Affleck–Dine baryogenesis in anomaly-mediated supersymmetry breaking

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Abstract. It is known that in the anomaly-mediated supersymmetry breaking model, Affleck–Dine baryogenesis does not work due to trapping of the Affleck–Dine field into charge breaking minima. We show that when the finite-temperature effect is properly taken into account and if the reheating temperature is relatively high, the problem of falling into charge breaking global minima can be avoided and hence Affleck–Dine baryogenesis works. Moreover, for the $LH_u$ flat direction we obtain a constraint on the mass of the neutrino.

Keywords: baryon asymmetry, physics of the early universe

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1. Introduction

The origin of baryon asymmetry, or matter–anti-matter asymmetry, is one of the most interesting topics in both cosmology and particle physics. From WMAP three-year results [1],

$$\eta = \frac{n_B}{n_\gamma} \simeq (6.1 \pm 0.2) \times 10^{-10}. \quad (1)$$

is obtained. Primordial big-bang nucleosynthesis explains the observed light element abundances for about the same value of $\eta$ [2]. To explain this value, various baryogenesis mechanisms have been considered.

Supersymmetry (SUSY) [3] is the most attractive candidate for providing physics beyond the standard model. Thus it is worthwhile to construct a baryogenesis model based on SUSY. The Affleck–Dine mechanism [4] is one of the most studied baryogenesis scenarios in the framework of the supersymmetric standard model [5]. This uses the dynamics of flat directions existing in the minimal supersymmetric standard model (MSSM), which is constructed of the scalar fields having flat potential in the supersymmetric limit at the renormalizable level. The angular motion of flat directions...
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\[ n = \text{i}N(\dot{\phi}^* \phi - \phi^* \dot{\phi}) = N|\phi|^2 \dot{\theta}, \]  

where \( N \) is the constant determined by the flat direction, and we have defined \( \phi = |\phi|e^{i\theta}. \) How this angular motion is generated or what amount of baryon asymmetry can be created depends on the type of flat direction, the temperature of the universe and the mechanism of supersymmetry breaking. Finite-temperature effects [7, 8] are another important correction, which can significantly affect the whole dynamics.

Furthermore, formation of \( Q \)-balls makes the usual Affleck–Dine scenario complicated [9, 10]. The property of \( Q \)-balls strongly depends on the SUSY breaking mechanism. In the gravity-mediation scenario, the \( Q \)-ball is unstable against decay into nucleons and the created baryon number is finally converted to ordinary matter [11, 12]. In the gauge-mediation scenario, however, the \( Q \)-ball is stable and only evaporation and diffusion effects can extract the baryon number from \( Q \)-balls [13]–[15]. On the other hand, the \( Q \)-ball can explain naturally why the dark matter and baryon number density are roughly of the same order in the universe in some specific models [16]. Therefore, when considering the Affleck–Dine baryogenesis scenario, one must carefully trace the dynamics of flat directions taking into account various effects.

Anomaly-mediation models [17] have attractive features from the phenomenological point of view. In this scenario, SUSY breaking effects in the hidden sector are transmitted to the observable sector through the super-Weyl anomaly. This predicts model-independent generic features of low energy physics insensitive to physics at the high energy scale. As a consequence, the flavour problem existing in the usual gravity-mediation scenario is relaxed, and the new possibility of wino-like dark matter is provided [18, 19]. Gravitino mass becomes two orders of magnitude larger than that for the gravity-mediation case, which also solves the gravitino problem. However, as explained in the next section, Affleck–Dine baryogenesis in anomaly-mediation models has been revealed to be difficult [20, 19].

In this paper, we study the Affleck–Dine mechanism in anomaly-mediation models including finite-temperature effects, which were missed in the previous literature. It is shown that when the finite-temperature effect is properly taken into account, Affleck–Dine baryogenesis works. We also discuss \( Q \)-ball formation and its consequences.

This paper is organized as follows. First, in section 2, potentials for flat directions including finite-temperature effects are given. In section 3 we describe the dynamics of \( n = 4 \) flat directions when finite-temperature effects are taken into account. The case with the \( n = 6 \) flat direction is discussed in section 4. In section 5 the effects of \( Q \)-ball formation are described and we conclude in section 6.

2. Potential of the Affleck–Dine field in the anomaly-mediation model

2.1. Zero-temperature potential

First, we summarize the standard scenario for Affleck–Dine baryogenesis neglecting the finite-temperature effect. Potentials of flat directions in the MSSM are exactly flat in the renormalizable and supersymmetric limit, but lifted by non-renormalizable terms and the supersymmetry breaking effect. If we parametrize a flat direction \( \phi \), the non-
renormalizable superpotential
\[ W = \frac{\phi^n}{nM^{n-3}} \] (3)
generates the potential for \( \phi \),
\[ V = \frac{|\phi|^{2(n-1)}}{M^{2n-6}}. \] (4)

where \( M \) denotes some cut-off scale\(^1\). All MSSM flat directions are known to be lifted up to \( n = 9 \) gauge-invariant superpotentials. Including supersymmetry breaking effects, the potential would be
\[ V = (m^2 - cH^2)|\phi|^2 + \left\{ a_m \frac{m_3/2\phi^n}{nM^{n-3}} + a_H \frac{H\phi^n}{nM^{n-3}} + \text{h.c.} \right\} + \frac{|\phi|^{2(n-1)}}{M^{2n-6}}, \] (5)

where \( H \) is the Hubble parameter, \( c, a_m \) and \( a_H \) are O(1) constants. Initially, the \( \phi \) field is trapped at the minimum determined by the negative Hubble mass term \(-H^2|\phi|^2\) and the highest term \(|\phi|^{2(n-1)}/M^{2n-6}\) is
\[ |\phi| \simeq (HM^{n-3})^{1/(n-2)}. \] (6)

