Broadband Homonuclear Decoupling

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We present a solution to the problem of broadband decoupling of a coupled homonuclear two-spin system. We describe a pulse sequence that creates an effective field perpendicular to the coupling interaction with a magnitude proportional to the chemical shift of the spins over a broad range of chemical shifts. When the chemical shift difference is imprinted on the perpendicular field is greater than the coupling between the spins, we get effective decoupling. The pulse sequence may be useful in various NMR applications.

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Coupled spin systems are ubiquitous in nuclear magnetic resonance (NMR) \[1\][2]. In NMR, the coupling between spins causes multiplets in the spectrum of the Free Induction Decay (FID) signal \[2\], which complicates the interpretation of the data. For example to determine the structure of a large protein, it is desirable that these multiplets be collapsed into singlets, thereby increasing the resolution and the intensity of the peaks.

If the coupled spins belong to different spin species (heteronuclear spins), their Larmor frequencies are well separated. The underlying concept in many commonly used decoupling techniques \[3\][4][7] is inverting one spin rapidly while observing the other spin. Because the Larmor frequencies are very different, the radio frequency (rf) radiation can invert one spin species without affecting the other.

Larmor frequencies for Homonuclear spins are only separated by the chemical shifts, which are caused by different chemical environments around each spin that alters the strength of the magnetic field at their locations. Even at high magnetic fields used in the state of the art NMR experiments, the smallest chemical shift difference can be of the same order as the strength of the couplings between the spins. Therefore, it is very hard to selectively invert one spin without disturbing its neighbors, especially when the precise chemical shifts are unknown and are the parameters to be determined. There is some success in the use of band-selective shaped pulses for decoupling of a coupled IS homonuclear spin system, where spin $I$ belongs to a band of frequencies which is well separated from spin $S$ that lies in a distinct band (see \[8\], [9], [10]). This may be the case in many practical applications. However, when there is no information about the location of the chemical shifts, it is not clear how to do the selective pulses. Here, assuming almost no knowledge about the chemical shifts, we present a new pulse sequence that decouples homonuclear spins without the need for selective inversion over a broad range of frequencies.

The Hamiltonian $H_0$ of a system of two spins $I$ and $S$ coupled together in the rotating frame at the Larmor frequency of the spin species is

$$ H_0 = \omega_I I_z + \omega_S S_z + 2\pi J I_z S_z $$

(1)

where $\omega_I$ and $\omega_S$ are the chemical shifts of the spins $I$ and $S$. $I_\beta$ and $S_\beta$, $\beta = x, y, z$ are the spin operators. $J$ is the strength of the scalar coupling. This coupling, also named indirect interaction due to shared electrons in the bonding of different atoms, has the general form $2\pi J (I \cdot S) = 2\pi J (I_z S_z + I_y S_y + I_z S_z)$. However, under the condition

$$ 2\pi J < |\omega_I - \omega_S| $$

(2)

it is truncated to $2\pi J I_z S_z$. This is called high-field, or weak-coupling approximation \[11\]. For this paper we assume that we have the Ising coupling as in Eq. (1) and present a pulse sequence that decouples this coupling assuming no knowledge of the chemical shifts, except the relation (2) and that the chemical shifts fall in a certain range. We then show that even in the presence of general isotropic interaction $2\pi J (I \cdot S)$, we can create an effective $zz$ interaction to which our pulse sequence can be applied.

We will attempt to create an effective Hamiltonian

$$ H_1(t) = \frac{\theta}{2} (\omega_I I_y + \omega_S S_y) + 2\pi J I_z S_z $$

(3)

