Improved TOPSIS Based on IAHP and Entropy for Threat Assessment

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Abstract. Aiming at the problem of threat assessment for ballistic missile, a comprehensive assessment method of ballistic missile based on Improved TOPSIS is proposed to assess and rank the threat of incoming ballistic missile, which provides the basis for early warning resource scheduling, development of interception strategy, implementation of fire interception and other operations. First of all, the paper analyzes the shortcomings of TOPSIS method, and improves it in two aspects: one is the improvement of data standardization, the other is the improvement of index weight calculation method. The experimental results show that, compared with the typical methods such as fuzzy set and Bayesian network, the comprehensive threat assessment method of ballistic missile in this paper is more scientific and reasonable.

1. Introduction
With the development of ballistic missile technology, all countries in the world regard ballistic missile as the priority of air-to-air attack weapons, and it brings unprecedented threat to air-to-air defense operations. In antimissile operations, a key link is to evaluate the threat of the incoming ballistic missile. Ballistic missile threat comprehensive assessment is a multiple attribute decision making problem. At present, the main methods to solve this problem are Bayesian network method [1], neural network method [2], fuzzy set method [3].

Aiming at the characteristics and problems of ballistic missile threat assessment, combined with the existing assessment methods, this paper selects the technique for order preference by similarity to ideal solution (TOPSIS) [4], improves it, and puts forward a comprehensive threat assessment method for ballistic missile based on Improved TOPSIS.

2. Principle of TOPSIS
TOPSIS is a method for multiple attribute decision making analysis of limited schemes in system engineering, which converts the preference of target value into Euclidean distance and measures it, calculates the relative closeness between each scheme and positive solutions and negative ideal solutions, and then obtains the ranking of evaluation schemes. Its basic idea is to determine the best scheme and the worst one for normalized original data matrix, and then to find out the distance between each evaluated scheme and the best scheme and the worst one, so as to get the approach degree between the scheme and the best scheme, which is the basis for evaluating the merits and demerits of each evaluated object.
3. Improvement of TOPSIS

There are two problems in using TOPSIS to evaluate ballistic missile threat. One is the processing of ballistic missile data, the other is the calculation of the weight of each evaluation index. Aiming at these two problems, this paper improves the TOPSIS method, and proposes a comprehensive threat assessment method of ballistic missile based on the improved TOPSIS.

3.1. Standardization of Ballistic Missile Threat Assessment Data

With n evaluation indicators \( f_j (1 \leq j \leq n) \), m ballistic missiles \( a_i (1 \leq i \leq m) \), the matrix \( \mathbf{X} = (x_{ij})_{mn} \) composed of n indicators of m ballistic missiles is called evaluation matrix. In the matrix \( \mathbf{X} = (x_{ij})_{mn} \), for benefit type indicator \( f_j \), make

\[
y_j = \frac{x_{ij} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}}, \quad (1 \leq i \leq m, 1 \leq j \leq n)
\]

For cost indicator \( f_j \), make

\[
y_j = \frac{\max_i x_{ij} - x_{ij}}{\max_i x_{ij} - \min_i x_{ij}}, \quad (1 \leq i \leq m, 1 \leq j \leq n)
\]

3.2. Combination Weighting for Ballistic Missile Threat Assessment Indicator

The methods of determining index weight contain subjective and objective methods. In order to fully combine advantages of subjective and objective methods, we propose a combined method for the improvement of TOPSIS evaluation method.

