Beyond the tree level
in the AdS/CFT Correspondence

Mattia Jona-Lasinio*
Dipartimento di Fisica, Università di Pisa
INFN, Sezione di Pisa
Via Buonarroti 2, 56127 Pisa, Italy

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Abstract

The loop expansion of the effective action is used to evaluate quantum corrections to a scalar field theory (massive $\phi^4$ model) on $AdS_{n+1}$. We evaluate one loop corrections and show that they preserve conformal invariance of the boundary theory as conjectured by AdS/CFT correspondence.

*email: jona@df.unipi.it
1 Introduction

It was at the end of 1997 that Maldacena [1] conjectured a correspondence between the large $N$ limit of certain Yang-Mills superconformal field theories and supergravity on $AdS_{n+1}$. However in his article he didn’t explain how to do calculations with it and several authors tried to make the correspondence more precise. In [2] a possible formulation within string theory can be found, but it was mainly Witten [3] that explained how to construct explicitly the two point Green’s function in a general frame. He considered a simple model like the scalar field theory and using heuristic arguments, wrote an expression for the Green’s function in the massless and massive case.

Basically the idea is to identify the partition function of the bulk ($AdS_{n+1}$) theory with the generating functional for the boundary conformal theory. Obviously the partition function involves a functional integration so we must impose suitable boundary conditions to restrict this integration. The boundary value of the bulk field is to be identified as a source term for the boundary theory. In this formulation the correspondence seems to be true more generally and not restricted to string theory. In this paper we consider a massive scalar field theory.

Suppose to have the $\phi$ field defined on $AdS_{n+1}$ (which we shall denote $\Omega$) and consider the $\phi_0$ field defined as the value of the $\phi$ field on the boundary $\partial \Omega$. At the same time consider a set of operators $O$ which belong to the boundary theory and assume a coupling of the form

$$\int_{\partial \Omega} d^n x \, O(x) \phi_0(x)$$

We can define the generating functional for the boundary theory as

$$Z[\phi_0] = \langle e^{\int_{\partial \Omega} \phi_0 O} \rangle_{CFT}$$

but we have no idea on how to evaluate the $\langle \rangle_{CFT}$ expectation value. Consider now the bulk scalar theory and its partition function $Z_{AdS}[S; \phi_0]$ whose expression is

$$Z_{AdS}[S; \phi_0] = \int_{\Omega, \phi_0} D\phi \, e^{-S[\phi]}$$

The AdS/CFT correspondence states that we can identify the two last expressions

$$\langle e^{\int_{\partial \Omega} \phi_0 O} \rangle_{CFT} \equiv \int_{\Omega, \phi_0} D\phi \, e^{-S[\phi]}$$

provided that we restrict the functional integration on the right hand side to the $\phi$ fields that satisfy the $\phi_0$ boundary condition. In this way we have a prescription on how to evaluate the $\langle \rangle_{CFT}$ expectation value.

A large number of papers on this subject has been produced starting from [1] and [3]. See for example [4] - [8] for specific arguments. A detailed introduction to the AdS/CFT correspondence.

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$N$ is the dimension of the gauge group.
correspondence can be found in the first part of [9] and for a general reference on conformal field theories see [10].

To my knowledge the correspondence has always been considered at tree level, evaluating the partition function of the bulk theory only over the classical solution. The basic requirement is that the derivatives with respect to \( \phi_0 \) evaluated at \( \phi_0 = 0 \) of the right hand side of (1) are covariant under the conformal group. It is then interesting to push further our calculation by considering the one loop approximation in order to see whether the correspondence remains true. This is what we do in this paper. We emphasize that our calculations are formal in the sense that we do not worry about possible infinities in our expressions. Nevertheless infinities are a real problem which several authors discussed (for example [11], [12]) but we think that a formal verification of the transformation properties under the conformal group is a necessary prerequisite for any further development. In section 2 we propose a brief review of the tree level case and the basic results. In section 3 we calculate the one loop corrections for the correlation functions and we study their transformation rules.

