SETAR (Self-exciting Threshold Autoregressive) Non-linear Currency Modelling in EUR/USD, EUR/TRY and USD/TRY Parities

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Abstract  In economies that are open to foreign markets the numerical value of the currencies as a macroeconomic variable is of great importance especially when the mutual dependency among the economies is concerned. When it is considered in terms of political economy, the targeted level of the currencies have vital importance especially in economies that have the characteristics of export-driven growth and in economies that struggle not to disrupt the picture in macroeconomic design. When it is considered that each time series has a structure that is sensitive to its own internal dynamics (sometimes these dynamics are expressed as the time series components), these dynamics provide us with coordinates for estimations and may eliminate the compulsory dependency on the outsourced variables at a serious level. This is exactly what has been done in this study. First of all, the non-linear time series analyses are examined in terms of linearity tests, and the linearity tests are applied for all parties and for different time periods. Then, the SETAR Modelling, which is the title of the study, has been applied in order to explain the non-linear pattern in detail. The SETAR Modelling process and other definitions statistical analyses of this model have been applied in relevant parities for separate time periods. The SETAR model, which is one of the TAR Group modeling, shows a better performance than many other linear and non-linear modeling. In this study, the secondary purpose is to express that the SETAR model performance is superior to the other models by considering the observation values of the parities.

Keywords  Currency, Linearity Tests, Non-linear Time Series Analysis, SETAR Model

1. Introduction and the Literature

The TAR (Threshold Autoregressive) Model Family, which is important in non-linear time series modelling because of being more practical, keeps its popularity in academic literature. The basic logic of TAR Modelling is allowing different regimes that allow autoregressive analysis of different levels. TAR Modelling was first handled by Tong (1978) in [21] in the literature. Then, parallel studies were conducted by Tong and Lim (1980) in [19]. Again, [23] presented the SETAR (Self Excited Autoregressive Models), which has a popular usage area, to the literature, and with the help of this a light was cast on many non-linear analyses. For example, the SETAR Modelling was used by Tong and Yeung (1991)1 in [20], who examined the data on the assets market prices; by Tahir Ismail and Zaidi (2006) in [12], who examined the currency incomes by using the currencies of Malaysia, Singapore and Thailand; by Krager and Kugler (1993) in [11], who studied currencies, by Chappell, Mistry, Padmore, Ellis (1996) in [7], who examined the real currencies; by Feng and Liu (2002) [8] in a study conducted on the GDP of Canada covering the period between 1965 and 2000 period; and by P. Clements and Smith (1997) [14] in a study that was based on currencies, GNP and on the estimation performance among the other variables. Chan and Tsay (1998) in [6], Potter (1995) in [16], Tong (1990) in [23], Tiao and Tsay (1994) in [17], Hansen (1997) in [9] also have studies conducted on these group modellings. The definition, estimation and interpretation in TAR models are simpler when compared with the other non-linear models. Bratckoviene (2012) in [4] conducted a study on Real Harmonized Consumer Price Index in Lithuania, and used the adapted SETAR models for January/1996 - December/2009 period. He reported that the properties or the characteristics of the adapted SETAR of time series with complex nature were observed. Aydın and Güneri (2015) in [1] conducted a comparative study for Export Volume Index and Domestic Producer Price Index Series in Turkey, and

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1 SETAR-type non-linearity tests were included in the articles of the same authors and the article was published in Applied Statistics Magazine in 1991.
evaluated the performances of Hybrid AR, SETAR and ARM models. As a result of the evaluations, they reported that the best modeling for the time series in question was the AAR-SETAR modeling. Boero and Lampis (2016) in [3] conducted a study for 4 major European countries and applied SETAR modeling by using Industrial Production Index (IPI) values, and investigated the belief that claimed that the dynamic determination process increased the prediction performance (Teräsvirta et al. (2005)). However, the literature is not rich in terms of the properties of the sampling on this model group and in terms of test statistics (Hansen, 1997).

2. TAR Group Models

It is possible to show a TAR Model with k regime as follows; (Montgomery and others (1998) in [13])

\[
Y_t = \beta_0(i) + \beta_1(i)Y_{t-1} + \ldots + \beta_p(i)Y_{t-p} + \epsilon_t
\]

\[
\tau_{i-1} < x_{t-d} < \tau_i, \quad i = 1, \ldots, k
\]

(1)

Here, \(\beta_1(i), \beta_2(i), \ldots, \beta_p(i)\) are the autoregressive parameters of \(i^{th}\) regime, \(Y_{t-1}, Y_{t-2}, \ldots, Y_{t-p}\) variables are the ones that are believed to influence the \(Y_t\) dependent variable in each regime, and \(\epsilon_t\) is the error term. \(d\) and \(p\) denotes a positive integer (d shows the delay parameter here), \(k\) denotes the number of the regimes in the model; \(x_{t-d}\) denotes the threshold variable, and \(\tau\) denotes a real number (\(-\infty < \tau < +\infty\)). The model may be determined as based on the position of the observance values according to the \(x_{t-d}\) variable.

