Rall gauge trajectory, macro and micro profile of tracks

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Abstract. The article discusses some of the possibilities of formalizing (creating a digital clone) of one of the most important subsystems of the railway – rail gauge. It is proposed to evaluate it from the point of view of the safety of the wheel – rail interaction.

1. The rail trajectory

For the trajectory of the rail [1], let’s take the position in the space of the track axis. The railroad wheel track is the part of the upper surface of the rail head on which it rolls. The rail trajectory is characterized by a plan and a profile, which can change under the influence of passing trains. The track surface “abounds” with various deviations from the norms, which in the profile are called irregularities (subsidence, distortions), and in terms of “straightening”. The abstraction of these deviations is the trajectory of the rail.

In [1], “the rail trajectory” – (RT-1) is the intersection of the calculated plane with the working face of the rail head. “The horizontal longitudinal plane located 10 mm below the surface of the wheel rolling along the rail is taken as the calculated one.” As you can see, the concept of “rail trajectory” – “RT-1” differs from that adopted in this text by a height lower by 10 mm and is shifted into the track by half the width of the rail head. “RT-1” is easy to build based on the results of the work of the car – the TsNII-4MD track measuring station or a hardware-software complex developed at the Novosibirsk State University of Railway Engineering – Agro-industrial complex “Profile”.

TsNII-4MD measures the geometry of rail tracks and its changes in time, lateral wear and roughness on the track. The maximum speed of the car is 140 km/h. The measurements are continuous taking into account the dynamic load on the lines of the rails. AIC “Profile” weighs 45 kg and measurements are made with a pedestrian speed of 5÷7 km/h, i.e. no dynamic load.

With the participation of the authors at the Department of “Path and Railway Construction” of Ural State Transport University, algorithms and computer programs have been developed that make it possible to construct rail trajectories using the measurement results of TsNII-4 and AIC “Profile” (Figure. 1).

We take the displacement of the lower point of the middle rolling circle for the trajectory of the wheel movement along the rail [5]. Unfortunately, the trajectory of the wheel movement does not always coincide within the permissible limits with the rail trajectory. This discrepancy can be considered as a criterion for the quality of the railway track.

The contact of the wheel with the rail can be one and two-point (usually in curved sections of the track). In the first approximation, we accept a single point contact.
Figure 1. Algorithm for calculating [1] the rail trajectory according to the results of monitoring the track of TsNII-4 and the AIC “Profile”.

2. Tracks of railway wheels
As already mentioned, the track surface “abounds” with various deviations from the norm, which we will call “irregularities”. The abstraction of these irregularities is the "rail trajectory". A specific section of such a track will be considered a realization of a random surface. The successive sum of these implementations represents the topography of the track, which is the primary concept for determining indicators or patterns of irregularities. The “topography” of the track, from the condition of the wheel moving along the rail, can be supplemented with parameters such as elastic deformations of the rail, coefficient of friction and adhesion.

A longitudinal section of the surface of a track is called a profile. The totality of the implementation of such sections is a longitudinal profile, which characterizes the profile of the track surface as a random process.

For specifics, of all possible longitudinal sections, for the profile, we take the section along the vertical axis of the rail. A two-track profile will be considered as a vector (or multidimensional) random
process. For specific models, the track surface topography is represented in the form of a grid, which corresponds to a discrete representation of a random process.

The profile and topography of the rail track are taken depending on the linear coordinate (CM, PC+), i.e. as a function of distance – S. The profile can be represented as a regular, normal stationary process. In general, this process is not stationary, because it has a tendency to develop in time on a certain segment of it. When the observed changes fit into the accuracy of the calculation, it can be considered stationary.

The topography can be represented as a homogeneous isotropic normal two-dimensional scalar random field. This field can be “built” according to the results of video recording of the contact when the wheel moves along the rail [11].

Actually, the topography of the track is an anisotropic field: along and across its form and dynamics of change are often not the same. But, as a first approximation, for simplicity we take it isotropic.

