Simulation and Optimization of an Airfoil with Leading Edge Slat

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Abstract. A gradient-based optimization is used in order to improve the shape of a leading edge slat upstream of a DU 91-W2-250 airfoil. The simulations are performed by solving the Reynolds-Averaged Navier-Stokes equations (RANS) using the open source CFD code OpenFOAM. Gradients are computed via the adjoint approach, which is suitable to deal with many design parameters, but keeping the computational costs low. The implementation is verified by comparing the gradients from the adjoint method with gradients obtained by finite differences for a NACA 0012 airfoil. The simulations of the leading edge slat are validated against measurements from the acoustic wind tunnel of Oldenburg University at a Reynolds number of $Re = 6 \cdot 10^5$. The shape of the slat is optimized using the adjoint approach resulting in a drag reduction of 2%. Although the optimization is done for $Re = 6 \cdot 10^5$, the improvements also hold for a higher Reynolds number of $Re = 7.9 \cdot 10^6$, which is more realistic at modern wind turbines.

1. Introduction
Turbulence and fluctuating inflow may lead to increased fatigue loads of rotor blades and complete wind turbines. In order to control these small scale loads, it is proposed to use active or passive smart blades, wherein one of the possible technologies is the use of a leading edge slat upstream of the main airfoil. These devices are known from the field of aviation and the investigation of their use in wind energy is part of the common German research project “Smart Blades” by Research Alliance Wind Energy (“Forschungsverbund Windenergie”, FVWE).

The flow simulations in this work are based on the Reynolds-Averaged Navier-Stokes equations (RANS). These are solved by using the open source CFD code OpenFOAM [1], which can be modified or extended by the user himself and is thus convenient for research purposes.

Nemec et al. [2] and Anderson et al. [3] used the discrete adjoint approach for optimizing a trailing edge flap as a high-lift device for aircraft take-off, but OpenFOAM is more suitable for the use of continuous equations, which is why the continuous adjoint approach is used here. This approach can be widely found in literature [4, 5, 6] and Kim et al. [7] used it for a high-lift design including leading edge slat and trailing edge flap. They use a compressible solver

1 The Research Alliance Wind Energy comprises the partners German Aerospace Center (“Deutsches Zentrum für Luft- und Raumfahrt”, DLR), ForWind – Center for Wind Energy Research, and Fraunhofer Institute for Wind Energy and Energy System Technology.
and do not include the adjoints to the turbulence model, which is a common approach denoted as “frozen turbulence” assumption [8]. In this work, the Spalart-Allmaras turbulence model without transition is used [9] and the adjoints to this model are also included in the gradient computation as derived by Zymaris et al. [10].

In the following chapters two and three, the implemented mesh motion and the general adjoint approach is described. The gradients of the adjoint approach are verified against gradients by finite differences in chapter four and the numerical set-up for simulations of airfoils with leading-edge slats is validated against experiments in chapter five. Finally, chapter six shows the results of the optimization of the leading edge slat upstream of a DU 91-W2-250.

2. Mesh motion
The optimization in the following sections will use the surface points of the slat as design variables, which is a direct parametrization (in contrast to parameters such as leading edge radius, overlap length or similar). This allows the highest degree of freedom for finding the optimal shape, but it also can result in jumps or kinks in the shape. In order to avoid such kinks a simple averaging of the gradients is used by including the values of the direct neighbours of each surface point.

The mesh motion solvers in OpenFOAM can produce overlapping cells or even for small movements a deformation of the boundary layer, which can lead to wrong gradients and hence to bad optimization results. That is why a new mesh motion algorithm is implemented in the code, which is based on the principle from Jameson and Reuther [11].

