Prognosis and Health Management using Energy Activity.

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Abstract: Accurate detection of faults in a dynamic system is very beneficial as this information can be used in a wide variety of ways by the machine operators or designers. This advantage becomes many folds when regarding the future condition i.e. time to failure, named remaining useful life, is available in addition to that of the present condition. Thus, prognosis is one of the most useful tools to improve the working of a machine as many critical decisions can be made. Prognosis can be critical for applications that risk loss of life and property. In this paper, a hybrid method, utilizing bond graph and artificial intelligence, is proposed for system health estimation (SHE) and prognosis. The Bond Graph model is used to calculate Energy Activity, which is used as a common metric for both SHE and prognosis. The proposed method is checked by simulation on a spring mass damper system undergoing a fault.

Keywords: Bond Graph, Prognosis and Health Management, Energy Activity, Maintenance

1. INTRODUCTION

Maintenance is one of the most important and one of the most under appreciated aspect for the proper operation of any system. In most of the industries, the maintenance strategy falls under one of the two approaches i.e. Preventive maintenance and Corrective maintenance. Under the preventive maintenance approach, maintenance action is implemented after some fixed intervals of time. The objective in such approach is to perform maintenance action before the occurrence of the failure itself. On the other hand, in corrective maintenance approach, the maintenance action is performed once the system has failed. Both the above mentioned approaches have certain disadvantages. While there is a high monetary cost associated with the preventive maintenance due to frequent maintenance actions, high cost is also associated to corrective maintenance approach on account of the loss in working time of machine under failure. Therefore, a new Condition Based Maintenance approach is gaining popularity. Under this approach, the operating conditions of the system are continuously monitored. Whenever a fault (not failure) is detected, i.e. system is in a degraded state but still in a functioning condition, further action is suggested by the monitoring system. Condition based maintenance is achieved using Prognostics and Health Management (PHM). The sequence of steps involved in PHM are shown in figure 1 and are in detail as follows (Atamuradov et al. [2017]):

• Signal acquisition: This step deals with extracting proper information from the system. It is performed by sensors which measure the variations in physical properties of the system (temperature, flow etc.).
• Signal Processing: Signal processing prepares the acquired signal for subsequent analysis or control. For this the signal is first cleaned by removing the noise and then features of the clean signal are extracted. These features are defined by the requirements of the analysis or control system.
• System health estimation (SHE): System health estimation groups the FDI procedures allowing to estimate the health state of the system (faulty or non faulty). These procedures consist in three steps.
  • Fault Detection: Fault detection investigates the consistency between the actual values of the system outputs provided by the sensors and the predicted values of these outputs obtained from the reference system model.
  A fault is detected as soon as this consistency, expressed on the form of mathematical expressions called residuals, is not respected. Equation 1 gives the general form of a residual where $Y_{measured}$ is the actual value of the system output Y and $Y_{estimated}$ is its estimated value predicted by the model or other reference tables.

\[
Residual = Y_{measured} - Y_{estimated} \quad (1)
\]

For example, in an vehicle transmission system, the r.p.m of the wheel can be measured for fault diagnosis. The residual can be fixed as the difference between the measured r.p.m and expected r.p.m (calculated from the provided input and model describing the behavior of machine). A near zero
residual indicates that the actual system behavior conforms to it’s expected behavior and hence, an absence of fault. However, if the residual is non-zero, this indicates the presence of a shift in the system behavior from expected, hence, the presence of a fault.

- **Fault Isolation**: Fault isolation consists in finding the faulty component using sensor information and, for examples, logic procedures, signal processing or reference tables.

  Continuing from the previous example, once it is confirmed that there is come fault in the system, the fault isolation is performed to pin point the location of the fault. The fault can be in any of the bearings, or shafts, or the gears.

- **Diagnosis**: Diagnosis gives an interpretation of the nature and the cause of the fault.

