Heavy-fermion superconductivity in the Kondo-lattice system CeCu\textsubscript{2}Si\textsubscript{2}

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Abstract. The formation of "heavy fermions" (HFs), i.e., charge carriers with largely enhanced effective mass, in Ce-based Kondo lattice (KL) systems is recalled. In contrast to the disastrous pair-breaking effect of magnetic Ce\textsuperscript{3+} impurities in certain superconducting Kondo alloys, a periodic dense array of Ce\textsuperscript{3+} ions is a prerequisite for the generation of HF superconductivity (SC) below $T_c = 0.6$ K in the KL system CeCu\textsubscript{2}Si\textsubscript{2}. Subsequent to this discovery in 1979, more than 35 other Ce-, Pr-, Yb-, U-, Np- and Pu-based HF superconductors (with maximum $T_c = 18.5$ K) have been found. These materials exhibit a variety of fascinating properties, e.g., even- or odd-parity Cooper pairs, multi-phase diagrams or the coexistence with magnetic order. In particular, the interplay between HF SC and quantum criticality is of high timely interest. CeCu\textsubscript{2}Si\textsubscript{2} has emerged as a prototype system showing a three-dimensional (3D) spin-density-wave (SDW) quantum-critical point. Recent results of inelastic-neutron-scattering experiments on this compound are briefly discussed. They lend strong support to the perception of Cooper pairs mediated by nearly quantum-critical 3D-SDW fluctuations covering a wide frequency range.

1. Heavy fermions in Kondo-lattice systems

In 1964, Kondo showed [1] that the anomalous increase of the electrical resistivity towards low temperatures, known to exist in certain dilute magnetic alloys for many years, originates in an unusual scattering process of the conduction electrons from isolated paramagnetic impurities. This is a consequence of both its dynamic nature and the Pauli exclusion principle, combined with a narrowing of the Fermi surface. The Kondo effect invokes a screening of the local paramagnetic moments by the spins of the conduction electrons well below the Kondo temperature $T_K \sim T_F \exp(-1/N_F J_K)$ [1]: Fermi temperature; $N_F$: conduction-electron density of states (DOS) at the Fermi energy $E_F = k_B T_F$; $J_K > 0$: coupling constant of the antiferromagnetic (AF) exchange interaction between the effective local spin and the conduction-electron spins.

The resulting local Kondo singlets give rise to Kondo resonances in the DOS near $E_F$ and have been ascribed by Nozières [2] to the quasiparticles of a local Fermi liquid theory. The latter explains

- the typical Curie-Weiss behaviour found in the magnetic susceptibility [3], which invokes the effective local moment to vanish as $T \to 0$, and the
- linear-in-$T$ impurity contribution, $\Delta C = \gamma T$, to the specific heat at $T \ll T_K$. For example, $\gamma = 16$ J/K\textsuperscript{2}mole-Cr and 1 J/K\textsuperscript{2}mole-Fe for CuCr and CuFe (figure 1a), respectively [4].
These values are of the same order as $\gamma \approx 1 J/K^2$mole-Ce observed for certain intermetallic compounds of trivalent Ce. Ce$^{3+}$ (4f$^1$) has a $j = 5/2$ Hund's rule multiplet whose degeneracy is partially lifted by the crystal field (CF), which frequently results in an $S_{\text{eff}} = 1/2$ Kramers doublet ground state. Dilute alloys containing Ce$^{3+}$ impurities often show properties typical for the Kondo effect. The periodic, dense arrangement of Ce$^{3+}$-ions in certain intermetalics realizes a Kondo lattice (KL). Assuming only one conduction band the KL-Hamiltonian reads

$$H_{\text{KL}} = \sum_{ij} t_{ij} c_{i\sigma}^+ c_{j\sigma} + \sum_{ij} J_{ij} S_i S_j + (1/2) \sum_i J_K S_i \cdot c_i^+ \sigma c_i.$$ 

It contains a conduction-electron band, $c_{\text{in}}$, whose hopping matrix $t_{ij}$ specifies a band dispersion $\varepsilon_k$. The spins of the conduction electrons, $s_{\text{cl}} = (1/2) c_i^+ \sigma c_i$ are coupled to the spin of the local 4f shell, $S_i$, through the AF Kondo-exchange interaction $J_K$.

