3D Radiative Transfer with PHOENIX

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Collaborators

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Supernovae

- A supernova occurs once a second out to $z \approx 1$
- Supernovae responsible for galactic nucleosynthesis
- Supernovae inject energy into ISM, trigger star formation
- SNe Ia make good standardizable candles
Two Supernova Mechanisms

(a) Type-I Supernova
- Binary star system
- White dwarf
- Planetary nebula
- Accretion disk
- Red giant
- Growing white dwarf
- Detonation

(b) Type-II Supernova
- Helium, carbon, Hydrogen
- Normal star fusion
- Iron core
- Massive star imploding
- Core rebound
- Remnant core
- Shock wave
- Explosion
SNe Ia Can be Found Far Away
Light Curve Shape → Standardization

-20

0

20

40

60

-15

-16

-17

-18

-19

-20

B Band

as measured

Calan/Tololo SNe Ia

Kim, et al. (1997)

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SNe Discover Dark Energy

![Graph showing the relationship between dark energy fraction ($\Omega_\Lambda$) and matter fraction ($\Omega_M$). The graph includes contour lines from the High-Z SN Search Team and the Supernova Cosmology Project, with regions labeled for no Big Bang, accelerating, decelerating, expanding to infinity, and collapsing. The graph illustrates the observational constraints on dark energy and the role of supernovae in cosmology.](image-url)
Spectra Suggest Distant SNe Same as Nearby
Riess et al. 1998

![Graph showing relative flux vs. rest wavelength for various SNe](image-url)
Modest Change with Redshift?
Sullivan et al. 2009

Phase range: \(-1d < \tau < +8d\)

High-\(z\) (R07): \(N=11; \tau=3.5d; z=1.16; s=1.09\)
Inter-\(z\) (E08): \(N=18; \tau=2.0d; z=0.48; s=1.04\)
Low-\(z\) (M08): \(N=15; \tau=2.8d; z=0.02; s=1.01\)
Low-\(z\) (UV): \(N=3; \tau=4.1d; z=0.01; s=0.91\)
Diversity due to Nickel Mass ↔ Spectroscopic Sequence (Höflich et al. 1993; Nugent et al. 1995)
Spectroscopic Indicators

Spectroscopic Indicators for Maximum Brightness $\mathcal{R}(\text{SiII})$

$\mathcal{R}_{\text{SiS}}$ can be used by JDEM (Bongard et al. 2006)
Spectroscopic Indicators Correlated with Light Curve Shape

(Garnavich et al. 2001)
Hubble Diagram
S. Bailey et al. (SNfactory) 2009

SNfactory Preliminary

\[ \Delta \mu_B(R) \]
\[ \sigma(\Delta \mu_B) = 0.128 \pm 0.012 \]

\[ \Delta \mu_B(R', c) \]
\[ \sigma(\Delta \mu_B) = 0.119 \pm 0.011 \]

\[ \Delta \mu_B(x, c) \]
\[ \sigma(\Delta \mu_B) = 0.161 \pm 0.015 \]
Type Ia are basically round, but explosion asymmetries lead to compositional and ionization inhomogeneities.

Core-collapse almost certainly come from asymmetric engine. Observed asymmetry depends on intactness of envelope.

GRB are beamed and highly relativistic.

AGN.
3D Hydro Models
Gamezo et al. (2003)

Velocity \( \times 10^3 \text{ km/s} \)

A

B

C

D

E

F

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3D Transfer
Generalized Model Atmosphere Code
Static or Moving Flows in Full Relativity
Well Calibrated on many astrophysical objects: Planets/BDs, Cool Stars, Hot Stars ($\beta$CMa, $\epsilon$CMa), $\alpha$-Lyra, Novae, SNe (Iabc, IIP, IIb)
3-D Test: Sphere in a box

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3D Transfer
Toy Solar Model PBC
Caffau et al. 2007; $\epsilon = 1$, $\theta = 0$, $\phi = 25$

Continuum

Line Center

Line Wing

Composite

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3D Transfer
Scattering in Toy Solar Model

