Classification Evaluation of Loess Slope Stability Based on the Combination Weight of Game Theory

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Abstract. Rational classification evaluation of loess slope stability has a guiding significance on the prevention and control of geological hazards. The key of classification evaluation of loess slope stability is to construct evaluation indices system and determine the indices weights, which can be obtained through various approaches currently. Therefore, according to the characteristics of slope hazards and geo-environmental conditions of Mizhi County, the evaluation indices system was constructed by six indices, namely, slope shape, slope gradient, slope height, types of geomorphic unit, lithology and average annual precipitation. And based on the thought of game theory, the combination weights of indices were figured out by different expert weights. Taking the loess slope located at Xinchengqu Village as an example, stability classification of the loess slope was analyzed using the methods mentioned above, and the main results showed that stability classification of the loess slope was unstable whether before sliding or after sliding.

1. Introduction
Slope hazard, of which basic patterns include land sliding, falling and lateral spreading, is one of the most common geological hazards in loess area [1]. Currently, the evaluation of slope stability has been a critical research issue, which has a profound influence on the engineering activities and people's life and property safety. The general approaches to slope stability assessment contain natural history analysis [2], engineering geologic analogy [3], limit equilibrium method [4], probabilistic analysis [5] and so on. However, to adopt the assessment approaches mentioned above, we have to obtain detailed characteristics and parameters of slopes, which may reduce work efficiency. Therefore, various models of synthetic classification evaluation of slope stability have been proposed. Zhang [6] established the fuzzy comprehensive evaluation model of rock slope stability, and rationality of the model was verified by 142 high rock slopes along Changde-Jishou Highway. Huang [7] built the evaluation model of rock slope stability, which integrated multilevel uncertainty measure and set pair analysis. Liu [8] introduced ideal point method into the model of rock slope stability evaluation, and the natural properties of rock and exterior disturbance were considered comprehensively in the process of modeling. And the cloud model theory [9], FAHP [10] and CSMR [11] were also applied in classification evaluation of rock slope stability. Nevertheless, in terms of loess slopes, the relevant researches are rare. In this case, taking Mizhi County as study area, the evaluation indices system of loess slope stability was established, and the combination weights of indices were determined based
on game theory.

2. Construction of evaluation indices system
In accordance with existing research achievements [12], slope shape, slope gradient, slope height, types of geomorphic unit, lithology and average annual precipitation were considered as the evaluation indices of loess slope stability. All the values of indices were divided into several sections, and the corresponding evaluation value of each section can be calculated by equation (1).

\[
v_{ij} = \frac{N_{ij}}{N}
\]

(1)

where \(v_{ij}\) represents the evaluation value of the \(j\)-th section of the \(i\)-th index. \(N_{ij}\) is the total number of geological hazards covered in the \(j\)-th section of the \(i\)-th index (PS. Geological hazards include landslides, collapse and unstable slopes in this study). \(N\) is the total number of geological hazards in the whole study area.

In the light of the detailed survey data of Mizhi County, the statistical results of geological hazards in different sections of indices were shown in figure 1.
Figure 1. The statistical results of geological hazards of various indices

In accordance with equation (1), the evaluation values of various sections of all the indices were figured out, and the ultimate evaluation indices system was listed in table 1.

Table 1. The evaluation values of various sections of all the indices

| Indices         | Sections                  | Evaluation values |
|-----------------|---------------------------|-------------------|
| Slope shape     | Convex slope              | 0.66              |
|                 | Straight slope            | 0.15              |
|                 | Concave slope             | 0.10              |
|                 | Ladder slope              | 0.09              |
|                 | 80°~90°                   | 0.03              |
|                 | 70°~80°                   | 0.11              |
|                 | 60°~70°                   | 0.05              |
|                 | 50°~60°                   | 0.16              |
|                 | 40°~50°                   | 0.39              |
|                 | <40°                      | 0.26              |
|                 | >50 m                     | 0.21              |
|                 | 40 m~50 m                 | 0.28              |
|                 | 30 m~40 m                 | 0.32              |
|                 | 20 m~30 m                 | 0.13              |
|                 | 10 m~20 m                 | 0.05              |
|                 | <10 m                     | 0.01              |
| Slope height    | Loess hilly region        | 0.82              |
|                 | River valley region       | 0.18              |
| Geomorphic unit | Middle and upper Pleistocene loess | 0.97 |
|                 | Bedrock                   | 0.03              |
|                 | >450 mm                   | 0.34              |
|                 | 425 mm~450 mm             | 0.16              |
| Lithology       | 400 mm~425 mm             | 0.03              |
|                 | 375 mm~400 mm             | 0.32              |
| Average annual precipitation | 350 mm~375 mm | 0.11              |
|                 | <350 mm                   | 0.04              |

3. Determination of indices weights

Traditional weighting methods can be classified as subjective weighting methods and objective weighting methods. Lacking ample measured data of loess slopes in Mizhi County, we had to employ the expert scoring method, one of the most classic subjective weighting methods, to compute indices weights. In this case, five experts were invited to determine the indices weights, and the results were listed in table 2.
In view of the differences of experiences, knowledge and subjective thoughts among experts, we need to find out the optimal weights through game theory. Suppose a basic weight set is $\omega_k = \{\omega_{k1}, \omega_{k2}, \ldots, \omega_{kn}\}$ ($k=1, 2, \ldots, L$), and $L$ is the total number of basic weight sets. A certain linear combination of all the basic weight sets can be expressed as equation (2).

$$\omega = \sum_{k=1}^{L} \beta_k \omega_k^T$$  

(2)

where $\omega$ is a certain linear combination of all the basic weight sets. $\beta_k$ is the combination coefficient set.

