On the QCD dipole content of hard photon and gluon probes

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A gluon forward jet playing the rôle of a deep probe in high energy scattering, we analyze its infinite momentum QCD wave function in terms of dipole (color-singlet $q\bar{q}$) configurations using $k_T$-factorization properties. The comparison is made with virtual photon $q\bar{q}$ configurations. Some implications for hard processes with forward jets at Hera and Tevatron are suggested.

1. Photons and gluons as hard probes

The studies on deep-inelastic scattering experiments by virtual photon probes have emphasized the interest of using the formalism of $q\bar{q}$ wave-functions of the photon in the infinite momentum frame [1]. Indeed, the theoretical predictions for small-$x$ structure functions at the leading-log approximation can be expressed in terms of the probability of finding the $q\bar{q}$ configurations inside the photon probe [2] convoluted with the high-energy dipole-dipole cross-section [3,4]. This is to be compared to the $k_T$-factorization scheme [5] where the photon vertex function is convoluted with the unintegrated off-shell gluon structure function solution of the Balitskii, Fadin, Kuraev, Lipatov (BFKL) equation [6]. Interestingly enough, both schemes can be shown equivalent [7]. More recently, the photon wave-functions proved to be very useful in the investigations of the so-called “hard diffraction” processes characterized by a large rapidity gap between particles produced at the photon and proton vertices at Hera. Indeed, the physical process in which a color singlet $q\bar{q}$ configuration of the photon interacts [7,8] with the proton seems to give an interesting and original insight on the dynamics of “hard diffraction” processes initiated by a photon probe. The interaction of various configurations can also be treated in the dipole model framework [7-9] using the evolution with energy of the initial color singlet $q\bar{q}$ configuration of the photon into a cascade of colorless dipoles.

In other reactions of interest, the hard probe is not furnished by a virtual photon. This is the case at the Tevatron for instance, where an energetic forward jet with high transverse momentum $k_T$ is used as the hard QCD probe in various studies such as the Mueller-Navelet process [10] or “hard diffraction” processes at Tevatron. The gluon jet as a hard probe is also present in forward jet studies at Hera where it allows for a completely perturbative QCD prediction for the hard gluon vs. hard photon scattering at high energy where one looks for a clear signal of the BFKL Pomeron [11-14]. However, if a treatment of the gluon vertex following the $k_T$-factorization scheme is well known, the corresponding formalism using color-singlet $q\bar{q}$ wave-functions has not been either derived or even discussed. In fact, there exist some reasons why it is interesting to discuss a hard probe using this formalism: first, it provides a description of the hard probe in the configuration space. This configuration space is spanned by the variables $\vec{r} \equiv re^{i\phi}$, $z$ where $\vec{r}$ is the transverse vector coordinate between $q$ and $\bar{q}$ and of the energy fraction $z$ (resp. $1-z$) brought by the quark (resp. the antiquark). Second, it is interesting to check the validity and limitations of the vertex factorization properties in configuration space. Third, “hard diffraction” has been formulated using the wave-function formalism and leads to quite non-trivial formulae (including the presence of interference terms [7,9]) which are not yet known for the gluon jet probe, e.g. at the Tevatron.

We present here a report on a new [15] derivation and discussion of the color-singlet dipole content of a gluon jet probe. The starting point of this work starts from the result of the color-singlet
factorization scheme for the photon and its equivalence with the $k$-factorization scheme in deep inelastic scattering. Using a parallel to the photon approach for the color singlet content of a hard gluon jet, the configuration space distributions are derived in terms of a definite combination of squared of Bessel functions depending on the variable $rk_T$, where $k_T$ of the jet gives the hard scale. The validity of the color-singlet scheme is then checked both for the Mueller-Navelet [10] and DIS forward jet formulae including the azimuthal asymmetries [13,14]. The resulting functions can be discussed in terms of absorption of the gluon partial waves in the dipole center-of-mass frame. Some distinctive (w.r.t. a photon probe) features of the resulting states, such as the presence of both virtual and real contributions and the long distance behaviour of the wave-functions are typical features of the gluon jet which may have a significant impact on the phenomenology of hard reactions at high energy initiated by a forward jet probe.

