NEUTRINO FLAVOR GONIOMETRY BY HIGH ENERGY ASTROPHYSICAL BEAMS

Sandip Pakvasa

Department of Physics and Astronomy
University of Hawaii, Honolulu, HI 96822

Abstract

It is shown how high energy neutrino beams from very distant sources can be utilized to learn about many properties of neutrinos such as lifetimes, mass hierarchy, mixing, minuscule pseudo-Dirac mass splittings and other exotic properties; in addition, the production mechanism of neutrinos in astrophysical sources can also be elucidated.
1 Introduction

We make several basic assumptions which are reasonable. The first one is that distant neutrino sources (e.g. AGN’s and GRB’s) exist; and furthermore with detectable fluxes at high energies (up to and beyond PeV). The second one is that in the not too far future, very large volume, well instrumented detectors of sizes of order of KM3 and beyond will exist and be operating; and furthermore will have (a) reasonably good energy resolution and (b) good angular resolution (\( \sim 1^0 \) for muons). The first is motivated by the fact that cosmic rays are observed all the way to \( 10^5 \) PeV and gamma rays to 100 TeV and these sources would presumably produce neutrinos as well. We further assume that a neutrino signal will be seen with a reasonable event rate. Finally, we assume that neutrino flavors can be distinguished. At the moment we know how to do this for H2O detectors[1]; however, extending such flavor identification to other types of detectors such as ones based on air shower arrays[2] or the Askaryan effect[3] remains a task for future.

2 Neutrinos from Astrophysical Sources

If these two assumptions are valid, then there are a number of uses these detectors can be put to[4]. Here I want to focus on those that enable us to determine some properties of neutrinos: probe neutrino lifetimes to \( 10^4 s/\text{eV} \) (an improvement of \( 10^8 \) over current bounds), pseudo-Dirac mass splittings to a level of \( 10^{-18} \text{eV}^2 \) (an improvement of a factor of \( 10^6 \) over current bounds) and in case of very small pseudo-Dirac mass differences measure cosmological parameters such as red-shift in neutrinos. There is the possibility of potentially measuring quantities such as \( |U_{e3}| \) and the phase \( \delta \) in the MNSP matrix[5].
3 Astrophysical neutrino flavor content

In the absence of neutrino oscillations we expect a very small $\nu_\tau$ component in neutrinos from astrophysical sources. From the most discussed and the most likely astrophysical high energy neutrino sources[6] we expect nearly equal numbers of particles and anti-particles, half as many $\nu'_e$s as $\nu'_\mu$s and virtually no $\nu'_\tau$s. This comes about simply because the neutrinos are thought to originate in decays of pions (and kaons) and subsequent decays of muons. Most astrophysical targets are fairly tenuous even compared to the Earth’s atmosphere, and would allow for full muon decay in flight. (There could be flavor independent fluxes from cosmic defects and exotic objects such as evaporating black holes. Observation of tau neutrinos from these would have great importance.) A conservative estimate[7] shows that the prompt $\nu_\tau$ flux is very small and the emitted flux is close to the ratio $1 : 2 : 0$. The flux ratio of $\nu_e : \nu_\mu : \nu_\tau = 1 : 2 : 0$ is certainly valid for those AGN(or GRB) models in which the neutrinos are produced in beam dumps of photons or protons on matter, in which mostly pion and kaon decay(followed by the decay of muons) supply the bulk of the neutrino flux.

This flavor mix of 1:2:0 is only approximate; a more careful estimate shows the actual result is 1:1.85: $\epsilon$ where $\epsilon$ is rather small (less than 0.001)[8]. The precise mix also depends on the energy spectrum at injection. The sources in which the primary process is $\gamma p$ rather than pp are distinguished by a lack of $\bar{\nu}'_e$s in the initial flavor mix; this is due to the fact that $\gamma p$ scattering produces dominantly $\pi^+$ which does not have a $\bar{\nu}_e$ amongst its decay products.

There are two other initial flavor mixes possible. One is the so called damped muon case when the $\mu'$s lose energy (via interaction with strong magnetic fields or with matter[9]). The lower energy of the muon makes the $\nu_e$ have much lower energy than the $\nu_\mu$ from $\pi$ decay, and hence effectively the flavor mix becomes $\nu_e : \nu_\mu : \nu_\tau = 0 : 1 : 0$. Again it should be emphasized that this is not exact; the $\nu_e$ content is never exactly zero and the actual
flavor mix is more like $\eta : 1 : 0$ where $\eta$ may be a few (2 to 4\%)%

A third case is of sources which emit dominantly neutrons originating in photo-
dissociation of nuclei[10]. Decay of neutrons leads to an initial pure "$\beta$-beam" of $\bar{\nu}_e$ with
flavor mix of 1:0:0; again contaminated by $\nu_\mu$ at a few % level.

It is also of interest to consider the flavor content of the very highest energy neutrinos,
sometimes called "the GZK neutrinos", which were predicted[11] soon after the original
observation of the GZK cutoff mechanism[12]. These are the neutrinos emitted following
the scattering of highest energy cosmic rays on the microwave background photons. The
dominant process is the production of the resonance $\Delta^+$ which decays into a neutron and a
$\pi^+$. Below about 100 PeV, the neutrinos from neutron decay dominate, resulting in the flavor
mix $1 : 0 : 0$ whereas above 100 PeV, pion decays dominate, resulting in the conventional
flavor mix of $2 : 1 : 0$ [13].

