Dissipative phenomena in chemically non-equilibrated quark gluon plasma

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Abstract

The dissipative corrections to the hydrodynamic equations describing the evolution of energy-momentum tensor and parton densities are derived in a simple way using the scaling approximation for the expanding quark gluon plasma at finite baryon density. This procedure has been extended to study the process of chemical equilibration using a set of rate equations appropriate for a viscous quark gluon plasma. It is found that in the presence of dissipation, the temperature of the plasma evolves slower, whereas the quark and gluon fugacities evolve faster than their counterparts in the ideal case without viscosity.

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1. INTRODUCTION

A phase transition from normal nuclear matter to a quark gluon plasma (QGP) is expected in ultrarelativistic heavy ion collisions planned to be carried at the future colliders of RHIC and LHC. If the QGP is formed during the collision, according to the usual scenario, it is thought that a local thermal equilibrium will set in over a typical time scale of 1 fm/c \cite{1}. The plasma would then expand hydrodynamically and cool down until it reaches a temperature $T_c$ when the confinement phase transition starts. However, some recent studies \cite{2,3} have suggested that the plasma may attain kinetic equilibrium in a very short time scale of $\tau = 0.2 - 0.7$ fm/c, but it may not be chemically equilibrated. Earlier theoretical studies \cite{4,5} have revealed that a chemically non-equilibrated plasma cools faster than the equilibrated plasma, when the plasma is assumed to be ideal i.e. non dissipative. But in a realistic scenario the dissipative phenomena, in principle, should play an important role \cite{7}. It needs to be mentioned here that the dissipative effects in fluid hydrodynamics can be
neglected when the Reynold’s number ($\approx L/\lambda$, L is the dimension of the medium and $\lambda$ is the mean free path of the constituents) is quite large. However, in nucleus-nucleus collisions, as the dimension of the plasma is not large compared to the mean free path of the partons, dissipative effects are expected to be important during the hydrodynamic expansion of the plasma. The formalism of dissipative hydrodynamics is, in general, complicated since the form of dissipative corrections depends on the choice of the rest frame [7–10]. The effects of dissipative phenomena in quark gluon plasma have been studied in ref [7], where the results were reported for a chemically equilibrated baryon-free plasma. It was shown that the dissipative effect enhances the entropy production leading to a dynamical path intermediate between idealized isentropic and isoergic ones. The energy density and temperature of the plasma also decrease at slower rate with the inclusion of dissipative effects. This would lead to greater yields of direct probes such as photons and dileptons, which are sensitive to the thermal history of the reaction. However, the hydrodynamic expansion for a viscous quark gluon plasma undergoing chemical equilibration either at zero or at finite baryon density has not been investigated so far.

In this work, we study the dissipative effects in quark gluon plasma with an emphasis on the results at finite baryon density. We derive a set of hydrodynamical equations for the evolution of the energy-momentum tensor and parton densities based on simple scaling laws. This set of equations has been applied to study the hydrodynamical evolution of the viscous plasma undergoing chemical equilibration. It is found that due to viscous heating, the temperature evolves slower, whereas the parton fugacities evolve faster than the ideal plasma (without viscosity). This is contrary to the expectation that chemical equilibration consumes more energy which makes the plasma cool faster.

The present work has been organized as follows. In section II, we begin with a brief review of the dissipative hydrodynamics and derive a set of hydrodynamical equations using Landau definition of rest frame. Using this set of equations, the chemical equilibration of a viscous baryon-free plasma is studied in section III. Finally, in section IV, we discuss the results and present the conclusions. In the derivations, we have used simple scaling law for the dissipative corrections, for which a more formal justification has been given in Appendix A using Boltzmann equation for the evolution of the parton densities. The viscosity coefficient is shown to be proportional to $\epsilon/T$ which is also consistent with the finite temperature QCD calculations as shown in Appendix B.
II. HYDRODYNAMICS FOR A DISSIPATIVE SYSTEM

Energy dissipation in a moving fluid occurs due to internal friction (viscosity) and heat exchange between different parts (thermal conduction) of the fluid. If the mean free path of the constituent particles is comparable to the fluid dimension, the collective quantities like pressure, energy density, number density, velocity etc. may vary over a distance of mean free path. This will lead to decay of fluid kinetic energy as heat energy. In nucleus-nucleus collisions, as the dimension of the plasma is not very large compared to the mean free path of the partons, dissipative effects may be quite important [7].

In the presence of dissipation, the energy-momentum tensor $T^{ik}$ and number density $n^i$ can be decomposed into an ideal and a dissipative part [8,9]

$$T^{ik} = [(\epsilon + P)u^i u^k - Pg^{ik}] + \tau^{ik},$$

$$n^i = nu^i + \nu^i.$$  \hspace{1cm} (1)

where $\epsilon$, $P$ and $n$ are the energy density, pressure and particle number density and $u^i = \gamma(1, \vec{v})$ is four-velocity in terms of local fluid velocity $\vec{v}(x)$ and $\tau^{ik}$ and $\nu^i$ are the dissipative corrections.

The relativistic hydrodynamics is based on the local conservation laws:

$$\partial_i T^{ik} = \partial_i n^i = 0.$$ \hspace{1cm} (3)

In the present context $n^i$ represents the baryon four vector and the equation $\partial_i n^i = 0$ signifies the conservation of baryon number. However, if there is no net particle production i.e. chemical equilibration is complete, the above equation is applicable to the individual partons as well.

