Stable and deterministic quantum key distribution based on differential phase shift

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We present a stable and deterministic quantum key distribution (QKD) system based on differential phase shift. With three cascaded Mach-Zehnder interferometers with different arm-length differences for creating key, its key creation efficiency can be improved to be 7/8, more than other systems. Any birefringence effects and polarization-dependent losses in the long-distance fiber are automatically compensated with a Faraday mirror. Added an eavesdropping check, this system is more secure than some other phase-coding-based QKD systems. Moreover, the classical information exchanged is reduced largely and the modulation of phase shifts is simplified. All these features make this QKD system more convenient than others in a practical application.

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I. INTRODUCTION

Quantum key distribution (QKD)\textsuperscript{1} supplies a novel way for generating a private key securely between two legitimate users, say the sender Alice and the receiver Bob. Its security is based on the laws in quantum mechanics such as noncloning theorem, coherence of entangled systems, and quantum measurement, but not the computation difficulty with a limited computation power. As an unknown quantum state cannot be cloned\textsuperscript{2}, the action done by a vicious eavesdropper, say Eve will inevitably disturb the quantum system and leave a trace in the outcome obtained by the two legitimate users. Alice and Bob can detect the eavesdropping by analyzing the error rate of a subset of instances chosen randomly. Since Bennett and Brassard published an original protocol in 1984 (BB84)\textsuperscript{1}, QKD attracts a great deal of attention\textsuperscript{3-8}.

The experimental implementation of long-distance QKD over an optical fiber channel requires the two legitimate users to control the influence of the fluctuation of the birefringence which alters the polarization state of photons. For overcoming this noise, several elegant QKD schemes have been developed with some unbalanced Mach-Zehnder interferometers (MZIs), such as the QKD scheme based on the phase difference of single photon\textsuperscript{9-11}, the "plug and play" system\textsuperscript{12} and its modifications\textsuperscript{13-16}, the QKD system based on faithful qubit distribution with additional qubits\textsuperscript{17}, and the QKD system with faithful single-qubit transmission\textsuperscript{18}. With the development of technology, the QKD system based on faithful qubit distribution\textsuperscript{17} seems to be perfect if there is an ideal single-photon source which can produce two photons in a deterministic time, although the success probability of this system for BB84 protocol is no more than 1/16 in principle in a passive way. The QKD system with the faithful single-qubit transmission technique\textsuperscript{18} is more efficient than that in Ref.\textsuperscript{17} as it only requires one photon in each signal time and the qubit transmitted can reject the error arisen from collective noise by itself. Its success probability for generating a private key is 1/4 in theory with BB84 protocol in an absolutely passive way. With some cascaded unbalanced Mach-Zehnder interferometers (MZIs), a special encoder and a special decoder\textsuperscript{19}, one-way QKD can be implemented against collective noise (with which the fluctuation is so slow in time that the alteration of the polarization is considered to be the same over the sequence of several photons or wavepackets\textsuperscript{17}) in a passive way with a success probability approaching the intrinsic one in BB84 QKD protocol\textsuperscript{1}.

In those QKD protocols with phase coding\textsuperscript{9-11,12,13,14,15,16}, the two legitimate users need not share a reference frame for choosing the common polarization bases, which makes these protocols more convenient than those with polarization coding in a practical application. The influence of the birefringence effect in fibers on the one-way QKD systems\textsuperscript{9,10,11} is far more severe than that on the two-way "plug and play" QKD systems\textsuperscript{12,13}. Thus "plug and play" QKD systems\textsuperscript{14,15,16} based on differential phase shift (DPS)\textsuperscript{11} were proposed for increasing the key

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creating efficiency with some unbalanced MZIs and Faraday mirrors. These systems are stable as the Faraday mirrors can be used to automatically compensate for birefringence and polarization-dependent losses in the transmission fiber [15]. Although these systems [14, 15, 16] are better than some others, there are some spaces for improving. First, most faint laser pulses split by the front Mach-Zehnder interferometer (MZI) are combined again by the next ones, which makes the key creation efficiency improved be limited. Second, in order to compensate for the amplitude differences caused by the overall interference of pulses travelling through different paths, phase shifters should be inserted in the long arms of MZIs, which increases the complexity and decreases stability of the system. Moreover, as there is no eavesdropping check process in the first transmission, the systems are vulnerable to an eavesdropping technique known as the intercepting-resending attack [3]. As pointed out in Ref. [20], for each block of transmission, an eavesdropping checking is inevitable for secure communication no matter what is transmitted with a quantum channel.

