A Restoring Force Model for Prefabricated Concrete Shear Walls with Built-In Steel Sections

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Featured Application: This paper provides an innovative design of prefabricated shear wall that is suitable for industrialized construction. A restoring force model that reflects the hysteretic performance of the prefabricated shear walls with built-in steel sections is developed and a calculation method for cracking stiffness for prefabricated shear walls based on test fitting, mechanical calculation, and test modifications is proposed and evaluated. A calculation method of the ultimate load for prefabricated shear walls based on an ultimate equilibrium equation is developed. The results of this paper can be applied in developing design codes for prefabricated shear walls and serve as a reference for practical applications.

Abstract: Prefabricated shear walls have been widely used in engineering structures. Vertical connection joints of the walls are the key to ensure the safety of the structures. Steel–concrete composite structures have been proved to have a good bearing capacity and ductility. In this paper, a new type of prefabricated structure is proposed, in which vertical wall members are connected together through built-in steel sections and cast-in-place concrete. This paper studies the seismic performance of the proposed prefabricated concrete shear wall structure. Hysteretic curves and skeleton curves of the shear wall are obtained based on experimental analyses. A dimensionless skeleton curve model is developed using the theory of material mechanics and the method of regression analysis. A stiffness calculation method for different loading stages is obtained and a restoring force model is proposed. The proposed innovative prefabricated shear wall structure provides good resistance to seismic performance and the related analysis provides a fundamental reference for studies of prefabricated shear wall structures.

Keywords: prefabricated shear wall; built-in steel section; seismic performance; stiffness calculation; restoring force model

1. Introduction

Shear wall structures have been widely used in multistory and high-rise residential and commercial buildings. To promote standardized construction, it has become a new trend to use prefabricated shear wall structures suitable for industrialization standards.

The connections of prefabricated concrete shear wall structures have been the focus of research. To improve the seismic performance of prefabricated shear wall connections, reinforcements, steel pipes, steel angles, or other types of steel sections are placed in edge restraint zones of shear walls. Shear walls with built-in steel sections have advantages including easy installation and a clear load transmission path. Because of the importance of the connections of fabricated wall components, studies on the performance of prefabricated shear walls with different types of connections are reviewed and summarized below with...
a focus on shear walls reinforced with steel sections and restoring force models. In seismic engineering, the resilience of structures has been evaluated using restoring force models that reflect the relationship between the load and displacement of structures under cyclic loading and measure the ability of shear walls to recover their original shape after an earthquake event.

Research has been carried out on concrete walls reinforced with steel sections. It has been proved that steel sections can improve the bending resistance and ductility of the walls [1]. Liu et al. [2] performed a study on edge restraint zones that are strengthened with steel sections, which have good ductility and can effectively reduce the corner damage of the test wall. Wu et al. [3] tested five specimens strengthened with steel sections. The results showed that this type of connection has a good bearing capacity and good ductility. The experimental and numerical results are in good agreement.

Zhao, J [4] developed a cyclic restoring force model of fiber-reinforced concrete shear walls in high-rise building structures. Five steel fiber-reinforced concrete shear walls were tested under a horizontal reverse cyclic load combined with a constant axial load. Clough et al. [5] considered the stiffness degradation under repeated loading and proposed a bilinear model with a stiffness degradation input. Saiidi et al. [6] simplified the “Clough” model and developed a new model that reflects the bending deformation of reinforced concrete beam-column members and is simpler and more practical. Park et al. [7] presented static (monotonic) and dynamic (cyclic) test data to evaluate the statistics of the appropriate parameters of the proposed damage model. The uncertainty in the ultimate structural capacity was also examined.

Wallace [8] presented modeling of both flexural and shear responses of tall reinforced concrete core wall buildings and studied the potential impact of coupled flexure–shear behavior. Thomson et al. [9] proposed a simplified model for simulating the damage of squat RC shear walls under lateral loads. Jalali et al. [10] investigated the effectiveness of a simple macroscopic model in predicting the nonlinear response of slender reinforced concrete shear walls.

Dan et al. [11] studied composite steel–concrete structural shear walls with different encased steel shapes. Ma et al. [12] carried out low-cycle repetitive tests of eight steel-reinforced concrete shear walls. A four-line load-displacement restoring force model was proposed, in which the stiffness degradation is considered. Bai et al. [13] developed a restoring force model for a steel high-performance concrete composite structural wall and demonstrated the application of nonlinear dynamics analysis as well as static structural calculations.

