LET THE SPIN AND THE CHARGES UNIFY

NORMA MANKOČ BORŠNIK *

Department of Physics, University of Ljubljana, Jadranska 19
J. Stefan Institute, Jamova 39 1001, Slovenia

ABSTRACT

In space of d ordinary and d Grassmann coordinates, with \( d \geq 15 \), the charges unify with the spin: the Lorentz group \( SO(1, d-1) \) in Grassmann space manifests under certain conditions as \( SO(1, 3) \) (in \( d = 4 \) subspace) times \( SU(3) \times SU(2) \times U(1) \) (in the rest of the space), accordingly the symmetry group of the S-matrix, which is approximately unitary in \( d = 4 \) ordinary subspace, manifests as the direct product of the Poincaré group in \( d = 4 \) subspace and the groups describing charges.

The fact that ordinary space-time is not sufficient to describe dynamics of our world was first recognized in 1925, when in addition to the infinite dimensional vector space, spanned over ordinary coordinate space, a space of two vectors - the internal space of the fermionic spin, was introduced. Since then the internal space of fermions was enlarged to be able to describe particle - antiparticle degrees of freedom or their handedness (in 1928), the weak charge (1932) and the colour charge (1964). The unification of electromagnetic and weak interactions makes clear that also the electromagnetic charge originates in the internal space. The internal space of bosons grew more or less parallel to the internal space of fermions.

Theories connect all symmetries or the corresponding properties, appearing in physics, with appropriate groups and define accordingly quantum numbers: The spin is connected with the Lorentz group \( SO(1, 3) \). Charges are connected with the group \( U(1) \) (the electromagnetic symmetry), \( SU(2) \) (the weak symmetry) and \( SU(3) \) (the colour symmetry).

In ordinary space time only vectorial types of representations for the Lorentz group are possible: the generators of the infinitesimal transformations \( L^{mn} = (x^mp^n - x^np^m) \) define integer angular momenta. For the internal spaces two types of representations for either the Lorentz group or the groups describing charges are required: the fundamental and the adjoint. Fundamental representations are used to describe the internal space of fermions. Adjoint representations are used to describe the internal space of bosons - the gauge vector fields. To each type of representations singlets are added, in order to describe fermions and bosons which don’t manifest the colour or the weak charge.

Generators of the infinitesimal transformations of the Lorentz group \( M^{mn} \) and the groups defining charges \( \tau^{Ai} \)

\[
M^{mn} = L^{mn} + \begin{cases} 
S^{mn} \\ S_{mn}
\end{cases}, \quad m, n \in \{0, 1, 2, 3\}, \quad (1)
\]

\[
\tau^{Ai} = \begin{cases} 
T^{Ai} \\ T^{Ai}
\end{cases}, \quad A \in \{1, 2, 3\}, i \in \{1, n_A\}.
\]

*Talk presented at XIX Triangular Meeting on Recent Development in Quantum Theories, Rome, March 1996 and at IWCQIS 96, Dubna, July 1996.
define representations, with \( n_A = N_A^2 - 1 \) for \( SU(N_A) \) and \( n_A = N_A^2 \) for \( U(N_A) \).

For later convenience we introduce the generalized commutation relations

\[
\{A, B\} := AB - (-1)^{n_{AB}} BA, \quad n_{AB} = \begin{cases} +1, & \text{if } A \text{ and } B \text{ have Grassmann odd character} \\ 0, & \text{otherwise.} \end{cases}
\]

(3)

The infinitesimal generators of the Lorentz group \( SO(1, 3) \): \( M^{mn} \) fulfil the commutation relations

\[
\{M^{ab}, M^{cd}\} = i(M^{ad}\eta^{bc} + M^{bc}\eta^{ad} - M^{ac}\eta^{bd} - M^{bd}\eta^{ac}),
\]

(4)

while \( \tau^{Ai} \), which are the infinitesimal generators of the groups \( SU(3) \), \( SU(2) \) and \( U(1) \), fulfil the commutation relations

\[
\{\tau^{Ai}, \tau^{Bj}\} = if^{Aijk}\delta^{AB}\tau^{Ak}.
\]

(5)

All operators in Eqs.(4,5) have an even Grassmann character.

