Thermal conductivity in the doped two-leg ladder antiferromagnet Sr$_{14-x}$Ca$_x$Cu$_{24}$O$_{41}$

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Within the t-J model, the heat transport of the doped two-leg ladder material Sr$_{14-x}$Ca$_x$Cu$_{24}$O$_{41}$ is studied in the doped regime where superconductivity appears under pressure in low temperatures. The energy dependence of the thermal conductivity $\kappa_c(\omega)$ shows a low-energy peak, while the temperature dependence of the thermal conductivity $\kappa_c(T)$ is characterized by a broad band. In particular, $\kappa_c(T)$ increases monotonously with increasing temperature at low temperatures $T < 0.05J$, and is weakly temperature dependent in the temperature range $0.05J < T < 0.1J$, then decreases for temperatures $T > 0.1J$, in qualitative agreement with experiments. Our result also shows that although both dressed holons and spinons are responsible for the thermal conductivity, the contribution from the dressed spinons dominates the heat transport of Sr$_{14-x}$Ca$_x$Cu$_{24}$O$_{41}$.

74.25.Fy,74.62.Dh,74.72.Jt

In recent years the two-leg ladder material Sr$_{14}$Cu$_{24}$O$_{41}$ has attracted great interest since its ground state may be a spin liquid state with a finite spin gap$^1$. This spin liquid state may play a crucial role in superconductivity of doped cuprates as emphasized by Anderson$^2$. When carriers are doped into Sr$_{14}$Cu$_{24}$O$_{41}$, such as the isovalent substitution of Ca for Sr, a metal-insulator transition occurs$^{3,4}$, and further, this doped two-leg ladder material Sr$_{14-x}$Ca$_x$Cu$_{24}$O$_{41}$ is a superconductor under pressure in low temperatures$^{3,4}$. Apart from the observation of superconductivity under pressure in Sr$_{14-x}$Ca$_x$Cu$_{24}$O$_{41}$, the particular geometrical arrangement of the Cu ions provides a playground for magnetic and transport studies of low-dimensional strongly correlated materials$^1$. All cuprate superconductors found up now contain square CuO$_2$ planes$^3$, whereas Sr$_{14-x}$Ca$_x$Cu$_{24}$O$_{41}$ consists of two-leg ladders of other Cu ions and edge-sharing CuO$_2$ chains$^{1,3,4}$, and is the only known superconducting copper oxide without a square lattice. Experimentally, it has been shown by virtue of the nuclear magnetic resonance and nuclear quadrupole resonance, particularly inelastic neutron scattering measurements that there is a region of parameter space and doping where Sr$_{14-x}$Ca$_x$Cu$_{24}$O$_{41}$ in the normal state is an antiferromagnet with commensurate short-range order$^{1,6,7}$. Moreover, transport measurements on Sr$_{14-x}$Ca$_x$Cu$_{24}$O$_{41}$ in the same region of parameter space and doping indicate that the resistivity is linear with temperatures$^8$, one of the hallmarks of the exotic normal state properties found in doped cuprates on a square lattice$^5$. These unusual normal state properties do not fit in the conventional Fermi-liquid theory$^8$, and may be interpreted within the framework of the charge-spin separation$^2$, where the electron is separated into a neutral spinon and a charged holon, then the basic excitations of the system are not fermionic quasiparticles as in other conventional metals with charge, spin and heat all carried by one and the same particles.

The heat transport is one of the basic transport properties that provide a wealth of useful information on carriers and phonons as well as their scattering processes$^9$–$^{12}$. In the conventional metals, the thermal conductivity contains both contributions from carriers and phonons$^9$. The phonon contribution to the thermal conductivity is always present in the conventional metals, while the magnitude of the carrier contribution depends on the type of material because it is directly proportional to the free carrier density. For the underdoped cuprates on a square lattice, the phonon contribution to the thermal conductivity is strongly suppressed$^{13,14}$. Recently, an unusual contribution to the thermal conductivity of the doped two-leg ladder material Sr$_{14-x}$Ca$_x$Cu$_{24}$O$_{41}$ has been observed$^{10}$–$^{12}$. It has been argued that this unusual contribution may be due to an energy transport via magnetic excitations$^{10,15}$, and therefore can not be explained within the conventional models of phonon heat transport based on phonon-defect scattering or conventional phonon-electron scattering. In particular, in the doped regime where superconductivity appears under pressure in low temperatures, this unusual contribution dominates the thermal conductivity of Sr$_{14-x}$Ca$_x$Cu$_{24}$O$_{41}$. This is a challenge issue since it is closely related to the doped Mott insulating state that forms the basis for superconductivity$^1$. We$^{16}$ have developed a charge-spin separation fermion-spin theory to study the physical properties of doped Mott insulators, where the electron operator is decoupled as the gauge invariant dressed holon and spinon. Within this theory, we$^{17,18}$ have discussed charge transport and spin response of Sr$_{14-x}$Ca$_x$Cu$_{24}$O$_{41}$. It has been shown that the charge transport is mainly governed by the scattering from the dressed holons due to the dressed spinon fluctuation,
while the scattering from the dressed spinons due to the dressed holon fluctuation dominates the spin response. In this paper, we apply this successful approach to discuss the heat transport of Sr$_{14-x}$Ca$_x$Cu$_2$O$_{11}$. Within the $t$-$J$ model, we show that although both dressed holons and spinons are responsible for the heat transport, the contribution from the dressed spinons dominates the thermal conductivity of Sr$_{14-x}$Ca$_x$Cu$_2$O$_{11}$ in the doped regime where superconductivity appears under pressure in low temperatures.

