TRANSFORMATION OF NETWORK MODELS CONSIDERING THEIR TOPOLOGICAL PROPERTIES AND WEIGHT CHARACTERISTICS

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Abstract

This paper proposes a method of transformation of network models, taking into account their topological properties and weight characteristics. The method is intended for the networks with a large number of vertices. Social information networks, mustering millions of users, should be considered as the most illustrative...
example of such. Simulation of similar networks structures takes tremendous calculation expenditures and, therefore, the authors set themselves a task to transform the initial network by means of size reduction, yet retaining its properties. Since modern corporate and global networks are suspended (all vertices and arcs have different weights – specific traffic) and heterogeneous (the number of vertices bonds varies significantly), therefore, the authors aim to (when transforming the graph) preserve all above-mentioned topological properties and weight characteristics of the analyzed network. Some equivalent transformations are formalized in the form of algorithm of the researcher’s actions. Software based on this algorithm confirmed efficiency of the proposed approach. Adduced examples illustrate the peculiarities of the proposed algorithm of transformation of networks models. Emphasized results of the research are the following: for the first time an algorithm of similarity transformation offers an opportunity to reduce an initially large network into a considerably smaller network that is convenient to use in the analysis of social networks and epidemic processes of content distribution; resulting assessments of metrics and characteristics of suspended networks, in contrast to analogues, give an opportunity to consider weight properties of the network and present an apparatus for studying properties of harmful content distribution in suspended heterogeneous social networks; in this case, discrete macro-models of the epidemic process differ from the analogues, they specifically simulate a suspended heterogeneous social network, including filler of the vertices (agents quality) and network bandwidth (the traffic that passes along the communication lines of the network).

**Keywords:** Suspended graph, vertex, traffic, and algorithm.

### I. Introduction

Modern social information networks include millions of users. Social networks are abstractedly represented in the form of graph models with millions of nodes and milliards of bonds among the nodes. Both social networks and modern corporate and global networks are suspended (all vertices and arcs have different weight – traffic) and heterogeneous (the number of bonds of vertices varies significantly). All of them are suspended and inhomogeneous, where the vertices are randomly distributed in space, and at that, each vertex and each edge has its own weight. The social network virtually represents a combination of agents that are distributed in the information space, each having weight, while information acts as filler. The more valuable information belongs to the agent, the greater weight it possesses (Chebotarev & Shamis, 1997; Dekker, 2006). Such networks are widely spread and, therefore, their study is an urgent task.

Earlier, the networks models were built on the supposition that the number of friends is the main parameter of the network agent (Dries, Nijssen & Raedt, 2009; Monjardet & Raderinirina, 2001). However, when assessing modern networks, the following parameters become more relevant than the number of vertex’s bonds with the neighbor’s:

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1) quality of vertices, that is, the volume and the value of the filler that is stored and processed in them;
2) capacity of edges, that is, the volume and the value of the filler that is pumped through them per time unit.

Different metrics are used to assess the characteristics of social networks; the basic ones include: degree of the suspended central vertex, degree of centrality of the whole network, a density of centrality, density of the suspended centrality, centrality as a mediation, and suspended equivalence of vertices (Bailey, Craswell & Hawking, 2003; Bourqui, Gilbert, Simonetto, Zaidi, Sharan & Jourdan, 2009; Chebotarev & Shamis, 1997; Dekker, 2006; Dries, Nijssen & Raedt, 2009; Monjardet & Raderinirina, 2001; Newman, 2006; Pfaltz, 2013; Pfaltz, 2006; Richards & Seary, 2000; Toivonen, Kumpula, Saramäki, Kertész & Kaski, 2007; Oystein, 1946; Wax & Sheinvald, 1997).

