Transverse modulation of the positron beam density by using the laser standing wave

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Abstract. Recently it was shown that charged particles motion in the field of standing electromagnetic wave can undergo the features similar to the particles channeling in crystals. When a charged particle enters the channels formed by electromagnetic standing waves at a small angle to the node (anti-node) planes its motion represents namely the oscillations between two neighboring planes. The phenomenon is mostly known as channeling in a lattice of the standing waves. Obviously, this effect can be used to handle beams in accelerator physics, more general, for the beam shaping with the specific properties. The advantage of the plane wave channeling is the absence of inelastic scattering that takes place in a crystal. The possibility to re-distribute the current density of particles in the beam by means of the laser standing wave is demonstrated.

1. Introduction

The recent achievements in laser techniques open new areas for investigations in many fields of research. In particular, high intensity laser waves can find their applications in accelerator physics for the beam shaping: the beam deflection, focusing, micro-bunching etc (see, for example, in [1]). In [2] it was shown the standing laser wave could be also used for beam managing due to the channeling effect. The channeling in the laser standing wave has the similar nature as the famous channeling in crystals (see, for example, in [3,4] and references therein). As example of modern application of the channeling one can point out beam channeling in a bent crystal (see, for example, in [5,7] and references therein), when channelled projectiles follow the channel along bent crystallographic planes and, hence, they can be deflected outward the beam core. This scheme is suggested for beam collimation in modern hadron accelerators. In principle, the system of standing wave channels could be used instead of crystal channeling in all possible applications. The use of the laser standing wave has some evident advantages. Scattering of the beam by a crystal matter as well as the nuclear interactions between particles of the beam and crystal nuclei are also absent.

First, in the manuscript the short theoretical overview of the channeling in a standing wave is given. Further we investigate the evolution of the beam phase space to demonstrate the possibility of the beam focusing by the laser standing wave. After that the possible application of the focusing for the beam current re-distribution is analyzed. The simulations are carried out for 400 MeV positron beam (positron beam energy available at LNF INFN [7]). The laser has the wavelength \(\lambda = 800\) nm and the intensity \(I = 10^{17}\) W/cm\(^2\). It will be shown such
intensity produces the potential barriers which are close to those in crystals at charged particles channeling. At the same time, the transverse space dimension of a laser channel, $a = \lambda/2$, is a few orders bigger than the width of a crystal channel.

2. Channeling in electromagnetic standing wave

Let consider the standing wave formed by two linear polarized counterpropagating (along the $X$-direction, see in figure 1) plane waves. The nodes (antinodes) of a standing wave form planes that are parallel to the YZ-plane. The bunch of relativistic charged particles moves by the way when the reference orbit coincides with the $Z$-direction. Particles penetrate into the wave field at small angles to the node (antinode) planes. In this case the particle motion could be described using averaged ponderomotive potential. Here we follow the work [2]. The ponderomotive forces of the standing laser wave acting on the projectile were considered also in [8,9]. The ponderomotive potential forms the channels along the $Z$-direction whereas in the $XZ$-cross-section the periodic cos-like potential takes place. The ponderomotive potential defines the smooth oscillating trajectory similar to the particle trajectory at crystal channeling. Namely, the channeled particle moves in the $XZ$-plane along the channel and oscillates between two neighboring node planes (or between antinode planes, this depends on the particle’s velocity, see [2] for details). So, here we will not consider the motion in the $Y$-direction (orthogonal to the plane of figure 1). The small angle of particle incidence in the field with respect to the $Z$-axis is denoted by $\theta_0$. The difference between real interactions and the interaction described by the averaged potential leads to small rapid oscillations along the smooth trajectory. These small oscillations are not considered in this work.

**Figure 1.** The geometry of computer experiment. The standing wave (b) is formed by two counterpropagating laser waves. The bunch of particles penetrates into the field along the $Z$-axis and particles enter the field at small glancing angle $\theta_0$ to node planes (that are parallel to $Z$-axis and simultaneously are orthogonal to the figure plane). The ponderomotive potential forms cos-like structure in transverse $X$-direction with the wall $2|U_{am}|$. Possible types of motion are: 1 — channeling, oscillations within single channel between node lines, 2 — quasichanneling, particle can cross channel borders.
For the case of linear polarized counterpropagating waves the averaged potential has the form

\[ U(x) = -U_{am} \cos(2kx), \]

(1)

where the potential amplitude is defined by the expression (here the projectile with the charge number \( \pm 1 \) is assumed):

\[ U_{am} = \frac{4\pi\alpha h I}{\omega^2 \gamma_z m} \left( 2\beta_z^2 - 1 \right). \]

