Do Transformers use variable binding?

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Abstract

Increasing the explainability of deep neural networks (DNNs) requires evaluating whether they implement symbolic computation. One central symbolic capacity is variable binding: linking an input value to an abstract variable held in system-internal memory. Prior work on the computational abilities of DNNs has not resolved the question of whether their internal processes involve variable binding. We argue that the reason for this is fundamental, inherent in the way experiments in prior work were designed. We provide the first systematic evaluation of the variable binding capacities of the state-of-the-art Transformer networks BERT and RoBERTa. Our experiments are designed such that the model must generalize a rule across disjoint subsets of the input vocabulary, and cannot rely on associative pattern matching alone. The results show a clear discrepancy between classification and sequence-to-sequence tasks: BERT and RoBERTa can easily learn to copy or reverse strings even when trained on task-specific vocabularies that are switched in the test set; but both models completely fail to generalize across vocabularies in similar sequence classification tasks. These findings indicate that the effectiveness of Transformers in sequence modelling may lie in their extensive use of the input itself as an external “memory” rather than network-internal symbolic operations involving variable binding. Therefore, we propose a novel direction for future work: augmenting the inputs available to circumvent the lack of network-internal variable binding.

1 Introduction

The ability of connectionist systems to conduct symbolic computation is a topic of longstanding controversy [Fodor and Pylyshyn, 1988, Marcus, 2001, 2020, Hummel, 2011]. Deep neural networks (DNNs) based on attention [Luong et al., 2015, Vaswani et al., 2017] have shown strong performance in characteristically symbolic tasks, such as linguistic reasoning [Petroni et al., 2019, Clark et al., 2020], detecting grammatical structure [Jawahar et al., 2019, Tenney et al., 2019], and mathematical problem solving [Kaiser and Sutskever, 2016, Lample and Charton, 2020]. At the same time, they appear to be prone to errors on novel datapoints [Zhang et al., 2019], using irrelevant information [Mickus et al., 2020], lexical biases [Kurita et al., 2019], and susceptibility to adversarial examples [Li et al., 2020]. The question of whether DNNs conduct symbolic reasoning still remains open.

A critical task in resolving this question – given the propensity of DNNs to resort to superficial heuristics [Gururangan et al., 2018, McCoy et al., 2019] – is to distinguish between the associative memorization of feature correlations in the training set, and the genuine internalization of abstract symbolic rules. The computational capacities of DNNs have been investigated across many prima facie symbolic tasks [Lake and Baroni, 2018, Kassner et al., 2020, Hupkes et al., 2020, Clark et al., 2020, Tafjord et al., 2021]. However, prior results remain ambiguous since they do not properly rule out possible non-symbolic explanations of model performance. We argue that this problem arises from two (related) sources: the lack of baseline information on how well a non-symbolic model would perform in the relevant task, and the lack of

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a proper definition of what kind of computation can be deemed “symbolic” (Section 2).

We propose that an appropriate criterion for implementing symbolic computation is the distinction between a read-write memory and a processor (Sections 2-3). This is needed for variable binding, which is central to von Neumann architectures (via location-based memory addressing) and has also been argued to have a crucial role in biological cognition [Marcus, 2001, 2020, Gallistel and King, 2010, Hummel, 2011]. Variable binding involves linking the input to an abstract system-internal variable, on which computational operations are performed. Since the variable has no intrinsic connection to the value, the link needs to be explicitly memorized. This requires a read-write memory and permits central symbolic operations such as copying, which are applicable to any input regardless of its specific identity.

We present the first experimental evaluation of variable binding in the state-of-the-art Transformer networks BERT [Devlin et al., 2019] and RoBERTa [Liu et al., 2019]. The experiment pipeline is based on the requirement of generalizing a rule across disjoint vocabularies (Section 4). We call this vocabulary generalization. We train the model with distinct vocabularies for different types of target outputs, and then measure its generalization performance when the vocabularies and target output types are flipped in the test set. Success in the test set thus requires generalizing the rule beyond the vocabulary-task pairs seen during training.

Our results are twofold (Section 5). First, we train BERT and RoBERTa on the sequence-to-sequence (seq2seq) task of either copying or reversing the input string. These are separated by a sequence-initial task-marker token. In the seq2seq experiments, both models succeed perfectly at vocabulary generalization in the test set. Second, we train the models to classify sequences or sequence pairs across three tasks: distinguishing between copies and non-copies, recognizing the repetition of some token in the sequence, and distinguishing between copies and reversals. In the classification experiments, both models completely fail at vocabulary generalization.

We explain the striking contrast by the seq2seq task allowing the use of the input itself as an “external memory” (Section 6). Transformers attend to certain input positions based on other aspects of the input, such as the task marker token. This resembles location-based memory addressing: the task marker gives the network an instruction to look up the value of a certain other input position. Here, the encoder-decoder network has the role of a processor in a larger computational system which includes the input and output as external memory components. Our classification tasks do not permit the use of the input in such a way, instead restricting all computational steps to model-internal operations. The failure of vocabulary generalization in the classification tasks thus indicates the lack of model-internal variable binding.

Our findings illustrate both the challenge and promise of using Transformers for symbolic computation that requires variable binding, depending on how well the input is usable as an external memory. Echoing recent theoretical results concerning the Turing-completeness of attention [Pérez et al., 2021], we propose that future work should focus on expanding intermediate inputs, which would give the model further information about the input or its own prior states. We discuss both the technical aspects of this approach and its ramifications for concrete use cases.

We list our main contributions below. We provide all source code for our experiments as supplementary material, and will make it available as open-source.

- We demonstrate that prior work on the use of DNNs for symbolic computation is inconclusive, for fundamental reasons (Section 2).
- We defend the following theoretical claims:
  1. symbolic computation is based on a distinction between a memory and a processor (Section 2).
  2. the capacity for variable binding is a sufficient criterion for symbolic computation (Section 3).
  3. variable binding is operationalizable via vocabulary generalization (Section 4).
- We provide the first generic experiment pipeline for studying vocabulary generalization in sequence modelling tasks (Section 4).
- We show that BERT and RoBERTa generalize across vocabularies in our seq2seq
tasks but not our sequence classification tasks (Section 5).

• We explain our results by the availability of the input itself as an external memory in the seq2seq tasks but not the classification tasks (Sections 6.1–6.2).

• We propose that the variable binding capacities of Transformers could be expanded through the use of model-external intermediate inputs (Section 6.3).

