The Importance of Weak Boson Emission at LHC

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Abstract

We point out that gauge bosons emissions should be carefully estimated when considering LHC observables, since real \( W \)s and \( Z \)s contributions can dramatically change cross sections with respect to tree level values. Here we consider observables that are fully inclusive respect to soft gauge boson emission and where a certain number of nonabelian isospin charges in initial and/or final states are detected. We set up a general formalism to evaluate leading, all order resummed electroweak corrections and we consider the phenomenologically relevant case of third family quark production at the LHC. In the case of \( b \bar{t} \) production we find that, due to the interplay between strong and weak interactions, the production cross section can become an order of magnitude bigger than the tree level value.

1 Introduction

It is by now well established that one loop electroweak corrections are not sufficient to keep under control Standard Model predictions at the TeV scale. The reason for this is the sharp growth with energy of these kind of corrections, that reach the 10 % level at 1 TeV. More in detail, this growth is related to the infrared structure of the theory, so that one loop contributions are proportional to \( \log^2 \frac{s}{M_{W,Z}^2} \), the gauge bosons masses \( M_{W,Z} \) acting as infrared regulators [1]. In order to cope with the expected precision of hadronic (LHC) and lepton (ILC) colliders, fixed-order one and two loop corrections [2] and resummation of leading effects [3] have been considered by various groups in the last few years. Broadly speaking, two kinds of observables have been considered. In first place the exclusive observables where only virtual electroweak effects need to be considered [2, 3]. In second place, observables including \( W, Z, \gamma \) emissions have been considered [4]. A prototype of these kind of observables is \( e^+ e^- \rightarrow \text{hadrons} \): the two final jets are detected, while any other object in the final state is summed over, including the final decay products of \( Ws \) and \( Zs \) [4]. In this case, the collider provides two initial nonabelian charges, and due to this fact the outcome is surprising: even though fully inclusive, the cross section is sensitive to the infrared cutoff \( M_W \) and affected by big \( \log^2 \frac{s}{M_W^2} \) terms. This effect, baptized “Bloch-Nordsieck violation”, has been shown to occur only in broken gauge theories, including the abelian case [5]. Recently, electroweak evolution equations, which are the analogous of QCD DGLAP equations, have been derived [6].

The aim of this work is to considered another class of observables, which we might call “partially inclusive”. That is, gauge boson radiation in is still summed over, however we retain the possibility to observe nonabelian charges in both the initial and/or final states. The cross sections we consider are, generally speaking, identified by 4 hard partons, 2 in the initial and 2 in the final state. However in order to compute the leading logs related to virtual and real \( W, Z, \gamma \) emission we can reduce to 2 or 3 relevant legs. Suppose for instance that the flavor of leg 4 is undetected (see fig.1) This could be the case if we consider leg 4 to be a final jet and we do not isolate the jet’s flavor. Then, by unitarity, since we are summing over real and virtual corrections [4], we need only to consider the electroweak corrections
related to legs 1,2,3. This is showed in fig. 1 where the overlap matrix with the remaining three legs is depicted.

2 General formalism

Our starting point, as in previous works, is the SU(2) isospin structure of the overlap matrix, defined in terms of the S-matrix as follows

\[
\langle \beta_1 \beta_2 \ldots \beta_n | S \dagger S | \alpha_1 \alpha_2 \ldots \alpha_n \rangle = O_{\beta_1 \alpha_1, \beta_2 \alpha_2, \ldots, \beta_n \alpha_n} \tag{1}
\]

and the observable cross sections are related to the above definition by:

\[
d\sigma_{\alpha_1 \alpha_2 \ldots \alpha_n} = O_{\alpha_1 \alpha_1, \alpha_2 \alpha_2, \ldots, \alpha_n \alpha_n} \tag{2}
\]

Notice that we use a “generalized S matrix” formalism, such that all the states over which we are inclusive appear on the left of S and all the detected nonabelian charges 1, 2...n appear on the right. So for instance a detected final (outgoing) antiquark is seen as an initial (ingoing) quark state. Namely, for the case \( n = 2 \) this means that we describe cross sections with two electroweak charges in the initial states or systems with one charged particle in the initial and one in the final state, or the case of two final charges.

The SU(2) generators \( t^a, a = 1, 2, 3 \) act on the overlap matrix as follows

\[
(t^a_i \mathcal{O})_{\beta_1 \alpha_1, \ldots, \beta_n \alpha_n} = \sum_{\delta_i} t^a_{\alpha_i \delta_i} \mathcal{O}_{\beta_1 \alpha_1, \ldots, \beta_i \delta_i, \ldots, \beta_n \alpha_n} \quad (t^a_i \mathcal{O})_{\beta_1 \alpha_1, \ldots, \beta_n \alpha_n} = \sum_{\gamma_i} t^a_{\beta_i \gamma_i} \mathcal{O}_{\beta_1 \alpha_1, \ldots, \gamma_i \alpha_i, \ldots, \beta_n \alpha_n} \tag{3}
\]

where the generators \( t^a \) depend on the representation of the considered \( i-th \) particle.

