Local spin alignment of vector mesons in relativistic heavy-ion collisions

Xiao-Liang Xia,1,‡ Hui Li,2,† Xu-Guang Huang,1,2,‡ and Huan Zhong Huang2,3,§

1Department of Physics and Center for Field Theory and Particle Physics, Fudan University, Shanghai 200433, China
2Key Laboratory of Nuclear Physics and Ion-beam Application (MOE), Fudan University, Shanghai 200433, China
3Department of Physics and Astronomy, University of California, Los Angeles, California 90095, USA

We derive the spin density matrix for vector mesons in the case that quarks and anti-quarks are polarized along arbitrary directions, and investigate the spin alignment of vector mesons arising from locally polarized quarks and anti-quarks (local spin alignment). We found that $\rho_{00} \neq 1/3$ does not signal the global polarization along the direction of orbital angular momentum, but may also originate from local spin polarization. Such local spin polarization could be induced by local vorticity arising from anisotropic expansion of the fireball in heavy-ion collisions. We explore the features of the local spin alignment and propose observables that can distinguish between the local and global spin alignments. These features can be used to probe the vorticity pattern and shed light on the puzzles in local $\Lambda$ polarization and $\phi$ and $K^{*0}$ spin alignments in heavy-ion collision experiments.

Introduction.— In noncentral relativistic heavy-ion collisions, when two nuclei collide at a finite impact parameter, a large orbital angular momentum (OAM) of the order of $10^5$–$10^7\hbar$ could be generated [1–3]. It has been proposed [4–8] that such an OAM can be partially transferred to the spin of quarks and anti-quarks in the produced quark-gluon plasma (QGP) due to spin-orbit coupling. Statistical mechanics and kinetic theory further show that the OAM can manifest itself in the form of fluid vorticity and polarize the particles in the system [9–12]. As a result, hadrons emitted from the QGP would have a net spin polarization along the OAM direction. This phenomenon is referred to as the global polarization. Recently, the global polarization of $\Lambda$ hyperon in Au+Au collisions was observed by STAR Collaboration at RHIC [13, 14]. The data are well described by various theoretical calculations based on the vorticity interpretation of the polarization, see e.g. Refs. [15–21], revealing that QGP processes a vorticity of the order of $10^{22}$ s$^{-1}$, surpassing the vorticity of all other known fluids in nature [13].

Besides the global $\Lambda$ polarization, another remarkable effect of the OAM is the global spin alignment of vector mesons [22–25]. Following the idea of the global polarization, if quarks and anti-quarks in QGP are globally polarized along the OAM direction, vector mesons produced during the hadronization processes will have different probabilities to occupy spin states $S_y = 1, 0, \text{and} -1$. Here the $y$ axis is along the OAM direction, which is perpendicular to the $z$-$x$ plane (the reaction plane) with $z$ axis along the colliding beams and $x$ axis along the impact parameter. In Ref. [22], it was found that the 00-th element of the spin density matrix of the vector meson is related to the spin polarization of quarks and anti-quarks through the following equation:

$$
\rho_{00} = \frac{1 - P_y^s P_y^{\bar{s}}}{3 + P_y^s P_y^{\bar{s}}},
$$

where $P_y^s$ and $P_y^{\bar{s}}$ are the spin polarization of quarks and anti-quarks along the $y$ axis, respectively. According to Eq. (1), $\rho_{00} < 1/3$ if quarks and anti-quarks are both polarized in $y$ direction by the OAM while $\rho_{00} > 1/3$ if quarks and anti-quarks are oppositely polarized along $y$ and $-y$ axes, respectively, which may be caused by quark fragmentation [22], magnetic field [23] or vector meson field [24, 25]. In both cases, $\rho_{00} \neq 1/3$ is expected to signal a nontrivial global polarization pattern of quarks and anti-quarks. Recently, the STAR and ALICE Collaborations reported the experimental results of $\rho_{00}$ for $\phi$ and $K^{*0}$ mesons which indeed deviates from $1/3$ in a wide range of centrality [26–28] but with an unexpectedly large magnitude that has not been understood.