If we define \( \tilde{M}^{\pm}_{ij} = \frac{1}{2}(\varepsilon_{ijk}M^{jk} \pm iM^{0i}) \), \( i, j, k \in \{1, 2, 3\} \), we find \( \{\tilde{M}^{\pm}_{i}, \tilde{M}^{\pm}_{j}\} = i\varepsilon_{ijk}M^{jk}, \{\tilde{M}^{\pm}_{i}, \tilde{M}^{\mp}_{j}\} = 0 \), which demonstrates the \( SU(2) \times SU(2) \) structure of the group \( SO(1, 3) \). It follows that \( M^{ij} = \varepsilon_{ijk}(M^{k+} + M^{k-}) \), \( M^{0i} = -i(M^{k+} - M^{k-}) \). Operators \( M^{k+} \) and \( M^{k-} \) define the left handed and the right handed representations of the Lorentz group of either fundamental or adjoint types.

Let \( S^{mn} \) and \( T^{Ai} \) stand for the operators, defining the fundamental representations of the Lorentz group and the group \( SU(N_A) \), respectively, and let \( S^{mn} \) and \( T^{Ai} \) stand for operators defining the adjoint representations of the corresponding groups. Both types of operators fulfill the algebra of Eqs.(4,5), respectively. Operators of the adjoint representations are determined by the structure constants of the groups. To see this for the Lorentz group \( SO(1, 3) \), we take into account the \( SU(2) \times SU(2) \) structure of this group, presented above. We find \( (S^{ij})_{lm} = i\varepsilon_{ijk}\varepsilon_{klm}, (S^{0i})_{lm} = i\varepsilon_{ilm} \). The operators, defining the adjoint representations of the group \( SU(N_A) \), are: \( (T^{Ai})_{jk} = -if^{Aijk} \). The operators \( S^{mn} \) (as well as \( S^{k}_{\pm} \)) are the Pauli 2 \( \times \) 2 matrices, while \( S^{mn} \) (as well as \( S^{k}_{\pm} \)) are 3 \( \times \) 3 matrices. Similarly, operators \( T^{Ai} \) are \( N_A \times N_A \) matrices, while \( (T^{Ai})_{jk} = -if^{Aijk} \) are \( N_A \times N_A \) matrices.

The spin operators in the internal space and the angular momentum operators in the ordinary space time fulfill the same algebra (\( M^{ab} \) from Eq.(4) is equal to either \( S^{mn} \), or to \( S^{mn} \), or to \( L^{mn} \)). In order that the corresponding group transformations are coupled, theories assume the same parameters for the corresponding group elements.

Modern theories try to unify the internal spaces of charges, but they don’t unify the internal spaces of charges with the internal space of spins, although it seems tempting to generalize Eq.(1) in a d-dimensional space-time as follows:

\[
M^{ab} = L^{ab} + \begin{cases} S^{ab} \\ S_{ab} \end{cases}, \quad a, b \in \{0, 1, 2, 3, 5, \ldots, d\},
\]

(7)

* We use in this paper units in which \( c = 1 = \hbar \).

† \( \varepsilon_{a_1a_2\ldots a_n} = \varepsilon_{a_1a_2\ldots a_n} \) is the totally antisymmetric tensor with \( \varepsilon_{123\ldots n} = 1 \).
recognizing that for energies $\ll \frac{1}{x^h}$, where $<x^h>$, $h \in \{5,6,\ldots,d\}$, is the radius of the $h-th$ coordinate, the contribution of $L^{hk}$ to $M^{hk}$, $h,k \in \{5,6,\ldots,d\}$ is nonnoticeable. Since we do not observe more then four ordinary coordinates, d-4 coordinates should be compactified. At low enough energies then Eq.(1) manifests approximately as Eqs.(1,2), with

$$
\tau^{Ai} = c^{Ai}_{hk} M^{hk} = c^{Ai}_{hk} \begin{cases} S^{hk} \end{cases}, \quad h,k \in \{5,\ldots,d\}, \ A \in \{1,2,3\}, \quad i \in \{1,\ldots,n_A\}, \quad c^{Ai}_{hk} = -c^{Ai}_{kh}
$$

and with coefficients $c^{Ai}_{hk}$ which fulfil the equation

$$
-4c^{Ai}_{hk}c^{Bjk}l - \delta^{AB} f^{Ai jk} c^{Ak}_{hl} = 0,
$$

so that for the operators $\tau^{Ai}$ the commutation relations of Eq.(5) are valid.

