THE EFFECT OF NON-RADIAL MOTIONS ON THE CDM MODEL PREDICTIONS

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Abstract

In this paper we show how non-radial motions, originating from the tidal interaction of the irregular mass distribution within and around protoclusters, can solve some of the problems of the CDM model. Firstly the discrepancy between the CDM predicted two-points correlation function of clusters and the observed one. We compare the two-points correlation function, that we obtain taking account of non-radial motions, with that obtained by Sutherland & Efstathiou (1991) from the analysis of Geller & Huchra’s (1988) deep redshift survey and with the data points for the APM clusters obtained by Efstathiou et al. (1992). Secondly the problem of the X-ray clusters abundance over-production predicted by the CDM model. In this case we compare the X-ray temperature distribution function, calculated using Press-Schechter theory and Evrard’s (1990) prescriptions for the mass-temperature relation, taking also account of the non-radial motions, with Henry & Arnaud (1991) and Edge et al. (1990) X-ray temperature distributions for local clusters. We find that in both cases the model is in good agreement with experimental data. Finally we calculate the bias coefficient using a selection function that takes into account the effects of non-radial motions, and we show that the bias so obtained can account for a substantial part of the total bias required by observations on cluster scales.

keywords: cosmology: theory-large scale structure of Universe - galaxies: formation
1 Introduction

Although at his appearance the standard form of CDM was very successful in describing the observed structures in the universe (galaxy clustering statistics, structure formation epochs, peculiar velocity flows) (Peebles, 1982; Blumenthal et al. 1984; Bardeen et al. 1986; White et al. 1987; Frenk et al. 1988; Efstathiou 1990) recent measurements have shown several deficiencies of the model, at least if any bias of the distribution of galaxies relative to the mass is constant with scale. Some of the most difficult problems to reconcile with the theory are the strong clustering of rich clusters of galaxies, $\xi_{cc}(r) \approx \left(\frac{25h^{-3}Mpc}{r}\right)^{-2}$, far in excess of CDM predictions (Bahcall & Soneira 1983), the X-ray temperature distribution function of clusters, over-producing the observed cluster abundances (Bartlett & Silk 1993), the conflict between the normalisation of the spectrum of the perturbation which is required by different types of observations. Alternative models with more large-scale power than CDM have been introduced in order to solve the latter problem. Several authors (Peebles 1984; Efstathiou, Sutherland & Maddox 1990, Turner 1991, White, Efstathiou & Frenk 1993) have lowered the matter density under the critical value ($\Omega_m < 1$) and they have added a cosmological constant in order to retain a flat universe ($\Omega_m + \Omega_\Lambda = 1$). The spectrum of the matter density is specified by the transfer function, but its shape is affected because of the fact that the epoch of matter-radiation equality (characterized by a redshift $z_{eq}$) is earlier, $1 + z_{eq}$ being increased by a factor $1/\Omega_m$. Around the epoch $z_\Lambda$, where $z_\Lambda = (\Omega_m/\Omega_\Lambda)^{1/3} - 1$, the growth of the density contrast slows down and ceases after $z_\Lambda$. As a consequence the normalisation of the transfer function begins to fall, even if its shape is retained. Mixed dark matter models (MDM) (Bond et al. 1980; Shafi & Stecker 1984; Valdarnini & Bonometto 1985; Schaefer et al. 1989; Holtzman 1989; Schaefer 1991; Shaefer and Shafi 1993; Holtzman & Primack 1993) increase the large-scale power because neutrinos free-streaming damps the power on small scales. Alternatively changing the primeval spectrum several problems of CDM are solved (Cen et al. 1992). Finally it is possible to assume that the threshold for galaxy formation is not spatially invariant but weakly modulated ($2\% - 3\%$ on scales $r > 10h^{-1} Mpc$) by large scale density fluctuations, with the result that the clustering on large-scale is significantly increased (Bower et al. 1993).

Here we propose a different solution to several of the CDM model problems connected to the non-radial motions developing during the protocluster evolution.

It has long been speculated that angular momentum could have a fundamental role in determining the fate of collapsing proto-structures and several models have been proposed in which the galaxy type can be correlated with the angular momentum per unit mass of the structure itself (Faber 1982; Kashlinsky 1982; Fall 1983). Some authors (see Barrow & Silk 1981, Szalay & Silk 1983 and Peebles 1990) have proposed that non-radial motions would be expected within a developing proto-cluster due to the tidal interaction of the irregular mass distribution around them, typical of hierarchical clustering models, with the neighboring proto-clusters. The kinetic energy of this non-radial motions opposes the collapse of the proto-cluster, enabling the same to reach statistical equilibrium before the
final collapse (the so called previrialization conjecture by Davis & Peebles 1977, Peebles 1990). This effect may prevent the increase of the slope of the mass autocorrelation function at separations given by $\xi(r, t) \simeq 1$, expected in the scaling solution for the growth of $\xi(r, t)$ but not observed in the galaxy two-point correlation function. The role of non-radial motions has been pointed by several authors (see Davis & Peebles 1983, Gorski 1988, Groth et al. 1989, Mo et al. 1993, Weygaert & Babul 1994, Marzke et al. 1995 and Antonuccio & Colafrancesco 1997). Antonuccio & Colafrancesco derived the conditional probability distribution $f_{pk}(v|\nu)$ of the peculiar velocity around a peak of a Gaussian density field and used the moments of the velocity distribution to study the velocity dispersion around the peak. They showed that regions of the proto-clusters at radii, $r$, greater than the filtering length, $R_f$, contain predominantly non-radial motions. Non-radial motions change the energetics of the collapse model by introducing another potential energy term. In other words one expects that non-radial motions change the characteristics of the collapse and in particular the turn around epoch, $t_m$, and consequently the critical threshold, $\delta_c$, for collapse. One expects that non-radial motions produce firstly a change in the turn around epoch, secondly a new functional form for $\delta_c$, thirdly a change of the mass function calculable with the Press-Schechter (1974) formula and consequently of the predicted X-ray temperature distribution function of clusters and finally a modification of the two-point correlation function. Moreover this study of the role of non-radial motions in the collapse of density perturbations can help us to give a deeper insight on the so called problem of biasing. As pointed out by Davis et al. (1985), unbiased CDM suffers of several problems: pairwise velocity dispersion larger than the observed one, galaxy correlation function steeper than observed (see Liddle & Lyth 1993 and Strauss & Willick 1995). The remedy to these problems is the concept of biasing (Kaiser 1984), i.e. that galaxies are more strongly clustered than the mass distribution from which they originated. The physical origin of such biasing is not yet clear even if several mechanisms have been proposed (Rees 1985; Dekel & Rees 1987; Dekel & Silk 1986; Carlberg 1991; Cen & Ostriker 1992; Bower et al. 1993; Silk & Wyse 1993). Recently Colafrancesco, Antonuccio & Del Popolo (1995, hereafter CAD) have shown that dynamical friction delays the collapse of low-$\nu$ peaks inducing a bias of dynamical nature. Because of dynamical friction under-dense regions in clusters (the clusters outskirts) accrete less mass with respect to that accreted in absence of this dissipative effect and as a consequence over-dense regions are biased toward higher mass (Antonuccio & Colafrancesco 1995 and Del Popolo & Gambera, 1996). Non-radial motions acts in a similar fashion to dynamical friction: they delay the shell collapse consequently inducing a dynamical bias similar to that produced by dynamical friction. This dynamical bias can be evaluated defining a selection function similar to that given in CAD and using Bardeen, Bond, Szalay and Kaiser (1986, hereafter BBKS) prescriptions. The plan of the paper is the following: in §2 we obtain the total specific angular momentum acquired during expansion by a proto-cluster. In §3 we find the effect of non-radial motion on the critical density threshold, $\delta_c$. In §4 we derive a selection function for the peaks giving rise to proto-structures while in §5 we calculate some values for the bias parameter, using the selection function derived, on three relevant filtering scales. In §6 we find the effects
of non-radial motions on the X-ray temperature distribution function and then we compare this prevision to the X-ray observed data. In § 7 we study how non-radial motions influence the clusters two-points correlation function. §8 is devoted to conclusions and discussions.

