Abstract

The thermally driven confinement-deconfinement transition exhibited by lattice quantum electrodynamics in two space dimensions is re-examined in the context of the statistical gauge-fields common to anyon superconductors and to spin-liquids. Particle-hole excitations in both systems are bound by a confining string at temperatures below the transition temperature $T_c$. We argue that $T_c$ coincides with the actual critical temperature for anyon superconductivity. The corresponding specific-heat contribution, however, shows a smooth peak just below $T_c$ characteristic of certain high-temperature superconductors.

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The discovery of high-temperature superconductivity has generated a number of new theoretical approaches to the problem of strongly interacting Fermi systems in two dimensions. Among these, the proposals of spin-liquids in two-dimensional (2D) Mott insulators and of anyon superconductivity in doped 2D Mott insulators are perhaps the most novel. The former state is characterized by short-range spin correlations while the latter is essentially a 2D superconductor that generally breaks time-reversal and parity symmetries. In addition, both the spin-liquid state and the anyon superconductor have a gap for Fermi excitations.

Parallel technical developments have evolved in the arena of $U(1)$ gauge-field descriptions of strongly correlated electron systems in two dimensions. In particular, the 2D spin-1/2 antiferromagnet has been treated within both the Schwinger-boson scheme and the corresponding long-wavelength description given by the $CP_1$ model, where the gauge-field that appears in such case measures chiral spin-fluctuations. In addition, the random-phase approximation (RPA) results for anyon superconductors have been recovered by Chern-Simons (CS) gauge-theories in the mean-field approximation, where a statistical gauge-field is introduced to describe the flux-tube attached to each fermion by the CS term. Very similar gauge-field descriptions also exist for anyon superconductors in the context of doped 2D antiferromagnets ($t-J$ model).

In this paper, we study the thermal properties of such statistical gauge-fields common to anyon superconductors and to spin-liquids on the square lattice. Specifically, we re-examine the confinement-deconfinement (CD) transition at non-zero temperature experienced by particle-hole excitations about such groundstates using an effective lattice (compact) QED lagrangian for the statistical gauge-field. The CD transition is found to be dual to the 2D Coulomb-gas (CG) transition in the weak-coupling regime, like in the strong-coupling regime. In fact, both the string-tension and the inverse of the confinement length-scale vanish exponentially as the transition temperature, $T_c$, is approached.
from low-temperature at both the strong and weak-coupling limits. The specific-heat is also computed, where it is characterized by a smooth bump anomaly below $T_c$ reminiscent of certain high-temperature superconductors.\textsuperscript{18} Last, we argue that $T_c$ coincides with the actual transition temperature for anyon superconductivity.

The action for compact QED\textsuperscript{3} at temperature $T \neq 0$ is defined by

$$S = \frac{1}{2g_0^2} \sum_{x_3/a=1}^\beta \sum_x \sum_{\mu,\nu=1,2,3} \{1 - \cos[\Delta_\mu a_\nu(x) - \Delta_\nu a_\mu(x)]\},$$

(1)

where $a_\nu(x)$ represents the statistical gauge-field, $\Delta_\mu$ denotes the lattice difference operator, and where $x = (x_1,x_2,x_3)$ is the three-vector that spans the cubic-lattice space-time with lattice constant $a$. Quantum statistical mechanics requires that the gauge-field be periodic in the time-like direction\textsuperscript{19} $x_3 = ic_0 t$, where $c_0$ is the zero-sound speed for anyon superconductors and the spin-wave velocity for spin-liquids; i.e., $a_\nu(x_3 + \beta a) = a_\nu(x_3) \mod 2\pi$, where $\beta = \hbar \omega_0/k_B T$ is presumed to be a positive integer, with $\omega_0 = c_0 a^{-1}$ as the Debye frequency-scale of the theory. Polyakov has shown that compact QED\textsuperscript{3} is confining at zero-temperature in the weak-coupling regime,\textsuperscript{20,21} $g_0 \ll 1$. Below, we extend his instanton-gas analysis to the present case of non-zero temperature, and find that a CD-transition occurs at $k_B T_c \sim g_0 \hbar \omega_0$.

