Mesonic Content of the Nucleon and the Roper Resonance

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We investigate colorless (mesonic) $3q - q\bar{q}$ components in the nucleon and the Roper resonance $N^*(1440)$. Starting from constituent quarks and gluons we estimate the excitation of $q\bar{q}$ pairs in a gluon exchange model with the strong coupling constant extracted from a comparison with a non-perturbative resonating group calculation of the $\pi, \eta, \rho$ and $\omega$ content of the nucleon. Applying the same model to the Roper resonance, we find as the most striking result a very strong scalar-isoscalar $\sigma$-content of the $N^*(1440)$ with a strength of the $3q - q\bar{q}$ configurations comparable to the $3q$ component itself.

Keywords: Constituent quark model, $q\bar{q}$ admixture, Roper resonance
PACS: 12.39.Jh, 12.39.Pn, 14.20.Gk, 21.45.+v

Among the excited states of the nucleon, i.e. the baryon resonances in the continuum, the Roper resonance $N^*(1440)$ as the first excited state with the quantum numbers of the nucleon ($J^P, T$) = ($\frac{1}{2}^+, \frac{3}{2}$), plays a particular role. On the one side, parameterizing conventionally the Roper resonance as a system of three constituent quarks, most quark models predict the position of the Roper resonance as generally several hundred MeV above its experimental value [1-3]. On the other side, the $N^*(1440)$ seems to dominate the dynamics of coherent near threshold $2\pi$ production, both in $pp \rightarrow pp\pi\pi$ [4-7] and $\pi A \rightarrow \pi\pi A$ collisions [8,9], through the coupling to a virtual $\sigma$-meson (which subsequently then decays into two pions). Thus it is hoped that from the study of the Roper-induced dynamics ultimately shed light on the $\sigma$-degree of freedom - which is generally phrased as a $2\pi$ resonance with a mass and width of $m_\sigma \sim \Gamma_\sigma \sim 550 MeV$ [10] - both in the NN and in nuclear systems, where the $\sigma$-meson provides the medium range attraction; in addition, it might provide information of a partial restoration of chiral symmetry in dense nuclear matter.

*Supported in part by Forschungszentrum FZ Jülich (COSY)
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‡preprint FAU-TP3-06/Nr. 08
through a strong reduction of the effective $\sigma$-meson mass down to twice the pion mass \[11,12\].

One source of the problem in describing the Roper resonance - and, in general, all unstable baryon resonances - in quark models, is the restriction of their quark content to three constituent quarks. Already the experimentally verified strong decay of the Roper resonance with a width of $\Gamma \sim 240 - 450 \text{MeV}$ into $N\pi$ and $N\pi\pi$ final states \[10\], signals a strong next order $4q - \bar{q}$ component in the Fock expansion

$$|N^*\rangle = (1 - \alpha)|3q\rangle + \alpha|3q(q\bar{q})\rangle$$

(1)

where in color-singlet channels the $(q\bar{q})$ content can be classified along experimentally known mesons \[13-17\].

In this note we investigate the mesonic structure of the Roper resonance, with the emphasis in estimating its scalar $\sigma$-content. In more detail, we would like to pursue the following route. Starting from a Hamiltonian which includes the one gluon exchange (OGE), harmonic confinement and in addition, $q\bar{q}$ pair creation and annihilation, the mesonic content of the component $3q - (q\bar{q})$ of the nucleon is calculated in a non-perturbative resonating group calculation for $\pi, \eta, \rho$ and $\omega$ from minimizing \[18,19\]

$$\delta \left( \frac{\langle \Psi |H - E|\Psi \rangle}{\langle \Psi |\Psi \rangle} \right) = 0$$

(2)

We then simulate this result in a perturbative one-loop calculation, with $q\bar{q}$ pair creation induced by constituent gluon exchange \[18-23\]. With the strength of the effective $qqg$ vertex given, we then deduce the $\sigma$-content of the nucleon and extent the model into the continuum, to estimate the mesonic content of the Roper resonance in $N^*(1440) \rightarrow (N, \lambda)$ transitions ($\lambda$ denotes the corresponding meson). In comparing meson creation by one gluon exchange with the $^3P_0$ vacuum pair creation model \[24,25\], the basic difference to eq. (14) below is its simplified spin structure $\sim \vec{\sigma}\vec{q}$.