This kind of unification of spins and charges was proposed in ref. within the approach that space time has d ordinary commuting ( $x^a x^b - x^b x^a = 0$, $a,b \in \{0,1,2,3,5,\ldots,d\}$) and d Grassmann anticommuting ( $e^a e^b + e^b e^a = 0$, $a,b \in \{0,1,2,3,5,\ldots,d\}$) coordinates $\tilde{a}$ with $d \geq 15$, and that consequently all symmetries of a system are connected with only coordinate transformations in ordinary and Grassmann space.

It turns out that in Grassmann space there exist two kinds of operators of the Lorentz transformations: one defining spinorial kind of representations, which include what is known as fundamental representations, the other defining vectorial kind of representations, which include what is known as adjoint representations. They are therefore appropriate to describe spins and charges for fermions and bosons, respectively, unifying spins with charges for each kind of representations separately.

In this paper we comment on fermionic and bosonic representations, defined by these two kinds of operators, from the point of view of the Electroweak Standard Model and on a ”no go” theorem $\tilde{a}$. A more elaborate version will be published in a separate paper $\tilde{a}$.

Let us briefly present the approach.

A linear vector space spanned over a Grassmann coordinate space of d coordinates has the dimension $2^d$. If monomials $\theta^{a_1} \theta^{a_2} \ldots \theta^{a_m}$ are taken as a set of basic vectors with $a_j \neq a_k$, half of the vectors have an odd (those with an odd m) and half of the vectors an even (those with an even m) Grassmann character. Any vector in this space may be represented as a linear superposition of monomials $f(\theta) = \alpha_0 + \sum_{i=1}^{d} \alpha_{a_1 a_2 \ldots a_i} \theta^{a_1} \theta^{a_2} \ldots \theta^{a_i}$, $a_k < a_{k+1}$, where constants $\alpha_0, \alpha_{a_1 a_2 \ldots a_i}$ are complex numbers.

On this linear space we define the following linear operators $\tilde{a}$,

$$
\tilde{p}^a : = i \theta^a, \quad \tilde{a}^a : = i(\tilde{p}^a - i\theta^a), \quad \tilde{\tilde{a}}^a : = -(\tilde{p}^a + i\theta^a).
$$

According to Eqs.(3,6) we find

$$
\{\tilde{p}^a, \tilde{p}^{\tilde{b}}\} = 0 = \{\theta^a, \theta^b\}, \{\tilde{p}^{\tilde{a}}, \tilde{p}^{\tilde{b}}\} = -i \eta^{\tilde{a} \tilde{b}}, \{\tilde{a}^a, \tilde{a}^b\} = 2 \eta^{ab} = \{\tilde{\tilde{a}}^a, \tilde{\tilde{a}}^b\}, \{\tilde{\tilde{a}}^a, \tilde{\tilde{a}}^b\} = 0.
$$

The metric tensor $\eta_{ab} = \text{diag}(1, -1, -1, -1, \ldots, -1)$ lowers the indices of a vector $\{\theta^a\} = \{\theta^0, \theta^1, \ldots, \theta^d\}$, $\theta_a = \eta_{ab}\theta^b$. Linear transformation actions on vectors $(\alpha\theta^a + \beta x^a) = L^a_b(\alpha\theta^b + \beta x^b)$, which leave forms $(\alpha\theta^a + \beta x^a)(\alpha\theta^b + \beta x^b)\eta_{ab}$ invariant, are called the Lorentz transformations.
Operators denoted by a special name \( \gamma \) and \( \tilde{\gamma} \) denote the spins of fermionic (\( S \)) and bosonic fields. The generators of properties \( SO \) containing subgroups \( SU \). Eqs. (2) define what we call the spinorial representations of the Lorentz group and of subgroups of the Lorentz group. We use these representations to describe spins and charges of fermionic fields. In this paper we shall make use of only \( \tilde{\gamma} \). We shall therefore omit the sign \( \tilde{\gamma} \). The operators

