Theoretical analysis on flow characteristics of melt gear pump

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Abstract. The relationship between Geometric parameters and theoretical flow of melt gear pump is revealed, providing a theoretical basis to melt gear pump design. The paper has an analysis of meshing movement of melt gear pump on the condition of four different tooth numbers, stack movement law and flow ripple. The regulation of flow pulsation coefficient is researched by MATLAB software. The modulus formula of melt gear pump is proposed, consistent with actual situation.

1. Introduction
Common external gear pump is hard to be applied for large flow pulsation and large radial force or other defects [1-3] in the situations that require high uniformity such as melt extrusion. The foreign gear pump has compact structure and one import with several export. Simultaneously, it can reduce in-homogeneity of flow largely, improving quality of flow.

In domestic, the research concentrates on common gear pump. Yulong Li etc. have an analysis of relationship among volume oil, the trap pressure and unloading groove parameters of external gear pump [4-5]. Xianliang Xu etc. have an analysis of traffic character and radial force [6-9].

The paper has a research on meshing displacement and superposition motion law about symmetric melt gear pump on the condition of four different teeth numbers by mathematical derivation. The four theoretical flows are derived, according to which, the theoretical flows of the model pumps are calculated.

2. The Related Explain and Definition
2.1. the structure explain of melt gear pump
Melt gear pump principle structure is shown in Figure 1, the driving wheel gear is Z₁=68, the pinion gear is Z₂=32, modulus is m=0.75, the breadth of tooth is b=12.8 mm, the speed is n=60r/min, the displacement is 8x2ml. The first layer is studied assuming the same flow characteristics of each export to simplified analysis. An approximate individual external gear pump is formed between the driving wheel and each driven wheel. The total import of the pump is connected, each sub-pump inlet and outlet independent, thus forming four parallel pinion pump [10].
2.2 the regulations about the related serial number

As showed in Figure 1, four driven wheel centers O₁, O₂, O₃, O₄ and the driving wheel center are connected at 90°. The pumps with center located O₁, O₂, O₃, O₄ on the driven wheels are defined driven wheels 1, 2, 3, 4. The respective external gear pump is 1, 2, 3, 4 with nodes P₁, P₂, P₃, P₄. Driving wheel counterclockwise rotation is assumed, and the initial time is defined when a front tooth line medium point and p₁ coincident and the pump is defined Z₁. The other pumps are defined 1, ..., (Z₁-1) in turn in counterclockwise rotation. For the driven wheel 1, the tooth which is meshed with the tooth Z₁ of the driving wheel is defined as tooth 1. The other teeth are defined as 2, ..., Z₂ in turn in clockwise[11].

![Diagram](image)

1-bottom cover; 2-meshing cavity; 3-intermediate cover; 4-front shroud
5-driving shaft; 6-driven shaft; 7-driving wheel; 8-driven wheel

Fig.1 Melt gear pump model

3. The analysis of meshing point displacement

According to the above analysis, the 4 meshing points of the driving wheel locates at P₁, P₂, P₃, P₄ respectively at Z₁=4K, t=0. The 4 meshing points move synchronously, the flow pulsation cycle angle is defined as α. According to the gear mesh, there is

\[ f_i(\phi) = f \left( -\frac{p_b}{2} \leq f \leq \frac{p_b}{2} \right) \]

Where,
- \( R_b \) - base circle radius of driving wheel, \( R_b = mZ_1 \cos 20° \)
- \( \phi_i \) - angular displacement that meshing points with respect to nodes P₁, P₂, P₃, P₄
- \( p_b \) - coxa, \( p_b = mZ_1 \cos 20° \)

At \( Z₁=4K+1 \) and \( t=0 \), we have

\[ \phi_1(0) = 0, \phi_2(0) = -\alpha/4, \phi_3(0) = -\alpha/2, \phi_4(0) = \alpha/4 \]
\[ f_1(0) = 0, f_2(0) = -p_b/4, f_3(0) = -p_b/2, f_4(0) = p_b/4 \]

Considering the meshing discipline and the initial displacement of each meshing point, meshing displacement and alternate of each pinion are obtained. Meshing displacement can be obtained from the formulas:

\[ f_1(\phi) = f \left( 0 \leq f \leq \frac{p_b}{8} \right) \]
\[ f_2(\phi) = f - \frac{p_b}{4} \left( 0 \leq f \leq \frac{p_b}{8} \right) \]
\[ f_3(\phi) = f - \frac{p_b}{2} \left( 0 \leq f \leq \frac{p_b}{8} \right) \]
\[ f_4(\phi) = f + \frac{p_b}{4} \left( 0 \leq f \leq \frac{p_b}{8} \right) \]

By the same method motioned above, if it is \( Z₁=4K+1 \) and \( t=0 \), the distribution condition meshing displacement absolute values of gear pump 1, 2, 3, 4.

