Multidimensional signal interpolation based on autocorrelation models for HGI compression

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Abstract. In this article, I develop interpolation algorithms for compression methods of multidimensional signals. These compression methods are based on hierarchical grid interpolation (HGI). I modify the well-known approach to the development of interpolators. This approach is based on the model of the autocorrelation function (ACF) of the signal. Based on this model, I optimize the parameters of the interpolator. In this paper, I modify this approach for use in special hierarchical grids of signal samples. I develop adaptive interpolation algorithms based on this approach. I embed the proposed interpolators in compression methods based on HGI. Experimental part of the research study is carried out with natural signals. The experimental results prove that the efficiency of the hierarchical compression method is enhanced by using the proposed adaptive interpolator.

1. Introduction

At present, a data size of digital multidimensional signals is growing faster than a capacity of memory devices. The data size of multi- and hyperspectral signals [1-3] is growing particularly fast. Due to restrictions on available resources (weight, power supply, etc.), this problem is especially acute for onboard (remote) systems for digital signals sensing placed on aircraft, including satellites and atmospheric drones. In all of these situations, it is necessary to use signal compression methods to reduce the data size.

To date, a large number of compression methods of digital signals have been developed [4-9]. Fractal compression methods [9] have a high compression ratio but are used very rarely due to the high computational complexity and unacceptable signal distortions like strange artifacts. Compression methods based on wavelet transform [10], in particular, JPEG-2000 [11], are the most preferred in the efficiency-to-complexity ratio and have the broadest possible scope of use. Methods based on two-dimensional discrete orthogonal transformations (DOP) [12], in particular, JPEG [13], have similar advantages.

However, most of signal compression methods, based on resource-demanding transformations (DCT, wavelet, Fourier, etc.) have a high computational complexity, which significantly complicates their application in onboard systems and real-time systems. Besides, these systems often require quality control of compressed signals, but it is problematic for the transformation-based compression methods since a compression error of these methods must be controlled in a transformed space.

Thus, when resources are limited, and the quality control is needed, compression methods have to perform in a spatial domain (not spectral domain). These compression methods provide quality control of compressed information and low computational complexity. These methods include
compression methods based on hierarchical grid interpolation (HGI) [14-15]. HGI methods also have many significant advantages when used in ground-based signal processing systems (scale-independent access speed to a scaled signal fragment, etc.).

HGI-compression is based on the interpolation of the signal samples by resampled versions of the same signal and subsequent entropy [16] coding of post-interpolation residues. Thus, the most critical stage of these compression methods is the interpolator, so the topical task is to improve signal interpolation algorithms suitable for use in the HGI method. In this paper, we propose an interpolator based on optimization of the interpolation coefficients for each signal sample. This interpolation is performed using a model of the local autocorrelation function (ACF) [6-7] of the signal (an inseparable exponential ACF model is used), in the same way as in the construction of optimal predictors for differential [4, 6, 8] methods of signal compression.

2. The compression method of multidimensional digital signals based on HGI

Consider the non-redundant hierarchical representation [14, 17-18] of the multidimensional signal. Let \( X = \{x(c)\} \) be the original multidimensional signal, and \( c \) be the vector of its arguments. Consider a hierarchical representation of this signal as a set of several hierarchical levels \( X_l \):

\[
X = \bigcup_{l=0}^{L-1} X_l, \quad X_{L-1} = \{x_{L-1}(c)\}, \quad X_l = \{x_l(c)\} \cup \{x_{l+1}(c)\}, \quad l < L-1
\]

where \( L \) is the number of hierarchical levels \( X_l \), \( x_l(c) \): is the signal resampled with step \( 2^l \):

\[
x_l(c) = x(2^l c)
\]

During compression based on hierarchical grid interpolation (HGI), hierarchical levels (1) are processed sequentially, from the highest (most decimated) level \( X_{L-1} \) to the lower levels: \( X_{L-2}, X_{L-3}, ..., X_1, X_0 \). With \( L > 3 \), the data size of the highest level \( X_{L-1} \) is already pretty small. Therefore a compression algorithm of this level is unimportant. Next, we describe the compression algorithm (see also Figure 1) for an arbitrary "non-highest" hierarchical level \( X_l, l < L-1 \).

1) Interpolation.

