**Energy Loss of Charm Quarks in the Quark-Gluon Plasma: Collisional vs Radiative**

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Considering the collisional energy loss rates of heavy quarks from hard light parton interactions the total energy loss of a charm quark for a static medium has been computed. For the energy range \( E \sim (5 - 10) \text{ GeV} \) of charm quark, it is found to be almost same order as that of radiative ones estimated to a first order opacity expansion. The collisional energy loss will become much more important for lower energy charm quarks and this feature could be very interesting for phenomenology of hadrons spectra. Using such collisional energy loss rates we estimate the momentum loss distribution employing a Fokker-Planck equation and the total energy loss of a charm quark for an expanding quark-gluon plasma under conditions resembling the RHIC energies. The fractional collisional energy loss is found to be suppressed by a factor of 5 as compared to static case and does not depend linearly on the system size. We also investigate the heavy to light hadrons \( D/\pi \) ratio at moderately large \((5 - 10) \text{ GeV/c} \) transverse momenta and comment on its enhancement.

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**I. INTRODUCTION**

In the initial stage of ultra-relativistic heavy-ion collisions energetic partons are produced from hard collisions between the partons of the nuclei. Receiving a large transverse momentum, these partons will propagate through the fireball which might consist of a quark-gluon phase for a transitional period of a few \( \text{fm}/c \). These high-energy partons will manifest themselves as jets leaving the fireball. Owing to the interaction of the hard partons with the fireball these partons will lose energy. Hence jet quenching will result. The amount of quenching might depend on the state of matter of the fireball, i.e., quark-gluon plasma (QGP) or a hot hadron gas, respectively. Therefore jet quenching has been proposed as a possible signature for the QGP formation \[1\]. Indeed, first results from \( Au + Au \) at RHIC have shown a suppression of high-\( p_{\perp} \) hadron spectra \[2\] which could possibly indicate the quenching of light quark and gluon jets \[2, 4, 5, 6, 7, 8\]. On the other hand the data \[9\] from light ion interactions \( D + Au \) at RHIC indicate no evidence of suppression in high \( p_{\perp} \) hadron spectra implying the absence of jet quenching as there is no formation of extended dense medium in the final state in such light ion interactions. However, this information from the light ion interactions in turn lends a strong circumstantial support that the observed suppression in \( Au + Au \) is due to the final state energy loss of jets in the dense QGP matter.

Hadrons containing heavy quarks are important probes of strongly interacting matter produced in heavy ion collisions and has also excited a considerable interest. Heavy quark pairs are usually produced at early on a time scale of \( 1/2M_C \approx 0.07 \text{fm}/c \) from the initial fusion of partons (mostly from \( gg \rightarrow c\bar{c} \), but also from \( q\bar{q} \rightarrow c\bar{c} \)) and also from quark-gluon plasma (QGP), if the initial temperature is high enough. There is no production at late times in the QGP and none in the hadronic matter. Thus, the total number of charm quarks get frozen very early in the history of collision which make them a good candidate for a probe of QGP, as one is then left with the task of determining the \( p_{\perp} \) distribution, whose details may reflect the developments in the plasma. The momenta distribution of \( c \) quarks are likely to be reflected in the corresponding quantities in \( D \) mesons as the \( c \) quarks should pick up a light quark, which are in great abundance and hadronize. The first PHENIX data \[17\] from RHIC in \( Au + Au \) collisions at \( \sqrt{s} = 130 \text{AGeV} \) on prompt single electron production are now available, which gives an opportunity to have an experimental estimate of the \( p_{\perp} \) distribution of the heavy quarks. Within the admittedly large experimental error the data indicate the absence of a QCD medium effects. We hope that the future experimental study will provide data with improved statistics and wider \( p_{\perp} \) range, which could then help us to understand the effect of medium modifications of the heavy quarks spectra.

In order to see the effect of medium modifications on the final states, the energy loss of hard partons in the QGP has to be determined. There are two contributions to the energy loss of a parton in the QGP: one is caused by elastic collisions among the partons in the QGP and the other by radiation of the decelerated color charge, i.e., bremsstrahlung of gluons. The energy loss rates due to collisional scatterings among partons were estimated extensively \[11, 12, 13, 14, 15, 16, 17, 18\] in the literature. Using the Hard-Thermal-Loop (HTL) resummed perturbative QCD at finite temperature \[19\], the collisional energy loss of a heavy quark could be derived in a systematic way \[20, 21, 22, 23, 24\].
It was also shown [12, 17, 18, 24] that the drag force can be related to the elastic scatterings among partons in a formulation based on the Fokker Planck equation which is equivalent to the treatment of HTL approximations [21]. From these results also an estimate for the collisional energy loss of energetic gluons and light quarks could be derived [25], which was rederived later using the Leontovich relation [26, 27].

The energy loss due to multiple gluon radiation (bremsstrahlung) was estimated and shown to be the dominant process. For a review on the radiative energy loss see Ref. [28]. Recently, it has also been shown [29] that for a moderate value of the parton energy there is a net reduction in the parton energy loss induced by multiple scattering due to a partial cancellation between stimulated emission and thermal absorption. This can cause a reduction of the light hadrons quenching factor as was first anticipated in Ref. [8], though the most of the earlier studies insisted that the collisional energy loss is insufficient to describe the medium modification of hadronic spectra. These studies, however, were limited to the case of massless energetic quarks and gluons.

