SUPERLUMINOUS LIGHT CURVES FROM SUPERNOVAE EXPLODING IN A DENSE WIND

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ABSTRACT

Observations from the last decade have indicated the existence of a general class of superluminous supernovae (SLSNe), in which the peak luminosity exceeds $10^{44}$ erg s$^{-1}$. Here we focus on a subclass of these events, where the light curve is also tens of days wide, so the total radiated energy is of order $10^{51}$ erg. If the origin of these SLSNe is a core-collapse-driven explosion of a massive star, then the mechanism that converts the explosion energy into radiation must be very efficient (much more than in typical core-collapse SNe, where this efficiency is of order 1%). We examine the scenario where the radiated luminosity is due to efficient conversion of kinetic energy of the ejected stellar envelope into radiation by interaction with an optically thick, pre-existing circumstellar material, presumably the product of a steady wind from the progenitor. We base the analysis on analytical derivations of various limits, and on a simple, numerically solved, hydrodynamic diffusion model, which allows us to explore the regime of interest, which does not correspond to the analytical limits. In our results, we identify the qualitative behavior of the observable light curves, and relate them to the parameters of the wind. We specifically show that a wide and superluminous supernova requires the mass of the relevant wind material to be comparable to that of the ejected material from the exploding progenitor. We find the wind parameters that explain the peak luminosity and width of the bolometric light curves of three particular SLSNe, namely, SN 2005ap, SN 2006gy, and SN 2010gx, and show that they are best fitted with a wind that extends to a radius of order $10^{15}$ cm. These results serve as an additional indication that at least some SLSNe may be powered by interaction of the ejected material with a steady wind of similar mass.

Key words: circumstellar matter – shock waves – supernovae: general – supernovae: individual (SN 2005ap, SN 2006gy, SN 2010gx)

Online-only material: color figures

1. INTRODUCTION

The discovery of extremely luminous transients in the last years led to their classification as superluminous supernovae, or SLSNe (Quimby et al. 2007, 2011; Pastorello et al. 2010; Smith et al. 2010; Chatzopoulos et al. 2012; Gal-Yam 2012). In many of these events, the light curve is tens of days wide, so the total radiated energy is of order $\sim 10^{51}$ erg. The radius of the photosphere at peak luminosity is $R_{\text{ph}} \gtrsim 10^{13}$ cm, as inferred from the observed temperature, assuming blackbody emission. These high values of radiated energy challenge our understanding of the energy source and the conversion mechanism of available energy to radiated energy.

The standard model of core-collapse supernovae is that the explosion initiates a shock wave that propagates through the progenitor and deposits about half of the explosion energy as thermal energy and half as kinetic energy of the ejecta (for a strong shock in an ideal gas; see Zel’dovich & Raizer 1966). Supergiant supernovae progenitors have a typical initial radius of $R_{\text{p}} \sim 10^{13}–10^{14}$ cm. For such a progenitor, as the shock reaches the stellar surface the star is highly opaque, so most of the thermal energy is transformed to kinetic energy of the ejecta via adiabatic expansion, and the fraction of thermal energy that escapes as radiation when the expanding ejecta becomes transparent is roughly $R_{\text{e}}/R_{\text{ph}}$ (Arnett 1996). Other progenitors are even more compact. Therefore, in order to explain the radiated energy in SLSNe, we must either assume an energy source much larger than $\sim 10^{51}$ erg (typical for core-collapse SNe) or find a more efficient mechanism of transforming the explosion energy to radiation.

There have been several suggestions of efficient mechanisms for converting the explosion energy into emerging radiation (Smith & McCray 2007; Woosley et al. 2007; Chevalier & Irwin 2011; Chatzopoulos et al. 2012). These suggestions are all based upon extensive pre-explosion mass loss—a wind—by the progenitor star that expels matter to large radii, comparable with $R_{\text{p}}$. In principle, this can be the result of a burst of mass loss or of a steady wind. This matter contributes to the radiated energy in two complimentary manners. The extended mass defines an effective radius $R_{\text{eff}} > R_{\text{ph}}$, over which the shock deposits the explosion energy (essentially, creating a “bloated” star). With this larger radius, less of the thermal energy is lost through adiabatic expansion, and a larger fraction $R_{\text{ph}}^3/R_{\text{eff}}^3$ of this energy emerges as radiation. In addition, this material interacts with the expanding ejecta. This interaction converts kinetic energy of the expanding ejecta back to thermal energy via shocks propagating forward through the wind material and backward through the ejecta (Chevalier 1982; Chevalier & Fransson 1994; Gal-Yam 2012). In the roughest approximation, this interaction between the ejecta and the wind can be regarded as a plastic collision between an ejecta mass $M_{\text{ej}}$ moving at a certain velocity and a “stationary” wind mass $M_{\text{w}}$. In this approximation, a fraction $\sim M_{\text{ej}}/(M_{\text{ej}}+M_{\text{w}})$ of the ejecta kinetic energy will be converted to shock energy. If these shocks deposit the energy at a radius $\sim R_{\text{ph}}$, then it will escape as radiation without suffering significant adiabatic losses. We conclude that an excess mass of $M_{\text{w}} \sim M_{\text{ej}}$ extending to a radius $\sim R_{\text{ph}}$ can convert a substantial portion of the explosion energy to radiated energy and therefore explain the high luminosities of SLSNe with a conventional $\sim 10^{51}$ erg energy source.
Our focus here is on the particular case of supernova explosions in dense surroundings due to a steady mass loss, so the wind has an $r^{-2}$ density dependence. We do not necessarily advocate this scenario in terms of the late stages of stellar evolution, although some arguments in its favor have been made (see Quataert & Shiode 2012). The main advantage of the steady wind case is that it has only two parameters—the wind mass and its cutoff radius—and thus is well defined and can be examined thoroughly. Most of the previous works on supernova explosions in a dense, steady wind (Smith & McCray 2007; Smith et al. 2010; Ofek et al. 2010; Balberg & Loeb 2011; Chevalier & Irwin 2011; Chatzopoulos et al. 2012) used simple approximations that allow analytical order-of-magnitude estimations. Some of these works also drew conclusions from the self-similar solutions of Chevalier (1982), which describe the interaction of an expanding ejecta with a wind in the limit where the wind mass is small, $M_{ej} \gg M_\infty$, so only the outer layers of the ejecta interact with the wind, and $M_{ej}$ does not create a natural scale in the dynamics. For a more accurate relation between parameters of the star–wind system and the observed light curve, a self-consistent radiation hydrodynamic calculation is required. Early numerical calculations of supernova explosion into a dense wind have been carried out by Falk & Arnett (1973, 1977) and more systematically by Moriya et al. (2011). These works do not cover well the regime $M_\infty \sim M_{ej}$, which is of interest in the current work, in order to achieve a high efficiency in terms of generating a luminous light curve. One exception is the very recent work of Moriya et al. (2012), who conducted several numerical calculations of a light curve for the specific case of SN 2006gy: They considered the relevant mass regime for a similar but different case of a shell set at a standoff distance $R_{ph}$ from an exploding star.

We use hydrodynamic diffusion calculations to conduct a general survey of the relations between the light curve and the progenitor and wind parameters. We focus on the particular case of a steady wind generated during the last stages of the progenitor star’s evolution, so the wind essentially extends from the surface of the star. Our goal is to keep the model as simple as possible, and therefore we adopt many simplifying approximations, which while still sufficient to investigate the key features of light curves in the interacting ejecta and wind scenario, are also simple enough to be instructive and the results are easily understood. The simplifications we applied are detailed below.

