Constraining the mixing matrix for Standard Model with four generations:
time dependent and semi-leptonic CP asymmetries in $B_0^0$, $B_s$ and $D^0$

Soumitra Nandi$^1$ and Amarjit Soni$^2$

$^1$Physique des Particules, Université de Montréal,
C.P. 6128, succ. centre-ville, Montréal, QC, Canada H3C 3J7
$^2$Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA

Using existing experimental information from K, B and D decays as well as electroweak precision tests and oblique parameters, we provide constraints and correlations on the parameters of the 4X4 mixing matrix for the Standard Model with four generations (SM4). We emphasize that some correlations amongst the parameters have important repercussions for key observables. We work with a particular representation of this matrix which is highly suited for extracting information from B-decays. Implications of the resulting constraints for time dependent and semileptonic CP asymmetries for $D^0$, $B^0$ and for $B_s$ are also given. While we show that the semi-leptonic asymmetries may be significantly enhanced in SM4 over the SM, there are important constraints and correlations with other observables. In this context we suggest that existing data from B-factories taken on $\Upsilon(4S)$ and $\Upsilon(5S)$, and in the relevant continuum be used to constrain the semi-leptonic asymmetries for $B_d$, $B_s$ as well as their linear combination. Of course, the data from the Tevatron and LHCb experiments can provide non-trivial tests of SM4 as well.

I. INTRODUCTION

In the past few years a number of tensions in the CKM fits for the Standard Model (SM) with 3 generations have been revealed [1-5]. There are quite serious indications that the “predicted” value of $\sin 2\beta$ is larger compared to the value measured directly via the “gold-plated” $\psi K_S$ mode by as much as $\approx 3.3\sigma$ [6]. Of course, the value of $\sin 2\beta$ determined from the penguin dominated modes tends to be even smaller compared to that from the $\psi K_s$ mode and therefore that constitutes even a larger deviation from the SM predicted value [2]. There are other anomalies as well that appear related. The difference in the partial rate asymmetries between $B^0 \to K^+\pi^-$ and $B^+ \to K^+\pi^0$ is also too large [7] to understand [2], though QCD complications do not allow us to draw compelling conclusions in this regard [8]. But with the backdrop of the hint of presence of a new CP-odd phase in the $\Delta S = 1$ penguin dominated modes, it is highly suggestive that the direct CP problem in K $\pi$ modes is receiving, at least in part, contribution from the same new physics source.

There are also some indications from the CDF and DO experiments at the Tevatron [9]. While
the earlier indication of possible non-standard effects in $B_s \to \psi\phi$ seem to have weakened somewhat at the higher luminosity around 6/fb now being used. D0 has announced a surprisingly large CP-asymmetry in the same sign dimuons which they attribute primarily to originate from $B_s \to X_s\mu\nu$. From a theoretical standpoint if new physics exists in $\Delta S = 1$ B-decays, then it becomes highly unnatural for it not to exist in $\Delta S = 2$, $B_s$ mixings as well.

A simple extension of the SM with four generations (SM4) can readily account for such anomalies. Of course, even without these anomalies, SM4 is an interesting extension of the SM worth study. The two extra phases that it possesses can give rise to a host of non-standard CP asymmetries and in fact SM4 can significantly ameliorate the difficulties with regard to baryogenesis that SM3 has. Besides, the heavier quarks and leptons of the 4th generation may well lead to dynamical electroweak symmetry breaking and thereby become useful in addressing the hierarchy problem without the need for supersymmetry at the weak scale.

Motivated by these considerations we will continue our investigations of the physical implications of SM4. In particular we will use all the known experimental constraints such as $B \to X_s\gamma$, $B \to X_s l^+ l^-$, $\Delta M_{B_s}$, $\Delta M_{B_d}$, $K^+ \to \pi\nu\nu$, electroweak precision constraints from $Z \to b\bar{b}$ as well as oblique corrections as in our previous work. However, we will now use an explicit representation of the 4X4 CKM matrix of given long ago. We make this particular choice as it is very well designed to extract constraints from B decays since it was shown in a series of papers that SM4 is highly susceptible to those decays.

We will provide constraints and many correlations amongst the 6 real parameters and the 3 phases that enter the SM4. We will then apply this framework to study mixing induced CP asymmetries $S(B_d \to \psi K_s)$, $S(B_s \to \psi\phi)$, $S(D^0 \to f)$ (where $f$ may be any self conjugate final state such as $K_s\pi^0$, $K_s\omega$, $K_s\rho^0$, $\pi^0\pi^0$, $K^+K^-$, $\pi^+\pi^-$ etc.) and semi-leptonic asymmetries in $D^0$, $B^0$, and in $B_s$.

In obtaining these constraints and implications we will allow $m_\nu$ to range from 375 to 575 GeV as suggested by current hints from study of B-decays. An interesting aspect of SM4 is that it is rather well constrained already. Thus, for example, while the semi-leptonic asymmetry in $B_s$ ($a_{sl}^s$) can be enhanced by as much as a factor of about 300 over SM3 it still cannot account for the central value of the recent D0 result. Of course, that observation has only about 2-$\sigma$ significance and therefore rather large errors but improved experimental results could certainly rule out or confirm SM4, since the predicted range in SM4 for $a_{sl}^s$ is between about (0.006) to (-0.006); also its sign has to be the same as $S_{\psi\phi}$. Furthermore, for $B_d$, $a_{sl}^d$ can only be larger by around a factor of four over SM3. These semi-leptonic asymmetries also have interesting correlations with
$S(B_s \rightarrow \psi\phi)$ and $S(B_d \rightarrow \psi K_s)$ respectively that should be testable.

As mentioned above one of the key difficulty for the CKM-paradigm of SM3 uncovered in recent years is that the predicted value of $\sin 2\beta$ is too large compared to the measured one \[1, 6\]. We will show here that SM4 tends to alleviate this tension appreciably but at the same time then it allows to place an important bound on $a_{sl}^d$ through the correlation mentioned in the previous para.

B-factories placed a bound on $a_{sl}^d$ [10] some years ago but by now they have considerable more data. So an improved bound would be extremely worthwhile. In the past couple of years BELLE also took substantial data on $\Upsilon$ 5S [31]. In fact that data could provide a very clean study of $a_{sl}^s$ as well as on $A^b_{sl}$, which is defined as the linear combination of $a_{sl}^s$ and $a_{sl}^d$ [12], since that sample provides a valuable source of this combination as well as an enriched sample of $B_s$. CDF, D0 and LHCb should be able to provide very useful results on these semi-leptonic asymmetries. In fact whereas the Tevatron $p\bar{p}$ collider allowed D0 to yield the sum of $a_{sl}^d$ and $a_{sl}^s$, the $pp$ collider at LHC cannot do that, but LHCb should be able to study the difference of these two asymmetries [32].

