The estimation of the neutrino flux produced by pep reactions in the Sun

B F Irgaziev¹, V B Belyaev² and Jameel-Un Nabi¹

¹ GIK Institute of Engineering Sciences and Technology, Topi, Pakistan
² Joint Institute for Nuclear Research, Dubna, Russia

E-mail: irdagiev@yahoo.com

Received 3 September 2013, revised 22 April 2014
Accepted for publication 24 April 2014
Published 11 July 2014

Abstract
The result of the experiment performed to measure the solar neutrino flux at one AU produced by the \( \nu^{++} \rightarrow + + e^{-} \) (pep) reaction was announced for the first time in 2012 by the Borexino collaboration. This neutrino flux was significantly greater than the flux predicted by Bahcall and May, who used two-body approaches for the calculation of this reaction. We have used the three-body model for the proton-proton-electron system in the continuous spectrum of energy to determine the rate of the pep reaction, and have estimated the neutrino flux. Our result for the neutrino flux is 25–40% more than the value predicted by Bachall et al and depends on the shape of the nucleon–nucleon (NN) potential. Moreover, the calculated flux lies within the confidence interval of the experimental data in the case of purely attractive potentials as well as when the potential is repulsive at small distances between nucleons.

Keywords: pep reactions, solar neutrino flux, solar models, three body problem, NN potential

(Some figures may appear in colour only in the online journal)

1. Introduction
The measurement of the neutrino flux from the Sun presents us with the possibility of solving several important problems. A careful study of the solar neutrino flux penetrating the Earth provides us with the possibility of understanding many characteristics of the neutrino, including the effect of neutrino oscillation, an estimation of the mixing angle and neutrino mass. The neutrino flux, along with its spectral properties, gives us a useful insight into nuclear reactions which cannot otherwise be observed under terrestrial conditions. The main reaction, called the pep reaction, going on in the interior of the Sun and determining the luminosity, is

\[ p + p \rightarrow d + e^{+} + \nu_{e}. \] (1)

Furthermore, as underlined by John N Bahcall in his series of papers and his book [1–6], solar neutrinos bring the signals from processes in the core of the star, and allow one to make comparisons of the data with solar structure models. Unfortunately it is impossible to directly measure the neutrino flux from this reaction in the laboratory. The reaction

\[ p + p + e^{-} \rightarrow d + \nu_{e}, \] (2)

which is called the pep reaction, stimulates interest because an emitted neutrino is monoenergetic even though pep reaction plays no essential role in hydrogen burning in the Sun. The energy of a neutrino from this reaction is \( E_{\nu} = 1.44 \text{ MeV} \). The parameters of the standard solar model can be estimated from the measurement of the neutrino flux coming from the pep reaction.

Bahcall and May [7] used the two-body approach for considering the three-body pep reaction [8]. In the abstract of [9] the authors presented some qualitative estimation of the three-body effect for the pep reaction and came to the conclusion that the previous conventional estimation of the pep solar neutrino flux could have been underestimated. Therefore the solution of the three-body equation without applying two-body approximation was required.

The purpose of this work is to treat the initial state of the reaction (2) as a purely three-body state. It is a well known fact that low energy nucleon-nucleon data is insensitive to the form of NN potential in the two-body case. However, the three-body system is sensitive to the type of potential. The
idea behind this work is to check the sensitivity of neutrino flux to a used $NN$ potential, within different standard solar models, and most importantly, to extend our previous results [10] by using a realistic $NN$ potential. It was more than fifty years from the start of the solar neutrino problem until the announcement of the first experimental observation of process (2) [11]. Further, a three-body description of the initial state in process (2) with different types of $NN$ potential was not available in the literature. These were the main motivations for presenting our current calculation.

In section 2 we briefly introduce our weak Hamiltonian and the two types of nucleon-nucleon potentials. We determine the probability and the astrophysical $S_{\text{pep}}$ factor of the reaction (2) in section 3. The reaction rate and calculation of solar neutrino flux is presented in section 4. We finally conclude and summarize our calculation in section 5.