**Weak-coupling.** In the limit $g_0 \ll 1$, the Villain substitution for the partition function, $Z = \int \mathcal{D}a_\mu e^{-S}$, is then valid,\textsuperscript{20–22} yielding the factorization $Z = Z_{\text{gauss}}Z_{\text{inst}}$, where $Z_{\text{gauss}}$ represents the gaussian (“spin-wave”) approximation to the action (1), and where the corresponding instanton contribution is

$$Z_{\text{inst}} = \sum_{\{n(\bar{x})\}} \exp \left\{ -\frac{g_0^2}{2} \sum_{\bar{x}} \left[ \Delta_\mu n(\bar{x}) \right]^2 \right\}.$$

(2)

Above, $n(\bar{x})$ is an integer-field on the dual cubic-lattice, such that $n_{\mu\nu}(x) = \epsilon_{\mu\nu\lambda} \Delta_\lambda n(\bar{x})$ is the antisymmetric integer-field dual to the minima of the periodic energy functional (1) at a given plaquette $(\mu, \nu)$. [The link associated with $\Delta_\lambda n(\bar{x})$ passes perpendicularly through the plaquette associated with $n_{\mu\nu}(x)$.] Again, the field $n(\bar{x})$ must be periodic in the time-like direction with period $\beta a$; i.e., the sum over time $\bar{x}_3$ in Eq. (2) is restrict to a slab of thickness $\beta a$. After employing the Poisson summation formula on the configuration sum above (2), and then integrating over the continuous field corresponding to $n(\bar{x})$, we
arrive at the following instanton-gas ensemble:

\[ Z_{\text{inst}} = \sum_{\{m(\vec{x})\}} \exp \left[ -\frac{1}{2g_0^2} (2\pi)^2 \sum_{\vec{x},\vec{x}'} m(\vec{x}) G(\vec{x})^2 (\vec{x} - \vec{x}') m(\vec{x}') \right], \]  

(3)

where \( G(\vec{x}) = (2\pi)^{-d} \int_{BZ} d^d k e^{i k \cdot \vec{x}} [2d - 2 \sum_{\mu=1}^d \cos(k_\mu a)]^{-1} \) is the Greens function for the \( d \)-dimensional hypercubic lattice, and where \( m(\vec{x}) \) is the integer charge-field of the instanton-gas, which is also periodic in the time-like direction. Consider now two instantons of charge \( m(\vec{x}) \) and \( m(\vec{x}') \) centered at space-time points \( \vec{x} = (\vec{r}, \vec{x}_3) \) and \( \vec{x}' = (\vec{r}', \vec{x}_3') \), respectively. Furthermore, suppose that they are widely spaced, such that \( |\vec{r} - \vec{r}'| \gg \beta a \).

Then since both configurations have period \( \beta a \) along the time-like direction, their mutual action (3) per elementary time-slice is proportional to \( \beta^{-1} m(\vec{r}) G(\vec{r} - \vec{r}') \beta^{-1} m(\vec{r}') \), where \( G(\vec{r}) \) is the Greens function for the square-lattice and \( \beta^{-1} m(\vec{r}) \) is simply the linear charge density along time. After multiplying the latter action-density by \( \beta \) to get the action per period, we arrive at the following 2D CG partition function for the periodic instanton gas ensemble (3):

\[ Z_{\text{inst}} = \sum_{\{m(\vec{r})\}} \beta^{N_m} \exp \left[ -\frac{1}{2g_0^2} (2\pi)^2 \sum_{\vec{r},\vec{r}'} m(\vec{r}) G(\vec{r} - \vec{r}') m(\vec{r}') \right], \]  

(4)

where the pre-factor of \( \beta^{N_m} \) above results from counting all possible instanton configurations fixed at \( N_m \) sites in space. Hence, we obtain a transition at \( k_B T_c \simeq k_B T_0 \simeq 0.44 g_0^2 h \omega_0 \) that is dual to the 2D CG transition, where \( T_0 \) denotes the temperature at which the chemical potential vanishes in the CG ensemble (4). For temperatures below \( T_c \) free instantons exist, while instanton-anti-instanton pairs are bound above \( T_c \). Note that the exponentially small number of free instantons that exist near the dual CG transition validates the previous assumption of diluteness.

