The Impact of Ultraviolet Regularization on the Spectrum of Curvature Perturbations During Inflation

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Inflationary predictions based on the linear theory of cosmological perturbations are related to the two point function of a (second quantized) real scalar free field during the accelerated stage. Such a two point function is finite, in contrast with its coincidence limit, which is divergent due to the ultraviolet divergences proper of field theory. We therefore argue that predictions of most of the inflationary models do not necessarily need a regularization scheme to leading order, i.e. tree level, which is required instead for non-linear corrections or calculations involving the energy-momentum tensor. We also discuss unpleasant features of the "would be" regularized spectrum obtained using the traditional fourth order adiabatic subtraction.

I. INTRODUCTION

Inflationary predictions rely on the heritage of quantum field theory in cosmological space-times, pioneered by Parker at the end of the sixties [1]. The nearly scale invariant spectrum of curvature perturbations predicted by successful inflationary models is at present associated with the concept of amplification of vacuum fluctuations by the geometry.

The calculation of the two point function in the coincidence limit and of the energy-momentum tensor requires a renormalization scheme, just as in ordinary Minkowski space-time. Ultraviolet divergences due to fluctuations on arbitrarily short scales are common in field theory. In Minkowski space-time, infinities in a free theory are removed by the subtraction of the vacuum expectation value of the energy (also called normal ordering), the physical justification being that these vacuum contributions are unobservable. In cosmology, according to the inflationary paradigm, the large scale fluctuations we see derive from quantum vacuum fluctuations. However, a prescription similar to the one used in Minkowski is used in order to regularize infinities in curved space-times. The idea is to subtract from a bare (infinite) quadratic quantity its counterpart evaluated for an adiabatic change of the geometry. This procedure is therefore called adiabatic subtraction [2, 3] and in this way the infinities of field theory are removed.

Although adiabatic subtraction has been so far only a tool to remove infinities from divergent quantities, it has been argued recently [4] that a "regularized" spectrum may differ substantially from the bare one, leading to a rethinking of the inflationary predictions. The "regularized" spectrum proposed in [4] is the bare one minus the first two adiabatic terms, i.e. a minimal subtraction scheme for the coincidence limit of the two-point function.

The correlation function of different sets of observations - such as the pattern of anisotropies in the cosmic microwave background radiation (CMBR) or a catalogue of galaxies - is related to the one describing the curvature fluctuation for adiabatic initial conditions. In the simplest picture, long wavelength curvature perturbations remain constant after their exit from the Hubble radius during inflation until their re-entry in the Hubble radius during the radiation or matter dominated stage. During inflation, a curvature perturbation is proportional to the gauge invariant (i.e. with respect to coordinate transformation) inflaton fluctuation. Therefore, in the simplest picture, the correlation function of the primordial curvature fluctuations we derive from observations is related to the two-point function of inflaton fluctuations taken at different space (or space-time) points. We remark that this two point function of a
free scalar field during inflation is finite, in contrast with its coincidence limit, and no regularization scheme seems necessary to leading order. Although Ref. [4] proposes a minimal subtraction scheme for the coincidence limit of the two-point function, we show that if one adopts the traditional fourth order adiabatic subtraction - which is needed for consistency in the regularization of objects evaluated in the coincidence limit - in order to obtain a "regularized" spectrum, this latter quantity becomes negative for a range of wavelengths.

Our paper is organized as follows. In sections II we illustrate the two-point function of a real minimally coupled scalar field in de Sitter space-time. We discuss its coincidence limit and its associated adiabatic subtraction scheme in section III. We discuss general inflationary models and, in particular, the case of power-law inflation in section IV. We then conclude in Section V.