A two-variable TAR Model, on the other hand, is expressed as follows in Hansen (1997);

\[
Y_t = (\phi_0 + \phi_1Y_{t-1} + \ldots + \phi_pY_{t-p})I(q_{t-1} \leq \gamma) +
+
(\phi_{2,0} + \phi_{2,1}Y_{t-1} + \ldots + \phi_{2,p}Y_{t-p})I(q_{t-1} > \gamma) + \epsilon_t
\]

(2)

Here, \(q_{t-1}\) denotes the threshold variable. \(I(.)\) denotes the indicator function, \(q_{t-1} - q(Y_{t-1}, \ldots, Y_{t-p})\) denotes the functional structure, and \(\gamma\) denotes the threshold parameter or the threshold value. The threshold variable is expressed as \(Y_{t-d}\) (\(d\) is an integer).

When expressed in a different way, the following model may be considered; (Hansen (1997))

\[
Y_t = X_t(\gamma) \omega + \epsilon_t
\]

(5)

Here, if the relevant model is described by separating it into terms, the \(\omega\) parameter will be as follows;

\[
\omega = (\alpha' \beta'), \quad (\alpha = (\alpha_0, \alpha_1, \ldots, \alpha_p)) \quad \text{and} \quad \beta = (\beta_0, \beta_1, \ldots, \beta_p)
\]

(6)

The expression \(X_t(\gamma)\), on the other hand, will be expressed as follows in Hansen (1997)

\[
X_t(\gamma) = (X_t^I(Y_{t-d} \leq \gamma))X_t^I(Y_{t-d} > \gamma)'
\]

(7)

As a consequence, the relevant model will be shown as follows, as it is expressed in the study of Hansen (1997) in [9].

According to the different forms of the \(x_{t-d}\) threshold variable, the TAR group models also show differences. In this sense, the SETAR Modelling will be examined in the next title.

2.1. Self-excited Autoregressive Models (SETAR)

Although there are few origin studies in the field of threshold autoregressive models, several studies attract attention. [2] and [21] conducted studies and expressed the following procedure about the SETAR models and parameter estimations, which we will explain.

2.1.1. Hansen’s Approach

The Structure of the Model

These are the models that are valid in case the threshold variable in the TAR modelling is determined as the delayed value of the relevant time series.

A Double-Regime SETAR Model may be shown in a simple way as follows. (Yılancı, (2007), in [26])

\[
Y_t = \begin{cases} 
\beta_1^1Y_{t-1} + \epsilon_t^1 & Y_{t-1} < \gamma \\
\beta_2^2Y_{t-1} + \epsilon_t^2 & Y_{t-1} \geq \gamma 
\end{cases}
\]

(3)

Or it may be shown as follows based on the notation given in the study of Hansen (1997) in [9];

\[
Y_t = (\alpha_0 + \alpha_1Y_{t-1} + \ldots + \alpha_pY_{t-p})I(Y_{t-d} \leq \gamma) +
+(\beta_0 + \beta_1Y_{t-1} + \ldots + \beta_pY_{t-p})I(Y_{t-d} > \gamma) + \epsilon_t
\]

(4)

Here, again \(p\) denotes the autoregressive level; \(\gamma\) denotes the threshold parameter or the threshold value. The threshold variable is expressed as \(Y_{t-d}\) (\(d\) is an integer).

2 The problem of the level of the delay here will be solved in further parts.
\[ Y_t = x_t'\alpha I(Y_{t-d} \leq \gamma) + x_t'\beta I(Y_{t-d} > \gamma) + \epsilon_t \quad (8) \]

When the (5) model is considered, the method for estimating the \( \omega \) parameter is the Successive Least Squares Method because the model parameters are non-linear (Hansen, 1997)

**The Estimation of the Parameters**

The following formulas are given by Hansen (1997) in [9] for the estimation of the \( \omega \) parameter and for the inclusion.

\[ \hat{\omega} = \left( \sum_{i=1}^{n} X_i(\gamma)X_i(\gamma) \right)^{-1} \left( \sum_{i=1}^{n} X_i(\gamma)Y_i \right) \quad (9) \]

\[ \hat{e}_i(\gamma) = Y_i - X_i(\gamma)\hat{\omega}(\gamma) \quad (10) \]

The inclusion variance that is dealt with in the Successive Least Squares Method is as follows; (Hansen, (1997))

\[ \sigma^2_n(\gamma) = \frac{1}{n} \sum_{i=1}^{n} \hat{e}^2(\gamma) \quad (11) \]

The search in (12) is important for the selection of the threshold parameter. The possible threshold parameter in the time series being more in number makes the error variance to be obtained become various. For this reason, it would be proper to select the threshold parameter (\( \gamma \)) that will minimize the inclusion variance (12) as shown. Right at this point, the process will follow an algorithmic system for selecting the threshold variable \( Y_{t-d} \) and the threshold parameter (\( \gamma \)), which will minimize the error variance.\(^3\)

### 2.1.2. Tong’s Approach

**The Structure of the Model**

\[ Y_t = \begin{cases} \beta_1^{(1)}Y_{t-1} + \epsilon_t^{(1)} & Y_{t-d} \geq \gamma \\ \beta_1^{(2)}Y_{t-1} + \epsilon_t^{(2)} & Y_{t-d} \geq \gamma \end{cases} \quad (13) \]

Tong (1983) in [22] took the above model as the basis and recommended the 3 steps explained as follows.

