SPHALERONS IN THE MSSM

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Abstract

We construct the sphaleron solution, at zero and finite temperature, in the Minimal Supersymmetric Standard Model as a function of the supersymmetric parameters, including the leading one-loop corrections to the effective potential in the presence of the sphaleron. At zero temperature we have included the one-loop radiative corrections, dominated by the top/stop sector. The sphaleron energy $E_{\text{MSSM}}$ mainly depends on an effective Higgs mass $m_h^{\text{eff}} = \lim_{m_A \gg m_W} m_h$, where $m_h$ is the lightest CP-even Higgs mass and $m_A$ the pseudoscalar mass. We have compared it with the Standard Model result, with $m_h^{\text{SM}} = m_h^{\text{eff}}$, and found small differences (1-2%) in all cases. At finite temperature we have included the one-loop effective potential improved by daisy diagram resummation. The sphaleron energy at the critical temperature can be encoded in the temperature dependence of the vacuum expectation value of the Higgs field with an error $\lesssim 10\%$. The light stop scenario has been re-examined and the existence of a window where baryon asymmetry is not erased after the phase transition, confirmed. Although large (low) values of $m_h$ ($m_A$) are disfavoured by the strength of the phase transition, that window (along with LEP results) allows for $m_h \lesssim 80$ GeV, $m_A \gtrsim 110$ GeV and $A_t \lesssim 0.4 m_Q$.

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1 Introduction

't Hooft observation [1] that baryon, and lepton, number is not conserved at the quantum level in the Standard Model (SM) of electroweak interactions, due to the presence of an SU(2) triangular anomaly, opened up the exciting possibility of baryon number non-conservation at low-energy. Even if the probability for $\Delta B \neq 0$ processes at zero temperature is exponentially suppressed by a factor $\sim \exp(-16\pi^2/g^2)$, where $g$ is the SU(2) gauge coupling, such suppression factor is absent at high temperatures [2]-[6] and, more precisely, at temperatures of the order of 100 GeV, triggering the hope of generating the baryon asymmetry of the Universe [8] at the electroweak phase transition [9].

The existence of sphalerons, static and unstable solutions to classical field equations in the SU(2) gauge theory with Higgs fields in the fundamental representation, has been known since long ago [10]. A key observation was that of Klinkhamer and Manton [2] who interpreted the sphaleron solution as a saddle point of the energy barrier separating gauge-inequivalent classical vacua. Detailed numerical calculations of the sphaleron solutions and energy in the SM were subsequently examined by many authors [2, 4, 5]. The precise value of the sphaleron energy is of the utmost importance for mechanisms aiming to explain the generation of the baryon asymmetry of the Universe [11]-[14]. In particular the sphaleron transition rate for temperatures below the phase transition temperature [6]

$$\Gamma_{sph} \sim \exp \left\{ \frac{E_{sph}(T)}{T} \right\}, \tag{1.1}$$

along with the bound obtained by comparing Eq. (1.1) with the expansion rate of the Universe at the temperature of the electroweak phase transition [12, 13]

$$\frac{E_{sph}(T_b)}{T_b} \geq 45 \tag{1.2}$$

where $T_b$ denotes the transition, or bubble formation temperature, allows to put upper bounds on the Higgs boson mass and thus discard models of electroweak symmetry breaking [14] on the basis of baryon asymmetry generation.

Even though the SM has all the required properties for the generation of the baryon asymmetry (CP violation, baryon number violating processes and non-equilibrium processes generated by a first-order phase transition) they are not strong enough to generate the necessary amount of baryon asymmetry. In particular, the CP-violating Cabibbo-Kobayashi-Maskawa phases strongly restrict the possible baryon number generation [16], though the latter remains as a very controversial subject. Even disregarding this point, the phase transition is not strongly first order enough and, moreover, condition (1.2) translates into an upper bound on the Higgs boson mass [17, 18, 19] which is well below the present experimental lower bound of 65 GeV.

Imposing the requirement of baryon asymmetry generation at the electroweak phase transition translates into the requirement of new physics at the weak scale. As a matter of fact, when extending the SM at the weak scale and probing the baryogenesis capability of the extended model the detailed calculation of the sphaleron energy should be an essential piece of information. It has been proved that there are extra sources of
CP violation, suitable for baryogenesis, in minimal extensions of the SM, in particular in
the SM with a gauge singlet \[ \Phi \] (with CP-violating non-renormalizable couplings
arising from integrated out extra fields), and in two-Higgs doublet models \[ \Phi \] (with
extra phases in the Higgs sector), and that the phase transition is strongly first order
enough not to wipe out, after the phase transition, the previously generated baryon
asymmetry \[ \Phi \] for values of the Higgs mass beyond the experimental bounds. In
both cases the sphaleron energy has been computed, Refs. \[ \Phi \] and \[ \Phi \] respectively, and
can support the baryogenesis achievements of the corresponding models. However, the
sphaleron solutions have been evaluated in these models using the tree-level potential
at zero temperature, so that the sphaleron energy at the phase transition temperature
relies on the assumption of the scaling law

\[ E_{sph}(T) = E_{sph}(0) \frac{\langle \phi(T) \rangle}{\langle \phi(0) \rangle} \]  

(1.3)

which has only been proved to hold approximately in the SM case \[ \Phi \], a similar calcu-
lation being absent in the above extensions of the SM.

The possibility that new physics at the electroweak scale be provided by the Minimal
Supersymmetric Standard Model (MSSM) \[ \Phi \] is a very appealing one on the theoretical
front, since it can technically solve the hierarchy problem. The MSSM is also the natural
candidate for an effective theory from a more fundamental theory valid at the high scale
(\[ \lesssim M_{Pl} \]), as e.g. string theory. Finally LEP electroweak precision measurements predict
in the MSSM the unification of gauge couplings at a scale \[ \sim 10^{16} \text{ GeV} \], which is the
only present ‘evidence’ for Grand Unification and new physics beyond the SM. All
of that has generated a plethora of experimental searches for new physics within the
MSSM framework in present and planned colliders \[ \Phi \]. Exploring the capability
of the MSSM of generating the observed baryon asymmetry of the Universe at the
electroweak phase transition is therefore of the highest interest, mainly in order to
confront the experimental searches of supersymmetry with possible regions of the space
of supersymmetric parameters where baryogenesis might be possibly produced. In this
sense new sources of CP violation are present in the MSSM \[ \Phi \], which can serve
to overcome the strong SM suppression factors mentioned above \[ \Phi \]. On the other
hand, the strength of the first order phase transition has been extensively studied in
the MSSM \[ \Phi \]-\[ \Phi \], and proved that the weak phase transition in the SM (driven by
the gauge coupling) can be strengthened in the MSSM (driven by the top Yukawa
coupling) in the presence of light supersymmetric partners of the top quark (stops) and
small values of \[ \tan \beta \] \[ \Phi \]. However a detailed study of the sphaleron solutions and the
sphaleron energy in the MSSM was missing.