We should emphasize that in this series of studies on the 4th generation [13, 14], for simplicity, and for definiteness, we have been making a tacit assumption that a heavy charge $2/3$ and $-1/3$ quark doublet has weak interaction just like the previous three families allowing us to incorporate these readily into a 4X4 mixing-matrix resulting from an immediate generalization of the 3X3 case. Clearly if and when such a doublet of quarks is observed we will need to make detail tests on the weak interaction properties of the new quarks to verify that this assumption is correct.

The paper is arranged as follows. After the introduction, in Sec. II A and II B we provide information regarding the parametrisation and the constraints on the 4$\times$4 CKM matrix by incorporating oblique corrections along with experimental data from important observables involving Z, B and K decays as well as $B_d$ and $B_s$ mixings etc. In Sec. II C we present the estimates of many useful observables in the SM4. Finally in Sec. III we present our summary.

II. NUMERICAL ANALYSIS

A. Parametrisation of $V_{CKM4}$

We use the parametrisation of the SM4 mixing matrix from [28], then the elements of fourth row such as $V_{t'd}$, $V_{t's}$ and $V_{t'b}$, which are more relevant for the discussion of $b$ physics, will be rather
simple. Defining

$$V_{us} = \lambda, \quad V_{cb} = A \lambda^2, \quad V_{ub} = A \lambda^3 C e^{-i\delta_{ub}},$$

$$V_{t's} = -Q A \lambda^2 e^{i\delta_{t's}}, \quad V_{t'd} = -P A \lambda^3 e^{i\delta_{t'd}}, \quad V_{t'b} = -r \lambda$$

the generalised 4 × 4 mixing matrix $V_{SM4}$ is given in eq. (3). With the inputs $|V_{ub}| = (32.8 \pm 3.9) \times 10^{-4}$ and $|V_{cb}| = (40.86 \pm 1.0) \times 10^{-3}$ taken at 1σ, constraints obtained on $A$ and $C$ are given by

$$0.825 \leq A \leq 0.865, \quad 0.32 \leq C \leq 0.42,$$

while $\lambda = 0.2205 \pm 0.0018$. The phase of $V_{ub}$ i.e $\delta_{ub}$ can be taken as the CKM angle $\gamma$ of SM3.

$$
\begin{pmatrix}
1 - \frac{\lambda^2}{2} & \lambda & A \lambda^3 C e^{-i\delta_{ub}} & P A \lambda^3 e^{-i\delta_{t'd}} \\
-\lambda & 1 - \frac{\lambda^2}{2} & A \lambda^2 & Q A \lambda^3 e^{-i\delta_{t's}} \\
A \lambda^3 (1 - C e^{i\delta_{ub}}) & -A \lambda^2 & 1 - \frac{\lambda^2}{2} & r \lambda \\
-\frac{1}{2} A C \lambda^2 e^{i\delta_{t'd}} & -Q r A \lambda^3 e^{i\delta_{t's}} & +A \lambda^2 & 1 - \frac{\lambda^2}{2}
\end{pmatrix}
$$

B. Inputs

In our earlier papers [13, 14], to find the limits on some of the $V_{CKM4}$ elements, we concentrated mainly on the constraints that will come from non-decoupling oblique corrections, vertex correction to $Z \rightarrow b \bar{b}$, $BR(B \rightarrow X_s \gamma)$, $BR(B \rightarrow X_s l^+ l^-)$, $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixing, $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ and the indirect CP violation in $K_L \rightarrow \pi \pi$ described by $\epsilon_K$; we did not consider $\epsilon'/\epsilon$ as a constraint because of the large hadronic uncertainties, in the evaluation of its matrix elements. With the inputs given in Table II we have made the scan over the entire parameter space by a flat random number generator and obtained the constraints on various parameters such as, $P$, $Q$, $r$, $\delta_{t'd}$ and $\delta_{t's}$ of the 4×4 mixing matrix.

From direct searches at the Tevatron, it follows that $m_{t'} > 335$ GeV [33]. Taking into account the limits from electroweak precision tests [34, 37], perturbativity [38] and indications from our
TABLE I: Inputs that we use in order to constrain the SM4 parameter space, when not explicitly stated, we take the inputs from Particle Data Group [7]; for the lattice input see also [6].

| Parameter | Value |
|-----------|-------|
| $B_K$     | $0.740 \pm 0.025$ [39–41] |
| $f_{bd} \sqrt{F_{bd}}$ | $0.224 \pm 0.015$ GeV [42, 43] |
| $\xi$     | $1.232 \pm 0.042$ [42, 43] |
| $\eta_c$  | $1.51 \pm 0.24$ [44] |
| $\eta_b$  | $0.5765 \pm 0.0065$ [45] |
| $\eta_{ct}$ | $0.494 \pm 0.046$ [46] |
| $\Delta M_s$ | $(17.77 \pm 0.12)$ ps$^{-1}$ |
| $\Delta M_d$ | $(0.507 \pm 0.005)$ ps$^{-1}$ |
| $|\epsilon_k| \times 10^3$ | $2.32 \pm 0.007$ |
| $\kappa_e$ | $0.94 \pm 0.02$ [47] |
| $R_{kb}$   | $0.216 \pm 0.001$ |
| $|V_{ub}|$  | $(32.8 \pm 2.6) \times 10^{-4}$ a |
| $|V_{cb}|$  | $(40.86 \pm 1.00) \times 10^{-3}$ |
| $\gamma$   | $(73.0 \pm 13.0)^\circ$ |
| $BR(B \to X_s \gamma)$ | $(3.55 \pm 0.25) \times 10^{-4}$ |
| $BR(B \to X_s \ell^+\ell^-)$ | $(0.44 \pm 0.12) \times 10^{-6}$ |
| $BR(K^+ \to \pi^+\nu\bar{\nu})$ | $(0.147^{+0.130}_{-0.089}) \times 10^{-9}$ |
| $BR(B \to X_s \ell\nu)$ | $(10.61 \pm 0.17) \times 10^{-2}$ |
| $T_\alpha$ | $0.11 \pm 0.14$ |
| $m_t(m_t)$ | $(163.5 \pm 1.7)$ GeV |

It is the weighted average of $V_{ub}^{\text{ext}} = (40.1 \pm 2.7 \pm 4.0) \times 10^{-4}$ and $V_{ub}^{\text{ext}} = (29.7 \pm 3.1) \times 10^{-4}$, error on the weighted average is increased by 50% because of the appreciable disagreement between the two measurements.

We tacitly assume that $\kappa_e$ in SM4 is approximately the same as in SM3.

