INTRODUCTION

Nowadays, energy harvesting becomes one of the most essential and favorite research topics. One of the primary sources of mechanical vibration energy is the car suspension system as a damper for car vibrations.\(^1,2\) These unwanted vibrations could be converted into useful energy to be reused in the vehicles as for example, storing it back into batteries for later use, running the controller for the active suspension system, or power different electrical microsystems. Abdelkareem et al.\(^3\) reported that the harvested power from a medium-size vehicle was around 350 W, while for the off-road vehicle, the possible harvested power could reach over to 1 kW. In car suspension vibration, there are many ways of harvesting the vibration energy such as piezoelectric,\(^4-18\) electromagnetic,\(^19-23\) electrostatic transducers,\(^24\) linear electromagnetic shock absorber, MR electromagnetic regenerative damper, hydraulic, rack-pinion, ball-screw, and cable/pulleys.\(^3,25-29\)
Piezoelectric transduction is one of the most used harvesters due to its simplicity, ability to operate in the wide frequency range, good electromechanical coupling effect, and providing the highest power density.\textsuperscript{30-34} Lead Zirconate Titanate (PZT) has become the most dominant piezoelectric type to be utilized in energy harvesting studies. The PZT film coefficient is at least two times greater than the other types which helps in providing high power output.\textsuperscript{35}

Utilizing piezoelectric harvesters for energy conversion from ambient vibrations has attracted some researchers; however, more consideration must be paid to this topic. Recently, few researchers have been studying in implementing the piezoelectric energy harvesters in a car suspension system. There are several locations of installing the piezoelectric material in the car suspension system. Piezoelectric patches could be implemented in the car suspension springs, and from the road irregularities, they will strain and then produce an electric charge across their terminals.\textsuperscript{4,15} Namuduri et al\textsuperscript{4} built a piezoelectric harvester that was attached to the vehicle spring assembly. A layer of piezoelectric composite fiber was attached on the top of the four surfaces of the leaf springs. Each piezoelectric element was coupled to a rectifier to convert the AC output voltage to DC that is used in powering multiple devices in the vehicle or storing it in a battery or capacitor. They reported that the generated electric charge is proportional to the level of stress $\sigma$ or strain $S$ applied to the springs, more deformations resulting in a more generated voltage across the piezoelectric element. Wang et al\textsuperscript{5} modeled the harvesting system as a quarter-car model attached with a piezoelectric material and connected in parallel with the spring of the suspension system. From the theoretical analysis method, the harvesting system was excited by a sine wave acceleration of 1 g and produced a power of 2.84 W. The efficiency of the energy harvesting system is affected by the tire stiffness, suspension spring stiffness, and a suspension damping coefficient. The theoretical analysis results were validated experimentally by modeling the car body and tires as aluminum blocks. The maximum output voltage was 0.33 V at a frequency of 38.58 Hz. However, the experimental harvested power was not considered in the paper. Also, different characteristics of the piezoelectric material such as type (cantilever or stack), dimensions (area $A_p$ and thickness $t_p$), piezoelectric stiffness coefficient $K_p$, charge constant $d_{33}$, dielectric permittivity $\varepsilon_{33}$, and the modulus of elasticity $E_p$ were not clearly stated. In contrast, Al-Yafeai et al\textsuperscript{6} compared the results of the harvested energy from Wang’s quarter model\textsuperscript{5} with their proposed half-car model by inserting a piezoelectric in the front and rear suspension system. The findings show that there was an increase in the harvested voltage and power by about 77% and 57%, respectively. The output voltage and power from Wang et al\textsuperscript{5} and Al-Yafeai et al\textsuperscript{6} models were calculated without considering the stiffness coefficient of the piezoelectric element which will profoundly affect the magnitude of both harvested voltage and power. Considering the value of the piezoelectric stiffness coefficient and installing it in parallel with the suspension system will increase the overall stiffness of the system, and this instantaneously decreases significantly the harvested voltage and power from W to mW due to the less deformation of the piezoelectric material.

Mounting the piezoelectric harvester on the suspension wheels could be other possible location in the vehicles to produce electricity. Xie and Wang\textsuperscript{7} built a piezoelectric harvester placed on the wheels of the car suspension system. The mathematical model was subjected to different classes of road roughness classified by ISO/TC108/SC2N67 starting from very smooth to an inferior road surface. From a road profile of poorest road surface (class D), the system had a potential power of 738 W that could be attained by inserting more than four piezoelectric harvesters to the system. However, the actual harvested power via a piezoelectric material was not considered. They have mentioned that the resultant power was influenced by several factors such as the input car velocity, road roughness irregularity, and the dimensions of the piezoelectric element. Likewise, Zhao et al\textsuperscript{18} designed a piezoelectric harvester attached to the suspension system that subjected to random and pulse road excitations. From random excitation, the harvesting system generated power of 18.83 W, while for the pulse excitation, a power of 102.24 W was harvested at a car speed of 30 km/h. Their findings were also similar to the one mentioned by Xie and Wang\textsuperscript{7} in which they stated that the harvested power is affected by different parameters like vehicle speed and the piezoelectric material cross-sectional area.

Piezoelectric materials can also be mounted in the inner circumference of the tires and connected to the capacitor for storing the electrical charge. Esmaeeli et al\textsuperscript{8} modeled a modified shape of cymbal piezoelectric harvester to be installed in the car tire. The regenerative tire produced a voltage of 3.5 V and a power of 2.8 mW that was able to power two sensors. The findings also demonstrated that increasing the piezoelectric layer thickness to 0.8 mm could be capable of powering up to five sensors. Behera\textsuperscript{9} connected 32 piezoelectric modules of type PZT-5A that were arranged in three strips connected in series. The model was able to harvest the power of 14 mW per wheel rotation per second at velocity travel of 40 km/h, whereas Lafarge et al\textsuperscript{10} assembled a cantilever piezoelectric beam of type PZT-27 into the wheels of the car. The generated power from the quarter-car model was evaluated with two different loads resistance: 22 and 222 k$\Omega$. The peak values were reported as 1 and 1.4 mW, respectively, as it can be used for supplying sensors for monitoring applications.

Additionally, piezoelectric material could be designed as a new type of shock absorber system by combining the piezoelectric material with a tubular shock absorber to produce electrical energy from the variation of fluid pressure due to
piston displacement. Lee et al.\textsuperscript{11} installed two parallel plates of piezoelectric material of type PZT 4 in the suspension shock absorber. Exciting the system at 10 Hz resulted in an output voltage and power of 20 V and 1.2 mW, respectively. However, Lafarge et al.\textsuperscript{12} investigated the effect of the location of piezoelectric material in the shock absorber, whether to be stacked on the damper’s surface ($d_{31}$) or placed between two surfaces ($d_{33}$). At speed of 30 km/h, the outcomes illustrated that PZT-5H harvested a higher power (6 mW) when it was in the mode $d_{33}$, while the power of 3 mW was recorded in the mode $d_{31}$.

