Submergence of the Sidebands in the Photon-assisted Tunneling through a Quantum Dot Weakly Coupled to Luttinger Liquid Leads

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Abstract

We study theoretically the photon-assisted tunneling through a quantum dot weakly coupled to Luttinger liquids (LL) leads, and find that the zero bias dc conductance is strongly affected by the interactions in the LL leads. In comparison with the system with Fermi liquid (FL) leads, the sideband peaks of the dc conductance become blurring for $\frac{1}{2} < g < 1$, and finally merge into the central peak for $g < \frac{1}{2}$ ($g$ is the interaction parameter in the LL leads). The sidebands are suppressed for LL leads with Coulomb interactions strong enough, and the conductance always appears as a single peak for any strength and frequency of the external time-dependent field. Furthermore, the quenching effect of the central peak for the FL case does not exist for $g < \frac{1}{2}$.

PACS numbers:73.63.-b,71.10.Pm, 73.40.Gk
The photon-assisted tunneling (PAT) through a quantum dot weakly coupled to two Fermi liquid leads (FL-QD-FL) has been studied intensively in the last decade. When two time-dependent fields with the same form $V \cos(\Omega t)$ are applied on the leads symmetrically, a series of additional peaks (called sidebands) occurs in the curve of the zero bias dc conductance vs. the gate voltage. These peaks locate at energies $\delta E = n\hbar\Omega$ away from the original resonant level. The $n$th sideband will be quenched as $\alpha$ equal to zeros of the Bessel function $J_n$, because the height of the $n$th peak is proportional to $J_n^2(\alpha)$ (where $\alpha = eV/\hbar\Omega$ is a dimensionless variable). Physically, all these features of PAT can be ascribed to characters of the electron occupations in the leads, or equivalently the characters of the density of states and the Fermi distribution of the electrons in the leads.

One dimensional interacting electron systems behave as Luttinger liquid (LL), for that theory predicts the power-law behavior of the density of states (DOS) of the electrons. What will happen for the sidebands features if the two FL leads are replaced by two LL leads? We expect that these features may be strongly affected by the specific characters of the DOS in LL leads. With this idea in mind, we study in this letter the PAT through a QD weakly coupled to two LL leads, i.e., a LL-QD-LL system.

In contrary with the FL-QD-FL system, we find that the sideband peaks of the dc conductance become blurring for $\frac{1}{2} < g < 1$, and finally merge into the central peak for $g < \frac{1}{2}$. The sidebands are suppressed for LL leads with Coulomb interactions strong enough, and the conductance appears as a single peak for any strength and frequency of the external time-dependent field. We call this the submergence of the sidebands. The contributions of each PAT process to the corresponding sidebands are diminished by the suppression of the DOS near Fermi energy, and shared due to the unusual DOS of the LL leads. Furthermore, in strong interaction case ($g < \frac{1}{2}$), the central peak consists not only of the direct tunneling process but also many other type PAT processes, so that it does not exist a single parameter $\alpha$ by which the central peak can be quenched.

We use the nonequilibrium-Green-function technique to calculate the average current through the quantum dot. Under the adiabatic approximation, the electron occupations in each lead remains unchanged. Thus the exact time-dependent Green functions in the uncoupled leads are

$$g_{L/R}^{r,a,<,>}(t_1, t_2) = \exp(-i \int_{t_2}^{t_1} \Delta_{L/R}(t) dt) g_{0,L/R}^{r,a,<,>}(t_1, t_2),$$

where $g_{0,L/R}^{r,a,<,>}(t_1, t_2)$ are the Green functions of the uncoupled leads without the time-
dependent field, and \( \Delta_{L/R}(t) = eV_{L/R} \cos(\Omega t) \). We take the units of \( \hbar = 1 \) hereafter.