We point out that alternative scenarios have been suggested for the SLSN light curves. These include pair instability models (Barkat 1967; Rakavy & Shaviv 1967; Smith et al. 2007), magnetar spin-down models (Maeda et al. 2007; Kasen & Bildsten 2010; Woosley 2010), and Quark-nova scenarios (Leahy & Ouyed 2008; Ouyed et al. 2012). We do not discuss these alternatives in the current work.

The outline of the paper is as follows. In Section 2, we present our model for general SLSN progenitor systems. In Section 3 we consider the qualitative relations between the system parameters and the observed light curve, and analytically examine various limits of the problem. The hydro-diffusion code is described in Section 4 and numerical results are presented in Section 5. The observed light curves of three specific candidates for an interacting ejecta and wind scenario, SN 2005ap, SN 2006gy, and SN 2010gx are analyzed and reproduced in Section 6. We summarize our conclusions in Section 7.

![Figure 1. Calculated light curves for different polytropic indices—$n = 3/2$ (solid black line) and $n = 3$ (dashed blue line). The plotted light curves were calculated with the parameters $R_\infty = 2.5 \times 10^{15}$ cm, $K = 10^{44}$ g cm$^{-1}$, and $E = 2 \times 10^{51}$ erg. (A color version of this figure is available in the online journal.)](image)

2. PROGENITOR SYSTEMS

In our scenario the progenitor system is composed of two parts: the gravitationally bound compact star and the outer material that is created by pre-explosion mass loss. Both are characterized by their total masses and respective density profiles. Since the supernova explosion induces temperatures much higher than the initial star temperatures, and ejecta velocities much higher than the initial wind velocity, both the star and wind are approximated as initially stationary and cold. The explosion can then be treated as an instantaneous release of thermal energy $E$ (a “thermal bomb”) at the center of the star ($r = 0$).

For systems in which the photosphere lies at $R_{ph} \gg R_\infty$ and $M_{ej} \sim M_\infty$, the details of the structure of the progenitor star are generally unimportant in terms of the resulting light curve (see Figure 1), since the star can be considered as a point object. None the less, we do want to allow also for calculations of systems where the wind mass is much smaller than the ejected mass, mainly for comparison with other works that investigated these cases (see, e.g., Moriya et al. 2011). We follow Matzner & McKee (1999) by modeling the progenitor star by a polytrope with radius $R_\infty$, mass $M_\infty$, and polytropic index $n = 3/2$ suitable for convective envelopes, like those in red supergiants, while $n = 3$ is used to describe radiative envelopes, such as those in blue supergiants. We approximate the star by applying the Lane–Emden equation to the whole star, not only to the outer envelope. Thus, $R_\infty$, $M_\infty$, and $n$ completely determine the initial density profile of the star. Note that since the Lane–Emden equation is applicable only for the envelope, $M_\infty$ actually denotes the envelope mass, and not the star mass. All these approximations are valid since the inner structure of the mantle has little effect on the shock wave propagation. The mantle can be considered as a point mass that is added artificially.
(see Matzner & McKee 1999). Thus, in terms of size, a 15 $M_{\odot}$ progenitor should actually be modeled by a $M_{\ast} \sim 10 M_{\odot}$ structure, to allow for the $\sim 5 M_{\odot}$ mantle.

For the wind density profile, we follow Balberg & Loeb (2011) and Chevalier & Irwin (2011) and consider a steady wind with mass loss rate $\dot{M}$ and wind velocity $v_{w}$. The resulting density profile is

$$\rho(r) = \frac{\dot{M}}{4\pi r^2 v_{w}} \equiv Kr^{-2}. \quad (1)$$

We impose this wind density profile by continuously attaching it to the polytropic structure of the star at a radius where the polytrope and Equation (1) coincide. For the parameters chosen in this work, this wind base radius is essentially at $R_{\ast}$.

The wind is assumed to continue to an outer radius $R_{w}$ so that its total mass is

$$M_{w} = \int_{R_{w}}^{R_{w}} 4\pi r^2 Kr^{-2} dr = 4\pi K (R_{w} - R_{s}). \quad (2)$$

For $R_{w} \gg R_{s}$, which is the case of interest in this work, Equation (2) is reduced to

$$M_{w} \approx 4\pi K R_{w}. \quad (3)$$

A discrete wind cutoff radius is consistent with the assumption of a high mass loss rate that develops during the latest stages of the star’s evolution. There may be some observational evidence in the case of SN 2006jc (Itagaki et al. 2006; Ofek et al. 2012). It is both more physical and numerically convenient to smooth the wind edge at $R_{w}$. Instead of cutting the wind abruptly at $R_{w}$, we extend the wind to a larger radius $r = \alpha R_{w}$ with $\alpha > 1$ (we usually choose $\alpha \sim 1.2$) and change the wind density profile from Equation (1) to

$$\rho(r) = Kr^{-2} e^{-(r/R_{w})^k} \quad (4)$$

with $k \gtrsim 10$ providing a sharp power law. The details of this smoothing have a minor effect on the results.

Since the light curve is dominated by the structure of the wind material, we mostly survey the wind parameters and fix the parameters of the star. We choose a standard progenitor star model with $R_{\ast} = 10^{13}$ cm, $M_{\ast} = 15 M_{\odot}$, and $n = 3/2$. These values are representative of typical supergiants. Since we study the case $R_{w} \gg R_{\ast}$, the progenitor star is approximately a point object in the wind. In this regime the exact value of $R_{\ast}$ has little effect on the light curve, and the same applies for the value of $n$ (3/2 or 3; see Figure 1). However, the value of $M_{\ast}$ does have a significant effect on the light curve, and the choice of 15 $M_{\odot}$ is arbitrary. Some consequences of changing $M_{\ast}$ are discussed in Section 6.

The motivations described in Section 1 lead to the choice $R_{w} \sim R_{ph} \sim 10^{13}$ cm (the relation $R_{w} \sim R_{ph}$ is justified in Section 3) and $M_{w} \sim M_{ej} \sim M_{\ast}$, which, using Equation (3), sets $K \sim 10^{18}$ g cm$^{-1}$. It is noteworthy that this combination of parameters implies a large mass loss rate of order $10^{-1} M_{\odot}$ yr$^{-1}$ (assuming that the mass is lost at velocities of order $10^{6}$ cm s$^{-1}$). Most likely, sustaining a steady wind at this rate requires very specific conditions, which may explain why SLSNe are so rare. Connecting the specifics of the progenitor history and conditions to a well-defined progenitor system is beyond the scope of this work. We note that luminous blue variables have been suggested (Smith et al. 2007) as possible progenitors, in view of the large periodic mass loss rates observed in some cases, but several complications exist (see, e.g., Dwarkadas 2011).

To summarize, our progenitor system is thus modeled by four parameters: the star mass $M_{\ast}$, the explosion energy $E$, the wind outer radius $R_{w}$, and the wind density coefficient $K$ from Equation (1). After fixing the star mass, we are left with three parameters that determine the properties of the light curve.

3. QUALITATIVE PICTURE AND EXTREME LIMITS

The supernova explosion initiates with a strong shock wave, which, when close to the edge of the star, becomes radiation dominated. Diffusion of energy carried by radiation causes the shock to develop a finite width with optical depth $\delta \tau \sim \beta_{sh}^{-1} \equiv c/v_{sh} \; (Weaver 1976)$, where $v_{sh}$ is the shock velocity and $c$ the speed of light. This optical depth can be intuitively understood by equating the hydrodynamical timescale and the diffusion timescale over the shock front. At large optical depths from the surface, $\tau \gg \beta_{sh}^{-1}$, the shock wave can still be treated as an ideal discontinuity, and diffusion can be neglected. When the shock approaches the surface, and $\tau \sim \beta_{sh}^{-1}$, energy can escape by diffusion to the surface, and the shock dissolves; the purely hydrodynamical (with diffusion neglected) solutions are no longer valid. At this stage the shock is said to “breakout.”