Xiao [14] established a restoring force model of a fabricated double-slab concrete shear wall. Through dimensionless processing of skeleton curves, the key points of skeleton curves and the calculation methods are determined. Tang [15] developed a modified method for strength and stiffness calculation of fabricated concrete shear wall models based on the bending calculation theory and established its restoring force mathematical model. Wang et al. [16] developed a simplified method to assess the seismic behavior of reinforced concrete columns that failed in different modes. Zou et al. [17] developed a cyclic restoring force model for high-strength foamed concrete (HFC)-filled cold-formed thin-walled steel (CFS) shear walls based on experimental studies, which investigated the characteristics of hysteretic and skeleton curves. Hideo ONO et al. [18] analyzed the restoring force characteristics of shear walls subjected to horizontal two-directional loading. Based on the results of static loading tests of the box type and cylindrical type shear walls, it was verified that the resultant shear force–total deformation angle relationship under two-directional loading is analogous to that under one-directional loading.

It is observed that although stiffness calculation methods of different types of shear walls have been developed, including research on the resilience models of prefabricated concrete shear walls, no research has been conducted on restoring force models of prefabricated concrete shear walls with built-in steel sections. Furthermore, even general research on the behavior of steel-section-reinforced shear walls is very limited. Steel sections can
provide temporal support to the wall components before completion of the connections and make the assembling process smooth during the construction. The steel sections also contribute significantly to the load resistance of the connections. Therefore, it is worth further investigation on the performance of prefabricated shear walls strengthened with steel sections. This paper proposes a new shear wall structure that is prefabricated and field-connected with steel sections and reinforcement bars. This paper presents an experimental and numerical study on the proposed wall structure and develops a restoring force model based on the test results. The performance of the connections at different heights of the walls are compared. This study provides basic information on the seismic performance of steel-section-reinforced shear walls based on their restoring force models that can serve as experimental and theoretical support for further formulation of the relevant standards and specifications for prefabricated composite shear walls.

2. Layout of the Prefabricated Concrete Shear Walls with Built-In Steel Sections

Four specimens of prefabricated concrete shear wall with built-in steel sections were made by connecting two prefabricated parts, and low-cycle repeated tests were carried out to obtain the hysteretic curves of each specimen. On this basis, the restoring force model was established. The completed wall specimens are 960 mm × 2000 mm × 120 mm (length × height × thickness), as shown in Figure 1.

![Figure 1](image)

Figure 1. Schematic of the test wall specimens.

The plan view and the reinforcement layout of the fabricated concrete shear walls is shown in Figure 2. An I-shaped steel section is embedded in each end of the precast reinforced concrete shear wall and the corresponding locations of the footing. The I-shaped steel section runs through the whole height of the wall and the footing. The depth of the I-shaped steel section is 100 mm, the web thickness is 4.5 mm, the flange width is 63 mm, and the flange thickness is 7.6 mm, as shown in Figure 2. The prefabricated wall specimens have two rows of vertical reinforcement bars (D = 6.5 mm), one in the front and another in the back of the wall, uniformly spaced at 100 mm. Horizontal reinforcement bars (D = 6.5 mm) are placed in the wall to provide shear capacity. The interval of the horizontal bars is shown in Figures 3 and 4. There is a stirrup of 120 × 80 mm (D = 6.5 mm) at the end of each layer of the horizontal rebars, as shown in Figure 2.
Figure 2. Plan view and the layout of the reinforcements of the shear wall specimens (unit: mm). (Φ is the diameter of bars, @ is the center-to-center distance of bars).

Figure 3. Layout of the wall specimen with the connection at the bottom of the wall (named XZW1 and 2). Unit: mm. RG represents “strain gauge attached on the reinforcing bar”, and SG “strain gauge attached on the I-steel section” (Φ is the diameter of bars, @ is the center-to-center distance of bars.)
Part of the wall and the footing were precast with a portion of the rebars, and steel sections protruded out of the concrete for field connection. The precast wall and the footing are connected with a layer of 200 mm thick cast-in-place concrete, as shown in Figures 3 and 4. The field connection is convenient. First, the protruded steel sections are welded to connect the two precast structural components, which provides temporal support to the precast components before casting concrete in the connection layer. Next, the protruded rebars are connected, then the formwork and cast concrete are set in the connection layer. To test the effect of the different connection configurations, two different connection locations are considered. The first is connecting the precast wall and the footing at the bottom of the wall stem, as shown in Figure 3. The other is connecting the precast wall and the footing at the location 400 mm above the bottom of the wall stem, as shown in Figure 4. The bottom of the wall receives the largest moment from the lateral load applied on the top of the wall; therefore, it is the worst case for connection. If the connection at the wall bottom works well and is comparable to that at the intermediate height, it can be verified that the connection design is good. During the installation of the wall, the I-shaped steel sections from the wall and from the footing are connected with butt welds. The vertical reinforcements of the wall and the footing are extended and bent into the connection layer in 45 degrees and are connected there.

