Driven dust vortex characteristics in plasma with external transverse and weak magnetic field

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Abstract
The two-dimensional hydrodynamic model for bounded dust flow dynamics in plasma is extended for analysis of driven vortex characteristics in presence of external transverse and weak magnetic field (B) in a planner setup and parametric regimes motivated by recent magnetized dusty plasma (MDP) experiments. This analysis has shown that shear in the B can produce a sheared internal field (E_a) in between electrons and ions due to the E × B and ∇B × B-drifts that cause rotation of dust cloud levitated in the plasma. The flow solution demonstrates that neutral pressure decides the dominance between the ions-drag and the E_a-force. The shear ions-drag generates an anti-clockwise circular vortical structure, whereas the shear E_a-force is very localized and gives rise to a clockwise D-shaped elliptical structure which turns into a meridional structure with decreasing B. Effect of the strength of B, shear mode numbers, and the sheath field are analyzed within the weak MDP regime, showing noticeable changes in the flow structure and its momentum. In the regime of high pressure and lower B, the E_a-force becomes comparable or dominant over the ion drag and peculiar counter-rotating vortex pairs are developed in the domain. Further, when the B is flipped by 180⁰-degree, both the drivers act together and give rise to a single strong meridional structure, showing the importance of B-direction in MDP systems. Similar elliptical/meridional structures reported in several MDP experiments and relevant natural driven-dissipative flow systems are discussed.

1. Introduction

Dusty plasma is a partially ionized and low-temperature plasma consisting of additional suspended micron-sized charged dust particles, which is known for being an ideal domain for experimental and theoretical studies of basic collective behaviors in relevant nonequilibrium flows system of nature [1–7]. When the suspended dust particles are numerous, the interaction with the background plasma and neutral gas constitutes a range of interesting fluid-phases phenomena such as waves, instabilities, vortices, void, and other nonlinear structures [2, 8–11]. In particular, the vortices are widespread from small scale biological complexity to large scale planetary surface [4, 12, 13, 7, 14]. The vortex structure in the nonequilibrium systems evolves subjected to various driving fields and the dynamical regimes as in the dusty plasma [4, 5, 15]. The vortical structure has been observed in different shapes, sizes, numbers, orientations, and strength [12, 15, 16]. For examples, the elliptical flow structures are observed in magnetized dusty plasma (MDP) systems [9, 15], which is also common in magnetic confinement configurations in a tokamak, FRC (field reverse configuration) [17, 18], and in neutral fluid flows such as the sedimentation of dust in a lid-driven cavity at low to moderate Reynolds number [19, 20, 5]. Further, the vortex structure plays an important role in the fluid mixing and transport process in laminar and turbulent flows [4, 12]. Due to the ubiquitous vortex flow in nature, it has been a topic of active research in finding the fundamental sources of the vorticity and how the vortical structure evolves with system parameters in various complex flow systems. Using the dusty plasma, numbers of our past studies have investigated the characteristic features of the vortex in nonequilibrium systems [6, 7, 14]. However, the underlying physics of elliptical vortical structures are yet to be fully understood.
Recently, the elliptical vortical structures are reported in several MDP experiments [15, 9], and there is a resurgence of interest in the studies of dust collective behaviors in the MDP because of its importance and varieties of applications in studies of astrophysical star formation, space, laboratory discharge, fusion devices, and industrial processing [21–24]. For example, Mark Kushner [24] and Barnat et al [25] analyzed the RF discharge of the industrial etching process under the influence of a transverse magnetic field, showing the plasma becomes more resistive to electrons and the distribution becomes skewed toward the \((\mathbf{E} \times \mathbf{B})\)-drift. The mobility of electrons and ions are different, therefore, charge separation is created along with the \((\mathbf{E} \times \mathbf{B})\)-drift in presence of the collisions. Thus, a new internal electric field called ambipolar electric field \(\mathbf{E}_a\) is arisen in between the magnetized electrons and the ions. In space science, atmospheric escape remains an open issue, and Cohen [22] reported the transport of ionospheric constituents to Earths (as well as other in other planets) magnetosphere by the similar ambipolar \(\mathbf{E}_a\) generated in the system. Therefore, a detailed analysis of the \(\mathbf{E}_a\) and Lorentz force acting on the dust particles other than the usual ions and neutral drags is crucially important for understanding the relevant physical process in nature.

One main difficulty in the MDP is that the domain is widely diverse starting from strongly magnetized to weakly magnetized systems, from dominant longitudinal to transverse magnetic fields, and many more. In most of the past MDP-studies where dust vortices are experimentally observed, the magnetic field is considered parallel to the sheath electric field \(\mathbf{E}_s\) i.e., \(\mathbf{B}||\mathbf{E}_s\), and both the fields are directed along with the gravity [26, 27, 9, 15]. In these experiments, dust is not magnetized, however, the \((\mathbf{E} \times \mathbf{B})\)-drift of the magnetized ions excited the rotational motion of the dust particles. In recent years, a lot of efforts have been made to fully magnetize heavy dust in MDP laboratories [28, 29, 15], although it is a big challenge as the dust is relatively heavy and so requires a strong magnetic field (\(B \geq 4\) Tesla) [28]. On the other, there has been less discussion in MDP with the transverse magnetic field where the \(B.\mathbf{E}_a\).

Yeng and Maemura et al [30, 31] discussed the transport of the dust particles raised by the \((\mathbf{E} \times \mathbf{B})\)-drift of electrons in the system. Samsonov et al [32] analyzed the enhancement of dust levitation by an inhomogeneous magnetic field \((\nabla \mathbf{B})\). Also, the inhomogeneity of density \((\nabla n)\) is common in realistic low-temperature plasma with dust particles [33]. Further, in the experiment by Puttscher and Melzer et al [34], the dynamics of dust particle displacement are investigated in presence of the transverse and weak \(\mathbf{B}\) i.e., only electrons are magnetized. They observed the displacement of single particles and the whole dust clusters by the \(\nabla \mathbf{B}\)-drift and ambipolar \((\mathbf{E} \times \mathbf{B})\)-drift due to the magnetization of the electrons. Further, In their subsequent work [35, 36], they discussed the competition between the ambipolar \((\mathbf{E} \times \mathbf{B})\)-drift and ion/neutral drag using a force balance model that decides the displaced direction of the dust cluster. However, there are many queries as to whether 2D vortex of the dust cloud can be developed in the same configuration. What will be the effect of diamagnetic-drift and the gradient-drift which are very common in realistic laboratory plasma [37]? Most importantly what will be the characteristics of dust vortex structure in the setup with transverse and weak \(\mathbf{B}\)? To know the physics insight of the above queries, systematic studies using a theoretical and numerical analysis of the MDP system are required as attempted in the present work.