$H_{\text{KL}}$ describes the competition of the indirect Rudermann-Kittel-Kasuya-Yoshida (RKKY) exchange interaction between the 4f-derived local moments, mediated by the conduction electrons, with the local Kondo interaction [5]. If the RKKY interaction prevails at low temperature (7), a magnetically ordered ground state (in most cases AF order) forms, CeAl$_2$ being an exemplary material [6]. In the case of dominating Kondo interaction, either a paramagnetic ground state like in CeAl$_3$ [7], or even superconductivity form like in CeCu$_2$Si$_2$ [8].

![Figure 1](image_url)

**Figure 1.** Low-temperature incremental specific heat due to magnetic ions. (a) $\Delta C = C - C_{\text{Cu}}$ for Cu$_{1-c}$Fe$_c$ with $c = 0.000081$ as $\Delta C/cT$ vs $T$ (on a logarithmic scale) for zero field as well as 3.0 and 3.8 T (from Ref. 4) (b) Calculated Ce-derived specific heat $\Delta C$, being almost identical to the measured specific heat $C$, as $C/T$ vs $T$ for CeAl$_2$ in the presence and absence (dashed line) of a molecular field describing AF order (from Ref. 10).

The paramagnetic low-$T$ phase in CeAl$_3$ can be considered a global Kondo singlet between the local moments and the spins of the conduction electrons. The quasiparticles, described by Kondo resonances, are charge carriers with a very large effective mass $m^*$ which exceeds the bare electron mass $m_0$ by up to a factor of $10^3$ ["heavy electrons" or "heavy fermions" (HFs)]. This is inferred from the huge Sommerfeld coefficient of low-$T$ electronic specific heat, $\gamma_0 = C/T$ ($= 1.62$ J/K$^2$mole in case of CeAl$_3$ [7]). Strongly renormalized charge carriers appear to exist in antiferromagnetically ordered KL systems as well. The specific heat of CeAl$_2$ well below its Néel temperature $T_N = 3.9$ K follows a dependence $C = \gamma T + \beta T^3$ [9]. While the cubic term is due to acoustic magnons, $\gamma T$ with $\gamma = 0.135$ J/K$^2$mole represents an enhanced electronic contribution. Bredl et al. [10] succeeded to explain the low-$T$ specific heat of CeAl$_2$ semi-quantitatively by (i) describing the Kondo effect by a (single-impurity) resonance level model [11], (ii) neglecting the actual spin arrangement.
and, instead, assuming a simple type-I AF order which (iii) is treated within a mean-field approximation. As seen in Fig. 1b, the (upon cooling) rapidly increasing specific-heat coefficient $\gamma(T) = C(T)/T$ reaches 1.74 J/K$^2$mole at zero temperature, very similar to $\gamma_0 = 1.62$ J/K$^2$mole observed for paramagnetic CeAl$_3$ [7]. Comparison with the results for the impurity-derived $\gamma(T)$ of dilute Kondo alloys (Fig. 1a) strongly suggests that the global Kondo resonance in Ce-based HF metals is predominantly built up by the individual Kondo resonances at the Ce$^{3+}$ sites. A direct consequence of the Kondo entanglement between the effective 4f-derived local spin and the spins of the conduction electrons is a "large" Fermi surface which exceeds the one formed by the conduction electrons alone, because it incorporates all individual Kondo resonances [12]. The low-$T$ electrical resistivity of such a "heavy" Fermi liquid follows a dependence $\rho(T) = \rho_0 + AT^2$, where $\rho_0$ is the residual resistivity and $A \sim (m^*)^2$ assumes a huge value as, in fact, observed for CeAl$_3$ [7]. In contrast, the low-$T$ resistivity of dilute Kondo alloys follows $\rho(T) = \rho_0 - AT^2$ [13], characteristic of a local Fermi liquid [2].

