Fate of Kaluza-Klein Black Holes: Evaporation or Excision?

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We study evaporation process of black strings which are typical examples of Kaluza-Klein black holes. Taking into account the backreaction of the Hawking radiation, we deduce the evolution equation for the radion field. By solving the evolution equation, we find that the shape of the internal space is necked by the Hawking radiation and the amount of the deformation becomes large as the evaporation proceeds. Based on this analysis, we speculate that the Kaluza-Klein black holes would be excised from the Kaluza-Klein spacetime before the onset of the Gregory-Laflamme instability and therefore before the evaporation.

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I. INTRODUCTION

In the conventional 4-dimensional spacetime, a black hole with the horizon size \( r_H \) is considered as a blackbody which emits the Hawking radiation \( L \) with the luminosity

\[
L \sim T_{BH}^2
\]

where \( T_{BH} = \frac{1}{4\pi r_H} \) is the temperature of the black hole. Because of this radiation, the size of the black hole horizon gradually shrinks as

\[
\frac{dr_H}{dt} \sim -\frac{G}{r_H^2}
\]

where \( G \) is the Newton constant. It is believed that the black hole evaporates eventually.

From the point of view of the superstring theory, it is natural to consider the higher dimensional spacetime. In the higher dimensional spacetime, the event horizon of the black hole would be the direct product of the usual event horizon and the compact internal space. The size of internal space could be much larger than the Planck length in the large extra dimension scenario. The internal space must be stabilized with some mechanism so that the theory does not contradict experiments. Therefore, naively, we envisage the same picture as the 4-dimensional one for the evaporation process.

However, things are not so simple. Let us consider a black string, which is the simplest example of Kaluza-Klein black holes, with a circle \( S^1 \) as the compact internal space. The evaporation processes of the black strings could be different from the 4-dimensional one due to the Gregory-Laflamme instability (see also and references therein). In fact, it is known that the black string is unstable when the horizon radius is smaller than the length scale of compactification. This instability changes the spacetime structure.

Soon after the onset of the Gregory-Laflamme instability, it is believed that the higher dimensional spherically symmetric black hole, the so-called Myers-Perry black hole, is formed. Once this transition is assumed, the subsequent process is the 5-dimensional one. The evaporation process taking into account this instability is depicted in Fig. 1. However, the detail of the spacetime dynamics after the Gregory-Laflamme instability is not well known. Hence, we will only consider the evaporation process before the Gregory-Laflamme instability in this paper.

Moreover, the dynamics of the internal space, the so-called radion, can not be neglected all the time. It is reasonable to assume that the radion is constant at low energy. However, for sufficiently small black holes, this would not be true. Indeed, when the mass scale of the stabilized radion becomes less than \( 1/r_H \), the radion is effectively free. Hence, in contrast to the conventional assumption that the radion is always fixed during the evaporation process, we take an attitude that the radion

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is crucial in understanding the evaporation process of the black string. In fact, the backreaction of the Hawking radiation could change the radion. It is therefore natural to expect the radion plays an important role in the evaporation process of the black strings. Previously, we have investigated the interplay between the radion and the Gregory-Laflamme instability \cite{6, 7, 8}. To the best of our knowledge, however, the interplay between the Hawking radiation and the radion dynamics has not been considered at all (except for our preliminary report \cite{3}).

In this paper, we study the role of the radion dynamics in the evaporation process of the black string and show that the internal space is deformed inhomogeneously by the Hawking radiation. Based on this result, we can speculate the fate of the Kaluza-Klein black holes as follows. The black hole shrinks due to Hawking radiation. At the same time, the internal space is necked. This process continues quasi-stationary. And, the spacetime is eventually pinched at the radius close to the black hole horizon. Consequently, assuming the resolution of the singularity, the black string would be excised from the spacetime. We stress that the black string excision from the spacetime could occur before the evaporation through the conventional process.

The organization of this paper is as follows. In section II, we introduce the Kaluza-Klein black hole solutions. In order to make the problem tractable, we do dimensional reduction to 2-dimensional problem. In section III, we derive the evolution equation for the radion field with the backreaction of the Hawking radiation. In section IV, we analyze the radion dynamics and speculate the fate of the Kaluza-Klein black holes. The final section is devoted to the conclusion.

