Physical models within the framework of the Randall-Sundrum scenario

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Abstract

The Randall-Sundrum scenario with non-factorizable geometry and fifth dimension $y$ being an orbifold, is studied. It has two branes located at fixed points of the orbifold. The four-dimensional metric is multiplied by a warp factor $\exp[\sigma(y)]$. Recently, a new general expression for $\sigma(y)$ was derived which has the orbifold symmetry $y \rightarrow -y$ and reproduces jumps of its derivative on both branes. It has an explicit symmetry with respect to the branes. The function $\sigma(y)$ is determined by the Einstein-Hilbert’s equations up to a constant $C$. In the present paper we demonstrate that different values of $C$ result in quite different physical schemes. Three schemes are considered, among which are: (i) the RS1 model; (ii) the RSSC model with a small curvature of the five-dimensional space-time; (iii) the “symmetric” scheme with $C = 0$. The latter scenario is studied in detail.

1 Introduction

The Randall-Sundrum (RS) scenario [1] is a framework with one extra dimension in a slice of the AdS$_5$ space-time restricted by two 3D branes. Contrary to the ADD model [2]-[4], it solves the hierarchy problem due to the higher-dimensional curvature rather than the volume of the extra dimension.

The RS1 model [1] predicts an existence of heavy Kaluza-Klein (KK) excitations (massive graviton resonances) with the lightest mass around few
TeV. The phenomenological implications of the RS1 model were explored in ref. [5] and next publications. For the time being, graviton resonances are intensively searched for at the LHC (see, for instance, [6], [7]).

Our main goal is to show that the RS scenario admits a variety of models with different spectra of the KK gravitons, and, consequently, diverse collider phenomenology. In fact, the RS1 model is a particular physical scheme within the framework of the RS scenario. The interesting scheme (RSSC model) is realized when a curvature of the five-dimensional space-time is much smaller than the 5-dimensional Planck scale [8]-[10]. It predicts a continuous spectra of the KK gravitons. In the present Letter we study in detail the other scheme which was not yet considered by other authors. All the schemes has quite different experimental signature in searching for warped extra dimension.

2 General solution for the warp factor

The RS scenario is described by the AdS$_5$ background warped metric of the form

$$ ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2 , $$

where $\eta_{\mu\nu}$ is the Minkowski tensor with the signature $(+, -, -, -)$, and $y$ is the 5-th coordinate. It is assumed that the periodicity condition $y = y + 2\pi r_c$ is imposed and the points $(x, y)$ and $(x, -y)$ are identified. As a result, we get the orbifold $S^1/Z_2$. Note, that this orbifold can be described as two non-trivial $Z_2$ reflections, one which acts around $y = 0$ and the other around $y = \pi r_c$ [11]. After orbifolding, the coordinate $y$ varies within the limits $0 \leq y \leq \pi r_c$. We consider the scenario with two 3D branes located at the fixed points $y = 0$ and $y = \pi r_c$. The SM fields are constrained to the second of these branes (TeV brane).

The classical action of the Randall-Sundrum scenario [1] is given by

$$ S = \int d^4x \int dy \sqrt{G} \left( 2\tilde{M}_5^3 R - \Lambda \right) + \int d^4x \sqrt{\left| g^{(1)} \right|} (\mathcal{L}_1 - \Lambda_1) + \int d^4x \sqrt{\left| g^{(2)} \right|} (\mathcal{L}_2 - \Lambda_2) , $$

where $G_{MN}(x, y)$ is the 5-dimensional metric, with $M, N = 0, 1, 2, 3, 4$, $\mu = 0, 1, 2, 3$. The quantities

$$ g^{(1)}_{\mu\nu}(x) = G_{\mu\nu}(x, y = 0) , \quad g^{(2)}_{\mu\nu}(x) = G_{\mu\nu}(x, y = \pi r_c) $$


are induced metrics on the branes, \( \mathcal{L}_1 \) and \( \mathcal{L}_2 \) are brane Lagrangians, \( G = \det(G_{MN}) \), \( g^{(i)} = \det(g_{\mu\nu}^{(i)}) \). Here and below the reduced mass scales are used:

\[
\bar{M}_{\text{Pl}} = M_{\text{Pl}}/\sqrt{8\pi} \simeq 2.4 \cdot 10^{18} \text{ GeV}, \quad \bar{M}_5 = M_5/(2\pi)^{1/3} \simeq 0.54 \bar{M}_5.
\]

