Parametric Instability in Advanced Laser Interferometer Gravitational Wave Detectors

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Abstract. High frequency parametric instabilities in optical cavities are radiation pressure induced interactions between test mass mechanical modes and cavity optical modes. The parametric gain depends on the cavity power and the quality factor of the test mass internal modes (usually in ultrasonic frequency range), as well as the overlap integral for the mechanical and optical modes. In advanced laser interferometers which require high optical power and very low acoustic loss test masses, parametric instabilities could prevent interferometer operation if not suppressed. Here we review the problem of parametric instabilities in advanced detector configurations for different combinations of sapphire and fused silica test masses, and compare three methods for control or suppression of parametric instabilities—thermal tuning, surface damping and active feedback.

1. Introduction
Advanced laser interferometer gravitational wave detectors plan to use very high optical power in the arm cavities to reduce shot noise. It was first pointed out by Braginsky et al [1,2] that with such a high power, the nonlinear coupling between acoustic modes of the test masses and the cavity optical modes can cause parametric instabilities. The carrier power in high power cavities can excite high order cavity modes $\omega_1$ (Stokes modes) below the operating cavity frequency $\omega_0$, or high order modes $\omega_a$ (anti-Stokes modes) above the operating cavity frequency. If the test masses acoustic modes $\omega_m$ coincide with the optical modes $\omega_0-\omega_1$, instability can occur. In the case of anti-Stokes modes being resonant with acoustic modes ($\omega_m \sim \omega_0-\omega_1$), acoustic modes are damped. Usually, Stokes and anti-Stokes modes are not symmetrically distributed around the carrier. Thus while Stokes and Anti-Stokes processes may both occur, they happen independently and are rarely coincident. In addition, the mode shape overlap between acoustic modes and Stokes/anti-Stokes modes are often quite different. Thus, Stokes mode contributions to instability are not generally compensated by anti-Stokes modes contributions. This contrasts to the situation of low frequency parametric instability where the unstable mechanical mode is within the resonance linewidth of the cavity. In this case, which corresponds to a degenerate parametric device, Stokes and anti-Stokes contributions may occur symmetrically on either side of the carrier resonance within the carrier mode linewidth. Under these circumstances either gain or damping can be achieved by simple tuning. A more detailed study has shown that multiple Stokes modes can contribute to single mechanical modes.

In previous analysis [3], only up to second optical mode orders are considered in parametric instability for advanced detectors. We show that optical modes with order greater than 2 still
contribute to the instability and that multiple optical modes can contribute to the stability of a single mechanical mode.

It was pointed out by Zhao et al [3] that by changing the radius of curvature of the test mass mirrors, one can tune the Stokes modes away from test mass acoustic modes and thus reduce the parametric instability gain. It showed that sapphire test masses have advantages over fused silica test masses in term of parametric instability because sapphire test masses have a lower density of internal acoustic modes and also a faster thermal response for thermal tuning. We show here a fused silica inboard test mass and sapphire outboard mass combination offers certain advantages over an all-silica system. This combination allows reduction in the number of parametrically unstable modes, and rapid thermal tuning to the minimum instability operating point, while avoiding optical absorption problems associated with a sapphire input test mass.

We used Adv/LIGO parameters as an example to demonstrate that parametric instability can occur in advanced laser interferometer detectors. We compare the parametric instability situation of sapphire/fused silica systems with all fused silica systems, and discuss possible methods of control and suppression of parametric instability.

2. Parametric Instability

Parametric instability is characterized by instability gain \( R > 1 \) as defined by Braginsky et al [2].

\[
R = \frac{4PQ_m}{McLw_{pr}^2} \left( \frac{Q_1A_1}{\delta_{pr}^2} \right) \left( 1 + \frac{\Delta \omega_1}{\delta_{pr}^2} \right) \left( 1 + \frac{\Delta \omega_{1a}}{\delta_{pra}^2} \right) > 1,
\]

Here \( P \) is optical power inside the cavity, \( M \) is the mass of the test mass, \( c \) is the speed of light, \( L \) is the arm cavity length, \( R_1, R_2 \) are the radii of curvature of cavity mirrors, \( \omega_m, \omega_0, \omega_1, \omega_{1a} \) are frequency of the mechanical and optical modes respectively, \( Q_m, Q_1, Q_{1a} \) are the mechanical quality factor of the test mass and optical quality factors of the optical modes respectively, \( \delta_{pr} \) and \( \delta_{pra} \) are the linewidth of the Stokes and anti-Stokes modes in a FP cavity with power recycling. The factors \( A_1 \) and \( A_{1a} \) contains geometrical overlapping factors, which measure the overlap between the electromagnetic field pattern and the acoustic displacement pattern, and the effective mass.