We use the SM predictions $\phi$, leading order in $\alpha_s$ we do not have appreciable SM4 contribution to $\Delta\Gamma_q$; we use the SM predictions for $\Delta\Gamma_q$ [48] to find out the allowed ranges for the semileptonic asymmetries.

Detailed mathematical formulas for the above mentioned observables (Table I) can be seen from one of our earlier papers [14]. In this paper, we do not impose $S_{\psi K_s} = \sin 2\beta_{\text{eff}}$ as a constraint, we show SM4 prediction for $S_{\psi K_s}$ and its correlation with the semileptonic asymmetry $a_{s,l}^d$. The mathematical expression for the semileptonic asymmetry is given by

$$a_{s,l}^d = \frac{\Delta\Gamma_q}{\Delta M_q} \tan \phi_q, \quad (q = d, s),$$

where $\phi_q$ ($q = d, s$) is defined as

$$\phi_q = \text{Arg} \left[ \frac{M_{12}^q}{\Gamma_{12}} \right].$$

Therefore the semileptonic asymmetry is the function of the CP phase $\phi_q$ and the width difference $\Delta\Gamma_q$ between the heavy and light mass eigenstates; $\Delta M_q$ are known with at least 1% accuracy [41]. SM prediction for $\Delta\Gamma_q$ has an overall impact on the results for semileptonic asymmetry. At leading order in $\alpha_s$ we do not have appreciable SM4 contribution to $\Delta\Gamma_q$; we use the SM predictions for $\Delta\Gamma_q$ [48] to find out the allowed ranges for the semileptonic asymmetries.
In addition, we study $D^0 - \bar{D}^0$ mixing in the presence of a fourth generation of quarks. In particular, we calculate the size of the allowed CP violation, which could be large compared to the SM, and show its parametric dependence on CKM4 elements.

Within the SM, $D^0 - \bar{D}^0$ mixing proceeds to an excellent approximation only through the box diagrams with internal $b$ and $s$ quark exchanges. In the case of four generations there is an additional contribution to $D^0 - \bar{D}^0$ mixing coming from the virtual exchange of the fourth generation down quark $b'$.

The short distance (SD) contributions to the matrix element of the $\Delta C = 2$ effective Hamiltonian can be written as

$$\langle \bar{D}^0 | H_{\Delta C=2}^{\text{eff}} | D^0 \rangle_{\text{SD}} \equiv | M_{12}^D | e^{i\phi_D} = (M_{12}^D)^*, \quad (7)$$

where

$$M_{12}^D = \frac{G_F^2}{12\pi^2} f_D B_D m_D M_W^2 \bar{M}_{12}^D, \quad (8)$$

with

$$\bar{M}_{12}^D = \lambda_s^{(D)*} \eta_{cc}^{(K)} S_0(x_s) + \lambda_b^{(D)*} \eta_{cc}^{(K)} S_0(x_b) + \lambda_{b'}^{(D)*} \eta_{tt}^{(K)} S_0(x_{b'})$$

$$+ 2\lambda_b^{(D)*} \lambda_s^{(D)} \eta_{cc}^{(K)} S_0(x_b, x_s) + 2\lambda_b^{(D)*} \lambda_s^{(D)} \eta_{ct}^{(K)} S_0(x_s, x_s) + 2\lambda_{b'}^{(D)*} \lambda_b^{(D)} \eta_{ct}^{(K)} S_0(x_{b'}, x_b), \quad (9)$$

where

$$\lambda_i^{(D)} = V_{ci}^* V_{ui} \quad (i = s, b, b'). \quad (10)$$

For the QCD corrections we will use the approximate relations

$$\eta_{b'b'}^{(D)} \approx \eta_{tt}^{(K)}, \quad \eta_{b'b}^{(D)} \approx \eta_{tt}^{(K)} \approx \eta_{ct}^{(K)}, \quad \eta_{bs}^{(D)} \approx \eta_{bb}^{(D)} \approx \eta_{bs}^{(K)} \approx \eta_{ct}^{(K)}. \quad (11)$$

Including the long distance part the full matrix elements are given by,

$$\langle \bar{D}^0 | H_{\Delta C=2}^{\text{eff}} | D^0 \rangle = (M_{12}^D + M_{12}^{LD})^* - \frac{i}{2} \Gamma_{12}^{LD*}, \quad (12)$$

$$\langle D^0 | H_{\Delta C=2}^{\text{eff}} | \bar{D}^0 \rangle = (M_{12}^D + M_{12}^{LD}) - \frac{i}{2} \Gamma_{12}^{LD}. \quad (13)$$

Here $\Gamma_{12}^{LD}$ and $M_{12}^{LD}$ stand for long distance (LD) contributions with the former arising exclusively from SM3 dynamics. These contributions are very difficult to estimate reliably; we scan flatly over the intervals \([49, 50]\).

$$-0.02 \text{ps}^{-1} \leq M_{12}^{LD} \leq 0.02 \text{ps}^{-1}, \quad (14)$$

$$-0.04 \text{ps}^{-1} \leq \Gamma_{12}^{LD} \leq 0.04 \text{ps}^{-1}. \quad (15)$$
\( D^0 - \bar{D}^0 \) oscillations can be characterised by the normalised mass and width differences

\[
x_D \equiv \frac{\Delta M_D}{\Gamma}, \quad y_D \equiv \frac{\Delta \Gamma_D}{2\Gamma}, \quad \bar{\Gamma} = \frac{1}{2}(\Gamma_1 + \Gamma_2),
\]

with

\[
\Delta M_D = M_1 - M_2 = 2 \text{Re}\left[ \frac{q}{p} (M_{12}^D - \frac{i}{2} \Gamma_{12}^D) \right] \\
= 2 \text{Re}\sqrt{|M_{12}^D|^2 - \frac{1}{4} |\Gamma_{12}^D|^2 - i \text{Re}(\Gamma_{12}^D M_{12}^{D*})},
\]

\[
\Delta \Gamma_D = \Gamma_1 - \Gamma_2 = -4 \text{Im}\left[ \frac{q}{p} (M_{12}^D - \frac{i}{2} \Gamma_{12}^D) \right] \\
= -4 \text{Im}\sqrt{|M_{12}^D|^2 - \frac{1}{4} |\Gamma_{12}^D|^2 - i \text{Re}(\Gamma_{12}^D M_{12}^{D*})},
\]

where

\[
\frac{q}{p} \equiv \sqrt{\frac{M_{12}^{D*} - \frac{i}{2} \Gamma_{12}^{D*}}{M_{12}^D - \frac{i}{2} \Gamma_{12}^D}}.
\]