Macro-models are basic for studying suspended heterogeneous social networks (Islamgulova, Ostapenko, Radko, Babadzhanov & Ostapenko, 2016). Various matrices serve as the basis for such models, for example, the vertex matrix or the matrix of layer-wise coherence (Islamgulova, Ostapenko, Radko, Babadzhanov & Ostapenko, 2016). But this basis is insufficient for description of suspended social networks, having such parameters as the vertex filler volume and edge capacity. Therefore, suspended networks must be built with the use of the matrix of networks suspension, the elements of which are weights of arcs and vertices. At that, the vertices weights are in the diagonal of this matrix, and the weights of ingoing and outgoing arcs are correspondingly in the columns and the lines. The matrix of the layer-wise coherence is also a basis for building suspended social networks. Thus, to build macro-models of suspended heterogeneous social networks, the matrix of layer-wise coherence and the matrix of the network suspension should be utilized as the basis (Islamgulova, Ostapenko, Radko, Babadzhanov & Ostapenko, 2016).

Building a micro-model of infection of the vertex (network agent) along with the study of transitions of the vertex into different states is another direction of research of the epidemic processes in heterogeneous suspended networks. The idea of building a micro-model is not new; however, the new step is introduction of discrete changes of the model structure into the model by time intervals. Thus, when describing the epidemic process in the model, not only the vertex changes its state, but the micro-model structure also changes. Correspondingly, along with the change of the structure, the probabilities of transition of the vertex from one state into another alter.

Practical research of epidemiological processes in real social networks requires an automated toolkit of simulation (Ostapenko, Plotnicov, Makarov, Tikhomirov & Yurasov, 2013; Ostapenko, Parinova, Belonozhkin, Bataronov & Simonov, 2013; Ermakov, Zavorykin, Kolenbet, Ostapenko & Kalashnikov, 2014; Radko, Ostapenko, Mashin, Ostapenko & Gusev, 2014; Radko, Ostapenko, Mashin, Ostapenko & Avdeev, 2014; Ostapenko, Bursa, Ostapenko & Butrik, 2014; Islamgulova, Ostapenko, Radko, Babadzhanov & Ostapenko, 2016). Such a toolkit must implement all the functions that are necessary for modelling suspended
heterogeneous social networks, and include the whole set of the harmful content known at the moment (Ostapenko, Plotnikova & Guzev, 2016). For all that, it is important to choose the type of model that describes the network under study to the fullest degree.

When studying suspended networks, it is the processes of distribution of the content that are of primary interest, rather than the networks themselves (Islamgulova, Ostapenko, Radko, Babadzhanov & Ostapenko, 2016). Thus, the study and management of epidemic processes in the suspended networks are urgent tasks of high priority.

In this case, mathematical apparatus of the graph theory is applied as a main research tool, where suspended graphs act as social networks (Ostapenko, Plotnicov, Makarov, Tikhomirov & Yurasov, 2013; Ostapenko, Parinova, Belonozhkin, Bataronov & Simonov, 2013; Ermakov, Zavorykin, Kolenbet, Ostapenko & Kalashnikov, 2014; Radko, Ostapenko, Mashin, Ostapenko & Gusev, 2014; Radko, Ostapenko, Mashin, Ostapenko & Avdeev, 2014; Ostapenko, Bursa, Ostapenko & Butrik, 2014; Islamgulova, Ostapenko, Radko, Babadzhanov & Ostapenko, 2016; Ostapenko, Plotnikova & Guzev, 2016).

At the same time, simulation of similar network structures is connected with the tremendous calculation expenditures, and, therefore, the authors set a task to transform the initial network by reduction of its size, while retaining its basic qualities. That is, the proposed transformations of graph models of the networks must retain basic topological properties and weight characteristics of the analyzed network. This will permit to factor capacities of the arcs and volumes of the processed content, taking into consideration the network clustering.

