Baryon-baryon interactions in large $N_C$ chiral perturbation theory

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Abstract

Interactions of two baryons are considered in large $N_C$ chiral perturbation theory and compared to the interactions derived from the Skyrme model. Special attention is given to a torus-like configuration known to be present in the Skyrme model.

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I. INTRODUCTION

Chiral perturbation theory relates various low-energy properties of hadronic systems by means of effective actions. Such effective lagrangians are defined in terms of the degrees of freedom manifest in elementary excitations of the system. Constraints imposed on the effective action originate from the symmetry requirements only, as deduced from the observed particle spectra, from which fact the generality of the method may be understood \[1\]. In the absence of baryonic degrees of freedom this calculational scheme has lead to an impressive amount of statements on properties of mesonic systems, which commonly have been derived and thus are valid up to fourth chiral order, i.e. to one-loop level \[2,3\]. Inclusion of the baryons into the scheme unfortunately introduces complications due to the fact that a counting scheme based on chiral orders no longer limits the number of terms appearing at a definite chiral order \[4\]. Baryons included, the perturbative expansion looses much of its usefulness unless further criteria of smallness are introduced from outside. One such criterion is the order in an expansion in terms of the number of colors \(N_C\) appearing in the underlying more fundamental theory, quantum chromodynamics \[5,6\].

Restricted to purely mesonic systems the effective lagrangians of chiral perturbation theory are identical to those of the Skyrme model \[7\], if parameters are chosen accordingly: both involve the Goldstone bosons of spontaneously broken chiral symmetry as principle degrees of freedom. In the presence of baryons the similarity of the two approaches seems to disappear, since the lagrangian of chiral perturbation theory introduces baryons explicitly coupling them to the Goldstone bosons in \(U = \exp \mathbf{i} \tau \cdot \pi / f_\pi\) via a vertex

\[
\mathcal{L}_{\pi N} = \frac{1}{2} \hat{g}_A \bar{N} \gamma_\mu \gamma_5 \tau_a N \cdot \frac{i}{2} \text{tr} \tau_a \sqrt{U^\dagger} \partial^\mu U \sqrt{U^\dagger}
\]

(1)
of order \(\sqrt{N_C}\), whereas the Skyrme model has no such couplings (\(\hat{g}_A = 0\)): baryons in the Skyrme model only appear as topological knots in the meson fields. More recent developments \[8,9\], however, are now suggesting that even in the presence of baryons both approaches, at least to leading order in \(N_C\), are identical in the limit that the extension of
the bare baryon tends to zero. In section 1. I will repeat this suggestion thus introducing formalism and notations for the present work, which otherwise is concerned with the interactions of two baryons.

Systems with baryon number $B = 2$ have had a somewhat peculiar status in the $SU(2)$ Skyrme model since the minimal energy configuration is of torus-like structure \cite{10,11} as far as energy and baryon number density are concerned thus displaying only a very remote resemblance to two interacting baryons. The natural question to ask then is: if large $N_C$ chiral perturbation theory leads to the Skyrme model for systems containing one baryon, does the interaction of two baryons lead to torus-like structures in chiral perturbation theory? Since the foundations of chiral perturbation theory are firmly established, the answer to the question posed is of principle importance. I will attempt to answer the question in two steps. In section 2. I first will show, that the interaction between two baryons in large $N_C$ chiral perturbation theory at large separations is identical to the Skyrme model expressions. In section 3. I will examine the short distance behaviour, which is only accessible numerically, displaying the results as to make clear, that large $N_C$ chiral perturbation theory will indeed lead to torus-like configurations when two baryons are approaching one another adiabatically. Latter assumption is, of course, inherent at leading order in $N_C$.