The two-leg ladder is defined as two parallel chains of ions, with bonds among them so that the interchain coupling is comparable in strength to the couplings along the chains, while the coupling between the two chains that participates in this structure is through rungs. It has been argued that the essential physics of the doped two-leg ladder antiferromagnet is contained in the two-leg ladder, and can be effectively described by the $t$-$J$ model,

$$ H = -t \sum_{i,i' \alpha} C_{i\alpha}^\dagger C_{i'\alpha} - t \sum_{i} (C_{i1\sigma}^\dagger C_{i2\sigma} + C_{i2\sigma} C_{i1\sigma}) $$

$$ - \mu \sum_{i} C_{i\alpha}^\dagger C_{i\alpha} $$

$$ + J \sum_{i,i' \alpha} S_{i\alpha} \cdot S_{i'\alpha} + J \sum_{i} S_{i1} \cdot S_{i2}, \quad (1) $$

supplemented by a single occupancy local constraint

$$ C_{i\alpha} C_{i\alpha} \leq 1, \text{ where } \hat{\eta} = \pm cQ \hat{x}, \text{ and } cQ \text{ is the constant of the two-leg ladder lattice, which is set as unity hereafter, } i \text{ runs over all rungs, } \sigma(=\uparrow, \downarrow) \text{ and } a(=1, 2) \text{ are spin and leg indices, respectively, } C_{i\alpha}^\dagger (C_{i\alpha}) \text{ is the electron creation (annihilation) operator, } S_{i\alpha} = C_{i\alpha}^\dagger 3C_{i\alpha}/2 \text{ is the spin operator with } \sigma = (\sigma_x, \sigma_y, \sigma_z) \text{ as the Pauli matrices, and } \mu \text{ is the chemical potential. In the materials of interest, the exchange coupling } J \text{ close to the exchange coupling } J \text{ along the legs is the same as the hopping } t \text{ along the legs and the rung hopping strength } t. \text{ Therefore in the following discussions, we will work with the isotropic system } J = J = J, \text{ and } t = t. \text{ The strong electron correlation in the } t$-$J$ model manifests itself by the single occupancy local constraint, and thus the crucial requirement is to impose this local constraint. This local constraint can be treated properly within the charge-spin separation fermion-spinon theory, $C_{i\alpha}^\dagger = h_{i\alpha}^\dagger S_{i\alpha}^{-}, \text{ where the spinful fermion operator } h_{i\alpha} = e^{-\Phi_{i\alpha}} h_{i\alpha} \text{ describes the charge degree of freedom together with some effects of the spin configuration rearrangements due to the presence of the hole itself (dressed holon), while the spin operator } S_{i\alpha} \text{ describes the spin degree of freedom (dressed spinon), then the electron local constraint for the single occupancy, } \sum_{\sigma} C_{i\alpha}^\dagger C_{i\alpha} = S_{i\alpha}^\dagger h_{i\alpha}^\dagger S_{i\alpha} - S_{i\alpha} h_{i\alpha} S_{i\alpha} = h_{i\alpha} S_{i\alpha}^\dagger S_{i\alpha} - S_{i\alpha} S_{i\alpha} = 1 - h_{i\alpha}^\dagger h_{i\alpha} \leq 1, \text{ is satisfied in analytical calculations, and the double spinful fermion occupancy, } h_{i\alpha} h_{i\alpha}^\dagger h_{i\alpha}^\dagger h_{i\alpha} e^{-\Phi_{i}} = 0, \text{ are ruled out automatically. It has been emphasized that this dressed holon } h_{i\alpha} \text{ is a spinless fermion } h_{i\alpha} \text{ incorporated a spinon cloud } e^{-\Phi_{i}} \text{ (magnetic flux), and then is a magnetic dressing. In other words, the gauge invariant dressed holon carries some spinon messages, i.e., it shares its non-trivial spinon environment. Although in common sense } h_{i\alpha} \text{ is not a real spinful fermion, it behaves like a spinful fermion. In this charge-spin separation fermion-spinon representation, the low-energy behavior of the } t$-$J$ ladder (1) can be expressed as $^{16}$,

