Relational Thinking Skills of Junior High School Students and Their Relationship with Creativity in Solving Mathematical Problems

Budi Usodo¹, Mardiyana¹, Ikrar Pramudya¹, Sutopo¹, Rubono Setiyawan¹
¹Mathematics Education Departement, Universitas Sebelas Maret, Surakarta, Indonesia

Email: budi_usodo@yahoo.com

Abstract. Relational thinking skills are the ability of children to see and use possible variations between numbers in one number sentence. This study aims to describe relational thinking skills and their relationship with the creativity of junior high school students in solving mathematical problems. This research uses quantitative and qualitative descriptive research methods. The subjects of this study were several 8th grade students in one of junior high schools in the city of Surakarta. Data collection methods used are test and interview methods. Analysis of the data used is quantitative and qualitative descriptive analysis. The results of the study showed that the relational thinking ability of junior high school students was less than 12.5%. Most students only use computational thinking to solve math problems. From the interview results show that the low ability of students' relational thinking, because the students have never been taught the use of relational thinking to solve mathematical problems. After giving several examples of the use of ways of thinking in solving problems, students can solve this problem in more than one way. The interview results show that students have creativity in solving mathematical problems.

1. Introduction

Learning mathematics in junior high school is a transition from number sense to symbol sense. Mastery of symbols is used to study mathematics which is abstract knowledge. Junior high school students begin to be taught algebraic concepts which are a part of mathematics that are presented a lot symbolically. Thus junior high school students are expected to be able to understand basic algebraic concepts. For this reason, the seventh grade mathematics curriculum is given material which is a transition from arithmetic to algebra, namely integer material. Presentation of material about integers is designed so that the construction of knowledge will occur symbols using mastery of numbers. Thus the presentation of integer material in junior high school students is expected to bring students from an understanding of arithmetic to algebra.

Although the material on integers in the junior high school mathematics curriculum has been designed in such a way, but mathematics learning will bring from arithmetic knowledge to algebra, from number sense to symbol sense is not easy. Systematic errors in algebra are still common, for example, some students still say \( 2x = 2 + x, x^2 = x2, 2x = 5x, 3x + 2y = 5xy \) and many other forms of error. This is certainly due to the concept of operation and the concept of similarity which are still not well understood by junior high school students when they are still learning the operation of integer numbers and their properties.

Understanding the concept of similarity is needed by students in learning algebra in junior high. Understanding the concept of similarity and understanding the sign "same" is very important to learn the equation. [1,2]. The equal sign in an equation is interpreted as a symbol that states the relation or equality of two equations. But most students who have studied arithmetic, think that the equal sign is a command to operate rather than a relation that connects two equations [3-6]. This can result in students experiencing a misconception of the equal sign. The results of a study explain that in solving equations, students already know that the same sign is a symbol of similarity, but they have not understood the concept of similarity correctly. Students always move segments in solving...
equations with addition and subtraction operations as well as doing multiplication in solving equations with multiplication or division operations [7]. Whereas students who see the right side of an equation as an answer and solve a problem by moving a segment, do not understand the similarity between the right side and the left side of the equation [8].

Besides that the basis for changing arithmetic thinking to algebraic thinking lies in how students can abstract key concepts including things related to similarity and relationships. For students the process of abstracting students' structural numerical thinking towards algebraic thinking is enough to understand the sign equally rationally as a symbol of similarity ( = ) which means the same as [2, 9]. In learning mathematics so far, if students are given a problem like the following example: Calculate 169 + 289 = .. then what most students do is to do it by stacking downwards. Usually only a few or even no students do 169 + 289 = ... by doing the following calculation: 169 + 289 = 170 + 290 - 2 = 460 - 2 = 458. Another example, count 78 x 9. The way students often use it during this is by stacking down. Other methods that can be used are as follows: (78 x 10) - 78 = 780 - 78 = 780 - 80 + 2 = 702

Compared to the way it is arranged downward, this other method is easier and more accurate.

The process of working in a collapsed way down process involves computational thinking. The execution in the other way is based more on how the results of the operation are obtained without having to rely on formal methods. The formal methods that are usually taught since elementary school, emphasize more on how to use step by step which only requires procedural thinking skills. While solving the problem in another way does not only require the ability to think procedurally but also requires the ability to think conceptually. Using these other methods requires an understanding of the relationships between the numbers involved, so it’s not just the ability to do operations on numbers.