The form of the dissipative terms $\tau^{ik}$ and $\nu^i$ depend on the definition of what constitutes the local rest frame of the fluid. The four velocity $u^i$ should be defined such a way that in a proper frame of any given fluid element, the energy and the number density are expressible in terms of other thermodynamic quantities by the same formulae as when dissipative processes are not present. It is also necessary to specify whether $u^i$ is the velocity of energy transport or particle transport. Accordingly, there exists two definitions for the rest frame; one due to Landau and other due to Eckart. In Landau approach, $u^i$ is taken as the velocity of the energy transport so that energy three flux $T^{0i}$ vanishes in a comoving frame. The Landau definition requires $u^i \tau^{ik} = u_i \nu^i = 0$ so that $T^{00} = \epsilon$ is the energy density and $n^0 = n$ is the number density in the proper frame of the fluid. In the Eckart definition, $u^i$ is taken as
the velocity of the particle transport and the particle three current, rather than the energy
three flux vanishes in the fluid rest frame. So in the Eckart definition of rest frame, the
particle four vector can be written as $n^\mu = (n, \vec{0})$, whereas in the Landau definition of rest
frame $n^\mu = (n, \vec{v})$. Therefore, the two frames are related by a Lorentz transformation with
a boost velocity $\vec{v}/n$.

It is found that due to ill defined boost velocity [7], the energy three flux in the Eckart
frame, which involves heat conductivity $\kappa$ is not well defined as it diverges in the limit
of chemical potential $\mu \to 0$. On the other hand, in the Landau definition heat conduction
enters as a correction to baryon flux. It was shown that, despite the divergence of $\kappa$, the
correction to the baryon flux $\nu^i$ is finite [7]. We use the Landau definition for the subsequent
study of the evolution of the energy momentum tensor and parton densities of the quark
gluon plasma.

Under the assumption that the dissipative terms $\tau^{ik}$ and $\nu^i$ are of first order in the
gradients, the requirement that the entropy increases with time ($\partial_\alpha \sigma^\alpha > 0$, $\sigma^\alpha = \sigma u^\alpha - \mu \nu^\alpha/T$ with chemical potential $\mu$ and temperature $T$) leads to (in the Landau definition)

$$
\tau^{\alpha\beta} = \eta (\nabla^\alpha u^\beta + \nabla^\beta u^\alpha - \frac{2}{3} \Delta^{\alpha\beta} \nabla^\rho u^\rho) + \xi \Delta^{\alpha\beta} \nabla^\rho u^\rho, \quad (4)
$$

$$
\nu^\alpha = \kappa \left[ \frac{nT}{\epsilon + P} \right]^2 \nabla^\alpha \left[ \frac{\mu}{T} \right], \quad (5)
$$

where $\Delta^{\beta}_{\beta} = g^{\beta}_{\beta} - u^\alpha u_\beta$ and $\nabla^\alpha = \Delta^{\alpha\beta} \partial_\beta$, $\eta$ and $\xi$ are the coefficients of the shear and bulk viscosities and $\kappa$ is the coefficient of heat conduction. As mentioned before, the heat conductivity does not enter in the energy flux $T^{0i}$, but rather as a finite baryon current $\nu^i \propto \partial^i (\mu/T)$ in the rest frame of the fluid. In the presence of dissipation, the conservation
equations (3) need to be solved with the above correction terms. Using one-dimensional
boost-invariant scaling law [4] and Eq. (4) for $\tau^{ik}$ the identity $u_\mu \partial_\mu T^{ik} = 0$, resulting from the
energy momentum conservation, takes the simple form [7]

$$
\frac{\partial \epsilon}{\partial \tau} + \frac{\epsilon + P}{\tau} = \frac{4}{3} \frac{\eta + \xi}{\tau^2}. \quad (6)
$$

The evaluation of $\partial_\tau \nu^i = 0$ is not straightforward as one needs to know the space time dependence of $\mu/T$ and also care has to be taken so that $\nu^i$ is finite although $\kappa$ diverges in the limit $\mu \to 0$ [4]. However, we will follow a simple procedure to obtain the evolution of the baryon
density directly without invoking the baryon current (hence the conductivity) as given by
Eq. (5). It may be mentioned here that in addition to the baryon number conservation, the
parton densities (quark, anti-quark and gluon) also evolve satisfying the relation $\partial_t n^i = 0$ in absence of any particle production. Clearly, the evolution of the parton densities will also get modified due to the dissipative corrections. We estimate these dissipative corrections in a simple way from the following considerations. Since the shear viscosity scales as $T^3$, while the bulk viscosity $\xi$ vanishes for a quark gluon plasma, one can define an acceptable range for the shear viscosity $\eta$ given by [7]:

$$2T^3 \leq \eta \leq \frac{1}{4} \epsilon \tau.$$  

(7)

so that Navier-Stokes scaling theory is applicable to the expansion of the plasma. A baryon-free non-viscous plasma ($\mu=0$, $\eta=0$) leads to $\epsilon \tau^{4/3} = \epsilon_0 \tau_0^{4/3}$. Using the upper bound of viscosity as $\eta = \epsilon \tau / 4$ and substituting $\epsilon = 3P$ in Eq.(6), we get $\epsilon \tau = \epsilon_0 \tau_0$. This means that $T$ scales as $\tau^{-1/3}$ for $\eta = 0$ and as $\tau^{-1/4}$ for $\eta$ approaching the upper bound. Therefore, it is reasonable to assume that $T$ scales as $\tau^{-\gamma}$ for any value of $\eta$ in the range $1/3 \leq \gamma \leq 1/4$.

