On the compatibility of a proposed explanation of the Pioneer anomaly with the cartography of the solar system

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Abstract

We analyze here the reasons why an explanation of the Pioneer anomaly proposed by the authors is fully compatible with the cartography of the solar system. First, this proposal posits that the phenomenon is an apparent acceleration, not a real one, caused by a progressive desynchronization of the astronomical and the atomic clock-times, after they had been synchronized at a previous instant. The desynchronization could be caused by a coupling between the background gravitation and the quantum vacuum. Therefore, the standard argument for the incompatibility of the Pioneer acceleration and the values of the planets’ orbits radii cannot be applied. Second, this proposal gives exactly the same results for radar ranging observations as standard physics. Hence, it cannot be in conflict with the very precise cartography of the solar system determined by NASA’s Viking mission. Otherwise stated, while this proposal predicts apparent changes in the velocities of the spaceships and in the frequencies of Doppler observations, as really observed, it does not affect the values of the distances in any way whatsoever. Note that an acceleration between the astronomical and the atomic clock-times (i.e. a progressive desynchronization) can not be discarded a priori as long as we will lack a theory of quantum gravity.

1 Introduction: the Pioneer anomaly

The purpose of this note is to analyze the reasons why an explanation of the Pioneer anomaly proposed by the authors is fully compatible with the cartography of the solar system [1, 2]. In fact it gives exactly the same values as NASA’s Viking mission for the distances from any planet to the others and to the sun. This is important since the failure to find a model with this compatibility has been a major difficulty to explain this intriguing phenomenon, reported in 1998 by Anderson et al., although they had been studying it since 1980 [3, 4]. It consists in an adiabatic frequency blue drift of the two-way radio signals from the Pioneer 10 and 11 (launched in 1972 and 1973), manifest in a residual Doppler shift that increases linearly with time as

$$\frac{\dot{\nu}}{\nu} = 2a_t, \text{ or } \nu = \nu_0 [1 + 2a_t(t - t_0)],$$

where $t_0$ is an initial time and $2a_t = (5.82 \pm 0.88) \times 10^{-18} \text{ s}^{-1} \simeq (2.53 \pm 0.38)H_0$, $H_0$ being the Hubble constant (overdot means time derivative). It
must be stressed that the signal found by Anderson et al. is very clean and well-defined. Because of its linearity in time, the residual frequency $\nu$ of the ship appears as a straight line in a plot $(\nu, t)$ [3]. This strongly suggests that the anomaly is the signature of a new phenomenon, probably with a cosmological origin, so that the curve $\nu(t)$ can be accurately approximated by its tangent for a scale of just a few decades.

The simplest interpretation of eqs. (1) is that the ship was submitted to a constant force directed towards the sun. However this would be in conflict with the cartography of the solar system and with the equivalence principle. In view of this difficulty, the discoverers held the view that “the most likely cause of the effect is an unknown systematics”. This systematics was not found, however, in spite of several different analysis of the data. More recently, some researchers put their hopes in the so called thermal model, which assumes that the effect is due to non isotropic radiation by the spaceship. But it is difficult to imagine that such radiation could give a simple and neat straight line in the diagram $(\nu, t)$. In fact, the phenomenon is still unexplained many years after its discovery.

2 Summary of our proposal

We showed in reference [2] that the analysis of a coupling between the quantum vacuum and the background gravitation presented in [1] gives a solution to the anomaly, which is free of internal contradictions and does not conflict with any established physical law or principle. The reader is referred to [1] for the details. In this section, we give just a terse summary of our proposal as follows: i) a coupling between the quantum vacuum and the background gravity that pervades the universe is unavoidable because of the long range and universality of the gravity; ii) the fourth Heisenberg relation implies then that this coupling must cause a progressive desynchronization of the astronomical and the atomic clocks, after they had been synchronized at any arbitrary previous instant, in such a way that the former decelerate adiabatically with respect to the later; iii) since gravitational theories use astronomical time, say $t_{\text{astr}}$, and observers use atomic time, say $t_{\text{atom}}$ (they are using devices based on quantum physics), this desynchronization necessarily causes a discrepancy between theory and observation. The consequence is that the observed velocity of the spaceship is smaller than the predicted one, in such a way that the Pioneer seems to lag behind its expected position. In
our proposal, therefore, the anomaly is a cosmological effect.

We must underscore that the possibility that the two times are accelerating with respect to one another cannot be discarded a priori, as long as we will lack a unified theory of gravitation and quantum physics. Also that although the best election for $t_{\text{astr}}$ is probably the ephemeris time with relativistic corrections, any other time based on the motion of celestial bodies could be used instead.