2 Tidal torques in clusters evolution.

The explanation of galaxies spins gain through tidal torques was pioneered by Hoyle (1949) in the context of a collapsing protogalaxy. Peebles (1969) considered the process in the context of an expanding world model showing that the angular momentum gained by the matter in a random co-moving *Eulerian* sphere grows at second order in proportion to $t^{5/3}$ (in a Einstein-de Sitter universe), since when the proto-galaxy is still a small perturbation, while in the nonlinear stage the growth rate of an oblate homogeneous spheroid decreases with time as $t^{-1}$. More recent analytic computations (White 1984, Hoffman 1986, Ryden 1988) and numerical simulations (Barnes & Efstathiou 1987) have re-investigated the role of tidal torques in originating galaxies angular momentum. In particular White (1984) expanded an analysis by Doroshkevich (1970) showing that the angular momentum of galaxies grows to first order in proportion to $t$ and that the result of Peebles is a consequence of the spherical symmetry imposed in the model. White showed that the angular momentum of a Lagrangian sphere does not grow either in first or in second order while the angular momentum of a non-spherical volume grows to first order in agreement to Doroshkevich’s result. Hoffman (1986) has been much more involved in the analysis of the correlation of the growth of angular momentum with the density perturbation $\delta(r)$. He found an angular momentum-density anticorrelation: high density peaks acquire less angular momentum than low density peaks. One way to study the variation of angular momentum with radius in a galaxy is that followed by Ryden (1988). In this approach the protogalaxy is divided into a series of mass shells and the torque on each mass shell is computed separately. The density profile of each proto-structure is approximated by the superposition of a spherical profile, $\delta(r)$, and a random CDM distribution, $\varepsilon(r)$, which provides the quadrupole moment of the protogalaxy. To first order, the initial density can be represented by:

$$\rho(r) = \rho_b [1 + \delta(r)] [1 + \varepsilon(r)]$$  \hspace{1cm} (1)

where $\rho_b$ is the background density and $\varepsilon(r)$ is given by:

$$\langle |\varepsilon_k|^2 \rangle = P(k)$$  \hspace{1cm} (2)

being $P(k)$ the power spectrum, while the density profile is (Ryden & Gunn 1987):

$$\langle \delta(r) \rangle = \frac{\nu \xi(r)}{\xi(0)^{1/2}} - \frac{\partial(v \gamma, \gamma)}{\gamma(1 - \gamma^2)} \left[ \gamma^2 \xi(r) + \frac{R_*^2}{3} \nabla^2 \xi \right] \cdot \xi(0)^{-1/2}$$  \hspace{1cm} (3)

where $\nu$ is the height of a density peak, $\xi(r)$ is the two points correlation function, $\gamma$ and $R_*$ are two spectral parameters (BBKS, Eq. 4.6a, 4.6d)
while \( \vartheta(\gamma \nu, \gamma) \) is a function given in BBKS (Eq. 6.14). As shown by Ryden (1988) the net rms torque on a mass shell centered on the origin of internal radius \( r \) and thickness \( \delta r \) is given by:

\[
\langle |\tau|^2 \rangle^{1/2} = \sqrt{30 \left( \frac{4\pi}{5} G \right) \left[ \langle a_{2m}(r)^2 \rangle \langle q_{2m}(r)^2 \rangle - \langle a_{2m}(r)q_{2m}^*(r) \rangle^2 \right]^{1/2}} \tag{4}
\]

where \( q_{lm} \), the multipole moments of the shell and \( a_{lm} \), the tidal moments, are given by:

\[
\langle q_{2m}(r)^2 \rangle = \frac{r^4}{(2\pi)^3} M_{sh}^2 \int k^2 dk P(k) j_2(kr)^2 \tag{5}
\]

\[
\langle a_{2m}(r)^2 \rangle = \frac{2\rho_0^2 r^{-2}}{\pi} \int dk P(k) j_1(kr)^2 \tag{6}
\]

\[
\langle a_{2m}(r)q_{2m}^*(r) \rangle = \frac{r}{2\pi^2 \rho_0 M_{sh}} \int kdk P(k) j_1(kr) j_2(kr) \tag{7}
\]

where \( M_{sh} \) is the mass of the shell, \( j_1(r) \) and \( j_2(r) \) are the spherical Bessel function of first and second order while the power spectrum \( P(k) \) is given by:

\[
P(k) = Ak^{-1} [\ln (1 + 4.164k)]^2 \\
(192.9 + 1340k + 1.599 \times 10^5 k^2 + 1.78 \times 10^5 k^3 + 3.995 \times 10^6 k^4)^{-1/2} \tag{8}
\]

(Ryden & Gunn 1987). The normalization constant \( A \) can be obtained imposing that the mass variance at \( 8h^{-1} Mpc, \sigma_8 \), is equal to unity. Filtering the spectrum on cluster scales, \( R_f = 3h^{-1} Mpc \), we have obtained the rms torque, \( \tau(r) \), on a mass shell using Eq. (4) then we obtained the total specific angular momentum, \( h(r, \nu) \), acquired during expansion integrating the torque over time (Ryden 1988 Eq. 35):

\[
h(r, \nu) = \frac{1}{3} \left( \frac{3}{4} \right)^{2/3} \frac{\tau_o t_0}{\delta_o} \frac{\bar{\delta}_o^{5/2}}{M_{sh}} \int_0^\pi \frac{(1 - \cos \theta)^3}{(\theta - \sin \theta)^{4/3}} f_2(\vartheta) f_1(\vartheta) - f_2(\vartheta) f_1(\vartheta) d\vartheta \tag{9}
\]

where \( \tau_o, \delta_o \) and \( \bar{\delta}_o \) are respectively the torque, the mean overdensity and the mean overdensity within a sphere of radius \( r \) at the current epoch \( t_0 \). The functions \( f_1(\vartheta) \), \( f_2(\vartheta) \) are given by Ryden (1988 - Eq. 31):

\[
f_1(\theta) = 16 - 16 \cos \theta + \sin^2 \theta - 9 \theta \sin \theta \tag{10}
\]

\[
f_2(\theta) = 12 + 12 \cos \theta + 3 \sin^2 \theta - 9 \theta \sin \theta \tag{11}
\]

where \( \theta \) is a a parameter connected to the time, \( t \), through the following equation:

\[
t = \frac{3}{4} t_0 \bar{\delta}_o^{-3/2} (\theta - \sin \theta) \tag{12}
\]
The mean overdensity within a sphere of radius $r$, $\delta(r)$, is given by:

$$\delta(r, \nu) = \frac{3}{r^3} \int_0^r dx x^2 \delta(x)$$

(13)

In fig. 1 we show the variation of $h(r, \nu)$ with the distance $r$ for three values of the peak height $\nu$. The rms specific angular momentum, $h(r, \nu)$, increases with distance $r$ while peaks of greater $\nu$ acquire less angular momentum via tidal torques. This is the angular momentum-density anticorrelation showed by Hoffman (1986). This effect arises because the angular momentum is proportional to the gain at turn around time, $t_m$, which in turn is proportional to $\delta(r, \nu)^{-\frac{3}{2}} \propto \nu^{-3/2}$.

