Secret key rates of free-space optical continuous-variable quantum key distribution

Laszlo Gyongyosi1,2,3 | Sandor Imre2

1School of Electronics and Computer Science, University of Southampton, Southampton, UK
2Department of Networked Systems and Services, Budapest University of Technology and Economics, Budapest, Hungary
3MTA-BME Information Systems Research Group, Hungarian Academy of Sciences, Budapest, Hungary

Correspondence
Laszlo Gyongyosi, School of Electronics and Computer Science, University of Southampton, Southampton, SO17 1BJ, UK.
Email: lasgy_ph@yahoo.com

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Summary
In this letter, we derive the maximal achievable secret key rates for continuous-variable quantum key distribution (CVQKD) over free-space optical (FSO) quantum channels. We provide a channel decomposition for FSO-CVQKD quantum channels and study the SNR (signal-to-noise ratio) characteristics. The analytical derivations focus particularly on the low-SNR scenarios. The results are convenient for wireless quantum key distribution and for the quantum Internet.

KEYWORDS
cryptography, networking, quantum cryptology, quantum key distribution, security

1 | INTRODUCTION

Free-space optical (FSO) quantum links1-6 provide a tool to implement quantum communications via wireless telecommunication6-10 network infrastructures. As an integrated component of future quantum Internet11-16 and long-distance quantum communications,11,17-29 the FSO quantum channels could play a significant role in the global-scale practical implementations of quantum communications and quantum key distribution (QKD).1-3,30-36,44-49 QKD systems allow us to utilize the fundamentals of quantum mechanics to realize unconditionally secure communications for legal users. QKD protocols can be decomposed into discrete-variable (DV) and continuous-variable (CV) counterparts.11,17-27 Continuous-variable quantum key distribution (CVQKD) schemes enable parties to use standard telecommunication
devices for experimental implementations. The multicarrier CVQKD has been recently introduced through the adaptive quadrature division modulation (AMQD) scheme. The multicarrier CVQKD injects several additional degrees of freedom into the transmission, which are not available for a standard, single-carrier CVQKD setting. The achievable secret key rates in a multicarrier CVQKD setting have been proven in Gyongyosi and Imre. The secret key rates confirm the multimode bounds determined in Pirandola et al.

The FSO systems bring several new attributes to both the theoretical and experimental side of CVQKD. An FSO quantum link’s special characteristics require a specific mathematical description. The channel characteristics of the FSO quantum links are approachable via the mathematical framework of the GG (gamma-gamma) distribution. The secret key rates for CVQKD schemes over FSO links and the performance of free-space quantum links in diverse environmental conditions raise several questions and call for further examination. Another interesting problem is the private classical capacity of a GG link. Without loss of generality, the private classical capacity measures the amount of classical information that can be privately transmitted from a sender (Alice) to a receiver (Bob) in the presence of an eavesdropper (Eve). For further information on the rate-loss scaling in quantum optical communications, we suggest the derivations in Pirandola et al. Our results on the private classical capacity also confirm the bounds of on private quantum communications in a CVQKD setting.

The private classical capacity imposes a theoretical upper bound on the achievable secret key rates in QKD implementations. Since practical QKD implementations operate in the low-SNR regime, we will analyze the behavior of private classical capacity in the low-SNR domain for CVQKD over FSO (referred to as FSO-CVQKD). By theory, the DVQKD and CVQKD implementations require different channel models. For DVQKD, the resulting channel noise distribution is analogous to the binary-symmetric channel (BSC), while for the DVQKD setting, the resulting noise is Gaussian. Another important difference from traditional crypto systems is that the correlation measure functions are non-traditional. In the theoretical analysis, it is assumed that the legal parties and the eavesdropper have quantum memories and can perform joint measurements, therefore the Holevo information is the appropriate correlation measure function for deriving the private classical capacity.

Several works have focused on implementations of CVQKD in diverse networking conditions, but the question of the achievable secret key rates over FSO quantum links remains open. Consequently, we have chosen to investigate the FSO-CVQKD case because of its significant benefits over DVQKD in practical implementations.

