the gyrowheel model with centroid offset, the absolute velocity and acceleration of M body, G body and R body are required, firstly; Secondly, it is necessary to establish the moment balance equation by the D’Alembert principle[6].
Considering the rotation of R body to point \( O_r \), the equilibrium equation of R body can be obtained, as shown in the formula:

\[
\int \mathbf{r}_r \times \mathbf{a}_r \, dm_r = \mathbf{T}_{re},
\]

where \( \mathbf{T}_{re} \) is the sum of the external torque vectors on the R body. Then the following expression can be obtained:

\[
\int \mathbf{r}_r \times \mathbf{a}_r \, dm_r = \int \mathbf{r}_r \times \alpha_r \cdot \mathbf{r}_r \, dm_r + \int \mathbf{r}_r \times \left( \omega^r \times \mathbf{r}_r \right) \, dm_r + \int \mathbf{r}_r \times \left[ \left( \omega^r \times \omega^r \right) \times \mathbf{r}_r \right] \, dm_r = \mathbf{T}_{re}
\]

(2)

According to the vector calculation relationship, the second item on the right in equation (2) can be obtained, and the third item can be transformed and written as shown in the following.

\[
\int \mathbf{r}_r \times \left( \alpha_r \cdot \mathbf{r}_r \right) \, dm_r = \mathbf{e}_r^T \left( \mathbf{r}_r \cdot \mathbf{r}_r \mathbf{E}_r - \mathbf{r}_r \cdot \mathbf{r}_r \right) \, dm_r = \alpha_r \cdot \omega^r = \mathbf{J}_r \cdot \omega^r
\]

(3)

\[
\int \mathbf{r}_r \times \left[ \omega^r \times \mathbf{r}_r \right] \, dm_r = \omega^r \times \int \mathbf{r}_r \times \left( \omega^r \times \mathbf{r}_r \right) \, dm_r = \omega^r \times \mathbf{J}_r \cdot \omega^r
\]

(4)

By introducing equation (3) and equation (4) into equation (2), we can get:

\[
\int \mathbf{r}_r \times \mathbf{a}_r \, dm_r = \mathbf{r}_r \times \mathbf{a}_r m_r + \mathbf{J}_r \cdot \omega^r + \omega^r \times \mathbf{J}_r \cdot \omega^r = \mathbf{T}_{re}
\]

(5)

Based on the derivation of equations (1) - (5), the inertia moments of G body to fixed point \( O_g \) and M body to fixed point \( O_m \) can be obtained, which can be sorted and written as follows:

\[
\int \mathbf{r}_g \times \mathbf{a}_g \, dm_g = \mathbf{r}_g \times \mathbf{a}_g m_g + \mathbf{J}_g \cdot \omega^g + \omega^g \times \mathbf{J}_g \cdot \omega^g = \mathbf{T}_{re}
\]

(6)

\[
\int \mathbf{r}_m \times \mathbf{a}_m \, dm_m = \mathbf{r}_m \times \mathbf{a}_m m_m + \mathbf{J}_m \cdot \omega^m + \omega^m \times \mathbf{J}_m \cdot \omega^m = \mathbf{T}_{re}
\]

(7)

3. Establishment of moment balance equation of gyrowheel

(1) Taking the M + R + G body as a whole, the moment balance equation of the M + G + R body is established by using the D'Alembert principle, and the following can be obtained:

\[
\int \mathbf{r}_r \times \mathbf{a}_r \, dm_r = \mathbf{r}_r \times \mathbf{a}_r m_r + \mathbf{J}_r \cdot \omega^r + \omega^r \times \mathbf{J}_r \cdot \omega^r = \mathbf{T}_{re} + T_m + T_g + \sum_{i=m,g,r} \mathbf{r}_c \times \mathbf{g} \cdot \mathbf{m}_i
\]

(8)

Taking the equation (5), equation (6) and equation (7) into equation (8), the following can be deduced:

\[
\sum_{i=m,g,r} \mathbf{r}_c \times \mathbf{a}_i \cdot \mathbf{m}_i + \mathbf{J}_i \cdot \omega^i + \omega^i \times \mathbf{J}_i \cdot \omega^i = \mathbf{T}_{re} + T_m + T_g + \sum_{i=m,g,r} \mathbf{r}_c \times \mathbf{g} \cdot \mathbf{m}_i
\]

(9)