There is a multitude of algorithms of transforming networks into similar ones (Kuwata, 2009; Park, Moore & Bader, 2010; Zhao, Kumar, Harrison & Yen, 2011), which are frequently based on *a priori* definition of some parameters used for subsequent isolation of communities or strongly connected components in the graph (Bourqui, Gilbert, Simonetto, Zaidi, Sharan & Jourdan, 2009; Newman, 2006). Such transformations are unacceptable from the perspective of supporting the permanent topological properties of the initial network. The absence of support of weight characteristics and properties of the networks is another important disadvantage, which is unacceptable in the context of study of social networks that are suspended by their nature (Islamgulova, Ostapenko, Radko, Babadzhanov & Ostapenko, 2016).

Thus, there arises the necessity to build the algorithm of concentration of the content, isolation of vertices subjected to combination, and formation of the combined edges for the purpose of preserving basic bonds among the combined vertices (hubs). Subsequently, this will permit to apply existing mechanisms of risk-analysis and topological analysis of the networks to assessment of the transformed network (Ostapenko, Plotnicov, Makarov, Tikhomirov & Yurasov, 2013; Ostapenko, Parinova, Belonozhkin, Bataronov & Simonov, 2013; Ermakov, Zavorykin, Kolenbet, Ostapenko & Kalashnikov, 2014; Radko, Ostapenko, Mashin, Ostapenko & Gusev, 2014; Radko, Ostapenko, Mashin, Ostapenko & Avdeev, 2014; Ostapenko, Bursa,
II. Materials and Methods

Today, there exist a significant number of networks with pronounced clustering. Their description is frequently based on *a priori* definition of some parameters that are used for subsequent isolation of communities and strongly connected components in the graph. An important disadvantage of these algorithms is the absence of support of weight characteristics and network properties, which is unacceptable in the context of social networks that are suspended by their nature (Ostapenko, Plotnikova & Guzev, 2016). Hence, there is a need to build an algorithm of isolation of concentrating hubs and retention of bonds among them. For this, it is appropriate to use the following procedures:

1) The number of vertices that is necessary for preserving the properties of the initial network with a specified matching probability is assessed with the use of statistic sufficiency criteria, according to the existing statistical data on topology of the analysed network. Choice of the necessary number of vertices in the resulting graph depends, in many respects, on the type of the considered network and can be implemented with regard to the required accuracy of description.

2) Next, the most central vertices are isolated, and concentrators are built around them. Such metrics as a degree of the suspended centrality of the network can be used for this purpose (Ostapenko, Plotnikova & Guzev, 2016). Thus, a combination can be created on the basis of the degree of the vertex and, also, on the basis of the volume of the content circulating through it. Suspended centrality of the vertex is a key metric characteristic that helps to isolate highly demanded vertices in the network. This characteristic has an imperative significance in the context of social networks as it provides substantiated data for determination of the probable damage. The degree of the suspended centrality of the network is calculated as a sum of arc resources (in both directions), adjacent to vertex $x_i$:

$$D_{wc}(x_i) = R_{es}(s) = \sum_f \left( R_{es}(a_{ij}) + R_{es}(a_{ji}) \right).$$

The concept of the resource (Ostapenko, Plotnikova & Guzev, 2016) includes qualitative and quantitative parameters of the content that is pumped through the arc per time unit. Such data for social networks are available on open access.

At that, the multitude of vertices $A_{suff}$ must meet the following conditions:

$$\forall a_i \in A_{suff}, D_{wc}(x_i) > D_{wcrit};$$

that is $|A_{suff}| \geq N_{suff}$, where $N_{suff}$ is necessary number of vertices, determined at stage 1; $D_{wcrit}$ is critical value of centrality that results from the initial multitude of vertices $A_{init}$ by sorting vertices in the descending order of suspended centrality.

As an illustration of the stated above, consider the example of the network topology shown in Figure 1.
Vertices 1, 10 and 21 possess maximum centrality; let us accept $N_{suff} = 3$. Note that vertex 1 has a bond with vertices 2, 3, 4, 5, 6, 7, 8, which are ‘satellite vertices’ of vertex 1. However, rough combination of these vertices leads to the loss of weight characteristics, which is unacceptable in the context of the analysis of the suspended networks. Therefore, it is appropriate to combine vertex 1 with vertices 2, 3, 4, 5, 6, 7, 8 and let the weight of such vertex be total centrality of combined edges. After such transformation, the network takes the form shown in Figure 2.

