A Hamilton-Jacobi Reachability-Based Framework for Predicting and Analyzing Human Motion for Safe Planning

Somil Bansal*, Andrea Bajcsy*, Ellis Ratner*, Anca D. Dragan, Claire J. Tomlin

Abstract—Real-world autonomous systems often employ probabilistic predictive models of human behavior during planning to reason about their future motion. Since accurately modeling human behavior a priori is challenging, such models are often parameterized, enabling the robot to adapt predictions based on observations by maintaining a distribution over the model parameters. Although this enables data and priors to improve the human model, observation models are difficult to specify and priors may be incorrect, leading to erroneous state predictions that can degrade the safety of the robot motion plan. In this work, we seek to design a predictor which is more robust to misspecified models and priors, but can still leverage human behavioral data online to reduce conservatism in a safe way. To do this, we cast human motion prediction as a Hamilton-Jacobi reachability problem in the joint state space of the human and the belief over the model parameters. We construct a new continuous-time dynamical system, where the inputs are the observations of human behavior, and the dynamics include how the belief over the model parameters change. The results of this reachability computation enable us to both analyze the effect of incorrect priors on future predictions in continuous state and time, as well as to make predictions of the human state in the future. We compare our approach to the worst-case forward reachable set and a stochastic predictor which uses Bayesian inference and produces full future state distributions. Our comparisons in simulation and in hardware demonstrate how our framework can enable robust planning while not being overly conservative, even when the human model is inaccurate. Videos of our experiments can be found at the project website.

Fig. 1: The robot models the human as walking towards one of the two goals shown in red. However, the human actually walks straight in between them. The predictions from our Hamilton-Jacobi reachability framework (center, in magenta) approximates the full probability distribution (right, in teal) while being more robust to model misspecification and less conservative compared to the naive full forward reachable set (left, in grey).

I. INTRODUCTION

Planning around humans is critical for many real-world robotic applications. To effectively reason about human motion, practitioners often couple dynamics models with human decision-making models to generate predictions that are used by the robot at planning time. Such predictors often model the human as an agent maximizing an objective [1–5] or they learn human behavioral structure from data [6]. When coupled with robot motion planners, these human decision-making models have been successfully used in a variety of domains including manipulation [7–11], autonomous driving [12], and navigation [13, 14] (see [15] for a survey).

Such predictive models are often parameterized to encode variations in decision-making between different people. However, since modeling human behavior and how people make decisions a priori is challenging, predictors can maintain a belief distribution over these model parameters [16, 17]. This stochastic nature of the predictor naturally incorporates the model parameter uncertainty into future human state uncertainty [4, 18, 19]. Importantly, once deployed, the robot can observe human behavior and update the distribution over the model parameters to better align its predictive model with the observations.

However, there are two key challenges with such stochastic predictors. First, to update the distribution and to generate predictions, stochastic predictors rely on priors and on observation models. Although these two components enable data and prior knowledge to improve the model, observation models are difficult to specify and the priors may be incorrect. When either is erroneous, then the state predictions will be erroneous as well, and the robot may confidently plan unsafe motions. Secondly, there exist limited computational tools to generate these stochastic predictions in continuous time and state, which may be important for safety-critical applications.

To address the former challenge, researchers in the robust control community have looked into full forward reachable set predictors, which compute the set of all possible states that the human could reach from their current state [12, 20]. Unfortunately, because these predictors consider worst-case human behavior, the resulting predictions are overly conservative, do not typically leverage any data to update the human model, and they significantly impact the overall efficiency of the robot when used in practice.