**The Estimation of the Parameters**

**Step 1**

It is assumed that the \( d \) and \( \gamma \) values are known as the first step. Based on these assumptions, the observation values are separated into small sub-groups, and the AIC data criterion for each sub-group is calculated with the \( p_i \) (\( i = 1, \ldots, k \)) level.\(^4\)

This is shown as follows;

\[ AIC(p_i) = \min[AIC(k_i)] \quad i = 1, 2 \quad (14) \]

In this situation, firstly the \( p_i \) value of each regime is obtained (by using \( \min[AIC(k_i)] \)) in return for the constant \( d \) and \( \gamma \) values.

**Step 2**

In the second step, the \( d \) value is kept constant (it is assumed that it corresponds to a certain value; in other words, it is known), this time, the threshold parameters that will minimize the \( AIC \) data criterion value are tested. In other words, the \( \gamma \) value that minimizes the \( AIC(d, \gamma) \) value is selected from among the other threshold parameters. This situation is shown in Tong (1983) in [22] as follows.

\[ AIC(d_0, \gamma) = \min[AIC(d_0, \gamma)] \quad (15) \]

**Step 3**

In the first two steps, the \( p_i \) and \( \gamma \) values are determined. In the remaining third step, the \( d \) value will be determined. The \( d \) value that minimizes the \( NAIC(d) \) value will be found from the \( d \) choice in \( k \) number.\(^5\)

After the 3 steps that are evaluated by using the data criterion, the model will be estimated by depending on the abovementioned parameters.

### 2.1.3. Tsay’s Approach

Tsay (1989) presented the successive regression and local estimation as a different method to the SETAR Modelling. Tsay explained this estimation procedure both in his own study in 1989 and in the study about the assets volatility he conducted with [2]. The modelling steps in the study of Tsay (1989) were presented as follows;

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\(^3\) When \( Y_{t-d} \) variable is considered, it is important to select the delay length. Hansen (1997) expressed in his study that the \( d \) delay length might not be known, and for this reason, the iterations would be followed as if the \( d \) numerical value were known.

\(^4\) Although Tong suggested the maximum level in the regimes as \( L = n^{\alpha} (\alpha < (1 / 2)) \), the selection of this value is optional.

\(^5\) Here, since the values of the variable \( d \) will influence the number of the observations (\( n \)) in different regimes, the NAIC value will be used instead of the usual \( AIC \). The \( NAIC \) here is shown as \( NAIC(d) = \frac{AIC(d)}{n - n_d} \).
1. Determining the $AR$ level ($p$) (Here, two possible methods are mentioned. The first one is using the $AIC$ data criterion, and the second is making use of the $PACF$.)

2. Applying the threshold linearity test corresponding to the $p$ and possible $d$, which are data. (If the process is decided to be non-linear, the delay value that gives the highest $F$ test value for the relevant $p$ value will be selected from among the threshold variables. Actually, the linearity test recommended here has been expressed as the combination of several linearity tests, as it is also expressed in Tsay (1989) in [25]. The linearity tests mentioned here are the linearity tests belonging to Petrucelli and Davies (1986) in [15], Keenan (1985) in [10], and Tsay (1986) in [24]).

3. Determining the threshold values by using the distribution graphics. (Here, the purpose is determining the place of the threshold parameter by making use of distribution graphics visually for the threshold variable of some statistical parameters.) The statistical parameters in question are as follows:
   - Inclusion estimations
   - Standardized inclusion estimations
   - The $t$ value of the successive estimations of the autoregressive coefficients

In practice, mostly the $t$ values of the recursive estimations of the autoregressive (AR) coefficients in distribution regressions are the subject matters. The significant $t$ values converge towards a certain value as the repetitive (successive) regressions are established. When the unknown threshold parameter is encountered, the parameter estimations and therefore the $t$ values begin to be distracted. When the distribution graphics of the inclusion estimations are considered, it is necessary that the dependency relation between the internal variables and the inclusion estimations from the graphics. When the graphical verticalization between the dependent variables and the inclusion estimations disappears, the model starts to alternate. This situation in return changes the regime in the location where the change happens. Eventually, the observation value at which changes the regime will be $selected$ as the threshold parameter.

4. Determining the autoregressive level of each regime with linear auto-regression techniques.

In summary, Tsay (1989) conducted a study and determined firstly the AR level in modeling process, and then applied threshold linearity test for predefined $p$ and probable $d$ value. Through this Linearity Test, he decided whether the process was linear or not, and selected the delay value ($d$) that gave the highest $F$ test value for the relevant $p$ value, which was selected as the threshold value. After the $(Y_{t-d})$ value of the threshold variable was determined, the threshold value was determined by making use of the distribution graphics. The distribution graphics residual estimations mentioned here are the $t$ values of the standardized residual estimations and consecutive estimations of the model coefficients. After this stage, the autoregressive level of each regime was determined with the linear modeling approaches.