There is another approach to increase the harvested energy from the suspension system by using multilayer piezoelectric stack as it was discussed by several researchers.\textsuperscript{13-16} Arizti\textsuperscript{15} implemented a multilayer PZT stack in the top part of the piston in the shock absorber. Due to the significant disturbance in the front wheel compare it to the rear one, the system was able to generate a voltage of 17.69 mV from the front wheel per one bump in the road. However, the harvested power from their proposed system was not mentioned. They have reported that the effectiveness of a multilayer PZT stack could be improved by increasing the thickness of the piezoelectric layers or increasing the number of layers. Hendrowati et al.\textsuperscript{16} modeled a harvesting system with multilayer piezoelectric mechanism to enhance the performance of the piezoelectric harvester. This mechanism was able to convert the suspension displacement from vertical to the horizontal direction in order to minimize the relative displacement and magnify the spring force. The PZT stack structure was connected in series with the suspension’s spring, and this led to rising the power 7.2 times larger than implementing the PZT stack without this mechanism. Unlike Wang et al.\textsuperscript{5} and Al-Yafeai et al.\textsuperscript{6} models, Hendrowati et al.\textsuperscript{16} considered the stiffness coefficient of the piezoelectric stack in the calculations. However, the values of the stiffness coefficient, the piezoelectric dielectric permittivity $\varepsilon_{33}$, and the resistance $R$ were not clearly specified.

In the light of the previous literature, it can be concluded that most of the previous studies were performed considering only the quarter-car model (2 DOF) which does not imitate the real functions of the car. Additionally, the researches on harvesting energy from the car suspension system were implemented by modifying the shock absorbers in the system, and this will affect the car suspension performance from different aspects in road handling and ride quality. The current work aims to develop a quarter- and a half-car models with attaching a piezoelectric stack by taking into consideration its stiffness and connect it in series with the suspension’s springs. The harvested energy from the half-car model is compared with the quarter-car model. A parametric study of the factors that affect the resultant harvested energy, such as the piezoelectric parameters, as well as the suspension parameters, is performed. Different road amplitudes are also considered in evaluating the harvested energy. The harvesting system is developed and analyzed using MATLAB/Simulink. Finally, full experimental study of a quarter-car model is conducted in order to examine the theoretical results.

2 | MATHEMATICAL MODEL

2.1 | Mechanical system without piezoelectric stack

A quarter-car model is commonly used in modeling the car suspension system. It examines only the vertical vibrations of the car body (bouncing). However, a half-car model is more practical as it provides two different motions: bouncing and pitching motion. In this work, both two and four degrees of freedom model are built and illustrated in Figure 1. The

![Figure 1](image_url)
All symbols used in the equations are presented in the nomenclature.

The road profile is approximated by harmonic acceleration excitation that excites the vehicle front tire $\ddot{Y}_{Rf}(t)$ and the rear tire with the following delayed function $\ddot{Y}_{Rr}(t)$:

$$\ddot{Y}_{Rr}(t) = \ddot{Y}_{Rf}(t - \tau) \quad \text{where} \quad \ddot{Y}_{Rf}(t) = a \sin(\omega t) \quad (7)$$

In the above equation, $a$ is the sine wave amplitude, $\omega$ is the angular frequency, and $\tau$ is the time delay given by $\tau = L/u$, where $L$ and $u$ are the vehicle wheelbase and velocity, respectively.

## 2.2 Electrical system

A piezoelectric stack is made of numbers of piezoelectric sheets layers, $N$, placed on top of each other. These layers are connected electrically in parallel and mechanically in series, which means that each layer will have the same voltage. The piezoelectric stack is operated in mode 33 where the force is applied along the polarization axis while the electric current is accumulated on the surface perpendicular to the polarization axis. The piezoelectric stack stiffness value is very huge (up to hundred newtons per micrometers). Accordingly, in this study, the piezoelectric stack element will be connected in series with the suspension stiffness in order to not affect the mechanical performance of the car suspension system. The equivalent stiffness in the series connection will be approximately equal to the lowest stiffness which is the suspension spring. On the other hand, connecting a piezoelectric stack in parallel with the suspension spring increases the equivalent suspension system stiffness by summing each of their stresses and consequently affects the ride quality and comfortability.

### Table 1: Equivalent stiffness different connections

| Connection | Series | Parallel |
|------------|--------|----------|
| Equivalent stiffness | $K_{eq} = \frac{K_p}{K_p + K_s} \approx K_s$ | $K_{eq} = K_p + K_s$ |

![Diagram of electrical system connections](image)
The mathematical representation of the piezoelectric element is mainly expressed as the resorting force in the piezoelectric stack element and the output current from the piezoelectric material $i_p$. The restoring force in Equation (8) consists of the mechanical and electrical forces with neglecting the effect of piezoelectric damping. The electric circuit is included in order to collect and store the energy converted from the vibration energy. The integrated circuit shown in Figure 2 composed of the piezoelectric material that is modelled as a piezoelectric capacitance $C_p$ and resistance $R_p$ with the output AC voltage $V_p$ connected to the external circuit. The external rectified circuit consists of the bridge rectifier made of four ideal silicon diodes to obtain the output DC voltage $V$ connected to the smoothing capacitor $C_e$ and external load $R_e$. The smoothing capacitor is utilized to reduce the ripple of the output AC-DC signal.

For the piezoelectric element, the relationship between the electrical and mechanical variables can be derived as follows:\textsuperscript{37}:

$$F_p = K_p Y_p(t) + a V_p(t)$$  \hspace{1cm} (8)

$$i_p = a \dot{Y}_p - C_p V_p$$  \hspace{1cm} (9)

In which the featuring quantities are the short circuit stiffness of piezoelectric stack material $K_p$, the displacement $Y_p$, and the force factor $a$. The mathematical expressions of these quantities are illustrated in Equations (10-12) with the piezoelectric parameters illustrated in Table 2.\textsuperscript{36,38}

$$K_p = \frac{E_p A_p}{l_p}$$  \hspace{1cm} (10)

$$a = N d_{33} E_p \frac{A_p}{l_p}$$  \hspace{1cm} (11)

$$C_p = N \frac{\varepsilon_{33} A_p}{h_p}$$  \hspace{1cm} (12)

where $A_p$ and $h_p$ are the surface area and thickness of one layer of the piezoelectric stack. The thickness of the electrode layers alternatively inserted between the piezoelectric layers is neglected, which is reasonable because they are very thin ($\sim \mu m$) and has already been experimentally verified by Qain et al\textsuperscript{39} while the total length of the PZT stack is defined by the following equation.

