Theory of quantum Hall effect and high Landau levels

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The angular momentum model which couples the spin and charge is discussed as a pos-
sible theory of the quantum Hall effect. The high Landau level filling fractions $5/2$, $7/3$
and $8/3$ are understood by this model. It is found that $7/3$ and $8/3$ are the particle-hole
conjugates and $5/2$ arises due to a limiting level at $1/2$ with Landau level number $n = 5$
which makes the fraction as $5/2$. 
Recently, Eisenstein et al\textsuperscript{1} have found that at large values of the Landau level quantum number, $n$, there is much less structure in the diagonal resistivity as a function of field than at low values of $n$. It was pointed out by Lilly et al\textsuperscript{2} that the rich structure of fractions of the fractionally quantized Hall effect, FQHE, found at small values of $n$ is virtually absent for large values of $n$; only fragile and poorly understood states at Landau filling fractions, $\nu = 7/3, 5/2$ and $8/3$ are seen in the best samples. In a recent letter\textsuperscript{3} we have shown that our earlier representation\textsuperscript{4} of the quantum Hall states works very well for the understanding of the particle-hole symmetry of the quantum Hall states. In particular we are able to understand the measured equivalence in the $g$ factor and effective masses of the quasiparticles. We have shown that $4/5$ and $6/5$ have equal masses because they are particle-hole conjugates. Yeh et al\textsuperscript{5} also find that the ratio of the effective mass to the square root of the perpendicular magnetic field is equal for some of the fractions in agreement with our calculations.

In the present letter, we interpret the fractions $7/3$, $5/2$ and $8/3$ seen in the measurement carried out by Eisenstein et al\textsuperscript{1}. We report that $7/3$ and $8/3$ are particle-hole conjugates and $5/2$ is the $n = 5$ states of the level at $1/2$.

Our earlier model\textsuperscript{4} has several features which are in agreement with experimental measurements. The fractions predicted by us are the same as those experimentally found by Eisenstein and Stormer\textsuperscript{6}. The fractions occur in two groups. One of the groups belongs to spin $+1/2$ and another to $-1/2$. Therefore, the groups represent time reversed states. The grouping of fractions in our theory is the same as experimentally found in the measurement of Eisenstein and Stormer\textsuperscript{6}. Our model also gives $\nu = 1/2$ for very large values of $l$ and hence there is a limiting value at $n/2$ where $n$ is the Landau level quantum number. In our model one of the series is,

$$\nu = \frac{l}{2l + 1}$$

which predicts one group of fractions, $0, 1/3, 2/5, 3/7, 4/9, 5/11$, etc. which are also observed by Willett et al\textsuperscript{7}. Another group of fractions is predicted\textsuperscript{4} by the expression,

$$\nu = \frac{(l + 1)}{(2l + 1)}$$

which are $1, 2/3, 3/5, 4/7$, etc. in complete agreement with the experimental measurement\textsuperscript{6,7}. When $l = \infty$ both the above series approach $1/2$ except that one series approaches from the right hand side and the other from the left hand side exactly.
as observed\textsuperscript{6}. The left and the right side approaches arise from the Kramers conjugate states and the predicted approach is exactly as observed\textsuperscript{6}. Since the limit is involved there is a Fermi surface at 1/2. However, we can shift the Fermi surface to higher values when higher Landau levels are occupied. The fraction 1/2 becomes \( n/2 \) with \( n \) as the Landau level quantum number. The 1/2, 3/2, 5/2, 7/2, 9/2, etc. become allowed. This predicted feature with an odd numerator with 2 in the denominator is also exactly as observed. Thus for \( n = 1 \), we have two series one merging from left while the other merging from right at 1/2 and the same picture is repeated for different values of \( n \). The entire pattern of pairwise series is observed exactly as predicted. Many fractions with 2 in the denominators have been observed by Lilly et al\textsuperscript{8}.

In the C.F. model\textsuperscript{9} the spin and charge are decoupled but one of the series is the same as our series. Another series in the C.F. model is,

\[
\nu_{CF} = \frac{m}{(2m-1)} .
\]  

(3)

We point out that the above series is the same as in eq.(2) which is given in the earlier dated theory of ref. [4]. We equate (2) and (3) so that,

\[
\frac{(l + 1)}{(2l + 1)} = \frac{m}{(2m-1)}
\]  

(4)

which can be simplified to yield \( m = l+1 \). Therefore, the series of (3) of C.F. is identical to that of ref. [4] except for the shift of the integer by one. However, in the theory of ref. [4] the spin and charge are coupled whereas in the C.F. model these are decoupled. In the C.F. theory 1/3 and 2/3 are treated at par. Their spin can have any value, up or down. On the other hand in the theory\textsuperscript{4} 1/3 has spin down and 2/3 has spin up. The polarization experiment\textsuperscript{10} demands \( \gamma_e \sim 1 \) for 1/3, 0.5 for 2/3 and 0.3 for 3/5. This means that 1/3 is in one group with spin down whereas 2/3 and 3/5 are in another group with spin up. This feature agrees better with our model of ref. [4] than with C.F. model. It also means that the experiment discriminates between 1/3 and 2/3 but the C.F. model does not. Our model of ref. [4] discriminates 1/3 and 2/3 by virtue of spin, being up in one case and down in another. The need for a term which has spin raising and lowering operators along with a function of orientation of the magnetic field is also clear. The effective charge in the theory\textsuperscript{4} is \( (1/2)g_\pm e = \nu e \) and the gap is \( \omega_c = \nu B/mc \) but there is spin-charge coupling as 1/3 has spin down and 2/3 has spin up. Thus charge and spin are coupled in the theory of ref. 4 as they should be but in the C.F.
model spin and charge are decoupled. According to one suggestion\textsuperscript{11} spin and charge are decoupled but in our theory fixing the value of the spin, such as 1/2, automatically fixes the charge. Spin-charge decoupling can be obtained if $l$ and $s$ belong to two different particles. According to a recent study\textsuperscript{12-14} spin and charge are coupled so that as the spin tilts, charge also moves and there is a phase transition in such a system, which forms spin textures along with a charge density wave. However, formation of charge-density waves by the symmetry breaking in the electron-phonon interaction does not depend on the spin. In the ordinary electron-phonon interaction the electrons are scattered without change in spin.