If the profile (topography) and the plan are taken depending not on distance, but on time, then we will call such a function “disturbance” - f(t), where t is time.

If the speed of the train (crew) is not constant, then the disturbances, in order for the process to remain stationary, should be applied in the function not of time, but of the path φ(s).

In problems in studies of dynamic loads, when determining disturbances, instead of a topography (profile), one should use a microtopography (microprofile), which differs from the macroprofile in that it does not have the lowest-frequency component: descents, rises, i.e. as a first approximation, macroprofile elements are not taken into account over the existing rail head, the length of which usually exceeds 100 m: for railroad design feeds, depending on the category of railroad, the mode of train movement and its length - 200÷400 m [2]. The recommended length of the longitudinal macroprofile is not less than half the length of the train.

In case of two-point contact, the topography of the lateral surface of the rail, which is formed due to friction of the wheel flange against the lateral plane of the rail, should be kept in mind. As the traffic flow increases, this topography becomes more “obvious”. Periodic rail grinding “smoothes” it. The same thing happens with the topography when grinding along the track. The combination of these two processes is called volume grinding of the rail head.

3. Microprofile

The railway profile, as mentioned, can be divided into two components: macroprofile and microprofile. The macroprofile is reflected in a document called “Longitudinal Rail Profile”. The longitudinal profile of the railway may consist of a "straight longitudinal profile" (elements, as already mentioned, more than 100 m, of "curved longitudinal profile" - the length of the elements 25÷100 m). For a smoother “articulation”, adjacent elements of the longitudinal profile are connected between themselves with a curve in a vertical plane – a vertical curve, the radius of which, depending on the speed of the train, can be in the range of 3-20 thousand meters.

The influence of the vertical curve on the conditions of train movement can be seen in [9].

A macroprofile consisting of long smooth irregularities (wavelengths greater than 100m) often does not cause the railway crew to oscillate on springs, but it very much affects the train operating mode, longitudinal forces in the coupler, at wheel-rail contact points, i.e. on the dynamics of the railway element.

However, during the passage of fractures of the longitudinal profile, which the train overcomes along a vertical curve, vertical centrifugal forces arise (permissible vertical accelerations $a_\theta \leq 0,1÷0,4$ m/s$^2$), leading to vertical vibrations. Due to the possible asymmetry of spring suspension resilience and track resilience, this can contribute to other, for example, transverse oscillations of the crew.

But it is more natural to use a microprofile as a disturbance.

Such an assumption: a microprofile, and not a macroprofile, has several advantages:

- the microprofile can be considered a stationary random process with a decreasing regularity index, because it does not contain a slowly changing component, compared with the macroprofile. This is of no small importance for the statistical analysis of realizations: some segments of the microprofile,
spaced several tens of meters apart, can be considered independent. Macroprofile is a process with stationary increments.

- for a macroprofile, the difference in elevations can reach tens or even hundreds of meters, and for a microprofile the amplitude is limited to mm.

The microprofile \( f(t) \) based on its frequency composition can be found as a linear transformation \( f(t) = \mathcal{H}(t) \) of the profile \( h(t) \) performed by a filter with a rectangular transfer characteristic \( \mathcal{H}(\lambda) \):

\[
\mathcal{H}(\lambda) = \begin{cases} 
0, & \text{at } |\lambda| > \lambda_{\max} \\
1, & \text{at } \lambda_{\min} < |\lambda| < \lambda_{\max} \\
0, & \text{at } |\lambda| < \lambda_{\min}
\end{cases}
\]

\[ \lambda_{\min} = \frac{2\pi}{L_{\max}}; \quad \lambda_{\max} = \frac{2\pi}{L_{\min}} \]

Thus, without distortion, the filter passes a section of road frequencies

\[ \lambda_{\min} < |\lambda| < \lambda_{\max} \]

With such a determination of the microprofile, \( L_{\min} \) and \( L_{\max} \) are taken according to the actual assessment of the rail geometry state.