Two wall specimens were made for each connection option: XZW 1 and 2 for the connection at the bottom of the wall (Figure 3), and XZW 3 and 4 for the connection 400 mm above the bottom of the wall (Figure 4).
The cast-in-place concrete connection layer runs through the whole horizontal section of the wall specimen with a dimension of 960 mm × 200 mm × 120 mm (length × height × thickness). The horizontal reinforcements are spaced at 100 mm for the top 300 mm of the wall and in the connection layer and spaced at 200 mm anywhere else. A closer spacing of rebars on the top 300 mm and in the connection is used to ensure sufficient strength because the test loading is applied on the top of the wall. The property parameters of the steel and the layout of the wall and the reinforcements are summarized in Tables 1 and 2, respectively.

Table 1. Property parameters of the steel bars and steel sections.

| Type        | Size (mm) | Yield Strength (MPa) | Ultimate Strength (MPa) | Modulus of Elasticity (MPa) |
|-------------|-----------|----------------------|-------------------------|-----------------------------|
| Steel bars  | 6.5 (diameter) | 363                  | 454                     | $2.12 \times 10^5$         |
| Steel sections | 100 × 68 × 4.5 × 7.6 | 278                  | 382                     | $1.92 \times 10^5$         |

Table 2. Reinforcements of the walls.

| Wall Code | Wall Size (mm) | Horizontal Rebar | Vertical Rebar | Stirrups | Axial Compression Ratio * |
|-----------|----------------|------------------|----------------|----------|--------------------------|
| XZW1      | 2000 × 960 × 120 | $\Phi 6.5@100/200$ | $\Phi 6.5@100$ | $\Phi 6.5@100/200$ | 0.05                     |
| XZW2      | 2000 × 960 × 120 | $\Phi 6.5@100/200$ | $\Phi 6.5@100$ | $\Phi 6.5@100/200$ | 0.15                     |
| XZW3      | 2000 × 960 × 120 | $\Phi 6.5@100/200$ | $\Phi 6.5@100$ | $\Phi 6.5@100/200$ | 0.05                     |
| XZW4      | 2000 × 960 × 120 | $\Phi 6.5@100/200$ | $\Phi 6.5@100$ | $\Phi 6.5@100/200$ | 0.15                     |

1. spaced @100 mm on the top 300 mm of the wall and in the connection at the bottom of the wall; 2. Spaced @100 mm on the top 300 mm of the wall and in the connection 400 mm above the bottom of the wall ($\Phi$ is the diameter of bars, @ is the center-to-center distance of bars.)  
* Axial compression ratio: the ratio of the vertical pressure applied on the wall to the compressive strength of the wall.

The compressive strength test of concrete cubes (3 cubes) was carried out on a universal testing machine with an electro-hydraulic servo controlled by a microcomputer (WAW-2000), and the mean value of the axial compressive strength of concrete cubes was 35.39 MPa on the 28th day with moist curing.

3. Experimental Plan

Low-cyclic horizontal loading was applied through an MTS electro-hydraulic servo actuator fixed on the reaction wall with a range of ±250 mm (±1000 kN). The vertical load was applied through an electro-hydraulic servo jack fixed on a portal reaction frame. A rolling guide rail was set between the portal frame and the loader to reduce the friction so the top of the wall specimen could move freely in the horizontal direction. During the test, the shear wall specimens were placed vertically at the designated position, and the wall footing was fixed on the laboratory floor by ground anchor bolts. A distribution steel beam slightly longer than the wall length was placed on the top of the wall to distribute the vertical load. One end of the actuator was fixed to the steel plate on the reaction wall by high-strength bolts, and the other end applied the load on the shear wall through a pair of steel plates fixed on the top side of the wall. The layout of the tests is shown in Figure 5. The actuator acted on the top side of the wall specimen over an area of 300 mm × 120 mm. The actuator was connected with a computer system, and the load and displacement on the top of the wall specimen can be obtained in real-time with the sensors on the actuator and on the top of the wall as well as on both sides of the footing, as shown in Figure 5. Strain gauges were glued on the reinforcing bars and on the outer flanges of the I-shaped steel sections at the bottom section of the wall, as shown in Figures 3 and 4, to measure the strains at those locations. All the strain gauges are placed along the longitudinal direction of the steel section flange or reinforcement bars.
5. Strain gauges were glued on the reinforcing bars and on the outer flanges of the I-sections at the bottom section of the wall, as shown in Figures 3 and 4, to measure the strains at those locations. All the strain gauges are placed along the longitudinal direction of the steel section flange or reinforcement bars.

A vertical pressure was applied on top of the specimen according to the preset axial pressure ratio to evaluate the effect of the vertical pressure. After the vertical load stabilized, low cyclic horizontal loading was applied. A vertical pressure of 5% of the compressive strength of the wall was applied to XZW 1 and 3 while a vertical pressure of 15% of the compressive strength was applied to XZW 2 and 4.

In the process of loading, the vertical load was first applied to the top of the specimen, then a low-cyclic horizontal reciprocating load was applied after the vertical load was stabilized. When the actuator pushes out, the load is defined as positive. When the actuator pulls back, the load is negative. In each loading cycle, a positive load was first applied, then switched to a negative load with the same magnitude. Then, it was switched to a positive load again with an increased increment (10 KN increase for each cycle), then switched back to negative load. After cracking of the wall, the loading was switched to displacement control. The low-cycle repeated loading was continued until the specimen failed. The loading process is demonstrated in Figure 6 and summarized in Table 3.