From action (2) 5-dimensional Einstein-Hilbert’s equations follow

\[
\sqrt{|G|} \left( R_{MN} - \frac{1}{2} G_{MN}R \right) = -\frac{1}{4\bar{M}_5^3} \left[ \sqrt{|G|} G_{MN} \Lambda + \sqrt{|g^{(1)}|} g_{\mu\nu}^{(1)} \delta_M^\mu \delta_N^\nu \delta(y) \Lambda_1 + \sqrt{|g^{(2)}|} g_{\mu\nu}^{(2)} \delta_M^\mu \delta_N^\nu \delta(y - \pi r_c) \Lambda_2 \right].
\] (4)

For the 5-dimensional background metric (1), the Einstein-Hilbert’s equations are reduced to the following set of equations:

\[
6 \sigma'^2(y) = -\frac{\Lambda}{4\bar{M}_5^3},
\] (5)

\[
3 \sigma''(y) = \frac{1}{4\bar{M}_5^3} \left[ \Lambda_1 \delta(y) + \Lambda_2 \delta(\pi r_c - y) \right].
\] (6)

As usual, we ignore the backreaction of the brane terms on the space-time geometry.

The first solution of these equations was presented by Randall and Sundrum [1],

\[
\sigma(y) = \kappa |y|,
\] (7)

with the fine tuning relations

\[
\Lambda = -24\bar{M}_5^3 \kappa^2,
\] (8)

\[
\Lambda_1 = -\Lambda_2 = 24\bar{M}_5^3 \kappa.
\] (9)

The parameter \( \kappa \) defines the magnitude of the 5-dimensional scalar curvature \( R^{(5)} \) in the region \( 0 < y < \pi r_c \), where \( R^{(5)} = -20\kappa^2 \). The solution (7) is consistent with the orbifold symmetry \( y \to -y \). But it reproduces a jump of \( \sigma'(y) \) on one of the branes only. In other words, it is not symmetric with respect to the branes.

Later on, an explicit expression which makes the jumps of \( \sigma'(y) \) on both branes was given in [12],

\[
\sigma(y) = \kappa \{ y[2 \theta(y) - 1] - 2(y - \pi r_c) \theta(y - \pi r_c) \} + \text{constant}.
\] (10)

\(^1\)Here and in what follows, the prime denotes the derivative with respect to variable \( y \).
However, this expression is neither symmetric in variable $y$ nor invariant with respect to an interchange of the branes.

Recently, a general solution of eqs. (5), (6) was derived in refs. [13], [14],

$$\sigma(y) = \frac{\kappa}{2}(|y| - |y - \pi r_c|) + C, \quad (11)$$

where the constant $C$ can depend on a modulus of $\kappa$. The 5-dimensional and brane cosmological constants were found to be

$$\Lambda = -6\bar{M}_5^2\kappa^2[\varepsilon(y) - \varepsilon(y - \pi r_c)]^2, \quad (12)$$

$$\Lambda_1 = -\Lambda_2 = 12\bar{M}_5^3\kappa, \quad (13)$$

where $\varepsilon(x) = \theta(x) - \theta(-x)$. Note that the cosmological constant $\Lambda$ has discontinuities at the fixed points $y = 0$ and $y = \pi r_c$ [12]. The same is true for $\sigma'(y)$, while $\sigma''(y)$ is unambiguously defined.

This solution (11) has the following properties: (i) it has the orbifold symmetry $y \rightarrow -y$; (ii) the jumps of $\sigma'(y)$ are correctly reproduced on both branes; (iii) it is symmetric with respect to the interchange of the branes.

