Quantum Oscillations of Electrons and of Composite Fermions in Two Dimensions: Beyond the Luttinger Expansion

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Quantum oscillation phenomena, in conventional 2-dimensional electron systems and in the fractional quantum Hall effect, are usually treated in the Lifshitz-Kosevich formalism. This is justified in three dimensions by Luttinger’s expansion, in the parameter \( \omega_c/\mu \). We show that in two dimensions this expansion breaks down, and derive a new expression, exact in the limit where rainbow graphs dominate the self-energy. Application of our results to the fractional quantum Hall effect near half-filling shows very strong deviations from Lifshitz-Kosevich behaviour. We expect that such deviations will be important in any strongly-interacting 2-dimensional electronic system.

71.18, 73.40.Hm

Quantum oscillation (QO) phenomena (in which Landau quantisation causes all thermodynamic and transport properties of conductors to oscillate with \( 1/B \), where \( B \) is the sample induction) have been for four decades amongst the most powerful tools in solid state physics. The CF cyclotron frequency \( \omega_c \) is the mean “statis-'

\[ \frac{\Sigma_{osc}}{\Sigma} \sim \left( \frac{\omega_c}{\mu} \right)^{3/2} \]  

and also \( \Phi(\Sigma)+\beta^{-1}\text{Tr}(\Sigma \tilde{G})=0 \) at least to \( \sim O((\omega_c/\mu)^3) \). Thus writing \( \Omega = \Omega_0 + \Omega_{osc} \), we have that the leading oscillatory contribution up to \( \sim O((\omega_c/\mu)^{3/2}) \) is contained in

\[ \Omega \sim -\frac{1}{\beta} \text{Tr}[\log \tilde{G}^{-1}] \]

where \( \mu \) is defined as the zero of the energy, \( n \) labels the Landau levels and \( \sigma \) is a spin index. Eq. (6), which contains the non-oscillatory self-energy \( \Sigma \), provides the fundamental justifi-

\[ \omega \sim -\frac{1}{\beta} \sum_{i\omega_m,n,\sigma,\bar{k}_z} \log \left[ i\omega_m - \epsilon_{\sigma nk_z} + \bar{\Sigma}(i\omega_m, \epsilon_{\sigma nk_z}) \right] \]

with

\[ \epsilon_{\sigma nk_z} = \epsilon + \left( n + \frac{1}{2} \right) \hbar \omega_c + \frac{\hbar^2 k_z^2}{2m} - \mu \]

where \( \mu \) is the upper cut-off, equal to the Fermi energy (including the highly singular \( \Sigma_{osc} \)) must be used.

In this paper we show that

(a) Luttinger’s expansion fails in any interacting 2D electronic system; however

(b) an alternative expansion can be found under certain circumstances (see below), in which now the full self-energy (including the highly singular \( \Sigma_{osc} \)) must be used.

(c) This new expansion can give results sharply different from the previous one.

To show the practical importance of these results, we will apply them to CF’s: however they are relevant in principle to any 2D electronic system.

(i) Failure of Luttinger’s Expansion: We first repeat the analysis which yields Eqs. (2,3), but now in two dimensions. We shall find quite generally that

\[ \frac{1}{\beta} \text{Tr}[\log \tilde{G}^{-1}] \sim \left( \frac{\omega_c}{\mu} \right)^2 \]
where \( \hat{\text{der}} \) (2nd-order) contribution to \( \Omega \), and (e) shows the lowest graph sum of such terms. In (d) we have the lowest order “RPA” term, (c) a self-consistent “rainbow self-energy graphs include (a) the Hartree-Fock term, (b) higher graphs. The graph in Fig. 1 (a) has the real-space dependent phase factor \( \Sigma_{osc} \), and \( \Sigma \) again. In fact the scaling property \( \Sigma_{osc}/\Sigma \) as a function of \( \omega_c/\mu \), depends only on the dimensionality of the graph (as well as the presence of at least one internal fermion line \( \Sigma \)), and is true of all higher graphs.

(ii) Alternative Expansions: There are two cases for which a simple alternative to \( \Sigma \) can be found for \( \Omega_{osc} \).

The first is where vertex corrections to the usual Schwinger-Dyson/Nambu-Eliashberg self-energy (Fig. 1(c)) can be neglected. Then \( \Sigma = \lambda^2 \int \mathcal{G} \mathcal{D} \), where \( \mathcal{G} \) and \( \mathcal{D} \) are given self-consistently in terms of \( \Sigma \), thus summing over all “rainbow graphs”. The relevant skeleton graph \( \Phi_2 \) (Fig. 1(d)) then exactly cancels \( \beta^{-1} \text{Tr}[\Sigma \mathcal{G}] \) in \( \Sigma \), and \( \Omega = \Omega + \Omega_{osc} \) is given, to all orders in \( \omega_c/\mu \), by

\[
\begin{align*}
\Omega = -\frac{1}{\beta} \text{Tr}[\log \mathcal{G}^{-1}] \\
= -\frac{1}{\beta} \sum_{\omega_m, n, \sigma} \log[i\omega_m - \epsilon_{\sigma n} - \Sigma(i\omega_m, \epsilon_{\sigma n})]
\end{align*}
\]

