The effects of a fast-turning trajectory in multiple-field inflation

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Abstract. The latest results from \textit{Planck} impose strong constraints on features in the spectrum of the curvature perturbations from inflation. We analyse the possibility of particle production induced by sharp turns of the trajectory in field space in inflation models with multiple fields. Although the evolution of the background fields can be altered by particle production, we find rather modest changes in the power spectrum even for the most extreme case in which the entire kinetic energy of the scalar fields is converted into particles.

Keywords: inflation, cosmological perturbation theory

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1 Introduction

After an era characterised by scarcity and inaccuracy of data, cosmology is now thriving with observations and measurements, which make our understanding of the universe remarkably precise. A key feature of our present picture of the early universe is the presence of small inhomogeneities of energy density. Whilst their exact origin cannot be directly inferred from the data, it certainly lies within the quantum realm. An almost universally accepted opinion is that they appeared as quantum fluctuations of one or more scalar fields and were subsequently amplified during a period of inflation to become seeds for the anisotropy pattern in the microwave sky and the large structures in the universe [1–3].

The latest and most accurate data, obtained by the Planck collaboration [4–6], strongly support the paradigm of inflation, which generically predicts a non-scale-invariance of the spectrum of the primordial perturbations. Present observations rule out a scale-invariant spectrum at a significance level exceeding 5 standard deviations. In spite of this progress, the data suggest a large degeneracy between predictions of different inflationary models. This should be, to some extent, expected as the power spectrum represents the lowest order statistics, and necessarily loses a vast amount of information (in particular, about the interactions of these fields). Some exceptions exist, namely in models of inflation in which the effects of a heavy scalar field can be accommodated by a non-trivial speed of sound of perturbations for a massless one (see, for example, refs. [7–10]).

Whereas this might indicate that single-field models can be acceptable toy models for inflationary phenomenology, on a theoretical level these models lack a well-defined setup.
This is because in a single-field model there is a lack of control of the quantum-mechanically induced radiative corrections, which can change the shape of the potential, thereby changing predictions for observables. This issue can be ameliorated if we take an effectively single-field action as arising after integrating out heavy degrees of freedom, which couple to the light field. Moreover, in high-energy models of particle physics one expects a number of light scalar degrees of freedom, with masses not much larger (and often smaller) than the Hubble scale. The fluctuations of these fields can, in principle, act as sources of primordial perturbations. This feature has led to a significant investment in searching for characteristic signatures in models with multiple light scalar fields. One of the key aspects of such models is that, unlike in single-field inflation scenarios, the primordial curvature perturbation need not be frozen on super-horizon scales [11, 12], which makes the phenomenology of such models potentially richer and more interesting.

There are a number of possible effects, which result from the presence of multiple active scalar degrees of freedom during inflation. One well-studied example is the coupling between the fields, either non-minimal or through the metric itself, which might cause the inflationary trajectory to deviate from a geodesic line in field configuration space. Such bending can then induce a change in the power spectrum of perturbations, which ought to be properly taken into account for a rigorous comparison to observations [9, 13, 14]. In addition, some of the usual slow-roll parameters can become large, which requires keeping track of higher-order corrections [14, 15].

On the other hand, the coupling between different fields can also give rise to observable non-gaussianity with quite a distinctive statistical signature, making it possible to test these models, especially with the current stringent constraints [4]. Nevertheless, it is fair to argue it will be possible to choose a region in the parameter space for which the predicted non-gaussianity is well within the observational constraints. This possibility makes the degeneracy between different inflationary models much broader and extends it beyond the two-point statistics [16].

Even in the absence of directly observable effects, understanding the possibly rich phenomenology in multiple-field inflation can also provide an important toolkit for grasping the detailed physical picture of the inflationary mechanism. This is somewhat analogous to the relatively recent motivation behind analysing theories of modified gravity in the IR (see, e.g., Clifton et al. [17] for a review). Whilst the theory of general relativity agrees very well with a vast array of observations, studying various modifications of this theory allows gaining more insight into theories of gravity as effective field theories.\footnote{\emph{The currently observed accelerated expansion of the universe can be an indication of GR breaking down in the IR, which adds to the interest in this research area. In the inflationary cosmology case, on the other hand, there is still no empirical hint for a breakdown of the single-clock picture.}}

For these reasons it is important to investigate the potentially interesting physical effects arising in these models. In this regard, there is a noticeable gap in the existing literature on the subject. Whenever the evolution of the background fields is non-adiabatic, particle production can occur (see, e.g., refs. \cite{18-23} for interesting realizations of this concept); this effect may drain the energy from the scalar field sector and transform it to a component of the total energy density with a different evolution. The non-adiabaticity can manifest itself...
in fast changes of the masses of the field(s) or, in multiple-field models, in a fast change of
the direction of the mass eigenstates in field space. Strikingly, despite the fact that relevant
theoretical tools are readily available [24], only the first of the two possibilities has been
examined in the context of inflation, in setups known as moduli trapping scenarios. Filling
this gap will constitute a step towards a better understanding of the phenomenology in
multiple-field models of inflation.

In this paper, we shall therefore analyse the consequences of particle production in the
simplest multiple-field models of inflation in which the inflationary trajectory exhibits a fast
turn. In particular, transferring energy into particles during inflation can affect the homo-
geneous background, thereby changing the inflationary trajectory as well as the evolution
of the parameters, such as the Hubble parameter. Consequently, the predictions of many
important cosmological observables (power spectrum of primordial perturbations, normal-
ization and shape of potential non-gaussianities) may be modified. Obtaining such accurate
predictions with particle production fully taken into account will allow for a better insight
into the phenomenology of multiple-field models of inflation. The primary purpose of this
paper is to provide an advance in this direction.

Outline. This paper is organised as follows. In section 2 we analyse the behaviour of
perturbations in the vicinity of a trajectory turn in field configuration space, whilst ignoring
the possibility of particle production. We drop this simplifying assumption in section 3,
and use the mode mixing technique to compute the Bogolyubov coefficients as a means to
determine the efficiency of particle production. We provide both analytical and numerical
estimates by investigating a particular two-field inflation model in which a sharp turn of the
inflationary flow occurs. We also study the backreaction of particle production on the classical
inflationary trajectory. In section 4 we elaborate on the interpretation of the occupation
number in particles on the basis of the Bogolyubov coefficients, and review the reasons why
the concept of particle production in a time-dependent setting is unclear from the field theory
viewpoint. We summarize and discuss the implications of our results in section 5.

Notation. We use units in which the Planck mass, \( M_P^2 = 1/8\pi G \), is set to unity, and
\( c = \hbar = 1 \). We work with the mostly plus metric signature and take the cosmological
background to be given by the Friedmann-Lemaître-Robertson-Walker line element. We use
primes to denote derivatives with respect to conformal time, \( \tau = \int a(t) \, dt \), where \( a \) is the
scale factor, while overdots denote differentiation with respect to cosmic time, \( t \). Lorentz
indices are contracted using the metric \( g_{\mu\nu} \) under Einstein’s convention and in which Greek
indices denote spacetime coordinates, whilst Roman letters specify the spatial components
only. Capital Roman letters are reserved to specify the particle species. We adopt the usual
terminology that the dimensionless power spectrum, \( P \), can be obtained from the traditional
power spectrum, \( \mathcal{P} \), by applying the rule \( \mathcal{P} = k^3 \mathcal{P}/2\pi^2 \).

2 Sharp turns without accounting for particle production

Consider a model with multiple, canonically normalised scalar fields described by the action
\[
S = \frac{1}{2} \int d^4 x \sqrt{-g} \left\{ R - (\partial_\alpha \phi)^2 - (\partial_\alpha \chi)^2 - 2V(\phi, \chi) \right\},
\]  
(2.1)
The classical inflationary trajectory in this case can be decomposed into infinitesimal displacements of each of these fields in configuration space, \( \{ \delta \phi, \delta \chi \} \). This trajectory will not be, in general, a straight line since the fields will attempt to minimise the potential. This can be intuitively understood by describing the inflationary flow by a bundle of geometrical rays which respond to Huygens equation \([25]\) — a light ray bends owing to a varying refractive index, which can be shown to be related to the shape of the potential (in particular, to the slow-roll parameter \( \varepsilon \equiv -\dot{H}/H^2 \)).

How are the predictions for the power spectrum of the curvature perturbations modified after a sharp turn of the inflationary trajectory? In what follows we shall refer to a sharp turn if the rate of turning of the trajectory is much larger than the Hubble parameter, so that the turn completes in a fraction of an e-fold. Said differently, the rate of turn is the largest scale entering the equations of motion of the inflationary perturbations (or it is at least of the same order as the largest mass parameter).

