Supersymmetric CP Violation in $B \to X_s l^+ l^-$
in Minimal Supergravity Model

Chao-Shang Huang, Liao Wei

Institute of Theoretical Physics, Academia Sinica, P.O.Box 2735,
Beijing 100080, P.R. China

Abstract

Direct CP asymmetries and the CP violating normal polarization of lepton in inclusive decay $B \to X_s l^+ l^-$ are investigated in minimal supergravity model with CP violating phases. The contributions coming from exchanging neutral Higgs bosons are included. It is shown that the direct CP violation in branching ratio, $A_{CP}^1$, is of $O(10^{-3})$ for $l = e, \mu, \tau$. The CP violating normal polarization for $l = \mu$ can reach 0.5 percent when $\tan \beta$ is large (say, 36). For $l = \tau$ and in the case of large $\tan \beta$, the direct CP violation in backward-forward asymmetry, $A_{CP}^2$, can reach one percent, the normal polarization of $\tau$ can be as large as a few percent, and both are sensitive to the two CP violating phases, $\phi_{\mu}$ and $\phi_{A_0}$, and consequently it could be possible to observe them (in particular, the normal polarization of $\tau$) in the future B factories.
1 Introduction

CP violation has so far only been observed in K system. It is one of the goals of the B factories presently under construction to discover and examine CP violation in the B system. CP violation is originated from the CKM matrix in the standard model (SM) and new sources of CP violation may appear in extensions of SM. In the constrained minimal supersymmetric standard model, i.e. the minimal supergravity model (mSUGRA), besides the standard CP violating phase $\delta_{\text{CKM}}$, there are two new CP violating phases, which may be chosen as the phase of $\mu$ ($\phi_\mu$) and the phase of $A_0$ ($\phi_{A_0}$), that can’t be rephased away when the universality of soft terms is assumed at unification scale.

It is well-known that the supersymmetric (SUSY) CP violating phases are constrained by the experiments on the electric dipole moments (EDMs) of neutron and electron. SUSY contributions to EDMs will exceed the current experimental limits on the EDMs of neutron and electron unless either the SUSY phases are rather much smaller ($\leq 10^{-2}$) or sfermion masses of the first and second generations are very large ($> 1$ TeV). However, large sfermion masses may be incompatible with bounds on the relic density of a gaugino-type LSP neutrilino. Recently, a third possibility to evade the EDM constraints has been pointed out. That is, various contributions to EDM cancel with each other in significant regions of the parameter space, which allows SUSY phases to be of order one and sparticles are relatively light. It is found that in the mSUGRA with small $\tan \beta (\leq 3)$, the phase $|\phi_\mu| \leq \frac{\pi}{10}$ while the phase $\phi_{A_0}$ remains essentially unconstrained, by combining cosmological and EDM constraints. Similar results have also been obtained in Refs. In this paper we shall investigate effects of SUSY phases on $B \to X_s l^+ l^-$ assuming the third possibility to evade the EDM constraints (i.e choosing the parameters in the region of the parameter space where cancellations happen). We extend the analyses of the EDM constraints to the large $\tan \beta (\geq 20)$ case. It is found that the cancellations are insufficient to make the EDMs of electron and neutron satisfy the experimental limits if $\phi_\mu$ is of order one and the sparticle spectrum is below $O (1$ Tev). That is, in the large $\tan \beta$ case $\phi_\mu$ must be $\leq 10^{-2}$ in order to satisfy the experimental limits of EDMs of electron and neutron and have a relatively light sparticle spectrum.

Effects of SUSY CP violating phases on the branching ratio of $B \to X_s l^+ l^-$ have been examined. In this paper we study direct CP asymmetries and the CP violating normal polarization of lepton in $B \to X_s l^+ l^-$ ($l=\text{e, } \mu, \tau$) in mSUGRA with CP violating phases. The direct CP asymmetry of this mode in the SM is unobservably small. Thus, an observation of CP violation in this mode would signal the presence of physics beyond SM.

