Astrophysical S-factors, Thermonuclear Rates, and Electron Screening Potential for the 3He(d,p)4He Big Bang Reaction via a Hierarchical Bayesian Model

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Abstract

We developed a hierarchical Bayesian framework to estimate S-factors and thermonuclear rates for the 3He(d,p)4He reaction, which impacts the primordial abundances of 3He and 7Li. The available data are evaluated and all direct uncertainties are taken into account in our analysis for which we can estimate separate uncertainties for systematic and statistical effects. For the nuclear reaction model, we adopt a single-level, two-channel approximation of R-matrix theory, suitably modified to take the effects of electron screening at lower energies into account. In addition to the usual resonance parameters (resonance location and reduced widths for the incoming and outgoing reaction channel), we include the channel radii and boundary condition parameters in the fitting process. Our new analysis of the 3He(d,p)4He S-factor data results in improved estimates for the thermonuclear rates. This work represents the first nuclear rate evaluation using R-matrix theory embedded into a hierarchical Bayesian framework, properly accounting for all known sources of uncertainty. Therefore, it provides a test bed for future studies of more complex reactions.

Key words: methods: statistical – nuclear reactions, nucleosynthesis, abundances

1. Introduction

The big bang theory rests on three observational pillars: big bang nucleosynthesis (BBN; Gamow 1948; Cyburt et al. 2016), the cosmic expansion (Riess et al. 1998; Peebles & Ratra 2003), and the cosmic microwave background radiation (Spergel et al. 2007; Planck Collaboration et al. 2016). The BBN takes place during the first 20 minutes after the big bang, at temperatures and densities near 1 GK and 10^{-5} g cm^{-3}, and is responsible for the production of the lightest nuclides, which play a major role in the subsequent history of cosmic evolution.

Primordial nucleosynthesis provides a sensitive test of the big bang model if the uncertainties in the predicted abundances can be reduced to the level of the uncertainties in the observed abundances. The uncertainties for the observed primordial abundances of 4He, 2H (or D), and 7Li have been greatly reduced in recent years and amount to 1.6%, 1.2%, and 20%, respectively (Sbordone et al. 2010; Aver et al. 2015; Cooke et al. 2018). At present, the uncertainties in the predicted abundances of 3He, 2H (or D), 7Li, and 7Be amount to 0.07%, 1.5%, 2.4%, and 4.4%, respectively (Pitrou et al. 2018). The primordial abundances predicted by the big bang model are in reasonable agreement with observations, as shown in Figure 1, except for the 7Li/H ratio, where the predicted value (Cyburt et al. 2016) exceeds the observed one (Sbordone et al. 2010) by a factor of ≈3. This long-standing “lithium problem” has not found a satisfactory solution yet (see, e.g., Cyburt et al. 2016, for a review).

Twelve nuclear processes of interest take place during BBN, as illustrated in Figure 2. Among these are the weak interactions that transform neutrons into protons, and vice versa, and the p(n,γ)d reaction whose cross section can be calculated precisely using effective field theories (Savage et al. 1999; Ando et al. 2006). The 10 remaining reactions, d(p,γ)3He, d(d,n)3He, d(d,τ), (d,n)4He, (t,α)7Li, 3He(d,p)4He, 4He(n,τ,γ)3He, 3He(α)γ7Be, 2Li(α)γ4He, and 7Be(n,p)Li, have been measured directly in the laboratory at the energies of astrophysical interest. Nevertheless, the estimation of thermonuclear reaction rates from the measured cross-section (or S-factor) data remains challenging. Results obtained using χ^2 optimization are plagued by a number of problems, including the treatment of systematic uncertainties, and the implicit assumption of normal likelihoods (see, e.g., Andrae et al. 2010). Therefore, we have attempted to derive statistically sound BBN reaction rates using a hierarchical Bayesian model. Bayesian rates have recently been derived for d(p,γ)3He, 3He(α,γ)7Be (Iliadis et al. 2016), d(d,n)3He, and d(d,τ)4He (Gómez Iñesta et al. 2017). These studies adopted the cross-section energy dependence from the microscopic theory of nuclear reactions, while the absolute cross-section normalization was found from a fit to the data within the Bayesian framework.

We report here the Bayesian reaction rates for a fifth big bang reaction, 3He(d,p)4He. This reaction marginally impacts the primordial deuterium abundance, but sensitively influences the primordial abundances of 3He and 7Li. For example, a reaction rate uncertainty of 5% at big bang temperatures translates to a 4% variation in the predicted abundance of both 3He and 7Li (Coc & Vangioni 2010). Because of its low abundance, 3He has not been observed outside of our Galaxy. However, the next generation of large telescope facilities will likely enable the determination of the 3He/4He ratio from observations of extragalactic metal-poor H II regions (Cooke 2015). Therefore, although a revised 3He(d,p)4He reaction rate will not solve the 7Li problem, a more reliable rate is nevertheless desirable for improving the predicted BBN abundances.

The 3He(d,p)4He reaction has been studied extensively over the past few decades, not only because of its importance to
BBN, but also because of its relevance to understanding the electron screening effect (e.g., Barker 2007). At the lowest energies, the measured astrophysical $S$-factor shows a marked increase caused by electron screening effects. The data at those energies were used to derive the electron screening potential, but the results depended strongly on the data sets analyzed and the nuclear model applied.

To enable a robust treatment of different sources of uncertainties, we first evaluate the available cross-section data and take only those experiments into account for which we can quantify the separate contributions of statistical and systematic effects to the total uncertainty. This leaves us with seven data sets to be analyzed. For the underlying nuclear model we assume a single-level, two-channel approximation of $R$-matrix theory (Descouvemont & Baye 2010). As will be discussed below, we include both the channel radii and boundary conditions as fit parameters into the statistical model. This has not been done in most $R$-matrix studies because of the difficulty of obtaining acceptable fits via $\chi^2$ minimization. We will demonstrate that these parameters can be included naturally in our hierarchical Bayesian model. Compared to previous works (Iliadis et al. 2016; Gómez Iñesta et al. 2017), the present Bayesian model is more sophisticated and therefore, represents an important test bed for future studies of more complex systems.

In Section 2, we summarize the reaction formalism. Our Bayesian model is discussed in Section 3, including likelihoods, model parameters, and priors. In Section 4, we present Bayesian $S$-factors and screening potentials. The thermonuclear reaction rates are presented in Section 5. A summary and conclusions are given in Section 6. Details about our data evaluation are provided in Appendix A.

2. Reaction Formalism

The $^4\text{He} + d$ low-energy cross section is dominated by an $s$-wave resonance with a spin-parity of $J^p = 3/2^+$, corresponding to the second excited level near $E_x = 17$ MeV excitation energy in the $^6\text{Li}$ compound nucleus (Tilley et al. 2002). This level decays via emission of $d$-wave protons. It has a mainly $^3\text{He} + d$ structure, corresponding to a large deuteron spectroscopic factor (Barker 1997).

