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COVID-19 pandemic, health risks, and economic consequences:
Evidence from China

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ABSTRACT

This paper introduces the Susceptible-Infectious-Recovered model into the Bewley-type incomplete market model and uses it to study the impact of the coronavirus disease (COVID-19) pandemic on China’s macroeconomics. The calibrated model predicts that the average propensity to consume household wealth will decline, while the demand for money will increase, and these predictions are consistent with the data. Monetary policy is effective because it provides enough liquidity for households to buffer health risks. Monetary stimulus is more effective in an economy with greater health risks and consumption uncertainty. Counterfactual experiments show that abandoning the containment policy too early would avoid a sharp drop in output and employment in the short term, but it would greatly increase mortality and ultimately lead to a decline in social welfare.

1. Introduction

Coronavirus disease 2019 (COVID-19) is an infectious disease caused by severe acute respiratory syndrome coronavirus. It was first identified in December 2019 in Wuhan, Hubei, China, and has resulted in an ongoing pandemic. As of August 30, 2020, more than 25 million cases were reported across 188 countries and territories, resulting in more than 843,000 deaths. The World Economic Outlook, updated by the International Monetary Fund in June, predicts that the global economy will shrink sharply by 4.9% in 2020, which is far more severe than the 2008–09 financial crisis.

Due to differences in population structure, residents’ health levels, medical systems, and epidemic prevention policies, the impact of the COVID-19 pandemic on each country is different. As a country with a large population and a developed economy, China has successfully resisted the COVID-19 epidemic and the economy is gradually recovering. Fig. 1 plots the number of newly confirmed cases in China as well as the share of cases in Hubei province from January 20 to July 29. It is clear that most of the newly confirmed cases were in Hubei province. The spread of COVID-19 was quickly contained within Hubei province and the number of newly confirmed cases in China as well as the share of cases in Hubei province from January 20 to July 29. It is clear that most of the newly confirmed cases were in Hubei province. The spread of COVID-19 was quickly contained within Hubei province and the number of newly confirmed cases in Hubei dropped to zero on March 19 for the first time since the outbreak of the pandemic.1

One of the most pressing issues at present is to predict the economic consequences of COVID-19 and design policies to reduce the loss of life and wealth caused by the pandemic. Many economic studies combine representative agent macroeconomic models and...
epidemiological models to provide policy recommendations to curb the spread of COVID-19 and reduce the damage to the economy and health. Atkeson (2020) uses the Susceptible-Infectious-Recovered (SIR) model developed by Kermack and McKendrick (1927) to predict the progression of COVID-19 in the United States over the next 12–18 months. Eichenbaum, Rebelo, and Trabandt (2020) extend the classic SIR model to study the equilibrium interaction between economic decisions and epidemic dynamics. One of the important features in their model is that economic activity affects the rates of infection. Therefore, the competitive equilibrium is not Pareto optimal because people who are infected with the virus do not fully internalize the effect of their consumption. This leaves room for the government to intervene. Krueger, Uhlig, and Xie (2020) distinguish goods by the degree to which they can be consumed at home rather than in a social (and thus possibly contagious) context. They argue that the severity of the economic crisis in Eichenbaum et al. (2020) is much smaller if individuals can endogenously adjust the sectors in which they consume or work.

A few quantitative studies emphasize heterogeneity across households. Glover, Heathcote, Krueger, and Rios-Rull (2020) study the heterogeneous impact of government mitigation policies on different age groups and sectors. They investigate the optimal mitigation and redistribution policy by a utilitarian equal-weights social planner. Kaplan, Moll, and Violante (2020) conduct a quantitative analysis of the trade-offs between economic and health outcomes associated with alternative policy responses to the COVID-19 pandemic, with a focus on distributional implications. Similar to Glover et al. (2020) and Kaplan et al. (2020), this paper also features heterogeneous risks faced by households. We focus on the heterogeneity of health and analyze the impact of this heterogeneity on social welfare and the effect of monetary policy.

There are huge differences in the level of health between people, and the level of health is closely related to households decision-making. Finkelstein, Luttmer, and Notowidigdo (2013) find that a one-standard-deviation increase in the number of chronic diseases is associated with a 10%–25% decline in the marginal utility of consumption relative to the marginal utility when the individual has no chronic diseases. The outbreak of COVID-19 has exposed the differences in health between people. According to the World Health Organization (WHO), most people (about 80%) can recover from COVID-19 without special treatment, and for most people (especially children and young people), the disease caused by COVID-19 is usually minor. However, for some people, it can cause serious illness. Therefore, when studying the impact of the COVID-19 pandemic on economic activities and households' behavior, it is important to take into account the heterogeneity of health status.

This paper introduces the off-the-shelf epidemiological SIR model from Kermack and McKendrick (1927) into an analytically tractable Bewley-type incomplete market model to quantify the impact of the COVID-19 pandemic on China's macroeconomics. The model is based on the framework of Wen (2009), Wen (2015), and Wen and Dong (2017), which is a stochastic general equilibrium model.

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2 Table 1 in Finkelstein et al. (2013) shows that on average, a person in the Health and Retirement Survey (HRS) data has 1.95 diseases; the within-person standard deviation in number of diseases is 0.625.

3 According to Li et al. (2020), the main clinical symptoms of COVID-19 patients are fever (88.5%), cough (68.6%), myalgia or fatigue (35.8%), expectoration (28.2%), and dyspnea (21.9%). Minor symptoms include headache or dizziness (12.1%), diarrhea (4.8%), and nausea and vomiting (3.9%). Laboratory results show that lymphocytopenia (64.5%), increase of C-reactive protein (44.3%), increase of lactic dehydrogenase (28.3%), and leukocytopenia (29.4%) are more common.
version of the model in Bewley (1983) and Lucas (1980). The key friction in the Bewley-Lucas model is the no-short-sale constraint on nominal balances, so agents cannot completely smooth idiosyncratic shocks by engaging in mutually lending and borrowing directly among themselves.

The main mechanism of the model is as follows. The impact of COVID-19 has worsened the health of infectious households and reduced their marginal utility. This has made them choose to reduce consumption and increase their demand for money, to withstand the potential increase in liquidity demand in the future (after recovery). As the only households in the model facing mortality risk, infectious households need to obtain the highest liquidity premium. However, it is not only the infectious households that increase their demand for money in the model. For the remaining households (households that are susceptible or recovered), the expected decline in the inflation rate (regardless of whether the government maintains a constant money supply growth rate or temporarily increases it) makes them also increase their demand for money. Finally, all households increase their demand for money to buffer health risks. When the nominal interest rate falls, the money demand of infectious households grows fastest, because their liquidity premium responds most to the nominal interest rate.

To quantify the economic impact of COVID-19 and the importance of health risks, we calibrate the model to match the development of the epidemic and economic losses in Hubei province, China. The benchmark model predicts that as more people are infected and nominal interest rates fall, households' demand for money will increase, especially for infectious households, and the ratio of consumption to wealth will decrease. Similar to Eichenbaum et al. (2020) and Krueger et al. (2020), in our model, COVID-19 affects the aggregate economy by simultaneously affecting supply and demand. However, unlike their model, our model also emphasizes the impact of the COVID-19 pandemic on capital investment and money demand.

We conduct some counterfactual experiments to study the role of monetary policy and containment policies in China. The complete abolition of the government's containment policy would prevent a sharp decline in production, but it would also increase the death toll by 22 times and cause substantial losses in social welfare.4 In another counterfactual experiment, we study the impact of expansionary monetary policy on economic activity and social welfare. Robustness analysis finds that monetary policy stimulus to economic activities and social welfare is proportional to the variance of the country's household health distribution.

The main contributions of this paper are as follows:

First, the paper analyzes for the first time how COVID-19 interacts with heterogeneous health risks and affects economic activity. We find that households' health risks have a quantitatively important impact on money demand. The paper analyzes an additional transmission mechanism of monetary policy: monetary policy can help households during the COVID-19 pandemic to meet their money demand and reduce the risk of future liquidity shortages. Therefore, monetary policy will be more effective in an economy where households face greater health risks and consumption uncertainty.

Second, the structural model allows us to quantify the impacts of the COVID-19 pandemic on household consumption, investment, money demand, and labor supply. So far, as far as we know, the existing macro literature does not pay much attention to capital investment and money demand (Kaplan et al. (2020) is an exception). This paper complements the literature in this field and can be used to analyze the short-term dynamics as well as the long-term impacts of the COVID-19 pandemic.

Third, after taking heterogeneity into account, the welfare effects of policies can be evaluated more accurately. We find that the welfare gains of monetary policy are positively correlated with the health risks faced by households.

Various other empirical papers study the spread of COVID-19 and its impact on the Chinese economy. Fang, Wang, and Yang (2020) quantify the causal impact of human mobility restrictions, finding that the lockdown was very effective, and provide estimates of diffusion under different scenarios. Qiu, Chen, and Shi (2020) study the impacts of social and economic factors on the transmission of COVID-19 in China and, through counterfactual analyses, quantify the effects of different public health measures in reducing the number of infections. Chen, He, Hsieh, and Song (2020a) and Chen, He, Hsieh, and Song (2020b) document that the impacts of the lockdown in China on various economic activities were immediate and dramatic. The official statistics suggest that there has been a quick recovery in manufacturing; however, small businesses were hit much harder. (Dai et al., 2020) document the impact of coronavirus on small and medium-size enterprises (SMEs), using data from the Enterprise Survey for Innovation and Entrepreneurship in China and follow-up interviews. Zhang and Wang (2020) propose how SMEs can resume production without compromising pandemic control in China.

The rest of the paper is organized as follows: Section 2 describes the spread of the pandemic in China and the government's policy response. Section 3 builds a structural model and uses it to analyze the impact of the COVID-19 pandemic on the macro economy. This section also presents the results of counterfactual experiments and robustness tests. Section 4 concludes.

2. The COVID-19 pandemic in China

In this section, we first summarize several key time points in the development of the COVID-19 pandemic in China. Then we describe the government's response to COVID-19, including fiscal and monetary policies. Finally, we describe the changes in some macroeconomic variables during the epidemic, including changes in the money market and the real economy.

4 As in Glover et al. (2020), the trade-off between health and wealth is difficult. We also construct a utilitarian social welfare function to evaluate the policy effects.
2.1. Early responses to the COVID-19 in China

On December 27, 2019, Hubei Provincial Hospital of Integrated Chinese and Western Medicine reported cases of pneumonia of unknown cause to the Wuhan Jianghan Center for Disease Prevention and Control. Preliminary laboratory test results showed that they were cases of a viral pneumonia. On January 3, 2020, The Wuhan City Health Commission issued an Information Circular on Viral Pneumonia of Unknown Cause, reporting a total of 44 cases. Two days later, China sent a situation update to the WHO. The WHO released its first briefing on cases of pneumonia of unknown cause in Wuhan.

On January 9, the National Health Commission's expert evaluation team made a preliminary judgment that a new coronavirus was the cause. Immediately after that, Wuhan began to test all relevant cases admitted to local hospitals to screen for the new coronavirus. Three days later, the China Center for Disease Control and Prevention submitted to the WHO the genome sequence of the novel coronavirus (2019-nCoV), which was shared globally.

On January 19, after careful examination, a high-level national team of senior medical and disease control experts determined that the new coronavirus was spreading between humans. The next day, the National Health Commission held a press conference announcing that the virus could transmit from human to human.

2.2. Containing the spread of COVID-19

The situation became most pressing with the rapid increase in newly confirmed cases in China. On January 22, the State Council Information Office held its first press conference on the novel coronavirus.

On January 23, Wuhan decided to close the city's outbound routes at its airports and railway stations at 10 a.m. The Ministry of Transport issued an emergency circular suspending passenger traffic into Wuhan from other parts of the country by road or waterway. On the same day, many other cities in Hubei province also closed outbound traffic. From January 23 to 29, all provinces and equivalent administrative units on the Chinese mainland activated a level-1 public health emergency response.

In response to the epidemic, the government mobilized additional medical personnel from all over the country to Hubei, initiated large-scale screening, extended the Spring Festival holiday, postponed the start of school, and established 16 mobile hospitals. On February 19, for the first time in Wuhan, newly cured and discharged cases outnumbered newly confirmed ones. On February 21, all provinces began to downgrade their public health emergency response levels and gradually lift traffic restrictions. On March 11, the daily increase in the number of domestic cases on the Chinese mainland dropped to single digits.

On March 25, Hubei lifted outbound traffic restrictions and removed all health checkpoints on highways across the province except in Wuhan. With the exception of Wuhan, work and life gradually returned to normal in the whole province, and people could now leave Hubei if they had a “green” health code to show that they were not infected. On April 8, Wuhan lifted its 76-day outbound traffic restrictions and local work and daily life began to return to normal.

2.3. Fiscal and monetary policy reaction

This subsection discusses some of the fiscal and monetary policies that the government and central bank put in place. Since the outbreak, the Chinese government has introduced many tax relief measures. These include cutting the value-added tax, consumption tax, and corporate and individual income taxes, as well as waiving employers' payments to various social insurance schemes.

