Implication on Two Higgs Doublet Models from the Latest $b \rightarrow s \gamma$ Measurement and Top Discovery\(^1\)

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Abstract

With the fresh experimental bounds of $b \rightarrow s \gamma$ issued by CLEO and the value of the top-quark mass by CDF and D0, the constraints on the two Higgs doublet models (2HDM) are re-examined. In the examination an effective Lagrangian is used, which contains all of the energy scale evolution not only from the W boson mass to the bottom-quark mass but also from the top-quark mass to W boson mass for the QCD corrections. As a result, it is implied that the constraint on the mass of the charged Higgs boson emerging in the concerned models turns to be more stringent than that achieved by the earlier analyses.

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It is known that the experimental bounds of $b \rightarrow s \gamma$ set strong constraints on the two Higgs doublet models (2HDM), a kind of extensions of the minimal standard model (MSM).

Of the MSM, searching for the neutral Higgs and investigation of possible extensions certainly are two of the important current topics. As for the topic of the model extension, it is interesting to apply the fresh experimental results such as those on $b \rightarrow s \gamma$ (Ref. [1]) and the top-quark mass $m_t$ (Ref. [2]) etc. to re-examine the implication on the 2HDM so as to upgrade the allowed area of the parameter space of the model.

In the earlier analyses\(^3\) of the adopted effective Lagrangian, the QCD correction corresponding to the energy scale evolution from the top-quark mass $m_t$ down to that of the W boson $M_W$ was ignored because the heavy-top effects were not so emphasized as now. Naively this piece of the correction itself is expected to be not large since $\alpha_s$ is small in the region. However, theoretically the ignorance makes the QCD corrections incompletely, when we are interested in the heavy-top effects, and practically we need to know the quantitative consequence of the ignorance too. To relate to the measured branching ratio for the concerned process $b \rightarrow s \gamma$, the effective Lagrangian should involve the whole evolution of the QCD correction from the 'start point' (as we are considering the heavy-top effects, it is certain to be better to 'set' the 'start point' of the evolution at $m_t$ but not at $M_W$ since $m_t > M_W$) down to the energy scale where the concerned process takes place, i.e. that of the bottom-quark mass $m_b$ for the $b$-quark decays. Due to the asymptotic freedom nature of QCD, general speaking, it is better to take the 'start point' of the evolution at $m_t$ than that at $M_W$ since $m_t \geq M_W$. Specially, we are to focus lights on the effects relevant to the heavy-top quark later in the paper and consider that sometimes the naive expectation is not so correct, the final consequence of 'ignoring' the piece of the evolution from $m_t$ down to $M_W$ should be computed quantitatively. Indeed by a quantitative analysis and with the fresh value of the top mass we did find the consequence being substantial. Namely with the fresh value of $m_t$, the effective Lagrangian containing the whole energy evolution for QCD affects the constraint on the charged Higgs mass in the 2HDM quite substantially and we will report it in the paper briefly below.

Let us explain how to obtain the effective Lagrangian, which is used for the re-examination. To be the leading order perturbative QCD correction, the effective Lagrangian for the decay $b \rightarrow s \gamma$, the method of operator product expansion (OPE) and renormalization group (RG) are applied, and the $\beta$ function as well as the anomalous dimension matrix of the operators,

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affecting the effective Lagrangian, are computed up to one-loop level. Whereas the QCD evolution involves the whole energy scale range from $m_t$ to $m_t$. To achieve the effective Lagrangian at the energy scale of top mass $m_t$, i.e., the 'start point' of the evolution for the requested effective Lagrangian, the 'large top mass $m_t$ expansion' is applied and only the operators in dimension less or equal six in the expansion are kept. It is because that we are interested only in the leading order accuracy so far, the higher dimension operators do not mix with the concerned operators (lower dimension) at all in the whole range of the QCD evolution thus the higher dimension ones are irrelevant to the computation on the QCD leading corrections.

