Resolving information loss paradox with Euclidean path integral*  

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The information loss paradox remains unresolved ever since Hawking’s seminal discovery of black hole evaporation. In this essay, we revisit the entanglement entropy via Euclidean path integral (EPI) and allow for the branching of semi-classical histories during the Lorentzian evolution. We posit that there exist two histories that contribute to EPI, where one is information-losing that dominates at early times, while the other is information-preserving that dominates at late times. By so doing we recover the Page curve and preserve the unitarity, albeit with the Page time shifted significantly towards the late time. One implication is that the entropy bound may thus be violated. We compare our approach with string-based islands and replica wormholes concepts.

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Introduction

The information loss paradox \cite{1} remains an unresolved problem in modern theoretical physics since Hawking’s seminal discovery of black hole evaporation in 1975 \cite{2}. This problem reveals a potential inconsistency between unitary quantum mechanics and general relativity.

According to Hawking’s computation \cite{2}, a black hole will emit radiation and evaporate completely, where the radiation depends only on mass, charge, and angular momentum of the black hole. This implies the loss of information. If this indeed happens, then one would lose the predictable power of quantum mechanics. On the other hand, if one believes that unitarity should not be violated, then the information loss paradox must be resolved and explained.

To resolve this paradox, one typically invokes one or more of the following widely accepted assertions \cite{3}: (1) unitarity of the black hole evaporation process, (2) general relativity, at least far from the singularity, (3) local quantum field theory in a semi-classical black hole background spacetime, and (4) equivalence of the Bekenstein-Hawking entropy and the Boltzmann entropy. There have been arguments, however, that not all of these assertions are consistent \cite{3, 4}. In particular, a soft spot appears to be the notion of entanglement entropy and the monogamic nature of the maximal entanglement \cite{5}. Evidently, entanglement entropy, and thus Assertion (4), is the prime suspect.

In this essay, we propose a new resolution to the information loss problem a la the notion of wave function of the universe cast in the Euclidean path integral formulation. By insisting the multi-history condition and the late-time dominance condition, which will be explained later, we manage to recover the Page curve, albeit being necessarily modified with the Page time shifted significantly towards the late-time of black hole evaporation. Surprisingly, our resolution is a natural outcome of the most conservative approach to quantum gravity, i.e., canonical quantum gravity \cite{6}, without resorting to exotic means.

Entanglement entropy and Page time

To quantitatively describe the flow of information, the introduction of the entanglement entropy is found very useful \cite{7}. Let us consider a system composed of two subsystems $A$ and $B$, and a pure state given by $|\Psi\rangle$. The density matrix of the system is $\rho \equiv |\Psi\rangle\langle\Psi|$. The reduced density matrix for the subsystem $A$ is given by tracing the subsystem $B$ out, i.e., $\rho_A \equiv \text{tr}_B \rho$. Likewise,
the reduced density matrix for the subsystem $B$ is given by $\rho_B \equiv \text{tr}_A \rho$. Then the von Neumann entropy of the subsystem $A$ is $S_B(A) \equiv -\text{tr}_A \rho_A \ln \rho_A$. This is known as the entanglement entropy of a subsystem $A$, which is the same as that of its complementary, $S_A(B)$, if the state is pure.

Let us consider quantum states of a black hole. We divide the system into the interior of a black hole, denoted by $A$, and the Hawking radiation in the exterior, denoted by $B$. We assume that initially all degrees of freedom were in $A$, and, as time goes on, they are monotonically transmitted from $A$ to $B$ by the Hawking radiation. According to the analysis by Page [8], by assuming a typical pure state with a fixed number of total degrees of freedom in the beginning, the entanglement entropy is almost the same as the Boltzmann entropy of the radiated particles. However, when the original entropy of the black hole decreased to approximately its half value, the entanglement entropy of the radiation begins to decrease (left of Fig. 1). This turning time is called the Page time. If one further assumes that the Boltzmann entropy of the black hole is the same as the Bekenstein-Hawking entropy, one can compute the value of the Page time, which is approximately $\sim M^3$ (in Planck units), where $M$ is the black hole mass; it is evident that even at this time, the black hole remains semi-classical.

Trouble with entanglements

Based on general relativity (2) and semi-classical quantum field theory (3), the Hawking radiation can be interpreted as a particle-antiparticle pair creation process, where the particle and antiparticle are maximally entangled [9]. On the other hand, for a pure and random system with fixed degrees of freedom (1), after the initial Bekenstein-Hawking entropy decreased to its half-value (4), the entanglement entropy should begin to decrease. In other words, the Hawking particles after the Page time starts to be entangled, in addition, with the previously emitted Hawking particles.