![Figure 5. Layout of test loading.](image)

![Figure 6. Layout of test loading (left: loading-controlled ΔP = 10 KN; Right: displacement-controlled Δ = 3 mm.](image)
### Table 3. Loading scheme.

| Loading Steps | Load Magnitude and Direction | Loading Steps | Displacement Magnitude and Direction |
|---------------|------------------------------|---------------|--------------------------------------|
| 1             | +10 kN                       | 7             | +3 mm                                |
| 2             | −10 kN                       | 8             | −3 mm                                |
| 3             | +20 kN                       | 9             | +6 mm                                |
| 4             | −20 kN                       | 10            | −6 mm                                |
| 5             | +30 kN                       | 11            | +9 mm                                |
| 6             | −30 kN                       | 12            | −9 mm                                |
|               | Crack occurs and switch to   |               |                                      |
|               | displacement control         |               |                                      |
|               | Continue until the wall fails|               |                                      |

4. Results of the Low-Cycle Loading Test

As mentioned earlier, a vertical load was applied first to reach the vertical pressure calculated by the axial pressure ratio defined in the design scheme. Then, a cyclic horizontal load was applied step by step, and the load and corresponding displacement were recorded. All the specimens showed similar failure patterns, which were bending-shear failure. The bending cracks developed obviously in the early and middle stages of loading, and cross oblique cracks appeared in the later stage.

The yield load of the steel section, which was measured by a pre-mounted strain gauge on the steel sections (Figures 3 and 4), was taken as the yield load of the wall specimen, and the maximum load that can be reached in the process of horizontal loading was taken as the peak load. After passing the peak load, the displacement on the top of the specimen continued to increase gradually while the load decreased to 75% of the peak load, which is considered the failure load.

The load and displacement at concrete cracking, at yielding, and the maximum load (peak load) and the corresponding displacements, as well as the load and displacement at failure are summarized in Table 3.

The displacement ratio (horizontal displacement of the top of the wall/height of the specimen) increased with the load until the specimens failed. Due to the restraint effect of the steel sections, the specimens did not collapse.

Under the same axial compression ratio ((vertical load applied to the top of the wall)/(measured concrete strength × area of the wall section)), the cracking load and deformation of the specimens with different connection locations (bottom of the wall vs. 400 mm above the wall footing) are close to each other. With the increase of the axial compression load, the maximum lateral load resistance increases. Comparing the walls of XZW1 to 2, and 3 to 4, with a 10% increase of the axial compression load, the maximum lateral load resistance increased by about 25%.

Comparing the wall specimens with different connection locations, the walls with a connection 400 mm above the footing have a slightly higher ultimate loading capacity and a higher ductility. This indicates that the connection at the worst location worked well.

5. Mathematical Fitting of the Restorative Force Curve Models

5.1. Determination of Dimensionless Skeleton Curves

(1) Mathematical expression of dimensionless skeleton curves

The skeleton curves of the specimens (Figure 7) are obtained from the quasi-static tests of the four specimens. It can be seen that although the curves vary in values due to factors including the axial pressure ratio, the shapes of the curves are similar. During the test, the cracking load and cracking displacement were recorded at the occurrence of cracking (close to the base), which was visually observed. The yield load and displacement were obtained by a commonly used secant line method with reference to the strains recorded at the flange of the steel sections. The failure load and ultimate displacement were obtained by taking
the values corresponding to the point when the horizontal force dropped to 85% of the maximum load. The maximum load and the corresponding displacement were obtained by the measured force of the actuator and the displacement sensor, which are the actual measured values.

![Skeleton curves of specimens.](image)

**Figure 7.** Skeleton curves of specimens.

Because all of the specimens have similar initial stiffness, and before yielding, the deformation and fracture development of specimens are small, no stiffness change is significant. The curves of all specimens coincide before the yielding point, indicating that all the specimens had a relatively large stiffness. After yielding, the stiffness of different specimens varies greatly due to the different axial compression ratios. As a result, the curves shift away from each other, especially after the maximum load point.

The dimensionless skeleton curve (Figure 8) is established by normalizing the load values by the maximum load and displacement; that is, the values at the cracking point, yielding point, maximum load point, and failure point are all divided by the value of the maximum load point. In the same dimension, the curves of each specimen converge and have similar changing characteristics. Therefore, on this basis, a changing broken line can be used to approximately express the change rule of the restoring force curve.

![Dimensionless skeleton curves](image)

**Figure 8.** Dimensionless skeleton curves (loads are normalized by the maximum load).
Through the analysis of the dimensionless curves, the curves are transformed into a four-line curve, as shown in Figure 9. Because the curves in the positive and negative directions are similar, only the positive part of the curves are used.

