Semileptonic $B_c$ decays and Charmonium distribution amplitude

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Abstract

In this paper we study the semileptonic decays of the $B_c$ meson in the Light-Cone Sum Rule (LCSR) approach. The result for each channel depends on the corresponding distribution amplitude of the final meson. For the case of $B_c$ decaying into a pseudoscalar meson, to twist-3 accuracy only the leading twist distribution amplitude (DA) is involved if we start from a chiral current. If we choose a suitable chiral current in the vector meson case, the main twist-3 contributions are also eliminated and we can consider the leading twist contribution only. The leading twist distribution amplitudes of the charmonium and other heavy mesons are given by a model approach in the reasonable way. Employing this charmonium distribution amplitude we find the cross section $\sigma(e^+e^- \rightarrow J/\psi + \eta_c) \approx 22.8$ fb which is consistent with Belle and BaBar’s data. Based on this model, we calculate the form factors for various $B_c$ decay modes in the corresponding regions. Extrapolating the form factors to the whole kinetic regions, we get the decay widths and branching ratios for various $B_c$ decay modes including their $\tau$ modes when they are kinematically accessible.

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I. INTRODUCTION

The $B_c$ meson has been observed by the CDF and D0 groups in different channels \[1,2,3\]. The semileptonic decays of $B_c$ was studied in Ref. \[4\] using the BSW(Bauer, Stech, Wirbel) model \[5\] and the IGSW(Isgur, Grinstein, Scora, Wise) model \[6\], and in the framework of the Bethe-Salpeter equation in Ref. \[7\] and in the relativistic constituent quark model in Ref. \[8\]. Alongside the small differences in the partial decay widths in these models, the first estimates made on the basis of the three points (3P) QCD sum rules (SR) \[9\] are significant smaller. The reason was supposed to be the valuable role of Coulomb corrections, which implied the summation of $\alpha_s/v$ corrections significant in the $B_c$ \[10\]. It is suggested that the discrepancy observed between the QCD sum rules and the quark models can be eliminated by including these higher QCD corrections.

However, the 3PSR inherits some problems when describing heavy-to-light transitions, the main one being that some of the form factors have a nasty behavior in the heavy quark limit \[11\]. The reason is, when almost the whole momentum is carried by one of the constituents, the distribution amplitude of the final meson can be described by the short-distance expansion. Moreover, the calculation for the form factors is valid only at the point $q^2 = 0$, and a pole approximation has to be employed to study the semileptonic decays. These limit the applicability of QCD sum rules based on the short-distance expansion of a three-point correlation function to heavy-to-light transitions and calls for an expansion around the light-cone, as realized in the light-cone sum rule approach. In this paper we will try to study the semileptonic decays of the $B_c$ meson in this approach and compare the results with the traditional sum rule approach.

The semileptonic decays of the $B_c$ meson involve the transition $B_c \rightarrow \eta_c, J/\psi, D, D^*, B, B^*, B_s, B_s^*$. For the case of $B_c$ decaying into a pseudoscalar meson, to twist-3 accuracy only the leading twist distribution amplitude (DA) is involved if we start from a chiral current. If we choose a suitable chiral current in the vector meson case, the main twist-3 contributions are also eliminated and we can consider the leading twist contribution only. The result depends on the corresponding distribution amplitude (DA) of the final meson. We have to construct realistic models for describing the heavy quarkonium and other heavy mesons. In particular, the behavior of $\eta_c$ and $J/\psi$ DA’s is an interesting subject by the Belle result for the cross section $\sigma(e^+e^- \rightarrow J/\psi + \eta_c)$. Hence we pay more attention to discussing the
heavy quarkonium DA. We calculate the cross section $\sigma(e^+e^- \rightarrow J/\psi + \eta_c)$ by employing our charmonium distribution amplitude and the result is consistent with the experiment data. Based on a phenomenological model for the leading twist DA, we calculate the form factors for various $B_c$ decay modes in the corresponding regions. Then we extrapolate the form factors to the whole kinetic regions, and get the decay widths and branching ratios for various $B_c$ decay modes including their $\tau$ modes when they are kinematically accessible.