However, the above analysis based on Eq. (1) is not the entire story in realistic heavy-ion collisions, because the global OAM is not the only source of vorticity. In fact, the anisotropic expansion of the QGP can generate complicated local structure of the vorticity which does not contribute to the global OAM. The particles in QGP can thus be polarized locally and lead to specific momentum-space distribution of $\Lambda$ polarization. This phenomenon (called local $\Lambda$ polarization) has already been examined by recent theoretical [29–36] and experimental studies [37], though remarkable puzzles regarding the azimuthal-angle dependence remain to be resolved.

In this Letter, in accordance with the local vortical structure, we propose a scenario of local spin alignment, in which the spin alignment of vector mesons composed by the locally polarized quarks and anti-quarks is considered. We will show that, even in the situation of zero global spin polarization, the local polarization of quarks and anti-quarks can still drive $\rho_{00}$ of vector mesons to deviate from $1/3$. Therefore, measurement of the average value of $\rho_{00}$ cannot distinguish the global and local scenarios of spin alignment. In the following, we will discuss the characteristics of the local spin alignment and propose specific observables to separate it from the global one. These observables may also help us understand the puzzles in local $\Lambda$ polarization. We will use $\hbar = c = 1$.

Spin density matrix of the vector meson.— The spin state of a vector meson can be described by a $3 \times 3$ spin density matrix $\rho^V$:

$$
\rho^V = \begin{pmatrix}
\rho_{11} & \rho_{10} & \rho_{1-1} \\
\rho_{01} & \rho_{00} & \rho_{0-1} \\
\rho_{-11} & \rho_{-10} & \rho_{-1-1}
\end{pmatrix},
$$

where the indices 1, 0, and $-1$ label the spin component of the vector meson along the spin-quantization axis. Throughout
this Letter, we take the OAM direction (i.e., the $y$ axis) in noncentral collisions as the spin-quantization axis.

In heavy-ion collisions, vector mesons are produced by multiple mechanisms, such as quark combination and quark fragmentation [22]. In this study, we mainly consider the vector mesons produced by quark combination. This mechanism is dominant in the low and intermediate $p_T$ and mid-rapidity regions which are the main kinematic regions covered by current experiments [38]. We consider that quarks and antiquarks are polarized along arbitrary directions and their spin polarization vectors are

$$\mathbf{P}^{\rho q} = (P^{\rho q}_x, P^{\rho q}_y, P^{\rho q}_z).$$

(3)

Here all the spatial components of the polarization vectors can be nonzero. This is different from the scenario of global spin alignment in which only $P^{\rho q}_z$ are considered to be nonzero. Given the above polarization vector, the spin density matrix of quarks and anti-quarks can be written as

$$\rho^{\rho q} = \frac{1}{2} \left( \begin{array}{ccc} 1 + P^{\rho q}_z & P^{\rho q}_+ & -i P^{\rho q}_- \\ P^{\rho q}_+ & P^{\rho q}_+ + i P^{\rho q}_y & P^{\rho q}_- \\ -i P^{\rho q}_- & P^{\rho q}_- + i P^{\rho q}_y & 1 - P^{\rho q}_z \end{array} \right).$$

(4)

Note that here the $y$ axis is the spin-quantization axis.

In the quark combination mechanism, the produced vector meson is a triplet state composed of its constituent quark and anti-quark. To obtain the spin density matrix of the vector meson, we first make a direct product of $\rho^q$ and $\rho^{\bar{q}}$ and then project it to the spin-triplet state [22, 39]:

$$\rho^V = \frac{U \rho^q \otimes \rho^{\bar{q}} U^\dagger}{\text{tr}(U \rho^q \otimes \rho^{\bar{q}} U^\dagger)},$$

(5)

where the transform matrix $U$ is

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ 0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 \end{pmatrix}.$$  

(6)