The novel contributions of our paper are as follows:

- We derive an upper bound for the secret key rates of wireless CVQKD over FSO channels.
- We provide a decomposition model for the FSO quantum channel in a CVQKD setting.
- We investigate the SNR attributes of the GG-channel and the complementary channel for the information leakage.

This paper is organized as follows: Section 2 provides the channel model for FSO-CVQKD. Section 3 focuses on the private classical capacity of an FSO link in a CVQKD setting. Finally, Section 4 concludes the results.

## 2 SYSTEM MODEL

In this section, the general formulas and equations are briefly summarized.

The private classical capacity identifies the maximum rate at which classical information can be transmitted privately (i.e., an eavesdropper has no knowledge about the original message) over a quantum channel. To derive the private classical capacity of the FSO setting, the physical quantum link is divided into logical channels. Logical channel denotes information transmission through the GG quantum channel between Alice (A) and Bob (B). Logical channels and are complementary channels that model the information leakage from Alice to eavesdropper Eve and from Bob to Eve, respectively. In our setting, is relevant for the DV case, while is important in the reverse reconciliation of CVQKD (Bob starts the reconciliation to minimize information leakage).

Let

$$|\varphi_i\rangle = |x_i + ip_i\rangle,$$  (1)
identify a \( j \)th input coherent state (Gaussian state) in the phase space \( S \), with \( i.i.d. \) Gaussian random position and momentum quadratures \( x_j \in \mathcal{N} \left( 0, \sigma_{x_j}^2 \right) \) and \( p_j \in \mathcal{N} \left( 0, \sigma_{p_j}^2 \right) \), where \( \sigma_{x_j}^2 \) is the modulation variance. The coherent state \( |\varphi_j\rangle \) in the phase space \( S \) can be modeled as a zero-mean, circular symmetric complex Gaussian random variable

\[
z_j \in \mathcal{CN} \left( 0, \sigma_{\varphi_j}^2 \right),
\]

with variance

\[
\sigma_{\varphi_j}^2 = \mathbb{E} \left[ |z_j|^2 \right] = \mathbb{E} \left[ x_j^2 + p_j^2 \right] = 2\sigma_{x_j}^2,
\]

and with \( i.i.d. \) real and imaginary zero-mean Gaussian random components, \( \text{Re} \left( z_j \right) \in \mathcal{N} \left( 0, \sigma_{\text{Re} z_j}^2 \right), \text{Im} \left( z_j \right) \in \mathcal{N} \left( 0, \sigma_{\text{Im} z_j}^2 \right) \).

The transmission of this complex variable over the Gaussian quantum channel \( \mathcal{N} \) can be characterized by the \( T \left( \mathcal{N} \right) \) \( \in \mathbb{C} \) normalized complex transmittance variable

\[
T \left( \mathcal{N} \right) = \text{Re} T \left( \mathcal{N} \right) + i \text{Im} T \left( \mathcal{N} \right),
\]

where \( 0 \leq \text{Re} T \left( \mathcal{N} \right) \leq 1 / \sqrt{2} \) is the transmission of the position quadrature and \( 0 \leq \text{Im} T \left( \mathcal{N} \right) \leq 1 / \sqrt{2} \) is the transmission of the momentum quadrature. During the evaluation of the private classical capacity for an FSO setting, we assume that the CVQKD protocol operates in the low-SNR regime, \( \text{SNR} \to 0 \), which is precisely the situation for practical CVQKD scenarios.

Utilizing an FSO channel for the transmission of a given input \( z_j \), logical channel \( \mathcal{N}_{AB} \) is defined as

\[
\mathcal{N}_{AB} : T \left( \mathcal{N} \right) z_j + \epsilon_j,
\]