4 Effect of Oscillations

The current knowledge of neutrino masses and mixing can be summarized as follows[14].
The mixing matrix elements are given to a very good approximation by the so-called tri-bi-
maximal matrix[15]. The bound on the element $|U_{e3}|$ comes from the CHOOZ experiment[16]
and is given by $|U_{e3}| < 0.17$. The mass spectrum has two possibilities: normal or inverted.
The mass differences are given by $|\delta m^2_{32}| \sim 2.4.10^{-3} eV^2$ (with the + sign corresponding to
normal hierarchy and - sign to the inverted one) and $\delta m^2_{21} \sim +7.6.10^{-5} eV^2$. Since $\delta m^2 L/4E$
for the distances to GRB’s and AGN’s (even for energies up to and beyond PeV) is very large
($> 10^7$) the oscillations have always averaged out and the conversion(or survival) probability
is given by

\[ P_{\alpha\beta} = \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2 \]  

Assuming no significant matter effects en-route, it is easy to show that the tri-bi-maximal mixing matrix leads to a simple propagation matrix \( P \), which, for any value of the solar mixing angle, converts a flux ratio of \( \nu_e : \nu_\mu : \nu_\tau = 1 : 2 : 0 \) into one of \( 1 : 1 : 1 \). Hence the flavor mix expected at arrival is simply an equal mixture of \( \nu_e, \nu_\mu \) and \( \nu_\tau \) as was observed long ago[7, 17]. If this universal flavor mix is confirmed by future observations, our current knowledge of neutrino masses and mixing is reinforced and conventional wisdom about the beam dump nature of the production process is confirmed as well. However, it would be much more exciting to find deviations from it, and learn something new. How can this come about? I give below a shopping list of variety of ways in which this could come to pass, and what can be learned in each case.

5 Deviations from Canonical Flavor Mix

There are quite a few ways in which the flavor mix can be changed from the simple universal mix.

The first and simplest is that initial flavor mix is NOT \( 1 : 2 : 0 \). The damped muon case in which the initial flavor mix is \( 0 : 1 : 0 \), the final result after the averaged out oscillations becomes \( 0.57 : 1 : 1 \) on arrival. The “beta” beam which starts out as \( 1 : 0 : 0 \) initially becomes \( 2.5 : 1 : 1 \) on arrival. These are sufficiently different from the universal mix so that the nature of the source can be easily distinguished from such observations. The two kinds of production processes which both lead to the initial flavor mix of \( 1 : 2 : 0 \), namely the pp and \( \gamma p \) can also be distinguished from each other, at least in principle[18]. In the former case the flux of \( \bar{\nu}_e \) relative to the total neutrino flux is given by \( 1/6 \), whereas in the
latter case it is given by $2/27$; the $\bar{\nu}_e$ flux can be measured at an incident energy of 6.3 PeV by the showers due to the “Glashow” resonance as was first stressed in Ref. 7.

Small deviations from 1:1:1 can be used to determine deviations from the tri-bimaximal neutrino mixing, e.g. a non-zero value for $U_{e3}[19]$. A non-zero $U_{e3}$ leads to a flavor mix of $(1 + 2\Delta) : (1 - \Delta) : (1 - \Delta)$ where $\Delta = \sqrt{2}/3U_{e3}\cos(\delta)$. Also, for a pure $\nu_e$ beam, the ratio $R = \mu/(e + \tau)$ is given approximately by $2/7 - 0.14\Delta$, and hence is also a measure of $|U_{e3}|[20]$. Here it is assumed that $\theta_{23}$ is exactly $\pi/4$; one can also attempt precision measurement of $\theta_{23}[21]$. In principle, one can also envisage a damped muon beam to measure $\Delta$, as the ratio $R$ becomes $7/11 + 0.42\Delta$. If the element $|U_{e3}|$ is known by the time this measurement is made, one can hope to get a handle on the CPV phase $\delta[22]$. Unfortunately, these measurements not only need to measure small deviations but are made even more difficult by the impure nature of the initial flavor mixes as has been discussed recently[8]. Another way in which small deviations can arise is from small mixing with sterile neutrino states[23].

The possibility that the mass differences between neutrino mass eigenstates are zero in vacuum (and become non-zero only in the presence of matter) has been raised[24]. If this is true, then the final flavor mix should be the same as initial, namely: $1 : 2 : 0$. However, very recently, analysis of low energy atmospheric neutrino data by Super-Kamiokande has ruled out a wide variety of models for such behavior[25].

Neutrino decay is another important possible way for the flavor mix to deviate significantly from the democratic mix[26]. We now know that neutrinos have non-zero masses and non-trivial mixing, based on the evidence for neutrino mixing and oscillations from the data on atmospheric, solar and reactor neutrinos.

Once neutrinos have masses and mixing, then in general, the heavier neutrinos are
expected to decay into the lighter ones via flavor changing processes[27]. The only remaining questions are (a) whether the lifetimes are short enough to be phenomenologically interesting (or are they too long?) and (b) what are the dominant decay modes. Since we are interested in decay modes which are likely to have rates (or lead to lifetimes) which are phenomenologically interesting, we can rule out several classes of decay modes immediately. For example, the very strong constraints on radiative decay modes and on three body modes such as $\nu \to 3\nu$ render them uninteresting.

