LOCAL-GLOBAL KNOWLEDGE DISTILLATION IN HETEROGENEOUS FEDERATED LEARNING WITH NON-IID DATA

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ABSTRACT

Federated learning enables multiple clients to collaboratively learn a global model by periodically aggregating the clients’ models without transferring the local data. However, due to the heterogeneity of the system and data, many approaches suffer from the “client-drift” issue that could significantly slow down the convergence of the global model training. As clients perform local updates on heterogeneous data through heterogeneous systems, their local models drift apart. To tackle this issue, one intuitive idea is to guide the local model training by the global teachers, i.e., past global models, where each client learns the global knowledge from past global models via adaptive knowledge distillation techniques. Coming from these insights, we propose a novel approach for heterogeneous federated learning, namely FEDGKD, which fuses the knowledge from historical global models for local training to alleviate the “client-drift” issue. In this paper, we evaluate FEDGKD with extensive experiments on various CV/NLP datasets (i.e., CIFAR-10/100, Tiny-ImageNet, AG News, SST5) and different heterogeneous settings. The proposed method is guaranteed to converge under common assumptions, and achieves superior empirical accuracy in fewer communication runs than five state-of-the-art methods.

1 INTRODUCTION

Clients collaboratively learn a global model without transferring their local data in federated learning (FL). The data distribution on clients are often non-IID (independent and identically distributed) in practice, also known as data heterogeneity, which is a challenge in FL and has drawn much attention in recent studies (Karimireddy et al., 2019; Wang et al., 2020; 2021). Moreover, heterogeneity can be caused by the empirical sampling size of clients data, the joint data-label distribution, communication efficiency and capacities of computation, and many others (Kairouz et al., 2019). Since the local data on each client are not sampled from the global joint distribution of all clients (Li et al., 2019), the local objectives are different and may not share common minimizers. Even every communication starts from the same global model, clients’ local models will drift towards the minima of their local objectives, and the aggregated global model may not be the optimum of the global objective. Such “client-drift” phenomenon not only degrades performance, but also increase the number of communication rounds (Karimireddy et al., 2019). Thus, it is desirable to explicitly handle the heterogeneity during training.
Table 1: Comparison of federated learning approaches.

| Method          | Add. Inf. Sharing | Proxy Data | Model Modification |
|-----------------|-------------------|------------|--------------------|
| FedAvg (McMahan et al., 2017) | x                 | x          | x                  |
| FedProx (Li et al., 2018)      | x                 | x          | x                  |
| FedDistill (Seo et al., 2020)  | ✓                 | x          | x                  |
| FedGen (Zhu et al., 2021)      | ✓                 | x          | ✓                  |
| FedDF (Lin et al., 2020)       | x                 | ✓          | x                  |
| MOON (Li et al., 2021)         | x                 | x          | ✓                  |
| FEDGKD (Ours)                | x                 | x          | x                  |

In light of the client-drift problem, many efforts have been made mainly from two aspects. (1) One focuses on using additional data information to address the model drift issue induced by non-IID data. As illustrated in Tab. 1, FedDistill (Seo et al., 2020) shares local output logits on training dataset, and FedGen (Zhu et al., 2021) shares local label count information. While the additional information sharing methods may face the risk of privacy leakage. FedDF (Lin et al., 2020) requires additional proxy data in server for ensemble distillation. Though the efficacy of model aggregation is improved, the additional proxy dataset may not always be available. And the inherent heterogeneity among local models is not fully addressed by only refining the global model, which may affect the quality of the knowledge ensemble, especially when the data distribution shift exists (Khoussainov et al., 2005). (2) Another line of work aims at local-side regularization. FedProx (Li et al., 2018) adds a proximal term in the local objective, and SCAFFOLD (Karimireddy et al., 2019) uses control variate to correct the client-drift. FedDyn (Acar et al., 2020) proposes a dynamic regularization to ensure the local optima is consistent. By injecting a projection head to the model, MOON (Li et al., 2021) uses the model-level contrastive learning method to correct the local training of individual clients. In this paper, we try to use knowledge distillation technique to solve the client-drift problem, in which does not need to share additional information, use additional proxy data, or modify the model structure.

Motivated by the aforementioned problems, we propose a novel ensemble-based global knowledge distillation method, named FEDGKD, which fuses the knowledge from past global models to tackle the client drift in training. Specifically, FEDGKD combines multiple classifiers (partial knowledge) to form a compact representation of global knowledge and transfer it to clients, guiding local model training. A knowledge distillation loss based on compact global knowledge representation is imposed on each client to mitigate the client-drift issue. Overall, the primary contributions of this paper are as follows.

- We introduce an ensemble-based knowledge distillation technique to transfer the information of historical global models to local model training for preventing over-biased local model training on non-IID data distribution.
- We provide a generalized and simple method for federated knowledge distillation, which does not require additional information sharing, proxy data, or model modification. The communication cost and training overhead are at the same complexity level as that of FedAvg. Moreover, the proposed method is compatible with many privacy protection methods like differential privacy in federated learning.
- We use extensive experiments and analysis on various datasets (i.e. CIFAR-10/100, Tiny-ImageNet, Ag News and SST-5) and settings (i.e. different level of non-IID data distribution and participation ratio) to validate the effectiveness of the proposed FEDGKD, and compare FEDGKD with several state-of-the-art methods.