Cross-section (or $S$-factor) data for the $^3\text{He}(d,p)^4\text{He}$ reaction have been fitted in the past using various assumptions, including polynomials (Krauss et al. 1987), a Padé expansion (Barbui et al. 2013), $R$-matrix expressions (Barker 2007), and hybrid models (Prati et al. 1994). Since we are mostly interested in the low-energy region, where the $s$-wave contribution dominates the cross section, we follow Barker (1997) and describe the theoretical energy dependence of the cross section (or $S$-factor) using a one-level, two-channel $R$-matrix approximation, suitably modified to take electron screening at low energies into account.

The inclusion of higher-lying levels and background poles in the formalism will likely not impact our results significantly. Hale et al. (2014) showed (see their Figure 4) that in the analog $^3\text{He}(d,n)^4\text{He}$ reaction the deviation in $S$-factors between a multi-level, multi-channel $R$-matrix fit and a single-level $R$-matrix fit is less than 1.5% over the majority of the energy range shown. This deviation should be regarded as a worst case scenario, because the channel radii for these two fits were not the same.
In the one-level, two-channel $R$-matrix approximation, the integrated cross section of the $^3\text{He}(d,p)^4\text{He}$ reaction is given by

$$\alpha_{dp}(E) = \frac{2J + 1}{k_d^2 (2J + 1)(2J_s + 1)} |S_{dp}|^2,$$

where $k_d$ and $E$ are the deuteron wavenumber and center-of-mass energy, respectively, $J = 3/2$ is the resonance spin, $j_1 = 1/2$ and $j_2 = 1$ are the ground-state spins of $^3\text{He}$ and deuteron, respectively, and $S_{dp}$ is the scattering matrix element. The corresponding bare-nucleus astrophysical $S$-factor of the $^3\text{He}(d,p)^4\text{He}$ reaction, which is not affected by electron screening, is given by

$$S_{\text{bare}}(E) = E e^{2\pi\eta/\alpha_{dp}(E)},$$

where $\eta$ is the Sommerfeld parameter. The scattering matrix element (Lane & Thomas 1958) can be expressed as

$$|S_{dp}|^2 = \frac{\Gamma_d \Gamma_p}{(E_0 + \Delta - E)^2 + (\Gamma/2)^2},$$

where $E_0$ represents the level eigenenergy. The partial widths of the $^3\text{He} + d$ and $^4\text{He} + p$ channels ($\Gamma_d, \Gamma_p$), the total width ($\Gamma$), and the level shift ($\Delta$) are given by

$$\Gamma = \sum_c \Gamma_c, \quad \Gamma_c = 2\gamma^2_c P_c,$$

$$\Delta = \sum_c \Delta_c, \quad \Delta_c = -\gamma^2_c (S_c - B_c),$$

where $\gamma^2_c$ is the reduced width and $B_c$ is the boundary condition parameter. Note that we are not using the Thomas approximation (Thomas 1951). Therefore, our partial and reduced widths are “formal” $R$-matrix parameters. Use of the Thomas approximation necessitates the definition of “observed” $R$-matrix parameters, which has led to significant confusion in the literature.

The energy-dependent quantities $P_c$ and $S_c$ denote the penetration factor and shift factors, respectively, for channel $c$ (either $d + ^3\text{He}$ or $p + ^4\text{He}$), which are computed from the Coulomb wave functions, $F_i$ and $G_i$, according to

$$P_c = \frac{k_d a_c}{F_i^2 + G_i^2}, \quad S_c = \frac{k_d a_c (F_i F'_i + G_i G'_i)}{F_i^2 + G_i^2}.$$

The Coulomb wave functions and their derivatives are evaluated at the channel radius, $a_c$; the orbital angular momentum for a given channel is denoted by $\ell$.

The $R$-matrix channel radius is usually expressed as

$$a_c = r_0 (A_1^{1/3} + A_2^{1/3}),$$

where $A_1$ and $A_2$ are the mass numbers of the two interacting nuclei; $r_0$ denotes the radius parameter, with a value customarily chosen between 1 and 2 fm. Fitted $R$-matrix parameters and cross sections have a well-known dependence on the channel radii (see, e.g., deBoer et al. 2017), which arises from the truncation of the $R$-matrix to a restricted number of poles (i.e., a finite set of eigenenergies). The radius of a given channel has no rigorous physical meaning, except that

### Table 1

| Parameter | Prior |
|-----------|-------|
| $E_0$     | Uniform(0.1, 0.4) |
| $E_0^b$   | TruncNormal(0, 1.0$^5$) |
| $\gamma_{d,2}^2$ | TruncNormal(0, (3$^2_{\text{mod}})^2$) |
| $\gamma_{p,2}^2$ | TruncNormal(0, (3$^2_{\text{mod}})^2$) |
| $a_d$     | Uniform(2.0, 10.0) |
| $a_p$     | Uniform(2.0, 10.0) |
| $U_c$     | TruncNormal(0, 0.1$^5$) |
| $\sigma_{\text{ew}}$ | TruncNormal(0, 5$^2$) |
| $\zeta$   | LogNormal(0, 0.2$^2$) |

**Notes.**

$^a$ Units for energies and reduced widths are in MeV, while those for the channel radii are in fm.

$^b$ $E_0$ denotes the energy at which the level shift is zero according to $B_c = S_c (E_0)$ (see Equation (5) and Section 2).

$\alpha_{\text{ew}} \equiv \ln(1 + \alpha_{\text{ew}}).$

The chosen values do not exceed the sum of the radii of the colliding nuclei (e.g., Descouvemont & Baye 2010). For the $^3\text{He}(d,p)^4\text{He}$ reaction, previous choices for the channel radii ranged between 3 fm and 6 fm (e.g., Barker 2002; Descouvemont et al. 2004). In Section 4, we will discuss the impact of the channel radii on our derived $S$-factors by including them as parameters in our Bayesian model. This method ensures that correlations between model parameters are fully taken into account, contrary to the previously applied procedure of performing fits with two different, but fixed values of a channel radius. Note that a few previous works have also treated the channel radii as fit parameters in their $R$-matrix analysis (e.g., Woods et al. 1988; Hale et al. 2014).