(2) Taking the G + R body as a whole, the moment balance equation of the G + R body is established by using the D'Alembert principle, as follows:

\[
\int \mathbf{r}_r \times \mathbf{a}_r \, dm_r = \mathbf{T}_{re} + T_g + \sum_{i=g,r} \mathbf{r}_c \times \mathbf{g} \cdot \mathbf{m}_i
\]

(10)

Taking equation (5) and equation (6) into equation (10) and sort it out to obtain:

\[
\sum_{i=g,r} \mathbf{r}_i \cdot \mathbf{a}_i \cdot \mathbf{m}_i + \mathbf{J}_i \cdot \omega^i + \omega^i \times \mathbf{J}_i \cdot \omega^i = \mathbf{T}_{re} + T_m + \sum_{i=g,r} \mathbf{r}_c \times \mathbf{g} \cdot \mathbf{m}_i
\]

(11)

(3) Taking R-body as a whole, the moment balance equation of R-body is established as follows:

\[
\int \mathbf{r}_r \times \mathbf{a}_r \, dm_r = \mathbf{T}_{re} + T_m + T_g + \mathbf{r}_c \times \mathbf{g} \cdot \mathbf{m}_i
\]

(12)

Bring equation (5) into equation (12), and we can get:

\[
\mathbf{r}_c \times \mathbf{a}_r \cdot \mathbf{m}_i + \mathbf{J}_i \cdot \omega^i + \omega^i \times \mathbf{J}_i \cdot \omega^i = \mathbf{T}_{re} + T_m + \sum_{i=g,r} \mathbf{r}_c \times \mathbf{g} \cdot \mathbf{m}_i
\]

(13)

In order to obtain the radial dynamic equation of the gyrowheel, the vector equations (11) and (13) are transformed into the component form under the balance ring coordinate system by using the Newton Euler method from whole to part, as shown in equations (14) and (15):
\[ C_{gr} \left( \mathbf{a}_{ro}^{(c)} \times \mathbf{m}_r \right) + \left( \mathbf{J}_g^{(s)} \cdot \omega_g^{(s)} + \omega_g^{(c)} \times \mathbf{J}_g^{(s)} \cdot \omega_g^{(s)} \right) + C_{gr} \left( \mathbf{a}_{ro}^{(c)} \times \mathbf{m}_r \right) + C_{gr} \left( \mathbf{J}_r^{(c)} \cdot \omega_r^{(c)} + \omega_r^{(c)} \times \mathbf{J}_r^{(c)} \cdot \omega_r^{(c)} \right) = C_{gr} \mathbf{T}_r^{(c)} + C_{gr} \left( \mathbf{r}_{rc}^{(c)} \times \mathbf{g}^{(c)} \cdot \mathbf{m}_r \right) \]

Due to the rotor, balance ring, motor support point \((O_r, O_s, O_m)\), motor and balance ring centroid\((C_m, C_p)\) respectively coincide with the centroid, so the following can be shown:

\[ \mathbf{a}_{ro}^{(c)} = a_g^{(c)} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T \]

After defining the centroid offset, the rotor moment of inertia tensor is expressed as:

\[ \mathbf{J}_r = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix}, \quad \mathbf{J}_g^{(s)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & I_{gy} & 0 \\ 0 & 0 & I_{gz} \end{bmatrix} \]

Bringing equations (16) and (17) into two-component equations (14) and (15) respectively, and take the X and Y-axis directions to obtain the radial dynamic equation of gyrowheel considering centroid offset, as shown in the following formulas:

