Mirror Families in Electro-weak Symmetry Breaking

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Abstract

We study symmetry breaking in a left-right symmetric extension of the Standard Model with mirror fermions, one for each Standard-Model fermion. The new particles assist a top-quark condensate in breaking electro-weak symmetry. Half of the fermions acquire electro-weak-invariant masses at around 500 GeV and would be probably accessible at future high-energy experiments like LHC or NLC. The contributions to the $S$ and $T$ parameters are small and negative in accordance with electro-weak precision data.

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1 Introduction

High-energy experiments have given so far data consistent with the Standard Model described by the gauge group structure $SU(3)_C \times SU(2)_L \times U(1)_Y$. However, it is well known that the conventional Higgs mechanism implemented for the $SU(2)_L$ symmetry breaking has a naturalness problem, in that it is hard to keep a mass of a fundamental scalar at energies as low as the weak scale. One possible solution is to consider the Higgs particle as a composite state of new strongly-interacting fermions as in technicolor theories. Such approaches have however lost their popularity because they tend to give large positive contributions to the electro-weak $S$ and $T$ parameters inconsistent with experimental data coming from LEP and SLC, except for special cases [1].

Another dynamical symmetry breaking scenario has its origin in that the top quark has turned out to be very massive, and in fact quite close to the electro-weak scale. This could indicate that the Higgs mechanism is closely related to a top-quark condensate. Models in this direction have provided interesting insights in the problem of electro-weak symmetry breaking, but they are usually plagued by various problems. Originally they were formulated in terms of four-fermion interactions of unspecified origin [2]. In the minimal version they either do not solve the fine-tuning problem or they predict a top mass which is much too large [3]. In extensions of the minimal scenario the top mass is also too large, except for the supersymmetric or left-right-symmetric extensions. This could be an indication that, even though the top quark is an important factor in $SU(2)_L$ breaking, it is not the only one. A possible combination of top-mode electro-weak symmetry breaking and technicolor introduces again the usual problems with the electro-weak parameters [4].

An interesting approach which solves these problems goes in the direction of introducing new fermions with large electro-weak-invariant masses [5], [6]. These assist the top-quark condensate in breaking $SU(2)_L$ and simultaneously lead to acceptable contributions to the electro-weak parameters due to the decoupling theorem [7]. This paper studies a left-right symmetric model with extra flavor symmetries which possesses these features, with the additional motivation that it can be readily incorporated into unification schemata which can in principle produce specific fermion mass hierarchies and CKM angles.
2 The model

It was recently shown in a general context [5] that electro-weak-invariant fermion masses could help in keeping contributions of new physics to the $S$ and $T$ parameters under control. These masses can appear naturally in the theory by introducing, along with new fermions, “mirror” fermions with the same quantum numbers but opposite handedness. In [5] these were introduced in a technicolor context, but in the present study a left-right and flavor symmetric direction is taken. Specifically, mirror families to the ordinary Standard Model fermion families are introduced, after extending their quantum numbers in a left-right symmetric way. First ideas in this direction appeared quite early [8], but not in conjunction with gauge-invariant masses.

In particular, the gauge group structure $SU(3)_C \times SU(2)_L \times SU(2)_R \times SU(3)_F \times U(1)_F \times U(1)_{B-L}$ is considered, unbroken at scales on the order of $10^3 - 10^4$ TeV. The magnitude of these scales, as will become clear later, is constrained from below due to flavor-changing neutral currents (FCNC) and from above due to the magnitude of the lightest-family masses. The group $SU(3)_F$ unifies the three Standard-Model families and the role of the abelian $U(1)_F$ group is explained in the following. The gauge structure and the new fermions introduced have the advantage that, apart from restoring the left-right quantum-number symmetry missing in the Standard-Model fermions, they can be easier embedded in unification schemata, as will be discussed later.

Under the above groups, the following left-handed fermion representations are introduced:

| Families | Mirror families |
|----------|-----------------|
| $q_{1L}$ : $(3, 2, 1, 3, \kappa, 1/3)$ | $q_{2L}$ : $(3, 1, 2, \bar{3}, -\kappa, 1/3)$ |
| $l_{1L}$ : $(1, 2, 1, 3, \kappa, -1)$ | $l_{2L}$ : $(1, 1, 2, 3, -\kappa, -1)$ |
| $q_{1R}^c$ : $(\bar{3}, 1, 2, 3, -\kappa, -1/3)$ | $q_{2R}^c$ : $(\bar{3}, 2, 1, 3, \kappa, -1/3)$ |
| $l_{1R}^c$ : $(1, 1, 2, 3, -\kappa, 1)$ | $l_{2R}^c$ : $(1, 2, 1, 3, \kappa, 1)$ |

where the subscripts 1,2 indicate whether a fermion is of Standard-Model type or its mirror, $q$ and $l$ denote quarks and leptons respectively, $\kappa > 0$ is the $U(1)_F$ charge, and the superscript $c$ denotes charge conjugation.
The $U(1)_{R-L}$ anomalies are canceled between quarks and leptons and the $U(1)_{F}$ anomalies between the fermions and their mirrors. Moreover, the absence of other chiral anomalies in models having such a fermion content has been discussed in [9]. In principle, fermions in such representations could acquire large gauge-invariant masses on the order of the GUT scale. However, the $U(1)_{F}$ coupling $\kappa$ is taken to be large enough to prohibit the initial formation of large $SU(2)_L \times SU(2)_R$ invariant fermion masses. It is worth noting here that the proposed doubling of the fermionic content in a left-right symmetric context is typical of models proposed to provide a solution to the strong CP problem [10].