2 N=1 supergravity and CP violation phases

In mSUGRA it is assumed that the soft SUSY breaking terms, which are originated from the gravitational interaction, are universal at the high energy (GUT or Planck) scale. So there are only five free parameters at the high energy scale: $M_{1/2}$, the mass of gauginos; $A_0$, the trilinear couplings; $B$, the bilinear couplings; $M_0$, the universal masses for all scalars, as well as $\mu$, the Higgs mass parameter in superpotential ($\mu$ and $A_0$ are defined as in). In general $A_0$, $B$, $\mu$ and $M_{1/2}$ are complex. However, not all the phases are physical. It is possible to rephase away the phase of $M_{1/2}$ and to make $B \mu$ real by redefinition of the fields and by R transformation. So there are only two physically independent phases left, which can be chosen as $\phi_{A_0}$ and $\phi_\mu$. The
breakdown of electroweak symmetry via radiative effect allows one to determine the magnitude of $\mu$ and $B$ at electroweak scale. Therefore, one has four real parameters ($M_0, M_{1/2}, |A_0|, \tan\beta$) and two phases ($\phi_\mu, \phi_{A_0}$) finally.

Mass spectra of sparticles, flavor mixing, and soft term parameters at the EW scale can be determined by solving the renormalization group equations (RGEs) running from GUT scale to EW scale. In order to see the running of the phases we record the equations for trilinear terms and $\mu$ (we neglect the effects of $A_i$ of 1st and 2nd generation because of their corresponding very small Yukawa couplings) [13]:

\[
\begin{align*}
\frac{dA_u}{dt} &= \frac{1}{4\pi} \left( \frac{16}{3} \alpha_3 M_3 + 3\alpha_2 M_2 + \frac{13}{15} \alpha_1 M_1 + 3Y^t A_t \right), \\
\frac{dA_d}{dt} &= \frac{1}{4\pi} \left( \frac{16}{3} \alpha_3 M_3 + 3\alpha_2 M_2 + \frac{7}{15} \alpha_1 M_1 + 3Y^b A_b + Y^\tau A_\tau \right), \\
\frac{dA_e}{dt} &= \frac{1}{4\pi} \left( 3\alpha_2 M_2 + \frac{9}{5} \alpha_1 M_1 + 3Y^b A_b + Y^\tau A_\tau \right), \\
\frac{dA_t}{dt} &= \frac{1}{4\pi} \left( \frac{16}{3} \alpha_3 M_3 + 3\alpha_2 M_2 + \frac{13}{15} \alpha_1 M_1 + 6Y^t A_t + Y^b A_b \right), \\
\frac{dA_b}{dt} &= \frac{1}{4\pi} \left( \frac{16}{3} \alpha_3 M_3 + 3\alpha_2 M_2 + \frac{7}{15} \alpha_1 M_1 + Y^t A_t + 6Y^b A_b + Y^\tau A_\tau \right), \\
\frac{dA_\tau}{dt} &= \frac{1}{4\pi} \left( 3\alpha_2 M_2 + \frac{9}{5} \alpha_1 M_1 + 4Y^\tau A_\tau + 3Y^b A_b \right), \\
\frac{d\mu}{dt} &= \frac{\mu}{8\pi} \left( -\frac{3}{5} \alpha_1 - 3\alpha_2 + Y^\tau + 3Y^b + 3Y^t \right)
\end{align*}
\]

where $\alpha_i = \frac{g_i^2}{4\pi}, Y^i = \frac{y_i^2}{4\pi} (i = t, b, \tau), g_i$ are the gauge coupling constants, $y_i$ are Yukawa couplings and $t = \ln(Q^2/M_{GUT}^2)$. It is explicit from the equations that the phase of $\mu$ does not run and both the magnitudes and phases of $A_i$ evolve with $t$.

3 Constraints on the parameter space from EDMs of electron and neutron and $B \to X_s \gamma$

The cancellation mechanism for suppression of the EDMs of electron and neutron in mSUGRA with SUSY phases have been pointed out. In the small $\tan\beta$ (say, $\tan\beta=3$) case the region of the parameter space in which the EDMs of electron and neutron satisfy the experimental limits [14]

\[
|d_e| < 4.3 \times 10^{-27} \text{ecm}, \tag{2}
\]

and

\[
|d_n| < 6.3 \times 10^{-26} \text{ecm} \tag{3}
\]

