Load Balancing User Association in Millimeter Wave MIMO Networks

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Abstract—User association is necessary in dense millimeter wave (mmWave) networks to determine which base station a user connects to in order to balance base station loads and maximize throughput. Given that mmWave connections are highly directional and vulnerable to small channel variations, user association changes these connections and hence significantly affects the user's instantaneous rate as well as network interference. In this paper, we introduce a new load balancing user association scheme for mmWave MIMO networks which considers this dependency on user association of user's transmission rates and network interference. We formulate the user association problem as mixed integer nonlinear programming and design a polynomial-time algorithm, called Worst Connection Swapping (WCS), to find a near-optimal solution. Simulation results confirm that the proposed user association scheme improves network performance significantly by moving the traffic of congested base stations to lightly-loaded ones and adjusting the interference accordingly. Further, the proposed WCS algorithm outperforms other generic algorithms for combinatorial programming such as the genetic algorithm in both accuracy and speed at several orders of magnitude faster, and for small networks where exhaustive search is possible it reaches the optimal solution.

Index Terms—User association; mmWave MIMO systems; load balancing; MINLP; polynomial-time algorithm.

I. INTRODUCTION

Bandwidth shortage is a major challenge for current wireless networks. Most of the available spectrum at microwave frequencies is occupied while there is a pressing need for higher throughputs and larger bandwidths [1]. During the past few years, millimeter wave (mmWave) frequencies have attracted the interest of academia and industry due to the capability of multi-Gbps data rates and the huge amount of bandwidth available at frequencies between 30 - 300 GHz. Millimeter wave communication for the new 5th generation (5G) cellular systems is a promising solution to the current spectrum shortage and ever-increasing capacity demand [2], where its feasibility has been demonstrated not only for backhaul but also for mobile access [3]-[5].

User association plays an important role in resource allocation for cellular networks. While user association has been studied in the context of heterogeneous networks (HetNets) [6], massive multiple-input multiple-output (MIMO) systems [7], and mmWave Wi-Fi networks [8], the problem in mmWave cellular networks is significantly different. Because of the higher path loss and sensitivity to blockage, mmWave signals can only propagate a much shorter distance than the current RF signals, making it necessary for the mmWave cellular network to be dense. The base station (BS) density is expected to be higher by an order of magnitude [9], and each user equipment (UE) is likely to be surrounded by a number of BSs, where the channel to each of these BS can vary significantly given the erratic nature of mmWave propagation with probabilistic line-of-sight (LOS), non-LOS (NLoS) and outage events. In addition, mmWave systems require the use of MIMO beamforming at both the transmitter and receiver ends [5], different from massive MIMO which has beamforming at only the BS. Hence the formulation of a user association problem for mmWave is different from other systems and requires a new approach.

A. Background and Related Work

Max-SINR user association is a traditional method of using the highest signal to interference plus noise ratio (SINR) or largest received power to associate an UE with a BS [10]. This technique has been working well in cellular networks at microwave frequencies where all cells are homogeneous macro cells. The emergence of HetNets with smaller, lower power pico, femto and relay BSs requires a different look at user association to ensure load balancing across the network, avoiding the case that all users connect to only a macro BS because of its strongest SINR and overload it.

Cell breathing is a technique that balances the load in a single tier network consisting of BSs of the same type by allowing each BS to adjust its transmit power, based on its load, to change its coverage area [11]-[12]. If the BS is underloaded (overloaded), the transmit power increases (decreases) to expand (contract) the coverage region. HetNets also requires balancing the network load among different tiers. A conventional approach is biasing in which the transmit power of BSs in a low-power lightly loaded tier is artificially biased in order to seem more attractive than congested BSs [13]. Although biasing can be done both within and among tiers, it cannot precisely control the load on each BS [14]-[15].

A load balancing user association scheme for HetNets uses an optimization theoretic approach to...
achieve the balance among all BSs across different tiers in the network [6]. This reference presented a novel user association technique with integer constraints on association coefficients and showed that this unique association problem is a combinatorial program, which is NP-hard. Considering a heuristic approach, the researchers first relaxed the unique association constraints to solve for a joint association problem (allowing a UE to be served by multiple BSs), and then rounded the relaxed solution. This relaxation reduced the complexity of the problem and provided an upper performance bound for the original unique association.

In a similar work to [6], the problem of user association in HetNets with massive MIMO BSs and single-antenna UEs is studied in [7]. Because of the channel hardening effect in massive MIMO, the considered association approach is independent of the user instantaneous rate or instantaneous channel state information (CSI), but only depends on converging deterministic rate values as a function of the system parameters and large-scale CSI. Even though unique association is considered, the association coefficients are not necessarily integers, but represent the limit of the fraction of time slots in which each UE is served by one BS. Using this interpretation of fractional user association as long-term time average, the researchers then formulated user association as a convex network utility maximization problem and solved it using Lagrangian duality.

Optimizing user association is also studied in a 60-GHz mmWave Wi-Fi network [8]. The optimization problem in [8] is similar to the one appeared in [6], but instead of maximizing network throughput it is formulated to minimize the maximum per-BS load subject to demanded user data rates. At this high atmospheric-absorption frequency, the user instantaneous rates converge to deterministic values and the interference is assumed to be negligible due to directional steerable antenna arrays. This assumption simplifies the user association problem compared to the case when we must take into account the effect of interference, as in a dense mmWave cellular network at the proposed frequencies (28, 38, and 73 GHz), for which interference has been noted for outdoor environments [3]-[4], [16].

A number of other existing works also considered the joint problem of user association, beamforming design, and power allocation [17]-[19]. This joint problem is shown to be NP-hard, and researchers usually proposed iterative algorithms to achieve near-optimal solutions. Algorithms solving such an NP-hard joint optimization problem may be too complex to be implemented in practice. As such, we take the alternative approach of fixing the beamforming designs and optimize for user association alone.

Most of recent works on user association ignore the effect of small-scale fading (instantaneous CSI) on the user instantaneous rate and consider a scalar channel model with only large-scale fading [20]-[25], hence the SINR used for user association is only a function of large-scale parameters (e.g. distance) regardless of small-scale channel variations. Specifically, the SINR term in the capacity formula is not a matrix but a scalar value indicating that the mmWave MIMO small-scale variations were ignored and the system is usually simplified by only considering directional gains for mmWave antenna. This consideration reduces the complexity of the analysis, but this approach is not suitable for mmWave systems where the channel variation can be fast and the instantaneous CSI may change rapidly [9]. Our simulation results confirm that considering both large-scale and small-scale CSI significantly improves the average network spectral efficiently.

