Special frequencies in reflection spectra of Bragg multiple quantum well structures

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We have studied theoretically optical reflection spectra from the Bragg multiple quantum well structures. We give an analytical explanation of the presence of two special frequencies in the spectra at which the reflection coefficient weakly depends on the quantum well number. The influence of the exciton nonradiative damping on the reflection spectra has been analyzed. It has been shown that allowance for the dielectric contrast gives rise to the third special frequency at which the contributions to the reflectivity related to the dielectric contrast and the exciton resonance mutually compensate one another.

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1. INTRODUCTION

The resonant Bragg structures have been first considered theoretically in Ref. 1 and then investigated experimentally in systems based on semiconductor compounds $A_2B_6$ and $A_3B_5$. The further progress in understanding of optical properties of resonant Bragg structures has been achieved in a series of theoretical studies. In resonant Bragg structures without the dielectric contrast, i.e., with the coinciding dielectric constants, $\varepsilon_b \equiv n_b^2$, of the barrier material and the background dielectric constant, $\varepsilon_a \equiv n_a^2$, of the quantum well (QW), the optical reflection spectrum for small enough number $N$ of wells is described by a Lorentzian with the halfwidth $N\Gamma_0 + \Gamma$, where $\Gamma_0$ and $\Gamma$ are, respectively, exciton radiative and nonradiative damping rates. For a very large number of wells the reflection coefficient is close to unity within the forbidden gap for exciton polaritons and rapidly decreases near the gap edges $\omega_0 - \Delta$ and $\omega_0 + \Delta$, where $\omega_0$ is the exciton resonance frequency and $\Delta = \sqrt{2\omega_0\Gamma_0/\pi}$. In Refs. 14,15 the reflection spectra are calculated for arbitrary values of $N$, including the intermediate region where $N\Gamma_0$ and $\Delta$ are comparable. The calculations have demonstrated an existence of two particular frequencies: near these special frequencies a value of the reflection coefficient from the resonant Bragg structure with the matched dielectric constants is practically independent of the QW number and very close to that for reflection from a semi-infinite homogeneous medium with the refractive index $n_b$. In the present paper we give an analytical interpretation for this effect. Moreover, we analyze the role of dielectric contrast $\varepsilon_a -\varepsilon_b \neq 0$ in the formation of the special frequencies in the reflection spectra.

2. STRUCTURES WITH MATCHED DIELECTRIC CONSTANTS

The structure under consideration is schematically depicted in Fig. 1: it borders vacuum in the left and contains the cap layer of the thickness $b'$ made from the material B, $N$ QWs made from the material A, each of the width $a$, separated by the barriers of the thickness $b$ and the semi-infinite medium B. Under normal incidence (from vacuum) of the light wave of the frequency $\omega$, the amplitude reflection coefficient can be written in the following form:

$$r(N) = \frac{r_0 + \tilde{r}_N e^{2i\phi}}{1 + r_0\tilde{r}_N e^{2i\phi}}. \tag{1}$$

Here $r_0 = (1 - n_b)/(1 + n_b)$ is the reflection coefficient “vacuum – semi-infinite medium B”, $\phi = k_b(b' - b/2)$, $k_b = n_b(\omega/c)$, $c$ is the light velocity in vacuum, $\tilde{r}_N$ is
the reflection coefficient from the structure with \( N \) QWs placed between the infinite barriers. It is convenient to refer the phase of the latter coefficient to the plane shifted by \( (a + b)/2 \) from the center of the leftmost QW. Then this coefficient is given by\(^{10,17}\)

\[
\tilde{r}_N = \frac{\tilde{r}_1}{1 - \tilde{t}_1 \frac{\sin(N-1)Kd}{\sin NKd}}, \tag{2}
\]

where the complex coefficients \( \tilde{r}_1, \tilde{t}_1 \) describe the reflection from and transmission through the layer of the thickness \( d = a + b \) containing a QW in its center, \( K \) is the wave vector of an exciton polariton at the frequency \( \omega \) in an infinite regular QW structure. For a structure without the dielectric contrast one has\(^{11,17}\)

\[
\tilde{r}_1 = e^{iKd} r_1, \quad \tilde{t}_1 = e^{iKd}(1 + r_1), \quad r_1 = \frac{i\Gamma_0}{\omega_0 - \omega - i(\Gamma_0 + \Gamma)} . \tag{3}
\]

