Static and non-static vector screening masses

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Inverse correlation length over which an electric field is screened in the medium
Vector Screening Mass

1. Inverse correlation length over which an electric field is screened in the medium

2. Test of perturbation theory
   \[ \Rightarrow \text{probing effective potential which enters calculation of dilepton production rate} \]

B.B.Brandt, A.Francis, M.Laine, H.B.Meyer, arXiv:1404.2404; H.B.Meyer, arXiv:1512.06634, PoS Lattice 2015
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   ⇒ probing effective potential which enters calculation of dilepton production rate
   B.B.Brandt, A.Francis, M.Laine, H.B.Meyer, arXiv:1404.2404;
   H.B.Meyer, arXiv:1512.06634, PoS Lattice 2015

3. Transport properties of the QCD plasma
   ⇒ screening pole (Euclidean correlator) analytically connected to diffusion pole (retarded correlator)
   B.B.Brandt, A.Francis, M.Laine, H.B.Meyer, arXiv:1408.5917, PoS QM 2014
in QED

\[ k^2 + \Pi_{00}(0, k)\big|_{k^2 = -m_E^2} = 0 \]  

defines the static screening Debye mass as the pole of the longitudinal static photon self-energy

hep-ph/9408273, E.Braaten, A.Nieto
The Debye Screening Mass

in QED

\[ k^2 + \Pi_{00}(0, k)|_{k^2 = -m_E^2} = 0 \quad (1) \]

defines the static screening Debye mass as the pole of the longitudinal static static photon self-energy

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in QCD the correct observable (has to be odd under Euclidean time-reversal) would be

\[ \text{Im} (\text{tr} [P]) \quad (2) \]

where \( P \) is Polyakov loop \( \Rightarrow \) chromo-electric Debye mass

P.Arnold, L.G.Yaffe, hep-ph/9508280
Q: how well is a $U(1)$ electric field screened in QCD plasma?
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- there will be an effective vector screening mass $M_V \leftrightarrow$ screening length of a $U(1)$ electric field
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A: vector correlators; here: flavour non-singlet

- there will be an effective vector screening mass $M_V \leftrightarrow$ screening length of a $U(1)$ electric field
- quark blob will contain chromo-electric Debye mass $m_E \leftarrow$ dealt with by EFT and LQCD
for thermal gluons there are IR problems in the colour-magnetic dof
Effective Approach

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- therefore they cannot be treated perturbatively
Effective Approach

- for thermal gluons there are IR problems in the colour-magnetic dof
- therefore they cannot be treated perturbatively
- make use of scale hierarchy $g^2 T \ll gT \ll 2\pi T$
  naturally arising in a thermal description

$$m_E^2 = g^2 T^2 \left( \frac{N_C}{3} + \frac{N_f}{6} \right)$$
Implementation of our problem (dimensional reduction)

the vector current screening correlator

\[ G^{(k_n)}_{\mu\nu}(z) = \int_0^\beta d\tau e^{ik_n\tau} \int_x \left\langle \left( \bar{\psi} \gamma_\mu \psi \right)(\tau, x, z) \left( \bar{\psi} \gamma_\mu \psi \right)(0) \right\rangle \]  \hspace{1cm} (3)

\[ \rightarrow G^{(k_n)}_{\mu\nu}(z) = T \int_x \left\langle V^{(k_n)}(x, z) V^{(-k_n)}(0) \right\rangle, \]  \hspace{1cm} (4)

\[ x = (x_1, x_2)^T \rightarrow \text{transverse plane} \]

\[ \bar{\psi}(\tau) = T \sum_{p_n} e^{-ip_n\tau} \bar{\psi}_{p_n} \]

\[ \psi(\tau) = T \sum_{p_n} e^{ip_n\tau} \psi_{p_n} \]

\[ V^{(k_n)}(x, z) = T \sum_{p_n} \bar{\psi}_{p_n}(x, z) \gamma_\mu \psi_{p_n-k_n}(x, z) \]