The re-analyzed results (the values and, especially, the tendencies), achieved with the fresh experimental values and the novel effective Lagrangian, should be worth while to be reported, because the changes are quite substantial and at least they may be used as some valuable references in phenomenological studies on 2HDM.

For the convenience later on, let us repeat some points and list some useful formulas of 2HDM (from Ref. [4] mainly) for the analyses briefly.

To avoid tree-level flavor changing neutral current (FCNC), there are two kinds of 2HDM. The first (model I) is that only one of the two Higgs doublets couples to the two types of quarks: u- and d-type, while the other doublet, forbidden by certain symmetries, does not couple to the quarks at all. The second (model II, the SUSY inspired one) is different from the first by other symmetries else to make one of the Higgs doublets couple to u-type quarks while the other couple to d-type quarks respectively.

The Yukawa coupling involving the charged Higgs is
\[ \mathcal{L} = \frac{1}{\sqrt{2}} M_W \left( \frac{v_2}{v_1} \right) R M_U V \left( \begin{array}{c} d_L^i \\ \xi (\bar{u} \tilde{c}) L \end{array} \right) R M_D \left( \begin{array}{c} d_R^i \\ \bar{v} \end{array} \right) H^+ + \text{h.c.}, \]

where $V$ represents the $3 \times 3$ unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix, $M_U$ and $M_D$ denote the diagonalized quark mass matrices, and the subscript $L$ and $R$ denote left-handed and right-handed chiralities, respectively. For model I, $\xi = v_2/v_1$; while for model II, $\xi = -v_1/v_2$. $v_1$ and $v_2$ are the vacuum expectation values of two Higgs doublets.

To achieve the requested effective Lagrangian for $b \to s \gamma$, which involves all the QCD correction is based on OPE and RG. The first step to achieve it at the energy scale of $m_t$ is to integrate out the heavy-top degree of freedom. It, in fact, is the 'start point' to obtain the final effective Lagrangian for the present analyses. As a result, the relevant effective Lagrangian (the one at the energy scale $\mu = m_t$) is obtained:
\[ \mathcal{L}_{\text{eff}} = 2 \sqrt{2} G_F V_{tb} V_{t\mu}^* \sum_i C_i(\mu) O_i(\mu). \]

As in Ref. [4], here the operator set $\{O_i\}$ is truncated up to dimension six in the large top mass $m_t$ expansion, and the coefficients $C_i(\mu)$, $\mu = m_t$, can be calculated by means of the electroweak corrections up to one-loop accuracy but renormalizing at the top mass $m_t$. We should note here the fact that the charged Higgs is indicated very heavy by several indirect precise analyses, we assume the mass of the charged Higgs $M_\phi \geq m_t$ (here the physical charged Higgs in 2DHM is denoted by $\phi$) in the electroweak loop calculations.

To pursuing the final effective Lagrangian for the concerned process, the next step is to achieve the effective Lagrangian at energy scale $M_W$ in a way that the coefficients $C_i(\mu)$ corresponding to operators $O_i(\mu)$ are calculated with renormalization group equation (RGE):
\[ \mu \frac{d}{d\mu} C_i(\mu) = \sum_j (\gamma^\mu)_{ij} C_j(\mu), \]