Indeed, $\sigma(y)$ (11) is symmetric in variable $y$ due to the periodicity condition which says that the points $y - \pi r_c$ and $y + \pi r_c$ are identified. It is also symmetric with respect to the branes, since under the replacement $y \rightarrow \pi r_c - y$, the positions of the branes are interchanged (the fixed point $y = 0$ becomes the fixed point $y = \pi r_c$, and vice versa), while under the replacement $\kappa \rightarrow -\kappa$ their tensions (13) are interchanged. Note that the original RS solution (7) does not match conditions (ii) and (iii).

### 3 Physical models in the RS scenario

In this section we will demonstrate that not only the brane warp factors, but hierarchy relations and graviton mass spectra depend drastically on a particular value of the constant $C$ in eq. (11). Correspondingly, the parameters of the model, $\bar{M}_5$ and $\kappa$, differ significantly for different $C$.

Let us define

$$\sigma_1 = \sigma(0), \quad \sigma_2 = \sigma(\pi r_c). \quad (14)$$

It follows from (11) that

$$\Delta \sigma \equiv \sigma_2 - \sigma_1 = \kappa \pi r_c. \quad (15)$$
The r.h.s of the hierarchy relations,
\[ M_{Pl}^2 = \frac{\bar{M}_5^3}{\kappa} e^{-2\sigma_1} (1 - e^{-2\Delta \sigma}) \text{ \bigg|}_{\pi r_c \gg 1} \simeq \frac{\bar{M}_5^3}{\kappa} e^{-2\sigma_1} , \]
depends on the size of the extra dimension except for the case \( C = \kappa \pi r_c / 2 \).

There exist relations between physical parameters which look the same for any \( C \). The masses of the KK graviton excitations \( h_{\mu \nu}^{(n)} \) on the TeV brane are defined from the equation
\[ J_1(a_{1n}) Y_1(a_{2n}) - Y_1(a_{1n}) J_1(a_{2n}) = 0 , \quad n = 1, 2, \ldots \]  
where \( J_\nu(x), Y_\nu(x) \) are Bessel functions, and \( a_{in} = (m_n/\kappa) \exp(\sigma_i) \). As a result, we get
\[ m_n = x_n \left( \frac{\kappa}{M_5} \right)^{3/2} \frac{\bar{M}_{Pl}}{\sqrt{e^{2\Delta \sigma} - 1}} \text{ \bigg|}_{\pi r_c \gg 1} \simeq x_n \left( \frac{\kappa}{M_5} \right)^{3/2} \bar{M}_{Pl} e^{-\pi \kappa r_c} , \]  
for all \( m_n \ll \bar{M}_{Pl}(\kappa/\bar{M}_5)^{3/2} \). Here \( x_n \) are zeros of the Bessel function \( J_1(x) \).

The interactions of the gravitons with the SM fields on the TeV brane are described by the effective Lagrangian
\[ \mathcal{L}_{\text{int}} = -\frac{1}{\bar{M}_{Pl}} h_{\mu \nu}^{(0)}(x) T_{\alpha \beta}(x) \eta^{\mu \alpha} \eta^{\nu \beta} - \frac{1}{\Lambda_\pi} \sum_{n=1}^{\infty} h_{\mu \nu}^{(n)}(x) T_{\alpha \beta}(x) \eta^{\mu \alpha} \eta^{\nu \beta} , \]  
were \( T^{\mu \nu}(x) \) is the energy-momentum tensor of the SM fields, \( h_{\mu \nu}^{(0)} \) is the field of the massless graviton. The coupling constant of the KK gravitons is
\[ \Lambda_\pi \simeq \frac{\bar{M}_{Pl}}{\sqrt{e^{2\Delta \sigma} - 1}} \text{ \bigg|}_{\pi r_c \gg 1} \simeq \bar{M}_{Pl} e^{-\pi \kappa r_c} . \]

The very expressions (18), (20) do not depend on \( C \). Nevertheless, by taking different values of \( C \) in (11), we come to quite diverse physical scenarios. To demonstrate this, let us consider the following three cases. From now on, it will be assumed that \( \kappa > 0 \) and \( \pi \kappa r_c \gg 1 \).