In this paper we will construct the sphaleron solutions in the MSSM at zero and
finite temperature, including in all cases the leading one-loop radiative corrections. The
planning of this paper is as follows: In Section 2 we will construct the sphaleron so-
lutions and the sphaleron energy in the MSSM at zero temperature, including the full
set of one-loop radiative corrections to the effective potential in the presence of the
sphaleron Higgs. They are dominated by the top Yukawa coupling, and we will consis-
tently neglect the bottom Yukawa coupling, a reasonable approximation for \[ \tan \beta \lesssim 15 \],
which are the values we will consider. In fact, as we will show, only for \[ \tan \beta \lesssim 3 \]
the baryogenesis scenario is feasible. We have compared in every case the sphaleron energy in the MSSM, with arbitrary sets of supersymmetric parameters predicting the supersymmetric and Higgs mass spectra, with the sphaleron energy in the SM with an effective Higgs mass. The deviations are typically \( \lesssim 1.5\% \). In Section 3 we will construct the sphaleron solutions and energy in the MSSM at finite temperature. We have included the one-loop effective potential at finite temperature, in the presence of the sphaleron Higgs, improved by the resummation of daisy diagrams. We have checked that the approximation given by Eq. (1.3) is accurate with an error less than a few percent for the cases where the phase transition is weak. For the cases studied in Section 4, where we have applied the results of Sections 2 and 3 to the recently proposed case of light stops, the error can be as large as \( \sim 10\% \). We have qualitatively confirmed the results of Ref. [36]. In particular we have found a baryogenesis window for \( m_\tilde{\tau} \lesssim m_t \), \( m_h \lesssim 80 \text{ GeV} \), \( \tan \beta \lesssim 3 \) and \( m_A \gtrsim 110 \text{ GeV} \), although not all the previous inequalities can be simultaneously saturated. Finally Section 5 contains our conclusions.

We have adopted, throughout this paper, the approximation \( g' = 0 \), where \( g' \) is the \( U(1)_Y \) gauge coupling, in order to use a spherically symmetric ansatz for the sphaleron. The corrections of \( \mathcal{O}(g'^2) \) to the sphaleron energy have been evaluated in Ref. [2], computed in Ref. [39] for the SM sphaleron, and proved to be negative and \( \lesssim 1\% \). Also we have kept in the effective potential only one-loop corrections in the presence of the sphaleron Higgs, which are dominated by the top/stop sector and the top Yukawa coupling. We have neglected one-loop radiative corrections in the presence of the sphaleron-W. This procedure should provide a rather accurate one-loop approximation because the leading finite temperature \( \mathcal{O}(T^2) \) corrections to the effective potential in the presence of the sphaleron-W vanish \([1]\). Subleading finite temperature corrections to the effective potential are suppressed by powers of \( g^n, \ n > 2 \), and can therefore be neglected as compared to those proportional to powers of the top Yukawa coupling. The approximation of considering only radiative corrections to the effective potential (or effective action) in the presence of the sphaleron Higgs, and neglecting those generated in the presence of the sphaleron-W, is consistent only in cases where the former are dominated by a large coupling (e.g. in our case by the top Yukawa coupling) while the latter are always provided by extra powers of the gauge coupling constants. In other words, the approximation should work in theories where putting \( g = 0 \) in radiative corrections is a good approximation. An example of this kind of theories is the MSSM, where radiative corrections are dominated by the top Yukawa coupling. Unlike the MSSM, the finite temperature radiative corrections in the SM are dominated by the gauge coupling and making this approximation would miss some terms similar to those considered \([27]\).

\footnote{This is due to the fact that the sphaleron solution is non-zero only for transverse degrees of freedom whose (magnetic) Debye mass vanishes to all orders in perturbation theory \([40]\). This is not the case for longitudinal degrees of freedom, so that a non-vanishing component along \( W_0 \) would trigger, through the one-loop electric mass, sizeable corrections.
2 Sphalerons at zero temperature

In this section we will compute the static unstable solutions of classical equations of motion in the MSSM. As we have said in Section 1 we will work in the approximation of taking \( g' = 0 \) so that the \( U(1)_Y \) gauge field \( B_\mu \) can be consistently set to zero. This will allow a spherically symmetric ansatz, as follows. The lagrangian density for the \( SU(2) \) gauge fields \( W_\mu \) and the Higgs system

\[
H_1 = \begin{bmatrix} H_1^0 \ \\
H_1^+ \end{bmatrix} \quad (2.1)
\]

\[
H_2 = \begin{bmatrix} H_2^+ \\
H_2^0 \end{bmatrix}
\]

is given by

\[
\mathcal{L} = -\frac{1}{4} W_\mu^a W^{a\mu} + (D_\mu H_1) ^\dagger (D^\mu H_1) + (D_\mu H_2) ^\dagger (D^\mu H_2) - V_{\text{eff}}(H_1, H_2) \quad (2.2)
\]

where the \( SU(2) \) field strength is defined as,

\[
W_\mu^a = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g \epsilon^{abc} W^b_\mu W^c_\nu \quad (2.3)
\]

the covariant derivatives are

\[
D_\mu \equiv \partial_\mu - \frac{g}{2} W^a_\mu \sigma^a \quad (2.4)
\]

\( \sigma^a \) being the Pauli matrices, and \( V_{\text{eff}} \) is the effective potential where we will consider one-loop corrections.

We can expand the effective potential (at zero temperature) as

\[
V_{\text{eff}} = V_0(H_1, H_2) + V_1(H_1, H_2) + \cdots \quad (2.5)
\]

where \( V_0 \) is the tree-level potential and \( V_1 \) contains the one-loop radiative corrections. They are determined from the supersymmetric structure of the MSSM, with superpotential

\[
W = h_t Q_L \cdot H_2 U'_L + \mu H_1 \cdot H_2 \quad (2.6)
\]

and from the soft-breaking terms. In particular, the tree-level potential is given by

\[
V_0(H_1, H_2) = m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 + m_3^2 (H_1 \cdot H_2 + \text{h.c.}) + \frac{g^2}{8} \left[ (H_1^\dagger H_1 - H_2^\dagger H_2)^2 + 4 (H_1^\dagger H_2)(H_2^\dagger H_1) \right] \quad (2.7)
\]

and the one-loop corrections, in the 't Hooft-Landau gauge and in the \( \overline{\text{DR}} \) renormalization scheme, by

\[
V_1(H_1, H_2) = \sum_i \frac{n_i}{64\pi^2} m_i^4 (H_1, H_2) \left[ \log \frac{m_i^2 (H_1, H_2)}{Q^2} - \frac{3}{2} \right] \quad (2.8)
\]
where $Q$ is the renormalization scale, that we are taking, for definiteness, as $Q^2 = m_t^2$, 
$m_i^2(H_1, H_2)$ is the field dependent mass of the $i$th particle in the background $H_1, H_2$, and $n_i$ is the corresponding number of degrees of freedom, which is taken negative for fermions. The relevant degrees of freedom for our calculation are the gauge bosons $(W, Z)$, the top quark $(t)$ and its supersymmetric partners $(\tilde{t}_1, \tilde{t}_2)$, with

$$n_t = -12, \ n_W = 6, \ n_Z = 3, \ n_{\tilde{t}_1} = n_{\tilde{t}_2} = 6.$$  \ (2.9)

Choosing the temporal, radial gauge 

$$W_0 = 0, \ x_iW_i^a = 0 \ (i = 1 - 3, \ a = 1 - 3)$$  \ (2.10)

we can write the static, spherically symmetric ansatz

$$W_j^a(\vec{x}) = \frac{2f(r)}{gr^2} \epsilon_{ajk}x_k$$

$$H_1(\vec{x}) = h_1(r) i \sigma \cdot \frac{x}{r} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$H_2(\vec{x}) = h_2(r) i \sigma \cdot \frac{x}{r} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$  \ (2.11)

for the sphaleron in the MSSM, where $r^2 = x^2 + y^2 + z^2$.

