Censored regression for modelling small arms trade volumes and its ‘Forensic’ use for exploring unreported trades

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Abstract
In this paper, we use a censored regression model to investigate data on the international trade of small arms and ammunition provided by the Norwegian Initiative on Small Arms Transfers. Taking a network-based view on the transfers, we do not only rely on exogenous covariates but also estimate endogenous network effects. We apply a spatial autocorrelation gravity model with multiple weight matrices. The likelihood is maximized employing the Monte Carlo expectation maximization algorithm. Our approach reveals strong and stable endogenous network effects. Furthermore, we find evidence for a substantial path dependence as well as a close connection between exports of civilian and military small arms. The model is then used in a ‘forensic’ manner to analyse latent network structures and thereby to identify countries with higher or lower tendency to export or import than reflected in the data. The approach is also validated using a simulation study.

KEYWORDS
gravity model, latent variable, Monte Carlo EM algorithm, network analysis, spatial autocorrelation, zero inflated data
1 INTRODUCTION

The Small Arms Survey Update 2018 indicates transfers of small arms in 2015 amounting to 5.7 billion (Holtom & Pavesi, 2018, p. 19) with a major share and highest increases in ammunitions (Holtom & Pavesi, 2018, p. 22). Given the often fatal consequences—civilian or military—of the availability of these arms for intrastate conflict and shootings as well as for interstate war, the absence of empirical evidence for supplier-recipient networks is surprising. A major reason behind this research gap are the notorious data deficiencies due to non-reporting and illicit trafficking (see Holtom & Pavesi, 2018, p. 29–46). Based on the only large-scale data base for small arms (Marsh & McDougal, 2016), we aim to analyse for the first time the small arms trading network. To do so, we combine gravity models with network statistics to apply a ‘forensic’ statistical analysis.

Starting with the seminal work of Tinbergen (1962), the gravity model was established as a valuable tool of empirical trade research in empirical economics. The success of the model stems from its intuitive interpretation as well as its surprisingly strong empirical validity, see, for example Head and Mayer (2014). In its most simplistic version, the gravity model is based on a loose analogy to Newtonian mechanics and relates the dyadic volumes of commercial trade to the geographical distance between the exporter and the importer and their respective market sizes. More generally, it aims to explain the volume of a trade flow based on factors related to the exporter, the importer and relational covariates. This basic model is sometimes enriched by heterogeneity effects and extensions guided by economic theory, for example multilateral resistance terms as introduced by Anderson and Van Wincoop (2003).

The recent literature in econometrics related to the gravity models goes beyond spatial dependence in terms of geographical distance and shows the need to account for endogenous (network) dependencies among transfers. See, for example Balazsi et al. (2018) and LeSage and Fischer (2019a, b). More general, Fagiolo et al. (2009, 2010) have shown that the binary network topology resulting from trade flows incorporates dependencies that constitute valuable additional information which is not captured by the valued flows (see also more recently Almog et al., 2019). As a consequence, it appears to be necessary to enrich the gravity framework with valued network statistics, derived from binary topologies. Especially in the domain of international arms trade, it is very plausible that network effects play a very central role and are even more important than spatial dependencies. However, while the gravity model is already used for empirical research in arms trading, the focus seldom lies on network dependencies.

An early example for the application of gravity models to arms trade research is the work of Bergstrand (1992). Although he doubted the suitability of the model for arms trade because of the strong political considerations in this area, the approach was taken up more recently. Akerman and Seim (2014) and Thurner et al. (2019) use a binary gravity model in order to explain the exchange of major conventional weapons (MCW) in a network context. Martinez-Zarzoso and Johannsen (2017) rely on the framework of Helpman et al. (2008) to investigate the influence of economic and political variables on the so-called extensive and intensive margin of MCW trade. The interplay between oil imports and arms exports is determined using a gravity model in Bove et al. (2018). While the papers above focus on the exchange of MCW, we are investigating data on transfers of small arms and ammunition (SAA) provided by the Norwegian Initiative on Small Arms Transfers (NISAT). This data is arguably even better suited for a gravity model since small arms are potentially less dependent on political decision making and many more trade occurrences are recorded.

We propose a network perspective on international SAA trade and conceptualize countries as nodes and transfers between them as directed, valued edges. Although gravity models are a
standard tool for the analysis of network data (Kolaczyk, 2009), endogenous network effects are rarely incorporated in these models. We do so by connecting the idea of gravity models with the spatial autoregressive (SAR) model adjusted to network data. Especially in sociology, SAR models are regularly used in a network context since the early eighties (Doreian, 1989; Doreian et al., 1984; Dow et al., 1982). They are called network autocorrelation models in this strand of literature. More recently, network autocorrelation models became popular in political science applications, see for example Franzese and Hays (2007), Hays et al. (2010) and Metz and Ingold (2017). Here, it is assumed that actors with certain characteristics are embedded in a network and this embedding leads to contagion and/or spillover effects transmitted through the edges that relate the actors (Leenders, 2002). Hence, one presumes that the characteristics of actors are correlated because a specific social, political or economic mechanism is connecting them. Note that the design of these models is different as compared to the usual setup of gravity models since the outcome is related to the nodes, and the edges only represent indicators for node dependence.

In this paper, we are interested in the dependencies among the transfers (instead of the actors) and account for outdegree, indegree, reciprocity, time dependencies and exogenous covariates. A similar model in a non-network context is the spatial gravity model (LeSage & Pace, 2008) that accounts for spatial dependence of the exporter, the importer as well as for the spatial importer–exporter dependencies.

Similar, but even more severe as compared to commercial trade data, we observe a high degree of reported non-trade in SAA. In other words, the trade data exhibits a large percentage of zero entries. For many valued network models, as for example the generalized exponential random graph model (GERGM, see Denny et al., 2016; Desmarais & Cranmer, 2012) or latent space models (e.g. Hoff et al., 2002), this constitutes a major problem because these models cannot deal with zero inflation. For gravity models, this problem is often solved using Tobit estimators (see, e.g. Eaton & Kortum, 2001; Eaton & Tamura, 1994 and more generally Wooldridge, 2010). This estimator approaches zero inflated data via a latent utility assumption which solves the problem with a censored regression model. We are following this basic idea and accommodate the zero inflation problem by employing a censored SAR model that can be fitted using the Monte Carlo expectation maximization (MCEM) algorithm (Dempster et al., 1977; Wei & Tanner, 1990). There are already several similar EM-based approaches that have been pursued. For instance Suesse and Zammit-Mangion (2017) use the EM algorithm in spatial econometric models, Schumacher et al. (2017) apply an EM-based application to a censored regression model with autoregressive errors, and Vaida and Liu (2009) utilize EM estimation in a censored linear mixed effects model. In Augugliaro et al. (2018), a similar estimation procedure is used in the context of fitting a graphical LASSO to genetic networks.

While the application of the model per se provides new insights into SAA trading, a second objective of this paper is to make use of the model to explore the validity of reported zero trades. This reflects a ‘forensic’ analysis, that is, we estimate, whether unreported trades are likely to have happened based on the fitted model. Despite this idea is in line with forensic statistics and forensic economics (Aitken & Taroni, 2004; Zitzewitz, 2012) our goal is apparently less ambitious. We do not aim to provide statistical evidence that some states are under-reporting but we do want to investigate potential under-reporting by utilizing the fitted network model.

The paper is organized as follows. After presenting the data in Section 2, we explain the model and show how to proceed with estimation and inference in Section 3. In Section 4 we apply the model to the arms trade data and Section 5 provides the ‘forensic’ analysis, accompanied by a simulation study. Section 6 concludes the paper. In the Supplementary Materials we provide further investigations and more details on estimation and inference.
DATA DESCRIPTION

Since 2001, the Geneva-based Small Arms Survey specializes on documenting the international flows of the respective products. However, only the Norwegian Initiative on Small Arms Transfers (NISAT, see Marsh, 2017) provides truly relational data necessary for applying network analysis. The NISAT database contains relational information on the trade of small arms, light weapons and ammunition (see also Marsh & McDougal, 2016). This information is collected from different sources as described in Haug et al. (2002). Although NISAT represents the most reliable source of data regarding the exchange of small arms and light weapons, there is nevertheless an enormous amount of uncertainty inherent to arms trading data. This is especially true for light weapons where data quality and availability is partly very poor (Herron et al., 2011). Therefore, we restrict our analysis to small arms and the associated ammunition (SAA). See Table A1 in Annex A for the types of small arms and ammunition included in the data set. Note, that the NISAT database also contains data on sporting guns, which we excluded from the data set since we are particularly interested in the export of small arms with potential military value. Actually, we will rely on transferred sporting guns volumes later as an explanatory variable. In the remaining data set, more than 86,000 SAA transfers are recorded for the years 1992–2014, providing the exporting country, the importing country as well as the transferred arms category. The value of the export is measured in constant 2012 USD. In order to make estimation feasible, we restrict our analysis to a subnetwork and select those countries that account for the major share of the SAA trade activity. The resulting 59 countries (see Annex A, Table A2) account for 73%–91% (depending on the year) of the total transfer volume and have participated in arms trade at least once in each year under study. Hence, we investigate the ‘core’ of the international small arms trade network, balancing the trade-off between the number of countries included, the share of trade volume and the density of the subnetworks. In Figure 1, we show two binary networks for 1992 and 2014, with the countries represented as nodes and the arms transfers as directed edges among them.

