Appliance of Inertial Gas-Dynamic Separation of Gas-Dispersion Flows in the Curvilinear Convergent-Divergent Channels for Compressor Equipment Reliability Improvement

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Abstract. The new methods of vibration and inertial gas-dynamic separation of gas-condensate and dusty flows and the corresponding separation devices are proposed in order to avoid emergencies and premature wear of parts and components of the compressor equipment. The formation of the gas flow and disperse particles in the curvilinear convergent-divergent channels are investigated. The optimizing hydrodynamic profiling of a geometrical configuration of curvilinear separation channels with rigid and flexible walls of baffles is carried out.

1. Introduction

Application of the highly effective gas-separation equipment for cleaning gas-condensate and dusty flows is one of the important and necessary techniques for increasing the reliability of compressor equipment for compression and pumping of gases. The occurrence of dropped liquid and mechanical impurities in gas flow leads to emergencies and premature wear of rotor parts, impellers of compressors and superchargers. In terms of evaluation of specific energy consumption and separation efficiency, ways of inertial and inertial-filtering separation are optimal [1,2]. Herewith, it is necessary to consider that the secondary processes accompanying the main process of separation have the significant impact on efficiency and hydraulic resistance of separation devices [2-4].

Due to complexity of secondary processes and their diversity, they remain insufficiently studied despite of their decisive practical importance. Most often secondary processes have a negative impact on the efficiency and intensity of the main separation process, and decrease hydraulic and separation characteristics of the compressor equipment. However, the secondary processes in inertial-filtering separation devices change the separation degree of gas-liquid flow due to various factors, such as co-deposition caused by manifestation of adhesion effects, when drops are settled mainly on already deposited ones, which form an irregular liquid film on the surface of channels; coagulation of...
dispersed particles, when highly dispersed particles move to further coalesce into larger drops with following accumulation of fluid in cavities of blinds and crossing fiber filter; mechanical vibrations and deformations, which impacton the effectiveness of the inertial collision and hydraulic resistance; capillary phenomena and mechanical destruction of fiber filters due to solid particles; secondary entainment of particles from the curved inertia-filtering separation channels by turbulent gas flow [3,5,6].

Notwithstanding the foregoing, along with the negative impact of secondary processes, positive effects can be obtained. For example, in the inertial gas-dynamic separation devices with curvilinear convergent-divergent separation channels, the separation degree can be increased due to coagulation and flocculation of the weighed liquid drops and solid particles in a turbulent gas flow [8,13].

Consequently, implementation of processes of inertial gas-dynamic separation is accompanied by setting as the aim of mutual avoidance of negative impact and using positive impact of secondary processes for an intensification and increasing efficiency of the main separation process, as well as improvement of hydraulic and separation parameters of the equipment.

To achieve the abovementioned aim, two approaches are proposed:

1) implementation of the method of imposing acoustic vibrations on the gas-disperse flow, which causes an intensive mechanical vibration of the high-disperse weighed liquid drops and solid particles in gas-disperse flow for sharp increasing the number of collisions and coagulation / flocculation of droplets / particles due to acoustic coagulation into drops of significantly larger sizes, which can be captured more efficiently due to mechanisms of inertial trapping, which allows increasing the efficiency and intensity of the separation process [7,9,14];

2) application the curvilinear separation channels with elastic and mobile elements, such as channel borders or directing baffles to the design of inertial gas-dynamic separation devices [10-12] for dynamic regulation of the flow cross-section in curvilinear convergent-divergent channel, the velocity of a gas-disperse flow, as well as for deflection of flow trajectories or change the attack angle of separation elements for increasing the efficiency of separation in a wide range of loading of a gas phase.

Combining the abovementioned approaches in order to create an effect, similar to the acoustic coagulation as a source of vibration is proposed for application of mechanical vibrations directly to the borders of convergent-divergent channels [15].

Furthermore, the objective of this work is the mathematical formulation and solving the problem of modeling the gas-dispersion flow motion in curvilinear separation channels with curved borders, which oscillate in the transverse direction.