2 BACKGROUND AND PRELIMINARIES

**Federated Learning** (FL) is a communication-efficient distributed learning framework where a subset of clients performs local training before aggregation (Konečný et al., 2016; McMahan et al., 2017), which can train a machine learning model without sharing clients’ private data (Bonawitz et al., 2017), therefore the data privacy will be preserved. In FL, one critical problem is on handling the unbalanced, non-independent and identically distributed (non-IID) data, which are very common in the real world (Zhao et al., 2018; Sattler et al., 2019; Li et al., 2019). At the application level,
FL has been applied to a wide range of real applications, such as biomedical [Brisimi et al., 2018], healthcare [Xu et al., 2021], finance [Yang et al., 2019], and smart manufacturing [Hao et al., 2019].

Suppose there are $N$ clients, denoted as $C_1, \ldots, C_N$. Client $C_k$ stores a local dataset $D_k$ with a distribution $D_k$. In general, FL aims to learn a global model weight $w$ over the dataset $D = \bigcup\{D_k\}_{k=1}^{n_K}$, where $n_K$ is the number of participants in each training round. The objective of FL is to solve the optimization problem [McMahan et al., 2017]:

$$\min_{w \in \mathbb{R}^d} f(w) := \sum_{k=1}^{K} p_k F_k(w), \quad (1)$$

where $p_k = n_k / \sum_{k=1}^{K} n_k$, $n_k = |D_k|$ and $F_k(w) = \frac{1}{n_k} \sum_{i=1}^{n_k} \mathcal{L}(h_k(w; x_{ki}), y_{ki})$ is the local objective function at the $k$-th client. Here, $h_k : \mathbb{R}^d \times \mathbb{R}^d \rightarrow \mathbb{R}^c$ represents the local model and $\mathcal{L} : \mathbb{R}^c \times \mathbb{R}^c \rightarrow \mathbb{R}$ is the loss function. Moreover, $(x_{ki}, y_{ki})$ corresponds to a sample point from the $k$-th client for all $i \in [n_k]$ and $k \in [K]$.

**Knowledge Distillation (KD)** is referred as teacher-student paradigm that a cumbersome but powerful teacher model transfers its knowledge through distillation to a lightweight student model [Bucilua et al., 2006]. KD aims to minimize the discrepancy between the logits outputs from the teacher model $w^T$ and the student model $w^S$ with a dataset $D$ [Hinton et al., 2015]. The discrepancy could be measured by Kullback-Leibler divergence:

$$\min_{w^S} \mathbb{E}_{x \sim D}[h(w^T; x)||h(w^S; x)] .$$

KD has shown great success in various FL tasks. First, many studies focus on data heterogeneity (non-IID) problem. FedDistill [Seo et al., 2020] performs KD to refine the local training by sharing local logits. FedGen [Zhu et al., 2021] learns a global generator to aggregate the local information and distills global knowledge to users. FedDF [Lin et al., 2020] uses the averaged logits of local models on proxy data for aggregation. This approach treats the local models as teachers and transfers their knowledge into a student (global) model to improve its generalization performance. Second, some works aim to reduce the communications cost. CFD [Sattler et al., 2020] sends the soft-label predictions on a shared public data set to server to save communication cost. Besides these, FedLSD [Lee et al., 2021] and FedWeIT [Yoon et al., 2021] try to solve continual learning problem in FL. In this paper, we focus on non-IID data scenarios.

In general, previous works require either additional local information [Seo et al., 2020; Zhu et al., 2021] or proxy data [Lin et al., 2020; Sattler et al., 2021; 2020]. However, as discussed in [McMahan et al., 2017; Bonawitz et al., 2017; Li et al., 2018], the partial global sharing of local information violates privacy, and using globally-shared proxy data should be cautiously generated or collected. Comparing with previous methods, our approach does not require any local information sharing or proxy data.

## 3 **LOCAL-GLOBAL KNOWLEDGE DISTILLATION IN HETEROGENEOUS FEDERATED LEARNING**

### 3.1 Motivation

FedGKD is motivated by an intuitive idea: the global model is able to extract a better feature representation than the model trained on a skewed subset of clients. More precisely, under non-IID scenarios, local datasets across clients are nonidentical distributed, so local data fail to represent the overall global distribution. For example, given a model trained on airplane and bike images, we cannot expect the features learned by the model to recognize birds and dogs. Therefore, for non-IID data, we should control the drift and bridge the gap between the representations learned by the local model and the global model. KD has been verified to be a powerful knowledge transferring method [Hinton et al., 2015]. Additionally, the ensemble [Polyak & Juditsky, 1992] of multiple historical global models can further enhance the power of global knowledge to handle the local model drift problem by providing a more comprehensive view of the global data distribution.
3.2 **FedGKD**

Our core target is to reduce local model drift through global model distillation. An overview of this procedure is illustrated in Fig. 1. We first introduce a base method that uses the latest global model to guide local training. Then we extend the method to use multiple historical global models for guidance.

**Global Knowledge Distillation** FedGKD is proposed for transferring the global knowledge to local model training, where each client’s data has different data distribution. Due to the nature of non-IID data distribution, the local model trained may be biased towards its own dataset, i.e., over-fitting its local dataset, which causes the complexity and difficulty of global aggregation. In order to mitigate the data and information gaps among local models, a novel idea stems from the knowledge distillation technique that regularizes local training with global model output. Suppose the \( w_{t} \) is the parameter of the global model at round \( t \), we formulate the local objective of FedGKD for the \( k \)th client at \( t + 1 \) round as

\[
\min_w F_k(w) + \gamma \sum_{i=1}^{n_k} \text{CE loss} + \sum_{i=1}^{n_k} \text{KL loss},
\]

where \( \gamma > 0 \).