This paper is organized as follows. In the following section we derive the LCSRs for the form factors for various $B_c$ decay modes. A discussion of the DA models for charmonium and other heavy mesons is given in section III. In section IV the cross section $\sigma(e^+e^- \rightarrow J/\psi + \eta_c)$ is calculated by using our charmonium distribution amplitude. Section V is devoted to the numerical result for the semileptonic $B_c$ decays and comparison with other approaches. The last section is reserved for summary.

II. LCSRS FOR THE $B_c$ SEMILEPTONIC FORM FACTORS

According to the definition, the weak transition matrix element $B_c \rightarrow P(V)$ can be parametrized in term of the form factors in the following way:

$$<P(p_2)|\bar{q}\gamma_\mu Q|B_c(p_1)> = f_+(q^2)(p_1 + p_2)_\mu + f_-(q^2)q_\mu, \quad (1)$$

$$<V(p_2)|\bar{q}\gamma_\mu(1-\gamma_5)Q|B_c(p_1)> = -ie_\mu^\star(m_{B_c} + m_V)A_1(q^2) + i(p_1 + p_2)_\mu(e^\star q)\frac{A_+(q^2)}{m_{B_c} + m_V}$$
$$+ iq_\mu(e^\star q)\frac{A_-(q^2)}{m_{B_c} + m_V} + \epsilon_\mu^{\alpha\beta\gamma}e^\star q^\beta(p_1 + p_2)\gamma^\alpha \frac{V(q^2)}{m_{B_c} + m_V}, \quad (2)$$

where $q = p_1 - p_2$ is the momentum transfer, $e_\mu^\star$ is the polarization vector of the vector meson.

For $B_c \rightarrow P\ell\bar{\nu}$ we follow Ref. [12] and consider the correlator $\Pi_\mu(p,q)$ with the chiral current,

$$\Pi_\mu(p,q) = i\int d^4xe^{iqx}<P(p)|T\{\bar{q}(x)\gamma_\mu(1+\gamma_5)Q_1(x), \bar{Q}_1(0)i(1+\gamma_5)Q_2(0)\}|0>$$
$$= \Pi_+(q^2, (p + q)^2)(2p + q)_\mu + \Pi_-(q^2, (p + q)^2)q_\mu. \quad (3)$$

A standard procedure, concentrating on $\Pi_+(q^2, (p + q)^2)$, results in the following LCSR for $f_+(q^2)$:

$$f_+(q^2) = \frac{m_1(m_1 + m_2)f_P e^{m_{B_c}^2/M^2}}{m_{B_c}^2 f_{B_c}} \int_{\Delta^\nu} du \frac{\phi(u)}{u} \exp \left[-\frac{m_1^2 - \bar{u}(q^2 - um_2^2)}{u M^2}\right] + \text{higher twist terms} \quad (4)$$
with $\bar{u} = 1 - u$ and

$$\Delta_F = \frac{[\sqrt{(s_0^m - q^2 - m_{P}^2)^2 + 4m_{P}^2(m_F^2 - q^2)} - (s_0^m - q^2 - m_{P}^2)]}{2m_{P}^2},$$

(5)

where $m_1$ is the mass of the decay quark $Q_1$, $m_2$ the mass of the spectator quark $Q_2$, and $s_0$ and $M^2$ denote the corresponding threshold value and the Borel parameter respectively. In deriving Eq. (4) the following definition of the leading twist distribution amplitude (DA) $\varphi(u)$ of the pseudoscalar meson has been used:

$$<P(p)|T\bar{q}(x)\gamma_\mu\gamma_5Q(0)|0> = -ip_\mu f_P \int_0^1 du e^{iux} \varphi(u) + \text{higher twist terms},$$

(6)

with $u$ being the momentum fraction carried by $\bar{q}$. It has been pointed out in Ref. [12] that all the twist-3 contributions have been eliminated so those DA’s entering the higher twist terms in Eq. (4) are of at least twist 4. By repeating the procedure for $\Pi_-(q^2, (p + q)^2)$ we find a simple relation between $f_+(q^2)$ and $f_-(q^2)$ up to this accuracy:

$$f_-(q^2) = -f_+(q^2)$$

(7)