II. LARGE $N_C$ CHIRAL PERTURBATION THEORY AND THE SKYRMION

The gap between chiral perturbation theory and the Skyrme model is bridged by the observation \cite{12,13} that the $\pi N$-scattering amplitude, which is of order one in $N_C$-counting, can only emerge once all order $N_C$ diagrams, which are present due to the coupling of the baryonic axial current to the mesonic one (order $\sqrt{N_C}$) in (11), have cancelled. Without such cancellations $\pi N$-scattering would be of order $N_C$. The cancellation requires an infinite tower of baryonic states, all degenerate at order $N_C$, having their spins equal to their isospins. The generalization of the $\pi N$-coupling (1) to the whole tower is obtained by substitution

$$\gamma_\mu \gamma_5 \tau_a \rightarrow X^{a i}$$
of the spin-isospin matrix elements for $I = J = \frac{1}{2}$ representations, by those for the whole tower. The couplings which lead to a cancellation of the order $N_C$ scattering amplitude have been determined in ref. [12,13]:

$$\langle I' = J', I_3, J_3 | X^{\alpha \beta} | I = J, I_3, J_3 \rangle =$$

$$-\sqrt{(2I' + 1)(2I + 1)}(-)^{I' - I_3} \begin{pmatrix} I' & 1 & I \\ -I_3 & \alpha & I_3 \end{pmatrix} (-)^{J' - J_3} \begin{pmatrix} J' & 1 & J \\ -J_3 & \beta & J_3 \end{pmatrix} =$$

$$\int d[A] \sqrt{\frac{2I' + 1}{8\pi^2}}(-)^{I' - I_3} D^{*}_{-I_3 I_3}(A)(-)^{\alpha} D^{-\alpha \beta}(A) \sqrt{\frac{2I + 1}{8\pi^2}}(-)^{I - I_3} D_{-I_3 I_3}(A).$$

Note, that spherical indices have been used here. The second part of the equation is just an identity for the D-functions of matrices $A \in SU(2)$. This identity will prove to be useful, once we have rotated the infinite tower of degenerate baryon states $|I = J, I_3, J_3\rangle$ to a basis classified according to the orientations $A$ of a baryon in isospace:

$$|A \rangle = \sum_{I, I_3, J_3} \sqrt{\frac{2I + 1}{8\pi^2}}(-)^{I - I_3} D^{*}_{I_3 I_3}(A) |I = J, I_3, J_3\rangle.$$

The new basis of degenerate baryons diagonalizes the pion-baryon coupling

$$X^{\alpha i} = \int d[A] D_{ai}(A) |A \rangle \langle A|$$

as may easily be seen by insertion of (3) into (4). Thus, in leading order in $N_C$ baryons do not change their orientation in isospace when interacting with pions. For similar reasons baryons do not move in space upon interaction with the mesons since their velocities are of order $1/N_C$: baryons behave like a static source of fixed orientation $A$ and position $X$ for the pion fields. The large $N_C$ interactions of pions and baryons are then summarized by the following lagrangian, the leading $N_C$-dependence of which has been factored to the front of the lagrangian:

$$\mathcal{L} = N_C \left[ \mathcal{L}^{(\text{meson})}(U) + \mathcal{L}^{(\text{source})}(U; A; X) \right]$$

$$\mathcal{L}^{(\text{meson})} = \frac{f_2^2}{4N_C} \mathrm{tr} \partial_\mu U \partial^\mu U^\dagger + \frac{f_2^2 m_\pi^2}{4N_C} \mathrm{tr} (U + U^\dagger - 2) + \cdots$$

$$\mathcal{L}^{(\text{source})} = -\frac{3}{2N_C} \delta_A \Delta(x = X) D_{ai}(A) \cdot \frac{i}{2} \mathrm{tr} \tau_a \sqrt{U^\dagger \partial_i U \sqrt{U^\dagger}}.$$
$\mathcal{L}^{(\text{source})}$ and $\mathcal{L}^{(\text{meson})}$ are of order $(N_C)^0$. For spin=isospin=$\frac{1}{2}$ states the matrix elements of the $D$-function in (3) are given by $D_{ai} \to -\frac{1}{3} \tau_a \sigma_i$, so the $\pi N$-coupling implicit in (3) coincides with the one given earlier in (1).

