The $\Delta I = 1/2$ Rule for Kaons

Joaquim Prades
Departamento de Física Teórica y del Cosmos, Universidad de Granada
Campus de Fuente Nueva, E-18002 Granada, Spain.

Abstract
We report on recent advances at understanding the $\Delta I = 1/2$ rule for
kaons. We get reasonable matching between short– and long–distances for
scales between between 0.6 and 1.0 GeV and reproduce the $\Delta I = 1/2$ rule
huge enhancement in the chiral limit. A detailed analysis of the different
contributions to the relevant octet and 27-plet couplings is done. For
the $B_6^{(1/2)}(\mu) \equiv \langle (\pi\pi)_0|Q_6|K \rangle / \langle (\pi\pi)_0|Q_6|K \rangle_{\Lambda_c}$ parameter, we get in the
chiral limit $B_6^{(1/2)}(\mu) = 2.2 \pm 0.5$ for scales $\mu \in [0.6, 1.0]$ GeV.

August 1999

1Work supported in part by CICYT, Spain (Grant No. AEN-96/1672) and by Junta de Andalucía, (Grant No. FQM-101), and by the European Union TMR Network EURODAPHNE (Contract No. ERBFMX-CT98-0169). Invited talk at “High Energy Euroconference on Quantum Chromodynamics (QCD ’99)”, 7-13 July 1999, Montpellier, France.
1 Introduction

Understanding the $\Delta I = 1/2$ rule for kaons within QCD has been a continuous challenge, see [1] for a review. Here, we report on recent work and advances at understanding this empirical rule in the chiral limit [2]. Due to the lack of space, we would like to concentrate on two main issues. The first one is the heavy $X_i$–bosons technique and the the matching procedure. The second one is the anatomy of the different contributions to the octet coupling enhancement. We also give results on the penguin operator $Q_6$ which are relevant for $\varepsilon'/\varepsilon$.

Within the Standard Model (SM), $K \to \pi\pi$ decay amplitudes can be decomposed into definite isospin 0 and 2 amplitudes as follows $|A| = -iT$,

$$A[K_S \to \pi^0\pi^0] \equiv \frac{2}{\sqrt{2}}A_0 - \frac{2}{\sqrt{3}}A_2,$$

$$A[K_S \to \pi^+\pi^-] \equiv \frac{2}{\sqrt{2}}A_0 + \frac{1}{\sqrt{3}}A_2,$$

$$A[K^+ \to \pi^+\pi^0] \equiv \frac{3}{2}A_2.$$ (1)

Where we have included the final state interaction phases $\delta_0$ and $\delta_2$ into the amplitudes $A_0$ and $A_2$ as follows

$$A_{0(2)} \equiv -ia_{0(2)}e^{i\delta_{0(2)}}.$$ (2)

Performing a fit to experimental data on $K \to \pi\pi$ and $K \to \pi\pi\pi$ up to Chiral Perturbation Theory (CHPT) order $p^4$, in ref. [3] obtained

$$\left| \frac{A_0}{A_2} \right|^{(2)} = 16.4$$ (3)

to lowest order. Unfortunately no fit uncertainties were quoted. This is the so–called $\Delta I = 1/2$ rule for kaons.

At $O(p^2)$ in CHPT, $|\Delta|S = 1$ amplitudes can be described in terms of three couplings in octet symmetry,

$$L_{\Delta S=1}^{(2)} = -\frac{3G_F}{5\sqrt{2}}V_{ud}V^*_{us}F_0^4 \left[ G_8 \langle u_\mu u^\mu \Delta_{32} \rangle 
+ G'_8 \langle \chi(+)\Delta_{32} \rangle 
+ G_{27} t^{ij,kl} \langle u_\mu \Delta_{ij} \rangle \langle u_\mu \Delta_{kl} \rangle \right] + h.c.$$ (4)