For practical purposes it is sufficient to consider the time-dependent CP asymmetry \( S_f \) as

\[
\frac{\Gamma(D^0(t) \to f) - \Gamma(\bar{D}^0(t) \to f)}{\Gamma(D^0(t) \to f) + \Gamma(\bar{D}^0(t) \to f)} = S_f(D) \frac{t}{2\tau_D},
\]

which is given by

\[
\eta_f S_f(D) \approx -\left[ y_D \left( \left| \frac{q}{p} \right| - \left| \frac{p}{q} \right| \right) \cos \varphi - x_D \left( \left| \frac{q}{p} \right| + \left| \frac{p}{q} \right| \right) \sin \varphi \right],
\]

where \( \eta_f = \pm 1 \) is the CP parity of the final state \( f \). The SM3 prediction for \( \eta_f S_f(D) \) is

\[
[\eta_f S_f(D)]_{\text{SM3}} \approx -2 \cdot 10^{-6}.
\]

Finally, the semileptonic asymmetry is defined as

\[
a_{\text{SL}}(D) \equiv \frac{\Gamma(D^0(t) \to \ell^- \bar{\nu} K^{+}(s)) - \Gamma(\bar{D}^0(t) \to \ell^+ \nu K^{-}(s))}{\Gamma(D^0(t) \to \ell^- \bar{\nu} K^{+}(s)) + \Gamma(D^0(t) \to \ell^+ \nu K^{-}(s))} = \frac{|q|^4 - |p|^4}{|q|^4 + |p|^4} \approx 2 \left( \left| \frac{q}{p} \right| - 1 \right).
\]

The world averages based on data from BaBar, Belle and CDF are given by

\[
x_D = (0.98^{+0.24}_{-0.26}) \%, \quad y_D = (0.83 \pm 0.16) \%,
\]

\[
\left| \frac{q}{p} \right| = (0.87^{+0.17}_{-0.15}) \, , \quad \phi = (-8.5^{+7.4}_{-7.0})^\circ, \quad \left| \frac{p}{q} \right| = (0.87^{+0.17}_{-0.15}) \, , \quad \phi = (-8.5^{+7.4}_{-7.0})^\circ,
\]

\[
\eta_f S_f(D) = (-0.248 \pm 0.496) \% ,
\]

with \( \phi \) being the phase of \( q/p \) and the asymmetry \( \eta_f S_f(D) \) defined in \( \text{[20]} \).

In addition to Table I the relevant input parameters for \( D^0 - \bar{D}^0 \) mixing are given in Table II.
TABLE II: Values of the input parameters for $D$ mesons used in our analysis.

| parameter | value | parameter | value |
|-----------|-------|-----------|-------|
| $m_D$     | (1.86484 ± 0.00017)GeV | $\bar{f}_D$ | (0.4101 ± 0.0015)ps |
| $f_D$     | (0.212 ± 0.014)GeV | $m_c(m_c)$ | (1.268 ± 0.009)GeV |
| $\bar{B}_D$ | $1.18^{+0.07}_{-0.05}$ [16, 55] | $m_b(m_b)$ | (4.20$^{+0.17}_{-0.07}$)GeV |

TABLE III: Allowed ranges of the CKM4 parameters obtained from our analysis.

| parameter | allowed range | parameter | allowed range |
|-----------|---------------|-----------|---------------|
| $\lambda$ | 0.2205 ± 0.0018 | $|V_{t'b}|$ | < 0.12 |
| $C$       | 0.32 → 0.42 | $|V_{t'd}|$ | < 0.05 |
| $A$       | 0.825 → 0.865 | $|V_{t's}|$ | < 0.11 |
| $\gamma$  | (73 ± 13)$^\circ$ | $|V_{ub'}|$ | < 0.05 |
| $r$       | < 0.5 | $|V_{cb'}|$ | < 0.11 |
| $P$       | < 5.0 | $|\lambda_{db'}^t|$ | < 0.002 |
| $Q$       | < 2.5 | $|\lambda_{sb'}^t|$ | < 0.01 |
|           |       | $|\lambda_{uc'}^t|$ | < 0.0025 |

C. Results

Allowed ranges for different CKM4 parameters/elements are given in Table III. Constraint on $V_{t'b}$ or equivalently on the new parameter $r$ (i.e $V_{t'b} = -r\lambda$) is obtained from non-decoupling oblique corrections ($T_4$) and vertex corrections to $Z \to b\bar{b}$. We also note the allowed ranges for the product of the different CKM4 elements, $|\lambda_{db'}^t| = |V_{t'd}^s V_{t'b}|$, $|\lambda_{sb'}^t| = |V_{t's}^s V_{t'b}|$ and $|\lambda_{uc'}^t| = |V_{ub'}^s V_{cb'}|$, obtained from our analysis; these are relevant to $B_d^0$, $B_s$ and $D^0$ oscillations. Allowed ranges for the corresponding phases and their correlations with the magnitude of the product couplings are shown in Fig. [1]. We note that values of $|\lambda_{sb'}^t|$ larger than 0.002 correspond to very narrow regions of the phase $\delta_{t's}$ (left panel, Fig. [1]) close to 90$^\circ$ or 270$^\circ$, whereas that for $\delta_{uc'}^t = \delta_{cb'} - \delta_{ub'}$ (right panel) is close to zero when $|\lambda_{uc'}^t| > 0.0008$. $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixing are sensitive to the new parameters $[P, \delta_{t'd}]$ and $[Q, \delta_{t's}]$ respectively whereas $K^0 - \bar{K}^0$ and $D^0 - \bar{D}^0$ mixing are sensitive to all these four new parameters and their parametric dependencies are given by,
FIG. 1: Fourth generation parameter space; left panel shows the variation of $|\lambda'_{ts}|=|V'_{ts}V'_{tb}|$ with the phase $\delta'_{ts}$ of $V'_{ts}$, whereas right panel shows it for $|\lambda'_{uc}|=|V'_{ub}V'_{cb}|$ with $\delta'_{uc}$, the phase difference between the phase of $V'_{ub}$ and $V_{cb}$.

respectively. In this framework it is quite natural to expect that there is a strong correlation between $K^0-K^0$ and $D^0-D^0$ mixing, as pointed out in the case of purely left-handed currents [49, 58], $D^0-D^0$ mixing is also correlated with the observables from $B_d$ and $B_s$ mixing and decays. So the constraints obtained on the new parameters from the inputs given in Table I, especially $\epsilon_K$ and $Br(K^+ \rightarrow \pi^+\nu\bar{\nu})$, are helpful to find the allowed parameter space for $\lambda'_{uc}$ and the corresponding phase difference $\delta'_{uc}$.