In this paper, we propose a stable and deterministic QKD system based on DPS with a high key creation efficiency approaching 100% by using the least cascaded MZIs with different arm-length differences. Compared with that in Ref. [15], the key generation efficiency is 7/8 in principle, more than 3/4, when the two parties exploit three MZIs for creating their private key. Its security is much higher than the latter. Moreover, this system is stable and deterministic. Any birefringence effects and polarization-dependent losses in the long-distance fiber are automatically compensated with a Faraday mirror. The modulation of the phase shifts is also more easier than others, and the classical information exchanged is reduced largely.

II. STABLE AND DETERMINISTIC QKD SYSTEM

Fig.1 shows the setup of our QKD system based on DPS. It is made up of three cascaded MZIs (MZI1, MZI2, and MZI3) in Bob’s site for creating a private key, and a Faraday mirror and a phase modulator in Alice’s site. Similar to Ref. [13, 14, 15, 16], the MZIs, with long and short arms connected by 50%-50% fiber couplers C1 - C5, are designed to have different arm-length differences so that each pulse split by the front MZI does not overlap with others in the process of preparing the quantum signal. The time delays between the long and the short arm of MZI1, MZI2, and MZI3 are 4T, 2T, and T, respectively. The Faraday mirror is used to automatically compensate for birefringence and polarization-dependent losses in the transmission fiber. The phase modulator is used to encode the key.

![FIG. 1: Stable and deterministic DPS-based QKD system with a key creation efficiency of 7/8. LD represents laser diode. C1-C7 represent 50%-50% couplers. MZI1, MZI2, MZI3, and MZI4 are four Mach-Zehnder interferometers with different arm-length differences 4T, 2T, T, and T, respectively. L1-L4 and S1-S4 are the long (L) arms and the short (S) arms of MZIs, respectively. D1-D4 are four avalanche photon detectors. PM, FM, and A represent a phase modulator, a Faraday mirror, and an attenuator, respectively. For eavesdropping check, Alice samples a subset of quantum signals with beam splitter (BS) and measured them with another MZI (MZI4) by choosing two bases (φ′A ∈ {0, π/2}).](image)

An original pulse $\psi_1 = e^{-i\phi_0}|t_1\rangle$ from a laser diode is split into two sequential pulses with a time interval 4T by passing through MZI1, i.e., $\psi_2 = e^{-i\phi_0}|t_1\rangle$ and $\psi_3 = e^{-i\phi_0}|t_5\rangle$ are combined at the second coupler C2 at time instances $t_1$ and $t_5$. Where $\phi_0$ is the initial phase factor and the subscript 2 in $\psi_2$ is used to label the second coupler. We neglect the global factor in each pulse in this paper. Subsequently, each of the pulses is split into two sequential pulses by MZI2 with a time interval 2T, i.e., there are four pulses described as

$$
\begin{align*}
\psi_3^1 &= e^{-i\phi_0}|t_1\rangle, \\
\psi_3^2 &= e^{-i\phi_0}|t_3\rangle, \\
\psi_3^3 &= e^{-i\phi_0}|t_5\rangle, \\
\psi_3^4 &= e^{-i\phi_0}|t_7\rangle.
\end{align*}
$$

