Axial Current Generation from Electric Field: Chiral Electric Separation Effect

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We study a relativistic plasma containing charged chiral fermions in an external electric field. We show that with the presence of both vector and axial charge densities, the electric field can induce an axial current along its direction and thus cause chirality separation. We call it the chiral electric separation effect (CESE). On a very general basis, we argue that the strength of CESE is proportional to $\mu_V \mu_A$ with $\mu_V$ and $\mu_A$ the chemical potentials for vector charge and axial charge. We then explicitly calculate this CESE conductivity coefficient in thermal QED at leading-log order. The CESE can manifest a new gapless wave mode propagating along the electric field. Potential observable effects of CESE in heavy-ion collisions are also discussed.

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Introduction.—It was discovered and understood a long time ago that an external electric field, when applied to any conducting matter, induces a vector current, as described by Ohm’s law

$$j_V = \sigma E, \quad (1)$$

where $\sigma$ is the electric conductivity of the matter, and we use the convention that the electric current is $e j_V$. In quantum field theories such as the quantum electrodynamics (QED) and the quantum chromodynamics (QCD), one has not only the vector current $j_V^\mu$ but also the axial current $j_A^\mu$ when there are charged chiral fermions. A very interesting question is then, in addition to the above conducting vector current under applied electric field, what are the other possible current generations in response to externally applied Maxwell electric and/or magnetic fields.

Recently, the QCD axial anomaly has been found to induce the following two phenomena in the high-temperature deconfined phase of QCD, the quark-gluon plasma (QGP), with the presence of an external magnetic field: the chiral magnetic effect (CME) and the chiral separation effect (CSE). The CME is the generation of vector current and thus the electric charge separation along the axis of the applied magnetic field in the presence of nonzero axial charge density arising from fluctuating topological charge [1–5]. With an imbalance between the densities of left- and right-handed quarks, parametrized by an axial chemical potential $\mu_A$, an external magnetic field induces the vector current

$$j_V = \sigma_5 \mu_A B, \quad (2)$$

with chiral conductivity $\sigma_5 = \frac{N_c e}{2 \pi^2}$. A “complementary” effect also arising from the axial anomaly is the CSE which predicts the generation of an axial current, $j_A^\mu = \langle \bar{\psi} \gamma^i \gamma_5 \psi \rangle$, and thus separation of axial charges along the external $B$ field at nonzero vector charge density (parametrized by its chemical potential $\mu_V$) [6, 7],

$$j_A = \sigma_5 \mu_V B. \quad (3)$$

It should be emphasized, though, the $\mu_A$ (unlike $\mu_V$) is not associated with any conserved charge and can only be treated as an external parameter arising from external dynamics in the slowly varying limit, e.g., via effective axion dynamics $\mu_A \sim \partial \theta / \partial t \ll Q_{\text{QCD}}$; see detailed discussions in [2, 8]. This approach is justified for the study in the present Letter as also in previous studies [1–8].

There is robust evidence for both CME and CSE from kinetic theory, hydrodynamics, and holographic QCD models in strong coupling regime as well as in lattice QCD computations [9–21]. Experimentally, the hot QGP is created in high-energy heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC). In such collisions, domains of QGP with nonzero chirality ($\mu_A \neq 0$) can arise from topological transitions in QCD, and there are also extremely strong transient $E$ and $B$ fields [22–26], so the CME and CSE effects can occur. There have been measurements of charge asymmetry fluctuations motivated by CME predictions from the STAR [27] and PHENIX [28] Collaborations at RHIC, as well as from the ALICE [29] at LHC. The precise meaning of these data is under investigations (see, e.g., [30]). It has also been proposed that the combination of the CME and CSE leads to a collective excitation in QGP called chiral magnetic wave (CMW) [31]. The CMW induces an electric quadruple of QGP that can be measured via elliptic flow splitting between $\pi^- / \pi^+$ [32], with supportive evidence from STAR measurements [33].