2 Scalar field theory on \( AdS_{n+1} \) : a review

In the following we will use some notations introduced in several recent papers ([3], [9]). Consider a \((n+2)\)-dimensional pseudoeuclidean space \( E^{n+2} \) with coordinates

\[
Y \equiv (y_0, \ldots, y_{n+1}) \quad \quad ds^2 = dy_0^2 - dy_{n+1}^2 - \sum_{i=1}^{n} dy_i^2
\]

In this frame the euclidean \( AdS_{n+1} \) equation turns out to be

\[
y_0^2 - y_{n+1}^2 - \sum_{i=1}^{n} y_i^2 = 1
\]

Define \( u \equiv y_0 + y_{n+1}, \quad v \equiv y_0 - y_{n+1}, \quad x_i \equiv \frac{y_i}{u} \) with \( i = 1, \ldots, n \) and \( x_0 \equiv u^{-1} \). In this way we obtain the well known \( AdS_{n+1} \) metric

\[
ds^2 = \frac{1}{x_0^2} \sum_{i=0}^{n} dx_i^2
\]

The \( y \) set of coordinates should not be confused with the \( x \) set since the former is defined in the \( E^{n+2} \) space while the latter is defined on the \( AdS_{n+1} \) manifold. The \( AdS_{n+1} \) manifold \( \Omega \) is represented by the upper half plane with equation \( x_0 > 0 \) while its boundary \( \partial \Omega \) is represented by the hyperplane \( x_0 = 0 \) plus the single point \( x_0 = \infty \).

\footnote{Here the “time” coordinate is represented by \( y_{n+1} \), so the euclidean \( AdS_{n+1} \) corresponds to considering \( y_{n+1} \rightarrow iy_{n+1} \); the minkowskian version of \( AdS_{n+1} \) would have been

\[
y_0^2 + y_{n+1}^2 - \sum_{i=1}^{n} y_i^2 = 1
\]
The action $S[\phi]$ for the scalar $\phi^4$ model is

$$S[\phi] = \int_{\Omega} d^{n+1}x \sqrt{g} \left\{ \frac{1}{2} \left[ (\partial_\mu \phi \partial^{\mu} \phi) + m^2 \phi^2 \right] + \frac{\lambda}{4!} \phi^4 \right\}$$

which yields the field equation

$$S[\phi] = \int_{\Omega} d^{n+1}x \sqrt{g} \{ 1 \} + \frac{1}{\sqrt{g}} \left[ (\partial_\mu \phi \partial^{\mu} \phi) + m^2 \phi^2 \right]$$

where $\nabla^2 \equiv \frac{1}{\sqrt{g}} \partial_\mu (\sqrt{g} \partial^\mu)$. Here the $\sqrt{g}$ factor is the square root of the metric determinant and makes the global measure $\{ d^{n+1}x \sqrt{g} \}$ invariant under general coordinates transformations. By means of the Green’s formula we transform (3) into an integral equation

$$\phi(x) = \int_{\partial \Omega} d^n y \sqrt{h} n_\mu \partial G(x,y) \phi(y) + \frac{\lambda}{3!} \int_{\Omega} d^{n+1}y \sqrt{g} G(x,y)(\phi(y))^3$$

Here $h$ is the determinant of the induced metric on $\partial \Omega$ and the covariant Green’s function satisfies the equation

$$(\nabla^2 - m^2) G(x,y) = \frac{\delta^{n+1}(x-y)}{\sqrt{g}}$$

with the boundary condition $G(x,y) |_{x \in \partial \Omega} = 0$. The tree level approximation is then $\phi(x) \simeq \phi(0)(x)$ and the action takes the form

$$S[\phi] = \frac{1}{2} \int_{\partial \Omega} d^n x \sqrt{h} n_\mu \partial \phi^{(0)} \partial^\mu \phi^{(0)} + \frac{\lambda}{4!} \int_{\Omega} d^{n+1}x \sqrt{g} (\phi^{(0)})^4$$

