Three Generations or More
for an Attractive Gravity?

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Abstract

We calculate the induced Einstein action from supersymmetric models in general space-time. Supersymmetric models consist of two kinds of supermultiplets, called scalar supermultiplets and vector supermultiplets, respectively. We show that the vector multiplets generate a negative Newton constant, while the scalar multiplets a positive one. Then we find that the positivity of Newton constant depends on the ratio of the number of scalar multiplets to one of vector multiplets. If we apply this result to two hopeful supersymmetric unified models: one is minimal SUSY standard model and another is minimal SUSY SU(5) GUT, we are led to the conclusion that we need more than or equal to three generations of quark and lepton multiplets to have a positive Newton constant, i.e., an attractive gravity.
1 Introduction

A long-standing unsolved problem in the field of high energy physics is how one can describe the quantum theory of gravity satisfactorily. In such a situation, about a quarter century ago some people began to consider that a gravity is only induced through the vacuum fluctuations of matter and gauge fields \([1, 2, 3]\), i.e., not an elementary field. People call such a theory of gravity induced gravity or, more tasteful name, pregeometry. In this model both the Newton constant and the cosmological constant are determined by a physical cut-off of some underlying theory. The quartically divergent cosmological constant, however, is opposite to our observations. With this fact in mind we introduce supersymmetry (SUSY) to the underlying theory, because it is well known that SUSY gives rise to cancellations between curvature independent vacuum fluctuations of fermions and those of bosons \([4, 5]\), so leads to vanishing cosmological constant. Besides, SUSY, more precisely supergravity, decides the coefficient \(\xi\) of conformally coupled term \(-\xi R A A^*\), which the induced Einstein action drastically depends on via Seeley-DeWitt coefficients, of some scalar field \(A\). As a result there is a predictability in our result. Also from the viewpoint of particle physics, SUSY makes unified theories very hopeful as ‘beyond the standard model’.

In what follows we consider that the candidates of an underlying theory of induced gravity are supersymmetric models with other three forces, strong and electroweak interactions. Then we calculate the vacuum fluctuations of both scalar multiplets and vector ones which are ingredinets of supersymmetric unified models, and see what their induced Einstein actions are. Finally, we apply these results to the two
supersymmetric unified models and conclude that we need three or more generations
to obtain an attractive induced gravity.

This paper is organized as follows. In chapter 2 we review the DeWitt-Schwinger
technique for the evaluation of the effective action. In chapter 3 and 4 we evaluate the
induced Einstein action from the scalar multiplets and the vector ones, respectively.
In chapter 5 we study the induced Einstein action from the underlying theory with
both scalar and vector multiplets. Then we apply this to two unified models and
obtain the suggestion that we need three or more generations to have an attractive
gravity.

2 Effective Action and DeWitt-Schwinger Technique

First we review how to evaluate effective actions, vacuum fluctuations in our case,
using DeWitt-Schwinger technique [6, 7]. Note that we calculate the effective action
in Euclidean curved space-time in order to regularize this action by proper time
cut-off [3–6, 8]. A merit of proper time cut-off regularization is that it does not
break SUSY so long as we are concerning about divergent part, as we shall see
below. And in the regularization we can naturally identify the quartically divergent
and curvature independent term with cosmological constant. Likewise, we identify
quadratically divergent term linear in curvature with Einstein action.

We begin with the generating functional $Z[g_{mn}]$:

$$Z[\phi, g_{mn}] = \int D\phi e^{-S[\phi, \bar{\phi}, g_{mn}]}; \tag{1}$$
where \( g_{mn} \) is the background gravitational field, and \( \phi \) and \( \bar{\phi} \) represent all the quantum fields and classical ones but the background gravitational field, respectively. Then the effective action \( W[\bar{\phi}, g_{mn}] \) is defined as follows:

\[
Z[\bar{\phi}, g_{mn}] = e^{-W[\bar{\phi}, g_{mn}]}.
\]

(2)