In this paper, we applied the same notation for the quantities as in our previous article [10].

2. Inputs

Electron capture by nuclei emitting electron neutrinos is considered by the application of the effective Hamiltonian (nonrelativistic) describing weak interaction presented in [12]. Taking into account the smallness of the neutrino energy in the reaction $\text{pep} \rightarrow d + \nu_e$, as well as the ansatz that the transition satisfies the Gamow–Teller selection rule, we reduce the weak transition operator to the form:

\[ H_w = \tau^{(\pm)} G_\alpha \sum_{i=1}^{3} t_i^{(\pm)} \sigma \cdot \sigma_i \delta (r-r_i), \tag{3} \]

where $\sigma$ and $\sigma_i$ are spin operators for the lepton and $i$th nucleon; $r$ and $r_i$ are the space coordinates of the lepton and an $i$th nucleon; $\tau^{(\pm)}$, $t_i^{(\pm)}$ are the isobaric-spin operators transforming a lepton electron state into a lepton neutrino state and an $i$th nucleon proton state into an $i$th nucleon neutron state, respectively. We used axial vector coupling constant $G_\alpha$ equal to $G_\alpha/(\hbar c)^3 = -1.454 \times 10^{-11}$ GeV$^{-2}$ [13].

In our previous calculation [10] we used the simplest potentials (Gauss and Yukawa types) which are purely attractive in nature. In this work we apply one more simple exponential potential and, in addition, a realistic $NN$ potential (Malfliet-Tjon potential [14]) with repulsion at small distance, to check sensitivity of the flux from the reaction (2) to the shape of potential.

The fitted parameters to these potentials listed below correctly determine the low energy $NN$ scattering data and binding energy of the deuteron.

The parameters of the exponential potential

\[ V^N (r) = -V_0 \exp (-r/R_0) \tag{4} \]

describing the low energy $NN$ data at the singlet $(s)$ and triplet $(t)$ states are

\[ V_0^s = 98.10 \text{ MeV}, \quad R_0^s = 0.744 \text{ fm}; \]
\[ V_0^t = 184.08 \text{ MeV}, \quad R_0^t = 0.683 \text{ fm}. \]

These parameters define the following scattering lengths and effective ranges

\[ 'a_{pp}^s = -7.874 \text{ fm}, \quad 'r_{pp}^s = 2.804 \text{ fm}, \]
\[ 'a_{pp}^t = 5.403 \text{ fm}, \quad 'r_{pp}^t = 1.716 \text{ fm}. \tag{5} \]

The M-T potential is

\[ V^N (r) = \frac{V_A}{r/R_A} \exp (-r/R_A) + \frac{V_R}{r/R_R} \exp (-r/R_R). \tag{6} \]

Fitting of $NN$ low energy data gives the following parameters of the M-T potentials:

\[ V_A = -898.75 \text{ MeV}, \quad R_A = 0.617 \text{ fm}, \]
\[ V_R = 4319.85 \text{ MeV}, \quad R_R = 0.325 \text{ fm}, \]
for the singlet state and

\[ V_A = -945.50 \text{ MeV}, \quad R_A = 0.645 \text{ fm}, \]
\[ V_R = 4476.845 \text{ MeV}, \quad R_R = 0.322 \text{ fm}, \]
for the triplet state. Here we omitted indexes $s$ and $t$ to simplify entries. These parameters determine the scattering lengths and effective ranges:

\[ 'a_{pp}^s = -7.88 \text{ fm}, \quad 'r_{pp}^s = 2.69 \text{ fm}, \]
\[ 'a_{pp}^t = 5.52 \text{ fm}, \quad 'r_{pp}^t = 1.89 \text{ fm}. \tag{7} \]

The indexes $s$ and $t$ mean the singlet and triplet state, respectively.