By inserting Eq. (4) into Eq. (5), we finally obtain the spin density matrix of the vector meson, of which the diagonal and off-diagonal elements are

$$\rho_{11} = \frac{1 + P^{\rho q}_z(1 + P^{\rho q}_y)}{3 + \mathbf{P}^q \cdot \mathbf{P}^{\bar{q}}},$$

(7)

$$\rho_{00} = \frac{1 - P^{\rho q}_z + P^{\rho q}_+ + P^{\rho q}_y P^{\rho q}_z}{3 + \mathbf{P}^q \cdot \mathbf{P}^{\bar{q}}},$$

(8)

$$\rho_{-11} = \frac{1 - P^{\rho q}_z}{3 + \mathbf{P}^q \cdot \mathbf{P}^{\bar{q}}},$$

(9)

and

$$\rho_{10} = \rho^*_{01} = \frac{(1 + P^{\rho q}_z)(P^{\rho q}_z - i P^{\rho q}_y) + (P^{\rho q}_z - i P^{\rho q}_y)(1 + P^{\rho q}_z)}{\sqrt{2}(3 + \mathbf{P}^q \cdot \mathbf{P}^{\bar{q}})}.$$  

(10)

$$\rho_{0-1} = \rho^*_{-10} = \frac{(1 - P^{\rho q}_z)(P^{\rho q}_z - i P^{\rho q}_y) + (P^{\rho q}_z - i P^{\rho q}_y)(1 - P^{\rho q}_z)}{\sqrt{2}(3 + \mathbf{P}^q \cdot \mathbf{P}^{\bar{q}})},$$

(11)

$$\rho_{-1-1} = \rho^*_{-11} = \frac{(P^{\rho q}_z - i P^{\rho q}_y)(P^{\rho q}_z - i P^{\rho q}_y)}{3 + \mathbf{P}^q \cdot \mathbf{P}^{\bar{q}}}.$$  

(12)

Equations (7-12) provide the relation between the spin density matrix of the vector meson and the spin polarization of quarks and anti-quarks. From these equations, we observe that (i) the element $\rho_{00}$, which is the crucial variable in the previous studies on the spin alignment, does not only receive contribution from the polarization component $P^{\rho q}_z$, but also from $P^{\rho q}_x$ and $P^{\rho q}_y$, and (ii) in the presence of $P^{\rho q}_z$ and $P^{\rho q}_y$, the off-diagonal elements in $\rho^V$ can also be nonzero.

Measurable elements in spin density matrix.— In heavy-ion collision experiments [26–28, 38], the spin information of vector mesons such as $f$ and $K^{*0}$ is extracted by analyzing their strong decays $f \rightarrow KK$ and $K^{*0} \rightarrow K\pi$. In these decay processes, the angular distribution of the decay products is measurable elements in spin density matrix appearing in Eq. (13). Here $\mathbf{p}^*$ is the momentum of one of the decay products in rest frame of the vector meson.

$$\frac{\text{d}^2N}{d(\cos \theta^*) d\varphi^*} = \frac{3}{8\pi} \left[ (1 - \rho_{00}) + (3\rho_{00} - 1) \cos^2 \theta^* - \sqrt{2}(\text{Re}\rho_{10} - \text{Re}\rho_{0-1}) \sin(2\theta^*) \cos \varphi^* + \sqrt{2}(\text{Im}\rho_{10} - \text{Im}\rho_{0-1}) \sin(2\theta^*) \sin \varphi^* - 2\text{Re}\rho_{-1-1} \sin \theta^* \cos(2\varphi^*) + 2\text{Im}\rho_{-1-1} \sin \theta^* \sin(2\varphi^*) \right].$$  

(13)

where $\theta^*$ and $\varphi^*$ are the polar and azimuthal angles of one of the decay products in the rest frame of the vector meson. Their definitions are shown in Fig. 1, namely, $\theta^*$ is the angle between the $y$ axis (the OAM direction) and the momentum of the decay product and $\varphi^*$ is between the $z$ axis (the beam direction) and the projection of the same momentum on the $z$-$x$ plane.