### II. DIMENSIONAL REDUCTION OF KALUZA-KLEIN BLACK HOLES

In this section, first of all, we introduce the black string solutions as the simplest example of the Kaluza-Klein black holes. Then, we show that the black string system can be approximated by the 2-dimensional effective action where we can easily incorporate the backreaction of the Hawking radiation.

Let us consider the $D$-dimensional Einstein gravity with a massless scalar field $f$, $g$ where $D$ is the number of spacetime dimensions.

\begin{equation}
S = M_D^{D-2} \int d^Dx \sqrt{-g(D)} R^{(D)} + \int d^Dx \sqrt{-g(D)} \left[-\frac{1}{2} (\nabla f)^2\right], \tag{3}
\end{equation}

where $g_{MN}^{(D)}$ $(M, N = 0, 1, \ldots, D - 1)$ and $R^{(D)}$ are $D$-dimensional metric and Ricci scalar, respectively. Here, $M_D$ denotes the $D$-dimensional Planck mass. The equations of motion of the action (3) are given by

\begin{equation}
G_{MN} = \frac{1}{M_D^{D-2}} \left[ \nabla_M f \nabla_N f - \frac{1}{2} g_{MN} (\nabla f)^2 \right], \tag{4}
\end{equation}

\begin{equation}
\Box f = 0. \tag{5}
\end{equation}

In the vacuum case $f = 0$, it is easy to find a black string solution, which is an example of Kaluza-Klein black holes. The result is given by

\begin{equation}
ds^2 = -V dt^2 + V^{-1} dr^2 + r^2 d\Omega_{n-2}^2 + dy^2, \quad V = 1 - \left(\frac{r H}{f} \right)^{n-3}, \quad n \geq 4, \tag{6}
\end{equation}

where $d\Omega_{n-2}$ is the line element of the $(n-2)$-dimensional sphere $S^{n-2}$. Here, we defined $n = D - 1$, that is, $n$ is the dimension of the external space. The direction of $y$ is compactified with the length $L$. We would like to consider the evaporation process of this black string. Unfortunately, it is difficult to perform semi-classical analysis without any approximation. Therefore, we reduce the degrees of freedom of the system by dimensional reduction. After that, we quantize the system.

First of all, we show that the system can be reduced to the 3-dimensional one where we can show the existence of the Gregory-Laflamme instability. The black string spacetime (6) has $SO(n-1)$ symmetry. We assume the evaporation proceeds keeping this symmetry. In other words, we assume the functional form of the scalar field as $f = f(x^\mu)$ and the metric as

\begin{equation}
ds^2 = g^{(3)}_{\mu\nu}(x^\mu) dx^\mu dx^\nu + M_D^{-2} e^{-\frac{n}{2} \phi(x^\mu)} d\Omega_{n-2}^2, \tag{7}
\end{equation}

where $\mu, \nu = 0, 1, 2$. Then, in the action (3), we can carry out the integration of angular variables and the result becomes

\begin{equation}
S = \omega_{n-2} M_D \int d^3x \sqrt{-g^{(3)}} e^{-2\phi} \times \left[ R^{(3)} + \frac{n-3}{n-2} (\nabla \phi)^2 + M_D^2 (n-2)(n-3) e^{-\phi} \right]
\end{equation}

\begin{equation}
+ \int d^3x \sqrt{-g^{(3)}} e^{-2\phi} \left[-\frac{1}{2} (\nabla f)^2\right], \tag{8}
\end{equation}

where $R^{(3)}$ is the 3-dimensional Ricci scalar and $\omega_{n-2}$ is the area of $(n-2)$-dimensional unit sphere,

\begin{equation}
\omega_{n-2} = \frac{2\pi(n-1/2)}{\Gamma((n-1)/2)}. \tag{9}
\end{equation}

Thus, we have obtained the 3-dimensional effective action. Note that the formal limit $n \to \infty$ gives the model similar to that proposed by Callan et al \cite{10} as is shown in \cite{11}. We have numerically analyzed the classical stability using this action and confirmed the existence of the Gregory-Laflamme instability \cite{3, 8}.