3. Method and Findings

3.1. Application on Currencies

In our study, the parities that are expressed as major parties, which are many in number among the currency parities, and the EUR/USD parity, which is active in operations in spot currency market, and the USD/TRY, EUR/TRY parities, which are convertible into TL will be taken as the basis. The parities will be examined for descriptive statistics in 3-period intervals as D1 (1-day), W1 (1-week), MN (1-month); and for linearity tests, the examination will be made as D1 (1-day), W1 (1-week), MN (1-month). Since the time intervals of the observation values were taken from the history section of the Forex platform that was provided to the users by intermediary institution acting in Forex market, and since the maximum observation number was limited with the data given here, they were expressed as given in the table. In addition, since investors are inclined to take long-term position, daily (D1), weekly (W1) and monthly (MN) data were taken. The parities are given as follows with the relevant intervals and the start and end dates. If we express the justification of using the EUR/USD, USD/TRY and EUR/TRY parities in this paper; firstly, the EUR/USD parity was used because it is the most frequently processed parity in Forex Market. The USD/TRY and EUR/TRY parities, on the other hand, were used because of the volatility experienced in the Turkish Lira in recent years. Especially the exchange policy and monetary policy of the economy administrator in Turkey require more sensitive investigation in terms of mathematical modeling, and therefore SETAR, which is a more sensitive modeling tool, was used.
Table 1. Parities and the Number of the Observation of the Parities (n)

| Parities | Time Period | Start Date   | End Date     | Number of Obsrv. (n) |
|----------|-------------|--------------|--------------|----------------------|
| EUR/USD  | DI          | 31.12.1999   | 26.02.2015   | 3944                 |
|          | WI          | 26.12.1999   | 22.02.2015   | 792                  |
|          | MN          | 1.12.1999    | 1.02.2015    | 183                  |
| EUR/TRY  | DI          | 31.05.2010   | 26.02.2015   | 1171                 |
|          | WI          | 30.05.2010   | 22.02.2015   | 247                  |
|          | MN          | 1.05.2010    | 1.02.2015    | 58                   |
| USD/TRY  | DI          | 31.05.2010   | 26.02.2015   | 1202                 |
|          | WI          | 30.05.2010   | 22.02.2015   | 248                  |
|          | MN          | 1.05.2010    | 1.02.2015    | 58                   |

In the light of the abovementioned currency categories, the relevant observation values of the given intervals are obtained from Integral Trader Version: 4.00 Build 765 (12 Dec 2014) (Copyright 2001-2014, MetaQuotes Software Corp.) platform.

The descriptive statistics of linear time series, modellings and diagnostic methods will be applied separately to the observation values mentioned in this study.

It is important that before the SETAR modelling is applied to the datasets, the issue of whether they are linear or not must be tested. For this purpose, firstly, the linearity tests will be dealt with, and then the SETAR modellings will be given together with parameter estimations.

4. Linearity Tests

Although the linearity tests for non-linear time series are various, they show parallelism with the method used in relevant studies. The most popular and qualified linearity tests will be explained in this study; and eventually, the Likelihood Ratio Linearity Test, which tests the threshold linearity hypotheses, will especially be dealt with.

4.1. Keenan’s One-Degree Test for Nonlinearity

It denotes the linearity test against Second-Level Volterra Expansion. In this context, the Keenan Test examines and tests the Quadratic Nonlinearity Hypothesis and provides information on threshold nonlinearity (Keenan, 1985 in [10]). This situation refers to the F test in the following model:

\[ Y_t = \mu + \sum_{u=0}^{\infty} a_u \varepsilon_{t-u} + \sum_{u,v=0}^{\infty} a_{uv} \varepsilon_{t-u} \varepsilon_{t-v} \]  

(16)

In case the 3rd Term of the model is zero, it gives proof on linearity. In this situation, the null hypothesis will express the situation in which the 3rd Term in Model (16) is equal to zero.

In [10]’s test, first of all a regression is formed between \( Y_t \) and \( (1, Y_{t-1}, Y_{t-2}, \ldots, Y_{t-p}) \) at \( p \) level; and based on this, the \( \hat{\varepsilon}_t \) inclusions are obtained. Then, a regression is established between \( Y_t^2 \) and \( (1, Y_{t-1}, Y_{t-2}, \ldots, Y_{t-p}) \), and the following is formed with the \( \hat{\sigma}_t^2 \) inclusions (Keenan, 1985),

\[ \tilde{\eta}_t^2 = \frac{\sum_{t=p+1}^{n} \hat{\varepsilon}_t^2 \hat{\sigma}_t^2}{\sum_{t=p+1}^{n} \hat{\sigma}_t^2} \]  