In the left panel of Figure 2, we show the aggregated exports for the most important exporters United States (USA), Germany (DEU) and Italy (ITA) together with the exports of the 56 other countries (other). On the right-hand side of Figure 2, we present the density, defined as the sum of existent edges divided by the number of potential edges. Although the network can be described as a dense one

FIGURE 1 Binary SAA trade network for the 59 most relevant countries in 1992 (left) and 2014 (right). Countries are indicated by grey nodes and transfers by edges in black.
(as compared to the density of typical social networks), the density is smaller than 0.2 in the beginning and remains below 0.4 in the subsequent recent years.

3 | REGRESSION MODEL

3.1 | General model

In the following, we will formalize a gravity model of trade with network autocorrelation effects with a special focus will on the fitting procedure with a tobit-type estimation strategy. Also note that the model itself is introduced as a static model but Section 4.1 shows how the model can be used to describe dynamic networks. See also the Supplementary Materials for a comparison of this model with other network models and standard gravity approaches.

Let \( Y=(Y_{ij}) \in \mathbb{R}^n \times n \) represent a network of transfers between \( n \) nodes (countries), with diagonal elements \( Y_{ii} \) left undefined. Note, that we do not have records for trades for all potential \( N=n(n-1) \) directed edges (transfers). Instead, it holds that \( Y_{ij} > 0 \) if a transfer is recorded while \( Y_{ij} = 0 \) refers to no recorded network transfer. In order to cope with the non-recorded transfers, we assume that for each dyad from country \( i \) to country \( j \), there exists a latent utility for the potential transfer. Let \( Z=(Z_{ij}) \in \mathbb{R}^n \times n \) denote the network of latent utility and assume that we observe a realized trade \( Y_{ij} \) only if \( Z_{ij} > c \) for some threshold value \( c \). We further assume the elements of \( Z \) to be normally distributed and the utility \( Z_{ij} \) materializes to an observed transfer through

\[
Y_{ij} = \begin{cases} 
0 & \text{for } Z_{ij} < c \\
\exp(Z_{ij}) & \text{for } Z_{ij} \geq c ,
\end{cases}
\]

with \( c = \min \{ \log(Y_{ij}); Y_{ij} > 0 \} \). Hence, we assume \( Z_{ij} \) to be normal and \( \exp(Z_{ij}) \) to be log-normal distributed (and as a consequence \( Y_{ij} \) to be truncated log-normal). For the latent network \( Z \), we rearrange the

![Figure 2](image-url)
entries of $Z$ and set $\tilde{Z} = \text{vec}(Z) \in \mathbb{R}^N$ as the row wise vectorization of $Z$, excluding the diagonal elements. Based on that, we model the latent utility $\tilde{Z}$ with a regression model

$$\tilde{Z} = \sum_{k=1}^{q} \rho_k W_k \tilde{Z} + X\beta + \epsilon, \quad \epsilon \sim \mathcal{N}_N(0, \sigma^2 I_N)$$

(1)

with $\beta$ being a $p$-dimensional parameter vector for the design matrix $X$. The matrices $W_k$ are row-normalized weight matrices representing linear endogenous network effects (see Section 4.2 for concrete examples), with parameters $\rho_k$ measuring their strength. Model (1) is usually known as SAR model and we refer to LeSage and Pace (2009) for a more detailed discussion and to Lacombe (2004) or LeSage and Pace (2008) for similar models with multiple weight matrices. Standard software implementations that allow for a likelihood based estimation of the model are mostly restricted to the special case with $q = 1$, for example in the R package spdep (Bivand & Piras, 2015; Bivand et al., 2013). The package tnam by Leifeld et al. (2017) allows for multiple weight matrices but is based on pseudo-likelihood estimation and therefore valid only if the weight matrices exclusively apply to exogenous covariates. Another possibility to estimate similar models is given by the package ARCensReg (Schumacher et al., 2017), initially designed to fit models with autoregressive errors. Because of the similar mathematical structure, the package could be used to fit models with spatially dependent errors known as spatial error models (SEM). In the given case, however, the network structure is assumed to influence the response directly which prevents us from using the package.

Model (1) can be rewritten as

$$\tilde{Z} = \left( I_N - \sum_{k=1}^{q} \rho_k W_k \right)^{-1} (X\beta + \epsilon) = (A(\rho))^{-1} (X\beta + \epsilon) = B(\rho)(X\beta + \epsilon),$$

where the dependence on the $q$-dimensional parameter vector $\rho = (\rho_1, ..., \rho_q)^T$ is made explicit for notational clarity. Similar as in Besag (1974) and assuming that all $n(n-1)=N$ edges in the network are observed, their distribution is given by

$$P(\tilde{Z}|X; \theta) = \frac{1}{(2\pi\sigma^2)^{N/2}} |A(\rho)| \exp \left\{ -\frac{(A(\rho)\tilde{Z} - X\beta)^T (A(\rho)\tilde{Z} - X\beta)}{2\sigma^2} \right\}.$$

(2)

The parameter space of this model is restricted such that $A(\rho)$ is non-singular, which is ensured if the eigenvalues of $A(\rho)$ are real valued and greater than zero.

### 3.2 Censored regression model

Note, that Equation (2) holds only for fully connected networks but we do not observe $Z_{ij}$ for $Z_{ij} < c$. That is, the observed zero transfers $Y_{ij} = 0$ do not provide exact information on $Z_{ij}$ but only provide the information that $Z_{ij} < c$. For all observed positive transfers $Y_{ij} > 0$ on the other hand, we have $Z_{ij} = \log(Y_{ij})$. We rearrange the vector value entries $\tilde{Z}$ and decompose it into the observed component...
denoted by \( \tilde{Z}_o \) and the unobserved, and hence missing, components \( \tilde{Z}_m \). A reordering according to the observational pattern of \( \tilde{Z} \) gives

\[
\tilde{Z} = \begin{pmatrix} \tilde{Z}_o \\ \tilde{Z}_m \end{pmatrix} \sim \mathcal{N}_N \left( \begin{pmatrix} \mu_o \\ \mu_m \end{pmatrix}, \begin{pmatrix} \Sigma_{oo} & \Sigma_{om} \\ \Sigma_{mo} & \Sigma_{mm} \end{pmatrix} \right),
\]

where censored information is available for \( \tilde{Z}_m \) in that \( \tilde{Z}_m < c \). Since the density of the network (see Figure 2) is roughly between 0.2 and 0.4 in all years, the number of missing transfers \( N_m \) is always substantially larger than the number of observed ones (\( N_o \)). The mean-covariance structure is given by

\[
B(\rho)X\beta = \begin{pmatrix} \mu_o \\ \mu_m \end{pmatrix}, \quad B(\rho)(B(\rho))^T\sigma^2 = \begin{pmatrix} \Sigma_{oo} & \Sigma_{om} \\ \Sigma_{mo} & \Sigma_{mm} \end{pmatrix}.
\]

In the following, we will denote all reordered matrices in the notation with double subscripts, that is, \( A_{oo} \) refers to the submatrix of \( A \) where only interactions of observed variables \( \tilde{Z}_o \) enter.