In a very readable recent publication Manohar [8] demonstrates that an $N_C$-independent regularization of the functional integral constraining the effective lagrangian (3) to its range of validity, i.e. to small momentum scales, can be achieved by giving the baryon source a finite extension $R_0 \sim 1\text{GeV}^{-1}$

$$\Delta(x - X) = \left(\frac{4\pi}{3} R_0^3\right)^{-1} \Theta(R_0 - |x - X|). \quad (6)$$

In this case a factor $N_C/\hbar$ multiplies the exponent in the integrand of the functional integral for which the leading terms in an expansion in powers of $1/N_C$ therefore turn out to be equivalent to the leading terms in a semiclassical expansion in powers of $\hbar$. Thus, the leading terms in $N_C$ of the pion-baryon interactions are obtained by solution of classical equations of motion for the pion cloud around a static baryon source of fixed position and isospin orientation!

The structure of the pion cloud around such a fixed source which satisfies the classical Euler-Lagrange equations has the form

$$U = A e^{i\tau \cdot \hat{x} \chi(x)} A^\dagger \quad (7)$$

and the cloud is completely determined by solution of a second order radial differential equation for the remaining chiral angle $\chi$

$$\partial_x (x^2 \partial_x \chi) - \sin 2\chi - m_{\pi}^2 x^2 \sin \chi + \cdots = \frac{3g_A}{2f^2_{\pi}} \left[ \partial_x \Delta(x) - \frac{2}{x} (1 - \cos \chi) \Delta(x) \right]. \quad (8)$$

The dots stand for higher order terms from $\mathcal{L}^{(\text{meson})}$ in (3).

In figure 1. we display the chiral angle of the cloud as a function of distance from the center of the baryon source. Coming from large distances the chiral angle has the one-pion tail

$$\chi(x) \xrightarrow{x \to \infty} \frac{3g_A}{8\pi f^2_{\pi}} m_{\pi}^2 (1 + \frac{1}{m_{\pi} x}) \frac{1}{m_{\pi} x} e^{-m_{\pi} x} \quad (9)$$
where $\tilde{g}_A = f_\pi/M g_{\pi N N}$ is the physical $\pi N$-coupling. $\tilde{g}_A$ differs from the axial charge $g_A$ by terms of order $O(m_\pi^2)$. At $x = R_0$ the source enforces a discontinuity in the derivatives proportional to the bare $\pi N$-coupling $\hat{g}_A$. The discontinuity is adjusted, of course, by making the total solution regular at the origin.

For a large source, i.e. a small cutoff, the lowest order terms (w.r.t. $\chi$) of the lagrangian (5) are sufficient. The physical $\pi N$-coupling equals

$$\tilde{g}_A = \frac{-3}{64 \pi f_\pi^2 m_\pi^2 R_0^6} (1 + m_\pi R_0) \left[ (1 - m_\pi R_0) - (1 + m_\pi R_0)e^{-2m_\pi R_0} \right] (10)$$

and the mass shift $\delta M$ of the baryon due to the cloud may also be calculated analytically as

$$\delta M = \frac{81 \hat{g}_A^2}{64 \pi f_\pi^2 m_\pi^2 R_0^6} (1 + m_\pi R_0) \left[ (1 - m_\pi R_0) - (1 + m_\pi R_0)e^{-2m_\pi R_0} \right] (11)$$

The first two (for $R_0 \to 0$ singular) cutoff-dependent shifts may be absorbed into the constants of the bare lagrangian of chiral perturbation theory: term one into the chiral invariant baryon mass term, term two into the quark mass contribution to the nucleon mass which is proportional to $m_q \sim m_\pi^2$. The third term is non-analytic in the quark masses and cannot be reabsorbed into the bare lagrangian, where no such terms are present: the third term, independent of the cut-off, is a genuine finite correction and identical to the one-loop correction to the baryon mass as calculated in standard chiral perturbation theory [4,14] with intermediate nucleon and isobar states.