Studies of steady-state dust vortex characteristics in a non-magnetized and incompressible fluid-phase regime of dusty plasmas have been extensively done in a series of our theoretical-simulation work [14, 7, 6, 38]. In the current work, we extend the existing model in presence of external transverse and weak \(\mathbf{B}\) in a planner setup and parametric regimes motivated by recent magnetized dusty plasma(MDP) experiments [35, 36, 34, 26, 27, 39]. This formulation reveals the conditions for sustaining a 2D vortex of the dust cloud in a cross-section parallel \((\omega_c)\) and perpendicular \((\omega_ic)\) to the \(\mathbf{B}\) as highlighted in figure 1. Shear in the \(\mathbf{B}\) can produce a shear internal field \(\mathbf{E}_s\) in between electrons and ions due to the combined \(\mathbf{E}_s \times \mathbf{B}\) and \(\nabla \mathbf{B} \times \mathbf{B}\)-drifts that causes a rotation of the dust cloud levitated in the plasma. Effect of system parameters including the strength of shear \(\mathbf{B}\) and the \(\mathbf{E}_s\) are analyzed within the weak \(\mathbf{B}\) regime \((\omega_ic \gg \nu_{ic} \gg \nu_{ic})\) to \(\nu_{ic} \gg \omega_{ic}\), where, \(\omega_{ic}\) is the cyclotron frequency and \(\nu_{ic}\) is collision frequency of \(i\)-species with neutrals), demonstrating the role of \(\mathbf{B}\)-direction and formation of peculiar counter-rotating vortex pairs developed in the same setup.

This paper is organized as follows. In section 2, we discuss the extended 2D hydrodynamic model for a bounded dust flow in a MDP setup, deriving the general equation of the shear field \(\mathbf{E}_s\) and associated vorticity sources. The 2D dust vortex solutions are characterized concerning specific driving fields and parametric regimes in section 3. The circular anti-clockwise structure due to the dominant ion-drag force is analyzed in section 3.2, whereas the elliptical/meridional structure due to a dominant shear \(\mathbf{E}_s\)-force is discussed in section 3.3. Further, a new condition for the formation of counter-rotating vortex pairs and the impact of the \(\mathbf{E}_s\) are discussed in section 3.4. Summary and conclusions are presented in section 4.

2. Hydrodynamical model of dust dynamics in a magnetized plasma

As discussed above, the present work is motivated by several new dusty plasma experiments [30, 31, 34–36] which have studied the behavior of dust particle dynamics applying transverse \(\mathbf{B}\) near the sheath region of a glow discharge plasma. Therefore, we consider a similar system of dust cloud/cluster suspended near the sheath region of magnetized plasma. The schematic cross-section of the system is shown in figure 1, which highlights
dust particles are trapped close to the sheath region, using an electrostatic potential \( \phi \) and a transverse magnetic field \( \mathbf{B} \) perpendicular to the applied sheath field \( \mathbf{E}_s ( - \hat{y} ) \). The main driving forces are ion-drag force \( \mathbf{F}_i ( - \hat{y} ) \) and the internal ambipolar field \( \mathbf{F}_a \) throughout the dust domain.

Figure 1. Schematic representation of dust cloud levitated above the electrode by an electrostatic potential \( \phi \), in presence of sheath field \( \mathbf{E}_s ( - \hat{y} ) \), transverse magnetic field \( \mathbf{B} \) having shear \( \nabla \mathbf{B} \cdot \hat{y} \). The main driving forces are ion-drag force \( \mathbf{F}_i ( - \hat{y} ) \) and the internal ambipolar field \( \mathbf{F}_a \) throughout the dust domain.

Now, we extend the hydrodynamic model for the dust flow from our previous work considering the weak transverse \( \mathbf{B} \) [7]. For a bounded dust cloud that satisfies incompressible, isothermal conditions, and has a finite viscosity, the dynamics in the magnetized plasma can be model by the simplified continuity equation and Navier–Stokes momentum equation as follows [40, 7, 41],

\[
\nabla \cdot \mathbf{u} = 0, \tag{1}
\]

\[
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla \phi_b + \frac{q_d}{m_d} \mathbf{E}_a + \frac{1}{\rho} (\mathbf{J}_d \times \mathbf{B}) - \frac{\nabla P}{\rho} + \mu \nabla^2 \mathbf{u} - \xi (\mathbf{u} - \mathbf{v}) - \nu (\mathbf{u} - \mathbf{w}). \tag{2}
\]

Here \( \mathbf{u}, \mathbf{v}, \) and \( \mathbf{w} \) are the flow velocities of the dust, ion, and neutral fluids, respectively. \( \phi_b \) is the effective confining potential (from all the conservative fields including gravity), \( q_d \) is dust charge, \( m_d \) is dust mass, and \( \mathbf{J}_d = n_g \mathbf{j}_d \mathbf{u} \) is dust current density. \( \mathbf{E}_a \) is the internal field due to charge separation between electrons and ions of the background plasma due to the magnetization [33]. \( P \) and \( \rho \) are the pressure and mass density of the dust fluid, respectively, \( \mu \) is the kinematic viscosity, \( \xi \) is the coefficient of ion–drag acting on the dust, and \( \nu \) is the coefficient of friction generated by the background neutral fluid [42–44]. The above equation (2) describes that the dust cloud can be confined near the sheath region of the discharge by balancing all the conservative fields \( \phi_b \), and it can be driven either by the \( \mathbf{E}_a \)-force or by the interaction with the background ion or neutral drag present in the system. The proposed model consisting of various force terms is motivated by recent works in magnetized dusty plasma [9]. We know that during the longer time scale on which the steady dust flow is maintained, all the other highly mobile fluids such as electrons and ions have already been in thermal equilibrium even though it interacts with the confined dust fluid and drive it. Therefore, the steady-state profile of \( \mathbf{E}_a \) of electron magnetization and ions velocity field \( \mathbf{v} \) in equation (2) is valid for the analysis of steady dust flow characteristics in the weakly magnetized dusty plasma.