have been analyzed [3, 4]. We make similar analyses and confirm their results. We extend the analyses to the large $\tan\beta$ ($\geq 20$) case. It is found that the cancellations are insufficient to make the EDMs of electron and neutron satisfy the experimental limits if $\phi_\mu$ is of order one and the sparticle spectrum is below $O(1 \text{TeV})$. Thus we have to give up the phase $\phi_\mu$ of order one and search for cancellations for $\phi_\mu \leq 10^{-2}$ in the large $\tan\beta$ case if
we stick to the low sparticle spectrum. We scan the parameters $M_0, M_{1/2}, |A_0|$ and $\phi_{A_0}$ in the range of $300 \leq M_0, M_{1/2} \leq 800, 100 \leq |A_0| \leq 1200,$ and $0 \leq \phi_{A_0} \leq 2\pi$ for fixed values of $\tan \beta$ and $\phi_{\mu}$ in the range of $\tan \beta \geq 20$ and $\phi_{\mu} \leq 10^{-2}$. It is found that there are significant regions in which sparticle spectrum are below $\mathcal{O}$ (1 TeV) and the EDM constraints are satisfied. The result for a set of typical values of the parameters $M_0, M_{1/2}, |A_0|$ and $\phi_{\mu}$ ($M_0 = 400, M_{1/2} = 550, |A_0| = 750, \tan \beta = 36$ and $\phi_{\mu} = \pm \pi/1000$) is shown in fig.1, where EDME and EDMN represent the EDMs of electron and neutron respectively. The bounds between the two horizontal lines in the figures represent the experimental limits of the EDMs of electron and neutron. One can see from the figure that the electron EDM (fig.1a) imposes a constraint on $\phi_{A_0}$ and the neutron EDM (fig1b) imposes no constraints on $\phi_{A_0}$. The experimental constraint on the electron EDM is more stringent than that on the neutron EDM when there exist cancellations between different components, a conclusion similar to the small $\tan \beta$ case. The EDMs of electron and neutron as functions of $\phi_{A_0}$ in the small $\tan \beta$ case have also been given in fig.1. There are constraints on $\phi_{A_0}$, which is different from ref. [7]. The main reason is that we use the new data of the EDMs of electron and neutron which are quite smaller than the old data they used.

Because we shall pay attention to the case of large $\tan \beta$ we should also consider the contributions arising from Barr-Zee mechanism [10]. It is found that [11]

$$\langle \frac{dF}{e} \rangle_{BZ} = Q_f \frac{3\alpha}{64\pi^2} \frac{R_fm_f}{m_A^2} \sum_{q=t,b} \xi_q Q_q[F(\frac{m_{\tilde{q}}^2}{m_A^2}) - F(\frac{m_q^2}{m_A^2})],$$

(4)

where $R_f = \cot \beta (\tan \beta)$ for $I_{3f} = 1/2$ (-1/2), and

$$\xi_t = \frac{\sin \theta_t m_t Q_t (\mu e^{i\delta})}{\sin \beta v^2}, \quad \xi_b = \frac{\sin \theta_b m_b Q_b (\mu e^{i\delta})}{\sin \beta \cos \beta v^2}$$

(5)

with $\delta_f = \text{arg}(A_f + R_f \mu^*)$, and $F(x)$ can be found in Ref. [11], due to the two loop diagram contributions. The parameters chosen in our numerical calculations of $B \rightarrow X_s l^+ l^-$ satisfy the constraints from EDMs including the above two loop contributions.

It is well-known that $B \rightarrow X_s \gamma$ puts a stringent constraint on the parameter space of mSUGRA without SUSY CP violating phases [13, 17]. With SUSY CP violating phases, we calculate $B_r(B \rightarrow X_s \gamma)$. The branching ratio as function of $\phi_{A_0}$ is shown in fig.2, for the values of other SUSY parameters same as those in fig.1. As can be seen from the fig.2, region in the range $0 \leq \phi_{A_0} \leq 2\pi$ is allowed by the experimental limit of $B_r(B \rightarrow X_s \gamma)$. Since under the choices given here results are almost the same for sign of $\phi_{\mu}$ switched and other parameters unchanged, we only show the cases of positive $\phi_{\mu}$ in fig.2.