2. General Problem Description and Mathematical Model
The system of Navier-Stokes and continuity differential equations can be applied for describing the motion of a continuous phase in curvilinear channels (Figure 1) with oscillating deformable borders. Analytical solving this system is realized under the following assumptions and simplifications:

a) two-dimensional isothermal viscous incompressible liquid flow in the curvilinear channel of infinite length;

b) small oscillations of deformable borders;

c) neglecting the convective terms of the Navier-Stokes equation.
Figure 1. The design model of louver unit and the separate sections of curvilinear separation channels: (a) – curvilinear inertial separation channels; (b) – considered section.

Considering the abovementioned simplifications and assumptions allows rewriting these equations:

– the law of mass conservation as the continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(1)

– the system of Navier-Stokes equations:

$$\begin{cases}
\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \\
\frac{\partial v}{\partial t} = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)
\end{cases}$$

(2)

where

- $u, v$ – components of flow velocity in the directions of axes $x, y$ respectively, m/s;
- $p$ – pressure at the considering point, N/m$^2$.

It should be noted that in such problem shaping, the components $u$ and $v$ and pressure $p$ are function of coordinates $x, y$ and time $t$.

To solve differential equations (1), (2), the initial and boundary conditions are taken into account:

$$\begin{cases}
u(0, y, t) = u_0(y) & u(x, 0, t) = 0 & u(x, h, t) = 0 \\
\nu(0, y, t) = 0 & \nu(x, 0, t) = \nu(x, t) & \nu(x, h, t) = 0 \\
p(0, y, t) = p_0
\end{cases}$$

(3)

where

- $u_0(y)$ – flow velocity profile in the inlet section of the channel, m/s;
- $h$ – characteristic linear dimension (height, width) of the channel, m;
- $f(x)$ – function of wall oscillation form;
- $\Phi(t)$ – time function for oscillating border;
- $p_0$ – inlet pressure, Pa.

3. Solving the Equations

For solving the equations (1), (2) under the boundary conditions (3) the Laplace transform is used with respect to coordinate $x$:
\[ U_f(y, t) = L_x[u(x, y, t)] = \int_0^\infty u(x, y, t) e^{-st} \, dx, \quad U(y) = L_y[U_f(y, t)] = \int_0^\infty U_f(y, t) e^{-st} \, dt \]

\[ V_f(y, t) = L_x[v(x, y, t)] = \int_0^\infty v(x, y, t) e^{-st} \, dx, \quad V(y) = L_y[V_f(y, t)] = \int_0^\infty V_f(y, t) e^{-st} \, dt \]

\[ P_f(y, t) = L_x[p(x, y, t)] = \int_0^\infty p(x, y, t) e^{-st} \, dx, \quad P(y) = L_y[P_f(y, t)] = \int_0^\infty P_f(y, t) e^{-st} \, dt \]

where

\[ U_f, V_f, P_f, U, V, P \text{ – Laplace integrals;} \]

\[ s, \tau \text{ – parameters of transformation.} \]

Consequently, the continuity equation (1), the system of Navier-Stokes equations (2) and boundary conditions (3) take the following form:

\[
\begin{align*}
\mathbf{s} U - \frac{U_y}{\tau} + V' &= 0 \\
\mathbf{s} P - \frac{p_y}{\tau} + \rho \tau U &= \mu \left( s^2 U - \frac{U_y}{\tau} + V^2 \right) \\
P' + \rho \tau U &= \mu \left( s V + V^2 \right) \\
U(0) &= 0 \quad U(h) = 0 \quad V(0) = L_x \quad V(h) = 0
\end{align*}
\]

where

\[ \ll \text{-- sign for the differentiation with respect to coordinate } y; \]

\[ L_x, L_t \text{ – Laplace integrals of functions } f(x) \text{ and } \phi(t) \text{ in terms of appropriate parameters.} \]