\[
\tilde{S}^{ab} := -\frac{i}{4}[\tilde{a}^a, \tilde{b}^b], \quad \tilde{S}^{\dot{a}\dot{b}} := -\frac{i}{4}[\tilde{a}^a, \tilde{b}^b], \tag{8a}
\]

with \([A, B] := AB - BA\), close the algebra of the Lorentz group \( SO(1, d-1) \) (Eq.(4)) and define what we call the vectorial representations of the Lorentz group and of subgroups of this group. We use these representations to describe spins and charges of bosonic fields. For the vectorial operators the symbol is used, which we introduced for operators, describing the fundamental representations.

The operators

\[
S^{ab} := (\theta^a p^b - \theta^b p^a) \tag{8b}
\]

close the algebra of Eq.(4) and define what we call the vectorial representations of the Lorentz group \( SO(1, d-1) \) and of subgroups of this group. We use these representations to describe spins and charges of bosonic fields. For the vectorial operators the symbol is used, which was introduced for operators of the adjoint representations.

Both kinds of operators are, according to Eqs.(6), bilinear forms of differential operators.

It can be proved for \( d = 2n \), where \( n \) is an integer, that \( M^2 \), \( M^2 = \frac{1}{2} M^{ab} M_{ab} \), and

\[
\Gamma, \quad \Gamma = \frac{i}{(2\pi)^n} \varepsilon_{a_1 a_2 \ldots a_{2n}} M^{a_1 a_2 \ldots M^{a_{2n-1} a_{2n}},}
\]

are among invariants of the Lorentz group: \( \{M^2, M^{ab}\} = 0 \), \( \{\Gamma, M^{ab}\} = 0 \).

According to Eqs.(1, 2a, 2b) the algebra of the group \( SO(1, 14) \) contains as subalgebras the algebras of subgroups \( SO(1, 4) \) and \( SO(10) \). The group \( SO(1, 4) \) then the group \( SO(1, 3) \) will be used to describe the spin of fermionic and bosonic fields, the group \( SO(10) \), containing subgroups \( SU(3), SU(2), U(1) \), will be used to describe the charges of fermionic and bosonic fields. The generators of \( SO(1, 3) \), which is the subgroup of \( SO(1, 4) \), determine spins of fermionic (\( S^{mn} \), \( m, n \in \{0, 1, 2, 3\} \)) and of bosonic (\( S^m m \), \( m, n \in \{0, 1, 2, 3\} \)) fields. The remaining generators of group \( SO(1, 4) \), that is \( M^m m \), \( m \in \{0, \ldots, 3\} \), will be denoted by a special name \( \gamma^a := -2i M^a \). In the case of generators of spinorial character, \( \gamma^m = -2i S^{mn} a^a a^m \) may be recognized as the Dirac \( \gamma^m \) matrices, with all the desired properties.

Looking for the \( SU(3) \times SU(2) \times U(1) \) structure of the group \( SO(10) \) in accordance with Eqs.(2a, 2b), where operators \( \tau^1, \tau^2, \tau^3 \) close the subalgebras according to Eq.(5) and the coefficients \( f^{ijk}, \varepsilon^{ijk} \) are the structure constants of the groups \( SU(3), SU(2) \), respectively, one finds:

\[
\tau^1 := \frac{1}{2} (M_{69} - M_{78}), \quad \tau^2 := \frac{1}{2} (M_{68} + M_{79}), \quad \tau^3 := \frac{1}{2} (M_{67} - M_{89}),
\]

\[
\tau^4 := \frac{1}{2} (M_{611} - M_{710}), \quad \tau^5 := \frac{1}{2} (M_{610} + M_{711}), \quad \tau^6 := \frac{1}{2} (M_{811} - M_{910}),
\]

