THE DEBATE OF GALAXY CORRELATIONS AND ITS THEORETICAL IMPLICATIONS

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We discuss the problem of galaxy correlations by considering the various methods by which this information can be obtained. We focus in particular on the volume limited three dimensional samples and discuss a new way to increase the scale of their statistical validity. From our previous results and the most recent ones on all the available catalogues we conclude that galaxy correlations show fractal properties with dimension $D \approx 2$ up to the present observational limits ($200 - 800h^{-1}Mpc$ depending on the catalogue) without any tendency towards homogenization. We also discuss the compatibility of this result with the galaxy counts as a function of magnitude and the angular catalogues. A new picture emerges which changes the standard ideas about the properties of the universe and requires a corresponding change in the theoretical concepts that one should use to describe it.

1. INTRODUCTION AND HISTORICAL PERSPECTIVE

The first galaxy catalogues were only angular, namely they defined the two angles corresponding to the galaxy positions in the sky (1). These angular distributions show structures at small scales but appear rather smooth at large angular scales. This situation was therefore fully satisfactory with respect to the theoretical expectations of a homogeneous universe (2). In the late seventies however the first redshift measurements became available and permitted the identification of the absolute distance of galaxies. In this way it was possible to obtain the complete three dimensional distribution of galaxies. These distributions showed a more irregular structure with respect to the angular data with the appearance of superclusters and large voids. At first these large structures were considered as accidental or due to experimental incompleteness. But more and more data showed that the structures are all over and the voids do not fill with better measurement. This three dimensional picture did not show any more a clear tendency towards homogenization and was in contrast with the angular data. This conflictual situation found an apparent solution with the first statistical analysis of the CfA1 galaxy catalogue (3). In fact this analysis identified a small correlation length of only $\approx 5h^{-1}Mpc$ in a catalogue
that showed structures of much larger sizes. The statistical analysis appeared therefore to provide a way out such that large structures can be compatible with small correlation lengths.

In the following years the situation evolved in a dramatic way because deeper and deeper surveys showed larger and larger structures that appeared difficult to reconcile with such a small correlation length. In addition the catalogues of clusters gave a correlation length of $\approx 25h^{-1}\text{Mpc}$, five times larger than the one of galaxies, even though the clusters are made themselves of galaxies. This situation led to a great confusion and different authors looked for different possible solutions to the problem. At this stage various hypotheses were formulated like the luminosity segregation, the clustering richness relation that leads to the biased theory of structure formation etc. In the end the most popular picture is that clusters and galaxy structures require different theories because their correlation show different amplitudes. A linear theory is appropriate for clusters while a non linear theory should be adopted for galaxies. The large scale structures can be compatible with small correlation lengths and with a large scale homogeneity because the amplitudes of the structures becomes smaller the larger is the structure. Finally a clear evidence of homogeneity cannot yet be obtained because the present samples are not yet fair.

In the past years we have challenged this picture (4) by showing that it arises just by a mathematical inconsistency in the characterization of the galaxy and cluster correlations. Our main result is that a correct analysis of the data shows fractal correlations up to the present observational limits. The galaxy-cluster mismatch disappears and the visible universe is characterized by a multifractal distribution of matter when the galaxy masses are also included. This requires a radical change of perspective for the properties of the universe and for the theoretical methods that one should use to describe it. In this lecture we present a colloquial discussion of these subjects including the most recent results (5-8).

2. GALAXY CORRELATIONS

The information about galaxy correlation is supposed to arise from a variety of facts:
- The Cosmological Principle
- The isotropy of the background radiation
- The properties of angular catalogues
- The galaxy number counts
- The N-body simulations
- The three dimensional galaxy catalogues

The information obtained by these different sources is often conflictual so it is important to establish a hierarchy of validity and strength between the different points mentioned above. For example, suppose that the three dimensional distribution of galaxies turns out to be not homogeneous, what shall we do? Throw away the telescopes or try to change the Cosmological Principle? And what about the background radiation? Should we disregard the inhomogeneous galaxies or the isotropic radiation? In order to clarify the situation it is useful to distinguish between conceptual and technical questions.

A. Conceptual Questions:

The Cosmological Principle corresponds to a reasonable requirement of democracy: we should try to avoid models that imply that we are in a very special point in the universe. Its interpretation in terms of isotropy and homogeneity for the whole
space is however too strong. For example a fractal structure is locally isotropic, in
the sense that all points (galaxies) have about the same environment, but it is not
homogeneous (4,9). This occurs because isotropy implies homogeneity only for a
regular (analytical) structure. A fractal is non-analytical everywhere so this relation
does not hold and we can have a coexistence of democracy with non homogeneity.
The asymmetry in such a structure is between occupied and empty points, but this
is a perfectly acceptable asymmetry.

The background radiation and the observation of galaxy positions are two dif-
ferent experimental facts. If these two observations appear as conflictual this means
that we do not have the correct theory to relate one to the other and not that we
should eliminate one of the two or manipulate the data so that the disagreement is
eliminated. In this respect it may be useful to try to separate clearly the bare exper-
imental facts from the theoretical results. Generic statements of ”consistency” in
which the theory is used directly in the data analysis have usually led to confusion.