At this stage, the discrete $L - R$ parity is assumed to be already spontaneously broken in such a way that the gauge coupling $g_R$ corresponding to $SU(2)_R$ is larger than the $SU(2)_L$ coupling $g_L$. Such models where $SU(2)_R$ and $L - R$ parity break independently have already been considered in the literature [11]. On the other hand, the family group is assumed to spontaneously break at high energy scales sequentially down to an abelian group, a process which will induce effective four-fermion operators. It is then imagined that at a scale $\Lambda_R \approx 500$ GeV the group $SU(2)_R$ becomes strongly coupled and breaks the abelian gauge group which prevented the formation of gauge-invariant masses. The fermions which are doublets under $SU(2)_R$ acquire therefore dynamically gauge-invariant masses. The $SU(2)_L$ coupling remains meanwhile weak. At lower energies around the electro-weak scale, the most attractive of the effective four-fermion operators mentioned above becomes critical, leading thus to the breaking of the $SU(2)_R$ and $SU(2)_L$ gauge symmetries. One therefore has a scenario where $SU(2)_R$ breaks at a low energy scale, after it has become strongly coupled. The sequence of gauge-symmetry breakings envisaged is graphically shown in Fig.1.

A more detailed study of the scenario outlined above is now presented. In a first step, $SU(3)_F$ breaks down to $SU(2)_F$ at a scale $\Lambda_{3F}$, separating one fermion family from the other two. It will turn out later that the singlet family under $SU(2)_F$ is the first and lightest family. The scale $\Lambda_{3F}$ should be on the order of $10^3 - 10^4$ TeV as already explained, in order to avoid too large FCNC and to get reasonable first generation fermion masses, since the massive bosons corresponding to the broken generators of $SU(3)_F$ are expected to feed masses down to first-family fermions.
SU(3)_C × SU(2)_L × SU(2)_R × U(1)_{B-L} × SU(3)_F × U(1)_F

\[ \Lambda_{3F} \approx 10^3 \text{ TeV} \]

\[ \Lambda_F \approx 40 \text{ TeV} \]
(origin of 4f inter.)

\[ \Lambda_R \approx 0.5 \text{ TeV} \]
(origin of strong inter.)

\[ \Lambda_L \approx 0.3 \text{ TeV} \]

After this breaking, the Standard-Model families, together with their mirror partners, transform with respect to \( SU(3)_C \times SU(2)_L \times SU(2)_R \times SU(2)_F \times U(1)_F \times U(1)_{B-L} \) as

| Families | Mirror families |
|----------|----------------|
| \( q_{1L} \): (3, 2, 1, f, \kappa, 1/3) | \( q_{2L} \): (3, 1, 2, f, -\kappa, 1/3) |
| \( l_{1L} \): (1, 2, 1, f, \kappa, -1) | \( l_{2L} \): (1, 1, 2, f, -\kappa, -1) |
| \( q_{1R}^c \): (\bar{3}, 1, 2, f, -\kappa, -1/3) | \( q_{2R}^c \): (\bar{3}, 2, 1, f, \kappa, -1/3) |
| \( l_{1R}^c \): (1, 1, 2, f, -\kappa, 1) | \( l_{2R}^c \): (1, 2, 1, f, \kappa, 1) |

where \( f = 2 \) for the two heavier families and \( f = 1 \) for the lightest one.

At a lower scale \( \Lambda_F \), the symmetry \( SU(2)_F \times U(1)_F \) should break sponta-
neously via a non-zero vacuum-expectation value having the right quantum numbers according to the pattern $SU(2)_F \times U(1)_F \rightarrow U(1)_{F'}$. At this point the high-energy physics which generates this breaking (for instance by means of a fundamental Higgs mechanism or of a fermionic composite operator) are left unspecified. At $\Lambda_F$ one has then physics producing effective four-fermion operators involving second and third family fermions. It will turn out later that, in order to get the correct electroweak symmetry breaking scale, one should have $\Lambda_F \approx 40$ TeV. What should be kept in mind, however, is that the new physics producing this four-fermion term is independent of the consequences this term implies for lower energy physics and that alternative ways to produce it would not affect the phenomenological results of this work.

