Non-Thermal Dark Matter from Cosmic Strings

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Abstract

Cosmic strings can be created in the early universe during symmetry-breaking phase transitions, such as might arise if the gauge structure of the standard model is extended by additional $U(1)$ factors at high energies. Cosmic strings present in the early universe form a network of long horizon-length segments, as well as a population of closed string loops. The closed loops are unstable against decay, and can be a source of non-thermal particle production. In this work we compute the density of WIMP dark matter formed by the decay of gauge theory cosmic string loops derived from a network of long strings in the scaling regime or under the influence of frictional forces. We find that for symmetry breaking scales larger than $10^{10}$ GeV, this mechanism has the potential to account for the observed relic density of dark matter. For symmetry breaking scales lower than this, the density of dark matter created by loop decays from a scaling string network lies below the observed value. In particular, the cosmic strings originating from a $U(1)$ gauge symmetry broken near the electroweak scale, that could lead to a massive $Z'$ gauge boson observable at the LHC, produce a negligibly small dark matter relic density by this mechanism.
1 Introduction

Cosmic strings are one-dimensional topological defects \[1, 2, 3\]. They can be formed during symmetry breaking phase transitions in the early universe \[4\], such as might arise if the gauge symmetry group of the standard model (SM) is enlarged by additional \(U(1)\) factors at high energies \[5, 6, 7\]. Such gauge factors appear naturally in many models of physics beyond the SM. A typical cosmic string network consists of horizon-length long strings as well as smaller closed string loops. The long strings carry a net conserved topological charge and are stable, while closed string loops do not have a net charge and can decay away. In the present work we study the production of dark matter by the decays of cosmic string loops \[8, 9\].

The evolution of cosmic string networks in the early universe has been studied extensively. It is found that the long strings in the network stretch with the Hubble expansion, as well as form closed string loops by the process of reconnection when they intersect themselves or each other. After an initial transient, these two processes balance out such that the string network reaches a scaling regime, in which the total energy density of the network makes up a small, constant fraction of the dominant matter or radiation energy density of the universe \[10, 11, 12, 13, 14, 15\]. This string energy fraction is largely independent of the initial conditions, and is about \(G\mu\) relative to the critical density, where \(G = 1/(8\pi M_{\text{Pl}}^2)\) is Newton’s constant and \(\mu\) is the string tension. In terms of the scale of spontaneous symmetry breaking \(\eta\) from which the strings arise, the tension is on the order of

\[
\mu \simeq \eta^2. \tag{1.1}
\]

Cosmic string scaling can be modified if the strings have significant interactions with the thermal background that lead to an effective frictional force on the strings. The importance of friction to the evolution of a string network depends on the pattern of symmetry breaking as well as the temperature of the surrounding plasma.

A possible decay product of the string loops formed by a scaling cosmic string network is dark matter (DM) \[16\]. Indirect evidence for cold dark matter has been obtained from a number of sources. Together, they indicate a dark matter density of \[17\]

\[
\Omega_{DM} h^2 = 0.1143 \pm 0.0034 \tag{1.2}
\]

This dark matter density can be explained by the existence of a new weakly-interacting stable particle with a mass on the order of the electroweak scale (WIMP) \[16\]. Such particles are common in extensions of the SM that stabilize the electroweak scale against quantum corrections such as supersymmetry \[18\], universal extra dimensions \[19\], and little Higgs models with \(T\)-parity \[20\].

String loops can produce DM and other particles in a number of ways. For local cosmic strings, corresponding to the spontaneous breakdown of a gauge symmetry, the direct emission of particles by the strings is thought to be suppressed \[21\]. Instead, loops lose most of their energy by oscillating and emitting gravitational radiation \[23\]. However, when

\[1\] However, see Refs. \[14, 22\] for arguments to the contrary.
the loop radius shrinks down to the order of the string width, it will self-annihilate into its constituent fields \[8\]. The subsequent decays of these states can produce dark matter. An even larger source of string annihilation and particle production by closed loops is the formation of *cusp*[s] \[24\]. These are points on a loop where the string segment folds back on itself and briefly approaches the speed of light \[25, 26\]. In the vicinity of a cusp, a small portion of the string will self-annihilate creating particles \[24, 27\]. In many simple solutions for the motion of a string loop, a cusp is generally found to occur about once per loop oscillation period \[25, 26, 28, 29, 30, 31\].

The amount of dark matter created by a cosmic string network depends on the initial size of the string loops that are formed by the network. While the evolution of the long horizon length strings is well understood, the details of loop formation are less clear. These details are closely related to the spectrum of small fluctuations on long strings. Significant advances have been made recently in this direction, both in numerical simulations \[32, 33, 34\], as well as in analytic models Ref. \[35, 36, 37, 38, 39, 40\]. In the present work we will mostly adopt the results of Refs. \[36, 37, 38, 39\] to characterize the initial loop size spectrum.

In the picture of loop formation (in the scaling regime) that emerges from Refs. \[36, 37, 38, 39\], fluctuations on long strings are created near the horizon scale \(d_H \sim t\), which is also the scale that characterizes the long string network in the scaling regime. After they are formed, these fluctuations grow less quickly than the horizon, and thereby shrink relative to the characteristic scale of the long string network. The fluctuation spectrum that emerges is a power law in the fluctuation size that increases going to smaller scales. This power law is eventually cut off well below the horizon by gravitational radiation damping, which erases very small fluctuations. The cut-off occurs when the fluctuation size falls below the gravitational radiation scale \[36\]

\[
l_{GW} = \Gamma (G\mu)^{1+2\chi} t
\]

where \(\Gamma \simeq 50\) is a constant, \(t\) is the cosmic time, and \(\chi = 0.10\) during radiation and 0.25 during the matter era. The small fluctuation spectrum on long strings is therefore peaked near \(l_{GW}\). This peak implies that \(l_{GW}\) sets the typical initial loop size \(\ell_i\) \[36, 37, 38, 39\],

\[
\ell_i \simeq l_{GW}.
\]

Both the number and the energy density of the loops formed are dominated by loops of this initial size\[3\]. However, recent simulations (and the analytic model of Ref. \[40\]) also point toward a significant loop population near the horizon scale, with \(\ell_i \simeq (0.1) t_i\) \[32, 33, 34\]. We will therefore consider larger loops as well in our analysis.

The primary goal of this paper is to compute the dark matter density generated by a network of gauge theory cosmic strings in the scaling regime. We also calculate the dark matter generated when *frictional* interactions of the strings with the background plasma modify the evolution of the network. A necessary condition for the phenomenological

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2For smaller values of \(\eta\) and at very early times, the gravitational damping length \(l_{GW}\) can fall below the width of the string, \(w \sim \eta^{-1}\). In this case, we expect that small fluctuations are cut off at \(\ell \sim w > l_{GW}\) by direct particle emission as suggested in Ref. \[14, 22\].
viability of a cosmic string network is that the dark matter density it generates not exceed the observational bound. Our results therefore provide a constraint on theories of new physics that lead to the formation of cosmic strings, such as models containing new $U(1)$ gauge groups that arise frequently in superstring theory constructions and in models of grand unification [5, 6, 7]. Our results also motivate certain classes of gauge extensions of the standard model that are able to generate the observed dark matter relic density by cosmic string loop decay. While we concentrate on the dark matter produced by a network of cosmic strings, our methods can also be applied to computing the densities of other cosmologically interesting particles arising from cosmic string loop decays, such as moduli and gravitinos.