  Continuing from the previous example, suppose that the fault isolation process indicates the fault at the bearing, the fault diagnosis process indicates the nature of the fault. The fault in the bearing can be due to degradation, or crack, or misalignment.

- **Prognosis**: Prognosis is a dynamic estimate of the degradation of the system. This deals with calculation of the End of Life of a system, a point in time at which the fault increases to its maximum limit resulting in system failure. Remaining Useful Life (RUL) of the system is expressed by equation 2 where the predicted time where the system cannot continue to operate due to complete failure and is the time at which the RUL is calculated.

\[
\text{RUL} = t_{\text{failure}} - t_{\text{current}}
\]  

This RUL is represented on figure 2.

The RUL depends highly on the degradation model, which in-turn usually depends on the nature of the fault. However, it should also be noted that the prognosis indicator can be different from the residual used in SHE.

Continuing from the previous example, if the fault is diagnosed in a bearing due to the presence of a crack, then crack propagation model can be used to estimate the RUL.

- **Decision making**: Detecting the occurring fault and estimating the RUL of the system can help in both protecting the system components, the system environment and/or ensuring the continuity of service when possible.

  Decision making can range from immediate human intervention to implementation of fault tolerant control by putting in priority users safety measures, system protection and continuity of service.

Prognosis is the most important step of PHM and has attracted the attention from researchers all over the world. The quick and accurate prediction of RUL is important because the subsequent action after the appearance of fault depends on the RUL. The various approaches introduced for prognosis fall under one of the following:

1. **Model-based approach**: These approaches use either a deterministic or stochastic model of the system and physics based degradation models to perform prognostics. (Kordestani et al. [2019])
2. **Data-based approach**: These approaches use the existing data records and pattern recognition techniques for prognosis. (Zhong et al. [2019])
3. **Hybrid approach**: Many times the complete model is not known so to fill in the gaps in the model, data based techniques are used. Such an approach using both elements from both model based and data based approaches is called hybrid approach. (Liu et al. [2016])

Irrespective of the approach used for prognosis, certain challenges must always be addressed. These include the representation of degradation model and failure criteria in terms of the prognostic parameter. This is a big challenge because degradation model and failure criteria can be more easily set during the design stage of the system. However, the design stage deals only with numeric value of system parameters and not the prognostic parameter. Therefore, a prognostic parameter which can be express the degraded state in terms of system component values can be very useful.

Another challenge to the prognosis process arises due to the complex nature of systems themselves. Systems now a days are a complex amalgamation of different domains of physics like electrical, chemical, mechanical etc. These domains have different laws that govern their dynamics and require different prognostic parameters. For example, the State of Charge of battery (Hu et al. [2015],Dong et al. [2018]), a very common prognostic indicator an electro-chemical systems is of no use in a mechanical system. However, all of different domains of physics do follow the law of conservation of energy. Hence, energy is the common currency of exchange between the different domains. Therefore, a prognostic parameter based on energy is beneficial as it can be used in systems with any combination of domains. Energy based prognostic parameters are developed in (Jouin et al. [2016]). It was shown that power is a good indicator for systems under constant load, but for system under and known load, the cumulative energy, on account of it’s always increasing nature, is more suited for prognosis. A major drawback of the proposed metric is that it is calculated for a system a whole. The individual components are not differentiated and therefore, any know information about degradation model and failure criteria can not be exploited.

In addition to the challenges, an opportunity is observed from the nature of the PHM process itself. It is evident from the previous discussion about PHM that SHE and prognosis are closely related to each other. Therefore a common parameter for SHE and prognosis is desired. This is well recognised and attempts have been made to extend know SHE indicators for prognosis (Jha et al. [2016]). However, a fundamental difference between the two lies in the knowledge of initial conditions. In systems where only SHE is required, the initial conditions are usually considered as unknown. However, for systems incorporating prognosis, a full or partial knowledge of initial conditions or non-faulty conditions is assumed (Medjaher and Zerhouni [2009]). Therefore if eventual prognosis is to be performed, then the known initial conditions can be incorporated in the SHE process.
In this paper the above mentioned challenges and opportunity is addressed and a common parameter i.e. Energy Activity, is proposed for PHM, on account for the following:

- It works at component level and therefore can be used for with available degradation models and failure criteria.
- It is energy based and always increasing.
- It can exploit initial conditions at SHE stage.

