Polynomial inflation models after BICEP2

Tatsuo Kobayashi

Department of Physics, Hokkaido University, Sapporo 060-0810, Japan

Osamu Seto

Department of Life Science and Technology, Hokkai-Gakuen University, Sapporo 062-8605, Japan

Abstract

Large field inflation models are favored by the recent BICEP2 that has detected gravitational wave modes generated during inflation. We study general large field inflation models whose potential contains (constant,) quadratic and quartic terms of inflaton field. We show, in this framework, those inflation models can generate the fluctuation with the tensor-to-scalar ratio of 0.2 as well as the scalar spectral index of 0.96, those are very close to the center value of the tensor-to-scalar ratio reported by BICEP2 as well as Planck. Finally, we briefly discuss particle physics model building of inflation.
I. INTRODUCTION

Inflationary cosmological model is the standard paradigm of modern cosmology, because an inflationary expansion in the very early Universe solves various problems in the standard Big Bang cosmology and also provides the seed of large scale structure in our Universe from the quantum fluctuation of an inflaton field $\phi$. The property of the generated density fluctuation from a single field slow roll inflation model, namely adiabatic, Gaussian, and its almost scale invariant spectrum, is quite consistent with various cosmological observations.

As the scalar perturbation is generated from the inflaton’s quantum fluctuation during inflationary expansion, the tensor perturbation also is generated from graviton’s one. The tensor perturbation induces $B$-mode polarization of the temperature anisotropy in the cosmic microwave background radiation (CMB) and is important for inflationary cosmology, because the tensor perturbation directly tells us the energy scale of inflation.

Recently, BICEP2 collaboration reported the detection of the tensor mode through the $B$-mode polarization with the corresponding tensor-to-scalar ratio \( r_T = 0.20^{+0.07}_{-0.05} \).\(^1\) Its cosmological implications also have been studied. It has been well-known that theoretically such a large tensor-to-scalar ratio can be generated only by so-called large field models, where its potential, as of a polynomial function of $\phi$, is convex and the variation of the inflaton field value during inflation $\Delta \phi$ is as large as of Planck scale.\(^2\) On the other hand, the Planck satellite has reported the scalar spectral index as \( n_s \simeq 0.96 \).\(^3\)

Now, if we compare the central values of Eqs. (1) and (2) with the predicted values of well-studied potential models such as $V \propto \phi^2$ or $\phi^4$, there is a discrepancy. Namely, $r_T$ from $V \propto \phi^2$ is too low and that from $\phi^4$ is too high.\(^4\)

In this paper, we extend analysis to general polynomial models of inflation whose potential is expressed as

\[ V = c_1 + c_2 \phi^2 + c_4 \phi^4, \] \(^3\)

\(^1\) $V \propto \phi^3$ potential would well agree with data. However, naively, this potential is pathological because the potential is not bounded from below. Note, however, an effective realization would be possible with a field redefinition from $\mathcal{L} \sim \phi^2 (\partial \phi)^2 - \phi^6$.\(^5\)
to examine whether this form of potential can reconcile the mismatch mentioned above, by taking the latest BICEP2 data into account. Since it is not easy to control so many parameters, in practice, we will consider terms up to $\phi^4$, which might be motivated by the renormalizability of quantum field theory.

II. POLYNOMIAL POTENTIAL MODEL

We study canonical single field inflation models with a polynomial potential. Within this framework, the power spectrum of the density perturbation, its spectral index and the tensor-to-scalar ratio are expressed as

$$P_\zeta = \left(\frac{H^2}{2\pi|\phi|}\right)^2 = \frac{V}{24\pi^2\epsilon},$$

$$n_s = 1 + 2\eta - 6\epsilon,$$

$$r_T = 16\epsilon,$$

respectively by using slow roll parameters

$$\eta = \frac{V_{\phi\phi}}{V},$$

$$\epsilon = \frac{1}{2}\left(\frac{V_\phi}{V}\right)^2,$$

in the unit with $8\pi G = 1$. Here, a subscript $\phi$ and dot denote a derivative with respect to $\phi$ and time respectively, and $H$ is the Hubble parameter of the Universe.

