Cumulative effect of Weibel-type instabilities in counterstreaming plasmas with non-Maxwellian anisotropies

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Abstract

Counterstreaming plasma structures are widely present in laboratory experiments and astrophysical systems, and they are investigated either to prevent unstable modes arising in beam-plasma experiments or to prove the existence of large scale magnetic fields in astrophysical objects. Filamentation instability arises in a counterstreaming plasma and is responsible for the magnetization of the plasma. Filamentationally unstable mode is described by assuming that each of the counterstreaming plasmas has an isotropic Lorentzian (kappa) distribution. In this case, the filamentation instability growth rate can reach a maximum value markedly larger than that for a plasma with a Maxwellian distribution function. This behaviour is opposite to what was observed for the Weibel instability growth rate in a bi-kappa plasma, which is always smaller than that obtained for a bi-Maxwellian plasma. The approach is further generalized for a counterstreaming plasma with a bi-kappa temperature anisotropy. In this case, the filamentation instability growth rate is enhanced by the Weibel effect when the plasma is hotter in the streaming direction, and the growth rate becomes even larger. These effects improve significantly the efficiency of the magnetic field generation, and provide further support for the potential role of the Weibel-type instabilities in the fast magnetization scenarios.

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The electromagnetic instabilities of Weibel-type are driven by the velocity anisotropy of the plasma particles, whether it resides in a plasma with an electron temperature anisotropy (Weibel instability \[1, 2\]), or in a counterstreaming plasma system (filamentation instability \[3\]). Davidson \[4\], has combined for the first time these two sources of free energy showing that the Weibel-type instabilities release the excess of free energy stored in "the relative directed motion between plasma components and/or an anisotropy in plasma kinetic energy" \[4\]. Moreover, in order to describe a counterstreaming plasma with temperature anisotropies, Davidson \[4\] defined a new counterstreaming distribution function including a bi-Maxwellian electron temperature distribution function, and derived a dispersion relation, which admits purely growing solutions propagating perpendicular to the electron streaming direction. In a counterstreaming plasma with electron temperature anisotropies, the both free energy sources are present. Their contributions cumulate leading either to enhancing or quenching the electromagnetic instability, depending on the plasma kinetic energy perpendicular to the propagation direction of the instability, if it exceeds or not the parallel kinetic energy \[5, 6, 7\].

The stability properties of the counterstreaming plasma have been extensively investigated \[5, 6, 7, 8, 9, 10, 11, 12, 13\], and the general observation is that the effect of a finite plasma temperature has been revealed only for a Maxwellian electron distribution function. Further investigations have been carried out in order to examine these instabilities in a magnetized counterstreaming plasma with a bi-Maxwellian electron temperature anisotropy \[14, 15, 16\]. As expected, the presence of a dc magnetic field in the streaming direction inhibits the Weibel-type instabilities \[8, 17, 18\]. We therefore ignore any influence of an ambient magnetic field in the present investigation.

Space plasmas are observed to possess a non-Maxwellian particle distribution function with a high energy tail \[19, 20\]. The most common distribution function used to model such natural plasmas is the generalized kappa distribution \[21\]

\[
f_\kappa(v) = \frac{1}{\pi^{3/2}\theta^3} \frac{\Gamma(\kappa + 1)}{\kappa^{3/2}\Gamma(\kappa - 1/2)} \left(1 + \frac{v^2}{\kappa\theta^2}\right)^{-\kappa-1},
\]

where \(\kappa\) is the spectral index (for typical space plasmas, \(\kappa\) generally lies in the range 2–6), and thermal velocity \(\theta\) is related to the particle temperature by \(\theta^2 = [(2\kappa - 3)/\kappa] v_T^2\) and \(v_T^2 = (k_BT)/m\). The Maxwellian and kappa distributions differ substantially in the high energy tail (\(|v/\theta| \gg 1\)), but in the limit \(\kappa \to +\infty\), \(f_\kappa\) degenerates into a Maxwellian
distribution function. The above \( \kappa \)-distribution has been used to explain different waves and instabilities and their related effects in isotropic plasmas where the velocity distribution function is assumed to deviate from the Maxwellian thermal equilibrium due to the presence of high energy tails \[22, 23\].