B.B.Brandt, A.Francis, M.Laine, H.B.Meyer, arXiv:1404.2404

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Static and non-static vector screening masses
1-gluon exchange potential

non-relativistic auxiliary fields in the transverse plane

\[ \psi = \frac{1}{\sqrt{T}} \begin{pmatrix} \chi \\ \phi \end{pmatrix} \]  

(5)

remember: quarks carry momenta \( \sim \pi T \gg gT \gg g^2 T \)

B.B.Brandt, A.Francis, M.Laine, H.B.Meyer, arXiv:1404.2404
1-gluon exchange potential

dimensional reduction through Matsubara formalism

\[ V_{\text{LO}}^+(y) = \frac{g_E^2 C_F}{2\pi} \left[ \log \left( \frac{m_E y}{2} \right) + \gamma_E + K_0(m_E y) \right] \]

with \( g_E^2 = g^2 T \) effective coupling of dimensionally reduced theory, \( m_E \) Debye mass and \( K_0 \) modified Bessel function
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with \( g_E^2 = g^2 T \) effective coupling of dimensionally reduced theory, \( m_E \) Debye mass and \( K_0 \) modified Bessel function

the same effective potential enters the calculation of the dilepton production rate (cf. B.B.Brandt,A.Francis,M.Laine,H.B.Meyer, arXiv:1404.2404; H.B.Meyer, arXiv:1512.06634, PoS Lattice 2015 and refs. therein)
Extracting screening masses

The radial homogeneous part of the S-eqn. to be solved reads

\[
\left\{ -\frac{d^2}{d\bar{y}^2} - \frac{1}{\bar{y}} \frac{d}{d\bar{y}} + \frac{l^2}{\bar{y}^2} + \rho \left( \frac{2\pi V^\pm}{g_E^2 C_F} - \hat{E}^{(l)} \right) \right\} R_l = 0
\]

\[
\Rightarrow \left\{ -\frac{d^2}{d\bar{y}^2} - \frac{1}{\bar{y}} \frac{d}{d\bar{y}} + \frac{l^2}{\bar{y}^2} + \rho \left( \hat{V}^\pm - \hat{E}^{(l)} \right) \right\} R_l = 0
\]

with dimensionless quantities

\[
\bar{y} = m_E y, \quad \rho = \frac{g_E^2 C_F M_r}{\pi m_E^2}
\]

and \( g_E^2 = g^2 T, \ C_F = \frac{N_C^2 - 1}{2N_C} \)
Extracting Screening Masses

\[ E_{\text{full}} = M_{cm} + \frac{g_E^2 C_F}{2\pi} \hat{E}(l) \]

\[ M_{cm} = k_n + \frac{m_\infty^2}{2M_r}, \quad m_\infty^2 = \frac{g^2 T^2 C_F}{4}, \quad M_r = \left( \frac{1}{p_n} - \frac{1}{k_n - p_n} \right)^{-1} \]  \hspace{1cm} (7)

\( E_{\text{full}} \) can be understood as screening masses

B.B.Brandt,A.Francis,M.Laine,H.B.Meyer, arXiv:1404.2404
the screening correlator exhibits as a long-distance behaviour; with a proper ansatz for the radial S-eqn. one finds

\[ - \frac{G_{00}^{(k_n)}(z)}{T^3} \approx \frac{N_c m^2_E A_0^+}{\pi T^2} e^{-|z|E_0^{(l=0)}} \]

\[ - \frac{G_T^{(k_n)}(z)}{T^3} \approx \frac{N_c m^4_E A_1^+}{\pi T^2} \left[ \frac{1}{p_n^2} + \frac{1}{(k_n - p_n)^2} \right] e^{-|z|E_0^{(l=1)}} \]

with

\[ A_0^+ = \frac{|R_0(0)|^2}{\int_0^\infty dy y |R_0(y)|^2}, \quad A_1^+ = \frac{|R_1'(0)|^2}{\int_0^\infty dy y |R_1(y)|^2} \]  \hspace{1cm} (8)