The second and third family quantum numbers under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{F'} \times U(1)_{B-L}$ are then given by

| Mirrors of 2nd & 3rd families | 2nd & 3rd families |
|-------------------------------|---------------------|
| $q_{1L}$: $(3, 2, 1, \kappa_{\pm}, 1/3)$ | $q_{2L}$: $(3, 1, 2, -\kappa_{\pm}, 1/3)$ |
| $l_{1L}$: $(1, 2, 1, \kappa_{\pm}, -1)$ | $l_{2L}$: $(1, 1, 2, -\kappa_{\pm}, -1)$ |
| $q_{1R}^c$: $(\bar{3}, 1, 2, -\kappa_{\pm}, -1/3)$ | $q_{2R}^c$: $(\bar{3}, 2, 1, \kappa_{\pm}, -1/3)$ |
| $l_{1R}^c$: $(1, 1, 2, -\kappa_{\pm}, 1)$ | $l_{2R}^c$: $(1, 2, 1, \kappa_{\pm}, 1)$ |

where $\kappa_{\pm} = (\kappa \pm 1)/2$ correspond to the $U(1)_{F'}$ charge $Q_{F'} = T_{3F} + Q_F/2$ of the second and third families respectively, where $T_{3F}$ is an $SU(2)_F$ generator and $Q_F$ is the $U(1)_F$ charge.

As will be seen in the following, the $SU(2)_F$ gauge symmetry between the second and third family plays a role analogous to the one the QCD-like gauge groups play in top-color models [3]. Its breaking induces effective four-fermion operators that will later be responsible for the $SU(2)_R$ and $SU(2)_L$ gauge symmetry breakings. After Fierz rearrangement, such a four-fermion term for the quarks of the second and third generation and their mirrors is

$$F_{(1,2)} = \frac{\lambda}{\Lambda_F^2}(\bar{q}_{(1,2)R}q_{(1,2)L})(\bar{q}_{(1,2)L}q_{(1,2)R})$$

plus the same term with $L$ and $R$ subscripts interchanged, where $\lambda/\Lambda_F^2$ is an effective
four-fermion coupling. The fermion bilinears in both parentheses transform under $SU(2)_L \times SU(2)_R$ like a $(2, 2)$.

The next step is connected to the assumption made at the beginning, namely that at some high energy scale the left-right parity is broken and that the gauge coupling $g_R$ is stronger than $g_L$, where $g_{L,R}$ correspond to $SU(2)_{L,R}$ respectively. In fact, it was assumed that at energy scales close to the $SU(2)_R$ characteristic scale $\Lambda_R \approx 500$ GeV, the $SU(2)_R$ coupling becomes strong enough to break $U(1)_{F'}$ via fermionic condensates.

In order to prevent these condensates from breaking QCD, one has to assume that only two-quark operators like $<\bar{q}_R q_L> \approx \Lambda^3_R/(4\pi)^2 \approx 1$ GeV for the $U(1)_F$ charge normalization is such that both $\kappa_+$ and $\kappa_-$ are positive, i.e. $\kappa > 1$. This prohibits also the formation of condensates involving simultaneously second and third generation leptons.

The abelian symmetry which protected the fermions from acquiring a mass is broken by these condensates. Therefore, gauge-invariant mass terms of the form $\bar{q}_{2R}q_{1L}$ will also appear in the theory. However, they will be induced mainly from the relevant four-fermi operators and will be on the order of $\Lambda^3_R/\Lambda^2_F \approx 0.1$ GeV for the

\footnote{The $U(1)_{F'}$ charge normalization is such that both $\kappa_+$ and $\kappa_-$ are positive, i.e. $\kappa > 1$. This prohibits also the formation of condensates involving simultaneously second and third generation leptons.}

$SU(2)_R \times SU(2)_L$ invariant dynamical masses on the order of the $SU(2)_R$ scale $M \approx \Lambda_R \approx 500$ GeV, while the other half remain so far massless. These dynamical masses are equivalent to the constituent quark masses in ordinary QCD. The fermion masses get also small contributions from the effective four-fermion interactions originating from $SU(2)_F$ and $SU(3)_F$.

Note that these masses are not constrained from above by considerations concerning Yukawa couplings becoming non-perturbative \cite{12}, since their origin is dynamical and not connected with a symmetry breaking. This is novel as regards studies of models involving mirror fermions and their phenomenological implications \cite{8}. It will be interesting to see in the next section how the smallness of the measured $T$ parameter is related to the value of the dynamical mass $M$.
second and third generation, and even smaller for the first generation. Interesting
mass contributions for the light fermions are thus obtained, which will be studied
elsewhere. If these light masses are ignored, the mass matrix takes the following
form

\[
\begin{pmatrix}
q_1^e \\
q_2^e \\
q_1^\nu \\
q_2^\nu
\end{pmatrix} =
\begin{pmatrix}
0 & 0 \\
0 & M
\end{pmatrix}
\]

for the quarks of the two heavier generations and a similar form for the leptons and
first-generation fermions. The fields in this matrix are ordered in a way that will
allow later the direct use of the formalism of \[5\], i.e. diagonal entries are SU(2)_L
invariant and the off-diagonal SU(2)_L breaking.