At one-loop level the effective action is formally written as follows:

\[
W[\bar{\phi}, g_{mn}] = S[\bar{\phi}, g_{mn}] \pm \frac{1}{2} \ln \text{Det} \Delta[\bar{\phi}, g_{mn}],
\]

(3)

where

\[
\Delta[\bar{\phi}, g_{mn}] = -\Box + m^2[\bar{\phi}, g_{mn}] - \xi R.
\]

(4)

\( m \) and \( \xi \) are a mass parameter and a conformal coupling to the background gravitational field, respectively, and \( R \) is the scalar curvature of background gravitational field. The signs above correspond to fermion and boson, respectively. Using the identity

\[
\frac{1}{\Delta} = \int_0^\infty ds \ e^{-s\Delta}
\]

(5)

we obtain

\[
\pm \frac{1}{2} \text{Tr} \ln \Delta = \pm \frac{1}{2} \int_0^\infty \frac{ds}{s} \text{Tr} e^{-s\Delta} \pm \text{field independent constant}.
\]

(6)

We set this field independent constant zero.\(^1\) The trace part of the integrand in eq.(6) is expressed in terms of heat kernel as follows

\[
\text{Tr} e^{-s\Delta} = \int d^4x \sqrt{g} K(x, x; s),
\]

(7)

\(^1\)We mean auxiliary field in the underlying theory by background field.

\(^2\) In SUSY case these constants cancel one another.
where the heat kernel $K(x, x'; s) = \langle x | e^{-s\Delta} | x' \rangle$ and satisfies the equations

$$\frac{\partial}{\partial s} K(x, x'; s) + \Delta K(x, x'; s) = 0$$
$$K(x, x'; 0) = \delta(x, x'). \quad (8)$$

General solution of the equations (8) can be written in terms of asymptotic series

$$K(x, x'; s) = \frac{1}{(4\pi s)^{d/2}} e^{-m^2 s - \sigma(x, x')/2s} \sqrt{D(x, x')} \sum_{n=0}^{\infty} E_n s^{n/2}, \quad (9)$$

where $\sigma(x, x')$ is the half of square of geodesic length between $x$ and $x'$, and $D(x, x')$ is the covariant Van Vleck-Morette determinant defined by

$$D(x, x') = \frac{1}{\sqrt{g}} \det \{-\partial_m \partial_n' \sigma(x, x')\} \sqrt{g}. \quad (10)$$

In the coincidence limit $\sigma(x, x') \to 0$ and $D(x, x') \to 1$, one can find one-loop part of the effective action, the second term of r.h.s. of eq. (3):

$$W_{1-loop} = \pm \int d^4x \sqrt{g} \frac{1}{2(4\pi)^2} \sum_{n=0}^{\infty} \int_0^{\infty} ds \ e^{-m^2 s} s^{n/2-3} E_n. \quad (11)$$

We separate the one-loop part of the effective action into the divergent part $\mathcal{L}_D$ and the finite part $\mathcal{L}_F$:

$$W_{1-loop} = \pm \int d^4x \frac{1}{2(4\pi)^2} (\mathcal{L}_D + \mathcal{L}_F). \quad (12)$$

In order to regularize $\mathcal{L}_D$ and give it physical meanings, let us introduce an ultraviolet cut-off $\Lambda$, then we obtain

$$g^{-1/2} \mathcal{L}_D = \sum_{n=0}^{4} E_n \int_{1/\Lambda^2}^{\infty} ds \ e^{-m^2 s} s^{n/2-3} \quad (13)$$

$$g^{-1/2} \mathcal{L}_F = \sum_{n=5}^{\infty} \Gamma\left(\frac{n}{2} - 2\right) E_n (m^2)^{2-n/2}. \quad (14)$$
We can easily evaluate the divergent part, especially in the limit $\Lambda \to \infty$

$$g^{-1/2}L_D = \frac{1}{2}E_0 \left[ \Lambda^4 - 2m^2\Lambda^2 - m^4 \left( \gamma - \frac{3}{2} - \ln \frac{\Lambda^2}{m^2} \right) \right]$$

$$+ \frac{2}{3}E_1(\Lambda^3 - 3\Lambda m^2 + 2\sqrt{\pi}m^3) + E_2 \left[ \Lambda^2 + m^2(\gamma - 1) - m^2 \ln \frac{\Lambda^2}{m^2} \right]$$

$$+ 2E_3(\Lambda - \sqrt{\pi}m) + E_4 \left( \ln \frac{\Lambda^2}{m^2} - \gamma \right).$$