The State Administration of Taxation and the Ministry of Human Resources and Social Security issued guidance allowing enterprises outside/within Hubei province to make catch-up employer social security contributions within a period of three/five months following the containment of the COVID-19 outbreak without adversely affecting employee rights to social security benefits. Under the pandemic, local governments faced great fiscal pressure due to the increased fiscal expenditures needed to fight against the pandemic and the decreased economic activities as a result of a suspension of work and production. To support local governments, the Ministry of Finance allowed them to retain 5% more tax revenue from March to June 2020, which was estimated to increase total local revenue by RMB 110 billion. The Ministry of Finance also issued 1 trillion yuan in special treasury bonds from mid-June to the end of July. All the funds raised were transferred to the local governments and required direct access to the city and county grassroots to support the local governments in fulfilling the “six guarantees” task and fighting the epidemic. The Ministry of Finance disclosed that the anti-pandemic special Treasury bonds were mainly invested in infrastructure construction and anti-pandemic-related

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5 A total of 346 medical teams composed of 42,600 medical workers and 965 public health workers from across the country and the armed forces were dispatched to Hubei and Wuhan. On January 26, the General Office of the State Council issued the decision to extend the Chinese New Year holiday of 2020 and postpone the opening of all universities, secondary schools, elementary schools, and kindergartens. On February 2, Wuhan conducted mass screenings to identify people with infections. Since February 5, Wuhan has set up and put into operation 16 mobile cabin hospitals and operated over 2,000 patients with new coronary pneumonia. On February 10, a mechanism was established to organize pairing assistance from other provinces to Hubei's cities other than Wuhan for treatment of the infected. Assistance from 19 provinces was provided to 16 cities in Hubei.

6 In addition, certain local authorities have introduced policies in the respective city/province to support local enterprises during the outbreak. These include deferring adjustments to the social security contribution base, adjusting the employer contribution rate for certain social security plans, extending payment of employer social security contributions, and relaxing restrictions on applying for refunds of unemployment insurance.

7 The “six guarantees” task aims to ensure security in jobs, basic living needs, operations of market entities, food and energy security, stable industrial and supply chains, and the normal functioning of primary-level governments.
expenditures.\footnote{Among them, infrastructure construction is specifically subdivided into 12 areas, namely public health system construction, epidemic prevention and control system construction, food security, energy security, emergency material support, industrial chain transformation and upgrading, urban old community transformation, ecological environment governance, transportation infrastructure construction, municipal infrastructure construction, regional planning infrastructure construction, and other infrastructure construction. Anti-pandemic-related expenditures are subdivided into six areas, namely rent reduction and exemption subsidies, corporate loan interest discounts, entrepreneurial guaranteed loan interest discounts, subsidies for supporting enterprises and stabilizing jobs, basic living allowances for people in need, and other anti-pandemic-related expenditures.}

To cushion the economic blow of the coronavirus outbreak, the People's Bank of China (PBoC) adopted several expansionary monetary policies, including lowering the reserve requirement rate, loan prime rate (LPR), and re-lending and rediscount rates. These policies effectively reduced the financing cost of business, especially for SMEs.

On February 3 and 4, the PBoC pumped in 1.7 trillion yuan through open market operations. On February 20, the PBoC lowered the one-year LPR by 10 basis points and the five-year LPR by 5 basis points. On March 13, the PBoC said the reserve requirement rate would be lowered by 50 to 100 basis points for all banks that meet certain criteria for lending to SMEs and private companies, under its inclusive finance program. Joint-stock banks that meet other criteria will receive an additional reduction of 100 basis points in their reserve requirement rate. On April 3, the PBoC announced that it would cut the targeted reserve requirement rate by 0.5 percentage point on April 15 and again on May 15, amounting to a full 1 percentage point cut. The PBoC also announced that it would lower the interest rate paid for excess reserves from 0.72% to 0.35%.

Fig. 2 exhibits some facts about the money market in China during the pandemic. The growth rate of the money supply, measured by M1 or M2, has grown at a much faster pace than last year. The nominal interest rate, measured by the interbank lending interest rate, decreased rapidly until April 2020 and then gradually came back. The core Consumer Price Index, which excludes food and energy prices, came down by 0.7 percentage point from January to March 2020 and then gradually increased.\footnote{The price of food fluctuated much more, mainly due to the steep climb in the prices of pork and other meats after African swine fever killed a large portion of China's pigs. The price of pork more than doubled year-on-year in October 2019. However, the core inflation rate, which excludes food and energy prices, has remained modest. The Chinese government has been implementing multi-pronged measures to boost the supply of meat, including increasing subsidies to restore hog production, releasing frozen pork reserves, and expanding pork imports. Economic planning authorities have promised land permits, loans, and subsidies to pig farmers to stoke production. The Transport Ministry doled out free toll passes for trucks carrying pigs. The global price of oil has also decreased during the pandemic. Therefore, we use the core Consumer Price Index instead to isolate the fluctuations due to other shocks.}

Fig. 3 plots the impact of COVID-19 on the real economy. Panel a shows that the largest fall in the annual growth rate of the cumulative gross domestic product (GDP) was in the first quarter of 2020. In the first quarter, the total GDP of Hubei province fell by nearly 40%, far exceeding the 6.8% decline in national GDP. Panel b shows that investment in fixed assets fell sharply, across the country and in Hubei province. The decline in fixed asset investment was more than twice the decline in GDP. Panel c plots the response of household consumption. Compared with the same period last year, household nominal consumption expenditure in Hubei province fell by 15% in the first half of the year, and the growth rate of national household consumption expenditure was about zero.

Finally, we summarize these stylized facts in the following points:

1. A substantial increase in the stock of money was accompanied by a decline in nominal interest rates and inflation rates.
2. Investment fell the most, followed by total output and household consumption.
3. The average propensity to consume household income has dropped sharply.
4. Demand deposits in the private sector rose significantly.
5. The number of newly created jobs in urban areas has fallen sharply, but the urban unemployment rate has increased slightly.

Later, we will build a model to explain facts 1–5, but our model will remain silent on the unemployment rate in fact 5.

3. Model

In the previous section, we introduced the spread of COVID-19 in China and the response of the country's macroeconomic variables. In this section, we construct a general equilibrium structural model to quantify the impact of COVID-19 on the economy and conduct some counterfactual experiments.

Fig. 3, panel d, plots the year-on-year change in the average propensity to consume household income. Whether in Hubei province or the whole country, the average propensity to consume dropped by about 6 percentage points in the first half of 2020.

Fig. 3, panel e, shows that the number of new urban jobs created in February fell by 40% year-on-year and then slowly recovered. But as of the end of June, the amount of new employment was far from reaching the level during the same period last year. In stark contrast to the sharp decline in new urban jobs, the urban unemployment rate rose by only 1 percentage point in February and then slowly declined. This situation is reminiscent of the characteristics of China's employment volatility, as described by Storesletten, Zhao, and Zilibotti (2019). Since the agriculture sector absorbed a large portion of the reduced urban employment, the overall unemployment rate is not high.

Fig. 3, panel f, plots the nominal growth rate of demand deposits in the private sector in 2019 and January-July 2020.\footnote{The demand deposits in the private sector are the domestic demand deposits of the household sector plus the domestic demand deposits of non-financial companies. Unfortunately, we do not have statistics on the stock of demand deposits in the private sector in Hubei province.} Except for January, the growth rate of demand deposits in 2020 is significantly higher than the level during the same period in 2019. Further, we will build a model to explain facts 1–5, but our model will remain silent on the unemployment rate in fact 5.
3.1. SIR model in discrete time

The SIR model developed by Kermack and McKendrick (1927) is widely used to describe the spread of COVID-19 (Atkeson (2020); Eichenbaum et al. (2020); Krueger et al. (2020); Glover et al. (2020)). We assume that the birth rate in the model is zero, and the
initial total population size is standardized to 1. The households in the model include four types: susceptible $S_t$, infectious $I_t$, recovered $R_t$ and deceased $D_t$. Given the initial distribution of the population $(S_0, I_0, R_0, D_0)$, the dynamic adjustment of population composition is given by the following nonlinear equations:

$$\Delta S_t = -\pi_t^s S_t (1 - I_t) I_t$$  \hspace{1cm} (1)

$$\Delta I_t = \pi_t^s S_t (1 - I_t) I_t - (\pi_t^r + \pi_t^d) I_t$$  \hspace{1cm} (2)

Fig. 3. Characteristics of the Real Economy during the COVID-19 Pandemic in China. Panel (a) plots the year-on-year growth rate of Hubei’s and China’s cumulative GDP. Panel (b) plots the year-on-year growth rate of cumulative investment in fixed assets. To avoid the impact of the Spring Festival, this statistical information is not reported for January of each year. Panel (c) plots the cumulative growth rate (at current prices) of household expenditure in Hubei province and China. Panel (d) plots the annual change in the average propensity to consume household disposable income. Panel (e) plots the year-on-year growth rate of cumulative private sector demand deposits. Panel (f) plots the year-on-year growth rate of new urban jobs created and the urban unemployment rate in urban areas. Data source: WIND.
\[ \Delta R_t = \pi^r_t I_t \]
\[ \Delta D_t = \pi^d_t I_t \]

\( \pi^r_t \) reflects the average number of susceptible people contacted by each infected individual during the infection process. \( S_t \) of these contacts is susceptible to infection. \( \Gamma_t \in [0, 1] \) is a proxy variable for the government’s containment policy: individuals must work in a company with probability \( 1 - \Gamma_t \) and they must work from home with probability \( \Gamma_t \).

We assume that the infection only occurs when people work in a company, so that the spread of the epidemic has nothing to do with the number of people working from home.\(^{11}\) Glover et al. (2020) consider a similar government containment policy (they call it a mitigation policy). Different from their assumption, we assume that people working from home still earn the same wage rate, while in Glover et al. (2020), people who are forced to stay home do not earn wage income and need to rely on government subsidies to survive.\(^{12}\)

Therefore, \( \pi^r_t \frac{N(1 - \Gamma_t) \times h(1 - \Gamma_t)}{1 - \Gamma_t} = \pi^r_t (1 - \Gamma_t) I_t \) will become the newly infected population in the next period. Let \( \pi^r_t \) and \( \pi^d_t \) denote the possibility of recovery and the possibility of death, respectively. The implicit assumption of the SIR model is that recovered individuals are immune to the COVID-19 virus.\(^{13}\)

### 3.2. Firm sector

The production function in the model has a standard Cobb-Douglas form:

\[ Y_t = K^\alpha_t ((1 - \Gamma_t)N_t + A\Gamma_t N_t)^{1-\alpha} \]

The total labor input \( N_t \) consists of two parts: workers working onsite (proportion \( 1 - \Gamma_t \)) and workers working from home (proportion \( \Gamma_t \)). Parameter \( A \) measures the productivity of workers who work from home relative to workers who work on site. When \( \alpha = 0 \), the production function becomes a special case of Krueger et al. (2020). Following Glover et al. (2020), the elasticity of labor substitution between different types of labor is assumed to be infinite.

The following first-order conditions are obtained by solving the profit maximization problem of the firm:

\[ (1 - \alpha) \frac{Y_t}{N_t} = W_t \]
\[ \frac{\alpha Y_t}{K_t} = r_t + \delta \]

where \( W_t \) represents the hourly wage rate, and \( r_t \) represents the capital rental rate.

### 3.3. Households sector

We extended the setting of Wen (2015) in modeling the household sector, so that we can easily aggregate heterogeneous households.

The surviving households in each period are divided into three different categories, denoted by \( j, j = S, I, R \). Households in category \( j \) randomly sample their health level \( \theta^j_t, i \) according to their health distribution (the cumulative distribution function is \( F^j(\theta^j_t, i) \)) in each period. The larger is \( \theta^j_t, i \), the better is health. Without loss of generality, we assume that the health distributions of the infected and recovered are the same, \( F^I = F^R \).\(^{14}\)

The health-level distribution function satisfies the following conditions

\[ E_i(\theta^j_t, i) < E_i(\theta^s_t, i) = E_i(\theta^r_t, i) \]

where

\[ E_i(\theta^j_t, i) = \int \theta^j_t dF^j(\theta^j_t, i) \]

**Condition 7** indicates that the average health level of infectious households is lower than that of the other two types of households.

Households’ utility is given by

\(^{11}\) See Krueger et al. (2020) for a similar assumption about infections-at-workplace. However, they do not consider the possibility of working from home.

\(^{12}\) Another explanation of \( \Gamma_t \) is the friction that causes employees not to return to work in time. According to the Survey of Chinese Innovation and Entrepreneurship Enterprises, Dai and Zhang (2020) find that as of February 10, 2020, the proportion of employees unable to return to work in heavy industry companies was the highest, close to 30%. In other industries, such as light industry, accommodation services, and business services, the proportion of people unable to return to work was above 20%.