There, however, remain time-independent gauge transformations which can be used to rotate the background fields in (2.11) into a more convenient form for our calculation. In particular, the gauge transformation $U$

$$U(\vec{x}) = \exp \left[-i \frac{\pi}{2} \tilde{\sigma} \cdot \frac{\vec{x}}{r} \right]$$  \ (2.12)

transforms the ansatz (2.11) into the background fields

$$W_j^a(\vec{x}) = \frac{2[1 - f(r)]}{gr^2} \epsilon_{ajk}x_k$$

$$H_1(\vec{x}) = h_1(r) \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$H_2(\vec{x}) = h_2(r) \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$  \ (2.13)

which should smoothly approach a vacuum in the unitary gauge as $r \to \infty$.

In the presence of the background (2.13) the tree-level potential reads as

$$V_0(h_1, h_2) = m_1^2 h_1^2(r) + m_2^2 h_2^2(r) + 2 m_3 h_1(r) h_2(r) + \frac{g^2}{8} \left[ h_1^2(r) - h_2^2(r) \right]^2$$  \ (2.14)

while the one-loop corrections are given by Eq. (2.8) with background dependent masses. The relevant masses are the top quark mass, given by

$$m_t^2(h) = h_t^2 h_2^2(r),$$  \ (2.15)
the gauge boson masses, by

$$m_W^2(h) = m_Z^2(h) = \frac{1}{2} g^2 \left[ h_1^2(r) + h_2^2(r) \right],$$  \hspace{1cm} (2.16)

and the stop squared mass matrix given by

$$\mathcal{M}_t^2 = \begin{pmatrix} m_{t_L}^2 & m_{t_{LR}}^2 \\ m_{t_{LR}}^2 & m_{t_R}^2 \end{pmatrix},$$

with entries

$$m_{t_L}^2(h) = m_t^2 + m_Z^2(h) + \frac{1}{2} g^2 [ h_1^2(r) - h_2^2(r) ],$$

$$m_{t_R}^2(h) = m_t^2 + m_Z^2(h)$$  \hspace{1cm} (2.18)

$$m_{t_{LR}}^2(h) = h_t ( A_t h_2(r) + \mu h_1(r) ),$$

the mass eigenstates being defined by the diagonalization of matrix (2.17),

$$m_{t_{1,2}}^2(h) = \frac{m_{t_L}^2(h) + m_{t_R}^2(h)}{2} \pm \sqrt{ \left[ \frac{m_{t_L}^2(h) - m_{t_R}^2(h)}{2} \right]^2 + \left[ m_{t_{LR}}^2(h) \right]^2 },$$  \hspace{1cm} (2.19)

with $h \equiv (h_1, h_2)$. 

By minimizing the effective potential (2.5) with respect to $h_1, h_2$, and imposing the minimum of the potential at $(v_1, v_2)$, with $v = \sqrt{v_1^2 + v_2^2} = 171.1$ GeV, and $\tan \beta = v_2/v_1$, fixed, we can eliminate $m_1^2$ and $m_2^2$ in favour of the other parameters of the theory, as [35, 41]

$$m_1^2 = -m_3^2 \tan \beta - \frac{m_Z^2}{2} \cos 2\beta - \sum_i \frac{n_i}{64\pi^2} \left[ \frac{\partial m_i^2}{\partial h_1} \left. \frac{m_i^2}{h_1} \left( \log \frac{m_i^2}{Q^2} - 1 \right) \right] \bigg|_{v=(v_1,v_2)}$$

$$m_2^2 = -m_3^2 \cot \beta + \frac{m_Z^2}{2} \cos 2\beta - \sum_i \frac{n_i}{64\pi^2} \left[ \frac{\partial m_i^2}{\partial h_2} \left. \frac{m_i^2}{h_2} \left( \log \frac{m_i^2}{Q^2} - 1 \right) \right] \bigg|_{v=(v_1,v_2)}$$  \hspace{1cm} (2.20)

while $m_3^2$ can be traded in favour of the one-loop corrected squared mass $m_A^2$ of the CP-odd neutral Higgs boson, as [41]

$$m_3^2 = -m_A^2 \sin \beta \cos \beta$$

$$- \frac{3 g^2 m_t^2 \mu A_t}{32 \pi^2 m_W^2 \sin^2 \beta} \left[ \log(m_{t_1}^2/Q^2) - 1 \right] - \frac{m_{t_2}^2}{m_{t_1}^2 - m_{t_2}^2} \left[ \log(m_{t_2}^2/Q^2) - 1 \right] \bigg|_{v=(v_1,v_2)}$$  \hspace{1cm} (2.21)

The energy functional corresponding to the ansatz of Eq. (2.13) can be written as:

$$E_{\text{static}} = 4\pi \int dr \left\{ \frac{4}{g^2} \left[ (\partial_r f)^2 + \frac{2}{r^2} f^2 (1 - f)^2 \right] + r^2 \left[ (\partial_r h_1)^2 + \frac{2}{r^2} h_1^2 (1 - f)^2 \right] + r^2 \left[ (\partial_r h_2)^2 + \frac{2}{r^2} h_2^2 (1 - f)^2 \right] + r^2 \left[ V_{\text{eff}}(h_1, h_2) - V_{\text{eff}}(v_1, v_2) \right] \right\}$$  \hspace{1cm} (2.22)
where we have subtracted the vacuum energy. The sphaleron is described by the functions \( f(r), h_1(r) \) and \( h_2(r) \) which are the solutions to the equations of motion

\[
\left( \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} \right) h_1(r) - \frac{2}{r^2} h_1(r) (1 - f(r))^2 = \frac{1}{2} \frac{\partial V_{\text{eff}}(h_1, h_2)}{\partial h_1} \tag{2.23}
\]

\[
\left( \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} \right) h_2(r) - \frac{2}{r^2} h_2(r) (1 - f(r))^2 = \frac{1}{2} \frac{\partial V_{\text{eff}}(h_1, h_2)}{\partial h_2} \tag{2.24}
\]

\[
\frac{\partial^2}{\partial r^2} f(r) - \frac{2}{r^2} f(r) (1 - f(r)) (1 - 2f(r)) = -\frac{g^2}{2} (1 - f(r)) \left[ h_1^2(r) + h_2^2(r) \right] \tag{2.25}
\]

subject to the boundary conditions,

\[
h_1(0) = h_2(0) = f(0) = 0 \tag{2.26}
\]

\[
h_1(\infty) = v_1, \ h_2(\infty) = v_2, \ f(\infty) = 1
\]

which guarantee that the solutions of (2.23-2.25) have finite energy and the correct vacuum behaviour when \( r \to \infty \).