In this work we seek a marriage between stochastic prediction and robust control: a predictor which is more robust to
misspecified models and priors, but can still leverage human behavioral data online to reduce conservatism in a safe way. To do this, we cast human motion prediction as a Hamilton-Jacobi (HJ) reachability problem \cite{20, 22} in the joint state space of the human and the belief over the predictive model parameters. Observations of the human are treated as continuous “input” into our new dynamical system and how the belief over the model parameters changes based on this data are part of the “dynamics” of the system. However, unlike stochastic predictors, we do not rely on the exact probability of an observation during the prediction. Rather, we divide the possible human observations under the current model in two disjoint sets of likely and unlikely observations, and compute the possible future human states by treating all likely enough observations as equally probable. Since we no longer rely on the exact observation probabilities, the resulting predictor exhibits robustness to incorrect priors, while still leveraging human behavioral data to reduce conservatism online in a safe way. In doing so, our algorithm provides a bridge between stochastic predictors and predicting the full forward reachable set. Additionally, our reachability formulation allows us to leverage the computational tools developed for reachability analysis to predict future human states in continuous time and state \cite{23, 24}.

Interestingly, because HJ reachability is rooted in optimal control, our novel formulation also allows us to analyze human models which change online based on data. For example, we can now ask (and answer) questions like “How long will it take the predictor to reach a desired level of confidence in its human model?” As long as the robot does not have enough confidence in the human model, this information can be used to plan safe maneuvers. Once the confidence in the human model is high enough, the robot can plan more aggressive maneuvers to improve performance. To summarize, our key contributions in this work are:

- a Hamilton-Jacobi reachability-based framework for human motion prediction which provides robustness against misspecified observation models and priors;
- a demonstration of how our framework can be used to analyze human decision-making models that are parameterized by \( \lambda^t \);
- a demonstration of our approach in simulation and on hardware for safe navigation around humans.

II. PROBLEM STATEMENT

We study the problem of safe motion planning for a mobile robot in the presence of a single human agent. In particular, our goal is to compute a control sequence for the robot which moves it from a given start state to a goal state without colliding with the human or the static obstacles in the environment. We assume that both the robot and human states can be accurately sensed at all times. Finally, we also assume that a map of the static parts of the environment is known; however, the future states of the humans are not known and need to be predicted online. Consequently, we divide the safe planning problem into two subproblems: human motion prediction and robot trajectory planning.

A. Human Motion Prediction with Online Updates

To predict future human states, we model the human as a dynamical system with state \( x_H \in \mathbb{R}^{m_H} \), control \( u_H \in \mathbb{R}^{m_H} \), and dynamics

\[
\dot{x}_H = f_H(x_H, u_H).
\]

(1)

Here, \( x_H \) could represent the position and velocity of the human, and \( f_H \) describes the change in their evolution over time. To find the likely future states of the human, we couple this dynamics model with a model of how the human chooses their actions. In general, this is a particularly difficult modeling challenge and many models exist in the literature (see \cite{15}). In this work, we primarily consider stochastic control policies that are parameterized by \( \lambda^t \):

\[
u_H^t \sim P(u_H^t | x_H^t; \lambda^t).
\]

(2)

In this model, \( \lambda^t \) can represent many different aspects of human decision-making from how passive or aggressive a person is \cite{25} to the kind of visual cues they pay attention to in a scene \cite{5}. The specific choice of parameterization is often highly problem specific and can be hand-designed or learned from prior data \cite{4, 26}. Nevertheless, the true value of \( \lambda^t \) is most often not known beforehand and instead needs to be estimated after receiving measurements of the true human behavior. Thus, at any time \( t \), we maintain a distribution \( P^t(\lambda) \) over \( \lambda^t \), which allows us to reason about the uncertainty in human behavior online based on the measurements of \( u_H \).

Running example: We now introduce a running example for illustration purposes throughout the paper. In this example, we consider a ground vehicle that needs to go to a goal position in a room where a person is walking. We consider a planar model with state \( x_H = [h_x, h_y] \), control \( u_H = \theta \), and dynamics \( \dot{x}_H = [v_H \cos(\theta), v_H \sin(\theta)] \). The model parameter \( \lambda^t \) can take two values and indicates which goal location the human is trying to navigate to. The human policy for any state and goal is given by a Gaussian with mean pointing in the goal direction and a variance representing uncertainty in the human action:

\[
u_H^t | x_H^t \sim \begin{cases} N(\mu_1, \sigma_1^2), & \text{if } \lambda^t = g_1 \\ N(\mu_2, \sigma_2^2), & \text{if } \lambda^t = g_2 \end{cases},
\]

(3)

where \( \mu_i = \tan^{-1} \left( \frac{g_i(y) - h_y}{g_i(x) - h_x} \right) \) and \( \sigma_i = \pi/4 \) for \( i \in \{1, 2\} \). \((g_1(x), g_2(y))\) represents the position of goal \( g_i \).