**Historical Global Knowledge Distillation** Motivated by [Xu et al., 2020], we further utilize the averaged parameters of historical global models for ensemble. We define \( \overline{w}_t = \frac{1}{M} \sum_{m=1}^{M} w_{t-m+1} \) as the averaged weight of the global, where \( M \) is the buffer size. In round \( t + 1 \), \( M \) latest global models are averaged to a fused model \( \overline{w}_t \) on server and send to (sampled) clients. As shown in Fig. 1, the local model \( w_{t+1}^k \) is trained under the guidance of the latest \( M \) round global models to avoid local bias training. The modified local objective function for the \( k \)th client is defined as

\[
\min_w F_k(w) + \gamma \sum_{i=1}^{n_k} \text{KL loss},
\]

We also develop the FedGKD-VOTE for regularizing the local models with past \( M \) models output logits and the local objective is given as

\[
\min_w F_k(w) + \sum_{m=1}^{M} \gamma_m \sum_{i=1}^{n_k} \text{KL loss},
\]
Algorithm 1 FedGKD: Global Knowledge Distillation in Federated Learning

1: \textbf{Notations.} total number of clients \( K \), server \( S \), total communication rounds \( T \), local epochs \( E \), fraction of participating clients \( C \), learning rate \( \eta \), buffer size \( M \), \( B \) is a set holding client’s data sliced into batches of size \( B \).

2: //On the server side

3: \textbf{procedure} SERVER\_EXECUTION \hspace{1cm} \textcircled{\texttt{Server Model Aggregation}}

4: \hspace{1cm} Initial global weight \( w_0 \)

5: \hspace{1cm} for each communication round \( t = 1, \ldots, T \) do

6: \hspace{1cm} \hspace{1cm} \( K^t \leftarrow \) random sample a set of \( C \cdot K \) clients

7: \hspace{1cm} \hspace{1cm} if FedGKD-VOTE then

8: \hspace{1cm} \hspace{1cm} \hspace{1cm} Send the global weight \( w_t, \ldots, w_{t-M+1} \) to the selected clients

9: \hspace{1cm} \hspace{1cm} else

10: \hspace{1cm} \hspace{1cm} \hspace{1cm} Send the global weight \( w_t \) and ensembled global weight \( \overline{w}_T \) to the selected clients

11: \hspace{1cm} Buffer the global model with \( M \) size

12: \hspace{1cm} for each client \( k \in K_t \) in parallel do

13: \hspace{1cm} \hspace{1cm} \( w_{k+1}^t \leftarrow \text{CLIENT\_UPDATE}(k, w_t) \)

14: \hspace{1cm} \hspace{1cm} \( w_{t+1} \leftarrow \sum_{k=1}^{K} \frac{M}{K} w_{k+1}^t \)

15: //On the data owners side

16: \textbf{procedure} CLIENT\_UPDATE\( (k, w_t) \) \hspace{1cm} \textcircled{\texttt{Local Model Training}}

17: \hspace{1cm} \hspace{1cm} \( w \leftarrow w_t \)

18: \hspace{1cm} \hspace{1cm} for each local epoch \( i = 1, \ldots, E \) do

19: \hspace{1cm} \hspace{1cm} \hspace{1cm} for each batch \( b \in B \) do

20: \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \( w \leftarrow w - \eta \cdot \nabla L(w, b) \) \hspace{1cm} \textcircled{\texttt{Update Local Weights via Eq[4] or Eq[5]}}

21: \textbf{Return} \( w \) back to server

where \( \gamma_1, \ldots, \gamma_M \) are the distillation coefficients for different models. Note that in numerical experiments, we found that using the global model from the last round can already achieve a good performance, which is also easier to set up the hyper-parameter.

In Algorithm[1] we give a detailed description for FedGKD and FedGKD-VOTE. Both of FedGKD and FedGKD-VOTE need to buffer the global models on the server, and FedGKD average the global models weights directly so that only \( \overline{w}_T \) and \( w_t \) should be sent to the clients. The communication cost of FedGKD is twice compared to FedAvg if \( M > 1 \) and the same as FedAvg if \( M = 1 \). For FedGKD-VOTE, the communication cost would be \( M \) times. Consider the communication rounds \( T \gg M \) in realistic, the communication cost of FedGKD-VOTE is still \( O(T) \). Specially, for NLP fine-tuning tasks, we’d suggest set \( M \) as 1.

4 FedGKD Analysis

In this section, we provide multiple perspectives to understand our proposed approach.

4.1 Convergence Analysis

Before formally state the convergence result, we introduce following notations. For any positive integer \( a \geq 0 \), denote \( [a] = \{1, 2, \ldots, a\} \). For any vector \( v \), \( [v]_j \) represents its \( j \)-th coordinate. To perform convergence analysis, assumptions made on the properties of the client functions and the Algorithm are summarized in Assumption[1] and Assumption[2] respectively.

Assumption 1.

