An Improved Chaotic Detection System for Metal Detectors

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Abstract. This paper first applies a chaotic system-Duffing oscillator to a metal detector, and uses RHR algorithm to compute two Lyapunov characteristics exponents of the Duffing system. In this way, the two Lyapunov characteristic exponents can help to judge the Duffing system being chaotic or not quantitatively. And also help to get the threshold value more accurately. Then a simulation model of Duffing system fit for detectors is established by Matlab. Simulation results indicate that an suitable Duffing system can improve the sensitivity of a detector effectively.

1. Introduction
This paper mainly studies metal detectors based on Faraday's law of electromagnetic induction in industry [1,2]. A detector response relies on the probe design technology and the subsequent weak signal processing. If eddy current induced in the probe can be detected by advanced signal processing system, sensitivity will be improved. The Duffing system is sensitive to periodic signal and immune to noise to some extent. We applied Duffing oscillator to metal detection[3]. This method is the first time used in metal detection as far as we know. Recently with extensive application of chaotic theory, Duffing chaotic system is applied to sinusoidal weak signal measurement.

The frequency of a detector probe is usually more than 10KHz. But present research work about periodic signal detection of Duffing system is mainly about a few hertz. As for as we know, materials about hundreds of Hertz are not available[4]. New research work at high frequency is necessary. So here the Duffing system is adopted a frequency 31.8KHz (namely \(\omega\) is 10^5 rad/s) that a detector often use. Then at this frequency we use RHR algorithm to derive and compute Duffing system’s Lyapunov characteristics exponents (LCE). We use LCEs to obtain the critical value of the state transform and judge the system state quantitatively. This method is more precise than that to determine the system state by direct observation of the phase trajectory especially it becomes deformed. Besides, it may be applied to an automatic discrimination device instead of judgement by our eyes. We also make simulation by the Simulink of Matlab based on above analysis.

2. Typical configuration
Figure 1 gives the typical configuration of a metal detector[6]. The key parts are probe (coils) and signal processing system. The driving coil is in the middle, and two identical balance coils are placed with the same distance to it. The detected objects(such as food, cotton,etc.) passes parallel through the common axis of the three coils. A sinusoidal AC current of tens of kHz is applied to the driving coil, therefore, it produces a AC magnetic field that causes voltage in balance coils[6]. If products without metal passing through the probe, the two balance coils have equal induced voltages and offset in the PDA[1]. When products with metal appear in the probe, an induced current will be generated in the metal and creates a voltage difference between the balance coils. Thus it will be sent to PDA. The
signal processing system mainly includes a chaotic detection system and a differential amplifier in this paper. They can identify the signal from the probe and alarm.

The phase locked loop (PLL) synthesizer produces sinusoidal signals which will be input to the driving coil through an medium-frequency(MF) power amplifier[1].

**Figure 1. A common configuration.**

### 3. Chaotic system for detection

Duffing equation is

\[
\begin{align*}
\frac{dy_1}{dt} &= \omega y_2 \\
\frac{dy_2}{dt} &= \omega (r \cos(\omega t) + n(t) - ky_2 + y_1^3 - y_1^5)
\end{align*}
\]

Where \(k\) is the damping coefficient greater than zero(usually \(k\) is set to 0.5), \(-y_3^3+y_5^5\) is a nonlinear function term, \(r \cos(\omega t)\) is a periodic external force, and \(n(t)\) represents Gaussian white noise. If \(k\) remains constant, when we raise \(r\) gradually, the system will experience a series of different states. And finally it will be the chaotic critical and large scale periodic states[5]. Through identifying this two completely different states of the system, we may know whether a sinusoidal signal appears or not[4].

Here we use \(r_d\) as the critical amplitude of the internal reference signal \(r_d \cos(\omega t)\). If products without metal are in the probe, the system will still be chaotic. If products with metal are in the probe, a signal from the probe will be input to the signal processing system. It will make the system transform into a large-scale periodic state from chaos. Thus Through distinguishing the phase transition, we know products with metal are in the probe.

We use RHR algorithm to deduce and calculate LCEs, so as to provide a theoretical basis for quantitatively judging the system state and calculating the critical amplitude \(r_d\).

### 4. RHR algorithm for LCEs computation of Duffing oscillator

As a main standard, LCEs are key to discriminate a system being chaotic or not. We can calculate two LCEs of one Duffing oscillator. Less than zero value of the larger LCE shows that it is in a periodic state. And greater than zero value of the larger LCE indicates that it is chaotic. In this way, by calculating LCEs, we may accurately know its state of every moment clearly[7].

The QR factorization of fundamental solution matrix is crucial in calculation of LCEs. Ragarajang proposed an new algorithm(RHR algorithm.)for QR factorization[7]. Here Differential equations for LCEs calculation are deduced by this algorithm[7].