Very often plasma particles have a preferred direction of motion in space (e.g. a high temperature direction, a stationary magnetic field direction, or a streaming direction) and their velocity distribution function is anisotropic. In order to describe the anisotropic plasmas with high-energy tails, one can use a two-temperature bi-kappa distribution function \[21\]

\[
f_\kappa(v_x, v_y, v_z) = \frac{1}{\pi^{3/2}\theta_y^2} \frac{\Gamma(\kappa + 1)}{\kappa^{3/2}\Gamma(\kappa - 1/2)} \left( 1 + \frac{v_x^2 + v_z^2}{\kappa\theta_x^2} + \frac{v_y^2}{\kappa\theta_y^2} \right)^{-\kappa-1},
\]

which approaches the bi-Maxwellian distribution function as \( \kappa \to +\infty \). The temperature anisotropy is defined as \( A = (T_y/T) - 1 \), with

\[
\theta^2 = \left( \frac{2\kappa - 3}{\kappa} \right) v_T^2; \quad \theta_y^2 = \left( \frac{2\kappa - 3}{\kappa} \right) v_T^2,
\]

and

\[
v_T^2 = v_T^2 = \frac{k_B T}{m}; \quad v_T^2 = \frac{k_B T_y}{m}.
\]

The bi-kappa distribution function has been earlier attributed to magnetized plasmas providing a useful tool for a more realistic characterization of the turbulent fields and acceleration mechanisms in the planetary environment \[24, 25, 26\]. Moreover, Hellberg and Mace (2002) have generalized the plasma dispersion function for a kappa-Maxwellian distribution, and investigated the electrostatic waves arising in such an anisotropic plasma. The kappa-Maxwellian distribution function combines a one-dimensional \( \kappa \) distribution along a preferred direction in space, and a two-dimensional Maxwellian in the plane perpendicular to this direction. This choice is justified physically by the fact that in many instances there is equilibration and isotropization in the perpendicular plane, but there is preferential acceleration, leading to a power-law distribution along another preferred direction \[27\].

Recently, a bi-kappa distribution function of form \[2\] has been used to evaluate the Weibel instability rates in an unmagnetized plasma \[2\]. For small values of \( \kappa \), the growth rate is reduced, but as \( \kappa \) is increased, the growth rate enhances approaching the results obtained with a bi-Maxwellian distribution. However, it has already been shown \[3, 6\].
that, in a counterstreaming plasma with bi-Maxwellian temperature anisotropies, the (electro)magnetic instabilities of the Weibel-type can be markedly faster when the plasma is hotter along the streaming direction, or they can be stabilized by a higher transverse plasma temperature. Moreover, there are many space plasma systems in which colliding plasma shells can be described as counterstreams (e.g. relative motion of fully ionized gaseous matters in intergalactic medium, interaction of internal or external fireballs with surrounding interstellar medium in gamma-ray burst sources, flares in the solar winds). We are therefore entitled to extend here the investigation on the aperiodic electromagnetic instabilities in counterstreaming plasmas with non-Maxwellian anisotropies.