2 Background and related work

Fodor and Pylyshyn [1988] originally raised the question of whether connectionist models display systematicity, where certain computational abilities are reliably linked with others. For example, a systematic cognitive system able to understand John saw Mary could also understand Mary saw John. Controversially, Fodor and Pylyshyn asserted that systematicity requires a Turing architecture that encodes symbolic rules, which simple connectionist systems (e.g. MLPs) lack. The matter has remained a topic of long-lasting debate without a general consensus [Aizawa, 2003, McLaughlin, 2009, Kiefer, 2019]. Since the industrial deep learning revolution of the 2010s, similar fundamental questions about computational capacities have resurfaced for state-of-the-art DNNs: LSTMs [Hochreiter and Schmidhuber, 1997], GRUs [Cho et al., 2014], and Transformers [Vaswani et al., 2017].

DNN research and applications have largely followed an end-to-end approach that focuses on improving model performance rather than understanding it theoretically [Church, 2017, Church and Liberman, 2020]. However, increasing emphasis has recently been placed on model explainability [Danilevsky et al., 2020, Angelov et al., 2021]. This development is manifested by the common use of human-readable symbolic rules for analyzing the results of procedures like attention visualization [Thorne et al., 2019, Vig, 2019] or structural probing [Hewitt and Manning, 2019, Chen et al., 2021]. For example, multiple NLP studies have interpreted BERT as constructing classical linguistic representations such as syntax trees, dependency graphs, or semantic roles [Jawahar et al., 2019, Kovaleva et al., 2019, Tenney et al., 2019, Manning et al., 2020]. These are paradigm examples of symbolic representations.

Such work relies on the idea that DNNs implement symbolic processes similar to those used by traditional rule-based AI systems and theoretical analyses in related scientific disciplines (e.g. linguistics, mathematics, or logic). This view resembles implementational connectionism as opposed to radical connectionism, where the former sees neural networks as implementing symbolic computation on a sub-symbolic level while the latter sees them as replacing the symbolic paradigm on the whole [Horgan and Tienson, 1996].

The shift in DNN technology from an end-to-end approach toward “explainable AI” [Angelov et al., 2021] thus correlates with a shift in the general orientation toward symbolic rules: reliance on “black boxes” reflected radical connectionism, whereas the increasing aim for greater explainability reflects implementational connectionism. The theoretical considerations have remained largely implicit in empirical research, being embodied in the respective practices of either (i) ignoring symbolic rules or (ii) relying on them when interpreting model performance.

Conjectures about the symbolic capacities of DNNs have also been challenged by empirical research. Lake and Baroni [2018] illustrate the difficulties of LSTMs in tasks that require systematic inference. Goodwin et al. [2020] argue that LSTMs and GRUs encode input tokens in context-sensitive ways that discourage systematic generalization. Hupkes et al. [2020] evaluate LSTMs and Transformers with respect to various capacities related to compositionality [Montague, 1970, Fodor and Lepore, 2002], and deem that none of the models they experiment on reliably exhibits all of them. Kassner et al. [2020] evaluate BERT’s abilities on six symbolic reasoning tasks, with only partial success. DNNs have a general tendency to rely on surface heuristics [Gururangan et al., 2018, McCoy et al., 2019, Mickus et al., 2020], which can lead to biases [Kurita et al., 2019, Nadeem et al., 2021] and susceptibility to adversarial examples [Li et al., 2020]. These considerations indicate that models often prefer associative pattern matching to abstract symbolic rules. However, this does not yet mean that they could not learn symbolic rules with appropriate training.
There are two critical challenges in drawing conclusions about the symbolic capacities of DNNs based on prior work: (i) the lack of baseline information on how a non-symbolic system would perform; and (ii) the lack of general agreement of what constitutes “symbolic” computation.

The first problem can be illustrated by the quote below:

“It has been shown that such a language model [as BERT] contains certain degrees of syntactic [Goldberg, 2019], semantic [Clark et al., 2019], common-sense [Cui et al., 2020] and logical reasoning [Clark et al., 2020] knowledge.” (Liu et al. 2021, p. 13392; our emphases; references reformatted by us)

The quote is unproblematic if “knowledge” is understood in the procedural sense of “knowing how” rather than the propositional sense of “knowing that” [Ryle, 1949]: BERT has e.g. common-sense knowledge if it performs sufficiently well in end-to-end tasks that have been defined as common-sense reasoning. In contrast, the jump from procedural to propositional knowledge requires the crucial additional assumption that procedural knowledge could only be achieved via propositional knowledge. Applied to AI systems, the inference from successful task performance to the internalization of symbolic information relies on the assumption that such internalization is necessary for the performance. Summarized below, the argument from premises P1–P2 to the conclusion is only valid with the additional premise P3:

P1: task T can be described by the symbolic rule R
P2: model M succeeds in T (in an end-to-end setting)
P3: M would fail in T without internalizing R

⇒ M has internalized R

Studies cited above – both those advocating the symbolic capacities of DNNs and those expressing scepticism – have treated P3 as a tacit assumption without providing baseline information to support it. Model success/(failure) at a task that can be described via a symbolic rule has been treated as direct evidence for/(against) the model having internalized the rule. This is insufficient since we lack the knowledge on how well the model could perform in the task without having internalized the rule.

The second major problem for studying the symbolic capacities of DNNs is characterizing “symbolic” computation. Often this is left unexplained: e.g. Hoehndorf and Queralt-Rosinach (2017) state that symbolic systems “represent things (...) through physical symbols, combine symbols into symbol expressions, and manipulate symbols and symbol expressions” (p. 27; our emphases). Below, we raise five characteristics commonly assigned to symbols.

Model generalization. The end-to-end focus of DNN research has resulted in symbolicity being given mostly operational definitions. Kassner et al. (2020) consider the criterion for symbolic reasoning to be the ability to “infer knowledge not seen explicitly during pretraining” (p. 552). However, since this applies to any machine learning task with a train-test split, it is not limited to symbolic reasoning. A more specific idea is that the models should generalize beyond the training distribution [Bengio, 2014; Marcus, 2020]. This, too, is inadequate without further elaboration, as non-symbolic associative processes could also generalize across certain aspects of training and test distributions.

Semantics. In the field of semiotics, symbols are taken to be signs that bear an arbitrary relation to their referents, based on social convention rather than on resemblance as in iconic signs, or factual relation as in indexical signs [Peirce, 1868]. Alternatively, the word “symbol” can be used of anything that has a semantic interpretation, basically assimilating it to the semiotic notion of sign. However, semantic definitions of “symbol” can apply to a wide range of entities, not restricted to units or structures in computational systems. The necessity of semantic interpretation for computational symbols has also been challenged [Pylyshyn, 1984, Egan, 2014; Piccinini, 2015]. The relationship between formal computation and semantic content deserves more detailed scrutiny in future NLP research (cf. Bender and Koller 2020).

