Instability of cyclonic convective vortex

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Abstract. Localized heating in the rotating layer of fluid leads to the formation of intensive cyclonic vortex. Cyclonic vortex becomes unstable at low values of viscosity and fast rotation of the experimental model. The instability of the vortex is tightly connected with a structure of the radial inflow. For moderate values of rotational Reynolds number $Re$ the radial flows consist of several branches which transport angular momentum to the center of the model. When $Re$ exceeds critical value (about 23) radial inflow changes its structure and appears as one wide branch which does not reach the center. As a result of strong anisotropy of radial inflow the cyclonic vortex is formed at some distance from the center. Further increase of $Re$ leads to chaotic state with several vortices which appears at different locations near the periphery of the heating area. The map of regimes with stable and unstable vortices is presented.

1. Introduction

The present study is focused on the stability of laboratory vortex over localized source of heat in a rotating layer. There are only few laboratory realizations of cyclonic vortices driven by localized heating in rotating layers. The main motivation of these studies is a problem of formation of large-scale vortices in atmosphere. Laboratory hurricane model with an analog of latent heat release was proposed in [1] where resulting vortical flow was similar to the hurricane eye wall. Convection from isolated source at the bottom for low values of Rossby number was experimentally studied in [2, 3]. These experiments were mainly directed to the study of small eddies formation in a buoyancy source region, their properties and evolution. Only short remark was made about formation of the single cyclonic vortex in the case of high values of convective Rossby number (more than 1). A qualitative experimental study of convective flow driven by a finite-sized circular heating plate at the bottom of a horizontal fluid layer, both with and without background rotation, was carried out in [4]. The studies [2-4] showed that in the case of water, for relatively low value of Prandtl number ($Pr \approx 7$) cyclonic motion in the lower layer exists only for slow rotation of the vessel.

Successful laboratory model of hurricane-like vortex was proposed in [5, 6]. The important differences of this experimental approach were more viscous fluids (in comparison with water) and using of a shallow layer. Later, series of the experiments [7] were done for the same configuration using PIV system for velocity measurements. The main focus of [7] was on integral characteristics of the azimuthal flows such as angular momentum and kinetic energy. Detailed study of different constraints of the steady-state hurricane-like vortex was carried out in [8]. PIV measurements showed that general structure of laboratory vortex is similar to the observational data [9]. The three main dimensional parameters that define the vortex structure
for a fixed geometry – heating flux, rotation rate and viscosity were varied independently. It was found that viscosity is one of the main parameters that define the steady-state vortex structure. Increasing of kinematic viscosity may substantially suppress the cyclonic motion for fixed values of buoyancy flux and rotation rate. Strong competition between buoyancy and rotation provides the optimal ratio of the heating flux and rotation rate for achieving cyclonic vortex of maximal intensity.

The transitional stage when cyclonic vortex loses its stability and a system of smaller vortices appears instead was not studied before. The nature and description of this instability is our main goal. The paper is organized as follows. In section 2 we describe the experimental setup and measurement technique. Experimental results are presented in section 3 and conclusions are given in section 4.

2. Experimental setup

Experimental model is a cylindrical vessel of diameter $D = 300$ mm, and height $H = 40$ mm (figure 1(a)). The sides and bottom were made of plexiglass with a thickness 3 mm and 20 mm respectively. There was no cover or additional heat insulation at the sidewalls. The heater is a brass cylindrical plate mounted flush with the bottom. The diameter of the plate $d$ is 104 mm, and its thickness is 10 mm. The brass plate is heated by an electrical coil placed on the lower side of the disc. Massive heater provides uniform heating which is optimal for vortex excitation. Cylindrical vessel was placed on a rotating horizontal table. Silicon oils with different values of kinematic viscosity, PMS-20, PMS-10 and PMS-5 (20, 10 and 5 cSt at at $T = 25^\circ$C) were used as working fluids. In all experiments, the depth of the fluid layer $h$ was 30 mm and the surface of the fluid was open. The room temperature was kept constant by air-conditioning system, and cooling of the fluid was provided mainly by the heat exchange with surrounding air on the free surface and some heat losses through sidewalls. The velocity field measurements were made with a 2D particle image velocimetry (PIV) system Polis and the software package Actual Flow. The PIV velocity measurements were accurate to within 5%, estimated from calibration experiments in solid body rotation and long time series. Along with the dimensional parameters (heating flux, rotation rate and kinematic viscosity) we use the set of the non-dimensional parameters. These are the flux Grashof number $Gr_f$, rotational Reynolds number $Re$ and Prandtl number $Pr$:

\[
Gr_f = \frac{g\beta h^3 q}{c\rho\kappa
}
\]

\[
Re = \frac{\Omega h^2}{\nu}
\]

\[
Pr = \frac{\nu}{\kappa}
\]

where $g$ is the gravitational acceleration, $h$ is the layer depth, $\beta$ is the coefficient of thermal expansion, $c$ is the thermal capacity, $\rho$ is the density, $\nu$ is the coefficient of kinematic viscosity and $\kappa$ is the thermal diffusivity, $q$ is a heat flux ($q = P/S_h$, $P$ is the power of the heater and $S_h$ is the heater’s surface area).

The parameters of experiments which are used for description of the flow are presented in Table 1.

3. Results

Detailed description of the basic flow structure can be found in [8] and here we present only brief description of the general structure of the large-scale flow. The heat flux in the central part of the bottom is a source of the intensive upward motion above the heater. Warm fluid cools at
Figure 1. a - experimental model, dimensions and location of the coordinate system. T - thermocouple for control of the mean temperature; b - scheme of large-scale circulation.