The energy, $E_0$, entering Equation (3) is the energy eigenvalue, and is not necessarily equal to the “energy at the center of the resonance” (Lane & Thomas 1958). For a relatively narrow resonance, the scattering matrix element (and the $S$-factor and the cross section) peaks at the energy where the first term in the denominator of Equation (3) is equal to zero. Therefore, the resonance energy, $E_r$, is frequently defined as

$$E_0 + \Delta(E_r) - E_r = 0.$$

The boundary condition parameter is then chosen as

$$B_c = S_c (E_r),$$

so that the level shift becomes zero at the center of the resonance. This assumption, which corresponds to setting the eigenenergy equal to the resonance energy ($E_r = E_0$), has also been adopted in most previous studies of the $^3\text{He}(d,p)^4\text{He}$ reaction (e.g., Barker 2007). Note, however, that for broad resonances this procedure predicts values of $E_r$ that do not coincide with the $S$-factor maximum anymore. This can be seen by comparing in Barker’s Table 1 the fitted values of $E_r$ (between 307 and 427 keV) with the measured energies of the $S$-factor peak, $E_m$ (between 200 and 241 keV, depending on the analyzed data set). In fact, the low-energy resonance in the $^3\text{He}(d,p)^4\text{He}$ reaction is so broad that the scattering matrix element, the cross section, and the $S$-factor peak at different energies, therefore the definition of an “energy at the center of the resonance” becomes ambiguous.

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5 Interestingly, deBoer et al. (2017) reported in their Table XIX that the channel radius provides the third largest source of uncertainty, among 13 considered sources, in the $^{16}\text{O}(\alpha,\gamma)^{20}\text{Ne}$ $S$-factor at 300 keV.
The assumption of $E_r = E_0$ made in previous studies of the $^3\text{He}(d, p)^4\text{He}$ reaction fixes the boundary condition parameters and thereby reduces the number of fitting parameters. However, as its name implies, the boundary condition parameter represents a model parameter and it is our intention to include it in the fitting. We believe that previous studies neither included the channel radii nor the boundary condition parameters in the fitting because of the difficulty of achieving acceptable fits. We will demonstrate below that acceptable fits can indeed be obtained using a hierarchical Bayesian model.

In this work, instead of providing the boundary condition parameters, $B_{\epsilon}$, directly, we find it more useful to report the equivalent results for the energy, $E_B$, at which the level shift is zero according to $B_{\epsilon} = S_r(E_r)$ (see Equation (5)). We use the notation $E_B$ instead of $E_r$ to emphasize that the value of $E_B$ does not correspond to any measured “resonance energy,” because such a quantity cannot be determined unambiguously for a broad resonance.

In laboratory experiments, electrons are usually bound to the interacting projectile and target nuclei. These electron clouds effectively reduce the Coulomb barrier and give rise to an increasing transmission probability. We perform the $S$-factor fit to the data using the expression (Assenbaum et al. 1987; Engstler et al. 1988)

$$S(E) \approx S_{\text{bare}}(E) e^{\pi \theta (U_r/E)},$$

where $U_r$ is the energy-independent electron screening potential.

Note that the measurement can be performed in two ways, depending on the identity of the projectile and target. The situation is shown in Figure 3. The notation $^3\text{He}(d, p)^4\text{He}$ refers to a deuterium ion beam (without atomic electrons) directed onto a neutral $^3\text{He}$ target, while $d(^3\text{He}, p)^4\text{He}$ refers to a $^3\text{He}$ ion beam directed onto a neutral deuterium target. These two situations result in different electron screening potentials, $U_r$.

The distinction is particularly important at center-of-mass energies below 50 keV, as will be shown below.

### 3. Bayesian Model

#### 3.1. General Aspects

Bayesian probability theory provides a mathematical framework to infer, from the measured data, the degree of plausibility of model parameters (Jaynes & Bretthorst 2003). It allows an update to current knowledge of a set of model parameters, $\theta$, in view of newly available information. The updated state of knowledge of $\theta$ is described by the posterior distribution, $p(\theta|y)$, i.e., the probability of the parameters, $\theta$, given the data, $y$. At the foundation of the theory lies Bayes’ theorem:

$$p(\theta|y) = \frac{\mathcal{L}(y|\theta)p(\theta)}{\int \mathcal{L}(y|\theta)p(\theta)d\theta}. \tag{11}$$

The numerator on the right side of Equation (11) represents the product of the model likelihood, $\mathcal{L}(y|\theta)$, i.e., the probability that the data, $y$, were obtained given the model parameters, $\theta$, and the prior distribution, $p(\theta)$, which represents our state of knowledge before considering the new data. The normalization factor appearing in the denominator, called the evidence, describes the probability of obtaining the data considering all possible parameter values.

In the Bayesian framework, the concept of hierarchical Bayesian models is of particular interest when accounting for the different effects and processes that impact the measured data (Parent & Rivot 2012; de Souza et al. 2015, 2016; Hilbe et al. 2017; González-Gaitán et al. 2019). The underlying idea is to decompose higher-dimensional problems into a number of probabilistically linked lower-dimensional substructures. Hierarchical Bayesian models allow for a consistent inclusion of different types of uncertainties into the model, thereby solving inferential problems that are not amenable to traditional statistics (e.g., Gelman & Hill 2006; Parent & Rivot 2012; Loredo 2013; Long & de Souza 2018).

#### 3.2. Likelihoods and Priors

We will next discuss how to construct likelihoods to quantify the different layers of uncertainty affecting the $^3\text{He}(d, p)^4\text{He}$ and $d(^3\text{He}, p)^4\text{He}$ data, namely extrinsic scatter, statistical effects, and systematic effects.

Throughout this work, we evaluate the Bayesian models using an adaptive Markov Chain Monte Carlo (MCMC) sampler called differential evolution MCMC with a snooker update (ter Braak & Vrugt 2008). The method exploits different sub-chain trajectories in parallel to optimally explore the multidimensional parameter space. The model was implemented using the general purpose MCMC sampler BAYESIAN TOOLS (Hartig et al. 2018) within the R language (Team R Development Core 2010). Computing a Bayesian model refers to generating random samples from the posterior distribution of model parameters. This involves the definition of the model, likelihood, and priors, as well as the initialization, adaptation, and monitoring of the Markov chain. We evaluated the theoretical $S$-factor model for the $^3\text{He}(d, p)^4\text{He}$ reaction based on the $R$-matrix formalism (Section 2), including the effect of electron screening.

#### 3.2.1. Extrinsic Scatter

We will first discuss the treatment of the statistical uncertainties with unknown variance, which we will call
extrinsic scatter. This extra variance encodes any additional sources of random uncertainty that were not properly accounted for by the experimenter. Suppose an experimental S-factor, \( S_{\text{exp}} \), is free of systematic and statistical measurement uncertainties (\( \epsilon_{\text{sys}} = 0 \), and \( \epsilon_{\text{stat}} = 0 \)), but is subject to an unknown additional scatter (\( \epsilon_{\text{extr}} \neq 0 \)). If the variable under study is continuous, one can assume in the simplest case a Gaussian noise, with a standard deviation of \( \sigma_{\text{extr}} \). The model likelihood connecting experiment with theory, assuming a vector of model parameters, \( \theta \), is then given by

\[
L(S_{\text{exp}}|\theta) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi \sigma_{\text{extr}}^2}} \exp \left\{ \frac{-(S_{\text{exp},i} - S_i(\theta))^2}{2\sigma_{\text{extr}}^2} \right\},
\]

where \( S_i(\theta) \) is the model S-factor (obtained from R-matrix theory), while the product runs over all data points, \( N \), labeled by \( i \). In symbolic notation, this expression can be written as

\[
S_{\text{exp},i} \sim \text{Normal}(S_i(\theta), \sigma_{\text{extr}}^2),
\]

implying that the experimental S-factor datum, \( i \), is sampled from a normal distribution, with a mean equal to the true value, \( S_i(\theta) \), and a variance of \( \sigma_{\text{extr}}^2 \).