One should note that the strong SU(2)_R interactions produce mass terms
mixing the fermion generations, so the mass matrices take the above form after
diagonalization in fermion family space. This should produce FCNC for the heavy
partners of the Standard-Model fermions, which can in principle be fed down to
the known SM particles via four-fermion operators. The scales of these effective
operators are however large enough, in order to avoid problems with FCNC origin-
ating from the broken family groups. Therefore, they are also large enough to
avoid FCNC in the SM sector coming from the broken SU(2)_R group.

For a last step some dynamics are needed close to the SU(2)_R scale \(\Lambda_R\)
which leads to the spontaneous breaking \(SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y\), where
\(Y\) is the usual hypercharge given by \(Q_Y = 2T_{3R} + Q_{B-L}\), with \(T_{3R}\) an SU(2)_R
generator and \(Q_{B-L}\) the \(U(1)_{B-L}\) charge. This breaking can be achieved by a non-
zero vacuum-expectation value of either a fundamental or a composite field which
is a doublet under SU(2)_R and charged under U(1)_{B-L}. One of these possibilities
will be discussed later, namely how the breaking of SU(2)_R could be due to a
fermionic condensate. Moreover, since \(g_R\) grows fast at energy scales close to \(\Lambda_R\), it
is expected to be much larger than the \(B-L\) coupling \(g_{B-L}\) there. Therefore, the
hypercharge gauge coupling \(g_Y\) at \(\Lambda_R\) will be approximately equal to \(g_{B-L}\), since
\(g_Y = \frac{g_{3R}g_{B-L}}{\sqrt{g_R^2 + g_{B-L}^2}}\). This relation should constrain the breaking scale and the strength
of the coupling of the unifying group from which \(U(1)_{B-L}\) possibly originates.

The third family quantum numbers under \(SU(3)_C \times SU(2)_L \times U(1)_Y\),
where the star is a reminder that the gauge group is broken, are then

| 3rd family | Mirror of 3rd family |
|------------|----------------------|
| $Q_{1L}$ : $(3, 2, \kappa_+, 1/3)$ | $U_{2L}$ : $(3, 1, -\kappa_+, 4/3)$ |
| $D_{2L}$ : $(3, 1, -\kappa_+, -2/3)$ |
| $L_{1L}$ : $(1, 2, \kappa_+, -1)$ | $N_{2L}$ : $(1, 1, -\kappa_+, 0)$ |
| $E_{2L}$ : $(1, 1, -\kappa_+, -2)$ |
| $U_{1R}^c$ : $(\bar{3}, 1, -\kappa_+, -4/3)$ | $Q_{2R}^c$ : $(\bar{3}, 2, \kappa_+, -1/3)$ |
| $D_{1R}^c$ : $(\bar{3}, 1, -\kappa_+, 2/3)$ |
| $N_{1R}^c$ : $(1, 1, -\kappa_+, 0)$ | $L_{2R}^c$ : $(1, 2, \kappa_+, 1)$ |
| $E_{1R}^c$ : $(1, 1, -\kappa_+, 2)$ |

where $Q_{1L,2R} = \left( \begin{array}{c} U_{1L,2R} \\ D_{1L,2R} \end{array} \right)$ and $L_{1L,2R} = \left( \begin{array}{c} N_{1L,2R} \\ E_{1L,2R} \end{array} \right)$, while the ones for the second family are the same except for the $U(1)_F$ charges which are $\kappa_-$ instead of $\kappa_+$.

Finally, the four-fermion operators $F_{(1,2)}$ involving the 3rd-family up-type quarks have to be chosen critical, to form $<\bar{U}_{1R} U_{1L}>=<\bar{U}_{2R} U_{2L}>\neq 0$ condensates and break electro-weak symmetry along the standard pattern $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$, which requires of course a large initial $U(1)_F$ coupling. These fermions therefore acquire $SU(2)_L$ breaking masses, which, in order to reproduce the top quark mass and the weak scale correctly, should be on the order of $m \approx 300$ GeV, as will be seen in the next section. The gap equations corresponding to the dynamical masses $m$ and $M$ are diagrammatically shown in Fig.2. The source of the mass $m$ is the effective four-fermion coupling $\lambda/\Lambda_F^2$, which is assisted by the $QCD$, hypercharge and $U(1)_F$ couplings in a sense of a gauged Nambu-Jona-Lasinio mechanism [13]. The source of the mass $M$ are, as has already been seen, the strong $SU(2)_R$ interactions.