1. \( C = \kappa \pi r_c / 2 \). Then we get from (11)
\[ \sigma(y) = \frac{\kappa}{2} (|y| - |y - \pi r_c| + \pi r_c) . \]
It means that $\sigma_1 = 0$, $\sigma_2 = \kappa \pi r_c$. The hierarchy relation looks like [1]

$$\bar{M}_{P1}^2 = \frac{M_5^3}{\kappa} \left(1 - e^{-2\kappa \pi r_c}\right) \simeq \frac{M_5^3}{\kappa}.$$  \hspace{1cm} (22)

It requires $\bar{M}_5 \simeq \kappa \simeq \bar{M}_{P1}$. Then the masses of the KK excitations are defined by eq. (18) to be

$$m_n \simeq x_n \bar{M}_{P1} e^{-\kappa \pi r_c} \simeq x_n \kappa e^{-\kappa \pi r_c}.$$ \hspace{1cm} (23)

Thus, the original RS1 model [1] is realized in this case. The KK spectrum of the model is a set of heavy resonances with the lightest one around few TeV, if $\Lambda_\pi$ is chosen to be about one TeV.

2. $C = 0$. This scheme was not yet considered by other authors. The symmetry between the branes become very clear if a new variable $z = y - \pi r_c/2$ is introduced, that results in

$$\sigma(z) = \frac{\kappa}{2} \left(\left|\frac{\pi r_c}{2} + z \right| - \left|\frac{\pi r_c}{2} - z \right|\right).$$ \hspace{1cm} (24)

Note, that $\sigma_1 = -\sigma_2 = -\kappa \pi r_c/2$. According to eq. (16), the hierarchy relation is given by

$$\bar{M}_{P1}^2 = \frac{2M_5^3}{\kappa} \sinh(\pi \kappa r_c),$$ \hspace{1cm} (25)

while the masses of the KK excitations are

$$m_n \simeq x_n \kappa e^{-\kappa \pi r_c/2}.$$ \hspace{1cm} (26)

As for the coupling constant of the massive gravitons (20), it looks like

$$\Lambda_\pi = \frac{M_5^3}{\kappa \bar{M}_{P1}}.$$ \hspace{1cm} (27)

Then the hierarchy relation (25) can be rewritten as $\bar{M}_{P1} = 2\Lambda_\pi \sinh(\pi \kappa r_c)$.

Two different physical frameworks can be considered within scheme 2: 2a) $\bar{M}_5 = \kappa$. Then, according to (27), the parameters of the model have the following large values

$$\bar{M}_5 \simeq \kappa \simeq 4.9 \cdot 10^{10} \left(\frac{\Lambda_\pi}{\text{TeV}}\right)^{1/2} \text{GeV}.$$ \hspace{1cm} (28)

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[2] Under simultaneous replacements $(z \rightarrow -z, \kappa \rightarrow -\kappa)$, one gets $\sigma_1 \leftrightarrow \sigma_2$ and $\sigma \rightarrow -\sigma$. 

6
The graviton masses are given by

\[ m_n = x_n \Lambda_\pi . \] (29)

For \( \Lambda_\pi = 1 \text{ TeV} \), one gets from the hierarchy relation (43) that \( \kappa r_c \simeq 11.28 \). Thus, this scenario leads to a model with heavy graviton resonances, as in the RS1 model.

2b) \( \bar{M}_5 \gg \kappa \). This case leads to quite different collider phenomenology. For instance, suppose that \( \bar{M}_5 = 2 \cdot 10^9 \text{ GeV} \) and \( \kappa = 10^4 \text{ GeV} \). Then we get from eq. (27) that \( \Lambda_\pi \simeq 3.3 \cdot 10^5 \text{ GeV} \). The KK gravitons with the masses

\[ m_n = x_n \kappa \left( \frac{\Lambda_\pi}{M_{Pl}} \right)^{1/2} \] (30)

form almost continuous spectrum,

\[ m_n \simeq 3.7 x_n \text{ MeV} . \] (31)

Note that \( \kappa r_c \simeq 9.43 \) in this case.