Shock breakout through the outer layers of a bare star has been discussed in several works. These rely on the self-similar solution found by Sakurai (1960) for a planar shock propagating through the steeply declining density profile at the edge of the star; see Matzner & McKee (1999) for a complete derivation. Sapir et al. (2011) expanded this self-similar solution to include diffusion and thus calculate a light curve under these conditions, again—for the plane-parallel case. In the presence of an extended wind, due to a pre-explosion mass loss from the progenitor, shock breakout must be considered in spherical symmetry. As the outgoing ejecta plows through the wind and slows down, it drives a forward shock through the wind and a reverse shock that propagates back into the ejecta, and it is the combined shock profile that eventually breaks out as it reaches a low optical depth region in the wind. Our focus is on the case where $M_{w} \sim M_{ej}$, so the entire ejecta participates in the shock breakout, and the light curve is the result of the breakout through the opaque wind (Chevalier & Irwin 2011).

While no self-similar solution exists for the general case, we can still quantify several important consequences about the typical timescale of the light curve. We can also draw some insight from the two extreme limits where $M_{w} \gg M_{ej}$ and $M_{w} \ll M_{ej}$. If the wind mass is much larger than the ejecta mass, the dynamics are similar to the Sedov–Taylor explosion, but with a power-law ambient density instead of a constant one. In the opposite extreme limit ($M_{w} \ll M_{ej}$), the self-similar solution of Chevalier (1982) can be used to model the combined forward and reverse shock. For such a configuration the result of shock breakout through the wind (Balberg & Loeb 2011) will only be the initial light curve, followed by the main light curve driven by the internal energy held by the bulk of the ejecta. In the following, we examine the estimates that can be made about the resulting light curve from these solutions and other considerations; the qualitative picture we draw serves to clarify the numerical results presented in the later sections.

3.1. Breakout Radius

As the forward shock propagates through the wind, it breaks out when it reaches optical depth $\tau \sim c/v_{sh}$. We adopt the
assumption of constant opacity, which is appropriate for electron (Thompson) scattering (a similar assumption was adopted by Arnett 1996; Chevalier & Irwin 2011; Balberg & Loeb 2011; Moriya et al. 2011). The value of \( \kappa \) is, of course, composition dependent, ranging between \( \kappa \approx 0.2 \) cm\(^2\) g\(^{-1}\) for hydrogen-free matter and \( \kappa \approx 0.4 \) cm\(^2\) g\(^{-1}\) for pure hydrogen (we usually used \( \kappa \approx 0.34 \) cm\(^2\) g\(^{-1}\), which is appropriate for a 70% hydrogen composition). In this case of constant opacity the optical depth from a radius \( r \) to the edge of the wind is given by

\[
\tau(r) = \int_r^{R_w} \kappa \rho dr = \int_r^{R_w} \kappa K r^{-2} dr = \kappa K \left( \frac{1}{r} - \frac{1}{R_w} \right). \tag{5}
\]

Therefore, the shock breaks out at radius \( R_{sh} \), which satisfies

\[
\frac{c}{v_{sh}} \approx \kappa K \left( \frac{1}{R_{sh}} - \frac{1}{R_w} \right). \tag{6}
\]

Following Chevalier & Irwin (2011) we denote

\[
R_d = \frac{\kappa K v_{sh}}{c}, \tag{7}
\]

and rewrite Equation (6) as

\[
\frac{1}{R_{sh}} \approx \frac{1}{R_w} + \frac{1}{R_d}. \tag{8}
\]

Chevalier & Irwin (2011) discussed the breakthrough at the two limits:

\[
R_{sh} \approx \begin{cases} R_w & R_w \ll R_d \\ R_d & R_w \gg R_d. \end{cases} \tag{9}
\]

Our focus is on the intermediate case of \( R_d \approx R_w \), which appears to be motivated by observations. Note that as long as the wind mass is of order the ejecta mass, the shock velocity naturally tends to \( v_{sh} \sim \sqrt{E/M_e} \sim 3 \times 10^8 \) cm s\(^{-1}\), so for the parameters chosen in Section 2, \( R_d \sim 10^{15} \) cm.

3.2. Luminosity and Timescale

The main features of the observed light curve are total radiated energy, the peak luminosity, and the typical width. The three are connected, of course, through the emission of the thermal energy at shock breakout by photon diffusion through the wind. Being an integral quantity, the total emitted energy, \( E_{rad} \) is expected to follow the plastic collision picture described in Section 1, giving the simple relation

\[
E_{rad} \propto E \frac{M_w}{M_j + M_w} \propto E \frac{M_w}{M_e + M_w}. \tag{10}
\]

The typical timescale in the light curve must depend not only on integral quantities, but on the details of the wind profile as well. We estimate the time it takes the shock energy to diffuse to the surface following breakout as follows. First, we note that the photons do not diffuse all the way from \( R_{sh} \) to the surface, but rather to the radius of the photosphere \( R_{ph} \), which is located at \( \tau \sim 1 \) (specifically \( \tau = 2/3 \) for Eddington’s approximation).

Using Equation (5), \( R_{ph} \) is given by

\[
\frac{1}{R_{ph}} \approx \frac{1}{R_w} + \frac{1}{\kappa K}. \tag{11}
\]

For the case \( R_w \ll R_d < \kappa K \), Equation (11) yields \( R_{ph} \approx R_w \). According to Equation (8), \( R_{sh} \approx R_w \) in this case as well, so the shock breaks out very close to the edge of the wind and we can assume a constant diffusion coefficient

\[
D \sim \frac{c}{\kappa \rho} \sim \frac{c R_w^2}{\kappa K} \tag{12}
\]

and a diffusion distance (see Equations (8), (11), and assuming \( v_{sh} \ll c \) of

\[
\Delta R = R_{ph} - R_{sh} \approx R_w - R_{sh} = \frac{R_w^2}{R_w + R_d} \approx \frac{R_w^2}{R_d}, \tag{13}
\]

which results in a diffusion time

\[
t_d \approx \frac{\Delta R^2}{D} \approx \frac{R_w^2}{(\kappa K/c) v_{sh}^2}. \tag{14}
\]

In the opposite limit, \( R_w \gg R_d \), and so \( R_{sh} \approx R_d \approx R_w \). Since the shock velocity is far from relativistic, we also have \( R_{sh} \approx \kappa K v_{sh}/c \ll \kappa K \). These relations imply, according to Equation (11), that \( R_{sh} \ll R_{ph} \). In this case shock breakout evolves through the diffusion of radiation to the photosphere, and the diffusion time can be estimated by considering the change of the density-dependent diffusion coefficient

\[
t_d \approx \int_{R_{sh}}^{R_{ph}} \frac{d(r-R_{sh})^2}{D(r)} \approx \int_{R_{sh}}^{R_{ph}} \frac{(r-R_{sh}) \kappa \rho dr}{c}. \tag{15}
\]

After substituting a \( \rho = K r^{-2} \) density profile the integration yields

\[
t_d \approx \frac{K}{c} \left( \ln \frac{R_{ph}}{R_{sh}} + \frac{R_{sh}}{R_{ph}} - 1 \right). \tag{16}
\]

Combining this result with Equations (8) and (11) gives a general expression for the diffusion time for any \( R_w \). By substituting \( R_{sh} \approx R_d \) from Equation (7) and \( R_{ph} \) from Equation (11), and using \( R_{sh} \ll R_{ph} \), we summarize the diffusion time at both limits:

\[
t_d \approx \begin{cases} \frac{R_w^2}{(\kappa K/c) v_{sh}^2} & R_w \ll R_d \\ \frac{\kappa K}{c} \left[ \ln \left( \frac{c}{v_{sh}} \right) - 1 \right] & R_w \gg R_d. \end{cases} \tag{17}
\]