The use of the second method involves the ability to think relational.

Mathematics is one of the most important tools for developing the thinking skills that individuals need to solve their daily life problems. One type of thought process is relational thinking [10]. The definition of relational thinking is a strategy for solving problems related to numbers by understanding the relationship between numbers. For example, to solve 47 + 25 can be transformed into 50 + 22 by adding 3 to the number 47 and subtracting 3 from the number 25 [11]. Another definition, relational thinking is the ability to understand real analogies of different objects or events and the ability to apply abstract rules in new situations [12]. Some of the definitions that have been put forward by the experts above are only limited to the subject of arithmetic. The definition different from relational thinking in mathematics is relational thinking in mathematics as a skill and a tendency to use deductive reasoning and logic to make connections between abstract mathematical concepts and their specific examples [13]

Students who successfully use relational thinking to solve equality problems can also identify directions that will change the shape of the equation while maintaining existing equations. For example to solve the problem 8 + 4 = - + 5 is changed to (7 + 1) + 4 = - + 5 and becomes 7 + (1 + 4) = - + 5. The results of the study, show that there is evidence of a relationship between elementary school students' understanding of the equal sign (=) and their ability to understand the concept of similarity in an equation. The results of the study explain that students who define equal signs as symbols of equality in mathematical sentences, not only know that sentence numbers 2 × [] + 15 = 31 and 2 × [] + 15-9 = 31-9 have the same solution, but also can explain that the two mathematical sentences have the same solution without completing the two mathematical sentences [2]. Thus teaching numbers operations to students learning by using relational thinking in addition to increasing the ability to operate on integers will also help solve mathematical learning problems that are still felt difficult, namely bringing children from arithmetic thinking to algebraic thinking [2, 14-16].

On the other hand, creativity is a way of learning that allows students to make connections between unrelated elements, identify important problems, ask questions that come from curiosity, be open to new ideas, unwillingness to accept regular norms, along with flexibility and originality, new categorization, and regulation of these norms [16]. From the definition of creativity, it appears that there is a relationship between relational thinking skills with creativity in solving mathematical problems. Some research on relational thinking skills that have been done include most students in grades six and seven who use their own methods of calculation to solve problems with relational thinking. [17].

In connection with the description above, the purpose of this study is to describe the relational thinking skills and their relationship with the creativity of junior high school students in solving mathematical problems.
2. Method

This research uses a quantitative and qualitative approach. The quantitative approach is used to describe the results of students' relational thinking skills tests. A qualitative approach is used to describe the relational thinking skills of several research subjects. The subjects in this study were eighth-grade students of junior high school students in the city of Surakarta. 32 students were subjected to the test, then 3 subjects were selected to determine their relational thinking skills. The three subjects were selected with the following conditions: subject 1 is a student whose test results have many errors and does not use the relational thinking skill in completing the test, subject 2 is a student who can do it correctly but does not use the relational thinking skill and subject 3 is a student who has used relational thinking skills in answering tests.

To obtain the data needed in this study, test techniques and interview guidelines were used. The test used in this study is a test of relational thinking skill and interview guidelines that are used to explore the think relational skill. The test is as follows:

Table 1. The test of relational thinking skill

| No. | Item | Questions |
|-----|------|-----------|
| 1.  | For the questions below, write down the numbers in each box to make the statement true. Explain how you work in finding these numbers. Look for as many different ways as you can to fill the box. |
|     | a.  | 79 + 84 = □ + 86 |
|     | b.  | 79 + □ = 74 + 86 |
|     | c.  | □ + 93 = 64 + 95 |
|     | d.  | 89 - □ = □ - 86 |
|     | e.  | □ - 123 = 82 - 125 |
|     | f.  | 746 - 262 + □ = 747 |
|     | g.  | □ - 341 + 456 = 459 |
|     | h.  | 79 x 8 =□ |
| 2.  | Given the following data: 4, 12, -8, -7, 5, 6, 8, 10 Calculate the average of the data in the way you think is easiest |
| 3.  | Mrs. Ani shopping at a shop in a traditional market with the total price of goods purchased Rp 76,500.00. If Mrs. Ani pays with Rp 100,000 denomination, how much is the change, and explain how the trader counts the change. |

The data analysis technique used is quantitative descriptive analysis and qualitative descriptive analysis. Quantitative descriptive analysis uses descriptive percentage techniques, while qualitative descriptive analysis uses steps consisting of data reduction, data presentation, and interpretation.

