Strong Constraints on Fuzzy Dark Matter from Ultrafaint Dwarf Galaxy Eridanus II

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The fuzzy dark matter (FDM) model treats DM as a bosonic field with astrophysically large de Broglie wavelength. A striking feature of this model is $O(1)$ fluctuations in the dark matter density on time scales which are shorter than the gravitational timescale. Including for the first time the effect of core oscillations, we demonstrate how such fluctuations lead to heating of star clusters, and thus an increase in their size over time. From the survival of the old star cluster in Eridanus II we infer $m_a \gtrsim 0.6 \rightarrow 1 \times 10^{-19}$ eV within modelling uncertainty if FDM is to compose all of the DM, and derive constraints on the FDM fraction at lower masses. The subhalo mass function in the Milky Way implies $m_a \gtrsim 0.8 \times 10^{-21}$ eV to successfully form Eridanus II. The window between $10^{-21}$ eV $\lesssim m_a \lesssim 10^{-20}$ eV is affected by narrow band resonances, and the limited applicability of the diffusion approximation. Some of this window may be consistent with observations of Eridanus II and more detailed investigations are required.

A wide variety of astrophysical observations require the existence of non-baryonic dark matter (DM) \[1-4\]. At two extreme ends of the model space lie primordial black holes (PBHs) with masses as large as $M_{PBH} \approx 10 M_\odot$ and “fuzzy” DM (FDM) composed of particles (possibly axions) as light as $m_a \approx 10^{-22}$ eV \[5-8\]. The fraction of DM allowed in heavy PBHs is severely constrained by the dynamics of stars in ultrafaint dwarf galaxies \[9-10\]. Two body relaxation and gravitational scattering between the PBHs leads to heating of stars in the DM potential. This causes star clusters to grow in size on length and time scales incompatible with their observed sizes and ages, excluding a range of PBH parameter space, $(M_{PBH}, \Omega_{PBH}/\Omega_d)$, where $\Omega_{PBH}$ is the local fraction of the critical density in PBHs. In the following we will show that, somewhat remarkably, the very same observations of old star clusters in ultrafaint dwarf (UFD) galaxies place strong constraints on the FDM parameter space, $(m_a, \Omega_a/\Omega_d)$.

FDM is modelled as a coherent bosonic field, $\phi$, and in the minimal non-interacting case has the potential $V(\phi) = m_a^2 \phi^2/2$. At late cosmological times, $H < m_a$, the field oscillates in the potential minimum. This coherence leads to fluctuations on two distinct time scales. Firstly, relativistic Compton scale fluctuations of order $m_a^{-1} \approx m_a^{-1}$ month, where $m_{22} = m_a/10^{-22}$ eV. These lead to pressure perturbations, and in turn metric fluctuations and can be searched for by a variety of techniques including pulsar timing arrays \[11\], binary pulsar orbits \[12\], “scalar gravitational waves” \[13\], and other relativistic effects \[14\]. They also underlie methods of direct detection of FDM \[15\]. In the present work we neglect Compton fluctuations since the time scale and amplitude are not relevant to the dynamics of star clusters in DM halos.

In the WKB approximation in the non-relativistic environments of DM halos, the Compton scale is integrated out (see e.g. Ref. \[7\]). However, fluctuations remain on the de Broglie scale, $\lambda_{dB} = 2\pi/m_a v$ (we use units $\hbar = c = 1$), with oscillation period $\tau_{osc} = 2\pi/m_a v^2$. In linear theory, these fluctuations manifest as the FDM Jeans scale $\lambda_{Jeans} \approx a_0/\lambda_{dB}$. The Jeans scale suppresses FDM structure formation relative to CDM. This drives constraints on FDM from e.g. CMB lensing \[17\], high-$z$ galaxy formation \[18, 21\], and the Lyman-alpha forest \[22, 23\], leading to the bound $m_a \gtrsim 10^{-22} \rightarrow 10^{-21}$ eV depending on the data and modelling. In terms of the halo mass function, numerical \[19, 21\] and semi-analytical \[6, 20\] calculations predict that the abundance of halos in FDM is severely reduced relative to CDM for masses less than $M_{cut} \approx 3 \times 10^5 m_{22}^{-3/2} M_\odot$.