For a manifold without boundary $E_{2n+1} = 0$ and $E_{2n} = a_n$ where $a_n$ are well-known Seeley-DeWitt coefficients\cite{6}. Non-vanishing first two coefficients are

$$E_0 = a_0 = 1$$

$$E_2 = a_1 = (\xi - 1/6)\mathcal{R}$$

a degree of freedom. Here we find that in a manifold without boundary the coefficients of logarithmically divergent terms are same as that of divergent terms in dimensional regularization, and curvature independent terms of quartic divergence and quadratic one vanish through Bose-Fermi cancellations. After all it gives supersymmetric results as far as we are concerning on cosmological constant and Einstein action.

### 3 Induced Einstein Action from Scalar Multiplets

In order to see the induced Einstein action purely from scalar multiplets, we deal with a curved space-time version of Wess-Zumino model\cite{9} which consists of scalar multiplets and an auxiliary gravity multiplet. We want to treat unified models far below so called Planck scale ($M_P = 1/\sqrt{8\pi G}$) and, as we can see from the eqs.\cite{15} and \cite{17}, to obtain the cosmological constant and Einstein action as fourth and second power of cut-off, Planck scale, respectively. So we can omit the terms of
\(O(1/M_P)\) from the starting Lagrangian. With this fact in mind we start from the renormalizable part of an N=1\(^3\) supergravity (locally supersymmetric) Lagrangian without terms as follows:\(^4\):

\[
\frac{1}{8\pi G} \left[ -\frac{1}{2} R - \frac{1}{3} MM^* + \frac{1}{3} b^a b_a \right] + \frac{i}{2} \epsilon^{kmn} \bar{\psi}_k \gamma_5 \gamma_l \bar{D}_m \psi_n \\
- c \left[ \frac{1}{8\pi G} M - \bar{\psi}_{La} \sigma^{ab} \psi_{Lb} \right] - c^* \left[ \frac{1}{8\pi G} M^* - \bar{\psi}_{Ra} \sigma^{ab} \psi_{Rb} \right],
\]

(18)

where \(c\) is a complex constant, \(M(M^*)\) and \(b_a\) are auxiliary fields and \(\psi_n\) is the gravitino which is the gauge field of SUSY.\(^5\) We may neglect \(b_a \sim O(1/M_P^2)\) in the above. Above terms (18) include the kinetic terms of graviton and gravitino, and cosmological terms of supergravity, which are supersymmetric extension of Einstein action with cosmological constant. Starting from the supergravity Lagrangian without above terms (18), we obtain as the quantum corrections the induced action of kinetic terms of graviton and gravitino, and (probably vanishing) cosmological constant. In other words, through the quantum fluctuations of elementary fields we obtain the same kinds of terms as those (18), which we discarded at the underlying Lagrangian level. Note that \(G\) is would-be Newton constant.

We are here interested only in bosonic part of the induced Einstein supergravity action, so we can set auxiliary gravitino field zero. In other words, we can set the background space-time bosonic. In calculating the induced Einstein action, we can confine ourselves in free theory because we are only interested in vacuum fluctuations.

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\(^3\) N=1 means that there is only one Majorana spinor field as gravitino.

\(^4\) This separation is justified when we treat the theory far below Planck scale and superpotential without linear term.