\(^{13}\) Recently, Hong Kong has experienced the world’s first case of re-infection with the virus in a survivor of COVID-19, but so far, there have been few such cases. We do not consider this possibility here.

\(^{14}\) In other words, the patient can fully recover without sequela.
where health status $\theta_{j,t}$ affects the households' marginal utility. This assumption follows the study by Finkelstein et al. (2013), which finds that the marginal utility declines as health deteriorates. The disutility of working $\psi$ is inversely correlated with health status, $\psi(\theta_{j,t}) = \psi(\theta_{j,t})^{-\alpha}$, $\alpha > 0$. To derive the analytical solutions, we set $\alpha = 1$: 

$$u(c_{j,t}', n_{j,t}') = \mathcal{E}_{j,t}(\log c_{j,t}' - \psi(\theta_{j,t}')), j = S, I, R$$

On the one hand, good health reduces the cost of work. On the other hand, good health increases the marginal utility of consumption and leisure. When $\alpha = 1$, the two effects cancel each other out and the marginal utility of leisure becomes independent of the level of health.

The timing of economic activities in the model is as follows. At the beginning of period $t$, households will be sampled and randomly become one of the $j = S, I, R, D$. Households $j$ begins with $m_{j,t}'$ amount of real cash holdings of and $k_{j,t}'$ amount of capital stocks. For simplicity, we assume that the government will collect the wealth of deceased households to finance its own consumption $G_n$, which does not generate any utility flows to households.

We assume that households decide their labor supply $n_{j,t}$ and capital stock $k_{j,t}$ in the next period before they know their own health level $\theta_{j,t}$ and whether they will be infected with COVID-19 in period $t+1$. Such a hypothesis can greatly simplify the solution of the model, and it is a bit close to reality: many infectious households have no symptoms at first.

When making a decision, households regard the possibility of infection, the possibility of recovery, and the possibility of death as exogenously given. When the government implements a containment policy, households must work onsite with probability $\Gamma_t$ and work from home with probability $1 - \Gamma_t$. No matter where they work, their wage rate is $W_t$. After households draw their health levels $\theta_{j,t}$, they continue to make decisions on consumption $c_{j,t}'$ and cash holdings $m_{j,t} + \mu_{j,t}'$.

We can formulate the households' problem recursively. First, we define the households' real wealth as the sum of cash holdings $m_{j,t} + \mu_{j,t}'$, labor income $W_t n_{j,t}$, capital income $(r_t + \delta_t) k_{j,t}'$, wealth gains from the sale of capital $(1 - \delta_t) k_{j,t}' - k_{j,t+1}$, and transfers from the government $\tau_{j,t}$.\footnote{Following Wen (2015), we assume that the central bank returns all income from currency issuance to households.}

$$X_{j,t}' = \frac{m_{j,t}' + \mu_{j,t}'}{P_t} + W_t n_{j,t}' + (1 + \delta_t) k_{j,t}' - k_{j,t+1} + \tau_{j,t}, j = S, I, R$$

The problem of susceptible households can be written as

$$V^S = \max_{m_{j,t}', k_{j,t}'} \left\{ \mathcal{E}_{j,t}(\log m_{j,t}' - \psi(\theta_{j,t}')), \right. \left. \mathcal{E}_{j,t}(\log k_{j,t}' - \psi(\theta_{j,t}')), \mathcal{E}_{j,t}(\log c_{j,t}' - \psi(\theta_{j,t}')), \mathcal{E}_{j,t}(\log c_{j,t}') \right\}$$

subject to the following constraints:

$$(1 + \tau_{S,t}) c_{S,t}' + \frac{m_{S,t+1}'}{P_t} = X_{S,t}'$$

where $\bar{E}$ denotes the expectation without knowing the health level $\theta_{j,t}$. We assume that the probability of susceptible households being infected depends not only on the total probability $n_t'(1 - \Gamma_t)H_t$ but also on the individual's health level $\theta_{S,t}$, where $E\bar{\psi}(\theta_{S,t}) = 1$ and $\bar{\psi}(\theta_{S,t}) < 0$, that is, healthy households are less likely to be infected.\footnote{The exact functional form of $\bar{\psi}(\theta_{S,t})$ will not affect the results. For details, please refer to Appendix 5.1.} $1 + \tau_{c,t}$ is the time-varying consumption tax, which is collected by the government to cover the medical expenses of infectious households.\footnote{Or, we can assume that labor income taxes are used to collect income to pay for medical expenses. This will not affect our results.}

Similarly, we can write down the problem of infectious households as:

\begin{align*}
\end{align*}
where \( X_{t,i} \) represents the per capita medical expenses of infectious households. We treat the medical expenses of infectious households as part of government expenditures, which are completely funded by government transfers, i.e., \( X_{t,i} = u_{t,i}t \). Another interpretation is that the government maintains a medical insurance system that collects medical insurance premiums \( r_{c,t} \) from everyone and pays the infectious households. Susceptible households and recovered households have no medical expenses, so the transfer payment from the government is zero. The budget of the medical insurance fund is balanced period by period:

\[
\tau_{c,t} C_t = \int X_{t,i} dF_t^i
\]

We assume that the probability of recovery for the infectious households depends not only on the aggregate probability \( \pi_t \), but also on the individual health level \( \theta_{i,t} \), where \( E\phi(\theta_{i,t}) = 1 \) and \( \frac{d\phi(\theta_{i,t})}{d\theta_{i,t}} > 0 \), i.e., healthy households are more likely to recover.

In the end, the recovered households solve the optimization problem:

\[
v^R_t \left( \frac{m_{R,t}^i}{P_t}, k_{R,t}^i \right) = \max_{\theta_{R,t},k_{R,t}^i} \left\{ \left[ \frac{R_t}{P_t} \max_{\theta_{R,t},k_{R,t}^i} \left\{ \phi_{R,t}^i \log c_{R,t}^i + \beta E_t^R v^R_{t+1} \left( \frac{m_{R,t+1}^i}{P_{t+1}}, k_{R,t+1}^i \right) \right\} - \psi_{R,t}^i \right\} \right. 
\]

subject to the constraint

\[\left( 1 + \tau_{c,t} \right) C_{t} + \frac{m_{R,t+1}^i}{P_t} = X_{R,t}^i\]

### 3.4. Recursive competitive equilibrium

**Definition 1.** Given the exogenous monetary supply \( \{M_t, \mu_t\}_{t=1}^\infty \), government containment policy \( \{\Gamma_t\}_{t=1}^\infty \), medical insurance system \( \{r_{c,t}, u_{t,i}, \theta_{i,t}\}_{t=1}^\infty \), initial capital stocks \( K_t \), and initial composition of households \( S_0, I_0, R_0 , D_0 \), the recursive competitive equilibrium consists of the sequence of allocations \( \{G_t, X_{t,i}, 0, c_{t,i}^R, k_{t,i}^R, m_{t,i}^R, n_{t,i}^R, \theta_{t,i}\}_{t=1}^\infty \), prices \( \{P_t, r_t, W_t\}_{t=1}^\infty \), and threshold health levels \( \{\theta_t, \theta_t\}_{t=0}^\infty \), \( j = S, I, R \), such that.

1. Given prices \( \{P_t\}_{t=0}^\infty \), households decisions \( \{c_{t,i}^R, X_{t,i}, 0, m_{t,i}^R, k_{t,i}^R, \theta_{t,i}, n_{t,i}^R, \theta_{t,i}\}_{t=0}^\infty \), \( j = S, I, R \) solve households’ optimization problems 8, 10, and 13.
2. The central bank’s budget constraint holds:
   \[M_{t+1} = M_t + \mu_t\]
3. The government’s budget is balanced each period

\[3.4. \text{Recursive competitive equilibrium} \]

**Definition 1.** Given the exogenous monetary supply \( \{M_t, \mu_t\}_{t=1}^\infty \), government containment policy \( \{\Gamma_t\}_{t=1}^\infty \), medical insurance system \( \{r_{c,t}, u_{t,i}, \theta_{i,t}\}_{t=1}^\infty \), initial capital stocks \( K_t \), and initial composition of households \( S_0, I_0, R_0 , D_0 \), the recursive competitive equilibrium consists of the sequence of allocations \( \{G_t, X_{t,i}, 0, c_{t,i}^R, k_{t,i}^R, m_{t,i}^R, n_{t,i}^R, \theta_{t,i}\}_{t=1}^\infty \), prices \( \{P_t, r_t, W_t\}_{t=1}^\infty \), and threshold health levels \( \{\theta_t, \theta_t\}_{t=0}^\infty \), \( j = S, I, R \), such that.

1. Given prices \( \{P_t\}_{t=0}^\infty \), households decisions \( \{c_{t,i}^R, X_{t,i}, 0, m_{t,i}^R, k_{t,i}^R, \theta_{t,i}, n_{t,i}^R, \theta_{t,i}\}_{t=0}^\infty \), \( j = S, I, R \) solve households’ optimization problems 8, 10, and 13.
2. The central bank’s budget constraint holds:
   \[M_{t+1} = M_t + \mu_t\]
3. The government’s budget is balanced each period

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\[\text{In the data, household consumption includes out-of-pocket medical expenses. In the model, we abstract from other health-related expenditure and focus on medical expenditure due to COVID-19 only. We assume that families infected with COVID-19 do not have to pay for medical treatment, which is largely consistent with the empirical facts. Thus, household consumption in the model does not include medical expenditure on COVID-19 and expenditure on COVID-19 is regarded as government expenditure, which is balanced by transfer payments.}\]
\[ \pi^d I^t X^t = G^t \]

4. The medical insurance budget holds

\[ \tau^t C^t = \int x^t dF^t \]

5. All markets clear:

(a) Money supply equals money demand:

\[ \frac{M^t}{P^t} = S^t X^t_s H^t(\theta^t_s) + L^t (1 - \pi^d) X^t_i H^t(\theta^t_i) + R^t X^t_R H^t(\theta^t_R) \]

where

\[ H^t(\theta^t_j) \equiv \int \max \left\{ \frac{\theta^t_j - \theta^t_j}{\theta^t_j}, 0 \right\} dF^t(\theta^t_j), j = S, I, R \]

(b) The supply of capital equals the aggregate demand for capital:

\[ S^t \int k^t \tau^t_{t+1} dF^t(\theta^t_s) + I^t \int k^t \tau^t_{t+1} dF^t(\theta^t_i) + R^t \int k^t \tau^t_{t+1} dF^t(\theta^t_R) = K^t_{t+1} \]

(c) The supply of labor equals the aggregate demand for labor:

\[ S^t \int n^t \tau^t_{t+1} dF^t(\theta^t_s) + I^t \int n^t \tau^t_{t+1} dF^t(\theta^t_i) + R^t \int n^t \tau^t_{t+1} dF^t(\theta^t_R) = N^t \]

(d) The supply of goods equals the demand for goods:

\[ K^t + (1 - \delta) K^t = K^t_{t+1} + C^t + G^t \]

In the model, because money does not enter the utility function and is not required as a medium of exchange, there exists equilibrium where money is not valued, i.e., \( P_t = + \infty \). In the following analysis, we focus on the case \( 0 < P_t < + \infty \).

Following Wen (2015), we focus on the case when money is used as a buffer against health risks for households.

We have the following proposition characterizing the solution to the households’ problem.

**Proposition 2.** The decision rules for consumption, money demand, and real wealth are determined by

\[ c^t_{j,t} = \frac{1}{1 + \tau^t_{j,t}} \min \left\{ 1, \frac{\theta^t_j - \theta^t_j}{\theta^t_j}, X^t_{j,t} \right\} \]

\[ \frac{m^t_{j,t+1}}{P^t} = \max \left\{ \frac{\theta^t_j - \theta^t_j}{\theta^t_j}, 0 \right\} X^t_{j,t} \]

\[ X^t_{j,t} = \frac{1}{\psi^t_{j,t}} \psi^t(\theta^t_{j,t}) W^t \]  \hspace{1cm} (15)

where the cutoff \( \theta^t_{j,t} \), \( j = S, I, R \) is determined by the following equations:

\[ \frac{1}{W^t} = \beta E^t \left( \frac{1}{W^t_{t+1}} \right) \psi^t(\theta^t_{S,t}) \]  \hspace{1cm} (16)

\[ \frac{1}{W^t} = \beta (1 - \pi^d) E^t \left( \frac{1}{W^t_{t+1}} \right) \psi^t(\theta^t_{I,t}) \]  \hspace{1cm} (17)

\[ \frac{1}{W^t} = \beta E^t \left( \frac{1}{W^t_{t+1}} \right) \psi^t(\theta^t_{R,t}) \]  \hspace{1cm} (18)

where \( \psi^t(\theta^t_{j,t}) \equiv \int \max \left\{ 1, \frac{\theta^t_j}{\theta^t_j} \right\} dF^t \)  \hspace{1cm} (19)

**Proof.** See Appendix 5.1.

**Proposition 2** describes the solutions. Eqs. (16)–(17) are Euler’s equations about money. \( \psi^t(\theta^t_{j,t}) \) Eq. (19) measures the rate of return to money, which we refer to as the liquidity premium (Wen 2010a).
Take the Eq. (16) as an example. The left side represents the opportunity cost of holding one more unit of real wealth, and the right side measures the income from holding one more unit of real wealth. It consists of two parts: 
\[
\beta E_i \left( \frac{1}{W_{t+1}} \right)^{\theta_j} \mathbb{P}(\theta_j > \theta_j^*) + \beta E_i \left( \frac{1}{W_{t+1}} \right)^{\theta_j} \int_{\theta_j}^{\theta_j^*} \theta_j^d \mathbb{P}(\theta_j) d\theta_j. 
\]
The first term represents the marginal utility of consumption when the liquidity constraint is not binding, and its probability of occurrence is \( \mathbb{P}(\theta_j^* < \theta_j^*) \). The second term represents the marginal utility of consumption when the liquidity constraint is binding, and the probability is \( \mathbb{P}(\theta_j > \theta_j^*) \). Therefore, \( q^*(\theta_j^*) \) can be regarded as the rate of return to money or the liquidity premium. As long as the probability of being constrained is positive, \( q^*(\theta_j^*) \) is strictly greater than 1. The intuition is that the option value of a dollar is greater than 1, because it provides liquidity when there is an urgent need to increase consumption.