The first estimate of heavy quark radiative energy loss was found to dominate [30] the average energy loss rate and subsequently the charmed hadron [31] and the dilepton [32] spectra had a strong dependence on the heavy quark radiative energy loss. Most recent studies of the medium modifications of the charm quark spectrum have computed by emphasizing only the energy loss of heavy quarks by gluon bremsstrahlung [33, 34, 35, 36]. In Ref. [33] it was shown that the appearance of kinematical dead cone effect due to the finite mass of the heavy quarks leads to a large reduction in radiative energy loss and affects significantly the estimation of the quenching of charm quarks and D/π ratio. Also in Refs. [34, 35, 36] a surprising degree of reduction in radiative energy loss for heavy quarks was obtained by taking into account the opacity expansion with and without the Ter-Mikayelian (TM) effect. In Ref. [33] it was shown that the collisional energy loss is insufficient to describe the medium modification of hadronic spectra. These studies, however, were limited to the case of massless energetic quarks and gluons.

Rest of the paper is organized as follows: in Sec. III the collisional energy loss for charm quarks is compared with the radiative ones computed in Refs. [31, 35]. The quenching of hadron spectra in a medium is briefly reviewed in Sec. III. The charm quark in a thermally evolving plasma is modeled in Sec. IV by an expanding fireball created in relativistic heavy-ion collisions: we first obtain Fokker-Planck equation for a Brownian particle from a generic kinetic equation (Sec. IV A); we, next, compute analytically the momentum loss distribution for charm quarks using the elastic perturbative cross section implemented into a Fokker-Planck equation (Sec. IV B); the total energy loss for charm quark is obtained for an expanding plasma (Sec. IV C); and then the quenching of hadron spectra and D/π is obtained for RHIC energies (Sec. IV D). We conclude in Sec. V with a brief discussion.

II. HEAVY QUARKS IN A STATIC QUARK-GluON PLASMA

At leading order in strong coupling constant, $\alpha_s$, the energy loss of a heavy quark comes from elastic scattering from thermal light quarks and gluons. The energy loss rate of heavy quarks in the QGP due to elastic collisions was estimated in Ref. [21]. In the domain $E \ll M^2/T$, it reads

$$- \frac{dE}{dL} = \frac{8\pi\alpha_s T^2}{3} \left(1 + \frac{n_f}{6}\right) \left[\frac{1}{v} - \frac{1 - v^2}{2v^2} \ln \left(\frac{1 + v}{1 - v}\right)\right] \ln \left[2^{\frac{n_f}{v}} B(v) \frac{ET}{m_g M}\right]$$

where $n_f$ is the number of quark flavors, $\alpha_s$ is the strong coupling constant, $m_g = \sqrt{1 + n_f/6} g T/3$ is the thermal gluon mass, $E$ is the energy and $M$ is the mass of the heavy quarks. $B(v)$ is a smooth velocity function, which can be taken approximately as 0.7. Following [12], one can now estimate the static energy loss for heavy quarks at the energies (temperatures) of interest.

On the other hand, heavy quarks medium induced radiative energy loss [33, 35] to all orders in opacity expansion, $L/\lambda_g$ ($L$ is the length of the plasma, $\lambda_g$ is the mean free path of the gluon), has been derived by generalizing the massless case [35] to heavy quarks with mass in a QCD plasma with a gluon dispersion characterized by an asymptotic plasmon mass. This also provides the estimate of the influence of a plasma frequency cut-off on a gluon radiation (Ter-Mikayelian effect) and thus shields the collinear singularities ($k_L \rightarrow 0$) those arise due to massless quarks.

The medium induced radiative energy loss [35] for charm quark in first order opacity expansion, has been computed with a fixed Debye screening mass, $\mu = 0.5$ GeV with $\alpha_s = 0.3$, and a static plasma length, $L = 4$ fm with $\lambda_g = 1$ fm.
FIG. 1: Left panel: The scaled static energy loss of a charm quark $\Delta E/E$ as a function of energy $E$ for a given length of the plasma, $L = 4$ fm. The collisional one is represented by solid line with plasma parameters for RHIC energy. The radiative energy losses according to Ref. [35] are also plotted in first order opacity expansion with (dotted) and without (dashed) the Ter-Mikayelian (TM) effect at a plasma length, $L = 4$ fm and a fixed Debye screening mass, $\mu = 0.5$ GeV (see text for details).

Right panel: The effective shift of the scaled collisional (solid curve) and radiative (dashed) energy loss $\Delta E/E$ as a function of distance $L$ for a charm quark of energy $E = 10$ GeV. The scaled energy loss was found to obey a linear Bethe-Heitler like form, $\Delta E/E \mid_{rad} \propto L \sim CL$, where $C$ is constant of proportionality per unit length. The differential energy loss follows as $\frac{d(\Delta E)}{dL} \mid_{rad} \sim CE$. Now, $C$ can be estimated from right panel of Fig. 1 (also from Fig. 2 of Ref. [35]), as $C \sim \Delta E/E \mid_{L} \sim 0.15$ fm$^{-1}$ at a plasma length $L = 4$ fm, $\mu = 0.5$ GeV and $\alpha_s = 0.3$. For a charm quark with energy $E = 10$ GeV, the differential radiative energy loss is estimated as $\frac{d(\Delta E)}{dL} \mid_{rad} \sim 0.375$ GeV/fm. The Debye screening mass is given as $\mu = T \sqrt{4\pi\alpha_s(1 + n_f)} = 2.2415T$, for two light flavors, $n_f = 2$ and $\alpha_s = 0.3$. The Debye screening mass, $\mu = 0.5$ GeV corresponds to a temperature, $T = 0.225$ GeV. With the plasma parameters corresponding to $\mu = 0.5$ GeV, the differential collisional energy loss for a 10 GeV charm quark in a static medium can also be estimated from $d(\Delta E)/dL \mid_{coll} \sim 0.36$ GeV/fm, and it is found to be of same order as that of radiative ones in Ref. [35].

