Tuning of PID controller using optimization techniques for a MIMO process

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Abstract: In this paper, two processes were considered one is Quadruple tank process and the other is CSTR (Continuous Stirred Tank Reactor) process. These are majorly used in many industrial applications for various domains, especially, CSTR in chemical plants. At first mathematical model of both the process is to be done followed by linearization of the system due to MIMO process and controllers are the major part to control the whole process to our desired point as per the applications so the tuning of the controller plays a major role among the whole process. For tuning of parameters we use two optimizations techniques like Particle Swarm Optimization, Genetic Algorithm. The above techniques are majorly used in different applications to obtain which gives the best among all, we use these techniques to obtain the best tuned values among many. Finally, we will compare the performance of the each process with both the techniques.

1. Introduction

Many chemical industries process are of dynamic and highly nonlinear due to many process and manipulated variables. This type of MIMO (Multi Input and Multi Output) process are difficult to control. There are many methods to control the MIMO process. In this paper, we proposed the design of feedback control for controlling the levels of a quadruple tank system and also for controlling temperature and concentration of the chemical in a CSTR process. Which has two input and output variables, also known as 2 X 2 MIMO process[2][3]. It is a dynamic system and also with high nonlinearities. So, at first the mathematical model of the four tank system [1], and CSTR system will be done by using the principle of linearization and the proposed system state space model can be represented by using Jacobian matrix.

RGA is a powerful tool that has been employed in the multivariable control to choose controlled variable and manipulated variable pairs that produces desirable response.[2][3]

In highly complex multivariable processes which involves large number of feedback control loops, Decouplers are added in order to reduce interaction between minor process variables[3]. In this prototype two pumps are used in delivering the water to the tanks from the reservoir. Tanks are connected in such a way that change in either of the inlet will have an effect on both the process variables. And also the input flow of the any liquid, the inlet temperature will effect the temperature of the CSTR and also the PID tuning controllers are majorly and commonly used technique for process control in many industries.
because they are easy to employ and cost benefit.[10]. Due to its simplicity, it has the capability of providing the satisfactory performances [5]. In addition to that, there are many tuning methods. It also has the additional functions which will increase the performance of the process [7][10][11]. For the tuning of the PID parameters there are many optimization techniques are available like Zeigler Nicholas method, Ant Colony algorithm, Particle Swarm Optimization, Genetic Algorithm. Among them in this project we are going to use two optimizations such as Particle Swarm Optimization (PSO), Genetic Algorithm (GA).

The systems with tuned parameters from the above optimization techniques is given to the controllers, which is designed in the SIMULINK and assess the performance by comparing the response of two methods with two proposed techniques (PSO and GA). This paper comprises of the section-2 deals with the modeling of both the systems, section-2 deals with the tuning of PID with two techniques for both systems, section-3 discusses about parameters tuning, section-4 for simulation and results.

2. Modelling of Systems

2.1 Modelling of Quadruple Tank

In this proposed work one of the process we have taken is quadruple tank system as a process model which is shown in the figure 1. Tank 1 and tank 4 are fed by pump 1, similarly tank 2 and tank 3 are fed from Pump 2. The Pump outlet is split into two halves by simple hand operated valves. The dimensions of the system are given in the table 1.
Table 1. Quadruple Tank Process tank dimensions

|                          |       |
|--------------------------|-------|
| $A_i (cm^2)$-area of tank i | 28    |
| $a_i (cm^2)$-area of drain in tank i | 0.16,0.13,0.16,0.13 |
| $\gamma_i$-ratio of flows in the valves | 0.5,0.5 |
| $k_i$-Pump proportionality constant | 0.67,0.74 |
| $g (cm/s^2)$ – Gravitational constant | 9800 |

The process is modeled by writing mass balance equation for individual tanks by considering both inlets simultaneously.

Thus, mass balance equations for individual tanks are,

\[
\frac{dh_1}{dt} = -a_1 \sqrt{2gh_1} + a_3 \sqrt{2gh_3} + \gamma_1 k_1 v_1 \\
\frac{dh_2}{dt} = -a_2 \sqrt{2gh_2} + a_4 \sqrt{2gh_4} + \gamma_2 k_2 v_2 \\
\frac{dh_3}{dt} = -\frac{a_3}{A_3} \sqrt{2gh_3} + \frac{(1-\gamma_2)k_2 v_2}{A_3} \\
\frac{dh_4}{dt} = -\frac{a_4}{A_4} \sqrt{2gh_4} + \frac{(1-\gamma_1)k_1 v_1}{A_4} 
\]

Where $h_i$ is the level in tank $i$.

