In this paper, a parametric method is introduced to solve fuzzy transportation problem. Considering that parameters of transportation problem have uncertainties, this paper develops a generalized fuzzy transportation problem with fuzzy supply, demand, and cost. For simplicity, these parameters are assumed to be fuzzy trapezoidal numbers. Based on possibility theory and consistent with decision-makers' subjectiveness and practical requirements, the fuzzy transportation problem is transformed to a crisp linear transportation problem by defuzzifying fuzzy constraints and objectives with application of fractile and modality approach. Finally, a numerical example is provided to exemplify the application of fuzzy transportation programming and to verify the validity of the proposed methods.
importance when a decision-maker is confronted with uncertainty or fuzziness.

This paper has five sections. Section 1 is an introduction. In section 2, some basic concepts and possibility theory are introduced. Section 3 is the main part of this paper, where we proposed fuzzy transportation problem and defuzzied it into four crisp linear programming problems by utilizing the fractile or modality approach. In section 4, a numerical example is provided to illustrate the methods developed in this paper. And section 5 concludes.

Preliminaries

In this section the basic concepts of transportation problem, fuzzy number, trapezoidal fuzzy number, possibility theory and their properties are recalled.

2.1 The crisp transportation problem

Mathematically a transportation problem can be stated as follows:

$$\min z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij}$$ (1)
subject to
\[ \sum_{i=1}^{m} c_{ij} x_{ij} = c_j, \quad j = 1, 2, \ldots, n \] (supply constraints)
\[ \sum_{j=1}^{n} x_{ij} = b_i, \quad i = 1, 2, \ldots, m \] (destination constraints) (2)

where \( c_{ij} \) is the cost of transportation of a unit from the \( i \)th source to the \( j \)th destination, and the quantity \( x_{ij} \) is some non-negative number, which is to be transported from the \( i \)th origin to the \( j \)th destination. An obvious necessary and sufficient condition for the linear programming problem given in (1) to have a solution is that the total available is equal to the total required. If it is not true, a fictitious origin or destination can be added. It should be noted that the problem has a feasible solution if and only if condition (2) is satisfied. Now the problem is to determine \( x_{ij} \), in such a way that the total transportation cost is minimum.

### 2.2 Generalized fuzzy number and its \( \alpha \)-cut

A fuzzy number is a generalization of a regular, real number in the sense that it does not refer to a single value but rather to a connected set of possible values, where each possible value has its own weight between 0 and 1. This weight is called the membership function. A fuzzy number is thus a special case of a convex, normalized fuzzy set of the real line.

Mathematically a generalized fuzzy number \( A \), conventionally represented by \( A = (a_1, a_2; \beta, \gamma) \), i.e., \([ \text{left point, right point; left spread, right spread}]\), is a normalized convex fuzzy subset on the real line \( R \) if

i) \( a_1 - \beta < a_1 \leq a_2 < a_2 + \gamma \);

ii) the membership function is of the following form:

\[
\mu_A(x) = \begin{cases} 
  f(x), & \forall x \in [a_1 - \beta, a_2] \\
  1, & \forall x \in [a_1, a_2] \\
  h(x), & \forall x \in [a_2, a_2 + \gamma]
\end{cases}
\]

where \( f(x) \) and \( h(x) \) are the monotonic increasing and decreasing functions in \([a_1 - \beta, a_2]\) and \([a_2, a_2 + \gamma]\) respectively;

iii) \( \mu_A(x) \) is an upper semi-continuous function;

iv) \( Supp(A) = \{ x \in X | \mu_A(x) > 0 \} \) is a closed and bounded interval, i.e., \([a_1 - \beta, a_2 + \gamma]\).

Specifically an LR-type fuzzy number is obtained from a generalized fuzzy number if the shape functions \( f(x) \) and \( h(x) \) are approximated by \( L \left( \frac{x - a_1}{\beta} \right) \) and \( R \left( \frac{x - a_2}{\gamma} \right) \) respectively. The \( \alpha \)-cut of \( A \) is an interval number denoted by \([ A_\alpha ] = [ A^l_\alpha, A^u_\alpha ] \), which is explicitly shown for a LR-type fuzzy number, i.e., \([ A^l_\alpha, A^u_\alpha ] = [a_1 - \beta L^{-1}(\alpha), a_2 + \gamma R^{-1}(\alpha)] \) for all \( \alpha \in [0, 1] \).