On the other hand, the second family has a smaller charge under $U(1)_F$ than the third one and it is assumed that its four-fermion interactions are not large enough to drive the corresponding gap equations to criticality. The same goes for

\[\text{This should not pose a problem in principle with a Landau pole, since } U(1)_F \text{ is embedded at not too distant energy scales into a non-abelian group.}\]
the down-type quarks and the leptons of the third generation, which have smaller hypercharge and no color respectively. Lighter family fermions and down type fermions in general are expected to acquire their masses subsequently by effective operators induced by the broken $SU(3)_F$, $SU(2)_F$ and $SU(2)_R$ groups respectively.

The same fermion condensate that breaks $SU(2)_L \times U(1)_Y$ could also be responsible for the original $SU(2)_R \times U(1)_{B-L}$ breaking. However, because the characteristic scale of this condensate is somewhat smaller than the $SU(2)_R$ scale, non-perturbative contributions push the dynamical masses of the gauge bosons of $SU(2)_R$ up to its characteristic scale $\Lambda_R$. This could in principle be an economical way of $SU(2)_R \times U(1)_{B-L}$ breaking avoiding not only too small $SU(2)_R$-boson masses but also the introduction of additional gauge-symmetry breaking mechanisms. In such a scenario the $U(1)_Y$ symmetry would never be essentially realised, since it would only be an intermediate technical step between $U(1)_{B-L}$ and $U(1)_{EM}$. It is also worth noting that, after inspecting the $U(1)_{EM}$ quantum numbers of the mirror families one could qualify them as “anti-matter”.

The mass matrix for the up-type quarks of the third generation and their mirrors, denoted by $M_U$, takes now the form

$$
M_U = \begin{pmatrix}
U_{1L} & U_{2L} \\
U_{1R} & U_{2R}
\end{pmatrix}
\begin{pmatrix}
U_{1L} & U_{2L} \\
U_{1R} & U_{2R}
\end{pmatrix} =
\begin{pmatrix}
0 & m & m & M
\end{pmatrix},
$$

while for the mass matrix of the down-type quarks and their mirrors, denoted by
\( \mathcal{M}_{D} \), one has as before

\[
\begin{pmatrix}
D_{1L}^2 & D_{1R}^2 \\
D_{2L}^2 & D_{2R}^2
\end{pmatrix}
\begin{pmatrix}
0 & 0 \\
0 & M
\end{pmatrix}.
\]  \tag{8}

After diagonalization therefore, in which the lighter mass eigenstates are identified with the Standard Model fermions, a see-saw mechanism \cite{6} produces small masses for the SM particles and large masses for their partners, in a way that their condensation reproduces the weak scale and the top mass correctly. The large gauge invariant masses of these partners are expected to damp their contributions to the electro-weak parameters, as will be seen in the following.

3 Phenomenology

For such a model to be phenomenologically viable, it should first of all be able to reproduce the known mass hierarchies of the Standard Model fermions and be consistent with present experimental bounds on new exotic particles. The new particles introduced should decay fast enough so that cosmological problems are avoided. Their decays could however produce interesting signals in upcoming experiments like LHC and NLC. Moreover, the proposed mechanism should reproduce the weak scale and not give too large contributions to FCNC and to the S and T parameters.

The mass \( m \) breaks the electro-weak symmetry at a scale \( v \). A rough calculation of the weak scale gives

\[
v^2 \approx \frac{3}{2\pi^2} m^2 \ln \left( \frac{\Lambda_F}{M} \right),
\]  \tag{9}

so for \( \Lambda_F \approx 40 \text{ TeV}, M \approx 500 \text{ GeV} \) and \( m \approx 300 \text{ GeV} \) one gets \( v \approx 246 \text{ GeV} \), as is required. The values of \( M \) and \( m \) are chosen in a way that produces the correct top quark mass and simultaneously does not introduce problems with the \( T \) parameter, as will be seen in the following. Note that the factor multiplying the logarithm is twice as large as usual, since there are two, independent but equal, electro-weak-breaking masses \( m \). It is also worth mentioning that such a high scale of four-fermion interactions requires a fine-tuning of about \( (m/\Lambda_F)^2 \approx 10^{-4} \) which is not explained here, but which is typical of similar scenarios \cite{6}.
The diagonalization of the mass matrix in Eq. (7) gives the eigenvalues \( m_{U1} \approx -m^2/M \) and \( m_{U2} \approx M \) for \( m \ll M \). These correspond to the eigenvectors \( t \) and \( t' \), where \( t \) is identified with the usual top quark, in which case \( m_t = -m_{U1} \), while \( t' \) is a heavier partner it mixes with, having \( m_{t'} = m_{U2} \). For \( m \approx 300 \text{ GeV} \) and \( M \approx 500 \text{ GeV} \) one gets \( m_t \approx 175 \text{ GeV} \).