Now the total collisional energy loss can simply be evaluated from $d(\Delta E)/dL \mid_{coll}$. The scaled collisional (solid line) and the radiative ones with (dotted) and without (dashed line) TM effect of a charm quark as a function energy $E$ are displayed in the left panel of Fig. 1 for a static plasma of length, $L = 4$ fm, with parameters $T = 0.225$ GeV ($\mu = 0.5$ GeV) and $\alpha_s = 0.3$. As discussed earlier that the radiative energy loss is proportional to $E$, resulting the scaled energy loss to be almost constant in $E$. In the energy range, $E \sim (5-10)$ GeV, the scaled collisional energy loss is found to be almost similar as that of radiative ones [35] but decreases with $E$ as the differential rates in [12, 35] have dependence on a log factor involving $E$. Therefore, the collisional energy loss will become much more important for lower energy range, and this feature itself will be quite interesting for phenomenology of particle spectra.

In the right panel of Fig. 1 we display the scaled effective energy loss of a charm quark due to collisional (solid curve) and radiative (dashed curve) ones in a static medium as a function of its thickness, $L$ for a given charm quark energy $E = 10$ GeV. The thickness dependence of the scaled collisional energy loss for a given $E$ is linear like the radiative case [7, 35] whereas the earlier calculations [28, 37, 38] show a quadratic form. This scaling clearly reflects a random walk in $E$ and $L$ as a fast parton moves in the medium [7, 35] with some interactions resulting in an energy gain and others in a loss of energy.

In the energy range $(5-10)$ GeV, which is much of the experimentally measured range ($\gamma_v \leq 4$), the charm quark is not very ultra-relativistic and the collisions are found to be one of the the most dominant energy loss mechanisms.
In the weak coupling limit bremsstrahlung [24] is the dominant energy loss mechanism if the charm quark is ultra-relativistic ($\gamma v \geq 4$). Though the collisions have a different spectrum than radiation, the collisional rather than radiative energy loss should in principle determine the medium modifications of the final state hadron spectra. In the following we would study the suppression of heavy quark spectra.

III. QUENCHING OF HADRON SPECTRA

We will follow the investigations by Baier et al. [6] and Müller [7], using the collisional instead of the radiative parton energy loss. Following Ref. [6] the $p_\perp$ distribution is given by the convolution of the transverse momentum distribution in elementary hadron-hadron collisions, evaluated at a shifted value $p_\perp + \epsilon$, with the probability distribution, $D(\epsilon)$, in the energy $\epsilon$, lost by the partons to the medium by collisions, as

$$
\frac{d\sigma^{\text{med}}}{d^2p_\perp} = \int d\epsilon D(\epsilon) \frac{d\sigma^{\text{vac}}(p_\perp + \epsilon)}{d^2p_\perp} = \int d\epsilon D(\epsilon) \frac{d\sigma^{\text{vac}}}{d^2p_\perp} + \int d\epsilon D(\epsilon) \frac{d\sigma^{\text{vac}}}{d^2p_\perp} \cdot \epsilon \cdot d\sigma^{\text{vac}}(p_\perp + \Delta E).$$

Here $Q(p_\perp)$ is suppression factor due to the medium and the total energy loss by partons in the medium is

$$\Delta E = \int \epsilon D(\epsilon) d\epsilon.$$  

We need to calculate the probability distribution, $D(\epsilon)$, that a parton loses the energy, $\epsilon$, due to the elastic collisions in the medium. This requires the evolution of the energy distribution of a particle undergoing Brownian motion, which will be obtained in the following Sec. IV.

IV. CHARM QUARK IN AN EXPANDING PLASMA

A. Generic Kinetic Equation, Fokker-Planck Equation, Drag and Diffusion Coefficients

The operative equation for the Brownian motion of a test particle can be obtained from the Boltzmann equation, whose covariant form can be written as

$$p^\mu \partial_\mu D(x,p,t) = C\{D\},$$

where $p^\mu(E,p)$ is the four momentum of the test particle, $C\{D\}$ is the collision term and $D(x,p,t)$ is the distribution due to the motion of the particle. If we assume a uniform plasma, the Boltzmann equation becomes

$$\frac{\partial D}{\partial t} = \frac{C\{D\}}{E} = \left( \frac{\partial D}{\partial t} \right)_{\text{coll}}.$$  

We intend to consider only the elastic collisions of the test parton with other partons in the background. The rate of collisions $w(p,k)$ is given by

$$w(p,k) = \sum_{j=q,g} w^j(p,k),$$

where $w^j$ represents the collision rate of a test parton $i$ with other partons, $j$, in the plasma. The expression for $w^j$ can be written as

$$w^j(p,k) = \gamma_j \int \frac{d^3q}{(2\pi)^3} D_j(q) v_{\text{rel}} \sigma^j,$$

where $\gamma_j$ is the degeneracy factor, $v_{\text{rel}}$ is the relative velocity between the test particle and other participating partons $j$ from the background, $D_j$ is the phase space density for the species $j$ and $\sigma^j$ is the associated cross section. Due to this scattering the momentum of the test particle changes from $p$ to $p - k$. Then the collision term on the right-hand side of (8) can be written as

$$\left( \frac{\partial D}{\partial t} \right)_{\text{coll}} = \int d^3k \left[ w(p+k,k)D(p+k) - w(p,k)D(p) \right].$$
where the collision term has two contributions. The first one is the gain term where the transition rate \( w(p + k, k) \) represents the rate that a particle with momentum \( p + k \) loses momentum \( k \) due to the reaction with the medium. The second term represents the loss due to the scattering of a particle with momentum \( p \).

