Swing amplification and global modes reciprocity
in models with cusps

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Abstract. Using 3D N-body simulations we analyse an onset of the bar in
cuspy models, and argue that role of swing amplification is twofold. Amplified
shot noise due to disc discreteness hampers bar formation, while induced res-
onance perturbations allow bar amplitude to overcome shots. A bar pattern
speed and a growth rate obtained in N-body simulations agree well with global
mode analysis.

Key words: galaxy: formation – galaxy: evolution

1. INTRODUCTION

Among different possibilities for a physical mechanism that is responsible for
bar formation in disc galaxies there are two competing especially fiercely.

The first one is a local approach known now as swing amplification developed
in works by Goldreich & Lynden-Bell (1965), Julian and Toomre (1966), Toomre
(1981). Randomly emerging short leading wave packets propagate outward to a
forbidden zone boundary near corotation, where they swing into trailing ones.
This process is accompanied by amplification of the packets that eventually travel
back to the centre. If the packets can freely reflect from the centre, we have
an instability mechanism for formation of spiral and barred structures: they are
assumed to piece together from the amplified transient waves. However, no one is
succeeded yet to calculate earnestly the pattern speed \( \Omega_p \) or the growth rate \( \omega_I \)
of the structures in this frame.

The second possibility is unstable global modes of axisymmetric stellar discs.
Weak bars appear as very open trailing spirals. Here the situation is opposite:
at small amplitudes mode’s characteristics can be found from linear perturbative
analysis using matrix equations (Kalnajs, 1977; Polyachenko 2005) but a physical
mechanism hasn’t been widely discussed. Lynden-Bell & Kalnajs (1972) showed
that the presence of a trailing wave lowers angular momentum in the inner part
of the disc and increases it in the outer parts. The mechanism is basically similar
to inverse Landau damping. A steady wave with pattern speed greater than the
maximum precession rate of orbits is supported by the central part of the disc
(Polyachenko 2004). In the presence of resonances, the wave becomes unstable
(Polyachenko 2005). Contrary to the local approach in which an outer Lindblad
resonance (OLR) doesn’t play any role, for the global modes OLR sometimes is
more important than corotation in a sense that transfer of angular momentum to
OLR dominates over corotation region (Polyachenko & Just, 2015).

The bar forming mechanism should explain appearance of numerical bars in
N-body simulations. When using a quiet start technique, which reduces a shot
noise (Sellwood & Athanassoula, 1986), a bar appears as exponentially growing
density wave with fixed pattern speed $\Omega_p$ and growth rate $\omega_I$ (e.g., Polyachenko
2013). However, disc discreteness induces the shot noise and secular orbital diffu-
sion (Fouvry et al. 2015). Although not captured by the collisionless Boltzmann
equation, they are important ingredients of disc dynamics. In particular, secular
changes of the disc DF can lead to instability of initially stable models (Sellwood
2012).

Using 3D N-body tree-code simulations with $N_d = 6$ M randomly distributed
disc particles in live and rigid cuspy halo/bulge surrounding we analyse an onset
of the bar. A key feature that triggers the bar formation in a system with the shot
noise is a wave occurring relatively rarely at radii of the order of the disc scale,
most likely due to swing amplification of some induced resonance perturbation.
We refer to this wave as high wave. It opposes to low waves, i.e. more frequent
low amplitude waves that are swing amplified shot noise. The high wave increases
a bar amplitude to the level invulnerable to the shot noise, where the amplitude
starts to grow exponentially fast in accordance with linear perturbation theory
based on the pure collisionless Boltzmann equation.

2. CALCULATIONS AND RESULTS

The N-body model we use here is a 3-component model consisting of the stellar
disc, Sérsic cuspy bulge, and NFW dark matter halo with parameters described
in Polyachenko et al. (2016ab). The bar pattern speed is obtained from an angle
of the rotation of the main axes of the inertia ellipsoid. The bar with $\Omega_p \approx 55$
km/s/kpc shows up approximately exponentially. Its growth rate $\omega_I$ is obtained
from slopes of the bar strength $B(t)$ or the bar amplitude $A_2/A_0$ (Figure 1), where

$$B(t) = 1 - I_{yy}/I_{xx}, \quad I_{xx} = \sum_j \mu_j x_j^2, \quad A_m(t) = \sum_j \mu_j e^{-im\theta_j}.$$  (1)

($\mu_j$ and $\theta_j$ are mass and polar angle of star $j$; $j$ spans particles within some
fixed radius, e.g., typical radial scale length; expression for $I_{yy}$ is analogous to
$I_{xx}$). In live simulations, the e-folding time $t_e$ is 230 Myr, but in rigid halo/bulge
simulations it increases to 530 Myr.