The Hamiltonian of the system can be split into three parts: \( H = H_{\text{leads}} + H_d + H_T \), where \( H_{\text{leads}} = H_L + H_R \) denotes the left and right LL leads, \( H_d = \varepsilon d^+ d \) is the Hamiltonian of the QD, with \( \{d^+, d\} \) the creation/annihilation operators of the electron in the dot and only one energy level is considered. \( H_T \) is the tunneling Hamiltonian and can be written as:

\[
H_T = \sum_{\lambda=L,R} (t_\lambda d^+ \psi_\lambda + h.c),
\]

in which \( \{\psi_\lambda^+, \psi_\lambda\}(\lambda = L/R) \) are the Fermi operators at the end points of the left/right lead; \( \{t_\lambda\} \) is the tunneling constant. The spin indices have been suppressed since they are not important in the following discussion.

Using nonequilibrium-Green-function technique, the current through the quantum dot can be formulated as:

\[
J(t) = J_L(t) = -2e \text{Re} \int dt_1 \left[ t_L^\ast (G^r(t, t_1) g_L^\ast(t_1, t) + G^< (t, t_1) g_L^a(t_1, t)) t_L \right],
\]

where we have defined \( G^r(t, t_1) = -i\theta(t - t_1) \langle \{d(t), d^+ (t_1)\} \rangle_H \) and all the other Green functions with the standard forms.

Supposing \( J(t) = \sum_n J_n \exp(i n\Omega t) \), then

\[
J_n = -2e |t_L|^2 \text{Re} \sum_m \int \frac{d\omega}{2\pi} \left( G_{m,0}^r(\omega) g_{L,n,m}^\ast(\omega) + G_{m,0}^<(\omega) g_{L,n,m}^a(\omega) \right),
\]

in which the functions \( F_{n,m}(\omega) \equiv F_{n-m}(\omega + m\Omega) \), and \( F_n(\omega) \) is the double Fourier transformation of function \( F(t, t') \):

\[
F(t, t') = \sum_n \exp(i n\Omega t') \int \frac{d\omega}{2\pi} e^{-i\omega(t-t')} F_n(\omega).
\]

The retarded Green function of the QD in equation (4), \( G^r(t, t_1) \), can be calculated by using Dyson equation

\[
G^r(t, t_1) = g^r(t, t_1) + \int d\tau d\tau' g^r(t, \tau) \Sigma^{r/a} (\tau, \tau') G^r(\tau', t_1),
\]

where \( \Sigma^{r/a} (\tau, \tau') = \sum_{\lambda=L,R} |t_\lambda|^2 g^{r/a}_{\lambda}(\tau, \tau') \) is the irreducible self-energy, \( g^r(t, t_1) \) is the free retarded Green function of isolated dot. Notice that the Green functions of the QD depend on two time variables, not the time difference; so one should take the double Fourier transform, and obtain:
\[ G_{n,m}^r(\omega) = g_{n,m}^r(\omega) + \sum_{l_1,l_2} g_{n,l_1}^r(\omega) \Sigma_{l_1,l_2}^r(\omega) G_{l_2,m}^r(\omega). \]  

(7)

Neglecting the off-diagonal elements of \( \Sigma_{l_1,l_2}^r(\omega) \), we have

\[ G_{n,m}^r(\omega) = \frac{\delta_{n,m}}{\omega - \varepsilon + n\Omega - \sum_{l,\lambda} J_{l,m}^2(\alpha_l) |t_l|^2 g_{0,\lambda}^r(\omega - l\Omega)}, \]

(8)
in which \( J_l(\alpha) \) is the Bessel function, \( \varepsilon \) is the energy level in the dot and \( \alpha_{L/R} = \frac{eV_{L/R}}{2\hbar}. \) The lesser Green function can be calculated using Keldysh formalism:

\[ G_{n,m}^<_{l_1,l_2}(\omega) = \sum_{l_1,l_2} G_{n,l_1}^r(\omega) \Sigma_{l_1,l_2}^<(\omega) G_{l_2,m}^a(\omega). \]

(9)

Substituting equation (8) and (9) into equation (4), we can obtain the average current as:

\[ \langle J \rangle = e |t_L t_R|^2 \int \frac{d\omega}{2\pi} \left( \frac{\sum_m J_m^2(\alpha L) g_{0,L}^r(\omega - m\Omega)}{(\omega - \varepsilon)^2 + \left(\sum_m J_m^2(\alpha L) g_{0,L}^r(\omega - m\Omega)\right)^2} \right). \]