When a piezoelectric voltage is equal to the output rectified voltage, the diodes conduct, and the current $i_p$ flows from the piezo stack into the external rectifier circuit that can be identified as:

$$i_p = \begin{cases} C_e \dot{V} + \frac{V}{R_e} & \text{if } V_p = V \\ -C_e V - \frac{V}{R_e} & \text{if } V_p = -V \\ 0 & \text{if } |V_p| < V \end{cases}$$  \hspace{1cm} (14)

The piezoelectric harvesting circuit shown in Figure 2 is made of resistors and capacitors that are connected in parallel. As the piezoelectric resistance is much higher than the external load resistance, the equivalent resistance $R$ will be the external resistance $R \approx R_e$. Comparing the smoothing capacitor $C_e$ with the piezoelectric capacitance $C_p$, the highest capacitance is provided with the circuit smoothing capacitor $C_e$ and the DC rectified voltage is approximated as a constant over a cycle.\textsuperscript{41,42} The equivalent capacitance equals to the piezoelectric capacitance $C \approx C_p$. The external load is modelled as a constant current source, and the four diodes are assumed to exhibit the ideal behavior which means the diode drop is negligible.\textsuperscript{43} As a result, the piezoelectric harvesting circuit can be simplified in Equation (15) with the values of $R = 10 \text{ k} \Omega$ and $C = 60 \text{ nF}$.

$$\frac{V}{R} = a \dot{Y}(t) - C \dot{V}(t)$$  \hspace{1cm} (15)
TABLE 2 Piezoelectric parameters

| Property          | Symbol | Value | Unit |
|-------------------|--------|-------|------|
| Elastic constant  | $E_{33}$ | 53    | GPa  |
| Piezoelectric constant | $d_{33}$ | 600   | pm/V |
| Dielectric constant | $\varepsilon_{33}$ | 39.975 | nF/m |

2.3 Mechanical system with piezoelectric stack

The governing equations of the electromechanical model having piezoelectric harvester stacks mounted in series with suspension spring in both models (see Figure 3) are derived.

Quarter-car model:

The equation of vertical motion of the unsprung mass:

$$M_u \ddot{Y}_u = -K_{us} (Y_{us} - Y_R) - b_{us} (\dot{Y}_{us} - \dot{Y}_R) + K_s (Y_p - Y_{us}) + b_s (\dot{Y}_s - \dot{Y}_{us})$$  \hspace{1cm} (16)

The equation of vertical motion of the quarter sprung mass:

$$I_s \ddot{\theta}_s = L_s (\dot{Y}_s - L_s \dot{\theta}_s - \dot{Y}_af) + K_p (Y_s - L_s \dot{\theta}_s - Y_{pf}) + aV_{pf} - L_r (b_{af} (\dot{Y}_{af} + L_r \dot{\theta}_s - \dot{Y}_af) + K_{pr} (Y_s + L_r \dot{\theta}_s - Y_{pr}) + aV_{pr})$$  \hspace{1cm} (17)

The governing equations for the equivalent electrical system as derived in Equation (15) can be written for voltage and power as:

$$V = aR (\dot{Y}_s - \dot{Y}_p) - CRV$$  \hspace{1cm} (18)

Half-car model:

The equation of vertical motion of the front unsprung mass:

$$M_{af} \ddot{Y}_{af} = -b_{af} (\dot{Y}_{af} - \dot{Y}_{Rf}) - K_{af} (Y_{af} - Y_{Rf}) + b_{sf} (\dot{Y}_s - \dot{Y}_{af}) + K_s (Y_p - Y_{af})$$  \hspace{1cm} (20)

The equation of vertical motion of the rear unsprung mass:

$$M_{ur} \ddot{Y}_{ur} = -b_{ur} (\dot{Y}_{ur} - \dot{Y}_{Rr}) - K_{ur} (Y_{ur} - Y_{Rr}) + b_{sf} (\dot{Y}_s - \dot{Y}_{ur}) + K_s (Y_p - Y_{ur})$$  \hspace{1cm} (21)

The equation of vertical motion of the half-sprung mass:

$$M_s \ddot{Y}_s = -b_{sf} (\dot{Y}_s - L_s \dot{\theta}_s - \dot{Y}_{af}) - b_{ur} (\dot{Y}_s + L_r \dot{\theta}_s - \dot{Y}_{ur}) - K_{pf} (Y_s - L_s \dot{\theta}_s - Y_{pf}) - K_{pr} (Y_s + L_r \dot{\theta}_s - Y_{pr}) - (a)V_{pf} - (a)V_{pr}$$  \hspace{1cm} (22)

The equation of angular motion of the half-sprung mass:

$$L_s (\dot{Y}_s - L_s \dot{\theta}_s - \dot{Y}_af) + K_p (Y_s - L_s \dot{\theta}_s - Y_{pf}) - L_r (b_{af} (\dot{Y}_{af} + L_r \dot{\theta}_s - \dot{Y}_af) + K_{pr} (Y_s + L_r \dot{\theta}_s - Y_{pr}) + aV_{pr})$$  \hspace{1cm} (23)

The equation of the front and rear piezoelectric elements:

Front piezoelectric stack:

$$M_{pf} \ddot{Y}_{pf} = -K_{pf} (Y_{pf} - Y_{af}) + K_{pf} (Y_s - L_s \dot{\theta}_s - Y_{pf}) + (a)V_{pf}$$  \hspace{1cm} (24)

Rear piezoelectric stack:

$$M_{pr} \ddot{Y}_{pr} = -K_{sf} (Y_{pr} - Y_{ur}) + K_{pr} (Y_s + L_r \dot{\theta}_s - Y_{pr}) + (a) V_{pr}$$  \hspace{1cm} (25)

FIGURE 3 Car suspension models with PZT stacks. A, Quarter-car model. B, Half-car model.
The governing equations for the equivalent electrical system as derived in Equation (15) can be written as:

\[ V_f = aR(Y_s - L_s \dot{Y}_s - Y_{pt}) - CRV_t \]

(26)

\[ P_f = \frac{V_f^2}{R} \]

(27)

The rear harvested voltage and power:

\[ V_r = aR(Y_s + L_r \dot{Y}_s - \dot{Y}_{pr}) - CRV_t \]

(28)

\[ P_r = \frac{V_r^2}{R} \]

(29)

3 | RESULTS AND DISCUSSION

3.1 | Simulation results

The simulation results from both quarter and half-car models were obtained using MATLAB/Simulink under harmonic excitation to evaluate the harvested voltage and power from piezoelectric stack elements installed in series with the suspension springs. Different car suspension system and the piezoelectric parameters were examined for a half-car model.