Interpolation of samples \( \{x_l(c)\} \) of the current hierarchical level \( X_l \) is based on the samples \( \{\tilde{x}_k(c), k > l\} \) of already processed (compressed and then decompressed) hierarchical levels \( \{\tilde{X}_k, k > l\} \):

\[
\tilde{x}_l(c) = P\left(\bigcup_{k=l+1}^{L-1} \tilde{X}_k\right) = P\left(\{\tilde{x}_k(c)\}\right),
\]

where \( \tilde{x}_l(c) \) are interpolating values and \( P(..) \) is interpolation function.

2) Calculation of difference signal.

The differences between the initial and interpolating values of the level samples are calculated:

\[
f_l(c) = x_l(c) - \tilde{x}_l(c).
\]

3) Quantization.

A quantization of the difference signal (3) is performed. In this paper we use the quantizer with a uniform scale ([..] is the integer part of the real number):

\[
q_l(c) = \text{sign}(f_l(c)) \left[ \frac{|f_l(c)| + e_{\text{max}}}{2e_{\text{max}} + 1} \right],
\]

which provides control of maximum error [19] \( e_{\text{max}} \):

\[
|f_l(c)| = \left| x(c) - \tilde{x}(c) \right| \leq e_{\text{max}}.
\]
A statistical encoder then compresses the quantized signal (4). Then the encoded signal is placed in a communication channel or an archive file.

4) Recovering.
Calculation of the restored (decompressed) values of signal samples is performed:

\[ \tilde{x}_i(c) = q_i(c)(1+2\epsilon_{\text{max}}) + \hat{x}_i(c) \]

They are needed for interpolation (2) of the following hierarchical levels \( \{X_k, k < l \} \).

**Figure 1.** Compression of the «non-highest» hierarchical level \( \{X_k, k < L - 1 \} \) by the HGI method.

3. **Averaging interpolation for compression based on HGI**
Trivial averaging over the nearest counts of more resampled hierarchical levels is usually used [20-21] for interpolation during HGI compression. This trivial averaging makes it possible to reduce computational complexity.

To simplify the discussion, consider the averaging interpolators for a two-dimensional input signal \( X = \{x(c)\} = \{x(m,n)\} \).

Let us consider two types of signal samples: "central" samples with indices \((2m + 1, 2n + 1)\) and "edge" samples with indices \(x_i(2m + 1, 2n)\) and \(x_i(2m, 2n + 1)\). We write down the averaging interpolation of signal samples of both types.

Interpolation of the central samples is based on the restored values of samples of previous (more resampled) scale levels:

\[ \tilde{x}_i(2m + 1, 2n + 1) = \frac{1}{4}(\tilde{x}_{i+1}(m,n) + \tilde{x}_{i+1}(m+1,n) + \tilde{x}_{i+1}(m,n+1) + \tilde{x}_{i+1}(m+1,n+1)) \]

Then, for these "central" samples, the reconstructed values are calculated. Interpolation of "edge" samples is based on two signal samples of the more resampled hierarchical level and two "central" samples of the same hierarchical level:

\[ \tilde{x}_i(2m + 1, 2n) = \frac{1}{4}(\tilde{x}_{i+1}(m,n) + \tilde{x}_{i+1}(m+1,n) + \tilde{x}_i(2m + 1, 2n - 1) + \tilde{x}_i(2m + 1, 2n + 1)) \]

The averaging interpolator has relatively low computational complexity, but insufficiently high efficiency since it does not take into account any local characteristics of the signal.

4. **Interpolation based on ACF model**
In this paper, we propose an interpolator that takes into account local statistical characteristics of the signal due to using a model of the local autocorrelation function (ACF) of the signal (a similar approach is used in differential compression methods [4, 6, 8] for predictors development).
This interpolator is constructed as a linear reconstruction filter [6, 8] with the finite impulse response \( h_{rec}(k,l) \) (FIR filter):

\[
\hat{x}(m,n) = \sum_{(k,l) \in D} h_{rec}(k,l) \cdot x(m-k,n-l)
\]

where \( D \) is an area of nonzero values of the impulse response \( h_{rec}(k,l) \) (see Figure 2), which has the following form for interpolation of central and edge signal samples respectively:

\[
D_c: \{(-1,-1), (-1,0), (1,-1), (1,0)\},
\]

\[
D_e: \{(0,-1), (-1,0), (0,1), (1,0)\}.
\]

Compact designations \((a_1,a_2,a_3,a_4)\) for four non-zero values of the reconstructing (interpolating) impulse response \( h_{rec} \) are shown in Figure 2.