for S-wave \((l = 0)\) and P-wave \((l = 1)\) channels, respectively

B.B.Brandt,A.Francis,M.Laine,H.B.Meyer, arXiv:1404.2404
the situation is very similar for the static case

keep in mind: roles of transversal and longitudinal part of the correlator are reversed
Lattice setup

\[ N_T \cdot N_x^3 = 16 \cdot 64^3 \] lattice with \( a \approx 0.024\text{fm} \)
corresponding to

\[ T = \frac{1}{N_T a} \approx 508\text{MeV} \]
Lattice setup

\(N_\tau \cdot N_x^3 = 16 \cdot 64^3\) lattice with \(a \approx 0.024\text{fm}\) corresponding to

\[
T = \frac{1}{N_\tau a} \approx 508\text{MeV}
\]

\[
\frac{6}{g_0^2} \approx 6.038, \quad \kappa = 0.136238, \quad c_{sw} = 1.51726
\]
Lattice setup

\[ N_\tau \cdot N_x^3 = 16 \cdot 64^3 \] lattice with \[ a \approx 0.024\text{fm} \] corresponding to

\[ T = \frac{1}{N_\tau a} \approx 508\text{MeV} \]

\[ \frac{6}{g_0^2} \approx 6.038, \quad \kappa = 0.136238, \quad c_{SW} = 1.51726 \]

with \( N_f = 2 \mathcal{O}(a) \)-improved Wilson-type fermions generated in Mainz on 'Clover' (using MP-HMC) exploiting \( N_{\text{cfg}} = 345 \) and \( N_{\text{src}} = 64 \) rnd src
Lattice formulation

\[ G^{(k_n)}_{\mu\nu}(z) = \sum_{n=1}^{2} A_n \frac{\cosh[M_n(z - L_z/2)]}{\sinh[M_nL_z/2]} \]
Lattice formulation

\[ G_{\mu\nu}^{(k_n)}(z) = \sum_{n=1}^{2} A_n \frac{\cosh[M_n(z - L_z/2)]}{\sinh[M_n L_z/2]} \]

\[ M_1 = M_{\text{eff}}, \quad M_2 = M_{\text{exc}} \]  (9)
A previous study

Figure: Screening masses at $T = 254\text{MeV}$ (left panel) and $T = 338\text{MeV}$ (right panel).
The fits, $n = 0$

**Figure:** The correlator and two-state fit for the static case. Left: transversal channel. Right: longitudinal channel.
The fits, $n = 1$

**Figure:** The correlator and two-state fit for the non-static case. Left: longitudinal channel. Right: transversal channel.
$T = 508\,\text{MeV}$, screening mass

**Figure:** Static ($n=0$) transversal ($S$-wave) screening mass.
$T = 508\, \text{MeV}$, screening mass

**Figure:** Static ($n=0$) longitudinal (P-wave) screening mass.
$T = 508\,\text{MeV}$, screening mass

**Figure:** Non-static ($n=1$) longitudinal (S-wave) screening mass.
$T = 508\text{MeV}$, screening mass

**Figure**: Non-static ($n=1$) transversal (P-wave) screening mass.
$T = 508\text{MeV}$, masses

**Figure:** Comparison of the masses at different $T$. The graph illustrates the masses at $T=508\text{MeV}$ for different values of $n$. The data points are denoted by different markers for $n=0$ and $n=1$, with distinct lines for static and non-static vector screening masses.
$T = 508\text{MeV}$, masses

$T = 254\text{MeV}$  $T = 338\text{MeV}$  $T = 508\text{MeV}$
\( T = 254\text{MeV} \quad T = 338\text{MeV} \quad T = 508\text{MeV} \)

**Figure:** Comparison of the masses at different \( T \).

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Static and non-static vector screening masses
$T = 508\text{MeV}$, amplitudes