Due to the multivaluedness of the lagrangian, the requirement of regularity of the energy density only demands the chiral angle to be some multiple of $\pi$ at the origin. Returning to figure 1. we see, that the bare pion-baryon coupling $\hat{g}_A \to 0$ if the chiral angle, coming from large distances where it is fixed, just reaches a multiple of $\pi$ at the origin. In such a case we have a finite renormalized pion-baryon coupling $\tilde{g}_A$ in a purely mesonic theory since the bare pion-baryon coupling $\hat{g}_A$ now is zero: this configuration of the cloud is identical to the chiral field of the skyrmion [9].
III. ASYMPOTOTIC INTERACTIONS

The presence of two baryons in large $N_C$ chiral perturbation theory is realized by placing two baryonic sources, one at $X/2$ with orientation $A$ and the other at $-X/2$ with orientation $B$. As long as the separation of the two sources $X$ is greater than twice the radius $R_0$ of the source the interaction proceeds through meson exchange only, given by a trivial generalization of the lagrangian (8) for the sources:

$$\mathcal{L} = N_C \left[ \mathcal{L}^{(\text{meson})}(U) + \mathcal{L}^{(\text{sources})}(U; A, B; X) \right]$$

From the lagrangian (12) we may deduce the classical Euler-Lagrange equations in order to calculate cloud effects to leading order in $N_C$.

The restriction of the equations of motion to the case of large sources simplifies matters appreciably. Then it is sufficient to keep terms which are maximally linear in the chiral angles $\chi_b$ parametrizing the matrix $U$. In this case the two sources only appear as inhomogeneous terms in the equations of motion, independent of the chiral angles. The solution to such a linear inhomogeneous differential equation is, of course, a superposition of the chiral fields for each of the sources separately, as they have emerged from eq.(8) (in its linearized form):

$$\chi_b = D_{bi}(A) \hat{x}_- \chi_- + D_{bi}(B) \hat{x}_+ \chi_+$$

where

$$\chi_- = \chi(| x_- |), \quad \chi_+ = \chi(| x_+ |).$$

The mass shift of the two baryons may be deduced from

$$\delta M_{B=2} = \frac{1}{2} \int d^3x \left\{ \frac{3}{2} \delta_A \Delta(x_-) D_{bi}(A) \partial_i \chi_b - \frac{3}{2} \delta_A \Delta(x_+) D_{bi}(B) \partial_i \chi_b \right\}$$

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where the equations of motion for $\chi_b$ have been used to eliminate the contributions from the purely mesonic parts of the energy density. Inserting the chiral fields given in (13) the mass shift in (15) contains the mass shifts of the individual baryons given in (11) and an interaction term

$$V_{\text{asy}} = \frac{3}{4} g_A \int d^3 x \left\{ \Delta(x_-) D_{bi}(A) D_{bj}(B) \partial_i \dot{x}_{+j} \, \chi_+ + \Delta(x_+) D_{bi}(B) D_{bj}(A) \partial_i \dot{x}_{-j} \, \chi_- \right\} \quad (16)$$

$$= \frac{3}{4} g_A \int d^3 x \left\{ D_{ij}(A^\dagger B) \dot{x}_{+j} \, \chi_+ \partial_i \Delta(x_-) + D_{ij}(B^\dagger A) \dot{x}_{-j} \, \chi_- \partial_i \Delta(x_+) \right\}$$

$$= \frac{3}{4} g_A D_{ij}(A^\dagger B) \int d^3 x \left\{ \dot{x}_{+j} \, \chi_+ \partial_i \Delta(x_-) + \dot{x}_{-j} \, \chi_- \partial_i \Delta(x_+) \right\},$$

For the last step I have used the fact that the $D$-functions w.r.t. Cartesian indices are real.

Eq. (16) is valid for large sources and consequently large separations $X$ between the sources. In the integrand each source multiplies the chiral field of the other source so the asymptotic form of the chiral angles from (9), (10) may safely be inserted leading to the final result

$$V_{\text{asy}} = \frac{9}{16 \pi f^2} g_A^2 \int d^3 x \left\{ \Delta(x_-) D_{bi}(A) D_{bj}(B) \partial_i \partial_j \frac{1}{X} \exp(-m_X R_0) \right\}, \quad (17)$$

where the derivatives now act on $X$.

