Spectroscopic Evidence of the Aharonov-Casher effect in a Cooper Pair Box

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We have observed the effect of the Aharonov-Casher (AC) interference on the spectrum of a superconducting system containing a symmetric Cooper pair box (CPB) and a large inductance. By varying the charge \( n_g \) induced on the CPB island, we observed oscillations of the device spectrum with the period \( \Delta n_g = 2e \). These oscillations are attributed to the charge-controlled AC interference between the fluxon tunneling processes in the CPB Josephson junctions. Total suppression of the tunneling (complete destructive interference) has been observed for the charge \( n_g = e(2n + 1) \). The CPB in this regime represents the \( 4\pi \)-periodic Josephson element, which can be used for the development of the parity-protected superconducting qubits.

The Aharonov-Casher (AC) effect is a non-local topological effect: the wave function of a neutral particle with magnetic moment moving in two dimensions around a charge acquires a phase shift proportional to the charge \( n_e \). This effect has been observed in the experiments with neutrons, atoms, and solid-state semiconductor systems (see, e.g., \[2–4\] and references therein). Similar effects exist in superconducting networks of nanoscale superconducting islands coupled by Josephson junctions. For example, the wave function of the flux vortices (fluxons) moving in such a network should acquire the phase that depends on the charge on superconducting islands \[5\]. Oscillations of the network resistance in the flux-flow regime have been observed as a function of the gate-induced charges \[6\]; these oscillations have been attributed to the interference associated with the AC phase. However, this attribution is not unambiguous, because qualitatively similar phenomena can be produced by the Coulomb-blockade effect due to the quantization of charge on the superconducting islands of the devices \[7\]. Suppression of the superconductivity in one-dimensional (1D) Josephson chains by quantum fluctuations is also affected by the AC phase \[8–10\]. Indeed, the quantum phase slips (QPS) in the junctions can be viewed as the charge-sensitive fluxon tunneling provided the conditions discussed below are satisfied. More recently, the microwave experiments \[11\] demonstrated the dephasing of a fluxonium, a small Josephson junction shunted by a 1D Josephson chain, due to the effect of fluctuating charges on the QPS in the chain. Applications of the AC effect in classical Josephson devices have been discussed in Refs. \[11,12\].

In this Letter we describe the microwave experiments which provide direct evidence for the charge-dependent interference between the amplitudes of fluxon tunneling processes. We have studied the microwave resonances of the device consisting of two identical Josephson junctions separated by a nanoscale superconducting island (the so-called Cooper-pair box, CPB) and a large inductance. A similar device with even greater kinetic inductance provides a physical implementation of the fault-tolerant qubit (see below and Ref. \[13\]). The spectrum of the device is determined by the QPS rate in the CPB junctions, which depends on the charge of the superconducting island. The abrupt change of the phase difference across each junction by \( 2\pi \) can be considered as adding/subtracting a single fluxon to the superconducting loop formed by the CPB and the superinductor. We have observed almost complete suppression of the fluxon tunneling due to the destructive AC interference for the charge on the central CPB island \( q = e(2n + 1) \). Our results obtained for this well-controlled system allow for direct quantitative comparison with the theory.

The studied device (Fig. 1) consists of a superconducting loop that includes a Cooper pair box and a superconducting inductor with a large Josephson inductance \( L_{SL} \), the so-called superinductor \[11\]. The magnetic flux \( \Phi \) in this “phase” loop controls the phase difference across the CPB. The design of our superinductor has been described in Ref. \[14\]: the superinductor used in this experiment...
consisted of 36 coupled cells, each cell represented a small superconducting loop interrupted by three larger and one smaller Josephson junctions (Fig. 1b). The inductance \( L_{SL} \) reaches its maximum when the unit cell is threaded by the magnetic flux \( \Phi_{SL} = \Phi_0/2 \). In this regime of full frustration, \( L_{SL} \) exceeds the Josephson inductance of the CPB junctions by two orders of magnitude.