Representing the above governing equation in state space,

\[
x' = \begin{bmatrix} -1/T_1 & 0 & a_1 & 0 \\ 0 & -1/T_2 & 0 & a_2 \\ 0 & 0 & -1/T_3 & 0 \\ 0 & 0 & 0 & -1/T_4 \end{bmatrix} x + \begin{bmatrix} \gamma_1 k_1 \\ \gamma_2 k_2 \\ (1-\gamma_1) k_1 \\ (1-\gamma_2) k_2 \end{bmatrix} u \\
y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x
\]

where, $x=[h_1 \ h_2 \ h_3 \ h_4]$ and $u=[v_1 \ v_2]$
\[ T_i = \frac{A_i}{a_i} \sqrt{\frac{2h_i^0}{g}} \]

\[
\begin{bmatrix}
H_1(s) \\
H_2(s)
\end{bmatrix} = G(s) \begin{bmatrix}
U_1(s) \\
U_2(s)
\end{bmatrix}
\]

\[ G(s) = \mathcal{C} (sI - A)^{-1} + D \]

\[ G(s) = \begin{bmatrix}
G_{11}(s) & G_{12}(s) \\
G_{21}(s) & G_{22}(s)
\end{bmatrix} \]

(7)

\[ G_{11} = \frac{2.57}{62.7s + 1}, G_{12} = \frac{1.5}{(23.8s + 1)(62s + 1)} \]

\[ G_{21} = \frac{1.4}{(30s + 1)(90s + 1)}, G_{22} = \frac{2.8}{90s + 1} = \frac{H_2}{U_2} \]

\[ H_1 = G_{11}U_1 + G_{12}U_2 \]

\[ H_2 = G_{21}U_1 + G_{22}U_2 \]

Controller-1 forces \( H_1 \) towards its set point, \( U_1 \) and \( U_2 \) also affects \( H_1 \) through \( G_{12} \). Similarly Controller-2 adjusts \( H_2 \) towards its set point, \( U_2 \) and \( U_1 \) affects \( H_2 \) through \( G_{21} \).

2.2 CSTR Modeling

The CSTR process schematic diagram is in figure 2. The input flow rate and outlet flow rate of a reactant \( A \) is \( F \) with concentration of \( C_{Af} \) at temperature \( T_f \) with concentration \( C_A \) at temperature \( T \). And also the coolant jacket with input flow rate of coolant at temperature \( T_j \) in and \( T_j \) out. The dimensions of the system are given in the table 2.

![Figure 2. CSTR Process diagram](image-url)
Table 2. Process parameters of CSTR

| Parameters                        | Values            |
|-----------------------------------|-------------------|
| F/V, hr⁻¹                        | 1                 |
| Ko, hr⁻¹                          | 9703x3600         |
| (-\(\Delta H\),\(\text{K}_{\text{cal}}\)/\(\text{K}_{\text{mol}}\)) | 5960              |
| \(E, \text{K}_{\text{cal}}/\text{K}_{\text{mol}}\) | 11843             |
| \(\rho C_p, \text{K}_{\text{cal}}/\text{m}^3 \cdot ^\circ \text{c}\) | 500               |
| \(T_f, ^\circ \text{c}\)          | 25                |
| \(C_{af}, \text{K}_{\text{cal}}/\text{m}^3\) | 10                |
| \(UA/V, \text{K}_{\text{cal}}/\text{m}^3 \cdot ^\circ \text{c} \cdot \text{hr}\) | 150               |
| \(T_j, ^\circ \text{c}\)          | 25                |

Linearization of dynamic equations

\[
F1(CA,T) = \frac{dCA}{dt} = \frac{F}{V(CAF-CA)} - Ko \exp \left( -\frac{E}{RT} \right) CA
\]

(8)

\[
F2(CA,T) = \frac{dT}{dt} = \frac{F}{V(T_f-T)} + \left( -\frac{\Delta H}{\rho C_p} \right) Ko \exp \left( -\frac{E}{RT} \right) CA - \frac{UA}{\nu C_p(T-T_f)}
\]

(9)

\[X = \begin{bmatrix} CA & -CAS \\ T & -TS \end{bmatrix}\]

\[U = \begin{bmatrix} F & -FS \\ Tj & -TjS \end{bmatrix}\]

