Gravitational lensing effects on the gamma-ray burst Hubble diagram

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Gamma-Ray Bursts (GRBs) offer a potential way to extend the Hubble diagram to very high redshifts and to constrain the nature of dark energy in a way complementary to distant type Ia supernovae. However, gravitational lensing systematically brightens distant GRBs through the magnification bias, in addition to increasing the dispersions of distance measurements. We investigate how the magnification bias limits the cosmological usage of GRBs. We perform Monte-Carlo simulations of \textit{Swift} GRBs assuming a cosmological constant dominated universe and then constrain the dark energy equation of state neglecting gravitational lens effects. The originally assumed model is recovered with 68% confidence limit even when the dispersion of inferred luminosities is comparable to that of type Ia supernovae. This implies that the bias is not so drastic for \textit{Swift} GRBs as to change constraints on dark energy and its evolution. However, the precise degree of the bias in cosmological parameter determinations depends strongly on the shape of the luminosity function of GRBs. Therefore, an accurate determination of the shape of the luminosity function is required to remove the effect of gravitational lensing and to obtain an unbiased Hubble diagram.

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I. INTRODUCTION

The expansion history of the universe provides invaluable information about the energy budget. Observations of distant type Ia supernovae (SNeIa) revealed that the universe is accelerating\(^1\), implying that the significant amount of dark energy is present. The nature of dark energy is still unknown and is one of central issues in modern cosmology. It is expected that detailed measurements of the expansion history leads to understanding the nature of dark energy, because the expansion rate of the universe is related with the equation of state of the dark energy.

In addition to SNeIa, gamma-ray bursts (GRBs) can serve as another probe of the expansion history of the universe. Although luminosities of different long duration GRBs are not quite similar, they can be inferred from observables such as variability\(^2\), spectral-lag\(^3\), peak energy\(^4, 5\), and jet opening angle\(^6\). An advantage of GRBs is that GRBs are observed at higher redshifts than SNeIa; the average redshift of GRBs discovered by \textit{Swift} satellite is \(z \sim 3\)\(^7\), and GRB with redshift as high as \(z = 6.3\) was indeed discovered\(^8\). The higher mean redshift implies that we may obtain useful information on the expansion history from observations of GRBs, in a complementary way to SNeIa\(^9\). In fact, attempts to construct the Hubble diagram from observed GRBs and to constrain cosmological parameters have already been made\(^10\). Given the fact that the number of GRBs is now rapidly growing, GRBs are expected to offer unique insight into the nature of dark energy.

However, it should be kept in mind that gravitational lensing has sometimes a great impact on high-redshift objects. While the effect of gravitational lensing on SNeIa is modest, effectively just introducing additional dispersion (e.g.,\(^11\)), gravitational lensing is expected to affect GRBs much more drastically because of the following two reasons. First, the effect of lensing is a strong function of the redshift. At higher redshifts, the probability distribution functions (PDFs) of lensing magnification has much larger dispersions, and also deviates from the Gaussian distribution more significantly. Second, and more importantly, their apparent isotropic luminosity could range over three orders of magnitudes, although they may be standardized through the use of luminosity indicators to some (uncertain) level. In this situation, the magnification bias, i.e., the effect of observing highly magnified GRBs selectively whose fluxes without lensing effect are below a flux limit of GRB detections, becomes quite significant.

In this paper, we study the effect of gravitational lensing on the GRB Hubble diagram and the determination of the dark energy equation of state. We pay particular attention on the non-Gaussian nature of magnification PDFs and the magnification bias. First, we derive the amount of magnification bias analytically, and then we perform Monte-Carlo simulations to show the impact of gravitational lensing on the cosmological parameter determination. As a fiducial cosmological model, we consider a model with the matter density \(\Omega_M = 0.27\), the dark energy density \(\Omega_D = 0.73\), the dark energy equation of state \(w(z) = -1\), the dimensionless Hubble constant \(h = 0.72\). Below we refer the model as the \(\Lambda\)CDM.

II. MAGNIFICATION PROBABILITY DISTRIBUTION

In this section, we derive the magnification PDF which is used to examine the effect of gravitational lensing on GRBs. We adopt a compound method that we derive the PDFs at around the peak and tail separately and combine them. First, for the PDF at the peak, we construct it
from the PDF of lensing convergence $\kappa$. We adopt a modified log-normal model \[12\] for the convergence PDF:

$$P(\kappa) d\kappa = \frac{N}{\sqrt{2\pi} \omega} \exp \left[ -\frac{\{\ln(1 + \kappa/|\kappa_{\text{min}}|) + \omega^{2}/2\}^{2} (1 + A/(1 + \kappa/|\kappa_{\text{min}}|))}{2\omega^{2}} \right] \frac{d\kappa}{\kappa + |\kappa_{\text{min}}|},$$  \tag{1}