It moves the points in the near- and far-field according to the movement of the surface points. For this purpose each surface point is considered as the starting point of a spline going from the airfoil to the end of the domain as shown in figure 1. In order to keep the far-field borders of the domain constant, the motion is linearly interpolated with a factor $\gamma = 0...1$:

$$x_{\text{new}} = x_{\text{old}} + \gamma(x_{\text{new}S} - x_{\text{old}S}),$$  (1)

where $x$ are the point positions and the index $S$ stands for the surface of the airfoil. This motion technique, as it is implemented here, works only for hexahedral meshes, but it ensures the same quality of the boundary layer close to the geometry, which is an essential base for a good optimization.

3. The adjoint approach
The derivation of the adjoint approach can be found in literature [3, 4, 5, 7, 10] and for the sake of brevity only the basic principles are described here according to the notation of Soto and Löhner [6].

Let $I$ be the objective function, which should be optimized, and $R = (R^u, R^p)$ be the steady-state incompressible Navier-Stokes equations, where $R^p$ is the continuity and $R^u$ are the momentum equations. Then the optimization can be written as:

$$\min I(u,p,\beta) \text{ w.r.t. } R(u,p,\beta) = 0 \text{ in } \Omega,$$  (2)

where $I$ and $R$ depend on the flow variables $(u,p)$ and the design variables $\beta$. $\Omega$ is the flow domain and a combination of objective and equality constraints can be done via a Lagrange
function L:

\[ L = I + \int_\Omega \Psi \cdot R \, d\Omega , \]  

(3)

where \( \Psi \) are the Lagrangian multipliers or as they are often called the adjoint variables. A variation of the Lagrangian leads to:

\[
\delta L = \left( \frac{\partial I}{\partial \beta} + \int_\Omega (\Psi^u, \Psi^p)^T \cdot \frac{\partial R}{\partial \beta} \, d\Omega \right) \delta \beta \\
+ \left( \frac{\partial I}{\partial u} + \int_\Omega \Psi^u \cdot \frac{\partial R^u}{\partial u} \, d\Omega \right) \delta u \\
+ \left( \frac{\partial I}{\partial p} + \int_\Omega \Psi^p \cdot \frac{\partial R^p}{\partial p} \, d\Omega \right) \delta p ,
\]

(4)

where \( \Psi = (\Psi^u, \Psi^p)^T \) denote the adjoints to velocity and pressure.

The adjoint variables can now be defined in such a way that the last two terms become zero, which gives a new set of PDEs for the adjoint variables and which makes the gradient independent from any variation of the flow field. Since the flow field does not have to be computed for each design variable again, this basically means that the gradient computation becomes independent from the amount of design parameters. Finally, the geometry can be parametrized by every surface grid point without an increase in the computational costs, which is the strength of the adjoint approach.

OpenFOAM comes already with a solver using the adjoint approach for the optimization of ducted flows [5], which are simulated by inserting cells into the domain with a high, blocking porosity. This solver is extended to external aerodynamics by the authors, so the approach of porous cells is replaced by walls and the complete gradient calculation is redefined, since the gradients are computed on the walls. Also the objective function for airfoil optimization is different than in most duct optimizations, which has to be considered in the gradient calculation. After some mathematical derivations and a few simplifications the gradient on the wall is then computed as in the publication by Soto and Löhner [6]:

\[
\frac{\partial L}{\partial \beta} = \frac{\partial I}{\partial \beta} - \int_S \Psi^p \, n \cdot (n \cdot \nabla u) \, dS - \int_S \nu \, n \cdot (\nabla \Psi^u + (\nabla \Psi^u)^T) \cdot (n \cdot \nabla u) \, dS ,
\]

(5)

where \( S \) is the domain boundary at the airfoil. The objective function in this case goes into the boundary conditions for the adjoint equations and could be the drag or lift force or a combination of both.

4. Verification of the gradients

Since the derivation of the final adjoint equations can be challenging and errors could be implemented, it is common in literature to verify the correct implementation of the gradients against gradients by finite differences (FD). This is also done in this work and because the gradients by FD are computationally expensive, the verification is done for a flow at low Reynolds number, which is still laminar.