where $(\gamma^\mu)_{ij}$ is the anomalous dimension matrix, and a set of boundary conditions of the RGE, i.e., the matching conditions: the values of $C_i(\mu)$ at $\mu = m_t$, will be taken from the lowest order values of electroweak theory. Note that in practice some extra approximation is made when calculating the anomalous dimension matrix $(\gamma^\mu)_{ij}$ and the $\beta$ function as well, and indeed here for all the formulas (as in Ref. [4]) only one-loop level accuracy for QCD is
Due to the threshold effects of the particles, we need to consider the thresholds individually in the procedure of the QCD evolution. To consider the effects of the W boson threshold, which will appear unavoidably in the way of the evolution, we will integrate out further the W boson freedom, thus the effective Lagrangian at the energy scale $\mu = M_W$ is obtained and the coefficients $C_i(\mu)$ and just above $\mu = M_W$ are those obtained by the evolution from $m_t$ to $M_W$. Moreover six more relevant four-quark operators should be added to Eq. (2) if one would like to make the operator set 'complete'. Once more by the RGE with the anomalous dimension matrix $(\gamma_i^T)_{ij}$ of an enlarged set of the operators and the matching conditions at $M_W$, i.e. to set the coefficients $C_i$ being equal to each other just below and above $M_W$, the requested effective Lagrangian corresponding to the energy scale $m_b$ is achieved for the re-examination usage.

To write the formulas precisely and briefly, we will not list all the operators but only those being relevant:

$$\mathcal{O}_7 = \frac{\alpha_S(M_W)}{\alpha_S(M_t)} \{ e^2 U_t^{1/23} \} \delta'(m_t) - C_2 (m_t) \delta'(m_t) \left[ \frac{\alpha_S(M_W)}{\alpha_S(M_t)} \right]^{1/23} \right] + \left[ \frac{9}{2} C_3 (m_t) - \frac{9}{4} C_{PL,t} (m_t) + \frac{9}{4} C_{PL,s} (m_t) \right] \times \left[ 1 - \frac{8}{9} \left( \frac{\alpha_S(M_W)}{\alpha_S(m_t)} \right)^{2/3} \right] - \frac{1}{4} C_{PL,0} (m_t) + \frac{9}{23} 16 \pi^2 \pi^2 C_W (m_t) \left[ 1 - \frac{\alpha_S(m_t)}{\alpha_S(M_W)} \right] \right| - \frac{23}{36} \right.$$  

with the coefficients of the operators $C_{W_L}, C_{O_{L,R}^0}, C_{O_{L,R}^0}'$, $C_{P_{L}}$, $C_{P_{L}'}$, $C_{P_{L}'}$ and $C_{P_{L}'}$ at $\mu = m_t$ (the precise expressions may be found in Ref. [4], and we would not repeat them here only due to their lengthiness).

Finally the coefficients $C_i(\mu)$ of the operators, with the renormalization point $\mu$ from $M_W$ to $m_b$ at one-loop level, are well described in Refs [7]-[12]. The relevant coefficients of the operators at $\mu = m_b$ now read

$$C_7^{\text{eff}}(m_b) = \eta^{16/23} C_7(M_W) + \frac{8}{3} (\eta^{14/23} - \eta^{16/23}) C_8(M_W) + C_2(M_W) \sum_{i=1}^{8} h_i \eta^{a_i},$$  

where $\eta = \alpha_S(M_W)/\alpha_S(m_b)$,

$$h_i = \left( \frac{5.266926 + 5.266926}{2}, \frac{5.266926 - 5.266926}{2}, -0.6494, -0.0380, -0.0186, -0.0057 \right),$$

$$a_i = \left( \frac{0.14}{23}, \frac{0.16}{23}, \frac{0.4086}{23}, -0.4230, -0.8994, 0.1456 \right).$$

Following Refs [7], [8] and [10]-[12],

$$\frac{\text{BR}(B \to X_s \gamma)}{\text{BR}(B \to X_c \gamma)} \approx \frac{\Gamma(b \to s\gamma)}{\Gamma(b \to c\gamma)}$$  

Then

$$\frac{\text{BR}(B \to X_s \gamma)}{\text{BR}(B \to X_c \gamma)} \approx \frac{|V_{ts} V_{tb}^*|^2}{|V_{cb}^*|^2} \frac{6 \alpha_{\text{QED}}}{\pi g(m_c/m_b)} \left| C_{\gamma}^{\text{eff}}(m_b) \right|^2,$$
where the phase space factor $g(z)$ is given by
\[ g(z) = 1 - 8z^2 + 8z^6 - z^8 - 24z^4 \log z. \] (10)