Since $\epsilon$ scales as $\tau^{-4\gamma}$, the LHS of Eq.(6) can be expressed in terms of $\gamma$ as,

$$\frac{\partial \epsilon}{\partial \tau} + \epsilon + \frac{P}{\tau} = \frac{4}{3}(1 - 3\gamma)\frac{\epsilon}{\tau}.$$  

(8)

On similar grounds, the evolution of parton density $n_i$ ($i=q, \bar{q}, g$) can be described in one dimension as (since $n_i$ scales as $\tau^{-3\gamma}$),

$$\frac{\partial n_i}{\partial \tau} + \frac{n_i}{\tau} = (1 - 3\gamma)\frac{n_i}{\tau}.$$  

(9)

For an ideal plasma, $\gamma = 1/3$ and the RHS of Eqs. (8) and (9) vanish, while for other values of $\gamma$ the RHS is non-zero which can be interpreted as the correction due to dissipation. Comparing RHS of Eq.(8) with Eq.(6), we can identify

$$\eta = (1 - 3\gamma)\epsilon \tau.$$  

It is also interesting to note that $\eta = \epsilon \tau / 4$ for $\gamma = 1/4$ is consistent with the upper limit of viscosity. In terms of $\eta$, Eq. (9) can be written as

$$\frac{\partial n_i}{\partial \tau} + \frac{n_i}{\tau} = \frac{\eta n_i}{\epsilon \tau^2}.$$  

(10)

The dissipative correction appearing in Eq. (10) is derived for a baryon-free plasma. However, as shown in Appendix A, the above correction is also valid for $\mu \neq 0$ if $\epsilon$ and $n_q(\bar{q})$ are defined appropriately for a baryon-rich plasma. Finally, the evolution of baryon density can be obtained from Eqs.(9) and (10) as

$$\frac{\partial n_B}{\partial \tau} + \frac{n_B}{\tau} = (1 - 3\gamma)\frac{n_B}{\tau} = \frac{\eta n_B}{\epsilon \tau^2}.$$  

(11)
where \( n_B = n_q - n_{\bar{q}} \). It may be noted here that although the baryon conductivity vanishes for \( \mu = 0 \), the dissipative corrections for the quark and anti-quark densities are finite as seen from Eq.(10). It may be mentioned here that for a baryon-symmetric matter, the energy momentum equation given by Eq.(6) is sufficient to describe the dynamical evolution of the plasma. However, Eq.(10) has been derived for consistency and will be useful to study the density evolution in presence of particle production particularly when the chemical equilibration is not complete.

The dynamical evolution of a baryon-rich plasma can be studied by solving Eqs.(6) and (11) numerically. For the plasma phase, we consider only the u and d quarks for which the energy and baryon densities are given by

\[
\epsilon = T^4 \left( \frac{37\pi^2}{30} + 3x^2 + \frac{3}{2\pi^2}x^4 \right),
\]

\[
n_B = \frac{2}{3} T^3 (x + \frac{x^3}{\pi^2}).
\]

where \( x = \mu/T \). The shear viscosity \( \eta \) depends on both \( T \) and \( \mu \) in a complicated way \[11\]. However, as shown in Appendix B, we can approximate \( \eta \approx \eta_0 \epsilon/T \) which scales as \( T^3 \) in case of a baryon free plasma. It is found that \( \eta_0 \) weakly depends on \( T \) and \( \mu \). Therefore, we will treat it as a constant parameter for the subsequent calculations.

Figures. 1(a) and 1(b) show the time evolution of the ratio of energy density \( \epsilon/\epsilon_0 \) and total entropy \( (\sigma \tau)/(\sigma_0 \tau_0) \) for a plasma at \( x_0(\mu/T)=1.0 \), energy density \( \epsilon_0=9.0 \text{ GeV/fm}^3 \) (corresponding to an initial temperature of \( T_0=0.258 \text{ GeV} \) ) with different values of viscosity coefficients \( \eta_0=0.0, 0.5 \) and 1.0. As seen from the Figures 1(a) and (b), with increase in viscosity, the energy density evolves faster than that corresponding to the isentropic expansion \( (\eta_0 = 0) \) and also extra entropy is produced when the medium is viscous. Figure 1(b) also shows that most of the entropy is produced in the first few fm/c and the asymptotic value of \( \sigma \tau \) is approached quickly. So these results are consistent with the findings of \[6\] except that the present work has been extended to finite baryon density. As mentioned before, the shear viscosity may have an upper bound as given by Eq. (7). However, in the above calculations, we only carry out a parametric study to see the effect of viscosity on the evolution of the energy density and entropy production although some of the \( \eta_0 \) values may exceed the upper limit.