Let us now assume that there is an observer who detects a Hawking particle, say $\alpha$, after the Page time. The observer will notice that the particle $\alpha$ is entangled with the previously emitted Hawking particle $\beta$, following the entanglement distillation protocol. On the other hand, this $\alpha$ must be maximally entangled with its antiparticle partner, say $\alpha'$, if the infalling observer sees a smooth horizon. This is contradictory because $\alpha$, which is maximally entangled with $\alpha'$, cannot be entangled with $\beta$ due to the monogamy nature of maximal entanglements (1) [5]. How can this inconsistency be resolved?
Wave function of the universe and superposition of states

According to canonical quantum gravity [6], the entire information of the universe is included by the wave function of the universe $\Psi$, which is a functional of the three-geometry $h_{\mu\nu}$ and a matter field configuration, say $\phi$, on top of $h_{\mu\nu}$. This wave function should satisfy the quantum Hamiltonian constraint equation, the so-called Wheeler-DeWitt (WDW) equation. Since this is the fundamental equation of quantum gravity, unitarity must be manifest.

One can assume that the in-state of the wave function of the universe is given by $|\text{in}\rangle \equiv |h_{\mu\nu}^{(\text{in})}, \phi^{(\text{in})}\rangle$, where we assume that this in-state is a fixed classical configuration. This configuration will evolve to an out-state, say $|\text{out}\rangle \equiv |h_{\mu\nu}^{(\text{out})}, \phi^{(\text{out})}\rangle$. The WDW equation will determine the wave function of the universe for a given initial condition.

In order to understand the black hole evaporation process, we need to choose a proper out-state. It should be such that the observer at future infinity will see a semi-classical spacetime. However, the final out-state is not necessarily a unique classical spacetime; rather, it can be a superposition of states corresponding to each classical spacetime [10]. This follows from the observation that the Hawking radiation can be interpreted as a result of quantum tunneling [11]. Thus

$$|\text{out}\rangle = \sum_{i, \alpha} c_{\alpha, i} |\alpha; i\rangle ,$$

(1)
FIG. 2: Left: the causal structure of the usual semi-classical black hole, where the green curve is the trajectory of the collapsing matter, the red curve is the apparent horizon, and the blue line is the event horizon. Middle: the causal structure after a quantum tunneling at the time slice $t$. After the tunneling, matter or information (red curve) is emitted and the black hole structure disappears. Right: $h_1$ is the information-losing history, while $h_2^{(1,2)}$ are the information-preserving histories. Tunneling may happen either early time (A) or late time (B); the tunneling probability must be dominated at the late time.

where $|\alpha; i\rangle$ is a quantum state associated with a semi-classical state labeled by $i$, while $\alpha$ represents microscopic quantum degrees of freedom in the semi-classical state, and $c_{\alpha,i} = \langle \alpha; i | i n \rangle$ (right of Fig. 1).

**Essential conditions for unitary entanglement entropy**

As mentioned in the above, a natural consequence from the picture based on canonical quantum gravity is that the out-state is a superposition of semi-classical states. Let us assume that one can categorize the semi-classical states into two distinct classes. The first class is those with *information-losing histories*, where the black hole keeps existing and loses information by the Hawking radiation, and therefore the entanglement entropy monotonically increases up to the end point (left of Fig. 2). The second class is those with *information-preserving histories*, which appear as a result of quantum tunneling, where there is no black hole, hence no singularity nor event horizon. Hence the entanglement entropy is zero for this class of histories (middle of Fig. 2).
When the in-state is dominated by information-losing histories, the (semi-classical) observer outside of the black hole cannot have access to the degrees of freedom inside of the black hole, which leads to the increase of the entanglement entropy. After the Page time, if the out-state is dominated by information-preserving histories, the observer can now have access to all degrees of freedom which would not have been measured in the information-losing histories. Hence, the entanglement entropy will vanish in the end. To summarize, in order to obtain a unitary Page curve, what one needs to justify is the following two essential conditions (right of Fig. 2) [12]:

1. **Multi-history condition**: existence of multiple information-preserving and non-preserving histories;

2. **Late-time dominance condition**: dominance of the information-preserving history at late-time.

For simplicity and without losing generality, let us assume that there are only two histories, one information-losing and the other information-preserving. Then one can approximately evaluate the entanglement entropy as

$$S_{\text{ent}} \simeq p_1 S_1 + p_2 S_2,$$

where 1 denotes the information-losing history, 2 denotes the information-preserving history, $p_i$ and $S_i$ ($i = 1, 2$) are the probability and the entanglement entropy for each history, respectively. In the beginning, the history 1 dominates, and hence explains the increasing phase of the entanglement entropy. However, at late times, the history 2 dominates as $S_2 = 0$. The total entanglement entropy will eventually decrease to zero.

