Nonzero cosmological constant
and the many vacua world

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Abstract

The idea of the quantum state of the Universe described by some density matrix, i.e. mixture of at least two vacua, the trivial symmetric and the nontrivial one with spontaneously broken symmetry is discussed. Nonzero cosmological constant necessarily arises for such a state and has the observable value if one takes the axion mass for the vacuum expectation value. The Higgs model, Nambu’s model and discrete symmetry breaking are considered. Human observers can observe only the world on the nonsymmetric vacuum, the world on the other vacuum is some dark matter. Gravity is due to action of two worlds. Tachyons nonobservable for visible matter can be present in the dark matter, leading to some effects of nonlocality in the space of the Universe.

1. Introduction

Any theory of spontaneously broken symmetry, the most known being the Higgs model, leads to the possibility of at least two vacua. One vacuum corresponds to the trivial solution, the other to the nontrivial one with nonzero vacuum expectation value of the Higgs field. If one considers the vacuum expectation value of the quantized stress energy tensor and puts it into the right hand side of Einstein’s equations one will see that two vacua are different by the nonzero cosmological constant appearing for the nontrivial vacuum. The observable value of the cosmological constant can be obtained by taking vacuum expectation value of the order of one thousand’s of eV, i.e. the mass of some axion. So if one puts the hypothesis, expressed by the author many years ago, that the quantum state of the Universe is not a pure state but the mixture of two vacua — the normal and the superfluid one, nonzero cosmological constant necessarily is observed. Human observers can observe only the world on the nontrivial vacuum but gravitation is influenced by worlds on both vacua. Different vacua correspond to unitary
non equivalent representations of commutation relations and to orthogonal Hilbert spaces of states. So differently from other situations in physics in cosmology if the volume is infinite no tunneling is possible from one vacuum to the other.

Three models of spontaneous breaking of symmetry are considered: discrete symmetry breaking with one real scalar field, the Higgs model for the complex scalar field and the Nambu model with spinor particles. In the first model to the massive real field on one vacuum correspond tachyons on the other.

For Higgs model one predicts existence of the relativistic dark matter composed of massless vector particles and tachyons. For the Nambu model to massive fermions and Goldstone massless particles on one vacuum correspond massless fermions of hidden matter on the other. Differently from the domains idea these vacua exist everywhere due to translational invariance and no domain walls are present in the Universe.

Tachyons are nonobservable in the world where human observers exist, they are massive Higgs bosons in this world. However their existence in the other vacuum world due to gravitational effects can lead to correlations on spacelike intervals for ordinary matter which can solve some Friedmann cosmology paradoxes. The main result of the mixture of vacua hypothesis is that if dark matter correspond to the world on the other vacuum then the nonzero cosmological constant must be observed.

2. Nonzero cosmological constant in spontaneously broken symmetry theories

Observations show that the cosmological constant in the observable Universe is some positive nonzero number. The widespread opinion is that nonzero cosmological constant can arise due to special properties of the vacuum for quantum particles in the Universe. What are these properties and how they can lead to the observable value of the cosmological constant? There are two different ways to obtain this constant from the properties of the quantum vacuum.

1. If one calculates the vacuum expectation value of the stress-energy tensor for free quantized field in curved space-time one obtains some divergent expression. The leading divergent term is proportional to the fourth degree of the momentum and has the form of the cosmological term in Einstein’s equations. This term is present also in flat Minkowski space-time and describes quantum vacuum oscillations. The cosmological constant occurs to
be positive for bosons and negative for fermions. This gives hope that due to some supersymmetry for bosons and fermions the divergence can be cancelled and some finite value for the cosmological constant can occur. However there is no quantitative method to obtain this value, so the whole reasoning has only some qualitative meaning.

2. The other way to obtain the nonzero value of the cosmological constant as it was shown by the author in 1967 [2] is to consider the models with spontaneous breaking of discrete and continuous symmetries in quantum field theory in curved space-time and consider the quantum state of the Universe to be not a pure but a mixed state. Instead of taking the so called wave function of the Universe as a pure state one can consider some mixture of vacua, the trivial and the nontrivial one described by some density matrix with weights for elements of the mixture.