Let us further reduce the system into the 2-dimensional one where we can easily incorporate the backreaction of
the Hawking radiation. We assume that the horizon radius \( r_H \) is much larger than the length scale of compactification \( L \). In that case, the Gregory-Laflamme instability is not relevant. and, we can ignore the Kaluza-Klein modes. Then we can take the functional form of the scalar field as \( f = f(x^a) \) and the metric as
\[
g^{(3)}_{\mu \nu} dx^\mu dx^\nu = g_{ab}(x^a) dx^a dx^b + e^{-2x(x^a)} dy^2 ,
\]
where \( a, b = 0, 1 \). The radion is denoted by \( \chi \), which describe the dynamics of the internal space. Then, in the 3-dimensional action \( \mathcal{S} \), we can carry out the integration of \( y \). The resultant action is
\[
S = \omega_{n-2} M_D L \int d^2 x \sqrt{-g} e^{-2\phi - \chi} \times \left[ R + 4 \frac{n-3}{n-2} (\nabla \phi)^2 + 4 \nabla \phi \cdot \nabla \chi \\
+(n-2)(n-3)M_D^2 e^{-2\chi} \right] \\
+ \int d^2 x \sqrt{-g} e^{-2\phi - \chi} \left[ -\frac{1}{2} (\nabla f)^2 \right] ,
\]
where \( R \) is the 2-dimensional Ricci scalar. Here, we have rescaled the field \( f \) to absorb the volume factor. The solutions of the equations of motion derived from the action \( \mathcal{S} \) can be read off from the black string solution \( \mathcal{B} \) as
\[
g_{ab} dx^a dx^b = -V dt^2 + V^{-1} dr^2 ,
\]
\[
M_D^{-1} e^{-\frac{n-3}{2} \phi} = \kappa ,
\]
\[
\chi = 0 ,
\]
\[
f = 0 .
\]
We shall study the evaporation process of the black string with the above 2-dimensional action \( \mathcal{B} \). Of course, the approximation we performed may not give the quantitatively correct answer. However, we believe the qualitative result will not be altered.

III. BACKREACTION OF HAWKING RADIATION

In this section, we derive the master equation for the radion field with the backreaction of the Hawking radiation.

The Hawking radiation is the quantum effect of the scalar field in Eq. \( \mathcal{B} \),
\[
S_m[\phi, \chi, f] = \int d^2 x \sqrt{-g} e^{-2\phi - \chi} \left[ -\frac{1}{2} (\nabla f)^2 \right] .
\]
The other fields are treated as classical ones. This semiclassical approximation can be implemented by considering the effective action
\[
W[\phi, \chi] = -i \ln \left[ \mathcal{D} f e^{iS_m[\phi, \chi, f]} \right] .
\]
What we need to do is to replace the action \( \mathcal{B} \) by the above effective action \( \mathcal{W} \) and derive equations of motion. Let us define
\[
T_{ab} = -2 \frac{\delta W}{\sqrt{-g} \delta g^{ab}} ,
\]
\[
X = \frac{1}{\sqrt{-g}} \frac{\delta W}{\delta \phi} ,
\]
\[
Y = \frac{1}{\sqrt{-g}} \frac{\delta W}{\delta \chi} .
\]
Fortunately, the explicit functional forms of \( X \) and \( Y \) are not necessary, because they disappear in the final equation. The variation with respect to the metric gives
\[
2 \nabla_a \nabla_b \phi + \nabla_a \nabla_b \chi \\
- \frac{4}{n-2} \nabla_a \phi \nabla_b \phi - \nabla_a \chi \nabla_b \chi + g_{ab} \{ -2 \nabla^2 \phi \\
- \nabla^2 \chi + 2 \frac{n-1}{n-2} (\nabla \phi)^2 + 2 \nabla \phi \cdot \nabla \chi + (\nabla \chi)^2 \\
- \frac{1}{2} (n-2)(n-3)M_D^2 e^{-\frac{n-1}{2} \phi} \} = \kappa e^{2\phi + \chi} X_{ab} ,
\]
where \( \kappa^{-1} = 2 \omega_{n-2} M_D L \). Notice that the Einstein tensor in 2-dimensions vanish. From the variation with respect to the dilaton \( \phi \), we obtain
\[
R + 4 \frac{n-3}{n-2} \nabla^2 \phi + 2 \nabla^2 \chi \\
n - \frac{4}{n-2} (\nabla \phi)^2 - 4 \frac{n-3}{n-2} \nabla \phi \cdot \nabla \chi - 2 (\nabla \chi)^2 \\
+ (n-3)(n-4)M_D^2 e^{-\frac{n-1}{2} \phi} = \kappa e^{2\phi + \chi} X ,
\]
and, finally, from the variation with respect to the radion \( \chi \), we have
\[
R + 4 \nabla^2 \phi - 4 \frac{n-1}{n-2} (\nabla \phi)^2 \\
+ (n-2)(n-3)M_D^2 e^{-\frac{n-1}{2} \phi} = 2 \kappa e^{2\phi + \chi} Y .
\]
The above five equations determines the five variables \( g_{ab}, \phi \) and \( \chi \).