$$ H = t \sum_{i,i' \alpha} (h_{i+\alpha}^\dagger h_{i\alpha} + h_{i\alpha} h_{i+\alpha}) $$

$$ + t \sum_{i} (h_{i\alpha} h_{i\alpha} S_{i\alpha}^\dagger - h_{i\alpha} S_{i\alpha} S_{i\alpha}^\dagger) $$

$$ + \mu \sum_{i} h_{i\alpha}^\dagger h_{i\alpha}, \quad (2) $$

with $J_{\text{eff}} = J(1 - \delta)^2, \text{ and } \delta = \langle h_{i\alpha}^\dagger h_{i\alpha} \rangle = \langle h_{i\alpha}^2 \rangle \text{ is the hole doping concentration. In this case, the kinetic part has been expressed as the dressed holon-spinon interaction, and therefore reflect a competition between the kinetic energy and magnetic energy. This competition dominates the essential physics since the dressed holon and spinon self-energies are ascribed purely to the dressed holon-spinon interaction.}

Now we follow the linear response theory, and obtain the thermal conductivity along the ladder,

$$ \kappa(\omega, T) = -\frac{1}{T} \frac{\Im \Pi_Q(\omega, T)}{\omega}, \quad (3) $$

with $\Pi_Q(\omega, T)$ is the heat current-current correlation function, and is defined as

$$ \Pi_Q(\tau - \tau') = -\langle T_{\tau} j_Q(\tau) j_Q(\tau') \rangle, \quad (4) $$

where $\tau$ and $\tau'$ are the imaginary times and $T_{\tau}$ is the order operator, while the heat current density is obtained within the $t$-$J$ ladder Hamiltonian (2) by using Heisenberg’s equation of motion as

$$ j_Q = i \sum_{a,b} \sum_{i,j} R_{ij} [H_{ia}, H_{j\beta}] = j_Q^{(b)} + j_Q^{(s)}, \quad (5a) $$

$$ j_Q^{(b)} = -i(\chi || t)^2 \sum_{i,i' \alpha} \eta \eta_{\alpha}^* h_{i+\alpha}^\dagger h_{i\alpha}^\dagger h_{i\alpha} h_{i+\alpha} $$

$$ + i \chi \chi \sum_{i,i' \alpha} \sum_{\alpha} [R_{i2} - R_{i1} - \eta] h_{i+\alpha}^\dagger h_{i+\alpha} $$

$$ (R_{i2} - R_{i1} + \eta) h_{i+\alpha}^\dagger h_{i+\alpha} $$

$$ - i \mu \sum_{i,i' \alpha} \eta h_{i+\alpha}^\dagger h_{i+\alpha} $$

$$ + i \mu \sum_{i,i' \alpha} [R_{i2} - R_{i1}] (h_{i+\alpha}^\dagger h_{i+\alpha} - h_{i+\alpha}^\dagger h_{i+\alpha}) $$

$$ (R_{i2} - R_{i1}) (h_{i+\alpha}^\dagger h_{i+\alpha} - h_{i+\alpha}^\dagger h_{i+\alpha}) $$
\[ j_Q^{(s)} = i \frac{1}{2} \xi_j J_{\text{eff}}^2 \sum_{i\eta} (\eta I - \eta I) \]

\[
\left[ \epsilon I I_{i\eta} (S^z_{i+\eta} S^z_{i+\eta} - S^z_{i+\eta} S^z_{i+\eta}) - S^z_{i+\eta} S^z_{i+\eta} (S^z_{i+\eta} S^z_{i+\eta}) + (S^z_{i+\eta} S^z_{i+\eta} - S^z_{i+\eta} S^z_{i+\eta})(S^z_{i+\eta}) \right] + \frac{i}{2} J_{\text{eff}}^2 \sum_{i\eta} \left[ \tilde{q}_I (S^z_{i_1} S^z_{i_2} - S^z_{i_2} S^z_{i_1}) + S^z_{i_1} S^z_{i_2} \tilde{S}_I - S^z_{i_2} S^z_{i_1} \tilde{S}_I \right] + \frac{i}{2} J_{\text{eff}}^2 \sum_{i\eta} (\tilde{S}^z_{i_1} S^z_{i_2} - S^z_{i_2} \tilde{S}^z_{i_1}) \right]