For $B_c \rightarrow Vl\bar{\nu}$ we choose the following correlator as our starting point:

$$\Pi_\mu(p, q) = -i \int d^4xe^{iqx} <V(p)|T\{\bar{q}(x)\gamma_\mu(1 - \gamma_5)Q_1(x), \bar{Q}_1(0)(1 + \gamma_5)Q_2(0)|0>$$

$$\Gamma^1 e^\mu - \Gamma_+(e^*q)(2p + q)q_\mu - \Gamma_-(e^*q)q_\mu + i\Gamma^V \varepsilon_{\mu\alpha\beta\gamma} e^{*\alpha} q^\beta p^\gamma.$$  

(8)

Also we take the standard definition of the twist-2 and twist-3 distribution amplitudes of the vector meson (see, e.g., Ref. [13]), and neglect higher twist DA’s which is supposed to be less important in comparison with those written below:

$$<V(p)|\bar{q}_\beta(x)Q_\alpha(0)|0> = \frac{1}{4} \int_0^1 du e^{iux} \{f_V m_V e^{*\mu} (u) + \tilde{p} (e^{*x} u)(\phi_{\parallel}(u) - \phi_{\perp}(u))\}$$

$$- if_V \varepsilon_{\mu\nu\rho\sigma} e^{*\mu} p^\nu \phi_{\perp}(u) + \frac{m_V}{4} (f_V - f_V m_q + M_Q) \varepsilon_{\mu\nu\rho\sigma} e^{*\mu} p^\nu x^\rho \phi_{\perp}(u)$$

(9)

In Eq. (9) $u$ is also the momentum fraction of $\bar{q}$, and $m_q(M_Q)$ is the mass of $\bar{q}(Q)$.

Similarly one can obtain the following sum rules for $A_1(q^2)$, $A_\pm(q^2)$ and $V(q^2)$ in Eq.(2):

$$A_1(q^2) = \int_0^1 \frac{du}{\Delta_V} \exp \left[ - \frac{m_1^2 - (1 - u)(q^2 - um_2^2)}{uM^2} \right] - \frac{m_1^2 - q^2 + u^2m_2^2}{u} \phi_\perp(u),$$

(10)
\[
A_+(q^2) = \frac{f_T^f(m_1 + m_2)(m_{Bc} + m_V)}{f_{Bc} m_{Bc}^2} e^{m_{Bc}^2/M^2}
\int_{\Delta V}^1 du \frac{du}{u} \exp \left[ -\frac{m_T^2 - (1 - u)(q^2 - um_V^2)}{u M^2} \right] \phi_\perp(u),
\]

(11)

\[
A_-(q^2) = -A_+(q^2),
\]

(12)

\[
V(q^2) = A_+(q^2)
\]

(13)

with

\[
\Delta V = \sqrt{\left(s_0^V - q^2 - m_V^2\right)^2 + 4m_V^2(m_1^2 - q^2) - (s_0^V - q^2 - m_V^2)} / (2m_V^2),
\]

(14)

and also \(m_1\) the mass of the decay quark \(Q_1\), \(m_2\) the mass of the spectator quark \(Q_2\).

III. THE DISTRIBUTION AMPLITUDES OF THE CHARMONIUM AND OTHER HEAVY MESONS

The leading twist distribution amplitude for the heavy quarkonium, as defined in the previous section, can be related to the light-cone wave function \(\psi_M^f(x, k_\perp)\) as:

\[
\varphi_M(x) = \frac{2\sqrt{6}}{f_M} \int \frac{d^2k_\perp}{16\pi^3} \psi_M^f(x, k_\perp)
\]

(15)

where \(f_M\) is the decay constant. In the non-relativistic case, the distribution amplitude \(\varphi_M(x)\) goes to the \(\delta\)-like function and the peak is at the point \(x = 1/2\). For heavy quarkonium, \(\eta_c\), the DA should be wider than the \(\delta\)-like function since the \(c\) quark is not heavy enough. Of course, it goes to \(\delta\)-function as the heavy quark mass \(m_c^* \to \infty\).

