Improved implementation of nonclassicality test for a single particle

Giorgio Brida, Ivo Pietro Degiovanni, Marco Genovese, Fabrizio Piacentini, Valentina Schettini, Sergey V. Polyakov, and Alan Migdall

1Istituto Nazionale di Ricerca Metrologica, Strada delle Cacce 91, 10135 Torino, Italy
2Group of Applied Physics, University of Geneva, CH-1211 Geneva 4, Switzerland
3Optical Technology Division, National Institute of Standards and Technology, 100 Bureau Drive, Gaithersburg, MD 20899-8441 and Joint Quantum Institute, Unv. of Maryland, College Park, MD 20742

*Corresponding author: i.degiovanni@inrim.it

Recently a test of nonclassicality for a single qubit was proposed [R. Alicki and N. Van Ryn, J. Phys. A: Math. Theor. 41, 062001 (2008)]. We present an optimized experimental realization of this test leading to a 46 standard deviation violation of classicality. This factor of 5 improvement over our previous result was achieved by moving from the infrared to the visible where we can take advantage of higher efficiency and lower noise photon detectors.

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INTRODUCTION

A simple test of nonclassicality at the single qubit level was proposed [1, 2] to show that some quantum states in a two dimensional Hilbert space cannot be classical. This test looks very appealing for its simplicity compared to other tests of quantumness (see [3] and references therein) and could represent a useful tool for various applications in the fields of quantum information, fundamental quantum optics, foundations of quantum mechanics, etc.

As this is a test of single-particle states, it is not relevant to the question of locality, rather it is a more fundamental test of nonclassicality with respect to the possibility of an underlying hidden variable theory (HVT). Furthermore, we understand that it does not apply to every conceivable HVT like Bell’s inequalities, but only to a restricted class of HVTs [4]. We also note that the precise identification of this class and whether and if it maps to any physical system at all remains to be determined. In addition, another question has been raised about this test by Zukowski [5], who suggests that the Alicki’s classicality criterion is equivalent to the von Neumann theorem.

The criterion for classicality in Alicki’s model is summarized by the following statement: for any pair of observables \( \hat{A} \) and \( \hat{B} \) that satisfy the condition

\[
\langle \hat{B} \rangle > \langle \hat{A} \rangle > 0
\]

for all states of the system, it must be always true that

\[
\langle \hat{B}^2 \rangle > \langle \hat{A}^2 \rangle.
\]

For quantum systems, one can find pairs of observables \( \hat{A}, \hat{B} \) such that the minimum eigenvalue of \( \hat{B} - \hat{A} \) (minimized over all possible states) is greater than zero, while for certain quantum states

\[
\langle \hat{B}^2 \rangle < \langle \hat{A}^2 \rangle.
\]

This sharp difference between classicality and nonclassicality in Alicki’s model can be tested experimentally at the single-qubit level [4].

One possible pair of operators \( \hat{A} \) and \( \hat{B} \) are of the form [4]

\[
\hat{A} = \frac{a + \hat{Z}}{2}
\]

\[
\hat{B} = \frac{1 + r \cos \beta \hat{Z} + r \sin \beta \hat{X}}{2},
\]

\[
\hat{A}^2 = \frac{a^2 + 2a + 1 + \hat{Z}}{4}
\]

\[
\hat{B}^2 = \frac{1 + 2r \cos \beta \hat{Z} + r^2 \hat{Z} + r^2 \sin \beta \hat{X} + r^2 \sin ^2 \beta \hat{X}}{4}.
\]
The quantum objects we use to implement this test are linearly polarized single-photons (|Ψ⟩ = cos ψ|H⟩ + sin ψ|V⟩) produced by a heralded single-photon source based on parametric down conversion (PDC). The main difference of this experiment with respect to the previous one is that in this case both the heralded and the heralding photons are in the visible, while in the previous case the heralded photon was at a telecom wavelength [4]. This allows us to use more efficient and lower noise detection systems that significantly reduce the experimental uncertainty. We also note that in this experiment measuring the two operators requires manually changing the waveplate angle rather than switching between the two measurements in an automated fashion, an inconsequential difference.

The experimental apparatus is sketched in Fig. 1. The PDC source is a 5 mm long LiIO₃ bulk crystal, pumped by 400 nm light, that produces pairs of correlated photons at 800 nm. The pump light is obtained by doubling the frequency of the output of a mode-locked laser (with a repetition rate of ≈80 MHz) pumped by a 532 nm laser. Two interference filters (IF) with spectral bandwidth full-width-half-maximum of 20 nm are placed in both the heralded and heralding arms to reduce background light. Microscope objectives (20x) collect the light into multi-mode fibers (MMF) and the photons are finally counted by Si-Single-Photon Avalanche Diodes (Si-SPADs) operating in Geiger mode. A half-wave plate (λ/2) and polarizing beamsplitter (PBS) are used for our polarization projective measurements.

