Can Two Quantum Chesire Cats Exchange Grins?

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A common-sense perception of a physical system is that it is inseparable from its physical properties. The notion of Quantum Chesire Cat challenges this, as far as quantum systems are concerned. It shows that a quantum system can be decoupled from its physical property under suitable pre and postselections. However, in the Quantum Chesire Cat setup, the decoupling is not permanent. The photon, for example, and its polarization can be decoupled from each other and made to travel separately through the two arms. This echoes the description of certain events in the novel Alice in Wonderland where Alice remarks, “Well! I’ve often seen a cat without a grin,…. but a grin without a cat! Its the most curious thing I ever saw in my life!” [2].

Quantum Chesire Cat has opened up new understanding of quantum systems and attracted a lot of debates and discussions [3-7, 8, 14]. It pertains not only to photons and their polarizations but can, in principle, be observed with any quantum system and its property, such as neutron and its magnetic moment, electron and its charge and so on. Experimental verifications of the phenomenon with neutron as the cat and its magnetic moment as the grin have been conducted [11, 12]. The phenomenon has also been observed experimentally in the context of photon and polarization [13]. Further developments on the idea of the Quantum Chesire Cat include the proposal of a complete Quantum Chesire Cat [14] and twin Quantum Chesire Cats [5]. The effect has been used to realize the three box paradox [6] and has been studied in the presence of decoherence [9]. Recently, a protocol has been developed using which the decoupled grin of a Quantum Chesire Cat has been teleported between two spatially separated parties without the cat [10].

The premise of the realization of the Quantum Chesire Cat involves weak measurements and weak values [15]. The development of the concept weak measurements and weak values stemmed from the limitation posed by the measurement problem in quantum mechanics, in acquiring knowledge about the value of an observable of a quantum system. Given a quantum system in a general pure state $|\Psi\rangle$, if a measurement of an observable $A$ is performed, then the outcome is an arbitrary eigenvalue $a_i$ of $A$. Measurement of $A$ on an ensemble of states, shows that the measurement outcomes are indeterministic and probabilistic, the probability of an outcome $a_i$ in any given run being $|\langle a_i | \Psi \rangle|^2$, where $|a_i\rangle$ is the eigenstate, corresponding to the eigenvalue $a_i$. Thus in quantum mechanics, one actually measures the expectation value $\langle A \rangle$ which is the ensemble average of the outcomes. In addition, in a run, the state of the system collapses to $|a_i\rangle$. As a result, there is no general consensus among physicists as to whether the measurement of an observable actually reveals a property of the system or is an artifact of the measurement process itself. The proponents of weak value tried to circumvent this problem of wavefunction collapse by cutting off the disturbance caused to the initial state. This is achieved by weakly measuring the observable $A$, causing minimal disturbance to the state. We briefly recapitulate the process of weak measurement below.

Consider a quantum system preselected in the state $|\Psi_i\rangle$. Suppose an observable $A$ is weakly measured by introducing a small coupling between the quantum system and a suitable measurement device or a meter. A second observable $B$ is thereafter measured strongly and one of its eigenstates $|\Psi_f\rangle$ is postselected. For all successful postselections of the state $|\Psi_f\rangle$, the meter readings corresponding to the weak measurements of $A$ are taken into consideration while the others are discarded. The shift in the meter readings, on an average, for all such postselected systems is $A_w$ which is known as the weak value of $A$ [15]. Mathematically the weak value of $A$ is defined as

$$A_w = \frac{\langle \Psi_f | A | \Psi_i \rangle}{\langle \Psi_f | \Psi_i \rangle}. \quad (1)$$

The weak value is therefore to be interpreted as the value the observable $A$ takes between the preselected state $|\Psi_i\rangle$ and the postselected state $|\Psi_f\rangle$. The measurement of $A$ must be weak to preserve the initial probability distribution for the final postselected state. Weak values via weak measurements have been observed experimentally [17]. The weak value of an observable can...
be complex [20] or can take up large values that lie outside the eigenvalue spectrum [15, 16, 18]. The latter has led to the use of weak measurements as a tool for signal amplification [22, 23]. A geometrical interpretation of weak values can be found in Ref. [19]. Among the myriad applications of weak measurements are observation of the spin Hall effect [23], resolving quantum paradoxes [26, 28], quantum state visualization [27], quantum state tomography [28, 29], direct measurement of wavefunction [30, 31], probing contextuality [32], measuring the expectation value of non-Hermitian operators [33, 34] and quantum precision thermometry [35].