(17)

and then the F test statistics will be expressed as follows in Keenan (1985):

\[ F = \frac{(n-2-p-2)\tilde{\eta}_t^2}{(HKT - \tilde{\eta}_t^2)} \]  

(18)

Here, HKT denotes the sum of the error squares expressed in the first model. \( F \sim F_{1,n-2,p-2} \)

4.2. Tsay’s Test for Nonlinearity

Tsay’s (Tsay, 1986) linearity test is based on recursive auto regression and destructive term estimators, and firstly, the recursive auto regressions are established starting from \( b \), observation value in return for the \( p \) and the relevant \( d \) values with \( AR \) level, and then the model is established between the \( \hat{\varepsilon}_t \) values and \( (1, Y_{t-1}, Y_{t-2}, \ldots, Y_{t-p}) \). Then the following test is obtained among the inclusions of the model formed

\[ \text{Here, } b \text{ is expressed as } b = (n/10) + p. \]
with $\hat{e}_t$:

$$F(p,d) = \frac{\sum \hat{e}_{t}^2 - \sum \hat{e}_{t}^2}{(p+1)} / \frac{\sum \hat{e}_{t}^2 / (n-d-b-p-h)}$$  \hspace{1cm} (19)

Here, it is expressed that $h = \max(1, p+1-d)$. In the F test of Tsay, the null hypothesis claiming that the relevant time series is $AR(p)$. The recursive regression mentioned here is the modeling approach that was focused on the change in the model parameters. The sharp changes in the numerical values of the parameters in the model that occur as a result of each additional iteration, degenerations or refractions are considered in recursive regression.

4.3. Likelihood Ratio Test for Threshold Nonlinearity

Keenan’s and Tsay’s quadratic nonlinear tests above are insufficient in dealing with the threshold non-linearity hypothesis. Therefore, Tong (1986) in [18] developed a different approach to test the opposite hypotheses.

- The null hypothesis: “Model is an autoregressive process. In other words $AR(p)$”, and
- The opposite hypothesis “Model is the TAR model with constant variance at $p$ level with two regimes”.

In their studies, Tong (1986) dealt with the probability ratio that focused the numerical approach. When the excessiveness of the observation value of the dataset in question is considered, it is inevitable that the threshold autoregressive processes and the iterative mathematical processes that must be performed about the modelling of these processes are done in a package program that includes statistical notations. Especially the “R” statistical package program has been used because of its having open-source systematic and its being used in a great extent that cannot be underestimated in the field of statistics in the literature. This program provides vast opportunities in terms of data and graphics performance. It was necessary to develop the commands or the codes that were dealt with in time series to be able to form the threshold autoregressive model with R, which is the subject matter of this study. These codes include both the time series analysis commands and the more up-to-date structures in the developing academic literature.

The linearity tests that will be dealt with in the further parts of the study were found by finding the model hyper parameters; and the descriptive statistics part and the SETAR modelling results were found with R Project 3.1.2 Version.

5. Applying the Linearity Tests to the Relevant Parameters

When the significance values in Table 2 are examined, it is observed that the null hypothesis in the 3 parities in the Keenan’s (1985) test and Tsay’s (1986) test are not denied. The null hypothesis was denied in the three parities that were examined in the Likelihood Ratio test except for the EURUSD D1, EURTRY D1 and EURTRY MN parities. Therefore, the expression that may be uttered about linearity is as follows; “Some of the parities in different time periods show linear characteristics, while some others show non-linear pattern. The linearity tests of Keenan (1985) and Tsay (1986) give similar results in relevant parities. It is necessary to point out that, here, while the null hypothesis is tested against a non-alternative hypothesis in the linearity tests of Keenan and Tsay, the null hypothesis is tested against a specific alternative hypothesis in the Likelihood Ratio test”.

5.1. Applying the SETAR Modelling to Parities

In this part of the study, the SETAR modelling in Tong’s approach will be applied separately in the relevant time period for each parity, and the findings will be discussed in the light of descriptive statistical notations. For this purpose, firstly, the hyper parameters of the SETAR modelling (SETAR hyper parameters) of the relevant parities will be given with the R Project (version 3.1.2) printout.

### Table 2. Keenan (1985), Tsay (1986) and Likelihood Ratio linearity tests

| Parity  | Keenan’s (1985) One-Degree Test For Nonlinearity | Tsay’s (1986) Test For Nonlinearity | Likelihood Ratio Test for Threshold Nonlinearity |
|---------|-------------------------------------------------|-----------------------------------|-----------------------------------------------|
| EURUSD  | Test Statistics | P value | Test Statistics | P value | Test Statistics | P value |
| D1      | 3.424          | 0.064   | 2.513          | 0.113   | 5.016           | 0.207   |
| W1      | 3.448          | 0.063   | 2.698          | 0.100   | 2421.08         | 0.000   |
| MN      | 3.655          | 0.057   | 3.015          | 0.084   | 528.438         | 0.000   |
| EURTRY  | Test Statistics | P value | Test Statistics | P value | Test Statistics | P value |
| D1      | 2.792          | 0.094   | 0.679          | 0.410   | 9.744           | 0.000   |
| W1      | 2.666          | 0.103   | 0.381          | 0.537   | 690.584         | 0.000   |
| MN      | 2.513          | 0.118   | 0.336          | 0.564   | 8.908           | 0.000   |

| USDTRY  | Test Statistics | P value | Test Statistics | P value | Test Statistics | P value |
| D1      | 0.023          | 0.877   | 0.010          | 0.919   | 2989.552        | 0.000   |
| W1      | 0.110          | 0.739   | 0.070          | 0.790   | 606.222         | 0.000   |
| MN      | 0.104          | 0.747   | 0.243          | 0.623   | 145.833         | 0.000   |