II. TWO POINT FUNCTION IN DE SITTER SPACE-TIME

Let us consider scalar fluctuations with mass $m$ and non minimal coupling to the curvature $\xi$ propagating in a cosmological (rigid) space-time, whose Fourier modes satisfy the following equation

$$\left(a\varphi_k\right)'' + \Omega_k^2 (a\varphi_k) = 0, \quad \Omega_k^2 = k^2 + m^2 a^2 + \left(\xi - \frac{1}{6}\right) a^2 R,$$

(1)

where the prime denotes the derivative with respect to the conformal time $\eta$. Once these fluctuations are quantized in terms of creation and annihilation operators $\hat{b}^\dagger$, $\hat{b}$:

$$\hat{\varphi}(\eta, x) = \frac{1}{(2\pi)^{3/2}} \int dk \left[ \varphi_k(\eta) e^{i k \cdot x^\dagger} + \varphi_k^*(\eta) e^{-i k \cdot x} \right]$$

(2)

we obtain the relation between the two point correlation function of a scalar fluctuation evaluated at different space points and the spectrum $P_{\varphi}$:

$$\langle \hat{\varphi}(\eta, x) \hat{\varphi}(\eta, x') \rangle = \int_\ell^{+\infty} \frac{dk}{k} \int \frac{dk}{k} \left| k^3 |\varphi_k(\eta)|^2 \right| \int_\ell^{+\infty} \frac{dk}{k} \left| k^3 |\varphi_k(\eta')|^2 \right| = \int_\ell^{+\infty} \frac{dk}{k} \left| k^3 |\varphi_k(\eta)|^2 \right| = \int_\ell^{+\infty} \frac{dk}{k} \left| k^3 |\varphi_k(\eta)|^2 \right| P_{\varphi}(k, \eta).$$

(3)

The presence of a non-trivial infrared cut-off $\ell \neq 0$, related to the beginning of inflation [5, 6], becomes important if the spectrum of $\varphi$ is scale invariant or steeper (i.e. $d \ln P_{\varphi}/d \ln k \leq 0$) and does not alter the argument of this paper related to ultraviolet divergencies.

During the inflationary expansion fluctuations are stretched and amplified from initial vacuum fluctuations:

$$|\varphi_k| \sim \frac{1}{a\sqrt{2\Omega_k}} \text{ for } k/a >> H.$$  

(4)

In order to evaluate the correlation function above (of Hadamard kind) one should keep in mind that it may be written as the time coincidence limit of the more general Green’s function, $\langle \hat{\varphi}(\eta, x) \hat{\varphi}(\eta', x') \rangle$, which is a solution of the homogeneous equation of motion [7, 8] (see also [9]). The latter may be computed from the Wightman functions, again solutions of the homogeneous equations of motion, determined by the usual prescription which gives a dependence in $\left[(\eta - \eta' - i\epsilon)^2 - |x - x'|^2\right]$ and is compatible with the Minkowsky limit form in suitable cases. Indeed this fact can be seen easily for a de Sitter space-time [9, 10], which is conformally flat. In the time coincidence limit the Hadamard function $\langle \hat{\varphi}(\eta, x) \hat{\varphi}(\eta, x') \rangle$ is well defined and depends on $|x - x'|^2$. The above mentioned prescription allows for the convergence of the momentum integral in the ultraviolet region (see the integral in Eq. [4] where the mode behaviour in [1] is considered), which otherwise would be characterized by an oscillating integrand in the infinite momentum region.

Although de Sitter space-time does not support Gaussian inflationary perturbations, it admits an exact solution for test scalar fields - with arbitrary masses and couplings to the curvature - which for our present purposes is similar to that for inflaton fluctuations in a nearly de Sitter space-time. We here just consider a minimal coupling. For an evolution of the scale factor given by

$$a(\eta) = -\frac{1}{H \eta}, \quad -\infty < \eta < 0,$$

(5)

the solution for $\varphi_k$ agreeing with the adiabatic vacuum for short wavelengths is

$$\varphi_k = \sqrt{\frac{\pi}{4Ha^3 H_1^{(1)}}} \left( \frac{k}{aH} \right)^\nu, \quad \nu = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}.$$  