The warp extra dimension can be searched for in the processes mediated by \( s \)-channel exchanges of the KK gravitons.\(^3\) The universal part of their matrix elements is defined by the sum [10]

\[ S(s) = \frac{1}{\Lambda_\pi^2} \sum_{n=1}^{\infty} \frac{1}{1 - s - m_n^2 + i m_n \Gamma_n} , \] (32)

where \( \Gamma_n \simeq \eta m_n^3/\Lambda_\pi^2 \) (with \( \eta = 0.09 \)) denotes the total width of the graviton with the KK number \( n \) and mass \( m_n \). By doing calculations analogous to those from ref. [10], with the use of relation [13], we obtain

\[ S(s) = -\frac{\bar{M}_5^3}{2\kappa^3\Lambda_\pi^4} \frac{1}{\sqrt{1 - 4i\eta s \Lambda_\pi^2}} \left[ \frac{J_2(z_1)}{z_1 J_1(z_1)} - \frac{J_2(z_2)}{z_2 J_1(z_2)} \right], \] (33)

where

\[ z_{1,2}^2 = \frac{1}{2i\eta} \left( \frac{\bar{M}_5}{\kappa} \right)^3 \left[ 1 \mp \sqrt{1 - 4i\eta s \Lambda_\pi^2} \right] . \] (34)

\(^3\)For instance, \( aa \to G^{(n)} \to b\bar{b} \), where \( a/b = l, g, q, g \), and \( G^{(n)} \) is the KK graviton.
Taking into account that $|z_2| \gg |z_1| \gg 1$, we get

$$S(s) \simeq -\frac{1}{2\Lambda_\pi^3 \sqrt{s}} \left( \frac{\bar{M}_5}{\kappa} \right)^{3/2} \frac{J_2(z_1)}{J_1(z_1)} ,$$  \hspace{1cm} (35)

where

$$z_1 \simeq \left( \frac{\bar{M}_5}{\kappa} \right)^{3/2} \frac{\sqrt{s}}{\Lambda_\pi} \left[ 1 + \frac{i\eta}{2} \left( \frac{\sqrt{s}}{\Lambda_\pi} \right)^2 \right] .$$  \hspace{1cm} (36)

Thus, in spite of the large value of the coupling constant $\Lambda_\pi \simeq 330$ TeV, we come to a TeV physics, since

$$|S(s)| = \frac{F}{(1\text{TeV})^3 \sqrt{s}} ,$$  \hspace{1cm} (37)

where $F = O(1)$ for our values of the parameters $\bar{M}_5$, $\kappa$ and $\Lambda_\pi$.

By using asymptotic behavior of the Bessel functions, we obtain

$$S(s) = -\frac{1}{4\Lambda_\pi^3 \sqrt{s}} \left( \frac{\bar{M}_5}{\kappa} \right)^{3/2} \frac{\sin 2A + i \sinh(2\varepsilon)}{\cos^2 A + \sinh^2 \varepsilon} ,$$  \hspace{1cm} (38)

where

$$A = \sqrt{s} \left( \frac{\bar{M}_5}{\kappa} \right)^{3/2} , \quad \varepsilon = \frac{\eta}{2} \left( \frac{\sqrt{s}}{\Lambda_\pi} \right)^3 \left( \frac{\bar{M}_5}{\kappa} \right)^{3/2} .$$  \hspace{1cm} (39)

For the chosen values of $\bar{M}_5$ and $\kappa$, we find that

$$A \simeq 2.7 \cdot 10^5 \left( \frac{\sqrt{s}}{\text{TeV}} \right) , \quad \varepsilon \simeq 0.1 \left( \frac{\sqrt{s}}{\text{TeV}} \right)^3 .$$  \hspace{1cm} (40)