It is convenient to define the isospin generator referred to a single leg \( i \) as \( T_i = t_i + t'_i \). Since we consider energy scales of the order of 1 TeV and beyond, we take all particles to be massless. In other words we work in the high energy limit in which the \( SU(2) \otimes U(1) \) symmetry is recovered; then the overlap matrix is invariant under a symmetry transformation:

\[
T^a_{\text{tot}} = \sum_i T^a_i \quad \exp[\alpha^a T^a_{\text{tot}}] \mathcal{O} = \exp[\alpha \cdot T_{\text{tot}}] \mathcal{O} = \mathcal{O} \quad \Rightarrow \quad T_{\text{tot}} \mathcal{O} = 0 \tag{4}
\]

This property allows to write the overlap matrix as a sum of projectors with definite isospin properties and gives various relations between the apriori independent cross sections (see next sections).
The dressing of the hard overlap matrix $O^H$ to obtain the evolved one $O$ through exchange of virtual and real soft $W$ quanta is described by the external line insertion of the eikonal current:

$$J^\mu(k) = g_w \sum_{i=1}^n T_i \frac{p_i^\mu}{p_i \cdot k},$$

$k$ being the momentum of the emitted soft gauge boson, $p_i$ the $i$-th leg momentum and $g_w$ the SU(2) gauge coupling. Notice that the part of the current proportional to $g'$ is absent altogether because of the cancellation of the abelian components for inclusive observables [4].

By squaring the eikonal current one obtains, in the limit where all invariants are of the same order $2p_i \cdot p_j \approx s$, the insertion operator:

$$I(k) = g_w^2 \frac{p_1 p_2}{(k p_1)(k p_2)} \sum_{i<j}^n T_i \cdot T_j$$

The resummed expression for the overlap matrix is finally given by the following expression involving the insertion operator:

$$O(s) = \exp[L_W \sum_{i<j}^n T_i \cdot T_j] O^H$$

where we have defined the eikonal radiation factor for $W$ exchange:

$$L_W(s) = \frac{g_w^2}{2} \int_M^E \frac{d^3 k}{2\omega k (2\pi)^3} \frac{2 p_1 p_2}{(k p_1)(k p_2)} = \frac{\alpha_w}{16\pi} \log \frac{2s}{M^2}, \quad \alpha_w = \frac{g_w^2}{16\pi}$$

It is useful to rewrite (7) by using

$$\sum_{i<j}^n T_i \cdot T_j = \frac{1}{2} \sum_{i=1}^n T_i \cdot (T_{tot} - T_i) = -\frac{1}{2} \sum_{i}^n T_i^2$$

where we used $T_{tot} O = 0$, so that

$$O(s) = \exp \left[ -\frac{1}{2} L_W \sum_{i=1}^n T_i^2 \right] O^H$$

Eqn. (10) shows the single particle property of inclusive emission, i.e. the fact that corrections are calculated by considering an exponential factor for each leg in a definite total isospin state. In the following we systematically adopt the procedure of writing the overlap matrix as a sum of projectors on total isospin eigenstates and then applying (10) to obtain the “EW BN” corrected overlap matrix. The hard overlap matrix is therefore first decomposed as follows:

$$O^H = \sum_{t_1 t_2 \ldots t_n} O_{t_1 t_2 \ldots t_n}^H P_{t_1 t_2 \ldots t_n}$$

where $O, P_{t_1 t_2 \ldots t_n}$ are operators acting on the $n$ external legs indices, and $O_{t_1 t_2 \ldots t_n}$ are the coefficients of the expansion. The projectors satisfy, by definition:

$$T_j P_{t_1 t_2 \ldots t_n} = t_j P_{t_1 t_2 \ldots t_n}, \quad j = 1 \ldots n, \quad T_{tot} P_{t_1 t_2 \ldots t_n} = 0$$

Then we apply (10) in order to obtain the all order resummed values. Due to property (12) this is particularly simple, since it amounts to the substitution:

$$O_{t_1 t_2 \ldots t_n}^H \rightarrow O_{t_1 t_2 \ldots t_n}(s) = \exp \left[ -\frac{1}{2} L_W(s) \sum_{i=1}^n t_i(t_i + 1) \right] O_{t_1 t_2 \ldots t_n}^H$$
In next section we give the explicit form of the projection operators for the cases of two and three external legs.

To end with, we want to compare the above describe “BN EW” corrections with “Sudakov EW” corrections, i.e. EW corrections given only by the virtual contributions without weak bosons emissions. The latter depend on how the observable is defined, namely on which cutoff is decided on real photon emission: a certain degree of inclusiveness on photons is mandatory in order to render the observable infrared finite. For definiteness and in order to compare with the BN corrections, we choose for the photon a cutoff of the order of the weak scale; the result is an effective SU(2) \( \otimes \) U(1) theory with all gauge bosons at a common mass \( M_W \approx M_Z \) [3]. In this limit Sudakov corrections are in fact rather simple: the resummed cross section is obtained from the hard one by multiplying each external leg by an exponential factor:

\[
\sigma^{Sud}(s) = \exp[-L_W(s) \sum_i (t_i(t_i + 1) + y_i^2 \tan^2 \theta_W)]\sigma_H
\]

where \( \theta_W \) is the Weinberg angle, \( t_i \) is the i-th leg isospin and \( y_i \) its hypercharge [3]. Despite the similarities between (13) and (14), the inclusive (BN) and exclusive (Sudakov) case are of course very different and give rise to significantly different patterns of radiative corrections. Namely:

- in (14) \( t_i \) is the external leg isospin (e.g., \( \frac{1}{2} \) for a fermion) while in (13) \( t_i \) is obtained by composing two single-leg isospins (see fig. 1)
- no correction proportional to \( y^2 \) is present in the “BN” inclusive case, since contributions proportional to the U(1) coupling \( g_Y \) cancel out [4].
- There is a factor 2 of difference in the argument of the exponential (compare (14), (13)).
- while Sudakov corrections always depress the tree level cross section, BN ones can be negative or positive (see section 4).

