Action in the Entropic Revolution of Newtonian Gravity

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Abstract

The theory of gravity has undergone somewhat of a revolution lately. Gravity is no longer a fundamental force it seems, but rather an effect of holographic entropy. Building on the works by Jacobsson, Padmanabhan and Verlinde we review the concept of Newtonian gravity as an entropic force and discuss a possible general action approach to Verlinde’s theory. We also discuss some open problems and future prospects of Verlinde’s approach.

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1 Introduction

In a remarkable paper Erik Verlinde recently proposed a framework for gravity as an entropic force [20]. This theory, which was built on the works of Jacobsson [8] and Padmanabhan [13] [14] [15], showed that Newtonian gravity could be obtained with very simple tools and utilizing practically only one assumption. His assumption was that the holographic principle holds and that space is emergent [20]. Perhaps the most important concept he used was the concept of holographic entropy, which was discovered in the 1970s by Bekenstein [1], as the defining component of the gravitational force [20]. Thoughts similar to Verlinde’s had been put forward before by Jacobsson [8] and Padmanabhan [13], where they utilized Rindler space time and

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reversed the framework to get the Einstein field equations as well as the Lanchos-Lovelock equations for gravity [13]. This research entirely reversed the view on physics and in particular the view on gravity. All of a sudden the fundamental laws were the result of a deeper theory that had been derived from a more advanced one, this almost rendered physics either circular or a strange loop [7]. Verlinde concluded that the change of entropy was linked to the change of the Newtonian potential, this led to the conclusion that inertia might be equivalent to the lack of entropy gradients [20]. He states the following in his seminal paper [20]:

The holographic principle has not been easy to extract from the laws of Newton and Einstein, and is deeply hidden within them. Conversely, starting from holography, we find that these well known laws come out directly and unavoidably.

His paper attracted quite some attention and several papers from various fields of theoretical physics, including cosmology, Loop Quantum Gravity and quantum mechanics, have been published relating to its topic [4, 5, 12, 16, 19, 21]. We shall briefly review some parts of Verlinde’s theory and discuss a possible action for his theory.

2 The holographic principle

In the holographic view space is mainly a storage place of information, which is associated with positions, movements and mass of matter [2, 12, 13, 14, 17, 18, 20]. This information is displayed on a surface, a holographic screen [20]. The information is stored in discrete bits on the screen and since the number of bits is limited we get holographic effects. This is the holographic principle in practice. Thus the dynamics on the screen is governed by some unknown rules which then only can utilize the information on the screen. Since information is stored on a screen this means that space is emergent in the normal direction of the screen [20]. The microstates may be thought of having all sorts of physical attributes such as energy, temperature etc. This is then related, via entropy, to the information associated with the system [20].
3 Entropy as a gravitational force

Bekenstein related the area of a black hole to the entropy of it by assuming that all information lost down a black hole must still be conserved and is therefore contained in some measure \[1, 6\]. If we now in the gravitational situation consider a small piece of a holographic screen and a particle with mass \(m\) approaching it from the side at which time has already emerged, then Verlinde concluded (utilizing Bekenstein’s arguments) that the entropy associated with this process should be Bekenstein entropy with an extra factor of \(2\pi\) \[20\]:

\[
\Delta S = 2\pi k_B \frac{mc}{\hbar} \Delta x.
\]

(1)

Here \(k_B\) is Boltzmann’s constant and the factor \(2\pi\) was added by Verlinde for reasons to be clear in connection with the gravitational force. An entropic force \(F\) is defined as \[20\]:

\[
F \Delta x = T \Delta S
\]

(2)

where \(T\) is temperature. It should also be noted that an entropic force is a macroscopic force that originates in a system with many degrees of freedom by the universe’s statistical tendency to maximize its entropy \[20\]. In order to relate the entropy to the screen the maximum number of bits \(N\) that can be associated with a screen is then assumed to be:

\[
N = \frac{Ac^3}{G\hbar} = \frac{4\pi R^2 c^3}{G\hbar},
\]

(3)

where \(R\) is the radius and \(A = 4\pi R^2\) is the area of the screen. The temperature can be determined from the equipartition rule:

\[
E = \frac{1}{2} Nk_B T,
\]

(4)

which is the the average energy per bit on the screen \[13, 14\]. We shall also assume that the mass-energy relation holds:

\[
E = Mc^2.
\]

(5)

In a straightforward way these equations yields the gravitational force:

\[
F = G\frac{Mm}{R^2}.
\]

(6)