### 3.3 Monte Carlo EM estimation

In order to estimate the unknown parameter vector \( \theta = (\rho, \beta, \sigma^2) \in \mathbb{R}^{q+p+1} \), we employ the EM algorithm (Dempster et al., 1977). The complete log-likelihood \( \ell_{\text{comp}}(\theta) \) is simply derived from Equation (2). We are interested in maximizing the observed log-likelihood \( \ell_{\text{obs}}(\theta) = \ell_{\tilde{Z}_o}(\theta) + \ell_{\tilde{Z}_m|\tilde{Z}_o}(\theta) \), where the first part is simply the multivariate normal density of the observed transfers. The second part equals

\[
\ell_{\tilde{Z}_m|\tilde{Z}_o}(\theta) = \log \left\{ \int_{(-\infty,c)^{N_m}} \frac{1}{\sqrt{(2\pi)^{N_m} |\Sigma_{m|o}|}} \exp \left\{ -\frac{(U - \mu_{m|o})^T\Sigma_{m|o}^{-1}(U - \mu_{m|o})}{2} \right\} dU \right\},
\]

where \( \mu_{m|o} \) and \( \Sigma_{m|o} \) are the first and second conditional moments. Because \( N_m \) is greater than 2000 in each year, the observed log-likelihood is numerically hard to evaluate (and even more so to maximize) with state of the art software implementation. As a solution, we apply the EM algorithm and maximize \( Q(\theta|\theta_0) = \mathbb{E}_{\theta_0}[\ell_{\text{comp}}(\theta) | \tilde{Z}_o, X, \mathcal{M}] \) iteratively. The observation space is given by

\[
\mathcal{M} = \{ \tilde{Z}_m: \tilde{Z}_{m,1} < c, \ldots, \tilde{Z}_{m,N_m} < c \}. \tag{3}
\]

#### 3.3.1 E-Step

The E-Step essentially boils down to calculating the first two moments of a multivariate normally distributed variable \( \tilde{Z}^e \)

\[
\tilde{Z}^e \sim \mathcal{N}_{N_m}(\mu_m + \Sigma_{mo}\Sigma_{oo}^{-1}(\tilde{Z}_o - \mu_o), \Sigma_{mm} - \Sigma_{mo}\Sigma_{oo}^{-1}\Sigma_{om}) \tag{4}
\]
with restriction $\mathcal{M}$ from Equation (3) applied to $\tilde{Z}$. Let those truncated moments be $\mu^c_{m/o}$ and $\Sigma^c_{m/o}$ and define

$$
\tilde{Z}^* = \begin{pmatrix}
\tilde{Z}_o \\
\mu^c_{m/o}
\end{pmatrix}
$$

(5)

as the vector that contains the observed values as well as the conditional expectation of the non-observed ones. Given the two moments, we can calculate the conditional expectation of the quadratic form (see Mathai & Provost, 1992):

$$
S^e(\rho) = \mathbb{E}_{\tilde{\theta}}[\tilde{Z}^T(A(\rho))^T A(\rho) \tilde{Z} | \tilde{Z}_o, X, \mathcal{M}]
$$

= \text{tr} \left( (A_{mm}(\rho))^T A_{mm}(\rho) \Sigma^c_{m/o} \right) + (\tilde{Z}^*)^T(A(\rho))^T A(\rho) \tilde{Z}^*.
$$

(6)

Then, the function to maximize in the M-step is given by

$$
Q(\theta | \theta_0) = -\frac{N}{2} \log(2\pi \sigma^2) + \log(|A(\rho)|) - \frac{(S^e(\rho) - 2\beta^T X^T A(\rho) \tilde{Z}^* + \beta^T X^T X \beta)}{2\sigma^2}.
$$

(7)

In order to find the first and second moment of a truncated multivariate normally distributed variable, Vaida and Liu (2009) use the results of Tallis (1961) on the moment generating function to provide closed form expressions of the E-Step. This, however, is not practicable in our setting as (a) software implementations of a multivariate normal distribution function are overstrained by the high dimension of our problem (the standard package in R, mvtnorm by Genz et al. (2016) is not able to process dimensions higher than 1000) and (b) as noted by Schumacher et al. (2017), even if the distribution function could be evaluated, the closed form solution is computationally very expensive which leads to infeasible convergence times in applications with a high number of non-observed values. The same is true for the direct calculation using the moment generating function implemented in R by Manjunath and Wilhelm (2012).

A practicable alternative consists in using the MCEM algorithm (Wei & Tanner, 1990) where intractable expectations are replaced by sample-based approximations. In our specific case, we use the R package TruncatedNormal by Botev (2017) in order to draw 1000 samples from the truncated multivariate normal distribution in order to estimate its unknown moments. An alternative would be to enrich the E-Step with a stochastic approximation step (SAEM algorithm, see Schumacher et al., 2017 for a detailed description) which reduces the number of simulations needed and is very efficient if the M-step is faster than the E-Step. In our specific application, the computational bottleneck comes with the M-Step and simulations showed that the SAEM converges more slowly than the MCEM algorithm.

3.3.2 | M-Step

It is numerically more efficient to reduce the log-likelihood to a profile log-likelihood by first maximizing with respect to $\beta$ and $\sigma^2$ and then with respect to $\rho$. Using the derivatives of Equation (7) with respect to $\beta$ and $\sigma^2$ and defining $\hat{\beta}(\rho)$ and $\hat{\sigma}^2(\rho)$ as the solutions of the score equations as functions of $\rho$ it follows that

$$
\hat{\beta}(\rho) = (X^T X)^{-1} X^T A(\rho) \tilde{Z}^*,
$$

$$
\hat{\sigma}^2(\rho) = \frac{S^e(\rho) - \tilde{Z}^{*T} (A(\rho))^T H A(\rho) \tilde{Z}^*}{N},
$$

(8)
where $H = X(X^T X)^{-1}X^T$ is the hat matrix. With $k$ being a constant we can write the profiled function $\hat{Q}(\cdot)$ as

$$\hat{Q}(\rho | \theta_0) = k + \log(|A(\rho)|) - \frac{N}{2} \log \left( S^*(\rho) - \hat{Z}^*^T (A(\rho))^T HA(\rho) \hat{Z}^* \right).$$  \tag{9}$$

The expressions $(A(\rho))^T A(\rho)$ and $(A(\rho))^T HA(\rho)$ have derivatives

$$\frac{\partial (A(\rho))^T A(\rho)}{\partial \rho_k} = -W_k - W_k^T + 2\rho_k W_k^T W_k + \sum_{l \neq k} \rho_l (W_l^T W_l + W_k^T W_l) =: R_k(\rho),$$

$$\frac{\partial (A(\rho))^T HA(\rho)}{\partial \rho_k} = -HW_k - W_k^T H + 2\rho_k W_k^T HW_k + \sum_{l \neq k} \rho_l (W_l^T HW_l + W_l^T W_k) =: H_k(\rho).$$

Now define

$$R^*_k(\rho) = \text{tr}(R_{k,mn}(\rho) \Sigma^c_{m|o}) + (\hat{Z}^*)^T R_k(\rho) \hat{Z}^*$$

which gives

$$\frac{\partial \hat{Q}(\rho | \theta_0)}{\partial \rho_k} = -\text{tr}(B(\rho) W_k) - \frac{N}{2} \frac{R^*_k(\rho) - \hat{Z}^*^T H_k(\rho) \hat{Z}^*}{S^*(\rho) - \hat{Z}^*^T (A(\rho))^T HA(\rho) \hat{Z}^*}. \tag{10}$$

Iteration between the E- and the M-step provide the final estimate $\hat{\theta}$. The variance of $\hat{\theta}$ can be calculated using Louis (1982) formula. Based on an approximation of the Hessian matrix, this allows to obtain asymptotically valid confidence intervals. For more details on the practical implementation and the residuals (including a check of their distribution and an investigation of remaining transitive structure), see the Supplementary Materials.

4 | APPLICATION TO THE DATA

In the following, we apply the model to the arms trade data. We observe a series of 23 networks $Y_t = (Y_{t,ij})$ for the years $t=1992, \ldots, 2014$. We apply the natural logarithm to the observed part of the data in order to transform the truncated log-normal data to truncated normally distributed data. We define $d_t = \min\{ \{ Y_{t,ij} > 0 \} \}$ as the lowest strictly positive value in the network at a given year and set $c_t = \log(d_t)$. That is, the threshold $c_t$ is defined such that at a given year all transfers below the smallest observed log-transformed transfer in that sample are censored. Utility below the threshold $c_t$ implies that no transfer was carried out or was not recorded. (See the Supplementary Materials for the distribution of the log-transformed response).

4.1 | Time dependencies

Because the data under study comes as a time series of networks, we need to deal with time-dependencies. This can be achieved by estimating each time period separately. This relaxes the unrealistic assumption of time-constant effects for more than 20 years and reduces the computational effort. From Equation (6), it can be seen that model fitting requires the first two (and, as shown in the Supplementary Materials, for inference the first four) moments of a high-dimensional truncated
multivariate normal distribution with a non-diagonal covariance matrix. This is possible using a simulation-based MCEM approach.