This Hamiltonian represents an effective $y$ field which is different on two spins proportional to the chemical shift difference and perpendicular to the $zz$ coupling Hamiltonian. If we transform to the interaction frame of this field, we average the coupling. To synthesize the Hamiltonian $H_1$, we evolve rf Hamiltonian $A(I_z + S_z)$ for first 2 $\Delta t$ units of time and $-A(I_z + S_z)$ for next 2 $\Delta t$ units of time, where $\Delta t$ is appropriately chosen. We make the chemical shift Hamiltonian evolve as $\omega_I I_z + \omega_S S_z$, $-\omega_I I_z - \omega_S S_z$, $-\omega_I I_z - \omega_S S_z$ and $\omega_I I_z + \omega_S S_z$, for $\Delta t$ units of time each. We now observe how the rf Hamiltonian evolves in the interaction frame of the chemical shift Hamiltonian. This can be seen from the Fig. [1].
Assuming $A\Delta t$ is a small flip angle, the $x$ component of the rf Hamiltonian cancels while the $y$ component adds at the end of the cycle. If $\alpha = \omega_I \Delta t$ is a small tip angle then the $y$ component evolves as $\sim A\omega_I \Delta t$ which over a period $\Delta t$ integrates to $\frac{\Delta t}{2}\omega_I \Delta t = \frac{\theta}{2}\omega_I \Delta t$. This produces an effective Hamiltonian $\frac{\theta}{2}(\omega_I I_y + \omega_S S_y)$. In the interaction frame of the chemical shift, the coupling terms by themselves simply do not evolve. This then creates a field perpendicular to the couplings carrying the chemical shift difference which helps to decouple the spins. Chemical shifts can be inverted by simply applying a hard $\pi$ pulse. This does not affect the coupling Hamiltonian. Fig. 1b shows the basic pulse sequence with $\pi$ pulses with $x$ phase that inverts the chemical shift Hamiltonian and produces the cycle as described above. Similar cycles of the rf Hamiltonian have been recently proposed to design small tip angle–excitation pulses that are robust to rf inhomogeneity [12].

We wish to find the effective Hamiltonian

$$U = e^{-i(\Delta t)H_{\text{eff}}}$$

(6)

Using the Baker Campbell Hausdorff formula and keeping terms up to the third order in $\Delta t$, where $\sqrt{B^2 + A^2} \Delta t \ll 1$, we have

$$U = \exp(-4i(\Delta t)2\pi J I_y S_z - 2i(\Delta t)^2 A(w_I I_y + w_S S_y)$$

$$- 4i(\Delta t)^3 A^2 2\pi J (I_y S_z + I_z S_y) + 2i(\Delta t)^3 A^2 (w_I I_z + w_S S_z)$$

$$- 16i(\Delta t)^3 A^2 2\pi J (I_y S_y + I_z S_z) + O(\Delta t)^4 )$$

(7)

Compare the above equations, we have

$$H_{\text{eff}} = 2\pi J I_z S_z + \frac{1}{2}\theta(\omega_I I_y + w_S S_y)$$

$$+ \theta^2 J (I_y S_z + I_z S_y) - \frac{1}{2}\theta^2(\omega_I I_z + w_S S_z)$$

$$+ \frac{4}{3}\theta^2 2\pi J (I_y S_y - I_z S_z)$$

(8)

where as before $\theta = \Delta t A$. Ideally, we only want the first two terms: the $J$ coupling stays in the $z$ direction, while the main chemical shifts being scaled down by $\frac{\theta}{2}$ effectively point in the $y$ direction. There is a residual chemical shift still remaining along the $z$ direction and therefore we get an effective field in the direction $y’$ which makes an angle $\gamma$ to the $y$ axis. We resolve the coupling parallel and perpendicular to this effective field. The perpendicular part is truncated and the parallel component reduces the magnitude of residual coupling from $\frac{1}{4}\theta^2 2\pi J$, to $\frac{1}{4}\theta^2 2\pi J$, as described below. We write the full Zeeman Hamiltonian as $\frac{\theta}{2}(w_I I_{y’} + w_S S_{y’})$, where we neglect small change in $\theta$ arising from adding $\theta^2$ in quadrature. Then $I_y = I_{y’} \cos \gamma + I_z \sin \gamma, I_z = I_{y’} \cos \gamma - I_{y’} \sin \gamma$ where $\tan \gamma = \frac{\Delta t}{\frac{\theta}{2}} = \theta$. Replacing $y$ and $z$ in terms of $y’$ and $z’$, the coupling term in the $y’$ direction becomes

$$= J \sin^2 \gamma - \theta^2 2\pi J (\sin \gamma \cos \gamma + \cos \gamma \sin \gamma)$$

$$+ \frac{1}{3}\theta^2 2\pi J (\cos^2 \gamma - \sin^2 \gamma)$$

$$= 2\pi J (1 - \cos^2 \gamma) - 2\theta^2 2\pi J \tan \gamma \cos \gamma$$

$$+ \frac{4}{3}\theta^2 2\pi J (2 \cos^2 \gamma - 1)$$

$$= 2\pi J \left(1 - \frac{1}{1 + \theta^2}\right) - 2\theta^2 2\pi J \theta \frac{1}{1 + \theta^2}$$

$$+ \frac{4}{3}\theta^2 2\pi J \left[\frac{2}{1 + \theta^2} - 1\right]$$

$$\approx \frac{\theta^2 J (1 - 2 + 4/3)}{1 + \theta^2} = \frac{1}{3}\theta^2 2\pi J$$