There is, however, one more possibility that has not been previously discussed, namely the generation of an axial current in the electric field. We find this to occur when the matter has both nonzero $\mu_V$ and nonzero $\mu_A$, 

$$j_A = \chi \mu_V \mu_A E, \quad (4)$$

which can be called a chiral electric separation effect.
compute explicitly the leading-log order CESE conductivity for thermal QED plasma using the Kubo formula under the condition $\mu_V, \mu_A \ll T$. The extension of this computation to QCD is straightforward and will be presented elsewhere.

Let us denote $\sigma_e = \chi_e \mu_V \mu_A$. Starting with the retarded vector-vector and vector-axial correlators $G^R_{VV}$ and $G^R_{AV}$ (as diagrammatically shown in Fig. 1), the $\sigma$ and $\sigma_e$ are given via the Kubo formulas as

$$
\sigma = \sum_{i=1}^{3} \lim_{\omega \to 0} \lim_{k \to 0} \frac{i}{3\omega} G^R_{VV}(\omega, k),
$$

$$
\sigma_e = \sum_{i=1}^{3} \lim_{\omega \to 0} \lim_{k \to 0} \frac{i}{3\omega} G^R_{AV}(\omega, k),
$$

In Fig. 1, the shaded circle represents an effective vertex (see Fig. 2). This effective vertex represents a resummation of a set of ladder diagrams which contribute to the same order owing to the pinching singularity when $\omega, k \to 0$ [37]. The dotted line in Fig. 2 represents the hard thermal loop (HTL) resummed propagator. All other kinds of diagrams (e.g., the box diagrams which are vanishing identically due to Furry theorem at zero chemical potentials but finite at nonzero chemical potentials) are sub-leading-log order.

**FIG. 1:** The Feynman diagrams for retarded correlators $G^R_{VV}$ (left) and $G^R_{AV}$ (right).

One can write down the retarded correlators explicitly,

$$
G^R_{VV} = -e \int_P \text{Tr} \left[ \Gamma^i(P + K, P)S(P)\gamma^j S(P + K) \right],
$$

$$
G^R_{AV} = -e \int_P \text{Tr} \left[ \Gamma^i(P + K, P)S(P)\gamma^j \gamma_5 S(P + K) \right].
$$

The effective vertex in the above is given by Fig. 2,

$$
\Gamma^\mu(P + K, P) = \gamma^\mu + e^2 \int_Q \gamma^\rho S(P + K + Q) \times \Gamma^\mu(P + K + Q, P + Q)S(P + Q)\gamma^\sigma D_{\sigma\rho}(Q).
$$

The $D_{\sigma\rho}(Q)$ is the HTL propagator for the photon, while $S(P)$ is the electron propagator at nonzero $\mu_V$ and $\mu_A$,

$$
S(P) = \frac{-1}{\gamma_0 (P_0 + \mu_V + \mu_A \gamma_5) - \gamma \cdot P - \gamma^\mu \Sigma_\mu(P)}
$$

$$
= \sum_{s=\pm} \frac{\gamma \cdot \left( P_s - \Sigma_s \right)}{\left( P_s - \Sigma_s \right)^2},
$$

where $\Sigma_\pm = (1 \pm \gamma_5)/2$ is the chirality projection, $P^\mu_s = (P^0_s, p)$ with $P^0_s = P_0 + \mu_\pm$ and $\mu_\pm = \mu_V \pm \mu_A$. Substituting all these into the Kubo formulas and following
essentially the steps as in Refs. [38, 39], we finally obtain
\[ \sigma = \sum_{s,a=\pm} \sigma_{sa}, \]
(12)
\[ \sigma_e = \sum_{s,a=\pm} 8s\sigma_{sa}, \]
(13)
\[ \sigma_{sa} = -\frac{e^3}{3} \int \frac{d^3p}{(2\pi)^3} n_F(p - a\mu_s) \chi^s_\alpha(p), \]
(14)
with \( n_F(x) = 1/\exp(x/T) + 1 \) and \( \chi^s_\alpha(p) = D^s_\alpha(p)/\Gamma^s_\alpha \). The \( D^s_\alpha \) is defined through \( 2p^a D^s_\alpha(p) = \bar{u}_s(p) \Gamma^s_\alpha(p) u_s(p) \). The \( \Gamma^s_\alpha \) is the decay width for fermions (+) and antifermions (−) of chirality \( s \).