Since we are uncertain about the true value of \( \lambda^t \), we update \( P^t(\lambda) \) online based on the measurements of \( u_H^t \). This observed control may be used as evidence to update the robot’s prior belief \( P^t(\lambda) \) about \( \lambda^t \) over time via a Bayesian update to obtain the posterior belief:

\[
P^t_\lambda(u_H^t | x_H^t) = \frac{P(u_H^t | x_H^t; \lambda^t)P^t(\lambda)}{\sum_\lambda P(u_H^t | x_H^t; \lambda^t)P^t(\lambda)}.
\]

(4)

Given the human state \( x_H^t \), the dynamics \( f_H \) in (1), the control policy in (2), and the distribution \( P^t(\lambda) \), our goal

\footnote{This formulation is easily amenable to deterministic policies where \( P(u_H^t | x_H^t; \lambda^t) \) is the Dirac delta function.}
is to find the likely human states at some future time, \( t + \tau \):
\[
    K^\epsilon_t(\tau) = \{ x^*_{H}^{\tau} \mid P(x^*_{H}^{\tau} \mid x_H) > \epsilon \},
\]
where \( \epsilon \geq 0 \) is the desired safety threshold and is a design parameter. When \( \epsilon = 0 \), we drop the subscript \( \epsilon \) in \( K^\epsilon_t(\tau) \). Using this set of likely human states, our robot will generate a trajectory that at each future time step \( t + \tau \) avoids \( K^\epsilon_t(\tau) \).

Note that the requirement to compute \( K^\epsilon_t(\tau) \) is subtly different from computing the full state distribution, \( P(x^t_{H} \mid x_H) \). For computing the full distribution, one can explicitly integrate over all possible future values of \( \lambda \), state, and action trajectories. Alternatively, one can use the belief space to keep track of trajectories. Alternatively, one can use the belief space to keep track of all possible future values of \( \lambda \), state, and action trajectories. This in turn affects what human action distribution the robot will predict for the following timestep, and so on. To simultaneously compute all possible future beliefs over \( \lambda \) and corresponding likely human states, we consider the joint dynamics of \( P^t(\lambda) \) and the human:
\[
    \dot{z}^t = [\dot{x}^t_{H}, \dot{p}^t(\lambda)] = f(z^t, u_H^t).
\]
At any state \( z \), the distribution over the (predicted) human actions is given by
\[
    u_H \sim P(u_H \mid z) = \sum_{\lambda} P(u_H \mid x_H; \lambda)P(\lambda).
\]

To derive the dynamics of \( P^t(\lambda) \) in \( G_3 \), we note that the belief can change either due to the new observations (via the Bayesian update in \( G_3 \)) or the change in human behavior (modeled via the parameter \( \lambda \)) over time. This continuous evolution of \( P^t(\lambda) \) can be described by the following equation:
\[
    \dot{P}^t(\lambda) = \gamma \left( P^t_{\lambda+}(\lambda \mid u_H^t, x_H^t) - P^t(\lambda) \right) + k \left( P^t(\lambda) \right).
\]
Here, the function \( k \) represents the intrinsic changes in the human behavior, whereas the other component captures the Bayesian change in \( P^t(\lambda) \) due to the observation \( u_H^t \). Note that the time derivative in \( G_3 \) is pointwise in the \( \lambda \) space.