We will use $m_c/m_b = 0.316$ in the numerical calculations. Thus we obtain the decay branching ratio for $B \to X_s \gamma$ with the normalization to the well-established semileptonic decay one $\text{BR}(B \to X_c e \nu)$. If we take the experimental value $\text{BR}(B \to X_c e \nu) = 10.8\%$,\textsuperscript{[13]} the branching ratio of $B \to X_s \gamma$ is found to be
\[ \text{BR}(B \to X_s \gamma) \approx 10.8\% \times \frac{|V_{ts} V_{tb}|^2}{|V_{cb}|^2} \frac{6 \alpha_{\text{QED}}}{\pi g(m_c/m_b)} |C_T| \] (11)

Note that in the above expressions the top mass $m_t$ is kept as a parameter so that we may apply them to computing the values with a given experimental $m_t$ as needed.

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**Fig. 1.** Coefficients of operators $O_T$, $O_S$, as functions of $\tan \beta$ in model I (a) and model II (b). Solid and dashed lines correspond to $C_T$ with and without QCD running from $m_t$ to $M_W$; dashed-dotted and dotted lines correspond to $C_S$ with and without QCD running from $m_t$ to $M_W$.

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**Fig. 2.** Branching ratios of $b \to s \gamma$ depicted as functions of $\tan \beta$, with different masses of the charged Higgs $M_\phi$ in model I (a), and model II (b). The CLEO experiment's central value, upper and lower limits of 95\% C.L. for this decay are also depicted as solid lines.

To illustrate the effects of the top quark which are ignored in the literature, in Fig. 1 we plotted the coefficients of the most relevant operators $O_T$ and $O_S$ at $\mu = M_W$ versus $\tan \beta$ with and without the QCD correction corresponding to the energy scale running from $m_t$ (with the latest experimental value $m_t = 176$ GeV) to $M_W$ for both model I and model II. Here $m_b = 4.8$ GeV, $M_W = 80.22$ GeV and the QCD coupling $\alpha_s(m_Z) = 0.125$ were taken.\textsuperscript{[13]} Owing to the fact that the mixing of all the relevant operators is small, one can see that the effects of the QCD correction are roughly within ten percent and do not depend on $\tan \beta$ very much. However, the effect, of the ten percent variation only, makes a substantial change of the constraints on the parameter space of 2HDM finally.
In addition to the effects from the novel effective Lagrangian, in order to exhibit the effects due to the experimental uncertainties \( \text{BR}(b \to s\gamma) \), the top mass \( m_t \) and the strong coupling constant \( \alpha_s(m_Z^2) \) as well, we plotted the branching ratio \( \text{BR}(b \to s\gamma) \) (including the central value and the three standard deviations of the CLEO data) versus \( \tan \beta \) so as to show the constraints on \( \tan \beta \) and on the mass of charged Higgs \( M_\phi \) for model I (Fig. 2a) and model II (Fig. 2b). We also plotted the bands in the parameter space of \( \tan \beta \) versus \( M_\phi \) to show the constraints by taking a fixed \( m_t = 176 \text{ GeV} \) but two different values of the strong coupling constant \( \alpha_s(m_Z^2) = 0.12 \) and 0.13 (the dashed lines), and a fixed \( \alpha_s(m_Z^2) = 0.125 \) but two different values of the top mass \( m_t = 163 \text{ GeV} \) and 189 GeV (the solid lines) for model I (Fig. 3a) and model II (Fig. 3b) respectively. We should note here for the potential interest that, one cannot see very clearly in Fig. 3b but from the formulas, when \( \tan \beta \) approaches to infinity, the dashed lines will approach to 350 GeV and 390 GeV respectively whereas the solid lines to 310 GeV and 430 GeV respectively. All lines in Fig. 3 were drawn by taking the 95% C.L. experimental values given by CLEO, \( 1.0 \times 10^{-4} < \text{BR}(B \to X_s\gamma) < 4.2 \times 10^{-4} \).\(^{[1]} \)

It is shown from the figures that the parameter space is more sensitive to changing of \( m_t \) than to \( \alpha_s \), especially in model II.