Fig. 2 shows the reflection spectra from the resonant Bragg structure with matched dielectric constants. The spectra are calculated in the absence of the nonradiative damping (a) and for \( \hbar \Gamma = 100 \ \mu eV \) (b). The rest parameters are indicated in the figure caption. In agreement with\(^{14,15}\), one can see in the spectra presented in Figs. 2a and 2b that for two frequencies labelled \( \omega_+ \) and \( \omega_- \) the reflection coefficient \( R_N = |r(N)|^2 \) is indeed close to \( r_{01}^2 \) and almost independent of \( N \), at least for \( N < 100 \).

![FIG. 1: Schematic representation of light reflection from an N-QW structure. \( E_0 \) is the electric field amplitude of the incident light wave, and the reflection coefficient \( r(N) \) is defined as a ratio of the reflected amplitude to \( E_0 \).](image1.png)

![FIG. 2: Spectral dependence of the reflection coefficient \( R_N \) from N-QW structure with the matched dielectric constants of compositional materials \( A \) and \( B \). The calculation is performed for the background refractive index \( n_b = 3.45 \), the exciton resonance frequency and radiative damping rate defined by \( \hbar \omega_0 = 1.533 \ \text{eV}, \hbar \Gamma_0 = 50 \ \mu \text{eV}, \) \( b' = (a/2) + b \) and the nonradiative damping rate \( \hbar \Gamma = 0 \) (a) and \( \hbar \Gamma = 100 \ \mu \text{eV} \) (b). Curves correspond to six structures with different number of QWs indicated at each curve. The reflection spectrum for the structure with infinite number of QWs, \( N \rightarrow \infty \), is labelled by the symbol \( \infty \). The special frequencies are tied to the exciton-polariton forbidden-gap edges \( \omega_0 \pm \Delta \) in such a way that

\[
\varepsilon_+ \equiv \omega_+ - (\omega_0 + \Delta) \ll \Delta
\]

and \( \varepsilon_- \equiv (\omega_0 - \Delta) - \omega_- \ll \Delta . \tag{4} \)

On the other hand, the reflection coefficient \( R_\infty \) from the semi-infinite structure (curves \( \infty \) in Figs. 2a and 2b) shows abrupt change from values close to unity at \( \Gamma \neq 0 \) (or equal unity at \( \Gamma \rightarrow +0 \)) inside the forbidden gap to values \( R_\infty < r_{01}^2 \) in the adjoining allowed bands. For large but finite values of \( N \) exceeding 1000, the reflection spectrum \( R_N(\omega) \) from the structure with \( \hbar \Gamma = 100 \ \mu eV \) is close to \( R_\infty(\omega) \) while at \( \Gamma = 0 \) the spectrum strongly oscillates outside the gap region with the period decreasing as \( N \) increases.