Because of the relatively large optical linewidth, there can be many optical modes resonant with a single acoustic mode. Generally, each mechanical mode will have multiple optical modes that can contribute to the instability condition.

3. Instability for advanced detectors

It can be seen from equation (1) that a system having high optical power and high test masses mechanical Q-factor, which are the parameters advanced detectors required, is likely to have high parametric instability. Using Adv/LIGO nominal parameters [4], we can calculate the parametric instability gain value of unstable mode for fused silica test mass and sapphire test mass as shown in Figure 1. Here we assumed sapphire has mechanical Q-factor of \( 2 \times 10^8 \) while fused silica has frequency dependent Q-factor [5] ranging from \( 1.5 \sim 7 \times 10^7 \) in the frequency range of 80kHz to 10kHz, respectively. There is no data available to indicate that the Q-factor of sapphire is also frequency dependent at very high frequency.

Of the 9 unstable modes in fused silica test mass, 5 of them are due to contributions from optical modes with orders higher than 2, while in the sapphire test mass case, 5 out of the 6 unstable modes are due to optical modes higher than order 2. Thus the contribution of higher order modes to the instability is not negligible.
Figure 1 Unstable modes and parametric instability gain for fused silica test mass and sapphire test mass with adv/LIGO nominal parameters. Radii of curvature of the mirrors are 2076m. Sapphire test mass Q-factor is $2 \times 10^8$ and fused silica test mass has frequency dependence (ranging from $1.5-7 \times 10^7$).

Figure 1 is only for a fixed radius of curvature of mirrors of 2076m. Since higher order mode frequency in the cavity are a strong function of radius of curvature of the mirrors, a small deviation from the nominal parameter can result in change of resonant condition. This can change the parametric gain greatly as shown in next section.

4. Parametric instability suppression

4.1. Thermal tuning.
Zhao et al [3] proposed to thermally tune the radius of curvature of the cavity mirror to change the parametric instability condition. Figure 2 shows the change of the highest unstable mode parametric gain with different radius of curvature. We model changes of only one of the mirrors radii of curvature while the other mirror has a fixed curvature of 2076m.

It can be seen that with a small deviation of nominal radius of curvature, the parametric gain changes dramatically to $R=460$ at ROC=2074m. There are some regions where change of radius of...
curvature does not result in dramatic change in parametric gain. However, the inaccuracy of mirror radius of curvature due to manufacturing or thermal lensing effects, as well as the inaccuracy of acoustic mode frequencies due to finite element modelling makes it difficult to pre-design mirrors so that they lie within the low parametric gain “valley”. Thermally tuning the radius of curvature of the test masses from high parametric gain peak is then a valuable method to suppress parametric instability.

In practice one might consider slowly increasing laser power until instability is first observed. Then it would be advantageous if instability detuning could be implemented within the ring up time before the amplitude of oscillation makes the interferometer dysfunctional. The ring up time is proportional to the mechanical loss of the acoustical mode and the parametric gain factor $R$. For example, an acoustic mode $\sim 35\text{kHz}$, with a $R$ factor of $\sim 100$ and mechanical Q-factor $\sim 5 \times 10^7$, the ring up time to amplitude of $10^{-9} \text{m}$ is about 100s. Figure 3 compares the radius of curvature thermal tuning response time for fused silica and sapphire test masses. It can be seen that the fused silica tuning time is 10 times slower than sapphire due to its low thermal conductivity.

![Figure 3 Comparison of radii thermal response time for curvature change of fused silica and sapphire test masses.](image)

In reference [6], we proposed to replace end test masses with sapphire test masses. Thermal tuning of the cavity mode discussed above could be achieved in a time sufficient to detune the instability before the interferometer becomes dysfunctional. This configuration has the advantage of less unstable modes than all fused silica test masses cavity, fast thermal tuning while avoiding the disadvantage of high optical absorption in a sapphire inboard test mass.

For the parameters used here, all the parametric gain $R>1$ over the entire radius of curvature range shown in Figure 2, it is clear that thermal tuning cannot completely eliminate instability. Other methods of suppression instability must be used.

4.2. Reduce Q-factor of test masses

Parametric instability gain is directly proportional to the mechanical Q-factor of the test mass acoustic modes. Thus it is advantageous to have a low acoustic mode Q-factor. This needs not necessarily greatly reduce the thermal noise performance of the detector. Using Levin’s direct approach [7] for calculating thermal noise of an interferometer detector, it has been shown [8] that losses in test masses far away from the central area where laser beam passes contribute little to the thermal noise. Thus it is possible to introduce some loss around the edge of the test masses to reduce the Q-factor of the acoustic mode without greatly degradation of the thermal noise. Gras et al [9] has shown that by applying lossy coatings at the side of the test mass, the parametric gain can be greatly suppressed with the cost of 10% rise of thermal noise floor. There could be even some optimized position [10] on the side of the test masses that will degrade most of the acoustic mode Q-factors with even less noise degradation.

