Stationary spherical-symmetric configuration of dark matter

M. Abishev1,2*, N. Beissen1,2, B. Karsimanov1, I. Uspanov1, and M.O. Alimkulova1

1Al-Farabi Kazakh National University, 71, Al-Farabi Ave., 050040, Almaty, Kazakhstan
2Dulaty University, 7, Suleimenov Str., 080012, Taraz, Kazakhstan
*e-mail: abishevme@gmail.com

The article is devoted to the consideration of a model problem, where mass flow of dark matter which take as only gravitational interacting spherical continuous medium without pressure and with radial opposite motion with equilibrium ascending and descending components, starting free fall from boundary of the sphere and passing through the center, stopping and fall back at the boundary of the spherical mass distribution, which give us stationary spherical-symmetric configuration of dark matter. The presence of an opposite motion makes the configuration we are considering stationary. Limited mass and dimensions of the dark matter distribution gives us possibility to modelling galaxy galo and dark stars. This configuration is considered in the classical gravity. As a result of the study, we obtained a singular density distribution in the center and in the shell of sphere and corresponding velocity field with singular center. The resulting profile can be used to construct arbitrary distributions, corresponding to the observed profiles.

Key words: dark matter, gravitational interaction, stationary configuration, density profile, velocity field.
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1 Introduction

At present, the problem of distribution of dark matter in galaxies and other local inhomogeneities is one of the most actual problems of astrophysics. Evidence for the existence of dark matter is provided by the rotation curves of galaxies, which show that there is no decrease in the rotation speed at the periphery of stellar disks (the simplest explanation for this effect is the presence of massive invisible halos in galaxies, which make a large contribution to their masses); dynamics and morphology of satellite galaxies and globular clusters near massive galaxies (small satellite galaxies move around large ones, obeying the same laws as stars on the periphery of ordinary galaxies, thus being test bodies of the same kind, but on a larger scale, which allows us to draw conclusions about the distribution of the gravitational potential of such massive galaxies); analysis of data for our and other galaxies confirmed that the total mass of each galaxy is several times greater than the total mass of its stars; dynamics of galaxy systems from binary galaxies to galactic clusters, etc. [1-5]

Thus, it is revealed that dark matter is present at all levels of the galactic hierarchy, and its share increases with increasing scale: in binary systems, it exceeds the contribution of visible matter by several times, and in clusters of galaxies (consisting of hundreds or thousands of objects) – by tens or hundreds of times [6-12].

2 Stationary and static distributions

Various profiles, such as the Burkert profile, are used to describe the density distribution of dark matter in the galactic halo [3]:

$$\rho = \frac{\rho_0 r_0^3}{(r + r_0)(r^2 + r_0^2)}$$

(1)

Such of distributions of dark matter rather fine describes the rotation curves of galaxies [13-18]. However, despite of static dark matter configurations are mainly used to describe the distribution of dark matter, we will consider stationary configurations of
it, which is possible in the presence of dynamic stability of system.

We will consider simple model problem with equations of system in classical gravity and motion

\[ \Delta U = 4\pi \gamma \rho, \]
\[ \frac{\dot{\gamma}}{d\dot{t}} = -\gamma U, \]
\[ \frac{\partial \rho}{\partial t} + \left( \nabla \left( \rho \dot{v} \right) \right) = 0, \]  

(2)

where \( U \) – gravitational potential inside of dark matter, \( \gamma \) – gravitational constant, \( \rho \) - density of the dark matter, \( \dot{v} \) – field of velocity of dark matter. In every point inside dark matter motion is radial and opposite, i.e. there exist two components: ascending and descending. Radius and mass of sphere is \( R \) and \( M \).

In the case of stationary distribution of dark matter, we can take all functions as only radius dependent. In spherical coordinates we will take

\[ rU'' + 2U = -4\pi \gamma \rho r, \]
\[ \dot{v} = -U', \]
\[ r\rho' + r\rho v' + 2\rho v = 0, \]  

(3)

Let's rewrite (3) by velocity

\[ r^2 v^2 v'' + \left( 13r^2 + 7r^2 v' \right) v'' + \left( 14v^2 + 15r^2 v + r^2 v^2 \right) v' = 0. \]  

(4)

This equation cannot be solved analytically or approximately. It is possible reformulate this problem in terms of N-body simulation or to try to solve it directly by numerical methods. Velocity at the border of sphere must be zero. Potential at the border correspond to the point mass \( M \) potential at the distance \( R \). Additional conditions for system (3) is insufficient, so this problem has one free parameter. We take it in normalized \([-1,1]\) interval, which correspond to physical restrictions of our problem.

Numerical solution of equation (4) give us velocity field of stationary distribution of dark matter (Figure 1). Using third expression in (3)

\[ r\rho' + r\rho v' + 2\rho v = 0, \]  

(5)

we can find density distribution. For density distribution we can find numerically next type of profile (Figure 2):
3. Conclusions

Analyzing numerical solutions with various additional conditions, we find that there takes places two singularities of density: in the center and on the surface of spherical volume and one singularity of velocity in the center. In any case such of distribution give us good alternative of static distributions of dark matter. Further development of this investigation is consideration of non radial moving dark matter stationary distribution, which must be without singularities.

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