For example, with wave-like wear of the rail head, the lengths of irregularities are divided [2] into:

1. Short (SI) 0.03m÷0.25m;
2. Medium (MI) 0.25m÷1.5m;
3. Long (LI) 1.5m÷3.5m.

The amplitude of the wave (the depth of the irregularity) is 2.5 mm. Oscillations of the wheel will depend [5] on the speed of the train. The higher the speed, the shorter the period and the greater the frequency.

The \( \mathcal{H}(\lambda) \) filter is easier to receive not with a rectangular transfer characteristic, but with a simple fractional rational transfer characteristic.

The \( \mathcal{H}(\lambda) \) transformation is defined as the microprofile transformation, which is desirable to satisfy the following requirements:

1. Fluctuations of the crew should have minimal differences on the microprofile and profile;
2. The amplitude of the microprofile and the maximum frequency of the microprofile signal should be limited.

We find the form of the optimal transfer characteristic \( \mathcal{H}(\lambda) \) in the low-frequency region.

Let \( \Delta_i \) be the deflection of the spring suspension of the railway crew or another indicator of oscillations on the profile \( i \), and the transformation \( \bar{A} \) corresponding to the crew.

Then \( \Delta_i \Delta f_i \). We assume that \( \Delta_i \) is the same indicator of crew oscillations when moving along microprofile \( f \). Then \( \Delta f = \bar{A} f \).

Suppose that the microprofile \( f \) is fixed by some transformation \( \mathcal{H}(\lambda) \) of profile \( i \), i.e. \( f = \mathcal{H}_i \). To determine \( \mathcal{H}_i \) in the low-frequency region, we will bear in mind the limitations:

\[ |f| \leq f_{\max}; \]
\[ |\Delta_i - \Delta f| = \text{min.} \]

Here \( f_{\max} \) is a given value.

If we assume that \( i(t) \) is a normal stationary generalized random process, and \( \bar{A} \) and \( \mathcal{H} \) are linear transformations, then this variational problem can be solved relatively easily, see, for example, [8].

The microprofile can conditionally be:
1. “Static”, the “recording” of which is performed when the axle load that is close to zero and the movement speed along rail gauge is $< 7 \, \text{km/h}$ (pedestrian speed), i.e. the dynamic impact on the track is excluded, for example, a track measuring trolley with continuous recording and the use of satellite navigation AIC "Profile".

2. “Dynamic”, the “recording” of which is carried out by the crew with a load on the axles and speeds $V \geq 50÷80 \, \text{km/h}$. (for example TsNII-4).

By “recording” the microprofile, one can determine its spectral density or other statistical characteristics, as well as use as a disturbance in the simulation to determine the wheel-rail interaction forces, for example, in the UM computer complex.

To analyze deviations, the profile under consideration for each line of rail is preliminarily divided into homogeneous (“calculated”) “segments” so that within their limits the profile could be considered an implementation of a normal stationary process.

In this case, as already mentioned, it is possible to consider the static microprofile, in which the influence of the stiffness (elasticity) of the track and the dynamic, on which the stiffness (elasticity) can have a significant effect, are excluded. A dynamic microprofile characterizes the uniformity of the track in terms of stiffness (elasticity).

The "nature" of a dynamic microprofile depends on the existing stiffness and geometry by the load on the axle and the speed of the train.

As the studies of our department [4] showed, the movement of the train is often carried out with non-gradual speed: in some directions, the probability of the train moving at a constant speed in a section 0.5 km long is 0.19. The criterion for the uneven movement of the train, as shown in [4], is "acceleration noise". Thus, when dividing into homogeneous (calculated) "segments", acceleration noise should be taken into account.

The section is divided into homogeneous (calculated) segments with sufficient length to calculate the statistical characteristics of each "segment".

It should be noted that the characteristic of the microprofile of individual “homogeneous statistical segments” can be attributed to the primary ones. Secondary characteristics can be attributed to parameters (characteristics) that determine the alternation of these segments, their length and the distribution of individual deviations on them.