To demonstrate that the preceding is in fact a CD-transition, we now compute the confinement length-scale (string-tension), and show that it diverges (vanishes) exponentially as \( T_c \) is approached from low-temperature. Adapting Polyakov’s zero-temperature calculation for the auto-correlation function of the electromagnetic fields in compact QED\(_3\) to the present case at \( T \neq 0 \) reduces to the sine-Gordon reformulation of the dual 2D CG ensemble (4). A straightforward generalization of his methods then yields that the corresponding static auto-correlation function for the statistical electrodynamic field is
given by \( g_0^{-2}\beta \langle b_i(\vec{k})b_j(-\vec{k}) \rangle = \delta_{ij} - k_i k_j / [k^2 + \xi_X^{-2}(T)] \), where the magnetic fields in the spatial directions \( j = 1, 2 \) are related to the electric field \( e_i(x) = F_{0,i}(x) \) by \( b_j(x) = i\epsilon_{ij}e_l(x) \), and where \( \xi_X(T) \) denotes the Debye screening-length of the dual CG ensemble (4). Hence, the confinement length scale \( \xi_X(T) \) is infinite above \( T_c \) in the bound-instanton phase, while it diverges below \( T_c \) like

\[
\xi_X(T) = aB \exp[A/(1 - T/T_c)^{1/2}]
\]

in the free instanton phase, with \( A \) and \( B \) being non-universal numerical constants.\(^{23}\) A corresponding adaptation of Polyakov’s calculation of the zero-temperature string-tension\(^{20,21}\) \( \sigma \) also yields

\[
\sigma(T) \sim g_0^2\hbar\omega_0 \xi_X^{-1}(T).
\]

In conclusion, correlations between the statistical gauge field have a finite range below \( T_c \) indicating confinement, while they are of infinite-range above \( T_c \) implying gaussian \([n_{\mu\nu}(x) = 0]\) electrodynamics consistent with the absence of instantons. This observation suggests that the specific-heat contribution of the gauge-fields in the weak-coupling limit of compact QED\(_3\) follows a gaussian (photon) \( T^2 \) law above \( T_c \), while the temperature-scale below which activated behavior sets in the confined phase is on the order of \( \sigma(0)a \). Here, the zero-temperature string tension \( \sigma(0) \) is given by (6) with a zero-temperature confinement length-scale\(^{20,21}\) of \( \xi_X(0) = a(g_0/2\pi)e^{\text{const}}/g_0^2 \). The former energy-scale is exponentially small in the present weak-coupling limit, however, indicating that the specific heat is given essentially by the same gaussian \( T^2 \) law below \( T_c \).

**Strong-coupling.** Following Polyakov and Susskind,\(^{14,15,21}\) the Hamiltonian formulation of compact QED\(_3\) (1) reduces to the kinetic energy \( H_0 = \frac{1}{2}g_0^{-2}\hbar\omega_0 \sum \vec{x} e_i^2(\vec{x}) \), in the regime \( g_0 \gg 1 \), where the statistical electric field operator \( e_i(\vec{x}) = -ig_0^2\partial / \partial a_i(\vec{x}) \) satisfies Maxwell’s Eq., \( \Delta_i e_i = 0 \), in vacuum. (The temporal gauge \( a_3 = 0 \) has implicitly been chosen.) Because of the compact nature of the vector potential, the field-strength operator \( g_0^{-2} e_i(\vec{x}) \) has integer eigenvalues \( n_i(\vec{x}) \) satisfying \( \Delta_i n_i(\vec{x}) = 0 \). Since the latter constraint can be easily solved by taking \( n_i(\vec{x}) = \epsilon_{ij} \Delta_j n(\vec{r}) \), where \( n(\vec{r}) \) is an integer-field on the dual square-lattice, we arrive at the discrete gaussian (DG) model for surface roughening\(^{24-26}\) given by

\[
Z = \sum_{\{n(\vec{r})\}} \exp \left\{ -\frac{1}{2} \beta g_0^2 \sum_{\vec{r}} [\Delta_i n(\vec{r})]^2 \right\}.
\]
This model is dual to the 2D CG ensemble,\textsuperscript{24} with a transition temperature \( k_B T_c \cong 0.73g_0^2\hbar\omega_0 \). Notice also that the present DG model (7) is equal to the weak-coupling instanton action (2) for the special case of static configurations \( \Delta_3n(\vec{x}) = 0 \). Below we demonstrate that \( T_c \) marks the boundary for the CD transition that also exists in the strong-coupling limit.\textsuperscript{16,17}