It is worth emphasizing that a large magnitude of \( L_{SL} \) and, thus, a small value of the superinductor energy \( E_{SL} = \left( \frac{2 \pi}{20} \right)^2 \frac{1}{L_{SL}} \) is essential for the observation of the AC effect in our experiment. Indeed, the classification of the device states by the discrete values of the phase \( \varphi = 2\pi m \) can be justified only if \( E_{SL} \ll E_J \) (for more details see Supplementary Materials [15]). In this respect, the studied device resembles the fluxonium [16], in which a single junction is shunted by a superinductor. Large inductance \( L_{SL} \) is an important distinction of our device from the structure proposed in Ref. [7] for the observation of suppression of macroscopic quantum tunneling due to the AC effect. In the small-\( L_{SL} \) case considered in Ref. [7], the phase weakly fluctuates around the value \( 2\pi m/\Phi_0 \) and the phase slips should be completely suppressed (cf. Ref. [17]). Large \( L_{SL} \) values are also important for the spectroscopic measurements: the superinductor reduces the device resonance frequency down to the convenient-for-measurements 1-10 GHz range.

For the dispersive measurements of the device resonances, a narrow portion of the phase loop with the kinetic inductance \( L_{sh} \) was coupled to the read-out lumped-element \( LC \) resonator (for details of the readout design, see [18, 19]). The global magnetic field, which determines the fluxes in both the phase loop, \( \Phi \), and the unit cells of the superinductor, \( \Phi_{SL} \), has been generated by a superconducting solenoid. The offset charge on the CPB island was varied by the gate voltage \( V_g \) applied to the microstrip transmission line (Fig. 1b).

Table I. Parameters of Josephson junctions in the representative device. The values of \( E_J \) and \( E_C \) correspond to the fitting parameters; the \( E_J \) values are close (within 15%) to the respective energies estimated on the basis of the Ambegaokar-Baratoff relationship using the resistance of the test junctions fabricated on the same chip.

| Juncions     | In-plane areas, \( \text{nm}^2 \) | \( E_J, \text{GHz} \) | \( E_C, \text{GHz} \) |
|--------------|-----------------|-----------------|-----------------|
| CPB          | 114 × 114       | 14              | 22              |
| Superinductor large | 300 × 300 | 94              | 3.3             |
| Superinductor small     | 160 × 160       | 25              | 11              |

The device, the readout circuits, and the microwave (MW) transmission line (Fig. 1b) were fabricated using multi-angle electron-beam deposition of Aluminum through a lift-off mask (for fabrication details, see Refs. [18, 19]). Six devices have been fabricated on the same chip; they could be addressed individually due to different resonance frequencies of the read-out resonators.

Figure 2. Panel (a): The transmitted microwave power \( |S_{21}|^2 \) at the first-tone frequency \( f_1 \) as a function of the second-tone frequency \( f_2 \) and the gate voltage \( V_g \) measured at a fixed value of \( \Phi_{SL} = 0.5\Phi_0 \). The power maxima correspond to the resonance excitations of the device \( \langle f_2 = f_{01} \rangle \), the superinductor \( \langle f_{SL} \rangle \), and the read-out resonator \( \langle f_R \rangle \). Note that the resonance measurements could not be extended below \( \sim 1 \text{GHz} \) because of a high-pass filter in the second-tone feedline. Panel (b): The frequency dependence of the transmitted microwave power measured at \( V_g = 0 \text{V} \) and \( \Phi_{SL} = 0.5\Phi_0 \).

The parameters of the CPB junctions were nominally the same for all six devices, whereas the maximum inductance of the superinductor was systematically varied across six devices by changing the in-plane dimensions of the small junctions in the superinductors [14]. Below we discuss the data for one representative device: Table I summarizes the parameters of junctions in the CPB junctions and superinductor (throughout the Letter all energies are given in the frequency units, \( 1K \approx 20.8 \text{GHz} \)).