An existence of the special frequencies \( \omega_\pm \) in the reflection spectra can be understood if one notices that in
the vicinity of the edge frequencies \( \omega_0 \pm \Delta \), where
\[
|N(Kd - \pi)| \ll 1,
\]
the ratio of the sine functions in (2) can be approximated by
\[
\frac{\sin(N - 1)Kd}{\sin NKd} \approx -\frac{N - 1}{N}. \tag{5}
\]
Therefore, the reflection coefficient \( \tilde{r}_N \) and, hence, the reflection coefficient \( \tilde{r}_N \) can be presented as the Möbius or linear-fractional transformation
\[
r(N) = \frac{\alpha N + \beta}{\gamma N + \delta}, \tag{6}
\]
with
\[
\alpha = r_{01}(1 + \tilde{t}_1) + e^{2i\phi} \tilde{r}_1, \quad \beta = -r_{01} \tilde{t}_1, \tag{7}
\]
\[
\gamma = 1 + \tilde{t}_1 + e^{2i\phi} r_{01} \tilde{f}_1, \quad \delta = -\tilde{t}_1.
\]
It should be noted that this equation is valid for an arbitrary regular structure with \( N \) periods if \( \tilde{r}_1, \tilde{t}_1 \) are considered as the reflection and transmission coefficients for a single layer of the thickness \( d \) embedded between the semi-infinite media with the refractive index \( n_b \). In the particular case of an \( N \)-QW structure with the matched dielectric constants we have instead of (7)
\[
\alpha = r_{01}(1 + e^{i\phi_d})(\omega_0 - \omega - i\Gamma) + i\Gamma_0(e^{i(\phi_d + 2\phi)} - r_{01}), \tag{8}
\]
\[
\beta = r_{01}\delta, \quad \delta = -e^{i\phi_d}(\omega_0 - \omega - i\Gamma),
\]
\[
\gamma = (1 + e^{i\phi_d})(\omega_0 - \omega - i\Gamma) + i\Gamma_0(r_{01}e^{i(\phi_d + 2\phi)} - 1),
\]
where \( \phi_d = k_dd \) and, for the sake of convenience, the coefficients used here differ from those in (7) by the common factor \( \omega_0 - \omega - i(\Gamma_0 + \Gamma) \), which makes no change in the transformation Eq. (6).

Let us continue analytically the dependence \( r(N) \) to the whole complex plane \( z = z' + iz'' \) and take into account that the linear-fractional transformation \( r(z) \) sends a circle to a circle, a straight line can be considered as a circle of the infinite radius, and the points \( z = 1, 2, \ldots, N, \ldots \) lie on the real axis. It follows then that the complex values \( r(N) \) lie on the circle of some radius \( \rho \) centered at some point \( w_0 \) so that one has
\[
r(N) = w_0 + \rho e^{i\phi_N}, \tag{9}
\]
where only the phase \( \phi_N \) is \( N \)-dependent. The values of \( w_0 \) and \( \rho \) are related to \( \alpha, \beta, \gamma \) and \( \delta \) by
\[
w_0 = \frac{i}{2} \frac{\alpha \delta^* - \beta \gamma^*}{\Im(\gamma^* \delta)}, \quad \rho = \left| \frac{\alpha}{\gamma} - w_0 \right|. \tag{10}
\]
According to (9) we have
\[
R_N \equiv |r(N)|^2 = |w_0|^2 + \rho^2 + 2\rho \Re(w_0^* e^{i\phi N}). \tag{11}
\]
If there exists a frequency \( \omega \) where \( w_0 \) vanishes (or is very close to zero) then the reflection coefficient \( R_N \) at this frequency is independent (or almost independent) of \( N \) and equal (or almost equal) to \( |\alpha/\beta|^2 \). If this frequency satisfies the condition \( |\omega - \omega_0| \gg \Gamma_0 \), the exciton contribution to the reflectivity is negligible for small \( N \) and values of \( R_N \) are mainly determined by the reflectivity on the boundary between vacuum and material B. Thus, for small \( N \) the inequality \( |R_N(\omega) - r_{01}^2| \ll 1 \) is valid. Since \( |w_0(\omega)| \) is negligible the same condition is satisfied not only for small \( N \) but for all those \( N \) which allow the representation of \( r(N) \) in the form of linear-fractional transformation (6).