In the limit \( \mu_s \rightarrow 0 \), our result for \( \sigma \) is in agreement with \( \sigma \approx 15.6964T/e^3 \ln(1/e) \) obtained in Ref. [40].

**Coupled evolution of the two currents.**— As seen in Eq.(5), with the presence of external electromagnetic fields, the vector and axial currents mutually induce each other and get entangled in a nontrivial way. It is of great interest to understand the coupled evolution of small fluctuations of the two currents. A very good example is the aforementioned CMW [31, 32] in which the fluctuations of vector and axial currents are coupled together by external \( B \) field to form a propagating wave. Now the new CESE effect introduces nonlinearity (through the \( \mu_V \mu_A \)) and makes the problem more nontrivial.

Let us consider a thermal QED or QCD plasma in the static and homogeneous external \( E, B \) fields and study the coupled evolution of the small fluctuations in vector and axial charge densities. The presence of vector density and current will induce additional electromagnetic fields so that \( \mathbf{E}_{\text{tot}} = \mathbf{E} + \delta \mathbf{E} \) and \( \mathbf{B}_{\text{tot}} = \mathbf{B} + \delta \mathbf{B} \) with \( \delta \mathbf{E} \propto \mathbf{e} j^0_\mathbf{V} \) and \( \delta \mathbf{B} \propto \mathbf{e} j^0_\mathbf{A} \). As in the case of CMW, one can first replace the chemical potentials in Eq.(5) with the corresponding charge densities, \( \mu_{V,A} = \alpha_V \alpha_A J^0_{V,A} \), where the \( \alpha_V, \alpha_A \) are the susceptibilities defined as \( \alpha_V, \alpha_A = \partial \mu_{V,A}/\partial \mu_0^{\pm} \). These relations are valid as long as the chemical potentials are small compared with temperature \( T \). Then, combining Eq.(5) with the currents’ continuity equations \( \partial_t J^0_{V,A} + \mathbf{v} \cdot j^0_{V,A} = 0 \) and Maxwell’s equation \( \mathbf{v} \cdot \mathbf{E}_{\text{tot}} = j^0_\mathbf{V} \) and \( \mathbf{v} \cdot \mathbf{B}_{\text{tot}} = j^0_\mathbf{A} \), one can obtain
\[ \partial_t j^0_\mathbf{V} + \sigma_0 e j^0_\mathbf{V} + \sigma_5 \alpha_\mathbf{V} (\mathbf{B} \cdot \nabla) j^0_\mathbf{V} \]
\[ + 2 \sigma_2 \alpha_\mathbf{V} \mathbf{e} \cdot \mathbf{B} j^0_\mathbf{V} = 0, \]
\[ \partial_t j^0_\mathbf{A} + \sigma_0 e j^0_\mathbf{A} + \sigma_5 \alpha_\mathbf{A} (\mathbf{B} \cdot \nabla) j^0_\mathbf{A} \]
\[ + 2 \sigma_2 \alpha_\mathbf{A} \mathbf{e} \cdot \mathbf{B} j^0_\mathbf{A} = 0, \]
(19)
where we have introduced \( \sigma_0 \) and \( \sigma_2 \) defined through the small chemical potential expansion of \( \sigma, \sigma = \sigma_0 + \sigma_2 (\mu_0^2 + \mu_A^2) \). In arriving at the above, we have made the following approximations: first, we keep only terms up to \( \hat{O}(j^2_{V,A}) \) order (such that terms like \( j^0 \mathbf{e} \cdot \nabla j^0 \sim \hat{O}(j^3) \)) and \( \sigma_2 \mu_0^2 j^0_\mathbf{V} \) are dropped; second, we assume external fields are extremely strong such that the magnetic field feedback terms \( \sim \sigma_5 \alpha_\mathbf{V} \cdot \nabla \sim \sigma_5 \alpha_\mathbf{A} \cdot \nabla \) are negligible to other \( \hat{O}(j^2) \) terms \( \sim \nabla \sigma_0 \mathbf{e} \cdot \mathbf{B} \cdot \nabla \). We will next linearize the above equations by considering fluctuations on top of small uniform background vector and axial densities \( n_V \) and \( n_A \), so that \( j^0_{V,A} = n_V J^0_{V,A} + \delta j^0_{V,A} \) with \( \delta j^0_{V,A} \ll n_V J^0_{V,A} \) being fluctuations. Note that \( n_{V,A} \) themselves must still be small (compared to \( T^3 \)) to ensure the linear relations between \( J^0_{V,A} \) and \( \mu_{V,A} \). Strictly speaking, only a uniform axial density \( n_A \) is a static solution of the above equations, while a uniform vector density \( n_V \) suffers from the damping term \( e\sigma_0 j^0_\mathbf{V} \) and is only approximately static on time scale shorter compared with \( 1/e\sigma_0 \). Keeping only linear...
terms in $\delta j^0_A$, we obtain