1. (Lower-bounded eigenvalue) For all \( k \in [K], F_k(w) \) is \( L \)-smooth, i.e., \( \| \nabla F_k(w) - \nabla F_k(w') \| \leq L \| w - w' \| \) for some \( L > 0 \) and all \( (w, w') \in \mathbb{R}^d \times \mathbb{R}^d \). Furthermore, we assume the minimal eigenvalue of the Hessian of the client loss function \( \nabla^2 f_k(w) \) is uniformly bounded below by a constant \( \lambda_{\text{min}} \in \mathbb{R} \).

2. (Bounded dissimilarity) For all \( k \in [K] \) and any \( w \in \mathbb{R}^d \), \( E_k[\| \nabla F_k(w) \|^2] \leq B^2 \| \nabla f(w) \|^2 \), where the expectation is taken with respect to the client index \( k \).
3. For all $k \in [K]$, $h_k$ outputs a probabilistic vector, i.e., $[h_k(;.;.);]_j \geq 0$ and $\sum_{j=1}^{C}[h_k(;.;.);]_j = 1$ for all $j \in [C]$. Furthermore, we assume for any data point $x \in \mathbb{R}^d$, $w \in \mathbb{R}^d$, and $k \in [K]$, there exits a constant $\delta > 0$ such that $\min_{j \in \{1,..,C\}}[h_k(w, x);]_j \geq \delta > 0$. And there exits a constant $L_h > 0$, $|h_k(w, x) - h_k(w', x)| \leq L_h \|w - w'\|$ for any $(w, w') \in \mathbb{R}^d \times \mathbb{R}^d$.

We remark that Assumption 1.1 and Assumption 1.2 are also made in (Li et al., 2018). Assumption 1.3 is not stringent. For example, consider $h_k$ being a neural network for a classification task, and let the last layer being softmax, which is 1-Lipschitz. As long as other layers (see Virmaux & Scaman, 2018) for examples) are Lipschitz, then by the fact that finite composition of Lipschitz continuous function gives the Lipschitz function, $h_k$ is also Lipschitz. Indeed $F_k$ is already assumed to be L-smooth, which implies $h_k$ is locally Lipschitz and we only require $h_k$ to be Lipschitz along the iterate sequence generated by the algorithm.

**Assumption 2.** At the $t$-th round, the $k$-th client (if selected) solves the optimization problem $w_{k,t+1}^{*} \approx \arg \min_{w \in \mathbb{R}^d} m(w; w_t)$ approximately with

$$m(w; w_t) := F_k(w) + \frac{\gamma}{2n_k} \sum_{i=1}^{n_k} KL[h_k(w_t, x_{ki}) || h_k(w, x_{ki})],$$

such that

$$\left\| \nabla F_k(w_{k,t+1}^{*}) + \frac{\gamma L_h}{\delta} (w_{k,t+1}^{*} - w^*) \right\| \leq \eta \left\| \nabla F_k(w_t) \right\|,$$

where $\eta \in [0, 1]$ and $\gamma > 0$.

The Lemma 4 in the supplementary justifies the well-posedness of the Assumption 2. The following theorem gives the rate of convergence to a stationary point.

**Theorem 3.** Let Assumption 1 and Assumption 2 hold. Assume for each round, a subset of $S_t$ clients are selected, with $|S_t| = S$ and the $k$-th client is selected with the probability $p_k$. If $\gamma, \eta, \text{ and } S$ are chosen to satisfy:

1. $\kappa := \frac{\gamma L_h}{\delta} + \lambda_{\min} > 0$;
2. $\rho > 0$, where

$$\rho = \frac{\delta}{\gamma L_h} \left( 1 - \eta B - \frac{LB(1 + \eta)}{\kappa} \right) - \frac{1}{\kappa} \left( \sqrt{2B(1 + \eta)} + \frac{L(1 + \eta)^2 B^2}{2\kappa} + \frac{(2\sqrt{2S} + 2)LB^2(1 + \eta)^2}{\kappa S} \right),$$

then after $T$ rounds,

$$\min_{t \in [T]} \mathbb{E}[\|\nabla f(w^t)\|] \leq \frac{f(w^0) - f(w^*)}{\rho T}.$$

**4.2 Global Knowledge Distillation for Biased Local Model Regularization**

As shown in Fig. 2a, we compare the local model and the global model of FedAvg, FedProx, and FedGKD at the last round trained on CIFAR-10 by visualizing the penultimate layer representations of the test dataset. As the features are clustered by classes in T-SNE, a good model(classifier) can give more clear separation in the feature points. We notice that in Fig. 2a the local model can barely classify the entire classes due to the non-IID data distribution. And the global model is affected by the local model as shown in Fig. 2a since the boundary of different classes is vague due to the model drift. A clearer boundary can be observed in FedGKD as shown in Fig. 2b, where the improvement is brought by the knowledge distillation from global ensemble model, and we observe the improvement of local models leads to a better feature representation the global model of FedGKD as shown in Fig. 2c.
Figure 2: T-SNE visualizations of feature of model on CIFAR-10 test dataset (Model of 100 communication rounds under $\alpha=0.1$ setting). The global model accuracy of FedAvg, FedProx and FedGKD is 39.99%, 64.08% and 65.98%, test accuracy of the local model is 16.02%, 32.11% and 44.73%, respectively.

Figure 3: Top: Visualization of the non-IID ness on CIFAR-10 dataset. Bottom: Performance of FedAvg and FedGKD (M=1) with ResNet-8 on CIFAR-10 under different $\alpha$ and participation ratio setting.

