HOT WHITE DWARF DONORS IN ULTRACOMPACT X-RAY BINARIES

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ABSTRACT

The discovery of two accreting millisecond X-ray pulsars in binaries with ≈43 minute orbital periods allows for a new probe of the donor’s structure. For XTE J1751−305, only a hot white dwarf (WD) can fill the Roche lobe. A cold He WD is a possible solution for XTE J0929−314, although I will show that evolutionary arguments make a hot WD more likely. In addition to being larger than the $T = 0$ models, these finite entropy, low-mass ($M_c < 0.03 M_\odot$) WDs have a minimum mass for a fixed core temperature. If they remain hot as they lose mass and expand, they can “evaporate” to leave an isolated millisecond radio pulsar. They also adiabatically expand upon mass loss at a rate faster than the growth of the Roche radius if the angular momentum deposited in the disk is not returned to the donor. If the timescale of the resulting runaway mass transfer is shorter than the viscous timescale in the outer disk, then the mass transfer instability of Ruderman & Shaham for He WDs would be realized. However, my estimates of these timescales still make the instability unlikely for adiabatic responses. I close by noting the possible impact of finite temperature WDs on our understanding of AM CVn binaries.

Subject headings: accretion, accretion disks — binaries: close — stars: low-mass, brown dwarfs — stars: neutron — white dwarfs — X-rays: binaries

1. INTRODUCTION

The recent discovery by the Rossi X-Ray Timing Explorer of two accreting millisecond pulsar transients, XTE J1751−305 ($P \approx 435$ Hz; Markwardt et al. 2002) and XTE J0929−314 ($P \approx 185$ Hz; Galloway et al. 2002), has allowed us to learn about the donors in these ultracompact binaries. At $P_{orb} \approx 43$ minute orbital periods, H-rich donors are ruled out (Nelson, Rapaport, & Joss 1986) and He-rich stars (see Polskiadlowski, Rapaport, & Pfahl 2002 for an updated discussion of the range of H/He ratios) or white dwarfs (WDs) are filling the Roche lobe (RL). If it is a degenerate WD, then the orbital period increases as mass is transferred at the rate set by angular momentum losses from gravity waves (GWs), $J_{GW}$ (see Verbunt 1993).

The measured pulsar orbital parameters yield the RL-filling companion’s mass, $M_c$, and the minimum mass transfer rate, $\dot{M}_{crit} = 3 M_c J_{GW}/2 J$, which are shown in Figure 1 for neutron stars (NSs) of $M_X = 1.4 - 2.0 M_\odot$. The low $M_c$’s are likely cause for the transient behavior, as the steady state outer disk temperature is below these elements’ ionization temperature (Tsupawa & Osaki 1997; Menou, Perna, & Hernquist 2002), even if X-ray heating is included at the rate inferred in other X-ray binaries (Dubus, Hameury, & Lasota 2001). If identical WDs are the donors in both of these binaries, then their identical orbital periods would require the same RL-filling solutions. This would constrain the inclination for XTE J0929−314 to less than 37° and $M_c > 0.013 M_\odot$ for both XTE J0929−314 and XTE J1751−305.

The dotted lines in Figure 2 show the mass-radius relation for RL-filling donors of XTE J1751−305 (lower line) and XTE J0929−314 (upper line). The solid and dashed lines (which exhibit a maximum radius owing to the onset of Coulomb physics) are the cold ($T = 0$) WDs of Zapolsky & Salpeter (1969, hereafter ZS) for pure He and C, respectively. A cold He WD will fill the RL for XTE J0929−314 (Galloway et al. 2002), whereas there are no cold WD solutions that fill the RL for XTE J1751−305 (Markwardt et al. 2002).

This evidence for finite $T_c$ WDs motivated my calculations shown in Figure 2 by the solid and dashed lines that diverge at low $M_c$. These models (see § 2) are for He at $T_c = 10^3$ and $10^6$ K and C at $T_c = 10^6$ K and $T_c = 3 \times 10^6$ K. These provide RL-filling solutions but do not a priori differentiate between He or C or any other element that might dominate the donor star. I consider C WDs throughout this Letter, as Schulz et al. (2001) have inferred a high Ne-to-O ratio in the matter transferred onto NSs in ultracompact X-ray binaries (confirmed by Juett, Psaltis, & Chakrabarty 2001; Homer et al. 2002; Juett & Chakrabarty 2002). These measurements led Schulz et al. (2001) and Juett et al. (2001) to suggest that the donors in these binaries are the cores of previously crystallized C/O WDs.