At the beginning, we will change formula (1) to the following form.
\[
\begin{align*}
\frac{dy_1}{dt} &= \omega y_2, \\
\frac{dy_2}{dt} &= \omega (r \cos y_3 + n(t) - 0.5y_2 + y_1^3 - y_1^5), \\
\frac{dy_3}{dt} &= \omega
\end{align*}
\]

and there is
\[
y_1(0) = y_{10}, \quad y_2(0) = y_{20}, \quad y_3(0) = 0
\]

Formula (2) can be expressed as
\[
j(t) = F \left( y(t) \right), \quad Y(0) = I_1
\]

As for Formula (4), its variational form can be expressed as follows.
\[
Y'(t) = J(t)Y(t), \quad Y(0) = I_1
\]

Where \(J(t)\) is the Jacobin matrix of this oscillator and \(I_3\) is 3 times 3 identity matrix, then \(Y(t)\) is a 3 × 3 matrix. \(J(t)\) can be written as
\[
J(t) = \begin{bmatrix}
0 & \omega & 0 \\
0 & -0.5\omega & -\omega \sin y_1 \\
0 & 0 & 0 \\
\end{bmatrix}
\]

The elements \(Y_{31}, Y_{32}, \) and \(Y_{33}\) of \(Y(t)\) are invariant, for \(J(t)\) being a singular matrix with the last row are zero. we get the last row elements \(Y_{31}=0, Y_{32}=0, Y_{33}=1\) from \(Y(0)=I_3\). \(Y(t)\) is factorized into the following formula.
\[
\begin{bmatrix}
Y_{11} & Y_{12} & Y_{13} \\
Y_{21} & Y_{22} & Y_{23} \\
0 & 0 & 1 \\
\end{bmatrix} = \begin{bmatrix}
Q_{11} & Q_{12} & 0 \\
Q_{21} & Q_{22} & 0 \\
0 & 0 & 1 \\
\end{bmatrix} \begin{bmatrix}
R_{11} & R_{12} & R_{13} \\
R_{21} & R_{22} & R_{23} \\
0 & 0 & 1 \\
\end{bmatrix}
\]

LCEs can be obtained by \(R_{11}, R_{22}, R_{33}\). The formula for their variation over time is as follows.
\[
g_i(t) = \ln R_{ii}(t)
\]

Where \(i=1, 2, 3\). Three LCEs can be calculated by
\[
g_i(t) = \lim_{t \to 0} g_i(t)
\]

Due to \(R_{33}=1\), it is certain that one of the three LCEs is zero. \(R_{11}(t)\) and \(R_{22}(t)\) are used to calculate the other two LCEs. In this way, we only need to write variational equations of a two-dimensional subsystem. Its form is
\[
\dot{Y}_1(t) = J_1(t)Y_1(t), \quad Y(0) = I_2
\]

Where \(J_1(t)\) is its Jacobin matrix and \(I_2\) is 2 times 2 identity matrix, then \(Y_1(t)\) is 2 × 2 matrix. \(J(t)\) can be written as follows.
\[
J_1(t) = \begin{bmatrix}
0 & \omega \\
\omega (3y_1^2 - 5y_1^4) & -0.5\omega \\
\end{bmatrix}
\]

\(Y_1(t)\) is factorized into the following formula.
\[
Y_1(t) = Q_1(t)R_1(t)
\]

Substituting (12) into (10), we obtain the variational equation
\[
Q_1 \cdot R_1 + \dot{Q}_1 \cdot R_1 = J_1 \cdot Q_1 \cdot R_1, \quad Q_1(0) \cdot R_1(0) = I_2
\]

Equation (13) Left-multiplied by \(Q_1^T\) and right-multiplied by \(R_1^{-1}\) becomes
\[
Q_1^T \cdot \dot{Q}_1 + \dot{R}_1 \cdot R_1^{-1} = Q_1^T \cdot J_1 \cdot Q_1, \quad Q_1(0) = I_2, \quad R_1(0) = I_2
\]

By RHR algorithm, we may write the orthogonal matrix \(Q_1(t)\) of the Duffing system by an angle variable function. So \(Q_1(t)\) can be written as by one angle \(\theta(t)\)
4

\[ Q(t) = \begin{bmatrix} \cos \theta(t) & \sin \theta(t) \\ -\sin \theta(t) & \cos \theta(t) \end{bmatrix} \]  

(15)

\[ R_i(t) \], an upper triangular matrix, may be expressed as the following form.

\[ R_i(t) = \begin{bmatrix} e^{\theta_i(t)} & r_{i2} \\ 0 & e^{	heta_i(t)} \end{bmatrix} \]  

(16)

