COUPLING COEFFICIENTS OF ROTATING RECTANGULAR DIELECTRIC RESONATORS WITH RECTANGULAR WAVEGUIDE

Alexander A. Trubin
Institute of Telecommunication Systems
Igor Sikorsky Kyiv Polytechnic Institute, Kyiv, Ukraine

Background. A further increase in the speed of information transfer is determined by more stringent requirements for the elements of communication devices. One of the most important components of such devices is various filters, which are often made on dielectric resonators. Calculation of the parameters of multi-section filters is impossible without further development of the theory of their design. The development of filter theory is based on electrodynamic modeling, which involves calculating the coupling coefficients of dielectric resonators in various transmission lines.

Objective. The aim of the research is to calculate and study the coupling coefficients of rectangular dielectric resonators with a rectangular metal waveguide when their axes rotate. Investigation of new effects to improve the performance of filters and other devices based on them.

Methods. Methods of technical electrodynamics are used to calculate and analyze the coupling coefficients. The end result is to obtain new analytical formulas for new structures with rectangular dielectric resonators, which make it possible to analyze and calculate their coupling coefficients.

Results. New analytical expressions are found for the coupling coefficients of dielectric resonators with the rotation of their axes in a rectangular waveguide.

Conclusions. The theory of designing filters based on new structures of dielectric resonators with rotation of their axes in metal waveguides has been expanded. New analytical relationships and new patterns of change in the coupling coefficients are found.

Keywords: dielectric filter; rectangular dielectric resonator; rotation; coupling coefficients.

Introduction

Band-pass and band-stop filters based on dielectric resonators of different shapes are used in access devices of various telecommunication systems [2-3]. Dielectric filter theory is well developed. Further improvement of filter characteristics can be achieved by using less traditional structures, in particular, dielectric resonators with rotation of their axes relative to each other and the transmission line [1, 3]. To calculate and analyze the characteristics of such filters, it is required to calculate the coupling coefficients of the DR located at an arbitrary angle with respect to the waveguide axis.

Statement of the problem

Rectangular DRs are widely used in filter design [2-6]. The purpose of this article is to calculate and study the coupling coefficients of the rectangular DRs with the fundamental wave \( H_{10} \) of a rectangular metal waveguide.

Calculation of radiation fields during rotation of dielectric resonators in a waveguide.

The field of natural oscillations \((\vec{e}, \vec{h})\) of a rectangular resonator with magnetic oscillations \(H_{nml}\) in local DR coordinate system \((x', y', z')\) (Fig. 1, a) we represented in the form:

\[
\begin{align*}
\vec{e}_x & = -\frac{1}{h} \left( \sin \beta_x x' \right) \left( \cos \beta_y y' \right) \left( \cos \beta_z z' \right) ; \\
\vec{e}_y & = \frac{1}{h} \left( \sin \beta_y y' \right) \left( \cos \beta_x x' \right) \left( \cos \beta_z z' \right) ; \\
\vec{e}_z & = 0 ; \\
\vec{h}_x & = \frac{1}{h} \left( \cos \beta_x x' \right) \left( \sin \beta_y y' \right) \left( \cos \beta_z z' \right) ; \\
\vec{h}_y & = \frac{1}{h} \left( \cos \beta_y y' \right) \left( \sin \beta_x x' \right) \left( -\sin \beta_z z' \right) ; \\
\vec{h}_z & = \frac{1}{h} \left( \cos \beta_z z' \right) \left( \sin \beta_x x' \right) \left( \cos \beta_y y' \right) ; \\
\end{align*}
\]
where \((\beta_x, \beta_y, \beta_z)\) are the wave numbers; 
\(k_0 = \omega \sqrt{\mu \varepsilon_0} \), \(k_1 = \omega \sqrt{\mu_1 \varepsilon_1} \); \(h_1\) - is the amplitude; \(\omega\) - is the circular frequency; \(\mu_0\) - is the magnetic permeability; \(\varepsilon_0\); \(\varepsilon_1\) - is the dielectric permittivity of the external space and resonator, respectively.

To obtain formulas for the coupling coefficients, we need to calculate the integral

\[
\mathcal{S} = \iint_S \left[ [\mathbf{e}, \mathbf{n}] \mathbf{H}^+ \right] + \left[ \mathbf{n}, \mathbf{h} \right] \mathbf{E}^+ \mathbf{d}S \quad (2)
\]

over the resonator surface \(S\). Here \((\mathbf{E}^+, \mathbf{H}^+)\) - electric and magnetic fields of the eigenwaves of a rectangular waveguide, respectively.

For a rectangular resonator, it is more convenient to use an approximate representation (2) that is performed with an accuracy \(1/Q_\Sigma\):

\[
c^\pm = -1/2 \mathcal{S} \left[ [\mathbf{e}, \mathbf{n}] \mathbf{H}^+ \right] + \left[ \mathbf{n}, \mathbf{h} \right] \mathbf{E}^+ \mathbf{d}S \quad (3)
\]

Where \(Q_\Sigma\) is the Q-factor of DR radiation; \(V\) - resonator volume.