In Fig. 2 (upper-left panel) we show the correlation between $P$ and $Q$, larger values of $P$ corresponds to lower value of $Q$ and vice versa. We obtain such a correlation mainly due to the constraints from $\epsilon_K$ and $Br(K^+ \rightarrow \pi^+\nu\bar{\nu})$, although the upper bound on $P$ and $Q$ is coming from the other $B_d$ and $B_s$ data (see Table I). The expressions for $\epsilon_K$ and $Br(K^+ \rightarrow \pi^+\nu\bar{\nu})$ are sensitive to $\lambda'_{ds}$ i.e using these inputs we will get direct constraint on $\lambda'_{ds}$, as indicated in eq. $\lambda'_{ds}$ is proportional to the product of $P$ and $Q$. Therefore we will get direct constraint on the product not on individual $P$ or $Q$ and this is the reason why they follow the correlations shown. Similar correlation is possible between $V'_{t'd}$ and $V'_{ts}$ since they are proportional to $P$ and $Q$ respectively. We also show the correlations between some other CKM4 elements $^1$; the plot of $|V'_{ts}|$ as a function of

$^1$ Flavour data allows us to get direct constraints on various products of CKM4 elements. The bounds on individual
The mathematical expressions for the product of CKM4 elements $|\lambda'_{db}|$ and $|\lambda'_{sb}|$ are given by

$$|\lambda'_{db}| = |V'_{td}V'_{tb}| = Pr\lambda^4$$

$$|\lambda'_{sb}| = |V'_{ts}V'_{tb}| = Qr\lambda^3, \quad (26)$$

whereas that for $|\lambda'_{uc}|$ can be obtained from eq. (25) by taking its modulus, and we see that when $P << Q$ it is $\approx Q^2\lambda^5$. In Fig. 3 we show the correlations between the products of CKM4 elements; upper-left panel shows the correlation between $|\lambda'_{db}|$ and $|\lambda'_{sb}|$ which is similar to the correlation between $P$ and $Q$ (upper-left panel Fig. 2), as expected since the slope of the curve is given by $Q^2/\rho_X$. In the upper-right panel of Fig. 3 we show the correlation between $|\lambda'_{db}|$ and $|\lambda'_{uc}|$ and note that $|\lambda'_{uc}|$ could be as large as 0.0025 when $|\lambda'_{db}|$ is very small (say < 0.0005) i.e when $P << Q$ and vice versa. The most interesting one is the correlation between $|\lambda'_{sb}|$ and $|\lambda'_{uc}|$ (lower-panel Fig. 3); it shows an almost linear relationship between them which is prominent for larger values of $|\lambda'_{sb}|$ i.e for larger values of $Q$ due to strong $Q^2$ dependence of $|\lambda'_{uc}|$. It plays an important role in understanding the correlations between the CP asymmetries in $B_s$ and $D$ system; later we will discuss it in detail. The final remark from these discussions is that the allowed parameter space for the new CKM4 parameter space are highly correlated; random choices of the CKM4 parameters are not allowed, while doing so one has to be careful and the chosen values should be consistent with the appropriate correlations.

Let us move to next part of our discussion where we show the effect of the fourth generation on different observables related to $B_d$, $B_s$ and $D$ system. In Fig. 4 (upper-left panel) we plot CP asymmetry $S_{\psi K_s}$ as a function of $\lambda'_{db}$ and note that $S_{\psi K_s}$ can go down to $\approx 0.4$ or can
reach around 0.9 for large values of the product coupling $|\lambda'_{t'd}|$; so appreciable deviation from the present experimental measurement is, in principle, possible. We do not get any noticeable correlation between $S_{\psi K_s}$ with the phase $\delta_{t'd}$ of $\lambda'_{t'd}$. In the upper-right panel of Fig. 4 we show the semileptonic asymmetry $a_{d}^{d}$ ($B_d$ system) as a function of $S_{\psi K_s}$. In SM4 the present experimental bound on $S_{\psi K_s}$ allows $a_{d}^{d} \gtrsim -0.001$, whereas SM has a bound, $(-4.8_{-1.2}^{+1.0}) \times 10^{-4}$ as shown by the black band in the Fig. 4 (upper-right panel).

In the lower-left panel of Fig. 4 we are showing the allowed regions for the CP asymmetry
FIG. 3: Correlations between different CKM4 product couplings, $|\lambda'_{tb}| = |V_{t'd}^* V_{tb}|$ and $|\lambda'_{sb}| = |V_{t's}^* V_{tb}|$ (upper left panel), $|\lambda'_{db}|$ and $|\lambda'_{uc}| = |V_{ub'}^* V_{cb'}|$ (upper right panel), $|\lambda'_{sb}|$ and $|\lambda'_{uc}|$ (lower panel).

$S_{\psi\phi}$ in $B_s \rightarrow \psi\phi$ as a function of $|\lambda'_{sb}|$, for $375 \text{ GeV} < m_{t'} < 575 \text{ GeV}$; $S_{\psi\phi}$ is bounded by $-0.50 \lesssim S_{\psi\phi} \lesssim 0.50$, the explicit dependence on $m_{t'}$ has been shown in our earlier papers [13, 14]. It is also interesting to note that its magnitude increases with $|\lambda'_{sb}|$; precise measurements of $S_{\psi\phi}$ will be helpful to put tighter constraints on $|\lambda'_{sb}|$ and the corresponding phase. Recently CDF and DO have updated their measurement of the CP-violating phase with data sample corresponding to an integrated luminosity of 5.2 $fb^{-1}$ and 6.1 $fb^{-1}$ respectively. The allowed 68% C.L ranges are $[59, 60]$

$$\phi^s_{\psi\phi} \in [-0.04, -1.04] \cup [-2.16, -3.10], \quad CDF$$

$$\in -0.76^{+0.38}_{-0.36} \text{(stat)} \pm 0.02 \text{(syst)} \quad DO. \quad (27)$$

The corresponding 1σ ranges for $S_{\psi\phi} = \sin \phi^s_{\psi\phi}$ are given in Table IV.
FIG. 4: Various correlations in SM4 are shown. Variation of CP asymmetries with the magnitude of the product couplings; $S_{\psi K_s}$ as a function of $|\lambda'_{db}|$ (upper-left panel), $S_{\psi K_s}$ as a function of $|\lambda'_{db}|$ (lower-left panel). Correlations between the time dependent mixing induced CP asymmetries and semileptonic CP asymmetries for $B_d$ and $B_s$ are shown in upper and lower right panel respectively. Blue horizontal and vertical bands are the corresponding experimental ranges. In the upper-right panel SM allowed band (thick dark) is shown in $S_{\psi K_s}$ vs $a_{sl}^{d}$ plane. In the lower-right panel the blue and red regions correspond to $\Delta \Gamma_s$ with the uncertainties taken at $1\sigma$ and $2\sigma$ respectively and grey horizontal band corresponds to the experimental range for $a_{sl}^{d}$ and the vertical band is that for CP asymmetry.