As in Wen (2015), optimal consumption and money demand are piecewise linear functions of wealth. The threshold level \( \theta_j^* \) depends on the real interest rate, inflation rate, and policy variables. Households whose health state is below the threshold level choose to hold a positive amount of cash, because when they suffer a health shock in the future, their liquidity constraint may tighten. We further use the first-order condition to obtain the following corollary:

**Corollary 3.** Given \( \pi_j^d > 0 \) and the condition \( E_i(\theta_j^*) < E_i(\theta_{j,R}^*) = E_i(\theta_{j,S}^*) \) holds, then
\[
0 = k_{j,t+1} < k_{j,t+1} = k_{R,J,t+1} \\
X_{j,t} < X_{j,t} = X_{R,J,t} \\
\theta_{j,t} < \theta_{j,t} = \theta_{R,J,t}.
\]

**Proof.** See Appendix 5.2.

**Corollary 3** states that the infectious households will choose zero amount of capital stock in the next period. They also choose a smaller target level of wealth and are more likely hit by the liquidity constraint than the other two groups are. To understand why the infectious households behave differently, we focus on Euler Eqs. (20)–(21). For \( j = S, R \)
\[
\frac{1}{W_j} = \beta E_i \left( \frac{1}{W_{t+1}} \right)^{\theta_j} \mathbb{P}(\theta_j^*) \tag{20}
\]
\[
\frac{1}{W_j} = \beta E_i \left( 1 + \pi_j^d \right) \frac{1}{W_{t+1}} \tag{21}
\]
Therefore, we have
\[
E_i(1 + \pi_j^d) \frac{1}{W_{t+1}} = E_i \left( \frac{1}{W_{t+1}} \right)^{\theta_j} \mathbb{P}(\theta_j^*), j = S, R
\]
which indicates that the expected return to holding money equals the expected return to investing one more unit of capital for the household. For the infectious households, this is because
\[
E_i(1 + \pi_j^d) \frac{1}{W_{t+1}} = \beta (1 - \pi_j^d) E_i \left( \frac{1}{W_{t+1}} \right)^{\theta_j} \mathbb{P}(\theta_j^*)
\]
Hence,
\[
q^*(\theta_j^*) = \frac{\mathbb{P}(\theta_j^*)}{1 - \pi_j^d}, j = S, R \tag{22}
\]

Compared with the other two types of households, money brings higher expected returns to infectious households Eq. (22). Infectious households do not want to invest in capital, but some of them want to hold money. To persuade some infectious households to hold money, the liquidity premium of money must make up for the loss due to the risk of death. However, susceptible households and recovered households do not have this risk of death, so the liquidity premium of their money holdings is low.

In equilibrium, the expected capital gains will equal the liquidity premium of money held by susceptible households or recovered households, but it will be less than the liquidity premium of infectious households. When the nominal interest rate decreases, compared with the other two types of households, the money demand of infectious households increases the fastest. Infectious households have the lowest health threshold and will be more likely to hit the liquidity constraint. Expansionary monetary policy will help ease households’ liquidity constraint, especially for infectious households.

### 3.5. Aggregation

Now we calculate the aggregate variables in the model economy. Aggregate consumption \( C_t \) is given by
\[
C_t = \frac{1}{1 + \tau_{c,t}} (S_j X_{j,t} D^j(\theta_j^*) + I_t (1 - \pi_j^d) X_{j,t} D^j(\theta_j^*) + R_i X_{R,J,t} D^R(\theta_{R,J,t}))
\]
where
The steady-state wage rate is 
\[
\frac{1}{\left(1 + \frac{\beta}{\delta}\right)}
\]
Aggregate money demand is given by
\[
\frac{M_{t+1}}{R} = S_tX_{S,t}H^S(\theta_{*,t}^j) + I_t(1 - \pi_t^d)X_{I,t}H^I(\theta_{*,t}^j) + R_tX_{R,t}H^R(\theta_{*,t}^j)
\]
where
\[
H^j(\theta_{*,t}^j) = \int \max \left\{ \frac{\theta_{*,t}^j - \theta_{*,t}^j}{\theta_{*,t}^j}, 0 \right\} dF^j, j = S, I, R
\]
Aggregate labor supply \(N_t\) is given by
\[
N_t = \frac{1}{W_t} \left[ S_t \left( X_{S,t} - \int \frac{m_{I,t}^s dF^S + \mu_t}{R_t} + k_{S,t+1} - (1 + \eta_t)k_{S,t} \right) \right.
+ I_t \left( X_{I,t} - \int \frac{m_{I,t}^d dF^I + \mu_t}{R_t} + k_{I,t+1} - (1 + \eta_t)k_{I,t} \right) \\
\left. + R_t \left( X_{R,t} - \int \frac{m_{R,t}^s dF^R + \mu_t}{R_t} + k_{R,t+1} - (1 + \eta_t)k_{R,t} \right) \right]
\]
where the threshold levels \(\{\theta_{*,t}^j\}_{t=0}^\infty\), \(j = S, I, R\) are determined by Eq. (16)–(17); the degenerate wealth distribution \(\{X_t, \theta_t\}_{t=0}^\infty\), \(j = S, I, R\) is defined in Eq. (19).

3.6. Steady states

**Pre-COVID Steady State** Before the outbreak of COVID-19, all households are susceptible. Suppose the constant money growth rate is given by \(\sigma > \beta - 1\), i.e., \(\mu_t = \sigma M_t\), then the steady-state interest rate will be \(\Delta P_t/P_t = \sigma\). We normalize the price level \(P^* = 1\). From the Euler equation, the steady-state interest rate equals \(r^* = \beta - 1\). The containment policy is not implemented and \(\Gamma_t = 0\). Because \(r_t + \delta = \alpha(K_t/N_t)^{\alpha-1}\), we can solve for the steady-state capital-labor ratio \(k^* = [(1/\beta - 1 + \delta)/\alpha]^{\alpha-1}\). The steady-state wage rate is \(W^* = (1 - \sigma)\beta\alpha/(1 - \beta + \delta\beta)\), and the capital-output ratio is given by \((K/Y)^* = \beta\alpha/(1 - \beta + \delta\beta)\). The consumption-output ratio is \((C/Y)^* = 1 - \delta\beta\alpha/(1 - \beta + \delta\beta)\). The liquidity premium of money is given by \(\rho^*(\theta_{*,t}^s) = (1 + \alpha)/\beta\) and we can solve for \(\theta_{*,t}^s\). Therefore,

\[
\begin{align*}
X^* &= \frac{\theta_{*,t}^s \rho^*(\theta_{*,t}^s) W^*}{\psi} \\
C^* &= D(\theta_{*,t}^s) X^* \\
Y^* &= C^*[1 - \delta\beta\alpha/(1 - \beta + \delta\beta)] \\
K^* &= \beta\alpha/(1 - \beta + \delta\beta) Y^* \\
N^* &= (1 - \sigma) Y^*/W^* \\
M^* &= H(\theta_{*,t}^s) X^*
\end{align*}
\]
although the growth rate of money does not affect the key ratios, such as capital/income \((K/Y)\) and consumption/income \((C/Y)\), it determines the levels of consumption, output, capital stock, and labor supply.

**Post-COVID Steady State** In the new steady state, we assume that all time-varying coefficients in the SIR model will converge: \(\lim \pi_t^s = \pi^s\), \(\lim \pi_t^d = \pi^d\) and the containment policy has been abandoned, \(\lim \Gamma_t = 0\). Because
\[
\frac{\Delta S_t}{S_t} = -\pi^s(1 - \Gamma_t)\pi_t^s \leq 0
\]
Therefore, if \(\lim S_t = S^* > 0\), then \(\lim \Delta S_t \to 0\), which implies that \(\lim \Gamma_t = 0\) and \(\lim D_t = 1 - S^*\). We assume that the steady-state growth rate of money after COVID-19 is still \(\sigma\). Therefore, the steady-state inflation rate is also \(\sigma\). The new steady-state interest rate will return to its pre-crisis level. The economy has the same \(K/Y\) and \(C/Y\) ratios as before COVID-19.
Due to death, the total population declines. Although per capita consumption and output return to their steady-state levels before the COVID-19 outbreak, total consumption and output are smaller.

3.7. Calibration

In this subsection, we use Chinese data to calibrate the parameters of the model. The results of the calibration are shown in Table 1. We calibrate the parameters of the SIR model based on the recorded numbers of infected people, those who died, and recovered people in Hubei province from January 21 to April 8, 2020. The start date of the sample corresponds to the date when Hubei started to lock down the city.

We normalize the number of households of each type \(j = S, I, R, D\) based on the total population of Hubei province (59.27 million). Using the SIR model Eqs. (1)–(4), we directly calculate the time series \((\pi_s^t(1 - \Gamma_t), \pi_t^s, \pi_t^d)\) based on the data. We assume
that the long-term steady-state values of these parameters equal the values on April 8, 2020.

Fig. 4, panel a, plots the actual data. By construction, the calibrated SIR model exactly matches the time path from January 21 to April 8. Fig. 4, panel b, plots the estimated parameters, where we need more information to separate the impact of the containment policy on the labor force $1 - \Gamma_t$ from the parameters $\pi_t^S$. We can see that $(1 - \Gamma_t)\pi_t^S$ falls dramatically from week 1. An important reason is that Hubei’s early nucleic acid testing capabilities could not keep up, resulting in the actual number of infections being higher than the official statistics. Over time, the nucleic acid testing capabilities greatly improved, thereby reducing the gap between the actual number of infections and the official figures.19 We do not intend to perform detailed modeling of the screening process for COVID-19 (such as nucleic acid detection), but take this probability $\pi_t^S$ as exogenously given.

To separate $1 - \Gamma_t$ from $\pi_t^S$, we use the Baidu within-city travel intensity index in Chen et al. (2020a) as a proxy for $1 - \Gamma_t$. Fig. 5, Panels a and b, plot the Baidu weighted within-city travel index for cities in Hubei province and cities outside Hubei Province, respectively. The first week here refers to the first week of the Chinese New Year.

Three days before the Chinese New Year in 2020, the lockdown of Hubei province began. Starting from the first week of the Chinese New Year, we can see a significant decline in the travel intensity index. There was hardly any decline in the travel intensity index for the same period in 2019.

The travel intensity index in week 1 relative to week 0 is measured by $\Gamma_t$. The estimated $\Gamma_t$ is plotted in Fig. 5, panel c. Since Hubei citizens could leave Hubei freely after April 8, 2020, we assume that $\Gamma_t$ linearly decreases to 0 from week 9 to week 12.

The calibration of the production function $Y_t = K_t^\alpha((1 - \Gamma_t)N_t + A_tN_t)^{1-\alpha}$ involves two steps. First, we calibrate the capital income share $\alpha$ to 0.53, to match the capital income share estimated Bai and Qian (2010).20 Second, we calibrate the productivity parameter $A = 0.06$, to match the 39% drop in output in Hubei province in the first quarter of 2020.

One period in the model is one week. We set the annual capital depreciation rate at 5%, as in Bai and Qian (2010); therefore, the weekly depreciation rate $\delta = 0.05 * \frac{7}{365}$. We set the annual discount factor to 0.87, to match the fixed-price capital-output ratio, which was 2.7 in 2012 (Cheremukhin, Golosov, Guriev, and Tsyvinski (2015)). Therefore, the weekly discount factor becomes $\beta = 0.87^{\frac{7}{365}}$. According to a report by the State Council Information Office in China, as of May 31, a total of 162.4 billion yuan had been allocated for epidemic prevention and control funds from all levels of finance across the country.21 Assuming that the actual utilization rate of the national epidemic prevention and control funds was 50%, then the expenditure on epidemic prevention and control funds allocated to each person infected with COVID-19 was 1.4 million RMB.22 This is equivalent to 54 times the per capita consumption expenditure of Chinese residents in 2019.