In this section we have introduced the set of supersymmetric parameters which the sphaleron solutions and sphaleron energy depend upon. They are \( \tan \beta \equiv v_2/v_1, \ m_A \), the mass of the CP-odd Higgs boson, \( \mu \) the supersymmetric higgsino mass, \( m_Q^2 \) and \( m_U^2 \), the soft supersymmetry breaking squared mass terms for the left-handed doublet and right-handed singlet squarks

\[
Q_L = \left( \begin{array}{c} U_L \\ D_L \end{array} \right), \ U_L^c
\]

and \( A_t \) the soft breaking term for the third generation of squarks corresponding to the tri-linear coupling in the superpotential (2.6). We will consider \( \tan \beta \lesssim 15 \) and therefore we can neglect the bottom Yukawa coupling \( h_b \), and thus the corresponding tri-linear soft term \( A_b \) will have no influence on the result, and can be neglected too. As for the top Yukawa coupling, we will fix \( m_t = 175 \text{ GeV} \) throughout this paper. This value of \( m_t \) is inside the experimental range \[12, 13\] and we have decided to fix it in view of the large number of parameters in the model. Variations of the results in this paper corresponding to variations of \( m_t \) inside the experimental error band will not dramatically modify the final conclusions.

In Fig. 1 we have illustrated the functions \( h_1(r)/v, h_2(r)/v \) and \( f(r) \) [solid lines] for a particular value of supersymmetric parameters: \( \tan \beta = 1.5, \ m_Q = 500 \text{ GeV}, \ m_A = 100 \text{ GeV} \) and \( m_U = A_t = \mu = 0 \). In Fig. 2 we plot the sphaleron energy given by Eq. (2.22) [in units of \( m_W^2/\alpha_W \), \( \alpha_W = g^2/4\pi \)] as a function of \( \tan \beta \) for different values of the supersymmetric parameters, where we have varied \( m_U = 0 - 400 \text{ GeV} \) and \( m_A = 100 - 500 \text{ GeV} \). Comparison of curves (a) and (b), and (c) and (d), shows that the variation of \( E_{\text{sph}} \) with \( m_A \), within the considered range, is tiny, while comparison

\[2\text{Notice that } m_t \text{ is the MS running top mass determined at the scale } Q = M_t. \text{ It is related to the pole top mass } M_t \text{ by the QCD correcting factor } m_t = M_t/[1 + 4\alpha_s(M_t)/3\pi]. \text{ The on-shell mass } m_t = 175 \text{ GeV} \text{ corresponds to } M_t = 183 \text{ GeV.} \]
of curves (a) and (c), and (b) and (d), exhibits the influence of the parameter $m_U$ in the value of $E_{\text{sph}}$, being $\lesssim 3\%$. We can see from Fig. 2 that $E_{\text{sph}}$ varies $\sim 10\%$ over the whole range of $\tan \beta$.

A similar exercise to that performed in Fig. 2 has been done in Fig. 3 exchanging the variable $\tan \beta$ with the variable $m_A$. In Fig. 3 we plot $E_{\text{sph}}$ as a function of $m_A$ for different values of the other supersymmetric parameters where we have varied $\tan \beta = 2 - 15$ and $m_U = 0 - 400$ GeV. The large variation with $\tan \beta$ is explicit by comparing curves (a) and (b), and (c) and (d). The variation with $m_U$, comparison of curves (a) and (c), and (b) and (d), being smaller. Finally the variation with $m_A$ is tiny, as previously noticed, for the considered range of supersymmetric parameters.

The behaviour of $E_{\text{sph}}$ in Figs. 2 and 3 as a function of the different supersymmetric parameters can be understood in a simple way. The Higgs boson mass spectrum is determined in the MSSM from the tree-level potential (2.7) and the radiative corrections (2.8). In particular the CP-even Higgs boson masses $m_{h,H}$ can be determined from the supersymmetric parameters $|44| - |46|

$$m_{h,H} = m_{h,H}(m_A, \tan \beta, m_Q, m_U, A_t, \mu, \ldots)$$

(2.28)

where the first two parameters determine the tree-level expression while the rest enter into the radiative corrections. Now, the field in the ($h_1, h_2$) plane which acquires a vacuum expectation value (VEV) is along the direction $h_1 \cos \beta + h_2 \sin \beta$, and the squared mass (curvature) of the potential along that direction is

$$\left(m_{h}^{\text{eff}}\right)^2 = \sin^2(\alpha - \beta) m_h^2 + \cos^2(\alpha - \beta) m_H^2$$

(2.29)

where $\alpha$ is the mixing angle in the Higgs sector, whose determination, as for the masses $m_h$ and $m_H$ in (2.28), involves radiative corrections. The simplest way of evaluating (2.29), with radiative corrections provided by (2.8), is by observing that it does not depend on $m_A$, and therefore, using the limit (when $m_A \to \infty$), $\sin^2(\alpha - \beta) \to 1$, one obtains

$$m_{h}^{\text{eff}} = \lim_{m_A \to \infty} m_h .$$

(2.30)

Since the sphaleron solution asymptotically points towards $h_1 \cos \beta + h_2 \sin \beta$ one expects that $E_{\text{sph}}$ will mainly depend on $m_{h}^{\text{eff}}$. Notice that, from (2.30), $m_{h}^{\text{eff}}$ and $m_h$ will coincide only for very large values of $m_A$. This behaviour is exhibited in Fig. 4 where we plot $E_{\text{sph}}$ as a function of $m_{h}^{\text{eff}}$ for the same cases as in Fig. 2 and compare it with $E_{\text{sph}}^{\text{SM}}$ for $m_{h}^{\text{SM}} = m_h^{\text{eff}}$. We can see that the spreading is small ($\sim 1\%$) and due to the fact that the direction which does not acquire a VEV also contributes to the sphaleron functional.

Finally the dependence of $E_{\text{sph}}$ with the stop mixing parameters in Eq. (2.18), $A_t$ and $\mu$, are shown in Figs. 5 and 6. We can see from Fig. 5 that the variation of $E_{\text{sph}}$ with $A_t$ is $\lesssim 3\%$ for generic values of the supersymmetric parameters, while the variation with $\mu$ is smaller, as can be seen from Fig. 6. This fact can be understood from the fact that the effective stop mixing parameter is provided by

$$\tilde{A}_t = A_t + \mu/\tan \beta$$

(2.31)

\(^3\text{We thank J.R. Espinosa for making this observation to us.}\)
so that the effect of the $\mu$ parameter is suppressed by $1/\tan \beta$. For large values of $\tan \beta$, $E_{\text{sph}}$ does not depend on the $\mu$ parameter, as can be seen from curve (b) in Fig. 3.

