Left-Handed-Material-like behavior revealed by arrays of dielectric cylinders

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Abstract

We investigate the electromagnetic propagation in two-dimensional photonic crystals, formed by parallel dielectric cylinders embedded a uniform medium. The transmission of electromagnetic waves through prism structures are calculated by the standard multiple scattering theory. The results demonstrate that in certain frequency regimes and when the propagation inside the scattering media is not considered, the transmission behavior mimics that expected for a left-handed material. Such feature may illusively lead to the conclusion that a left-handed material is fabricated and it obeys Snell’s law of negative refraction. We also discuss possible ambiguities that may be involved in previous experimental evidence of left-handed materials.

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The concept of the so-called Left Handed Material (LHM) or Negative Refraction Index Material (NRIM) was first proposed by Veselago many years ago [1]. In the year 2000, Pendry proposed that a lens made of LHM can overcome the traditional limitation on the optical resolution and therefore make “perfect” images [2]. Since then, the search for LHM and the study of the properties of LHM have been skyrocketing, signified by the rapid growth of the related literatures.

An earlier realization of LHM consisted of composite resonator structures of metallic wires and rings operated in the regime of microwaves [3]. The measurements involved refraction of microwaves by a prism. As pointed out in [4], however, while this work is revolutionary, there are a few unaddressed questions. These include problems associated with such as near field effects due to either rapid dispersion along the propagation direction [5] or smaller amount of attenuation on the shorter side of the prism [6]. Moreover, the experiment had only measured a single angled prism; this is sufficient only if the \textit{a priori} assumption that the material is refractive is valid. These shortcomings allow for alternative interpretation of the reported experimental data, as discussed in Ref. [5, 6].

Ever since its inception, theoreticians have questioned the concept of LHM and the perfect lenses made of LHM. Such a challenge can be classified into two categories. First, scientists have questioned whether left-handed materials are genuinely possible. Second, even if they existed, whether LHM would really make perfect lenses is also debatable [7, 8, 9, 10]. The principle behind the first inquiry is that although peculiar phenomena observed for some artificial materials may be in conflict with our immediate intuition, if they can be still explained in the framework of current knowledge, the resort to negative refraction or LHM is at least not a definite necessity.

Recently, new sets of experimental measurements have been reported to provide affirmative evidences on LHM. For instance, Houck et al. [4] measured two dimensional profiles of collimated microwave beams transmitted through composite wire and split-ring resonator prisms. The authors used two angled prisms, in an attempt to refute alternatives posed in the criticisms on earlier measurements, to obtain a rather consistent negative refractive index. The data appeared to obey Snell’s law. The experimental setup in Ref. [4] represents a common method of deducing the refractive index of LHM. Based upon our rigorous simulations, however, we believe that these experimental results are not conclusive. They may not be sufficient to confirm the existence of LHM. As a matter of fact, we found that
the apparent abnormal refraction that has been thought to be the negative refraction may be well explained in the context of the partial-gap effects or the related Bragg scattering, which are common for all kinds of wave propagation in periodic structures. In this Letter, we present some key simulation results to support our claim.

Here we consider the transmission of electromagnetic (EM) waves in photonic crystals. To make it easy to reproduce our results, we will use the photonic crystal structures that have been commonly used in previous simulations, such as those in [12]. The systems are two dimensional photonic crystals made of arrays of parallel dielectric cylinders placed in a uniform medium, which we assume to be air. The solution for the wave scattering and propagation in such systems can be obtained by the multiple scattering theory. This theory is exact and was first formulated systematically by Twersky [13], then has been reformulated and applied successfully to optical, sonic and water wave problems [14, 15, 16]. The results show that an apparent ‘negative’ refraction is indeed possible, in line with the experimental observation [4]. But, such a negative refraction is not sufficient in proving that the materials are left handed.

The multiple scattering theory (MST) used in our simulation can be summarized as follows. Consider an arbitrary array of dielectric cylinders in a uniform medium. The cylinders are impinged by an optical source. In response to the incident wave from the transmitting source and the scattered waves from other scatterers, each scatterer will scatter repeatedly waves and thus scattered waves can be expressed in terms of a modal series of partial waves. Regarding these scattered waves as the incident wave to other scatterers, a set of coupled equations can be formulated and computed rigorously. The total wave at any spatial point is the summation of the direct wave from the source and the scattered waves from all scatterers. The intensity of the waves is represented by the modulus of the wave field. The details about MST can be found in Ref. [17].

For brevity, we only consider the E-polarized waves, that is, the electric field is kept parallel to the cylinders. The following parameters are used in the simulation. (1) The dielectric constant of the cylinders is 14, and the cylinders are arranged in air to form a square lattice. (2) The lattice constant is $a$ and the radius of the cylinders is 0.3$a$; in the computation, all lengths are scaled by the lattice constant, so all the lengths are dimensionless. (3) A variety of the outer shapes of the arrays is considered, including the prism structures and the slab.
First we consider the propagation of EM waves through the prism structures of arrays of dielectric cylinders, by analogy with those shown in [4]. Two sizes of the prisms are considered and illustrated by Fig. 1. We have used two types of sources in the simulations: (1) plane waves; (2) collimated waves by guiding the wave propagation through a window before incidence on the prisms, mimicking most experiments; (3) a line source. The results from these three scenarios are similar. In this Letter, we only show the results from the line source which is located at some distance beneath the prisms. We note here that when using a plane wave, effects from the prism edges play a role and should be removed.

