Measuring switching processes in financial markets with the Mean-Variance spin glass approach

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Abstract

In this article we use the Mean-Variance Model in order to measure the current market state. In our study we take the approach of detecting the overall alignment of portfolios in the spin picture. The projection to the ground-states enables us to use physical observables in order to describe the current state of the explored market. The defined magnetization of portfolios shows cursor effects, which we use to detect turmoils.

1 Introduction

There are different methods in order to estimate switching in market behavior. It reaches from Markov switching processes Hamilton [1989], Schaller and Norden [1997] to newer methods where physical phase transitions are used to describe a switching process Preis et al. [2011] as well as utilizing search engine queries Preis et al. [2013, 2010], Kristoufek [2013b,a], Moat et al. [2013]. These techniques rely on the assumption, that market moves or social transitions can be detected by observing the behavior of online queries. These queries represent precursor effects for phase transitions. All approaches have a similar goal: Identifying risks within the economy.

Our aim in this article is to identify switching processes within the Mean-Variance Model introduced by Markowitz [1952]. We use the portfolios of the efficient frontier under the assumption that these hold important information about the current market state.

We demonstrate that the optimal portfolios, in the magnetization picture with free spins aligning to the overall external market field, detect financial turmoils. This measure shows a not decaying risk of market switching since the Euro crisis. This can be seen in other studies containing the systemic risk Billio et al. [2012], Jurczyk et al. [2015].
2 The Mean-Variance Model

We consider the Mean-Variance Model, introduced by Markowitz in 1952 [Markowitz 1952]. The objective is to maximize the average return

\[ \mathbb{R}(P) = \sum_{i=1}^{N} R_i \sigma_i \]  

(1)

and minimize the risk defined by

\[ \text{var}(P) = \sum_{i,j} \sigma_i C_{ij} \sigma_j \]  

(2)

where \( P \) is the portfolio vector for a specific historical time window. \( \sigma_i \) is the weight of an asset \( i \), \( R_i \) is the average return of asset \( i \) for that time window, \( C_{ij} \) is the corresponding covariance matrix and \( N \) the number of considered assets. Since an investor has limited resources, the weights need to fulfill the constraint

\[ \sum_{i} \sigma_i \frac{1}{|} = 1 \]  

(3)

Therefore \( \sigma_i \) can range from \([-1,1]\) and short-selling is allowed. An investor can sell assets which he does not own and rebuy later for a lower price to benefit from a price drop. This gives us the analogy to spins, which can point in any direction and projected on an arbitrary direction which can be seen as a magnet model [Rosenow et al. 2002]. As Jahan, our objective function is regarded as a physical Hamiltonian

\[ \mathcal{H}_\lambda(P) = -\lambda \cdot \mathbb{R}(P) + (1 - \lambda) \cdot \text{var}(P) \]  

(4)

where \( \lambda \in [0,1] \) is responsible for balancing the two objectives [Jahan and Akbarzadeh-Totonchi 2010].

2.1 Identifying the switching process

For each \( \lambda \in [0,1] \) an optimal portfolio \( P_0 \) exists, which balances the risk and return, resulting in an efficient frontier. Each optimal portfolio \( P_0 \) on the efficient frontier has a “magnetization” value \( m = \sum_i \sigma_i \), which is a risk indicator, since the magnetization depends on the weighing parameter \( \lambda \) from Eq. (4) at each each measurement at time \( t \).

The magnetization \( m \) can range from all assets are sold-short \((m = -1)\) to a pure buying suggestion \((m = 1)\). These two market stages are called bearish or bullish market.
In a bullish market the magnetization $m$ ranges from approximately zero for $\lambda = 0$ to positive values $m > 0$. The opposite is the case for a bearish market.

In Figure 1 we used the assets of the German index DAX 30 in order to measure the market state. The magnetization $m$ clearly shows the two main crisis of the last ten years. The financial crisis of 2008 and the Euro crisis of 2011. With increasing $\lambda$ the magnetization $m$ starts to decrease in September 2007, which reflects the uncertainty of a Mean-Variance portfolio of the market direction. When the Lehman bankruptcy news hit the market, the magnetization finally flips in September 2008 to a bearish phase.