2. Superconductors containing magnetic impurities

Classical, i.e., phonon-mediated, superconductors with s-wave ($L = 0$), spin-singlet ($S = 0$, in the presence of strong spin-orbit-scattering: even-parity) Cooper pairs are extremely sensitive to the presence of magnetic impurities. This was demonstrated in 1958 by Matthias et al. [14], who registered the superconducting transition temperature $T_c$ of La doped with 1 at% of all (trivalent) rare-earth (RE) metals, cf. Fig. 2. They noticed that the unique depression of $T_c$ (Fig. 2a) is unambiguously determined by the quantum number of the spin rather than the effective magnetic moment of the 4f shell (Fig. 2b). This remarkable result was explained microscopically by Abrikosov and Gor'kov [15] who treated the exchange interaction between the local spin, $S$, and spin of the conduction electrons, $s$, in Born approximation. This interaction, described by the Hamiltonian $H_{\text{int}} = 2JSs$ ($J$: exchange constant), invokes spin-flip processes which break up the spin pairing of the Cooper pairs. The strength of this effect is governed by the pair-breaking parameter $\alpha \sim xS(S + 1)$, where $x$ denotes the concentration of the magnetic scattering centers.

![Figure 2. Pair breaking by spin-exchange scattering. (a) Superconducting transition temperature of 1 at% rare-earth solid solutions in lanthanum. (b) Spins and effective magnetic moments of the rare-earth elements (from Ref. 14).](image)

As seen in Fig. 2a, compared to the overall trend found for all other RE$^{3+}$ impurities, Ce$^{3+}$ has a much stronger pair-breaking effect - an observation which has to be related to the Kondo scattering. The effect of Kondo impurities on superconductivity was theoretically studied by Müller-Hartmann and Zittartz [16]. They predicted that the strongly $T$-dependent pair breaking parameter $\alpha \sim \pi^2S(S+1)[\ln(T/T_K)+\pi S(S+1)]^{-1}$ in a "Kondo superconductor" may lead to two or even three $T_c$s (see Fig. 3). Their prediction was verified soon after by Riblet and Winzer [17] and, independently, by Maple at al. [18] who discovered re-entrant superconductivity (SC) as shown in Figs. 3a and c for (La$_{1-x}$Ce$_x$)Al$_3$ with $x$ in the vicinity of 0.6 at%. This "intermediate" SC was found to be a true bulk
effect [19] (Fig. 3b). Experimental evidence for the theoretically predicted three-$T_c$ behavior was observed in 1977 by Winzer for $(\text{La}_{0.8}\text{Ce}_{0.2})_3\text{Y}_2$ [20].

Figure 3. (a) AC susceptibility $\chi$ vs $T$ (on a logarithmic scale) [17] and (b) specific heat as $(C_S-C_N)/T$ vs $T$ (S: superconducting state, N: normal state) [19] (b) for polycrystalline $\text{La}_{1-x}\text{Ce}_x\text{Al}_2$, $x = 0.0064$. (c) Reduced transition temperature vs Ce-impurity concentration $n$ for the $\text{La}_{1-n}\text{Ce}_n\text{Al}_2$ system (from Ref. 18).