B. Our contributions

In this paper we introduce a new user association problem formulation suitable for mmWave networks. Specifically, unlike the typical approach in user association for HetNets [6], massive MIMO [7], and mmWave Wi-Fi [8] which accounts only for large-scale CSI, we consider both the large-scale and instantaneous CSI in our formulations. This consideration is suitable for mmWave networks where the channels can vary significantly even during two consecutive time slots. In the case that instantaneous CSI is unavailable, however, we can still use the proposed user association scheme but only with large-scale CSI which will provide a much longer time span. Adaptation to instantaneous CSI will further improve the network performance. As our user association scheme is independent of channel model, the ideal assumption of having both large-scale and instantaneous CSI can be considered as a performance upper bound. In practical scenarios we may use the proposed user association scheme with only large-scale CSI or with both types of CSI depending on the availability.

Furthermore, previous works ignore the effect of user association on network interference; consequently in the downlink for example, each user suffers full interference from all other BSs, regardless of their association. While this assumption makes sense in an omnidirectional transmission, it no longer holds in a directional transmission as is the case in mmWave, where the interference will change substantially depending on which UE beamforms with which BS. The network interference structure in mmWave is highly dependent on user association and we need to consider this dependency while computing the user rates. Not adapting the interference to user association unfortunately degrades the network spectral efficiency severely.

The main contributions of this paper are summarized as follows:

- We propose a new load balancing user association scheme which takes into account the dependency
of network interference on user association. In particular, we formulate the user instantaneous rate as a function of user association, as is the case in mmWave. Consequently, the total interference coming from other BSs (while serving other UEs) also depends on association coefficients.

- We introduce an Activation Matrix which specifies the BS each UE connects to during each time slot, from which association coefficients are derived as its time average. The activation matrix allows per-time-slot user association which is necessary because the instantaneous CSI of a mmWave channel may change significantly even during two consecutive time slots.

- For the proposed user association scheme, we formulate an optimization problem as a mixed integer nonlinear programming (MINLP), which is known to be NP-hard due to its non-convex nonlinear structure and presence of integer variables. This optimization problem is similar in form to those derived in [6]-[7], but instead of being a linear function of the user association coefficients, its objective function is highly dependent on the network interference structure and user association in a non-linear fashion. This makes the optimization problem highly non-convex and challenging, which in turn makes our proposed user association scheme suitable for mmWave cellular networks and reveals its novelty.

- We design a polynomial-time specialized efficient algorithm, called Worst Connection Swapping (WCS), to solve the formulated MINLP and find a near-optimal solution for the user association. This algorithm is based on the intuition that swapping the worst UE-BS connection is likely to provide the UE a stronger link to another BS and/or reduce the interference, which consequently improves the user's rate. Complexity analysis reveals that our proposed algorithm's runtime grows as \( K^2 \log(K)M^2 \) with \( K \) as the number of UEs and \( M \) as the number of antennas at each BS.

- Our simulation results show that considering instantaneous CSI significantly improves the network spectral efficiency. Further, the proposed load balancing scheme outperforms max-SINR and other load balancing schemes by including the dependency of interference on user association. Numerical results also confirm the analysis of algorithm complexity and show that the WCS algorithm has fast convergence with the number of iterations growing linearly with the number of UEs, and the runtime growing quadratically, which is several orders of magnitude faster than genetic algorithm (GA) and exhaustive search.

- Our proposed user association scheme can be integrated with any mobility model since it is designed to be performed per-time slot. As an illustration, we provide simulation results of our scheme applied to a simple user mobility model where the UEs have fixed locations during each time slot, and in the consecutive time slot each UE moves to a new random location within a given range from its location in the previous time slot. Our simulations show the effect of mobility on the association results, for example, a UE with strong signal from a particular BS tends to be associated with that BS even as it moves around within a vicinity, whereas a UE with comparable signals from several BSs can switch association frequently as it mobilizes.

C. Paper Organization

The rest of the paper is organized as follows. We start with the channel and system model in Section II. In Section III, we introduce our load balancing user association scheme and define the activation and association matrices. The user association optimization problem is formulated in Section IV. In Section V, we introduce the WCS algorithm and analyze its complexity. We present the numerical results in Section VI, and Section VII contains our conclusion.

D. Notation

Throughout this paper, scalars are represented by lowercase letters, vectors are denoted by lowercase boldface letters and matrices by uppercase boldface letters. Superscript \((\cdot)^\ast\) denotes the conjugate transpose, \(\log(\cdot)\) stands for base-2 logarithm, \(\mod(\cdot)\) represents the modulo operation, and big-O notation \(O(\cdot)\) expresses the complexity. \(I_N\) is the \(N \times N\) identity matrix, and we use \(\text{tr}(A)\) and \(|A|\) to denote the trace and determinant of matrix \(A\), respectively. The distribution of a complex Gaussian vector \(x\) with mean \(\mu\) and covariance matrix \(Q\) is denoted by \(x \sim \mathcal{CN}(\mu, Q)\).

II. CHANNEL AND SYSTEM MODEL

A. mmWave Channel Model

The mmWave channel has completely different characteristics compared to the typical microwave model of a rich scattering independent and identically distributed (i.i.d.) channel. The channel model considered in this paper is based on the clustered channel model introduced in [26] and the 3GPP-style 3D channel model proposed for the urban micro (UMi) environments in [27], which is based on the measurements obtained in New York City. This channel model has \(C\) clusters with \(L\) rays per cluster, and it can be expressed as

\[
H = \frac{1}{\sqrt{CL}} \sum_{c=1}^{C} \sum_{l=1}^{L} \sqrt{\gamma_c} a(\phi_{c,l}^{\text{UE}}, \theta_{c,l}^{\text{UE}}) a^\ast(\phi_{c,l}^{\text{BS}}, \theta_{c,l}^{\text{BS}}) \tag{1}
\]

where \(\gamma_c\) is the power gain of the \(c\)th cluster. The parameters \(\phi_{c,l}^{\text{UE}}, \theta_{c,l}^{\text{UE}}, \phi_{c,l}^{\text{BS}}, \theta_{c,l}^{\text{BS}}\) represent azimuth angle of arrival (AoA), elevation angle of arrival (EoA),
azimuth angle of departure (AoD), and elevation angle of departure (EoD), respectively. These parameters are generated randomly based on different distributions and cross correlations as given in [27, Tables 1-3]. The vector \( \mathbf{a}(\phi, \theta) \) is the antenna array response vector, which depends on antenna geometry, including uniform linear array (ULA) or uniform planar array (UPA). In order to enable beamforming in the elevation direction (3D beamforming), we use the uniform \( U \times V \) planar array given by [26]

\[
\mathbf{a}(\phi, \theta) = [1, \ldots, e^{jkd_x(u \sin(\phi) \sin(\theta) + v \cos(\theta))}, \ldots, e^{jkd_y((U-1) \sin(\phi) \sin(\theta) + (V-1) \cos(\theta))}]^T
\]  

(2)

where \( d_x \) is the distance between antenna elements, and \( u \in \{1, \ldots, U\} \) and \( v \in \{1, \ldots, V\} \) are the indices of antenna elements.