### 3.2.2. Statistical Effects

Any experiment is subject to measurement uncertainties, which are the consequence of the randomness of the data-taking process. Suppose the measurement uncertainties are solely given by statistical effects of known variance, which differs for each data point of a given experiment (also known as heteroscedasticity), and that the likelihood can be described, in the simplest case, by a normal probability density. The likelihood for such model can be written as

\[
S_{\text{exp},i} \sim \text{Normal}(S_i(\theta), \sigma_{\text{stat},i}^2),
\]

where \( \sigma_{\text{stat},i}^2 \) represents the variance of the normal density for datum \( i \). Statistical uncertainties can be reduced by improving the data collection procedure.

When both statistical uncertainties, with known variance, and extrinsic scatter, with unknown variance, are present, the model can be easily extended to accommodate both effects. The likelihood for such a model is given by the nested expressions:

\[
S'_i \sim \text{Normal}(S_i(\theta), \sigma_{\text{extr}}^2),
\]

\[
S_{\text{exp},i} \sim \text{Normal}(S'_i, \sigma_{\text{stat},i}^2).
\]

These expressions provide an intuitive view of how a chain of probabilistic disturbances can be combined into a hierarchical structure. First, a homoscedastic scatter, quantified by the standard deviation \( \sigma_{\text{extr}} \) of a normal probability density, perturbs the theoretical value of a given S-factor datum, \( S_i(\theta) \), to produce a value of \( S'_i \); second, the latter value is in turn perturbed by the heteroscedastic statistical uncertainty, quantified by the standard deviation \( \sigma_{\text{stat},i}^2 \) of a normal probability density, to produce the measured value \( S_{\text{exp},i} \). The above demonstrates how an effect impacting the data can be implemented in a straightforward manner into a Bayesian data analysis.

### 3.2.3. Systematic Effects

Systematic uncertainties are usually not reduced by combining the results from different measurements or by collecting more data. Reported systematic uncertainties are based on assumptions made by the experimenter, are model-dependent, and follow vaguely known probability distributions (Heintrich & Lyons 2007). In a nuclear counting experiment, systematic effects impact the overall normalization by shifting all points of a given data set into the same direction, and they are often quantified by a multiplicative factor.

A useful distribution for normalization factors is the lognormal probability density:

\[
f(x) = \frac{1}{\sigma_{L} x \sqrt{2\pi}} \exp \left\{ -\frac{(\ln x - \mu_{L})^2}{2\sigma_{L}^2} \right\},
\]

which is characterized by two quantities, the location parameter, \( \mu_{L} \), and the shape parameter, \( \sigma_{L} \). The median value of the lognormal distribution is given by \( e^{\mu_{L}} \), while the factor uncertainty, \( f.u., \) for a coverage probability of 68%, is \( e^{\pm \sigma_{L}} \). For example, if the systematic uncertainty for a given data set is reported as \( \pm 10\% \), then \( f.u. \) amounts to 1.10. In our model, we include a systematic effect as an informative lognormal prior with a median of 1.0, i.e., \( \mu_{L} = 0 \), and \( f.u. \) estimated from the systematic uncertainty.

In our model, we can include systematic uncertainties using the nested expressions:

\[
S'_i \sim \text{Normal}(S_i(\theta), \sigma_{\text{extr}}^2),
\]

\[
S_{\text{exp},i} \sim \text{Normal}(\xi_i S'_i, \sigma_{\text{stat},i}^2),
\]

where \( \xi_i \) denotes the normalization factor for data set \( j \), which is drawn from a lognormal distribution.

### 3.2.4. Priors

Our choices of priors are summarized in Table 1. Previous estimates of the deuteron reduced width are of order \( \approx 1.0 \) MeV (Coc et al. 2012), which corresponds to values close to the Wigner limit (WL; Wigner & Eisenbud 1947; Dover et al. 1969). We use a broad prior for the reduced widths, which is restricted to positive energies only, i.e., truncated normal distributions (TruncNormal) with a zero mean and standard deviation given by the WL \( \gamma_{WL}^2 \equiv \hbar / (\mu_{c} a_{c}^2) \), where \( \mu_{c} \) is the reduced mass of the interacting pair of particles in channel \( c \), and \( \hbar \) is the reduced Planck constant.

Lane & Thomas (1958) recommended choosing the boundary condition, \( B_{c} \), in the one-level approximation so that the eigenvalue \( E_{0} \) lies within the width of the observed resonance. Thus, we chose for \( E_{0} \) a uniform prior between 0.1 and 0.4 MeV. For the energy, \( E_{0} \), at which the level shift is zero according to Equations (5) and (9), we chose a broad TruncNormal prior with zero mean, and a standard deviation of 1.0 MeV.

Woods et al. (1988) performed an R-matrix fit of experimental line shapes in the \(^4\text{He}(^5\text{Li},^7\text{Li})^4\text{He} \) and \(^4\text{He}(^6\text{Li},^7\text{He})^4\text{Li} \) stripping reactions and reported a best-fit value, for the channel radii, of \( 5.5 \pm 1.0 \) fm for the \(^5\text{He} \) system. We then adopt for the channel radii uniform priors in the range of 2.0–10.0 fm.

As for the electron screening potential, Aliotta et al. (2001) obtained values of \( U_{e} = 146 \pm 5 \) eV for the \(^d(^3\text{He},p)^4\text{He} \) reaction, and \( U_{e} = 201 \pm 10 \) eV for the \(^3\text{He}(d,p)^4\text{He} \) reaction. For \( U_{e} \), we
will adopt a weakly informative prior given by a TruncNormal distribution with zero mean, and a standard deviation of 100 keV for both scenarios.

As already pointed out above, lognormal priors are adopted for the normalization factors, \( \xi_j \), of each experiment, \( j \). For a given systematic uncertainty \( \sigma_{\text{sys}} \), the factor uncertainty, \( f.u. \), is given by \( 1 + \sigma_{\text{sys}} \), so if \( \sigma_{\text{sys}} = 2.5\% \), then \( f.u. = 1.025 \). For a coverage probability of 68%, we have \( f.u. = e^{ \epsilon \%} \). The priors are then given by

\[
\xi_j \sim \text{LogNormal}(0, \ln^2(f.u.))
\]

where \( f.u. \) denotes the factor uncertainty for experiment \( j \). Finally, for the extrinsic scatter, we employed a broad TruncNormal distribution with zero mean, and a standard deviation of 5 MeV.