We have pulled out the Fermi coupling constant, $G_F$, and the relevant Cabibbo-Kobayashi-Maskawa matrix elements $V_{ij}$. $U \equiv uu \equiv e^{i\sqrt{2}\Phi/F_0}$ with $\Phi$ a SU(3) matrix collecting the lowest pseudo-scalar meson $\pi$, $K$, and $\eta_8$ fields; $F_0$ is the
chiral limit value of the pion decay constant $f_\pi \simeq 92.4$ MeV; $D_\mu U$ is the co-
variant derivative acting on $U$ and $u_\mu \equiv iu^\dagger(D_\mu U)u$; $\chi(+) \equiv u^\dagger \chi u^\dagger + u\chi^\dagger u$ with $\chi \equiv 2B_0\mathcal{M}$, $\mathcal{M}$ is a $3 \times 3$ matrix collecting the light quark masses and $B_0$ is propor-
tional to the quark condensate in the chiral limit, $B_0 \equiv -\langle 0|\bar{q}q|0 \rangle/F_0^2$. The symbols $\Delta_{ij}$ and 27-plet tensor $t^{ij,kl}$ take into account for the correct flavour combinations and were defined in [4]. At this order

$$|A_0|^{(2)} = \sqrt{2} \left( \frac{9G_8 + G_{27}}{10G_{27}} \right).$$

(5)

At leading order in $1/N_c$, $G_8 = G_{27} = 1$ and

$$|A_0|^{(2)} = \sqrt{2};$$

(6)

i.e. more than a factor ten lower that the experimental number!

2 The Heavy $X_i$-Bosons Method: Matching Short– and Long–Distances

We analyse $|\Delta S| = 1$ off–shell two-point Green functions

$$\Pi^{ij}(q^2) \equiv i \int d^4x e^{iqx} \langle 0 | T\{P^i(0)^\dagger P^j(x) e^{i\Delta S=1} \}|0 \rangle$$

(7)

in the presence of strong interactions. These Green functions were studied in CHPT to $O(p^2)$ in [4] and to $O(p^4)$ in [5]. $P^i(x)$ are external pseudo-scalar sources that couple to pion, kaon, and $\eta_8$ fields. The $\Delta S = 1$ Standard Model effective action at some scale $\mu$ below the charm quark mass, can be written as

$$\Gamma_{\Delta S=1} \equiv -\frac{G_F}{\sqrt{2}} V_{ud}V_{us}^* \sum_i C_i(\mu) \int d^4y Q_i(y)$$

(8)

with $C_i(\mu)$ Wilson coefficients which are known to two-loops and $Q_i(y)$ are four–quark local operators inducing $\Delta S = 1$ transitions. The list of relevant operators is given in [2].

The effective action $\Gamma_{\Delta S=1}$ is generated by virtual $W$–boson exchanges. This makes necessary the intervention of strong interactions at all scales between 0 and $\infty$ to calculate weak matrix elements. Matching long– and short–distances is the big challenge of calculating weak matrix elements. The procedure we propose is to use an effective field theory which reproduces the physics of the four–quark
The heavy $X_i$–boson exchange effective field theory, the basic non–leptonic interaction for physics below $\mu_L$ is given by

$$\sim g_1^+ g_1' \int \frac{d^4 r}{(2\pi)^4} \int e^{i q \cdot x} \frac{g_{\mu\nu}}{M_{X_1}^2} J_{1,\mu}^+(x) J_{1,\nu}'(0). \quad (10)$$

Now, we can calculate analogously to what one does for the $\gamma$–exchange contribution to $\pi^+–\pi^0$ or $K^+–K^0$ mass difference. We can separate the long– and short–distance pieces using an Euclidean cut–off $\mu$

$$\int d^4 r_E \rightarrow \int d\Omega \left[ \int^\mu_0 d|r_E| + \int^\infty_\mu d|r_E| \right]. \quad (11)$$

The ENJL model doesn’t confine and does have a wrong high energy behaviour at high energies. We smear out these bad features by calculating far off–shell with very small momenta and using only fits up to order five or six at most, see more details in [2]. There are good prospects to eliminate to a large extent the bad high energy behaviour and enlarge beyond 1 GeV the matching between short– and long–distances using the model in [8].
3 The $\Delta I = 1/2$ Rule

Here we give the main conclusions of our work. Penguin–like diagrams with $Q_2$ dominate the octet coupling $G_8$ (around 63 %) in the whole range of scales studied [between 0.5 GeV and 1. GeV] producing the observed huge enhancement. The penguin–operator $Q_6$ contribution to $G_8$ is around 12 %.