Figures 2(a) and 2(b) show the variation of \( T/T_0 \) and \( \sigma \tau/(\sigma_0 \tau_0) \) as a function of \( \tau \) for different baryon densities of \( x_0=0.0, 1.0 \) and 2.0 at fixed \( \eta_0=1.0 \) and \( \epsilon_0=9.0 \text{ GeV/fm}^3 \). For a given temperature \( T \), the viscosity of the plasma at finite baryon density is higher than the
baryon-free case (since $\epsilon$ increases with $\mu$). This is true even if the energy density is held fixed since the plasma will have lower initial temperature at finite baryon density (resulting in higher viscosity). Since the viscosity increases with $x$, the temperature $T$ evolves slower and more entropy is produced as compared to the baryon free case. It is also interesting to note that whatever be the value of viscosity, both $T$ and $\mu$ scale with the same powerlaw as $\tau^{-\gamma}$ (see the solid circles in Figure 2a for $\mu/\mu_0$ which overlap with the curve for $T/T_0$.) This means the ratio of $\mu$ and $T$ (i.e. $x$) remains independent of $\tau$, consistent with the assumption made in appendix A. This is true even in case of an ideal ($\eta_0 = 0$) baryon-rich plasma where both $\mu$ and $T$ scale as $\tau^{-1/3}$ i.e. both obey Bjorken’s scaling. However, we have found that $x$ decreases with $\tau$ if the hydrodynamical expansion proceeds along with the chemical equilibration [12]. The $\tau$ dependence of the chemical potential $\mu$ and the temperature $T$ will be different due to parton production during chemical equilibration.

III. EFFECT OF VISCOSITY ON CHEMICAL EQUILIBRATION FOR BARYON FREE PLASMA

The plasma in equilibration means there is no net parton production and the momentum distribution of the partons can be described either by Fermi-Dirac (FD) or by Bose-Einstein (BE) distribution function. The picture emerging from the numerical simulations of parton cascades supports the formation of a plasma at RHIC and LHC energies which is initially gluon-rich. Rapid gluon scattering at a high initial temperature will lead to substantial thermalization of the plasma in a very short time while it will be far off from chemical equilibrium. Therefore, process of chemical equilibration will proceed along with the dynamical evolution of the plasma. The process of chemical equilibration will accelerate the cooling of the plasma further and the plasma may not achieve equilibrium by the time temperature drops to the critical value [4–6]. However, all these calculations assume an ideal plasma where the effects of dissipative processes are neglected. The chemical evolution of a viscous plasma will be different than its ideal counterpart due to extra entropy production and viscous heating of the plasma.

The distribution functions for quarks (anti-quarks) and gluons appropriate for a plasma which has achieved kinetic equilibrium but is far off from chemical equilibrium are given by

$$f_q(q) = \frac{\lambda_q(q)}{1 + e^{(p+\mu)/T}}; \quad f_g = \frac{\lambda_g}{e^{p/T} - 1}. \quad (14)$$

where $\lambda_i$’s give the measure of the deviation of the distribution functions from the equilibrium values. The chemical equilibrium is said to be achieved when $\lambda_i \to 1$ and the baryo-chemical
potential $\mu$ attains an equilibrium value $\mu_{eq}$ consistent with the baryon current conservation. It may be mentioned here that although the plasma is dissipative, we use the FD and BE distribution functions for quarks, anti-quarks and gluons. However, the temperature and the chemical potential will evolve as per viscous hydrodynamics and will differ from their ideal counterparts. As discussed in Appendix A, the present formalism of viscous hydrodynamics is not valid for a baryon rich plasma undergoing chemical equilibration. Since $x$ depends on $\tau$, additional corrections are needed in the rate equations describing the evolution of the viscous plasma. Therefore, in the following, we have studied the process of chemical equilibration for a viscous plasma only at zero baryon density.

We restrict our considerations to the following two reaction mechanisms for the equilibration of the parton flavours $gg \leftrightarrow ggg$ and $gg \leftrightarrow q\bar{q}$. The evolution of the parton densities are governed by the master equations

\begin{align}
\partial_\mu(n_\mu^g) &= (R_{2 \to 3} - R_{3 \to 2}) - (R_{g \to q} - R_{q \to g}), \\
\partial_\mu(n_\mu^q) &= \partial_\mu(n_\mu^{\bar{q}}) = (R_{g \to q} - R_{q \to g}),
\end{align}

where $R_{2 \to 3}$ and $R_{3 \to 2}$ denote the rates for the process $gg \to ggg$ and its reverse, and $R_{g \to q}$ and $R_{q \to g}$ are for the process $gg \to q\bar{q}$ and its reverse respectively. Note that the density four vector $n^i = nu^i + \nu^i$ now includes a dissipative correction $\nu^i$. The rate equations are in general quite complicated to solve. For baryon-free plasma ($\mu=0$), the authors in [4], have used a simple factorization for the RHS of the above equations based on a classical approximation i.e. using the Boltzmann distribution function for quark, anti-quark and gluon and eliminating the Pauli blocking and Bose enhancement factors in the final states. However, it was shown by us recently [12] that the same factorization can be used, even by including full quantum statistics, particularly when the plasma is highly unsaturated. Using the viscous hydrodynamics as derived in the previous section and with the factorization given in Ref [4], the Eqs. (15) and (16) can be written as

\begin{align}
\frac{\partial n_g}{\partial \tau} + \frac{n_g}{\tau} &= n_g R_2 (1 - \lambda_g) - 2n_g R_3 (1 - \frac{\lambda_g \lambda_q}{\lambda_g^2}) + \frac{n_g \eta}{\epsilon \tau^2}, \\
\frac{\partial n_q}{\partial \tau} + \frac{n_q}{\tau} &= n_g R_2 (1 - \frac{\lambda_g \lambda_q}{\lambda_g^2}) + \frac{n_q \eta}{\epsilon \tau^2}, \\
\frac{\partial n_{\bar{q}}}{\partial \tau} + \frac{n_{\bar{q}}}{\tau} &= n_g R_2 (1 - \frac{\lambda_g \lambda_q}{\lambda_g^2}) + \frac{n_q \eta}{\epsilon \tau^2}.
\end{align}
The above set of equations differ from the corresponding equations of ref.[4] with the last terms containing the viscosity factors. The density weighted cross sections $R_3$ and $R_2$ are given by

\[ R_3 \approx 2.1 \alpha_s^2 T \sqrt{2\lambda_g - \lambda_g^2}; \quad R_2 \approx .24N_f \alpha_s^2 \lambda_g T \ln\left(\frac{1.65}{\alpha_s \lambda_g}\right). \]  