### 4. Analysis of \( ^3\text{He}(d,p)^4\text{He} \)

The current status of the available data for the \( ^3\text{He}(d,p)^4\text{He} \) and \( d(^3\text{He},p)^4\text{He} \) reactions is discussed in detail in Appendix A. For the present analysis, we adopt the results of Zhichang et al. (1977), Krauss et al. (1987), Möller & Besenbacher (1980), Geist et al. (1999), Costantini et al. (2000), and Aliotta et al. (2001), because only these data sets allow for a separate estimation of statistical and systematic uncertainties. In total, our compilation includes 214 data points at center-of-mass energies between 4.2 and 471 keV.

#### 4.1. Results

Our model contains 16 parameters: six \( R \)-matrix parameters, seven normalization factors \( (\xi_1, \ldots, \xi_7) \), the extrinsic scatter \( (\sigma_{\text{extr}}) \), and the two screening potentials \( (U_{e1}, U_{e2}) \) for the \( ^3\text{He}(d,p)^4\text{He} \) and \( d(^3\text{He},p)^4\text{He} \) reactions, respectively. The full hierarchical Bayesian model can be summarized as

\[
\begin{align*}
S' & \sim \text{Normal}(S(\theta) e^{-\gamma_{\text{stat}}^2}), \sigma^2_{\text{stat}}), \\
S_{\text{Exp},ij} & \sim \text{Normal}(\xi_j S', \sigma^2_{\text{stat}}), \\
R-\text{matrix Parameters:} & \\
\theta & \equiv (E_0, E_B, \gamma_d, \gamma_p, a_d, a_p). \\
\text{Priors:} & \\
E_0 & \sim \text{Uniform}(0.1, 0.4), \\
E_B & \sim \text{TruncNormal}(0, 1.0^2), \\
\gamma_d & \sim \text{TruncNormal}(0, (\gamma_{\text{WL}}^2)^2), \\
\gamma_p & \sim \text{TruncNormal}(0, (\gamma_{\text{WL}}^2)^2), \\
(a_d, a_p) & \sim \text{Uniform}(2, 10), \\
U \text{-}\text{c} & \sim \text{TruncNormal}(0, 0.1^2), \\
\sigma_{\text{extr}} & \sim \text{TruncNormal}(0, 0.5^2), \\
\xi_j & \sim \text{LogNormal}(0, \ln^2(f.u.)).
\end{align*}
\]

The first layer accounts for the effects of an inherent extrinsic scatter and electron screening, while the second layer describes the effects of systematic and statistical uncertainties. The indices denote the number of data points \( (i = 1, \ldots, 214) \), data sets \( (j = 1, \ldots, 7) \), and the two possibilities for electron screening \( (k = 1 \text{ or } 2) \), depending on the kinematics of the experiment.

We randomly sampled all parameters of interest using three independent Markov chains, each of length \( 2 \times 10^5 \), after a burn-in phase of \( 10^5 \) steps. This ensures convergence of all chains according to the Gelman–Rubin convergence diagnostic (Gelman & Rubin 1992). The fitted \( S \)-factor and respective residuals, \( \Phi \), are displayed in Figure 4. The residual analysis highlights the agreement between the model and the observed values with prediction intervals enclosing \( \approx 98.6\% \) of the data within 99.7% credibility. The colored bands show three solutions and their respective 68% and 95% credible intervals: the bare-nucleus \( S \)-factor (purple), i.e., the estimated \( S \)-factor without the effects of electron screening, and the \( S \)-factors for \( d(^3\text{He},p)^4\text{He} \) (red) and \( ^3\text{He}(d,p)^4\text{He} \) (gray). The inset magnifies the region where the effects of electron screening become important.

Figure 5 presents the one- and two-dimensional marginalized posterior densities of the \( R \)-matrix parameters and the electron screening potentials. The yellow, green, and purple regions in the diagonal panels depict the 68%, 95%, and 99.7% credible intervals of the marginal posterior, while the lower left triangle displays the pairwise joint posteriors color-coded according to the local density of sampled points. The dashed contours highlight the 68% and 95% joint credible intervals.

Summary statistics, i.e., the 16th, 50th, and 84th percentiles, for our model parameters are presented in Table 2, together with previous estimates (Aliotta et al. 2001; Descouvemont et al. 2004; Barker 2007; Coc et al. 2012). For the \( R \)-matrix parameters, we obtain values of \( E_0 = 0.202^{+0.0184}_{-0.0157} \text{ MeV}, E_B = 0.516^{+0.176}_{-0.133} \text{ MeV}, \gamma_p = 0.0350^{+0.0063}_{-0.0049} \text{ MeV}, \) and \( \gamma_d = 0.362^{+0.048}_{-0.044} \text{ MeV} \). These uncertainties amount to less than a few percent. The deuteron reduced width is much larger than the proton reduced width, consistent with the structure of the \( 3/2^- \) level (Section 2). Note that these results cannot be directly compared to the values of Coc et al. (2012), who used fixed boundary conditions and channel radii, and did not provide uncertainties from their \( \chi^2 \) fit.

Table 2 also displays our estimate for the \( S \)-factor at zero energy, \( S_0 \), which agrees with the value of Descouvemont & Baye (2010). However, our uncertainty is smaller by about a factor of 3, which is likely caused by the differences in methodologies and adopted data sets. For the extrinsic scatter, we find a value of \( \sigma_{\text{extr}} = 0.566 \pm 0.029 \text{ MeV} \). This average value is mainly determined by the data sets from Zhichang et al. (1977) and Möller & Besenbacher (1980), which have few significant outliers.

Figure 6 shows the correlations of the predicted \( S \)-factor at zero energy, \( S_0 \), with the two-channel radii. A stronger correlation is observed with \( a_d \) compared to \( a_p \), resulting in a much smaller uncertainty for the former radius. We find values of \( a_d = 3.25^{+0.21}_{-0.26} \text{ fm} \) and \( a_p = 5.77^{+0.67}_{-0.72} \text{ fm} \). It is interesting to compare our results for the channel radii with the fixed, but arbitrary values that were used in previous analyses. For \( a_d \), we rule out the previously adopted values of 5 fm (Barker 2007), and 6 fm (Descouvemont et al. 2004) with 95% credibility, while the previously adopted values of \( a_p = 5 \text{ fm} \) (Descouvemont et al. 2004; Barker 2007), and \( a_p = 5.5 \pm 1.0 \text{ fm} \) from Woods et al. (1988) are consistent with our results.