Let’s consider now the free field case. Solving equation (3) by Fourier-transform methods, we find two linearly independent solutions

$$x_0^+ e^{-i k \cdot x} I_\alpha(kx_0) \quad x_0^- e^{-i k \cdot x} K_\alpha(kx_0)$$

where $I_\alpha$ and $K_\alpha$ are modified Bessel functions and $\alpha = \sqrt{\frac{n^2}{4} + m^2}$; $k$ is an $n$ dimensional vector while $k = |k| \equiv \sqrt{\sum_{i=1}^{n} k_i^2}$. The covariant Green’s function is

$$G(x,y) = - (x_0 y_0)^{\frac{n}{2}} \int \frac{d^n k}{(2\pi)^n} e^{-i k \cdot (x-y)} \left[ I_\alpha(kx_0) I_\alpha(ky_0) \theta(x_0 - y_0) + I_\alpha(kx_0) K_\alpha(ky_0) \theta(y_0 - x_0) \right]$$

In these coordinates the operator $(\nabla^2 - m^2)$ is explicitly given by

$$\left( x_0^2 \sum_{i=0}^{n} \frac{\partial^2}{\partial x_i^2} - x_0(n-1) \frac{\partial}{\partial x_0} - m^2 \right)$$

$I_\alpha(z)$ and $K_\alpha(z)$ satisfy the differential equation

$$z^2 \frac{d^2 w}{dz^2} + z \frac{dw}{dz} - (z^2 + \alpha^2) w = 0$$

\[^3\text{In these coordinates the operator } (\nabla^2 - m^2) \text{ is explicitly given by}
\[^4\text{In these coordinates the operator } (\nabla^2 - m^2) \text{ is explicitly given by}
where θ is the Heaviside step function. This expression can be integrated explicitly and the result is

\[ G(x, y) = -\frac{c}{2\alpha} \xi^{-\Delta} F \left( \frac{n}{2}, \Delta; \alpha + 1; \xi^{-2} \right) \]  
(7)

where

\[ \Delta = \frac{n}{2} + \alpha \quad c = \frac{\Gamma(\Delta)}{\pi^{\frac{n}{2}} \Gamma(\alpha)} \]

and \( F(a, b; c; z) \) is the hypergeometric function; the expression for \( \xi \) is

\[ \xi = \frac{1}{2x_0 y_0} \left\{ \frac{1}{2} \left[ |x - y|^2 + |x - y^*|^2 \right] + \sqrt{|x - y|^2 |x - y^*|^2} \right\} \]

where \( x \equiv (x_0, x), \ x^* \equiv (-x_0, x) \) and \( |u|^2 \equiv \sum_{i=0}^{n} u_i^2 \).

After a rather lengthy calculation it is possible to arrive at the following expression for the action \( S[\phi] \) at tree level

\[ S[\phi] = -\alpha c \int_{\partial \Omega} d^n x \, d^n y \, \frac{\phi_0(x) \phi_0(y)}{|x - y|^{2\Delta}} + \frac{\lambda c^4}{4!} \int_{\partial \Omega} d^n x_1 \ldots d^n x_4 \, \phi_0(x_1) \ldots \phi_0(x_4) \, I_4(x_1, \ldots, x_4) \]  
(8)

where \( I_4(x_1, \ldots, x_4) \) is expressed by

\[ I_4(x_1, \ldots, x_4) = \int_{\Omega} d^{n+1} y \, \left( \frac{y_0}{y_0^2 + |y - x_1|^2} \right)^{\Delta} \ldots \left( \frac{y_0}{y_0^2 + |y - x_4|^2} \right)^{\Delta} \]  
(9)

and \( \phi_0 \) represents the boundary value of the \( \phi \) field. The AdS/CFT correspondence states then that two and four point functions for the boundary theory must have the form

\[ \langle O(x)O(y) \rangle \propto \frac{1}{|x - y|^{2\Delta}} \]  
(10)

\[ \langle O(x_1)O(x_2)O(x_3)O(x_4) \rangle \propto f \left( \frac{x_{12}x_{34}}{x_{13}x_{24}}, \frac{x_{12}x_{34}}{x_{14}x_{23}} \right) \prod_{i,j=1;i<j}^{4} \frac{1}{x_{ij}^{\Delta}} \]  
(11)

as dictated by conformal invariance. In [14] it is shown that this is the case and the argument is generalized to the \( n \) point functions at tree level. Here \( f \) is an arbitrary function and \( x_{ij} \equiv |x_i - x_j| \).