The first equation of the system (5) allows obtaining Laplace integral \( U \):

\[ U = \frac{u_0}{s} - \frac{V'}{s} \quad (6) \]

and Laplace integral \( P \) can be obtained from the second equation (5):

\[ P = \frac{1}{s} \left[ \frac{p_0}{\tau} + \mu \left( -s V' + \frac{u_0^2}{s} - \frac{V^2}{s} \right) - \rho \tau \left( \frac{u_0}{s} - \frac{V'}{s} \right) \right] \quad (7) \]

as well as the third equation of the system (5) due to formulas (6) and (7) after equivalent transformations takes the form:

\[ V'' + 2s^2(1 - k) JV' + s^4(1 - 2k) JV = \frac{u_0^2}{\tau} - \frac{u_0'}{v} \quad (8) \]

where \( k = \tau/(2vs^2) \) – dimensionless parameter, which takes into account the influence of local inertia forces.

It should be noted that that in case of \( \kappa \gg 1 \), solution of the general equation (8) satisfied the boundary conditions is proportional to \( \sinh(s(h-y))/\sinh(sh) \), and the original function \( v = 0 \) is contrary to the physical meaning of the problem. Further consideration of equation (8) for the case of \( \kappa \ll 1 \) corresponds to the quasi-stationary problem statement due to the final value theorem.

In case of parabolic flow velocity profile in the channel

\[ u_0 = \frac{6q_0}{h^2} h y (h - y) \quad (9) \]

where \( q_0 \) - mass flowrate of a fluid, allows determining the flow rate:
q_0 = \int_0^h u(y) \, dy \quad (10)

Due to taking into consideration the velocity profile, equation (8) can be given in the form of inhomogeneous equation biquadratic equation:

\[ V'''' + 2s^2V''' + s^4V = -\frac{u_0'}{v} \quad (11) \]

the general solution of which

\[ V(y) = C_1 \sin sy + C_2 \cos sy + sy(C_3 \sin sy + C_4 \cos sy) - \frac{u_0'}{v s^3} \quad (12) \]

Substitution formula (12) to equation (6) allows obtaining the resulting expression in the form:

\[ U(y) = \frac{u_0(y)}{s\tau} - C_1 \cos sy + C_2 \sin sy - C_3(\sin sy + sy \cos sy) - C_4(\cos sy - sy \sin sy) \quad (13) \]

The constants of integration \( C_{1,2,3,4} \) are determined from the boundary conditions \( u_0(0) = u_0(h) = 0 \) as the result of algebraic linear equations solutions:

\[ C_1 = -C_4 = -\frac{sh + \frac{1}{2} \sin 2sh}{s^2 h^2 - \sin^2 sh} s L_i L_i + \frac{2 \sin^2 sh}{sh + \sin sh} \frac{u_0'}{v s^3} \]

\[ C_2 = L_i L_i + \frac{u_0'}{v s^3} \]

\[ C_3 = -\frac{\sin 2sh}{s^2 h^2 - \sin^2 sh} s L_i L_i + \frac{\sin sh}{sh + \sin sh} \frac{u_0'}{v s^3} \]

and correspondingly can be written as

\[ U(y) = U_1(\tau) + L_i L_i F_1(y) + \frac{u_0'}{v} F_2(y) \]

\[ V(y) = L_i L_i F_1(y) \]

\[ P(y) = P_0 \frac{s\mu}{s\tau} \left[ \frac{u_0''(y)}{v} + \frac{u_0'(y)}{v} \right] + \mu L_i L_i s \left[ F_1'(y) \right] \]

where Laplace integrals are introduced:

\[ F_1(y) = \frac{1}{s} \left[ \frac{sh + \frac{1}{2} \sin 2sh}{s^2 h^2 - \sin^2 sh} \sin sy + \frac{\sin^2 sh}{s^2 h^2 - \sin^2 sh} (\sin sy + sy \cos sy) + \sin sy \right] \]

\[ F_2(y) = \frac{1}{s} \left[ \frac{1}{s} \frac{\sin sy}{sh + \sin sh} \frac{\sin sy}{sh + \sin sh} + \sin sy \right] \]