We consider two link states for each channel, LoS and NLoS, and use the following probability functions obtained based on the New York City measurements in [28]

\[
p_{\text{LoS}}(d) = \min\left(\frac{d_{\text{bp}}}{d}, 1\right), \left(1 - e^{-\frac{d}{d_0}} + e^{-\frac{d}{\eta}}\right)^2
\]

\[
p_{\text{NLoS}}(d) = 1 - p_{\text{LoS}}(d)
\]

(3)

(4)

where \( d \) is the 3D distance between UE and BS in meters, \( d_{\text{bp}} \) is the breakpoint distance at which the LoS probability is no longer equal to 1, and \( \eta \) is a decay parameter. The obtained values based on measurements for these parameters are \( d_{\text{bp}} = 27 \) m and \( \eta = 71 \) m.

Moreover, we use the following path loss model for LoS and NLoS links [27]

\[
PL(\text{dB}) = 20 \log_{10}\left(\frac{4\pi d_0}{\lambda}\right) + 10n \log_{10}\left(\frac{d}{d_0}\right) + X_{\text{SSF}}
\]

(5)

where \( \lambda \) is the wavelength, \( d_0 \) is the reference distance, \( n \) is the path loss exponent, and \( X_{\text{SSF}} \) is the lognormal random variable with standard deviation \( \sigma_{\text{SSF}} \) (dB) which describes the shadow fading. These parameters vary depending on LoS of NLoS propagation at 73 GHz, the path loss exponents and the shadowing factors are \( n_{\text{LoS}} = 2 \), \( n_{\text{NLoS}} = 3.4 \), \( \sigma_{\text{SSF,LoS}} = 4.8 \) dB, and \( \sigma_{\text{SSF,NLoS}} = 7.9 \) dB.

In 4G cellular networks, pilot signals are used to estimate CSI at the receiver. Once the CSI is available at the receiver, it can be shared with the transmitter via limited feedback or channel reciprocity. However, in 5G dense HetNets these conventional approaches are inapplicable due to network densification and the limited amount of pilot resources [29]. To address the new challenges emerging from network densification, a promising radio access technology, called cloud radio access network (C-RAN), has been proposed [30]. In this radio network, CSI at both the transmitter and the receiver can be estimated through new CSI acquisition schemes and shared via C-RAN for centralized signal processing, coordinated beamforming, and resource allocation in 5G new radio [31], [32]. In this paper, we assume that the CSI is estimated for the aforementioned applications and we can utilize it for the purpose of user association. We note that C-RAN is an architecture that enables this shared CSI and centralized user association, but it may not be the only one, and the CSI required for beamforming in 5G can be used for user association. Further, distributed algorithms can also be developed for the same problem formulation which we will consider here, but do not require C-RAN because of the distributed nature.

Different user association schemes make use of different types of channel models: when an algorithm uses the detailed clustered channel model in (1), we call this instantaneous CSI, and when an algorithm uses only path loss, shadowing and LOS/N-LOS channel models as in (3)-(5), we call this large-scale CSI. Our proposed user association scheme as discussed in Section III utilizes both large-scale and instantaneous CSI as given in (1). The scheme, however, is also applicable if the channel model instead contains only large-scale fading as considered in [8, eq. (1)]. In the simulation section, we show examples of both cases and provide comparison with other schemes which mostly rely on large-scale CSI.

**B. System and Signal Model**

We consider a downlink scenario in a cellular mmWave MIMO network with \( J \) BSs and \( K \) UEs. \( M_j \) and \( N_k \) are the number of antennas at BS \( j \) and UE \( k \), respectively, where we assume \( M_j \geq N_k \) as handsets usually have fewer antennas than BSs. Let \( \mathcal{J} = \{1, \ldots, J\} \) denotes the set of BSs and \( \mathcal{K} = \{1, \ldots, K\} \) represents the set of UEs. Each UE \( k \) aims to receive \( n_k \) data streams from its serving BS such that \( 1 \leq n_k \leq N_k \), where the upper inequality comes from the fact that the number of data streams for each UE cannot exceed the number of its antennas.

Thus, we can define the total number of downlink data streams sent by BS \( j \) as

\[
D_j = \sum_{k \in \mathcal{Q}_j(t)} n_k
\]

(6)

We define \( \mathcal{Q}_j(t) \) as the Activation Set which represents the set of active UEs in BS \( j \) within time slot \( t \), such that \( \mathcal{Q}_j(t) \subseteq \mathcal{K} \) and \( |\mathcal{Q}_j(t)| = Q_j(t) \leq K \). Note that the total number of downlink data streams sent by each BS should be less than or equal to its number of antennas, i.e., \( D_j \leq M_j \). For notational simplicity, we drop the time index \( t \) in definition of \( D_j \), and only keep the time index for \( Q_j(t) \) due to its importance.

The \( M_j \times 1 \) transmitted signal from BS \( j \) is given by

\[
x_j = F_j d_j = \sum_{k \in \mathcal{Q}_j(t)} F_{k,j} s_k
\]

(7)

where \( s_k \in \mathbb{C}^{n_k} \) is the data stream vector for UE \( k \) consists of mutually uncorrelated zero-mean symbols, with \( \mathbb{E}[s_k s_k^H] = I_{n_k} \). The column vector \( d_j \in \mathbb{C}^{D_j} \) represents the vector of data symbols of BS \( j \), which
is the vertical concatenation of the data stream vectors $s_k$, $k \in Q_j(t)$, such that $E[d_s^*d_s] = I_{D_j}$. Matrix $F_{k,j} \in \mathbb{C}^{M_j \times n_k}$ is the linear precoder for each UE $k$ associated with BS $j$, and $F_j \in \mathbb{C}^{M_j \times D_j}$ is the complete linear precoder matrix of BS $j$ which is the horizontal concatenation of all $F_{k,j}, k \in Q_j(t)$.

The power constraint at BS $j$ can be described as

$$E[x_j^*x_j] = \sum_{k \in Q_j(t)} \text{Tr}(F_{k,j}F_{k,j}^*) \leq P_j \quad (8)$$

where $P_j$ is the transmit power of BS $j$. Now we can express the $N_k \times 1$ received signal at UE $k$ antennas as

$$y_k = \sum_{j \in \mathcal{J}} H_{k,j}x_j + z_k, \quad (9)$$

and the post-processed signal received by each UE is

$$\tilde{y}_k = \sum_{j \in \mathcal{J}} W_{k,j}^*H_{k,j}x_j + W_k^*z_k \quad (10)$$

where $W_k \in \mathbb{C}^{N_k \times n_k}$ is the linear combiner matrix of UE $k$, $H_{k,j} \in \mathbb{C}^{N_k \times M_j}$ represents the channel matrix between BS $j$ and UE $k$, and $z_k \in \mathbb{C}^{N_k}$ is the white Gaussian noise vector at UE $k$, with $z_k \sim \mathcal{CN}(0, N_0I_{N_k})$. The presented signal models (7)-(10) are applicable for all types of transmit beamforming and receive combining. Later for user association optimization, we will specify the transmit and receive beamforming schemes. Further, based on CRAN architecture as discussed earlier, each BS knows its channels to all UEs and can share that CSI with all other BSs for both beamforming design and user association purposes.