According to Eq. (13), by measuring the angular distribution of the decay products, one is able to determine the elements of the spin density matrix appearing in Eq. (13). These elements are related to the spin polarization of quarks and anti-quarks by

$$- \sqrt{2}(\text{Re}\rho_{10} - \text{Re}\rho_{0-1}) = - \frac{2(P^{\rho q}_y P^{\bar{q}}_z + P^{\rho q}_z P^{\bar{q}}_y)}{3 + \mathbf{P}^q \cdot \mathbf{P}^{\bar{q}}},$$

(14)

$$\sqrt{2}(\text{Im}\rho_{10} - \text{Im}\rho_{0-1}) = - \frac{2(P^{\rho q}_y P^{\bar{q}}_z + P^{\rho q}_z P^{\bar{q}}_y)}{3 + \mathbf{P}^q \cdot \mathbf{P}^{\bar{q}}},$$

(15)

$$- 2\text{Re}\rho_{-1-1} = - \frac{2(P^{\rho q}_y P^{\bar{q}}_z - P^{\rho q}_z P^{\bar{q}}_y)}{3 + \mathbf{P}^q \cdot \mathbf{P}^{\bar{q}}},$$

(16)
with local regions. quarks and anti-quarks are globally polarized along the azimuthal angle $\Delta \phi$ transverse polarization in the azimuth direction $q$ write down the polarization vector of a particle $q$, thus can survive even after averaging over many events. These two kinds of local polarization are uniformity of the radial flow along the $x$ axis; and $z$ polarized along the $y$ axis, therefore, according to Eq. (8), $\rho_{00}$ at $\Delta \phi = 0$ and $\pi$ is smaller than 1/3. Contrarily, the particles at $\Delta \phi = \pi/2$ and $3\pi/2$ are polarized along the $x$ axis, thus the corresponding $\rho_{00}$ is larger than 1/3. By analyzing Eq. (8), one can also find that $P_{l}^y P_{l}^z + P_{l}^z P_{l}^x < 2P_{l}^x P_{l}^y$ is the condition for $\rho_{00} < 1/3$. Solving this condition with the local-polarization configuration in Fig. 2 (b), one finds that the $\Delta \phi$ range for $\rho_{00} < 1/3$ is wider than that for $\rho_{00} > 1/3$. As a result, after taking the average, ($\rho_{00}$) is smaller than 1/3.

In order to understand the oscillating behavior of $\rho_{00}$ more intuitively, we plot $\rho_{00}$ as a function of $\Delta \phi$ according to Eq. (21) in the upper panel of Fig. 3. Considering the configuration in Fig. 2 (b), the particles at $\Delta \phi = 0$ and $\pi$ are $F_{y} = F_{s}$ and Eq. (22) is obtained when $F_{l}$ is small. From Eqs. (21-22), we observe that the local polarization of quarks and anti-quarks leads to not only a deviation of $\rho_{00}$ from 1/3 but also a harmonic oscillation of $\rho_{00}$ in $\Delta \phi$.

After integrating out $\Delta \phi$ in Eq. (22), we obtain the average value of $\rho_{00}$:

$$
\langle \rho_{00} \rangle \equiv \frac{1}{2\pi} \int_{0}^{2\pi} \rho_{00}(\Delta \phi)d(\Delta \phi) \approx \frac{1}{3} - \frac{F_{l}^2}{9},
$$

We see that $\langle \rho_{00} \rangle$ is smaller than 1/3, though the average polarization of quarks and anti-quarks is zero, i.e., $\langle P_{00}^{y} \rangle = 0$, in the most central collisions.