3 The case of two and three external legs

In this section we give the explicit forms of the projectors in the case of two and three external legs. This allows to calculate the EW BN corrections by simply inserting the appropriate values of the hard cross sections, as we explain in next section. Of course not all of the possible values of \( t_1, t_2 \ldots t_n \), appearing in (11) are allowed. In fact the total isospin must be 0 due to isospin invariance, so for instance in the case of two legs there is no \( P_{10} \) term, since no isospin invariant can be constructed from an isospin 0 and an isospin 1 objects. In the following we present tables with the allowed values for \( t_1 \ldots t_n \). The coefficients \( O_{t_1 t_2 \ldots t_n} \) are also called “form factors” since they are \( s \)-dependent and receive the exponential factor [13]. Here we limit ourselves to fermions, antifermions and transversely gauge bosons in the external legs. Therefore in the following by “boson” we always mean “transversely polarized (weak) gauge boson”.

We now consider the “two external legs” case. Notice that by this we do not mean that only two external particles are present. Rather, we mean a process with an arbitrary number of external particles, but in which only two non abelian weak charges are detected. Therefore, the process \( gg \rightarrow q\bar{q} \) belongs to this category since gluons do not carry weak charges; also the process \( e^+ e^- \rightarrow jets + X \) falls in this case since this process is fully inclusive in the final state: no weak charge is singled out. The case of two initial external charged legs has been widely discussed from various point of view: in [3] the BN violation was put in evidence and more refined studies were done in [6] including the next to leading corrections. Here we extend the same procedure to processes with \textit{two external charged legs} irrespective of their position of initial or final states.

The possible values for the isospin of the two external legs labeled by \( t_1, t_2 \) are given by (f.f.=form factors):
The explicit form of the projection operators is given below for the relevant case of fermions and (transverse) gauge bosons:

- two external fermions

\[ O(\alpha_1, \beta_1; \alpha_2, \beta_2) = O_{00} \delta_{\alpha_1 \beta_1} \delta_{\alpha_2 \beta_2} + O_{11} t_{\alpha_1}^a t_{\alpha_2}^a \]  

(15)

- one fermion and one external gauge boson

\[ O(\alpha_1, \beta_1; a_2, b_2) = O_{00} \delta_{\alpha_1 \beta_1} \delta_{a_2 b_2} + O_{11} t_{\alpha_1}^a T_{a_2 b_2}^a \]  

(16)

- two external gauge bosons

\[ O(a_1, b_1; a_2, b_2) = O_{00} \delta_{a_1 b_1} \delta_{a_2 b_2} + O_{11} T_{a_1 b_1}^a T_{a_2 b_2}^a + O_{22} \mathcal{P}_2(a_1 b_1; a_2 b_2) \]  

(17)

where \( t^a(T^a) \) are the SU(2) generators in the fundamental (adjoint) representation and the isospin 2 projector is defined by:

\[ \mathcal{P}_2(a_1, b_1; a_2, b_2) = \frac{1}{4} \left[ \{T^c, T^d\}_b \{T^c, T^d\}_a T^a_{b_2 a_2} - \frac{16}{3} \delta_{b_1 a_1} \delta_{b_2 a_2} \right] \]  

(18)

The case with three external particles charged under SU(2) is more complicated because the product of three isospins generates many invariant with definite total isospin. As shown in the table below in order to describe a system with three external fermion in the fundamental representation five gauge invariant form factors are needed; a system with two fundamental fermions and one boson (in the adjoint representation ) needs six form factors and for a system with one fermion plus two bosons we have to write nine form factors.

| fermion | fermion | Number of f.f. | fermion | fermion | boson | Number of f.f. |
|---------|---------|----------------|---------|---------|-------|----------------|
| \( t_1 = 0 \) | \( t_2 = 0 \) | 2 | \( t_1 = 0 \) | \( t_2 = 0 \) | 2 | \( t_1 = 0 \) | \( t_2 = 0 \) | 3 |
| \( t_1 = 1 \) | \( t_2 = 1 \) | 2 | \( t_1 = 1 \) | \( t_2 = 1 \) | 2 | \( t_1 = 1 \) | \( t_2 = 1 \) | 3 |
| \( t_1 = 1 \) | \( t_2 = 1 \) | 2 | \( t_1 = 1 \) | \( t_2 = 1 \) | 2 | \( t_1 = 1 \) | \( t_2 = 1 \) | 3 |
| \( t_1 = 1 \) | \( t_2 = 1 \) | 2 | \( t_1 = 1 \) | \( t_2 = 1 \) | 2 | \( t_1 = 1 \) | \( t_2 = 1 \) | 3 |

For the case with three bosons there are 15 form factors. The form factors are gauge invariant combinations of physical cross sections (that correspond to the diagonal elements of the overlap matrix). Reversing the problem, any physical cross section is a combination of form factors. It is interesting to note that the form factors \( O_{ijk...} \) whose sum \( i + j + k + ... = \text{odd number} \) do not contribute to physical cross sections. In practice this means that the degrees of freedom of the overlap matrix projected on the physical space of the cross sections are diminished. In the above examples, with three external particles the form factors \( O_{111} \) and \( O_{122} \) are unphysical. The final result is that, at the level of physical cross sections, the system with three fermions has 4 degrees of freedom: once we know 4 cross sections any
other one is fixed as a combination of these ones. The system of two fermions and one boson has 5 degrees of freedom and finally the one fermion and two bosons only 7.