4 Formulas for $B \rightarrow X_s l^+ l^-$

Neglecting the strange quark mass, the matrix element governing the process $B \rightarrow X_s l^+ l^-$ is given as follows [16, 17]:

$$M = \frac{G_F \alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \left[ C_8 \epsilon_{\mu} \bar{s}_L \gamma_{\mu} b_L \bar{\tau} \gamma_{\mu} \gamma^5 \tau + C_9 (m_b \bar{s}_L \gamma_{\mu} b_L \bar{\tau} \gamma_{\mu} \gamma^5 \tau - 2C_7 (m_b) m_b \bar{s}_L i \sigma_{\mu\nu} q^\nu b_R \bar{\tau} \gamma_{\mu} \gamma^5 \tau + C_{Q1} (m_b) \bar{s}_L b_R \bar{\tau} \gamma_5 \tau + C_{Q2} (m_b) \bar{s}_L b_R \bar{\tau} \gamma_5 \tau \right]$$

(6)
where

\[ C_{S}^{eff} = C_{S}(m_{b}) + (3C_{1}(m_{b}) + C_{2}(m_{b})) \left[ g(\frac{m_{b}}{m_{b}}, \hat{s}) + \lambda_{a}(g(\frac{m_{b}}{m_{b}}, \hat{s}) - g(\frac{m_{b}}{m_{b}}, \hat{s})) \right] \]

\[ + \frac{3}{\alpha_{s}^{2}} K \sum_{\nu_{i}=\psi} \frac{\pi M_{M} \Gamma(\nu_{i} \rightarrow \nu^{+} \nu^{-})}{M_{\nu_{i}} \chi_{\nu_{i}} - i M_{\nu_{i}} \Gamma_{\nu_{i}}} \]  

\[ g(z, \hat{s}) = -\frac{4}{9} \ln z^{2} + \frac{8}{27} + \frac{16}{9} \left( \frac{1}{\hat{s}} \right) \left( 2 + \frac{4z^{2}}{\hat{s}} \right) \left[ \ln \left( \frac{1 + \sqrt{1 - 4z^{2}/\hat{s}}}{1 - \sqrt{1 - 4z^{2}/\hat{s}}} \right) i \pi \right], \quad 4z^{2} < \hat{s} \]

\[ \frac{4}{9} \left( \frac{1}{\hat{s}} \right) \left( 2 + \frac{4z^{2}}{\hat{s}} \right) \arctan \left( \frac{1}{\sqrt{4z^{2}/\hat{s} - 1}} \right), \quad 4z^{2} > \hat{s} \]  

with \( q = p_{t+} + p_{t-}, \hat{s} = q^{2}/m_{b}^{2} \) and \( \lambda_{a} = \frac{V_{ab} V_{*a}^{*}}{V_{tb} V_{t*}} \). The final two terms in eq. (6) come from exchanging neutral Higgs bosons (NHBs).

The QCD corrections to coefficients \( C_{i} \) and \( C_{Q_{i}} \) can be incorporated in the standard way by using the renormalization group equations. Since no NLO corrections to \( C_{Q_{i}} \) have been given, we use the leading order corrections to \( C_{i} \) and \( C_{Q_{i}} \), although the NLO corrections to \( C_{i} \) have been calculated. They are given as below \[ 16 \]:

\[ C_{7}(m_{b}) = \eta^{-\frac{16}{3}} C_{7}(m_{W}) + \frac{8}{3} \left( \eta^{-\frac{2}{3}} - \eta^{-\frac{16}{27}} \right) C_{CG}(m_{W}) + C_{2}(m_{W}) \sum_{i=1}^{8} h_{i} \eta^{i} - 0.012 \eta^{-\frac{16}{27}} C_{Q_{3}}(m_{W}), \]

\[ C_{8}(m_{b}) = C_{8}(m_{W}) + \frac{4\pi}{\alpha_{s}(m_{W})} \left( \frac{8}{87} (1 - \eta^{-\frac{2}{3}}) - \frac{4}{33} (1 - \eta^{-\frac{2}{3}}) \right) C_{2}(m_{W}), \]

\[ C_{9}(m_{b}) = C_{9}(m_{W}), \]

\[ C_{Q_{i}}(m_{b}) = \eta^{-\gamma_{Q}/\beta_{0}} C_{Q_{i}}(m_{W}), \quad i = 1, 2, \]

\[ C_{1}(m_{b}) = \frac{1}{2} (\eta^{-\frac{6}{27}} - \eta^{-\frac{12}{27}}) C_{2}(m_{W}), \]

\[ C_{2}(m_{b}) = \frac{1}{2} (\eta^{-\frac{6}{27}} + \eta^{-\frac{12}{27}}) C_{2}(m_{W}) \]  