3 Non-radial motions and the density critical threshold.

One of the consequences of the angular momentum acquisition by a mass shell of a proto-cluster is the delay of the collapse of the proto-structure.
As shown by Barrow & Silk (1981) and Szalay & Silk (1983) the gravitational interaction of the irregular mass distribution of proto-cluster with the neighbouring proto-structures gives rise to non-radial motions, within the protocluster, which are expected to slow the rate of growth of the density contrast and to delay or suppress collapse. According to Davis & Peebles (1977) the kinetic energy of the resulting non-radial motions at the epoch of maximum expansion increases so much to oppose the recollapse of the proto-structure. Numerical N-body simulations by Villumsen & Davis (1986) showed a tendency to reproduce this so called previrialization effect. In a more recent paper by Peebles (1990) the slowing of the growth of density fluctuations and the collapse suppression after the epoch of the maximum expansion were re-obtained using a numerical action method. In the central regions of a density peak ($r \leq 0.5R_f$) the velocity dispersion attain nearly the same value (Antonuccio & Colafrancesco 1997) while at larger radii ($r \geq R_f$) the radial component is lower than the tangential component. This means that motions in the outer regions are predominately non-radial and in these regions the fate of the infalling material could be influenced by the amount of tangential velocity relative to the radial one. This can be shown writing the equation of motion of a spherically symmetric mass distribution with density $n(r)$:

$$\frac{\partial}{\partial t} n\langle v_r \rangle + \frac{\partial}{\partial r} n\langle v_r^2 \rangle + \left(2\langle v_r^2 \rangle - \langle v_\theta^2 \rangle \right) \frac{n}{r} + n(r) \frac{\partial}{\partial t} \langle v_r \rangle = 0$$ (14)

where $\langle v_r \rangle$ and $\langle v_\theta \rangle$ are, respectively, the mean radial and tangential streaming velocity. Eq. (14) shows that high tangential velocity dispersion ($\langle v_\theta^2 \rangle \geq 2\langle v_r^2 \rangle$) may alter the infall pattern. The expected delay in the collapse of a perturbation may be calculated using a model due to Peebles (Peebles 1993).

We consider an ensemble of gravitationally growing mass concentrations, we suppose that the material in each system collects within the same potential well with inward pointing acceleration given by $g(r, t)$. We indicate with $dP = f(L, rv_r, t)dLdv_dr$ the probability that a particle can be found in the proper radius range $r, r + dr$, in the radial velocity range $v_r, v_r + dv_r$ and with angular momentum $L = rv_\theta$ in the range $dL$. The radial acceleration of the particle is:

$$\frac{dv_r}{dt} = \frac{L^2(r, \nu)}{M^2r^3} - g(r)$$ (15)

where $g(r)$ is the acceleration. Eq. (15) can be derived from a potential and then from Liouville’s theorem follows that the distribution function, $f$, satisfies the collisionless Boltzmann equation:

$$\frac{\partial f}{\partial t} + v_r \frac{\partial f}{\partial r} + \frac{\partial f}{\partial v_r} \cdot \left[ \frac{L^2}{r^3} - g(r) \right] = 0$$ (16)

Using Gunn & Gott’s (1972) notation we write the proper radius of a shell in terms of the expansion parameter, $a(r_i, t)$, where $r_i$ is the initial radius:

$$r(r_i, t) = r_i a(r_i, t)$$ (17)
and remembering that $M = \frac{4\pi}{3} \rho(r_i, t) a^3(r_i, t) r_i^3$, that $\frac{3H_i^2}{8\pi G} = \rho_{ci}$, where $\rho_{ci}$ and $H_i$ are respectively the critical mass density and the Hubble constant at the time $t_i$, and assuming that no shell crossing occurs so that the total mass inside each shell remains constant, $(\rho(r_i, t) = \frac{\rho(r_i, t_i)}{a^3(r_i, t)})$ Eq. (15) may be written as:

$$\frac{d^2a}{dt^2} = -\frac{H_i^2(1 + \delta)}{2a^2} + \frac{4G^2L^2}{H_i^4(1 + \delta)^2r_i^{10}a^3}$$

where $\delta = \frac{\rho - \rho_{ci}}{\rho_{ci}}$, or integrating the equation once more:

$$\left(\frac{da}{dt}\right)^2 = H_i^2 \left[\frac{1 + \delta}{a}\right] + \int \frac{8G^2L^2}{H_i^4r_i^{10}(1 + \delta)^2} \frac{1}{a^3} da - 2C$$

where $C$ is the binding energy of the shell. Integrating once more we have:

$$t_{ta} = \int_0^{a_{max}} \frac{da}{\sqrt{H_i^2 \left[\frac{1 + \delta}{a} - \frac{1 + \delta}{a_{max}}\right] + \int_{a_{max}}^a \frac{8G^2L^2}{H_i^4r_i^{10}(1 + \delta)^2} a^3}}$$

Using Eqs (19) and (20) it is possible to find the linear overdensity at the turn-around epoch, $t_{ta}$. In fact solving Eq. (20), for some epoch of interest, we may obtain the expansion parameter of the turn-around epoch. This is related to the binding energy of the shell containing mass $M$ by Eq. (19) with $\frac{da}{dt} = 0$. In turn the binding energy of a growing mode solution is uniquely given by the linear overdensity, $\delta_i$, at time $t_i$. From this overdensity, using linear theory, we may obtain that of the turn-around epoch. We find the binding energy of the shell, $C$, using the relation between $v$ and $\delta_i$ for the growing mode (Peebles 1980) in Eq. (19) and finally the linear overdensity at the time of collapse:

$$\delta_c(\nu) = \delta_{co} \left[1 + \frac{8G^2}{\Omega_0^2 H_0^6 r_i^{10} \delta(1 + \delta)^2} \int_0^{a_{max}} L^2 \cdot da \right]$$

where $\delta_{co} = 1.68$ is the critical threshold for a spherical model, while $H_0$ and $\Omega_0$ are respectively the Hubble constant and the density parameter at the current epoch $t_0$. We may find the angular momentum, $L$, needed to calculate $\delta_c$ using Eq. (9).

The mass dependence of the threshold parameter, $\delta_c(\nu)$, can be found as follows: we calculate the binding radius, $r_b$, of the shell using Hoffmann & Shaham’s criterion (1985):

$$T_c(r, \nu) \leq t_0$$

where $T_c(r, \nu)$ is the calculated time of collapse of a shell and $t_0$ is the Hubble time. We find a relation between $\nu$ and $M$ through the equation $M = 4\pi \rho_0 r_b^3/3$. We so obtain $\delta_c(\nu(M))$. In Fig. 2 we show the variation of the threshold parameter, $\delta_c(M)$, with the mass $M$. Non-radial motions influence the value of $\delta_c$ increasing its value for peaks of low mass while leaving its value unchanged for high mass peaks. As a consequence, the structure formation by low mass peaks is inhibited. In other words, in agreement with the cooperative galaxy formation theory (Bower et al. 1993), structures form more easily in over-populated regions.
Figure 2: The threshold $\delta_c$ in function of the mass $M$, for a CDM spectrum ($\Omega_0 = 1, \ h = 1/2$) with $R_f = 3h^{-1}Mpc$, taking account of non-radial motions.

