The QCD Mixed Phase: Shaken, not Stirred

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Abstract

Baryon fluctuations exceeding Poisson expectations can signal a nearly first order phase transition at RHIC. We show how these fluctuations can be measured, and apply a dissipative-hydrodynamic formulation used in condensed matter physics to simulate their evolution.

If the phase transition from quark matter to hadron gas is first order, matter at the appropriate temperatures and densities can exist as a mixed phase consisting of plasma droplets in equilibrium with a surrounding hadronic fluid. If formed in ion collisions, this mixed phase can produce large event-by-event fluctuations as the system hadronizes. Extraordinary baryon fluctuations [1] can accompany a first order transition at high baryon density [2] and, possibly, a near transition at zero baryon density [3], [4]. In ref. [1], we argued that baryon number conservation and rapid longitudinal expansion limits the extent to which post-hadronization interactions can erode fluctuations in a rapidity interval. Here, we further explore the rise and fall of these fluctuations using real-time lattice simulations [5].

At high baryon density, QCD with two massless flavors can exhibit a first order transition whose coexistence curve culminates in a tricritical point at temperature $T_c$ and baryon chemical potential $\mu_c$ [2]. For $T > T_c$ and $\mu < \mu_c$, a second order phase transition breaks/restores chiral symmetry. If the quark masses are sufficiently large, the second order transition is replaced by a smooth transformation (chiral symmetry is explicitly broken). The first order line remains, however, with the tricritical point replaced by a critical point in the same universality class as a liquid–gas transition.

At RHIC, baryon density may also serve as an approximate order parameter for the nearly first order transition at small net baryon density. Lattice simulations [3] and general arguments [4] show that the baryon susceptibility $\chi$ at $\mu = 0$ can increase suddenly as temperature is increased near $T_h \sim 150$ MeV, where the chiral order parameter and the energy density change sharply. Jumps in the susceptibility commonly accompany first order transitions. For a liquid–gas transition, $\chi = \partial \rho / \partial \mu$ is proportional to the compressibility: steam is much more compressible than water.

Large fluctuations in baryon number occur during phase separation in a first order transition. Figure 1 (left) shows the phase diagram in the $T - \rho$ plane, where $\rho$
is the baryon density. A uniform system quenched into the outer parabolic region below $T_c$ will separate into droplets at high baryon density $\rho_q$ surrounded by matter at density $\rho_h$. The net baryon number $N_B$ in a sub-volume of the system varies depending on the number of droplets in the sub-volume. The variance of the baryon number $V = \langle N_B^2 \rangle - \langle N_B \rangle^2$ can exceed the equilibrium expectation by an amount

$$\Delta V \approx f(1 - f)(\Delta N_B)^2,$$

where $f$ is the fraction of the high density phase in the sub-volume $V$ and $\Delta N_B = (\rho_q - \rho_h)V$. We will argue that nonequilibrium evolution in ion collisions can allow these fluctuations to survive post-hadronization evolution.

We stress that a super-poissonian variance such as (1) is straightforward to test experimentally by measuring

$$\Omega_p = (V_p - \langle N_p \rangle)/\langle N_p \rangle^2,$$

where $N_p$ is the number of protons in a rapidity interval and $V_p$ is its variance. This quantity vanishes in equilibrium and is related to the more familiar scaled variance $\omega_p = \langle N_p \rangle(1 + \Omega_p)$. Significantly, we find that $\Omega_p$ equals the total $\Omega_B$ – which includes unseen neutrons – for a range of thermal and Glauber models that respect isospin symmetry. Specifically, $\Omega_p = \Omega_B$ because the probability of $N_p$ satisfies $p(N_p) = \sum_{N_B} p(N_B)p(N_B|N_p)$ with a binomial distribution $p(N_B|N_p)$ for $N_p$ at fixed baryon number $N_B$. Isospin fluctuations can alter $p(N_B|N_p)$ near the tricritical point or in the presence of a disoriented chiral condensate, but that will be evident from pion measurements.