In the model, we assume that the per capita medical expenditure of infectious households is

| Parameters             | Value       | Target                        |
|------------------------|-------------|-------------------------------|
| $\alpha$               | Capital income share | 0.53 | Bai and Qian (2010) |
| $A$                    | Relative productivity | 0.06 | −39% drop in output |
| $\delta$               | Depreciation rate    | $0.05 * \frac{7}{365}$ | Bai and Qian (2010) |
| $\varphi$              | Disutility of labor | 1.0 | normalization |
| $\beta$                | Discount factor     | $0.87^{\frac{7}{365}}$ | Huber’s CDC data |
| $D_t$                  | Initial death      | $1.01e^{-10}$ | Huber’s CDC data |
| $I_0$                  | Initial infection  | $4.03e^{-6}$ | Huber’s CDC data |
| $R_0$                  | Initial recovery   | $4.32e^{-7}$ | Huber’s CDC data |
| $\Pi$                  | Health cost function | 530 | See text |
| $\kappa$               | Health cost function | 0.28 | See text |
| $\sigma$               | CDF of S/R households | 2.0 | See text |
| $\eta$                 | CDF of S/R households | 1 | Normalization |
| $\zeta$                | CDF of I households | 2.0 | See text |
| $\eta_2$               | CDF of I households | 0.68 | See text |
| $\varpi$               | Steady-state money growth | (1.025)^{\frac{7}{365}} - 1 | See text |

19 Fang et al. (2020) find that the number officially reported cases on January 23, 2020 was only 58% of their estimated infection cases. The gap between their estimates and the official data is gradually narrowing.

20 Table 1 in Bai and Qian (2010). We use the value from 2007.

21 Epidemic prevention and control funds are mainly used to subsidize patients’ medical treatment expenses, temporary work subsidies for frontline medical staff, prevention and control equipment, and material procurement.

22 The National Audit Office requires all parts of the country to conduct special audits of funds for the prevention and control of the new crown pneumonia epidemic and donations. Recently, many provinces have published the results of special audits on the epidemic. The audit results announced by Zhejiang province show that there are problems in the local area where funds for epidemic prevention and control are allocated and settled in time. In 20 cities and counties, the expenditure of special funds has been slow, and of the allocated 465 million yuan in funds, only 174 million yuan have been used. Thus, 37% of the epidemic prevention and control funds were used. The audit results announced by Hainan province show that special fund expenditures in 10 cities and counties are less than 50%.
and \( \bar{h} > 0, 0 < \kappa < 1, \kappa < 1 \) indicate that medical resources are public goods, and their production and provision have economies of scale.\(^{23}\) We calibrated the two parameters and \( h \) at the same time, so that: 1. household consumption in the model has an average decline of \(-15\%\) in the first two quarters; and 2. per capita expenditure of people infected with COVID-19 is equivalent to 54 times the national per capita consumption level. The results of the calibration are \( \bar{h} = 530 \) and \( \kappa = 0.28 \).

We assume that health risks follow the Pareto distribution. The cumulative distribution functions of the health risks of susceptible and recovered households are given by

\[ \chi_t = \bar{h}(I_t)^{\kappa - 1} \]

\(^{23}\)For example, the medical resources can be used to build a temporary shelter hospital or purchase computerized tomography equipment.
According to the definition of $\varrho_j(\theta_j^*, t^*)$, $H_j(\theta_j^*, t^*)$, and $D_j(\theta_j^*, t^*)$, $j = S, I, R$ Eq. (19), (24), and (23)), we have the following analytical solution:

$$F^j(\theta_j, \gamma) = 1 - \left(\frac{\theta_j}{\gamma}\right)$$, $j = S, I, R$

According to the definition of $q_j(\theta_j^*, \gamma^*)$, $H_j(\theta_j^*, \gamma^*)$, and $D_j(\theta_j^*, \gamma^*)$, $j = S, I, R$ Eq. (19), (24), and (23)), we have the following analytical solution:
Table 2
Regression Results on Non-health Consumption.

| Variables                  | (1)          | (2)          | (3)          |
|---------------------------|--------------|--------------|--------------|
| Log non-health consumption|              |              |              |
| Bad health                | −0.156*** (0.0267) | −0.119*** (0.0246) | −0.0718*** (0.0226) |
| Log total asset           | 0.196*** (0.00492) | 0.147*** (0.00528) | 0.148*** (0.00484) |
| Age, age²                 | YES          | YES          | YES          |
| Year fixed effect         | YES          | YES          | YES          |
| Households fixed effect   | NO           | YES          | YES          |
| Other controls            | NO           | NO           | YES          |
| Observations              | 11,806       | 11,806       | 11,604       |
| R-squared                 | 0.29         | 0.30         | 0.49         |
| Number of households      | 5423         | 5423         | 5372         |
| R-squared                 | 0.29         | 0.30         | 0.49         |
| Number of households      | 5423         | 5423         | 5372         |

Standard errors are in parentheses.
***p < 0.01, **p < 0.05, *p < 0.1

To simplify the problem, we assume that the health distribution functions of recovered and susceptible households are exactly the same: \( \sigma^R = \sigma^S = \sigma, \eta^R = \eta^S = \eta \). Then we normalize \( \eta \) to 1 and calibrate the rest of the parameters \( \sigma, \sigma', \eta' \) using empirical data.

We use the panel data from a nationally representative, biannual longitudinal household-level survey, the China Households Panel Studies 2010–2018 (CFPS). We first calculate household per capita non-health consumption (household consumption after deducting health expenses) using data from the CFPS. The CFPS data records the self-reported health levels of the households, with values ranging from 1 to 5 (larger values indicate lower levels of health). We divide families into a good health group (health level between 1 and 4) and a poor health group (health level 5). We keep households whose heads are over age 16 years and discard observations that lack household registration information or health status. In the end, we have a sample of 11,604 observations (5372 households), of which the heads of 860 households reported poor health status.

We perform panel regression on the logarithmic non-health consumption of these two groups of households. The regression model is specified as:

\[
\log c_i^t = \alpha' + \beta_H I_{\text{bad health} = 1} + \beta_\eta \log X_i^t + Z_i^t + \epsilon_i^t
\]

(25)

where \( c_i^t \) denotes per capita non-health consumption. \( I_{\text{bad health} = 1} \) denotes the indicator for poor health. \( X_i^t \) represents the households’ total net worth. \( Z_i^t \) is a vector of controls. \( \alpha' \) denotes the households fixed effects.

The regression results are shown in Table 2. It can be seen from the table that bad health shocks have caused a decline in non-health-related consumer spending. Compared with the results of simple ordinary least squares regression (column 1), after controlling for household fixed effects (column 2), the impact of health on consumption becomes smaller. In the last column, we control a large number of household characteristics (occupation, household registration status, years of education, household size, work status, and so forth) and finally obtain that the impact of poor health on non-health consumption is about −7.2%.

Next, we obtain the error term from panel regression, and calculate the variance of the error term \( \epsilon_i^t \) grouped by health status. The variance of the good health group is 0.23, and the variance of the poor health group is 0.20. Although the difference is not large, the assumption that the two variances are the same can be rejected at the 95% confidence level. This level is comparable to results for the U.S. consumption variance data (Blundell, Pistaferri, and Preston (2008)).

Therefore, we chose \( \sigma = 2.0 \) to match the variance of the logarithmic consumption of susceptible households in steady state to 0.23. We chose \( \sigma' = 2.0 \) so that the consumption variance of the infectious households in the model in steady state (facing 2% mortality rate) is 0.20. Finally, we chose \( \eta' = 0.68 \) so that the average log consumption level of infectious households in steady state is 0.072 lower than that of susceptible households. Given these calibrated parameters, \( \frac{\eta'(0)}{\eta'(\infty)} = 0.68 \), which means that the health level of the infectious households is on average 68% of that of the susceptible and recovered.

We assume that the central bank set the weekly growth rate of money to be \( \varpi = 1.025 \sqrt{1 - \pi} - 1 \) in the steady state. Hence, the steady-state inflation rate is \( \pi \).

After the outbreak of the COVID-19 pandemic, the country adopted a series of relief measures, as described in Section 2.3. In the

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24 Non-health consumption includes expenditure on food, clothing, household equipment and daily necessities, housing, transportation and communication, culture, education, entertainment, other consumer items.
benchmark model, we used the actual year-on-year growth rate of China’s M2 as the growth rate of the money supply in the model. We assume that the currency growth rate remains unchanged after reaching 11.6%. We assume that in the first period of the pandemic, residents fully expected the government’s future currency growth rate.

We use the reverse-shooting algorithm to solve the entire transitional dynamics of the model. See Section 5.3 for details.

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We linearly interpolate the monthly year-over-year (y-o-y) M2 growth rate to get the weekly y-o-y M2 growth rate. The results will be similar if we match the y-o-y growth rate of aggregate financing.

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Fig. 6. Simulation Results of Baseline Model (I). D = deceased; I = infectious; R = recovered; SS = steady state.
3.8. Simulation

Figs. 6–7 exhibit the numerical simulation results. Fig. 6, panel a, plots the model’s predicted path of the composition of the population. In the limit, the deceased account for $5.44 \times 10^{-3}$% of the population in Hubei. Fig. 6, panel b, plots the year-on-year growth rate of money supply in the model. As a result of monetary stimulus, the total amount of money increases by about 11.6%. By construction, the baseline model fits the COVID-19 dynamics and nominal money stocks perfectly.

Fig. 6, panel c, plots the responses of the inflation rate and nominal interest rate. The money growth rate is about 2.2% in week 2, but the inflation rate is only about 0.3%, which is far less than the money growth rate. This is mainly due to two reasons. First, because households fully anticipated the increase in the government’s currency, prices had already risen before the increase in the money supply. The inflation rate in the first week was as high as 8.3% (not shown in the figure), which was still less than the increase in the total money stock. Second, although there is no price stickiness, money is not neutral in the model. After the first period, the expected inflation rate continues to fall, and the nominal interest rate also falls. This will increase household demand for money, ease liquidity constraints, and stimulate consumption. Recall stylized fact 1, summarized in Section 2.3; our model successfully predicts the declines in the inflation rate and nominal interest rate. However, the model over-predicts the decline in the inflation rate in the first quarter, which may be related to the lack of price stickiness in the model. The monthly nominal interest rate in the first quarter falls by approximately 0.20%. But in the data, the nominal interest rate dropped rapidly, by 1%, and then slowly recovered.

Fig. 6, panel d, shows that after the outbreak of the COVID-19 pandemic, real interest rates first fell and then rose. The marginal products of labor and capital have been reduced due to the government’s containment policy. When the containment policy gradually faded out, the real interest rate gradually returned to the previous steady-state level.

Fig. 7, panel a, shows that the cumulative money demand per capita of the three types of households has increased, and the money demand of infectious households has increased the fastest. The reason is that the expected decline in inflation will lead to a decline in the liquidity premium, which requires more households to hold money to achieve this. The money demand of infectious households is growing faster for two reasons. First, only infectious households face the risk of death, so their money holdings have the highest liquidity premium and their target wealth level is the lowest (see Corollary 5). These factors together lead to a small amount of money demand from infectious households at the beginning. Second, when the nominal interest rate drops, the liquidity premium of infectious households drops even more, which causes the threshold value of holding money $\theta_I^*$ to rise faster. As a result, the money demand of infectious households grows faster.26

Fig. 7, panel b, plots the velocity of money, which is defined as $V_t = \frac{Y_t}{M_{t+1}}$. The decline in output and the increase in money demand together cause a decline in the velocity of money circulation.

We compare the average propensity to consume in the model with the actual data (stylized fact 3, summarized in Section 2.3). In the data, the consumption-to-disposable income ratio decreases by about 6% after the COVID-19 shock. In our closed economy model, $S = I$, and the investment-to-output ratio falls both in the data and in the model. Since households own all the capital, the aggregate household saving rate $\frac{S}{Y}$ in our model must decrease. Therefore, unless there is a clearer distinction between corporate savings and household savings, our model cannot explain the declining consumption-income ratio. Instead, we focus on the consumption-to-wealth ratio. The average consumption-to-wealth ratio in the steady-state model is 11%, and in the CFPS data it is 16%. The two are relatively close, which is not our direct calibration target. Fig. 7, panel c, shows that the average consumption-to-wealth ratio has dropped by 3 percentage points, which accounts for 50% of the actual decline in the data. The decline in consumption is mainly due to the distortion of consumption caused by the increase in medical expenditures. Another reason is that infectious households have increased money demand and reduced consumption. The model’s prediction of the decline in average propensity to consume is only half of the actual decline. This may be due to other reasons not captured by the model. For example, in the data, households’ risks also come from other aspects, such as idiosyncratic income risk. Persistent downward income risks can lead to a further decline in consumption.