Now under the Landau approximation, i.e., most of the quark and gluon scattering is soft which implies that the function \( w(p, k) \) is sharply peaked at \( p \approx k \), one can expand the first term on the right-hand side of (10) by a Taylor series as

\[
w(p + k, k)D(p + k) \approx w(p, k)D(p) + k \cdot \frac{\partial}{\partial p}w(D) + \frac{1}{2} k_j k_j \frac{\partial^2}{\partial p_i \partial p_j}w(D) + \cdots .
\]

Combining (6), (9) and (10), one obtains a generic kinetic equation of the form

\[
\frac{\partial D}{\partial t} = \frac{\partial}{\partial p_i} [T_{1i}(p)D] + \frac{\partial^2}{\partial p_i \partial p_j} [B_{ij}(p)D],
\]

where the transport coefficients for momentum dispersion are given as

\[
T_{1i}(p) = \int d^3k w(p, k) k_i = \int d^3k w(p, k) (p - p')_i,
\]

\[
B_{ij}(p) = \frac{1}{2} \int d^3k w(p, k) k_i k_j = \frac{1}{2} \int d^3k w(p, k) (p - p')_i (p - p')_j.
\]

These transport coefficients in (12,13) depend on the distribution function, \( D \), through the transition probability, \( w(p, k) \) in (9), and can have different values depending upon the problem. The kinetic equation in (11) is the well known Landau equation [39], a non-linear integro-differential equation, which describes, in general, collision processes between two particles. It should therefore depend, in a generic sense, on the states of two participating particles in the collision process and hence on the product of two distribution functions making it non-linear in \( D \). Therefore, it requires to be solved in a self-consistent way, which is indeed a non-trivial task.

However, the problem can be simplified if one considers a large amount of weakly coupled particles in thermal equilibrium at a temperature, \( T \), constituting the heat bath in background and due to the fluctuation there can be some non-thermal but homogeneously distributed particles constituting foreground. It is assumed that the overall equilibrium of the bath will not be disturbed by the presence of such a few non-thermal particles. Because of their scarcity, one can also assume that these non-thermal particles will not interact among themselves but only with particles of the thermal bath in the background. This requires to replace the phase space distribution functions of the collision partners from heat bath appearing in (9) by time independent or thermal distribution, \( f_j(q) \). This will reduce the generic Landau kinetic equation (11), a non-linear integro-differential equation, to Fokker-Planck (FP) equation, a linear differential equation for the Brownian motion of the non-thermal particles in the foreground.

Now, one can write the transport coefficients in (12,13) for such a FP equation in terms of the two body matrix elements, \( M \), between a foreground and a background particles [12,17]:

\[
T_{1i}^{FP}(p) = \frac{1}{2E_p} \int \frac{d^3q}{(2\pi)^32E_q} \int \frac{d^3q'}{(2\pi)^32E_{q'}} \int \frac{d^3p'}{(2\pi)^32E_{p'}} \frac{1}{\gamma_c} \sum |M|^2 (2\pi)^4 \delta^4(p + q - q' - p') \times [p_i - p'_i] f(q) \tilde{f}(q) \equiv \langle \langle (p - p')_i \rangle \rangle,
\]

\[
B_{ij}^{FP}(p) = \frac{1}{2} \langle \langle (p - p')_i (p - p')_j \rangle \rangle.
\]

In our case, the incoming particle is a heavy quark which is different from the background. So, \( p' \) and \( q' \) represent the momenta of the incoming (outgoing) charm- and background light-quark/gluon, respectively. For each background species, there is a similar additive contribution to the collisional integral in (14). \( \gamma_c \) is the spin and color degeneracy factor of the foreground particle arising due to initial reaction channels. \( f(q) \) is the particle distribution of the thermal background, and \( \tilde{f}(q) = [1 \pm f(q)] \) corresponds a Bose enhancement/Pauli suppression factor for scattered background particles, as appropriate. Because of this thermal \( f(q) \) the contents of (12,13) are different from (14,15) and the charm quark in the foreground of a weakly coupled system is driven by a Brownian motion mechanism [12,17,18,21,41,42,43].

We are now set to study the momentum distribution of a charm quark undergoing Brownian motion and its relation with the transport coefficients. In absence of vectors other than \( p \) the values of \( T_{1i}^{FP} \) and \( B_{ij}^{FP} \), which depend functionally on \( p \) and the background temperature \( T \), must be of the form in Langevin theory [12,17,39]:

\[
T_{1i}^{FP}(p, T) = p_i A(p^2, T),
\]

\[
B_{ij}^{FP}(p, T) = \left( \delta_{ij} - \frac{p_i p_j}{p^2} \right) B_0(p^2, T) + \frac{p_i p_j}{p^2} B_1(p^2, T),
\]
where, $p^2 = p_i^2$, summation convention is always implied. $A$ is the drag, $B_0$ is the transverse diffusion and $B_1$ is the longitudinal diffusion coefficients. In terms of microscopic reaction amplitudes these functions are obtained \[12\, 17\] as

$$
A(p^2, T) = \langle\langle 1 \rangle\rangle - \frac{\langle\langle (\mathbf{p} \cdot \mathbf{p}') \rangle\rangle}{p'^2},
$$

(18)

$$
B_0(p^2, T) = \frac{1}{4} \left[ \frac{\langle\langle p'^2 \rangle\rangle - \langle\langle (\mathbf{p} \cdot \mathbf{p}')^2 \rangle\rangle}{p'^2} \right],
$$

(19)

$$
B_1(p^2, T) = \frac{1}{2} \left[ \frac{\langle\langle (\mathbf{p} \cdot \mathbf{p}')^2 \rangle\rangle}{p'^2} - 2\langle\langle (\mathbf{p} \cdot \mathbf{p}') \rangle\rangle + p^2 \langle\langle 1 \rangle\rangle \right].
$$

(20)

The averaging, $\langle\langle \cdots \rangle\rangle$, defined in \[14\] can further be simplified \[17\], by solving the kinematics in the centre-of-mass frame of the colliding particles, as

$$
\langle\langle F(p') \rangle\rangle = \frac{1}{512\pi^4 c E_p} \int_0^\infty \frac{d q}{E_q} \int_{-1}^1 d (\cos \chi) \sqrt{(s + M_C^2 - m^2_{g(q)})^2 - 4 s M_C^2} f(E_q) \int_{-1}^1 d \cos \Theta_{c.m.} \times \sum |M|^2 \int_0^{2\pi} d \phi_{c.m.} e^{i \mathbf{E}_{q'} \cdot \mathbf{F}(p')} F(p'),
$$