The only decay modes which can have interestingly fast decay rates are two body modes such as $\nu_i \to \nu_j + x$ where $x$ is a very light or massless particle, e.g. a Majoron. In general, the Majoron is a mixture of the Gelmini- Roncadelli[28] and Chikasige-Mohapatra-Peccei[29] type Majorons. The effective interaction is of the form:

$$g\bar{\nu}_\beta^\alpha (a + b\gamma_5)\nu_\alpha x$$  \hspace{1cm} (2)

giving rise to decay:

$$\nu_\alpha \to \bar{\nu}_\beta \text{ (or } \nu_\beta) + x$$  \hspace{1cm} (3)

where $\nu_\alpha$ and $\nu_\beta$ are mass eigenstates which may be mixtures of flavor and sterile neutrinos. Explicit models of this kind which can give rise to fast neutrino decays have been discussed[30]. The models with $\Delta L = 2$ are unconstrained by $\mu$ and $\tau$ decays which cannot be engendered by such couplings. Both($\Delta L = 2$ and $\Delta L = 0$) kinds of models with couplings of $\nu_\mu$ and $\nu_e$ are constrained by the limits on multi-body $\pi$, K decays, and on $\mu - e$ universality violation in $\pi$ and K decays[31], but these bounds allow fast neutrino decays.

There are a number of interesting cosmological implications of such couplings. The details depend on the mass spectrum of neutrinos and the scalars in the model, and on the strength of the couplings. For example, when all the neutrinos are heavier than the scalar; for sufficiently strong coupling($g > 10^{-5}$) the relic neutrino density vanishes today, and the
neutrino mass bounds from CMB and large scale structure are no longer operative, in the sense that potentially strong cosmological bounds can be violated by future measurements in the laboratory which find a non-zero result for a neutrino mass[32]. If the scalars are heavier than the neutrinos, there are signatures such as shifts of the $n$th multi-pole peak (for large $n$) in the CMB[33]. There are other implications as well, such as the number of relativistic degrees of freedom (or effective number of neutrinos) being different at the BBN and the CMB eras. The additional degrees of freedom should be detectable in future CMB measurements. The CMB data show preference for some free streaming neutrino components (but not all species) to be present during the photon decoupling era. If this could be established for all three flavors, very stringent limits on the couplings and hence on neutrino lifetimes can be derived, although this is not possible at present[34].

Direct limits on such decay modes are rather weak. Current bounds on such decay modes are as follows. For the mass eigenstate $\nu_1$, the limit is about

$$\tau_1 \geq 10^5 \text{sec/eV}$$  \hspace{1cm} (4)

based on observation of $\bar{\nu}_e$'s from SN1987A [35] (assuming CPT invariance). For $\nu_2$, strong limits can be deduced from the non-observation of solar anti-neutrinos in KamLAND[36]. A more general but similar bound is obtained from an analysis of solar neutrino data[37]. This bound is given by:

$$\tau_2 \geq 10^{-4} \text{sec/eV}$$  \hspace{1cm} (5)

For $\nu_3$, one can derive a bound from the atmospheric neutrino observations of upcoming neutrinos[38]:

$$\tau_3 \geq 10^{-10} \text{sec/eV}$$  \hspace{1cm} (6)

The strongest lifetime limit is thus too weak to eliminate the possibility of astrophysical neutrino decay by a factor about $10^7 \times (L/100 \text{ Mpc}) \times (10 \text{ TeV}/E)$. It was noted that the
disappearance of all states except $\nu_1$ would prepare a beam that could in principle be used to measure elements of the neutrino mixing matrix\cite{39}, namely the ratios $|U_{e1}|^2 : |U_{\mu 1}|^2 : |U_{\tau 1}|^2$. The possibility of measuring neutrino lifetimes over long baselines was mentioned in Ref.\cite{40}, and some predictions for decay in four-neutrino models were given in Ref.\cite{41}. The particular values and small uncertainties on the neutrino mixing parameters allow for the first time very distinctive signatures of the effects of neutrino decay on the detected flavor ratios. The expected increase in neutrino lifetime sensitivity (and corresponding anomalous neutrino couplings) by several orders of magnitude makes for a very interesting test of physics beyond the Standard Model; a discovery would mean physics much more exotic than neutrino mass and mixing alone. Because of its unique signature, neutrino decay cannot be mimicked by either different neutrino flavor ratios at the source or other non-standard neutrino interactions.

