De-tumbling control of uncooperative targets through space tether system

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Abstract. Due to the increase in launch missions and the cascade effect of debris collision, space debris already starts to threaten the space activities. Active debris removal is the only choice to mitigate the debris level, in which the tumble of debris objects is one of the most challengeable part. This paper proposes a new de-tumbling method for the uncooperative target through space tether system, by releasing the tether after capture of the target at short range, the fast rotation of the target evolves into the slow rotation of the whole space tether system, and thus tumbling mitigation is realized. The dynamics model considers the elastic of the tether and attitude motions are derived, and friction release law and attitude damping control strategy is then designed. Simulation results demonstrate the effectiveness of the de-tumbling strategy.

1. Introduction

Space debris especially large abandoned satellites not only take up precious orbital resources, but also pose threats to human space activities and the satellite in use. In addition, due to collision cascade effect, debris generated by collisions would participate in a new collision and eventually more and more fragments are produced. According to the recent research, active debris removal (ADR) is the only method to mitigate and reduce space debris [1].

The tumble is a movement form of space target. The uncontrollable spacecraft with residual angular momentum under the torques imparted by space environment would emerge complex rotational motion, and eventually starts to tumble [2]. The tumble increases the risk of collision as well as operation difficulty since any mistake would lead to devastating consequences.

De-tumbling methods of uncooperative targets mainly contain deceleration brush [3], mechanical pulse [4], space tether robot [5], gas shock [6], electrostatic force [7] and electromagnetic force [8], the former three are contact and the latter three are non-contact. They both need position keeping or flying-around in the process of racemization, the contact way could provide larger torques while the latter is safer.

Space tether system provides a promising way to capture the target by a tether rather than direct contact, thus prevent collisions and improve safeness of the platform. The harpoon capture device is proposed in [9], and underground test has validated the feasibility, its typical capture precision reaches 8cm over 10m. The procedure of de-tumble the uncooperative target using space tether system involves: (1) service spacecraft approaching target with initial velocity; (2) launch harpoon to capture target at certain distance; (3) release tether to mitigate tumble of the target.

Compared with the attitude control of conventional satellites [10-12], space tether system [13-16] are more complex in its dynamics and control, especially when the elastic and attitude motion is taken into consideration. In this paper, we investigate a new de-tumbling control problem through space
tether system. After capture of the target at short distance, the service spacecraft begins to release the tether, and then the fast rotation of the target gradually evolves into the slow rotation of the whole system. Afterwards, a controller is designed to stabilize the service satellite during releasing process, and a friction release law is proposed to control release rate and deplete the rotational kinetic energy. This de-tumbling method would not only increase security of service satellite and reduce operation time, but also save fuel consumption for position keeping and maneuver.

2. Dynamics of space tether system

The target is located in GEO orbit and we firstly establish the following Cartesian coordinate systems shown in figure 1: (1) Inertial coordinate system $I$, with its origin located at earth centre, with $x$-axis points to vernal equinox, $z$-axis points to the North Pole, and $y$-axis decided by right hand rule; (2) body-fixed coordinate system $S$, with its origin at the mass centre of service spacecraft (donated by $S$) and the three axes are decided by its layout; (3) body-fixed coordinate system $T$, with its origin at the mass centre of target (donated by $T$), and the three axes are decided by its layout.

Based on Newton-Euler method, we derive attitude dynamics equations as follows

$$I_S \mathbf{\omega}_S + \omega_T^2 I_S \mathbf{\omega}_S = T_u - r_S^T A_{ST} l_n f_t$$

(1)

$$I_T \mathbf{\omega}_T + \omega_S^2 I_T \mathbf{\omega}_S = r_T^T l_n f_t$$

(2)

where $\omega_S, \omega_T \in \mathbb{R}^{3 \times 1}$ are angular velocities, $I_S, I_T \in \mathbb{R}^{3 \times 3}$ are inertia matrices, $r_S, r_T \in \mathbb{R}^{3 \times 1}$ are tether connection points, $l_n \in \mathbb{R}^{3 \times 1}$ is the unit direction vector of tether (expressed in $T$ coordinate system for convenience), $f_t > 0 \in \mathbb{R}$ is the tether tension, $T_u \in \mathbb{R}^{3 \times 1}$ is the control torque of service spacecraft, $r_S^c$ denotes the cross product matrix and can be written as $r_S^c = \begin{bmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{bmatrix}$, $A_{ST} \in \mathbb{R}^{3 \times 3}$ is the coordinate transformation matrix, $-r_S^c A_{ST} l_n f_t$ and $r_T^c l_n f_t$ are torques imposed on spacecraft $S$ and $T$ by tether tension, respectively.