![Figure 2. Areas of non-zero values (crosses) of the impulse response \( h_{rec} \) when interpolating central (a) and edge (b) signal sample (circle).](image)

As the impulse response \( h_{rec} \), we use a function that minimizes the root mean square (RMS) deviation (interpolation error) between the original and the interpolating signals. From the theory of linear filtration it is known that this impulse response is the solution of Wiener-Hopf equations [6, 8]:

\[
\sum_{(k,l) \in D} h_{rec}(k,l) R_{\tau}(m-k,n-l) = R_{\tau}(m,n), \quad (m,n) \in D,
\]

\[
h_{rec}(m,n) = 0, \quad (m,n) \notin D,
\]

where \( R_{\tau} \) is the reconstructed signal ACF \( B_{\tau}(m,n) \) normalized to dispersion \( \sigma_x^2 \):

\[
R_{\tau}(m,n) = \frac{B_{\tau}(m,n)}{\sigma_x^2}.
\]

Using the above notation, the Wiener-Hopf equations system for the central signal samples takes the form:

\[
\begin{align*}
&a_1 + a_2 R_{\tau}(0,2) + a_3 R_{\tau}(2,0) + a_4 R_{\tau}(2,2) = R_{\tau}(1,1), \\
&a_1 R_{\tau}(0,2) + a_2 + a_3 R_{\tau}(2,2) + a_4 R_{\tau}(2,0) = R_{\tau}(1,1), \\
&a_1 R_{\tau}(2,0) + a_2 R_{\tau}(2,2) + a_3 + a_4 R_{\tau}(0,2) = R_{\tau}(1,1), \\
&a_1 R_{\tau}(2,2) + a_2 R_{\tau}(2,0) + a_3 R_{\tau}(0,2) + a_4 = R_{\tau}(1,1),
\end{align*}
\]

Accordingly, the Wiener-Hopf equations system for the edge signal samples is written as follows:

\[
\begin{align*}
&a_1 + a_2 R_{\tau}(1,1) + a_3 R_{\tau}(0,2) + a_4 R_{\tau}(1,1) = R_{\tau}(0,1), \\
&a_1 R_{\tau}(1,1) + a_2 + a_3 R_{\tau}(1,1) + a_4 R_{\tau}(2,0) = R_{\tau}(1,0), \\
&a_1 R_{\tau}(0,2) + a_2 R_{\tau}(1,1) + a_3 + a_4 R_{\tau}(1,1) = R_{\tau}(0,1), \\
&a_1 R_{\tau}(1,1) + a_2 R_{\tau}(2,0) + a_3 R_{\tau}(1,1) + a_4 = R_{\tau}(1,0).
\end{align*}
\]
5. Experimental research of interpolation algorithms

A software implementation of the proposed adaptive interpolator based on ACF is developed. Then this interpolator was built into the HGI compression method. Computational experiments were performed on natural signals to research the effectiveness of the proposed interpolation algorithm. Some test signals are shown in Figure 4. As a measure of efficiency, the relative gain in the size of the archive file was used. This gain is achieved through the use of the proposed interpolation algorithm instead of the averaging interpolation algorithm (7-8) within the framework of the HGI compression method:

\[ \Delta = \left( 1 - \frac{S_{\text{ACF}}}{S_{\text{averaging}}} \right) \cdot 100\% \]

where \( S_{\text{averaging}} \) and \( S_{\text{ACF}} \) are sizes of the archive files when signal have been compressed by the HGI method using the averaging and proposed interpolation algorithms, respectively. Some results for the given test signals are shown in Figure 5.

From the received experimental results, it is evident, that the proposed algorithm provides the gain (up to 2%) on the archive size.
6. Conclusion

Development of adaptive interpolators for the HGI signal compression was considered. An interpolator was proposed that adapts to local neighborhood of each signal sample by taking into account local statistical characteristics. The estimated local ACF was used as the considered statistical characteristic (a similar approach is usually used in differential compression methods for predictor development). The proposed interpolator was developed for an exponential inseparable ACF model. The software implementation of the proposed adaptive interpolator was developed. This interpolator was integrated into the compression method of multidimensional signals based on HGI. The advantage of the proposed interpolator over the averaging interpolator according to the archive size was shown within the framework of the HGI compression method.

7. References

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