This is, as Verlinde points out, a surprising result considering it practically comes from first principles \[20\]. Verlinde also showed another way to set up
the force of gravity as an entropic force by identifying the Unruh temperature arising in Rindler space time as the result of the gradient of the Newtonian potential $\phi$ [20]:

$$k_B T = \frac{1}{2\pi} \frac{\hbar a}{c} = \frac{\hbar}{2\pi} \frac{\nabla \phi}{c}.$$  (7)

The number of bits on a screen (3) can be put in a differential form:

$$dN = \frac{c^3}{G\hbar} dA.$$  (8)

The equipartition rule (4) can also be formulated on integral form:

$$E = \frac{1}{2} k_B \int_S T dN,$$  (9)

where $S$ is the screen enclosing the particle. If we just insert (8) and (5) in (9) and use the Unruh temperature (7) we get the expression:

$$M = \frac{1}{4\pi G} \int_S \nabla \phi \cdot dA.$$  (10)

This is Gauss law of gravity. Verlinde points out that this should hold for arbitrary screens given by equipotential surfaces $S$ [20]. Gauss law of gravity also transforms into Poisson’s equation for gravity:

$$\nabla^2 \phi = 4\pi G \rho.$$  (11)

Verlinde then utilized Killing vectors in order to obtain a relativistic version of the field, an approach which was similar to the previous approach by Jacobsson [8]. For strong fields the relativistic version of the theory turns out to be equivalent to Einstein field equations, see [8, 13, 20] for more information. Verlinde’s general conclusion is that inertia is due to the lack of entropy gradients, and conversely that gravity is due to the presence of them [20].

4 Action formulation of Verlinde’s gravity

In the context of Verlinde’s theory any physical theory will possess gravity naturally and unavoidably if there are entropy gradients present [20]. This means in effect that the action should be based on a Lagrangian that has the extra term of entropic energy. We have the energy from the entropic force on the integral form:

$$E = \int F dx = \int T dS.$$  (12)
The entropic energy should be considered a potential energy. Thus any physical system described by the Lagrangian $\mathcal{L}$ that does not a priori contain gravity (and it should not) is then reexpressed with gravity by the Lagrangian $\mathcal{L}'$:

$$\mathcal{L}' = \mathcal{L} + \int T dS. \quad (13)$$

This amounts to the action $\mathcal{A}$ when integrated over time:

$$\mathcal{A} = \int \left( \mathcal{L} + \int T dS \right) dt. \quad (14)$$

This approach to action from the entropic force is akin to Padmanabhan’s approach [15], but here applied directly to Verlinde’s non-relativistic theory [20]. The extra addition of the entropic force energy to the action may be seen as surface terms in many field theories, which are ignored in the conventional approach [15]. If we assume the differential form of the entropy equation (1) by letting $\Delta S \to dS$ then we can for a small infalling mass $m$ with the use of the temperature from the equipartition law (4) compute the entropy energy:

$$\int T dS = -\frac{GMm}{r} = -m\phi. \quad (15)$$

This is equal to the Newtonian potential energy with negative sign. If one works out the variational principle ($\delta \mathcal{A} = 0$) [11] for the single particle with $\mathcal{L} = \frac{1}{2}mv^2$ one naturally obtains Newton’s law of gravity in this approach:

$$m\ddot{x} = -m\nabla \phi. \quad (16)$$

The role of the action in the relativistic version of Verlinde’s theory is left open. Such a generally covariant action should correspond to the Einstein-Hilbert action of general relativity for strong fields [8, 15, 20]. The weak field limit of such an action, just as in Verlinde’s field equations, should then be of particular interest [20].

5 Discussion

There are many open problems and areas in need of investigation in Verlinde’s view of gravity. One may conclude that many papers have already been published on the basis of Verlinde’s theory, and among these papers a number of approaches to a quantum mechanical origin of holographic entropy have been proposed [5, 10, 12, 19]. Indeed, it is perhaps the quantum
dominated situations in physics that will provide the most interesting results in Verlinde’s gravitational theory. The search for a viable such theory is still ongoing, but the direction of research is different than in most quantum gravity theories, in fact Jacobsson states the following [8]:

This perspective suggests that it may be no more appropriate to canonically quantize the Einstein equation than it would be to quantize the wave equation for sound in air.

Thus the quantum theory of gravity is perhaps just quantum mechanics with the addition of gravity as a potential caused by the entropic force [5, 10, 12, 19]. Another interesting feature of Verlinde’s theory ought to be the general nature of the weak gravitational field, since his relativistic field equations only correspond to the Einstein field equations for strong fields. Perhaps the investigation of gravitomagnetism as a form of post-newtonian approximation could provide interesting observables. In addition to this one might finally conclude the true nature of gravitational radiation and how it may accurately be observed with such a linearized theory.

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