3. Results and Discussion

3.1 Relational Thinking Skills Test Results

From the results of the relational thinking ability test given to 32 students the following results were obtained, in questions number 1 (a, b, c, d, e, f, g, h) many students were able to answer correctly and use more than 1 method only 4 students (12.5%). From 32 students there are also around 10% to 20% who have the wrong calculation results. In questions number 2 and 3 only 3 students out of 32 (9.4%) students worked in unusual ways and the answers were correct, while 29 students (90.6%) worked in the normal way and among you the answers were wrong. This is possible because most students in working on the test questions that are thought about are to get the correct answer not the many ways that can be used in solving the problem. These results are in line with results in line with the results of studies in Brazil, when students in grades 7 and 8 are asked to complete mathematical sentences using four arithmetic operations, most students used computational methods [18]. To find out why students do not look for as many ways as possible to solve the problem is done by conducting in-depth interviews with selected subjects.
3.2. Relational Thinking Skills

3.2.1 Subject 1 (S1)
Subject 1 is a student who still have errors in calculating operations. The results of written answers from subject 1 are as follows:

![Figure 1](image1.png)

**Figure 1.** Subject 1’s written answers

From the answer written subject 1 in Figure 1, shows that subject 1 have many errors in solving problems. In addition to not using any other method than the method used as usual, subject 1 have a lot of mistakes for not understanding the concept of similarity. For example answer no 1 b, subject 1 made a mistake that is the contents of the box should be 81 but written -81. Likewise in question 1 f. Subject 1 made many mistakes in answering questions, so there was no indication of the ability to think relational. Therefore no interviews were conducted to explore their relational thinking skills.

3.2.2. Subject 2 (S2)
Subject 2 is a student who has no errors in counting operations but only does the problem in the usual way. The results of the written answers to subject 2 are presented in Figure 2 below:

![Figure 2](image2.png)

**Figure 2.** Students written answers about operations on integers
In the written answer subject 2 question no 1 in Figure 2 shows that in answering questions, the subject only presents one way for all questions and uses the commonly used method, namely computational thinking. This happens if the subject has no other way to get an answer. For this reason, the results of the interview are in Table 2 below.

Table 2. Result of interview with subject 2 about operations on integers

| Code | Interview Result |
|------|------------------|
| R    | Why is the method you use only 1, even though the command is to find the contents of the box in many ways. |
| S21  | Usually if the problem is like that the method used is only this way. |
| R    | The thing is you were asked to find as many ways as possible to answer the question, didn't you think of finding another way. |
| S22  | I'm confused about finding another way. |
| R    | To fill the box, for example for number 1a, we can see 84 and 86 are at odds 2 so that the left and right segments are the same, 79 with the contents of the box are also the same. So the contents of the box are 79 - 2 = 77. Do you understand? |
| S23  | (subject is silent for a moment). O ... that's how you do it. |
| R    | Can you find another way for number 1b? |
| S24  | means the way is the same, 79 with 74 at odds 5 so to fill in the box means 86 - 5 = 81. |
| R    | Can find a way other than that. |
| S25  | (The subject is silent, seems to think). For number 1a, for example, can I do this or not? 84 to 86 will increase by 2, so that the left-hand segment is equal to the right-hand side, it means 79 must decrease by 2. So the box answer is 77. |
| R    | Right, why don't you answer like that? |
| S26  | I do not think to answer in that way. I only know how to answer it just like that. |
| R    | Roughly to answer another question, can you find another different way? |
| S27  | It looks like it can, I'll try it. |
| R    | Now number 1h. Can you find other ways besides that? |
| S28  | Usually multiplying it like that. If you answer 79 x 8 = (70 + 9) x 8 = 560 + 72 = 632, can I? |
| R    | Do you think it's OK? |
| S29  | If the important thing is to find the answer, I think you can. |
| R    | Are there other ways besides these methods? |
| S210 | Yes, that means 79 x 8 = (80 - 1) x 8 = 640 - 8 = 632. |
| R    | Yeah right. In your opinion, from the way that you wrote the answer in the way that you just used which is better. |
| S211 | If the important thing is to find the answer, yes this way before (not the way in the written answer) |

