Quantum speedup in stoquastic adiabatic quantum computation

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Quantum computation provides exponential speedup for solving certain mathematical problems against classical computers. Motivated by current rapid experimental progress on quantum computing devices, various models of quantum computation have been investigated to show quantum computational supremacy. At a commercial side, quantum annealing machine realizes the quantum Ising model with a transverse field and heuristically solves combinatorial optimization problems. The computational power of this machine is closely related to adiabatic quantum computation (AQC) with a restricted type of Hamiltonians, namely stoquastic Hamiltonians, and has been thought to be relatively less powerful compared to universal quantum computers. Little is known about computational quantum speedup nor advantage in AQC with stoquastic Hamiltonians. Here we characterize computational capability of AQC with stoquastic Hamiltonians, which we call stoqAQC. We construct a concrete stoqAQC model, whose lowest energy gap is lower bounded polynomially, and hence the final state can be obtained in polynomial time. Then we show that it can simulate universal quantum computation if adaptive single-qubit measurements in non-standard bases are allowed on the final state. Even if the measurements are restricted to non-adaptive measurements to respect the robustness of AQC, the proposed model exhibits quantum computational supremacy; classical simulation is impossible under complexity theoretical conjectures. Moreover, it is found that such a stoqAQC model can simulate Shor’s algorithm and solve the factoring problem in polynomial time. We also propose how to overcome the measurement imperfections via quantum error correction within the stoqAQC model and also an experimentally feasible verification scheme to test whether or not stoqAQC is done faithfully.

I. INTRODUCTION

The Hamiltonians with nonpositive off-diagonal elements in a standard basis are called stoquastic Hamiltonians \cite{1}. Important quantum statistical models are included in this class. The quantum Ising model with a transverse field, the antiferromagnetic Heisenberg model on bipartite graphs, and the Bose-Hubbard model with negative hoppings are such examples. For this class of models, the ground state has real and positive coefficients in the standard basis. This allows a Monte-Carlo method on a classical computer, called quantum Monte-Carlo, to sample the ground state, while efficient convergence is not always guaranteed.

Another important aspect of stoquastic Hamiltonians is quantum annealing (QA) \cite{2}, which is a heuristic algorithm to solve combinatorial optimization problems approximately by adiabatically (or even non-adiabatically) changing the parameters. In the adiabatic case, QA is included in an adiabatic quantum computation (AQC) \cite{3}, which is known to be universal for quantum computation when nonstoquastic Hamiltonians are employed \cite{4}. When it is restricted to stoquastic Hamiltonians, its computational power has been speculated to be less powerful compared to universal quantum computation \cite{5,6}. From an experimental viewpoint, stoquastic Hamiltonians are relatively easy to be implemented. This is why QA with the quantum Ising model with a transverse field has already implemented with a larger number of qubits on a superconducting system \cite{7,9}, while the quality of the qubits are relatively poor compared to the standard circuit-based approaches \cite{10,11}. Unfortunately, there has been little theoretical or experimental solid evidence of quantum speedup in AQC with stoquastic Hamiltonians \cite{5,6} except for the oracle problems \cite{12-14}.

Computational complexity of stoquastic Hamiltonians has been investigated so far in various aspects \cite{1,15,16}. Local Hamiltonian problems of stoquastic Hamiltonians have been shown to be stoqMA-complete \cite{15}. Moreover, the local Hamiltonian problem of the quantum Ising model with a transverse field on degree-3 graphs is complete in the sense that it is equivalent modulo polynomial reductions to the local Hamiltonian problems of stoquastic Hamiltonians \cite{15,16}. StoqMA is the class of problems efficiently verifiable by reversible (unitary) classical computation, with an X-basis measurement, whose input consists of a quantum state provided by a prover as the proof and ancilla qubits prepared in the Pauli X and Z bases. While stoqMA is not so powerful as QMA \cite{17,18}, stoqMA includes MA and hence NP, which are efficiently verifiable problems by probabilistic and deterministic classical computations, respectively. This implies that computational complexity of the ground state energy of stoquastic Hamiltonians does not directly reflect computational power of AQC with stoquastic Hamiltonians; the former is thought to be much harder than the latter.