Now, we derive the evolution equation for the radion. The trace of Eq. \( \mathcal{B} \) becomes
\[
-2 \nabla^2 \phi + \nabla^2 \chi + 4 (\nabla \phi)^2 + 4 \nabla \phi \cdot \nabla \chi + (\nabla \chi)^2 \\
- (n-2)(n-3)M_D^2 e^{-\frac{n-1}{2} \phi} = \kappa e^{2\phi + \chi} T_{ab} .
\]
From Eq. \( \mathcal{B} \) and \( \mathcal{W} \), we can eliminate the curvature as
\[
-2 \nabla^2 \phi + (n-2) \nabla^2 \chi + 4 (\nabla \phi)^2 - 2(n-3) \nabla \phi \cdot \nabla \chi \\
- (n-2)(\nabla \chi)^2 - (n-2)(n-3)M_D^2 e^{-\frac{n-1}{2} \phi} \\
= \frac{(n-2)\kappa}{2} e^{2\phi + \chi} (X - 2Y) .
\]
From Eq. (24) and (25), we obtain

\[(\nabla \chi)^2 - \nabla^2 \chi + 2\nabla \phi \cdot \nabla \chi = \frac{(n-2)\kappa}{(n-1)} e^{2\phi + \chi} T^a_a - X + 2Y \cdot \] (26)

The fields \(\phi\) and \(\chi\) appear in the classical action \(S_{\phi} = S_{\phi}(g, f, 2\phi + \chi)\). Hence, they have to come into the effective action with the same combination, \(W = W(g, 2\phi + \chi)\). Then, from Eq. (11) and (20), we get the relation \(X = 2Y\). Thus, from Eq. (26), we obtain the equation

\[(\nabla \chi)^2 - \Box \chi + 2\nabla \phi \cdot \nabla \chi = \frac{\kappa}{(n-1)} e^{2\phi + \chi} T^a_a \cdot \] (27)

As Eq. (27) is the nonlinear equation, it is difficult to solve it exactly. Now, we consider the case \(r_H \gg M_D^{-1}\). That implies the Hawking temperature \(T_{BH}\) is low and the Hawking flux is weak. Then, we can treat the back reaction of the Hawking radiation as the perturbation and regard \(T_{oh}\) as the source of perturbation. We take Eqs. (12) \(\sim\) (14) as the background solution. Then, Eq. (27) becomes

\[- \Box \delta \chi + 2\nabla \phi \cdot \nabla \delta \chi = \frac{\kappa}{(n-1)} e^{2\phi + \chi} T^a_a \cdot \] (28)

Here, it should be noted that \(\phi\) takes the background value. The above Eq. (28) is a master equation for the radion perturbation \(\delta \chi\). Note that the perturbed radion is gauge invariant since the background vanishes, \(\chi = 0\). Therefore, the gauge mode cannot appear in the master equation (28). It is worth to note that, the master equation (28) is a wave equation with a source term. Since the classical action of the matter field (16) is Weyl invariant, the trace part of the energy-momentum tensor \(T^a_a\) should be zero classically. However, Weyl symmetry has the anomaly in the quantum theory. The trace anomaly is well known and given by

\[T^a_a = \frac{R}{24\pi} \cdot \] (29)

It should be stressed that this trace anomaly is intimately related to the Hawking radiation [12]. Thus, it turned out that the Hawking radiation induces the radion dynamics.