The \( \phi \) field traces this minimum until \( H \) becomes less than \( m_\phi \) and it begins to oscillate around the origin. The angular direction of \( \phi \) is determined by the Hubble-induced \( A \)-term up to this epoch. If \( m_\phi \sim m_3/2 \) as expected in gravity mediation, at the same time as \( \phi \) begins to oscillate, the soft \( A \)-term begins to dominate over the Hubble-induced \( A \)-term, and this causes the kick in the angular direction. Finally the \( \phi \) field shows the \( U(1) \) conserving elliptical motion around the minimum. At this stage, the baryon number created is conserved. This is the standard scenario for creating baryon asymmetry in the Affleck–Dine mechanism.

In anomaly-mediation models, however, there is a subtlety. Since soft mass is loop suppressed in anomaly mediation, we expect \( m_\phi \sim m_3/2/(8\pi^2) \). If we assume that soft masses should be 100–1000 GeV, the natural order of gravitino mass is estimated to be 10–100 TeV, two orders of magnitude larger than that of the gravity-mediation case. This is a good feature for avoiding the gravitino problem. For such large gravitino mass, its lifetime naturally becomes shorter than 1 s, which does not much affect BBN. In fact, even if its hadronic branching ratio is order 1, there is no upper bound on the reheating temperature if the gravitino mass is as large as 100 TeV [21]. However, this invalidates the usual Affleck–Dine baryogenesis scenario because the potential of the flat direction (5) has charge and/or colour breaking global minima with
\[ |\phi|_{\text{min}} \sim \left( \frac{|a_m|}{n-1} m_3/2 M^{n-3} \right)^{1/(n-2)}. \] (7)
The minimum value of the potential becomes
\[ V(|\phi|_{\text{min}}) \sim -\frac{n-2}{n} M^4 \left( \frac{|a_m|}{n-1} m_3/2 M \right)^{(2n-2)/(n-2)}. \] (8)

\(^1\) We use the same symbol \( \phi \) for a chiral superfield and for its scalar part.
which is always negative for \( n \geq 4 \). This is not a problem if the relevant fields sit at the origin initially, since the rate of decay of the false vacuum into a true charge breaking minimum is sufficiently small and the lifetime is longer than the age of the universe [20]. But in the Affleck–Dine set-up, the corresponding flat direction should have a large field value tracking the minimum (6), and finally fall into charge breaking minima (7). This is a fundamental problem when applying the Affleck–Dine mechanism to anomaly-mediation models.

There has been an attempt to obtain Affleck–Dine baryogenesis in anomaly-mediation models based on gauged \( U(1)_{B-L} \) symmetry [22]. This uses the fact that the \( U(1)_{B-L} \) breaking effect stops the \( \phi \) field moving beyond the \( U(1)_{B-L} \) breaking scale \( v \) due to the \( D \)-term contribution. If \( v \) is smaller than the field value corresponding to the hill of the potential (5),

\[
|\phi|_{\text{hill}} \sim \left( \frac{m_\phi^2 M^n}{m_{3/2}} \right)^{1/(n-2)},
\]

we do not need to worry about falling into the unphysical global minima. This is an appealing feature, but this model relies on the hypothesis of gauged \( U(1)_{B-L} \) symmetry. In the following, we show that the finite-temperature effect enables us to obtain baryon asymmetry and avoid the charge breaking minima in anomaly-mediation models without any further assumption beyond the MSSM.

2.2. Finite-temperature effect

Thermal effects modify the potential of flat directions. First, coupling of flat directions with other particles \( \psi \) yields a thermal mass term [7] given by

\[
\sum_{f_k|\phi|<T} c_k f_k^2 T^2 |\phi|^2,
\]

where \( c_k \) is a constant of \( O(1) \) and \( f_k \) denotes gauge or Yukawa coupling relevant for the flat direction. Note that when \( f_k|\phi| > T \), \( \psi \) receives a large mass of order \( f_k|\phi| \) and cannot be thermalized, and hence \( \phi \) does not feel thermal mass.

It was also pointed out that the following form of the potential [8]:

\[
V \sim a \alpha(T)^2 T^4 \log \left( \frac{|\phi|^2}{T^2} \right)
\]

should be included for the potential of flat directions, where \( a \) is an order 1 constant determined by a two-loop finite-temperature effective potential. For the \( LH_u \) direction, \( a = 9/8 \). When this term dominates, it is possible that the flat direction begins to oscillate due to the thermal logarithmic term. The epoch of the onset of oscillation is determined by

\[
H_{\text{OS}}^2 \sim m_\phi^2 + \sum_{f_k|\phi|<T} c_k f_k^2 T^2 + a \alpha(T)^2 T^4 \left( \frac{T^4}{|\phi|^2} \right).
\]

Details of the dynamics depend on corresponding flat directions and are somewhat complicated [23,24]. Now let us investigate this for \( n = 4 \) and 6 cases.
3. The case of the $n = 4$ flat direction

In this section, we describe the dynamics of the AD field for $n = 4$ based on the potential given in the previous section, concentrating on the $LH_u$ direction particularly. This is because the Affleck–Dine baryogenesis cannot work successfully for other $n = 4$ directions (see section 3.3).

