A distributed algorithm for demand-side management: Selling back to the grid

Milad Latifi a, Azam Khalili a, Amir Rastegarnia a,*, Sajad Zandi a, Wael M. Bazzi b

a Department of Electrical Engineering, Malayer University, Malayer, 65719-95863, Iran
b Electrical and Computer Engineering Department, American University in Dubai, Dubai, United Arab Emirates

* Corresponding author.
E-mail address: rastegarnia@malayeru.ac.ir (A. Rastegarnia).

Abstract

Demand side energy consumption scheduling is a well-known issue in the smart grid research area. However, there is lack of a comprehensive method to manage the demand side and consumer behavior in order to obtain an optimum solution. The method needs to address several aspects, including the scale-free requirement and distributed nature of the problem, consideration of renewable resources, allowing consumers to sell electricity back to the main grid, and adaptivity to a local change in the solution point. In addition, the model should allow compensation to consumers and assurance of certain satisfaction levels. To tackle these issues, this paper proposes a novel autonomous demand side management technique which minimizes consumer utility costs and maximizes consumer comfort levels in a fully distributed manner. The technique uses a new logarithmic cost function and allows consumers to sell excess electricity (e.g. from renewable resources) back to the grid in order to reduce their electric utility bill. To develop the proposed scheme, we first formulate the problem as a constrained convex minimization problem. Then, it is converted to an unconstrained version using the segmentation-based penalty method. At each consumer location, we deploy an adaptive diffusion approach to obtain the solution in a distributed fashion. The use of adaptive diffusion makes it possible for consumers to find the optimum energy consumption schedule with a small number of information exchanges. Moreover, the proposed method is able to track drifts resulting from changes in the price parameters and consumer preferences. Simulations and numerical results show that our framework can reduce the total

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load demand peaks, lower the consumer utility bill, and improve the consumer comfort level.

Keywords: Energy, Systems engineering, Electrical engineering

1. Introduction

The smart grid is a novel concept that was introduced to improve the stability, reliability, efficiency, and quality of service of the traditional power system. This paradigm focuses on integrating information and communication technologies (ICT) and advanced metering infrastructure (AMI) in current power grids to enable bidirectional, automated, and intelligent interaction among all system components [1, 2]. Unlike the conventional power grid, in which consumers are considered as passive consumption points, smart grids treat end users as dynamic entities, which participate in the grid operations and affect programs implemented throughout the system. The smart grid implementation enables electric power to be generated, transmitted, distributed, and consumed in a reliable and efficient manner and with high quality and environmental friendliness. Moreover, through proper planning and management of suppliers and consumers, smart grids are able to learn and adapt accurately and rapidly to the inherent uncertainties in the power system demand and supply process [3, 4].

In this regard, the demand side management (DSM) programs have been employed to manage factors such as load shaping and end users consumption behavior and to provide inelastic consumers with flexibility in their electric energy consumption. These programs are used to achieve a predetermined target such as improving consumer satisfaction and the overall operation efficiency and reliability of the power system. This is accomplished by integrating renewable energy resources and emerging technological appliances (e.g., PHEV), and reducing the total system peak-to-average load demand (PAS) as well as the cost of supplying energy [5, 6].

From the perspective of the utility company (energy provider), the DSM helps reducing the need to construct new fossil fuel power plants to compensate for the energy shortage caused by the rising energy demand. However, the utility company has to provide some incentives to assure that consumers follow the proposed program. We mention here two examples of DSM program implementations. One example is direct load control (DLC) programs offer rebates and financial incentives [7]. The other example is related to incentive based programs such as real time pricing and time of use pricing [8].

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1 Plug-in hybrid electric vehicles (PHEVs) and renewable energy sources (RESs) are the major origins of making uncertainty in management of the demand side and supply side of the future smart grid, respectively.
1.1. Related research

There have been several studies in the literature on DSM implementations using incentive based programs. The proposed methods are based on deriving an optimal consumption schedule according to which household appliances are used [2, 4, 8, 9, 10, 11, 12, 13, 14, 15, 16]. For example, in [2] Mohsenian-Rad et al. proposed an autonomous and decentralized demand side management program using game theory to minimize the peak to average ratio (PAR) and the consumer energy payment. They formulated the DSM problem as an energy consumption scheduling game, where the players are the consumers and their strategies are the daily load schedules. They showed that for a common scenario, with a single utility company serving multiple consumers, the global optimal performance in terms of minimizing the consumer energy cost is achieved at the Nash equilibrium of the formulated energy consumption scheduling game.

Samadi et al. [10] modeled consumer preferences and energy consumption patterns by an analytical utility function and used a Vickrey–Clarke–Groves (VCG) mechanism to maximize the social welfare criterion. The criterion was constructed by subtracting the total energy cost imposed on the utility company from the aggregate consumers utility functions. In [8], a real-time pricing method for reducing the PAR was proposed. In this method, the global optimization problem of the smart grid was divided into a two-stage optimization process. At demand side, when the price signal is received, each consumer seeks to maximize his payoff (i.e., the difference between the consumers quality of usage and the payment to the utility company) either in closed form or through an iterative algorithm with pricing parameters. At the supply side, the utility company designs the real-time pricing system using consumer behavior forecasts and a simulated annealing based price control (SAPC) algorithm.

In [11], the authors proposed a real-time energy consumption scheduling algorithm which takes into account load uncertainties and minimize consumers payment. To reduce the complexity of solving the load scheduling optimization problem, an approximate dynamic programming approach was utilized. Unlike common DSM algorithms which assume perfect knowledge of consumers load demand, their method required only some estimates of the future demand. In [17], Soliman et al. studied the DSM problem in which consumers have access to energy storage devices. They proposed two games, a non-cooperative game played between consumers and a Stackelberg game played between the utility company and the consumers. To allow consumers to sell back energy to the grid, a strictly convex logarithmic function was used. The considered Stackelberg game was shown to be a general case of the PAR minimization problem. The authors in [4] used game theory and formulated the energy consumption scheduling problem as a mixed integer programming
(MIP) problem to optimize the residential consumers objective functions. The scheme integrated local renewable energy resources to minimize dependency on the conventional fossil fuel energy resources which suffer higher energy production cost.

