Charged anisotropic models for quark stars

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Abstract We perform a detailed physical analysis for a class of exact solutions for the Einstein-Maxwell equations. The linear equation of state consistent with quark stars has been incorporated in the model. The physical analysis of the exact solutions is performed by considering the charged anisotropic stars for the particular nonsingular exact model obtained by Maharaj, Sunzu and Ray. In performing such an analysis we regain masses obtained by previous researchers for isotropic and anisotropic matter. It is also indicated that other masses and radii may be generated which are in acceptable ranges consistent with observed values of stellar objects. A study of the mass-radius relation indicates the effect of the electromagnetic field and anisotropy on the mass of the relativistic star.

1 Introduction

The Einstein-Maxwell equations describe charged gravitating matter which are important in relativistic astrophysics, and they model compact objects such as neutron stars, gravastars, dark energy stars and quark stars. In the study of such astrophysical compact objects, the Einstein-Maxwell field equations in static spherical spacetimes provide the basis of investigation, and have therefore attracted the attention of many researchers. With the help of these equations, researchers have discovered different structures and properties of relativistic stellar bodies relevant in astrophysical studies. For example, the solutions to the field equations in static spacetimes obtained by Thirukkanesh and Maharaj [2008] describe realistic compact anisotropic spheres whose properties are relevant to stellar bodies such as SAXJ1808.4-3658. These solutions contain masses and central densities that correspond to realistic stellar bodies. Recently the models for charged matter generated by Rahaman et al. [2012] describe ultra compact astrophysical objects. The solutions obtained by Kalam et al. [2012] describe charged compact objects and are compatible with well known stars. This indicates that the Einstein-Maxwell field equations have many applications in the study of relativistic astrophysical objects.

Pressure anisotropy is an important ingredient in many stellar systems in the absence of charge. Since the pioneering paper by Bowers and Liang [1974], who were the first to consider pressure anisotropy in the study of anisotropic spheres in general relativity, there has been extensive research in this direction. It was established by Dev and Gleiser [2002] that pressure anisotropy has a significant effect on the structure and properties of stellar spheres. In particular it was shown that both the maximum mass and the redshift vary with the magnitude of the pressure anisotropy. For a positive measure of anisotropy, the stability of the sphere is enhanced when compared to isotropic configurations, and anisotropic distributions are stable for smaller adiabatic index values as shown by Gleiser and Dev [2004] and Dev and Gleiser [2003]. Other uncharged anisotropic models with spherical symmetry include Mak and Harko [2003, 2005], Harko and Mak [2002], Karmakar et al. [2007], Maharaj and Chaisi [2006a,b],
and Chaisi and Maharaj (2005, 2006a,b). It is interesting to note the paper of Ivanov (2010) which showed that anisotropic models with heat flow can absorb the addition of charge, viscosity and convert null fluids to a perfect fluid.

It is important for many applications to include the electric field in stellar models. In particular, models with electric field permit causal signals over a wide range of parameters as illustrated by Sharma et al. (2001). It has been shown by Ivanov (2002) that the presence of the electric field significantly affects the redshift, luminosity and mass of the compact object. Most of the models that include an electromagnetic field distribution are isotropic; these include the new classes of solutions obtained by Maharaj and Komathiraj (2007), Komathiraj and Maharaj (2007a,b), Thirukkanesh and Maharaj (2006, 2009) and Maharaj and Thirukkanesh (2009a). Other stellar models that describe charged bodies with isotropic pressures are given by Chattopadhyay et al. (2012), Gupta and Maurya (2011a,b) and Bijalwan (2011). There are fewer research papers that include both anisotropic pressures and electromagnetic field distributions. The presence of pressure anisotropy with an electric field enhances the stability of a configuration under radial adiabatic perturbations compared to the matter with isotropic pressures. Stellar models containing both pressure anisotropy and electric field include compact objects admitting a one-parameter group of conformal motions of Esculpi and Aloma (2010), the generalized isothermal models of Maharaj and Thirukkanesh (2009b), the stellar models of Thirukkanesh and Maharaj (2008), and the regular compact models of Mafa Takisa and Maharaj (2013a). Other charged anisotropic models are those of Rahaman et al. (2012) and Maurya and Gupta (2012). However most of these models have the anisotropy parameter always present, and they do not contain isotropic solutions as a special case. It is important to build physical stellar models in which the anisotropy vanishes for an equilibrium configuration.