The asymptotic interaction behaves smoothly as the cutoff is removed and then precisely equals the expression for the asymptotic interaction of two skyrmions derived by Skyrme [7] thirty years ago. Taking its matrixelements for baryons $A$ and $B$, both with spin=isospin=$\frac{1}{2}$, yields the well known one-pion exchange potential for two nucleons, because then $D_{bi}(A) D_{bj}(B) \rightarrow \frac{1}{9} \tau^A \cdot \tau^B \, \sigma_i^A \sigma_j^B$.

**IV. SHORT RANGE INTERACTIONS**

The exploration of the short range behaviour of baryon interactions at leading order in $N_C$ introduces several speculative elements with respect to the precise form of the effective action and several uncertainties in precision, because the investigation has to be performed numerically, as I will explain. Nevertheless, I believe that the main ingredients and the main conclusions are under control.
Since we wish to calculate cloud energies at small separations $X$ of two baryons we must ensure that the sources do not overlap, i.e. the radius of the source must obey $R_0 < \frac{1}{2}X$. Therefore the volume of the source is small and due to its normalization to unit baryon charge, eq.(6), this will lead to strong meson fields close to the source. Clearly, the situation can no longer be handled using the linearized classical Euler-Lagrange equations and higher order terms in the chiral angles $\chi_b$ are required. Then, of course, the chirally lowest order terms quoted explicitly in the lagrangian (3) are no longer sufficient either.

For the purpose of the present investigation we add one further fourth order term to the mesonic lagrangian, namely the fourth order stabilizing term of the Skyrme model:

$$L_{\text{Skyrme}} = \frac{1}{32e^2} \text{tr} [U^\dagger \partial_\mu U, U^\dagger \partial_\nu U][U^\dagger \partial^\mu U, U^\dagger \partial^\nu U].$$  \hspace{1cm} (18)

It naturally appears as the larger of the two chirally symmetric terms in next (i.e. fourth) order chiral perturbation theory [4].

The truncation to fourth order, which we will apply - also for simplicity -, is a prejudice. Nevertheless, it is motivated by the experience, that other higher order terms in the Skyrme model do not change the details of the meson cloud beyond say .25fm and that the Skyrme term is phenomenologically - almost - sufficient [15].

In order to make a meaningful comparison between the $B = 2$ sectors of the Skyrme model and large $N_C$ chiral perturbation theory, we choose $e = 4$ for the Skyrme parameter, which together with $f_\pi = 93$MeV yields a good phenomenological description of baryon and baryon-meson systems in the former [13]. For the latter, we fix the bare $\pi N$-coupling to $\varphi_A = 1.72$ in which case the cloud of the skyrmion is identical to the cloud around a sharp baryon source of radius $R_0 = .25$fm. Due to numerical problems explained later, we will actually use a smoother source

$$\Delta(x) = \left(\frac{4\pi}{3}R_0^3\right)^{-1} \left(1 + \left(\frac{x}{R_0}\right)^{20}\right)^{-1}. \hspace{1cm} (19)$$

for which $\varphi_A = 2.01$ will make the meson cloud agree with the one around the skyrmion in the outside region. In figure 1. I have displayed the three chiral angles for the cases skyrmion, sharp source, smooth source as calculated with the parameters quoted here.
Three separations, $X = .7$ fm, 1.4 fm and 2.1 fm, will be considered for the two sources. Placed at the smallest separation, two skyrmions with a relative isospin orientation of $A^\dagger B = i\tau_2$ easily deform to a torus-like configuration, as has been shown by numerical minimizations [16–18] on finite three dimensional lattices. An essential point of these numerical calculations is that the transition from two solitons separated along say the $z$-axis leads to a torus with a symmetry axis perpendicular to the $z$-axis such that axial symmetry cannot be maintained all time during the transition. Therefore, the numerical minimization of such configurations requires a general three dimensional lattice. One immediate consequence is that in three dimensions the lattice cells will be rather coarse, if one wishes to keep the computational effort in reasonable limits. Hence, a sharp source is problematic on a mesh with a rather low point density and has motivated its substitution by the smoother counterpart in (19).