3. Heavy-fermion superconductors

Because of the disastrous effect of local magnetic moments on SC, the 1979 discovery of superconductivity in $\text{CeCu}_2\text{Si}_2$ was met by astonishment. On the other hand, this discovery was not fully accidental, as it followed the earlier discoveries of superfluidity in $^3\text{He}$ [21] and of HF phenomena in $\text{CeAl}_3$ [7]. In view of the apparent similarity of the latter material's low-$T$ Fermi-liquid phase with the (charge-neutral) normal-fluid phase of $^3\text{He}$ [22], the obvious question arose whether a paramagnetic KL system like $\text{CeAl}_3$ could possibly show unconventional superconductivity similar to the magnetically driven superfluid phases of $^3\text{He}$ [22]. In fact, when the first indications of SC were noticed in low-quality polycrystalline samples of $\text{CeCu}_2\text{Si}_2$ [23], it was the anticipation of an analogue to superfluid $^3\text{He}$ which called for the preparation of cleaner samples. This eventually led to the discovery of bulk SC in $\text{CeCu}_2\text{Si}_2$ [8], see Fig. 4.

All superconductors known before 1979 lose their SC when doped with tiny amounts (a few at% at most) of magnetic ions. By contrast, in $\text{CeCu}_2\text{Si}_2$ a periodic dense array of magnetic $\text{Ce}^{3+}$ ions is prerequisite for SC to be formed. In Fig. 4a, the 4f-derived specific heat of $\text{CeCu}_2\text{Si}_2$ is displayed over four decades in $T$ as $C_4f/T$ vs $T$ [24]. The normal (n) to superconducting phase transition at $T_c = 0.6$ K manifests itself by a giant mean-field-type anomaly, which scales with the n-state value of $\gamma \approx 1$ J/K$^2$mole obtained by extrapolation to $T = 0$. This value is of the same size as the one measured for $\text{CeAl}_3$ and estimated for $\text{CeAl}_2$ in its putative paramagnetic low-$T$ phase (Fig. 1b). It indicates (i) that $\gamma(T) = C_4f(T)/T$ has the meaning of a $T$-dependent Sommerfeld coefficient of the electronic specific heat and (ii) that well below $T_K \approx 15$ K, electronic quasiparticles with huge effective mass $m^*$ are generated which (iii) form massive Cooper pairs below $T_c$.

The strongly enhanced effective mass, $m^* \approx 10^3 m_e$, is equivalent to an effective Fermi velocity $v_F^*$ which is correspondingly reduced compared to $v_F$ of the quasi-free conduction electrons at room
temperature. In fact, as inferred from Fig. 4a, \( \gamma(300 \text{ K}) \) is of the order of a few mJ/K^2 mole only, which is typical for transition metals (in the case of Ce-based intermetallics, 5d electrons contribute to the conduction band). The pronounced increase of \( \gamma(T) \) with decreasing \( T \) proves that the formation of the huge effective mass is a true low-temperature phenomenon that cannot be explained by one-electron theory. Since, in contrast to classical BCS superconductors, \( v_F^* \) is of the order of the velocity of sound only, the electron-phonon coupling in CeCu_2Si_2 is not retarded, i.e., the heavy charge carriers cannot escape their own polarization cloud. Therefore, phonons had been discarded as superconducting glue from the outset [8] and, instead, magnetic Cooper pairing mechanisms were proposed [25, 26] for CeCu_2Si_2 and its subsequently discovered U-based counterparts UBe_13 [27], UPt_3 [28] and URu_2Si_2 [29].

Figure 4. Heavy-fermion superconductivity in CeCu_2Si_2. (a) 4f-derived specific heat as \( C_{4f}/T \) vs \( T \) (on a logarithmic scale) over almost four decades. Solid line is a guide to the eye (from Ref. 24). (b) Upper critical field \( B_{c2} \) vs \( T \) for \( B \parallel \) and \( \perp \) to the basal tetragonal plane (from Ref. 30).