(10)

Equation (10) is the main result of our work. For the model discussed in this letter, the Green functions \( g_{0,L/R}^{<,>\rangle} \) are taken from [12]:

\[ g_{0,L}^{<\rangle}(\varepsilon) = \pm i \frac{T}{|t_L|^2} e^{\mp \frac{i\varepsilon}{2\pi}} \gamma_L(\varepsilon), \]

\[ g_{0,R}^{\langle\rangle}(\varepsilon) = \pm i \frac{T}{|t_R|^2} e^{\pm \frac{eV_b}{2\pi}} \gamma_R(\varepsilon - eV_b), \]

(11)

where \( T \) is the temperature, and \( V_b \) the dc bias voltage. \( \gamma_{L/R}(\varepsilon) \) is defined as \( \gamma_{L/R}(\varepsilon) = \frac{\Gamma_{L/R}}{2\pi|t_{L/R}|} \left( \frac{\pi T}{\Lambda} \right)^{1/g_{L/R} - 1} \frac{\left| \Gamma(1/2g_{L/R} + is/2\pi T) \right|^2}{\Gamma(1/g_{L/R})}, \) here \( \Gamma(x) \) is the Gamma function, \( g_{L/R} \) is the interaction parameters characterizing the left/right LL liquids, \( \Gamma_{L/R} \) describes the effective level broadening of the dot, proportional to \( |t_{L/R}|^2 \), and \( \Lambda \) is the high-energy cutoff or a band width. The Hamiltonian can describe a quantum dot coupled to the end points of either two half-infinite quantum wires or two edge states of fractional quantum Hall liquids with filling factor \( \nu = \frac{1}{2m+1} = g. \) The average current formula (Eq. (10)) covers a wide range of interaction in the leads, from noninteracting limit \( (g = 1) \) to strong interacting LL (small \( g) \).

What we are interested in is the dc conductance at zero bias. We consider the symmetry case, namely \( t_L = t_R = t, g_L = g_R = g, \) and \( \alpha_L = \alpha_R = \alpha. \) If the external fields are not very strong, equation (10) can be further simplified. Under the condition \( \frac{\Gamma_{L/R}}{2\pi} \left( \frac{\pi T}{\Lambda} \right)^{1/g - 1} \ln \left( \frac{\varepsilon}{2\pi T} \right) \ll \frac{\pi T}{\lambda} \)

(10)
we obtain the zero bias conductance after a contour integral (all the subscript $L/R$ are omitted):

\[
G = e^2 |t|^2 \frac{\left( \sum_m J_m^2(\alpha) g_0^> (\varepsilon - m\Omega) \right) \left( \sum_m J_m^2(\alpha) g_0^< (\varepsilon - m\Omega) \right) - (\leftrightarrow)}{\left( \sum_m J_m^2(\alpha) g_0^< (\varepsilon - m\Omega) \right) - (\sum_m J_m^2(\alpha) g_0^> (\varepsilon - m\Omega))}
\]

(12)

In the following, we use current formula (12) to study the zero bias conductance vs. the energy level of the QD, $G/G_{\text{max}}$ vs. $\frac{\varepsilon}{T}$, numerically. We consider the whole range of interaction of the leads, from noninteracting limit ($g = 1$) to strong interaction case ($g < \frac{1}{2}$), with much attention on the latter. Fig.1 presents the results for $\frac{\Omega}{T} = 5$, $\alpha = \frac{eV}{\Omega} = 1$ and $g = 1.0, 0.8, 0.5, 0.2$. The sideband peaks appears clearly for $g = 1$; become blurring for $g = 0.8$; merge into a single central peak for $g = 0.5$; and finally become narrower for $g = 0.2$.