5 EXPERIMENTS

5.1 EXPERIMENTAL SETUP

Datasets. We compare with previous FL methods on both CV and NLP tasks and use architectures of ResNet (He et al., 2016) and DistillBERT (Sanh et al., 2019) respectively. For CIFAR-10 and CIFAR-100 (Krizhevsky, 2009), we conduct image classification tasks using ResNet-8. For Tiny-ImageNet dataset, we train the ResNet-50. For all these CV tasks, we use 90% of the data as the training dataset. For NLP tasks, we perform text classification tasks on a 5-class sentiment classification dataset SST-5 and a 4-class news classification dataset AG News. We use 50% and 100% of the data respectively as the training datasets. The datasets correspond to each task are summarized in Tab. 2.

Following prior work (Lin et al., 2020; Hsu et al., 2019), we use the Dirichlet distribution $\text{Dir}(\alpha)$ to sample disjoint non-IID client training data, where $\alpha$ denotes the concentration parameter, and the smaller $\alpha$ indicates higher data heterogeneity. Fig. 2 visualizes the sampling results on CIFAR-10 in our experiments.
Table 2: Statistics of training datasets.

| Dataset     | Task     | Train Data Size | Class | clients |
|-------------|----------|-----------------|-------|---------|
| CIFAR-10    | image    | 45,000          | 10    | 20      |
| CIFAR-100   | image    | 45,000          | 100   | 20      |
| Tiny-Imagenet | image   | 90,000          | 200   | 20      |
| SST-5       | text     | 8,544           | 5     | 10      |
| AG News     | text     | 60,000          | 4     | 20      |

Implementation Details  
The proposed FedGKD and baselines are all implemented in PyTorch and evaluated on a Linux server with four V100 GPUs. MPI is used as communication backend to support distributed computing.

Considering Batch Normalization (BN) fails on heterogeneous training data (Hsieh et al., 2020) due to the statistics of running mean and variance for the local data, we replace BN by Group Normalization (GN) to produce stabler results and alleviate some of the quality loss brought by BN. It is worth noting that on tiny-imagenet, if GN is used in ResNet-50 and a 2-layer MLP is added, the model will not converge. To compare with MOON (Li et al., 2021), we use the BN on ResNet-50 in MOON and FedGKD +.

For all the CV tasks, we set the channels of each group as 16. The optimizer of client training used is SGD, we tune the learning rate in {0.1, 0.05, 0.01} for FedAvg and set the learning rate as 0.05, weight decay is set 1e-5, momentum is 0.9. We run 100 global communication rounds on CIFAR-10/100 datasets, and 30 communication rounds on Tiny-Imagenet dataset, the local training epochs is set as 20, and the default participant ratio is set as C=0.2.

For all the NLP fine-tuning tasks, Adam is used as local optimizer, learning rate is set 1e-5, weight decay is set 0. Considering the fine-tuning process could be done in a few epochs, the communication round is set 10, local epoch is set 1 on AG News, 3 on SST-5. The clients participated each round is set 4.

The training batch size on all the tasks is set B=64. We run all the methods three trials and report the mean and standard derivation.

Parameter Setting  
As stated in MOON (Li et al., 2021), the projection head is necessary for the method, for fair comparison, we add the experiments with projection head. The projection layer setting is same as SimCLR (Chen et al., 2020), 2-layer MLP with output dimension 256. To compare with MOON, we set the coefficient \(\mu=5,5,1,0.1\) on CIFAR-10,CIFAR-100, Tiny-imagenet and NLP datasets. As for the temperature, we set it 0.5 as reported in (Li et al., 2021).

To compare with FedProx, we set \(\mu=0.01,0.001,0.001,0.001\) on CIFAR-10,CIFAR-100, Tiny-imagenet and NLP datasets.

The distillation temperature of FedDistill +, and for the distillation coefficient, we set it as 0.1 for all the datasets.

To compare with FedGen, we follow the global generator training setting on server as (Zhu et al., 2021), and tune the local regularization coefficient from \{0.1, 1, 10\}. The hidden dimension of 2-layer MLP generator is set 512,2048,768 for ResNet-8, ResNet-50, DistilBERT.

For FedGKD, we set \(\gamma=0.2\) for ResNet-8 and DistilBERT, \(\gamma=0.1\) for ResNet-50. For FedGKD-VOTE, the \(\gamma_1 \ldots \gamma_M\) are determined by validation performance.

For FedGKD-VOTE, we set the default buffer size as default 5 on CV tasks, 3 on NLP tasks, the parameters \(\gamma_1 \ldots \gamma_M\) are set by validation performance follows the calculation: \(\frac{\gamma}{\gamma_i} = \frac{\lambda e^{-\beta L_i}}{\sum_{i=1}^{M} e^{-\beta L_i}}\), where \(L_i\) is the validation loss of the \(i\)th global model in the buffer, \(\beta\) denotes the temperature and \(\lambda\) is the coefficient to control the scale, we set \(\beta\) as \(\frac{1}{M}\) for simplicity and \(\lambda\) is set 0.1.

Evaluation Methods  
We compare our proposed FedGKD with the following state-of-the-art approaches.

- **FedAvg**: FedAvg (McMahan et al., 2017) is a classic federated learning algorithm, which directly uses averaging as aggregation method.
• **FedProx**: FedProx [Li et al., 2018] improves the local objective based on FedAvg. It adds the regularization of proximal term for local training.

