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A wave energy system based on peristaltic pumping of air by sea waves

D Ünsalan¹, K T Gürsel¹, , and K İzet-Ünsalan²

¹ Dokuz Eylül University Institute of Marine Sciences and Technology, Baku Blv 100, Inciraltı, İzmir 35340, Turkey
² Ovidius University of Constanța, Faculty of Mechanical, Industrial and Maritime Engineering, Bd Mamaia 124, Constanța 900527, Romania

Abstract. Oscillating water column type of wave energy converters have attracted researchers and engineers working on the field of renewable energy systems, despite the problems caused by the alternating direction of air flow through the turbines. This problem has been circumvented by the use of single direction of rotation turbines such as Wells, Denniss-Auld and omnidirectional impulse turbines, albeit with rather low efficiencies. The authors have considered the usage of near-sinusoidal (cnoidal) form of sea waves as the drivers for the linear peristaltic pumping of air along a channel. The conceived device is an inverted U-shaped channel on a barge, aligned in the direction of wave and serves as a channel for the progress of waveform. Air is driven through the channel by peristaltic action to achieve a unidirectional air flow at the leeward end of the channel. An end-wall operated by a float experiencing heaving and surging motions at the leeward side of the channel prevents the escape of pumped air, which instead is directed to an upward duct leading to a turbine. Since the air flow is unidirectional, the use of more convenient air turbines compared to the ones used in oscillating water column devices are enabled. Air flow parameters with wave amplitudes exceeding and less than channel height above the calm water line are analysed using the Airy wave to demonstrate the feasibility of the proposed system analytically. It was found that the optimum solution was achieved when the channel top is at the calm water level

1. Introduction

Energy from ocean waves is a challenging area of engineering that has attracted researchers for more than two centuries, first such system patented in 1799 [1]. Various wave energy systems have been proposed, some of them based on the wave-induced motions of floating objects, some on wave-induced motions of water and some on wave-induced motions of air [1],[2].

The devices that employ wave-induced motions of air are mainly of oscillating flow column (OWC) type, based on the reciprocating flow of air into and out of a chamber freely connected to the sea. The challenge lies in the selection of turbines to drive electric generators from the flow of air: Either two separate turbines, one for inward flowing air and one for the air expelled out, or omni-directional
turbines of Wells type, Denniss-Auld type or symmetric impulse type[1]. Those options are inferior in efficiency compared to the unidirectional turbines that operate with more uniform streams of air.

On the other hand, the wave itself can be used to induce a flow by the so-called “peristaltic” flow effect. Allison [3] has experimented with a bottom-located flexible water bag aligned in the direction of a surface wave with two edges connected and has reported positive results. Recently Ünsalan et al [4] have presented a system based on conveying the heaving motions of surface buoys to the diaphragm of a peristaltic pump to obtain a flow capable of driving a water turbine.

The present paper is also about the peristaltic pumping by near-sinusoidal form of waves, but this time of air. It is deemed that the unidirectional and less oscillating flow obtained by this system is more suitable for air turbines, thereby overcoming the disadvantages of OWC devices.

Figure 1. General outline of the proposed wave energy system.

2. Analysis

For the purpose of this paper, an inverted U-shaped channel with the upper plate at a distance $H$ above the calm water surface and aligned in the direction of wave propagation is studied. It is further assumed that the device is without any trim or loll and is motionless. The waves are assumed to be linear Airy waves with amplitude $\zeta_{\text{max}}$. Depending on the ratio $\zeta_{\text{max}}/H$, three cases are considered:

Case 1: $H < \zeta_{\text{max}}$ (Wave amplitude larger than channel height)

Case 2: $H > \zeta_{\text{max}}$ (Waveform totally inside channel, with some clearance)

Case 3: $H = \zeta_{\text{max}}$ (Waveform totally inside channel, with zero clearance)

For each case, estimating equations for air flow rate and exit pressure shall be derived.

Case 1 – Wave amplitude larger than channel height ($H < \zeta_{\text{max}}$)
Figure 2. Waveform inside the channel, channel height \( H \) less than the wave amplitude, \( \zeta_{\text{max}} \).

In this case, referring to Figure 2, the waveform can be modelled by the equation,

\[
z(x,t) = \zeta_{\text{max}} \left[ 1 + \cos(kx - \omega t) \right]
\]

(1)

where the \( x \)-axis is placed at the wave crest, \( \zeta_{\text{max}} \) is the wave amplitude, \( k = \frac{2\pi}{\lambda} \), \( \lambda \) being the wavelength, and \( \omega \) is the angular frequency of the wave. Taking time \( t \) arbitrarily as \( t = 0 \), and denoting \( kx = \theta \),

\[
z(x,0) = \zeta_{\text{max}} \left( 1 + \cos \theta \right)
\]

(2)