We find that for symmetry breaking scales $\eta$ larger than $10^{10}$ GeV, the decays of cosmic string loops derived from a network of long cosmic strings that are scaling or dominated by friction can potentially generate (more than) enough cold dark matter to account for the observed relic density. The precise density depends on the typical initial loop size and the effective branching fraction of the loop decays into cold dark matter, in addition to $\eta$. Note that $\eta = 10^{10}$ GeV corresponds to $G\mu \simeq 10^{-18}$, for which the standard cosmic string signatures such as gravitational radiation and gravitational lensing are expected to be unobservably weak [2, 3]. When $\eta$ is much smaller than $10^{10}$ GeV, including those scales for which the massive $Z'$ gauge boson associated with the symmetry breaking could be visible at the LHC, the density of non-thermal dark matter produced by this mechanism is well below the observed value. Thus, such models are not constrained by dark matter overproduction from scaling string loops.

This paper is organized as follows. In Section 2 we derive a general formula for the production of dark matter by cosmic string loops. In Section 3 we apply this formula to compute the amount of dark matter produced by string loops in the scaling regime. In Section 4 we extend our results to compute the dark matter density from cosmic string loops when friction plays an important role in the evolution of the string network. Finally, Section 5 is reserved for our conclusions.

Before proceeding, let us point out that the production of dark matter by the decays of cosmic string loops was considered previously for ordinary Abelian Higgs model cosmic strings in Ref. [8], for cosmic strings derived from a supersymmetric flat direction in Ref. [41], and for cosmic strings associated with aspects of superstring theory in Refs. [9, 42, 43]. We concentrate on ordinary cosmic strings in the present work, and we update and extend the results of Ref. [8] in a number of ways. Most importantly, we focus on cusping as the primary source of particle production by cosmic strings. We also make use of the recent results on the size distribution of string loops when they are formed from Refs. [36, 37, 38, 39], and we consider the evolution of cosmic string networks both with and without friction. When friction is relevant, we extend Ref. [8] by using the analytic model of Ref. [13] to describe the long string network.
2 A Formula for Dark Matter from Cosmic Strings

Let us denote the initial \textit{invariant} length of a string loop formed at cosmic time \( t_i \) by \( \ell_i \). The invariant length \( \ell \) of a loop is defined in relation to its energy in the cosmological frame by

\[
E_{\text{loop}} = \mu \ell.
\]  

(2.1)

If the loop is boosted with speed \( \nu \), the invariant length \( \ell \) will exceed the proper length of the loop in its rest frame by a factor of \( \gamma = 1/\sqrt{1 - \nu^2} \).

The key quantity characterizing loop formation is

\[
r(\ell_i, t_i) d\ell_i dt_i,
\]  

(2.2)

the number density of string loops formed in the time interval \((t_i, t_i + dt_i)\) with initial length in the range \((\ell_i, \ell_i + d\ell_i)\). The form of the function \( r(\ell_i, t_i) \) is constrained by the evolution of the long string network. In particular, the total rate at which energy is transferred from the long string network to loops is typically a known quantity \[13\]. We can use this fact to impose the constraint

\[
\frac{d\rho_{\text{loop}}}{dt_i} = \int d\ell_i \; \mu \ell_i r(\ell_i, t_i),
\]  

(2.3)

where \( d\rho_{\text{loop}}/dt \) is the rate of energy transfer into loops from the long string network.

Consider the evolution of a loop of initial size \( \ell_i \) formed at time \( t_i \). The length of this loop is described by the function

\[
\ell(t; \ell_i, t_i)
\]  

(2.4)

which is the solution to the equation

\[
\mu \frac{d\ell}{dt} = -P_{\text{tot}}, \quad \text{with} \quad \ell(t_i; \ell_i, t_i) = \ell_i,
\]  

(2.5)

where \( P_{\text{tot}} \) is the total rate of energy loss from the loop. We will assume that \( P_{\text{tot}} \) is positive so that \( \ell \) is monotonically decreasing in time. Under this assumption, the loop will eventually decay away completely at the time \( t_{\text{co}}(\ell_i, t_i) \), defined implicitly by

\[
0 = \ell(t_{\text{co}}; \ell_i, t_i).
\]  

(2.6)

While the loop is decaying, it will emit a fraction of its energy in the form of (cold) dark matter. We will write this fraction as \( P_{\text{DM}} \leq P_{\text{tot}} \).

To find the total rate of dark matter production by the collection of string loops, consider loops with initial size in the range \((\ell_i, \ell_i + d\ell_i)\) formed in the time interval \((t_i, t_i + dt_i)\). There are \( r(\ell_i, t_i) d\ell_i dt_i \) such loops per unit volume initially. As time goes on, this collection of loops is diluted by the cosmic expansion by a factor of \( a^{-3} \). This is the only modification of their density until they decay away completely at time \( t_{\text{co}}(\ell_i, t_i) \). At time \( \bar{t} \), this collection
of loops will produce dark matter at the rate $P_{DM}$. The dark matter will thermalize if it is produced before the freeze-out time $\tilde{t} < t_{fo}$, and redshift as $a^{-3}$ after this time. Thus, we have

$$
\Delta \rho_{DM} = \int d\ell_i \int_{t_\eta}^{t_0} dt_i \int_{t_{fo}}^{\ell_{co}(\ell_i, t_i)} d\tilde{t} \ r(\ell_i, t_i) \left[ \frac{a(t_i)}{a(\tilde{t})} \right]^3 P_{DM} \left[ \frac{a(\tilde{t})}{a(t_0)} \right]^3 \Theta(t_{co} - t_{fo}). \quad (2.7)
$$

The integral over $t_i$ runs between the string network formation time $t_\eta$ and the present time $t_0 \approx 4 \times 10^{41} \text{ GeV}^{-1}$. Typically, $t_\eta$ is on the order of $t_\eta \approx \frac{M_{Pl}}{\eta^2}$, (2.8) where $\eta$ is the scale of spontaneous symmetry breaking. The integration over the decay time $\tilde{t}$ runs from the freeze-out time $t_{fo}$ to either $t_{co}$ or $t_0$, whichever is smaller. Thus, we have defined $\tilde{t}_{co}(\ell_i, t_i)$ by

$$
\tilde{t}_{co}(\ell_i, t_i) = \begin{cases} 
  t_{co}(\ell_i, t_i) & : \ t_{co} < t_0 \\
  t_0 & : \ t_{co} \geq t_0
\end{cases}. \quad (2.9)
$$

Any restriction on the integration limits of $\ell_i$ are encoded in the support of the function $r(\ell_i, t_i)$.