• **MOON**: MOON [Li et al., 2021] learns from contrastive learning (Chen et al., 2020), injecting the projection layer for the original model. The local previous model features are seen as negative and global model features are seen positive samples.

• **FedDistill**: FedDistill [Seo et al., 2020] shares the logits of user-data and adds the distillation regularization term for clients. We further develop it with model parameters sharing and name it FedDistill⁺.

• **FedGen**: FedGen [Zhu et al., 2021] shares the local label count information to train a global generator and regularizes the local training with this light-weighted generator.

• **FEDGKD**: The proposed method in Fig. 4 sends the ensemble of global models to participants and updates local weights via optimizing Eq. 4.

• **FEDGKD-VOTE**: The server sends M models to participants and updates local weights via optimizing Eq. [5]

• **FEDGKD⁺**: To compare with MOON, we add the projection layer for FEDGKD and develop it as FEDGKD⁺.

### Table 3: Top-1 Test Accuracy overview given different data settings for ResNet on CV Datasets (20 clients with C=0.2, 100 communication rounds, buffer length for FEDGKD-VOTE is set 5). **Bold number denotes the best result and underline denotes the second best result.**

| Dataset          | Setting | FedAvg | FedProx | MOON | FedDistill | FedGen | FEDGKD | FEDGKD-VOTE | FEDGKD⁺ |
|------------------|---------|--------|---------|------|------------|--------|--------|-------------|--------|
| CIFAR-10         | α=1     | 37.38±0.72 | 36.39±0.59 | 78.17±0.81 | 77.96±0.51 | 76.44±0.34 | 79.14±0.32 | 78.80±0.39 | 79.42±0.37 |
|                  | α=0.5   | 75.15±0.44 | 75.02±0.74 | 77.24±0.90 | 75.85±0.63 | 73.62±0.20 | 77.15±0.28 | 77.44±0.60 | 77.38±0.60 |
| CIFAR-100        | α=0.5   | 42.22±0.41 | 41.97±0.56 | 40.36±0.04 | 42.38±0.24 | 39.93±0.58 | 43.82±1.39 | 42.91±0.22 | 40.99±0.41 |
|                  | α=0.1   | 40.42±0.58 | 40.08±0.24 | 35.89±1.12 | 40.97±1.12 | 39.50±0.64 | 42.44±0.32 | 42.34±0.31 | 37.91±0.63 |
| Tiny-ImageNet    | α=1     | 34.81±1.57 | 35.64±1.67 | 42.20±0.60 | 39.99±1.18 | 34.79±1.15 | 37.50±0.43 | 37.79±1.04 | 43.78±1.18 |
|                  | α=0.5   | 32.48±1.59 | 35.41±0.64 | 34.15±1.76 | 34.02±0.73 | 34.47±0.63 | 37.28±0.86 | 37.35±0.65 | 40.56±2.34 |
|                  | α=0.1   | 29.02±0.65 | 31.93±0.85 | 33.58±0.16 | 28.95±0.10 | 32.84±0.99 | 34.64±0.21 | 33.71±1.74 | 35.72±0.71 |

### Table 4: Performance overview given different data settings for DistilBERT (4 participants each round, 10 communication rounds, buffer length for FEDGKD-VOTE is set 3).

| Dataset          | Setting | FedAvg | FedProx | MOON | FedDistill | FedGen | FEDGKD | FEDGKD-VOTE | FEDGKD⁺ |
|------------------|---------|--------|---------|------|------------|--------|--------|-------------|--------|
| AG News          | α=0.1   | 89.19±0.87 | 89.19±0.87 | 89.25±0.48 | 89.35±0.85 | 89.28±0.77 | 89.60±0.44 | 89.04±0.22 | 89.48±0.18 |
| SST-5            | α=0.1   | 40.39±0.50 | 40.42±0.49 | 41.48±0.23 | 41.23±0.03 | 41.00±3.23 | 43.24±1.80 | 41.92±2.05 | 41.23±1.20 |

5.2 **EXPERIMENTAL RESULTS**

**Results Overview** In Fig. [3] we give a brief comparison for FedAvg and FEDGKD under various participation ratio setting, FEDGKD constantly outperforms FedAvg. We compare the top-1 test accuracy of the different methods as reported in Tab. [3] and Tab. [4] the proposed method FEDGKD outperforms previous state-of-the-art methods Under the α = 0.1 (the hyper-parameter for Dirichlet distribution) setting, FEDGKD can outperform FedAvg by 5.6% on Tiny-ImageNet, 1.9% on CIFAR-100, 3.0% on CIFAR-10, 0.4% on AG News, and 2.8% on SST-5. Moreover, we observe FEDGKD brings the smoothness for learning procedure compared with the FedAvg as shown in Fig. [4] and FEDGKD constantly outperforms FedAvg and FedProx in the training procedure. We present the advantage of FEDGKD to mitigate local drifts in Fig. [2] the local model of FedAvg achieves only 16.02% accuracy on CIFAR-10 test dataset, and the global model fails to classify the image features due to the local drifts, the global accuracy falls to 39.99%. FEDGKD improves the test accuracy of the local model by 28.7%, and over 25% on global model test accuracy. FedProx improves the global accuracy to 64.08% but the test accuracy of local model is 12.6% lower than FEDGKD.