(1)
After MZI3 with a time interval T, there are eight sequential pulses arriving at the coupler C5. These eight pulses have the same amplitude and the delay between two near pulses is T (shown in Fig.2). These eight pulses can be described as

\[
\begin{align*}
\psi_1^4 &= e^{-i\phi_B} |t_1\rangle, \\
\psi_2^3 &= e^{-i\phi_B} |t_2\rangle, \\
\psi_3^2 &= e^{-i(\phi_A+\phi_B)} |t_3\rangle, \\
\psi_4^1 &= e^{-i(\phi_A+\phi_B)} |t_4\rangle, \\
\psi_5^7 &= e^{-i\phi_B} |t_5\rangle, \\
\psi_6^8 &= e^{-i(\phi_A+\phi_B)} |t_6\rangle, \\
\psi_7^7 &= e^{-i(\phi_A+\phi_B)} |t_7\rangle, \\
\psi_8^8 &= e^{-i(\phi_A+\phi_B)} |t_8\rangle,
\end{align*}
\]  

(2)

where \(\phi_B\) is a phase shift added by Bob for preparing the quantum signal. Bob chooses randomly one of the four phase shifts \(\{0, \pi/2, \pi, 3\pi/2\}\) for each original pulse \(\psi_1 = e^{-i\phi_B} |t_1\rangle\) in the quantum communication. That is, Bob chooses four nonorthogonal phase-coding states to carry the message transmitted, which is similar to the way in polarization coding that the two legitimate users choose four nonorthogonal states \(\{|0\rangle, |1\rangle, (|0\rangle + |1\rangle)/\sqrt{2}, (|0\rangle - |1\rangle)/\sqrt{2}\}\) to complete a deterministic quantum communication [21, 22]. This feature can forbid an eavesdropper to eavesdrop the QKD system freely with intercepting-resending attack [3].

When the eight pulses arrive at Alice’s site, she first attenuates the signal with a variable attenuator and then encodes her random key on the odd pulses with the same phase shift \(\phi_A \in \{0, \pi\}\). 0 and \(\pi\) represent the bit values in key string 0 and 1, respectively. After the coding performed by Alice, the quantum signal becomes

\[
\begin{align*}
\psi_A &= e^{-i(\phi_A+\phi_B)} |t_1\rangle + e^{-i(\phi_A+\phi_B)} |t_2\rangle + e^{-i(\phi_A+\phi_B)} |t_3\rangle \\
&\quad + e^{-i(\phi_A+\phi_B)} |t_4\rangle + e^{-i(\phi_A+\phi_B)} |t_5\rangle + e^{-i(\phi_A+\phi_B)} |t_6\rangle \\
&\quad + e^{-i(\phi_A+\phi_B)} |t_7\rangle + e^{-i(\phi_A+\phi_B)} |t_8\rangle.
\end{align*}
\]  

(3)

Alice reflects the quantum signal to Bob by a Faraday mirror, same as Ref. [12, 13, 14, 15, 16].

When the pulses are reflected back after coding, 7/8 of the pulses interfere at the coupler C4 and can be used to generate the private key, higher than 3/4 in Ref. [15].

Bob can read out the key with success probability 7/8 by using two detectors \(D_1\) and \(D_2\). When \(\phi_A = 0\), the pulse train clicks the detector \(D_1\) at the time instances \(t_2, t_4, t_6,\) or \(t_8\); Otherwise, it clicks the detector \(D_2\) at one of these four time instances. At the other three time instances \(t_1, t_3,\) and \(t_5\), the detector clicked depends on both the phase shifts \(\phi_A\) and \(\phi_B\), shown in Eq. (4). In detail, when \(2\phi_B \equiv 2\pi = \phi_A\), the pulse train clicks the detector \(D_1\) at the time instances \(t_3, t_5,\) or \(t_7\); Otherwise, it clicks the detector \(D_2\). In a word, Bob can obtain the key with his phase shift \(\phi_B\) at the time instances \(t_2, t_3, t_4, t_5, t_6, t_7,\) or \(t_8\) in a deterministic way. At the time instances \(t_1\) and \(t_9\), no interference takes place and the pulse train will click one of the two detectors randomly, which happens with the probability 1/8. Bob discards these useless time instances. In quantum communication, Bob need only tell Alice the fact that he
detects a photon or not in the useful time instances, which will reduce the classical information exchanged largely, compared with that in Ref.\[15,16\] as Bob does not announce the detailed time slots.

