Spontaneous coherent cyclotron emission from photo-injector electron bunches: superradiation and two-frequency regime

Yu S Oparina¹ and D S Pershin¹

¹Institute of Applied Physics of RAS, 46 Ul'yanov st., Nizhny Novgorod, Russia

E-mail: YuliaOparina1993@yandex.ru

Abstract. Short dense electron bunches are attractive from the viewpoint to use them for the implementation of powerful and compact radiation source based on the spontaneous mechanism of radiation. The regime of spontaneous radiation is realized, when the bunch phase size is smaller than 2π. Modern photo-injectors insure formation of mildly relativistic dense electron bunches with duration less than 1 ps. Such bunches are “ready” to radiate waves at terahertz frequencies in the spontaneous regime. However, the repulsion of particles caused by a strong Coulomb field inside the dense electron bunch strictly limits the duration of the radiation process due to the increase in the bunch length. We propose two mechanism of cyclotron radiation to solve this problem. The first one is the regime of group synchronism of particles with the radiated wave; in this case compensation of Coulomb repulsion in the phase space takes place. The second proposed solution is to use compression of bunch by radiated electromagnetic fields.

1. Introduction

Photo-injector accelerators ensure formation of dense electron bunches of less than 1 picosecond (ps) durations with charges of up to 1 nC, and with particles energy of 3-7 MeV [1-3]. Photo-injector electron bunches are demanded by important physical applications. We will focus on the spontaneous emission of powerful coherent terahertz radiation. The spontaneous radiation process is realized, when the effective phase size of the electron bunch with respect to the radiated wave is small enough (≪ 2π), so that the wave packets emitted by each of the electrons add up basically in phase. Therefore, bunches with durations less than 1 ps are initially well-bunched with respect to radiated wave and the emission occurs in the spontaneous regime.

A key problem here is a stabilization of longitudinal Coulomb repulsion in dense bunches, which leads to the increase of the bunch length. If the “operating” radiation mechanism of the source is based on the longitudinal electron bunching [4] (free-electron lasers and Cherenkov masers), then axial expansion of the bunch leads automatically to an increase in the bunch phase size with respect to the radiated wave (see figure 1.a). This results in the saturation of the process of spontaneous emission. Thus, in the case of undulator and Cherenkov masers, special methods providing control of the axial length of the bunch [5,6] (and, possibly, the compression of the bunch [7-9], [19-21]) are required. In the case of cyclotron radiation, the situation is more complicated due to the two-dimensional character of the electron phase with respect to the wave (see figure 1, b). Another words, it is possible to provide conditions to the bunch stretching along the 2D helix of the constant electron phase with respect to the wave. In this case, an increase in the bunch length does not lead to an increase of the phase size of the
bunch. We have shown that such effect takes place at definite conditions (namely, when the group wave velocity is equal to the axial electron velocity, see figure 2, b), [10,11].

In addition, we propose the two-frequency regime, when the velocity of the high-frequency wave slightly exceeds the bunch longitudinal velocity, and the group velocity of the low-frequency wave slightly smaller than the bunch velocity (see figure 2, b). For the low-frequency wave it is possible to suppress the generation by “placing” close to the zero-wave phase. The such bunch place is optimal from the viewpoint of the cyclotron radiation self-compression [10,12].

2. Spontaneous superradiant sub-THz coherent cyclotron emission

As was said, essential to the realization of a THz source based on the spontaneous coherent radiation is the presence of the mechanism of stabilization of a bunch phase size with respect to the radiated wave. In the case of the cyclotron maser, the electron bunching has a longitudinal component proportional to the electron axial velocity change, and a transverse component, which is inversely proportional to the change in electron energy. The Coulomb repulsion leads to opposite contributions of these components, and it is possible to provide the compensation of its influence in the phase space. This effect takes place in the group resonance regime [10], where the group velocity of the radiated wave is close to the axial electron velocity (see figure 2, c). Such regime is attractive from the viewpoint of the wave emission process, as the maximal growth rate of the cyclotron instability is achieved in this regime. This is due to the radiated wave does not “slip” along the electron bunch. Therefore, during the radiation process, the wave field is accumulated in the region close to the bunch [10,11].

Figure 1. (a): In the case of either ubitron or Cherenkov masers, axial expansion of the bunch leads to an increase in the bunch phase size with respect to the radiated wave. (b): In the cyclotron maser, the bunch is stretched along the 2D helix of the constant electron phase with respect to the wave.

