Dynamical instability of the electric transport in strongly fluctuating superconductors.

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Theory of the influence of the thermal fluctuations on the electric transport beyond linear response in superconductors is developed within the framework of the time dependent Ginzburg - Landau approach. The I - V curve is calculated using the dynamical self - consistent gaussian approximation. Under certain conditions it exhibits a reentrant behaviour acquiring an S - shape form. The unstable region below a critical temperature $T^*$ is determined for arbitrary dimensionality ($D = 1, 2, 3$) of the thermal fluctuations. The results are applied to analyse the transport data on nanowires and several classes of 2D superconductors: metallic thin films, layered and atomically thick novel materials.

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I. INTRODUCTION

In most electrical transport phenomena in condensed matter the current in a conductor is a monotonic function of the applied voltage. Moreover at small current densities the I-V curve is nearly linear (see the dark green line in Fig.1 representing a normal metal), so that only the linear response theory is generally needed to describe the transport via conductivity $\sigma_{\text{n}}$. However in certain types of materials the linearity does not extend to higher current densities. In superconductors close to the normal state (when temperature for example is just below critical, see the purple curve in Fig.1), at small currents the I-V curve slope $\alpha$ is very large $\alpha >> \sigma_{\text{n}}$, however at higher currents it diminishes and then smoothly approaches the normal line.

It turns out that under certain conditions (for example at yet lower temperatures, the solid cyan curve in Fig.1), the initial slope is even steeper and moreover at certain current density the differential resistivity becomes negative signalling a dynamical instability.

This possibility was envisioned theoretically by Gorkov and Masker, Marcelja and Parks before strongly fluctuating superconductors like the high $T_c$ cuprates were discovered. The arguments required strong fluctuations that enhance conductivity of a metal, beyond the parameter range in which the coherent condensate is not formed.

The theory in the one-dimensional geometry was discussed in a comprehensive paper by Tucker and Halperin. Different versions of the dynamical Hartree-Fock approximation were critically compared. The focus on wires (one dimensions, 1D) was justified, since low $T_c$ superconductors have very small Ginzburg number $G_1$ and the fluctuations are detectable only when the dimensionality is reduced (or strong magnetic fields applied). The Tucker and Halperin conclusion was that the approximation is probably inapplicable for currents for which differential resistivity is negative, but qualitatively the phenomenon should be observable in 1D. Later several experiments indeed appeared both in 1D (thin metallic nanowires and in 2D both in thin metallic film and layered high $T_c$ materials that have a much larger Ginzburg number, so that thermal fluctuations in them are much easier to observe. Moreover recently purely 2D superconductors (with thickness of just one or very few unit cells) appear and similar phenomenon was observed.

It is important to note that, due to experimental reasons, only in the first two experiments the voltage drive was used, so that the full I-V curve including the “unstable” parts was observed. In rest of experiments the current drive was employed, so one observed that at certain current the voltage “jumped” over the unstable state. Many more experiments observing instability (with jumps due to the current drive) were performed in superconducting films and wires under strong magnetic field. In the presence of magnetic field in type II superconductors the dynamical problem becomes more complex due to effects of the vortex pinning and theoretical explanations invoke thermal transport (hot spots). The experiment on 1D nanowires was qualitatively explained using dynamics of the condensate, rather then utilizing Tucker-Halperin theory. As was noted early on, the dynamical instability, is firmly established, can leads to dynamical phase separation patterns and other phenomena and applications.

In this paper we revisit and expand the self-consistent theory of the nonlinear response in superconductors and demonstrate that the old and the new experiments on the dynamical instability can in fact be explained by it, not just qualitatively, but quantitatively. The conditions for the instability are derived in D=1,2 and even D=3 (in which case these are almost impossible to observe even for the most “fluctuating” materials). It seems that a covariant version of the dynamical gaussian approximation in the framework of the Ginzburg-Landau phenomenological approach is precise and universal enough to quantitatively describe the phenomenon including the unstable regions.