Discreteness. Symbols are sometimes assimilated to discrete/digital units or structures, set against the contiguous/analogical representations used by DNNs (e.g. Lample and Charton 2020, Cartuvvels et al. 2021). However, this is problematic in both directions. Discrete representations and operations can also be used in non-symbolic systems, such as classical perceptrons [Rosenblatt, 1958]. Moreover, symbolic computation can be
either digital or analog \cite{Gallistel:2010,Piccinini:2015}; see Section 3).

Symbolic vs. numerical mathematics. Mathematical expressions are symbolic if they contain variables instead of specific numerical values. This is insufficient to define symbolic computation more broadly, which can apply beyond mathematical expressions to e.g. natural language. However, variables have a crucial role in our experiment design (Sections 3–4). As we will argue, the capacity to use variables relies on a fundamental architectural constraint.

Memory-processor distinction. A prevalent computational criterion for symbolic processing is the presence of a read-write memory which is accessed by a separate processor that enacts read and write operations \cite{Gallistel:2010}. Symbols are units stored in the memory and manipulated by the processor. In Turing machines \cite{Turing:1937} the memory-processor distinction corresponds to the division between the tape and the read/write head, and in von Neumann architectures \cite{von-Neumann:1945} to the division between the memory unit and the processor.

Based on this discussion, we endorse the last characterization of symbolic computation and develop it further in Section 3.

3 Problem statement

Treating computation as symbolic by virtue of the memory-processor distinction is an architectural criterion. It accounts for the symbolicity of standard digital computers as they instantiate von Neumann architectures. On the other hand, it does not restrict computation to digital units or operations. Both the symbols in memory and the read/write operations can also be analog, i.e. real-valued. This raises the possibility that (some) DNNs could be such analog symbolic systems. What matters for symbolicity is not the discreteness of representations but the internal organization of the implementational system.

Our research question is thus: how to assess whether a DNN manifests a memory-processor distinction in its internal structure, such that stages of the forward pass implement read/write operations on structures held in memory?

We aim to approach the research question experimentally. The design for an experimental task should fulfil the following two criteria:

1. success in the task requires meeting a sufficient condition for model-internal symbolic computation, i.e. the memory-processor distinction; and
2. failure in the task can be readily explained without assuming the memory-processor distinction

Following \cite{Marcus:2001, Marcus:2018, Marcus:2020}, we consider variable binding to be a sufficient criterion for symbolic computation, and one of the most central aspects of real-world symbolic systems. Variable binding requires the system to implement binding relations that link variables to instances, and computational operations involving variables rather than only the instances. Both the instance and variable are symbols held in memory. In von Neumann architectures, variable binding is implemented via location-based addressing and pointer architectures. Similar ideas have also been adopted in cognitive psychology \cite{Blouw:2016, Green:2017, Quilty-Dunn:2020}.

Crucially, the memory-processor distinction is required to allow the indefinite maintenance of the binding relation in the memory across changes to the rest of the system during computation.

The most crucial part in experiment design is ensuring that variable binding is strictly needed. The model should learn a rule that (i) abstracts away from input token identities, but also (ii) produces different outputs from different inputs. The first condition ensures the abstractness of the rule, and the second ensures that the original input is retained in memory. Informally, the model should learn to apply rule \( R \) on input string \( X \) whatever \( X \) is (within the known vocabulary) but without forgetting what \( X \) is. A simple example is detecting whether two tokens are identical. The fact that \( a a \) and \( b b \) contain two identical tokens is clearly dependent on identities of the tokens \( a \) and \( b \) in the sense that they are not interchangeable. On the other hand, the general rule is independent of their identities: the string \( x y \) contains identical tokens if \( x = y \), regardless of the specific identities of \( x \) and \( y \). Our experiment design is built for detecting this capacity.

4 Experiment design

To test the presence of variable binding, we evaluate whether the model learns to generalize rules
Algorithm 1 Experiment design.

**Arguments:**
- vocabulary voc
- task classes \( \{ C_1, C_2 \} \)
- training set size per task class \( n^{tr} \)
- test set size per task class \( n^{test} \)
- mix ratio \( \text{MIX} \in [0, 0.5] \)
- model architecture \( M \)

\( \text{divide voc to disjoint subsets } V_1 \text{ and } V_2 \)

\( \text{TRAIN} = \emptyset \)

\( \text{for } i \in \{1, \ldots, n^{tr}\} \text{ do} \)

- randomly sample \( r \) from \([0, 1]\)
- generate \((i_1^{tr}, o_1^{tr})\) and \((i_2^{tr}, o_2^{tr})\)
  such that:
  - every token of \( i_1^{tr} \) is from \( V_1 \)
  - every token of \( i_2^{tr} \) is from \( V_2 \)
  - \( o_1^{tr} \) and \( o_2^{tr} \) are target outputs of \( i_1^{tr} \) and \( i_2^{tr} \)
  - if \( r < \text{MIX} \) then
    - \( i_1^{tr} \in C_2 \) and \( i_2^{tr} \in C_1 \)
  - else
    - \( i_1^{tr} \in C_1 \) and \( i_2^{tr} \in C_2 \)

\( \text{TEST} = \emptyset \)

\( \text{for } i \in \{1, \ldots, n^{test}\} \text{ do} \)

- generate \((i_1^{test}, o_1^{test})\) and \((i_2^{test}, o_2^{test})\)
  such that:
  - every token of \( i_1^{test} \) is in \( V_2 \)
  - every token of \( i_2^{test} \) is in \( V_1 \)
  - \( o_1^{test} \) and \( o_2^{test} \) are target outputs of \( i_1^{test} \) and \( i_2^{test} \)
  - \( i_1^{test} \in C_1 \) and \( i_2^{test} \in C_2 \)

\( \text{TRAIN} = \text{TRAIN} \cup \{(i_1^{tr}, o_1^{tr}), (i_2^{tr}, o_2^{tr})\} \)

\( \text{TEST} = \text{TEST} \cup \{(i_1^{test}, o_1^{test}), (i_2^{test}, o_2^{test})\} \)

\( \text{Train}(M, \text{TRAIN}) \)

\( \text{Test}(M, \text{TEST}) \)

across disjoint subsets of the vocabulary: \( V_1 \) and \( V_2 \). We call this \textit{vocabulary generalization}. We split inputs into two \textit{task classes}: \( C_1 \) and \( C_2 \). In classification experiments, these are based on disjoint subsets of target classes. In seq2seq experiments, they represent two distinct tasks that are marked with a special task-marker token in the beginning of each input string. In the basic case, we use \( V_1 \) for \( C_1 \) and \( V_2 \) for \( C_2 \). The trained model is then tested with flipped vocabularies: \( V_2 \) for \( C_1 \) and \( V_1 \) for \( C_2 \). Algorithm 1 shows the generic experiment pipeline.

