The model of energy conversion process in a single-circuit electromagnetic system

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Abstract. The model of energy conversion process is proposed. The model is based on the relationships between electrical and magnetic energies during their conversion. Using Hamilton's principle of minimal action, the trajectory of movement of the operating point on the energy plane corresponding to the process of electromagnetic energy conversion is obtained. Division of the dynamic trajectory of the operating point into successive sections where the corresponding energy conversions take place gives the opportunity to consider the energy conversion as a process of successive conversion from one form of energy to another. The essence of the research is the model including the electrical voltage balance equation for a single-circuit electromagnetic system where the energy conversion process is described by electromagnetic and magnetic components. The created model makes it possible to evaluate the efficiency of energy conversion in a single-circuit electromagnetic system.

1. Introduction

Linear electromagnetic converters (LECs) are widely used as auxiliary drives for instrumentation and other industries [1-4]. Improving the positioning accuracy of LECs is associated with the study of the process of energy conversion in dynamic modes.

The development of a model of the energy conversion process in the LEC is a variational problem. Using the provisions of the calculus of variations based on the Hamilton minimum action principle, it becomes possible to find equations describing the trajectory of the working point in the phase planes of generalized coordinates reflecting the process of electromechanical energy conversion. The spatial coordinate and the electric charge flowing through the transformer winding can be chosen as these coordinates [5]. Direct methods for solving such problems are known [6], one of which is the Euler method. According to it, the desired trajectory of the working point on the phase plane can be described by a polyline passing through the start and end points of the trajectory. Moreover, the
integral of the action along the polyline approaches the value of the integral of the action along the
ttrue, desired dynamic trajectory, which is arbitrarily small in magnitude.

In this paper, we propose an algorithm for constructing the desired polyline that describes the
dynamics of energy conversion in the most general form. The choice of directions in which the
components of the polyline under consideration are deposited allows us to divide the process of the
conversion into two components that describe the energy conversion between electric and magnetic
forms of energy, as well as between magnetic and mechanical forms of energy.

As the result of the analysis, the expression for the balance of electrical voltages in the considered
electromagnetic system, describing the dynamics of the energy conversion process, is given [7, 8].

2. Energy conversion process model

When moving the operating point on the "current-flux linkage" plane (Figure 1), the magnetic energy
conversion power is determined by the expression:

\[ P_{\text{mag}} = \lim_{\Delta t \to 0} \frac{W_{\text{mag},1} - W_{\text{mag},0}}{\Delta t}, \]

where \( W_{\text{mag},0} \) is the initial value of the magnetic energy at the moment of time, for example, at the
point \( A \) (Figure 1) with coordinates \((i_0, \psi_0)\); \( W_{\text{mag},1} \) is the final value of the magnetic energy at the
moment of time at the point \( C \) with coordinates \((i_1, \psi_1)\); \( \Delta t \) is a time increment. When striving, there is
an aspiration of points.

Figure 1. Dynamic trajectory of the development of the process of electromechanical energy
conversion in the coordinates "current - flux linkage"

The magnitude of the magnetic energy is a function of current and flux linkage [5, 9-11]:

\[ W_{\text{mag}} = \frac{1}{2} \psi i, \]

where \( \psi \) – the flux linkage associated with the circuit in the current flows.

The vector connecting the points \( W_{\text{mag},0}(i_0, \psi_0) \) and \( W_{\text{mag},1}(i_1, \psi_1) \) on the plane in the coordinates of
the current and flux linkage is parallel to the tangent to the true trajectory of the working point \( A \)
(Figure 1).

Let us assume that the elementary segments of the broken line describing the real trajectory of the
change in the parameters of the dynamic system are the segments \( AB \) and \( BC \) (Figure 1) in a vector
form that determine the expression:
\[ \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} . \] (3)

The point \( B \) determined by the vector \( \overrightarrow{AB} \) directed along the tangent \( \delta \) to the magnetization line of the magnetic system at the angle \( \beta \) that takes into account the degree of saturation of the magnetic circuit and determines the value of the differential static inductance:

\[ L_{sd} = \tan(\beta) . \] (4)

When the working point moves along the vector \( \overrightarrow{AB} \), the following condition is met:

\[ (\overrightarrow{AB} \cdot \nabla) W_{mag} = dW_{mag} , \] (5)

where \( \nabla = \frac{\partial}{\partial \psi_0} \overrightarrow{\psi_0} + \frac{\partial}{\partial i} \overrightarrow{i}_0 \) is the linear operator; \( \overrightarrow{\psi}_0, \overrightarrow{i}_0 \) are the basic components of the vector of the plane described by the axes of the current and flux linkage, respectively; \( dW_{mag} \) is the increment in the energy of the magnetic field, determined according to the expression:

\[ dW_{mag} = \frac{1}{2} id\psi + \frac{1}{2} d\psi i + \frac{1}{2} did\psi , \] (6)

where \( d\psi_{mag}, di_{mag} \) are the projections of the vector \( \overrightarrow{AB} \) onto the flux linkage and current axes, respectively.

Substituting (2) into (5), taking into account (6) and neglecting the term \( \frac{1}{2} did\psi \) of the second degree of smallness, we find:

\[ id\psi_{AB} + \psi di_{AB} = \frac{1}{2} id\psi + \frac{1}{2} d\psi i . \] (7)

Denoting \( d\psi_{AB} = d\psi_{mag} \) and \( di_{AB} = di_{mag} \), we transform expression (7) to the form:

\[ d\psi_{mag} - \frac{1}{2} d\psi = L \left( \frac{1}{2} di - di_{mag} \right) , \] (8)

where \( L = \psi / i \) is the instantaneous value of its own inductance.