A characteristic feature of decay is its strong energy dependence: $\exp(-Lm/E\tau)$, where $\tau$ is the rest-frame lifetime. For simplicity, we will consider the case that decays are always complete, i.e., that these exponential factors vanish. The simplest case (and the most generic expectation) is a normal hierarchy in which both $\nu_3$ and $\nu_2$ decay, leaving only the lightest stable eigenstate $\nu_1$. In this case the flavor ratio is\cite{39} $|U_{e1}|^2 : |U_{\mu 1}|^2 : |U_{\tau 1}|^2$. Thus, if $|U_{e3}| = 0$ we have

$$\phi_{\nu e} : \phi_{\nu \mu} : \phi_{\nu \tau} \simeq 4 : 1 : 1,$$

where we used the propagation matrix derived from the tri-bi-maximal mixing. Note that this is an extreme deviation of the flavor ratio from that in the absence of decays. It is difficult to imagine other mechanisms that would lead to such a high ratio of $\nu_e$ to $\nu_\mu$. In the case of inverted hierarchy, $\nu_3$ is the lightest and hence stable state, and so\cite{26} we have instead

$$\phi_{\nu e} : \phi_{\nu \mu} : \phi_{\nu \tau} = |U_{e3}|^2 : |U_{\mu 3}|^2 : |U_{\tau 3}|^2 = 0 : 1 : 1.$$
If $|U_{e3}| = 0$ and $\theta_{\text{atm}} = 45^0$, each mass eigenstate has equal $\nu_\mu$ and $\nu_\tau$ components. Therefore, decay cannot break the equality between the $\phi_{\nu_\mu}$ and $\phi_{\nu_\tau}$ fluxes and thus the $\phi_{\nu_e} : \phi_{\nu_\mu}$ ratio contains all the useful information.

When $|U_{e3}|$ is not zero, and the hierarchy is normal, it is possible to obtain information on the values of $|U_{e3}|$ as well as the CPV phase $\delta$[42]. The flavor ratio $e/\mu$ varies from 4 to 10 (as $|U_{e3}|$ goes from 0 to 0.2) for $\cos \delta = +1$ but from 4 to 2.5 for $\cos \delta = -1$. The ratio $\tau/\mu$ varies from 1 to 4 ($\cos \delta = +1$) or 1 to 0.25 ($\cos \delta = -1$) for the same range of $U_{e3}$.

If the decays are not complete and if the daughter does not carry the full energy of the parent neutrino; the resulting flavor mix is somewhat different but in any case it is still quite distinct from the simple 1 : 1 : 1 mix[26]. There is a very recent exhaustive study of the various possibilities[43].

If the path of neutrinos takes them thru regions with significant magnetic fields and the neutrino magnetic moments are large enough, the flavor mix can be affected[44]. The main effect of the passage thru magnetic field is the conversion of a given helicity into an equal mixture of both helicity states. This is also true in passage thru random magnetic fields[45]. It has been shown recently that the presence of a magnetic field of a few (10 or more) Gauss at the source can make the neutrinos decohere as they traverse cosmic distances[46].

If the neutrinos are Dirac particles, and all magnetic moments are comparable, then the effect of the spin-flip is to simply reduce the overall flux of all flavors by half, the other half becoming the sterile Dirac partners. If the neutrinos are Majorana particles, the flavor composition remains 1 : 1 : 1 when it starts from 1 : 1 : 1, and the absolute flux remains unchanged.

What happens when large magnetic fields are present in or near the neutrino production region? In case of Dirac neutrinos, there is no difference and the outgoing flavor ratio
remains 1 : 1 : 1, with the absolute fluxes reduced by half. In case of Majorana neutrinos, since the initial flavor mix is no longer universal but is $\nu_e : \nu_\mu : \nu_\tau \approx 1 : 2 : 0$, this is modified but it turns out that the final (post-oscillation) flavor mix is still 1 : 1 : 1!

Other neutrino properties can also affect the neutrino flavor mix and modify it from the canonical 1 : 1 : 1. If neutrinos have flavor (and equivalence principle) violating couplings to gravity (FVG); then there can be resonance effects which make for one way transitions (analogues of MSW transitions) e.g. $\nu_\mu \rightarrow \nu_\tau$ but not vice versa[47, 48]. In case of FVG for example, this can give rise to an anisotropic deviation of the $\nu_\mu/\nu_\tau$ ratio from 1, becoming less than 1 for events coming from the direction towards the Great Attractor, while remaining 1 in other directions[47]. If such striking effects are not seen, then the current bounds on such violations can be improved by six to seven orders of magnitude.

Another possibility that can give rise to deviations of the flavor mix from the canonical 1 : 1 : 1 is the idea of neutrinos of varying mass (MaVaNs). In this proposal[49], by having the dark energy and neutrinos (a sterile one to be specific) couple, and track each other; it is possible to relate the small scale ($2 \times 10^{-3}$ eV) required for the dark energy to the small neutrino mass, and furthermore the neutrino mass depends inversely on neutrino density, and hence on the epoch. As a result, if this sterile neutrino mixes with a flavor neutrino, the mass difference varies along the path, with potential resonance enhancement of the transition probability into the sterile neutrino, and thus change the flavor mix[50]. For example, if only one resonance is crossed en-route, it can lead to a conversion of the heaviest (mostly) flavor state into the (mostly) sterile state, thus changing the flavor mix to $1 - |U_{e1}|^2 : 1 - |U_{\mu 1}|^2 : 1 - |U_{\tau 1}|^2 \approx 0.4 : 1 : 1$, in case of inverted hierarchy and to $1 - |U_{e3}|^2 : 1 - |U_{\mu 3}|^2 1 - |U_{\tau 3}|^2 \approx 2 : 1 : 1$ in case of normal hierarchy.