Conceptually, the year-wise estimation can be legitimated by assuming that the distributions of the latent utility networks \( Z_t \) for \( t=1993, \ldots, 2014 \) are independent after conditioning on the network effects (see Section 4.2), the exogenous covariates (see Section 4.3) and the lagged trade volumes of the previous years. This approach allows us to apply the static model (1) to the sequence of arms trade networks over time. To be specific, we take the first network in 1992 to be given and assume independence for each utility network \( Z_t \) in a given year \( t \) conditionally on the previous utility networks \( Z_{t-1}, \ldots, Z_{t-5} \). However, in fact we do not fully observe the latent utility \( Z_t \) but only the realized utility \( Y_t \) (the observed trade), leading us to the following dependence structure

\[
P(Z_T, \ldots, Z_1|X_T, \ldots, X_1; \theta) = P(Z_T|Y_{T-1}, \ldots, Y_{T-5}, X_T; \theta) \cdot P(Z_{T-1}|Y_{T-2}, \ldots, Y_{T-6}, X_{T-1}; \theta) \cdot \ldots \cdot P(Z_6|Y_5, Y_4, Y_3, Y_2, Y_1, X_4; \theta) \cdot P(Z_5|Y_4, Y_3, Y_2, Y_1, X_5; \theta) \cdot \ldots \cdot P(Z_2|Y_1, X_2; \theta) \cdot P(Z_1|X_1; \theta),
\]

where \( X_t \) denotes exogenous information and the indices 1, \ldots, \( T \) represent the years 1992, \ldots, 2014. Furthermore, for those years, where less than five previous networks are available, we assume that the Markov order can be shortened accordingly. This type of Markov structure is similar to standard autoregressive models and not unusual for modelling dependencies over time. We operationalize the dependence on the past 5 years by a moving average by incorporating a term called path dependency, that is defined as the average logarithmic transfers in the last 5 years given that there was trade in all of these 5 years. As a consequence, the path dependency is a measure for persistent trade relationships over time. Furthermore it also has a substantial meaning. Path dependency, that is, frequently repeated trading, leads to inertias that arises because of diminishing transaction costs, trust relations, security aspects and potentially interoperability. The variable is a very important determinant in the MCW trade network (Thurner et al., 2019). In the years with less than five lagged periods available, the moving average is shortened accordingly.

4.2 | Network structure

Next, we specify the network-specific effects represented by matrices \( W_k \) in model (1) (note that the weighting matrices are independent of time). We include three effects which are explained subsequently and visualized in Figure 3.

(a) Reciprocity

(b) Exporter Effect

(c) Importer Effect

FIGURE 3 Schematic representation of linear network effects. The focal edge in dashed grey
Reciprocity: The reciprocity effect measures whether the export volume from country $i$ to country $j$ increases in the export volume from $j$ to $i$. This can be achieved by defining the binary matrix $W_{\text{reciprocity}}$ such that in each row that corresponds to the transfer $Z_{t,ij}$ the column that corresponds to $Z_{t,ji}$ is equal to one. In the given context it is a plausible assumption that countries tend to specialize in certain types of small arms and/or ammunition and therefore complement each other with their products. Mutual trade is likely to be encouraged by political partnerships and indicates strategic elements, induced by bilateral agreements. The measure is also investigated in the context of commercial trade (e.g. Barigozzi et al., 2010; Garlaschelli & Loffredo, 2005; Ward et al., 2013). In the arms trade literature, reciprocity is specified by Thurner et al. (2019), with the finding that this is rather unusual in the context of MCW.

Exporter and Importer Effect: The exporter and the importer effect have their analogies in binary networks and can be interpreted as the valued versions of the outdegree and the indegree. The corresponding weighting matrices $W_{\text{exporter}}$ and $W_{\text{importer}}$ are defined such that the row of $W_{\text{exporter}}$ that corresponds to $Z_{t,ij}$ has entries $1/(n-2)$ for all columns that correspond to $Z_{t,ju}$ with $u\neq j$. For $W_{\text{importer}}$ all columns $Z_{t,uj}$ with $u\neq i$ in the row that is related to $Z_{t,ij}$ are set to $1/(n-2)$. The coefficient of the exporter effect measures whether the transfers going out from a certain exporter $i$ are correlated. A positive effect indicates the presence of ‘super-exporters’. Contrary, the importer effect measures whether the imports of a certain importer $j$ are related, with a positive effect indicating ‘super-importers’. The degree structure is a crucial feature of the SAA network because a rather small number of countries accounts for the major share of the trade volume, while a small share of (potentially identical) importing countries accounts for a great amount of the import volume.

4.3 Exogenous covariates

Node-Specific Variables: Following standard applications (Egger & Staub, 2016; Head & Mayer, 2014; Thurner et al., 2019; Ward et al., 2013), we control for the logarithmic real GDP in constant 2010 USD as a measure for the market size of the exporting and importing country. The data are provided by the World Bank (2017). For the 2 years 1993–1994, no reliable GDP data are available for Serbia, Croatia, Estonia, Latvia, Lithuania and Slovenia, we therefore assume that the GDP remained constant in the first three years for this countries. In order to control for the potential influence of intrastate conflicts, we insert a binary variable that is one if there is an intrastate conflict in the receiving country in the respective year and zero otherwise. The corresponding data are available from the webpage of the Uppsala Conflict Data Program (UCDP, 2019).

Edge-Specific Distance Measures: Because of the strong empirical evidence that geographical distance is a relevant factor in trade (Disdier & Head, 2008), we control for the logarithmic distance between capital cities in kilometres (Gleditsch, 2013). In recent applications of the gravity model to arms trade (Akerman & Seim, 2014; Bove et al., 2018; Martinez-Zarzoso & Johannsen, 2017; Thurner et al., 2019), it is argued that political distance measures in terms of regime dissimilarity must also be inserted in the gravity equation. We use the absolute difference of the polity IV index (Marshall, 2017) between two countries, ranging from 20 (highest ideological distance) to 0 (no ideological distance). Additionally, we include a dummy variable for formal alliances between the exporting and importing country, being one if the two countries have a formal alliance. The data are available from Correlates of War Project (2017) until 2012 and we assume that the alliances stay constant for the years 2013 and 2014.

Edge-Specific Trade Measures: We enrich the model with a 5-year moving average of logarithmic civilian weapon transfers. The intuition behind that is that exports of SAA for military
usage and civilian usage might be correlated. This is plausible because countries that export massive amounts of civilian arms also have the capabilities to produce military arms. The data is also provided by NISAT (Marsh & McDougal, 2016). Furthermore it seems plausible that there is a connection between the volume of small arms traded and the volume of MCW. MCW transfers are recorded by the Stockholm International Peace Research Institute (SIPRI) and measured in so called trend indicator values (TIV). This measure represents the military value and the production costs of the transferred products. For detailed explanation of the data and the TIV see SIPRI (2017a, b) and Holtom et al. (2012). We use a dummy variable that is one if there was an MCW transfer from country $i$ to $j$ in the actual year or in the four preceding years, zero otherwise. Additionally, we use the logarithmic sum of the exported TIV volumes in the actual year and the four preceding ones.

4.4 | Final model

Given these specifications, the final model is now given by

$$Z_{t,ij} = X_{t,ij}^T \beta_t + \rho_{t,1} Z_{t,ji} + \rho_{t,2} \frac{1}{n-2} \sum_{u \neq j} Z_{t,iu} + \rho_{t,3} \frac{1}{n-2} \sum_{u \neq i} Z_{t,uj} + \epsilon_{t,ij},$$

where the path dependence is added as a variable in $X_{t,ij}$.

4.5 | Results: Coefficients

In Figure 4, we show the time series of coefficients obtained from fitting each year individually against time for the years 1993–2014 as explained above. The whiskers around the coefficients give two standard error bands based on an approximation of the Hessian matrix as outlined by Louis (1982). The resulting confidence bands do not correspond to point-wise confidence intervals but only give a confidence measure for the given parameters as estimated at the respective time points.

*Exogenous Covariates:* The exogenous covariates in the first row and in the second row on the right represent the standard gravity variables logarithmic GDP of the exporter and the importer as well as the geographical distance between them (second row, right panel). Overall, the expected results of the gravity equation hold, except for the logarithmic GDP of the exporter with coefficients close to zero and the zero-line included in the confidence bands in the most recent years. This is an interesting result, because it highlights the fact, that market size is not a prerequisite for producing and exporting internationally competitive SAA. This finding is in stark contrast to the insights on MCW by Thurner et al. (2019).

The effect of the geographical distance strongly negative and the zero-line is never included in the confidence bands which is similar in other applications of the gravity model.