### III. mmWave Load Balancing User Association

In the literature, when computing the instantaneous rate for a specific UE (connected to a BS), the interference coming from other UE-BS connections is assumed to be both independent of the user association and present all the time (full interference) [6]-[8]. This assumption is not realistic in mmWave systems due to the use of beamforming and results in lower instantaneous user rates. In mmWave systems, the network interference highly depends on user association and we need to consider this while computing user rates. Moreover, user association depends on channel realizations which is highly directional and can vary fast in mmWave frequencies. Thus, we cannot use the full interference structure in mmWave systems.

In this section, we introduce a new user association model for mmWave MIMO networks. First, we need to introduce our definition of a time slot throughout this paper. Each time slot $t$ is a duration of time comparable to channel coherence time such that the instantaneous CSI remains relatively the same within it and only changes from one time slot to another. During time slot $t$, each UE is connected to exactly one BS. Because of beamforming, the interference structure in each time slot depends on the user association on that specific slot. This interference structure is appropriate for mmWave channels where the channel is probabilistic and can vary fast. Moreover, we assume it is possible to split the data streams of each UE and transmit them in different time slots. Considering above definitions, we study two association approaches in this paper: (i) instantaneous user association, which is performed within each time slot and results in unique association (each UE can be associated with only one BS during each time slot), and (ii) fractional (joint) user association, which is obtained by averaging the instantaneous association over $T$ time slots (for an arbitrarily large $T$). At each time slot, the mmWave channels are generated independently based on the channel model presented in Section II-A. We consider both approaches to evaluate the performance of our proposed user association model and compare it with existing user association schemes.

It is worth mentioning that our proposed per-time-slot user association can be carried out under the synchronizations requirement of modern cellular systems. In LTE TDD networks, the basic synchronization requirement for any pair of cells with overlapping coverage areas which operate on the same frequency is $3 \mu s$ [33], and it would be in the order of $1 \mu s$ or below for 5G systems [34]. Moreover, based on the work in [35], a time slot of duration $T_{\text{slot}} = 100 \mu s$ is considered as a sufficiently short time duration to ensure channel coherence at mmWave systems. Since the synchronization time is relatively small compared to the time slot duration, synchronization will not be an issue for performing per-time-slot user association.

We differentiate between user association and user scheduling. Note that user association determines the UE-BS connections among multiple BSs, while user scheduling allocates the available resources (in this case, data streams) at each BS to its associated UEs. Thus, user scheduling can be considered as a post-association stage. In the literature, it is shown that the joint problem of user association and user scheduling has high computational complexity and researchers usually decouple the joint optimization problem into sub-problems to find the optimal solutions [6], [36]. In this paper, we assume that the BSs’ loads are determined based on their available resources. Specifically, we consider the multi-user MIMO scenario where at each time slot each BS can simultaneously transmit to multiple UEs by constraining the available number of data streams at each BS to be smaller than the number of its antennas. Therefore, at each time slot, the BS can serve all its associated users simultaneously, by beamforming simultaneously to multiple users using multiple beams, and there are enough spatial degrees of freedom for all the associated users as long as the aforementioned constraint is satisfied. As such, in this
case, there is no need to scheduling among the associated users. In the case that the number of required data streams at each time slot is larger than the number of BS antennas, then user scheduling is inevitable. In such a case, a simple scheduling policy (e.g. round robin scheduling [36]) or more efficient scheduling schemes (e.g. proportional fairness scheduling [6] or CDF-based scheduling [37], [38]) can be implemented to allocate resources of each BS among its associated users.

We start by defining the **Activation Matrix** as

\[
B \triangleq \begin{bmatrix}
\beta(1) & \cdots & \beta(T)
\end{bmatrix}
\]

where \(\beta(t)\) is called the **Activation Vector** at time slot \(t\), and each element of \(B\) is called an **activation factor** and is the index of BS to whom user \(k\) is associated with during time slot \(t\), i.e., \(\beta_k(t) \in J\) with \(k \in K\) and \(t \in T = \{1, \ldots, T\}\). Considering above definition, the relationship between the activation set of BS \(j\) and the activation factors can be described as

\[
Q_j(t) = \{k : \beta_k(t) = j\}. \tag{12}
\]

As stated earlier, we assume each UE can be associated with only one BS at any time slot \(t\), i.e.,

\[
Q_j(t) \cap Q_i(t) = \emptyset, \quad j \neq i \tag{13}
\]

where (14) indicates that during each time slot all UEs are served by the BSs.

The elements of activation matrix should satisfy the following conditions

\[
\sum_{j=1}^{J} \beta_k(t)(j) \leq 1, \quad \forall k \in K \tag{15}
\]

\[
\sum_{k \in K} \beta_k(t)(j).n_k \leq D_j, \quad \forall j \in J \tag{16}
\]

where the indicator function is defined as

\[
1_{\beta_k(t)}(j) \triangleq \begin{cases} 
1, & \beta_k(t) = j \\
0, & \beta_k(t) \neq j 
\end{cases} \tag{17}
\]

The activation constraints in (15) reflect the fact that each UE cannot be associated with more than one BS in each time slot, and the resource allocation constraints in (16) denote that the sum of data streams of UEs served by each BS cannot exceed the total number of available data streams on that BS. Note that \(1_{\beta_k(t)}(j)\) is equal to one only if \(\beta_k(t) = j\) or equivalently, \(k \in Q_j(t)\). Thus, the summation in (16) is actually over the set of active UEs in BS \(j\).

Next, we define the **Association Matrix** \(A\) as follows

\[
A \triangleq \begin{bmatrix}
\alpha_{1,1} & \cdots & \alpha_{1,J} \\
\vdots & \ddots & \vdots \\
\alpha_{K,1} & \cdots & \alpha_{K,J}
\end{bmatrix} \tag{18}
\]

where \(\alpha_{k,j} \in [0,1]\) is the (fractional) association coefficient representing the average connectivity of UE \(k\) to BS \(j\). If \(\alpha_{k,j} = 0\), we say UE \(k\) is not associated with BS \(j\) over time. In this model, association coefficients are considered as a fraction of time. Specifically, \(\alpha_{k,j}\) is the average fraction of time during which UE \(k\) is connected to BS \(j\). The relationship between association coefficients and activation factors is given by

\[
\alpha_{k,j} = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \beta_{k}(t)(j). \tag{19}
\]

According to (19), given the activation matrix \(B\), one can easily obtain the association matrix \(A\). For this reason, we formulate our user association problem in terms of the activation factors, which are integer variables representing the serving BS indices.