**Impact of Participation Ratio** As shown in Tab. [5] we give a more comprehensive description to exhibit the advantage of our proposed method. With the participants number growing, our proposed methods constantly outperform the other methods, FEDGKD shows its greater superiority over other methods when there are fewer participants. For FedGen, it is not trivial to manage the generator training procedure, and it’s even difficult to learn global features when the participation ratio is low. FedDistill⁺ sends the local logits of each client directly to server, treating the averaged logits as global logits, while the quality of global logits also suffers from the low participation ratio. Compared with
Table 5: Comparison of different FL methods in different sampling ratio, we evaluate on CIFAR-10 with ResNet-8, $\alpha=0.5$.

| Method     | C=0.1      | C=0.2      | C=0.3      | C=0.4      |
|------------|------------|------------|------------|------------|
|            | Best       | Final      | Best       | Final      | Best       | Final      |
| FedAvg     | 73.35±0.10 | 73.31±0.35 | 75.15±0.44 | 72.70±0.78 | 74.18±0.21 | 77.74±0.61 | 74.45±0.52 |
| FedProx    | 72.59±0.66 | 72.59±0.66 | 78.02±0.74 | 72.08±0.92 | 76.36±0.64 | 74.55±0.48 | 76.65±0.76 | 74.03±0.61 |
| MOON       | 73.47±0.47 | 72.57±0.51 | 77.24±0.90 | 74.48±1.01 | 77.88±0.26 | 75.00±0.33 | 78.20±0.57 | 75.05±0.44 |
| FedDistill | 73.75±0.35 | 73.51±0.35 | 75.85±0.63 | 73.63±0.26 | 77.31±0.65 | 74.38±0.19 | 78.22±0.45 | 74.92±0.47 |
| FedGen     | 73.39±0.34 | 73.39±0.34 | 73.62±0.20 | 72.27±0.42 | 76.73±0.53 | 73.75±0.35 | 77.05±0.28 | 73.96±0.58 |
| FedGKD     | 74.30±0.96 | 74.30±0.96 | 78.30±0.10 | 78.10±0.77 | 75.45±0.34 | 75.32±0.40 | 75.39±0.67 |
| FedGKD-VOTE| 74.41±0.56 | 74.25±0.58 | 78.44±0.60 | 73.96±0.86 | 77.22±0.51 | 75.74±0.31 | 78.08±0.58 | 75.75±0.75 |
| FedGKD$^+$| 75.16±0.67 | 74.46±0.54 | 79.38±0.80 | 74.55±0.78 | 78.02±0.45 | 74.54±0.42 | 77.28±0.16 | 75.24±0.33 |

Figure 4: Test accuracy of FedAvg, FedProx and FedGKD on CV datasets in different number of communication rounds. From top to bottom, the dataset is CIFAR-10, CIFAR-100, Tiny-ImageNet. From left to right, the $\alpha$ ranges from 0.1 to 1.

the model-modification methods, we find FedGKD$^+$ consistently outperforms MOON. FedGKD and FedGKD-VOTE benefit from the historical global models ensemble and distillation to produce a more accurate result.

Impact of Data Heterogeneity  As shown in Tab. 5, FedGKD, FedGKD-VOTE and FedGKD$^+$ outperform other methods and show their generality in different $\alpha$ setting. FedGKD shows the superiority when the data heterogeneity increases. FedGKD$^+$ benefits from the modification on model structure on CIFAR-10 and Tiny-Imagenet. For FedProx, if the coefficient $\mu$ is set as a small number, the performance is close to FedAvg, otherwise it harms the convergence when the data heterogeneity decreases. While for FedGKD, setting the regularization coefficient as 0.2 is helpful in most cases. FedGen regularizes the local model mainly for the classifier layer based on the light-weighted generator, which limits its performance.

Robustness of the Methods  As shown in Fig. 4, we observed the learning curve of FedGKD constantly outperforms FedAvg and FedProx on CIFAR datasets. FedGKD performs worse than
We conduct the experiments on different buffer length as shown in Tab. 7. For F\textsuperscript{ED}Distill\textsuperscript{+}, the communication cost would be constant regardless of the buffer size $M$, since the ensemble is proceeded on the server. Setting $M = 5$ performs best on $\alpha = 1$ and $\alpha = 0.1$, which are higher than $M = 1$ over 0.5\% on CIFAR-10. For F\textsuperscript{ED}Distill\textsuperscript{VOTE}, we find there is no more benefit by increasing the buffer length to 7, and it does not help significantly when the non-IID-ness is not a major issue or communication rounds are few.

We conduct the experiments on different buffer length for F\textsuperscript{ED}Distill\textsuperscript{+} on CIFAR-10, C=0.2, and find the best buffer length varies under different settings as shown in Tab. 8.