(2)

In these expressions \( v_z = \beta_z c \) is the longitudinal velocity of the particle (in the task considered here \( v_z = \text{const} \)), \( c \) is the speed of light, \( \gamma_z = (1 - \beta_z^2)^{-1/2} \), \( m \) is the positron’s mass, \( \alpha \) is the fine structure constant, \( h \) is the Planck constant, \( \omega = 2\pi c/\lambda \) is the angular frequency of the laser wave, \( k = \omega/c \) is the wave number. The amplitude (2) defines the barrier \( 2|U_{am}| \) of ponderomotive potential as shown in figure 1(c).

As in the case of channeling in a crystal, we can define the critical angle of projectile’s penetration in the field

\[ \theta_{lim} = \sqrt{\frac{4|U_{am}|}{\gamma \beta^2 mc^2}} \]

(we write the general expression which can be used at any energies). The channeling motion is impossible if \( \theta_0 > \theta_{lim} \). Nevertheless the particle’s motion is still governed by averaged potential while \( \theta_0 \) is of the order of a few \( \theta_{lim} \). This type of motion is known as the quasichanneling regime, when the particle can cross channel borders (see samples of trajectories in figure 1(b)).

In general case the beam has the nonzero divergence, and both channeling and quasichanneling take place at the beam passage through the electromagnetic field. In this work we deal with non-divergent beam where all particles are captured in the channeling regime of motion.

The parameters of channeling for the case considered here are: \( \gamma = 781, U_{am} = 15.3 \) eV, \( \theta_{lim} = 0.4 \) mrad. Another important parameter is the oscillation length \( l_{osc} \) that is the length along the channel when a particle makes the complete transverse oscillation. The potential is not harmonic. Hence, this length depends on the initial particle transverse position in the channel (see in figure 2). For the aims of estimation it is convenient to evaluate \( L_{osc} = l_{osc}(0) \) for a particle initially penetrating into the field along the Z-axis near the channel center. For 400 MeV positrons this estimation gives \( L_{osc} = 2.0 \) mm. The figure 2 demonstrates the oscillation length \( l_{osc} \) increases rapidly when the initial transverse coordinate approaches \( a/2 \). Nevertheless, particles with initial transverse coordinates within the range \((-0.37a, 0.37a)\) (i.e. about of 70\% of particles in the beam) have \( l_{osc} < 1.5L_{osc} \). Therefore, the parameter \( L_{osc} \) is the characteristic parameter that defines the longitudinal dimension of the standing wave lattice for the observation of channeling-related effects. Namely, it defines the channel length when the effects of beam focusing could be observable. It should be noticed the dimensions of the lattice formed by the laser standing wave is a few order bigger than the crystal lattice. Hence, in principle, some thin features in the beam density re-distribution which are not observable by using the crystal channeling could be found with the standing wave lattice.

### 3. Evolution of the transverse phase space with the penetration depth

The idealized positron beam is considered in this section to demonstrate the main features of the standing wave channeling. The beam has no divergence, all particles penetrate into the field along the Z-axis. The transverse beam dimension coincides with the channel width \( a \). figure 3 demonstrates the transverse phase space evolution in dependence on the penetration depth \( L \). In figure 3 \( x \) is the transverse coordinate. The angle between velocity \( v \) and Z-axis is used instead of transverse velocity \( v_x \) as the parameter conjugate with the coordinate \( x \). All particles are channeled.
As one can see from figure 3 the beam shape could be controlled by the varying of the field length $L$. One of the interesting task of the accelerator physics is the possibility of beam focusing by a standing wave. If the potential will be harmonic, this beam is precisely focused at distances $L = 0.25L_{osc}$, $L = 0.75L_{osc}$, $L = 1.25L_{osc}$ etc. In the case of potential (1) the precise focusing is, of course, not possible. The best conditions for the beam focusing is at $L = 0.25L_{OSC}$. After that, at $L = 0.75L_{osc}$, $L = 1.25L_{osc}$ and at successive "focused points" the defocusing takes place because of the potential inharmonicity.