Let us first determine the temperature dependence of the confinement length-scale for compact QED\(_3\) in the present strong-coupling limit. Given the identity \( g_0^{-2}\langle e_k(\vec{x})e_l(\vec{x}')\rangle = g_0^2\epsilon_{k_1}\epsilon_{l_1}\langle \Delta_i n(\vec{r})\Delta_j n(\vec{r}')\rangle \), and the fact that the latter is “proportional” to the height-height correlation function of the surface-roughening model (7),\textsuperscript{24} we find that the spatial Fourier transform of the auto-correlation function for the statistical electric-field is given by \( g_0^{-2}\langle \vec{e}(k)\cdot \vec{e}(k)\rangle = \beta^{-1}\epsilon_{k_1}\epsilon_{l_1}\epsilon_{k_2}\epsilon_{l_2}/[k^2 + \xi^2(T)] \), where \( \xi(T) \) denotes the Debye screening length of the dual CG ensemble. Hence, we recover precisely the same pole obtained in the prior weak-coupling analysis, with a confinement length-scale \( \xi(T) \) that diverges exponentially (5) at \( T_c \), but that vanishes linearly with temperature at zero temperature.\textsuperscript{23} Note that \( g_0^{-2}\beta\langle \vec{c}(0)\cdot \vec{c}(0)\rangle \) jumps from zero to unity upon heating through the transition.

Confinement properties are also directly probed by the string-tension. To compute this quantity, it is convenient to return to the original strong-coupling Hamiltonian that is known to be dual to the 2D XY-model. Specifically, Polyakov and Susskind have shown that the effective potential energy between opposing unit statistical charges separated by a distance \( R \) is given by \( V(R) = -k_B T \ln\langle \exp i[\phi(0) - \phi(\vec{R})]\rangle \), where \( \phi(\vec{R}) \) denotes the phase variable of the dual XY-model.\textsuperscript{14,15} Yet since the correlation function for this model is given by \( \langle \exp i[\phi(0) - \phi(\vec{R})]\rangle = (r_0/R)^{1/4}\exp(-R/\xi) \) near \( T_c \), we have that the string-tension [defined by \( \lim_{R \to \infty} V(R) = \sigma R \)] is \( \sigma(T) = k_B T_c \xi^{-1}(T) \) in this region. [This result agrees with Eq. (6).] On the other hand, the zero-temperature string-tension in the strong-coupling limit is given by \( \sigma(0) = \frac{1}{2}g_0^2\hbar\omega_0a^{-1} \), since a confining flux-tube corresponds to a smooth step in the DG-model for surface-roughening (7). In fact, it is evident that the step free-energy per length of this model is equivalent to the string-tension in general. The temperature-dependence of the former quantity is well known,\textsuperscript{25,26} and agrees with the results just cited.

Last, the specific-heat per site for the interface roughening model (7) obtained from Monte-Carlo simulations\textsuperscript{25} is reproduced in Fig. 1. It is qualitatively similar to the
weak-coupling result discussed previously, with the exception that (i) the strong-coupling specific-heat saturates to the classical value of $\frac{1}{2}k_B$ above the transition because $k_B T_c \gg \hbar \omega_0$, and that (ii) the energy-scale for activated behavior is on the order of $\sigma(0)a \approx 0.69k_B T_c$. The latter observation confirms the fact that the elementary excitations in compact QED$_3$ at low-temperature are strings of electric flux.

**Anyon Superconductors and Spin-liquids.** We now apply these results to the spin-1/2 anyon superconductor,$^{11}$ which is known to be a large-$N$ saddle-point of the 2D $t-J$ model near half-filling.$^{12}$ Consider then ideal spin-1/2 pseudo-fermions on the square-lattice with a statistical flux-tube of one flux-quantum attached to each. The corresponding long-wavelength limit CS lagrangian is given by

$$L = \sum_i \left\{ \frac{i}{2} (\dot{f}_{i\sigma} f_{i\sigma} - \text{c.c.}) - (\tilde{t} e^{i[A_0+a]_\mu(i)} f_{i\sigma} f_{i+\bar{\mu},\sigma} + \text{c.c.}) + (2\pi)^{-1} \epsilon_{\mu\nu} a_\mu(i) \dot{a}_\nu(i) + (8\pi)^{-1} \epsilon_0 \dot{a}_\mu^2(i) - (8\pi \mu_0)^{-1} [\Delta_1 a_2(i) - \Delta_2 a_1(i)]^2 \right\} \tag{8}$$