Since the proposed user association scheme is carried out per-time-slot, it can result in frequent handover, especially in the case that the UE is mobile and has comparable signal strengths from several surrounding BSs. While we do not consider handover cost explicitly in this work, we note that the proposed user association can be applied using only large-scale CSI which will result in a longer time span for each association and lessen handover. In Section VI, we consider a simple mobility model where the location of each UE is fixed during each time slot, and changes in the subsequent time slot to a new random location within a fixed range from its previous location. The user association is then performed at each time slot to assess the effect of mobility on association and handover.

**IV. USER ASSOCIATION OPTIMIZATION PROBLEM**

In this section, we first evaluate the instantaneous and average per-user throughputs by processing the received signal at each user. Then, we formulate an optimization problem to find the optimal activation factors that maximize the throughputs while satisfying load balancing constraints on the BSs.

**A. Formulation of instantaneous user rate**

Considering (7), the received signal by UE \(k\) in (10) can be decomposed as

\[
y_k(t) = W_k^H F_{k,j} s_k + \sum_{l \in Q_j(t) \backslash k} W_{l,j}^H s_l + \sum_{l \in \mathcal{J}} H_{k,l} F_{l,j} s_l + W_k^H z_k + \sum_{l \in \mathcal{J}} H_{k,l} z_l \tag{20}
\]

where the first term is the desired received signal from the serving BS \(j\), the second term represents the interference coming from the same BS \(j\) by signals...
intended for its other active UEs, the third term is
the interference coming from other BSs $i \neq j$ by
signals sent to their active UEs, and the last term is
the thermal noise at UE $k$. The activation sets $Q_j(t)$
and $Q_i(t)$ appeared in the interference terms indicate
that the interference is highly dependent on the user
association, which highlights the novelty of this work.
Again, we note that all vectors and matrices in (20)
are time-dependent, and the time index $t$ is dropped
for the sake of notational simplicity.

When UE $k$ is connected to BS $j$ in time slot $t$, its
instantaneous rate can be obtained as [39]

$$R_{k,j}(t) = \log_2 |I_{n_k} + (V_{k,j}(t))^{-1}W^*_k H_{k,j} F_{k,j} F^*_j H^*_k W_k|$$

(21)

where $V_{k,j}$ is the interference-plus-noise covariance
matrix given as

$$V_{k,j} = W^*_k H_{k,j} \left( \sum_{l \in Q_j(t)} F_{l,j} F^*_l \right) H^*_k W_k + W^* \left( \sum_{i \in J} \sum_{i \neq j} H_{k,i} F_{i,i} F^*_i H^*_k \right) W_k + N_0 W^*_k W_k.$$  

(22)

The instantaneous rate given in (21) is a function of
activation sets $Q_j(t)$. Thus, the instantaneous per-user
throughput at time slot $t$ can be expressed as

$$r_k(t) = \sum_{j \in J} 1_{\beta_k(t)}(j) \times R_{k,j}(t),$$

(23)

and the average per-user throughput is given by

$$r_k = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} r_k(t).$$

(24)

B. Optimization Problem

As stated before, the channel variation can be fast
and unpredictable in mmWave frequencies and the
clustered channel as modeled in (1) may change signifi-
cantly even during two consecutive time slots. Thus, we
need to perform the user association in each time slot,
which is comparable with the channel coherence time.
Defining the instantaneous user throughput vector $r(t) = (r_1(t), ..., r_K(t))$, we wish to find the optimal activ-
ation vector $\beta(t)$ which maximizes an overall network
utility function. This utility function should be concave
and monotonically increasing. Our proposed user asso-
ciation scheme can implicitly incorporate user fairness
via the choice of network utility function including the
family of utility functions defined in [7].

In this paper, we consider two well-known and
widely used utility functions. The sum-rate utility
function defined as

$$U_s(r(t)) = \sum_{k \in K} r_k(t)$$

(25)

and the min-rate utility function given by

$$U_m(r(t)) = \min_{k \in K} r_k(t).$$

(26)

Maximizing the first utility function corresponds to
achieving the highest network throughput, while max-
imizing the second utility function guarantees a fair
user association and improves the throughput for cell-
edge UEs who usually suffer from low data rates.

Thus, the user association optimization problem at
time slot $t$ can be written as

$$\begin{align}
\text{maximize}_{\beta(t)} & \quad U(r(t)) \\
\text{subject to} & \quad \sum_{j \in J} 1_{\beta_k(t)}(j) \leq 1, \forall k \in K \\
& \quad \sum_{k \in K} 1_{\beta_k(t)}(j), n_k \leq D_j, \forall j \in J
\end{align}$$

(27a)

(27b)

(27c)

This is an optimization problem with integer variables
$\beta_k(t) \in J$. Here, we use equal power allocation scheme
to split the BS power among its active users. Thus,
the power constraint in (8) is automatically satisfied
and can be ignored. Note that the set of constraints in
(27c) allows our user association scheme to limit each
BS’s load separately, and assuming the number of data streams ($D_j$) is chosen based on the available
resources at each BS, this set of constraints guarantees
that each BS can serve all its associated UEs simulta-
neously and thus scheduling is not required.

As discussed earlier, we adopt the approach of fixing
the beamforming scheme and only optimize for user
association. For beamforming in the ideal condition
of instantaneous CSI and absence of interference, the
optimal beamforming vectors of a channel from UE $k$
to a BS $j$ are the eigenvectors corresponding to the first
$n_k$ largest singular values, where $n_k$ is the number of
data streams intended for UE $k$ [40]. This ideal case
provides an upper bound on beamforming performance
in a realistic network. Considering (21), it is clear
that our formulation of the user instantaneous rate
includes the precoder and combiner matrices of all UEs
and BSs which are applicable using any beamforming
method. Thus, it is plausible that a user association
scheme based on singular value decomposition (SVD)
beamforming is compatible with user association based
on other beamforming approaches and can achieve
near-optimal results.