\( \odot \) Operators \( a^a \) are Grassmann odd operators. Operating on spinors they change fermions to bosons, changing the Grassmann character from odd to even, and therefore \( a^m \) can not be recognized as Dirac \( \gamma^m \) matrices. One also finds \( \Gamma^{(4)} = i a^a a^b a^c a^d = i\gamma^a \gamma^b \gamma^c \gamma^d \), where exponent (4) of \( \Gamma^{(4)} \) denotes the four dimensional subspace.
\[ \tau^{1^7} := \frac{1}{2} (M_{810} + M_{911}), \quad \tau^{1^8} := \frac{1}{2\sqrt{3}} (M_{67} + M_{89} - 2M_{1011}), \]

\[ \tau^{2^1} := \frac{1}{2} (M_{1215} - M_{1314}), \quad \tau^{2^2} := \frac{1}{2} (M_{1214} + M_{1315}), \quad \tau^{2^3} := \frac{1}{2} (M_{1213} - M_{1415}), \quad (9) \]

\[ \tau^{3^1} := \sqrt{\frac{3}{5}} \left[ -\frac{1}{3} (M_{67} + M_{89} + M_{1011}) + \frac{1}{2} (M_{1213} + M_{1415}) \right]. \]

Operators \( \tau^{Ai} \), \( A \in \{1, \ldots, 3\} \), \( i \in \{1, n_A\} \), define either spinorial (\( M^{hk} = S^{hk} \), \( \tau^{Ai} = T^{Ai} \)) or vectorial (\( M^{hk} = S^{hk} \), \( \tau^{Ai} = T^{Ai} \)) representations.

To find the irreducible representations of the group \( SO(1,14) \) in terms of subgroups \( SO(1,4) \times SU(3) \times SU(2) \times U(1) \), the eigenvalue problem for the Casimir operators and all the commuting operators for each of subgroups has to be solved:

\[ \langle \theta | A_i | \phi \rangle = a_i^f < \theta | \phi >, \quad \langle \theta | A_i | \phi \rangle = a_i^b < \theta | \phi >, \quad i = \{1, r\}, \quad (10) \]

where \( A_i \) and \( A_i \) stand for \( r \) commuting operators of spinorial and vectorial character, respectively and \( a_i^f \) and \( a_i^b \) for the corresponding eigenvalues.

To solve Eqs.(10), one has to express the operators in the coordinate representation and write the eigenvectors as polynomials \( \parallel \) of \( \theta^a \), \( a = 1, 15 \). We assume that spinorial representations have an odd and vectorial an even Grassmann character, respectively.

According to Eqs.(1, 2a, 2b), one can first solve the eigenvalue problem separately in each of subspaces in which generators of the groups \( SO(1,4), SU(3) \) and \( SU(2) \) operate, respectively, and then get the representations in the hole space as the direct product of representations in different subspaces.

In the spinorial case one finds eight bispinors, four left and four right handed. The operators \( \gamma^m \) connect them into four four spinors. Each can be a triplet or a singlet with respect to \( SU(3) \), a doublet or a singlet with respect to \( SU(2) \), while the possible values for the \( U(1) \) charge are presented on Table I.

| \( \langle \theta | \phi^a \rangle \) | \( \langle \theta | \phi^b \rangle \) | \( T^{31} \times \sqrt{\frac{5}{3}} \) |
|---|---|---|
| triplets | doublets | \( \pm \frac{1}{\sqrt{5}} \) |
| triplets | singlets | \( \pm \frac{1}{2}, \pm \frac{1}{2} \) |
| singlets | doublets | \( \pm \frac{1}{2} \) |
| singlets | singlets | 0, \pm 1 |

Table I. The eigenvalues of the spinorial operator \( T^{31} \) forming the algebra of the group of \( U(1) \) (Eq.(9)), with \( M^{ab} = S^{ab} \), for the representations, which are the direct products of the spinorial representations of the group \( SU(3) \) and the group \( SU(2) \). Index \( a \) runs over different triplets or singlets belonging to the group \( SU(3) \) and index \( i \) runs within the same triplet. Index \( b \) runs over different doublets or singlets of the group \( SU(2) \) and \( j \) within the same doublet.