**Realization: Euclidean path integral approach**

To confirm these essential conditions, one needs to compute the transition element $\langle i | \text{in} \rangle$, where $| i \rangle$ is a state representing a classical history. Here we have omitted the label $\alpha$ for the microscopic degrees of freedom in each classical history for notational simplicity. The probability of each history is given by $p_i = |\langle i | \text{in} \rangle|^2$. If we recover the label $\alpha$, then we have $p_i = \sum_{\alpha} |\langle \alpha; i | \text{in} \rangle|^2$.

Although we do not yet have a final formulation for quantum gravity, at the semi-classical level the Euclidean path integral can provide a good approximation that captures the essence of a bona fide full-blown quantum gravity theory [13]:

$$\langle i | \text{in} \rangle = \langle h^{(i)}_{\mu\nu}, \phi^{(i)} | h^{(\text{in})}_{\mu\nu}, \phi^{(\text{in})} \rangle = \int \mathcal{D}g_{\mu\nu} \mathcal{D}\Phi \ e^{-S_E[g_{\mu\nu}, \Phi]}, \quad (2)$$
where \( S_E \) is the Euclidean action and all Euclidean geometries that connect \(|h^{(\text{in})}_{\mu\nu}, \phi^{(\text{in})}\rangle\) to \(|h^{(i)}_{\mu\nu}, \phi^{(i)}\rangle\) are summed over. This path integral can be well approximated by summing over on-shell solutions, i.e., either Lorentzian classical solutions or Euclidean instantons.

To evaluate the evolution of the entanglement entropy, the following two technical observations are important:

- To interpret the Hawking radiation as a quantum tunneling process, one can consider the perturbation of a free scalar field in the Euclidean Schwarzschild background \([11]\). The tunneling probability is given by \( p \simeq e^{-2B} \), where \( B = S_E(\text{solution}) - S_E(\text{background}) \). If we impose the reality condition on the scalar field at future infinity, the solution becomes complex-valued. Nevertheless, there is some evidence that we may accept such a complex instanton as a legitimate classical solution that dominates the path integral \([14]\).

Assuming that the emitted energy \( \omega \) is negligible compared to the black hole mass \( M \), the probability is approximately found as \( e^{-\omega/T} \) with the Hawking temperature \( T = 8\pi M \) \([11]\). This shows that the Hawking radiation may indeed be regarded as a quantum tunneling process. If we extrapolate this result to the case when the whole mass is emitted, then the spacetime transits to Minkowski spacetime, with its probability given by \( e^{-S} \) where \( S = 4\pi M^2 \). Namely, there exists a tunneling channel to a trivial geometry. In fact, one can construct such an instanton by considering a thin-wall scalar field configuration \([15]\).

- Adopting the above thin-shell toy model as the tunneling channel to a trivial geometry, one realizes that there are instantons with multiple periods in the Euclidean time as long as one can correctly perform analytic continuation to the future infinity. Hence, these multiple period solutions should be taken into account for the computation of the tunneling probability from the information-losing history to the information-preserving history. One obtains the following:

\[
2B_n = n ((\text{bulk term of solution}) + (\text{boundary term of solution}))
- (\text{boundary term of background}),
\]

(3)

where \( n = 1, 2, 3 \cdots \).

It is now straightforward to evaluate the entanglement entropy. The tunneling probability is
FIG. 3: Left: Probabilities of the information-losing history \((p_1, \text{ blue curve})\) and the information-preserving history \((p_2, \text{ black curve})\). Right: Entropy of emitted radiation \(S_{\text{rad}}/S_0\) vs. entanglement entropy \(S_{\text{ent}}/S_0\), where \(S_0 = 3\) (black), 10 (blue), 50 (red), respectively. The thin red dashed curve is the location of the Page time, i.e., \(dS_{\text{ent}}/dS_{\text{rad}} = 0\).

Given by [12]

\[
\frac{p_2}{p_1} = \sum_{n=1}^{\infty} e^{-(2n-1)S} = \frac{1}{e^S - e^{-S}}. \tag{4}
\]

Interestingly, although the \(n = 1\) solution makes the most dominant contribution, the multi-period instanton contributions become important as \(M\) decreases. If we impose the normalization condition \(p_1 + p_2 = 1\), we obtain

\[
p_1 = \frac{e^S - e^{-S}}{1 + e^S - e^{-S}}; \quad p_2 = \frac{1}{1 + e^S - e^{-S}}. \tag{5}
\]

This successfully explains that \(p_1\) initially dominates and \(p_2\) is exponentially suppressed, while at the late times, \(p_2\) eventually dominates over \(p_1\) as \(S\) decreases. This explains the late-time dominance condition (left of Fig. 3).