In these models one has the Lagrangian and Hamiltonian invariant under some symmetry transformations but there exist two different vacua, the trivial invariant one and the other noninvariant vacuum. For the continuous symmetry breaking one has the degenerate noninvariant vacuum and infinite number of different vacua with the same energy can be obtained by the symmetry transformation. In this case Goldstone massless bosons exist which however can acquire nonzero mass if some explicit breaking of symmetry in the Lagrangian is taken into account.

a) Take the simplest model of one real scalar or pseudoscalar field with selfinteraction with the Lagrangian

\[ L = -\frac{1}{2} \left( \frac{\partial \Phi}{\partial x_\mu} \right)^2 + \frac{1}{2} \mu_0^2 \Phi^2 - \frac{\lambda}{4} \Phi^4. \]  

(1)

The wrong sign for mass squared means that the free field corresponds to some tachyons, which is the usual case for Goldstone models. The potential

\[ V(\Phi) = \frac{1}{2} \mu_0^2 \Phi^2 - \frac{\lambda}{4} \Phi^4 \]  

(2)

has two extremal points defined by the equation

\[ \mu_0^2 \Phi - \lambda \Phi^3 = 0 \]  

(3)

so that besides the trivial zero solution the nontrivial solution also exists. In quantum field theory these two solutions correspond to two different vacua, one with the zero vacuum expectation value, the other one with the nonzero value \( \Phi_0 = \frac{\mu_0}{\sqrt{\lambda}} \). For pseudoscalar particles nonzero value means that \( P \)-invariance of the Lagrangian is broken for the nontrivial vacuum. Quantization on the nontrivial vacuum is made by going to new fields obtained
by the shift on the vacuum expectation value of the field, so that in terms of
the new fields taking into account the equation for the vacuum expectation
value one gets a new Lagrangian

\[ L = -\frac{1}{2} \left( \partial_{\mu} \phi \right)^2 - \frac{1}{2} \mu_0^2 \phi^2 - \lambda \Phi_0 \phi^3 - \frac{\lambda}{4} \phi^4 + C \]  

(4)

where for the constant \( C \) one has \( C = \mu_0^4/(4 \lambda) \). If one considers vacuum
expectation value of stress-energy tensor in the normal form of the quanti-
tized scalar (pseudoscalar) field due to translational invariance of vacuum
(in curved space in homogeneous space-time instead of Minkowski transla-
tions one must consider elements of the group of homogeneity) this vacuum
expectation value is equal with the negative sign to the vacuum expectation
value of the potential. So for the nontrivial vacuum one obtains in the right
hand side of Einstein’s equations after multiplying on the gravitational con-
stant some positive cosmological constant \( \Lambda = G \). Its numerical value is
calculated if one knows the values of the mass of the boson and the self-
interaction constant. For the trivial vacuum this cosmological constant is
zero. However one can speculate that the Lagrangian and the Hamiltonian
are defined up to some constant, so it may be that this arbitrary constant is
defined so that it just compensates the new constant arising on the nontrivial
vacuum. The idea expressed by the author in [2] was that in cosmology one
can think that both vacua are realized in the infinite Universe with some
weights in the density matrix of the Universe. Hilbert spaces in which unit-
ary nonequivalent representations of commutation relations for the scalar
field can be realized are orthogonal. So no quantum physical process can
connect these states. For the actually infinite volume of the Universe (how-
ever it can be that this volume must be much larger than the volume inside
the horizon!) no tunneling from one vacuum to the other is possible. This is
the difference of the situation in the Universe and that arising for any finite
volume manybody system when tunneling with nonzero probability is possible.
That is the reason that no domains or space regions with one and the
other vacua can coexist. Both vacua are translational invariant in our case.
There are two disconnected different worlds. No physical interaction exists
between them. However our hypothesis is that both worlds gravitate and it
is only due to the gravity (quantization of which is still nonexistent) that
they interact. One of the worlds is considered from the point of the other
as some dark matter and the nonzero cosmological constant must be neces-
sarily present in Einstein’s equations! If one takes selfinteraction constant
for scalar particles in our model to be of the order of unity, then observa-
tional value of the cosmological constant is consistent with the mass of our
field of the order of \( 10^{-3} \) eV, i.e. of the axion in some models [3]. In the
result for our simple model one can come to the following conclusions. We as human observers live on one nontrivial vacuum where no tachyons can be observed, instead we can observe massive scalar (pseudoscalar) particles with the axion mass. These particles can interact with other particles which have interactions with the electromagnetic field forming the observable Universe. However there exists the other world on the trivial vacuum, nonobserved by us through electromagnetic interaction and where we cannot have our ordinary body made from usual particles. So anthropic principle makes this world unseen and we cannot be conscious of it due to the usual nervous mechanism. In this other world tachyons as some dark matter exist. Tachyon theorists [4] claim that tachyons can account for some correlations on spacelike intervals.