For the vectorial case, one finds a scalar, a pseudoscalar and two three vectors, one left and one right handed, and two four vectors. Each of them can be an octet, a triplet or a

\[ ^{\parallel} \text{To orthonormalize vectors, the inner product has to be defined.} \]

\[ ^{\ast\ast} \text{In the canonical quantization of fields spinorial representations should quantize to fermions, vectorial to bosons.} \]
singlet with respect to $SU(3)$, or a triplet, a doublet or a singlet with respect to $SU(2)$, while the possible values for the $U(1)$ charge are presented on Table II.

For $d = 15$, Grassmann space offers all the representations needed in the Electroweak Standard Model to describe fermions, gauge fields and Higgs scalars.

We find left handed spinors, which are $SU(3)$ triplets and $SU(2)$ doublets with $U(1)$ charge equal to $\pm \frac{1}{6}$ and right handed spinors, which are $SU(3)$ triplets and $SU(2)$ singlets with $U(1)$ charge equal to $\pm \frac{2}{3}$ and $\mp \frac{1}{3}$, needed to describe quarks, left handed spinors, which are $SU(3)$ singlets and $SU(2)$ doublets with $U(1)$ charge equal to $\mp \frac{1}{2}$ and right handed spinors, which are $SU(3)$ singlets and $SU(2)$ singlets with $U(1)$ charge equal to $\mp 1$, needed to describe leptons. Since there are four four spinors, the approach predicts, if quarks and leptons are elementary fields, four rather than three families.

\begin{table}[h]
\begin{center}
\begin{tabular}{|l|l|l|}
\hline
$\langle \theta | \phi^a_i \rangle$ & $\langle \theta | \phi^b_j \rangle$ & $T^{31} \times \sqrt{\frac{5}{3}}$ \\
\hline
octets & triplets & 0 \\
\hline
octets & doublets & $\pm \frac{1}{2}$ \\
\hline
octets & singlets & 0, $\pm 1$ \\
\hline
triplets & triplets & $\pm \frac{1}{3}, \pm \frac{2}{3}$ \\
\hline
triplets & doublets & $\pm \frac{1}{3}, \pm \frac{2}{3}$ \\
\hline
triplets & singlets & $\pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{1}{3}, \pm \frac{2}{3}$ \\
\hline
singlets & triplets & 0 \\
\hline
singlets & doublets & $\pm \frac{1}{2}$ \\
\hline
singlets & singlets & 0, $\pm 1$ \\
\hline
\end{tabular}
\end{center}
\caption{Table II. Eigenvalues of the vectorial operator $T^{31}$ from Eqs.(9), with $M^{ab} = S^{ab}$, for the vectorial representations which are the direct product of representations of the group $SU(3)$ and the group $SU(2)$. Index $a$ runs over different octets, triplets and singlets, index $i$ within the octet and the triplet. Index $b$ runs over different triplets, doublets and singlets, $j$ within the triplet or the doublet.}
\end{table}

We find left and right handed three vectors, which are $SU(3)$ octets and $SU(2)$ singlets with $U(1)$ charge equal zero, needed to describe gluons, left handed three vectors, which are $SU(3)$ singlets and $SU(2)$ triplets with $U(1)$ charge equal zero, needed to describe weak bosons and left and right handed three vectors, $SU(3)$ and $SU(2)$ singlets with $U(1)$ charge equal zero, needed to describe the $U(1)$ field.

We find also scalars, which are $SU(3)$ singlets and $SU(2)$ doublets, needed to describe Higgs fields.

In this approach there is more representations than needed in the Electroweak Standard Model. The detailed study of those will be presented elsewhere. The structure of the Grassmann space, with the limited number of vectors, however, limits the possible representations allowed by the group theory, offering only representations of groups, which are subgroups of the group $SO(1, d - 1)$.

Let us point out that among the representations of $SO(1, 14) \supset SO(1, 3) \times SU(3) \times SU(2) \times U(1)$ one can find no bosinos, which would be $SU(3)$ octets or $SU(2)$ triplets, required by supersymmetric extensions of the Electroweak Standard Model. These models assume the existence of fermions, which are in the adjoint representations with respect to groups determining charges. They require also the existence of sfermions, that is bosons, which are in the fundamental representations with respect to groups determining charges.