It is often the case that $P_{DM}$ and $P_{tot}$ depend on $\tilde{t}$ only through $\tilde{\ell} = \ell(\tilde{t}; \ell_i, t_i)$. If so, it is convenient to change the integration variable from $\tilde{t}$ to $\tilde{\ell}$. The corresponding Jacobian is simply $(\partial \tilde{\ell} / \partial \tilde{t})^{-1} = -\mu / P_{tot}$, where $P_{tot}$ is the total power released by a loop of length $\tilde{\ell}$. It follows that

$$
\Delta \rho_{DM} = \int d\tilde{\ell_i} \int_{t_\eta}^{t_0} dt_i \int_{t_{fo}(\ell_i, t_i)}^{\tilde{t}_{co}(\ell_i, t_i)} d\tilde{\ell} \ r(\ell_i, t_i) \left[ \frac{a(t_i)}{a(\tilde{\ell})} \right]^3 \frac{\mu P_{DM}}{P_{tot}} \quad (2.10)
$$

The limits on the $\tilde{\ell}$ integration depend on the functions

$$
\tilde{t}_{fo}(\ell_i, t_i) = \begin{cases} 
  \ell(t_{fo}; \ell_i, t_i) & : \ t_i < t_{fo} \\
  \ell_i & : \ t_i > t_{fo}
\end{cases}, \quad (2.11)
$$
as well as

$$
\ell_x(\ell_i, t_i) = \begin{cases} 
  0 & : \ t_{co}(\ell_i, t_i) < t_0 \\
  \ell(t_0; \ell_i, t_i) & : \ t_{co}(\ell_i, t_i) \geq t_0
\end{cases}. \quad (2.12)
$$

Eqs. (2.7) and (2.10) are our main results. We will apply them to two interesting special cases below.

3 Dark Matter Production from a Scaling Network

As a first application of our main result, Eq. (2.7), we compute the dark matter density produced by a network of cosmic strings in the scaling regime due to the cusping of closed string loops. To simplify the analysis, we focus on monochromatic loop formation distributions with

$$
\ell_i = \alpha t_i. \quad (3.1)
$$
This relation implies \( r(\ell_i, t_i) \propto \delta(\ell_i - \alpha t_i) \). More general distributions can be obtained by introducing a weight function and summing the final result over different values of \( \alpha \).

In the string scaling regime, the rate at which the long string network transfers energy into loops is \[ \frac{d\rho_{\text{loop}}}{dt_i} = \zeta \mu t_i^{-3}, \] with \( \zeta \approx 10 \) a constant characterizing the mean properties of the long string network. Imposing the constraint of Eq. (2.3) on the loop formation rate \( r(\ell_i, t_i) \), we obtain

\[ r(\ell_i, t_i) = \frac{\zeta}{\alpha} t_i^{-4} \delta(\ell_i - \alpha t_i). \] (3.3)

All that remains to do is to specify the evolution of the loop length and the power emitted as dark matter, and apply Eq. (2.7).

Cosmic string loops lose energy to gravitational radiation as well as cusping, and shrink as a result. The rate of energy emission into gravity waves is \[ P_{\text{grav}} = \Gamma G \mu^2, \] where \( \mu \) is the string tension, \( G = 1/(8\pi M^2_{\text{Pl}}) \), and \( \Gamma \approx 50 \) is a dimensionless constant \[ 28\].

Loops also lose energy when they form cusps, which are points on the loops that briefly fold back upon themselves and approach the speed of light. Near the apex of a cusp, a portion of the string overlaps itself and self-annihilates. Summing over many cusps, the net rate at which a loop loses energy to cusping is given by \[ 27\]

\[ P_{\text{cusp}} = \mu p_c \sqrt{\frac{w}{\ell}}, \] (3.5)

where \( w \sim \eta^{-1} \) is the string width, and \( p_c \) is the probability for a cusp to form per period of loop oscillation. Several studies suggest \( p_c \approx 1 \) \[ 26\] \[ 28\] \[ 29\] \[ 30\] \[ 31\]. Comparing the cusp power of Eq. (3.5) to the gravitational wave power of Eq. (3.4), we see that cusping is the dominant energy-loss mechanism by loops when they are shorter than \( \ell < \ell_e \) with

\[ \ell_e = w \left( \frac{p_c}{\Gamma G \mu} \right)^2. \] (3.6)

The particles produced by cusping can decay into dark matter. Provided \( p_c \gtrsim \Gamma G \mu \), we expect cusping to be the dominant source of particle production and dark matter from cosmic strings.

If loops lose energy to gravitational radiation and cusping, the loop length evolves according to

\[ \mu \frac{d\ell}{dt} = -P_{\text{tot}} = -\Gamma G \mu^2 - \mu p_c \sqrt{\frac{w}{\ell}}. \] (3.7)
The solution of this equation subject to the initial condition \( \ell(t_i) = \ell_i \) is given implicitly by

\[
t - t_i = \frac{\ell}{\Gamma G \mu} \left[ \left( \frac{\ell_i - \ell}{\ell} \right) - 2 \left( \sqrt{\frac{\ell_i}{\ell}} - \sqrt{\frac{\ell}{\ell}} \right) + 2 \ln \left( \frac{1 + \sqrt{\ell_i/\ell}}{1 + \sqrt{\ell/\ell}} \right) \right].
\] (3.8)

Suppose that a fraction \( \epsilon_{\text{cusp}} \) of the energy emitted by a string loop cusp (eventually) takes the form of cold dark matter. It follows that

\[
P_{\text{DM}} = \epsilon_{\text{cusp}} P_{\text{cusp}}.
\] (3.9)

Putting this into Eq. (2.10) and making use of Eq. (3.3), we find the dark matter density due to cusping to be

\[
\Delta \rho_{\text{DM}} = \int_0^\infty \frac{d\ell}{\Gamma G \mu} \int_0^{\ell_fo(t_i)} dt_i \int_{\ell_x(t_i)}^{\ell_fo(t_i)} d\ell \, r(\ell_i, t_i) \left( \frac{a_i}{a_0} \right)^3 \mu \frac{\epsilon_{\text{cusp}} P_{\text{cusp}}(\ell)}{P_{\text{tot}}(\ell)} \left( \frac{a_i}{a_0} \right)^3 2 \epsilon_{\text{cusp}} \mu \ell \right]^{\ell_fo(\alpha t_i, t_i)}_{\ell_x(\alpha t_i, t_i)} \Theta(\ell_fo) \Theta(\ell_x).
\] (3.10)

The \( \Theta \) functions in this expression account for the fact that only those loops that decay after \( t_fo \) and before \( t_0 \) can contribute to the dark matter density. Numerically, we find that the integrand of Eq. (3.10) is a steeply falling function of \( t_i \) that is cut off at small \( t_i \) by the \( \Theta(\ell_fo) \) condition. It follows that the dominant contribution to the dark matter density comes from loops formed at the smallest possible value of \( t_i \) such that they decay shortly after \( t_fo \). In this sense, the dark matter density is created nearly instantaneously at \( t_fo \).

To be concrete, we evaluate Eq. (3.10) assuming a weakly-interacting massive dark matter particle with a freeze-out time of \( t_fo = 2 \times 10^{16} \text{ GeV}^{-1} \). This corresponds approximately to a freeze-out temperature of 5 GeV, which is in the range expected for a stable, weakly-interacting particle with a mass on the order of 100 GeV. We take the universe to be radiation dominated prior to \( t_fo \), and we assume a standard cosmological evolution afterward. We also set the loop network parameter to \( \zeta = 10 \), the cusping probability to \( p_c = 1 \), and the branching fraction into dark matter equal to unity, \( \epsilon_{\text{cusp}} = 1 \). In realistic models \( \epsilon_{\text{cusp}} \) can be much smaller than unity. Our results can therefore be interpreted as providing an upper bound on \( \epsilon_{\text{cusp}} \) within the underlying gauge theory model.

