STUDY OF SPARSITY-AWARE REDUCED-DIMENSION BEAM-DOPPLER SPACE-TIME ADAPTIVE PROCESSING

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ABSTRACT

Existing reduced-dimension beam-Doppler space-time adaptive processing (RD-BD-STAP) algorithms are confined to the beam-Doppler cells used for adaptation, which often leads to some performance degradation. In this work, a novel sparsity-aware RD-BD-STAP algorithm, denoted Sparse Constraint on Beam-Doppler Selection Reduced-Dimension Space-Time Adaptive Processing (SCBDS-RD-STAP), is proposed. This approach formulates the filter design as a sparse representation problem and enforcing most of the elements in the weight vector to be zero (or sufficiently small in amplitude). Simulation results illustrate that the proposed SCBDS-RD-STAP algorithm outperforms the traditional RD-BD-STAP approaches with fixed beam-Doppler localized processing.

Index Terms— Space-time adaptive processing, reduced-dimension, sparsity-aware, clutter suppression.

1. INTRODUCTION

Space-time adaptive processing (STAP) is a leading technology candidate for improving detection performance of phased-array airborne radar [1] and other related approaches. However, STAP techniques often suffer from the lack of snapshots for training the receive filter, especially in nonhomogeneous environments, which is a crucial concern in the development of STAP algorithms [1][2][3].

In the past decades, many related works have been investigated to improve the clutter mitigation performance in scenarios with a number of snapshots (see [1][2][3][5][6][8][9][10][11][63] and the references therein). For instance, the auxiliary channel receiver (ACR) [4], the joint domain localized approach (JDL) [5][6], the space-time multiple-beam (STMB) [7] are three kinds of effective reduced-dimension (RD) algorithms in the beam-Doppler domain. However, the filter design in [4][5][6][7] relies on fixed beam-Doppler cells and cannot provide optimal selection, suffering significant performance degradation in the presence of sensor array errors. To overcome this issue, the studies in [8] and [9] proposed sequential methods that reduce the required partially adaptive dimension in the transformed domain.

Motivated by the rank deficiency in clutter suppression, sparsity-aware beamformers have been proposed to improve the convergence by exploiting the sparsity of the received data and filter weights [10][11]. The studies in [12] and [13] developed a Min-Max STAP strategy based on the selection of an optimum subset of antenna-pulse pairs that maximizes the separation between the target and the clutter trajectory. Both the sparsity-aware beamformers and the Min-Max STAP strategy are in the antenna-pulse domain. The former is a data-dependent strategy and the latter is a data-independent strategy which requires prior knowledge of the clutter ridge. By drawing inspiration from compressive sensing, recently reported sparsity-based STAP algorithms have formulated the STAP problem as a sparse representation that exploits the sparsity of the entire observing scene in the whole angle-Doppler plane [63]. However, this kind of approach suffers from high computational complexity due to the large dimension of the discretized angle-Doppler plane. Previous works imply that the degrees of freedom (DoFs) used for STAP filters required to mitigate the clutter are much smaller than the full dimension, and different selection strategies have resulted in various levels of performance.

In this work, we introduce the idea of sparse selection in the beam-Doppler domain and formulate the STAP filter design as a sparse representation problem. Unlike the sparsity-based STAP [63], the proposed Sparse Constraint on Beam-Doppler Selection Reduced-Dimension STAP (SCBDS-RD-STAP) algorithm does not discretize the angle-Doppler plane into a large number of grids, but only transforms the received data into a same-size beam-Doppler domain. Differently from the sparsity-aware beamformers [10][11] or the Min-Max STAP strategy [12][13], the proposed SCBDS-RD-STAP algorithm designs the filter in the beam-Doppler domain and automatically selects the best beam-Doppler cells used for adaptation by solving a sparse representation problem. In addition, an analysis of the complexity is performed for the proposed algorithm. Simulation results show the effective-
ness of the proposed algorithm.

This paper is structured as follows: Section 2 describes the signal model of a pulse Doppler side-looking airborne system and states the problem. Section 3 details the proposed SCBDS-RD-STAP algorithm along with approximate solutions and their computational complexity. Section 4 presents and discusses simulation results while Section 5 provides the concluding remarks.

2. SIGNAL MODEL AND PROBLEM FORMULATION

In this section we describe the signal model of a pulse Doppler side-looking airborne radar system and state the problem of designing a beam-Doppler STAP.