The effects of such a sharp turn of the inflationary trajectory have been addressed by a number of authors (see, for example, refs. \([9, 13, 26]\)) . Interestingly, a couple of analyses found a correction to the power spectrum which does not vanish in the limit of infinitely large comoving wavenumber \( k \). In ref. \([27]\), Shiu & Xu have argued that the correction to the dimensionless power spectrum can be approximated as

\[
P_\zeta/P_0 \simeq 1 + 2\Delta \theta \sin(2k/k_{\text{turn}}),
\]

where \( P_0 = H_*^4/8\pi^2\varepsilon_*^2 \) is the (dimensionless) power spectrum in an effectively single-field inflation model. Here, \( H_* \) and \( \varepsilon_* \) are the Hubble and the slow-roll parameters evaluated at Hubble crossing for the reference mode, which is taken to be the mode that leaves the Hubble radius exactly at the turn; \( \Delta \theta \) is the deflection angle of the inflationary trajectory.

In contrast, Gao, Langlois & Mizuno \([28]\) found that this correction can be schematically written as

\[
P_\zeta/P_0 \simeq 1 + F_l + F_h + F_{lh},
\]

and, for a turn of negligible duration and a mass parameter of the heavy field much larger than \( H \), they found \( F_l = 0 \) and \( F_h \sim \Delta \theta^2 \), while the contribution \( F_{lh} \) was left unevaluated.\(^2\) Given the disparity of these results and the fact that they predict or suggest a non-vanishing effect for all modes with \( k > k_{\text{turn}} \), it is interesting to take a closer look at the evolution of the perturbations at the turn. As we will argue at the end of this section, the principal object of interest is the full expression for the power spectrum, and therefore each of the individual contributions to its formula should be carefully tracked, since it is possible that cancellations might occur. In particular such care will be essential for the scales we want to focus on in this paper when the turn occurs.

Our approach to this problem will be two-fold: we first discuss analytically the solutions to the equations of motion for the perturbations in the sub-horizon regime, and then corroborate our findings in comparison to numerical estimates for a simple model. Our estimates will not rely on the effective field theory treatment because the sharpness of the turn can be sufficient to induce oscillations of the heavy field, which can make the effective field theory

\(^2\)The definition of the correction factors \( F_l, F_h \) and \( F_{lh} \) is given in the appendix.
description break down. Instead, we proceed with the analysis of the perturbations for both fields directly, keeping both degrees of freedom (light and heavy fields) manifest throughout the calculation.

2.1 Evolution of the perturbations at the turn: analytical discussion

In multiple field modes the dynamical evolution is sometimes best described in terms of curvature (adiabatic) and isocurvature (entropy) perturbations [29]. These can be obtained by performing an instantaneous rotation with respect to the field perturbations themselves, and can be interpreted as the perturbations defined along and perpendicularly to the inflationary trajectory. We can then introduce the gauge-invariant Mukhanov-Sasaki variables [30, 31], which we shall denote by $Q_\sigma$ and $\delta s$, respectively. On super-Hubble scales, these represent the instantaneous curvature and isocurvature perturbations, $\zeta = Q_\sigma/\sqrt{2\varepsilon M_P}$ and $S = \delta s/\sqrt{2\varepsilon M_P}$.

The general equations of motion for these variables are usually rather lengthy, and their derivation can be found, for example, in ref. [32]. They can, however, be significantly simplified under the assumption that the slope and the curvature of the inflationary potential change negligibly around the portion of the inflationary trajectory where the turn takes place. We also assume that at a given instant the trajectory momentarily changes direction, and follows a straight line in field space after the turn. Moreover, we only consider models for which $\varepsilon$ is very small and nearly constant.

Subject to these simplifying approximations, the equations of motion for the rescaled comoving perturbations, $v_\sigma = aQ_\sigma$ and $v_s = a\delta s$, assume a much simpler form. Up to slow-roll corrections, they can be written as

$$v_\sigma'' + \left(k^2 + \frac{\mu_\sigma^2 - \rho^2 - 2}{\tau^2}\right)v_\sigma + \left(\frac{2\rho}{\tau}v_s\right)' - \frac{2\rho}{\tau^2}v_s = 0 \quad (2.4)$$

$$v_s'' + \left(k^2 + \frac{\mu_s^2 - \rho^2 - 2}{\tau^2}\right)v_s - \frac{2\rho}{\tau}v_\sigma' - \frac{2\rho}{\tau^2}v_\sigma = 0 \quad (2.5)$$

Above we have parametrised the rate of turn by introducing

$$\rho \equiv \frac{\dot{\theta}}{H} = \frac{d\theta}{dN},$$

where $N = \int H dt$ is the number of e-folds. Away from the turn $\rho$ is approximately zero. In eqs. (2.4) and (2.5) $\mu_{\sigma,s}^2$ are the mass parameters of the perturbations, given in Hubble units, which can be read from the full equations of motion. We shall assume that the mass of the variable associated with the curvature perturbation $v_\sigma$, denoted by $\mu_\sigma$, is negligible in comparison to the Hubble scale, $\mu_\sigma^2 \ll 1$. We notice the presence of a gravitational mass (squared) term, $-2/\tau^2$, which results from the dynamical spacetime background. Such term would be absent in Minkowski spacetime.

In the following, we parametrize the localised sharpness of the turn by assigning $\rho$ a Dirac delta distribution, as done by Shiu & Xu [27]. This is an appropriate description of the trajectory’s feature: it starts with a straight line in field space and rapidly changes its
direction to continue as a straight line after the turn. More specifically, we assume that the turn rate, $\theta'$, can be described by

$$\theta'(\tau) = \Delta \theta \delta(\tau - \tau_{\text{turn}})$$  \hspace{1cm} (2.7)

where $\tau_{\text{turn}}$ corresponds to the value of the conformal time at the turn. In our analysis we will be mainly interested in the dynamics of modes which are still sub-horizon when the turn happens. Therefore, we can restrict our attention to the case $|k\tau_{\text{turn}}| \gg \mu_s$, for which the mass parameters $\mu_\sigma$ and $\mu_s$, as well as the gravitational mass term, are comparatively negligible.

It appears that the ansatz (2.7) is problematic as in eqs. (2.4) and (2.5) we can find a square and a derivative of $\theta'$. This is, however, just an artifact of working with a time-dependent basis for the perturbations. Indeed, $v_\sigma$ and $v_s$ represent perturbations along and perpendicular to the inflationary trajectory, respectively, a notion that becomes somewhat ill-defined at the instantaneous turn.

Therefore, it will be helpful to rewrite eqs. (2.4) and (2.5) by performing a field rotation such that the new perturbations correspond to fixed directions in field space. A general rotation by a constant angle $\theta_0$ reads $v = R u$, where $v = (v_1, v_2)^T$, $u = (u_1, u_2)^T$ and $R$ is a time-dependent rotation matrix

$$R = \begin{pmatrix} \cos(\theta - \theta_0) & -\sin(\theta - \theta_0) \\ \sin(\theta - \theta_0) & \cos(\theta - \theta_0) \end{pmatrix}.$$  \hspace{1cm} (2.8)

Since this matrix satisfies \cite{32, 34}

$$R' = \frac{\rho}{\tau} E R,$$  \hspace{1cm} (2.9)

where $E$ is an antisymmetric $2 \times 2$ matrix with $E_{21} = 1$, then eqs. (2.4) and (2.5) expressed in terms of $u$ become

$$u'' + \left( k^2 - \frac{1}{\tau^2} R^T Q R \right) u = 0,$$  \hspace{1cm} (2.10)

in which

$$Q = \begin{pmatrix} 0 & 3\rho - \rho' \tau \\ 3\rho - \rho' \tau & 0 \end{pmatrix}.$$  \hspace{1cm} (2.11)

We note that the passage to the new variables $u$ made potentially troublesome terms proportional to $\rho^2$ in eqs. (2.4) and (2.5) vanish, and that the effective ‘mass matrix’ of the perturbations, $R^T Q R$, is now symmetric. Using $\frac{3\rho}{\tau^2} - \frac{\rho'}{\tau} = \tau^2 \frac{d}{d\tau} \frac{\theta'}{\tau^2}$, we can cast eq. (2.10) in the form

$$u'' + \left[ k^2 - \tau^2 \frac{d}{d\tau} \left( \frac{\theta'}{\tau^2} \right) \right] u = 0,$$  \hspace{1cm} (2.12)

with the orthogonal matrix $R_2$ being given by

$$R_2 = R^T \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} R = \begin{pmatrix} \sin 2(\theta - \theta_0) & \cos 2(\theta - \theta_0) \\ \cos 2(\theta - \theta_0) & \sin 2(\theta - \theta_0) \end{pmatrix}.$$  \hspace{1cm} (2.13)

