Stationary layered vector fields and their divergence functions

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Abstract. A scalar – vector approach to the study of general properties of scalar functions and vector fields is formulated. This approach is based on taking into account the general properties of their scalar and vector characteristics, as well as taking into account their invariants. The scalar – vector form of writing layered vector fields and an alternative version of this form of writing are discussed. New invariant characteristics of multidimensional scalar functions are introduced. For the name of these characteristics, it is proposed to use the terms - the Hamilton derivative and the Laplace derivative. A local volume approximation of a scalar function is introduced. It is essential that the nonlinear part of this approximation consists of two qualitatively different components. For the names of these components, it is proposed to use the terms-parabolic component and harmonic component. A factorized form of writing the divergence functions of a stationary layered vector field is obtained.

1. Introduction

Currently, two approaches to describing the diffusion phenomenon are well known. One of these approaches can be defined as a physical approach [1], and the second approach – as a hydrodynamic approach [2]. The physical approach is based on the concepts of the speed of thermal motion of molecules and the free-path length and allows us to find the value of the diffusion coefficients. The hydrodynamic approach is based on the concepts of multi-speed continuum and interpenetrating motion and allows us to find the value of the rate of diffusion transport of matter.

However, an alternative approach to obtaining the value of the diffusion transfer rate of a substance is possible, which can be defined as a mechanical approach. In this approach, the phenomenon of diffusion is considered as the phenomenon of diffusive transfer of the amount of motion or momentum. This approach is an alternative to the mentioned hydrodynamic approach. The reason for the conceptual difference between the hydrodynamic and mechanical approaches is due to the fundamental difference in the methods used for dividing a multi-speed continuum into single-speed components. In contrast to the hydrodynamic approach, the mechanical approach takes into account the interpenetrating movements that occur within each component of an inhomogeneous gas – liquid mixture.

In the mechanical approach, the well-known kinematic method of dividing a chemically homogeneous substance into two parts is of decisive importance [3]. The kinematic method of dividing a chemically homogeneous substance into components is the basis of the theory of superfluidity. This method allows you to correctly divide liquid helium in the superfluid state into normal and superfluid parts. The distinctive feature of the superfluid part is that its velocity field belongs to the potential type of vector fields.
When studying the diffusion phenomenon, the feasibility of using a kinematic method for dividing a chemically homogeneous substance into parts is due to the possibility of correctly dividing the component of an inhomogeneous mixture into two parts. For the names of these two parts, we suggest using the terms normal component and diffusant. A distinctive feature of a diffusant is that its velocity field belongs to a set of layered vortex vector fields.

Naturally, the normal component and the diffusant exchange matter and momentum. For the name of this type of exchange of matter and momentum, it is proposed to use the term \( \text{fractional type of exchange} \).

When obtaining a ratio that allows us to quantify the intensity of this fractional exchange, it is important to take into account the general properties of the divergence functions of layered vector fields.

When studying the general properties of layered vector fields and divergence functions, it is important to introduce two new invariant characteristics of multidimensional scalar functions. To name these new invariant characteristics of multidimensional scalar functions, we suggest using the terms "Hamilton derivative" and "Laplace derivative".

One of the main results obtained using these invariant characteristics is the obtaining an alternative form of writing the so-called local volume approximation of the scalar function. This approximation allows us to approximate the value of the considered scalar function in the vicinity of the considered point in three-dimensional space with the second order of accuracy. Using the local volume approximation of a scalar function, we can obtain a factorized form of writing the divergence function of a stationary layered vector field.

Important practical value of obtaining a factorized form of writing functions layered divergence of a vector field is that, using this notation, we can significantly expand the understanding of the physical nature of diffusion phenomena of mass transfer and momentum. The fact is that this notation allows us to say that in fact the transfer phenomena consist of two interrelated, but qualitatively different from each other phenomena. One of these phenomena is the well-known phenomenon of transfer of matter and momentum, and the second phenomenon is the so-called phenomenon of fractional exchange of matter and momentum.

