Identification of electrical equivalent circuits of research objects by phase-frequency characteristics

Svetlana N Medvedeva¹,²,⁵, Vladimir I Chernetsov², Mikhail V Chernetsov³ and Margarita M Chernetsova⁴

¹ Penza State University, Department of Power and Electrical Engineering, 440026, 40 Krasnaya St., Penza, Russia
² Penza Cossack Institute of Technology (branch) of K.G. Razumovsky Moscow State University of Technologies and Management (the First Cossack University), Department of Technical Service and Power Engineering, 440039, 11a Gagarin St., Bldg. 12, Penza, Russia
³ Penza State Technological University, Department of Technical Quality Management, 440039, 1a Baidukov Ave./11 Gagarin St., Penza, Russia
⁴ Bauman Moscow State Technical University, Department of Computer Software and Information Technologies, 105005, 5 2nd Baumanskaya St., Bldg. 1, Moscow, Russia

⁵ E-mail: sn-medvedeva@yandex.ru

Abstract. The paper discusses methods for identifying electrical equivalent circuits of the studied object based on the analysis of phase-frequency and differential phase-frequency characteristics of the output signal in a wide frequency range, using MathCAD software. As a result, the following possibilities are revealed: establishing a method for connecting several electrical links of the same type in a single electrical circuit; identification of topology of bipolar electrical equivalent circuits for parametric sensors of various types, both separately and as part of bipolar electrical circuits; identification of equivalent circuits of metal-insulator-semiconductor (MIS) structures described by equivalent amplitude-frequency characteristics. A technique for the analysis of electrical equivalent circuits is proposed, based on the study of phase characteristics in the coordinates of phase-frequency and differential-frequency dependencies, called by the authors as “phase portraits” of equivalent circuits of the parametric research object, which allowed determining changes in the values of the equivalent circuit elements. This approach is promising from a metrological point of view, since in measuring phase-frequency parameters, frequency being the most stable parameter measured with very high accuracy compared to other analog values, can be used as a measure to increase the objectivity of the analysis result.

1. Introduction
Identification of a structural model for the equivalent circuit of objects can be performed by experimental data processing. The following methodology of mathematical and algorithmic modeling is generally accepted: “physical phenomenon”→“mathematical description”→“measuring experiment”→“substantiation of physical and mathematical models and their practical application” [1, 2].
There is an extensive series of tasks in which information about the structure is unknown or insufficiently studied, for example, sensors used in biomedical research, when studying the properties of MIS structures, various substances and other physical objects [3, 4].

The development of methods that increase identification reliability of electrical equivalent circuits (EEC) of research objects (RO) is very relevant, since the quality of phenomena and pattern description, characterizing RO is improved, and the choice of a reasonable plan for a measurement experiment is simplified when determining static and dynamic properties, etc.

2. Materials and methods

In this regard, to identify EEC, it is proposed to use methods based on the analysis of phase-frequency characteristics (PFC) \( \Phi(\omega) \) and differential phase-frequency characteristics (DPFC) \( \Phi_d(\omega) = \frac{d\Phi(\omega)}{d\omega} \) [5]. Note that an additional advantage of these methods is the possibility of using analytical and graphical capabilities offered by the MATLAB package for the analysis of PFC and DPFC [6].

Let us study a bipolar electrical circuit (BEC), which has two poles and one zero, described by the transfer function (TF) of the form:

\[
W(s) = \frac{s \cdot \omega_0 / Q}{s^2 + \omega_0 / Q + \omega_0^2},
\]

where \( Q \) is the quality factor of the system, \( \omega_0 = 2\pi f_p \) is the resonant frequency, \( s \) is the Carson-Heaviside operator. The PFC and amplitude-frequency characteristic (AFC) graphs combined with normalized DPFC corresponding to this BEC obtained for the values \( Q=16 \) and \( \omega_0 = 62.83 \) are presented in figure 1a and 1b, respectively.

![Figure 1](image.png)

**Figure 1.** Graphs of a) PFC and b) AFC, combined with normalized DPFC.

It follows from the analysis of figure 1b that the DPFC is no less informative than the generally accepted AFC. To measure the quality factor by the DPFC, one can use the same formula as for the AFC [7]

\[
Q_A = \omega_0 / (\omega_2 - \omega_1),
\]

but calculating \( Q_A \) by the AFC, \( \omega_1 \) and \( \omega_2 \) values are taken at the level of \( 1/\sqrt{2} \), and when determining \( Q_p \) by the DPFC, \( \omega_1 \) and \( \omega_2 \) values are taken at the level of 0.5 normalized DPFC. The relative error in the calculation of the quality factor by the DPFC in comparison with calculation by the AFC decreases with increasing \( Q_A \), and this error can be neglected at \( Q_A \geq 14 \) as shown in [8].