Any probing of position or a component of polarization of the photon for the observation of the Quantum Chesire Cat effect must necessarily be weak. This is because projective measurements tend to destroy the original state of a system while extracting information. Weak measurements, on the other hand, minimally disturb the system while gaining small information about it. The state cannot be disturbed in a Quantum Chesire Cat setup as any alteration of the original state will lead to altered probabilities of the postselected state.

In this paper we explore yet another counterintuitive aspect of the Quantum Chesire Cat. We design a setup where we can not only decouple the grin from the Quantum Chesire Cat, but can replace it with a grin originally belonging to another Chesire Cat. Our setup comprises of two modified and overlapping Mach-Zehnder interferometers. We try to show that the notion that a property of a physical system does not uniquely belong to that system in the quantum domain and can be replaced by the same property from another physical system. Some indications of this phenomenon were obtained in an earlier work [10] where the decoupled circular polarization has been used for teleportation between two spatially separated parties who are not in possession of the photon.

The paper is arranged as follows. In Section II, we describe the Quantum Chesire Cat protocol in some details. Next, in Section III we present our recipe for the exchange of the grins of two Quantum Chesire Cats. We conclude in Section IV with discussions on some of the implications of our findings.

II. THE QUANTUM CHESIRE CAT

The phenomenon of Quantum Chesire Cat can be realized by a scheme that is based on a Mach Zehnder interferometer, first presented in Ref. [1]. A source sends a linearly polarized single photon towards a 50:50 beam-splitter $BS_1$ that channels the photon into a left and right path. Let $|L\rangle$ and $|R\rangle$ denote two orthogonal states representing the two possible paths taken by the photon, the left and the right arm, respectively. If the photon is initially in the horizontal polarization state $|H\rangle$, the photon after passing through the beam-splitter $BS_1$ can

\begin{align}
|\Psi\rangle &= \frac{1}{\sqrt{2}} (|L\rangle + |R\rangle) |H\rangle,
\end{align}

where the relative phase factor $i$ is picked up by the photon traveling through the left arm due to the reflection by the beam splitter. The postselection block, conducting the process of projective measurement and eventual postselection, comprises of a half-waveplate (HWP), a phase-shifter (PS), beam-splitter $BS_2$, a polarization beam- splitter PBS that transmits polarization states $|H\rangle$ and reflects state $|V\rangle$ and three detectors $D_1$, $D_2$ and $D_3$. Let the postselected state be

\begin{align}
|\Psi_f\rangle &= \frac{1}{\sqrt{2}} (|L\rangle |H\rangle + |R\rangle |V\rangle),
\end{align}

where $|V\rangle$ refers to the vertical polarization state orthogonal to the initial polarization state $|H\rangle$. The HWP flips the polarization of the photon from $|H\rangle$ to $|V\rangle$ and vice versa. The phase-shifter (PS) adds a phase factor of $i$ to the beam. The beam-splitter $BS_2$ is such that when a photon in the state $\frac{1}{\sqrt{2}}(|L\rangle + i |R\rangle)$ is incident on it, the detector $D_2$ never clicks. In other words, in such cases, the photon always emerges towards the PBS. The PBS is chosen such that it always transmits the horizontal polarization $|H\rangle$ and always reflects the vertical polarization $|V\rangle$. The above arrangement thus ensures that only a state given by $|\Psi_f\rangle$, before it enters the postselection block, corresponds to the click of detector $D_1$. Any clicking of the detectors $D_2$ or $D_3$ implies a different
state entering the postselection block. Therefore, selecting the clicks of the detector $D_1$ alone and discarding all the others leads to the postselection onto the state $|\Psi_f\rangle$.

Suppose we want to know which arm a photon, prepared in the state $|\Psi\rangle$ and was ultimately postselected in the state $|\Psi_f\rangle$, passed through. This can be effected by performing weak measurements of the observables $\Pi_L = |L\rangle \langle L|$ and $\Pi_R = |R\rangle \langle R|$ by placing weak detectors in the two arms. Similarly, the polarizations can be detected in the left and the right arms by respectively performing weak measurements of the following operators

$$\sigma^L_z = \Pi_L \otimes \sigma_z, \quad \sigma^R_z = \Pi_R \otimes \sigma_z,$$

where

$$\sigma_z = |+\rangle \langle +| - |-\rangle \langle -|,$$

the circular polarization basis $\{ |+\rangle , |−\rangle \}$ itself defined as

$$|+\rangle = \frac{1}{\sqrt{2}} (|H\rangle + i |V\rangle),$$

$$|−\rangle = \frac{1}{\sqrt{2}} (|H\rangle - i |V\rangle).$$

The weak values of the photon positions are measured to be

$$(\Pi_L)^w = 1 \text{ and } (\Pi_R)^w = 0$$

which implies that the photon in question has traveled through the left arm. The measured weak values of the polarization positions, on the other hand, turn out to be

$$(\sigma^L_z)^w = 0 \text{ and } (\sigma^R_z)^w = 1.$$  \hspace{1cm} (8)