So the most important terms in the effective Hamiltonian are

$$H_{\text{eff}}' = \frac{\theta}{2}(\omega_I I_{y’} + w_S S_{y’}) + \frac{\theta^2}{3} 2\pi J I_{y’} S_{y’}$$

(9)
That means, the chemical shifts are scaled down as \( \theta \), while the coupling strength is scaled down as \( \theta^2 \). This, as shown in the simulation below is a big improvement in enhancing the spectrum. The smaller the value of \( \theta \), the more effective the decoupling is. Scaling down the chemical shifts too much is not preferable as it degrades the resolution. Note that the chemical shift information of the spins is still retained and since the tilt \( \theta \) is well defined (assuming no rf-inhomogeneity) we can make the effective Zeeman interaction point along \( z \) axis by appropriate rf rotations.

Figures 2 and 3 show the results obtained by simulating the decoupling sequence starting with an initial density operator \( \rho(0) = I_x + S_x \). We simulate the evolution of \( s(t) = \langle I_x(t) + S_x(t) \rangle \). The simulation parameters have been chosen to reflect true experimental conditions. In absence of decoupling the original signal has an envelope \( \cos(\pi J I t) \), that of the decoupled signal decays much more slowly: \( \approx \cos[(\Delta B) t^2] \). This however gives a frequency dependence which is the basic building block of our pulse sequence. The couplings are scaled by a factor \( \frac{\tau_1 + \tau_2}{\tau_1} \), \( \kappa_1 \) and \( \kappa_2 \) depend on \( \omega_1 - \omega_2 \) and time \( \tau_1 \) and \( \tau_2 \) as shown in Fig.[[4]]. By choice of different \( \tau_1's \) and \( \tau_2's \), over various building blocks, we can make \( \kappa_1 \sim 0 \) over a broad range of \( \omega_1 - \omega_2 \). Similarly we can make \( \kappa_2 \sim 0 \) (or invert the sign of \( \kappa_2 \) with a \( y \) phase pulse to reduce scaling of the zz couplings).

The decoupling sequence scales down the chemical shifts by a factor \( \theta \). This obviously degrades the resolution but because we can decouple effectively, we can make spins precess for longer periods and restore resolution. The most ideal setting to do this is to transfer the spin...
FIG. 3: Results using decoupling sequence $\omega_I = 2\pi 120$, $\omega_S = 2\pi 100$, $J = 1$, $A = 10^3$, $\Delta t = 2 \times 10^{-4}$. (a) Signal that decays much more slowly with the envelope of $\cos(\frac{1}{2}(A\Delta t)^2\pi Jt)$, (time in units of $\frac{1}{J}$) (b) Spectrum obtained by truncating the signal at time $20$ $\pi$ pulses are assumed to be delta pulses.

coherence after it acquires a phase under chemical shift evolution on to a coupled spin for readout, as in the indirect evolution periods in multi-dimensional NMR. The basic setup is as shown in Fig. 4b where spins precess for constant time $d_1 + d_2$. Moving the center $\pi$ pulse causes chemical shift evolution as function of difference $d_1 - d_2$. Since it is a constant time evolution, the relaxation loss is constant and hence does not effect resolution. Effective decoupling ensures that $d_1 + d_2$ can be made large.

Since the chemical shifts are scaled by a factor $\theta = A\Delta t$ which depends on the strength of the applied rf field, in the presence of rf inhomogeneity, the observed spectrum is blurred by the point spread function $\chi(f)$. The observed spectrum is therefore

$$Z(f) = \sum_i X(f_i)\chi(f - f_i) + N(f)$$

where the true spectrum comprises of spikes at $f_i$. $N(f)$ is the noise spectrum. We wish to deconvolve the observed spectrum $Z(f)$ to recover $X(f_i)$. To do this, first calibrate $\chi(f)$ from an isolated peak. $\chi(f)$ reflects the inhomogeneity of the sample. We can then deconvolve using least square estimator.

In conclusion, we have introduced a new pulse sequence to decouple homonuclear spins with Ising interactions and proposed how the solution can be extended to Isotropic couplings.

FIG. 4: Panel a shows how an isotropic coupling Hamiltonian may be averaged to an Ising coupling by free precession periods, where the chemical shift difference between the spins evolves the planar part $xx + yy$ of the isotropic interaction. Panel c shows the evolution of the planar Hamiltonian $xx + yy$ as result of the building block in Panel a. Panel b shows the application of the decoupling sequence in the indirect evolution period, where spins precess for constant time. The center $\pi$ pulse refocusses the chemical shift so there is chemical shift evolution for time $d_1 - d_2$.

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