where \( R_{i1} \) and \( R_{i2} \) are lattice sites of leg 1 and leg 2, respectively, \( \xi = 1 + 2t_{i\eta} / J_{\text{eff}} \), \( \xi = 1 + 4t_{i\eta} / J_{\text{eff}} \), the dressed spinon correlation functions \( \chi = \langle S^z_{i_1} S^z_{i_2} \rangle \), and \( \chi = \langle S^z_{i_1} S^z_{i_2} \rangle \), and the dressed holon particle-hole order parameters \( \phi = \langle h_{i\eta} \hat{h}_{i\eta} \rangle \), \( \phi = \langle h_{i\eta} \hat{h}_{i\eta} \rangle \). Although the total heat current density \( j_Q^{(s)} \) has been separated into two parts \( j_Q^{(h)} \) and \( j_Q^{(s)} \), with \( j_Q^{(h)} \) is the dressed holon heat current density, and \( j_Q^{(s)} \) is the dressed spinon heat current density, the strong correlation between dressed holons and spinons is still included self-consistently through the dressed spinon’s order parameters entering in the dressed holon’s propagator, and the dressed holon’s order parameters entering in the dressed spinon’s propagator. In this case, the heat current-current correlation function of the two-leg ladder system can be obtained in terms of the full dressed holon and spinon Green’s functions \( g_0(k, \omega) \) and \( D(k, \omega) \). Because there are two coupled \( t-J \) chains in the two-leg ladder system, the energy spectrum has two branches. Therefore, the one-particle dressed holon and spinon Green’s functions are matrices and can be expressed as \( g_0(i - j, \tau - \tau) = G_{\text{holo}}(i - j, \tau - \tau) + \sigma z g_{\text{tr}}(i - j, \tau - \tau) \) and \( D(i - j, \tau - \tau) = D_{\text{holo}}(i - j, \tau - \tau) + \sigma z D_{\text{tr}}(i - j, \tau - \tau) \), respectively, where the longitudinal and transverse parts are defined as \( g_{\text{holo}}(i - j, \tau - \tau) = -\langle T_I h_{i\eta}(\tau) h_{j\eta}(\tau) \rangle \), \( D_{\text{holo}}(i - j, \tau - \tau) = -\langle T_I S^z_{i\eta}(\tau) S^z_{j\eta}(\tau) \rangle \), and \( g_{\text{tr}}(i - j, \tau - \tau) = -\langle T_I h_{i\eta}(\tau) h_{j\eta}(\tau) \rangle \), \( D_{\text{tr}}(i - j, \tau - \tau) = -\langle T_I S^z_{i\eta}(\tau) S^z_{j\eta}(\tau) \rangle \), with \( a \neq a \). Following the discussions of the charge transport, we can obtain the thermal conductivity of the doped two-leg ladder antiferromagnet as,

\[ \kappa_c(\omega, T) = \kappa_c^{(h)}(\omega, T) + \kappa_c^{(s)}(\omega, T), \]

\[ \kappa_c^{(h)}(\omega, T) = -\frac{1}{N} \sum_{k} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \times \left\{ \text{A}_1 [A_T^{(h)}(k, \omega' + \omega) A_T^{(h)}(k, \omega')] + A_L^{(h)}(k, \omega' + \omega) A_L^{(h)}(k, \omega') + A_T^{(h)}(k, \omega' + \omega) A_T^{(h)}(k, \omega') + \text{A}_3 [A_T^{(h)}(k, \omega' + \omega) A_T^{(h)}(k, \omega')] \right\} \times \frac{n_F(\omega') - n_F(\omega)}{T \omega}, \]