It looks like that unification of spins and charges doesn’t support the simplified version...
of a supersymmetry, unless bosinos are constituent fields, which would mean that all known fields are constituent fields as well.

The proposed approach does manifest supersymmetry: there are equal number of Grassmann odd \((2^d-1)\) and Grassmann even representations\((2^d-1)\).

Gravitons and gravitinos appear as tensor fields.

The approach, in which spins and charges unify, suggests the unification of all interactions: Yang-Mills fields with gravity. To see this, let us look at the equation of motion for a massless spinorial particle in \(d\) dimensional ordinary and \(d\) dimensional Grassmann space in the presence of a gravitational field:

\[
\gamma^a p_{0a} = 0, \quad p_0^a p_{0a} = 0,
\]

with

\[
p_{0a} = f^\mu_a p_{0\mu}, \quad p_{0\mu} = p_\mu - \frac{1}{2} S^{ab} \omega_{ab\mu}, \quad \omega_{ab\mu} = \frac{1}{2} (e_{ab\mu} - e_{b\mu a}).
\]

Vielbeins \(e^a_\mu\) and their inverses \(f^\mu_a\), \(e^a_\mu f^\mu_b = \delta^a_b\), \(f^\mu_a e^\nu_a = \delta^\mu_\nu\), and spin connections \(e^c_{\mu b}\) depend on ordinary on Grassmann coordinates. The detailed derivation of the above equation is presented in refs.

Under special conditions, when vielbeins have a block structure

\[
\begin{pmatrix}
e^m_\alpha \\
0 \\
0 \\
e^h_\sigma
\end{pmatrix}, \quad \alpha, m \in (0, ..., 3), \quad \sigma, h \in (5, ..., d),
\]

and depend only on \(x^\alpha\) and \(\theta^a\), \(\alpha \in \{0, ..., 3\}\), while spin connections \(\omega_{ab\alpha}\) fulfil the equations \(\omega_{hka} = 2 c^{A_i}_{hk} A^{A_i}_\alpha\), \(h, k \in \{6, ..., d\}\), \(\alpha \in \{0, ..., 3\}\), which according to Eqs.(2) means that most of \(\omega_{hka}\) are equal to zero, while those nonzero values are expressible with gauge fields \(A^{A_i}_\alpha\) (the number of these fields is 12 for each \(\alpha\): 8 for \(SU(3)\), 3 for \(SU(2)\) and 1 for \(U(1)\)), the gravitational field in \(d\) dimensional ordinary and \(d\) dimensional Grassmann space time manifests in the four dimensional (sub)space time as an ordinary gravity and Yang-Mills gauge fields.

We find

\[
\gamma^a f^\mu_a p_{0\mu} = \gamma^m f^\alpha_m (p_{\alpha} - \frac{1}{2} S^{mn} \omega_{mna} + A_\alpha), \quad \text{where} \quad A_\alpha = \sum_{A,i} T^{A_i} A^{A_i}_\alpha,
\]

with \(\sum_{A,i} T^{A_i} A^{A_i}_\alpha = \frac{1}{2} c^{hk}_i \omega_{hka}, \quad h, k = 6, ..., d\).

For \(e^m_\alpha = \delta^m_\alpha\) one easily sees that Eq.(11) manifests the Dirac equation for a particle, whose spin is determined by \(S^{mn}\), \(m, n \in \{0, ..., 3\}\) and whose Yang-Mills charges are determined by \(S^{hk}\), in the presence of only gauge fields.

In the proposed theory charges as well as spins are determined by the generators of the Lorentz transformations in Grassmann space. Spin connections (Eq.(11c)) rather than vielbeins determine gauge fields.

†† Let us point out that the vielbein structure from Eq.(11b) is not the one proposed by ordinary Kaluza-Klein theories. In Kaluza-Klein theories the nondiagonal vielbeins determine Yang-Mills fields.
Since this paper suggests the unification of spins and charges within the group $SO(1, 14)$, and since the paper of Colemann and Mandula together with the paper of Haag, Lopuszański and Sohnius speaks against it, convincing the physical community that it is no hope for this kind of unification, let us comment on this "no go" theorem.