Friedmann cosmology for the early Universe has the well known problem of causality which inflation tries to solve. But if tachyons exist on the other vacuum they can provide another solution of the problem. In our reasoning we for simplicity took both weights for vacua of the order of unity. However one can take one of these weights much smaller than the other, so that the mass of the scalar particle can be much larger. In nonstationary Friedmann Universe these weights can depend on time which opens the way to new theoretical possibilities.

b) The Higgs model. One has the complex scalar field (or equivalently two real fields) with tachyonic mass (negative sign of the mass squared) interacting with massless vector field and with selfinteraction term as in the previous case with the Lagrangian

\[ L = -\frac{1}{2} (\nabla \varphi_1)^2 - \frac{1}{2} (\nabla \varphi_2)^2 - V(\varphi_1^2 + \varphi_2^2) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \]

where

\[ \nabla_\mu \varphi_1 = \partial_\mu \varphi_1 - e_0 A_\mu \varphi_2 , \quad \nabla_\mu \varphi_2 = \partial_\mu \varphi_2 + e_0 A_\mu \varphi_1 , \]

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu . \]

Looking for the stationary points of the potential

\[ V(\varphi_1^2 + \varphi_2^2) = \mu_0^2 (\varphi_1^2 + \varphi_2^2) - \frac{1}{6} \lambda (\varphi_1^2 + \varphi_2^2)^2 \]

one obtains two solutions

\[ \varphi_1 = \varphi_2 = 0 ; \quad \varphi_1 = 0 , \quad \varphi_2 = \varphi_0 \neq 0 . \]

The solution leading to noninvariant nonzero vacuum expectation value describes one scalar field of the Higgs boson with the mass, the other scalar
component forms the degree of freedom of the vector massive boson so that
one comes to equations

\[ (\Box - 4\phi_0^2 V''(\phi_0^2)) \tilde{\phi}_2 = 0, \quad (10) \]

\[ \partial_\mu F^{\mu\nu} - e_0^2 \phi_0^2 B_\mu = 0, \quad (11) \]

\[ B_\mu = A_\mu - (e_0\phi_0)^{-1} \partial_\mu \tilde{\phi}_1 \quad (12) \]

As in the model discussed by us previously the nonzero cosmological constant
is equal to

\[ \Lambda = G \frac{1}{6} \lambda \phi_0^4. \quad (13) \]

c) Nambu’s model for the spinor field. The Lagrangian is

\[ L = \bar{\psi} i \partial_\mu \nabla^\mu \psi + g \left[ (\bar{\psi} \psi)^2 - (\bar{\psi} \gamma_5 \psi)^2 \right]. \quad (14) \]

There are two different vacua for this chiral invariant Lagrangian. Following [5] define

\[ m_S(x) = 2g i \text{Tr} S_F(x,x), \quad m_P(x) = 2g \text{Tr} (\gamma_5 S_F(x,x)), \quad (15) \]

\[ \Phi(x) = m_S(x) + im_P(x). \quad (16) \]

Then one obtains the analog of the Ginzburg-Landau equation for the superconductor wave function

\[ \Box \Phi - 2m_\infty^2 \Phi(x) + 2|\Phi(x)|^2 \Phi(x) = 0 \quad (17) \]

\[ \lim_{x \to \infty} \Phi(x) = m_\infty. \quad (18) \]

where

As it is well known in Minkowski space-time one must introduce some
cutoff parameter for the theory to be consistent. However in curved space-
time with the negative curvature of space one can have nontrivial vacuum
without the cutoff [6]. Two vacua, the trivial and the nontrivial one are
different by the cosmological constant

\[ \Lambda = G \langle 0'| g \left[ (\bar{\psi} \psi)^2 - (\bar{\psi} \gamma_5 \psi)^2 \right] | 0' \rangle. \quad (19) \]

This constant can have different sign depending on the values of different
vacuum expectation values \( m_S, m_P \). This means that there is the possibility
of compensation of different cosmological constants occurring due to Higgs
bosons etc and fermions in the Nambu model. Two worlds are different in
the sense that massless fermions with exact chiral symmetry exist in one world, while in the other world there exist massive fermions and Goldstone pseudoscalar particles. If one takes the observable value of the cosmological constant then for close to unity weights in the density matrix for vacua the mass of the fermion must be close to the axion mass discussed before, so that one can identify it as some neutrino particle. Can we make some estimates on weights in the density matrix for vacua? If the world on the trivial vacuum is identified with some dark matter then it seems that in the present form our model must lead to the identification of cold dark matter with massive tachyons and to the difference in weights consistent with observations not larger than one percent. This excludes the possibility of taking instead of the axion mass the Higgs boson mass of the electroweak model. For Higgs model the weight of the nontrivial vacuum at the modern epoch must be very small.

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