Different forms of the barotropic equation of state have been applied together with the field equations to find exact models that govern relativistic compact-gravitating objects such as dark energy and quark strange stars (hybrid stars). Thirukkanesh and Rago (2012) have found exact solutions for the uncharged anisotropic sphere with the polytropic equation of state for particular choices of the polytropic index. Mafa Takisa and Maharaj (2013b) used the same general polytropic equation of state, and obtained exact solutions for field equations in the presence of the electromagnetic field and anisotropic pressures. Shibata (2004) studied the stability of rotating bodies and Lai and Xu (2008) indicated that large amounts of gravitational energy are released in the gravitational collapse of polytropes. Other treatments on polytropes include the results of Tooper (1964), Nilsson and Uggla (2001), Kinasiewicz and Mach (2007) and Heinzel et al. (2003). Maharaj and Mafa Takisa (2013) and Ferriz-Meyer and Siddiqui (2011) found exact solutions of the Einstein-Maxwell field equations for charged anisotropic stars using a quadratic equation of state. There have been many anisotropic and charged exact models with a linear equation of state: Mafa Takisa and Maharaj (2013a) generated compact exact models with regular distributions. Thirukkanesh and Maharaj (2008) found models consistent with dark energy stars and quark stars. Maharaj and Thirukkanesh (2009a) generated anisotropic isothermal models, Sharma and Maharaj (2007) found models consistent with quark matter, and Esculpi and Aloma (2010) generated conformally invariant spheres. However, in general, most of these models do not regain charged isotropic models. Some analytical solutions to the field equations with a linear quark equation of state for charged isotropic stars were found by Komathiraj and Maharaj (2007a). Using the same equation of state, Sotani and Harada (2003), Sotani et al. (2004), and Bombaci (2000) analyzed quark stars with isotropic pressures. There has been an extension of the linear quark equation to include anisotropic pressures in modeling the behaviour of strange stars by Rahaman et al. (2012), Kalan et al. (2013), and Mak and Harko (2002).

It should be noted that the microscopic effects of the strange quark matter coming from strong interactions of QCD (e.g., see Dong et al. (2013) and Dev et al. (1998)) are all encrypted in the final form in the equation of state of matter. We study the general relativistic behaviour of these equations of states, by employing a linear approximation for these strange matter equation of states. Such approximations of the equation of states can be found in the literature in the study of various properties of compact stars. The linear approximation of the strange quark matter equation of state has been used by Zdunik (2000) to study the quasi periodic oscillation (QPO) frequencies in the Lower Mass X-ray Binaries (LMXBs). Gondek-Rosinaka et al. (2000) used the linear approximation of strange matter to compute the mass shedding limit of strange stars.

Recently a class of exact isotropic solutions of Einstein’s equations for non-rotating relativistic stars has also been studied by Murad and Paul (2014). They also comment that as strange stars are not purely gravitationally bound; they are bound by strong interactions. Study of the same in the light of modified gravity theories should not produce any difference in
the mass-radius relation. In this context, although Astashenok et al. (2013) showed that there is an increase in the mass of neutron stars in the $f(R) = R + R (e^{-R}/r^2 - 1)$ gravity model, Ganguly et al. (2014) showed that for the $f(R) = R + \alpha R^2$ model (and subsequently many other $f(R)$ models where the uniqueness theorem is valid) the existence of compact astrophysical is highly unnatural. This is because the equation of state of a compact star should be completely determined by the physics of nuclear matter at high density, and not only by the theory of gravity.

The objective of this paper is to perform a detailed physical analysis of the particular exact solutions to the Einstein-Maxwell system of equations with a linear quark equation of state for charged anisotropic stars obtained by Maharaj et al. (2014). In performing such a physical analysis we seek to regain models with masses and radii obtained by other researchers, and show that other masses contained in our model are in acceptable ranges. We also seek to compare masses and radii by considering anisotropic and isotropic pressures. This analysis shows that the relevant class of exact solutions with a quark equation of state has astrophysical significance.