To assure the reader that there is no contradiction between the proposed unification and the "no go" theorem it is only needed to say that the dynamics of fields in our approach is defined in a $d$ dimensional ordinary (and $d$ dimensional Grassmann) space so that the scattering matrix, defined in a similar way as in Ref. 8, is unitary in $d$ rather then in four ordinary dimensions, as assumed in the "no go" theorem. If all the coordinates but four are compactified, as we already have assumed, then at energies (of scattering particles in a center of mass coordinate system), low compared to the inverse radii of the subspace of $d$-4 dimensions, the $S$ - matrix manifests approximately as an unitary matrix in a four dimensional subspace, and as an analytic function of only the four momenta $p^m$, $m \in \{0, 1, 2, 3\}$, with the connected symmetry group isomorphic to the direct product of the Poincaré group in four dimensions and the groups defining charges in $d$-4 Grassmann dimensions $\dagger\dagger$. At such energies the Poincaré group in a $d$ dimensional space manifests as the direct product of the Poincaré group in a four dimensional subspace and the groups describing charges in the way we have commented above. This is what the theorem of Colemann and Mandula states for: If the Poincaré transformations in four dimensional space should not transform one charge degree of freedom into another (Coleman and Mandula speak about particle types or different irreducible representations of the Poincaré group), the generators of the Poincaré group and the internal group should commute.

With the growing energy, however, not only the $S$ matrix would start to manifest the unitarity in the $d$ dimensional space, but charges and spins start to manifest as a part of the Lorentz group in the $d$ dimensional ordinary and Grassmann space.

1. Acknowledgement.

This work was supported by Ministry of Science and Technology of Slovenia. The author appreciates fruitful discussions with H.B. Nielsen.

2. References

1. G.E. Uhlenbeck and S. Goudsmit, Naturwiss. 13, 953 (1925),
2. P.M. Dirac, Proc. Roy. Soc., A 117, 601 (1928), A 118, 351 (1928),
3. W. Heisenberg, Zeitschrift für Phy. 77, 1(1932),
4. M. Gell-Mann, Phys. Lett. 8, 214 (1964),
5. S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967), A. Salam and J. C. Ward, Phys. lett. 13 168, (1964), S.L. Glashow, Nucl. Phys. 22 , 579 (1961),
6. N. Mankoč -Borštnik, Phys.Lett. B 292, 25 (1992), Il Nuovo Cimento A 105, 1461 (1992) , J. of Math. Phys. 34, 8 (1993), Int. J. of Mod. Phys. A 9, 1731 ( 1994), J. of Math. Phys. 36(4) 1593 (1995),

$\dagger\dagger$ Situation is similar to the nonrelativistic approach in the four dimensional space-time: Instead of the Poincaré group, the group of rotations $SU(2)$, the group of translations in the three dimensional subspace, the internal group of charges and the Galilean symmetry are assumed to define the exact symmetry of the $S$ matrix, which is assumed to be unitary and an analytic function of the center of mass energy of a two particle state and in which time plays a role of parameter rather then a coordinate. The manifestation of the generators $M^0_i$, $i \in \{1, 2, 3\}$, is supposed to be negligible.
7. N. Mankoč Borštnik, *Modern Phys. Lett.* A **10**, 587 (1995), N. Mankoč Borštnik, Unification of spins and charges in Grassmann space enables unification of all interactions, to appear in the Proceedings of ICSMP, Dubna, July 95, [hep-th/9512050].
N. Mankoč Borštnik, S. Fajfer, [hep-th/9506175](http://arxiv.org/abs/hep-th/9506175), to appear in *J.of Phys. A, Math. and General*, A. Borštnik, N. Mankoč Borštnik, Standard Electroweak Model and Beyond and Grassmann space, in preparation, N. Mankoč Borštnik, Matjaž Poljšak, The ”no go” theorem and unification of spins and charges in Grassmann space, in preparation.
8. S. Coleman, J. Mandula, Phys. Rev. **159**, 1251 (1967),
9. R. Hagg, J.T. Lopuszanski, M. Sinius, Nucl. Phys. B**88**, 257 (1975),
10. E. Witten, Nucl. Phys. B**186**, 412 (1981)