A pool-based demand response exchange (DRX) model in which economic demand response (DR) is traded among DR participants to manage the variability of renewable energy sources (RESs) is discussed in [18]. The paper describes load curtailment bids by individual DRX participants and the DRX is cleared by maximizing the total social welfare, which is subject to supply-demand balance and individual bidders’ intertemporal operation constraints. The problem of load scheduling and power trading in systems with high penetration of RESs was studied in [19]. In the considered scenario, customers can sell their excess power generation to other users or to the utility company and an approximate dynamic programming approach is used to schedule energy consumption of houses hold appliances.

The DSM method introduced in [20] deals with mitigating the voltage rise problem and considers high penetration of rooftop photovoltaic units in the future smart grid systems. In [21], a two-tier cloud-based DSM implementation was used to control the residential load of customers equipped with local power generation and storage facilities as auxiliary sources of energy. The proposed framework considers a power system consisting of multiple regions and equipped with a number of micro-grids. Then, an edge cloud is utilized to find the optimal power consumption schedule for customer appliances in each region. Zazo et al. have proposed a realistic model that accounts for uncertainties in smart grids and calculates a robust price for all users in the system [22]. Another energy consumption scheduling scheme dealing with the RESs is presented in [23]. The scheme proposes an alternative one-day-ahead energy dispatch solution to achieve the economic benefits of meeting the demand while minimizing electricity purchase cost by optimizing the demand generation and storage utilization efficiency in the presence of real-time pricing (RTP).

1.2. Contributions

One main drawback of the reviewed methods is that none of them is scalable in terms of real time adaptation nor is robust to additive noise channels or link failures. In this paper, we propose a fully distributed framework wherein, each consumer in a neighborhood minimizes his electric bill while maximizing his service level in a cooperative mode and through local information exchange. Unlike earlier research efforts, this work requires consumers to share only the estimate of optimal energy consumption pattern with their neighbors, without the need to know any of their own information and preferences. Another difference between our method and previous techniques is its ability learn and track drifts resulting from changes in
the wholesale market price parameter and consumer preferences. In addition, due to using the diffusion strategy and modeling the smart grid as an adaptive network, our framework is scale-free and an increase in the number of consumers does not increase the system calculations nor the communication burden significantly. Besides, as our proposed algorithm relies on the diffusion strategy, we expect that the framework will also work in the presence of imperfect communication and link failures [24, 25, 26]. Moreover, the theoretical models of the most important renewable energy resources (solar and wind) and the new objective function enable consumers to sell any excess energy back to the main grid at the right time and best price to make profit.

The rest of this paper is organized as follows. We first present the smart grid model in Section 2. Subsequently, the DSM problem formulation and the distributed algorithm is proposed in Section 3. In Section 4, numerical results are presented and discussed. Conclusions are drawn in Section 5.

2. Model

Similar to common models used in the smart grid DSM literature, we consider a power system with a number of residential consumers obtaining their electric energy needs from a utility company. In this model, a consumer interacts with the utility company and other consumers through a local area network (LAN) protocol. The utility company, in turn, is connected to the wholesale market through a wide area network (WAN) for buying electricity and selling it to consumers. Embedded smart meters at the consumer side are responsible for communication and data transfer between the consumer and the utility company, as well as controlling, managing, and monitoring activities of household appliances. Electric appliances of each consumer are connected to the smart meter through a simple home area network protocol (e.g., ZigBee).

Furthermore, we assume that all the smart meters have the ability to calculate and optimize their objective functions through the energy consumption scheduler (ECS). Indeed, the ECS is embedded in the smart meter and is responsible for scheduling the energy consumption of household appliances using a suitable objective function and the price signal information sent from the utility company. The block diagram of the smart grid model is shown in Figure 1. In the present paper, our focus is on the consumer side; we seek to provide a framework in which the smart meters interact among each other and with the utility company to reduce their payment and increase the consumer satisfaction level as much as possible. To this end, we adopt a discrete model for DSM scheduling and use vector analysis for operation of the appliances, as explained in the following section.
2.1. Demand-side model

In the considered smart grid model, we denote the set of consumers (i.e., home owners) as \( \mathcal{K} \), the set of time slots for a scheduling horizon \( H \) as \( \mathcal{H} = \{1, 2, \ldots, H\} \), and the set of household appliances for consumer \( k \in \mathcal{K} \) as \( A_k \). Let \( K = |\mathcal{K}| \), \( A_k = |A_k| \) denote the total number of consumers and the total number of consumer \( k \)'s appliances, respectively. Thanks to considering energy storage devices (e.g., battery), each consumer can buy and store electricity in his battery at low price times, and consume and/or sell the stored energy back to the utility company at high price times (i.e., peak load demand). Moreover, the storage devices prevent creation of sub-peak load demand, help to create better balancing between supply and demand, and reduce the fluctuations in power consumption patterns of the consumers.

Let \( x_{k,a}^h \) denotes the power schedule for appliance \( a \in A_k \) of consumer \( k \in \mathcal{K} \) at slot \( h \in H \). In this way, we show the power consumption vector scheduled by consumer \( k \)'s ECS for appliance \( a \) in a scheduling horizon\(^2\) to be bought from the utility company as follows:

\[
x_{k,a} \triangleq [x_{k,a}^1, x_{k,a}^2, \ldots, x_{k,a}^H]
\]

In this way, the power consumption vector scheduled to be taken from the consumer’s battery is:

\[
z_{k,a} \triangleq [z_{k,a}^1, z_{k,a}^2, \ldots, z_{k,a}^H]
\]

where \( z_{k,a}^h \) is the amount of power scheduled for appliance \( a \) of consumer \( k \) to be taken from battery at slot \( h \). We further denote by \( s_k \) the power storage vector scheduled for battery of consumer \( k \) as:

\[
s_k \triangleq [s_k^1, s_k^2, \ldots, s_k^H]
\]

\(^2\) Without loss of generality, we can choose the scheduling horizon as one day ahead with each time slot equal to one hour by setting \( H = |\mathcal{H}| = 24 \).
where $s^h_k$ is the power scheduled to be bought from the utility company and stored in the battery of consumer $k$ at slot $h$. Using (1), (2), and (3), the total power demand of consumer $k$ at slot $h$ is calculated as:

$$
I^h_k = \sum_{a \in A_k} x^h_{k,a} + s^h_k, \quad \forall \ h \in H
$$

(4)

According to (4), the total daily schedule vector for this consumer is:

$$
I_k = [I^1_k, I^2_k, \ldots, I^H_k]
$$

(5)

Finally, the utility company calculates the total load demand of all the consumers at slot $h$ as follows:

$$
L^h = \sum_{k \in \mathcal{K}} I^h_k, \quad \forall \ h \in H
$$

(6)

2.2. Appliances and equipment characteristics

Although, the most efficient schedule for energy consumption of the power system is a pattern in which the total load demand curve is flat, nonetheless, due to the different features and characteristics, the appliances have limited flexibility in charging their power consumption pattern. In general, all the household appliances have different specifications and characteristics. However, one can classify these appliances into specific groups according to appropriate criteria. In this regards and in order to model the constraints in our DSM program, we divide appliances into three categories as outlined in the following section.