Fig. 7, panel d, plots the growth rate of per capita output, consumption, investment, and employment relative to the steady state. The government’s containment policies reduced the marginal output of labor and capital, resulting in a sharp decline in employment and investment in the economy. The expansionary monetary policy temporarily increased the growth rate of money, causing the expected inflation rate to fall, and to a certain extent reduced the negative impact of the containment policies on employment and investment. Our model successfully predicts the responses of output and consumption in the first quarter, which are our calibration targets. Quantitatively, the model over-predicts the decline in investment (the decline in investment in the first quarter in the model was $-140\%$, while in the data it was $-80\%$), which may be related to the absence of investment adjustment costs.27

3.9. Policy experiment

In this section, we investigate the results of four policy experiments and compare them with the benchmark model. In experiment A, we keep the growth rate of money at the steady-state level ($\mu_t = \bar{\mu}$), but we do not change the government’s containment policy. In experiment B, we temporarily increase the growth rate of money, so that the stock of money increases by 40%. More specifically, we

26 In the model, households’ total demand for money equals the total supply of money, which increases by 11.6%. In the data, demand deposits in the private sector have increased by 8%, which is closer to the growth rate of M1.

27 The decline in employment in the model is basically the same as that in output, which was $-35\%$ in the first quarter. In the data, the decline in newly created urban employment was $-30\%$. But the two are not directly comparable.
assume the following exogenous money growth process:

\[ \mu_{t+1} - \pi^* = \rho (\mu_t - \pi^*) , \quad t \geq 2. \]

where we set \( \mu_1 = 0.088 \) and \( \rho = 0.75 \) such that the stock of money will increase by 40% permanently. Experiments C and D are equivalent to abolishing the government's containment policy under the conditions of experiments A and B, respectively. The results of the numerical simulations are shown in Table 3.

Since the government's containment policy has not changed in both cases, the death rate in experiment A has not changed. Compared with the benchmark model, experiment A shows that the 11.6% increase in the stock of money does not have a strong stimulus effect on output, employment, and consumption. The 11.6% increase in the stock of money brought a 0.03 percentage point

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**Fig. 7.** Simulation Results of Baseline Model (II) In panel a, I = infectious households; R = recovered households; S = susceptible households. In panel d, C = consumption; I = investment; N = employment; Y = output. APC = average propensity to consume; SS = steady state.
increase in output, 0.3 percentage point increase in fixed investment, and 1.03 percentage points increase in real household wealth \(X_t\). Because

\[ X_{t+1} = \frac{m_{t+1}^j + \mu_t}{\beta} + W_t n_{t+1}^j + (1 + r_t)k_{t+1}^j - k_{t+1} + \tau_{t+1,j} \]

we can decompose the growth of total household wealth into four parts: the growth of monetary assets \(\frac{m_{t+1}^j + \mu_t}{\beta}\), the growth of labor income \(W_t n_{t+1}^j\), the growth of capital gains \((1 + r_t)k_{t+1} - k_{t+1}\), and the growth of transfer income \(\tau_{t+1,j}\). We find that the main increase in household wealth comes from the growth of monetary assets, followed by government transfers. Household labor income and capital gains have suffered losses. This is consistent with stylized fact 4 summarized in Section 2.3.

For further analysis of the welfare effects of the policy, we follow the approach of Wen, 2015 and Glover et al., 2020, and constructed a utilitarian social welfare function, in which all households have equal weights.

Let the social welfare function under the monetary policy \(\{\mu_t, \pi_t\}\) and the containment policy \(\{\Gamma_t\}\) be \(W(\{\mu_t, \pi_t\}, \Gamma_t)\)

\[
\begin{align*}
W & = \frac{\pi_t}{\beta} \mathbb{E} \left[ \int_{\tilde{S}(\delta, \Gamma)} \int_{\tilde{S}(\delta, \Gamma)} \log(c_{j+1}) c_{j+1} d\tilde{F}_{j+1} \right] \\
& = \frac{\pi_t}{\beta} \mathbb{E} \left[ S E_t \left( \frac{A}{\rho} \log(c_{j+1}) \right) + I_t E_t \left( \frac{A}{\rho} \log(c_{j+1}) \right) + R_t \left( \frac{A}{\rho} \log(c_{j+1}) \right) - \psi N_t \right] \\
& \text{(26)}
\end{align*}
\]

We calculate the level of social welfare on the transition path from the initial steady state to the new steady state. For comparison, we use consumption variation (CV) to measure the welfare loss of various models relative to the steady state before the COVID-19 outbreak.

Since all households in the steady state before the epidemic are susceptible, the social welfare function can be expressed as

\[
W^* = \frac{\pi_t}{\beta} E_t \left( \frac{A}{\rho} \log(c_{j+1}^*) \right) - \psi N^* \\
\text{(27)}
\]

where \(c_{j+1}^*\) represents the allocation of consumption in the steady state.

We denote the consumption variation by \(\Delta\), which can be calculated as the solution to the following equation:
After some algebraic derivation, we can finally solve $\Delta$, which is given by the following expression

$$\Delta = \exp \left( W(\mu, \kappa, \{I\}) - W^* \right) - 1 \tag{28}$$

We calculate social welfare according to Eq. (28), and the result is shown in the last column of Table 3. The welfare loss in the benchmark model caused by the COVID-19 pandemic is about $-0.74$ percentage point. Policy experiment A shows that if the monetary policy in the benchmark model were not implemented, social welfare would have been reduced by an additional $0.06$ percentage point.

In policy experiment B, we temporarily increase the growth rate of money, and the stock of money increases by 40%. Not surprisingly, compared with the results of policy experiment A, a substantial increase in the stock of money is a greater stimulus for consumption, investment, output, employment, and household wealth. Although the model shows that a temporary increase in the growth rate of money helps to increase social welfare, a permanent increase in the growth rate of money will bring about a permanent increase in the inflation rate, which will reduce the level of social welfare (Wen (2015)).

In policy experiment C, the government abolishes the containment policy, but the increase in infectious households leads to huge social costs. The number of infected people increases by 35 times, and the death rate increases by 22 times. Although we assume that there are economies of scale in medical treatment, due to rising medical expenses, distortions in household consumption are further aggravated and household consumption is further reduced. Due to the slow change in capital stock, the sharp drop in consumption in the short term causes households to increase investment temporarily. When the number of infections drops, investment will drop again.

After weighing economic output and mortality, the social welfare loss of experiment C has greatly increased. These calculations do not take into account the possibility of aggravating the spread of COVID-19 due to the abolition of the containment policy (Eichenbaum et al. (2020), Krueger et al. (2020)), nor the additional value of survival (Glover et al. (2020)). Therefore, our calculations can be interpreted as a conservative estimate of welfare loss.

### 3.10. The effectiveness of monetary policy

A key variable in the benchmark model is $\sigma$, whose size determines the health risks that households face. Following Wen (2009) and Wen (2015), we tried different values of $\sigma$ as a robustness test. The results are presented in Table 3. We choose $\sigma = 1.5$ from Wen (2009) and $\sigma = 2.5$ from Wen (2015) respectively as our robustness test. These two values also correspond to the two endpoints of the interval in the literature (Clementi and Gallegati (2005); Levy and Levy (2003); Nirei and Souma (2007)). To simplify the discussion, we keep the sigma of all households equal, $\sigma = \sigma^I$. The results are shown in Table 3.

When $\sigma = \sigma^I = 1.5$, the social welfare of policy experiments B and D improves relative to the level pre-COVID-19, and the effectiveness of monetary policy increases. This is because when sigma is small, a small drop in the interest rate will trigger a strong increase in household money demand. If the government can increase the money supply accordingly, it will enable those households...
to ease their liquidity constraints.

To illustrate this point, we plot the demand for money as a function of the nominal interest rate in Fig. 8. Lucas (2000) interprets the downward-sloping curve as the “money demand” curve, which is the inverted velocity in our model. Fig. 8 shows that when the $\sigma$ rises, the absolute value of the slope becomes smaller, which means that the elasticity of money demand with respect to interest rates becomes smaller. This suggests that monetary policy will be more effective if households face greater health risks (given the same decline in the nominal interest rate, output increases more relative to the real money balance when $\sigma$ is smaller). If we further use ordinary least squares to estimate the elasticity of money demand, we get elasticities of $-9$ and $-12$, respectively (corresponding to $\sigma = 2.5$ and $\sigma = 1.5$, respectively). If money demand is converted into annual data, the elasticities are estimated to be 2 and 3, respectively.

We could estimate the elasticity of China’s money demand to interest rates and use it to recalibrate the $\sigma$. The regression model is specified as:

$$\log \frac{M_t}{P_t Y_t} = \alpha + \beta_0 \log(1 + i_t) + Z_t + \epsilon_t$$

The dependent variable $\frac{M_t}{P_t Y_t}$ is the currency-output ratio. The money supply $M_t$ is measured by the stock of M2, and nominal output $P_t Y_t$ is quarterly GDP. Here we can overwhelmingly reject the null hypothesis of a unit root at all common significance levels. The interest rate $i_t$ comes from the weighted interbank lending interest rate (three months). The sample interval is selected from 2011 to the present, to exclude the impact of the economic crisis in 2008–09. The estimation results, in Table 4, show that the elasticity $\beta_0$ using China’s quarterly data since 2011 is between $-7.4$ and $-5.0$. If we use this moment condition alone to estimate $\sigma$, ignore its impact on the variance of household consumption, then the smallest value we can get for $\sigma$ is 3.7. This value is a large outlier in the literature. The fact that monetary demand is less elastic with respect to interest rates may be related to the insufficient progress of China’s interest rate marketization.

### Table 4

The Elasticity of Money Demand to Interest Rates.

| Variables | (1) | (2) | (3) |
|-----------|-----|-----|-----|
| $\log \frac{M_t}{P_t Y_t}$ | $-5.893^{***} (1.818)$ | $-7.379^{**} (2.743)$ | $-4.985^{**} (2.430)$ |
| $\log(1 + i_t)$ | | | |
| L.$\log \frac{M_t}{P_t Y_t}$ | $0.123 (0.172)$ | $0.196 (0.170)$ | |
| L.$\log(1 + i_t)$ | $2.894 (2.891)$ | $2.529 (2.472)$ | |
| Time, time$^2$ | NO | NO | YES |
| Observations | 38 | 37 | 37 |
| R-squared | 0.226 | 0.259 | 0.492 |

Standard errors are in parentheses.

$***p < 0.01$, $**p < 0.05$, *p < 0.1

4. Conclusion

This paper introduced the existing epidemiological SIR model into an incomplete market model and used it to study the impact of the COVID-19 pandemic on China’s macroeconomics. We calibrated the model to match the decrease in actual output and the variance in household consumption. The benchmark model predicts that as more and more people are infected, the average propensity to consume household wealth will drop significantly, while the total demand for money will increase. The COVID-19 pandemic has led to a decline in the health of infectious families, and families have increased their cash reserves to alleviate these health risks. Monetary policy will be more effective in the economy with greater household consumption uncertainty. Expansionary monetary policy can mitigate the negative impact of the pandemic on the economy to a certain extent. Abandoning the containment policy too early will avoid a sharp drop in output and employment in the short term, but it will also increase the mortality rate by 22 times and ultimately lead to a decline in social welfare. Robustness analysis found that the effectiveness of monetary policy is affected by household health risks. If households generally face greater health risks, the central bank will increase the provision of liquidity, which will greatly improve social welfare.

5. Appendix

5.1. Proof of Proposition 2

Write down the Bellman equation as
The FOCs are

\[
\max_{k_{j+1}} \log c_{j,t} + \beta (1 - \eta) \frac{q(t)}{1 - \eta}\left(1 - \frac{c_{j,t}^{1}}{L_{t+1}^{1}} k_{j+1}^{1}\right) \\
+ \beta \eta q^{k}(\frac{1 - \eta}{1 - \eta}) \left(1 - \frac{c_{j,t}^{1}}{L_{t+1}^{1}} k_{j+1}^{1}\right) \\
+ \psi c_{j,t} \left(\frac{m_{j,t}^{1} + (1 + \tau)_{j,t}^{1} + \eta_{j,t}^{1}}{R_{j,t}^{1}} - \frac{m_{j,t+1}^{1}}{R_{j,t+1}^{1}} - \kappa_{j,t+1}^{1}\right) \\
+ \gamma_{j,t}^{1} m_{j,t+1}^{1} \\
+ \gamma_{j,t}^{1} k_{j,t+1}^{1}
\]

\[
\max_{k_{j+1}^{1}} (m_{j,t}^{1} + (1 + \tau)_{j,t}^{1} + \eta_{j,t}^{1}) R_{j,t+1}^{1} k_{j,t+1}^{1}
\]
where \( \hat{m} \equiv \frac{m}{T} \) and the envelope conditions yield

\[
\frac{\partial V^S}{\partial k_t} = \frac{1 + \eta_t E_t}{1 + \tau_t} \left( \frac{\partial S_t}{\partial c_{S,t}} \right)
\]

Similarly, we have

\[
\frac{\partial V^S}{\partial m_t} = \frac{1}{1 + \tau_t} E_t \left( \frac{\partial S_t}{\partial c_{S,t}} \right)
\]

and Envelope conditions yield

\[
\frac{\partial V^I}{\partial k_t} = \frac{1 + \eta_t E_t}{1 + \tau_t} \left( \frac{\partial I_t}{\partial c_{I,t}} \right)
\]

Similarly, we have

\[
\frac{\partial V^I}{\partial m_t} = \frac{1}{1 + \tau_t} E_t \left( \frac{\partial I_t}{\partial c_{I,t}} \right)
\]

and

\[
\frac{\partial V^R}{\partial k_t} = \frac{1 + \eta_t E_t}{1 + \tau_t} \left( \frac{\partial R_t}{\partial c_{R,t}} \right)
\]

Similarly, we have

\[
\frac{\partial V^R}{\partial m_t} = \frac{1}{1 + \tau_t} E_t \left( \frac{\partial R_t}{\partial c_{R,t}} \right)
\]

and

5.1.1. Susceptible households

Case A: \( v_{S,t} = 0 \).