Equation (17) gives a diffusion time which is monotonically increasing with \( R_w \) in both limits, but more mildly at large \( R_w \). For \( R_w \gg \kappa K \gg R_d \) the diffusion time reaches an asymptotic value of

\[
t_d \rightarrow \frac{\kappa K}{c} \left[ \ln \left( \frac{c}{v_{sh}} \right) - 1 \right] \approx \frac{\kappa K}{c} \ln \left( \frac{c}{v_{sh}} \right). \tag{18}
\]

The expression \( t \sim \kappa K/c \), sometimes with the logarithmic correction mentioned, was used in previous works (Ofek et al. 2010; Balberg & Loeb 2011; Chevalier & Irwin 2011) to estimate the order of magnitude of the light curve timescale. Indeed, for \( K \approx 10^{18} \) g cm\(^{-1}\) (see Section 2), this timescale is \( \sim 100 \) days, which is consistent with the timescale in the relevant observations (Quimby et al. 2007, 2011; Pastorello et al. 2010; Smith et al. 2010). However, since we are interested in the regime where \( R_d \) and \( R_w \) are of the same order of magnitude, a more careful analysis is necessary and we must take into account the dependence of \( t_d \) on \( R_w \), which is evident in Equation (17). Note that the parameters chosen in Section 2 do dictate that \( R_w \ll \kappa K \), which means, according to Equation (11), that the radius of the photosphere \( R_{ph} \) is roughly the wind radius \( R_w \).
Thus, our choice of $R_w \sim 10^{15}\text{ cm}$, motivated by the observed photosphere radius (see Section 1) is self-consistent.

In general, the shock velocity changes with radius, and we can roughly estimate that in the regime $M_w \sim M_{ej}$

$$v_{sh} \propto \left(\frac{E}{M_{ej} + M_{sh}}\right)^{1/2} \approx \left(\frac{E}{M_{ej} + 4\pi K R_{sh}}\right)^{1/2},$$

with $M_{sh} \propto K R_{sh}$ is the accumulated mass enclosed by $R_{sh}$. When $v_{sh}$ is a function of $R_{sh}$, and therefore a function of $R_w$ the functional dependence of $t_d$ on $R_w$ can be more complex than presented in Equation (17). Nonetheless, we can gain significant insight by relating $v_{sh}$ to the progenitor system parameters in the limits where self-similar solutions exist. The fundamental point is that for a fixed value of $K$, the limit $R_w \gg R_d$ also corresponds to $M_w \gg M_{ej}$, while in the opposite case of $R_w \ll R_d$ we also have a low-mass wind, $M_w \ll M_{ej}$.

In a very massive wind dimensional analysis similar to the Sedov–Taylor problem (see Zel’dovich & Raizer 1966) can be applied, leading to a shock radius and shock velocity which evolve as

$$R_{sh} \propto \left(\frac{E^2}{K}\right)^{1/3}; \quad v_{sh} = \frac{R_{sh}}{t_d} \propto \left(\frac{E}{K R_{sh}}\right)^{1/2}.$$  

However, since in this limit breakout occurs at $R_{sh} \approx R_d$, independent of $R_w$, the shock velocity at breakout is also independent of $R_w$. Equation (17) provides a qualitative understanding of the dependence of $t_d$ on $K$ and $E$; neglecting the logarithmic correction, we have (for $R_w \gg R_d$)

$$t_d \propto K^{1/3} E^0.$$  

For a very low mass wind, $v_{sh}$ can be found with the alternative self-similar solution of Chevalier (1982), who studied the interaction of an ejecta, with the unshocked density profile $\rho_0(r, t) \propto r^{-m} t^{n-3}$, and wind with density profile $\rho_{in}(r) \propto r^{-3}$. In our case $s = 2$ and $m$ can be related (see Matzner & McKee 1999; Rabinik & Waxman 2011) to the polytropic index $n$: $m \approx 10$ for $n = 3$ or $m \approx 12$ for $n = 3/2$. The shock propagates in time with radius

$$R_{sh} \propto E^{5/3} K^{-1/3} t^{(5/3)-(m-3)/(m-5)},$$  

and with velocity given by

$$v_{sh} = \frac{R_{sh}}{t_d} \propto E^{1/2} K^{-1/3} R_{sh}^{(3-3)/(m-3)}.$$  

Combining Equation (23) with Equation (17) results in an inverted dependence of the diffusion time on $K$ in the regime $R_w \ll R_d$:

$$t_d \propto E^{-1} K^{2/(m-3)-1} \approx \begin{cases} E^{-1} K^{-5/7} m \approx 10 & (n = 3) \\ E^{-1} K^{-7/9} m \approx 12 & (n = 3/2). \end{cases}$$  

In this regime $R_{sh} \approx R_w$ and, using Equation (23), we have

$$v_{sh} \propto R_w^{-1/(m-3)} \approx \begin{cases} \frac{R_w^{-1/7}}{m} & m \approx 10 & (n = 3) \\ \frac{R_w^{-1/9}}{m} & m \approx 12 & (n = 3/2). \end{cases}$$  

This is a weak dependence on $R_w$, which would result in a small deviation from the relation $t_d \propto R_w^{2}$ of Equation (17). The inversion of the dependence of $t_d$ on $K$ between limits (linear in $K$ for $R_w \gg R_d$ and inverse for $R_w \ll R_d$) suggests a weak dependence on $K$ for the intermediate regime. We note that, in theory, if the Sedov–Taylor solution were applied to the $R_w \ll R_d$ case, by combining Equation (20) with Equation (17), we would also have the $t_d \propto E^{-1}$ dependence, implying the robustness of this result.

We conclude that when identifying the limits of $R_w \gg R_d$ and $R_w \ll R_d$ with $M_w \gg M_{ej}$ and $M_w \ll M_{ej}$, respectively, then at both limits $v_{sh}$ is approximately constant (a different constant for each limit), justifying Equation (17). We expect the region $M_w \sim M_{ej}$ to exhibit some deviation from these timescale estimates.

4. THE CODE

We have written a one-dimensional Lagrangian computer program in order to calculate the shock propagation and light curve. In this section we describe the code in brief. The code uses the standard von Neumann and Richtmyer staggered mesh method (von Neumann & Richtmyer 1950; Richtmyer & Morton 1967) to solve the nonrelativistic equations of motion. The energy equation is solved implicitly, and the radiative flux is added to the hydrodynamics assuming local thermal equilibrium (LTE) and in the diffusion approximation (see Zel’dovich & Raizer 1966). More specifically, the code solves the following energy equation (implicitly, solving a tridiagonal equation system):

$$\frac{\partial e}{\partial t} + \frac{\partial}{\partial t} \frac{1}{\rho} \nabla F = 0 \quad (26a)$$

$$F = -D \nabla (a T^4) \quad (26b)$$

where $e$ is the specific energy, $V = 1/\rho$ is the specific volume, and $\rho$ is the pressure. $F$ denotes the radiative flux, with $D = \kappa c/3$ as the diffusion coefficient, and $\alpha$ as the radiation constant. The temperature $T$ in Equation (26b) is the temperature of the fluid (a result of the LTE assumption). In the case of constant opacity, the mean-free path satisfies $\lambda = 1/\kappa \rho$. Gravitation can be neglected for the description of the shock propagation and breakout, since $GM_{w}^{2}/R_{w} \ll E$.