3.1.1 | Quarter vs. half-car models

To examine the performance of the quarter and half-car models with built-in piezoelectric stack, MATLAB/Simulink model was investigated to evaluate the harvested energy in the time and frequency domains simulation. The suspension parameters applied in the both mathematical models are shown in Table 3.44,45

For harmonic excitation of any dynamic model, it is essential to calculate the natural frequencies of the mechanical system to determine the locations of the resonance. Studying the relation between the displacement amplitude of each system with the input road displacement amplitude in frequency domain helps in finding the resonant frequencies of the system. The quarter-car model (2 DOF) has two resonant frequencies that can be found from the peak values shown in Figure 4A. However, half-car model (4 DOF) has four resonant frequencies that are illustrated in Figure 4B. The four peaks correspond to the pitch and bounce motions of the half-car body and the other bounce motion of the front and rear unsprung masses.

In this work, the piezoelectric stack of type PZT-5H was used since it is the best choice in energy harvesting applications because of its high value of piezoelectric constant $d_{33}$, availability, and low cost.46 The piezoelectric stacks were mounted in series with the suspension’s springs in quarter- and half-car models. Their mounting in this way will not affect the dynamic performance/response of the car suspension system. The recommended natural frequency of the car sprung mass is below 1.5 Hz. A value of more than 1.5 Hz is not expected since it increases the acceleration of the car body and causes discomfort to the passengers. Also, the natural frequency of the pitch mode should be close to the bounce motion of the sprung mass and should not be affected during the pitch motion. Moreover, the unsprung masses natural frequencies should be not less than 8 Hz since the human body

| Model          | Property                  | Symbol | Value  | Unit  |
|---------------|---------------------------|--------|--------|-------|
| Quarter-car model | Sprung mass               | $M_s$  | 260    | kg    |
|                | Unsprung mass             | $M_{us}$ | 40     | kg    |
|                | Suspension spring stiffness | $K_s$  | 26 000 | N/m   |
|                | Tire stiffness             | $K_{us}$ | 130 000 | N/m  |
|                | Suspension damping coefficient | $b_s$ | 520 | N.s/m |
|                | Tire damping coefficient   | $b_{us}$ | 264.73 | N.s/m |
| Half-car model | Sprung mass               | $M_s$  | 520    | kg    |
|                | Moment of inertia         | $I_s$  | 2000   | kg.m² |
|                | Front/rear unsprung masses | $M_{uf}, M_{ur}$ | 40     | kg    |
|                | Front/rear suspension stiffness | $K_{uf}, K_{ur}$ | 26 000 | N/m   |
|                | Front/rear tire stiffness  | $K_{uf}, K_{ur}$ | 130 000 | N/m  |
|                | Front/rear suspension damping | $b_{uf}, b_{us}$ | 520 | N.s/m |
|                | Front/rear tire damping   | $b_{uf}, b_{us}$ | 264.73 | N.s/m |
|                | Distance from CG to the front axle | $L_f$ | 1.23  | m     |
|                | Distance from CG to the rear axle | $L_r$ | 1.65  | m     |
is more sensitive to the vertical vibration motion in the frequencies ranging from 4 Hz to 8 Hz. The natural frequencies of the both energy harvesting systems presented in Table 4 matched the ranges that were recommended by Hossain and Chowdhury.47

For harmonic input excitation, the power that is available for harvesting is the amount of the power dissipated by the car suspension dampers. Thus, the average potential power that can be harvested from the suspension dampers in one oscillation is the product of the damping force with the relative velocity and it can be calculated for quarter- and half-car model as 48:

\[
P_{\text{avg, quarter}} = \frac{b_s\omega^2 (Y_s - Y_u\theta)^2}{2}
\]

\[
P_{\text{avg, half}} = \frac{b_s\omega^2 (Y_s - Lf \theta - Y_u\theta)^2}{2} + \frac{b_r\omega^2 (Y_s + Lr \theta - Y_u\theta)^2}{2}
\]

According to the ISO 13473-1,49 the road texture was divided into four main groups based on the value of the texture wavelength \( \lambda \), i—microtexture: \( \lambda < 0.5 \) mm, ii—macrotexture: \( 0.5 \) mm \(< \lambda < 50 \) mm, iii—megatexture: \( 50 \) mm \(< \lambda < 0.5 \) m, and iv—unevenness: \( 0.5 \) m \(< \lambda < 50 \) m. The common unevenness texture wavelength used in the literature was 5 m.50

The vehicle traveling velocity \( u \) over a road is affected by the excitation frequency \( f \) and the road wavelength \( \lambda \) as shown in the following equation:

\[
u = \frac{\lambda f}{2}
\]

Since most of the vehicles’ rigid body excitation frequency within the range [0.5-15 Hz], so that the corresponding vehicle velocity is varied between 9 and 270 km/h.51 Accordingly, the results from Figure 5 showed the two highest dissipated power were occurred for the quarter-car model at the velocities of 26.28 and 179.28 km/h. On the other hand, the four
highest peaks were recorded at the velocities of 19.26, 26.64, and 179.28 km/h. First dissipated power for the half-car model was recorded at a car velocity of 19 km/h. The second corresponding peak that revealed to the sprung bouncing mode had a power dissipation of 2990 W. However, the average power increased significantly to 31 kW at the front and rear unsprung resonant frequency of 9.68 Hz with a velocity of 174.24 km/h.

Inserting the piezoelectric stack that was made of 40 layers and with a surface area and length of 49 mm² and 40 mm, respectively, was utilized in harvesting the dissipated power from the suspension systems. The steady-state responses for evaluating the harvested voltage and power were modeled by exciting the car by sinusoidal acceleration excitation input with a velocity of 9.17 rad/s and an amplitude of 0.5 g (4.9 m/s²). It can be observed from both quarter model (see...
Figure 6A) and half-car model (see Figure 6B) that, at the beginning of time simulation, there is a transient response to the excitation acceleration. The transient phase ends within very short period, and after this phase, the maximum harvested voltage and power could be measured. The maximum harvested voltage and power for the 2 DOF model were around 19.11 V and 36.74 mW, respectively, at the steady-state stage (see Figure 6A). The root mean square (RMS) was then determined for the harmonic acceleration excitation through dividing the peak values by the square root of 2. Consequently, the RMS voltage and power values were found to be 13.51 V and 18.26 mW, respectively. For half-car model, two piezoelectric stacks were installed in series with the front and rear suspension springs. At the steady-state phase in Figure 6B, the maximum harvested voltage and power were 33.56 V and 56.35 mW, respectively. The previous findings of the half-car model revealed that there is an increase of 75.6% and 53.38% for the generated voltage and power when compared with the quarter-car model.