The isospin decomposition of the overlap matrix in the various cases is given below. Fermionic and antifermionic legs are treated on equal grounds, so for instance the case of one antifermionic and two fermionic legs belongs to the “3 fermionic legs” case.

- 3 fermionic legs

\[
\mathcal{O}(\alpha_1, \beta_1; \alpha_2, \beta_2; \alpha_3, \beta_3) = O_{000} \delta_{\alpha_1 \beta_1} \delta_{\alpha_2 \beta_2} \delta_{\alpha_3 \beta_3} + O_{101} t^a_{\alpha_1 \beta_1} t^a_{\alpha_2 \beta_2} t^a_{\alpha_3 \beta_3} + O_{111} f_{abc} t^a_{\alpha_1 \beta_1} t^b_{\alpha_2 \beta_2} t^c_{\alpha_3 \beta_3}
\]

- 2 fermionic, 1 bosonic

\[
\mathcal{O}(\alpha_1, \beta_1; \alpha_2, \beta_2; a_3, b_3) = O_{000} \delta_{\alpha_1 \beta_1} \delta_{\alpha_2 \beta_2} \delta_{a_3 b_3} + O_{101} t^a_{\alpha_1 \beta_1} \delta_{\alpha_2 \beta_2} T_{a_3 b_3} + O_{111} \delta_{\alpha_1 \beta_1} t^a_{\alpha_2 \beta_2} T_{a_3 b_3} + O_{112} t^a_{\alpha_1 \beta_1} t^b_{\alpha_2 \beta_2} T_{a_3 b_3}
\]

- 1 fermionic, 2 bosonic

\[
\mathcal{O} = O_{000} \delta_{\alpha_1 \beta_1} \delta_{\alpha_2 \beta_2} \delta_{a_3 b_3} + O_{022} \delta_{\alpha_1 \beta_1} T_{a_2 b_2} + O_{101} t^a_{\alpha_1 \beta_1} T_{a_2 b_2} + O_{112} t^a_{\alpha_1 \beta_1} T_{a_2 b_2} + O_{112} t^b_{\alpha_1 \beta_1} T_{a_2 b_2} + O_{121} t^a_{\alpha_1 \beta_1} T_{a_2 b_2} + O_{121} t^b_{\alpha_1 \beta_1} T_{a_2 b_2} + O_{122} f_{abc} t^a_{\alpha_1 \beta_1} f^b_{a_2 b_2} T_{a_3 b_3}
\]

The SU(2) symmetry encoded into eqns. \[15\] \[21\] gives various relations between the apriori independent overlap matrix elements, and therefore between the various cross sections. For instance in the case of \[12\] the overlap has \(2^6 = 64\) values in principle; however there are only 4 independent projectors, and therefore only 4 independent values. The most general relation is the following: let us give the index assignments:

\[
1 = \nu, 2 = e \quad \text{for fermions} \quad 1 = W^+, 2 = W^-, 3 = W^3 \quad \text{for gauge bosons}
\]

then we have \(\sigma_{abc} = \sigma_{a'bc'}\) where the set \(a'bc'\) is obtained from \(abc\) by the exchange \(1 \leftrightarrow 2, 3 \leftrightarrow 3\). So \(\sigma_{112} = \sigma_{221}, \sigma_{331} = \sigma_{332}\) and so on. It is easy to realize that the described exchange corresponds to a unitary transformation with the matrix \(i\sigma_2\). The dressing of the overlap matrix, i.e. resumming leading electroweak double logs at all orders, is done by applying eq. \[10\]. The above formula will be useful in order to evaluate the BN corrections to the high energy cross section for third family quarks at LHC.

### 4 Third generation quarks production at LHC

In this section we will apply some of the above formulas for LHC cross sections partially inclusive over soft W and Z emission. The idea is to analyze the third family (top and bottom) quark production at LHC for very large momentum transfer. In order to give an idea of the size of the corrections we will also compute the resummed Sudakov corrections at leading order. In such a way we can compare processes without any emission of W and processes with the same hard final states but with the possibility to emit soft W bosons.

As is well known we can write the heavy quark production mechanism at LHC as a convolution of the luminosity functions \(L_{ij}\) for the partons \(p_i\) and \(p_j\) times the partonic cross sections \[7\]:

\[
\frac{d\sigma_{pp \rightarrow QQ}}{ds} = \frac{1}{s} \sum_{i,j} L_{ij}(\hat{s}) d\sigma_{p_i p_j \rightarrow QQ}(\hat{s}) \quad L_{ij}(\hat{s}) = \int_{s_0}^{s} \frac{dx}{x^2} f_{p_i} (x) f_{p_j} \left( \frac{\hat{s}}{x} \right)
\]
This implies that the partonic process structure functions of the anti-up and anti-down quarks and of the remaining sea quarks at fixed energy. Not a four legs one, since the $\bar{q}$ proton is a flavour singlet state. To corroborate our statement we show in fig. 2 the SU(2) point of view, implies automatically that, with a reasonable approximation, the sea quarks of a is almost the same of the antiquark down and so on for the other sea families. This statement, from the SU(2) point of view, implies automatically that, with a reasonable approximation, the sea quarks of a proton is a flavour singlet state. To corroborate our statement we show in fig. 2) the $x$ dependence of structure functions of the anti-up and anti-down quarks and of the remaining sea quarks at fixed energy. This implies that the partonic process $q\bar{q} \rightarrow Q\bar{Q}$ is, with a good approximation, a three leg process and not a four legs one, since the $\bar{q}$ leg is summed over SU(2) quantum numbers, and therefore receives no inclusive EW corrections (see also fig. 1).

