Experimental multi-location remote state preparation

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Transmission of quantum states is a central task in quantum information science. Remote state preparation (RSP) has the same goal as teleportation, i.e., transferring quantum information without sending physically the information carrier, but in RSP the sender knows the state which is to be transmitted. We present experimental demonstrations of RSP for two and three locations. In our experimental scheme Alice (the preparer) and her three partners share four and six photon polarization entangled singlets. This allows us to perform RSP of two or three copies of a single qubit state, a two qubit Bell state, and a three qubit W, or WW state. A possibility to prepare a two-qubit non-maximally entangled and GHZ states is also discussed. The ability to remotely prepare an entangled states by local projections at Alice is a distinguishing feature of our scheme.

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Theoretical studies in quantum information predict existence of various types of entangled states, which could be useful in many communication situations, and information processing, for example, quantum key distribution [1], teleportation [2], etc. Correlations between entangled systems are so strong that they cannot be modeled by any classical means [3]. In theory we can study entangled states of very many qubits, and complicated quantum protocols. But experimental practice shows that protocols involving many qubits are very difficult to demonstrate in the laboratory. In order to see to what extent theoretical quantum information science talks about experimentally controllable phenomena, one has to keep on testing the limits of the range of feasibility of such schemes, and keep extending such limits. With this in mind, we present realizations of several remote state preparation (RSP) protocols, [1–7], using tools of advanced multiphoton quantum interferometry [8].

The aim of teleportation and RSP is to take the advantage of entanglement to prepare a desired state at a distant location. In teleportation protocol, Alice’s task is to prepare an unknown, given to her, quantum state at Bob’s location. In the case of RSP Alice knows which state she wants to prepare at Bob’s location. The basic, most elementary scheme runs as follows. Alice and Bob share a maximally entangled state of two qubits, say singlet. Alice performs a projective measurement in a basis, which contains the state which is to be remotely prepared. If her measurement locally projects onto the state orthogonal to the one she wants to prepare, Bob’s sub-system collapses into the required state. She sends a single bit [4] announcing whether or not her projection measurement was successful. Such an experiment was realized with polarization qubits [5] and with a photon-atom system [10]. Note, that such a protocol must be probabilistic. Alice has a probability of $\frac{1}{2}$ of projecting onto the required state. In the case of failure Bob obtains a state orthogonal to the intended one. Because of the impossibility of a universal NOT gate, such a state cannot be corrected without the knowledge of the basis to which belongs. Nevertheless, if Alice is choice restricted to e.g. states from the equator of the Bloch sphere, the protocol becomes deterministic. Simply, given the bit from Alice, Bob may perform the $\sigma_z$ operation, which acts as the NOT gate on the equatorial plane.

In this letter present a more general scheme, allowing Alice to remotely prepare a large class of symmetric states, including entangled ones. For this purpose we will utilize rotationally invariant multi-qubit singlet states.

We begin with a brief description of the experimental set-up which allows to prepare such generalized singlet states, using methods of multiphoton interferometry. The setup consists of a non-linear crystal allows an efficient down-conversion process (non-collinear type-II PDC). Photons from a pulsed laser pumping field can spontaneously, with a low probability, fission into a pair of photons with orthogonal polarizations, in two conjugate propagation modes. If pumping is strong enough one can observe multi-fold emissions of such kind form a single pulse. The state can be expressed as

$$|PDC\rangle = \frac{1}{\text{cosh}^2 K} \sum_{p=0}^{\infty} \tanh^p K \sum_{m=0}^{p} e^{im\phi}$$