A verification case for a symmetric NACA 0012 is created at Re=2,000 and an angle of attack of \( \text{AoA} = 3^\circ \). Because of the costs for the finite differences, only a few design parameters are considered, which are shown in figure 2. There are 26 selected design points in total, 13 on suction and pressure side each.

**Figure 2.** Design parameters of the verification case.
The convective term is discretized by a second order upwind scheme and the finite differences as well as the adjoint gradients are of first order. Higher order gradients are usually more precise, which can lead to a faster or more stable convergence in the optimization and thus they can be beneficial in complex optimization problems. Still, gradients of higher order are computationally more expensive when using finite differences. And a higher accuracy of the adjoint gradients is not possible without discretization schemes with higher order in the flow computation, as it is discussed by Castro et al. [12].

The resulting gradients via the adjoint approach and by finite differences are shown in figure 3, where the objective is the drag coefficient (left) and the lift coefficient (right), respectively. The gradients are computed with respect to the point motion in normal direction and for simplicity the absolute values of the gradients are plotted.

![Figure 3. Comparison of gradients by finite differences (labelled as “FD StSz”) and gradients via the adjoint approach for drag objective (left) and lift objective (right). Different step sizes are used for the finite differences (FD).](image)

In general there is a good agreement between the two methods, although some minor differences are visible. The mean relative difference in case of the drag objective is 9 % with a standard deviation of 19 %, which seems to be very high, but it can also be seen that the gradients of the finite differences depend on the chosen step size. This is a known drawback of using FD and thus their values also have to be regarded with caution. A better agreement could be possible with a higher accuracy of the gradients, but as mentioned this is only possible by using higher order discretization schemes. The mean relative difference in case of the lift objective is 2 % with a standard deviation of 3 %, which is smaller than in case of the drag objective. Concluding this verification, the implementation of the adjoint approach is expected to be accurate enough for this work.

Interestingly, the gradients of the lift coefficient have a higher absolute value than the gradients of the drag coefficient. This could lead to problems when both coefficients shall be optimized at the same time, because the lift may have a higher influence to the optimal result, although this also depends on the definition of the objective function.

5. Validation of the simulations

Before optimizing the more complex geometry of an airfoil with leading edge slat, the numerical set-up is validated against experimental data. The geometry of the slat upstream of a DU 91-W2-250 airfoil is shown in figure 4. The airfoil was designed at the TU Delft by Timmer and van Rooij [13] and the initial slat was designed within the “Smart Blades” project by the DLR. The measurements in that project were conducted by ForWind in the acoustic wind tunnel of Oldenburg University at a Reynolds number of $Re = 600,000$, with the chord length of the main airfoil as reference length.
The steady-state simulations are performed using incompressible computational fluid dynamics (CFD) with a two-dimensional mesh and a non-dimensional wall distance of $y^+ < 1$. The RANS equations are closed via the Spalart-Allmaras turbulence model without transition [9]. Figure 5 shows the lift and drag coefficients over the angle of attack, compared with the results from the CFD simulations. There is a good agreement in the linear range, but some differences in the stall. Since the steady-state simulations are done with a two-dimensional mesh, the numerical results could be improved by using quasi-2D grids\(^2\) or transient computations.

Nevertheless, the focus of this work is the optimization and thus the validation of the simulations is considered to be good enough.

6. Results of the optimization

As a last point, the discussed methods and implementations are now applied to an optimization of the leading edge slat. The gradients are computed via the adjoint approach and because of a simple implementation the method of steepest descent is used for finding the minimum. A more advanced optimization could be used in principle, but due to small changes in the final geometry, this simple method is sufficient enough and from the authors’ experience it even can be more robust than other optimization methods. Note that a gradient-based optimization can only find the local optimum near the initial configuration, which may only be global for certain problems.