To verify the single photon nature of our source, which is critical for our test, we quantify the possibility of having more than one photon in the heralded arm after detecting the heralding photon. For this we use the same setup as
for the main experiment (Fig. 1), but we substitute into the heralded arm the multi-mode fiber with an integrated 50:50 beam-splitter that sends the photons to two Si-SPADs. The purity of a single-photon source can be described by means of the two parameters \( \gamma_1 = \theta(1)/\theta(0) \) and \( \gamma_2 = \theta(2)/\theta(1) \), where \( \theta(0) \), \( \theta(1) \), and \( \theta(2) \) are the probabilities of the heralded arm producing 0, 1, or 2 counts for each heralding count, respectively.

In general, a heralding detection announces the arrival of a “pulse” containing \( n \) photons at the heralded channel. The probability that neither of the Si-SPADs fire for a heralded optical pulse containing \( n \) photons is

\[
\theta(0|n) = \sum_{m=0}^{n} \frac{(1-\tau_A)^m (1-\tau_B)^{n-m}}{(1 - \frac{\tau_A + \tau_B}{2})^n} B(m;n;p = 0.5)
\]

where \( p \) represents the BS splitting ratio \( (p = 0.5) \), \( B(m;n;p) = n!/[m! (n-m)!]^{-1} p^m (1-p)^{n-m} \) is the binomial distribution representing the splitting of \( n \) photons towards the two Si-SPADs, and \( \tau_A \) and \( \tau_B \) are the detection efficiencies of each Si-SPAD (that includes all collection and optical losses in the detection channel). Analogously, the probability of observing 1 or 2 counts due to a heralded optical pulse with \( n \) photons are respectively

\[
\theta(1|n) = \sum_{m=0}^{n} \frac{[(1-\tau_A)^m (1-\tau_B)^{n-m} + (1-\tau_A)^m (1-(1-\tau_B)^{n-m})] B(m;n;p = 0.5)}{(1 - \frac{\tau_A + \tau_B}{2})^n} ,
\]

\[
\theta(2|n) = \sum_{m=0}^{n} \frac{[1-\tau_A]^m [1-(1-\tau_B)^{n-m}] B(m;n;p = 0.5)}{(1 - \frac{\tau_A + \tau_B}{2})^n}.
\]

From these we get \( \theta(0) = \sum_n \theta(0|n) \mathcal{P}(n) \), \( \theta(1) = \sum_n \theta(1|n) \mathcal{P}(n) \), and \( \theta(2) = \sum_n \theta(2|n) \mathcal{P}(n) \) for \( \mathcal{P}(n) \) being the general probability distribution of the number of photons in a heralded optical pulse. Assuming both Si-SPADs have the same detection efficiencies \( (\tau_A = \tau_B = \tau) \), we list in the Table I the parameters for an ideal single-photon source \( (\mathcal{P}(n) = \delta_{n,1}) \) and a Poissonian source \( (\mathcal{P}(n) = \mu^n e^{-\mu}/n!) \) with, on average, \( \mu \) photons per pulse. Comparing the measured results to the theoretical values for the two types of sources, we see that while our source differs from an ideal single-photon source that emits one photon per pulse, the very small \( \gamma_2/\gamma_1 \), supports the point that conditional single-photon output does dominate our source’s output.

The expectation values \( \langle \hat{A} \rangle \), \( \langle \hat{B} \rangle \), \( \langle \hat{A}^2 \rangle \), \( \langle \hat{B}^2 \rangle \) can be obtained experimentally by projecting the heralded photons
onto the linear polarizations states as the operators $\hat{A}$ and $\hat{B}$ can be rewritten as

$$\hat{A} = a\hat{P}_0,$$
$$\hat{B} = b\left(\frac{1+r}{2}\hat{P}_\phi + \frac{1-r}{2}\hat{P}_\psi\right).$$

where $\hat{P}_0$ is the projection operator on the state $|s(\theta)\rangle = \cos\theta|H\rangle + \sin\theta|V\rangle$.

If we choose the parameter values $a = 0.74$, $b = 1.2987$, $r = 3/5$, $\beta = 2/9\pi$ (we note that, with this choice, the condition in Eq. (9) is satisfied), and $\psi = -11/36\pi$ the results for $\langle \hat{B}^2 \rangle - \langle \hat{A}^2 \rangle$, and $\langle \hat{B} \rangle - \langle \hat{A} \rangle$ predicted by quantum theory are those presented in Table 2, while the minimum eigenvalue of $\hat{B} - \hat{A}$ is $d_- = 0.0189$ (satisfying the $\hat{B} - \hat{A} > 0$ requirement for the Alicki test), where

$$d_- = \frac{1}{2}\left(b - a - \sqrt{a^2 + b^2}r - 2\sin\frac{\beta}{2}\right).$$

The experimental results are presented in Table II. From the value of $\langle \hat{B}^2 \rangle - \langle \hat{A}^2 \rangle$ we see a very large violation, $\approx 46$ standard deviations, of the classical limit of $\langle \hat{B}^2 \rangle - \langle \hat{A}^2 \rangle > 0$.

### CONCLUSION

In conclusion, we have presented a very simple and efficient experimental implementation of Alicki’s proposed nonclassicality test, that results in a large (46.1 standard deviations) and low noise violation of the Alicki classicality condition. This 5x improvement over our previous result was achieved by moving from the infrared to the visible, where we can take advantage of higher-efficiency and lower-noise photon detectors.

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