In Figure 1 we fix the orientation for the counterstreams and the filamentation mode we are looking for: the electric field, \( \mathbf{E} \parallel \mathbf{v}_{1,2} \), is along the streaming direction, and the wave vector, \( \mathbf{k} \perp \mathbf{v}_{1,2} \), is perpendicular to the streams. This electromagnetic mode will be solution of the dispersion relation

\[
\frac{k^2c^2}{\omega^2} = \epsilon_{yy},
\]

where \( \omega \) and \( k \) are respectively, the frequency and the wave number of the plasma modes, \( c \) is the speed of light in vacuum, and \( \epsilon_{yy} \) is the dielectric tensor which describes the oscillatory properties of the plasma system. From the linearized Vlasov-Maxwell equations, one simply obtains

\[
\epsilon_{yy} = 1 - \sum_a \frac{\omega^2_{p,a}}{\omega^2} \left[ 1 - k \int_{-\infty}^{+\infty} d\mathbf{v} \frac{v_y^2}{\omega - kv_x} \frac{\partial f_{a,0}}{\partial v_x} \right],
\]

where the unperturbed velocity distribution function \( f_{a,0}(\mathbf{v}) \) for the particles of species \( a \)
is considered normalized by \( \int d\mathbf{v} f_{ab}(\mathbf{v}) = 1 \), and \( \omega_{p,a} = (4\pi n_a e^2/m_a)^{1/2} \) is the plasma frequency.

We now introduce the distribution function

\[
f_\kappa = \frac{1}{2\pi^{3/2}\theta^2_y} \frac{\Gamma(\kappa + 1)}{\kappa^{3/2}\Gamma(\kappa - 1/2)} \left\{ \left[ 1 + \frac{v_x^2 + v_z^2}{\kappa \theta^2} + \frac{(v_y - v_0)^2}{\kappa \theta^2_y} \right]^{-\kappa-1} + \left[ 1 + \frac{v_x^2 + v_z^2}{\kappa \theta^2} + \frac{(v_y + v_0)^2}{\kappa \theta^2_y} \right]^{-\kappa-1} \right\},
\]

which describes the electron plasma counterstreams with bi-Kappa temperature anisotropies. This distribution function has to be added to the list of Lorentzian-type particle distribution functions provided in Table I of Ref. [21]. For the sake of simplicity, we assume symmetric counterstreams with equal densities, \( \omega_{p,e,1} = \omega_{p,e,2} = \omega_{p,e} \), and equal temperature anisotropies, \( A_1 = A_2 = A = (T_y/T) - 1 \), and moving with the same velocities, \( |v_1| = v_2 = v_0 \). Moreover, we assume that imobile ions form the neutralizing background, and the electron plasma counterstreams are considered homogeneous, and charge and current neutralized.

Inserting (7) into (6) and the resultant expression into (5) we derive the dispersion relation

\[
\frac{k^2 c^2}{\omega^2} = \epsilon_{yy}^0 + 2 \frac{\omega_{p,e}^2}{\omega^2} \frac{v_0^2}{\theta^2} \left[ (1 - \frac{1}{2\kappa}) + \frac{\omega_{p,e}}{k \theta} Z_\kappa \left( \frac{\omega_{p,e}}{k \theta} \right) \right],
\]

where

\[
\epsilon_{yy}^0 = 1 + \frac{\omega_{p,e}^2}{\omega^2} \left[ A + (A + 1) \frac{\omega_{p,e}}{k \theta} Z_\kappa^0 \left( \frac{\omega_{p,e}}{k \theta} \right) \right]
\]

has been obtained in Ref. [2] for an anisotropic plasma with a bi-kappa distribution function without \( v_0 \), where the simple Weibel instability arises. Otherwise, for \( A = 0 \) Eqs. (8)–(9) simplify to

\[
\frac{k^2 c^2}{\omega^2} = 1 + \frac{\omega_{p,e}}{\omega k \theta} Z_\kappa \left( \frac{\omega_{p,e}}{k \theta} \right) + 2 \frac{\omega_{p,e}^2}{\omega^2} \frac{v_0^2}{\theta^2} \left[ (1 - \frac{1}{2\kappa}) + \frac{\omega_{p,e}}{k \theta} Z_\kappa \left( \frac{\omega_{p,e}}{k \theta} \right) \right],
\]

which describes the filamentation instability arising from counterstreaming plasmas with the same isotropic kappa distribution function. The dispersion equation (8) contains the modified plasma dispersion function, defined in Ref. [21],