The second major crisis started in 2010, when several Euro states were unable to place national bonds. Until September 2011 the market was not concerned but then suddenly switched to a negative trend. This was due to the fact that in this time a fear of a Euro crash was amongst many investors.

![Figure 1](image.jpg)

Figure 1: On the left the data for the Mean-Variance Model has a time-frame of one year. One right the time-frame was set to two years. Both measurements rely on monthly returns of the assets, which belong to the DAX 30. The financial crisis of 2008 has a build up starting in September 2007, while the Euro crisis suddenly changes. (blue corresponds to $m > 0$, red $m < 0$)

The integral

$$M = \int_{0}^{1} \lambda \cdot m(\lambda)$$

is proportional to the tendency of the optimal portfolios to be bearish or bullish. The value of $M$ ranges between $-1$ and $1$. While these are the upper bounds and real data are not achieved since for small values of $\lambda$ a distribution of assets is wanted and leads to $m$ close to $0$. In figure 2 we show the $M$ in respect to the time.

In both figures 2 and 1 the stability of the ground-states starts to decay in 2008. This shows that the optimal portfolios of the efficient frontier become aware of the
risk in the market and can only compensate that risk by shorting certain assets.

2.2 Cursor effects in the switching process

The uncertainty within the dataset is represented by the magnetization as described above. We observed two effects within the magnetization before the two crisis. The first is that we noticed cumulated events of the form

$$\mathcal{E} = \begin{cases} \lambda_0^{crit} & \exists \lambda_0^{crit} | m(\lambda_0^{crit}) = 0 \\ 0 & \text{else} \end{cases}$$

which translates to: Although there is emphasize on the portfolio to generate return, the discussion to invest in a bearish or bullish trend is not given. Hence the magnetization $m = 0$. We also measured the smallest $\lambda$, where $m(\lambda)$ reaches its absolute maximum (Event $\mathcal{E}'$). In figure 3 and 4 we show that these two events accumulate before the crisis in 2008 and 2011. The upbuilding process can be demonstrated by a cumulative averaged rolling mean (CARM) [Jurczuk et al. 2014]

$$CARM(t) = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=0}^{i} \frac{\mathcal{E}(t - j \cdot \delta)}{i}$$

where $N \cdot \delta$ is the maximum time-window one considers of the event $\mathcal{E}$ at time $t$.

In both events one can identify the two crisis. The rise in the $CARM(\mathcal{E})$ takes place in January 2008 ($\Delta = 1y$) and June 2008 ($\Delta = 2y$). This references to a higher risk within the DAX 30 at that time. The peak in September 2008 finishes the upbuilding switching process (see figure 1).
A similar structure can be observed with the $\text{CARM}(\mathcal{E}')$. A surge occurs in January 2008. The Lehman crash in September 2008 leads to the distinct peak of the crisis.

The Euro crisis has a different from. A first switch from bullish to bearish took place in September 2011. But both events show distinct spires since 2011. This circumstance leads to the conjecture, that the portfolios chosen by the Mean-Variance Model do not settle for a phase.

Figure 3: (left) $\text{CARM}$ of the event in Eq. 6 with a time window of one year. (Right) A time window of two years is depicted. Note that since the beginning of the Euro crisis in both time frames the $\text{CARM}$ does not settle down and keep surging. The measurements used monthly returns.

Figure 4: The event $\mathcal{E}$ is defined by the smallest $\lambda$ before $(\lambda)$ reaches its absolute maximum. (left) $\text{CARM}$ with a time window of one year. (Right) $\text{CARM}$ with a time window of two years. Notice that the CARM was normalized by the median $Q$
3 Conclusion

We showed that the Mean-Variance Model with a short-selling option can be used to measure the current state of an underlying market by utilizing the portfolios of the efficient frontier as a spin grid under the constraints of risk and returns. There are three event types, which are proportional to the minimal risk within the underlying market, the integral over the magnetization and the two cursor events. Therefore we connected the overall market state to the ground-states of the portfolio given by the efficient frontier of the Mean-Variance Model. By introducing a memory measure CARM we detect upbuilding risk bubbles.

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