4.3. Active control

Another approach to stabilisation is by feedback methods. Braginsky et al proposed [11] the use of external high finesse cavity to “tranquilise” each test mass. This method, which utilizes the tunable
properties of low frequency parametric interactions, has the disadvantage of adding more cavities to
the already complicated nested cavities systems. An alternative is direct radiation pressure force
feedback to the test masses. In this case, the feedback must take into account the fact that unstable
modes cannot be identified with individual test masses, so error signals must be applied to all test
masses. It could be worth considering deliberately making the dimensions of the test masses different
to break the near degeneracy. It will require intensive modeling to determine the benefits of such
design change. Feedback may also be achieved by the injection of optical modes into the cavity,
which act to damp the instabilities. A very small amount of power needs to be injected into the
appropriate higher order modes (with the appropriate phase shift) to achieve suppression. All these
methods need further careful analysis and investigation.

5. Discussion
We have shown that parametric instability imposes a serious potential problem in operating advanced
interferometer gravitational wave detectors, if not quenched. Several methods could be used for the
control/suppression of the instabilities. Thermal tuning can be used to tune the system away from
regions where parametric gain is high. The use of sapphire end test masses can make thermal tuning
easier. However for the parameters used here thermal tuning alone is not sufficient to eliminate
instability. Passive method such as applying lossy coatings to the test mass surfaces can be used to
reduce the test mass acoustic Q-factor as a means to suppress the parametric gain. More study is
needed to optimize the coating position and the amount of coating to reduce all the acoustic modes Q-
factors without greatly sacrificing thermal noise performance. If Q-reduction does not prove to be
sufficient, active control schemes will be necessary to suppress instability. There are several possible
means that can be used. All the proposed methods need further investigation.

Our analysis assumes cylindrically symmetric thermal deformation of test masses as well as
cylindrical symmetry of laser beam position. Lack of cylindrical symmetry changes the overlap
between optical and acoustic modes, and hence changes the parametric gain. We also used simple
clipping losses for high order optical modes. In reality, diffraction losses of high order optical modes
are more complicated and will differ from the simple clipping loss. All these effects act to reduce the
parametric instability gain. However, the entire parameter space is so large that much further
investigation will be required to fully understand the problem. In particular, it is very important to
understand how differences between the arms of an interferometer influence the occurrence of
instabilities.

In Gingin, Western Australia, the Australian Consortium for Interferometric Gravitational
Astronomy (ACIGA) is building a high optical power facility, consisting of an 80m power recycled F-
P interferometer. This facility is intended to investigate high power related issues for advanced
detector and will be an ideal test bed for observing parametric instabilities and investigation of the
instability suppression and control strategies [12].

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References
[1] Braginsky V B, Strigin S E and Vyatchanin S P 2001 Phys. Lett. A 287 331
[2] Braginsky V B, Strigin S E and Vyatchanin S P 2002 Phys. Lett. A 305, 111
[3] Zhao C, Ju L, Degallaix J, Gras S and Blair D G 2005 Phys. Rev. Lett. 94, 121102
[4] http://www.ligo.caltech.edu/~ligo2/scripts/l2refdes.htm
[5] Penn S D, Ageev A, Busby D, Harry G M, Gretarsson A M, Numata K and Willems P 2005
  Frequency and surface dependence of the mechanical loss in fused silica Preprint
  http://www.ligo.org/pdf_public/techpapers_penn.pdf
[6] Ju L, Zhao C, Gras S, Degallaix J, Blair D G, Munch J, Reitze D H 2005 Comparison of Parametric Instabilities in Different Test Mass Materials for Advanced Gravitational Wave Interferometers Preprint http://www.ligo.org/pdf_public/techpapers_ju02.pdf
[7] Levin Y 1998 Phys. Rev. D 57 659
[8] Gras S, Blair D G and Ju L 2004 Phy. Lett. A, 333, 1
[9] Gras S, Zhao C, Ju L and Blair D G 2005 Optimal mirror design for parametric instability suppression This proceedings
[10] DeSalvo R 2005 Private communications
[11] Braginsky V B, Vyatchanin S P 2002 Phys. Lett. A, 293 228
[12] Zhao C, et al 2005 Gingin High Optical Power Research Facility, This proceedings