This problem does not appear if the delta-like singularity can be somehow resolved. Ref. \cite{33} gives an excellent example of such a solution; however, it follows from eqs. (2.4) and (2.5) that working in the curvature-isocurvature (‘kinetic’) basis does not remove the singularity.
In the equation above there remains the subtlety of having to deal with the derivatives of the Dirac delta distribution. This issue is solved when we rewrite the components of eq. (2.12) as

\begin{align}
  u''_1 - \theta'' [\sin 2(\theta - \theta_0) u_1 + \cos 2(\theta - \theta_0) u_2] + \ldots &= 0 \\
  u''_2 - \theta'' [\cos 2(\theta - \theta_0) u_1 - \sin 2(\theta - \theta_0) u_2] + \ldots &= 0,
\end{align}

where the ellipses represent terms which do not affect regularity of the solutions. Integrating eqs. (2.14) and (2.15) over an infinitesimally small interval \((\tau_-, \tau_+)\) around \(\tau_{\text{turn}}\), and applying the identity \(\int_{\tau_-}^{\tau_+} \delta'(\tau) f(\tau) \equiv -\int_{\tau_-}^{\tau_+} \delta(\tau) f'(\tau)\), we find that there are solutions corresponding to a vanishing jump of \(u'_1\) and \(u'_2\) across \(\tau_{\text{turn}}\). If the rate of turn is the largest scale in the dynamics, these are

\begin{align}
  u^{(1)} = \begin{pmatrix} \cos 2(\theta - \theta_1) \\ -\sin 2(\theta - \theta_1) \end{pmatrix} \quad \text{and} \quad u^{(2)} = \begin{pmatrix} \sin 2(\theta - \theta_1) \\ \cos 2(\theta - \theta_1) \end{pmatrix},
\end{align}

where \(\theta_1\) is an arbitrary constant. A choice of \(\theta_1 = \theta_0\) is particularly convenient, as it corresponds to \(u^{(j)} = \delta^j_I\) as the initial condition. This gives

\begin{align}
  v^{(1)} = \begin{pmatrix} \cos \theta \\ -\sin \theta \end{pmatrix} \quad \text{and} \quad v^{(2)} = \begin{pmatrix} \sin \theta \\ \cos \theta \end{pmatrix}.
\end{align}

Hence, the main effect of a sudden turn of the inflationary trajectory consists in a rotation of the components of the perturbations for a range of sub-horizon modes. Since such an orthogonal transformation leaves the correlation functions invariant, we conclude that after the turn the final power spectrum of the curvature perturbations \(P_\zeta\) is equal to \(P_0\), the value that the power spectrum would have attained in the absence of a turn.

We have so far dealt with the case in which \(\theta'\) is the dominant scale in the problem. It is much simpler to solve eqs. (2.4) and (2.5) for very short wavelengths, still assuming a fast turn, \(k/\theta_{\text{turn}} \gg \theta' \gg \{\mu_s, \mu_\sigma\}\). In this case, we can neglect all terms which do not contain a derivative of a wavefunction in eqs. (2.4) and (2.5), except for the gradient terms

\begin{align}
  v''_\sigma + k^2 v_\sigma - 2\theta' v'_\sigma &= 0 \\
  v''_s + k^2 v_s + 2\theta' v'_s &= 0.
\end{align}

It is easy to check that the approximate solutions to eqs. (2.18) and (2.19) read

\begin{align}
  v^{(1)} = \begin{pmatrix} \cos \theta \\ -\sin \theta \end{pmatrix} e^{-ik\tau} \quad \text{and} \quad v^{(2)} = \begin{pmatrix} \sin \theta \\ \cos \theta \end{pmatrix} e^{-ik\tau}.
\end{align}

Again we can see that the main effect of a fast turn consists in an orthogonal transformation of the modes. Hence the power spectrum does not change with respect to the single-field slow-roll case. This conclusion is not surprising — during the turn the large-\(k\) modes are deep inside the Hubble and mass radii, and they behave as massless fields in Minkowski space.
Note that this is different from what happens with respect to modes which are super-horizon when the turn takes place.\(^\text{4}\)

## 2.2 Evolution of the perturbations at the turn: numerical study

Gao, Langlois & Mizuno \cite{Gao2014} put forth a simple two-field potential which allows studying the effects of a sudden turn of the inflationary trajectory. The potential between the two canonically normalised fields, \(\phi\) and \(\chi\), reads

\[
V(\phi, \chi) = \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{2} M^2 \cos^2 \left( \frac{\Delta \theta}{2} \right) \left[ \chi - (\phi - \phi_0) \tan \Xi(\phi) \right]^2, \tag{2.21}
\]

with \(\Xi(\phi) = \Delta \theta \arctan [s(\phi - \phi_0)]\). Here, \(\Delta \theta\) is a constant parameter corresponding to the net angle variation during the turn, while adjusting the constant parameter \(s\) allows to control the sharpness of the turn. In the following, we fix the parameters of the potential as

\[
m_\phi = 10^{-7} M_P, \quad M = 2 \times 10^{-4} M_P, \quad \phi_0 = -100\sqrt{6} M_P, \quad \delta \theta = \frac{\pi}{10}, \quad s = 5000\sqrt{3} M_P^{-1}, \tag{2.22}
\]

in which we reintroduced the Planck mass for clarity. The shape of the potential corresponding to this choice is shown in figure 1. Indeed, it corresponds to a slowly descending valley with steep slopes exhibiting a turn. With the parameters choice (2.22), the mass of the heavy perturbation (or equivalently, the curvature of the potential across the bottom of the valley) is twenty times larger than the Hubble parameter. It is in this sense that we can refer to the turn as \textit{sharp}.

In order to discuss the evolution of the perturbations numerically, it is convenient to adopt yet another time-dependent basis for the perturbations. Following the same prescription as Gao, Langlois & Mizuno, we will track the evolution of the perturbations, schematically denoted by \(\varphi_I\), in the directions determined by the eigenvectors of the mass matrix \cite{30, 31, 37}

\[
M_{IJ} = V_{IJ} - \frac{1}{a^3} \frac{d}{dt} \left( \frac{a^3}{H} \dot{\varphi}_I \dot{\varphi}_J \right) = V_{IJ} + (3 - \varepsilon) \dot{\varphi}_I \dot{\varphi}_J + \frac{1}{H} (V_I \dot{\varphi}_J + V_J \dot{\varphi}_I), \tag{2.23}
\]

where \(V_I = \frac{\partial V}{\partial \varphi_I}\) and \(V_{IJ} = \frac{\partial^2 V}{\partial \varphi_I \partial \varphi_J}\). The mass matrix is related with the expansion tensor of the inflationary flow via \(U_{IJ} = -M_{IJ}/3H^2\) \cite{25}, and it is often more useful to work with its eigenvalues. Indeed, denoting by \(v_l\) and \(v_h\) the perturbations corresponding to the smaller and larger mass eigenvalues, \(m_l^2\) and \(m_h^2\), respectively, we obtain the following equations of motion

\[
v_l'' + \left( k^2 + a^2 m_l^2 - \vartheta'^2 - \frac{a''}{a} \right) v_l - \vartheta'' v_h - 2 \vartheta' v_h' = 0 \tag{2.24}
\]

\[
v_h'' + \left( k^2 + a^2 m_h^2 - \vartheta'^2 - \frac{a''}{a} \right) v_h + \vartheta'' v_l + 2 \vartheta' v_l' = 0, \tag{2.25}
\]

\textit{A notable example is that of double quadratic inflation (see, for example, Vernizzi & Wands \cite{35}). It is well known that, in this case, there are appreciable changes to the power spectrum, as well as other observables, such as the scalar spectral index and local non-gaussianity as measured by \(f_{NL}\), (see, for example, Dias & Seery \cite{36}).}
Figure 1. A sketch of the potential (2.21) with the parameter choice (2.22) (except that the value \( \Delta \theta = \frac{\pi}{4} \) was chosen to visually make the turn more pronounced).

where \( \vartheta \) is the angle of the rotation that diagonalizes \( M \). Since by assumption \( |\vartheta'| \to 0 \) in the regime of very early and very late times (well before and well after the turn), at late times the light and the heavy modes naturally correspond to the curvature and the isocurvature perturbations, respectively.

