Representation of the synoptic spectra of atmospheric turbulence by sums of spectra of coherent structures

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Abstract. It is shown that the known experimental synoptic spectra of the atmospheric turbulence (spectrum of Van der Hoven, 1957; spectrum of Kolesnikova, Monin, 1965) represent the sum of the solitary spectra of coherent structures with various sizes (with variety outer scales). The spectrums registered by us near to a mirror of the Baikal Solar Vacuum Telescope (BSVT) of the Baikal astrophysical observatory and in a under dome room of the Big Alt-Azimuth Telescope (BTA) of the Special astrophysical observatory are presented too. Apparently, the good coincidence of theoretical spectrums with experimental data (from both parties from a micrometeorological maximum) is observed. This spectrum well approximates the experimental spectrums not only in the field of a micrometeorological interval of frequencies, but also on frequencies of lower, than a micrometeorological maximum where the observational spectrums have decrease.

1. Introduction

In [1, 2] it is shown that at measuring of turbulent parameters it is enough, that the interval of the registered frequencies fluctuations overlapped the most part of a micrometeorological interval of a spectrum of atmospheric turbulence as in a micrometeorological interval the basic energy of turbulent fluctuations of a surface layer is concentrated. At measuring of the meteorological parameters related to low-frequency daily and seasonal changes of a meteorological situation, often it is already not enough width of one micrometeorological interval. In this case averaging time at the parameter recording is usually incremented from 2-3 mines to 10 mines (and more), at the expense of expansion of an interval of measured frequencies.

In the range of continuances from minutes till several o'clock the field of a mesometeorological minimum is observed in a spectrum of atmospheric turbulence ([1], Fig. 6, 7 see also below). Spectra again start to increasing in low-frequency field (at decrease of frequency), transiting through the intermediate 12 hour maximum, and reach a synoptic maximum (a continuance of fluctuations about 4-6 days), and then again decrease.

Synoptic spectrums viewed in given article (synoptic - summary, from Greek: syn - together, optimai - see) cover a wide frequency band, including the micrometeorological, mesometeorological and synoptic intervals.

As it is known [1, 8], at heights above an boundary layer of friction of the atmosphere at big external scales for empirical spectra \( W(f) \) of fluctuations of temperature and wind speed the frequency dependence \( W(f) \sim f^{-3} \) is often shown. Similar decrease of spectra in an inertial interval of small-scale turbulence \( W(f) \sim f^{-8/3} \) (as opposed to the Kolmogorov model \( W(f) \sim f^{-5/3} \)) is registered in our experimental observations in the atmosphere of an boundary layer of high-mountain observatories.
In [3-7] it is shown that such behaviour of spectra is the main sign of presence at the turbulent atmosphere of coherent structures. Hydrodynamic coherent structure are called [4] compact formation including long-living spatial vortex structure (cell) resulting from long action of thermodynamic gradients and products of its discrete coherent cascade disintegration. In expanded understanding the coherent structure is the solitonic solution of hydrodynamical equations (a topological three-dimensional soliton, a lonely wave). This is or one-solitonic decision, or one soliton in the multisolitonic decision. The coherent structure contains both large-scale, and small-scale turbulence.

The products of disintegration of a main vortex within a coherent structure are in-phase (coherent) with the primary family vortex. Usually there are different coherent structures with different (aliquant, incommensurable) frequencies of the primary vortex in the atmosphere. At mixing of various coherent structures the disintegration vortices will be asynchronous (incoherent) with such elements of another family. Therefore, turbulence originating during mixing of coherent structures with primary vortices of different sizes, in reality will be incoherent turbulence [7].