At that, if a vertex with centrality above the critical value turns out to be related to another vertex with centrality also above the critical value, their combination is unacceptable. Rules of vertices combination are summarized as follows:

$$a_i \in A_{init}, a_i \notin A_{suff}, \quad \exists a_{ij}, a_j \in A_{suff},$$
where \( a_i \) is graph vertex; \( A_{init} \) is multitude of vertices of the initial graph; \( A_{suff} \) is multitude of the most needed vertices.

After combination of vertices, multiple edges may appear in the transformed graph, and, they can be combined taking into account the network suspension. Consider the resulting graph shown in Figure 3.

![Fig. 3: Combined graph of the network.](image)

Successive execution of the algorithm for all vertices from multitude \( A_{suff} \) reduces the model network to representation in the form of the transformed graph of the network, shown in Figure 4.

![Fig. 4: Transformed graph of the network.](image)

The resulting graph is convenient for the analysis. A researcher can operate with vertices-concentrators, instead of specified vertices, which in majority have little significance (in view of the demonstrative law of distribution of the degrees of vertices in social networks) and do not exert influence on the content distribution.

### III. Results

The results of transformation largely depend on the type of the network under study. If one considers social networks that are inherently scaleless, the transformation seems quite logical and applicable.

The case with a small diameter and the demonstrative law of distribution of degrees of network vertices is of the greatest practical interest. The smallness of the diameter means that the network is strongly connected; hence, the content in such networks will spread swiftly. Strong connectedness of the network will allow one to reduce
significantly the size of the network without the loss of its properties. In case of the
demonstrative law of distribution of the degrees of vertices, an extremely small
number of vertices have a very large number of bonds, and an overwhelming majority
of vertices has a very small number of bonds. A similar property permits to
substantially reduce the volume of representative sampling by means of the proposed
transformation. It is worth noting that after combining, the situation when multiple
edges appear in the transformed graph is possible; however, in view of suspension of
the network, they can also be combined.

The algorithm of transformation of the suspended networks represents the following
sequence of actions:

1) introduction of the minimum permissible volume of representative sampling,
determined based on the resource of the initial network and its topological properties;
2) to determine the maximum permissible resource of the vertex, below which
the vertex must be combined with others, a critical value of the resource of vertex
$\delta_{\text{crit}}$ is determined as $\delta_{\text{crit}} = \delta_{\text{Aper}}$, where initial sampling $A_{\text{init}}$ is sorted by the
principle:

$$D_{\text{we}}(a_i) > D_{\text{we}}(a_{i+1});$$

3) for each vertex $x_i \in V_{\text{per}}$, a set of combined vertices $A_{aj}$ is determined such
that $\forall a_{xjk} \in A_{aj}$, $a_{ajk} \notin A_{\text{suff}}$. Thus, a set of vertices, which must be combined, is
determined. The resulting combined vertices are the concentrators of the content and
have a significant number of edges. Consideration of such vertices allows studying
the network in a more compact way. At that, both the weight of initial nodes of the
network and bonds among them are taken into account:

4) all edges that are available between $a_j$ and $A_{aj}$; their resources are summed
and supplemented to the weight of vertex $a_j$;
5) the weights of parallel unidirectional edges are summed.

The graph, obtained by means of the similar transformation, preserves the basic
properties of the initial one; however, in contrast to it, it is convenient for conducting
risk-analysis, determination of a probable damage (Ostapenko, Plotnicov, Makarov,
Tikhomirov & Yurasov, 2013; Ostapenko, Parinova, Belonozhkin, Bataronov &
Simonov, 2013; Ermakov, Zavorykin, Kolenbet, Ostapenko & Kalashnikov, 2014;
Radko, Ostapenko, Mashin, Ostapenko & Gusev, 2014; Radko, Ostapenko, Mashin,
Ostapenko & Avdeev, 2014; Islamgulova, Ostapenko, Radko, Babadzhanov &
Ostapenko, 2016), and treatment by other algorithms that are of interest to a researcher (Evin &
Habibullin, 2012; Ostapenko, Plotnikova & Guzev, 2016; Kuwata, 2009; Park, Moore &
Bader, 2010; Zhao, Kumar, Harrison & Yen, 2011).