(21)

where $M_C$ is the mass of a charm quark, $s = (E_p + E_q)^2 - (\mathbf{p} + \mathbf{q})^2$, $E_{q'} = E_p + E_q - E_{q'}$ and $p'$ is a function of $\mathbf{p}$, $\mathbf{q}$ and $\Theta_{c.m.}$. $\mathcal{M}^2$ is the matrix elements \[12\] for scattering processes $Qg, Qq$, and $Q\bar{q}$, where $Q$ is a heavy quark and $g(q)$ is gluon (light quarks with 2-flavors). The expression in (21) is larger by a factor of 2 than the ones originally derived in (3.6) of Ref. \[12\]. Apart from this we have also introduced the thermal masses of quarks ($m_q$), gluons ($m_g$) and the quantum statistics, as appropriate.

The momentum and temperature dependence of the $A, B_0$ and $B_1$ are summarised in Figs. 1, 2 and 3 in Ref. \[17\] and we do not repeat them here and refer the reader to this work for details. The main finding is that these coefficients are momentum independent up to $p = 5$ GeV/c and beyond this there is a weak momentum dependence. Note that the detailed studies of the dynamics of charm quark, as discussed in Refs. \[12\], may only depend on $A$ and $B_0$, but perhaps not on $B_1$ in the phenomenological relevant momentum range. In the present calculation we are interested in the kinematical domain of $p = (5 - 10)$ GeV/c, for which one needs to solve the FP equation considering the momentum dependence of drag and diffusion coefficients. This will require to solve the FP equation numerically.

Instead, we assume the momentum independence of these coefficients in \[18\, 19\, 20\], which will correspond to a scenario where a particle travels through an ideal heat bath and undergoes linear damping (Rayleigh’s particle). So, these transport coefficients are expected to be largely determined by the properties of the heat bath and not so much by the nature of the test particle \[39\]. This is also a fairly good approximation which we will justify in the next section. Under this approximation, the transport coefficients in \[16\, 17\] become

$$
\mathcal{T}_{ij}^{FP} = p_i A,
$$

(22)

$$
\mathcal{B}_{ij}^{FP} = \delta_{ij} B_0 \equiv \delta_{ij} \mathcal{T}_{ij}^{FP},
$$

(23)

where $\mathcal{B}_0(\mathbf{p} \to 0, T) = \mathcal{B}_1(\mathbf{p} \to 0, T) \equiv \mathcal{T}_2^{FP}$. This could be viewed as a course-grained picture in which the finer details of the collisions have been averaged out over a large number of macroscopic situation (or over an ensemble). Then combining \[11\] and \[22\, 28\] one can write the FP equation as

$$
\frac{\partial D}{\partial t} = \frac{\partial}{\partial p_i} \left[ \mathcal{T}_{ij}^{FP} D \right] + \mathcal{T}_2^{FP} \left( \frac{\partial}{\partial \mathbf{p}} \right)^2 D.
$$

(24)

In the next Sec. \[14\, 13\] we obtain the time evolution of FP equation in a thermally evolving QGP.

### B. Time Evolution of Fokker-Planck Equation, Drag and Diffusion Coefficients in an Expanding Plasma

We assume that the background partonic system has achieved thermal equilibrium when the momenta of the background partons become locally isotropic. At the collider energies it has been estimated that $t_0 = 0.2 - 0.3$ fm/c. Beyond this point, the further expansion is assumed to be described by Bjorken scaling law \[43\]

$$
T(t) = t_0^{1/3} T_0/t^{1/3},
$$

(25)
FIG. 2: The momentum averaged \( \langle A(t) \rangle \) in (31) and momentum dependence \( A(p, t) \) in (30) of the drag coefficient of a charm quark in an expanding QGP with plasma parameters (see text) suitable for RHIC energy.

where \( T_0 \) is the initial temperature at which background has attained local thermal equilibrium.

We consider, for simplicity, the one dimensional problem, for which FP equation in (24) reduces to

\[
\frac{\partial D}{\partial t} = \frac{\partial}{\partial p} \left[ T_1^{FP} D \right] + T_2^{FP} \frac{\partial^2}{\partial p^2} D ,
\]

(26)

and as discussed in the previous Sec. [IV A] that the coupling between the Brownian particle and the bath is weak, the quantities \( T_1^{FP} \) and \( T_2^{FP} \) can also be written using the Langevin formalism \[39\] as

\[
T_1^{FP}(p) = \int dk \, w(p, k) k = \frac{\langle \delta p \rangle}{\delta t} = \langle F \rangle = pA
\]

(27)

\[
T_2^{FP} = \frac{1}{2} \frac{\langle (\delta p)^2 \rangle}{\delta t} \approx T T_1^{FP}.
\]

(28)

Now the work done by the drag force, \( T_1^{FP} \), acting on a test particle is

\[-dE = \langle F \rangle \cdot dL = T_1^{FP} \cdot dL ,\]

(29)

which can be related to the energy loss \[21, 25\] of a particle as

\[-\frac{dE}{dL} = T_1^{FP} = pA .\]

(30)

The drag coefficient is a very important quantity containing the dynamics of elastic collisions and it has a weak momentum dependence. Then one can average out the drag coefficient as

\[\langle A(p, T(t))\rangle \equiv A(T(t)) = \left\langle -\frac{1}{p} \frac{dE}{dL} \right\rangle ,\]

(31)

implying that the dynamics is solely determined by the collisions in the heat bath and independent of the initial momentum of the Brownian particle.