All shape points of the slat are used as individual design parameters, which results in 480 design points. In order to avoid kinks or jumps in the geometry the gradients are averaged by including their direct neighbour values. High gradients in the flow field arise behind the trailing edge of the slat, which either have to be limited or strongly averaged over the entire trailing edge. This can produce errors and may lead to bad optimization results. Besides, the adjoint approach, as it is implemented here, cannot handle geometrical discontinuities, which occur at the trailing edge corners [14]. Thus the trailing edge is fixed and only the rest of the slat is deformed, which further reduces the number of real design parameters.

The high flexibility of the shape deformation can in theory lead to bumps in the geometry, which may be mathematically correct, but could be useless for real applications. That is why only small deformations and thus small changes in the force coefficients are considered. Then

\(^2\) In this context “quasi-2D” is meant to be an extrusion of the numerical 2D grid into the spanwise direction.
the objective is to minimize the drag to an aimed value, a few per cent smaller than the initial one, but keeping the lift above the initial lift:

\[
\min \frac{1}{2}(c_d - c_{d,\text{aim}})^2 \text{ w.r.t. } c_l \geq c_{l,0} \text{ at } \text{AoA} = 13^\circ
\] (6)

Here, a single-point optimization is done at \( \text{AoA} = 13^\circ \), which is the angle with the highest glide ratio and thus could be the operating point of a wind turbine.

![Figure 6](image)

**Figure 6.** Numerical lift and drag coefficients of the optimized and initial geometry over the angle of attack at Reynolds number of \( Re = 600,000 \).

The objective of Eq. 6 leads to a shape which has a 2% smaller drag and a slight decrease in lift of 0.3%, so the lift constraint is not fully fulfilled. The resulting polars are shown in figure 6 and although the differences at \( \text{AoA} = 13^\circ \) are not visible, the changes are not only affecting the behaviour at the design angle of attack, but also at a wide range near this angle; this is strongly visible near the stall region. The maximum lift and its angle of attack is increased, although this is a single-point optimization at a lower angle. This could be explained by the principle of the slat as a high-lift device, where even “minor” changes in the geometry (as it is shown with the presented case) may lead to strong effects in the near-stall region.

Figure 7 shows the original and optimized shape and the optimization basically leads to a slat which is a little smaller and thinner than before. Since the trailing edge of the slat was fixed a-priori, the optimal and initial shape at the trailing edge do not differ.

In order to investigate whether the improvements of the optimization at \( Re = 0.6 \cdot 10^6 \) also hold at higher Reynolds numbers, which are more realistic for modern wind turbines, the same geometry is also simulated at \( Re = 7.9 \cdot 10^6 \). The general set-up of the case is similar as before and only the boundary conditions as well as the dimensionless wall distance change according to the higher Reynolds number.

The resulting polars are shown in figure 8 and as at the previous polars in figure 6 the most visible changes appear in the stall region. Also the known effect of the Reynolds number can be seen, which leads to an increase in the maximum lift with higher Reynolds number.
The lift at $\text{AoA} = 13^\circ$ decrease by 0.5%, but the drag reduces by more than 2%, which leads to an overall improved aerodynamic behaviour as in the case at lower Reynolds number.

7. Conclusions and future work
Two-dimensional steady-state simulations of a DU 91-W2-250 airfoil with leading edge slat were performed using the open source CFD code OpenFOAM and were validated against wind tunnel measurements. The adjoint approach was deployed within a gradient-based optimization, which was used here for optimizing the shape of the slat in order to improve the aerodynamic behaviour.
within a few percent. The optimal shape led to improved force coefficients in a wide range next to the design angle of attack, but especially in the stall, although a single-point optimization at a lower angle of attack was done. A further investigation at a higher Reynolds number of \(Re = 7.9 \cdot 10^6\) showed that the improvements, generated at a Reynolds number of \(Re = 0.6 \cdot 10^6\), also led to an improved aerodynamic behaviour at high velocities.

The working principle of the adjoint approach in optimization has been shown and the presented set-up can be considered as a proof of concept. The implementation will be used for further investigations on various geometries and with different objective functions, where also other optimization methods will be tested.

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