In Fig. 1 we show the dark matter relic density due to cusping as a function of the initial loop size parameter \( \alpha \), normalized to \( \Gamma G \mu \), with the other parameter values as described above. We normalize \( \alpha \) to \( \Gamma G \mu \) because, in the absence of cusping, a loop is long-lived relative to the Hubble time at formation (\( \sim t_i \)) provided \( \alpha/\Gamma G \mu > 1 \), and short-lived otherwise. From this plot, we see that increasing the initial loop size \( \alpha \) well above \( \Gamma G \mu \) increases the final dark matter density. The reason for this is that the integration over \( t_i \) in Eq. (3.10) is dominated by the earliest times for which loops produced at \( t_i \) decay after \( t_fo \). Since larger loops are longer-lived, loops contributing to the dark matter density can be formed at earlier times when the integrand of Eq. (3.10) is larger. As \( \alpha/\Gamma G \mu \) decreases, the dark matter density curves flatten out for lower values of the VEV \( \eta \). In these flat regions, cusping dominates the
evolution of the loops contributing to the dark matter, and these loops are short-lived even though $\alpha/\Gamma G\mu > 1$. When the VEV is larger, $\eta \gtrsim 10^{13}$ GeV, gravitational radiation becomes more important than cusping. Loops then become short-lived only when $\alpha/\Gamma G\mu < 1$. When $\alpha$ becomes very small, cusping again takes over and the curves flatten out once more.

In Fig. 2 we show the dependence of the dark matter density from string cusping as a function of the symmetry breaking VEV $\eta$. The other string model parameters are as described above. This plot shows a general increase in the dark matter density up to large values of $\eta$, and then a fall-off. Short-lived loops, with $\alpha \leq \Gamma G\mu$, and smaller values of $\eta$ decay mostly through cusping at time $t_{fo}$. Using Eq. (2.10), it is possible to show that $\Omega_{DM}$ increases as $\eta^2$ in this case. As $\eta$ is increased further, gravitational radiation becomes more important than cusping, and loops lose most of their energy to gravity waves instead of dark matter. Thus, the dark matter density falls for these very large values of $\eta$, with $\Omega_{DM} \propto (\alpha \eta)^{-1/2}$ in this regime. The precise value of $\eta$ at which the cross-over occurs depends on the value of $\alpha$, with smaller initial loops being more prone to cusping. For very large initial loops, $\alpha = 0.1$, a similar cross-over occurs.

In general, we expect the loop cusping probability $p_c$ to be on the order of unity. However, it is also interesting to look at smaller values of $p_c$, such as might arise on large string loops with many kinks. Fig. 3 shows the effect of reducing $p_c$ for small $\alpha = \Gamma G\mu$ and large $\alpha = 0.1$ initial loop sizes, and a range of values of the symmetry breaking VEV $\eta$.

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3The backreaction from string annihilation at a cusp can suppress its reoccurrence, but this need not prevent the formation of new cusps.
Figure 2: Dark matter density due to loop cusping for $\epsilon_{cusp} = 1$, $\zeta = 10$, $p_c = 1$, and $t_{fo} = 2 \times 10^{16}$ GeV$^{-1}$ as a function of the symmetry breaking VEV $\eta$. The various curves correspond to different values of the initial loop size parameter $\alpha$. The black dashed horizontal line denotes the observed dark matter relic density.

breaking VEV $\eta$. All other parameters are as in the previous plots. For $\alpha = \Gamma G\mu$ the loops are necessarily short-lived. In this case, cusping is the dominant energy loss mechanism at time $t_{fo}$ for $\eta \lesssim 10^{12}$ GeV, and remains so even for lower values of $p_c$. Thus, reducing $p_c$ does not alter the resulting dark matter density. When $\eta$ is larger, gravitational radiation dominates the energy loss at time $t_{fo}$, and reducing $p_c$ therefore decreases the fraction of loop energy released as dark matter. With $\alpha = 0.1$ string loops can be long-lived. For smaller values of $\eta \lesssim 10^{12}$ GeV, lowering $p_c$ can enhance the dark matter density. This occurs because cusping is the dominant energy loss mechanism around time $t_{fo}$ for these loops. Reducing $p_c$ thus further increases the lifetime of these long-lived loops which, as discussed above, enhances the final dark matter density. For larger values of $\eta$, gravitational radiation dominates at time $t_{fo}$, so that reducing $p_c$ only decreases the fraction of energy released as dark matter without significantly altering the loop lifetime.

We have only computed here the contribution to the dark matter density from loops formed in the scaling regime. Loops created in the string-forming phase transition at time $t_\eta$ and while the string network was evolving towards scaling will give an additional contribution to the dark matter density if they decay after $t_{fo}$. The approach of a string network to scaling depends on the details of the phase transition, and a study of this process is beyond the scope of the present work. However, we can derive the condition for loops formed in the phase transition to decay before $t_{fo}$. We obtain

$$\eta \gtrsim p_c^{-2/5} \left( \frac{\alpha}{0.1} \right)^{3/5} \left( 7 \times 10^3 \text{ GeV} \right).$$

(3.11)
Figure 3: Dark matter density due to loop cusping for $\epsilon_{\text{cusp}} = 1$, $\zeta = 10$, and $t_{\text{fo}} = 2 \times 10^{16} \text{GeV}^{-1}$ as a function of the cusp formation probability $p_c$ for several values of the symmetry breaking scale $\eta$. The panel on the left corresponds to small initial loops with $\alpha = \Gamma G\mu$, while the panel on the right corresponds to large initial loops with $\alpha = 0.1$. The black dashed horizontal line indicates the observed dark matter relic density.

If this bound is satisfied and scaling is attained quickly after the network is formed at $t_{\eta}$, loops formed in the scaling regime will give the dominant contribution to the dark matter density. This bound also indicates that our results are applicable to the phenomenologically interesting case of a $U(1)$ gauge symmetry broken only slightly above the electroweak scale provided $p_c \sim 1$ and $\alpha \ll 0.1$.

Before moving on, let us mention two additional effects that we have not yet taken into account. The first is that we have treated the loops as having no net motion relative to the thermal background, even though it is found in Refs. [36, 37, 38, 39] that small loops ($\alpha < \Gamma G\mu$) can be highly boosted. This boost does not alter the cusping power, although the expression for this power in Eq. (3.5) picks up a factor of $\gamma^{1/2}$ (for short-lived loops), where $\gamma = 1/\sqrt{1 - \nu^2}$ is the boost factor. The decay products will also be boosted. Unless the loops are very tiny, however, this boost is not expected to greatly modify our results.

A second and potentially much larger effect that we have not yet included is that the decay products from a loop cusp can be very highly boosted, even if the loop is initially at rest. The size of this boost $\gamma$ was estimated in Ref. [27] to be

$$\gamma = \sqrt{\frac{\ell}{w}}.$$  \hspace{1cm} (3.12)

On account of this boost, the cusp decay products including the dark matter might not thermalize immediately upon production. If not, they may thermalize at a later time, or
even remain as hot dark matter.