\[
Z_\kappa(f) = \frac{1}{\pi^{1/2} \kappa^{1/2}} \frac{\Gamma(\kappa)}{\Gamma(\kappa - 1/2)} \int_{-\infty}^{+\infty} dx \frac{(1 + x^2/\kappa)^{-(\kappa+1)}}{x - f}, \quad \Im(f) > 0,
\]
and that differs only slightly from the new modified plasma dispersion function

\[ Z_0^0(f) = \frac{1}{\pi^{1/2} \kappa^{1/2}} \frac{\Gamma(\kappa)}{\Gamma(\kappa - \frac{1}{2})} \int_{-\infty}^{\infty} dx \frac{(1 + x^2/\kappa)^{-\kappa}}{x - f}, \quad \Im(f) > 0 \]  

introduced recently in Ref. [2]. It is simple to demonstrate that the kappa dispersion functions from above are related to each other by

\[ Z_0^\kappa(f) = \left(1 + \frac{f^2}{\kappa}\right) Z_\kappa(f) + \frac{f}{\kappa} \left(1 - \frac{1}{2\kappa}\right), \]

and therefore, both of them approach the plasma dispersion function [29] in the limit of \( \kappa \to \infty \). We should also remark that, in this limit, Eq. (8) becomes identical to the dispersion relation of cumulative filamentation-Weibel mode in a counterstreaming plasma with a bi-Maxwellian anisotropic distribution function (see Eq. (8) in Ref. [7]). It is clear now that the general dispersion relation given by (8) and (9), allows us to extend the investigation of purely growing filamentation instability by including the Weibel-type enhancing effects due to the existence of a non-Maxwellian temperature anisotropy in the counterstreaming plasmas.

An analytical expression for the aperiodic solutions, \( \Im(\omega) = \omega_i, \Re(\omega) = 0 \), of the general dispersion relations (8) and (9), can be further obtained only by using the asymptotic forms of the modified plasma dispersion functions provided in Refs. [2] and [21] for very large or very small arguments. Here, we keep the accuracy of a general approach and evaluate exactly the numerical growth rates for several representative cases.

We first assume that the plasma is hotter in the streaming direction, e.g. \( T_y > T \), that means the anisotropy is positive. Thus, the aperiodic solutions of Eq. (8), which correspond to the cumulative filamentation-Weibel mode, are plotted with solid lines in Fig. 2 and Fig. 3 for two different values of the temperature anisotropies, \( A = 1 \) and \( A = 0.5 \), respectively. In Ref. [2] it was shown that the growth rates of the Weibel instability in a thermally anisotropic plasma with a bi-kappa distribution function decrease in comparison to those obtained for the case of a bi-Maxwellian electron distribution function. The Weibel growth rates are plotted here with dashed line. On the other hand, in counterstreaming plasmas with sufficiently large temperature anisotropies we observe the opposite effect: the growth rates of the cumulative filamentation-Weibel instability obtained for bi-kappa anisotropies increase and become much higher than those obtained for counterstreaming plasmas with bi-Maxwellian anisotropies. This is the case in Fig. 2 (a) and Fig. 3 (a), provided a
FIG. 2: The growth rates of the filamentation instability (solid lines) obtained from (8) for two values of relative streaming velocity: (a) $v_0/c = 0.1$ and (b) $v_0/c = 0.05$, and which cumulate the Weibel instability effect due to a bi-kappa temperature anisotropy, $A = 1$. The coordinates are scaled as $W = \omega_i/\omega_{pe}$ and $K = k_c/\omega_{pe}$. For comparison, with dashed lines are shown the growth rates of simply Weibel modes ($v_0 = 0$).