(6)
On considering \( m \neq 0 \) the infrared divergence is absent and the infrared cutoff \( l \) can be set to zero. The de Sitter result for Eq. (6,7,9) is given by

\[
\langle \hat{\varphi}(\eta, \mathbf{x})\hat{\varphi}(\eta, \mathbf{x}') \rangle_{\text{ds}} = \frac{H^2 \sec(\pi \nu)}{16\pi^2} \left( \frac{1}{4} - \nu^2 \right) \, _2F_1 \left[ \frac{3}{2} + \nu, \frac{3}{2} - \nu; 1; 1 - \frac{|\mathbf{x} - \mathbf{x}'|^2}{4\eta^2} \right].
\] (7)

We note that this result can be simply obtained from Eq. (6) by an analytic continuation obtained on performing a general integration in \( d \) space dimensions and afterwards taking the \( d \to 3 \) limit. The analytic structure of the hypergeometric function provides a covering of the different space-time regions, as is evident if one considers the more general correlation function at different times (in such a case the last argument in the hypergeometric function is \( 1 + [ (\eta - \eta')^2 - |\mathbf{x} - \mathbf{x}'|^2] / (4\eta' \nu) \)). In particular in the space-like region the argument ranges from 1 (deep inside the Hubble radius) to \(-\infty\) (fully outside the Hubble radius), while for time-like regions the argument is greater than 1.

In the space-like region the asymptotic infrared (large distance) behavior (see Fig. 1) is given by

\[
\langle \varphi(\eta, \mathbf{x})\varphi(\eta, \mathbf{x}') \rangle_{\text{as}} \sim -H^2 \sec(\pi \nu) \frac{2^{2\nu-5}}{\pi^{\nu/2}} \frac{\Gamma(\nu)}{\Gamma(\nu - 1/2)} \left( \frac{|\mathbf{x} - \mathbf{x}'|^2}{4\eta^2} \right)^{-\nu - \frac{5}{2}}.
\] (8)

Inflationary models without a classical time dependent homogeneous background value for the inflaton, such as a test field in a de Sitter background, were also proposed as models with non-gaussian (\( \chi^2 \) distributed) isocurvature perturbations with a blue spectral index \([11,12]\). The experimentally interesting quantities are related to correlation functions of such objects and at a quantum level in such a simple free case one has from Wick’s theorem

\[
\langle \varphi^2(\eta, \mathbf{x})\varphi^2(\eta, \mathbf{x}') \rangle - \langle \varphi^2(\eta, \mathbf{x}) \rangle^2 = 2 \langle \varphi(\eta, \mathbf{x})\varphi(\eta, \mathbf{x}') \rangle^2.
\] (9)

Therefore, in this case the subtraction (which implements a trivial renormalization) is included in the definition of a connected Green’s function.

### III. REGULARIZATION OF THE COINCIDENCE LIMIT OF THE TWO POINT FUNCTION

Let us try to understand why one may be tempted to introduce a deformation of the spectrum although there is no necessity to do it. When one is interested in the vacuum expectation value of composite quantum operators such as the energy-momentum tensor, the appearance of divergences of ultraviolet origin calls for a renormalization procedure which normally consists of two steps: regularization (in order to work with finite quantities) and subtraction (to remove the divergent pieces). The simplest of such operators is \( \varphi^2(x) \). In order to have a regularized quantity for its vacuum average one may consider point-splitting \([10]\), also other approaches, such as the dimensional regularization, are often considered. As stated before a possible way of removing the infinities is given by the adiabatic subtraction \([2,3]\). To be in agreement with the renormalized effective action results \([10]\), which also determine the anomalous contribution to the trace of the energy momentum tensor averaged over the vacuum, a fourth order adiabatic subtraction is needed. In fact, this is also needed to remove the ultraviolet divergence for the energy-momentum tensor in a general space-time (as can be seen, in the case of inflationary models with scalar metric fluctuations, from the general adiabatic expansion described in \([13]\).

