AdS$_5$ and the 4d Cosmological Constant

Christof Schmidhuber

CERN, Theory Division, 1211 Genève 23

Abstract

The hypothesis is discussed that our universe is really 5–dimensional with a nonzero cosmological constant that produces a large negative curvature. In this scenario, the observable flat 4–dimensional world is identified with the holographic projection of the 5–dimensional world onto its own boundary.

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schmidhu@mail.cern.ch
“Very often a source of strong poetry and strong science is a good metaphor. My favorite one is Plato’s cave: the parable of men sitting in a dark cave, watching the moving shadows on its wall. They think that the shadows are “real” and not just projections of the outside world. It seems to me that the latest stages of the ongoing struggle to understand interactions of elementary particles create a picture stunningly close to this parable.”

A. M. Polyakov in [1]

1. Motivation

It has recently been conjectured that various conformally invariant 4–dimensional gauge theories including $\mathcal{N} = 4$ supersymmetric $SU(N)$ Yang Mills theory with ‘t Hooft coupling $\lambda = g^2_{YM} N$ have a strong coupling description in terms of type IIB superstring theory compactified on a 5-dimensional Einstein manifold $E^5$. The dilaton is constant, $\Phi = \Phi_0$, and the string coupling constant is related to the gauge coupling constant by $\kappa = e^{\Phi_0} = g^2_{YM}$. The Einstein manifold has constant curvature radius

$$L \sim \lambda^\frac{1}{2},$$

and $N$ units of electric and magnetic Ramond-Ramond flux flow through it. The geometry of the remaining 5-dimensional spacetime is that of anti-de Sitter space $AdS_5$ with the same curvature radius $L$:

$$ds^2 = d\phi^2 + e^{-2\phi} dx^2 |x||.$$ 

$x|$ parametrizes the 4-dimensional space in which the gauge theory lives, and $\phi$ can be regarded as “renormalization group time” in the sense that $4d$ scale transformations

$$x| \rightarrow x| e^\tau$$

can be absorbed by the shift

$$\phi \rightarrow \phi + L \tau.$$

The AdS boundary at $\phi = -\infty$ represents the UV limit of the gauge theory. $\phi$ is a new physical coordinate, e.g. in the sense that various objects in the gauge theory, such as
instantons or “Pfaffian particles” in the case of gauge group $SO(N)$ are localized both in the $x_{\parallel}$ and the $\phi$ directions \[11, 12\].

String loop corrections are proportional to $\frac{1}{N^2}$, while $\alpha'$-corrections are proportional to $\frac{1}{\sqrt{\lambda}}$. In particular, the strong-coupling limit $\lambda \to \infty$ can, for large $N$, be investigated in the classical supergravity approximation. It is hoped that non-supersymmetric Yang-Mills theory and perhaps QCD have a similar description in terms of a 5-dimensional string theory on some different background \[12, 13\].

One immediate question might be: why don’t we see the fifth dimension? If it is true that nonabelian gauge theory at large scales (large enough for the coupling to be strong) is best described by a dual string theory in a 5-dimensional curved background, why then does the real world – which contains $SU(3)$ gauge theory – look 4-dimensional and flat, rather than 5-dimensional and curved? In the case of $\mathcal{N} = 4$ SYM, the answer is: even though $AdS_5$ is strongly curved, it has a flat 4-dimensional Minkowskian boundary, and at scales much larger than the curvature radius $L$ the 5-dimensional world essentially reduces to its own projection onto this boundary\[\footnote{This is similar to Kaluza-Klein reduction on a circle of radius $L$, where at scales much larger than $L$ the Kaluza-Klein modes drop out and an effective 4d theory is seen \[15\]. In our case the circle is replaced by the noncompact coordinate $\phi$, so things are more subtle, but the conclusion comes out essentially the same.}.

Given this answer, it is very tempting to use it for a seemingly unrelated question that can already be asked purely in the context of Einstein gravity and a priori has nothing to do with the AdS/CFT correspondence: why does our 4-dimensional world look flat and large, given that there should naturally be a huge cosmological constant which should strongly curve space-time, with a tiny curvature radius?