For the massive quark-antiquark system, Ref. \[14\] provides a good solution \(\psi_{C,M}(\vec{q}^2) = A \exp(-b^2\vec{q}^2)\) of the bound state by solving the Bethe-Salpeter equation with the the harmonic oscillator potential in the instantaneous approximation. Then one can apply Brodsky-Huang-Lepage (BHL) prescription \[15\]:

\[
\psi_{C,M}(\vec{q}^2) \leftrightarrow \psi_{LC} \left( \frac{k_\perp^2 + m_Q^2}{x(1 - x)} - M^2 \right)
\]

(16)

and get the momentum space LC wave function:

\[
\psi_M(x, k_\perp) = A_M \exp \left[ -b_M^2 \frac{k_\perp^2 + m_Q^2}{x(1 - x)} \right]
\]

(17)

where \(m_Q^*\) is the heavy quark mass and \(M\) is the mass of the quarkonium. Furthermore, the spin structure of the light-cone wave function should be connected with that of the instant-form wave function by considering the Wigner-Melosh rotation. As a result, the full form of
the light-cone wave function should be
\[ \psi^f_M(x, k_\perp) = \chi_M(x, k_\perp) \psi_M(x, k_\perp) \] (18)
with the Melosh factor
\[ \chi_M(x, k_\perp) = \frac{m_Q^*}{\sqrt{k_\perp^2 + m_Q^*}} \] (19)
After integrating out \( k_\perp \), the leading-twist distribution amplitude of the heavy quarkonium becomes
\[ \Phi^f_M(x) = \sqrt{6} A_M m_Q^* \sqrt{x(1-x)} \left[ 1 - E_r f \left( \frac{b_M m_Q^*}{\sqrt{x(1-x)}} \right) \right], \] (20)
where \( E_r f(x) = \frac{2}{\pi} \int_0^x \exp(-t^2) dt \). As \( m_Q^* \to \infty \), \( \Phi_M(x) \) goes to the \( \delta \)-like function certainly.
This model of the \( \eta_c \) distribution amplitude has been used to study the large-\( Q^2 \) behavior of \( \eta_c - \gamma \) and \( \eta_b - \gamma \) transition form factors in Ref. [16]. The parameters \( A_M \) and \( b_M^2 \) in Eq. (17) can be determined by two constraints on them completely. One constraint is from the leptonic decay constant \( f_M \)
\[ \int_0^1 dx \int \frac{d^2k_\perp}{16\pi^3} \chi_M(x, k_\perp) \psi_M(x, k_\perp) = \frac{f_M}{2\sqrt{6}} \] (21)
and another one from the probability of finding the \( |Q\bar{Q}> \) state in the heavy quarkonium,
\[ \int_0^1 dx \int \frac{d^2k_\perp}{16\pi^3} |\psi_M(x, k_\perp)|^2 = P_M. \] (22)
with \( P_M \approx 1 \) for heavy quarkonium. Inputting the constituent mass \( m_c^* \approx 1.5 \text{ GeV} \), and the decay constant \( f_{\eta_c} \approx 0.40 \text{ GeV} \), We get the corresponding parameters for \( \eta_c \):
\[ A_{\eta_c} = 128.1 \text{ GeV}, b_{\eta_c} = 0.427 \text{ GeV}^{-1} \] (23)
Then the behavior of the leading twist \( \eta_c \) DA can be given and the comparison with the model from the QCD sum rule analysis [17] and the model in Ref. [13] is plotted in Fig. 1.
The moments of these models are given in Tab. 1. All the wave functions and corresponding moments are defined at the soft scale \( \mu^* \approx 1 \text{ GeV} \). However, the appropriate scale \( \mu \) for the wave functions entering the LC sum rules will be \( \mu \approx m_b \) for \( b \)-quark decays and \( \mu \approx m_c \) for \( c \)-quark decays with \( m_b \) and \( m_c \) the one loop pole masses. Since \( \mu \) is not far from \( \mu^* \), this
\[ \text{1 The value of } f_{J/\psi} \text{ is taken from the leptonic decay of } J/\psi: \Gamma(J/\psi \to e^+e^-) = (16\pi\alpha^2/27)(|f_{J/\psi}|^2/M_{J/\psi}), f_{J/\psi} \approx 0.41 \text{GeV}. \] The one loop corrections \( (\sim \alpha_s/\pi) \) to the ratio \( \Gamma(\eta_c \to 2\gamma)/\Gamma(J/\psi \to e^+e^-) \) indicate that \( f_{\eta_c} \) is slightly smaller than \( f_{J/\psi} \) and we take \( f_{\eta_c} \approx f_{J/\psi} \approx 0.40 \text{GeV} \) on average.
TABLE I: The moments of our model for the $\eta_c$ distribution amplitude, compared with that in Ref. [13] and Ref. [17].