Let us consider the coincidence limit of Eq. (6)

\[
\langle \varphi^2(\eta, \mathbf{x}) \rangle = \int_{-\infty}^{+\infty} \frac{dk}{k} \frac{k^3 |\varphi_k(\eta)|^2}{2\pi^2} = \int_{-\infty}^{+\infty} \frac{dk}{k} P_{\varphi}(k, \eta).
\] (10)

This quantity is not defined since it is divergent (\(|\mathbf{x} - \mathbf{x}'|^2 \neq 0 \) can be seen as a regulator in the point-splitting version). Nevertheless, on using the adiabatic subtraction, the renormalized value can be obtained in different ways: one can

\[
FIG. 1: \langle \hat{\varphi}(\eta, \mathbf{x})\hat{\varphi}(\eta, \mathbf{x}') \rangle_{\text{ds}} \text{ in units of } H^2 \text{ for } m^2/H^2 = 10^{-2} \text{ as a function of } \log_{10} \frac{|\mathbf{x} - \mathbf{x}'|^2}{4\eta^2}.
\]
first perform a dimensional regularization of the divergent integral and then subtract the fourth order adiabatic value, safely removing the regularization afterwards to obtain the finite renormalized expectation value:

\[
\langle \hat{\varphi}^2 \rangle_{\text{REN}} = \lim_{d \to 3} \left( \langle \varphi^2 \rangle - \langle \varphi^2 \rangle_{(4)} \right) = \lim_{d \to 3} \left\{ \frac{1}{(2\pi)^d \Gamma(d/2)} \left[ \int_{\ell}^{\infty} dk k^{d-1} |\varphi_k|^2 - \int_{\ell}^{\infty} dk k^{d-1} |\varphi_k^{(4)}|^2 \right] \right\}.
\]

Or else one may subtract the fourth adiabatic order before performing the integrals without using an additional regularization procedure:

\[
\langle \hat{\varphi}^2 \rangle_{\text{REN}} = \frac{1}{2\pi^2} \int_{\ell}^{\infty} dk k^2 \left[ |\varphi_k|^2 - |\varphi_k^{(4)}|^2 \right].
\]

These two ways of proceeding give, with the presence of a non zero infrared cut-off \( l \), always the same result. Without a non zero infrared cut-off there are some interesting exceptions, for the particular case of a de Sitter space-time with \( m = \xi = 0 \), for instance, those two methods fail to reproduce the right value for \( \langle \varphi^2 \rangle_{\text{REN}} \) (given by Eq. 11) while for the renormalized value of the energy-momentum tensor, which has no infrared divergence, only the second method is applicable, on proceeding as described in \[15\], obtaining the correct Allen-Folacci value \[13\].

We now go back to our particular case of a de Sitter space-time, the \( \varphi_k^{(4)} \) expansion up to the fourth adiabatic order is given by (using the result of \[16\]):

\[
|\varphi_k^{(4)}| = \frac{1}{a \sqrt{2\Sigma_k}} \left\{ 1 + \frac{a^2 R}{24} \frac{1}{\Sigma_k} + \frac{5 a^4 R^2}{32 \Sigma_k} + \frac{1}{8 \Sigma_k^2} \left[ (a^\alpha + a^\beta) \left( m^2 - \frac{R}{6} \right) + \frac{5}{16 \Sigma_k^4} \right] \right. \\
\left. - \frac{3}{64 \Sigma_k^2} \left( a^\alpha + a^\beta \right) m^2 - \frac{65}{64 \Sigma_k} a^\alpha m^4 + \frac{5}{32} \epsilon_2 - \frac{1}{4} \epsilon_4 \right\}
\]

with

\[
\epsilon_2 = -\frac{1}{2} \frac{\Sigma_k'}{\Sigma_k} + \frac{3}{4} \frac{\Sigma_k'^2}{\Sigma_k}, \quad \epsilon_4 = -\frac{1}{4} \frac{\Sigma_k'}{\Sigma_k} - \frac{1}{4} \frac{\Sigma_k''}{\Sigma_k}
\]

Let us note that the adiabatic terms are defined over the whole \( k \) range.