Thus if we assume that the entanglement entropy of the history 1 (information-losing history) monotonically increases and the entanglement entropy of the history 2 (information-preserving history) is zero, we obtain the expectation value of the entanglement entropy, or that from the point of view of the entire wave function, as

\[
S_{\text{ent}} = p_1 \times (S_0 - S) + p_2 \times 0 = (S_0 - S) \left( \frac{e^S - e^{-S}}{1 + e^S - e^{-S}} \right); \tag{6}
\]

\(\text{FIG. 3: Left: Probabilities of the information-losing history (} p_1, \text{ blue curve) and the information-preserving history (} p_2, \text{ black curve). Right: Entropy of emitted radiation } S_{\text{rad}}/S_0 \text{ vs. entanglement entropy } S_{\text{ent}}/S_0, \text{ where } S_0 = 3 \text{ (black), 10 (blue), 50 (red), respectively. The thin red dashed curve is the location of the Page time, i.e., } dS_{\text{ent}}/dS_{\text{rad}} = 0.\)
where $S_0$ is the initial Boltzmann entropy of the black hole (right of Fig. 3).

Now we can explain the unitary evolution of the black hole system: The entanglement entropy starts from zero, monotonically increases for a while, and eventually decreases to zero. An important, interesting observation is that the turning point is far beyond the moment when the Bekenstein-Hawking entropy decreased to its half value. It occurs when $S \sim \log S_0$. In other words, the equivalence of the Bekenstein-Hawking entropy and the Boltzmann entropy is violated. However, one has to realize that this equivalence is not a result of any fundamental principles. In fact, an intriguing counter example has recently been pointed out in [16], which is perfectly consistent with both local field theory and semi-classical general relativity. In our case, it is also important to note that the turning point occurs while the semi-classical approximation is still perfectly valid provided that the initial black hole is macroscopic, $S \sim \log S_0 \gg 1$. (For example, even for a black hole of a fairly small mass $M \sim 10^5$g, $S_0 \sim 10^{20}$ and hence $S \sim 46$.)

**Recent developments based on string theory**

It is interesting to compare our approach with the recent developments based on string theory, where the entanglement entropy was investigated by evaluating several quantum extremal surfaces (QES). It was found [17, 18], in two-dimensional gravity, that a saddle of QES without an island is dominant before the Page time, which induces the increase of the entanglement entropy. After the Page time, QES is dominated by the other saddle with an island, which results in the decrease of the entanglement entropy. It was argued in [19] that such a role played by the island can also reproduce the Page curve in higher dimensional black holes.

The string-based islands approach and ours share two essential features. One, there exist more than two contributions to the evolution of the entanglement entropy, where one is dominant at early-time and the other at late-time. Two, the final entanglement entropy is dominated by the late-time condition. That is, the multi-history condition and the late-time dominance condition in our Euclidean path integral approach are analogous to the spirit of the QES computations for black hole.

However, the physical interpretation of the islands conjecture is still not very clear. One reason is that the entanglement entropy computed in QES is based on the density matrix, instead of the quantum states, which is the standard quantum field theory approach and what we have followed. It is therefore natural to ask, what is the implication of the islands or the replica wormholes
to the more orthodox, state-level path integral in Lorentzian signatures, and vice versa?

**Conclusion and future prospects**

We have argued that the canonical quantum gravity with the Euclidean path integral approach can provide a consistent picture to resolve the information loss paradox. By computing the wave function of the universe with the Euclidean path integral, we successfully justified the two essential conditions, that is, the *multi-history condition* and the *late-time dominance condition*, and eventually obtained a modified Page curve that preserves the unitarity but with the Page time shifted significantly towards the late-time.

Note that the entanglement entropy of a black hole can never exceed its Boltzmann entropy. Therefore, if one insists on Assertion (4) (equivalence between the Bekenstein-Hawking entropy and the Boltzmann entropy), then the entanglement entropy cannot exceed the Bekenstein-Hawking entropy. In contrast, one salient outcome of our computation is that there exists a moment where the entanglement entropy is greater than the Bekenstein-Hawking entropy. Assertion (4) is thus violated in our approach. This necessarily implies that the number of states inside the horizon must have been accumulated during the black hole evaporation, although such an accumulation is strictly bounded. We emphasize that this violation of Assertion (4) is not in contradiction with basic principles of physics [16], nor the first three assertions.

In our picture, the turning point of the Page curve, though shifted significantly towards the end-life of the black hole evaporation, is still in the semi-classical regime of quantum gravity as we have shown. Hence there might be a way to experimentally investigate our notion. If our model can be examined not only by theoretical means, but also by experimental methods [22], then the synergy between theory and experiment may hopefully lead us to the ultimate understanding of the information loss paradox.

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