Inflection system of a language as a complex network

Henryk Fukś
Department of Mathematics
Brock University
St. Catharines, ON, Canada
Email: hfuks@brocku.ca

Abstract—We investigate inflection structure of a synthetic language using Latin as an example. We construct a bipartite graph in which one group of vertices correspond to dictionary headwords and the other group to inflected forms encountered in a given text. Each inflected form is connected to its corresponding headword, which in some cases in non-unique. The resulting sparse graph decomposes into a large number of connected components, to be called word groups. We then show how the concept of the word group can be used to construct coverage curves of selected Latin texts. We also investigate a version of the inflection graph in which all theoretically possible inflected forms are included. Distribution of sizes of connected components of this graphs resembles cluster distribution in a lattice percolation near the critical point.

I. INTRODUCTION

Vocabulary of human languages can be viewed as a large and complex network or graph, in which individual vertices represent words or families of words, and edges represent relationships between words. Many such models have been studied in recent years, including networks of co-occurrences of words in sentences [1], thesaurus graphs [2], [3], [4], WordNet database graphs [5], and many others [6], [7], [8], [9], [10].

Much of the aforementioned work has been done in the context of the English language, which, among other characteristic properies, exhibits only a minimal inflection, especially if compared to other Indo-European languages. In analytic languages like English, grammatical categories and relations are handled mostly by the word order, and not by the inflection.

In contrast to this, synthetic languages such as Latin, Greek, Polish, or Russian make an extensive use of inflection, and one word in these languages can appear in great many forms, reflecting grammatical categories such as tense, mood, person, number, gender, case, etc. In the past, there was relatively little work done on modelling of inflected languages using the paradigm of complex networks, and the goal of this paper is to present some initial findings of the author in this area.

Given the abundance of synthetic languages, one faces the issue of selecting one of them for detailed analysis. The language which has been most heavily studied and for which the largest body of literature exists is a natural choice – and there is no doubt that Latin must be chosen given these criteria.

Latin literature stretches for over 20 centuries ranging from the literature of ancient Rome right up to the 21st century. Latin served as lingua franca for Western civilization for many centuries, so there is no shortage of Latin texts, and a vast number of them is available in electronic form.

In addition to this, Latin inflectional system has been studied so extensively that literally every single aspect of this system is exceptionally well documented. Software tools which “understand” Latin inflection system are also readily available, including an excellent open-source WORDS program written by W. Whitaker [11].

II. MOTIVATION

One of the motivation of this work was the problem of vocabulary size. Let us suppose that we want to count how many distinct words a given work contains – for example, for the purpose of comparing two works and deciding which one is more “difficult” as long as vocabulary is concerned. How do we do this in a language like Latin, where one dictionary headword can have as many as hundreds of different forms? In addition to this, in some cases, one inflectional form can correspond to more than one dictionary headword, and one must deduce from the contexts which one to choose.

The earliest qualitative approach to Latin vocabulary can be found in the Ph.D. thesis of Paul Bernard Diederich [12] from...
1939, who performed a count of Latin headwords occurring in a selection of texts from 200 Latin authors totaling over 200,000 words. He did this entirely by hand – a hardly attractive proposition in today’s computerized world.

If one wants to perform computerized counts of words, and wants to count various inflectional forms of the same headword as one entry, one has to understand the relationship between inflected forms and headwords. This can be done by introducing the concept of an inflection graph.

### III. Inflection Graph

The inflection graph for a given text is a bipartite graph which is constructed as follows. First, we create a set of vertices, to be denoted by $B$, corresponding to all distinct words of the text. If a word occurs in the text more than once, it is represented by one vertex nevertheless. We then go over all vertices in $B$, and check which headwords can possibly correspond to each of words in $B$. These headwords form another set of vertices, to be called $A$. In practice, headwords may be obtained by using W. Whitaker’s WORDS program. As in the case of $B$, elements of $A$ are unique, so that each headword appears in $A$ only once.

If an element of $A$ is a headword corresponding to some element of $B$, then these two are connected. Obviously, for most headwords in $A$, there are many corresponding inflected forms in $B$, so an element of $A$ is typically connected to many elements of $B$. For example, *dicunt* (they say) and *dixit* (he said) are both inflected forms of the verb *dico*, thus we will have a vertex in $A$ corresponding to *dico* connected to vertices in $B$ corresponding to *dicunt* (they say) and *dixit*.

