Mutual-radiation efficiency estimation of vibration modes by finite element method and boundary element method software

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Abstract
When the structure is forced to vibrate and multiple modes exist simultaneously, each vibration mode contributes to the total acoustic power radiated by the structure. Moreover, the acoustic radiation of each vibration mode is not independent of each other, and the coupling between them will also have an impact on the total radiated power. Mutual-radiation efficiency is an important metric to measure the ability of the coupling of different modes to radiate. In this note, for both real and complex modes, a method to calculate the mutual-radiation efficiencies by finite element method and boundary element method software is presented. Numerical examples are presented to illustrate and validate the approach.

Keywords
Acoustic radiation, complex vibration modes, mutual-radiation efficiency, acoustic power

Introduction
In engineering industries, researchers pay much attention to the study of structural vibration and sound radiation characteristics in recent years. References 1 and 2 derived formulations to analyze the relative panel acoustic contributions of a vibrating structure, and the relative acoustic contributions of a scaled vehicle cabin are ranked. Reference 3 proposed an approach for structural response reconstruction based on the modal superposition method in the presence of closely spaced modes. When considering the vibration of a structure, it can be analyzed in the vibration modal space. The modes of each order are independent of each other, and the modal coordinate represents the contribution of the corresponding vibration mode to the total vibration response. However, when the acoustic radiation of a structure is analyzed by vibration modes, the modes that dominate the vibration response are not necessarily the modes that dominate the acoustic response. Moreover, the vibration modes are not independent of each other, that is, not only they themselves contribute to the sound power, but also the couplings between the modes influence the total radiation. Self-radiation efficiency and mutual-radiation efficiency are two important metrics when studying the contribution of vibration modes to acoustic radiation, and they respectively represent the ability of a mode itself and the coupling of different modes to radiate. It is of great significance to calculate self-radiation and mutual-radiation efficiencies accurately in both active and passive control of noise reduction. Reference 5 determined the self-radiation efficiencies of baffled beams theoretically, with both hinged and clamped supports considered. References 6 and 7 took beams as research objects and implied that the interaction between modes is significant for the total sound power well below the coincidence wave number ratio. Reference 8 derived the formula to calculate the total acoustic power of a rectangular simply supported panel using modal radiation efficiencies. Reference 9 studied the vibration of a simply supported rectangular plate which is reinforced by springs, the result indicates that the coupling effect between modes is normally meaningful. Reference 10 studied the mutual-radiation resistances of simply supported plates, the result shows that modal interaction should not always be neglected at resonant frequencies, and some
spurious peaks may be generated at some frequency points if the coupling effect between modes is ignored. So it is hard to determine when the modal interaction could be neglected. Reference 11 presented an analytical solution for the self- and mutual-radiation resistances of a simply supported plate in the form of power series. References 12 and 13 studied the calculation methods of intermodal radiation impedance for fluid-loaded plates. Reference 14 presented a partially coupled modal contribution assumption to identify the dominant noise-contribution mode. Analytical and numerical methods are commonly used to analyze the acoustic radiation of a plate. As one of the numerical methods, the elementary radiator approach is an effective and convenient method. This approach is summarized and introduced in References 15 and 16. Reference 16 used elementary radiator approach to calculate the radiation efficiencies of a simply supported beam and a simply supported plate. Modal radiation efficiencies of complex modes for plates were determined by elementary radiator approach in References 17 and 18.

It is noted that most of the research results on modal radiation efficiencies have been aimed at real modes. When the damping matrix of the vibration system does not satisfy the Caughey and O’Kelly condition,\(^*\) the modes of the system are complex modes. As described in Reference 20, the distribution of non-proportional or inhomogeneous damping within a structure will lead to the generation of complex modes, if the complexity of the modes is too high, the reliability and accuracy of the analysis using real modes could be adversely affected. References 17, 18, and 21 studied the acoustic radiation efficiencies of complex modes. In Reference 18, inhomogeneous damping was applied to plate structures to generate complex modes, and it was found that the acoustic radiation behavior of complex modes is different from that of real modes. For example, in the case of a uniform rectangular plate, only the modes of similar type can be coupled in the condition of real modes, that is, odd-even modes can only be coupled with odd-even modes, but not with odd–odd, even–even, or even–odd modes. But in the condition of complex modes, the interaction of different types of modes may also occur. Reference 21 shows that the traveling wave component in complex modes could greatly improve the self-radiation efficiency of even-order modes. It can be seen that in the acoustic radiation analysis, if the complexity of the modes is too high, it is necessary to study the acoustic radiation based on complex modes.