$$\begin{align*}
\partial_t \delta j^0_A + e\sigma_0 \delta j^0_A + \sigma_5 \alpha_A (B \cdot \nabla) \delta j^0_A \\
+ 2\sigma_2 n_A \nabla (E \cdot \nabla) \delta j^0_A + 2\sigma_2 \alpha_A n_A (E \cdot \nabla) \delta j^0_A = 0, \\
\partial_t \delta j^0_A + \sigma_5 \alpha_A (B \cdot \nabla) \delta j^0_A + \chi_e \alpha_V \alpha_A n_A (E \cdot \nabla) \delta j^0_A \\
+ \chi_e \alpha_V \alpha_A n_A (E \cdot \nabla) \delta j^0_A = 0. \tag{20}
\end{align*}$$

In order to find possible normal modes, we make the Fourier transformation of these equations. Using $\delta j^0_A = \int \omega_k C_V (\omega, k) \exp (-i\omega t - k \cdot x)$, we obtain from Eqs. (20) the following relations:

$$\begin{align*}
\omega V_C + i e\sigma_0 C_V - \sigma_5 \alpha_A (B \cdot \kappa) C_A \\
- 2\sigma_2 \alpha_A n_v (E \cdot \kappa) C_V - 2\sigma_2 \alpha_A n_A (E \cdot \kappa) C_A = 0, \\
\omega C_A - \sigma_5 \alpha_A (B \cdot \kappa) C_V - \chi_e \alpha_V \alpha_A n_A (E \cdot \kappa) C_V \\
- \chi_e \alpha_V \alpha_A n_A (E \cdot \kappa) C_A = 0. \tag{21}
\end{align*}$$

Without loss of generality, we can always assume $B$ is along the $z$-axis, i.e., $B = B\hat{z}$ while $E = E\hat{e}$. The dispersion relation obtained from Eq. (21) can be expressed as

$$\omega = -\frac{1}{2} [i e\sigma_0 - v_+ (\hat{e} \cdot \kappa)]$$

$$\pm \frac{1}{2} \sqrt{(i e\sigma_0 - v_- (\hat{e} \cdot \kappa))^2 + 4 A_\chi (k),} \tag{22}$$

where $v_\pm = v_e \pm v_a$ with $v_e = 2\sigma_2 \alpha_A^2 n_v E$ and $v_a = \chi_e \alpha_V \alpha_A n_A E$, and

$$A_\chi (k) = [\sigma_5 \alpha_A B (\hat{z} \cdot \kappa) + 2\sigma_2 \alpha_A n_A E (\hat{e} \cdot \kappa)]$$

$$\times \{\sigma_5 \alpha_A B (\hat{z} \cdot \kappa) + \chi_e \alpha_V \alpha_A n_A E (\hat{e} \cdot \kappa)\}. \tag{23}$$