### 2.1. Vorticity-stream function formulation

Our main goal is to study the characteristics of a 2D vortex or circulating motion of a dust cloud/cluster confined in a weak magnetized dusty plasma. Therefore, taking the curl of the above equation (2), the steady dust flow in a 2D plane either in \( \mathbf{xy} \) or \( \mathbf{xz} \) can be written in term of dust stream function (\( \psi \)) and corresponding vorticity (\( \omega \)) as follows [6, 7],

\[
\nabla^2 \psi = -\omega, \tag{3}
\]

\[
\frac{\partial \omega}{\partial t} + (\mathbf{u} \cdot \nabla) \omega = \mu \nabla^2 \omega - (\xi + \nu) \omega + \xi \omega_x + \beta \omega_y. \tag{4}
\]

Where, \( \mathbf{u} = \nabla \times \psi, \omega = \nabla \times \mathbf{u} \), and the direction of the \( \psi \) and \( \omega \) is always perpendicular to the chosen 2D plane. The curl of terms having conservative fields such as \( P \) and \( \phi \) goes to zero. It means conservative forces do not
Contribute to any vortex dynamics and their other roles are absorbed in the stream function and vorticity fields. \( \omega_i \) is the external vorticity source from the curl of \( \nu \) and \( \nu i \) terms, i.e., from the shear nature of unbounded background flow fields. For simplicity, the neutral velocity field is assumed to be stationary or we are on the neutral frame of reference. In a general electrostatic fluid constituted by charged particles, the coefficients \( \xi \) and \( \nu \) depend on the state variables such as temperature, densities, velocity, and charge distribution \cite{42, 43, 44}. As a consequence, the \( \omega_i \) in the equation (4) represents any non-zero fields, such as \( \nabla \times \mathbf{u}_{i(n)} \), \( \nabla Q_d \times \mathbf{E} \), \( \nabla u_{i(n)} \times \nabla n_{i(n)} \), or their effective combinations as the external driving mechanism for the dust vorticity in a non-magnetized dusty plasma \cite{14}. Where \( Q_d \) is dust charge, \( E \) is an effective electric field along with the streaming ions with velocity \( u_i \) and density \( n_i \). The subscript \( i(n) \) represents the background ions ( neutrals). Further, \( \omega_i \) is the additional vorticity source from the curl of magnetization forces, and \( \beta \) is the co-efficient of magnetization. The detailed derivation of \( \beta \omega_i \) is discussed in the following section 2.2.

### 2.2. Vorticity sources \((\beta \omega_i)\) in a weakly magnetized plasma

From the above equations (2) and (4), the addition vorticity source \( \beta \omega_i \) can be written as follow,

\[
\beta \omega_i = \nabla \times \left[ \frac{q_i}{m_i} \mathbf{E}_a + \frac{1}{\rho} (\mathbf{J}_d \times \mathbf{B}) \right]
\]

In the weakly magnetized dusty plasma, the last term, i.e., Lorentz force on the dust particles is neglected for the rest of the present analysis. Then, for the two-dimensional plasma flow across \( B \), it is known that charge separation usually takes place due to available drifts (such as \( \mathbf{E} \times B \), \( \nabla n_i \times B \), and \( \nabla B \times B \) present in the system) \cite{37}, and a charge separation field \((\mathbf{E}_a)\) occurs in the system. To estimate the \( \mathbf{E}_a \), we start from the perpendicular component of the flow equation of motion of electrons and ions as follows,

\[
m_j \frac{du_{j\perp}}{dt} = q_j n_j (\mathbf{E}_a + u_{j\perp} \times \mathbf{B}) - K_B T_j \nabla n_j + m_j n_j \nu_{jm} u_{j\perp}
\]

Where the subscript \( j \) represents the background electrons and ions with mass \( m_j \), density \( n_j \), and drift velocities \( u_{j\perp} \) across the \( \mathbf{B} \). Assuming isothermal and weakly ionization plasma, the corresponding \( u_{j\perp} \) of the \( j \) th species can be derived \cite{37} as,

\[
u_{jm} = \pm \mu_j E_a \frac{\nabla n_j}{n_j} + v_{jh} + v_{jd} + \frac{v_{jD} E_a}{|B|^2} + \frac{v_{jB} B}{|B|^2}
\]

Where \( v_{jD} = \frac{m_j}{1 + \omega_{ji}^2 \tau_{jm}} \), \( v_{jB} = \frac{m_j}{\tau_{jm}} \), \( v_{jh} = \frac{q_j n_j q_i}{|q_i| B} \), and \( m_j v_{jD}^2 / 2 \approx K_B T_j \). Where, \( \mu_j, v_{jm} \) and \( D_j \) are mobility and diffusivity across the magnetic field respectively, \( \omega_{ji} \) is Larmour frequencies and \( \tau_{jm} \) is collision time of \( j \) species with background neutrals, \( v_{jB} \) is the \((\mathbf{E} \times B)\)-drift across the plane containing \( \mathbf{E} \) and \( \mathbf{B} \), \( v_{jD} \) is the diamagnetic-drift across the magnetic field, and \( v_{jB} \) is the gradient-drift of the magnetic field. Both \( v_{jD} \) and \( v_{jB} \) are charge-dependent, whereas \( v_{jB} \) is independent of charge. In equation (6), the first term and second term on the right side are immediately recognizable as drift along the gradients in potential and density which are reduced by the factor \((1 + \omega_{ji}^2 \tau_{jm})\) due to the magnetization. The third term consisting of \( v_{jB} \), \( v_{jD} \) and \( v_{jB} \) are drift across the gradients in potential and density which are reduced by a different factor \((1 + \omega_{ji}^2 \tau_{jm})\), and all the contributions are slowed down by collisions with neutrals and magnetization. Ambipolar diffusion is not a trivial problem in presence of inhomogeneous \( \nabla n \) and \( \nabla B \). In highly magnetized plasma \cite{15, 9}, it would be expected that ions move faster than electrons across the magnetic field. However, in the weak magnetization, electrons still move faster than ions due to their mobility difference and collision with neutrals. Therefore, for the present 2D flow analysis in presence of a weak magnetic field, we consider the condition for ambipolar diffusion \( n_i u_{i\perp} = n_e u_{e\perp} \) and quasi-neutrality condition \( n_i = n_e + Z_p q_i n_i \) in a dusty plasma. Then the corresponding expression for ambipolar field \( E_a \) can be obtained as follow,

\[
(n_i \mu_i + n_e \mu_e) E_a = -D_{e\perp} \nabla n_e + D_{n\perp} \nabla n_i + n_i \frac{v_{jD} + v_{jB} + v_{jB} \nabla E_a}{1 + (v_{ji}^2 / \omega_{ji}^2)}
\]