$\epsilon = 10^{-4}$, $\theta = 0$, $\phi = 25$
Solution of ALI Equation

\[ J_{\text{new}} = \Lambda^* S_{\text{new}} + (\Lambda - \Lambda^*) S_{\text{old}}. \]

This relation can be written as

\[ [1 - \Lambda^*(1 - \epsilon)] J_{\text{new}} = J_{fs} - \Lambda^*(1 - \epsilon) J_{\text{old}}, \]

- Inverse of a banded matrix is full
- Band matrix solvers require large amounts of memory. Same is true for parallelized band matrix solvers
- Iterative methods (Jordan and Gauss-Seidel) work well and use little memory.
- Ng acceleration is still very useful
ALI Works as expected

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3D Transfer
Formal Solution Along Characteristics
1D Gustafsson et al.
Formal Solution Along Characteristics
3D Cartoon

a

b
SOLUTION OF THE COMOVING-FRAME EQUATION OF TRANSFER IN SPHERICALLY SYMMETRIC FLOWS.

VI. RELATIVISTIC FLOWS

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ABSTRACT
Within the framework of special relativity, radiative transport equations describing exactly, i.e., to all orders in \(v/c\), the interaction of radiation and matter in spherically symmetric flows are derived. The full angle- and frequency-dependent transfer equation, frequency-dependent moment equations, and frequency-integrated moment equations are obtained. The frequency-integrated moment equations are shown to be precisely equivalent to the equations obtained from the four-divergence of the radiation stress-energy tensor, and all three sets of equations recover the results obtained earlier by Castor in the limit \(v/c \ll 1\). A method of solution of the steady-flow transfer equation is discussed, and two illustrative examples are presented.

Subject headings: line formation — radiative transfer — relativity — stars: atmospheres

I. INTRODUCTION

The interaction of radiation with high-velocity material presents a problem of considerable astrophysical interest. In certain phenomena, for instance in the post-blast-wave flow in supernova envelopes (see, e.g., Colgate and White 1966) or compact extragalactic radio sources (Shapiro 1979), in the material surrounding quasars, and in the early phases of the cosmic expansion when the material is still optically thick, the matter emitting, absorbing, and scattering photons is moving at relativistic velocities. In such cases the interplay between the radiation and the material should, ideally, be described relativistically, and this is done most easily if both the material and radiation field can be evaluated in the comoving frame, i.e., the moving frame in which a given element of material is at rest.

The comoving-frame transfer equation was first posed by McCrea and Mitra (1936), and solved under a number of simplifying assumptions by Chandrasekhar (1945). More recently, work by Noordlinger and Rybicki (1974) and earlier papers of this series (II-V) have presented methods for solving the comoving-frame line-transfer problem with a fairly high degree of physical realism. With the exception of Paper III of this series, essentially all treatments of the comoving-frame equations account only for the first-order effect of Doppler-shifting of photons and ignore other terms, described classically as aberration and advection, despite the fact that these terms are also formally of order \(v/c\). For line-transfer problems the Doppler-shift terms actually do provide the dominant effect because the characteristic frequency-width over which the line-profile changes significantly is small; hence these terms are, in effect, amplified to order \(v/c\) (where \(v_{\text{typical}}\) sets the line width). An evaluation of the aberration and advection terms was made in Paper III using Castor's (1972) comoving-frame transfer equation, which includes all terms consistently to order \(v/c\); it was found there that for line-transfer problems in the physical regime appropriate to stellar winds \((v/c \approx 0.01)\), the effects of aberration and advection can be neglected. In contrast, for continuum-formation the Doppler-shift terms enjoy no special advantage, and all three effects are of equal importance; but because the velocities normally considered are relatively small, it has usually been assumed that all terms of order \(v/c\) may be neglected. (In fact, it is far from obvious that this is true, and this point needs to be investigated further, particularly for large optical depth.)