A qualitative argument ensuring the emergence of the dynamical instability for superconductors that possess large enough thermal fluctuations is as follows. The superconducting fluctuations contribution to the voltage has the following form, see dashed lines in Fig.1. It rises very fast at small currents and gradually decreases to zero when the virtual Cooper pairs are broken by the electric field. The negative slope at some point becomes equal or larger then the normal electrons conductivity that is roughly independent of the transport current (dark green line in Fig.1). The appearance of the S-shaped I-V curves at certain value of temperature (the blue curve) is the crossover temperature that will be determined in the paper.

The rest of the paper is organized as follows. In Section II the time dependent Ginzburg-Landau model incorporating the effects of thermal fluctuations is specified, while in Section III the I-V curves are derived in $D=1,2,3$ within the gaussian approximation. The instability is analyzed in Section IV by considering a quantum wire experiment and several 2D materials ranging from thin films to layered superconductors and few atomic thick new materials. Section V contains conclusions.

II. THERMAL FLUCTUATIONS AND ELECTRIC FIELD IN THE TIME-DEPENDENT GL MODEL

Unlike in many other second order transitions in condensed matter, some superconductor-normal transitions exhibit a wide thermal fluctuation region. Since the discovery of high $T_c$ superconductors, the superconducting fluctuations
have been demonstrated to be the prime cause of many interesting phenomena. For example, fluctuations broaden the critical region of resistivity in the vicinity of the transition temperature [19], lead to large diamagnetism [20] and Nernst effect [21] far above \(T_c\) etc. The influence is especially enhanced under strong magnetic fields.

A. The model

While it is impossible at the present level of our understanding of superconductivity in these materials to describe the effect of thermal fluctuations on transport within a microscopic model, the Ginzburg-Landau (GL) phenomenological description in terms of the order parameter field \(\Psi\) is a method of choice [17,18] for that purpose. To describe the thermal fluctuations of the order parameter in \(D\) - dimensional superconductors a starting point is the GL free energy as a functional of the order parameter field \(\Psi\):

\[
F_{\text{GL}} = A \int d^D r \frac{\hbar^2}{2m^*} |\nabla \Psi|^2 + \alpha (T - T_\Lambda) |\Psi|^2 + \frac{b}{2} |\Psi|^4. \tag{1}
\]

For low dimensional superconductors the cross - section “area” is indeed area, \(A = L_y L_z\), for \(D = 1\), while in \(D = 2\) it is the sample effective thickness \(A = L_z\). In the GL potential term, \(T_\Lambda\) is the mean-field critical temperature, that can be significantly larger than the measured critical temperature \(T_c\) due to strong thermal fluctuations on the mesoscopic scale [22] and \(m^*\) is effective Cooper pair mass.

For strong fluctuation superconductors away from both the critical range and the gaussian fluctuations regime at very low temperatures, one have to take the quartic term in GL free energy into account. The other two parameters \(\alpha\) and \(b\) determine the two characteristic length scales, the coherence length \(\xi^2 = \hbar^2/(2m^* \alpha T_\Lambda)\) and the penetration depth \(\lambda^2 = b c^2 m^*/(16\pi e^2 \alpha T_\Lambda)\).

The relaxational dynamics and thermal fluctuations of the superconducting order parameter in the presence of electric field \(E\) are conveniently described by the gauge-invariant time-dependent GL (TDGL) equation [23] with the Langevin white noise:

\[
\Gamma_0^{-1} \left( \frac{\partial}{\partial \tau} - i \frac{e^* \varphi}{\hbar} \right) \Psi = -\frac{1}{A} \frac{\delta F_{\text{GL}}}{\delta \Psi^*} + \zeta (r, \tau). \tag{2}
\]

Here the order parameter relaxation time is given by \(\Gamma_0^{-1} = \hbar^2 \gamma/(2m^*)\), where the inverse diffusion constant \(\gamma/2\), controlling the time scale of dynamical processes via dissipation, is assumed to be real [24] \(e^* = 2 |e|\). The scalar
potential for constant homogeneous electric field (assume to be applied along the x axis) is \( \varphi = -E x \). The white-noise forces, which induce the thermodynamical fluctuations, satisfy the fluctuation-dissipation theorem

\[
\langle \zeta^*(r, \tau) \zeta(r', \tau') \rangle = \frac{2T}{A \Gamma_0} \delta(r - r') \delta(\tau - \tau').
\] (3)