Figure 9. Four-fold skeleton curve model.

In Figure 9, $P_c$, $P_y$, $P_m$, and $P_u$ respectively represent the cracking load, yielding load, maximum load, and failure load; and $P_c = 0.295 \ P_m$, $P_y = 0.822 \ P_m$, and $P_u = 0.85 \ P_m$ according to the average test results from Table 4.

### Table 4. Test results.

| Specimens | Cracking Load (KN) | Cracking Deformation (mm) | Yield Load (KN) | Yield Deformation | Maximum Load (KN) | Displacement at Maximum Load (mm) | Failure Load (KN) | Ultimate Displacement (mm) | Ductility Factor $^*$ |
|-----------|--------------------|--------------------------|----------------|------------------|-------------------|----------------------------------|------------------|-----------------------------|-----------------------|
| XZW1      | 58                 | 3.2                      | 203            | 10.2             | 245.6             | 24.2                             | 208.8            | 39.4                        | 3.86                  |
| XZW2      | 109                | 3.0                      | 259            | 11               | 307.1             | 22.3                             | 261.0            | 36.1                        | 3.28                  |
| XZW3      | 60                 | 3.1                      | 195.9          | 9.2              | 252.3             | 27.0                             | 215.1            | 37.5                        | 4.08                  |
| XZW 4     | 110                | 3.3                      | 265            | 10.5             | 313.0             | 25.9                             | 268.0            | 34.6                        | 3.29                  |

$^*$ Ductility factor: The ratio of the ultimate displacement to the yield displacement of the specimen.

The slope of the four lines is the stiffness of each part, labelled as $K_1$, $K_2$, $K_3$, $K_4$. Their numerical relations can be obtained from the geometric relations, $K_2 = 0.821 \ K_1$, $K_3 = 0.145 \ K_1$, $K_4 = 0.129 \ K_1$, $D_c$, $D_y$, $D_m$, $D_u$ are the cracking displacement, yielding displacement, maximum-load displacement, and failure displacement, respectively, which are obtained from the average test results from Table 4.

The obtained four-fold-line skeleton curve is expressed as Equation (1), and the relationships between load and stiffness are defined in Equations (2)–(5):

$$ P(x) = \begin{cases} 
K_1 x & 0 \leq x \leq D_c \\
K_2 (x - D_c) + P_c & D_c < x \leq D_y \\
K_3 (x - D_y) + P_y & D_y < x \leq D_m \\
K_4 (x - D_m) + P_m & D_m < x \leq D_u 
\end{cases} \quad (1) $$

$$ D_c = \frac{P_c}{K_1} = \frac{0.295 \ P_m}{K_1} \quad (2) $$

$$ D_y = \frac{P_c}{K_1} + \frac{P_y - P_c}{K_2} = \frac{0.295 \ P_m}{K_1} + \frac{0.527 \ P_m}{0.821 \ K_1} = \frac{0.936 \ P_m}{K_1} \quad (3) $$
(2) Relationship between the theoretical initial stiffness and cracking stiffness

The specimens studied in this paper are all straight shear walls, which are like vertically placed cantilever beams. By observing the skeleton curves of the specimens, it is close to an elastic state before cracking.

The theoretical initial stiffness is derived by applying the basic theory of mechanics of materials. According to the design code [19], the rigidity of a steel–concrete composite wall is similar to that of a reinforced concrete wall of the same size. The steel sections can improve the flexural bearing capacity of the structure but make little contribution to its rigidity. Thus, the stiffness of a reinforced concrete shear wall with steel sections can be calculated approximately according to the stiffness of a reinforced concrete shear wall with the same section, and the corresponding shear deformation stiffness can be calculated with:

\[ K_0 = \left( \frac{H^3}{3E_cI_c} + \frac{\mu H}{G_c A} \right)^{-1} \]  

where \( H \) is the height of the wall, which is 1850 mm for the walls in this study (the height from the center of the actuator loading pad to the bottom of the wall);

\( E_c, I_c \)—The elastic modulus of concrete and the moment of inertia of the rectangular cross section of the wall, respectively. \( E_c \) is obtained from the concrete test results;

\( G_c \)—The shear modulus of concrete, generally 0.4 of the elastic modulus of concrete;

\( \mu \)—The non-uniformity coefficient of shear stress, for a rectangular section, is 1.2;

\( A \)—Section area of the concrete wall (mm\(^2\)).

Substituting the stiffness value into Equation (2), the theoretical cracking displacement is calculated as 1.75 mm, which is smaller than that obtained in the test (about 3 mm, as listed in Table 4).

(3) Calculation of the peak load

For shear walls without embedded steel sections, the ultimate compressive strain of concrete in the constrained edge members is increased because of the stirrup constraints.