Using the distribution functions given by (14) for quarks, anti-quarks and the gluons, the energy and number densities can be written as:

\[ \epsilon = T^4[a_2\lambda_g + b_2(\lambda_q + \lambda_{\bar{q}})]. \]  

with $a_2 = \frac{8\pi^2}{15}$; $b_2 = N_f(7\pi^2/40)$; where $N_f$ is the dynamical quark flavours. Similarly, the number densities for gluon, quark and antiquark are:

\[ n_g = \lambda_g a_1 T^3; \quad a_1 = \frac{16}{\pi^2} \zeta(3), \]  

\[ n_q = \lambda_q b_1 T^3; \quad b_1 = \frac{9}{2\pi^2} \zeta(3)N_f, \]  

\[ n_{\bar{q}} = \lambda_{\bar{q}} b_1 T^3. \]  

The above set of Eqs. (17-19) along with Eq. (6) were solved using fourth order Runge-Kutta method, to obtain the time dependence of the temperature $T$, the quark and anti-quark fugacities $\lambda_g$, $\lambda_q$ (=$\lambda_{\bar{q}}$). The initial conditions were taken from the HIJING calculations both for RHIC and LHC energies [4]. We take $T_0= 0.57$ GeV, $\lambda_{g0} = 0.09$, $\lambda_{q0} = 0.02$ at $\tau_0 =0.31$ fm for RHIC energy and $T=0.83$ GeV, $\lambda_{g0} = 0.14$, $\lambda_{q0} = 0.03$ and $\tau_0 =0.23$ fm for LHC energy. Since, energy density depends on fugacities and chemical potential, viscosity will also depend on them implicitly.

Figures (3) and (4) show the variation of $T$, $\lambda_g$ and $\lambda_q$ with proper time $\tau$ for $\eta_0=0.0$ and 0.2. As mentioned above, these calculations correspond to a non-equilibrated plasma at zero baryon density. In the absence of dissipation, i.e. $\eta_0=0$, the plasma cools faster than that predicted by the Bjorken scaling (see the curves with solid circles in Figure 3(a) and 4(a)), since additional energy is consumed in the process of chemical equilibration. A further rise in equilibration rate (i.e rise in fugacities) will result in faster cooling of the plasma. The reverse happens in presence of viscosity. As seen from the Figures (3) and (4), during chemical equilibration, the quark and gluon equilibration rates become faster when the medium becomes more viscous. This, however, does not result in a faster cooling of the plasma as expected. So in the case of a viscous plasma, the chemical equilibration
becomes faster and the temperature drops more slowly due to generation of heat. Since the presence of viscosity makes the equilibration faster and also increases the life time of the plasma phase, it would be interesting to see whether chemical equilibration can be achieved when the medium is viscous. While this requires the precise knowledge of viscosity, the short-dashed curve in Figures (3) and (4) shows the result that can achieved with the upper limit of viscosity $\eta = (\epsilon \tau)/4$ (i.e. $\eta_0 = \frac{\tau_0}{4}$). These calculations show that even for the limiting case, the temperature may still drop faster than the Bjorken scaling and the plasma may not achieve chemical equilibration by the time $T$ drops to $T_c \approx 0.2$ GeV, provided the initial conditions taken from HIJING calculations are prevalent at RHIC and LHC energies.

The above results are obtained for the chemical evolution of a viscous plasma at zero baryon density. In a recent paper, we have shown that in case of a non-viscous plasma at finite baryon density, the rate of chemical equilibration slows down particularly for gluons resulting in slower cooling of the plasma. Therefore, in the presence of baryon density, the cooling rate of the viscous plasma is expected to slow down further. However, it is difficult to conclude about the net gluon equilibration rate since it is suppressed at finite baryon density.

**IV. SUMMARY AND CONCLUSION**

We have studied the properties of a viscous quark gluon plasma at finite baryon density. A set of equations that describes the space time evolution of the energy-momentum and parton densities has been derived based on a simple scaling law as well as by solving Boltzmann approximation under boost invariant scenario. The total entropy is conserved in case of an ideal plasma and the temperature and chemical potential evolve as per the Bjorken’s scaling. However, for a viscous plasma additional entropy is produced due to viscosity both at zero and finite baryon density. In fact, finite baryon density makes the plasma more viscous as energy density increases with $\mu$. Even at a fixed energy density, viscosity rises due to the lower initial temperature. Therefore, plasma at finite baryon density produces more entropy as compared to the baryon free case. We have also studied the chemical equilibration of the plasma at zero baryon density using the rate equations appropriate for a viscous plasma. It is found that due to viscosity, the temperature evolves more slowly, while the fugacities evolve faster than the ideal plasma. The slow cooling of the plasma and the fast equilibration rates of gluon and quark (anti-quark) will affect the thermal photon and di-lepton yields which are the important probes to study the dynamical evolution of the quark gluon plasma. The thermal open charm production will also be enhanced particularly at high $P_T$ or at high
invariant mass regions. The additional entropy which is generated due to viscosity will also
reflect on an increased multiplicity distributions of the final particles. Further studies in
this direction will be important to understand the behavior of the dynamical evolution of
the quark gluon plasma produced in relativistic heavy ion collisions.