(9)

where \( \gamma_{Q} = -4 \) is the anomalous dimension of \( \sum_{L} b_{R} \) \[ 14 \], \( \beta_{0} = 11 - 2n_{f}/3, \eta = \alpha_{s}(m_{b})/\alpha_{s}(m_{W}), \]

\( C_{2}(m_{W}) = -1 \), and \( C_{i}(m_{W}) \) (i=7,8,9) and \( C_{Q_{i}}(m_{W}) \) (i=1,2,3) can be found in Refs. \[ 20 \] and \[ 17 \], respectively (since flavor changing contributions from gluino-downtype squark loop and neutrino-downtype squark loop are very small compared to those from chargino-uptype squark loop in mSUGRA \[ 20 \], \[ 17 \], we neglect them in this paper).

With the matrix element (eq. (6)), it is easy to derive the invariant dilepton mass distribution as follows \[ 16 \]:

\[ \frac{d\Gamma(B \to X_{s} \tau^{+} \tau^{-})}{d\hat{s}} = B(B \to X_{s}) \frac{\alpha^{2}}{4\pi^{2} f(\frac{m_{W}}{m_{b}})} (1 - \hat{s})^{2} \left( 1 - \frac{4t^{2}}{\hat{s}} \right)^{\frac{1}{2}} \left| V_{tb} V_{ts}^{*} \right|^{2} D(\hat{s}), \]

\[ D(\hat{s}) = 4\left[ C_{7}^{2} \left( 1 + \frac{2}{\hat{s}} \right) \left( 1 + \frac{2\hat{s}}{\hat{t}} \right) + \left| C_{S}^{eff} \right|^{2} (2\hat{s} + 1)(1 + \frac{2\hat{t}}{\hat{s}}) + \left| C_{9} \right|^{2} (2\hat{s} + 1 - 4\hat{s}) \frac{2\hat{t}^{2}}{\hat{s}} \right] \]

\[ + 12 \Re \left( C_{S}^{eff} C_{7}^{*} \right) (1 + \frac{2\hat{t}}{\hat{s}}) + \frac{3}{2} \left| C_{Q_{1}} \right|^{2} (1 - \frac{4\hat{t}^{2}}{\hat{s}}) \hat{s} + \frac{3}{2} \left| C_{Q_{2}} \right|^{2} \hat{s} + 6 \Re \left( C_{9} C_{Q_{2}}^{*} \right) t \]  

(10)

where \( t = m_{\tau}/m_{b}, B(B \to X_{s}) \) is the branching ratio, and \( f(x) \) is the phase space factor: \( f(x) = 1 - 8x^{2} + 8x^{6} - x^{8} - 24x^{4} \ln x \). Backward-forward asymmetry can also be calculated to be:

\[ A(\hat{s}) = \frac{\int_{0}^{1} dz \frac{d\Gamma}{d\hat{s} dz} - \int_{-1}^{0} dz \frac{d\Gamma}{d\hat{s} dz}}{\int_{0}^{1} dz \frac{d\Gamma}{d\hat{s} dz} + \int_{-1}^{0} dz \frac{d\Gamma}{d\hat{s} dz}} = \frac{E(\hat{s})}{D(\hat{s})}, \]
\[
E(\hat{s}) = 3\sqrt{1 - \frac{4t^2}{s}} \text{Re}(C_{8}^{eff}C_{9}^{*}\hat{s} + 2C_{7}C_{9}^{*} + C_{8}^{eff}C_{Q}^{*}t + 2C_{7}C_{Q}^{*}t)
\]  

(11)

