Fractional Electric Charge and Massive Quasiparticles on the Domain Wall between Topological Insulators and Spin Ice Compounds

I Kanazawa, E Imai, T Sasaki, R Maeda
Department of Physics, Tokyo Gakugei University, Koganeishi, Tokyo 184-8501, Japan
E-mail: kanazawa@u-gakugei.ac.jp

Abstract. We have proposed the mass-creation mechanism on massless Dirac fermions on the domain wall between topological insulators and spin ice compounds through the interaction between the massless Dirac fermion and excited magnetic monopoles.

1. Introduction

The quantum Hall state gives the first example of topological states of matter which have topological quantum numbers different from ordinary states of matters [1, 2]. In the presence of many-body interactions and disorder, the Berry curvature and the Chern number can be defined over the space of twisted boundary condition [3]. The quantum Hall effect of the (2+1)-dimensional time reversal-breaking insulator has been generalized to time reversal-invariant insulators in various dimensions. Time reversal- invariant topological insulators have been classified in (3+1) dimensions [4]. Recently the three-dimensional topological insulator Bi$_{1-x}$Sb$_x$ in a certain range of composition [5] has been predicted. Really massless Dirac fermions on the surface of three-dimensional topological insulator have been observed by angle-resolved photoemission spectroscopy [6]. Spin ices are frustrated magnets. The spin ice family [Dy$_2$Ti$_2$O$_7$ and Ho$_2$Ti$_2$O$_7$] are predicted to support sharply defined magnetic monopole excitations [7, 8]. In addition, spin ices have residual entropy at low temperature, which is well-approximated by the Pauling entropy for water ice [9]. One(I.K) of the present authors has reported the importance of the hole-induced domain wall and the hedgehog-like magnetic soliton in magnetoresistance in diluted magnetic semiconductors [10, 11, 12] and doped manganites [13, 14]. In addition, one(I.K) of the present authors has proposed exotic quasiparticle with fractional charges in semiconductor-dot from collectively induced-charge effect on a domain wall shell [15, 16]. Recently the present authors [17, 18] have proposed that there might be emergent quasiparticles with fractional electronic charge such dyons on the domain wall between topological insulators and spin ice compounds through the Witten effect. In this study, we shall propose the mass-creation mechanism of Dirac fermions on the domain wall between topological insulators and spin ice components through the interaction between Dirac fermions and excited magnetic monopoles.
2. Exotic particle and mass-creation on the domain wall between the topological insulator and spin ice

The time reversal-invariant (TRI) topological insulator in (3+1) dimension can be obtained as descendants from the fundamental TRI insulator in (4+1) dimension through a dimensional reduction procedure. The effective topological field theory and $Z_2$ topological classification for the TRI insulators in (3+1) dimension are naturally obtained from this procedure [19]. The effective action of gauge field $A^\mu$ is obtained by the following path integral,

$$e^{iS_{eff}} = \int D[c]D[c^+]\exp\left[i\int dt\sum_m e^+_{m\alpha}(i\partial_t)c_{m\alpha} - H(A)\right]$$

which determines the response of the fermionic system through the equation,

$$\mathcal{J}_\mu(x) = \frac{\delta S_{eff}}{\delta A_\mu(x)}$$

In the case of (4+1)-dimensional insulator, a topological term is in general present in the effective action, which is the second Chern-Simons term. To study the response properties of the (3+1) dimensional system, the effective action $S_{3D}(A,\theta)$ with an adiabatic parameter $\theta$ can be defined as

$$e^{iS_{3D}(A,\theta)} = \int D\psi D\bar{\psi} \exp\left[i\int dt \sum_{\vec{x}} \bar{\psi}_{\vec{x}}(i\partial_\tau - A_{\vec{x}\theta})\psi_{\vec{x}} - H[A,\theta]\right].$$