3 Sphalerons at finite temperature

In order to introduce finite temperature effects we have to modify the effective potential of Eq. (2.5) by adding the thermal corrections. We will consistently work at the level of the one-loop thermal corrections improved by the resummation of daisy diagrams. The effective potential can be written, to this level of approximation, as

$$V_{\text{eff}} = V_0(h) + V_1(h) + \Delta V_1(h, T) + \Delta V_{\text{daisy}}(h, T)$$  (3.1)

where $V_0$ and $V_1$ are the tree-level and one-loop potentials at zero temperature, given by Eqs. (2.14) and (2.8), respectively, $\Delta V_1$ the one-loop correction at finite temperature, and $\Delta V_{\text{daisy}}$ the resummation of daisy diagrams. They are given by:

$$\Delta V_1(h, T) = \frac{T^4}{2\pi^2} \left\{ \sum_i n_i J_i \left[ \frac{m_i^2(h)}{T^2} \right] \right\}$$  (3.2)

where $n_i$ is given in Eq. (2.9), the masses $m_i(h)$ are defined in Eqs. (2.15-2.19), and the thermal function $J_i = J_+(J_-)$, if the $i$th particle is a boson (fermion), with

$$J_{\pm}(y^2) = \int_0^\infty dx x^2 \log \left( 1 \mp e^{-\sqrt{x^2+y^2}} \right).$$  (3.3)

As for the term $\Delta V_{\text{daisy}}$, it is given by,

$$\Delta V_{\text{daisy}}(h, T) = -\frac{T}{12\pi} \sum_B n_B \left[ \overline{m}_B^2(h, T) - m_B^3(h) \right]$$  (3.4)

where the sum is extended to scalar bosons and the longitudinal degrees of freedom of gauge bosons, with $n_B$ given by Eq. (2.9) for $\tilde{t}_{1,2}$ and

$$n_{WL} = 2, \ n_{ZL} = 1.$$  (3.5)

The thermal masses $\overline{m}_B^2(h, T)$ are obtained from $m_B^2(h)$ by adding the leading $T$ dependent self-energy contributions, which are proportional to $T^2$. In particular, the stop squared thermal mass matrix is given by

$$\overline{M}_\tilde{t}^2 = \begin{pmatrix} m_{\tilde{t}L}^2 + \Pi_{\tilde{t}L}(T) & m_{\tilde{t}LR}^2 \\ m_{\tilde{t}LR}^2 & m_{\tilde{t}R}^2 + \Pi_{\tilde{t}R}(T) \end{pmatrix}.$$  (3.6)

The self-energies are given by

$$\Pi_{\tilde{t}L}(T) = \left[ \frac{4}{9} g_s^2 + \frac{1}{12} \left( \Theta_{\tilde{t}L} + \sin^2 \beta + \cos^2 \beta \Theta_A \right) h_t^2 \right] T^2$$

$$\Pi_{\tilde{t}R}(T) = \left[ \frac{4}{9} g_s^2 + \frac{1}{6} \left( \Theta_{\tilde{t}L} + \sin^2 \beta + \cos^2 \beta \Theta_A \right) h_t^2 \right] T^2$$  (3.7)
where $g_s$ is the strong gauge coupling. Only loops of gauge bosons, Higgs bosons and third generation quark/squark have been included, assuming that all the remaining supersymmetric particles are heavy and decouple by Boltzmann suppression factors. In particular the gluinos, if light, would provide contributions to the self-energies which are given by $2/9 g_s^2 T^2$. They constitute the main missing contribution to the thermal masses and would weaken the strength of the phase transition. We are considering them heavy as in Ref. [35]. We also have introduced explicit step-$\Theta$ functions for the contribution of left-handed and right-handed stops and pseudoscalar bosons, with the convention: $\Theta(X) = 1 \left[ 0 \right]$ for $m_X < \sim \pi T \left[ m_X > \sim \pi T \right]$.

The thermal $W_L$-mass is given by

$$\tilde{m}^2_{W_L}(h, T) = m^2_{W}(h) + \Pi_{W_L}(T)$$ (3.8)

where the self-energy is

$$\Pi_{W_L}(T) = \frac{5}{2} g^2 T^2$$ (3.9)

and of course, since we are taking $g' = 0$, $\tilde{m}^2_{W_L}(h, T) = m^2_{W_L}(h, T)$.

Up to now we have considered one-loop corrections to the effective potential in the sphaleron background (ansatz) corresponding to diagrams with $h_1(r)$ and $h_2(r)$ as external legs. Strictly speaking we should also consider diagrams with $W^a_j$ as external legs. They should contribute to the effective action as

$$-\frac{1}{2} \Pi^a_{\mu \nu} W^a_{\mu} W^b_{\nu} + \cdots = -\frac{5}{4} g^2 T^2 W^a_0 W^a_0 + \cdots.$$ (3.10)

While the leading term in Eq. (3.10) cancels for the sphaleron ansatz of Eq. (2.11), the ellipsis denotes terms which are suppressed by powers of the gauge coupling constant and by inverse powers of the temperature and we will not consider them explicitly in this paper. As previously stated this procedure is self-consistent since in the MSSM the radiative corrections are dominated by the top Yukawa coupling and the top/stop sector.

Now, at finite temperature, the energy of the sphaleron, the equations of motion and the boundary conditions for functions $h_1(r, T), h_2(r, T)$ and $f(r, T)$, are given by Eqs. (2.22–2.26), where $V_{\text{eff}}(h_1, h_2, T)$ is provided by Eq. (3.1) and $v_1 = v_1(T)$, $v_2 = v_2(T)$ is the minimum of the effective potential (3.1) at the temperature $T$. We define the critical temperature $T_c$, as in Ref. [35], as the temperature at which the determinant of the second derivatives of $V_{\text{eff}}(h, T)$ at $h = 0$ vanishes $\Box$, i.e.

$$\det \left[ \frac{\partial^2 V_{\text{eff}}(h, T_c)}{\partial h_i \partial h_j} \right]_{h_1 = h_2 = 0} = 0.$$ (3.11)

Explicit formulae for computing the critical temperature can be found in Ref. [35]. For illustrative purposes we have plotted in Fig. 1 the functions $h_1(r, T_c)/v$, $h_2(r, T_c)/v$ and

$^4$The temperature $T_c$, as defined in (3.11), is usually called lower metastability temperature or lower spinodial decomposition point. We thank M. Shaposhnikov for pointing out this to us. We call it critical temperature just for simplicity.
$f(r, T_c)$ (dashed lines) for $\tan \beta = 1.5$, $m_A = 100$ GeV, $m_Q = 500$ GeV, $m_U = 0$ and $A_t = \mu = 0$.