| $\xi^n$ | This work | [13] | [17] |
|---------|-----------|------|------|
| $n=2$   | 0.21      | 0.13 | 0.07 |
| $n=4$   | 0.053     | 0.040| 0.012|
| $n=6$   | 0.018     | 0.018| 0.003|

Scale dependence can be neglected in our calculations for simplicity. From Tab. I it can be found that the moments of the model (20) is similar to that in Ref. [13], but much larger than that in Ref. [17]. Obviously the Melosh factor $\chi_M(x, k_\perp) \to 1$ in the heavy quark limit $m_Q^* \to \infty$. If we neglect this factor and integrate $k_\perp$ from Eq.(17), we get the corresponding distribution amplitude which has a much simple form:

$$\varphi_M(x) = \frac{\sqrt{3}A_M}{8\pi^2 f_M b_M^2} x (1-x) \exp \left[ -\frac{b_M^2 m_Q^* x}{x(1-x)} \right].$$

(24)

Actually this is just the wave function proposed in Ref. [17] based on the QCD sum rule analysis.

FIG. 1: The model for the leading twist distribution amplitude for $\eta_c$ (in solid line), in comparison with the one in Ref. [13] (dashed line) and Ref. [17] (dash-dotted line).
For the vector charmonium, $J/\psi$, it is expected that the behavior of the transverse distribution amplitude is the same as that of the longitudinal DA since there is no light quark in the charmonium system, i.e.:

$$\phi_\parallel(x) = \phi_\perp(x) = \varphi_{\psi_c}(x)$$

which is confirmed by the moment calculation in the QCD sum rules [18].

For the $D$, $B$ and $B_s$ meson, which are composed by one heavy ($\bar{Q}_1$) and one light quark ($Q_2$), According to the BHL prescription one takes the following connection:

$$\psi_{C.M}(q^2) \leftrightarrow \psi_{L.C} \left( \frac{k_1^2 + m_1^*}{x} + \frac{k_2^2 + m_2^*}{1-x} - M_P^2 \right).$$

with $m_1^*(m_2^*)$ the constituent quark mass of $\bar{Q}_1(Q_2)$, $x$ the momentum fraction carried by $\bar{Q}_1$. Also the Melosh factor should be modified as

$$\chi_P(x, k_\perp) = \frac{(1-x)m_1^* + xm_2^*}{\sqrt{k_\perp^2 + ((1-x)m_1^* + xm_2^*)^2}}.$$

From which we get the light-cone wave function for pseudoscalar meson

$$\psi_P^f(x, k_\perp) = A_P \chi_P(x, k_\perp) \exp \left[ -b_P^2 \left( \frac{k_1^2 + m_1^*}{x} + \frac{k_2^2 + m_2^*}{1-x} \right) \right].$$

and the corresponding distribution amplitude

$$\varphi_P(x) = \frac{\sqrt{6}A_P y}{8\pi^{3/2}f_P b_P} \sqrt{x(1-x)[1 - Erf \left( \frac{b_P y}{\sqrt{x(1-x)}} \right) \exp \left[ -b_P^2 \frac{(xm_2^* + (1-x)m_1^* - y^2)}{x(1-x)} \right]}}.$$

where $y = xm_2^* + (1-x)m_1^*$. Similarly, there are two constraints Eq. (21) and Eq. (22) to determine the unknown parameters. We take $P_D \approx 0.8$, $P_B \approx P_{B_s} \approx 1.0$ as suggested in Ref. [21]. Inputting the decay constants (We use the least-squares fit values of the results reported by the CLEO Collaboration [22] and lattice simulations [23, 24, 25, 26, 27, 28], see Tab. II) and the constituent quark masses $m_u^* = 0.35$ GeV, $m_s^* = 0.5$ GeV, $m_c^* = 1.5$ GeV, $m_b^* = 4.7$ GeV, we get the parameters:

$$A_D = 116 \text{ GeV} \quad b_D = 0.592 \text{ GeV}^{-1},$$
$$A_B = 1.07 \times 10^4 \text{ GeV} \quad b_B = 0.496 \text{ GeV}^{-1},$$
$$A_{B_s} = 2.65 \times 10^4 \text{ GeV} \quad b_{B_s} = 0.473 \text{ GeV}^{-1}. \quad (30)$$