### 2.3 Trapezoidal fuzzy number

A fuzzy number \( T \) is a trapezoidal fuzzy number denoted by \( T = (t_1, t_2; \beta, \gamma) \) where \( t_1, t_2, \beta, \gamma \) are real number and its membership function \( \mu_T(x) \) is given below.

\[
\mu_T(x) = \begin{cases} 
  0, & x \leq t_1 - \beta \\
  \frac{x - t_1}{\beta}, & t_1 - \beta \leq x \leq t_1 \\
  1, & t_1 \leq x \leq t_2 \\
  \frac{x - t_2}{\gamma}, & t_2 \leq x \leq t_2 + \gamma \\
  0, & x \geq t_2 + \gamma
\end{cases}
\]

The \( \alpha \)-cut of the fuzzy number \( T \) which can be denoted by \( T_\alpha = [ T^l_\alpha, T^u_\alpha ] \), is shown in the Fig. 1.

\[
T^l_\alpha = t_1 - (1 - \alpha) \beta, \quad T^u_\alpha = t_2 + (1 - \alpha) \gamma
\]

### 2.4 Possibility theory

Possibility theory is an uncertainty theory devoted to handle incomplete information. As such, it complements probability theory. It differs from the latter by using of a pair of dual set-functions (possibility and necessity measures) instead of only one. This feature makes it easier to capture partial ignorance. Besides, it is not additive and makes sense on ordinal structures. The name Theory of Possibility was coined in Negoita and Zadeh [10], inspired by Gaines [11]. In Zadeh’s view, possibility distributions were meant to provide a graded semantics to natural language statements. However, possibility and necessity measures can also be the basis of a full-fledged representation of partial belief that
If $\alpha$ and $\beta$ are two fuzzy sets and a fuzzy event $\omega \in \alpha$, the possibility and necessity of $\omega \in \beta$ are defined as:

$$II_{\alpha}(\beta) = \sup_i \min \{\mu_{\alpha}(r), \mu_{\beta}(r)\}$$

$$N_{\alpha}(\beta) = \inf_i \max \{1 - \mu_{\alpha}(r), \mu_{\beta}(r)\}$$

If $\tilde{a}$ and $\tilde{b}$ are two fuzzy numbers and their membership function are $\mu_a(x)$ and $\mu_b(y)$ respectively, the measure of possibility and necessity are defined as:

$$\text{Pos}(\tilde{a} \ast \tilde{b}) = \sup \{\min (\mu_a(x), \mu_b(y))|x, y \in R, x \ast y\}$$

$$\text{Nes}(\tilde{a} \ast \tilde{b}) = \inf \{\max (1 - \mu_a(x), \mu_b(y))|x, y \in R, x \ast y\}$$

If $\tilde{b}$ is a crisp number, then we can achieved that

$$\text{Pos}(\tilde{a} \leq \tilde{b}) = \sup \{\mu_a(x)|x \in R, x \leq b\}$$

$$\text{Nes}(\tilde{a} \leq \tilde{b}) = \sup \{1 - \mu_a(x)|x \in R, x > b\}$$

Generally speaking, the possibility degree evaluates to what extent an event is consistent with the knowledge $\Pi$, while the necessity degree evaluates to what extent an event is certainly implied by the knowledge.

**Mathematical formulation of fuzzy transportation problem**

In this section, we first present the Fuzzy Transportation Problem (FTP). Then, based on possibility theory, treatment of constraints and objective are discussed in order to develop the four formulations of the FTP.