In order to study the quenching behavior, we chose $\alpha$ equal to the smallest positive root of the zero order Bessel function, $J_0$. For the noninteracting limit case ($g = 1$), the central peak is completely quenched as expected. However, with the increase of the interaction or the decrease of $g$, the situation changes significantly. For $g = 0.8$, the relative heights of the sideband peaks are almost unchanged, but the valleys lifted. Both are shown in Fig.2(a). Most surprisingly, the quenching effect disappears completely for the strong interaction case (shown in Fig.2(b)): a broader peak with a flat plateau for $g = 0.5$; and a narrower single central peak for $g = 0.2$.

To further understand the PAT in LL-QD-LL for strong interaction case, we calculate $G/G_{\text{max}}$ vs. $\frac{\varepsilon}{T}$ for $g = 0.2$, with different strengths and frequencies of the external fields, shown in Fig.3. One can clearly see that all sideband peaks disappear. And the width of the single central peak becomes broader with the increase of the strength or the frequency of the fields, originated from the contributions of the multi-photon-assisted tunneling processes.

All features mentioned above can be ascribed to the power-law behavior of the DOS and the suppression of the electron tunneling near the Fermi energies of LL leads. In order to understand the underlying physics, we label $(m, n)$ to indicate two pairs of processes: (1) the electron tunneling through the dot with $m$ photons absorbed in the left and $n$ photons emitted in the right lead, and its reversal process; (2) same processes as in (1) except $m \rightleftharpoons n$. As an analogy of the $(m, n)$, one can consider the tunneling currents under two different finite bias voltages $(m\Omega + eV_b) - n\Omega$ and $(n\Omega + eV_b) - m\Omega$, without the time-dependent fields. The difference of these two net currents is approximately equal to the net
current of \((m, n)\), except the effect due to difference of the width of the energy level in QD. In the following, we shall discuss the PAT qualitatively with the help of this analogy.

It is well-known that for FL-QD-FL, the current under finite bias change abruptly at the Fermi energies in the two leads due to the abrupt change of the electron occupations near Fermi energies in the leads; thus two sharp peaks at \(\varepsilon = m\Omega\) and \(n\Omega\) obtained from the difference of the currents under two finite bias voltages, provided \(V_b\) is small. Similarly, the conductance from \((m, n)\) has the same peaks at \(\varepsilon = m\Omega\) and \(n\Omega\). Conductance at other energies is strongly suppressed due to the uniform distribution of the electrons.

However, for LL-QD-LL, the DOS of the leads behaves as \(\rho(\omega) \propto |\omega|^{\frac{1}{g-1}}\) as \(T \to 0\), the so-called power-law behavior of the DOS, so \(\rho'(\omega) \propto |\omega|^{\frac{1}{g-2}}\). For the weak interaction (\(g > \frac{1}{2}\)), \(\rho'(\omega) \to \infty\) as \(\omega \to 0\), thus there is also an abrupt change in the electron occupation near Fermi energy; two peaks at \(\varepsilon = n\Omega\) and \(m\Omega\) still remain, but the valley between them is lifted due to the nonuniform of the DOS. The contributions from all different \((m, n)\) processes give the total conductance with sideband peaks at \(\varepsilon = \pm \Omega, \pm 2\Omega\ldots\), and the valleys are lifted with the decrease of \(g\), consistent with the curves for \(g = 1, 0.8\) in Fig.1 and Fig.2(a).

For the strong interaction case (\(g < \frac{1}{2}\)), \(\rho'(\omega) \to 0\) as \(\omega \to 0\), thus no abrupt change in the electron occupation exists, and the difference of the currents under two finite bias voltages rises slowly at the outset and finally appears as a single peak centered at \(\frac{m+n}{2}\Omega\) approximately, so that the \((m, n)\) processes do not contribute significantly at \(\varepsilon = m\Omega\) and \(n\Omega\), instead, a peak at \(\varepsilon = \frac{m+n}{2}\Omega\) is produced. Furthermore, our calculation shows that processes such as \((m, m)\), always producing a single peak at \(\varepsilon = m\Omega\) for any values of \(g\), are not important, due to the fact that the direct tunneling near Fermi energy is strongly suppressed for strong interaction case. In Fig.4, we present the results from processes \((1, 1)\) and \((2, 2)\), showing a contribution almost 6 orders of magnitude smaller than the main processes. Moreover, one can see from Fig.4 that the main contributions to the conductance are originated from processes \((m, -n)\), with \(m, n = 0, 1, 2\ldots\), especially from that of \(m = n\), which make significant contribution at \(\varepsilon = 0\) but little at the sideband energies. That is why for the strong interaction case only one single peak at \(\varepsilon = 0\) exists, while all the sidebands are suppressed completely, as shown in Fig.3.