It is worth noting that the research objects above are all simple structures such as planar beams or plates. As for complicated structures in practice, finite element method (FEM) and boundary element method (BEM) software is generally used in acoustic analysis. When calculating the mutual-radiation efficiencies of a structure, there is no corresponding module in FEM and BEM software, thus the mutual-radiation efficiencies cannot be obtained directly after the model is built. In this note, for both real and complex modes, a method to calculate the mutual-radiation efficiencies by FEM and BEM software is presented, so that the mutual-radiation efficiencies of complicated structures can be obtained. By using the method, we can calculate the mutual-radiation efficiencies of complicated structures in engineering. It should be emphasized that the commercial software is only used to obtain the radiated acoustic power. This note explains which acoustic power values need to be calculated and how to process them. Numerical examples are presented to illustrate and validate the approach. In addition, the meaning of the mutual-radiation efficiencies in the condition of complex modes is discussed.

**Theory**

**Radiation efficiencies of vibration modes**

The relationship between the radiated sound power \( W \) by a structure and the sound pressure \( p(x,y,z) \), the normal velocities \( v_n(x,y,z) \) can be expressed as

\[
W = \frac{1}{2} \int_S \text{Re} [p(x,y,z)v_n^*(x,y,z)] dS
\]

(1)

where \( \text{Re} [\cdot] \) denotes the real part, the superscript * represents the complex conjugate.

The relationship between the sound pressure and the velocities on the structural surface can be obtained by discretizing the Helmholtz integral equation,\(^22\) then be written as

\[
Ep = Dv_n
\]

(2)

in which \( E \) and \( D \) are coefficient matrices. Then, the relationship can be written as

\[
p = Zv_n
\]

(3)

where \( Z = E^{-1}D \) is the acoustic impedance matrix. Equation (1) can be written in the form of matrices

\[
W = \frac{1}{2} \text{Re} [v_n^H DZv_n] = v_n^H Rv_n
\]

(4)
where the superscript \(^{H}\) denotes the Hermitian transpose (complex conjugate transpose). \(A = \int_{S}^{\overline{}} N^{T} N dS\), where \(N\) is the vector of interpolation functions. \(R = (A/2) \Re [Z]\) is defined as radiation resistance matrix, and it is real, symmetric, and positive definite.\(^{16}\)

Using the matrix of eigenvectors \(\Phi\) and modal coordinates \(a, v_{n}\) can be expressed as:

\[ v_{n} = \Phi a \] (5)

Equation (4) can be rewritten as:

\[ W = a^{H} M a = \sum_{i=1}^{N} \sum_{j=1}^{N} M_{ij} (a_{i}^{*} a_{j}) = \sum_{i=1}^{N} \sum_{j=1}^{N} W_{ij} \] (6)

where \(M = \Phi^{H} R \Phi\) is the power transfer matrix.\(^{15}\) If \(\Phi\) is a matrix of real mode shapes, \(M\) is real and symmetric. If \(\Phi\) is a matrix of complex mode shapes, then \(M\) is Hermitian.\(^{16,18}\) Moreover, the element of \(M\) is \(M_{ij} = \phi_{i}^{H} R \phi_{j}\), where \(\phi_{i}\) and \(\phi_{j}\) are the \(i\)-th and \(j\)-th column vectors of \(\Phi\), respectively.

Radiation efficiency is generally defined as:

\[ \sigma = \frac{W}{\rho_{0} c S_{T} \langle v_{n}^{2} \rangle} \] (7)

where \(\rho_{0}\) is the density of the medium, \(c\) is the sound speed in the medium, \(S_{T}\) is the total area, \(\langle v_{n}^{2} \rangle\) is the space average mean-square velocity, and it is given by:

\[ \langle v_{n}^{2} \rangle = \frac{1}{2 S_{T}} \int_{S} |v_{n}(x,y,z)|^{2} dS \] (8)

Suppose that a single vibration mode is excited, that is, the velocity vector is \(v_{n} = a_{i} \phi_{i}\), according to equation (6) we know \(W = |a_{i}|^{2} M_{ii}\). If the rules to normalize the eigenvectors is given by:

\[ \frac{1}{S_{T}} \int_{S} |\phi_{i}(x,y,z)|^{2} dS = 1 \] (9)

so \(\langle v_{n}^{2} \rangle = |a_{i}|^{2}/2\), the modal self-radiation efficiency is written as:

\[ \sigma_{ii} = \frac{2 M_{ii}}{\rho_{0} c S_{T}} \] (10)