Take the inertial coordinate system as reference, and by adopting quaternion to describe the attitude of spacecraft $S$ and $T$, we have the following attitude kinematics equations

$$\mathbf{\dot{q}}_S = \frac{1}{2} \left[ -q_{Sx} \mathbf{\omega}_S \right]$$

(3)
\[
\dot{q}_T = \frac{1}{2} \begin{bmatrix} -q_{T0}^T \\ q_{T0} + q_{T0}E \end{bmatrix} \omega_T 
\]

where \( q_{s0} \in \mathbb{R} \), \( q_{t0} \in \mathbb{R} \), \( q_{s0} \in \mathbb{R}^{3_1} \), \( q_{t0} \in \mathbb{R}^{3_1} \) and \( q_T = \begin{bmatrix} q_{T0} \\ q_{T0} \end{bmatrix} \), \( q_s = \begin{bmatrix} q_{s0} \\ q_{s0} \end{bmatrix} \) are quaternion of service spacecraft and target spacecraft, \( E \in \mathbb{R}^{3_3} \) is an identity matrix.

The motion dynamics of the system can be written as

\[
\begin{align*}
\ddot{R}_s &= -\frac{\mu}{R_s^3} R_s + \frac{F_s - F_t}{m_s} \\
\ddot{R}_t &= -\frac{\mu}{R_t^3} R_t + \frac{F_t}{m_t}
\end{align*}
\]

where \( \mu \) is gravitational constant of the earth, \( m_s \) and \( m_t \) are mass of \( S \) and \( T \) respectively, \( F_t = A_T l_t f_t \) is the tether force vector, \( F_s \in \mathbb{R}^{3_1} \) is the control force of spacecraft \( S \). Define \( \rho = R_s - R_t \), according to Eq.(5) and (6), we have

\[
\dot{\rho} = -\frac{\mu}{R_t^3} \rho - \frac{F_t}{M} + \frac{F_s}{m_s}
\]

where \( M = \frac{m_s m_t}{m_s + m_t} \). Ignore the mass of the tether, the tension can be calculate as

\[
f_t = \begin{cases} 
\frac{E_{mat} S_t}{l_0} (l - l_0) + b l & l - l_0 \geq 0 \\
0 & l - l_0 < 0
\end{cases}
\]

where \( E_{mat} \) is the Young's modulus of tether material, \( S_t \) is the cross section of tether, \( l_0 \) is natural length of tether, and \( b \) is damping ratio.

In order to reflect the relation of tether and the two spacecrafts, we define two angles as follows

\[
\begin{align*}
\theta_{ts} &= \arccos \frac{-r_s \cdot A_T l_t}{|r_s| |l_t|} \\
\theta_{tt} &= \arccos \frac{r_t \cdot l_t}{|r_t| |l_t|}
\end{align*}
\]

where \( \theta_{ts} \) is the angle between tether and \( r_s \), and \( \theta_{tt} \) is the angle between tether and \( r_t \), so if \( \theta_{ts} > \frac{\pi}{2} \) or \( \theta_{tt} > \frac{\pi}{2} \), the tether would have the risk of tangle.

