Calculation of the Shortest Path for Robot to Avoid Obstacles

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Abstract. Obstacle avoidance path was widely used in real life, such as rescue and search, through battle field, robot through obstacles. In this paper, the obstacle avoidance problem of robot was studied. By establishing a mathematical model and considering the speed change of robot when turning, the selection principle and calculation of the shortest path for robot to pass through the obstacle area were discussed.

1. Introduction
With the progress and rapid development of science and technology, the application of robot was more and more common in our life, especially in some advanced fields [1-2]. Although the degree of automation and performance of the robot was constantly improving, how to make the robot find a safe and efficient walking path to complete a specific task in its working range was an important problem for us. There were many ways to deal with the problem of robot obstacle avoidance, but from the perspective of practical application, a simple model was better than a complex model [3]. This paper mainly studied the shortest path problem of robot bypassing obstacles to reach the designated destination in a scene, and gave a more convenient method to solve it.

2. Calculation of the shortest path length to avoid obstacles
When a robot passed through an obstacle, its path consists of a straight line and an arc. The arc was the turning path of the robot. The turning path was composed of a section of arc tangent to the straight line path. The turning path could also be composed of two or more tangent arc paths, but the radius of each arc was at least 10 units, the robot could not to turn a corner in broken line. In order not to collide with obstacles, at the same time, the nearest distance between the robot's walking line and obstacles was required to be 10 units. Otherwise, there would be a collision. If there was a collision, the robot would not be able to continue walking.

The maximum speed of the robot when walking in a straight line was $v_0 = 5$ units / second. When the robot turned, the maximum turning speed was $v(\rho) = \frac{V_0}{1 + e^{10-0.1\rho}}$, where was the turning radius. If the speed was exceeded, the robot would roll over and could not continue to walk.

No matter how many obstacles there were between two points, the shortest path should be composed of several line circle structures. In the case of obstacles, the dangerous boundary of the obstacles at the turning point was an arc with a distance of 10 units from the obstacle boundary.
2.1. Calculation model of tangent point coordinates

If there was only one obstacle between two points, the obstacle avoidance path from one point to another was required to pass through two straight-line path and one circular arc path. The path was divided into two situations.

Tangent point $C$, $D$ and center $O$ are on the same side of line $AB$.

Suppose point $A(x_1, y_1)$ was the initial point, point $B(x_2, y_2)$ was the target point, point $C(x_3, y_3)$ and point $D(x_4, y_4)$ were the tangent points of the robot passing through the inflection point and the corner arc of the collision area respectively, and the center $O(x_5, y_5)$ was, as shown in (a) in Figure 1.

Here, $\angle AOB = \alpha, \angle AOC = \beta, \angle BOD = \gamma, \angle COD = \theta$, the length of $AB$ was $a$, the length of $AO$ was $b$, the length of $BO$ was $c$, the arc length of $CD$ was $l$.

To calculate the path length $S_{AB}$ from point $A$ to point $B$, it was necessary to calculate the straight-line path length of $AC$ segment, the straight-line path length of $DB$ segment and the arc path length of $CD$ segment. Use the distance formula between two points to find:

$$a = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}, \quad b = \sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2}, \quad c = \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2}$$

In $\triangle AOC$, $\beta = \arccos \frac{\rho}{b}$, $\gamma = \arccos \frac{\rho}{c}$, in $\triangle AOB$, according to cosine theorem:

$$\alpha = \arccos \left( \frac{b^2 + c^2 - a^2}{2bc} \right)$$

$$S_{AB} = \sqrt{b^2 - \rho^2 + \sqrt{c^2 - \rho^2 + \rho \theta}} \quad \theta = 2\pi - \alpha - \beta - \gamma$$

or

$$S_{AB} = \sqrt{b^2 - \rho^2 + \sqrt{c^2 - \rho^2 + \rho \theta}} \quad \theta = \alpha - \beta - \gamma$$

Tangent point $C$, $D$ and center $O$ are on the opposite side of the line $AB$.

When there were multiple obstacles between the starting point and the target point, there would be multiple turns. As shown in Figure 2, we could not directly use the line circle structure for analysis and solution, but need to make a simple transformation.