The equation of state (EOS) is that of a perfect gas, with radiation terms added to the pressure and energy:

$$p(\rho, T) = \Gamma \rho T + \frac{a T^4}{3} \quad (27)$$

$$e(\rho, T) = \frac{\Gamma T}{\gamma - 1} + \frac{a T^4}{\rho} \quad (28)$$

where $\Gamma = R/\mu$ is the gas constant divided by the molar mass. In the examples shown in this section we choose $\gamma = 5/3$, suitable for monoatomic gas and $\mu = 0.6$, which corresponds to a fully ionized mixture of hydrogen and helium with primordial ratios. In the context of shock breakout the energy and pressure of the fluid are dominated by radiation after the shock passage and during the period of adiabatic expansion that follows, so these terms have a minimal effect on the EOS and the light curve.

We treat the interaction of the radiation field with the gas through the diffusion approximation and complete thermal equilibrium. Both assumptions are reasonable for nonrelativistic shocks (Katz et al. 2010). We do note that near the photosphere during shock breakout (Nakar & Sari 2010; Rabinik & Waxman 2011) transport, rather than diffusion, is a more appropriate description of the photon propagation, which we will apply in future work. Hence, we also do not attempt to include...
Figure 2. Calculated temperature profiles at different times following explosion. The profiles were obtained with the parameters $R_w = 2.5 \times 10^{15}$ cm, $K = 10^{18}$ g cm$^{-1}$, and $E = 5 \times 10^{51}$ erg. The plotted profiles are from 5 (solid black line), 25 (dashed blue line), and 50 (dot-dashed red line) days following the explosion. (A color version of this figure is available in the online journal.)

Figure 3. Calculated velocity profiles at different times following explosion. The profiles were obtained with the same model as Figure 2. Line styles/colors are as in Figure 2. (A color version of this figure is available in the online journal.)

Figure 4. Calculated density profiles at different times following explosion. The profiles were obtained with the same model as Figure 2. Line styles/colors are as in Figure 2. (A color version of this figure is available in the online journal.)

An example calculation is shown in Figures 2–5. Figure 2 shows the temperature profiles at different times. The transition from a discrete shock front to breakout at $\tau \sim c/v_{sh}$, as discussed in Section 3, is evident in the late time (50 days after explosion) profile. Velocity profiles at the relevant times are presented in Figure 3, showing the forward-reverse shock structure and the homologous expansion of the ejecta. Density profiles are plotted in Figure 4, showing (at the early time profile—5 days after explosion) the forward-reverse shock structure (see Chevalier 1982), including the contact discontinuity, and the part of the ejecta that the reverse shock has not reached yet. The sharpening of the forward shock density profile at later times is a feature of similar radiation hydrodynamics codes (see Ensman 1994). The corresponding light curve of this example is presented in Figure 5, it is readily seen that the emergence of a light curve and its duration correspond to the shock breakout timescales.

4.1. Comparison with Moriya et al. (2011)

As a final test, we compared our numerical results with those of Moriya et al. (2011), who also studied the effect of a wind on light curves. Their work focused on less dense winds and nonthermal cooling, which can be important during breakout (see, e.g. Chevalier & Irwin 2012). Deviations from spherical symmetry, such as the Rayleigh–Taylor instability are not taken into account by the code. Such deviations should have a minor effect on the general properties of the bolometric light curve (see Moriya et al. 2012 for discussion of the implications).

As a code check, we calculated several test problems with our program and compared the results to known solutions. Among the test problems we considered are the self-similar interaction of ejecta and wind (Chevalier 1982), Elliot’s extension to the Sedov–Taylor explosion, which includes radiative flux (Elliot 1960), and planar shock breakout (Sapir et al. 2011). We present our results for the planar shock breakout in the Appendix.

Using a more complex numerical model, which we do not try to reproduce. However, we can compare our calculations to the progenitor-wind model that is the most relevant to the regime studied in the current work: model s15w2r20m1e3, marked as $10^{-1} M_\odot$ yr$^{-1}$ in Figures 3 and 4 of Moriya et al. (2011), which has a wind mass of 6.5 $M_\odot$. In Figure 6, we show a comparison between the light curve calculated with our code and their results. Our light curve was calculated using the progenitor...
Figure 5. Calculated light curve. The light curve was calculated with the same model as Figure 2.

Figure 6. Comparison of the light curve calculated by our code (solid black line) to the one calculated by Moriya et al. (2011) for the model s15w2r20m1e3 (dashed blue line). For the comparison we used the same progenitor profile as Moriya et al. (2011). (A color version of this figure is available in the online journal.)

profile given in Figure 3 of Moriya et al. (2011), which is slightly different from our standard $M_\star = 15 M_\odot$ progenitor (see Section 2). The resulting relevant light curve parameters (peak luminosity, time duration, and total radiated energy) agree up to a few percent.

5. NUMERICAL PARAMETER SURVEY

In this section, we study the light curves calculated with the code and conduct a parameter survey in order to map the dependence on the important parameters. We also verify that the analytical limits presented in Section 3 are recovered, and use them to gain insight to the results.

5.1. Comparison with Analytical Limits

Following a convergence test, we model the compact star part of the progenitor system with $n_\star = 50$ cells, geometrically decreasing in size $\Delta r$ toward the outer edge with a quotient $q_\star = 0.98$. The wind is divided into $n_w = 250$ cells, geometrically increasing in size (with a constant quotient $q_w > 1$) toward the outer wind edge. The innermost wind cell is of the same size as the outermost star cell. The size of the innermost wind cell, together with $n_w$, determine $q_w$. In these calculations we use the smoothing parameters $\alpha = 1.25$ and $k = 15$ (see Section 2).

Generally, we set $R_\star = 10^{13}$ cm, except for very low values of $R_w$, for which we had to reduce the star radius, $R_\star$, in order to remain in the regime $R_\star \ll R_w$. Additionally, we had to change the resolution for convergence and to disable the wind edge smoothing (see Section 2). These changes have an effect only at very low $R_w$.

We relate the diffusion timescale $t_d$ with the full width at half-maximum (FWHM) of the light curve. This choice is independent of the low-luminosity "tail" at late times, which arises in part from the continued interaction of the shock wave and the wind (Chevalier & Irwin 2011), and is sensitive to the shape of the wind cutoff profile. The results, for $E = 5 \times 10^{51}$ erg, $K = 10^{18}$ g cm$^{-1}$, and a wide range of $R_w$ are shown in Figures 7 and 8.

Both figures demonstrate that the calculated light curves do indeed reproduce the analytic timescales at the appropriate limits. The fit is very good at low values of $R_w$, while at high values there is some deviation from Equation (17). The reason is that the simplified treatment of diffusion in the code extends...
all the way to $R_w$, whereas our analytic estimates were based on emission from a $\tau = 1$ (or $\tau = 2/3$) surface. A more exact fit of the numerical results is therefore found with a revised analytical estimate of the diffusion time using Equation (16), and substituting $R_{ph} = R_w$:

$$t_d \approx \frac{\kappa K}{c} \left[ \ln \frac{R_w}{R_3} - 1 \right] = \frac{\kappa K}{c} \left[ \ln \left( \frac{c}{v_{ph} \kappa K} \right) - 1 \right].$$

Unlike Equation (17), Equation (29) does not reach an asymptotic value at $R_w \gg \kappa K$. Correspondingly, our numerical calculations, which do not take into account the photosphere at $\tau \sim 1$, are inexact in this sense. However, since we deal with the regime $R_w \ll \kappa K$, the difference between Equations (17) and (29) is small (see Figure 8, which even approaches $R_w \sim \kappa K$), so the numerical treatment of the photosphere is sufficient for our purposes in the current work.