\footnote{This model has been used in Refs. [19, 20] for the $D$ meson distribution amplitude with the different parameters. There was a misprint of the factor $\sqrt{2}$ with the decay constant in Ref. [19].}
TABLE II: Leptonic decay constants (MeV) used in the least-squares fit for our model parameters.

|        | This work | other                                      |
|--------|-----------|--------------------------------------------|
| $f_D$  | 223       | $222.6 \pm 16.7^{+2.8}_{-3.4}$ CLEO [22]   |
|        |           | $201 \pm 3 \pm 17$ MILC LAT [23]           |
|        |           | $235 \pm 8 \pm 14$ LAT [24]                |
|        |           | $210 \pm 10^{+17}_{-16}$ UKQCD LAT [25]    |
|        |           | $211 \pm 14^{+2}_{-12}$ LAT [26]           |
| $f_B$  | 190       | $216 \pm 9 \pm 19 \pm 4 \pm 6$ HPQCD LAT [27] |
|        |           | $177 \pm 17^{+22}_{-22}$ UKQCD LAT [25]    |
|        |           | $179 \pm 18^{+34}_{-9}$ LAT [26]           |
| $f_{B_s}$ | 220       | $259 \pm 32$ HPQCD LAT [27]                |
|        |           | $204 \pm 16^{+36}_{-0}$ LAT [26]           |
|        |           | $260 \pm 7 \pm 26 \pm 8 \pm 5$ LAT [28]   |
|        |           | $204 \pm 12^{+24}_{-23}$ UKQCD LAT [25]    |

The distribution amplitudes of these heavy-light mesons are plotted in Fig. 2. The distribution amplitudes of the corresponding vector mesons are treated in the same way as $J/\psi$.

IV. THE CROSS SECTION $\sigma(e^+e^- \rightarrow J/\psi + \eta_c)$

Following Ref. [13], the cross section $\sigma(e^+e^- \rightarrow J/\psi + \eta_c)$ can be calculated by using the distribution amplitudes (20) and (25). We neglect the complicated but slow logarithmic evolution of wave function forms, and account only for the overall renormalization factors of the local tensor and pseudoscalar currents and for running of the quark mass, as in Ref. [13]. One obtains

$$\sigma(e^+e^- \rightarrow J/\psi + \eta_c) \simeq 22.8 \text{ fb}$$

which is consistent with Belle and BaBar’s measurements [29, 30] of this cross section:

$$\sigma(e^+e^- \rightarrow J/\psi + \eta_c) = 25.6 \pm 2.8 \pm 3.4 \text{ fb (Belle)}.$$
\[ \sigma(e^+e^- \rightarrow J/\psi + \eta_c) = 17.6 \pm 2.8^{+1.5}_{-2.1} \text{ fb} \ (\text{BaBar}). \] (32)

The value given by Eq. (31) is the same order of the numerical result in Ref. [13] and much larger than the standard non-perturbative QCD (NRQCD) calculation. The reason is that the DA behavior of the charmonium in our paper and Ref. [13] is much wider than \( \delta \)-like function due to the relativistic effect. Also our result confirms the observation by Ref. [31]. It may be expected that the large disagreement between the experimental data and the standard NRQCD calculation can be resolved by combining the light-cone wave function with relativistic effect and radiative corrections [32].

V. NUMERICAL RESULT FOR SEMILEPTONIC \( B_c \) DECAYS

For the decay constant of the \( B_c \) meson, we recalculate it in the two-point sum rules using the following correlator

\[ K(q^2) = i \int d^4x e^{iqx} < 0| \bar{c}(x)(1 - \gamma_5)b(x), \bar{b}(0)(1 + \gamma_5)c(0)|0 >, \] (33)