The output harvested voltage and power were also studied in the frequency domain both models as presented in Figure 7A and B. The only changing parameter was the excitation frequency, while the car suspension parameters demonstrated in Table 3 were constant. At each excitation frequency [0-20 Hz] and at the steady-state stage, the maximum value of both voltage and power was taken and plotted. The results for the both models showed that the highest values of the harvested voltage and power were recorded at 1.46 Hz at which it matched with the bouncing motion of the sprung mass. The harvested voltage was 19.11 V for the quarter car and 33.56 V for the half car, while the harvested power both quarter and half-car models were 36.74 and 56.35 mW for, respectively. In contrast, the lowest values were found at the bouncing motion of the unsprung masses at 9.68 Hz. The harvested voltage and power for the quarter-car model were 1.427 V and 0.204 mW, while for the half car, the lowest peaks were 2.815 V and 0.396 mW. Furthermore, at the sprung pitch motion of the half-car model (f = 91.06 Hz), the second-high peaks were occurred (17.82 V and 31.74 mW). Therefore, it is clearly shown that (as expected), the resonant frequencies generate more harvested energy if compared with that of the nonresonant frequencies. The use of sensors and vibration modal analysis are beneficial for similar studies.52,53

It can be clearly noticed that, the harvested power is very small compare it to the whole power dissipated from the car's dampers. From the simulation parameters utilized in the analysis, the harvested power from two piezoelectric stacks installed in series with the front and rear suspension springs was calculated. However, there are many ways to increase the harvested power as discussed by Al-Yafeai et al.54 Car suspension parameters, piezoelectric stack parameters, sensor location, and the road roughness are highly affecting the amount of the harvested power.

Furthermore, it is worth noting that the harvested voltage and power from a proposed car suspension system with the PZT stack cannot be compared with the other systems found in the literature. The harvested systems are different in the configuration (cantilever10-12 and stack13-16), location of the piezoelectric element in the suspension system (springs,4-6,16,17 shock absorbers,12-14 and wheels,7-10), the suspension system model (quarter5,7,10,12,16 or half-car model6,17), and the road input excitation (harmonic5,6,13,16 or random7,10,12,14,17).

3.1.2 Effect of different parameters on the Half-Car Model

The harvested voltage and power from the piezoelectric stacks attached in series with the suspension springs were examined according to road input excitation, piezoelectric parameters, and car suspension parameters. The effect of sine wave acceleration amplitude on the harvested voltage and power at the excitation frequency of 1.46 Hz are depicted in Figure 8. The output voltage from the piezoelectric stack raised linearly from 6.7 V to 67.03 V with the road acceleration amplitude of 0.1 g up to 1 g, while the output power increased in a quadratic relationship with minimum and maximum harvested power of 2.25 mW and 225 mW, respectively. It can be observed that the output voltage and power increased with the amplitude road unevenness. This is due to the change of the linear momentum of the vehicle,
which means an increase of the vehicle velocity and consequently increases the harvested voltage and power with respect to the unevenness amplitude.

The influence of the piezoelectric stack parameters on the performance of the energy harvesting model was also investigated. The analysis was done for multiple numbers of layers up to 100 layers. The results demonstrated in Figure 9A show that more PZT stack layers increase linearly the harvested voltage and exponentially the harvested power. Since the piezoelectric stack layers were connected electrically in parallel, the total displacement of the stack was expressed in references 36,38 as:

\[ Y_{stack} = N Y_p = \frac{1}{K_p} F_p + N d_{33} V \]  

(33)

This means that the overall displacement of the stack increases with the number of layers which will consequently decrease the stiffness of the piezoelectric stack. Besides, the total charge in the piezoelectric stack is the summation of the output charge of all layer. This observation is supported by Equation (34). Accordingly, the voltage and power from the proposed piezoelectric stack with 40 up to 100 layers increased by 54 V and 0.3 W.

\[ Q_{stack} = N Q_p = N d_{33} F_p + C_p V \]  

(34)

The effect of another parameter of the piezoelectric stack that can be studied was the area to thickness ratio. This parameter will affect different piezoelectric parameters such as the force factor, stiffness, and capacitance. The ratio was examined within a range of 1-1000 mm. As shown in Figure 9 (b), increasing the area will significantly increase the harvested power by 0.3 W due to the increase of the electric generating capacity of the piezoelectric stack.

The influence of the car selected parameters on the performance of the half-car piezoelectric energy harvesting model was examined as well. This was done by varying the values of the selected parameter and keeping the other car parameters in Table 3 constant. The influence of the sprung and unsprung stiffness coefficients is demonstrated in Figure 10A and B. The findings showed that both the sprung and unsprung resonant frequencies increase with increasing the stiffness coefficient. This result is validated by the relationship between the natural frequencies of the system and the stiffness coefficient constant (ie, \( \omega = \sqrt{K/M} \)). In contrast, increasing the tire stiffness will decrease the resonance power. This means that stiffer tires result in a harder ride and less suspension movement, whereas increasing the suspension stiffness will increase the relative displacement between the sprung and unsprung masses. Accordingly, this will rise the harvested power from the piezoelectric stack.

The influence of the sprung and unsprung damping coefficient is demonstrated in Figure 10C and D. The findings showed that increasing the damping coefficient for both masses will significantly reduce the output resonance power. However, this will not affect the sprung resonant frequency. It was noticed that less suspension damping will provide higher harvested
power by allowing more stress to be applied to the piezoelectric material. Conversely, having lightly damped or soft suspension will reduce car handling performance and stability. The effect of the sprung mass (Figure 10E) and unsprung mass (Figure 10F) show that increasing both masses will increase the output resonance power. Although the sprung bouncing resonant frequency mode for the sprung mass was decreased, the output power was kept increasing. The results also show that the sprung mass is only affecting the sprung bounce frequency, while the unsprung mass is only affecting the unsprung bounce frequency.

The above results show that there is a significant potential for harvesting energy from the car suspension system. The vibrations generated from the road excitation are transmitted to the car suspension system, and the produced stress/deformation will be applied to the piezoelectric stack material and create a significant amount of electric charges. Without these harvesters, the vibration energy is wasted and dissipated into heat. Moreover, the vehicle suspension system and piezoelectric stack parameters have shown an essential impact on the amount of harvested energy.