\[ F_3(y) = \frac{1}{s} \left[ \frac{\sin sy}{sh + \sin sh} \frac{\sin sy}{sh + \sin sh} - \frac{\sin^2 sh}{s^2 h^2 - \sin^2 sh} \sin sy + \cos sy \right] \]

\[ F_4(y) = \frac{1}{s} \left[ \frac{\sin sy}{sh + \sin sh} + \sin sy + sy \cos sy \right] \]

\[ F_5(y) = \frac{1}{s} \left[ \frac{2 \sin^2 sh}{sh + \sin sh} (\sin sy + sy \cos sy) + \frac{\sin sh}{sh + \sin sh} \sin sy + \cos sy - 1 \right] \]

\[ F_6(y) = F_1(y) + \frac{F_1''(y)}{s^2} \quad F_7(y) = u_0' F_2(y) + \frac{[u_0' F_2'(y)]''}{s^2} \]
Analytical definition of original functions by Riemann-Mellin’s formulas is not possible. In this case due to the conditions of absolute convergence of \( F_{U_{1,2}} \), \( F_{V_{1,2}} \), the original functions can be determined as auxiliary images by the residue theorem:

\[
f_{u_{1,2}}(x, y) = \text{Re} s(e^{s} F_{U_{1,2}}) = \frac{1}{(n-1)!} \lim_{s \to s_{0}} \sum_{l=0}^{n-1} \frac{d^{n-l}}{ds^{n-l}} \left[(s-s_{0})^{n} e^{s} F_{U_{1,2}}\right]
\]

\[
f_{v_{1,2}}(x, y) = \text{Re} s(e^{s} F_{V_{1,2}}) = \frac{1}{(n-1)!} \lim_{s \to s_{0}} \sum_{l=0}^{n-1} \frac{d^{n-l}}{ds^{n-l}} \left[(s-s_{0})^{n} e^{s} F_{V_{1,2}}\right]
\]

\[
f_{p_{1,2}}(x, y) = \text{Re} s(e^{s} F_{p_{1,2}}) = \frac{1}{(n-1)!} \lim_{s \to s_{0}} \sum_{l=0}^{n-1} \frac{d^{n-l}}{ds^{n-l}} \left[(s-s_{0})^{n} e^{s} F_{p_{1,2}}\right]
\]

where

\( s_{0} = 0 \) – the only root of equation \( s^{2}h^{2} - \sin^{2}sh = 0 \) and \( s^{2}(sh + \sin sh) \) of the order \( n = 4 \).

Application of the L’Hôpital’s rule for regular functions of a complex variable \( s \), which turn into uncertainty \{0/0\} allows receiving dependences (17) in an explicit form. Taking into account the abovementioned dependences for auxiliary images, it can be obtained:

\[
f_{u_{1}}(x, y) = 6 \left(\frac{h - y}{h}\right) \delta(x) 
\]

\[
f_{v_{1}}(x, y) = \left(1 - \frac{y}{h}\right)^{2} \left(1 + \frac{2y}{h}\right) \delta(x) 
\]

\[
f_{p_{1}}(x, y) = \frac{3x^{2}}{h^{2}} 
\]

which corresponds to Laplace integrals:

\[
F_{U_{1}}^{*}(y) = \frac{6}{h^{3}} \delta(x) 
\]

\[
F_{V_{1}}^{*}(y) = \left(1 - \frac{y}{h}\right)^{2} \left(1 + \frac{2y}{h}\right) \delta(x) 
\]

\[
F_{p_{1}}^{*}(y) = \frac{6}{h^{3}} 
\]

where \( \delta(x) \) – Dirac delta function. The sign \( \ast \) shows the alternative variant of Laplace integral notation.