$H(A,\theta)$ is the (3+1) dimensional Hamiltonian of Dirac model coupled to an external $U(1)$ gauge field $A$ [10]. $\psi$ is the wavefunction of the Dirac electron. Where $\vec{x}$ stands for the three-dimensional coordinates. A Taylor expansion of $S_{3D}$ can be carried out around the gauge field configuration $A_4(\vec{x},t)$, whose $s = 1,2,3$ stands for the x,y,z directions, $\theta(\vec{x},t) \equiv \theta_0$, which contains a nonlinear-response term derived from the (4+1) dimensional Chern-Simon action,

$$S_{3D}(A,\theta) = \frac{G_3(\theta_0)}{4\pi} \int d^4x dt \varepsilon^{\mu\nu\sigma\tau} \delta\theta \partial_\mu A_\nu \partial_\sigma A_\tau.$$

The field $\delta\theta(\vec{x},t) = \theta(\vec{x},t) - \theta_0$ plays the role of $A_4$ in the (4+1) dimension, and the coefficient $G_3(\theta_0)$ is determined by Goldstone and Wilczek-type Feynman diagram [11] in Fig. 1. Consequently, $G_3(\theta_0)$ can be calculated as follows.

$$G_3(\theta_0) = -\frac{\pi}{6} \int \frac{d^3k}{(2\pi)^3} \frac{d\omega}{(2\pi)^2} Tr_{\varepsilon^{\mu\nu\sigma\tau}} \left[G^{-1}_0(\partial G^{-1}_0)\left(G^{-1}_0\partial G^{-1}_0\right)\left(G^{-1}_0\partial G^{-1}_0\right)\left(G^{-1}_0\partial G^{-1}_0\right)\right]$$

where $q^\mu = (\omega, k_x, k_y, k_z)$ and $G(q_\mu) = [\omega + i\delta - h(k_i)]^{-1}$ is the single-particle Green’s function. In addition, $G_3(\theta_0)$ is determined from the Berry phase curvature as

$$G_3(\theta_0) = \frac{1}{8\pi^2} \int d^3k \varepsilon^{ijk} Tr[f_i f_j k]$$

in which the Berry phase gauge field is defined in the four-dimensional space $(k_x, k_y, k_z, \theta_0)$, i.e.,

$$a_{i}^{\alpha \beta} = -i \left\langle \vec{k}, \theta_0; \alpha \right| \left(\frac{\partial}{\partial k_i}\right) \left| \vec{k}, \theta_0; \beta \right\rangle$$

and

$$a_{\theta}^{\alpha \beta} = -i \left\langle \vec{k}, \theta_0; \alpha \right| \left(\frac{\partial}{\partial \theta_0}\right) \left| \vec{k}, \theta_0; \beta \right\rangle.$$
A generalized polarization \( P_3(\theta_0) \) can also be defined in (3+1) dimensions so that \( G_3(\theta_0) = \frac{\partial P_3(\theta_0)}{\partial \theta_0} \), with

\[
P_3(\theta_0) = \int d^3k K_\theta = \frac{1}{16\pi^2} \int d^3k \varepsilon^{ijk} T_F \left\{ \left( f_{ij} \right) - \frac{1}{3} [a_i, a_j] \right\} \cdot a_k.
\]

The effective action for the (3+1)-dimensional system is finally written as

\[
S_{3D}(A, \theta) = \frac{1}{16\pi^2} \int d^3x dt \theta_0(\vec{x}, t) \varepsilon^{\mu \nu \sigma \tau} \partial_\mu A_\nu \partial_\sigma A_\tau
\]

In the case of the (3+1) dimensional topological insulator, \( \theta_0(\vec{x}, t) \) correspond to \( \pi \). Here we shall consider the domain wall between the topological insulator and spin ice as shown in Fig. 1. We must consider the (3+1) dimensional Dirac fermion with a domain wall configuration of the \( \theta_0(z) \) field given by

\[
\theta_0(z) = \frac{\pi}{2} \left[ 1 - \tanh \left( \frac{z}{4\xi} \right) \right]
\]

which has the asymptotic behavior \( \theta_0(z \to -\infty) = \pi, \theta_0(z \to \infty) = 0 \). The domain wall width is \( \xi \).