Then, the above equation can be simplified for \( E_a \) as follow,

\[
E_a = \eta_j \left( D_{e\perp} \nabla n_e - D_{n\perp} \nabla n_i \right) + \frac{q_i}{(1 + \omega_{ji}^2 / \omega_{ji}^2)} \left[ \left( \frac{E_a \times B}{|B|^2} \right) \right] + \frac{K_B T_e \nabla (\nabla n_e \times B)}{|B|^2} + \frac{n_j K_B T_e \nabla B \times B}{q_e |B|^3}
\]

Where \( \eta_j = 1/(n_i \mu_i + n_e \mu_e) \) is the plasma resistivity. In the weakly magnetized plasma, \( 1 + (v_{ji}^2 / \omega_{ji}^2) \) is an important quantity. It shows the influence of magnetization depends on the ratio of the collision and cyclotron frequencies. Taking curl, the above equation can be expressed as,
\[ \nabla \times E_a = \frac{\eta \eta_d}{(1 + \nu_m^2/\omega_m^2)} \left[ \eta_d \nabla \times \frac{E \times B}{|B|^2} + \frac{K_t T_e}{q_e} \nabla \times \frac{(\nabla n_e \times B)}{|B|^2} \right] \]

\[ + \frac{n_e K_t T_e}{q_e} \nabla \times \frac{(\nabla B \times B)}{|B|^3} \] \hspace{1cm} (8)

Now, using the standard vector identity,
\[ \nabla \times (A \times B) = A(\nabla \cdot B) - B(\nabla \cdot A) + (A \cdot \nabla)B - (A \cdot B)\nabla, \] the 1st-term of equation (8) can be expressed as,
\[ \nabla \times \frac{(E \times B)}{|B|^2} = E \left( \nabla \cdot \frac{B}{|B|^2} \right) - \frac{B}{|B|^2} (\nabla \cdot E) + \left( \frac{B}{|B|^2} \cdot \nabla \right) E - \left( E \cdot \nabla \right) \frac{B}{|B|^2} \] \hspace{1cm} (9)

It is to be noted that, the first term on the right-hand side is negligible as the B is divergenceless and directed only along 2. Similarly, the 2nd-term and 3rd-term of equation (8) can be expressed as follows,
\[ \nabla \times \frac{(\nabla n_e \times B)}{|B|^3} = -\frac{B}{|B|^2} (\nabla^2 n_e) + \left( \frac{B}{|B|^2} \cdot \nabla \right) \nabla n_e - (\nabla n_e \cdot \nabla) \frac{B}{|B|^2}. \] \hspace{1cm} (10)
\[ \nabla \times \frac{(\nabla B \times B)}{|B|^3} = -\frac{B}{|B|^2} (\nabla^2 B) + \left( \frac{B}{|B|^2} \cdot \nabla \right) \nabla B - (\nabla B \cdot \nabla) \frac{B}{|B|^3}. \] \hspace{1cm} (11)

Putting equations (9), (10), and (11) in equation (8), further from equation (8) and equation (5), we can write for the vorticity source as,
\[ \beta \omega_B = \frac{\eta \eta_d q_d}{m_d(1 + \nu_m^2/\omega_m^2)} \left[ -n_e \left( \frac{B}{|B|^2} (\nabla \cdot E) - \left( \frac{B}{|B|^2} \cdot \nabla \right) E \right. \right. \]
\[ \left. \left. + (E \cdot \nabla) \frac{B}{|B|^2} - \frac{K_t T_e}{q_e} \left( \frac{B}{|B|^2} (\nabla^2 n_e) \right) \right. \right. \]
\[ \left. \left. - \left( \frac{B}{|B|^2} \cdot \nabla \right) \nabla n_e - (\nabla n_e \cdot \nabla) \frac{B}{|B|^2} \right) \right. \]
\[ \left. \left. - n_e K_t T_e \left( \frac{B}{|B|^2} (\nabla^2 B) - \left( \frac{B}{|B|^2} \cdot \nabla \right) \nabla B + (\nabla B \cdot \nabla) \frac{B}{|B|^3} \right) \right] \right. \] \hspace{1cm} (12)

In a more specific way, we focus on the effect of the weak magnetization on the xy-plane having the \( E_x(-\hat{y}), \) \( \nabla B \hat{y} \) across the transverse \( \hat{z}, \) then the above equation (12) is reduced as,
\[ \beta \omega_B = \frac{\eta \eta_d q_d}{m_d(1 + \nu_m^2/\omega_m^2)} \left[ -n_e \left( \frac{B}{|B|^2} \frac{\partial E_z}{\partial y} + E_x \frac{\partial B}{\partial y} \right) \right. \]
\[ \left. - \frac{K_t T_e}{q_e} \left( \frac{B}{|B|^2} \frac{\partial^2 n_e}{\partial y^2} + \frac{\partial n_e}{\partial y} \frac{\partial B}{\partial y} \right) \right. \]
\[ \left. \left. - n_e K_t T_e \left( \frac{B}{|B|^2} \frac{\partial^2 B}{\partial y^2} + \frac{\partial B}{\partial y} \frac{\partial B}{\partial y} \frac{B}{|B|^3} \right) \right] \right. \] \hspace{1cm} (13)

In the above equation (13), the 1st term represents the contribution from the variation of the sheath field \( E_z \) and the 2nd-term represents the contribution from shear nature of the \( B \) along with the \( E_z. \) The \( \partial^2 / \partial y^2 \) in the 3rd and the 5th-terms represent the measure of the concavity of the density/magnetic fields across the field. The 4th and the 6th-terms represent the contribution from shear nature of \( B \) along with the shear \( \partial n_e / \partial y \) and \( \partial B / \partial y \) respectively.