Inside DM halos the de Broglie fluctuations are observed in simulations as granular structure in the outer halo resulting from wave interference \[26, 27\]. It is the central insight of Ref. \[8\] that these fluctuations can be treated statistically as short-lived quasiparticles, and lead to heating effects and relaxation in a similar way to MACHOs and PBHs. The relaxation time is estimated as:

$$t_{relax} \approx \frac{3}{10^{10} \text{yr}} m_{22}^3 \left( \frac{v}{100 \text{ km s}^{-1}} \right)^2 \left( \frac{r}{5 \text{ kpc}} \right)^4$$

(1)

The effect of FDM fluctuations on stellar dynamics in the Milky Way (MW) region has been investigated extensively in the recent literature. The systems so far investigated impose constraints on the FDM mass of order $m_a > 0.6 \rightarrow 1.5 \times 10^{-22}$ eV from the thickening of the disk \[28\] and stellar streams \[29\] respectively.

FDM simulations also point to the existence of a central DM core on the de Broglie scale, known as a soliton, or axion star \[26\]. In recent zoom-in simulations of FDM galaxies \[27\] it was observed that the central soliton is not stationary, as was previously thought, but undergoes quasi-coherent oscillations in its central density, with a relative amplitude of $O(30\%)$ and period $O(\tau_{osc})$. The present work presents the first study of the effect of core oscillations on stellar dynamics.
FDM solitonic cores are observed to form in simulations of dwarf galaxies with $M \approx 10^{10} M_\odot$ when $m_a \approx 10^{-22}$ eV. They form by direct collapse almost instantaneously when the halo virialises. For larger FDM masses, however, it is not clear whether soliton formation in dwarf galaxies will proceed in the same way, since the length scales involved are much longer than the de Broglie wavelength. The time scale for soliton formation by wave condensation increases at larger particle masses \cite{30}, and thus solitons may not have had time to form in all halos for all FDM masses.

Assuming it forms, the central soliton has the density profile of the ground state of the Schrödinger-Poisson equation. The solution $\rho_{\text{sol}}(r)$ is a one parameter family described by the core radius, $r_c$, and has a flat central density, $\partial_r \rho_{\text{sol}} = 0$. The soliton mass within the core radius is observed to follow a scaling relation with the host halo mass, which at $z = 0$ is given by \cite{31}:

$$M_{\text{sol}} = \frac{M_0}{4} \left( \frac{M_h}{M_0} \right)^{1/3},$$

(2)

where the scale $M_0 \approx 4.4 \times 10^7 m_{22}^{-3/2} M_\odot$ is approximately the Jeans mass. The relation Eq. (2) can be used to fix $r_c$ in terms of $M_h$:

$$r_c = 740 \left( \frac{m_a}{10^{-21} \text{ eV}} \right)^{-1} \left( \frac{M_h}{10^2 M_\odot} \right)^{-1/3} \text{ pc}.$$  

(3)

The central soliton has some favourable consequences, e.g., its stabilising effect on the cold clump in Ursa Minor \cite{42}, a possible explanation for cored density profiles in dSphs \cite{20, 33, 34} and UFDs \cite{35}, help alleviating the "too big to fail" problem \cite{6, 36}, and an explanation for excess mass in the centre of the MW \cite{37} (though the cusp-core problem in $M \approx 10^{11} M_\odot$ galaxies is exacerbated \cite{36}). These observations, as well as other hints from the small-scale structure of DM \cite{6, 8, 38}, point to a preferred FDM mass $m_{22} = O(\text{few})$.