However, the opposite can also be true: in some instances, a word can be an inflected form of more than one headword, so that elements of $B$ are sometimes connected to more than one element of $A$. As an example, consider the word *sublatus*, which could be a form of *tollo* (lift, raise) or *suffero* (bear, endure), thus a vertex of $B$ corresponding to *sublatus* will be connected to vertices of $A$ corresponding to *tollo* and *suffero*.

The bigraph obtained using the aforementioned procedure is typically quite large but not very dense. For example, for the classic work of Julius Caesar *De bello Gallico* (published in 50s or 40s BC), consisting of 51,300 words, this bigraph has 5,377 of vertices in $A$, 10,977 vertices in $B$, and 15,349 edges. Figure 1 shows a visualization of this graph done by Walrus [13], a software tool for visualizing large graphs using 3D hyperbolic geometry and a fisheye-like distortion. Degree distribution for vertices of type A (headwords) for *De bello Gallico*, as well as some other works (to be discussed later) is shown in Figure 3. The distribution seems to have features of a power law. We also observe that the degree of a headword represents the number of inflected forms of that headword appearing in the text – and, as one can see from the figure, this number can easily approach 100.

#### A. Connected components

An important feature of the inflection graph is that it is, obviously, not connected, and that it has a large number of disjoint components – in the case of *De bello Gallico*, 3,740 components. Each of these components will be called a word group. Example of such a component is shown in Figure 2. It consists of four headwords (written in uppercase) and 18 inflected forms (written in lowercase).
In most cases, words within one group are closely related semantically, but not always – occasionally words with quite different meaning may belong to the same group, as in the case of tollo and suffero mentioned above. This is especially true for the largest group, as we will see later on. Nevertheless, word groups usually closely correspond to what linguists call word families. The main advantage of word groups lies in the fact that they can be easily determined using well known algorithms for computing connected components of a graph. We used python package NetworkX [14] for this purpose.

B. Rank-frequency distribution for groups

Having the concept of the word group defined, we can now label all groups with distinct labels, for example, with consecutive integers $i$. If a given word from the text belongs to a group labelled $i$, we will say that it is an occurrence of the group $i$. Obviously, some groups occur more often than others, so we can sort all groups in decreasing order of occurrences in the text. Position of a group on this list will be denoted by $r$ (rank), and the number of occurrences of that group in the text will be denoted by $n_g(r)$. Similar rank-frequency function for individual words will be denoted by $n_w(r)$. Figure 4 shows log-log plots of $n_g(r)$ versus $r$ for two very different works, namely the aforementioned De bello Gallico and for the Latin Bible translation of St. Jerome known as Vulgate (AD 390 to 405). In both cases non-Zipfian behavior is very clear, that is, the resulting curves a not straight lines.

C. Coverage curves

A helpful concept used to describe statistical properties of texts is the so called coverage curve. If we assume that the reader of the text knows $k$ top-ranking words, then the text coverage, or the fraction of known words in the text is defined as

$$C_w(k) = \frac{\sum_{r=1}^{k} n_w(r)}{\sum_{r=1}^{N} n_w(r)},$$

where $N$ is the total number of distinct words in the text. The graph of $C_w(k)$ versus $k$ is known as the coverage curve. Analogously, for groups we can define

$$C_g(k) = \frac{\sum_{r=1}^{M} n_g(r)}{\sum_{r=1}^{M} n_g(r)},$$

where $M$ is the total number of word groups in the text.