The block-diagram of the algorithm is shown in Figure 5.
Fig. 5: Block diagram of the algorithm of network graph transformation.

The algorithm was brought to software implementation and is used in the simulation of processes of content distribution in social networks.

IV. Discussion

In the first example, the working capacity of the proposed algorithm is illustrated by the graphs of distribution of the degrees of vertices for the social network according to interests (users of the geolocation service “Locata”) before making the transformation (Figure 6) and after it (Figure 7).
Application of the algorithm under consideration allows reducing the degrees of the most central vertices up to two orders, having preserved in general topological properties of the graph model of the analyzed network. Total volume of the content, pumped among combined vertices, is practically adequate to the content, circulating in the initial model of the network. There is a direct evidence of a significant reduction of sizes of the network, and, as a consequence, a reduction of the subsequent calculation expenditures on analysis of the corresponding social network.

Consider another example, illustrating the functioning of the proposed algorithm in the context of the dynamic resource (traffic). The initial network, shown in Figure 8, can be reduced to the graph (Figure 9) on the basis of pronounced concentrators, that is, vertices 4, 8, 12 and 18. At that, the inter-cluster traffic is retained:

\[
\begin{align*}
Res(4', 8') &= Res(2,6) + Res(4,6) + Res(4,7); \\
Res(4', 12') &= Res(4,11) + Res(4,5) + Res(4,14); \\
Res(12', 8') &= Res(12,7) + Res(14,7) + Res(14,8); \\
Res(12', 18') &= Res(14,15) + Res(14,18) + Res(12,17); \\
Res(18', 8') &= Res(15,7) + Res(15,9) + Res(16,9); \\
Res(18', 4') &= Res(15,4) + Res(16,4) + Res(15,5); 
\end{align*}
\]
where $\text{Res}(i, j)$ characterises the traffic of the arc, connecting vertices $i$ and $j$ of the network, that is, the amount of its filler, pumped among them per time unit.

In case the secondary vertices are of no interest and attention can be focused on concentrators that in many aspects determine the functioning of the network, the proposed approach can be extremely productive. Such situation of clustering around high-degree vertices is fairly widespread in modern network structures and this can be used during their study. All the more so because today, the dimension of many networks is fairly high and its significant reduction by the method, proposed in the present paper, can reduce calculation expenditure manifold when modeling the processes, proceeding in them.

Fig. 8: Example of network, consisting of four clusters I, II, III, IV, formed by concentrators 4, 8, 12, 18.

Fig. 9: Graph of the transformed network.

As a result of making the proposed transformation, a graph, preserving all basic weight characteristics and topological properties of the initial model of the social network, was obtained.

Such graph is convenient for calculation of different metrics and conducting risk-analysis (Ostapenko, Plotnicov, Makarov, Tikhomirov & Yurasov, 2013; Ostapenko, 2015).
Parinova, Belonozhkin, Bataronov & Simonov, 2013; Ermakov, Zavorykin, Kolenbet, Ostapenko & Kalashnikov, 2014; Radko, Ostapenko, Mashin, Ostapenko & Gusev, 2014; Radko, Ostapenko, Mashin, Ostapenko & Avdeev, 2014; Ostapenko, Bursa, Ostapenko & Butrik, 2014; Islamgulova, Ostapenko, Radko, Babadzhanov & Ostapenko, 2016). In this case it is possible to operate with combined vertices (concentrators), which determine the overall behavior of the network, instead of specific vertices, which in majority are of small significance (in view of the demonstrative law of distribution of the degrees of vertices in social networks) and do not exert influence on content distribution.