Let’s consider the motion of a particle along a helical trajectory in a circular waveguide immersed in the uniform magnetic field. For the description the Coulomb interaction inside the bunch we represent the bunch as a discrete set of $n$ fractions (thin disks) [8]. We assume that the particle interacts with the lowest $\text{TE}_{11}$ circular-polarized transverse mode of the waveguide. The equations describing electron-wave interaction in the such system are written as follows (in more details [10]):

\[
\begin{align*}
\frac{1}{\gamma_{z,0}} \frac{\partial \gamma}{\partial \tau} &= -\text{Re}[\chi_1 a * \exp(i\theta)] - F_c, \\
\frac{1}{\gamma_{z,0}} \frac{\partial p_z}{\partial \tau} &= -\beta_{gr} \text{Re}[\chi_1 a * \exp(i\theta)] - F_c/\beta_z, \\
\frac{1}{\gamma_{z,0}} \frac{\partial \theta}{\partial \tau} &= -\beta_{gr}(\beta_z - \beta_{z,0}) - \frac{b\Delta \gamma}{\gamma} + \left(1 - \beta_{z,0}\beta_{gr}\right) \text{Im} \frac{a * \exp(i\theta)}{p_\perp} + \Delta,
\end{align*}
\]
\[ a = \frac{G}{2} \sqrt{\frac{\pi}{\tau}} \int_0^\tau \left( \chi_1 \exp(-i\theta) \phi_0 \exp \left( -i \epsilon^2 \frac{\tau - \tau'}{2} \right) \right) d\tau'. \]  

Here, \( \tau = \omega(t - V_{z,0}z/c^2) \), \( \omega \) is the radiation frequency, which is determined by the formula describing the Doppler up-转化 of the nonrelativistic cyclotron frequency

\[ \omega = \frac{\Omega_c}{\gamma^2(1 - \beta_{gz}^2)}, \]

\( \gamma = \sqrt{1 - \beta^2} \) - the relativistic gamma-factor, \( \beta = V/c \) - the normalized electron velocity, \( \beta_z = V_z/c \) is the normalized longitudinal velocity, \( \beta_{gz} \) - the wave group velocity.

Equation (1) for the change in the electron relativistic gamma factor determines the electron energy change, equation (2) describes change in the axial electron momentum \( p_z = \gamma \beta_z \). Coefficient \( \chi_1 \approx \beta_1/2 \) is the electron-wave coupling factor, \( F_c \) is the Coulomb field contribution. For an electron placed in the \( j \)th disk

\[ F_c = \frac{H e \lambda \beta_z}{l_a S} \sum_{i \neq j} |\alpha_j - z_i| \left( 1 - \frac{|\Delta z_{ij}| \gamma_{z,i}}{\sqrt{R^2 + (\Delta z_{ij})^2}} \right) f_q(z_{i0}), \]

here, \( l \) - the bunch current, \( L_e \) - the bunch length, \( S = \pi R^2 \), \( R \) is the radius of the electron bunch, \( \lambda \) is the radiated wave length, and \( f_q(z_{i0}) \) is the function of the charge distribution.

Equation (3) describes the change in the electron resonance phase \( \vartheta = \omega t - k_z z - \varphi \). Here, \( k_z = \beta_{gz} \kappa_0 \) is the longitudinal wavenumber, \( \kappa_0 \) is the wave number of the radiated wave, \( \Delta = 1 - \beta_{gz} \beta_{z,0} - b/\gamma_0 \) is the mismatch of the electron-cyclotron resonance, here \( b = \Omega_c/\omega \).

In the integral equation for the wave excitation (4) \( a \) is the normalized «slow» amplitude of the wave, \( G = \kappa_0 H e \beta_{z,0} 2(1 - \beta_{gr}^2) \gamma_{z,0}^2 / l_a \gamma \eta \beta_{gr} \) is the factor of the wave excitation, \( eN \) is the mode norm, \( \epsilon \approx \beta_z - \beta_{gr} \) is the slippage parameter.

**Figure 2.** (a): Cross section of the cavity. (b): Longitudinal section of the cavity. (c): Dispersion characteristic for the cases of the electron velocity is close to the group velocity of the radiated wave (“grazing”, \( G \)), and for the two waves at low (L) and at high (H) frequencies generation regime.

Let’s consider equation (3) for the electron phase. In order to describe the contribution of the electron-wave interaction and of the Coulomb repulsion into the phase change, it is convenient to split the energy change into two components, \( \Delta \gamma = \Delta \gamma_c + \Delta \gamma_w \). Coming from equations (1) and (2), for the respective longitudinal velocity changes one obtains

\[ \Delta \beta_{z,c} \approx \frac{\Delta \gamma_c}{\beta_{z,0} \gamma_0^2 \gamma_0}, \]

\[ \Delta \beta_{z,w} \approx \frac{\Delta \gamma_w}{\gamma_0} (\beta_{gr} - \beta_{z,0}). \]
In the case of cyclotron resonance and weak influence of the force bunching, equation (3) comes down to the following equation:

$$\frac{1}{y_{z,0}^2} \frac{\partial \theta}{\partial \tau} = \frac{\beta_{z,0} \beta_{gr}}{\gamma_0} \left[ (1 - \beta_{z,0} \beta_{gr}) - \frac{\beta_{gr}}{\beta_{z,0} y_{z,0}^2} \right] + \frac{\Delta y_w}{\gamma_0} \left[ (1 - \beta_{z,0} \beta_{gr}) - \beta_{gr} (\beta_{gr} - \beta_{z,0}) \right].$$

(9)

Such form of the phase change equation makes obviously, that in the case of group synchronism the Coulomb repulsion doesn’t affect the phase change.