We use SVD beamforming to obtain the precoder
and combiner matrices. To this end, we first need to
decompose the channel matrix $H_{k,j} \in \mathbb{C}^{N_k \times M_j}$ as

$$H_{k,j} = \Phi \Sigma \Gamma^*$$

(28)

where $\Phi \in \mathbb{C}^{N_k \times r}$ is the unitary matrix of left singular
vectors, $\Sigma \in \mathbb{C}^{r \times r}$ is the diagonal matrix of singular
values (in decreasing order), $\Gamma \in \mathbb{C}^{M_j \times r}$ is the unitary
matrix of right singular vectors, and \( r = \text{rank}(H_{k,j}) \). Then, we partition the channel matrix as
\[
H_{k,j} = \begin{bmatrix} \Phi_1 & \Phi_2 \\ \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} \Gamma_1^* \\ \Gamma_2^* \end{bmatrix} = \Phi_1 \Sigma_1 \Gamma_1^* + \Phi_2 \Sigma_2 \Gamma_2^*
\]
where \( \Phi_1 \in \mathbb{C}^{N_k \times n_k}, \Sigma_1 \in \mathbb{C}^{n_k \times n_k}, \) and \( \Gamma_1 \in \mathbb{C}^{M_j \times n_k} \).

The above partitioning is done to extract the precoder and combiner of appropriate sizes to support the number of data streams of each user \( k \). Specifically, each precoder \( F_{k,j} \) and combiner \( W_k \) must be of size \( M_j \times n_k \) and \( N_k \times n_k \), respectively. Then, the SVD precoder and combiner can be obtained as
\[
F_{k,j} = \Gamma_1
\]
\[
W_k = \Phi_1.
\]

The optimization problem in (27) is a highly non-convex MINLP, which is known to be NP-hard due to its non-convex and nonlinear structure and presence of integer variables [41]. The non-convexity and nonlinearity come from the fact that interference structure contains association variables, specifically the indicator function appeared in (23), and constraints (27b-c).

Suboptimal and often heuristic methods exist for solving such an MINLP. For example, genetic algorithm (GA) is a such tool which can be effective for solving combinatorial optimization problems. GA is a method based on natural selection which simulates biological evolution. It iteratively generates and modifies a population of individual solutions. After successive generations, the population eventually evolves toward a near-optimal solution [42]. In [43], we used the GA solver provided in Global Optimization Toolbox of MATLAB to solve the optimization problem in (27). We showed that the GA works effectively for small networks. However, we found that GA does not work well when the network size increases, taking an exceedingly long time to run and still not reaching a good (close to optimal) solution. In this paper, we propose an efficient algorithm to find a near-optimal solution for our MINLP.

Algorithm 1: WCS User Association Algorithm

1. initialization:
2. - Generate an arbitrary feasible activation vector \( \beta^{(1)} \);
3. - Find the initial worst connection \( (k^{(1)}, j^{(1)}) = \arg \min_{k,j} R_{k,j}(\beta^{(1)}) \);
4. repeat
5. \( \text{Swap} \) the \( k^{(i)} \) th element (activation factor) with all other elements of \( \beta^{(i)} \) to obtain \( (K-1) \) new vectors \( \beta^{(i)}_n \) with \( n \in K, n \neq k^{(i)} \);
6. Find the best activation vector \( \beta^{(i+1)} = \arg \max_{\{\beta^{(i)}_n | n \in K, n \neq k^{(i)} \}} U(r(\beta)) \);
7. Find the worst connection \( (k^{(i+1)}, j^{(i+1)}) = \arg \min_{k,j} R_{k,j}(\beta^{(i+1)}) \);
8. if \( \beta^{(i+1)} = \beta^{(i)} \) then
9. \( l^{(m)} \leftarrow \text{mod}(m - 1, K) + 1 \);
10. \( \text{Switch} \) the \( k^{(i+1)} \) th and \( l^{(m)} \) th elements of \( \beta^{(i+1)} \) to obtain \( \beta^{(i+1)}_{sw} \);
11. \( \beta^{(i+1)}_{sw} = \beta^{(i+1)} ;\)
12. \( m \leftarrow m + 1 \);
end
9. Stopping criterion: Break if the best \( \beta \) does not change after \( K \) consecutive swapping iterations;
10. \( i \leftarrow i + 1 \);
11. until stopping criterion achieved;

V. WORST CONNECTION SWAPPING (WCS) ALGORITHM

In the previous section, we formulated the optimization problem in (27) which is an NP-hard MINLP. In this section, we introduce an efficient iterative algorithm which converges to a near-optimal solution in a polynomial time. The algorithm is based on the rational that a user association throughput may be suboptimal because of the weakest UE suffering from a weak direct link or high interference from other BSs in the network. Thus, swapping the worst UE-BS connection is likely to provide the UE a stronger link (better connection to another BS) and/or reduce the interference, which consequently improves user instantaneous rates and hence the throughput. This reasoning leads us to design an algorithm to swap the BS in the worst UE-BS connection with all other BSs and then take the best configuration (activation vector) which results in the highest network utility function. Next, we describe the algorithm in detail.

A. Algorithm Description

The proposed algorithm is summarized in Algorithm 1 and works as follows. It starts with a random feasible initial activation vector \( \beta^{(1)} \) which satisfies the constraints in (27b-c), and an initial worst UE-BS connection \( k^{(1)} \) obtained from \( \beta^{(1)} \) as described later in (33). The algorithm has two main steps: i) Swapping step, ii) Switching step. In what follows, we describe these steps in detail.

1) Swapping step: At each iteration \( i, \) we swap the worst connection \( \beta^{(i)}(k^{(i)}) \), which is the index of the BS with worst UE-BS connection in current activation vector \( \beta^{(i)} \), with all other connections (i.e. other BS entries in the same activation vector \( \beta^{(i)} \)). Since the
size of an activation vector is \(K\), this swapping results in \(K-1\) new activation vectors \(\beta_n^{(i)}\) with \(n \in \mathcal{K}, n \neq k^{(i)}\). Then, the network utility function, defined in (25)-(26), is computed for the current and new activation vectors, and the one corresponding to the largest network utility function is chosen as the best activation vector for the next iteration, i.e.,

\[
\beta^{(i+1)} = \arg \max_{\{\beta^{(i)}|n \in \mathcal{K}, n \neq k^{(i)}\}} U(r(\beta))
\]  

(32)

Next, we find the worst UE-BS connection in \(\beta^{(i+1)}\) by comparing the user instantaneous rates as

\[
(k^{(i+1)}, j^{(i+1)}) = \arg \min_{k,j} R_{k,j}(\beta^{(i+1)}).
\]  

(33)

and then repeat the process until no improvement in the utility function is achieved. Note that the algorithm is applied for any given time slot \(t\) to obtain the optimal activation vector \(\beta(t)\). Here, we dropped the time index \(t\) for the ease of notation.

**Proposition 1:** The sequence of network utility function values produced by Algorithm 1 is non-decreasing.

**Proof:** Since the user instantaneous rate \(R_{k,j}\) and the network utility function \(U(r)\) are both functions of the activation vector, at each iteration based on (32), the network utility function either remains the same (when \(\beta^{(i+1)} = \beta^{(i)}\)) or improves (when \(\beta^{(i+1)} \neq \beta^{(i)}\)).