where \( \epsilon_j \) is a zero-mean, circular symmetric complex Gaussian random variable that identifies the Gaussian noise added by Eve; \( \epsilon_j \in \mathcal{CN} \left( 0, \sigma_{\epsilon_j}^2 \right) \), with variance \( \sigma_{\epsilon_j}^2 = \mathbb{E} \left[ |\epsilon_j|^2 \right] = 2\sigma_{\epsilon_j}^2 \), and with \( i.i.d. \) real and imaginary zero-mean Gaussian random components, \( \text{Re} \left( \epsilon_j \right) \in \mathcal{N} \left( 0, \sigma_{\text{Re} \epsilon_j}^2 \right), \text{Im} \left( \epsilon_j \right) \in \mathcal{N} \left( 0, \sigma_{\text{Im} \epsilon_j}^2 \right) \), while the \( T \left( \mathcal{N} \right) \) complex transmittance coefficient is as

\[
T \left( \mathcal{N} \right) = \eta I + i\eta I,
\]

where \( \eta \) is the effective photocurrent conversion ratio of the receiver, while \( I \) is the normalized irradiance with the gamma-gamma (GG) probability density\(^7\)-\(^10\)

\[
f \left( I \right) = \frac{2(ab)^{\frac{a+b}{2}}}{\Gamma \left( a \right) \Gamma \left( b \right)} I^{\frac{a+b}{2}-1} K_{a-b} \left( 2\sqrt{ab}I \right),
\]

where \( K_{\cdot} \left( \cdot \right) \) is the modified Bessel function of the second kind and of order \( v, \Gamma \left( \cdot \right) \) is the Gamma function, while \( a \geq 0 \), and \( b \geq 0 \) are the distribution-shaping parameters expressed as

\[
a = \left\lfloor \exp \left( \frac{0.49\delta^2}{\left( 1 + 0.18d^2 + 0.56\delta^{12/5} \right)^{7/6}} \right) - 1 \right\rfloor^{-1},
\]

and

\[
b = \left\lfloor \exp \left( \frac{0.51\delta^2}{\left( 1 + 0.9d^2 + 0.62\delta^{12/5} \right)^{5/6}} \right) - 1 \right\rfloor^{-1},
\]

where

\[
\delta^2 = 1.23 C^2 \left| x_j \right|^{7/6} I^{11/6} = 1.23 C^2 \left| p_j \right|^{7/6} I^{11/6}
\]
is the Rytov variance, $C^2$ is the altitude-dependent turbulence strength, $l$ is the length of the link, and $|k|$ is the optical wave number, while $d = \sqrt{|k|^2D^2/4l}$, where $D$ is the receiver's aperture diameter. The complementary channel $N_{BE}$ for a reverse reconciliation in the CVQKD case is defined as

$$N_{BE} : \mathcal{N} \left( 0, \sigma_{BE}^2 \right),$$

where $\sigma_{BE}^2$ is the noise variance for $N_{BE}$. Assuming that the parties have quantum memories and can perform joint measurement on their quantum registers in the QKD protocol run, the appropriate correlate measure functions for the logical channels $N_{AB}$, $N_{AE}$, and $N_{BE}$ are the Holevo quantities $\chi_{AB}$, $\chi_{AE}$, and $\chi_{BE}$.

Without loss of generality, for the $S(N)$ secret key rate over a quantum channel $N$, the following relation holds:

$$S(N) \leq \lim_{n \to \infty} \frac{1}{n} P(N),$$

where $P(N)$ is the private classical capacity of $N$. Assuming reverse reconciliation in CVQKD with GG channel $N_{AB}$ and Eve’s Gaussian channel $N_{BE}$, $P(N)$ is

$$P(N) = \lim_{n \to \infty} \frac{1}{n} \max_{\rho \in \mathcal{D}} \left( f \chi_{AB} - \chi_{BE} \right),$$

where $f$ is the reconciliation efficiency, $\chi_{AB}$ is the Holevo information between Alice and Bob

$$\chi_{AB} = S(N_{AB}(\rho_{AB})) - \sum_i p_i S(N_{AB}(\rho_i))$$

and $\chi_{BE}$ is the Holevo information between Bob and Eve

$$\chi_{BE} = S(N_{BE}(\rho_{BE})) - \sum_i p_i S(N_{BE}(\rho_i))$$

are the Holevo quantities between Alice and Bob and Bob and Eve; $S(\rho) = -\text{Tr} (\rho \log(\rho))$ is the von Neumann entropy, while $\rho_{AB} = \sum_i p_i \rho_i$ and $\rho_{BE} = \sum_i p_i \rho_i$. The quantity $\chi_{BE}$ is the Holevo information between Bob and Eve, which plays a role in a reverse reconciliation CVQKD. We also use this approach to derive $P(N)$ for the CVQKD case, since reverse reconciliation is proved to minimize the eavesdropper’s Holevo information compared with the direct-reconciliation case.

