Modeling for Time-variant Reliability of Mechanism

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Abstract. A novel time-variant reliability model is developed after studying the traditional dynamic performance of mechanism. The expression of reliability is given based on stochastic process. A two-dimensional description of the error generating process is established by studying mechanism output kinematic accuracy. The reliability of mechanism output kinematic accuracy can be obtained according to this description. The model proposed can be used to calculate the reliability of mechanism directly, if the distribution of mechanism output kinematic accuracy is known.

Introduction

A mechanism is able to work continually if it is operating below the static and dynamic strength. However, it is already in failure state if certain factors which influence its motion can not meet the predetermined performance levels. The key issue of such problems is kinematic accuracy of mechanism. Making kinematic accuracy of mechanism to meet the stipulated requirements, mechanism must follow a particular law of motion to pass the movement in the composition of components. Therefore, kinematic accuracy, timeliness, continuity and coordination are important conditions for the system \cite{1}. If one motion fails, it will inevitably lead to system failure.

There exit many factors which impact kinematic accuracy, including the power source, design errors, manufacturing errors, assembly errors and behavior errors, friction and wear, dead spots, corrosion and lubrication. Such operating environment and parameters mostly change with time, so the dynamic performance of a mechanism changes with running-time \cite{2, 3}.

Traditionally, the institutional model of dynamic performance reliability is carried out under the following basic assumptions: Firstly, The mechanism has sufficient stiffness and precision, which means each component of the elastic deformation and co-location of components on the output gap would be negligible impact; Secondly, the size of the movements processing error is random variables fitting for the normal distribution; Besides, the output error of the component is random variables fitting for the normal distribution. However, by such calculation, dynamic performance reliability is only valid under the basic assumptions, and as one result can only calculate the reliability at a time, rather than reflects reliability over time \cite{4-7}.

In this paper, the displacement of the mechanism is used as the object of the analysis and calculation of reliability. Modeling theory and method of time-variant reliability is researched by using random process and the stress-strength interference model. According to the integral operator by using the stress-strength interference model, the reliability calculation model of mechanism output kinematic accuracy is given. Meanwhile, the variation, which reflects the reliability of mechanism output kinematic accuracy, will be investigated over time.
The description for the uncertainty of mechanism output kinematic accuracy

Mechanism output kinematic accuracy includes displacement error, velocity error, acceleration error, and so on. For the mechanism under the same loading the condition, the output kinematic accuracy is still uncertain, and the uncertainty may exist in the power source, friction and wear, corrosion, lubrication and other circumstances. The motion process of the mechanism can be used to describe the random process. In the whole of the mechanism’s motion process, the output kinematic accuracy tested for each time is bound to contain two features: first is the time when the output error is tested; second is the size of the error. So it is that the mechanism has a two-dimensional nature of the output error. Therefore, only the probability density function to characterize the output error method does not accurately reflect the characteristics of time. Below, as the case of displacement error of the unit target amount, the method of two-dimensional description for the uncertainty output kinematic displacement accuracy is created by using the Poisson stochastic process and probability density function, as is shown in Fig.1. Specifically, in the time dimension (i.e., the horizontal axis direction), the displacement error of the output variation over time is described with Poisson stochastic process. In the range dimension (i.e., the vertical axis direction), the size of the displacement error is described with probability density function $f_{error}$.

![Fig.1 Two-dimensional description of mechanism output kinematic displacement accuracy](image1)

![Fig.2 Displacement accuracy distribution simple measured using the Poisson stochastic process](image2)

Fig. 2 Displacement accuracy distribution simple measured using the Poisson stochastic process

Fig. 2 shows a normal distribution of the displacement accuracy magnitude. The mean value is 0mm and the standard deviation is $0.64 \times 10^{-3}$mm. The relationship between the frequency of the measurement and time obeys Poisson stochastic process ($\lambda(t) = 0.5m/s^{-1}$). The formation of the displacement accuracy of the course can be simulated with the Monte Carlo method.

The model of time-variant reliability with the same permissible error

When the process of error is described as Poisson stochastic process, $N(t)$, which is the number of measurements, obeys the Poisson stochastic process, $\lambda(t)$. The probability of measurement times of displacement accuracy at the time of $t$ can be expressed as:
Below is an example of building a time-variant reliability model of displacement in the case of displacement accuracy values exceeding the allowable value.

Time-variant reliability model of mechanism can be inferred by parts, the cumulative distribution function of displacement error is \( F_s(error) \), probability density function is \( f_s(error) \), when measurement time is \( w \), the reliability is:

\[
R(w) = \int_{error_{\min}}^{error_{\max}} w(F_s(error))^{w-1} f_s(error) d(error),
\]

where \( error_{\max} \) and \( error_{\min} \) are the upper and lower limits of the displacement accuracy of the mechanism.

Combining equation (1) and equation (2) at the moment of \( t \), the reliability of the mechanism can be expressed as:

\[
R(t) = \sum_{n=0}^{+\infty} P(N(t) - N(0) = w)R(w) = \sum_{n=0}^{+\infty} \left[ \int_{0}^{t} \lambda(t)d(t) \right]^{w} w! e^{-\int_{0}^{t} \lambda(t)d(t)} \cdot \int_{error_{\min}}^{error_{\max}} w(F_s(error))^{w-1} f_s(error) d(error).
\]

Using the Taylor expansion, equation (3) can be simplified as:

\[
R(t) = e^{(F_s(error) -1) \int_{0}^{t} \lambda(t)d(t)} \mid_{error_{\min}}^{error_{\max}}.
\]

**Example**

The reliability of dynamic performance will be calculated with displacement accuracy which has the same target. Variation of reliability over time will also be investigated. Between the permitted limits of error [-0.0025mm, 0.0025mm], the frequency and time of measurement obey Poisson random process, where \( \lambda(t) = 0.5 ms^{-1} \). The displacement error obeys normal distribution, with the mean value of 0mm and the standard deviation of 0.64×10^{-3}mm. The change of reliability of dynamic performance with time in the first 10ms and then, respectively, is shown in Figs. 3 and 4.
As can be seen in the Fig. 3 and Fig. 4, with the same allowable error, the reliability of the dynamic performance of the mechanism will increase rapidly with time, and then decrease slowly. The research shows the initial state of not full lubrication and other factors cause this phenomenon. The performance of institutions achieves a steady state after running, and the mechanism has the highest reliability meanwhile. However, the reliability of the mechanism will fall slows after it reaches the maximum, due to friction, wear and other reasons.

**Conclusions**

1) This paper investigates the distribution of the output of performance parameters, and establishes a two-dimensional description of the error generating process.

2) A reliability model of output parameters of dynamic performance is established based on stochastic processes. It can be applied to calculate the reliability of the mechanism at any time.

3) Calculations show that the reliability of this mechanism reaches the maximum (0.9995) at 20 ms, and then decreases slowly to 0.9976 at 100 ms.

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