The quench dynamics of stoquastic Hamiltonians is as powerful as universal quantum computation. Quantum computational supremacy of quantum approximated op-
timization algorithm [19], which is a digitalized version of QA, has been argued [20] by using the fact that Ising type commuting interactions on the eigenstates of the transverse field can simulate non-universal model, so-called IQP [21][24] (instantaneous quantum polynomial time computation). Even translation invariant quench dynamics on one-dimensional quantum Ising model can simulate universal quantum computation [24]. Note that, quench dynamics of stoquastic Hamiltonians can generate negative (even complex) coefficients in the standard basis. Moreover, quench dynamics does not inherit the robustness of AQC, i.e., protection as a ground state. Regarding adiabatic dynamics, the computational power of stoquastic Hamiltonians has not yet fully understood. Neither quantum computational supremacy nor the capability of universal quantum computation has been addressed so far.

Here we characterize computational capability of AQC with stoquastic Hamiltonians, which we call stoqAQC, and show that stoqAQC exhibits strong quantum speedup. Based on AQC using the Feynman-Kitaev Hamiltonian [4][17][25], we construct a stoqAQC model whose lowest energy gap is always lower bounded by the inverse of a polynomial function in the size of the system. This guarantees that the final state can be faithfully obtained with a polynomial time. Specifically, we consider both non-adaptive and adaptive single-qubit measurements in the Pauli bases on the final state of the stoqAQC. While this contains a non-standard basis, it is not so difficult to perform non-standard basis measurements, if the actual quantum machine works coherently.

In the case of the adaptive single-qubit measurements, we can successfully show that stoqAQC can simulate universal quantum computation only Pauli basis measurements. However, such adaptive single-qubit measurements take time for the sequential measurements, and the final state would decohere during the measurements. Therefore, non-adaptive measurements might be relevant to characterize an actual stoqAQC machine. Even in the case of the non-adaptive measurements, we can show that stoqAQC can perform non-universal quantum computation, which exhibits quantum speedup. More precisely, stoqAQC can simulate two types of non-universal models for quantum computational supremacy, IQP [21][22] and HC1Q (Hadamard-classical circuit with one-qubit) [20] models. Moreover, we can show that stoqAQC can also simulate Simon’s algorithm [27] and Shor’s factorization algorithm efficiently [28][29]. To this end, we slightly modify the phase estimation so that the phase is obtained with non-adaptive measurements without quantum Fourier transformation. In this way, we find strong evidence that an ideal stoqAQC machine can provide plenty of quantum speedup.

Yet, this result does not mean that the state-of-the-art d-wave quantum annealer designed to solve optimization problems readily exhibits quantum speedup, since it consists of relatively poor qubits and only employs the standard basis measurements. In our construction, the final state is a highly entangled state and the measurements are done in non-standard bases. Therefore noise or imperfection in the system would affect the output crucially. Moreover, the standard basis measurements only result in classical randomized computation in our construction. Therefore, coherence in the final state is crucially important to gain quantum speedup in our construction, though this would always be the case for any quantum computing device.

We further show that our construction is robust against measurement imperfections by showing how to embed quantum error correction within the stoqAQC model. We also argue how to verify whether or not the proposed stoqAQC is faithfully done experimentally.

II. STOQUASTIC ADIABATIC QUANTUM COMPUTATION

We adopt the following definition of AQC:

Definition 1 (AQC [4][5]) A k-local adiabatic quantum computation is specified by two k-local Hamiltonians $H_{initial}$ and $H_{final}$. The ground state of $H_{initial}$ is unique and is a product state. The output is a state that is $\epsilon$-close in $l_2$-norm to the ground state of $H_{final}$. Let $s(t) : [0, t_f] \to [0, 1]$ (the schedule) and let $t_f$ be the smallest time such that the final state of an adiabatic evolution generated by $H(s) = [1 - s(t)]H_{initial} + s H_{final}$ for time $t_f$ is $\epsilon$-close in $l_2$-norm to the ground state of $H_{final}$. Then, arbitrary single-qubit measurements can be done adaptively or non-adaptively on the final state.

Definition 2 (StoqAQC [5]) Stoquastic adiabatic quantum computation (stoqAQC) is the special case of AQC restricted to k-local stoquastic Hamiltonians.