2. The number of set functions as a numeric invariant and a distinctive hallmark of layered vector fields

In [1], a method for geometric and numeric classification of vector fields was proposed. This classification allowed us to divide the vortex vector fields on a vortex layered vector fields and a vortex spiral vector fields. By definition, the vortex spiral vector fields \( \mathbf{a}^{\text{spir}} \) are vector fields satisfying conditions of the form

\[
\nabla \times \mathbf{a}^{\text{spir}} \neq 0, \\
\n\nabla \times \mathbf{a}^{\text{spir}} \neq 0.
\]

(2.1)

(2.2)

While the vortex layered vector fields \( \mathbf{a}^{\text{lay}} \) are vector fields satisfying conditions of the form

\[
\nabla \times \mathbf{a}^{\text{lay}} = 0, \\
\n\n\n\nabla \times \mathbf{a}^{\text{lay}} = 0.
\]

(2.3)

(2.4)

Relations (2.1) describe a common property of spiral \( \mathbf{a}^{\text{spir}} \) and layered \( \mathbf{a}^{\text{lay}} \) vector fields – their belonging to the set of vortex vector fields \( \mathbf{a}^{\text{rot}} \). While relations (2.1) and (2.3) describe their difference, which is that layered vector fields \( \mathbf{a}^{\text{lay}} \) have the property of orthogonality to their vortex fields \( \nabla \times \mathbf{a}^{\text{lay}} \), and spiral vector fields do not have this property. Expression (2.3) is a mathematical form of writing one of the main distinguishing hallmarks of layered vector fields \( \mathbf{a}^{\text{lay}} \).

It is possible to show [3] that any spiral vector field \( \mathbf{a}^{\text{spir}} \) can be written in the form

\[
\n\mathbf{a}^{\text{spir}} = f \mathbf{\phi} + \nabla F
\]

(2.5)

where \( f, \phi, \) and \( F \) are differentiable scalar functions. It can also be shown that any layered vector field \( \mathbf{a}^{\text{lay}} \) can be written as

\[
\n\mathbf{a}^{\text{lay}} = f \mathbf{\phi} + \nabla F
\]
It follows from expression (2.5) that to set the spiral vector field $a^{spir}$, it is necessary and sufficient to set three scalar functions, and to set the layered vector field $a^{lay}$, it is necessary and sufficient to set two scalar functions. The number of scalar functions that must be set to define a layered vector field is an important quantitative invariant characteristic of this type of vortex vector fields.

An alternative form of writing a layered vector field $a^{lay}$ is possible, which looks like

$$a^{lay} = f \nabla \phi$$

(2.6)

where $\phi = f\partial_\xi$. Expression (2.7) can be easily obtained from expression (2.6) using a well-known vector identity having the form

$$\nabla(f\phi) = f\nabla\phi + \phi \nabla f$$

(2.8)

It is known [3] that an arbitrary vector field $a$ can be written as

$$a = \alpha \nabla \beta + \nabla \gamma$$

(2.9)

where $\alpha$, $\beta$, and $\gamma$ are three fairly smooth scalar functions. For the name of the notation of an arbitrary vector field $a$ in the form of expression (2.9), we suggest using the term – scalar-vector notation.

By comparing the scalar-vector the form of writing (2.5) of the spiral vector field $a^{spir}$ and alternative scalar-vector the form of writing (2.7) - layered vector field $a^{lay}$, you can set an important distinguishing hallmark of the spiral $a^{spir}$ and layered $a^{lay}$ vector fields. For layered vector fields, the vectors $\nabla f$, $\nabla \phi$ and $\nabla F$ are coplanar vectors, while for spiral vector fields, the vectors $\nabla f$, $\nabla \phi$ and $\nabla F$ do not have this property. It should also be noted that there is another alternative form of writing a layered vector field $a^{lay}$, which has the form

$$a^{lay} = (f |\nabla \phi|/K)(K|\nabla f|)$$

(2.10)

where $K$ is a real number and $e_\phi$ is the directing unit vector of the gradient vector field $\nabla \phi$ ($e_\phi = \nabla \phi/|\nabla \phi|$).

3. Numeric method for classifying unit vector fields

The numerical method proposed in [1] for classifying arbitrary vector fields $a$ can also be used for classifying single vector fields $e$. According to this numerical classification, potential unit vector fields $e^{pot}$ are unit vector fields $e$, for which it is enough to set one scalar function, and vortex unit vector fields $e^{rot}$ are unit vector fields $e$, for which it is necessary to set two scalar functions. The expression, which in this paper is considered as a proposed method for determining the unit potential vector field $e^{pot}$, has the form

$$e^{pot} = a^{pot} / |a^{pot}|$$

(3.1)

where $a^{pot} = \nabla f$ is a potential vector field (where $f$ is a differentiable scalar function).