That is, it seems tempting to assume that our world is really 5-dimensional rather than 4-dimensional. Let us further assume that the 5$d$ cosmological constant $\Lambda_5$ is huge as it should be. But let us assume it comes out such that the sign of the 5$d$ curvature is negative. So the 5$d$ space becomes anti-de Sitter, with curvature radius

$$L^2 \propto \frac{1}{\kappa_5^2 \Lambda_5^5},$$

and so it acquires a boundary ($\kappa_5^2$ is the 5$d$ Newton constant). Although the curvature radius
of $AdS_5$ is small, its 4$d$ boundary is infinitely extended. In this sense we can consider field theory on this $AdS_5$ at scales much larger than $L$. For exactly the same reason as in the case of SYM theory, all we would see at low energies should be the projection of the 5$d$ world onto its own flat boundary. Can we identify this projection with the observable flat 4$d$ world?

2. Hidden cosmological constant

Before discussing some obvious objections against this scenario, let us demonstrate in more detail why in this setup the cosmological constant $\Lambda_4$ of the 4–dimensional world can be zero: the 5–dimensional cosmological constant $\Lambda_5$ does not induce a 4–dimensional cosmological constant but gets absorbed in the $AdS_5$ curvature radius. This is essentially a new version of a mechanism discussed long ago by Rubakov and Shaposhnikov \[19\]. Very similar suggestions by E. and H. Verlinde have also just appeared in \[20\].

Let us first consider Einstein gravity. Also, we start with the case where the 4$d$ metric $g_{\mu\nu} = e^{2\alpha(\phi)}\hat{g}_{\mu\nu}$ has only an overall $\phi$–dependence:

$$ds^2 = d\phi^2 + e^{2\alpha(\phi)}\hat{g}_{\mu\nu}(x)dx^\mu dx^\nu .$$

$\hat{g}$ is some background metric which for now we allow to be non–flat.\[2\]

The Einstein equations imply (we redefine $\Lambda_5$ by a factor)

$$R^{(5)}_{\mu\nu} = g_{\mu\nu} \Lambda_5$$

with $\Lambda_5$ assumed negative. The LHS of (2.1) can be written in terms of the 4$d$ curvature tensor $\hat{R}^{(4)}_{\mu\nu}$ of the metric $\hat{g}$ and the “shifted dilaton” (a slight misnomer here)

$$\varphi \equiv - \log(\sqrt{g}) = - 4\alpha(\phi) + \text{const.}$$

in the form

$$R^{(5)}_{\mu\nu} = \hat{R}^{(4)}_{\mu\nu} + \frac{1}{4} e^{-\frac{\phi}{2}}\hat{g}_{\mu\nu}(\dddot{\varphi} - \dot{\varphi}^2) ,$$

$^2$as the author found out after this paper was written.

$^3$In general, $g(x, \phi)$ will not factorize into a $\phi$–dependent piece and an $x$–dependent piece. In this case one should define $\alpha$ via the logarithm of the 4$d$ volume: $e^{4\alpha(\phi)} \equiv \{ \int d^4x \sqrt{g^{(4)}(x, \phi)} \}/\{ \int d^4x \}$.
where “dot” means “d/dφ”. From this and (2.1) we read off that \( \hat{g} \) is an Einstein metric with effective 4d cosmological constant \( \Lambda_4 \) (also redefined by a factor):

\[
\hat{R}^{(4)}_{\mu\nu} = \hat{g}^{(4)}_{\mu\nu} \Lambda_4 ,
\]

where

\[
\Lambda_4 = e^{-\hat{\varphi}/2} [\Lambda_5 - \frac{1}{4}(\hat{\varphi} - \hat{\varphi}^2)]
\]

(2.4)
is \( \phi \)-independent.