where $\kappa_{\text{min}}$ is the convergence of the empty beam, and $N$, $A$, $\omega$ are determined from the normalization and the conditions $\int \kappa P(\kappa) d\kappa = 0$ and $\int \kappa^{2} P(\kappa) d\kappa = \langle \kappa^{2} \rangle$. Although the model was intended to describe the PDF for each lens plane, we regard it as the PDF for convergence projected along all line-of-sight by adopting the projected variance for $\langle \kappa^{2} \rangle$; Taruya et al. \[13\] showed that this prescription well reproduces the non-Gaussianity of the PDF at around the peak, $\kappa/\langle \kappa^{2} \rangle^{1/2} < 10$. The projected variance of lensing convergence, $\langle \kappa^{2} \rangle$ is computed by a standard method using non-linear power spectrum. Since GRBs are point sources, we adopt sufficiently small smoothing angle $\theta_{s} = 0.1''$ in computing the variance. We convert it to the magnification PDF by neglecting shear and adopt a relation

$$\mu = \frac{1}{(1 - \kappa)^{2}},$$  \tag{2}

where $\mu$ denotes a magnification factor. On the other hand, we compute PDFs at high-magnification tails with a halo approach assuming an NFW profile for dark halos (see, e.g., \[12\]). Specifically, we compute the cross section, i.e., the area with the magnification larger than $\mu$, $\sigma(> \mu)$, assuming an NFW profile \[12\], and then they are summed up with a mass function of dark halos $dn/dM$:

$$P(\mu) d\mu = -\frac{d}{d\mu} \left[ \int dzl \int dM \sigma(> \mu) \frac{d\ell}{dzl} \frac{dn}{dM} \right].$$  \tag{3}

For the mass function, we adopt a form proposed by Sheth & Tormen \[14\]. These PDFs are computed assuming the $\Lambda$CDM model and $\sigma_{8} = 0.8$. We connect both PDFs where they intersect, which occurs at $\mu \sim 2$. We show the resulting PDFs in Figure 1. As seen, the widths of the PDFs are wider for higher redshifts. The probabilities for relatively high magnifications are becoming very high at high redshifts. The peaks shift to smaller magnification factor $\mu$ with increasing redshift to assure that the mean magnification is unity. These behaviors clearly indicate that effects of gravitational lensing is more prominent at high redshifts. The PDFs are roughly consistent with those in literature (e.g., \[11\]). We note that the detailed shape of the magnification PDF is not very important for our purpose; our qualitative result will not be affected by a slight change of the PDF.

In this section, we estimate the impact of the magnification bias effect analytically. This can be done by combining the magnification PDF we derived above and GRB luminosity function in the following way. The GRB luminosity function $\phi(L)$ has been constrained from the number count of GRBs as a function of fluxes and the redshift distribution of GRBs \[8\] \[17\] \[18\]. We adopt a luminosity function derived by Firmami et al. \[18\] which includes the luminosity evolution of GRBs. Specifically we adopt a model of a double power-law luminosity function with no correlation of peak energy with luminosity (a model named 'DPE' in \[18\]). The slopes of the luminosity function are $-1.53$ and $-3.4$, and the break luminosity $L_{b}$ evolves with redshift such that $L_{b} \propto (1 + z)^{1.2}$. They also constrained the GRB formation rate from the data, and pointed out that it resembles the observed cosmic star formation rate.

Given the shape of luminosity function, we can estimate the effect of the magnification bias. The mean magnification factor for observed GRBs are calculated from

![FIG. 1: The magnification PDFs for source redshift $z = 1$ (dotted), 3 (solid), and 7 (dashed) are plotted as a function of the magnification factor $\mu$. The inset shows expanded view of the PDFs around the peaks.](image)
lensing decreases the distance modulus by Hubble diagram from GRBs. First of all, the degree of parameters are biased.

\[ \langle \mu \rangle = \int \frac{\mu dp/\mu f(\mu)d\mu}{\int dp/\mu f(\mu)d\mu}, \]
\[ f(\mu) = \frac{\int_{L_{\text{min}}}^{\infty} \phi(L/\mu)dL/\mu}{\int_{L_{\text{min}}}^{\infty} \phi(L)dL}, \]

where \( L_{\text{min}} \) is the minimum luminosity corresponding to the flux limit of observations and is a function of the redshift. As a specific example, we consider the Swift satellite as a GRB detector. Since all GRBs with the redshifts measured have observed photon flux larger than \( \sim 0.4 \text{ cm}^{-2}\text{s}^{-1} \) \( (15-150 \text{ keV}) \) \( [11] \), in what follows we adopt the flux limit \( P_{\text{lim}} = 0.4 \text{ cm}^{-2}\text{s}^{-1} \). The energy band is converted correctly using a spectral energy distribution of GRBs presented in [18]. The magnification factor due to lensing affects the estimate of a distance modulus by \( -2.5 \log \mu \). Thus, we expect that the Hubble diagram derived from GRBs is on average shifted from the true one by \( -2.5 \log \langle \mu \rangle \). Hence this quantity provides a good estimate of how much constraints on cosmological parameters are biased.