In order to illustrate the difference between $C_w(k)$ and $C_g(k)$, we constructed these coverage curves for five different texts. These texts, in addition to already mentioned De bello Gallico and Vulgate, include Cicero’s Philippicae (written 44-43 BC) and collection of medieval stories Gesta Romanorum (13th-14h century). We also wanted to include some longer contemporary text of considerable length, which proved to be difficult due to scarcity of such texts. Finally we somewhat artificially produced a text by combining two shorter documents. The resulting file is titled Encyclicals and consists of two encyclicals of John Paul II, Ut Unum Sint and Evangelium Vitae. Both of these were issued in the same year (1995), thus they are sufficiently similar in style to consider them as parts of one document. Texts of encyclicals were obtained from Vatican repository [33], and the remaining texts from The Latin Library [16]. In order to make sure that differences in text size do not interfere with our analysis, all texts have been truncated so that they have the same length as De bello Gallico, that is, 51,300 words. The coverage curves were obtained by the following procedure:

- The text was converted to lowercase, all punctuation marks and digits were removed.
- A list of words was created, together with number of occurrences of each word.
- For each word in the above list, we used Whitaker’s WORD program to find corresponding headwords.
- The inflection graph was constructed, and its connected components determined using Python/NetworkX script.
- The frequency of occurrence of each word group was computed, and coverage curve plotted.
Figures 5 and 6 show coverage curves for words and groups for all five sample texts. Comparing these figures one can immediately notice two things. First of all, the coverage converges to 100% faster or slower, depending on the text. For words coverage, Vulgate and Gesta Romanorum are clearly converging faster than both classical Latin texts of Caesar and Cicero and contemporary encyclicals. This agrees with the general consensus of latinists who consider medieval texts “easier” than classical one.

Another observation is the difference between words coverage and group coverage. Much smaller number of groups than individual words is needed to achieve the same coverage, as one would naturally expect. Table I lists the number of words and groups required to obtain 95% and 98% coverage in all five sample texts. These two numbers have been used because one can define the normalized coverage as 100% coverage, immediately notice two things. First of all, the coverage for all five sample texts. Comparing these figures one can see that Cicero used the inflection system of the Latin language much more skillfully requiring larger number of word groups to achieve the same coverage. This shows that Cicero used the inflection system of the Latin language much more skillfully than the case of Vulgate. On the other hand, Vulgate requires larger number of word groups to achieve the same coverage. This shows that Cicero used the inflection system of the Latin language much more skillfully – he used fewer word groups than St. Jerome, yet from these he obtained a larger number of inflected forms!

**D. Normalized coverage**

In order to make coverage curves independent of N and M, one can define the normalized coverage as

\[ c_w(x) = C_w(Nx), \]

where \( x \in [0, 1] \). If the exact form of \( n_w(r) \) or \( n_g(r) \) was known, the corresponding coverage could be computed. For example, in the case of the Zipf law, \( n_w(r) = A/r \) where A is a normalization constant. In such a case, sums required in eq. (1) can be computed in closed forms, yielding

\[ c_w(x) = \frac{\Psi(Nx + 1) + \gamma}{\Psi(N + 1) + \gamma}, \]

where \( \Psi \) is the digamma function, and \( \gamma = 0.57721566... \) is the Euler-Mascheroni constant.

Unfortunately, in the case of the group coverage, we do not know what is the form of \( n_g(r) \), thus a similar formula cannot be produced. It is possible, however, to obtain empirical fit with small number of parameters. This can be readily understood if we plot \( c_g(x) - x \) as a function of \( x \), as shown in Figure 7 where, in order to avoid clutter, only coverage data for one data set (Encyclicals) is presented. Symmetry of this figure suggests that \( c_g(x) - x \) may be approximated by the graph of \( x^\alpha(1-x)^\beta \), where \( \alpha, \beta \) are parameters of the fit. We, therefore, fitted the function

\[ f(x) = x + x^\alpha(1-x)^\beta \]

to the normalized coverage data \( c_g(x) \). The fit, although good, was less than ideal, so we introduced another two parameters,
TABLE II
FIT PARAMETERS

|                       | $\alpha$ | $\beta$ | $\gamma$ | $\delta$ |
|-----------------------|----------|---------|----------|---------|
| De bello Gallico      | 0.3688   | 1.3583  | 0.3445   | 0.5877  |
| Philipicæ             | 0.3491   | 1.3707  | 0.3249   | 0.5537  |
| Vulgate               | 0.3045   | 1.3188  | 0.2781   | 0.4206  |
| Gesta Romanorum       | 0.3433   | 1.3134  | 0.3196   | 0.5199  |
| Encyclicals           | 0.3615   | 1.2899  | 0.3407   | 0.5047  |

fitting the function

$$f(x) = x^\gamma + x^\alpha (1 - x^\delta)^\beta.$$  \(7\)