In order to prevent Eve from eavesdropping with intercepting-resending attack strategy, we add a phase shift \(\phi_B \in \{0, \pi/2, \pi, 3\pi/2\}\) in Bob’s site. Moreover, a eavesdropping check is designed in Alice’s site. That is, Alice should sample some quantum signals randomly and measure them with two nonorthogonal phase bases, shown in Fig.1, same as Ref.\[21,22\]. Without these two tricks, this DPS QKD protocol is insecure. We give the detail of a special intercepting-resending attack strategy to demonstrate the necessity of the procedure of eavesdropping check. This strategy works in the protocols in Refs.\[14,15,16\], which means these two QKD protocols is insecure in principle. For obtaining the phase shifts performed by Alice, Eve first intercepts all the eight pulses travelling through the coupler \(C_5\) and then stores them. She prepares another pulse \(a\) and splits it into two parts, \(b\) and \(c\). For evading the energy check done by Alice, Eve controls the intensity of her pulses \(b\) and \(c\) to be equal to each of the eight pulses \(|t_1\rangle-|t_8\rangle\) sent by Bob. Eve sends the part \(c\) to Bob, instead of each of the eight pulses. No matter what the phase shift \(\phi_A \in \{0, \pi\}\) is chosen by Alice, Eve can in principle get this information by interfering the part \(b\) with \(c\) when it is reflected from Alice. In this way, Eve can pretend Alice and encodes Alice’s phase shift on the original eight pulses and resends them to Bob. This attack will in principle leave nothing in the outcome obtained by the two legitimate users. The same way can be used to attack the quantum communication in Refs.\[14,15,16\]. For example, for the pulse train \(P_1\) in Ref.\[15\], Eve replaces the original one with the part \(c\) and sends it to Alice. No matter what the phase shift is chosen by Alice from the two values \(\{\pi/3, 4\pi/3\}\), Eve need only add a phase shift \(-2\pi/3\) on the part \(b\) and then interfere it with the part \(c\) reflected by Alice. Obviously, Eve can determine that the phase shift performed by Alice is \(\pi/3\) or \(4\pi/3\) in principle. The phase shifts on the other three pulse train \(P_2, P_3,\) and \(P_4\) can also be obtained in the same way. In essence, this insecurity comes from the lack of the eavesdropping check done by Alice. As pointed out in Ref.\[20\], for each block of transmission, an eavesdropping checking is inevitable for secure communication no matter what is transmitted with a quantum channel. This principle can be used to ensure the security of our DPS QKD system. If the influence of the birefringence effect in fiber is large enough, Alice should prepare a subset of nonorthogonal pulses with which she replace some signal pulses with a probability \(p_d\) for preventing Eve from measuring the pulses reflected by Alice. She can produce her nonorthogonal pulses by randomly adding one of the two phase shifts \(\phi_A \in \{0, \pi/2\}\) on some signal pulses. In this time, Bob should modulate her another phase shift \(\phi_B \in \{0, \pi/2\}\) besides \(\phi_B\) with a small probability. For improving the key creation rate, Alice and Bob can use two biased phase bases to prepare and measure the nonorthogonal pulses, same as that in Ref.\[6\].

### III. SUMMARY

In summary, we have proposed a stable and deterministic QKD system based on DPS. The most specific feature of this DPS QKD system is the high key creation efficiency and the high security. By using three MZIs with different arm-length differences for creating key (the fourth MZI is used to check eavesdropping), the efficiency is improved to be \(7/8\), more than \(3/4\) in Ref.\[15\]. Moreover, this system is expansible. With \(n\) MZIs for creating the key, the key creation efficiency can be added up to \(\frac{2^n-1}{4^n}\), higher than \(\frac{n}{n+1}\) in Ref.\[13\] (much higher than that in Ref.\[16\]). When \(n\) is large enough, the efficiency approaches \(100\%\). Another feature is the high security. With eavesdropping check and nonorthogonal pulses, this QKD system is obviously more secure than that in Ref.\[15\]. Moreover, this system is stable and deterministic, and the classical information exchanged is reduced largely.

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