Similarly, when we focus on the \( xx \)-plane along with the transverse \( B, \) i.e., the cross-section parallel to the electrode surface, the above equation (12) can be simplified as follow,
\[ \beta \omega_B = \frac{\eta \eta_d q_d}{m_d(1 + \nu_m^2/\omega_m^2)} \left[ n_e \left( \frac{B}{|B|^2} \frac{\partial E_x}{\partial z} \right) E_a \right. \]
\[ \left. + \frac{K_t T_e}{q_e} \left( \frac{B}{|B|^2} \frac{\partial n_e}{\partial z} \right) \frac{\partial B}{\partial y} + n_e K_t T_e \frac{B}{|B|^2} \frac{\partial B}{\partial z} \right] \] \hspace{1cm} (14)

In the above equation (14), the 1st term represents the contribution from \( E_x \) shear along the \( B. \) The 2nd and the 3th-terms represent the contribution from shear nature of \( \partial n_e / \partial y \) and \( \partial B / \partial y \) along the \( B \) respectively. Both the terms are contributed only when diamagnetic and gradient drift is significant. Again, there is no contribution of shear
We assume that the bounded dust cloud is driven by the shear nature of the background ion-"drag and the B\textsubscript{fi} from the shear E\textsubscript{fi} derived in equation (13). As in the previous work [14], ions are considered streaming downward throughout the dust cloud, having a monotonic shear profile given by a single natural mode of the cartesian plane as given by,

\[
v_i(-\hat{y}) = U_i + \frac{k_y}{L_x} \left( x - x_i \right),
\]

(15)

Here, \( U_i \) represents an offset, and \( \frac{k_y}{L_x} \) is the strength of the shear variation of the ion flow. The mode number \( k_y = \pi n/2, n = 0.25, 0.5, \ldots \) represents the zeros of the corresponding monotonic ion velocity profile coinciding with the external boundary location \( L_x \). The corresponding \( \omega_i \) in the \( xy \)-plane is found to be

\[
\omega_i = \nabla \times v_i = -U_0 \frac{k_y}{L_x} \sin \left( \frac{k_y}{L_x} \left( x - x_i \right) \right).
\]

(16)

The entire analysis is done using the same driver velocity \( v_i(x, y) \), and the corresponding vorticity \( \omega_i(x, y) \). However, the similar vorticity \( \omega_i(x, y) \) can be achieved for ions flow \( v_i(-\hat{x}) \) with shear variation along the \( y \)-direction. For the \( B\hat{z} \), the horizontal \( B \) \( \hat{z} \) is also considered having a monotonic shear profile over the bounded dust domain given by single natural mode of the cartesian plane as given by,

\[
B\hat{z} = B_0 \sin(y), \quad yy = k_y \frac{y - y_i}{L_y - y_i}.
\]

(17)

The profile maintains minimum \( B \) at the lower boundary \( y = y_1, (y_1 \approx 0) \) and a maximum at the upper confining boundary \( y = L_y \). The mode number \( k_y = m\pi/2, m = 0.25, 0.5, \ldots \) represents the strength of the monotonic shear \( B \) that can be varying through the \( k_y \). In equation (7) and equation (13), the diamagnetic drift contribution follows a similar form with that of magnetic-gradient drift when both the shear profile are natural modes of the cartesian plane. As in the previous work [14], the simplified \( \beta \omega_{B\parallel} \) in the \( xy \)-plane is found to be

\[
\beta \omega_{B\parallel} = -\frac{\eta}{m_i(1 + \nu^2/m_i^2)} \frac{E_i}{B_0} \left( \frac{k_y}{L_y - y_i} \right)^2 \cos^2(y),
\]

(18)

It is noted that \( \omega_{B\parallel} \) consists of two main parts, i.e., the \( E \times B \)-drift of both electrons and ions toward \( -\hat{x} \)-direction and the \( \nabla B \times B \)-drift of electron toward the \( \hat{x} \)-direction while ions drift in the opposite direction. However, the shear nature of both the drifts acts together as the vorticity source. The last-term is three orders stronger than the first-term and six orders larger than the second-term which is almost negligible.

Now, for defining the co-efficient \( \beta, \xi, \nu, \) and \( \mu_i \), we consider a typical laboratory glow discharge argon plasma having dust density \( n_d \approx 10^{15} \text{ cm}^{-3} \), ions density \( n_i \approx 10^{16} \text{ cm}^{-3} \), neutral density \( n_n \approx 10^{15} \text{ cm}^{-3} \) (corresponds to \( p \approx 12.4 \text{ Pascal of neutral pressure} \) [45]), electron temperature \( T_e \approx 3 \text{ eV} \), ion temperature \( T_i \approx 1 \text{ eV} \), and shear ions are streaming with \( U_i \) cm/sec a fraction of the ion-acoustic velocity \( c_{ai} = \sqrt{T_e/m_i} \approx 10^5 \text{ cm/sec} \) while the dust acoustic velocity \( c_{ad} \approx 12 \text{ cm/sec} \). Here, \( c_{ad} = \sqrt{Z_d^2 (n_d/n_i) (T_i/m_d)} \), \( Z_d \approx 10^4 \) is dust charge number, \( m_d \approx 10^{-14} \text{ kg} \) is dust mass, and other notations are all conventional [46]. Using dust mass \( m_d \) dust charge \( q_d \), the...
width of the confined domain $L_x(\approx 10 \text{ cm})$, and streaming shear ions velocity strength $U_{d0}$ as the ideal normalization units, the corresponding value of the system parameters/co-efficient are $\xi \approx 10^{-3} U_{d0}/L_x$, $\nu \approx 10^{-2} U_{d0}/L_y$, and $\mu \approx 2.5 \times 10^{-4} U_{d0}L_x$ respectively. We estimate the range of $\mu$ keeping in view that dust fluid flow is incompressible $u < c_{s,d}$ and the associated Reynolds number is very small (Re $\approx 1$). This leads to a linear limit of the formulation [6,38] and closely agrees with that of Yukawa systems [47,48]. In the present studies, we consider the specific range of the $\mu$ and $\nu$ that are supported by several laboratory dusty plasma experimental systems and previous work [14,45,46]. Further, for a weak magnetized system of $B_0 = 4-500 \text{ G}$ (corresponds to $\omega_{ke} \geq \nu_{e} \text{ to } \nu_{i} \geq \omega_{ke}$ [35,36]), the normalized values of other system parameters are estimated as $B_0 \approx 1.6 \times 10^{-3} [m_d U_{d0}^2/q_d L_x]$ for $B_0 = 10 \text{ G}$, the sheath electric field $E_s(=m_d q_d^2/d_{e}) \approx 9.8 \times 10^{-3} [m_d U_{d0}^2/q_d L_x]$ for $E_s \approx 61.25 \text{ V/m}$, $n_e \approx 9.0 \times 10^7 \text{ cm}^{-3}$ satisfying the quasiinflatibility condition, $(1 + \nu_{s,e}^2/\omega_{pe}^2) \approx 1$, and $\beta(=\eta n_e q_d^2/m_d (1 + \nu_{s,e}^2/\omega_{pe}^2)) \approx 4.5 \times 10^{-11} [U_{d0}/L_x]$ respectively. In the calculation of resistivity $\eta$, we used the standard Spitzer model with two orders higher concerning the presence of dust and neutral particles [49,37].