4 Tidal field and the selection function

According to biased galaxy formation theory the sites of formation of structures of mass $\sim M$ must be identified with the maxima of the density peak smoothed over a scale $R_f$ ($M \propto R_f^3$). A necessary condition for a perturbation to form a structure is that it goes nonlinear and that the linearly extrapolated density contrast reaches the value $\delta(r) \geq \delta_c = 1.68$ or equivalently that the threshold criterion $\nu_\ell > \delta_c/\sigma_o(R_f)$ is satisfied, being $\sigma_o(R_f)$ the variance of the density field smoothed on scale $R_f$. When these conditions are satisfied the matter in a shell around a peak falls in toward the cluster center and virializes. In this scenario only rare high $\nu$ peaks form bright objects while low $\nu$ peaks ($\nu \approx 1$) form under-luminous objects. The kind of objects that form from nonlinear structures depends on the details of the collapse. Moreover if structures form only at peaks in the mass distribution they will be more strongly clustered than the mass. Several feedback mechanisms has been proposed to explain this segregation effect (Rees 1985, Dekel & Rees 1987). Even if these feedback mechanisms work one cannot expect they have effect instantaneously, so the threshold for structure formation cannot be sharp (BBKS). To take into account this
effect BBKS introduced a threshold or selection function, \( t(\nu/\nu_t) \). The selection function, \( t(\nu/\nu_t) \), gives the probability that a density peak forms an object, while the threshold level, \( \nu_t \), is defined so that the probability that a peak form an object is 1/2 when \( \nu = \nu_t \). The selection function introduced by BBKS (Eq. 4.13), is an empirical one and depends on two parameters: the threshold \( \nu_t \) and the shape parameter \( q \):

\[
t(\nu/\nu_t) = \frac{(\nu/\nu_t)^q}{1 + (\nu/\nu_t)^q}
\]  

(23)

If \( q \to \infty \) this selection function is a Heaviside function \( \vartheta(\nu - \nu_t) \) so that peaks with \( \nu > \nu_t \) have a probability equal to 100% to form objects while peaks with \( \nu \leq \nu_t \) do not form objects. If \( q \) has a finite value sub-\( \nu_t \) peaks are selected with non-zero probability. Using the given selection function the cumulative number density of peaks higher than \( \nu \) is given, according to BBKS, by:

\[
n_{pk} = \int_{\nu}^{\infty} t(\nu/\nu_t)N_{pk}(\nu)d\nu
\]  

(24)

where \( N_{pk}(\nu) \) is the comoving peak density (see BBKS Eq. 4.3). A form of the selection function, physically motivated, can be obtained following the argument given in CAD. In this last paper the selection function is defined as:

\[
t(\nu) = \int_{\delta_c}^{\infty} p[\delta, \langle \delta \rangle(r_{Mt}, \nu), \sigma_\delta(r_{Mt}, \nu)]d\delta
\]  

(25)

where the function

\[
p[\delta, \langle \delta \rangle(r)] = \frac{1}{\sqrt{2\pi}\sigma_\delta} \exp \left( -\frac{|\delta - \langle \delta \rangle(r)|^2}{2\sigma_\delta^2} \right)
\]  

(26)

gives the probability that the peak overdensity is different from the average, in a Gaussian density field. The selection function depends on \( \nu \) through the dependence of \( \delta(r) \) from \( \nu \). As displayed the integrand is evaluated at a radius \( r_{Mt} \) which is the typical radius of the object that we are selecting. Moreover the selection function \( t(\nu) \) depends on the critical overdensity threshold for the collapse, \( \delta_c \), which is not constant as in a spherical model (due to the presence, in our analysis, of non-radial motions that delay the collapse of the proto-cluster) but it depends on \( \nu \). Known \( \delta_c(\nu) \) and chosen a spectrum, the selection function is immediately obtainable through Eq. (23) and Eq. (26). The result of the calculation, plotted in Fig. 3, for two values of the filtering radius, \( (R_f = 2, 3\ h^{-1}\ Mpc) \), shows that the selection function, as expected, differs from an Heaviside function (sharp threshold). The value of \( \nu \) at which the selection function \( t(\nu) \) reaches the value 1 (\( t(\nu) \approx 1 \)) increases for growing values of the filtering radius, \( R_f \). This is due to the smoothing effect of the filtering process. The effect of non-radial motions is, firstly, that of shifting \( t(\nu) \) towards higher values of \( \nu \), and, secondly, that of making it steeper. The selection function is also different from that used by BBKS (Tab. 3a). Finally it is interesting to note that the selection function defined by Eq. (23) and Eq. (26) is totally general, it does not depend on the presence or absence of non-radial motions. The latter influence the selection function form through the changement of \( \delta_c \) induced by non-radial motions itself.
Figure 3: The selection function, $t(\nu)$, for $R_f = 3h^{-1}\text{Mpc}$ ($\delta_c = 1.68$, solid line; $\delta_c$ function of $\nu$, dotted line) and for $4h^{-1}\text{Mpc}$ ($\delta_c = 1.68$, short dashed line; $\delta_c$ function of $\nu$, long dashed line).

5 The bias coefficient

A model of the Universe in which light traces the mass distribution accurately (unbiased model) is subject to several problems. As pointed out by Davis et al. (1985) an unbiased CDM produces a galaxy correlation function which is steeper than observed and a pairwise velocity dispersion larger than that deduced from redshift surveys. A remedy to this problem can be found if the assumption that light trace mass is relaxed introducing the biasing concept, i.e. that galaxies are more clustered than the distribution of matter in agreement to the concept of biasing inspired by Kaiser’s (1984) discussion of the observation that clusters of galaxies cluster more strongly than do galaxies, in the sense that the dimensionless two-point correlation function, $\xi_{cc}(r)$, is much larger than the galaxy two-point function, $\xi_{gg}(r)$. The galaxy two-point correlation function $\xi_{gg}(r)$ is a power-law:

$$\xi_g(r) = \left(\frac{r}{r_{0,g}}\right)^{\gamma}$$  \hspace{1cm} (27)

with a correlation length $r_{0,g} \simeq 5h^{-1}\text{Mpc}$ and a slope $\gamma \simeq 1.8$ for $r \leq 10h^{-1}\text{Mpc}$ (Davis & Peebles 1983; Davis et al. 1985; Shanks et al. 1989), (some
authors disagree with this values; for example Strauss et al. 1992 and Fisher et al. 1993 find $r_{0,g} \simeq 3.79h^{-1}\text{Mpc}$ and $\gamma \simeq 1.57$). As regards the clusters of galaxies the form of the two-point correlation function, $\xi_{cc}(r)$, is equal to that given by Eq. (27). Only the correlation length is different. In the case of clusters of galaxies the value of $r_{0,c}$ is uncertain (see Bahcall & Soneira 1983; Postman et al. 1986; Sutherland 1988; Bahcall 1988; Dekel et al. 1989; Olivier et al. 1990 and Sutherland & Efstathiou 1991) however it lays in the range $r_{0,c} \simeq 12 \div 25h^{-1}\text{Mpc}$ in any case larger than $r_{0,g}$. One way of defining the bias coefficient of a class of objects is that given by (BBKS):

$$b(R_f) = \frac{\langle \tilde{\nu} \rangle}{\sigma_0} + 1$$

(28)

where $\langle \tilde{\nu} \rangle$ is:

$$\langle \tilde{\nu} \rangle = \int_0^\infty \left[ \nu - \frac{\gamma \theta}{1 - \gamma^2} \right] t\left(\frac{\nu}{\nu_t}\right) N_{pk}(\nu) d\nu$$

(29)

from Eq. (29) it is clear that the bias parameter can be calculated once a spectrum, $P(k)$, is fixed. The bias parameter depends on the shape and normalization of the power spectrum. A larger value is obtained for spectra with more power on large scale (Kauffmann et al. 1996). In this calculation we continue to use the standard CDM spectrum ($\Omega_0 = 1, h = 0.5$) normalized imposing that the rms density fluctuations in a sphere of radius $8h^{-1}\text{Mpc}$ is the same as that observed in galaxy counts, i.e. $\sigma_8 = \sigma(8h^{-1}\text{Mpc}) = 1$. The calculations have been performed for three different values of the filtering radius ($R_f = 2, 3, 4h^{-1}\text{Mpc}$). The values of $b$, that we have obtained, are respectively, in order growing of $R_f$, 1.6, 1.93 and 2.25.