To find the neutrino flux some standard solar model (SSM) must be used. The results of Bahcall et al [4] showed that the sensitivity of the neutrino flux from the $pp$ and $pep$ reactions to the type of SSM was very weak. In previous calculations we used the parameters of the BS2005(OP) model presented on website [15]. For the case of the exponential potential we use additionally the BP2000 solar standard model [15] to check sensitivity of the flux to the type of SSM.

3. The probability of the $\text{pep} \rightarrow d + \nu_e$ reaction

It should be noted that Bahcall and May [7] used the adiabatic approximation for the wave function of the $pep$ system. In this approach the $pep$ wave function was represented by the product of the wave function of the electron moving relative to the center of mass of the two protons, and the wave function of the relative motion of the two protons. Unfortunately, such a description is not adequate even in the region where the distance from the center of mass of the protons to the electron far exceeds the size of the nucleon-nucleon system due to the nature of the Coulombic force [16]. Moreover, we cannot use such a factorized wave function in the region of the small distances between the particles where it is necessary to know the wave function with sufficient accuracy.
to perform a precise calculation of the transition matrix element of the process $pp \rightarrow d\bar{u}$.

In our paper [10] we showed the use of the hyperspherical harmonics method [17, 18] to directly solve the 3-body Schrödinger equation for the $pp$ system. The deduction of one-dimensional coupled radial equations can be found in our article [10]. Due to the small energy of the colliding particles we restrict ourselves to considering the radial equation assuming that all quantum numbers are zero. Therefore we solve the following radial equation:

$$\frac{d^2 U(\rho)}{d\rho^2} + \frac{1}{\rho} \frac{dU(\rho)}{d\rho} - \left( V(\rho) + \frac{4}{\rho^2} - \kappa^2 \right) U(\rho) = 0,$$

where $\kappa^2 = 2\mu_2 E/\hbar^2 > 0$ ($E$ is the total energy of the colliding particles in the $pp$ system);

$$V(\rho) = V^N(\rho) + V^C(\rho),$$

$$V^C(\rho) = \frac{32\mu_2}{3\pi \hbar^2} \left( a_1 + a_2 + a_3 \right) \frac{2\eta \kappa}{\rho}.$$ \hspace{1cm} (9)

Here $\eta$ is the 3-body Coulomb parameter (analogue of the Sommerfeld parameter) which is defined as

$$\eta = \frac{16\mu_2^3}{3\pi \hbar^2 \kappa} (a_1 + a_2 + a_3),$$ \hspace{1cm} (11)

$$a_1 = -\frac{m_2 m_3}{\mu_2 (m_2 + m_3)} e^2 \equiv e^2,$$ \hspace{1cm} (12)

$$a_2 = -\frac{m_1 m_3}{\mu_2 (m_1 + m_3)} e^2 \approx -\frac{2m_2}{m_3} e^2,$$ \hspace{1cm} (13)

$$a_3 = -\frac{m_1 m_2}{\mu_2 (m_1 + m_2)} e^2 \approx -\frac{2m_3}{m_2} e^2.$$ \hspace{1cm} (14)

$V^N(\rho)$ and $V^C(\rho)$ are the reduced nuclear and Coulomb potentials resulting due to integration by applying the hyperspherical function of zero value of the hypermoment. These potentials can be more easily calculated analytically for the considered potentials.

To find a numerical solution to equation (8) we use the following boundary condition at the point $\rho_b$ close to the origin:

$$U(\rho_b) = I_2(\kappa_0 \rho_b), \quad U'(\rho_b) = \kappa_0 J_2(\kappa_0 \rho_b),$$ \hspace{1cm} (15)

in the case $V^N(\rho_b) < 0$ (purely attractive potential) or

$$U(\rho_b) = I_2(\kappa_0 \rho_b), \quad U'(\rho_b) = \kappa_0 J_2(\kappa_0 \rho_b),$$ \hspace{1cm} (16)

