Baryon axial currents

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Abstract

The baryon axial currents are calculated at one–loop order in heavy baryon chiral perturbation theory by employing both a cutoff and dimensional regularization. Data from the semileptonic baryon decays is used to perform a least–squares fit to the two axial couplings \( D \) and \( F \) of the effective Lagrangian. Predictions for the momentum dependent axial form factors are made and the two different regularization schemes are compared. We also include the spin–3/2 decuplet in the effective theory.
1 Introduction

In a recent work [1] a cutoff regularization was introduced in the framework of SU(3) heavy baryon chiral perturbation theory. Therein, it was demonstrated that short distance effects, arising from propagation of Goldstone bosons over distances smaller than a typical hadronic size, are model-dependent and lead to a lack of convergence in the SU(3) chiral expansion when regularized dimensionally. The use of a cutoff removes such effects in a chirally consistent way and improves the convergence problems which arise from large loop effects. A simple dipole regulator was employed in that work, however, the specific form of the cutoff is irrelevant for the cases discussed there – a consistent chiral expansion can be carried out.

We will analyze the SU(3) breaking corrections to the baryon axial currents both by employing dimensional regularization and a cutoff. First, this calculation serves as a comparison between both regularization schemes. In addition, this generalizes other investigations carried out in dimensional regularization [2, 3]. We will present the results for all three momentum-dependent axial form factors. So far, only the leading nonanalytic corrections to one of the three axial form factors at zero momentum transfer were calculated in the limit of vanishing light quark masses $m_u = m_d = 0$. The chiral logarithmic corrections to the axial currents were as big as the lowest order values, and the authors came to the conclusion that the axial couplings $D$ and $F$ of the lowest order effective Lagrangian cannot be reliably extracted from hyperon semileptonic decays [3].

Another complication arises from the closeness of the spin–3/2 decuplet resonances which are separated only by 231 MeV in average from the octet baryons which is considerably smaller than the kaon and the η mass. These resonances are, therefore, expected to play an important part at low energies. It has been suggested [4] to include the decuplet explicitly. There, it is shown that the spin–3/2 decuplet partially cancels the large spin–1/2 octet contribution. The average octet–decuplet mass splitting was chosen to be $\Delta = 0$ MeV. One has to reinvestigate this topic in the cutoff scheme and also the impact of the parameter $\Delta$ to the decuplet contributions.

The present work is organized as follows. In the next Section the definitions for the axial form factors in the framework of heavy baryon chiral perturbation theory are presented and related to the conventional relativistic formulation. Results for the three axial form factors are shown. Sec. 3 deals with the inclusion of the decuplet. A least–squares fit for $D$ and $F$ is performed in Sec. 4. The chiral expansions of the hyperon semileptonic decays are presented. We conclude with a summary in Sec. 5. The computation of the integrals is relegated to the Appendix.

2 Axial form factors

The hadronic axial current for the decay $B_i \rightarrow B_j \bar{l} \nu_l$ can be written in the form

$$< B_j | A_\mu | B_i > = \bar{u}(p_j) \left( g_1(q^2) \gamma_\mu \gamma_5 - \frac{i g_2(q^2)}{M_i + M_j} \sigma_{\mu\nu} q^\nu \gamma_5 + \frac{g_3(q^2)}{M_i + M_j} q_\mu \gamma_5 \right) u(p_i),$$

where $q = p_i - p_j$ is the momentum transfer. The form factor $g_3$ is usually neglected since after contraction with the leptonic current it is multiplied by a small lepton mass and therefore difficult to observe. In this work, we calculate the three form factors in the framework of heavy baryon chiral perturbation theory both using dimensional regularization and in a momentum dependent cutoff scheme.
The pseudoscalar Goldstone fields \((\phi = \pi, K, \eta)\) are collected in the \(3 \times 3\) unimodular, unitary matrix \(U(x)\),

\[
U(\phi) = u^2(\phi) = \exp\{2i\phi/\hat{F}\}
\]  

with \(\hat{F}\) being the pseudoscalar decay constant (in the chiral limit), and

\[
\phi = \frac{1}{\sqrt{2}} \left( \begin{array}{ccc}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^+ & K^+ \\
\pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & K^0 \\
K^- & K^0 & -\frac{2}{\sqrt{6}} \eta
\end{array} \right)
\]  

Under \(SU(3)_L \times SU(3)_R\), \(U(x)\) transforms as \(U \rightarrow U' = LUR^\dagger\), with \(L, R \in SU(3)_{L,R}\). One forms an object of axial–vector type with one derivative

\[
\nabla_{\mu}U = \partial_{\mu}U - ia_{\mu}U - iUa_{\mu}
\]  

with \(\nabla_{\mu}\) being the covariant derivative of \(U\) and \(a_{\mu}\) generates the Green functions of the axial current. The matrix \(B\) denotes the baryon octet,

\[
B = \left( \begin{array}{ccc}
\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & \Sigma^+ & p \\
\Sigma^- & -\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & n \\
\Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}} \Lambda
\end{array} \right)
\]  

The matrices \(u_{\mu}\) and \(B\) transform under \(SU(3)_L \times SU(3)_R\) as any matter field, \(e.g.,\)

\[
B \rightarrow B' = K B K^\dagger
\]

with \(K(U, L, R)\) the compensator field representing an element of the conserved subgroup \(SU(3)_V\).

In the heavy baryon formulation the baryons are described by a four–velocity \(v_{\mu}\) and relativistic corrections appear as \(1/\hat{M}\) corrections where \(\hat{M}\) is the average octet baryon mass in the chiral limit. A consistent chiral counting scheme emerges, \(i.e.,\) a one–to–one correspondence between the Goldstone boson loops and the expansion in small momenta and quark masses. To this end, one constructs eigenstates of the velocity projection operator \(P_v = (1 + \eta)/2\)

\[
B_v(x) = e^{i\hat{M}v \cdot x} P_v B(x)
\]  

The Dirac algebra simplifies considerably. It allows to express any Dirac bilinear \(\bar{B}_v \Gamma_\mu B_v (\Gamma_\mu = 1, \gamma_\mu, \gamma_5, \ldots)\) in terms of the velocity \(v_{\mu}\) and the spin operator \(2S_{\mu} = i\gamma_5 \sigma_{\mu\nu} v^\nu\). The latter obeys the relations (in \(d\) space–time dimensions)

\[
S \cdot v = 0, \quad S^2 = \frac{1}{4} - \frac{d}{4},\quad \{S_{\mu}, S_{\nu}\} = \frac{1}{2}(v_{\mu}v_{\nu} - g_{\mu\nu})\quad \text{[}S_{\mu}, S_{\nu}\text{]} = i\epsilon_{\mu\nu\alpha\beta}v^\alpha S^\beta
\]  

Using the convention \(\epsilon^{0123}_+ = 1\), one can rewrite the Dirac bilinears as:

\[
\begin{align*}
\bar{B}_v \gamma_\mu B_v & = v_{\mu} \bar{B}_v B_v, \quad \bar{B}_v \gamma_5 B_v = 0, \quad \bar{B}_v \gamma_\mu \gamma_5 B_v = 2\bar{B}_v S_{\mu} B_v, \\
\bar{B}_v \sigma_{\mu\nu} B_v & = 2\epsilon_{\mu\nu\alpha\beta} v^\alpha \bar{B}_v S^\beta B_v, \quad \bar{B}_v \gamma_5 \sigma_{\mu\nu} B_v = 2i(v^\mu \bar{B}_v S^\nu B_v - v^\nu \bar{B}_v S^\mu B_v)
\end{align*}
\]
In the following, we will drop the index $v$. In the heavy baryon formulation the hadronic axial current can be decomposed into

$$< B_j | A_\mu | B_i > = \tilde{H}(q_j) \left( G_1(q^2) S_\mu + \frac{G_2(q^2)}{M_i + M_j} v_\mu S \cdot q + \frac{G_3(q^2)}{(M_i + M_j)^2} q_\mu S \cdot q \right) H(q_i) \quad , \quad (10)$$

with $2H = (1 + q') u$ being the large–component field and $q_k = p_k - \hat{M} v$ the baryon off–shell momenta in the heavy baryon formulation. We prefer to work in the rest frame of the heavy baryon and with $v_\mu = (1,0,0,0)$. The momenta of the baryons are then given by

$$p_i = \hat{M} v + q_i = M_i v , \quad p_j = \hat{M} v + q_j = M_j v - q . \quad (11)$$

In this frame the $G_i$ are related to the $g_i$ via

$$G_1(q^2) = 2g_1(q^2) + \frac{g_2(q^2)}{M_i + M_j} \left( -v \cdot q + \frac{(v \cdot q)^2 - q^2}{M_j + E_j} \right)$$

$$G_2(q^2) = 2 \frac{M_j + M_i}{M_j + E_j} g_1(q^2) + 2g_2(q^2) \left( 1 - \frac{v \cdot q}{M_j + E_j} \right)$$

$$G_3(q^2) = 2 \frac{M_j + M_i}{M_j + E_j} \left( g_2(q^2) - g_3(q^2) \right) \quad , \quad (12)$$

with

$$E_j = M_i - v \cdot q = M_i - \frac{1}{2M_i} \left( q^2 + M_i^2 - M_j^2 \right) \quad (13)$$

being the energy of the outgoing baryon. There are contributions from $g_2$ to $G_1$ which have been neglected in previous works [3, 4].