\[ \kappa_c^{(s)}(\omega, T) = -\frac{1}{2N} \sum_{k} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \times \left\{ \text{A}_1 [A_L^{(s)}(k, \omega' + \omega) A_L^{(s)}(k, \omega')] + A_T^{(s)}(k, \omega' + \omega) A_T^{(s)}(k, \omega') + A_T^{(s)}(k, \omega' + \omega) A_T^{(s)}(k, \omega') + \text{A}_3 [A_L^{(s)}(k, \omega' + \omega) A_L^{(s)}(k, \omega')] \right\} \times \frac{n_B(\omega') - n_B(\omega)}{T \omega}, \]
the fermion and boson distribution functions, respectively, and the longitudinal and transverse spectral functions of the dressed holon and spinon are obtained as \(A_L^{(h\sigma)}(k, \omega) = -2\text{Im}gL_{\sigma}(k, \omega), A_L^{(s)}(k, \omega) = -2\text{Im}D_L(k, \omega)\) and \(A_T^{(h\sigma)}(k, \omega) = -2\text{Im}gL_{\sigma}(k, \omega), A_T^{(s)}(k, \omega) = -2\text{Im}D_T(k, \omega)\), respectively, where the full dressed holon and spinon Green’s functions have been obtained in Refs.17,18, and are expressed as, \(g_{\sigma}^{-1}(k, \omega) = g_{\sigma}^{(0)}^{-1}(k, \omega) - \Sigma^{(h)}(k, \omega)\) and \(D^{-1}(k, \omega) = D^{(0)}^{-1}(k, \omega) - \Sigma^{(s)}(k, \omega)\), with the longitudinal and transverse second-order dressed holon and spinon Green’s functions \(g_{\sigma}^{L}(k, \omega) = 1/2 \sum_q \frac{1}{(\omega - \xi_q^{(\nu)})}, D^{(0)}_{L}(k, \omega) = \frac{1}{2} \sum_q B_{k}^{(\nu)}/(\omega^2 - (\omega^{(\nu)})^2)\) and \(\Sigma^{(h)}_{T}(k, \omega) = 1/2 \sum_{\nu}(-1)^{\nu+1}/(\omega - \xi_k^{(\nu)})\), \(D^{(0)}_{T}(k, \omega) = 1/2 \sum_{\nu}(-1)^{\nu+1}B^{(\nu)}_{k}/(\omega^2 - (\omega^{(\nu)})^2)\), where \(\nu = 1, 2\), and the longitudinal and transverse second-order dressed holon and spinon self-energies are obtained by the loop expansion to the order \(\lambda^{16,17,18}\) as

\[
\Sigma^{(h)}_{L}(k, \omega) = \left(\frac{\xi}{\gamma}\right)^2 \sum_{pq} \sum_{\nu,\nu'} \Xi^{(h)}_{\nu\nu',\nu'} (k, q, p, q, \omega), \tag{8a}
\]

\[
\Sigma^{(h)}_{T}(k, \omega) = \left(\frac{\xi}{\gamma}\right)^2 \sum_{pq} \sum_{\nu,\nu'} \Xi^{(h)}_{\nu\nu',\nu'} (k, q, p, q, \omega), \tag{8b}
\]

\[
\Sigma^{(s)}_{L}(k, \omega) = \left(\frac{\xi}{\gamma}\right)^2 \sum_{pq} \sum_{\nu,\nu'} \Xi^{(s)}_{\nu\nu',\nu'} (k, q, p, q, \omega), \tag{8c}
\]

\[
\Sigma^{(s)}_{T}(k, \omega) = \left(\frac{\xi}{\gamma}\right)^2 \sum_{pq} \sum_{\nu,\nu'} \Xi^{(s)}_{\nu\nu',\nu'} (k, q, p, q, \omega), \tag{8d}
\]

where \(\Xi^{(h)}_{\nu\nu',\nu'} (k, p, q, \omega)\) and \(\Xi^{(s)}_{\nu\nu',\nu'} (k, p, q, \omega)\) are given by

\[
\Xi^{(h)}_{\nu\nu',\nu'} (k, p, q, \omega) = \frac{B^{(\nu)}_{p+k}B^{(\nu)}_{q}}{2 \omega^{(\nu)}_{p+k} \omega^{(\nu)}_{q}} \left\{ \left[ Z_{\gamma q+p+k} + (-1)^{\nu+\nu'} \right]^2 \right. + \left[ Z_{\gamma q-k} + (-1)^{\nu+\nu'} \right]^2 \right. \times \left( \frac{F^{(1)}_{\nu\nu',\nu'} (k, p, q, \omega)}{\omega + \omega^{(\nu)}_{p} - \omega^{(\nu)}_{q} - \xi_{p+k}^{(\nu)}} \right. \right.
\]
\[
+ \frac{F^{(2)}_{\nu\nu',\nu'} (k, p, q, \omega)}{\omega - \omega^{(\nu)}_{p+k} + \omega^{(\nu)}_{q} - \xi_{p+k}^{(\nu)}} \right. \right.
\]
\[
+ \left. \frac{F^{(3)}_{\nu\nu',\nu'} (k, p, q, \omega)}{\omega + \omega^{(\nu)}_{p+k} + \omega^{(\nu)}_{q} - \xi_{p+k}^{(\nu)}} \right) \left( \frac{\xi_{p+k}^{(\nu)}}{\omega - \omega^{(\nu)}_{p+k} - \omega^{(\nu)}_{q} - \xi_{p+k}^{(\nu)}} \right) \right\}, \tag{9a}
\]