Following Ref. [3], we here use the term QA when stoquastic Hamiltonians are employed to solve combinatorial optimization problems either adiabatically or non-adiabatically. Our construction employs a stoquastic version of AQC using the Feynman-Kitaev Hamiltonian [4][17][25]. We consider a composite system $H_{work} \otimes H_{clock}$ of the working system $H_{work} = (\mathbb{C}^2)^{\otimes(n+m)}$ and the clock system $H_{clock} = \mathbb{C}^{T+1}$, where $T = \text{poly}(n,m)$. Later, the clock system is replaced by a $(T + 1)$-qubit system by using the domain wall clock construction, $|t\rangle = |11\ldots10(t+1)\ldots\rangle$ [17][18].

The Hamiltonian is given by

$$H(s) = (1 - s)H_{initial} + s H_{final},$$

where

$$H_{initial} = H_{in} + (I_2 - |0\rangle\langle 0|)_c,$$

$$H_{final} = H_{in} + \sum_{t=1}^{T} \frac{1}{2} \left[ |t\rangle\langle t|_c + |t - 1\rangle\langle t - 1|_c - (U_c|t\rangle\langle t - 1|_c + \text{h.c.}) \right].$$
with the energy penalty term for the initial state of the working system,
\[ H_{\text{in}} = \sum_{i=1}^{n} |1\rangle \langle 1| \otimes |0\rangle \langle 0| + \sum_{j=n+1}^{m+n} |–\rangle \langle –| \otimes |0\rangle \langle 0|, \]

imposing the initial state \(|0\rangle^{\otimes n+1}^{\otimes m}\). The unitary operator \(U_i\) acting on the working system consists only of Toffoli, CNOT, and \(X\). This guarantees that \(H(s)\) is stoquastic for \(0 \leq s \leq 1\) in the standard basis
\[ \{|0\rangle, |1\rangle\}^{(n+m)} \otimes \{|t\rangle\}_{t=0}^T. \quad (5) \]

The parameter \(s\) is adiabatically changed from \(s = 0\) to \(s = 1\). The ground state of the initial Hamiltonian \(H_{\text{initial}}\) is
\[ |\eta_0\rangle = |0\rangle^{\otimes n+1}^{\otimes m}|0\rangle_c. \quad (6) \]

The ground state of the final Hamiltonian \(H_{\text{final}}\) is
\[ |\Psi\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^{T} |\eta_t\rangle, \quad (7) \]

where
\[ |\eta_t\rangle = U_t \cdots U_1 |0\rangle^{\otimes n+1}^{\otimes m} |t\rangle_c. \quad (8) \]

Furthermore, regarding the minimum energy gap \(\Delta\) between the ground and first excited state of \(H(s)\) \((0 \leq s \leq 1)\), we can make the same argument as Ref. [4] for AQC with general Hamiltonians. This guarantees \(\Delta\) is lower bounded by \(O(1/T^2)\).

By virtue of the adiabatic theorem, there exist a certain constant \(c(k)\) for any constant \(k\) such that if the computation time is sufficiently long
\[ t \geq c(k) \frac{|H_{\text{final}} - H_{\text{initial}}|^{1+1/k}}{\epsilon^{k/\Delta^{2+1/k}}}, \quad (9) \]

then the final state \(|\Psi_{\text{ad}}\rangle\) is \(\epsilon\)-close to the exact ground state \(|\Psi\rangle\) in \(l_2\)-norm [11, 5, 30].

On the current QA machine, measurements are done only in the standard computational basis, in which the Hamiltonian is stoquastic. Here we slightly relax this condition, and assume that arbitrary single-qubit measurements can be done on the final state either adaptively or non-adaptively. (Note that the measurement basis is not restricted in Defs. 1 and 2 as is also stated in Ref. [5].) Suppose the clock state is measured, and the state \(|T\rangle\) is obtained, which occurs with a high probability \(\geq 1/\text{poly}(n,m)\). It is known that this probability can be further amplified to a constant value \(1/2\), say, by inserting the identity gates \((T - 1)\) times using the clock system \(|\{t\rangle\}_{t=0}^{2T-1}\). The ground state is given by
\[ |\Psi\rangle = \frac{1}{\sqrt{2T}} \sum_{t=0}^{2T-1} |\eta_t\rangle, \quad (10) \]

where \(U_t = I\) for \(T + 1 \leq t \leq 2T - 1\). Any clock states in the range \(T \leq t \leq 2T - 1\) allow us the desired computation.