In lower-right panel Fig. 4 we show the correlation $^2$ between $S_{\psi\phi}$ and $a_{sl}^{d}$ (eq. 5) with $^{48}$

$$\Delta \Gamma_s^{SM} = 0.096 \pm 0.036 \quad 68\% \ C.L \ ,$$

(28)

taken at $1\sigma$ (blue) and $2\sigma$ (red). We note that its magnitude increases with $S_{\psi\phi}$ as well as with

$^2$ The plot corresponds to negative solution for $S_{\psi\phi}$, we do not show the points corresponding to the positive solution of $S_{\psi\phi}$ for which one should get a region symmetric to that shown in the figure.
\[ \Delta \Gamma_s, \text{ as expected from eq. 5, the maximum allowed ranges are given by} \]
\[ a_{sl}^s \gtrsim -0.004, \quad \Delta \Gamma_s^{SM} @1\sigma, \]
\[ \gtrsim -0.006, \quad \Delta \Gamma_s^{SM} @2\sigma. \] (29)

In Table IV we summarise the allowed ranges for different CP observables in SM4, it includes time dependent CP asymmetries in \( B_d \to \psi K_s, B_s \to \psi \phi \) as well as the semileptonic asymmetries associated with \( B_d \) and \( B_s \) system (eq. 5). We also mention the corresponding experimental ranges and SM3 predictions obtained with the inputs given in Table I.

| CP observable | SM3          | Exp          | SM4 ranges                        |
|---------------|--------------|--------------|-----------------------------------|
| \( S_{\psi K_s} = \sin 2\beta_d \) | 0.739 ± 0.049 | 0.67 ± 0.02  | 0.40 → 0.90                       |
| \( S_{\psi \phi} = \sin \phi_{s}^{\phi} \) | -0.04 ± 0.002 | [-0.04,-0.86] | CDF                              |
| \( a_{sl}^d \) | \((-4.8 \pm 1.2) \times 10^{-4}\) | -0.0047 ± 0.0046 | > -0.002                         |
| \( a_{sl}^s \) | \((2.1 \pm 0.6) \times 10^{-5}\) | -0.0146 ± 0.0075 | -0.006 → 0.006                    |
| \( \eta_f S_{CP}(D) \) | \( \approx -2 \cdot 10^{-6}\) | -0.248 ± 0.496% | -0.01 → 0.01                     |
| \( a_{sl}(D) \) | \( \approx 1 \cdot 10^{-4}\) | -0.6 → 0.6   |                                   |

TABLE IV: Allowed ranges of different CP observables related to \( B_d, B_s \) and \( D^0 \) systems in SM3 and SM4; current experimental status is also given.

In Fig. 5 we show the correlation between the real and imaginary part of the short distance contribution to \( D^0 - \bar{D}^0 \) mixing. Note that the magnitude of \( \text{Im}(M_{12}^D) \) could be as high as 0.6%, which could be negative or positive; very small number of points are allowed for \( \text{Re}(M_{12}^D) < 0 \), however, it could be as high as 0.032. These findings are in good agreement with Ref. [16].

In Fig. 6 we plot real (left panel) and imaginary (right panel) part of \( M_{12}^D \) as a function of \( |\lambda_{uc}'| \) and note that in both the cases its magnitude increases with the product coupling. In the case of the real part almost all the allowed points are for \( \text{Re}(M_{12}^D) > 0 \), however, in case of imaginary part we have both positive and negative solutions. As we noticed before (Fig. 3), \( |\lambda_{uc}'| \) has a linear relationship with \( |\lambda_{sb}'| \); a tighter constraints on \( |\lambda_{sb}'| \), which is possible to get by reducing the errors in the measurements of \( B_d \) or \( B_s \) observables, will be helpful to put tighter constrain on \( D^0 - \bar{D}^0 \) mixing.

In Fig. 7 we plot the time dependent CP asymmetry \( \eta_f S_{CP}(D) \) (eq. 21) and the semileptonic asymmetry \( a_{sl}(D) \) (eq. 23) in the \( D \) system as a function of the phase of \( q_p \) (eq. 19) and \( |q_p| \) respectively; it could be directly compared with the correlations shown in [16]. We note that with
FIG. 5: Correlation between the real and imaginary part of the SD contribution to $M_{12}^{D}$.

FIG. 6: Real (left panel) and imaginary (right panel) part of $M_{12}^{SD}$ as a function $|\lambda_{b'uc}^{\psi\phi}|$.

the present experimental bound on the phase of $\frac{q}{p}$ (eq. 24), the magnitude of $\eta_f S_{CP}(D)$ could be enhanced up to the present experimental bound. On the other hand with the present constraint on $|\lambda_{b'uc}^{\psi\phi}|$, $a_{sl}(D)$ could be reduced to $-0.6$; again these results are also in agreement with Buras et. al [16].

In Fig. 8 we plot $S_{CP}(D)$ and $a_{sl}(D)$ as a function of $|\lambda_{b'uc}^{\psi\phi}|$ and note that the magnitude of both may increase with $|\lambda_{b'uc}^{\psi\phi}|$; SM3 predictions and the allowed ranges in SM4 for the corresponding observables are summarised in Table IV. As discussed before (Fig. 4), the magnitude of $S_{\psi\phi}$ increases with the corresponding product coupling, we also noticed that $|\lambda_{b'uc}^{\psi\phi}|$ increases with $|\lambda_{sb}^{t'}|$ which indicates a definite correlation between $S_{\psi\phi}$ and $\eta_f S_{CP}(D)$ [16]. In the near future if we are
FIG. 7: Time dependent CP asymmetry for $D$ system, $\eta_f S_{CP}(D)$, as a function of the phase $\phi$ of $\frac{f}{p}$ (left panel), semileptonic CP asymmetry, $a_{sl}(D)$, as a function of $|\frac{q}{p}|$ (right panel) the corresponding SM value is $O(10^{-4})$. The grey horizontal and vertical bands represent the corresponding experimental ranges.

FIG. 8: $\eta_f S_{CP}(D)$ as a function of $|\lambda'_{uc}|$ (left panel), $a_{sl}(D)$ as a function $|\lambda'_{uc}|$ (right panel).

able to put tighter constraints on $|\lambda'_{ub}|$, we will be able to get strong limit on $\eta_f S_{CP}(D)$ and $a_{sl}(D)$ due to fourth generation effects.

