Radiative Corrections for SMC Measurements

Arif A. Akhundov

King Fahd University of Petroleum and Minerals,
P.O. Box 1803, Hafr Al-Batin 31991, Saudi Arabia
and
Institute of Physics, Azerbaijan Academy of Sciences,
H. Cavid ave. 33, Baku 370143, Azerbaijan

ABSTRACT

A numerical study of QED radiative corrections for the SMC experiment at SPS is been performed. The semi-analytical program POLRAD is been used to get the size of the radiative corrections for the proton to get the size of the radiative corrections for the measurements of the proton spin dependent structure functions $g_1^p(x)$ and $g_2^p(x)$. A brief description of the program POLRAD is given.

1. Introduction

The asymmetry of deep inelastic scattering of polarized leptons on polarized nucleons

$$l(k_1) + N(p_1) \rightarrow l(k_2) + X(p_2),$$

(1.1)
gives important information about the spin dependent structure functions of the nucleons $g_1(x,Q^2)$ and $g_2(x,Q^2)$.

The experimental study of the asymmetry by the Spin Muon Collaboration (SMC) is based on the measurements of the scattering of polarized muons on polarized deuterons and protons.\(^2\)-\(^4\) In the data analysis of this experiment the Fortran program POLRAD\(^5\) is used for the calculation of the radiative corrections (RC) from higher order QED processes (Fig. 1) for the cross section asymmetries

$$A_{\parallel} = \frac{\sigma^{\uparrow\downarrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\uparrow\downarrow} + \sigma^{\uparrow\uparrow}}$$

(1.2)

and

$$A_{\perp} = \frac{\sigma^{\uparrow\rightarrow} - \sigma^{\uparrow\leftarrow}}{\sigma^{\uparrow\rightarrow} + \sigma^{\uparrow\leftarrow}}$$

(1.3)

where $\sigma^{\uparrow\downarrow}, \sigma^{\uparrow\uparrow}$ ($\sigma^{\uparrow\rightarrow}, \sigma^{\uparrow\leftarrow}$) are the cross sections for inclusive deep inelastic scattering of longitudinally polarized muons off longitudinally (transversely) polarized nucleons.
The radiative corrections are important for the higher statistics experiments and are dependent on the methods used to extract the wanted cross section from the data.

The fixed target experiments on deep inelastic lepton-nucleon scattering mainly use the energy \( E' \) and the angle \( \theta \) of the scattered lepton to determine the kinematics of the events, i.e. the analysis of deep inelastic events is based on the measurement of the familiar leptonic variables

\[
Q^2_l = -(k_1 - k_2)^2, \quad y_l = p_1(k_1 - k_2)/p_1 k_1, \quad x_l = Q^2_l/(S y_l),
\]

with

\[
S = (k_1 + p_1)^2 = 2 E E. \tag{1.5}
\]

Here \( E \) is the energy of the incident lepton in the laboratory system, \( \vec{p}_1 = 0 \), and \( M \) is the mass of the target.

The methods for the calculation of the RC for deep inelastic scattering of leptons on unpolarized nucleons in terms of the leptonic variables are well known.\(^6,7\) Both methods are based mainly on model-independent approaches. The model-independent treatment of the RC uses the phenomenological structure functions of nucleons, which are defined in the Born approximation. A detailed comparison\(^8\) has shown that the agreement between the two methods is better than 2 \%.

The model-independent radiative corrections in terms of the variables \( x_l, Q^2_l \) to the cross section of deep inelastic scattering of polarized leptons by longitudinally polarized nucleons were calculated in.\(^9\) The formulae derived in these papers were generalized in\(^10\) to take into account the case of a transversely polarized target and the scattering on polarized spin-1 targets.

A new development for the model-independent approach in the calculations of the RC for deep inelastic scattering has been arised for the experiments at HERA where for the physical analysis of deep inelastic events one could use not only the leptonic variables (1.4) but also the kinematical variables derived from the hadron measurements

\[
Q^2_h = -(p_2 - p_1)^2, \quad y_h = p_1(p_2 - p_1)/p_1 k_1, \quad x_h = Q^2_h/(S y_h), \tag{1.6}
\]

or some mixture of both.

The theoretical investigations of the radiative corrections for the experiments at HERA have been summarized in the Proceedings of the Workshop ”Physics at HERA”\(^11\), and the different programs for the calculation of the RC have been cross-checked.\(^12\)

The model-independent treatment\(^13\) of leptonic QED corrections for neutral current \( ep \)-scattering, including the bremsstrahlung process

\[
l(k_1) + N(p_1) \to l(k_2) + X(p_2) + \gamma(k), \tag{1.7}
\]

the elastic radiative tail

\[
l(k_1) + N(p_1) \to l(k_2) + N(p_2) + \gamma(k). \tag{1.8}
\]
and QED vertex corrections have been done in terms of several variables - leptonic, hadronic, mixed and using the Jacquet-Blondel variables.  