This paper is organised as follows. Section 1 gives a basic introduction to Prognosis and Health Management and its various processes and challenges. In Section 2 the concept of Energy Activity is introduced and developed in the context of bond graph modelling. Section 3 gives the detailed methodology of the using energy activity for for prognosis and health management. In Section 4, the proposed method is simulated using a spring mass damper system and the simulation results are presented. The paper is concluded in Section 5.

2. ENERGY ACTIVITY FOR PROGNOSIS

2.1 Bond Graph

Bond Graph (BG) (Mukherjee et al. [2006]) is a technique based on the law of conservation of energy, used to model the dynamic behavior of systems.

In bond graph technique, power($p$) in every component is measured as a product of an agent bringing about the change, called generalised effort $e$ (electric potential, force, torque etc), and the rate of observed change (current, linear velocity, angular velocity etc), called generalised flow $f$. The power flow is represented by the bond and the positive sense of this power flow by a half arrow on the bond. The cross-stroke at one end of a bond (named causality) represents the cause effect relationship between variables (i.e. which unknown variables can be calculated from which known variables). This information is essential to generate the Analytical Redundancy Relations useful for FDI procedures Bouamama et al. [2003]. The components in the system can be either energy storing elements or energy dissipators. Components storing generalised kinetic and potential energy are represented as $I$ and $C$ elements respectively. The energy dissipators are represented as $R$ elements. The system can also have energy conversion elements i.e. Transformers $TF$ and Gyrators $GY$. The different components can interact with each other and the system in general, through either a 1 or 0 junction. Components sharing a common 1 junction have same generalised flow whereas those sharing common 0 junction have a common generalised effort. External input to the system can be either an effort source $SE$ or a flow source $SF$.

Figure 3 gives an exemple of Electrical system, the associated BG and examples of equations that can be deduced.

The constitutive relationships i.e. the equations governing the interaction of components with the system are given as:
The structural relationship at the junctions i.e. equations representing the structural relation between various elements are given as:

1 - junction \( e_1 - e_2 - e_3 - e_4 = 0 \)

\( f_2 = f_3 = f_4 \)

0 - junction \( f_4 - f_5 - f_6 = 0 \)

\( e_4 = e_5 = e_6 \)

As shown by the BG model, all dynamic components, in any dynamic system, irrespective of the domain interact with energy. The components can either absorb energy during certain phases and release the absorbed energy during the other. Some components just dissipate energy. The amount of energy that they interact with depends on the component values of the system. Component values are the numerical values of elements R, C, I, TF, GY of the BG. When a system is under fault, the component value, and hence the energy interaction for that of the system component changes. As energy can not be created or destroyed, the whole system experiences a redistribution of energy as a consequence of the fault. Also, for a system to work properly, every component should play its role in energy interaction within certain limits. Monitoring this energy interaction in the various components of the system can help in PHM. This is the reason why, we propose energy-based modeling metrics, named Energy Activity and Energy Activity Index to monitor energy interactions between components of a same system. As the BG explicitly shows energy exchanges between system components, it is a logical and convenient tool to evaluate energy activity.

