The DKP oscillator with a linear interaction in the cosmic string space-time

Mansoureh Hosseinpour\textsuperscript{1,}\textsuperscript{*}, Hassan Hassanabadi\textsuperscript{1,†} and Fabiano M. Andrade\textsuperscript{2,‡}

\textsuperscript{1} Physics Department, Shahrood University of Technology, Shahrood, Iran, P.O. Box 3619995161-316

\textsuperscript{2} Departamento de Matemática e Estatística, Universidade Estadual de Ponta Grossa, 84030-900 Ponta Grossa, Paraná, Brazil

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Abstract

We study the relativistic quantum dynamics of a DKP oscillator field subject to a linear interaction in cosmic string space-time in order to better understand the effects of gravitational fields produced by topological defects on the scalar field. We obtain the solution of DKP oscillator in the cosmic string background. Also, we solve it with an ansatz in presence of linear interaction. We obtain the eigenfunctions and the energy levels of the relativistic field in that background.

Key Words: DKP oscillator, cosmic string, linear potential, ansatz solution.
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\textsuperscript{*}E-mail:hosseinpour.mansoureh@gmail.com
\textsuperscript{†}E-mail:hha1349@gmail.com
\textsuperscript{‡}E-mail:fmandrade@uepg.br
1 Introduction

The Dirac equation including the linear harmonic potential was initially studied by Ito et al. [1], Cook [2] and Ui et al.[3]. This system was latterly called by Moshinsky and Szczepaniak as Dirac oscillator [4], because it behaves as an harmonic oscillator with a strong spin-orbit coupling in the non-relativistic limit. This model is based on the dynamics of a harmonic oscillator for spin-1/2 particles by introducing a nonminimal prescription into free Dirac equation [4]. Physically, it can be shown that the Dirac oscillator interaction is a physical system, which can be interpreted as the interaction of the anomalous magnetic moment with a linear electric field [5, 6].

As a relativistic quantum mechanical system, the Dirac oscillator has been widely studied. Because it is an exactly solvable model, several investigations have been developed in the context of this theoretical framework in the last years. Although the Dirac oscillator is normally introduced within the context of many body theory, relativistic quantum mechanics and quantum chromodynamics (in particular as an interquark potential and also as the confining part of the phenomenological Cornell potential). The interest in this issue appears in different contexts such as quantum optics [7–9], supersymmetry [5, 10, 11], nuclear reactions [12], the hadronic spectrum using the two-body Dirac oscillator[13, 14] a new representation for its solutions using the Clifford algebra [15, 16], noncommutative space[17, 18], thermodynamic properties [19] Lie Algebra symmetries [20], supersymmetric (non-relativistic) quantum mechanics [21] the super symmetric path integral formalism [22] the chiral phase transition in presence of a constant magnetic field [8], the relativistic Landau in presence of external magnetic field [23] the Aharonov-Bohm effect [24, 25] condensed matter physical phenomena and graphene [26].

The DKP oscillator is an analogous to Dirac oscillator [4]. The DKP oscillator in the (1+2)-dimensional noncommutative phase space for spin-zero particles has been investigated in the work of Guo et al. [27]. Yang et al. studied the DKP oscillator with spin-zero in three-dimensional noncommutative phase space [28]. A generalized bosonic oscillator within the minimal length quantum mechanics has been analyzed in [29]. De Melo et al. released a higher-dimensional formulation of Galilean covariance to consider the noncommutative DKP oscillator [30]. Falek and Merad presented both spin-zero and spin-one DKP equations in noncommutative phase in the (1 + 3)-dimensional case [31]. Recently, there has been an increasing interest on the so-called DKP oscillator [32–41]. The DKP oscillator considering minimal length [29, 42], noncom-
mutative phase space [27, 28, 31, 43] and topological defects [44].

The DKP oscillator embedded in a magnetic cosmic string background has inspired a great deal of research in last years. A cosmic string is a linear defect that changes the topology of the medium when viewed globally. The spacetime around a cosmic string is locally flat but not globally. The theory of general relativity predicts that gravitation is manifested as curvature of spacetime. This curvature is characterized by the Riemann tensor. There are connections between topological properties of the space and local physical laws. The non-trivial topology of spacetime, as well as its curvature, leads to a number of interesting gravitational effects. For example, it has been known that the energy levels of an atom placed in a gravitational field will be shifted as a result of the interaction of the atom with spacetime curvature. Therefore, we have to consider the topology of the spacetime in order to describe completely the physics of system.