Considering that the differential static inductance determines the relationship between the "magnetic" increments of current and flux linkage, that is \( L_{sd} = d\psi_{mag} / di_{mag} \), from expression (8) we can write the following dependencies between the true increments of current and flux linkage:

\[ \begin{cases} 
  di = 2 di_{mag} \frac{L_{sd} + L}{L_{sd} + L} \\
  d\psi = 2 d\psi_{mag} \frac{L_{sd} + L}{L_{sd} + L} \frac{L_{sd}}{L_{sd}} .
\end{cases} \] (9)

where

\[ L_{sd} = d\psi / di , \] (10)

where \( L_{sd} \) is the differential inductance.

Similarly, when the operating point moves along a vector \( \overrightarrow{BC} \) tangential to the line of the magnetic energy level, the following relation will be fulfilled

\[ (\overrightarrow{BC} \cdot \nabla) W_{mag} = 0 . \] (11)

Substituting (2) into (11), after the transformation we get the expression
Denoting \( d\psi_{AB} = d\psi_{\text{mech}} \) and \( di_{AB} = -di_{\text{mech}} \), we transform expression (12) to the form

\[
d\psi_{\text{mech}} = \frac{\psi}{\ell} di_{\text{mech}} = Ldi_{\text{mech}}. \tag{13}
\]

Thus, when describing the dynamic trajectory of the working point of the energy conversion process by broken lines constructed according to the vector sum (3), the following conditions are met:

1) the directional segment \( AB \) corresponds to the process of changing the value of the magnetic energy accumulated in the electromechanical converter;

2) the directional segment \( BC \) corresponds to the process of changing the value of the converted mechanical energy;

3) the directed segment \( AC \) will characterize the true current increments in the electrical circuit of the electromagnetic converter and the flux linkage associated with this circuit, that is

\[
0 \odot di_{\text{mech}} = \psi \odot i_{\text{mech}}. \tag{14}
\]

The derivative of the value of magnetic energy (2), according to expression (11), from the condition of fulfilling the law of conservation of energy is equal to the increment of mechanical power, that is, you can write:

\[
dW_{\text{meh}} = id\psi_{\text{mech}} - \psi d\psi_{\text{mech}}. \tag{15}
\]

It is taken into account here that to redirect the vector \( BC = di_{\text{mech}} \odot i_0 + d\psi_{\text{mech}} \odot \psi_0 \) from the level line to the magnetic energy gradient line, it is necessary to change the sign of its projection onto the current axis \( di_{\text{mech}} \odot i_0 \), and the derivative (11) will become numerically equal to the mechanical energy increment.

Based on the law of conservation of energy, you can write the equality:

\[
dW_e = dW_{\text{mag}} + dW_{\text{meh}}, \tag{16}
\]

where \( dW_e \) is the increment of electrical energy, equal to the power flow \( P_e \) of electrical energy between the circuit and the source supplying it, multiplied by the time differential:

\[
dW_e = P_e dt = i \frac{d\psi}{dt} dt = id\psi. \tag{17}
\]

By dividing the dynamic trajectory of the operating point into successive sections, within which the corresponding transformations of energies occur, it can be concluded that it is possible to separately consider the processes of converting energy from an electric form to a magnetic one, and from a magnetic form to a mechanical one [7-12].

The linearization of the properties of the magnetic system when considering small terms of the "magnetic" \( AB \) and "mechanical" \( BC \) increments is consistent with the provisions on the need for preliminary linearization of the properties of the magnetic system when determining the electromagnetic forces (and, hence, the mechanical power developed by the transducer) [9, 10].

The proposed mathematical model forms the following equation of Kirchhoff’s law, which describes the balance of voltages during electromagnetic energy conversion

\[
u = R \int_0^t L_d (u - iR) dt + 2 \int_0^t \left( \frac{F_{el,m} - F_{\text{mop}}}{{L_{ed}} + L} \right) L_{ed}^2 + L \int_0^t \frac{F_{el,m} - F_{\text{mop}}}{m} dt, \tag{18}
\]

where \( u \) is the external electrical voltage present in the electrical circuit of the converter; \( R \) is the active resistance of the electrical circuit of the converter; \( t \) is the time; \( F_{el,m} \) is the electromagnetic...
force (moment) developed by the converter; $F_{\text{sop}}$ is the effort (moment) of counteracting the movement of the movable element of the converter; $\delta$ is the spatial linear (angular) coordinate of the moving element of the converter; $m$ is the mass (or moment of inertia with rotational motion) of the moving element of the converter.

The solution of expression (18) is possible with respect to an unknown quantity $L_0$, given that: $R$ is determined by the design of the converter; $L, L_{nd}, F_{\text{sop}}$ are depend on the converter design and the instantaneous value of the current; $F_{\text{sop}}$ is the initially known function of the coordinate and its derivatives; $u$ is the set mode parameter.

3. Conclusion
The model of electromagnetic energy conversion in the form of the electric voltage balance equation for the electromagnetic system has been obtained. The model based on the theory of the classical dynamics describes the dynamics of the process of energy conversion and agrees with the well-known theory of energy conversion.

The model makes it possible to estimate the efficiency of energy conversion in a single-circuit electromagnetic system and to improve the accuracy of LECs positioning in control and measuring electronic devices.

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