We briefly sketch the main lines of our calculation, deferring, however, technical details to a forthcoming comprehensive publication. The basic formulae for calculating the admixture of a nucleon and a meson $\lambda$ in the nucleon itself or the Roper resonance is given as

$$\langle B|B \rangle = (1 - \alpha)^2\langle 3q|3q \rangle + \alpha^2\langle 3q(q\bar{q})|3q(q\bar{q}) \rangle$$

(3)

where the admixture probability $\alpha^2$ is given in the one-loop approximation as \[26,27\]

$$\alpha^2 = \mathbf{P} \int \frac{|M_{B\rightarrow N\lambda}(\vec{q})|^2d\vec{q}}{2\omega_{\lambda}(\vec{q})E_{\lambda}(\vec{q}) + EN(\vec{q}) - MB)^2}d(\vec{q})$$

(4)

where the numerator contains the $B \rightarrow N\lambda$ transition amplitude $M_{B\rightarrow N\lambda}(\vec{q})$, while $MB$ represents the $N$ or $N^*$ mass, respectively (we assume the initial baryon to be at rest); $\omega_{\lambda}(\vec{q})$ and $E_N(\vec{q})$ are the total energies of the meson and the nucleon; The
symbol \( P \) denotes the principal value of the integral: for \( m_\lambda + M > M_B \) it yields a standard integral without singularity. Note that for \( m_\lambda + M \leq M_B \) the principal value is conveniently evaluated from the relation

\[
P \int_0^{\infty} \frac{f(q) \, dq}{q^2 - q_0^2} = \int_0^{\infty} \frac{f(q) - f(q_0)}{q^2 - q_0^2} \, dq + \frac{i\pi}{q_0} f(q_0)
\]

where \( q_0 \) denotes the position of the singularity.

The dynamics of the admixture is contained in the transition amplitude \( M_\lambda(q) \) given as (in an obvious notation)

\[
M_\lambda(q) = \langle N(3q; \vec{q})|\lambda(q\bar{q}, \vec{q})|V_{B\rightarrow N\lambda}(\vec{q})|N_B(3q)\rangle
\]

More explicitly \( M_\lambda(q) \) factorizes into a \( \vec{q} \)-dependent piece times spin, flavor and color coefficients, i.e.

\[
M_\lambda(q) = T_\lambda(q)\lambda_S\lambda_F\lambda_C
\]

for all quarks in relative \( s \)-states; in case of the \( \sigma \) meson the orbital angular momentum \( l = 1 \) is coupled with the \( q\bar{q} \) (see eq. (13) below).

For an economical evaluation we introduce two additional simplifications: we represent the relative motion between the \( 3q \) and \( q\bar{q} \) clusters by a plane wave (through a full resonating group calculation yields a self consistent relative wave function between the clusters) and we model the nucleon and the \( N^* \) as a quark - (scalar - isoscalar) diquark system in a relative \( 0s \) and \( 1s \) state for the nucleon and the Roper resonance, respectively; both assumptions simplify in particular antisymmetrization among the quarks and the evaluation of the color matrix elements significantly). This diquark picture reflects the strong \( qq \) correlations in the scalar - isoscalar channel and is supported by numerous investigations, such as on the lattice [28,29], Dyson - Schwinger studies [30,31] and from the non relativistic quark model [32,33]. This yields then for the complete wave function

\[
\Phi_\lambda(|q\bar{q})\rangle = \Phi_\lambda(\rho)\frac{1}{\sqrt{2}}[\frac{1}{2}1]\lambda\frac{1}{\sqrt{2}}[\frac{1}{2}1]\lambda[(10)(01)]\lambda_{00}
\]

for the \( \pi, \eta, \rho \) and \( \omega \) mesons and correspondingly

\[
|(3q)_B\rangle = \Phi_B(r)\frac{1}{\sqrt{2}}[\frac{1}{2}1]m\frac{1}{\sqrt{2}}[\frac{1}{2}1]M[(10)(01)]_{00}
\]

for the baryons. Above, the bracket \([ab]^{JM}\) denotes appropriate spin and isospin coupling; the color wave function, which in our diquark picture becomes identical for all mesons and baryons, is easily reduced from the Elliot notation [34] to

\[
[(10)(01)]_{00} = \delta_{ab} / \sqrt{3}
\]