Representing the above governing equation in state space matrices as,

\[
A = \begin{bmatrix} -\frac{F}{V} - Ks & -CasKs\left(\frac{\Delta E}{RTs}\right) \\ \left( -\frac{\Delta H}{\rho C_p} \right) Ks & -\frac{F}{V} - \frac{UA}{\nu C_p} + \left( -\frac{\Delta H}{\rho C_p} \right) Ks\left(\frac{\Delta E}{RTs}\right) \end{bmatrix}
\]

\[
B = \begin{bmatrix} (Caf-Cas)V & 0 \\ (T_f-T)sV & UA/\nu C_p \end{bmatrix}\]

\[
C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

\[
G_{11} = \frac{1.4364s + 1.5393}{(s-0.580)(s-0.88)} \quad G_{12} = \frac{-0.0249}{(s-0.580)(s-0.88)}
\]
2.3 Selection of Controlled and Manipulated Variables

As seen from the above schematic diagram in figure 3, the process is considered as MIMO model which consists of 2 input variables \((U_1 \text{ and } U_2)\) and 2 output variables \((H_1 \text{ and } H_2)\). Change in either of the input variables will affect both output variables. So, totally 4 combinations of controlled and manipulated variable pair is possible \((U_1 \text{ Vs. } H_1 \text{ and } U_2 \text{ Vs. } H_2)\) or \((U_1 \text{ Vs. } H_2 \text{ and } U_2 \text{ Vs. } H_1)\).

But practically it is not possible to design control loops for all the pairings. Relative Gain Array (RGA) is a tool that measures the interaction between the controlled and manipulated variable. It recommends the possible pairing of controlled and manipulated variable which produces optimum results. RGA matrix is,

\[
G_{21} = \frac{-13.171s - 12.5517}{(s-0.580)(s-0.88)} \quad G_{22} = \frac{0.3s + 0.347}{(s-0.580)(s-0.88)}
\]

\[
\begin{align*}
\lambda &= \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} \\
&= \frac{1}{1 - \frac{K_{21}K_{12}}{K_{22}K_{11}}} \\
\lambda_{12} &= \lambda_{21} = 1 - \lambda_{11} \\
\lambda_{11} &= \lambda_{22}
\end{align*}
\]

Where \(K_{ij}\) is the steady state gain of \(G_{ij}\).

For Quadruple Tank system the RGA matrix is given by

\[
\lambda = \begin{bmatrix} 1.412 & -0.412 \\ -0.412 & 1.412 \end{bmatrix}
\]

For CSTR system the RGA matrix is given by

\[
\lambda = \begin{bmatrix} 2.3 & -1.3 \\ -1.3 & 2.3 \end{bmatrix}
\]
As seen from the above matrix, $\lambda_{12} < 0$ and $\lambda_{21} < 0$ opening or closing of either of the loops will have an adverse effect on other loop which may produce oscillatory response. So, $H_1$ should not be paired with $U_2$ and also $H_2$ should not be paired with $U_1$. Thus $H_1-U_1$ and $H_2-U_2$ pairing gives the effective results.

2.4 Decouplers

After selecting major controlled and manipulated variables by inferring RGA matrix, minor interactions between $H_2-U_1$ and $H_1-U_2$ are eliminated by adding decouplers in the process.

$$T_{21} = \frac{-g_{21}}{g_{22}}$$
$$T_{12} = \frac{-g_{12}}{g_{11}}$$

For Quadruple tank ,

$$T_{21} = \frac{-0.5}{30s + 1}, T_{12} = \frac{-0.5769}{23s + 1}$$

For CSTR System,

$$T_{12} = \frac{-0.0249}{1.4364s + 1.5393}, T_{21} = \frac{-0.13171s + 12.5517.5769}{23s0.3s + 0.347 + 1}$$

![Figure 4. Decouplers](image-url)
3. Controller Tuning

Tuning of PID controller parameters are made with many methods but for this above two process we have taken two optimization techniques such as Particle swarm Optimization (PSO), Genetic Algorithm (GA)

3.1 Particle Swarm Optimization

This algorithm works with having population (swarm) of candidate solution (particle). Every particle is a candidate solution to optimize the problem. The best solution particles are moved around in the search-space. The movements of the particles are guided by their own best known position in the search-space as well as the entire swarm's best known position. In this technique a set of particles are put in search space with randomly choosing velocity and position. The algorithm for the proposed models is shown in figure 5. And implementing this PSO in the MATLAB using the cost function for the respective models. The tuned PID parameters are tabulated in following tables 3,4 and 7,8 for QTP and CSTR process respectively.