2 The model

We model the stellar interior with quark matter in general relativity. The spacetime geometry is spherically symmetric and given by

$$ds^2 = -e^{2\nu(r)} dt^2 + e^{2\lambda(r)} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2),$$

(1)

where $\nu(r)$ and $\lambda(r)$ are the gravitational potentials. The Reissner-Nordstrom line element

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2),$$

(2)

describes the exterior spacetime. The quantities $M$ and $Q$ define the total mass and charge of the star, respectively. The energy momentum tensor is defined by

$$T_{ij} = \text{diag} \left( -\rho - \frac{1}{2} E^2, p_r - \frac{1}{2} E^2, p_t + \frac{1}{2} E^2, \right.$$  

$$p_t + \frac{1}{2} E^2),$$

(3)

in the presence of charge and anisotropy. The energy density ($\rho$), the radial pressure ($p_r$), the tangential pressure ($p_t$), and the electric field intensity ($E$) are measured relative to a comoving fluid four-velocity $u^a (u^a u_a = -1)$.

The Einstein-Maxwell field equations are given by

$$\frac{1}{r^2} \left(1 - e^{-2\lambda}\right) + \frac{2\nu}{r} e^{-2\lambda} = \rho + \frac{1}{2} E^2,$$

(4a)

$$-\frac{1}{r^2} \left(1 - e^{-2\lambda}\right) + \frac{2\nu}{r} e^{-2\lambda} = p_r - \frac{1}{2} E^2,$$

(4b)

$$e^{-2\lambda} \left(\nu'' + \nu' - \nu\lambda'\right) + \frac{\nu'}{r} - \frac{\lambda'}{r} = p_t + \frac{1}{2} E^2,$$

(4c)

$$\sigma = \frac{1}{r^2} e^{-\lambda} (r^2 E)',$$

(4d)

where primes denote differentiation with respect to the radial coordinate $r$. The function $\sigma$ represents the proper charge density. The equation of state is linear and of the form

$$p_r = \frac{1}{3} (\rho - 4B),$$

(5)

where $B$ is a constant related to the surface density of the stellar body representing a sharp surface. If we consider the MIT bag model for quark stars, then $B$ can also be identified with the bag constant.

We introduce a new independent variable $x$ and define the metric functions $Z(x)$ and $y(x)$ as

$$x = Cr^2, \quad Z(x) = e^{-2\lambda(r)} A^2 y^2(x) = e^{2\nu(r)},$$

(6)

where $A$ and $C$ are arbitrary constants following Durrsenal and Banerjied (1983). With this transformation the line element in (1) becomes

$$ds^2 = -A^2 y^2 dt^2 + \frac{1}{4x CZ} dx^2 + \frac{x}{C} (d\theta^2 + \sin^2\theta d\phi^2).$$

(7)

The Einstein-Maxwell field equations become

$$\rho = 3p_r + 4B,$$

(8a)

$$\frac{p_r}{C} = \frac{Z\dot{y}}{y} - \frac{\dot{Z}}{2} - \frac{B}{C},$$

(8b)

$$\frac{p_t}{C} = p_r + \Delta,$$

(8c)

$$\Delta = \frac{4x CZ\ddot{y}}{y} + C \left(2x \ddot{Z} + 6\dot{Z}\right) \frac{\ddot{y}}{y}$$

$$+ C \left(2 \left(\dot{Z} + \frac{B}{C}\right) + \frac{Z - 1}{x}\right),$$

(8d)

$$\frac{E^2}{2C} = \frac{1 - Z}{x} - \frac{3Z\dot{y}}{y} - \frac{\dot{Z}}{2} - \frac{B}{C},$$

(8e)

$$\sigma = 2\sqrt{\frac{ZC}{x}} \left(x \dot{E} + E\right),$$

(8f)

where dots represent derivatives with respect to the variable $x$. The quantity $\Delta = p_t - p_r$ is called the
measure of anisotropy. We introduce the mass function given by

$$M(x) = \frac{1}{4C^2} \int_0^x \sqrt{\omega} \left( \rho_* + E^2 \right) \, d\omega,$$

where

$$\rho_* = \left( \frac{1 - Z}{x} - 2Z \right) C,$$

is the energy density when the electric field $E = 0$.