3.1. Dynamics of the $n = 4$ flat direction

From equation (12) for $n = 4$, early oscillation due to the thermal log term occurs when

$$T_R \gtrsim \frac{m_\phi}{\alpha(T)} \left( \frac{M}{M_P} \right)^{1/2},$$

(13)

where $T_R$ is the reheating temperature after inflation, and in this case the Hubble parameter at the start of the oscillation is

$$H_{OS} \sim \alpha T_R \left( \frac{M_P}{M} \right)^{1/2}.$$  

(14)

Here we have used $T \sim (T_R^2 H M_P)^{1/4}$ and $|\phi| \sim (HM)^{1/2}$. For a natural range of cut-off scale $M$, early oscillation naturally takes place unless the reheating temperature $T_R$ is unnaturally low.

It should be noticed that, for sufficiently high reheating temperature, this thermal logarithmic potential can hide the unwanted valley of the potential. Substituting $n = 4$ into equation (8), the minimum of the zero-temperature potential is given by

$$V(|\phi|_{\text{min}}) \sim -\frac{1}{54} m_3^{3/2} M.$$  

(15)

In order to avoid falling into this minimum, at least $\alpha^2 T^4 \gtrsim |V(|\phi|_{\text{min}})|$ is required at the beginning of the oscillation. This leads to

$$T_R \gtrsim \alpha^{-1} m_3^{3/2} \left( \frac{M}{M_P} \right)^{1/2}.$$  

(16)

If we assume $M \sim M_P$, $T_R \gtrsim 10 m_3^{3/2} \sim 10^{5-6}$ GeV is needed to satisfy the above condition. Note that thermal mass term cannot dominate over the thermal logarithmic term at this epoch. We have checked numerically that the above condition is almost sufficient to drive the Affleck–Dine field into the origin. As another condition for avoiding global minima, one may require $|\phi| < |\phi|_{\text{hill}}$ at $H \lesssim m_3/2$, which is the epoch where the soft $A$-term begins to dominate over the Hubble-induced $A$-term. This is achieved when the following condition is satisfied:

$$T_R \gtrsim \alpha^{-1} \frac{m_3^{3/2}}{m_\phi} \left( \frac{M}{M_P} \right)^{1/2}.$$  

(17)

But in fact this condition is too strong. This condition is sufficient, but not always necessary. Numerical calculation shows that the condition (16) is almost sufficient.

Thus, for high enough reheating temperature, Affleck–Dine baryogenesis can work irrespective of the charge breaking minima of the potential. To confirm this statement,
we have performed numerical calculations with a full scalar potential including the finite-temperature effect. For simplicity, we set $M = M_P$ (in next section, we see that this choice is valid for obtaining a proper amount of baryon asymmetry). As explained above, $T_R \gtrsim 10^6$ GeV is needed to obtain appropriate motion of the flat direction. In figure 1, we show the result when $T_R = 10^6$ GeV and $m_{3/2} = 100$ TeV. Clearly one can see that the $\phi$ field falls into the origin with angular motion, which indicates that the Affleck–Dine baryogenesis works. The resultant baryon asymmetry is analysed in the next section.

On the other hand, in figure 2 the result for $T_R = 10^5$ GeV is shown. In this case, the finite-temperature effect is insufficient to take the $\phi$ field into the origin, and finally it is trapped at the charge breaking displaced minimum (7). Therefore, in order to make the
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Affleck–Dine scenario successful in anomaly-mediation models, a reheating temperature of at least $T_R \gtrsim 10^6$ GeV is needed, though this value varies depending on the cut-off scale $M$.

3.2. Baryon number generation

Let us estimate the baryon number created in this process. Actually the $LH_u$ direction generates lepton number, but the electroweak sphaleron process quickly converts it into baryon number [25, 26]. From equation (2) and the equation of motion of $\phi$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi^*} = 0,$$

we obtain

$$\dot{n} + 3Hn = 2N\text{Im}\left(\frac{\phi}{\phi^*} \frac{\partial V}{\partial \phi}\right).$$

Figure 2. Upper: typical motion of the field $\phi$ in the complex plane when $T_R = 10^5$ GeV. The dotted line represents the valley of the potential from the soft $A$-term. Lower: the motion near the minimum is magnified.
The baryon number per comoving volume is almost fixed at the beginning of the oscillation. Integrating the above expression, we obtain

\[ n(t_{OS}) \sim N \delta_e m_{3/2} (H_{OS} M^{n-3})^{2/(n-2)}, \]  

(20)

where \( \delta_e = \sin(\arg a_m + n \arg \phi) \) represents the degree of CP violation, which is naturally expected to be of order 1. Notice that the reheating in which the inflaton decays completely takes place after the AD field starts oscillation. After reheating, the baryon-to-entropy ratio is estimated as

\[ \frac{n}{s} = \frac{n(t_R)}{s(t_R)} \sim N \delta_e m_{3/2} T_R \frac{H_{OS}^2 M_P^2}{H_{OS}^n M^{3-n-2}}. \]  