3 Beyond the tree level: one loop corrections

In the following we discuss in detail the two and four point one loop corrections and we show that the argument can be extended to the \( n \) point case straightforwardly.
3.1 Effective action

Let’s define \( \phi_J(x) \) to be the classical solution, which obeys the classical field equation \( \frac{\delta S}{\delta \phi(x)} \bigg|_{\phi=\phi_J} \equiv J(x) \) and satisfies \( \phi_J(x)|_{J=0} = 0 \). Define the fluctuation field \( \xi \equiv \phi - \phi_J \) and perform a Taylor expansion of \( S[\phi] \) around the stationary point \( \phi = \phi_J \). Denoting the generating functional by \( Z[J] \) we have

\[
Z[J] \approx e^{-S[\phi_J]} + \int J \phi_J \left( \frac{\int D\xi e^{-\frac{1}{2} \int d^n x \, d^n y \, \sqrt{g(x)} \sqrt{g(y)} \frac{\delta^2 S}{\delta \xi(x) \delta \xi(y)} \bigg|_{\xi=0}}{\int D\xi e^{-\frac{1}{2} \int d^n x \, d^n y \, \sqrt{g(x)} \sqrt{g(y)} \frac{\delta^2 S}{\delta \xi(x) \delta \xi(y)} \bigg|_{\xi=0} \xi(x)\xi(y)} \right) \quad (12)
\]

Note that \( \frac{\delta^2 S}{\delta \phi(x) \delta \phi(y)} \bigg|_{\xi=0} = \frac{\delta^2 S}{\delta \phi(x) \delta \phi(y)} \bigg|_{\phi=\phi_J} \) and the integrals involved in (12) are Gaussian, so we have

\[
Z[J] \approx e^{-S[\phi_J]} + \int J \phi_J \sqrt{\det \left( \frac{\delta^2 S}{\delta \phi(x) \delta \phi(y)} \bigg|_{\phi=0} \right)} \sqrt{\det \left( \frac{\delta^2 S}{\delta \phi(x) \delta \phi(y)} \bigg|_{\phi=\phi_J} \right)} \quad (13)
\]

The effective action \( \Gamma \) is

\[
\Gamma[\phi_J] = S[\phi_J] + \frac{1}{2} \ln \det \left( \frac{\delta^2 S}{\delta \phi(x) \delta \phi(y)} \bigg|_{\phi=\phi_J} \right) = S[\phi_J] + \frac{1}{2} \ln \det \left( A[\phi(0)] \right) \quad (14)
\]

where \( \phi(J; x) \equiv \frac{1}{\sqrt{g(x)}} \frac{\delta \ln Z[J]}{\delta J(x)} \) and we have defined

\[
A[\phi(0)] = \frac{x_0^2 \sum_{i=0}^{n} \frac{\partial^2}{\partial x_i^2} - x_0(n-1) \frac{\partial}{\partial x_0} - m^2 - \frac{\Delta}{2} (\phi(0)(x))^2}{x_0^2 \sum_{i=0}^{n} \frac{\partial^2}{\partial x_i^2} - x_0(n-1) \frac{\partial}{\partial x_0} - m^2} \quad (15)
\]

3.2 One loop corrections

The one loop corrections for the two and four point functions are

\[
\langle O(y)O(z) \rangle = \frac{a \lambda c^2}{2} \int \frac{d^{n+1}x}{x_0^{n+1}} \left( \frac{x_0}{x_0^2 + |x-y|^2} \right)^\Delta \left( \frac{x_0}{x_0^2 + |x-z|^2} \right)^\Delta \quad (16)
\]

\[
\langle O(y)O(z)O(v)O(w) \rangle = \frac{3}{2} \lambda^2 c^4 \int \frac{d^{n+1}x \, d^{n+1}x'}{x_0^{n+1} \, x_0^{n+1}} G^2(x, x') \times \left( \frac{x_0}{x_0^2 + |x-y|^2} \right)^\Delta \left( \frac{x_0}{x_0^2 + |x-z|^2} \right)^\Delta \left( \frac{x_0}{x_0^2 + |x'-v|^2} \right)^\Delta \left( \frac{x_0}{x_0^2 + |x'-w|^2} \right)^\Delta \quad (17)
\]
We have here defined $a \equiv G(x, x)$ because it is easy to check that $G(x, x)$ is a constant.

