Numerical Simulations of Microwave Heating of Liquids: Enhancements using Krylov Subspace Methods

M R Lollchund¹, K Dookhitram², M S Sunhaloo² and R Boojhawon³

¹Department of Physics, University of Mauritius, Republic of Mauritius
²Department of Applied Mathematical Sciences, University of Technology, Mauritius, Republic of Mauritius
³Department of Mathematics, University of Mauritius, Republic of Mauritius

E-mail: r.lollchund@uom.ac.mu

Abstract. In this paper, we compare the performances of three iterative solvers for large sparse linear systems arising in the numerical computations of incompressible Navier-Stokes (NS) equations. These equations are employed mainly in the simulation of microwave heating of liquids. The emphasis of this work is on the application of Krylov projection techniques such as Generalized Minimal Residual (GMRES) to solve the Pressure Poisson Equations that result from discretisation of the NS equations. The performance of the GMRES method is compared with the traditional Gauss-Seidel (GS) and point successive over relaxation (PSOR) techniques through their application to simulate the dynamics of water housed inside a vertical cylindrical vessel which is subjected to microwave radiation. It is found that as the mesh size increases, GMRES gives the fastest convergence rate in terms of computational times and number of iterations.

1. Introduction

Microwave heating of liquids is a very complex process and numerical simulations are becoming indispensable tools in helping researchers to understand the phenomenon, predict and control its behaviours. Moreover, using a numerical model, fully empirical design of microwave heating system and expensive prototype equipment for testing the design can be avoided [1]. The simulation process involves solving Maxwell’s electromagnetic equations using a numerical scheme such as the Finite Difference Time Domain (FDTD) or Finite Element Method (FEM) in order to obtain the steady-state electromagnetic (EM) field distributions within the computational domain [2]. The volumetric dissipative electromagnetic power is then utilized as input to the heat transport and the Navier-Stokes (NS) equations for computations of temperature and fluid flow respectively. These equations are generally solved using a semi-implicit projection scheme for faster and reliable solutions [3]. This scheme results into a Poisson-type equation for solving the coupled pressure-velocity fields of the heated liquid. It should be noted that such type of Pressure-Poisson equation (PPE) is also obtained when other schemes such as SIMPLE, PISO or Fractional Step Method are employed to solve the NS equations. The temperature variation during the heating process can cause a change in the complex permittivity
and/or permeability of the liquid, which will affect the space and time variation of the EM field [1]. Therefore, the updated EM parameters need to be re-injected into the solver for Maxwell’s equations and the whole simulation process is repeated until the desired heating time or temperature is reached.

It is well known that in simulations involving heating of liquids, the major computational cost is mainly influenced by finding iterative solutions of the PPE [3]. This motivates the present work, where we aim to reduce the computational time and memory requirements of our in-house developed numerical model for microwave heating simulations by employing an appropriate solver for the PPE. A comparative computational study is carried out to determine the best performing algorithms from the existing traditional solvers Gauss Seidel (GS) and Point Successive over Relaxation (PSOR), and the Krylov Subspace method Generalized Minimal Residual (GMRES) [4], to solve the PPE. Our model is applied in the study of the dynamics of water housed inside a vertical cylindrical vessel which is subjected to microwave radiation.

2. Methodology

2.1. The Problem Studied

The problem studied in this section was first investigated in [5]. It consists of a closed vertical cylindrical vessel of radius $R = 85$ mm and height $H = 101$ mm, completely filled with water. It is assumed that microwave radiation of intensity $Q_0 = 1.125 \times 10^6$ Wm$^{-3}$ are incident on the sides of the vessel and that the power distribution inside the vessel is radially symmetrical and is given by Lambert’s law as

$$ Q(r) = Q_0 \exp \left( \frac{r - R}{\delta} \right), $$

where $\delta$ is the penetration depth of the wave. The change in temperature will lead to changes in the complex permittivity of the material and hence to a change in the penetration depth. In the range 20 °C to 50 °C, the penetration depth can be approximated as a linear function of temperature as $\delta = 0.004408 + 0.0005752T$, where $T$ is in °C and $\delta$ is in meters [5].

The characteristics of the fluid flow are obtained by solving the continuity and momentum equations which are commonly termed as Navier-Stokes (NS) equations. The heat transfer is modeled by the energy equation. The mathematical formulations of these equations are given elsewhere [6]. The no-slip boundary conditions are used for the velocity components at the side, top and bottom walls of the vessel which are also taken to be thermally insulated. The buoyancy force term in the NS equation is given by Boussinesq approximation [6]. The thermal properties of the water sample are given elsewhere [5, 6]. Initially, the temperature throughout the vessel is set at 25 °C and the liquid velocity is zero. The simulation is run for a total of 180 seconds of microwave heating.