---

6 The prediction interval is the region in which a given observation should fall with a certain probability. It is not to be confused with the credible interval around the mean, which is the region in which the true population mean should fall with a certain probability.
For the screening potentials, we obtain values of $U_e^1 = 213 \pm 13$ eV for the $^3$He($d,p)^4$He reaction, and $U_e^2 = 121 \pm 12$ eV for the $d(^3$He,$p)^4$He reaction. Our result for $^3$He($d,p)^4$He is consistent with previous estimates (Aliotta et al. 2001; Descouvemont et al. 2004; Barker 2007) within 95% credibility. Our estimated screening potential for $d(^3$He,$p)^4$He also agrees with Aliotta et al. (2001) and Descouvemont et al. (2004) within the uncertainties. Note that the present and previous estimated values are still larger than the called adiabatic limit, i.e., the difference in electron binding energies between the colliding atoms and the compound atom, $U_e = 120$ eV for $^3$He($d,p)^4$He, and $U_e = 65$ eV for $d(^3$He,$p)^4$He (see, e.g., Aliotta et al. 2001), and this discrepancy is still not understood.

The posterior densities for the data set normalization factors are displayed in Figure 7 and their summary statistics is presented at the bottom of Table 2. The gray areas depict the 68%, 95%, and 99.7% credible intervals. The vertical dashed red line represents zero systematic effects (i.e., a normalization factor of unity). The fitted normalization factors are consistent with unity within 95% credibility, except for the data set of Costantini et al. (2000), for which the median value of $\xi$ differs from unity by 6.3%.

### 5. Bayesian Reaction Rates

The thermonuclear reaction rate per particle pair is given by Iliadis (2008):

$$N_s \langle \sigma v \rangle = \left( \frac{8}{\pi \mu} \right)^{\frac{3}{2}} \frac{N_A}{(k_B T)^{\frac{3}{2}}} \int_0^\infty e^{-2\pi S(E)} e^{-\frac{\pi \Delta}{k_B T}} dE, \quad (22)$$

where $S(E)$ is the bare-nucleus $S$-factor at the center-of-mass energy $E$, according to Equation (2); $\mu = m_a m_b / (m_a + m_b)$ is the reduced mass, with $m_a$ and $m_b$ denoting the masses of target and projectile, respectively; $k_B$ is the Boltzmann constant, $N_A$ is Avogadro’s constant, and $T$ is the temperature. For calculating the $^3$He($d,p)^4$He reaction rate, we integrate Equation (22) numerically. The $S$-factor is calculated from the samples obtained with our Bayesian model, which contains the effects of varying channel radii and boundary condition parameters.
Our rate is based on 5000 MCMC samples, randomly chosen from the complete set of S-factor samples, which ensures that Monte Carlo fluctuations are negligible compared to the rate uncertainties. Our lower integration limit is 10 eV. We estimate the reaction rates on a grid of temperatures from 1 MK to 1 GK. Recommended rates are computed as the 50th percentile of the probability density, while the rate factor uncertainty, \( f.u. \), is found from the 16th and 84th percentiles. Numerical reaction rate values are listed in Table 3. Our rate uncertainties amount between 1.1% and 1.7% over the entire temperature region of interest.

Figure 8 compares our present rates with previous estimates. Our 16th and 84th percentiles, divided by our recommended rate (50th percentile), are shown as the gray band. The “lower,” “adopted,” and “upper” rates of Descouvemont et al. (2004) are displayed as the purple band. The solid line depicts the ratio of the previously “adopted” rate and our median (50th percentile) rate. Compared to the present rates, the previous results are slightly higher at temperatures below 10 MK and lower at \( \approx 300 \) MK. Previous and present results agree within the uncertainties for the entire temperature range. We emphasize that we achieved a significant reduction of the reaction rate uncertainties, from...
Figure 6. Two-dimensional projections of the posterior probability densities for the predicted S-factor at zero energy (S₀) vs. the channel radii (a₀, a₀). The panels and colors have the same meaning as in Figure 4. The correlation with the deuteron channel radius, a₀ (left panel), is larger compared to the correlation with a₀ (right panel).

Table 2
Results of the Bayesian Fit

| Parameter | Present | Previous |
|-----------|---------|----------|
| E₀ (MeV) | 0.202±0.034 | 0.35779² |
| E₁ (MeV) | 0.516±0.137 | 0.35779² |
| γ₁² (MeV) | 0.0350±0.0097 | 0.025425² |
| γ₁³ (MeV) | 0.362±0.044 | 1.0085³ |
| a₀ (fm) | 5.77±0.67² | 5.0 d, 5.5 ± 1.0² |
| a₀ (fm) | 3.25±0.3³ | 6.0², 5.0² |
| S₀ (MeV b) | 5.729±0.08² | 5.9 ± 0.3², 6.7³ |
| σ_{text} (MeV b) | 0.566 ± 0.029 | x |
| U₁₁² (eV) | 213 ± 13 | 201 ± 10¶, 194¹, 219 ± 7² |
| U₁₁² (eV) | 121 ± 12 | 146 ± 5, 109 ± 9² |

Notes. The associated uncertainties in our estimates correspond to a 68% credible interval around the median.

² From Coc et al. (2012), who assumed fixed channel radii and boundary conditions; no uncertainty estimates were given.
³ E₀ denotes the energy at which the level shift is zero according to B_e = S_e (E₀) (see Equation (5) and Section 2).
⁴ From Descouvemont et al. (2004).
⁵ From Barker (2007), Table 2, row C, who assumed fixed channel radii.
⁶ From Woods et al. (1988).
⁷ Bare S-factor at zero energy.
⁸ From Aliotta et al. (2001); no uncertainties estimates were given.
⁹ x ≡ Not available.
¹ For the ¹He(d,p)³He reaction.
¹² From Aliotta et al. (2001).
¹³ For the d¹He(p)³He reaction.

a previous average value of 3.2% (Descouvemont et al. 2004) to 1.4% in the present work.

6. Summary and Conclusions

BBN represents a milestone in the evolution of the universe, marking the production of the first light nuclides. Accurate

Figure 7. Posteriors of the normalization factors, ξ, for each of the seven data sets. The gray areas depict the 68%, 95%, and 99.7% credible intervals, respectively. For the data set labels, see Figure 4.