1. positive quadratic and quartic

The first example is the inflation driven by the potential

$$V = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4.$$  \hspace{1cm} (9)

For this potential, the slow roll parameters are given by

$$\eta = \frac{m^2 + 3\lambda\phi^2}{\frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4},$$

$$\epsilon = \frac{1}{2}\left(\frac{m^2\phi + \lambda\phi^3}{\frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4}\right)^2.$$  \hspace{1cm} (11)
When inflaton reaches $\phi^2 = 2$, then $\eta$ becomes unity and inflation ends. By solving slow roll equation $3H\dot{\phi} + V_\phi = 0$, we obtain

$$N = \int_{\phi_e}^{\phi} \frac{V}{V_\phi} d\phi \simeq \frac{\phi^2}{8} + \frac{m^2}{\lambda} \ln \left( \frac{m^2 + \lambda \phi^2}{m^2} \right),$$

(12)

with $N$ being the number of e-folds during inflation, where $\phi_e = \sqrt{2}$ is the field value when inflation ends and dropped in Eq. (12) due to the smallness compared to the others.

We show the contours of inflationary observables in the $m^2 - \lambda$ plane. Figure 1 is for the case that the observed cosmological scale $k_* = 0.002$ Mpc is assumed to correspond to $N = 60$. The double curve depicts the amplitude of the power spectrum of the density perturbation $P_\zeta = (22.42 - 21.33) \times 10^{-10}$ quoted from Table. 3 (ΛCDM) in Ref. [8]. Red lines with numbers are contours of $n_s$. Blue lines with numbers enclosed by a square are contours of $r_T$, whose values are those noted in Eq. (1). For $N = 60$, the predicted $r_T$ without $\lambda$ lies outside of the error bar reported by BICEP2. However, including a small quartic term with $\lambda \simeq 1 \times 10^{-13}$, we obtain $r_T \simeq 0.2$ and $n_s \simeq 0.96$. Figure 2 is for $N = 50$ and shows that $\lambda \simeq 0.8 \times 10^{-13}$ provides the best fitting to $r_T$.

2. negative quadratic and quartic

Next, let us consider the case with a negative quadratic term, in other words, this is double well potential. We introduce a constant term as well in order to realize the vanishing cosmological constant at the true minimum. The potential is given by

$$V = V_0 - \frac{1}{2}m^2 \phi^2 + \frac{\lambda}{4} \phi^4.$$  

(13)

From the stationary condition and the vanishing cosmological constant condition at the minimum, we find the vacuum expectation value (VEV) of $\phi$ and $V_0$ as

$$\langle \phi \rangle^2 = \frac{m^2}{\lambda},$$  

(14)

$$V_0 = \frac{m^4}{4\lambda}.$$  

(15)

The slow roll parameters are given by

$$\eta = \frac{-m^2 + 3\lambda \phi^2}{V_0 - \frac{1}{2}m^2 \phi^2 + \frac{\lambda}{4} \phi^4},$$  

(16)

$$\epsilon = \frac{1}{2} \left( \frac{-m^2 \phi + \lambda \phi^3}{V_0 - \frac{1}{2}m^2 \phi^2 + \frac{\lambda}{4} \phi^4} \right)^2.$$  

(17)
FIG. 1: Various contours for $N = 60$; black double curve is for the amplitude of the density perturbation, blue lines are for the tensor-to-scalar ratio, and red lines are the spectral index $n_s$. To reproduce the central value $r_T \simeq 0.2$, we need $\lambda \simeq 1 \times 10^{-13}$.