### 3.2 Experimental results

The experiment work was conducted in order to assess the validity of the theoretical results. Quarter-car model with piezoelectric stack was developed and connected to the shaker as shown in Figure 11.

Quarter-car system shown in Figure 11 (Detailed A) consists of three masses, or plates that are made of Poly(methyl methacrylate) (PMMA). The plates from the bottom represent the road input, unsprung mass, and sprung mass with dimensions of 30 cm × 15 cm. The thickness of the sprung plate is 6 mm, while for the road and unsprung plates, it is 4 mm. There are four recesses at the bottom of the sprung plate to set in the piezoelectric stacks. Four bottom springs (eg, the spring store PC1880) installed between the bottom and middle plates represent the tire stiffness, each with a free length of 41.4 mm, outer diameter of 22.2 mm, and stiffness coefficient of 1550 N/m. However, the four springs (eg, the spring store PC1575) installed between the sprung and unsprung masses represent to the suspension springs with a free length of 57.15 mm, outer diameter of 22.6 mm, and stiffness coefficient of 1550 N/m. Without these harvesters, the vibration energy is wasted and dissipated into heat.
There are two shock absorbers (e.g., Amazon Rc Car 1/10 Shock Absorber Damper pair) that represent the suspension dampers each with a viscous damping coefficient of 25 Ns/m. Tipped masses of 1 and 0.6 kg are located at the center of the upper and middle plates that correspond to the sprung and unsprung masses, respectively. Four piezoelectric stacks are installed in series with the suspension springs through aluminum cylinders. The physical, electrical, and electromechanical properties of the piezoelectric stacks are presented in Table 5.

The quarter-car model with the new experiment parameters was tested theoretically and experimentally. The $K_s$ is the total stiffness values of the four springs located between the sprung and unsprung plates, and $K_{us}$ is the sum of the stiffness values of the four springs located between the unsprung and road plates. The $b_s$ is the total viscous damping coefficient of the two shock absorbers connected between the sprung and unsprung plates. Moreover, the other suspension parameters utilized in the experiment are illustrated in Table 6.

The resonant frequencies of the system were calculated theoretically from the roots of the Equation (35).

\[
(K_{us} - M_{us} \omega^2)(K_s - M_s \omega^2) - K_s M_s \omega^2 = 0 \tag{35}
\]

The sprung and unsprung resonant frequencies were found to be 7 and 20 Hz, respectively. These resonant frequencies were validated experimentally from the peak values of the acceleration plot for both sprung and unsprung masses as shown in Figure 12A.

The road input acceleration was measured by the accelerometer located on the first bottom plate. The excitation amplitude acceleration was kept at 0.05 g. The simulated and

| Property          | Symbol | Value | Unit  |
|-------------------|--------|-------|-------|
| Length            | $L$    | 40    | mm    |
| Area              | $A$    | 49    | mm$^2$|
| Mass              | $M$    | 15    | g     |
| Density           | $\rho$ | 7.8   | g/cm$^3$|
| Stiffness         | $K$    | 51    | MN/s  |
| Capacitance       | $C$    | 4.5   | pF    |
| Piezoelectric constant | $d_{33}$ | 600 | pm/V    |
| Elastic constant  | $E_{33}$ | 53  | GPa   |
| Dielectric constant | $\varepsilon_{33}$ | 30.975 | nF/m |
| Electromechanical coupling coefficient | $k$ | 0.65 | NA |

| Type             | Parameter | Value | Unit |
|------------------|-----------|-------|------|
| Sprung mass      | $M_s$     | 1     | kg   |
| Unsprung mass    | $M_{us}$  | 0.6   | kg   |
| Sprung stiffness  | $K_s$     | 6200  | N/m  |
| Unsprung stiffness| $K_{us}$ | 2800  | N/m  |
| Viscous damping coefficient | $b_s$ | 50  | N.s/m |

![Figure 11: Experimental setup of quarter-car model with piezoelectric stack](image)
Experimental output voltage and power were studied at different excitation frequencies. The simulated output voltage matched well with the experimental voltage from piezoelectric stack as shown in Figure 12B. The results show that the greatest voltage output for both simulation and experimental studies was occurred at the sprung resonant frequency of 7 Hz. At this resonant mode, the corresponding simulated voltage was 0.14 V which was slightly higher than the experimental voltage that was recorded as 0.12 V. This small difference with a percentage error of 14% was due to some experimental errors. The errors may be due to not considering the damping coefficient of the PZT stacks in the simulation. Also, the mutual friction between the moving could have an effect. However, the second peak was found to be at the unsprung resonant frequency of 20 Hz. At this frequency, the simulated and experimental voltages were almost equal with a value of 0.02 V.

In order to calculate the harvested power from the PZT stack, a circuit should be connected to the system. It is worth noting here that Wang et al. who evaluated the harvested voltage from the quarter-car model is the only experimental work available in the literature. In this study, since the harvested voltage is relatively small, each piezoelectric stack was connected to the electrical circuit (PDU100B) to magnify the output voltage. As shown in Figure 12C, the peak values of 6 and 1 V are recorded at the resonant frequencies. These voltage values were magnified by a factor of 50 due to the configuration used in this circuit which is the unipolar input with bipolar output type. The output power was also calculated by having the built-in impedance circuit of 100 MΩ for unipolar input type. The results show that the maximum harvested power from piezoelectric stack was found to be 0.36 mW. However, at the unsprung resonant frequency, the output power was 0.05 mW.

It can be observed that the harvested voltage and power are relatively low due to the small-scale experimental parameters. In real case/scale, the parameters are more significantly, and consequently, the harvested power will be higher than the small-scale size. Besides, there will be other factors that are affecting the harvested power in real case situations such as the time of traveling, road roughness, speed of the car, the suspension parameters, the type of piezoelectric material, and the number of installed piezoelectric elements.

4 | CONCLUSION

Mathematical models of quarter and half-car systems with a piezoelectric stack inserted between the sprung and unsprung systems were developed to evaluate the dissipated and harvested voltage and power. The piezoelectric stacks were attached in series with the springs of the sprung mass system. The maximum harvested voltage and power were found at the resonant frequencies. When the car was subjected to a harmonic excitation with a frequency of 1.46 Hz and an acceleration amplitude of 0.5 g, the peak values of the harvested voltage and power in quarter-car model were 19.11 V and 36.74 mW, respectively. These results were approximately increased by 2 for the harvested voltage and power of the
half-car model. The variation of the voltage and power vs. frequency showed a similar trend to that of the car velocity. This is because of the direct correlation between the velocity and frequency.

