Optimization of refueling times in fast neutron reactors

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Abstract. The importance of solving issues of increasing energy efficiency and energy saving for the development of the Russian economy was emphasized. The main directions of increasing the energy efficiency of the nuclear power industry (increasing the efficiency of using primary fuel and reducing the shutdown time of the power unit for repair and reloading of nuclear fuel) are highlighted. The importance of route optimization in solving applied problems of nuclear power is updated, in particular, in minimizing the time of nuclear fuel reloading, increasing the utilization factor of the installed capacity of NPP power units. The necessity of periodic reloading of nuclear fuel in reactors has been substantiated. The features of nuclear fuel reloading in thermal and fast neutron reactors are presented. The necessity of using in fast neutron reactors a system for guiding a capture on a fuel assembly using eccentrically located rotary plugs is substantiated. The necessity and importance of optimization of the process of guidance and movement of the overload mechanism in order to reduce the time to stop the power unit is shown. Mathematical models for the mechanisms of reloading fuel assemblies have been built. Time-optimal algorithms for the operation of overload mechanisms with two and three rotating plugs are proposed.

1. Introduction

"Energy efficiency, energy saving, nuclear power" is one of the priority directions of the development of science, technology and technology in the Russian Federation. There are various ways to improve the energy efficiency of nuclear power plants. For NPPs with thermal reactors (VVER, RBMK), the thermal efficiency is approximately 33%, with fast reactors - more than 40%. This means that most of the thermal energy released in the reactor core as a result of the nuclear fission chain reaction is released into the atmosphere. The output of useful energy can be increased, for example, by utilizing low-grade waste heat and nuclear heat supply [1], [2], [3].

Another way is to increase the utilization factor of the installed capacity of the power unit by reducing its downtime for repairs and reloading of nuclear fuel. The duration of scheduled repairs can be reduced by careful planning of work, their optimization, and the introduction of high-performance repair equipment. The critical path determines the duration of the shutdown of the power unit and includes operations for reloading nuclear fuel. Most of these operations are strictly regulated in accordance with nuclear safety rules (for example, the speed of raising and lowering fuel assemblies, moving them in a horizontal plane). But there is a potential in reducing the time of pointing the mechanism to certain coordinates and rearranging the fuel assemblies through route optimization. Work on the use of
route optimization as applied to applied problems in nuclear power has been carried out by specialists from the departments of nuclear power, applied mathematics and the IMM UB RAS since the early 2000s. The results of these studies have shown their effectiveness [4], [5], [6], [7].

In VVER reactors, refueling of fuel assemblies is carried out using a refueling machine (RM) under a layer of water in the refueling basin, which performs the functions of radiation protection and removal of residual heat. The RM is a bridge moving over the reactor and the spent fuel pool, along which the cart with the reloading mechanism moves, i.e. guidance of the gripper of the reloading machine is carried out along two mutually perpendicular coordinates.

The features of route optimization in relation to such a scheme for guiding the mechanism of reloading (capturing PM) of VVER-type reactors, which form the basis of nuclear power in Russia and the world, are discussed in detail in the monograph [8]. The problem of reloading nuclear fuel for a VVER-1000 reactor contains a large number of various, poorly formalized restrictions. To solve this problem P.A. Chentsov constructed an efficient greedy algorithm, which was implemented in the form of a standard computer program.

Russia has many years of experience in the construction and operation of sodium-cooled fast reactors (RBR), which represent the innovative development of nuclear power. For 40 and 5 years, the world's most powerful fast-neutron power reactors BN-600 and BN-800 have been reliably and safely operated at the Beloyarskaya NPP. The design of the BN-1200 reactor has been developed, which by its characteristics meets the requirements of the fourth generation of safety [9], [10].

By virtue of the design feature of BN-type reactors, the guidance of the grip of the reloading mechanism on the fuel assembly to be reloaded is carried out by rotating two or three eccentrically located rotary plugs, on the smaller of which the gripper is located.

In the presented work, the ways of solving the optimization problem for the mechanisms of reloading fast neutron reactors with sodium coolant are considered.