Table 1. The values of the main parameters.

| Experiment | $P$ (W) | $\Omega$ (rad s$^{-1}$) | $\nu$ (cSt) | $Gr_f$ | $Re$ | $Pr$ |
|------------|--------|-------------------------|-------------|--------|------|------|
| 1          | 19     | 0.048                   | 5.1         | $6 \times 10^6$ | 9.3  | 57   |
| 2          | 19     | 0.081                   | 5.1         | $6 \times 10^6$ | 14.7 | 57   |
| 3          | 19     | 0.11                    | 5.1         | $6 \times 10^6$ | 19.9 | 57   |
| 4          | 19     | 0.17                    | 5.1         | $6 \times 10^6$ | 30.6 | 57   |
| 5          | 13.5   | 0.13                    | 2.8         | $1.4 \times 10^7$ | 46   | 34   |

Figure 2. Mean radial (a) and azimuthal (b) velocity fields in a vertical cross-section over heating area, $Gr_f = 1.4 \cdot 10^7$, $Re = 15.6$. Thick solid white line shows the border between positive and negative values of velocity.

the free surface and moves toward the periphery where the cooled fluid moves downward along the side wall. Large-scale advective flow occupies the whole vessel (figure 1(b)).

The cyclonic vortex formation in the laboratory system can be described by following scenario. Large-scale radial circulation leads to the angular momentum transport and the angular momentum exchange on the solid boundaries. Convergent flow in the lower layer brings the fluid parcels with large values of angular momentum from the periphery to the center and produces cyclonic motion (figure 1(b)), lower horizontal cross-section). In the upper layer situation is opposite - divergent flow takes the fluid with low values of angular momentum to the periphery resulting in anticyclonic motion (figure 1(b), upper horizontal cross-section).
Friction in the viscous boundary layers leads to the sink of angular momentum in the part of the bottom occupied by cyclonic flow and produces source of angular momentum on the sidewalls when anticyclonic flow comes to the periphery. Zero net angular momentum flux on the solid boundaries is the necessary condition for the steady-state regime [7]. The general structure of laboratory vortex (figure 2) is similar to the observational data [9] excluding the side wall area.

Figure 3. Instantaneous vector velocity fields (upper panel) and streamline flow fields (lower panel), left - experiment 2, right - experiment 4, \( z = 3 \) mm.

Series of experiments [6,8] showed that stable localized cyclonic vortex exist in a short range of governing parameters. Vortex becomes unstable at low value of viscosity and fast rotation of the model. Examples of instantaneous vector velocity field and streamline flow fields for the stable and unstable cases are shown in figure 3. In the case of developed cyclonic vortex there are several branches of radial inflow that comes very near to the center of the model figure 3 (left). Radial inflow transports angular momentum from the large radii to the small radii and its symmetry or asymmetry strongly influences the vortex structure. This result is
in a good agreement with numerical simulation [10]. Increasing of rotational Reynolds number (2) by decreasing of viscosity or increasing of rotation rate leads to the instability of radial inflow. It changes its structure as shown in figure 3 (right). Instead of several branches which provide inflowing from different directions it is transformed into one big spiral branch that does not reach the center. As a result of this asymmetrical flux of angular momentum a localized cyclonic vortex is formed at some distance from the center. Radial inflow and cyclonic vortex slowly precessed around axis of rotation and in the mean vector field it appears as a vortex ring. The instantaneous vorticity fields for different values of Reynolds number (Grf and Pr are fixed) are shown in figure 4 and in figure 5. We see that transformation of the vortex structure is a gradual process, when we increase Re from 15.6 to 21.1 the radial inflow has already the dominant direction but still comes close to the center. After further increasing of Re up to 32.3 we achieve a state with localized at some radius vortex (figure 5). The radial inflow changes its direction (not periodically) so location of the vortex shown in figure 5 also changes. Figure 6
Figure 6. Locations of the cyclonic vortex center, triangles - experiment 2, circles - experiment 4, $z = 3$ mm.

Figure 7. Instantaneous streamline flow fields (a,b) for experiment 5, $z = 3$ mm.

shows that locations of the center of the vortex for the stable case are concentrated near the axis of rotation but for unstable case they are dispersed at larger radii. For illustration of the flow structure at higher values of $Re$ and $Gr_f$ we used working fluid with lower viscosity (2.8 cSt instead of 5.1 cSt as in most of described experiments). For the fixed rotation rate and heating power it leads to increasing of $Re$ and $Gr_f$ and further destabilization of the flow. The flow becomes more chaotic with one or several vortices of different size (figure 7). Concluding the description of the vortex instability we want to emphasize two important features of the vortex evolution. The first one is that the instability of the vortex is tightly connected with changing structure of the radial inflow so the study of this process requires 3D approach. Another essential
feature of the described evolution of the cyclonic vortex is its strong dependence on $Re$. We present map of regimes in figure 8 which clearly shows that there is critical value of rotational Reynolds number $Re \approx 23$ for the cyclonic vortex instability.

4. Conclusions
Present study showed that instability of the vortex is tightly connected with a structure of the radial inflow. Up to moderate values of rotational Reynolds number the radial flows consist of several branches which transport angular momentum to the center. When $Re$ exceeds the critical value radial inflow changes its structure and appears as one wide branch which does not reach the center. As a result the vortex which slowly moves around the center is formed instead of the vortex localized in the center. Further increase of $Re$ leads to the chaotic state with several vortices which appears at different locations near the periphery of the heating area. The short interval of existence of localized intensive cyclonic vortex explains small number of laboratory studies of large-scale atmospheric vortices such as tropical cyclones. Also we assume that if described instability takes place in real tropical cyclones than it can be a reason why from a multiple cases of large-scale vortical disturbances in a favourable environment only a few can become a tropical cyclone.

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