In § 2, I justify (on evolutionary grounds) that hot WDs are expected, describe the construction of their $R_c (M_c)$ relations, and point out the existence of a minimum $M_c$ solution for a given $T_c$. I put the models in the context of mass donors in § 3 and calculate

$$n_{ad} = \frac{d \ln R_c}{d \ln M_c},$$

the adiabatic exponent needed to evaluate the stability of mass transfer. Although uncertainties remain regarding the ability of the accretion disk to store angular momentum on long timescales (Verbunt & Rappaport 1988), my initial work finds that the expansion of an He WD donor under mass transfer can exceed that of the RL, possibly allowing for the Ruderman & Shaham (1983) instability.

2. FINITE ENTROPY WHITE DWARFS AND EVAPORATION

The material that fills the RL in these systems today was once the deep interior of an initially more massive WD, so I begin by discussing the extent to which prior evolution (before RL filling) can affect the temperature of the current low-mass WDs. There are a few evolutionary scenarios that place an He or C/O WD in a tight enough orbit (following a common-envelope–induced spiral-in) about an NS that GW emission will place it in contact within 5–10 Gyr (Rasio, Pfahl, & Rapaport 2000; Yungelson, Nelemans, & van den Heuvel 2002).
In this case, the WD has had some time to cool before initiating RL overflow. Nelemans et al. (2001) discuss a similar scenario for the origin of AM CVn binaries.

The rate at which they cool differentiates He and C/O WDs. The larger specific heat of an He WD slows its cooling (Althaus & Benvenuto 1997; Hansen & Phinney 1998). For example, a 0.20 $M_\odot$ He WD would have $T_i = 10(3.3) \times 10^6$ K at $\approx 1.0(4.0)\text{ Gyr}$ (Althaus & Benvenuto 1997) and be liquid when RL filling occurs. The smaller specific heat of a 0.6 $M_\odot$ C/O WD allows it to cool to $2.5 \times 10^6$ K in 4 Gyr and begin crystallization (Salaris et al. 2000). Hence, most C/O WDs would be solid prior to RL filling (this is needed for the enhancement of $^{22}$Ne by fractionation; Schulz et al. 2001) and might melt only if heated during the GW in-spiral (Iben, Tutukov, & Fedorova 1998). The other differentiation between such WDs comes from their mass, as Yungelson et al. (2002) argue that the mass transfer at the onset of RL filling will be unstable if $M_\text{f} > 0.44 M_\odot$, excluding traditional C/O WDs from ever reaching short orbital periods. However, Yungelson et al. (2002) note that “hybrid” WDs with C/O cores can have $M_\text{f} < 0.44 M_\odot$ and thus evolve to lower mass systems filling the RL at 20–40 minutes.

The inability for the donor WDs to cool on the approximately gigayear timescale of the mass transfer phase (Rappaport et al. 1987) makes it clear that the initial entropy is the minimum value attainable. All of the entropy is in the liquid ions, which have an adiabatic scaling, $T \propto \rho^{0.5-0.6}$ (Hernanz et al. 1988), less steep than for an ideal gas ($T \propto \rho^{2/3}$). Since $\rho \propto M_\text{f}^2$, a $\approx 0.013 M_\odot$ He WD made by adiabatically expanding the deep interior of a 0.20 $M_\odot$ WD would be a factor of $\approx 15$ cooler, or $T_i \approx 6.6(2.2) \times 10^6$ K for the initial range of $T_i$'s discussed earlier. Of course, any tidal heating will increase $T_i$ (see § 3).

So I have constructed $M_\text{f} < 0.03 M_\odot$ WDs of finite $T_i$ that contain the Coulomb physics (from Chabrier & Potekhin 1998) that causes the turnover to constant density “rocks” in the mass-radius relation at $\approx 10^{-3} M_\odot$ seen in the ZS models in Figure 2. I have imposed an arbitrary (but convectively stable) temperature profile of $T = T_i \left(P/P_i\right)^{1/5}$ while integrating hydrostatic balance and mass conservation. I used the electron equation of state of Paczyński (1983) and halt integrations at the point where the pressure has fallen to $10^{-5}$ of the central value, which avoids the need for an envelope model. The results are shown in Figure 2 and make clear that the temperatures expected from adiabatic evolution of an He WD are adequate to provide the slight radius expansion needed to fill the RL for these accreting millisecond pulsars. This is easier for He WDs because of the extra number of ions per electron compared to C/O WDs. White dwarfs made of C/O will need some tidal heating to fill these RLs.