Typically, the Bayesian update is performed in discrete time when the new observations are received; however, in this work, we reason about continuous changes in \( P^t(\lambda) \) and the corresponding continuous changes in the human state. We omit a detailed derivation, but intuitively, to relate continuous-time Bayesian update to discrete-time version, \( \gamma \) in \( G_3 \) can be thought of as the observation frequency. Indeed, as \( \gamma \uparrow \infty \), i.e., observations are received continuously, \( P^t(\lambda) \) instantaneously changes to \( P^t_{\lambda+}(\lambda \mid u_H^t, x_H^t) \). On the other hand, as \( \gamma \downarrow 0 \), i.e., no observation are received, the Bayesian update does not play a role in the dynamics of \( P^t(\lambda) \).

Given the joint state at time \( t \), \( z^t \), and the control policy in \( G_2 \), we are interested in computing the following set:
\[
    \mathcal{V}(\tau) = \{ z^\tau \mid P(z^\tau \mid z^t) > 0 \}, \tau \in [t, t + T].
\]

Intuitively, \( \mathcal{V}(\tau) \) represents all possible states of the joint system, i.e., all possible human states and beliefs over \( \lambda \), that are reachable under the dynamics in \( G_3 \) for some sequence of human actions. We refer to this set as Belief Augmented Forward Reachable Set (BA-FRS) from here on. Given a BA-FRS, we can obtain \( K^\epsilon_t(\tau) \) by projecting \( \mathcal{V}(\tau) \) on the human state space. In particular,
\[
    K^\epsilon_t(\tau) = \bigcup_{z^\tau \in \mathcal{V}(\tau)} \Pi(z^\tau), \quad \tau \in [t, t + T],
\]
where \( \Pi(z) \) denotes the human state component of \( z \). Consequently, the probability of any human state can be obtained states and beliefs, given a stochastic human model as in \( G_2 \). Let the current time be \( t \) with the current (known) human state \( x_H^t \) and a belief \( P^t(\lambda) \). Different control actions \( u_H^t \) that the human might take next will induce a change in both the human’s physical state and the robot’s belief over \( \lambda \). This in turn affects what human action distribution the robot will predict for the following timestep, and so on. To simultaneously compute all possible future beliefs over \( \lambda \) and corresponding likely human states, we consider the joint dynamics of \( P^t(\lambda) \) and the human:

}\]
as $P(x_H^T) = \sum_{z' \in \mathcal{V}(\tau)} P(z' | z^T)$, if $x_H^T = \Pi(z^T)$ (and 0 otherwise) which can be used to obtain $K_{\epsilon}^f(\tau)$ for any $\epsilon \geq 0$.

Since the control policy in (7) is stochastic, the computation of $\mathcal{V}(\tau)$ is a stochastic reachability problem. However, when the prior $P^h(\lambda)$ is not correct, safeguarding against $\mathcal{V}(\tau)$ is not sufficient. Moreover, even when the prior is correct, computing stochastic reachable sets can be computationally demanding [28]. To overcome these challenges, we recast the computation of $\mathcal{V}(\tau)$ as a deterministic reachability problem. We next discuss HJ-reachability analysis for computing the BA-FRS thanks to modern computational tools [23][24], and discuss how we can cast $\mathcal{V}(\tau)$ as a deterministic reachability problem.

B. Background: Hamilton-Jacobi Reachability

HJ-reachability analysis [20][22][29] can be used for computing a general Forward Reachable Set (FRS) $\mathcal{V}(\tau)$ given a set of starting states $\mathcal{L}$. Intuitively, $\mathcal{V}(\tau)$ is the set of all possible states that the system can reach at time $\tau$ starting from the states in $\mathcal{L}$ under some permissible control sequence. The computation of the FRS can be formulated as a dynamic programming problem which ultimately requires solving for the value function $V(\tau, z)$ in the following initial value Hamilton Jacobi-Bellman PDE (HJB-PDE):

$$D_{\tau}V(\tau, z) + H(\tau, z, \nabla V(\tau, z)) = 0, \quad V(0, z) = l(z),$$

where $\tau \geq 0$. Here, $D_{\tau}V(\tau, z)$ and $\nabla V(\tau, z)$ denote the time and space derivatives of the value function respectively. The function $l(z)$ is the implicit surface function representing the initial set of states $\mathcal{L} = \{z: l(z) \leq 0\}$. The Hamiltonian, $H(\tau, z, \nabla V(\tau, z))$, encodes the role of system dynamics and control, and is given by

$$H(\tau, z, \nabla V(\tau, z)) = \max_{u_H \in \mathcal{U}} \nabla V(\tau, z) \cdot f(z, u_H).$$