Coherent structures were registered in all measurements of the turbulence parameters executed by authors during more than a decade. Measurements are performed at different times, in various geographical areas and climatic conditions: in mountain regions of observatories of the South Siberia (Baikal astrophysical and Sayan solar observatories), in mountains of Kolyvansky ridge and the North Caucasus. In a surface layer over territories of observatories the existence of areas of coherent turbulence [4-7] in which one coherent structure has the prevailing effect is established. The lifetime of areas of coherent turbulence in the atmosphere makes from 2 - 4 min to 20 - 120 min. Such coherent structures are found and described by us [4-7] also in air the closed specialized rooms of telescopes in which turbulence, comparable on intensity with turbulence in the free atmosphere is registered. In the presence in the atmosphere of coherent turbulence the effect of decrease of phase and amplitude fluctuations of light radiation is observed. It is shown in essential decrease of fluctuations of jitter of astronomical images and leads to increase of their quality [4].

2. Theoretical micrometeorological spectra

On the basis of theoretical Karman model [1, 2] of a three-dimensional spectrum of the temperature fluctuations $\Phi(\omega)$ in [3, 5] the theoretical spatial one-dimensional spectrum of a single coherent structure $V(\omega)$ has been found:

$$V(\omega) = C_f \omega_0 (\omega_0^2 + \omega^2)^{-4/3},$$

$$C_f = 0.514 C_T^2, \quad C_T^2 = 2.96 \sigma_f^2 L_0^{-2/3},$$

where $\omega_0 = 2\pi/L_0$, $L_0$ - Karman outer scale in coherent turbulence, $C_T^2$ is the structural characteristic of a temperature fluctuations, $\sigma_f$ - variance of a temperature fluctuations ($\sigma_f^2 = 1.15 C_T^2 \omega_0^{-2/3}$ [5]). By means of a known relation [1, 2]

$$W(f) = (4\pi/ \nu) V(2\pi f/ \nu),$$

where $\nu$ - the module of a vector of a wind velocity, in [3, 5] a theoretical frequency spectrum $W(f)$ of a single coherent structure (on the plus frequencies) are constructed. These spectrums $W(f)$ are agreed [5] with the experimental spectrums of single coherent structures registered by us (Fig. 1, 2).

Use of such spectrum has allowed us to show [3-7] that the experimental spectrums of a really observable atmospheric turbulence (with frequencies from a micrometeorological interval, including Kolmogorov turbulence), represent the sums of spectrums of the separate coherent structures of the different sizes (with various outer scales).

As well as any Karman spectrum, a spectrum (1) remains to constant values on frequencies, close to zero, $V(0) = \text{const}$. It does not influence the description of an inertial interval of Kolmogorov turbulence by the sum of spectrums of the different coherent structures [5]. Besides, the total energy of fluctuations from a micrometeorological interval (integral on all frequencies from spectrum $W(f)$) weakly depends on behavior of a spectrum in a low-frequency band of a micrometeorological interval
This fact also allows using a Karman spectrum for the theoretical description of atmospheric turbulence.

3. The modified spectrum of single coherent structure

At the same time real observational spectrums of fluctuations at sufficiently large time of averaging for a micrometeorological interval decrease in zero (Fig. 3). Therefore the theoretical spectrum which is more carefully featuring experiment should have decrease in zero. Such spatial one-dimensional spectrum of a single coherent structure, in details corresponding to an experiment, is constructed in the present article. It is generalization of a spectrum (1). The generalized spatial one-dimensional spectrum \( V(\omega) \) for single coherent structure it is possible to present in the view

\[
V(\omega) = C_3 \cdot \omega_0^{-5/3} \left(1 + \omega^2 \omega_0^{-2}\right)^{-4/3} \left[1 - \exp(-\omega^2 \omega_0^{-2})\right],
\]

The function \( V(\omega) \) (3) has a maximum at a frequency \( \omega_M \approx (7/8)^{1/2} \omega_0 = 0.938 \omega_0 \). As appears from comparison of expressions (1) and (3), function \( V(\omega) \) in (3) differs from function (1) by additional factor in square brackets in (3). This factor gives a common decrease of a spectrum at \( \omega/\omega_0 \ll 1 \) and is necessary for approximation of an experimental data (spectrum decrease on low frequencies, Fig. 3).

