Modeling and Maneuverability Simulation for Vertical Plane of Autonomous Underwater Vehicle in Current

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Abstract. As the large diving depth Autonomous underwater vehicle (AUV) developed, modeling and maneuverability simulation for AUV in vertical plane under current disturbance are increasingly important. The REMUS model is adopted to build the mathematical model of AUV. The observation data of the current in South China Sea are analysed to build the current model varied with depth, then the relative motion method is adopted to build the model of AUV in current. The simulation system of maneuverability is established with MFC and some simulation experiments including without current and in current are carried out. The results show that the motion of AUV can be well simulated and some suggestions are given for vertical plane control of AUV.

1. Introduction
The large diving depth Autonomous underwater vehicle (AUV), especially the full sea depth AUV, is currently a hot research field. The maneuverability of AUV in vertical plane is the main factor affecting the efficiency of the diving.

The accurate mathematical model is the basis of the maneuverability analysis. The precise dynamic model is convenient for the designer to grasp the motion characteristics of AUV. It also can help understand the law for movement and maneuverability of AUV and improve the stability and robustness of the controller. The models currently used include DTNSRDC (1979) standard submarine equations of motion \cite{1} and REMUS model \cite{2}.

The research for maneuverability of AUV in vertical plane concentrates on the mobility without current. Hao discussed the effect of the longitudinal distance between barycenter and buoyant centre and the buoyancy to the maneuverability of the vertical plane based on the analysis of the motion equation of the vertical plane and the response equation of the pitch angle \cite{3}. Liu et al carried out the unpowered diving prediction for "CR-02" AUV, and the accuracy is verified by sea trial \cite{4}.

In this paper, the maneuverability of AUV in the current varied with depth is studied.
2. Modelling of the AUV in vertical plane
The research object of this paper is a underactuated AUV with rudders developed by Harbin Engineering University, China. The AUV belongs to a propeller-rudder-driven AUV. The dimensions and specifications of AUV are shown in table 1.

| Table 1. Dimensions and specifications of AUV |
|-----------------------------------------------|
| Length $L$ | 5600mm | Diameter of the hull $D$ | 1000mm |
| Weight $m$ | 2150kg | Maximum speed | 5kn |
| Cruising speed | 3kn |
| Gravity center $[x_g \ y_g \ z_g]'=[0 \ 0 \ 0]'$ |
| Buoyancy center $[x_b \ y_b \ z_b]'=[-0.094 \ 0 \ -0.0223]'$ |
| Inertia moment $I_{xx}=5.42 \times 10^3 \text{kg} \cdot \text{m}^2, I_{yy}=7.58 \times 10^3 \text{kg} \cdot \text{m}^2, I_{zz}=7.62 \times 10^3 \text{kg} \cdot \text{m}^2$ |

2.1. Coordinate system
The inertial coordinate system $\{I\}$ is established with the Earth defined as the origin, and the body-fixed coordinate system $\{B\}$ with origin chosen to coincide with the centre of mass of the AUV, as shown in figure 1.

![Figure 1. Coordinate system of the AUV in vertical plane](image)

The AUV is equipped with a thruster and a pair of horizontal rudder. They help in producing the vertical plane control input for the three degrees of freedom including surge, heave and pitch.

2.2. Kinematics model
The coordinate transformation between inertial coordinate system and body-fixed coordinate system can be expressed as:

$$
\begin{bmatrix}
\dot{x}_G \\
\dot{y}_G \\
\dot{z}_G \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & \sin \theta & 0 \\
-\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
u \\
w \\
q
\end{bmatrix}
$$

(1)

Where, $\dot{x}_G, \dot{y}_G$ and $\dot{z}_G$ and $\dot{\theta}$ are the barycentre coordinates and pitch angle in inertial coordinate system, $u$, $w$ and $q$ are the linear velocity and angular velocity in body-fixed coordinate system.

2.3. Dynamics model
Based on the rigid momentum theorem, the nonlinear equations of motion for the vertical plane of AUV in matrix form can be expressed as follows [5]:

2
the force $X, Z$ and $M$ for the degrees of freedom surge, heave and pitch can be decomposed according to the source of force, including the forces and moments of hydrostatics, hydrodynamics, thruster and rudders in each direction. So they can be expressed as:

$$
\begin{align*}
\sum X &= X_{wq}[u] + (X_{wq} - m)wq + (X_{wq} + m)q^2 + X_{prop} - (W - B) \sin \theta + X_{rudder} \\
\sum Z &= Z_{wq}[w] + Z_{q}[q] + (Z_{wq} + m)uq + Z_{wu}uW + mz_q q^2 + (W - B) \cos \theta + Z_{rudder} \\
\sum M &= M_{wq}[w] + M_{q}[q] + (M_{wq} - mx_q)uq + mz_q wq + M_{wu}uW - (z_g W - z_g B) \sin \theta - (x_g W - x_g B) \cos \theta + M_{rudder}
\end{align*}
$$

(3)

Where, $X_{wq}$, $X_{wq}$, $X_{wu}$, $Z_{wq}$, $Z_{wu}$, $M_{wq}$, $M_{wu}$ and $M_{wu}$ denote the Hydrodynamic Coefficients of AUV, $W$ denotes the weight of vehicle, $B$ is the buoyancy.