2. Features of nuclear fuel reloading
A feature of the operation of a nuclear reactor (the occurrence of a self-sustaining chain reaction of fission) is the presence of a critical mass of nuclear fuel in the core. Since the concentration of fissile isotope nuclei (uranium-235 or plutonium-239) decreases during operation of a nuclear reactor, an excess of fissile material above the critical mass is loaded into the core, which is compensated by special neutron absorbers of the control and protection system (CPS). At present, the loading of fresh nuclear fuel ensures the operation of the NPP power unit with VVER reactors until the next refueling within 12 months, for new power units - 18 months. For the BN-600 reactor, the operating time at rated power between overloads is up to 160 days [11].

In general, the reloading of nuclear fuel is as follows: spent 3-4-5 years fuel assemblies (depending on the duration of the fuel campaign for VVER-type reactors) are unloaded from the central part of the core into the spent fuel pool; partially burnt fuel assemblies move (rearrange) in the core from the periphery to its center; fresh fuel assemblies are loaded (installed) on the periphery of the core. The procedure for rearranging the fuel assemblies is strictly regulated and is carried out taking into account the observance of nuclear safety (preventing the occurrence of an uncontrolled fission chain reaction).

In the core of fast breeder reactors (RBR), the power density is about 500 MW / m3, which is 50 times more than in RBMK reactors and 5 times more than in VVER. Sodium is used to ensure reliable heat dissipation. Being a highly efficient heat carrier, sodium has a number of disadvantages, first of all it is high chemical activity with respect to water and air. The presence of sodium in the reactor vessel requires its reliable isolation from the environment in all operating modes, including refueling. Therefore, in modern fast reactors with a sodium coolant, a system is used to guide the gripper of the refueling mechanism of fuel assemblies to the required coordinates of the core, breeding zone, in-reactor storage or lift slots by rotating two (three for BN-800) eccentrically located rotary plugs (columns), which are at the same time a sealing lid of the reactor and biological shielding (Fig. 1). Rotation tightness is ensured with the help of a water seal filled with a tin-bismuth alloy with a low melting point [11].
Some of the first such reactor plants (RU) had a loop layout (for example, BN-350 - [12]), others - integral, when all the equipment of the primary circuit is located in one vessel-vessel of the reactor. Currently, the integral layout is recognized as the most optimal.

In this paper, we consider the problem of the fastest possible guidance of a gripper located on a smaller plug on a given fuel assembly under the assumptions: for the BN-600 reactor, that plugs can rotate simultaneously, and for the BN-800 reactor, that three plugs will rotate sequentially, i.e. only one plug can be turned at a time.

To unload a burned-out fuel assembly from the reactor, the reloading machine must move to its coordinates, lower the gripper, perform coupling, lift the gripper from the fuel assembly, transfer it to the spent fuel pool, lower it, release and raise the gripper. These operations are performed for each unloaded fuel assembly, regardless of the reloading order. In the reverse order, fresh fuel assemblies are loaded. The time spent on the operation of lowering-lifting the gripper of the reloading machine, coupling-uncoupling the gripper from the fuel assembly, can be ignored in the calculation algorithm, since it is a parameter that cannot be optimized in this context. As a result, the problem can be represented as flat, while some of the displacements are not considered [8]. The considered problem of optimizing the sequence of operations for rearranging fuel assemblies consists in minimizing the time for replacing nuclear fuel and, accordingly, reducing the downtime of the NPP power unit.

3. Mathematical models of overload mechanisms and optimal control algorithms

Reloading mechanism for the BN-600 reactor consists from two rotating plugs. The big plug is a disk of radius $R_1$ with a geometric center fixed during movement. The large plug has an eccentric circular cutout of radius $R_2$, into which a small plug is placed - a disk of radius $R_2$, the center of mass of which is at a distance $e_2$ from its geometric center (Fig. 2). When describing the interaction of plugs, we neglect the friction forces. The movement of the mechanical system is flat. The axial moment of inertia of the large plug is $I_1$ and of the small plug relative to its center of mass - $I_2$, the mass of the second plug is $m_2$. The distance between the geometric centers of the large and small plugs is $e_1$. 

![Figure 1. BN-600 reactor: 1 - core; 2 - fuel assembly in the liner of the inclined lift; 3 - large swivel plug; 4 - small swivel plug (central swivel column); 5 - circulation pump.](image-url)
Figure 2. Scheme of the overload mechanism of the BN-600 reactor.