\[
\Xi^{(s)}_{\nu\nu',\nu'} (k, p, q, \omega) = \frac{B^{(\nu)}_{p+k}B^{(\nu)}_{q}}{16 \omega_{p+k+q}} \left\{ \left[ Z_{\gamma q+p+k} + (-1)^{\nu+\nu'} \right]^2 \right. \\
+ \left. \left[ Z_{\gamma q-k} + (-1)^{\nu+\nu'} \right]^2 \right\} \times \left( \frac{G^{(1)}_{\nu\nu',\nu'} (k, p, q, \omega)}{\omega + \omega^{(\nu)}_{p} - \omega^{(\nu)}_{q} - \omega_{p+k}^{(\nu)}} \right. \right.
\]
\[
- \left. \frac{G^{(2)}_{\nu\nu',\nu'} (k, p, q, \omega)}{\omega + \omega^{(\nu)}_{p+k} - \omega^{(\nu)}_{q} + \omega_{p+k}^{(\nu)}} \right) \right\}, \tag{9b}
\]

with \(B^{(\nu)}_{k} = B_{k} - J_{\text{eff}}(\chi_{\perp} + 2\chi_{\parallel}(-1)^{\nu}[(\epsilon_{\parallel} + (-1)^{\nu}], B_{k} = \lambda(2\epsilon_{\perp} + \chi_{\perp} - (\chi_{\perp} + 2\chi_{\parallel})), \lambda = 2ZJ_{\text{eff}}, \) and

\[
F^{(1)}_{\nu\nu',\nu'} (k, p, q, \omega) = n_F(\xi^{(\nu')}_{p+k})[n_B(\omega^{(\nu')}_{q+p}) - n_B(\omega^{(\nu')}_{q+p})] + n_B(\omega^{(\nu')}_{q+p})[1 + n_B(\omega^{(\nu')}_{q+p})] \tag{10a}
\]

\[
F^{(2)}_{\nu\nu',\nu'} (k, p, q, \omega) = n_F(\xi^{(\nu')}_{p+k})[n_B(\omega^{(\nu')}_{q+p}) - n_B(\omega^{(\nu')}_{q+p})] + n_B(\omega^{(\nu')}_{q+p})[1 + n_B(\omega^{(\nu')}_{q+p})] \tag{10b}
\]

\[
F^{(3)}_{\nu\nu',\nu'} (k, p, q, \omega) = n_F(\xi^{(\nu')}_{p+k})[1 + n_B(\omega^{(\nu')}_{q+p})], \tag{10c}
\]

\[
G^{(4)}_{\nu\nu',\nu'} (k, p, q, \omega) = n_F(\xi^{(\nu')}_{p+k})[1 + n_B(\omega^{(\nu')}_{q+p})] - n_F(\xi^{(\nu')}_{p+k})[1 + n_B(\omega^{(\nu')}_{q+p})]. \tag{10d}
\]

and the mean-field dressed holon and spinon excitations,

\[
\xi^{(\nu)}_{k} = Zt\chi_{\parallel}\kappa_{\parallel} + \mu_{\parallel} + \chi_{\perp}(-1)^{\nu+1}, \tag{11a}
\]

\[
\omega^{(\nu)}_{k} = \alpha_{\parallel}\lambda^{2}\chi_{\parallel} + \epsilon_{\parallel}\chi_{\parallel}^{2} - \epsilon_{\parallel}^{2}\chi_{\parallel}^{2}/2 \tag{11b}
\]

where the dressed spinon correlation functions \(\chi_{\parallel}^{2} = \langle S_{a\parallel}^{z}S_{a\parallel}^{z} \rangle, \chi_{\perp} = \langle S_{a\parallel}^{1}S_{a\parallel}^{2} \rangle, C_{\perp} = (1/2Z^{2})\sum_{p_{\parallel}}\langle S_{a\parallel+p_{\parallel}}^{z}S_{a\parallel+p_{\parallel}}^{z} \rangle, \) and
In order to satisfy the sum rule for the correlation function \( \langle S^+_{a\delta}S_{\delta\delta}^- \rangle = 1/2 \) in the absence of the antiferromagnetic long-range-order, a decoupling parameter \( \alpha \) has been introduced in the mean-field calculation, which can be regarded as the vertex correction. All the above mean-field order parameters, decoupling parameter \( \alpha \), and chemical potential \( \mu \), have been determined by the self-consistent calculation.