IV. FATE OF KALUZA-KLEIN BLACK HOLES

In this section, we analyze the master equation (28) and show how the radion is deformed. Based on this linear analysis, we speculate the fate of the black strings.

In the Schwarzschild coordinates \((t, r)\), the master equation (28) becomes

\[\delta \chi_{,tt} - V^2 \delta \chi_{,rr} + (2V\phi_{,r} - V_{,r})V \delta \chi_{,r} = F(r) \cdot \] (30)

where \(F(r)\) is the external force,

\[F(r) \equiv \frac{\kappa}{(n-1)} V e^{2\phi} T^a_a - \frac{\kappa}{24(n-1)\pi} e^{2\phi} V V_{rr} \cdot \] (31)

It is convenient to define the tortoise coordinate,

\[r_* = \int \frac{dr}{V} \cdot \] (32)

and rewrite the master equation (28) as

\[\delta \chi_{,tt} - \delta \chi_{,rr} + 2\phi_{,r} \delta \chi_{,r} = F(r_*) \cdot \] (33)

where \(r_* \equiv \partial / \partial r_* \) and \(F(r_*) \equiv F(r(r_*))\).

It is useful to see the explicit functional form of the source term. Notice that the asymptotic forms of the \(r_*(r)\) is

\[r_* \sim \begin{cases} r \\ \frac{1}{n-2} r_H \ln \left( \frac{r}{r_H} \right) \end{cases} \quad \left( r \to \infty \right) \quad \left( r \sim r_H \right) \cdot \] (34)

and its inverse function \(r(r_*)\) is given by

\[r \sim r_* \sim \frac{\kappa}{(n-3) \pi} r_*/r_H \quad \left( r_* \to \infty \right) \quad \left( r_* \to -\infty \right) \cdot \] (35)

Hence, from Eq. (31) and (33), the asymptotic form of the \(F(r_*)\) can be deduced as

\[F(r_) \sim \frac{(n-2) (n-3) \kappa}{24\pi(n-1) M_D^{n-2} r_H^n} \left( \frac{r_H}{r_*} \right)^{n-3} \quad \left( r_* \to \infty \right) \cdot \] (36)

and

\[F(r_*) \sim \frac{(n-2) (n-3)^2 \kappa}{24\pi(n-1) M_D^{n-2} r_H^n} e^{(n-3) r_*/r_H} \quad \left( r_* \to -\infty \right) \cdot \] (37)

From the above expression, we can depict the figure of the function \(F(r_*)\) as in Fig. 2. We can see that this function has the height \(\sim \kappa / (M_D^{n-2} r_H^n)\) and width \(\sim r_H\).
The effective mass term in Eq. (40) can be calculated as

$$\delta \chi(t = 0, r_*) = \delta \chi(t = 0, r_*) = 0.$$  \hspace{1cm} (38)

Then, from Eq. (38), we can deduce the radion dynamics for small $t$,

$$\delta \chi = \frac{1}{2} F(r_*) t^2.$$  \hspace{1cm} (39)

We gave a schematic picture of the radion dynamics in Fig. 3. However, this is just the initial motion. To see what occurs subsequently, let us define a new variable $u = e^{-\phi} \delta \chi$. In terms of the new variable, we can write the master equation (33) as

$$u_{,tt} - u_{,ss} + m^2_{\text{eff}} u = e^{-\phi} F(r_*) .$$  \hspace{1cm} (40)

The effective mass term in Eq. (40) can be calculated as

$$m^2_{\text{eff}} = e^\phi (\gamma e^{-\phi},)_{,ss}$$

$$= \left( \frac{n}{2} - 1 \right) \frac{V}{r^2} \left[ (n - 3) - \left( \frac{n}{2} - 1 \right) V \right].$$  \hspace{1cm} (41)

The point is that the effective mass $m^2_{\text{eff}}$ is positive everywhere. The order of magnitude can be estimated as $m^2_{\text{eff}} \sim 1/r_H^2$. From Eq. (40), we see that the mass term prevent the motion of the radion. As a consequence, the radion approaches the static equilibrium solution. The amount of the deformation can be evaluated as

$$\delta \chi = \frac{F(r_*)}{m^2_{\text{eff}}}$$

$$= \frac{(n - 3)\kappa (1 - V)}{6\pi(n - 1)(M_D r)^{n - 2} [2(n - 3) - (n - 2)V]}.$$  \hspace{1cm} (42)