Once the value function $V(\tau, z)$ is computed, the FRS is given by $\mathcal{V}(\tau) = \{z: V(\tau, z) \leq 0\}$.

C. An HJ Reachability-based framework for prediction and analysis

In this section, we build on the reachability formalism for prediction in Sec. [III-A] to obtain a framework which we will use to both generate robust and faster predictions, as well as to enable planners to answer important analysis questions about Bayesian predictors. Our framework is based on one key idea: rather than using a probability distribution over human actions as in [7], we will use a deterministic set of allowable human actions at every step. Very importantly, this set will be state-dependent, and therefore belief-dependent:

$$u_H \in \mathcal{U}(z), \quad \mathcal{U}(z) = \{u_H: h(u_H, z) \geq \delta\} \quad (13)$$

where $h$ is a function allowed to depend on both the control and the state $z = (x_H, \lambda)$, and threshold $\delta$. Using a control set rather than a distribution allows us to convert the stochastic reachability problem in Sec. [III-A] to a deterministic reachability problem, which can be solved using the HJB-PDE formulation in Section [III-B]. We now illustrate how different instantiations of $h$ in our framework enable both robust prediction and predictor analysis.

**Prediction.** We generate a predictor using our framework by instantiating the set of allowable human actions to be only those with sufficient probability under the belief:

$$u_H \in \mathcal{U}(z), \quad \mathcal{U}(z) = \{u_H: P(u_H | z) \geq \delta\} \quad (14)$$

Now, instead of associating future states with probabilities, we maintain a set of feasible states $z$ at every time step. Over time, this set still evolves via (6), but now all actions that have too low probability are excluded, and actions that have high probability are all treated as equally likely. However, because of the coupling between future belief and allowable actions, we may approximate $K_{\epsilon}^f(\tau)$ via a $K_{\epsilon}^f(\tau)$, using a non-zero $\delta$. This has two advantages: (a) when the prior is correct, this allows us to compute an approximation of $K_{\epsilon}^f(\tau)$ using the computational tools developed for reachability analysis, and (b) when prior is incorrect, this allows the predictions to be robust to such inaccuracies, since computation of $K_{\epsilon}^f(\tau)$ no longer relies on the exact action (or observation) probabilities. We further discuss these aspects in Sec. [IV].

**Analysis.** Suppose we have a prior (or current belief) over $\lambda$; however, the prior might be wrong, i.e. $\arg \max_{\lambda'} P(\lambda') \neq \lambda^*$, with $\lambda^*$ being some hypothesized ground truth value for the human internal state. A reasonable question to ask in such a scenario would be “how long it would take the robot to realize that the value of the internal state is $\lambda^*$, i.e. to place enough probability in its posterior on $\lambda^*$?” A different instantiation of our framework can be used to answer such questions: we now want to compute the BA-FRS under allowed human actions that are modeling the hypothesized ground truth, and compute how long it takes to attain the desired property on the belief (we discuss this further in Sec. [V]). Thus, the allowed control set is:

$$u_H \in \mathcal{U}(z), \quad \mathcal{U}(z) = \{u_H: P(u_H | x_H, \lambda^*) \geq \delta\} \quad (15)$$

Overall, by choosing $h$ appropriately, we can generate a range of predictors and analyses. The two examples above seemed particularly useful to us, and we detail them in the following sections.