### 3.1 Fuzzy transportation problem

Mathematically the FTP can be described as follows.

$$\min \tilde{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij}x_{ij} \quad (4)$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, \cdots, m \quad (\text{supply constraints})$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, \cdots, n \quad (\text{destination constraints}) \quad (5)$$

In the constraints of FTP, the left side is a crisp number while the right side is a fuzzy number. Hence, the constraints produce a fuzzy set. To determine whether a crisp number is in a fuzzy set, the classical method is to use the membership function. If the membership function value of the crisp number is positive, then it can be sure that it belongs to the fuzzy set, and the larger is the value, the higher is the extent. As for the objective function, it is a linear non-negative combination of fuzzy variables. So the objective of FTP is a fuzzy variable too. To minimize the objective, a ranking method of fuzzy numbers should be introduced.

Therefore, in order to solve FTP, we should introduce a measure to determine whether a crisp number belongs to a fuzzy set and a ranking index to rank fuzzy numbers. Generally speaking, let the feasible fuzzy set of the FTP is denoted by $\tilde{X}$, and all possible objective values is in the fuzzy set $\tilde{Z}$. And there is a measure system $\mathcal{M}$ to determine whether a crisp number belongs to a fuzzy set, where $0 \leq M(a \in \tilde{X}) \leq 1$. Then the FTP can be described as follows conceptually.

$$\max \mathcal{M}(z(x) \in \tilde{Z})$$
We think that how to solve FTP depends on the decision-maker’s attitude to the fuzziness, so it is hard to define which model should be used. Decision-maker can choose any proposed method to solve FTP just according to his attitude to the fuzziness. We propose possibility theory to describe decision-maker’s attitude. As FTP consists of fuzzy constraints and objective, this paper deals with them individually as shown in Figure 2. With respect to fuzzy constraints, we utilize possibility theory to convert them into crisp and bounded intervals. As for fuzzy objective, by applying fractile and modality approach we convert it into four formations from the view of necessity and possibility respectively. Then by recombining the converted constraints and objective, four models are established.

3.2 Treatment of constraints

As constraints of FTP are equations, and according to the definition of possibility and necessity, we can get

\[
\text{Pos}(\hat{a} = b) = \mu_h(b), \quad \text{Nes}(\hat{a} = b) = 0
\]

Hence, the treatment of constraints can be considered from the view of possibility exclusively.

It is reasonable and frequently-used that when decision makers are confronted with uncertainty, they can only make sure that the constraints are satisfied to some extent. For the \( f^b \) origin \( A_i \) in FTP, the decision-maker expects that the \( \sum_{j=1}^{m} x_{ij} = \hat{a}_i \) should be satisfied as possible because the production is uncertain. According to the definition of possibility, we can set that the possibility of the constraint be true is larger than a given level \( \xi \), where \( 0 \leq \xi \leq 1 \), which reflects the extent how the constraint is satisfied.

\[
\text{Pos}\left( \sum_{j=1}^{m} x_{ij} = \hat{a}_i, i = 1,2,\cdots,m \right) \geq \xi_i
\]

According to the definition of possibility, the equation (6) is depicted in the Figure 3, where \( P_1 \) and \( P_2 \) is the left and right bound of \( \hat{a}_i \)’s \( \alpha \)-cut. To ensure the equation (6) can be satisfied, the point \( O \) should be above of \( P_1 \) and \( P_2 \), which converts the constraint \( \sum_{j=1}^{m} x_{ij} = \hat{a}_i \) to

\[
a_{i1} - (1-\xi_i)L_i = a_{iL}^r \leq \sum_{j=1}^{m} x_{ij} \leq a_{iR}^l \leq (1-\xi_i)R_i, i = 1,2,\cdots,m
\]

And the possibility and necessity measure are adopted as ranking index.

### Table 1. Data of the example.

|   | \( B_1 \) | \( B_2 \) | \( B_3 \) | \( B_4 \) | \( B_5 \) | Sup. |
|---|---|---|---|---|---|---|
| \( A_1 \) | (8,9,3,2) | (2,3,1,5) | (2,5,1,6) | (10,16,4,8) | (7,12,5,5) | (22,28,20,30) |
| \( A_2 \) | (3,6,1,8) | (8,11,4,3) | (12,16,5,8) | (3,5,1,6) | (2,6,1,7) | (28,35,25,36) |
| \( A_3 \) | (13,16,5,8) | (3,8,1,10) | (10,18,4,9) | (4,10,1,1) | (12,20,5,2) | (16,24,12,28) |
| \( A_4 \) | (18,23,3,6) | (12,18,5,9) | (11,15,4,8) | (30,36,6,4) | (6,9,2,1) | (28,36,25,40) |
| Dem. | (20,30,12,30) | (22,32,15,30) | (32,40,25,40) | (12,28,6,18) | (28,36,18,40) |   |