We consider electron bunches with the total charge of 0.5 nC, 1 mm diameter, the electron energy is 6 MeV (which corresponds to the relativistic Lorentz factor electrons $y_0 = 13$), and the duration 0.25 ps. These parameters are quite typical for modern photo injectors [1]; in particular, they are close to the expected parameters of the photoinjector the Israeli THz radiation source being constructed at Ariel University [13]. The initial transverse velocity $\beta_L = 1/y_0$ is chosen from the viewpoint of maximization of the electron efficiency predicted by the cyclotron maser theory [14]. This transverse velocity together with the group resonance condition results in the following formula describing the Doppler up-conversion of the non-relativistic cyclotron frequency $\omega = \Omega_e y_0/2$ for the cyclotron radiation at a wave frequency of 0.4 THz, which corresponds to the operating magnetic field $B_0 = 2.2$ T (the waveguide diameter 4 mm). Figure 3 illustrates simulations of spontaneous emission from the bunch in the “grazing” regime. It shows the electron efficiency, the change in electron energy averaged over all particles in the bunch normalized to the initial kinetic electron energy $(y_0 - \gamma)/(y_0 - 1)$, the efficiency of electron bunching (which is described by formula $\rho = \langle \exp(-i\theta) \rangle$), and the relative increase in the bunch length $L_e/L_{e,0}$.

![Figure 3](image)

**Figure 3.** Spontaneous emission from the short bunch with the total charge 0.5 nC and with the energy 6 MeV in the regime of “grazing”, solid curves accord to the case without initial electrons angle spread, dashed curves accord to the case of initial transverse momentum spread $\sim 0.5 y_0$. Efficiency of the electron-wave interaction $(y_0 - \gamma)/(y_0 - 1)$ (a), electron bunching efficiency $\rho = \langle \exp(-i\theta) \rangle$ (b), and the effective bunch axial length normalized to the initial length $L_e/L_{e,0}$ (c) versus the axial coordinate (cm).

The saturation is achieved at relatively short lengths $\sim 30$ cm, and the saturated values of the electron efficiency are relatively high ($\sim 15\%$). If there is the initial particle angular spread $\sim 0.5/y_0$ the efficiency of the electron-wave interaction is slightly lower.

### 3. Two-frequency regime

In this section we consider the case, when the magnetic field slightly exceed the resonance value. This corresponds to the situation of two wave (at low and high frequency) generation, see figure 2, c. The wave group low-frequency radiated wave is slightly smaller than the longitudinal velocity of electrons, and the the high-frequency wave “overtakes” the e-bunch.

In the recent works we have mentioned [10] and briefly described [12] that it is possible to provide the situation when the bunch is placed close at the “zero” phase with respect to the radiated wave, see figure 4, a (L). Obviously, the efficiency of radiation process in this case is relatively low. However, this case can be way to provide self-compression of the bunch by the radiated field (similar to the undulator radiation self-compression considered in Ref. [7]). This is realised if the group velocity of the radiated wave slightly smaller, than the bunch longitudinal velocity. Consequently, in the case of
two waves generation regime for the low-frequency wave generation this situation takes place. This is possible to use, firstly, for the suppression of the low-frequency wave generation, and, secondly, for the bunch longitudinal stabilization.

Let’s consider the integral equation for the amplitude (4). In the case of the group synchronism, its solution \( a \approx \sqrt{\pi} \beta \rho \), the bunch “tail” is placed close to the “zero” wave phase, the electrons of the bunch head in the decelerating phase, see figure 4, a (G). Otherwise, the slippage parameter is not equal to zero and the solution in approximation of the constant bunching parameter \( \rho \) as follows

\[
a \approx \frac{G \beta_0 \rho}{2 \varepsilon} \sqrt{\frac{2}{\pi}} \Gamma \left( \frac{1}{2} \right) \exp \{ i \phi (\varepsilon) \} = \frac{G \beta_0 \rho}{2 \varepsilon} \ast \exp \{ i \phi (\varepsilon) \},
\]

Figure 4, b illustrates results of numerical simulations for the wave phase offset \( \phi (\varepsilon) \) as a function of the slippage parameter. It is possible to realize the two-frequencies regime, when the electron bunch is placed close to the “zero” phase of the low frequency wave [see figure 4, a (L)], and the efficiency of generation at low frequency could be significant, despite the parameter of the bunching with respect to this wave sufficiently high. At the same time, the bunch is slightly shifted in the accelerating wave phase, but substantial part of particles stays at the decelerating wave phase [see figure 4, a (H)].