The mutual-radiation efficiency between modes is written as:

\[ \sigma_{ij} = \frac{2 M_{ij}}{\rho_{0} c S_{T}} \] (11)

A baffled panel could be divided into \(I\) small pistons of the same size according to the elementary radiator approach summarized in References 15 and 16. The vibration of each piston is specified by the velocity at its central position. If the area of each elementary radiator is \(S_{E}\), the sound power is given by:

\[ W = \left(\frac{S_{E}}{2}\right) \Re [v_{n}^{H} p] \] (12)

And \(R\) is a matrix of \(I \times I\) elements, it is given by:

\[
R = \frac{\omega^{2} \rho_{0} S_{E}^{2}}{4 \pi c} \begin{bmatrix}
1 & \sin(\kappa_{r_{12}})/\kappa_{r_{12}} & \cdots & \sin(\kappa_{r_{1I}})/\kappa_{r_{1I}} \\
\sin(\kappa_{r_{21}})/\kappa_{r_{21}} & 1 & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
\sin(\kappa_{r_{I1}})/\kappa_{r_{I1}} & \cdots & \cdots & 1
\end{bmatrix}
\] (13)
here, $k = \omega / c$, $r_{ij}$ is the distance from element $i$ to element $j$, obviously $r_{ij} = r_{ji}$. Thus, the radiation resistance matrix $R$ can be obtained conveniently for planar structures.

**Mutual-radiation efficiency estimation from finite element method and boundary element method software**

When calculating the self-radiation efficiency by software, the key is to obtain $M_{ii} = \phi_i^H R \phi_i$, that is the sound power when $v_i = \phi_i$. Thus we can input $\phi_i$ as velocity conditions on the structure in the software, then calculating the radiated acoustic power $W_{ii}$. Note that $W_{ii} = M_{ii}$, the self-radiation efficiency could be obtained by equation (10).

It should be emphasized before introducing the calculation method of mutual-radiation efficiencies that the values of the mutual-radiation efficiencies are real in the case of real modes and complex in the case of complex modes. In the condition of complex modes, suppose that two vibration modes are excited, and $a_1$, $a_2$ are their modal coordinates respectively, according to equation (6), the total sound power radiated by the structure is given by

$$W = a_1^H M_{11} a_1 + a_2^H M_{22} a_2 + a_1^H M_{12} a_2 + a_2^H M_{21} a_1$$

(14)

where $a_1^H M_{12} a_2$ and $a_2^H M_{21} a_1$ are conjugate complex. The overall contribution of the coupling of the modes is given by

$$W_m = a_1^H M_{12} a_2 + a_2^H M_{21} a_1 = 2 \text{Re}[a_1^* M_{12} a_2]$$

(15)

Let $a_1 = a_{1r} + i \cdot a_{1i}$, $a_2 = a_{2r} + i \cdot a_{2i}$, where $i$ is the imaginary unit, then consider the relationship between $\sigma_{12}$ and $M_{12}$ in equation (11), equation (15) is rewritten as

$$W_m = \rho_0 c S_T \cdot \text{Re}[\sigma_{12}] \cdot (a_{1r} a_{2r} + a_{1i} a_{2i}) + \rho_0 c S_T \cdot \text{Im}[\sigma_{12}] \cdot (a_{1r} a_{2i} - a_{1i} a_{2r})$$

(16)

According to equation (16), if the modal coordinates of structural vibration are known, the radiated acoustic power contributed by mode coupling can be calculated accurately only when both the real part and the imaginary part of $\sigma_{12}$ (or $M_{12}$) are obtained. In addition, it is reasonable to use the magnitude of $\sigma_{12}$ as a metric to measure the ability of the coupling of complex modes to radiate. Because if we only use the real part as the metric, a situation may occur that $\text{Re}[\sigma_{12}]$ is equal to zero, that is, the metric indicates that there is no acoustic coupling between the two modes, but the sound power contributed by the coupling is not zero (e.g. when $\text{Re}[\sigma_{12}] = 0$, $\text{Im}[\sigma_{12}] \neq 0$ and $(a_{1r} a_{2r} - a_{1i} a_{2i}) \neq 0$). Apparently, it is unreasonable. Similarly $\text{Im}[\sigma_{12}]$ should not be used as the metric alone. It can be seen that in the condition of complex modes, both the real and the imaginary part should be obtained when calculating the mutual-radiation efficiencies.