2.2 Energy Activity

Energy Activity was introduced in Louca et al. [2010] as a tool for physical model reduction. EA is the total energy interaction that a component has with the system over a time period. The major difference between energy and energy activity is that while the energy associated with a component can increase or decrease, the energy activity of the component always increases with time. This makes energy activity a more suited component for prognosis than energy. The equations for energy and energy activity are given by equation 3 and 4 respectively. Here, \( P(t) \) is the power, \( e(t) \) is the generalized effort, \( f(t) \) is the generalized flow, \( \Delta t \) is the duration of the observation period beginning at time \( a \).

\[
\text{Energy} = \int_a^{a+\Delta t} P(t) \, dt \quad (3)
\]

\[
P(t) = e(t) \cdot f(t)
\]

Energy Activity Index of a component during a time interval is the ratio of the energy activity of that component to the total energy activity of the system during this time. Therefore, energy activity index analysis provides the relative picture of the activity of different components on the system. The expression for calculation of energy activity index of component \( i \) in a system with \( n \) components is given by equation 5.

\[
EAI_i = \frac{EA_i}{\sum_{i=1}^{n} EA_i} \quad (5)
\]

2.3 EA Computation

To calculate EA, power must be known. As shown in table 1, power can be calculated using the constitutive equations of bond graph elements, given in differential or integral causality and either the flow or effort information.

For more convenience, the general eq 6 is used as follows to calculate the power.

\[
P(t) = S(t).\phi.g(S(t)) \quad (6)
\]

In eq 6, \( S(t) \) is the known effort or flow variable received at the component input. The constitutive equation associated with the element is given by \( \phi.g(S(t)) \). The constitutive equation depends upon the component value \( \phi \) and the nature of the component (R, C or I), and allows to compute the unknown power variable (flow or effort) from the known power variable (effort of flow respectively).

From eq 6 the EA can be expressed by eq 7.

\[
EA = \int_a^{a+\Delta t} |S(t).\phi.g(S(t))| \, dt \quad (7)
\]

3. PHM PROCESS USING ENERGY ACTIVITY

A general overview of PHM process using Energy Activity is shown in figure 5.

3.1 Data Processing

This process includes the different steps shown in figure 1. A simulated system in fault-free conditions is run in parallel with the real system. Signal acquisition on the real system is performed using real sensors. Noise reduction is applied on sensed signals in order to provide clean measured outputs. In parallel, similar outputs are calculated from the simulated system. Calculated outputs are used to calculate reference EA and reference EAI. The de-noised measured outputs are used to calculate real EA and real EAI. Subsequently, fault detection is performed using the residuals obtained from the difference between reference and real EA.

3.2 Health Estimation

The process of filtering can modify signal phase & magnitude. This introduces error in the residuals. These errors
can be easily removed by analysing the residuals in frequency domain. As frequency of residual signal changes when system moves from a fault-free state to faulty state, residuals are analysed using Short Time Fourier Transformation technique. The results from Short Time Fourier transformation are compared with a pre-trained Neural Network to achieve fault isolation.

A fault is simulated by changing the value of a component parameter. Therefore to have an exhaustive database, a range of fault for every component is decided. A fault-free system is simulated in parallel with the faulty system. The generated residual is the difference of energy activity indexes from faulty and fault-free system. Fourier transformation is performed on the residuals to have a frequency picture of the residual. The location of the peaks is recorded in the frequency domain. The noisy peaks i.e. peaks at high frequency and low amplitude are removed. The remaining peaks are added to the neural network training data.

As discussed earlier, for fault isolation, Fourier transformation is performed on the residuals. However, in a real system, fault can occurs after some period of fault-free operation. Hence, the residuals change from zero to non-zero after some time. In such a case, a direct fourier transformation does not properly capture the change in behavior of residuals. Therefore, short time fourier transformation is used. In short time fourier transformation (Liu et al. [2016]), the residual signal is divided into small windows of equal duration, and subsequently fourier transformation is applied on it. The signal processing used for training the neural network is also applied to the results in the time-frequency map.

The window for short time fourier transformation should be equal to the simulation time used for training the neural network. This assures that the neural network is able to recognise the pattern properly. The window of short time fourier transformation should also be more than the time \( \Delta t \) used in equation 7 to calculate the energy activity.