The problem is that the resonator field and the waveguide field have a simple form in different coordinate systems. Therefore, to calculate (3), we represent the waveguide field [7] in the coordinate system of the dielectric resonator \((x', y', z')\).

After calculating (3), the coupling coefficient of the resonator with the waveguide is represented according to the normalization of the waveguide field:

\[
\bar{k}_s = \frac{c^2}{\omega W} \quad (3)
\]
where \( W \) - energy stored in the dielectric of the resonator.

**Rotation of the dielectric resonator relative to the x-axis.**

The description of the resonator rotation in the general case leads to rather cumbersome expressions, therefore, for simplicity, let us single out several simple cases of rotation about the fixed axes of the waveguide coordinate system \((x, y, z)\).

Let us first consider the rotation of a rectangular resonator relative to the x-axis of the waveguide (Fig. 2, a) for initial position shown on Fig. 1, d. In this case, the coupling coefficient of the DR with the \( H_{10} \) wave of a rectangular waveguide has the form:

\[
\kappa_k = \frac{\kappa_0}{\sqrt{\varepsilon}} \cdot \sin \chi_{1x} x_0 \left( \frac{\cos \chi_{1x} x_0}{\left(1 + \cos^2 \chi_{1x} x_0 \right)^{1/2}} \right) \cdot \\
\text{Where} \\
\kappa_0 = 2 (\varepsilon_r - 1)^2 \cdot \frac{k_0^2 \varepsilon}{|ab|} (\beta_r, \gamma_r) \cdot \frac{(\alpha_0, \beta_0, \gamma_0)}{\varepsilon} \cdot \\
\varepsilon_0 = \varepsilon \cdot \frac{1}{\varepsilon_0} \cdot \beta_r \cdot a_0 \cdot 2; \quad p_r = \beta_r b_0 / 2; \quad p_z = \beta_z L / 2; \quad \chi_{1x} = \pi / a; \quad (x_0, y_0, z_0) \text{ is the coordinates of the center of the resonator in the waveguide}; \quad \Gamma = \sqrt{k_0^2 - (\chi_{1x})^2} \text{ the wave numbers of a rectangular waveguide with a cross section } a \times b \text{ for the fundamental wave of a magnetic type } H_{10} \text{ (Fig. 1, b) [7].}
\]

Here and below

\[
\omega_r (\xi) = \frac{1}{p_r^2 - (q_r, \xi)^2}; \\
\left( -i p_r \cos \alpha_r, \sin \alpha_r \right) - q_r \xi \sin \alpha_r, \cos \left( q_r \xi \right) \right); \\
\omega_s (\xi) = \frac{1}{p_s^2 - (q_s, \xi)^2}; \\
\left( -i p_s \sin \gamma_s, \cos \gamma_s \right) - q_s \xi \sin \gamma_s, \cos \left( q_s \xi \right) \right);
\]

\( v = (x, y, z); \quad q_x = k_0 a_0 / 2; \quad q_y = k_0 b_0 / 2; \quad q_z = k_0 L / 2. \)

**Rotation of the dielectric resonator relative to the y-axis.**

The coupling coefficient of a dielectric resonator rotated relative to the y-axis with the \( H_{10} \) wave of a rectangular waveguide at the initial position shown in Fig. 1, d, has the form:

\[
\kappa_k = \frac{\kappa_0}{\sqrt{\varepsilon}} \cdot \sin \chi_{1y} y_0 \left( \frac{\cos \chi_{1y} y_0}{\left(1 + \cos^2 \chi_{1y} y_0 \right)^{1/2}} \right) \cdot \\
\text{Where} \\
\kappa_0 = 2 (\varepsilon_r - 1)^2 \cdot \frac{k_0^2 \varepsilon}{|ab|} (\beta_r, \gamma_r) \cdot \frac{(\alpha_0, \beta_0, \gamma_0)}{\varepsilon} \cdot \\
\varepsilon_0 = \varepsilon \cdot \frac{1}{\varepsilon_0} \cdot \beta_r \cdot a_0 \cdot 2; \quad p_r = \beta_r b_0 / 2; \quad p_z = \beta_z L / 2; \quad \chi_{1y} = \pi / a; \quad (x_0, y_0, z_0) \text{ is the coordinates of the center of the resonator in the waveguide}; \quad \Gamma = \sqrt{k_0^2 - (\chi_{1y})^2} \text{ the wave numbers of a rectangular waveguide with a cross section } a \times b \text{ for the fundamental wave of a magnetic type } H_{10} \text{ (Fig. 1, b) [7].}
\]
Rotations of the dielectric resonator relative to the $z$-axis.