As shown, the value of the bias parameter tends to increase with $R_f$ due the filter effect of $t(\nu)$. As shown $t(\nu)$ acts as a filter, increasing the filtering radius, $R_f$, the value of $\nu$ at which $t(\nu) \simeq 1$ increases . In other words when $R_f$ increases $t(\nu)$ selects density peaks of larger height. The reason of this behavior must be searched in the smoothing effect that the increasing of the filtering radius produces on density peaks. When $R_f$ is increased the density field is smoothed and $t(\nu)$ has to shift towards higher value of $\nu$ in order to select a class of object of fixed mass, $M$.

6 The X-ray temperature function

The PS theory provides an analytical description of the evolution of structures in a hierarchical Universe. In this model the linear density field, $\rho(x,t)$, is an isotropic random Gaussian field, the non-linear clumps are identified as over-densities (having a density contrast $\delta_c \sim 1.68$ - Gunn & Gott 1972) in the linear density field, while a mass element is incorporated into a non-linear object of mass $M$ when the density field smoothed with a top-hat filter of radius $R_f$, exceeds a threshold $\delta_c (M \propto R_f^3)$. The
probability distribution for fluctuations is given by:

\[ p[\delta(M)] = \frac{1}{\sqrt{2\pi}\sigma(M)} \exp[-(\delta(M))^2/2\sigma(M)^2] \]  

(30)

The probability that an object of mass \( M \) has formed is obtained integrating Eq. (30) from the threshold value \( \delta_c \) to infinity and the comoving number density of non-linear objects of mass \( M \) to \( M + dM \) is given simply by differentiating the integral with respect to mass and is given by:

\[ N(M,t)dM = -\rho_b \sqrt{\frac{2}{\pi}} \nu \exp \left( -\frac{\nu^2}{2} \right) \frac{1}{\sigma} \frac{d\sigma}{dM} \frac{dM}{M} \]  

(31)

where \( \rho_b \) is the mean mass density, \( \sigma(M) \) is the rms linear mass overdensity evaluated at the epoch when the mass function is desired and \( \nu = \frac{\delta_c}{\sigma(M)} \).

The redshift dependence of Eq. (31) can be obtained remembering that \( \nu = \frac{\delta_c(z)D(0)}{\sigma_o(M)D(z)} \)  

(32)

being \( D(z) \) the growth factor of the density perturbation and \( \sigma_o(M) \) the current value of \( \sigma(M) \). In Eq. (31) PS introduced arbitrarily a factor of two because \( \int_0^\infty dF(M) = 1/2 \), so that only half of the mass in the Universe is accounted for. Bond et al. (1991) showed that the "fudge factor" 2 is naturally obtained using the excursion set formalism in the sharp \( k \)-space while for general filters (e.g., Gaussian or "top hat") it is not possible to obtain an analogous analytical result. As stressed by Yano et al. 1996, the factor of 2 obtained in the sharp \( k \)-space is correct only if the spatial correlation of the density fluctuations is neglected. In spite of the quoted problem, several authors (Efstathiou et al. 1988; Brainerd & Villumsen 1992; Lacey & Cole 1994) showed that PS analytic theory correctly agrees with N-body simulations. In particular Efstathiou et al. (1988), showed that PS theory correctly agrees with the evolution of the distribution of mass amongst groups and clusters of galaxies (multiplicity function). Brainerd & Villumsen (1992) studied the CDM halo mass function using a hierarchical particle mesh code. From this last work results that PS formula fits the results of the simulation up to a mass of 10 times the characteristic 1\( \sigma \) fluctuation mass, \( M_* \), being \( M_* \approx 10^{15} b^{-6/(n_l+3)} M_\odot \), where \( b \) is the bias parameter and \( n_l \) is the local slope of the power spectrum. PS theory has proven particularly useful in analyzing the number counts and redshift distributions for QSOs (Efstathiou & Rees 1988), Lyman \( \alpha \) clouds (Bond et al. 1988) and X-ray clusters (Cavaliere & Colafrancesco 1988).

Some difficulties arise when PS theory is compared with observed distributions. To estimate the multiplicity function of real systems it is in fact required a knowledge of the temperature-mass (T-M) relation in order to transform the mass distribution into the temperature distribution. Theoretical uncertainty arises in this transformation because the exact relation between the mass appearing in the PS expression and the temperature of the intracluster gas is unknown. Under the standard assumption of the Intra-Cluster (IC) gas in hydrostatic equilibrium with the potential well...
of a spherically symmetric, virialized cluster, the IC gas temperature-mass relation is easily obtained by applying the virial theorem and for a flat matter-dominated Universe we have that (Kaiser 1986, Evrard 1990):

$$T = (6.4 h^{2/3} \text{keV}) \left( \frac{M}{10^{15} M_\odot} \right)^{2/3} (1 + z)$$ (33)

The assumptions of perfect hydrostatic equilibrium and virialization are in reality non completely satisfied by clusters. Clusters profile may have departure from isothermality, with slight temperature gradients throughout the cluster. The X-ray weighted temperature can be slightly different from the mean mass weighted virial temperature. In any case the scatter in the T-M relation given by Eq. (33) is of the order of $\approx 10\%$ (Evrard 1991). As shown by Bartlett & Silk (1993) the X-ray distribution function obtained using a standard CDM spectrum over-produces the clusters abundances data obtained from Henry & Arnaud (1991) and Edge et al. (1990). The discrepancy can be reduced taking into account the non-radial motions that originate when a cluster reaches the non-linear regime. In fact, the PS temperature distribution requires specification of $\delta_c$ and the temperature-mass relation T-M. The presence of non-radial motions changes both $\delta_c$ and the T-M relation. To get the temperature distribution it is necessary to know the temperature-mass relation. This can be obtained using the virial theorem, energy conservation and using Eq. (19) (Bartlett & Silk 1993). From the virial theorem we may write:

$$\langle K \rangle = \frac{GM}{2 r_{\text{eff}}} + \int_{0}^{r_{\text{eff}}} \frac{L^2}{2M^2r^3} dr$$ (34)

while from the energy conservation:

$$- \langle K \rangle + \frac{GM}{r_{\text{eff}}} + \int_{0}^{r_{\text{eff}}} \frac{L^2}{M^2r^3} dr = \frac{GM}{r_{ta}} + \int_{0}^{r_{ta}} \frac{L^2}{M^2r^3} dr$$ (35)

Eq. (34) and Eq.(35) can be solved for $r_{\text{eff}}$ and $\langle K \rangle$. We finally have that:

$$T = (6.4 h^{2/3} \text{keV}) \left( \frac{M \cdot h}{10^{15} M_\odot} \right)^{2/3} \left[ 1 + \frac{\eta \psi \int_{0}^{r_{ta}} \frac{L^2 dr}{M^2r^3}}{(G^2 H^2 \Omega b^2 / 2M^2)^{1/3}} \right]$$ (36)

where $\eta$ is a parameter given by $\eta = r_{ta}/x_1$, being $r_{ta}$ the radius of the turn-around epoch, while $x_1$ is defined by the relation $M = 4\pi\rho_0 x_1^3/3$ and $\psi = r_{\text{eff}}/r_{ta}$ where $r_{\text{eff}}$ is the time-averaged radius of a mass shell. Eq. (36) was normalised to agree with Evrard’s (1990) simulations for $L = 0$.