(21)

In the present \( LH_u \) case, \( H_{OS} \) is given by equation (14),

\[ n \sim 10^{-10} \delta_e \left( \frac{0.1}{\alpha} \right) \left( \frac{m_{3/2}}{50 \text{ TeV}} \right) \left( \frac{10^{-7} \text{ eV}}{m_\nu} \right)^{3/2} \]  

(22)

where we have used the fact that \( m_\nu \) is given by

\[ m_\nu \sim \left( \frac{H_u}{M} \right)^2 = \frac{\sin^2 \beta}{M} (174 \text{ GeV})^2 \]  

with \( \sin \beta \sim 1 \). Interestingly, the baryon-to-entropy ratio (22) is independent of the reheating temperature [24]. This is because lowering the reheating temperature tends to generate a small baryon-to-entropy ratio, but on the other hand, the epoch of oscillation due to the thermal logarithmic term becomes late and leads to larger baryon number. As a result, these two effects cancel and the baryon-to-entropy ratio becomes independent of the reheating temperature, as long as just the requirement \( T_R \gtrsim 10^5 \text{ GeV} \) is satisfied. The mass of the neutrino should be less than \( \sim 10^{-7} \text{ eV} \) for successful baryogenesis. Note that due to the largeness of \( m_{3/2} \), the constraint on the neutrino mass for successful baryogenesis is weaker than that of the usual gravity-mediation case [24].

3.3. Some comments

Some comments are in order.

First, we comment on the instability of AD condensate and \( Q \)-ball formation in our scenario. If \( Q \)-balls are formed, almost all charges are trapped into them [12] and the subsequent evolution becomes complicated. When the finite-temperature effect is neglected, whether the relevant AD condensate is stable or not is determined by the quantum correction to the mass squared:

\[ m^2 \left\{ 1 + K \log \left( \frac{|\phi|^2}{M^2} \right) \right\}. \]  

(24)

If \( K < 0 \), instability develops and finally \( Q \)-balls are formed. For the \( LH_u \) direction, \( K \) is positive and \( Q \)-balls are not formed [27]. But when the oscillation of AD condensates occurs due to the finite-temperature effect, instability can develop and result in the formation of \( Q \)-balls. However, the resultant charges of \( Q \)-balls are so small that they cannot survive until the temperature becomes lower than the electroweak scale, as shown in section 5. Thus, \( Q \)-ball formation does not have any non-trivial cosmological consequence, and we do not bother to worry about complications due to \( Q \)-ball formation.
Then, what about \( n = 4 \) flat directions other than \( LH_u \)? It is known that other \( n = 4 \) directions in MSSM conserve \( B - L \) [28]. Thus sphaleron effects completely wash out the created baryon asymmetry. Although a sufficiently large \( Q \)-ball can protect baryon asymmetry from the sphaleron effect, such large \( Q \)-balls do not seem to be created in the presence of early oscillation as described above.

Finally, we comment on the dark matter candidate in our scenario. The thermal relic of the wino LSP in anomaly-mediation models cannot explain the observed dark matter density due to the large annihilation cross section [29], unless it is as heavy as 2 TeV. The non-thermally produced wino from \( Q \)-ball decay is a possible candidate [19], but in our model large \( Q \)-balls are not formed, so this possibility is excluded. Thus, we cannot explain in this model both the baryon asymmetry and dark matter simultaneously, and we need another particle such as an axion to account for the present density of matter in the universe.

4. The case of the \( n = 6 \) flat direction

Similar analysis can be applied for the \( n = 6 \) flat direction. There are some flat directions in MSSM which are lifted by \( n = 6 \) non-renormalizable superpotentials. The most interesting direction is the \( udd \) direction, which is responsible for \( B \)-ball baryogenesis [11]. In the usual gravity-mediation case, \( Q \)-balls associated with the AD condensate corresponding to the \( n = 6 \) flat direction can survive below the freeze-out temperature of the LSP, and subsequent decay produces baryon number and the non-thermal LSP. In some models, this can naturally explain both the baryon asymmetry and dark matter [16,19].

However, in our model, to avoid charge or colour breaking minima, early oscillation due to the thermal logarithmic term is needed. Since the \( udd \) direction is expected to have a negative coefficient for the thermal logarithmic term (\( a < 0 \) in equation (11)), early oscillation is unlikely to occur. Thus, an argument similar to that for the \( LH_u \) case cannot be applied to this direction. But for other \( n = 6 \) directions, e.g. the \( LLe \) direction, it may be possible that AD baryogenesis in anomaly-mediation models works. In this section, we investigate this possibility.

4.1. Dynamics of the \( n = 6 \) flat direction

First, we study the condition for early oscillation to occur. Using \( |\phi| \sim (HM^3)^{1/4} \), we obtain

\[
T_R \gtrsim \frac{1}{\alpha} \left( \frac{m_\phi^3 M^3}{M_p^6} \right)^{1/4} \approx 3 \times 10^5 \text{ GeV} \left( \frac{m_\phi}{100 \text{ GeV}} \right)^{3/4} \left( \frac{M}{10^{15} \text{ GeV}} \right)^{3/4}.
\]

Compared with the \( n = 4 \) flat direction case, a higher reheating temperature or smaller cut-off scale is needed.