\[ \begin{align*}
&I_{gx} \omega_{gx} - I_{gy} \omega_{gy} - I_{gz} \omega_{gz} + \left( I_{gx} - I_{gy} \right) \omega_{gx} \omega_{gy} + \left( I_{gy} \omega_{gy} - I_{gz} \omega_{gz} \right) \omega_{gy} + I_{gz} \left( \omega_g^{2} - \omega_g^{2} \right) \\
&+ C_{\theta_g} \left[ I_{rx} \omega_{rx} - I_{ry} \omega_{ry} - I_{rz} \omega_{rz} + \left( I_{rx} - I_{ry} \right) \omega_{rx} \omega_{ry} + \left( I_{ry} \omega_{ry} - I_{rz} \omega_{rz} \right) \omega_{ry} + I_{rz} \left( \omega_r^{2} - \omega_r^{2} \right) \right] \\
&+ S_{\theta_g} \left[ I_{rx} \omega_{rx} - I_{ry} \omega_{ry} - I_{rz} \omega_{rz} + \left( I_{rx} - I_{ry} \right) \omega_{rx} \omega_{ry} + \left( I_{ry} \omega_{ry} - I_{rz} \omega_{rz} \right) \omega_{ry} + I_{rz} \left( \omega_r^{2} - \omega_r^{2} \right) \right] \\
&= T_{g_x}^{(c)} + \mathbf{T}_g + \left( R_{gy}^{(c)} \mathbf{g}_y^{(c)} - R_{gz}^{(c)} \mathbf{g}_z^{(c)} \right) \mathbf{C}_{\theta_g} + \left( R_{gy}^{(c)} \mathbf{g}_y^{(c)} - R_{gz}^{(c)} \mathbf{g}_z^{(c)} \right) \mathbf{S}_{\theta_g} \\
&I_{gy} \omega_{gy} - I_{gx} \omega_{gx} - I_{gz} \omega_{gz} + m_r \left[ I_{rx} \omega_{rx} \omega_{ry} + \left( I_{rx} \omega_{rx} - I_{ry} \omega_{ry} \right) \omega_{ry} + \left( I_{gz} - I_{gy} \right) \omega_{gy} \right] = T_{g_y}^{(c)} + T_{g_y} \\
&+ m_r \left[ \left( R_{gy}^{(c)} \mathbf{g}_y^{(c)} - R_{gz}^{(c)} \mathbf{g}_z^{(c)} \right) \mathbf{C}_{\theta_g} \mathbf{S}_{\theta_g} + \left( R_{gy}^{(c)} \mathbf{g}_y^{(c)} - R_{gz}^{(c)} \mathbf{g}_z^{(c)} \right) \mathbf{C}_{\theta_g} \mathbf{S}_{\theta_g} + \left( R_{gy}^{(c)} \mathbf{g}_y^{(c)} - R_{gz}^{(c)} \mathbf{g}_z^{(c)} \right) \mathbf{C}_{\theta_g} \mathbf{S}_{\theta_g} \right] \\
\end{align*} \]

4. The Influence analysis of rotor centroid offset

According to the mathematical description of rotor centroid offset given in the previous section, the disturbance torque generated by the rotor centroid offset of gyrowheel can be expressed as formula (19) in the shell system:

\[ \mathbf{T}_u^{(c)} = \mathbf{r}_{rc}^{(c)} \times \mathbf{g}^{(c)} \cdot \mathbf{m}_r = \begin{bmatrix} I_{ux}^{(c)} \\ I_{uy}^{(c)} \\ I_{uz}^{(c)} \end{bmatrix} = \begin{bmatrix} R_{rx}^{(c)} \mathbf{g}_x^{(c)} - R_{ry}^{(c)} \mathbf{g}_y^{(c)} \\ R_{ry}^{(c)} \mathbf{g}_y^{(c)} - R_{rz}^{(c)} \mathbf{g}_z^{(c)} \\ R_{rz}^{(c)} \mathbf{g}_z^{(c)} - R_{rx}^{(c)} \mathbf{g}_x^{(c)} \end{bmatrix} \cdot \mathbf{m}_r \]

Where

\[ \begin{align*}
R_{rx}^{(c)} &= \begin{bmatrix} C_{\theta_g} \left( R_{yx}^{(c)} \mathbf{C}_{\theta_g} + R_{yz}^{(c)} \mathbf{S}_{\theta_g} \right) - S_{\theta_g} \left( R_{yz}^{(c)} \mathbf{C}_{\theta_g} + \left( R_{yx}^{(c)} \mathbf{S}_{\theta_g} - R_{yz}^{(c)} \mathbf{C}_{\theta_g} \right) \mathbf{S}_{\theta_g} \right) \end{bmatrix} \\
R_{ry}^{(c)} &= S_{\theta_g} \left( R_{yx}^{(c)} \mathbf{C}_{\theta_g} + R_{yz}^{(c)} \mathbf{S}_{\theta_g} \right) + C_{\theta_g} \left( R_{yx}^{(c)} \mathbf{C}_{\theta_g} + \left( R_{yx}^{(c)} \mathbf{S}_{\theta_g} - R_{yz}^{(c)} \mathbf{C}_{\theta_g} \right) \mathbf{S}_{\theta_g} \right) \\
R_{rz}^{(c)} &= R_{ry}^{(c)} \mathbf{S}_{\theta_g} - \left( R_{rx}^{(c)} \mathbf{S}_{\theta_g} - R_{ry}^{(c)} \mathbf{C}_{\theta_g} \right) \mathbf{C}_{\theta_g} \end{align*} \]
$R_{rx}^{(r)}, R_{ry}^{(r)}, R_{rz}^{(r)}$ and $R_{rx}^{(c)}, R_{ry}^{(c)}, R_{rz}^{(c)}$ are the centroid offset of rotor in body coordinate system and shell coordinate system, respectively. $g_i^{(c)}, g_i^{(r)}$ are respectively the projection of gravity in the triaxial direction of the shell. $\theta_1, \theta_2$ are the inner and outer torsion bar angle. $\theta_3$ is the motor rotation angle.