The direct CP asymmetries in decay rate and backward-forward asymmetry for \(B \to X_s l^+ l^-\) and \(\bar{B} \to \bar{X}_s l^+ l^-\) are defined by

\[
A_{CP}^B(\hat{s}) = \frac{d\Gamma/d\hat{s} - d\Gamma/d\hat{s}^*}{d\Gamma/d\hat{s} + d\Gamma/d\hat{s}^*},
\]

\[
A_{CP}^{\bar{B}}(\hat{s}) = \frac{A(\hat{s}) - \overline{A}(\hat{s})}{A(\hat{s}) + \overline{A}(\hat{s})}
\]

(12)

(13)

In SM the direct CP violation can only arise from the interference of non-trivial weak phases which are contained in CKM matrix elements. Therefore, it is suppressed by the ratio of CKM matrix elements, \(\frac{V_{ts}V_{tb}}{V_{ts}V_{tb}} \sim O(10^{-2})\). The CP asymmetry in the branching ratio is predicted to be of the order of \(10^{-3}\) [22], which is unobservably small. Thus, an observation of CP violation in this mode would signal the presence of new physics. In mSUGRA without new CP violating phases, the alignment of masses of the first two generation squarks causes a cancellation and only contributions proportional to \(\frac{m_\mu}{m_W}\) or \(\frac{m_\tau}{m_W}\) (coming from couplings of chargino-right handed up type squarks-down type quarks) remain for the imaginary parts of Wilson coefficients. But as the mixings of left and right handed squarks of 1st and 2nd generations are negligibly small, imaginary parts of Wilson coefficients can not be large. So without new CP violating phases, mSUGRA will induce CP violating effects in the same order as those in SM in these processes. In mSUGRA with new CP violating phases, one may expect larger CP asymmetries due to the presence of new phases of order one.

Another CP violating observable in \(B \to X_s l^+ l^-\) is the normal polarization of the lepton in the decay, \(P_N\), which is the T-violating projection of the lepton spin onto the normal of the decay plane, i.e \(P_N \sim \hat{s}_i \cdot (\vec{p}_\ell \times \vec{p}_{\ell^*})\) [21]. A straightforward calculation leads to

\[
P_N = \frac{3\pi}{4} \sqrt{1 - \frac{4t^2}{s}} \frac{\hat{s}}{s} \text{Im} \left[2C_8^{charged}C_9^{*}t + 4C_9C_7^{*}t + C_8^{charged}C_Q^{*} + 2C_7C_{Q_1} + C_8C_{Q_2}\right] / D(\hat{s})
\]

(14)

\(P_N\) have been given in the ref.[23], where they give only two terms in the numerator of \(P_N\).

We may find from eq.(12) that contributions to \(P_N\) are of imaginary parts of product of two Wilson coefficients, i.e the product of real part of one Wilson coefficient and imaginary part of another one. So compared to \(A_{CP}^B\), \(P_N\) can be larger when the imaginary parts of some relevant Wilson coefficients have significant values. Because the normal polarization is proportional to the mass of lepton (we remind that \(C_{Q_i} (i=1,2)\) is proportional to \(m_l\) [17]), it will be unobservably small for \(l=e\). However, for \(l=\mu, \tau\), when \(C_{Q_i} (i = 1, 2)\) get significant values, \(\text{Im}(C_8^{charged}C_{Q_1})\) and \(\text{Im}(C_9C_{Q_2})\), as well as \(\text{Im}(C_8^{charged}C_9)\), will be main contributions, which can make \(P_N\) large.

5 numerical results

In the numerical calculations in mSUGRA the SUSY parameters are taken as

\[M_0 = 800\text{GeV}, M_{1/2} = 180\text{GeV}, |A_0| = 350\text{GeV}, \tan\beta = 2, \phi_\mu = \pm \pi/30;\]
\[ M_0 = 400 \text{GeV}, \ M_{1/2} = 550 \text{GeV}, |A_0| = 750 \text{GeV}, \tan\beta = 36, \phi_\mu = \pm \pi/1000 \]

with \( \phi_{A_0} \sim O(1) \) which satisfy the constraints from EDMs of neutron and electron, as well as the constraint from \( b \to s\gamma \). We follow Ref. \([14]\) for detailed procedures of calculation.