The electric current density includes two components, \( J = J_n + J_s \), where \( J_n = \sigma_n E \) is the current density contributed by the Ohmic normal part, and \( J_s \) is fluctuation supercurrent density given by

\[
J_s = \frac{ie^* \hbar}{2m^*} (\Psi^* D \Psi - \Psi D \Psi^*).
\] (4)

### B. Characteristic scales and dimensionless variables

In order to facilitate the following discussion and fitting of experimental I-V curves, let us use characteristic units of length, the coherence length \( \xi \), time, the Ginzburg-Landau “relaxation” time \( \tau_{GL} = \gamma_2 \xi^2 / 2 \). The order parameter is normalized by \( \Psi^2 = (2\alpha T_\Lambda / b) \psi^2 \) and electric field by \( E = E_{GL} E \), where

\[
E_{GL} = \frac{2\hbar}{\gamma e^* \xi^3}.
\] (5)

The fluctuation strength is conveniently characterised by the parameter \( \omega \),

\[
\omega = \sqrt{2Gi\pi},
\] (6)

related to the \( D \)-dimensional Ginzburg number (consistent with the original definitions in \( D = 2 \)) by

\[
Gi_D = 2 \left( \frac{T_\Lambda e^* \lambda^2}{\hbar^2 \lambda^2 D - 2} \right)^2.
\] (7)

The TDGL Eq. (2), written in dimensionless units reads,

\[
\left( D_\tau - \frac{1}{2} \nabla^2 \right) \psi + \frac{t}{2} - \frac{1}{2} |\psi|^2 \psi = \bar{\zeta},
\] (8)

where \( t \equiv T / T_\Lambda \), \( D_\tau = \frac{\partial}{\partial \tau} + iE y \) and \( \bar{\zeta} = \bar{\zeta} (2\alpha T_\Lambda / b)^{3/2} / b^{1/2} \), the white noise correlation takes a dimensionless form:

\[
\langle \bar{\zeta}^*(r, \tau) \bar{\zeta}(r', \tau') \rangle = 2\omega \delta(r - r') \delta(\tau - \tau').
\] (9)

Finally, the dimensionless current density \( j_s = J_s / J_{GL} \), with \( J_{GL} = cH_2 \xi / 2\pi \lambda^2 \) as the unit of the current density, is

\[
j_s = \frac{i}{2} (\psi^* D \psi - \psi D \psi^*).
\] (10)

The problem is clearly nonperturbative, so that one should rely on methods of a variational nature that are outlined next. The relevant unit of conductivity is therefore \( \sigma_{GL} \equiv J_{GL} / E_{GL} = c^2 \gamma \xi^2 / 4\pi \lambda^2 \).

### III. THE SELF-CONSISTENT APPROXIMATION CALCULATION OF THE I-V CURVE

A sufficiently simple nonperturbative method is the Hartree-Fock type self-consistent Gaussian approximation (SCGA). It has already been applied to other fluctuations phenomena like magnetization and conductivity above \( T_c \).

#### A. Dynamical gaussian approximation

The TDGL in the presence of the Langevin white noise, Eq. (6), is nonlinear, so cannot generally be solved. Since we will need only the thermal averages of quadratic in \( \psi \) quantities, like the superfluid density and the electric current, a sufficiently simple and accurate approximation (similar in nature to the Hartree-Fock approximation in the fermionic
models) is the gaussian approximation\textsuperscript{22,25,26}. The nonlinear $|\psi|^2 \psi$ term in the TDGL Eq. (8) is approximated by a linear one $2 \langle |\psi|^2 \rangle \psi$ (there are two possible contractions between $\psi^*$, $\psi$ in $|\psi|^2 \psi$, see discussion of this point in\textsuperscript{15}):

\[
\left(D_\tau - \frac{1}{2} \nabla^2 + \frac{t-1}{2} + 2 \langle |\psi|^2 \rangle \right) \psi(\mathbf{r}, \tau) = \zeta(\mathbf{r}, \tau).
\] (11)