Under the condition of obtaining a successful clock state, the computation, which we can perform on the final state, is specified to be
\[ U_T \cdots U_1 (I^{\otimes n} \otimes H^{\otimes m}) |0\rangle^{\otimes (n+m)} \quad (11) \]
followed by arbitrary non-adaptive or adaptive single-qubit measurements as shown in Fig. 1(a). If the measurements in Fig. 1 are restricted to the computational basis, it corresponds to classical randomized computation. When measurements on \(xy\)-plane are additionally available, it belongs to Fourier hierarchy (FH) of the second level, \(\text{FH}_2\) [31, 32]. If the initial states and measurements are restricted into \(|0\rangle^{\otimes n}\rangle^+\rangle^m\rangle\) and \(X\) and \(Z\) basis measurements on them, the circuit in Fig. 1 corresponds to HClQ model [29]. Then, for arbitrary single-qubit measurements, it is included in \(\text{FH}_3\).

Since \(U_T \cdots U_1\) contains CNOT gates, this readily tells us that an arbitrary CSS (Calderbank-Shor-Steane) state [33, 54] can be prepared from the final state via the measurement on the clock. More generally, we have

**Theorem 1** An arbitrary state that is generated by a polynomial number of \(X\), CNOT, and Toffoli gates from \(|0\rangle^{\otimes n} \rangle^+ \rangle^m\) can be prepared efficiently by stoqAQC.

If we are allowed to perform arbitrary single-qubit measurements, we can show that universality with adaptive measurements using measurement-based quantum computation [35] and quantum computational supremacy with non-adaptive measurements [21, 22] as follows.

### III. UNIVERSALITY WITH ADAPTIVE MEASUREMENTS

Let us consider a more elaborate and concrete model with a 6-local Hamiltonian. Let us consider a union jack lattice as shown in Fig. 2 where the qubits in the working and clock systems are located on the vertices (white and red circles) and face centers (blue circles) of the triangles, respectively. The qubits on the working system are prepared \(|0\rangle\) and \(|\pm\rangle\) on the white and red colored qubits, respectively. By introducing the domain wall...
white and red qubits act as the controls and target, respectively, as shown in Fig. 2. Under the condition of projecting the clock state to $|11\ldots1\rangle^c$ on the final state, the union jack state is obtained. The union jack state is known to be a universal resource for measurement-based quantum computation with the Pauli basis measurements \[37\]. Therefore, stoqAQC with adaptive Pauli measurements is universal.

Theorem 2 6-local StoqAQC with adaptive Pauli basis measurements can simulate universal quantum computation.

The 6-local interactions can be reduced to 2-local perturbatively by using the mediator qubits \[1\].

IV. QUANTUM COMPUTATIONAL SUPREMACY WITH NON-ADAPTIVE MEASUREMENTS

The advantage of AQC would its protection against decoherence by the Hamiltonian. However, the adaptive measurements considered above would deteriorate this good property, since the resource state might decohere during the measurements. (Later, we will also see how to make stoqAQC robust against the measurement imperfections.) If we consider a robust physical implementation, non-adaptive measurements on the final state would be preferred. Even in such a case, we can show strong evidence of quantum speedup of stoqAQC as follows.

Measurement-based quantum computation on the union jack lattice and HC1Q are both known to be universal under postselection \[26, 37\]. Therefore, non-adaptive measurements on the exact ground state $|\Psi\rangle$ are as powerful as postBQP \[38\] under postselection. This indicates that classical (non-adaptive) sampling in the Pauli bases with a constant $l_1$ additive error is impossi-

Based on anti-concentration conjecture and average v.s. worst case conjecture of HC1Q or IQP with the union jack state, efficient classical simulation of 6-local stoqAQC with non-adaptive Pauli basis measurements with a small constant $l_1$ additive error implies the collapse of polynomial hierarchy to the third level.