IV. A NEW HJ REACHABILITY-BASED PREDICTOR

Our reachability-based framework enables us generate a new predictor by computing an approximation of BA-FRS. In this section, we analyze the similarities and differences
between this predictor and the one obtained by solving the full stochastic reachability problem.

**Prediction as an approximation of \( \mathcal{K} \).** Since the stochastic reachability problem needs to explicitly maintain the state probabilities, it might be challenging to compute \( \mathcal{K} \) compared to \( \tilde{\mathcal{K}} \). However, this advantage in computation complexity is achieved at the expense of losing the information about the human state distribution, which can be an important component for several robotic applications. However, when the full state distribution is not required, as is the case in this paper, \( \tilde{\mathcal{K}} \) provides a very good approximation of \( \mathcal{K} \). In fact, it can be shown that \( \tilde{\mathcal{K}}(\tau) = \tilde{\mathcal{K}}(\tau) \).

**Running example:** For simplicity, consider the case when the intrinsic behavior of the human does not change over time, i.e., \( \tilde{\mathcal{K}}(\lambda) = 0 \). Since in this case \( \lambda \) takes only two possible values, the joint state space is three dimensional. In particular, \( z = [h_x, h_y, p_1] \), where \( p_1 := P(\lambda = g_1) \). \( P(\lambda = g_2) \) is given by \( (1 - P(\lambda = g_1)) \) so we do not need to explicitly maintain it as a state. \( P(u_H | z) = p_1 N(\mu_1, \sigma_1^2) + (1 - p_1) N(\mu_2, \sigma_2^2) \) which can be used to compute the set of allowable controls \( \mathcal{U} \) for different \( \delta \) as per (14). We use the Level Set Toolbox [22] to compute the BA-FRS, starting from \( x_H = (0, 0) \). The corresponding likely human states, \( \tilde{\mathcal{K}}(T) \), for different initial priors and \( \delta \)s are shown in Fig. 3 in magenta. For comparison purposes, we compute \( \mathcal{K}(T) \) (teal), as well as the “naive” FRS obtained using all possible human actions (gray). \( \epsilon \) for \( \mathcal{K} \) is picked to capture 95% area of the set.

As evident from Fig 3, \( \tilde{\mathcal{K}} \) is an over-approximation of \( \mathcal{K} \), but at the same time it is not overly conservative unlike the naive FRS. This is primarily because even though the proposed framework doesn’t maintain the full state distribution, it still discards the unlikely controls during the BA-FRS computation. It is also interesting to note that BA-FRS is not too sensitive to the initial prior for low \( \delta \)s. This property of BA-FRS allows the predictions to be robust to incorrect priors as we explain later in this section.

We also show the full 3D BA-FRS, as well as the projected \( \mathcal{K}(T) \) sets over time for an initial prior, \( p_1 = 0.8 \), in Fig. 2. When \( \delta \) is high, both the belief as well as the human states are biased towards the goal \( g_1 \) over time. When \( \delta \) is high, only the actions that steer the human towards \( g_1 \) will be initially contained in the control set for the BA-FRS computation. Moreover, propagating the current belief under these actions further reinforces the belief that the human is moving towards \( g_1 \). As a result, the beliefs in the BA-FRS shift towards a higher \( p_1 \) over time. On the other hand, when \( \delta = 0.1 \), the human actions under \( g_2 \) are also contained in the control set, which leads to the belief and the human state shift in both directions (towards \( g_1 \) and \( g_2 \)).

**Prediction as robust to incorrect priors and misspecified models.** The set \( \tilde{\mathcal{K}}(T) \) depends heavily on the prior \( P(\lambda) \). When the initial prior for the human motion prediction is not accurate enough, using \( \tilde{\mathcal{K}}(T) \) for planning might lead to unsafe behavior as it can be too optimistic. This issue is particularly exacerbated when the true (unknown) parameter of the human, \( \lambda^* \), is not within the support of \( \lambda \) considered in the model, i.e., when the model is misspecified. For example, when the exact goal of the human may not be any of the goals specified in the model. In such scenarios, a full Bayesian inference may fail to assign sufficient probabilities to the likely states of the human, which can lead to unsafe situations. On the other hand, using the full FRS, i.e., the set of all states the human can reach under any possible control, will ensure safety but can impede robot plan efficiency.