The Lagrangian can be decomposed into a purely mesonic part $\mathcal{L}_\phi$ and a piece $\mathcal{L}_{\phi B}$ in which the pseudoscalar Goldstone bosons are coupled to the baryon fields

$$\mathcal{L} = \mathcal{L}_{\phi B} + \mathcal{L}_\phi \quad . \quad (14)$$

For the mesonic part one has [3]

$$\mathcal{L} = \frac{\bar{\phi}^2}{4} \text{tr}[u_\mu u^\mu] + \frac{\bar{\phi}^2}{4} \text{tr}[\chi_+] \quad , \quad (15)$$

with $\chi_+ = 2B_0(u^\dagger M u^\dagger + u M u)$ being proportional to the quark mass matrix $M = \text{diag}(m_u, m_d, m_s)$. Also, $B_0 = - \langle 0 | \bar{q} q | 0 \rangle / \bar{\phi}^2$ is the order parameter of the spontaneous symmetry violation, and we assume $B_0 \gg \bar{\phi}$. In the following we will work in the isospin limit $m_u = m_d$.

The lowest order baryon meson Lagrangian $\mathcal{L}_{\phi B}^{(1)}$ includes the two axial–vector couplings $D$ and $F$

$$\mathcal{L}_{\phi B}^{(1)} = i \text{tr} \left( B [v \cdot D, B] \right) + D \text{tr} \left( B S_\mu \{ u^\mu, B \} \right) + F \text{tr} \left( B S_\mu \{ u^\mu, B \} \right) \quad , \quad (16)$$

and the superscript denotes the chiral order. The covariant derivative $D_\mu$ on the baryon fields includes the external gauge field $a_\mu$

$$[D_\mu, B] = \partial_\mu B + [\Gamma_\mu, B]$$

$$\Gamma_\mu = \frac{1}{2} [u^\dagger, \partial_\mu u] - \frac{i}{2} (u^\dagger a_\mu u - u a_\mu u^\dagger) \quad (17)$$
with $\Gamma_\mu$ the so-called chiral connection. If only this effective Lagrangian would be used to compute the chiral corrections in the effective theory, there would be no contributions to the form factor $G_2$. From the second equation in (12) this would lead to $g_1[\pi n](0) = -g_2(0)[\pi n] \simeq 1.26$ for the decay $n \rightarrow p$. This is in sharp contradiction with the value $g_2[\pi n](0) = 0$ assuming $G$–parity. Therefore, we will also include the terms from the baryon meson Lagrangian at second chiral order that contribute at tree level to the form factors

$$L_{\phi B}^{(2)} = \frac{iD}{2M}\text{tr}\left([D^\mu, \bar{B}]S_\mu\{v \cdot u, B\}\right) + \frac{iF}{2M}\text{tr}\left([D^\mu, \bar{B}]S_\mu[v \cdot u, B]\right)$$

$$- \frac{iD}{2M}\text{tr}\left(\bar{B}S_\mu\{v \cdot u, [D^\mu, B]\}\right) - \frac{iF}{2M}\text{tr}\left(\bar{B}S_\mu[v \cdot u, [D^\mu, B]]\right).$$

These are relativistic corrections to the lowest order Lagrangian and the only terms contributing to the axial currents at this order. In the initial–baryon restframe one has the relation $S \cdot q_1 = 0$ and the last two terms do not contribute.

Corrections from the Goldstone boson loops contribute at third chiral order together with counterterms from the Lagrangian $L_{\phi B}^{(3)}$. The Lagrangian $L_{\phi B}^{(3)}$ contains numerous counterterms which contribute to the axial form factors [3]. Performing the calculation with the complete Lagrangian up–to–and–including one–loop order one has, of course, no predictive power. The theoretical predictions contain considerably more low–energy constants than there are experimental results. One can resort to model dependent estimations of these constants, e.g. via resonance saturation. This method works very accurately in the meson sector [7], but in the baryon case it gives only a rough estimate of the LECs and there is still a sizeable uncertainty in these parameters [8]. Since such LECs renormalize the part of the loops analytic in the quark masses, no reliable estimate can be given for the analytic pieces at this order. We will therefore restrict ourselves to the nonanalytic pieces of the one–loop integrals and completely neglect the local counterterms at this order.

The leading nonanalytic corrections from the Goldstone boson loops are usually evaluated by using dimensional regularization [2, 3, 4]. The nonanalytic part of a typical integral in this analysis has for the case of zero momentum transfer and $d$ dimensions the form

$$\int \frac{d^dl}{(2\pi)^d} \frac{i^3 (S \cdot l)^2}{[l^2 - m_\phi^2 + i\epsilon][v \cdot l + i\epsilon]^2} = -\frac{3}{64\pi^2} m_\phi^2 \ln \frac{m_\phi^2}{\lambda^2},$$

where $m_\phi$ is the meson mass and $\lambda$ the scale introduced in dimensional regularization. The integral grows with increasing meson mass. We expect the long distance portion of the integral to be larger for small meson masses since for small momenta the meson propagator can be approximated by $1/m_\phi^2$. This indicates that in the dimensionally regularized integral there are significant contributions from short distance physics which cannot be described appropriately by chiral symmetry. Therefore, one has to employ other regularization schemes that emphasize long distance effects of the integrals and reduce short distance contributions. In [1] it was shown that a simple dipole regulator fulfills these requirements.

For the evaluation of the Goldstone boson loops in the cutoff scheme we will employ the dipole regulator

$$R = \left(\frac{\Lambda^2}{\Lambda^2 - l^2}\right)^2,$$
where \( l \) is the loop momentum. Inserting this regulator into the integral in Eq. (19) leads to

\[
I_\Lambda = \int \frac{d^4l}{(2\pi)^4} \frac{i^3 (S \cdot l)^2}{[l^2 - m_\phi^2 + i\varepsilon][v \cdot l + i\varepsilon]^2} \left( \frac{\Lambda^2}{\Lambda^2 - l^2} \right)^2
\]

\[
= -\frac{3}{64\pi^2} \frac{\Lambda^4}{(\Lambda^2 - m_\phi^2)^2} \left( \Lambda^2 - m_\phi^2 + m_\phi^2 \ln \frac{m_\phi^2}{\Lambda^2} \right). 
\]

(21)

The introduction of the additional scale \( \Lambda \) spoils the one-to-one correspondence between the meson loops and the expansion in the quark masses and the integral depends strongly on the value of the cutoff \( \Lambda \). However, this does not mean that to the order we are working the resulting physics will depend on \( \Lambda \), since one is able to absorb the effects of \( \Lambda \) into a renormalization of the couplings \( D \) and \( F \) as we will show later.

We can now proceed in writing down the results both for the case of dimensional regularization and the regularization with the cutoff. Note, that in [2, 3] only the leading nonanalytic pieces for the form factor \( g_1(0) \) at zero momentum transfer have been calculated.

At tree level the contributing diagrams are shown in Fig. 1. The diagrams 1.a and 1.b contribute to \( G_1, G_2 \) and \( G_3 \), respectively. The coefficients for \( G_1 \) read

\[
G_1^{+12}[pn] = \alpha_{pn} = 2(D + F)
\]

\[
G_1^{+12}[\Lambda \Sigma^-] = \alpha_{\Lambda \Sigma^-} = \frac{4}{\sqrt{6}} D
\]

\[
G_1^{+12}[\Xi^0 \Xi^-] = \alpha_{\Xi^0 \Xi^-} = 2(D - F)
\]

\[
G_4^{+15}[\mu \Lambda] = \alpha_{\mu \Lambda} = -\frac{2}{\sqrt{6}}(D + 3F)
\]

\[
G_4^{+15}[\Lambda \Xi^-] = \alpha_{\Lambda \Xi^-} = -\frac{2}{\sqrt{6}}(D - 3F)
\]

\[
G_4^{+15}[n \Sigma^-] = \alpha_{n \Sigma^-} = 2(D - F)
\]

\[
G_4^{+15}[\Xi^0 \Xi^-] = \alpha_{\Xi^0 \Xi^-} = \sqrt{2}(D + F) = \frac{1}{\sqrt{2}} G_1^{+12}[\Sigma^+ \Xi^0]. 
\]

(22)

For \( G_2 \) one obtains contributions only from \( \mathcal{L}^{(2)}_{\phi B} \) with

\[
G_2[ij] = G_1[ij],
\]

where we replaced the appearing prefactor \( (M_i + M_j)/\hat{M} \) in each of the decays by 2 which is consistent to the order we are working. At tree level we obtain from Eq. (12), e.g., \( g_2[pn](0) = 0 \) which is consistent with the assumption of \( G \)-parity. The relativistic corrections at next-to-leading order in the baryon meson Lagrangian play an important role for the form factors \( g_2 \).

For \( G_3 \) the results read

\[
G_3[ij](q^2) = \frac{(M_i + M_j)^2}{m_\phi^2 - q^2} G_1[ij]
\]

(24)

where \( m_\phi = m_\pi \) for the decays \( [ij] = [pn], [\Lambda \Sigma^-], [\Xi^0 \Xi^-] \) and \( m_\phi = m_K \) otherwise. Here, we replaced the lowest order expression for the meson masses in the propagator by their physical value. The difference shows up at second chiral order for the form factors, i.e. the same order as the loop contributions, and can be incorporated into these, as we will show. We also neglected
the contributions from $\mathcal{L}^{(2)}_{\phi B}$ to $G_3$ since they are proportional to $v \cdot q$ and amount to analytical corrections at loop order after employing Eq. (13). It is these forms for $G_1$ which are used in SU(3) fits to hyperon beta decay.