* The null hypothesis may be rejected at 5% significance level. (Since many statistical analyses take the analysis significance value as 5%, the significance value ($\alpha$) was taken as 0.05 in this study.)
Table 3. EUR/USD - SETAR Model Hyper parameters

| Parity | m | Threshold Delay | mL | mH | Threshold Value | Pooled AIC | Number of Possible Threshold Value | Number of Threshold Values tested with Hyper Parameters |
|--------|---|-----------------|----|----|-----------------|------------|-----------------------------------|------------------------------------------------------|
| EURUSD | D1 | 2               | 0  | 1  | 1               | -2713.74   | 1954                             | 3908                                                 |
|        | W1 | 2               | 0  | 1  | 1               | -4183.613  | 498                              | 996                                                  |
|        | MN | 2               | 0  | 1  | 1               | -678.8609  | 124                              | 248                                                  |

*m: The autoregressive level of the whole model

**Threshold Delay: The delay level of the Self-excited model

***mL: Low regime level, mH: High regime level

Graphic 1. EUR/USD, D1 threshold and AIC values

Graphic 2. EUR/USD, W1 threshold and AIC values
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in EUR/USD, EUR/TRY and USD/TRY Parities

Graphic 3. EUR/USD, MN threshold and AIC values

The delay (d) of the threshold value of the model and the threshold value are defined in Graphic 1-2-3 (within relevant time range) where the pooled-AIC value is at the minimum level.

Table 4. EUR/USD - SETAR Model Estimation and Coefficient Statistics

| Parity | Low Regime | High Regime | Threshold Value | Residuals Variance |
|--------|------------|-------------|-----------------|--------------------|
|        | Constant   | phiL.1      | phiL.2          | Constant           | phiL.1    | phiL.2 |
| EUR/SD | D1         | 0.008094    | 0.991197        | -                  | 0.004810  | 0.996353 | -     |
|        | W1         | 0.03896     | 0.957703        | -                  | 0.02339   | 0.98226 | -     |
|        | MN         | 0.15693     | 0.829344        | -                  | 0.11794   | 0.91075 | -     |

Coefficient Statistics

| Parity | Estimation | Standard Error | p Value | Estimation | Standard Error | p Value |
|--------|------------|----------------|---------|------------|----------------|---------|
| D1     | Constant   | 0.00809        | 0.00566 | 0.004810   | 0.004810       | 0.00172 |
|        | phiL.1     | 0.99119        | 0.00615 | 0.99635    | 0.99635        | 0.00131 |
|        | phiL.2     | -              | -       | -          | -              | -       |
| W1     | Constant   | 0.03896        | 0.028179| 0.02339    | 0.02339        | 0.00859 |
|        | phiL.1     | 0.95770        | 0.03060 | 0.98226    | 0.98226        | 0.00653 |
|        | phiL.2     | -              | -       | -          | -              | -       |
| MN     | Constant   | 0.15693        | 0.13341 | 0.11794    | 0.11794        | 0.03743 |
|        | phiL.1     | 0.82934        | 0.14555 | 0.91075    | 0.91075        | 0.0284  |
|        | phiL.2     | -              | -       | -          | -              | -       |

***: Very close to zero, **: Below 0.001, *: Below 0.01, ( ) : Below 0.05, ( ) : Very close to 1

The descriptive properties of the SETAR model of the relevant parities are given in D1 time periods in the following tables. The “$ff” expression in the graphics is written for ease of encoding, and denotes the relevant time series.
Graphic 4. EUR/USD, D1 data and inclusions

Graphic 5. EUR/USD, D1- Regime-sensitive Scatter Diag
When especially Graphic 5 and Graphic 6 are examined, it attracts attention that polarization in the regimes disappear as the delay intervals of the parity values, which are the subject matter of diaphragms, expands.
Graphic 8.  EUR/USD, D1- Threshold value graphics

Graphic 7 expresses the red pattern which is over the threshold value and the black patterns which is over the threshold value. When Graphic 8 is examined the first graphic threshold value indicates the observation values. Again, when the second graphic is examined in Graphic 8, it is observed that the position of the threshold value may be determined provided that 15% of the observation values from both ends are kept hidden.