From the diagonalization one gets

\[
\begin{pmatrix}
t_{L,R} \\
t'_{L,R}
\end{pmatrix} = V_{U(L,R)} \begin{pmatrix}
U_{1L,1R} \\
U_{2L,2R}
\end{pmatrix}
\tag{10}
\]

while

\[
\begin{pmatrix}
b_{L,R} \\
b'_{L,R}
\end{pmatrix} = V_{D(L,R)} \begin{pmatrix}
D_{1L,1R} \\
D_{2L,2R}
\end{pmatrix}.
\tag{11}
\]

In the above, \( V_{UL} = V_{UR} \approx \begin{pmatrix} 1 & -m/M \\ m/M & 1 \end{pmatrix} \), \( V_{DL} = V_{DR} = 1 \), and

\[\mathcal{M}_{U,D} = V_{U,R,DR}^d \begin{pmatrix}
m_{U1,D1} & 0 \\
0 & m_{U2,D2}
\end{pmatrix} V_{U,L,DR},\]

where the notation of [3] is followed closely. For simplicity the bottom quark mass has been taken equal to zero, so the corresponding mass matrix is already diagonal with eigenvalues \( m_{D1} = 0 \) and \( m_{D2} = M \). It is therefore important to note that, in contrast to the lighter fermion eigenstates, the top-quark eigenstate has a non-negligible \( SU(2)_L \)-invariant component which could in principle be detectable in future experiments.

For \( S \) and \( T \) one obtains then [3]

\[
S = \frac{N}{6\pi} \frac{m^2}{M^2} \left( -\frac{4}{3} \ln \left( \frac{M^2}{m^2} \right) - 6\chi(m^2/M, M) - M^2/m^2 + 2 \right)
\]

\[
T = \frac{N}{8\pi \sin^2 \theta_w m_w^2} \frac{m^2}{M^2} \left( \theta(M,0) - \theta(M, m^2/M) \right),
\tag{12}
\]

where \( N \) is the number of contributing new fermion doublets, \( m_{w,z} \) are the usual \( W^\pm \) and \( Z^0 \) boson masses and the functions \( \theta \) and \( \chi \) are defined in [3]. Note that, in accordance to the decoupling theorem, both \( S \) and \( T \) tend to zero as \( m/M \) goes to zero. A recent fit of experimental data involving \( S \) and \( T \) gave [14]:

\[
S = -0.4 \pm 0.55
\]

\[
T = -0.25 \pm 0.46.
\tag{13}
\]

Therefore, one can adjust \( M \) in the present model so that it gives results consistent with present electro-weak data.
Expanding the $\theta$ and $\chi$ functions in powers of $m/M$, one finally finds for the $S$ parameter, in leading order and for $M \approx 500$ GeV and $N = 12$ new $SU(2)_L$ doublets:

$$ S = \frac{N m_t}{3 \pi M} \left( \ln(M^2/m^2_t) - 2 \ln(M^2/m^2_z)/3 - 2/3 \right) \approx -0.44 . \quad (14) $$

Note that there are sizable corrections to this result since $m$ is not much smaller than $M$, but one should not expect qualitative changes of the results when these are included.

The $T$ parameter is in leading order given by

$$ T = \frac{3 m^2_t/m^2_w}{4 \pi \sin^2 \theta_w} (2 - \ln(M^2/m^2_t)) \approx -0.45 . \quad (15) $$

Here it is assumed that $N = 3$, i.e. only the three (from the three QCD colors) “mirror” doublets of the top and bottom quarks contribute. This is expected, since all other Standard Model quarks have very small masses, while their “mirrors” have all masses of order $M$, so their contributions to the $T$ parameter are vanishingly small, as is easily seen from Eq.[13]. One can check that, for a given top-quark mass $m_t$, values of $M$ too far away from about 500 GeV would yield unacceptable values for the $T$ parameter. The relative lightness of the mirror particles that ensues from this fact provides therefore an accessible test for the proposed mechanism in experiments like LHC or NLC. It is important to note at this point that the fermion content used can naturally produce negative values for the $S$ and $T$ parameters.

We turn now to further phenomenological considerations. First of all, it is noted that the unification scale of the first family with the two heavier ones at about $10^3 - 10^4$ TeV, as well as the unification scale of the second and third families at $\Lambda_F \approx 40$ TeV is too high to produce detectable effects like FCNC in present experiments. However, the bosons corresponding to the broken $U(1)^*_F$ and $SU(2)^*_R$ could give effects just on the border of present experimental constrains on their masses. Moreover, in this model all symmetries are gauged, so there are no light pseudo-goldstone bosons one should worry about.

Cosmological problems in this scenario are not expected, since the mirror families should decay rapidly enough. Similar to the top quark, mirror quarks are expected to decay before they have the time to hadronize via four-fermion opera-
tors originating from the broken gauge groups. They would however be copiously produced at colliders like LHC or NLC.