The N-body simulations correspond to computer experiments in which the re-
sult is crucially dependent on the assumptions for the meaning of the initial and final
elements, the starting situation and the dynamical evolution. In order to appreciate
the subtleties and possible complications of this type of simulations it is useful to
mention the present state of the art in fully developed turbulence. Most authors
believe that this phenomenon is certainly contained into Navier-Stokes equations,
so in some sense one knows the correct theory. However even the most powerful
simulations that have been performed with these equations do not seem to prop-
erly reproduce the phenomenon. This is because many different length scales are
involved in the energy transfer and dissipation and even the most powerful comput-
ers do not seem to be able to fully describe this phenomenon. In this respect one
should be extremely careful in the interpretation of these simulations with respect
to the physical reality.

**B. Technical Questions**

Clearly the most direct information about galaxy correlations comes from three
dimensional volume-limited samples. The limited amount of data may pose a prob-
lem of statistical validity. However, within the limits of their statistical validity,
these samples represent directly the real properties of galaxy correlations while all
the other measurements can only lead to consistency arguments. In case of dis-
agreement the $3-d$ catalogues should be used as a test for the other measurements.
The $3-d$ volume limited catalogues have usually been analyzed in terms of the
so called two point correlation function $\xi(r)$ (10). The characteristic correlation
length is defined by the distance at which this function equals unity ($\xi(r_0) = 1$).
In the past we have discussed in detail the inconsistency of this approach (4) and
the fact that the characteristic lengths obtained represented just a fraction of the
sample volume and not the real correlation length. A new analysis of the data with
more general methods leads in fact to the result that galaxy and cluster catalogues
show fractal correlations up to their limits without any evidence for homogeniza-
tion (4). The statistical limits of these analyses correspond roughly to the radius of
the largest sphere that can be contained in the sample. For galaxies this is about
20–30 $h^{-1} \text{Mpc}$ and for clusters about 80 $h^{-1} \text{Mpc}$. These studies already allowed us
to show that the claimed correlation lengths of 5 and 25 $h^{-1} \text{Mpc}$ for galaxies and
clusters respectively were spurious results due to the inconsistent analysis. This situation eliminates the apparent inconsistency between galaxy and cluster correlations and made the correlation analysis compatible with the observation of large scale structures.

Some authors tried to push the statistical limits of these catalogues beyond the above limits by using various types of weighting schemes (11). In our opinion this is a very risky procedure because, in one way or another, all the weighting schemes imply some homogeneity hypothesis. If this would not be so, then from the observation of a few galaxies, one would be able to reconstruct the properties of the entire universe.

In order to push the limiting length of statistical validity for the 3-d samples we have followed instead a different path that again does not involve any a priori assumption. The basic idea is that if one has a large volume limited sample and one integrates from the observation point the galaxies at progressively larger distances one should observe a power law with exponent 3 for the homogeneous case or a lower exponent for a fractal distribution. Clearly this is what is done from each galaxy in the complete correlation studies. However in that case one uses spherical shells for the integration and from this comes the previous limitation. The integration from the origin uses instead the conic sample that can be properly defined only from this point. In this way the statistical fluctuations, especially at small scale, are very large but the advantage is that, in the favourable situations, the effective depth can be extended by more than a factor of four.

We have performed this type of analysis in the Perseus-Pisces survey, in CfA1 (just as a test) and in the ESP survey. We report below a comprehensive summary of our correlation studies of both types for the various catalogue:

- CfA1 and CfA2. For CfA1 the analysis are limited to $20 h^{-1} Mpc$ and show fractal properties up to this limit. This trend continues in CfA2 in view of the shift of $r_0$ with depth and on the properties of its power spectrum (4,7). Therefore both CfA1 and 2 show fractal correlations up to their limits. The fractal dimension is somewhat smaller at small scales and it becomes about 2 for CfA2.

- Perseus-Pisces. For the correlation function the limit is $30 h^{-1} Mpc$ while for the counting from the origin one can extend this limit up to $130 h^{-1} Mpc$. Both data are consistent with a fractal distribution with dimension $D = 2.$ (6).

- LEDA database. Correlation analysis up to $150 h^{-1} Mpc$. Fractal with $D = 2$. Scaling of $r_0$ clearly defined up to $60 h^{-1} Mpc$ (8).

- ESP survey (12)(preliminary results). Correlation analysis is impossible because of the too small solid angle. Integral from the vertex shows fractal properties up to $800 h^{-1} Mpc$ (5).

A summary of the behavior of the conditional density as a function of distance for the various catalogues is shown in Fig.1, with the exception of the ESP data that are still preliminary.