![Fig. 3. \( \tan \beta-M_\phi \) plane to show excluded region of parameters of model I (a) and model II (b). Every two solid lines denote the allowed and excluded region(s) with \( \alpha_s(m_Z^2) = 0.125 \) and \( m_{\text{top}} = 163 \text{ GeV} \); Every two dashed lines correspond to those of \( m_{\text{top}} = 176 \text{ GeV} \) and \( \alpha_s(m_Z^2) = 0.13 \) or \( \alpha_s(m_Z^2) = 0.12 \).](image)

One may see from Figs 2a and 3a that for model I, two bands in the \( \tan \beta-M_\phi \) plane are excluded by our analyses with the latest measurements on \( b \to s\gamma \) and \( m_t \). As for another aspect of the analyses, we plotted the \( \tan \beta \) versus \( M_\phi \) in Fig. 4 to show the excluded region of the parameters for model II at 95% C.L.. The solid line corresponds to the favored result of the present analyses with the central experimental values of all the parameters (\( m_t, \alpha_s \) and the branching ratio of the semileptonic decay of \( b \) quark etc.); the dashed line corresponds to \( |V_{ts}V_{tb}|^2/|V_{cb}|^2 = 0.99 \) rather than \( |V_{ts}V_{tb}|^2/|V_{cb}|^2 = 0.95 \), which is widely adopted for all the earlier analyses;\(^{[2]} \) and the dotted-dashed line corresponds to the results obtained by other authors,\(^{[3]} \) which do not include the QCD corrections from \( m_t \) to \( M_W \).

In conclusion, with the fresh experimental values for the top mass \( m_t \), the strong coupling constant \( \alpha_s(m_Z^2) \) and the bounds for \( b \to s\gamma \), the effective Lagrangian involving the QCD correction evolution from \( m_t \) to \( M_W \) and the uncertainties for all of them as well, the constraints on 2HDM are modified substantially. For instance, the lower bound of the mass of

\(^{[2]} \)In fact, there are uncertainties from the measurements and the formulas on the branching ratio of the inclusive semileptonic decay of the \( B \) meson \( \text{BR}(B \to X_s\ell\nu) \) (see Eq. (10)) as well as those from the determination of the CKM matrix elements. Concerning the facts of the uncertainties and the possibility for more than three generations of elementary fermions, we try to change the value of \( |V_{ts}V_{tb}|^2/|V_{cb}|^2 \), so as to let one see the tendency for the constraints when the value becomes greater (equivalent to that the value of \( \text{BR}(B \to X_c\ell\nu) \) becomes smaller than 10.8% etc.)
the charged Higgs is pushed up about by 150 GeV for model II. Since the value $M_W^2/m_t^2$ is
not very small, how accuracy of the approximation of the large top mass expansion at $\mu = m_t$
for the adopted effective Lagrangian should have additional studies (we would leave it for our
further study elsewhere), at the moment the obtained resultant values for the constraints still
need to be confirmed by the studies, nevertheless at least the direction (or the tendency) of
the modification for the constraints obtained by the analyses without doubt is worthy to be
an important reference for experimental studies in 2HDM.

![Diagram](image)

Fig. 4. $\tan \beta - M_\phi$ plane to show excluded region of parameters of model II. Solid lines correspond to
all central values of parameters; dashed line corresponds to $|V_{ts}^* V_{td}/V_{cb}|^2 = 0.99$; and dotted-dashed
line corresponds to results without QCD corrections from $m_t$ to $M_W$, obtained by other authors.

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