The new formulae were implemented in the program package TERAD91. The program TERAD91 can be applied to calculate the RC factor of the order $\mathcal{O}(\alpha)$ to the measured differential cross section, $d^2\sigma^{\text{meas}}/dxdQ^2$. After the acceptance and smearing corrections it reduces to

$$d^2\sigma^{\text{meas}}/dxdQ^2 = d^2\sigma^{\text{Born}}/dxdQ^2 (1 + \delta(x, Q^2)).$$

(1.9)

The semi-analytical program TERAD91 has been used to evaluate the radiative corrections for the first measurement of the proton structure function $F_2(x, Q^2)$ in the range $x = 10^{-2} - 10^{-4}$ and $5 \text{ GeV}^2 < Q^2 < 80 \text{ GeV}^2$ using the electron and the mixed kinematical variables.

The radiative correction factor $\delta(x, Q^2)$ for the inclusive cross section of deep inelastic scattering is defined by the equation

$$\delta(x, Q^2) = \frac{d^2\sigma^{\text{theor}}/dxdQ^2}{d^2\sigma^{\text{Born}}/dxdQ^2} - 1,$$

(1.10)

where $d^2\sigma^{\text{theor}}/dxdQ^2$ is the theoretical approximation for the measured cross section, $d^2\sigma^{\text{meas}}/dxdQ^2$, which contains contributions from higher order QED and electroweak processes, and $d^2\sigma^{\text{Born}}/dxdQ^2$ is the Born cross section of the process (1.1).

In this note we describe the semi-analytical program POLRAD which has been used to calculate the radiative corrections for the measurements of the spin dependent proton structure functions $g_1^p(x)$ and $g_2^p(x)$ by the SMC experiment.

### 2. The semi-analytical program POLRAD

For $Q^2 \ll M_z^2$ only electromagnetic processes are relevant, and the unpolarized part of the Born cross section of the deep inelastic scattering (1.1) is determined by two structure functions $F_2(x, Q^2)$ and

$$2xF_1(x, Q^2) = \frac{(1 + \rho^2)F_2(x, Q^2)}{1 + R(x, Q^2)},$$

(2.1)

$$\frac{d^2\sigma_0^{\text{unpol}}}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4x} \left[ 1 - y + \frac{y^2}{2(1 + R(x, Q^2))} \right] F_2(x, Q^2),$$

(2.2)

Here and below

$$x \equiv x_l, \quad y \equiv y_l, \quad Q^2 \equiv Q_l^2,$$

(2.3)

and

$$\rho^2 = \frac{Q^2}{\nu^2} = \frac{2Mx}{Ey}.$$  

(2.4)
The spin dependent part of the Born cross section has contributions from the structure functions $g_1$ and $g_2$ as:

$$\frac{d^2\sigma_{\uparrow\downarrow}^0}{dx dQ^2} = \frac{8\pi\alpha^2}{Q^4} y \left( 1 - \frac{y}{2} - \frac{\rho^2 y^2}{4} \right) g_1(x) - \frac{\rho^2 y^2}{2} g_2(x) , \quad (2.5)$$

when the beam and target spins are collinear, and

$$\frac{d^2\sigma_{\uparrow\rightarrow\downarrow}^0}{dx dQ^2} = \frac{8\pi\alpha^2}{Q^4} y\rho \sqrt{1 - y - \frac{\rho^2 y^2}{4}} \left[ \frac{y}{2} g_1(x) + g_2(x) \right] , \quad (2.6)$$

when the spin directions of beam and target are orthogonal.

The program POLRAD contains the formulae for leptonic QED corrections to the cross sections (2.2),(2.5) and (2.6). These corrections include photonic bremsstrahlung from leptons, vertex corrections (see Fig.1, only $\gamma$-exchange ) and vacuum polarization of the order $O(\alpha)$ with soft-photon exponentiation. This set of the higher order Feynman graphs is standard for the model-independent treatment of the QED RC to deep inelastic scattering\cite{7} and gives the numerically largest contribution.