There is a large cancellation between the $B_K$–like diagrams contribution to $G_8$ from $Q_1$ and $Q_2$. The relatively large positive contribution from $Q_1$ is canceled by $B_K$–like diagrams from $Q_2$ to give in total less than 7 % of $G_8$ from $B_K$–like diagrams. Factorizable contributions plus $B_K$–like contributions are around 23 % of $G_8$. The sum of the rest of operators contributes by less than 5 % and decreases $G_8$ up to its final value. More than 75 % of the value of $G_8$ comes from penguin–like diagrams. We show in Figure 1 the matching obtained for the three $O(p^2)$ couplings and in Figure 2 the relative contributions of $Q_1$, $Q_2$, and $Q_6$ to $G_8$.

At the same time, there are no penguin–like contributions to $G_{27}$ and $B_K$–like diagrams for $Q_1$ and $Q_2$ decrease the 27–plet coupling from one to a value between one half and one third. There is a large cancellation between the contributions of $Q_1$ and $Q_2$. In summary, penguin–like diagrams with $Q_2$ dominate largely the enhancement of $G_8$ and $B_K$–like diagrams for $Q_1 + Q_2$ produce the small value of $G_{27}$. These two facts are responsible for the $\Delta I = 1/2$ rule in (3).

Experimentally

$$G_8 = 6.2 \pm 0.7; \quad G_{27} = 0.48 \pm 0.06.$$  \hspace{1cm} (12)

Here, we have only included the uncertainty from the value of the pion decay constant in the chiral limit $F_0 = (86 \pm 10)$ MeV, since no uncertainties from the
Figure 2: The contributions of $Q_1$, $Q_1 + Q_2$, $Q_1 + Q_2 + Q_6$ to $G_8$. The final result for $G_8$ using short–distance at two–loops with the scheme dependence removed [2, 3] is also shown.

The fit procedure were quoted in [3]. We get

$$4.3 < G_8 < 7.5; \quad 0.8 < G'_8 < 1.1;$$

$$0.25 < G_{27} < 0.40$$ \quad (13)

and

$$15 < \left| \frac{A_0}{A_2} \right|^2 < 40.$$ \quad (14)

This last result is somewhat large mainly because of the small value we get for $G_{27}$. Notice that this calculation is to next-to-leading in $1/N_c$ and to all orders in Chiral Perturbation Theory. One can expect, therefore non-negligible $1/N_c^2$ corrections typically of the order of (30~40) %, but the main $\Delta I = 1/2$ enhancement is there. We want to stress that these are parameter free predictions, the three input values we need were fixed in [8] from low energy phenomenology in the strong sector. We believe there are good prospects at obtaining predictions on $\Delta S = 1$ transitions and $\varepsilon'/\varepsilon$ [10].

4 The $Q_6$ Penguin Operator

In the chiral limit, the contribution of $Q_6$ to $G_8$ is proportional to the quark condensate squared. At leading order in $1/N_c$, the scale dependence of $\langle \pi \pi | Q_6 | K \rangle$ is exactly canceled by the Wilson coefficient $C_6(\mu)$ [12]. We have shown in [3] that the scale dependence is also canceled at next-to-leading in $1/N_c$ for the factorizable part. Then, as for the rest of $Q_1(y)$ operators, the matching between short- and long-distances becomes an affair of non-factorizable contributions.
Outside the chiral limit, the strange quark condensate squared does not factorize and most of its quark mass corrections produce actually the coupling $G'_8$ which does not contribute to $K \to \pi \pi$. Both $G_8$ and $G'_8$ are still proportional to the chiral limit value of the quark condensate squared, and kaon and pion masses enter in higher order CHPT corrections. Therefore, the contribution to $G_8$ from $Q_6$ cannot be proportional to $1/m_s^2$ and the usual parameterization being inversely proportional to the strange quark mass squared is very misleading. In addition, the VSA result of $\langle (\pi\pi)_0|Q_6|K\rangle$ has an IR divergence as shown in [2]. In view of all these problems, we propose to quote directly values of matrix elements as we did in [2]. We give in Table 1 the results for the contribution of $Q_6$ to $G_8$.