For stationary homogeneous processes considered here, the superfluid density $\langle |\psi|^2 \rangle$ is just a constant. Now it takes a form,

\[
\left[D_\tau - \frac{1}{2} \nabla^2 + \varepsilon \right] \psi(\mathbf{r}, \tau) = \zeta(\mathbf{r}, \tau),
\] (12)

where the excitations energy gap\textsuperscript{17} is,

\[
\varepsilon = -\frac{1-t}{2} + 2 \langle |\psi|^2 \rangle.
\] (13)

The solution therefore can be written via the Green’s function,

\[
\psi(\mathbf{r}_1, \tau_1) = \int d\mathbf{r}_2 \int d\tau_2 G(\mathbf{r}_1, \tau_1; \mathbf{r}_2, \tau_2) \zeta(\mathbf{r}_2, \tau_2).
\] (14)

Then the superfluid density, using the noise correlator, Eq. (9), can be expressed via the Green’s function as,

\[
\langle |\psi(\mathbf{r}_1, \tau_1)|^2 \rangle = 2\omega t \int d\mathbf{r}_2 \int d\tau_2 G^*(\mathbf{r}_1, \tau_1; \mathbf{r}_2, \tau_2) G(\mathbf{r}_1, \tau_1; \mathbf{r}_2, \tau_2),
\] (15)

and is a function of the parameter $\varepsilon$ which is determined self consistently by Eq. (13).

\section{B. Green’s function for a homogeneous constant electric field}

To calculate the response of the system, one needs the well known Green’s function in the presence of electric field:

\[
G(\mathbf{r}_1, \mathbf{r}_2, \tau) = \theta(\tau) \frac{1}{(2\pi \tau)^{D/2}} \exp \left[ -\varepsilon \tau - \frac{E^2 \tau^3}{24} - \frac{iE}{2} \tau \left( x_1 + x_2 \right) - \frac{(\mathbf{r}_1 - \mathbf{r}_2)^2}{2\tau} \right].
\] (16)

The invariance with respect to the time translations is already taken into account by setting $\tau = \tau_1 - \tau_2$. Using these expressions, the superfluid density of Eq. (15) takes a form,

\[
\langle |\psi(\mathbf{r}, \tau)|^2 \rangle = \frac{\omega t}{2^{D-1} \pi^{D/2}} \int_0^\infty \frac{d\tau}{\tau^{D/2}} \exp \left[ -2\varepsilon \tau - \frac{E^2 \tau^3}{12} \right].
\] (17)

The integrand in Eq. (17) is divergent as $1/\tau$ when $\tau \to 0$ when $D > 1$. The cutoff $\tau_{\text{cut}}$ is thus required to account for the inherent UV divergence of the Ginzburg-Landau theory and it will be addressed below.

Finally the gap equation takes a form

\[
\varepsilon = -\frac{1-t}{2} + \frac{\omega t}{2^{D-2} \pi^{D/2}} \int_{\tau_{\text{cut}}}^\infty \frac{d\tau}{\tau^{D/2}} \exp \left[ -2\varepsilon \tau - \frac{E^2 \tau^3}{12} \right].
\] (18)

After (numerical) solution for the energy gap $\varepsilon$, we turn to calculation of the supercurrent. While the upper limit of the integration in Eq. (18) is safe (both terms in exponent are positive), the lower limit (UV) depends on dimensionality.

In Ref.\textsuperscript{22} it was shown that $\tau_{\text{cut}}$ in time dependent Ginzburg Landau and the energy cutoff $\Lambda$ in static Ginzburg Landau theory are related by

\[
\tau_{\text{cut}} = \frac{\hbar^2}{2m^* \xi^2 \Lambda E}\gamma_E.
\] (19)

where $\gamma_E$ is Euler constant and $\Lambda$ is the energy cutoff\textsuperscript{22,25,26}. Our calculation show that taking value $\tau_{\text{cut}}$ from 0.1 to 10, the physical quantities is essentially unchanged, and is taken as $\tau_{\text{cut}} = 1$ in what follows.
C. The electric current density