FIG. 3: The same as in Fig. 2 but for a smaller anisotropy $A = 0.5$.

sufficiently large relative streaming velocity, $v_0$, is used. Such behaviour is typical for the simple filamentations instability, which develops in isotropic plasma counterstreams ($A = 0$). In this case, the filamentation instability growth rates are solutions of Eq. (10), and they are shown in Fig. 4 (a) and (b) for two different streaming velocities, $v_0/c = 0.1$ and $v_0/c = 0.05$, respectively.

Otherwise, if the streaming velocity, $v_0$, is small the kappa growth rates are lower than
FIG. 4: The growth rates of the filamentation instability \((A = 0)\) obtained from \([10]\) for two different values of relative streaming velocity: (a) \(v_0/c = 0.1\) and (b) \(v_0/c = 0.05\). On the ordinate and abscissa are \(W = \omega_i/\omega_{pe}\) and \(K = kc/\omega_{pe}\), respectively.

the Maxwellian, and this is shown in Fig. (2) (b).

Due to a finite temperature in both plasma counterstreams the purely growing solutions appear to resemble a Weibel regime and therefore their existence is limited to the small wave numbers less than a cutoff value, \(k \leq k_c\), which is the nontrivial solution of \((8)\) to the limit of \(\omega_i = 0\)

\[
k_c = \frac{\omega}{c} \left[ A + \frac{v_0^2}{\theta^2} \left( 2 - \frac{1}{\kappa} \right) \right]^{1/2}.
\]

Finally, when the contrastreaming plasma is colder in the streaming direction, i.e., \(T_y < T\), the temperature anisotropy becomes negative, \(-1 < A < 0\). In this case, the effective velocity anisotropy imposed to the plasma particles by the relative motion of the counterstreaming plasmas, is reduced, and this explains the lower growth rates obtained in Fig. 5 for the negative temperature anisotropies. For a small streaming velocity, but a sufficiently large negative anisotropy, the instability is suppressed and instead natural damped modes occur. Further improvement of stabilization is observed for those modes corresponding to the larger kappa indices, which approach the Maxwellian modes.

To summarize, we have investigated the properties of purely growing electromagnetic instabilities in an anisotropic counterstreaming plasma and propagates perpendicular to the streaming direction. Specifically, we have considered for the first time the filamentation instability in counterstreaming plasmas with isotropic kappa distributions, and derived the
FIG. 5: The growth rates of the filamentation instability (solid lines) obtained from \((8)\) for negative temperature anisotropies: (a) \(v_0/c = 0.1, A = -0.5\), (b) \(v_0/c = 0.1, A = -0.95\), and (c) \(v_0/c = 0.07, A = -0.5\). On the ordinate and abscissa are \(W = \omega_i/\omega_{pe}\) and \(K = kc/\omega_{pe}\), respectively.

dispersion relation \((10)\), and displayed its numerical solutions in Fig. 5. We found that the aperiodic instability can reach maximum growth rates significantly larger than those calculated for the plasma with a Maxwellian distribution function. This is contrary to the behaviour of the Weibel instability in a bi-kappa plasma, which has growth rates smaller than those obtained for a bi-Maxwellian plasma.

Moreover, we have generalized the theoretical approach assuming that each of the counterstreaming plasmas exhibits a temperature anisotropy of a non-Maxwellian type, namely, a bi-kappa distribution function. In this case, the instability is driven equally by the free energy stored in the relative motion of counterstreaming plasmas and the temperature anisotropy of the plasma particles. Comparing Figures 2 and 3 with Figure 5, we observe that the filamentation instability growth rate is enhanced in the presence of the positive temperature anisotropies, when plasma is hotter in the streaming direction. Higher growth rates are obtained for larger temperature anisotropies. This is the first effect of the so-called cumulative filamentation-Weibel instability, and it improves significantly the efficiency of the magnetic field generation, and provides further support for the potential role of the Weibel-type instabilities in the fast magnetization scenarios, e.g. gamma-ray bursts, etc.