Table 3
Recommended ³He(d,p)⁴He Reaction Rates²

| T (GK) | Rate | f.u. | T (GK) | Rate | f.u. |
|-------|------|-----|-------|------|-----|
| 0.001 | 3.4959E−19 | 1.017 | 0.07 | 1.0933E+04 | 1.014 |
| 0.002 | 6.0351E−13 | 1.017 | 0.08 | 2.1937E+04 | 1.014 |
| 0.003 | 6.2480E−10 | 1.016 | 0.09 | 3.9560E+04 | 1.014 |
| 0.004 | 4.9259E−08 | 1.016 | 0.1 | 6.5782E+04 | 1.014 |
| 0.005 | 1.0932E−06 | 1.016 | 0.11 | 1.0274E+05 | 1.013 |
| 0.006 | 1.1578E−05 | 1.016 | 0.12 | 1.5260E+05 | 1.013 |
| 0.007 | 7.6001E−05 | 1.016 | 0.13 | 2.1759E+05 | 1.013 |
| 0.008 | 3.5811E−04 | 1.016 | 0.14 | 2.9999E+05 | 1.013 |
| 0.009 | 1.3253E−03 | 1.016 | 0.15 | 4.0190E+05 | 1.013 |
| 0.01 | 4.0848E−03 | 1.016 | 0.16 | 5.2554E+05 | 1.012 |
| 0.011 | 1.0917E−02 | 1.016 | 0.18 | 8.4624E+05 | 1.012 |
| 0.012 | 2.6040E−02 | 1.016 | 0.2 | 1.2778E+06 | 1.012 |
| 0.013 | 5.6610E−02 | 1.016 | 0.25 | 2.9262E+06 | 1.011 |
| 0.014 | 1.1397E−01 | 1.016 | 0.3 | 5.4933E+06 | 1.011 |
| 0.015 | 2.1514E−01 | 1.016 | 0.35 | 9.0208E+06 | 1.011 |
| 0.016 | 3.8445E−01 | 1.016 | 0.4 | 1.3448E+07 | 1.011 |
| 0.018 | 1.0727E+00 | 1.016 | 0.45 | 1.8653E+07 | 1.011 |
| 0.02 | 2.5929E+00 | 1.016 | 0.5 | 2.4479E+07 | 1.011 |
| 0.025 | 1.5146E+01 | 1.016 | 0.6 | 3.7358E+07 | 1.011 |
| 0.03 | 5.8024E+01 | 1.015 | 0.7 | 5.0858E+07 | 1.01 |
| 0.04 | 4.0895E+02 | 1.015 | 0.8 | 6.4133E+07 | 1.012 |
| 0.05 | 1.6354E+03 | 1.015 | 0.9 | 7.6622E+07 | 1.012 |
| 0.06 | 4.7026E+03 | 1.015 | 1.0 | 8.7989E+07 | 1.012 |

Note.² In units of cm³ mol⁻¹ s⁻¹, corresponding to the 50th percentiles of the rate probability density function. The rate factor uncertainty, f.u., corresponds to a coverage probability of 68% and is obtained from the 16th and 84th percentiles.
Figure 8. Present $^3$He(d,p)$^4$He thermonuclear reaction rates compared to the evaluation of Descouvemont et al. (2004). The gray shaded area corresponds to our new Bayesian rates, while the purple shaded area depicts the previous rates. The bands depict 68% coverage probabilities. All rates are normalized to the our new Bayesian rates, while the purple shaded area depicts the previous rates.

estimates of the rates of the nuclear reactions occurring during this period are paramount to predict the primordial abundances of the first elements. Inferring reaction rates from the measured $S$-factors requires a proper quantification and implementation of the different types of uncertainties affecting the measurement process. If these effects are not taken properly into account, the reaction rate estimate will be biased. The main results of our analysis can be summarized as follows:

1. This work represents the first implementation of a hierarchical Bayesian $R$-matrix formalism, which we applied to the estimation of $^3$He(d,p)$^4$He $S$-factors and thermonuclear reaction rates.
2. The single-level, two-channel $R$-matrix approximation was incorporated into an adaptive MCMC sampler for robustness against multi-modality and correlations between the $R$-matrix parameters.
3. The Bayesian modeling naturally accounts for all known sources of uncertainties affecting the experimental data: extrinsic scatter, systematic effects, imprecisions in the measurement process, and the effects of electron screening.
4. We include the $R$-matrix parameters (energies, reduced widths, channel radii, and boundary conditions), the data set normalization factors, and the electron screening potential in the fitting process.
5. Thermonuclear reaction rates and associated uncertainties are obtained by numerically integrating our Bayesian $S$-factors. We achieved a significant reduction of the reaction rate uncertainties, from a previous average value of 3.2% (Descouvemont et al. 2004) to 1.4%. This implies that the big bang $^3$He abundance can now be predicted with an uncertainty of only 1.0%. Future observations of the primordial $^3$He abundance will thus provide strong constraints for the standard big bang model (Cooke 2015).

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| $E_{cm}$ ± $\Delta E_{cm}$ (MeV) | $S$ ± $\Delta S$ (a) (MeV b) | $E_{cm}$ ± $\Delta E_{cm}$ (MeV) | $S$ ± $\Delta S$ (a) (MeV b) |
|-------------------------------|--------------------------------|-------------------------------|--------------------------------|
| 0.1344 ± 0.0048               | 10.074 ± 0.060                  | 0.2559 ± 0.0042                | 15.950 ± 0.095                 |
| 0.1422 ± 0.0047               | 12.080 ± 0.072                  | 0.2676 ± 0.0042                | 14.505 ± 0.087                 |
| 0.1506 ± 0.0047               | 12.544 ± 0.075                  | 0.2736 ± 0.0041                | 14.194 ± 0.085                 |
| 0.1650 ± 0.0046               | 12.375 ± 0.074                  | 0.2748 ± 0.0041                | 14.231 ± 0.085                 |
| 0.1677 ± 0.0045               | 14.590 ± 0.087                  | 0.2949 ± 0.0041                | 12.757 ± 0.076                 |
| 0.1692 ± 0.0045               | 15.314 ± 0.092                  | 0.3186 ± 0.0041                | 10.505 ± 0.063                 |
| 0.1746 ± 0.0045               | 15.760 ± 0.095                  | 0.3468 ± 0.0040                | 9.188 ± 0.055                  |
| 0.1800 ± 0.0044               | 15.987 ± 0.096                  | 0.3822 ± 0.0040                | 6.773 ± 0.040                  |
| 0.1854 ± 0.0044               | 17.415 ± 0.104                  | 0.4110 ± 0.0039                | 5.863 ± 0.035                  |
| 0.1914 ± 0.0044               | 18.116 ± 0.109                  | 0.4314 ± 0.0038                | 5.071 ± 0.030                  |
| 0.1974 ± 0.0044               | 17.763 ± 0.107                  | 0.4500 ± 0.0037                | 4.695 ± 0.028                  |
| 0.2040 ± 0.0044               | 17.728 ± 0.106                  | 0.4710 ± 0.0037                | 4.411 ± 0.026                  |
| 0.2100 ± 0.0044               | 17.825 ± 0.107                  | 0.5070 ± 0.0036                | 3.761 ± 0.022                  |
| 0.2247 ± 0.0043               | 17.502 ± 0.105                  | 0.5466 ± 0.0035                | 3.002 ± 0.018                  |
| 0.2400 ± 0.0043               | 16.640 ± 0.099                  | 0.5772 ± 0.0034                | 2.999 ± 0.018                  |
| 0.2520 ± 0.0042               | 15.987 ± 0.096                  | 0.6135 ± 0.0033                | 2.364 ± 0.014                  |

Note.

* Systematic uncertainty: 3.4%.