Table 5.  EUR/TRY - SETAR Model Hyper parameters.

| Parity | m | Threshold Delay | mL | mH | Threshold Value | Pooled AIC | Number of Possible Threshold values | Number of Threshold Values Tested with Hyper parameters |
|--------|---|-----------------|----|----|-----------------|------------|-------------------------------------|-------------------------------------------------------|
| EURTRY |   |                 |    |    |                 |            |                                     |                                                        |
| D1     | 2 | 0               | 1  | 1  | 2.3985          | -6493.028  | 738                                 | 1476                                                  |
| W1     | 2 | 0               | 1  | 1  | 2.5729          | -977.2308  | 168                                 | 336                                                   |
| MN     | 2 | 0               | 1  | 1  | 2.4898          | -154.7870  | 38                                  | 76                                                    |

*m: The autoregressive level of the whole model

**Threshold Delay: The delay level of the Self-excited model

***mL: Low regime level, mH: High regime level
Results of the grid search

Graphic 9. EUR/TRY, D1 threshold and AIC values

Results of the grid search

Graphic 10. EUR/TRY, W1 threshold and AIC values
The delay \((d)\) of the threshold value of the model and the threshold value are defined in Graphic 9-10-11 (within relevant time range) where the pooled-AIC value is at the minimum level.

**Table 6.** EUR/TRY - SETAR Model Estimation and Coefficient Statistics

| Parity | Low Regime | High Regime | Threshold Value | Residuals Variance |
|--------|------------|-------------|-----------------|--------------------|
|        | Constant   | phiL.1      | phiL.2          | Constant           | phiL.1 | phiL.2 |
| EUR    |            |             |                 |                    |        |        |
| TRY    | 0.01015    | 0.99591     | -               | 0.06689            | 0.97666| -      |
|        | 0.04638    | 0.98144     | -               | 0.27188            | 0.9052 | -      |
| MN     | 0.192976   | 0.92151     | -               | 0.78153            | 0.72827| -      |

**Coefficient Statistics**

| Parity | Estimation | Standard Error | p Value |
|--------|------------|----------------|---------|
| EUR    |            |                |         |
| TRY    | Constant   | 0.00765        | 0.18488 |
|        | phiL.1     | 0.00334        | 2.2e-16 **|
|        | phiL.2     | -              | -       |
|        |            |                |         |
| W1     | Constant   | 0.03352        | 0.16778 |
|        | phiL.1     | 0.0146         | 2.2e-16 ***|
|        | phiL.2     | -              | -       |
| MN     | Constant   | 0.1370         | 0.1649  |
|        | phiL.1     | 0.06021        | 2.2e-16 ***|
|        | phiL.2     | -              | -       |

***: Very close to zero, **: Below 0.001, *: Below 0.01, (.) : Below 0.05, ( ) : Very close to 1
SETAR (Self-exciting Threshold Autoregressive) Non-linear Currency Modelling in EUR/USD, EUR/TRY and USD/TRY Parities

Graphic 12. EUR/TRY, D1 data and inclusions

Graphic 13. EUR/TRY, D1- Regime-sensitive Scatter Diag
When especially Graphic 13 and Graphic 14 are examined, it attracts attention that polarization in the regimes disappear as the delay intervals of the parity values, which are the subject matter of diaphragms, expands.
Graphic 16. EUR/TRY, D1-Threshold value graphics

Graphic 15 expresses the red pattern which is over the threshold value and the black patterns which is over the threshold value. When Graphic 16 is examined the first graphic threshold value indicates the observation values. Again, when the second graphic is examined in Graphic 16, it is observed that the position of the threshold value may be determined provided that 15% of the observation values from both ends are kept hidden.

Table 7. USD/TRY - SETAR Model Hyper parameters

| Parity | m | Threshold Delay | mL | mH | Threshold Value | Pooled AIC | Number of Possible Threshold Values | Number of Threshold Values Tested with Hyper Parameters |
|--------|---|-----------------|----|----|-----------------|------------|-------------------------------------|-----------------------------------------------|
| USDTRY | D1 | 2               | 0  | 1  | 2.1722          | -7107.389  | 719                                 | 1438                                          |
|        | W1 | 2               | 0  | 1  | 1.8269          | -1091.849  | 162                                 | 324                                           |
|        | MN | 2               | 0  | 1  | 1.7685          | -156.5893  | 38                                  | 76                                            |

*m: The autoregressive level of the whole model

**Threshold Delay: The delay level of the Self-excited model

***mL: Low regime level, mH: High regime level
Graphic 17. USD/TRY, D1 threshold and AIC values

Graphic 18. USD/TRY, W1 threshold and AIC values
The delay (d) of the threshold value of the model and the threshold value are defined in Graphic 17-18-19 (within relevant time range) where the pooled-AIC value is at the minimum level.