To estimate the effect of this possibly very large boost, we adopt a simple picture of the cusp decay products as consisting of a bunch of light states (including WIMP dark matter) moving together with a boost $\gamma$ relative to the thermal background. In the rest frame of the bunch we assume very little relative motion, and that a fraction $\epsilon_1$ of the total bunch energy consists of dark matter particles of mass $m \simeq 100$ GeV. The dark matter in the bunch will thermalize when $\sigma n_{\text{tot}} \gtrsim H$, where $n_{\text{tot}} \sim T^3$ is the total plasma number density of light states and $\sigma$ is the cross-section for the dark matter to scatter off the thermal background. To estimate the thermalization time, we take this cross-section to be

$$\sigma = \frac{1}{8\pi} \frac{1}{s} = \frac{1}{8\pi} \frac{1}{\gamma^2 m^2}. \quad (3.13)$$

With this cross-section, the dark matter decay products from a cusp on a small loop with $\alpha = \Gamma G\mu$ formed at time $t_{f_0}$ will reach kinetic equilibrium nearly instantaneously after the cusp occurs when the symmetry breaking scale lies below $\eta \lesssim 10^{12}$ GeV. In this case, our previous estimates for the dark matter density remain valid.

If the cusp decay products do not thermalize instantaneously, they may still thermalize at a later time $t_T$ as their momentum redshifts. To compute the final density of cold dark matter in this case, it is only necessary to add a factor of $a(\tilde{t})/a(t_T)$ to the integrand of Eq. (2.7) and modify the limits of integration such that $t_T > t_{f_0}$ rather than $\tilde{t} > t_{f_0}$. Doing so, we find that the resulting dark matter density coincides with what we found previously (without the boost) up to an overall factor of order unity even when instantaneous thermalization does not occur. The additional redshift $a(\tilde{t})/a(t_T)$ is cancelled nearly exactly by the delay in thermalization, which allows loops contributing to the dark matter density to be formed at earlier times when the rate of loop production was larger. Similar to before, we also find that the dominant contribution to the cold dark matter density comes from loop decay products that thermalize shortly after $t_{f_0}$.

The prospect of highly boosted decay products also raises several new issues. Since a large fraction of the energy of the cusp typically goes into the kinetic energy of the decay products, the fraction of this energy that ends up as cold dark matter can be suppressed. If kinetic thermalization proceeds exclusively through elastic collisions, we would have $\epsilon_{\text{cusp}} = \epsilon_1/\gamma(t_T)$, where $\gamma(t_T)$ is the boost factor at the time of thermalization. The actual value of $\epsilon_{\text{cusp}}$ can be larger than this if inelastic collisions during thermalization generate additional dark matter particles, but in general we expect a significant suppression of $\epsilon_{\text{cusp}}$ relative to the non-boosted case. Highly boosted loop decay products can also generate relics that have not thermalized by the present time (or the time of matter-radiation equality). In general, the resulting densities of hot dark matter are suppressed by factors of $G\mu$ and are safely small for lower values of the symmetry breaking scale. However, a reliable precise estimate is complicated by the fact that for extremely large boosts, the cusp decay products will naively have super-Planckian momenta. A full analysis of this issue is beyond the scope of the present work.

In summary, our results indicate that the density of WIMP cold dark matter generated by string loops from cusping with $p_c = 1$ in the scaling regime is safely below the observed value...
for $\eta \lesssim 10^{10} \text{GeV} \ (G\mu \approx 10^{-18})$. For values of the symmetry-breaking scale larger than this, the string-induced dark matter density depends on the initial loop size. Loops of initial size $\alpha = \Gamma (G\mu)^{1.2}$, motivated by the results of Refs. [36, 37, 38, 39], can create as much or more than the observed dark matter density for $\eta \gtrsim 5 \times 10^{12} \text{GeV} \ (G\mu \approx 10^{-13})$ if the branching fraction $\epsilon_{\text{cusp}}$ is on the order of unity. Larger loops, with $\alpha \sim 0.1$, can lead to a much greater dark matter density. If such large loops are typical, the branching fraction into dark matter $\epsilon_{\text{cusp}}$ must be significantly less than unity if the cosmic strings are to avoid overproducing dark matter. This is natural for large loops, whose cusps generate very highly boosted decay products, leading to much of the cusp energy being lost to kinetic energy that is transferred to the thermal bath rather than cold dark matter. Let us also emphasize that for models with a new $U(1)$ gauge symmetry broken only slightly above the electroweak scale, the cold dark matter produced by the corresponding cosmic string network in the scaling regime is negligibly small for both large and small initial loop sizes.

4 Dark Matter Production with Friction

The evolution of a cosmic string network can be modified if the strings have significant interactions with the thermal background. As we will discuss below, the relevance of these interactions to the string network depends on the details of the symmetry breaking from which the cosmic strings arise. Such interactions, when present, tend to slow the motion of the strings by creating an effective frictional force on them [44, 45, 13]. This in turn changes the density and rate of growth of the network. The frictional forces on strings decrease as the universe cools, and eventually become unimportant relevant to the Hubble damping from the expansion of spacetime. Frictional effects also change the way cosmic string loops form and decay, and can enhance the total rate of loop formation. In this section we apply the result of Eq. (2.7) to compute the density of dark matter created by a cosmic string network evolving under the influence of frictional forces.

For local (gauge) cosmic strings, the dominant interaction between the strings and the thermal background comes from Aharonov-Bohm scattering [46]. This results from the phase change experienced by the charged particle as it is transported around the string. The effective frictional force induced by this scattering can be characterized by a friction length $\ell_f$. Friction becomes unimportant when $\ell_f(t) \gtrsim H(t)$ grows larger than the Hubble length. The friction length due to Aharonov-Bohm scattering is given by [44, 13]

$$\ell_f = \frac{\mu}{\beta T^3},$$

(4.1)

where the dimensionless quantity $\beta$ is [44]

$$\beta = \frac{2\zeta(3)}{\pi^2} \sum_a b_a \sin^2(\pi \nu_a),$$

(4.2)

with the sum running over relativistic degrees of freedom, and $b_a = 1 (3/4)$ for bosons (fermions). The value of $\nu_a$ is related to the charge $Q_a$ of the light particle species $a$. If
the underlying $U(1)$ gauge symmetry (subgroup) is broken by the condensation of a field with charge $Q$, it is equal to $Q_a/Q$. Therefore we expect $\beta$ to be of order unity when some of the $\nu_a$ are non-integer, and zero otherwise. When Aharonov-Bohm scattering vanishes, Everett scattering of charged particles off the strings will be the dominant source of frictional interactions \[^4\]. The corresponding friction length is similar to that for (non-trivial) Aharonov-Bohm scattering but is enhanced by a factor of $\ln^2(T/\eta)$. Frictional interactions will be completely irrelevant when all the light states in the theory are uncharged under the broken gauge group. In the present section, we will assume that Aharonov-Bohm scattering is the dominant source of frictional interactions with $\beta = 1$.

Frictional effects on the long string network decouple when the friction length grows larger than the Hubble length. This occurs at the time $t_*$ defined by the relation

$$H(t_*) = 1/\ell_f(t_*). \quad (4.3)$$

Friction is only relevant for the long string network when $t < t_*$. For $\beta = \mathcal{O}(1)$ and radiation domination, $t_*$ has the parametric size\[^4\]

$$t_* \sim \frac{M_{Pl}^3}{\eta^4}, \quad (4.4)$$

where $\eta$ denotes the symmetry breaking VEV. This is parametrically larger than the typical (radiation-era) formation time $t_\eta \sim M_{Pl}/\eta^2$.