5.2. Progenitor-system–Light-curve Relation

As is obvious from the discussion in Section 3 and from Figures 7 and 8, the range of $R_w$ of interest in this work ($R_w \sim 10^{15}$ cm) cannot fit well with any of the analytical limits. Henceforth, we conduct a numerical parameter survey in this range. We begin by examining the relation between the diffusion time, $t_d$, and the parameters of the progenitor system, namely $E$, $K$, and $R_w$, whereas for simplicity we fix $M_* = 15 M_\odot$. We study the dependence of $t_d$ on $R_w$ for a nominal calculation with $E = E_0 \equiv 5 \times 10^{51}$ erg and $K = K_0 \equiv 10^{18}$ g cm$^{-1}$. The dependence on $E$ and $K$ is studied by repeating the calculations with other values of $E \equiv E/E_0$ and $K \equiv K/K_0$. The results are shown in Figures 9 and 10.

The increase of $t_d$ with increasing $R_w$ is understood qualitatively by Equation (17). Quantitatively, we see that $t_d$ is strongly dependent on $R_w$ throughout the relevant range, so the approximation $t_d \approx \kappa K/c$ (Ofek et al. 2010; Balberg & Loeb 2011; Ginzburg & Balberg, 2012).
Chevalier & Irwin (2011) can serve only as an order of magnitude estimate. This behavior is also evident in Figure 9 of Moriya et al. (2011). As can be seen in Figure 9, the dependence of the diffusion time on $K$ is in good agreement with the analysis of Section 3.2. At large $R_w$, the diffusion time increases roughly linearly with $K$, but for lower $R_w$ the dependence becomes weaker and is finally inverted, as expected in low-mass wind (see Equations (21) and (24)), yielding smaller diffusion times for larger values of $K$. We also recover the $t_d \sim E^{-1}$ relation expected at low $R_w$ (again, see Equation (24)). The dependence becomes weaker at larger $R_w$, as might be expected from Equation (21).

The other feature in the light curve that we relate to the parameters of the progenitor system is the total radiated energy. Figure 11 shows the dependence of the radiated energy on $K$ for different values of $\hat{E}$ and for a fixed $R_w = 2.5 \times 10^{15}$ cm. We calculate the radiated energy by integrating the light curve until the luminosity drops to 0.1% of the maximum luminosity. The behavior exhibited in Figure 11 is qualitatively understood by the plastic collision relation (Equation (10)), thereby saturating when $M_w \lesssim M_\ast$. Using Equation (3), $M_w \approx M_\ast$ for $\hat{K} = 1$.

We digress and discuss the shape of the light curve because the total radiated energy is not always reliably observed. When observations are limited to the vicinity of peak magnitude and do not track low luminosities, the shape of the light curve must be modeled theoretically to assess its total energy. One such model uses the observed peak luminosity $L_{\text{max}}$ and the observed FWHM of the light curve, and, assuming a Gaussian light curve (Arnett 1996), $E_{\text{rad}} \approx (0.5\sqrt{\pi}/\ln 2)L_{\text{max}} \times \text{FWHM}$. We plot this estimate of the radiated energy (using the calculated peak luminosity and the FWHM of the calculated light curve) as dashed lines in Figure 11. It is obvious from Figure 11 that as the wind becomes more dense, this simple estimate deviates more from the total radiated energy (compare the solid and dashed lines in Figure 11). The reason is that the late time “tail” strongly deviates from a Gaussian form, and includes much more energy than predicted by a Gaussian approximation. This tail is partially powered by continued interaction of the forward shock wave with the wind, and the reverse shock wave with the ejecta (Chevalier & Irwin 2011), and thus becomes more prominent and contains more energy. We can conclude that total energy estimates based on observations should be done carefully, and that a Gaussian fit must be treated as a lower limit if only the vicinity of the peak region of the light curve is observed.

### 5.3. Timescale Constraints

As mentioned above, we focus on winds with a radius $R_w \sim 10^{15}$ cm, which is motivated by the observed photosphere radius $R_{\text{ph}} \sim 10^{15}$ cm and the relation $R_w \sim R_{\text{ph}}$, the latter being valid for $M_w \sim M_\ast$ (see Section 3). One of the conclusions of the numerical results is that the observed light curve timescale gives another constraint on $R_w$. We demonstrate this point in Figure 12, where we show the light curves calculated for three combinations of $K$ and $R_w$, all of which satisfy $K R_w = 2.5 \times 10^{33}$ g, so that the total wind mass is kept constant. The intermediate model with $R_w = 2.5 \times 10^{15}$ cm is used below to fit SN 2010gx (see Section 6), while the other two are more compact and extended winds (note that for the extended wind, the approximation $R_w \ll \kappa K$ that our code assumes is marginal). By keeping the total wind mass constant, the efficiency of converting a given explosion energy to radiated energy is fixed (for a given progenitor star mass) and so the light curve timescale becomes a direct indicator of the wind radius.
It is obvious that for timescales of ~50 days, as is the case of SN 2010gx, the models with the more compact and more extended winds are ruled out. Due to uncertainties in other parameters, in particular the exact density profile, at present we can use this timescale constraint only as an order of magnitude estimate for the value of $R_w$.

6. OBSERVATIONS

In this section we relate our model to SLSNe observations. Specifically, we focus on three events: SN 2010gx, SN 2006gy, and SN 2005ap. Our goal is to examine whether a steady wind model can provide a viable explanation for the observed SLSNe light curves, and to constrain the likely parameters of the progenitor system. In general, previous works that considered a star–wind system for these objects (Ofek et al. 2007, 2010; Smith & McCray 2007; Smith et al. 2010; Chevalier & Irwin 2011) provided only order of magnitude correlation between the steady wind model and the observations. Very recently, Moriya et al. (2012) presented a specific numerical model for SN 2006gy where the ejecta interacts with a a distant shell (rather than a steady wind), and we comment on the similarities and differences regarding this specific object below.

6.1. SN 2010gx

We take the data for SN 2010gx from Pastorello et al. (2010). The measured luminosity (which is taken with zero bolometric correction) and temperature imply a blackbody radius $\sim 3 \times 10^{15}$ cm at peak luminosity. Since the photosphere lies close to the wind edge (see Section 3), we choose a model with $R_w = 2.5 \times 10^{15}$ cm (the smaller radius is due to the finite width of the wind edge in our density profiles, and also due to the expansion following the passage of the shock). We find that the light curve is recovered well when setting the other parameters to be $K = 10^{18}$ g cm$^{-1}$ and $E = 2 \times 10^{51}$ erg. The quality of the fit is shown in Figure 13, which compares the calculated and observed light curves. It is noteworthy that we do include a correction for the difference in arrival times of photons originating from different positions on the photosphere (Katz et al. 2012). This effect on the timescales is $\sim R_{ph}/c$, which is about one day, and therefore negligible in the cases considered here.

The corresponding blackbody temperature for the calculated light curve is compared with temperatures inferred from observations in Figure 14. We emphasize that the calculated blackbody temperatures are recovered using the luminosity, $L(t)$, and identifying the photosphere, $R_{ph}(t)$, with the position where the optical depth is $\tau = 2/3$,

$$L = 4\pi R_{ph}^2 \sigma T^4. \quad (30)$$

Clearly this determination of the blackbody temperature is a crude one, and, in fact, the blackbody assumption must generally be considered only as an approximation (Nakar & Sari 2010; Rabinak & Waxman 2011; Chevalier & Irwin 2011). Therefore, we view the fit between calculated and observed temperatures as indicative that our model is compatible with observations.