I also found that there is a minimum mass solution for a WD of fixed $T_i$. This is related to the electron Fermi energy of the nearly constant density solutions at low masses and is reflected in the radius divergence of the models in Figure 2 at low $M_\text{f}$. My semianalytic modeling finds that these finite $T_i$ solutions eventually “turn over” to follow a track of $R \propto M_\text{f}$, as expected for an ideal gas polytrope (where $T_i \propto M_\text{f}/R_i$). Since there is no way to reach these solutions of higher $M_\text{f}$ while mass is being lost, these “evaporative” endpoints where $d \ln R_i/d \ln M_\text{f}$ diverges can be reached only if tidal heating is adequate to keep these models hot while the central density is dropping. More work remains to actually show that

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**Fig. 1** — Companion masses and GW-driven mass transfer rates for XTE J1751–305 and XTE J0929–314 as a function of cosine of the inclination angle. The hatched regions (upper is XTE J1751–305 and lower is XTE J0929–314) are bounded below by $M_\text{f} = 1.4 M_\odot$ and above by $M_\text{f} = 2.0 M_\odot$. It was presumed that the donor responds to mass transfer with, only a small $n_{c,0}$ difference from $n_{c,0}$.

**Fig. 2** — RL-filling mass-radius relations for XTE J1751–305 and XTE J0929–314 compared to WDs. The dotted lines denote the RL-filling donors of XTE J1751–305 (lower line) and XTE J0929–314 (upper line). The solid (dashed) lines are $T = 0$ WDs of ZS for pure He (C) using the Rappaport & Joss (1984) correction to ZS. The lines that diverge at small $M_\text{f}$ are hot WDs. The He WDs have $T_i = 10^6$ and $10^7$ K, and the C WDs have $T_i = 10^6$ K and $T_i = 3 \times 10^5$ K.
this will eliminate the companion and lead to an isolated millisecond radio pulsar.

3. MASS TRANSFER INSTABILITIES AND QUIESCENT EMISSION

Barring tidal heating adequate to reach the evaporative endpoint alluded to in § 2, I will now discuss a possible mass transfer instability. Matter leaving the WD takes angular momentum with it as it settles into the accretion disk. This angular momentum is returned to the WD via tidal torques once there has been enough time for viscosity to move the material outward from the splash point. The timescale for this angular momentum loop to be closed has been considered to be long enough (or uncertain enough) that Ruderman & Shaham (1983) raised the possibility that a mass transfer instability could occur when the expansion of a low-mass donor \((M_\text{d} \ll M_\text{c})\) due to mass loss (measured by \(n_{\text{ad}}\) of eq. [1]) exceeds that of the RL, measured by

\[
 n_R = \frac{d \ln R_R}{d \ln M} = \frac{d \ln a}{d \ln M} + \frac{1}{3} \frac{1}{\frac{d \ln a}{d \ln M}} \tag{2}
\]

where \(a\) is the orbital separation. The value of \(n_R\) presuming the angular momentum “sink” of the disk (using the fitting formula of Verbunt & Rappaport 1988) is shown in Figure 3 by the dotted lines for (from left to right) \(M_\text{c} = 0.6, 1.0, 1.4, \) and 1.8 \(M_\odot\).

Ruderman & Shaham (1983) presumed \(n_{\text{ad}} = -\frac{1}{2}\), which was shown to be an overestimate by Hut & Paczyński (1984) and Bonsema & van den Heuvel (1985), who found \(n\) from the ZS relation (shown in Fig. 3 by the solid [dashed] lines for He [C]). These authors noted that a ZS model moved the crossing (\(n_{\text{ad}} < n_R\)) to such low \(M_c\) that the binary might not reach it in a Hubble time. However, for finite \(T\) He WDs, I show that this conclusion is altered. The filled circles are my evaluation of \(n_{\text{ad}}\) for the \(T = 10^5\) K (set closest to the solid line) and \(T = 10^6\) K (bottom set) He WDs of Figure 2 and show that the finite entropy model sits between the \(V\) value and the “naive” \(n = -\frac{1}{2}\) guess for a perfect gas. For an \(M_c = 1.4\) \(M_\odot\) accretor, the instability question is first raised (since \(n_{\text{ad}} < n_R\)) when a \(10^5\) K He WD has \(M_c \approx 0.01\) \(M_\odot\). This will occur about a gigayear after mass transfer has started presuming just GW emission. My evaluations of \(n_{\text{ad}}\) for C WDs at the \(T\) of Figure 2 never found such a crossing.

The instability question is thus raised for He WDs and requires some extra entropy from tidal heating. The amount of heating is small, as for liquid He to reach \(T \approx 10^6\) K from about half that value (which it had initially) requires \(3 k_b (0.5 T_\odot) 4 m_p \approx 3 \times 10^{31}\) ergs g\(^{-1}\) (the specific heat in the liquid state is \(\approx 3 k_b T\) per ion). This energy is comparable to the current rotational energy per gram for a tidally locked WD at the 40 minute orbital period. Hence, only a fraction of the higher rotational energy per gram from the tidally locked WD at a shorter orbital period needs to be deposited to yield \(T \approx 10^6\) K today.