The calculation shows that, for the structure characterized by the parameters indicated in the caption to Fig. 2, the special frequencies correspond to \( \varepsilon_\pm \approx 0.045\Delta \), and the ratio \( |w_0(\omega)/r_{01}| \) reaches a minimal value of \( \approx 10^{-2} \) for \( \Gamma = 0 \) and \( \approx 10^{-3} \) for \( \Gamma = 100 \mu \text{eV} \). At the same time \( \rho/r_{01} \) differs from unity less than by \( 10^{-3} \). In order to derive approximate analytical expressions for the frequencies \( \omega_\pm \) we can expand the coefficients \( \alpha, \beta, \gamma, \delta \) in powers of small parameters \( \Delta/\omega_0, \Gamma_0/\Delta, \varepsilon-/\Delta \) or \( \varepsilon+/\Delta \). Substituting the obtained approximate equations into (10) and solving the equation \( w_0 = 0 \) we find
\[
\omega_\pm = \omega_0 \pm \Delta \left( 1 + \frac{1}{2n_b^2 - 3} \right) \quad \text{or} \quad \varepsilon_\pm = \frac{\Delta}{2n_b^2 - 3}. \tag{12}
\]
For \( n_b = 3.45 \) the ratio \( \varepsilon_\pm/\Delta \) following from this equation differs from the numerical result only by 8%. Therefore, the closeness of the frequencies \( \omega_\pm \) to the gap edges is determined by a large value of the dielectric constant \( n_b^2 \). We see that the approximation of \( r(N) \) by a linear-fractional transformation (6) and a smallness of the min-
FIG. 3: The dependence of the reflection coefficient $R(\omega)$ on the number of QWs at the frequency $\omega_+ = \omega_0 + 1.045\Delta$. The parameters coincide with those indicated in the caption to Fig. 2. The exact values for the structure with $\Gamma = 0$ and $h\Gamma = 100\;\mu\text{eV}$ are shown by squares and triangles, respectively. Solid and dotted curves represent the analytical approximation for $\Gamma = 0$ and $h\Gamma = 100\;\mu\text{eV}$.

In contrast to the absolute value of the reflection coefficient $r(N, \omega_{\pm})$ which is practically independent of the QW number, the phase of the reflected wave $\phi_N(\omega_{\pm})$ appreciably changes with increasing $N$ and is described with a good accuracy by the linear function

$$\phi_N(\omega_{\pm}) \approx \pi \pm \frac{4n_b\Gamma_0 N}{\Delta(n_b^2 - 1)}. \quad (13)$$

In Fig. 3 squares show the exact values of $R_N$ at the frequency $\omega_+$ and the solid curve shows the approximate dependence $R_N(\omega_+)$ obtained by the substitution of (13) into (11). One can see that the approximate formula reproduces the results of numerical calculation with high accuracy.

Fig. 2b presents the spectra $R_N(\omega)$ calculated with allowance for exciton nonradiative damping $h\Gamma = 100\;\mu\text{eV}$. One can see that the nonradiative damping leads to a decrease of the reflection coefficient near the center of the forbidden gap but makes no remarkable effect on the position of the special frequencies $\omega_{\pm}$. Near these frequencies the phase of the complex coefficient $r(N)$ is also described by Eq. (13). However the dependence of $R_N(\omega_{\pm})$ on $N$ for $h\Gamma = 100\;\mu\text{eV}$ shown in Fig. 3 by triangles essentially differs from that for $\Gamma = 0$. For the description of this dependence it is not enough to take into account the variation of $\phi_N$ given by Eq. (11). The reason is that, for nonzero nonradiative damping, the condition for validity of the approximation (13) is violated at noticeably smaller $N$ than in the case of vanishing $\Gamma$. As a result one has to keep the next nonvanishing term in the expansion of the ratio of sine functions in powers of the variable $Kd - \pi$, namely,

$$\sin (N - 1)Kd \quad \sin NKd = \frac{N - 1}{N}F(N),$$

$$F(N) = 1 + 2\frac{N - 1}{6} (Kd - \pi)^2. \quad (14)$$

This expansion is valid for $N \leq 100$. Since the factor $F(N)$ differs from unity the dependence $r(N)$ obtained in the approximation (14) is not, strictly speaking, linear-fractional: the coefficients $\alpha, \beta, \gamma, \delta$ become functions of $N$ and, therefore, the transformation $r(N)$ no more sends the real axis to a perfect circle. Formally, the expression $r(N)$ can be presented in the form (9). Moreover, the dependence of $\phi_N$ on $\Gamma$ can be neglected. However, both $w_0$ and the real factor $\rho$ are now functions of $N$. Triangles and dotted curve in Fig. 3 present results of exact and approximate calculations of the coefficient $R_N$ at the frequency $\omega_+$ for $h\Gamma = 100\;\mu\text{eV}$. The approximate calculation is carried out according to Eq. (11), but with modified $w_0$ and $\rho$. For nonzero nonradiative-damping rate, values of $K$ become complex even in the allowed bands. The analysis shows that it is the imaginary part of $K$ which gives rise to the difference between the solid and dotted curves in Fig. 3.