Complete quantum decoherence would give rise to a flavor mix given by 1 : 1 : 1,
which is identical to the case of averaged out oscillations as we saw above. The distinction is
that complete decoherence always leads to this result; whereas averaged out oscillations lead
to this result only in the special case of the initial flavor mix being 1 : 2 : 0. To find evidence
for decoherence, therefore, requires a source which has a different flavor mix. One possible
practical example is the “beta” beam source with an initial flavor mix of 1 : 0 : 0. In this
case decoherence leads to the universal 1 : 1 : 1 mix whereas the averaged out oscillations
lead to 2.5 : 1 : 1\[51\]. The two cases can be easily distinguished from each other.

Violations of Lorentz invariance and/or CPT invariance can change the final flavor
mix from the canonical universal mix of 1 : 1 : 1 significantly. With a specific choice of the
change in dispersion relation due to Lorentz Invariance Violation, the effects can be dramatic.
For example, the final flavor mix at sufficiently high energies can become 7 : 2 : 0\[51\].

If each of the three neutrino mass eigenstates is actually a doublet with very small
mass difference (smaller than 10^{-6}eV), then there are no current experiments that could
have detected this. Such a possibility was raised long ago\[52\]. It turns out that the only way
to detect such small mass differences (10^{-12}eV^2 > \delta m^2 > 10^{-18}eV^2) is by measuring flavor
mixes of the high energy neutrinos from cosmic sources. Relic supernova neutrino signals
and AGN neutrinos are sensitive to mass difference squared down to 10^{-20}eV^2 [53].

Let (\nu^+_1, \nu^+_2, \nu^+_3; \nu^-_1, \nu^-_2, \nu^-_3) denote the six mass eigenstates where \nu^+ and \nu^- are a
nearly degenerate pair. A 6x6 mixing matrix rotates the mass basis into the flavor basis. In
general, for six Majorana neutrinos, there would be fifteen rotation angles and fifteen phases.
However, for pseudo- Dirac neutrinos, Kobayashi and Lim\[54\] have given an elegant proof
that the 6x6 matrix \( V_{KL} \) takes the very simple form (to lowest order in \( \delta m^2/m^2 \)):

\[
V_{KL} = \begin{pmatrix} U & 0 \\ 0 & U_R \end{pmatrix} \cdot \begin{pmatrix} V_1 & iV_1 \\ V_2 & -iV_2 \end{pmatrix},
\]

(9)
where the $3 \times 3$ matrix $U$ is just the usual mixing (MNSP) matrix determined by the atmospheric and solar observations, the $3 \times 3$ matrix $U_R$ is an unknown unitary matrix and $V_1$ and $V_2$ are the diagonal matrices $V_1 = \text{diag}(1, 1, 1)/\sqrt{2}$, and $V_2 = \text{diag}(e^{-i\phi_1}, e^{-i\phi_2}, e^{-i\phi_3})/\sqrt{2}$, with the $\phi_i$ being arbitrary phases.

As a result, the three active neutrino states are described in terms of the six mass eigenstates as:

$$\nu_{\alpha L} = U_{\alpha j} \frac{1}{\sqrt{2}} \left( \nu^+_j + i\nu^-_j \right).$$

(10)

The flavors deviate from the democratic value of $\frac{1}{3}$ by

$$\delta P_e = -\frac{1}{3} \left[ \frac{3}{4} \chi_1 + \frac{3}{4} \chi_2 \right],$$

$$\delta P_\mu = \delta P_\tau = -\frac{1}{3} \left[ \frac{1}{8} \chi_1 + \frac{3}{8} \chi_2 + \frac{1}{2} \chi_3 \right]$$

where $\chi_i = \sin^2(\delta m^2_i L/4E)$. The flavor ratios deviate from 1 : 1 : 1 when one or two of the pseudo-Dirac oscillation modes is accessible. In the ultimate limit where $L/E$ is so large that all three oscillating factors have averaged to $\frac{1}{2}$, the flavor ratios return to 1 : 1 : 1, with only a net suppression of the measurable flux, by a factor of 1/2. As a bonus, if such small pseudo-Dirac mass differences exist, it would enable us to measure cosmological parameters such as the red shift in neutrinos (rather than in photons) [40, 53].

## 6 Experimental Flavor Identification

It is obvious from the above discussion that flavor identification is crucial for the purpose at hand. In a water (or ice) cerenkov detector flavors can be identified as follows.
The $\nu_\mu$ flux can be measured by the $\mu'$s produced by the charged current interactions and the resulting $\mu$ tracks in the detector which are long at these energies. $\nu_e'$s produce showers by both CC and NC interactions. The total rate for showers includes those produced by NC interactions of $\nu'_\mu$ and $\nu'_e$ as well and those have to be (and can be) subtracted off to get the real flux of $\nu'_e$'s. Double-bang and lollipop events are signatures unique to tau neutrinos, made possible by the fact that tau leptons decay before they lose a significant fraction of their energy. A double-bang event consists of a hadronic shower initiated by a charged-current interaction of the $\nu_\tau$ followed by a second energetic shower from the decay of the resulting tau lepton[7]. A lollipop event consists of the second of the double-bang showers along with the reconstructed tau lepton track (the first bang may be detected or not). In principle, with a sufficient number of events, a fairly good estimate of the flavor ratio $\nu_e : \nu_\mu : \nu_\tau$ can be reconstructed, as has been discussed recently. Deviations of the flavor ratios from 1 : 1 : 1 due to possible decays are so extreme that they should be readily identifiable[55]. Future high energy neutrino telescopes, such as Icecube[56], will not have perfect ability to separately measure the neutrino flux in each flavor. However, the situation is salvageable. In the limit of $\nu_\mu - \nu_\tau$ symmetry the fluxes for $\nu_\mu$ and $\nu_\tau$ are always in the ratio 1 : 1, with or without decay. This is useful since the $\nu_\tau$ flux is the hardest to measure.