doi:10.1371/journal.pone.0105142.t001
Similarly, for the $j$th destination $B_j$, the constraint should be satisfied with the possibility larger than $\beta_j (0 \leq \beta_j \leq 1)$, i.e.,

$$\text{Pos} \left( \sum_{i=1}^{m} x_{ij} \equiv \tilde{b} \right) \geq \beta_j, j = 1, 2, \cdots, n$$  \hspace{1cm} (8)

And it can be converted to

$$b_1 - (1 - \beta_j)L_j = b_{\tilde{b}_j} \leq \sum_{i=1}^{m} x_{ij} \leq b_{\tilde{b}_j} = b_2 + (1 - \beta_j)R_j, j = 1, 2, \cdots, n$$  \hspace{1cm} (9)

In such ways, the constraints of FTP are transformed into crisp and bounded intervals.

3.3 Treatment of objective

According to the operation algorithm of fuzzy numbers, because the unit cost of FTP is a linear non-negative combination of $m \times n$ trapezoidal numbers, the objective is a trapezoidal numbers too. Let $\tilde{C} = (C_1, C_2; L, R)$ be the objective value. When $x_{ij} \geq 0$, there exists

$$C_1 = \sum_{j=1}^{n} \sum_{i=1}^{m} c_{ij}x_{ij}, \quad C_2 = \sum_{j=1}^{n} \sum_{i=1}^{m} c_{ij}x_{ij}$$

$$L = \sum_{j=1}^{n} \sum_{i=1}^{m} L_{ij}x_{ij}, \quad R = \sum_{j=1}^{n} \sum_{i=1}^{m} R_{ij}x_{ij}$$

There are two approaches to treat the objective: fractile and modality approach.

3.3.1 Fractile approach. A fractile approach corresponds to the Kataoka’s model of a stochastic programming problem [14,15]. Geoffrion [16] calls the Kataoka’s model the fractile criterion approach. By the definition in statistics, $p$-fractile is the value $u$ which satisfies

$$\text{Prob}(X \leq u) = p$$

where $X$ is a random variable. In this definition, $p$-fractile does not generally exist for all $p \in (0, 1)$. That is why we define $p$-fractile as the smallest value $u_p$ of $u$ which satisfies

$$\text{Prob}(X \leq u) \geq p$$

From the viewpoint of Dempster-Shafer theory of evidence [17], it is known that $\text{Pos}(X \leq u)$ and $\text{Nes}(X \leq u)$ can be regarded as the upper and lower bounds of $\text{Prob}(X \leq u)$ (see [18]). In this sense, $p$-possibility fractile is defined as the smallest value of $u$ which satisfies

$$\text{Pos}(X \leq u) \geq p$$

and $p$-necessity fractile is defined as the smallest value of $u$ which satisfies

$$\text{Prob}(X \leq u) \geq p$$

### Table 2. The crisp transportation problem originated from Model I.

| $\gamma$ | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $B_5$ | Sup. |
|----------|-------|-------|-------|-------|-------|------|
| 0.95     | 10.90 | 7.75  | 10.70 | 23.60 | 16.75 | [19.0, 32.0] |
|         | 13.60 | 13.85 | 23.60 | 10.70 | 12.65 | [25.5, 38.6] |
|         | 23.60 | 17.50 | 26.55 | 10.95 | 21.90 | [13.6, 29.6] |
|         | 28.70 | 26.55 | 22.60 | 10.95 | 9.95  | [23.0, 44.0] |
| Dem.     | [17.0, 37.5] | [20.5, 35] | [30.75, 42] | [11.1, 30.7] | [24.4, 44] |
| 0.05     | 9.10  | 3.25  | 5.30  | 16.40 | 12.25 | [19.0, 32.5] |
|         | 6.40  | 11.15 | 16.40 | 5.30  | 6.35  | [25.5, 38.6] |
|         | 16.40 | 8.50  | 18.45 | 10.05 | 20.10 | [13.6, 29.6] |
|         | 23.30 | 18.45 | 15.40 | 36.20 | 9.05  | [23.0, 44.0] |
| Dem.     | [17.0, 37.5] | [20.5, 35] | [30.75, 42] | [11.1, 30.7] | [24.4, 44] |