The general formula for the radiative cross sections expressed in terms of leptonic variables can be written as a sum of two parts:

$$\frac{d^2\sigma^{O(\alpha)}}{dx dQ^2} = \frac{d^2\sigma^{\text{Born}}}{dx dQ^2} (1 + \delta^{VR}(x, Q^2)) + \int \int dx_h dQ^2_h H(x, Q^2, x_h, Q^2_h) \frac{d^2\sigma^{\text{Born}}}{dx_h dQ^2_h} \quad (2.7)$$

The first part of (2.7), proportional to $\delta^{VR}$, contains the contributions from vertex corrections, vacuum polarization and from the soft part of the real photon radiation. The second part accounts for the bremsstrahlung of hard photons. It depends on the structure functions, not only in a given $F(x, Q^2)$ point, but in the range of $(x_h, Q^2_h)$ defined by kinematics of Fig. 2a.

For radiative events

$$x \leq x_h \leq 1, \quad M^2 \leq M_h^2 \leq W^2 , \quad (2.8)$$

were the square of the invariant mass of the hadronic final state $M_h^2$ can be defined as

$$M_h^2 = M^2 + Q_h^2 \left( \frac{1 - x_h}{x_h} \right) , \quad (2.9)$$

and the invariant mass

$$W^2 = M^2 + Q^2 \left( \frac{1 - x}{x} \right) , \quad (2.10)$$

corresponds to the hadronic jet without radiation.

From the experience of the studies of the RC for deep inelastic scattering\cite{7,12} it is known that the large radiative corrections are mainly due to the emission of hard
photons (the second part of (2.7)). For hard photon bremsstrahlung in the process (1.7) and (1.8) the kinematic minimum of $Q_h^2$ is

$$(Q_h^2)_{\text{min}} \simeq x^2 M^2.$$  \hfill (2.11)

This formula shows that it is impossible to calculate the RC without the knowledge of the nucleon structure functions in the region $Q^2 \to 0$.

The program POLRAD, instead of using the usual $M^2_h$ and $Q^2_h$, or $x_h$ and $Q^2_h$, (Fig. 2a) is using other invariant kinematical variables (Fig. 2b):

$$R = W^2 - M^2 + Q^2 - Q_h^2, \quad \tau = \frac{Q_h^2 - Q^2}{W^2 - M^2 + Q^2 - Q_h^2},$$  \hfill (2.12)

for the calculation of the two-dimensional integral in (2.7) – the contribution of the inelastic radiative tail. The boundaries of these variables are:

$$0 \leq R \leq \frac{W^2 - M^2}{1 + \tau},$$  \hfill (2.13)

and

$$\tau_{\text{max, min}} = \frac{S y}{2 M^2} (1 \pm \sqrt{1 + \frac{M^2 x}{S y}}).$$  \hfill (2.14)

The point $F(x, Q^2)$ of the Fig.2a in which we want to calculate the RC has been transformed into the line $R = 0$ from $\tau_{\text{min}}$ to $\tau_{\text{max}}$ of the Fig. 2b, and the Born kinematics of the process (1.1) is defined on this line. The contribution of the elastic radiative tail (1.8) is determined by the integral on the line $R = (W^2 - M^2)/(1 + \tau)$.

Now we briefly explain how the program POLRAD works. For any point $F(x, Q^2)$ of deep inelastic scattering the POLRAD routine CONKIN evaluates the Born kinematics (2.12)-(2.14) and routine BORNIN gives the Born cross sections (2.2)-(2.6). DELTAS calculates the factorized part of the RC $\delta^{VR}$ which is independent on the polarization of the beam or target. The two-dimensional integral in (2.7) is calculated in terms of the variables (2.12) by the routines QQT, QQINT, TAILS and FFU. The routines STRF and COMPST contain the parameterizations of the structure functions $F_{1,2}$, $g_{1,2}$ and also elastic and quasi-elastic form factors for the calculation of their contribution to the process (1.8). In the last two routines there is the possibility of putting other parameterizations of structure functions and form factors. The main routine POLRAD calculates the final results – the RC to the asymmetries.

3. Numerical results and conclusion

In this section we will give the results of the calculations of the RC with the help of the program POLRAD to the longitudinal $A^\parallel_p$ and transverse $A^\perp_p$ asymmetries in inclusive deep inelastic scattering of polarized muons off polarized protons.