Equations (7) and (8) together reveal that the photon traveled through the left arm but its circular polarization traveled through the right arm. This means the two degrees of freedom of a single entity can, in fact, be decoupled. That is, property of a quantum system can exist independent of its existence in that region.

III. SWAPPING OF GRINS

In the previous section we have seen that the grin of a cat can be separated from the cat itself under suitable choices of pre and postselection. Here we consider two such Quantum Cheshire Cats and exchange their grins. In terms of physical realization, we take two linearly polarized photons, decouple their circular polarizations and then recouple them with the other photon.

To see this effect let us consider the setup shown in Fig. 2. There are two sources of linearly polarized photons, near the unprimed and the primed halves of the arrangement, on the left and the right, respectively. The input polarization in the unprimed half is $|H\rangle$ while that in the primed half is $|H'\rangle$. The setup allows one to prepare a preselected state given by

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (i |L\rangle + |R\rangle) \otimes \frac{1}{\sqrt{2}} ((|L'\rangle + i |R'\rangle) |H'\rangle).$$  \hspace{1cm} (9)

where $|L\rangle$ and $|R\rangle$ indicate the states of a photon in the left and right arms of the unprimed half of the apparatus and $|L'\rangle$ and $|R'\rangle$ indicate the states of a photon in the left and right arms of the primed half of the apparatus. The right arm of the unprimed half is connected to the output of the primed half and the left arm of the primed half is connected to the output of the unprimed half.

The photons and their polarizations are postselected in the state as given by

$$|\Psi_f\rangle = \frac{1}{2} (|L\rangle |H\rangle |R'\rangle |H'\rangle + |R\rangle |V\rangle |R'\rangle |H'\rangle + |L\rangle |H\rangle |L'\rangle |V'\rangle - |R\rangle |V\rangle |L'\rangle |V'\rangle).$$  \hspace{1cm} (10)

This a four qubit maximally entangled state and is one of the cluster states $|\mathcal{E}\rangle$. The postselection is realized using the following setup. It is clear that this postselected state $|\Psi_f\rangle$ demands that there is entanglement between the path degrees of freedom of the two halves of the interferometric arrangement. Suppose that the HW $P$ and the $HWP'$ cause the transformations $|H'\rangle \leftrightarrow |V'\rangle$ and $|H\rangle \leftrightarrow |V\rangle$, respectively, and $PS$ and $PS'$ add a phase-factor $i$, in continuation with the previous arrangement. Now let the beam-splitters $BS_2$ and $BS'_2$ are chosen such that if a state $|L\rangle |L'\rangle$ is incident on $BS_2$ or a state $|R\rangle |R'\rangle$ is incident on $BS'_2$, then the photons emerge towards $PBS$ and $PBS'$, respectively. The $PBS$ and the $PBS'$, once again, allow only polarizations $|H\rangle$ and $|H'\rangle$ to be transmitted and other polarizations to be reflected. Now if any other states are incident on $BS_2$ and $BS'_2$, they proceed towards $PBS_3$ or $PBS'_3$. These polarization beam splitters, once again, allow polarizations $|H'\rangle$ and $|H\rangle$ to be transmitted, towards $BS_3$ and $BS'_3$, respectively, and reflect polarizations $|V'\rangle$ and $|V\rangle$ towards $D_6$ and $D'_6$, respectively. Next, the beam-splitter $BS_3$ is so chosen that the state $|L\rangle$ is transmitted towards the detector $D_3$ and any other state is reflected towards $BS_1$. In conjuction with this, the beam-splitter $BS'_3$ is chosen to transmit the state $|R'\rangle$ towards $D'_3$ and reflects any other state towards $BS'_4$. Thus, the simultaneous clicks of the detectors $D_3$ and $D'_3$ would mean a postselection of the state $|L\rangle |R'\rangle$. Using a similar reasoning and appropriate choice of beam-splitters, $BS_4$ and $BS'_4$, the state $|R\rangle |L'\rangle$ can be postselected using simultaneous clicks of the detectors $D_4$ and $D'_4$. This means any clicking of the detectors $D_2$, $D_5$, $D_6$, $D'_2$, $D'_5$, or $D'_6$ would indicate an unsuccessful postselection. On the other hand if and only if there are simultaneous clicks of all the detectors $D_1$, $D'_1$, $D_3$, $D'_3$, $D_4$ and $D'_4$ we get a successful postselection.
FIG. 2. Swapping of the grins of two Quantum Chesire Cats. The desired postselection for observing the swapping of the grin is obtained by selecting only the cases for which there are simultaneous clicks of $D_1$, $D_3$, $D_4$, $D'_1$, $D'_3$ and $D'_4$. The mirror $M''$ is used just to accommodate the detectors in a compact space.