In this work, we examine the relativistic quantum dynamics of DKP oscillator in the presence of the linear interaction, on the curved spacetime of a cosmic string. From the corresponding DKP equation, we analyze the influence of the topological defect on the equation of motion, the energy spectrum and the wave-function. In Sec. 2, we introduce the covariant DKP equation. In Sec. 3 we present the covariant DKP oscillator in cosmic string background and obtain the solution of DKP oscillator In Sec. 4, we present solution of DKP oscillator presence linear interaction. In the next section we present our conclusions.

2 Covariant form of the DKP equation in the cosmic string background

The cosmic string spacetime with an internal magnetic field in cylindrical coordinates is described by the line element (units such that $\hbar = c = 1$) [45, 46]

$$ds^2 = -dt^2 + dr^2 + \alpha^2 r^2 d\varphi^2 + dz^2,$$

with $-\infty < z < \infty$, $\rho \geq 0$ and $0 \leq \varphi \leq 2\pi$. The angular parameter $0 < \alpha < 1$ is related to the linear mass density $\mu$ of the string as $\alpha = 1 - 4\mu$. From the geometrical point of view, the metric (1) describes a Minkowski space-time with a conical singularity.
The DKP equation in the cosmic string spacetime (1) reads [47–49]

\[(i\beta^\mu \nabla_\mu - M)\psi = 0. \] (2)

The covariant derivative in (2) is given by [50]

\[\nabla_\mu = \partial_\mu - \Gamma_\mu (x), \] (3)

where \(\Gamma_\mu\) are the spinorial affine connections given by

\[\Gamma_\mu = \frac{1}{2} \omega_{\mu ab} \left[ \beta^a, \beta^b \right]. \] (4)

The matrices \(\beta^a (x)\) are the standard Dirac matrices in Minkowski spacetime and \(\omega_{\mu ab}\) represents the spin connection given by

\[\omega_{\mu\bar{a}\bar{b}} = \eta_{\bar{a}\bar{c}} \bar{e}_{\nu}^\bar{c} \Gamma_\mu^\nu - \eta_{\bar{a}\bar{c}} \bar{e}_{\nu}^\bar{c} \bar{e}_{\nu}. \] (5)

The nonnull components of the spin connection are

\[\omega_{\varphi}^{12} = -\omega_{\varphi}^{21} = 1 - \alpha. \] (6)

We can build the local reference frame through a non-coordinate basis with \(e_{\mu}^a\) where \(e_{\mu}^a\) and \(e_{a}^\mu\) are transformation matrices. The components of the non-coordinate basis \(e_{\mu}^a\) are called tetrads or vierbeins that form our local reference frame. With the line element (1), we can use tetrads \(e_{\mu}^a\) and \(e_{a}^\mu\) as follows

\[
e_{\mu}^a = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \varphi & \sin \varphi & 0 \\ 0 & -\sin \varphi & \cos \varphi & 0 \\ 0 & \frac{\alpha r}{\varphi} & \frac{\alpha r}{\varphi} & 0 \end{pmatrix}, \quad e_{a}^\mu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \varphi & -\alpha r \sin \varphi & 0 \\ 0 & \sin \varphi & \alpha r \cos \varphi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \] (7)

The vierbeins form our local reference frame that satisfy the orthonormality conditions

\[e_{\mu}^a (x) e_{a}^\nu (x) = \delta_\nu^\mu, \quad e_{\mu}^a (x) e_{b}^\mu (x) = \delta_a^b. \] (8)

And satisfy

\[g_{\mu\nu} (x) = e_{\mu}^{(a)} (x) e_{\nu}^{(b)} (x) \eta_{ab}. \] (9)
The Kemmer matrices in curved spacetime are related to their Minkowski counterparts via

\[ \beta^\mu (x) = e^\mu_a \beta^a. \]  

(10)

In terms of the Minkowski flat spacetime coordinates, these matrices can be cast into the form

\[
\beta^0 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \beta^1 = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},
\]

(11)

\[
\beta^2 = \begin{pmatrix} 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \beta^3 = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{pmatrix}.
\]

(12)