In general, the evolution of the transverse phase space can be explained as follows. The initial beam is almost space-uniform and all particles have the same, zero, transverse velocity as shown in the panel (a). The phase points in the channel center and at the channel borders are stationary (see equation (1) and figure 1(c)). So, the line shown in the panel (a) is curving to keep its continuity and to keep the stationary points fixed while the beam propagates. During the beam motion the line is whirling around the channel center and it forms two-branch helix. For both the space and transverse velocity distributions this leads to the forming of peak in the center of distribution with the periodicity $L = 0.5L_{osc}$. The peak for the space distribution means the focusing penetration depth. The peak for the transverse velocity distribution means the minimal divergence at this moment. In general, when the beam is mostly focused, at this moment the beam demonstrates the maximal divergence (as in panel (b)), and vice versa, when the beam obtains the minimal divergence its transverse space tends to become uniform (see in panel (c)). The central peaks is smeared while the beam propagates. Finally, at $L \gg L_{osc}$ (the panel (d)) the beam shape does not depend on $L$. So, the best conditions for beam handling are provided at $L \leq L_{osc}$.

4. Transverse modulation of the beam current density
From figure 3(b) one can conclude there is a possibility to produce the kind of modulation in transverse beam cross-section if the beam is much wider than the channel width $a$. The channel width $a$ defines the modulation dimension. Of course, this transverse modulation should take place also for crystal channeling, but the modulation dimension in that case is about of several Å, and it is too small to be observed. Moreover, scattering in the crystal target could prevent appearance of the modulation. On the contrary, for the case of laser standing wave the transverse modulation, perhaps, can be resolved by existing detectors; it depends on the laser wavelength $\lambda$ used.

To simulate the transverse modulation the non-divergent Gaussian-shaped beam with the RMS width $\sigma_x = 5a$ was considered. The field length $L = 0.25L_{osc}$ was chosen to satisfy the best focusing condition for every channel. Results of simulations presented in figure 4 confirm the suggestion on the beam transverse modulation. Every channel gives the one peak of transverse
Figure 3. The transverse phase space evolution while the initially non-divergent beam propagates in the standing wave field. Panels demonstrate the phase space at different penetration depth $L$. The left pictures are the phase space, the central pictures are space distribution, the right pictures are the angular distributions. Following from the top picture to bottom one can trace the evolution of phase space, space distribution and angular distribution correspondingly. (a) Initial beam, (b) $L = 0.25L_{\text{osc}}$, (c) $L = 0.5L_{\text{osc}}$, (d) $L = 2\text{ cm}$. 

\[ \text{Angle } \phi = \arctan(v_x/v_z), \text{ mrad} \]
beam density, so the beam splitting is evident. The full beam width remains the same as before the standing wave field. Hence, the peak beam intensity is higher than the maximal beam intensity before the field (the intensity increase is about of 50%). It should be mentioned the modulation might be quickly dissolved after the field because at $L = 0.25L_{osc}$ the beam obtains a strong divergence (see in figure 3(b), this figure presents also the angular distribution of particles for the case considered in this section).

Now to obtain the transverse modulation of the beam the mask with the set of slits is used [10]. The mask leads to the significant particle loss (95% in [10], for example) whereas the suggested method simply re-distributes particles in the beam. The significant loss takes place because usually the distance between slits (1 mm in [10]) is several orders bigger than the slit width (50 um in [10]). The channel width in the scheme considered in the paper is defined by the laser wave length. It could be made significantly less than the slit width. The space between channels is absent. These features could make the plane wave channeling attractive for certain applications in comparison with masks. However, the plane wave channeling could increase the beam divergence, as shown in figure 3.

5. Conclusion
The main goal of this work was to demonstrate the possible applications of the beam channeling in the standing laser wave for the beam shaping tasks. It was pointed out that this effect can have some possible advantages in comparison with the crystal channeling. It was shown the potential wall of the standing wave lattice can get the same values as one of the crystal lattice. Hence, the laser standing wave channeling is effective such as the crystal channeling. Simultaneously, the channel width of the standing wave is few order bigger than the channel width of the crystal channel. Also, the scattering of particles by the solid target is absent in the case of the standing wave lattice. Therefore, the channeling in the standing wave can open new possibilities for the beam shaping that are inaccessible for the case of crystal channeling. Moreover, the use of the tunable laser sources enables the ”dynamic” beam shaping during the bunch passage through the field.

This work deals with the light particles channeling as well as its application for the beam handling. The positron beam is considered here but the ponderomotive potential (1) does not
depend on the charge sign. Hence, there is no difference in the motion of electron or positron beams at the same energies. The examples with simple model beams demonstrate the beam focusing as well as the transverse modulation of the beam current density. Nevertheless, the problem of particle’s radiation influence [2] on the feasibility of light particles beam handling by the electromagnetic standing wave needs additional investigation.

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