Next, in order to avoid the unphysical minima, the thermal log must hide the valley of the zero-temperature potential. From equation (8) with \( n = 6 \),

\[
V(|\phi|_{\min}) \simeq -\frac{2}{75\sqrt{5}} m_{3/2}^{5/2} M^{3/2}.
\]
We require $\alpha^2 T^4 \gtrsim |V(\phi_{\text{min}})|$ at the beginning of oscillation. This leads to

$$T_R \gtrsim \alpha^{-1} M^{3/4} M_P^{-1/2} m_{3/2}^{3/4} \approx 8 \times 10^5 \text{ GeV} \left( \frac{0.1}{\alpha} \right) \left( \frac{M}{10^{15} \text{ GeV}} \right)^{3/4} \left( \frac{m_{3/2}}{10 \text{ TeV}} \right)^{3/4}. \quad (27)$$

Thus, $T_R \gtrsim 10^6 \text{ GeV}$ is necessary for $M \sim 10^{15} \text{ GeV}$ and $m_{3/2} \sim 10 \text{ TeV}$. When this constraint on the reheating temperature is satisfied, the dynamics of the flat direction is similar to that of the $LH_u$ direction studied in the previous section. Until $H$ becomes lower than $H_{\text{OS}}$, the radial component of the AD field is trapped by the instant minimum determined by the negative Hubble mass term and the non-renormalizable term. Then, the AD field begins to oscillate around their origin, and receives an angular kick. The baryon-to-entropy ratio is almost fixed at this epoch. Although this reheating temperature seems rather high, the gravitino mass is large enough for decay before the BBN epoch in anomaly-mediation models. So a high reheating temperature is not a problem. We have checked numerically that for $T_R \gtrsim 10^6 \text{ GeV}$, $\phi$ rolls down to the origin without being trapped by the displaced minima.

### 4.2. Baryon number generation

If the constraint on the reheating temperature (27) is satisfied, the Affleck–Dine mechanism can work with no more difficulty. From equation (21),

$$n/s \sim \frac{1}{9} \delta_e m_{3/2} T_R M_P^{-2} \left( \frac{M}{H_{\text{O}}} \right)^{3/2} \quad (28)$$

where $H_{\text{OS}}$ is given by

$$H_{\text{OS}} \sim \left( \frac{a \alpha^2 T_R^2 M_P M^{-3/2}}{M_{10}} \right)^{2/3} \quad (29)$$

from equation (12). Substituting this into equation (28), we obtain

$$n/s \sim \frac{\delta_e m_{3/2} M^3}{9 a^2 T_R M_P^3} \quad (30)$$

$$\sim 10^{-10} \delta_e \left( \frac{0.1}{\alpha} \right)^2 \left( \frac{m_{3/2}}{10 \text{ GeV}} \right) \left( \frac{10^8 \text{ GeV}}{T_R} \right) \left( \frac{M}{10^{16} \text{ GeV}} \right)^3. \quad (31)$$

Thus, we can obtain a proper amount of baryon asymmetry with parameters consistent with the constraint (27).

### 5. $Q$-ball formation

In the previous section, we have shown that the Affleck–Dine mechanism can create a sizable baryon asymmetry in anomaly-mediation models with rather high reheating temperature. But it is non-trivial matter whether the created baryon asymmetry actually provides the baryon density of the universe which is required by BBN or CMB anisotropy. This is because $Q$-balls may be formed and almost all baryon charge is trapped into them. If the charge of $Q$-balls is large and they are stable, the evolution of the AD field and...
resultant baryon asymmetry of the universe may be changed significantly. In this section, we see that $Q$-ball formation in our models has no great importance in cosmology.

Generally, perturbations to the AD fields $\phi$ grow when their potential is less steep than $\phi^2$ due to the negative pressure. In our models, oscillation of the AD fields is controlled by the thermal logarithmic term, and hence the instability develops into formation of $Q$-balls. Since this is similar to the gauge-mediation-type $Q$-ball case, the same analysis as for the gauge-mediation case can be applied.

First note that although the whole dynamics of $Q$-ball formation is highly non-linear, the radius of $Q$-balls is determined by the fastest growing mode in perturbative analysis. This is checked by numerical calculation based on lattice simulation [12]. Roughly speaking, the wavelength of the fastest growing mode is comparable to the Hubble radius at the epoch of oscillation of AD fields,

$$\frac{|k|}{a} \sim H_{\text{OS}} \sim \frac{T_{\text{OS}}^2}{|\phi(t_{\text{OS}})|},$$  \hspace{1cm} (32)

where $T_{\text{OS}}$ denotes the temperature at the beginning of oscillation and is given by $T_{\text{OS}} \sim (T_R^3 H_{\text{OS}} M_F)^{1/4}$. Thus, we expect that early oscillation due to the thermal effect tends to yield smaller $Q$-balls. The resultant charge inside $Q$-balls $Q$ is given by $Q \sim H_{\text{OS}}^{-3} n_B(t_{\text{OS}})$. Here we note that equation (20) can be written in the convenient form $n_B(t_{\text{OS}}) \sim \epsilon H_{\text{OS}} |\phi(t_{\text{OS}})|^2$, where $\epsilon$ denotes the ellipticity of the orbit of the AD field, $\epsilon = m_3/2/H_{\text{OS}}$. The result is

$$Q \sim \epsilon \frac{|\phi(t_{\text{OS}})|^4}{T_{\text{OS}}^4} \sim \epsilon \left(\frac{M}{T_{\text{OS}}}\right)^{4((n-3)/(n-1))}.$$  \hspace{1cm} (33)