The loop contributions for $G_1(q^2)$ are depicted in Fig. 2. They have the form

$$\delta G_1[ij](q^2) = \frac{1}{\Lambda \chi} \sum_{\phi=\pi, K, \eta} \left( \left[ \beta_{ij}^\phi - \frac{1}{2} \alpha_{ij} (\lambda_i^\phi + \lambda_j^\phi) \right] I^\phi(0) + \gamma_{ij}^\phi I^\phi \left( -\frac{q^2}{2M_i^2} \right) \right) + \alpha_{ij} \delta_j(q^2) \quad (25)$$

with $\Lambda \chi = 4\pi F_\pi$ and we have also included the wavefunction renormalization factors of the external baryons proportional to $\alpha_{ij}$. Furthermore, one has to account for the contributions of the heavy components of the external baryons to their $Z$-factors, see [9]. In the rest frame of the heavy baryon they vanish for the decaying baryon. For the light baryon with the mass $M_j$ we get a term $\delta_j$ which is to lowest order proportional to $q^2/(4M_j^2)$. Also, $\hat{F}$ has been replaced by $F_\pi$ which is consistent to the order we are working, and $I^\phi$ is the integral appearing in the calculation. The pertinent coefficients read

$$\begin{align*}
\beta_{pm}^\pi &= -2(D + F) \quad , \quad \beta_{pm}^K = -(D + F) \quad , \quad \beta_{pm}^\eta = 0 \\
\beta_{p\Lambda}^\pi &= \frac{\sqrt{6}}{8} (D + 3F) \quad , \quad \beta_{p\Lambda}^K = \frac{\sqrt{6}}{4} (D + 3F) \quad , \quad \beta_{p\Lambda}^\eta = \frac{\sqrt{6}}{8} (D + 3F) \\
\beta_\Lambda^{\pi \Sigma -} &= -\frac{4}{\sqrt{6}} D \quad , \quad \beta_\Lambda^{K \Sigma -} = -\frac{2}{\sqrt{6}} D \quad , \quad \beta_\Lambda^{\eta \Sigma -} = 0 \\
\beta_{\Xi^- \Xi^-}^{\pi \Xi^-} &= -2(D - F) \quad , \quad \beta_{\Xi^- \Xi^-}^{K \Xi^-} = -(D - F) \quad , \quad \beta_{\Xi^- \Xi^-}^{\eta \Xi^-} = 0 \\
\beta_{n \Sigma -}^{\pi \Sigma -} &= -\frac{3}{4} (D - F) \quad , \quad \beta_{n \Sigma -}^{K \Sigma -} = -\frac{3}{2} (D - F) \quad , \quad \beta_{n \Sigma -}^{\eta \Sigma -} = -\frac{3}{4} (D - F) \\
\beta_{\Xi^0 \Xi^-}^{\pi \Xi^-} &= \frac{\sqrt{6}}{8} (D - 3F) \quad , \quad \beta_{\Xi^0 \Xi^-}^{K \Xi^-} = \frac{\sqrt{6}}{4} (D - 3F) \quad , \quad \beta_{\Xi^0 \Xi^-}^{\eta \Xi^-} = \frac{\sqrt{6}}{8} (D - 3F) \\
\beta_{\Sigma^0 \Xi^-}^{\pi \Xi^-} &= -\frac{3\sqrt{2}}{4} (D + F) \quad , \quad \beta_{\Sigma^0 \Xi^-}^{K \Xi^-} = -\frac{3\sqrt{2}}{4} (D + F) \quad , \quad \beta_{\Sigma^0 \Xi^-}^{\eta \Xi^-} = -\frac{3\sqrt{2}}{8} (D + F) \\
\beta_{\Sigma^0 \Xi^-}^{\phi \Xi^-} &= \frac{\sqrt{2}}{8} \beta_{\Sigma^0 \Xi^-}^{\pi \Xi^-} \quad . \quad (26)
\end{align*}$$

$$\gamma_{pm}^\pi = \frac{1}{2} (D^3 + F^3 + 3D^2 F + 3F^2 D) \quad , \quad \gamma_{pm}^K = \frac{2}{3} D^3 - \frac{2}{3} F D^2 + 2D F^2 - 2F^3 \quad ,$$

$$\gamma_{pm}^\eta = -\frac{1}{6} D^3 + \frac{5}{6} F D^2 - \frac{1}{2} D F^2 - \frac{3}{2} F^3 \quad , \quad \gamma_{p\Lambda}^\pi = \frac{1}{\sqrt{6}} (-3D^3 + 3D F^2) \quad ,$$

$$\gamma_{p\Lambda}^K = \frac{1}{\sqrt{6}} \left( \frac{5}{3} D^3 - 5D F^2 - 3F^2 D + 9F^3 \right) \quad , \quad \gamma_{p\Lambda}^\eta = \frac{1}{\sqrt{6}} \left( \frac{1}{3} D^3 - 3D F^2 \right) \quad ,$$

$$\gamma_{\Lambda \Sigma -}^\pi = \frac{4}{\sqrt{6}} \left( \frac{1}{3} D^3 + 2DF^2 \right) \quad , \quad \gamma_{\Lambda \Sigma -}^K = \frac{2}{\sqrt{6}} (D^3 - DF^2) \quad ,$$

$$\gamma_{\Lambda \Sigma -}^\eta = \frac{4}{3\sqrt{6}} D^3 \quad , \quad \gamma_{\Xi^- \Xi^-}^{\Xi^- \Xi^-} = \frac{1}{2} (D^3 - F^3 - 3D^2 F + 3F^2 D) \quad ,$$

$$\gamma_{\Xi^- \Xi^-}^{K \Xi^-} = \frac{2}{3} D^3 + \frac{2}{3} F D^2 + 2D^2 F + 2F^3 \quad , \quad \gamma_{\Xi^- \Xi^-}^{\eta \Xi^-} = -\frac{1}{6} D^3 - \frac{5}{6} F D^2 - \frac{1}{2} D F^2 + \frac{3}{2} F^3 \quad ,$$

$$\gamma_{n \Sigma -}^\pi = \frac{1}{3} D^3 - \frac{2}{3} D^2 F + DF^2 + 2F^3 \quad , \quad \gamma_{n \Sigma -}^K = F^3 + DF^2 + \frac{1}{3} D^2 F + \frac{1}{3} D^3 \quad ,$$

$$\gamma_{n \Sigma -}^\eta = \frac{1}{3} D^3 - \frac{2}{3} D^2 F + DF^2 + 2F^3 \quad , \quad \gamma_{n \Sigma -}^\eta = F^3 + DF^2 + \frac{1}{3} D^2 F + \frac{1}{3} D^3 \quad ,$$

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mass depend on the cutoff $\Lambda$. To this end, one expands the result in Eq.\((30)\) in terms of the meson momentum transfer squared $q^2$ consistent to the order we are working. Then, the integral reads in the cutoff scheme for zero $\delta_j$ have the form

$$
\delta_j = -\frac{q^2}{8M_j^2} .
$$

In dimensional regularization we obtain for the nonanalytic part of the integral

$$
I^\phi_\text{dim}(q^2 = 0) = m^2_\phi \ln \frac{m^2_\phi}{\Lambda^2} .
$$

The more general cases for $q^2 = 0$ can be found in App. A.

As we mentioned above, the integral in Eq.\((30)\) depends on $\Lambda$. But the physics does not depend on the cutoff $\Lambda$. To this end, one expands the result in Eq.\((30)\) in terms of the meson mass $m_\phi$

$$
I^\phi_{m_\phi^2 << \Lambda^2} \rightarrow \Lambda^2 + m^2_\phi \ln \frac{m^2_\phi}{\Lambda^2} + \ldots
$$

The second term delivers the nonanalytic contribution in dimensional regularization. The contributions quadratic in $\Lambda$ can be absorbed into renormalizations of the lowest order axial couplings $D$ and $F$ via

$$
\begin{align*}
D^r &= D - \frac{3}{2} D (3D^2 + 5F^2 + 1) \frac{\Lambda^2}{16\pi^2 F^2}, \\
F^r &= F - \frac{1}{6} F (25D^2 + 63F^2 + 9) \frac{\Lambda^2}{16\pi^2 F^2} .
\end{align*}
$$
Since such coefficients are determined empirically the analysis with small meson masses becomes identical to that of the dimensionally regularized case. That the renormalization can occur involves a highly constrained set of conditions and the fact that they are satisfied is a significant verification of the chiral invariance of the cutoff procedure.

When employing any regularization scheme that introduces a dimensionful parameter, the usual power counting will be upset. This is manifest in the results quoted above, in which the lower order chiral parameters, $D$ and $F$, are shifted by the loop correction. However, since these shifts are just the renormalization of phenomenological parameters, they do not influence the physics. One can use the small mass limit to set up the chiral expansion. In this limit the loops will renormalize the chiral parameters and the power counting for the order of the residual loop effects remains the same as in the standard regularization, so that the same loop diagrams should be included to a given order. Taking the meson masses to their physical values, which are in general not small compared to the cutoff, the short distance parts of the loops will be discarded. In the calculation this amounts to a partial resummation of higher order terms in the chiral expansion, i.e. higher powers of $m_\phi/\Lambda$.