in the temporal gauge $a_3(i) = 0$, where $\tilde{t}$ denotes the hopping matrix-element for the pseudo-fermion ($f_{i\sigma}$) and $\dot{a}_\nu(i) = \hbar \partial a_\nu(i)/\partial t$, etc. Also, $\epsilon_0$ and $\mu_0$ respectively denote the vacuum dielectric constant and the magnetic permeability for fluctuations $a_\mu(i)$ of the statistical gauge-field,$^{11,12}$ while $\vec{A}_0$ describes the statistical magnetic field due to the flux-tubes averaged over the entire lattice.$^{9,10}$ After integrating out the pseudo-fermions, one finds that the cancellation of the resulting Hall conductance against that of the CS term leads to the action (1) for compact QED$_3$ at zero-temperature,$^{12,27}$ with a zero-sound velocity $c_0 = \hbar^{-1}(\mu_0^{-1} + \mu_f^{-1})^{1/2}/(\epsilon_0 + \epsilon_f)^{1/2}$ and a coupling-constant $g_0^{-2} = (2\pi)^{-1}(\epsilon_0 + \epsilon_f)\hbar \omega_0$. Here, $\epsilon_f \sim \Delta_f^{-1}$ and $\mu_f^{-1} \sim \Delta_f \xi_f^2$ are the pseudo-fermion contribution to the dielectric constant and the magnetic permeability, respectively, with $\Delta_f$ and $\xi_f$ denoting the relevant (Hofstadter) energy-gap and magnetic length of the pseudo-fermions in the mean-field approximation.$^{11}$ Now suppose that we extend this result above zero-temperature. Then no free statistical charge can exists below $T_c$ because of confinement, implying that the Hall conductance generated by the pseudo-fermions is temperature-independent. Due to the exact cancellation of the latter against the CS-term, we (i) recover the assumption that the statistical gauge-field is described by vacuum QED$_3$ below $T_c$, and find (ii) that the electromagnetic response is that of a superconductor in this regime.$^{11,12}$ Hence, $T_c$ is in
fact the critical temperature for lattice anyon-superconductors. This is in sharp contrast
to straight-forward extensions of RPA methods to non-zero temperature, which find that
the critical temperature vanishes due to a spurious temperature dependence acquired by
the psuedo-fermion Hall conductance. Zero-temperature RPA calculations find, however, that
the London penetration length varies as $\lambda_L \propto (\epsilon_0 + \epsilon_f)^{1/2}$. By extension, $\lambda_L^{-1}$
should have an s-wave-like temperature dependence at low-temperature, while it should
jump to zero at the transition. In conclusion, particle-hole excitations in lattice anyon
superconductors are bound by a confining string. The similarity between the duality
of this transition to that of the 2D XY model and the duality of conventional su-
perconductors in the presence of fluctuating magnetic fields to the three-dimensional XY
model is striking. Also, if we assume that $k_B T_c \ll \Delta_f$, then the specific-heat should
be dominated by the gauge-field contribution shown in Fig. 1. Such “rounding” of the
specific-heat anomaly actually occurs in certain high-temperature superconductors.

In the case of the quantum-disordered (spin-liquid) phase of the 2D quantum antifer-
romagnet, on the other hand, $CP_1$ calculations indicate that chiral spin-fluctuations are
also described by the action (1) for vacuum QED, where $c_0$ is identified here with the spin-
wave velocity. The coupling-constant is given by $g_0^2 = 12\pi \Delta_f / \hbar \omega_0$ in such case, where
$\Delta_f$ denotes the spin-gap. Hence, spinons are confined below $T_c$, where the elementary
excitations are again strings of statistical electric flux. Also, the existence of a universal-
jump in the long-wavelength autocorrelation for the electric fields implies a corresponding
jump $\Delta \langle Q^2 \rangle / N = (2\pi)^{-2} g_0^2 / \beta_c$ in the fluctuation per $\langle N \rangle$ site of the total number of
skyrmions (quanta of chiral spin) $Q_\chi = (2\pi)^{-1} \sum_\vec{x} \epsilon_{ij} \Delta_i a_j (\vec{x})$ at the transition. Last,
the gauge-fields will contribute a smooth bump anomaly to the specific heat below $T_c$ (see
Fig. 1), just as in the previous case.