For negative temperature anisotropies the effective velocity anisotropy is reduced, and therefore the growth rates of the filamentation instability decrease, but they remain larger than those for the bi-Maxwellian plasmas. The unstable modes with largest kappa indices can be evenly suppressed. This is the second cumulative effect of the counterstreaming
plasmas and the (negative) temperature anisotropy, which contributes in this case to the stabilization of the plasma system.

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[1] E.S. Weibel, Phys. Rev. Lett. 2, 83 (1959).
[2] S. Zaheer and G. Murtaza, Phys. Plasmas 14, 022108 (2007)
[3] B.D. Fried, Phys. Fluids 2, 337 (1959).
[4] R.C. Davidson, in Handbook of Plasma Physics, edited by M.N. Rosenbluth and R.Z. Sagdeev (North-Holland, Amsterdam, 1983), Vol. 1, p. 519
[5] A. Bret and C. Deutsch, Phys. Plasmas 13, 022110 (2006)
[6] M. Lazar, R. Schlickeiser and P.K. Shukla, Phys. Plasmas 13, 102107 (2006).
[7] A. Stockem and M. Lazar, Phys. Plasmas 15, 014501 (2008).
[8] J.R. Cary, L.E. Thode, D.S. Lemons, M.E. Jones and M.A. Mostrom, Phys. Fluids 24, 1818 (1981).
[9] T.-Y. B. Yang, Y. Gallant, J. Arons and A.B. Langdan, Phys. Fluids B 5, 3369 (1993).
[10] J. Krall and G. Joyce, Phys. Plasmas 2, 1326 (1995).
[11] L.O. Silva, R.A. Fonseca, J.W. Tonge, W.B. Mori, and J.M. Dawson, Phys. Plasmas 9, 2458 (2002).
[12] E.A. Startsvev and R.C. Davidson, Phys. Plasmas 10, 4829 (2003).
[13] R.C. Tautz and R. Schlickeiser, Phys. Plasmas 12, 072101 (2005).
[14] K.F. Lee and J.C. Armstrong, Phys. Rev. A 4, 2087 (1971).
[15] S.S. Aggarwal and S.P. Talwar, Astrophys. Space Sci. 23, 315 (1973); S.S. Aggarwal, Astrophys. Space Sci. 33, 259 (1975)
[16] R.C. Tautz and R. Schlickeiser, Phys. Plasmas 12, 122901 (2005); Phys. Plasmas 13, 062901 (2006).
[17] C.B. Hededal and K.-I. Nishikawa, Astrophys. J. **623**, L89 (2005).
[18] R.C. Tautz and J.-I. Sakai, Phys. Plasmas **14**, 012104 (2007).
[19] V.M. Vasyliunas, J. Geophys. Res. **73**, 2839 (1968).
[20] A.T.Y. Lui and S.M. Krimigis, Geophys. Res. Lett. **10**, 13 (1983).
[21] D. Summers and R.M. Thorne, Phys. Fluids **B** 3, 1835 (1991).
[22] M. Maksimovic, V. Pierrard and J.F. Lemaire, Astron. Astrophys. **324**, 725 (1997).
[23] M.A. Hellberg, R.L. Mace and F. Verheest, in *Waves in Dusty, Solar and Space Plasmas*, *Leuven, 2000*, edited by F. Verheest, M. Goosens, M.A. Hellberg, and R. Bharuthram (AIP, Melville, NY, 2000), p. 348.
[24] M.P. Leubner, J. Geophys. Res. **88**, 469 (1983).
[25] D. Summers and R.M. Thorne, J. Geophys. Res. **97**, 16827 (1992).
[26] R.L. Mace, J. Geophys. Res. **103**, 14643 (1998).
[27] M.A. Hellberg and R.L. Mace, Phys. Plasmas **9**, 1495 (2002).
[28] G. Kalman, C. Montes and D. Quemada, Phys. Fluids 11, 1797 (1968).
[29] B.D. Fried, & S.D. Conte, *The Plasma Dispersion Function*, (Academic Press, New York 1961).