In the two-leg ladder material \( \text{Sr}_{1-x}\text{Ca}_x\text{Cu}_{24}\text{O}_{41} \), the most remarkable expression of the nonconventional physics is found in the doped regime where the hole doping concentration on ladders at \( x = 8 \sim 12 \) is corresponding to \( \delta = 0.16 \sim 0.20 \) per Cu ladder. In this doped regime, superconductivity appears under pressure in low temperatures, and the value of \( J_\parallel \) has been estimated as \( J = J_\parallel \approx 90 \text{ meV} \approx 1000\text{K} \). Therefore in the following discussions, we focus on this doped regime. In Fig. 1, we present the results of the thermal conductivity \( \kappa_c(\omega) \) in Eq. (6a) as a function of frequency at \( \delta = 0.16 \) (solid line) and \( \delta = 0.20 \) (dashed line) for \( t/J = 2.5 \) with \( T = 0.05J \). Although \( \kappa_c(\omega) \) is not observable from experiments, its features will have observable implications on the observable \( \kappa_c(T) \). Our results in Fig. 1 show that the frequency dependence of the thermal conductivity spectrum is characterized by a rather sharp low-energy peak. The position of this low-energy peak is doping dependent, and is located at a finite energy \( \omega \approx 0.1t = 0.25J \approx 23 \text{ meV} \). This low-energy peak is corresponding to the peak observed in the conductivity spectrum. Moreover, we also find from the above calculations that although both dressed holons and spinons are responsible for the thermal conductivity \( \kappa_c(\omega) \), the contribution from the dressed spinons is much larger than that from the dressed holons, i.e., \( \kappa_c(s)(\omega) \gg \kappa_c(h)(\omega) \), and therefore the thermal conductivity of the doped two-leg ladder antiferromagnet in the hole doped regime \( \delta = 0.16 \sim 0.20 \) is mainly determined by its dressed spinon part \( \kappa_c(s)(\omega) \).

Now we turn to discuss the temperature dependence of the thermal conductivity \( \kappa_c(T) \), which is observable from experiments and can be obtained from Eq. (6a) as \( \kappa_c(T) = \lim_{\omega \to 0} \kappa_c(\omega, T) \). The results of \( \kappa_c(T) \) at \( \delta = 0.16 \) (solid line) and \( \delta = 0.20 \) (dashed line) for \( t/J = 2.5 \) are plotted in Fig. 2 in comparison with the corresponding experimental results taken on \( \text{Sr}_{14-x}\text{Ca}_x\text{Cu}_{24}\text{O}_{41} \) (inset), where the hole doping concentration on ladders at \( x = 12 \) is \( \delta = 0.20 \) per Cu ladder. The present results indicate that the temperature dependence of the thermal conductivity in the normal-state exhibits a broad band, i.e., \( \kappa_c(T) \) increases monotonously with increasing temperature at low temperatures \( T < 0.05J \approx 50\text{K} \), and is weakly temperature dependent in the temperature range \( 0.05J \approx 50\text{K} < T < 0.1J \approx 100\text{K} \), then decreases for temperatures \( T > 0.1J \approx 100\text{K} \), in qualitative agreement with the experimental data. For \( T < 0.01J \approx 10\text{K} \), the system is a superconductor under pressure. In this case, the thermal conductivity of the doped two-leg ladder antiferromagnet is under investigation now.

In the above discussions, the central concern of the heat transport in the doped two-leg ladder material \( \text{Sr}_{14-x}\text{Ca}_x\text{Cu}_{24}\text{O}_{41} \) is the charge-spin separation, then the contribution from the dressed spinons dominates the thermal conductivity. Since \( \kappa_c(s)(\omega, T) \) in Eq. (6c) is obtained in terms of the dressed spinon longitudinal and transverse Green’s functions \( D_L(k, \omega) \) and \( D_T(k, \omega) \), while these dressed spinon Green’s functions are evaluated by considering the second-order correction due to the dressed holon pair bubble, then the observed unusual frequency and temperature dependence of the thermal conductivity spectrum of the doped two-leg ladder antiferromagnet is closely related to the commensurate spin response. The dynamical spin structure factor of the doped two-leg ladder antiferromagnet has been obtained
within the charge-spin separation fermion-spin theory\(^\text{18}\) in terms of the dressed spinon longitudinal and transverse Green's functions as,

\[
S(k, \omega) = -2[1 + n_B(\omega)][2\text{Im}D_L(k, \omega) + 2\text{Im}D_T(k, \omega)]
\]

\[
= [1 + n_B(\omega)][A_L^{(s)}(k, \omega) + A_T^{(s)}(k, \omega)]
\]

\[
= -\frac{4[1 + n_B(\omega)][B_k^{(1)}\text{Im}\Sigma_L^{(s)}(k, \omega)]}{|W(k, \omega)|^2 + |B_k^{(1)}\text{Im}\Sigma_L^{(s)}(k, \omega)|^2},
\]

(12a)

\[
W(k, \omega) = \omega^2 - (\omega_k^{(1)})^2 - B_k^{(1)}\text{Re}\Sigma_L^{(s)}(k, \omega),
\]