The basic experiment setting relates the vocabularies and task classes with a 1–1 mapping, flipped in the test set. This ensures that test set success would require rule generalization from one vocabulary to the other. However, it might also encourage a simple vocabulary-specific “shortcut” during training: mapping inputs from \( V_1 \) to one kind of target and inputs from \( V_2 \) to another, regardless of their other properties. Since this is in line with all input-target pairs in the training and evaluation sets, the model does not receive evidence against it in the basic setting.

To steer the model away from the vocabulary-specific shortcut during training, we introduce random mixing of vocabularies and task classes regulated by a \textit{mix ratio} \( \text{MIX} \). This is the probability of switching the vocabulary-task pairing in any training datapoint. If \( \text{MIX} = 0 \), no such switching occurs. If \( \text{MIX} = 0.5 \), both \( V_1 \) and \( V_2 \) are equally likely to appear in both task classes. Using a low but non-zero value of \( \text{MIX} \) (e.g., 0.01) gives the model “hints” that the rule should be applied across vocabularies, while still maintaining a strong bias toward the link between vocabularies and task classes. If the model failed when \( \text{MIX} = 0 \) but succeeded when \( \text{MIX} \) is a low non-zero value, this would indicate that even only a few counterexamples to the vocabulary-task link can make the model “realize” the generic rule.

We experimented on four sequence modelling tasks, listed in Table 1 and described below. We trained both BERT and RoBERTa on each task with four values of \( \text{MIX} \): 0, 0.01, 0.1, and 0.5. We used the vocabulary size of 15 for both \( V_1 \) and \( V_2 \) (hence 30 overall), and 20000 datapoints in both training and test sets with a 20% evaluation split. Each datapoint was a randomly generated sequence (of length 1–10) from the dedicated vocabulary, with ensured token repetition or the lack of it used for the repetition detection task classes. We trained each model for 10 epochs and applied the best-performing variant in the evaluation set to the test set. (Appendix A provides the complete hyperparameters.)

The four tasks are (see Table 1):

**Copy detection.** This sequence pair classification task requires mapping two input sequences to Boolean markers of whether they are identical (\( C_1 \)) or not (\( C_2 \)).

**Repetition detection.** In this sequence classification task we map sequences to Boolean markers of whether at least one token is repeated (\( C_1 \)) or no token is repeated (\( C_2 \)).

**Copy/reverse detection.** This task is similar to copy detection, except that each sequence pair consists either of identical sequences (\( C_1 \)) or a sequence and its reversal (\( C_2 \)).

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1 A similar technique of gradually introducing test-like datapoints to the training set was used by Lake and Baroni 2015 for LSTMs in systematic inference tasks. In their experiment settings, small mixing did not significantly improve model performance across vocabulary-task combinations. This is in line with our results on BERT and RoBERTa (Section 4).

2 To avoid ambiguity between the two task classes, we discarded palindromes in both training and test sets. We also did this for the copying/reversal seq2seq task.
Examples were derived from $V_1 \cup V_S$ and $V_2 \cup V_S$ instead of only $V_1$ and $V_2$ (cf. Algorithm 1). Increasing $V_S$ should encourage vocabulary generalization, since the model receives more evidence of the task classes being independent of specific token identities.

Keeping the overall vocabulary size constant (30), we re-ran each experiment with the $V_S$ sizes of 10

Table 1: Sequence modelling tasks (input $\rightarrow$ output).

| Task                  | Examples | task class | input   | output  |
|-----------------------|----------|------------|---------|---------|
| Copy detection        | $C_1$    | (ab, ab)   | True    |         |
|                       | $C_2$    | (ab, cd)   | False   |         |
| Repetition detection  | $C_1$    | aba        | True    |         |
|                       | $C_2$    | abc        | False   |         |
| Copy/reverse detection| $C_1$    | (ab, ab)   | Copy    |         |
|                       | $C_2$    | (ab, ba)   | Reverse |         |
| Copying/reversal      | $C_1$    | Copy ab    | ab      |         |
|                       | $C_2$    | Reverse ab  | ba      |         |

Copying/reversal. This seq2seq task requires mapping strings either to themselves ($C_1$) or their reversals ($C_2$), based on a task marker token as an input prefix.

5 Results

We summarize our main experimental findings here, and provide the complete data in Appendix B. Neither BERT nor RoBERTa generalized across vocabularies in any classification task. In contrast, both models generalized perfectly in the copying/reversal seq2seq task even with $\text{mix} = 0$.

5.1 Task-specific results

We review each task with respect to BERT and RoBERTa’s overall performance and the impact of $\text{mix}$.

Copy detection. Evaluation F1-scores were consistently high (0.96 – 1.00) regardless of $\text{mix}$, which illustrates that both models learned to recognize copies in vocabulary-specific settings. This is further demonstrated by the high test scores with $\text{mix} = 0.5$ (0.97 – 0.98). With $\text{mix} = 0.1$, BERT’s test score stayed relatively high (0.77) while RoBERTa’s was lower but still clearly above zero (0.37). However, test scores remained $\leq 0.04$ with $\text{mix} = 0.01$ and $\leq 0.01$ with $\text{mix} = 0$, showing no vocabulary generalization. Thus, both models were able to learn copy detection, but only in a vocabulary-specific manner.

Repetition detection. BERT’s evaluation F1-score was consistently high across all $\text{mix}$ variants (0.91 – 1.00), and RoBERTa’s was similar except slightly lower with $\text{mix} = 0.5$ (0.86). Similarly, test scores with $\text{mix} = 0.5$ were 0.99 for BERT and 0.87 for RoBERTa. Hence, both models were able to learn the task in vocabulary-specific settings. However, test scores were $\leq 0.06$ with all other values of $\text{mix}$, showing no vocabulary generalization. As with copy detection, both models could learn the task but did not generalize across vocabularies.