Figure 2: Parton distributions for the sea of the proton at $Q = 4000 GeV$ as a function of $x$, the fraction of momentum carried by the parton. Left: $s = \bar{s}, c = \bar{c}, b = \bar{b}$. Right: $u, d$.

where $f_{pi}(x)$ is the distribution of parton $i$ inside the proton, $\sqrt{s}$ is the partonic c.m. energy and $\sqrt{s} = 14$ TeV the hadronic one. For each channel we will decompose the hard partonic cross sections in isospin defined form factors whose EW BN corrections can be directly computed with eq. (13).

At this point analyzing the luminosity functions of the sea quarks of the proton we can obtain a quite reasonable simplification for the evaluation of the BN corrections to the hard partonic cross sections in isospin. Lower: $s = 1$ for down type initial quarks, $\alpha_2$ for up and down type initial quarks, $\alpha_3$ for $t, b$ final states and $\alpha_3$ for $\bar{t}, \bar{b}$ antifermion final states. The SU(3)$\otimes$SU(2)$\otimes$U(1) couplings are given respectively by $g_s, g_w, g_y$. In the massless limit chirality is conserved, so we can label the cross sections with the chirality of the initial and final fermions: the chiralities of corresponding antifermions are unambiguously fixed. The overlap matrix receives various contributions from $s$-channel exchange of electroweak gauge bosons and gluons*:

\[
\frac{d\sigma_H^{LL}(\alpha_1, \alpha_2, \alpha_3)}{d\cos \theta} = \frac{\hat{u}^2}{8\pi s^3} \left[ (g_y^4 g_{La1}^2 g_{La2}^2 + 2 g_y^2 g_w^2 g_{La1} g_{La2} t_{a1a1}^3 t_{a2a3}^3) \delta_{a2a3} + g_w^4 t_{a3a2}^c (t^{c\alpha}_{a1}) \right] \delta_{a2a3} \tag{24}
\]

\[
\frac{d\sigma_H^{LR}(\alpha_1, \alpha_2, \alpha_3)}{d\cos \theta} = \frac{\hat{t}^2}{8\pi s^3} \left[ g_y^4 g_{La1}^2 g_{Ra2}^2 + \frac{2}{9} g_s^4 \right] \delta_{a2a3} \tag{25}
\]

\[
\frac{d\sigma_H^{RL}(\alpha_1, \alpha_2, \alpha_3)}{d\cos \theta} = \frac{\hat{t}^2}{8\pi s^3} \left[ g_y^4 g_{Ra1}^2 g_{La2}^2 + \frac{2}{9} g_s^4 \right] \delta_{a2a3} \tag{26}
\]

\[
\frac{d\sigma_H^{RR}(\alpha_1, \alpha_2, \alpha_3)}{d\cos \theta} = \frac{\hat{u}^2}{8\pi s^3} \left[ g_y^4 g_{Ra1}^2 g_{Ra2}^2 + \frac{2}{9} g_s^4 \right] \delta_{a2a3} \tag{27}
\]

*We do not consider here $t$-channel contributions from initial sea heavy quarks like $b\bar{t} \rightarrow b\bar{t}$, a process initiated by the excitations of bottom and top quarks from the gluon sea. However for a full consistent calculation a careful evaluation of these kind of processes has to be included.
where $\hat{t} = -\hat{s}/(1 - \cos \theta)$ and $\hat{u} = -\hat{s}/(1 + \cos \theta)$ are the Mandelstam variables in the partonic c.m. frame. Notice the absence of $q_g^2 g_w^2$ and $g_g^2 g_w^2$ terms since electroweak and strong amplitudes do not interfere due to the color structure. The values of the corresponding hard overlap matrix elements in the LL channel can be obtained by equating the general form (15) to the values of the hard cross sections. Namely, from $O_{LL}^H(\alpha_1, \alpha_1; \alpha_2, \alpha_2; \alpha_3, \alpha_3) = \frac{\partial d\sigma^H_{LL}(\alpha_1, \alpha_2, \alpha_3)}{\partial \cos \theta}$ and defining $\alpha_i = \frac{q_i^2}{\hat{s}}$ one obtains:

$$O_{000}^H = \frac{2 \pi \hat{u}^2}{9 \hat{s}^3} (\alpha_s^2 + 27 \alpha_s^2 + \frac{1}{288} \alpha_y^2), \quad O_{110}^H = \frac{\pi \hat{u}^2}{8 \hat{s}^3} (\alpha_w^2 + \frac{1}{9} \alpha_w \alpha_y) \quad (28)$$

$$O_{011}^H = \frac{8 \pi \hat{u}^2}{9 \hat{s}^3} (\alpha_s^2 - \frac{9}{32} \alpha_w^2 + \frac{1}{288} \alpha_y^2), \quad O_{101}^H = -\frac{\pi \hat{u}^2}{2 \hat{s}^3} (\alpha_w^2 - \frac{1}{9} \alpha_w \alpha_y) \quad (29)$$