We now show that these expressions satisfy the constraints imposed by conformal invariance. It is clear from the very structure that they are invariant under the Poincaré subgroup, so all we have to do is to verify their covariance under dilations and SCT (Special Conformal Transformations).

Under a dilation $u \rightarrow \alpha u$ we have

$$\langle \mathcal{O}(\alpha y)\mathcal{O}(\alpha z) \rangle = \frac{a\lambda c^2}{2} \int \frac{d^{n+1}x}{x_0^{n+1}} \left( \frac{x_0}{x_0^2 + |x - \alpha y|^2} \right)^\Delta \left( \frac{x_0}{x_0^2 + |x - \alpha z|^2} \right)^\Delta$$

(18)

Changing the integration variable $x \rightarrow \alpha x$ we have

$$\langle \mathcal{O}(\alpha y)\mathcal{O}(\alpha z) \rangle = \frac{a\lambda c^2}{2} \frac{\alpha^{2\Delta}}{\alpha^{4\Delta}} \int \frac{d^{n+1}x}{x_0^{n+1}} \left( \frac{x_0}{x_0^2 + |x - y|^2} \right)^\Delta \left( \frac{x_0}{x_0^2 + |x - z|^2} \right)^\Delta$$

$$= \alpha^{-2\Delta} \langle \mathcal{O}(y)\mathcal{O}(z) \rangle$$

(19)

The dilation is performed on an n-vector while the change in the integration variable is on an (n+1)-vector, but we can always interpret $u \equiv (0, u)$.

Let’s consider now the SCT transformation $u \rightarrow u' = \frac{u - bu^2}{1 - 2(b \cdot u) + b^2u^2}$ and define

$$\gamma(u) \equiv \left| 1 - 2(b \cdot u) + b^2u^2 \right|$$

$$\gamma(u) \equiv \left| 1 - 2(b \cdot u) + b^2u^2 \right|$$

where the dot stands for the canonical scalar product.

We have

$$\langle \mathcal{O}(y')\mathcal{O}(z') \rangle = \frac{a\lambda c^2}{2} \int \frac{d^{n+1}x}{x_0^{n+1}} \left( \frac{x_0}{x_0^2 + |y' - b\gamma(y)|^2} \right)^\Delta \left( \frac{x_0}{x_0^2 + |z' - b\gamma(z)|^2} \right)^\Delta$$

Note that $\gamma(u) \neq \gamma(u)$ so that the $\gamma$ factors cannot be reabsorbed by changing the integration variable. In this case the verification requires three preliminary remarks:

- an n dimensional SCT transformation with parameter $b$ performed on n dimensional vectors $u$ is equivalent to an n+1 dimensional SCT transformation with parameter $b = (0, b)$ performed on n+1 dimensional vectors $u = (0, u)$;

- an n+1 dimensional SCT transformation with parameter $b$ performed on n+1 dimensional vectors $u = (u_0, u)$ is explicitly given by

$$\begin{cases} u_0 \rightarrow \frac{u_0}{\gamma(u)} \\ u \rightarrow \frac{u - bu^2}{\gamma(u)} \end{cases}$$

so that the “time” component is only rescaled;
• equation (16) can be written as

\[
\langle O(y)O(z) \rangle = \frac{a\lambda c^2}{2} \int \frac{d^{n+1}x}{x_0^{n+1}} \left( \frac{x_0}{|x-y|^2} \right)^{\Delta} \left( \frac{x_0}{|x-z|^2} \right)^{\Delta}
\]

(20)

where \( y = (0, y), z = (0, z) \).