We solve eqs. (2.24) and (2.25) assuming the initial state of the curvature and isocurvature perturbations is the Bunch-Davies vacuum. For clarity, we focus on a single mode of wavenumber \( k \) much larger than the wavenumber \( k_{\text{turn}} \) of the mode that leaves the Hubble radius at the turn — namely, we take the modes that leave the Hubble radius 3, 4 and 6 e-folds after the turn. The results are shown in figure 2. As our initial conditions are \( (v_{\ell}^{(1)}, v_{h}^{(1)}) \propto (1, 0) \) and \( (v_{\ell}^{(2)}, v_{h}^{(2)}) \propto (0, 1) \), we refer to \( v_{\ell}^{(1)} \) as the *initially* light perturbation, while \( v_{\ell}^{(2)} \) is the *induced* light perturbation; the *total* light perturbation arises from adding these contributions in quadratures. Similar nomenclature is applied to refer to the heavy field perturbations.

In figure 2 we show for several modes the initial, induced and the total light perturbation, as well as the total heavy perturbation. The instantaneous power spectra are normalised to \( P_0(\tau) \), which is the instantaneous power spectrum of a single massless mode with elementary wavefunction \( v_k(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}} \left( 1 - \frac{i}{k\tau} \right) \), corresponding to the same values of the Hubble parameter and the field velocity at the Hubble radius exit. Figure 2 shows that the evolution of the modes closely follows the estimate (2.17)—this estimate works well if the turn happens when the mode is still within its mass horizon, for which \( k = am_h \). Small wiggles appearing immediately after the turn result from the fact that it takes a small finite amount of time before the true inflationary trajectory settles at the bottom of the potential valley. We also checked that the deviations of \( P_l \) from \( P_0 \) do not exceed 1\% for the modes which are within their mass horizons at the turn; this conclusion does not change if the mass of the heavy field is increased or reduced by a factor of 2.
Figure 2. The evolution of the power spectra \( P_l \) and \( P_h \) of the light and the heavy modes, respectively, that leave the Hubble radius 3, 4 and 6 e-folds after the turn. Lighter (darker) shaded regions denote times at which the largest-\( k \) of these modes is outside the mass radius \( k = am \) (Hubble radius \( k = aH \)). Different lines of the same colour correspond to the various contributions to the light mode (the perturbation that remained of the light-only initial condition, the perturbation that was induced by the heavy-only initial condition and the total perturbation, defined as the sum of squares of amplitudes of the components), or to the total power spectrum of the heavy mode. Dashed lines show the behaviour of \( \cos^2 \theta \) and \( \sin^2 \theta \).

Comparison with previous results. At this point, it is relevant to understand why our result is different from the corrections reported in refs. [27, 28]. The authors of ref. [27] used equations of motion different from eqs. (2.4) and (2.5), which results in a different estimate of the correction to the power spectrum compared to numerical estimates.

On the other hand, the authors of ref. [28] based their estimate on a perturbative calculation, with \( \vartheta' \) as the expansion parameter. Applying our approximation of \( \vartheta' \) as a Dirac delta distribution in the formalism of ref. [28] would lead to singularities due to the presence of terms involving \( \vartheta'^2 \) in integrals. To circumvent this problem, we regularize our treatment of the fast turn, choosing the profile of \( \vartheta' \) either as a Gaussian or a product of two step functions. The respective formulae and the resulting expressions for \( F_l \), \( F_h \) and \( F_{lh} \) are given in table 1. To obtain these results, we have used the fact that the wave functions for the modes inside the mass or Hubble horizon can be expressed as \( v_{l,h} \approx \frac{1}{\sqrt{2k}} e^{-ik\tau} \). This is justified since we are assuming a perturbative correction to the power spectrum and therefore it suffices to use the lowest-order result for the wavefunction. It is clear that for both parametrizations of \( \vartheta \) the corrections \( F_l \) and \( F_{lh} \) diverge in the fast-turn limit when \( \Delta \tau \to 0 \), but the sum of
Table 1. Corrections $F_l$, $F_h$, $F_{lh}$ to the power spectrum calculated for two different parametrizations of the turn rate $\vartheta'$.

the three corrections entering eq. (2.3) is zero, which is in effect the physical estimate for the corrections to the power spectrum. Hence the actual prediction of ref. [28] for the small-scale power spectrum is that the power spectrum is to a good accuracy indistinguishable from one obtained in the single-field case. This conclusion complements calculations performed by Noumi & Yamaguchi [33], who studied analytically and numerically the evolution of the perturbations with $H < k/a < M$; their numerical results also suggest the null effects of the turn for large $k$.

3 Particle production at sharp turns

So far we have analysed the dynamics of the field perturbations when they encounter a turn in the classical trajectory in field configuration space. For the preceding analysis we have dismissed the possibility of producing particles and the induced displacement of the heavy field from its local vacuum, as a consequence of the turn. However, if the turn is non-adiabatically sharp we should expect particle production to be effective. In this section, we shall focus on such an event of particle production.

3.1 Particle production in single-field inflation

Before proceeding with the analysis of production of quanta of particles induced by the turn, it is instructive to recapitulate the issue of particle production in a simpler case. We follow the work by Mijić [38], and consider a model with a single, massive, canonically normalised scalar field, $\varphi$, in a de Sitter background ($H = $ const). Notice that giving the scalar field a mass, breaks conformal invariance and introduces a scale in the problem to be compared to the scale associated with horizon exit.

---

5One can try to estimate how fast the effects of the turn decouple, by expanding the wave function of the heavy mode in powers of the finite mass corrections. It is easy to check that the leading correction is $\mathcal{O}\left(\frac{M^2}{H^2 k^2}\right)$, i.e. it decreases as $k^{-2}$. 

---

Table 1. Corrections $F_l$, $F_h$, $F_{lh}$ to the power spectrum calculated for two different parametrizations of the turn rate $\vartheta'$. 

| Function | $\vartheta'$ profiles |
|----------|-----------------------|
| $F_l$    | $\Delta \vartheta^2 \exp \left(-\frac{(\tau - \tau_{\text{turn}})^2}{\Delta \tau^2}\right)$ | $\Delta \vartheta^2 \Theta(\tau - \tau_{\text{turn}})\Theta(\tau_{\text{turn}} + \Delta \tau - \tau)$ |
| $F_h$    | $\frac{\Delta \vartheta^2}{\sqrt{2\pi k \Delta \tau}} e^{-\frac{1}{2} k^2 \Delta \tau^2} \sin (2k/k_{\text{turn}})$ | $\Delta \vartheta^2 \left[ \frac{\sin (2k/k_{\text{turn}})}{k \Delta \tau} + \cos (2k/k_{\text{turn}}) \right]$ |
| $F_{lh}$ | $-\Delta \vartheta^2 - \frac{\Delta \vartheta^2}{\sqrt{2\pi k \Delta \tau}} e^{-\frac{1}{2} k^2 \Delta \tau^2} \sin (2k/k_{\text{turn}})$ | $-\Delta \vartheta^2 \left[ 1 + \frac{\sin (2k/k_{\text{turn}})}{k \Delta \tau} + \cos (2k/k_{\text{turn}}) \right]$ |
The equation of motion for the elementary comoving wavefunction \( u_k(\tau) = a(\tau)\varphi_k(\tau) \) associated with this field has the well-known form\(^6\)

\[
0 = u'' + \left( k^2 + m^2 a^2 - \frac{a''}{a} \right) u = u'' + \left[ k^2 + \frac{1}{\tau^2} \left( \frac{m^2}{H^2} - 2 \right) \right] u, \tag{3.1}
\]

where we used the de Sitter relations \( aH\tau = -1 \) with constant \( H \), which is a good approximation if \( \varepsilon \ll 1 \). The solution to eq. (3.1) reduces in the early times regime, \( \tau \to -\infty \), to a positive-frequency plane-wave

\[
u i(\nu-1/2)/2 \sqrt{\pi/2} \sqrt{-k\tau} H^{(1)}_{\nu}(-k\tau), \tag{3.2}
\]

where \( H^{(1)}_{\nu} \) is the Hankel function of the first kind and order \( \nu = \sqrt{9/4 - m^2 H^2} \).