![Figure 8. The relationship between supply and demand.](doi:10.1371/journal.pone.0105142.g008)
As for the objective of FTP, from the point of necessity it can convert to
\[
\min C \quad \text{s.t.} \quad \text{Nes}(C \leq C) \geq \gamma \quad (10)
\]
where \(\gamma\) is a parameter to reflect to what extent the decision-maker is certain about the lower bound of the fuzzy event \(C \leq C\). Eq. (10) expresses that the decision-maker minimizes the total cost at a given necessity level.

As shown in Figure 4, we can get
\[
\text{Nes}(C \leq C) = \frac{C - C_s}{R}
\]
then the constraint in (10) can be converted to
\[
\frac{C - C_s}{R} \geq \gamma \Rightarrow C_s + \gamma R \leq C
\]

Hence, Eq. (10) turns to
\[
\min C \quad \text{s.t.} \quad C_s + \gamma R \leq C \quad (11)
\]

It is easy to find that the optimal solution of (11) is \(C = C_s + \gamma R\).
Therefore, the objective function of FTP can be substituted by
\[
\min C_2 + \gamma R = \sum_{i=1}^{m} \sum_{j=1}^{n} (c_{ij} + \gamma R_{ij})x_{ij} \quad (12)
\]

And with the constraints (7) and (9), the FTP turns to the following model, which is a crisp linear programming problem.

Model I:
\[
\begin{align*}
\min & \quad \sum_{i=1}^{m} \sum_{j=1}^{n} (c_{ij} + \gamma R_{ij})x_{ij} \\
\text{s.t.} & \quad \text{Eq. (7)} \\
& \quad \text{Eq. (9)}
\end{align*}
\]

From the point of the possibility and by the fractile approach, the objective of FTP means that
\[
\min C \quad \text{s.t.} \quad \text{Post}(C \leq C) \geq \theta \quad (13)
\]
i.e., the decision-maker expects that the total cost should be minimized as the possibility that total cost is not larger than some given level \(C\).
As shown in Figure 5, we can get

$$\text{Pos}(\tilde{C} \leq C) = \frac{C - C_1 + L}{L} \geq \theta$$

or

$$C_1 - (1 - \theta)L \leq C$$

Hence, (13) can be converted to

$$\min C \quad \text{s.t.} \quad C_1 - (1 - \theta)L \leq C$$

(14)

It is easy to find that the optimal solution of (14) is $C_1 - (1 - \theta)L$, i.e.,

$$C_1 - (1 - \theta)L = \sum_{i=1}^{m} \sum_{j=1}^{n} (c_{ij} - (1 - \theta)L_{ij})x_{ij}$$

(15)

And with the constraints (7) and (9), the FTP turns to the following model.

Model II

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} (c_{ij} - (1 - \theta)L_{ij})x_{ij}$$

s.t. \begin{align*}
\text{Eq. (7)} \\
\text{Eq. (9)}
\end{align*}

3.3.2 Modality approach. A modality optimization model corresponds to the minimum-risk approach to a stochastic programming problem [15]. The minimum-risk approach is also called the maximum probability approach [14] or the aspiration criterion approach [16]. A modality optimization approach is a dual approach to the fractile optimization one. In this approach, the decision-maker puts more importance on the certainty degree comparing to the fractile approach.