Our model accepts the following phenomenological hypothesis: the empty space can be considered as a substratum, a transparent optical medium, characterized by a permittivity and a permeability due to the sea of virtual pairs that are continuously created and destroyed. As a consequence, therefore, of the coupling between the quantum vacuum and the background gravity, the existence must be admitted of some kind of adiabatic progressive modification of the structure of the quantum vacuum in the expanding universe. Let $\Psi(t)$ be the background gravitational potential that pervades the universe, in the approximation that all the mass-energy is uniformly distributed. Assuming that $\Psi = 0$, it follows from the fourth Heisenberg relation that the average lifetime of a pair with energy $E$ can be taken to be $\tau_0 = \hbar/E$. On the other hand, if $\Psi \neq 0$, the pairs acquire an extra energy $E\Psi$ so that their lifetime and number density must depend on the potential as

$$
\tau_\Psi = \hbar/(E + E\Psi) = \tau_0/(1 + \Psi); \quad N_\Psi = N_0/(1 + \Psi)
$$

(2)

(see [1], section 4.) As is seen, the fourth Heisenberg relation implies that the gravitational potential affects the density of the quantum vacuum in such a way that the more negative is the potential, the larger is the number density of pairs. As a consequence the optical properties of empty space must depend on the potential, i.e. on time, including its permittivity and permeability. As shown in [1], it follows that the astronomical time $t_{\text{astr}}$ and the atomic time $t_{\text{atom}}$, which are equal in the absence of gravity, accelerate with respect to one another, so that

$$
dt_{\text{atom}}/dt_{\text{astr}} = u(t_{\text{astr}}) \neq 0,
$$

(3)

$u$ being a function of time which we call the “march” of $t_{\text{atom}}$ with respect to $t_{\text{astr}}$. This variation must be very small, otherwise it would have been detected before, so that $u$ is very close to one. At first order in the variation of time, this march can be expressed as

$$
u = 1 + a(t_{\text{astr}} - t_{\text{astr},0}), \quad a = \frac{d^2 t_{\text{atom}}}{dt_{\text{astr}}^2},
$$

(4)
where $a$ is a positive inverse time depending on the potential that corresponds to $2a_t$ in [3] and $t_{\text{astr},0}$ is an initial instant at which the two times are synchronized. Since $a$ must be very small, the phenomenon is adiabatic. Note that, at first order, it is not necessary to specify the kind of time in the RHS of the first equation (4). As a consequence, the speed of light is constant if defined or measured with atomic time but increases with $t_{\text{astr}}$. In fact, since $v_{\text{astr}} = \frac{d\ell}{dt_{\text{astr}}}$ and $v_{\text{atom}} = \frac{d\ell}{dt_{\text{atom}}}$,

$$v_{\text{astr}} = uv_{\text{atom}}; \quad c_{\text{astr}} = uc. \quad (5)$$

Because $u > 0$ for $t > t_0$, it happens that $v_{\text{atom}} < v_{\text{astr}}$ for $t > t_0$, so that the atomic velocity is smaller than the astronomical one, this explaining why the ship seems to lag behind its expected position.

A comment is suitable here: Our proposal might seem strange at first sight but note that the effect remains unexplained more than 25 years after its discovery, so that it would seem stranger yet that, once its solution is found, it didn’t seem strange.

### 3 The problem of the compatibility with the cartography of the solar system

From the very beginning it was clear that the phenomenon poses some difficult problems. First of all and since a similar effect was not observed in the planets, it seemed that it could affect a small body as the Pioneer but had no action on more massive ones, what is incompatible with the equivalence principle, a cornerstone of gravitation theories. Second, it was thought that the Pioneer acceleration $a_P$ should be due to an attractive force towards the Sun, so that the radii of the planets’s orbits should be smaller than their well known values. Third, any explanation of the effect should face a difficult proof: to agree with the extremely precise determination of the solar system cartography achieved by Viking’s mission.

Let $a_P$ be a real acceleration. In that case, the radial equation of a planet should be

$$\ddot{r} = -\frac{GM}{r^2} + \frac{J^2}{r^3} - a_P, \quad (6)$$

with standard notation [5]. In the limit case of circular orbits the RHS vanishes so the radius changes to $r + \Delta r$ with

$$\Delta r = -r a_P/a_N, \quad (7)$$
at first order in $a_P$, where $a_N$ is the Newtonian acceleration. For non circular orbits, one has $\Delta r = -J^6 a_P/(GM)^4$. This is about $-21$ km and $-76$ km for Earth and Mars, respectively, so that the radii of the orbits of these two planets should be 21 km and 76 km smaller than their accepted values. However, this effect is not observed. More precisely, Anderson et al. conclude than any unmodelled radial acceleration acting on Earth and Mars and larger than $0.1 \times 10^{-10}$ m/s$^2$ is unacceptable. Indeed, it would be in conflict with NASA's Viking data which determine the difference and the sum of the Earth and Mars orbital radii to about 100 m and 150 m accuracy, respectively, as the discoverers of the anomaly underline in their first paper [3]. Note, however, that this irreproachable argument assumes as a necessary condition that the Pioneer effect is an anomalous but real acceleration, i.e. an anomalous but real force ($\Delta r$ is calculated by adding $a_P$ to the planets’ radial equation of motion). Otherwise, the phenomenon would not include necessarily a negative correction to the radii of the orbits. In any case, this caused somehow a misunderstanding that probably has been hindering the solution of the riddle: that it follows plainly from the Pioneer data that the effect should necessarily imply a reduction of the planets’ radii. This was loosely interpreted sometimes as an indication that no mechanical model can explain the anomaly and fostered the search for non mechanical alternatives.