All this allows:
1) retaining weigh characteristics and topological properties of the initial network;
2) accelerating the calculation of metrics and their use in risks management;
3) implementing further the simulation of the epidemic process of content distribution, proceeding in the social network;
4) implementing the search of the most important network elements (critical for risk management), including partition of the graph for operating with the whole layers of concentrators.

Such approach offers entirely new prospects of studying social networks (Radko, Ostapenko, Mashin, Ostapenko & Avdeev, 2014; Ostapenko, Bursa, Ostapenko & Butrik, 2014; Islamgulova, Ostapenko, Radko, Babadzhanov & Ostapenko, 2016; Evin & Habibullin, 2012; Ostapenko, Plotnikova & Guzev, 2016), especially from the viewpoint of ensuring their safety. Unfortunately, today social networks are the space of the violent information confrontation, which forces the researchers to assess various risks by means of weights of the attacked vertices and arcs.

V. Conclusion

The proposed method is, first of all, aimed at multi-dimensional networks, where the abundance of vertices and arcs generates substantial difficulties of simulation of the processes, proceeding in these networks. At that, the authors consider in the first place suspended networks, that is, structures, having different weights of their elements (for instance, from the viewpoint of traffic, the network filler, pumped through them). In this respect, the so-called heterogeneous networks (Ostapenko, Plotnicov, Makarov, Tikhomirov & Yurasov, 2013; Ostapenko, Parinova, Belonozhkin, Bataronov & Simonov, 2013; Ermakov, Zavorykin, Kolenbet, Ostapenko & Kalashnikov, 2014; Radko, Ostapenko, Mashin, Ostapenko & Gusev, 2014; Radko, Ostapenko, Mashin, Ostapenko & Avdeev, 2014; Ostapenko, Bursa, Ostapenko & Butrik, 2014; Islamgulova, Ostapenko, Radko, Babadzhanov & Ostapenko, 2016), having a significant repeatability error of vertices degrees, deserve special attention. Transformation of such networks according to the algorithm, proposed in the paper, significantly reduces the sizes of the network under study, and, consequently, calculation expenditures during its simulation. This is very important when solving the tasks of ensuring information safety and analysis (Ostapenko, Plotnicov, Makarov, Tikhomirov & Yurasov, 2013; Ostapenko, Parinova,
Belonozhkin, Bataronov & Simonov, 2013; Ermakov, Zavorykin, Kolenbet, Ostapenko & Kalashnikov, 2014; Radko, Ostapenko, Mashin, Ostapenko & Gusev, 2014; Radko, Ostapenko, Mashin, Ostapenko & Avdeev, 2014; Ostapenko, Bursa, Ostapenko & Butrik, 2014; Islamgulova, Ostapenko, Radko, Babadzhanov & Ostapenko, 2016) of risks, emerging when viruses and other malicious software attack the information networks. Solution of such tasks is problematic even for the most productive contemporary computer aids. Therefore, the preliminary transformation of the network graph, which significantly reduces its sizes, is very useful. The fact that the authors, in this case, base on equivalent topological simplifications, permits, in many respects, to preserve structural properties and weight characteristics of the initial model of the network, as well as to obtain adequate results of its simulation.

Novelty and significance of the results of the paper consist, first of all, in the facts that:

1) the algorithm of similarity transformation offers a researcher an opportunity to turn the initial multidimensional network into a new network of significantly less sizes, which is convenient to use in the analysis of social networks and epidemic processes, proceeding in them;

2) in this case, calculation of metrics and characteristics of the suspended networks, in contrast to analogues, takes into account weigh properties of the network and provides a concise apparatus for studying the processes of distribution of the harmful content in the suspended heterogeneous social networks;

3) the models of the epidemic process, resulting from this, differ from analogues by the fact that namely the suspended heterogeneous social network is simulated, including both the vertices filler (agent quality) and the capacity of network arcs (traffic-filler along network lines).

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