As the ratio \(v/c\) approaches unity, all three effects mentioned above become large and it is essential that all three be treated fully consistently, ideally to all orders in \(v/c\). In this high-velocity limit, line transfer becomes less interesting because one expects the Sobolev approximation to be valid everywhere: the line is simply smeared out and no longer traps photons for more than a single scattering. Primary attention then shifts to the problem of continuum formation because continua have, in effect, an infinite bandwidth and can interact with photons over an enormous range of frequencies. We therefore ask whether we can derive, and solve, equations that account correctly for the full angle- and frequency-dependent relativistic transport of radiation, and that yield relativistically correct moment equations.

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Want to work in the “Co-moving Frame”

The emissivity $\eta$ and opacity $\chi$ depend upon the angle as well as frequency in the inertial frame because of Doppler shifts, aberration, and advection induced by the motion of the material in the frame. The goal of this section is to rewrite equation (2.1) with all material and radiation-field quantities measured in the comoving frame; in that frame both the opacity and emissivity are isotropic, and can be related directly to proper variables that specify the thermodynamic state of the material. Furthermore, in that frame both the scattering properties of the material and the rate equations describing the mechanisms populating and depopulating its internal energy states are most easily defined.

In our analysis we shall, however, leave both the space and time variables in the inertial frame, as this is the only frame in which synchronism of clocks can be effected, and further this choice obviates the need to develop a metric for accelerated fluid frames (Castor 1972) which in general can only be done approximately. With this choice of frame we can write exact Lorentz transformations for all the material and radiation-field quantities and use these to develop a transfer equation that will remain valid for relativistic flow in the limit as $v/c \to 1$.

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3D Transfer
Characteristic Equations

$$\frac{dr}{ds_M} = \gamma (\mu + \beta),$$

$$\frac{d\mu}{ds_M} = \gamma (1 - \mu^2) \left[ 1 + \mu \frac{\beta}{r} - \gamma^2 (\mu + \beta) \frac{d\beta}{dr} \right].$$
Fig. 1.—Characteristic rays of equations (3.4)-(3.7) for an expanding envelope with core radius $r_c = 1$ (inner semicircle) and surface radius $R = 11$ (outer semicircle). The velocity law is linear (left panels) or quadratic (right panels) as given by eq. (4.2). Each figure is labeled with the value of $\beta_\infty = \frac{v}{c_{\text{in}}}$ of the maximum expansion velocity at the outer boundary. These results, which follow from exact equations, may be compared with those in Fig. 1 of Pyper III, which were obtained from equations that are correct only to $O(\beta)$; the earlier results are at least qualitatively correct for $\beta_\infty \leq 0.5$, but yield incorrect ray forms at higher speeds.
Formal Solution Along Characteristics
Affine Method (Chen et al. 2007)
Formal Solution Along Characteristics
Affine Method (Baron, Hauschildt, Chen 2009)

\[ \frac{\partial I_{\lambda}}{\partial s}|_{\lambda} + a(s)\lambda \frac{\partial I_{\lambda}}{\partial \lambda} = -[\chi_{\lambda} f(s) + 5a(s)]I_{\lambda} + \eta_{\lambda} f(s). \quad (1) \]

**Funny Frame**

*\( \mathbf{r} \) and *\( \mathbf{n} \) in Observer’s Frame

*\( \lambda \) is measured by a comoving observer
Need $J_\lambda$ in co-moving frame (in momentum space)

So just integrate RTE equation over $d\Omega$ (where $d\Omega$ is measured by a comoving observer), but since $I_\lambda$ is known in the funny frame:

Use Chain Rule

$$
d\Omega = (\gamma [1 - \beta \cdot n])^{-2} d\Omega_0
= f(s)^{-2} d\Omega_0.
$$

$$
J_\lambda = \Lambda S_\lambda
$$
The results of 1-D calculations are compared with 3-D calculations for 
$\beta_{\text{max}} = (0.03, 0.33, 0.67, 0.87)$. 
\[ \epsilon = 0.1, \beta_{\text{max}} = 0.03. \]
Wall-clock time for $\epsilon = 0.1$ as a function of angular resolution/number of CPUs.
have working, tested 3-D RT code with homologous flows
including arbitrary flows is a computational, not algorithmic challenge
next step is to go beyond test problems to production code
expect progress from intercomparison of new datasets of nearby supernovae with 3-D hydro and synthetic spectroscopy