By the approximate assumption $\sin \theta_i \approx \theta_i, \cos \theta_i \approx 1, i = x, y$, the second-order small quantity is kept, and equation (20) is converted to shell coordinate system, we can get:

$$
\begin{bmatrix}
R_{rx}^{(c)} \\
R_{ry}^{(c)} \\
R_{rz}^{(c)}
\end{bmatrix} \approx
\begin{bmatrix}
\phi_y R_{rx}^{(r)} + R_{rx}^{(r)} C_{\phi_1} - R_{ry}^{(r)} S_{\phi_1} + \frac{1}{4} R_{rx}^{(r)} \left[ \left( \phi_y^2 - \phi_z^2 \right) \left( C_{\phi_1} - C_{3\phi_1} \right) + 2 \phi_y \phi_z \left( S_{\phi_1} - S_{3\phi_1} \right) \right] \\
- \phi_y R_{rx}^{(r)} + R_{rx}^{(r)} S_{\phi_1} + R_{ry}^{(r)} C_{\phi_1} + \frac{1}{4} R_{rx}^{(r)} \left[ \left( \phi_y^2 - \phi_z^2 \right) \left( S_{\phi_1} + S_{3\phi_1} \right) + 2 \phi_y \phi_z \left( C_{\phi_1} + C_{3\phi_1} \right) \right] \\
R_{rx}^{(c)} + R_{ry}^{(r)} \left( \phi_y C_{\phi_1} + \phi_z S_{\phi_1} \right) - R_{rx}^{(c)} \left( -\phi_y S_{\phi_1} + \phi_z C_{\phi_1} \right)
\end{bmatrix}
$$

(21)

where $\phi_1, \phi_2$ is the rotation angle of the gyrowheel in the radial direction in the shell coordinate system.

According to formula (19), there are three frequency components in the formula: DC, frequency doubling and frequency doubling.

Since $m_i$ and $g_i^{(c)}, i = x, y, z$ in equation (19) are constants, when equation (21) is brought into equation (19), the radial disturbing moments $T_{ax}^{(c)}, T_{aw}^{(c)}$ caused by centroid offset must have three torque components: DC, frequency doubling and frequency doubling.

When these torques act on the rotor along the radial direction of the shell, the DC disturbance torque will cause gyro drift of the gyrowheel. Because the first and third frequency belong to high frequency, it will not cause gyro drift of the gyrowheel, but will stimulate and produce high-frequency vibration of one and three times the speed frequency as the rotor tilting motion.

5. Simulation

The gyrowheel rotor mass is set to $m_r = 1kg$, the center of mass offset $R_r^{(c)} = [1, 1, 1]^T mm$, and the initial inclination angles of the x-axis of the gyrowheel are $\phi_{x0} = 3^\circ, \phi_{y0} = 0^\circ$, the rotating speed is $\omega = \dot{\theta}_z = 398 rad/s$, and other simulation conditions remain unchanged relative to the ideal model of gyrowheel. The analysis of the tilting motion frequency of the rotor along the x-axis of the shell can be obtained as shown in the figure below.
According to the analysis of the above figure, compared with the ideal state, when the rotor has centroid offset, the frequency component of the tilt motion response of the gyrowheel rotor increases the frequency of one and three times of the speed. Before and after the existence of centroid offset, the precession frequency of the system changes from 0.061hz to 0.064hz, which is caused by the DC torque component caused by the rotor centroid offset. It can be seen that this part of DC torque will drift the rotor, which will affect the accuracy of spacecraft attitude angular velocity measurement based on gyrowheel. The double frequency, nutation frequency, difference frequency and quadruple frequency all change slightly in the same order of magnitude before and after the centroid shift.

6. Conclusion
In this paper, two major contributions of this paper can be summarized as follows: (1) the model of gyrowheel considering the rotor centroid offset is established by Newton-Eular method; (2) The non-coplanarity of the rotor centroid offset has evident impact on vibration frequency, which means that research on the compensation of this non-ideal factor for gyrowheel is needed in the future work.

Acknowledgments
This work is supported by Shanghai Natural Science Foundation under Grant 19ZR1455600.

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