4 Why our proposal is fully compatible with the cartography of the solar system

The previous argument on the decrease of the radii of the planets’ orbits cannot be applied to our work because what we affirm is that the Pioneer anomaly has the same observational fingerprint, and could well be the same thing, as the effect of the desynchronization of the astronomical and the atomic clock-times. In other words: we propose that the Pioneer anomaly effect is an apparent, non real, acceleration, not caused by any force. In our view, the Pioneer did not suffer any extra acceleration but followed faithfully the equations of standard gravitation theories. These theories use astronomical clock-time and predict that the ship’s trajectory must be given by a certain function parameterized by time $r = r(t_{astr})$, which is the one followed by the Pioneer. However, as the observers use devices based on quantum physics, they use quantum clock-time $t_{atom}$, so that what they observe is the same trajectory,
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although parameterized by a different time and given by a different function $r' = r'(t_{\text{atom}})$. The two are related as $r'(t_{\text{atom}}) = r(t_{\text{astr}})$. If the observers are unaware of the difference between the two times, they would interpret their data as the fingerprint of an unmodelled anomalous acceleration, i.e. they would see an anomaly.

In fact, if the observers are unaware that there are two times which accelerate with respect to one another and, consequently, use only one variable $t$, there must be necessarily an anomaly because of the difference between the observed and the calculated distances from the ship to the sun $\Delta r = r_{\text{obs}} - r_{\text{theor}} = r'(t) - r(t) \neq 0$. Then as (i) $t_{\text{atom}} > t_{\text{astr}}$, after the synchronization of $t_{\text{atom}}$ and $t_{\text{astr}}$ at a previous time $t_0$, and (ii) both $|r|$ and $|r'|$ are increasing functions of time since the ship is receding from the sun, it happens that

$$|r'(t_{\text{atom}})| = |r(t_{\text{astr}})| < |r(t_{\text{atom}})|. \quad (8)$$

Since the observers accept standard gravitation and use atomic time, they expected to measure $|r(t_{\text{atom}})|$, i.e. the theoretical prediction for the distance to the sun expressed in terms of $t_{\text{atom}}$, but obtained instead $|r'(t_{\text{atom}})|$. Otherwise stated: the observed distance of the ship to the sun is always shorter than the distance calculated with standard gravitation theories, see [2], pp. 10-12 and figure 1. In our view the anomaly boils down to that. There is no real acceleration, just an apparent one. This argument shows also that there is no problem with the equivalence principle in our model.

Moreover, it is easy to see that our model can have no conflict with the very precise cartography of the solar system obtained by NASA’s Viking mission. In a radar ranging experiment, the observers send a light ray from an initial point $P_1$ at time $t_{\text{atom},1}$ and detect its arrival to $P_2$, which can be the same one as $P_1$, at time $t_{\text{atom},2}$. The distance traveled by the light ray is simply $d = c(t_{\text{atom},2} - t_{\text{atom},1}) = \int_{t_{\text{atom},1}}^{t_{\text{atom},2}} v_{\text{atom}} \, dt_{\text{atom}}$. It turns out then that, in our model, the value of this distance does not depend on which time is used. In fact, changing the time variable and since $v_{\text{atom}} = v_{\text{astr}}/u$ and $dt_{\text{atom}} = u \, dt_{\text{astr}}$ (eqs. (3) and (5)),

$$d = \int_{t_{\text{atom},1}}^{t_{\text{atom},2}} c_{\text{atom}} \, dt_{\text{atom}} = \int_{t_{\text{astr},1}}^{t_{\text{astr},2}} c_{\text{astr}} \, dt_{\text{astr}}. \quad (9)$$

This shows that the predictions of our model on the cartography of the solar system are exactly the same as in standard physics, independently of which time is used. The model is thus fully compatible with the results obtained by
the Viking mission because, although it predicts changes in the velocities of
the spaceships and the frequencies in Doppler observations, it does not affect
the distances in any way whatsoever.

To summarize, our proposal (i) is free from internal contradictions; (ii)
is not in conflict with any established physical law or principle, at least
as long as we lack a working unified theory of gravitation and quantum
physics; and (iii) does not affect the accepted cartography of the solar system.
Consequently it does offer a solution of the Pioneer anomaly and must be
further investigated, therefore, as a candidate for the solution.

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