With these choices the direct CP violation in branching ratio is of the order \( O(10^{-3}) \) for \( l=e, \mu \) and \( \tau \), i.e. the same order as that in SM, thus is hard to be observed. However, as pointed out in ref. \([8]\), \( \text{Im} C_7 \) can be as large as \( C_7 \) of SM at some values of the parameters and consequently a CP violation significantly larger than the result may probably be obtained. The results for \( A_{CP}^2 \) at \( s = 0.76 \) and \( P_N \) which is experimentally measured, are shown in Figs. 3 and 4, where we do not show the regions of \( \phi_{A_0} \) in which the constraints from EDMs of electron and neutron and \( B_r(B \to X_s\gamma) \) are satisfied (the regions can be easily read from figs.1,2). One can see from fig.3 that in the large \( \tan\beta \) case the direct CP asymmetry in backward-forward asymmetry, \( A_{CP}^2 \), for \( l = \tau \) can be 0.5 to 1 percent in the most of range of \( \phi_{A_0} \) and sensitive to \( \phi_{A_0} \), which is not easy to be observed. \( A_{CP}^2 \) for \( l=e, \mu \) is as small as \( A_{CP}^1 \). This large difference between \( \tau \) and \( e, \mu \) is due to that the contributions of exchanging NHBs are proportional to the mass square of lepton (see eq. \([11]\) ).

Fig. 4 shows the CP violating normal polarization. The polarization is almost equal to zero for \( l=e \) no matter how \( \tan\beta \) is large or small because of negligible smallness of electron mass. For \( l=\mu \), it can reach only 0.5 percent and is sensitive to \( \phi_\mu, \phi_{A_0} \) when \( \tan\beta \) is large. For \( l=\tau \), it is close to 2 percent and not sensitive to \( \phi_\mu, \phi_{A_0} \) for small \( \tan\beta \) and can be as large as 4 percent and sensitive to \( \phi_\mu, \phi_{A_0} \) for large \( \tan\beta \), which could be observed in the future B factories with \( 10^8 - 10^{12} \) B hadrons per year \([24]\). The reason is that \( C_{Q_1} \) can be neglected for \( \tan\beta = 2 \) and \( C_9, C_{8}^{\text{eff}} \) depend on \( \phi_\mu, \phi_{A_0} \) weakly. But for \( \tan\beta = 36, C_{Q_1} \)'s have large imaginary parts and strongly depend on \( \phi_\mu, \phi_{A_0} \). There is a cancelation between contributions from \( \text{Im}(C_{8}^{\text{eff}}sC_{Q_1}) \) and \( \text{Im}(C_{8}^{\text{eff}}C_{Q_1}) \) in the large \( \tan\beta \) case, since when \( \phi_\mu \) is around 0 the real part of \( C_{Q_1} \) is of the opposite sign of the real part of \( C_9 \). Combined with the suppression coming from the enhancement of \( D(s) \) induced by large \( C_{Q_1} \) and \( C_{Q_2} \), this cancellation may cause even a lower value of \( P_N \) (for some values of \( \phi_{A_0} \)) than that in SM, as can be seen from fig.4. The sensitivity of the normal polarization to \( \phi_\mu, \phi_{A_0} \) can be used to discriminate small \( \tan\beta \) from large \( \tan\beta \).

In summary, we have analyzed EDMs and \( B \to X_s\gamma \) constraints on the parameter space in mSUGRA with CP violating phases, in particular, in the case of large \( \tan\beta \). we have calculated the direct CP asymmetries and CP violating normal polarization for the rare decays \( B \to X_s l^+l^- \) (\( l=e, \mu, \tau \)) in the model. When \( \phi_\mu \) and \( \phi_{A_0} \) are of \( O(0.1) \) and \( O(1) \) respectively, the direct CP asymmetries in branching ratio for \( l=e, \mu, \tau \) are about \( 10^{-3} \), i.e the same order as that in SM. So it is hopeless to observe \( A_{CP}^1 \) in mSUGRA even with large CP violating phases no matter how \( \tan\beta \) is large or not. For \( l=e, \mu \), \( A_{CP}^2 \) is as small as \( A_{CP}^1 \). In the case of large \( \tan\beta \) the CP violating normal polarization of muon in \( B \to X_s \mu^+\mu^- \) can reach 0.5 percent. For \( B \to X_s \tau^+\tau^- \), \( A_{CP}^2 \) can reach one percent and the normal polarization of \( \tau \) can be as large as a few percent when \( \tan\beta \) is large (say, 36), and consequently it is possible to observe the normal polarization. Thus, a few percent CP asymmetry would be discovered in \( B \to X_s \tau^+\tau^- \) if the nature gives us a CP violating SUSY with large (say, \( \geq 30 \)) \( \tan\beta \).