2.2.1. Inelastic load

Inelastic appliances include refrigerators, lighting, TV, PC, and similar devices. The set of all inelastic loads for consumer $k \in \mathcal{K}$ is denoted as $A^m_k$. These appliances have always a strict and non-schedulable\(^3\) energy consumption pattern. So, there is no elasticity in adjusting the operation time, and they must start work as soon as needed. Because of stochastic operation nature of most of these appliances, a stochastic model needs to be used to capture the total hourly power consumption of these appliances. As the start and operation time of the appliances are stochastic and independent of the previous time-slots, one can use the discrete-time Markov chain for modeling the energy consumption behavior of these appliances [27]. Let’s consider a random-integer decision variable $x^h_{k,a} \in \{1(\text{on}), 0(\text{off})\}$ representing the state of the appliance $a \in A^m_k$ at slot $h$ with the state probability row vector $P^h_{k,a} = (p^h_{k,a}, p^h_{k,a}) = (P[x^h_{k,a} = 0], P[x^h_{k,a} = 1])$. In this vector, $p^h_{k,a}$, $p^h_{k,a}$ are the

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\(^3\) As is clear from Figure 1 (a), the ECS embedded in the smart meter does not has any authority on the inelastic loads.
probabilities of no change in operational status, and $p_{k,a}^{h,fo}, p_{k,a}^{h,of}$ are time varying transition probabilities from state off to state on and vice versa, respectively. The dynamic evolution in time of the state probability $p_{k,a}^{h+1}$ and the random energy consumption $x_{k,a}^h$ [kWh] at each slot $h$ can be described as:

$$I_{a,k} : \begin{cases} p_{k,a}^{h+1} = p_{k,a}^h M_{k,a}^h \\ x_{k,a}^h = x_{k,a}^{rat} M_{k,a}^h \end{cases} \quad (7)$$

where the time varying right stochastic matrix $M_{k,a}^h$ is the Markov transition probability matrix and $x_{k,a}^{rat}$ is the rated power consumption of the related appliance when it is on. Given $M_{k,a}^h$ and the initial state probability vector $P_0^h$, we can replace the random energy consumption $x_{k,a}^h$, in (7), with its deterministic value $\hat{x}_{k,a}^h$ taking expectation ($\mathbb{E}[-]$) over its probability distribution as follows:

$$\mathbb{E}[x_{k,a}^h] = \hat{x}_{k,a}^h = x_{k,a}^{rat} P_0^h M_{k,a}^h [0 \ 1]^T, \ \forall \ a \in A_{k}^{in} \quad (8)$$

where $M_{k,a,h} = M_{k,a}^1 \cdot M_{k,a}^2 \cdots M_{k,a}^{h-1}$ [28]. It is clear that the total energy consumption profile of all the inelastic appliances are also stochastic. As we can not change the original energy consumption pattern of these appliances, it is sufficient to know how much energy they consume throughout the scheduling horizon $H$. In this way, from (8), we can calculate the total energy consumption of these appliances at slot $h \in H$ as $\hat{E}_{k,in}^h = \sum_{a \in A_{k}^{in}} \hat{x}_{k,a}^h$, and the total energy consumption profile of these appliances becomes $\hat{E}_{k,in} = \{ \hat{E}_{k,in}^1, \hat{E}_{k,in}^2, \cdots, \hat{E}_{k,in}^H \}$.

### 2.2.2. Elastic-shiftable load

Elastic-shiftable appliances include PHEV, pool pump, iron, etc. The set of these appliances is denoted as $A_{k}^{sh}$. The operation time of each appliance $a \in A_{k}^{sh}$ can be changed in its operation window $H_{k,a} = \{ a_{k,a}, \cdots, b_{k,a} \}$, where $a_{k,a}$ and $b_{k,a}$ are the earliest time slot the appliance $a$ belonging to consumer $k$ can be turned on and the latest time slot before it, that appliance must finish its task, respectively. Although the operation time of these appliances can be managed, they must consume a specified amount of energy before the end of scheduling horizon for accomplishing their task. So, we must consider the following constraints:

$$\sum_{h=a_{k,a}}^{b_{k,a}} (x_{k,a}^h + z_{k,a}^h) = E_{k,a}, \ \forall \ a \in A_{k}^{sh} \quad (9)$$

$$x_{k,a}^h, z_{k,a}^h = 0, \ \forall \ h \notin H_{k,a} \text{ and } a \in A_{k}^{sh} \quad (10)$$
where $E_{k,a}$ in (9), is the total amount of energy appliance $a$ need to accomplish its task. Besides, the allowed range of power consumption of these appliances is shown as:

$$\gamma_{k,a}^{\min} \leq x_{k,a}^{h} + z_{k,a}^{h} \leq \gamma_{k,a}^{\max}, \quad \forall \ h \in H_{k,a} \text{ and } a \in \mathcal{A}_{k}^{h}$$

(11)

where $\gamma_{k,a}^{\min}$ and $\gamma_{k,a}^{\max}$ are minimum standby and maximum amount of power which can be consumed by appliance $a$ at each time slot $h \in H_{k,a}$.

### 2.2.3. Elastic-curtailable load

In a similar way, for an elastic-curtailable appliance $a \in \mathcal{A}_{k}^{cu}$, such as an air conditioner and ventilators, the following conditions are considered:

$$E_{k,a}^{\min} \leq \sum_{h=\ell_{l,a}}^{\ell_{u,a}} (x_{k,a}^{h} + z_{k,a}^{h}) \leq E_{k,a}^{\max}, \quad \forall \ a \in \mathcal{A}_{k}^{cu}$$

(12)

$$\bar{u}_{k,a}^{h} \leq x_{k,a}^{h} + z_{k,a}^{h} \leq \overline{u}_{k,a}^{h}, \quad \forall \ a \in \mathcal{A}_{k}^{cu}$$

(13)

where $E_{k,a}^{\min}$, $E_{k,a}^{\max}$ in (12) are the minimum and maximum tolerable energy requirement of appliance $a$ for finishing the assigned task, and $\bar{u}_{k,a}^{h}$, $\overline{u}_{k,a}^{h}$ in (13) are the minimum and maximum acceptable amount of power consumption determined based on consumer $k$’s preferences. It is clear that, for this kind of appliances we always have $\gamma_{k,a}^{\min} \leq \bar{u}_{k,a}^{h} \leq \overline{u}_{k,a}^{h} \leq \gamma_{k,a}^{\max}$. Moreover, constraint (10) must hold for the curtailable appliances out of their feasible operation window $H_{k,a}$ either.