The same procedure allows to find the values for the RL channel (in this case we have two different overlaps: one for the initial $u_R$ and one for $d_R$, distinguished by the hypercharge contribution):

$$O_{001}^H = \frac{2 \pi \hat{u}^2}{9 \hat{s}^3} (\alpha_s^2 + \frac{1}{8} \alpha_y^2 y_R), \quad O_{111}^H = \frac{8 \pi \hat{u}^2}{9 \hat{s}^3} (\alpha_s^2 + \frac{1}{8} \alpha_y^2 y_R) \quad (30)$$

The dressed overlap matrix can be now obtained by using the values for the hard overlap matrix of eqs. (28) and applying the rule of eq. (13). So for instance

$$O_{101}(s) = O_{101}^H \exp[-2L_W] = -\frac{\pi \hat{u}^2}{8 \hat{s}^3} (\alpha_w^2 - 4 y_L^2 \alpha_w \alpha_y) \exp[-2L_W] \quad (31)$$

and so on. On the other hand, the LR, RR channels do not receive BN EW corrections.

The gluon gluon hard cross section ($gg \to Q\bar{Q}$) can be decomposed giving the chirality of the final states:

$$\frac{d\sigma^H}{d\cos \theta} = \frac{d\sigma^H}{d\cos \theta} + \frac{d\sigma^H}{d\cos \theta}; \quad \frac{d\sigma^H}{d\cos \theta} = \frac{d\sigma^H}{d\cos \theta} \quad (32)$$

It is clear that the contributions coming from initial gluons, from the point of view of SU(2), are two-charged-leg final states only when left handed heavy quarks are produced, while the overlap of the process $gg \to Q_R \bar{Q}_R$ is singlet under SU(2) decomposition. The isospin structure of the process $gg \to Q_L \bar{Q}_L$ is given by

$$\frac{d\sigma^H}{d\cos \theta}(\alpha_1, \alpha_2) = \frac{\pi \alpha_s^2}{4 \hat{s}} \left( \frac{\hat{t}^2 + \hat{u}^2}{6\hat{u}} - \frac{3(\hat{t}^2 + \hat{u}^2)}{8\hat{s}^2} \right) \delta_{\alpha_1 \alpha_2} \quad (33)$$

corresponding to an overlap $O^H$ given by the coefficients

$$O_{00}^H = \frac{1}{2} \frac{d\sigma^H}{d\cos \theta} \quad O_{11}^H = 2 \frac{d\sigma^H}{d\cos \theta} \quad (34)$$

The evolved overlap matrix and the respective dressed cross sections with the all order resummed virtual and real EW corrections, can now be obtained by using eqn. (10) applied to all the hard overlap form factors. An important feature of such a channel is the fact that, being a mixture of s and t-channels, its angular dependence is different from the $qg$ s-channel cross section. This fact is important not only for $t\bar{t}$, $bb$ production but mainly for $tb$, $\bar{b}t$ production, where the tree level $\alpha_s^2$ cross section proceeds only through s-channel annihilation. The outcome is that the BN corrected angular distribution is different from the tree level one (fig. 4).

Finally, in fig. 6 we plot the differential cross section for the process $PP \to b\bar{t} + X$ for the BN and Sudakov cases.

We can obtain a rather simple formula for the BN corrections if we evaluate the hard cross section in the limit $g_y, g_w \to 0$; this is a reasonable approximation since the contributions proportional to $g_y$ are the bulk of the hard cross sections. It is easy to check that in this case the same correction is obtained
for the $q\bar{q}$ and $gg$ channels, allowing the factorization of the BN corrections with respect the hard QCD cross section:

$$
\frac{d\sigma^{\text{BN}}(PP \rightarrow t\bar{t})}{d \cos \theta} \approx \frac{d\sigma^{\text{BN}}(PP \rightarrow b\bar{b})}{d \cos \theta} \approx \frac{d\sigma_{\text{QCD}}^{H}(PP \rightarrow t\bar{t})}{d \cos \theta} \frac{1}{4} (3 + \exp[-2 L_W(\hat{s})])
$$

(35)

$$
\frac{d\sigma^{\text{BN}}(PP \rightarrow t\bar{b})}{d \cos \theta} \approx \frac{d\sigma^{\text{BN}}(PP \rightarrow b\bar{t})}{d \cos \theta} \approx \frac{d\sigma_{\text{QCD}}^{H}(PP \rightarrow t\bar{t})}{d \cos \theta} \frac{1}{4} (1 - \exp[-2 L_W(\hat{s})])
$$

(36)

Let us now comment on our final results for the cross sections $PP \rightarrow Q\bar{Q} + X$ where $Q\bar{Q} = t\bar{t}, t\bar{b}, b\bar{t}, b\bar{b}$, summarized in figs. 3,4. We recall again that we consider two kinds of observables:

"Sudakov" : $(PP \rightarrow \text{tagged final state} + X)$ with $W, Z \notin X$

"BN" : $(PP \rightarrow \text{tagged final state} + X)$ with $W, Z \in X$

Sudakov corrections always depress the tree level cross section [8], while in BN case the sign of the corrections can be positive or negative. The results for resummed BN and Sudakov corrections to $t\bar{t}$ hadronic cross section are shown in fig. 3. Both corrections are negative in size and more pronounced in the Sudakov case. The $b\bar{b}$ case has a very similar behavior: in fact, in the limit $\alpha_Y \rightarrow 0$ the $t\bar{t}$ and $b\bar{b}$ partonic cross sections are equal; the same holds in the $t\bar{b}$, $b\bar{t}$ case. Notice however that the $t\bar{b}$ and $b\bar{t}$ hadronic cross sections are very different, due to the different luminosities involved. For instance at tree level the hadronic cross section for $t\bar{b}$ depends on $L_{ud}$ while the one for $b\bar{t}$ depends on $L_{d\bar{u}}$, which is smaller.