4. The analysis of flow characteristic
4.1. The analysis of instantaneous flow

At \( Z_1 = 4k \), the 4 meshing points are synchronous, \( f(\varphi) = f \ ( -p_s / 2 \leq f \leq p_s / 2) \), instantaneous flow is defined as \( q_{vs} \):

\[
q_{vs} = 4a_1 - a_2 \sum_{i=1}^{4} f_i^2 = b\omega_1 (4a_1 - 4a_2 f^2)
\]

At \( Z_1 = 4k+1, 4k+3 \), the meshing point displacement change at \( f = 0 \sim p_b / 8 \), theory flow is defined as \( q_E \):

\[
q_{vs} = b\omega_1 [4a_1 - a_2 (f_1^2 + f_2^2 + f_3^2 + f_4^2)] = b\omega_1 [4a_1 - a_2 (4f^2 - fp_b + 3p_b^2 / 8)]
\]

At \( Z_1 = 4k+2 \), the meshing point displacement change at \( f = 0 \sim p_b / 2 \), theory flow is defined as \( q_E \):

\[
q_{vs} = b\omega_1 [4a_1 - a_2 (f_1^2 + f_2^2 + f_3^2 + f_4^2)] = b\omega_1 [4a_1 - a_2 (4f^2 + 3fp_b / 2 + 5p_b^2 / 16)]
\]

\[
f_i(\varphi) = f \ ( -p_b / 2 \leq f \leq p_b / 2),
\]

Comparative analysis about the four kinds of melt gear pump is conducted \( k = 17 \). For the driving wheels, \( Z_1 \), the number of teeth is 68, 69, 70, 71 respectively. The other parameters are defined as: \( Z_2 = 32, m = 0.75, b = 12.8 \text{mm}, n = 60 \text{r/min} \).

The flow characteristics of the melt gear pumps are simulated by MATLAB (Fig.2, \( \varphi \in (0, 3\alpha / 2) \)). According to equations (1)~(3), the flow fluctuation coefficient \( \delta \) and ripple frequency \( f \) are obtained, shown as Tab.1.
Fig. 2 simulation results of four different melt gear pumps

| Category | δ(%) | Flow pulsation cycle angle | f (Hz) |
|----------|------|----------------------------|--------|
| Z₁=68   | 4.885 | α                         | 32     |
| Z₁=69   | 0.491 | α/8                       | 32*8   |
| Z₁=70   | 1.109 | α/4                       | 32*4   |
| Z₁=71   | 0.477 | α/8                       | 32*8   |

Shown as Tab.1 and Fig.2, at Z₁=68, the flow fluctuation coefficient (δ) and ripple frequency (f) are same as common external gear pump and these of gear pumps (Z₁=69 and Z₁=71) are basically same, their flow fluctuation coefficients are improved obviously compared with the common one and their ripple frequencies are 8 times that of the common external gear pump; when Z₁ is 70, the flow fluctuation coefficient (δ) is improved obviously and the ripple frequency (f) is 4 times that of the common external gear pump.

3.2. The analysis of theoretical displacement

The analysis of theoretical displacement is as follows:

\[ dv = dt \cdot q_{vs} \]

\[ dv = dt \cdot q_{vs} = 0.5 \alpha dt b \left[ 4a_1 - 4a_2 f^2 \right] \]

\[ = 0.5 \beta dt b \left[ 4a_1 - 4a_2 f^2 \right] \]

Where

\[ d\varphi = df / R_{b_1} \]

\[ R_1 \] —base circle radius of driving wheel

\[ dv = \frac{0.5 \beta dt b \left[ 4a_1 - 4a_2 f^2 \right]}{R_{b_1}} \]

The first teeth pair starts to discharge fluid at \( f_1 = 0.5p_b \) and ends at \( f_2 = 0.5p_b \), the drainage volume of one teeth pair is:

\[ v = \int_{f_1}^{f_2} \frac{0.5b \left[ 4a_1 - 4a_2 f^2 \right]}{m_z \cos 20^\circ} df \]

The equation (6) depicts the drainage volume of pump with \( z_2 \) teeth.

\[ V = z_2 \int_{f_1}^{f_2} \frac{0.5b \left[ 4a_1 - 4a_2 f^2 \right]}{m_z \cos 20^\circ} df \]
\[ V = \frac{[8p_a a_1 - 2p_a a_2 / 3] b z_2}{z_1 \cos 20^\circ} \] (4)

Similarly, the results of the other model pumps can be obtained:

4k+1、4k+3:
\[ V = \frac{[8p_a a_1 - 2p_a a_2 / 3] b z_2}{z_1 \cos 20^\circ} \] (5)

4k+2:
\[ V = \frac{[8p_a a_1 - 5p_a a_2 / 12] b z_2}{z_1 \cos 20^\circ} \] (6)

The equations of theoretical displacement deduced provide theoretical basis for the number of teeth and modulus of the melt gear pumps. The more direct relationship can be obtained from the above equations, as follows.

\[ V = 50.24m^3 + 3.45m^3 \left( \frac{1}{z_1} + \frac{1}{z_2} \right) \] (7)

at 4k+2,

\[ V = 50.24m^3 + 6.86m^3 \left( \frac{1}{z_1} + \frac{1}{z_2} \right) \] (8)

According to (9)、(10), it can be estimated:

\[ 41.15m^3 < V < 56.86m^3, \]

\[ m = (0.26 - 0.29) \sqrt[3]{V} \] (9)

The displacement of the driving wheel in a circle is:2.718×4×68/32=18.513ml

The value range of the modulus (m) is 0.68-0.767, taking it as 0.75. The inverse solution of modulus is consistent with the real value, proving the equation (9) to calculate modulus in designing melt gear pump.

5. Conclusions

(1) At Z1=4k, the four driven wheels mesh with the driving wheel synchronously, flow unevenness coefficient and flow fluctuation frequency are consistent with those of common external gear pumps.

(2) At Z1=4k+1, Z1=4k+3, the flow fluctuation coefficient and ripple frequency are basically same, their flow fluctuation coefficients are improved obviously compared with the common one and their ripple frequencies are 8 times that of the common external gear pump; at Z1=4k+2, the flow fluctuation coefficient is improved obviously and the ripple frequency is 4 times that of the common external gear pump.

(3) The theoretical displacement value deduced is slightly higher than that of real displacement, proving the derivation proper, which provides theoretical basis for the number of teeth and modulus of the melt gear pumps. The calculation equations of modulus of melt gear pump are proposed.

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