in the case $V^N(\rho_b) > 0$ ($NN$ potential is repulsive near the origin). Here $\kappa_0 = \sqrt{\kappa^2 + |V^N(\rho_b)|}$ ($\kappa_0 = \sqrt{V^N(\rho_b) - \kappa^2}$ if $V^N(\rho_b) > 0$), $J_i(\chi)$ is the Bessel function and $I_i(\chi)$ is the modified Bessel function. Such a boundary condition follows from the behavior of the radial function near the origin. It is clear to see that $U(\rho) \sim \rho^2$ if $\rho \rightarrow 0$ for any type of $NN$ potential without hard core.

The asymptotic of the radial function $U(\rho)$ at large distance ($\rho \rightarrow \infty$) is:

$$U(\rho) \sim e^{i\delta} \cos \delta \left( F_\infty(\kappa \rho) - \tan \delta G_\infty(\kappa \rho) \right),$$ \hspace{1cm} (17)

where $\delta_3$ is the three-body nuclear scattering phase shift modified by the Coulomb interactions. At the considered low energy range the phase $\delta_3 \sim 0$ and according to [17] the three-body Coulombic phase shift is given by

$$\delta_3 = \arg \left[ \Gamma \left( \frac{5}{2} + i\eta \right) \right].$$ \hspace{1cm} (18)

and $F_\infty(\kappa \rho)$ and $G_\infty(\kappa \rho)$ are the three-body regular and irregular Coulomb wave functions [17]. A reader can find the representations of these functions in our paper [10].

Equation (8) was solved using the boundary conditions (15) or (16) depending on the behavior of the $NN$ potential near the origin. Equating the logarithmic derivative of the numerical solution at a distance where the nuclear potential is negligible to the logarithmic derivative of the asymptotic solution [equation (17)] we find the three-body phase shift $\delta_3$. The result for $U(\rho)$ obtained with the Gaussian potential was shown in figure 1 of [10]. We got the same results with the Yukawa and exponential potentials. The limit of $U(\rho)/|\kappa \rho|^2$ goes to a nonzero value. However, near the origin, the behavior of the curve describing the solution applying the M-T potential differs from the solution applying simple potentials. In this case the limit of $U(\rho)/|\kappa \rho|^2$ approaches a value close to zero due to the repulsive term in the M-T potential which becomes very large at small distances. However, the behavior of the radial function is almost the same at large distances as for the simplest potentials having only an attraction term. At sufficiently large distances, the ratio of the unnormalized solution of equation (8) to the asymptotic function given by equation (17) becomes constant for all considered cases. It allowed us to obtain the normalized wave function [10]. The matching distance depends on the type of potential and lies at a distance $\sim 35$ fm for the case of the simple $NN$ potentials, and in the case of M-T potential, the matching distance is $\sim 50$ fm owing to the repulsive part of its interaction. It should be noted that the wave function of the $pp$ system can already be replaced by its asymptotics from a distance of $\sim 5$ fm. For this reason, the rate of the $pp$ reaction is insensitive to the choice of potential. At the same time, in the calculation of the $pp$ reaction the wave function of the $pp$ system can be replaced by its asymptotics from a distance of only $\sim 35-50$ fm, but for distances of less than 35 fm we must use the exact three-body wave function. The three-body wave function must be calculated with sufficient accuracy in the range $0 < \rho < 35$ fm because the deuteron wave function decays exponentially to zero at around 35 fm. Interested readers can study the dependence of the $pp$ three-body wave function on the hyperradius in figure 1 of our paper [10].
Applying the weak Hamiltonian (equation (3)) the matrix element for the pep → dv transition can be written as

\[ H_f = G_A < q_\nu | \sigma \tau^\dagger | q_\nu > \times \sum_{i=1}^2 < \Psi_\nu^i | \sigma \tau^\dagger | \Psi_{pep} > , \]  

(19)

where \( q_\nu \) and \( q_\nu \) refer to the spin functions of the neutrino and the electron, respectively; the wave function of the deuteron is denoted by \( \Psi_d \), and \( \Psi_{pep} \) is the pep wave function. We take the neutrino wave function \( \Psi_\nu \) as a plane wave which can be replaced by unity owing to cutoff of the integration interval in the calculation of the matrix element (equation (19)). Due to the zero-range weak interaction of the electron with the proton, the electron coordinate in the wave function \( \Psi_{pep} \) is taken at the point where either of the protons is located.