Effect of different factors on the harvested voltage and power was examined for a half-car model. The findings demonstrated that, the more road unevenness amplitude, the higher the harvested power. Furthermore, the parameters of the piezoelectric stack (eg, number of layers and the area to thickness ratio) and the car suspension parameters (eg, sprung and unsprung stiffness, damping coefficients, and masses) were found to influence significantly the harvested power.

The simulation approach of the quarter-car model was verified experimentally. By fixing the input acceleration at 0.05 g, the output voltages were matched at the second resonant frequency. However, at the first resonant frequency, there was a slight increase in the simulated voltage when compared to the experimental one. To maximize the output voltage, the piezoelectric stacks were connected directly to the electrical circuits. This led to an increase in the output voltage from 0.12 to 6 V. A value of 0.36 mW for the harvested power was also recorded at the sprung resonant frequency.

**Nomenclature**

**English symbols**

| Symbol | Description                  |
|--------|------------------------------|
| A      | Surface Area (m²)           |
| a      | Amplitude (m)               |
| b      | Damping Coefficient (N.s/m)  |
| C      | Capacitor (F)               |
| d₃₃    | Strain Coefficient (m/V)    |
| E      | Young's Modulus (N/m²)      |
| F      | Force (N)                   |
| f      | Natural Frequency (Hz)      |
| h      | Thickness (m)               |
| I      | Moment of Inertia (kg.m²)   |
| i      | Current (A)                 |
| K      | Spring Coefficient (N/m)    |
| k      | Electromechanical Coupling Coefficient |
| L      | Wheelbase (m)               |
| l      | Total Length (m)            |
| M      | Mass (kg)                   |
| N      | Number of Layers            |
| P      | Power (W)                   |
| Q      | Electric Charge (C)         |
| R      | Resistor (Ω)                |
| S      | Strain (m/m)                |
| t      | Time (s)                    |
| u      | Vehicle Velocity (m/s)      |
| V      | Voltage (V)                 |
| Y      | Vertical Displacement (m)   |

**Creek symbols**

| Symbol | Description                  |
|--------|------------------------------|
| α      | Force Factor (N/V)           |
| ε₃₃    | Dielectric Permittivity (F/m)|
| λ      | Wavelength (m)               |
| ω      | Angular Frequency (rad/s)    |
| ρ      | Density (g/cm³)              |
| σ      | Stress (N)                   |
| τ      | Time Delay (s)               |
| θ      | Angular Displacement (rad)   |

**Subscripts**

| Subscript | Description |
|-----------|-------------|
| avg       | Average     |
| e         | External    |
| f         | Front       |
| p         | Piezoelectric |
| R         | Road        |
| r         | Rear        |
| s         | Sprung      |
| us        | Unsprung    |

**CONFLICT OF INTEREST**
The authors declare no conflict of interest.

**AUTHOR CONTRIBUTIONS**

Abdel-Hamid I. Mourad and Tariq Darabseh involved in conceptualization. Doaa Al-Yafeai involved in data curation. Doaa Al-Yafeai, Abdel-Hamid I. Mourad, and Tariq Darabseh involved in formal analysis. Doaa Al-Yafeai, Tariq Darabseh, and Abdel-Hamid I. Mourad involved in investigation. Tariq Darabseh, Abdel-Hamid I. Mourad, and Doaa Al-Yafeai involved in methodology. Abdel-Hamid I. Mourad and Tariq Darabseh involved in resources. Abdel-Hamid I. Mourad and Tariq Darabseh involved in supervision. Abdel-Hamid I. Mourad and Tariq Darabseh involved in visualization; Doaa Al-Yafeai involved in writing-original draft. Abdel-Hamid I. Mourad and Tariq Darabseh involved in writing-review and editing.

**ORCID**

Abdel-Hamid I. Mourad [https://orcid.org/0000-0002-8356-0542](https://orcid.org/0000-0002-8356-0542)

**REFERENCES**

1. Graves KE, Iovenitti PG, Toncich D. Electromagnetic regenerative damping in vehicle suspension systems. *Int J Veh Des*. 2000:24(2-3):182-197.
2. Segel L, Lu X. Vehicular resistance to motion as influenced by road roughness and highway alignment. *Aust Road Res*. 1982;12(4):211-222.
3. Abdelkareem MA, et al. Vibration energy harvesting in automotive suspension system: a detailed review. *Appl Energy*. 2018;229:672-699.
4. Namuduri CS, Li Y, Talty TJ, Elliott RB, McMahon N. Harvesting energy from vehicular vibrations using piezoelectric devices. 2012.
5. Wang X. Frequency Analysis of Vibration Energy Harvesting Systems. United States: Academic Press; 2016.

6. Al-Yafeai D, Darabseh T, Mourad A-HI. Quarter vs. Half Car Model energy harvesting systems. In: 2019 Advances in Science and Engineering Technology International Conferences (ASET). 2019:1-5.

7. Xie XD, Wang Q. Energy harvesting from a vehicle suspension system. Energy. 2015;86:385-392.

8. Esmaeeli R, et al. Design, modeling, and analysis of a high performance piezoelectric energy harvester for intelligent tires. Int J Energy Res. 2019;43(10):5199-5212.

9. Behera MM. Piezoelectric energy harvesting from vehicle wheels. Int J Eng Res Technol. 2015;4(05):1-4.

10. Lafarge B, Grondel S, Delebarre C, Cattan E. A validated simulation of energy harvesting with piezoelectric cantilever beams on a vehicle suspension using Bond Graph approach. Mechatronics. 2018;53:202-214.

11. Lee H, Jang H, Park J, Jeong S, Park T, Choi S. Design of a piezoelectric energy-harvesting shock absorber system for a vehicle. Integr. Ferroelec. 2013;141(1):32-44.

12. Lafarge B, Delebarre C, Grondel S, Curea O, Hacala A. Analysis and optimization of a piezoelectric harvester on a car damper. Phys Procedia. 2015;70:970-973.

13. Ali SF, Adhikari S. Energy harvesting dynamic vibration absorbers. J Appl Mech. 2013;80(4):041004.

14. Madhav C, Ali SF. Harvesting energy from vibration absorber under random excitations. IFAC-Pap. 2016;49(1):807-812.

15. Ariziti M. Harvesting energy from vehicle suspension. 2010.

16. Hendrowati W, Guntur HL, Sutantra IN. Design, modeling and analysis of implementing a multilayer piezoelectric vibration energy harvesting mechanism in the vehicle suspension. Engineering. 2012;4(11):728.

17. Al-Yafeai D, Darabseh T, Mourad A-HI. Energy harvesting from car suspension system subjected to random excitation. In: 2020 Advances in Science and Engineering Technology International Conferences (ASET). 1-5.