3.2.1. Determining Subjective Weight Based on IAHP.

IAHP is an extension of AHP in the interval number. Let \( \mathbf{A} = (\tilde{a}_{ij})_{mn} \) be an interval Matrix, \( \mathbf{A} = [\mathbf{A}^l, \mathbf{A}^u] \), note \( \mathbf{A}^l = (\tilde{a}_{ij}^l)_{mn} \), \( \mathbf{A}^u = (\tilde{a}_{ij}^u)_{mn} \), and note \( \mathbf{A}^l = [\mathbf{A}^l^l, \mathbf{A}^l^u] \). Also for interval vectors \( \mathbf{x}^l = (x_{1}^l, x_{2}^l, \cdots, x_{n}^l)^t \), \( \mathbf{x}^u = (x_{1}^u, x_{2}^u, \cdots, x_{n}^u)^t \), and note \( \mathbf{x} = [\mathbf{x}^l, \mathbf{x}^u] \). For a given interval number judgment matrix \( \mathbf{A} = [\mathbf{A}^l, \mathbf{A}^u] \), IAHP determines the weight of the indicator by the following calculation steps:

Step 1: The normalized feature vector \( \mathbf{x}^l \), \( \mathbf{x}^u \) with positive components corresponding to the maximum eigenvalues of \( \mathbf{A}^l \) and \( \mathbf{A}^u \) are respectively calculated using the power method.

Step 2: By \( \mathbf{A}^l = (\tilde{a}_{ij}^l)_{mn}, \mathbf{A}^u = (\tilde{a}_{ij}^u)_{mn}, \) \( \alpha \) and \( \beta \) are calculated according to the following formula.

\[
\alpha = \left[ \frac{1}{n} \sum_{j=1}^{n} \frac{1}{\mathbf{A}^l_{jj}} \right]^{\frac{1}{2}} \quad \beta = \left[ \frac{1}{n} \sum_{j=1}^{n} \frac{1}{\mathbf{A}^u_{jj}} \right]^{\frac{1}{2}}
\]

Step 3: Subjective weight vector \( \omega = [\alpha \mathbf{x}^l, \beta \mathbf{x}^u] \).

3.2.2. Determination of objective weight based on entropy.

Entropy was originally a thermodynamic concept. C.E. Shannon introduced the entropy into information theory, it is called information entropy. It has been widely used in engineering technology, social economy and other fields [6].

The steps for entropy weight empowerment are generally as follows:

Step 1. Data standardization
Standardized processing of data for each indicator. For the problem of threat assessment for ballistic missile, the decision matrix \( A = (a_{ij})_{n \times m} \) is constructed, and \( n \) is number of ballistic missiles, \( m \) is number of threat indicators, and \( a_{ij} \) is attribute value of the first target under the \( j \)-th threat indicator. After matrix \( A \) is standardized with formula (1) to (5), a standardized matrix \( R = (Y_{ij})_{n \times m} \) is obtained.

Step 2. Compute information entropy for each indicator
The information entropy of a set of data is:

\[
E_j = -\ln \left(\frac{1}{n}\right) \sum_{i=1}^{n} p_{ij} \ln p_{ij}
\]

(4)

Where

\[
p_{ij} = \frac{Y_{ij}}{\sum_{i=1}^{n} Y_{ij}}
\]

(5)

If \( p_{ij} = 0 \), define \( \lim_{p_{ij} \to 0} p_{ij} \ln p_{ij} = 0 \).

Step 3. Determination of weights for all indicators
According to the information entropy calculation method, it is calculated as \( E_1, E_2, \ldots, E_k \). The weight vector \( \omega = (w_1, w_2, \ldots, w_k) \) is calculated by the information entropy.

Where

\[
w_i = \frac{1-E_i}{k-\sum E_i}, \quad (i = 1, 2, \ldots, k)
\]

(6)

3.2.3. Combination of subjective and objective weight. Let \( w_i^s \) be the subjective weight obtained by IAHP method, \( w_i^o \) is the objective weight obtained by entropy weight method, \( w_i \) is the weight of the \( i \)-th index after combination of the two methods, \( w_i \) can be expressed as a linear combination of \( w_i^s \) and \( w_i^o \).

\[
w_i = \lambda w_i^s + (1-\lambda)w_i^o
\]

(7)

There, \( \lambda \) is the proportion coefficient of index weight in combination weight calculated by IAHP method, and \( 1-\lambda \) is the proportion coefficient of index weight in combination weight calculated by entropy weight method. In order to obtain the best combination weight, the subjective weight and objective weight are balanced to minimize the deviation from the best weight. This is an optimization problem, and an optimization model can be established

\[
\min z = \sum [(\lambda w_i^s + (1-\lambda)w_i^o - w_i)^2 + (\lambda w_i + (1-\lambda)w_i^o - w_i)^2]
\]

(8)

The optimal value of \( \lambda \) can be obtained by making the derivative of \( \lambda \) be equal to 0.