III. CONCLUSION

This paper represents a continuation of our study of some of the properties of SM4, Standard Model with four generations. Herein we choose a specific representation for the 4X4 mixing matrix and obtain constraints and correlations on its elements using available data from K, B and D decays as well as electroweak precision tests and oblique corrections and allowing the $m_{t'}$ mass to range from 375 to 575 GeV. Constraints obtained are then used to study the mixing induced and semi-leptonic CP asymmetries in $B_d$, $B_s$ and in $D^0$. We find that SM4 allows $S(B_d \to \psi K_s)$ to be
closer to experiment thus alleviating a key difficulty for SM3 that has been found in recent years. SM4 allows $a_{sl}^d$ to be bigger by a factor of $O(3)$. The B-factories have a lot more data since they studied this asymmetry some years ago; it would be very worthwhile to update this bound. On the other hand, $a_{sl}^s$ can be a lot bigger in SM4, and of opposite sign, than in SM3 where it is essentially negligible. It would also be very useful to constrain this asymmetry as well as the linear combination $(A^b_{sl})$ of the two. Interestingly, the large same sign dimuon asymmetry recently discovered by D0 implies a rather large $a_{sl}^s$. This has the same sign as in SM4 though the central value of the D0 result is somewhat larger than the expected range in SM4; however, the significance of the D0 result is only about $2 \sigma$ on $a_{sl}^s$. These asymmetries should be a high priority target for experiments at the Tevatron as well as at LHCb. In recent years Belle also has taken appreciable data at the $\Upsilon(5S)$ which should be used for placing bounds on these asymmetries. In the future, these asymmetries should also be a very useful target at the Super-B factories.

Note Added: Very recently Ref presented constraints on SM4 using a completely different representation of the 4X4 mixing matrix.

Acknowledgments

SN thanks Theory Division of Saha Institute of Nuclear Physics (SINP) for Hospitality. SN’s work is financially supported by NSERC of Canada. The work of AS is supported in part by the US DOE grant # DE-AC02-98CH10886(BNL).

[1] E. Lunghi and A. Soni, Phys. Lett. B 666, 162 (2008) [arXiv:0803.4340 [hep-ph]].
[2] E. Lunghi and A. Soni, JHEP 0908, 051 (2009) [arXiv:0903.5059 [hep-ph]].
[3] A. Lenz et al. [CKMfitter Group], arXiv:1008.1503 [hep-ph].
[4] M. Bona et al. [UTfit Collaboration], Phys. Lett. B 687, 61 (2010) [arXiv:0908.3470 [hep-ph]].
[5] E. Lunghi and A. Soni, Phys. Rev. Lett. 104, 251802 (2010) [arXiv:0912.0002 [hep-ph]].
[6] E. Lunghi and A. Soni, arXiv:1010.6069 [hep-ph].
[7] K. Nakamura et al. [Particle Data Group], J. Phys. G 37, 075021 (2010).
[8] see for example, M. Gronau and J. L. Rosner, Phys. Lett. B 644, 237 (2007) [arXiv:hep-ph/0610227]; H. Y. Cheng and C. K. Chua, Phys. Rev. D 80, 114008 (2009) [arXiv:0909.5229 [hep-ph]].
[9] T. Aaltonen et al. [CDF Collaboration], Phys. Rev. Lett. 100, 161802 (2008); [arXiv:0712.2397 [hep-ex]]; V. M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. 101, 241801 (2008) [arXiv:0802.2255 [hep-ex]].

[10] The Heavy Flavor Averaging Group et al., [arXiv:1010.1589 [hep-ex]].

[11] V. M. Abazov et al. [D0 Collaboration], Phys. Rev. D 82, 032001 (2010) [arXiv:1005.2757 [hep-ex]].

[12] V. M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. 105, 081801 (2010) [arXiv:1007.0395 [hep-ex]].

[13] A. Soni, A. K. Alok, A. Giri, R. Mohanta and S. Nandi, Phys. Lett. B 683, 302 (2010) [arXiv:0807.1971 [hep-ph]].

[14] A. Soni, A. K. Alok, A. Giri, R. Mohanta and S. Nandi, Phys. Rev. D 82, 033009 (2010) [arXiv:1002.0599 [hep-ph]].

[15] A. J. Buras, B. Duling, T. Feldmann, T. Heidsieck, C. Promberger and S. Recksiegel, JHEP 1009, 106 (2010) [arXiv:1002.2126 [hep-ph]].

[16] A. J. Buras, B. Duling, T. Feldmann, T. Heidsieck, C. Promberger and S. Recksiegel, JHEP 1007, 094 (2010) [arXiv:1004.4565 [hep-ph]].

[17] W. S. Hou and C. Y. Ma, Phys. Rev. D 82, 036002 (2010) [arXiv:1004.2186 [hep-ph]].

[18] See also, M. Bobrowski, A. Lenz, J. Riedl and J. Rohrwild, Phys. Rev. D 79, 113006 (2009) [arXiv:0902.4883 [hep-ph]]; O. Eberhardt, A. Lenz and J. Rohrwild, Phys. Rev. D 82, 095006 (2010) [arXiv:1005.3505 [hep-ph]].

[19] G. W. S. Hou, [arXiv:0810.3396 [hep-ph]].

[20] For earlier related works see, C. Jarlskog and R. Stora, Phys. Lett. B208, 288 (1988); F. del Aguila and J. A. Aguilar-Saavedra, Phys. Lett. B386, 241 (1996); F. del Aguila and J. A. Aguilar-Saavedra and G. C. Branco, Nucl. Phys. B510, 39, 1998.

[21] J. Carpenter, R. Norton, S. Siegemund-Broka and A. Soni, Phys. Rev. Lett. 65, 153 (1990). In passing, we note that the quark mass needed for dynamical electroweak symmetry breaking in this work, translated to the fourth family quasi-degenerate doublet, gives $m_{t'} \sim 500$ GeV and $m_H \approx \sqrt{2} m_{t'} \sim 700$ GeV.

[22] See the proceedings of the First International Symposium on the fourth family of quarks and leptons, Santa Monica, CA, Feb 1987, published by the NY Academy of Sciences; eds D. Cline and A. Soni; see also the proceedings of the Second International Symposium on the fourth family of quarks and leptons, Santa Monica, CA, Feb 1989, published by the NY Academy of Sciences; eds D. Cline and A. Soni.

[23] B. Holdom, Phys. Rev. Lett. 57, 2496 (1986) [Erratum-ibid. 58, 177 (1987)]; W. A. Bardeen, C. T. Hill and M. Lindner, Phys. Rev. D 41, 1647 (1990); C. T. Hill, M. A. Luty and E. A. Paschos, Phys. Rev. D 43, 3011 (1991); P. Q. Hung and G. Isidori, Phys. Lett. B 402, 122 (1997) [arXiv:hep-ph/9609518].