Copy/reverse detection. This task was exceptional in failing at both evaluation and test performance with $\text{mix} = 0.5$. Hence, the models did not learn the task even in vocabulary-specific settings. The most straightforward interpretation is that they only learned to detect tokens of $V_1$ or $V_2$ rather than whether the two input sequences were copies or reversals of each other. This is corroborated by the results with $\text{mix} \leq 0.1$, where evaluation F1-scores clearly tracked vocabulary ratios: 1.00 with $\text{mix} = 0$, 0.99 with $\text{mix} = 0.01$, and 0.90 with $\text{mix} = 0.1$. In each of these cases, test F1-score was $\leq 0.03$. With $\text{mix} = 0.5$, the models could no longer rely on a vocabulary-task bias (as this was removed by 50% – 50% mixing), which resulted in both models simply always predicting the same output class. This gave an average precision of 0.25 and average recall of 0.5 in both evaluation and test sets, yielding the F1-score of 0.33 (Table 1 rows 7 – 8, columns 9 – 10). Despite being above zero, this score thus does not indicate proper success in the task.

Copy/reversal. In stark contrast to all classification tasks, both models performed perfectly on this seq2seq task, irrespective of $\text{mix}$. On both evaluation and test sets, accuracy was always 1.00.

5.2 Impact of additional shared vocabulary

We also experimented on another method of sharing training resources between the task classes, via an additional shared vocabulary $V_S$. Here, training examples were derived from $V_1 \cup V_S$ and $V_2 \cup V_S$ instead of only $V_1$ and $V_2$ (cf. Algorithm 1). Increasing $V_S$ should encourage vocabulary generalization, since the model receives more evidence of the task classes being independent of specific token identities.

Keeping the overall vocabulary size constant (30), we re-ran each experiment with the $V_S$ sizes of 10

\[ \text{Basic accuracy (the rate of correct predictions) is rarely an appropriate performance metric for seq2seq tasks, as it does not differentiate between different kinds of imperfect candidates. We use it in this exceptional case since every prediction was identical with its target.} \]
Table 2: Model performance: F1-score for classification (clf); accuracy for seq2seq;
red + bold indicates model failure to learn the task;
\(|\text{TRAIN}| = |\text{TEST}| = 20000 (10000 per class); 20\% of \text{TRAIN} used as evaluation data (eval).

| Task (type)       | Data | mix = 0       | mix = 0.01  | mix = 0.1      | mix = 0.5          |
|-------------------|------|---------------|-------------|---------------|-------------------|
|                   |      | BERT | RoBERTa | BERT | RoBERTa | BERT | RoBERTa | BERT | RoBERTa |
| Copy detection (clf) | EVAL | 1.00 | 1.00   | 1.00 | 1.00   | 0.96 | 0.96    | 0.99 | 0.99   |
|                   | TEST | 0.00 | 0.01   | 0.00 | 0.04   | 0.77 | 0.37    | 0.97 | 0.98   |
| Repetition detection (clf) | EVAL | 1.00 | 1.00   | 0.99 | 0.99   | 0.91 | 0.91    | 0.99 | 0.86   |
|                   | TEST | 0.00 | 0.05   | 0.05 | 0.05   | 0.06 | 0.05    | 0.99 | 0.87   |
| Copy/reverse detection (clf) | EVAL | 1.00 | 1.00   | 0.99 | 0.99   | 0.90 | 0.90    | 0.33 | 0.33   |
|                   | TEST | 0.00 | 0.00   | 0.00 | 0.00   | 0.00 | 0.03    | 0.33 | 0.33   |
| Copying/reversal (seq2seq) | EVAL | 1.00 | 1.00   | 1.00 | 1.00   | 1.00 | 1.00    | 1.00 | 1.00   |
|                   | TEST | 1.00 | 1.00   | 1.00 | 1.00   | 1.00 | 1.00    | 1.00 | 1.00   |

and 20. Increasing \( V_S \) never made either model successful in classification tasks with \( \text{mix} \in [0,0.1] \). Thus, BERT and RoBERTa showed no evidence of vocabulary generalization in classification tasks, even when “nudged” to this direction by increasing \( V_S \). In the copying/reversal seq2seq task, model performance remained perfect irrespective of \( V_S \), (See Appendix \[B\] for complete data and further discussion.)

5.3 Summary

Our most salient experimental result is the disparity between the three classification tasks and the copying/reversal seq2seq task. The latter succeeded in all experiment settings irrespective of \( \text{mix} \), while vocabulary generalization between \( V_1 \) and \( V_2 \) was never learnt for classification. BERT and RoBERTa did not differ in this respect.

Increasing \( \text{mix} \) to 0.01 had practically no effect: only showing “hints” of vocabulary-generality was insufficient to induce generalization. With \( \text{mix} = 0.1 \), model performance increased in copy detection but not other tasks. Even here, performance was significantly lower with \( \text{mix} = 0.1 \) than with \( \text{mix} = 0.5 \), indicating that the increase was not due to a vocabulary-general rule but instead vocabulary-specific rules derived from the mixed training data-points.

As expected, test performance with \( \text{mix} = 0.5 \) closely corresponded to evaluation performance. All tasks except copy/reverse detection were learnt well in this setting. Copy/reverse detection was the only task that the models failed to learn, as model performance simply reflected the training distribution of \( V_1 \) and \( V_2 \).

6 Discussion

We provide an account of our experimental results (Section 6.1), relate them to theoretical considerations on the memory-processor distinction (Section 6.2), and discuss prospects for future work (Section 6.3).

6.1 Accounting for experimental results

Neither BERT nor RoBERTa generalized from \( V_1 \) to \( V_2 \) in classification tasks. Evidently, however, vocabulary generalization can be possible even when \( \text{mix} = 0 \), as illustrated by the copying/reversal task. To explain this divergence, we begin with how the latter can be learnt, and then observe that this method is unavailable in the classification tasks.

Transformers apply \textit{multi-head attention} to input positions for producing contextual embeddings of input tokens [Vaswani et al., 2017]. At each step of copying/reversal, the model thus needs to first attend to the input position of the relevant token, and then replicate this token in the output. Attention can be learned via the task marker token: the encoder should begin with the sequence-initial position and increase the position by one at each step when copying; and the converse when reversing. Token replication is a one-to-one mapping task learnt separately for each token.