Eridanus II (Eri II) is a UFD with a centrally located star cluster. Its properties are inferred from observations reported in Refs. \cite{10, 29}. Eri II is located at a distance of 370 kpc from the centre of the MW. The mass within the half-light radius is estimated as $M_{\text{EII}} = 1.2^{+0.4}_{-0.3} \times 10^7 M_\odot$, 1D velocity dispersion $\sigma_v = 6.9^{+1.2}_{-0.9} \text{ km s}^{-1}$, and central DM density $\rho_{DM} = 0.15 M_\odot \text{ pc}^{-3}$. The central star cluster has a half light radius $r_h = 13 \text{ pc}$, age $T_{\text{EII}} = 3 \rightarrow 12 \text{ Gyr}$ and mass $M_s = 2000 M_\odot$.

We can use these basic properties of Eri II to assess the relevant FDM scales. The total number of MW subhalos in the 2$\sigma$ range around $M_{\text{EII}}$ ($M_{\text{low}} = 4 \times 10^6 M_\odot$, $M_{\text{up}} = 2 \times 10^7 M_\odot$) is

$$n_{\text{EII}}(m_a) = \int_{M_{\text{low}}}^{M_{\text{up}}} \text{d} M \frac{dn_{\text{sub}}(m_a)}{d \ln M},$$

(4)

where $dn_{\text{sub}}/d \ln M$ is the subhalo mass function (see Fig. 1). We estimate the FDM subhalo mass function with the fits of Ref. \cite{10}, which uses the methods of Refs. \cite{6, 8, 11, 22} applied to numerical merger trees \cite{43}. The exclusion on $m_a$ implied by the existence of Eri II is found by solving $n_{\text{EII}}(m_a) = 1$, which gives the approximate bound $m_a \gtrsim 8 \times 10^{-22}$ eV. As a comparison we also test the subhalo mass function of Refs. \cite{14, 16} computed using the sharp-k filtering method \cite{44}. This gives the stronger bound $m_a \gtrsim 8 \times 10^{-21}$ eV. We take the weaker bound as more conservative given the large theoretical uncertainty in the subhalo mass function. A similar bound would apply to the sharp-k filter if the total mass of Eridanus II is significantly larger than the mass contained within the half-light radius. A more crude estimate based on the mass function cut-off alone implies $m_a \gtrsim 10^{-21}$ eV. At the limit $m_a = 10^{-21}$ eV we find that the soliton mass is of order $M_{\text{EII}}$, and the UFD is a single core remnant (see also Ref. \cite{17}). For larger values of $m_a$, Eri II will have a granular outer halo in addition to the core.

The stability of the star cluster in Eri II can be taken to imply the existence of a DM core with radius $r_c \geq r_h$. We estimate the FDM mass preferred by a core in Eri II by setting $\rho_{\text{sol}}(r_h) = \rho_{DM}$, which implies $m_a \approx 10^{-20}$ eV. Note that this is significantly larger than the FDM mass required for cored profiles in UFDs Draco II or Triangulum III \cite{35}, or dSphs Fornax and Sculptor \cite{33, 34} to be explained by the presence of a soliton. Assuming that the total mass of Eri II is given by $M_{\text{EII}}$, using Eq. (3) with $M_h = M_{\text{EII}}$ we can fix $r_c = r_h$ and solve for $m_a$ to find the highest possible FDM mass consistent with the star cluster residing within the soliton.

![FIG. 1. Number of subhalos in the range of the Eri II half-light mass as a function of FDM mass $m_a$. Solid: from merger trees, modified barrier and core stripping; dotted: no core stripping; dashed: sharp-k filter. We demand FDM produce at least one subhalo in the Eri II region (black dotted horizontal line), and take the weaker bound as more conservative given the mass function uncertainties. The horizontal red lines show the CDM prediction, which FDM converges to in the limit $m_a \rightarrow \infty$.](image-url)
core, \( m_\alpha \approx 10^{-19} \) eV. For \( m_\alpha \lesssim 10^{-20} \) eV the Eri II star cluster is guaranteed to be inside the soliton core. For \( 10^{-20} \) eV \( \lesssim m_\alpha \lesssim 10^{-19} \) eV(within the observational uncertainty on the location of the star cluster, and theoretical uncertainty on the formation time of the soliton) it is possible for the star cluster to lie either inside or outside the soliton.