Suppose that the coordinates of the centers of the two circles were $O(x_1, y_1)$ and $O'(x_2, y_2)$, the radius was $\rho$, and the coordinates of the points $M$ was $(x_3, y_3)$, so

$$x_3 = \frac{x_1 + x_2}{2}, \quad y_3 = \frac{y_1 + y_2}{2}$$
Combined with the above method, to calculate the distance of $ACDMEFB$, we can calculate it in two sections. First, calculate the distance of $ACDM$, then calculate the distance of $MEFB$. To calculate the length of line segment and arc, the final calculation is the coordinates of arc tangent point, which is divided into three situations:

When the center coordinate was any point, as shown in Figure 3. In the new coordinate system $y'ox$, the equation of a circle was $x'^2 + y'^2 = \rho^2$, the coordinate of the point $M$ was $(x_0', y_0') = (x_0 - a, y_0 - b)$, the coordinate of tangent point $A$

$$\{x', y'\} = \frac{\rho^2}{x_0'^2 + y_0'^2} \{x_0', y_0'\} \pm \frac{\rho \sqrt{x_0'^2 + y_0'^2 - \rho^2}}{x_0'^2 + y_0'^2} \{y_0', -x_0'\}$$

(4)

$$\{x - a, y - b\} = \frac{\rho^2}{(x_0 - a)^2 + (y_0 - b)^2} \{x_0 - a, y_0 - b\}$$

$$\pm \frac{\rho \sqrt{(x_0 - a)^2 + (y_0 - b)^2 - \rho^2}}{(x_0 - a)^2 + (y_0 - b)^2} \{y_0 - b, -x_0 + a\}$$

(5)

So,

$$\{x, y\} = \{a, b\} + \frac{\rho^2}{(x_0 - a)^2 + (y_0 - b)^2} \{x_0 - a, y_0 - b\}$$

$$\pm \frac{\rho \sqrt{(x_0 - a)^2 + (y_0 - b)^2 - \rho^2}}{(x_0 - a)^2 + (y_0 - b)^2} \{y_0 - b, -x_0 + a\}$$

(6)

The plus and minus signs in formula (2) represent the coordinates of two tangent points $A$ and $B$ respectively.

The following is the formula of coordinates of tangent points of two common tangents which are separated from each other, as shown in Figure 4.

Suppose the equation of circle $O_1$ was $(x - x_1)^2 + (y - y_1)^2 = \rho^2$, the equation of circle $O_2$ was $(x - x_2)^2 + (y - y_2)^2 = \rho^2$, so $O_1O_2 = \{x_2 - x_1, y_2 - y_1\}$.

The unit vector perpendicular to $O_1O_2$ was $\pm \frac{\{y_1 - y_2, x_2 - x_1\}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$

so $O_1A_2$, $O_1B_2$ were $\pm \rho \frac{\{y_1 - y_2, x_2 - x_1\}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$

That is

$$\{x - x_1, y - y_1\} = \pm \rho \frac{\{y_1 - y_2, x_2 - x_1\}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

(7)

The coordinates of $A_1$ and $B_1$ was $\{x, y\}$

$$\{x, y\} = \{x_1, y_1\} \pm \rho \frac{\{y_1 - y_2, x_2 - x_1\}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

(8)

In the same way: the coordinates of $A_2$ and $B_2$

$$\{x, y\} = \{x_2, y_2\} \pm \rho \frac{\{y_1 - y_2, x_2 - x_1\}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

(9)
The following gives the coordinates and other formulas of the tangent points of the common
tangents in the two separated equal circles, as shown in Figure 5.

Intersection of two tangents point $M$ was also the midpoint of $O_1, O_2$, the coordinate of $M$ was

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

the coordinates of tangents point $A_1, B_1$.

$$\{x, y\} = \{x_1, y_1\} + \frac{2\rho^2}{(x_2 - x_1)^2 + (y_2 - y_1)^2} \{x_2 - x_1, y_2 - y_1\} \pm \frac{4\rho \sqrt{(x_1 + x_2 - x_1)^2 + (y_1 + y_2 - y_1)^2 - \rho^2}}{(x_2 - x_1)^2 + (y_2 - y_1)^2} \left\{\frac{y_1 + y_2}{2} - y_1, -\frac{x_1 + x_2}{2} + x_1\right\}$$

That is,

$$\{x, y\} = \{x_1, y_1\} + \frac{2\rho^2}{(x_2 - x_1)^2 + (y_2 - y_1)^2} \{x_2 - x_1, y_2 - y_1\} \pm \frac{\rho \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 - 4\rho^2}}{(x_2 - x_1)^2 + (y_2 - y_1)^2} \{y_2 - y_1, -x_2 + y_1\}$$
The same can be obtained the coordinates of $A_2, B_2$

$$
\begin{align*}
\left\{ \begin{array}{l}
x = x_1 + \frac{2\rho^2}{(x_2 - x_1)^2 + (y_2 - y_1)^2} (x_2 - x_1) - \rho \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} (y_2 - y_1) \\
y = y_1 + \frac{2\rho^2}{(x_2 - x_1)^2 + (y_2 - y_1)^2} (y_2 - y_1) - \rho \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} (x_2 - x_1)
\end{array} \right.
\end{align*}
$$