[24] P. Q. Hung and C. Xiong, [arXiv:0911.3892].

[25] M. Hashimoto and V. A. Miransky, [arXiv:0912.4453].

[26] M. S. Chanowitz, Phys. Rev. D 79, 113008 (2009) [arXiv:0904.3570 [hep-ph]].
[27] M. S. Chanowitz, Phys. Rev. D , 035018 (2010) [arXiv:1007.0043 [hep-ph]].
[28] W. S. Hou, A. Soni and H. Steger, Phys. Lett. B 192, 441 (1987).
[29] W. S. Hou, R. S. Willey and A. Soni, Phys. Rev. Lett. 58, 1608 (1987) [Erratum-ibid. 60, 2337 (1988)].
[30] W. S. Hou, A. Soni and H. Steger, Phys. Rev. Lett. 59, 1521 (1987).
[31] R. Louvot et al. [Belle Collaboration], Phys. Rev. Lett. 102, 021801 (2009) [arXiv:0809.2526 [hep-ex]].
A. Drutskoy, arXiv:hep-ex/0605110.
[32] Ulrich Uwer, talk at the conference “Flavor Physics in the LHC Era”, November 8-12 2010, Singapore, http://www.ntu.edu.sg/ias/upcomingevents/FlavorPhysics/Pages/default.aspx
[33] CDF collaboration, CDF note 10110, http://www-cdf.fnal.gov/physics/S10CDFResults.html
[34] H. J. He, N. Polonsky and S. f. Su, Phys. Rev. D 64, 053004 (2001) [arXiv:hep-ph/0102144].
[35] V. A. Novikov, L. B. Okun, A. N. Rozanov and M. I. Vysotsky, Phys. Lett. B 529, 111 (2002) [arXiv:hep-ph/0111028].
[36] G. D. Kribs, T. Plehn, M. Spannowsky and T. M. P. Tait, Phys. Rev. D 76, 075016 (2007) [arXiv:0706.3718 [hep-ph]].
[37] J. Erler and P. Langacker, arXiv:1003.3211 [hep-ph].
[38] P. Q. Hung and M. Sher, Phys. Rev. D 77, 037302 (2008) [arXiv:0711.4353 [hep-ph]].
[39] D. J. Antonio et al. [RBC Collaboration and UKQCD Collaboration], Phys. Rev. Lett. 100, 032001 (2008) [arXiv:hep-ph/0702042].
C. Kelly, talk at Lattice 2010, June 14-19 2010, Sardinia, Italy, http://www.infn.it/Lattice2010.
[40] Y. Aoki et al., arXiv:1012.4178 [hep-lat].
[41] T. Bae et al., arXiv:1008.5179 [hep-lat].
[42] C. Aubin, J. Laiho and R. S. Van de Water, Phys. Rev. D 81, 014507 (2010) [arXiv:0905.3947 [hep-lat]].
[43] E. Gamiz, C. T. H. Davies, G. P. Lepage, J. Shigemitsu and M. Wingate [HPQCD Collaboration], Phys. Rev. D 80, 014503 (2009) [arXiv:0902.1813 [hep-lat]].
[44] E. Gamiz, private communication.
[45] S. Herrlich and U. Nierste, Nucl. Phys. B 419, 292 (1994) [hep-ph/9310311].
[46] A. J. Buras, M. Jamin and P. H. Weisz, Nucl. Phys. B 347, 491 (1990).
[47] S. Herrlich and U. Nierste, Phys. Rev. D 52, 6505 (1995) [hep-ph/9507262].
[48] A. J. Buras, D. Guadagnoli and G. Isidori, Phys. Lett. B 688, 309 (2010) [arXiv:1002.3612 [hep-ph]].
[49] A. Lenz and U. Nierste, JHEP 0706, 072 (2007) [arXiv:hep-ph/0612167].
[50] I. I. Bigi, M. Blanke, A. J. Buras, and S. Recksgiel, JHEP 07 (2009) 097, 0904.1545.
[51] Heavy Flavor Averaging Group (HFAG) Collaboration, E. Barberio et al., Averages of b-hadron properties at the end of 2006, 0704.3575. Updates available on http://www.slac.stanford.edu/xorg/hfag.
[52] Heavy Flavor Averaging Group Collaboration, E. Barberio et al., Averages of b−hadron and c−hadron Properties at the End of 2007, 0808.1297.
[53] A. J. Schwartz, 0911.1464.
[54] V. Lubicz and C. Tarantino, *Nuovo Cim.* **123B** (2008) 674–688, [0807.4605](https://arxiv.org/abs/0807.4605).

[55] C. W. Bernard, T. Draper, G. Hockney and A. Soni, Phys. Rev. D **38**, 3540 (1988); R. Gupta, T. Bhattacharya and S. R. Sharpe, Phys. Rev. D **55**, 4036 (1997) [arXiv:hep-lat/9611023]; L. Lellouch and C. J. D. Lin [UKQCD Collaboration], Phys. Rev. D **64**, 094501 (2001) [arXiv:hep-ph/0011086]; H. W. Lin, S. Ohta, A. Soni and N. Yamada, Phys. Rev. D **74**, 114506 (2006) [arXiv:hep-lat/0607035].

Although these results are obtained in the quenched approximation, we know now that for B-parameters quenching effects are rather small; see Ref. 39.

[56] J. Laiho, R. S. Van de Water, and E. Lunghi, [0910.2923](https://arxiv.org/abs/0910.2923).

[57] **HPQCD** Collaboration, I. Allison et al., *Phys. Rev.* **D78** (2008) 054513, [0805.2999](https://arxiv.org/abs/0805.2999).

[58] K. Blum, Y. Grossman, Y. Nir and G. Perez, Phys. Rev. Lett. **102**, 211802 (2009) [arXiv:0903.2118 [hep-ph]].

[59] CDF collaboration, CDF note 10206, [http://www-cdf.fnal.gov/physics/new/bottom/bottom.html](http://www-cdf.fnal.gov/physics/new/bottom/bottom.html).

[60] **DØ** collaboration, Conference note 6098-CONF, [www-d0.fnal.gov/Run2Physics/WWW/results/prelim/B/B60/](http://www-d0.fnal.gov/Run2Physics/WWW/results/prelim/B/B60/).

[61] A. K. Alok, A. Dighe and D. London, [arXiv:1011.2634](https://arxiv.org/abs/1011.2634) [hep-ph].

[62] C. S. Kim and A. S. Dighe, Int. J. Mod. Phys. E **16**, 1445 (2007) [arXiv:0710.1681 [hep-ph]].