The new T-M relation, Eq. (36), differs from Eq. (33) for the presence of the term:

$$\frac{\eta \psi \int_{0}^{r_{ta}} \frac{L^2 dr}{M^2r^3}}{(G^2 H^2 \Omega b^2 / 2M^2)^{1/3}}$$ (37)

This last term changes the dependence of the temperature from the mass, $M$, in the T-M relation. Moreover the new T-M relation depends on the
angular momentum, $L$, originating from the gravitational interaction of the quadrupole moment of the protocluster with the tidal field of the matter of the neighboring protostructures. In Fig. 4 the X-ray temperature distribution, derived using a CDM model with $\Omega_0 = 1$, $h = 1/2$ and taking into account of non-radial motions, is compared with Henry & Arnaud (1991) and Edge et al. (1990) data and with a pure CDM model with $\Omega_0 = 1$, $h = 1/2$. As shown the CDM model that does not take account of the non-

![Figure 4: X-ray temperature distribution function. The solid line gives the temperature function for a pure CDM model ($\Omega_0 = 1$, $h = 1/2$), with $R_f = 3h^{-1}Mpc$. The dotted line is the same distribution but now taking account of non-radial motions. The data are obtained by Edge et al. 1990, and Henry & Arnaud 1991](image-url)

radial motions over-produces the clusters abundance. The introduction of non-radial motions gives a more careful description of the experimental data. As we have seen the X-ray temperature distribution function obtained taking account of non-radial motions is different from that of a pure CDM model for two reasons:

1) the variation of the threshold, $\delta_c$, with mass, $M$. This is due to the changement of the energetics of the collapse model produced by the introduction of another potential energy term ($\frac{L(r,\nu)^2}{M^{2+\nu}}$) in Eq. (15); 
2) the modification of the T-M relation produced by the alteration of the partition of energy in virial equilibrium.
For values of mass \( M = 0.5M_\odot \) the difference between the two theoretical lines in Fig. 4 is due to the first factor for a \( \approx 59\% \) and this value increases with increasing mass. The uncertainty in our model fundamentally comes from the uncertainty of the T-M relation whose value has been previously quoted.

One of the objection to the result of this paper may that the effect described has not been seen in some hydrodynamic simulations (Evrard & Crone 1992). The answer to this objection is that our model is fundamentally based on the previrialization conjecture (Davis & Peebles 1977; Peebles 1990), (supposing that initial asphericities and tidal interactions between neighboring density fluctuations induce significant non-radial motions, which oppose the collapse) and while some N-body simulations (Vilumsen & Davis 1986; Peebles 1990) appear to reproduce this effect, other simulations (for example Evrard & Crone 1992) do not. An answer to this controversy was given by (Lokas et al. 1996). The problem is connected to spectral index \( n \) used in the simulations. The ”previrialization” is seen only for \( n > -1 \). While Peebles (1990) used simulations with \( n = 0 \), Evrard & Crone (1992) assumed \( n = -1 \). Excluding this particular case generally the ensemble properties of clusters like their optical and X-ray luminosity functions, or their velocity and temperature distribution functions, are difficult to address directly in numerical simulations because the size of the box must be very large in order to contain a sufficient number of clusters; an then analytical approach remains an effective alternative.

### 7 Non-radial motions and the clusters correlation function

Supposing that the Universe at some early epoch can be described using the same assumptions of the previous section the probability \( p(M_1, M_2, r) \) per unit volume per unit masses of finding two collapsed objects of mass \( M_1 \) and \( M_2 \) separated by a distance \( r \) is obtained by integrating both variables of the bivariant Gaussian distribution in \( \delta(M_1) \) and \( \delta(M_2) \), with correlation \( \rho(r) \) from \( \delta_c \) to infinity and then taking the partial derivatives with respect to both masses. The correlation function for collapsed objects is simply obtained from:

\[
\xi_{MM} = p(M_1, M_2, r)/p(M_1)p(M_2) - 1
\]  

which for equal masses and weak correlations \( \rho \ll 1 \) is (Kashlinsky 1987):

\[
\xi_{MM} = [\delta_c^2/\sigma_\rho(M)^2]\rho(r)
\]  

where \( \sigma_\rho(M) \) is the variance of the mass fluctuation and \( \rho \) is the correlation function of the matter density distribution when the density fluctuations had small amplitude. Eq. (39) shows that the correlation of collapsed objects may be enhanced relative to that of the underlying mass fluctuations. This condition is usually described by the bias parameter \( b \) which is sometimes defined as \( \left[\xi_{MM}(r)/\rho(r)\right]^{1/2} = \delta_c/\sigma_\rho(M) \).

Studies of clustering on scales \( \geq 10h^{-1}\text{Mpc} \) have shown that the correlation
function given in Eq. (39) is different from that obtained from observations. The most compelling data are angular correlation functions for the APM survey. These decline much less rapidly on large scales than the CDM prediction (Maddox et al. 1990). As discussed in the introduction there are two ways to reduce the quoted discrepancy: either with a modification of CDM theory involving the physics of early Universe, or with a modification of CDM theory involving the physics of galaxy formation. This discrepancy can be reduced, similarly to the problem of over-abundance of clusters, taking into account the non-radial motions that originate when a cluster reaches the non-linear regime. In fact, the calculation of the correlation function requires the specification of $\delta_c$ which is changed by non-radial motions and whose new value is given by Eq. (21). In Fig. 5 the correlation function derived taking into account non-radial motions is compared both with the two-point correlation function obtained by Sutherland & Efstathiou (1991) from the analysis of Huchra’s et al. (1990) deep redshift survey as discussed in Geller & Huchra (1988) and with the data points for the APM clusters computed by Efstathiou et al. (1992). As shown the discrepancy between pure CDM previsions and experimental data is remarkable. The CDM model seems to have trouble in re-producing the behaviour of the data. In fact, the predicted two-point cluster function is too steep and rapidly goes nearly to zero for $r \approx 30 h^{-1} \text{Mpc}$, while the data show no significant anticorrelation up to $r \approx 60 h^{-1} \text{Mpc}$ (see Borgani 1990). The introduction of non-radial motions gives a more accurate description of the experimental data. The result obtained is in agreement with that of Borgani (1990) who studied the effect of particular thresholds (erfc-threshold and Gaussian-threshold) on the correlation properties of clusters of galaxies. The fundamental difference between our and Borgani’s approach is that our threshold function is physically motivated: it is simply obtained from the assumptions of a Gaussian density field and taking account of non-radial motions. Borgani’s threshold functions (erfc and Gaussian threshold) are ad-hoc introduced in order to reduce the discrepancy between the observed and the CDM predicted two points correlation functions of clusters of galaxies. The connection with the quoted non-sphericity effects, even if logical and in agreement with our results, is only a posteriori tentative to justify the choice made.

8 Conclusions

In these last years many authors have shown the existence of a strong discrepancy between the observed properties of clusters of galaxies and that predicted by the CDM model. To reduce this discrepancy several alternative models have been introduced but no model has considered the role of the non-radial motions. Here we have shown how non-radial motions may reduce some of these discrepancies: namely a) the discrepancy between the observed $\xi(r)$ of cluster of galaxies and that predicted by CDM model; 
b) the discrepancy between the measured X-ray temperature distribution function and that predicted by the CDM model.