2.1. Signal Model

Considering a pulse Doppler side-looking airborne radar with a uniform linear array (ULA) consisting of \( M \) elements. The radar transmits a coherent burst of \( N \) pulses at a constant pulse repetition frequency (PRF) \( f_r \). Generally, for a range bin with the space-time snapshot \( x \), target detection can be formulated as a binary hypothesis problem and expressed as

\[
H_0 : x = x_u \\
H_1 : x = \alpha_t s + x_u,
\]

where \( H_0 \) and \( H_1 \) denote the disturbance only and the target plus disturbance hypotheses, respectively, \( \alpha_t \) is a complex gain, \( s \) is the \( NM \times 1 \) target space-time steering vector and \( x_u \) denotes the clutter-plus-noise vector which encompasses the clutter and the thermal noise \( \mathbf{I} \).

The STAP filter based on a minimum variance distortion-less response (MVDR) approach by minimizing the clutter-plus-noise output power while constraining a unitary gain in the direction of the desired target signal is expressed as \( \mathbf{w}_\text{opt} \)

\[
\mathbf{w}_\text{opt} = \frac{\mathbf{R}^{-1}s}{\mathbf{s}^H \mathbf{R}^{-1} \mathbf{s}},
\]

where \( \mathbf{R} = E[\mathbf{x}_u \mathbf{x}_u^H] \) denotes the clutter-plus-noise covariance matrix. Approaches to compute the beamforming weights include [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62].

2.2. The Beam-Doppler STAP Approaches

The beam-Doppler STAP approaches firstly transform the data \( x \) in the antenna-pulse domain to the beam-Doppler domain, denoted as \( \tilde{x} \), where "\~" above \( x \) signifies the beam-Doppler domain. This procedure can be represented by

\[
\tilde{x} = T_{\text{LP}}^H x,
\]

where \( T_{\text{LP}} \) denotes the transformation matrix. The common idea under the beam-Doppler STAP approaches is to choose a localized processing (LP) region, or equivalently, the matrix \( T_{\text{LP}} \), corresponding to a set of beam-Doppler responses, for adaptive processing. The optimal beam-Doppler STAP filter can be represented by

\[
\mathbf{w}_\text{opt} = \frac{\mathbf{R}^{-1} \mathbf{s}}{\mathbf{s}^H \mathbf{R}^{-1} \mathbf{s}},
\]

where \( \mathbf{R} = T_{\text{LP}}^H \mathbf{R} T_{\text{LP}} \) and \( \mathbf{s} = T_{\text{LP}}^H \mathbf{s} \).

Fig. 1. The LP region selections in different beam-Doppler LP approaches. \( \circ \) denotes the selected beam-Doppler cell, and \( \times \) denotes the target beam-Doppler cell.

Observing [4], the key challenge is how to efficiently select the LP region. The ACR method [4] suggests to select the LP region placed along the clutter ridge, as shown in Fig 1(a). The JDL method [5, 6] chooses the beam-Doppler cells around the target cell, which turns out to be a rectangular shape, as shown in Fig 1(b). Unlike ACR and JDL, STMB [7] chooses the beam-Doppler cells with a "cross" shape centered at the target cell, as shown in Fig 1(c). All these approaches can reduce the STAP filter dimension, resulting in improved convergence and steady-state performance in a small training data set. However, the ACR requires the knowledge of the clutter ridge, and there is no rule to determine the optimum size of the chosen beam-Doppler LP region for JDL and STMB. The optimum choice of the beam-Doppler region should be related to the scenario or the data rather than just fixed.

3. PROPOSED SCBDS-RD-STAP ALGORITHM

In this section, we detail the proposed SCBDS-RD-STAP algorithm, how to design the receive filter and discuss the computational complexity.

3.1. Proposed SCBDS-RD-STAP Scheme

The core idea of the proposed SCBDS-RD-STAP scheme is based on a transformation matrix and a filter with sparse constraints. The received space-time data vector \( x \) is first mapped by an \( NM \times NM \) transformation matrix \( T \) into an \( NM \times
1 beam-Doppler domain data vector. Here, $\mathbf{T}$ can be constructed as

$$
\mathbf{T} = \begin{bmatrix} \mathbf{s} & \mathbf{T}_{\text{aux}} \end{bmatrix},
$$

where $\mathbf{T}_{\text{aux}}$ is an $NM \times (NM - 1)$ matrix, given by

$$
\mathbf{T}_{\text{aux}} = \begin{bmatrix}
(\mathbf{v}_d(f_d, t) \otimes \mathbf{v}_s(f_s, t + \frac{1}{M}))^T
& \vdots & 
(\mathbf{v}_d(f_d, t) \otimes \mathbf{v}_s(f_s, t + \frac{M-1}{M}))^T