Some solutions to the system, applicable to quark matter, were presented in Maharaj et al. (2014). In that model it was assumed that:

$$y = (a + x^m)^n,$$

$$\Delta = A_0 + A_1 x + A_2 x^2 + A_3 x^3.$$}

For particular choices of the parameters $m$ and $n$ it is possible to integrate the Einstein-Maxwell system exactly. The choice of anisotropy ensures isotropic pressures can be regained. To ensure that the anisotropy vanishes at the stellar centre we should set $A_0 = 0$.

Here we consider a particular solution of Maharaj et al. (2014) that enables us to perform a detailed physical analysis. The particular solution that we can utilize can be written in terms of analytical functions, and this is presented in the Appendix. Note that this generalized class of models with a quark equation of state contains the nonsingular solutions of Komathiraj and Maharaj (2007d) with isotropic pressures.

### 3 Stellar masses

Our exact solutions are more general than earlier treatments and have the flexibility of allowing for the fine-tuning of the parameters. The right choice of parameters in the multi-dimensional parameter space enables us to regain the stellar masses of compact bodies previously identified by many other research groups. To start with, we make the following transformations:

$$\tilde{A}_1 = A_1 R^2, \quad \tilde{A}_2 = A_2 R^2, \quad \tilde{A}_3 = A_3 R^2,$$

$$\tilde{B} = B R^2, \quad \tilde{C} = C R^2, \quad \tilde{a} = a R^2,$$

where $R$ takes the same unit as $x$, and in order to match with the realistic units, it is renormalised by a factor of 43.245, i.e., $R = 43.245x$. In the literature, we find many observed and analysed charge-neutral compact star masses, varying from $0.9 M_\odot$ to $2.01 M_\odot$. The studies of charged stars however allow for more mass in the stable configuration. In our present study, we aim to regain masses of some of the observed compact stellar bodies for the uncharged cases identified to be strange stars, thereby narrowing the parameter ranges in our model for the existence of such objects. For the charged cases, we follow the same exercise to regain the values of the theoretically obtained masses for the charged stars.

In particular, for the electrically charged strange quark stars, we have regained the mass $M = 2.86 M_\odot$ with radius $r = 9.46$ km consistent with mass and radius obtained by Mak and Harko (2004), the mass $M = 2.02 M_\odot$ with radius $r = 10.99$ km consistent with the object found by Negreiros et al. (2009), and the mass $M = 1.433 M_\odot$ with radius $r = 7.07$ km consistent with the particular results obtained by Thirukkanesh and Maharaj (2008) and Mafa Takisa and Maharaj (2013a).

Charged compact stars have been identified as quark stars: the mass $M = 1.67 M_\odot$ with radius of 9.4 km consistent with the star PSR J1903+327 is discussed by Freire et al. (2011) and Gangopadhyay et al. (2013), and the mass $M = 1.433 M_\odot$ with radius of 7.07 km was found by Dev et al. (1998) in their strange star models. Parameter values which give these masses and radii in our model are given in Table I.

### 4 Physical analysis

Although numerically we could regain the values of masses and radii of many previously obtained stellar models, a systematic study of the variation of the anisotropic parameters in our model is also necessary. To this end, we study the effect of anisotropic parameters $\tilde{A}_1$, $\tilde{A}_2$ and $\tilde{A}_3$ on masses and radii of stellar bodies, by varying one parameter at a time and keeping the others fixed. Also, to make the effects more pronounced, we have chosen a few sets of parameters so as to give a value of the mass-radius relation in the acceptable range. The surface of the anisotropic star is considered to be a point of vanishing radial pressure. Also, in our most general solution we have set $\tilde{A}_1 = \tilde{A}_2 = \tilde{A}_3 = 0$ so as to obtain the isotropic model.