for consistency. The calculation is performed to leading order in QCD, since the QCD radiative corrections to the sum rule for the form factors are not taken into account. We also neglect the higher power correction corresponding to the gluon condensates. The value of the threshold parameter \( s_0 \) is determined by requiring the experimental value of \( B_c \) be obtained in the reduced sum rule after taking the derivative of the logarithm of the SR with
respect to $1/M^2$. The quark mass parameters entering our formulas are the one-loop pole masses for which we use $m_b = 4.7$ GeV and $m_c = 1.3$ GeV (cf. Tab. 3 and Tab. 4 in the review [33] and references therein). To get the experimental value $m_{B_c} = 6.286$ GeV [2], we find $s_0$ should be $s_0 \approx 42.0$ GeV$^2$, which is smaller than the threshold value input in the ordinary sum rule [34]. This will ensure in some sense that the scalar resonances will make less contribution in our sum rule. The corresponding value of $f_{B_c}$ is $f_{B_c} = 0.189$ GeV, which is smaller than that in Ref. [34] since we do not include the $\alpha_s$ corrections. The same set of parameters will be used in the LCSRs for the form factor in order to reduce the quark mass dependance. Take the derivative of the logarithm of the LCSR for the form factors with respect to $1/M^2$, we get a sum rule for the mass of the $B_c$ meson. Requiring this sum rule to be consistent with the experiment value at $q^2 = 0$, we can determine $M^2$ for each LCSR. This results in $M^2(B_c \to \eta_c) = 25.8$ GeV$^2$, $M^2(B_c \to D) = 11.6$ GeV$^2$, $M^2(B_c \to B) = 112$ GeV$^2$, $M^2(B_c \to B_s) = 111$ GeV$^2$. It seems that the Borel parameters for $B_c \to B(B_s)$ are somewhat large. However, the LCSRs are quite stable in the large region of the Borel parameter $50$ GeV$^2 < M^2 < 150$ GeV$^2$ actually and we just use the above value for explicit calculation. For the vector meson we simply use the same $M^2$ as the corresponding pseudoscalar meson just as we do for the DA’s. Also we make the assumption that $f_{T}^V = f_{V} = f_{P}$.

With all the parameters chosen, we can proceed to calculate all the form factors involved. The results of the form factors at $q^2 = 0$ are given in Tab. III in comparison with those from other approaches. Notice that in our calculation we always have:

$$f_+(q^2) > 0, f_-(q^2) < 0, A_1(q^2) > 0$$
$$A_+(q^2) > 0, A_-(q^2) < 0, V(q^2) > 0.$$  \hspace{1cm} (34)

In the 3PSR approach the same relations can be obtained, but only in the case of nonrelativistic description for both initial and final meson states, eg., $B_c \to J/\psi(\eta_c)$. In these decay modes the QM results show the same signature pattern, as can be seen in Tab. III.

Our calculations for the form factors only valid in limited regions where the operator product expansion (OPE) goes effectively. For $b$-quark decays, the LCSR is supposed to be valid in $0 < q^2 < m_b^2 - 2m_b\Lambda_{QCD} \simeq 15$ GeV and for $c$-quark decays $0 < q^2 < m_c^2 - 2m_c\Lambda_{QCD} \simeq 0.4$ GeV. It turns out that the calculated form factors for can be fitted excellently by the
TABLE III: The values of the form factors at $q^2 = 0$ in comparison with the estimates in the three points sum rule (3PSR) (with the Coloumb corrections included) [10] and in the quark model (QM) [8].

| Mode                   | $f_+(0)$ | $f_-(0)$ | $A_1(0)$ | $A_+(0)$ | $A_-(0)$ | $V(0)$ |
|------------------------|---------|---------|---------|---------|---------|--------|
| $B_c \rightarrow \bar{c}c[1S]$ | This work | 0.87   | -0.87   | 0.75    | 1.69    | -1.69  | 1.69   |
| 3PSR [10]              | 0.66    | -0.36   | 0.63    | 0.69    | -1.13   | 1.03   |
| QM [8]                 | 0.76    | -0.38   | 0.68    | 0.66    | -1.13   | 0.96   |
| $B_c \rightarrow B_s^{(*)}$ | This work | 1.02   | -1.02   | 1.01    | 9.04    | -9.04  | 9.04   |
| 3PSR [10]              | 1.3     | -5.8    | 0.69    | -2.34   | -21.1   | 12.9   |
| QM [8]                 | -0.61   | 1.83    | -0.33   | 0.40    | 10.4    | 3.25   |
| $B_c \rightarrow \bar{c}c[1S]$ | This work | 0.90   | -0.90   | 0.90    | 7.9     | -7.9   | 7.9    |
| 3PSR [10]              | 1.27    | -7.3    | 0.84    | -4.06   | -29.0   | 15.7   |
| QM [8]                 | -0.58   | 2.14    | -0.27   | 0.60    | 10.8    | 3.27   |
| $B_c \rightarrow D^{(*)}$ | This work | 0.35   | -0.35   | 0.32    | 0.57    | -0.57  | 0.57   |
| 3PSR [10]              | 0.32    | -0.34   | 0.43    | 0.51    | -0.83   | 1.66   |
| QM [8]                 | 0.69    | -0.64   | 0.56    | 0.64    | -1.17   | 0.98   |