Application of the inverse Laplace transformation to the formulas (16) with respect to parameter \( \tau \), and taking into account the dependencies (19) for \( t \gg 0 \) allows obtaining:

\[
U_{1}(y, t) = \frac{u_{0}(y)}{s} + L_{\phi}(t)F_{U_{1}}(y) 
\]

\[
V_{1}(y, t) = L_{\phi}(t)F_{V_{1}}(y) 
\]

\[
P_{1}(y, t) = \frac{p_{0}}{s} + \mu \left[\frac{u_{0}}{s} + sL_{\phi}(t)F_{p_{1}}(y)\right] 
\]

Using the Duhamel’s integral and theorem of convolution for Laplace integrals

\[
f(x) \ast \delta(x) = \int_{0}^{x} f(x - \xi) \delta(\xi) d\xi = f(x) 
\]

\[
L_{\ast}^{t}\left[\frac{L_{\ast}^{t}}{s} \right] = \int_{0}^{t} f(\xi) d\xi = F_{1}(x) 
\]

\[
L_{\ast}^{t}\left[\frac{L_{\ast}^{t}}{s} \right] = \int_{0}^{t} f(\xi) d\xi = F_{2}(x) 
\]

allows determining the initial original function.
\begin{equation}
\begin{aligned}
\mathbf{u}(x, y, t) &= u_o(y) + \frac{6y}{h^3} F'(x) \phi(t) \\
\mathbf{v}(x, y, t) &= \left(1 - \frac{y}{h}\right)^2 \left(1 + \frac{2y}{h}\right) f(x) \phi(t) \\
p(x, y, t) &= p_o + \mu \left[ u_o''(y) x - \frac{12}{h^3} F_2(x) \phi(t) \right] + \frac{6 \mu y (h - y)}{h^3} f(x) \phi(t)
\end{aligned}
\end{equation}

Taking into account the sinusoidal form of borders of curvilinear channel with the wavelength $L_0$, and under assumption of mono-harmonic oscillations with amplitude $a$ and vibration frequency $\omega_0$,

\begin{equation}
f(x) = \sin(\lambda x) \quad \phi(t) = a \sin(\omega_0 t)
\end{equation}

where $\lambda = \frac{2\pi}{L_0}$ – introduced parameter, the following expressions for the pressure field and components of flow velocity can be obtained:

\begin{equation}
\begin{aligned}
\mathbf{u}(x, y, t) &= \frac{6y}{h^3} q_o + \frac{a}{h^3} \left(1 - \cos\lambda x\right) \sin\omega_0 t \\
\mathbf{v}(x, y, t) &= \left(1 - \frac{y}{h}\right)^2 \left(1 + \frac{2y}{h}\right) a \sin(\lambda x) \sin(\omega_0 t) \\
p(x, y, t) &= p_o + \frac{12 \mu}{h^3} q_o x + \frac{\lambda x - \sin(\lambda x)}{\lambda^2} + \frac{6 \mu y (h - y)}{h^3} a \sin(\lambda x) \sin(\omega_0 t)
\end{aligned}
\end{equation}

Due to the fact, that the elastic borders of the channel are used as a source of vibrations, the spatial position of which is changed by the flow, it is necessary to investigate not only the flow motion in these channels, but wall deformations.

Two-dimensional problem of deformation of a curvilinear elastic element under the influence of pressure in stationary formulation is considered in the work [10]. The fields of velocities is defined, and analytical dependences for determination of the maximum deflection of elastic borders is obtained, which allows improving the geometry of borders and sections of curvilinear convergent-divergent channels in the inertial gas-dynamic separation devices as a result of their deformation.

4. Conclusions
A possibility of mutual avoiding of negative impact and using of positive effects of the secondary processes on the main process of inertial gas-dynamic separation is obtained for improvement of hydraulic and separation parameters. Ways for implementation of vibration coagulation and gas-dynamic separation are proposed.

Expressions for the flow velocity and pressure fields in case of isothermal incompressible flow on the channel of a sinusoidal form with small oscillating borders have been defined analytically. For achieving the main aim of the research, unification of abovementioned approaches and using the elastic walls of curvilinear channels as the source of vibration is offered. Theseparation of gas-disperse flows was considered as a way to make possible carrying out coagulation not only of high-disperse fogs, but also effective cleaning of dusty flows.