When calculating the mutual-radiation efficiency, we need to know $M_{ij} = \phi_i^H R \phi_j$ it cannot be obtained directly. Consider

$$(\phi_i + \phi_j)^H R (\phi_i + \phi_j) = \phi_i^H R \phi_i + \phi_j^H R \phi_j + \phi_i^H R \phi_j + \phi_j^H R \phi_i$$

(17)

let $W_{T1} = (\phi_i + \phi_j)^H R (\phi_i + \phi_j)$, it can be obtained by inputting $(\phi_i + \phi_j)$ as velocity conditions on the structure in the software and solving the sound power. Then calculate $W_{T1}$ together with $W_{ij}$ in the condition of real modes, $\phi_i^H R \phi_j = \phi_j^H R \phi_i$, so

$$\sigma_{ij} = \frac{(W_{T1} - W_{ij} - W_{ji})}{\rho_0 c S_T}$$

(18)

hence the mutual-radiation efficiency of real modes can be obtained.

In the condition of complex modes, $\phi_i$ and $\phi_j$ are complex vectors, so $\phi_i^H R \phi_j$, $\phi_j^H R \phi_i$ are conjugate values. Let $\phi_i^H R \phi_j = a + i \cdot b$, then $\phi_j^H R \phi_i = a - i \cdot b$, so $\phi_i^H R \phi_j + \phi_j^H R \phi_i = 2a$. It can be seen that the real part of $\sigma_{ij}$ is

$$\text{Re}[\sigma_{ij}] = \frac{(W_{T1} - W_{ij} - W_{ji})}{\rho_0 c S_T}$$

(19)

Consider that

$$(\phi_i - i \cdot \phi_j)^H R (\phi_i - i \cdot \phi_j) = \phi_i^H R \phi_i + \phi_j^H R \phi_j - i \cdot \phi_i^H R \phi_j + i \cdot \phi_j^H R \phi_i$$

(20)

in which $-i \cdot \phi_i^H R \phi_j + i \cdot \phi_j^H R \phi_i = 2b$. Let $W_{T2} = (\phi_i - i \cdot \phi_j)^H R (\phi_i - i \cdot \phi_j)$, it can be obtained by inputting $(\phi_i - i \cdot \phi_j)$ as velocity conditions on the structure in the software and solving the sound power. The imaginary part of $\sigma_{ij}$ is
\[ \text{Im}[\sigma_v] = \frac{(W_{T2} - W_{T1} - W_{P})}{(\rho_0 c S_T)} \] (21)

Thus the mutual-radiation efficiency of complex modes can be obtained by combining equations (19) and (21).

**Numerical results**

**Example 1**

This example is to verify the method in the condition of real modes. The structure is a rectangular aluminum plate, it has a dimension of 348 × 304.8 × 0.762 mm. Aluminum material properties (\(\rho = 2700 \text{ kg/m}^3, E = 72.0 \text{ GPa}, \nu = 0.3\)) are defined. The structural damping ratio is not taken into account, the damping ratio \(\zeta\) is zero for the whole plate. The edges of the plate are clamped. The sound speed in the medium is 344 m/s and the density of the medium is 1.21 kg/m³. The plate is modeled by Shell 181 elements in ANSYS software and the number of elements is 64 × 64 × 1. The first nine natural frequencies and corresponding mode shapes are listed in Table 1. In Table 1, the modes of the rectangular plate are expressed in the form of \((m, n)\), where \(m\) and \(n\) represent the number of anti-nodes (wave peaks or valleys) in the direction of plate length and width in the mode shape, respectively.

A fluid-structure coupling model is established in ANSYS, and the radius of the fluid is 5 m. as shown in Figure 1. For complicated structures, we can just model the part in contact with the fluid of the structure to reduce the computational amount. Looping statement of APDL language is applied to input the eigenvector \(\phi_i\) as velocity conditions on the structure, and then the sound pressure at each position can be calculated. The total radiated acoustic power can be obtained by equation (1).

For mode \((3, 1)\) and \((3, 3)\), the self-radiation efficiency results calculated by elementary radiator approach and software respectively are shown in Figure 2. The mutual-radiation efficiency results of mode \((1, 1)\) with \((3, 1)\), mode \((1, 2)\) with \((3, 2)\) are shown in Figure 3.