To conclude, larger transverse momentum associated with the hydrodynamic expansion
would be reduced as collective flow velocities are dissipated into heat. The dissipative effects
may dampen the fluctuations which otherwise are expected to serve as signatures of unusual
phenomena of QGP phase transitions. Since most proposed observables of the plasma are
sensitive to the full space-time history of the reaction, dissipative phenomena must be taken
into account if quantitative predictions are to be made. In future, we plan to estimate the
dilepton and photon yields from QGP phase with the inclusion of these dissipative effects.

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Appendix A

The dissipative corrections $\tau^{ik}$ and $\nu^i$ in Eqs. (4) and (5) can be evaluated using the
Boltzmann equation $p^\mu \partial_\mu f_a = C_a(f)$, which describes the evolution of the Wigner densities
$f_a(x,p)$ of the parton of type $a$ in terms of the collision integrals $C_a$. At equilibrium,
the collision integral $C(f)$ vanishes and the distribution function $f_0(p,\mu,T)$ becomes Bose-
Einstein or Fermi-Dirac type. However, in the presence of dissipation, the hydrodynamical
solution assumes only local equilibrium where

$$f_H = f_0(p_\mu u^\mu(x),\mu(x),T(x)). \quad (A.1)$$

with the four velocity $u$, chemical potential $\mu$ and temperature $T$ being functions of $x_\mu$.
However, $p^\mu \partial_\mu f_H \neq 0$, while $C(f_H) = 0$. So $f_H$ cannot be a solution of the Boltzmann
equation and there has to be a correction to $f_H$ by an amount $\delta f$ that is first order in
gradients of $u$, $\mu$ and $T$. Writing $f = f_H \pm \delta f$, the Boltzmann equation can be written as

$$p^\mu \partial_\mu f_H = p^\alpha u_\alpha \frac{\delta f}{\tau_c}. \quad (A.2)$$

where $\tau_c$ is the relaxation time which depends on time. In terms of $\delta f$, the dissipative
corrections are given by [1]
\[ \tau^{ik} = \int \frac{d^3p}{(2\pi)^3} \frac{p^i p^k}{p^0} (\delta f_g + \delta f_q + \delta f_{\bar{q}}), \quad (A.3) \]

\[ \nu^i = \int \frac{d^3p}{(2\pi)^3} \frac{p^i}{p^0} (\delta f_q - \delta f_{\bar{q}}). \quad (A.4) \]

The Boltzmann equation (A.2) in (1+1) dimension has the simple form:

\[ \frac{\partial f_H}{\partial t} + v_z \frac{\partial f_H}{\partial z} = \frac{\delta f}{\tau_c}. \quad (A.5) \]

where \( v_z \) is the parton velocity. As in [13], we further assume that the central rapidity regime is initially approximately Lorentz invariant under longitudinal boost so that the conditions at point \( z \) at time \( t \) are the same as those at \( z = 0 \) at proper time \( \tau = (t^2 - z^2)^{1/2} \). Hence the left-hand side of Eq. (A.5) at \( z = 0 \) becomes simply the time derivative at constant \( p_z t \)

\[ \frac{\partial f_H(p_\perp, p_z, t)}{\partial t} \bigg|_{p_z t} = \frac{\delta f(p_\perp, p_z, t)}{\tau_c}. \quad (A.6) \]

Therefore, the integral equations A(3) and A(4) can be evaluated using \( \delta f \) given by

\[ \delta f = \tau_c \frac{\partial f_H(p_\perp, p_z, t)}{\partial t} \bigg|_{p_z t} \]

\[ = -\tau_c f_0 \bar{f}_0 \left( \frac{p}{T} \right) \frac{\partial}{\partial t} \bigg|_{p_z t}. \quad (A.7) \]

where \( \bar{f}_0 = (1 \mp f_0) \) for fermions and bosons. An ideal plasma in one dimension expands following the Bjorken’s scaling where the temperature \( T \) decreases as \( t^{-1/3} \). In presence of dissipation, there is an extra entropy production and the plasma cools slowly. However, we can still assume a power law dependence for \( T \propto t^{-\gamma} \) where \( 1/4 \leq \gamma \leq 1/3 \). Thus, the time derivative of \( (p/T) \) can be evaluated keeping \( p_z t \) constant

\[ \delta f = \tau_c f_0 \bar{f}_0 \frac{p}{T t} (\cos^2 \theta - \gamma). \quad (A.8) \]