\(^5\) Accurately we mean no more than dimension-four parts of the supergravity by SUSY, supersymmetry, in this paper.
at one-loop order. In practice we invoked supergravity only to decide conformal coupling $-\frac{1}{6}\mathcal{R}AA^*$. The decision of conformal coupling of scalar field is equivalent to that of Einstein action because the conformal coupling of other fields (spinor and vector fields) to gravity is known that $\xi = 1/4$. In Euclidean bosonic curved space-time, Lagrangian written in terms of a scalar multiplet with supersymmetric mass $m$ are \cite{9}

$$
\mathcal{L} = \partial_m A^* \partial^m A + \frac{1}{2} \bar{\Psi} \gamma^m \mathcal{D}_m \Psi - FF^* + \frac{1}{3} (MA^*F + M^*AF^*) - \left( \frac{1}{6} \mathcal{R} + \frac{1}{9} MM^* \right) AA^*
$$

$$
- m \left\{ \left( AF + A^* F^* \right) - \frac{1}{2} \bar{\Psi} \Psi - \frac{1}{2} (M^* A^2 + MA^{*2}) \right\}
$$

where $A$ is a complex scalar, $\Psi$ is a Majorana spinor, and $F$ is a complex auxiliary scalar field. And $\mathcal{D}_m$ is covariant derivatives, $\mathcal{R}$ is the scalar curvature and $M$ is auxiliary field of gravitational field. Here we take the term with $mM$, $mM^*$ and $M^*M$ for mass terms like one with $m^2$. From the Lagrangian above we find the first two Seeley-DeWitt coefficients which are

$$
a_0 = 2, \quad \quad \quad (20)
$$

$$
a_1 = 0, \quad \quad \quad (21)
$$

for a complex scalar, and

$$
a_0 = 2 \quad \quad \quad (22)
$$

$$
a_1 = \frac{1}{6} \mathcal{R} \quad \quad \quad (23)
$$

for a Majorana spinor.
Then, for a Wess-Zumino model, we can easily evaluate the vacuum fluctuation using the above coefficients. The induced Einstein action in Euclidean curved space-time is

\[ W_S = \int d^4x \sqrt{-g} \frac{1}{2(4\pi)^2} \left[ \frac{1}{6} R \Lambda^2 \right]. \]  

(24)

The sign of curvature term is same as one of [3]. We find that, without gauge fields we always obtain a positive Newton constant.

4 Induced Einstein Action from Vector (Gauge) Multiplets

We will next study the induced action from an SU(N) or a U(1) gauge multiplet in general space-time, because it is also contained in realistic unified models. As we mentioned in the previous chapter, we are interested only in vacuum energy density at one-loop order, so only non-interacting parts are relevant here. The pure supersymmetric Yang-Mills (renormalizable) Lagrangian in Wess-Zumino gauge [9] is

\[ g^{-1/2} L = \frac{1}{4} F_{mn}^{(a)} F^{(a)mn} + \frac{1}{2} \bar{\lambda}^{(a)} \gamma^m \tilde{D}_m \lambda^{(a)} - \frac{1}{2} D^{(a)2} \]  

(25)

where \( D^{(a)} = 0 \) from the equation of motion and \( \tilde{D}_m \) is general coordinate and gauge covariant derivatives. The non-interacting parts of Lagrangian (25) is

\[ g^{-1/2} L_{\text{non-int}} = \frac{1}{4} \left[ \partial_m \psi^{(a)}_n - \partial_n \psi^{(a)}_m \right] \left[ \partial^m \psi^{(a)n} - \partial^n \psi^{(a)m} \right] + \frac{1}{2} \bar{\lambda}^{(a)} \gamma^m D_m \lambda^{(a)}. \]  

(26)

Then, the non-interacting Lagrangian (26) is the Lagrangian of \((N^2 - 1)\) independent supersymmetric electromagnetisms. Next step is to quantize this theory.
In order to do that we add the gauge fixing term and corresponding ghost term to Lagrangian (26) as everyone always does.

\[ g^{-1/2}L_{tot} = \frac{1}{4}\left[ \partial_m v^{(a)}_n - \partial_n v^{(a)}_m \right]\left[ \partial^m v^{(a)m} - \partial^n v^{(a)n} \right] + \frac{1}{2}\bar{\lambda}^{(a)}\gamma^m D_m \lambda^{(a)} \]

\[ + \frac{1}{2\alpha} \left[ D^m v^{(a)}_m \right]^2 - \bar{\eta}^{(a)} \Box \eta^{(a)} \]  