2. Color-singlet versus $k$-factorization at the photon vertex

As mentioned in the introduction, the factorization properties of (resummed at the leading logs) perturbative QCD in the high-energy regime can be put into two equivalent forms [8]. One form is using the $k$-factorization property [9] which relates the $\gamma^*$-dipole cross-section to the product of the impact factors $V_{T,L}$ by a $g^*$-dipole cross-section where $g^*$ is an off-mass-shell gluon ($T,L$ are for the Transverse, Linear polarizations of the photon). The equivalent form is obtained through the photon wave-function in terms of $q\bar{q}$ configurations [9] convoluted with the $q\bar{q}$-dipole cross-section. The target dipole is considered to be small (i.e. massive) in order to justify the (resummed) perturbative QCD calculations. This equivalence is sketched in Fig.1, taking as an example the transverse helicity contributions from the virtual photon.

The light-cone wave functions of the photon for photon helicity $\pm 1,0$ are [10]

$$\Psi^+_T(z,r,Q^2) = C_z \ e^{i\xi} \hat{Q} K_1(\hat{Q}r)$$

$$\Psi^-_T(z,r,Q^2) = C_1 (1-z) \ e^{-i\xi} \hat{Q} K_1(\hat{Q}r)$$

where $K_{0,1}$ are the known Bessel functions of second kind. By definition $\hat{Q} \equiv Q \sqrt{1-z}$ and the normalization is $C^2 = \frac{\alpha_m N_c e_q^2}{4\pi \alpha_s}$ with $e_q$ the quark charge.

For a given target $t$ (e.g. the massive dipole $d$ in Fig.1), the transverse and longitudinal photon-target total cross sections $\sigma^T_{T,L}$ obtained by high-energy QCD in the leading logs approximation read

$$\sigma^\gamma_{T,L}(r,z,Q^2) = \int d^2 r_d dz_1 \times \Phi^T(r,z;Q^2) \Phi'(r_1,z_1;Q^2_1) \sigma_d(r,r_1;Y),$$

where, by definition, $\Phi_T \equiv |\Phi^T|^2 + |\Phi^-|^2$ for the transversely polarized photon, $\Phi_L \equiv |\Phi^L|^2$ for the longitudinally polarized one and $\Phi'(r_1,z_1)$ defines the probability distribution of initial dipoles in-
side the target (e.g. $\delta(r_t-1/Q_t)\delta(z_t-1/2)$ for a massive dipole at scale $Q_t$).

Concerning the $q\bar{q}$ dipole cross section $\sigma_d$, it can be expressed using a Mellin transform and reads 8:

$$\sigma_d(r_t,r_t;Y) = 4\pi \int \frac{d\gamma}{2i\pi} \left(r^2\right)^{1-\gamma} \left(r_t^2\right)^\gamma \times e^{\frac{\alpha_s N_c}{\pi} \chi(\gamma) Y} A_{el}(\gamma), \quad (3)$$

where

$$\chi(\gamma) = 2\Psi(1) - \Psi(\gamma) - \Psi(1-\gamma), \quad (4)$$

stands for the BFKL kernel 9.

$$A_{el}(\gamma) = \frac{\alpha_s^2}{16\gamma^2(1-\gamma)^2} = \alpha_s^2 v(\gamma) v(1-\gamma) \quad (5)$$

is the first order elastic two-gluon exchange amplitude while the factor

$$v(\gamma) = \frac{2^{-2\gamma-1} \Gamma(1-\gamma)}{\gamma \Gamma(1+\gamma)} \quad (6)$$

gives the coupling of the virtual gluon to the massive dipole (related to $\sigma_{\gamma-d}$ in Fig.1 by Mellin transform 10).