Under the assumption that the geometric center of mass of the small plug coincides with its center of mass (i.e., the parameter $e_z = 0$), according to [13, 14], the mathematical model describing the movement of the overload mechanism will have the form:

$$\dot{\phi}_1 = \frac{1}{\Delta} J_2 (u_1 - u_2),$$
$$\dot{\phi}_2 = \frac{1}{\Delta} (-J_2 u_1 + (J_1 + J_2 + m_2 e_1^2) u_2),$$

where

$$\Delta = J_2 (J_1 + m_2 e_1^2).$$

The control moments applied to the large and small plugs have restriction

$$|u_1| \leq \mu_1,$$
$$|u_2| \leq \mu_2.$$  \hfill (2)

Let

$$a_1 = \frac{1}{J_1 + m_2 e_1^2},$$
$$a_2 = \frac{J_1 + J_2 + m_2 e_1^2}{J_2 (J_1 + m_2 e_1^2)}.$$

Then the system of equations (1) takes the form

$$\dot{\phi}_1 = a_1 (u_1 - u_2),$$
$$\dot{\phi}_2 = -a_1 u_1 + a_2 u_2.$$  \hfill (3)
Let us rewrite system (3) in normal form. As a result, we get
\[
\begin{align*}
\dot{\varphi}_1 &= \omega_1 \\
\dot{\omega}_1 &= a_1 (u_1 - u_2) \\
\dot{\varphi}_2 &= \omega_2 \\
\dot{\omega}_1 &= -a_1 u_1 + a_2 u_2
\end{align*}
\] (4)

The coordinates of a point in a fixed coordinate system \( r \) and \( \theta \) are related to the variables \( \varphi_1 \) and \( \varphi_2 \) using the formulas
\[
R = \sqrt{(R_1 - R_2)^2 + R_2^2 - 2(R_1 - R_2)R_2 \cos(\pi - \varphi_2)},
\]
\[
\theta = \varphi_1 + \arccos \frac{R^2 + (R_1 - R_2)^2 - R_2^2}{2(R_1 - R_2)R} \text{sign}(\varphi_2).
\]

Let the start point have coordinates \( x^0 = (\varphi_1^0, 0, \varphi_2^0, 0)^T \), and the end point have coordinates \( x^f = (\varphi_1^f, 0, \varphi_2^f, 0)^T \). The phase vector \( x(t) = (\varphi_1(t), \omega_1(t), \varphi_2(t), \omega_2(t))^T \) is described by the system of equations (4). Consider the problem of the fastest movement of the phase point from the initial position to the final one subject to constraints (2). This problem was solved in [13, 14] using the L.S. Pontryagin maximum principle [15]. Let's introduce the functions \( \varphi = \varphi_1 + \varphi_2 \) and \( \chi = \varphi_1 + \frac{a_1}{a_2} \varphi_2 \). Let's \( \Delta \varphi = \varphi_1^f - \varphi_1^0 - \varphi_2^0 \) and \( \Delta \chi = \varphi_1^f + \frac{a_1}{a_2} \varphi_2^f - \varphi_1^0 - \frac{a_1}{a_2} \varphi_2^0 \). Then, according to [14,15], if the condition
\[
a_2 |\Delta \chi| \leq \frac{\mu_1}{\mu_2} a_1 |\Delta \varphi|,
\] (5)
then the optimal control solving the time optimal problem will have the form
\[
\begin{align*}
u_1(t) &= \frac{4a_2 (\Delta \varphi_1 + \frac{a_1}{a_2} \Delta \varphi_2)}{a_1 (a_2 - a_1) \varphi_2^f} \text{sign} \left( t - \frac{\varphi}{2} \right), \\
u_2(t) &= \mu_2 \text{sign} (\varphi(\theta) - \varphi(0)) \text{sign} \left( \frac{\varphi}{2} - t \right).
\end{align*}
\]
if the condition
\[
a_2 |\Delta \chi| \geq \frac{\mu_1}{\mu_2} a_1 |\Delta \varphi|,
\] (6)
then
\[
\begin{align*}
u_1(t) &= \mu_1 \text{sign} (\chi(\theta_1) - \chi(0)) \text{sign} \left( \frac{\varphi}{2} - t \right), \\
u_2(t) &= \frac{4 \Delta \varphi}{(a_2 - a_1) \varphi_1^f} \text{sign} \left( \frac{\varphi_1}{2} - t \right).
\end{align*}
\]