In order to understand the oscillating behavior of $\rho_{00}$ more intuitively, we plot $\rho_{00}$ as a function of $\Delta \phi$ according to Eq. (21) in the upper panel of Fig. 3. Considering the configuration in Fig. 2 (b), the particles at $\Delta \phi = 0$ and $\pi$ are polarized along the $y$ axis, therefore, according to Eq. (8), $\rho_{00}$ at $\Delta \phi = 0$ and $\pi$ is smaller than 1/3. Contrarily, the particles at $\Delta \phi = \pi/2$ and $3\pi/2$ are polarized along the $x$ axis, thus the corresponding $\rho_{00}$ is larger than 1/3. By analyzing Eq. (8), one can also find that $P_{l}^{y} P_{l}^{z} + P_{l}^{z} P_{l}^{x} < 2P_{l}^{x} P_{l}^{y}$ is the condition for $\rho_{00} < 1/3$. Solving this condition with the local-polarization configuration in Fig. 2 (b), one finds that the $\Delta \phi$ range for $\rho_{00} < 1/3$ is wider than that for $\rho_{00} > 1/3$. As a result, after taking the average, ($\rho_{00}$) is smaller than 1/3.

Substituting Eqs. (18-20) into Eqs. (14-17) with $P_{\text{global}} = F_{z} = 0$, we find that the following quantities are not vanishing:

$$
\sqrt{2}(\mathrm{Im}\rho_{10} - \mathrm{Im}\rho_{01}) = \frac{2F_{l}^2}{3 + F_{l}^2} \sin(2\Delta \phi),
$$

$$
-2\mathrm{Re}\rho_{11} = \frac{2F_{l}^2}{3 + F_{l}^2} \sin^2(\Delta \phi).\n$$

The existence of these nonzero off-diagonal elements of $\rho^{y}$ indicates that the angular distribution (13) of the vector-meson decay has a non-trivial shape not only in $\theta$ but also in $\phi$. Similar to $\rho_{00}$, $\sqrt{2}(\mathrm{Im}\rho_{10} - \mathrm{Im}\rho_{01})$ and $-2\mathrm{Re}\rho_{11}$ also oscillate in $\Delta \phi$ as illustrated in the lower panel of Fig. 3.

It should be pointed out that there is no experimental data yet to constrain $F_{l}$. Therefore, when we plot Fig. 3, the value of $F_{l}$ is chosen to be 0.1 according to the theoretical
calculation of the transverse local $\Lambda$ polarization given in Ref. [31]. Different theoretical models may give different $F_\perp$ and thus different values for $\rho_{00}$, $\sqrt{2}(\text{Im} \rho_{10} - \text{Im} \rho_{0-1})$, and $-2\text{Re} \rho_{1-1}$. However, the $\Delta \psi$ dependence as shown in Fig. 3 is a robust feature which is insensitive to the actual value of $F_\perp$.

Let us turn to noncentral collisions. In this case, $F_{\text{global}}$ and $F_{\perp}$ are nonzero, and $F_x$ and $F_y$ are unequal. Accordingly, the results of $\rho_{00}$ and other elements of $\rho^V$ will change from that in the central-collision case. One can study these changes using Eqs. (18-20) and Eqs. (8,14-17). However, to our best knowledge, the values of $P_{\text{global}}$, $F_{\perp}$ and $F_x - F_y$ are small compared to the mean value of $F_x$ and $F_y$ [20, 31], thus the changes are likely to be small. Thus, the $\Delta \psi$-modulation features of $\rho_{00}$, $\sqrt{2}(\text{Im} \rho_{10} - \text{Im} \rho_{0-1})$, and $-2\text{Re} \rho_{1-1}$ are expected to remain.

To end this section, we discuss the impact of strong magnetic field on our results. In the presence of the magnetic field, quarks and anti-quarks acquire additional spin polarization due to the coupling between their magnetic moments and the magnetic field. Hence, $P^{\vec{\psi}_{\parallel}}$ read

$$P^{\vec{\psi}_{\parallel}} = P^{\vec{\omega}_{\parallel}} \pm P^{\vec{\theta}_{\parallel}},$$

