Measuring the Higgsplosion Yield: Counting Large Higgs Multiplicities at Colliders

James S. Gainer

Dept. of Physics and Astronomy, University of Hawaii, Honolulu, HI 96822, USA

Abstract
Recent work has brought renewed attention to the possibility that the cross section for producing \( n \) Higgs bosons grows large with \( n \) at a sufficiently energetic hadron collider. In particular, this “Higgsplosion” mechanism has been suggested as a solution to the hierarchy problem. We investigate the phenomenology of this scenario. Discovery is trivial, so we consider several variables for use in measuring large Higgs multiplicities and evaluate their effectiveness. We find that a 10% level measurement of the number of Higgs bosons is possible with a handful of events, using the scalar sum of jet transverse momenta, but determining the exact multiplicity may take a few thousand events, depending on the degree of statistical significance desired for the measurement. While this situation may be acceptable given the potentially large cross sections in this scenario, future research to improve the measurement is warranted.

1. Introduction
The cross section for producing \( n \) Higgs bosons appears to grow exponentially large in perturbative calculations due to the scaling of the number of diagrams (which goes as \( n! \)). This fact has long been noted and has been much discussed [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. It is at present unclear whether this apparent scaling reflects reality or an incomplete calculation; it could also be that this scaling of the cross section with \( n \), which is calculated from only the interactions of the scalar particles, is violated by loops involving heavy fermions [21].

Recently the asymptotic behavior of the multiple Higgs production cross section with respect to \( n \) has been invoked as a solution to the hierarchy problem [20]. In this work, the production of a large number of Higgs bosons is referred to as “Higgsplersion”. This work also, importantly, pointed out a “Higgsplersion” mechanism, via which the exponential growth in Higgs cross sections is tamed via the momentum-dependent width for a off-shell Higgs decaying to...
$n$ Higgs bosons becoming large. This prevents the amplitude from violating unitarity at some sufficiently large value of $n$. Thus Higgsplosion is a viable possibility, though, as noted above, by no means a definite prediction.

Ref. [20] suggests that Higgsplosion may also solve the hierarchy problem. The argument goes roughly as follows. Any new heavy particle which couples to the Higgs has a large partial width for decay to $n$ Higgses. This large width cuts off integrals involving the heavy particle; therefore quadratic corrections to the Higgs mass from new heavy particles are cut off at a scale far below the Planck scale. Ref. [20] suggests this scale may be as low as $\sim 25$ TeV. This interesting behavior is reminiscent of classicalization [22, 23, 24] with the $n$-Higgs boson state playing the role of the classicalon.

As far as we are aware, the phenomenological consequences of the Higgsplosion scenario have not been considered in detail. Yet the exotinest of the scenario raises interesting questions. Events with $O(100 - 200)$ Higgs bosons, which we will term “Higgsplosion events” in the remainder of this letter, are easy to detect. For example, none of our Monte Carlo simulated Higgsplosion events had fewer than 20 jets with $p_T > 50$ GeV; in general the jet multiplicities are even larger and many of these jets have much higher $p_T$. We therefore do not need to consider standard model (SM) backgrounds. In the event that Higgsplosion is realized in nature, discovery would only require enough events for us to convince ourselves that some strange detector malfunction were not to blame.

We therefore turn our attention to what we can measure about Higgsplosion events. The most obvious property is the number, $n$, of Higgs bosons produced in the event. This measurement proves to be not entirely straightforward, as the large number of Higgs bosons, and consequently of decay products, provide challenges that would be absent for a small or intermediate Higgs multiplicity. In the remainder of this letter we will examine variables that could be used to measure the Higgs multiplicity in Higgsplosion events. Future research on this issue should consider potential improvements from (a) multivariate analyses and (b) the use of variables related to jet substructure to reduce the number of events needed for this measurement.

2. Procedure

We will consider three benchmark scenarios, namely 150 or 200 Higgs bosons at a 100 TeV collider and 100 Higgs bosons at the 14 TeV CERN Large Hadron Collider (LHC). While Ref. [20] does not suggest that the 100-Higgs boson events will be visible at the LHC, we consider this scenario to have something relevant to the current experimental situation. (Also the “theory error bars” are large in these suggestive arguments about the large $n$ Higgs cross sections.) Ref. [20] suggests that cross sections reach observable levels for $n \gtrsim 130$ at future colliders; so $n = 150$ and $n = 200$ should be seen as the more realistic scenarios.

In each of these scenarios we will follow the approach of Higgsplosion calculations and consider Higgs bosons as being produced at threshold. This is sensible also because the exponential scaling of the $n$-Higgs production cross
section with \( n \) means that producing an extra Higgs is generally favored over having additional phase space available, up to the point where suppressions from parton distribution functions (pdfs) or Higgspersion cuts off the cross section.

We construct the “parton level” events explicitly by writing Les Houches Accord Event (LHE) files \([25]\) with \( n \) Higgs bosons at rest with respect to each other. Each event has an overall value of longitudinal momentum found from the distribution we calculate using the NNPDF23_lo_as_0130_qed pdf \([26]\) as implemented in LHAPDF \([27]\). We note, however that even at 100 TeV, since \( p_Z \sim 20 - 50 \text{ GeV} \) for individual Higgs bosons in the event, the Higgses are non-relativistic in the lab frame and the effect of this longitudinal boost (or likewise a transverse boost from ISR) will be minimal. For this reason the choice of collider in the event simulation is essentially irrelevant.