where \( p_z = p \cos \theta \) and \( p^2 = p_\perp^2 + p_z^2 \). Here we have assumed a baryon-free plasma ( \( \mu = 0 \)). However, for finite baryon density, \( p \) in Eq. (A.7) needs to be replaced by \( (p \pm \mu) \) for anti-quark or quark distribution function respectively. Consequently, there will be an extra contribution arising from the term \( \partial x/\partial t \) where \( x = \mu/T \). In case of a non-viscous plasma, by solving the equations for the conservation of entropy (\( \sigma t = \text{const} \)) and of baryon number (\( n_B t = \text{const} \)), it can be shown that both \( \mu \) and \( T \) decrease as \( t^{-1/3} \) even though the
plasma is baryon-rich. This implies that the ratio \( x \) remains independent of \( t \). Therefore, it is reasonable to assume that \( x \) will also remain independent of \( t \) even for a viscous plasma at finite baryon density. This would mean both \( \mu \) and \( T \) should follow the same power law as \( t^{-\gamma} \). In fact, this assumption is consistent with the results that we find in section II (see solid circles for \( \mu/\mu_0 \) in figure 2a). Since \( x \) does not depend on \( t \), there would not be any extra correction and Eq. (A.8) can be used for \( \delta f \) with distribution function \( f_0 \) (for the quark and anti-quark) defined appropriately for a plasma at finite baryon density. The above assumption is not valid for a plasma undergoing chemical equilibration. It is shown in a recent work \[12\] that \( x \) decreases with \( t \) due to parton production during chemical equilibration, although the time dependence can be neglected for small values of \( x \). Since \( x \) varies with \( t \), Eq. (A.8) is not valid in case of a chemically equilibrating plasma at finite baryon density except when \( x = 0 \) or very small.

Using \( \delta f \) given by Eq. A.8 and BE or FD distribution for \( f_0 \), the dissipative corrections for baryon-rich plasma, can be evaluated from Eqs. (A.3) and (A.4). Consider the case \( i = k = 0 \), for which \( \tau^{00} \) and \( \nu^0 \) can be written as

\[
\tau^{00} = \frac{1}{T t} \int \frac{d\phi}{(2\pi)^3} \int p^4 \left( f_q \bar{f}_g + f_q \bar{f}_q + f_q \bar{f}_q \right) dp \int d\theta \left( \cos^2 \theta - \gamma \right) \sin \theta, \tag{A.9}
\]

\[
\nu^0 = \frac{1}{T t} \int \frac{d\phi}{(2\pi)^3} \int p^3 \left( f_q \bar{f}_q - f_q \bar{f}_q \right) dp \int d\theta \left( \cos^2 \theta - \gamma \right) \sin \theta. \tag{A.10}
\]

Replacing the \( \theta \) integration by \((2/3)(1 - 3\gamma)\) and using

\[
\int_0^\infty p^n f_0 \bar{f}_0 dp = nT \int_0^\infty p^{n-1} f_0 dp,
\]

Eqs. (A.9) and (A.10) can be written as

\[
\tau^{00} = \frac{4\tau_c}{3t} (1 - 3\gamma) \epsilon, \tag{A.11}
\]

\[
\nu^0 = \frac{\tau_c}{t} (1 - 3\gamma) (n_q - n_{\bar{q}}). \tag{A.12}
\]

where \( \epsilon = \epsilon_q + \epsilon_{\bar{q}} + \epsilon_g \) is the total energy density and \( n_q \), \( n_{\bar{q}} \) are the quark and anti-quark densities. Similarly, it can be shown that \( \tau^{0z} \) and \( \nu^z \) will vanish due to vanishing \( \theta \) integration. Recall that the terms which need to be added to the hydrodynamic Eqs. (3) are \( u_k \partial_i \tau^{ik} \) and \( \partial_i \nu^i \). Since, we consider the expansion in (1+1) dimension around the central rapidity regime at \( z = 0 \), the terms which are of interest to us are \( \partial_0 \tau^{00} \) and \( \partial_0 \nu^0 \). Although, gluons equilibrate much faster than the quarks and anti-quarks, we will assume an effective
relaxation time for gluon, quark and anti-quark given by $\tau_c^{-1} \propto T^{2} \alpha_s \ln(\alpha_s^{-1})$ [14]. Neglecting time dependence of $\alpha_s$ one can have

$$\partial_0 \tau^{00} = -\frac{4}{3}(3\gamma + 1)(1 - 3\gamma) \frac{\epsilon}{T^2 l^2},$$

(A.13)

$$\partial_0 \nu^0 = -(2\gamma + 1)(1 - 3\gamma) \frac{n_B}{T l^2}.$$  

(A.14)

Therefore, in the presence of dissipation, terms proportional to $\epsilon/(T l^2)$ and $n_B/(T l^2)$ are to be added to the Eq. (3) describing the evolution of the energy momentum and parton densities. It is also interesting to note that for an ideal plasma $\gamma = 1/3$ so that the dissipative correction vanishes. However, comparing Eq. A(13) with RHS of Eq. (6) which has been derived from Eq.(4) directly, we can write

$$\partial_0 \tau^{00} = -\frac{4}{3} \eta \frac{\epsilon}{l^2},$$

(A.15)

where $\eta = (3\gamma + 1)(1 - 3\gamma)\epsilon/T$.