For averaging over the momentum the Boltzmann distribution and the differential energy loss rates \[12\] were used.

The time dependence of the drag coefficient comes from assuming a temperature, \( T(t) \), decreasing with time as the...
system expands, according to Bjorken scaling law \[43\] given in (25). We consider the initial temperature \( T_0 = 0.5 \) GeV, initial time \( t_0 = 0.3 \) fm/c and \( \alpha_s = 0.3 \) of the plasma for RHIC energy. In Fig. 2 the momentum averaged as well the momentum dependence of drag coefficient of a charm quark in the QGP phase of the expanding fireball is shown as a function of the time. As can be seen that the behavior of the momentum averaged drag coefficient (solid curve) is dominated by \( T^2/p \sim t^{-1/3} \) according to the scaling law. Now, it can also be seen that upto \( p = 10 \) GeV/c there is no significant difference between momentum averaged, \( \langle A(T(t)) \rangle \), and momentum dependence, \( A(p, T(t)) \), of drag coefficient and it has only a weak \( p \) dependence beyond \( p = 10 \) GeV/c. Since it decreases with moderately high values of \( p (\geq 15 \) GeV/c), the momentum averaged approximation of drag coefficient, \( \langle A(T(t)) \rangle \), would overestimate the actual \( A(p, T(t)) \) in this high momentum range. In our phenomenological approach the momentum independence of drag coefficient in (31) is a good approximation upto a moderate value of momentum \( p \leq 15 \) GeV/c.

Now, combining (27) and (28) we can write the diffusion coefficient as

\[ T_{FP}^2 = T \langle T_{FP}^1 \rangle = T A p. \]  

(32)

Once the drag coefficient is averaged out using the properties of heat bath, one can approximate \( p \) by the temperature \( T \) of the bath (as discussed earlier that it is independent of initial momentum of the Brownian particle) and \( A \) by its average value given in (31). This leads

\[ T_{FP}^2 = A(T(t))^2(t), \]  

(33)

which is also known as the Einstein relation \[39\] between drag and diffusion coefficients. In the left panel of Fig. 3 the diffusion coefficient obtained in (32) represented by solid line is displayed. It is found to have agreed quite well with the momentum independent diffusion coefficient (dashed line), \( B_0(p \to 0) \), in (19) with a factor 1/3 multiplied with it, because we consider the 1-dimensional scenario. As evident that momentum independence of diffusion coefficient is also a fairly good approximation.

Alternatively, one can also calculate the \( T_{FP}^2 \) in (32) by substituting \( A \) from (30) and averaging out the momentum dependence as

\[ \langle T_{FP}^2 \rangle = T \left\langle -\frac{dE}{dL} \right\rangle. \]  

(34)

In the right panel of Fig. 3 the average diffusion coefficient computed in (34) (filled triangle) is displayed. The momentum dependence transverse diffusion coefficient, \( B_0(p^2, T(t)) \), in (19) is also plotted for different momenta.
It can be seen that there is weak momentum dependence in $B_0$ in the momentum range $p = 5 - 20$ GeV/c. The momentum averaged values are in agreement with $B_0$ for higher momenta $p \geq 10$ GeV/c whereas it overestimates lower momenta $p < 10$ GeV/c. We will use both the approximations for diffusion coefficient in (33,34) in our purpose to obtain the momentum distribution below.

Combining (26) and (30), we find

$$\frac{\partial D}{\partial t} = A \frac{\partial}{\partial p} (pD) + D_F \frac{\partial^2 D}{\partial p^2},$$

(35)

where $D_F$, in general, has been used as the diffusion coefficient corresponding to (33,34), and $A$ is the averaged drag coefficient in (31).

Next we proceed with solving the above equation with the boundary condition

$$D(p, t) \big|_{t \to t_0} = \delta(p - p_0).$$

(36)

The solution of (35) can be found by making a Fourier transform of $D(p, t)$,

$$D(p, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{D}(x, t) e^{ipx} dx,$$

(37)

where the inverse transform is

$$\tilde{D}(x, t) = \int_{-\infty}^{+\infty} D(p, t) e^{-ipx} dp.$$  

(38)

Under the Fourier transform the corresponding initial condition follows from (36) and (38) as

$$\tilde{D}(x_0, t = t_0) = e^{-ip_0x_0}$$

(39)

where $x = x_0$ at $t = t_0$ is assumed.

Replacing $p \to i \frac{\partial}{\partial x}$ and $\frac{\partial}{\partial p} \to ix$, the Fourier transform of (35) becomes

$$i \frac{\partial \tilde{D}}{\partial t} + Ax \frac{\partial \tilde{D}}{\partial x} = -D_F x^2 \tilde{D}.$$  

(40)

This is a first order partial differential equation which may be solved by the method of characteristics [44]. The characteristic equation corresponding to (40) reads

$$\frac{\partial t}{\partial \tau} = \frac{\partial x}{\partial \tau} = -\frac{\partial \tilde{D}}{D_F x^2 \partial \tau}. $$

(41)

Using the boundary condition in (39) the solution of (40) can be obtained as

$$D(p, L) = \frac{1}{\sqrt{\pi \mathcal{W}(L)}} \exp \left[ -\left( \frac{p - p_0 e^{-\int_{t_0}^{t} A(t') dt'}}{\mathcal{W}(L)} \right)^2 \right],$$

(42)

where $\mathcal{W}(L)$ is given by

$$\mathcal{W}(L) = \left(4 \int_0^L D_F(t') \exp \left[ 2 \int_{t'}^{t''} A(t'') dt'' \right] dt' \right) \left[ \exp \left( -2 \int_0^L A(t') dt' \right) \right],$$

(43)

which is the probability distribution in momentum space. Since the plasma expands with the passage of time, we have used the length of the plasma, $L$ as the maximum time limit for the relativistic case ($\gamma v \sim 1$).