18. Zhen Z, Tie W, Jinhong S, et al. Analysis and application of the piezoelectric energy harvester on light electric logistics vehicle suspension systems. Energy Science & Engineering. 2019;7(6):2741-2755.

19. Zuo L, Zhang P-S. Energy harvesting, ride comfort, and road handling of regenerative vehicle suspensions. J Vib Acoust. 2013;135(1):011002.

20. Fang Z, Guo X, Xu L, Zhang H. An optimal algorithm for energy recovery of hydraulic electromagnetic energy-regenerative shock absorber. Appl Math Inf Sci. 2013;7(6):2207.

21. Gopalakannan S, Kumar SP, Premragar V, Pradeep TR. Design, fabrication and testing of regenerative shock absorber (linear alternator type). Int J Appl Eng Res. 2015;10(8):6133-6137.

22. Scully B, Zuo L, Shestani J, Zhou Y. Design and characterization of an electromagnetic energy harvester for vehicle suspensions. In: ASME 2009 International Mechanical Engineering Congress and Exposition. 2009;1007-1016.

23. Kim YB, Hwang WG, Kee CD, Yi HB. Active vibration control of a suspension system using an electromagnetic damper. Proc Inst Mech Eng Part J Automob Eng. 2001;215(8):865-873.

24. Mitcheson P, Yeatman E. Energy harvesting for pervasive computing. PerAda Mag. 2008:1-3.

25. Mićka P. Energy-harvesting potential of automobile suspension. Veh Syst Dyn. 2016;54(12):1651-1670.

26. Zhang R, Wang X, John S. A comprehensive review of the techniques on regenerative shock absorber systems. Energies. 2018;11(5):1167.

27. Eriksson J, Piroi S. Review of methods for energy harvesting from a vehicle suspension system. 2016.

28. Jin-gui Z, Zhi-zhao P, Lei Z, Yu Z. A review on energy-regenerative suspension systems for vehicles. Proc World Congress Eng. 2013;3:3-5.

29. Amer NH, Ramli R, Isa HM, Mahadi WNL, Abidin MAZ. A review of energy regeneration capabilities in controllable suspension for passengers’ car. Energy Educ Sci Technol Energy Res. 2012;30(1):143-158.

30. Xiao H, Wang X. A review of piezoelectric vibration energy harvesting techniques. Fuel Cells Methanol. 2014:280:28.

31. Roundy SJ. Energy scavenging for wireless sensor nodes with a focus on vibration to electricity conversion. PhD Thesis, University of California, Berkeley Berkeley, CA; 2003.

32. Worthington E. Piezoelectric energy harvesting: Enhancing power output by device optimisation and circuit techniques. 2010.

33. Priya S. Advances in energy harvesting using low profile piezoelectric transducers. J Electroceramics. 2007;19(1):167-184.

34. Wang C, Zhao J, Li Q, Li Y. Optimization design and experimental investigation of piezoelectric energy harvesting devices for pavement. Appl Energy. 2018;229:18-30.

35. Ambrosio R, Jimenez A, Mireles J, Moreno M, Monfil K, Heredia H. Study of piezoelectric energy harvesting system based on PZT. Integr Ferroelecit. 2011;126(1):77-86.

36. Lee DJ. Engineering Analysis of Smart Material Systems. New Jersey: John Wiley & Sons; 2007.

37. Lefevre E, Bodel A, Richard C, Petit L, Guyomar D. A comparison between several vibration-powered piezoelectric generators for standalone systems. Sens Actuators Phys. 2006;126(2):405-416.

38. Qian F, Xu T-B, Zuo L. Design, optimization, modeling and testing of a piezoelectric footwear energy harvester. Energy Convert Manag. 2018;171:1352-1364.

39. Qian F, Xu T-B, Zuo L. Material equivalence, modeling and experimental validation of a piezoelectric boot energy harvester. Smart Mater Struct. 2019;28(7):075018.

40. PiezoDrive. PiezoDrive 200V Stack Actuators.

41. Jiang X, Li Y, Li J, Wang Y, Yao J. Piezoelectric energy harvesting from traffic-induced pavement vibrations. J Renew Sustain Energy. 2014;6(4):043110.

42. Ramadass YK, Chandrakasan AP. An efficient piezoelectric energy harvesting interface circuit using a bias-flip rectifier and shared inductor. IEEE J Solid-State Circuits. 2010;45(1):189-204.

43. Kazmierski TJ, Beeby S. Energy Harvesting Systems. United States: Springer; 2014.

44. Xiao H, Wang X, John S. A dimensionless analysis of a 2DOF piezoelectric vibration energy harvester. Mech Syst Signal Process. 2015;58:355-375.

45. Aghairkaki A, Sabet GS, Barouz A. Simulation and analysis of passive and active suspension system using quarter car model for different road profile. Int J Eng Trends Technol. 2012;3(5):636-644.

46. Lin C-Y. Material characterization and modeling for piezoelectric actuation and power generation under high electromechanical driving levels. PhD Thesis, Massachusetts Institute of Technology; 2002.

47. Hessaini MZ, Chowdhury MNA. Ride comfort of a 4 DOF nonlinear heavy vehicle suspension. ISESCO J Sci Technol. 2012;8:80-85.
48. De Silva CW, Khoshnoud F, Li M, Halgamuge SK. Mechatronics: Fundamentals and Applications. United States: CRC Press; 2015.
49. ISO. 13473–1. Characterization of pavement texture by use of surface profiles–Part 1: Determination of mean profile depth. Eur Stand ICS 1714030 Eur Comm Stand Bruss. 2004:1997.
50. Ahlin K, Granlund NJ. Relating road roughness and vehicle speeds to human whole body vibration and exposure limits. Int J Pavement Eng. 2002;3(4):207-216.
51. Tyan F, Hong Y-F, Tu S-H, Jeng WS. Generation of random road profiles. J Adv Eng. 2009;4(2):1373-1378.
52. Fouad H, Mourad A-HI, Alshammari BA, Hassan MK, Abdallah MY, Hashem M. Fracture toughness, vibration modal analysis and viscoelastic behavior of Kevlar, glass, and carbon fiber/epoxy composites for dental-post applications. J Mech Behav Biomed Mater. 2020;101:103456.
53. Elgandor E, Kolkailah FA, Mourad A-HI. Sensors location effect on the dynamic behavior of the composite structure with flaw detection. In: Proceedings of the 44th International SAMPE (Society for the Advancement of Material and Process Engineering) Symposium and Exhibition; 1999:439-358.
54. Al-Yafeai D, Darabseh T, Mourad A-HI. A state-of-the-art review of car suspension-based piezoelectric energy harvesting systems. Energies. 2020;13(9):2336.