**Table 8. USD/TRY - SETAR Model Estimation and Coefficient Statistics**

| Parity | SETAR Model | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Parity | Low Regime | High Regime | Threshold | Residuals |
| | | Constant | phiL.1 | phiL.2 | Constant | phiL.1 | phiL.2 | Value | Variance |
| USD TRY | D1 | -0.01599 | 1.01086 | - | -0.0052 | 1.002866 | - | 1.775 | 0.0001724 |
| | W1 | -0.05090 | 1.0351 | - | -0.0291 | 1.015810 | - | 1.785 | 0.000739 |
| | MN | -0.28829 | 1.19714 | - | -0.12986 | 1.07049 | - | 1.768 | 0.002994 |

| Coefficient Statistics | Estimation | Standard Error | p Value | Estimation | Standard Error | p Value |
|---|---|---|---|---|---|---|
| D1 | Constant | -0.01599 | 0.01025 | 0.1191 | -0.00521 | 0.00474 | 0.2713 |
| | phiL.1 | 1.01086 | 0.00639 | 2e-16 *** | 1.0028 | 0.00237 | 2e-16 *** |
| | phiL.2 | - | - | - | - | - | - |
| W1 | Constant | -0.05090 | 0.03967 | 0.2007 | -0.0291 | 0.0229 | 0.2046 |
| | phiL.1 | 1.03519 | 0.0245 | 2e-16 *** | 1.0158 | 0.0114 | 2e-16 *** |
| | phiL.2 | - | - | - | - | - | - |
| MN | Constant | -0.28829 | 0.19580 | 0.1467 | -0.12986 | 0.098387 | 0.1924 |
| | phiL.1 | 1.19711 | 0.12202 | 1.3e-13 *** | 1.0704 | 0.04921 | 2.2e-16 *** |
| | phiL.2 | - | - | - | - | - | - |

***: Very close to zero. **: Below 0,001, *: Below 0,01, (): Below 0,05, ( ) : Very close to 1
Graphic 20. USD/TRY, D1 data and inclusions

Graphic 21. USD/TRY, D1- Regime-sensitive Scatter Diag
When especially Graphic 21 and Graphic 22 are examined, it attracts attention that polarization in the regimes disappear as the delay intervals of the parity values, which are the subject matter of diaphragms, expands.
The SETAR Model, which is one of the TAR Group modeling, shows a better performance than many other linear and non-linear modeling. In this study, the purpose is to express that the SETAR model performance is superior to the other models by considering the observation values of the parities. In this context, the AIC (Akaike Information Criteria) values of the SETAR Model and the other models were compared, and it was revealed that the SETAR model is more influential in terms of the relevant parities.

### Table 9. Some Models and AIC Values

| Models | AIC       |
|--------|-----------|
| Linear | -38136.37 |
| SETAR  | -38136.77 |
| LSTAR  | -38136.01 |
| NNETTs | -38123.69 |
| AAR    | -38107.12 |

**Graphic 24.** USD/TRY, D1- Threshold value graphics

The SETAR Model, which is one of the TAR Group modeling, shows a better performance than many other linear and non-linear modeling. In this study, the purpose is to express that the SETAR model performance is superior to the other models by considering the observation values of the parities. In this context, the AIC (Akaike Information Criteria) values of the SETAR Model and the other models were compared, and it was revealed that the SETAR model is more influential in terms of the relevant parities.
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Here, in the other modeling in question;

**Linear Modeling (Linear):** In this modeling, AR(p)-Type modeling was selected; and the delay length m=2 was selected.

**Logistic Soft-Transition Autoregressive Modeling (LSTAR):** It is the type of modeling which is considered within a continuous indicator function.

**Artificial Neural Networks Modeling (NNETTs):** In a neural network problem with hidden unit and linear output with activation function, the embedded dimension was selected as m=2.

**Additive Autoregressive Models (AAR):** Non-parametric additive model form was handled when m=2.

6. Conclusions

In this study, self-excited threshold autoregressive models have been used in order to be able to model the volatile changes in currency market. The linearity of the currency parities, which were dealt with before the SETAR modelling approach was applied, have been examined with the help of various approaches (Tsay, Keenan, Likelihood Ratio), and then the SETAR modelling was applied to all parities no matter whether they are linear or not in D1 (daily), W1 (weekly) and MN (monthly) time intervals. In applying the modelling, firstly the model hyper parameters were determined for each parity and each time period, and then in the light of these hyper parameters the SETAR model was estimated.

The SETAR model performance is superior to the other models by considering the observation values of the parities. (Table 9 – Graphic 25)

When the regime-sensitive scatter diagram are examined, it attracts attention that polarization in the regimes disappear as the delay intervals of the parity values, which are the subject matter of diaphragms, expands.

(The observation values in red are expressed as the High Regime; and the ones in black are expressed as Low Regime).

In this study, a structure integrated with two different linear models was used in the context of single threshold while the non-linear structures of the time series of the parities were examined. Although not used commonly in the macro-economic literature, there are some situations in which the number of the threshold value is more than one (in such a situation, the number of the threshold value being more will also increase the number of the regimes estimated). The non-linear structure in currencies may be investigated with modelling in which there are a number of threshold values.

Even, the chaos estimation of the relevant datasets may also be performed because the non-linear structure in the parities might be handled with chaotic approach (which the other linearity tests lose their values in such a situation, and the linearity tests, which include the basic hypothesis that investigate chaos, are made use of instead).

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