In contrast, classification tasks cannot rely on attention to different input positions at each decoding step, since there is \textit{only one} decoding step. Copy detection and copy/reverse detection involve com-
paring two encodings, while repetition detection involves finding two or more instances of the same token in the input. Comparing two encodings is more challenging if they are very close. In copy/reverse detection they only differ in token positions, which likely explains why both BERT and RoBERTa had trouble learning it even with \( \text{mix} = 0.5 \). Otherwise, vocabulary-specific rules allowed the models to conduct the tasks based on specific input values. However, vocabulary generalization would require the model to map inputs to abstract placeholders (i.e. variables) without relying on vocabulary-specific decoding.

6.2 Model-internal vs. model-external memory

In Sections 2–3 we argued that symbolic computation is based on a memory-processor distinction, and manifested especially in variable binding. Concerning the question of whether Transformers can conduct symbolic computation, our results point to a more complex answer than merely “yes” or “no”. We suggest that they can occupy the role of a processor in a larger computational architecture that contains the input and output as external memory components. Even if they lack their own internal memory-processor distinction, they can still participate in genuinely symbolic computation within the whole input(-output)-network structure.

Crucially, the copying/reversal task allows avoiding model-internal variable binding, since both attention and token replication are vocabulary-specific (attention being based on the task marker). However, attention to input positions bears important resemblance to location-based memory addressing. The encoder attends to certain positions in the input, which is independent of their content. Based on this, the encoder then reads the input in the attended position. This mirrors the symbolic process of reading the content of a memory address. Like there, the content-independence of the address is crucial to allow the productivity of rules: the same computation is performed on the content of a location whatever this content is. Similarly, Transformers can attend to an input position regardless of what it contains.

Thus, instead of contending that BERT and RoBERTa are incapable of symbolic computation, we suggest broadening the perspective from the network alone to the whole network-input-output structure. While the models do not display evidence of internal read-write memories, they can function as processors performing read/write operations on the input/output used as an external memory, based on positional attention and vocabulary-specific decoding. Here, they are genuine components of symbolic computation.

6.3 Ramifications for future work

When analyzing the behavior of Transformer networks (e.g. Rogers et al. 2020) or training them to perform a dedicated task, it is vital to understand whether or not the task would require model-internal variable binding to achieve vocabulary generalization. Such considerations are critical for improving model generalizability across data distributions, mitigating model biases [Kurita et al., 2019, Nadeem et al., 2021] and increasing resistance to adversarial inputs [Li et al., 2020]. Sufficient vocabulary mixing cannot always be guaranteed during training, especially in the development of real-world applications. It therefore needs to be evaluated whether vocabulary generalization in the task could be achieved by the combination of positional attention and vocabulary-specific decoding. As witnessed by our experimental results, this is far from self-evident especially in classification tasks.

While we affirm the symbolic nature of the network-input-output structure, it is important to recognize the limits of using the input as a symbolic memory. Standard encoder-decoder networks read the static input (including the prior output) and write to fixed output positions. Compared to most symbolic systems, they lack intermediate inputs(outputs) that are used only within the computation and can be dynamically modified. Incorporating these to Transformers is a challenging venue for future work, given the absence of ground truth targets for supervised learning.

We identify two potential techniques for adding intermediate inputs to Transformers. First, model-external rule-based operations can be applied to the input (and/or prior output) to provide alternative variants as additional input. As a simple example, both aa and bb are of the form xx, where x is some token. Via a simple rule, both could thus turned to e.g. xx, concatenated (or otherwise combined) with the standard input from the raw string.
This technique would allocate the symbolic computation itself to network-external rules, but provide additional symbolic information to the network that could help it recognize symbolic relations. Its main challenge is devising the optimal rules for obtaining the intermediate inputs, as these are likely to be task-specific.

Another possibility echoes recent theoretical work on the Turing-completeness attention [Pérez et al., 2021], which utilizes outputs that function as markers of model states. Given some clustering of the model’s internal states into discrete groups, the model could output symbols denoting these states to a separate output sequence. This would allow it to access not only prior outputs, but additional (highly condensed) markers of its own prior states. This resembles a symbolic form of recurrence, which is currently lacking in Transformers. A challenge here would be finding appropriate model state clusters and their symbolic notation.

Without making predictions about the concrete utility of these suggested techniques as of yet, we stress the importance of addressing the lack of model-internal variable binding via the expanded use of model-external memory, which is currently limited to only the raw input and output. Without intermediate outputs the range of symbolic capacities remains limited to very simple read- and write operations. We consider tackling this challenge to be vital for extending the computational capacities of Transformers.

7 Conclusions and current work

Irrespective of whether Transformers could learn model-internal variable binding in principle, our empirical findings illustrate that they at least shun it in practice. However, the combination of attention and vocabulary-specific decoding allows mimicking location-based memory addressing with the input itself as the external memory component. As the basis of symbolic computation, this memory-processor distinction allows Transformers to occupy a central role in a symbolic system: the processor.

Our main message for future work is that the possibilities of using Transformers for symbolic computation (as processors) depends fundamentally on the kind of input available, and the range of computational steps that can be used. Simple classification limits the latter to the minimum (one step), whereas seq2seq tasks allow far more elaborate uses of the input as an external memory via attention.

A major challenge for enhancing the symbolic capacities of Transformers concerns their current inability for symbol manipulation, as both the input and prior output are static. We therefore see the need for intermediate outputs that could be both read and (re)written in the course of decoding. The main problem here is the lack of ground truth for their supervised learning, and we suggest two possible techniques for tackling it: adding material via symbolic post-processing and outputting markers of model states. We are currently working on such procedures with a particular focus on NLP, where the need for symbolic computation is especially prominent.

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A Appendix: Implementation details and experiment hyperparameters

We used Python 3.9.7. for programming the experiments, with Pytorch\(^4\) (1.10.1) as the deep learning framework and simpletransformers\(^5\) (0.63.3) for obtaining BERT and RoBERTa. We conducted GPU computation on a GeForce RTX 2080 Ti. We provide our source code as supplementary material, and will release it as open-source.