In the case of rotation of the dielectric resonator relative to $z$-axis at the initial position shown in Fig. 1, d, the coupling coefficient takes the form:

$$
\tilde{k}_s = \tilde{k}_y \left[ \beta_1 \sin \alpha_1 \sigma_y (x_{1z} / k_0 \cos \alpha_1 + \Gamma / k_0 \sin \alpha_1) \right]^2 + \\
\beta_1 \cos \alpha_1 \sigma_y (x_{1z} / k_0 \cos \alpha_1) \sigma_y (x_{1z} / k_0 \sin \alpha_1) + \\
\sin^2 \chi_{1z} x_0.
$$

(7)
Calculation and analysis of coupling coefficients

Relations (4–7) were used for calculations the coupling coefficient dependences.

In Fig. 2-3, 5 depending on the coupling coefficients on the angles of rotation and transverse coordinates are plotted for the resonator with \( \varepsilon_{1r} = 36; \ a_0 = b_0; \ L / a_0 = 0.4 \). The cross section of the waveguide was \( a \times b = 35 \times 15 \text{ mm} \); frequency of the fundamental magnetic oscillation of the resonator \( H_{111} \) \( (h_z = h_x \cos \beta_x x' \cos \beta_y y' \cos \beta_z z' \text{ in (1) } f = 7 \text{ GHz}.) \)

The most interesting dependences were obtained for the rotation of the DR relative to the y-axis (Fig. 3, b, c). In the first case (Fig. 3, b, curve 2), we see that the coupling coefficient does not depend on the orientation of the DR axis at the point of "circular polarization" of the magnetic field of the waveguide:

\[
x_0 = \frac{1}{\chi_{1x}} \arctan \frac{\Gamma'}{\chi_{1x}}.
\]

In the second case (Fig. 3, c), the coupling coefficient does not depend on the transverse coordinates when the axis of the DR is rotated by an angle:

\[
x_C = \arctan \frac{\Gamma'}{\chi_{1x}}.
\]

In other cases, the dependences of the coupling coefficients on the angles of rotation change according to non-harmonic laws. In limiting cases \( \alpha_1, \beta_2 = 0, \pm \pi / 2 \), expressions (4-7) coincide with the coupling coefficients for standard positions (Fig. 1, b-d) of the resonator in the waveguide.

Discussion and Conclusion

In the presented work, new analytical expressions are obtained for the coupling coefficients of rectangular DRs with a regular rectangular waveguide, when the DR is rotated relative to the waveguide axis.

In the case of rotation about an axis located parallel to the narrow wall of the waveguide, the coupling coefficients do not depend on the angles at the point of circular polarization of the magnetic field, and in the case of rotation of the DR by an angle determined only by the longitudinal and transverse wave numbers, they do not depend on coordinates.

In all cases, the dependences of the coupling coefficients on the angles of rotation change according to non-harmonic laws.

References

1. Tomassoni C., Bastioli S., Snyder R.V. Propagating Waveguide Filters Using Dielectric Resonators // IEEE Trans. on Microwave Theory and Techniques. 2015. Vol. 63, No. 12, pp. 4366-4375.
Коэффициенты связи прямоугольных диэлектрических резонаторов с прямоугольным волноводом при вращении их осей

Труби́н О.О.

Коэффициенты связи прямоугольных диэлектрических резонаторов с прямоугольным волноводом при вращении их осей

Проблематика. Подавляющее большинство передачи информации в настоящее время осуществляется на основе сигналов с высокой скоростью. Это определяется большим спросом на высокоскоростные каналы передачи информации. Одной из важнейших составных частей передающей системы является фильтр, который позволяет разделить сигналы на необходимые частоты. Фильтры могут быть выполнены на основе диэлектрических резонаторов, которые позволяют получить высокую степень разделения частот.

Мета досліджень. Метою досліджень є розрахунок та дослідження коефіцієнтів зв'язку прямоугольних диелектричних резонаторів з прямоугольним металевим хвилеводом при застосуванні нових структур з обертанням осей резонаторів. Дослідження нових ефектів, що дозволяють покращувати характеристики розсіювання фільтрів та інших пристріїв на їх основі.

Методика реалізації. Дослідження проведено за допомогою чисельного моделювання. Для розрахунку та аналізу коефіцієнтів зв'язку використовувалися методи технічної електродинамики. Конструкційним результатом є використання нових аналітичних формул для розрахунку коефіцієнтів зв'язку в різноманітних лініях передачі.

Результати досліджень. Найдено нові аналітичні вирази для розрахунку коефіцієнтів зв'язку діелектричних резонаторів з обертанням их осей в прямоугольному волноводі.

Ключові слова: діелектричний фільтр; пряма прямокутний діелектричний резонатор; обертання; коефіцієнти зв'язку.
Выводы. Расширена теория конструирования фильтров на новых структурах диэлектрических резонаторов с вращением их осей в металлических волноводах. Найдены новые аналитические соотношения и новые закономерности изменения коэффициентов связи.

Ключевые слова: диэлектрический фильтр; прямоугольный диэлектрический резонатор; вращение; коэффициенты связи.