Figure 4. (a): The bunch places according to the single-frequency regime of “grazing” (G) of dispersion characteristics, to emission at a low frequency (L) with a weak efficiency, and at a high frequency (H). (b): The phase offset as a function of “slippage” parameter.

Figure 5 illustrates results of numerical simulation for the spontaneous cyclotron emission of the bunch in the two-frequency regime. The value of operating magnetic field 4 T, which corresponds to the high-frequency radiation at the 1.35 THz and to the low-frequency suppressed wave at 0.12 THz. For the bunch with the considered parameters the effect of the radiative self-compression is not significant. It is more appropriate to talk about the bunch length stabilization. In this case the process of radiation is rapid, and a spread is not a influence on the electron-wave interaction.

Figure 5. Spontaneous emission from the bunch with the charge 0.5nC and the energy 6 MeV in the two-waves regime (blue dashed curves accord the low frequency 0.12 THz; purple solid curves accord to the high frequency 1.35THz). Efficiency of the electron-wave interaction (a), electron bunching efficiency (b), and the effective bunch length normalized to the initial length \( L_e/L_e,0 \) (c) versus the axial coordinate (cm).
4. Conclusion
In this paper, we have developed the theory of electron cyclotron masers to the situation when a short dense electron bunch produces spontaneous radiation of a short-wave packet. This type of emission is an attractive way to realize a relatively short, simple, and efficient source of THz radiation. We propose two mechanisms of the cyclotron radiation. The first one is relatively low-frequency regime of the group synchronism in this situation, along with the solution of the problem of Coulomb repulsion, one more the wave is emitted in the so-called superradiant regime. The second mechanism is the two-frequency regime, when the generation of the low-frequency wave is suppressed, also it stabilizes the bunch phase size, whereas the efficiency of high-frequency wave is relatively high. It turns out, that two-frequency radiation of short dense electron bunches is more effective in the negative-mass regime.

It is important to note, that in the contrast with the undulator two-frequency source [7,19], no need for additional short-period undulator for the THz-frequency radiation, or the high-frequency radiation transverse structure accords to the lowest mode TE_{11} (consequently the expected radiation efficiency is higher).

The work is supported by Russian Foundation for Basic Research Project 18-32-00351.

References
[1] Power J G 2010 AIP Conf. Proc 1299 20
[2] Bartnik A et al B 2015 Phys. Rev. ST Accel. Beams 18 083401
[3] Stephan F et al 2010 Phys. Rev. ST Accel. Beams 13 020704
[4] Bratman V L et al 1979 Opt. Commun 30 4
[5] Lurie Y, Bratman V L and Savilov A V 2016 Phys. Rev. Accel. Beams 19 050704
[6] Balal N et al 2015 Appl. Phys. Lett. 107 163505
[7] Bandurkin I V, Oparina Yu S and Savilov A V 2017 Appl. Phys. Lett. 110 263508
[8] Bandurkin I V, Kurakin I S and Savilov A V 2017 Phys. Rev. Accel. Beams 20 020704
[9] Savilov A V, Bandurkin I V and Oparina Yu S 2017 EPJ Web of Conferences 149 05008
[10] Oparina Yu S and Savilov AV 2019 Phys. Rev. Accel. Beams 22 030701
[11] Oparina Yu S and Savilov A V 2017 EPJ Web of Conferences 149 05019
[12] Oparina Yu S et al 2018 J. Phys.: Conf. Ser. 1135 012018
[13] Friedman A et al 2015 Phys. Rev. ST Accel. Beams 18(7) 070701
[14] Bratman V L, Ginzburg N S, Nusinovich G S, Petelin M I and Strelkov P S 1981 Int. J. Electron 51(4) 541
[15] Nielsen C and Sessler A 1959 Rev. Sci. Instrum. 30 80
[16] Kolomensky A A and Lebedev A N 1959 At. Energ. 7 549
[17] Bratman V L 1976 Tech. Phys 46 2030
[18] Bratman V L and Savilov A V 1995 Phys. Plasmas 2 557
[19] Bandurkin I V et al 2018 Evestiya Rossiiskoi Akademii Nauk, Seriya Fizicheskaya 82(12) 1754
[20] Bandurkin I V et al 2017 IEEE 64(9) 8002605
[21] Savilov A V 1997 Phys. Plasmas 4 2276