4. Ballistic Missile Threat Assessment Based on Improved TOPSIS
The threat assessment steps of ballistic missile based on improved TOPSIS are as follows.
Step 1. Obtain the indicator values of each ballistic missile
According to the ballistic missile threat evaluation indicator, the corresponding evaluation indicator values of each incoming ballistic missile are collected.
Step 2. Building standardized evaluation matrix
According to the relevant index information of the incoming missile, the evaluation matrix \( A = \left( a_{ij} \right)_{nm} \) is constructed. Through the threat evaluation index system, the evaluation matrix is standardized, and the standardized evaluation matrix \( R = \left( r_{ij} \right)_{nm} \) is obtained.

Step 3. Determination of subjective weight by IAHP

The interval number judgment matrix is constructed by consulting experts, and the interval number weights of each threat assessment index are obtained according to the calculation steps of IAHP.

Step 4. Determination of objective weight by entropy weight method

According to the normalized evaluation matrix, objective weight of threat evaluation index is obtained according to the calculation method of entropy weight method.

Step 5. Combination weight

The combination of subjective and objective weights is used in the calculation formula of combined weights, and the weight factors are optimized by using optimization theory.

Step 6. Computation of weighted standardized matrix

From the combination of weights and standardized evaluation matrix, the weighted standardized evaluation matrix is calculated.

Step 7. Determining positive ideal solution and negative ideal one

First, the maximum and minimum values of each evaluation index are calculated:

\[
\begin{align*}
  y_i^+ & = \max_{y_{ij}} | i \in (1 \square m), j \in (1 \square n) | \\
  y_i^- & = \min_{y_{ij}} | i \in (1 \square m), j \in (1 \square n) |
\end{align*}
\]

(9)

(10)

The maximum value of each index forms positive ideal solution \( F^+ \), and the minimum value of each index forms negative ideal solution \( F^- \):

\[
\begin{align*}
  F^+ & = [y_1^+, y_2^+, \cdots, y_n^+] \\
  F^- & = [y_1^-, y_2^-, \cdots, y_n^-]
\end{align*}
\]

(11)

(12)

Step 8. Calculating the relative closeness of ballistic missiles

First, calculate distance between ballistic missile and positive ideal solution or negative ideal solution

\[
\begin{align*}
  S^+ & = \sqrt{\sum_{i=1}^{n} (y_i^* - y_i^+)^2} \\
  S^- & = \sqrt{\sum_{i=1}^{n} (y_i^* - y_i^-)^2}
\end{align*}
\]

(13)

(14)

The relative closeness is calculated according to the following formula

\[
S = \frac{S^-}{S^+ + S^-}
\]

(15)

Step 9. Threat ranking of ballistic missile

5. Results And Analysis

One day, the authorities of state a plan to launch a joint fire attack on state B. first, the missile operation group will attack the key targets such as the key command center of state B, and then the air operation group will launch an action to focus on the important targets such as the airport. State B focuses on intercepting "high value targets" with great harm to weaken the enemy's air control capability. According to the real-time detection information of the early warning radar, the ballistic missile targets that fall into the air defense area are screened out, and the targets are ranked according to the ranking results, so as to smash the enemy's false intention of fire suppression through ballistic missiles.
5.1. Threat Assessment Based on Improved TOPSIS

