The free-fall mode experiment on LISA Pathfinder: first results

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Abstract. The LISA Pathfinder space mission is testing the critical experimental challenge for LISA by measuring the differential acceleration between two free-falling test masses inside a single co-orbiting spacecraft at a level of sub-femto-g for frequencies down to 0.1 mHz. In LPF it is necessary that one test mass (TM) is electrostatically forced to follow the orbit of the other TM. This force represents a noise source in differential acceleration at frequencies below 1 mHz. The free-fall mode experiment has been performed in order to reduce this source of noise: the actuation is limited to short impulses on one TM, so that it is in free fall between two successive kicks, while the other TM is drag-free. The free-fall mode thus provides a different technique for measuring the differential TM acceleration without the added force noise and calibration issues introduced by the actuator. Data analysis challenge is related to the presence of the kicks: they represent a high-noise contribution and need to be removed, thus leaving short gaps in data. This article presents preliminary data of the LPF free-fall measurement campaign and describes the three data analysis techniques developed to mitigate the presence of gaps.

1. Introduction
LISA Pathfinder (LPF) is an European Space Agency mission launched on December 3rd 2015 with the objective of measuring the nearly-pure free-fall motion close to the level needed for future space-based gravitational wave observatories, like LISA [1]. The main observable of LPF is the differential acceleration between two TMs measured along the \(x\) sensitive degree of freedom joining the TMs and arising from spurious random forces, \(\Delta g\) (see Figure 1). While the TMs in LISA will be hosted separately in two spacecrafts on average 2.5 million kilometers apart [2], in LPF the acceleration measurement is evaluated between two test masses that are enclosed in the same spacecraft. To fulfill the specifications, the differential acceleration noise on LPF must be maintained within 30 fm s\(^{-2}\) Hz\(^{-1/2}\) at 1 mHz, in terms of Amplitude Spectral Density (ASD).

Because it is not possible, for a single spacecraft, to follow both the trajectories of the TMs along the same sensitive degree of freedom \(x\), one TM (TM2) must be electrostatically controlled to follow the other. The electrostatic control on TM2 is based on the differential interferometric readout \(o_{12}\) and is principally needed to compensate the gravity imbalance experienced between the TMs along the sensitive \(x\) axis. The continuous control along \(x\) for TM2 is typical of the standard science measurement in LPF and it will not present in LISA. Actually, in LPF all degrees of freedom for both TMs, except \(x\) for the free-falling TM (TM1), are controlled.

The electrostatic suspension loop acts at frequencies \(\leq 1\) mHz and is based on a capacitive sensor composed of 18 electrodes surrounding the TMs. Each set of opposite electrodes controls
the TM motion along two degrees of freedom (i.e. $x_2$ and $\phi_2$, as depicted in Figure 1). Unavoidable fluctuations in the applied voltages induce fluctuations in the force exerted along $x_2$ which add acceleration noise to the TM2 motion and thus to $\Delta g$. The resulting noise in force for a given relative fluctuation of voltage on a certain electrode is proportional to the force applied by that electrode:

$$F \propto V_{act}^2 \Rightarrow \frac{\delta F}{F} \propto 2 \frac{\delta V_{act}}{V_{act}}$$

where $V_{act}$ is the voltage applied to the electrode [3]. If the actuation were limited to a single force to compensate the DC differential acceleration $\Delta g_{DC}$, applied with a single electrode, we would have a noise contribution due to actuation of $S^{1/2}_{\Delta g_{act}} \approx 2\Delta g_{DC}S^{1/2}_{V_{act}/V_{act}}$ in ASD terms. The true actuation noise is complicated by the simultaneous application of actuation torques in $\phi$ and the “constant stiffness” actuation scheme [4] which pulls on both sides of the TM. The ASD of the relative voltage amplitude measured on ground and confirmed in flight, takes a value in a range between 3 and 8 ppm/$\sqrt{\text{Hz}}$ at 1 mHz [1, 4]. Considering the requirement levels for $\Delta g_{DC} < 0.65 \text{nm s}^{-2}$ and rotational $\phi$ angular acceleration below 2 nrad s$^{-2}$, the pre-flight estimates of actuation noise were roughly 7.5 fm s$^{-2}/\sqrt{\text{Hz}}$ at 1 mHz [4]. This source of noise is thus expected to be the dominant contribution at low frequency in LPF.

