Resolvent analysis of turbulent channel flow with manipulated mean velocity profile

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Abstract
Using the resolvent analysis, we investigate how the near-wall mode primarily responsible for the friction drag is amplified or suppressed depending on the shape of the mean velocity profile of a turbulent channel flow. Following the recent finding by Kühnen et al. (2018), who modified the mean velocity profile to be flatter and attained significant drag reduction, we introduce two types of artificially flattened turbulent mean velocity profiles: one is based on the turbulent viscosity model proposed by Reynolds and Tiederman (1967), and the other is based on the mean velocity profile of laminar flow. A special care is taken so that both the bulk and friction Reynolds numbers are unchanged, whereby only the effect of change in the mean velocity profile can be studied. These mean velocity profiles are used as the base flow in the resolvent analysis, and the response of the wavenumber-frequency mode corresponding to the near-wall coherent structure is assessed via the change in the singular value (i.e., amplification rate). The flatness of the modified mean velocity profiles is quantified by three different measures. In general, the flatter mean velocity profiles are found to result in significant suppression of near-wall mode. Further, increasing the mean velocity gradient in the very vicinity of the wall is found to have a significant importance for the suppression of near-wall mode through mitigation of the critical layer.

Keywords : Flow control, Resolvent analysis, Drag reduction, Flatness, Mean velocity profile

1. Introduction

Reduction of turbulent friction drag is of significant importance for energy saving and environmental conservation. Flow control methods for friction drag reduction are classified into passive and active control methods. In the passive control, e.g., riblets (Walsh, 1983), only the initial setup is required, so that it is considered easier to implement. However, due to some problems such as the little drag reduction effect exceeding the maintenance cost, few passive control methods have been put into practice. On the other hand, some active control methods are attracting attention due to their large drag reduction effects, although they require power input from the external system.

The active control methods are further classified into feedback and predetermined control methods. In the feedback control methods that numerically succeeded in turbulent friction drag reduction, e.g., the opposition control (Choi et al., 1994; Hammond et al., 1998) and the suboptimal control (Lee et al., 1998; Iwamoto et al., 2002), the control input was always determined based on the information related to the near-wall structures represented by low- and high-speed streaks and quasi-streamwise vortices. Therefore, the sensors and actuators used for the feedback control should be as small as they can sense and actuate such small-scale structures (Kasagi et al., 2009). Although there were some attempts to manipulate large-scale structure (e.g., Fukagata et al., 2010; Canton et al., 2016), there is no method so far that can reduce the drag by actuations from the wall only. Unlike the feedback control, the control input in the predetermined control, e.g., uniform blowing/suction (Stevenson, 1963; Sumitani and Kasagi, 1995; Kametani and Fukagata, 2011) and streamwise traveling wave (Min et al., 2006; Nakanishi et al., 2012; Mamori et al., 2014), is determined beforehand irrespective of the flow field. In addition, compared with the feedback control, larger drag reduction effect such as relaminarization has been
confirmed in the numerical simulations. Moreover, the recent experiments have also confirmed the large drag reduction
effects of the uniform blowing (Eto et al., 2019) and the streamwise traveling wave (Suzuki et al., 2019).

Very recently, Kühnen et al. (2018) and Scarselli et al. (2019) numerically and experimentally demonstrated that
controls aiming at decelerating the flow in the center of a channel and accelerating it in the near-wall region lead to
relaminarization of turbulent flow. In other words, making the turbulent mean velocity profile ‘flattened’ successfully
suppresses the turbulence intensities. Although these results are striking, the drag reduction mechanism and the detailed
conditions leading to significant friction drag reduction are not sufficiently understood.

Although many effective control methods have been proposed so far, most previous studies have relied on direct
numerical simulation (DNS), which has a drawback in that computational costs are relatively high. Recently, in order
to assess the control effect at a low computational cost, modal analysis techniques have attracted great interests from
researchers in fluid mechanics. Resolvent analysis, proposed by McKeon and Sharma (2010), is known to be one of the
powerful modal analysis methods. This method enables us to evaluate the performance of a given control on physically-
important turbulent structures at a low computational cost. This analysis interprets the turbulent velocity field as an output
generated through a linear transfer function forced by the nonlinear term in the Navier-Stokes equation. By using the
resolvent analysis, we can investigate linear amplification mechanisms for each structure within turbulent flow under a
certain mean velocity profile.