Generally speaking, nothing prevents us from defining a potential unit vector field $e^{pot}$ as

$$e^{pot} = a^{lay} / |a^{lay}|$$

(3.2)

where $a^{lay} = f \nabla \phi$. Given the expressions (2.6), (3.1) and (3.2) we can define potential unit vector fields $e^{pot}$ as monogradient unit vector fields $e^{mono}$, that can be defined as vector fields, for which it is sufficient to set a single scalar function. The expression, which in this paper is considered as a method for determining vortex unit vector fields $e^{rot}$, has the form

$$e^{spir} = a^{spir} / |a^{spir}|$$

(3.3)

Using the scalar-vector form of writing (2.5) of the spiral vector field $a^{spir}$, it is easy to obtain an expression of the form

$$e^{rot} = (f |\nabla \phi| / |a^{spir}|) e_1^{pot} + (|\nabla F| / |a^{spir}|) e_2^{pot}$$

(3.4)

where $e_1^{pot} = \nabla \phi /|\nabla \phi|$, and $e_2^{pot} = \nabla F/|\nabla F|$. Given that the vortex unit field $e^{rot}$ can be represented as a linear combination of two potential unit vector fields $e_1^{pot}$ and $e_2^{pot}$, it is possible to define the vortex unit vector fields $e^{rot}$ as poly-gradient unit
vector fields $\mathbf{e}_\text{poli}$, that is, they can be defined as vector fields that require to set of two scalar functions.

Thus, it was shown how the numerical approach to the classification of arbitrary vector fields $\mathbf{a}$ proposed in [1] can be used in the classification of arbitrary unit vector fields $\mathbf{e}$.

4. Numeric method for classifying geometric and vector lines

The proposed numeric method for classifying arbitrary vector fields $\mathbf{a}$ and unit vector fields $\mathbf{e}$ can also be used in geometry when classifying geometric lines $\mathbb{L}$ and vector lines $\mathbb{L}(\mathbf{a})$. Recall that vector lines $\mathbb{L}(\mathbf{a})$ are geometric lines $\mathbb{L}$, at each point of which the tangents to these lines are parallel to the vectors $\mathbf{a}$ coming from these points [4].

It is assumed that the potential type of geometric lines $\mathbb{L}_\text{pot}$ includes geometric lines $\mathbb{L}$, to set which you need to set one the scalar function. While the screw type of geometric lines $\mathbb{L}_\text{screw}$ includes geometric lines $\mathbb{L}$, to set which you need to set two of the scalar functions. It is also assumed that potential vector lines $\mathbb{L}_\text{pot}(\mathbf{a})$ are vector lines $\mathbb{L}(\mathbf{a})$ belonging to the set of potential geometric lines $\mathbb{L}_\text{pot}$. While screw vector lines $\mathbb{L}_\text{screw}(\mathbf{a})$ are vector lines that belong to the set of screw helical geometric lines $\mathbb{L}_\text{screw}$.

Thus, new important geometric distinguishing features of various types of vector fields are obtained. It is shown that an important common distinguishing geometric feature of potential $\mathbb{L}_\text{pot}$ and layered $\mathbb{L}_\text{lay}$ vector fields is that their vector lines $\mathbb{L}(\mathbf{a}_\text{pot})$ and $\mathbb{L}(\mathbf{a}_\text{lay})$ belong to the number of potential $\mathbb{L}_\text{pot}$ geometric lines. While an important distinguishing geometric feature of spiral vector fields $\mathbb{L}_\text{spir}$ is that their vector lines $\mathbb{L}(\mathbf{a}_\text{screw})$ belong to the number of helical geometric lines $\mathbb{L}_\text{screw}$.

5. Dimension of space as a numeric distinctive hallmark of vector fields

We now show that another distinctive numerical hallmark of vector fields is the dimension of the space in which these fields are defined. For example, potential vector fields $\mathbb{a}_\text{pot}$ may exist in a one-dimensional space, but layered $\mathbb{a}_\text{lay}$ and spiral vector fields $\mathbb{a}_\text{spir}$ cannot exist. This means that transport phenomena cannot be studied in the one-dimensional approximation if layered vector fields are necessary to describe them.