The next obstacle one is confronted with in the numerical minimization comes from parametrizations using three chiral angles, where the multivaluedness of the angular functions quickly leads to numerical instabilities on a finite three dimensional mesh. To overcome the problem, I have switched to a non-unitary parametrization of the chiral fields

$$U = \Phi_0 + i\tau \cdot \Phi,$$  

(20)

where unitarity is enforced by a constraint

$$C = \int d^3x \lambda(x) (\Phi_0^2 + \Phi^2 - 1)^2$$  

(21)

on the four functions.

The non-unique non-unitary extension of the energy functional in terms of these four functions was chosen as

$$M^{(\text{meson})} = \int d^3x \left\{ \frac{f_\pi^2}{2} \Lambda_i^a \Lambda_i^a + \frac{1}{4e^2} \varepsilon_{abc} \varepsilon_{ade} \Lambda_i^b \Lambda_j^c \Lambda_i^d \Lambda_j^e + f_\pi^2 m_\pi^2 (1 - \Phi_0) \right\}$$  

(22)

for the purely mesonic parts where the abbreviation

$$\Lambda_i^a = \Phi_0 \partial_i \Phi_a - \Phi_a \partial_i \Phi_0 + \varepsilon_{abc} \Phi_b \partial_i \Phi_c$$  

(23)
has been used. The source terms require a non-unitary extension of the the square root of $U$. I have used the following form which in contrast to other possibilities is numerically non-singular when $\Phi_0 \to -1$:

$$M^{(\text{sources})}(A, B, X) = -\frac{3}{2} \bar{\partial}_A \int d^3 x \left\{ \Delta(x - \frac{1}{2} X) \delta_{ab} + \Delta(x + \frac{1}{2} X) D_{ab}(A^\dagger B) \right\} \times$$

$$\left\{ \Phi_0 \partial_b \Phi_a - \Phi_a \partial_b \Phi_0 + (\sqrt{\Phi_0^2 + \Phi^2} - \Phi_0) \Phi \partial_b \hat{\Phi}_a \right\}. \quad (24)$$

$\hat{\Phi}$ is the unit vector of the fields $\Phi$ and $\Phi = | \Phi |$. A global isospin rotation $A$ has been performed on the whole $B = 2$ configuration. Due to this global rotation $A^\dagger B$ appears as relative isospin orientation between the two sources in the functional. Of course, because of isospin symmetry the global rotation does not affect the energy density, $M^{(\text{sources})}(A, B, X) = M^{(\text{sources})}(1, A^\dagger B, X)$.

Once the unitarity constraints are satisfied exactly, different extensions of the energy functional would, of course, yield identical answers. However, since the constraints are only obeyed approximately in a numerical minimization on a finite mesh, different extensions lead to differing numbers.

The minimization, finally, varies the four functions $(\Phi_0, \hat{\Phi})$ at every mesh point independently lowering their contribution to the sum $M^{(\text{meson})} + M^{(\text{sources})} + C$ for some fixed large non-negative function $\lambda$ till no further decrease in this sum occurs. The sum without the contribution of the constraint, $M^{(\text{meson})} + M^{(\text{sources})}$, is then interpreted as the minimal energy of the configuration.

The difference between Skyrme model and large $N_C$ chiral perturbation theory has been reduced to the magnitude of the bare pion-baryon coupling and the boundary conditions on the meson cloud, here. Thus, for both cases numerical minimizations may be performed using the same program and the same three dimensional lattice which is an advantage in a direct comparison of the two.

A lattice with randomly distributed points, the density of which is roughly proportional to the expected energy density, has been used here. It proved to be superior in precision and stability relative to an equidistant one, once the same number of points is involved. The
price to be paid for such an advantage is that the energy of a given configuration depends on the position of where it has been placed on the lattice. I have tested this dependence for the case of a single smooth source. Its exact energy as determined from the solution of the differential equations of motion is $M = -7568 \text{MeV}$ (using the parameters quoted already). Putting this configuration on the lattice at positions where the two sources will be located later overestimates the energy by 8% at $\frac{1}{2}X = .35 \text{fm}$, 15% at $\frac{1}{2}X = .7 \text{fm}$, and 20% at $\frac{1}{2}X = 1.05 \text{fm}$. The reason is understood from the errors in derivatives calculated from finite differences in regions where the source changes rapidly: further away from the origin of the lattice the density of points drops.