In the two-tone measurements, the microwaves at the second-tone frequency \( f_2 \) excited the transitions between the \( |0\rangle \) and \( |1\rangle \) quantum states of the device, which resulted in a change of its impedance. This change was registered as a shift of the resonance of a coupled \( LC \) resonator probed with microwaves at the frequency \( f_1 \). The microwave set-up used for these measurements has been described in Refs. [13, 18, 19]. The resonance frequency \( f_{01} \) of the transition between the \( |0\rangle \) and \( |1\rangle \) states was measured as a function of the charge \( n_g \) and the phase difference across the CPB. The \( f_{01} \) measurements could not be extended below \( \sim 1 \text{GHz} \) because of a high-pass filter in the second-tone feedline. The results discussed below have been obtained in the magnetic fields that correspond to \( \Phi_{SL} \approx \Phi_0/2 \) where \( L_{SL} \) reaches its maximum [14]. Because the phase loop area \( (\sim 1,850\mu\text{m}^2) \) was much greater than the superinductor unit cell area \( (15\mu\text{m}^2) \), the phase across the chain could be varied at an approximately constant value of \( L_{SL} \). All measurements have been performed at \( T = 20 \text{mK} \).

The resonances corresponding to the \( |0\rangle \rightarrow |1\rangle \) transition are shown in Fig. 2b as a function of the gate voltage \( V_g \) at a fixed value of the magnetic field that is close to the full frustration of the superinductor unit.
cells ($\Phi_{SL} \approx 0.5\Phi_0$). The dependence $f_{01}$ ($V_g$) is periodic in the charge on the CPB island, $n_g$, with the period $\Delta n_g = 1$ (here and below the charge is measured in units $2e \mod 2e$). Note that no disruption of periodicity by the quasiparticle poisoning [20] was observed in the data in Fig. 2a that were measured over 80 min. Significant suppression of quasiparticle poisoning was achieved due to the gap engineering and shielding of the device from infrared photons [20]. Figure 2 also shows the resonance of the read-out resonator at $f_r = 6.45\text{GHz}$ and the self-resonance of the superinductor $f_{SL} \approx 5.5\text{GHz}$. All three resonances are shown in Fig. 2b for $n_g \approx 0.47(V_g = 0)$ and $\Phi_{SL} \approx 0.5\Phi_0$. Weaker resonances observed at $f_2 \approx 3, 4.8\text{GHz}$ at $V_g = -1.5\text{V}$ correspond to the multi-photon excitations of the higher modes of the superinductor.

The expected flux dependence of the energy levels of the device is shown in Fig. 3a. This flux dependence can be understood by noting that in the absence of fluxon tunneling ($n_g = 0.5$ and identical CPB junctions) different states are characterized by a different number $m$ of fluxons in the phase loop. At $E_J \gg E_L$ the energies of these states are represented by crossing parabolas $E_{SL}(m, \Phi) = \frac{1}{2}E_{SL}(m - \frac{\Phi}{\Phi_0})^2$. The phase slip processes mix the states with different number of fluxons and lead to the level repulsion.

Figure 3b shows the main result of this Letter: the dependences of the resonance frequency $f_{01}$ on the flux across the CPB for the charges $n_g = 0$ and 0.5. In line with the level modeling, at $n_g = 0$ the frequency $f_{01}$ periodically varies as a function of phase, but never approaches zero. On the other hand, when $n_g = 0.5$, the amplitudes of fluxon tunneling across the CPB junctions acquire the Aharonov-Casher phase difference $\pi$. Provided that the CPB junctions are identical, the destructive interference completely suppresses fluxon tunneling, coupling between the states $|m\rangle$ and $|m \pm 1\rangle$ vanishes and the avoided crossing disappears. Since the difference $E_{SL}(m, \Phi) - E_{SL}(m \pm 1, \Phi)$ is linear in $\Phi$, the spectrum at $n_g = 0.5$ should acquire the sawtooth shape. This is precisely what has been observed in our experiment. Fitting allowed us to extract all relevant energies (see the caption to Fig. 3).