with \( a \) and \( b \) referring to the \( q \), diquark or \( q, \bar{q} \) color content, respectively. The radial dependence is then parameterized in a translational invariant normalized Gaussian
form as
\[ \Phi(r) = \sum_i N_i e^{-\frac{r^2}{2b_i^2}} \]  
(11)
where for the mesons \( \pi, \eta, \rho \) and \( \omega \) the sum over \( i \) includes only a single Gaussian form, which is related via
\[ b_i = \frac{2}{\sqrt{3}} r_{r.m.s.} \]  
(12)
to the mean rms-radius of the mesons, whereas for the parameterization of the baryons 3 Gaussians with different width parameters are included ([19], compare Table 1). Only for the \( \sigma \)-meson with positive parity the radial structure is different: the \( q\bar{q} \) pair is in a relative \( l = 1 \) orbit [35], yielding
\[ \Phi_\sigma(r) = N_\sigma e^{-\frac{r^2}{2b_\sigma^2}} \left| Y_1 \left( \frac{1}{2} \right) \right|^2 \]  
(13)
Finally we specify the \( B \to N \lambda \) transition operator. Here we focus on an effective one-gluon exchange for the excitation of the \( q\bar{q} \) pair. This yields in momentum space [18,19] obtain in momentum space (Fig. 1)
\[ V_{B \to N \lambda}(q) = 4\pi \frac{\alpha_s}{m_q} \frac{\lambda_i \lambda_j}{4} \left( \hat{\sigma}_i \times \hat{\sigma}_j \right) \frac{q - 2\hat{\sigma}_j \hat{p}_i}{q^2 + m_g^2} \]  
(14)
(with quark and gluon mass \( m_q \) and \( m_g \), respectively; \( \hat{p}_i \) is the momentum of the initial quark, from which the \( q\bar{q} \) pair is exited), which can be cast for \( m_g^2 \gg q^2 \) into a Gaussian form in coordinate space
\[ V_{B \to N \lambda}(r) = -i \left( \frac{\alpha_s}{8\sqrt{\pi}} \frac{m_g^3}{m_q} \frac{\lambda_i \lambda_j}{4} \left( \hat{\sigma}_i \times \hat{\sigma}_j \right) \frac{-m_g^2}{q^2} \hat{r}_{ij} + i 4e^{-\frac{m_g^2}{q^2}} \hat{r}_i \right) \]  
(15)
The strong coupling constant \( \alpha_s \sim 2 \) is kept as a parameter, which is adjusted to reproduce in our one-loop approximation the admixture probabilities obtained from a corresponding non-perturbative resonating group calculation later on.

The formalism, developed in a Gaussian representation given above, allows a very transparent evaluation of the admixture probability: the complete transition amplitude, involving multiple Gaussian integrals, can be evaluated analytically; only the evaluation of the principal value in eq. 5 has to be performed numerically.

For the final result we specify our model parameters as follows: we use \( m_q = m_g/2 = 330\text{MeV} \) for the constituent quark and gluon masses [36,37]; the Gaussian parameters for the participating particles are given in Table 1 [19].

The effective coupling strength \( \alpha_s \), if fitted to the probabilities of RG calculation in the nucleon with
\[ P_\pi = 0.045, P_\eta = 0.005; P_\rho = 0.03, P_\omega = 0.004 \]  
(16)
Figure 1: Schematical representation of a mesonic $q\bar{q}$ excitation in an (effective) one-gluon exchange model.

Table 1: Parameters for the $N, N^*$ and mesons $\lambda$ (given in [fm])

|    | $\beta_1$ | $\beta_2$ | $\beta_3$ | $b_1$ | $b_2$ | $b_3$ |
|----|-----------|-----------|-----------|-------|-------|-------|
| $N$ | 1.1712    | 1.393     | 0.0121    | 0.440 | 0.659 | 0.911 |
| $N^*$| 4.625     | 0.418     | -0.822    | 0.440 | 0.659 | 0.911 |
| $\pi$|          |           |           |       |       |       |
| $\eta$|           |           |           |       |       |       |
| $\sigma$|           |           |           |       |       |       |
| $\rho$|           |           |           |       |       |       |
| $\omega$|           |           |           |       |       |       |

| $b_\lambda$ | $b_\lambda$ | $b_\lambda$ | $b_\lambda$ | $b_\lambda$ | $b_\lambda$ |
|-------------|-------------|-------------|-------------|-------------|-------------|
| 0.5         | 0.5         | 0.7         | 0.75        | 0.6         | 0.6         |
leads qualitatively to $\alpha_s \approx 1.8$. Then, as a first result, we estimate the $\sigma$-content in the nucleon as $P_\sigma = 0.025$ (Fig.2). In going then over to the mesonic admixtures in the Roper resonance, the admixture probabilities and their relative importance are changed in part dramatically. For the $\pi, \rho$ and $\omega$ the probabilities remain qualitatively close to the content in the nucleon. This is not surprising, as for the pion with $m_\pi + M < M^*$, the singular piece in the principal value is strongly canceled, while the $\rho$ and $\omega$ admixture are still fairly small due to $m_{\rho,\omega} + M - M^* \sim 280\text{MeV}$. For the $\sigma$ and the $\eta$, however, the situation is quite different: here, from $m_{\sigma,\eta} + M - M^* \sim 50\text{MeV}$, already the squared energy denominators enhance the admixture significantly; explicitly we find an admixture probability of typically 50 percent for the $\sigma$ meson and of 10 percent for the $\eta$ meson, respectively; these characteristic values are fairly stable against a moderate variation of the Gaussian model parameters: the maximal sensitivity of the result is found with respect to the root mean square radii of the mesons (Fig.3).