![Figure 1](image_url)

Figure 1: Schematic representation of the coordinate systems and the capacitive actuation for the $x$-$\phi$ configuration in LPF. $g_1$ and $g_2$ represent the stray acceleration experimented by the TM1 and TM2 respectively.

It is important to underline that the actuation noise depends also on the force/torque authorities. As a consequence, the noise changes with the actuation configuration. On LPF the authority was reduced with respect to the nominal values. Indeed, the time-series of $\Delta g$ measured in flight within the first months of science operations, was characterized by a quasi-static part, of a few tens of pm s$^{-2}$, with fluctuations of $(5.57 \pm 0.04)$ fm s$^{-2}/\sqrt{\text{Hz}}$ for frequencies between 0.7 and 20 mHz [1]. Because of this low static gravitational acceleration level compared with the prediction of 650 pm s$^{-2}$, the maximum force along $x_2$ has been reduced from 2200 pN, typical of the Nominal authority, to 50 pN. The corresponding lower actuation configuration is called URLA (Ultra Reduced Low Authority).

As anticipated, the sensitive axis ($x_2$) actuation control, that potentially could limit sensitivity in LPF, is not needed in LISA, where each TM is drag-free along its sensitive interferometry axis. However, an experimental investigation of the actuation force noise contributes to the characterization of the capacitive sensor with effect on the LISA instrument design, especially for the $x$-$\phi$ scheme as in LISA the $\phi$ degree of freedom will be controlled. In this context, the LPF free-fall mode is a control scheme that aims at eliminating the noise
contribution due to \( x \)-axis actuation: the electrostatic control on the coordinate \( x_2 \) is applied by means of short electric impulses. In this way the actuation is limited to brief kicks, so that TM2 is in free fall between two successive kicks. The actuation-free motion is then analyzed for the remaining sources of acceleration noise. In fact this mode solves, at least partially, the problem of actuation noise, as the \( \phi \) control remains. However, this experiment can provide an upper limit on the LISA sensitivity, by assuming the same \( \phi \)-gravitational balance level of LPF. In any case, the free-fall experiment constitutes an alternative technique for measuring the differential TM acceleration without the added force noise and calibration issues introduced by the actuator.

It is possible to predict the actuation noise contributions, in standard science and free-fall mode, on basis of a full actuation noise model and a dedicated actuation noise test campaign that will be discussed in detail in a separate article [5]. The projections, for both URLA and Nominal authorities, are depicted in Figure 2, together with the first LPF results in URLA and the LISA and LPF requirements [1]. The figure evidences that, in Nominal case, the implementation of an intermittent control on LPF is expected to subtract a considerable contribution to the total noise measured in a standard run. Differently, in the lowest authority configuration, a small fraction of actuation noise is subtracted (roughly 15\% in ASD terms, as shown in Figure 2). This result derives from the fact that actuation is already dominated by \( \phi \)-control in URLA standard measurement and removing \( x \)-actuation does not reduce much noise at low frequency. However, if the predictions in URLA are confirmed, the free-fall data could validate the accuracy of the actuator calibration achieved in the standard measurement and reported in [1]. Indeed, in this case, an agreement between the intermittent and the continuous control is expected, in terms of low frequency performance [5]. It is necessary to remark, finally, that this result is related to the low control levels effectively applied in flight, thanks to the unexpected low gravity imbalance measured on the spacecraft.