The probability \( P_i \) of reaction per unit time was calculated using the first-order perturbation theory in the weak interaction. We define the probability \( P_i \) as

\[ P_i = \frac{2\pi}{\hbar} | H_{ei} | \rho (E_i) , \]  

(20)

where the density of neutrino states is

\[ \rho (E_i) = \frac{E_i^2}{2\pi^2 \hbar^2 c^3} . \]  

(21)

Here \( E_i = 1.44 \text{ MeV} \) is the neutrino energy and \( c \) is light speed.

Finally, we obtain the equation for the probability \( P_3 \) of the pep → dv reaction in the form:

\[ P_3 = \frac{3E_i^2 G_A^2}{\pi \hbar^2 c^3} \int \Psi_d^* (x) \Psi_{pep} (x, y_0) d^3x . \]  

(22)

where \( y_0 \) is a Jacobi coordinate of the electron at the point where it contacts with one of the protons.

The overlap integral of equation (22) can be reduced to the one dimensional integral:

\[ \int \Psi_d^* (x) \Psi_{pep} (x, y_0) d^3x = \frac{16\sqrt{\pi}}{k^2} \times \int_0^\infty x^{-1} \chi_d (x) U (x) dx . \]  

(23)

Here the function \( U (\rho) \) is taken at the point \( \rho = \sqrt{x^2 + y_0^2} \approx x \quad (x \gg y_0) \) and the radial wave function \( \chi_d (x) \) of the deuteron is normalized to unity. Figure 1 shows the results of the dependence of the integrand of the matrix element (23) calculated with the exponential and M-T potentials. We see that the integrands depend on whether the NN potential has repulsion at small distances or whether it is a purely attractive potential.

In nuclear astrophysics the rate of the binary reaction is calculated using the astrophysical S factor [19]. The cross section of such reactions is related to the astrophysical S factor as

\[ \sigma (E) = \frac{S(E)}{E} e^{-2\eta} , \]  

(24)

where \( \eta = Z_1 Z_2 e^2 / (\hbar v) \) is the Sommerfeld parameter, \( Z_1 e \) and \( Z_2 e \) are charges of colliding nuclei with the relative velocity \( v \). The factor \( e^{2\eta} \) is proportional to the probability of the penetration of a charged particle through the Coulomb barrier. In the pep reaction we can adopt a similar parameter because the three-body pep radial wave function \( U (\rho) \) encloses the factor \( e^{2\eta} \) owing to the Coulomb interaction between colliding particles. Therefore we can identify the Coulombic factor in the probability \( P_i \) and define the astrophysical \( S_{pep} \) factor for the pep reaction as

\[ R_i (E) = G_0 (E) S_{pep} (E) , \]  

(25)

\[ G_0 (E) = \frac{2\pi e^{-2\eta}}{1 + e^{-2\eta}} \left( \frac{1}{z} + \eta_0^2 \right) \left( \eta_0 + \eta_0^2 \right) . \]  

(26)

For the range of energy close to zero, our results show that \( S_{pep} (E) \) varies almost linearly with energy \( E \). Its limit is not zero when the energy goes to zero. Also we note that \( G_0 (E) \) is dimensionless therefore the \( S_{pep} (E) \) dimension coincides with the unit of \( R_i (E) \) according to our definition of this quantity. Due to the linear property of \( S_{pep} (E) \) at low energies we can expand it into a series and restrict ourselves to the first three terms, so we write

\[ S_{pep} (E) = S_0 + S_1 E + S_2 E^2 , \]  