According to the ballistic missile information, the threat assessment matrix is

\[
A = \begin{bmatrix}
900 & 2.60 & 310 & 1 \\
1000 & 2.80 & 335 & 1 \\
1800 & 3.45 & 535 & 2 \\
2600 & 4.25 & 780 & 3 \\
3000 & 3.8 & 824 & 3 \\
\end{bmatrix}
\]

After normalization, the normalized evaluation matrix is

\[
A = \begin{bmatrix}
0.5714 & 0.6400 & 0.9933 & 0.9 \\
0.6000 & 0.6700 & 0.9767 & 0.9 \\
0.6800 & 0.7386 & 0.8433 & 0.8 \\
0.7600 & 0.8071 & 0.7100 & 0.7 \\
0.8000 & 0.7686 & 0.6880 & 0.7 \\
\end{bmatrix}
\]

The combination weight \( \vec{\omega} \) of threat assessment index is obtained by combination weight method

\[
\vec{\omega} = (0.169, 0.169, 0.4614, 0.2005)
\]

From the combination of weights and normalized evaluation matrix, the weighted normalized evaluation matrix can be calculated:

\[
A = \begin{bmatrix}
0.0966 & 0.1082 & 0.4583 & 0.1805 \\
0.1014 & 0.1133 & 0.4506 & 0.1805 \\
0.1150 & 0.1249 & 0.3891 & 0.1604 \\
0.1285 & 0.1364 & 0.3276 & 0.1404 \\
0.1352 & 0.1299 & 0.3174 & 0.1404 \\
\end{bmatrix}
\]

It is found that the positive and negative ideal solutions are

\[
F^+ = [0.1352, 0.1364, 0.4583, 0.1805]
\]

\[
F^- = [0.0966, 0.1082, 0.3174, 0.1404]
\]

From the distance between ballistic missiles and positive and negative ideal solutions, the relative closeness is calculated

\[
S = [0.7537, 0.7696, 0.5087, 0.2424, 0.2322]
\]

According to the relative closeness, the result of ballistic missile threat ranking is 2 > 1 > 3 > 4 > 5.

5.2. Results Comparison and Analysis

For comparison, the rough set method and Bayesian network method are implemented in this paper. The sorting results are shown in Table 1.

| Threat ranking method        | Threat ranking results |
|------------------------------|------------------------|
| Fuzzy set method             | 1, 2, 4, 3, 5          |
| Bayesian network method       | 1, 2, 3, 5, 4          |
| Our method                   | 2, 1, 3, 4, 5          |

Table 1. Comparison of threat ranking results
From table 1, it can be seen that the conclusions of the three threat ranking methods are quite different. By observing threat assessment matrix, it can be found that the remaining flight time of targets 101 and 102 is significantly less than the other three targets, and the priority of the security targets of targets 101 and 102 is level 1, but the other two targets 101 are lower than 102. Therefore, target 101 and 102 can be ranked in the first two places, and the remaining two targets 102 are. The threat degree is slightly higher than that of target 101, the remaining flight time of the remaining three targets and the priority of the target to be protected are successively decreasing, and the difference between range and shutdown point speed as secondary evaluation indexes is not too obvious, so the threat degree of target 103, 104 and 105 is successively decreasing, which shows that the threat evaluation ranking obtained by this algorithm is more scientific and reasonable.

6. Conclusion
Ballistic missile threat assessment plays an important role in antimissile operations. It is an important basis for early warning resource management, interception strategy formulation and other operations. Threat assessment results directly affect the effectiveness of antimissile operations. Aiming at the problems of TOPSIS threat assessment method, this paper improves TOPSIS method, which includes two aspects: one is the data processing of ballistic missile threat assessment; the other is the weight calculation of threat assessment index for ballistic missile. On this basis, a new method of ballistic missile threat assessment based on Improved TOPSIS is proposed. Through scenario experiments, it is proved that the method of ballistic missile threat assessment in this paper is more scientific and reasonable than the typical methods such as fuzzy set and Bayesian network.

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