The usefulness of resolvent analysis has been confirmed for various manipulated turbulent flows, e.g., opposition
control (Luhar et al., 2014), suboptimal control (Nakashima et al., 2017), streamwise-varying steady transpiration (Gómez
et al., 2016), and a channel flow under spanwise rotation (Nakashima et al., 2019). In particular, resolvent analysis reveals
which mode is amplified or suppressed in the spectral space when a specific control is applied. Based on such knowledge,
Kawagoe et al. (2019) proposed a modified feedback control law and confirmed its performance by DNS. Therefore,
application of resolvent analysis will provide additional physical insight also for the case where the mean velocity profile
is flattened similarly to Kühnen et al. (2018).

Based on this background, the objective of the present study is to investigate the effect of mean velocity profile on
the near-wall structure, which is primarily responsible for the friction drag, via the resolvent analysis. We first propose
two functions which can represent artificially flattened mean velocity profiles in a turbulent channel flow. Then, we
investigate how the near-wall mode is amplified or suppressed depending on the ‘flatness’ of the mean velocity profile.
The remainder of the paper is organized as follows. In Sec. 2, we explain the resolvent analysis and introduce ’flattened’
mean velocity profiles. The influence of the mean velocity profile on the amplification rate of the near-wall mode is
investigated and discussed in Sec. 3. Finally, conclusions are drawn in Sec. 4.

2. Methods

2.1. Resolvent analysis

In the present study, we assume a fully developed turbulent channel flow governed by the incompressible continuity
and Navier-Stokes equations, i.e.

\[ \nabla \cdot \mathbf{u} = 0, \]

\[ \frac{\partial \mathbf{u}}{\partial t} = -\nabla p - \mathbf{u} \cdot \nabla \mathbf{u} + \frac{1}{Re_s} \nabla^2 \mathbf{u}. \]

Here, \( \mathbf{u} = [u, v, w]^T \) denotes the streamwise (x), wall-normal (y) and spanwise (z) velocity components; \( t \) is the time; \( p \) is
the pressure. All variables are made dimensionless by the fluid density, the channel half-width \( \delta \), and the friction velocity
\( u_*^f \) (the superscript \# denotes dimensional quantities). The friction Reynolds number is set at \( Re_s = u_*^f \delta / \nu^f = 180 \), where \( \nu^f \)
is the kinematic viscosity. Hereafter, the superscript \# denotes the wall units based on \( u_*^f \) and \( \nu^f \).

The velocity and pressure fields can be expressed as the sum of harmonic modes by applying the Fourier transform in
x, z and t directions:

\[
\begin{bmatrix}
    u(x, y, z, t) \\
    p(x, y, z, t)
\end{bmatrix}
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty}
\begin{bmatrix}
    u_k(y) \\
    p_k(y)
\end{bmatrix}
 e^{i(k_x x + k_z z - \omega t)} dk_x dk_z d\omega,
\]

where each wavenumber-frequency combination \( \mathbf{k} = (k_x, k_z, \omega/k_z) \) represents a propagating wave with the streamwise
wavelength \( \lambda_x = 2\pi/k_z \), the spanwise wavelength \( \lambda_z = 2\pi/k_z \), and the phase speed \( c = \omega/k_z \), and \( i = \sqrt{-1} \). For each
Fourier mode, Eqs. (1) and (2) can be re-arranged as a matrix form:

\[
\begin{bmatrix} u_k \\ pk \end{bmatrix} = \begin{pmatrix} -i\omega & I \\ 0 & -L_k \nabla_k^T \end{pmatrix}^{-1} \begin{pmatrix} I \\ 0 \end{pmatrix} f_k = H_k f_k,
\]

where the nonlinear term in the Navier-Stokes equation \( f_k = -(u \cdot \nabla u)_k \) is treated as the forcing term (i.e., input) to the system represented by the resolvent operator \( H_k \), as shown in Fig. 1. The linear Navier-Stokes operator \( L_k \) in Eq. (4) is defined as

\[
L_k = \begin{bmatrix} -ik_x U + Re^{-1} \nabla_k^2 & -(\partial U/\partial y) \\ 0 & -ik_x U + Re^{-1} \nabla_k^2 \\ 0 & 0 \end{bmatrix},
\]

where \( U(y) \) is the mean velocity which will be introduced in sections 2.2 and 2.3, \( \nabla_k^2 = -(k_x^2 + k_y^2) + \partial^2/\partial y^2 \), and the discretization in \( y \) direction is done by using the Chebyshev collocation method (Weideman and Reddy, 2000). Although this analysis is essentially similar to the linear analysis of the Navier-Stokes equation, it should be emphasized that the nonlinear term \( f_k \) is explicitly retained as the external forcing term.