**Figure**: Comparison of the amplitudes.
vector screening masses as probes of PT
EFT for 1-gluon exchange potential between quarks coupling to a photon in the medium
lattice ansatz
comparison
FUTURE: Diffusion coefficient from analytic continuation
The Debye Screening Mass

thermal gluon propagator in Fourier space

\[ \langle \tilde{A}^a_\mu(K)\tilde{A}^b_\nu(Q) \rangle \propto \left( \frac{\delta^{ab}\delta_{\mu\nu}}{K^2 + \delta_{\mu0}\delta_{\nu0}m_E^2} \right) \]
The Debye Screening Mass

thermal gluon propagator in Fourier space

\[
\langle \hat{A}_\mu^a(K)\hat{A}_\nu^b(Q) \rangle \approx 0 \propto \frac{\delta^{ab}\delta_{\mu\nu}}{K^2 + \delta_{\mu0}\delta_{\nu0}m_E^2}
\]

\[
m_E^2 = g^2T^2 \left( \frac{N_C}{3} + \frac{N_f}{6} \right)
\]
The Debye Screening Mass

thermal gluon propagator in Fourier space

\[ \langle \tilde{A}_\mu^a(K)\tilde{A}_\nu^b(Q) \rangle \approx 0 \propto \delta^{ab}\delta_{\mu\nu} \]

\[ m_E^2 = g^2 T^2 \left( \frac{N_C}{3} + \frac{N_f}{6} \right) \]

at leading order:
color-electric fields get screened in a thermal plasma

color-magnetic fields are not screened

Basics of Thermal Field Theory, M.Laine, A.Vuorinen
Charge density correlator

\[ G_{00}^{(k_n)}(z) = \lim_{y,y' \to 0} T \int_x \left\langle V_0^{(k_n)}(x, z; y) V_0^{(-k_n)}(0; -y') \right\rangle \]

with the point-splitting

\[ V_0^{(k_n)}(x, z; y) \equiv \sum_{0 < p_n < k_n} \phi^\dagger(x + \frac{y}{2}, z) W_{y,z} \phi_{p_n-k_n}(x - \frac{y}{2}, z) \]
Charge density correlator

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with the point-splitting

\[ V_0^{(k_n)}(x, z; y) \equiv \sum_{0 < p_n < k_n} \phi^\dagger(x + \frac{y}{2}, z) \mathcal{W}_{y,z} \phi_{p_n-k_n}(x - \frac{y}{2}, z) \]

and non-relativistic auxiliary fields

\[ \psi = \frac{1}{\sqrt{T}} \begin{pmatrix} \chi \\ \phi \end{pmatrix} \quad (10) \]

remember: quarks carry momenta \( \sim \pi T \gg gT \gg g^2 T \)

B.B.Brandt,A.Francis,M.Laine,H.B.Meyer, arXiv:1404.2404

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Static and non-static vector screening masses
Effective Potential

after weak-coupling expansion and taking the $y'$-limit

$$G_{00}^{(k_n)}(z) = - \sum_{0 < p_n < k_n} 2N_c T \lim_{y \to 0} w_{\text{LO}}(z, y) + O(\alpha_s)$$

$$w_{\text{LO}}(z, y) = \int_q e^{-i\mathbf{q} \cdot \mathbf{y}} \left( M_{cm} + \frac{q^2}{2M_r} \right) |z|$$
after weak-coupling expansion and taking the $y'$-limit

$$G_{00}^{(k_n)}(z) = - \sum_{0 < p_n < k_n} 2N_c T \lim_{y \to 0} w_{\text{LO}}(z, y) + \mathcal{O}(\alpha_s)$$

$$w_{\text{LO}}(z, y) = \int_{q} e^{-i q \cdot y} (M_{cm} + \frac{q^2}{2M_r}) |z|$$

$$M_{cm} = k_n + \frac{m_\infty^2}{2M_r}, \quad m_\infty^2 = \frac{g^2 T^2 C_F}{4}, \quad M_r = \left( \frac{1}{p_n} - \frac{1}{k_n - p_n} \right)^{-1}$$
Motivation