### Choice of Regularizer

In this paper, the KL-divergence is used as the regularizer to control the discrepancy between the local logit and global logit distribution. We also consider the case when the regularization term being modified as means squared error (MSE) defined over the logit, and we tested the MSE regularizer on CIFAR-10, CIFAR-100 and SST-5 under $\alpha = 0.1$ with the buffer size $M = 1$. As shown in Tab. 9, both choices outperform the FedAvg. However, when using the MSE as
Table 7: Performance of different buffer length for FedGKD, $C = 0.2$.

| DataSet | Buffer Length | $\alpha=1$ | $\alpha=0.5$ | $\alpha=0.1$ |
|---------|---------------|------------|--------------|--------------|
| CIFAR-10| 1             | 78.58±0.61 | 77.15±0.28   | 71.73±0.79   |
|         | 3             | 78.63±0.19 | 76.91±0.05   | 72.12±0.41   |
|         | 5             | 79.14±0.72 | 76.54±0.17   | 72.27±0.84   |
|         | 7             | 78.72±0.73 | 77.02±0.43   | 72.24±0.21   |
| CIFAR-100| 1           | 42.77±0.37 | 41.76±0.06   | 36.50±0.12   |
|         | 3             | 42.92±0.19 | 42.33±0.21   | 36.82±0.24   |
|         | 5             | 43.82±1.39 | 42.29±0.30   | 36.83±0.20   |
|         | 7             | 43.15±1.37 | 42.44±0.32   | 36.65±0.64   |

Table 8: Performance of different buffer length for FedGKD-VOTE.

| Buffer Length | $\alpha=1$ | $\alpha=0.5$ | $\alpha=0.1$ |
|---------------|------------|--------------|--------------|
| 1             | 78.58±0.61 | 77.15±0.28   | 71.73±0.79   |
| 3             | 78.68±0.39 | 76.63±0.40   | 72.27±0.05   |
| 5             | 78.88±0.39 | 77.44±0.66   | 72.41±1.70   |
| 7             | 78.56±0.63 | 76.78±0.20   | 71.57±1.43   |

Table 9: Performance of different loss types, under $\alpha = 0.1$, $C = 0.2$ setting, buffer size is set as $M = 1$.

| Loss Type | CIFAR-10 | CIFAR-100 | SST-5   |
|-----------|---------|-----------|--------|
| None      | 69.22±0.91 | 34.91±1.00 | 40.39±0.50 |
| MSE       | 70.12±0.98 | 38.19±0.77 | 43.50±1.68 |
| KL        | 71.73±0.79 | 36.50±0.12 | 43.24±1.80 |

6 Conclusion

In this paper, we propose a novel global knowledge distillation method FedGKD that utilizes the knowledge learnt from past global models to mitigate the client drift issue. FedGKD benefits from the ensemble and knowledge distillation mechanisms to produce a more accurate model. We validate the efficacy of proposed methods on CV/NLP datasets under different non-IID settings. The experimental results show that FedGKD outperforms several state-of-the-art methods and demonstrates the effect of reducing the drift issue.

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7 Proof of Convergence Analysis

Lemma 4. The set \( \{ w \in \mathbb{R}^d | w \text{ satisfies (6)} \} \) is non-empty.

Proof. Note that

\[
m(w; w_t) = F_k(w) + \frac{\gamma}{2n_k} \sum_{i=1}^{n_k} \text{KL}(h_k(w_t, x_{ki}) || h_k(w, x_{ki}))
\]

\[
\leq F_k(w) + \frac{\gamma}{2n_k} \sum_{i=1}^{n_k} \| h_k(w_t, x_{ki}) - h_k(w, x_{ki}) \|^2 \\
\leq F_k(w) + \frac{\gamma L}{2\delta} \| w_t - w \|^2 \leq \hat{m}(w; w_t),
\]

where the first inequality follows from (Klemela, 2009, Lemma 11.6) and the second inequality follows from the Assumption 1.3. Notice that for any approximate solution \( w_{t+1} \) satisfies \( \hat{m}(w_{t+1}; w_t) \leq \hat{m}(w^t; w_t) \), then

\[
m(w_{t+1}; w_t) \leq \hat{m}(w^t; w_t) = m(w^t; w^t),
\]

which implies (6) will be satisfied at least by the minimizer of \( \hat{T}_k(w, w^t) \).

Proof of Theorem 3

Proof. By Assumption 2, for the \( k \)th client,

\[
\nabla F_k(w_{t+1}^k) + \frac{\gamma L}{\delta} (w_{t+1}^k - w^t) + e_{t+1}^k = 0 \\
\| e_{t+1}^k \| \leq \eta \| w^t \|.
\]

Provided \( \gamma, \eta \) and \( S \) are chosen as described, then one can use the same proof in (Li et al., 2018, Theorem 4) to show that

\[
E_{S_t}[f(w_{t+1})] \leq f(w_t) - \rho \| \nabla f(w_t) \|^2.
\]

Then take the total expectation with respect to all randomness and by telescoping, one reaches

\[
\rho \sum_{t=0}^{T-1} E[\| \nabla f(w_t) \|^2] \leq f(w_0) - f(w^*).
\]

Divide \( T \) on both sides, then

\[
\min_{t \in [T]} E[\| \nabla f(w_t) \|] \leq \frac{1}{T} \sum_{t=0}^{T-1} E[\| \nabla f(w_t) \|^2] \leq \frac{f(w_0) - f(w^*)}{\rho T}.
\]

8 Extended Experiments

Toy Example for FedAvg Limitation Illustration Fig.[5] provides a illustration of the limitation in FedAvg. We consider a 4-class classification task with a 3-layer MLP, the optimizer used is Adam. The 2-dim data points are randomly generalized in (-4,4). We notice that for FedAvg, the model overfits the local data and results a bias for the global model, while adding the global models distillation term would relieve the overfitting phenomenon.
Figure 5: The decision bound for FedGKD and FedAvg on toy examples.