The $T$-dependence of $E_{\text{sph}}$ is shown in Fig. 7 for three typical sets of supersymmetric parameters (solid lines). In all cases the pattern of curves is similar to that shown in Ref. [27] for the SM case. In particular $E_{\text{sph}}$ decreases with increasing temperatures and sharply goes to its minimal value at the temperature where the local minimum we are considering disappears. We have also probed the approximated scaling law,

$$E_{\text{sph}}(T) = E_{\text{sph}}(0) \frac{v(T)}{v} \quad (3.12)$$

where $v(T) = \sqrt{v_1^2(T) + v_2^2(T)}$, $v_1(T)$ and $v_2(T)$ being the vacuum expectation values of the fields $h_1$ and $h_2$ at finite temperature. We have plotted in Fig. 7 $E_{\text{sph}}(T)$ (dashed lines) for the three cases. We have found that Eq. (3.12) is an excellent approximation, for all values of supersymmetric parameters used in Fig. 7, with an error $\lesssim 3\%$. In fact we have found that the scaling law (3.12) is not controlled by any particular combination of supersymmetric parameters, but by the strength of the phase transition. The weaker the phase transition the better the scaling law. In fact for a second order phase transition the scaling law would be exact as happens in the SM. The reason of the little departure between solid and dashed lines in Fig. 7 is because of the weakness of the phase transition for the particular values of the chosen parameters. In cases where the phase transition is stronger, as we will discuss in Section 4, the departure corresponding to the approximation of Eq. (3.12) is greater.

Finally we have plotted in Figs. 8 and 9, for the cases considered in Figs. 2 and 3, $E_{\text{sph}}(T_c)/T_c$ as a function of $\tan \beta$ and $m_A$, respectively. We can see that the dependence of $E_{\text{sph}}(T_c)$ on the supersymmetric parameters, is qualitatively similar to that found for $E_{\text{sph}}(0)$ in Figs. 2 and 3. We can see from Figs. 8 and 9 that for the sample of generic cases considered up to now the phase transition does not satisfy condition (1.2) and therefore is not strong enough first order to generate baryon asymmetry. This problem was taken care in Ref. [36] where the order of the phase transition was considered and a ‘non-generic’ region of the space of supersymmetric parameters was proposed to trigger a strong first order phase transition. This case is characterized by light right-handed stops, heavy left-handed stops and low values of $\tan \beta$ and will be analysed in the next section.

4 The case of light right-handed stops

In the previous sections we have studied the sphaleron solutions and energy in the MSSM for generic values of supersymmetric parameters, as was done in analyses of the phase transition in the MSSM in the previous literature [33]-[35]. Our results on the value of the order parameter $E_{\text{sph}}(T_c)/T_c$, when compared with the general bound (1.2) confirm the former analyses based on the study of the phase transition: for generic

\footnote{We are making here the reasonable assumption that the rate of sphaleron mediated transitions in the MSSM is provided by (1.1) and, therefore, that the effect of the new physics in the MSSM is entirely encoded in the definition of $E_{\text{sph}}$. Under these conditions the bound (1.2) still applies.}
values of supersymmetric parameters the phase transition in the MSSM is not strong enough to generate the baryon asymmetry of the universe. These negative results have motivated a recent search [36, 38] of the region in the parameter space where the phase transition has better chances to allow generation of the baryon asymmetry, while being in agreement with precision electroweak measurements at LEP [17]. This region has been identified as follows:

- Large values of $m_Q^2$ guarantee small supersymmetric contributions to the oblique radiative corrections [17], while small or negative values of $m_U^2$ can help in enhancing the value of $R_b$ [18].

- Large values of $m_A$ and small values of the stop mixing parameters $A_t$ and $\mu$ favour the strength of the phase transition [34, 35].

That region was recently analysed in Ref. [36] from the point of view of the strength of the phase transition, that was found to be much stronger than in previous analyses. Our aim in this section is to study the sphaleron solutions and the sphaleron energy in the MSSM, in the above region of the supersymmetric parameter space, in order to refine (and eventually confirm) the positive results of Ref. [36]. Moreover we will relax at some point the condition of large $m_A$ (even though large values of $m_A$ are favoured by the strength of the phase transition) because some mechanisms of CP violation associated with supersymmetric particles require a sizeable variation of $\tan \beta(T_c)$ along the bubble wall in order to generate the necessary amount of baryon asymmetry at the electroweak phase transition [31].

The intuitive explanation of why negative values of $m_U^2$ (keeping the hierarchy $m_Q^2 \gg m_U^2$) favour the phase transition, along with the appearance of color breaking minima [49] which can endanger the stability of the standard electroweak minimum, can be found in Ref. [36]. We refer the reader, for a thorough explanation of this phenomenon, to Ref. [36] where all technical details can be found.

The standard vacuum of the potential (2.5), with radiative corrections provided by (2.8), has a depth which does not depend on $m_A$, as can be easily checked. Therefore the simplest procedure is to compute the depth at the standard vacuum for large values of $m_A$: in this limit the MSSM goes to the SM with a particular value of the Higgs mass, determined by the supersymmetric boundary condition of the quartic coupling, and the threshold effects due to the stop mixing parameters, at the scale of supersymmetry breaking ($\sim m_Q$). These effects can be encoded in an effective quartic coupling [46] and a particular value of the SM Higgs mass, in terms of which the depth of the SM minimum is:

$$V_{\text{eff}}(v) = -\frac{1}{4} m_h^2 v^2. \quad (4.1)$$

On the other hand, for $A_t = 0$ and negative values of $m_U^2$

$$\tilde{m}_U^2 = -m_U^2 \quad (4.2)$$

the potential along the direction $U_L^c$ has a minimum at [30]

$$\langle U_L^c \rangle^2 = \frac{3 \tilde{m}_U^2}{g_s^2} \quad (4.3)$$
with a depth

\[ V_{\text{eff}}(\langle U_L^c \rangle) = - \frac{3}{2} \frac{\tilde{m}_U^4}{g_s^2}. \]  

(4.4)

Comparison of (4.1) with (4.4) allows to determine a critical value of the parameter \( \tilde{m}_U \) such that the depth of the charge and color breaking minimum (4.3) does not exceed that of the standard electroweak minimum (4.1):

\[ m_{U}^{\text{crit}} = \left( \frac{g_s^2 m_h^2 v^2}{12} \right)^{1/4} \]  

(4.5)

For \( A_t \neq 0 \) we have numerically verified [36] that the condition \( \tilde{m}_U < m_{U}^{\text{crit}} \) also guarantees that the standard electroweak minimum is the global minimum if \( A_t \lesssim 0.8 \) \( m_Q \).

Notice however that such high values of the mixing parameters are uninteresting for our analysis since the phase transition will become very weak for those values, as we will see.

We have then fixed \( \tilde{m}_U = m_{U}^{\text{crit}} \) and repeated the analysis of Sections 2 and 3. We will also fix \( m_Q = 500 \) GeV, to be in good agreement with electroweak precision measurements, as we said above, and, for the moment, we will adhere to large values of \( m_A \) and low mixing: \( m_A = 500 \) GeV, \( A_t = \mu = 0 \), although we will relax the latter conditions later on. For these values of the supersymmetric parameters we have plotted \( E_{\text{sph}} \) at zero temperature as a function of \( \tan \beta \) in Fig. 10 [solid line] along with the comparison, for illustrative purposes, with \( E_{\text{SM}}^{\text{sph}} \) for the value of the Higgs mass given by Eq. (2.29) [dashed line], and \( E_{\text{sph}}(T_c)/T_c \) in Fig. 11 [solid line], also as a function of \( \tan \beta \). We also plot in Fig. 11 \( m_h \) [thin dashed line] as a function of \( \tan \beta \) for the corresponding values of supersymmetric parameters.