The crucial difference from \( \Sigma \) (apart from the suppression of \( k_z \)) is that \( \Sigma \) now includes \( \Sigma_{osc} \). Deviations from \( \Sigma \) arise from “crossed” graphs (Fig. 1(e)), and there are many physical cases in which these are unimportant. In the case of composite fermions the corrections from crossed “gauge fluctuation” graphs are not small but at low energy their main effect both in zero field and finite field, is simply to renormalise the vertices in the rainbow graph sum, without changing the functional form of \( \Sigma \). Thus this approximation actually works well even beyond the “ Migdal limit” in which crossed graphs are small. The difference between \( \text{Tr}[\log \mathcal{G}^{-1}] \) and \( \text{Tr}[\log \mathcal{G}] \) depends crucially on how big is \( \Sigma_{osc}/\Sigma \); even though formally this is \( \sim \mathcal{O}(\omega_c/\mu) \) for all 2D systems, its actual value, for a given \( \omega_c/\mu \), varies enormously between different systems.

The second case is of more academic interest; it arises when we may write \( \Omega \) in terms of a set of “statistical quasiparticle” (SQP) energies \( \epsilon_{\sigma \nu} \) as

\[
\Omega = -\frac{1}{\beta} \sum_{\nu, \sigma} \log(1 + e^{\beta(\mu - \epsilon_{\sigma \nu})})
\]

The contour \( \hat{C} \) encircles the negative real axis counterclockwise, the contours \( \hat{C}_l \) likewise encircle the points \( T_l = 2\pi i l/\beta \omega_c \), with \( l = \pm 1, \pm 2 \ldots \). The 3D function \( F_{3D}(r,t) \) differs from \( \Sigma \) by the factor \((2\pi \beta t)^{-1/2} \exp[-z^2/2\beta t] \), where \( z \) is the third dimension, perpendicular to \( r \) (cf. Ref. [16] Eq. (A.16)). It is this difference which yields \( \Sigma_{osc}/\Sigma \), instead of \( \Sigma \), upon integrating over \( t \) in (11) and (14).

Consider now graph \( \Sigma \) (b), assuming that the internal boson line represents either (i) a phonon, or a conventional “Fermi liquid” electronic fluctuation; or (ii) a singular gauge fluctuation \( \Sigma \). Using the known results for \( \Sigma_{osc} \) for these cases, one easily verifies (6) and (8) again. In fact the scaling property \( \Sigma_{osc}/\Sigma \) of \( \Sigma_{osc}/\Sigma \) as a function of \( \omega_c/\mu \), depends only on the dimensionality of the graph (as well as the presence of at least one internal fermion line \( \Sigma \)), and is true of all higher graphs.

\[
\frac{\Sigma_{osc}}{\Sigma} \sim \frac{\omega_c}{\mu}
\]

Thus the term \( \sim \mathcal{O}(\Sigma_{osc}^2) \) is as important as the “leading” term, and the whole expansion must be re-examined.

FIG. 1. The Feynman graphs discussed in the text. The self-energy graphs include (a) the Hartree-Fock term, (b) the lowest order “RPA” term, (c) a self-consistent “rainbow graph” sum of such terms. In (d) we have the lowest order (2nd-order) contribution to \( \Omega \), and (e) shows the lowest order “RPA” term, (c) a self-consistent “rainbow self-energy graphs include (a) the Hartree-Fock term, (b) higher graphs. The graph in Fig. 1 (a) has the real-space dependent phase factor \( \Sigma_{osc} \), and \( \Sigma \) again. In fact the scaling property \( \Sigma_{osc}/\Sigma \) as a function of \( \omega_c/\mu \), depends only on the dimensionality of the graph (as well as the presence of at least one internal fermion line \( \Sigma \)), and is true of all higher graphs.

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\end{align*}
\]
We now consider the new result (15) in more detail. Supressing the sum over spins, we rewrite (15) as an integral, with Σ(x) = Σ′(x) + iΣ″(x)

\[ \Omega = \frac{m \omega_c}{\Phi_0} \int_0^\infty \frac{dx}{\pi} n_f(x) \tan^{-1}(\phi(x, \epsilon)) \] (17)

where \( \phi(x, \epsilon) = \text{Im}G/\text{Re}G \). The prefactor \( \frac{m \omega_c}{\Phi_0} \) is the Landau level degeneracy. Defining \( \epsilon = n \omega_c - \mu \), the Poisson sum formula is used to separate the oscillatory components of Ω:

\[ \Omega = \frac{m \omega_c}{\Phi_0} \int_0^\infty \frac{dx}{\pi} n_f(x) \left[ \int_{-\mu}^{\infty} d\epsilon \tan^{-1}(\phi(x, \epsilon)) \right] - \frac{m \omega_c}{\Phi_0} \int_{-\mu}^{\infty} d\epsilon \int_0^\infty \frac{dx}{\pi} n_f(x) \sin \left( \frac{2 \pi k (\epsilon + \mu)}{\omega_c} \right) \] (18)

where in the oscillatory (i.e., \( k > 0 \)) terms in the Poisson sum we have extended the limits of the \( \epsilon \) integral to \( \pm \infty \) and integrated by parts.