The matrices \( \beta^\mu (x) \) in (6) are given more explicitly as

\[
\beta^t = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \beta^r = \begin{pmatrix} 0 & 0 & -\cos \varphi & -\sin \varphi & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -\cos \varphi & 0 & 0 & 0 & 0 \\ -\sin \varphi & 0 & 0 & 0 & 0 \end{pmatrix},
\]

(13)

\[
\beta^\varphi = \begin{pmatrix} 0 & 0 & \sin \varphi & -\cos \varphi & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \sin \varphi & 0 & 0 & 0 & 0 \\ -\cos \varphi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \beta^z = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{pmatrix}
\]

(14)

where \( \beta^z, \beta^r, \beta^\varphi \) and \( \beta^z \) are the general form of the Kemmer matrices in this spacetime.
3 Solution of DKP oscillator in cosmic string background

In this section, we concentrate our efforts in the interaction called DKP oscillator. For this external interaction we use the non-minimal substitution

\[ \partial_r \rightarrow \partial_r + M \omega r \eta^0, \quad (15) \]

where \( \omega \) is the oscillator frequency and \( \eta^0 = 2 \beta^0 \omega^2 - 1 \). Considering only the radial component, with the non-minimal substitution one gets

\[ \left[ i \beta^2 \partial_t + i \beta^r (\partial_r + M \omega r \eta^0) + i \beta^\phi (\partial_{\phi} - \Gamma_{\phi}) + i \beta^z \partial_z - M \right] \Psi(t, r, \phi, z) = 0. \quad (16) \]

As the interaction is time-independent one can write the spinor as

\[ \Psi(t, r, \phi, z) = e^{-i(E t - m \phi - k z)} \Psi(r), \]

where \( E \) is the energy of the scalar boson, and the five-component spinor can be written as \( \Psi(r) = (\psi_1(r), \ldots, \psi_5(r))^T \) and the DKP equation for scalar bosons becomes (for compactness of next equations, we momentarily drop the \( r \) dependence in the spinor components)

\[ -m \sin \phi \psi_1 - \alpha r M \psi_3 - i \alpha r \cos \phi [M \omega r \psi_1 + \psi'_1] = 0, \]
\[ m \cos \phi \psi_1 - \alpha r M \psi_4 - i \alpha r \sin \phi [M \omega r \psi_1 + \psi'_1] = 0, \]
\[ k_z \psi_1 - M \psi_5 = 0, \]
\[ E \psi_1 - M \psi_2 = 0, \]
\[ \alpha r \left[ -M \psi_1 + E \psi_2 + k_z \psi_5 + i \cos \phi(1 - \alpha + \alpha M \omega r^2) \psi_3 - im \psi_4 - \alpha r \psi'_3 \right] \]
\[ + i \sin \phi \left[ im \psi_3 + (1 - \alpha + \alpha M \omega r^2) \psi_4 - r \alpha \psi'_4 \right] = 0. \]

By solving the above system of equations in favour of \( \psi_1(r) \) we get

\[ \psi_2 = \frac{E \psi_1}{M}, \quad (17) \]
\[ \psi_3 = \frac{-m \sin \phi \psi_1 + i[-\alpha M \omega r^2 \cos \phi \psi_1 - \alpha r \cos \phi \psi'_1]}{\alpha r M}, \quad (18) \]
\[ \psi_4 = \frac{m \cos(\phi) \psi_1 + i[-\alpha M \omega r^2 \sin \phi \psi_1 - r \alpha \sin \phi \psi'_1]}{\alpha r M}, \quad (19) \]
\[ \psi_5 = \frac{k_z \psi_1}{M}. \quad (20) \]
Combining these results we obtain an equation of motion for the first component of the DKP spinor

\[ \psi''_1(r) + \frac{\alpha - 1}{\alpha r} \psi'_1(r) - \left( E^2 - M^2 + k_z^2 - \frac{(2\alpha - 1)M\omega}{\alpha} + \frac{m^2}{\alpha^2 r^2} + M^2 \omega^2 r^2 \right) \psi_1(r) = 0. \]  

(21)

In order to solve the above equation, we employ the change of variable: \( s = r^2 \), thus we rewrite the radial equation (21) in the form

\[ \psi''_1(s) + \frac{(2\alpha - 1)}{2\alpha s} \psi'_1(s) + \frac{1}{4s^2} \left( -\xi_1 s^2 + \xi_2 s - \xi_3 \right) \psi_1(s) = 0. \]  

(22)

where

\[ \xi_1 = M^2 \omega^2, \]  