Due to physical limitations on power transfer in transformers and cables, consuming and selling electricity per hour is constrained as follows:

$$l_{sel}^{\max} \leq I_{k}^{h} \leq l_{con}^{\max}, \quad \forall h \in H$$

(14)

where $I_{k}^{h}$ is defined in (5) and $l_{sel}^{\max} < 0$, $l_{con}^{\max} > 0$ are the maximum of selling and consuming energy threshold, respectively.

### 2.2.4. Storage devices

Storage devices such as battery banks and PHEV’s batteries have received considerable attention in DSP programs in recent years. However, the proposed frameworks did not include all implementation costs. One important factor is the battery degradation cost, which is directly related to the battery operations in the energy consumption scheduling program. Including effective factors in increasing the degradation cost of battery can be pointed to as; (1) number of charge/discharge cycles, (2) depth of discharge (DoD), (3) state of charge (SOC) in usage time, (4) current/power-rate in charge/discharge operations, and (5) the environmental
temperature. Based on the physical characteristics, each storage device has a limited capacity of energy storage. So, we must consider the upper ($B_k^{\text{max}}$) and lower ($B_k^{\text{min}}$) bound of the amount of energy which can be stored in the storage device in each slot $h \in H$:

$$B_k^{\text{min}} \leq \sum_{u=1}^{h} \left( s_{k,u} - \sum_{a \in A_k} z_{k,a}^u \right) \leq B_k^{\text{max}}$$

(15)

where the middle term represents the net inward power for battery of consumer $k$ during the start time of the scheduling horizon up to time $h$. Furthermore, for increasing the battery life time, the battery charging/discharging rate must be bounded as follows:

$$0 \leq s_k^h, 0 \leq z_k^h, \forall h \in H$$

(16)

where $s_k^{\text{max}}$ is the maximum charging rate and $z_k^{\text{max}}$ is the maximum discharge rate. The most important factor we must take into consideration in the proposed DSM framework is the increase in the degradation process with extra cycling under extreme ambient temperature and the change in DoD during charging/discharging periods [29]. So, we must provide some appropriate cost function for consumer $k$ to prevent numerous charging/discharging cycles during energy consumption scheduling as follows:

$$C_{k,h}^\text{bat}(L_{k,cyc}^h) = \frac{C_{k,\text{inv}}}{L_{k,cyc}^h \cdot B_k^{\text{max}} \cdot DOD_{k,\text{ref}}}$$

(17)

where $C_{k,\text{inv}}$ is the battery investment cost (e.g., in $\text{S}$), $L_{k,cyc}$ is the cyclic lifespan of battery determined based on the DoD at slot $h$, $B_k^{\text{max}}$ is the battery capacity in [kWh], and $DOD_{k,\text{ref}}$ is some constant reference of depth of discharge. DoD of consumer $k$’s battery is simply calculated as $DOD_k = 1 - B_k^{\text{max}}$, where $B_k$ is the battery energy level in [kWh]. However, the relationship between batteries life span $L_{k,cyc}$ (in the number of cycles) and the depth of discharge can be obtained using statistical data processing techniques such as maximum likelihood estimation methods (see [30] for detailed discussion).

### 2.2.5. Photovoltaic (PV) system

Power generation of PV depends on some factors; for instance, the number of cells, the direction of cells, the weather conditions and the temperature. Since solar irradiance is a stochastic variable and varies in different weather conditions, the generation of PV is also an uncertain variable. The best function to model the stochastic behavior of solar irradiance is Beta distribution [31]:

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\[ f(s) = \begin{cases} \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \cdot s^{\alpha-1} \cdot (1-s)^{\beta-1}, & 0 \leq s \leq 1, \; \alpha \geq 0, \; \beta \geq 0 \\ 0, & \text{otherwise} \end{cases} \] (18)

where,

\[ \Gamma(z) = \int_0^\infty x^{z-1} e^{-x} dx, \] (19)

\[ \alpha = \frac{\mu \cdot \beta}{1 - \mu}, \; \beta = (1 - \mu) \cdot \left( \frac{\mu \cdot (1 + \mu)}{\sigma^2} - 1 \right) \]

with the shape parameters \( \alpha, \beta \), the solar irradiance \( s \) in [kW/m\(^2\)], the gamma function \( \Gamma(\cdot) \), and \( \mu, \sigma^2 \) denoting the mean and variance of solar irradiance.

According to (18) and (19), the estimated generated power from the PV can be calculated as follows [31]:

\[ \hat{P}_{pv} = N \cdot F_{fill} \cdot V \cdot I, \]

\[ F_{fill} = \frac{V_{mpp} \cdot I_{mpp}}{V_{oc} \cdot I_{sc}}, \; V = V_{oc} - k_v \cdot T_{cell}, \]

\[ I = s \cdot (I_{sc} + k_c(T_{cell} - 25)), \; T_{cell} = T_a + \left(s \cdot \frac{T_N - 20}{0.8}\right) \] (20)

where \( N \) is the number of PV cells, \( F_{fill} \) is the fill factor, \( V_{mpp}, I_{mpp} \) are voltage and current at maximum power point, respectively, \( V_{oc} \) is the open-circuit voltage, and \( I_{sc} \) is the short-circuit current. \( k_v, k_c \) are the voltage/current temperature coefficients, and \( T_{cell}, T_a, T_N \) are the cell temperature, the ambient temperature and nominal operating temperature in [°C], respectively.