Thus, for all admissible starting and ending points, we have constructed optimal controls. The found optimal controls are relay ones with one switching. The optimal time for case (5) is determined by the formula
\[
\vartheta_2 = \sqrt{\frac{|\varphi(\theta) - \varphi(0)|}{\mu_2}}.
\]
and for case (6) is given by the formula

\[ \vartheta_1 = \sqrt{\frac{\alpha_2 |\chi(\vartheta_1) - \chi(0)|}{\alpha_1 \mu_2}} \]

In [16], a similar problem was considered for the BN-800 reactor. Let us introduce the notation. Let \( R_1 \) be the radius of the disk - the big plug, \( O_1 \) is the geometric center of the big plug, which remains stationary all the time. The large plug has an eccentric circular cutout of radius \( R_2 \) inside which is placed the middle plug - a disk of radius \( R_2 \), the geometric center \( O_2 \), which during movement remains stationary relative to the large plug. The middle plug has an eccentric circular cut of radius \( R_3 \), inside which is placed a small plug - a disk of radius \( R_3 \), the geometric center \( O_3 \), which during movement remains stationary relative to the middle plug (see Fig. 3).

![Figure 3](image-url). Scheme of the overload mechanism of the BN-800 reactor.

When describing the interaction of plugs, we neglect the friction forces. The movement of the mechanical system is flat. The mathematical model for the transfer mechanism with three plugs is as follows

\[
J_1 (\ddot{\varphi}_1 + \dot{\varphi}_1 + \dot{\varphi}_2) + J_3 (\dot{\varphi}_1 + \dot{\varphi}_2 + \dot{\varphi}_3) \\
+ m_2 (e_2^2 \ddot{\varphi}_1 + e_2 \alpha \cos(\varphi_2 + \alpha)(2\dot{\varphi}_1 + \dot{\varphi}_2) - e_2 \alpha \sin(\varphi_2 + \alpha)(2\dot{\varphi}_1 + \dot{\varphi}_2) \dot{\varphi}_2 \\
+ m_3 (e_3^2 \ddot{\varphi}_1 + e_3^2 (\ddot{\varphi}_1 + \ddot{\varphi}_2) + e_2 e_3 \cos(\varphi_2)(2\dot{\varphi}_1 + \dot{\varphi}_2) \\
- e_2 e_3 \sin(\varphi_2)(2\dot{\varphi}_1 + \dot{\varphi}_2) \dot{\varphi}_2) = u_1, \\
J_2 (\ddot{\varphi}_2 + \dot{\varphi}_2) + J_3 (\ddot{\varphi}_2 + \dot{\varphi}_2 + \dot{\varphi}_3) + m_2 e_2 a (\cos(\varphi_2 + \alpha) \ddot{\varphi}_1 + \sin(\varphi_2 + \alpha) \dot{\varphi}_2^2) \\
+ m_3 \left( e_3^2 (\ddot{\varphi}_1 + \ddot{\varphi}_2) + e_2 e_3 (\cos(\varphi_2) \ddot{\varphi}_1 + \sin(\varphi_2) \dot{\varphi}_1^2) \right) = u_2, \\
J_3 (\ddot{\varphi}_3 + \dot{\varphi}_3) = u_3
\]
As generalized coordinates, we select the angle of rotation of the large plug \( \varphi_1 \), the angle of rotation of the middle plug - \( \varphi_2 \), with respect to the large plug, and the angle of rotation of the small plug - \( \varphi_3 \), with respect to the middle plug. The generalized forces are the control moments \( u_1 \), \( u_2 \), and \( u_3 \), applied to the large, medium and small plugs, respectively.

Note that the mathematical model in the case of three rotating plugs is much more complicated than the model for two rotating plugs. In this regard, it was not possible to obtain an optimal solution under the assumption that all three plugs can rotate simultaneously.