2.2. The Pressure Poisson equation (PPE)

Upon employing a semi-implicit projection scheme and application of the above boundary conditions to the NS equations for calculating the flow fields, a Pressure-Poisson equation (PPE) is obtained. The interested reader is referred to [6] for a derivation of the PPE. Following the conventional process, the PPE can be written as the linear system

$$ Ax = b, \quad (1) $$

where the coefficient matrix $A$ is block tri-diagonal, the elements of the unknown vector $x$ consist of the pressure values that need to be computed at each point on the grid and the elements of vector $b$ are obtained from known terms in the NS equations [6].
2.3. The Iterative Linear Solvers

Many different iterative methods for solving equation (1) are known. In the present study we compare the performances of three well known iterative methods. The linear solvers considered are the traditional Gauss Seidel (GS) and Point Successive over Relaxation (PSOR) methods, and the Krylov projection scheme GMRES. We should mention that the GMRES method is rarely employed in the microwave heating simulation community due to its complex algorithm structure as compared to the GS and PSOR techniques [7]. Details about these solvers can be obtained from [3, 4, 6, 7]. The objective is to study their convergence rate and computational times for the problem studied using different grid sizes for different convergence tolerances. The convergence criteria for determining solution of the PPE is based on the absolute difference between two successive iterates:

\[ \sum_i |x_i^{(n)} - x_i^{(n-1)}| < \epsilon, \]

where \( x_i \) is the \( i \)-th element in vector \( x \), \( n \) is the iteration number and \( \epsilon \) is the tolerance value for convergence.

3. Results and Discussions

3.1. Temperature profiles

Figure 1 compares the temperature profiles obtained inside the vessel after various heating times (60 s, 120 s and 180 s) with the work presented in [5]. The left plot of figure 1 shows the radial temperature profiles at a height of 68 mm from the bottom of the vessel and the right plot of figure 1 depicts the variation of temperature with height (measured from the bottom of the vessel) at a radius of 51 mm. Good agreement between the results are observed.

![Temperature profiles](image1.png)

Figure 1. Comparison of temperature profiles, obtained using the present model with those published by Chatterjee et al. [5], at different times in the cylindrical vessel (Grid size: 31 × 19 × 62).

3.2. Performance of the iterative linear solvers

The numerical experiments are performed for different grids. In table 1 the computational times (cpu) involved in the computation of the PPE and the corresponding number of iterations involved (n) are displayed. It should be noted that all computations were carried out on an Intel Core 2 Quad desktop computer with 2.50 GHz clock speed and 2 GB RAM. It can be observed...
Table 1. Performance of three iterative schemes for solving the PPE, with different values of $\epsilon$.

| $\epsilon$ | Grid size | GS    | PSOR  | GMRES |
|------------|-----------|-------|-------|-------|
| $10^{-3}$  | 15 $\times$ 7 $\times$ 30 | 80    | 76    | 73    |
|            | 21 $\times$ 13 $\times$ 42 | 197   | 191   | 186   |
|            | 31 $\times$ 19 $\times$ 62 | 423   | 405   | 395   |
|            | 41 $\times$ 25 $\times$ 82 | 718   | 688   | 678   |
| $10^{-4}$  | 15 $\times$ 7 $\times$ 30 | 356   | 339   | 331   |
|            | 21 $\times$ 13 $\times$ 42 | 839   | 803   | 787   |
|            | 31 $\times$ 19 $\times$ 62 | 1801  | 1725  | 1698  |
|            | 41 $\times$ 25 $\times$ 82 | 3078  | 2956  | 2912  |
| $10^{-5}$  | 15 $\times$ 7 $\times$ 30 | 936   | 898   | 885   |
|            | 21 $\times$ 13 $\times$ 42 | 1875  | 1798  | 1771  |
|            | 31 $\times$ 19 $\times$ 62 | 4042  | 3879  | 3821  |
|            | 41 $\times$ 25 $\times$ 82 | 6975  | 6684  | 6589  |

from table 1 that the GMRES method is more efficient than both GS and PSOR schemes, especially when the domain size is increased. The number of iterations involved is nearly the same for all three methods considered but the cpu speedup for GMRES is more than twice faster than GS and PSOR in all the cases.

4. Conclusion

It was demonstrated in this work that the use of GMRES for solving the linear PPE is much more efficient than the traditional GS and PSOR schemes. In practice, one always uses a fine mesh to obtain a better accuracy of the required solution. We observed that for large grid size GMRES converges faster than the GS and PSOR methods. Moreover, the GMRES method is computationally much more efficient because it provides very accurate numerical results by using very little computational times and virtual storage than the GS and PSOR schemes. As future work, we will consider other versions of GMRES, namely the Simpler GMRES and its augmented variant [8].

References

[1] Zhao X, Yan L and Huang K 2011 Review of Numerical Simulation of Microwave Heating Process, Advances in Induction and Microwave Heating of Mineral and Organic Materials (Stanislaw Grundas (Ed.), ISBN: 978-953-307-522-8, InTech, DOI: 10.5772/13387)
[2] Taflove A 1999 Advances in Computational Electrodynamics: The Finite Differences Time Domain Method (Artech House Inc.)
[3] Johnston H and Liu J J. Comput. Phys. 199 221
[4] Saad Y and Schultz M H SIAM J. Sci. Stat. Comput. 7 856
[5] Chatterjee S, Basak T and Das S K J. Food Eng. 79 1269
[6] Lollchund M R 2007 Numerical Simulation of Heat and Mass transfer during Microwave Heating Processes: Study of a High Pressure Vessel Ph.D. thesis University of Mauritius
[7] Saad Y 1996 Iterative Methods for Sparse Linear Systems (U.S.A: PWS Publishing Company)
[8] Boojhawon R and Bhuruth M Future Gener. Comp. Sy. 20 389