The amplification rate of a specific wavenumber-frequency mode is obtained by singular value decomposition (SVD) of the resolvent operator \( H_k \) (McKeon and Sharma, 2010). The SVD yields a set of orthonormal forcing modes \( f_{k,m} \) and response modes \( u_{k,m} \) that are ordered based on the forcing-response gain \( \sigma_{k,m} \):

\[
H_k = \sum_m u_{k,m} \sigma_{k,m} f_{k,m}^*.
\]

Here, the superscript of * denotes the complex conjugate. McKeon and Sharma (2010) and Luhar et al. (2014) have shown that the first singular value \( \sigma_{k,1} \) is much larger than the rest (i.e. \( \sigma_{k,1} > ... > \sigma_{k,n} \), \( n > 1 \)), and many key statistical and structural features of wall-bounded turbulent flow can be captured only by this rank-1 mode. Following these studies, we also consider rank-1 mode only, and hereafter the subscript \( m \) is dropped: \( \sigma_k = \sigma_{k,1}, u_k = u_{k,1}, f_k = f_{k,1} \). Also, we assume unit forcing following these studies.

The validity of rank-1 assumption has been checked by the ratio of rank-1 and rank-2 singular values \( \sigma_{k,2}/\sigma_{k,1} \), as shown in Fig. 2. The rank-1 mode is about 5 times greater than the rank-2 mode in the uncontrolled case, which is in good agreement with the previous report (McKeon and Sharma, 2010), and this ratio is nearly unchanged in the controlled cases, too.
Re pressure gradient, as mentioned above. Accordingly, the Reynolds numbers are fixed at

\[ u = \frac{C u_0}{Re} \]

but also the bulk Reynolds number

\[ Re = \frac{U}{\nu} \]

using these conditions, near-wall structures. In the conventional study of drag reduction using DNS, in contrast, the constraint is of crucial importance for a fair comparison on the effect of mean velocity profile on the amplification rate of turbulence. 

2.2. Mean velocity profile for the uncontrolled case

The mean velocity profile \( U(y) \) in the linear Navier-Stokes operator \( L_k \) is given by the turbulent viscosity model proposed by Reynolds and Tiederman (1967) (RT model). Following van Driest’s wall law and Reichardt’s law, the turbulent kinematic viscosity \( \nu_T \) (nondimensionalized by \( \nu^* \)) is expressed as

\[
\nu_T = \frac{1}{2} \left( 1 + \left( \frac{\kappa Re}{3} \right) \left( 2y - y^2 \right) \left( 3 - 4y + 2y^2 \right) \left( 1 - \exp \left( \frac{(y - 1 - Re\nu)}{\alpha} \right) \right) \right)^{1/2} \exp \left( \frac{1}{2} \right),
\]

where \( y \) is the distance from the wall normalized by \( \delta^* \), \( \kappa \) is the Kármán constant, and \( \alpha \) is the constant in van Driest’s wall law. Then, the mean velocity profile of the RT model can be computed by using \( \nu_T \) as

\[
U^*(y) = -Re \int_0^y \frac{1 - y}{1 + \nu_T(y)} dy.
\]

We set \( \kappa = 0.426 \) and \( \alpha = 25.4 \) for the uncontrolled case (hereafter denoted as Case NC).

2.3. Artificially flattened mean velocity profiles

In the present study, we propose two functions (Case 1 and Case 2) for the flattened mean velocity profiles. A special care is taken here so that the mean velocity profile is modified while keeping not only the friction Reynolds number \( Re \), but also the bulk Reynolds number \( Re_b = \frac{U_b^2}{2\nu^*} \) (where \( U_b \) denotes the bulk mean velocity) unchanged. This constraint is of crucial importance for a fair comparison on the effect of mean velocity profile on the amplification rate of near-wall structures. In the conventional study of drag reduction using DNS, in contrast, \( Re_b \) increases when \( Re \) is fixed (i.e., constant pressure gradient condition), while \( Re \) decreases when \( Re_b \) is fixed (i.e., constant flow rate condition) (see, e.g., Frohnapfel et al., 2012), which makes it difficult to study the isolated effect of the shape of mean velocity profile on the turbulent structure.