For shear walls with embedded steel sections, in addition to the constraints from the stirrups and marginal longitudinal reinforcements, the steel sections also contribute to the edge constraint, and the stress–strain relationship of the restrained concrete is calculated by formulas given in the literature [20] as:

\[ f_{cc} = (1 + 0.5\lambda_t)f_c \]  
\[ \epsilon_{cc} = (1 + 2.5\lambda_t)\epsilon_c \]

where:

\( f_{cc} \)—Compressive strength of the confined concrete.

\( \epsilon_{cc} \)—Peak strain of the confined concrete.

\( \lambda_t \)—Constraint index \( \lambda_t = \mu_t f_y^f / f_c \).

\( \mu_t \)—The volume ratio of stirrups.

\( f_c \)—Design value of concrete axial compressive strength.

\( \epsilon_c \)—Elastic limit strain of concrete.

\( f_y^f \)—Design value of yield strength of stirrup.

For the shear wall specimens, when the concrete strain \( \epsilon_{cc} \) in the constrained region reaches the ultimate compressive strain under the constrained state, the steel sections and longitudinal bars in the pull and press edge members have yielded.
In this paper, in the prefabricated concrete shear wall with built-in steel sections, symmetrical reinforcements and steel sections are used to establish the equilibrium equation of the force under the ultimate state of the bearing capacity of the section [21], see Figure 10 and Equations (9) and (10). In the equilibrium equations, it is assumed that the reinforcement bars and steel sections in both the tension and compression sides have yielded under the ultimate state. As shown in Figure 10, under the axial load $N$ and the moment $M$ from the lateral load, the steel section ($A_a$) and the core reinforcements inside the stirrup ($A_s$) as well as the uniformly distributed vertical rebars in the tension zone provide tensile resistance, and the steel section ($A_a'$) and the core reinforcements ($A_s'$) as well as the concrete in the compression zone provide compressive resistance.

\begin{align}
N + b(h_0 - 1.5x)\rho_v f_{yv} &= r f_{cc}(bx - A_a) \\
M &= r(bx - A_a)f_{cc}(h_0 - 0.5x) + (f'_y A'_s + f'_a A'_a)(h_0 - a_a) - 0.5b(h_0 - 1.5x)^2 \rho_v f_{yv} - N(h_0 - 0.5h)
\end{align}

where:
- $\rho_v$—Vertical distributed reinforcement ratio.
- $f'_y$—Measured yield strength of the core reinforcements in compression.
- $f'_a$—Measured yield strength of the steel section in compression.
- $f_y$—Measured yield strength of the core reinforcements in tension.
- $f_a$—Measured yield strength of the steel section in tension.
- $a_a$—Distance of the steel compression force from the wall side.
- $f_{yv}$—Measured strength of the vertical distributed reinforcements.
- $x$—Height of the compression zone.
- $h_0$—Effective height of the shear wall stem section.
- $N$—Axial compression load.
- $M$—Bending moment caused by the peak load.
- $r$—The ratio of the core area to the constrained area is 0.8.
- $b$—The width of the wall.
- $A_a, A_a'$—The area of the embedded steel section.
- $f_{cc}$—Compressive strength of the confined concrete.
- $A_s, A_s'$—Area of the vertical reinforcements in compression.
- $A_s$—Area of the vertical reinforcements in tension.
- $h$—Height of the wall section.
- $\varepsilon_{cc}$—Peak strain of the confined concrete.

The above equations give the force–balance relationship of the wall section. The compressive strain of concrete in the confined area increases significantly, which is considered in the above equations.

For the whole compression area, the average stress should be taken. The effect of the stirrup reinforced concrete should be taken into account using a correction parameter [20]. The specimens studied in this paper have a small axial pressure ratio and the height of the compression zone is less than the range of the edge constraint zone. In this case, the above formulas can be used. If the compression zone exceeds the edge constraint zone, the calculation needs to be revised.

The horizontal force is calculated by $P_m = M_{\text{max}}/H$, where $H$ is the distance from the loading point to the bottom of the wall stem. In this paper, 1.85 m is taken.

The values of $K_1$ and $P_m$ calculated above for each specimen were substituted into the four-line skeleton curve relation equations to obtain a theoretical skeleton curve, which is compared with the experimental skeleton curves and shown in Figure 11. It can be seen from the figure that the two curves agree well with each other. The results verify that the theoretical stiffness should be adjusted in the design of composite shear walls.
Figure 10. The force equilibrium diagram of a prefabricated concrete shear wall with steel sections in the ultimate bearing capacity state.