| Scale (GeV) | One-Loop | Two-loops (SI) |
|-------------|-----------|----------------|
| 0.5         | 0.98      | 2.14           |
| 0.6         | 0.73      | 1.49           |
| 0.7         | 0.53      | 1.10           |
| 0.8         | 0.38      | 0.83           |
| 0.9         | 0.26      | 0.62           |
| 1.0         | 0.15      | 0.45           |

Table 1: The contribution of $Q_6$ to $G_8$ using short-distance to one-loop and to two-loops with the scheme dependence removed see [2, 3].

However, for the sake of comparison with other results in the literature which only quote $B_6$-parameters, we give our results for

$$B_6^{(1/2)}(\mu) \equiv \frac{\langle (\pi\pi)_0|Q_6|K\rangle}{\langle (\pi\pi)_0|Q_6|K\rangle|_{N_c}} \langle (\pi\pi)_0|Q_6|K\rangle|_{N_c}$$

where

$$\langle (\pi\pi)_0|Q_6|K\rangle|_{N_c} = -i \frac{32 G_F}{\sqrt{2}} V_{ud}^* V_{us} C_6(\mu)$$

$$\times F_0(m_K^2 - m_\pi^2) \left\langle \bar{q}q \right| \frac{(0|\bar{q}q|0)^2(\mu)}{F_0^2} \right| L_5(\nu) .$$

$$\langle 0|\bar{q}q|0 \rangle$$ is the quark condensate in the chiral limit which in a very good approximation we take to be the average of up and down quark condensate [13]. With this definition, we avoid the IR divergence in the VSA value of $\langle (\pi\pi)_0|Q_6|K\rangle$ [2]. The large $N_c$ result [14] contains still the ambiguity in the value of the scale $\nu$ which is only canceled by the IR divergent part. We fix $\nu = M_\rho$ and use $L_5(M_\rho) = (1.4 \pm 0.3) 10^{-3}$. 

6
To lowest CHPT order $p^2$ and next-to-leading in $1/N_c$, we get

$$B_6^{(1/2)}(\mu) = 0.76 \pm 0.20$$

(17)

in agreement with [1]. To this order the scale dependence $\mu$ is canceled exactly due to the need of canceling the IR divergence. Notice also that the value of $B_6^{(1/2)}$ is very near to one is due to the large cancellation between the two types of factorizable contributions [2]. This cancellation is exact at order $p^0$ and very large at order $p^2$ due to the cancellation of the IR divergence. It does not however protect the value of $B_6^{(1/2)}$ from higher CHPT order corrections. In fact, also in the chiral limit but to all orders in CHPT and next-to-leading in $1/N_c$, we get

$$B_6^{(1/2)}(\mu) = 2.2 \pm 0.5$$

(18)

for scales $\mu \in [0.6,1.0]$ GeV. The scale dependence is very mild. The importance of higher order CHPT corrections is manifest in this result. A large enhancement of $B_6^{(1/2)}$ was also obtained in [11] when $O(p^4)$ corrections were included.

We found that the $G_8$ enhancement is not due to the contribution of the penguin operator $Q_6$. The only relation between the dynamics underlying the value of $\varepsilon'/\varepsilon$ in the SM and the large value of $G_8$ is the type of dominant diagrams, namely, penguin–like diagrams. But, $\varepsilon'/\varepsilon$ is dominated by the penguin operators $Q_6$ and $Q_8$ while $G_8$ by the $Q_2$ operator.

This work has been done in an enjoyable collaboration with Hans Bijnens. It is a pleasure to thank Stephan Narison for the invitation to this very interesting conference.

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