We prefer to remove the asymptotic mass–independent component of the integral $I$ by setting

$$I = I - \Lambda^2$$  \hspace{1cm} (34)$$

and redefining the axial couplings $D$ and $F$. In the following we will work with the renormalized values of $D$ and $F$ and neglect the superscript $r$. The leading chiral corrections from the loops read then in the cutoff scheme

$$\delta G_1[ij](q^2) = \frac{1}{\Lambda^2} \sum_{\phi=\pi,K,\eta} \left( \frac{5}{3} \beta_{ij}^\phi - \frac{1}{2} \alpha_{ij}(\lambda_i^\phi + \lambda_j^\phi) \right) \tilde{I}^{\phi}(0) + \gamma_{ij}^\phi \tilde{I}^{\phi}\left(\frac{-q^2}{2M_i}\right) + \alpha_{ij} \delta_j(q^2) \ . \hspace{1cm} (35)$$

In order to keep the formulae compact we use the same notation in the case of dimensional regularization with

$$\tilde{I}^{\phi}_{\text{dim}} = I^{\phi}_{\text{dim}} \ . \hspace{1cm} (36)$$

The contributing loop diagrams to the form factor $G_3$ are shown in Fig. 3. The graph 3.d leads to the meson $Z$–factor and the renormalization of the meson mass in the propagator of the tree diagram 1.b. Due to this diagram we used the physical value of the meson mass at tree level. Here, we neglect the analytic corrections to the meson mass from the counterterms at fourth chiral order of the mesonic Lagrangian. In our case, their contribution to the form factors turns out to be negligible. One ends up with a similar form for the form factors $G_3$

$$\delta G_3[ij](q^2) = \frac{(M_i + M_j)^2}{m_\phi^2 - q^2} \left[ \frac{1}{\Lambda^2} \sum_{\phi=\pi,K,\eta} \left( \frac{5}{3} \beta_{ij}^\phi - \frac{1}{2} \alpha_{ij}(\lambda_i^\phi + \lambda_j^\phi) \right) \tilde{I}^{\phi}(0) + \gamma_{ij}^\phi \tilde{I}^{\phi}\left(\frac{-q^2}{2M_i}\right) \right.
\left. + \alpha_{ij} \left( \delta_j(q^2) + \zeta_{\tilde{\phi}} \right) \right] , \hspace{1cm} (37)$$

with $\tilde{\phi} = \pi$ for the decays $[ij] = [pn], [\Lambda\Sigma^-], [\Xi^0\Xi^-]$ and $\tilde{\phi} = K$ otherwise. The term $\zeta_{\tilde{\phi}}$ is due to the meson $Z$–factor and reads

$$\zeta_{\pi} = \frac{1}{96\pi F_\pi^2} \left( 4\tilde{I}_{\pi}(0) + 2\tilde{I}_{K}(0) \right)$$
$$\zeta_{K} = \frac{1}{96\pi F_\pi^2} \left( \frac{3}{2} \tilde{I}_{\pi}(0) + 3\tilde{I}_{K}(0) + \frac{3}{2} \tilde{I}_{\eta}(0) \right) \ . \hspace{1cm} (38)$$
There are no loop contributions to $G_2$ at the order we are working. Before presenting the numerical results we will include the decuplet in the next section.

## 3 Inclusion of the decuplet

In general it is assumed that baryon resonance states are much heavier compared to the lowest-lying baryon octet. In this case they can be integrated out and replaced by counterterms that do not include these resonance states explicitly. However, while this might be a reasonable procedure for heavier resonances like the Roper-octet, it is a questionable assumption for the decuplet. The low-lying decuplet is separated from the octet by only $\Delta = 231$ MeV in average which is much smaller than the $K$ or the $\eta$ mass. Furthermore, the $\Delta(1232)$ couples strongly to the $\pi N$ sector and its contribution plays an important role in the channels wherein this effect is possible. In the meson sector, the first resonance is the vector meson $\rho$ with a mass of 770 MeV which is considerably heavier than the Goldstone bosons. It was therefore argued in \[3\] to include the spin-3/2 decuplet as explicit degrees of freedom. In the framework of conventional heavy baryon CHPT it was shown that the spin-3/2 decuplet partially cancels the large spin-1/2 octet contribution for the form factor $g_1(0)$\[4\]. This calculation was performed in the $SU(6)$ limit by neglecting the octet-decuplet mass splitting $\Delta$ and also, $m_u = m_d = 0$ was assumed.

In this section we will include the decuplet fields both in dimensional regularization and in the cutoff scheme. We keep the average octet-decuplet mass splitting $\Delta$ at its physical value.

The pertinent interaction Lagrangian between the spin-3/2 fields – denoted by the Rarita-Schwinger fields $T^\mu$ –, the baryon octet and the Goldstone bosons reads

$$L_{\phi BT} = -i \bar{T}^\mu v \cdot DT^\mu + \Delta \bar{T}^\mu T^\mu + \frac{C}{2} \left( \bar{T}^\mu u_\mu B + \bar{B} u_\mu T^\mu \right) + H \bar{T}^\mu S_\nu u^\nu T^\mu$$ \hspace{1cm} (39)

where we have suppressed the flavor $SU(3)$ indices. In the heavy mass formulation the fields $T^\mu$ satisfy the condition $v \cdot T = 0$. The coupling constant $C = 1.2\ldots1.8$ can be determined from the strong decays $T \rightarrow B\pi$. The parameter $H$ is not determined experimentally. Thus a fit to semileptonic hyperon decays involves one additional parameter when intermediate decuplet states are included. After integrating out the heavy degrees of freedom from the relativistic Lagrangian there is still a remaining mass dependence which is proportional to the average octet-decuplet splitting $\Delta$ and does not vanish in the chiral limit. In the Feynman rules the mass splitting $\Delta$ is contained in the decuplet propagator

$$\frac{i}{v \cdot l - \Delta + i\epsilon} \left( v_\mu v_\nu - g_\mu_\nu - \frac{4d-3}{d-1}s_\mu s_\nu \right)$$ \hspace{1cm} (40)

in $d$ dimensions. The appearance of the mass scale $\Delta$ destroys in the case of dimensional regularization the one-to-one correspondence between meson loops and the expansion in small momenta and quark masses. No further complications arise in our case since the strict chiral counting scheme has already been spoilt by introducing the scale $\Lambda$.

The decuplet contributions to $G_1$ are presented in Fig. 4 and have the following form

$$\delta G_1[ij](q^2) = -\frac{C^2}{\Lambda^2} \sum_{\phi=\pi,K,\eta} \left( \frac{4}{3} \rho_{ij}^\phi J^\phi(-\Delta, -\frac{q^2}{2M_i}) + \frac{4}{3} \rho_{ij}^\phi J^\phi(0, -\frac{q^2}{2M_i} - \Delta) \right)$$

$$-10 \frac{H}{9} \sigma_{ij}^\phi J^\phi(-\Delta, -\frac{q^2}{2M_i} - \Delta) \alpha_{ij}(\epsilon_i^\phi + \epsilon_j^\phi) J^\phi(-\Delta, -\Delta) \right). \hspace{1cm} (41)$$
The integrals $J^\phi(x,y)$ can be found in App. A and we neglected again the baryon mass differences in the integrals by setting $v \cdot q_i = 0$ and $v \cdot q_j = -q^2/(2M_i)$. One also has to include the contributions from intermediate states to the baryon $Z$–factors. The last term with the coefficients $\epsilon_1^\phi$ takes this into account. The coefficients read

\begin{align}
\kappa_{pm}^\pi &= \frac{4}{3}(D + F) , \quad \kappa_{pm}^K = \frac{1}{2}D + \frac{1}{6}F , \quad \kappa_{pm}^\eta = 0 \\
\kappa_{\Lambda_\Sigma}^\pi &= -\frac{2}{3\sqrt{6}}D , \quad \kappa_{\Lambda_\Sigma}^K = \frac{1}{\sqrt{6}}(\frac{5}{3}D + 3F) , \quad \kappa_{\Lambda_\Sigma}^\eta = \frac{1}{\sqrt{6}}D \\
\kappa_{\Xi_\Sigma}^\pi &= -\frac{1}{6}(D - F) , \quad \kappa_{\Xi_\Sigma}^K = \frac{1}{6}(D + 5F) , \quad \kappa_{\Xi_\Sigma}^\eta = \frac{1}{6}(D + 3F) \\
\kappa_{p\Lambda}^\pi &= -\frac{3}{2\sqrt{6}}(D + F) , \quad \kappa_{p\Lambda}^K = -\frac{2}{\sqrt{6}}D , \quad \kappa_{p\Lambda}^\eta = 0 \\
\kappa_{\Lambda_\Xi}^\pi &= -\frac{1}{\sqrt{6}}D , \quad \kappa_{\Lambda_\Xi}^K = \frac{3}{2\sqrt{6}}(D - F) , \quad \kappa_{\Lambda_\Xi}^\eta = \frac{1}{\sqrt{6}}D \\
\kappa_{n_\Sigma}^\pi &= \frac{1}{3}(D + F) , \quad \kappa_{n_\Sigma}^K = \frac{2}{3}F , \quad \kappa_{n_\Sigma}^\eta = -\frac{1}{6}(D - 3F) \\
\kappa_{\Sigma_\Xi}^\pi &= \frac{1}{3\sqrt{2}}(D + 2F) , \quad \kappa_{\Sigma_\Xi}^K = \frac{1}{6\sqrt{2}}(7D + 5F) , \quad \kappa_{\Sigma_\Xi}^\eta = \frac{1}{3\sqrt{2}}D \\
\kappa_{\Sigma^0_\Xi}^\phi &= \sqrt{2}\kappa_{\Sigma^0_\Xi}^\phi \quad .
\end{align}