Figure 2 shows how gravitational lensing biases the Hubble diagram from GRBs. First of all, the degree of the bias again depends strongly on the redshift. At \( z = 5 \) lensing decreases the distance modulus by \( \sim -0.05 \), thus it cannot be ignored. Second, errors of slopes of luminosity functions introduce large uncertainties in the effect of gravitational lensing on the GRB Hubble diagram, in particular at high redshifts. The shallow luminosity function means that there are much more GRBs below the flux limit, thus we have much more GRBs that are beyond the flux limit just because of amplifications by gravitational lensing than those which are below the flux limit due to dimming by lensing, resulting in a large mean magnification. The dispersion introduced by lensing is \( \sim 0.3 \text{ mag at } z \sim 5 \), larger than the systematic shift by the magnification bias. In summary, we find that gravitational lensing systematically changes the shape of the GRB Hubble diagram particularly at high redshifts, not to mention introducing additional dispersions.

How does the bias affect the determination of the dark energy equation of state from the GRB Hubble diagram? To explore this, we consider the following two parameterizations of the equation of state \( w(z) \): One is \( w(z) = w_0 + [z/(1+z)]w_a \) and the other is \( w(z) = w_0 + w'z \), both of which have been widely adopted in dark energy studies. We note that in the latter parametrization \( w(z) \) increases rapidly with increasing redshift, which results in unphysical values of \( w(z) \) at high redshifts. We compute distance moduli for following two parameter sets, \( (\Omega_{DE}, w_0, w') = (0.75, -1.1, 0.4) \) and \( (\Omega_{DE}, w_0, w_a) = (0.74, -1.2, 1.0) \), which are plotted in Figure 2. As shown in the Figure, distance moduli of these evolving dark energy models behave just as that of the ΛCDM model biased by gravitational lensing. The increase of the equation of state as a function of \( z \) results in the enhancement of the expansion rate at high-redshifts and thus make distant objects brighter than those in no dark energy evolution model, which is just similar as gravitational lensing effect. It is interesting to note that the behavior more resembles dark energy parametrized unphysically as \( w(z) = w_0 + w'z \). Therefore the effect of gravitational lensing is expected to appear as an artificial evolving dark energy in determining cosmological parameters from the GRB Hubble diagram.

**IV. MONTE-CARLO SIMULATIONS**

We pursue the bias introduced by gravitational lensing further by performing Monte-Carlo simulations of GRB observations. Again, we adopt a model of Firmami et al. [13] for the cosmological distributions of GRBs, and also assume the ΛCDM model as the underlying cosmology. First we distribute GRBs in redshift-luminosity space according to the GRB formation rates and the best-fit luminosity function. We are conservative to restrict our attention to GRBs at \( z < 7 \), because it is still unclear whether we can really measure redshifts of GRBs at \( z > 7 \). For each event, we first compute an observed flux. After adding a measurement error which is modeled by a Gaussian error with \( \sigma = 0.05 \text{ mag} \), we compare it with the flux limit of Swift satellite \( P_{\text{lim}} = 0.4 \text{ cm}^{-2}\text{s}^{-1} \).
We show constraints on the equation of state in Figure 3. As expected, gravitational lensing shifts the best-fit models: The best-fit parameter set from fitting is \((w_0, w_a) = (-1.15, 0.6)\). However, the assumed model is still within 68% confidence limit for both large and small errors in estimating GRB absolute magnitudes, implying that gravitational lensing does not bias the parameter estimate very much for Swift GRBs. The cosmological constant model \((w(z) = -1)\) has \(\Delta \chi^2 \sim 0.5\) when the error is large and \(\Delta \chi^2 \sim 0.1\) when the error is small. We note that the effect of lensing is dependent on the shape of the luminosity function: Thus the effect could be much more significant if the luminosity function has a steep slope at high redshifts, as is clear from Figure 2.

In summary, the gravitational lensing is important systematic effect in constraining the nature of dark energy from the GRB Hubble diagram but is probably not so drastic as to totally change results from Swift GRBs.

V. CONCLUSION

In this paper, we have studied the effect of gravitational lensing on the determination of dark energy properties from distant GRBs. Gravitational lensing systematically brighten the apparent luminosity of observed GRBs, resulting in the modification of the Hubble diagram inferred from GRBs. Our Monte-Carlo simulations have shown that lensing does bias the best-fit model, but we can recover the assumed model within 68% confidence limit for Swift GRBs. Therefore, it does not degrade the cosmological usage of GRBs very much, at least within accuracies currently expected from Swift observations.

One of the reasons is that the constraint on dark energy from Swift GRBs are not very strong. We, however, emphasize that the amount of the bias introduced by lensing is quite sensitive to the shape of the luminosity function. Thus, we need an accurate measurement of the GRB luminosity function, in particular near the flux limit and beyond, in order to remove the effect of gravitational lensing and to obtain unbiased Hubble diagram. In addition, the strong influence of gravitational lensing on high-redshift GRBs implies that estimates of high-redshift GRB (or similarly star formation) rates and even the luminosity function itself may be biased. We plan to address these issues in a forthcoming paper.

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