In both Figures 2 and 3, between 20–100 Hz, the relative errors of radiation efficiencies calculated by software method and elementary radiator approach are below 11%. It can be seen that for real modes, the calculation method of radiation efficiencies by software is valid. If more accurate results are needed, or the frequency bandwidth needs to be wider, it can be achieved by reducing the size of grids or using BEM software to calculate the radiated acoustic power.

**Example 2**

In order to reduce noise emitted by vibrating structures additional damping treatments such as constraint layer damping or embedded elastomer layers can be used. To save weight and cost, the additional damping is often placed at some critical locations of the structure, which leads to spatially inhomogeneous distribution of damping. This inhomogeneous distribution of structural damping leads to an occurrence of complex vibration modes.\(^{18}\) This example is to verify the method in the condition of complex modes.

The size and material properties of the model, except for damping, are the same as those in example 1. The distribution of inhomogeneous damping is shown in Figure 4, in which the dark area is the location of damping. The damping ratio \(\zeta\) of the material is 0.2 in the dark area. The operation method of applying damping in ANSYS is as follows: based on the model in example 1, first the “NSEL” command is used to select the nodes in the damping area, then the “ESLN” command is used to select the corresponding elements, finally the “EMODIF” command is used to modify the elements’ material (the damping ratio of the new material is 0.2).

For mode \((1, 1)\) and \((2, 1)\), the self-radiation efficiency results calculated by elementary radiator approach and software respectively are shown in Figure 5. The mutual-radiation efficiency results of mode \((1, 1)\) with \((3, 1)\) are shown in Figure 6, and the mutual-radiation efficiency results of mode \((2, 2)\) with \((1, 3)\) are shown in Figure 7.

**Table 1.** The natural frequencies and mode shapes of the plate.

| Mode     | Frequency (Hz) |
|----------|----------------|
| \((1, 1)\) | 65             |
| \((2, 1)\) | 122            |
| \((1, 2)\) | 143            |
| \((2, 2)\) | 196            |
| \((3, 1)\) | 213            |
| \((1, 3)\) | 264            |
| \((3, 2)\) | 283            |
| \((2, 3)\) | 314            |
| \((4, 1)\) | 337            |
In Figures 5–7, the relative errors of radiation efficiencies calculated by software method and elementary radiator approach are below 11%. It can be seen that for complex modes, the calculation method of radiation efficiencies by software is valid.

Figure 8 shows the mode shapes of mode (2, 2) for the plates in both example 1 and example 2, and Figure 9 shows the mode shapes of mode (1, 3). The most intuitive difference between real mode shapes and complex mode shapes is that all elements of the modal vectors are real numbers in real modes, and the phase angle difference between each element is 0° or 180°, all nodes reach their equilibrium positions at the same time, however, as for complex modes, the elements are complex...
numbers and possess different phase angles, and the nodes of mode shapes do not pass through their equilibrium positions at the same time. Mode (2, 2) and mode (1, 3) are different types of modes, and in the case of real modes the mutual-radiation efficiency between them is zero. However, as seen in Figure 7, due to the generation of complex modes caused by inhomogeneous damping, the acoustic coupling between the two modes will exist and the mutual-radiation efficiency is no longer zero in this example. When study the acoustic power of a vibrating plate by modal method, there is no need to consider the coupling between different types of modes for real modes because the mutual-radiation efficiencies of even–even with odd–odd, even–odd, or odd–even order modes are zero, but for complex modes, the coupling between different types of modes is not zero and may not be neglected.

Figure 4. Distribution of inhomogeneous damping.

Figure 5. Self-radiation efficiencies of complex modes: (a) mode (1, 1) and (b) mode (2, 1).

Figure 6. Mutual-radiation efficiency of mode (1, 1) with mode (3, 1): (a) real part, (b) imaginary part and (c) amplitude.
Example 3

In this example we compute the radiation efficiencies of a benchmark model of a submarine. The model is much more complicated than those in example 1 and example 2. The size of the benchmark model is about $62.0 \times 7.5 \times 11.0$ m, and steel material properties ($\rho = 7850$ kg/m$^3$, $E = 210$ GPa, $\nu = 0.3$) are defined. The FEM model of the structure is of free boundary condition. Equation (18) is used to calculate the mutual-radiation efficiencies of the real modes in example 3, the three acoustic powers in the equation should be computed, and the velocity of the structural surface is used as the boundary condition when calculating each acoustic power by BEM. The specific operation process is as follows: First, the mode shapes of the structure are calculated by FEM, then the numerical values needed are applied to each node of the FEM model.
by looping statement in ANSYS. After the FEM model is imported into BEM software, BEM software can read the node information in the FEM model, convert the node information to the velocities of acoustic elements, then use the velocities as the boundary condition of BEM. Its shape is shown in Figure 10. The vibration modes under fluid loading are calculated by FEM, the radiated acoustic powers are calculated by BEM, then the mutual-radiation efficiencies of the modes are calculated by the method presented in this paper. The selected two vibration modes are shown in Figure 11, the self-radiation efficiencies of the modes and the mutual-radiation efficiencies between the modes are shown in Figure 12.