In formula 8, $\Phi_T \equiv |\Psi_T|^2 + |\Psi_{\bar{T}}|^2$ (resp. $\Phi_L \equiv |\Psi_L|^2$) are thus the probability distributions of $q\bar{q}$ configurations for transverse (resp. longitudinally) polarized photons. Note that there exists also subdominant BFKL contributions which depend 11 on the interference term $2\Re \{\Psi_T^\dagger \Psi_{\bar{T}}\}$. They play a rôle in the study of azimuthal correlations 13,14 beyond the leading BFKL terms. Introducing the Mellin-transforms of the photon probability distributions defined by:

$$\int \frac{d^2r}{2\pi} (rQ)^{-2\gamma} \int dz \Phi_{T,L}(r,z;Q^2) = \phi_{T,L}(\gamma) \quad (7)$$

and performing the integrations with respect to $r,z$, the equation 8 can be rewritten as

$$\sigma_{T,L}^\gamma = \frac{32\pi^2}{Q^2} \int \frac{d\gamma}{2i\pi} \left(\frac{Q}{Q_t}\right)^{2\gamma} e^{\frac{\alpha_s N_c}{\pi} \chi(\gamma) Y} \times \phi_{T,L}(\gamma) A_{el}(\gamma). \quad (8)$$

This equation is identical to the BFKL expression of the same cross-sections thanks to the following identity 12:

$$A_{el}(\gamma) \cdot \phi_{T,L}(\gamma) = \alpha_em \frac{\alpha_s N_c}{4\pi} v(\gamma) \cdot \frac{V_{T,L}(\gamma)}{\gamma}, \quad (9)$$

where

$$V_T(\gamma) = \frac{\alpha_s}{6\pi} (1+\gamma)(2-\gamma)2^{1-2\gamma} \Gamma(1+\gamma)\Gamma^3(1-\gamma) \quad (10)$$

$$V_L(\gamma) = \frac{V_T(\gamma)}{(1+\gamma)(2-\gamma)} \quad (10)$$

are the $k$-factorized impact factors 13 for transverse and longitudinal photons.

One finally obtains the relation between the color singlet distribution functions and the related impact factors for a virtual photon as:

$$\phi_{T,L}(\gamma) \equiv \alpha_em e^2 \frac{N_c}{4\pi\alpha_s} \frac{V_{T,L}(\gamma)}{\gamma} \frac{1}{\psi(1-\gamma)}. \quad (11)$$

The formula (11) gives the explicit relation between the high energy $k$ factorization and the wave-function formalisms. The (resummed) perturbative QCD cross section can thus be factorized equivalently in two ways: i) by the convolution of the photon gluon cross section times the gluon coupling to the dipole (right hand side), ii) by the probability distribution of a pair of quarks in the photon times the dipole-dipole elementary interaction (left hand side). Note however, that “soft” targets may lead to models differing only through the non perturbative extensions of the two formalisms, see, e.g. 14.

3. The gluon jet probe

In order to determine the color singlet $q\bar{q}$ content of a gluon jet, we shall follow an approach very similar to the previous one for the photon probe. Let us consider now the gluon-target cross-sections instead of the photon-target ones by writing

$$\sigma_{T,L}^g = \int d^2rdz \int d^2r_tdz_t \times \Phi_{T,L}^g(r,z;Q^2) \Phi^d(r_t,z_t;Q_t^2) \sigma_d(r_t,r_t;Y), \quad (12)$$

and using the same equations (8,11), one writes

$$\sigma_{T,L}^g = \frac{32\pi^2}{Q^2} \int \frac{d\gamma}{2i\pi} \left(\frac{Q}{Q_t}\right)^{2\gamma} e^{\frac{\alpha_s N_c}{\pi} \chi(\gamma) Y} \times \phi_{T,L}^g(\gamma) A_{el}(\gamma), \quad (13)$$

where now

$$\int \frac{d^2r}{2\pi} (rQ)^{2(1-\gamma)} \int dz \Phi_{T,L}^g(r,z;Q^2) = \phi_{T,L}^g(\gamma). \quad (14)$$
by definition. The BFKL equation for the same cross-sections is particularly simple since the impact factors of the gluon are just equal to one by definition and the gluon coupling to the dipole is given by formula (13). One thus get the following identities

$$A_d(\gamma) \cdot \Phi_{T,L}^g(\gamma) = \frac{\alpha_s^2}{4\pi} \frac{v(\gamma)}{(1-\gamma)}, \quad (15)$$

where the factor $(1-\gamma)^{-1}$ comes from the integration of transverse momentum scales with lower bound $Q$ at the upper vertex (the unintegrated spectrum, appearing in the BFKL formalism, is expressed without this factor).