**Diffusion Approximation: Star Cluster Heating**

The diffusion approximation applies for outer halo fluctuations and stochastic core oscillations as long as \( \tau_{osc} \) is less than the stellar orbital period, \( \tau_{orb} \). The typical oscillation frequency is \( \omega = m_\alpha \sigma_{3D}^2 \), with \( \sigma_{3D} = \sqrt{3} \sigma_{1D} \). Taking the stellar period to be the Keplerian period we find:

\[
\frac{\tau_{orb}}{\tau_{osc}} \sim \frac{m_\alpha}{10^{21}} \text{ eV}.
\]

(5)

We assume that the density fluctuations have a shot noise distribution produced by the granular interference structure of the scalar field, and relate the spatial and temporal fluctuations with the dispersion velocity \( v \) of the dark matter as \( r = vt \).

We derive the gravitational heating rate produced by a fluctuating density field from the force correlation function, closely following the arguments described in Ref. [48] for the case of a turbulent baryon field. The force correlation function is the Fourier transform of the force power spectrum, given by

\[
\langle F(0)F(r) \rangle = \frac{1}{2\pi^2} \int \mathcal{P}_F(k) \frac{\sin(kr)}{kr} k^2 dk
\]

assuming statistical isotropy. The force power spectrum produced by fluctuations of the gravitational potential, \( \Phi \), in the volume \( V \),

\[
\mathcal{P}_F(k) = V k^2 \langle |\Phi_k|^2 \rangle,
\]

(7)

is related to the power spectrum of density fluctuations

\[
\mathcal{P}_\delta(k) = V \langle |\delta_k|^2 \rangle,
\]

(8)

where \( \delta_k \) are the Fourier components of the density contrast \( \delta = \rho/\rho_0 - 1 \), by the Poisson equation \( k^2 \Phi_k = -4\pi G \rho_0 \delta_k \).

As stated above, we assume a \( k \)-independent shot noise density power spectrum, \( \mathcal{P}_\delta \sim n^{-1} \), with \( n \sim (l_c/2)^{-3} \) determined by the scalar field coherence length \( l_c = 2\pi/k_c \sim (mv)^{-1} \). In this case,

\[
\mathcal{P}_F(k) = (8\pi G \rho_0)^2 \mathcal{P}_\delta k^{-2}
\]

(9)

and

\[
\langle F(0)F(r) \rangle = \frac{C}{r} \int_{k_0}^{k_c} \frac{\sin(kr)}{kr} \, dk = \frac{C}{r} \text{ Si}_k k_c
\]

(10)

where \( k_0 \) corresponds to the largest fluctuation scale and \( C = 8G^2 \rho_0^2 \mathcal{P}_\delta \).

Following Ref. [48] (see also Ref. [49]), we compute the velocity variance induced by the force fluctuations on the trajectory of a star in the cluster during the time \( \tau \) as

\[
\langle (\Delta v)^2 \rangle = \frac{2C}{v} \int_0^{r \tau} (\tau - t) \langle F(0)F(t) \rangle \, dt
\]

\[
= \frac{2}{v^2} \int_0^{r \tau} (\tau - t) \langle F(0)F(t) \rangle \, dt,
\]

(11)

where we used the dark matter velocity dispersion \( v \) to relate the temporal fluctuations to the spatial ones as explained above. In the diffusion limit we demand that \( \tau \gg \tau_{osc} \), i.e. that the orbital period of the stars is greater than the fluctuation time scale.