\begin{align}
\rho_{pm}^\pi &= \frac{4}{3}(D + F) , \quad \rho_{pm}^K = \frac{1}{2}D + \frac{1}{6}F , \quad \rho_{pm}^\eta = 0 \\
\rho_{\Lambda_\Sigma}^\pi &= \frac{1}{\sqrt{6}}(D + 2F) , \quad \rho_{\Lambda_\Sigma}^K = \frac{1}{\sqrt{6}}(D + F) , \quad \rho_{\Lambda_\Sigma}^\eta = 0 \\
\rho_{\Xi_\Sigma}^\pi &= -\frac{1}{6}(D - F) , \quad \rho_{\Xi_\Sigma}^K = \frac{1}{6}(D + 5F) , \quad \rho_{\Xi_\Sigma}^\eta = \frac{1}{6}(D + 3F) \\
\rho_{p\Lambda}^\pi &= -\frac{4}{\sqrt{6}}D , \quad \rho_{p\Lambda}^K = \frac{1}{2\sqrt{6}}(D - 3F) , \quad \rho_{p\Lambda}^\eta = 0 \\
\rho_{\Lambda_\Xi}^\pi &= \frac{3}{2\sqrt{6}}(D - F) , \quad \rho_{\Lambda_\Xi}^K = 0 , \quad \rho_{\Lambda_\Xi}^\eta = 0 \\
\rho_{n_\Sigma}^\pi &= \frac{4}{3}F , \quad \rho_{n_\Sigma}^K = \frac{1}{6}(D + F) , \quad \rho_{n_\Sigma}^\eta = 0 \\
\rho_{\Sigma_\Xi}^\pi &= \frac{1}{3\sqrt{2}}(D - F) , \quad \rho_{\Sigma_\Xi}^K = \frac{4}{3\sqrt{2}}(D + F) , \quad \rho_{\Sigma_\Xi}^\eta = \frac{1}{6\sqrt{2}}(D + 3F) \\
\rho_{\Sigma^0_\Xi}^\phi &= \sqrt{2}\rho_{\Sigma^0_\Xi}^\phi \quad .
\end{align}

\begin{align}
\sigma_{pm}^\pi &= \frac{10}{9} , \quad \sigma_{pm}^K = \frac{2}{9} , \quad \sigma_{pm}^\eta = 0 \\
\sigma_{\Lambda_\Sigma}^\pi &= \frac{2}{3\sqrt{6}} , \quad \sigma_{\Lambda_\Sigma}^K = \frac{1}{3\sqrt{6}} , \quad \sigma_{\Lambda_\Sigma}^\eta = 0 \\
\sigma_{\Xi_\Sigma}^\pi &= \frac{1}{18} , \quad \sigma_{\Xi_\Sigma}^K = -\frac{2}{9} , \quad \sigma_{\Xi_\Sigma}^\eta = -\frac{1}{6} \\
\sigma_{p\Lambda}^\pi &= -\frac{2}{\sqrt{6}} , \quad \sigma_{p\Lambda}^K = -\frac{1}{\sqrt{6}} , \quad \sigma_{p\Lambda}^\eta = 0
\end{align}
\[
\sigma_{\Lambda \Xi^-}^\pi = \frac{1}{\sqrt{6}}, \quad \sigma_{\Lambda \Xi^-}^K = \frac{1}{\sqrt{6}}, \quad \sigma_{\Lambda \Xi^-}^\eta = 0
\]
\[
\sigma_{n \Sigma^-}^\pi = -\frac{2}{9}, \quad \sigma_{n \Sigma^-}^K = -\frac{1}{9}, \quad \sigma_{n \Sigma^-}^\eta = 0
\]
\[
\sigma_{\Sigma^0 \Xi^-}^\pi = \frac{2}{9\sqrt{2}}, \quad \sigma_{\Sigma^0 \Xi^-}^K = \frac{7}{9\sqrt{2}}, \quad \sigma_{\Sigma^0 \Xi^-}^\eta = \frac{1}{3\sqrt{2}}
\]
\[
\sigma_{\Sigma^0 \Xi^-}^\rho = \sqrt{2}\sigma_{\Sigma^0 \Xi^-}^\phi.
\] (44)

\[
\epsilon_N^\pi = 1, \quad \epsilon_N^K = \frac{1}{4}, \quad \epsilon_N^\eta = 0, \quad \epsilon_S^\pi = \frac{1}{6}, \quad \epsilon_S^K = \frac{5}{6}, \quad \epsilon_S^\eta = \frac{1}{4}
\]
\[
\epsilon_A^\pi = \frac{3}{4}, \quad \epsilon_A^K = \frac{1}{2}, \quad \epsilon_A^\eta = 0, \quad \epsilon_S^\pi = \frac{1}{4}, \quad \epsilon_S^K = \frac{3}{4}, \quad \epsilon_S^\eta = \frac{1}{4}.
\] (45)

There is an analogue formula for the form factor \( G_3 \) [Fig. 5]

\[
\delta G_3[ij](q^2) = -\frac{C^2}{\Lambda^2} \left( \frac{M_i + M_j}{\rho^2 - q^2} \right)^2 \sum_{\phi=\pi,K,\eta} \left( \frac{4}{3} \kappa_{ij}^\phi J^\phi(-\Delta, -\frac{q^2}{2M_i}) + \frac{4}{3} \rho_{ij}^\phi J^\phi(0, -\frac{q^2}{2M_i} - \Delta) \right)
\]
\[
-\frac{10}{9} H \sigma_{ij}^\phi J^\phi(-\Delta, -\frac{q^2}{2M_i} - \Delta) - \alpha_{ij}(\epsilon_i^\phi + \epsilon_j^\phi)J^\phi(-\Delta, -\Delta)
\] (46)

with \( m_\phi = m_\pi \) for the decays \([ij] = [pn], [\Lambda \Sigma^-], [\Xi^0 \Xi^-] \) and \( m_\phi = m_K \) otherwise.

## 4 Results and discussion

In this section we present the numerical results for the calculation of the hadronic axial form factors. We consider first the case with no resonances. The values for our parameters are \( F_\pi = 93 \) MeV, \( m_\pi = 138 \) MeV, \( m_K = 495 \) MeV, and for the mass of the \( \eta \) we use the GMO value for the pseudoscalar mesons \( m_\eta = 566 \) MeV. The scale in dimensional regularization is chosen to be \( \lambda = 1 \) GeV. The differences for \( F_\pi \) and \( m_\eta \) to \( F^- \) the pseudoscalar decay constant in the chiral limit– and to the physical mass of \( \eta \), respectively, appear only at higher orders. We will restrict ourselves to these central values of the parameters since a small variation in these parameters does only lead to some minor changes in the results.

In baryon chiral perturbation theory, the transition between short and long distance occurs around a distance scale of \( \sim 1 \) fermi, or a momentum scale of \( \sim 200 \) MeV. This corresponds to the measured size of a baryon. The effective field theory treats the baryons and pions as point particles. This is appropriate for the very long distance physics. However, for propagation at distances less than the separation scale, the point particle theory is not an accurate representation of the physics. The composite substructure becomes manifest below this point.

Of course, the cutoff \( \Lambda \) should not be taken too low in energy that it removes any truly long distance physics. Also, while it can in principle be taken much larger than the separation scale, this will lead to the inclusion of spurious short distance physics which can upset the convergence of the expansion. It is ideal to take the cutoff slightly above the separation scale so that all of the long distance physics, but little of the short distance physics, is included. Therefore, we will vary the cutoff in the range \( \Lambda \geq 1/ < r_B > \sim 300 - 600 \) MeV.
The two unknown axial couplings $D$ and $F$ have to be fixed from phenomenology. We will choose the semileptonic decays $n \rightarrow p$, $\Sigma^- \rightarrow \Lambda$, $\Lambda \rightarrow p$, $\Xi^- \rightarrow \Lambda$, $\Sigma^- \rightarrow n$ and $\Xi^- \rightarrow \Sigma^0$ to perform a least–squares fit for $D$ and $F$. In Table 1 the values for $D$ and $F$ for different values of the cutoff $\Lambda$ are compared to the fit in dimensional regularization and a fit at tree level. At tree level the least–squares fit leads to

$$D = 0.80, \quad F = 0.50 \quad (47)$$

Including the loop contributions, one obtains in the cutoff scheme the values

$$D = 0.59 \pm 0.06, \quad F = 0.36 \pm 0.05 \quad (48)$$

whereas in the case of dimensional regularization a fit delivers

$$D = 0.44, \quad F = 0.26 \quad (49)$$

In the latter case we neglected the analytic parts from the loops. The loop corrections lower in both cases the values for $D$ and $F$ but the change in $D$ and $F$ is larger in dimensional regularization. On the other hand, the ratio $F/D$ remains the same in all cases: $F/D \approx 0.61$.