(27)

The calculated results for the value of the coefficients are the following: the exponential potentials...
\[ S_0 = 2.49 \times 10^{10} \text{fm}^2 \text{ s}^{-1}, \]
\[ S_1 = 3.35 \times 10^{10} \text{fm}^2 (\text{MeV s})^{-1}, \]
\[ S_2 = 2.21 \times 10^{10} \text{fm}^2 (\text{MeV}^2 \text{ s})^{-1}; \]  
(28)

and the M-T potentials
\[ S_0 = 2.11 \times 10^{10} \text{fm}^2 \text{ s}^{-1}, \]
\[ S_1 = 3.70 \times 10^{10} \text{fm}^2 (\text{MeV s})^{-1}, \]
\[ S_2 = 1.76 \times 10^{10} \text{fm}^2 (\text{MeV}^2 \text{ s})^{-1}. \]  
(29)

The coefficients \( S_0 \), \( S_1 \) and \( S_2 \) for the Gaussian and the Yukawa potentials are given in [10]. We note that the difference between the values of the coefficients \( S_0 \) for the considered simple potentials (Gaussian, Yukawa, exponential) can reach \( \sim 3-6\% \), while the difference in the coefficient \( S_0 \) calculated with M-T potentials reaches \( \sim 15\% \) with respect to the simplest potentials. The difference in the coefficients \( S_1 \) and \( S_2 \) increases to much larger values. Fortunately the contribution of \( S_0 \) and \( S_2 \) to the neutrino flux is much less as compared to \( S_0 \). We would also like to mention that the analogue astrophysical \( S \) factor for the two-particle reaction (1) does not depend on the type of \( NN \) potential, in contrast to the three-particle astrophysical \( S \) factor defined by equation (26).

### 4. Rate and flux of the solar neutrinos

We define the rate constant of the pep reaction [10] by the following equation
\[ K_{\text{pep}} = \langle P_3 \rangle = \int_0^\infty \int_0^\infty \int_0^\infty \varphi(v_e) \varphi(v_{p_1}) \varphi(v_{p_2}) \]
\[ \times P_3(E) \text{d}v_e \text{d}v_{p_1} \text{d}v_{p_2}. \]  
(30)

After averaging over the Maxwell–Boltzmann distributions \( \varphi(v_i) \) describing the random motion of protons and electrons in the core plasma of the Sun, we get the simple formula for the rate constant expressed though the three-body astrophysical \( S_{\text{pep}} \) factor:
\[ K_{\text{pep}} = \frac{1}{(kT)^3} \int_0^\infty G_0(E) S_{\text{pep}}(E) e^{-E/kT} E^2 \text{d}E, \]  
(31)

where \( k \) is the Boltzmann constant and \( T \) is the temperature in the core of the Sun. The maximum of the integrand is reached at the energy
\[ E_{\text{max}} = \left( \frac{1}{2} kT \sqrt{E_G} \right)^{2/3}. \]  
(32)

Here \( E_G \) is the Gamow energy for the pep reaction [10].

The rate of reaction related to the rate constant is
\[ R_{\text{pep}} = K_{\text{pep}} n_p^2 n_e, \]  
(33)

where \( n_p \) and \( n_e \) are the density of the reactants (protons and electrons).

In our calculation of the \( pp \) and \( pep \) reaction rates we applied the BS2005(OP) SSM [6] for both types of \( NN \) potential. The \( pp \) and \( pep \) reaction rates do not depend much on the type of solar model (for details we refer to [5]). Additionally, we used the BP2000 SSM for calculating the rate with the exponential potentials to check the sensitivity of the results to the SSMs.

In Figure 2 we present the dependence of the rate of the \( pep \) and \( pp \) reactions on the solar interior calculated by the exponential and M-T potentials. The rate of the \( pp \) reaction differs within 2.4\% for all types of potentials used, including the potential used in [10]. Therefore in the figure the rate of the \( pp \) reaction is shown only for the exponential potential.