(23)

\[ \xi_2 = -E^2 - k_z^2 + M^2 + \frac{(2\alpha - 1)}{\alpha}M\omega, \]  

(24)

\[ \xi_3 = \frac{m^2}{\alpha^2}. \]  

(25)

which gives the energy levels of the relativistic Klein-Gordon equation from [51]

\[ \alpha_2 n - (2n+1)\alpha_5 + (2n+1)(\sqrt{\alpha_9} + \alpha_3 \sqrt{\alpha_8}) + n(n-1)\alpha_3 + \alpha_7 + 2\alpha_3 \alpha_8 + 2\sqrt{\alpha_8 \alpha_9} = 0, \]  

(26)

where

\begin{align*}
\alpha_1 &= \alpha - \frac{1}{2}, & \alpha_2 &= 0, & \alpha_3 &= 0, & \alpha_4 &= \frac{1}{2} \left( \frac{3}{2} - \alpha \right), & \alpha_5 &= 0, \\
\alpha_6 &= \xi_1, & \alpha_7 &= -\xi_2, & \alpha_8 &= \alpha_4^2 + \xi_3, & \alpha_9 &= \xi_1, & \alpha_{10} &= 1 + 2\sqrt{\alpha_8}, \\
\alpha_{11} &= 2\sqrt{\alpha_6}, & \alpha_{12} &= \alpha_4 + \sqrt{\alpha_8}, & \alpha_{13} &= -\sqrt{\alpha_6}.
\end{align*}

As the final step, it should be mentioned that the corresponding wave function is

\[ \psi_1(r) = N r^{2\alpha_12} e^{\alpha_{13} r^2} L_n^{\alpha_{10} - 1}(\alpha_{11} r^2). \]  

(27)

where \( N \) is the normalization constant.
4 Solution of the DKP oscillator in presence linear interaction

Let us now to analyse the situation when a DKP field interacts with a scalar potential \( U(r) \), which is introduced via the substitution \( M \to M + U(r) \). Thus, (16) becomes

\[
\{i\beta^0 \partial_t + i\beta^r (\partial_r + M\omega r^0) + i\beta^\phi (\partial_\phi - \Gamma_\phi) + i\beta^z \partial_z - [M + U(r)]\} \Psi(t, r, \phi, z) = 0.
\]

(28)

Here we are interested in studying the linear scalar potential:

\( U(r) = ar \).

(29)

In order to solve Eq. (28) we make the following change of variables:

\[
\psi_1(r) = r^{\frac{1-a}{2a}}(M + ar)^{\frac{1}{2}} R(r).
\]

(30)

This leads us to an equation without first-order derivative term

\[
R''(r) + \left[ - E^2 - k_z^2 + M \left( M + \omega - \frac{\omega a}{M} \right) + \frac{a^2 - 4m^2 - 1}{4a^2r^2} - \frac{a(1 - a)}{2Mar} + 2aMr + \frac{M^2\omega^2}{(M + ar)^2} - \frac{3a^2}{4(M + ar)^2} + \frac{a^2(1 - a) + 2M^3a\omega}{2Ma(M + ar)} \right] R(r) = 0.
\]

(31)

The next step, is to write \( R_{n,m}(r) \) in the following form

\[
R_{n,m}(r) = R_n(r) e^{g_m(r)}.
\]

(32)

Thus, the \( r \)-dependent terms in Eq. (32) suggest that we take \( g_m(r) \) as

\[
g_m(r) = b_1 r + b_2 r^2 + b_3 \log(r) + b_4 \log(M + ar).
\]

(33)

where the five constants \( b_1, \ldots, b_4 \) are to be expressed in terms of the physical constants \( \alpha, a, \omega, M, M' \), \( k_z \) and \( E \). For nodeless states, with \( n = 0 \), we have \( R_n(r) = 1 \), consequently [52–54]

\[
R_{0,m}(r) = e^{g_m(r)},
\]

(34)

In this manner, from Eq. (26) we have

\[
R''_{0,m}(r) + (-g''_m - g'^2_m) R_{0,m}(r) = 0.
\]