Theorem 3 Based on anti-concentration conjecture and average v.s. worst case conjecture of HC1Q or IQP with the union jack state, efficient classical simulation of 6-local stoqAQC with non-adaptive Pauli basis measurements with a small constant $l_1$ additive error implies the collapse of polynomial hierarchy to the third level.

Again, the 6-local interactions can be reduced perturbatively to 2-local \[1\].

Proof. Below we will first show that the sampling on the polynomial-time stoqAQC is sufficiently close to the output of the ideal circuit shown in Fig. 1(a) in $l_1$-norm. Then, a classical sampling of such stoqAQC with a constant $l_1$ additive error implies a classical simulation of the output of the ideal circuit with a constant $l_1$ additive error.

Due to the adiabatic theorem, the final state $|\Psi_{ad}\rangle$ satisfying

\[
\||\Psi_{ad}\rangle - |\Psi\rangle\|_2 < \epsilon
\]
Therefore, to achieve a polynomially small $l_1$ means that the final state of an accuracy $\epsilon$ is to be factorized $N$ be the target of the classical simulation. Therefore, classical conditional sampling $p(x)$ is the conditional probability distribution $p(x)$ on the working system satisfies

$$\|p(x) - \bar{p}(x)\|_1 < 2\eta/p_{\text{clock}} \quad (30)$$

$$< 2\eta/p_{\text{clock}} - 2\delta' \quad (31)$$

where we assume $2\delta' < p_{\text{clock}}$. Note that $2\delta' = \delta''p_{\text{clock}}/2$ is small enough. Moreover, $p_{\text{clock}}$ can be a constant, and we chose $p_{\text{clock}} = 1/2$. Then, the $l_1$ additive error between conditional probability distributions from classical sampling $p(x)$ and the ideal one $p(x)$ is bounded by a constant value

$$\|p(x) - p(x)\|_1 < \|p(x) - \bar{p}(x)\|_1 + \|\bar{p}(x) - p(x)\|_1 < 4\eta/(1 - 4\delta') + \delta'' \quad (32)$$

Both $\delta'$ and $\delta''$ can be made small by improving the accuracy $\epsilon$ of the stoqAQC as seen in Eq. (29), which is the target of the classical simulation. Therefore, classical conditional sampling $p(x)$ is constantly close to $p(x)$ with $l_1$ additive error. The probability distribution $\tilde{p}(x)$ contains IQP on the union jack lattice and HID model[26], both of which are shown to be postBQP-complete under postselection. Therefore, by assuming that the anti-concentration and average v.s. worst case conjectures are correct in these models, a classical efficient sampling of stoqAQC with a small constant $l_1$ additive error leads to the collapse of the polynomial hierarchy to the third level.

Here we should note that while a decision problem on stoqAQC with a single-qubit $X$-basis measurement on the working system and computational basis measurements on the clock system corresponds to stoqMA with a trivial proof $|0\rangle^{\otimes n}|+\rangle^{\otimes m}$, and hence is upper bounded by postBPP[3,39]. Although this has been thought to be a partial evidence of the weakness of computational power of stoqAQC, the sampling problem on stoqAQC with non-standard bases can be much harder leading to postBQP under postselection and exhibits quantum computation supremacy as seen above.

V. QUANTUM SPEEDUP WITH NON-ADAPTIVE MEASUREMENTS

Next we show that stoqAQC with non-adaptive single-qubit measurements can solve the factoring problem by simulating Shor’s algorithm. Since any classical reversible (unitary) computation can be done on the circuit shown in Fig. 1, the modular exponentiation can also be implemented. We employ the Kitaev’s original phase estimation[29], where adaptive single-qubit measurements are performed without the inverse quantum Fourier transformation. Furthermore, in order to avoid the adaptive measurements, we introduce a non-adaptive iterative phase estimation as shown in Fig. 3.