It should be noted that this choice of the equation does not have any particular meaning, it just has been observed that it produces a good fit, thus it is a convenient way to describe coverage curves. Sample fit produced using this equation is shown in Figure 7 (continuous line). Values of parameters for all five sample texts are shown in table III. From the form of eq. (7) one can see that the smaller of parameters $\alpha$ and $\gamma$ controls the initial steepness of the normalized coverage curve, and therefore we will define

$$\eta = \min\{\alpha, \gamma\}.$$  \(8\)

The value of $\eta$ can be given an interpretation related to the nature of the underlying text. If $\eta$ is large, it means that the normalized coverage curve is growing slowly – that is, high percentage of all word groups present in the text is needed to achieve, say, 95% text coverage. On the other hand, if $\eta$ is small, this means steep coverage curve, so that high coverage is reached quickly.

For that reason, one can say that the parameter $\eta$ tells us to what degree is the the inflection mechanism of the language used in order to provide high text coverage. Small $\eta$ means high reliance on the inflection mechanism. From Table III we can therefore conclude that Vulgate and Gesta Romanorum do not rely on the inflection as much as the classical works or modern encyclicals.

IV. INFLECTION GRAPH FOR A DICTIONARY

In previous sections, we were considering inflection graphs for individual literary works. It is possible to obtain such graph for the whole Latin language – or, to be more precise, for all words from a large dictionary. In Whitaker’s WORDS dictionary, there are 35,670 distinct headwords. Using WORDS program, one can construct all possible inflected forms for all these headwords, resulting in a list of 1,032,669 word forms. We shall note that those are all theoretically possible forms, and that not all of them are attested in the surviving corpus of Latin texts. For example, the form *coquor* is a theoretically possible passive, present tense, first person singular of *coquo* (to cook), yet in does not seem to be attested in Latin texts [17].

If we connect all headwords from the dictionary with all theoretically possible inflected forms, we obtain an inflection graph of the whole language. The resulting graph is very sparse, having 1,117,394 edges, that is, only 10% more than the number of edges. The reason fort his is that the vast majority of inflected forms correspond to only one dictionary headword. The number of connected components of this graph is 50,847, and their size distribution appears to follow a power law. This is illustrated in Figure 8. If $H(m)$ is the number of groups with $m$ headwords, then the line of the best fit shown in Figure 8 closely follows the power law

$$H(m) \sim m^{-\tau},$$  \(9\)

where the value of the exponent $\tau$ obtained by fitting a straight line to datapoints excluding several largest clusters is $\tau = 3.32$.

This phenomenon strongly resembles percolation. The scaling theory for percolation predicts that connected components exhibit close to power-law behavior near the percolation threshold. This seems to suggest that the sparse inflection graph discussed here may be close to its percolation threshold.

Let us now make some remarks regarding largest components, corresponding to data points lying above the fitted line on the right of Figure 8. This deviation could be caused by a finite size of the graph. Similar behavior is often observed in numerical simulations of percolation in finite systems.

Obviously, the headwords belonging to very large clusters cannot be all semantically related, and the fact that they are grouped together may be somewhat related to the inclusion of all theoretically possible inflected forms. A path joining two vertices may exist solely because it passes through some inflected form which is unattested. In Figure 9 the largest cluster of the dictionary inflection graph is shown. One can see from this picture that the cluster is composed of several one-level trees (stars) loosely connected via a number of bridges. Some of these bridges are likely “artificial” (unattested) forms, and removing them would most likely divide the big cluster into a number of smaller clusters.
Fig. 9. Visualization of the largest component of the dictionary inflection graph.