The same can be obtained the coordinates of $A_2, B_2$

$$
\begin{align*}
\left\{ \begin{array}{l}
x = x_2 + \frac{\rho^2}{(x_2 - x_1)^2 + (y_2 - y_1)^2} (x_1 - x_2) + \rho \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} (y_1 - y_2) \\
y = y_2 + \frac{\rho^2}{(x_2 - x_1)^2 + (y_2 - y_1)^2} (y_1 - y_2) + \rho \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} (x_1 - x_2)
\end{array} \right.
\end{align*}
$$

2.2. Calculation model of arc length

By observing the arcs in all paths of the robot, it is found that all arcs can be classified into the following three types: Figure 1, figure 4 and figure 5:

Type 1, there is only one obstacle between the starting point and the target point, as shown in Figure 1

$$\theta = 2\pi - \alpha - \beta - \gamma , \ l = \rho \theta$$

Type 2: there are two or more obstacles between the starting point and the target point, and there is a common tangent between the two obstacles, as shown in Figure 4. For this kind of line circle structure, it is necessary to make a simple transformation, use the midpoint formula to determine the midpoint coordinate of the tangent, convert type 2 to type 1, and calculate the length of the arc.

Type 3: there are two obstacles between the starting point and the target point. The two obstacles are connected by a common tangent line. Given the coordinates of the starting point, the target point and the obstacle point, the arc length can be calculated by the same method as type 1.

3. Judgment and selection of the shortest path

Judgment rule of the shortest path: when there is only one obstacle at the starting point and the end point, the smaller the radius of the arc around the obstacle, the shorter the path the robot will pass; the closer the position of the arc around to the line between the starting point and the end point, the shorter the path the robot will pass (the conclusion of the situation that the arc is on both sides of the line and the situation that it is on one side remains unchanged$^{[4-5]}$).

3.1. One obstacle between the starting point and the ending point

When there was only one obstacle at the starting point and the end point, as shown in Figure 6, the robot will move from point O to point a and pass through an obstacle.

There are two effective paths from point O to point A, they were $O \rightarrow V_1 \rightarrow A; O \rightarrow V_2 \rightarrow A$. The radius of turning arc of both paths at turning $V_1$ and $V_2$ was the minimum value of 10 units. It could be seen from the figure that $V_2$ was closer to the line between point O and point A than $V_1$. According to the judgment rule, the shortest path from point O to point A was $O \rightarrow V_2 \rightarrow A$.

3.2. Multiple obstacles between the starting point and the end point

If there are multiple obstacles between the starting point and the end point, the principle of proximity is adopted: the closer the position around the arc is to the line between the starting point and the end point, the shorter the path the robot passes. as shown in Figure 7.
To move from point $O$ to point $B$, 1, 2, 3 three obstacles should be bypassed. The two vertices of obstacle 1 are $V_1$ and $V_2$ respectively, The two vertices of obstacle 2 are $V_3$ and $V_4$ respectively, The two vertices of obstacle 3 are $V_5$ and $V_6$ respectively, There were multiple paths to target point B, We divide the path into four sections. When bypassing obstacle 1 from point $O$, $V_2$ was close to $OB$, so the path is $O \rightarrow V_2$; when bypassing obstacle 2, $V_2B$ was connected, and $V_4$ was close to $V_2B$, so the path is $V_2 \rightarrow V_4$; when bypassing obstacle 3, $V_4B$ was connected, and $V_5$ was close to $V_4B$, so the path is $V_4 \rightarrow V_5$; finally, $V_5B$ was connected, so the shortest path was $O \rightarrow V_2 \rightarrow V_4 \rightarrow V_5 \rightarrow B$.

This method is simple, but only applicable to the situation that the line between the starting point and the target point passes through the interior of the obstacle. For the situation that some obstacles do not pass through, another path needs to be planned and then the two paths are compared to get the shortest path.

4. summary
In this paper, through the analysis of the robot's obstacle avoidance walking route, the robot's obstacle avoidance walking route is designed to be composed of several straight lines and arcs. The coordinates of each tangent point and the length of each arc are calculated by mathematical modeling, and the geometric method is used for calculation. For the selection of the shortest path, this paper gives the method of the principle of proximity to judge, but it only applies to the situation that the line between the starting point and the target point passes through the interior of the obstacle, but for the situation that the obstacle does not pass through, other paths need to be planned for comparison.

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