(35)
Thus substituting Eq. (33) into Eq. (35), the latter equation becomes

\[
R''_{0,m}(r) + \left( -b_1^2 - 2b_2 - 4rb_1b_2 - 4b_2b_3 - 4b_2b_4 - 4b_2^2r^2 + \frac{b_3 - b_3^2}{r^2} - \frac{2Mb_1b_3 + 2ab_3b_4}{Mr} + \frac{a^2b_4 - a^2b_4^2}{(M + ar)^2} + \frac{4M^2b_2b_4 - 2Mab_1b_4 + 2a^2b_3b_4}{M(M + ar)} \right) R_{0,m}(r) = 0. 
\]

(36)

If we compare Eq. (36) with Eq. (31), we acquire the following six equations (displayed here along with their respective factors in (r):

\[
\begin{align*}
- E^2 - k_z^2 + M(M + \omega - \frac{\omega}{\alpha}) + b_1^2 + 2b_2 + 4rb_1b_2 + 4b_2b_4 + 4b_2b_3 &= 0, \\
- 2ab_1b_4 + 4Mb_2b_4 + 2a^2b_3b_4 + \frac{a^2(-1 + \alpha)}{2Ma} - M^2\omega &= 0, \\
\frac{2ab_3b_4}{M} + 2b_1b_3 + \frac{a(-1 + \alpha)}{2Ma} &= 0, \\
-1 + \frac{a^2 - 4m^2}{4\alpha^2} - b_3 + b_4^2 &= 0, \\
(b_4 - b_4^2) + \frac{3}{4} &= 0, \\
4b_2^2 + (a^2 - M^2\omega^2) &= 0, \\
4b_1b_2 + 2aM &= 0. 
\end{align*}
\]

(37)

These equations can be solved for \(b_1, b_2, b_3\) and \(b_4\) they also provide constraints on the physical parameters, in particular, the energy \(E\). Equation (37) admit the following solutions:

\[
\begin{align*}
 b_1 &= \pm \frac{aM}{\sqrt{-a^2 + M^2\omega^2}}, \\
 b_2 &= \pm \frac{1}{2} \sqrt{-a^2 + M^2\omega^2}, \\
 b_3 &= \pm \frac{\alpha^2 + \sqrt{\alpha^2 + 4\alpha^2m^2}}{2\alpha^2}, \\
 b_4 &= \frac{1 + 2}{2}. 
\end{align*}
\]

(38)

Finally, the solution for DKP oscillator interacting with a linear scalar potential can be written as

\[
\Psi_{n,m}(t, r, \varphi, z) = N_{n,m} e^{i(k_z + mp - Et)} e^{b_1r + b_2r^2} r^{b_3 + \frac{1}{4m}} (M + ar)^{b_4 + \frac{1}{2}},
\]

(39)
where $N_{n,m}$ is normalization constant.

5 Conclusion

In this contribution, we have investigated the relativistic quantum dynamics of DKP oscillator in the presence of the linear interaction, on the curved space-time of a cosmic string. From the corresponding DKP equation, we analyze the influence of the topological defect on the equation of motion, the energy spectrum and the wave function. We established and found solutions of covariant DKP oscillator in cosmic string background. We present solution of DKP oscillator presence linear interaction. We solved the DKP oscillator analytically by using a proper ansatz solution to the equation.

References

[1] D. Itô, K. Mori, and E. Carriere. An example of dynamical systems with linear trajectory. *Nuovo Cim. A*, 51(4):1119, 1967.

[2] P. A. Cook. Relativistic harmonic oscillators with intrinsic spin structure. *Lett. Nuovo Cimento*, 1(10):419–426, mar 1971.

[3] H. Ui and G. Takeda. Does accidental degeneracy imply a symmetry group? *Progr. Theor. Exp. Phys.*, 72(2):266–284, aug 1984.

[4] M Moshinsky and A Szczepaniak. The dirac oscillator. *J. Phys. A*, 22(17):L817–, 1989.

[5] R. P. Martínez y Romero and A. L. Salas-Brito. Conformal invariance in a dirac oscillator. *J. Math. Phys.*, 33(5):1831–1836, may 1992.

[6] M Moreno and A Zentella. Covariance, CPT and the foldy-wouthuysen transformation for the dirac oscillator. *J. Phys. A*, 22(17):L821, 1989.

[7] D. Dutta, O. Panella, and P Roy. Pseudo-hermitian generalized dirac oscillators. *Ann. Phys. (NY)*, 331:120–126, apr 2013.