where, $|nX_c\rangle$ denotes a Fock state with $n$ photons, of polarization $X = H, V$ in mode $c = a, b$. The parameter $K$ is a function of the non-linearity and length of the crystal, pump power and filtering bandwidth, and $\phi$ is the possible phase difference between horizontal and vertical polarization due to birefringence in the crystal [11]. The $n$-th order PDC emission corresponds to terms with $p = n$. The trick is to place $n - 1$ consecutive beam splitters in each of the two emission spacial modes, and observe $2n$-fold coincidences [12]. Correlations characteristic for four and six (polarization) qubit states ($|\Psi_k\rangle, k = 2, 3$)
The four-qubit state was reported in Ref. [13], while Rådmark et al. [14] observed the six-qubit one. The states are generalizations of singlets, that is they have the same form irrespective which pair of orthogonal (in general elliptic) polarizations is used to express the polarization of each and every qubit. This implies rotational symmetry: if each qubit is rotated by the same unitary transformation $U$, such that $\text{det} U = 1$, the states does not change, $U^\otimes k |\Psi^-_k\rangle = |\Psi^-_k\rangle$ just like the two qubit singlet $|\Psi^-_2\rangle$. This property can be used to circumvent some forms decoherence, [15]. If the interaction with the environment is symmetric under an exchange of systems, one can process information within a so-called decoherence subspace, [16] [17]. For a decoherence process, in a form a random rotation, acting of all qubits in the same way, such a space is spanned by singlet states. For four qubits such a decoherence-free subspace is spanned by two orthogonal four-qubit states invariant under such transformations. One of them describes the product of two two-qubit singlets $|\psi^-_2\rangle \otimes |\psi^-_2\rangle$ and the other one is $|\psi^-_4\rangle$. A decoherence-free operation in this subspace has been demonstrated experimentally in Ref. [18].

All reduced density operators of the subsystems in $|\Psi^-_k\rangle$ are also rotationally invariant:

$$\rho = \text{Tr}_S |\Psi^-_k\rangle \langle\Psi^-_k| = \text{Tr}_S (U^\otimes k |\Psi^-_k\rangle \langle\Psi^-_k| U^\otimes k)$$

where $S$ stands for the traced out part of the system and $U^\otimes k|S$ is a tensor product unitary acting only on non traced out qubits. In the third line we used the fact that trace operation is basis-independent.

Using such states Alice can, by projecting her half of the qubits (which originate from one of the propagation modes of the PDC radiation), efficiently change the state of remote qubits (from the other propagation mode, sent to her Partners). Here we consider remote state preparation with $|\Psi^-_2\rangle$, $|\Psi^-_4\rangle$, and $|\Psi^-_6\rangle$.

To make our discussion more transparent, we can put $|\Psi^-_2\rangle$, $|\Psi^-_4\rangle$ and $|\Psi^-_6\rangle$ as

$$|\Psi^-_2\rangle = \frac{1}{\sqrt{2}}(|\psi\rangle|\bar{\psi}\rangle - |\bar{\psi}\rangle|\psi\rangle),$$

$$|\Psi^-_4\rangle = \frac{1}{\sqrt{3}}(|\psi\psi\rangle|\bar{\psi}\bar{\psi}\rangle + |\bar{\psi}\bar{\psi}\rangle|\psi\psi\rangle)$$

$$|\Psi^-_6\rangle = \frac{1}{2}(|\psi\psi\psi\rangle|\bar{\psi}\bar{\psi}\bar{\psi}\rangle - |\bar{\psi}\bar{\psi}\bar{\psi}\rangle|\psi\psi\psi\rangle)$$

where $|W_3\rangle = \frac{1}{\sqrt{3}}(|\psi\psi\psi\rangle + |\bar{\psi}\bar{\psi}\bar{\psi}\rangle + |\psi\bar{\psi}\bar{\psi}\rangle)$, while $|\overline{W}_3\rangle = \frac{1}{\sqrt{3}}(|\psi\bar{\psi}\psi\rangle + |\bar{\psi}\psi\bar{\psi}\rangle + |\bar{\psi}\bar{\psi}\psi\rangle)$, and $|\Psi^+_2\rangle = (|\psi\rangle + |\bar{\psi}\rangle)$ Due to the rotational invariance guarantees that probability of three pair emission is lower by yet another order of magnitude. Such conditions allow high interferometric contrast (visibility) in two, four, and six fold coincidence detections (the interference occurs while one changes the polarization settings at final analyzers at each of the exit arms of the beam-splitter system), see [11]. Note, that lower pump rates, could make the contrast higher, but the count rates of six-fold coincidence detections would become prohibitively low. Thus a proper tuning of the pump strength must be made.