Let $N$ and $x$ be an integer to be factorized and its coprime respectively, and define $U_x = \sum_y |xy \mod N\rangle\langle y|$, where $y = 1, \ldots, N$. Let $r$ be the
An eigenstate of $U_x$ with a label $s$ ($0 \leq s \leq r - 1$) is given by

$$|u_s\rangle = \frac{1}{\sqrt{r}} \sum_{k=0}^{r-1} e^{-2\pi i (s/r)k} |x^k (\text{mod } N)\rangle. \quad (33)$$

While the measurements are done non-adaptively, we below employ an inductive argument for explanation. Let us first consider the measurements on $\bar{1}$ and $\bar{2}$. Since $j_k^{(s)}$ is either 0 or 1, we can easily determine $j_1^{(s)}$ from the $R$ measurement outcomes in each $X$ and $Y$ bases. The $R$ samples are enough to decide $j_1^{(s)} = 0, 1$ with an exponential accuracy by using Hoefting’s inequality, since expectation values of the angle $\theta_1 = \arctan((Y)/(X))$ is constantly separated by $\pi$ for $j_1^{(s)} = 0$ or = 1. We can think this process just as a tomography of

$$\left( |0\rangle + e^{(2\pi i)j_1^{(s)}} |1\rangle \right) \otimes \left( \cdots \left( |0\rangle + e^{(2\pi i)j_1^{(s)} \cdots j_l^{(s)}} |1\rangle \right) \otimes \cdots \right)$$

is again constantly separated by $\pi$ for $j_k^{(s)} = 0, 1$ cases. Therefore, from $2R$ measurement outcomes, we can determine $j_k^{(s)}$ with an exponential accuracy in the number of samples $R$ by using Hoefting inequality for $(X)$ and $(Y)$. Inductively, all binary bits, $j_1^{(s)}, \ldots, j_l^{(s)}$, are determined. Again, we should note that the measurements are non-adaptive and hence can be done simultaneously. From the obtained phase, the order $r$ is obtained by using the continued fraction as usual. Then, a non-trivial factor of $N$ is found with a high probability against a random choice of $x$.

The circuit shown in Fig. 3 consists only of $X$, CNOT, and Toffoli acting on the initial $|0\rangle$s and $|+\rangle$s. Therefore, including the measurements on the clock, we conclude that stoqAQC with non-adaptive single-qubit measurements can solve the factoring problem in polynomial time by simulating Shor’s algorithm. If measurement bases are restricted to $X$ and $Z$, still stoqAQC with non-adaptive measurements can solve Simon’s problem [27]. Since swap operations can be constructed from CNOT gates, the stoquastic Hamiltonian can be reduced to spatially two-local one on a two-dimensional lattice by localizing the domain wall clock [36] and applying perturbative approach with the mediator qubits [1].
VI. VERIFICATION AND MEASUREMENT
ERROR TOLERANCE

Our construction based on the Feynman-Kitaev construction naturally provides a verification protocol of whether or not the final state of stoqAQC is faithfully generated. Several verification protocols with single-qubit measurements have been proposed so far based on the Kitaev-Feynman construction \([40, 41]\) and can be readily applied to the present stoqAQC Hamiltonians.

The history state \(|\Psi\rangle\) is the unique ground state of the final Hamiltonian \(H_{\text{final}}\) and satisfies

\[
E_g = \langle \Psi | H_{\text{final}} | \Psi \rangle = 0.
\]

(41)

Any pure state can be expanded by the energy eigenstates \(\{|E_i\rangle\}\) with eigenvalue \(\{E_i\}\), respectively:

\[
|\psi\rangle = \alpha_0 |E_0\rangle + \sum_{i=1}^{(T+1)2^{(n+m)}-1} \alpha_i |E_i\rangle,
\]

where \(|E_0\rangle = |\Psi\rangle\) and \(|E_0\rangle = 0\). The energy expectation value becomes

\[
\sum_{i=0}^{(T+1)2^{(n+m)}-1} |\alpha_i|^2 E_i > (1 - |\alpha_0|^2) \Delta.
\]

(43)

(Its extension to the mixed states is straightforward.) Therefore, by measuring the energy expectation value \(E_{\text{exp}}\) of the final state within an additive error and by checking whether or not

\[
E_{\text{exp}} < \epsilon_{\text{ver}} \Delta
\]

(44)

is satisfied with polynomially small \(\epsilon_{\text{ver}} = 1/\text{poly}(n,m)\), we can verify that the overlap between the experimentally obtained state \(|\psi\rangle\) and the ideal final state \(|\Psi\rangle\) is sufficiently large \(|\alpha_0|^2 > 1 - \epsilon_{\text{ver}}\). Note that the fidelity is given by \(|\langle \psi | \Psi \rangle| = |\alpha_0|\). The measurement of the energy can be done by polynomially repetitive single-qubit measurements on the final state as done in variational quantum eigensolver \([42, 43]\). A more elaborated verification scheme as done in Ref. \([41]\) can also be employed.