The mean velocity profile in Case 1 is based on the RT model. The two coefficients, \( \kappa \) and \( \alpha \) in Eq. (7), are changed — this corresponds to varying the mixing length. The mean velocity profile in Case 2 is proposed here so that the flatness of the profile can be varied more flexibly:

\[
U^*(y) = A \left\{ 1 - \frac{\cosh(B(1 - y)) - 1}{\cosh B - 1} \right\} + Cy \exp(-Dy),
\]

where \( A, B, C \) and \( D \) are the arbitrary constants. In both cases, \( U(y) \) is manipulated under a constant flow rate and constant pressure gradient, as mentioned above. Accordingly, the Reynolds numbers are fixed at \( Re_b = 5600 \) and \( Re = 180 \). By using these conditions, \( A \) and \( C \) in Eq. (9) can be uniquely determined by \( B \) and \( D \) as

\[
A = \frac{\left( \frac{Re}{D} \exp(-ReD) + \frac{1}{D} \exp(-ReD) - \frac{1}{D} + ReU_b^2 \right) \left( \frac{Re}{D} \exp(-ReD) + \frac{1}{D} \exp(-ReD) - \frac{1}{D} \right)}{Re(\cosh B - 1) + Re - \frac{Re}{D} \sinh B + \frac{B \sinh B}{Re} \left( \frac{Re}{D} \exp(-ReD) + \frac{1}{D} \exp(-ReD) - \frac{1}{D} \right)},
\]

\[
C = 1 - \frac{AB \sinh B}{Re(\cosh B - 1)}.
\]
In order to quantify the flatness of the mean velocity profiles, we examine three different measures because the definition of ‘flatness’ is somewhat arbitrary. The first measure \( F_1 \) is the flatness factor conventionally used in statistics, defined as

\[
F_1 = \frac{\frac{r^2}{s^2}}{\frac{r^2}{s^4}}
\]  

(12)

where \( r(y) = U(y) - U_b \) is the deviation of \( U(y) \) from \( U_b \) and \( s = \sqrt{r^2} \) is the root-mean-square value of \( r \). The overbar denotes the average across the channel half-width, and the subscripts 0 and \( c \) denote the uncontrolled and controlled cases, respectively. The second measure \( F_2 \) is so-called the spectral flatness, which was originally proposed to quantify the flatness of power spectra (see, e.g., Johnston (1988)), and defined as the ratio of the geometric mean and the arithmetic mean of the profile:

\[
F_2 = \frac{\phi_c}{\phi_0}.
\]  

(13)

with

\[
\phi = \frac{\prod_{n=1}^{N} (U_n^+)^{s_n}}{\sum_{n=1}^{N} U_n^+},
\]  

(14)

where \( U_n^+ = U^+(y_n^+) \) and \( N \) is the number of equi-spaced bins in the channel half-width, i.e., \( y_n^+ = n\Delta y^+ \), with \( \Delta y^+ = R_Re/N \) being the constant spacing defined only for the computation of \( \phi \). We set \( N = 200 \) in the present study. The third measure \( F_3 \) is based on the velocity gradient weighted by the distance from the wall, and it is introduced to enable more physical understanding:

\[
F_3 = \frac{\frac{-y(dU/dy)}{c}}{\frac{-y(dU/dy)}{0}}.
\]  

(15)

For all these three measures, the values greater than unity \( (F_1, F_2, F_3 > 1) \) means that the mean velocity profile in the controlled case is flatter than that in the uncontrolled case, and vice versa for \( F_1, F_2, F_3 < 1 \).

Table 2 summarizes \( F_1, F_2, \) and \( F_3 \) in the representative cases presented above. From Fig. 3 and Table 2, it is clear that flatter mean profiles correspond to larger values of these flatness measures. For example, comparison between Case 2a and Case 2c with any measures can identify that Case 2c is flatter, i.e., \( F_1, F_2, \) and \( F_3 \) for Case 2c are larger than that for Case 2a. Namely, all three measures give consistent trends despite the different definitions of flatness.