Even when the tau events are not all identifiable, the relative number of shower events to track events can be related to the most interesting quantity for testing decay scenarios, i.e., the $\nu_e$ to $\nu_\mu$ ratio. The precision of the upcoming experiments should be good enough to test the extreme flavor ratios produced by decays. If electromagnetic and hadronic showers can be separated, then the precision will be even better[55]. Comparing, for example, the standard flavor ratios of 1 : 1 : 1 to the possible 4 : 1 : 1 (or 0 : 1 : 1 for inverted hierarchy) generated by decay, the higher(lower) electron neutrino flux will result in a substantial increase(decrease) in the relative number of shower events. The measurement
will be limited only by the energy resolution of the detector and the ability to reduce the atmospheric neutrino background (which drops rapidly with energy and should be negligibly small at and above the PeV scale).

7 Discussion and Conclusions

The flux ratios we discuss are energy-independent to the extent that the following assumptions are valid: (a) the ratios at production are energy-independent, (b) all oscillations are averaged out, and (c) that all possible decays are complete. In the standard scenario with only oscillations, the final flux ratios are \( \phi_{\nu_e} : \phi_{\nu_{\mu}} : \phi_{\nu_{\tau}} = 1 : 1 : 1 \). In the cases with decay, we have found rather different possible flux ratios, for example 4 : 1 : 1 in the normal hierarchy and 0 : 1 : 1 in the inverted hierarchy. These deviations from 1 : 1 : 1 are so extreme that they should be readily measurable.

If we are very fortunate, we may be able to observe a reasonable number of events from several sources (of known distance) and/or over a sufficient range in energy. Then the resulting dependence of the flux ratio \( \nu_e/\nu_{\mu} \) on \( L/E \) as it evolves from say 4 (or 0) to 1 can be clear evidence of decay and further can pin down the actual lifetime instead of just placing a bound[57].

To summarize, we suggest that if future measurements of the flavor mix at earth of high energy astrophysical neutrinos find it to be

\[
\phi_{\nu_e}/\phi_{\nu_{\mu}}/\phi_{\nu_{\tau}} = \alpha/1/1; \tag{11}
\]

then

(i) \( \alpha \approx 1 \) (the most boring case) confirms our knowledge of the MNSP[5] matrix and our prejudice about the production mechanism;
(ii) $\alpha \approx 1/2$ indicates that the source emits pure $\nu'_\mu$s and the mixing is conventional;

(iii) $\alpha \approx 3$ from a unique direction, e.g. the Cygnus region, would be evidence in favor of a pure $\bar{\nu}_e$ production as has been suggested recently[10];

(iv) $\alpha > 1$ indicates that neutrinos are decaying with normal hierarchy; and

(v) $\alpha \ll 1$ would mean that neutrino decays are occurring with inverted hierarchy;

(vi) Values of $\alpha$ which cover a broader range (2.5 to 10) and deviation of the $\mu/\tau$ ratio from 1 (between 0.2 to 4) can yield valuable information about $U_{e3}$ and $\cos \delta$. Deviations of $\alpha$ which are less extreme (between 0.7 and 1.5) can also probe very small pseudo-Dirac $\delta m^2$ (smaller than $10^{-12} eV^2$).

Incidentally, in the last three cases, the results have absolutely no dependence on the initial flavor mix, and so are completely free of any dependence on the production model. So either one learns about the production mechanism and the initial flavor mix, as in the first three cases, or one learns only about the neutrino properties, as in the last three cases. To summarize, the measurement of neutrino flavor mix at neutrino telescopes is absolutely essential to uncover new and interesting physics of neutrinos. In any case, it should be evident that the construction of very large neutrino telescopes is a “no lose” proposition.

8 Acknowledgments

This talk is based on work done in collaboration with John Beacom, Nicole Bell, Dan Hooper, John Learned, Werner Rodejohann and Tom Weiler. I thank them for a most enjoyable collaboration and Tom Weiler for a very careful reading of the manuscript. I would like to thank the organizers of COSPA 2007 for the opportunity to present this talk as well as their
hospitality and for providing a most stimulating atmosphere during the meeting. This work was supported in part by U.S.D.O.E. under grant DE-FG02-04ER41291.

References

[1] Descriptions of several KM3 detectors can be found at http://icecube.wisc.edu/ and at http://www.km3net.org/home.php

[2] The Auger detector is described at http://www.auger.org/; for the planned JEM-EUSO detector see http://jemeuso.riken.jp/ or http://aquila.lbl.EUSO/.

[3] Examples are ANITA:hep-ph/0503304, and SALSA:astro-ph/0412128.