A variant of the problem was considered when the plugs rotate sequentially. According to (7), if only the \( i \)-th plug rotates, then the equation of motion will have the form

\[
J_i'' \varphi_i = u_i, \quad i=1,2,3.
\] (8)

Here

\[
J_1'' = J_1 + J_2 + J_3 + m_2 e_2^2 + 2e_2 a \cos(\varphi_2 + \alpha) + m_3 \left( e_2^2 + e_3^2 + 2e_2 e_3 \cos(\varphi_2) \right),
\]

\[
J_2'' = J_2 + J_3 + m_3 e_3^2,
\]

\[
J_3'' = J_3.
\]

According to [13, 14], the optimal time for the reversal of one \( i \)-th plug through the angle \( \Delta \varphi_i \) is set by the value

\[
\theta_i = 2 \frac{J_i}{\mu_i} |\Delta \varphi_i| \quad i = 1,2,3.
\] (9)

Then the total time for the successive turn of three traffic jams is given by the value

\[
T = \theta_1 + \theta_2 + \theta_3 = 2 \frac{J_1}{\mu_1} |\Delta \varphi_1| + 2 \frac{J_2}{\mu_2} |\Delta \varphi_2| + 2 \frac{J_3}{\mu_3} |\Delta \varphi_3|.
\] (10)

In this case, the values \( \Delta \varphi_i \) \( (i = 1,2,3) \) are connected by the constraints

\[
x = R_3 \cos(\varphi_{10} + \varphi_{20} + \varphi_{30} + \Delta \varphi_1 + \Delta \varphi_2 + \Delta \varphi_3) + e_3 \cos(\varphi_{10} + \varphi_{20} + \Delta \varphi_1 + \Delta \varphi_2) + e_2 \cos(\varphi_{10} + \Delta \varphi_1),
\]

\[
y = R_3 \sin(\varphi_{10} + \varphi_{20} + \varphi_{30} + \Delta \varphi_1 + \Delta \varphi_2 + \Delta \varphi_3) + e_3 \sin(\varphi_{10} + \varphi_{20} + \Delta \varphi_1 + \Delta \varphi_2) + e_2 \sin(\varphi_{10} + \Delta \varphi_1).
\] (11)

As a result, we got a nonlinear programming problem: it is required to minimize functional (10) under constraints (11).

The problem of minimizing functional (10) under constraints (11) was solved in [16] numerically. It follows from (11) that in fact \( \Delta \varphi_i \) \( (i = 1,2) \) depend on \( \Delta \varphi_3 \). Below are the graphs of \( T \) dependence \( \Delta \varphi_3 \). Software has been developed that allows finding a solution to problem (10) - (11). Below are the graphs of the functions \( T = T (\Delta \varphi_3) \).
Figure 4. The boundary conditions for the problem, the solution of which is this in Figure, were as follows, $x = 0.4; y = 0.6; \Delta \phi_{10} = 1; \Delta \phi_{20} = 1; \Delta \phi_{30} = -1$.

Figure 5. The boundary conditions for the problem, the solution of which is this in Figure, were as follows, $x = 0.5; y = 0.5; \Delta \phi_{10} = -1; \Delta \phi_{20} = 1; \Delta \phi_{30} = 1$.

4. **Conclusion**
An optimal algorithm is proposed for the problem of the fastest guidance of the capture of the fuel re-fueling mechanism for the BN-600 reactor. For the BN-800 reactor, assuming that the plugs can rotate only sequentially, the time-optimal guidance algorithm is proposed. Application of the proposed algorithms will help to reduce the time of reloading nuclear fuel. And this, in turn, will lead to a reduction in the downtime of a reactor shut down for reloading nuclear fuel.