3. ALLOWANCE FOR THE DIELECTRIC CONTRAST

For $n_a \neq n_b$, the reflection and transmission coefficients for a single QW are described by (10).
\[ \tilde{r}_1 = e^{ikad} r_1, \tilde{t}_1 = e^{ikad} t_1, \]
\[ r_1 = r^{(0)} + r_{\text{exc}}, t_1 = t^{(0)} + r_{\text{exc}}, \]
(15)

Here \( r^{(0)} \) and \( t^{(0)} \) are the reflection and transmission coefficients calculated neglecting the exciton contribution and given as follows
\[ r^{(0)} = e^{-ik_a a} r_b a \frac{1 - e^{2ik_a a}}{1 - r_b a e^{2ik_a a}}, \]
\[ t^{(0)} = e^{ik_a a} \left( e^{-ik_a a} + r_b a r^{(0)} \right), \]
(16)

where \( r_{ab} = -r_{ba} = (n_a - n_b)/(n_a + n_b) \). The exciton contribution to \( r_1 \) and \( t_1 \) has the form
\[ r_{\text{exc}} = t^{(0)} \frac{i\tilde{r}_0}{\omega_0 - \omega - i(\Gamma + \bar{\Gamma})}, \]
where
\[ \tilde{r}_0 = 1 + r_{ab} e^{2ik_a a} \frac{1 - e^{2ik_a a}}{1 - r_{ab} e^{2ik_a a}} \bar{\Gamma}_0. \]
(18)

As well as in the case \( n_a = n_b \), the expression coefficient from an \( N \)-QW structure is expressed via those for a single QW according to Eqs. (1), (2). The period performed with allowance for the nonradiative damping \( \bar{\Gamma} = 100 \mu eV, a = 120 \text{ Å, } n_a = 3.59 \) and \( n_b = 3.45 \). The calculation is performed for \( \hbar \Gamma = 100 \mu eV, a = 120 \text{ Å, } n_a = 3.59 \) and \( n_b = 3.45 \). Curves are calculated for six structures containing different number, \( N \), of wells indicated at each curve. The symbol \( \infty \) corresponds to the structure with infinite \( N \).

![FIG. 4: The reflection spectra from Bragg QW structures with the dielectric contrast between the compositional materials A and B. The calculation is performed for \( \hbar \Gamma = 100 \mu eV, a = 120 \text{ Å, } n_a = 3.59 \) and \( n_b = 3.45 \). Curves are calculated for six structures containing different number, \( N \), of wells indicated at each curve. The symbol \( \infty \) corresponds to the structure with infinite \( N \).](image)

The forbidden-gap edges one can apply the approximation [13]. For \( \Gamma \neq 0 \), similarly to [14], it is necessary to take into account a term quadratic in the difference \( Kd - \pi \). As in the above case \( n_a = n_b \), an absolute value of the reflection coefficient \( r(N) \) at the frequencies \( \omega_{\pm} \) weakly depends on \( N \) whereas the phase appreciably varies with \( N \). The dependence \( \phi_N \) at a frequency \( \omega \) lying close to the upper or lower edge of the polariton gap can approximately be described by
\[ \phi_N(\omega) = \pi + \left( \frac{\bar{\Gamma}_0}{\omega - \omega_0} - 2\pi r_{ab} a \frac{a}{d} \right) \frac{4n_b N}{n_b^2 - 1}, \]
valid for \( |\omega - \omega_0| \gg \bar{\Gamma}_0, \Gamma \). In the particular case \( r_{ab} = 0 \) this equation transforms into Eq. (13).