Finally, using relation (13) the result reads:

$$\Phi_{T,L}^g(\gamma) = \frac{1}{4\pi} \frac{1}{(1-\gamma)} \frac{1}{v(1-\gamma)}, \quad (16)$$

where the normalization of $\Phi_{T,L}^g(r,z;Q^2)$ is 1 after integration. This formula is the basis of our derivation (17) of the gluon jet probe in terms of colorless $q\bar{q}$ configurations. All in all it consists in the fact that the reformulation of the impact factors of the gluon in terms of dipole configurations generate a definite prediction for the probability distribution of dipoles in a gluon probe.

It can be shown (13) that a parametrization of the probability distributions of dipoles inside the gluon probe can be written as follows:

$$\Phi_T^g(z,r,Q^2) = (z^2 + (1-z)^2) \hat{Q}^2 \phi_T^2(u), \quad (17)$$

$$\Phi_L^g(z,r,Q^2) = 4z(1-z) \hat{Q}^2 \phi_L^2(u), \quad (18)$$

where the functions $\phi_T^2$ and $\phi_L^2$ describe the configuration space features of the color singlet dipoles in the gluon probe and $u \equiv \hat{Q}r$. These functions can be derived and lead to an interpretation in terms of quantum states and wave-functions of these dipoles. Indeed, as discussed in [13], the parametrization (18) reflects the fact that the first-created dipole in the ordered in rapidity cascade of the QCD dipole model [13] brings almost the whole energy and shares approximately the same energy between its quark $z$ and antiquark $1-z$, than the quark and antiquark directly coupled to the incident gluon, while the color had flown away. From this assumption (18) and from the equation (10), and after some algebra [13], it is not too difficult to find the following solutions (valid within some approximation [13])

$$\Phi_T^g(z,r,Q^2) = \frac{2}{\pi} (z^2 + (1-z)^2) \hat{Q}^2$$

$$\times \left\{ \frac{J_1^2(u)}{(u)^2} + J_0^2(u) - J_1^2(u) - J_2^2(u) \right\}$$

$$\Phi_L^g(z,r,Q^2) = \frac{6}{\pi} z(1-z) \hat{Q}^2$$

$$\times \left\{ \frac{J_1^2(u)}{(u)^2} + J_0^2(u) - \frac{2}{3} J_1^2(u) - \frac{1}{3} J_2^2(u) \right\}, (19)$$

where $J_{0,1,2}$ are the usual Bessel functions of first kind.

4. gluon versus photon probes

The resulting formulae (11) call for some comments when compared to the photon wave functions (1) and may have interesting implications for hard processes with forward jets.

i) The dependence on the parameter $u$ is much softer (power-like) for the gluon than for the photon (exponential). Thus, the size fluctuations of the dipoles in configuration space for a given scale $\hat{Q}$ are expected to be more important.

ii) In the expression of the distributions $\Phi_{T,L}^g$, there are a terms with both signs. Thus, contrary to the photon case, there cannot be interpreted as probability distributions. Indeed, a formulation in terms of real and virtual intermediate dipole states can match with formulae (11). It amounts to consider that there are initial colorfull states in the description of the gluon probe, since the gluon is not locally connected with a colorless state as is the photon. In practice (13) this affects only the tail of the distributions in the variable $u$ and one satisfies the positivity condition on the observable cross-sections after integration.

Both points i,ii) may have interesting applications to the phenomenology of hard forward jets. On the one hand, it is expected to give specific predictions to the leading BFKL prediction for “hard scattering” cross-sections, since these predictions (especially for the small mass component (17)) depend quite strongly on the projectile wave functions in the configuration space. On
the other hand, the next-leading and non perturbative corrections to the BFKL prediction could be different for the gluon w.r.t. the photon due to the larger size fluctuations obtained in the former case. This clearly requires further interesting studies.

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Lev Lipatov (St.Petersburg Nuclear Physics Institute, Russia):
Do you consider the virtual gluon cross-section in the framework of the $k_T$-factorization, where the initial particle really is the reggeized gluon?

R. P.:
No, I consider a real gluon jet with large $k_T$ as for instance in the case of Mueller-Navelet jets [10], and thus the ordinary renormalization-group factorization at short distance.

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