Volume of air trapped between waveform and channel top, \( V_A \) can be found by the integral:

\[
V_A = 2W \left[ \left( H + \zeta_{\text{max}} \right) \left( \frac{\lambda}{2} - x' \right) \right] \frac{x}{x'} \int_0^{\lambda/2} z(x,0) \cdot dx
\]

(3)

where the angle \( \theta' \) is the angle corresponding to the ordinate \( x' \), as shown in Figure 2, and \( W \) is the width of the channel. Performing the integration,

\[
V_A = W \left\{ \left( H + \zeta_{\text{max}} \right) \left( \frac{\lambda}{2} - 2x' \right) - \frac{\lambda \zeta_{\text{max}}}{\pi} \int_0^{\pi/2} \left( 1 + \cos \theta \right) \cdot d\theta \right\}
\]

(4)

where the ordinate \( x' \) and corresponding angle \( \theta' \) can be expressed in terms of the ratio \( \eta = \frac{H}{\zeta_{\text{max}}} \):

\[
x' = \frac{\lambda}{2\pi} \cos^{-1} \eta \quad \theta' = \cos^{-1} \eta
\]

(5)
Finally, one can obtain the nondimensional flow rate:

$$\frac{V_A}{W\lambda\zeta_{\text{max}}} = \left[ \eta \left( 1 - \frac{1}{\pi} \cos^{-1} \eta \right) + \frac{\sqrt{1 - \eta^2}}{\pi} \right]$$

(6)

Figure 3. Nondimensional trapped air volume per wavelength for Case 1 ($\eta<1$).

Pressure of the entrapped air can be found by neglecting the dynamic effects and realizing that the volume center of the entrapped air is below the calm water free surface. This results in an increase in the pressure of the entrapped air. The downwards volume center shift of entrapped air can be found conveniently by the downward shift of the water volume center for a half wavelength. Referring to Figure 4,

$$z = \frac{M_{AC} + M_{CD} - M_{AB}}{V_{AC} + V_{CD} - V_{AB}}$$

(7)

Figure 4. Analysis of half cycle waveform
Using the notations above,

\[ V_{AC} = V_{CD} = \frac{W \lambda \xi_{\text{max}}}{2\pi} \]  

\[ V_{AB} = \frac{W \lambda \xi_{\text{max}}}{2\pi} \left( \sqrt{1-\eta^2} - \eta \cos^{-1} \eta \right) \]  

\[ M_{AC} = -M_{CD} \]  

\[ M_{AB} = W \int_{0}^{x'} (z - H) \left( H + \frac{z - H}{2} \right) \, dx \]  

\[ = \frac{W \lambda \xi_{\text{max}}^2}{4\pi} \int_{0}^{\varphi} \left( \cos^2 \theta - \eta^2 \right) \, d\theta = \frac{W \lambda \xi_{\text{max}}^2}{8\pi} \left[ \cos^{-1} \eta \cdot \left( 1 - 2\eta^2 \right) + \eta \sqrt{1-\eta^2} \right] \]  

Hence, from Equation 7, the shift of the center of water volume below the calm water level can be calculated.

\[ \bar{z} = \frac{\xi_{\text{max}}}{4} \left[ \cos^{-1} \eta \cdot \left( 1 - 2\eta^2 \right) + \eta \sqrt{1-\eta^2} \right] \]  

\[ \frac{\bar{z}}{\xi_{\text{max}}} \] is shown in Figure 5.

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Figure 5. Vertical shift of the center of water volume below the calm water level
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The pressure increase, \( \Delta p \) within the captured air volume shall be,

\[ \Delta p = \rho_{\text{water}} g \bar{z} \]  

\[ \]
Case 2 – Wave amplitude less than channel height ($H > \zeta_{\text{max}}$)

In this case, $\delta(x) = H - \zeta_{\text{max}} \cos (kx - \omega t)$. The amount of air between the wave surface and the channel top per wavelength can be found by:

$$V_A = W \int_0^\lambda \delta(x) \cdot dx = W \lambda \zeta_{\text{max}} \cdot \eta$$

(14)

The flow rate of air increases linearly with $\eta$, or channel height above water relative to wave amplitude. With the same assumptions and transformations as Case 1, $\delta(x) = H - \zeta_{\text{max}} \cos \theta$.