From the point of necessity and modality approach, the decision-maker in FTP expects that the total cost should not be greater than a given level $C_0$, i.e.,

$$\max \text{Nes}(\tilde{C} \leq C_0)$$

(16)

From Figure 6, we can get

$$\text{Nes}(\tilde{C} \leq C_0) = \frac{C_0 - C_2}{R}$$

hence, the objective of FTP can be converted to

$$\min \frac{C_0 - C_2}{R} = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij} - C_0}{\sum_{i=1}^{m} \sum_{j=1}^{n} R_{ij}x_{ij}}$$

(17)

Such that FTP turns to

$$\min \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij} - C_0}{\sum_{i=1}^{m} \sum_{j=1}^{n} R_{ij}x_{ij}}$$

s.t. \begin{align*}
\text{Eq. (7)} \\
\text{Eq. (9)}
\end{align*}

This is a fractional programming problem which can be transformed to a linear programming problem by the substitution

$$y = \frac{1}{\sum_{i=1}^{m} \sum_{j=1}^{n} R_{ij}x_{ij}}, \quad z_{ij} = x_{ij}y, \forall i, j$$

We obtain the following model.

Model III

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}z_{ij} - C_0y$$

s.t. \begin{align*}
\sum_{i=1}^{n} z_{ij} - (1 - \theta)L_{ij}y & \geq 0, \quad i = 1, 2, \ldots, m \\
\sum_{j=1}^{n} z_{ij} - (1 - \theta)L_{ij}y & \leq 0, \quad i = 1, 2, \ldots, m \\
\sum_{i=1}^{m} z_{ij} - (1 - \theta)L_{ij}y & \geq 0, \quad j = 1, 2, \ldots, n \\
\sum_{i=1}^{m} z_{ij} - (1 - \theta)L_{ij}y & \leq 0, \quad j = 1, 2, \ldots, n \\
\sum_{i=1}^{m} \sum_{j=1}^{n} R_{ij}z_{ij} & = 1 \\
y & \geq 0, z_{ij} \geq 0, j = 1, 2, \ldots, n; j = 1, 2, \ldots, n
\end{align*}

From the point of possibility, the decision-maker expects that the total cost should not be greater than a given level $C_0$, i.e.,

$$\max \text{Pos}(\tilde{C} \leq C_0)$$

(18)
### Table 4. Solution from Model II ($\theta = 0.95$).

|       | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $B_5$ | Sup. | Pos  |
|-------|-------|-------|-------|-------|-------|------|------|
| $A_1$ | 0     | 1.75  | 30.75 | 0     | 0     | 32.5 | 1.00 |
| $A_2$ | 17    | 0     | 0     | 11.1  | 1.4   | 29.5 | 1.00 |
| $A_3$ | 0     | 18.75 | 0     | 0     | 0     | 18.75| 1.00 |
| $A_4$ | 0     | 0     | 0     | 0     | 23    | 23   | 0.80 |
| Dem.  | 17    | 20.5  | 30.75 | 11.1  | 24.4  | 103.75|      |
| Pos   | 0.75  | 0.90  | 0.95  | 0.85  | 0.80  |      |      |
| Total cost $\tilde{C} = (346.35, 681.9; 126.75, 616.15)$

doi:10.1371/journal.pone.0105142.t004

### Table 5. Solution from Model III ($C_0 = 1000$).

|       | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $B_5$ | Sup. | Pos  |
|-------|-------|-------|-------|-------|-------|------|------|
| $A_1$ | 0     | 1.75  | 30.75 | 0     | 0     | 32.5 | 1.00 |
| $A_2$ | 17    | 5.15  | 0     | 11.1  | 0     | 33.25| 1.00 |
| $A_3$ | 0     | 13.6  | 0     | 0     | 0     | 13.6 | 0.80 |
| $A_4$ | 0     | 0     | 0     | 0     | 24.4  | 24.4 | 0.856|
| Dem.  | 17    | 20.5  | 30.75 | 11.1  | 24.4  | 103.75|      |
| Pos   | 0.75  | 0.90  | 0.95  | 0.85  | 0.80  |      |      |
| Total cost $\tilde{C} = (377.7, 701.55; 143.6, 571.70)$

doi:10.1371/journal.pone.0105142.t005
From Figure 7, we can get

\[ \text{Pos}(C \leq C_0) = \frac{C_0 - C_1 + L}{L} \]