Effective Approach

Lattice Approach

Results

Summary and Outlook

Effective Potential

after weak-coupling expansion and taking the $y'$-limit

$$G_{00}^{(k_n)}(z) = - \sum_{0 < p_n < k_n} 2N_c T \lim_{y \to 0} w_{LO}(z, y) + O(\alpha_s)$$

$$w_{LO}(z, y) = \int_q e^{-i q \cdot y - \left(M_{cm} + \frac{q^2}{2M_r}\right)|z|}$$

$$M_{cm} = k_n + \frac{m_\infty}{2M_r}, \quad m_\infty = \frac{g^2 T^2 C_F}{4}, \quad M_r = \left(\frac{1}{p_n} - \frac{1}{k_n - p_n}\right)^{-1}$$

$$\left(\partial_z + M_{cm} - \frac{\nabla^2}{2M_r}\right) w_{LO}(z, y) = 0,$$

$$w_{LO}(0, y) = \delta^{(2)}(y)$$
Effective Potential and Schrödinger Equation

after NLO corrections and suppressing $y'$

$$(\partial_z + M_{cm}) w_{NLO}(z, y) \xrightarrow{z \to \infty} - V_{LO}^+(y) w_{LO}(z, y)$$

$$V_{LO}^+(y) = \frac{g_E^2 C_F}{2\pi} \left[ \log \left( \frac{m_E y}{2} \right) + \gamma_E + K_0(m_E y) \right]$$

with $g_E^2 = g^2 T$ effective coupling of dimensionally reduced theory, $m_E$ Debye mass and $K_0$ modified Bessel function.
Effective Potential and Schrödinger Equation

after NLO corrections and suppressing $y'$

$$(\partial_z + M_{cm}) w_{NLO}(z, y) \overset{z \to \infty}{=} -V_{LO}^+(y) w_{LO}(z, y)$$

$$V_{LO}^+(y) = \frac{g_E^2 C_F}{2\pi} \left[ \log \left( \frac{m_E y}{2} \right) + \gamma_E + K_0(m_E y) \right]$$

with $g_E^2 = g^2 T$ effective coupling of dimensionally reduced theory, $m_E$ Debye mass and $K_0$ modified Bessel function

with initial condition and a Fourier transform

$\rightarrow z$-independent inhomogeneous Schrödinger eqn.

B.B.Brandt, A.Francis, M.Laine, H.B.Meyer, arXiv:1404.2404

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Static and non-static vector screening masses
Static Sector $k_n = 0$

$S$-wave contribution from the transverse part of $G_{\mu\nu}(z)$

$$G_T^{(0)} = \lim_{y,y' \to 0} T \sum_{i=1}^{2} \int_x \langle V_i^{(0)}(x, z; y) V_i^{(0)}(0; -y') \rangle$$

inducing now

$$V_{LO}^{-}(y) = \frac{g_E^2 C_F}{2\pi} \left[ \log \left( \frac{m_E y}{2} \right) + \gamma_E - K_0(m_E y) \right]$$

$$M_{cm} = 2p_n + \frac{m^2_\infty}{2M_r}, \quad M_r = \frac{p_n}{2}$$

B.B.Brandt,A.Francis,M.Laine,H.B.Meyer, arXiv:1404.2404
\[ - \frac{G_T^{(0)}(z)}{T^3} \approx \frac{4N_c m_E^2 A_0^{-}}{\pi T^2} e^{-|z|E_0^{(l=0)}} \]
\[ - \frac{G_{00}^{(0)}(z)}{T^3} \approx \frac{4N_c m_E^4 A_1^{-}}{\pi T^2 p_n^2} e^{-|z|E_0^{(l=1)}} \]

with

\[ A_0^{-} = \frac{|R_0(0)|^2}{\int_0^\infty dy \, y |R_0(y)|^2}, \quad A_1^{-} = \frac{|R'_1(0)|^2}{\int_0^\infty dy \, y |R_1(y)|^2} \] (11)

for \(S\)-wave \((l = 0)\) and \(P\)-wave \((l = 1)\) channels, respectively

(reverse order!)

B.B.Brandt, A.Francis, M.Laine, H.B.Meyer, arXiv:1404.2404