Using (3) in the relation (2), for a frequency spectrum of a single coherent structure \( W_T(f) \) (designation \( E_T \) is often applied to instead of \( W_T(f) \)) we deduce

\[
W_T(f) = 0.266 \sigma_T^2 \omega_0^{-1} f_0^{-1} \left(1 + f^2/f_0^2\right)^{-4/3} \left[1 - \exp(-f^2/f_0^2)\right],
\]

\[
f_0 = L_0^{-1} \nu, \quad f_M = \omega_M \nu / (2\pi).
\]
Apparently, key parameter of a spectrum (4) is the characteristic frequency \( f_0 \), corresponding to time of transport of turbulent inhomogeneities of atmosphere with medial velocity of a wind \( \nu \) through outer scale of turbulence \( L_0 \) (\( \tau = f_0^{-1} = L_0 / \nu \)). The spectrum (4) well describes real observational spectrums of single coherent structures.

On Fig. 4, 5 the result of approximation of the experimental frequency spectrums of the single coherent structures at use the theoretical frequency spectrum (4) offered by us is shown. The spectrums registered by us near to a mirror of the Large Solar Vacuum Telescope (LSVT) of the Baikal astrophysical observatory on July, 19th, 2012 (Fig. 4) and in a under dome room of the Large Altazimuth Telescope (LTA) of the Special astrophysical observatory on October, 29th, 2012 (Fig. 5) are given. Apparently, the good coincidence of theoretical spectrums with experimental data (from both parties from a micrometeorological maximum) is observed.

The sums of several spectrums of single coherent structures (a view (4), with different \( f_0 \)) allow to approximate real observational spectrums as well in fields of low frequencies, including a synoptic interval. Reviews of the available experimental spectrums observed in fields of low frequencies, are given, for example, in [1, 8].

On Fig. 6 the empirical energy spectrum \( fE_T(f) \) of a temperature fluctuations in a wide interval of frequencies according to the data from Kolesnikova and Monin (1965, [1]) is given. On a spectrum a synoptic maximum with a period in four days and also the sharp maximum corresponding to a period at 12 o’clock are well expressed. On Fig. 8 a approximation of this spectrum \( E_T(f) \) (dashed line) by the sums of spectrums of coherent structures with different outer scales is shown. Values of frequencies \( f_0 = L_0^{-1} \nu \) (i = 1-8) of single coherent structures in spectrums \( E_T(f) \) (Fig. 8) are resulted in tab. 1:
Table 1. Parameters of single spectra of coherent structures in a synoptic spectrum $E_T(f)$.

| $f_{0}$, Hz | 2.7·10^{-6} | 1.98·10^{-5} | 1.79·10^{-4} | 5.56·10^{-4} | 2.22·10^{-3} | 5·10^{-3} | 1.33·10^{-2} | 6.67·10^{-2} |
|------------|--------------|--------------|--------------|--------------|--------------|-------------|---------------|---------------|
| $\tau_i$ | 4.2 days     | 14 h         | 1.6 h        | 30 min       | 7.5 min      | 3.3 min     | 1.3 min      | 15 c          |
| $\alpha_M \approx (7/8)^{1/2} \alpha_0$ | 1.59·10^{-5} | 1.07·10^{-4} | 1.05·10^{-3} | 3.27·10^{-3} | 1.31·10^{-2} | 2.94·10^{-2} | 7.84·10^{-2} | 3.92·10^{-1} |
| (by $\nu = 1$ m/c) , m^{-1} | 8.85 | 6 | 0.9 | 1.5 | 1.25 | 1.27 | 1.25 | 3.5 |

On Fig. 7 the known experimental spectrum of a spectral density of a fluctuations energy of a horizontal component of a wind velocity $f E_u(f)$ according to Van der Hoven data (1957 [1]) is given. On Fig. 7, 9 the result of approximation of a Van der Hoven spectrum ($f E_u$ and $E_u$ accordingly) by the sums of spectrums (dashed line) is shown. Apparently, good coincidence of experimental data to theoretical curves takes place. The data of Fig. 7–9 show that known experimental synoptic spectrums (Kolesnikova and Monin, 1965; Van der Hoven, 1957) are the sums of spectrums of single coherent structures with the various sizes.