The velocity of spacecraft \( T \) and \( S \) relative to the common mass centre could be written as

\[
\begin{align*}
\nu_T &= -\frac{m_s}{m_s + m_t} \dot{\rho} \\
\nu_S &= \frac{m_s}{m_s + m_t} \dot{\rho}
\end{align*}
\]

Then the kinetic energy caused by the motion of mass centres of \( S \) and \( T \) around the common mass centre could be written as

\[
J_3 = \frac{1}{2} m_s \nu_S^T \nu_S + \frac{1}{2} m_t \nu_T^T \nu_T = \frac{1}{2} m_s m_t \rho^T \dot{\rho} = \frac{1}{2} M \dot{\rho}^T \dot{\rho}
\]

The total rotational kinetic energy of the system contains the rotational kinetic energy of \( T \) and \( S \) around their own mass centre and the energy due to entire rotation around the common mass centre, can be calculated as follows

\[
J_3 = \frac{1}{2} m_s \nu_S^T \nu_S + \frac{1}{2} m_t \nu_T^T \nu_T = \frac{1}{2} m_s m_t \rho^T \dot{\rho} = \frac{1}{2} M \dot{\rho}^T \dot{\rho}
\]
\[ J_s = \frac{1}{2} \omega_1^T I_1 \omega_1 + \frac{1}{2} \omega_2^T I_2 \omega_2 + J_s = \frac{1}{2} \omega_1^T I_1 \omega_1 + \frac{1}{2} \omega_2^T I_2 \omega_2 + \frac{1}{2} M \rho^T \rho \]  

(11)

The work done by the tether tension during the release procedure can be calculated as

\[ W_f = \int_0^T f_i \, dt \]  

(12)

3. De-tumbling strategy

In order to prevent collisions between the target and the service spacecraft, it is necessary to have an initial relative speed in the vertical direction of the tether (also perpendicular to the angular momentum of T to prevent twining). The centrifugal force of the relative speed would tighten the tether after capture.

Define the desired attitude quaternion \( q_d \), with \( r_s \) coincidence with \( l_s \), then define the error quaternion \( q_e = q_s - q_d \), and \( \omega_e = \omega_s \), assume the tether is always tight, and under the circumstance of small attitude deviation, the error quaternion dynamics can be simplified as

\[ \dot{q}_{ev} = \frac{1}{2} \omega_e, \quad \dot{\omega}_e = -I_s^{-1} r_s^T A_q l_n f_i \]  

(13)

Rearrange Eq. (13), we have

\[ \ddot{q}_{ev} = -2I_s^{-1} r_s^T f_i |q_{ev}| \]  

(14)

So, the attitude motion of service spacecraft under the tether tension is a free reciprocating oscillation, we design the damping torque to stabilize the spacecraft at the release process, and the controller can be expressed as

\[ T_a = -k_d \omega_e \]  

(15)

And closed loop system can be written as

\[ \ddot{q}_{ev} + 2k_d \dot{q}_{ev} + 2I_s^{-1} r_s^T f_i |q_{ev}| = 0 \]  

(16)

For any \( k_d > 0 \), the system is stable, and to get better dynamic performance, we choose \( k_d = \xi I_s^{-\frac{1}{2}} \sqrt{2|f_i|} \), where \( \xi \) is the damping ratio.

Another problem is to design the tension law, the condition of the relative motion to be a circle is the tether can provide the centripetal acceleration

\[ a_c = \frac{1}{\rho} \rho \times \dot{\rho} \times \dot{\rho} \]  

(17)

So, if we want the tether released continuously and without vibration,

\[ f_i \leq Ma_c \]

And we design the friction law as

\[ f_i = -c \dot{\rho} \]

In the initial period of time, the centrifugal force is very large and at the end of release, the centrifugal force is small, therefore, a constant damp ratio cannot satisfy the requirements of rapid release and no oscillation, we choose \( c \) as

\[ c = \frac{k_c}{l} \]  

(18)

where \( k_c \) is a design parameter. According to the law of conservation of angular momentum, we have

\[ I_r \omega_r(t) + M \rho(t) \times \dot{\rho}(t) = H_c \]  

(19)

where \( H_c \) is a constant vector, and is equal to the initial angular momentum of the system. Assume \( \dot{\rho} \approx \omega_r \times \rho \), means the angular velocity of the system is equal to \( \omega_r \), then we have
\[ \omega_t = \left[ I_r + M |\rho|^2 E \right]^{-1} H_c \] (20)