In fact, $Q$-ball formation begins slightly later than the oscillation of the AD field and the number of $Q$-balls per Hubble horizon is expected to be more than 1. In [30], it is found that the maximum charge of $Q$-balls is fitted by the formula when the ellipticity $\epsilon$ is much smaller than 1,

$$Q_{\text{max}} = \beta \left(\frac{\phi(t_{\text{OS}})}{T_{\text{OS}}}\right)^4$$  \hspace{1cm} (34)

with $\beta \sim 6 \times 10^{-4}$ from lattice simulation. This agrees with equation (33) except for a numerical factor which cannot be determined analytically. Equation (34) is independent of $\epsilon$ because anti-$Q$-balls are also produced, so the net baryon number is small. In the case of early oscillation, since one expects $\epsilon \ll 1$, we use equation (34) in the following. Now we estimate the charges of $Q$-balls in the $n = 4$ and 6 cases.

\section{5.1. The $n = 4$ case}

From equation (14), the temperature at the oscillation $T_{\text{OS}}$ is given by

$$T_{\text{OS}} \sim (\alpha T_R^3 M_F^{3/2} M^{-1/2})^{1/4}.$$  \hspace{1cm} (35)
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Thus, the charge of $Q$-balls is estimated as

$$ Q \sim \beta \left( \frac{M}{T_{OS}} \right)^{4/3} \sim 3 \times 10^9 \left( \frac{0.1}{\alpha} \right)^{1/3} \left( \frac{\beta}{6 \times 10^{-4}} \right) \left( \frac{10^6 \text{ GeV}}{T_R} \right)^2 \left( \frac{M}{M_P} \right)^{3/2}. \quad (36) $$

With the aid of equation (21), this can be rewritten as

$$ Q \sim 1 \times 10^9 \delta_e^{-1} \left( \frac{\alpha}{0.1} \right)^{2/3} \left( \frac{\beta}{6 \times 10^{-4}} \right) \left( \frac{10^6 \text{ GeV}}{T_R} \right) \left( \frac{50 \text{ TeV}}{m_{3/2}} \right) \left( \frac{n_B/s}{10^{-10}} \right). \quad (37) $$

As shown in the appendix, in order to survive evaporation in a high temperature plasma, $Q \gtrsim 10^{18}$ is needed. Thus, even if $Q$-balls are formed, they are expected to evaporate completely well above $T \sim 100$ GeV and the estimation of baryon asymmetry in section 3 need not be changed.

5.2. The $n = 6$ case

In this case, $H_{OS}$ is given by equation (29). Then, the temperature at the oscillation is

$$ T_{OS} \sim \alpha^{1/3} T_R^{5/6} M^{-1/4} M_P^{5/12}. \quad (38) $$

The charge of $Q$-balls in the $n = 6$ case becomes

$$ Q \sim \beta \left( \frac{M}{T_{OS}} \right)^{12/5} \sim 2 \times 10^{10} \left( \frac{0.1}{\alpha} \right)^{4/5} \left( \frac{\beta}{6 \times 10^{-4}} \right) \left( \frac{10^7 \text{ GeV}}{T_R} \right)^2 \left( \frac{M}{10^{15} \text{ GeV}} \right)^3. \quad (39) $$

Using equation (31), this is rewritten in terms of $n/s$ as

$$ Q \sim 2 \times 10^{12} \delta_e^{-1} \left( \frac{\alpha}{0.1} \right)^{6/5} \left( \frac{\beta}{6 \times 10^{-4}} \right) \left( \frac{10^7 \text{ GeV}}{T_R} \right) \left( \frac{10 \text{ TeV}}{m_{3/2}} \right) \left( \frac{n_B/s}{10^{-10}} \right). \quad (40) $$

This is also so small that $Q$-balls cannot survive evaporation.

6. Conclusion

We have investigated the Affleck–Dine mechanism in anomaly-mediated SUSY breaking models. In contrast to previous studies, ours has found that early oscillation due to finite-temperature effects can drive flat directions into the correct vacuum and a proper amount of baryon asymmetry can be generated for neutrino mass of about $m_\nu \lesssim 10^{-7}$ eV in the case of the $LH_u$ direction. Our scenario requires somewhat high reheating temperature, but this leads to no cosmological difficulties such as gravitino problems since the gravitino is heavy enough and decays before the onset of BBN in anomaly-mediation models.

We have also investigated the same mechanism for the $n = 6$ flat direction case. It is found that for natural range of parameters, the proper amount of baryon asymmetry can be obtained. Furthermore, we have also discussed consequences of $Q$-ball formation. It is shown that all $Q$-balls evaporate in high temperature plasma. Therefore, the $Q$-ball formation does not complicate the baryogenesis process.
Appendix. Evaporation of $Q$-balls

A $Q$-ball is a non-topological soliton whose stability is ensured by global $U(1)$ symmetry. But it can release its charge in some manner. Here we concentrate on the following two processes. The first is decay of the AD field into a pair of fermions or lighter bosons, and the second is evaporation and diffusion effects in a thermal bath. In this appendix, we give a rough estimation of the total amount of charge evaporated from $Q$-balls.