The flavor-breaking sequence leads in principle to interesting hierarchical Yukawa couplings. Future studies will show whether such a model can produce reasonable mass hierarchies, CKM matrix elements and CP violation, as well as reproduce correctly the measured value of $\sin \theta_w$. In fact, the experimentally known value of $\sin \theta_w$ should allow the prediction of the breaking scale of a possible unification group from which the assumed group structure resulted. Next, it would be interesting to check if the proposed fermion content could lead to some gauge coupling unification at higher energies. An investigation in this direction could be updated using the now known top-quark mass and considering heavier partners for the Standard-Model fermions, since their masses are gauge invariant, while at the same time trying to use a smaller unifying Pati-Salam breaking scale that can produce reasonably large masses for the leptons.

4 Discussion

We have extended the gauge structure and the fermion content of the Standard Model in a left-right and flavor symmetric way, by introducing $SU(2)_R$ and $SU(3)_F$ gauge groups and mirror fermions to the ordinary ones. A mechanism for electroweak symmetry breaking was then proposed, which reproduces correctly the weak scale and the top-quark mass. By giving gauge-invariant dynamical masses to half of the fermions due to strong $SU(2)_R$ dynamics, one is able to naturally produce values for the $S$ and $T$ parameters in good agreement with experimental data, a long-standing problem in dynamical symmetry breaking models. The masses of the new fermions and bosons are accessible to the next-generation high-energy experiments, providing therefore a concrete testing ground for the proposed model.

The starting point of the model appears even more appealing from a grand-unification perspective, since quark-lepton and family unifying groups give in principle the possibility to reproduce the observed fermion mass hierarchies. In fact, the fermion content used in this paper fits very nicely in unification schemata where fermions and vector bosons transform under the lowest-dimensional representation of an $E_8$ group, i.e. the adjoint $248$. Other groups could also be considered, but the
The attractiveness of $E_8$ comes e.g. from string theory \[16\]. If there are no other particles like fundamental scalars in other representations, $E_8$ is supersymmetric and asymptotically free. Under its maximal $SO(16)$ subgroup the particles transform under $128 + 120$, so if one assumes that $SO(16)$ breaks at Planck-scale energies down to its maximal $SO(10) \times SU(4)_F$ subgroup \[17\], where $SU(4)_F$ is a fermion family group, the fermions of interest in this paper transform under the new gauge structure like $(16, \bar{4})$ and $(\bar{16}, 4)$, i.e. one has four ordinary and four “mirror” (or “conjugate”) families. The appearance of mirror families is in this context therefore natural. Furthermore, it is intriguing to be able to relate the number of fermion families, via the appearance of an $SU(4)_F \approx SO(6)$ group, with the number of the (six) compactified dimensions in string theory \[16\].

In such a scenario $SO(10)$ then breaks down to $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$, where $SU(4)_{PS}$ is a Pati-Salam group \[18\] and $SU(4)_F$ down to $SU(3)_F \times U(1)_F$. To avoid fast proton decay, the $SO(10)$ breaking scale should be larger than about $\Lambda_{10} \approx 10^{16}$ GeV. The structure considered suggests the existence of a fourth fermion family and its mirror, which is assumed to acquire a large mass and decouple from the physics studied here. An example of how this can be achieved, together with giving Planck-scale masses also to the other fermions and vector bosons not needed in this discussion, is given in a similar discussion of Ref.\[17\]. After the breaking of the Pati-Salam group down to $SU(3)_C \times U(1)_{B-L}$ at around $10^3 - 10^4$ TeV, a scale that would allow for reasonable lepton masses to be fed down from the $SU(2)_L$-breaking up-type quark condensate, one gets the group structure and fermion representations assumed at the beginning. Other breaking sequences might also be possible, so this discussion provides only an example of how one could get elegantly the fermion content used, and it should not affect the conclusions drawn from the proposed mechanism of $SU(2)_L$ breaking.

In the past, a similar fermion content has been used in connection with a breaking of $SO(10)$ down to $SU(5)$, along with a usual Higgs mechanism \[19\], even though in that case the mirror fermions are assumed to have electro-weak-breaking masses. The motivation for using here the $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$ subgroup of $SO(10)$ instead of $SU(5)$ is that, apart from unifying quarks and leptons in a nice way, it introduces naturally a left-right symmetry which renders the generation of gauge-invariant masses possible. Moreover, in contrast to \[19\], the symmetry that
prohibits large gauge-invariant fermion masses is here flavor-diagonal, which is due to the sequential breaking of the family group, so there are no problems with FCNC induced by the groups $U(1)_{F,F'}$.

Since both $SO(10)$ and $SU(4)_F$ are asymptotically free, it is conceivable that they self-break via fermionic condensates and tumble down to the assumed gauge structure. It would be interesting if the right-handed Standard-Model neutrinos were involved in such condensates, because then they would acquire very large masses and the lightness of their left-handed partners would be explained by a seesaw mechanism. This mechanism would produce neutrino masses small enough to provide an MSW solution to the solar neutrino problem \[20\]. A thorough analysis of the attractive channels needed for such a symmetry breaking sequence goes however beyond the scope of this paper. On the other hand, a Higgs-based mechanism of such a spontaneous breaking sequence, albeit in a supersymmetric context, is considered for instance in Ref.\[11\].