Hence, the objective of FTP can be transformed to

\[
\max \quad \frac{C_0 - C_1 + L}{L} \Rightarrow \min \quad \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} - C_0}{\sum_{i=1}^{m} \sum_{j=1}^{n} L_{ij} x_{ij}} \tag{19}
\]

Such that the FTP turns to

\[
\min \quad C = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} - C_0}{\sum_{i=1}^{m} \sum_{j=1}^{n} L_{ij} x_{ij}}
\]

\[
\text{s.t.} \quad \begin{cases} 
\text{Eq. (7)} \\
\text{Eq. (9)}
\end{cases}
\]

Again, this is a fractional programming problem which can be transformed to a linear programming problem by the substitution

\[ y = \frac{1}{\sum_{i=1}^{m} \sum_{j=1}^{n} L_{ij} x_{ij}}, \quad z_{ij} = x_{ij} y, \forall i, j \]

We obtain the following model.

\[
\min \quad \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} z_{ij} - C_0 y \\
\text{s.t.} \quad \begin{cases} 
\sum_{i=1}^{m} a_{i1} z_{ij} - [a_{i1} - (1 - a_{i2}) L_{i}] y \geq 0 & i = 1, 2, \ldots, m \\
\sum_{i=1}^{m} a_{i2} z_{ij} - [a_{i2} + (1 - a_{i1}) L_{i}] y \leq 0 & i = 1, 2, \ldots, m \\
\sum_{j=1}^{n} b_{j1} z_{ij} - [b_{j1} - (1 - b_{j2}) L_{j}] y \geq 0 & j = 1, 2, \ldots, n \\
\sum_{j=1}^{n} b_{j2} z_{ij} - [b_{j2} + (1 - b_{j1}) L_{j}] y \leq 0 & j = 1, 2, \ldots, n \\
\sum_{j=1}^{n} L_{ij} z_{ij} = 1 \\
y \geq 0, z_{ij} \geq 0, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n
\end{cases}
\]

To sum up, from the view of possibility and necessity, and by fractile and modality approach, we proposed four ways to defuzzify FTP to four crisp linear programming problems. During the transformation process, some parameters are introduced to clarify the decision-maker’s subjectiveness about fuzziness, which makes the solutions more practical. As for constraints, we introduced \( z_i \) and \( b_j \) to reflect the decision-maker’s requirement on the extent how the constraint is satisfied in the view of possibility. With respect to the objective function, in the fractile approach we proposed \( y \) as the lower bound of necessity and \( \theta \) as the lower bound of possibility that the total cost is not larger than a given level set by the decision-maker. While in the modality approach, \( C_0 \) was introduced as the upper bound of total cost that decision-maker expected. In brief, these parameters reflect the decision-maker’s attitude...
to the fuzziness in FTP from the view of possibility or necessity.

**Numerical example**

In this section, numerical examples of FTP will be presented to demonstrate and verify the proposed approaches. All established models are originated from the above models based on the different parameter settings. All models are solved by the Lingo software, so the solving process is omitted for simplification. The description of transportation problem is standard tables, where the central part is the cost $c_{ij}$, the column "Supply" are $a_i$ and the row "Demand" are $b_j$. While in the solution tables, the central part is the quantities of transportation from $A_i$ to $B_j$.

Table 1 is a FTP with four origins $A_i$ ($i=1,2,3,4$) and five destinations $B_j$ ($j=1,2,3,4,5$) because the supply, demand and cost of unit are assumed to be fuzzy trapezoidal numbers. By adding up for origins' supply and five destinations' demand in Table 1, the total supply and demand are fuzzy trapezoidal numbers (94,123;82,134) and (114,166;76,158) depicted in the Figure 8.

To solve this FTP, firstly we set the value of $x_i, \beta_j, \gamma$ and $C_0$ as follows:

$$x_i : = (0.85,0.90,0.80,0.80); \quad \beta_j : = (0.75,0.90,0.95,0.85,0.80)$$

$$\gamma : = 0.95,0.05; \quad \theta : = 0.95; \quad C_0 : = 1000$$

Then according to (7) and (9), the supply and demand constraints can be converted to different bounded intervals as shown in Table 2 based on the given value of $x_i$ and $\beta_j$.