Table II shows different masses and radii for isotropic and anisotropic stars for different choices of parameters. This study shows that the masses and radii of the anisotropic stars are less than the corresponding quantities for isotropic stars for most of the values of the parameters chosen. However, the model indicates that there are also some values of the parameters which give the mass and radius of the anisotropic star greater than the corresponding value for the isotropic star. This is shown in the last row (R6) which indicates small values of radii but greater masses for both anisotropic and
Table 1  Various parameter values for various models of stellar objects

| $\tilde{a}$ | $B$ | $C$ | $A_1$ | $A_2$ | $A_3$ | radius | mass   | Model                          |
|------------|-----|-----|-------|-------|-------|--------|--------|--------------------------------|
| 52         | 12  | 1   | 20    | 25    | 20    | 9.46   | 2.86$M_\odot$ | Mak and Harko (2004)         |
| 350        | 12  | 1   | 250   | 280   | 290   | 10.99  | 2.02$M_\odot$ | Negreiros et al. (2009)       |
| 350        | 12  | 1   | 230   | 235   | 240   | 9.40   | 1.67$M_\odot$ | Gangopadhyay et al. (2013)    |
| 202        | 12  | 1   | 25    | 20    | 20    | 7.07   | 1.43$M_\odot$ | Dev et al. (1998)            |
| 350        | 12  | 1   | 289   | 200   | 260   | 7.07   | 0.94$M_\odot$ | Thirukkanesh and Maharaj (2008)|

Table 2  Masses and radii for isotropic and anisotropic stars for different choice of parameters

| Name | $\tilde{a}$ | $B$ | $C$ | $\tilde{A}_1$ | $\tilde{A}_2$ | $\tilde{A}_3$ | $r_{(\Delta \neq 0)}$ | $r_{(\Delta = 0)}$ | $\left(\frac{M}{M_\odot}\right)_{\Delta \neq 0}$ | $\left(\frac{M}{M_\odot}\right)_{\Delta = 0}$ |
|------|-------------|-----|-----|---------------|---------------|---------------|-------------------|-------------------|---------------------------------|---------------------------------|
| R1   | 285         | 12  | 1   | 25            | 20            | 25            | 6.84              | 6.85              | 1.28994                        | 1.31530                        |
| R2   | 100         | 12  | 1   | 20            | 5             | 10            | 6.67              | 6.68              | 1.56259                        | 1.56730                        |
| R3   | 260         | 10  | 1   | 35            | 25            | 30            | 7.59              | 7.61              | 1.58585                        | 1.61878                        |
| R4   | 260         | 10  | 1   | 20            | 30            | 20            | 7.60              | 7.61              | 1.60033                        | 1.61878                        |
| R5   | 200         | 10  | 1   | 40            | 30            | 40            | 7.57              | 7.59              | 1.66749                        | 1.69064                        |
| R6   | 35          | 12  | 1   | 25            | 10            | 15            | 5.78              | 5.77              | 1.73268                        | 1.72885                        |

Table 3  Variation of parameter $\tilde{A}_1$ for $\tilde{a} = 260$, $\tilde{B} = 10$, $\tilde{C} = 1$, $\tilde{A}_2 = 15$, $\tilde{A}_3 = 20$

| $\tilde{A}_1$ | $r_{(\Delta \neq 0)}$ | $r_{(\Delta = 0)}$ | $\left(\frac{M}{M_\odot}\right)_{\Delta \neq 0}$ | $\left(\frac{M}{M_\odot}\right)_{\Delta = 0}$ |
|---------------|----------------------|-------------------|---------------------------------|---------------------------------|
| 5             | 7.6100               | 7.6100            | 1.6140                          | 1.6100                          |
| 10            | 7.6100               | 7.6100            | 1.6078                          | 1.60349                         |
| 15            | 7.6100               | 7.6100            | 1.59981                         | 1.59612                         |
| 20            | 7.6100               | 7.6100            | 1.59243                         | 1.58874                         |
| 25            | 7.6100               | 7.6100            | 1.59243                         | 1.58874                         |
| 30            | 7.6100               | 7.6100            | 1.59243                         | 1.58874                         |
| 35            | 7.6100               | 7.6100            | 1.59243                         | 1.58874                         |
| 40            | 7.6100               | 7.6100            | 1.59243                         | 1.58874                         |
isotropic stars. The masses and radii indicated in Table 2 are in the acceptable range for the quark stars as studied by Gangopadhyay et al. (2013).