The dimensionless supercurrent density along the electric field direction $x$, defined by Eq. (10), expressed via the Green’s functions is

$$\langle j_s^x \rangle = i \omega t \int d\mathbf{r} d\tau' G^* (\mathbf{r}_1, \mathbf{r}_2, \tau - \tau') \frac{\partial}{\partial x} G (\mathbf{r}_1, \mathbf{r}_2, \tau - \tau') + c.c$$

Performing the integrals, one obtains,

$$\langle j_s^x \rangle = \omega t \frac{\varphi}{2D\pi D/2} \int d\tau \frac{\partial}{\partial \tau} \exp \left[ -2\varepsilon \tau - \left( \frac{E}{E_{GL}} \right)^2 \frac{\tau^3}{12} \right].$$

Returning to the physical units, the total electric current density reads

$$J_x = E \left\{ \sigma_n + \frac{\varphi}{2D\pi D/2T_A} \int d\tau \frac{\partial}{\partial \tau} \exp \left[ -2\varepsilon \tau - \left( \frac{E}{E_{GL}} \right)^2 \frac{\tau^3}{12} \right] \right\},$$

where $E_{GL}$ was defined in Eq. (5) and the dimensionless fluctuation stress parameter $\varphi$ in Eq. (6). The gap equation determining the dimensionless energy gap $\varepsilon$ in this units is

$$\varepsilon = -\frac{1}{2} - \frac{T}{T_A} + \frac{\varphi}{2D-\pi D/2T_A} \int d\tau \frac{\partial}{\partial \tau} \exp \left[ -2\varepsilon \tau - \left( \frac{E}{E_{GL}} \right)^2 \frac{\tau^3}{12} \right].$$

In general there is a factor $k$ relating the two conductivities: $k = \sigma_n/\sigma_{GL}$. The (obtained numerically) value of the energy gap $\varepsilon$ should be used. Illustrative results are presented and compared with experiments in the next section and discussed in the following one.

D. The dynamical instability point.

The dynamical instability transition temperature on the phase diagram, $T^*$, see Fig. 1, defined as a maximal temperature at which the instability appears. Mathematically is determined by vanishing of the first two derivatives, $\frac{dJ}{dE} = 0$ and $\frac{d^2J}{dE^2} = 0$. Differentiating the current, Eq. (22) (via chain rule of the gap equation), results in:

$$\frac{\sigma_n}{\sigma_{GL}T^*} + \frac{\varphi}{2D\pi D/2} \int d\tau \frac{\partial}{\partial \tau} \exp \left[ -2\varepsilon \tau - \left( \frac{E}{E_{GL}} \right)^2 \frac{\tau^3}{12} \right]$$

$$= E \frac{\varphi}{2D\pi D/2} \int d\tau \frac{\partial}{\partial \tau} \left( 2 \frac{\varepsilon}{6E_{GL}^2} \right) \exp \left[ -2\varepsilon \tau - \left( \frac{E}{E_{GL}} \right)^2 \frac{\tau^3}{12} \right]$$

$$+ \int d\tau \frac{\partial}{\partial \tau} \left\{ -\frac{E^2}{2E_{GL}} + \frac{E^2}{3E_{GL}^3} \right\} \exp \left[ -2\varepsilon \tau - \left( \frac{E}{E_{GL}} \right)^2 \frac{\tau^3}{12} \right] = 0$$

Together the gap equation (18), the dynamical instability transition temperature $T^*$ is determined numerically.

IV. COMPARISON WITH EXPERIMENTS AND DISCUSSION

The results are first applied to a one dimensional superconductors - metallic wires, and then for several qualitatively different types of 2D superconductors (as explained above, it is very difficult to observe the instability phenomenon in purely 3D materials, although in layered high $T_c$ cuprates close to $T_c$ the fluctuations become nearly 3D and the phenomenon was observed in magnetic field[15]).
A. I-V curves of 1D Sn nanowires

We start with 1D nanowires. Granular superconducting Pb and Sn nanowires of quite regular cross-section and length have been produced by electro-deposition in nanoporous membranes. It is important to note that the series of experiments of Ref. 5 on Pb and Sn nanowires is the only one (known to us) in which both the current and the voltage drives were employed. This allows a qualitative understanding of the important difference between the dynamical behaviour two. We focus on the voltage drive I-V curves of Sn.