At first glance the first term \( M_1 \) resembles the results of Fowler and Prange, and Engelsberg and Simpson, however it now involves the full \( \Sigma \) (including \( \Sigma_{osc} \)). The second term \( M_2 \) is formally of the same order in \( \omega_c/\mu \) as \( M_1 \), and quite new. Typically the term in \( \frac{\partial}{\partial \beta} \) dominates \( M_2 \), and we shall see below that in 2 dimensions it can be much larger than \( M_1 \).

Eqs. (15-21) are valid for any 2-dimensional charged system for which the full self-energy (including oscillatory contributions) can be written down.

(iv) Application to the Composite Fermion System: We now wish to demonstrate on a particular example that the deviations from orthodox behaviour can be rather large. We choose the CF gauge theory, for which the oscillatory self-energy for composite fermions interacting with gauge fluctuations was previously calculated. Here we assume unscreened Coulomb interactions (i.e., the dynamical exponent \( z = s = 2 \)). For numerical work it is convenient to use a Matsubara sum over \( \Sigma(x) \) evaluated at \( x = i \omega_l = i \pi (2l + 1)/\beta \); writing \( \text{Im} \Sigma(i \omega_l) \equiv \xi(i \omega_l) \), we have

\[ \xi(i \omega_l) = 2 \kappa_2 \left[ \frac{\omega_l}{\pi} \log \left( \frac{\omega_l}{\mu} \right) + \frac{4}{\beta} \sum_{k'=1}^{\infty} \sum_{\omega_m > 0} (-1)^{k'} \exp \left( \frac{-2 \pi k' \omega_m}{\omega_{CF}} \right) \log \left( \frac{\omega_l + \omega_m}{\omega_{CF}} \right) \cos \left( \frac{2 \pi k' \mu}{\omega_{CF}} \right) \right] \] (22)

where the coupling \( \kappa_2 \) is usually slightly less than one. In QO experiments one examines \( \log(A) \); in LK theory \( \log(A) \) should be a linear function of \( 1/B \) (the “Dingle plot”), as well as of \( T \) (the “mass plot”). Fig. 3 shows (for the example of CF fermions), the importance of \( M_2 \), as well as the considerable non-linearity shown in Dingle plots (which we also find in the mass plots, not shown here). Thus in this example a conventional analysis of QO phenomena, using either the LK formula, or its generalisation, clearly fails. We do not believe this example to be untypical (in fact if we choose screened short-range interactions between the CF systems with dynamical exponent \( s = 3 \), we get much worse deviations!). We thus believe that where strong violations of conventional behaviour are observed or where interaction effects are known to be strong, one should re-analyse the data using the results herein. In the context of the FQHE near half-filling, fits of QO results to LK theory should clearly be treated with caution.
FIG. 3. A numerical evaluation of $\log(A)$, where $A$ is the amplitude of the dHvA oscillations in $M$, as a function of $1/B$ for various fixed temperatures, for a system of CF quasi-particles (we assume an unscreened Coulomb interaction, and use the second-order result for $\Sigma$ derived earlier [14]). We assume a chemical potential of 180K, and a coupling constant $\kappa_2 = 0.8$. The dashed lines show $\log(A_1)$, and the solid lines show $\log(A_1 + A_2)$, where $A_1$ and $A_2$ are the amplitudes of oscillations of $M_1$ and $M_2$.

In summary, we have shown that the LK formalism (or its many-body generalisations [14]) for describing quantum oscillations breaks down in two dimensions. To remedy this, we have derived new results that can be applied when crossed diagrams may be neglected. We have applied these results to a problem of current interest, i.e., composite fermions interacting with gauge fluctuations (believed to give a good description of the fractional quantum Hall states, at least near half-filling). The results show radical departures from LK behaviour. Such departures should also exist in other strongly-interacting 2-dimensional electronic systems, whether or not they behave as Fermi liquids in zero field.

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23 For ordinary electrons in 2D, it is likely that $\Omega(\lambda c)$ has an essential singularity, as a function of the Coulomb interaction $V_{c}$, at $\lambda = 0$. For CF’s, both the convergence of the theory and its stability about the mean field starting point have yet to be clarified. In three dimensions it is often assumed that any non-perturbative breakdown of Fermi liquid theory, in the presence of Landau quantisation, must be negligible when $\omega_{c}/\mu \ll 1$. This is probably true, given the success of conventional dHvA experiments; but theoretical work (see eg., V. Yakovenko, Phys. Rev. B 47, 8851 (1993), and refs. therein) shows that a breakdown is inevitable.
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