We have plotted the transmitted intensity fields. The results are presented in Fig. 1. Here the incident waves are transmitted vertically from the bottom. The impinging frequency is $0.192 \times \frac{2\pi c}{a}$; the frequency has been scaled to be non-dimensional in the same way as in Ref. [12]. We note that at this frequency, the wave length is about five times of the lattice constant, nearly the same as that used in the experimental of [4]. The incidence is along the $[\cos 22.5^\circ, \sin 67.5^\circ]$ direction, that is, the incidence makes an equal angle of $22.5^\circ$ with regard to the [10] and [11] directions of the square lattice; the reason why we choose this direction will become clear from later discussions. Moreover, on purpose, the intensity imaging is plotted for the fields inside and outside the prisms on separate graphs which have been scripted by ‘1’ and ‘2’ respectively.

Without looking at the fields inside the prisms, purely from Fig. 1 (a1) and (b1) we are able to calculate the main paths of the transmitted intensities. The geometries of the transmission are indicated in the diagrams. The tilt angles of the prisms are denoted by $\phi$, whereas the angles made by the outgoing intensities relative to the normals of the titled interfaces are represented by $\theta$. According to the prescription outlined in Ref. [4], once $\phi$ and $\theta$ are determined, Snell’s law is applied to determine the effective refractive index: $n \sin \phi = \sin \theta$. If the angle $\theta$ is towards the higher side of the prisms with reference to the normal, the angle is considered positive. Otherwise, it is regarded as negative. In light of these considerations, the apparent ‘negative refractions’, similar to the experimental observations, indeed appear and are indicated by the black arrows in the graphs. After invoking Snell’s law, the negative refraction results are deduced. From (a1) and (b1), we obtain the negative refractive indices as $-0.84 \pm 0.17$ and $-0.87 \pm 0.21$ for the two prisms respectively. The inconsistency between the two values is less than 4%, which is close to that estimated in the experiment [4]. The overall uncertainty in the present simulation (up
to 24%) is less than that in the experiment (up to 36%) [4]. Therefore, a consistent negative refractive index may be claimed from the measurements shown in Fig. (a1) and (b1). This would have been regarded as an evidence showing that the photonic structures described by Fig. are left-handed materials, at least for the frequency considered. By the same token, we even found that with certain adjustments such as rotating the arrays or varying the filling factor, thus obtained index can be close to the perfect -1.

Though tempting, there are ambiguities in the above explanation of the transmission in the context of negative refraction that has led to the negative refraction index. This can be discerned by taking into consideration the wave propagation inside the prisms. As shown clearly by Fig. (a2) and (b2), the transmission inside the prisms has already been bent at the incidence interfaces, referring to the two white arrowed lines in (a2) and (b2). If Snell’s law were valid at the outgoing interfaces, it should also be applicable at the incident boundaries. Then with the zero incidence angle and a finite refraction angle, Snell’s law would lead to the absurd result of an infinite refractive index for the surrounding medium which is taken as air. Besides, when taking into account the bending inside the prism, the incident angle at the outgoing or the tilted surface is not \( \theta \) any more. Therefore the index value obtained above cannot be correct. These ambiguities cannot be excluded from the current experiments that have been claimed to support LHM using Snell’s law [4, 11].

In the experiments of Ref. [4], for example, the transmission fields inside the prisms are not shown. The authors also stated clearly that the negative index behavior can be only achieved by some adjustments, implying that the experimentally observed negative index behavior is not robust. We have checked this point by rotating the orientation of the arrays of the cylinders. The apparent ‘negative refraction’ may disappear, depending on the relative geometries between the incident direction and the orientation of the lattice.

In search for the cause of the apparent ‘negative refraction’ shown in Fig. we have carried out further simulations. For example, in Fig. we plot the EM wave transmission across two slabs of photonic crystals with two different lattice orientations: one is along the diagonal direction, i.e. the [11] direction, and the other is along the direction making an equal angle to the [11] and [10] directions. The incident frequency is again 0.192 \( \times 2\pi c/a \). Here, we see that when the incident wave is along the [11] direction, the transmission follows a straight path inside the slab. For the second case, the propagation direction is bent inside the slab: the major lobe tends to follow the [11] direction, and a minor lobe of transmission
appears to make an perpendicular angle with regard to the [11] direction, that is the [-11] direction. As aforementioned, if Snell’s law were used to obtain the refraction index, the absurd number of infinity would be deduced only ambiguously.