If an effective energy of colliding partons at the LHC $\hat{s}$ is large enough, namely, $\sqrt{\hat{s}} \gtrsim 2.8$ TeV, eq. (38) can be significantly simplified.\footnote{At $\sqrt{s} = 2.8$ TeV, one gets $\sinh(2\varepsilon) \simeq 2 \sinh^2 \varepsilon \simeq 40$.}

$$S_{\text{asympt}}(\hat{s}) = -\frac{i}{2\Lambda_\pi^3 \sqrt{\hat{s}}} \left( \frac{\bar{M}_5}{\kappa} \right)^{3/2} = -\frac{i}{\sqrt{\hat{s}}} \left( \frac{\kappa \bar{M}_5^2}{\bar{M}_5^5} \right)^{3/2} .$$  \hspace{1cm} (41)

In contrast, at ILC energies ($\sqrt{s} \lesssim 500$ GeV) the exact formula (38) should be used, since $\varepsilon \ll 1$ in this case, and a real part of $S(s)$ is comparable with (or larger than) an integer part.

\footnote{At $\sqrt{s} = 2.8$ TeV, one gets $\sinh(2\varepsilon) \simeq 2 \sinh^2 \varepsilon \simeq 40$.}
3. $C = -\kappa \pi r_c / 2$. In such a case, eq. (11) means that
\[
\sigma(y) = \frac{\kappa}{2} (|y| - |y - \pi r_c| - \pi r_c), \quad (42)
\]
and $\sigma_1 = -\kappa \pi r_c$, $\sigma_2 = 0$. Now the hierarchy relation is of the form
\[
\bar{M}_5^2 = \frac{\bar{M}_5^3}{\kappa} \left( e^{2\pi \kappa r_c} - 1 \right) \approx \frac{\bar{M}_5^3}{\kappa} e^{2\pi \kappa r_c}. \quad (43)
\]
Correspondingly, the masses of the KK gravitons appear to be proportional to the curvature $\kappa$
\[
m_n = x_n \kappa. \quad (44)
\]
By using hierarchy relation (43), the coupling constant of the massive gravitons on the TeV brane (20) can be rewritten as
\[
\Lambda_\pi = \left( \frac{\bar{M}_5^3}{\kappa} \right)^{1/2}. \quad (45)
\]
Again, two diverse physical frameworks can be considered:
3a) To get $m_1 \sim 1$ TeV, we can put $\bar{M}_5 \sim \kappa \sim 1$ TeV. In such a case, one obtains a series of massive graviton resonances in the TeV region which interact rather strongly with the SM fields, since $\Lambda_\pi \sim 1$ TeV [10].

3b) More interesting scenario with the small curvature is realized, if one takes $\kappa \ll \bar{M}_5 \sim 1$ TeV (RSSC model). For instance, suppose that the fundamental gravity scale $\bar{M}_5$ is of order few TeV, while the curvature $\kappa$ varies from hundreds MeV to tens GeV. Then the graviton spectrum is almost continuous (44), and it remains that in the model with one flat extra dimensions [2]-[4]. For the first time, this framework was considered in refs. [8], [9]. It was developed in our forthcoming publications (see, for instance, [10], [15], [16]).

It is often said that we have TeV physics in the original RS1 model [1], if the coupling constant $\Lambda_\pi$ is about one or few TeV. Indeed, it is enough to put $\kappa r_c \simeq 10.54$ in (20) to obtain $\Lambda_\pi = 1$ TeV. But at the same time, the hierarchy relation (22) requires $\bar{M}_5 \sim \kappa \sim \bar{M}_\text{Pl}$.\footnote{Correspondingly, the size of the extra dimensions is very small, $r_c \simeq 50 l_{\text{Pl}}$.}

In the RSSC model a situation is completely different. The point is that in the RSSC model a magnitude of all cross sections (after summing up virtual
or real gravitons) is defined by the fundamental gravity scale $\bar{M}_5$, not by $\Lambda_\pi$, provided $\kappa \ll \bar{M}_5$ [9]-[10]. In particular [10],

$$|S(s)| \sim \frac{1}{\bar{M}_5^3 \sqrt{s}}.$$  \hspace{1cm} (46)

An analogous mechanism takes place in the ADD model [2]-[4] in which the smallness of the graviton coupling to the SM fields ($\sim 1/\bar{M}_\text{Pl}$) is compensated by a huge number of gravitons.