From the results of the interview in the Table 2, the subject did not provide a different way in his written answers, because he did not know that there were other ways that could be used to answer question number 1. After being given some direction about other ways that can be used to solve the problem, the subject realized that the way to solve the problem is not just 1 way, even the subject can find other ways to solve the problem. This gives an illustration that the subject actually has the ability to think relational, but because learning habits are only limited to solving problems by teaching only by computational thinking so that learning is not accustomed to students thinking alternative and divergent thinking. Though alternative thinking and divergent thinking is part of creative thinking. These results are in line with the results of research conducted in Brazil, that when students in grades 7 and 8 are asked to complete mathematical sentences using four arithmetic operations, most students use computational methods. However, when students are asked to show their relational thinking, most students also do it even though it only reaches a developmental stage [18]. This result is also in line with the results of research conducted by other researchers that students are only able to answer problems but do not use methods that are different from the methods that have been taught in school [19].

To find out whether the subject is thinking about a more efficient way to solve the problem, the subject's answers can be seen in questions number 2 and 3. Subject's written answers are as follows:
Figure 3. Students' written answers from questions number 2 and 3

From the written answer number 2 in Figure 3 shows that the subject performs in the usual way. The subject does not seem to have thought to look for ways to answer it more easily, for example grouping numbers so that the number is a multiple of 10. In the written answer number 3 Figure 4, the subject also answers in the usual way used to answer the problem of a story that is changing in the form of a mathematical model and completing the model the math. To find out how to think of the subject, it can be seen in the results of the interview presented in the following Table 3.

Table 3. The results of interview subject 2 related answers to question number 2 and 3

| Code  | Interview Result |
|-------|------------------|
| R     | Now it's about number 2. Why do you answer it that way? |
| S212  | You are asked to calculate the average. The formula is the amount of data divided by a lot of data. I add the data and then I divide it with lots of data, which is 8. |
| R     | Yeah right. Is there no easier way to calculate it. For example, classifying numbers so that the number is a multiple of 10. If a multiple of 10 is easy to add them later. |
| S213  | O. yes. I did not think of using such methods. Because many numbers are small, it's not too complicated to calculate the amount of data. |
| R     | Now for the story, don't you think of finding an easier way than that? |
| S214  | What I do is usually like that. The mathematical model is then made and the mathematical model is answered. |
| R     | Have you ever been asked to go to the market or shop to buy something that has change? |
| S215  | I have |
| R     | How do sellers count their change to you? |
| S216  | Sorry I didn't pay attention when the seller counted his change. |

From the interview in the Table 3, the subject did not find other ways that might be easier, because they felt the way they had was able to solve the problem. So don't think of looking for another better way. The situation is more due to the habit in learning activities that never give students a thought to look for various ways to solve problems.

3.2.3. Subject 3 (S3)
Subject 3 is students who can solve problems using more than 1 method and using more efficient methods. The following are the results of the written answers to subject 3
Figure 4. The written answer to subject 3 in integer operations

From the answers of subject 3 in Figure 4 shows that, the subject has been able to find a different way to answer even though only 2 ways. The subject seems to have more ability to see the relationship between the left and right segments in number sentences. To find out how to think of the subject, it can be seen in the results of the interview presented in the following Table 4.

Table 4. Result of interview with subject 2 about operations on integers

| Code | Interview Result |
|------|------------------|
| R    | Try to explain the first method that you use |
| S31  | If you look at the problem, for example number 1a, because 84 and 86 are at odds 2, then 79 and the contents of the box must also be at odds 2. For the left hand side to be the same as the right hand, then the contents of the box must be 79 - 2 = 77. |
| R    | Notice the question number 1a, 84 becomes 86, does it increase 2. Why does the contents of the box not 79 + 2 = 81? |
| S32  | If the answer is so, then the left side will not be the same as the right. |
| R    | Why do you think using such methods? |
| S33  | Because the question was asked to have to find a different way. I think the important thing is there is a different program and the answer is correct. |
| R    | Have you ever worked on a similar problem like that? |
| S34  | Never before |
| R    | For question number 1h, where did you get the idea to work with? |
| S35  | At first I thought about it first, but finally I remembered the distributive nature. |