![Figure 2: Blue curve: ASD of \( \Delta g \) measured for 6.5 days after 127 days from launch in URLA authority \( (F_{\text{max}} = 50\,\text{pN}) \) and compared with LPF and LISA requirements (dashed curves) [1]. The ASD is the result of averaging 26 periodograms of 40000-s each with 50\% overlap, according to the standard Welch’s averaged periodogram method [6]. The figure includes the actuation noise contribution in URLA and Nominal \( (F_{\text{max}} = 2200\,\text{pN}) \) authority for both standard (red curves) and free-fall mode measurements (green curves).](image-url)
2. Eliminating the \( x \)-axis actuator: the free-fall mode experiment

The free-fall mode experiment has been implemented in LPF according to the configuration indicated in Figure 3a: the flight and kick durations are fixed to \( \sim 350 \) s and \( 1 \) s respectively [7]. These values have been set to limit the displacement to reduce non-linearity issues: in presence of the maximum gravitational imbalance predicted \( (\sim 0.65 \text{ nm s}^{-2}) \), the TM exceeds the sensing range \( (\pm 5 \mu \text{m} \text{ [8]}) \) in \( \sim 400 \) s. A kick controller is used to produce periodic kicks: it keeps track of the motion of TM2 during the free-phase and estimates the impulse needed to “kick it back” on the other side. The amplitude of the following kick is then set to apply this impulse and the kick-and-drift scheme is repeated. The amplitude of the kick force is thus strictly related to the DC gravity imbalance to be compensated but it is also set by the duty cycle of the experiment, \( \chi \equiv T_{\text{kick}}/T_{\text{exp}} \): \( F_{\text{kick}} = F_{\text{DC}}/\chi \), where \( T_{\text{kick}} \) is the kick duration and \( T_{\text{exp}} = T_{\text{flight}} + T_{\text{kick}} \) the experimental time.

To estimate the spectrum at frequencies below the experimental frequency, \( f_{\text{exp}} = 1/T_{\text{exp}} \approx 2.8 \text{ mHz} \), data from successive flights are combined. Figure 3b shows the differential displacement time-series measured by the interferometer \( o_{12} = x_2 - x_1 \), during the free-fall experiment performed in URLA in June 2016 and compared with the kick force.

![Figure 3](image_url)

In the period to which this article refer, this experiment has been executed four times on LPF (from June to August 2016). Here we present the June measurements only, performed in Nominal and URLA authority. In all cases the kick control has been achieved successfully and maintained stable over the mission.

3. Data analysis challenge and proposed strategies

The differential acceleration time-series measured during the free-fall mode experiment is characterized by periodic kicks which represent a high noise configuration, as shown in Figure 4a. The kicks reveal as spikes in the spectrum at frequencies multiple of the experimental frequency, \( f_{\text{exp}} = 1/T_{\text{exp}} = 2.85 \text{ mHz} \), right in the LPF measurement bandwidth (Figure 4b).
Because the purpose of the experiment is to consider only data relative to the free-phases, the kicks must be removed. As a result, $\Delta g$ data contain gaps. Gaps may corrupt the spectral estimation in form of spectral leakage both from low frequencies and high frequencies, thus introducing systematic bias in the underlying spectrum. The nature of this bias is related to the kick characteristics (duration, repetition) and to the original spectrum of the signal. The understanding of this effect is fundamental, as it might corrupt the spectrum especially at frequencies below 1 mHz, where actuation noise could be the main contribution to the total noise and needed to be investigated.

Three data analysis approaches, which will be described in the following, have been developed within the LPF collaboration to mitigate the presence of gaps. These methods are applied to $\Delta g$, which, in turn, is estimated after having “calibrated” the free-fall data, that is once the dynamical parameters of the experiment are extracted, such as the elastic couplings.