This function can be decomposed using a Bogolyubov transformation, also known as the mode mixing prescription, as follows

\[
u = 1/\sqrt{2\omega} (\alpha + \beta). \tag{3.3}
\]

The Bogolyubov coefficients \( \alpha \) and \( \beta \) stand for the WKB-like positive and negative frequency modes

\[
u = A \exp \left[ -i \int \omega(\eta) \, d\eta \right] \quad \text{and} \quad \beta = B \exp \left[ +i \int \omega(\eta) \, d\eta \right], \tag{3.4}
\]

and the dispersion relation reads

\[
u^2 = k^2 + \frac{1}{\tau^2} \left( \frac{m^2}{H^2} - 2 \right). \tag{3.5}
\]

The functions \( \alpha \) and \( \beta \) obey a system of coupled differential equations, which determines their time evolution, as follows:

\[
\begin{cases}
\alpha' = -i\omega\alpha + \frac{\omega'}{2\omega}\beta \\
\beta' = i\omega\beta + \frac{\omega'}{2\omega}\alpha.
\end{cases} \tag{3.6}
\]

Provided \( \omega^2 > 0 \), they satisfy the Wronskian condition \(|\alpha|^2 - |\beta|^2 = 1 = |A|^2 - |B|^2| \).

It may appear that eqs. (3.6) become singular for \( \omega \to 0 \). However, there is a simple prescription that allows a determination of \( \alpha \) and \( \beta \) once the solution for the wavefunction \( u \) is known. Indeed, using eqs. (3.3) and (3.6), we find

\[
u' = -i\sqrt{\omega/2} (\alpha - \beta). 
\]

Hence

\[
\alpha = \frac{1}{\sqrt{2}} \left( \sqrt{\omega} u + \frac{i}{\sqrt{\omega}} u' \right) \quad \text{and} \quad \beta = \frac{1}{\sqrt{2}} \left( \sqrt{\omega} u - \frac{i}{\sqrt{\omega}} u' \right). \tag{3.7}
\]

\(^6\)From now on we shall drop the subscript \( k \), unless it is necessary to distinguish between different modes.
The factor $|\beta|^2$, which can usually be interpreted as the occupation number of a given mode\(^7\) reads, for $\omega^2 > 0$ [41]

\[
|\beta|^2 = \frac{|u'|^2 + \omega^2 |u|^2}{2\omega} - \frac{1}{2}.
\]  

(3.8)

Far outside the horizon, we can expand eq. (3.2) for $k|\tau| \ll 1$, to find\(^8\)

\[
u_k(\tau, m) \approx \begin{cases} 
-\frac{i}{\sqrt{2k^{3/2}\tau}}, & \text{for } \frac{m}{H} \ll 1 \\
\sqrt{-\frac{H\tau}{2m}} \exp \left[ \frac{im}{H} \left( \ln \left( \frac{Hk\tau}{2m} \right) + 1 \right) \right], & \text{for } \frac{m}{H} \gg 1.
\end{cases}
\]

(3.9)

Substituting into eq. (3.8), we observe that if the field is very light compared to the Hubble scale, $m/H \ll 1$, the occupation number grows like $\sim (k\tau)^{-3}$ after Hubble radius crossing\(^9\) — the field fluctuations become semiclassical. On the other hand, for large $m/H$ we obtain the occupation number approaching a constant value

\[
\lim_{\tau \to 0^-} |\beta|^2 = \frac{H^2}{16m^2} \ll 1,
\]

indicating that a heavy field does not form a semiclassical condensate, in agreement with the findings of Haro & Elizalde [42].

3.2 Generalisation to the multiple-field case

The formalism outlined above can be readily applied to the case of multiple-field inflation. Following Nilles, Peloso & Sorbo [24], we now consider a system of $n$ canonically normalised scalar fields $\{\Xi_I\}_{I=1,...,n}$ governed by the action

\[
S = \frac{1}{2} \int d\tau d^3x \left[ \sum_{I=1}^{n} \partial^\mu \Xi_I \partial_\mu \Xi_I - \sum_{I,J=1}^{n} \Xi_I M^2_{IJ} \Xi_J \right],
\]

(3.11)

where the mass matrix $M^2$ can be written as

\[
M^2 = C^T M^2_d C,
\]

(3.12)

with $C$ being an orthogonal matrix and $M^2_d$ a diagonal matrix with eigenvalues $\{m_1^2, m_2^2, \cdots \}$.

---

\(^7\)Certainly, utmost care is required when one interprets $|\beta|^2 \neq 0$ as the presence of real particles [39, 40] in time-dependent backgrounds. We will leave the discussion of the ambiguities associated with this interpretation for section 4, which are nevertheless of no consequence to our results. In particular, we consider a very short non-adiabatic period, and focus on modes deep inside the Hubble radius (for which the Minkowski spacetime approximation works very well).

\(^8\)Here we used the asymptotic expansion of the Hankel function in (3.2), which for a massive field corresponds to an imaginary order $\nu = im/H$, and gives in the limit when $x \ll 1$ (that is, very late times)

\[
H^{(1)}_\nu(x) \to -(1+i) \frac{2im}{\sqrt{\pi x}} e^{-m(i+\pi/2-\ln m)} (-k\tau)^{-im}.
\]

\(^9\)Ref. [41] uses a Wigner function approach and finds a different, yet still divergent, dependence; the Lagrangian used for quantization there contains mixed $uu'$ terms, in contrast with our presentation.
One should recall that in a time-dependent background the matrices $C$ and $M_2^2$ generically depend on time too. In particular, as mentioned before, in an FLRW background these mass eigenvalues include the gravitational mass squared, $-a''/a$.

As there are many independent quantum fluctuations, instead of a single wavefunction one is now compelled to consider an $n \times n$ matrix, whose entries, $u_{IJ}$, should be interpreted as the amplitudes of the $I$-th mode resulting from initial conditions $u_{IJ} \sim \delta_{IJ} e^{-i k \tau \sqrt{2k}}$ for $\tau \to -\infty$. The Bogolyubov coefficients $\alpha$ and $\beta$ also become matrices, which obey the following differential equations:

$$
\begin{align*}
\alpha' &= -i\omega \alpha + \frac{1}{2} \omega' \omega^{-1} \beta - \mathbb{I} \alpha - \mathbb{J} \beta \\
\beta' &= i\omega \beta + \frac{1}{2} \omega' \omega^{-1} \alpha - \mathbb{I} \beta - \mathbb{J} \alpha .
\end{align*}
$$

(3.13)

Here, the matrices $\mathbb{I}$ and $\mathbb{J}$ are defined as [24]

$$
\mathbb{I} = \frac{1}{2} \left[ \sqrt{\omega} C^T C' \sqrt{\omega}^{-1} + \sqrt{\omega}^{-1} C^T C' \sqrt{\omega} \right]
$$

(3.14)

$$
\mathbb{J} = \frac{1}{2} \left[ \sqrt{\omega} C^T C' \sqrt{\omega}^{-1} - \sqrt{\omega}^{-1} C^T C' \sqrt{\omega} \right],
$$

(3.15)

in which the diagonal matrix $\omega$ is given by

$$
\omega^2 = k^2 + M_2^2.
$$

(3.16)

Solving eqs. (3.13) is, in general, rather complicated. In the remainder of this analysis, we shall simplify the discussion in three ways. First, we will only consider two-field inflation models. In most cases, models with two fields exhibit all the essential features of more general multiple-field inflationary models, but are much simpler to analyse. Our second assumption deals with the shape of the potential. We assume that before and after the turn the eigenvalues of the mass matrix $M_2^2$ are $m^2$ and $M^2$, and that at these times $\omega \frac{dC}{d\tau} = 0$. For successful inflation, the smaller of these eigenvalues, $m^2$, should be much smaller than $H^2$ and thus we take it to be negligible in our analysis. Finally, we assume that the turn is fast, i.e., that $|\dot{\vartheta}| \ll H$ except for a fraction of an e-fold. This will allow us to neglect the expansion of the universe for the duration of the turn and perform the calculations in Minkowski spacetime, not having to distinguish between the cosmic time $t$ and the conformal time $\tau$.

With these assumptions, the matrices $\mathbb{I}$ and $\mathbb{J}$ defined in eqs. (3.14) and (3.15) become very simple

$$
\mathbb{I} = \frac{1}{2} \dot{\vartheta} \begin{pmatrix} \sqrt{\vartheta} & \frac{1}{\sqrt{\vartheta}} \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix},
$$

(3.17)

$$
\mathbb{J} = \frac{1}{2} \dot{\vartheta} \begin{pmatrix} \sqrt{\vartheta} & \frac{1}{\sqrt{\vartheta}} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} ,
$$

(3.18)
where \( r = \sqrt{(k^2 + M^2)/(k^2 + m^2)} > 1 \) is the ratio of the eigenvalues of \( \omega \). In the fast turn approximation, \(|\dot{\vartheta}| \gg \{k, M, H, m\}\) (that is, \( \dot{\vartheta} \) is the largest scale in the problem),\(^{10}\) eqs. (3.13) can therefore be written as

\[
\frac{d}{dt}(\alpha \pm \beta) = -(I \pm J)(\alpha \pm \beta). \tag{3.19}
\]

Their solution takes the form

\[
(\alpha \pm \beta) = \exp \left( - \int t (I(t') \pm J(t')) dt' \right) (\alpha \pm \beta)_0, \tag{3.20}
\]

where the subscript ‘0’ refers to the initial conditions \( \alpha_0 = 1, \beta_0 = 0 \).