The optimal combination coefficient set can be obtained by solving equation (3), which is equivalent to equation (4).

$$\min_{g} \left\| \sum_{k=1}^{L} \beta_k \omega_k^T - \omega_k \right\|_2$$  

(3)

$$\begin{bmatrix} \omega_1 \omega_1^T & \omega_2 \omega_2^T & \ldots & \omega_L \omega_L^T \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_L \end{bmatrix} = \begin{bmatrix} \omega_1 \omega_1^T \\ \omega_2 \omega_2^T \\ \vdots \\ \omega_L \omega_L^T \end{bmatrix}$$  

(4)

### Table 2. The indices weights based on expert scoring method

| No. | Indices weights | Geomorphic unit | Lithology | Average annual precipitation |
|-----|-----------------|-----------------|-----------|------------------------------|
| Slope shape | Slope gradient | Slope height | | |
| 1 | 0.10 | 0.20 | 0.15 | 0.05 | 0.30 | 0.20 |
| 2 | 0.10 | 0.15 | 0.10 | 0.05 | 0.35 | 0.25 |
| 3 | 0.15 | 0.25 | 0.10 | 0.05 | 0.25 | 0.20 |
| 4 | 0.20 | 0.25 | 0.07 | 0.03 | 0.30 | 0.15 |
| 5 | 0.15 | 0.15 | 0.15 | 0.05 | 0.25 | 0.25 |

In conclusion, the optimal combination coefficient set is $\{0.9609, 1.0057, 0.9454, 0.9777, \text{and } 0.9271\}$. Then the ultimate weights can be figured out through substituting the normalized optimal combination coefficients into equation (2), and the ultimate weights were listed in table 3.

### Table 3. The ultimate indices weights based on game theory

| Indices weights | Geomorphic unit | Lithology | Average annual precipitation |
|-----------------|-----------------|-----------|------------------------------|
| Ultimate weights | Slope shape | Slope gradient | Slope height | | |
| 0.14 | 0.20 | 0.11 | 0.05 | 0.29 | 0.21 |

### 4. Engineering application

Suppose $V$ represents the comprehensive classification evaluation value which can be computed by equation (5).

$$V = \sum_{i=1}^{n} \omega_i v_i$$  

(5)

where $\omega_i$ is the ultimate weight of the $i$-th index. $v_i$ is the evaluation value of the $i$-th index.

In this study, the maximum of $V$ is 0.60 when the minimum of $V$ is 0.04. Therefore, the interval of $V$ is $[0.04, 0.60]$. As demonstrated in table 4, the interval can be divided evenly into four sections which correspond to four stability classification, namely, stable, basically stable, less stable and unstable.
Table 4. The classification list of loess slope stability

| Stability classification | Stable | Basically stable | Less stable | Unstable |
|--------------------------|--------|------------------|-------------|----------|
| $V$                      | 0.04-0.18 | 0.18-0.32     | 0.32-0.46 | 0.46-0.60 |

Applying the approaches mentioned above evaluated the stability of a loess slope located at Xinchengqu Village, Mizhi County. This loess slope slid in 28th July, 2017, which threatened the life and property safety of local residents. The length of slide body is 33m, average width is 14m and the average thickness is 2m. The main sliding direction of the landslide is about 77°. Photos of this loess slope hazard were shown in figure 2. The basic information of the loess slope was listed in table 5.

Figure 2. Photos of the landslide at Xinchengqu Village

Table 5. The basic information of the loess slope

| Indices | Slope shape | Slope gradient (°) | Slope height (m) | Geomorphic unit | Lithology | Average annual precipitation (mm) |
|---------|-------------|--------------------|------------------|-----------------|-----------|----------------------------------|
| Before  | Convex      | 59                 | 32               | Loess hilly region | Malan loess | 452.4                            |
| After   | Straight    | 47                 | 32               | Loess hilly region | Malan loess | 452.4                            |

Based on equation (5) and table 1-5, the stability classifications of the loess slope before sliding and after sliding were determined and the evaluation results were listed in Table 6.

Table 6. The evaluation values and classification results of the loess slope

| Evaluation values | Stability classification |
|-------------------|--------------------------|
| Slope shape       | Slope gradient | Slope height | Geomorphic unit | Lithology | Average annual precipitation | $V$ | Classification  |
| Before sliding    | 0.66         | 0.16         | 0.32          | 0.82       | 0.97       | 0.34          | 0.55 | Unstable        |
| After sliding     | 0.15         | 0.39         | 0.32          | 0.82       | 0.97       | 0.34          | 0.53 | Unstable        |

Before sliding, the loess slope was in unstable state, and the shallow sliding occurred in this slope under the action of short-time severe precipitation. After sliding, the slope shape and gradient changed, which slightly raised the loess slope stability, but the slope was still in unstable state. In thus, it is necessary to adopt comprehensive treatment measures to eliminate hidden danger of the loess slope.
5. Conclusions
(1) Based on the statistical analysis of geological hazards in Mizhi County, the evaluation indices system of loess slopes was constructed by six indices, namely, slope shape, slope gradient, slope height, types of geomorphic unit, lithology and average annual precipitation.
(2) Ultimate indices weights were determined through the expert scoring method and combination weighting method based on game theory. And the result showed that lithology was the most critical index in classification evaluation of loess slopes while geomorphic unit had the least influence on loess slope stability.
(3) In the light of the classification evaluation approaches in this paper, the stability of a loess slope located at Xinchengqu Village, Mizhi County was analyzed. Before sliding, the comprehensive classification evaluation value of this loess slope was 0.55, which indicated the loess slope was unstable. After sliding, the comprehensive classification evaluation value decreased to 0.53, and the slope was still unstable, which was closed to actual situation.

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