Regarding the political security measures we find that the presence of a conflict in the importing country (second row, left panel) has mostly a positive effect that can, however, statistically not be distinguished from zero in most years. The coefficient on the dissimilarity of political regimes (third row,
FIGURE 4  Time-series of annually estimated regression coefficients. The whiskers give ±2 standard errors left panel) is mostly negative but also often includes zero in the confidence bands. The coefficient on the dummy variable for formal alliances (third row, right panel) is positive in the beginning but almost permanently indistinguishable from zero from 2001 on.
The large coefficients of the path dependence (fourth row, left panel) illustrate an important feature of the network, namely path inertia. Intensive persistent transfer relationships in the past, strongly increase the export volume in the present. Similarly we find a strong connection between exporting civilian and military arms (fourth row, right panel). Looking at the relation between SAA and MCW trade we find that having traded MCW (fifth row, left panel) in the actual year or the in the four preceding ones has a strong positive effect—at least until the last two years. However, at the same time the effect of the logarithmic sum of the TIV values (fifth row, right panel) has a negative and effect with confidence bands that rarely include zero. That is, rather small transfers of MCW tend to coincide with SAA exports while dyads with high amounts of MCW exchange tend to transfer small arms to a relatively lower degree.

Network Structure: On the right panel in the sixth row of Figure 4, the coefficients for reciprocity are shown. The coefficients remain almost constant and positive with values between 0.02 and 0.06. Regarding the confidence bands we infer that there is at least a tendency that mutuality increases the volume of arms exchanged.

The strongest endogenous effect is the exporter effect (bottom row, left panel) with coefficients that are consistently positive. This indicates that the transfers stemming from the same exporter are indeed highly correlated and reflects the existence of ‘super-sellers’ like the United States, Germany, Brazil or Italy. On the other hand, we also find a stable, positive importer effect (bottom row, right panel). The fact that the two coefficients on the exporter and the importer effect are much higher than the reciprocity effect provides structural information about heterogeneity in the network. Being a strong exporter or sending to a strong importer increases the export volume more than simply having imported high amounts from the respective partner.

In the Supplementary Materials, we show that these results are in line with the result one obtains by applying more classical gravity models without endogenous network effects.

5 | ‘FORENSIC’ STATISTICAL ANALYSIS

5.1 | Under- and over-reporting

Our model rests on the assumption that the SAA network is determined by a latent utility network \(Z_t\). Based on the joint distribution (2), we can estimate the probability of \(Z_{t,ij}\) being greater than the censoring threshold \(c_t\), given the covariates, the endogenous effects and the rest of the network. In order to do so, let \(Z_{t-,ij}\) represent the \((N-1)\)-dimensional vector that contains the realized and the expected values of the latent variables, except the entry that corresponds to the transfer from \(i\) to \(j\). Because we are interested whether some latent transfers could have realized according to the model, we form the expectations without the restriction that the latent transfers must be smaller than \(c_t\). Based on this, we define the conditional probability of a specific latent transfer being greater than the threshold by

\[
\pi_{t,ij} = P(Z_{t,ij} > c_t | X_{t,ij}, PD_t, Z_{t-,ij}; \hat{\theta}_t).
\]

By construction (see the Supplementary Materials for the derivation), \(\pi_{t,ij}\) is high for transfers that are observed in the data set \((Y_{t,ij} > 0)\) and small for transfers that are not observed \((Y_{t,ij} = 0)\). However, we may calculate a high value of \(\pi_{t,ij}\) that is, a high probability for a realized transfer of arms, despite the data actually indicates \(Y_{t,ij} = 0\). We propose to consider this as potential under-reporting. Such a zero-record can happen due to random fluctuation, factors beyond the model as for example historical relationships, or because de-facto existent transfers have not been reported. More technically, under-reported trades can
be viewed as false positives. Vice versa, we may observe false negatives if we obtain a low value for $\pi_{t,ij}$ although $Y_{t,ij}$ is greater than zero. We label this as over-reporting. This label is not intended to suggest that potentially over-reported transfers in fact never happened, but highlights transfers where our model attaches a lower level of latent utility than manifested in the data. Naturally, our main ‘forensic’ interest is in uncovering potential under-reporting.

Apparently, this requires the fixation of a threshold value for the probabilities. Based on receiver-operating characteristic (ROC) curves, an optimal threshold value can be found using Youden’s $J$ statistic (Youden, 1950). This value is optimal in the sense that it allows for a separation such that both sensitivity and specificity are maximized. This defines the binary network

$$\Pi_t = (I(\pi_{t,ij} > J_t)).$$

This network is now set into relation with the observed binary SAA trade

$$\Gamma_t = (I(Y_{t,ij} > 0)).$$

Comparing $\Pi_t$ and $\Gamma_t$, we can define the ‘forensic’ network

$$\Omega_t = (\omega_{t,ij}) = \Pi_t - \Gamma_t,$$

which in turn creates two new binary networks

$$\Omega^+_t = (I(\omega_{t,ij} = 1))$$

$$\Omega^-_t = (I(\omega_{t,ij} = -1)).$$

For $\omega_{t,ij} = 1$, the model predicted a transfer that is not present in the data set, and for $\omega_{t,ij} = -1$, the model did not predict an actual transfer. Following our convention from above we label $\Omega^+_t$ as the under-reporting network and to $\Omega^-_t$ as the over-reporting network of unpredicted but realized transfers.

### 5.2 Simulation study of ‘forensic’ power

Before we apply our model in a ‘forensic’ matter to identify transfers with potential under-reporting, we demonstrate the behaviour of the model in a static simulation study (hence the dependency on time $t$ it omitted) to explore its detection properties. We use two different settings in order to investigating how well the proposed approach identifies under-reporting. The first setting builds on the following Data Generating Process (DGP1)

$$\rho = (0.1, 0.2, 0.3)^T, \quad \beta = (1, 2, 3, 4, 5)^T, \quad p = 5, \quad n = 20, \quad N = 380$$

$$X \sim \mathcal{N}_p(\mathbf{1}, \mathbf{I}_p)$$

$$Z \sim \mathcal{N}_N(B(\rho)X\beta, B(\rho)B(\rho)^T)$$

$$\tilde{Z}_{ij} = I(Z_{ij} > q_{0.75}(Z))Z_{ij}, \quad \text{for } i \neq j = 1, ..., n. \tag{11}$$

Here, $q_{0.75}(Z)$ denotes the 75% quantile and we are censoring the network towards an observed density of 0.25. Note, that DGP1 is not subject to under-reporting and all censored responses are in fact below the censoring threshold. The results of running DGP1 100 times and applying the estimation procedure
are summarized in the Supplementary Materials, indicating that the expected values approximate the latent variables very well and that we are able to find unbiased estimates despite the enormous amount of censoring.

In order to validate the ‘forensic’ power of the model, we run a second experiment (DGP2), being a modified version of DGP1. To be precise, we are censoring again 75% of the observations but only 65% correspond to the lowest ones, while the remaining 10% are randomly selected among the observations that are in fact higher than the threshold \( q_{0.65}(Y) \). This share of observations represents the under-reporting. Again we run DGP2 100 times.

In order to make the following evaluation transparent, we represent the evaluation scheme (e.g. Fawcett, 2006) for both DGPs in Table 1. On the left-hand side, we regard the simulation without under-reporting (DGP1). In this setting, we can investigate the false positive rate (FPR), being the sum of the false positives (FP) relative to the number of all observations that are in fact not under-reported. A low value for this measures means in DGP1 that in a setting without under-reporting a low share is classified as under-reporting. In DGP2, the measure tells us whether including true under-reporting in the simulation leads to an increase of misclassified under-reporting. The corresponding results are visualized on the top panel of Figure 5. The FPR shows a higher variability in DGP2 (right panel) and is slightly higher as compared to DGP1 (left panel). However, the results provide evidence for an overall low FPR in both setting.