### 2.2.6. Wind turbine

Power generation of WT depends on wind speed. Since wind speed is a stochastic variable, power generation of WT is also an uncertain variable. Some references show that wind speed profile in one area is conformed approximately to the Weibull distribution [32]

\[ f(w; \lambda, v) = \begin{cases} \frac{v}{\lambda^v} \left(\frac{w}{\lambda}\right)^{v-1} e^{-(w/\lambda)^v}, & w \geq 0 \\ 0, & \text{otherwise} \end{cases} \] (21)

where \( w \) is the wind speed, \( \lambda > 0 \) is the scale parameter, and \( v > 0 \) is the shape parameter of the distribution. As, the Weibull distribution is related to a number of other probability distributions; in particular, it interpolates between the exponential distribution (\( v = 1 \)) and the Rayleigh distribution (\( v = 2 \) and \( \lambda = \sqrt{2\sigma^4} \)), there are a...
lot of works that seek to provide an efficient framework to choose the most suitable parameter of this distribution to estimate the wind speed as accurate as possible. Meanwhile, it seems that the Rayleigh distribution is the most similar distribution to wind speed profiles [33]. Based on the estimated wind speed in (21), the generated power from the wind turbine can be calculated as follows:

\[
\hat{P}_w = \begin{cases} 
0, & v < v_{\text{in}} \cup v_{\text{out}} < v \\
prat \cdot \left( \frac{v - v_{\text{in}}}{v_{\text{rat}} - v_{\text{in}}} \right), & v_{\text{in}} \leq v \leq v_{\text{rat}} \\
prat, & v_{\text{rat}} < v \leq v_{\text{out}} 
\end{cases}
\]

where \( p_{\text{rat}} \) is the rated power output of wind turbine and \( v_{\text{in}}, v_{\text{out}} \), and \( v_{\text{rat}} \) are the cut-in, cut-out, and rated wind speeds, respectively [34].

### 2.3. Objective functions

To identify and clarify the purpose of implementing any optimization program, a rational objective function must be considered and modeled appropriately. In our proposed framework, we first define the cost and disutility functions, then, construct a weighted-sum multi-objective function using them.

#### 2.3.1. Cost function

Cost function represents the cost consumers have to pay (i.e., electric bill), and the profit earned by the utility company. This function is the most important tool for encouraging consumers to follow a proposed energy consumption pattern determined by the utility company. Indeed, cost functions are pricing tariffs the utility companies adopt to determine and broadcast the price signal to discourage consumers from consuming additional energy at peak load demands. Assume that \( C_h(l_k^h) \) denotes the payment of consumer \( k \) due to consuming \( l_k^h \) at slot \( h \). For modeling a legitimate cost function we must consider the following requirement:

- Cost function is an increasing function of total load demand, \( \partial C_h(l_k^h)/\partial l_k^h \geq 0, \forall h \in H \). Namely, more energy consumption results in a higher cost (i.e., \( C_h(l_k^h) \leq C_h(l_k^h) \forall l_k^h \leq l_k^h) [2, 5].
- Cost function is assumed to be strictly convex, \( \partial^2 C_h(l_k^h)/\partial l_k^h > 0, \forall h \in H \).
  This means that, for all \( h \in H \), we must have \( C(\theta l_k^h + (1 - \theta)l_k^h) < \theta C(l_k^h) + (1 - \theta)C(l_k^h) \), where \( \theta \) is real scalars such that \( 0 < \theta < 1 \).
- If consumer \( k \) wants to sell the stored energy in his battery back to the grid, we have \( l_k^h < 0 \). In this case, consumer \( k \) would like to make profit, therefore we must have \( (C(l_k^h) < 0 \forall l_k^h < 0) [17] \).
Because in our proposed model the consumers are seeking to sell electricity back to the grid, the quadratic and linear cost functions can not satisfy all the mentioned constraints. It seems that the widely used logarithmic barrier functions as a penalty function in the interior point methods in convex optimization problem is good choice [35]. So, we propose the following logarithmic cost function [17]:

\[
C_h(t^h_k) = -\nu_h \log \left(1 - \frac{t^h_k}{\lambda}\right), \quad \forall \ h \in H
\]  

(23)

where \(\nu_h\) is the price parameter determined by the wholesale market to entice customers to consume less energy at peak hours (i.e., higher \(\nu_h\) at peak and lower at off-peak). The parameter \(\lambda\) is adopted to give cost values that are very close to the values given by quadratic cost functions. By assuming that the consumers in the DSM program are price-tacker (i.e., their behavior does not affect the wholesale market price), it is proved that minimization of this cost function in a Stackelberg game model, is equal to minimization of the PAR (see proof in [17]). This means that, as objective function (23) is convex in \(t^h_k\) and concave (linear) in \(\nu_h\), from the theory of min–max optimization (the class of convex-concave functions [35]) with some manipulation on (23) and considering (6) we reach to the following same objective function [17]:

\[
\min_{L^h \forall h \in H} \max_{h \in H} H \cdot \frac{\sum_{k=1}^{K} t^h_k}{\sum_{h=1}^{H} \sum_{k=1}^{K} t^h_k} = \min_{L^h \forall h \in H} \max_{h \in H} H \cdot \frac{L^h}{\sum_{h=1}^{H} L^h}
\]  

(24)

which (24) is clearly the global PAR minimization problem. We have compared a quadratic [15], a two-step piecewise linear [2], and the presented logarithmic cost functions as in Figure 2.
2.3.2. Disutility function

We can model the characteristics and behavior of different consumers by adopting a proper disutility function. In fact, through considering disutility function $D_h(l_k^h)$, we can consider the comfort/satisfaction level obtained by consumer $k$ as a function of his total energy consumption at each slot $h \in H$. A legitimate disutility function has the following properties:

- It is a continuously decreasing ($\partial D_h(l_k^h)/\partial l_k^h > 0$) and convex ($\partial^2 D_h(l_k^h)/\partial (l_k^h)^2 \geq 0$) function.
- The sign of the function changes from positive to negative at the median level of energy demand (i.e., $m_k^h$)

\[
D_h(l_k^h) = \begin{cases} 
> 0, & \text{if } l_k^h \leq m_k^h \\
< 0, & \text{if } l_k^h \geq m_k^h \\
= 0, & \text{if } l_k^h = m_k^h 
\end{cases}
\]

We adopt the novel disutility function (presented in [36]) as follows:

\[
D_h(l_k^h) = e^{\omega_k^h \left(1 - l_k^h / m_k^h\right)} - 1, \quad \forall \ h \in H
\]  

(25)

where $\omega_k^h \geq 0$ is the priority factor determined by consumer $k$ for capturing the value of energy consumption at slot $h$, i.e., a slot with higher $\omega_k^h$ has a lower priority (value) for consuming energy. In this paper, parameter $\omega_k^h$ is assumed to be predetermined and constant.