Consider then the n-dimensional transformation with parameter \( b \) as a particular \( n+1 \) dimensional transformation with parameter \( b = (0, b) \). We shall note that under SCT we have

\[
|x_i - x_j| \rightarrow \frac{|x_i - x_j|}{\sqrt{\gamma(x_i)\gamma(x_j)}}
\]

so that equation (20) transforms into

\[
\langle O(y')O(z') \rangle = \frac{a\lambda c^2}{2} \int \frac{d^{n+1}x}{x_0^{n+1}} \left( \frac{x_0\gamma(x)\gamma(y)}{\gamma(x)|x-y|^2} \right)^{\Delta} \left( \frac{x_0\gamma(x)\gamma(z)}{\gamma(x)|x-z|^2} \right)^{\Delta}
\]

(21)

= \gamma(y)^{\Delta} \gamma(z)^{\Delta} \langle O(y)O(z) \rangle

but since we have assumed \( b = (0, b) \) and \( u = (0, u) \), we have \( \gamma(u) = \gamma(u) \) whence

\[
\langle O(y')O(z') \rangle = \gamma(y)^{\Delta} \gamma(z)^{\Delta} \langle O(y)O(z) \rangle
\]

(22)

accordingly to what was expected.

The four point case parallels the two point one. It is straightforward to show covariance under dilations and special conformal transformations: the argument is exactly the same. The only difference is the Green’s function \( G^2(x, x') \) which is now not constant but we show that in fact it is an invariant function. We remind its expression

\[
G(x, x') = -\frac{c}{2\alpha} \xi^{-\Delta} F \left( \frac{n}{2}, \Delta; \alpha + 1; \xi^{-2} \right)
\]

where

\[
\xi = \frac{1}{2x_0 x'_0} \left\{ \frac{1}{2} \left[ |x - x'|^2 + |x - x'|^2 \right] + \sqrt{|x - x'|^2|x - x'|^2} \right\}
\]

and \( x \equiv (x_0, x), x^* \equiv (-x_0, x) \).

Dilation invariance of the \( \xi \) variable is self evident. Under SCT, \( \xi \) transforms into

\[
\xi = \frac{\gamma(x)\gamma(x')}{2x_0 x'_0} \left\{ \frac{1}{2} \left[ \frac{|x - x'|^2}{\gamma(x)\gamma(x')} + \frac{|x - x'|^2}{\gamma(x)\gamma(x')} \right] + \sqrt{\frac{|x - x'|^2|x - x'|^2}{\gamma(x)\gamma(x')\gamma(x')}} \right\}
\]

but since \( b = (0, b) \), we have \( \gamma(u) = \gamma(u^*) \). We can conclude that the variable \( \xi \) is invariant under SCT too and write

\[
\langle O(\alpha y)O(\alpha z)O(\alpha v)O(\alpha w) \rangle = \alpha^{-4\Delta} \langle O(y)O(z)O(v)O(w) \rangle
\]

(23)

\[
\langle O(y')O(z')O(v')O(w') \rangle = \gamma(y)^{\Delta} \gamma(z)^{\Delta} \gamma(v)^{\Delta} \gamma(w)^{\Delta} \langle O(y)O(z)O(v)O(w) \rangle
\]

(24)
To generalize the argument it is not difficult to write down the 2N point function following the hint given by the two and four point case. We have

$$\left\langle O(x_{1a})O(x_{1b})\ldots O(x_{Na})O(x_{Nb}) \right\rangle \propto \lambda^N c^{2N} \int \frac{dz_1}{z_1^{n+1}} \ldots \frac{dz_N}{z_N^{n+1}} G(z_1, z_2) G(z_2, z_3) \ldots G(z_N-1, z_N) G(z_N, z_1) \times$$

$$\times \left( \frac{z_{10}}{z_{10}^2 + |z_1 - x_{1a}|^2} \right)^\Delta \left( \frac{z_{10}}{z_{10}^2 + |z_1 - x_{1b}|^2} \right)^\Delta \times \ldots$$

$$\ldots \times \left( \frac{z_{N0}}{z_{N0}^2 + |z_N - x_{Na}|^2} \right)^\Delta \left( \frac{z_{N0}}{z_{N0}^2 + |z_N - x_{Nb}|^2} \right)^\Delta \ (25)$$

Every step of our argument can be replicated, so we can say that also at one loop level the AdS/CFT correspondence holds in the sense that one loop corrections have the same transformation laws under the conformal group as the correlation functions at tree level.

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The number of points must be even because of the $\phi \rightarrow -\phi$ symmetry in the action. All functions with an odd number of points vanish.
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