\( \hat{\varphi} \) can be eliminated using the \( g_{\phi\phi} \)-equation of motion

\[
\hat{\varphi} - \frac{1}{4}\hat{\varphi}^2 = \Lambda_5 .
\]

(2.5)
The result is

\[
\Lambda_4 = \frac{3}{4} e^{-\hat{\varphi}/2} (\Lambda_5 + \frac{1}{4}\hat{\varphi}^2) .
\]

(2.6)

We see that there is a solution in which the 5d cosmological constant is completely cancelled by \( \hat{\varphi}^2 \), and the 4d space is flat:

\[
\hat{\varphi}^2 = -4\Lambda_5 = \rightarrow \Lambda_4 = 0 .
\]

This is \( AdS_5 \) (recall that \( \Lambda_5 \) is negative). Of course there are also classical solutions \( \varphi(\phi) \) with \( \Lambda_4 \neq 0 \). Those just correspond to different foliations of \( AdS_5 \) by 4d hypersurfaces, so in the present case the value of \( \Lambda_4 \) is in fact ambiguous. But in the next section, a particular foliation will be singled out such that \( \Lambda_4 \) is well-defined.

3. Breaking conformal invariance

Let us now come to two immediate objections against the proposal that our world is the projection of a 5–dimensional anti-de-Sitter world onto its flat boundary: the 4–dimensional world that this scenario predicts differs from the one we observe in at least two major aspects:

1. it is conformally invariant.
The first point reflects the fact that the SO(2,4) symmetry group of $AdS_5$ becomes the conformal group on the 4d boundary. To break conformal invariance, we can let the 5d geometry deviate from $AdS_5$ near its boundary. Such a deviation would seem to be natural for a physical system with a “surface”, such as the 5d universe with 4d boundary.

More precisely, we will assume that the 5d cosmological constant becomes $\phi$–dependent near the $AdS$–boundary:

$$\Lambda_5(\phi).$$

Note that this implicitly singles out a particular foliation of $AdS_5$ by 4–dimensional hypersurfaces: those of constant $\Lambda_5$. A $\phi$–dependent $\Lambda_5$ can arise when 5d Einstein gravity is embedded in string theory, in particular on perturbations of the $AdS_5 \times E^5$ backgrounds mentioned in the introduction. At low energies this corresponds to embedding Einstein gravity in 5d gauged supergravity. There, $\Lambda_5$ is related to the potential $V[\Phi^I(\phi)]$, where $\Phi^I$ are the scalar fields of gauged supergravity. Those fields in turn generally depend on $\phi$. The way in which they roll down the potential $V(\Phi)$ as a function of $\phi$ then encodes, i.p., the details of the breakdown of the conformal and other symmetries.

E.g., the 5d geometry might be the holographic dual of RG flows in 4d gauge theories that approach a fixed point in the IR (the interior of $AdS_5$) but not in the UV (the boundary of $AdS_5$). Explicit examples of 5d geometries with varying potential $V$ have been discussed e.g. in [15, 16]; although most of the examples correspond, in the dual picture, to RG flows between UV and IR fixed points, there are also flows without UV fixed points.

Assuming a constant dilaton $\Phi$, the equations of motion of section 2 including (2.6) remain the same even in the case of a $\phi$–dependent $\Lambda_5$, and the conclusion is also the same:

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4This scenario of a 4d universe whose conformal invariance is slightly broken might be related to the proposal [14] of conformal invariance at the TeV scale.

5In this way one can also add all kinds of 4d matter and gauge fields to the scenario, in terms of Kaluza–Klein modes on $E^5$ or branes wrapped around cycles in $E^5$.

6which is the case in the solutions in [15, 16]; the general case would require redefining $\varphi = 2\Phi - \log \sqrt{g}$ and also involves the dilaton equation of motion.
there exists a solution for $\varphi(\phi)$ with

$$\dot{\varphi}^2 = -4\Lambda_5(\phi) \quad \rightarrow \quad \Lambda_4 = 0$$

everywhere, despite of the fact that conformal symmetry is broken. The general solution for $\varphi(\phi)$ has a constant but nonzero $\Lambda_4$ in (2.3). $\Lambda_4$ is $\phi$–independent since $\dot{g}$ and $\dot{R}$ are $\phi$–independent in (2.3). But if $\Lambda_4$ is zero at one $\phi$, it is zero for all $\phi$.

So we only need to argue that $\Lambda_4$ is zero in the interior of $AdS_5$. At this point it might seem that we have merely replaced one fine–tuning problem by another one: now we have to fine–tune $\dot{\varphi}^2$ to exactly cancel the five–dimensional cosmological constant.