Plugging eqs. (3.17) and (3.18) into eq. (3.20), we find under these assumptions

\[
\alpha = \cos \Delta \vartheta \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2} \left( \sqrt{r} + \frac{1}{\sqrt{r}} \right) \sin \Delta \vartheta \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{3.21}
\]

\[
\beta = -\frac{1}{2} \left( \sqrt{r} - \frac{1}{\sqrt{r}} \right) \sin \Delta \vartheta \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{3.22}
\]

where \( \Delta \vartheta \) is the angle by which the inflationary trajectory turns. This means that both modes, light and heavy, can be produced at the turn with equal occupation numbers

\[
|\beta|^2 = \frac{1}{4} (\sqrt{r} - 1/\sqrt{r})^2 \sin^2 \Delta \vartheta. \tag{3.23}
\]

We note in passing that for \( r \to 1 \), which corresponds to the sub-mass-horizon limit \( k \gg \{M, H, m\} \), we have \(|\beta|^2 \approx \frac{1}{16} \left( \frac{H}{M} \right)^4\), while \( \alpha \) becomes a pure rotation matrix, in agreement with our discussion in section 2.1. Provided that \( \omega^2 \) is positively defined, the matrices \( \alpha \) and \( \beta \) obey the relation \( \alpha \alpha^\dagger - \beta \beta^\dagger = 1 \), as the coefficients of the Bogolyubov transformation should.

Finally, we can estimate the energy density in non-relativistic heavy particles produced at the turn, as

\[
\rho_h = \int \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + M^2} |\beta|^2 \approx \frac{M^4}{16\pi^2} \sin^2 \Delta \vartheta. \tag{3.24}
\]

To evaluate the integral in eq. (3.24), we have only included the modes for which \( H < k < M \), used the expansion \(|\beta|^2 \sim \frac{M}{k} \sin^2 \Delta \vartheta\) valid for \( k \ll M \) and neglected \( O(H^2/M^2) \) terms.\(^{11}\) The obtained energy density scales as \( M^4 \), which is a priori unrelated to the potential energy density during inflation, \( V \sim H^2 M^2 \). With sufficiently large \( M \), one can envision that the entire kinetic density of the fields corresponding to the velocity component orthogonal to

---

\(^{10}\)For simplicity, we assume that the eigenvalues of \( \omega^2 \) remain practically constant throughout the short time of the turn, as forces inducing the turn only require a transient presence of a first derivative of the potential in a specific direction in the field space. While in practice the deviations from this assumption can be sizeable, the relative change of the eigenvalues can be much smaller than \( \dot{\vartheta} \), which justifies neglecting the first two terms on the right-hand sides of eqs. (3.13).

\(^{11}\)The integral can be calculated without expanding \(|\beta|^2\), but the rather complicated final result differs from the estimate above by only \( \sim 10\% \). Therefore, we take the result (3.23) to provide a sufficient estimate of this effect.
Table 2. Parameter choices for the model described in section 2.2 used in the numerical study and with the model labels used in figure 4.

| model parameters | derived parameters | label |
|-------------------|---------------------|-------|
| $M/M_{Pl}$        | $sM_{Pl}$           | $M/H$ | $\Omega$ | $\max \left( \frac{\dot{\phi}}{H} \right)$ | width of $\frac{\dot{\phi}}{H}$ | |
| $10^{-4}$         | $5\sqrt{3} \times 10^3$ | 10    | 0.63      | 13      | 0.018     | 10   |
| $2 \times 10^{-4}$| $\sqrt{3} \times 10^3$ | 20    | 0.31      | 2.8     | 1.0       | 20   |
| $2 \times 10^{-4}$| $5\sqrt{3} \times 10^3$ | 20    | 0.31      | 14      | 0.018     | 20   |
| $2 \times 10^{-4}$| $\sqrt{3} \times 10^4$ | 20    | 0.31      | 27      | 0.010     | 20 fast |
| $4 \times 10^{-4}$| $5\sqrt{3} \times 10^3$ | 40    | 0.155     | 14      | 0.019     | 40   |

In order to corroborate our analytical results, we will now calculate $|\beta|^2$ numerically in the model discussed in section 2.2. This can be achieved by solving the equations for the wavefunctions of the perturbations, eqs. (2.24) and (2.25), and then employing a generalization of eq. (3.7) to read off $\beta$ from the wavefunction. The relation in question is

$$\beta = \frac{1}{\sqrt{2}} \sqrt{\omega} u - \frac{i}{\sqrt{2}} \sqrt{\omega}^{-1} \left[ u' + \vartheta' \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} u \right],$$

where $u$ is now to be understood as a $2 \times 2$ matrix with the following components

$$u = \begin{pmatrix} u_l^{\text{initial}} & u_l^{\text{induced}} \\ u_h^{\text{initial}} & u_h^{\text{induced}} \end{pmatrix}.$$

As before, the subscripts $l, h$ refer to the light and heavy components, respectively, and the definition of the initial and induced components is the same as in section 2.2. Values of $\beta$ were determined on an interval of 0.2 e-folds beginning 0.1 e-folds after the turn (when the rapid growth of $\beta$ has ceased); as the heavy mode interacts with the light one at the turn, its evolution is more complicated than that of a massive mode in the de Sitter spacetime, which leads to small oscillations in $\beta$. Hence, sampling is necessary for estimating the accuracy of the determined $|\beta|^2$.

In order to study how the results for $|\beta|^2$ depend on various parameters, we studied five different parameter choices shown in table 2. We chose three different values of the mass of the heavy field, satisfying $\frac{M}{M_{Pl}} = 10, 20$ and 40. In the absence of particle production and with the direction of the inflationary trajectory parametrised by an angle $\theta$, the function $\sin \theta$ expressed as a function of number of e-folds would exhibit damped harmonic oscillations, i.e., behave as $\sin(\Omega N + \psi_0)$ —the dimensionless frequency of these oscillations, $\Omega$, is shown...
Figure 3. Evolution of the occupation number of the light and the heavy mode (dashed and solid black lines, respectively) for several models listed in table 2. For reference, we also show the evolution of $\dot{\vartheta}/H$ (blue line) and the dashed red line corresponds to the analytical result (3.23) for the occupation number.

in table 2. The time dependence of the rotation angle $\vartheta$ diagonalizing the mass matrix is such that $\dot{\vartheta}/H$ is a Gaussian-like function (but with noticeable deviations from the exact Gaussian shape). In table 2, we give the maximal value of this function and the width of this function at half maximum in the number of e-folds.

In figure 3, we show the time evolution of the occupation number for the light and the heavy mode leaving the Hubble radius 3 e-folds after the turn. The buildup of the occupation number, which is very similar for the light and the heavy mode, takes place during the time at which the eigenvector of the mass matrix are fast changing (of the two eigenvalues of the mass matrix, the larger one is practically constant and the smaller one is always much smaller than $H$). The analytical result (3.23) is in qualitative agreement with the numerical solutions. Note that for the discussed modes our analytical result is on the verge of applicability, as the turn happens close to the mass radius crossing, $k = M a$, of the heavy mode.

In figure 4, we show the results for the occupation number of the heavy mode as a function of the comoving wavenumber $k$. Our analytical result agrees with the outcome of our numerical calculation within its range applicability: deviations are seen when the turn takes place after the mass horizon crossing of the heavy mode (i.e., for $M/H = 40$) or the turning rate is not the largest scale. The latter is visible in the behavior of $|\beta|^2$ in the large $k$ limit and for the mode with a reduced turn speed labeled 20 slow. Increasing the turn rate (model 20 fast) improves the agreement. For $k/k_{\text{turn}} \to 1$, where $k_{\text{turn}}$ is the comoving scale that exits the Hubble radius at the turn, we obtain a larger occupation number than given in (3.23); this can be attributed to the fact that the mass squared of the light field
Figure 4. Occupation numbers $|\beta|^2$ for different values of $k/k_{\text{turn}}$, where $k_{\text{turn}}$ is the comoving scale that exits the Hubble radius at the turn, and for three values $M/H = 10, 20, 40$. Solid lines correspond to the full numerical calculation outlined in section 3.3, while the dashed lines correspond to the analytical estimate (3.23). The shaded areas correspond to the de Sitter result $|\beta_{\text{dS}}|^2 = \frac{H^2}{16M^2}$, which is much smaller than $|\beta|^2$ in a wide range of $k$.

grows transiently negative. Modes with $k/k_{\text{turn}} < 1$ are already outside the Hubble radius, so they have already classicalised and they cannot be interpreted as particles. The occupation number of the light mode is very similar to the result for the heavy mode, therefore, we do not plot it.