The above procedure works effectively if the objective function is always increasing in all iterations, i.e. \(\beta^{(i+1)} \neq \beta^{(i)}, \forall i\). However, at some point, the current and best activation vectors may be equal. In this case, the algorithm repeatedly produces the same best activation vector (and consequently the same worst connection) in the following iterations. When that happens, swapping does not lead to any more changes in the connections of other users or the network utility function. Thus, we need to design another step, the switching step, in order to overcome this problem. Note that if \(\beta^{(i+1)} = \beta^{(i)}\), then the worst connections are equal, i.e. \(k^{(i+1)} = k^{(i)}\), but the reverse is not true in general.

2) **Switching step:** In order to resolve the above issue and find new possibility to improve the objective function, we introduce a switching step. At each iteration, we compare \(\beta^{(i+1)}\) and \(\beta^{(i)}\). If they are not equal, no further action is required and the algorithm proceeds to the next iteration. If they are equal, then we switch the worst connection \(k^{(i+1)}\) (which is equal to \(k^{(i)}\)) with another user to find a potentially better activation vector. To this end, at each switching step \(m\), we compute a UE index as \(l^{(m)} = \text{mod}(m-1, K) + 1\) and switch the \(k^{(i+1)}\)th and \(l^{(m)}\)th elements of the current \(\beta^{(i+1)}\) to obtain the switched activation vector \(\beta_{sw}^{(i+1)}\). Then, we use this vector as the initial activation vector for the next swapping iteration. Note that the switching process does not affect the non-decreasing nature of the algorithm, since in the swapping step of next iteration, the pre-switching activation vector \(\beta^{(i+1)}\) appears in \(\beta^{(i+2)}\) and therefore is included in the utility function comparisons (see (32)). Moreover, the switching process guarantees that the connection of all users is considered for improvement, since \(l^{(m)}\) sweeps the set of all users in order.

The algorithm stops when there is no further improvement in the network utility function (no change in optimal activation vector) after \(K\) consecutive swapping iterations. This criterion is reasonable because \(l^{(m)} = l^{(m+K)}\) means after \(K\) iterations the algorithm starts to switch the same worst connection with the same user. We note that the feasibility of the solution is also guaranteed since the algorithm starts with a feasible activation vector and at each iteration just swaps the elements of this vector to find the best activation vector.

**B. Complexity Analysis**

In this subsection we consider the sum-rate utility function and analyze the algorithm complexity by computing the number of floating multiplications. The cost of multiplication of two matrices, \(A \in \mathbb{C}^{n \times m}\) and \(B \in \mathbb{C}^{m \times p}\), is \(O(np)\). SVD decomposition of matrix \(A \in \mathbb{C}^{n \times m}\) has a cost of \(O(nm^2)\) \((m \geq n)\), matrix inversion by Jordan-Gauss elimination scales as \(O(n^3)\), and the cost of computing the matrix determinant is \(O(n^3)\). Finding \(\max\) or \(\min\) over a set of \(n\) variables, also known as sorting algorithms, requires \(n \log(n)\) comparisons [45]. When computing the multiplication of three matrices \(A \in \mathbb{C}^{n \times m}, B \in \mathbb{C}^{m \times p},\) and \(C \in \mathbb{C}^{p \times q}\), the order in which the product is parenthesized is important, and we take this into account to minimize the cost [44]. Based on these complexity orders, we can analyze the complexity of our WCS algorithm. We first look at the complexity order of a single swapping iteration, then examine the number of iterations. At each swapping iteration of the algorithm, Steps 6 and 7 have the highest computational cost because of the search/sort procedures, and according to above complexity bases, they exhibit similar computation order. Thus, we focus on computing the cost of Step 6 which involves two major operations: computing the network sum-rate \(U_r(r(t))\), and sorting the resulted sum-rate vector. Computing the network sum-rate involves computing the interference plus noise term \(V_{k,j}\) in (22) which is of order \(O(M_2^2)\), and this leads to the cost of computing user instantaneous rate \(R_{k,j}\) in (21) also as \(O(M_2^2)\). Thus, the cost of computing the network sum-rate in (25) is of the order \(O(M_2^2)\). Next, the cost of sorting and finding \(\max\) over the set of \(K - 1\) new activation vectors \(\beta_n^{(i)}\) and the previous activation vector \(\beta^{(i)}\) is \(O(K \log(K))\). Therefore, the total cost of computing Step 6 is \(O(K \log(K)M_2^2)\), and since Step 6 is computationally dominant in each iteration, this is also the order of the cost for a single swapping iteration. Based on the algorithm’s stopping criterion, the number of swapping iterations is between \(K\) and
cK iterations with c as a constant factor, 1 ≤ c ≪ K, and we can conclude that the number of iterations increases linearly with K. Thus, the final complexity of the algorithm is $O(K^2 \log(K) M^2_f)$.

VI. NUMERICAL RESULTS

In this section, we evaluate the performance of the proposed user association scheme in the downlink of a mmWave MIMO network with J BSs and K UEs operating at 73 GHz. In our simulations, we consider the network sum-rate utility function defined in (25), except when study user fairness and cell-edge users’ rate in Section VI.A.5. The mmWave links are generated as described in Section II-A, each link is composed of 5 clusters with 10 rays per cluster. Each BS is equipped with a $8 \times 8$ UPA antenna, and each UE is equipped with a $2 \times 2$ mmWave UPA antenna for 5G connections. The noise power spectral density is $-174$ dBm/Hz. Unless otherwise stated, all BSs transmit at the same power level $P_j$. Moreover, we assume that the network nodes are deployed in a region of size $300 \times 300$ m. The BSs are placed at specific locations and the UEs are distributed randomly within the given area. There are $n_k$ data streams for each UE and $Q_j(t)$ is the maximum number of allowed active users at each BS. Thus, the total number of data streams sent by each BS is $D_j = n_k Q_j(t)$. A BS is considered to be overloaded (congested) if more than $Q_j(t)$ UEs are associated with that BS.

We present our numerical results in two main thrusts. First is the performance of the proposed user association scheme for mmWave with respect to its novel features: (i) using both instantaneous and large-scale CSI, (ii) considering the dependency of interference on user association, and (iii) using MIMO beamforming at both ends. Second is the numerical analysis of our proposed WCS algorithm in terms of convergence and complexity to verify the derived theoretical complexity order.