**Table 1** The constants in each representative mean velocity profile.

| Case   | \( \kappa \) | \( \alpha \) | \( B \) | \( D \) |
|--------|--------------|--------------|-------|-------|
| Case 1a| 0.32         | 13.4         | –     | –     |
| Case 1b| 1.2          | 99.4         | –     | –     |
| Case 2a| –            | 9.5          | 1.0   | –     |
| Case 2b| –            | 16.2         | 1.0   | –     |
| Case 2c| –            | 28.5         | 1.0   | –     |

**Table 2** Three different flatness measures, \( F_1, F_2 \) and \( F_3 \), for some representative cases.

| Case   | \( F_1 \) | \( F_2 \) | \( F_3 \) |
|--------|-----------|-----------|-----------|
| Case NC| 1.00      | 0.995     | 1.02      |
| Case 1a| 1.00      | 0.927     | 1.00      |
| Case 1b| 1.00      | 0.995     | 1.02      |
| Case 2a| 1.00      | 0.994     | 1.02      |
| Case 2b| 1.00      | 0.994     | 1.02      |
| Case 2c| 1.00      | 0.948     | 1.12      |

It is based on the velocity gradient weighted by the distance from the wall, and it is introduced to enable more physical understanding.

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### 3. Results and discussion

#### 3.1. Amplification rate for the near-wall mode

In the present study, we focus on the near-wall coherent structures, which have been widely accepted as a dominant contributor to turbulent friction drag in wall-bounded flow. In the previous studies (Luhar et al., 2015; Nakashima et
al., 2017), the characteristics of the near-wall coherent structures have been reproduced and assessed using the resolvent mode with $(\lambda_x^*, \lambda_z^*, c^*) = (10^3, 10^2, 10)$, called the near-wall (NW) mode.

To check the validity of focusing on the NW mode only, let us present in Fig. 4 the distribution of singular value in the wavelength space $(\lambda_x^*, \lambda_z^*)$ at $c^* = 10$ for the uncontrolled case and a representative case of mode suppression (Case 2c). In Case 2c, while the singular value is significantly suppressed in the entire wavelength space, no significant difference can be observed in the shape of distribution as compared to that in the uncontrolled case. Although not shown here, we have also confirmed that the trend is similar for the other phase speeds. Hereafter, the amplification rate for the NW mode is defined here using the singular value in the controlled case $\sigma_{k_c}$ and that in the uncontrolled case $\sigma_{k_0}$, as

$$R = \sigma_{k_c}/\sigma_{k_0},$$

where $R < 1$ means suppression of the NW mode. Note that $R$ can be considered closely related to the friction drag according to the previous studies which compared the singular value and the drag reduction rate obtained in DNS, e.g., Luhar et al. (2014) on the opposition control and Nakashima et al. (2017) on the suboptimal control.

Figure 5 shows the amplification rate $R$ for Case 1 and Case 2 as functions of three different flatness measures. The

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gray scattered plots represent all examined cases by varying the parameters (i.e., 180 cases for Case 1 and 80 cases for Case 2), whereof the colored markers denote the representative cases mentioned above. Regardless of the choice of the flatness measure, the observed trend is similar: $R$ decreases as $F_1$, $F_2$, and $F_3$ increase.

Noteworthy here is that the plots in both control cases are seen on a master curve despite the fact that these two are generated from different functions. Besides, considering the fact that the NW mode is suppressed/amplified in Case 2c/2a, the measure $F_3$ suggests that increasing the mean velocity gradient in the vicinity of the wall substantially contributes to suppress the NW mode. Although this may sound somewhat counter-intuitive, as shown in Fig. 6 the increase of the mean velocity gradient takes place only in the region of $0 < y^+ < 8$ (i.e., below the critical layer discussed later), and this is considered to have less influence for increasing the production of Reynolds stress due to the small wall-normal velocity in the viscous sublayer.

3.2. Mechanism of mode responses under flattened mean velocity profiles

In this section, we investigate the mechanism of mode responses under the flattened mean velocity profiles. Figure 7 shows the magnitudes of the streamwise and wall-normal velocities for the NW mode, $\sigma_k |u_k|$, $\sigma_k |v_k|$, in Case 2a and Case 2c, where Case 2a is the case of mode amplification and Case 2c is the case of mode suppression. As shown in Fig. 7(a), the streamwise velocity in Case 2a is significantly amplified due to the modification of the mean velocity profile. Furthermore, the location where $\sigma_k |u_k|$ takes the maximum value moves away from the wall. On the other hand, no obvious change is observed for the magnitude and peak location of the wall-normal velocity. In contrast, for Case 2c, the magnitudes of the streamwise and wall-normal velocities significantly decrease. The reduction ratio relative to the uncontrolled case is larger for the streamwise component than for the wall-normal component. In addition, unlike Case 2a, the locations where $\sigma_k |u_k|$ and $\sigma_k |v_k|$ take the maximum values become closer to the wall.