The I-V curves, measured using the voltage drive at three temperatures, are shown in Fig. 2 (points). The voltage drive employed clearly demonstrates the non-monotonic character below the onset $T_c \approx T_\Lambda = 3.8K$ slightly above the bulk temperature of Sn ($3.72K$). The current drive experiment on the same sample (see Fig. 3b in Ref. 5) demonstrates the voltage jumps over unstable domains of the dynamical phase diagrams. The jumps are more pronounced in Pb, see Fig. 3a of Ref. 5. This is consistent with the existence of the dynamical instability and was observed in numerous experiments (see 2D examples below).

The experimental data are fitted by Eqs.22,23 for $D = 1$, see solid curves. The normal-state conductivity is given, $\sigma_n = 3.6 \cdot 10^4 (\Omega \ast m)^{-1}$. nanowires are $50\mu m$ long with $55nm$ in diameter. Measured material parameters are: coherence length $\xi = 210nm$, penetration depth $\lambda = 420nm$ and the normal conductivity was obtained from the red dotted line in Fig.2. The value of fitting parameters are: the fluctuation strength parameter $\omega = 0.0043$, corresponding to the Ginzburg number $Gt = 9.4 \cdot 10^{-7}$, consistent with one dimension Ginzburg number formula, $Gt_{D=1} = 2 (T_\Lambda e^2 \lambda^2 / Ac^2 h^2)^2 \approx 2.9 \ast 10^{-7}$ and the conductivity ratio $k = \sigma_n / \sigma_{GL} = 0.08$.

This experiment was already discussed in the framework of TDGL equations neglecting thermal fluctuations in Ref. 15 assuming the current drive. The dynamical equations were solved numerically and the focus was on the jumps. It seems to us that the origin of instability cannot ignore the thermal fluctuations, as explained above. Many more experiments were performed in 2D.

B. Instability in 2D

Several 2D superconductors exhibit the dynamical instability. We start with metallic thin films, then proceed to the customary layered materials in which the coupling between layers is sufficiently small to ensure that thermal fluctuations dimensionality is 2. Novel purely 2D materials are then mentioned. Of course in a 2D superconductor...
one should measure close to $T_c$ to be able to detect the thermal fluctuations effects like the dynamical instability. The only possible exceptions are high $T_c$ cuprates and novel 2D atomically thick superconductors.

1. Thin metallic films near $T_c$

In experiments on In thin films the voltage drive was applied in a narrow temperature range very close to $T_c$. For the low critical temperature superconductor, $T_A$ is very near the bulk critical temperature. The temperature range (only superconducting states for $0 = 1 - T/T_A < 2.1\%$ are replotted in Fig.3 as dots) nevertheless is wide enough to exhibit the dynamical instability, for $T/T_A = 0.9821$ and $T/T_A = 0.9793$. The coherence length is approximately $\xi = 300\text{nm}$, while the thickness $d$ of In thin films is ranged from $10\text{nm}$ to $300\text{nm}$ (less than $\xi$), therefore in this temperature range coherence length $\xi > d$ and the thermal fluctuations are 2D. The normal-state conductivity is $\sigma_n = 9 \times 10^4 (\Omega \cdot \text{cm})^{-1}$. The cross section area (perpendicular to the current direction) is approximately equal to $3.62 \times 10^{-7} \text{cm}^2$ in the fitting, which is consistent with the data provided in Ref. 6.

The calculated I-V curves according Eqs.(22,23) for $D = 2$, for different temperature are shown in Fig. 3 as solid curves. The experimental data are fitted best for the following values of parameters: $k = \sigma_n/\sigma_{GL} = 0.075$ and the fluctuation strength parameter $\omega = \sqrt{2G\pi} = 0.001$ corresponding to the Ginzburg number $Gi = 5.1 \cdot 10^{-8}$, consistent with $Gi_{D=2} = 2 (T_A e^{2} \lambda^2/L_z c^2 \hbar^2)^2 \approx 5 \cdot 10^{-8}$ with $\lambda = 296\text{nm}$ and $L_z = 24.1\text{nm}$. The fit is generally good except very low currents. The reason is obvious: critical current due to disorder on the mesoscopic scale is not present in the model.