The early studies in CeCu_2Si_2 were severely plagued by the fact that physical properties were strongly sample dependent. However, the situation improved substantially when the group of W. Assmus at the Goethe University of Frankfurt succeeded in growing high-quality CeCu_2Si_2 single crystals. In Fig. 4b the upper critical field, \( B_{c2}(T) \), curve is displayed for the field applied both within and perpendicular to the basal tetragonal plane [30]. In view of the layered (ThCr_2Si_2) structure of this compound, \( B_{c2}(T) \) exhibits an, at first glance, surprisingly minor anisotropy, which indicates that CeCu_2Si_2 is a three-dimensional (3D) superconductor. The giant initial slope \( B'_{c2} = (-dB_{c2}/dT)_{T_c} = 23 \) T/K signals the small (renormalized) Fermi velocity of the quasiparticles, which form the Cooper pairs. It has become a hallmark of HF SC. At \( T = 0 \), \( B_{c2} \) is found to be much lower than the orbital critical field expected from \( B'_{c2} \). This indicates strong Pauli limiting, i.e., even-parity (spin-singlet) SC - in contrast the odd-parity (spin-triplet) pairing in superfluid ^3He [22].

The astonishing afore-mentioned "sample dependences" observed in the physical properties of polycrystalline CeCu_2Si_2 were resolved many years later by C. Geibel and collaborators. They found a rather narrow homogeneity range of the 122 phase within the chemical Ce-Cu-Si phase diagram, which allows for a site exchange between Cu and Si atoms by not more than 1 at%, cf. Fig. 5a [31]. Later it was realized that different ground-state properties are closely situated inside this homogeneity range. These ground states occur in the generic physical phase diagram, shown in Fig. 5b, at varying strengths of the 4f-conduction electron hybridization, measured by a general coupling parameter \( g [32] \). The latter can be increased and SC be favored by a slight increase of the Cu concentration and/or by the application of pressure. On the other hand, the 4f-conduction electron hybridization can be weakened by a tiny Cu deficit and/or partial substitution of Si by Ge (corresponding to a lattice expansion). This causes the formation of an antiferromagnetically ordered state ("A-phase"). In these "A-type" samples it was observed that, well below the Néel temperature \( T_N \), SC forms smoothly and coexists with AF order [32]. In the so-called "A/S-type" samples which are close to the true stoichiometry point (\( T_N \approx T_c \)), SC was found to expel the A-phase. However, by applying a magnetic
field $B > B_{c2}$ apt to suppress SC, the AF phase transition is recovered. These data can be used to identify a quantum critical point (QCP) at which the A phase is continuously suppressed upon increasing the hybridization strength $g$ [32]. In "A/S-type" samples, for which $T_N > T_c$, the superconducting transition is of first order, and there is no microscopic coexistence between SC and phase A [32].

**Figure 5.** (a) Ternary chemical Ce-Cu-Si phase diagram in the neighborhood of the homogeneity range of CeCu$_2$Si$_2$ (hatched). Note that the "width" of this homogeneity range (associated with the Ce concentration) is limited by the resolution of the X-ray scattering experiments. Solid lines indicate foreign phases which form together with CeCu$_2$Si$_2$ in off-stoichiometric samples. Upon variation of the composition, the dashed-dotted lines separate from each other: Superconductivity "S" (full circles) and "phase A" (open circles), competition between them "AS" (half-filled circles) and (some disordered AF) "phase X" (squares). The true stoichiometric (1:2:2) composition is located in the "AS sector". (b) Generic phase diagram of CeCu$_2$Si$_2$ combining data obtained from undoped polycrystals within the homogeneity range (hatched) and from Ge-doped ones, CeCu$_2$(Si$_1-x$Ge$_x$)$_2$. Since $T_A$ increases linearly with the Ge concentration $x$, the coupling constant $g$ was assumed to be linear in $(1-x)$, i.e., $(1-T_A)$. Sectors I, II and III indicate samples of "type A", "AS" and "S", respectively (from Ref. 32). Sectors I, II and III indicate samples of "type A", "AS" and "S", respectively (from Ref. 32).