These changes can be explained by the location of the critical layer, also shown in Fig. 7, where the propagation...
speed of the mode is equal to the local mean velocity, i.e., \( c = U(y) \). The critical layer corresponds to a singular point of the linear dynamics in the inviscid limit. This singular behavior is smoothed by the viscosity, but it still remains also in viscous flows, whereby the velocity fluctuations are strongly amplified around the critical layer (Hreppirner and Fukagata, 2009; McKeon and Sharma, 2010). In Case 2a, the critical layer is slightly shifted upward from \( y^+ \approx 15 \) to \( y^+ \approx 17 \), where the viscous effect is weaker, to enhance amplification. On the other hand, in Case 2c, the critical layer shifts down to the region where the viscous effect is much stronger (\( y^+ \approx 8 \)), and the singular-like behavior is substantially mitigated.

Figure 8 shows the predicted velocity fields for the NW mode in the \( z^+ - y^+ \) cross-section. The contours represent the magnitude of the streamwise velocity, and the arrows show the wall-normal and spanwise velocities. As shown in Fig. 8(a), the velocity structure in the uncontrolled case is in good agreement with the well-known features of near-wall coherent structures: arrows represent the quasi-streamwise vortices, and high- and low-speed streaks exist among them (Robinson, 1991). It is found that both streaks and vortices in Case 2a are enhanced and the location of the vortex core shifts upward: the core exists at \( y^+ \approx 20 \) for the uncontrolled case, while at \( y^+ \approx 25 - 30 \) for Case 2a. This result is consistent with the change observed in Fig. 7(a) that the critical layer shifts away from the wall. On the other hand, as can be observed in Fig. 8(c), streaks and vortical structures in Case 2c are substantially suppressed, and the core of the vortex shifts closer to the wall (\( y^+ \approx 10 \)) corresponding to the downward shift of the critical layer.

In sum, flattening the mean velocity profile imposes the velocity gradient in the vicinity of the wall to increase under the constraint of constant bulk and Reynolds numbers. Due to this increase of velocity gradient, the critical layer is shifted down toward the wall, and the singular value is suppressed by the viscous effect to weaken the coherent structures contributing to the skin friction. Note that the movement of coherent structure toward the wall is usually considered to increase the drag. However, as discussed above, when the critical layer is further shifted down to approach the viscous sublayer, the singular behavior is substantially mitigated as is quantified by the significant suppression of the singular value.

4. Conclusions

We have investigated the effects of artificially flattened mean velocity profile on the near-wall (NW) mode responsible for turbulent friction drag via resolvent analysis. More specifically, we assessed the amplification rate of the NW mode under these flattened mean velocity profiles in a turbulent channel flow at \( Re_t = 180 \). Two functions (Case 1 and Case 2)
have been introduced in order to express the targeted profiles, and the flatness of the modified profiles are quantified by three different measures. A special care has been taken here so that both the bulk and friction Reynolds numbers are unchanged, whereby only the effect of change in the mean velocity profile can be studied.

Regardless of the choice of the flatness measure, the flattened mean velocity profile has been confirmed to directly suppress the NW mode. Subsequently, we have investigated the control effects of manipulating the mean velocity profile on the velocity fields and the near-wall coherent structures. The streamwise and wall-normal velocities are substantially suppressed by the flattened mean velocity profile.

This modification directly leads to the suppression of the near-wall vortices and streaks, which is considered to contribute to significant drag reduction. It has also been observed that the location where these structures exist has shifted toward the wall under the flattened mean velocity profiles.

The mechanism of the NW mode suppression by flattened mean velocity profiles can be summarized as follows. Flattening the mean velocity profile under the constraint of constant bulk and friction Reynolds numbers, which also corresponds to the initial condition of any controlled flows, imposes the velocity gradient in the region just above the wall to increase. Due to this change, the critical layer is shifted down toward the wall and the singular behavior of critical layer is mitigated due to the viscous effect, whereby the NW mode is suppressed.

Although we have used equations (10)–(12) to represent the flattened velocity profiles, there are many other candidates for the mathematical expressions. One can modify these expressions so that high Reynolds number flows or other control methods can be considered, which remains as future work.

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