In order to calculate the RC one has to use the realistic parameterizations of the structure functions $F_{1,2}$ and $g_{1,2}$ over the full range of $x$ and $Q^2$. Several different
parameterizations of the unpolarized structure functions, $F_2^p(x, Q^2)$, including the NA37 parametrization,\textsuperscript{19} have been used. For the region $Q^2 < 0.2$ GeV$^2$ the parameterizations of $F_2(x, Q^2)$ were taken at $Q^2 = 0.2$ GeV$^2$ and multiplied by the suppression factor\textsuperscript{20} $(1 - \exp(-aQ^2))$, where $a = 3.37$ GeV$^{-2}$. The $F_1^p(x, Q^2)$ is calculated by (2.1), where the values of $R(x, Q^2)$ were taken from the fit of the SLAC data.\textsuperscript{21}

The longitudinal spin structure function, $g_1^p$, can be obtained from the asymmetry $A_1^p$ by the relation

$$ g_1^p(x, Q^2) = \frac{A_1^p(x, Q^2)F_2^p(x, Q^2)}{2x[1 + R(x, Q^2)]}, \quad (3.1) $$

where $A_1^p$ is the virtual photon asymmetry which can be measured experimentally. But here we will use the model parameterizations for $g_1^p$ from\textsuperscript{22–24}

For the calculation of the transverse asymmetry, $A_1^p$, $g_2^p$ is needed. It can be estimated by equation (23) of ref.\textsuperscript{1} if we neglect the contribution of the last term of this equation.

Figures 3-9 shows the results for the quantities

$$ \frac{A_1^p}{D_\parallel}, \quad \frac{\Delta A_1^p}{D_\parallel}, \quad \frac{A_1^p}{D_\perp}, \quad \frac{\Delta A_1^p}{D_\perp}, \quad (3.2) $$

where $\Delta A_1^p$ is the difference between the asymmetries with and without the radiative corrections

$$ \Delta A_1^p = A_1^p(Born + QED \text{ RC}) - A_1^p(Born). \quad (3.3) $$

and $D_{\parallel,\perp}$ are the depolarization factors

$$ D_{\parallel} = \frac{y(2 - y)}{y^2 + 2(1 - y)(1 + R)}, \quad D_{\perp} = \frac{2y\sqrt{1 - y}}{y^2 + 2(1 - y)(1 + R)}. \quad (3.4) $$

The calculations presented here have been performed with the NA37 parametrization\textsuperscript{19} for $F_2(x, Q^2)$.

From Figures 5 and 6 it is seen that for the longitudinal polarization of the protons we get different results for $\Delta A_1^p/D_\parallel$ and we see a strong dependence of the RC on the choice of the $g_1^p$ parametrization. The RC are not small at low $x$: $|\Delta A_1^p/D_\parallel| \leq 0.025$. At low $x$ the influence of the hard bremsstrahlung from the processes (1.7) and (1.8) on the value of the RC is very big. For the results which were obtained from POLRAD in the case of the transverse polarization of the target we have used the $g_1^p$ parametrization from\textsuperscript{24} and have calculated $g_2^p$ from eq. 23 of.\textsuperscript{1} The transverse asymmetry with such input for $g_1^p, g_2^p$ in the Born approximation looks good for the who kinematical region. But for the QED corrected asymmetry $A_1^p/D_\perp$ (Fig. 8) and for absolute correction $\Delta A_1^p/D_\perp$ (Fig. 9) we get results which cannot be understood.
It is worth mentioning that the treatment of the radiative corrections should be crosschecked with the results of other programs. From this point of view the comparison of the results of the semi-analytical program POLRAD with the results of other existing programs is needed. For instance, with the Monte Carlo event generator HERACLES\textsuperscript{25} where the independent calculations have been implemented.\textsuperscript{26}

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Fig. 1. Feynman graphs contributing in the higher order to deep inelastic $lN$-scattering.
Fig. 2. Integration region of \((x_h, Q_h^2)\) in (2.7) and the correspondent region in the terms of \((\tau, R)\).
Fig. 3. Longitudinal asymmetry $A_{\parallel}/D_{\parallel}$ in the Born approximation for beam energy 190 $GeV^2$; $g_1^p$ from 23.
Fig. 4. QED corrected longitudinal asymmetry $A_{\parallel}^p/D_{\parallel}$ for beam energy 190 GeV$^2$; $g_1^p$ from $^{23}$.
Fig. 5. Absolute correction $\Delta A_p^\parallel / D_\parallel$ for beam energy 190 $GeV^2$; $g_1^p$ from $^23$.
Fig. 6. Absolute correction $\Delta A_{\parallel}^p/D_{\parallel}$ for beam energy $190 \text{ GeV}^2$; $g_1^p$ from $^{22}$
Fig. 7. Transverse asymmetry $A_{\perp}^p / D_{\perp}$ in the Born approximation for beam energy $100 \text{ GeV}^2$; $g_1^p$ from $^{24}$
Fig. 8. QED corrected transverse asymmetry $A_{\perp}^\mu / D_{\perp}$ for beam energy 100 GeV$^2$; $q_i^\mu$
from .

\[^{24}\]
Fig. 9. Absolute correction $\Delta A^p / D_\perp$ for beam energy 100 GeV$^2$; $g_1^p$ from $^24$. 