From the above interview excerpt in the Table 4, shows that the subject already has relational thinking skills. The relational thinking skills possessed by the subject are based on their ability to see
the number sentences that use the equal sign, by calling the left and right segments. In addition, the subject also has the ability to think alternatives and can justify these thoughts. The subject also has the ability to look for more efficient ways, for example in solving problems 1h by changing in the form of multiplications involving multiples of 10. The results of the interview illustrate that the skills of relational thinking will have an impact on the development of creative thinking skills and also more efficient thinking. These results are in line with the results of the research by Baiduri (2013) that in relational thinking requires creativity and accuracy in finding specific relationships to solve a problem [20]. Baiduri (2014) also in his research stated that the relational thinking process of high-ability students when faced with mathematical problems students find the relationships contained in the problems which are then used to determine how to calculate the results [21].

![Figure 5. The written answer to subject 3 from questions number 2 and 3](image)

From the written answer of the subject in the Figure 5, it shows that the subject sees the ease in answering questions by grouping the numbers in multiples of 10. In the written answer answers the subject completes by not making a mathematical model first, but as practiced directly between sellers and buyers. The method used is used by sellers in traditional markets. To find out how to think of the subject, it can be seen in the results of the interview presented in the following Table 5.

| Code | Interview Result |
|------|------------------|
| R    | *In the answer to question no. 2, why don't you add up the numbers directly from the problem, how come you have to change the order of the numbers?* |
| S36  | Let it be easier. Because if the result has zero (meaning multiples of 10) it's usually easier. |
| R    | Where do you get it from? |
| S37  | Usually when I add up, I first see the number that I want to add up. If it can be grouped so that it appears zero, first grouped. |
| R    | Now it's question number 3. Why do you solve it like that, why don't you make a mathematical model first. |
| S38  | Because the problem is not asked to determine the change but the way for merchants to give the change to the buyer. So I do it like when I shop at a stall, the way traders give their change like that. |
| R    | Do you think the way you do is easier or more difficult? |
| S39  | Easier |

Table 5. The results of interview subject 3 related answers to question number 2 and 3
From the interview in the Table 5, the subject has understood the concept of relational thinking, which can provide a way to solve a problem with the thought that whatever the important way the answer is correct and the subject can give a reason why the method used is correct. Besides that, the subject is also able to choose effective and efficient ways. This result is in line with what was conveyed by Bishara [16] that creativity in mathematics is expressed through the following: the formulation of new uncomplicated problems, the discovery of ways to solve problems, and the discovery of solutions to unusual problems. One way to create an original thought situation is to give students open-ended questions that require creative thinking and allow more than one possible answer [16].

4. Conclusion
From result of the research, it can be concluded that: most of the eighth grade students of SMP still have many who do not have relational thinking skills. On research subjects who still have errors in counting operations there is a tendency to have difficulty finding different ways, even though directions have been given. On subjects who can do the counting operations correctly but do not use different methods of solving problems because it is not known that there are other ways to solve the problems. After being given some direction on other ways that can be used to solve the problem, the subject is realized that the way to solve the problem is not just 1 way, even the subject can find other ways to solve the problem In subjects who can work on problems in more than 1 way and in different ways already have relational thinking skills. The relational thinking ability possessed by the subject is based on its ability to see the number sentences that use the equal sign, by referring to the terms are left segment and right segment. The subject has the ability to think alternatives and can justify these thoughts and the subject also has the ability to find more efficient ways. The results shown on the subject provide an explanation that subjects who have relational thinking skills also have creativity in solving mathematical problems.

Suggestions that can be delivered from the results of this study are the mathematics teachers are advised to teach the relational thinking skills to increase student creativity, advanced researchers are advised to develop teaching materials that can be used to build relational thinking skills and the education policy makers should begin to think that relational thinking skills can be part of the Indonesian mathematics curriculum.