The effect of the parameter $\tilde{A}_1$ on the mass and radius of the anisotropic star is shown in Table 3. As $\tilde{A}_1$ increases the mass of the anisotropic star decreases while the radius remains constant. The corresponding mass and radius for the isotropic case are $1.61878M_\odot$ and $7.61\text{ km}$ respectively.

However, variation of the parameter $A_2$ for fixed values of $\tilde{a} = 260$, $\tilde{B} = 10$, $\tilde{C} = 1$, $\tilde{A}_1 = 20$, $\tilde{A}_3 = 20$, did not result in any visible change of the mass and the radius. Carrying out the variation of $A_2$ up to two orders of magnitude (from 5 to 100), we have found that the mass of the anisotropic star is constant around $1.60033M_\odot$ with radius $7.60\text{ km}$ while the isotropic star has the mass $1.61878M_\odot$ with radius of $7.61\text{ km}$. The difference in the mass from the isotropic to the anisotropic case here is due to the presence of the other constant anisotropic parameters $A_1$ and $A_3$.

In Table 4 we see that the variation of the parameter $\tilde{A}_3$ does not affect the radius of the star. However, there is a decrease in the mass of the star with the increase of $\tilde{A}_3$. This decrease in the mass however still falls in the allowed range of the masses and radii for quark stars.

One noticeable feature of our results is that with the increase of the anisotropic parameters $\tilde{A}_1$ and $\tilde{A}_3$, there is decrease in the mass of the star, whereas the radius remain constant. This essentially means that the effective density of the star decreases with the increase of the anisotropic parameters, and hence the effective equation of state become stiffer.

In Fig. 1 and Fig. 2 we learn that the mass of the star decreases linearly with an increase of the anisotropic parameters $\tilde{A}_1$ and $\tilde{A}_3$ in the identified range of the constants. In order to compare the variation of the mass throughout the interior of anisotropic and isotropic stars, we have plotted in Fig. 3-8 the

| $A_3$ | $r(\Delta \neq 0)$ | $r(\Delta = 0)$ | $M(\odot)/M_{\odot}^{\Delta \neq 0}$ | $M(\odot)/M_{\odot}^{\Delta = 0}$ |
|-------|-----------------|-----------------|---------------------------------|---------------------------------|
| 5     | 7.6000          | 7.6000          | 1.60073                         | 1.60060                         |
| 10    | 7.6000          | 7.6000          | 1.60046                         | 1.60033                         |
| 15    | 7.6000          | 7.6000          | 1.60000                         | 1.59993                         |
| 20    | 7.6000          | 7.6000          | 1.60000                         | 1.59980                         |
| 25    | 7.6000          | 7.6000          | 1.60000                         | 1.59980                         |
| 30    | 7.6000          | 7.6000          | 1.60000                         | 1.59980                         |
| 35    | 7.6000          | 7.6000          | 1.60000                         | 1.59980                         |
| 40    | 7.6000          | 7.6000          | 1.60000                         | 1.59980                         |
| 100   | 7.6000          | 7.6000          | 1.60000                         | 1.59980                         |

Fig. 1 Variation of the mass with the parameter $\tilde{A}_1$, keeping other parameters constant

Fig. 2 Variation of the mass with the parameter $\tilde{A}_3$, keeping other parameters constant
Fig. 3  The mass-radius relation using parametric values indicated by $R1$. Here we see that there is an increase in the values for the isotropic case as compared to the anisotropic ones.

Fig. 4  The mass-radius relation using parametric values indicated by $R2$. Clearly, for these choices of parameter sets, there is not much difference between the anisotropic and the isotropic cases.

Fig. 5  The mass-radius relation using parametric values labelled by $R3$.

Fig. 6  The mass-radius relation using parametric values indicated by $R4$.

Fig. 7  The mass-radius relation using parametric values given by $R5$.

Fig. 8  The mass-radius relation using parametric values labelled by $R6$.

Mass against radial distance using the parameter values in Table 2. In general, we see that the masses for the isotropic cases are larger than their anisotropic counter-
parts, except for the parameter sets R2 and R6, where the two graphs appear to overlap.