Recently, it is shown that if gaugino masses at high energy scale are nonuniversal there exist two additional phases which can make cancellations happened easier than the universal case.
It is found that the phases may be large while certain approximate relations hold among the mass parameters and phases, resulting in cancellations in the calculation of the electron and neutron EDMs in the small $\tan \beta$ case. It is worth to extend the analysis to the large $\tan \beta$ case and investigate its effects on rare B decays.

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Fig. 1a. EDME($10^{-27}$) as functions of $\phi_{A_0}$ from 0 to $2\pi$. Lines labeled by 1 and 2 refer to $\phi_\mu = \pm \pi/1000$ with $\tan\beta = 36$. Lines labeled by 3 and 4 refer to $\phi_\mu = \pm \pi/30$ with $\tan\beta = 2$. We choose $M_0 = 400$, $M_{1/2} = 550$, $|A_0| = 750$ for large $\tan\beta$, and $M_0 = 800$, $M_{1/2} = 180$, $|A_0| = 350$ for small $\tan\beta$. 
Fig. 1b. EDMN($10^{-26}$) as functions of $\phi_{\lambda_0}$ from 0 to $2\pi$. Lines labeled by 1 and 2 refer to $\phi_\mu = \mp \pi/1000$ with $\tan \beta = 36$. Lines labeled by 3 and 4 refer to $\phi_\mu = \mp \pi/30$ with $\tan \beta = 2$. We choose $M_0 = 400$, $M_{1/2} = 550$, $|A_0| = 750$ for large $\tan \beta$, and $M_0 = 800$, $M_{1/2} = 180$, $|A_0| = 350$ for small $\tan \beta$. 
Fig. 2. $\text{Br}(b \rightarrow s \gamma)$ as functions of $\phi_{A_0}$ from 0 to $2\pi$. Line labeled by 1 refers to $\phi_\mu = \pi/1000$, $M_0 = 400$, $M_{1/2} = 550$, $|A_0| = 750$, $\tan \beta = 36$ and line labeled by 2 refers to $\phi_\mu = \pi/30$, $M_0 = 800$, $M_{1/2} = 180$, $|A_0| = 350$ and $\tan \beta = 2$. 
Fig. 3. $A_{CP}^2$ as functions of $\phi_{A_0}$ from 0 to $2\pi$. Lines labeled by 1, 3 and 5 refer to $\phi_\mu = -\pi/1000$, and lines labeled by 2, 4 and 6 refer to $\phi_\mu = \pi/1000$. As three groups, (1, 2), (3, 4) and (5, 6), lines correspond to $B \rightarrow X_s e^+ e^-$, $B \rightarrow X_s \mu^+ \mu^-$ and $B \rightarrow X_s \tau^+ \tau^-$ separately. Other parameters are chosen such that $M_0 = 400$, $M_{1/2} = 550$, $|A_0| = 750$ and $\tan \beta = 36$. Here $s = 0.76$ and lines labeled by 1, 2, 3 and 4 almost coincide with axis.
Fig. 4. $P_N$ as functions of $\phi_{A_0}$ from 0 to $2\pi$. Lines labeled by 1, 3 and 5 correspond to $\tan\beta = 36$, $M_0 = 400$, $M_{1/2} = 550$, $|A_0| = 750$ and $\phi_{\mu} = -\pi/1000$. Lines labeled by 2, 4 and 6 refer to $\tan\beta = 2$, $M_0 = 800$, $M_{1/2} = 180$, $|A_0| = 350$ and $\phi_{\mu} = -\pi/30$. As three groups, (1,2), (3,4) and (5,6), lines correspond to $B \to X_s e^+ e^-$, $B \to X_s \mu^+ \mu^-$ and $B \to X_s \tau^+ \tau^-$ separately. Here lines labeled by 1, 2 and 4 almost coincide with each other.