### 3.1. $\Delta g$ estimation with free-fall data

The method followed to estimate $\Delta g$ in this specific experiment consists in fitting the numerical second derivative of the relative displacement $\ddot{o}_{12}$, to the following model:

$$\ddot{o}_{12} = \Delta g - \omega_2^2 o_{12} - \Delta \omega^2 o_1 = \Delta g_{DC} + \dot{g}_0 t - \omega_1^2 o_{12} - \Delta \omega^2 o_1$$ (2)

where $o_{12} = x_2 - x_1$ and $o_1 = x_1 - x_{SC}$ are the interferometric readout, $\omega_1^2$ and $\omega_2^2 = \Delta \omega^2 + \omega_1^2$ the elastic couplings between each TM and the spacecraft and $\Delta g$ is disentangled into its static part $\Delta g_{DC}$, and its time-drift observed over the mission, $\dot{g}_0$ [1]. $\Delta g_{DC}$, $\dot{g}_0$, $\omega_1^2$ and $\Delta \omega^2$ are thus the fit parameters. This fit is performed in acceleration flight by flight, by means of a dedicated routine implemented in LTPDA. To reduce the high frequency noise, the telemetry channels included in the model are low-pass filtered ($\ddot{o}_{12}$, $o_{12}$, $o_1$). This is achieved by means of an anti-aliasing Finite Impulse Response filter with coefficients of a Blackmann-Harris window and with duration equal to $T_{win} = T_{exp}/4$ s. Next, to reduce the number of samples per flight time, the data are downsampled at $\sim 57$ mHz, which correspond to have 20 samples per flight. The parameter configuration of filtering and downsampling has been chosen to limit aliasing

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1 LTPDA (LISA Technology Package Data Analysis) is the MATLAB toolbox used for the data analysis within the LPF collaboration [9].
effects. At the end, as many sets of parameters as flights are obtained. Each parameter is then averaged over its N-values to get a single estimate. (N is the total number of flights in the experiment\textsuperscript{2}). Using the averaged dynamical parameters, the time-series of $\Delta g$ can be evaluated as follows from equation 2, with the original telemetry channels sampled at 10 Hz:

$$\Delta g(t) = \ddot{o}_{12}(t) + \omega^2_2\dot{o}_{12}(t) + \Delta \omega^2 o_1(t) - \dot{g}_0 t$$ (3)

The $\Delta g$ obtained in this experiment shows noisy data during the kicks, which must be removed. Similarly to what performed to the standard science data, the resulting time-series is corrected for centrifugal effects, as discussed in [1] and then analyzed by means of one of the following techniques proposed to fill the kick data by limiting the unavoidable bias that this operation would produce on spectral estimation.

3.2. Blackmann-Harris low-pass technique with kicks set to zero

The purpose of this method is to reduce the spectral leakage at frequencies within the LPF frequency band that derives from high frequency noise [10]. It essentially consists of a low-pass filtering and downsampling applied to $\Delta g$ data, after which the kick samples are set numerically to zero. A schematic of the method is depicted in Figure 5. The name of the technique refers to the shape of the low-pass filter chosen, that is a normalized Blackmann-Harris window.

The first step of this technique consists in setting up the correct number of samples per experimental time ($T_{exp} = 350.2$ s) once $\Delta g$ data, initially sampled at 10 Hz, are decimated. This number must be an integer factor of the total number of samples per experimental time in the original data such that each experimental segment still contains, after decimation, a fixed number of data points. It is straightforward that this number, $n_{tot}$, will fix the new sampling time $T_{samp}$ after the downsampling:

$$T_{samp} n_{tot} = T_{exp} = T_{flight} + T_{kick}$$ (4)

The flight duration considered in the analysis is reduced to $T_{flight} - 2T_{cut}$ ($T_{cut} = 2$ s) in order to avoid transients which may be close to the kicks. The low-pass filter length $T_{win}$, is set up in such a way as to have an integer number of finite windows per flight time:

$$T_{win} = T_{flight} - 2T_{cut} - (n_{keep} - 1)T_{samp}$$ (5)

where $n_{keep}$ is the number of samples maintained per flight time.