Considering the simplifications of our approach, we do not attempt to find a “best fit” model for the progenitor system, but rather demonstrate that a plausible range exists. Moreover, we emphasize that the wind parameters we choose to fit are not unique. If we fix $R_w = 2.5 \times 10^{15}$ cm, as implied by the temperature measurement, we have a considerable degree of freedom to choose $K$. The reason is the weak dependence of the diffusion time on $K$ for this $R_w$, as seen in Figure 9. The total radiated energy is also not strongly affected by an increase of $K$, as seen in Figure 11. This is because $M_w \approx 4\pi K R_w$ is close to the saturation region of Equation (10) and Figure 11.
More specifically, a different model with $K = 2 \times 10^{18}$ g cm$^{-1}$ (double the chosen value) results in a light curve with the same peak luminosity as the model in Figure 13 and FWHM wider by only 15%. In addition, we tested a model with $\kappa = 0.2$ cm$^2$ g$^{-1}$, which is appropriate for a hydrogen poor circumstellar material (CSM), as indicated by SN 2010gx observations (Pastorello et al. 2010). This change does not have a dramatic effect on the light curve and temperature (see Figures 13 and 14; though there is a time shift, and we moved the curves to fit at peak luminosity). The light curve depends weakly on $\kappa$ due to the change from inverse to linear dependence on $\kappa$ in Equation (17) (which suggests a weak dependence at $R_w \sim R_\odot$), and the temperature depends weakly on $\kappa$ due to the relation $R_{ph} \approx R_w$ (independent of $\kappa$) in our regime.

6.2. SN 2006gy

We take the data for SN 2006gy from Smith et al. (2010). The measured luminosity and temperature imply a blackbody radius $\sim 4.5 \times 10^{15}$ cm at peak luminosity. If we do not consider the temperature constraint, a model that fits the light curve can be found. The model with $R_w = 6.5 \times 10^{15}$ cm, $K = 0.9 \times 10^{18}$ g cm$^{-1}$, and $E = 5 \times 10^{51}$ erg, which reasonably fits the light curve except for the tail, is plotted in Figure 15 (solid black line). We shall distinguish this model as model A. The wind mass in this model is $M_w \approx 35 M_\odot$, which is similar to the estimates of previous works (Woosley et al. 2007; Smith et al. 2010; Chevalier & Irwin 2011).

Model A may fit the light curve, but since $R_w$ in this model is larger than the implied blackbody radius, it deviates from the temperature measurements, as can be seen in Figure 16. We find that a model in a different range of the progenitor parameters can be chosen to reproduce the observed temperatures (assuming a blackbody emission) and the peak luminosity, but at the expense of generating a light curve which is too narrow. One such model, distinguished as model B, is also plotted in Figures 15 and 16. In this model we set $R_w = 4 \times 10^{15}$ cm, $K = 10^{18}$ g cm$^{-1}$, and $E = 3.7 \times 10^{51}$ erg.

The light curve is narrow due to the strong dependence of the diffusion time on $R_w$, which allows little freedom (see Section 5.3). As a result, we cannot find a single model that fits well both the light curve and the measured temperature. This is easily understood by considering the analysis in Section 5.2. If we use the wind radius fixed by the observed peak luminosity and temperature, then for a given progenitor star only two free parameters remain: $K$ and $E$. Model B (which assumes the observationally inferred $R_w$) is adjusted to reproduce the observed peak luminosity, but it results in a narrow light curve, yielding a total radiated energy that is too low. The total radiated energy cannot be increased by increasing $K$, because at $K = 10^{18}$ g cm$^{-1}$ the efficiency of converting the explosion energy is already close to the asymptotic value (see Figure 11; we note that in this figure $R_w = 2.5 \times 10^{15}$ cm; for $R_w = 4 \times 10^{15}$ cm, asymptotic efficiency is reached for even smaller values of $K$, due to the larger wind mass). Stipulating a larger explosion energy $E$ can account for the total radiated energy, of course, but it is not a solution, since it leads to even narrower light curves, due to larger expansion velocities (see Figure 10). In theory, the desired effect can be obtained by increasing both the total energy and $K$ (the latter compensating for the narrowing of the light curve, since it generates a more massive wind). However, the dependence of the diffusion time on $K$ is weak for $R_w$ in the range of interest (see Figure 9), and we determine that in order to reproduce the observations, an unrealistically heavy wind mass (hundreds of solar masses) is required.

We note that the fit with observations cannot be improved by stipulating a larger mass for the progenitor star. While an
In essence, it appears difficult to reconcile both the observed light curve and temperature of SN 2006gy with a steady wind model, since the light curve time duration implies a wind radius $R_w$ that is different from the one inferred by the photosphere radius. The reason may be the blackbody interpretation (see Nakar & Sari 2010; Rabinak & Waxman 2011; Chevalier & Irwin 2011), which affects the calculated temperature, an inaccurate assumption of full ionization (which affects the opacity, and therefore the photosphere radius), or an indication that a different model, perhaps with a different CSM profile (see Moriya & Tominaga 2012, for example), is required.

Other, similar, models for SN 2006gy have been suggested in previous works. Chevalier & Irwin (2011) suggested a steady wind model with estimated parameters $R_w \sim 10^{15} \text{ cm}$, $E \sim 3 \times 10^{51} \text{ erg}$, and $K \sim 0.5 \times 10^{18} \text{ g cm}^{-1}$, resulting in $M_w \sim 30 M_\odot$. These parameters are similar to those we adjusted. Note that the large $R_w$ in this model results in a low blackbody temperature, as in our model A. Woosley et al. (2007) considered a pulsational pair instability scenario, in which a shell of $\sim 30 M_\odot$ was ejected to a radius of $\sim 10^{15} \text{ cm}$ prior to the explosion. In this model, the photosphere radius remains relatively low (this is due to the shell density profile and the large radii, which allow for a lower optical depth for the same CSM mass) but apparently not low enough to fit to the temperature measurement. Chatzopoulos et al. (2012) try to explain SN 2006gy with a semi-analytical model based on the self-similar solutions of Chevalier (1982). However, their model has a low wind mass ($5 M_\odot$ compared with $40 M_\odot$ ejecta mass), and a small CSM radius ($2.5 \times 10^{15} \text{ cm}$, albeit with a different power law), which does not allow us to recover the typical timescale in SN 2006gy. This discrepancy is noted by Moriya et al. (2012), who calculated the Chatzopoulos et al. (2012) model numerically (we do note that semi-analytical models such as this may provide a useful understanding of the parameter space and might be fitted to observations and calculations with a different choice of parameters). Moriya et al. (2012) suggest their own models for SN 2006gy, which include interactions with CSM shells that are not due to steady winds, but rather a finite shell situated at a standoff distance from the star (see their models D2 and F1). The CSM in these models extends to larger radii compared with our models ($1 \times 10^{16}$ to $2 \times 10^{16} \text{ cm}$), and contain less mass ($15–18 M_\odot$), which is compensated by a greater ejecta energy ($10 \times 10^{51} \text{ erg}$). A more detailed comparison with these different models is beyond the scope of the current work, but we emphasize that our model A fits the light curve with at least the same quality as the other numerical models. One of the important aspects of Moriya et al. (2012) is the comparison of effective and color temperatures (their Figure 8). The significant difference between the two temperatures may be the reason why a simple blackbody approximation cannot account for the observed temperature.