Substituting \( Q_1(t)^T, R_1(t)^{-1}, Q_1(t) \) and \( R_1(t) \) into (13), we obtain

\[ \begin{bmatrix} \cos \theta(t) - \sin \theta(t) \\ \sin \theta(t) \cos \theta(t) \end{bmatrix} \begin{bmatrix} -\sin \theta(t) & \cos \theta(t) \\ -\cos \theta(t) & -\sin \theta(t) \end{bmatrix} \begin{bmatrix} \vartheta + \frac{d\vartheta}{dt} \\ 0 \end{bmatrix} = \begin{bmatrix} e^{\theta(1)}(t) & 0 \\ 0 & e^{\theta(1)}(t) \end{bmatrix} \begin{bmatrix} -r_{i2} \\ 0 \end{bmatrix} \]  

(17)

Let the corresponding elements of the matrix on both sides of equation (17) be equal. And we can get the formula in the following form.

\[ \begin{align*} 
\frac{d\vartheta}{dt} &= \omega \left[ -0.5 \sin \theta(t) \right] \\
\frac{d\vartheta}{dt} &= \omega \left[ 0.5 \cos \theta(t) \right] \\
\frac{d\vartheta}{dt} &= \omega \left[ -0.25 \sin \theta(t) \right] 
\end{align*} \]  

(18)

We add and subtract the first two differential equations above and get two new differential equations. Together with the third differential equation, we obtain three new equations

\[ \begin{align*} 
\frac{dv_1}{dt} &= -0.5\omega \\
\frac{dv_2}{dt} &= \omega \left[ 0.5 \cos \theta(t) \right] \\
\frac{d\vartheta}{dt} &= \omega \left[ -0.25 \sin \theta(t) \right] 
\end{align*} \]  

(19)

Equation (19) is simpler for computation than (18). Then from

\[ \frac{dv_1}{dt} - \frac{dg_1}{dt} + \frac{dg_2}{dt}, \quad \frac{dv_2}{dt} - \frac{dg_1}{dt} - \frac{dg_2}{dt} \]  

(20)

We obtain

\[ g_1(t) = \frac{v_1(t) + v_2(t)}{2}, \quad g_2(t) = \frac{v_1(t) - v_2(t)}{2} \]  

(21)

The formula for their variation over time is as follows.

\[ g_i(t) = \lim_{t \to \infty} g_i(t) \]  

(22)

The LCEs are

\[ g_1 = \lim_{t \to \infty} \frac{g_1(t)}{t}, \quad g_2 = \lim_{t \to \infty} \frac{g_2(t)}{t} \]  

(23)

5. Simulations

According to the theoretical analysis above, we make simulation by matlab and get a chart Figure 2 from it, which includes a computation module of LCEs. In simulation, we set \( \omega = 10^5 \text{rad/s}, \ k_1 = 0.5\omega, \ k_2 = \omega^2, \ k_3 = 1/\omega, \ k = 0.5 \). It outputs \( y_1 \) to Matlab's Workspace. These data from Workspace are used to calculate LCEs by RHR algorithm in the M-file Module.

In simulation we first get \( r_d \) that is a critical amplitude in the chaotic critical state and also the amplitude of the internal reference signal. We can get \( r_d \) by the calculation of LCEs here.

When we gradually change the value of \( r \) from 0 to 1, the simulation model plots the curves of LCEs changing with \( r \) and outputs Figure 3. (The abscissa is \( r \) and the ordinate is LCE.). We can see
from Figure 3 that, when $r = 0.7311118V$, the larger LCE suddenly changes from positive to negative. It is at this point that the phase transition of the Duffing system will occur.

When $r=0.7311118V$, the larger LCE is $1.861 \times 10^4$. The system is in chaos. When $r$ is raised only $10^{-7}$ to $0.7311119V$, the larger LCE becomes negative $-2.169 \times 10^4$. At this time, both LCEs are negative, indicating that the system has changed from chaotic state to large-scale periodic state. Therefore, we can take 0.7311118 as the critical value of the system. Meanwhile, it is also used as the amplitude of the internal reference signal of the system. If a same frequency signal with an amplitude greater than or equal to $10^{-7}V$ appears, it can immediately make the system change from chaos to large-scale periodic state. Thus through discrimination of the transition we know the signal has appeared.

We set the system reference signal amplitude $r_d$ to 0.7311118V. The system is in chaos. In this case two LCEs are positive and negative respectively. If a same frequency signal with an amplitude greater than or equal to $10^{-7}V$ is input to the system, the system will immediately change phase and enter a large-scale periodic state. In other words, if the induced voltage by eddy current in metal particles is equal to or more than $10^{-7}V$, it can be detected and an alarm signal is sent.

6. Conclusion

In this paper, Duffing oscillator is applied to metal detectors. Satisfactory results have been obtained in improving the sensitivity of metal detectors. Because we have adopted the RHR algorithm to computer LCEs for threshold value obtaining and phase transition quantitative identification, the accuracy of chaotic detection system is improved. In addition, we also have gotten a first-hand data on chaotic detection system at high frequencies. Finally, simulation indicates that the chaotic system for metal detection can improve detector response, and has broad application prospects value for more in-depth research.

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