A numerical calculation of the interaction energies of two sources as a function of their separation only makes sense, if the result is compared to the sum of energies of single sources located at identical positions on the same lattice. The resulting difference is small and I estimate its errors to be much better than the $\sim 10\%$ deviations between exact and numerically determined absolute mass shifts. As may be seen from figure 2, the interaction energy determined this way shows a remarkably smooth dependence on the separation and actually approaches the analytically determined asymptotic interaction, also included in the figure.

Since the topological configuration has no sources, the position dependence of a given configuration is much smaller than for the case of explicit sources: for the $B = 1$ soliton we find $M_{\text{numerical}} = 1752 \text{MeV}$ relative to the exact result $M = 1756 \text{MeV}$ and for the torus configuration in the $B = 2$ sector we have $M_{\text{numerical}} = 3359 \text{MeV}$ relative to the exact result: $M = 3371 \text{MeV}$. Note, that the huge discrepancy in absolute masses between topological configurations and the one with explicit sources is irrelevent, since the latter still miss the unspecifed bare mass of the source to be added. Let me also emphasize, that the large soliton mass in the Skyrme model is of no concern, since the Casimir energies of the soliton appearing in next to leading order in $N_C$ yield the desired corrections [19] (at least for the parameters used here).

In figure 2. I have only displayed the interaction energy of two sources at a fixed rel-
ative orientation $A^\dagger B = i\tau_2$ which is the most attractive channel for skyrmion-skyrmion interactions. For the smallest separation $X = .7$fm, where the Skyrme model finds maximal attraction in a torus-like configuration, I have tested the isospin dependence of the interaction between explicit sources for two other cases $A^\dagger B = i\tau_3$ and $A^\dagger B = 1$: both orientations lead to a repulsive interaction of +780MeV in the first and +625MeV in the second case indicating that the most attractive orientation $A^\dagger B = i\tau_2$ with -290MeV interaction energy for the explicit sources is identical to the Skyrme model case. So there is a qualitative agreement between both, but quantitatively differences are rather large. Unfortunately, I have no possibility to check, whether the quantitative differences depend on the arbitrary extension of the source, since I cannot make it larger at separation $X = .7$fm without having the sources overlapping and I cannot make it smaller either, because the numerical problems become unmanageable.

There is, however, indirect evidence that the extension of the source plays a major role quantitatively: coming to the central point of the present investigation we now compare the minimal energy density of two sources separated by .7fm with a relative isospin orientation of $A^\dagger B = i\tau_2$ to the energy density of the Skyrme model’s torus.

This comparison is presented in the figures 3a,b - 5a,b which display these densities in three orthogonal planes with the origin as common point. Figures 3a,b show the plane orthogonal to the $y$-axis, figure 3a for explicit sources, figure 3b for the torus, which evidently has thus been cut perpendicular to its symmetry axis. Figure 4a,b show the corresponding cuts orthogonal to the $x$-axis and in figure 4b one sees the two bumps where the torus has been cut parallel to its symmetry axis. Figures 5a,b finally show the cut perpendicular to the $z$-axis. Due to the axial symmetry of the torus, this cut leads to an identical density distribution as the one in figure 4b. Since the explicit sources are separated in $z$-direction, the plane in figure 5a does not cut the sources and one only sees the positive definite energy density of the meson cloud arranged in a way very similar to the torus, albeit lower. Closer inspection of figure 4a, where now the sources have been cut, inside of which the energy density is high and negative, one may realize that the meson cloud outside is again very
similar to the case of the torus, figure 4b: low density between the sources leading to two isolated bumps, were it not for the holes punched into them by the sources.