In the limit \( k_0 v \tau \ll 1 \), Eq. (11) evaluates to

\[
\langle (\Delta v)^2 \rangle = \frac{2C}{v} \cdot (k_c v \tau)^{-1} \left( 1 - \cos(k_c v \tau) \right)
\]

\[
+ k_c v \tau 2 F_3 \left( \frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, - \left( \frac{k_c v \tau}{2} \right)^2 \right)
\]

\[
- \text{Si}(k_c v \tau),
\]

(12)

where \( 2F_3 \) is the generalized hypergeometric function. Considering the diffusion limit \( k_c v \tau \gg 1 \), we can neglect the first term in square brackets, the last one asymptotes to \( \pi/2 \), and the middle one gives approximately \( \pi/2 (\log(k_c v \tau) + 0.6) \). Together, we obtain

\[
\langle (\Delta v)^2 \rangle \simeq \frac{\pi C}{v} \log(k_c v \tau),
\]

(13)

where the logarithmic term can be identified with the Coulomb logarithm, i.e. the logarithm of the ratio of the largest and smallest relevant length scales of the system.

The relaxation time is defined as the time \( \tau \) for which the induced velocity variance equals the mean square velocity of the stars \( v^2_c \),

\[
t_{relax} = \frac{v^2_c}{\pi C \log(k_c v \tau)}.
\]

(14)

Finally, the diffusion coefficient for the gravitational heating of the star cluster is given by [50]

\[
D \left[ (\Delta v)^2 \right] = \frac{v^2}{t_{relax}} \approx \frac{\langle (\Delta v)^2 \rangle}{\tau}
\]

\[
\simeq \frac{8\pi G^2 \rho_0^2 \mathcal{P}_\delta}{v \tau} \log(k_c v \tau).
\]

(15)

It is interesting to compare Eq. (15) with the corresponding diffusion coefficient for gravitational heating by MACHOs [50] applied to the Eri II star cluster by Brandt [9]. Replacing the MACHO mass in Brandt’s Eq. (1) with the mass of granular quasiparticles [8], \( m_{qp} = \rho_0 (l_c/2)^3 = \rho_0 \mathcal{P}_\delta \), we obtain the identical result for the heating rate up to a factor of \( \sqrt{2} \) and the precise

1 The main difference to [48] is that density fluctuations are dominated by the smallest scale in our case as opposed to the largest scale in theirs.
definitions of the Coulomb logarithm which are $\lesssim \mathcal{O}(10)$ in both cases. This demonstrates that the quasiparticle model for FDM and shot noise density fluctuations produced by interference patterns of the scalar field make equivalent predictions for the diffusion coefficient (see [5] for an in-depth discussion of diffusion coefficients in FDM scenarios).

Using $dv^2/dt = D$ and the virial theorem, one can find an equation for the growth of the star cluster radius [9]. The half-light radius $r_h$ evolves as

$$\frac{dr_h}{dt} = C F \frac{8\pi G \rho_0 P_5}{v} \log(k_{\nu} \nu \tau) \left( \frac{M_*}{\rho_{h}^2} + 2\beta r_h \right)^{-1},$$

where $F = \Omega_a/\Omega_d$. For diffusion caused by the density granules in the outer halo we have $C = 1$, while for diffusion inside the core we take $C = 0.3$ to account for the amplitude of core density fluctuations found in simulations [27]. We use ten orbital periods of stars in the cluster to estimate $\tau$ in the Coulomb logarithm and set $\alpha = 0.4$, $\beta = 10$ as in [9]. We then use Brandt’s criteria on the evolution timescale of the cluster in Eri II to constrain the axion mass $m_a$: the time for $r_h$ to grow from 2 pc to 13 pc must be longer than the age of the cluster, 3 Gyr. The resulting exclusions on $(m_a, \Omega_a/\Omega_d)$ are shown in Fig. 2.