FIG. 2: Same as Fig. 1 but for $N = 50$. To reproduce the central value $r_T \simeq 0.2$, we need $\lambda \simeq 0.8 \times 10^{-13}$. 
\( \eta \) becomes unity and inflation ends, when the inflaton reaches
\[
\phi_e^2 = 6 + \frac{m^2 + 2\sqrt{\lambda(9\lambda + 2m^2)}}{\lambda}
\]  
(18)

By solving slow roll equation, we obtain
\[
N = \int_{\phi_e}^{\phi} \frac{V}{V'} d\phi \simeq \frac{\phi^2 - \phi_e^2}{8} - \frac{m^2}{8\lambda} \ln \left( \frac{\phi^2}{\phi_e^2} \right).
\]  
(19)

We show the contours of inflationary observables in the \( m^2/\lambda - \phi \) plane, with \( \phi \) being the field value during inflation given in Eq. (19). Red lines with numbers are contours of \( n_s \) with the error band reported by Planck. The blue line with the number 0.2 enclosed by a square is the central value contour of \( r_T \) by BICEP2. Without the \( m \) term, in other words in the \( V = \phi^4 \) potential limit, the predicted \( n_s \) lies outside of the uncertainty band reported by Planck, as is well known [8]. The black thick and solid line correspond to the \( \phi \) field value for \( N = 60 \) and 50 in Figs. 3 and 4, respectively. As increasing \( m \), those line approaches observed values of \( n_s \) as well as \( r_T \). From Fig. 3, we find that the \( r_T = 0.2 \) contour meets the \( \phi(N = 60) \) line at \( m^2/\lambda \simeq 170 \), which corresponds to
\[
\langle \phi \rangle \simeq 13
\]  
(20)

The double well potential with this VEV well reproduces those observed values. The double curve for \( P_\zeta = (22.42 - 21.33) \times 10^{-10} \) is drawn with \( \lambda = 4.5 \times 10^{-14} \) in Fig. 3.

Figure 4 is for \( N = 50 \), where we omit the \( P_\zeta \) contour for simplicity. We need \( m^2/\lambda = \mathcal{O}(10^3) \) to realize \( r_T = 0.2 \) for \( N = 50 \), that is far outside of the range in the figure 4.

III. SUMMARY AND DISCUSSION

We have shown that general polynomial inflation models are fit well to the observed data including the tensor-to-scalar ratio recently reported by BICEP2. In other words, after we know the size of \( r_T \), we are able to determine more parameters of inflation models. In fact, for \( V \sim +\phi^2 + \phi^4 \) with \( N = 60 \), we find \( m^2 \simeq 3 \times 10^{-11} \) and \( \lambda \simeq 1 \times 10^{-14} \). A double well potential also can be fit nicely to the data, then the VEV should be \( \langle \phi \rangle \simeq 13 \) and the self coupling constant is \( \lambda \simeq 4.5 \times 10^{-14} \) for \( N = 60 \).

Finally we note here possible directions of particle physics model construction for inflation. Being aware of the above size of self-coupling, an inflaton’s Yukawa coupling \( y \) to
FIG. 3: Various contours for $N = 60$; black double curve is for the amplitude of the density perturbation with $\lambda = 4.5 \times 10^{-14}$, the blue line is for the tensor-to-scalar ratio, and red lines are the spectral index $n_s$. To reproduce the central value $r_T \simeq 0.2$, we need $m^2/\lambda = \langle \phi \rangle^2 \simeq 170$.

fermion $\psi$, $\mathcal{L} = y \bar{\psi} \phi \psi$, should be smaller than $\mathcal{O}(10^{-4})$, because it induces a $\mathcal{O}(y^4)$ self-
coupling by radiative corrections. Supersymmetric construction of a large field inflation model has been challenging [10]. F-term inflation suffers from so-called “η problem” due to higher order terms from the Kahler potential. Imposing shift symmetry $\phi \to \phi + C$ is one of a few effective manners to overcome the problem [11]. While D-term inflation has been regarded an example of hybrid inflation [12], in fact, D-term chaotic inflation is also possible [13]. Such a model interestingly does not suffer from $\eta$ problem even if we consider a general Kahler potential, because the field redefinition to the canonical field absorbs higher order corrections and the Lagrangian is reduced to a quartic or double well potential. Since a self coupling is given by the square of a gauge coupling in D-term, we need to introduce a new gauge interaction with the gauge coupling constant of $\mathcal{O}(10^{-7})$. If we could realize a large VEV about $10$ by any means, as we have shown, such a model would easily reproduce the data.