There is a small difference in the rate of the \( pep \) reaction calculated by applying the simplest attractive potentials. However, in the case of M-T potential it differs within \( \sim 10-15\% \) from the rates calculated with the simplest potentials.

Finally we present, in Table 1, the results of our calculation of the fluxes of neutrinos at a distance of one astronomical unit (AU).

Note that whereas our results for calculated neutrino flux for the \( pp \) reaction are close to the Bahcall et al results, differences exist for the calculated results for the \( pep \) reactions.

If we take into account the neutrino flux measured by the Borexino collaboration (\( \Phi_{\text{pep}} = (1.6 \pm 0.3) \times 10^5 \text{cm}^{-2} \text{s}^{-1} \)) and the survival probability of the electron neutrino \( P = 0.62 \pm 0.17 \) at 1.44 MeV suggested by this group [11] we find that the neutrino flux at 1 AU should be equal to \( \Phi_{\text{pep}} = (1.33 \pm 0.36) \times 10^5 \text{cm}^{-2} \text{s}^{-1} \) for the exponential potential, and \( \Phi_{\text{pep}} = (1.13 \pm 0.31) \times 10^5 \text{cm}^{-2} \text{s}^{-1} \) for the M-T potential if the BS2005(OP) standard solar model is used. Applying the BP2000 standard solar model leads to the flux being equal to \( \Phi_{\text{pep}} = (1.30 \pm 0.36) \times 10^5 \text{cm}^{-2} \text{s}^{-1} \) in the case of the exponential potential.
Table 1. Predicted fluxes \( \Phi_\nu \) and \( \Phi_{\nu\gamma} \) (without survival probability), in units of \( 10^{10}(pp), 10^8(pep) \) cm\(^{-2}\)s\(^{-1}\).

| Standard solar model | \( \Phi_{pp} \) | \( \Phi_{\nu\gamma} \) | \( \Phi_{\nu\gamma}/\Phi_{pp} \) | References |
|----------------------|---------------|-----------------|-----------------|-----------|
| BS2005(OP)           | 6.20          | 2.04            | 304             | using Gauss potential [10] |
|                      | 6.05          | 1.99            | 304             | using Yukawa potential [10] |
|                      | 6.13          | 2.14            | 287             | using exponential potential (current work) |
|                      | 6.16          | 1.82            | 338             | using M-T potential (current work) |
| BP2000               | 6.02          | 2.10            | 288             | using exponential potential (current work) |
| BP04(Yale)           | 5.94          | 1.40            | 424             | [4] |
| BP04 (Garching)      | 5.94          | 1.41            | 421             | [4] |
| BS04                 | 5.94          | 1.40            | 424             | [4] |
| BS05(14N)            | 5.99          | 1.42            | 421             | [4] |
| BS05(OP)             | 5.99          | 1.42            | 421             | [4] |
| BS05 (AGS,OP)        | 6.06          | 1.45            | 418             | [4] |
| BS05 (AGS, OPAL)     | 6.05          | 1.45            | 417             | [4] |

We see that our calculated neutrino fluxes from the \( pep \) reaction lie within the confidence interval of the experimental data for all considered simplest potentials including the exponential potential, even if we multiply the calculated fluxes by the survival probability \( (P = 0.62 \pm 0.17) \) suggested by the Borexio group. The flux calculated using M-T potential lies at the lower limit of the confidence interval. If we multiply the neutrino fluxes from the \( pep \) reaction calculated by Bahcall et al by the value of the survival probability, all their results lie out of the confidence interval of the Borexio data.