To manifest the physical meaning of the solutions in (22), let us consider the following two special cases:

1. The case with only $B = B\hat{z}$ and $E = 0$.

   Equation (22) reduces to $\omega = \pm \sqrt{(v_0 k^2 z^2 - (e\sigma_0/2)^2)}$ with speed $v_0 = \sigma_5 \alpha_A B$. When $v_0 k^2 \gg e\sigma_0/2$ we get two well-defined propagating modes $\omega \approx \pm v_0 k^2 z - i (e\sigma_0/2)$. These are generalized CMWs which reduce to the CMW in [31] when $\sigma_0 = 0$ and $\alpha_V = \alpha_A$. When $v_0 k^2 \leq e\sigma_0/2$ the two modes become purely damped.

2. The case with only $E = E\hat{e}$ and $B = 0$.

   First, we consider a background without vector density, i.e., $n_v = 0$. In this case, we find two modes from (22),

   $$\omega = \pm \sqrt{(v_e k^2 z^2 - (e\sigma_0/2)^2)} - i (e\sigma_0/2) \tag{24}$$

   with $v_e = \sigma_2 \alpha_A n_A 2\sigma_2 \chi_e \alpha V \alpha_A E$. Similar to the CMWs, when $v_e k^2 \gg e\sigma_0/2$ there are two well-defined modes $\omega \approx \pm v_e k^2 z - i (e\sigma_0/2)$ from CESE that propagate along $E$ field and can be called the chiral electric waves (CEWs). They become damped when $v_e k^2 \leq e\sigma_0/2$.

   Second, if the background contains no axial density, i.e., $n_A = 0$, then we see that the vector and axial modes become decoupled, and Eq. (22) leads to

   $$\begin{align*}
   \omega_V (k) &= v_v k^2 z - i (e\sigma_0), \\
   \omega_A (k) &= v_a k^2 z. \tag{25}
   \end{align*}$$

   The first solution $\omega_V (k)$ represents a “vector density wave” (VDW) with speed $v_v = 2\sigma_2 \alpha_A^2 n_v E$ that transports vector charges along $E$ field but will be damped on a time scale $\sim 1/(e\sigma_0)$. The second solution $\omega_A (k)$ is a new mode arising from CESE and represents a propagating “axial density wave” (ADW) along $E$ with speed $v_a = \chi_e \alpha_V \alpha_A n_A E$ and without damping.

   **Summary and discussions.** —In summary, we have found a new mechanism for the generation of axial current by external electric field in a conducting matter with nonzero vector and axial charge densities, which we call the chiral electric separation effect. We have computed the CESE conductivity coefficient in a QED plasma and also studied possible collective modes arising from it.

**FIG. 3:** A schematic illustration for CESE-induced net charge distribution and correlation patterns in Cu + Au collision.

We end by discussing possible observable effects induced by CESE in heavy ion collisions. In the created hot QGP there can be both vector and axial charge densities from fluctuations and topological transitions. There are also very strong electric fields during the early moments of heavy ion collisions [22–24, 26]. One particularly interesting situation is in the Cu + Au collisions (see Fig. 3), where due to the asymmetric nuclei (rather than from fluctuations) there will be a strong $E$ field originating from the Au nucleus to the Cu nucleus [41]. In this case the $E$ field will lead to both an in-plane charge separation via (1) and an in-plane chirality separation via (4). The resulting in-plane axial dipole will then further separate charges via CME along the magnetic field in the out-of-plane direction, and cause an approximate quadrupole at certain angle $\Psi_q$ in between in- and out-of-plane. We therefore expect a highly nontrivial charge azimuthal distribution pattern $\delta N_{\pm} (\phi) \sim \delta \cos (\phi) + q \cos (2\phi - 2\Psi_q)$ with the dipole term due to usual conductivity and the quadrupole term due to CESE and CME effects. This pattern may possibly be measured either via charged pair correlations or the charged multiple analysis [24]. Quantitative predictions will require proper modeling of the QGP and solving the Eq. (20), which will be reported in a future work.
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