While in two-dimensional space can exist layered $\mathbb{a}_\text{lay}$ and potential $\mathbb{a}_\text{pot}$ vector fields, but spiral vector fields $\mathbb{a}_\text{spir}$ cannot exist. This means that physical phenomena that need to be described using spiral vector fields cannot be studied in a two-dimensional approximation. Finally, in three-dimensional spaces, there may be spiral $\mathbb{a}_\text{spir}$, layered $\mathbb{a}_\text{lay}$ and potential $\mathbb{a}_\text{pot}$ vector fields.

6. Geometric meaning of the potential component of a spiral vector field

We now discuss the specifics of the geometric meaning of the potential component of the VF of a spiral vector field

$$\mathbb{a}_\text{spir} = \mathbf{f} \nabla \Phi + \nabla \mathbf{F} \quad (6.1)$$

The simplest way to do this is by comparing it with the geometric meaning of the potential component $\mathbf{V} \Phi$, which is present on the right side of the alternative scalar – vector form of writing a layered vector field

$$\mathbb{a}_\text{lay} = \mathbf{f} \nabla \Phi + \nabla \mathbf{F} \quad (6.2)$$

It was mentioned above that for layered vector fields $\mathbb{a}_\text{lay}$, the vectors $\nabla \mathbf{f}$, $\nabla \Phi$ and $\nabla \Phi$ are coplanar vectors, while for spiral vector fields $\mathbb{a}_\text{spir}$, the vectors $\mathbf{f}$, $\nabla \Phi$ and $\nabla \mathbf{F}$ do not have this property. This means that the projections of the potential vector $\nabla \Phi$ on the direction determined by the vortex field
rota$^{\text{by}}$ are identically zero, while the projections of the potential vector VF on the direction determined by the vortex field rot $a^{\text{phi}}$ are non-zero.

It is easy to see that the potential component VF of the spiral vector field (6.1) cannot change the vector lines L(rot$^{\text{phi}}$) of its vortex field rot$^{\text{phi}}$=VF x $\nabla$ $\varphi$. However, it is this component that ensures the existence of the spiral vector field itself. In addition, it is this component that provides the screw nature of the vector lines of the spiral vector field.

7. Two new "multidimensional" invariant characteristics of the scalar function
The reason for using the adjective "multidimensional" in quotation marks in the title of this paragraph is due to the fact that when using a one-dimensional approximation, these characteristics, generally speaking, do not make sense. For the name of one of these characteristics, it is proposed to use the term – Hamilton derivative $f^{\prime}$. For the name of the other characteristic, it is proposed to use the term - Laplace derivative $f''_{\Delta}$. By definition, the value of the Hamilton derivative $f^{\prime}$ is equal to the maximum modulo value $\max |f^{\prime}|$ of the first derivatives $f^{\prime}$ of the function $f$ taken along all lines passing through the considered point of space ($f^{\prime} = \max |f^{\prime}|$).

While the relation, which should be considered as the definition of the Laplace derivative $f''_{\Delta}$, has the form

$$f''_{\Delta} = (1/n) \sum f''_{i} \quad (i = 1, \ldots, n) \quad (7.1)$$

where $n$ is the dimension of the space in which the function $f$ is defined, and the functions $f''_{i}$ are the second spatial derivatives of the function $f$ taken along the $i$-th coordinate.

It is easy to see that the value of the Laplace derivative $f''_{\Delta}$ of the function $f$ is equal to the average value of the second partial derivatives of the function $f(x)$ taken along all lines passing through the considered point of space. It is also easy to make sure that the value of the Laplace derivative $f''_{\Delta}$ of the function $f$ is a multiple of the scalar Laplacian $\Delta f$ of this function.

8. Local multidimensional approximation of a scalar function
A local $n$-dimensional approximation $f^{\delta}_{n}(x)$ of an $n$-dimensional scalar function $f_{n}(x)$ is an approximation that approximates the value of this function with second order accuracy in an infinitesimal neighborhood $\delta$ of the point $x = x_{0}$ of the $n$-dimensional space under consideration. An important special case of the local $n$-dimensional approximation $f^{\delta}_{n}(x)$ is the local three-dimensional or local volume approximation $f^{\delta}_{3}(x)$, which operates in three-dimensional Euclidean space.