Thus, $P(N)$ at a reverse reconciliation with reconciliation efficiency $f$ is evaluated as

$$P(N) = \lim_{n \to \infty} \frac{1}{n} \left( f \left( \max_{\rho \in \mathcal{D}} S(N_{AB} \left( \sum_i p_i (\rho_i) \right)) \right) - \sum_i p_i S(N_{AB}(\rho_i)) \right)$$

$$- S(N_{BE} \left( \sum_i p_i (\rho_i) \right)) + \sum_i p_i S(N_{BE}(\rho_i)),$$

where $N(\rho_i)$ represents the $i$th output density matrix.

Specifically, the $D(\cdot\|\cdot)$ quantum relative entropy function between density matrices $\rho$ and $\sigma$ is

$$D(\rho\|\sigma) = \text{Tr} (\rho \log(\rho)) - \text{Tr} (\rho \log(\sigma))$$

$$= \text{Tr} \left( \rho (\log(\rho) - \log(\sigma)) \right).$$

The Holevo quantity can be expressed by the quantum relative entropy function as $\chi = D(\rho_k\|\sigma)$, where $\rho_k$ denotes an optimal channel output state (for which the Holevo quantity will be maximal) and $\sigma = \sum p_k \rho_k$. The Holevo information $\chi$ can be derived in terms of $D(\cdot\|\cdot)$ as

$$\sum_i p_i S(N_{AB}(\rho_i)) \right)$$

$$- S(N_{BE} \left( \sum_i p_i (\rho_i) \right)) + \sum_i p_i S(N_{BE}(\rho_i)),$$
\[
\sum_k p_k D(\rho_k \| \sigma) = \sum_k \left( p_k \text{Tr} (\rho_k \log (\rho_k)) \right) - \text{Tr} \left( \sum_k \left( p_k \rho_k \log (\sigma) \right) \right) \\
= \sum_k \left( p_k \text{Tr} (\rho_k \log (\rho_k)) \right) - \text{Tr} (\sigma \log (\sigma)) \\
= S(\sigma) - \sum_k p_k S(\rho_k) = \chi.
\]

Therefore, \( \chi_{AB} \) is rewritten as
\[
\chi_{AB} = S \left( \mathcal{N}_{AB} \left( \sum_i p_i \rho_i \right) \right) - \sum_i p_i S \left( \mathcal{N}_{AB} (\rho_i) \right). \tag{19}
\]

The quantity \( \chi_{BE} \) measures the Holevo information leaked to Eve from Bob during a reverse reconciliation and is written as
\[
\chi_{BE} = S \left( \mathcal{N}_{BE} \left( \sum_i p_i \rho_i \right) \right) - \sum_i p_i S \left( \mathcal{N}_{BE} (\rho_i) \right). \tag{20}
\]

Using (19) and (20), \( P (\mathcal{N}) \) can be expressed as
\[
P (\mathcal{N}) = \lim_{n \to \infty} \frac{1}{n} \left( f \left( \min_{\rho} \max_{\sigma} D \left( \rho_k^{AB} \| \sigma^{AB} \right) \right) - \min_{\sigma} \max_{\rho} D \left( \rho_k^{BE} \| \sigma^{BE} \right) \right) \\
= \lim_{n \to \infty} \frac{1}{n} \left( f \left( \min_{\rho} \max_{\sigma} D \left( \rho_k^{AB-BE} \| \sigma^{AB-BE} \right) \right) \right), \tag{21}
\]

where \( \rho_k^{AB-BE} \) is the final optimal density matrix, while \( \sigma^{AB-BE} \) refers to the final output average density matrix. Figure 1 depicts the model of FSO-CVQKD channel \( \mathcal{N} \) used for the derivation of \( P (\mathcal{N}) \).
3 RESULTS

**Theorem 1.** (Scalability of the secret key rate). At a direct or reverse reconciliation with channels $\mathcal{N}_{AE}$ and $\mathcal{N}_{BE}$ between Alice and Eve and Bob and Eve, the $S(\mathcal{N})$ secret key rate over an FSO link $\mathcal{N}$ in any CVQKD protocol is scalable by the SNR$_{AB}$ of $\mathcal{N}_{AB}$.