The profile of the radion is inhomogeneous as can be seen from the formula (42). More accurately, we have solved Eq. (50) numerically and found that there is a maximum very close to the horizon. From Eq. (12), the maximal value of $\delta \chi$ can be estimated as $\delta \chi \sim (M_D r_H)^1 r_H/L$. Thus, we found that the backreaction of the Hawking radiation, which makes the black hole shrink, also deforms the radion inhomogeneously.

We now speculate the fate of the black string. The internal space is pushed by the Hawking radiation and tend to collapse. However, the effective mass term sustain the internal space. As a result, the internal space is slightly necked and stabilized there. In this quasi-stationary phase, the black hole is shrinking due to the loss of the mass in the Hawking radiation. As the horizon radius $r_H$ is decreasing, the size of the neck is also shrinking adiabatically as $1/r_H^{n-2}$. Therefore, at the end of the day, the internal space would be pinched at some radius close to the horizon. The topology of the resultant singular surface is a $(n-2)$-dimensional sphere. Assuming the resolution of the singularity, we can conclude that the black string is eventually excised from Kaluza-Klein spacetime. Therefore, we get the picture sketched in Fig. 4. Of course, there is a possibility the conventional evaporation ends before the excision. Even in that case, the excision will occur and the evaporating region is hidden behind the pinched point. Therefore, this excision process may add a complication to the information loss problem in black hole physics [13].

There are several caveats. Firstly, there may be a Gregory-Laflamme instability before excision depending on $L$. We have assumed $L$ is sufficiently small. Secondly, the radion is stabilized in the realistic models, although we analyzed as if the radion has not been stabilized at all. However, when Hawking temperature becomes much larger than the scale of the radion stabilization, the radion is effectively free. We are looking at this stage. Thirdly, we used the dimensional reduction method to simplify the problem. However, in quantization, the connection of 2-dimensional models with a real world becomes unclear due to the existence of the dimensional reduction anomaly [14]. For complete analysis, we should
calculate all of the anomalous terms and add them to the 2-dimensional effective action (17). We leave it for future work. Finally, we may have to worry about the fate of the bubble like spacetime after the excision of the black string. Although we do not have a conclusive answer to this important question, it would be reasonable to imagine such a tiny hole disappears and nothing remains there.

V. CONCLUSION

We have studied the evaporation process of the black strings as the simplest example of Kaluza-Klein black holes with focusing on the role of the internal space. We have obtained the master equation for the radion field with the backreaction of the Hawking radiation. It turned out that the internal space is deformed inhomogeneously (Fig.3). Based on this result, we speculated that the black string would be excised from the spacetime due to the non-trivial dynamics of the radion (Fig.4). This disappearance of the black string is different from the evaporation process naively considered so far. To further confirm the excision of black strings, we need to analyze the non-linear radion dynamics numerically.

We have considered the Kaluza-Klein black holes in the spacetime with the $S^1$ compactification. It is reasonable to believe that black holes in the spacetime with the more realistic compactification also give the same behavior as that obtained in the present analysis. So, it is intriguing to study general Kaluza-Klein black holes in the string theory. There may be a relation between our result and recently proposed mechanisms in the context of the closed tachyon condensation [15].

We should note that there may exist other different types of Kaluza-Klein black holes. In the context of the braneworld, the localized black hole on the brane would be possible (see the recent review [16] and references therein). In this context, a very interesting excision mechanism of the black hole is discussed in [17, 18, 19]. Another possible type of Kaluza-Klein black holes is the squashed Kaluza-Klein black hole [20]. The squashed Kaluza-Klein black hole looks like 5-dimensional black hole in the vicinity of the horizon, however, the spacetime far from the black hole is locally that of the black string. It is also of interest to consider the evaporation process of squashed Kaluza-Klein black holes [21].

The implication of our result in cosmology also deserve investigations. In particular, cosmological consequences of the evaporation of primordial black holes should be reconsidered in the light of our results.

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