Acknowledgments
Thank to the 8th grade students and mathematics teachers at SMPIT Nur Hidayah Surakarta

References
[1] Freiman V and Lee L 2004 Tracking primary students’ understanding of the equality sign. In M. Hoines, & A. Fuglestad (Eds.) Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education (Bergen: PME) 2 pp 415-22
[2] Knuth E, Stephens A, McNeil N and Alibali, M 2006 Does understanding the equal sign matter? Evidence from solving equations Journal for Research in Mathematics Education 36 pp 297-312
[3] Stephens M. 2006 Describing and exploring the power of relational thinking. In P. Grootenboer, R. Zevenbergen, & M. Chinnappan (Eds.), Identities, cultures and learning spaces. Proceedings of the 29th annual conference of the Mathematics Education Research Group of Australasia (Sydney: MERGA) pp 479-486
[4] Powell S R and Fuchs L S 2010 Contribution of equal-sign instruction beyond word-problem tutoring for third-grade students with mathematics difficulty Journal of Educational Psychology 102 pp 381-94
[5] Andrews P and Sayers J 2012 Teaching Linear Equations: Case Studies From Finland, Flanders and Hungary The Journal of Mathematical Behavior 31 pp 476-88
[6] Naphapun V 2012 Relational Thinking: Learning Arithmetic in order to Promote Algebraic Thinking Journal of Science and Mathematics 35 pp 84-101
[7] Kusuma N F, Subanti S and Usodo B 2018 Students’ Misconception on Equal Sign. Proceeding The First International Conference on Combinatorics, Graph Theory, and Network Topology (Jember: Universitas Negeri Jember)

[8] Aydin S, Celikkol E S, Of M and Mutlu C 2013 On the Two Connected Concepts in Secondary School Mathematics: Algebraic Expressions and Linear Equations Procedia-Social and behavioral Sciences 106 pp 1966-70

[9] McNeil N M and Alibali M 2005 Knowledge change as a function of mathematics experience: All contexts are not created equal Journal of Cognition and Development 6 pp 285-306

[10] Kızıltoprak A. Köse N Y 2017 Relational thinking: The bridge between arithmetic and algebra. International Electronic Journal of Elementary Education 10 pp 131-45

[11] Stephen M 2007 Exploring the Power of Rational Thingking Students Emerging Algebraic Thinking In The Middle School Years – Four Countries Workshop PMRI (Yogyakarta : Universitas Sanata Dharma Yogyakarta)

[12] Doumas L A A and Hummel J E 2005 Approaches to Modelling Human Mental Representations: What Works, What Doesn’t, and Why?, The Cambridge Handbook of Thinking and Reasoning: (Cambridge: University Press)

[13] Wright R D 2014 A Philosophical Approach to Relational Thinking in Mathematics Dissertation (Columbia: Columbia University)

[14] Irwin K and Britt M 2005 Algebraic thinking in the Numeracy project: Year two of a three-year study. In F. Ell, J. Higgins, K. Irwin, G. Thomas, T. Trinick, & J. Young-Loveridge (Eds.), Findings from the New Zealand Numeracy development project (Wellington: Learning Media) pp 46-54

[15] NCTM 2000 Principles and Standards for School Mathematics (Reston VA: NCTM)

[16] Bishara S 2016 Creativity in unique problem-solving in mathematics and its influence on motivation for learning Bishara Cogent Education 3 pp 1-14

[17] Alexandra N K and Helena P O 2018 The relationship between mental computation and relational thinking in the seventh grade Fields Math Educ J 3:6

[18] Stephens M and Riberio A 2012 Working Towards Algebra: The Importance of Relational Thinking Relime 15 pp 373-402

[19] Rahmah, Mubarokah and Aunillah 2015 Profil Pemahaman Relasional Siswa dalam Memecahkan Masalah Matematika Ditinjau dari Kemampuan Matematika Jurnal Pendidikan Matematika STKIP PGRI Sidoarjo 3

[20] Baiduri 2013 Analisis Proses Berpikir Relasional Siswa Sekolah Dasar Membuat Perencanaan Penyelesaian Masalah Matematika (Kasus Siswa Berkemampuan Matematika Rendah) Prosiding Konferensi Nasional Pendidikan Matematika V (Malang: Universitas Negeri Malang)

[21] Baiduri 2014 A Relational Thinking Process of Elementary School Students with High Capability Journal of Educational and Developmental Psychology 4