A.1. Decay into light particles

It is known that the energy per charge inside $Q$-balls of gauge-mediation type is proportional to $Q^{-1/4}$ \cite{13}. Thus, for large enough $Q$, $Q$-balls are stable against decay into nucleons. The energy per unit charge of a gauge-mediated-type $Q$-ball is given by

$$\frac{E_Q}{Q} \sim TQ^{-1/4}.$$  \hfill (A.1)

This leads to the stability condition

$$Q > \left(\frac{T}{m_N}\right)^4$$  \hfill (A.2)

where $m_N$ denotes the mass of nucleon, $m_N \sim 1$ GeV. Thus, for $T \lesssim Q^{1/4}$ GeV, $Q$-balls become stable against decay into nucleons, although decay into light neutrinos is possible for leptonic $Q$-balls ($L$-balls). But as temperature becomes low, as explained in the next subsection, gauge-mediated-type $Q$-balls dominated by the thermal logarithmic potential are deformed or converted into gravity-mediated-type ones. Even if $Q$-balls survive evaporation, eventually they decay into free fermions or bosons since gravity-mediated-type $Q$-balls have energy-to-charge ratio comparable to $m_\phi$, which is much larger than $m_N$.

Thus, we consider the process of decay of $Q$-balls into fermion pairs. This occurs only from the surface of the $Q$-ball since the Pauli exclusion principle forbids decay into fermions inside $Q$-balls \cite{31}. This sets an upper bound on the decay rate of $Q$-balls which can easily be saturated,

$$\left(\frac{dQ}{dt}\right)_{\text{fermion}} \leq \frac{A\omega^3}{192\pi^2},$$  \hfill (A.3)

where $A$ denotes the surface area of the $Q$-balls. Decay into pairs of bosons is possible and may be substantially enhanced compared with the case for fermions if scalar fields lighter than AD fields exist. However, since this bosons become heavy inside $Q$-balls, the decay into lighter bosons only occurs through loop diagrams suppressed by the large effective mass \cite{11}, which leads to the enhancement factor $f_s \lesssim 10^3$, defined by

$$\left(\frac{dQ}{dt}\right)_{\text{boson}} = f_s \left(\frac{dQ}{dt}\right)_{\text{fermion}}.$$  \hfill (A.4)
Using $A \sim 4\pi R_Q^2 \sim 4\pi |K|^{-1} m_\phi^{-2}$ for gravity-mediated-type $Q$-balls, we obtain the decay temperature of $Q$-balls:

$$T_d \sim 18 \text{ GeV} \sqrt{f_s \left( \frac{0.01}{|K|} \right)^{1/2} \left( \frac{m_\phi}{100 \text{ GeV}} \right)^{1/2} \left( \frac{10^{18}}{Q} \right)^{1/2}}.$$  (A.5)

For sufficiently large $Q$, this can become lower than the freeze-out temperature of dark matter, $T_f \sim m_{\text{DM}}/20$. If this is the case, wino dark matter, which is the natural consequence of the anomaly-mediation model, produced by $Q$-ball decay can amount to the desired abundance of dark matter [19]. But it should be noticed that the $udd$ direction is invalid because the coefficient of the thermal logarithmic term is expected to be negative and hence the early oscillation does not occur. Furthermore, if one wants to explain baryon asymmetry and dark matter in this scenario, the $LLe$ or other pure leptonic direction does not work either, since the created lepton number is protected from the sphaleron effect inside the $Q$-ball until they decay at a temperature below the electroweak scale. Other flat directions lifted by the $n=6$ superpotential are attractive candidates [28], but obtaining large $Q$ is difficult in our scenario (see equations (37) and (40)).

### A.2. Evaporation and diffusion

$Q$-ball formation is a non-adiabatic process, and almost all charges are trapped into the $Q$-balls. This configuration is energetically stable, but in a finite-temperature environment, this is not always the case. In a thermal bath there can exist free particles carrying charge surrounding $Q$-balls. The minimum of free energy is achieved when all charges are distributed in the form of $Q$-plasma. However, in an actual situation, the evaporation of charge from $Q$-balls is not sufficient on the cosmic timescale. Thus, a mixture of the plasma state and the $Q$-ball state is realized. Then, it is important to find out how and what amount of charge of $Q$-balls is released into the outer region.