5 Conclusion

We have studied the exact class of models with a quark equation of state found by Maharaj et al. (2014). A detailed analysis of the physical features of this class was performed. In the uncharged case we regained the masses of Dev et al. (1998) and Gangopadhyay et al. (2013). For the charged case we regained the models of Mak and Harko (2004), Negreiros et al. (2009), and Thirukkanesh and Maharaj (2008). For the anisotropic case we have also generated masses ranging from 1.28994M⊙ to 1.73268M⊙ with radius of the range 5.78 km to 7.61 km. For the isotropic case the masses generated are from 1.31530M⊙ to 1.72885M⊙ with radius varying from 5.77 km to 7.61 km. These particular models are good candidates for the astrophysical object SAXJ1808.4-3658. As the form of the measure of anisotropy on the mass of the star.

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Appendix

In this Appendix we consider an exact solution to the Einstein-Maxwell system (8) found by Maharaj et al. (2014). This class of exact solutions can be written in terms of elementary functions and is given by:

\[ e^{2 \nu} = A^2 (a + x)^4, \]

\[ e^{2 \lambda} = \frac{315(a + x)^2 (a + 3x)}{9(35a^3 + 35a^2x + 21ax^2 + 5x^3) - \frac{2Bx}{C} (105a^3 + 189a^2x + 135ax^2 + 35x^3) + \frac{315L(x)}{C}}, \]

\[ \rho = \frac{35(a + x)^3 (a + 3x)^2}{\Psi(x) + B \left[ (210a^5 + 798a^4x + 1476a^3x^2 + 2540a^2x^3 + 2090ax^4 + 630x^5) \right]} , \]

\[ p_r = \frac{C \left[ (140a^4 + 434a^3x + 318a^2x^2 + 150ax^3 + 30x^4) \right]}{35(a + x)^3 (a + 3x)^2} \]

\[ + \frac{\Psi(x) - B \left[ (70a^5 + 994a^4x + 3708a^3x^2 + \frac{16780}{3} a^2x^3 + \frac{11770}{3} ax^4 + 1050x^5) \right]}{105(a + x)^3 (a + 3x)^2} , \]

\[ p_t = \frac{C \left[ (140a^4 + 434a^3x + 318a^2x^2 + 150ax^3 + 30x^4) \right]}{35(a + x)^3 (a + 3x)^2} \]

\[ + \frac{\Omega(x) - B \left[ (168a^4x + 1296a^3x^2 + 6528a^2x^3 + 7280ax^4 + 2520x^5) \right]}{315(a + x)^3 (a + 3x)^2} , \]

where

\[ L(x) = A_1 \left( \frac{1}{5} a^3 x^2 + \frac{3}{7} a^2 x^3 + \frac{1}{3} ax^4 + \frac{1}{11} x^5 \right) + A_2 \left( \frac{1}{7} a^3 x^3 + \frac{1}{3} a^2 x^4 + \frac{3}{11} ax^5 + \frac{1}{13} x^6 \right) + A_3 \left( \frac{1}{9} a^3 x^4 + \frac{3}{11} a^2 x^5 + \frac{3}{13} ax^6 + \frac{1}{15} x^7 \right) , \]

\[ \Psi(x) = A_1 x \left( -21a^5 - 57a^4 x + 20a^3 x^2 + \frac{1360}{11} a^2 x^3 + 105a x^4 + \frac{315}{11} x^5 \right) + A_2 x^2 \left( \frac{45}{2} a^5 - \frac{185}{2} a^4 x - \frac{1145}{11} a^3 x^2 - \frac{315}{13} a^2 x^3 + \frac{7245}{286} ax^4 + \frac{315}{26} x^5 \right) - A_3 x^3 \left( \frac{70}{3} a^5 + \frac{3710}{33} a^4 x + \frac{2310}{13} a^3 x^2 + \frac{17206}{143} a^2 x^3 + \frac{392}{13} ax^4 \right) , \]