**Proof.** Let $\mathcal{N}_{AB}$ be the quantum channel between Alice and Bob. For the transmission of a given $z_i$, let the SNR of $\mathcal{N}_{AB}$ be evaluated as

$$\text{SNR}_{AB} = \frac{2\sigma^2_{m_0}}{\sigma^2_{\mathcal{N}_{AB}}}, \quad (22)$$

where $\sigma^2_{m_0}$ is the modulation variance of a quadrature component and $\sigma^2_{\mathcal{N}_{AB}} = 2\sigma^2_{\mathcal{E}_{\text{Eve}}}$ is the variance of the Gaussian noise. To evaluate the private classical capacity, we consider three scenarios$^8$ for the values of coefficients $a$ and $b$. First, assume that$^8$ $a + b = \frac{13}{2}$, which yields

$$\tau \text{SNR}_{AB} \approx e^{-2\sqrt{ab}(\frac{1}{\pi})^{1/4}}, \quad (23)$$

where

$$\tau = \frac{\Gamma(a) \Gamma(b) \Omega}{2\sqrt{\pi(ab)^2}}, \quad (24)$$

while coefficient $\Omega$ is as

$$\Omega = (\eta \mathbb{E}[I])^2 = \eta^2 \quad (25)$$

and $\lambda$ is a Lagrange multiplier.$^8$ Then, let $\lambda_{\text{SNR}}$ be a Lagrange multiplier at an average power constraint $c$,

$$c : 2\sigma^2_{m_0} = \text{SNR}_{AB} = \frac{1}{16(ab)^2} \Omega \text{SNR}_{AB} \log^4 \left( \frac{1}{\tau \text{SNR}_{AB}} \right), \quad (26)$$

where $2\sigma^2_{m_0}$ refers to the average input power$^8$ associated to a jth input $z_j$ and $\gamma = \eta^2 P$. The resulting modulation variance $2\sigma^2_{m_0}$ for $\mathcal{N}_{AB}$ with SNR$_{AB}$ is then

$$2\sigma^2_{m_0} \approx \text{SNR}_{AB} \sigma^2_{\mathcal{N}_{AB}} = 2\frac{1}{\tau}e^{-2\sqrt{ab}(\frac{1}{\pi})^{1/4}} \sigma^2_{\mathcal{N}_{AB}}, \quad (27)$$

which allows us to evaluate $\mathcal{P}(\mathcal{N})$ at $a + b = \frac{13}{2}$ via (13), where $\chi_{AB}$ is the Holevo information between Alice and Bob over $\mathcal{N}_{AB}$,

$$\chi_{AB} \geq \frac{\text{SNR}_{AB}}{16(ab)^2} \Omega \log^4 (\tau \text{SNR}_{AB}), \quad (28)$$

while $\chi_{BE}$ is the Holevo information derived via the Gaussian channel $\mathcal{N}_{BE}$ between Bob and Eve,

$$\chi_{BE} \approx \sum_{i=1}^{2} G \left( \frac{\lambda_i - 1}{2} \right) - \sum_{i=3}^{4} G \left( \frac{\lambda_i - 1}{2} \right), \quad (29)$$