3.2. Dust flow characteristics driven by the $\omega_i$ without the magnetic field $(\omega_{B||} = 0)$;

The steady-state converged solutions are obtained for the bounded dust dynamics represented by the above set of equations (3) and (4) in the rectangular domain $0 \leq x/L_x \leq 1$ and $0 \leq y/L_y \leq 1$, $L_y = L_x$ near the sheath region as highlighted in figure 1. We adopt no-slip boundary conditions for all the physical boundaries confining the dust fluid [41]. The series of structural changes in terms of the streamlines and velocities profile of the bounded dust flow are presented in figure 2, for a wide range of $\mu$ from $\mu = 10^{-4} U_{d0}L_x$ to $\mu = 5 \times 10^{-6} U_{d0}L_x$ and fixed other system parameters. Along with the structural change in figures, 2(a) to (d), the nonlinear structural bifurcation takes place through a critical $\mu^*$ and arises a new structure having a circular core region surrounded by weak and elongated vortices near the corners. The corresponding changes in velocities strength and slight variation in the boundary layer thickness are shown in figures 2(f) and (g). Boundary layers are the high shear region near the external no-slip boundaries [7]. It demonstrates that the steady-state dust flow structure changes from linear to a highly nonlinear regime with decreasing $\mu$, giving a new structure that retains more momentum and energy. In the previous work [7,14] without the magnetic field $(\omega_{B||} = 0)$, similar solutions are analyzed in a cylindrical rz-plane using different

![Streamlines for the steady bounded dust flow in the x-y plane for varying (a) $\mu = 10^{-4} U_{d0}L_x$, (b) $\mu = 5 \times 10^{-4} U_{d0}L_x$, (c) $\mu = 1 \times 10^{-3} U_{d0}L_x$, and (d) $\mu = 5 \times 10^{-3} U_{d0}L_x$, respectively having fixed other system parameters $\xi = 10^{-3} U_{d0}/L_x$, $\nu = 10^{-2} U_{d0}/L_y$, and $B = 0$. (e) Cross-section profile of driver ion’s velocity i.e., $\nu_\parallel(x,y) U_{d0}$ having sheared mode numbers $n = 1$. The corresponding dust flow velocity profiles through the static point $(x_n,y_n)$ are (f) $u_{\parallel}(x_n,y_n) U_{d0}$ and (g) $u_{\parallel}(x_n,y) U_{d0}$ respectively.](image-url)
can effects the momentum transfer from the ions to the dust and further reduces the strength of the dust circulation. Both the viscous diffusion and collision with the background neutral variation of boundary layer thickness in the series of structural changes in the present analysis in a cartesian setup.

The corresponding dust vortex characteristics are observed by comparing the series of parameters regimes, driving field, and boundary conditions. However, we reproduced it for comparison with the following new parametric effects in the present analysis in a cartesian setup.

Now, starting from the highly nonlinear flow structure shown in figure 2(d), a series of steady-state dust flow structure in term of streamlines and velocity profile for a reasonable range of neutral collision frequency $\nu$ (corresponds to the pressure of $\approx 0.05$–$10$ Pascal range) and fixed other system parameters is presented in figure 3. In the series of structural changes in figure 3(a) to (d), it is observed that neutral pressure takes a very sensitive role in determining the characteristic features of the bounded dust flow. The sensitivity of $\nu$ can be seen in the noticeable variation of boundary layer thickness in figure 3(f) and figure 3(g). The most probable reason appears in the model equation (2), in which the dust cloud is driven by the shear ion drag and the dissipative resistance is produced by both the viscous diffusion and collision with the background neutral fluid. Moreover, the neutral collision with ions can effects the momentum transfer from the ions to the dust and further reduces the strength of the dust circulation.

Thus, the following dust vortex characteristics are observed by comparing the series of flow structures in figures 2 and 3. First, in the weak flow regime (for higher $\mu$ and $\nu$), the forcing shear ions provide the rotary motion and its direction, while the confined domain determines its shape. The boundary layer is thick and its effects are distributed through the interior domain. Second, in the high flow regime (for lower $\mu$ and $\nu$), the nonlinear convective flow dominant over the diffusion, and the boundary effects are confined in thinner layers, so the interior flow responds to the monotonic forcing shear ion drag only. As a consequence, the vortices turn circular, and the rest of the domain is filled with several weaker vortical structures. Nevertheless, the size of secondary vortices increases as the nonlinearity increases and most of the circulating structure is co-rotating in nature following the shear scale of the driver ion’s field. Further, it is noted that, in the present studies of aspect-ratio ($L_x/L_y = 1$) and accessible parametric regime, the steady streamlines patterns display negligible variations while the cross-section velocity profile shows the significant changes in the flow strength and boundary layer thickness. Furthermore, figure 3(f) and figure 3(g) demonstrate that the ions drag-driven dust dynamics become negligibly small at higher $\nu \approx 7 \times 10^{-3} U_0/L_y$ (corresponds to neutral pressure of $\approx 10$ Pascal) even with the higher $\mu$ regimes as reported in the laboratory experiment [45].
3.3. Dust vortex characteristics driven by the $\omega_{Bz}$ in the weakly magnetized regime ($\omega_{le} \ll \nu_{in}$):