[4] S. Pakvasa, *9th International Symposium on Neutrino Telescopes*, Venice, Italy, 6-9 Mar 2001, Venice 2001, Neutrino Telescopes, ed. M. Baldo-Ceolin, Vol. 2, p. 603; hep-ph/0105127.

[5] Z. Maki, M. Nakagawa and S. Sakata, *Prog. Theoret. Phys.*** **28**, 870 (1962); B. M. Pontecorvo, *Zh.Eksp.Teor.Fiz.*** **53**, 1717(1967); V. N. Gribov and B. M. Pontecorvo, *Phys. Lett.*** **B28**, 493 (1969); B. W. Lee, S. Pakvasa, R. Shrock and H. Sugawara, *Phys. Rev. Lett.*** **38**, 937 (1977).

[6] J. G. Learned and K. Mannheim, *Ann. Rev. Nucl. Part. Sci.*** **50**, 603 (2000), and references therein.

[7] J. G. Learned and S. Pakvasa, *Astropart. Phys.*** **3**, 267 (1995), hep-ph/9405296.

[8] P.Lipari, M. Lusignoli and D. Meloni, *Phys. Rev.* **D75**, 123005 (2007), arXiv:0704.0718; S. Pakvasa, W. Rodejohann and T. J. Weiler, *JHEP* **02**, 05 (2008), arXiv:0711.4517.
[9] J. P. Rachen and P. Meszaros, *Phys. Rev.* **D58**, 123005 (1998); astro-ph/9802280; M.Kachelriess, S. Ostapchenko and R. Tomas, *Phys. Rev.* **D77**, 023007 (2008), arXiv:0708.3007.

[10] L. Anchordoqui, H. Goldberg, F. Halzen and T. Weiler, *Phys. Lett.* **B593**, 42 (2004), astro-ph/0311002.

[11] V. S. Berezinsky and G. T. Zatsepin, *Phys. Lett.* **B28**, 423 (1969).

[12] K. Greisen, *Phys. Rev. Lett.* **16**, 748 (1966); G. Zatsepin and V. Kuzmin, *JETP Lett.* **4**, 78 (1966).

[13] R. Engel, D. Seckel and T. Stanev, *Phys. Rev.* **D64**, 093010 (2001), astro-ph/0101216.

[14] M. C. Gonzalez-Garcia and M. Maltoni, arXiv:0704.1800.

[15] P.F. Harrison, D. H. Perkins and W. G. Scott, *Phys. Lett.* **B530**, 267 (2002), hep-ph/0202074.

[16] M.Applonio et al., *Eur.Phys.J.* **C2331** (2003), hep-ex/0301017.

[17] H. Athar, M. Jezabek and O. Yasuda, *Phys. Rev.* **D62**, 103007 (2000); hep-ph/0005104; L. Bento, P. Keranen and J. Maalampi, *Phys. Lett.* **B476**, 205 (2000); hep-ph/9912240.

[18] L. A. Anchordoqui, H. Goldberg, F. Halzen and T. J. Weiler, *Phys. Lett.* **B621**, 18 (2005), hep-ph/041003.

[19] Z-Z. Xing, *Phys. Rev.* **D74**, 013009(2006), hep-ph/ 065219; W. Rodejohann, *J. Cosmol. Astropart. Phys.* **01**, 029 (2007), hep-ph/0612047.

[20] P. D. Serpico and M. Kachelriess, *Phys. Rev. Lett.* **94**, 211102(2005).
[21] P. Serpico, Phys. Rev. D73, 047301 (2006), hep-ph/0511313; S. Choubey, V. Niro and W. Rodejohann, arXiv:0803.0423.

[22] K. Blum, Y. Nir and E. Waxman, arXiv:0706.2070.

[23] R. L. Awasthi and S. Choubey, Phys. Rev. D76, 113002 (2007), arXiv:0706.0399; H. Athar, M. Jezabek and O. Yasuda, Phys. Rev. D62, (2000), hep-ph/0005104.

[24] D. B. Kaplan, A. E. Nelson and N. Weiner, Phys. Rev. Lett. 93, 091801 (2004), hep-ph/0401099.

[25] Super-Kamiokande collaboration, arXiv:0801.0776.

[26] J. F. Beacom, N. Bell, D. Hooper, S. Pakvasa and T.J. Weiler, Phys. Rev. Lett. 91, 181301 (2003); hep-ph/0211305.

[27] S. Pakvasa, Physics Potential and Development of Muon Colliders, Mu 99, ed. D. Cline, San Francisco, AIP Conf. Proc. 542 (2000) 99, hep-ph/0004077.

[28] G. Gelmini and M. Roncadelli, Phys. Lett. B99, 411 (1981).

[29] Y. Chikasige, R. Mohapatra and R. Peccei, Phys. Rev. Lett. 45, 1926 (1980).

[30] A wide variety of models with $\Delta L = 2$ and/or $\Delta L = 0$ for decays of Majorana and Dirac neutrinos have been discussed in the literature: J. Valle, Phys. Lett., B131, 87 (1983); G. Gelmini and J. Valle, ibid. B142, 181 (1983); A. Joshipura and S. Rindani, Phys. Rev. D46, 3008 (1992); A. Acker, A. Joshipura and S. Pakvasa, Phys. Lett. B285, 371 (1992); A. Acker, S. Pakvasa and J. Pantaleone, Phys. Rev. D45, 1 (1992).