\[
\begin{align*}
\rho & \text{— Vertical distributed reinforcement ratio.} \\
y_f' & \text{— Measured yield strength of the core reinforcements in compression.} \\
a_f' & \text{— Measured yield strength of the steel section in compression.} \\
y_f & \text{— Measured strength of the vertical distributed reinforcements.} \\
a & \alpha & \text{— Distance of the steel compression force from the wall side.} \\
h_v & \text{— Effective height of the shear wall stem section.} \\
N & \text{— Axial compression load.} \\
M & \text{— Bending moment caused by the peak load.} \\
r & \text{The ratio of the core area to the constrained area is 0.8.} \\
b & \text{— The width of the wall.} \\
a_{AA} & \text{— The area of the embedded steel section.} \\
c_{cc} & \text{— Compressive strength of the confined concrete.} \\
s_{A} & \text{— Area of the vertical reinforcements in compression.} \\
s_{A} & \text{— Area of the vertical reinforcements in tension.} \\
h & \text{— Height of the wall section.} \\
\varepsilon_{cc} & \text{— Peak strain of the confined concrete.} \\
\end{align*}
\]

The above equations give the force–balance relationship of the wall section. The compressive strain of concrete in the confined area increases significantly, which is considered in the above equations.

For the whole compression area, the average stress should be taken. The effect of the stirrup reinforced concrete should be taken into account using a correction parameter [20]. The specimens studied in this paper have a small axial pressure ratio and the height of the compression zone is less than the range of the edge constraint zone. In this case, the above formulas can be used. If the compression zone exceeds the edge constraint zone, the calculation needs to be revised.

The horizontal force is calculated by \( P_m = \frac{M_{\text{max}}}{H} \), where \( H \) is the distance from the loading point to the bottom of the wall stem. In this paper, 1.85 m is taken.

The values of \( K_1 \) and \( P_m \) calculated above for each specimen were substituted into the four-line skeleton curve relations to obtain a theoretical skeleton curve, which is compared with the experimental skeleton curves and shown in Figure 11. It can be seen from the figure that the two curves agree well with each other. The results verify that the theoretical stiffness should be adjusted in the design of composite shear walls.

Figure 11. Cont.
5.2. Determination of Hysteresis Rules

Besides determining the skeleton curves of the restoring force model, the hysteretic rule should also be determined, including the selection of the stiffness at each stage and the path of positive and negative loading and unloading.

1) Post-yielding rigidity

The data of this test show that the post-yielding rigidity of the test structures is different from that before yielding. When the load exceeds the yield load, the post-yielding stiffness is no longer consistent with the loading stiffness, and each increase of the load level leads to a gradual decrease in the post-yielding stiffness of the specimen.

By measuring the wall-top deformation and the corresponding stiffness of each hysteresis loop after yielding, the ratio of the stiffness to the pre-cracking stiffness $K_1$ and the ratio of the wall-top deformation to the yield displacement is determined. According to the test results of the four wall specimens, the displacement and corresponding stiffness of each hysteresis loop apex after yielding were obtained for all hysteresis curves. In Figure 12, the ratios of the stiffness after yielding to that before cracking ($K_1$) was calculated as the $y$-axis values, and the ratio of the displacement at the apex of the hysteresis curve to the yielding displacement $D_y$ was calculated as the $x$-axis coordinate values. By a nonlinear curve fitting (Figure 12), an equation of the post-yielding stiffness of the specimen can be obtained (Equation (11)).
The post-yielding stiffness calculation formula obtained from the fitting curve:

\[ K_i = 1.07 \left( \frac{D_i}{D_y} \right) K_1 \]  

(11)

where:
- \( K_i \) — Unloading stiffness.
- \( K_1 \) — Cracking stiffness, taking the value of \( K_c \) in Figure 12.
- \( D_i \) — Displacement corresponding to a loading point.
- \( D_y \) — The displacement at the yield point.

(2) The stiffness and change law of each stage.

Before cracking, the prefabricated shear wall specimens are in the elastic stage. In this stage, there is no need to consider the stiffness degradation or residual deformation. The loading and reverse loading stiffness (forward and reverse loading applied on the top of specimens by the actuator) and unloading stiffness are the same, which is smaller than the cracking stiffness (\( K_1 \)).

From cracking occurrence to yielding, the loading stiffness is designated as pre-yield stiffness \( K_2 \); the corresponding unloading stiffness is close to \( K_1 \); and the unloading curve is parallel to the curve before cracking. In reverse loading, the pre-yield stiffness is \( K_2 \), while the unloading stiffness is taken as the pre-cracking stiffness \( K_1 \).

From yielding occurrence to peak load, when the load exceeds the yielding load, the loading stiffness of the curve is taken as the post-yield stiffness \( K_3 \), the unloading stiffness is \( K_5 \); and in the case of reverse loading, the post-yield stiffness is \( K_3 \), and the unloading stiffness is \( K_5 \).

Post peak load, when the load exceeds the peak load, the loading stiffness is taken as \( K_4 \), and the unloading stiffness is taken as \( K_5 \); and in the case of reverse loading, the loading stiffness is \( K_4 \), and the unloading stiffness is \( K_5 \).