Using the concepts of the Hamilton derivative $f^{\prime}$ and the Laplace derivative $f''_{\Delta}$ and assuming that the local volume approximation $f^{\delta}_{3}(x)$ is written in the vicinity of the point $x = 0$, we can easily obtain an expression of the form

$$f^{\delta}_{3}(x) = f(0) + f^{\prime}_{3} \mathbf{e}_{3}(0) \cdot \mathbf{x} + (1/2)f''_{\Delta3}(0) |\mathbf{x}|^{2} + f^{\delta}_{3\Delta}(x) \quad (8.1)$$

where $f(0) = \text{const}$ is a real number, the term $f^{\prime}_{3}(0) \mathbf{e}_{3}(0) \cdot \mathbf{x}$ is the linear part, and the terms $(1/2)f''_{\Delta3}(0) |\mathbf{x}|^{2}$ and $f^{\delta}_{3\Delta}(x)$ describe the nonlinear part of this approximation. It can be shown that the term $f^{\delta}_{3\Delta}(x)$ belongs to the set of harmonic functions. To do this, it is sufficient to apply the scalar Hamilton operator $\Delta$ to both parts of expression (8.1) and take into account that in the Cartesian reference system XYZ the scalar Laplacian $\Delta f(x, y, z)$ of the function $f(x, y, z)$ is written as

$$\Delta f(x, y, z) = f''_{x} + f''_{y} + f''_{z} \quad (8.2)$$

Using the concept of the Laplace derivative $f''_{\Delta}$ and the expression (8.2), we can easily obtain an expression that can be interpreted as the definition of the Laplace derivative $\Delta f_{\Delta}(x, y, z)$ of the harmonic function $f^{\delta}_{3\Delta}(x, y, z)$. This expression has the form

$$\Delta f^{\delta}_{3\Delta} = (f''_{x} - f''_{\Delta}) + (f''_{y} - f''_{\Delta}) + (f''_{z} - f''_{\Delta}) = 0. \quad (8.3)$$

Currently, functions that satisfy expression (8.3) are called harmonic functions. This is the reason for using the term-harmonic function for the name of the function $f^{\delta}_{3\Delta}$. The specificity of the nonlinear part of the approximation (8.1) is that one component of this nonlinear part is a harmonic function $f^{\delta}_{3\Delta}$,
and the second component \((1/2)f''_{\Delta 3}(0)\left| x \right|^2\) depends only on the distance from the considered point of space \(x\) to the point \(x=0\). For the name of the function \(f''_{\Delta 3}(0)\left| x \right|^2\) the term local parabolic function will be used below.

9. General properties of local parabolic functions

It is obvious that the local parabolic functions \(f''_{\Delta 3}(0)\left| x \right|^2\) mentioned in the previous paragraph belong to the set of scalar functions \(f^3(\delta r)\), which can be defined using an expression of the form

\[ f^\Omega(\delta r) = K\delta r^2 \quad (\delta r < \delta r_0) \quad (9.1) \]

where \(K\) is a real number. It is assumed that the domain of defining the function \(f^3(\delta r)\) is the sphere \(\Omega(\delta r_0)\) of infinitesimal radius \(\delta r_0\). When studying the general properties of the local volume approximation \(f^3(x)\), it should be taken into account that the functions \(f''_{\Delta 3}(0)\left| x \right|^2\) form the set \(\{f^3(\delta r)\}\) local scalar functions \(f^3(\delta r)\) with the following properties:

- the value of the functions \(K\delta r^2\) in the domain of their definition is of the second order of smallness,
- modules of gradients of functions \(K\delta r^2\) have the first order of smallness,
- the values of the functions \(K\delta r^2\) in the center of the domain of their definition, that is, at the point \(\delta r = 0\), are zero, that is, these points are the points of minimum,
- the functions \(K\delta r^2\) reach their maximum value at the boundary of the domain of their definition, that is, when \(\delta r = \delta r_0\).