The most hydrodynamic coefficients of AUV can be determined by the Planar Motion Mechanism (PMM) tests [6]. The dimensionless hydrodynamic coefficients obtained from PMM tests are shown in table 2. Then remaining hydrodynamic coefficients can be calculated from the empirical formula.

**Table 2.** The dimensionless hydrodynamic coefficients of AUV

|   | $X'_{uq}$ | $Z'_{wq}$ | $M'_{wq}$ |
|---|-----------|-----------|-----------|
| 1 | -1.58e-3  | -1.06e-1  | -2.63e-4  |
| 2 | -6.86e-3  | -8.02e-3  | -1.00e-5  |
| 3 | -7.63e-2  | -2.05e-3  |           |
| 4 | -1.19e-2  | -3.80e-3  |           |

The output force of the thruster $\bullet_{prop}$ can be obtained by interpolating the thrust curve.

The force and moment produced by rudder $\bullet_{rudder}$ can be obtained by calculating the force of lift and resistance [7].

### 3. Modeling in the current for the vertical plane

#### 3.1 The current model in the vertical plane

According to [8], observation results of the current in the South China Sea indicates that the current velocity is stable for a period of time and varies along depth direction. In this paper, it is assumed that the velocity of the current increases evenly between 100m and 400m, the average velocity of the current at 100m is 1kn, the current velocity increases 0.025cm/s with the depth of 1m and the current direction is positive along the $E^\xi$ axis.

Based on the previous analysis, the ocean current model varied with depth can be expressed as:

$$V' = 0.514 + 0.00025 \times (d - 100)$$

(4)

Where, $d$ denotes the depth from the surface of water.

#### 3.2 Modelling in the current

It is assumed that the current velocity in the inertial coordinate system $\{I\}$ and body-fixed coordinate system $\{B\}$, shown in figure 2, can be expressed as $U_C = (U_\xi \quad U_\zeta) = (U_\xi \quad 0)^T$ and $U_{CM} = (u_\xi \quad w_\zeta)$. 

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So, the coordinate transformation can be expressed as:

$$U_{cm} = \mathbf{R}^{-1} U_c = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}^{-1} U_c \tag{5}$$

The current acceleration in the body-fixed coordinate system $\{B\}$ can be expressed as:

$$\dot{U}_{cm} = (\mathbf{R})^{-1} \dot{U}_c + (\mathbf{R})^{-1} \ddot{U}_c \tag{6}$$

Then, the relative velocity and acceleration can be expressed as:

$$\begin{bmatrix} u_r \\ w_r \end{bmatrix} = \begin{bmatrix} u - u_c \\ w - w_c \end{bmatrix}, \quad \begin{bmatrix} \dot{u}_r \\ \dot{w}_r \end{bmatrix} = \begin{bmatrix} \dot{u} - \dot{u}_c \\ \dot{w} - \dot{w}_c \end{bmatrix} \tag{7}$$

Assuming that the hydrodynamic coefficients do not change, the relative motion method is adopted. Replacing $u, w, \dot{u}$ and $\dot{w}$ in formula (2) with $u_r, w_r, \dot{u}_r$ and $\dot{w}_r$, we can get the mathematical model of the AUV in vertical plane under the current disturbance.

4. The simulation system design and experiment

4.1 The simulation system design

The simulation system of maneuverability is established with MFC based on the mathematical model of the AUV. It contains five modules, which are the initialization module, the hydrodynamic coefficient module, the force module, the acceleration module and the state module. The calculation process is shown in figure 3.

4.2. Simulation experiment

4.2.1. No current. After several simulation experiments, the thruster input voltage needed is obtained when the speed is stabilized at 1-5kn. The thruster voltage is set to a specific value, when the speed is stable, AUV will dive after the rudder is set as a certain angle. $\delta=10^\circ$ and $\delta=15^\circ$ are selected as the
characteristic rudder angle to carry out the simulation experiment. The relationship curve of the maximum pitch angle and rudder angle at each speed is obtained, as shown in figure 4. From this figure, when the condition of the simulation experiment, the law of floating and diving for AUV of underwater vehicles is basically the same: When the speed is the same, a bigger rudder angle meant bigger the stable pitch angle of the AUV; The maximum pitch angle of the AUV is 31.2° when the speed is 5kn and the rudder angle is 15°. The motion performance of the vertical plane is good.