& \vdots 

(\mathbf{v}_d(f_d, t + \frac{1}{N}) \otimes \mathbf{v}_s(f_s, t + \frac{M-1}{M}))^T
& \vdots 

& 
(\mathbf{v}_d(f_d, t + \frac{N-1}{N}) \otimes \mathbf{v}_s(f_s, t + \frac{M-1}{M}))^T
\end{bmatrix}. 
$$

Denoting $d = \mathbf{s}^H \mathbf{x}$ and $\mathbf{\tilde{x}} = \mathbf{T}_{\text{aux}}^H \mathbf{x}$, we note that $d$ is the component at the target beam-Doppler cell (also called main channel), and elements of $\mathbf{\tilde{x}}$ are the components from otherwise beam-Doppler cells (also called auxiliary channels). Following the concept of GSC, we can expect to reduce the clutter in $d$ by employing a filter on the auxiliary channel data $\mathbf{\tilde{x}}$. Furthermore, based on the first three observations analyzed above, we do not need to use all auxiliary channel data but only a few of them. In order to realize this idea, we perform a sparse constraint on the STAP filter weight vector $\mathbf{\tilde{w}}$. Precisely, we design the filter $\mathbf{\tilde{w}}$ by solving the following optimization problem

$$
\min_{\mathbf{\tilde{w}}} E \left[ |d - \mathbf{\tilde{w}}^H \mathbf{\tilde{x}}|^2 \right] + \kappa \| \mathbf{\tilde{w}} \|_0, 
$$

where $\kappa$ is the regularization parameter that controls the balance between the sparsity and total squared error. Theoretically, the optimum choice can be determined by an algorithm that is properly designed for the task. To show an intuitive observation of the above idea, we will provide examples by simulations later on. I am not sure about the above but it would be useful to include a table with the pseudo-code of the SCBDS-RD-STAP algorithm here.

### 3.2. Approximate Solutions

Since the sparse regularization function is $l_0$-norm, it leads to an NP-hard problem. In the following, we use the relaxation penalty $l_p$-norm (where $0 < p \leq 1$) instead of the $l_0$-norm and rewrite (7) as

$$
\min_{\mathbf{\tilde{w}}} E \left[ |d - \mathbf{\tilde{w}}^H \mathbf{\tilde{x}}|^2 \right] + \kappa \| \mathbf{\tilde{w}} \|_0. 
$$

In practice, since the expectation in (8) cannot be obtained, we now modify (8) based on a least-squares type cost function. Let $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_L]$ denote the space-time data matrix formed by $L$ training snapshots, and let $\mathbf{d} = \mathbf{X}^2 \mathbf{s}^*$, and $\mathbf{\tilde{X}} = \mathbf{T}_{\text{aux}}^H \mathbf{X}$, then the least-squares type cost function is described by

$$
\min_{\mathbf{\tilde{w}}} \| \mathbf{d}^* - \mathbf{\tilde{X}}^H \mathbf{\tilde{w}} \|_2^2 + \kappa \| \mathbf{\tilde{w}} \|_p. 
$$

Note that (9) is a standard sparse representation problem and can be solved by the regularized focal underdetermined system solution (R-FOCUSS) algorithm.

It should be noted that the sensing matrix or the dictionary $\mathbf{\tilde{X}}^H$ of the optimization problem (9) of the proposed SCBDS-RD-STAP scheme is formed by the received data (i.e., snapshots from the beam-Doppler domain), and is different from that of the sparsity-based STAP approaches [63], which is composed of known space-time steering vectors from the discretized angle-Doppler plane. Furthermore, unlike the ACR, JDL, and STMB, which are performed with fixed beam-Doppler LP region, the proposed SCBDS-RD-STAP scheme provides an iterative approach to automatically select the beam-Doppler LP region aided by a sparse constraint. Additionally, the auxiliary channel data are formulated by a standard 2-D discrete Fourier transform with explicit physical meaning in the proposed SCBDS-RD-STAP scheme, whereas the auxiliary channel data are formulated by a signal blocking matrix in the sparsity-aware beamformer [10].