The remaining criticism against the mass transfer instability is one of relative timescales (Verbunt & Rappaport 1988; Priedhorsky & Verbunt 1988). The mass transfer instability most likely grows on a timescale \(\tau_c \approx M_c H M R, (\text{Verbunt} & \text{Rappaport} 1988), \) where \(H\) is the scale height in the WD atmosphere, fixed by the X-ray heating in quiescence from the hot NS (Bildsten & Chakrabarty 2001). The binary spends most of the time undergoing mass transfer to the outer disk but little to no accretion onto the NS, Bildsten & Chakrabarty (2001) showed that the X-ray emission always detected from NS binaries in quiescence (see Bildsten & Rutledge 2001 for an overview) heat the companion on the side facing the NS. Presuming the NS thermal emission is at the level predicted by Brown, Bildsten, & Rutledge (1998; giving \(L_\text{X} = 5 \times 10^{32}\) ergs s\(^{-1}\)), then the WDs in XTE J1751−305 and XTE J0929−314 will have \(T_{\text{eff}} \approx 5000−6600\) K.\(^1\) The scale height is \(H/R_\odot \approx 3 \times 10^{-4}\), yielding \(\tau_c \approx 10^6\) yr. This is still much longer than the time for viscosity to finally play a role in moving material outward in the outer accretion disk, most likely allowing for the angular momentum to get back to the donor and move it out, so that \(n_R = -5/3\). The adiabatic index is never less than \(-5/3\). However, the isothermal WD response is less than \(-5/3\) and mass transfer would be unstable for \(M_c < 4(9.5) \times 10^{-11} M_\odot\) in an He WD of \(T_c = 4(10) \times 10^6\) K as long as internal heating can keep the WD isothermal as mass is lost. These masses are near the values I noted earlier for the evaporation, namely, where the mass-radius relation turns over.

4. CONCLUSIONS

The timing of two accreting millisecond pulsars (Markwardt et al. 2002; Galloway et al. 2002) in ultracompact binaries has probed the WD donor properties to new levels and shown that they are of finite entropy. This motivated my calculations of low-mass WDs of finite \(T_c\) that allow for \(T\) to be constrained. For He WDs, the implied \(T_c\) are nearly that expected just from adiabatic expansion of the initially hot WD that filled the RL. Only a small amount of tidal heating is needed. More tidal heating is needed to make a C/O WD fill the RL.

\(^1\) These high \(T_{\text{eff}}\)’s should aid detection of the quiescent counterpart, as the luminosity (presuming \(R_\text{ NS} = 0.045 R_\odot\)) will be \(L_\odot = (0.5–1.0) \times 10^{-3} L_\odot\) just from reprocessing the NS’s thermal emission. Presuming \(T_{\text{eff}} \approx 6000\) K and using the WD radius implied by RL filling, I get \(M_c \approx 12\) for the heated WD face. For the high-latitude source XTE J0929−314 at \(d = 5–7\) kpc (Galloway et al. 2002), this gives \(V \approx 26\).
These finite $T$ solutions allowed for a reevaluation of Ruderman & Shaham’s (1983) scenario for making isolated millisecond radio pulsars via a mass transfer instability. I find that the adiabatic mass transfer instability can occur for a hot He WD as long as the angular momentum leaving the RL-filling star is not returned to the orbit. I have thus eliminated one criticism of their model, although the question of angular momentum elimination remains a serious one. The physics of hot, low-mass WDs also yields a minimum mass WD solution for a fixed $T_c$, so that a mass transfer instability can occur if the donor remains isothermal under mass loss. I thus speculate that an evaporative or mass transfer instability endpoint might occur as long as tidal (or other) heating persists at 40–80 minute orbital periods.

This work also impacts AM CVn binaries, where a low-mass He star donates material to a more massive WD (see Solheim 1995 for a review). The larger WD radii lead to more GW emission and a higher $\dot{M}_{\mathrm{orb}}$ than expected for a given $P_{\mathrm{orb}}$. Hence, models that track the WD entropy will fall between the degenerate and nondegenerate models in Nelemans et al. (2001) and depend on both the age of the system when RL filling occurs (as this fixes the initial WD entropy) and any tidal heating that occurs during the mass transfer. If either of the instabilities discussed above occur, then the endpoint of AM CVn’s could well be a DB WD (e.g., Tutukov & Yungelson 1996).

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