FIGURE 1. $\Gamma(r)$ for various catalogues. The amplitude in the various cases is not arbitrary, and it is normalized only to take into account the different luminosity distribution in different volume limited samples (6),(8)
The result is that in all cases galaxy correlation are well defined and extend up to the observational limits without any tendency towards homogenization. As a byproduct of these studies we can also make the following comments:

- The so-called luminosity segregation hypothesis can be definitely eliminated. The shifts of $r_0$ are clearly due to the sample depth as predicted for a fractal distribution.

- The use of weighting schemes should be avoided because it is now clear that they were responsible for the apparent trends towards homogenization \((11)\) that have been disproved by the new results of galaxy counts for volume limited samples.

Another way to gain information about the galaxy distribution without knowing the redshifts is the number count as a function of apparent magnitude and the corresponding angular catalogues. In these cases however the information is not direct and it requires a nontrivial interpretation. The galaxy count as a function of magnitude represents one of the very first elements that have been studied \((13)\) and gave rise to extensive problems that are still unclear today\((2,5,14)\). The point is that one can easily show that the exponent which characterizes this counting should be $\alpha = D/5$. This implies $\alpha = 0.6$ for a homogeneous distribution and a smaller value for a fractal one. The observation show the very puzzling behavior that $\alpha = 0.6$ for relatively small scales while for larger scales one has $\alpha = 0.4$ up to the limits. Eventually one would have expected just the opposite situation, namely fractal behavior at relatively small scales followed by homogeneity at very large scales. At the moment the traditional interpretation of these data is that the value $0.6$ corresponds to the much hoped homogeneity, while the smaller value at large scales is affected by evolutionary effects invoked for the occasion.

We now know for sure that this interpretation cannot be correct. In fact it is clear beyond any doubt \((from the 3 − d volume limited catalogues)\) that fractal correlations extend up to $\approx 100 h^{-1} Mpc$ and probably much more. At these relatively small scales evolutionary effects are certainly negligible. So we have the situation that a fractal distribution with dimension $D = 2$ leads to a counting vs. magnitude behavior that appears instead to correspond to $D = 3$. We have studied this effect in detail by considering both real catalogues and computer simulations with preassigned properties and our conclusion is that these counting are strongly affected by finite size effects \((15)\). Namely at relatively small scales one finds almost no galaxies because the total number is rather small. Then one enters in a regime dominated by finite size fluctuation effects. Finally the correct scaling behavior of the distribution is recovered. This means for example that if one has a fractal distribution, there will be first a raise of the conditional density, due to finite size effects because no galaxies are present before a certain characteristic length. Once one enters in the correct scaling regime the density will begin to decay with the correct power law as corresponding to the fractal correlations. So in this intermediate regime of raise and fall the will be a region in which the density can be roughly approximated by a constant value. This region will lead to an apparent dimension $D = 3$, which however is not real but just due to the finite size effects. We have checked that this is exactly what happens in the real counting in which the initial exponent $\alpha = 0.6$ corresponds to the finite size effects and the subsequent value $\alpha = 0.4$ is the real value, now perfectly consistent with the observed fractal dimension $D = 2$.

This counting problem is present as well in the calculation of the amplitude of the angular correlation function. This quantity also correspond to the integration
from a single point and therefore is also strongly affected by these finite size fluctuations. Therefore the scaling argument at the basis of the presumed homogeneity inferred from the angular catalogues corresponds to these finite size effects and not to the real scaling properties of the distribution. We hope this discussion clarifies now completely the statistical properties of galaxy and clusters and will permit to focus on the real theoretical problems.

3. AMPLITUDES VERSUS EXPONENTS

From what we have seen all we can say about the galaxy distribution is that it is a fractal up to the present observational limits. This means that it is not possible to define concepts like the average density of galaxies. Therefore also the amplitude of the correlation function has no physical meaning because one cannot define what is small and what is big. In the same way it is not possible to assign any meaning to the relative density fluctuation $\delta N/N$ because the normalization value $N$ is not intrinsic. Clearly $\delta N/N$ goes to zero at the sample limits also for a fractal structure, as shown in Fig. 2.

![Figure 2](image_url)

FIGURE 2. Behavior of $\delta N/N$ as a function of size $r$ in a portion of a fractal structure for various depths of the sample $R_s = 100, 200, 300 h^{-1} Mpc$. The average density is computed in the whole sample of radius $R_s$. The fact that $\delta N/N$ tends to zero does not mean that the fluctuations are small and an homogenous distribution has been reached. The distance at which $\delta N/N = 1$ scales with sample depth and has not physical meaning.

but this does not mean that the system is becoming homogeneous. This situation may appear strange but it is indeed quite common in various fields of physics where one deals with scale-invariant or self-similar structures (4). In these cases the relevant physical phenomenon that leads to the scale-invariant structures is characterized by the exponent and not by the amplitude. Correspondingly one has to change the theoretical framework into one that it capable of dealing with non analytical fluctuations. This means going from differential equations to something like the Renormalization Group for the study of the exponents.

If a crossover towards homogeneity would eventually be detected, this would not change the above discussion but it would simply introduce a crossover into it. The fractal nature of the observed structures would, in any case require this change of theoretical perspective.

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