The evolution of a cosmic string network in the presence of friction was studied in Ref. \[^{13}\]. In their analytic model, the long string network is characterized by an effective correlation length $L$ and a mean velocity $\nu$. The energy density of the network is given in terms of these variables as

$$\rho_\infty = \frac{\mu}{L^2}. \quad (4.5)$$

If the initial string density is larger than the scaling density, as would be expected if the symmetry breaking phase transition is second-order or weakly first-order, the string network evolves very quickly to the Kibble regime \[^2\] \[^{13}\]. In this regime, with the universe assumed to be radiation dominated, the string network variables $L$ and $\nu$ have the parametric dependences \[^{13}\]

$$L(t) \sim \left(\frac{t}{t_*}\right)^{1/4} t \quad (4.6)$$

$$\nu(t) \sim \left(\frac{t}{t_*}\right)^{1/4}. \quad (4.7)$$

The Kibble regime only lasts while $t < t_*$. From Eqs. (4.1) and (4.4) we see that the friction length grows as

$$\ell_f \sim \left(\frac{t}{t_*}\right)^{1/2} t. \quad (4.8)$$

\[^4\] Due to the many uncertainties involved in the description of cosmic strings within the friction-dominated regime, we only list and use here the leading parametric dependences of the string network parameters.
As $t$ approaches $t_*$, the friction length catches up to the long string length $L$ as well as the horizon, and the string network transitions into the usual scaling regime with $L \propto t$ and $\nu \sim 1$.

During the friction-dominated Kibble regime, the rate at which energy is transferred from the long string network to loops is on the order of

$$\frac{dp_{\text{loop}}}{dt_i} \simeq \mu \left( \frac{t_*}{t_i} \right)^{1/2} t_i^{-3}, \quad t_i < t_*.$$  \hfill (4.9)

This is parametrically larger than during the scaling regime, as can be seen by comparing with Eq. (3.2). Once a loop is formed, its subsequent evolution in the Kibble regime is also considerably different than during scaling. In particular, the typical initial loop size as well as the subsequent loop evolution are both strongly modified. Both of these effects can modify the resulting dark matter density.

### 4.1 Loop Production and Evolution with Friction

The evolution of linear perturbations on long strings and closed string loops in the presence of friction was studied in Refs. [44, 45]. In Ref. [45] it was found that linear fluctuations on a long string of wavelength larger than $\ell_f$ are overdamped and stretched. For wavelengths much smaller than $\ell_f$, the damping time is on the order of $\ell_f$, which is much longer than the typical period of oscillation but much shorter than the Hubble time. Hence these small fluctuations oscillate and lose energy to friction very quickly relative to the Hubble time. Given these results and the picture of loop formation of Ref. [39], we expect that the typical initial loop size during friction is on the order of $\ell_f$.

Fluctuations smaller than $\ell_f$ are damped out quickly, while those larger than $\ell_f$ grow more slowly than the long string correlation length $L$, and therefore shrink relative to $L$. Thus, we expect that fluctuations build up near $\ell_f$, which in turn sets the typical size of a loop when it is formed.

To model the evolution of string loops during friction, we will take the results of Ref. [45] for the evolution of a circular loop to be representative of the evolution of general loops. (Indeed, friction tends to make the loops more circular.) Ref. [45] finds that loops smaller than $\ell_f$ oscillate essentially freely and lose their energy to the thermal background according to

$$\mu \frac{d\ell}{dt}_{\text{friction}} \simeq -\mu \frac{\ell}{\ell_f}. \hfill (4.10)$$

It follows that such loops lose energy over the time scale $\ell_f$, much less than the Hubble time in the friction regime. We do not expect these interactions between the strings and the thermal background to be a significant source of dark matter.

The motion of loops larger than $\ell_f$ is overdamped. They evolve according to

$$R \dot{R} \simeq -\frac{\ell_f}{2a} \hfill (4.11)$$

---

5We emphasize however that the picture of loop formation obtained in Ref. [39] was developed under the assumption of long string scaling, and did not consider the effects of friction.
where $R$ is the comoving coordinate radius of the loop (and $aR$ corresponds to the physical loop radius). The solution of Eq. (4.11) implies that loops of initial size smaller than the long string scale $L$ shrink down to size $\ell_f$ in less than about a Hubble time. Once they do, the overdamping approximation of Eq. (4.11) breaks down, and the loops begin to oscillate and decay away.

In summary, loops are formed in the friction regime with a typical initial length close to $\ell_f$. The loop formation rate per unit volume per unit length is

$$r(\ell_i, t_i) \simeq \left( \frac{t}{t_i} \right)^{-4} \delta(\ell_i - \ell_f(t_i)), \quad t_i < t_*. \quad (4.12)$$

Any loop formed with initial length larger than $\ell_f$ will shrink down to length $\ell_f$ in less than about a Hubble time. Loops smaller than $\ell_f$ execute underdamped oscillations, transferring their energy to the thermal background and decaying away over the time scale $\ell_f$. The loop length $\ell$ in this regime evolves according to

$$\frac{d\ell}{dt} = -\ell_{\ell_f} - \Gamma G \mu - p_c \sqrt{w} \ell. \quad (4.13)$$

Here, the first term comes from friction, the second from gravitational radiation, and the third from cusping.

Before going on to compute the dark matter density generated by the decaying loops, let us make note of the fact that at the end of the friction-dominated Kibble regime, the long string network is smoothed out nearly all the way up to the Hubble scale. In the loop formation picture of Ref. [36, 37, 38, 39], small fluctuations on long strings giving rise to loops originate from fluctuations of Hubble size that have slowly shrunk. Thus it will take some time for small scale structure to build up on the long strings, and the typical initial loop size will be initially larger than the gravitational damping length $l_{GW}$. We follow Ref. [13] and model this transitional period by writing the initial typical loop size parameter as

$$\alpha_{eff}(t) = 2 + \alpha \left( \frac{t}{t_\ast} \right)^{\xi}, \quad t > t_*, \quad (4.14)$$

with the exponent $\xi \simeq 1$. A naïve application of the results of Refs. [36, 37, 38, 39] suggests that $\xi = 0.9$ in the radiation era. We will study a range of values of $\xi$.

### 4.2 Dark Matter with Friction

The discussion above provides all the ingredients needed to evaluate the dark matter density created by a string network in the presence of friction using Eq. (2.7). We do so here under the assumption that loop cusping is the dominant source of dark matter from the strings. Our results are presented in Fig. 4, where we show the dark matter density due to cosmic strings as a function of the symmetry breaking VEV $\eta$. In making this plot, we have also set $\zeta = 10$, $p_c = 1$, $\epsilon_{cusp} = 1$, and we have again taken the freeze-out time to be $t_{fo} = 2 \times 10^{16}$ GeV$^{-1}$. We have also fixed an overall prefactor in Eq. (4.12) (equal to $2\sqrt{\zeta/(2+\alpha)}$) to ensure that the
Figure 4: Dark matter density due to loop cusping for $\epsilon_{\text{cusp}} = 1$, $p_c = 1$, and $t_{fo} = 2 \times 10^{16}$ GeV$^{-1}$ as a function of the symmetry breaking VEV $\eta$ in the presence of friction. The various curves correspond to different values of the initial loop size parameter $\alpha$. The black dashed horizontal line indicates the observed dark matter relic density.