The number of known HF superconductors has steadily increased from 4 (1984) to 11 (1998) and almost 40 at present [33]. Interestingly, the vast majority of the recently discovered examples belong to two distinct groups of intermetallics: (i) systems of the type Ce$_n$Tm$_{3n}$In$_{3n+2m}$ ($T$: transition metal), like CeCoIn$_5$ [34], and (ii) non-centrosymmetric superconductors, like CePt$_3$Si [35]. The former compounds are quasi-2D variants of cubic CeIn$_3$ ($T_c = 0.2$ K [36]), formed by stacking alternating layers of CeIn$_3$ and $T$In$_2$ sequentially along the tetragonal $c$-axis. As theoretically predicted [37], the reduced dimensionality causes an increase of $T_c$, which amounts to more than one order of magnitude ($T_c = 2.3$ K for CeCoIn$_5$). A further substantial $T_c$-enhancement could be achieved by replacing Ce (with well localized 4f shell) by Pu (with less localized 5f shell). Presently, PuCoGa$_5$ is the HF superconductor with the highest $T_c$ (18.5 K) [38]. Both, PuRhGa$_5$ [39] and NpPd$_3$Al$_2$ [40] also exhibit strongly enhanced $T_c$ values.

Commonly, HF superconductors show an anisotropic even-parity order parameter, as inferred from non-exponential $T$-dependences (often simple power laws) of the specific heat and related properties [12] which make d-wave pairing very likely [41]. This type of HF SC can, however, exist only in sufficiently clean samples, i.e., if the quasiparticle mean-free path exceeds the superconducting coherence length. A few HF superconductors are candidates for odd-parity pairing [42, 43]. In all these cases, SC seems to coexist either with AF (e.g., UPt$_3$ [44]) or FM (e.g., UGe$_2$ [45]) order. Recently, non-centrosymmetric superconductors have attracted substantial interest. This is due to the fact that the lack of inversion symmetry allows for a mixing of even- and odd-parity pair states, the degree of mixing depending on the strength of the antisymmetric spin-orbit interaction [46, 47]. Non-centrosymmetric SC has even been discovered in classical, i.e., phonon-mediated, superconductors [48].
Among HF superconductors, UPt$_3$ is probably the most extensively studied one. In its $T$-$B$ phase diagram several superconducting phases exist [49], similar to the different superfluid phases of $^3$He [22]. Multi-phase diagrams were discovered also for Th-doped UBe$_{13}$ [50], CeCoIn$_5$ [51] and PrOs$_4$Sb$_{12}$ [52], the latter system being unique in that here, electric quadrupole rather than magnetic dipole fluctuations are believed to mediate the Cooper pairing.

SC in HF compounds often occurs close to some type of instability. CePd$_2$Si$_2$ is the prototype material exhibiting a narrow dome of SC (with maximum $T_c \approx 0.4$ K) near a critical pressure $p_c \approx 2.8$ GPa, where AF order becomes continuously suppressed [36]. For UGe$_2$ [45], URhGe [53] and UCoGe [54] SC forms inside a regime of ferromagnetic (FM) order. Pure CeCu$_2$Si$_2$ [55] as well as CeCu$_2$Ge$_2$ [56] are superconductors within surprisingly wide ranges of pressure (Fig. 6). Upon weakening SC in CeCu$_2$Si$_2$ via reducing the mean-free path of the heavy charge carriers, i.e., by doping with 10 at% Ge, two distinct superconducting domes emerge [57] (Fig. 6). The one at low pressure is centered at $p_c \approx 0$ where AF order smoothly disappears, resembling the case of CePd$_2$Si$_2$ [36]. SC underneath the second dome, which occurs near a weak (first-order) valence instability at $p \approx 5.5$ GPa, is considered to result from almost critical valence fluctuations [58].

Figure 6. Phase diagram of CeCu$_2$(Si$_{1-x}$Ge$_x$)$_2$ showing transition temperatures into the antiferromagnetically ordered ($T_N$, closed symbols) and the superconducting state ($T_c$, open symbols) vs relative pressure $\Delta p = p - p_c(x)$, which reflects the inverse volume. The $p_c(x)$ values are chosen so that the magnetic transition lines for $x = 0.1$ ($p_c = 1.5$ GPa, circles) and $x = 0.25$ ($p_c = 2.5$ GPa, squares) coincide. Pure CeCu$_2$Si$_2$ is assumed here to have $p_c = 0.4$ GPa (open triangles) (from Ref. 57).