The maxima and minima of the sawtooth shape could be rounded by very different processes. The minima rounding is due to incomplete cancellation of the single flux tunneling that can be due to slightly different $E_J$ values of two junctions in the CPB. We do not observe any sign of this rounding in the frequency range $f_2 > 1\text{GHz}$ which indicates a high degree of the CPB symmetry (less than 5% difference between $E_J$ values). The maxima rounding is more interesting. In the ideal device it is due to the level repulsion of the states that differ by two flux quanta, i.e. double fluxon tunneling. As explained in Ref. [13], the devices in which the rate of this tunneling is sufficiently high might exhibit protected states. However, this rounding can be also due to the charge noise, and it is likely that this effect dominated in the studied device.

The observed effect can be used for the design of the parity-protected Josephson-junction qubits for quantum computing [13]. In those qubits the single fluxon tunneling rate should be completely suppressed by the AC destructive interference. However, if the ratio $E_J/E_C$ is sufficiently small, the rate of the simultaneous tunneling
of pairs of fluxons, and, thus, the corresponding energy of double phase slips could be sufficiently high - indeed, for this process the AC phase is 2π and the interference is constructive. In this regime, the CPB represents a \( \cos(\phi/2) \) Josephson element which energy is 4π-periodic (see Supplementary Materials [15]). Such a “flux-pairing” qubit would be dual to a recently realized charge-pairing qubit [19]. For the proper operation of the flux-pairing qubit, not only the ratio \( E_J/E_C \) should be smaller than that in the studied device, but also the inductance of the superinductor should be further increased by approximately an order of magnitude. To satisfy the latter challenging requirement without reducing the superinductor resonance frequency, the parasitic capacitance of the superinductor should be significantly reduced.

To conclude, we have observed the effect of the Aharonov-Casher interference on the spectrum of the Cooper pair box (CPB) shunted by a large inductance. Large values of \( L_{SL}/(E_L \ll E_J) \) are essential for the observation of the AC effect with the Cooper pair box; in this respect our devices differ from the earlier proposed structures [7]. We have demonstrated that the amplitudes of the fluxon tunneling through each of the CPB junctions acquire the relative phase that depends on the CPB island charge \( n_g \). In particular, the phase is equal to 0 (mod 2π) at \( n_g = 2n_e \) and \( \pi \) (mod 2π) at \( n_g = e (2n + 1) \). The interference between these tunneling processes results in periodic variations of the energy difference between the ground and first excited states of the studied quantum circuit; the period of the oscillations corresponds to \( \Delta q = 2e \). The phase slip approximation provides quantitative description of the data and the observed interference pattern evidences the quantum coherent dynamics of our large circuit.

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SUPPLEMENTARY MATERIALS

A. Energy spectra of the Cooper pair box shunted by a large inductance.

The device studied in this paper consists of two very different elements: a Cooper pair box (CPB) and a superinductor. The Cooper pair box is described by the quantized value of the charge, or by a phase in the interval \( (0, 2\pi) \). At significant CPB charging energy it is convenient to use the former basis. In contrast, the superinductor is characterized by continuous conjugated variables, \( \phi \) (phase across) and \( q \) (charge). The Cooper pair tunneling to the CPB island changes its charge by \( \pm 1 \); in the symmetric gauge the phase at the inductor ends is \( \pm \phi/2 \), so the processes of tunneling from different superinductor ends acquire phase factors \( \exp(\pm \phi/2) \). Thus, the full Hamiltonian describing the CPB coupled to the superinductor is

\[
H = -E_J (a^+ + a^-) \cos(\phi/2) + 4E_C(n - n_g)^2 + 4E_{CL}q^2 + \frac{1}{2}E_L(\phi - 2\pi\Phi/\Phi_0)^2
\]

where \( a^\pm \) are operators that raise (lower) the charge of the CPB island, \( n_g \) is the charge induced by the gate, and \( E_{CL} \) is the effective charging energy of the superinductor.
that is due to the capacitors of its junctions and ground capacitance of the whole structure.