**Figure 1.** The bilayer of spin ice Dy\(_2\)Ti\(_2\)O\(_7\) and topological insulator Bi\(_1-x\)Sb\(_x\)

It is proposed that magnetic charges can exist in certain materials (spin ices) in the form of emergent excitations that manifest like point charges of magnetic monopoles. Spin ice is governed by a model dipolar Hamiltonian as follows,

\[
H = J_{ex}^{nm} \sum_{<ij>} S_i \cdot S_j + \frac{\mu_0}{4\pi} \sum_{i<j} \left[ \frac{S_i \cdot S_j}{r_{ij}^3} - \frac{3(S_i \cdot r_{ij})(S_j \cdot r_{ij})}{r_{ij}^5} \right]
\]

(1)

Where \( J_{ex}^{nm} \) is the exchange interaction truncated at the nearest-neighbor level, the spins \( S_i \) point parallel to the local [111] axis, and \( \mu_0 \) is the vacuum permeability. The rare-earth spins \( S_j \) have typical a dipole moment of approximately \( \mu \sim 10 \mu_B \) (\( \mu_B \) is Bohr magneton). The magnetic charge, \( g_M \), of excited magnetic monopole in the spin-ice takes the values \( \pm 2\mu/b \), \( \mu \sim 10 \mu_B \) being the dipole moment of a spin and \( b \) the distance between the center of adjacent tetrahedra. Now we shall introduce the effective gauge field \( A_{ice}(SU(2)) \), the scalar field \( \phi_{ice} \) (Higgs triplet),
in order to discuss in more detail the effect of excited magnetic monopoles in the spin-ice. That is,

\[ A_{i,\text{ice}}^{a,\text{cl}} = \frac{1}{g} \varepsilon^{aij} n^j (1 - F_{\text{ice}}(r)) \]  
\[ A_{0,\text{ice}}^{\text{cl}} = 0 \]  
\[ \varphi_{\text{ice}}^a = n^a \tau^a v (1 - H_{\text{ice}}(r)). \]

where \( g = \frac{1}{2 m} \), \( r = \sqrt{x^2} \), \( n = x/r \), \( F_{\text{ice}}(r) \) and \( H_{\text{ice}}(r) \) obey the following boundary conditions, \( F_{\text{ice}}(0) = H_{\text{ice}}(0) = 1 \), \( F_{\text{ice}}(\infty) = H_{\text{ice}}(\infty) = 0 \). We shall immediately write down the Dirac equation.

\[ (i \gamma^\mu D_\mu - \hbar \frac{\tau^a}{2} \varphi_{\text{ice}}^a) \psi = 0 \]

\( \psi \) is the wavefunction of the Dirac field on the domain wall between topological insulator and the spin ice. \( h \) is a real coupling constant between the excited magnetic monopole and massless Dirac fermion on the domain wall. It is noted that we have set explicit mass parameter equal to zero. The covariant derivative in eq.(5) is

\[ D_\mu = \partial_\mu - ig \frac{\tau^a}{2} A_\mu \]

In the covariant derivative (6), we retain the electromagnetic field \( A^3_\mu \) and we set \( \varphi_{\text{ice}}^a = v \delta^{a3} \). Then equation (5) will have the form

\[ [i \gamma^\mu (\partial_\mu - ig \frac{\tau^a}{2} A^3_\mu) - \hbar \frac{\tau^3}{2} v] \psi = 0 \]

The equation for the upper and lower components of SU(2)-doublet,

\[ \psi = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \]

decouple and have the form

\[ [i \gamma^\mu (\partial_\mu - ig \frac{A^3_\mu}{2}) - \hbar \frac{\tau^3}{2} v] \chi_1 = 0 \]
\[ [i \gamma^\mu (\partial_\mu + ig \frac{A^3_\mu}{2}) + \hbar \frac{\tau^3}{2} v] \chi_2 = 0 \]

That is, the fermions \( \chi_1 \) and \( \chi_2 \) have the same mass \( \hbar v/2 \). Taking into account the monopole density in the spin ice [21], we might be able to measure the mass of fermions on the domain wall between the spin ice in Dy₂Ti₂O₇ and the topological insulator Bi₁₋ₓSbₓ at \( ~1K \) by means of angle-resolved photoemission spectroscopy.

3. Conclusion

The mass-creation mechanism of massless Dirac fermions on the domain wall between topological insulators and spin ice compounds has been proposed. The interaction between excited magnetic monopoles and massless Dirac fermions is of significance in the mechanism.
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