In this calculation, we have not included the backreaction of the produced particles on the inflationary background; we shall return to this issue in section 4.2. Here we just note that due to backreaction the occupation numbers obtained above should be regarded as maximal, subject to energy conservation.

4 Particle production in an expanding universe

At this point we would like to make a short digression and review the ambiguities associated with the concept of particle production in a time dependent setting [43–47]. The quest for understanding the nature of this phenomenon has lasted for several decades now. When can we interpret $|\beta_k|^2$ as a measure of particle production?

This question is closely related with the concept of classicalisation of quantum perturbations in cosmology. It can be briefly phrased as follows. Deep inside the Hubble radius, the fluctuations are fundamentally quantum-mechanical and the solutions to the equations of motion for the field perturbations can be described by a linear combination of creation and
annihilation operators. These operators span the particle Hilbert space in which the vacuum is, by definition, annihilated by the destruction operator. A few e-folds after exiting the horizon, the scalar perturbations are said to classicalise, in the traditional sense that the position and momentum of the wavepacket of quantum fluctuations can be measured simultaneously. Said differently, the quantum uncertainty inherent to the scalar field becomes negligible.

Where such transition precisely occurs is still a matter of dispute. Recent work in the literature has focused on the location of the matching surface between quantum and classical eras [48–53]. This is undeniably of theoretical interest but perhaps even more importantly can induce an error in the precision with which one predicts the value of imprint of the curvature perturbation in the CMB radiation (see ref. [15], for example). Another approach to this question is to replace the discussion of the quantum to classical transition with a transition from hard to soft subprocesses using the methods of the renormalization group, which can be properly adapted to time-varying backgrounds [52].

Whatever the origin, there is little doubt that this classicalisation process ought to occur some e-folds after horizon crossing by the time when the decaying mode (in the context of inflation) has become negligible. Then, the perturbation is solely characterised by the surviving growing contribution, which for single-clock inflation can be related to the constant mode associated with the primordial fluctuation. In this regime, it is said the WKB classicality condition has been reached [54], and the commutation relations between two fields ascribed different momenta vanish.

4.1 Decoherence effects and particle production

The concern of where in the e-folding history the transition from quantum to classical regimes takes place is also sometimes phrased in terms of the decoherence phenomenon, after which one can refer to the particle state as a classical condensate [55]. Mathematically, we say that decoherence has occurred if the density matrix (which also includes the interaction Lagrangian in addition to the free action) is in the diagonal form. The origin of this process is, however, far from being understood. Nevertheless, once decoherence (or classicalisation) has occurred, the classical evolution of the perturbations is then determined by the separate universe approach (also known as the $\delta N$ technique) [25, 37, 56–62]. Strictly, this evolution is uniquely determined on super-horizon scales when the perturbations are smoothed over a scale much bigger than the size of the horizon, $H^{-1}$, but it is independent of the number of fields which contribute (or not) to the primordial curvature fluctuation.12

In addition to this quantum to classical transition ambiguity, the concept of particle itself is also unclear and in some sense related to this first issue. There are two reasons which make the discussion around particle production convoluted. On the one hand, to describe particle production one needs well-defined initial conditions which describe the vacuum state of the background [45, 46]. A time-dependent background will, however, violate in some region in parameter space the adiabaticity condition, which for modes $k$ with frequency $\omega$

---

12Recently, Mulryne [62] has extended this formalism to sub-horizon scales, allowing for the quantum statistical properties of the field fluctuations to be described at all scales by a unique set of differential equations.
reads
\[ \frac{\dot{\omega}_k}{\omega_k^2} \ll 1 . \]

If the condition above is not verified, then particle production occurs \[39\]. Consequently, the vacuum of the theory will have to be systematically adjusted. However, a more crucial problem arises as a direct result of the time-dependent background, which explicitly breaks Poincaré invariance — inertial observers do not agree on each other’s definitions of vacuum. This adds to the problematic interpretation of particle production, when compared to a clearer result in Minkowski spacetime.\(^\text{13}\) On the other hand, in such a setting the Heisenberg uncertainty relations prompt to an uncertainty in the particle number owing to the possibility of creating virtual pairs of particle-anti-particle.

Yet another factor to take into account is the detection of the produced particles, which can depend on the level of the coarsening of measurements themselves. This is usually a concern of the particle-wave duality, which is again related with the classicalisation process. As explained by Birrell & Davies \[39\], the detection of a particle is only unambiguous in static spacetimes — even in the case of asymptotically static spacetimes it is not clear how to define the volume in momentum space required to compute a meaningful (expectation value of the) particle number density.

For these reasons it is unclear how to define a particle or confirm the detection of a particle in curved spacetime. The discussion of the previous sections, however, evades this issue since by studying particle production related to sub-horizon scales owing to a sharp turn in field configuration, thereby ignoring the local curvature and taking spacetime as effectively Minkowski (in the vicinity of the turn). Moreover, if there were any effects on super-horizon scales, these would only add to the calculation presented before, and as such the results of the previous sections can be regarded as a conservative estimate of particle production.

### 4.2 Effects of the turn on observables

Having presented our arguments for particle production at a turn of the inflationary trajectory, we will now proceed to discussing the impact of this phenomenon on the power spectrum of the curvature perturbations. We first note that the turn — and the burst of particle production — lasts only for a fraction of an e-fold. The timescale of this event is therefore much smaller than the Hubble time, \(H^{-1}\), which allows for the application of the energy conservation law, as the gravitational effects can be neglected on such small timescales.

The energy emitted in the form of the particles is drained from the kinetic energy of the scalar fields. Immediately after the turn, the energy density of the universe consists of potential energy, and the subdominant components of kinetic energy of the scalar fields and a collection of particles. As the kinetic energy is reduced by particle production, the field velocity after the turn is smaller than the one inferred from the slow-roll relation, which was satisfied before the turn.

\(^{13}\)A possible resolution to this issue is to recall that in the infinite past, when the modes are deep inside the horizon, spacetime is effectively Minkowski, and there all inertial observers agree with the same vacuum state.
Particle production therefore changes the composition of the background energy density by introducing heavy particles, whose energy density redshifts as $a^{-3}$. However, classical oscillations of a heavy scalar field around a minimum of the potential also redshift as $a^{-3}$, so the energy transfer does not change the time dependence of the Hubble parameter.\(^{14}\) Light modes are produced at the turn, too, with a similar occupation number compared to the heavy ones. This can further reduce kinetic energy after the turn, transferring it into a component that redshifts as $a^{-4}$, but these changes in kinetic energy are also unimportant for small values of the slow-roll parameter $\varepsilon$.

There is, however, one effect which can have a non-trivial impact on the predictions for the power spectra of the perturbations. As it is clear from eqs. (2.24)–(2.25), the coupling between the light and the heavy modes depends on the time evolution of $\vartheta$, the rotation angle that diagonalizes the mass matrix of the perturbations.\(^{15}\) A sudden decrease of the kinetic energy by particle production affects $\vartheta$ in two ways: a sudden change in the field velocity results in a jump of $\vartheta'$ and a delta-like singularity in $\vartheta''$. The slowing down of the fields also increases the amount of time in which the fields spend at the turn of the inflationary potential, thereby lowering $\vartheta'$ after particle production with respect to the situation in which particle production is neglected.

A direct comparison of the kinetic energy in the model analysed numerically in section 3.3 (which is of order $\varepsilon H^2 \sim 10^{-14}$) with the estimate in eq. (3.24) of the maximal energy density in produced particles (which gives $\rho_h \sim 10^{-17}$ for $M/H = 40$) shows that for the models discussed in section 3.3 the backreaction of the produced particles onto the inflationary background is negligible, in accordance with the assumption therein. In order to illustrate the effect of particle production on the power spectrum, we need a model with a smaller inflaton velocity or a larger mass of the heavy field. Here, we chose that last option

\(^{14}\)While the evolution of the total energy density does not change, the kinetic energy of the scalar fields exhibits different behaviours: during the oscillations of the heavy field, the kinetic energy is periodically transferred into potential energy and vice versa; this effect is absent if the energy is transferred into particles. Such oscillation can be in resonance with the mode functions, see [63, 64]. Owing to the smallness of the slow-roll parameter $\varepsilon$, the effect is, however, very small in the models considered here.