In the the $b\bar{t}$, $t\bar{b}$ channels the BN corrections instead enhance the cross sections. From fig. 4 we see that the BN enhancement is dramatic: the cross section including gauge bosons emission is more than one order of magnitude bigger than the exclusive (Sudakov) one. This is due to an interesting interplay between strong and weak interactions. In fact in these channels, while the tree level cross sections are proportional to $\alpha_s^2$, when considering BN corrections they receive big contribution from the strong $O(\alpha_s^4)$ channel. Moreover, the tree level and the BN cross sections have different angular behavior (see fig. 4).

For heavy quark production the leading tree level (with no $W$ emission) cross sections are of order $O(\alpha_s^2)$ when diagonal in isospin (\(\sigma_{pp \rightarrow t\bar{t}}^H, \sigma_{pp \rightarrow b\bar{b}}^H\)), while the two isospin changing one \(\sigma_{pp \rightarrow t\bar{b}}^H, \sigma_{pp \rightarrow b\bar{t}}^H\) are $O(\alpha_s^4)$. We can summarize the pattern for the leading tree level and one loop EW corrected cross sections as follows:

| Cross sections | Isospin Structure | Tree Level $(X = 0)$ | BN corrections |
|----------------|------------------|---------------------|----------------|
| $pp \rightarrow Q_i\bar{Q}_j + X$ | $\delta_{ij}$ | $\alpha_s^4$ | $\alpha_s^4 \alpha_u \log^2 \hat{s}$ |
| | $i \neq j$ | $\alpha_w^2$ | $\alpha_w^2 \alpha_u \log^2 \hat{s}$ |

Finally, we expect analogous results for other observables in which nonabelian legs are detected, such as single top production.

In this work the CTEQ5M parton distributions [9] have been used.

5 Conclusions

The main point of this paper is that emission of real gauge bosons should be carefully examined when considering LHC observables. As we have seen, including it or not may result in cross sections differing by an order of magnitude.

As explained in the text, a number of simplifications have been made: we consider the proton sea to be an isospin singlet, only resummed double logs are computed and so on. Therefore our main results in formulae (35,36) and figs 3,4 have to be taken as first order estimates. However first, more detailed calculations are feasible and not too hard and second, we think that the outcome is already clear at this preliminary level: by considering observables that are inclusive, rather than exclusive, of weak bosons emissions, the pattern of radiative electroweak corrections changes significantly. In some cases the cross
sections that one wants to measure are dramatically enhanced (fig. 4), and also differential cross sections such as the angular distribution are significantly different (fig. 4).

While we reckon that it is not entirely clear what can, and will, be measured at the LHC with respect to "soft" final Ws and Zs emission, we think that it is worthwhile opening the physics case.

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References

[1] P. Ciafaloni and D. Comelli, Phys. Lett. B 446 (1999) 278.

[2] E. Accomando, A. Denner and S. Pozzorini, Phys. Rev. D 65 073003 (2002); S. Dittmaier and M. Kramer, Phys. Rev. D 65 033007 (2002); U. Baur, O. Brein, W. Hollik, C. Schappacher and D. Wackeroth, Phys. Rev. D 65, 033007 (2002); E. Maina, S. Moretti, M.R. Nolten and D.A. Ross, Phys. Lett. B 570 205 (2003); M. Beccaria, F. M. Renard and C. Verzegnassi, Phys. Rev. D 69, 113004 (2004), hep-ph/0405036; and Phys. Rev. D 71 (2005) 033005; U. Baur and D. Wackeroth, Phys. Rev. D 70, 073015 (2004); J. H. Kuhn, A. Kulesza, S. Pozzorini and M. Schulze, Phys. Lett. B 609, 277 (2005); E. Accomando, A. Denner and A. Kaiser, Nucl. Phys. B 706 325 (2005); E. Accomando and A. Kaiser, Phys. Rev. D 73 (2006) 093006; E. Accomando, A. Denner and C. Meier, arXiv:hep-ph/0509234; W. Hollik et al., Acta Phys. Polon. B 35, 2533 (2004); S. Moretti, M. R. Nolten and D. A. Ross, arXiv:hep-ph/0503152; arXiv:hep-ph/0509254; arXiv:hep-ph/0603083; M. Beccaria, P. Ciafaloni, D. Comelli, F. M. Renard, C. Verzegnassi, Phys. Rev. D 61 073005 (2000); Phys. Rev. D 61 011301 (2000); M. Beccaria, F. M. Renard, C. Verzegnassi, Phys. Rev. D 63 095010 (2001); Phys. Rev. D 63 053013 (2001); Phys. Rev. D 64, 073008 (2001); Nucl. Phys. B 663, 394 (2003); M. Beccaria, S. Prelovsek, F. M. Renard, C. Verzegnassi, Phys. Rev. D 64, 053016 (2001); M. Beccaria, M. Melles, F. M. Renard, C. Verzegnassi, Phys. Rev. D 65 093007 (2002); M. Beccaria, M. Melles, F. M. Renard, S. Trimarchi, C. Verzegnassi, Int. J. Mod. Phys. A 18, 5069 (2003); M. Beccaria, F. M. Renard, S. Trimarchi, C. Verzegnassi, Phys. Rev. D 68 035014 (2003); A. Denner and S. Pozzorini, Eur. Phys. J. C 18, 461 (2001) and Eur. Phys. J. C 21, 63 (2001); S. Pozzorini, arXiv:hep-ph/0201077; Hori, H. Kawamura, J. Kodaira, Phys. Lett. B 491 275 (2000); W. Beenakker, A. Werthenbach, Phys. Lett. B 489, 148 (2000); A. Denner and S. Pozzorini, Eur. Phys. J. C 18 461 (2001) and Eur. Phys. J. C 21 63 (2001); W. Beenakker and A. Werthenbach, Nucl. Phys. B 630 3 (2002); A. Denner, M. Melles, S. Pozzorini, Nucl. Phys. B 662, 299 (2003); U. Aglietti, R. Bonciani, Nucl. Phys. B 668, 3 (2003) and Nucl. Phys. B 698, 277 (2004); Feucht, J. H. Kuhn and S. Moeh, Phys. Lett. B 561, 111 (2003); B. Feucht, J. H. Kuhn, A. A. Penin and V. A. Smirnov, Phys. Rev. Lett. 93 101802 (2004); A. Denner and S. Pozzorini, Nucl. Phys. B 717, 48 (2005); S. Pozzorini, Nucl. Phys. B 692, 135 (2004); B. Jantzen, J. H. Kuhn, A. A. Penin and V. A. Smirnov, Phys. Rev. D 72 (2005) 051301; J. H. Kuhn, A. Kulesza, S. Pozzorini and M. Schulze, Nucl. Phys. B 727 (2005) 368 and arXiv:hep-ph/0508253; B. Jantzen, J. H. Kuhn, A. A. Penin and V. A. Smirnov, Nucl. Phys. B 731 (2005) 188; B. Jantzen and V. A. Smirnov, arXiv:hep-ph/0603133.