Here \( \alpha \) is the gauge parameter, and \( \eta^{(a)} \) and \( \bar{\eta}^{(a)} \) are a ghost and an anti-ghost, respectively. As we can find easily, even if we define that both the ghost and the anti-ghost are SUSY invariant, the gauge fixing term breaks the SUSY (accurately, SUSY in Wess-Zumino gauge) invariance of total Lagrangian (27). We know that the effective action (\( W[g_{mn}] \)) obtained from this Lagrangian is gauge parameter independent in Abelian case [10], however, so we could take the limit of \( \alpha \to \infty \). This means, if we take the limit which SUSY breaking gauge fixing term goes to zero in, then the renormalizable Lagrangian (27) is invariant under the SUSY-transformation.

After all, we must do is to sum up the contributions of massless vector fields with four degrees of freedom, massless Majorana spinor fields with two degrees of freedom, and a pair of minimal massless ghost and anti-ghost field.

For a vector field with a ghost and an anti-ghost (totally, a gauge field), the first two Seeley-DeWitt coefficients [7] are

\[ a_0 = 2 \]  

\[ a_1 = \frac{2}{3}R \]  

and for a Majorana spinor (gaugino) field

\[ a_0 = 2 \]
Finally we obtain the induced Einstein action from an SU(N) vector multiplet \( R \) which is equivalent to \((N^2 - 1)\) U(1) vector multiplets

\[
W_V = \int d^4x \sqrt{g} \frac{1}{2(4\pi)^2} \left[ -\frac{1}{2} \mathcal{R} \Lambda^2 \right] (N^2 - 1).
\]  

Then we find that with vector multiplets alone we always obtain negative Newton constant, which is opposite to scalar multiplet case.

5 Total Induced Einstein Action and Generations of Quarks, Leptons and Their Superpartners

Since we have no Einstein action at the original action, summing up the two kinds of contributions obtained in the previous two chapters, the total induced Einstein action is written as

\[
W = n_S W_S + n_V W_V
\]  

where \(n_S\) and \(n_V\) are non-negative integers. From this expression, again, we easily find that there is no contribution to the cosmological constant through the vacuum fluctuations of gauge and matter fields (including their superpartners). It just means SUSY, if it is global or local, works. However, the contribution to the curvature term is nonvanishing in general. It decides the induced Newton constant \(G_{\text{ind}}\).

Note that, while we studied theories with both gauge symmetries and SUSY unbroken in the previous chapter, even if we do the same with theories with gauge

\[\text{In this context we ignored the infrared divergences appearing as } m \to 0 \text{ limit of eqs.}\]
symmetry or SUSY spontaneously broken taking into account the couplings to other scalar multiplets, we obtain the same results. This is because the relation of mass squareds, which in general depend on classical fields, summed over all the species contained in supersymmetric Lagrangian:

\[
\text{Str} \ m^2 \equiv \sum_B E_{0B} m_B^2 - \sum_F E_{0F} m_F^2 = 0 \quad (34)
\]

where subscripts \(B\) and \(F\) denote all the kinds of bosons and fermions, respectively, holds even after spontaneous symmetry breaking \[9\]. In the above eq.(34) \(E_{0B}\) or \(E_{0F}\) is a coefficient appearing in eq.\((13)\). Then the above relation prevents mass dependent quadratically divergent term appearing\[8\] if gauge symmetry or SUSY is spontaneously broken or not.