Similarly, in terms of $\eta$, we can have

$$\partial_0 \nu_0 = -C \frac{n_B \eta}{\epsilon l^2}.$$  

(A.16)

Therefore, we can write the following evolution equations in terms of $\eta$

$$\frac{\partial \epsilon}{\partial t} + \frac{\epsilon + P}{t} = \frac{4}{3} \eta,$$

(A.17)

$$\frac{\partial n_B}{\partial t} + \frac{n_B}{t} = \frac{C n_B \eta}{\epsilon l^2}.$$  

(A.18)

where $C = (2\gamma + 1)/(3\gamma + 1)$. It is also possible to write the density evolution for quarks and anti-quarks separately as:

$$\frac{\partial n_q}{\partial t} + \frac{n_q}{t} = \frac{C n_q \eta}{\epsilon l^2},$$

(A.19)

$$\frac{\partial n_{\bar{q}}}{\partial t} + \frac{n_{\bar{q}}}{t} = \frac{C n_{\bar{q}} \eta}{\epsilon l^2}.$$  

(A.20)

The above equations have the same form as that of Eqs. (6), (10) and (11). The value of $C$ lies between 0.83 and 0.86 depending on the value of $\gamma$. Ideally, we expect $C=1$ from the requirement that in the absence of baryon density all the Eqs. A(17)-A(20) should give same solution for temperature. This deviation ($C < 1$) arises due to various approximations that are used in deriving $\delta f$. For example, we have used an effective relaxation time $\tau_c$ both for quarks and gluons although $\tau_q \geq \tau_g$. Similarly, we use $T = At^{-\gamma}$ where $A$ is taken as constant, although it may depend on $t$. However, the deviation is not too big and we will use $C=1$ for consistency in the subsequent studies. It is also important to note that the form of viscosity $\eta \propto \epsilon/T$ is also consistent with the results of [14] which was obtained using finite theory QCD (see Appendix B).
APPENDIX B

The shear viscosity co-efficient using the relativistic kinetic theory for a massless QGP under relaxation time approximation can be written as [11,15]

\[ \eta_i = \frac{4}{15} \epsilon_i \lambda_i. \]  

(B.1)

where \( \epsilon_i (i = q, \bar{q}, g) \) is the energy density of particle type \( i \) and \( \lambda_i \) is the mean free path, which is the inverse interaction rate, \( \lambda_i = 1/\Gamma_i \). Recently, Hou and Jiarong [11] have evaluated the interaction rates for quarks and gluons as for baryon rich plasma using finite temperature QCD

\[ \Gamma_q = T \nu, \quad \Gamma_g = \frac{9}{4} T \nu. \]  

(B.2)

with

\[ \nu = \frac{16 \pi}{27} \alpha_s^2 \left( \frac{4}{3} + \frac{1}{\pi^2 x^2} \right) \left( -\ln \alpha_s + \ln \left( \frac{1}{4\pi \left( \frac{4}{3} + \frac{1}{\pi^2 x^2} \right)} + \alpha_s \right) + D \right). \]  

(B.3)

where \( D \) is the gluon damping factor given by

\[ D \approx \frac{N_c^2 \alpha_s}{4\pi \left( \frac{4}{3} + \frac{1}{\pi^2 x^2} \right)} \ln \left( \frac{\alpha_s^2}{1 + N_c^2 \alpha_s^2} \right). \]  

(B.4)

Finally, the viscosity co-efficient \( \eta \) is given by

\[ \eta_q = \frac{\epsilon_q}{T \nu}, \quad \eta_g = \frac{4 \epsilon_g}{9 T \nu}. \]  

(B.5)

Therefore, the \( \mu \) dependence of the viscosity arises through \( \epsilon_q \) and \( \nu \). We have studied \( \nu \) as a function of \( \mu \) at various temperatures \( T \) using the following expression for the running coupling constant:

\[ \alpha(T, \mu) = \frac{12\pi}{(33 - 2N_f) \ln \left( T^2 \frac{0.8x^2 + 15.622}{\Lambda_s^2} \right)}, \]  

(B.6)

where \( \Lambda_s = 0.1 \) GeV and \( N_f = 2 \). It is found that \( \nu \) does not depend on \( \mu \) very strongly. Even the variation of \( \nu \) with temperature is also not very significant. The viscosity primarily depends on \( \epsilon/T \) where \( \epsilon \) is the total energy density of the plasma. Therefore, we can write

\[ \eta = \eta_0 \frac{\epsilon}{T}, \]  

(B.7)

where \( \eta_0 \) is treated as a constant although it may depend on \( T \) and \( \mu \) rather weakly.
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FIG. 1. (a) The ratio of the energy density $\epsilon/\epsilon_0$, and (b) the ratio of the total entropy $(\sigma\tau)/(\sigma_0\tau_0)$ as a function of proper time $\tau$. 
FIG. 2. (a) The ratio of the temperature $T/T_0$ and (b) the ratio of total entropy $(\sigma\tau)/(\sigma_0\tau_0)$ as a function of proper time $\tau$. The solid circles are for $\mu/\mu_0$ as discussed in the text.
FIG. 3. The evolution of (a) temperature $T$, (b) gluon fugacity $\lambda_g$ and quark (anti-quark) fugacities $\lambda_q$ as a function of proper time $\tau$ with initial conditions $T_0=0.57$ GeV, $\tau_0=0.31$ fm, $\lambda_{g0}=0.09$ and $\lambda_{q0}=\lambda_{\bar{q}0}=0.02$ for $\eta_0=0.0$ and 0.2. The short-dashed curves represent the upper limit when $\eta = \frac{4\tau}{c}$. The curve with solid circles in (a) is the Bjorken’s limit.
FIG. 4. Same as Figure 3, but for initial conditions at LHC, i.e. $T_0=0.83$ GeV, $\tau_0=0.23$ fm, $\lambda_{q0}=0.14$ and $\lambda_{\bar{q}0}=\lambda_{q0}=0.03$. 