Furthermore, DGP2 allows to evaluate the share of under-reported observations that is identified. This is assessed based on the true positive rate (TPR) and shown on the bottom left panel of Figure 5. In 50% of all simulation runs, we are able to identify at least 95% of the falsely censored observations and even in the simulation runs with the worst performance, the TPR does not fall below 74%. Additionally, we investigate the false discovery rate (FDR) that relates the observations that are wrongly classified to be under-reporting to the sum of all observations that are classified for under-reporting. A low value for this measure provides evidence, whether the model is able to keep the number of potential over-reporting that are in fact not under-reporting low. The corresponding results are shown in the south-east panel of Figure 5. We find a median share of less than 26% to be classified incorrectly.

| TABLE 1 | Schematic representation of the evaluation scheme used in the simulation study |
| TRUE | | | |
| UR | UR | ∑ |
| Estimated | 0 | FP | FP |
| 0 | TN | TN |
| 0 | 0.75N | 0.75N |

(b) Classifier evaluation DGP2.

| TRUE | | | |
| UR | UR | ∑ |
| Estimated | TP | FP | TP+FP |
| FN | TN | FN+TN |
| 0.1N | 0.65N | 0.75N |

True conditions in the columns and Estimated in the rows. UR denotes under-reporting and UR denotes censored observations that are not under-reported. Further abbreviations: True positive (TP), false positive (FP), false negative (FN) and true negative (TN).
We now turn back to the data and provide the development of the densities for the latent networks in Figure 6. In the real data, over- and under-reporting is certainly not random but potentially clustered among countries. We therefore evaluate node (i.e. country)-specific network topologies of $\Omega^+_t$ and $\Omega^-_t$ for each year $t$ and summarize the information in box-plots for each country, ordered according to the median of the respective feature. This is shown in Figure 7 for potential under-reporting and in Figure 8 for over-reporting. In the first row, we represent the eigenvector centrality scores. This

**FIGURE 5** Results of DGP1 and DGP2. The top panel shows boxplots for the false positive rate (FPR) in DGP1 (left) and DGP2 (right). On the bottom boxplots for the true positive rate (TPR) and the false discovery rate (FDR) are provided for DGP2.

**FIGURE 6** Densities of the under-reporting network $\Omega^-_t$ and the over-reporting network $\Omega^+_t$ over time [Colour figure can be viewed at wileyonlinelibrary.com]

**5.3 ‘Forensic’ analysis of arms trade data**

We now turn back to the data and provide the development of the densities for the latent networks in Figure 6. In the real data, over- and under-reporting is certainly not random but potentially clustered among countries. We therefore evaluate node (i.e. country)-specific network topologies of $\Omega^+_t$ and $\Omega^-_t$ for each year $t$ and summarize the information in box-plots for each country, ordered according to the median of the respective feature. This is shown in Figure 7 for potential under-reporting and in Figure 8 for over-reporting. In the first row, we represent the eigenvector centrality scores. This
FIGURE 7  Ordered box-plot representation of topological network features of the under-reporting networks $\Omega^+_t$ for $t=1993, \ldots, 2014$: Eigenvalue centrality (top), outdegree (middle) and indegree (bottom)

FIGURE 8  Ordered box-plot representation of topological network features of the over-reporting networks $\Omega^-_t$ for $t=1993, \ldots, 2014$: Eigenvalue centrality (top), outdegree (middle) and indegree (bottom)
measure is undirected and constructed such that the centrality of each country is proportional to the sum of the centralities of its trading partners. Hence, countries with high scores have many potentially under-reported (over-reported) import and export relations with many other countries that themselves have many under-reported (over-reported) import- and export-relations, see, for example Csardi and Nepusz (2006). In the middle row, we present the outdegree that is the number of potentially under-reported (over-reported) exports for a country. The bottom row in Figures 7 and 8 gives the indegree that is the number of potentially under-reported (over-reported) imports. All measures are scaled to take values between 0 and 1. Countries at the right hand side in the plots of Figure 7 are potentially under-reporting and in Figure 8, the right hand side of the plots mirrors high over-reporting.

To detect persistent patterns in the networks on a dyadic level, we check whether potential under-reporting or over-reporting occurs frequently, that is, counting instances of \( \omega^+_{t,ij} = 1 \) and \( \omega^-_{t,ij} = 1 \) for \( t \in T = \{1993, \ldots, 2014\} \). Denote the aggregated ‘forensic’ networks as

\[
\Omega^+_T = \sum_{t \in T} \Omega^+_t, \\
\Omega^-_T = \sum_{t \in T} \Omega^-_t.
\]

We look at the distribution of elements of \( \Omega^+_T \) and \( \Omega^-_T \), which is plotted in Figure 9. On the horizontal axis, we show the possible values of the matrix entries, that is the number years where transfers in the ‘forensic networks occur. This ranges from 1 (potential under-reporting or over-reporting in one year) to

![Figure 9](image-url)

**Figure 9** Frequency distribution of transfers in the aggregated under-reporting network (\( \Omega^+_T \), black ‘+’) and over-reporting (\( \Omega^-_T \), grey ‘−’) networks on the vertical axis. Number of years with under-reporting (\( \omega^+_{t,ij} \)) or over-reporting (\( \omega^-_{t,ij} \)) on the horizontal axis. Transfers with the most years predicted are indicated in the form ‘exporter–importer’ in black for \( \Omega^+_T \) and in grey for \( \Omega^-_T \).
22 (potential under-reporting or over-reporting in all years). The maximum entry of $\Omega^+_t$ is thereby less than 22, namely 21, while the maximum value of $\Omega^-_t$ is 15. On the vertical axis of Figure 9 we show the frequency of the entries of $\Omega^+_t$ and $\Omega^-_t$. Apparently for ‘forensic’ purposes, large values of $\Omega^+_t$ are of particular interest, since they report pairs of countries which are likely to under-report.

The line in solid black with the ‘+’ symbols represents $\Omega^+_t$ and the line in grey with the ‘−’ symbols represents the under-reporting network. Additionally, we indicate for both networks the pairs of countries (i.e. sender and receiver) which are of particular interest for ‘forensic’ purposes. This means for example for an element of $\Omega^+_t$ that has value 21, that the respective transfer from $i$ to $j$ is one of the four transfers appeared that appeared 21 times in the under-reporting network.

Under-reporting networks $\Omega^+_t$: Looking at the eigenvector centrality scores of Figure 7 on the top provides conclusive results about countries that are central in the network series $\Omega^+_t$. Among the countries where arms transfers are potentially under-reported, we find many Western European countries such as Belgium (BEL), Sweden (SWE), France (FRA), Spain (ESP) and Denmark (DNK). However, the list of presumed under-reporting is headed by Russia (RUS) and Turkey (TUR) but also Brazil (BRA), Israel (ISR) and China (CHN) have high scores. These countries also play a dominant role in Figure 9. In particular, exports from Brazil (BRA) to Russia (RUS), Hungary (HUN), Ukraine (UKR), China (CHN) and Japan (JPN) are likely to be frequently under-reported. Similarly, exports from Russia (RUS) to Cyprus (CYP), Denmark (DNK), Ireland (IRL), Turkey (TUR), Sweden (SWE), Portugal (PRT) and Greece (GRC) are listed. We also find imports of Israel (ISR) from Finland (FIN), Belgium (BEL) and Austria (AUT) as well as exports of Belgium (BEL) to India (IND), Ukraine (UKR), Russia (RUS), Lebanon (LEB), Colombia (COL) and China (CHN).

Over-reporting networks $\Omega^-_t$: Among the twelve countries with the highest Eigenvector centrality in Figure 8 is Croatia (HRV) as the only European country. There are, however, many countries from Asia such as Singapore (SGP), India (IND), Thailand (THA), Indonesia (IDN), Malaysia (MYS), South Korea (KOR) and Philippines (PHL). Furthermore, South Africa (ZAF), New Zealand (NZL), Australia (AUS) and Israel (ISR) are among the countries where trade activity is often over-reported. For Asian countries as well as for Australia and New Zealand this might mirror the fact that those countries export many SAA to Europe and the United States despite the strongly negative distance effect of the model. Furthermore, this network is very likely to be driven by bilateral agreements and historical developments not covered by the covariates. See for example in Figure 9 the number of over-reporting related to the Baltic countries Estonia (EST), Lithuania (LTU) and Latvia (LVA).

It remains to be emphasized that the constructions of the ‘forensic’ networks relies on our model with corresponding assumptions and admittedly high degrees of uncertainty. As a consequence, it does not allow for definite statements about actual hidden transfers. However, many of the dyads listed in Figure 9 indeed have either traded massive amounts of civilian arms (e.g. AUT-CHN, BRA-RUS, RUS-BEL, RUS-DNK) or had frequent MCW trade relations (e.g. RUS-CYP) but almost no documented small arms transfers for military usage. Additionally, many of the countries that take central positions in the ‘forensic’ networks are known for not being very transparent with respect to their SAA exports and imports (see, e.g. the small arms transparency barometer).