Although the subtraction of the adiabatic terms are required in order to obtain a finite answer for quantities integrated over (almost) the whole \( k \) range, it is worth investigating whether the subtracted adiabatic terms \( k \) by \( k \) do spoil \( P_\varphi \), which should be positive by physical reasons and whose shape is in agreement with observations and constitutes the major success of inflation. We shall therefore investigate

\[
P_\varphi^{(n)}(k, \eta) = P_\varphi(k, \eta) - P_\varphi^{(n)}(k, \eta),
\]

which is the most immediate "regularized" spectrum, given the adiabatic subtraction scheme illustrated in Eq. 12.

In \[4\] it has been chosen to make the subtraction only up to second order since, as is clear from Eqs. \[12, 13, 14\] for the particular case of a de Sitter space-time, this is enough to remove all the infinities in Eq. 12. But, as we state at the beginning of this section, although for \( \langle \varphi^2 \rangle \) the ultraviolet divergence are present only in the first two adiabatic orders, the fourth order adiabatic subtraction may be needed in order to have a self-consistent scheme.

For a de Sitter space time Eq. (15) gives:

\[
P_\varphi^{(4)}(k, \eta) = \frac{k^3}{4\pi^2} \left\{ \frac{\pi}{2H^2a^3} \left| H^{(1)} \left( \frac{k}{aH} \right) \right|^2 - \frac{1}{a^2 \Sigma_k} \right. \\
\left. - \frac{5 a^4 m^4 H^2}{8} \frac{1}{\Sigma_k^2} + \frac{3 a^2 m^2 H^2}{4} \frac{1}{\Sigma_k} + \frac{H^2}{\Sigma_k^3} \right\} + \left( \frac{1155 a^{10} m^8 H^4}{128} \Sigma_k^3 - \frac{693 a^8 m^6 H^4}{32} \Sigma_k^{11} + \frac{385 a^6 m^4 H^4}{32} \Sigma_k^9 + \frac{5 a^4 m^2 H^4}{2} \Sigma_k^7 \right).
\]

A key point in discussing Eq. 16 is the different time evolution of the mode \( \varphi_k \) and its adiabatic approximation. A nearly scale invariant spectrum is given by a minimally coupled scalar field with \( m << H \), whose Fourier mode \( \varphi_k \)
larger than the Compton one). The exact solution is: an equation for a massless minimally coupled scalar field (the same as the polarization amplitude for gravitational fluctuations should be taken into account with field fluctuations, leading to gauge-invariant curvature perturbations.

\( O \) is almost frozen in time after it has crossed the Hubble radius, although starting from the adiabatic vacuum when it is well within the latter. As is clear from Eq. (13), adiabatic terms decay at least as \( \mathcal{O}(1/a^2) \) for wavelengths much larger than the Compton one \( k/a \ll m(\ll H) \). Because of this differing behaviour in time, the infrared part of the spectrum is left unchanged by the subtraction in any case. Indeed, for scales well outside the Hubble radius \( k \ll aH \):

\[
\frac{k^3}{2\pi^2} \frac{\pi}{4H a^2} \left| H^{(1)}_\nu \right|^2 \approx \frac{k^3}{2\pi^2} \frac{\pi^2}{4H} \frac{\Gamma^2(\nu)}{2\nu} \left( \frac{2H}{k} \right)^{2\nu}
\]

which shows how the bare part weakly depends on time (\( \mathcal{O}(a^{-\frac{3+\nu}{2\nu}}) \) for \( m \ll H \)).

Fig. 2 shows the bare PS and its regularized forms for various values of the mass \( m \). Note that the fourth order regularized PS \( P^{(4)}_{\nu} \) becomes always negative for a different range of wavelengths for any value of the mass \( m \). Fig. 2 shows the same for the laplacian term on making an adiabatic expansion only for \( |\varphi| \).