[3] V. S. Fadin, L. N. Lipatov, A. D. Martin and M. Melles, Phys. Rev. D 61 094002 (2000); P. Ciafaloni, D. Comelli, Phys. Lett. B 476 49 (2000); M. Melles, Phys. Rev. D 63, 034003 (2001); Phys. Rev. D 64, 014011 (2001); Phys. Rev. D 64, 054003 (2001); Eur. Phys. J. C 24, 193 (2002); Phys. Rept. 375, 219 (2003); J. H. Kuhn, A. A. Penin and V. A. Smirnov, Eur. Phys. J. C 17, 97 (2000); J. H. Kuhn, S. Moeh, A. A. Penin, V. A. Smirnov, Nucl. Phys. B 616, 286 (2001) [Erratum-ibid. B 648, 455 (2003)].

[4] M. Ciafaloni, P. Ciafaloni and D. Comelli, Phys. Rev. Lett. 84, 4810 (2000); Phys. Lett. B 501, 216 (2001); Nucl. Phys. B 589 359 (2000); Nucl. Phys. B 613, 382 (2001); P. Ciafaloni, D. Comelli and A. Vergine, JHEP 0407, 039 (2004).
[5] M. Ciafaloni, P. Ciafaloni and D. Comelli, Phys. Rev. Lett 87, 211802 (2001).

[6] P. Ciafaloni, M. Ciafaloni and D. Comelli, Phys. Rev. Lett 88, 102001 (2002); P. Ciafaloni and D. Comelli, JHEP 0511 (2005) 022.

[7] “QCD and Collider Physics” (Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology) by R. K. Ellis, W. J. Stirling, B. R. Webber, T. Ericson (Series Editor), P. Y. Landshoff (Series Editor).

[8] M. Beccaria, S. Bentvelsen, M. Cobal, F. M. Renard and C. Verzegnassi, Phys. Rev. D 71 (2005) 073003; S. Moretti, M. R. Nolten and D. A. Ross, arXiv:hep-ph/0603083

[9] H. L. Lai et al. [CTEQ Collaboration], Eur. Phys. J. C 12 (2000) 375.
Figure 3: Sudakov and BN electroweak corrections at Leading Log order for the process $PP \rightarrow t\bar{t}$ (similar results hold for $PP \rightarrow b\bar{b}$) in the massless limit. Top: effects of radiative corrections on $\frac{d\sigma}{d\hat{s}}$ at $\theta = \frac{\pi}{2}$, $\hat{s}$ being the partonic c.m. energy (which is also the $t\bar{t}$ invariant mass) and $\theta$ the partonic reference frame scattering angle. Bottom: the same for $\frac{d\sigma}{d\hat{s}}$, integrated over $\theta$ for $p_\perp > 400$ GeV.
Figure 4: Top: angular dependence at fixed energy ($\sqrt{s} = 2000$ GeV in the partonic frame) of the LL BN cross section (red dashed line) and of the Sudakov one for $t\bar{b}$ production (blue continuous line). Bottom: $\sqrt{s}$ dependence of the ratio of BN and Sudakov cross sections for $p_{\perp} \geq 400$ GeV. Dashed (red) line for $b\bar{t}$, continuous (blue) line for $t\bar{b}$. 
Figure 5: The differential cross section for $b\bar{t}$ production, $\frac{d\sigma}{d\hat{s}}$, in pb/GeV$^2$, integrated over $\theta$ for $p_\perp > 400$ GeV. Dashed (red) line is for the fully inclusive LL BN case, while continuous (blue) line for the exclusive Sudakov case.