**Perturbation Theory: Star Cluster Resonances**

The star cluster evolution time scale caused by coherent density fluctuations inside the core can also be estimated using standard perturbation theory [52]. The DM mass contained within the half-light radius is $M_{DM}(r < r_h) \approx (4/3)\pi \rho_{DM} r_h^3 = 330M_\odot$, assuming the density is cored, giving $M_{DM} < M_\star$, suggesting that the star cluster is self-bound. Consider a star of mass $m_\star$ on a Keplerian orbit with semi-major axis $a_0 = r_h$ about the centre of mass of the star cluster, $V_0 = -G M_\star m_\star / r = -k/r$, in terms of the action-angle variables $(w_i, J_i)$ in the limit $m_\star \ll M_\star$. The unperturbed Hamiltonian is:

$$H_0 = \frac{2\pi^2 m_\star k^2}{J_3^2},$$

where $w_3 = t/\tau_{orb} + \text{const.}$ with $\tau_{orb} = 0.1$ Gyr is the Keplerian period. The semi-major axis $a_0 = J_3^2/4\pi^2 m_\star k$. The size fluctuations of the solitonic core [27] imply that stars within the core see a fluctuation of mass within the core radius. The perturbation Hamiltonian is:

$$\Delta H = C \frac{\Omega_a}{\Omega_d} V_0 \frac{M_{DM}}{M_\star} \cos \omega_{osc} t,$$

where $C \approx 0.3$ [27]. The time evolution of the semi-major
axis is given by
\[
\dot{a} = 2\Omega_0^2 \frac{M_{DM}}{a_0^2 \Omega_d} \frac{\sin \omega_{osc} t}{r},
\]
where \(r(t)\) is computed for a Keplerian orbit of eccentricity \(e\) and semi-major axis \(a_0\) given by the unperturbed solution. We fix \(e = 0.5\) and we have verified that the results are not strongly dependent on this value in the range \(0.1\) to \(0.9\).

The solution for \(a(t)\) is found by integrating Eq. (19) with initial condition \(a(0) = a_0\). The integrand oscillates on time scales much shorter than the lifetime of the star cluster, preventing a direct numerical solution. However, after one long cycle (the longer of \(\tau_{orb}\) and \(\tau_{osc}\)) the evolution settles down into a new periodic state around a different value of \(a\). We thus take the final value of \(a\) to be the average over the period \(\tau_{av} = 10 \times \text{Max}(\tau_{orb}, \tau_{osc})\), which can easily be computed numerically. Constraints are imposed by demanding the average orbit size does not double, \(a_{\text{final}}/a_0 < 2\). The perturbation analysis only excludes four orbital resonances in the range \(10^{−19} \text{ eV} \lesssim m_a \lesssim 10^{−16} \text{ eV}\) [53, 54], and masses larger than \(10^{−16} \text{ eV}\) are unlikely to have any observational signatures distinct from CDM on astrophysical scales. Any FDM masses of astrophysical relevance outside of our small window require going beyond the non-interacting model to be consistent with observations. Assuming a UV completion of FDM as an ultralight axion with a cosine instanton potential, the strength of interactions is set by the decay constant, \(f_a\). Masses \(m_a \lesssim 10^{−21} \text{ eV}\) tend to require lower decay constants based on the DM relic abundance [7]. Lower values of \(f_a\) increase the strength of axion self-interactions and relieve the constraints from black hole superradiance due to the Bosonova effect [55]. Lower values of \(f_a\) also lead to tachyonic instabilities during structure formation and relieve the Lyman-alpha forest bounds [56, 57].

The addition of significant self-interactions makes the phenomenology of FDM much more interesting: non-linear structure evolves differently [58], and axion stars are more unstable to collapse and nova [59, 60]. These phenomena could lead to as yet unexplored signatures of ultralight axions, but require simulations beyond those of vanilla FDM. Furthermore, at higher axion masses and lower decay constants the prospects for direct detection are greatly enhanced [61, 62].

Note added: While this work was in preparation, Ref. [51] appeared, which also derives relaxation effects produced by FDM halo fluctuations, applied to the cases of dynamical friction of very massive objects (satellites, supermassive black holes), and to the heating of early type galaxies.

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