### 3.3. Computational Complexity

We detail the computational complexity of the proposed SCBDS-RD-STAP algorithm, sparsity-aware beamformer [10], and JDL [6]/STMB [7], as shown in Table 1. Here, for the proposed algorithm, $K_{\text{foc}}$ is the total iteration number and $D_{\text{foc},q}$ is the number of elements above the preset threshold at the $q$th iteration, which is decided by the sparsity; for the JDL/STMB, $D$ is the number of selected beam-Doppler elements. From the table, we see that the computational complexity of the proposed algorithm is comparable or even lower than those of the sparsity-aware beamformer, and higher than those of the JDL and STMB. This is because the number of snapshots $L$ used in the proposed SCBDS-RD-STAP algorithm is much smaller than $NM$ (which can be seen in the simulations), the value of $D_{\text{foc},q}$ after several iterations will also be much smaller than $NM$, and the pseudo-inversion can be calculated by the conjugate gradient approach, which has low complexity [64].

### 4. SIMULATIONS

In this section, we assess the performance of the proposed SCBDS-RD-STAP algorithm and compare it with other existing algorithms, namely, the JDL ($3 \times 3$) [6], STMB ($8 + 4 + 1$)
Table 1. Computational Complexity

| Algorithm                | Complexity       |
|--------------------------|------------------|
| Sparsity-aware beamformer| $O(5L(NM)^2)$    |
| JDL/STMB                 | $O(LD^2 + D^3)$  |
| Proposed SCBDS-RD-STAP   | $O\left(\sum_{q=1}^{K_{foc}} D_{foc,q} L^2\right)$ |

and sparsity-aware beamformer \cite{10} in terms of the output signal-to-clutter-plus-noise-ratio (SCNR) loss \cite{1}, which is defined as

$$SCNR_{loss} = \frac{\sigma^2 |\hat{w}^H s|^2}{NM\hat{w}^H R\hat{w}},$$

(10)

where $\hat{w} = s - T_{aux}\tilde{w}$ is the corresponding filter weight vector in the original domain. We consider a side-looking ULA (half-wavelength inter-element spacing) airborne radar with the following parameters: uniform transmit pattern, $M = 12$, $N = 12$, carrier frequency $1.2$GHz, $f_r = 2$kHz, platform velocity $125$ m/s, platform altitude $8$ km, clutter-to-noise ratio (CNR) $45$dB. For the following examples: in the sparsity-aware beamformer, we set parameters as those in \cite{10}; in the proposed SCBDS-RD-STAP algorithm, we set the regularization parameter to $3$, the maximum iteration number is $500$, and the stopping criterion is decided by the preset limit relative change of the solution between two adjacent iterations $10^{-4}$.

In the first example, we examine the convergence performance (signal-to-clutter-plus-noise ratio (SCNR) loss against the number of snapshots) of the proposed SCBDS-RD-STAP algorithm, as shown in Fig.2. The true target is supposed to be boresight aligned with normalized Doppler frequency $-0.1667$. The curves show that the proposed Switched-SCBDS-RD-STAP algorithm converges to a higher SCNR loss with much fewer training snapshots compared to all the considered algorithms.

Fig. 3 illustrates the 2-D view of the weight vector, specifically, each element in the weight vector is represented by one grid point, and its amplitude is depicted by the grayscale of the grid. Note that, each element in the weight vector is associated to one auxiliary channel in the GSC, and a zero amplitude implies the associated auxiliary channel is not involved in the adaption. Apparently, most of the elements in the weight vector have zero amplitudes, which implies that the Switched-SCBDS-RD-STAP selects very few beam-Doppler cells for adaptation.

In the third example, we assess the performance of the proposed SCBDS-RD-STAP algorithm under different target Doppler frequencies, as depicted in Fig.4. Here, we set the number of snapshots for training used in the JDL \cite{30}, STMB

![Fig. 2. The SCNR loss against the number of snapshots for training.](image)

![Fig. 3. 2-D view of the weight vector of the Switched-SCBDS-RD-STAP](image)

![Fig. 4. The SCNR loss versus different target Doppler frequencies.](image)
5. CONCLUSIONS

This paper has proposed a novel STAP algorithm based on the beam-Doppler selection for clutter mitigation for airborne radar with small sample support. The SCBDS-RD-STAP algorithm transforms the received data into beam-Doppler domain, employs a sparse constraint on the filter weight for sparse beam-Doppler selection and formulates this selection as a sparse representation problem, where the sensing matrix is formed by the data matrix. Simulations have demonstrated the effectiveness of the proposed SCBDS-RD-STAP algorithm and shown its improvement in target detection over the existing algorithms, such as the JDL, STMB, and sparsity-aware beamformer both in absence and presence of array errors.

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