However, the situation does seem to have improved. Before, we had to fine-tune the 4d cosmological constant at high energies to a precise nonzero value, such that the low–energy cosmological constant ends up being exactly zero, which seemed absurd. Now, by contrast, we only have to find an argument why the hypersurfaces of constant $\Lambda_5$ defined by the 5d gauged supergravity solution should be flat. This seems much more natural. We will return to this issue in the future.

So here we have a “holographic mechanism” that keeps the visible 4–dimensional cosmological constant zero with the help of one extra dimension: the 5–dimensional cosmological constant $\Lambda_5$ may be huge and moreover may change as a function of $\phi$, as matter fields roll down some potential $V$ (as in the above example of 5d gauged supergravity). But $\Lambda_4$ remains zero: $\Lambda_5(\phi)$ is completely absorbed by the $\phi$–dependence of $\varphi$ (i.e. of the “warp factor”).

4. Adding 4d gravity

The second immediate objection mentioned above was that the 4d world that lives on the AdS–boundary has zero Newton constant. A related point is that the modes of the 5d graviton that would represent massless 4d gravitons are not normalizable: there is no dynamical 4d gravity.\footnote{I thank P. Horava for first making this point.} To recall why the 4d Newton constant is zero we use the form

$$ds^2 = \frac{L^2}{z^2}(dz^2 + dx^2)$$
of the $AdS$–metric. As in ordinary Kaluza–Klein compactification on a circle (instead of the noncompact coordinate $z$), integrating the 5d Hilbert-Einstein action over the coordinate $z$ yields a 4d Hilbert-Einstein action:

$$
\frac{1}{\kappa_5^2} \int dz \, d^4x_{\parallel} \sqrt{g^{(5)}} R^{(5)} \to \frac{1}{\kappa_4^2} \int d^4x_{\parallel} \sqrt{\hat{g}^{(4)}} \hat{R}^{(4)}
$$

with

$$
\frac{1}{\kappa_4^2} = \frac{1}{\kappa_5^2} \int_0^\infty \frac{L^3}{z^3} \, dz.
$$

The problem is that this integral diverges and therefore $\kappa_4$ is zero.

This point may also be overcome by letting the geometry deviate from $AdS_5$ near its boundary at $z = 0$. A radical deviation would be to simply cut off the 5d universe near its boundary following Randall and Sundrum [17]: we restrict $z$ to the range

$$
z \geq \epsilon.
$$

This corresponds to an explicit sharp UV cutoff in the dual gauge theory. Now there are propagating 4d gravitons and the 4d Newton constant is nonzero:

$$
\kappa_4^2 = \frac{1}{3} \kappa_5^2 \frac{\epsilon^2}{L^3}.
$$

Of course, in general the value of the 4d Newton constant is not universal: it will depend on how precisely $AdS_5$ is cut off near its boundary. E.g., instead of a sharp cutoff one might try to look for smooth modifications of the metric near the $AdS$–boundary such that the warp factor, instead of being proportional to $\frac{1}{z^2}$, converges to zero at $z = 0$. This would also lead to a non–vanishing Newton constant and to dynamical 4d gravity. One might ask at first what kind of gauge theory flow such a geometry would be dual to. But the presence of 4d gravity seems to suggest that such geometries have no pure gauge theory interpretation; rather, the gauge theory should be embedded in a string theory. In fact, if the warp factor goes to zero at the boundary, the metric $g_{\mu\nu}$ goes to zero. This suggests that one should perhaps think of the radial direction of $AdS_5$ as being compactified. In building a consistent compactification, presumably one then inevitably ends up with the type of string compactification studied in [18].
At this point the simple picture we started with becomes complicated. In particular, it seems to be no longer clear how to argue that the vanishing of $\Lambda_4$ inside $AdS_5$ implies the vanishing of $\Lambda_4$ near the “boundary”. Certainly the equations used in section 2 are no longer appropriate – there are $\alpha'$-corrections and string loop corrections.

In conclusion, the conjecture that our flat 4-dimensional world is only the flat projection of a strongly curved 5-dimensional world seems intriguing, but solving the cosmological constant problem requires, at the least, further thought.

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