[31] V. Barger, W-Y. Keung and S. Pakvasa, Phys. Rev. D25, 907 (1982); A. P. Lessa and O. L. G. Peres, Phys. Rev. D75, (2007), hep-ph/0701068.
[32] J.F. Beacom, N. Bell and S. Dodelson, *Phys. Rev. Lett.* **93**, 121304 (2004), hep-ph/0404585; see also P. Serpico, *Phys. Rev. Lett.* **98**, 171301 (2007), astro-ph/0701699.

[33] Z. Chacko, L.J. Hall, T. Okui and S.J. Oliver, *Phys. Rev.* **D70**, 085008 (2004), hep-ph/0312267.

[34] N. F. Bell, E. Pierpaoli and K. Sigurdson, *Phys. Rev.* **D73**, 063523 (2006), astro-ph/0511410; S. Hannestad and G. Raffelt, *Phys. Rev.* **D72**, 103514 (2005), hep-ph/0509278.

[35] K. Hirata et al., *Phys. Rev. Lett.* **58**, 1497 (1988); R.M. Bionta et al., ibid. 58, 1494 (1988).

[36] K. Eguchi et al; *Phys. Rev. Lett.* **92**, 071301 (2004), hep-ex/0310047.

[37] J. F. Beacom and N. Bell; *Phys. Rev.* **D65**, 113009 (2002), hep-ph/0204111; and references cited therein.

[38] V. D. Barger, J. G. Learned, S. Pakvasa and T. J. Weiler, *Phys. Rev. Lett.* **82**, 2640 (1999); hep-ph/9810121; V. Barger, J. G. Learned, P. Lipari, M. Lusignoli, S. Pakvasa and T. J. Weiler, *Phys. Lett.* **B462**, 104 (1999), hep-ph/9907421; Y. Ashie et al., *Phys. Rev. Lett.* **93**, 101801 (2004), hep-ex/0404034.

[39] S. Pakvasa, *Lett. Nuov. Cim.* **31**, 497 (1981); Y. Farzan and A. Smirnov, *Phys. Rev.* **D65**, 113001 (2002); hep-ph/0201105.

[40] T. J. Weiler, W. A. Simmons, S. Pakvasa and J. G. Learned, hep-ph/9411432.

[41] P. Keranen, J. Maalampi and J. T. Peltonieni, *Phys. Lett.* **B461**, 230 (1999), hep-ph/9901403.
[42] J. F. Beacom, N. Bell, D. Hooper, S. Pakvasa and T. J. Weiler, Phys. Rev. D69, 017303 (2004), hep-ph/0309267.

[43] M. Maltoni and W. Winter, arXiv:0803.2050.

[44] K. Enqvist, P. Keranen and J. Maalampi, Phys. Lett. B438, 295(1998), hep-ph/9806392.

[45] G. Domokos and S. Kovesi-Domokos, Phys. Lett. B410, 57 (1997), hep-ph/9703265.

[46] Y. Farzan and A. Yu. Smirnov, arXiv:0803.0495.

[47] H. Minakata and A. Yu. Smirnov, Phys. Rev. D54, 3698 (1996), hep-ph/9601311.

[48] V. D. Barger, S. Pakvasa, T. J. Weiler and K. Whisnant, Phys. Rev. Lett. 85, 5055 (2000), hep-ph/0005197.

[49] R. Fardon, A. E. Nelson and N. Weiner, J. Cosmol. Astropart. Phys. 410, 005 (2004), astro-ph/0309800; P. Q. Hung, hep-ph/00010126.

[50] P. Q. Hung and H. Paes, Mod. Phys. Lett. A20, 209 (2006), astro-ph/0311131.

[51] D. Hooper, D. Morgan and E. Winstanley, Phys. Lett. 609 206, (2005), hep-ph/0410094.

[52] L. Wolfenstein, Nucl. Phys. B186, 147 (1981); S. M. Bilenky and B. M. Pontecorvo, Sov. J. Nucl. Phys. 38, 248 (1983); S. T. Petcov, Phys. Lett. B110, 245 (1982).

[53] J. F. Beacom, N. Bell, D. Hooper, J. G. Learned, S. Pakvasa and T. J. Weiler; Phys. Rev. Lett., 92 (2004); hep-ph/0307151; see also P. Keranen, J. Maalampi, M. Myyrylainen and J. Riittinen, Phys. Lett. B574, 162 (2003), hep-ph/0307041 for similar considerations.

[54] M. Kobayashi and C. S. Lim, Phys. Rev. D64, 013003 (2001), hep-ph/0012266.
[55] J. F. Beacom, N. Bell, D. Hooper, S. Pakvasa and T. J. Weiler; *Phys. Rev.* **D68**, 093005 (2003), hep-ph/0307025; F. Halzen and D. Hooper, *Rept. Prog. Phys.* **65**, 1025 (2002), astro-ph/0204527.

[56] A. Karle, *Nucl. Phys. Proc. Supp.*, **118** (2003), astro-ph/0209556; A. Goldschmidt, *Nucl. Phys. Proc. Suppl.* **110**, 516 (2002).

[57] G. Barenboim and C. Quigg, *Phys. Rev.* **D67**, 073024 (2003), hep-ph/0301220.