Up to here we have used, for our calculations, the critical temperature \( T_c \), where the origin gets destabilized and the barrier between the origin and the finite temperature minimum disappears. However tunneling with formation of bubbles starts somewhat earlier, at a temperature \( T_b \) when the corresponding euclidean action \( B(T_b) \sim 140 \) and the phase transition can proceed sufficiently fast such that the whole universe can be filled with bubbles of the new phase. To compute \( B(T_b) \) we will use the analytical estimate obtained in Refs. [17, 50]. First of all we will define an approximated potential of the form

\[ V_{\text{app}} = D(T^2 - T_c^2)\phi^2 - E T \phi^3 + \frac{\lambda}{4} \phi^4 \]  

(4.6)

where

\[ \phi = \sqrt{2} (h_1 \cos \beta + h_2 \sin \beta), \]

and all the constants have been determined numerically. \( T_c \) is the critical temperature that we have been using throughout this paper, \( \lambda = m_h^2/(4v^2) \), where \( m_h \) is the lightest Higgs boson mass of the MSSM,

\[ E = \frac{\lambda}{3} \frac{\langle \phi(T_c) \rangle}{T_c} \]

where \( \langle \phi(T_c) \rangle \) is determined numerically, and

\[ D = \frac{3}{16v^2} \sum_i |n_i|m_i^2. \]
Secondly, we will use the analytical estimate of Refs. [17, 50]

\[ B(T_b) = \frac{38.8D^{3/2}}{E^2} \left( \frac{\Delta T}{T_b} \right)^{3/2} f \left[ \frac{2\lambda D}{E^2} \left( \frac{\Delta T}{T_b} \right) \right] \]  

(4.7)

where \( \Delta T = T_b - T_c \) and the function \( f \), defined in Ref. [50], equals 1 at zero value of its argument. Using (4.7) one can easily determine the value of the transition temperature, within the error inherent to the analytical estimate we are using. This degree of accuracy is enough for our purposes in this paper. The value of \( E_{\text{sph}}(T_b)/T_b \) is plotted in Fig. [[1] thick dashed line]. From Fig. [[1] we can see that the condition (1.2) translates into bounds on \( \tan \beta \) (\( \tan \beta \lesssim 3 \)) and on \( m_h \) (\( m_h \lesssim 80 \text{ GeV} \)). We can also see that one can translate the bound (1.2) at the transition temperature, into a conservative bound at the critical temperature \( T_c \),

\[ \frac{E_{\text{sph}}(T_c)}{T_c} \gtrsim 50. \]  

(4.8)

This bound can be useful in many phenomenological studies of phase transitions. Finally, the goodness of the scaling low (3.12) is shown, for the same values of the supersymmetric parameters, in Fig. [[2] where we see it is accurate with an error \( \lesssim 10\% \).

To obtain the results of Figs. [[1],[2] we have been using optimal values of the supersymmetric parameters, from the point of view of the phase transition. However, in particular not too large values of \( m_A \) might be required, for baryogenesis purposes, as we have said above, in order to generate the needed amount of CP violation at the bubble walls. In Figs. [[3] and [[4] we plot \( E_{\text{sph}} \) as a function of \( m_A \) for \( \tan \beta = 2.4, 2.5 \) and the same values of the other supersymmetric parameters. In Fig. [[3] we compare \( E_{\text{sph}} \) with \( E_{\text{sph}}^{\text{SM}} \) for a Higgs mass \( m_h^{\text{eff}} \). We see they are very close to each other. In genuine two-Higgs situations, when \( m_A \) is small, they can depart from each other by an amount \( \sim 1.5\% \). In Fig. [[4] we plot \( E_{\text{sph}}(T_c)/T_c \) as a function of \( m_A \). As expected we see that the order parameter \( E_{\text{sph}}(T_c)/T_c \) decreases with increasing \( m_A \). Using the bound (4.8) puts lower bounds on \( m_A \): \( m_A \gtrsim 100 \text{ GeV} \), for \( \tan \beta = 2.4 \), and \( m_A \gtrsim 110 \text{ GeV} \) for \( \tan \beta = 2.5 \), the corresponding bounds increasing for larger values of \( \tan \beta \). On the other hand, LEP results impose bounds in the \( (\tan \beta, m_A) \) plane. In particular, for the set of supersymmetric parameters in Fig. [[4] [[1]: \( m_A \gtrsim 120 \text{ GeV} \), for \( \tan \beta = 2.4 \), and \( m_A \gtrsim 110 \text{ GeV} \), for \( \tan \beta = 2.5 \), the corresponding bounds decreasing for larger values of \( \tan \beta \). We can conclude that for \( \tan \beta \sim 2.5 \), both (4.8) and LEP results translate into \( m_A \gtrsim 110 \text{ GeV} \), which is a very safe bound for generation of baryon asymmetry in the MSSM [[31]. As for the dependence of \( E_{\text{sph}}(T_c)/T_c \) on the stop mixing parameters, we have plotted in Fig. [[5] \( E_{\text{sph}}(T_c)/T_c \) as a function of \( A_t \), for \( \tan \beta = 1.7 \) and the same values of the other supersymmetric parameters. We see that, as expected, \( E_{\text{sph}}(T_c)/T_c \) decreases with increasing \( A_t \), and the bound (4.8) puts an upper bound on \( A_t \) as \( A_t \lesssim 0.4 \text{ m}_{Q} \).

\footnote{We thank P. Janot for sending us some original plots with present LEP bounds.}
5 Conclusion

We have constructed the sphaleron solution and computed its energy, at zero and finite temperature, in the MSSM for an arbitrary set of supersymmetric parameters. At zero temperature we have included the leading one-loop radiative corrections in the presence of the sphaleron. At finite temperature we have added the one-loop thermal corrections where all daisy diagrams are resummed. At zero temperature we have compared $E_{\text{sph}}$ with the corresponding one for the SM with an effective Higgs mass. The difference is small ($\lesssim 1.5\%$) even for small values of $m_A$. We have verified that, at finite temperature, the scaling law, where all finite temperature effects can be encoded into the temperature dependence of the vacuum expectation value of the Higgs fields, is accurate (with an error $\lesssim 3\%$), for the cases of weak first order phase transition. For the cases of strong first order phase transition (light stop scenario) it is somewhat larger ($\lesssim 10\%$). Finally our calculation supports the conclusion that for generic values of the supersymmetric parameters the MSSM is unable to keep any pre-existing baryon asymmetry after the phase transition. However we have confirmed the presence of a window for MSSM baryogenesis corresponding to heavy left-handed stops, light (lighter than the top) right-handed stops, light CP-even higgses ($m_h \approx 80$ GeV), not so heavy CP-odd higgses ($m_A \approx 110$ GeV), moderate stop mixing ($A_t \approx 0.4 m_Q$), and small values of $\tan \beta$ ($\tan \beta \lesssim 3$). These results are in qualitative agreement $[52]$ with recent non-perturbative calculations performed in the framework of the MSSM $[53]$, and the baryogenesis window will be probed at LEP2.