4. Numerical modeling

Theoretical researches of structure of turbulence by numerical modeling (a boundary-value problem for hydrodynamical equations of Navier-Stokes) also indicate presence at air of coherent structures. In our articles [9, 10] the structure of turbulent air movements in the closed volumes is in numerically investigated over non-uniform a heated surface. It is shown that over non-uniform a heated surface there are lonely toroidal whirlwinds (coherent structures or topological solitons). The quantity of whirlwinds and their internal structure depend on a form and the extent of heated surface. In case of simple forms of heating (uniform heating, one heated round spot) in volume the coherent turbulence resulting from coherent disintegration of whirlwinds is observed. For irregular shapes of heating (thermal diversity) toroidal whirlwinds are considerably deformed. Whirlwinds can be extended along a surface and have spiral streamlines. In the course of evolution coherent structures (whirlwinds) considerably mix up. Numerical calculations confirm (Fig. 10) that as a result of mixing of coherent structures with different close sizes (and with close frequencies of the main whirlwinds) Kolmogorov

Figure 10. Development of the pattern of medium motion over four spots. Solid lines show the streamlines. The simulation time is shown in the h:min:s format.
(incoherent) turbulence is formed. Also experimentally registered spectra of single coherent structures received earlier in the dome spaces of astronomical telescope are confirmed.

Figures 10 show the development of four coherent structures (toroidal convective cells) bounded by a cubic volume inside the closed volume. Each heated spot originates a proper coherent structure with the characteristic size and primary vortex frequency. The convective toroidal vortices mix insignificantly during the initial phases of cell formation. During further stages, the mixing becomes significant. Strong mixing is observed in upper layers of the cubic volume, and the interaction of the coherent structure with neighbor structures results in a significant distortion of its initial toroidal form (see streamlines from an isolated spot in the bottom row in Fig. 10). The turbulence spectrums of the mixture of four coherent structures (over four heated spots) have a longer Kolmogorov segment (with the $5/3$-power dependence) as compared to the Kolmogorov segment above two heated spots. Spectrum above separate spot has $8/3$-power dependence in inertial interval. Thus, the numerical solutions of the boundary problems also confirm the conclusion drawn earlier experimentally [3, 4, 7] and theoretically [4-7, 9, 10] that mixing of coherent structures with close sizes (and close primary vortex frequencies) results in a Kolmogorov turbulence.

5. Conclusions
Thus, in article, on the basis of gained by us earlier a frequency spectrum of single coherent structure in a micrometeorological interval of atmospheric turbulence, calculate expression for a spectrum of single coherent structure. This spectrum well approximates the experimental spectrums not only in the field of a micrometeorological interval of frequencies, but also on frequencies of lower, than a micrometeorological maximum where the observational spectrums have decrease. The sums of spectrums of coherent structures with various outer scales successfully approximate the experimental spectrums of atmospheric turbulence in a wide frequency band (including micrometeorological, mesometeorological and synoptic intervals). This proves that a real atmospheric turbulence is a noncoherent mixture of different coherent structures with incomparable frequencies of primary energy-carrying vortices. The results gained in the present work confirm and expand on the field of very low frequencies earlier made in our articles [3-7] a conclusion that, despite their complex internal structure, coherent structures are elemental components (fundamental particles) of atmospheric turbulence.

It is shown that known experimental synoptic spectrums of really observable atmospheric turbulence (including spectrums of small-scale turbulence, the spectrum by Van der Hoven, 1957; the spectrum by Monin and Kolesnikova, 1965) represent the sums of spectrums of single coherent structures with the different sizes (with various outer scales).

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