We can easily find from eq.(33) there are two cases depending on the number of scalar multiplets and one of vector multiplets. Note that we mean a multiplet of a complex scalar and a Majorana spinor\[9\] by a scalar multiplet, and that of a gauge field with two polarizations and a Majorana spinor by a vector multiplet. For example, we count three for an SU(2) vector multiplet. So the critical ratio of the number of scalar multiplets to one of vector multiplets where the coefficient of Newton constant vanishes is three to one\[10\]. We investigate each of two cases in the following two paragraphs.

\[7\]In this paper we mean QED or SU(N) gauge theory by gauge theory.

\[8\] See eq.(13)

\[9\] For scalar multiplets we can use chiral spinors instead of Majorana ones

\[10\] We do not care about the case that the number of scalar multiplets is just three times as many as one of vector multiplets, where the quadratically divergent contribution to the Newton constant also vanishes and the logarithmically divergent one becomes dominant, because it is too accidental and the coefficient of the induced Einstein action depends on masses of fields contained in some underlying theory (17).
If the number of scalar multiplets is larger than three times as that of vector multiplets, then
\[ \frac{1}{16\pi G_{\text{ind}}} = \text{positive const.} \times \Lambda^2. \] (35)

With \( \Lambda = M_P \approx 10^{19} \text{GeV} \) we obtain the positive induced Newton constant with same order of magnitude as ordinary Newton constant.

If the number of scalar multiplets is smaller than three times as that of vector multiplets, then
\[ \frac{1}{16\pi G_{\text{ind}}} = \text{negative const.} \times \Lambda^2 \] (36)

In this case, we obtain negative induced Newton constant which means repulsive gravity contrary to Newton’s theory of gravity.

We next apply this result to the supersymmetric unified models of elementary particle. We pick up the two models which are hopeful candidates for ‘beyond the standard model’. One is minimal supersymmetric standard model (MSSM) and another is minimal supersymmetric SU(5) GUT (MS SU(5) GUT), which is the famous model as ‘beyond the standard model’ because of the precise unification of three forces at so called GUT scale[11]. So, MS SU(5) GUT is important as a beyond the MSSM.

5.1 Minimal supersymmetric standard model

First, we count the number of scalar multiplets and vector multiplets contained in MSSM [12].

For scalar multiplets,
• quarks and squarks: (a left-handed SU(2) doublet plus two right-handed SU(2) singlets) times three colors a generation,
• leptons and sleptons: a left-handed SU(2) doublet plus a right-handed SU(2) singlet a generation,
• Higgses and Higgsinos: two SU(2) doublets.

For vector multiplets,

• gauge fields and gauginos: an SU(3) octet plus an SU(2) triplet plus a U(1) gauge multiplet.

Totally, for \( N_g \) generations there are \((15N_g+4)\) scalar multiplets and 12 vector multiplets. For \( N_g = 3, 4, 5, \ldots \) the case (1) realizes, otherwise the case (2) does. So, if \( N_g \) is equal to or larger than three, then gravity is attractive definitly.

5.2 Minimal supersymmetric SU(5) GUT

Next, we do the same in MS SU(5) GUT[13].

For scalar multiplets,

• quarks, leptons and their superpartners: a 10 and a \( \bar{5} \) of SU(5) a generation,
• Higgses and Higgsinos: a 24 and 5 + \( \bar{5} \) of SU(5).

For vector multiplets,

• gauge fields and gauginos: a 24 of SU(5).
Numbers in bold face represent the dimensionality of representations under SU(5), so they just mean the number of scalar multiplets and vector multiplets. Totally, for $N_g$ generations there are $(15N_g+34)$ scalar multiplets and 24 vector multiplets. The case of MS SU(5) GUT leads to the same result as MSSM case.

Finally, we conclude that we need three or more generations of quarks, leptons and their superpartners in order to make gravity attractive. Of course, we can apply the general result to any other models. For example, the extension of the above models with right-handed neutrinos and their superpartners also need three generations or more to have an attractive gravity.

6 Conclusion

In this paper we investigated the induced Einstein gravity, or bosonic part of induced Einstein supergravity, in supersymmetric unified models in general space-time. We considered that the gravity is purely induced from the general covariant and supersymmetric unified model with auxiliary gravity multiplet, namely neither the graviton nor the gravitino is elementary but the composite object of quarks, leptons, gauge fields etc.. Then we found that the sign of induced Newton constant depends on the ratio of the number of scalar multiplets to that of gauge (vector) multiplets. Applying the result to MSSM and MS SU(5) GUT, we concluded that we need three or more generations of quarks and leptons and their superpartners in order to make gravity attractive.
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