The free-fall data have been downsampled considering 34 samples per experimental time, as it is an integer factor of the total number of samples per experimental time ($N_{exp} = T_{exp} \cdot 10$ Hz = 3502). The decimation factor $N_{dec}$ is thus equal to 103. According to equations 4 and 5, it follows that the new sampling time is $T_{samp} = 10.3$ s and the filter length is $T_{win} = 98$ s, as $n_{keep}$ is set to 25. Data points corresponding to windows that overlap with the kick period are set to zero ($n_{gap} = n_{tot} - n_{keep} = 9$ samples).

The low-pass filtering is achieved by means of an anti-aliasing Finite Impulse Response filter, in order to not mix in the data in the gap, with coefficients of a Blackmann-Harris window (to be precise we use the “minimum 4-term Blackman-Harris window”, also known as “BH92”). The choice of such a window is justified by the requirements in terms of spectral leakage.

\textsuperscript{2} An alternative fit performed to estimate $\Delta g$, that can be applied to original 10 Hz data, is based on a time domain logarithmic likelihood function with the noise level being a parameter of the fit. After a first search using the Nelder-Mead simplex algorithm, an adaptive Markov Chain Monte Carlo algorithm (MCMC) which samples the parameter space with the Metropolis-Hastings algorithm, is applied to estimate proper errors.
performances\textsuperscript{3} \cite{6}. The filter is applied by sliding it over the $\Delta g$ time-series with a rate equal to the new sampling time: filtering and decimation are thus applied at the same time, as shown in Figure 5. After that, the spectrum can be estimated and normalized for the BH window transfer function. Finally it must be corrected for the bias related to the “gap ratio” $n_{\text{tot}}/n_{\text{keep}}$ because of presence of zeros in data\textsuperscript{4}.

![Figure 5: Block diagram of the BH low-pass approach. The figure on the right shows the effect of the technique: the BH filter is slid over the $\Delta g$ data stretch with a rate equal to the new sampling time, 10.3 s. The bold coloured windows are those contaminated by the kick interval, as it is shown in the filtered and decimated $\Delta g$ time-series below. The corresponding samples in will be set to zero. The vertical dashed lines delimit the resulting gap.](image)

3.3. Windowing technique

The basic idea of this approach is to zero out the kicks in $\Delta g$ by means of a spectral window \cite{11}. The concept is similar to spectral estimation in general, where the time-series is multiplied by a normalized windowing function before computing the spectrum. In this case the windowing ensures that the data stretch smoothly approaches zero at its ends, avoiding artifacts in the spectrum caused by the unavoidable truncation of the data series to a finite length.

In the approach proposed here, the choice of the window is crucial because it must guarantee two conditions: it has to take the value zero at both the extremities of the time-series and also during each gap and furthermore it has to minimize the spectral leakage in the LPF bandwidth. The window used here is an “high frequency” window, in the form of an Hahn spectral window, that smoothly reaches zero at each kick. The effect of the gap-windowing is visible in Figure 6, where the kick force time-series is compared with the window profile. The resulting $\Delta g$ data is thus multiplied by this window and then the spectrum is extracted. Note that this analysis is independent of the sampling frequency of the data. If desired, the data can be downsamplied prior to the analysis itself with standard LTPDA methods. The only quantity the user has to specify is the position or the duration of the kicks which can either be detected automatically or be derived from the time-series of the pulsed force. The estimation of the spectral bias, introduced by the data reduction procedure, is under investigation also for this approach.

\textsuperscript{3} 4-term Blackman-Harris window has a small lobe adjacent to the main peak in the transfer function (92 dB below the main peak) with the first zero located at frequency $f = \pm 4.00$ bins. Thus the power within the side lobes, which contributes to the spectral estimation, is reduced of 92 dB, making the BH92 one of the best-performing available window in terms of spectral leakage suppression.