Whereas most of the Ce-based HF superconductors exhibit non-Fermi-liquid (NFL) behavior at low $T$, several of their U-based counterparts are moderately heavy FLs below the critical temperature of AF (in the case of URu$_2$Si$_2$: "hidden" [59]) order. This FL phase apparently coexists with AF/hidden order and becomes unstable against the formation of HF SC below $T_c$. UPt$_3$ was the first of the HF SCs for which experimental evidence of magnetically driven SC could be provided from inelastic-neutron-scattering (INS) experiments: Here, the magnetic scattering intensity was found to drop upon cooling to below $T_c \approx 0.5$ K [44]. A combined analysis of quasiparticle tunneling [60] and INS results on UPd$_2$Al$_3$ [61] revealed strong-coupling SC ($T_c \approx 2$ K), which is mediated by an acoustic magnon at the AF ordering wave vector ("magnetic exciton").

Guided by the theoretical prediction of HF SC being mediated by AF spin fluctuations [26], several Ce-based HF antiferromagnets could be turned into superconductors under applied pressure. Early examples were CeCu$_2$Ge$_2$ [56] and CeRh$_2$Si$_2$ [62] for both of which it was found that AF disappears abruptly at a critical pressure $p_c$, and heavy FL behavior develops at sufficiently low $T$ for pressures $p \gtrsim p_c$. The majority of the $p$-induced Ce-based HF superconductors, however, show pronounced NFL phenomena near $p = p_c$ up to sometimes surprisingly high temperatures - the aforementioned CePd$_2$Si$_2$ being a prototypical material [36].

4. CeCu$_2$Si$_2$: d-wave superconductivity mediated by nearly quantum-critical spin-density-wave fluctuations

Usually a continuous quantum phase transition, or quantum critical point (QCP), which is in proximity to HF SC, occurs under pressure [63]. This makes it experimentally very difficult to probe both the nature of the magnetic order and the excitation spectrum near the QCP. CeCu$_2$Si$_2$ is ideally suited for such a study because here, d-wave SC [41] forms in the vicinity of an AF QCP at low and even
ambient pressure (Fig. 6). Neutron-diffraction measurements [64] showed AF order to be of the spin-density-wave (SDW) type, with a small ordered moment (0.1\(\mu_B/\text{Ce}\)) and an incommensurate propagation vector \(\mathbf{Q}_{\text{AF}}\). The latter could be identified as a nesting wave vector of the renormalized Fermi surface [64]. For a CeCu$_2$Si$_2$ single crystal located on the paramagnetic side of the QCP ("S-type"), pronounced NFL thermodynamic and transport properties were observed in the field-driven low-\(T\) normal state [65]. They strongly suggested the QCP in CeCu$_2$Si$_2$ to be of the 3D SDW variety. A recent pressure study on an A/S type CeCu$_2$Si$_2$ single crystal [66] revealed a line of QCPs in the normal state, i.e., in the \(T = 0\) magnetic field-pressure plane. This line hits the superconducting phase boundary \(B_{c2}(p)\) where the latter assumes its maximum. It can be extended into the superconducting state and, thus, demonstrates a close connection of magnetism and SC.

**Figure 7.** Results of inelastic-neutron scattering on S-type CeCu$_2$Si$_2$. (a) Energy scans at \(q = (0.215, 0.215, 1.458)\) at \(T = 0.07\) K in the superconducting state (\(B = 0\)) and in the normal state (\(B = 2\) T) (from Ref. 67). (b) Linewidth \(\Gamma\) vs. temperature \(T\) of the quasielastic magnetic response at \(\mathbf{Q}_{\text{AF}}\) in the normal state (at \(B = 1.7\) T) as obtained by fits to the data, similar to the ones shown in (a). Solid line \(\Gamma(T) = 0.30\) meV + 0.14 (meV/K$^{1.38}$)\(T^{1.38}\) is a best fit to the data. The exponent (1.38 ± 0.16) meets 1.5 as predicted for a 3D SDW QCP (from Ref. 68). (c) Dispersion of spin fluctuations from fits to \(q\) scans at \(T = 0.06\) K in the normal state (\(B = 0\)). Solid lines indicate linear fits (from Ref. 68).