We have restricted the polynomial potential in this paper. It would be important to add higher order terms $\phi^n$ as well as the cubic term $\phi^3$. We will study it elsewhere.

Acknowledgments

T.K. was supported in part by the Grant-in-Aid for Scientific Research No. 25400252 from the Ministry of Education, Culture, Sports, Science and Technology in Japan.

[1] A. A. Starobinsky, JETP Lett. 30, 682 (1979) [Pisma Zh. Eksp. Teor. Fiz. 30, 719 (1979)]; K. Sato, Mon. Not. Roy. Astron. Soc. 195, 467 (1981); A. H. Guth, Phys. Rev. D 23, 347 (1981); A. D. Linde, Phys. Lett. B 108, 389 (1982); A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982).

[2] S. W. Hawking, Phys. Lett. B 115, 295 (1982); A. A. Starobinsky, Phys. Lett. B 117, 175 (1982); A. H. Guth and S. Y. Pi, Phys. Rev. Lett. 49, 1110 (1982).

[3] A. A. Starobinsky, JETP Lett. 30, 682 (1979) [Pisma Zh. Eksp. Teor. Fiz. 30, 719 (1979)]; V. A. Rubakov, M. V. Sazhin and A. V. Veryaskin, Phys. Lett. B 115, 189 (1982);
L. F. Abbott and M. B. Wise, Nucl. Phys. B 244, 541 (1984);
B. Allen, Phys. Rev. D 37, 2078 (1988).

[4] P. A. R. Ade et al. [BICEP2 Collaboration], arXiv:1403.3985 [astro-ph.CO].

[5] K. Nakayama and F. Takahashi, arXiv:1403.4132 [hep-ph];
   K. Harigaya, M. Ibe, K. Schmitz and T. T. Yanagida, arXiv:1403.4536 [hep-ph];
   L. A. Anchordoqui, V. Barger, H. Goldberg, X. Huang and D. Marfatia, arXiv:1403.4578
   [hep-ph];
   M. Czerny, T. Kobayashi and F. Takahashi, arXiv:1403.4589 [astro-ph.CO];
   H. Collins, R. Holman and T. Vardanyan, arXiv:1403.4592 [hep-th];
   C. R. Contaldi, M. Peloso and L. Sorbo, arXiv:1403.4596 [astro-ph.CO];
   K. Harigaya and T. T. Yanagida, arXiv:1403.4729 [hep-ph];
   A. Kehagias and A. Riotto, arXiv:1403.4811 [astro-ph.CO];
   J. Lizarraga, J. Urrestilla, D. Daverio, M. Hindmarsh, M. Kunz, A. R. Liddle, arXiv:1403.4924
   [astro-ph.CO].

[6] D. H. Lyth, Phys. Rev. Lett. 78, 1861 (1997).

[7] P. A. R. Ade et al. [Planck Collaboration], arXiv:1303.5076 [astro-ph.CO].

[8] P. A. R. Ade et al. [Planck Collaboration], arXiv:1303.5082 [astro-ph.CO].

[9] For recent previous studies, see e.g., K. Nakayama, F. Takahashi and T. T. Yanagida, Phys.
   Lett. B 725, 111 (2013); JCAP 1308, 038 (2013).

[10] D. H. Lyth and A. Riotto, Phys. Rept. 314, 1 (1999);
   M. Yamaguchi, Class. Quant. Grav. 28, 103001 (2011).

[11] M. Kawasaki, M. Yamaguchi and T. Yanagida, Phys. Rev. Lett. 85, 2572 (2000).

[12] P. Binetruy and G. R. Dvali, Phys. Lett. B 388, 241 (1996);
   E. Halyo, Phys. Lett. B 387, 43 (1996).

[13] K. Kadota and M. Yamaguchi, Phys. Rev. D 76, 103522 (2007).