\(^{15}\)For modes with very large $k$, the actual direction of the field velocity or changes in this coupling are to a large extent irrelevant — as we argued in section 2.2, the oscillations of the inflationary trajectory around the flat valley of the potential do not affect the power spectrum for the modes which are well within their mass horizon, $k = aM$, at the turn.
and studied numerically models similar to that in table 2, but with a larger $M/H$ equal to 50, 100 and 200, assuming that the approximation (3.24) for the maximal energy density in produced particles holds. Then the amount of produced particles is sufficient to reduce the kinetic energy of the fields to zero ($M/H = 200$), to approximately one-third of its initial value ($M/H = 100$) or to a few per cent of its initial value ($M/H = 50$). The parameters of the models are given in table 3.

Figure 5 shows how the angle $\vartheta$ changes with and without particle production for the case $M/H = 200$. There is a clear difference between the two cases with $\vartheta$ changing more slowly after kinetic energy of the fields is transferred into particles. In this case, we can describe the evolution of $\vartheta'$ across the turn as

$$\vartheta' = \Delta \vartheta' \Theta(\tau - \tau_{\text{turn}}),$$

where $\Theta$ is the step function and $\Delta \vartheta'$ is the magnitude of the jump of $\vartheta'$. Solving the equations of motion (2.24)–(2.25) for the perturbations, we should therefore implement the following matching relations

$$v'_{\ell}(\tau_+) = v'_{\ell}(\tau_-) + \Delta \vartheta' v_h(\tau_-), \quad v'_{h}(\tau_+) = v'_{h}(\tau_-) - \Delta \vartheta' v_l(\tau_-),$$

where, as in section 2.1, an infinitesimally small interval $(\tau_-, \tau_+)$ contains $\tau_{\text{turn}}$. The results for $M/H$ are almost indistinguishable from those shown in figure 5; the only difference is that the small oscillatory features of $\frac{\dot{\vartheta}}{H}$ after particle production are less pronounced.
Applying the modified time evolution of \( \vartheta \) into the equations of motion for the perturbations leads to a modification of the power spectrum shown in figure 6. The most significant modification can be seen for \( k < k_{\text{turn}} \), i.e., for the modes which are already super-Hubble at the time of the turn. This can be understood in the following way, given our description of particle production. At the beginning of the turn the light mode acts as a source for the heavy mode, whose initial amplitude has already decayed. At the instant of particle production, there is a backreaction of this induced heavy mode on the light one, as described by the matching conditions (4.2). Thus, the light mode receives additional contribution, whose phase depends on the evolution of the heavy mode between the onset of the turn and the instant of particle production. This additional contribution can interfere either constructively or destructively with the initial light mode.

For \( k > k_{\text{turn}} \), the amplitude of the heavy mode is not so small as in the previous case and the interactions between the modes, which leads to oscillations in the power spectrum of the curvature perturbations, is not much affected by the altered evolution of \( \vartheta \). Therefore, the magnitude and \( k \)-dependence do not change qualitatively, as should be expected from the discussion above.
5 Summary

Inflationary phenomenology is an important branch in cosmology which aims at obtaining observable predictions which can probe and determine the nature of the inflationary period. The current constraints on higher correlation functions of the primordial fluctuations are compatible with a gaussian distribution [4]. This implies that non-gaussianity might not be a useful smoking-gun for the nature of the microphysics during the early universe. In particular, it raises the question of whether we might be able to ever break the degeneracy between inflation driven by a single scalar field, or multiple degrees of freedom.

When embedding an inflationary realisation within a high-energy theory demands dealing with a number of scalar-fields, all of which are, in general, active during inflation. In this paper we revisit multiple-field inflation models and reexamine the phenomenology of sharp turns in the inflationary trajectory, including the advent of particle production as a result. For simplicity we focus on a two-field inflation model and study the limiting case when one of the fields is very light, whilst the other is very heavy. We provide both analytical as well as numerical estimates for the particle numbers produced as a consequence of the turn. We focus on the modes which are sub-horizon when the turn happens, to avoid the ambiguities in the concept of particle production after horizon-crossing.

Using the generalised Bogolyubov coefficients, we are able to estimate the energy of the quanta produced during the turn. If the turn is very sharp, then most of the kinetic energy of the fields goes into particle production, while the potential energy remains approximately constant. Therefore, this process increases the amount of time spent by the fields at the turn of the trajectory and can induce a damping in the oscillations of the fields enveloping the inflationary flow, as they settle back into the local minima of the potential. Consequently, this will result in a modified power spectrum of perturbations.

On the other hand, if we were to ignore the production of quanta and focused on sub-horizon modes, there would be no change in the power spectrum of perturbations. We conclude that the rich phenomenology of multiple-field inflation can also lead to a set of parameters which might be largely unconstrained by observational measurements, which was also noted by Elliston et al. [16].

There are a number of ways in which this analysis can be further generalised. Under theoretical motivations, we might wish to contemplate the possibility of one or more fields having non-canonical kinetic terms. This, in principle, will change the effective potential between the fields. One could also imagine the effect of dropping the simplifying assumption of the initial vacuum state being Bunch-Davies, by considering a more generic, mixed state.

Another venue of study could focus on modes which are super-horizon when the turn happened. Consider, for example, double quadratic inflation [35, 48, 65–67]. In this toy model, two canonically normalised scalar fields, $\phi$ and $\chi$, which interact through gravity, obey the following action

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left\{ R - (\partial_\alpha \phi)^2 - (\partial_\alpha \chi)^2 - m_\phi^2 \phi^2 - m_\chi^2 \chi^2 \right\}. \tag{5.1}$$

It is well known that choosing appropriate initial conditions for the values of the fields, there is a turn of the inflationary trajectory in field configuration space. Despite the fluctuations
being parallel to the direction of the trajectory before and after the turn, resulting in a deflection angle of roughly $\pi/2$, the turn itself takes a sizeable number of e-folds to conclude. This will be the main limiting factor for an appreciable value of $|\beta_k|^2$. Using our formulae from the previous sections, we find that the magnitude of $|\beta_k|^2$ for super-horizon modes is below the numerical precision, and therefore particle production is not sizeable for such modes. Additionally, even if $|\beta_k|^2$ was appreciably large, one would raise the issue of whether one could interpret this object as a measure of particle production since one would expect classicalisation to have occurred.

To conclude, the analysis presented in this paper is a step forward towards understanding the phenomenology of turns in multiple-field inflation and their impact on lowest-order statistics. Our aim is purely exploratory, and we do not attempt to use the *Planck* data to constrain the parameters of the model we investigated. Rather, our analysis intends to illustrate as a point of principle the great level of complexity involved in a bottom-up approach, by which one would be able to learn about the microphysical Lagrangian from observations of the CMB.

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**A Definition of the correction factors $F_l$, $F_h$ and $F_{lh}$**

Here we collect the formulae for the factors $F_l$, $F_h$ and $F_{lh}$ introduced in ref. [28]. Let $v_l$ and $v_h$ be the wavefunctions of the light and heavy mode, respectively, in the absence of interaction between the modes, i.e. for $\vartheta =$-const. One then defines integrals:

\[
I_l = i \int d\tau (v_l(\tau) + v_l^*(\tau)) \vartheta'^2(\tau) u(\tau) \\
I_h = i \int d\tau (v_l(\tau) + v_l^*(\tau)) (\vartheta''(\tau) v_h(\tau) + 2\vartheta'(\tau) v_h'(\tau)) \\
J_{lh} = \int d\tau (v_l(\tau) + v_l^*(\tau)) (\vartheta''(\tau) v_h(\tau) + 2\vartheta'(\tau) v_h'(\tau)) \int d\tau' v_h'(\tau') (\vartheta''(\tau') v_l(\tau') + 2\vartheta'(\tau') v_l'(\tau')) + \\
- \int d\tau (v_l(\tau) + v_l^*(\tau)) (\vartheta''(\tau') v_h(\tau') + 2\vartheta'(\tau') v_h'(\tau')) \int d\tau' v_h'(\tau') (\vartheta''(\tau') v_l(\tau') + 2\vartheta'(\tau') v_l'(\tau'))
\]

and the correction factors:

\[
F_l = I_l + I_l^* , \quad F_h = |I_h|^2 , \quad F_{lh} = J_{lh} + J_{lh}^* .
\]
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