The chiral expansions for $g_1$ at zero momentum transfer read in the cutoff scheme for $\Lambda = 400$ MeV

$$g_1[pn](0) = 0.97 + 0.28 = 1.25 \quad (1.26)$$
$$g_1[\Lambda\Sigma^-](0) = 0.49 + 0.17 = 0.66 \quad (0.62)$$
$$g_1[p\Lambda](0) = -0.69 - 0.26 = -0.95 \quad (-0.92)$$
$$g_1[\Lambda\Xi^-](0) = 0.20 + 0.10 = 0.30 \quad (0.40)$$
$$g_1[n\Sigma^-](0) = 0.23 + 0.07 = 0.30 \quad (0.39)$$
$$g_1[\Sigma^0\Xi^-](0) = 0.69 + 0.30 = 0.99 \quad (0.97)$$
$$g_1[\Xi^0\Xi^-](0) = 0.23 + 0.09 = 0.32 \quad (50)$$

Using dimensional regularization we obtain

$$g_1[pn](0) = 0.70 + 0.46 = 1.16 \quad (1.26)$$
$$g_1[\Lambda\Sigma^-](0) = 0.36 + 0.28 = 0.64 \quad (0.62)$$
$$g_1[p\Lambda](0) = -0.50 - 0.48 = -0.98 \quad (-0.92)$$
$$g_1[\Lambda\Xi^-](0) = 0.14 + 0.17 = 0.31 \quad (0.40)$$
$$g_1[n\Sigma^-](0) = 0.18 + 0.14 = 0.32 \quad (0.39)$$
$$g_1[\Sigma^0\Xi^-](0) = 0.50 + 0.55 = 1.05 \quad (0.97)$$
$$g_1[\Xi^0\Xi^-](0) = 0.18 + 0.15 = 0.33 \quad (51)$$

The first number refers to the tree level contribution. The loop contributions are summarized in the second number. The numbers in the brackets are the experimental values. While the chiral series converge in the cutoff scheme, one cannot make a definite statement about the
convergence in dimensional regularization. Furthermore, we do not present the numerical results for the decay $\Xi^0 \rightarrow \Sigma^+$ since it is related to $\Xi^- \rightarrow \Sigma^0$ by a factor of $\sqrt{2}$. Also, the fit in dimensional regularization has $\chi^2/d.o.f. = 2.8$, whereas we have $\chi^2/d.o.f. = 0.6$ in the cutoff scheme for $\Lambda = 400$ MeV and $\chi^2/d.o.f. = 1.0$ for the tree level fit. Note, that we have increased the errors on the measurement of $g_A$ in the neutron decay to 0.03 to avoid biasing the fit to the $D + F$ value favored by this decay. The values for the three axial form factors $g_1, g_2$ and $g_3$ at zero momentum transfer in dimensional regularization and for different values of $\Lambda$ in the cutoff scheme are presented in Tab. 2.

Adding the decuplet, we set $\Delta = 231$ MeV, which is the average octet–decuplet mass splitting, and the value of the coupling constant $C$ is given by $C = 1.5$ from an overall fit to the decuplet decays [10]. The introduction of the decuplet leads to an additional parameter $H$ which has to be fixed from phenomenology. We will determine $H$ along with $D$ and $F$ by performing a least–squares fit to the form factors $g_1(0)$. One obtains in the cutoff scheme the values

$$ D = 0.55 \pm 0.11 \quad , \quad F = 0.46 \pm 0.05 \quad , \quad H = 3.0 \pm 5.0 $$

(52)

whereas in the case of dimensional regularization a fit delivers

$$ D = 0.43 \quad , \quad F = -0.14 \quad , \quad H = -3.5 \quad . $$

(53)

It turns out that there are significant changes in the fit in dimensional regularization after including the decuplet. The values for $D$ and $F$ in the cutoff scheme differ only slightly from the case without resonances. No reliable estimate of the parameter $H$ can be given since the uncertainty in the cutoff scheme is rather large and differs considerably from the value in dimensional regularization. The values of $D$, $F$ and $H$ for different values of $\Lambda$ are shown in Tab. 3. The chiral expansions in the cutoff scheme for $\Lambda = 400$ MeV read

$$ g_1[pm](0) = 1.03 + 0.34 - 0.13 = 1.24 \quad (1.26) $$
$$ g_1[\Lambda\Sigma^-](0) = 0.48 + 0.19 - 0.02 = 0.65 \quad (0.62) $$
$$ g_1[p\Lambda](0) = -0.78 - 0.33 + 0.18 = -0.93 \quad (-0.92) $$
$$ g_1[\Lambda\Xi^-](0) = 0.30 + 0.17 - 0.15 = 0.32 \quad (0.40) $$
$$ g_1[n\Sigma^-](0) = 0.14 + 0.04 + 0.13 = 0.31 \quad (0.39) $$
$$ g_1[\Sigma^0\Xi^-](0) = 0.73 + 0.36 - 0.08 = 1.01 \quad (0.97) $$
$$ g_1[\Xi^0\Xi^-](0) = 0.14 + 0.04 + 0.09 = 0.27 \quad . $$

(54)

Using dimensional regularization we obtain

$$ g_1[pm](0) = 0.29 + 0.21 + 0.70 = 1.20 \quad (1.26) $$
$$ g_1[\Lambda\Sigma^-](0) = 0.35 + 0.22 + 0.07 = 0.64 \quad (0.62) $$
$$ g_1[p\Lambda](0) = -0.01 - 0.02 - 0.80 = -0.83 \quad (-0.92) $$
$$ g_1[\Lambda\Xi^-](0) = -0.34 - 0.28 + 0.99 = 0.37 \quad (0.40) $$
$$ g_1[n\Sigma^-](0) = 0.56 + 0.51 - 0.73 = 0.34 \quad (0.39) $$
$$ g_1[\Sigma^0\Xi^-](0) = 0.20 + 0.18 + 0.75 = 1.13 \quad (0.97) $$
$$ g_1[\Xi^0\Xi^-](0) = 0.56 + 0.31 - 0.90 = -0.03 \quad . $$

(55)
The first and second number denote tree and loop contributions of the baryon octet, respectively. The third number is the loop contribution with intermediate decuplet states. The \( \chi^2/d.o.f. \) are \( \chi^2/d.o.f. = 0.4 \) and \( \chi^2/d.o.f. = 2.6 \) for the cutoff \( \Lambda = 400 \text{ MeV} \) and dimensional regularization, respectively. The contributions from the resonance loops are well behaved in the cutoff scheme, whereas they upset the behavior of the chiral series in dimensional regularization and in most cases their contribution dominates. A similar impact on the chiral series after the inclusion of the decuplet was observed in the calculation of the baryon \( \sigma \)-terms [11]. Furthermore, while in the cutoff scheme we can predict \( g_1[\Xi^0\Xi^-](0) = 0.30 \pm 0.03 \), the uncertainty in dimensional regularization for this decay is large. Setting the octet–decuplet mass splitting \( \Delta = 0 \text{ MeV} \) a least–squares fit delivers in the cutoff scheme for \( \Lambda = 400 \text{ MeV} \)

\[
D = 0.57 \quad , \quad F = 0.43 \quad , \quad H = 1.5
\]

whereas in the case of dimensional regularization we obtain

\[
D = 0.48 \quad , \quad F = 0.31 \quad , \quad H = -1.4
\]

The latter result is in agreement with [1] once one accounts for the vanishing pion mass in that calculation. Apparently, the fit for the three parameters in dimensional regularization depends strongly on the value of the mass splitting \( \Delta \) but in the cutoff scheme \( D \) and \( F \) are not altered significantly by setting \( \Delta = 0 \text{ MeV} \). The reason for this is the large contribution of the decuplet loops in dimensional regularization. Again, there is a large uncertainty in the parameter \( H \). The values for the relativistic form factors \( g_{1,2,3} \) can be found in Tab. 4. In the Figures 6 to 8 the \( g_{1,2,3} \) are shown for small values of the momentum transfer squared. By neglecting the counterterms from the Lagrangian \( L_{\phi B}^{(2)} \) one obtains similar results for the form factors \( g_{1,3} \) but there is a dramatic impact on \( g_2 \). While we have \( |g_2/g_1| \approx 0.3 \) for most decays in our calculation, dropping these counterterms leads to \( |g_2/g_1| \approx 1 \).

### 5 Summary

In this paper we have investigated the baryon axial currents both in dimensional regularization and in a cutoff scheme.

- First, we presented the relation of the axial form factors \( G_{1,2,3} \) in heavy baryon chiral perturbation theory with the corresponding relativistic amplitudes \( g_{1,2,3} \) in the convenient initial–baryon restframe. We calculated the chiral corrections to the axial currents by using both the lowest order effective Lagrangian and their relativistic corrections at next order in the heavy baryon formulation. The Goldstone boson integrals are evaluated both in dimensional regularization and by using a dipole regulator with a cutoff \( \Lambda \). We have given the expressions for the form factors for general momentum transfer. With our definitions one of the form factors \( G_2 \) obtains contributions only from the relativistic corrections of the meson baryon Lagrangian \( L_{\phi B}^{(2)} \) of second chiral order. Only these corrections, which have been neglected in previous investigations, ensure the vanishing of \( g_2[pn] \) at tree level. Otherwise, one would obtain \( g_2[pn] = -g_1[pn] \approx -1.26 \).