The dark matter density curves in Fig. 4 show three distinct regions, with the transitions between them occurring around $\eta = 10^8$ GeV and $\eta = 10^{12}$ GeV. For values of $\eta$ well above $10^{12}$ GeV, the curves coincide with those obtained in the absence of friction. Such large values of $\eta$ imply a value of $t_*$ that is much smaller than the freeze-out time $t_{fo}$, so that all loops generated while friction is relevant decay away before $t_{fo}$. On the other hand, when $\eta$ lies below $10^{12}$ GeV, the most important contribution to the dark matter comes from loops formed while friction dominates the network evolution, $t_i < t_*$. In the region $10^8$ GeV $\lesssim \eta \lesssim 10^{12}$ GeV most of the dark matter is produced by loops that are formed in the friction era, with $t_i < t_*$. As a result, the dark matter density is independent of the scaling value of $\alpha$ for this range of $\eta$. The largest contribution to the dark matter in this region comes from loops that are also long-lived, with $t_i < t_{fo}$. This enhances the amount of dark matter formed because, with the loop distribution function of Eq. (4.12), the integrand of the $t_i$ integral in Eq. (2.7) is a rapidly decreasing function of $t_i$. Increasing $\eta$ in this region leads to larger initial loop sizes that further extend the loop lifetime. However, the minimal value of $t_i$ for which loops decay after $t_{fo}$ decreases more slowly with increasing $\eta$ than $t_*$. The sharp transition near $\eta = 10^{12}$ GeV occurs when these two quantities become equal. When this happens, increasing $\eta$ further decreases the initial
loop size (see Eq. (4.14)) for the dominant DM loops, and these loops go quickly from being long-lived to being short-lived, with the dominant loops being formed near \( t_i = t_{fo} > t_* \). The transition is more gradual when the exponent \( \xi \) in Eq. (4.14) is less than \( \xi = 1.0 \), which can be seen by comparing with the curve for \( \xi = 0.7 \).

For \( \eta \lesssim 10^8 \) GeV, the initial loop size \( \ell_f \) becomes small enough that the loops are short-lived, decaying away within a Hubble time. Thus, the majority of the dark matter produced for these smaller values of \( \eta \) comes from loops formed near the freeze-out time, \( t_i \simeq t_{fo} \), with initial size close to \( \ell_f(t_{fo}) \). As \( \eta \) decreases below \( 10^8 \) GeV, we find that a larger fraction of the energy of each loop is lost to cusping, thereby increasing the dark matter density. Even so, the dark matter relic density generated by cosmic strings for \( \eta \sim 10^3 \) GeV is a negligibly small fraction of the observed value.

5 Conclusions

We have investigated the cold dark matter density created by the decays of loops of gauge cosmic strings. Dark matter is produced by string loops when they form cusps. At a cusp, a small portion of the string loop annihilates into its constituent fields, which can then cascade down to lighter states such as dark matter particles. Our results provide constraints on extensions of the gauge symmetry group of the standard model that give rise to cosmic strings in the early universe. The string loops that give decay to dark matter are themselves created continually by the network of long, horizon-length strings. We have studied the amount of dark matter generated when this network is in the scaling regime, and when its evolution is dominated by frictional forces. Both cases are physically relevant as the presence or absence of significant frictional interactions depends on the details of the symmetry breaking from which the strings originate.

The dark matter density generated by a string network in the scaling regime can potentially be enough to explain the observed relic density when the breaking VEV \( \eta \) exceeds \( 10^{10} \) GeV. The amount of dark matter produced by this mechanism is greatest when the initial loop size approaches a significant fraction of the cosmological horizon, although much smaller initial loop sizes can also generate sizeable densities of dark matter. However, larger loops have cusp decay products that are highly boosted, and this can reduce the fraction of the loop energy that becomes cold dark matter. For values of \( \eta \) below \( 10^{10} \) GeV, the amount cold dark matter created by a scaling string network is always well below the observed value.

String networks that are strongly influenced by frictional interactions can also generate a dark matter density that is equal to or greater than the observed value. This can occur if the symmetry breaking scale is greater than about \( 10^{11} \) GeV. For very large values of the symmetry breaking scale, above about \( 10^{13} \) GeV, frictional effects decouple well before the non-thermal dark matter is created; the frictional interactions do not alter the resulting dark matter density. When the symmetry breaking scale is much below \( 10^{11} \) GeV, the contributions to the dark matter density in the presence of friction are very small. This contrasts with and is less constraining than the result of Ref. [8], in which the effects of cusping on the decays of loops were not included in computing the dark matter density.
We conclude from our investigation that the non-thermal cold dark matter relic density is safely below the observed value when the symmetry breaking scale is less than $\eta \simeq 10^{10}$ GeV. For larger symmetry breaking scales, the non-thermal dark matter density produced by decaying string loops can potentially be as large or larger than the observed value. Thus, our results also put constraints on new gauge symmetries broken at very high scales based on the requirement that they do not overproduce dark matter. These constraints are model-dependent, in that they depend on the effective branching fraction of the decaying string loops into dark matter, but they can be more severe than the constraints from the more traditional cosmic string signatures such as gravitational radiation and gravitational lensing [2, 3]. For gauge symmetries broken at lower scales that could be probed by the LHC, we find that the contribution to the dark matter density from their corresponding scaling string loops is negligibly small.

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References

[1] H. B. Nielsen and P. Olesen, Nucl. Phys. B 61, 45 (1973).

[2] M. B. Hindmarsh and T. W. B. Kibble, Rept. Prog. Phys. 58, 477 (1995) [hep-ph/9411342].

[3] A. Vilenkin and E. P. S. Shellard, Cosmic Strings and Other Topological Defects, Cambridge University Press, Cambridge UK, 1994.

[4] T. W. B. Kibble, J. Phys. A 9, 1387 (1976).

[5] R. Blumenhagen, M. Cvetic, P. Langacker and G. Shiu, Ann. Rev. Nucl. Part. Sci. 55, 71 (2005) [hep-th/0502005]; P. Langacker, [hep-ph/9805486] P. Langacker and J. Wang, Phys. Rev. D 58, 115010 (1998) [hep-ph/9804428]; J. R. Espinosa, Nucl. Phys. Proc. Suppl. 62, 187 (1998) [hep-ph/9707541]; M. Cvetic and P. Langacker, Phys. Rev. D 54, 3570 (1996) [hep-ph/9511378]; M. Cvetic and P. Langacker, Mod. Phys. Lett. A 11, 1247 (1996) [hep-ph/9602424].

[6] P. Langacker, N. Polonsky and J. Wang, Phys. Rev. D 60, 115005 (1999) [hep-ph/9905252]; D. Suematsu, Phys. Rev. D 59, 055017 (1999) [hep-ph/9808409].

19
[7] J. L. Hewett and T. G. Rizzo, Phys. Rept. 183, 193 (1989); P. Langacker and J. Wang, Phys. Rev. D 58, 115010 (1998) [hep-ph/9804428]; G. C. Cho, K. Hagiwara and Y. Umeda, Nucl. Phys. B 531, 65 (1998) [Erratum-ibid. B 555, 651 (1999)] [hep-ph/9805448]; S. F. King, S. Moretti and R. Nevzorov, Phys. Rev. D 73, 035009 (2006) [hep-ph/0510419]; S. F. King, S. Moretti and R. Nevzorov, Phys. Lett. B 634, 278 (2006) [hep-ph/0511256]; R. Howl and S. F. King, JHEP 0801, 030 (2008) [0708.1451 [hep-ph]]; P. Langacker, [0801.1345 [hep-ph]].