To describe the evolution of the inhomogeneous mixed phase, we follow the standard condensed matter practice and write a Ginzburg-Landau free energy:

$$f = \kappa(\nabla \rho)^2/2 + f_0, \quad f_0 = -m^2(\rho - \rho_c)^2/2 + \lambda(\rho - \rho_c)^4/4$$

where $f_0(\rho)$ describes the excursions of the baryon density $\rho$ from its equilibrium value in the uniform matter. The $\kappa$ term describes the droplet surface tension, $\sigma \propto \kappa^{1/2}$. For $m^2 \propto T_c - T$ we find the correct liquid-gas critical exponents. The values $\rho_h$ and $\rho_q$ in fig. 1 (right) correspond to the equilibrium densities at $T < T_c$. To describe the dynamics of the system, we must account for the fact that baryon number is conserved. Furthermore, it is crucial to include dissipation to describe this strongly fluctuating system. The simplest equations that meet these criteria
Fig. 2. Order parameter in the transverse plane without (a) and with expansion (b).

are:

\[
\frac{\partial \rho}{\partial t} = M \nabla^2 \mu, \quad \mu = f'_0 - \kappa \nabla^2 \rho; \tag{4}
\]

model B in [6]. We identify \( D = 2m^2M \) as the baryon diffusion coefficient by linearizing (4) about \( \rho_h \).

To describe nuclear collisions, we extend (4) to include drift due to Bjorken longitudinal flow:

\[
\frac{\partial \rho}{\partial \tau} + \frac{\rho}{\tau} = M \nabla^2 \mu, \tag{5}
\]

where \( \tau \) is the proper time and \( \mu \) is given by (3, 4). The new term forces the average density to decrease as \( \langle \rho \rangle \propto \tau^{-1} \), driving the system through the phase coexistence region. Fluctuations grow when densities are near \( \rho_c \) (c.f. fig. 1).

Phase separation is most dramatic if the rapid expansion drives the system into the unstable region; i.e., the inner parabolic region in fig. 1 (left), corresponding to \( f''_0(\rho) < 0 \). Droplets form from runaway density fluctuations in a process known as spinodal decomposition. Linearizing near \( \rho_c \), we estimate the time scale for this process to be \( \tau_R = 8\xi^2/D \), where \( \xi = \kappa^{1/2}/m \) is the correlation length. For times \( t \gg \tau_R \) the system undergoes a nonlinear evolution in which droplets merge, reducing their surface energy.

Figure 2 compares 2+1 dimensional numerical simulations of (4) and (5) for \( \psi = (\rho - \rho_c)/\rho_c \) in the transverse plane. We take \( \tau_R \) and \( \xi \) to each be 1 fm as motivated in [1]. \( (D \sim 8 \text{ fm is consistent with calculations in [7])} \). The expanding system reaches \( \rho_c \) at \( \tau_0 = 5 \text{ fm}. \) Expansion shown in fig. 2b prevents droplets from merging as in fig. 2a. Because this is a dissipative system, we must apply thermal noise at each lattice site at \( \tau_0 \) to seed phase separation. The memory of the initial conditions is essentially lost for \( \tau - \tau_0 > \tau_R \).

Figure 3 shows the computed variance for two different initial times and for two rapidity intervals. The variance is computed from a sample of 5000 simulated events, each unique due to the thermal noise. We see that the super-poissonian fluctuations grow appreciably by \( \tau \sim 2\tau_0 \). This variance drops as the rapidity interval is
increased. We find that variance is governed by the ratio $\tau_0/\tau_R$, which compares the expansion and droplet-growth time scales.

We emphasize that these calculations include diffusion, which dampens the fluctuations once the system becomes stable. For (3) with $\langle \rho \rangle \propto \tau^{-1}$, the system is unstable only for $\tau < 2.3 \tau_0$. We extend the calculations to much longer times to demonstrate that the fluctuations in rapidity survive well past the freezeout time, of order 10–30 fm, in accord with [1]. We comment that convection, viscosity and collision-geometry effects can reduce $\Omega_p$ compared to fig. 3. Moreover, our phase transition effect may be compensated to some extent by the effect discussed by Koch and Asakawa in these proceedings, which owes to the difference between fluctuations in a plasma compared to a hadron gas. Nevertheless, the strength of the signal in our exploratory calculations invites further work.

We thank R. Bellwied, P. Braun Munzinger, K. Elder, M. Grant, J. Kapusta, G. Kunde, P. Keyes, B. Müller, I. Mishustin, C. Pruneau and S. Voloshin.

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