2.3.3. Estimation error penalty

In Sections 2.2.5 and 2.2.6, we introduced the output power estimation methods for the PV and wind turbine equipment. However, it is clear that there is always a difference between the actual power output and the estimated one. This difference creates some technical problems in the operation of power system, such as power quality reduction, increase in power losses, voltage and frequency deviation. So, it is vital to reflect the mentioned effects, by means of a penalty term that is added to the DSM objective function. It must be noted that as the investment, installation and maintenance costs of the renewable power production equipment are assumed to be constant, thus we only can consider the cost imposed to the grid because of non-programmable renewable energy sources as follows:

\[
C_h^{non}(P_{pu}^h, P_{pu}^h) = \left\| (P_{pu}^h + P_{pu}^h) - (\hat{P}_{pu}^h + \hat{P}_{pu}^h)^h \right\|^2_2
\]  

(26)
where \( P_x^h \) and \( P_z^h \) are the actual power output of the PV and wind turbine at slot \( h \), and \( \hat{P}_x^h, \hat{P}_z^h \) are the estimated quantities as slot \( h \) and are defined in (20) and (22), respectively.

Now, we construct our new weighted-sum multi-objective discomfort function at each consumer side using (17), (23), (25), and (26) as follows:

\[
J_k(t^h, t^h, P_x^h, P_z^h) = \sum_{h=1}^{H} \left( \sum_{i=1}^{\delta_1} C_h(t^h) + \sum_{j=1}^{\delta_2} D_h(t^h) + \sum_{k=1}^{\delta_3} C_{bat}(L^h) + \sum_{l=1}^{\delta_4} C_{non}(P_x^h, P_z^h) \right)
\]

where parameters \( \delta_1, \delta_2, \delta_3, \delta_4 \) denote the important (priority) of the corresponding functions to be optimized which satisfy \( \sum_{i=1}^{\delta_1} \theta_i = 1 \). For example, \( \delta_4 \) is adopted to reflect the importance of the accurate estimation of the power generation.

In the next section, we show that while each consumer seeks to minimize this function selfishly, the cooperation with other consumers make the proposed method to achieve social optimal solution in a fully distributed manner and without requiring any intervention from other part of the smart grid.

3. Methodology

In this paper we assume that the utility company is only regulated to provide energy for consumers with high quality and reliability and does not want to make profit. So, we seek to propose a fully distributed framework in which all the consumers can autonomously schedule their energy consumption profiles in order to minimize their discomfort without any need to the global information.

3.1. Problem formulation

The goal of proposed framework is to schedule the energy consumption pattern of all consumers such a way that the sum of all the discomfort functions of the consumers is minimized (i.e., socially optimal solution), while satisfying all the constraints distributed across the smart grid. To this end, we can formulate our DSM framework in a vector/matrix form as the following optimization problem:

\[
\begin{align*}
\min_{Y} & \quad g_{\text{glob}}(Y) = \sum_{k \in \mathcal{K}} J_k(y_k) \text{ s.t., } 1 \leq Y \leq u, \\
& \quad q \leq BY \leq b, \ r \leq AY \leq e, \ o \leq SY \leq d
\end{align*}
\]

where \( y_k \) is the vector of decision variables which is constructed by concatenating all variables \( x^h_{k,a}, z^h_{k,a}, \) and \( s^h_{k} \) on top of each other for all \( a \in \mathcal{A}_k \) and \( h \in \mathcal{H} \). In a similar way, \( Y \) is formed by concatenating all \( y_k \) for all \( k \in \mathcal{K} \). Moreover, \( l \) and \( u \)
denote the lower and upper bounds constructed of concatenating all the lower and upper bounds in (11), (13) and (16), respectively. B is considered to satisfy (15) for batteries of all consumers, and gives the batteries net inward flux, while b, q denotes the batteries charge and discharge capacity, respectively. Further, A captures the energy demand for all appliances of all consumers in equations (9) and (12) and r, e are the lower and upper bounds of total required energy for all appliances of all consumers, respectively. It is clear that for shiftable appliances we have r = e, which changes inequality r ≤ AY ≤ e to AY = e = r. The total amount of power consuming and selling (i.e., equation (14)) by all the consumers is captured by S and is bounded by d, o, respectively. As the cost function in (27) is strictly convex, the disutility is convex, and all the constraint are convex/affine, the global system objective minimization problem (27) become strictly convex and has a unique global optimum solution [35]. So, we can use any convex optimization method to solve it. However, due to the advantage of adaptive diffusion strategy mentioned in section 1, we use the fully distributed adaptive diffusion cost-augmentation method introduced in [37] to optimize all the strictly convex objective function distributed across the consumers side individually and in a cooperative and decentralized manner.

### 3.2. Diffusion strategy

To implement (solve) the DSM by diffusion strategy, as described in [37], we first need to convert the constrained global minimization problem (27) into an unconstrained version. Then, decompose the new problem into sub-problems where every sub-problem requires information available at that consumer. So, we consider the global optimization problem as follows:

\[
\min_{y_1, \ldots, y_K} J^g(Y) = \sum_{k=1}^{K} J_k(y_k),
\]

s.t. \( y_1 \in S_1, \ldots, y_K \in S_K \)

\[
S_k = \left\{ y_k : F_{k,f}(y_k) \leq 0 \text{ and } G_{k,g}(y_k) = 0 \right\}
\]

where in (29), \( G_{k,g}(y_k) \) and \( g_k \) denote the affine constraints and number of affine constraints, and \( F_{k,f}(y_k) \) and \( f_k \) denote the inequality (convex) constraint and number of inequality constraints for consumer \( k \), respectively. Using penalty method, the unconstrained version of (28) becomes:

\[
\min_Y J^g(Y)
\]

where
The structure of diffusion strategy (the network topology). Nodes (customers) that are connected by edges can interact with each other. The neighborhood of customer $k$ is highlighted in green.