**IV. INFLATIONARY EXAMPLES**

Inflationary models driven by a classical zero mode support Gaussian inflationary fluctuations. Scalar metric fluctuations should be taken into account with field fluctuations, leading to gauge-invariant curvature perturbations

\[
R = \frac{H}{\dot{\phi}} Q_{\nu}
\]

with \( Q_{\nu} \) being the Mukhanov variable. In the Uniform Curvature Gauge this coincide with the inflaton fluctuation and satisfies:

\[
\ddot{Q}_{\nu} + 3H \dot{Q}_{\nu} + \left[ \frac{k^2}{a^2} + V_{\varphi \varphi} + 2\frac{\dot{H}}{H} \left( 3H - \frac{\dot{H}}{H} + 2\frac{\dot{\phi}}{\dot{\phi}} \right) \right] Q_{\nu} = 0.
\]

Power-law inflation \([\ref{17}]\) with \( V(\phi) \propto e^{-\frac{\phi^\alpha}{m^2}} \) is one of the few cases for which solutions for inflaton fluctuations are known on the inflationary trajectory for which \( a(t) \sim t^p \) with \( p > 1 \). On the above trajectory Eq. (19) becomes an equation for a massless minimally coupled scalar field (the same as the polarization amplitude for gravitational waves). The exact solution is:

\[
\varphi_k = \sqrt{-\frac{\pi \eta}{4a^2}} H^{(1)}_{\alpha} (-k\eta), \quad \alpha = \frac{3}{2} + \frac{1}{p - 1}
\]
FIG. 4: Plots of the following power spectra in units of $H^2$ vs $\ln(k/aH)$ in a power-law model of inflation: $P_Q$ (thick line), $P_Q^{(0)}$ (solid line), $P_Q^{(2)}$ (short-dashed line), $P_Q^{(4)}$ (long-dashed line). From the left to the right plot, $p = 2, 10, 60, 100$. Note how the fourth order adiabatic spectrum $P_Q^{(4)}$ becomes negative for the larger values of $p$, even if for different values of $k$.

Although the above exact solutions are very similar to the ones in Eq. (6) for the de Sitter case, the two point correlation function for power-law inflation depends on the infrared cut-off since the spectrum for $Q_k$ is red-tilted [18]. Such an infrared divergence in the Fourier integral is general for all the inflationary models predicting a red-tilted spectrum of cosmological perturbations and of course persists in the coincidence limit. The final formula for the two point function will therefore be dependent on the infrared cut-off (which may be related to the beginning of inflation): in this case the power spectrum will not be simply the one obtained from Eq. (20), but it will be changed for very small $k$. Such a correction may be relevant in the case of a short inflationary stage and/or a very smooth beginning of inflation. In some extreme cases, such infrared modifications required by theory may lead to observable effects [19].

Also in such a simple case of inflation, $P_Q^{(4)}(k)$ can become negative. Proceeding as in the de Sitter case and using the result of [13] for the adiabatic part, $P_Q^{(4)}(k)$ is shown in Fig. 4 for different values of $p$. The adiabatic expansion for the gauge-invariant field fluctuation for a general potential $V(\phi)$ is given in [13], and therefore any model can be studied at tree-level in an analogous way.

V. CONCLUSIONS

We have discussed the necessity of introducing a deformation in the bare power spectrum of inflationary fluctuations due to regularization/renormalization prescriptions. We have shown that there is no necessity to concern oneself about ultraviolet divergencies for all inflationary models as long as the fluctuations are studied for free fields to lowest order: the two-point function (taken at different points) of fluctuations is finite. This conclusion applies also to inflationary models without a time dependent homogeneous value, as shown in Eq. (9). In contrast, the fluctuation two point function requires a prescription in the infrared for those red-tilted spectra which are in agreement with observations. To lowest order there is no ultraviolet renormalization to be invoked and we have shown that the deformation of the spectrum proposed in [4], if extended to a four order subtraction, may lead to additional problems, such as "regularized" spectra which are not positive in a certain range of $k$.

Beyond linear order, first corrections to correlation functions (of linear quantities) appear instead at quartic order in the fields. There can be different approaches which start from Einstein equations [20] or from the associated effective action for the fluctuations [21, 22]. In both cases it appears clear that the inclusion of perturbative corrections to the correlation functions requires taking into account a proper renormalization procedure, which removes the ultraviolet divergences arising from loop diagrams, and the consequent redefinition of the couplings.

VI. ACKNOWLEDGEMENTS

We wish to thank L. Parker for kind correspondence and useful comments on the manuscript.

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