From the FOC w.r.t \( m_{S,t+1} \):
\[
\frac{1}{1 + \tau_i} \frac{\partial e_{S,t}}{\partial e_{S,t}} = \beta (1 - \pi_t \phi^\beta (e_{S,t})(1 - \Gamma_t)) E_t \frac{\partial V_{it}}{\partial \hat{e}_{i,t+1}} \frac{R}{\hat{p}_{i,t+1}} + \beta \pi_t \phi^\beta (e_{S,t})(1 - \Gamma_t) I_t E_t \frac{\partial V_{it+1}}{\partial \hat{v}_{i,t+1}} \frac{R}{\hat{p}_{i,t+1}}
\]
\[
= \beta (1 - \pi_t \phi^\beta (e_{S,t})(1 - \Gamma_t)) E_t \left( \frac{1}{1 + \tau_i} \hat{e}_{i,t+1} \left( \frac{\partial g_{S,t+1}}{\partial e_{S,t+1}} \frac{R}{\hat{p}_{i,t+1}} \right) \right) + \beta \pi_t \phi^\beta (e_{S,t})(1 - \Gamma_t) I_t E_t \left( \frac{1}{1 + \tau_i} \hat{e}_{i,t+1} \left( \frac{\partial g_{S,t+1}}{\partial e_{S,t+1}} \frac{R}{\hat{p}_{i,t+1}} \right) \right)
\]
\[
= \beta (1 - \pi_t \phi^\beta (e_{S,t})(1 - \Gamma_t)) E_t \left( \frac{1}{1 + \tau_i} \hat{e}_{i,t+1} \left( \frac{\partial g_{S,t+1}}{\partial e_{S,t+1}} \frac{R}{\hat{p}_{i,t+1}} \right) \right) + \beta \pi_t \phi^\beta (e_{S,t})(1 - \Gamma_t) I_t E_t \left( \frac{1}{1 + \tau_i} \hat{e}_{i,t+1} \left( \frac{\partial g_{S,t+1}}{\partial e_{S,t+1}} \frac{R}{\hat{p}_{i,t+1}} \right) \right)
\]
\[
= \beta (1 - \pi_t \phi^\beta (e_{S,t})(1 - \Gamma_t)) E_t \left( \frac{1}{1 + \tau_i} \hat{e}_{i,t+1} \left( \frac{\partial g_{S,t+1}}{\partial e_{S,t+1}} \frac{R}{\hat{p}_{i,t+1}} \right) \right) + \beta \pi_t \phi^\beta (e_{S,t})(1 - \Gamma_t) I_t E_t \left( \frac{1}{1 + \tau_i} \hat{e}_{i,t+1} \left( \frac{\partial g_{S,t+1}}{\partial e_{S,t+1}} \frac{R}{\hat{p}_{i,t+1}} \right) \right)
\]
\[
= \beta E_t \left( \frac{\psi}{W_i + R_{i+1}} \right)
\]

Note that the exact form of \( f'(\theta_{S,t}) \) will not affect the FOC. We solve for optimal consumption

\[
c_{S,t} = \frac{1}{1 + \tau_i} \frac{\partial g}{\partial e_{S,t}} \left( E_t \left( \frac{\psi}{W_i + R_{i+1}} \right) \right)^{-1}
\]
\[
= \frac{1}{1 + \tau_i} \frac{\theta_{S,t}}{X_{S,t}}
\]

where we define

\[
\frac{\theta_{S,t}}{X_{S,t}} = \beta E_t \left( \frac{\psi}{W_i + R_{i+1}} \right)
\]

and \( X_{S,t} \) denotes the same amount of wealth level held by all Susceptible households. The intuition is that all susceptible households make decision about \( X_{S,t} \) before the realization of health shock, and all of them face the same distribution of idiosyncratic shocks. They will adjust their labor supply to achieve the same optimal level of wealth.

Because \( m_{S,t+1} \geq 0 \) implies \( 1 + \tau_i, c_{S,t} \leq X_{S,t} \), we must have \( c_{S,t} \leq \frac{1}{1 + \tau_i} X_{S,t} \), i.e.,

\[
\frac{1}{1 + \tau_i} \frac{\theta_{S,t}}{X_{S,t}} > \beta E_t \left( \frac{\psi}{W_i + R_{i+1}} \right) \frac{\theta_{S,t}}{X_{S,t}} = \frac{\theta_{S,t}}{X_{S,t}}
\]

In this case, we have \( \theta_{S,t} > \theta_{S,t} \).

Combining case A and case B, we can write down the optimal consumption as:

\[
\frac{1}{1 + \tau_i} \frac{\theta_{S,t}}{X_{S,t}} = \max \left( \beta E_t \left( \frac{\psi}{W_i + R_{i+1}} \right) \frac{\theta_{S,t}}{X_{S,t}} \right)
\]

Therefore,

\[
\frac{\psi}{W_i} = \int \left( \frac{1}{1 + \tau_i} \frac{\theta_{S,t}}{X_{S,t}} \right) dE^S
\]
\[
= \int \max \left( \beta E_t \left( \frac{\psi}{W_i + R_{i+1}} \right) \frac{\theta_{S,t}}{X_{S,t}} \right) dE^S
\]
\[
= \int \max \left( \frac{\theta_{S,t}}{X_{S,t}} \right) dE^S
\]

Hence

\[
\frac{\psi}{W_i} = \theta_{S,t} \int \max \left( \frac{\theta_{S,t}}{X_{S,t}} \right) dE^S
\]
\[
= \theta_{S,t} \phi^\beta (e_{S,t})
\]
where

$$\phi^s(\theta_{i,t}^*) \equiv \int \max \left\{ 1, \frac{\theta_{i,t}^*}{\theta_{i,t}} \right\} dF^S$$

Thus, for Susceptible households, the decision rules for consumption, money demand, and real wealth are given by

$$c_{i,t} = \frac{1}{1 + \tau_{i,t}} \min \left\{ 1, \frac{\theta_{i,t}^*}{\theta_{i,t}} \right\} X_{i,t}$$

$$\frac{m_{i,t+1}^I}{P_t} = \max \left\{ \frac{\theta_{i,t}^* - \theta_{i,t}}{\theta_{i,t}}, 0 \right\} X_{i,t}$$

$$X_{i,t} = \frac{1}{\psi} \theta_{i,t}^* \phi^s(\theta_{i,t}^*) W_t$$

Because

$$\frac{\theta_{i,t}^*}{X_{i,t}} = \beta \beta I \left( \frac{\psi}{X_{i,t}} \frac{P_t}{R_{t+1}} \right)$$

the cutoff $\theta_{i,t}^*$ is determined by

$$\frac{1}{W_t} = \beta \beta I \left( \frac{1}{X_{i,t}} \frac{P_t}{R_{t+1}} \right) \phi^s(\theta_{i,t}^*)$$

$\theta_{i,t}^*$ is independent of individual history. The FOC w.r.t. $\eta_{i,t}$ can also be simplified to

$$\int \max \left\{ \beta I \left( \frac{\psi}{X_{i,t}} \frac{P_t}{R_{t+1}} \right) \theta_{i,t}^* \phi^s(\theta_{i,t}^*) \right\} dF^S = \psi$$

**Infectious Households** Similarly, we consider the problem of the Infectious households,

**Case A:** $\nu_{i,t} = 0, m_{i,t+1}^I > 0, \theta_{i,t}^* \leq \theta_{i,t}^*$. In this case, the households will hold money as a store of value

$$\frac{1}{1 + \tau_{i,t}} \frac{\theta_{i,t}^*}{\theta_{i,t}} = \beta (1 - \alpha_{c,t}^d - \alpha_{c,t}^d \phi^d(\theta_{i,t}^*)) E_t \left( \frac{\psi_{t+1}^d}{\beta \beta I_{t+1} \left( \frac{P_t}{R_{t+1}} \right)} + \beta \alpha_{c,t}^d \phi^d(\theta_{i,t}^*) E_t \left( \frac{\psi_{t+1}^d}{\beta \beta I_{t+1} \left( \frac{P_t}{R_{t+1}} \right)} \right) \right)$$

$$= \beta (1 - \alpha_{c,t}^d - \alpha_{c,t}^d \phi^d(\theta_{i,t}^*)) E_t \left( \frac{1}{1 + \tau_{i,t}} \frac{\theta_{i,t}^*}{\theta_{i,t}} \frac{P_t}{R_{t+1}} \right)$$

$$+ \beta \alpha_{c,t}^d \phi^d(\theta_{i,t}^*) E_t \left( \frac{1}{1 + \tau_{i,t}} \frac{\theta_{i,t}^*}{\theta_{i,t}} \frac{P_t}{R_{t+1}} \right)$$

$$= \beta (1 - \alpha_{c,t}^d - \alpha_{c,t}^d \phi^d(\theta_{i,t}^*)) E_t \left( \frac{\psi}{X_{i,t}} \frac{P_t}{R_{t+1}} \right) + \beta \alpha_{c,t}^d \phi^d(\theta_{i,t}^*) E_t \left( \frac{\psi}{X_{i,t}} \frac{P_t}{R_{t+1}} \right)$$

$$= \beta (1 - \alpha_{c,t}^d) E_t \left( \frac{\psi}{X_{i,t}} \frac{P_t}{R_{t+1}} \right)$$

Again, the exact form of $f(\theta_{i,t}^*)$ do not affect the FOC. We define

$$\frac{\theta_{i,t}^*}{X_{i,t}} \equiv \beta (1 - \alpha_{c,t}^d) E_t \left( \frac{\psi}{X_{i,t}} \frac{P_t}{R_{t+1}} \right)$$

and

$$c_{i,t} = \frac{1}{1 + \tau_{i,t}} \frac{\theta_{i,t}^*}{\theta_{i,t}} \beta (1 - \alpha_{c,t}^d) E_t \left( \frac{\psi}{X_{i,t}} \frac{P_t}{R_{t+1}} \right)^{-1}$$

$$= \frac{1}{1 + \tau_{i,t}} \frac{\theta_{i,t}^*}{\theta_{i,t}} X_{i,t}$$

**Case B:** $m_{i,t+1}^I = 0, \nu_{i,t} > 0, \theta_{i,t}^* > \theta_{i,t}^*$. Households consumption $(1 + \tau_{i,t})c_{i,t} = X_{i,t}$, in this case, the marginal utility of consumption
The above two cases implies that
\[
\frac{1}{1 + \tau_{i,t}} \frac{\theta^*_t}{\theta^*_t} = \frac{\theta^*_t}{X_{i,t} > X_{i,t}}
\]

Because
\[
\frac{1}{1 + \tau_{i,t}} \frac{\theta^*_t}{\theta^*_t} = \max \left\{ \beta (1 - \pi^i_t) E_t \left( \frac{\psi}{W_{i+1} P_{i+1}} \right) \theta^*_t \right\}
\]

The decision rules for consumption, money demand, and real wealth are given by
\[
c^i_{i,t} = \frac{1}{1 + \tau_{i,t}} \min \left\{ 1, \frac{\theta^*_t}{\theta^*_t} \right\} X_{i,t}
\]

\[
m^i_{i,t+1} \frac{P_t}{P_{t}} = \max \left\{ \left( \theta^*_t \frac{\theta^*_t}{\theta^*_t} - \theta^*_t, 0 \right) \right\} X_{i,t}
\]

\[
X_{i,t} = \frac{1}{\psi} \frac{\psi}{\theta^*_t} \theta^*_t W_t
\]

Hence
\[
\frac{\theta^*_t}{X_{i,t}} = \beta (1 - \pi^i_t) E_t \left( \frac{\psi}{W_{i+1} P_{i+1}} \right) \frac{\theta^*_t}{X_{i,t}}
\]

where the cutoff \( \theta^*_t \) is determined by
\[
\frac{1}{W_t} = \beta (1 - \pi^i_t) E_t \left( \frac{1}{W_{i+1} P_{i+1}} \right) \theta^*_t
\]