To appreciate the working of this arrangement, let us define two circular polarization bases

$$|+\rangle = \frac{1}{\sqrt{2}}(|H\rangle + i|V\rangle),$$

$$|\rangle = \frac{1}{\sqrt{2}}(|H\rangle - i|V\rangle)$$

and

$$|+\rangle = \frac{1}{\sqrt{2}}(|H\rangle' + i|V\rangle'),$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|H\rangle' - i|V\rangle')$$

and two operators

$$\sigma_z = |+\rangle\langle+| - |\rangle\langle|-|,$$

$$\sigma'_z = |+\rangle\langle+|' - |\rangle\langle|-|'$$

To detect a photon in an arm of the interferometric setup we need to measure $\Pi_L = |L\rangle \langle L|$, $\Pi_R = |R\rangle \langle R|$, $\Pi_{L'} = |L'\rangle \langle L'|$ and $\Pi_{R'} = |R'\rangle \langle R'|$. In order that the original state is not disturbed due to these measurements, we need to perform them weakly. Subjected to a successful postselection of the state $|\Psi_f\rangle$, the corresponding weak values are measured as follows.

$$(\Pi_L)^w = \frac{\langle \Psi_f | \Pi_L | \Psi \rangle}{\langle \Psi_f | \Psi \rangle} = 1,$$

$$(\Pi_R)^w = \frac{\langle \Psi_f | \Pi_R | \Psi \rangle}{\langle \Psi_f | \Psi \rangle} = 0,$$

$$(\Pi_{L'})^w = \frac{\langle \Psi_f | \Pi_{L'} | \Psi \rangle}{\langle \Psi_f | \Psi \rangle} = 0,$$

$$(\Pi_{R'})^w = \frac{\langle \Psi_f | \Pi_{R'} | \Psi \rangle}{\langle \Psi_f | \Psi \rangle} = 1.$$
which means that the circular polarization of the photon at the unprimed input port must have traveled via the right arm of the unprimed half of the arrangement and the circular polarization of the photon at the primed input port must have journeyed via the left arm of the primed half of the setup for all final outcomes $|\Psi_f\rangle$. Equations \(\text{(14)}\) and \(\text{(15)}\) jointly demonstrate that, under the above postselection, the unprimed photon ended up at detector $D_1$ but its circular polarization ended up at the detector $D_1'$. Similarly, the primed photon finally reaches the detector $D_1'$ while its circular polarization goes to the detector $D_1$, for the postselected state $|\Psi_f\rangle$. Thus we have swapped the grins of two Quantum Chesire Cats.

IV. CONCLUSIONS

We have developed a thought experiment in which the circular polarizations of two photons can be swapped using an interferometric arrangement. The arrangement for executing this process is based on the original Quantum Chesire Cat setup where the circular polarization can be temporarily separated from the photon for suitable postselected states. Our method strives to decouple the polarization and the photon more permanently by replacing the original polarization with another that was previously associated with a different photon. This polarization in turn associates itself with the second photon. The effect is true only for a certain postselected state.

The implications for the swapping of photon polarization are significant. Firstly, it challenges the notion that a property must faithfully ‘belong’ to a particular physical system. In the realm of the quantum systems, this ‘belongingness’ is certainly very capricious with properties belonging to independent physical systems getting interchanged. The second point to note is that entanglement plays a crucial role in the realization of this swapping process. As discussed before, the swapping is successful only when a certain outcome is attained at the end. This so happens that this outcome is an entangled state. This implies that although the photon and its original polarization is permanently separated spatially, they are held together, along with the other photon and polarization as one global state.