V. CONCLUSION

We presented some preliminary findings regarding properties of inflection graphs. The concept of the word group defined as a connected component of the inflection graph appears to be useful in describing the vocabulary structure of the text. The parameter \( \eta \) could be used to characterize some aspects of the text difficulty, describing the balance between the diversity of vocabulary versus the diversity of inflected forms. Obviously, if one wants to categorize texts according to their perceived difficulty, vocabulary is not the only factor. Structure of sentences and word order are equally important, or sometimes even more important, thus such categorization scheme will most likely involve several parameters. The author hopes that an automated system classifying Latin texts is eventually constructed, since such system would be very beneficial to students of Latin. In languages such as English, sets of graduated books exist, and are routinely used in classrooms. Students start from very simple texts and seamlessly progress to texts of increasing difficulty. In Latin it is very unlikely that such set of books would be ever written, given the current status of the language. Nevertheless, texts of various levels of difficulty do exist in Latin – yet it is often hard for beginners to determine beforehand which texts (out of thousands available) should be read first, and which should be left till later. We plan to develop ideas presented here to create an automated system which could be used to “measure” text difficulty and eventually mine internet repositories to produce a list of Latin text forming a sequence with gradually increasing difficulty.

The similarity of the dictionary inflection graph to percolating network is also currently investigated. We are collecting a large corpus of Latin texts in order to remove from the graph all unattested words. Due to variation of spelling, especially in Medieval texts, this task cannot be fully automated, thus final results are not yet available. We also plan to find out whether other synthetic languages exhibit similar scaling phenomenon as the Latin inflection graph.

ACKNOWLEDGMENT

The author acknowledges partial financial support from the Natural Sciences and Engineering Research Council of Canada in the form of Discovery Grant.

REFERENCES

[1] R. F. i Cancho and R. V. Solé, “The small world of human language,” Proc. Roy. Soc. Lond. B, vol. 268, pp. 2261–2265, 2001.
[2] A. E. Motter, A. P. S. de Moura, Y. C. Lai, and P. Dasgupta, “Topology of the conceptual network of language,” Phys. Rev. E, vol. 65, 2002.
[3] O. Kinouchi, A. S. Martinez, G. F. Lima, G. M. Lourenco, and S. Risaguusman, “Deterministic walks in random networks: an application to thesaurus graphs,” Physica A, vol. 315, pp. 665–676, 2002.
[4] A. D. Holanda, I. T. Pisa, O. Kinouchi, A. S. Martinez, and E. E. S. Ruiz, “Thesaurus as a complex network,” Physica A-Statistical Mechanics And Its Applications, vol. 344, pp. 530–536, 2004.
[5] M. Sigman and G. A. Cecchi, “Global organization of the wordnet lexicon,” PNAS, vol. 99, no. 3, pp. 1742–1747, February 2002. [Online]. Available: http://dx.doi.org/10.1073/pnas.022341799
[6] J. Y. Ke and Y. Yao, “Analysing text development from a network approach,” Journal Of Quantitative Linguistics, vol. 15, pp. 70–99, 2008.
[7] S. M. G. Caldeira, T. C. P. Lobao, R. F. S. Andrade, A. Neme, and J. G. V. Miranda, “The network of concepts in written texts,” European Physical Journal B, vol. 49, pp. 523–529, 2006.
[8] A. Pomi and E. Mizzari, “Semantic graphs and associative memories,” Physical Review E, vol. 70, p. 066136, 2004.
[9] R. F. I. Cancho, R. V. Sole, and R. Kohler, “Patterns in syntactic dependency networks,” Physical Review E, vol. 69, p. 051915, 2004.
[10] L. Antisphere, M. G. V. Nunes, O. N. Oliveira, and L. D. Costa, “Strong correlations between text quality and complex networks features,” Physica A-Statistical Mechanics And Its Applications, vol. 373, pp. 811–820, 2007.
[11] W. Whitaker, “WORDS, Latin-English dictionary,” http://users.erols.com/whitak/cf/words.htm
[12] P. B. Diederich, “The frequency of Latin words and their endings,” Ph.D. dissertation, Columbia University, 1939. [Online]. Available: http://users.erols.com/whitak/cf/req.htm
[13] CAIDA, “Walrus – graph visualization tool,” http://www.caida.org/tools/visualization/walrus
[14] NetworkX, Python package for analysis of complex networks,” https://networkx.lanl.gov.
[15] Ioannes Paulus PP. II, “Litterae encyclicae,” http://www.vatican.va
[16] “The latin library,” http://www.thelatinlibrary.com
[17] R. E. Prior and J. Wolhberg, 501 Latin Verbs. Hauppauge, NY: Barron’s, 1995.