Unlike the frequencies \( \omega_{\pm} \), at the frequency \( \omega'_{+} \) both the absolute value and the phase of the reflection coefficient \( r(N) \) are practically independent of \( N \), i.e., not only \( R_N = |r(N)|^2 \approx r_{01}^2 \) but also \( r(N) \approx r_{01} \). This happens because the reflection of light at the frequency \( \omega'_{+} \) from a single QW sandwiched between the semi-infinite barriers is almost absent, namely, a sum of two terms \( r^{(0)}(\omega'_{+}) \) and \( r_{\text{exc}}(\omega'_{+}) \) in [10] is close to zero. In other words,
the contributions to the reflectivity of a QW due to the presence of the dielectric contrast and the exciton resonance cancel each other\cite{2}. According to \cite{2} the absence of reflection from one QW brings with it the vanishing reflection coefficient from $N$ such wells, i.e., $r_N(\omega'_+) = 0$. As a result, the amplitude reflection coefficient from the whole structure $r(N, \omega'_+) = r_{01}$ and is independent of $N$.

The frequency $\omega'_+$ satisfies the inequalities $|\omega'_+ - \omega_0| \gg \Gamma_0, \Gamma$. This allows one to neglect the damping rates $\Gamma_0$ and $\Gamma$ in the denominator of the expression \cite{14} for $r_{\text{exc}}$. Furthermore, because of closeness of the refractive indices $n_a$ and $n_b$ the inequality $|r_{ab}| \ll 1$ holds. Neglecting corrections of the order $r_{ab}^2$ in equations for $r^{(0)}$, $t^{(0)}$ and the difference of $\Gamma_0$ from $\Gamma_0$ we obtain that the condition $r_1(\omega'_+) = 0$ is satisfied at the frequency

$$\omega'_+ = \omega_0 + \frac{\Gamma_0}{2r_{ab} \sin k_a^{(0)} a}, \quad (19)$$

where $k_a^{(0)} = n_a \omega_0/c$. Obviously, the inequalities $|\omega'_+ - \omega_0| \gg \Gamma_0, \Gamma$ are realized for a small enough coefficient $r_{ab}$. In the limit $n_a \to n_b$ when $r_{ab} \to 0$, a value of $\omega'_+$ tends to infinity, i.e., this special frequency is absent for structures with the matched dielectric constants. For parameters of the structure used while calculating the spectra in Fig. 4, the frequencies $\omega_+$ and $\omega'_+$ accidentally turn out to be very close to each other.

4. CONCLUSION

For the resonant Bragg structures with the matched dielectric constants of the well and barrier materials, we have given an analytical explanation of an existence of the special frequencies $\omega_{\pm}$ in the optical reflection spectra at which the reflection coefficient $R_N$ is close to $r_{01}^2$ and almost independent of the number of QWs in the structure. Near these frequencies the amplitude reflection coefficient $r(N)$ can be approximately written in the form of an $N$-dependent linear-fractional function which sends points on the real axis to points in the complex plane lying on a circle centered near the coordinate origin. This means that the reflection coefficient $R(\omega_{\pm}) = |r(\omega_{\pm})|^2$ is indeed almost independent of $N$ although the phase of the reflected wave $r(N)$ is quite well approximated by a linear function of $N$. We have shown that, for nonzero exciton nonradiative damping $\Gamma$, the reflection spectra are also characterized by special frequencies but their values at these frequencies become more sensitive to $N$.

In optical reflection spectra from resonant Bragg structures with the dielectric contrast there are not two but three special frequencies. The origin of two of them can be interpreted in terms of a linear-fractional transformation in the same way as in the case $n_a = n_b$. The reflection coefficient from a single QW put between the semi-infinite barriers vanishes at the third frequency $\omega'_+$ because the contributions to the reflectivity resulting from the dielectric contrast and the exciton resonance mutually compensate one another. The consequence is that, at the frequency $\omega'_+$, the amplitude reflection coefficient from a structure containing an arbitrary number of such wells equals $r_{01}$ as if the structure contained no QWs at all.

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