References
[1] Sklabinskyi V, Liaposchenko O, Pavlenko I, Demianenko M 2015 Solution of the Navier–Stokes equations for the processes of inertial gas-dynamic separation in the curvilinear channels Proceedings of the International Symposium “Discrete Singularities Methods in Mathematical Physics”, Kharkiv, V. N. Karazin Kharkiv National University pp 232–235
Sklabinskyi V I, Liaposhchenko O O, Nastenko O V, Al-Rammahi M M 2014 Modeling and
designing of the inertial-filtering gas separators- sondensers for compressor units of oil and gas
industry Applied Mechanics and Materials, Problems of Mechanics in Pump and Compressor
Engineering 630 pp 117–123

[3] Liaposhchenko O 2016 The theoretical basics of inertial-filtering separation DSc. thesis, 05.17.08,
Lviv Polytechnic National University

[4] Motin A, Tarabara V V, Petty C A, Benard A 2017 Hydrodynamics within flooded hydrocyclones
during excursion in the feed rate Understanding of turndown ratio. Separation and Purification
Technology 185 pp 41–53

[5] Liaposhchenko O, Sklabinsky V, Logvin A 2009 Inertial-filtering separation equipment for
thermochemical processing of oil sludge of oil production Proceedings of the N Lviv Polytechnic
National University, Heat and power. Environmental Engineering. Automatization 659
pp 148–150

[6] Setnicky K, Sima V, Petrychkovych R, Reznickova J, Uchytil P 2016 Separation of gas mixtures
by new type of membranes Dynamic liquid membranes, Separation and Purification Technology
160 pp 132–135

[7] Sklabinsky V, Liaposhchenko O, Logvyn A, Al-Rammahi M 2014 Hydrodynamics modeling of the
gas separator’s inertial and filtering elements for natural gas fine cleaning Chemistry & Chemical
Technology 8 (4) pp 479–485

[8] Nastenko O, Broniarz-Press L, Liaposhchenko O 2016 Mathematical modelling of separation
process by coupled heat transfer in the inertial-filtering gas separator-condenser Engineering and
Chemical Equipment 2 pp 62–63

[9] Liaposhchenko O, Pavlenko I, Demianenko M 2015 Investigation of forced oscillations of gas-
dynamic separator baffles Scientific papers of the International Scientific and Technical
Conference “Young Engineer – the Basis of Scientific and Technical Progress”, Kursk, South-West
State University pp 262–265

[10] Liaposhchenko O, Pavlenko I, Nastenko O, Usik R, Demianenko M 2015 The method of capturing
highly dispersed dropped of gas-liquid flow Patent 102445, Ukraine, B01D 45/04 (2006.01),
u20150512420

[11] Sklabinsky V I, Liaposhchenko O O, Lohvyn A V, Skydanenko M S 2011 Device for capturing
highly dispersed liquid drops from the gas-liquid flow Patent 57386, Ukraine, B 01 D 45/04.
U2010094884

[12] Liaposhchenko O, Demianenko M, Pavlenko I, Sklabinsky V 2015 Solution of the Navier–
Stokes equations for the processes of inertial gas-dynamic separation in the curvilinear channels
Bulletin of V. N. Karazin Kharkiv National University, Series “Mathematical Modeling, Information
Technology. Automated Control Systems” 27 pp 53–64

[13] Liaposhchenko O, Nastenko O, Pavlenko I 2017 The model of crossed movement and gas-liquid
flow interaction with captured liquid film in the inertial-filtering separation channels Separation
and Purification Technology 173 pp 240–243

[14] Liaposhchenko O, Nastenko O 2015 Analysis of the conditions of phase equilibrium and
influence of the united heat and mass transfer on the effectiveness of separation in the inertial-
filtering separator Chemistry & Chemical Technology 9 (1) pp 125–130

[15] Jia W, Murad S 2005 Separation of gas mixtures using a range of zeolite membranes: A
molecular-dynamics study The Journal of Chemical Physics 122