The analysis is further simplified for large charging energies $E_C \gg E_J$ which is marginally satisfied in the studied device ($E_J = 0.66$ K, $E_C = 1.05$ K). In this case the Cooper pair tunneling is significant only in the vicinity of the full charge frustration, $n_g = N + 0.5$ where the only relevant charging states are $n = N, N + 1$. In the reduced space Hamiltonian becomes

$$H_R = -E_J \cos(\phi/2) \sigma^x + 4E_C(n_g - 0.5) \sigma^z + 4E_{CL} q^2
+ \frac{1}{2} E_L(\phi - 2 \pi \Phi/\Phi_0)^2$$

(2)

Away from the charge frustration ($n_g = 0.5$) the charge fluctuations are small, so that $\sigma^z \approx -1$ in all low energy states. Duality between phase and charge implies that in this situation the phase fluctuations are large. In this approximation the low energy states coincide with those of the oscillator with $\omega_0 = \sqrt{4E_{CL}E_L}$. Charge fluctuations lead to a weak flux dependence of the energies of these states. In the leading order to the perturbation theory the oscillator potential becomes

$$V = -\frac{E_J^2}{4E_C} \cos \phi + \frac{1}{2} E_L(\phi - 2 \pi \Phi/\Phi_0)^2$$

that leads to the weak dependence of the oscillator level on the flux $E_{01} = \omega_0 + \delta E \cos 2 \pi \Phi/\Phi_0$. This dependence is exactly the one observed experimentally (red data points in Fig. 3b).

In the opposite limit, close to the full frustration, at $|n_g - 0.5| \ll E_J/E_C$, the phase slip amplitude vanishes due to the destructive Aharonov-Casher interference. Formally in this limit one can treat the second term in (2) as perturbation. In the zeroth order of the perturbation theory one obtains two independent sectors characterized by $\sigma^z = \pm 1$ that we shall refer to as even/odd sectors below. Exactly at $n_g = 0.5$ these sectors are completely independent. At $E_J \geq E_{CL} \gg E_L$ in each sector the low energy wavefunctions are localized in the vicinity of points $\phi = 2 \pi m$, $m$ being the number of fluxons in the phase loop. The energy spectrum in this case is given by the set of parabolas shown in Fig. 3a. In this approximation the energy levels corresponding to different parabolas cross. The level repulsion between the levels represented by adjacent parabolas is due to the phase slips that vanish at $n_g = 0.5$.

The level repulsion between the levels represented by next nearest parabolas is due to the double phase slips. Formally it is described by the quantum tunneling in the effective potential

$$V(\phi) = \pm E_J \cos(\phi/2) + \frac{1}{2} E_L(\phi - 2 \pi \Phi/\Phi_0)^2$$

(3)

It occurs with amplitude

$$t = A(g) g^{1/2} \exp(-g) \omega_p$$

(4)

where $g = 4\sqrt{2E_J/E_{CL}}$, $\omega_p = \sqrt{2E_JE_{CL}}$ and $A(g) \approx 0.8$. This tunneling process changes $m \rightarrow m \pm 2$, but does not mix even and odd sectors. In the limit of large $E_J \gg E_{CL}$ the amplitude becomes exponentially small which was the case for the studied device ($g \approx 8$). In this case the transitions due to double phase slips are completely suppressed. The energies of the states with different $m$ are quadratic as a function of the flux, exactly at half flux quantum these energies cross. So the energy difference between the ground and first excited states is linear in $\Phi$ with zero intercept. This is exactly the behavior observed in the data (Fig. 3b).

Note that the classification of states by the discrete values of the phase is only possible if $E_L \ll E_J$. In the opposite limit $E_L \geq E_J$ the phase experiences small oscillations around $2 \pi \Phi/\Phi_0$ and the phase slips are completely suppressed. In the intermediate regime the phase is localized around the minima of the potential energy $V(\phi)$ that differ from each other by a non-integer fraction of $2 \pi$.