In such situations, the proposed framework presents a good middle-ground alternative to the two approaches – it does not rely heavily on the exact probability of an action while computing FRS since it leverages action probabilities only to distinguish between likely and unlikely actions. Yet, it still uses a threshold to discard highly unlikely actions under the current belief, ensuring the obtained FRS is not too conservative. This allows our framework to perform well in situations where the initial prior is not fully accurate but accurate enough to distinguish likely actions from unlikely actions. In particular, suppose the prior at time \( t \) is such that \( P(u_H | x_H; \lambda^*) \geq \delta \) \( \implies P(u_H | z^t) \geq \delta \) \( \forall \mu_H \), where \( \lambda^* \) is the true (unknown) human parameter, and \( z^t = (x_H^t, P(\lambda)) \). Intuitively, the above condition states that the prior at time \( t \) is accurate enough to distinguish the set of likely actions from the unlikely actions for the true human behavior; however, we do not have the knowledge of the true probability distribution of the actions. In such a case, it can be shown that any human state that is reachable under a control sequence consisting of at least \( \delta \)-likely controls will be contained within \( \tilde{\mathcal{K}}(T) \).

**Running example:** Consider the scenario where the actual human goal is \( g_3 \), midway between \( g_1 \) and \( g_2 \) (see Fig. 2). Thus, the current model does not capture the true goal of the human. Even though the human walks straight towards \( g_3 \), a full Bayesian framework fails to assign sufficient probabilities to the likely human states because of its over reliance on the model. Ultimately, this leads to a collision between the human and the robot. In contrast, since our
framework uses the model to only distinguish likely actions from unlikely actions, it recognizes that moving straight ahead is a likely human action. This is also evident from Fig. 3 where the states straight ahead of the human are contained within the BA-FRS even for a relatively high $\delta$ of 0.2. As a result, using the deterministic BA-FRS for planning leads to a safe navigation around the human.

These results are confirmed in our hardware experiments performed on a TurtleBot 2 testbed. As shown in Fig. 7 we demonstrate these scenarios for navigating around a human participant. We measured human positions at 200Hz using a VICON motion capture system and used on-board human participant. We measured human positions at 200Hz using a VICON motion capture system and used on-board TurtleBot odometry sensors for the robot state measurement. As discussed, our framework allows us to be robust to misspecified goals while not being overly conservative.

V. PREDICTION ANALYSIS

An interesting question that our framework can answer is how long does the predictor have to observe the human in order to determine the true human behavior for some prior. For simplicity, consider the scenario where $\lambda$ can take two possible values $b_1$ or $b_2$; however, the true human parameter is unknown. We also have an initial prior over $\lambda$, given by $P(0)$ (denoted as $\lambda^*_b = b_1$). Then we may pose the following questions: (1) What is the minimum and maximum possible time it will take to determine that $\lambda^*_h = b_1$ with sufficiently high probability (denoted as $T^{1}_{\min}$ and $T^{1}_{\max}$)? (2) What are the corresponding sequences of observations? Thus, we want to know what is the most and the least informative set of observations that the human can provide under $\lambda^*_h = b_1$. Similar questions can also be posed for $b_2$. The overall minimum and the maximum time to determine the true human behavior are then given by $T^{2}_{\min} = \min \{T^{1}_{\min}, T^{2}_{\min} \}$ and $T^{2}_{\max} = \min \{T^{1}_{\max}, T^{2}_{\max} \}$.

Once $T^{1}_{\min}$ and $T^{1}_{\max}$ are available, the robot trajectory can be planned to safeguard against both possibilities ($\lambda^*_h = b_1$ and $\lambda^*_h = b_2$) for $t \leq T^{1}_{\max}$. After a duration of $T^{1}_{\max}$, we will be able to determine the true human behavior so it is sufficient to safeguard against the likely states of the human under the belief $P^{T^{1}_{\max}}(\lambda)$ there on.