The analysis here is further extended in the high-pressure regime ($\nu = 10^{-2} U_0/L_x$ where the ions drag-driven source $\omega_c$ is negligibly small and the driving field $\omega_{Bz}$ influences the dust dynamics. A series of structural changes in terms of the streamlines and velocities profile of the bounded dust flow are presented in figure 4, for the case $k_y = \pi/4$, in the wide range of magnetic field $B_0 = 10$ G to 400 G, and fixed other system parameters in the nonlinear convective regime of $\mu = 5 \times 10^{-4} U_0/L_x$ and $\xi = 10^{-3} U_0/L_x$ respectively. It simply demonstrates that the $\omega_{Bz}$ due to the shear nature of $E_x$ can generate dust vortex dynamics. Further, in the higher $B_0$ regimes, the streamlines pattern in figures 4(a) to (b) and the corresponding velocity profiles in figures 4(f)–(g) shows that the dust cloud circulates very slowly in a $D$-shaped elliptical structure. With decreases in $B_0$, the flow is strengthened gradually, the interior static point $(x_0, y_0)$ convected toward the axial $\hat{x}$-direction, and the circulation turns into a small meridional structural as shown in figure 4(d). In the present analysis, the flow structure is determined by three main factors; the first one is the role of $E_x (\approx m q_B/\Omega_0)$ in finding the static point $(x_0, y_0)$ which will be discussed in detail in the following section 3.4. The second is the profile of $\omega_{Bz}$ which is inversely proportional to $B_0 (\sin(y))$, and the third is the incompressibility of the dust cloud. From equation (4) and equation (18), $\omega_{Bz}$ generates a localized but strong axial flow of the dust particles toward the $-\hat{x}$-direction. Because of the continuity, the localized flow is compensated by the whole clockwise circulation of the dust cloud in the $xy$-plane. Thus, the $D$-shaped elliptical structure is developed and it turns into a meridional structure with an increase in the nonlinear convective transport. The structural changes in the highly viscous linear regime $\mu \geq 1 \times 10^{-4} U_0/L_x$ is imperceptible in nature.

A more visible structural change is observed by varying the magnetic shear $k_y = m/2$ instead of the field strength $B_0$. In figure 5, a series of structural changes in terms of the streamlines and the velocities profile of the bounded dust flow are presented for $B_0 = 10$ G, a wide range of $m = 0.25$ to 1, and other system parameters remain unchanged. The cross-section profile of the magnetic field for the varying $k_y$ is shown in figure 5(e). In comparison to figure 4, this analysis demonstrates that the dust peak velocities, the convection of $(x_0, y_0)$, and meridional structural become more significant with a decrease in the $k_y$ than the decrease in $B_0$. Further, as in previous work of mode analysis [6], the present analysis also allows us to examine the formation of multiple counter-rotating vortices by using the non-monotonic higher magnetic mode number $m > 1$, says $m = 2, 3, or$.

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**Figure 4.** Streamlines for the steady bounded dust flow in the $x$-$y$ plane for varying (a) $B_0 = 400$ G, (b) $B_0 = 100$ G, (c) $B_0 = 50$ G, and (d) $B_0 = 10$ G, respectively having fixed other system parameters $\nu = 10^{-2} U_0/L_x$, $\mu = 5 \times 10^{-4} U_0/L_x$, and $\omega = 0$. (e) The corresponding $B$ profile for varying $B_0$ and fixed $k_y = \pi/4$. The dust flow velocity profiles through the interior static point $(x_0, y_0)$ are (f) $u_x(x_0, y_0)/U_0$ and (g) $u_y(x_0, y_0)/U_0$ respectively.
higher numbers in equation (17). Interestingly, the D-shaped elliptical flow characteristics have various similarities, for instance, the D-shaped elliptical structure is common in MHD solution of magnetic confinement configurations (tokamak) [17, 18] and in neutral flows in a lid-driven cavity at low to moderate Reynolds number [19, 20]. In more specific, the meridional structure of dust circulation displayed in figure 5(d) is observed in several magnetized [9, 15] and non-magnetized [16] dusty plasma experiments although the parametric regimes and driving fields are different. It justifies the fact that these structures are among the common nonlinear characteristic features of driven-dissipative flow systems and dusty plasma provides a domain for analysis of the associated basic underlying physics.

3.4. Dust vortex characteristics driven by both \( \omega \), the \( \omega_{\parallel} \) : In this section, we finally examine the dust flow structure in the presence of both \( \omega \) and \( \omega_{\parallel} \) in the same configuration. We note from the above analysis in section 3.2, that the ion-drags produce the dust cloud circulation in an anti-clockwise direction and its strength reduces as the neutral pressure increases. Again in section 3.3, weak magnetization generates dust cloud circulation in a clockwise direction, and the strength increases as \( B_0 \) and \( k_y \) decrease. Therefore, there are parametric regimes of high pressure and weak magnetization where both are comparable although ions-drags dynamics are dominant in most of the cases. Now, we present the steady-state dust flow structure in the specific parametric regime of \( \nu = 10^{-2} U_0 / L_x, \mu = 5 \times 10^{-6} U_0 L_x, \xi = 10^{-3} U_0 / L_x, B_0 = 10 \ G, \) and \( k_y = \pi / 2 \) where both \( \omega \) and the \( \omega_{\parallel} \) are significant as shown in figure 6(a).

It shows that unequal strength counter-rotating vortex pairs comprising of circular one driven by the ion-drags and localized elliptical structure driven by the weak magnetization can co-exist in the same plane. Further, in the case of lower \( k_y = \pi / 4 \), the strength and size of the elliptical structure increase while the circular one is relatively reduced as shown in figure 6(b). While the parametric evolution of the unequal vortex pairs and their stability is deserving a separate analysis, the present work shows the feasibility of existing steady-state counter-rotating vortex pair of unequal strength associated with different monotonic shear flow fields as reported in the recent dusty plasma experiments [15]. Such unequal strength vortex pair is common in vortex shedding dynamics behind aircraft wings [12]. Furthermore, when the direction of \( B \) is changed by 180\(^{\circ}\)-degree, both the sources \( \omega \) and the \( \omega_{\parallel} \) act in the same direction and generate circular dominant meridional structural as shown.
in figure 6(c). It simply illustrates that the direction of magnetization is a key factor in the MDP dynamics. The corresponding circulation with lower \( k_y = \pi/4 \) is again plotted in figure 6(d), showing an elliptical dominant strong meridional structural due to the strengthening of \( \omega_{BP} \).