We pass the LHE file to Pythia8 \([28]\) and the resulting HepMC \([29]\) file to DELPHES 3, which contains FastJet \([30]\) for jet algorithms, where we use the DELPHES 3 card described in Ref. \([31]\), but with leptons considered isolated if the activity in the detector within a \( \Delta R < 0.2 \) cone is less than the minimum of 5 GeV and 0.15 times the lepton \( p_T \). (While we provide the details of our simulation in the interest of transparency, we do not expect the details of our detector simulation to have a significant qualitative effect on our results.) We then use the LHCO files produced by DELPHES to calculate various observables.

### 3. Results

We use our simulated data to determine which variables will be most useful for measuring the multiplicity of Higgs bosons in Higgsplosion events. We first consider the number of jets in the final state, which is shown in Figure 1. Naturally, the jet multiplicity is proportional to the Higgs multiplicity, though the coefficient of proportionality is much less than one. For example, we find that the mean number of jets for a Higgs multiplicity of 100 is \( 36.2 \), while the mean jet multiplicity for 150 Higgs events is \( 42.8 \). Clearly decay products from many different Higgs bosons may contribute to the same jet. We do not consider \( b \)-tagging here because many jets in a given event include decay products from multiple high \( p_T \) \( b \) quarks. We feel a detailed analysis of \( b \)-tagging efficiencies in this situation, which is beyond the scope of this letter, would be required to draw even preliminary conclusions about the multiplicity of \( b \)-tagged jets. We do consider the number of isolated leptons, but find that it is a poor variable. Due to the isolation requirements the number of leptons observed actually decreases slightly with the increase in jet (and Higgs) multiplicity. Another variable of limited efficacy is the missing transverse momentum (\( E_T \)), which we plot in Figure 2. We see that this variable increases weakly with the increase in Higgs multiplicity, and, in general, takes on relatively low values.

A more useful variable for performing the Higgs multiplicity measurement is provided by the scalar sum of jet \( p_T \), the distributions for which are shown in Figure 3. The mean for the distribution of this variable is around 90 GeV times the Higgs multiplicity, while the standard deviation is between 2300 – 2700 GeV. Thus we can “translate” a measurement of this quantity by dividing by 90 GeV;
the 1σ error for a single event is then $\lesssim 30$ events. So a 10% measurement (an error on the multiplicity of $10-20$) is possible with ten or fewer events (treating the distributions as Gaussian, which is an approximation, but our conclusions should not be changed by a more detailed treatment). However reducing the error bar to the point where we are, e.g., able to distinguish a sample of 150 Higgs bosons from a sample of 151 Higgs bosons at 1σ will take $\approx 700-900$ events; more significant separations require more events in a straightforward way. It will likewise take thousands of events to determine the cross sections for various Higgs multiplicities when events of various Higgs multiplicities are produced (as we would expect).

While Higgsplosion cross sections may be at the picobarn level or higher [20], making this number of events trivial to obtain, we may still want to improve the measurement given the uncertainty on the cross sections. For example, assuming that the LHC has not currently seen any Higgsplosion events puts a 95% confidence level limit of $\lesssim 200$ expected Higgsplosion events in the lifetime of the 14 LHC (assuming the LHC will collect $\approx 60$ times the current integrated luminosity and ignoring the distinction between 13 and 14 TeV). We remind the reader that Higgsplosion events are not anticipated at the 14 TeV LHC [20].

As a first attempt to see whether more sophisticated kinematic variables can improve on the scalar sum of jet $p_T$, we consider the $\sqrt{\hat{s}_{\text{min}}}$ variable introduced in Ref. [22], in which one finds the minimum value of $\hat{s}$ consistent with the

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**Figure 1:** Multiplicity of $p_T > 50$ GeV anti-$k_T$ jets with $\Delta R < 0.4$ for 100, 150, and 200 Higgs events in the final state.
event if the missing energy in the event is due to two invisible particles with a set mass. (We set this mass to be zero in the analyses here.) In general, this is an appealing kinematic variable, as it does not require reconstruction of the event topology. However, as one can see from Figure 4, the distribution is rather broad with a long tail toward high values. It therefore will be a less effective variable than the scalar sum of jet $p_T$ for this particular application.

4. Conclusions

Higgsplosion represents an interesting possibility. Continuing theoretical research into whether it is a valid prediction of the standard model is clearly called for. We have found that for determining the Higgs multiplicity, the best variable, of those we consider, is the scalar sum of jet transverse momentum. The distributions are sufficiently narrow that a 10% measurement of the Higgs multiplicity takes only a few events, but determining the exact multiplicity of a sample of $n$ Higgs events will take at least $\sim 1000$ events. Similar large number of events will be required to measure $\sigma(n)$ in the case where a number of different, consecutive, values of $n$ contribute.

These analyses are therefore acceptable in the limit where picobarn cross sections for Higgsplosion events are observed. However, given the theoretical uncertainties, it would be useful to be able to perform better measurements with
Figure 3: Distribution of the scalar sum of $|p_T|$ for all $p_T > 50$ GeV anti-$k_T$ jets with $\Delta R < 0.4$ for 100, 150, and 200 Higgs events in the final state.

Figure 4: Distribution of $\sqrt{\hat{s}_{\text{min}}}$ for 100, 150, and 200 Higgs events in the final state.
fewer events. Multivariate analyses may be useful toward this end, though, un-
fortunately many of the variables considered here are correlated. Substructure
variables may also be useful in improving the Higgs multiplicity measurement, as
a given jet generally includes decay products from many different Higgs bosons.

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