(3) Positive and negative loading and unloading paths

The paths are shown in Figure 13. Before cracking, the specimen was in the elastic stage, loaded and unloaded along a straight line, and the stiffness was all \( K_1 \). Before yielding and after cracking, the pre-yield stiffness was taken as \( K_2 \) to point 1 for the loading stiffness, the unloading stiffness at the cracking stage is \( K_1 \) to point 2, reverse loading to point 3 as \( K_2 \), and the unloading stiffness before cracking is \( K_1 \) to point 4.

![Figure 13. Schematic diagram of loading and unloading paths.](image-url)
Before the peak load and after yielding, the loading stiffness of the curve is set as the post-yield stiffness $K_3$ to point 5, the unloading stiffness is $K_5$ to point 6, the post-yield stiffness is $K_3$ to point 7 in the reverse loading, and the unloading stiffness is $K_4$ to point 8 in the unloading process. If it intersects the skeleton curve in advance, the skeleton curve is extended to the load point, and then it unloads and returns.

After the peak load, the load stiffness is set as $K_4$ to point 9 and the unloading stiffness is $K_5$ to point 10. In the reverse loading, the loading stiffness is set as $K_4$ to point 11, and the unloading stiffness is set as $K_5$ to point 12.

5.3. Comparison between the Model Hysteresis Curve and the Test Hysteresis Curve

Based on the restoring force curves and hysteresis relationships obtained in Figure 13, the theoretical calculated hysteresis curves for wall specimens XZW 1 to 4 can be obtained by using the $D_m$ and $P_m$ values of each specimen from Table 4. The theoretical hysteresis curves of the restoring force model are compared with those from the test (Figure 14). From the comparison of each specimen, the four-line restoring force model proposed in this paper basically reflects the hysteresis characteristics of the specimens, as verified by the good fitting with the test results.

6. Discussions

This study is based on the test results of four wall specimens. The failure modes and hysteretic performances of the four test walls are similar, which reflects the repeatability of the tests and the consistency of the results. The connections placed at the bottom of the
walls received the largest stresses from the lateral loading, which worked well compared to the connections at an intermediate height of the wall. Due to the similar structure design, the pattern of the bending cracks developed in the early and middle stages of loading, and the oblique cracks appeared in the later stage of loading are also similar among the four wall specimens. All of the four wall specimens were damaged due to flexural and shear failure.

This is a pioneering preliminary study that provides a reference for further research in this field. To provide complete information for the development of specifications for composite shear wall design with steel sections, tests on more specimens should be conducted in the future to obtain statistically significant results.

7. Summary and Conclusions

(1) Four specimens of prefabricated concrete shear walls with built-in steel sections were tested under a constant vertical loading and pseudo-static horizontal cyclic loading. The test results and theoretical failure mechanics of the walls were compared and analyzed.

(2) Hysteretic and skeleton curves were developed based on the results of the pseudo-static test of prefabricated shear wall specimens. Based on the load and displacement of the walls, the stiffness of the walls at different loading stages was determined based on theoretical calculation and numerical regression. The establishment of a restoring force model provided theoretical support for the nonlinear analysis of such structures under low-cyclic loading.

(3) According to the skeleton curves obtained from the test, a dimensionless skeleton curve model was obtained by experimental fitting. It can be seen that the vertical load on the top of the wall has a great effect on the shear stiffness of the wall. With a 10% increase of the vertical pressure, a 25% increase of the maximum horizontal resistance can be reached. The connection location, though, has no significant effect on the resistance of the wall.

(4) Based on the test fitting method, the calculation method of the cracking stiffness for this kind of structure was proposed by means of mechanical calculation and test modifications. Then, the stiffness of yield point, limit point, and failure point were deduced.

(5) Based on the ultimate equilibrium equation, the calculation method of the ultimate load was proposed. Then, the load of the cracking zone, yield point, and failure point were deduced.

(6) The restoring force curve model of this kind of structure for nonlinear analysis was obtained. By numerical fitting the unloading stiffness after yielding, the hysteresis loop path was identified. The skeleton curve of the four-fold line considering stiffness degradation was obtained.

(7) The restoring force model proposed in this paper was verified by the test results, which can be used to reflect the hysteretic performance of prefabricated shear walls with built-in steel sections. It was observed that ductal behavior of shear walls can be achieved by embedding steel sections in the wall. This method can be used as a reference for the elastoplastic seismic response analysis of RC shear wall structures with built-in steel sections.

Author Contributions: Research protocol formation and implementation, T.W.; Assist in research proposal and formulation and analysis, R.J.; Experimental study, S.Y.; Experimental tests, K.Y.; Experimental tests, L.L.; Theoretical analysis, G.Z. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the Nature Fund program of Jilin, China (Grant No. 20200201227JC).

Data Availability Statement: The data presented in this study are available on request from the corresponding author.

Acknowledgments: The authors would like to thank Jilin Nature Fund program for the financial support and thank Changchun Institute of Technology for all the support during this research.

Conflicts of Interest: The authors declare no conflict of interest.
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