\[ \Omega(x) = A_1 x \left( 84a^5 + 888a^4 x + 3170a^3 x^2 + \frac{54490}{11} a^2 x^3 + 3570ax^4 + \frac{10710}{11} x^5 \right) + A_2 x^2 \left( \frac{165}{2} a^5 + \frac{1705}{11} a^4 x + \frac{33505}{13} a^3 x^2 + \frac{62474}{286} a^2 x^3 + \frac{998235}{286} ax^4 + \frac{44653}{26} x^5 \right) + A_3 x^3 \left( \frac{245}{3} a^5 + \frac{27475}{33} a^4 x + \frac{38640}{13} a^3 x^2 + \frac{673484}{143} a^2 x^3 + \frac{44653}{13} ax^4 + \frac{945x^5}{1} \right) . \]
\[ A(x) = A_1 x \left( 252a^5 + 2124a^4x + 6732a^3x^2 + \frac{100380}{11}a^2x^3 + \frac{63000}{11}ax^4 + \frac{15120}{11}x^5 \right) \\
+ A_2 x^2 \left( 225a^5 + 1845a^4x + \frac{63210}{11}a^3x^2 + \frac{113370}{143}a^2x^3 + \frac{55755}{11}ax^4 + \frac{16065}{13}x^5 \right) \\
+ A_3 x^3 \left( 210a^5 + \frac{18550}{11}a^4x + \frac{738360}{143}a^3x^2 + \frac{78624}{11}a^2x^3 + \frac{59934}{13}ax^4 + 1134x^5 \right). \]

With this exact solution the line element \( \text{(1)} \) becomes

\[ ds^2 = -A^2 (a + x)^4 \, dt^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \]

\[ + \frac{315(a + x)^2(a + 3x) dr^2}{9 (35a^3 + 35a^2x + 21ax^2 + 5x^3) - \frac{2Bx}{C} (105a^3 + 189a^2x + 135ax^2 + 35x^3) + \frac{315L(x)}{C^2}}. \]

The mass function \( \text{(9)} \) becomes

\[ M(x) = \left( \frac{1268}{96525}a - \frac{1}{30} x^2 - \frac{14}{555} ax \right) A_3 - \left( \frac{4}{91}x + \frac{74}{2145}a \right) A_2 - \frac{7}{110} A_1 \right) x^\mp \frac{2}{C^\pm} \]

\[ - \left( \frac{1}{9} B + \frac{2}{33}a A_1 - \frac{10}{429}a^2 A_2 + \frac{14}{1287}a^3 A_3 \right) \left( \frac{x}{C} \right)^\mp \]

\[ - \sqrt{a} \left( \frac{62}{105} aB + \frac{93}{35} aC + \frac{31}{385} a^2 A_1 - \frac{31}{1001} a^3 A_2 + \frac{31}{2145} a^4 A_3 \right) \arctan \sqrt{\frac{x}{a}} \]

\[ + \sqrt{3a} \left( \frac{188}{315} aB + \frac{129}{35} aC + \frac{59}{1155} a^2 A_1 - \frac{100}{9009} a^3 A_2 + \frac{157}{57915} a^4 A_3 \right) \arctan \sqrt{3x/a} \]

\[ + \left( \frac{76}{189} aB + \frac{8}{9} aC + \frac{52}{693} a^2 A_1 - \frac{934}{27027} a^3 A_2 + \frac{3088}{173745} a^4 A_3 \right) \sqrt{x} \frac{1}{C^3} \]

\[ - \left( \frac{6}{35} a^2 B + \frac{27}{35} aC + \frac{9}{85} a^3 A_1 - \frac{9}{1001} a^4 A_2 + \frac{3}{415} a^5 A_3 \right) \sqrt{x} \frac{1}{(a + x)C^2} \]

\[ - \left( \frac{4}{105} a^3 B + \frac{6}{35} a^2 C + \frac{2}{385} a^4 A_1 - \frac{2}{1001} a^5 A_2 + \frac{2}{2145} a^6 A_3 \right) \sqrt{x} \frac{1}{(a + x)^2 C^2} \]

\[ - \left( \frac{188}{945} a^2 B + \frac{43}{35} aC + \frac{59}{3465} a^3 A_2 - \frac{100}{27027} a^4 A_2 + \frac{157}{173745} a^5 A_3 \right) \sqrt{x} \frac{1}{(a + 3x)C^2}. \]