One more interesting characteristics of the driven-dust vortex structure is the effect of varying \( E_s \). The cases with a varying value of \( E_s \) is examined and the corresponding structural changes in terms of the streamlines are presented in figure 7, showing an increase in strength of the circulation, and the interior static point \((x_0, y_0)\) is convected toward the center of the xy-plane. The role of the \( E_s \) can be addressed in two ways. In the first, \( \omega_{BP} \) is directly proportional to \( E_s \) in equation (18), therefore, the strength of the circulation increase with increasing \( E_s \).

In the second, applying the force balance condition [36] at the static equilibrium point \((x_0, y_0)\) in equation (2), we
have

\[ 0 = -\nabla \phi_0 + \frac{q_d}{m_d} E_a - \frac{\nabla P}{\rho} + \mu \nabla^2 u + \xi v + \nu w. \]

Where \( E_a(\approx m_e g/q_d) \) along \(-\hat{y}\) is part of the confining potential \( \phi_0 \) against the gravity. From equation (7), the internal field \( E_a \) along \( \hat{x} \) is directly dependent on the \( E_a \). Therefore, \( E_a \) takes an important role other than \( \xi, \nu, \) and \( \mu \) in finding the steady-state dust flow structure in the confined domain. An increase in \( E_a \) means a rise in the levitation level of the dust cloud above the electrode. The other parametric effects for \( \xi, \nu, \) and \( \mu \) follow the same effect as discussed in the previous work [41, 14]. Most importantly, in all the cases of the above analysis, the max dust speed is \( u_d \approx 6.0 \times 10^{-3} U_0 \), i.e., \( u_d \approx 6.0 \text{ cm/ sec} \) while the dust acoustic speed is \( c_d \approx 12.65 \text{ cm/ sec} \). This velocity range is in good agreement with several laboratory dust plasma experiments which have observed dust acoustic waves of speed in the range of 12 cm/sec to 27.6 cm/sec [50, 46].

4. Summary and conclusions

The two-dimensional (2D) hydrodynamic model for a bounded dust flow dynamics in plasma from the previous work [7] is extended for analysis of driven vortex characteristics in presence of an external transverse and weak magnetic field \( (B) \) in a planner setup and parametric regimes motivated by recent magnetized dusty plasma (MDP) experiments. We have demonstrated that the weak magnetization (the only electron is magnetized) can produce an internal charge separation field \( E_a \) in between electrons and ions due to the combined effects of \( E \times B, \nabla n_a \times B, \nabla B \times B, \)-drifts, and collisions present in the system. Then the \( E_a \) can displace a dust particle or a dust cloud, whereas its shear can cause a rotation of the dust cloud. We derived the general equations for the \( E_a \) and associated 2D cross-section vorticity sources along with and across the \( B \) using the condition of 2D ambipolar diffusion in the dusty plasma. The formulation reveals that, in principle, vorticity can be developed on a plane parallel to the surface of electrodes in the setup, when there are variations of the \( E_a \) and magnetic/density shear along the \( B \). However, there is no contribution of the shear on the plane, therefore, difficult to achieve the condition in a real setup. This would be a probable reason why dust rotation was not observed in the real experiments by Puttscher and Melzer et al [35, 36].

Based on the proposed model, we have analyzed the characteristics of a 2D steady-state vortex developed on a plane perpendicular to the surface of the electrode and directed along the \( B(\hat{z}) \). The analysis is carried out assuming the ions flow profile of natural cosine mode, the magnetic field profile of natural cosine mode in the planner setup, and other parametric regimes are motivated by several recent MDP experiment. The analysis shows that the shear ions-drag force generates an anti-clockwise circular vortical structure and its characteristics are mainly reliant on the kinematic viscosity and neutral collision frequency. The neutral pressure takes a key role in deciding the dominance between the ions-drag and the \( E_a \)-force. When the neutral pressure is high, the \( E_a \)-force becomes comparable or dominant over the ion-drag force and generates a strong localized flow of the dust particles. Then the localized flow is compensated by the whole incompressible dust cloud dynamics developing a clockwise \( D \)-shaped elliptical structure. Further, a decrease in \( B \), shows a noticeable increase of dust flow strength, convection of the interior static point, and the structure turns gradually into a meridional structure. Such elliptical and meridional structural are reported in several MDP experiments and various driven natural flow systems [15, 9, 4, 12, 13].

Additionally, we have examined the dust vortex dynamics in the parametric regimes of high pressure and low \( B \), where both the ions-drag and the magnetizing force are comparable although ions-drag dynamics are dominant in most of the cases. It has shown that unequal strength counter-rotating vortex pairs can co-exist in the same domain and their characteristics depend on the driving fields and the parametric regimes. When the direction of \( B \) is flipped by \( 180^\circ \)-degree, both the drivers act together and give rise to a strong meridional structure. It simply points out the key role of \( B \)-direction in MDP systems. Moreover, we analyzed the role of the \( E_a(\approx m_e g/q_d) \), which stands for the levitation level of the dust cloud above the electrode. It reveals that both the confinement potential and the \( E_a \) are directly proportional to the \( E_a \), therefore, any increase in \( E_a \) sharply enhanced the flow strength of the meridional structural and the convection of the interior static point toward the center of the plane.

In conclusion, we stress that we have analyzed various mechanisms of dust vortex formation and its characteristics in a weakly magnetized dusty plasma. Specifically, we have interpreted the physics insight of elliptical vortex, meridional structural, and condition for co-existing unequal strength vortex pairs in several MDP, and also indicated the behaviors isomorphism with the relevant natural driven flow systems. This signifies the fact that dusty plasma can be an ideal domain for the study of various driven-dissipative flow systems. In the future, a more comprehensive quantitative relation can be established between a threshold \( B^* \) and other parameters that overtake the ion’s drag force. The formulated model can apply to a specific MDP setup and compare the theory, simulation, and experimental data.
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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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