To this aim we calculated the variation of the threshold parameter, $\delta_c$, as a function of the mass $M$, produced by the presence of non-radial motions in the outskirts of clusters of galaxies. We used $\delta_c(M)$ to calculate
Figure 5: Clusters of galaxies correlation function. The solid line gives the correlation function for a pure CDM model, with $R_f = 3h^{-1}Mpc$. The dashed line is the same distribution but now taking account of non-radial motions. The observational data refer to the two-point correlation function obtained by Sutherland & Efstathiou (1991) (*filled exagons*) from the analysis of Huchra’s et al. (1990) deep redshift survey and with the data points for the APM clusters computed by Efstathiou et al. (1992) (*dashed errorbars*).
the two-point correlation of clusters of galaxies and the X-ray temperature distribution function. We compared the prediction for the two point correlation function with that obtained by Sutherland & Efstathiou (1991) from the analysis of Huchra's et al. (1990) deep redshift survey as discussed in Geller & Huchra (1988) and with the data points for the APM clusters computed by Efstathiou et al. (1992). The prediction for the X-ray temperature function was compared with Henry & Arnaud (1991) and Edge et al. (1990) X-ray temperature distributions for local clusters. Our results (see Fig. 4 and Fig. 5) show how the non-radial motions change both the correlation length of the correlation function, making it less steep than that obtained from a pure CDM model where the non-radial motions are not considered, and the X-ray temperature function. In both cases our model gives a good agreement with the data.

Finally we calculated the bias coefficient using a selection function that takes into account the effects of non-radial motions, and we show that the bias so obtained can account for a substantial part of the total bias required by observations on cluster scales.

References

[1] Antonuccio-Delogu V., Colafrancesco S., 1994, ApJ, 427, 72
[2] Antonuccio-Delogu, V., Atrio, F., 1992, ApJ, 392, 403
[3] Antonuccio-Delogu, V., Colafrancesco, S., 1995, preprint Sissa astro-ph/9507002
[4] Babul A., White S.D.M., 1991, MNRAS, 253, 1P
[5] Bahcall N.A., 1988, A&A, 26, 631
[6] Bahcall, N.A., Soneira, R.M., 1983, ApJ 270, 20
[7] Bardeen J.M., Bond J.R., Kaiser N., Szalay A.S., 1986, ApJ, 304, 15 (BBKS)
[8] Barnes J., Efstathiou G., 1987, ApJ, 319, 575
[9] Barrow, J.D., Silk, J., 1981, ApJ 250, 432
[10] Bartlett J.G., 1997, Sissa preprint, astro-ph/9703090
[11] Bartlett, J.G., Silk, J., 1993, ApJ 407, L45
[12] Blumenthal, G.R., Faber S.G., Primack, J.R, Rees, M.J., 1984, Nat., 311, 517
[13] Bond J.R., Cole S., Efstathiou G., Kaiser N., 1991, ApJ, 379, 440
[14] Bond J.R., Efstathiou G., 1984, ApJ, 285, L45
[15] Bond J.R., Efstathiou G.P.E., Silk J., 1980, Phys. Rev. Lett. 45, 1980
[16] Bond, J.R., Szalay A.S., Silk J., 1988, ApJ, 324, 627
[17] Borgani, S., 1990, A&A 240, 223
[18] Bower R.G., Coles P., Frenk C.S., White S.D.M., 1993, ApJ, 405, 403
[19] Brainerd T.G., Villumsen J.V., 1992, ApJ, 394, 409
[20] Carlberg R.G., 1991, ApJ, 367, 385
[21] Cavaliere A., Colafrancesco S., 1988, ApJ, 331, 660
[22] Cen R.Y., Gnedin N.Y., Kofman L.A., Ostriker J.P., 1992 preprint
[23] Cen R.Y., Ostriker J.P., 1992, ApJ, 399, L113
[24] Colafrancesco S., Antonuccio-Delogu V., Del Popolo A., 1995, ApJ, 455, 32 (CAD)
[25] Coles P., 1993, MNRAS, 262, 1065-1075
[26] Croft R. A. C., Efstathiou G., 1994, MNRAS, 267, 390-400
[27] Crone, M.M., Evrard A.E., Richstone, D.O., 1994, ApJ 434, 402
[28] Davis M., Efstathiou G., Frenk C.S., White S.D.M., 1985, ApJ, 292, 371
[29] Davis M., Peebles P.J.E., 1977, ApJS., 34, 425
[30] Davis M., Peebles P.J.E., 1983, ApJ, 267, 465
[31] Davis M., Summers F.J., Schlegel D., 1992, Nat., 359, 393
[32] Davis, M., Peebles, P.J.E., 1977, ApJS 34, 425
[33] Dekel A., Bertshinger E., Yahil A. et al. 1992, IRAS galaxies verses POTENT mass: density fields, biasing and Ω, Princeton preprint IASSNS-AST 92/55
[34] Dekel A., Blumenthal G.R., Primack J.R., Olivier S., 1989, ApJ, 338, L5
[35] Dekel A., Rees M., 1987, Nat, 326, 455
[36] Dekel A., Silk J., 1986, ApJ, 303, 39
[37] Dekel, A., Aarseth, S.J., 1984, ApJ 238, 1
[38] Del Popolo A., Gambera M., 1996, A&A, 308, 373
[39] Doroshkevich A.J., 1970, Astrophysics 3, 320
[40] Edge A.C., Stewart G.C., Fabian A.C., Arnaud K.A., 1990, MNRAS, 245, 559
[41] Efstathiou G., Kaiser N., Saunders W. et al. 1990, MNRAS, 247, 10p

[42] Efstathiou G., 1990, in "The physics of the early Universe", eds Heavens A., Peacock J., Davies A., (SUSSP)

[43] Efstathiou G., Bond J.R., White S.D.M., 1992, MNRAS, 258, 1P

[44] Efstathiou G., Frenk C.S., White S.D.M., Davis M., 1988, MNRAS, 235, 715

[45] Efstathiou G., Rees M.J., 1988, MNRAS, 230, 5p

[46] Efstathiou G., Sutherland W.J., Maddox S.J., 1990, Nat., 348, 705

[47] Efstathiou, G., 1990, in "The physics of the early Universe", eds Heavens, A., Peacock, J., Davies, A., (SUSSP)

[48] Efstathiou, G., Dalton, G. B., Sutherland, W. J., Maddox, S. J., 1992, MNRAS 257, 125

[49] Evrard A.E., 1989, ApJ, 341, L71

[50] Evrard A.E., 1990, in Proc. STScI Symp.4, ed. W.R. Oegerle, M.J. Fitchett, & L. Danly (New York: Cambridge Univ. Press), 287

[51] Evrard A.E., 1991, in Clusters of Galaxies ed. M. Fitchett & W. Oegerle (Cambridge: Cambridge Univ. Press.)