The cutoff parameter induces an additional mass scale that does not vanish in the chiral limit and, therefore, destroys the strict chiral counting scheme. We are able to show that
to the order we are working the physics does not depend on \( \Lambda \), since one is able to absorb the effects of \( \Lambda \) into a renormalization of the coupling constants.

- The spin–3/2 decuplet is separated from the octet by \( \Delta = 231 \text{ MeV} \) in average which is smaller than the kaon or eta mass. Therefore, we proceeded by adding the decuplet to the effective theory. The appearance of the mass scale \( \Delta \) destroys in the case of dimensional regularization the one–to–one correspondence between meson loops and the expansion in small momenta and quark masses. No further complications arise in the cutoff scheme since the strict chiral counting scheme has already been spoilt by introducing the scale \( \Lambda \).

- We performed a least–squares fit to the semileptonic hyperon decays. The values for the two axial couplings \( D \) and \( F \) are at tree level \( D = 0.80 \) and \( F = 0.50 \). Including the chiral corrections from the loops without resonances we obtain \( D = 0.44 \), \( F = 0.26 \) in dimensional regularization and \( D = 0.59 \pm 0.06 \), \( F = 0.36 \pm 0.05 \) by using a cutoff. The uncertainty in the fit stems from the variation of the cutoff parameter. In our analysis the parameter \( \Lambda \) ranges from 300 to 600 MeV to account for all the long distance physics, but little of the short distance physics, which are not described appropriately by the effective theory, is included. The results are in good agreement with the experimental data and we have \( \chi^2/d.o.f. = 0.6 \) for the cutoff \( \Lambda = 400 \text{ MeV} \) to be compared with \( \chi^2/d.o.f. = 1.0 \) at tree level. In dimensional regularization, one obtains \( \chi^2/d.o.f. = 2.8 \). While the chiral expansions of the form factors converge in the cutoff scheme, one cannot make a definite statement about the convergence in dimensional regularization. After fixing \( D \) and \( F \) from experiment, results for the three momentum dependent axial form factors are presented.

The introduction of the decuplet leads to an additional parameter \( H \) which has to be fixed from phenomenology. Adding the loop contributions with intermediate decuplet states alter the results significantly in dimensional regularization. A least–squares fit to the semileptonic decays delivers the values \( D = 0.43 \), \( F = -0.14 \) and \( H = -3.5 \). The decuplet contributions tend to dominate the pieces from tree level and loops involving only baryon octet fields. In the cutoff scheme the contributions from the resonances behave moderate and one obtains \( D = 0.55 \pm 0.11 \), \( F = 0.46 \pm 0.05 \) and \( H = 3.0 \pm 5.0 \). No reliable estimate of the parameter \( H \) can be given since the uncertainty in the cutoff scheme is rather large and differs considerably from the value in dimensional regularization.

Setting the average octet–decuplet mass splitting \( \Delta = 0 \text{ MeV} \) leads to \( D = 0.57 \), \( F = 0.43 \), \( H = 1.5 \) and \( D = 0.48 \), \( F = 0.31 \), \( H = -1.4 \) using a cutoff \( \Lambda = 400 \text{ MeV} \) and in dimensional regularization, respectively. The latter case is in agreement with [4], once one accounts for the vanishing pion mass in that work.

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A Integrals

The fundamental integral in the calculation of the axial form factors is in the cutoff scheme

\[
J_\phi(x, y) = \frac{4}{3\pi^2} \int d^4l \frac{i^3 (S \cdot l)^2}{l^2 - m_\phi^2 + i\epsilon} [v \cdot l + x + i\epsilon][v \cdot l + y + i\epsilon] \left( \frac{\Lambda^2 - l^2}{\Lambda^2 - l^2} \right)^2
\]

\[
= -\frac{1}{3} \frac{\Lambda^4}{(\Lambda^2 - m_\phi^2)^2} \left( \Lambda^2 - m_\phi^2 + m_\phi^2 \ln \frac{m_\phi^2}{\Lambda^2} \right)
\]

\[
+ \frac{2}{3} x - y \frac{\Lambda^4}{(\Lambda^2 - m_\phi^2)^2} \left( \left[ y(m_\phi^2 - y^2) - x(m_\phi^2 - x^2) \right] \ln \frac{m_\phi^2}{\Lambda^2} \right)
\]

\[
+ \left[ \frac{3}{2} m_\phi^2 - x^2 - \frac{1}{2} \Lambda^2 \right] f(\Lambda, x) - \left[ m_\phi^2 - x^2 \right] f(m_\phi, x)
\]

\[
- \left[ \frac{3}{2} m_\phi^2 - y^2 - \frac{1}{2} \Lambda^2 \right] f(\Lambda, y) + \left[ m_\phi^2 - y^2 \right] f(m_\phi, y)
\]

(A.1)

with

\[
f(u, v) = 2 \sqrt{u^2 - v^2} \arccos \frac{v}{u} \quad ; \quad \text{for} \quad |u| > |v|
\]

\[
f(u, v) = 2 \sqrt{u^2 - u^2} \ln \left[ \frac{v}{u} + \sqrt{\frac{v^2}{u^2} - 1} \right] \quad ; \quad \text{for} \quad \frac{v}{u} > 1
\]

\[
f(u, v) = -2 \sqrt{v^2 - u^2} \ln \left[ -\frac{v}{u} + \sqrt{\frac{v^2}{u^2} - 1} \right] \quad ; \quad \text{for} \quad \frac{v}{u} < -1
\]

(A.2)

In dimensional regularization the nonanalytic part of the integral reads

\[
J_{\phi \text{dim}}(x, y) = \frac{64\pi^2}{3} \int \frac{d^d l}{(2\pi)^d} \frac{i^3 (S \cdot l)^2}{l^2 - m_\phi^2 + i\epsilon} [v \cdot l + x + i\epsilon][v \cdot l + y + i\epsilon] \left( \frac{\Lambda^2 - l^2}{\Lambda^2 - l^2} \right)^2
\]

\[
= -\left( m_\phi^2 - \frac{2}{3} \left[ x^2 + xy + y^2 \right] \right) \ln \frac{m_\phi^2}{\Lambda^2}
\]

\[
+ \frac{2}{3} x - y \cdot f(m_\phi, x) \quad + \frac{2 m_\phi^2 - y^2}{3} x - y \cdot f(m_\phi, y)
\]

(A.3)

with \( \lambda \) the scale introduced in dimensional regularization.

In Section 2 we use for the loop integrals involving only the baryon fields the notation

\[
I_\phi(x) = -J_\phi(0, x)
\]

(A.4)

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Table captions

Table 1 Given are the values for the couplings $D$ and $F$ and the pertinent $\chi^2/d.o.f.$ from a least–squares fit to the axial form factors $g_1(0)$ both in dimensional regularization and for various values of the cutoff $\Lambda$ in MeV.

Table 2 Given are the values for the three axial form factors $g_{1,2,3}$ at zero momentum transfer $q^2 = 0$ both in dimensional regularization and for different values of the cutoff $\Lambda$ in MeV.

Table 3 Given are the values for the couplings $D$, $F$ and $H$ and the pertinent $\chi^2/d.o.f.$ from a least–squares fit to the axial form factors $g_1(0)$ including contributions from the spin–3/2 decuplet both in dimensional regularization and for various values of the cutoff $\Lambda$ in MeV.

Table 4 Given are the values for the three axial form factors $g_{1,2,3}$ at zero momentum transfer $q^2 = 0$ including contributions from the spin–3/2 decuplet both in dimensional regularization and for different values of the cutoff $\Lambda$ in MeV.

Figure captions

Fig.1 Tree graphs to the form factors $G_{1,2}$ and $G_3$. Solid and dashed lines denote octet baryons and Goldstone bosons, respectively. The solid circle denotes a strong vertex and the solid square represents the axial current.

Fig.2 Loop contributions to $G_1$. Solid and dashed lines denote octet baryons and Goldstone bosons, respectively. The solid circle denotes a strong vertex and the solid square represents the axial current.

Fig.3 Loop contributions to $G_3$. Solid and dashed lines denote octet baryons and Goldstone bosons, respectively. The solid circle denotes a strong vertex and the solid square represents the axial current.

Fig.4 Loop contributions to $G_1$ with intermediate decuplet states. Solid and dashed lines denote octet baryons and Goldstone bosons, respectively. The double line represents the decuplet. The solid circle denotes a strong vertex and the solid square represents the axial current.

Fig.5 Loop contributions to $G_3$ with intermediate decuplet states. Solid and dashed lines denote octet baryons and Goldstone bosons, respectively. The double line represents the decuplet. The solid circle denotes a strong vertex and the solid square represents the axial current.