In Fig. 4 we show the momentum loss probability distribution $D(p, L)$ given in (42), of a charm quark with initial momentum, $p_0 = 5$ GeV/c, as a function of momentum $p$. The solid lines represent the distribution with the diffusion coefficient, $D_F = AT^2$, in (33) whereas the dashed lines with $\langle D_F \rangle$ in (34). Both set of curves are for two different expanded plasma lengths, $L = 1$ fm and 5 fm as indicated in Fig. 4. In general, the physical mechanism reflected in this Fig. 4 can be understood at initial time ($t_0$) or length of the plasma a momentum distribution is sharply peaked
at $p = p_0$, according to \[36\]. With passage of time (or distance traveled) the peak of the probability distribution is shifted towards smaller momentum, as a result of drag force is acting on the momentum of the charm quark indicating its most probable momentum loss due to elastic collisions in the medium. Moreover, the peak broadens slowly as a result of diffusion in momentum space, implying that a finite momentum dispersion sets in. As evident, with both the approximations in diffusion coefficient, only the momentum dispersion is affected while the peak positions remains unaltered, indicating that a drag force acting on the mean momentum of a charm quark is same. After plasma has expanded upto a length, $L = 1$ fm the charm quark loses 10% of its momentum whereas it is 25% at a expanded length of $L = 5$ fm. In the next subsec. IV C we will use this distribution to compute the total energy loss of a charm quark for an expanding plasma.

\section*{C. Energy-loss of a charm quark in an expanding plasma}

In the preceding subsec. IV B we obtain a momentum-loss distribution by solving the time evolution of FP equation in a thermally evolving plasma, which is modeled by an expanding fireball under conditions resembling central $Au-Au$ collisions at RHIC. The mean energy of a charm quark due to the elastic collisions in a expanding medium can be estimated as

$$\langle E \rangle = \int_0^\infty E \, D(p, L) \, dp . \quad (44)$$

The average energy loss due to elastic collisions in the medium is given by

$$\Delta E = \langle \epsilon \rangle = E_0 - \langle E \rangle , \quad (45)$$

where $E = m_{\perp} = \sqrt{p_{\perp}^2 + m^2}$ at the central rapidity region, $y = 0$.

Using the momentum loss distribution in (42) the total energy loss of a charm quark has been computed in (45). The numerical results for scaled collisional energy loss of a charm quark as a function of energy $E$ in an expanding plasma (solid line) is shown in the left panel of Fig. 5 for the plasma parameters $T_0 = 0.5$ GeV, $t_0 = 0.3$ fm/c, $\alpha_s = 0.3$ and expanded plasma length upto $L = 4$ fm. In the energy range, $E \sim 5 - 10$ GeV, the fractional collisional energy loss remains almost constant around a value 0.15 and the reason for which can be traced back to the momentum independence of the drag coefficient \[12, 17\] as discussed earlier. The corresponding scaled energy loss for a static
D. Quenching of Hadron Spectra in an expanding plasma

We assume that the geometry is described by a cylinder of radius $R$, as in the Boost invariant Bjorken model of nuclear collisions, and the parton moves in the transverse plane in the local rest frame. Then a parton created at a point $\mathbf{r}$ with an angle $\phi$ in the transverse direction will travel a distance

$$L(\phi) = \sqrt{R^2 - r^2 \sin^2 \phi} - r \cos \phi,$$

where $\cos \phi = \mathbf{\hat{v}} \cdot \mathbf{\hat{r}}$; $\mathbf{\hat{v}}$ is the velocity of the parton and $r = |\mathbf{r}|$. The value of the transverse dimension is taken as $R \sim 7$ fm.

The quenched spectrum convoluted with the transverse geometry of the partonic system can be written from as

$$\frac{dN_{\text{med}}}{d^2 p_\perp} = Q(p_\perp) \frac{dN_{\text{vac}}}{d^2 p_\perp} = \frac{1}{2\pi^2 R^2} \int_0^{2\pi} d\phi \int_0^R d^2 r \frac{dN(p_\perp + \Delta E)}{d^2 p_\perp}.$$

The $p_\perp$ distribution of charmed hadrons, $D$-mesons, produced in hadron collisions were experimentally found to be well described by the following simple parameterization as

$$\frac{dN_{\text{vac}}}{d^2 p_\perp} = C \left( \frac{1}{bM_C^2 + p_\perp^2} \right)^{n/2},$$

where $b = 1.4 \pm 0.3$, $n = 10.0 \pm 1.2$ and $M_C = 1.5$ GeV.
FIG. 6: Left panel: The ratio of charm to light quark quenching factors, $Q_H(\vec{p}_\perp)/Q_L(\vec{p}_\perp)$, as a function of transverse momentum $p_\perp$ with collisional energy loss. Right pane: The ratio of charm to light quark quenching factors, $Q_H(\vec{p}_\perp)/Q_L(\vec{p}_\perp)$, as a function of transverse momentum $p_\perp$ using both collisional and radiative energy losses.

The parameterization of the $p_\perp$ distribution exists in the literature which describes the first RHIC light hadroproduction data for moderately large values of $p_\perp$. In this case we consider the form given in Ref. 33 which reads as

$$dN_{\text{vac}} / d^2p_\perp = A \left( \frac{1}{p_0 + p_\perp} \right)^m,$$

(49)

where $m = 12.42$ and $p_0 = 1.71$ GeV/c.