Averaging the Bahcall et al fluxes from the \( pp \) reaction presented in table 1 gives us the value equal to \( \Phi_{pp} = 5.99 \times 10^{10} \) cm\(^{-2}\)s\(^{-1}\) and, with standard deviation, it is equal to \( 0.05 \times 10^{10} \) cm\(^{-2}\)s\(^{-1}\). Comparing the fitted low-energy parameters for the simplest potentials, we see that the difference between the effective range parameters may be 2%–7%; at the same time the neutrino fluxes from the \( pp \) reaction have a 3.5% maximal difference from the average value of the Bahcall result (in the case of the Gaussian potential, BS2005 (OP) SSM). This means the fluxes from the \( pp \) reaction is insensitive to the type of \( NN \) potential and to SSM. However, the difference between the fluxes of neutrinos from the \( pep \) reaction calculated by the simplest potentials and the potential having repulsion at small distances (M-T potential), may reach ~15%. Consequently our results from the \( pep \) reaction is expected to show dependence on the type of \( NN \) potential used. Also, we must note that the difference between our results and the Bahcall et al results for the \( pep \) process can reach up to 25% to 40%, and indicates strong dependence on the selection of the initial three-body state of the \( pep \) system.

5. Conclusion

The process \( pep \rightarrow d + \nu \) has been considered for the first time within the framework of the three-body description of the initial state employing the nucleon-nucleon potential with repulsion at small distances. The rate and the flux of neutrinos from \( pp \) and \( pep \) reactions has been calculated for specific conditions in the center of the Sun. The theoretical results are in satisfactory agreement with the reported measured data by the Borexino group, but different by (25–40)% from previous calculations made by Bahcall et al. For a better comparison between theory and measurement it is necessary to reduce uncertainty in the oscillation parameters of neutrinos and to perform a new calculation using other realistic nucleon-nucleon potentials with repulsive cores.

References

[1] Bahcall J N 1989 Neutrino Astrophysics (Cambridge: Cambridge University Press)
[2] Bahcall J N 2003 Nucl. Phys. Proc. Suppl. 118 77
[3] Bahcall J N and Pinsonneault M H 2004 Phys. Rev. Lett. 92 121301
[4] Bahcall J N, Serenelli A M and Basu S 2005 ApJ 621 L85 (arXiv:astro-ph/0412440).
[5] Bahcall J N, Serenelli A M and Basu S 2006 ApJ Suppl. 165 400
[6] www.sns.ias.edu/~jnb/
[7] Bahcall J N and May R M 1969 ApJ 155 501
[8] Belyaev V B, Levin S V and Yakovlev S L 2004 Phys. Rev. A 68 032101
[9] Kim Y E and Zubarev A L http://adsabs.harvard.edu/abs/1996APh....10...75K
[10] Irgaziev B F, Belyaev,and V B and Nabi J-U 2013 Phys. Rev. C 035804
[11] Bellini G et al (Borexino Collaboration) 2012 Phys. Rev. Lett. 108 051302
[12] Primakoff H 1959 Rev. Mod. Phys. 31 802
[13] Povh B, Rith K, Scholz C and Zetsche F 2008 Particles and Nuclei: An Introduction to the Physical Concepts 6th edn. (Berlin: Springer-Verlag)
[14] Malliet R A and Tjon J A 1969 Nucl. Phys. A127 161
[15] www.sns.ias.edu/~jnb/SNdata/Export/BS2005/bs05op.dat
[16] Alt E O and Mukhamedzhanov A M 1993 Phys. Rev. A 47 2004
[17] Djubiati R I and Shitikova K V 1993 Metod hipsfericheskikh funktsiy v atomnyy i yaderny fize (Method of Hyperspherical Functions in Atomic and Nuclear Physics) (Moscow: Energoatomizdat) in Russian
[18] Fabre de la Ripelle M 1987 The hyperspherical expansion method ed L S Ferreira, A C Fonseca and L Streit Models and Methods in Few-Body Physics (Lecture Notes in Physics vol 273) (Berlin: Springer) p 283
[19] Angulo C et al 1999 Nucl. Phys. A 656 3