\textsuperscript{4} In fact, this factor corrects the bias on the \textit{white} contribution to the spectrum only, as can be calculated analytically. In presence of a “coloured” noise spectrum, as that in LPF, the spectral bias estimation is more complicated. We refer this discussion to upcoming publications as it is strictly related to the technique calibration which is still under consolidation.
3.4. Constraint-Gaussian gap patching

Another strategy proposed to analyze the free-fall data consists of filling the gaps with proper random noise that has the same statistics as the rest of the free-fall data. The purpose of the approach is to generate a reconstructed data series with a spectrum as close as possible to that one would obtain if the data had been taken continuously but without the application of force kicks. To minimize the bias resulting from the patches, the data in the patches should have the same spectral content as the existing data. The technique, that is called “Constrained-Gaussian Gap Patching”, is described in detail in [11]. Once the gaps are filled with the synthetic noise, standard approaches can be followed to estimate the spectrum of the whole data set.

A comparison among the three approaches described above, in terms of $\Delta g$ with gaps filled, is shown in Figure 7. Data refer to the free-fall measurement executed in June in URLA authority.

Figure 6: Time-series of the window used to zero out the kicks in $\Delta g$ and comparison with the kick force.

Figure 7: Resulting $\Delta g$ data after having applied the three approaches.
4. First results of free-fall experiment and discussion

The free-fall experiment described in this article was performed in June 2016 and it lasted two days: it was implemented in Nominal authority on the first day, then the maximum forces and torques were reduced to URLA values and maintained until the end of the measurement.

As the initial mean value of the static differential acceleration was about $-2.5 \, \text{pm} \, \text{s}^{-2}$, according to what estimated in the preceding noise-only measurement (see Figure 8), the flights have a downward concavity and amplitude of the order of roughly 40 nm, as already shown in the previous figures. Moreover, because of the negative sign of $\Delta g_{DC}$, TM2 moves away from TM1. As a consequence, the kick force applied on TM2 is expected to be positive with respect to the TM2 reference frame (see Figure 1) as it forces TM2 toward TM1. We see a confirmation of this in Figure 3b.

The overall free-fall time-series estimated in June and analyzed for instance with the BH low-pass filtering approach, is compared with that measured in the preceding science measurement in Figure 8. The configuration of the free-fall measurements performed in June, in terms of duration, static and time-dependent differential acceleration, dynamic range in displacement and control force, is summarized in Table 1. A comparison with the previous standard noise measurement is included in the table.

![Image](image.png)

Figure 8: $\Delta g$ time-series measured in June 6-10 during a standard science measurement (blue) and the free-fall experiment performed in Nominal (red) and URLA (green) authority. The data in blue are sampled at 10 Hz and low-passed with a cut-off frequency of 0.6 mHz. The free-fall time-series are, for instance, the result of the BH low-pass approach, sampled at $\sim 97$ mHz before subtracting the time-drift: they correspond to the flight samples only (the kick samples are set numerically to zero).

To conclude, the free-fall mode control has been achieved and maintained successfully over the mission. The experiment dynamics is in agreement with what expected in terms of flight amplitude and kick force, considering the differential acceleration effectively measured between the TMs. In addition, the estimate of $\Delta g$ is consistent with that evaluated in the preceding science measurement, as shown in Figure 8.

As regards the data analysis, all the approaches described here allow to extract the power spectral density of $\Delta g$ measured in this specific experiment. However, the spectral bias estimation is still an ongoing analysis. The final results will be reported in a future publication.
Run DOY duration $\Delta g_{DC}$ $\dot{g}_0$ $\Delta x$ $F_{xx}$

|                              |    |       |      |      |      |      |
|------------------------------|----|-------|------|------|------|------|
| Standard noise URLA          | 158-160 | 54   | -1.7 | -0.6 | $\sim$ 0.04 | $\sim$ 0.010 |
| Free-fall Nominal            | 161 | 20    | -2.5 | -0.4 | 38    | 1.6  |
| Free-fall URLA               | 162 | 24    | -3.0 | -0.5 | 46    | 1.9  |

Table 1: Characteristic parameters of the standard science measurement and free-fall experiments executed in June 2016. The table reports the averaged value of $\Delta g_{DC}$ and $\dot{g}_0$, as well as the mean value of the dynamic range (flight amplitude in case of free-fall experiment) and control force.

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