(12b)

where \(\text{Im}\Sigma_L^{(s)}(k, \omega) = \text{Im}\Sigma_L^{(c)}(k, \omega) + \text{Im}\Sigma_L^{(s)}(k, \omega)\), \(\text{Re}\Sigma_L^{(s)}(k, \omega) = \text{Re}\Sigma_L^{(c)}(k, \omega) + \text{Re}\Sigma_L^{(s)}(k, \omega)\), while \(\text{Im}\Sigma_L^{(c)}(k, \omega)\) and \(\text{Re}\Sigma_L^{(c)}(k, \omega) \) are the imaginary and real parts of the second order longitudinal (transverse) spinon self-energy in Eqs. (8c) and (8d). The renormalized spin excitation \(E_k^{(1)} = (\omega_k^{(1)})^2 + B_k^{(1)}\text{Re}\Sigma_L^{(s)}(k, \omega)\) in \(S(k, \omega)\) is doping energy, and interchain coupling dependent. In the two-leg ladder system, the quantum interference effect between the chains manifests itself by the interchain coupling, i.e., the quantum interference increases with increasing the strength of the interchain coupling. In the materials of interest, \(J_\perp \sim J_\parallel\) and \(t_\perp \sim t_\parallel\), we have shown in detail in Ref.\(^\text{18}\), the dynamical spin structure factor in Eq. (12) has a well-defined resonance character, where \(S(k, \omega)\) exhibits the commensurate peak when the incoming neutron energy \(\omega\) is equal to the renormalized spin excitation \(E_k\), i.e., \(W(k, \omega) \equiv |\omega^2 - (\omega_k^{(1)})^2 - B_k^{(1)}\text{Re}\Sigma_L^{(s)}(k, \omega)|^2 = (\omega^2 - E_k^2)^2 \approx 0\) at antiferromagnetic wave vectors \(k_{AF}\), then the height of this commensurate peak is determined by the imaginary part of the spinon self-energy \(1/\text{Im}\Sigma_L^{(s)}(k, \omega)\). Since the incoming neutron resonance energy \(\omega_r = E_k\) is finite\(^\text{18}\), then there is a spin gap in the system. This spin gap leads to that the low-energy peak in \(\kappa_{\parallel}(\omega)\) is located at a finite energy \(\omega_{\text{peak}} \approx 23\text{meV}\). This anticipated spin gap \(\Delta_\parallel \approx 23\text{meV}\) is not too far from the spin gap \(\approx 32\text{meV}\) observed\(^\text{6}\) in \(\text{Sr}_{14-x}\text{Ca}_{x}\text{Cu}_{24}\text{O}_{41}\). Therefore the commensurate spin fluctuation of the doped two-leg ladder material \(\text{Sr}_{14-x}\text{Ca}_{x}\text{Cu}_{24}\text{O}_{41}\) is responsible for the heat transport, in other words, the energy transport via magnetic excitations dominates the thermal conductivity\(^\text{10}\). Based on the simple theoretical estimate\(^\text{11}\), the large thermal conductivity observed in the two-leg ladder material \(\text{Sr}_{14}\text{Cu}_{24}\text{O}_{41}\) has been interpreted in terms of the magnetic excitations in the system\(^\text{11}\). Our present explanation is also consistent with theirs\(^\text{11}\).

To conclude we have discussed the heat transport of the two-leg ladder material \(\text{Sr}_{14-x}\text{Ca}_{x}\text{Cu}_{24}\text{O}_{41}\) within the \(t-J\) ladder in the doped regime where superconductivity appears under pressure in low temperatures. It is shown that the energy dependence of the thermal conductivity spectrum \(\kappa_{\parallel}(\omega)\) shows a low-energy peak, and the position of this low-energy peak is doping dependent, and is located at a finite energy. The temperature dependence of the thermal conductivity \(\kappa_{\parallel}(T)\) is characterized by a broad band. \(\kappa_{\parallel}(T)\) increases monotonously with increasing temperature at low temperatures \(T < 0.05J\), and is weakly temperature dependent in the temperature range \(0.05J < T < 0.1J\), then decreases for temperatures \(T > 0.1J\). Our result of the temperature dependence of the thermal conductivity is in qualitative agreement with the major experimental observations of \(\text{Sr}_{14-x}\text{Ca}_{x}\text{Cu}_{24}\text{O}_{41}\) in the normal-state\(^\text{10}\). Our result also shows that although both dressed holons and spinons are responsible for the thermal conductivity, the contribution from the dressed spinons dominates the heat transport.

ACKNOWLEDGMENTS

The authors would like to thank Dr. Ying Liang, Dr. Tianxing Ma, and Dr. Yun Song for the helpful discussions. This work was supported by the National Natural Science Foundation of China under Grant Nos. 10125415 and 90403005, the Grant from Beijing Normal University, and the National Science Council.

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