6 | CONCLUSION

In this paper, we have modelled the volumes of international transfers of small arms and ammunition for the years 1992–2014 based on data provided by NISAT. As an analytical tool, we combined the gravity model of trade with a modified SAR model that allows to enrich the analysis by endogenous network dependencies, accounting for exporter-related, importer-related and reciprocal
dependencies among the transfers in the network. Using a censored normal regression model, we are able to include information provided by zero-valued transfers. The infeasible likelihood of the censored model is maximized using a MCEM algorithm. The fitted model shows strong and stable endogenous network effect, especially related to the sender effect and the receiver effect but also some evidence for reciprocity. Additionally, we find a high coefficient on path dependency and a close connection to the exports of civilian small arms. Conditional on that, the classical gravity hypothesis is confirmed with respect to the GDP of the importer and physical distance but only exceptionally with respect to political distance measures and the GDP of the exporter. This contrasts with the MCW network where distance plays no role, where political similarity and GDP of the exporter have a strong impact (see Thurner et al., 2019). Actually, this difference is plausible, as the technological requirements for the production of small and ammunition are relatively low, and strategic considerations of world-wide acting countries make geographical distances a negligible factor for MCW trade.

Building on our latent utility framework, we were able to explore latent utility networks. With the construction of under-reporting and over-reporting networks we perform for the first time a ‘forensic’ approach in this area highlighting especially potentially under-reported exports of Russia and Turkey. We refrain, of course, from making too far-reaching assertions. Note that we do not claim to provide unambiguous claims for intentional false reporting. However, we demonstrate that some zero entries in the SAA trading network tend to be not plausible.

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REFERENCES
Aitken, C.G.G. & Taroni, F. (2004) Statistics and the evaluation of evidence for forensic scientists. Chichester: John Wiley & Sons.
Akerman, A. & Seim, A.L. (2014) The global arms trade network 1950–2007. Journal of Comparative Economics, 42(3), 535–551.
Almog, A., Bird, R. & Garlaschelli, D. (2019) Enhanced gravity model of trade: Reconciling macroeconomic and network models. Frontiers in Physics, 7, 55.
Anderson, J.E. & Van Wincoop, E. (2003) Gravity with gravitas: A solution to the border puzzle. American Economic Review, 93(1), 170–192.
Augugliaro, L., Abbruzzo, A. & Vinciotti, V. (2018) ℓ1-Penalized censored Gaussian graphical model. Biostatistics. https://doi.org/10.1093/biostatistics/kxy043.
Balazsi, L., Matyas, L. & Wansbeek, T. (2018) The estimation of multidimensional fixed effects panel data models. Econometric Reviews, 37(3), 212–227. https://doi.org/10.1080/07474938.2015.1032164.
Barigozzi, M., Fagiolo, G. & Garlaschelli, D. (2010) Multinetwork of international trade: A commodity-specific analysis. Physical Review E, 81(4), 046104.
Bergstrand, J.H. (1992) On modeling the impact of arms reductions on world trade. In: Isard, C. & Anderton, W. (Eds.) Economics of arms reduction and the peace process. Amsterdam: Elsevier Science Publishing, pp. 121–142.
Besag, J. (1974) Spatial interaction and the statistical analysis of lattice systems. Journal of the Royal Statistical Society: Series B, 36(2), 192–236.
Bivand, R. & Piras, G. (2015) Comparing implementations of estimation methods for spatial econometrics. Journal of Statistical Software, 63(18), 1–36. Available at: http://www.jstatsoft.org/v63/i18/.
Bivand, R., Hauke, J. & Kossowski, T. (2013) Computing the Jacobian in Gaussian spatial autoregressive models: An illustrated comparison of available methods. *Geographical Analysis*, 45(2), 150–179. Available at: http://www.jstatsoft.org/v63/i18/.

Botev, Z.I. (2017) The normal law under linear restrictions: Simulation and estimation via minimax tilting. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 79(1), 125–148.

Bove, V., Deiana, C. & Nisticò, R. (2018) Global arms trade and oil dependence*. *The Journal of Law, Economics, and Organization*, 34(2), 272–299. https://doi.org/10.1093/jleo/ewy007.

Correlates of War Project. (2017) Formal interstate alliance dataset, 1648–2012, version 4.1. Available at: http://www.correlatesofwar.org/data-sets/formal-alliances. Accessed: 2017-02-06.

Csardi, G. & Nepusz, T. (2006) The igraph software package for complex network research. *InterJournal, Complex Systems*, 1695(5), 1–9.

Dempster, A.P., Laird, N.M. & Rubin, D.B. (1977) Maximum likelihood from incomplete data via the EM algorithm. *Journal of the Royal Statistical Society: Series B*, 39(1), 1–38.

Denny, M.J., Wilson, J.D., Cranmer, S.J., Desmarais, B.A. & Bhamidi, S. (2016) Gergm: Estimation and fit diagnostics for generalized exponential random graph models. Available at: https://cran.r-project.org/web/packages/GERGM/vignettes/getting_started.html. R package version 0.13.0.

Desmarais, B.A. & Cranmer, S.J. (2012) Statistical inference for valued-edge networks: The generalized exponential random graph model. *PloS One*, 7(1), e30136. https://doi.org/10.1371/journal.pone.0030136.

Doreian, P. (1989) Models of network effects on social actors. In: Freeman, L., White, D. & Romney, K. (Eds.) *Research methods in social network analysis*. Washington DC: George Mason University Press, pp. 295–317.

Doreian, P., Teunter, K. & Wang, C.-H. (1984) Network autocorrelation models: Some Monte Carlo results. *Sociological Methods & Research*, 13(2), 155–200.

Dow, M.M., Burton, M.L. & White, D.R. (1982) Network autocorrelation: A simulation study of a foundational problem. *Social Networks*, 4, 169–200.

Eaton, J. & Kortum, S. (2001) Trade in capital goods. *European Economic Review*, 45(7), 1195–1235.

Eaton, J. & Tamura, A. (1994) Bilateralism and regionalism in Japanese and US trade and direct foreign investment patterns. *Journal of the Japanese and International Economics*, 8(4), 478–510.

Egger, P.H. & Staub, K.E. (2016) GLM estimation of trade gravity models with fixed effects. *Empirical Economics*, 50(1), 137–175.

Fagiolo, G., Reyes, J. & Schiavo, S. (2009) World-trade web: Topological properties, dynamics, and evolution. *Physical Review E*, 79(3), 036115.

Fagiolo, G., Reyes, J. & Schiavo, S. (2010) The evolution of the world trade web: A weighted-network analysis. *Journal of Evolutionary Economics*, 20(4), 479–514.

Fawcett, T. (2006) An introduction to ROC analysis. *Pattern Recognition Letters*, 27(8), 861–874.

Franzese, R.J. & Hays, J.C. (2007) Spatial econometric models of cross-sectional interdependence in political science panel and time-series-cross-section data. *Political Analysis*, 15(2), 140–164.

Garlaschelli, D. & Loffredo, M.I. (2005) Structure and evolution of the world trade network. *Physica A: Statistical Mechanics and its Applications*, 355(1), 138–144.

Genz, A., Bretz, F., Miwa, T., Mi, X., Leisch, F., Scheipl, F. et al. (2016) mvtnorm: Multivariate normal and t distributions. Available at: http://CRAN.R-project.org/package=mvtnorm. R package version 1.0-5.

Gleditsch, K.S. (2013) Distance between capital cities. Available at: http://privatewww.essex.ac.uk/ksg/data-5.html. Accessed: 2017-04-07.

Haug, M., Langvandahl, M., Lumpe, L. & Marsh, N. (2002) Shining a light on small arms exports: The record of state transparency. Occasional Paper 4, Norwegian Initiative of Small Arms Transfers.

Hays, J.C., Kachi, A. & Franzese, R.J. (2010) A spatial model incorporating dynamic, endogenous network interdependence: A political science application. *Statistical Methodology*, 7(3), 406–428.

Head, K. & Mayer, T. (2014) Gravity equations: Workhorse, toolkit, and cookbook. In: Gopinath, G., Helpman, E. & Rogoff, K. (Eds.) *Handbook of international economics*, volume 4. Amsterdam: Elsevier Science Publishing, pp. 131–195. Available at: http://www.sciencedirect.com/science/handbooks/15734404.