Navier-Stokes equation for air volume within the tunnel in this case becomes [5],[6].

$$\mu \frac{\partial^2 u}{\partial z^2} = \frac{\partial p}{\partial x}$$

(15)

where $\mu$ is the viscosity of air, $u$ is the horizontal component of velocity, and $p$ is the pressure. Related boundary conditions are:

$$\frac{\partial u}{\partial z} = 0 \quad \text{when} \quad z = 0$$

$$u = c \quad \text{when} \quad z = \delta(x)$$

where $c$ is the wave celerity. The solution for $u$ shall thus becomes:

$$u = 1 - \frac{1}{2} \left( \frac{1}{\mu} \frac{\partial p}{\partial z} \right) (z^2 - \delta^2) + c$$

(16)

The air flow rate through the rectangular channel of width $W$,
\[
\frac{Q}{W} = \int_0^\delta \mu(x,z) \cdot dz = c \delta - \frac{1}{3} \left( \frac{1}{\mu} \frac{\partial p}{\partial x} \right) \delta^3
\]  
(17)

where \(\delta\) is a function of \(x\). Solving for the pressure gradient, and integrating for a full wavelength \(\lambda\), the pressure change(increase) \(\Delta p\) per wavelength \(\lambda\) can be found as:

\[
\Delta p = 3\mu \int_0^\delta \left( \frac{c}{\delta^2} - \frac{Q}{W\delta^3} \right) \cdot dx
\]  
(18)

Performing the integrations and re-arranging the terms:

\[
\frac{\Delta p}{\rho c^2} = 3 \left( \frac{1}{\zeta_{max}} \right) \left\{ \frac{1}{Re_\zeta} \right\} \left[ 1 - \frac{Q}{W\zeta_{max} c \left( \eta^2 + 1 \right)} \right]
\]  
(19)

where \(\Re_\zeta = \frac{c}{\eta c_{max}}\) is the Reynolds number related to flow. One word of caution is that this equation is based on the assumption that the air flow within the channel is turbulence-free, so Equation (19) is applicable to actual cases if the viscosity is replaced by turbulent viscosity in actual conditions. In any case, the pressure increase for Case II shall be negligible because of the high values of Reynolds number, \(\Re_\zeta\) to be encountered.

3. Optimum value for nondimensional channel height, \(\eta\)

The power output from an air turbine is equal to the difference in the enthalpies of air entering and leaving the turbine per unit time.

\[
P_{\text{turbine}} = \frac{d}{dt} \left( H_{in} - H_{out} \right) = \dot{m} c_v \Delta \Theta + \frac{d}{dt} \left( \rho_{air} \dot{V} V \right) \approx \rho_{air} \dot{V} \cdot \Delta p
\]  
(20)

where \(c_v\) is the specific heat of air for constant volume, and \(\Theta\) is the temperature difference of air between the entrance and exit.

For a quasi-steady flow, induced by waves of period \(T\), and using the approximation for sinusoidal deep water waves (Le Mehaute, 1976)

\[
T = \sqrt{\frac{2\pi}{\frac{g}{\lambda}}}
\]  
(20)

The power output shall become:

\[
\frac{P_{\text{turbine}}}{\frac{g^3}{2\pi} (\rho_{air} \rho_{water}) W \sqrt{\lambda} \zeta_{max}^2} = \left[ \eta \left( 1 - \frac{1}{\pi} \cos^{-1} \eta \right) + \sqrt{1 - \eta^2} \right] \frac{\zeta}{\zeta_{max}}
\]  
(21)

The plot of nondimensional turbine power is in Figure 7.
As can be observed, power obtainable from the peristaltic wave energy system has a maximum when \( \eta = 0 \), or, when the channel top is at calm water level.

Also, noting that the energy flux of a wave per unit width of wave crest perpendicular to the direction of propagation for a linear wave is expressed by the relation:

\[
\dot{E}_{\text{wave}} = \frac{\rho_w g^2 \sqrt{\zeta_{\text{max}}^2}}{8\sqrt{2\pi}}
\]

Hence, the power available to the turbine relative to the existing wave power shall be:

\[
\frac{P_{\text{turbine}}}{\dot{E}_{\text{wave}}} \approx 8\rho_{\text{air}} \left[ \eta \left( 1 - \frac{1}{\pi} \cos^{-1} \eta \right) + \frac{\sqrt{1-\eta^2}}{\pi} \right] \left[ \frac{\cos^{-1} \eta \left( 1 - 2\eta^2 \right) + \eta \sqrt{1-\eta^2}}{2 - \sqrt{1-\eta^2} + \eta \cos^{-1} \eta} \right]
\]

As a numerical example, assume a system with channel width 4 m, in a sea where the wave amplitude is 1.0 m and wavelength 25 m. Assuming air at standard atmospheric pressure and 20 °C, and also standard seawater density of 1025 kg/m\(^3\), the airflow rate shall be 31.8 m\(^3\) per wave, while the power of the incident wave shall be 32 kW, and the power available to turbine shall be about 12 kW. With a good turbine design, an electric power output of about 10 kW is possible.

### 4. Some design considerations

The system can have a reflector screen at the entrance, as can be seen in Figure 1. It can be anchored by a swivel or turret connection to the anchor cable(s) to orient itself to the wave direction. Second order and Stokes drift forces shall orient the system to the waves. Channel top shall preferably be at the calm water level (\( \eta = 0 \)), per Equation 21 and Figure 7. The air can be directed upwards to the turbine assembly by a floating air-trap at the leeward end, Figure 8.
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