When calculating the statistical characteristics of the “homogeneous segment” microprofile, its length can be from several hundred meters to several kilometers. If the process is considered normal, then the primary statistical characteristics may be limited by estimates of the correlation function or spectral density. When testing the hypothesis of normality and stationarity, other statistical characteristics should be found: mathematical expectation, distribution functions, moments of higher orders, etc.

Today, there are practically no data on the lengths of homogeneous segments, their interchangeability (alternation).

4. The spectral density of the profile

The wavelength of the “irregularities” of the macroprofile, depending on the terrain, can range from 200 m (for roads of categories I and II) to several km. The spectral density of the railway profile with long "irregularities" is approximately expressed

$$\varphi_\lambda(\lambda)=D_m \lambda^{-2} \quad (4)$$

Here $D_m$ is a coefficient depending on the terrain where the railway is traced. Let the increment of the mark of the macroprofile of the railway $\Delta h(L)$ over the mark of the nearest benchmark at a distance $L$.

Then the dispersion of this increment, taking into account (4), is determined:

$$D_{\Delta h}(L)=M\{|\Delta h|^2\} = \frac{2D_m L}{2\pi} \int_{-\infty}^{+\infty} \frac{\sin^2 \left(\frac{\lambda L}{2}\right)}{\left(\frac{\lambda L}{2}\right)^2} d\lambda = D_m L \quad (5)$$
The coefficient $D_m$ can be found by setting for the railways of the considered category, approximate values of $D_{\Delta h}(L)$ for various $L$.

$$D_m \approx \frac{1}{L} D_{\Delta h}(L)$$

(6)

Given that the maximum value is approximately three rms, then:

$$\frac{\Delta h_{max}}{3} \approx \sqrt{D_m L}$$

or

$$D_m = 0.1 \left( \frac{\Delta h_{max}}{L} \right)^2$$

(7)

Taking into account (4), it is possible to obtain from (7) $\Delta h_{max}$ for various values of $L$. With pronounced ups and downs in the region of “short irregularities” of the macroprofile with a wavelength of up to 200 m (on existing railways to the PC up to 100m) spectral density is determined:

$$\phi_1(\lambda) \approx \frac{D_4}{\lambda^4}$$

If shorter waves are excluded in the macroprofile, then the spectral density in the region of the remaining frequencies of the macroprofile can be set:

$$\phi_1(\lambda) = \frac{D_m \lambda^2}{\lambda^2 (\lambda^2 + \gamma_m^2)}$$

(8)

Here $D_m \lambda_m^2 = D_4$

The slope of the macroprofile element $i$ is determined

$$i(l) = \frac{d h(l)}{d l}$$

Then the spectral density of the slope (i) of the element of the longitudinal profile:

$$\phi(i) = \frac{D_m \lambda^2}{(\lambda^2 + \gamma_m^2)}$$

(9)

And its dispersion

$$D_i = M[\lambda^2] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_1(\lambda) d\lambda = \frac{D_m \lambda_m}{2}$$

(10)

If we evaluate $D_i$ for various railways, then

$$\lambda_m \approx \frac{2D_i}{D_m}$$

or

$$\lambda_m \approx 0.2 \left( \frac{\lambda_{max}}{D_m} \right)^2$$

(11)

(12)

Figure 2 shows the spectral densities of railways with different topography.

In the range of short and medium irregularities of the macroprofile:

1.2 – railways, I, II and III categories.

In the range of long macroprofile irregularities:

1 – flat terrain ($D_m = 10^{-4} \div 10^{-3}$m; $\Delta h_{max} = 0 \div 3$ m)

2 – hilly terrain ($D_m = 10^{-2} \div 10^{-1}$m; $\Delta h_{max} \leq 10 \div 30$ m).

If the wavelength is more than 1 km, the spectral density value depends practically only on the topography of the terrain, and not on the category of railway: In hilly terrain, reducing of the spectral density of the railway in comparison with the spectral density of the topography (surface) of the earth is carried out by appropriate laying of the railway track.
Figure 2. Spectral densities of the macroprofile of the railway with a different topography.

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