Initially we set the values of algorithm constants as:
Inertia weight factor, \( W = 0.3 \)
Acceleration constants, \( C_1, C_2 = 1.5 \)

Run the program with the PSO algorithm with 100 iterations in MATLAB and returned the final optimal fitness function value as “pbest” and global optimum solution as “Gbest”

3.2 Genetic Algorithm

It is a class of evolutionary algorithms, which generate solutions to optimization problems using techniques inspired by natural evolution, such as inheritance, mutation, selection, and crossover. The
algorithm for the propose models is shown in figure 6. And implementing this GA in the SIMULINK ‘Optimization’ tool box by using the cost function for the respective model. The tuned PID parameters are tabulated in following tables 5, 6 and 9, 10 for QTP and CSTR process respectively.

| Parameters | Tuned values |
|------------|--------------|
| Kp         | 39.4728      |
| Ki         | 20.1375      |
| Kd         | 13.3591      |

Iterations \((n)\)=100
Lower Limit of Controller parameters = [0 0 0]
Upper Limit of Controller parameters = [100 100 100]

4. Simulation and Results

After modeling the process, RGA matrix is formed as explained in the section 2 by which control and manipulated variable pairs are chosen. Then the process is splitted into two halves and tuned separately as explained in the section 2.

4.1 Quadruple Tank System Controller Parameters

Tuned PID parameters using PSO algorithm:

Table 3. Controller parameters for Loop 1

| Parameters | Tuned values |
|------------|--------------|
| Kp         | 39.4728      |
| Ki         | 20.1375      |
| Kd         | 13.3591      |

Table 4. Controller parameters for Loop 2

| Parameters | Tuned values |
|------------|--------------|
| Kp         | 47.1728      |
| Ki         | 20.7982      |
| Kd         | 30.1844      |
Tuned PID parameters using GA:

**Table 5.** Controller parameters for Loop 1

| Parameters | Tuned values |
|------------|--------------|
| Kp         | 93.239       |
| Ki         | 78.493       |
| Kd         | 82.478       |

4.2 CSTR System Controller Parameters

Tuned PID parameters using PSO algorithm:

**Table 7.** Controller parameters for Loop 1

| Parameters | Tuned values |
|------------|--------------|
| Kp         | 3.8048       |
| Ki         | 12.2341      |
| Kd         | 36.27        |

Tuned PID parameters using GA:

**Table 9.** Controller parameters for Loop 1

| Parameters | Tuned values |
|------------|--------------|
| Kp         | 89.827       |
| Ki         | 35.76        |
| Kd         | 0.013        |

**Table 6.** Controller parameters for Loop 1

| Parameters | Tuned values |
|------------|--------------|
| Kp         | 99.993       |
| Ki         | 92.734       |
| Kd         | 27.114       |

**Table 8.** Controller parameters for Loop 2

| Parameters | Tuned values |
|------------|--------------|
| Kp         | 1.3691       |
| Ki         | 9.944        |
| Kd         | 21.5722      |

**Table 10.** Controller parameters for Loop 2

| Parameters | Tuned values |
|------------|--------------|
| Kp         | 98.65        |
| Ki         | 38.383       |
| Kd         | 1.971        |

![Figure 7. Open loop Response of CSTR for and 2nd loop](image-url)
Figure 8. Open loop Response of CSTR for 1st loop

Figure 9. Closed loop Response of CSTR using PSO for 2nd loop
**Figure 10.** Closed loop Response of CSTR using PSO for 1st loop

**Figure 11.** Closed loop Response of CSTR using GA for 1st loop
Figure 12. Closed loop Response of CSTR using GA for 2nd loop
Figure 13. Open Loop Response of QTP 1st Loop

![Open Loop Response of QTP (Loop 2)](image)

Figure 14. Open Loop Response of QTP 2nd Loop

![Open Loop Response of QTP (Loop 2)](image)

Figure 15. Closed loop Response of QTP using PSO for 1st loop
**Figure 16.** Closed loop Response of QTP using PSO for and 2nd loop

**Figure 17.** Closed loop Response of QTP using GA for 1st loop
The response of CSTR model is shown in figure (7) – (12). Similarly the response of both the loops of QTP model is shown in figure (13) – (18).

5. Conclusions

In this work, we used two different optimization techniques for tuning of PID controller parameters for two process is discussed and we infer that, in CSTR the PSO based tuned values gives better response than that of GA. Whereas in QTP process both the optimization techniques gives almost similar response with slight variation in their peak Overshoot values.

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