To conclude, it has been demonstrated that the CD transition shown by compact
QED is dual to the 2D CG transition in both the weak and the strong coupling limits. It
has also been argued that the corresponding transition temperature coincides with the crit-
ical temperature of ideal anyon superconductors and of spin-liquids on the square-lattice.
The former constitutes a first step towards the thermodynamics of anyon superconductors,
which are perhaps the paradigm for 2D superconductivity in general.

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References

1. *The Physical Properties of High-Temperature Superconductors*, vol. 2, edited by D.M. Ginsberg (World Scientific, Singapore, 1990).
2. P.W. Anderson, Science **235**, 1196 (1987).
3. R.B. Laughlin, Phy. Rev. Lett. **60**, 2677 (1988).
4. R.B. Laughlin, Science **242**, 525 (1988).
5. D.P. Arovas and A. Aurbach, Phys. Rev. B**38**, 316 (1988).
6. N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989).
7. J.P. Rodriguez, Phys. Rev. B **41**, 7326 (1990).
8. X.G. Wen, F. Wilczek, and A. Zee, Phys. Rev. B **39**, 11413 (1989).
9. A. Fetter, C. Hanna, R.B. Laughlin, Phys. Rev. B **39**, 9679 (1989).
10. Y. Chen, F. Wilczek, E. Witten, and B.I. Halperin, Int. J. Mod. Phys. B **3**, 1001 (1989).
11. P.B. Wiegmann, Phys. Rev. Lett. **65**, 2070 (1990); J.P. Rodriguez. and B. Douçot, Phys. Rev. B **42**, 8724 (1990); (E) **43**, 6209 (1991).
12. J.P. Rodriguez and B. Douçot, Phys. Rev. B **45**, 971 (1992).
13. L.B. Ioffe and A.I. Larkin, Phys. Rev. B **39**, 8988 (1989).
14. A. Polyakov, Phys. Lett. **72B**, 477 (1978).
15. L. Susskind, Phys. Rev. D **20**, 2610 (1979).
16. I. Ichinose and T. Matsui, Nucl. Phys. **B394**, 281 (1993).
17. N. Nagaosa, Phys. Rev. Lett. **71**, 4210 (1993).
18. J.W. Loram et al., Phys. Rev. Lett. **71**, 1740 (1993); H. Wuhl et al., Physica C **185 - 189**, 755 (1991); A. Junod in ref. 1.
19. A.A. Abrikosov, L.P. Gorkov, and I.E. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics*, (Dover, New York, 1975).
20. A.M. Polyakov, Nucl. Phys. B**120**, 429 (1977).
21. A.M. Polyakov, *Gauge Fields and Strings* (Harwood, New York, 1987).
22. T. Banks, R. Myerson and J. Kogut, Nucl. Phys. B**129**, 493 (1977).
23. P. Minnhagen, Rev. Mod. Phys. **59**, 1001 (1987).
24. S.T. Chui and J.D. Weeks, Phys. Rev. B **14**, 4978 (1976); J.D. Weeks, in *Ordering in Strongly Fluctuating Condensed Matter Systems*, edited by T. Riste (Plenum, New York, 1977).
York, 1980).

25. R.H. Swendsen, Phys. Rev. B 15, 5421 (1977).

26. H. van Beijeren and I. Nolden, in Structure and Dynamics of Surfaces II, edited by W. Schommers and P. von Blanckenhagen (Springer, Heidelberg, 1987).

27. It is shown in ref. 12 that the effective action for the statistical gauge-field is time-reversal/parity symmetric to quadratic order in frequency and wave-number. This contrasts with the physical electromagnetic response, which exhibits a zero-field Hall effect within RPA.

28. J.E. Hetrick, Y. Hosotani, and B.H. Lee, Ann. Phys. (NY) 209, 151 (1991).

29. C. Dasgupta and B.I. Halperin, Phys. Rev. Lett. 47, 1556 (1981).
Figure Caption

Fig. 1. Shown are Monte-Carlo results obtained by Swendsen (ref. 25) for the specific heat of the DG model (7) on a $10 \times 10$ lattice. The specific-heat is plotted in units of $k_B$, while the temperature is plotted in units of $g_0^2 \hbar \omega_0 / k_B$. The circles represent extrapolations based on the solid-on-solid model (ref. 25).