### 3.3 Fault Prognosis

Once fault isolation is achieved, the mathematical form of Energy Activity can be used to calculate the real variation in the fault parameter, and furthermore the remaining useful life, assuming that the allowable limits of a parameter are known for failure.

In order to estimate the dynamics of the degradation (i.e. the time variation in the value of parameter), the time derivative of the EA is required.

As the fault (degradation) is due to a modification of the value of the component parameters in time, the Energy Activity for a faulty component can be expressed as a function of the component input signals, which itself is a function of the all the component parameters \( \phi \) (considered as time varying) and time \( t \). (See equation 8)

\[
EA = f(S(\phi), t) \quad (8)
\]

From equation 8 the following can be calculated

\[
dEA = \frac{\partial EA}{\partial S} \frac{\partial S}{\partial \phi} d\phi + \frac{\partial EA}{\partial t} dt \quad (9)
\]

\[
\frac{dEA}{dt} = \frac{\partial EA}{\partial S} \frac{\partial S}{\partial \phi} d\phi + \frac{\partial EA}{\partial t} \frac{\partial S}{\partial \phi} d\phi + \frac{\partial EA}{\partial t} dt \quad (10)
\]

- \( \frac{dEA}{dt} \) is the time derivative of the Energy Activity calculated from the real system.
- \( \frac{\partial EA}{\partial S} \) represents the variation of the Energy Activity only in time, i.e. due to no change in \( \phi \). This term can be calculated as the time derivative of the Energy Activity of the fault free system.
- \( \frac{\partial S}{\partial \phi} \) represents the variation of the input signal of the component due to the modification of component parameter value. As the dynamic model of the component is known as a pre-requisite this value can be calculated easily.
- \( \frac{\partial EA}{\partial \phi} \) depends on the nature of relation \( g \) in the equation 10.

Assume for example, that the constitutive equation for an R element is given by eq 11

\[
e(t) = R.f(t) \quad (11)
\]

The EA associated to this element can be calculated using eq 12.

\[
EA = \int_a^b |e(t).f(t)| dt = R \int_a^b f(t)^2 dt \quad (12)
\]

Using the general notations proposed be eq 8, eq 12 is written as eq 13
Therefore, for a generalised R element

$$\frac{\partial E_A}{\partial S} = R \int_{a}^{b} \frac{S^2}{S} dt = R \int_{a}^{b} S dt$$

Hence, the degradation rate of a faulty component, calculated as degradation rate of it’s component value $\phi$, can be expressed using both eq 10 and 14 as

$$\frac{\partial \phi}{\partial t} = \frac{\frac{\partial E_A}{\partial t} - \frac{\partial E_A}{\partial S}}{2R \int_{a}^{b} S dt \cdot \frac{\partial E_A}{\partial S}}$$

The eq 15 can be integrated as shown by eq 16 to estimate the evolution of component value (of the degrading component) over time, when the value of the said component is known in non-faulty condition ($\phi_0$).

$$\phi(t) = \int \frac{d\phi}{dt} dt + \phi_0$$

The continuous calculation can then be extrapolated according to a known degradation trend. If a degradation trend is unknown, then a polynomial equation can be used to extrapolate the component value. The Remaining Useful Life can be easily calculated if the safe limits of the component values are known beforehand (figure 2). The trend of component degradation is extrapolated to find the point in time when the component value reaches the allowable limit. This point is the End of Life. The time difference between present and end of life is the Remaining Useful Life.

4. APPLICATION

4.1 System

In order to illustrate the proposed methodology, a simulation is performed using a simple spring-mass-damper system. The system is shown in figure 6. The pre-requisites i.e. the dynamic model using bond graph, and the ideal & and safe working limits of components are given by figure 7, and table 2 respectively.

![Spring Mass Damper System](image)

Fig. 6. Spring Mass Damper System.