10. Factorized form of writing the function of divergence of a stationary layered vector field

The procedure for obtaining a factorized form for writing the function of divergence \(g^{lay}=\text{div}^{lay}\) of a stationary layered vector field \(a^{lay} = f\nabla \varphi\), (it is assumed that \(f > 0\)), is as follows. First, we write the standard form for writing the function of divergence \(g^{lay}(x)\) of a stationary layered vector field \(a^{lay}(x) = f(x)\nabla \varphi(x)\), which has the form

\[ g^{lay}(x) = f(x)\Delta \varphi(x) + \nabla f(x)\nabla \varphi(x) \quad (f(x) > 0) \quad (10.1) \]

where \(x = \{x_1,x_2,x_3\}\) is the coordinate of the point in question in a three-dimensional Cartesian reference system. Then recorded the alternate form of writing the expression (10.1), which has the form

\[ g^{lay}(x) = 3f(x)\varphi_{\Delta 3}(x) + \nabla f(x)\nabla \varphi(x), \quad (f(x) > 0) \quad (10.2) \]

where \(\varphi''_{\Delta 3}(x)\) - the Laplace derivative of the function \(\varphi(x)\). In the next step the value of the derivative Laplace \(\varphi''_{\Delta 3}(x)\), standing on the right side of expression (10.2) is written as a sum of two terms \(\varphi''_{\Delta 3\text{div}}\) and \(\varphi''_{\Delta 3\text{sol}}\), that is

\[ \varphi''_{\Delta 3} = \varphi''_{\Delta 3\text{div}} + \varphi''_{\Delta 3\text{sol}} \quad (10.3) \]

and willful manner, it is assumed that the value of the second term \(\varphi''_{\Delta 3\text{sol}}(x)\) standing on the right side of expression (10.3) can be obtained using expression of the

\[ \varphi''_{\Delta 3\text{sol}} = \nabla f \nabla \varphi / f \quad (10.4) \]

It is easy to see that the value of the component \(\varphi''_{\Delta 3\text{sol}}\) of the Laplace derivative \(\varphi''_{\Delta 3}\) is equal to such a value of this derivative that the value of the function of divergence \(g^{lay}\) of the layered vector field \(a^{lay} = f\nabla \varphi\) vanishes. Given that vector fields whose function of divergence is zero are called solenoidal vector fields, using the term "solenoidal component Laplace derivative" for the function name \(\varphi''_{\Delta 3\text{sol}}\) is logical. While for the name of the component \(\varphi''_{\Delta 3\text{div}}\) of the Laplace derivative \(\varphi''_{\Delta 3}\), it is proposed to use the term – divergent component of the Laplace derivative.

The validity of expressions (10.1) - (10.4) implies the validity of an expression of the form

\[ g^{lay}(x) = 3f(x)\varphi_{\Delta 3\text{div}}(x) \quad (f(x) > 0) \quad (10.5) \]

Expression (10.5) is the desired factorized form of writing the function of divergence \(g^{lay}(x) = \text{div}^{lay}(x)\) of a stationary layered vector field \(a^{lay}(x) = f(x)\nabla \varphi(x)\), if the condition \(f(x) > 0\) is met.
11. Applied value of the results obtained in the study of General properties of layered vector fields and their functions of divergence

It is known that the concepts of multi-speed continuum and interpenetrating motion are used in continuum mechanics when studying multicomponent inhomogeneous mixtures. Introduction to the consideration of these concepts is connected with the need to take into account the influence of transport phenomena on the evolution of multicomponent mixtures. One of the key hypotheses of the mechanics of multi-velocity continuums is the assumption that the motion of any chemically homogeneous component of a mixture can be described by a single velocity field. This hypothesis postulates that a priori there are no distinguishing features that allow dividing a chemically homogeneous component of a mixture into parts.

However, there is a well-known theory based on the kinematic method of dividing a chemically homogeneous substance into two parts. This is the theory of superfluidity, in which liquid helium in the superfluid state is divided into the superfluid part and the normal part. In this case, the distinctive kinematic feature of the superfluid part is that the velocity field describing its motion belongs to potential vector fields. Taking into account the methods used in this work for numerical and geometric classification of single vector fields, as well as vector lines and geometric lines, we can say that the distinctive kinematic feature of the superfluid part is that the vector lines of its velocity field belong to the mono-gradient vector lines \( L_{V^{\text{mono}}(a)} \).