Returning to figure 3a now we are no longer surprised to find, that outside of the holes made by the sources the meson cloud has arranged itself in form of a ring with an additional low positive density hole in the center, just as in figure 3b. It only appears, that the ring formed by the meson cloud is slightly deformed by the presence of the source, since the latter has a finite extension. I am confident, that sources of smaller extensions will lead to energy densities which will come even closer to the torus configuration, so I suspect that quantitative differences in the interaction energies are mainly due to the finite extension of the source.

There remains one interesting question unanswered: in contrast to the case with topological solitons, the distance between the explicit sources is a well defined quantity, so one can ask what happens, when the two sources approach each other even closer than the separation at which the torus forms from two initial $B = 1$ solitons? As emphasized already, the answer is, unfortunately, beyond the numerical abilities of the calculation outlined here.

V. CONCLUSIONS

The present investigation has been dealing with the interactions of two baryons in chiral perturbation theory at leading order in $N_C$. Such an investigation has become feasible due to an observation by Manohar [8] that the leading order interactions may be obtained by solution of classical Euler-Lagrange equations. These describe the pion fields around static baryon sources fixed at a definite position in space with a definite orientation in isospace. The situation is clearly reminiscent of the Skyrme model to the results of which we have made direct comparison.

Firm statements can be made for the long-range interaction, because in this case there are no uncertainties in the chirally lowest order terms of the effective action which are sufficient here. The long-range interaction turns out to be identical to the long-known
one-pion exchange interaction, a result that certainly is not unexpected. Furthermore, the long-range force is identical to the interaction derived from the Skyrme model, so in one more respect this suggests that large $N_C$ chiral perturbation theory and the Skyrme model are actually the same language.

If true, one must worry about a peculiar field configuration known in the $B = 2$ sector of the Skyrme model, namely a torus-like configuration of the meson cloud which represents the classical energy minimum located at small separations. Its reception among intermediate energy physicists has been ambivalent, ranging from 'looking at it as an artifact of the model' to 'accepting it as the origin of attraction between nucleons'.

I have tried to explore the case of the torus in the framework of large $N_C$ chiral perturbation theory, but in doing so, I had to add speculative elements to the investigation as far as chirally higher order terms of the effective action were concerned. Specifically, a simplifying assumption had to be made, that the main term of fourth chiral order, the well-known Skyrme stabilizer is sufficient to describe the physics of the meson cloud down to distances of a quarter of a fermi. Although this cannot be entirely correct quantitatively, corrections from other higher order terms will certainly not upset the outcome of this investigation, which is: a torus-like meson cloud also appears around explicit bare baryon sources in leading order $N_C$ just as in the Skyrme model. The configuration will be stable with respect to modifications in the effective action, because the torus in the Skyrme model has been stable against such changes.

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FIGURES

FIG. 1. The chiral angles of the meson cloud around a baryon source with sharp cut-off (full line), with smooth cut-off (dashed line) and for a topological configuration (thin full line).

FIG. 2. Energies of various baryon number $B = 2$ configurations as a function of their separation. The full line displays the interaction of two topological solitons in the product ansatz, which asymptotically equals the one-pion exchange force in equation (17). The dot marks the position of the topological torus configuration, the distance of which is defined by the separation in the product ansatz, which after minimization deforms to the torus. The vertical bars show the energy including an error estimate for two smooth explicit baryon sources with relative isospin orientation $A^\dagger B = i\tau_2$.

FIG. 3. Energy density in the plane containing the origin and perpendicular to the $y$-axis for (a) two smooth explicit baryon sources with relative isospin orientation $A^\dagger B = i\tau_2$ at .7fm separation along the $z$-axis, (b) the topological torus configuration.

FIG. 4. Energy density in the plane containing the origin and perpendicular to the $x$-axis for (a) two smooth explicit baryon sources with relative isospin orientation $A^\dagger B = i\tau_2$ at .7fm separation along the $z$-axis, (b) the topological torus configuration.

FIG. 5. Energy density in the plane containing the origin and perpendicular to the $z$-axis for (a) two smooth explicit baryon sources with relative isospin orientation $A^\dagger B = i\tau_2$ at .7fm separation along the $z$-axis, (b) the topological torus configuration.