Although $B(t)$ and $A_2/A_0$ slopes are nearly the same, the former curves are
more irregular. The $B(t)$ curve for 100 km/s central dispersion is raising starting
from 0.6 Gyr, the other curves − from 1 and 2 Gyr.

Raise of the bar amplitude $A_2/A_0$ is more steady (Figure 1, right panel). Linear
fits shown by dashed black lines are calculated using the second half of the raising
part and extrapolated to the first half. In all cases we see ‘jumps’ of the colour
lines, but they decay on time scales 100 ... 200 Myr. The jumps are absent when
the bar amplitude reaches $\sim 1\%$ of the axisymmetric background.

Bias of the fits from the origin, i.e. lags, are equal to 0.2, 0.7, 1.2 Gyr, corre-
spondingly. The less unstable model (smaller growth rate) or the number of disc
particles $N_d$ is larger, the lag in bar formation is larger. Our tree-code simulations
never show it in smaller runs with $N_d = 1.1$ M. The lag is seen also in works of
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Fig. 1. **Left panel:** the bar strength for live runs with central radial velocity dispersion $\sigma_{R0} = 100$ and 140 km/s, and rigid halo/bulge run with $\sigma_{R0} = 100$ km/s. The dash-dotted thin line shows a slope corresponding to the exponential growth with $t_e = 230$ Myr. **Right panel:** the bar amplitude $A_2/A_0$ for the same runs. The dashed black lines show slopes with $t_e = 230, 370, \text{and } 530$ Myr.

others (e.g., Dubinski et al. 2009), but it has been neglected.

Rigid halo/bulge models allow matrix calculation of eigen-modes. The obtained pattern speed $\Omega_p \approx 52$ km/s/kpc and the growth rate $\omega_I = 1.8 \ldots 1.9$ Gyr$^{-1}$—both agree well with results of N-body simulations.

Origin of the jumps can be traced by plotting Fourier amplitudes $A_2(R, t)$,

$$A_m(R, t) = \sum_j \mu_j e^{-im\theta_j},$$

where summation is taken over disc particles within a ring $\Delta R$ near $R$. Figure 2, left panel, shows an absolute value of $A_2(R, t)$ for $\sigma_{R0} = 100$ km/s live run. A shot noise exists in all runs (at level $\sim 200 \ldots 300$), but we show only perturbations with amplitudes higher than 400 by choosing proper colour mapping. A wave with an amplitude 1000–2000 appears at $t \approx 0.2$ Gyr and propagates inward reaching $R = 0.58$ kpc at $t \approx 0.47$ Gyr. The arrival is marked by a jump of the blue curve. The subsequent jumps are caused by a series of similar waves born at 4...6 kpc.

These waves are usually associated with swing amplified shot noise (e.g., Sellwood 2012). For given parameters $Q$ and $X \equiv k_{crit}R/m$ favouring swing amplification (Toomre 1981), maximum trailing/leading bias is $\sim 10$ within $2 < R < 7$ kpc, and shot noise $\sim (R \Sigma_d)^{1/2}$ is large. However, this bias is measured for the maximum of the trailing amplitude; the ratio for amplitudes far from amplification zone is much smaller. Besides, the given factor is for a full swing; e.g., for half-swing it is less than $\sim 3$. The live halo increases this value, but it is hardly possible to obtain values more than 1000. Typical values for such swing amplified noise well seen in Figure 3 (right) are 200–400. We denote these waves as low waves.

Another possibility is swing amplified induced perturbations. Such perturbations are possible as responses to the gravitation potential of the emerging weak bar in the centre (Polyachenko 2002) or spiral modes of the disc. In favour of this hypothesis, one can observe an increasing activity in formation of the high amplitude waves ($> 1000$) at $t > 0.6$ Gyr in Figure 2, and $t > 1$ and 1.5 Gyr at Figure 3. We denote these waves as high waves.

In Figure 2, right panel, real part of $A_2(R, t)$ is presented in a smaller range of time and radius. Before $t < 0.2$ Gyr we can see ordered perturbations in form of
Fig. 2. Maps $A_2(R, t)$ for the live run $\sigma_{R0} = 100$ km/s. Left panel: $\log |A_2(R, t)|$. The colour map is adjusted to show only values larger than 6 (blue). Right panel: $\Re A_2(R, t)$. Green colours show values near zero. A bar-like perturbation is seen as vertical blue and red stripes, while stripes with negative inclination show trailing spirals.