INS spectra on such an S-type single crystal revealed spin excitations, which are highly localized at \(\mathbf{Q}_{\text{AF}}\) in reciprocal space, but are broadly distributed in energy transfer and well described by a Lorentzian [67, 68]. As seen in Fig. 7a, out of this quasielastic excitation spectrum an inelastic line at 0.2 meV emerges in the superconducting state at very low \(T\). This was interpreted in terms of a shift of spectral weight from low excitation energies in the normal state to above the edge of a gap in the excitation spectrum in the superconducting state. Such a gap at \(\mathbf{Q}_{\text{AF}}\) does not open at the AF transition [64]. Since its \(T\)-dependence coincides with the scaled gap function of a weak-coupling d-wave superconductor, it has been related to the superconducting gap [67].

With increasing temperature in the normal state the magnetic response becomes weaker and broadens. The width of the Lorentzian can well be fitted by \(\Gamma = \Gamma_0 + \alpha T^{1.5}\) (Fig. 7b), which means a considerable slowing down of the (nearly isotropic) response upon cooling. This confirms the close vicinity of the paramagnetic S-type crystal to a 3D SDW QCP. Further on, the \(q\)-dependence of the magnetic response in the normal state, which for \(\omega = 0\) is well localized at \(\mathbf{Q}_{\text{AF}}\), reveals, upon increasing the energy transfer, two branches with linear dispersion (Fig. 7c). However, at \(\omega = (3 - 4)\omega_{\text{gap}}\) the magnetic intensity is completely lost, i.e., the broad quasielastic line of nearly quantum critical SDW fluctuations shows the \(q\)-dependence of a overdamped dispersive mode ("antiferromagnetic paramagnon"). In Fig. 7c, the opening of the spin gap at \(\mathbf{Q}_{\text{AF}}\) is clearly seen in the data for \(T < T_c\). From the mode velocity (\(v_P \approx 1040\) m/s [68]) one infers a substantial retardation of the
coupling of the heavy charge carriers (with $v_F^* \approx 8600 \text{ m/s}$ [69]) to the paramagnons, making the latter ideally suited to act as a glue for SC. This is supported by a surprisingly large reduction of the exchange energy, caused by the Kondo effect, when going from the normal to the superconducting state [67, 70].

CeCu$_2$Si$_2$ is the first superconductor for which nearly quantum-critical, broadly distributed SDW fluctuations could be identified as the major driving force for SC. However, this scenario may well prove to be more relevant for other pressure-induced HF superconductors like CePd$_2$Si$_2$ [36] as well as, e.g., high-$T_c$ cuprates [71]. For the electron-doped Fe-pnictide superconductors which crystallize in the same ThCr$_2$Si$_2$ structure as CeCu$_2$Si$_2$ (Ba(Fe$_{1-x}$Co$_x$)$_2$As$_2$ [72] being an example), $T_x$ or $T_p$ phase diagrams were established that are strikingly similar to those of CePd$_2$Si$_2$ [36] or CeCu$_2$Si$_2$ at low $p$, cf. Fig. 6. This suggests that nearly quantum critical SDW fluctuations may essentially contribute to the Cooper pairing in these new superconductors as well.

To which extent a Mott transition, like in cuprates [71] and organic charge-transfer salts [73], or an "orbital-selective" Mott transition in HF metals, like in CeRhIn$_5$ under pressure [74, 75] and, perhaps, $\beta$-YbAlB$_4$ [76], may be involved in the formation of unconventional SC remains an open question.

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