By using Model I, let $\gamma$ to be 0.95 and 0.05 respectively. And based on the Eq.(12), the cost of FTP can be transformed into a crisp value as shown in Table 2.

Solve the two crisp transportation problems by linear programming, the optimal solution is in Table 3, and the fuzzy total cost is depicted in Figure 9 for comparative analysis.

From Table 3 and Figure 9, it can be found that the distribution varies with the changing of $\gamma$. However, the total transportation, supply from different origins and demand to different destinations are same. And, the possibility requirement of supply and demand constraints are almost satisfied. From the point of fuzzy total cost, it gets smaller when $\gamma$ is smaller.

According to Model II, assume that the decision-maker expects that the lower bound of possibility that total cost is not larger than some given level $C$ is 0.95, i.e., let $\theta = 0.95$. The solution is in the Table 4.

In model III, let $C_0$ equals 1000, the solution is listed in Table 5.

From Table 5, we can easily get that

$$\text{Pos}(C \leq C_0 = 1000) = 1$$

Hence, the possibility that the total cost is not larger than given $C_0$ is satisfied. This is reasonable because $C_0$ is set to be the upper bound of the minimal total cost which a decision-maker can afford.

In model IV, similarly let $C_0$ equals 1000, and solution is listed in Table 6. From Table 6, the following relationship exist.

$$\text{Pos}(C \leq C_0 = 1000) = 1$$

Similar results can be achieved as shown in Model III.

**Conclusions**

In this paper, a simple but effective parametric method was introduced to solve fuzzy transportation problem. By using possibility theory in fractile and modality approach, the fuzzy transportation problem is transformed into four types of crisp linear programming problems. In the process of transformation, some parameters are introduced to reflect decision-maker's attitude to the uncertainty or fuzziness. The methods proposed in this paper can be used for all kinds of fuzzy transportation problem, whether triangular and trapezoidal fuzzy numbers with normal or abnormal data.

**Acknowledgments**

This research is supported by the Fundamental Research Funds for the Central Universities (2-9-2012-86) and supported by Key Laboratory of Carrying Capacity Assessment for Resource and Environment, MLR (CCA2012.05). The authors would like to heartily thank the Editor in Chief and anonymous reviewers for their careful reading of this paper and for their helpful comments and suggestions.

**Author Contributions**

Analyzed the data: RL. Contributed to the writing of the manuscript: DH PL. Developed models: DH. Designed the numerical examples: QH.

**References**

1. Chanas S, Kołodzieczyk W, Machaj A (1984) A fuzzy approach to the transportation problem. Fuzzy Sets and Systems 13: 211–221.
2. Chanas S, Delgado M, Verdegay J, Vila M (1993) Interval and fuzzy extensions of classical transportation problems. Transportation Planning and Technology 17: 203–218.
3. Chanas S, Kachta D (1996) A concept of the optimal solution of the transportation problem with fuzzy cost coefficients. Fuzzy Sets and Systems 82: 299–303.
4. Liu ST, Kao C (2004) Solving fuzzy transportation problems based on extension principle. European Journal of Operational Research 153: 661–674.
5. Gani A, Razak KA (2006) Two stage fuzzy transportation problem. Journal of Physical Sciences 10: 63–69.
6. Pandian P, Natarajan G (2010) A new algorithm for finding a fuzzy optimal solution for fuzzy transportation problems. Applied Mathematical Sciences 4: 79–90.
7. Basirzadeh H (2011) An approach for solving fuzzy transportation problem. Applied Mathematical Sciences 5: 1549–1566.
fuzzy approaches to multiobjective mathematical programming under uncertainty, Springer. pp. 71–101.

16. Geoffrion AM (1967) Stochastic programming with aspiration or fractile criteria. Management Science 13: 672–679.

17. Dempster AP (1967) Upper and lower probabilities induced by a multivalued mapping. The annals of mathematical statistics : 325–339.

18. Dubois D, Prade H (1987) The mean value of a fuzzy number. Fuzzy sets and systems 24: 279–300.