Fig. 3. Same as in the left panel of Figure 2, for the live run $\sigma_{R0} = 140$ km/s and the rigid run $\sigma_{R0} = 100$ km/s.

trailing waves with amplitudes 200–400. Two high waves appear during first 0.6 Gyr, which disappear after $t \gtrsim 0.8$ Gyr in the bar area (vertical stripes).

Less unstable models are given in Figure 3. The left panel shows a map for a live run with increased radial velocity dispersion. The first wave appears only at $t \approx 0.74$ Gyr and causes the first jump in the bar amplitude curve (Figure 1, green line). Note that no growth of the amplitude is seen before the jump, so the wave triggers the bar formation. Growth of the bar amplitude becomes approximately exponential after the second wave that forms at $\sim 1$ Gyr.

A rigid run with $\sigma_{R0} = 100$ km/s (right panel) presents even more irregular bar formation. The first wave appears at nearly the same time as in the live run (Figure 2), but it is not supported by the subsequent waves. This leads to a gradual decay of the bi-symmetric perturbation from 0.5 to 1.5 Gyr (Figure 1, red line) until the high waves do not restart process of bar formation.
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The time Fourier transform of $A_m$:

$$S_R(R, \Omega_p, T) = \int_{T-\Delta T}^{T+\Delta T} \mathfrak{F}_m(R, t) H(t - T) e^{im\Omega_p t},$$

(3)

with filter function $H$, produces a spectral map

$$M_R(R, \Omega_p, T) = \ln \frac{|S_R|^2}{2\pi}$$

(4)

providing locations of coherent structures in $(R, \Omega_p)$-plane.

Figure 4 shows $M_R$ of bi-symmetric ($m = 2$) harmonics. In the earliest panel we see 3 distinct maxima corresponding to modes with different pattern speeds. The top maximum corresponds to a bar mode, which is in fact an open spiral with average pitch angle $\sim 60^\circ$. Two other maxima, localised between 1.5 and 6 kpc and 2 and 8 kpc, can be spiral modes that seed the incoming waves. Indeed, these maxima look like modes, remaining fixed in the $(R, \Omega_p)$-plane almost until the bar is formed. Interference of these modes can locally form high enough over-density, that is eventually amplified by the swing mechanism. The same three maxima are seen in Figure 5 calculated for the rigid halo/bulge run. Note that bar mode is seen already in the first (upper left) panels of Figures 4 and 5.

3. DISCUSSION AND CONCLUSIONS

A real N-body system is a complex interplay of instability well described by collisionless Boltzmann equation, and the shot noise effects. Here we list our observations based on analysis of several N-body simulations:

- there is a shot noise background in the stellar disc;
- at some moment, a wave appears at $\sim 4...6$ kpc and propagates to the centre;
- plausibly, the swing mechanism amplifies a seed perturbation into the wave;
- spectral maps show several modes, including bar mode: these modes can provide the seed for the wave;
- arrival of the wave in the centre triggers bar formation;
- since the wave appears randomly, there is a random lag in bar formation;
- the wave causes a jump in the bar amplitude, which relaxes gradually;
- other waves appear randomly and cause similar jumps until it becomes about 1% of the axisymmetric background; afterwards the bar amplitude grows exponentially;
- the exponential growth rate agrees well with predictions based on the collisionless Boltzmann equation;
- the larger the growth rate and smaller the number of disc particles, the smaller the lag.
These observations fit to a following scenario. An emerging bar mode due to the global mode instability is kicked by two kinds of incident waves. Both kinds of waves are due to swing mechanism. Low amplitude waves that come from the shot noise are frequent and they are not in phase with the bar, so they hamper bar formation. High amplitude waves that induced by interference of modes are rare, but they can increase the bar amplitude. When the latter becomes high enough, it starts to grow uniformly due to usual global mode instability.

V.I. Korchagin reported (private communication) that introduction of annulus in the region within corotation wiping out all non-axisymmetric perturbations leads to faster bar formation. Our hypothesis explains this effect. Indeed, when all incoming waves (both low and high) are wiped out, bar forms as usual unstable mode with no obstruction.

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Fig. 5. Same as in Figure 4, for the rigid run.

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