When the swing is small in the process of release, then \( l \approx |\rho| - r \), where \( r = |r_f| + |r_s| \), according to Eq. (7), we have

\[ \dot{l} = -\frac{\mu}{R^2_c} (l + r) - \left( \frac{k_e}{m} l - a_t \right) \] (21)

Since \( \frac{\mu}{R^2_c} = 5.32 \times 10^{-9} \) is relative small, and assume that \( \dot{l} \approx 0 \) (this assumption is reasonable because release time is very long and the tether does not vibrate), then we have

\[ \dot{l} = \frac{Ml|a_t|}{k_e} = \frac{Ml(l + r)}{k_e} \left| \left[ M (l + r)^2 E + I_r \right]^{-1} H_c \right|^2 \] (22)

Then \( k_e \) can be decided by Eq. (22) to obtain appropriate \( \dot{l} \) from initial min length \( l_{\text{min}} \) to the max length \( l_{\text{max}} \).

4. Simulation results

The target is located in GEO orbit, and the remainder parameters are listed in table 1.

| Parameters | Value | Parameters | Value |
|------------|-------|------------|-------|
| \( m_s \) | 300 kg | \( E_m \) | 4.0841 \times 10^5 |
| \( m_r \) | 7878 kg | \( r_r \) | \( [5 \ 0 \ 0]^T \) m |
| \( I_s \) | \[ \begin{bmatrix} 350 & 3 & 4 \\ 3 & 270 & 0 \\ 4 & 0 & 190 \end{bmatrix} \] kg \cdot m^2 | \( I_r \) | \[ \begin{bmatrix} 17023.3 & 397.1 & -2171.4 \\ 397.1 & 124835.7 & 344.2 \\ -2171.4 & 344.2 & 12911.2 \end{bmatrix} \] kg \cdot m^2 |
| \( r_s \) | \( [-0.5 \ 0 \ 0]^T \) m | \( \rho \) | \( [10.5 \ 0 \ 0]^T \) m |
| \( \omega_t \) | \[ \begin{bmatrix} 1.15 \\ 1.72 \\ 22.9 \end{bmatrix} \] deg/s | \( \dot{\rho} \) | \[ \begin{bmatrix} 4 \\ -0.3 \\ 0 \end{bmatrix} \] m |
| \( l_0 \) | 5m | \( k_e \) | 10000 |

The simulation results are shown in figure 2-9.

**Figure 2.** Length change of the tether.

**Figure 3.** Rotational kinetic energy the work done by tether tension.
Figure 4. Relative position.

Figure 5. Angular velocity of service spacecraft.

Figure 6. Angular velocity of the target.

Figure 7. Control torque of the service spacecraft.

Figure 8. The tether angle $\theta_s$.

Figure 9. The tether angle $\theta_T$.

Figure 2 shows the change of tether length. It can be seen that the release rate decreases gradually because the tension provided by centrifugal force decreases, and the tether has small vibration after release is complete. Figure 3 gives the rotational kinetic energy of the system and the work done by
tether tension (see in Eq. (12)) during release procedure. From the resultant picture, we can conclude that the kinetic energy of the tethered system decreases because tether tension do negative work, and the energy is converted into internal energy by friction. Figure 4 shows the relative position, the system is spirally expanded. Figure 5-6 show the angular velocities of the service spacecraft and target respectively. We can find that the maximum angular velocity of the target at the end of simulation is less than $1\,\text{s}^{-1}$, which demonstrate the effectiveness of the de-tumbling strategy. It is worth to note that capture the target at a speed less than 1 degrees is pretty easy for current technology. Figure 7 gives the control torque. Figure 8-9 illustrate the tether angles with S and T, results show that both angles are less than 90°, which indicate that the risk of tangle is successfully avoided.

5. Conclusions
In this paper, we propose a new de-tumbling method for uncooperative spacecraft through space tether system, by releasing the tether with designed friction control law, the angular rate as well as rotational kinetic energy of the target is effectively reduced. The de-tumbling procedure has accomplished in about 1100s and when the deployment is complete, the tether has only a very small longitudinal vibration. Risk of tangle is also taken into account. Future research should be directed toward tether-assisted deorbit control, which is the next step of ADR missions.

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