3. Results and discussion

In order to bring the graphs to some logical uniformity, inverse DPFC images are used in the research.
Let us consider the case of identification of two potentially similar EEC that describe different modes of MIS structures: zero conductivity at a low frequency current (figure 2a), reduction of complex conductivity to zero at high frequencies (figure 2b) [6], and EEC of MIS capacitor [2]

![Figure 2. EEC of MIS structures in various modes.](image)

The impedances corresponding to the considered EEC are described by the formulas:

\[ Z_{R_pR_C}(s) = \frac{sR_C^2 + 1}{s^2RC_2 + s(C_1 + C_2)}; \]  
\[ Z_{C_pR_C}(s) = \frac{R}{sC_1} + \frac{sC_2}{R + \frac{sC_2}{sC_2}}. \]  

In practice, the parameter values differ insignificantly within the decimal order. Therefore, in the calculations, we assume that \( R = 100 \) Ohm, \( C_1 = 1 \) pF, \( C_2 = 10 \) pF [10].

The dependence graphs of BEC impedance modulus according to equations (3) and (4) for EEC of MIS structures presented in figure 2 are practically indistinguishable (figure 3a and 3b). But significant differences, both in the nature of functions and in the field of their determination, are manifested in the analysis of PFC and DPFC graphs presented in figure 4a and 4b, and figure 5a and 5b.

![Figure 3. Modulus graphs of BEC impedances.](image)

![Figure 4. PFC graphs of BEC impedances.](image)
Differences are more pronounced in the analysis of changes in phase characteristics at PFC and DPFC coordinates, i.e. when analyzing the so-called “phase portraits” of the EEC of MIS structures shown in figure 6-9 for various values of \( R, C_1, C_2 \) elements.

**Figure 5.** DPFC graphs of BEC impedances.

**Figure 6.** DPFC dependence on PFC if \( R=100 \) Ohm, \( C_1=1 \) pF, \( C_2=10 \) pF.

**Figure 7.** DPFC dependence on PFC if \( R=10 \) Ohm, \( C_1=1 \) pF, \( C_2=10 \) pF.

**Figure 8.** DPFC dependence on PFC if \( R=1000 \) Ohm, \( C_1=1 \) pF, \( C_2=10 \) pF.

**Figure 9.** DPFC dependence on PFC if \( R=100 \) Ohm, \( C_1=10 \) pF, \( C_2=1 \) pF.
The technique under consideration provides the opportunity to identify methods for connecting linear homogeneous electric circuits (sequentially or in parallel), as shown by the example of several variants of their connections in [8].

Additional advantages of the considered methods for identifying equivalent circuits are the ability to determine the topology of four-element circuits from RLC elements, which impedances are described by similar frequency characteristics in [9]. It is especially important when identifying complex circuits for connecting parametric sensors and measuring the parameters of the latter.

The proposed topology identification methods relate to linear equivalent circuits. In comparison with the known methods based, for example, on the use of spectral representation for object description (by spectral density, Fourier spectrum, etc.), they are more objective due to the use of frequency being a high-precision measure.

4. Conclusions
Thus, the presented results show that the methods for identifying EEC of a research object by PFC and DPFC make it possible to identify the nature of connection of elementary electrical circuits in one complex BEC, to simplify identification of BEC with a parametric sensor, to identify EEC of MIS structures with equivalent AFC, and to increase the reliability and accuracy of the result [11, 12].

References
[1] Myshkis A D 2007 *Elements of Mathematical Model Theory* (Moscow: Kom-Kniga) p 192
[2] Baltyansky S Sh 2000 *Measurement of Physical Objects Parameters Based on Identification and Synthesis of Electrical Models* (Penza: Penza State University Publishing House) p 176
[3] Doyle F J III, Pearson R K and Ogunnaike B A 2002 *Identification and Control Using Volterra Models* (London: Springer-Verlag) p 314
[4] Giannakis G B and Serpedin E A 2001 A bibliography on nonlinear system identification and applications in signal processing, communications and biomedical engineering *Signal Processing* 81(3) 533–80
[5] Chernetsov M V 2018 The use of phase-frequency characteristics for the identification of equivalent circuits of parametric sensors *Measuring. Monitoring. Management. Control* 2 26–33
[6] Gultyaev A K 2001 *MATLAB 5.3. Windows Simulation: A Practical Guide* (St. Petersburg: Korona) p 400
[7] Bessonov L A 1996 *Theoretical Foundations of Electrical Engineering* (Moscow: Higher School) p 638
[8] Chernetsov M V, Chernetsov V I and Medvedeva S N 2016 Identification of the structure of the objects of study characteristics “input-output” *Models, Systems, Networks in Economics, Engineering, Nature and Society* 3 188–202
[9] Turichin A M, Novitsky P V, Levshina E S et al. 1975 *Electrical Measurements of Non-Electric Values* (Leningrad: Energy) p 576
[10] Tsypin B V 2002 *Methods and Measuring Transducers for Monitoring and Diagnostics of Electronic Equipment in Production* (Penza: PhD Thesis) p 282
[11] Afonsky A A and Dyakonov V P 2009 *Digital Spectrum, Signal and Logic Analyzers* (Moscow: Solon-press) p 247
[12] Novitsky P V, Knorring V G and Gutnikov V S 1970 *Digital Devices with Frequency Sensors* (Leningrad: Energy) p 424