The interesting feature of the four-beam-splitter setup of Fig. 1 is that whenever Alice registers just a single count in one of her three detection stations, under the pumping conditions described above, the most probable events on the other side of the interferometer (shared by Bob, Charlie and David, each controlling one of exits) are no counts at all (due to an inefficiency of the detectors), or just a single count at one of the three exit arms (two photon event probability is lower by an order of magnitude times efficiency of the detectors, which is close to two orders of magnitude). Thus, Alice is able to remotely prepare any pure single-qubit state for her partners, but she does not have control who actually receives it. To prepare $|\psi\rangle$, Alice sets measurement stations, all three, to the $|\psi\rangle/|\bar{\psi}\rangle$-basis. Every time Alice gets the result $|\psi\rangle$ in one of her stations, while other two do not register.
anything, one her partners will have the state $|\psi\rangle$. However, if she registers in her all three measuring stations photons of the same polarization, she is (almost) sure that if all her partners register photons, then these will of the same polarization, orthogonal to the one she measured at each of the stations, see the fist term of $|\Psi_6^-\rangle$. If she gets just two counts, at different stations, with highest probability a two pair emission occurred, thus, we have the case of $|\Psi_4^-\rangle$, and she can be (almost) sure that a pair of her partners, if they register single photons, have qubits of polarization $|\psi\rangle$, but its is beyond her control who gets them.

Alice can also conditionally prepare a three-qubit entangled state $|W_3\rangle$ or $|\overline{W}_3\rangle$ for her partners to share. Alice measures at all her stations in the basis $\{\{|\psi\rangle,|\overline{\psi}\rangle\}$.

If she gets a count at each of her stations, consistent with three-qubit states $|\psi\psi\psi\rangle, |\overline{\psi}\overline{\psi}\overline{\psi}\rangle$ or $|\overline{\psi}\psi\psi\rangle$, the remote parties will be sharing $W$ state, provided each of them received just one photon. If she registers $|\psi\psi\psi\rangle, |\overline{\psi}\psi\psi\rangle$ or $|\overline{\psi}\overline{\psi}\psi\rangle$, the $\overline{W}$ state is remotely prepared (under the same proviso).

Similarly, if we have a two pair emission leading to $|\Psi_6^-\rangle$ state, Alice can prepare the Bell state $|\Psi^+_3\rangle$, shared by a pair of her Partners. It is so provided Alice measures $|\psi\psi\rangle$ or $|\overline{\psi}\overline{\psi}\rangle$ at a pair of her stations (and no counts at the third station), and two partners receive (register) photons.

It is important to notice that operating on $|\Psi_6^-\rangle$ Alice can prepare genuinely three-party entangled pure states $W$ and $\overline{W}$, by just using projections onto factorizable states. Interestingly, to prepare a Greenberger-Horne-Zeilinger state (GHZ), she needs to register one of her qubits in state $|\psi\rangle$, second in state $\cos \theta |\psi\rangle + \sin \theta |\overline{\psi}\rangle$, and the third one in $\cos \theta |\psi\rangle - \sin \theta |\overline{\psi}\rangle$, where $\theta = \pm \frac{\pi}{3}$. A back of an envelope calculation shows that in such a case, state $|\Psi_6^-\rangle$ collapses in such a way that Bob, Charlie, and David share $\frac{1}{2}(|\psi\psi\psi\rangle - |\overline{\psi}\overline{\psi}\overline{\psi}\rangle - |\psi\overline{\psi}\psi\rangle - |\overline{\psi}\psi\overline{\psi}\rangle)$. This is a GHZ state in the diagonal-antidiagonal basis $\frac{1}{\sqrt{2}}(|\psi\rangle \pm |\overline{\psi}\rangle)$.