where

$$G(x) = (x + 1) \log_2 (x + 1) - x \log_2 x, \quad (30)$$

and $\lambda_1^2 = \frac{1}{2} \left( A + \sqrt{A^2 - 4B} \right)$, $\lambda_2^2 = \frac{1}{2} \left( C + \sqrt{C^2 - 4D} \right)$, $\lambda_3^2 = 1$, $\lambda_4^2 = 2$, where $A = V^2 (1 - 2T) + 2T + T^2 (V + \kappa_{\text{line}})^2$, $V = \sigma^2_{m_0} + 1$, $\kappa_{\text{line}}$ is the total channel-added noise,$^{26}$ $B = T^2 (V \kappa_{\text{line}} + 1)^2$, $C = \frac{1}{(T(V + \kappa_{\text{line}}))} (A \kappa_M^2 + B + 1 + 2\kappa_M (V \sqrt{B} + T (V + \kappa_{\text{line}})) + 2T (V^2 - 1))$, while $\kappa_{\text{tot}}$ is the overall noise $\kappa_{\text{tot}} = \kappa_{\text{line}} + \kappa_M \frac{1}{T}$, $\kappa_M$ is the detector-added error,$^{27}$ $D = \left( \frac{V + \sqrt{B_M}}{T(V + \kappa_{\text{line}})} \right)^2$, with an SNR$_{BE}$ of the Gaussian channel $\mathcal{N}_{BE}$, as
Proof. Utilizing parameters of the FSO channel model yields the SNR, as given by (29). The proof is concluded here.

The next theorem focuses on the reverse reconciliation case.

**Theorem 2.** (Maximized secret key rate over an FSO link in CVQKD at a reverse reconciliation). The $\mathcal{P}(\mathcal{N})$ of an FSO channel $\mathcal{N}$ in a CVQKD setting for $\text{SNR}_{AB} \to 0$ of $\mathcal{N}_{AB}$ at a reconciliation efficiency $f$ is

$$
\mathcal{P}(\mathcal{N}) = \lim_{n \to \infty} \frac{1}{n} \max_{\mathcal{V}} \left( f \left( \frac{\Omega}{16(ab)^2 \text{SNR}_{AB}} \log_4 \left( \frac{1}{\text{SNR}_{AB}} \right) \right) - \chi_{BE} \right),
$$

where $\chi_{BE}$ is as given by (29). The proof is concluded here.

The next theorem focuses on the reverse reconciliation case.
with
\[
\text{SNR}_{AB} = \frac{2\sigma_h^2}{\sigma_N^2} = \frac{\sigma_{\text{sth}}^2}{\sigma_{\text{Eve}}^2} \approx \frac{1}{\tau} \frac{e^{ab - \frac{1}{2}}}{\sqrt{ab \tau / \Omega}} e^{-2\sqrt{ab \tau / \Omega}},
\]
and the \( \lambda_{\text{SNR}_{AB}} \) Lagrange multiplier is evaluated as
\[
\lambda_{\text{SNR}_{AB}} = \frac{1}{\tau} \frac{e^{ab - \frac{1}{2}}}{\sqrt{ab \tau / \Omega}} e^{-2\sqrt{ab \tau / \Omega}} \frac{1}{\text{SNR}_{AB}},
\]
where
\[
\tau = \frac{\Gamma(a) \Gamma(b) \Omega^{ab - \frac{1}{2}}}{2\sqrt{\pi (ab)^{ab - \frac{1}{2}}}}.
\]

The Holevo information \( \chi_{BE} \) of Eve is as given by (29), the proof is therefore concluded here.

\[ \square \]

4 | CONCLUSIONS

This letter studied the performance of CVQKD over free-space optical quantum channels. The analysis provided the private classical capacity of an FSO link for the CVQKD protocols at a reverse reconciliation. Our derivations were focused on the low-SNR setting. The results prove to be convenient for wireless quantum communications, wireless quantum key distribution and quantum Internet scenarios.

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STATEMENTS

ETHICS STATEMENT

This work did not involve any active collection of human data.

DATA ACCESSIBILITY STATEMENT

This work does not have any experimental data.

COMPETING FINANCIAL INTERESTS STATEMENT

We have no competing financial interests.

COMPETING INTERESTS STATEMENT

We have no competing interests.

AUTHORS’ CONTRIBUTIONS

L.GY. designed the protocol and wrote the manuscript. L.GY. and S.I. analyzed the results. All authors reviewed the manuscript.
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