The even denominators are indeed difficult to obtain. However, it should be noted that the first one to be discovered was 1/3 and later on other denominators were seen. For a long time only odd denominators were reported which shows that even denominators are weak. For some time odd denominator rule was established but such a rule is not correct because even denominators occur as well as even numerators with odd denominators are found. Later on even denominators were reported. Therefore, it is observed that the fractional quantum Hall effect plateaus corresponding to even denominators are weak compared with those of odd denominators. There is a limiting value of the series $l/(2l+1)$ which for $l \gg 1$, gives 1/2 which has an even denominator. Because of the limit there is a Fermi surface at 1/2. However, there are other even denominators which are not linked with a Fermi surface at 1/2. Besides, when higher Landau levels are populated the Fermi surface will have to be shifted from 1/2 to $n/2$. Thus there are two types of even denominators, type-I are those which are limits of the series which give odd denominators and the type-II which are not the limiting values. When $\nu$ is a fraction then $n\nu$ is also allowed. Therefore, several even denominators can be generated from 1/2 which include 1/2, 3/2, 5/2, 7/2, etc. However, there are other types of even denominators which require two-quasiparticle states or bound states of two quantum Hall states. We make an effort to combine a series of the type $l/(2l+1)$ with another series $(m+1)/(2m+1)$ in which both are the angular momenta series of ref. 4, such that an even denominator results,

$$\left[\frac{l}{(2l+1)}\right] + \frac{(m+1)}{(2m+1)} = \frac{1}{(2n)}.$$

The solution of which is,

$$m = \frac{[1 - 2n - l(6n - 2)]}{[8nl - 4l + 2n - 2]}.$$

For $n = 2, m = -(10l+3)/(12l+2)$ which gives $l = 0, m = -3/2, n = 2$ and hence $1/2n = \ldots$
1/4, so that even denominators can be generated if \( m \) can become a negative fraction. For \( l = 1, m = -13/14, n = 2 \) and \( 1/2n = 1/4 \) is also giving an even denominator of 4 but \( m \) has become a negative fraction. Thus we can obtain even denominators for fractional angular momentum. In the Pauli spin matrices, if 1/2 is replaced by some other fraction, the commutators remain unchanged and it is acceptable to have values other than 1/2 without any sacrifice of quantum mechanical rules. The occurrence of even denominators may be anisotropic and hence may depend on the rotation of matrices or on the angle, which the z-axis makes with the direction of the external magnetic field.

The cyclotron frequency is \( \omega_c = (1/2)geB/mc = \nu eB/mc \). Therefore, we can replace the charge by an effective charge, \( e_{eff} = (1/2)ge = \nu e \). Alternatively, we can replace the field by an effective field, \( B_{eff}(1/2)geB = \nu B \) so that the field is very much reduced. In the case of \( \nu = 1/3 \), the field experienced is \((1/3)B\) which is reduced from \( B \).

The series (1) and (2) above can be used to explain the high Landau levels easily. Eisenstein et al\(^1\) have found that at higher values of the Landau level quantum number, \( n \), the number of fractions observed is much less than at the lowest Landau level. At the magnetic field of 4 to 5 Tesla only a small number of fractions are observed, the strongest ones are at 8/3, 5/2 and 7/3. Since there is a charge versus Landau level quantum number degeneracy, it is not possible to distinguish large charge from a large Landau level quantum number. In the angular momentum (1) series, \( l/(2l+1) \) is the particle hole conjugate of \((l+1)/(2l+1)\). For \( l = 7 \) two values, 7/15 and 8/15 are predicted and \( l = \infty \) value is 1/2. When the same particle occurs in different levels its charge remains unchanged. We can multiply the values by \( n = 5 \) so that the predicted values of 1/2, 7/15 and 8/15 become 5/2, 7/3 and 8/3. These predicted values are exactly the same as those observed experimentally by Eisenstein et al. Thus 7/3 is the particle-hole conjugate of 8/3 as seen below for \( n = 5 \):

| \( l \) | \( l/(2l+1) \) | \((l+1)/(2l+1)\) | \( nl/(2l+1) \) | \( n(l+1)/(2l+1) \) |
|-------|----------------|----------------|----------------|----------------|
| \( \infty \) | 1/2 | 1/2 | 5/2 | 5/2 |
| 7 | 7/15 | 8/15 | 7/3 | 8/3 |

Thus the angular momenta series (1) and (2) given by ref. 4 explain the quantum Hall effect correctly.

Conclusions.

In conclusion, we have shown that the correct series of the quantum Hall effect is
derivable from angular momentum. The particle-hole symmetry recently observed in quantum Hall effect is understandable from the angular momentum and even denominators are predicted correctly. We are also able to understand the high Landau levels$^{15–16}$. 
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