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References

[1] Kostarev V S, Tashlykov O L and Klimova V A 2019 The increasing of the energy efficiency of nuclear power plants with fast neutron reactors by utilizing waste heat using heat pumps. *IOP Conf. Ser.: Mater. Sci. Eng.* **552**(1) 012022

[2] Tashlykov O L, Tolmachev E M, Semenov M Yu and Sapožnikov B G 2012 Snizhenie teplovy’x nagruzok AE’S na okruzhayushhuyu sredu putem ispol’zovaniya teplovy’x nasosov v sxene osnovnogo kondensatora paroturbinnoj ustanovki [Reducing heat loads NPP on the environment through the use of heat pumps in the circuit of the main steam turbine condenser] *Al’ternativnaya E’nergetika i E’kologiya* 3 16-21 [In Russian]

[3] Tashlykov O L, Koin I V and Kokorin V V 2012 Utilizatsiya nizkopotentsial’noy teploty AES s reaktorom na bystrykh neytronakh s ispol’zovaniem teplovoego nasosa [Disposal of low-grade heat from the nuclear reactor on fast neutrons with heat pump] *Al’ternativnaya E’nergetika i E’kologiya* 3 22-5

[4] Sesekin A N, Tashlykov O L, Shcheklein S Y and Chentsov A G 2014 Route optimization in the removal of radiation hazards *WIT Transactions on Ecology and the Environment* **190**(2) 919-26

[5] Petunin A A, Sesekin A N, Tashlykov O L and Chentsov A A 2017 Marshrutnaya optimizaciya na ob’ektax ispol’zovaniya yadernoj e’nergli i v mashin ostroneni [Route optimization on the nuclear objects and in mechanical engineering] *Mathematical modeling and information technologies* MMIT 2016 (Yekaterinburg: CEUR Workshop Proceedings) 69-79 [In Russian]

[6] Chentsov A A, Chentsov A G, Sesekin A N and Tashlykov O L 2019 Application of a generalized bottleneck routing problem to the task of adhering to acceptable doses of radiation during the dismantling of radiation hazardous objects *IFAC-Papers On Line* **52**(13) 2656-61

[7] Grigoryev A M and Tashlykov O L 2019 Rou te optimization during works in non-stationary radiation fields with obstacles *AIP Conf. Proc.* **2174**(1) 020216

[8] Korobkin V V, Sesekin A N, Tashlykov O L and Chentsov A A 2012 Metody’ marshrutizacii i ix prilozeniya v zadachax pov’ysheniya e’fektivnosti i bezopasnosti e’kspluatacii atomny’x stancij [Routing methods and their applications in improving the safety and efficiency of operation of nuclear power plants] (Moscow: Novye tehnologii) [In Russian]

[9] Tashlykov O L, Sheklein S E, Sesekin A N, Chentsov A A, Nosov Y V and Smyshlaeva O 2014 Ecological features of fast reactor nuclear power plants (NPPs) at all stages of their life cycle *WIT Transactions on Ecology and the Environment* **190**(2) 907-18

[10] Tashlykov O L, Sheklein S E, Nosov Y V and Smyshlaeva O 2016 Ecological foresight in the nuclear power of XXI century *Int. J. of Energy Production and Management* **1**(2) 133-140

[11] Beltukov I A, Karpenko A I, Poluyaktov S A, Tashlykov O L, Titov G P, Tuchkov A M and Shcheklein S E 2013 Atomnyye elektrostantsii s reaktorami na bystrykh neytronakh s natriyevym teplonositelem. V 2 chast’akh. Chast’ 1 [Nuclear power plants with fast neutron cooled reactors In 2 parts, part 1] ed S E Shcheklein and O L Tashlykov (Yekaterinburg: URFU) [In Russian]

[12] Nosov Y V, Rovneiko A V, Tashlykov O L and Shcheklein S E 2019 Decommissioning Features of BN-350, -600 Fast Reactors *Atomic Energy*. 2019 **125**(4) 219-223

[13] Dolgii Y F, Tashlykov O L, Sesekin A N and Zaynullina E Z 2018 Optimal control of the fuel reload mechanism *IFAC-PapersOnLine* **51**(32) 636-41

[14] Dolgii Y F, Petunin A A, Sesekin A N and Tashlykov O L 2018 Optimal control of the system of coupled cylinders *AIP Conf. Proc.* **2048** 020007

[15] Pontryagin L S, Boltyanskii V G, Gamkrelidze R V and Mishechenko E F 1962 *The mathematical theory of optimal processes* (New York: John Wiley & Sons)

[16] Dolgii Yu F, Sesekin A N, Tashlykov O L and Tran K T 2019 Sequential optimal control of the nuclear fuel reload mechanism *AIP Conf. Proc.* **2172** 070015