[52] Evrard A.E., Crone M.M., 1992, ApJ, 394, L1

[53] Faber S.M., 1982 in Astrophysical Cosmology: Proceedings of the Vatican Study Week on Cosmology and Fundamental Physics, ed. H.A. Brück, G.V. Coyne, and M.S. Longair (Rome: Specola Vaticana), p. 191

[54] Fall S.M., 1983, in IAU Symposium 100., International Kinematics and Dynamics of Galaxies, ed. E., Athanassoula (Dordrecht: Reiden), p. 391

[55] Filmore, J. A., Goldreich, P., 1984, ApJ 281, 1

[56] Fisher K.B., Davis M., Strauss M.A., Yahil A., Huchra J.P., 1993, ApJ, 402, 42

[57] Frenk C.S., White S.D.M., Davis M., Efstathiou G., 1988 ApJ, 327, 507

[58] Frenk C.S., White S.D.M., Efstathiou G., Davis M., 1990, ApJ, 351, 10

[59] Geller, M.J., Huchra, J.P., 1988, in Large Scale Motions in the Universe, eds. V. C. Rubin, G.V., Coyne, Princeton University Press, Princeton

[60] Gorski K., 1988, ApJ, 332, L7
[61] Groth E.J., Juszkiewicz R., Ostriker J.P., 1989, ApJ, 346, 558
[62] Gunn J.E., 1977, ApJ, 218, 592
[63] Gunn J.E., Gott J.R., 1972, ApJ, 176, 1 (GG)
[64] Hauser, M.G., Peebles, P.J.E., 1973, ApJ 185, 757
[65] Henry J.P., Arnaud K.A., 1991, ApJ, 372,410
[66] Hoffman Y., 1986, ApJ, 301, 65
[67] Hoffman, Y., Shalam, J., 1985, ApJ 297, 16
[68] Holtsmark, P.J., (1919), Phys. Z., 20, 162
[69] Holtzman J., 1989, ApJS, 71, 1
[70] Holtzman J., Primack J., 1990, Phys. Rev. D 43, 3155
[71] Holtzman J., Primack J., 1993, ApJ, 405, 428
[72] Hoyle F., 1949, in IAU and International Union of Theorehtical and Applied Mechanics Symposium, p. 195
[73] Huchra, J.P., Henry, J.P., Postman, M., Geller, M.J., 1990, ApJ 365, 66
[74] Kaiser N., 1984, ApJ, 284, L9
[75] Kaiser N., 1986, MNRAS, 219, 785
[76] Kaiser N., Efstathiou G., Ellis R. et al. 1991, MNRAS, 252, 1
[77] Kaiser N., Lahav O., 1989, MNRAS, 1989, 237, 129
[78] Kashlinsky A., 1982, MNRAS, 200, 585
[79] Kashlinsky, A., 1987, ApJ 317, 19
[80] Kauffmann G., Nusser A., Steinmetz M., 1996, submitted to MNRAS, and preprint MPA 900 see also preprint Sissa astro/ph 9512009
[81] Klypin, A.A., Kopylov, A.I., 1983, Soviet Astron. Lett., 9, 41
[82] Klypin, A. A., Nolthenius R., Primack J., 1997, ApJ, 474, 533
[83] Lacey C., Cole S., 1994, MNRAS, 271, 676
[84] Lahav O., Edge A., Fabian A.C., Putney A., 1989, MNRAS, 238, 881
[85] Lahav O., Rowan-Robinson M., Lynden-Bell D., 1988, MNRAS, 234, 677
[86] Liddle A.R., Lyth D.H., 1993, Phys. Rep., 231, n 1, 2
[87] Lilje P.B., 1990, ApJ, 351, 1
[88] Lokas E.L., Juszkiewicz R., Bouchet F.R., Hivon E., 1996, ApJ, 467, 1
[89] Maddox S.J., Efstathiou, G., Sutherland, W.J., Loveday, J., 1990, MNRAS 242, 43p
[90] Marzke R.O., Geller M.J., da Costa L.N., Huchra J.P., 1995, AJ 110, 477
[91] Mo H.J., Jing Y.P., Borner G., 1993, MNRAS, 217, 825
[92] Olivier, S., Blumenthal, G.R., Primack, J.R., Stanhill, D., 1990, ApJ 356, 1
[93] Oukbir J., Bartlett J.G., Blanchard A., 1996, Sissa preprint astro-ph/9611089
[94] Peacock J.A., 1991, MNRAS, 253, 1p
[95] Peacock J.A., Heavens A.F., 1990, MNRAS, 243, 133
[96] Peacock J.A., Nicholson D., 1991, MNRAS, 253, 307
[97] Peebles P.J.E., 1969, ApJ, 155, 393
[98] Peebles P.J.E., 1980, The large scale structure of the Universe, Princeton University Press
[99] Peebles P.J.E., 1982, ApJ, 258, 415
[100] Peebles P.J.E., 1984, ApJ, 284, 439
[101] Peebles P.J.E., 1990, ApJ, 365, 27
[102] Peebles P.J.E., 1993, Principles of Physical Cosmology, Princeton University Press
[103] Pen U., 1997, Sissa preprint astro-ph/9610147
[104] Postman M., Geller M.J., Huchra J.P., 1986, ApJ, 91, 1267
[105] Press, W.H., Schechter, P., 1974, ApJ 187, 425
[106] Quinn P.J., Salmon J.K., Zurek, W.H., 1986, Nat, 322, 329
[107] Quinn P.J., Zurek W.H., 1988, ApJ, 331, 1
[108] Rees M.J., 1985, MNRAS, 213, 75p
[109] Ryden B.S., 1988, ApJ, 329, 589
[110] Ryden B.S., Gunn J.E., 1987, ApJ, 318, 15
[111] Saunders, W., et al., 1991, Nature 349, 32
[112] Schaefer R.K., 1991, Int. J. Mod. Phys., A6, 2075
[113] Schaefer R.K., Shafi Q., 1993, Phys. Rev., D47, 1333
[114] Schaefer, R.K., Shafi, Q., Stecker, F., 1989, ApJ 347, 575
[115] Shafi Q., Stecker F.W., 1984, Phys. Rev. D 29, 187
[116] Shanks T., Hale-Sutton D., Fong R., Metcalfe N., 1989, MNRAS, 237, 589
[117] Silk J., Wyse R.F.G., 1993, Phys. Rep., 231, 293
[118] Smoot G.F., et al., 1992, ApJ, 396, L1
[119] Strauss M.A., Davis M., Yahil A., Huchra J.P., 1992, ApJ, 385, 421
[120] Strauss M.A., Willick J.A., 1995, Phys. Rept., see also preprint Sissa astro-ph/9502079
[121] Sutherland W.J., 1988, MNRAS 234, 159
[122] Sutherland W.J., Efstathiou G., 1991, MNRAS, 248, 159
[123] Szalay A.S., Silk J., 1983, ApJ, 264, L31
[124] Turner, M.S., 1991, Phys. Scr. 36, 167
[125] Valdarnini, R., Bonometto, S.A., 1985, A&A 146, 235
[126] van de Weygaert R.W., Babul A., 1994, ApJ, 425, L59
[127] Villumsen, J.V., Davis, M., 1986, ApJ, 308. 499
[128] Warren, M. S., Zurek, W. H., Quinn, P. J., Salmon, J. K., 1991, in After the First Three minutes, ed. S. Holt, V. Trimble, & C. Bennett (New York: AIP), 210
[129] White S.D.M., 1984, ApJ, 286, 38
[130] White S.D.M., Efstathiou, G., Frenk, C.S., 1993, The amplitude of the mass fluctuations in the Universe, Durham preprint
[131] White S.D.M., Efstathiou G., Frenk C.S., 1993, MNRAS, 262, 1023
[132] White S.D.M., Frenk C.S., Davis M., Efstathiou G., 1987, ApJ, 313, 505
[133] White, S.D.M., Davis, M., Efstathiou, G., Frenk C.S., 1987, Nature 330, 451
[134] Yano T., Masahiro N., Gouda N., 1996, ApJ, 466, 1
[135] Zaroubi, S., Naim, A., Hoffman, Y., 1996, ApJ 457, 50
[136] Zurek, W.H., Quinn, P.J., Salmon, J.K., 1988, ApJ 330, 519