The light hadron quenching using collisional energy loss rate was first anticipated to be of same order as that of radiative ones, though most of the previous studies insisted that the collisional energy loss is insufficient to describe the medium modifications of hadron spectra. We also here estimate the light hadron quenching (for details see Ref. 8) with the energy loss due to elastic collisions and the energy loss rate averaged over parton species reads

$$-\frac{dE}{dL} = \frac{4}{3} \left( 1 + \frac{9}{4} \right) \pi \alpha_s^2 T^2 \left( 1 + \frac{n_f}{6} \right) \log \left[ \frac{2^{n_f/2(6+n_f)} 0.92 \sqrt{ET}}{m_g} \right].$$

(50)

We now illustrate in the left panel of Fig. 6 the ratio of heavy to light quark quenching factors, $Q_H(\vec{p}_\perp)/Q_L(\vec{p}_\perp)$, as a function of transverse momentum $p_\perp$, using the collisional energy loss for plasma parameters $T_0 = 0.5$ GeV, $t_0 = 0.3$ fm/c, $\alpha_s = 0.3$ and the charm quark mass, $M_C = 1.5$ GeV. As discussed earlier this ratio may reflect heavy to light hadrons, $D/\pi$, ratio originating from the fragmentation of heavy and light quarks in heavy ion collisions. As shown the $D/\pi$ ratio is enhanced significantly as compared to $p-p$ collisions. The enhancement factor varies from 2.5 to 4 in the $p_\perp$ range (5-10) GeV/c, due to the uncertainties of different choices of parameter set to parameterize the heavy hadron spectra as depicted in (48). However, the ratio also strongly depends on the quenching of light quark jets. The numerical estimate shows that the quenching of charm quarks is about a half that of light quarks. The light quarks, for a given $p_\perp$, lose 10% of their energy after traversing a distance 1 fm and around 40% after 5 fm whereas the charm quark loses 5% at 1 fm and around 20% at 5 fm (Fig. 5). Because of the large mass of the charm quark the $D$ meson will be formed in shorter distance and hence charm quark would have less time to propagate in the medium before transforming into the $D$ meson. On the other hand, the light quarks would travel in the medium over a longer period and suffer more loss in energy than heavy quarks. The ratio is also found to be little more than that obtained earlier considering only the radiative energy loss with the appearance of the kinematical dead cone.
effect due to the finite heavy quark mass. This implies that the collision is one of the most dominant mechanisms of energy loss in the medium.

As shown that both collisional and radiative energy losses are of same order in magnitude it would be interesting to predict the $D/\pi$ ratio within our model considering radiative energy loss in addition to collisional one. Since neither the drag and diffusion coefficients have been calculated for other than collisional processes, it is not possible to infer the impact of radiative processes directly within our model. In order to circumvent our lack of knowledge of radiative processes in terms of the transport coefficients, we consider an alternative sets $K$ within our model, obtained by multiplying the transport coefficients by a factor $K=2$. It is our hope that the experimental data will allow us to fix an approximate value of $K$, if at all required. In the right panel of Fig. we plot the $D/\pi$ ratio for such a case with the same parameter set as before. As shown the ratio is little reduced compared to the collisional case and it varies from 2 to 3 within the $p_\perp$ range, (5-10) GeV/c. There is no striking change in the ratio mostly due to the cancellation of the introduced $K$ factor with reference to the collisional one for respective species. However, the small change can be attributed as the charm quark with the inclusion of radiative energy loss is relatively more quenched than that of the light quarks.

One may also add an interesting scenario, after the quark jet has hadronized to leading particles, they would scatter with hadronic matter before decoupling. Considering $\sigma_{D\pi} << \sigma_{\pi\pi}$, e.g., it is most likely that the heavy mesons would decouple quickly from the hadronic phase. Pions would interact with each other via resonance formation and also with other light hadrons. This might lead to further enhancement of $D/\pi$ ratio.

V. CONCLUSION

Apart from uncertainties in the various parameters describing the plasma and hadron spectra let us also have a look at some of the assumptions made in this work which may affect our findings. First, as discussed above, the momentum dependence of the drag and diffusion coefficients, containing the dynamics of the elastic collisions, has been averaged out. A major advantage of this is the simplicity of the resulting differential equation. Of course, this simplification can lead to some amount of uncertainty in the quenching factor. Secondly, the entire discussion is based on the one dimensional Fokker-Planck equation and the Bjorken model of the nuclear collision, which may not be a very realistic description here but can provide a useful information of the problem. However, extension to three dimension is indeed an ambitious goal, which may cause that many of the considerations of the present work will have to be revised.

In the present calculation the hadron spectra for both light and heavy quarks have directly been used to calculate the quenching factor, $Q(p_\perp)$. This is equivalent to assuming that a quark forms a hadron without much change in its energy. In order to calculate the effects of the parton energy loss on the quenching pattern of high $p_\perp$ partons in nuclear collisions, one should take into account the modification of the fragmentation function $F(x)$ of a leading quark resulting from many soft interactions of the hard partons in the medium. This also causes a significant energy loss of parton prior to hadronization and changes the kinematic variables of the fragmentation function $F$. This can also modify the quenching factor and thus $D/\pi$ ratio.

We show that the total collisional energy loss is almost same order as that of radiative energy loss for a static plasma. Considering the collisional energy loss rates we obtain a momentum loss distribution for charm quarks by solving the time evolution of Fokker-Planck equation for a thermally evolving plasma. The total energy loss for an expanding plasma is found to be reduced by a factor of 5 as compared to static case and does not depend linearly on the system size. A ratio of heavy to light hadrons $D/\pi$ ratio is also estimated. Though the collisions have different spectrum than radiation and therefore contribute in a similar way to the suppression factor than anticipated earlier.

We now eagerly wait for the experimental data and a detailed calculation have to be carried out before a realistic number for the value of the $D/\pi$ ratio can be presented. Nevertheless, the total collisional energy loss of a charm quark computed within our simplified model may imply that the collision is one of the important energy loss mechanisms in the medium for the energy range (5-10) GeV and this feature may be phenomenologically important. Also, the results for $D/\pi$ presented within this model is not definitive but can provide a very intuitive picture of the medium energy loss for partons in moderately large $p_\perp$. 

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