![Bond Graph of Spring Mass Damper System](image)

Fig. 7. Bond Graph of Spring Mass Damper System.

**Table 2. Ideal value of system components**

| Element               | Value in SI | Lower safe limit | Upper safe limit |
|-----------------------|-------------|------------------|-----------------|
| Force (F)             | 10          |                  |                 |
| Spring stiffness (k)  | 100         | 85               | 105             |
| Mass (M)              | 10          | 9                | 11              |
| Damping coefficient (b)| 0.5         | 0.4              | 0.6             |

4.2 Neural Network Training

The first step is to pre-train a neural network. The fault range for training the neural network are those given in...
Fault in Spring
Fault in Mass
Fault in Damper

Fig. 8. Neural Network Dataset.

Table 2. For creating the training data-set, the fault range for every component is divided into 20 equal intervals. The residual expressing the difference between the reference EA (calculated from the non faulty model) and the EA obtained in the faulty case is computed at the element i.e. mass. The frequency-amplitude graph used for training of neural network is shown in figure 8. The x-axis represents the frequency in Hz and the y-axis represents the amplitude. The cross marks of different colors represent the peaks observed at different magnitudes of faults. From the figures it is evident that peaks locations can provide a unique fault signature to the components and can be easily used to train a neural network. A default pattern recognition/classification algorithm provided in MATLAB is used with 5 hidden layers. 70% of the available database is used for training while 15% of database is used for validation and testing each.

4.3 Fault Isolation

During the simulation, a fault condition is indicated by deviation in the component value from ideal. For the purpose of the simulation, the fault magnitude is modelled as change in component value. Fault is introduced in the spring. The variation in spring stiffness is shown in figure 10. A fault is introduced at 5 seconds which continues to decrease the spring stiffness. At 50s from the start of simulation, a corrective action is simulated and spring stiffness starts to increase to recover its initial value at 100s. The Time-Frequency map obtained from the Short Time Fourier Transformation is applied on the obtained residual. The Time-Frequency map is shown in figure 9. The data entries corresponding to the each time interval are given as input to the neural network trained in the previous step. The neural network is able to correctly predict the fault location as spring.

4.4 Fault Prognosis

The equation 15 is used for evaluating the spring stiffness change rate. The change in spring stiffness $\dot{\phi}$ introduces a change in the damper input $S$, which affects the Energy Activity. The calculated stiffness change rate is passed through a median filter in order to remove sharp peaks due to numerical anomalies. The stiffness change rate after filtering is shown in figure 11. This change rate can be integrated to find the actual spring stiffness. The calculated variation of spring stiffness is shown in figure 12. The error in the calculated spring stiffness is shown in figure 13. From the figure it is evident that the spring stiffness is calculated with good accuracy.

At any time when the fault is observed the trend of the parameter variation can be extrapolated using a polynomial equation. For the current example a first order polynomial is used. The point of failure a.k.a. End of Life is reached when the extrapolation trend reaches the allowed limit of the component value. The Remaining Useful Life is continuously monitored. Once the corrective action is applied the calculation of Remaining Useful Life is continued. This represents the amount of time for which the corrective action can be applied before the component value overshoots the allowable limits. Calculation of End of Life is shown in figure 12.
5. CONCLUSION

In this paper a model based method for Prognosis and Health Management is proposed using Energy Activity. Both the diagnosis and prognosis processes are completed using variants of Energy Activity as a metric. Diagnosis is achieved by using a combination of Neural Network and Short Time Fourier Transformation. Given that the dynamic model of the system is known, the neural network is trained using fault simulations and does not require failure data. The prognosis process is completed using the mathematical nature of Energy Activity for energy dissipators. This can also be a limitation for the proposed process as the prognosis process can not utilize the energy storing elements. The proposed method is simulated for finding the end of life of a spring mass damper system undergoing a fault. The method is able to predict the fault location correctly and recreate the parameter values of the component under fault with good accuracy.

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