[8] A. Bermudez, M. A. Martin-Delgado, and A. Luis. Chirality quantum phase transition in the dirac oscillator. *Phys. Rev. A*, 77(6):063815, jun 2008.
[9] A. Bermudez, M. A. Martin-Delgado, and E. Solano. Exact mapping of the 2+1 dirac oscillator onto the jaynes-cummings model: Ion-trap experimental proposal. *Phys. Rev. A*, 76(4):041801, 2007.

[10] J. Bentez, R. P. Martinez y Romero, H. N. Núez-Yépez, and A. L. Salas-Brito. Solution and hidden supersymmetry of a dirac oscillator. *Phys. Rev. Lett.*, 64(14):1643–1645, 1990.

[11] O. Castaños, A. Frank, R. López, and L. F. Urrutia. Soluble extensions of the dirac oscillator with exact and broken supersymmetry. *Phys. Rev. D*, 43(2):544–547, jan 1991.

[12] J. Grineviciute and Dean Halderson. Dirac oscillators and the relativistic R-matrix. *Phys. Rev. C*, 80(4):044607, oct 2009.

[13] M. Moshinsky and Y.F. Smirnov. *The Harmonic Oscillator in Modern Physics*. Contemporary concepts in physics. Harwood Academic Publishers, 1996.

[14] M. Moshinsky and G. Loyola. Barut equation for the particle-antiparticle system with a dirac oscillator interaction. *Found. Phys.*, 23(2):197–210, feb 1993.

[15] R. de Lima Rodrigues. On the dirac oscillator. *Phys. Lett. A*, 372(15):2587–2591, apr 2008.

[16] James P. Crawford. The dirac oscillator and local automorphism invariance. *J. Math. Phys.*, 34(10):4428–4435, oct 1993.

[17] F. Vega. Oscillators in a (2+1)-dimensional noncommutative space. *J. Math. Phys.*, 55(3):032105, mar 2014.

[18] Shaohong Cai, Tao Jing, Guangjie Guo, and Rukun Zhang. Dirac oscillator in noncommutative phase space. *Int. J. Theor. Phys.*, 49(8):1699–1705, apr 2010.

[19] M.H Pacheco, R.R Landim, and C.A.S Almeida. One-dimensional dirac oscillator in a thermal bath. *Phys. Lett. A*, 311(2-3):93–96, may 2003.

[20] C Quesne and M Moshinsky. Symmetry lie algebra of the dirac oscillator. *J. Phys. A*, 23(12):2263–2272, jun 1990.
[21] J. Beckers and N. Debergh. Supersymmetry, foldy-wouthuysen transformations, and relativistic oscillators. *Phys. Rev. D*, 42(4):1255–1259, aug 1990.

[22] R. Rekioua and T. Boudjedaa. Path integral for one-dimensional dirac oscillator. *Eur. Phys. J. C*, 49(4):1091–1098, jan 2007.

[23] Bhabani Prasad Mandal and Shweta Verma. Dirac oscillator in an external magnetic field. *Phys. Lett. A*, 374(8):1021–1023, feb 2010.

[24] N. Ferkous and A. Bounames. Energy spectrum of a 2D dirac oscillator in the presence of the aharonov-bohm effect. *Phys. Lett. A*, 325(1):21–29, 2004.

[25] Fabiano M. Andrade and Edilberto O. Silva. Effects of spin on the dynamics of the 2D dirac oscillator in the magnetic cosmic string background. *Eur. Phys. J. C*, 74:3187, 2014.

[26] Emerson Sadurni. The dirac-moshinsky oscillator: theory and applications. *AIP Conf. Proc.*, 1334(1):249–290, 2011.

[27] Guangjie Guo, Chaoyun Long, Zuhua Yang, and Shuijie Qin. DKP oscillator in noncommutative phase space. *Can. J. Phys.*, 87(9):989–993, sep 2009.

[28] Zu-Hua Yang, Chao-Yun Long, Shuei-Jie Qin, and Zheng-Wen Long. DKP oscillator with spin-0 in three-dimensional noncommutative phase space. *Int. J. Theor. Phys.*, 49(3):644–651, jan 2010.

[29] M. Falek and M. Merad. A generalized bosonic oscillator in the presence of a minimal length. *J. Math. Phys.*, 51(3):033516, 2010.

[30] G R de Melo, M de Montigny, and E S Santos. Spinless duffin-kemmer-petiau oscillator in a galilean non-commutative phase space. *J. Phys.: Conf. Ser.*, 343:012028, feb 2012.