Extrapolate the calculated form factors to whole kinetic region using this parametrization, we can proceed to calculate the branching ratios of the simileptonic decays of $B_c$. The results is shown in Tab. IV together with those of other approaches, where we have used the following CKM-matrix elements:

$$V_{cb} = 0.0413, \quad V_{ub} = 0.0037,$$

$$V_{cs} = 0.974, \quad V_{cd} = 0.224.$$  \hspace{1cm} (36)

For the $b$-quark decay modes in the $B_c$ meson, our results for the branching ratios are much larger than the corresponding results in the 3PSR approach. In these decays the kinetic region is rather large, so the branching ratios depend slightly on the absolute value of the form factors at $q^2 = 0$. In the LCSR approach, the form factors always increase much faster.
TABLE IV: Branching ratios (in %) of simileptonic $B_c$ decays into ground state charmonium states, and into ground charm and bottom meson states. For the lifetime of the $B_c$ we take $\tau(B_c) = 0.45$ps.

| Mode   | This work | 3PSR [10] | QM [8] | [7] |
|--------|-----------|-----------|--------|-----|
| $\eta_c e\nu$ | 1.64      | 0.75      | 0.98   | 0.97|
| $\eta_c \tau\nu$ | 0.49      | 0.23      | 0.27   | —   |
| $J/\psi e\nu$ | 2.37      | 1.9       | 2.30   | 2.30|
| $J/\psi \tau\nu$ | 0.65      | 0.48      | 0.59   | —   |
| $D e\nu$ | 0.020     | 0.004     | 0.018  | 0.006|
| $D \tau\nu$ | 0.015     | 0.002     | 0.0094 | —   |
| $D^* e\nu$ | 0.035     | 0.018     | 0.034  | 0.018|
| $D^* \tau\nu$ | 0.020     | 0.008     | 0.019  | —   |
| $B e\nu$ | 0.21      | 0.34      | 0.15   | 0.16|
| $B^* e\nu$ | 0.32      | 0.58      | 0.16   | 0.23|
| $B_s e\nu$ | 3.03      | 4.03      | 2.00   | 1.82|
| $B_s^* e\nu$ | 4.63      | 5.06      | 2.6    | 3.01|

than the simple pole approximation required in the 3PSR analysis, which accounts for the discrepancy in these decays. For the $c$-quark decays in the $B_c$ meson, where the kinetic region is narrow enough, our results are consistent with the 3PSR approach roughly.

VI. SUMMARY

The semileptonic decays of the $B_c$ meson are studied in the Light-Cone sum rule approach. By using suitable chiral currents, we derive simple sum rules for various form factors, which depend mainly on the leading twist distribution amplitude of the final meson. A model with the harmonic oscillator potential for the light-cone wave function is employed. Special attention is payed to the leading DA of the charmonium. It has been found that our model is consistent with the QCD sum rule analysis. Also, the moments are found to be similar to the model proposed in Ref. [13]. Based on this model, we calculate the form factors for various $B_c$ decay modes in the corresponding regions. Extrapolating the form factors
to the whole kinetic regions, we get the decay widths and branching ratios for all the $B_c$ semileptonic decay modes. For the $b$-quark decay modes in the $B_c$ meson, where the kinetic regions are quite large, our results for the branching ratios are much larger than the 3PSR results. For the $c$-quark decays in the $B_c$ meson, they are consistent with each other in general.

It is a crucial point to construct a realistic model for the light cone wave function of the charmonium which is not a non-relativistic subject. Based on the solution of the relativistic Bethe-Salpeter equation in the heavy quark system, we provide a model in this paper by using the BHL prescription and the behavior of the charmonium DA is much wider than $\delta$-like function which was employed essentially by the approximation of NRQCD. Thus the cross section $\sigma(e^+e^- \to J/\psi + \eta_c)$ can be enhanced considerably and is about 22.8 fb.
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