\[ J_{\eta}^{glob}(Y) \triangleq \sum_{k=1}^{K} J_{k,\eta}(y_k) \]

\[ J_{k,\eta}(y_k) \triangleq J_k(y_k) + \eta \cdot P_k(y_k) \quad (31) \]

\[ P_k(y_k) \triangleq \sum_{g=1}^{g_k} \delta^{EP}(G_{k,g}(y_k)) + \sum_{f=1}^{f_k} \delta^{IP}(F_{k,f}(y_k)) \]

where $\eta > 0$ is a scalar parameter that controls the relative importance of the constraints and, $\delta^{EP}(\cdot)$ and $\delta^{IP}(\cdot)$ are penalty functions for affine and inequality constraints, respectively. The mentioned functions can be selected as $\delta^{SEP}(x) = x^2$ and $\delta^{SIP}(x) = \max(0, x^3/\sqrt{(x^2 + \rho^2)})$ where $\rho > 0$ [37]. Clearly, all the parameters of (31) are local and rely on individual consumer $k$’s information. Therefore, we can optimize objective function $J_{k,\eta}(y_k)$, at each consumer $k$ side, using only local information available at the neighborhood $\mathcal{N}_k$ of this consumer and personal constraint set $\mathcal{N}_k$. In fact, every consumer optimizes his personal objective function only according to its own constraints as follows:

\[
\begin{align*}
\min_{y_k} J_{k,\eta}(y_k) \quad & \text{s.t.} \quad \mathbf{l}_k \leq y_k \leq \mathbf{u}_k, \\
\mathbf{q}_k \leq \mathbf{B}_k y_k \leq \mathbf{b}_k, \quad & \mathbf{r}_k \leq \mathbf{A}_k y_k \leq \mathbf{e}_k, \quad \mathbf{o}_k \leq \mathbf{S}_k y_k \leq \mathbf{d}_k
\end{align*}
\]

The structure of diffusion strategy applied on a network (smart grid) with 11 nodes (customers) is presented as in Figure 3. For a suitably chosen step size $\mu$, the interactions between the consumers guarantees the convergence of the sum of objective functions (i.e. (30)). As a well-known strategy, each consumer can implement (run) the adapt-then-combine (ATC) cost augmented algorithm described in Algorithm 1 to solve (32) [38]. Note that Algorithm 1 uses constant step size $\mu$ for continuous learning and adaptation. Moreover, the non-negative combination coefficients $d_{rk}$ in Algorithm 1 are chosen in such a way that they satisfy the following conditions:
Algorithm 1 Adapt-then-combine (ATC) diffusion strategy.

1: repeat
2: \( \phi_{k,i} = y_{k,i-1} - \mu V_{k,i} p_i(y_{k,i-1}) \)
3: \( \psi_{k,i} = \phi_{k,i} - \mu \eta V_{k,i} p_k(\phi_{k,i}) \)
4: \( y_{k,i} = \sum_{i \in A} d_{r,k} \psi_{k,i} \)
5: until convergence

\[ d_{r,k} = \begin{cases} 
0 & \text{nodes } r \text{ and } k \text{ are not neighbors} \\
> 0 & \text{nodes } r \text{ and } k \text{ are neighbors} \\
\sum_{r=1}^{K} d_{r,k} = 1 
\end{cases} \]

We can choose these coefficients according to several ways such as the averaging rule, the relative variance rule, the Metropolis, etc. [39].

4. Results

4.1. Simulation setup

In the considered scenario, there are \( K = 15 \) residential consumers in the smart grid. Each consumer is assumed to have 3 inelastic, 3 elastic-shiftable, and 2 elastic-curtailable appliances, as well as a battery and PHEV with ideal characteristics. The parameters for different appliances are summarized in Table 1. Some customers is assumed to have PV and/or wind turbine and the aggregate power output of these renewable resources is randomly selected between 0 to 15 kW at each hour for each of them. To model a case where there are not any DSM program in the smart grid, we assume that each consumer uses his appliances at their preferred times (i.e., each appliance start its operation at time slot \( \alpha_{k,a} \)). The price parameters coming from the wholesale market is assumed to be \( \lambda = 50 \) and \( \nu_h = (1.5, 1.2, 1, 2, 3, 4.5, 4.5, 5, 6, 6, 6, 6, 5, 4, 4.5, 5, 6, 8, 9, 10, 11, 9, 7, 5) \).

In Algorithm 1, the combination coefficients \( d_{r,k} \) are selected according to the Metropolis rule, the constant step size is \( \eta = 8000 \) and other parameters are selected as \( \eta = 8000 \) and \( \rho = 100 \).

---

Table 1. Household appliances characteristics.

| Appliance name          | \( E^{\text{min}} \) (kWh) | \( E^{\text{max}} \) (kWh) | \( y_{a}^{\text{min}} \) (kW) | \( y_{a}^{\text{max}} \) (kW) | \( w_{a}^{\text{min}} \) (kW) | \( w_{a}^{\text{max}} \) (kW) | \( \alpha \) (h) | \( \beta \) (h) |
|------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------|-----------|
| Refrigerator           | 6               | 6               | 0.25            | 0.25            | -               | -               | 1:00      | 24:00     |
| Lighting               | 3               | 3               | 0.5             | 0.5             | -               | -               | 18:00     | 24:00     |
| PC                     | 1.6             | 1.6             | 0.2             | 0.2             | -               | -               | 8:00      | 24:00     |
| Washing machine        | 2.5             | 2.5             | 0               | 1.25            | -               | -               | 8:00      | 20:00     |
| Tumble dryer           | 1.5             | 1.5             | 0               | 0.5             | -               | -               | 20:00     | 7:00      |
| PHEV                   | 9               | 9               | 0               | 1.5             | -               | -               | 19:00     | 8:00      |
| Boiler                 | 10              | 10              | 0               | 2.5             | -               | -               | 15:00     | 7:00      |
| Air conditioner        | 12              | 24              | 0               | 1.75            | 0.5             | 1               | 1:00      | 24:00     |
| Ventilator             | 6               | 12              | 0               | 0.8             | 0.25            | 0.5             | 1:00      | 24:00     |
4.2. Results

The simulation results for the energy consumption pattern of the consumers in the cases with deployment of our framework vs without any DSM program are presented in Figure 4. We can see that our method reduces the fluctuation of load demand curve. The total system PAR with DSM (our method) is 1.0369, while without any scheduling the PAR is 1.6667 which shows almost 37.8% PAR reduction. The energy storage devices behavior and the price parameter $\nu_h$ trend are shown in Figure 4 in which the initial and final state of each consumer’s battery energy level is limited to be equal. For example, if the initial state of the charge of consumer k’s battery is 30%, the final state of the charge must be 30% either, to provide the same initial state for the next day. According to the aggregate storage devices profiles, we can find that the consumers are charged their batteries at time slots with low price and discharged them at peak load demand (high price) times.