In Figure 12 we can see that the self- and mutual-radiation efficiencies are of the same order of magnitude, which indicates that there is an obvious coupling of the two modes when radiating sound power. The example also shows that it is feasible to calculate the mutual-radiation efficiencies of complicated structures by the method presented in this note.

**Discussion**

In order to better understand the effect of the coupling between modes on the radiated sound power. First, we take the plate in example 1 as the study object, and assume that the damping ratio of the material is 0.01. We can get the actual modal coordinates of forced vibration by applying an excitation on the structure, and then analyze the results. Set the center of the plate as the origin of coordinates, apply a vertical force $F_0=1$ N at $(-0.087, -0.0762)$, calculate the first 24 real modes and
their modal coordinates at each frequency point. Figure 13 shows the radiated sound power between 200 Hz and 600 Hz with and without the coupling between vibration modes considered respectively, where 
\[ W = \sum_{i=1}^{N} \sum_{j=1}^{N} W_{ij} \] and 
\[ W_{vd} = \sum_{i=1}^{N} W_{ii}. \]

In Figure 13 we can find the coupling effect of modes may increase or decrease the total radiated acoustic power. The difference between \( W \) and \( W_{vd} \) is more than 20 dB at 242 Hz. The difference is about 3 dB at the peak of 557 Hz, which means the coupling effect should not always be neglected at resonant frequencies. This result is similar as that in Reference 10. \( W \) and \( W_{vd} \) are different at the peak of 557 Hz, because the force mainly leads to the vibration of mode (3, 4), but the resonant frequency of mode (5, 2) is near that of mode (3, 4), and the two modes are of similar type so the coupling exists, which influences the total acoustic power. As a result the interaction between modes should not be ignored when calculating the acoustic power at 557 Hz.

Furthermore, for complicated structures of large size, the interval between their natural frequencies is small, it is more likely that the two modes with close frequencies have strong acoustic radiation coupling. Therefore, the influence of the coupling effect between modes at the resonance frequencies may not be ignored.

In order to better understand the mutual-radiation efficiency of complex modes, we see the plate in example 2. We assume that, at 50 Hz, the modal coordinate of mode (1, 1) remains unchanged (\( a_1 = 1 \)). Assume the modal coordinate of mode (3, 1) has a unit amplitude, and its phase angle changes from 0° to 360° (\( a_2 = \cos \theta + i \sin \theta \)). Use equation (15) to calculate the sound power caused by the interaction of the two modes, and Figure 14 shows how \( W_m \) changes with \( \theta \).
From Figure 14 we can see that, when the structure is forced to vibrate and radiate sound, the coupling between vibration modes may increase or decrease the total radiated acoustic power, and the actual influence is greatly related to the specific values of modal coordinates $a_1$ and $a_2$. According to equations (11) and (15), then considering the algorithm for complex numbers that $|a_1^* M_{12} a_2| = |a_1^*| |M_{12}| |a_2|$, we know the physical meaning of $|\sigma_{12}|$ is the maximum value of $W_m/(\rho_0 c S_T)$ when the modal coordinates $a_1$ and $a_2$ are both of unit amplitude (their phase angle may varies from 0° to 360°). Therefore, in the case of complex modes, it is reasonable to use the amplitude of mutual-radiation efficiency as the index to measure the acoustic coupling ability between different modes. Then, compare the amplitudes of mutual-radiation efficiencies in Figures 6 and 7, we know that the acoustic coupling ability of mode (1, 1) with (3, 1) is stronger than that of mode (2, 2) with (1, 3).

Conclusions

In this note, the meaning of the mutual-radiation efficiencies in the condition of complex modes is discussed. A method for estimating the mutual-radiation efficiency between vibration modes is presented, which can be well combined with FEM and BEM software to calculate the efficiencies of complicated structures in practice. Examples are given to validate the method.

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