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Appendix A

Nuclear Data for the $^3$He(d,p)$^4$He Reaction

A.1. The $^3$He(d,p)$^4$He Data of Zhichang et al. (1977)

The cross-section data of Zhichang et al. (1977), as reported in EXFOR (2017), originate from a private communication by the authors, and were adopted from Figure 2 of the original article. The derived experimental $S$-factors, together with their statistical uncertainties ($\approx 0.6\%$), are listed in Table 4. The estimated systematic uncertainty is 3.4%, including the effects of target, solid angle, and beam intensity.

A.2. The $^3$He(d,p)$^4$He and d($^3$He,p)$^4$He Data of Krauss et al. (1987)

The Krauss et al. (1987) experiments took place at the ion accelerators in Münster and Bochum. Table 3 in Krauss et al. (1987) lists measured $S$-factors and statistical uncertainties. A normalization uncertainty of 6% originates from an absolute $^3$He(d,p)$^4$He cross-section measurement at a center-of-mass energy of 59.66 keV, to which a 5% uncertainty caused by variations in the alignment of beam and gas jet target profiles has to be added for the Münster data. Hence, we adopt a systematic uncertainty of 6.0% for the Bochum data, and following the authors, a value of 7.8% for the Münster data. Note that Krauss et al. (1987) did not report separately the results for $^3$He(d,p)$^4$He and d($^3$He,p)$^4$He. Since this distinction is important below a center-of-mass energy of 50 keV, we disregard all of their data points in this energy range. The data adopted in our analysis are listed in Table 5.

A.3. The d($^3$He,p)$^4$He Data of Möller & Besenbacher (1980)

In this experiment, the d($^3$He,p)$^4$He differential cross section was measured at two angles, relative to the d(d,p)$^4$H cross section. The total cross section was calculated using the angular distributions from Yarnell et al. (1953). The statistical error is
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Table 5
The \(^3\)He(d,p)\(^4\)He and d(\(^3\)He,p)\(^4\)He Data of Krauss et al. (1987)

| \(E_{cm}\) (MeV) | \(S \pm \Delta S_{\text{stat}}^a\) (MeV b) | \(E_{cm}\) (MeV) | \(S \pm \Delta S_{\text{stat}}^a\) (MeV b) |
|-----------------|------------------|-----------------|------------------|
| 0.04970 \(^d\)  | 7.24 \(\pm\) 0.07 | 0.0830          | 9.1 \(\pm\) 0.4  |
| 0.05369 \(^d\)  | 7.67 \(\pm\) 0.08 | 0.1007          | 9.9 \(\pm\) 0.5  |
| 0.05900 \(^d\)  | 8.4 \(\pm\) 0.4  | 0.1184          | 11.5 \(\pm\) 0.6 |
| 0.05952 \(^d\)  | 7.96 \(\pm\) 0.08 | 0.1360          | 13.1 \(\pm\) 0.6 |
| 0.05966 \(^d\)  | 8.26 \(\pm\) 0.08 | 0.1537          | 16.0 \(\pm\) 0.8 |
| 0.0653 \(^c\)    | 8.6 \(\pm\) 0.5  | 0.1713          | 17.2 \(\pm\) 0.9 |

Notes.

\(^a\) We disregarded all data points below a center-of-mass energy of 50 keV (see the text).
\(^b\) For some data points without reported statistical uncertainty, we assumed a value of 1%.
\(^c\) Münster data; S-factor uncertainty: 7.8%.
\(^d\) Bochum data; S-factor systematic uncertainty: 6.0%.

less than 3\%, except at the lowest energy (7\%). The systematic error, which arises from the normalization, the d(d,p)H cross section, and the anisotropy coefficient, amounts to 3.9\%. The EXFOR cross-section data were presumably scanned from Figure 3 of Möller & Besenbacher (1980). Our adopted S-factors are listed in Table 6.

A.4. The \(^3\)He(d,p)\(^4\)He and d(\(^3\)He,p)\(^4\)He Data of Geist et al. (1999)

The cross-section data are presented in Figure 5 of Geist et al. (1999) and are available in tabular form from EXFOR, as communicated by the authors. Three data sets exist, one for the \(^3\)He(d,p)\(^4\)He reaction and two for the d(\(^3\)He,p)\(^4\)He reaction. The data are listed in Table 7, with statistical uncertainties only. We do not distinguish between the \(^3\)He(d,p)\(^4\)He and d(\(^3\)He,p)\(^4\)He reactions because all data points were measured at center-of-mass energies above 50 keV, where electron screening effects can be disregarded. A systematic uncertainty of 4.3\% is obtained by adding quadratically the contributions from the d(\(^3\)He,p)\(^4\)He monitor cross-section scale and fitting procedure (1.3\% and 3\%, respectively), the incident beam energy and energy loss (2\%), and the beam interaction (2\%).

A.5. The d(\(^3\)He,p)\(^4\)He Data of Costantini et al. (2000)

We adopted the d(\(^3\)He,p)\(^4\)He results of Table 1 in Costantini et al. (2000), which lists the effective energy, S-factor, statistical uncertainty, and systematic uncertainty. The latter ranges from 8\% at the lowest energy (4.22 keV) to 3.0\% at the highest energy (13.83 keV). In our analysis, we assume an average systematic uncertainty of 5.5\%. Table 8 lists the S-factors adopted in the present work.

A.6. The \(^3\)He(d,p)\(^4\)He and d(\(^3\)He,p)\(^4\)He Data of Aliotta et al. (2001)

The S-factor data for the \(^3\)He(d,p)\(^4\)He and d(\(^3\)He,p)\(^4\)He reactions are taken from Tables 1 and 2, respectively, in Aliotta et al. (2001). The quoted uncertainties include only statistical (2.6\%) effects. The systematic uncertainty arises only from the target pressure, calorimeter, and detection efficiency, and amounts to 3.0\%. The S-factor data adopted in the present work are listed in Table 9.

A.7. Other Data

The following data sets were excluded from our analysis. Bonner et al. (1952) only presented differential cross sections measured at zero degrees. The cross-section data of Jarvis & Roaf (1953), obtained using photographic plates, are presented in their Table 1 for three bombarding energies. However, the origin of their quoted errors (6\%–14\%) is not clear. Yanell et al. (1953) reported the differential cross section at 86\° (see their Figure 6), but the bombarding energy uncertainty is large (3\%–14\%) and
The latter argument also holds for the data of Freier & Holmgren (2000) cannot be estimated individually from the information provided. The Astrophysical Journal, et al. 2005 (see their Table 3). We did not include the data of Engstler et al. (2007). The astrophysical Journal, 2000 provided cross factors obtained from d(d,n)He measurement.

Note.

\( a \) Average systematic uncertainty: 5.5%.

\( \Delta \) S-factors obtained from \(^3\text{He}(d,p)^4\text{He}\) measurement, unless noted otherwise.

\( b \) Systematic uncertainty: 3.0%.

\( c \) S-factors obtained from \(d(\text{He}^3,p)^4\text{He}\) measurement.

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