Fig. 2 clearly indicates that there are some favorable directions for waves to travel. This phenomenon is not uncommon for wave propagation in periodically structured media. It is well-known that periodic structures can modulate dramatically the wave propagation due to Bragg scattering, a feature fundamental to the X-ray imaging. In some situations, the systems reveal complete band gaps, referring to the frequency regime where waves cannot propagate in any direction. In some other occasions, partial band gaps may appear so that waves may be allowed to travel in some directions but not in some other directions. More often, the periodic structures lead to anisotropic dispersions in wave propagation. The whole realm of these phenomena has been well represented by the band structures calculated from Bloch’s theorem. The preferable directions for wave propagation are associated with the properties of band structures. In fact, the above abnormal ‘negative refraction’ phenomenon may be explained in the frameworks of the band structures calculated for the crystal structures considered.

Fig. 3 shows the band structure calculated for the square lattices used in the above simulations. A complete band gap is shown between frequencies of 0.22 and 0.28. Just below the complete gap, there is a regime of the partial band gap, in which waves are not allowed to travel along the [10] direction. The frequency $0.192 \times 2\pi c/a$ we have chosen for simulation lies within this partial gap area. Within this gap, waves are prohibited from transmission along the [10] direction. As a result, when incident along an angle that lies between the [11] and [10] direction, waves will incline to the [11] direction. This observation explains the abnormal refraction observed above. Due to the symmetry, the waves are also allowed to travel in the directions which are perpendicular to [11]. This explains why we have also seen an intensity lobe along [-11] from Fig. 2. We found that these results are also qualitatively similar for other frequencies within the partial band gap. Furthermore, we have checked other frequencies at which although there is no partial band gap, the dispersion is highly anisotropic, similar features may also be possible.

The phenomenon of the band structures may help discuss the abnormal transmission that has been interpreted as the onset of the negative index behavior in the experiments [4]. Up to date, all the claimed left-handed materials are artificially made periodic composite struc-
tures, in which the negative permittivity and permeability are obtained near a resonance frequency. From our experiences, near such a resonance frequency, either complete band gaps or partial band gaps are likely to appear. In other words, the wave propagation largely depends on the periodic structures. Without taking this fact into account, the transmission may be regarded as unconventional, and then the artifact conclusion that a LHM has been realized and observed may be resorted to. Due to the limited information available from the experiments, we cannot yet definitely come to the conclusion that the apparent ‘negative refractions’ observed in experiments are due to partial band gaps, highly anisotropic dispersions, or inhomogeneities. The present simulations, however, at least indicate that some of these effects must be addressed in order to be able to interpret the experimental data correctly.

Lastly, we would like to make a comment on the permittivity and permeability of composite materials. It is well-known that these quantities are only meaningful in the sense of the effective medium theory. A basic assumption is that the wave propagation should not be sensitive to details of the supporting media. Otherwise, whether the definitions of the permittivity and permeability are still meaningful is itself doubtful. If measurements of the two parameters could not be justified accordingly, it could be misleading to deliberately interpret wave propagation in terms of negative or positive refraction. In the above cases, although the wave length is much larger than the lattice constant, the wave propagation is obviously still sensitive to the lattice arrangements. Therefore, the flow of energy should be interpreted as mainly controlled by Bragg scattering processes, rather than as a propagation in an effective medium. In other words, the media act as a device that can bend the flow of optical energies, rather than as an effective refracting-medium.

In summary, we have simulated wave transmission through prism structures. The results have shown some ambiguities in interpreting the apparent abnormal behavior in the transmission as the onset of the negative refraction index behavior. It is suggested that periodic structures can give rise to peculiar phenomena which need not be regarded as a negative index behavior.

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FIG. 1: The images of the transmitted intensity fields. Here, the intensities inside and outside the prism structures are plotted separately, so that the images can be clearly shown with proper scales. The geometric measurements can be inferred from the figure. The tilt angles for the two prisms are $18^\circ$ and $26^\circ$ respectively, and have been labelled in the figures. For cases (a1) and (b1), we observe the apparent ‘negative refraction’ at the angles of $15\pm3^\circ$ and $21\pm5^\circ$ respectively. When applying Snell’s law, these numbers give rise to the negative refraction indices of $-0.84 \pm 0.17$ and $-0.87 \pm 0.21$ for (a1) and (b1) separately. In (a2) and (b2), the intensity fields inside inside the prisms are plotted. Here we clearly see that the transmission has been bent. In the plots, [10] and [11] denote the axial directions of the square lattice of the cylinder arrays.

FIG. 2: The imaging fields for slabs of photonic crystal structure. Two lattice orientations are considered: (a) The slab measures as about $56\times10$ and the incidence is along the [11] direction; (2) the slab measures as $50\times13$ and the incidence is along the direction that makes an equal angle to the [11] and [10] directions. The main lobes in the transmitted intensities are shown.

FIG. 3: The band structure a square lattice of dielectric cylinders. The lattice constant is $a$ and the radius of the cylinders is $0.3a$. The $\Gamma M$ and $\Gamma X$ denote the [11] and [10] directions respectively. A partial gap is between the two horizontal lines.
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