$Q$-balls emit their charge through two processes: evaporation [14] and diffusion [15]. Generally, as we see below, at high temperature the latter determines the rate of emission of charge from $Q$-balls. First we estimate the rate of evaporation from $Q$-balls. It occurs when the values of the chemical potential of $Q$-balls ($\mu_Q$) and that of the surrounding plasma ($\mu_p$) differ significantly. The evaporation rate is

$$\Gamma_{\text{evap}} = -4\pi R_Q^2 \xi (\mu_Q - \mu_p) T^2,$$  (A.6)

where $\xi$ is given by

$$\xi = \begin{cases} 1 & (T > m_\phi) \\ \left( \frac{T}{m_\phi} \right)^2 & (T < m_\phi). \end{cases}$$  (A.7)

But in fact around the edge of $Q$-balls, chemical equilibrium between plasma and $Q$-matter is achieved and charge inside the $Q$-ball cannot come out at high temperature. Therefore, the charge in the ‘atmosphere’ of the $Q$-ball should be taken away by diffusion in order for further charge evaporation to occur. In this situation, charge transfer from inside $Q$-balls into plasma is determined by diffusion effects rather than the above evaporation rate:

$$\Gamma_{\text{diff}} = -4\pi D R_Q \mu_Q T^2 \sim -4\pi a T.$$  (A.8)
where we have used $D = a/T$ with $a = 4–6$, $\mu_Q \sim T Q^{-1/4}$ and $R_Q \sim T^{-1} Q^{1/4}$. when $T > m_\phi$, $\Gamma_{\text{diff}} < \Gamma_{\text{evap}}$ and the charge transfer is controlled by diffusion effects.

In our model, the thermal logarithmic potential dominates when the AD field oscillates and $Q$-ball formation takes place. However, as the temperature becomes low, the thermal effect ceases to be the dominant component of the potential. At a rough estimate, this occurs when $T^4 \sim m_\phi^2 |\phi_{\text{hill}}|^2$, that is, $T \sim T_c \sim 10^6 \text{ GeV}$. If $|\phi| \ll |\phi_{\text{hill}}|$ at this epoch, the soft mass term determines the properties of $Q$-balls. If $K$ in (24) is negative, the $Q$-ball configuration is of gravity-mediation type, $R_Q \sim m_\phi |K|^{-1/2}$, $\mu_Q \sim m_\phi$. Otherwise, the $Q$-ball configuration is not stable any more and will collapse. In this case, $Q$-ball formation is irrelevant to baryogenesis. In the following, we consider the possibility that at $T < T_c$, the configuration of $Q$-balls is changed into a gravity-mediation-type one. In our model, the reheating temperature $T_R$ must be rather high as shown in previous sections. Thus, in the following, we assume $T_{\text{OS}} > T_R > T_c > m_\phi$. Then, we obtain

$$
\frac{d Q}{d T} = \begin{cases} 
\frac{32 \pi a T^2 R^2 M_P}{9 T^4} & (T > T_R), \\
\frac{16 \pi a M_P}{T^2} & (T_R > T > T_c), \\
\frac{16 \pi a M_P}{|K|^{1/2} T^2} & (T_c > T > T_s), \\
\frac{16 \pi M_P}{|K| m_\phi T} & (T_s > T > m_\phi), \\
\frac{16 \pi M_P T}{|K| m_\phi^3} & (T < m_\phi), 
\end{cases}
$$

(A.9)

where $T_s$ is defined by $T_s = a |K|^{1/2} m_\phi$. Although we have assumed $T_s > m_\phi$, this assumption does not affect the following analysis much. Now let us estimate the total amount of evaporated charge $\Delta Q$ and examine whether or not $Q$-balls can survive in our model. Integrating over the evaporation or diffusion rate, we obtain

$$
\Delta Q(T > T_R) \sim \frac{32 \pi a M_P}{9 T_R},
$$

(A.10)

$$
\Delta Q(T_R > T > T_c) \sim \frac{16 \pi a M_P}{T_c},
$$

(A.11)

$$
\Delta Q(T_c > T > T_s) \sim \frac{16 \pi a M_P |K|^{-1/2}}{T_s},
$$

(A.12)

$$
\Delta Q(T_s > T > m_\phi) \sim \frac{16 \pi M_P |K|^{-1}}{m_\phi},
$$

(A.13)

$$
\Delta Q(T < m_\phi) \sim \frac{8 \pi M_P |K|^{-1}}{m_\phi},
$$

(A.14)
which are estimated as

\[
\Delta Q(T > T_R) \sim 1 \times 10^{13} \left( \frac{10^7 \text{GeV}}{T_R} \right), \quad (A.15)
\]

\[
\Delta Q(T_R > T > T_c) \sim 2 \times 10^{14} \left( \frac{10^6 \text{GeV}}{T_c} \right), \quad (A.16)
\]

\[
\Delta Q(T_c > T > T_*) \sim 2 \times 10^{17} |K|^{-1/2} \left( \frac{10^3 \text{GeV}}{T_*} \right), \quad (A.17)
\]

\[
\Delta Q(T_* > T > m_\phi) \sim 2 \times 10^{17} |K|^{-1} \left( \frac{10^3 \text{GeV}}{m_\phi} \right), \quad (A.18)
\]

\[
\Delta Q(T < m_\phi) \sim 1 \times 10^{17} |K|^{-1} \left( \frac{10^3 \text{GeV}}{m_\phi} \right). \quad (A.19)
\]

From these, in order for \( Q \)-balls to survive evaporation,

\[
Q \gtrsim 10^{18} \quad (A.20)
\]

is required. Therefore, the charge transfer is enough to evaporate all charge from \( Q \)-balls for both \( n = 4 \) and \( 6 \) cases. Even if \( Q \)-balls are stable against decay into lighter particles, diffusion and evaporation effects can sufficiently transfer their charge into the outer environment. Since our model requires a high reheating temperature, this effect is unavoidable. After all, \( Q \)-balls cannot survive until temperature becomes lower than about 100 GeV, where the electroweak phase transition occurs.

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