The FOC w.r.t. \( n^i_t \) can also be simplified as
\[
\int \max \left\{ \beta (1 - \pi^i_t) E_t \left( \frac{\psi}{W_{i+1} P_{i+1}} \right) \theta^*_t \right\} W_t dF_t = \psi
\]

**Recovered Households** In the end, for the recovered households.

**Case A:**
\( \nu_{R, t} = 0, m_{R, t+1} \geq 0, \theta_{R, t} \leq \theta_{R, t}^* \). In this case, the households will hold money as a store of value
\[
\frac{1}{1 + \tau_{R, t}} \frac{\theta^*_t}{c^R_{R,t}} = \beta E_t \left( \frac{\psi}{W_{i+1} P_{i+1}} \right)
\]

Again, we define
\[ \frac{\theta_k^t}{X_{R,t}} = \beta E_k \left( \frac{\psi}{W_{t+1}^*} \frac{R}{R_{t+1}} \right) \]

In this case

\[ c_{k,t}^t = \frac{1}{1 + \tau_c} \theta_k^t \left[ \beta E_k \left( \frac{\psi}{W_{t+1}^*} \frac{R}{R_{t+1}} \right) \right]^{-1} \]

\[ = \frac{1}{1 + \tau_c} \theta_k^t X_{R,t} \]

**Case B:**

\[ v_t^R > 0, m_{R,t+1}^i = 0, \theta_{R,t} > \theta_{R,t^*}. \] Households consumption \((1 + \tau_c) c_{R,t}^i = X_{R,t}\) in this case, the marginal utility of consumption

\[ \frac{\theta_k^t}{(1 + \tau_c) c_{R,t}^i} = \frac{\theta_k^t}{X_{R,t}} \]

The above two cases implies that

\[ \frac{1}{1 + \tau_c} \frac{\theta_k^t}{c_{R,t}^i} = \max \left\{ \beta E_k \left( \frac{\psi}{W_{t+1}^*} \frac{R}{R_{t+1}} \right), \frac{\theta_k^t}{X_{R,t}} \right\} \]

Because

\[ \frac{\psi}{W_t} = \int \frac{1}{1 + \tau_c} \frac{\theta_k^t}{c_{R,t}^i} dF^R \]

\[ = \int \max \left\{ \beta E_k \left( \frac{\psi}{W_{t+1}^*} \frac{R}{R_{t+1}} \right), \frac{\theta_k^t}{X_{R,t}} \right\} dF^R \]

\[ = \int \max \left\{ \frac{\theta_k^t}{X_{R,t}}, \frac{\theta_k^t}{X_{R,t}} \right\} dF^R \]

Hence

\[ \psi \frac{X_{R,t}}{W_t} = \theta_{R,t}^* \int \max \left\{ 1, \frac{\theta_k^t}{X_{R,t}} \right\} dF^R \]

\[ = \theta_{R,t}^* \psi^\theta(\theta_k^t) \]

where

\[ \psi^\theta(\theta_k^t) \equiv \int \max \left\{ 1, \frac{\theta_k^t}{\theta_k^t} \right\} dF^R \]

Thus, for the recovered households, the decision rules for consumption, money demand, and real wealth are given,

\[ c_{R,t}^t = \frac{1}{1 + \tau_c} \min \left\{ \frac{1}{\theta_k^t}, X_{R,t} \right\} \]

\[ m_{R,t+1}^i = \max \left\{ \frac{\theta_k^t}{\theta_k^t}, 0 \right\} X_{R,t} \]

\[ X_{R,t} = \frac{1}{\psi} \theta_{R,t}^* \psi^\theta(\theta_k^t) W_t \]

where all recovered households supply labor to achieve the same level of \(X_{R,t}\) before the realization of \(\theta_{R,t}^i\). Because

\[ \frac{\theta_k^t}{X_{R,t}} = \beta E_k \left( \frac{\psi}{W_{t+1}^*} \frac{R}{R_{t+1}} \right) \]

\[ \frac{\theta_k^t}{X_{R,t}} = \frac{\psi}{W_t \psi^\theta(\theta_k^t)} \]

where the cutoff \(\theta_{R,t}^*\) is determined by the following equation

\[ \frac{1}{W_t} = \beta E_k \left( \frac{1}{W_{t+1}^*} \frac{R}{R_{t+1}} \right) \psi^\theta(\theta_k^t) \]

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The FOC w.r.t. $n_i$ becomes
\[
\int \max \left( \beta E_i \left( \frac{\psi}{W_{t+1}} - \frac{P_i}{R_{t+1}} \right) \frac{\partial \bar{h}_i}{\partial k_i} \right) dF^R = \psi
\]

5.2. Proof of Corollary 3

First of all, using the Envelop conditions, the FOC w.r.t $k_{S, t+1}$ becomes
\[
\frac{\psi}{W_t} = \beta (1 - \pi_i f^S (\theta_i) (1 - I_t I_t) E_t E_t \left( 1 + \frac{\pi_{t+1}}{1 + \tau_i} \frac{\bar{h}_{i+1}}{\bar{c}_{i+1}} \right)) + \beta \pi_i f^S (\theta_i) (1 - I_t I_t) E_t E_t \left( 1 + \frac{\pi_{t+1}}{1 + \tau_i} \frac{\bar{h}_{i+1}}{\bar{c}_{i+1}} \right) + \chi_{S,i}
\]

Plugging in the FOC w.r.t. $n_{i,t}$ into the above equation yields,
\[
\frac{\psi}{W_t} = \beta (1 - \pi_i f^S (\theta_i) (1 - I_t I_t) E_t E_t \left( 1 + \frac{\pi_{t+1}}{1 + \tau_i} \frac{\psi}{W_{t+1}} \right)) + \beta \pi_i f^S (\theta_i) (1 - I_t I_t) E_t E_t \left( 1 + \frac{\pi_{t+1}}{1 + \tau_i} \frac{\psi}{W_{t+1}} \right) + \chi_{S,i}
\]

\[
= \beta E_t \left( 1 + \frac{\pi_{t+1}}{1 + \tau_i} \frac{\psi}{W_{t+1}} \right) + \chi_{S,i}
\]

Similarly, we have
\[
\frac{\psi}{W_t} = \beta (1 - \pi_i f^S (\theta_i) - \pi_i^d) E_t E_t \left( \frac{\partial \varphi_i}{\partial k_{t+1}} \right) + \beta \pi_i f^S (\theta_i) E_t E_t \left( \frac{\partial \varphi_i}{\partial k_{t+1}} \right) + \chi_{S,i}
\]

\[
= \beta (1 - \pi_i f^S (\theta_i) - \pi_i^d) E_t E_t \left( 1 + \frac{\pi_{t+1}}{1 + \tau_i} \frac{\psi}{W_{t+1}} \right) + \beta \pi_i f^S (\theta_i) E_t E_t \left( 1 + \frac{\pi_{t+1}}{1 + \tau_i} \frac{\psi}{W_{t+1}} \right) + \chi_{S,i}
\]

\[
= \beta (1 - \pi_i f^S (\theta_i) - \pi_i^d) E_t \left( 1 + \frac{\pi_{t+1}}{1 + \tau_i} \frac{\psi}{W_{t+1}} \right) + \chi_{S,i}
\]

and
\[
\frac{\psi}{W_t} = \beta E_t \left( 1 + \frac{\pi_{t+1}}{1 + \tau_i} \frac{\psi}{W_{t+1}} \right) + \chi_{S,i}
\]

Because $\pi_i^d > 0$, $\chi_{S,i} \geq 0$, $\chi_i \geq 0$, $\chi_{R,i} \geq 0$, we must have $\chi_{R,i} \geq 0$, $\chi_i \geq 0$, $\forall i$, i.e., the infectious households will always choose $k_{S, t+1} = 0$.

To see why the S households and I households choose the same level of capital, i.e., $k_{S,t+1} = k_{R,t+1}$, let's assume that investment in capital incurs some adjustment cost, $\phi = \eta k_{j,t+1}$, $\eta > 0$. Then
\[
\frac{\psi}{W_t} (1 + \eta k_{j,t+1}) = \beta E_t \left( \frac{\psi}{W_{t+1}} (1 + \pi_{t+1}) \right) + \chi_{j,i}, i = S, R
\]

When we can see that $k_{j,t+1}$ only depend on the aggregate variables: the wage growth and the interest rate, not the individual history. The results holds if $\eta \to 0$. Hence, we have $0 < k_{S,t+1} = k_{R,t+1}$.
Given that \( \pi_t^d > 0 \), and the condition \( E_t(\theta_i, i') < E_t(\theta_R, i') \) holds, we must have \( \theta_{i,t} < \theta_{R,t} = \theta_{t}^* \).

**Proof.** From

\[
\int \max \left\{ \beta E_t \left( \frac{\psi}{W_{t+1}} \frac{R}{R_{t+1}} \right), \frac{\theta_{i,t}}{\theta_{R,t}} \right\} W_t dF_t = \psi
\]

\[
\int \max \left\{ \beta(1 - \pi_t^d) E_t \left( \frac{\psi}{W_{t+1}} \frac{R}{R_{t+1}} \right), \frac{\theta_{i,t}}{\theta_{R,t}} \right\} W_t dF_t = \psi
\]

\[
\int \max \left\{ \beta E_t \left( \frac{\psi}{W_{t+1}} \frac{R}{R_{t+1}} \right), \frac{\theta_{R,t}}{\theta_{R,t}} \right\} W_t dF_t = \psi
\]

we must have

\[ X_{t+1}^* < X_{t+1}^* = X_{t+1}^* \]

5.3. Algorithm to solve the model

In order to solve the transition, we assume a long enough transition period \( T = 500 \), after which the economy will reach the new steady states.

We first compute the \( S_t, I_t, R_t \) for the transition period using the Formulas (1)–(4).

Plug

\[ W_t = (1 - \alpha)k_t^{\alpha}(1 - I_t + AI_t)^{1-\alpha} \]

\[ r_t + \delta = \alpha k_t^{\alpha-1}(1 - I_t + AI_t)^{1-\alpha} \]

into the Euler equations and get

\[ k_t = [\beta(1 + \alpha k_t^{\alpha-1}(1 - (1 - A)I_t^{1-\alpha})^{1-\alpha} - \delta)]^{1/\alpha} \frac{1}{(1 - (1 - A)I_t^{1-\alpha})^{1-\alpha}} k_{t+1} \] \hspace{1cm} (30)

We can use backward shooting to solve the whole transitional path.

1. We make an initial guess about \( k_{T+1}, N_{T+1} \), which have to be close to the final steady state. Then we use the Eq. (30), we solve for \( \{k_t, W_t\}_{t=1}^{T} \).

2. Given the exogenous money growth rate \( \{1 + \mu_t\}_{t=1}^{T} \), we solve for the \( P_t, \{\pi_t\}_{t=1}^{T} \)

(a) Guess \( P_t \), and solve for \( \pi_2 \) such that \( \frac{M_{t+1}}{M_t} = \mu_2 \) using the following conditions

\[
(1 + \pi_t)(1 + \pi_{t+1}) = \phi'(\theta_{s,t}), \quad j = S, R
\]

\[
\frac{(1 + \pi_t)(1 + \pi_{t+1})}{1 - \pi_t^d} = \phi'(\theta_{s,t})
\]

\[
X_{s,t} = \frac{1}{\psi} W_t \phi'(\theta_{s,t}), \quad j = S, I, R
\]

\[
\frac{M_{t+1}}{P_t} = S_tX_s(H^S(\theta_{s,t}^*) + I_t(1 - \pi_t^d)X_tD^I(\theta_{s,t}^*) + R_tX_{R,t}H^R(\theta_{R,t}^*)
\]

(b) After solving \( \pi_2 \), we update \( P_2 \), and solve for \( \pi_3, \pi_4, \ldots \)

(c) Check if \( \pi_t \to \sigma \) under the initial guess \( P_t \), if not, update \( P_t \) in step 2.a

3. Solve for \( \{C_t, G_t\}_{t=1}^{T} \) using

\[ C_t = S_tX_S D^S(\theta_{s,t}^*) + I_t(1 - \pi_t^d)X_tD^I(\theta_{s,t}^*) + R_tX_{R,t}D^R(\theta_{R,t}^*) - \bar{H}_t^* \]
and

\[ G_t = N_t I_t |X_{t,t} | \]

From the resource constraint \( K_{t+1} + G_t + C_t = K_t (1 - \delta) + Y_t \), divide both sides by \( N_{t+1} \)

\[ \frac{N_t}{N_{t+1}} = \frac{k_t + c_t}{N_{t+1}} \]

Hence, given the guess for \( N_{t+1}, k_{t+1} \), we can solve for \( N_t, N_{t-1}, \ldots, N_1 \).

Check whether \( k_1 + N_1 \) equals \( K_1 = K^S \), if not, update initial guess in step 1

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