Fig.6 The results for the form factor $g_1(t)$ are presented: a) in the cutoff scheme with $\Lambda = 400$ MeV and ground state octet baryons only; b) in the cutoff scheme with $\Lambda = 400$ MeV and including the decuplet; c) in dimensional regularization and ground state octet baryons.
only; d) in dimensional regularization including the decuplet. The different lines refer to
the following decays: \( n \rightarrow p \): continuous line; \( \Lambda \rightarrow p \): dot–dashed; \( \Sigma^- \rightarrow \Lambda \): broken line;
\( \Sigma^- \rightarrow n \): dashed; \( \Xi^- \rightarrow \Lambda \): dotted; \( \Xi^- \rightarrow \Sigma^0 \): dot–dot–dashed; \( \Xi^- \rightarrow \Xi^0 \): dot–dash–
dashed.

Fig.7 The results for the form factor \( g_2(t) \) are presented: a) in the cutoff scheme with \( \Lambda = 400 \) MeV and ground state octet bayons only; b) in the cutoff scheme with \( \Lambda = 400 \) MeV and including the decuplet; c) in dimensional regularization and ground state octet baryons only; d) in dimensional regularization including the decuplet. The different lines refer to
the following decays: \( n \rightarrow p \): continuous line; \( \Lambda \rightarrow p \): dot–dashed; \( \Sigma^- \rightarrow \Lambda \): broken line;
\( \Sigma^- \rightarrow n \): dashed; \( \Xi^- \rightarrow \Lambda \): dotted; \( \Xi^- \rightarrow \Sigma^0 \): dot–dot–dashed; \( \Xi^- \rightarrow \Xi^0 \): dot–dash–
dashed.

Fig.8 The results for the form factor \( g_3(t) \) are presented: a) in the cutoff scheme with \( \Lambda = 400 \) MeV and ground state octet bayons only; b) in the cutoff scheme with \( \Lambda = 400 \) MeV and including the decuplet; c) in dimensional regularization and ground state octet baryons only; d) in dimensional regularization including the decuplet. The different lines refer to
the following decays: \( n \rightarrow p \): continuous line; \( \Lambda \rightarrow p \): dot–dashed; \( \Sigma^- \rightarrow \Lambda \): broken line;
\( \Sigma^- \rightarrow n \): dashed; \( \Xi^- \rightarrow \Lambda \): dotted; \( \Xi^- \rightarrow \Sigma^0 \): dot–dot–dashed; \( \Xi^- \rightarrow \Xi^0 \): dot–dash–
dashed.
| dim. | $\Lambda = 300$ | $\Lambda = 400$ | $\Lambda = 500$ | $\Lambda = 600$ |
|------|----------------|----------------|----------------|----------------|
| $D$  | 0.44           | 0.64           | 0.60           | 0.56           | 0.54           |
| $F$  | 0.26           | 0.40           | 0.37           | 0.35           | 0.33           |
| $\chi^2/d.o.f.$ | 2.8 | 0.5 | 0.6 | 0.8 | 1.0 |

Table 1

| dim. | $\Lambda = 300$ | $\Lambda = 400$ | $\Lambda = 500$ | $\Lambda = 600$ |
|------|----------------|----------------|----------------|----------------|
| $g_1[\rho\pi] (0)$ | 1.16           | 1.27           | 1.25           | 1.23           | 1.21           |
| $g_1[\Lambda\Sigma^-] (0)$ | 0.64           | 0.66           | 0.66           | 0.66           | 0.65           |
| $g_1[\Xi^0\Xi^-] (0)$ | 0.33           | 0.31           | 0.32           | 0.32           | 0.32           |
| $g_1[p\Lambda] (0)$ | -0.98          | -0.95          | -0.95          | -0.96          | -0.96          |
| $g_1[\Lambda\Xi^-] (0)$ | 0.31           | 0.30           | 0.30           | 0.30           | 0.31           |
| $g_1[n\Sigma^-] (0)$ | 0.32           | 0.30           | 0.30           | 0.30           | 0.31           |
| $g_1[\Sigma\Xi^-] (0)$ | 1.05           | 0.97           | 0.99           | 1.00           | 1.01           |
| $g_2[\rho\pi] (0)$ | -0.12          | -0.23          | -0.28          | -0.32          | -0.35          |
| $g_2[\Lambda\Sigma^-] (0)$ | -0.14          | -0.16          | -0.20          | -0.22          | -0.25          |
| $g_2[\Xi^0\Xi^-] (0)$ | 0.71           | 0.72           | 0.65           | 0.59           | 0.54           |
| $g_2[p\Lambda] (0)$ | 0.34           | 0.31           | 0.37           | 0.42           | 0.46           |
| $g_2[\Lambda\Xi^-] (0)$ | -0.12          | -0.11          | -0.13          | -0.15          | -0.16          |
| $g_2[n\Sigma^-] (0)$ | -0.12          | -0.10          | -0.12          | -0.14          | -0.15          |
| $g_2[\Sigma\Xi^-] (0)$ | -0.39          | -0.30          | -0.37          | -0.43          | -0.48          |
| $g_3[\rho\pi] (0)$ | -215.9         | -236.0         | -232.3         | -228.9         | -226.0         |
| $g_3[\Lambda\Sigma^-] (0)$ | -174.7         | -179.5         | -179.5         | -179.1         | -178.5         |
| $g_3[\Xi^0\Xi^-] (0)$ | -118.4         | -113.6         | -115.5         | -116.9         | -117.9         |
| $g_3[p\Lambda] (0)$ | 16.2           | 15.6           | 15.7           | 15.9           | 16.0           |
| $g_3[\Lambda\Xi^-] (0)$ | -7.2           | -6.8           | -7.0           | -7.0           | -7.1           |
| $g_3[n\Sigma^-] (0)$ | -5.6           | -5.1           | -5.2           | -5.3           | -5.4           |
| $g_3[\Sigma\Xi^-] (0)$ | -26.5          | -24.2          | -24.8          | -25.2          | -25.6          |

Table 2
| dim. | $\Lambda = 300$ | $\Lambda = 400$ | $\Lambda = 500$ | $\Lambda = 600$ |
|------|----------------|----------------|----------------|----------------|
| $D$  | 0.43           | 0.66           | 0.59           | 0.52           | 0.45           |
| $F$  | -0.14          | 0.48           | 0.44           | 0.43           | 0.46           |
| $H$  | -3.5           | 9.1            | 3.4            | 1.9            | 1.4            |
| $\chi^2/d.o.f.$ | 2.6 | 0.4 | 0.4 | 0.3 | 0.3 |

Table 3

| dim. | $\Lambda = 300$ | $\Lambda = 400$ | $\Lambda = 500$ | $\Lambda = 600$ |
|------|----------------|----------------|----------------|----------------|
| $g_1 [pn] (0)$ | 1.20 | 1.25 | 1.24 | 1.24 | 1.23 |
| $g_1 [\Lambda \Sigma^-] (0)$ | 0.64 | 0.65 | 0.65 | 0.65 | 0.65 |
| $g_1 [\Xi^0 \Xi^-] (0)$ | -0.03 | 0.28 | 0.27 | 0.26 | 0.23 |
| $g_1 [p\Lambda] (0)$ | -0.83 | -0.94 | -0.93 | -0.93 | -0.91 |
| $g_1 [\Lambda \Xi^-] (0)$ | 0.37 | 0.31 | 0.32 | 0.33 | 0.36 |
| $g_1 [n\Sigma^-] (0)$ | 0.34 | 0.31 | 0.31 | 0.31 | 0.32 |
| $g_1 [\Sigma^0 \Xi^-] (0)$ | 1.13 | 1.00 | 1.01 | 1.02 | 1.02 |
| $g_2 [pn] (0)$ | -0.33 | -0.21 | -0.28 | -0.33 | -0.37 |
| $g_2 [\Lambda \Sigma^-] (0)$ | -0.23 | -0.16 | -0.19 | -0.21 | -0.24 |
| $g_2 [\Xi^0 \Xi^-] (0)$ | 0.89 | 0.76 | 0.69 | 0.65 | 0.63 |
| $g_2 [p\Lambda] (0)$ | 0.30 | 0.30 | 0.35 | 0.38 | 0.40 |
| $g_2 [\Lambda \Xi^-] (0)$ | -0.24 | -0.13 | -0.15 | -0.19 | -0.23 |
| $g_2 [n\Sigma^-] (0)$ | -0.20 | -0.11 | -0.13 | -0.15 | -0.17 |
| $g_2 [\Sigma^0 \Xi^-] (0)$ | -0.60 | -0.34 | -0.40 | -0.45 | -0.48 |
| $g_3 [pn] (0)$ | -222.9 | -231.8 | -231.2 | -230.5 | -230.0 |
| $g_3 [\Lambda \Sigma^-] (0)$ | -175.9 | -178.1 | -177.2 | -176.5 | -176.9 |
| $g_3 [\Xi^0 \Xi^-] (0)$ | 11.2 | -100.6 | -98.6 | -93.5 | -84.3 |
| $g_3 [p\Lambda] (0)$ | 13.9 | 15.3 | 15.3 | 15.3 | 15.1 |
| $g_3 [\Lambda \Xi^-] (0)$ | -9.1 | -7.1 | -7.4 | -7.7 | -8.3 |
| $g_3 [n\Sigma^-] (0)$ | -5.7 | -5.3 | -5.4 | -5.6 | -5.9 |
| $g_3 [\Sigma^0 \Xi^-] (0)$ | -28.6 | -25.1 | -25.4 | -25.6 | -25.8 |

Table 4
Figure 6
Figure 7
Figure 8