In Figure 5, we have analyzed our proposed framework over 31 days in summer (from 8/1/2017 to 8/31/2017) with real wholesale day-ahead electricity price data from Pennsylvania–New Jersey–Maryland Interconnection (PJM) electricity market [40]. In the considered scenario, we have assumed that there are 10 customers with maximum 50 electric appliances. Each customer has 20 different appliances with inelastic characteristic, 20 different appliances with elastic-shiftable characteristic, 10 different appliances with elastic-curtailable characteristic, and one battery. Further, we have assumed that the network topology is changed at each day and each customer can adopt maximum 4 neighbors for data sharing as desired.

As is exhibited by Figure 5, our proposed strategy has lower power consumption fluctuation and achieves lower PAR in all the days compared with when there is not
any DSM strategy. However, the total average PAR reduction for each day in this analysis is almost 29% which is lower than the previous scenario. This is because the total PAR reduction ability is comparative and strictly depends on four factors:

1. **The original demand profile characteristic:** whatever the original demand profile has a large peak point and the ratio of elastic to inelastic appliances is also high (i.e., the demand profile is more flexible), the PAR reduction is more sensible.

2. **Efficiency of the applied price parameter:** The price parameters depend on the load demand; i.e. for higher load demand we must have higher price. As denoted in Figure 5, the price peak/valley and load demand peak/valley are not coincide. So, we can expect that the total PAR reduction is low, compared with the previous scenario.

3. **Efficiency and Capacity of the battery:** It is clear that battery with more capacity and efficiency provides more flexibility in the DSM. However, if the DSM strategy and the applied price signal are not appropriate, increasing battery usage can deteriorate the DSM strategy performance and create sub-peaks in the load demand profile [41].

4. **Synchronization of power production and consumption:** Suitable time synchronization between the generated power by renewable resources and power consuming, increases the efficiency of renewable power usage, which it turn reduces the total PAR.

**Figure 5.** Total system load demand with/without the proposed DSM method over 31 days.
Figure 6. Proposed DSM performance comparison through the scheduling horizon; (a) The disutility level of each customer, (b) total cost imposed of each customer.

By the same scenario considered for previous simulation, we have provided some results in Figure 6 to claim that our proposed DSM strategy is attractive, in terms of payment and satisfaction level, for all the customers and they will be self-interested to participated in our framework. In Figure 6 (b) we have depicted the consumers individual satisfaction level by choosing different weight factor $\theta_2$. As it is obvious, adapting larger amount for $\theta_2$ results in lower disutility level, meaning that the customers satisfaction level is improved. There are two bound on this figure; 1) minimum disutility level by choosing $\theta_2 = 1$, meaning that the customer do care only about his/her satisfaction and do not participate in the DSM program resulting in zero (p.u) disutility level, 2) maximum disutility by choosing $\theta_2 = 0$ considering the cost functions and seeking only to reduce the payments resulting in 1 (p.u) disutility level. However, the results for the customers are not the same due to different in the customers’ load demand profiles and preferences. In the same way, as much as the customers choose larger weight factor $\theta_1$, their payments become lower. However, there is always a trade-off for choosing the weight factors. As the weight factors are limited to add-up to one, choosing higher $\theta_1$ result in lower $\theta_2, \theta_3, \theta_4$ and so on. Meaning that, if one customer has good financial status, he/she can ignore cost functions $C_{th}(L_h^{th}), C_{k,h}(L_{k,cyc}^{th})$, and $C_{h}^{non}(P_{pv}^h, P_{w}^h)$ by lowering their weight factors and select higher $\theta_2$ to increase his/her satisfaction lever as much as desired.

The most important factor to satisfy the customers for participating in the DSM program is reducing their total payment. As it is shown in Figure 6 (b), all customers have reduced their payments by choosing some proper weight factors $\theta_2$. So, we can claim that our proposed scheme is attractive and all the customers are willing to
participate in it. The total system cost trend is presented in Figure 7. As illustrated in this figure, in the first iterations high oscillation occurs; however this oscillation is gradually attenuated.

In Figure 8 we see that the total PAR has monotonic decline and is not converged due to choice of constant step-size $\mu$. Indeed, this figure shows that the proposed diffusion-based method is able to track the changes in price parameter as well as the consumers conditions and preferences. The convergence of disutility function (25) is demonstrated in Figure 9. In the beginning of the proposed DSM method this
Figure 9. Convergence comparison of the total disutility function.

Figure 10. Effect of the consumers total load demand limit ($l_{con}^{max}$) on total PAR reduction percentage.

criteria is $-1$, which means that before adopting the scheduling program (without any DSM) the consumers are fully satisfied, because they are used their appliances when needed.

After running the diffusion ATC algorithm, the disutility is first increased but after a few iterations this function has downward trend along with cost reduction. Finally, the effect of maximum allowed power consumption ($l_{con}^{max}$) for each consumer $k \in \mathcal{K}$ on total PAR reduction with battery capacity $P_{k}^{max} = 3$ (kWh) is indicated in Figure 10. As is clear, if the utility company does not apply the total energy consumption restriction at each slots appropriately, the performance of the proposed method become worse and worse in terms of the PAR reduction. In this case there are some sub-peaks in total load demand curve at low price time slots, since the consumers attempt to consume more energy at low price slots to reduce their payment as much as possible.
5. Conclusion

In this paper, we proposed a fully distributed framework using diffusion strategy for minimizing the consumers’ payment and disutility simultaneously in smart grid. As the formulated problem is a constrained convex minimization problem, we used a cost augmented-based penalty method to obtain an unrestricted version of it. Then, we showed that the global optimization problem is sum of convex sub-problem, where each sub-problem requires information that are available by the corresponding consumer. In this sense, the consumers can autonomously, without any need to global information and with only local interaction with their neighbors minimize their discomfort level. The presented numerical results demonstrated that our framework converged in the acceptable number of iterations and achieved significant PAR and energy consumption cost reduction. Further, we showed that by using a constant step size the diffusion based algorithm continue learning and adaptation to track drift the location of optimal solution.

Declarations

Author contribution statement

Milad Latifi: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Amir Rastegarnia: Conceived and designed the experiments; Performed the experiments; Wrote the paper.

Wael Bazzi: Conceived and designed the experiments; Wrote the paper.

Azam Khalili: Analyzed and interpreted the data; Wrote the paper.

Sajad Zandi: Contributed reagents, materials, analysis tools or data.

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