As a result, we come to the TeV physics, if $\bar{M}_5$ is about few TeV\footnote{At the same time, $\Lambda_\pi \simeq 31.6 (\bar{M}_5/\text{TeV})^{3/2} (\text{GeV}/\kappa)^{1/2}$ TeV can be rather large.}. Thus, no new parameters of order $M_\text{Pl}$ have to be introduced in the RSSC scheme.

At the same time, the hierarchy relation (43) is satisfied due to the large warp factor $\exp(2\kappa \pi r_c)$. In particular, for $\bar{M}_5 = 1$ TeV and $\kappa = 100$ MeV, eq. (25) is valid if $\kappa r_c \simeq 9.81$. For $\bar{M}_5 = 10$ TeV and $\kappa = 1$ GeV, we get $\kappa r_c \simeq 9.08$.

For the time being, it is recognized that the hierarchy problem is not solved in the ADD model [2]-[4] in which the huge value of the Planck scale is explained in terms of a new large parameter which is a volume of extra dimensions. On the contrary, in schemes 2 and 3 considered above the Planck mass is defined by the large warp factors in the hierarchy relations (25), (43).

Let us stress that the RS1 scenario (case 1) differs from other scenarios, since it is the only scheme with $\sigma_1 = 0$. As a result, the coupling $\Lambda_\pi$ has no relation with the parameters $\bar{M}_5$ and $\kappa$ in the limit $\pi \kappa r_c \gg 1$, but it is entirely defined by the warp factor (20). At the same time, $\bar{M}_\text{Pl}$ depends very weakly on the warp factor, as one can see from (16). On the contrary, in the schemes with $\sigma_1 \neq 0$, the coupling $\Lambda_\pi$ can be related to the model parameters $\bar{M}_5$ and $\kappa$ via eqs. (16) and (20) (see, for instance, eqs. (27), (45)).

Note that a shift $\sigma \to \sigma - B$, where $B$ is a constant, is equivalent to a change of four-dimensional coordinates [17],

$$x^\mu \to x'^\mu = e^{-B} x^\mu.$$  \hspace{1cm} (47)

The effective gravity action on the TeV brane (with radion term omitted) looks like (see, for instance, [18])

$$S_{\text{eff}} = \frac{1}{4} \sum_{n=0}^{\infty} \int d^4 x \left[ \partial_{\mu} h^{(n)}_{\rho\sigma}(x) \partial_{\nu} h^{(n)}_{\lambda}(x) \eta^{\mu\nu} - m_n^2 h^{(n)}_{\rho\sigma}(x) h^{(n)}_{\lambda}(x) \right] \eta^{\rho\delta} \eta^{\sigma\lambda},$$  \hspace{1cm} (48)
The invariance of this action under transformation (47) needs rescaling of the graviton fields and their mass: \( h_{\mu\nu}^{(n)} \rightarrow h_{\mu\nu}^{(n')} = e^B h_{\mu\nu}^{(n)} \), \( m_n \rightarrow m_n' = e^B m_n \).

As an illustration, note that the transition from the RS1 model (case 1) to the RSSC model (case 3) means that \( \sigma \rightarrow \sigma - \pi \kappa r_c \). Correspondingly, eq. (23) transforms into eq. (44).

4 Conclusions

In the present Letter the RS scenario with two branes is studied. We used recently obtained general expression for the warp function \( \sigma(y) \) which has the explicit symmetry with respect to the branes. The Einstein-Hilbert’s equations define \( \sigma(y) \) up to an arbitrary constant \( C \). It was shown that, depending on a value of \( C \), one gets quite different physical schemes. The well-known RS1 model can be realized as one particular case. The other value of \( C \) corresponds to the scheme with the small curvature of the 5-dimensional space-time (RSSC model). Contrary to the RS1 model, it has the continuous spectrum of the KK gravitons. One more scheme is also suggested and studied which can give an interesting collider phenomenology in the TeV region.

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\(^7\)Of course, it does not mean that \( m_n \) becomes larger in the RSSC model, since very values of the parameter \( \kappa \) are quite different in these models.
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