Furthermore, as mentioned before, if the measurement basis is restricted to the \(xy\)-plane or the \(Z\) basis, stoqAQC belongs to FH\(_2\). Therefore, the output can be verified efficiently by using the fact that a decision problem in FH\(_2\) is in MA \([26, 32]\).

Finally, we consider the robustness of the proposed stoqAQC model against a measurement imperfection. Even if AQC is executed ideally, the final measurements, especially done in the non-standard Pauli bases, would cause an imperfect readout. Fortunately, the measurements are done all in the Pauli bases. The encoding circuit of CSS codes, such as the Steane 7-qubit code \([33]\), consists only of CNOT gates acting on \(|0\rangle\)s and \(|+\rangle\)s. Therefore, by adding the further working and clock qubits, we can encode each qubit in the final state into a self-dual CSS code as shown in Fig. \([1]\) (b). For the self-dual CSS code, like the Steane 7-qubit code, all logical Pauli basis measurements are done transversally by single-qubit Pauli basis measurements. Therefore quantum speedup of stoqAQC is at least robust against the measurement imperfections in the non-standard basis. While we employ non-fault-tolerant encoding circuit of the CSS code, the encoded state is prepared as a ground state in AQC, and hence it would be interesting to see whether or not this type of encoding improves the accuracy of computation against imperfections during adiabatic operations or finite temperature effects.

VII. DISCUSSION

Here we characterized computational power of stoqAQC with adaptive or non-adaptive single-qubit measurements in the non-standard bases. While our stoqAQC model employs the 6-local stoquastic Hamiltonian, it is straightforward to reduce it to a 2-local stoquastic Hamiltonian as shown in Ref. \([1]\). While the mediator qubits are added, the original Hamiltonian is simulated perturbatively on the original system. Hence single-qubit measurements are enough for our purpose. However, in the case of the reduction from the 2-local stoquastic Hamiltonian to a transverse Ising model (on degree-3 graphs) shown in Ref. \([10]\), each qubit is simulated by dual rail bosons, a hard-core dimer and an Ising spin (chain). This modifies the single-qubit measurements in the original model to non-local measurements on the mapped models. Therefore, the computational power of stoqAQC with single-qubit measurements in non-standard bases is open for the transverse Ising models.

Our result implies that non-stoquasticity is not necessarily required to get quantum speedup in an AQC machine. Instead, quantum coherence of the final state and non-standard basis measurements are key ingredients. Since the lowest energy gap closes inverse polynomially, a finite temperature effect would be crucial. A fault-tolerance theory would be further required to achieve scalability. An arbitrary CSS state can be prepared in an adiabatic way, which would be interesting itself as a robust entangled state generation scheme using only two-body stoquastic interactions. Moreover, our construction can also be viewed as measurement-based quantum computation on a ground state of two-body Hamiltonians \([44, 48]\). In such a context, thermal equilibrium states are also known as universal resources \([49, 51]\). These constructions might be helpful in a construction of a fault-tolerant theory of stoqAQC at finite temperature. In addition, the embedding of the CSS state would be also useful for this purpose.

Finally, it would be interesting to see whether or not quantum Monte-Carlo or other classical methods relevant for stoquastic Hamiltonians can cope with the final basis change at the measurements. It would be natural to conjecture that stoqAQC with a polynomially
bounded lowest energy gap can be classified into three classes by the types of the measurements: (i) classically simulatable with standard basis measurements, (ii) non-universal but quantum computational supremacy with non-adaptive non-standard basis measurements, and (iii) universal with adaptive non-standard measurements.

In the most models exhibiting quantum computational supremacy [22, 52–58], the sampled output itself is not so useful, except for the one-clean qubit model [59–61] and HC1Q [20]. The present result has pushed sto-qAQC to one of the most powerful intermediate model of quantum computation, which exhibits quantum computational supremacy and can solve a meaningful problem, such as the factoring problem.

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