Now let us explain why the quenching of the central peak disappears for strong interaction case. Notice that the main contributions of the central peak come mainly from processes \((0, m)\) with \(m \in Z\) for weak interaction (\(g > 1/2\)); but \((m, -n)\) with \(m \approx n\) for strong interaction case (\(g < 1/2\)). Given \(\alpha\) the zeros of \(J_0\), all the processes like \((0, m)\) become
zero and contribute nothing to the central peak for weak interaction case, but the processes in the strong interaction case are not affected much. Thus the central peak is quenched as usual in the former, but will be lack in the latter (see Fig.2). Furthermore, since the conductance at $\varepsilon = 0$ come from many PAT processes, it does not exist a single parameter $\alpha$ by which all the processes, having main contributions to the central peak, can be neglected; thus there is no quenching of the central peak in case of LL leads with $g < \frac{1}{2}$.

Formula (10) can also be applied to the case with asymmetric external fields. If only one lead is applied with an ac field, we find that the photon-electron pumping effect still occurs for LL case. However, the negative current becomes smaller and smaller with the decrease of $g$ and the maximum of the current shifts towards the negative $\varepsilon$ (not shown here).

In conclusion, we have studied the PAT in the system of LL-QD-LL for the whole range of interaction of the LL leads and the crossover behavior. We find that the zero bias conductance is strongly renormalized by the interaction. In particular, for the strong interaction case ($g < \frac{1}{2}$), all sidebands merge into a single peak (we called it submergence of the sidebands), and the quenching effect also disappears completely. These novel features of PAT have been ascribed to the power-law behavior of the DOS and the suppression of the electron tunneling near the Fermi energies of LL leads. Since recent experiments have confirmed that the carbon nanotubes (CNT) do have some properties of the LL, especially the power-law feature of the DOS, we suggest a setup consisted of CNT-QD-CNT to study the PAT of the system. We hope our theoretical predictions can be tested experimentally.

This work was supported by NSFC under grant No.10074001. One of the authors (T.H.Lin) would also like to thank the support from the State Key Laboratory for Mesoscopic Physics in Peking University.

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FIGURE CAPTIONS

Fig. 1 \( G/G_{\text{max}} \) vs. \( \varepsilon/T \) for \( g = 1.0, 0.8, 0.5, 0.2 \). \( G \) is the zero bias conductance, \( G_{\text{max}} \) is the maximum of \( G \), \( \varepsilon \) is the energy level in the QD, \( T \) is the temperature, and \( g \) is the interaction parameters of the leads. The parameters of the time-dependent fields are: \( \Omega/T = 5 \) and \( \alpha = \frac{eV}{\hbar} = 1 \).

Fig. 2 \( G/G_{\text{max}} \) vs. \( \varepsilon/T \) for (a) \( g = 1.0, 0.8 \); (b) \( g = 0.5, 0.2 \), with \( \Omega/T = 10 \) and \( \alpha \) the first positive root of \( J_0 \).
Fig. 3 $G/G_{\text{max}}$ vs. $\varepsilon/T$ for $g = 0.2$. The parameters are chosen in (a) $\alpha = 0, 1, 3, 5$ and $\Omega/T = 5$; (b) $\alpha = 1$ and $\Omega/T = 0, 5, 10, 15$.

Fig. 4 The contributions of some $(m, n)$ processes for $g = 0.2$, $\Omega/T = 10$, and $\alpha$ the first positive root of $J_0$. $G_{\text{max}}$ is the maximum value of the total conductance.
Fig. 1
Fig. 2
Fig. 3
Fig. 4