[31] M Falek and M Merad. DKP oscillator in a noncommutative space. *Commun. Theor. Phys.*, 50(3):587–592, sep 2008.

[32] N. Debergh, J. Ndimubandi, and D. Strivay. On relativistic scalar and vector mesons with harmonic oscillator-like interactions. *Z. Phys. C: Part. Fields*, 56(3):421–425, sep 1992.
[33] Y Nedjadi, S Ait-Tahar, and R C Barrett. An extended relativistic quantum oscillator for particles. *J. Phys. A*, 31(16):3867–3874, apr 1998.

[34] Y Nedjadi and R C Barrett. A generalized duffin-kemmer-petiau oscillator. *J. Phys. A*, 31(31):6717–6724, aug 1998.

[35] Y Nedjadi and R C Barrett. The duffin-kemmer-petiau oscillator. *J. Phys. A*, 27(12):4301–4315, jun 1994.

[36] A. Boumali and L. Chetouani. Exact solutions of the kemmer equation for a dirac oscillator. *Phys. Lett. A*, 346(4):261–268, oct 2005.

[37] I. Boztosun, M. Karakoc, F. Yasuk, and A. Durmus. Asymptotic iteration method solutions to the relativistic duffin-kemmer-petiau equation. *J. Math. Phys.*, 47(6):062301, jun 2006.

[38] A Boumali. One-dimensional thermal properties of the kemmer oscillator. *Phys. Scr.*, 76(6):669–673, nov 2007.

[39] F Yasuk, M Karakoc, and I Boztosun. The relativistic duffin–kemmer–petiau sextic oscillator. *Phys. Scr.*, 78(4):045010, oct 2008.

[40] A. Boumali. On the eigensolutions of the one-dimensional duffin–kemmer–petiau oscillator. *J. Math. Phys.*, 49(2):022302, feb 2008.

[41] Y. Kasri and L. Chetouani. Energy spectrum of the relativistic duffin-kemmer-petiau equation. *Int. J. Theor. Phys.*, 47(9):2249–2258, jan 2008.

[42] M. Falek and M. Merad. Bosonic oscillator in the presence of minimal length. *J. Math. Phys.*, 50(2):023508, feb 2009.

[43] H. Hassanabadi, Z. Molae, and S. Zarrinkamar. DKP oscillator in the presence of magnetic field in (1+2)-dimensions for spin-zero and spin-one particles in noncommutative phase space. *Eur. Phys. J. C*, 72(11):2217, nov 2012.

[44] Luis B. Castro. Quantum dynamics of scalar bosons in a cosmic string background. *Eur. Phys. J. C*, 75(6):287, jun 2015.

[45] Alexander Vilenkin. Cosmic strings and domain walls. *Phys. Rep.*, 121(5):263–315, may 1985.
[46] B. Linet. The static metrics with cylindrical symmetry describing a model of cosmic strings. *Gen. Relativ. Gravitation*, 17(11):1109–1115, nov 1985.

[47] N. Kemmer. The particle aspect of meson theory. *Proc. R. Soc. A*, 173(952):91–116, nov 1939.

[48] R. J. Duffin. On the characteristic matrices of covariant systems. *Phys. Rev.*, 54(12):1114–1114, dec 1938.

[49] G. Petiau. PhD thesis, 1936. Acad R. 1936. Belg. Cl. Sci. Mem. Collect 8: 16.

[50] A. Havare, T. Yetkin, and K. Sogut. On the equivalence of the massless dkp equation and maxwell equations in robertson-walker spacetime. *Chin. J. Phys*, 5:465–474, 2003.

[51] C. Tezcan and R. Sever. Dirac equation with vector and scalar cornell potentials and an external magnetic field. *Int. J. Theor. Phys*, 48:337–350, July 2009.

[52] Timothy Clifton and John D. Barrow. The existence of gödel, einstein, and de sitter universes. *Phys. Rev. D*, 72(12):123003, dec 2005.

[53] Reinaldo J Gleiser, Metin Gürses, Atalay Karasu, and Özgür Sarinodotoğlu. Closed timelike curves and geodesics of gödel-type metrics. *Class. Quantum Grav.*, 23(7):2653–2663, mar 2006.

[54] Troels Harmark and Tadashi Takayanagi. Supersymmetric gödel universes in string theory. *Nucl. Phys. B*, 662(1-2):3–39, jul 2003.