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[1] Y. Aharonov, S. Popescu, D. Rohrlich and P. Skrzypczyk, New Journal of Physics, 15, 113015 (2013).
[2] Carroll, L. Alices Adventures in Wonderland, Wisehouse Classics; 2016 ed. (1865).
[3] J. Bancal, Nature Physics, 10, 11 (2013).
[4] Y. Guryanova, N. Brunner and S. Popescu, ArXiv e-prints, 1203.4215 quant-ph, (2012).
[5] I. Ibnouhsein and A. Grinbaum, ArXiv e-prints, 1202.4894, quant-ph (2012).
[6] A. Matzkin and A. K. Pan, Journal of Physics A: Mathematical and Theoretical, 46, 315307 (2013).
[7] R. Corrêa, M. F. Santos, C. H. Monken and P. L. Saldanha, New Journal of Physics, 17, 053042 (2015).
[8] Q. Duprey and S. Kanjilal and U. Sinha and D. Home and A. Matzkin, Annals of Physics, 391, 1 (2018).
[9] M. Richter, B. Dziewit and J. Dajka, Advances in Mathematical Physics, 2018, 7060586 (2018).
[10] D. Das and A. K. Pati, arXiv:1903.01452 (2019).
[11] T. Denkmayr, H. Geppert, S. Sponar, H. Lemmel, A. Matzkin, J. Tollaksen and Y. Hasegawa, Nature Communications, 5, 4492 (2014).
[12] S. Sponar, T. Denkmayr, H. Geppert and Y. Hasegawa, Atoms, 4, 11 (2016).
[13] J. M. Ashby, P.D. Schwarz and M. Schlosshauer, Phys. Rev. A 94, 012102 (2016).
[14] D. P. Atherton, G. Ranjit, A. A. Geraci and J. D. Weinstein, Opt. Lett., 40, 879 (2015).
[15] Y. Aharonov, D. Z. Albert, and L. Vaidman, Phys. Rev. Lett. 60, 1351-1354 (1988).
[16] Y. Aharonov, S. Popescu and J. Tollaksen, Physics Today 63, 27-32 (2010).
[17] G. J. Pryde, J. L. O’Brien, A. G. White, T. C. Ralph and H. M. Wiseman, Phys. Rev. Lett. 94, 220405 (2005).
[18] I. M. Duck, P. M. Stevenson, and E. C. G. Sudarshan, Phys. Rev. D, 40, 2112–2117 (1989).
[19] M. Cormann, M. Remy, B. Kolaric, and Y. Caudano, Phys. Rev. A 93, 042124 (2016).
[20] R. Jozsa, Phys. Rev. A, 76, 044103 (2007).
[21] Y. Aharonov and L. Vaidman, Phys. Rev. A, 41, 11–20, (1990).
[22] P. B. Dixon, D. J. Starling, A. N. Jordan, N. Andrew and J. C. Howell, Phys. Rev. Lett., 102, 173601 (2009).
[23] A. Nishizawa, K. Nakamura and M. Fujimoto, Phys. Rev. A 85 (2012).
[24] D. Sokolovski, Quanta 2, 5057 (2013).
[25] O. Hosten and P. Kwiat, Science 319, 787-790 (2008).
[26] Y. Aharonov and S. Dolev, Springer Berlin Heidelberg, 283-297 (2005).
[27] H. Kobayashi, K. Nonaka and Y. Shikano, Phys. Rev. A, 89, 053816 (2014).
[28] H. F. Hofmann, Phys. Rev. A 81, 012103 (2010).
[29] S. Wu, Scientific Reports 3, 1193 (2013).
[30] J. S. Lundeen, B. Sutherland, A. Patel, C. Stewart and C. Bamber, Nature, 474, 188-191 (2011).
[31] J. S. Lundeen and C. Bamber, Phys. Rev. Lett. 108, 070402 (2012).
[32] J. Tollaksen, Journal of Physics: Conference Series 70, 012014 (2007).
[33] A. K. Pati, U. Singh and U. Sinha, Phys. Rev. A 92, 052120 (2015).
[34] G. Nirala, S. N. Sahoo, A. K. Pati and U. Sinha, Phys. Rev. A 99, 022111 (2019).
[35] A. K. Pati, C. Mukhopadhyay, S. Chakraborty and S. Ghosh, arXiv:1901.07415 (2019).
[36] H. J. Briegel and R. Raussendorf, Phys. Rev. Lett. 86, 910–913 (2001).