As discussed in Sec. III-C, an instantiation of the proposed reachability-based framework can be used to determine $T^{1}_{\min}$ and $T^{1}_{\max}$. In particular, given the initial human state $x^{1}_H$, and the initial prior $P(0)$, we can compute the BA-FRS with the control policy in (15), and the Hamiltonian $H(\tau, z, \nabla V(\tau, z)) = \max_{u \in U} \nabla V(\tau, z) \cdot f(z, u)$. Then, $T^{1}_{\min}$ can be obtained as the minimum time such that $P^{t}(\lambda)$ is contained in $V(\cdot)$ for some human state $x_H$. Here, $P^{t}(\lambda)$ is a distribution that assigns a sufficiently high probability to $\lambda = b_1$. Intuitively, this computation gives the minimum time it will take to reach the belief that $\lambda^*_h = b_1$ if the true human parameter is indeed $b_1$ and the human is giving us the most informative samples to discern its behavior. We can similarly compute $T^{2}_{\min}$ by computing a similar BA-FRS under the likely controls from $b_2$.

Similarly, $T^{1}_{\max}$ can be computed using a similar procedure, but instead of maximizing over the control in the Hamiltonian, we minimize over the control instead. This computation corresponds to finding the control sequence that is least informative at inferring $\lambda^*_h = b_1$ and can be obtained by: $u^*_H(\tau, z) = \arg \min_{u \in U} \nabla V(\tau, z) \cdot f(z, u)$. Intuitively, $u^*_H$ is the control observation that least differentiate between $b_1$ and $b_2$. Similarly, $u^*_H$ corresponding to the computation of $T^{1}_{\min}$ is the control observation that differentiates most between $b_1$ and $b_2$. This is closely related to prior work on legibility [30] and deception [31]: given a fixed horizon, our framework computes a sequence of controls that is maximally informative or maximally ambiguous cumulatively across all the time steps, which in general is nontrivial to compute.

Running example: Consider the planar pedestrian dynamics as before, but with the following human policy:

$$u^*_H \mid x^*_H \sim \begin{cases} \mathcal{N}(0, \sigma^2), & \text{if } \lambda = 0 \\ \text{Uniform}(-\pi, \pi), & \text{if } \lambda = 1, \end{cases} \quad (16)$$

where $\sigma = 0.1$. The human walks straight with a small variance when $\lambda = 0$ and move in a random direction when $\lambda = 1$, approximating an irrational human. We compute the minimum and maximum time to realize $\lambda^*_h = 0$ starting from a high initial prior on irrational behavior; (0.1, 0.9). We assume that we can confidently conclude that $\lambda^*_h = 0$ when all human trajectories reach a belief of at least 0.9 for $\lambda = 0$. The obtained $T^{1}_{\min}$ and $T^{1}_{\max}$ are 3.2s and 11.2s respectively. We also compute the control sequences that correspond to these times. The optimal control sequence for $T^{1}_{\min}$ is given by $u^*_H = 0$, since that is the most likely action under the rational behavior compared to irrational behavior. On the other hand, for $T^{1}_{\max}$, the optimal control sequences consist of an angle of 15 degrees, which is the least likely action that is above the $\delta$-threshold (0.3 for this example) for $\lambda = 0$.

VI. CONCLUSION

When robots operate in complex environments around humans, they often employ probabilistic predictive models to reason about human behavior. Even though powerful, such predictors can make poor predictions when the prior is incorrect or the observation model is misspecified. This in turn will likely cause unsafe behavior. In this work, we formulate human motion prediction as a Hamilton-Jacobi reachability problem. We demonstrate that the proposed framework provides more robust predictions when the prior is incorrect or human behavior model is misspecified, can perform these predictions in continuous time and state using the tools developed for reachability analysis, and can be used for the predictor analysis.
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