The various conclusions to be drawn from our estimate, are evident:

- the scalar-isoscalar $\sigma$-degree of freedom has a strong component in the Roper resonance \cite{38}; consequently, as the $\sigma$-meson strongly couples to the $\pi\pi$ channel, the $\sigma$ dominates hadron or photon induced double pion production in the $N^*(1440)$ regime \cite{39,40};

- the $\eta$-meson also strongly couples to the Roper resonance (in fact, in former tables of particle properties the decay $N^*(1440) \to N + \eta$ was listed with a fairly large strength). An important consequence might be, that the $\eta NN^*$ coupling strongly contributes to the near threshold production of the $\eta$-meson \cite{41}, where the threshold is only about $50\text{MeV}$ away;

- the large $q\bar{q}$ components in the Roper resonance suggest as a universal feature of baryon resonances in the continuum, that any quantitative treatment of $N^*$’s requires the inclusion of, at least, the next - to leading $3q(q\bar{q})$ component to the leading $3q$ piece in a corresponding Fock-expansion. As an immediate consequence we expect a significant shift of the (real part of the) $N^*$-masses of the order of the resonance widths, which would spoil the in part quantitative result obtained for baryon spectroscopy in various quark models.

Of course, for firmer conclusions, our model estimate in this note has to be improved, such as to include explicitly the $3q$-component of baryons (instead of a quark-diquark representation) or a consistent handling of the relative $N\lambda$- cluster wave function.

Beyond that the representation of the $\sigma$-meson as a correlated $\pi\pi$ system \cite{42} has to be investigated in detail. Even though this picture of a $\sigma$-meson as a $(q\bar{q})(q\bar{q})$ molecule is very different from viewing the $\sigma$ as a dominant $q\bar{q}$ system, we still expect simply from parity reasons, that $\sigma$ behaves quite different from the other mesons with masses below $1\text{GeV}$.
Figure 2: Admixture probability of various meson contributions (with masses below 1GeV) to the nucleon as a function of the mesonic root-mean-square radius $r_{rms}$.

Figure 3: As Fig. 2, however, for the Roper resonance $N^*(1440)$. 
Just to present the main argument. Assuming for simplicity identical root mean square radii for all mesons considered, leads to an identical product of the radial wave functions at the $N^*N\lambda$ vertices; thus the particular role of the $\sigma$ meson is related to its orbital - spin structure $[Y_1\frac{1}{2}\frac{1}{2}][1\frac{1}{2}]^{00}$ in contrast to $[Y_0\frac{1}{2}\frac{1}{2}]^{SM}$, with $S = 0,1$, for the $\pi, \eta$ and $\rho, \omega$, respectively. Consequently, recoupling the above invariants with the $q\bar{q}$ creation operator $[Y_1\Sigma][00]$ yields the production operator itself (with the angular momentum function $Y_1$) for $\pi, \eta, \rho, \omega$, whereas for the $\sigma$ meson

$$[Y_1\frac{1}{2}\frac{1}{2}][1\frac{1}{2}]^{00} = \sum_{L=0,2} \hat{L} [[Y_1\Sigma][1\frac{1}{2}][1\frac{1}{2}]^{L00}]$$

it results in a very different spin - orbit structure. This very difference with $L=0,2$, which is reflected in the large $\sigma$ content of the Roper resonance, is preserved for a $\pi\pi$ structure of the $\sigma$: recoupling $\sigma = [[\frac{1}{2}\frac{1}{2}][\frac{1}{2}\frac{1}{2}]^{00}]$ with twice the production operator yields immediately

$$[[Y_1\Sigma'][00][1\frac{1}{2}][1\frac{1}{2}]^{00} = \sum_{L=0,2} \hat{L} [[Y_1\Sigma][1\Sigma'][1\Sigma'][L00]]$$

i.e. the same structure as for the representation of $\sigma$ as a $q\bar{q}$ pair in eq. (15).

To substantiate our findings and conjectures and to extend the model predictions in a systematic way to other baryon resonances, detailed calculations are in progress; furthermore, more insight will be gained from detailed information on the internal structure of baryon resonances as under intensive study at modern hadron (COSY) [43] and electron accelerators (MAMI, ELSA,CEBAF)[44],[45],[46].

The authors thank H.M.Hofmann for fruitful discussions.

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