Hyperparameters were shared between all four tasks (copy detection, repetition detection, copy/reverse detection, and copying/reversal), as listed in Table 3. Aside of vocabularies, all hyperparameters were the same for different sizes of the shared vocabulary \(V_S\). We discarded palindromes in copy/reverse detection and copying/reversal to avoid ambiguity between task classes.

| \(V_1\) | \(|V_S| = 0\) | | \(|V_S| = 10\) | | \(|V_S| = 20\) |
|---|---|---|---|---|
| \(|V_S| = 0\) | abcdefghij01234 | abcdefghij | abcde |
| \(|V_S| = 10\) | klmnopqrstuvwxyz56789 | klmnopqrstuvwxyz | |
| \(|V_S| = 20\) | | fghij | |
| \(V_2\) | \(|V_S| = 0\) | | \(|V_S| = 10\) | | \(|V_S| = 20\) |
| \(|V_S| = 0\) | klmnopqrstuvwxyz56789 | klmnopqrstuvwxyz | |
| \(|V_S| = 10\) | | fghij | |
| \(|V_S| = 20\) | | | |
| \(V_S\) | \(|V_S| = 0\) | | \(|V_S| = 10\) | | \(|V_S| = 20\) |
| \(|V_S| = 0\) | 10123456789 | 10123456789 | |
| \(|V_S| = 10\) | | klmnopqrstuvwxyz0123456789 | |
| \(|V_S| = 20\) | | | |

| Data size | TRA1N | EVAL | TEST |
|---|---|---|---|
| | 16000 | 4000 | 20000 |
| Datapoint length | min. | 1 | |
| | max. | 10 | |
| Model training | batch size | 16 | |
| | epochs | 10 | |

Table 3: Experiment hyperparameters.

BERT and RoBERTa are first trained on a generic language modelling task, and the trained language model is then fine-tuned on the main input-output task. To ensure that the models could not use vocabulary-specific information obtained from elsewhere, we did not use pre-trained models but instead trained the language model from scratch on

\(^4\)https://pytorch.org/
\(^5\)https://simpletransformers.ai/
our own training data. We first trained the language model on the training set inputs, and then used the best-performing language model for initializing the fine-tuning task on input-output pairs. In both language model training and fine-tuning, we trained each model for a fixed number of epochs (10), and used the best-performing model (on the evaluation set) in further stages of the experiment.

B Appendix: All results

Tables 4–5 show the results on the four tasks with all values of \( V_1/V_2/V_S \) size and \( \text{mix} \).

If the model was able to learn genuinely vocabulary-general rules, we would expect it to prefer vocabulary generalization with larger ratios of \( V_S \) compared to \( V_1 \) and \( V_2 \). In contrast, if it cannot learn vocabulary generalization, increasing \( V_S \) is expected to have no effect on performance when \( \text{mix} = 0 \).

Increasing \( V_S \) never made either model successful in classification tasks with \( \text{mix} = 0 \). The only seeming exception to this was BERT's F1-score of 0.12 with \( |V_S| = 20 \) on the repetition detection task (Table 4, row 43, column 12). However, even here the model produced no true positives.

In copy detection with \( \text{mix} = 0.01 \), there was a noticeable increase in test F1-score when \( |V_S| \) was increased to 20 (0.32 for BERT and 0.14 for RoBERTa), although the ratio of correct predictions still remained clearly below the ratio of false ones. The most likely explanation for this is the reduction of \( V_1 \) and \( V_2 \) (given the constancy of the overall vocabulary), which allowed fewer training examples from the \( V_1-C_2 \) and \( V_2-C_1 \) combinations to improve test performance due to reduced vocabulary variation in the test set. Otherwise, model performance with \( \text{mix} = 0.01 \) remained 0.00 – 0.05 in all models regardless of \( V_S \).

In copy/reverse detection, increasing \( V_S \) to 10 differentiated the two models on the test set, with BERT remaining unsuccessful but RoBERTa achieving 0.83 – 0.95 F1-score with \( \text{mix} \in \{0.1, 0.5\} \). Increasing \( V_S \) to 20 retained RoBERTa’s relatively high performance with \( \text{mix} = 0.1 \) (0.78), but both models failed with \( \text{mix} = 0.5 \) (0.33; always predicting the False class). Hence, copy/reverse detection was at times learnable in vocabulary-specific settings; albeit this remained irregular and unpredictable.

In the copying/reversal seq2seq task, model performance remained perfect irrespective of \( V_S \). This conforms with our analyses (Section 6), and further illustrates the divergence between the classification and seq2seq tasks.