It would be nice if the vacuum expectation value that partially breaks $SO(10)$ breaks also the local $L-R$ discrete symmetry of $SO(10)$, explaining thus the large difference of the couplings $g_R$ and $g_L$ at low energies. The local character of this discrete symmetry and its breaking at scales higher than the $SU(2)_R$ breaking scale avoids also cosmological problems related to domain-wall formation. An additional group-theoretic argument supporting, but of course not proving, the simultaneous breaking of $L-R$ parity and $SO(10)$ is the twofold symmetry of the Dynkin diagram of $SO(10)$, which is a manifestation of the discrete $L-R$ symmetry and which does not exist in the Dynkin diagrams of the subgroups of $SO(10)$ considered here.

The above arguments all-together show that the specific model and the proposed symmetry-breaking sequence could very nicely fit into a larger, even more symmetric framework. Since there are many possibilities for the dynamics of the theory at such high energy scales however, one should consider the above just as pure speculation and only as a hint towards the origin of the gauge structure and new fermions introduced, the representations of which could just as well be taken ad hoc. In any case, it will be interesting to see how such a scenario develops as its implications are thoroughly investigated in future studies.

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**References**

[1] T. Appelquist and J. Terning, *Phys. Lett.* **B 315**, 139 (1993).

[2] V.A. Miransky, M. Tanabashi and K. Yamawaki, *Phys. Lett.* **B 221**, 177 (1989); W.A. Bardeen, C.T. Hill and M. Lindner, *Phys. Rev.* **D 41**, 1647 (1990); M. Lindner, *Int. J. of Mod. Phys.* **A 8**, 2167 (1993).

[3] C.T. Hill, *Phys. Lett.* **B 266**, 419 (1991).

[4] K. Lane and E. Eichten, *Phys. Lett.* **B 352**, 382 (1995); K. Lane, *Phys. Rev.* **D 54**, 2204 (1996); Preprint No. BUHEP-97-8, March 1997, [hep-ph/9703233](http://arxiv.org/abs/hep-ph/9703233).

[5] N. Maekawa, *Phys. Rev.* **D 52**, 1684 (1995); L. Lavoura and J.P. Silva, *Phys. Rev.* **D 47**, 2046 (1993).

[6] B.A. Dobresku and C.T. Hill, Fermilab Preprint No. FERMILAB-PUB-97/409-T, EFI-97-55, December 1997, [hep-ph/9712319](http://arxiv.org/abs/hep-ph/9712319).

[7] T. Appelquist and J. Carazzone, *Phys. Rev.* **D 11**, 2856 (1975).

[8] T.D. Lee and C.N. Yang, *Phys. Rev.* **104**, 254 (1956). For an interesting review, see J. Maalampi and M. Roos, *Phys. Rep.* **186**, 53 (1990).

[9] D.J. Gross and R. Jackiw, *Phys. Rev.* **D 6**, 477 (1972).

[10] S.M. Barr, D. Chang and G. Senjanovic, *Phys. Rev. Lett.* **67**, 2765 (1991).

[11] G. Senjanovic and R.N. Mohapatra, *Phys. Rev.* **D 12**, 1502 (1975); D. Chang, R.N. Mohapatra and M.K. Parida, *Phys. Rev. Lett.* **52**, 1072 (1974); R.N. Mohapatra, University of Maryland Preprint No. UMD-PP-98-71, January 1998, [hep-ph/9801235](http://arxiv.org/abs/hep-ph/9801235).

[12] F. Csiker, *Z. Phys.* **C 49**, 129 (1991).
[13] See for instance T. Appelquist, M. Soldate, T. Takeuchi and L.C.R. Wijewardhana, TeV Physics, Proceedings of the John Hopkins workshop on current problems in particle theory 12, p. 197, Eds. G. Domokos & S. Kovesi-Domokos, Baltimore 1988, and references therein.

[14] B.A. Dobresku and J. Terning, Phys. Lett B 416, 129 (1998).

[15] F. Csikor and I. Montvay, Phys. Lett. B 324, 412 (1994).

[16] M.B. Green, J.H. Schwarz and E. Witten, Superstring Theory, Cambridge University Press 1987.

[17] I. Bars and M. Gunaydin, Phys. Rev. Lett. 45, 859 (1980); M. Koca, Phys. Lett. B 107, 73 (1981).

[18] J.C. Pati and A. Salam, Phys. Rev. Lett. 31, 661 (1973); Phys. Rev. D 8, 1240 (1973).

[19] J. Bagger, S. Dimopoulos, E. Masso and M.H. Reno, Nucl. Phys. B 258, 565 (1985).

[20] S.P. Mikheev and A.Yu. Smirnov, Sov. J. Nucl. Phys. 42, 913 (1985); L. Wolfenstein, Phys. Rev. D 17, 2369 (1987).