A. Performance of the proposed user association scheme

1) Comparison of max-SINR and proposed load balancing user association schemes: First, we compare the conventional max-SINR user association scheme with our proposed load balancing user association scheme. In order to better reflect the capabilities of the proposed user association scheme, in this simulation study, we consider a two-tier HetNet with 1 macro BS (centered at the middle of the network), 3 pico BSs and 20 UEs. The transmit power of macro BS is 30 dBm, while pico BSs transmit at 20 dBm. The microwave links are i.i.d. Rayleigh fading MIMO channels, and we assume that each UE is also equipped with a single-antenna module designed for LTE/4G connections at microwave band in addition to their mmWave antennas. The macro BS can accommodate 8 UEs, and each pico BS can only serve 3 UEs. Fig. 1.a shows the result of max-SINR association in the network. As we can see from the figure, the macro BS is overloaded by 4 extra UEs, and BS #3 is overloaded with one extra UE. User association using our proposed scheme is shown in Fig. 1.b. It can be seen from the figure that the proposed method perfectly balances the BSs’ load by pushing the overloading UEs from the congested BSs into the lightly-loaded BSs.

2) Effect of network interference dependency on user association: Next, we compare throughput performance of the proposed user association method by comparing it with three other user association schemes: (i) load balancing user association [7], (ii) max-SINR user association with user drop, and (iii) max-SINR user association with resource (data stream) sharing and user drop. The first scheme is a load balancing user association which results in fractional association coefficients, because the integer association constraint is relaxed to solve the optimization problem [7]. In the second method, those UEs who overloaded the con-
Fig. 2. Average network spectral efficiency for user association schemes in a mmWave network with 3 BSs, 12 UEs, 8 × 8 UPA at each BS, and 2 × 2 UPA at each UE. The UEs are distributed randomly in an area of size 300 m × 300 m. All BSs transmit at the same power level, and the noise power spectral density is −174 dBm/Hz.

Fig. 3. Network spectral efficiency versus the number of antennas at each BS (upper subfigure) and at each UE (lower subfigure).

4) Effect of mobility on user association: We consider a simple mobility model where the location of UEs are fixed during each time slot. In the subsequent slot, each UE moves to a new random location within a 5m x 5m square box centered at its location in the previous time slot. The user association is performed at each time slot and the fractional user association is obtained by taking average over T = 1000 time slots. Fig. 4 depicts the network topology with random locations of UEs shown in small circles. Note that the number of random locations for each UE is 1000, but we only show 10 random locations (t = 1 : 100 : 1000) for each UE to avoid a messy figure. Fig. 5 depicts the average per-user throughputs and the fractional associations. As we can see from these simulations,
UE 4 is always moving around BS 1 and thus it is associated with BS 1 for about 87 percent of the time. The same result can be inferred for UE 1 which is connected to BS 2 for most of the time. UE 2, however, gradually moves away from BS 3 and gets closer to BS 2, and that is the reason this UE is associated with both BSs for about half of the time. UE 2 therefore could experience frequent handover in this case. These simulations confirm that our proposed user association is well-designed to be performed per-time slot, and it can be applied with any mobility model.

5) Effect of fairness on cell-edge users’ throughputs: Cell-edge users usually suffer the worst throughput in the network. In order to study the effect of fairness on cell-edge throughput, we compare the two utility functions defined in (25)-(26). Fig. 6 shows the probability density function (PDF) and cumulative distribution function (CDF) of users’ data rates in a mmWave network with 3 BSs and 15 UEs. We consider the sum-rate utility function in (25) for the “Max-throughput” case, and use the min-rate utility function in (26) to implement “Max-min Fairness”. By focusing on the low rate region (bottom left corner) of each figures, it can be inferred that compared to max-throughput, max-min fairness shows a lower probability of users having an extremely low (close to zero) rate. For example, in Fig. 6.b the probability of having users with data rate smaller than 1 bps/Hz is 5.1% for sum-rate utility function, while this probability is about 2.8% when using the max-min fairness. Since most cell-edge users would be represented in the low-rate region, this result implies that max-min fairness results in higher data rates for the cell-edge users in the network, especially ensuring that these users’ rates start at a reasonable non-zero value. This is in contrast to max-throughput in which the CDF figure shows a small portion of users actually having zero rates. This result confirms that max-min fairness improves the throughput for cell-edge users who usually suffer from low data rates.

B. Efficiency of the proposed WCS algorithm

1) Convergence performance: Fig. 7 compares the performance of the proposed algorithm with GA in a network with 12 UEs and 4 BSs. As a benchmark, exhaustive search result is included to provide the optimal solution. The “improvement” curve shows the increment of network sum-rate at each swapping iteration. It can be seen from the figure that the proposed WCS algorithm rapidly converges to the optimal solution in 32 iterations which execute in 1.08 seconds. However, the GA (with 50 stall generations) stops after 215 iterations and still does not reach the optimal solution. GA can, in theory, reach to the optimal solution by increasing the number of stall generations, but it takes drastically more time to reach this. Our simulations show that it takes 417 iterations which is about 86 seconds for GA (with 200 stall generations) to reach
the optimal solution, compared to the proposed WCS which takes only 1.08 seconds. Note that the GA stops if there is no improvement in the objective function for a sequence of consecutive generations of the length equal to the number of stall generations.

2) Complexity results: Comparing the convergence time of the proposed algorithm and GA in Fig. 7, one can easily confirm that the WCS algorithm has much lower complexity than GA. Fig. 8.a shows the number of swapping iterations of the proposed algorithm versus the number of UEs while the number of BSs is fixed \((J = 3)\). The average is taken over 500 channel realizations and polynomial data fitting is implemented. As expected, there is a linear relationship between the number of iterations and the number of UEs. Fig. 8.b depicts the actual elapsed time of the proposed algorithm. Based on our complexity analysis in Section V-B, we fit the elapsed time results with a function of the form \( f(K) = aK^2 \log(K) + bK^2 + cK + d \), where \( K \) is the number of UEs and \( a, b, c, \) and \( d \) are some constants. The result confirms the complexity analysis and shows that the fitting model matches well with the simulation results.

VII. Conclusion

In this paper we investigated the problem of optimal user association in a mmWave MIMO network. We first introduced the activation matrix and showed that the user instantaneous rate is a function of the activation factors. Then, we formulated a new association scheme in which network interference depends on user association. We designed an efficient polynomial-time algorithm to solve the formulated MINLP optimization problem. Simulation results showed that the proposed user association scheme outperforms existing association methods, which ignore the effect of interference on user association, confirming the fact that the network interference structure is highly dependent on user association. Moreover, we designed an efficient algorithm to solve the formulated MINLP optimization problem. Numerical results confirmed the polynomial complexity of the proposed algorithm and showed that it has significantly lower complexity than the genetic algorithm, and reaches a near-optimal solution in a reasonable time and number of iterations.

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Fig. 8. (a) Number of iterations and (b) elapsed time of the proposed algorithm in a network with 3 BSs and $K$ UEs. The error bars show the standard deviation.

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