In summary, while \( V_S \) impacted model performance in certain experiments, its effects (i) varied in whether they took model performance up or down, (ii) were never present with \( \text{mix} = 0 \), and (iii) were present only once with \( \text{mix} = 0.01 \). In the last case, test performance still remained very low (below random) and the performance increase most likely reflected the reduced test set vocabulary rather than the learning of an abstract vocabulary-independent rule. We therefore conclude that our results on increasing \( V_S \) ratio had no effect on our main claim: models did not succeed in vocabulary generalization on the classification tasks.
| Task (type) | Vocabulary size | mix | Model   | Precision | Recall | F1   |
|------------|-----------------|-----|---------|-----------|--------|------|
|            |                 |     |         | EVAL     | TEST   | EVAL | TEST |
|            |                 |     |         |          |        |      |      |
| Copy detection (clf) | | | | | | | |
|            | 15 15 0         | 0   | BERT    | 1.00     | 0.00   | 1.00 | 0.00 |
|            |                 |     | RoBERTa | 1.00     | 0.01   | 1.00 | 0.02 |
|            |                 | 0.01| BERT    | 1.00     | 0.00   | 1.00 | 0.00 |
|            |                 |     | RoBERTa | 1.00     | 0.04   | 1.00 | 0.04 |
|            |                 | 0.1 | BERT    | 0.96     | 0.77   | 0.96 | 0.77 |
|            |                 |     | RoBERTa | 0.96     | 0.37   | 0.96 | 0.34 |
|            |                 | 0.5 | BERT    | 0.99     | 0.97   | 0.99 | 0.97 |
|            |                 |     | RoBERTa | 0.99     | 0.98   | 0.99 | 0.98 |
|            | 10 10 10        | 0   | BERT    | 1.00     | 0.00   | 1.00 | 0.00 |
|            |                 |     | RoBERTa | 1.00     | 0.00   | 1.00 | 0.00 |
|            |                 | 0.01| BERT    | 1.00     | 0.04   | 1.00 | 0.05 |
|            |                 |     | RoBERTa | 0.99     | 0.02   | 0.99 | 0.02 |
|            |                 | 0.1 | BERT    | 0.98     | 0.83   | 0.98 | 0.75 |
|            |                 |     | RoBERTa | 0.99     | 0.87   | 0.99 | 0.86 |
|            |                 | 0.5 | BERT    | 1.00     | 0.97   | 1.00 | 0.97 |
|            |                 |     | RoBERTa | 0.99     | 0.96   | 0.99 | 0.96 |
|            | 5 5 20          | 0   | BERT    | 1.00     | 0.01   | 1.00 | 0.01 |
|            |                 |     | RoBERTa | 1.00     | 0.00   | 1.00 | 0.00 |
|            |                 | 0.01| BERT    | 1.00     | 0.32   | 1.00 | 0.37 |
|            |                 |     | RoBERTa | 0.99     | 0.12   | 0.99 | 0.16 |
|            |                 | 0.1 | BERT    | 1.00     | 0.84   | 1.00 | 0.76 |
|            |                 |     | RoBERTa | 0.99     | 0.83   | 0.99 | 0.74 |
|            |                 | 0.5 | BERT    | 1.00     | 0.97   | 1.00 | 0.97 |
|            |                 |     | RoBERTa | 0.95     | 0.95   | 0.95 | 0.95 |
| Repetition detection (clf) | | | | | | | |
|            | 15 15 0         | 0   | BERT    | 1.00     | 0.00   | 1.00 | 0.00 |
|            |                 |     | RoBERTa | 1.00     | 0.04   | 1.00 | 0.05 |
|            |                 | 0.01| BERT    | 0.99     | 0.04   | 0.99 | 0.05 |
|            |                 |     | RoBERTa | 0.99     | 0.04   | 0.99 | 0.05 |
|            |                 | 0.1 | BERT    | 0.91     | 0.06   | 0.91 | 0.06 |
|            |                 |     | RoBERTa | 0.91     | 0.04   | 0.91 | 0.05 |
|            |                 | 0.5 | BERT    | 0.99     | 0.99   | 0.99 | 0.99 |
|            |                 |     | RoBERTa | 0.86     | 0.87   | 0.86 | 0.87 |
|            | 10 10 10        | 0   | BERT    | 1.00     | 0.00   | 1.00 | 0.00 |
|            |                 |     | RoBERTa | 0.99     | 0.00   | 0.99 | 0.00 |
|            |                 | 0.01| BERT    | 0.99     | 0.05   | 0.99 | 0.05 |
|            |                 |     | RoBERTa | 0.99     | 0.04   | 0.99 | 0.05 |
|            |                 | 0.1 | BERT    | 0.91     | 0.05   | 0.91 | 0.05 |
|            |                 |     | RoBERTa | 0.92     | 0.06   | 0.92 | 0.07 |
|            |                 | 0.5 | BERT    | 1.00     | 1.00   | 1.00 | 1.00 |
|            |                 |     | RoBERTa | 0.61     | 0.67   | 0.61 | 0.62 |
|            | 5 5 20          | 0   | BERT    | 0.98     | 0.10   | 0.98 | 0.13 |
|            |                 |     | RoBERTa | 0.91     | 0.00   | 0.91 | 0.00 |
|            |                 | 0.01| BERT    | 0.99     | 0.05   | 0.99 | 0.05 |
|            |                 |     | RoBERTa | 0.93     | 0.02   | 0.93 | 0.02 |
|            |                 | 0.1 | BERT    | 0.98     | 0.72   | 0.98 | 0.58 |
|            |                 |     | RoBERTa | 0.86     | 0.05   | 0.86 | 0.05 |
|            |                 | 0.5 | BERT    | 1.00     | 1.00   | 1.00 | 1.00 |
|            |                 |     | RoBERTa | 0.76     | 0.76   | 0.55 | 0.55 |

Table 4: Results on copy detection and repetition detection.
| Task (type)                      | Vocabulary size | mix | Model     | Precision EVAL | Precision TEST | Recall EVAL | Recall TEST | F1 EVAL | F1 TEST |
|---------------------------------|-----------------|-----|-----------|----------------|---------------|-------------|-------------|---------|---------|
| Copy/reversal detection (clf)   | 15 15 0         | 0.0 | BERT      | 1.00           | 1.00          | 1.00        | 1.00        | 1.00    | 1.00    |
|                                 |                 | 0.01| BERT      | 0.99           | 0.99          | 0.99        | 0.99        | 0.99    | 0.99    |
|                                 |                 | 0.1 | BERT      | 0.90           | 0.90          | 0.90        | 0.90        | 0.90    | 0.90    |
|                                 |                 | 0.5 | BERT      | 0.25           | 0.25          | 0.30        | 0.30        | 0.33    | 0.33    |
|                                 | 10 10 10        | 0.0 | BERT      | 0.97           | 1.00          | 0.97        | 1.00        | 0.97    | 1.00    |
|                                 |                 | 0.01| BERT      | 0.97           | 0.97          | 0.97        | 0.97        | 0.97    | 0.97    |
|                                 |                 | 0.1 | BERT      | 0.89           | 0.89          | 0.99        | 0.83        | 0.89    | 0.99    |
|                                 |                 | 0.5 | BERT      | 0.49           | 0.49          | 0.49        | 0.03        | 0.47    | 0.03    |
|                                 | 5 5 20          | 0.0 | BERT      | 0.88           | 0.84          | 0.84        | 0.00        | 0.84    | 0.00    |
|                                 |                 | 0.01| BERT      | 0.85           | 0.84          | 0.84        | 0.00        | 0.83    | 0.00    |
|                                 |                 | 0.1 | BERT      | 0.80           | 0.77          | 0.77        | 0.00        | 0.76    | 0.00    |
|                                 |                 | 0.5 | BERT      | 0.25           | 0.25          | 0.50        | 0.50        | 0.33    | 0.33    |

| Task (type)            | Vocabulary size | mix | Model    | Accuracy EVAL | Accuracy TEST |
|------------------------|-----------------|-----|----------|---------------|---------------|
| Copying/reversing (seq2seq) | 15 15 0       | 0   | BERT     | 1.00          | 1.00          |
|                        |                 | 0.01| BERT     | 1.00          | 1.00          |
|                        |                 | 0.1 | BERT     | 1.00          | 1.00          |
|                        |                 | 0.5 | BERT     | 1.00          | 1.00          |
|                        | 10 10 10        | 0   | BERT     | 1.00          | 1.00          |
|                        |                 | 0.01| BERT     | 1.00          | 1.00          |
|                        |                 | 0.1 | BERT     | 1.00          | 1.00          |
|                        |                 | 0.5 | BERT     | 1.00          | 1.00          |
|                        | 5 5 20          | 0   | BERT     | 1.00          | 1.00          |
|                        |                 | 0.01| BERT     | 1.00          | 1.00          |
|                        |                 | 0.1 | BERT     | 1.00          | 1.00          |
|                        |                 | 0.5 | BERT     | 1.00          | 1.00          |

Table 5: Results on copy/reversal detection and copying/reversal.