Some unconsidered effects should tend to increase the sphaleron energy, and thus to improve the previous bounds, while others tend to decrease it and thus to worsen the bounds. As for the former, two loop effects at finite temperature have been recently proved to strengthen the order of the phase transition in the MSSM $[37]$ and thus they are expected to increase the sphaleron energy. As for the latter, first of all, we have worked within the approximation $g' = 0$ in order to consider a spherically symmetric ansatz. $O(g'^2)$ effects can be easily accounted for by linearizing the energy functional with respect to the $U(1)$ gauge field and neglecting the feed-back on the sphaleron $[4]$, or by solving explicitly an axially symmetric ansatz $[39]$: in both cases the SM result for the value of the sphaleron energy correction is tiny ($\sim 1\%$) as should be in the case of the MSSM. A second effect is characteristic of the nature of the MSSM due to the large number of scalar fields it contains. We have set all of them to zero in this work $[7]$. However, in some cases a non-vanishing configuration for these fields might minimize the energy functional leading to a modification of the results contained in this paper. In any case the variation of $E_{\text{sph}}$ should be negative. In particular, for those fields with negligible Yukawa couplings we expect the energy functional to be minimized by the zero configuration, since the combined effect of their kinetic terms and the large soft breaking masses, which are positive definite, will control the configuration of the energy functional. Only in the case of the third-generation squarks in Eq. (2.27), and for the case of relatively large values of $\tilde{m}_{U_{ij}}$ and/or $A_t$ has the energy functional a chance to be minimized for non-vanishing field configurations, even in the cases of no charge and

\footnote{Of course, the equations of motion for all scalars fields of the theory are identically satisfied for zero configurations.}
color breaking at the vacuum. We are at present investigating this issue and the results will be published elsewhere [74].

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Figure 1: Plots of $h_1(r)/v$, $h_2(r)/v$ and $f(r)$ at zero (solid lines) and the critical temperature $T = T_c$ (dashed lines) for $m_t = 175$ GeV and the values of supersymmetric parameters: $\tan \beta = 1.5$, $m_A = 100$ GeV, $m_Q = 500$ GeV, $m_W = 0$ and $A_t = \mu = 0$. 
Figure 2: Plot of $E_{\text{sph}}$ as a function of $\tan \beta$ for $m_Q = 500$ GeV, $A_t = \mu = 0$ and: a) $m_A = 100$ GeV, $m_U = 0$; b) $m_A = 500$ GeV, $m_U = 0$; c) $m_A = 100$ GeV, $m_U = 400$ GeV; and, d) $m_A = 500$ GeV, $m_U = 400$ GeV.
Figure 3: Plot of $E_{\text{sph}}$ as a function of $m_A$ for $m_Q = 500$ GeV, $A_t = \mu = 0$ and: 
a) $\tan \beta = 2$, $m_U = 0$; 
b) $\tan \beta = 15$, $m_U = 0$; 
c) $\tan \beta = 2$, $m_U = 400$ GeV; and, 
d) $\tan \beta = 15$, $m_U = 400$ GeV.
Figure 4: The same as in Fig. 2 but as a function of the effective Higgs boson mass, $m_{h}^{\text{eff}}$. The dashed line is $E_{\text{sph}}$ for the Standard Model with a Higgs boson with mass $m_{h}$. 
Figure 5: Plot of $E_{\text{sph}}$ as a function of $A_t$ for $m_A = m_Q = 500$ GeV, $\mu = m_U = 0$, and:
a) $\tan \beta = 2$; b) $\tan \beta = 15$. 
Figure 6: Plot of $E_{\text{sph}}$ as a function of $\mu$ for $m_A = m_Q = 500$ GeV, $A_t = m_U = 0$, and:
a) $\tan \beta = 2$; b) $\tan \beta = 15$. 
Figure 7: Solid [dashed] lines are plots of $E_{\text{sph}}(T)/v(0)$ as functions of $T$ for $m_Q = 500$ GeV, $\mu = m_U = 0$, and: a) $m_A = 100$ GeV, $A_t = 0$, $\tan \beta = 2$; b) $m_A = 100$ GeV, $A_t = 0$, $\tan \beta = 15$; and, c) $m_A = 500$ GeV, $A_t = 200$ GeV, $\tan \beta = 2$. 
Figure 8: Plot of $E_{\text{sph}}[T_c]/T_c$ as a function of $\tan \beta$ for the same values of supersymmetric parameters as in Fig. 2.
Figure 9: Plot of $E_{\text{sph}}[T_c]/T_c$ as a function of $m_A$ for the same values of supersymmetric parameters as in Fig. 3.
Figure 10: Plot of $E_{\text{sph}}$ at $T = 0$ as a function of $\tan \beta$ for $m_Q = m_A = 500$ GeV, $A_t = \mu = 0$ and $m_U = m_U^{\text{crit}}$. The dashed line is the Standard Model value for a Higgs mass equal to $m_h^\text{eff}$. 
Figure 11: Plot of $E_{\text{sph}}[T]/T$, for $T = T_c, T_b$, as function of $\tan \beta$ for the values of supersymmetric parameters of Fig. 10. The dashed line is a plot of the lightest Higgs boson mass for the corresponding values of supersymmetric parameters.
Figure 12: Solid [dashed] line is $E_{\text{sph}}(T) \ [E_{\text{sph}}(0)v(T)/v(0)]$ as function of $T$ for $\tan \beta = 2$ and $m_Q, m_A, \hat{A}_t, \mu$ and $m_U$ as in Fig. 10.
Figure 13: Plots of $E_{\text{sph}}$ at $T = 0$ as a function of $m_A$ for $m_Q = 500 \text{ GeV}$, $A_t = \mu = 0$, $\tan \beta = 2.4$ (lower solid), 2.5 (upper solid) and $m_U = m_U^{\text{crit}}$. The dashed lines are the corresponding Standard Model values for a Higgs mass equal to $m_h^{\text{eff}}$. 
Figure 14: Plots of $E_{\text{sph}}[T_c]/T_c$, as function of $m_A$ for the values of supersymmetric parameters of Fig. 13 [upper solid is for $\tan \beta = 2.4$ and lower solid for $\tan \beta = 2.5$]. The dashed lines are plots of the lightest Higgs boson mass for the corresponding values of supersymmetric parameters [upper dashed is for $\tan \beta = 2.5$ and lower dashed for $\tan \beta = 2.4$].
Figure 15: Plot of $E_{\text{sph}}(T_c)/T_c$, as function of $A_t$ for $\tan \beta = 1.7$ and $m_Q$, $m_A$, $\mu$ and $m_U$ as in Fig. 10.