Helpman, E., Melitz, M. & Rubinstein, Y. (2008) Estimating trade flows: Trading partners and trading volumes. *The Quarterly Journal of Economics*, 123(2), 441–487.
Herron, P., Marsh, N. & Schroeder, M. (2011) Larger but less known - authorized light weapons transfers. Available at: http://www.smallarmsurvey.org/fileadmin/docs/S-Trade-Update/SAS-Trade-Update-2018.pdf. Accessed: 2019-21-05.
Hoff, P.D., Raftery, A.E. & Handcock, M.S. (2002) Latent space approaches to social network analysis. Journal of the American Statistical Association, 97(460), 1090–1098.
Holtom, P. & Pavesi, I. (2018) Small arms survey - trade update 2018. Available at: http://www.smallarmssurvey.org/fileadmin/docs/S-Trade-Update/SAS-Trade-Update-2018.pdf. Accessed: 2019-06-02.
Holtom, P., Bromley, M. & Simmel, V. (2012) Measuring international arms transfers. Stockholm: International Peace Research Institute. Available at: https://www.sipri.org/sites/default/files/files/FS/SIPRIFS1212.pdf.
Kolaczyk, E.D. (2009). Statistical analysis of network data. Methods and models. New York: Springer Science & Business Media.
Lacombe, D.J. (2004) Does econometric methodology matter? An analysis of public policy using spatial econometric techniques. Geographical Analysis, 36(2), 105–118.
Leenders, R.T.A.J. (2002) Modeling social influence through network autocorrelation: Constructing the weight matrix. Social Networks, 24(1), 21–47.
Leifeld, P., Cranmer, S.J. & Desmarais, B.A. (2017) tnam: Temporal network autocorrelation models. Available at: https://cran.r-project.org/package=tnam. R package version 1.6.5.
LeSage, J.P. & Fischer, M.M. (2019a) Conventional versus network dependence panel data gravity model specifications. Available at: https://epub.wu.ac.at/6828/. Working Papers in Regional Science, 2019/02. WU Vienna University of Economics and Business, Vienna.
LeSage, J.P. & Fischer, M.M. (2019b) Cross-sectional dependence model specifications in a static trade panel data setting. Journal of Geographical Systems, 22, 1–42.
LeSage, J.P. & Pace, R.K. (2008) Spatial econometric modeling of origin-destination flows. Journal of Regional Science, 48(5), 941–967.
LeSage, J.P. & Pace, R.K. (2009) Introduction to spatial econometrics. Boca Raton: CRC Press.
Louis, T.A. (1982) Finding the observed information matrix when using the EM algorithm. Journal of the Royal Statistical Society: Series B, 44(2), 226–233.
Manjunath B.G., Wilhelm S. Moments calculation for the double truncated multivariate normal density. Social Science Research Network. https://doi.org/10.2139/ssrn.1472153
Marsh, N. (2017) Norwegian initiative on small arms transfers, firearms and ammunition trade data 1992–2014. Available at: http://www.nisat.prio.org. Accessed: 27.03.2017.
Marshall, M.G. (2017) Polity IV project: Political regime characteristics and transitions, 1800–2016. Available at: http://www.systemicpeace.org/inscr data.html. Accessed: 2017-06-02.
Martinez-Zarzoso, I. & Johannsen, F. (2017) The gravity of arms. Defence and Peace Economics. Available at: https://doi.org/10.1080/10242694.2017.1324722.
Mathai, A.M. & Provost, S.B. (1992) Quadratic forms in random variables: Theory and applications. London: Taylor & Francis.
Metz, F. & Ingold, K. (2017) Politics of the precautionary principle: Assessing actors’ preferences in water protection policy. Policy Sciences, 50(4), 721–743.
Schumacher, F.L., Lachos, V.H. & Dey, D.K. (2017) Censored regression models with autoregressive errors: A likelihood-based perspective. Canadian Journal of Statistics, 45(4), 375–392.
SIPRI. (2017a) Arms transfers database. Available at: https://www.sipri.org/databases/armstransfers. Accessed: 2017-03-10.
SIPRI. (2017b) Arms transfers database - methodology. Available at: https://www.sipri.org/databases/armstransfers/ background. Accessed: 2017-03-10.
Suesse, T. & Zammit-Mangion, A. (2017) Computational aspects of the EM algorithm for spatial econometric models with missing data. Journal of Statistical Computation and Simulation, 87(9), 1767–1786.
Tallis, G.M. (1961) The moment generating function of the truncated multi-normal distribution. Journal of the Royal Statistical Society: Series B, 23(1), 223–229.
Thurner, P.W., Schmid, C., Christian, S. J. & Kauermann, G. (2019) Network interdependencies and the evolution of international arms trade. Journal of Conflict Resolution, 63(7), 1736–1764. https://doi.org/10.1177/0022002718801965
Tinbergen, J. (1962) Shaping the world economy: An analysis of world trade flows. *New York Twentieth Century Fund*, 5(1), 27–30.

UCDP. (2019) UCDP. Available at: http://ucdp.uu.se/downloads/. Accessed: 2018-12-01.

Vaida, F. & Liu, L. (2009) Fast implementation for normal mixed effects models with censored response. *Journal of Computational and Graphical Statistics*, 18(4), 797–817.

Ward, M.D., Ahlquist, J.S. & Rozenas, A. (2013) Gravity's rainbow: A dynamic latent space model for the world trade network. *Network Science*, 1(1), 95–118.

Wei, G.C.G. & Tanner, M.A. (1990) A Monte Carlo implementation of the EM algorithm and the poor man’s data augmentation algorithms. *Journal of the American Statistical Association*, 85(411), 699–704.

Wooldridge, J.M. (2010) *Econometric analysis of cross section and panel data*. Cambridge: MIT Press.

World Bank. (2017) World bank open data, real GDP. Available at: http://data.worldbank.org/. Accessed: 2017-04-01.

Youden, W.J. (1950) Index for rating diagnostic tests. *Cancer*, 3(1), 32–35.

Zitzewitz, E. (2012) Forensic economics. *Journal of Economic Literature*, 50(3), 731–769.

**SUPPORTING INFORMATION**

Additional supporting information may be found online in the Supporting Information section.

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**APPENDIX A**

**DESCRIPTIVES**

**TABLE A1** Different arms types included in the NISAT data set with three digit arms category code, weapon type, subcategory and number of transfers in the data set

| Code | PRIO weapons type | Subcategories |
|------|-------------------|---------------|
| 200  | Small arms        |               |
| 210  | Pistols & revolvers |             |
| 230  | Rifles/shotguns (military) |           |
| 233  | Assault rifles     |               |
| 234  | Carabines         |               |
| 235  | Sniper Rifles      |               |
| 237  | Semi-automatic rifles (military) |       |
| 239  | Shotguns (military) |            |
| 240  | Machine guns       |               |
| 243  | Sub-machine guns   |               |
| 245  | Light machine guns |               |
| 247  | General purpose machine guns |     |
| 250  | Military weapons   |               |
| 260  | Military firearms  |               |
| 270  | Machine guns all types |         |
### TABLE A1  (Continued)

| Code | PRIOR weapons type | Subcategories          |
|------|--------------------|------------------------|
| 300  | Light weapons      |                        |
| 310  |                    | Heavy machine Guns <= 12.7 mm |
| 400  | Ammunition         |                        |
| 415  |                    | Small arms ammunition  |
| 417  |                    | Small calibre Ammunition <= 12.7 mm |
| 418  |                    | Shotgun cartridges     |

Source: nisat.prio.org.

### TABLE A2  The 59 major exporting and importing countries of the small arms and ammunition data set with ISO 3 country codes

| Country   | ISO3 code | Country    | ISO3 code | Country     | ISO3 code |
|-----------|-----------|------------|-----------|-------------|-----------|
| Argentina | ARG       | India      | IND       | Poland      | POL       |
| Australia | AUS       | Indonesia  | IDN       | Portugal    | PRT       |
| Austria   | AUT       | Ireland    | IRL       | Romania     | ROM       |
| Belgium   | BEL       | Israel     | ISR       | Russia      | RUS       |
| Brazil    | BRA       | Italy      | ITA       | Saudi Arabia | SAU      |
| Bulgaria  | BGR       | Japan      | JPN       | Serbia      | SRB       |
| Canada    | CAN       | Kenya      | KEN       | Singapore   | SGP       |
| Chile     | CHL       | South Korea | KOR     | Slovenia    | SVN       |
| China     | CHN       | Kuwait     | KWT       | South Africa | ZAF      |
| Colombia  | COL       | Latvia     | LVA       | Spain       | ESP       |
| Croatia   | HRV       | Lebanon    | LBN       | Sweden      | SWE       |
| Cyprus    | CYP       | Lithuania  | LTU       | Switzerland | CHE       |
| Denmark   | DNK       | Malaysia   | MYS       | Thailand    | THA       |
| Egypt     | EGY       | Mexico     | MEX       | Turkey      | TUR       |
| Estonia   | EST       | Netherlands | NLD     | Ukraine     | UKR       |
| Finland   | FIN       | New Zealand | NZL   | United Arab Emirates | ARE |
| France    | FRA       | Norway     | NOR       | United Kingdom | GBR |
| Germany   | DEU       | Pakistan   | PAK       | United States | USA |
| Greece    | GRC       | Peru       | PER       | Uruguay     | URY       |
| Hungary   | HUN       | Philippines | PHL   | —           | —         |