In a similar fashion, Alice can prepare non-maximally entangled state to two of her partners. She projects her two qubits on states $\cos \alpha |\psi\rangle \pm \sin \alpha |\overline{\psi}\rangle$ and $|\Psi_4^-\rangle$ collapses onto $(\cos^2 \alpha |\overline{\psi}\overline{\psi}\overline{\psi}\rangle - \sin^2 \alpha |\psi\psi\psi\rangle)/\sqrt{\cos^2 \alpha + \sin^2 \alpha}$.

In Table I, we give the probabilities (in the ideal cases) of the remote preparations of specific states. The preparation probabilities in Table I can be doubled, if the parties specify in advance that they want to remotely prepare qubit states on a specific great circle of the Bloch sphere. Then, if the remote qubits are $\psi$, the receivers can rotate their qubits to $\psi$ by applying $\sigma_z$ operations.

Table I: Probabilities of RSP for emissions of $|\Psi_k^-\rangle$, for $k = 2, 4, 6$.

| shared state | # qubits | prepared state | probability |
|--------------|----------|----------------|-------------|
| $|\Psi_5^-\rangle$ | 1 | $|\psi\rangle$ | 1/2 |
| $|\Psi_4^-\rangle$ | 2 | $|\psi\psi\rangle$ | 1/3 |
| $|\Psi_4^-\rangle$ | 2 | $|\Psi_2^+\rangle$ | 1/3 |
| $|\Psi_6^-\rangle$ | 3 | $|\psi\psi\psi\rangle$ | 1/4 |
| $|\Psi_6^-\rangle$ | 3 | $|W_3\rangle/\sqrt{2}$ | 1/4 |
| $|\Psi_6^-\rangle$ | 3 | $|GHZ\rangle$ | 1/4 |

In our experiment we use a frequency-doubled Ti:Sapphire laser (80 MHz repetition rate, 140 fs pulse length) yielding UV pulses with a central wavelength at 390 nm and an average power of 1300 mW. The pump beam is focused to a 160 µm waist in a 2 mm thick BBO (β-barium borate) crystal. Half wave plates and two 1 mm thick BBO crystals are used for compensation of longitudinal and transversal walk-offs. The emitted photons of non-collinear type-II PDC are then coupled to single mode fibers (SMFs), defining the two spatial modes at the crossings of the two frequency degenerated down-conversion cones. Upon exiting the fibers the down-conversion light passes narrow band (∆λ = 3 nm) interference filters (Fs) and is split into six spatial modes (a, b, c, d, e, f) by ordinary 50%–50% beam splitters (BS), followed by birefringent optics to compensate phase shifts in the BS’s. Due to the short pulses, narrow band filters, and single mode fibers the down-converted photons are temporally, spectrally, and spatially indistinguishable, see Fig. I. The polarization is being kept by passive fiber polarization controllers. Polarization analysis stations in each exit mode are implemented by a half wave plate (HWP), a quarter wave plate (QWP), and a polarizing beam-splitter (PBS). The outputs of the PBSs are lead to single photon silicon avalanche photo diodes (APDs) via multi mode fibers.
The APDs’ electronic responses, following photo detections, are being counted by a multi channel coincidence counter with a 3.3 ns time window. The coincidence counter registers any coincidence event between the 12 APDs as well as single detection events.