where the different sign in the second term is because quarks (+) and anti-quarks (−) have opposite magnetic moments. In the most central collisions, if QGP has a net electric charge, its longitudinal expansion will produce a magnetic field circling the $z$ axis at finite rapidity. Similar type of magnetic field may also be induced by collision remnants (i.e., unwounded protons) [40]. This magnetic field can lead to a transverse polarization in a similar pattern of Fig. 2 (b), but with opposite orientations for quarks and anti-quarks. In case that the magnetic field dominates over the vorticity, the value of $F_z^2$ in our analysis will have a minus sign, making $\rho_{00} - 1/3$ and Eqs. (24-25) flip the sign. In noncentral collisions, there exists a global magnetic field that is mainly along the $y$ axis [40–42]. If such a magnetic field is dominant, the resulting global polarization would be along $y$ and $-y$ axes for quarks and anti-quarks, respectively, and thus lead to $\langle \rho_{00} \rangle > 1/3$ [23]. Therefore, the detection of the local and global spin alignments may also provide useful information about the magnetic field in heavy-ion collisions which is complementary to other observables like the $y$-correlation for chiral magnetic effect.

**Global versus local spin alignments.**—From the analysis above, we find that a deviation of $\langle \rho_{00} \rangle$ from 1/3 in noncentral collisions can be caused by both the global and the local spin alignments. Below, we discuss the difference between these two scenarios of spin alignments, which will be helpful to distinguish which one is dominant in experiments.

1. Although $\langle \rho_{00} \rangle$ deviates from 1/3 in both the cases, the dependence of $\rho_{00}$ as a function of $\Delta \psi$ is different. If the local spin alignment dominates, $\rho_{00}$ oscillates in $\Delta \psi$ between values larger than 1/3 and smaller than 1/3; while if the global spin alignment dominates, the sign of $\rho_{00} - 1/3$ would be invariant versus $\Delta \psi$.

2. In the local spin alignment scenario, the variables in Eqs. (24-25) are nonzero. The amplitudes of their harmonic oscillations ($\approx 2F_z^2/3$) are twice that of $\rho_{00}$ ($\approx F_z^2/3$). On the contrary, those variables are zero in the global spin alignment scenario.

3. In all the above discussions, we have taken the $y$ axis as the spin-quantization axis. Nevertheless, one can study how the value of $\langle \rho_{00} \rangle$ changes with other choices of the spin-quantization axis. For example, let us choose the $x$ axis as the spin-quantization axis. In the experiment, this can be implemented by using the $y-z$ plane as the “event plane” and defining the $\theta^*$ angle with respect to the $x$ axis. In the local spin-alignment scenario, because the particle polarization in Fig. 2 (b) has approximately rotational symmetry around the $z$ axis, $\rho_{00}$ is actually independent from the choice of the event plane; see also the discussion in [43]. However, in the global spin-alignment scenario, $\langle \rho_{00} \rangle - 1/3$ will flip its sign if the event plane is rotated from the $z-x$ plane to the $y-z$ plane.

**Summary.**—We have studied the local spin alignment of vector mesons composed by quarks and anti-quarks which are locally polarized by the anisotropic expansion of the QGP fireball. We find that the local polarization of quarks and anti-quarks can cause $\rho_{00}$ of vector mesons to deviate from 1/3. As a characteristic feature of the local polarization, $\rho_{00}$ oscillates in vector meson’s azimuthal angle. After taking average over the azimuthal angle, $\langle \rho_{00} \rangle$ still deviates from 1/3. Therefore, both local spin alignment and global spin alignment may contribute to the deviation of $\langle \rho_{00} \rangle$ from 1/3.
We propose that the measurements of $\Delta \psi$ dependence of $\rho_{00}$, off-diagonal elements in $\rho^{\perp}$, and $\langle \rho_{00} \rangle$ with respect to different event planes can be used to separate the local spin alignment from the global one. Such measurements will shed important light on the local vorticity structure of the fireball and spin polarizations of quarks and anti-quarks in heavy-ion collisions.

Acknowledgments.— We thank Jinhui Chen, Chensheng Zhou, and Xin-Nian Wang for helpful discussions. This work is supported by National Natural Science Foundation of China through grants No. 11535012, No. 11675041, and No. 11835002. X.-L. X. and H. L. are also funded by China Postdoctoral Science Foundation through grants No. 2018M641909 and No. 2019M661333.