6.3. SN 2005ap

We take the data for SN 2005ap from Quimby et al. (2007). The light curve data also appears in Pastorello et al. (2010) for comparison with SN 2010gx. The measured luminosity and temperature imply a blackbody radius $\sim 2.5 \times 10^{15} \text{ cm}$ at peak luminosity. We choose a model with $R_w = 2.5 \times 10^{15} \text{ cm}$, $K = 1.5 \times 10^{18} \text{ g cm}^{-1}$, and $E = 2.8 \times 10^{51} \text{ erg}$. The calculated and observed light curves are plotted in Figure 17, and the calculated and observed temperatures are plotted in Figure 18. The fit of the model to the observations is marginal, and suffers from problems similar to the fit to SN 2006gy, but more mildly. The model is a bit too narrow and the calculated temperature is a bit too low. This implies some discrepancy between the wind radius imposed by the diffusion timescale and the blackbody radius imposed by the temperature and maximum luminosity. We note that as in Section 6.1, the results are not sensitive to the value of $\kappa$, which is set to be smaller due to lack of hydrogen (as indicated by Quimby et al. 2011). The discrepancy is again difficult to resolve by a different choice of parameters in the context of our model, but is more likely to be solved if we allow for corrections to the blackbody assumption and interpretation of observations. As in the case of SN 2006gy, if we do not constrain $R_w$ by the implied blackbody radius (because of the non-equilibrium conditions for example), it is quite easy to fit the light curve better by increasing $R_w$.

7. CONCLUSIONS AND DISCUSSION

In this work, we considered a scenario for SLSNe based on shock breakout from the interaction of an ejecta from an exploding star and a massive envelope of CSM, presumably emitted from the star prior to the explosion. We focused on a steady wind model for this CSM, which dictates a $\rho \sim r^{-2}$ density profile, and on massive winds, which have a total mass $M_w$ comparable with the ejected mass, $M_e$. The latter assumption allows for shock breakout in which the entire ejecta participates, leading to an efficient conversion of the explosion energy to bolometric luminosity. This efficiency is required...
find that the light curves of these objects can be understood in terms of a supernova in a heavy wind scenario, where the progenitor star has a mass of $M_\ast \approx 15 M_\odot$ and the wind mass is in the range $15$–$35 M_\odot$. The efficient conversion of the ejecta kinetic energy to radiation in this scenario enables us to reproduce the light curves with energies in the standard range of $2 \times 10^{51}$ to $5 \times 10^{51}$ erg, and, as mentioned above, with winds extending to radii of $2.5 \times 10^{15}$ to $6.5 \times 10^{15}$ cm. Assuming blackbody emission, our model reproduces the observations in SN 2010gx, but underestimates the temperatures measured in SN 2006gy, and to some extent in SN 2005ap as well. A similar trend was found in the other works mentioned above, and may be the result of a frequency-dependent (rather than constant) opacity, differences between the effective and color temperatures, or both.

We note that a separate subclass of SLSNe, where the peak luminosity is high but the timescales are shorter, such as SN 2008es (Gezari et al. 2009; Miller et al. 2009), may also be explainable with a steady wind model. In this case the wind mass must be significantly lower than the ejecta mass, so the initial wind optical depth may be small. We plan to investigate such systems numerically in future work.

We do not explicitly compare our results with an alternative, but similar, scenario, where the ejecta interacts with a single shell, situated at some distance from the star, presumably a result of a short period of enhanced mass loss. Clearly, a massive shell can be equally efficient in converting the explosion energy to radiation (Moriya et al. 2012). Differentiating between both scenarios must involve a wider parameter survey (especially since in the shell model its position and density structure can be assumed independently), and probably some additional physics in the numerical model as well.

Another issue that requires further work is a nonthermal component from shock breakout through the thick wind. Recently, several authors (Katz et al. 2011; Chevalier & Irwin 2012; Svirski et al. 2012) pointed out that during breakout the shock is likely to become collisionless, hence creating a higher energy, nonthermal component in the spectrum. These works vary considerably concerning their estimates regarding the fraction of the total nonthermal energy eventually emitted from breakout, but obviously, if this fraction is sizable, there will be some impact on the bolometric light curve.

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APPENDIX

COMPARISON TO PLANAR SHOCK BREAKOUT SOLUTION

Sakurai (1960) investigated the problem of a shock wave propagating through a nonuniform medium (ideal gas) of decreasing density and reaches a boundary where the density vanishes. For a planar initial density profile of $\rho(x) \propto x^\alpha$, where $x$ is the distance from the boundary, a self-similar solution exists. Sapir et al. (2011) derived an extension to the problem which includes radiative flux, in the diffusion approximation. At large optical depth from the boundary, the purely hydrodynamical (without diffusion) solution of Sakurai (1960) is applicable, but at $\tau \sim \beta_{\infty}^{-1}$, diffusion is significant and must be taken into account (see Section 3). Sapir et al. (2011) present a self-similar

Figure 18. Calculated and observed blackbody temperature of SN 2005ap. The calculated temperature (solid black line) was calculated with the same model as in Figure 17. The observed temperature (blue circles) is taken from Quimby et al. (2007). No estimate is given in Quimby et al. (2007) for the uncertainty in the two temperatures in the graph (only a general estimate of 16,000–20,000 K).

(A color version of this figure is available in the online journal.)

to explain the total energy observed in some SLSNe without resorting to high energy explosion mechanisms.

Our approach combines analytical limits and numerical hydro-diffusion calculations of the bolometric light curve. Thus, we improve upon previous works that considered a massive wind in the context of SLSNe with order of magnitude estimates (Smith & McCray 2007; Smith et al. 2010; Chevalier & Irwin 2011). We note that the numerical calculations are absolutely necessary for producing reliable results in a $M_\infty \approx M_\odot$ scenario, since the analytical limits cannot be applied. On the other hand, our numerical model includes only the fundamental physics of the problem, and so the principle trends are easily understood. Our calculations are in agreement with more complex numerical models (see, e.g., Moriya et al. 2011; Moriya & Tominaga 2012), and can be used to verify the validity of semi-analytical models (Chatzopoulos et al. 2012), which provide useful understanding of the parameter space.

Specifically, we related the main features of the observed light curve to the parameters of the progenitor system—the star and the wind. As expected, we explicitly find that a large wind mass does allow to efficiently convert the energy of the explosion into a bolometric light curve, and that this efficiency naturally saturates when $M_\infty \geq M_\odot$. However, we demonstrate that in this scenario the width of the light curve is strongly dependent on the cutoff distance of the wind, $R_\infty$. Quantitatively, we show that in order to recover a timescale of tens of days seen in the most energetic SLSNe, the wind outer radius must be of order a few $10^{15}$ cm. This result is consistent with the scenario of a steady mass loss of a few $10^{-1} M_\odot$ yr$^{-1}$ during the last 100 years of the star’s evolution (Quataert & Shiode 2012).

We applied our model to three SLSNe, namely SN 2010gx, SN 2006gy, and SN 2005ap, which exhibited luminosities as high as several $10^{44}$ ergs with timescales of tens of days. We...
numerical solution to the problem, assuming constant opacity and a radiation dominated gas. This problem, which can describe the planar phase of shock breakout in the absence of wind, served as one of several code checks for our program (see Section 4).

We use planar geometry, insert the appropriate density profile (with $n = 3$), and keep only the radiation terms of the EOS (see Section 4) for the comparison with Sapir et al. (2011). Our calculation was carried out by moving the inner boundary as a piston at a constant velocity. In addition, we deposited thermal energy in the innermost cell as an initial condition. The shock wave that arises in these conditions converges to the self-similar solution (Sakurai 1960) as it propagates through the gas. In Figure 19, we demonstrate the exact fit that we find between our calculated light curve and the self-similar solution of Sapir et al. (2011). Our light curve was normalized to breakout point $\tau = \rho v / v_0$ (for details see Sapir et al. 2011).

Our Code
Sapir et al.

Figure 19. Normalized (see Sapir et al. 2011) emitted energy flux as a function of normalized time relative to time of peak emitted energy flux, for a density profile with $n = 3$. The solid black line is the curve calculated by our code, and the blue circles are taken from Table 3 of Sapir et al. (2011). The right figure is a zoom of the left figure in a shorter time span near the peak.

(A color version of this figure is available in the online journal.)

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