The RSP protocol is implemented by projective measurements done by Alice on her qubits. The qubits in exit modes a, b, and c are given to Alice, and in each mode one has a polarization measuring station, see Fig. 1. The qubit in modes d, e, and f are given Bob, Charlie, and David, respectively. For example, if Alice like to prepare $|H\rangle$ for her three partners, she projects the state of her photons onto $|VVV\rangle$ implies that the remaining three photons are all $|HHH\rangle$. Hence Alice can in this manner probabilistically prepare qubits in the $|HHH\rangle$ state for her three partners. Due to the probabilistic nature of projective measurements on $|\Psi_0^-\rangle$, Alice also needs to send classical information indicating the success to each of her partners, informing them that the intended state has been remotely prepared for them. In the experiment, we have tested a possibility to prepare horizontally, diagonally, and left circularly polarized photons, as well as the two-qubit maximally entangled states. For two pairs emissions the states which we prepared were $|HH\rangle$, $|DD\rangle$, $|LL\rangle$, $|\psi^+\rangle$, as well as $\frac{1}{2}(|HH\rangle\langle HH| + |VV\rangle\langle VV|) + \frac{1}{2}|\psi^+\rangle\langle \psi^+|$. Finally, for three pair emissions we realized preparations of $|HHH\rangle$, $|DDD\rangle$, $|LLL\rangle$ and the mixture $\frac{1}{2}(|HHH\rangle\langle HHH| + |VVV\rangle\langle VVV|) + \frac{1}{2}(|W\rangle\langle W| + |\bar{W}\rangle\langle \bar{W}|)$. In figure 2 we show experimental results of three-location RSP of horizontally $H$, diagonally $D$, and left circularly $L$ polarized photons. The one qubit fidelities are $F_H = 0.98 \pm 0.02$, $F_D = 0.97 \pm 0.04$, and $F_L = 0.97 \pm 0.05$ respectively. In figures 3 and 4 we show experimental results of two-location RSP of horizontally $HH$, diagonally $DD$, left circularly $LL$ polarized photons, and the two-qubit entangled state $\psi^+$ with fidelities are $F_{HH} = 0.97 \pm 0.04$, $F_{DD} = 0.97 \pm 0.04$, $F_{LL} = 0.97 \pm 0.04$, and $F_{\psi^+} = 0.96 \pm 0.03$ respectively.

RSP of the three-qubit entangled $W$ or $\bar{W}$ states has been demonstrated by projections at Alice stations to $|HVV\rangle$, $|VHV\rangle$ or $|VVH\rangle$. Similarly, registrations of $|HHV\rangle$, $|HVV\rangle$ or $VHH\rangle$ were used to prepare $\bar{W}$. RSP of three copies of one qubit is obtained by projection of Alice qubits to $|VVV\rangle$. The results are given in figures 5 and 6. The three qubit states fidelities are $F_{HHH} = 0.97 \pm 0.07$, $F_{DDD} = 0.97 \pm 0.07$, $F_{LLL} = 0.96 \pm 0.07$, $F_W = 0.90 \pm 0.09$, and $F_{\bar{W}} = 0.91 \pm 0.09$.

The figures clearly show, that we have demonstrated a method to remotely prepare several types of states of one, two, or three qubits (product, $|\psi^+\rangle$, $W$, and GHZ). The states are produced by projective measurements on one half of rotationally invariant multipartite states, which are readily available in laboratories, via a simple beam-splitting method (which by avoiding interferometric overlaps leads to is stable configuration). Our scheme involves multi-photon interferometry using
FIG. 4: RSP of $|\Psi_2^+\rangle$ and a two qubit mixed state. Renormalized detection probabilities for two qubit detection events for Bob and Charlie, for the respective case of RSP.

FIG. 5: RSP of three identical qubit states ({$|\psi^-\rangle$ emissions}). Three photon detection probabilities for the case of $|H\rangle$, $|D\rangle$, and $|L\rangle$ at the three locations for Bob, Charlie, and David.

a pulsed PDC based source of entangled photons. The experimental data confirm the high precision, with which RSP can work using such experimental methods. Interestingly, this scheme works as a kind of symmetrizer of states. If Alice registers a projection on a product state, her partners obtain a symmetric superposition of the product of states orthogonal to ones, which she observed.

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FIG. 6: RSP of $W$ states. Renormalized detection probabilities for three qubit entangled states $|W\rangle$, $|\bar{W}\rangle$ shared between Bob, Charlie, and David after a successful RSP. The last graph represents the mixed state defined in the text.

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