Introduction the concept of layered vector field \( a^{lay} \) allows, when Fick's law \( J_{\rho_i} = -D_{\rho_i}\nabla \rho_i \) (\( D_{\rho_i} \) is the diffusion coefficient, and \( \rho_i \) is the density of the \( i \)-th component of the mixture) is valid, to divide a chemically homogeneous component of a multicomponent inhomogeneous mixture into two parts. One of these parts is the diffusant, or part of the component involved in the diffusion transfer of the substance. The remaining or normal part does not participate in the diffusion transfer of the substance. In this case, the distinctive kinematic feature of the diffusant is that its velocity field belongs to vector fields, the vector lines of which belong to the number of mono-gradient vector lines \( L_{V^{\text{mono}}(a)} \).

Naturally, there is an exchange of matter between the diffusant and the normal part of the component of an inhomogeneous mixture. For the name of this process of substance exchange, it is proposed to use the term – the process of fractional exchange of matter. Given the above, it can be argued that in fact the phenomenon of diffusion is a complex phenomenon that includes two phenomena. One of these phenomena is the phenomenon of diffusion transfer of matter, this phenomenon is subject to the action of Fick's law, which can be written as

\[
J_{\rho_i} = -D_{\rho_i}\nabla \rho_i \quad (11.1)
\]

where \( \rho_i \) is the density of the \( i \)-th component of the mixture) is valid, to divide a chemically homogeneous component of a multicomponent inhomogeneous mixture into two parts. One of these parts is the diffusant, or part of the component involved in the diffusion transfer of the substance. The remaining or normal part does not participate in the diffusion transfer of the substance. In this case, the distinctive kinematic feature of the diffusant is that its velocity field belongs to vector fields, the vector lines of which belong to the number of mono-gradient vector lines \( L_{V^{\text{mono}}(a)} \).

The second of these phenomena is the phenomenon of fractional exchange of matter, the intensity of which is described using an expression of the form

\[
I_{\rho_i} = -3D_{\rho_i}\rho_i^{\text{div} \Lambda_3} \quad (11.2)
\]

where \( \rho_i^{\text{div} \Lambda_3} \) is the divergent component of the Laplace derivative \( \rho_i^{\text{div} \Lambda_3} \) of the function \( \rho_i \). From the point of view of mathematics, expression (11.2) is a factorized form of writing the function of divergence \( g^{lay} \) of a layered vector field \( J_{\rho_i} \).

In the conclusion of this paragraph, it makes sense to pay attention to the following circumstances. Above, the form of writing of the local volume approximation \( f_3^{lay}(x) \), which has the form

\[
f_3^{lay}(x) = f(0) + \mathbf{e}_\gamma (0) \cdot x + (1/2)f_\gamma^{\text{div} \Lambda_3}(0) |x|^2 + f^{\text{div} \Lambda_3}(x) \quad (11.3)
\]

It follows from expression (11.3) that the intensity of the processes of diffusion transfer of matter \( J_{\rho_i} \) is determined by the linear part of this approximation, and the intensity of the processes of fractional exchange of matter \( I_{\rho_i} \) is determined by its nonlinear part.

Conclusions

The main results of this work are the following:
- alternative forms of writing vector fields are obtained and analyzed;
- it is shown that using a vector identity in which the gradient of a potential vector field is written as a superposition of two layered vector fields, an important alternative form of writing a layered vector field can be obtained;
- a method for numerical classification of geometric and vector lines, as well as single vector fields, is proposed;
- the concepts of "Hamilton derivative" and "Laplace derivative", which are invariant scalar characteristics of multidimensional scalar functions, are introduced;
- the concept of "local multidimensional approximation of a scalar function" is introduced. It is shown that the nonlinear part of this approximation can be represented as the sum of the harmonic and parabolic components;
- it is shown that the Laplace derivative can be represented as the sum of the divergent term and the solenoidal term;
- a factorized form of writing the divergence function of a layered vector field is obtained, which from the physical point of view is a ratio that allows us to quantitatively describe the intensity of fractional exchange of matter between the diffusant and the normal component of a component of an inhomogeneous gas-dynamic mixture.

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