The linearity of $E_{01}(\Phi)$ at $n_C = 0.5$ is disturbed by two factors. The first one is the difference in $E_J$ of two junctions comprising the CPB. Due to this difference the flux tunneling is not completely suppressed even at $n_g = 0.5$. This leads to the level repulsion at half integer flux at which neighboring parabolas intersect. This would lead to some rounding of $E_{01}(\Phi)$ around half integer flux. Within the experimental accuracy, the data do not show such effect which implies that the difference between two $E_J$ is small. The second factor is more interesting, it is due to a significant tunneling rate of two fluxons through the CPB. This leads to the level repulsion between the levels corresponding to next neighboring parabolas in Fig. 3a and rounding of the maxima of the spectra data at $n_g = 0.5$ (blue points in Fig. 3b). Some hint of this behavior can be seen in the data, we estimate that $t \approx 0.01 - 0.1$ GHz.

The analytical results obtained above become quantitatively correct in the regime $E_C \gg E_J$ but they remain qualitatively correct even for $E_C \geq E_J$. For more precise quantitative description of the experiment in this regime we used numerical diagonalization of the full Hamiltonian [1]. This allowed us to obtain the spectra for all values of the induced charges, $n_g$, and unambiguous data fit. We found that for the experimental device parameters it is sufficient to restrict oneself to the three lowest energy charging states and range of $\phi = (-12 \pi, 12 \pi)$. In this approximation the Hamiltonian becomes $3M \times 3M$ matrix where $M$ is the number of discrete values that were used to approximate the continuous variable $\phi$. Very accurate results can be achieved by using $0.2\pi$ increments (i.e. 20 steps for $4\pi$ period).
B. Quantum state protection expected for large inductance.

We now briefly discuss the behavior expected for significant double flux tunneling and very large inductance, at which, as we now show, one expects protection against both the charge and flux noise. Generally, tunneling between the states with different $m$ implies that the full wave function is the superposition of the states with different $m$ that can be found from the diagonalization of the discrete oscillator Hamiltonian

$$H_o = -t \left( |m\rangle \langle m+2| + |m+2\rangle \langle m| \right) + 2\pi^2 E_L (m - \Phi/\Phi_0)^2$$

(5)

where $m$ corresponds to either even or odd numbers. At large $t \gg E_L$ the low energy states in each sector are spread over many different $m$ and are almost indistinguishable. This implies the protection against the external noises. More quantitatively, the dependence on $m_0$ is given by

$$E (m_0) = 2A(G)G^{1/2} \exp(-G) \Omega \cos(2\pi m_0)$$

(6)

$$G = \frac{\pi}{2} \sqrt{\frac{2t}{E_L}}$$

(7)

$$\Omega = 4\pi \sqrt{2t E_L}$$

(8)

This dependence becomes exponentially small at $G \gg 1$ that indicates weak sensitivity to external flux.

The charge noise can be described as $n_g(t)$ variations. Non-zero value of $n_g - 0.5$ results in a small amplitude that mixes even and odd sectors. This amplitude is given by

$$t_{eo} \approx \frac{\pi}{3} \frac{1}{4} E_C (n_g - 0.5) \sqrt{\frac{t}{\omega_p}}$$

(9)

Exactly at $\Phi = \Phi_0/2$ the energies of the odd and even sectors become equal in the absence of $t_{eo}$. Non-zero $t_{eo}$ leads to level splitting but the effect is small in $n_g - 0.5$ and $(t/\omega_p)$.

The equations (6-9) allow one to derive the conditions for optimal charge and flux protection. Generally, the coupling to the charge noise is largest at $\Phi = \Phi_0/2$ because $E_{01} = 2t_{eo} \sim (n_g - 0.5)$ but this coupling becomes small at $t \ll \omega_p$. The flux noise affects the energy levels via $m_0(\Phi)$ dependence. This effect becomes small at $t \gg E_L$. Thus, the optimal protection against the noises is achieved for $\omega_p \gg t \gg E_L$. Notice that while the first inequality is easy to achieve, the second requires a large superinductance. For instance in this experiment $t/E_L \approx 0.05 - 0.5$. For significant protection against flux noise $t/E_L$ should be greater than 10, while for the good charge noise protection one needs $\omega_p/t \geq 10^2$ that results in the condition $\omega_p/E_L \gtrsim 10^3$. 