[8] R. Jeannerot, X. Zhang and R. H. Brandenberger, JHEP 9912, 003 (1999) [hep-ph/9901357].

[9] T. Matsuda, JCAP 0604, 005 (2006) [hep-ph/0509064].

[10] T. W. B. Kibble, Nucl. Phys. B 252, 227 (1985) [Erratum-ibid. B 261, 750 (1985)].

[11] D. P. Bennett and F. R. Bouchet, Phys. Rev. Lett. 60, 257 (1988); D. P. Bennett and F. R. Bouchet, Phys. Rev. D 41, 2408 (1990).

[12] B. Allen and E. P. S. Shellard, Phys. Rev. Lett. 64, 119 (1990).

[13] C. J. A. Martins and E. P. S. Shellard, Phys. Rev. D 53, 575 (1996) [hep-ph/9507335]; C. J. A. Martins and E. P. S. Shellard, Phys. Rev. D 54, 2535 (1996) [hep-ph/9602271]; C. J. A. Martins and E. P. S. Shellard, Phys. Rev. D 65, 043514 (2002) [hep-ph/0003298].

[14] G. R. Vincent, M. Hindmarsh and M. Sakellariadou, Phys. Rev. D 56, 637 (1997) [astro-ph/9612135]; G. Vincent, N. D. Antunes and M. Hindmarsh, Phys. Rev. Lett. 80, 2277 (1998) [hep-ph/9708427].

[15] V. Vanchurin, K. D. Olum and A. Vilenkin, Phys. Rev. D 74, 063527 (2006) [gr-qc/0511159].

[16] For reviews of dark matter, see:
G. Jungman, M. Kamionkowski and K. Griest, Phys. Rept. 267, 195 (1996) [hep-ph/9506380]; L. Bergstrom, Rept. Prog. Phys. 63, 793 (2000) [hep-ph/0002126]; K. A. Olive, astro-ph/0301505; G. Bertone, D. Hooper and J. Silk, Phys. Rept. 405, 279 (2005) [hep-ph/0404175].

[17] E. Komatsu et al. [WMAP Collaboration], 0803.0547 [astro-ph].

[18] For reviews, see for example H. E. Haber and G. L. Kane, Phys. Rept. 117, 75 (1985); S. P. Martin, [hep-ph/9709356]; D. J. H. Chung et al., Phys. Rept. 407, 1 (2005) [hep-ph/0312378]; M. A. Luty, [hep-th/0509029].

[19] T. Appelquist, H. C. Cheng and B. A. Dobrescu, Phys. Rev. D 64, 035002 (2001) [hep-ph/0012100]; G. Servant and T. M. P. Tait, Nucl. Phys. B 650, 391 (2003) [hep-ph/0206071]; H. C. Cheng, J. L. Feng and K. T. Matchev, Phys. Rev. Lett. 89, 211301 (2002) [hep-ph/0207125]; F. Burnell and G. D. Kribs, Phys. Rev. D 73, 015001 (2006) [hep-ph/0509118]; K. Kong and K. T. Matchev, JHEP 0601, 038 (2006) [hep-ph/0509119].

20
[20] H. C. P. Cheng and I. Low, JHEP 0408, 061 (2004) [hep-ph/0405243]; J. Hubisz and P. Meade, Phys. Rev. D 71, 035016 (2005) [hep-ph/0411264]. A. Birkedal, A. Noble, M. Perelstein and A. Spray, Phys. Rev. D 74, 035002 (2006) [hep-ph/0603077].

[21] M. Srednicki and S. Theisen, Phys. Lett. B 189, 397 (1987).

[22] S. Borsanyi and M. Hindmarsh, Phys. Rev. D 77, 045022 (2008) [0712.0300 [hep-ph]].

[23] A. Vilenkin, Phys. Lett. B 107, 47 (1981).

[24] R. H. Brandenberger, Nucl. Phys. B 293, 812 (1987).

[25] T. W. B. Kibble and N. Turok, Phys. Lett. B 116, 141 (1982).

[26] N. Turok, Nucl. Phys. B 242, 520 (1984).

[27] J. J. Blanco-Pillado and K. D. Olum, Phys. Rev. D 59, 063508 (1999) [gr-qc/9810005]; K. D. Olum and J. J. Blanco-Pillado, Phys. Rev. D 60, 023503 (1999) [gr-qc/9812040].

[28] T. Vachaspati and A. Vilenkin, Phys. Rev. D 31, 3052 (1985).

[29] C. J. Burden, Phys. Lett. B 164, 277 (1985).

[30] D. Garfinkle and T. Vachaspati, Phys. Rev. D 36, 2229 (1987).

[31] J. M. Quashnock and D. N. Spergel, Phys. Rev. D 42, 2505 (1990).

[32] V. Vanchurin, K. Olum and A. Vilenkin, Phys. Rev. D 72, 063514 (2005) [gr-qc/0501040]; V. Vanchurin, K. D. Olum and A. Vilenkin, Phys. Rev. D 74, 063527 (2006) [gr-qc/0511159]; K. D. Olum and V. Vanchurin, Phys. Rev. D 75, 063521 (2007) [astro-ph/0610419].

[33] C. Ringeval, M. Sakellariadou and F. Bouchet, JCAP 0702, 023 (2007) [astro-ph/0511646].

[34] C. J. A. Martins and E. P. S. Shellard, Phys. Rev. D 73, 043515 (2006) [astro-ph/0511792].

[35] X. Siemens and K. D. Olum, Nucl. Phys. B 611, 125 (2001) [Erratum-ibid. B 645, 367 (2002)] [gr-qc/0104085]; X. Siemens, K. D. Olum and A. Vilenkin, Phys. Rev. D 66, 043501 (2002) [gr-qc/0203006].

[36] J. Polchinski and J. V. Rocha, Phys. Rev. D 74, 083504 (2006) [hep-ph/0606205]; J. Polchinski and J. V. Rocha, Phys. Rev. D 75, 123503 (2007) [gr-qc/0702055].

[37] J. Polchinski, [0707.0888 [astro-ph]].

[38] J. V. Rocha, Phys. Rev. Lett. 100, 071601 (2008) [0709.3284 [gr-qc]].

[39] F. Dubath, J. Polchinski and J. V. Rocha, [0711.0994 [astro-ph]].
[40] V. Vanchurin, Phys. Rev. D 77, 063532 (2008) [0712.2236 [gr-qc]].

[41] Y. Cui, S. P. Martin, D. E. Morrissey and J. D. Wells, Phys. Rev. D 77, 043528 (2008) [0709.0950 [hep-ph]].

[42] T. Damour and A. Vilenkin, Phys. Rev. Lett. 78, 2288 (1997) [gr-qc/9610005].

[43] M. Peloso and L. Sorbo, Nucl. Phys. B 649, 88 (2003) [hep-ph/0205063].

[44] A. Vilenkin, Phys. Rev. D 43, 1060 (1991).

[45] J. Garriga and M. Sakellariadou, Phys. Rev. D 48, 2502 (1993) [hep-th/9303024].

[46] M. G. Alford and F. Wilczek, Phys. Rev. Lett. 62, 1071 (1989).

[47] A. E. Everett, Phys. Rev. D 24, 858 (1981).