![Figure 4. Relationship between the maximum pitch angle and rudder angle](image)

![Figure 5. Response for step steering (5kn)](image)

![Figure 6. trapezoidal steering motion simulation](image)

The simulation experiment for step steering on the typical stable speed 5kn is carried out. The response curve of pitch angle and depth about the time are obtained when the stable speed is 5kn and the rudder angle is ±10° and ±15°, as shown in figure 5. From the figures, under the typical speed, the AUV moves smoothly and its pitch angle is convergent, and the depth is divergent with time, which is consistent with the actual motion characteristics of AUV. When the rudder angle is same, a higher stable speed meant slower increase of pitch angle.

The trapezoidal steering motion simulation is carried out. The typical stable speed is 4kn and 5kn, rudder angle instruction is \( \delta_e = -8\degree \) , pitch angle instruction is \( \theta_e = -7\degree \) and rudder steering rate is \( 4\degree / s \). The response curve is shown in figure 6. From the figure, the execution time is about 4.1s and the transcendental pitch angle is about 3° when the stable speed is 5kn; the execution time is about 5s and the transcendental pitch angle is about 2.1° when the stable speed is 4kn. In this paper, the transcendental pitch angle of the AUV is larger. Enough attention should be paid to the steering in vertical plane.

4.2.2 With current. The simulation for step steering in the current is carried out. The thruster input voltage is 5V. The rudder angle is -10°. The current model uses the model described in 3.1 sections. From the figure 7, the speed of AUV is firstly rapidly increased then slowly increased. It will experience an unstable period at the beginning stage. Attention should be paid when controlling. From the figure 8, the curves of depth and pitch angle about the time in the current coincide with the curves without current, the final pitch angle converges to about 21°. This indicates that the current model built in this paper has little effect on the AUV pitch angle.

![Figure 7. Response of speed in the current](image)

![Figure 8. Response for step operation of the rudder in the current](image)

![Figure 9. trapezoidal steering motion simulation in the current](image)
The trapezoidal steering motion simulation in current is carried out. The thruster input voltage is 5V, the rudder angle instruction is $\delta_0 = -8^\circ$. The pitch angle instruction is $\theta_e = -7^\circ$. The response curve is shown in figure 9. From the figure, the response curves of the pitch angle and depth in the current are basically consistent with the curves without current, and the transcendental pitch angle has a slight decrease, and the depth decreases slightly, which indicates that the current model has little influence on the trapezoidal steering of the AUV.

5. Conclusion
In this paper, the model for AUV of the vertical plane in the current is built. And the simulation experiments are carried out including the no current and with current. The following conclusion can be drawn.
1) The simulation system built in this paper can be used to simulate the propeller-rudder-driven AUV motion in vertical plane well. The mathematical model is accurate and reliable.
2) The response of speed in the current varied with depth will experience an unstable period, attention should be paid when controlling. And the current model built in this paper has little effect on the AUV pitch angle.

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References
[1] Vollmayr A N, Sosnowski S, Urban S, et al. Snookie: An Autonomous Underwater Vehicle with Artificial Lateral-Line System // Flow Sensing in Air and Water 2014 Springer Berlin Heidelberg 521-562
[2] Timothy J. Prestero, Development of a Six-Degree of Freedom Simulation Model for the REMUS Autonomous Underwater Vehicle 2001 MIT/WHOI Joint Program in Oceanographic Engineering
[3] Hao L, Analysis and research of influencing factors on Autonomous Underwater Vehicle maneuverability in vertical plane 2013 Tianjin University
[4] Liu Z, PREDICTION OF AUV "CR-02" SUBMERGE WITHOUT POWER 2002 Robot 24:385-388
[5] Duan F, Pang S, Comparison of Two Six-Degree of Freedom Simulation Models for Mini Autonomous Underwater Vehicle 2012 ASME 2012 International Conference on Ocean, Offshore and Arctic Engineering 2012:281-287
[6] Gao T, Wang Y, Pang Y, et al, A time-efficient CFD approach for hydrodynamic coefficient determination and model simplification of submarine 2018 Ocean Engineering 154:16-26
[7] Liang X, Li Y, Peng Z, et al, Nonlinear dynamics modeling and performance prediction for underactuated AUV with fins 2016 Nonlinear Dynamics 84(1):237-249
[8] Yang Q X, Liang X F, Tian J W, et al, OBSERVATION ON THE CURRENT IN THE NORTHERN SOUTH CHINA SEA AND SPECTRUM ANALYSIS 2008 Oceanologia Et Limnologia Sinica