Sometimes the state close to “criticality” of the Berezinskii - Kosterlitz - Thouless variety is theoretically considered as a collection of the bound vortex - antivortex pairs. The critical current clearly seen in Fig.3 as associated with the pairs “pinning”. In fact in this 2D system strictly speaking critical current is zero (also seen in data), but it vanishes exponentially fast as $I \rightarrow 0$.

Much more common superconductors with 2D fluctuations are layered materials (will be discussed below).

2. Layered materials

Instability in the form of the voltage jumps was observed recently in FeSeTe thin film on Pb(MgNb)TiO substrate. Only the current drive was used, so that the S-shaped I-V curve cannot be determined. Only the voltage jumps were observed close to $T_c$. The thickness of FeSeTe thin films is $200\text{nm}$. The layer distance $L_z = 0.55\text{nm}$. Here, the normal-state conductivity is taken to be $\sigma_n = 1.3 \cdot 10^4 (\Omega \cdot \text{cm})^{-1}$.

The calculated I-V curves of the 2D FeSeTe thin film with different temperature are shown in Fig. 4 as solid curves. The experimental data of FeSeTe in a current driving setup from Ref. 10 are fitted best for the following values of parameters: $T_A = 8K$, $k = \sigma_n/\sigma_{GL} = 0.07$ and the fluctuation strength parameter $\omega = \sqrt{2G\pi} = 0.018$ corresponding to the Ginzburg number $Gi = 1.6 \cdot 10^{-5}$. According to $Gi_{D=2} = 2 (T_A e^{2} \lambda^2/L_z c^2 \hbar^2)^2$, we deduce...
FIG. 4. The I-V curves of FeScTe thin film at different temperatures. The points are the experimental data and the solid lines are the theoretical fitting results.

the sample’s effective penetration depth $\lambda = 123.8 \text{ nm}$ (we are not aware of an experimental determination of the penetration depth from a magnetic measurement).

The I-V curves clearly exhibit a re-entrant behavior for $T < T^* \approx 6K$. This is hard to observe directly in the current driving experimental setup. Experiments show that the current driving lead to the “jump” I-V Curve and the voltage driving lead to the re-entrant S-Shaped I-V Curve in the superconducting nanowires at low temperature.

3. Other layered materials

The instability in the ultra-thin granular $YBa_2Cu_3O_{7-\delta}$ nanobridges was clearly observed in a series of works in Ref. [8]. Unfortunately a 2D or a 3D model cannot quantitatively describe these I-V curves since the fluctuations in this layered material and the temperature range can be described by a more complicated Lawrence - Doniach model. The generalization is possible but was not attempted in the present work.

Also, the “jump” I-V curves in a current driving setup was also reported in BSCCO that is clearly 2D. Unfortunately I-V curve at zero magnetic field (actually field perpendicular to the layers) at one value of temperature ($76K$ for $T_c = 85.2K$) was measured. As noted above, the instability has been observed in numerous layered superconductors under strong magnetic field, but a quantitative interpretation requires additional parameters describing the magnetic vortex pinning.

V. CONCLUSIONS

In this paper, I - V curve of a $D$ dimensional superconductor including the thermal fluctuations effects is calculated in arbitrary dimension using the dynamical self consistent gaussian approximation method. An unstable region is found when currents flow through superconductor with temperature below a critical value $T^*$ at which the I-V curve become S-shaped. It is shown how the thermal fluctuations generate the instability. The results are applied to analyse the transport data on various materials that possess sufficiently strong fluctuations in 1D or 2D. While it is found that the unstable region can exist also in 3D, the S-Shaped I-V curve in realistic materials show only in 1D superconductors.

Let us stress that the majority of recent experiments on the resistive state are performed in the constant current (current driving) regime and at temperatures close to $T_c$. It would be very interesting to observe the whole S-shaped I-V curve using the voltage drive in novel atomically thick 2D materials as in extensively studied layered ones like
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