Some comments on the handbag approach to wide-angle exclusive scattering

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The handbag mechanism for wide-angle exclusive scattering reactions is discussed and compared to other theoretical approaches. The role of power laws in observables is critically examined. Applications of the handbag mechanism to Compton scattering and meson photoproduction are presented. The soft physics input to these processes are specific form factors which represent \(1/x\) moments of generalized parton distributions at zero skewness. A recent analysis of the nucleon form factors provides these GPDs and, hence, the new form factors.

1 Introduction

Recently a new approach to wide-angle Compton scattering off protons has been proposed [1, 2] where, for Mandelstam variables \(s, -t, -u\) that are large as compared to the square of a typical hadronic scale \(\Lambda\) (being of the order of \(1\ \text{GeV}^2\)), the process amplitudes factorize into a hard parton-level subprocess, Compton scattering off quarks, and in soft form factors which represent \(1/x\) moments of generalized parton distributions (GPDs) and encode the soft physics (see Fig. 1). Subsequently it has been realized that this so-called handbag mechanism also applies to a number of other wide-angle reactions such as meson photo- and electroproduction [3] or two-photon annihilations into pairs of mesons [4] or baryons [4, 5]. It should be noted that the handbag mechanism bears resemblance to the treatment of inelastic Compton scattering advocated for by Bjorken and Paschos [6] long time ago.

There are competing mechanisms which also contribute to wide-angle scattering besides the handbag which is characterized by one active parton, i.e. one parton from each hadron participates in the hard subprocess (e.g. \(\gamma q \rightarrow \gamma q\) in Compton scattering) while all others are spectators. First there are the so-called cat’s ears graphs (see Fig. 1) with two active partons participating in the subprocess (e.g. \(\gamma qq \rightarrow \gamma qq\)). It can be shown that in these graphs either a large parton virtuality or a large parton transverse momentum occurs. This forces the exchange of at least one hard gluon in the subprocess. Hence, the cat’s ears contribution is expected to be suppressed as compared to the handbag one. The soft physics in the case of the cat’s ears are hadronic matrix elements that describe higher order quark correlations inside the proton. Nothing is known about these matrix elements as yet.

The next class of graphs is characterized by three active quarks and, obviously, requires the exchange of at least two hard gluons. One can go on and consider four or more active partons. This way one obtains an expansion of the scattering amplitude which bears resemblance to the series of contributions from \(n\)-body operators appearing in non-relativistic many-body theory. In principle, all the different contributions have to be added coherently. In practice, however, this is a difficult, currently almost impossible task since each

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contribution has its own associated soft hadronic matrix elements which, as yet, cannot be calculated from QCD and are often even phenomenologically unknown.

2 Comments on the leading-twist contribution

At large momentum transfer the restriction to valence quarks is a reliable approximation to exclusive scattering. Exploiting it in the case of three active quarks, the big blob \(^1\) decays into two smaller blobs, see Fig. 1. These blobs describe the proton’s distribution amplitude for finding valence quarks in the hadron, each carrying some fraction \(x_i\) of the hadron’s momentum. This contribution is the so-called leading-twist contribution \(^2\) which is expected to dominate for asymptotically large momentum transfer but seems to be way below experiment for momentum transfer of the order of 10 GeV\(^2\). Actual calculations of Compton scattering \(^8\) and the proton form factor \(^9\) confirm this assertion. There are only very few exceptions where the leading-twist contribution is close to experiment. One of those is the \(\pi - \gamma\) transition form factor. This is a special case since here the handbag with one active quark and the leading-twist approximation - for which all valence quarks participate in the hard subprocess - fall together. Indeed there is general agreement in the literature that the leading-twist/handbag mechanism provides the bulk of the contribution to that form factor. The other exceptions are some exclusive charmonium decays into light hadrons where something like a (time-like version of the) handbag does not occur unless one allows for intrinsic charm in the light hadrons. Thus, for instance, the \(J/\psi\) decay into a baryon-antibaryon pair is dominated by the leading-twist contribution where the \(c\bar{c}\) pair from the \(J/\psi\) decay annihilates into three gluons which subsequently turn into light \(q\bar{q}\) pairs which form the final state hadrons. As has been shown \(^10\) this contribution is large enough to account for the measured decay width. I have however to remind the reader that there is a number of exclusive charmonium decays, characterized by non-conservation of hadronic helicity, which are not under control of the leading-twist mechanism. An example is set by the process \(J/\psi \to \rho \pi\), the famous \(\rho \pi\) puzzle.

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\(^1\) This holds for protons. In the case of, say, Compton scattering off pions the cat’s ears already include the leading-twist contribution.
A feature of the leading-twist contribution is the power behaviour of form factors and cross sections \cite{11}. These observables decrease by powers of the hard scale asymptotically. The powers are determined by the number of partons participating in the hard process \footnote{The asymptotic power laws hold if the hard scale is much larger than any soft scale available in the process (e.g. hadronic masses). In almost all data on exclusive reactions this is however not the case.}. It is to be stressed that the powers of the hard scale are accompanied by perturbative logs for which there is no experimental evidence in exclusive wide-angle scattering as yet in contrast to deep inclusive lepton-nucleon scattering where they are seen in experiment. Their role in the extraction of the parton distribution from the inclusive lepton-nucleon data cannot be overemphasized. Thus, an approximate power behaviour observed in an exclusive observable in a finite and typically rather limited range of the hard scale cannot be considered as evidence for the dominance of the leading-twist mechanism. This a premature: a careful analysis of the normalization and the role of the perturbative logs is mandatory. Furthermore an estimate of the size of contributions from alternative mechanisms like the handbag one, is required before this conclusion can be drawn.

An interesting example is set by the recent measurement of the ratio of the proton’s Pauli and Dirac form factor \cite{12}. The observation that \( \sqrt{-t} F_2 / F_1 \) exhibits a nearly constant behaviour for the largest measured momentum transfers \((2 - 6 \text{ GeV}^2)\), stirred a hot debate on the asymptotic behaviour of \( F_2 \). It is however more elucidating to isolate \( F_2 \) from the data and to look to its behaviour instead to the ratio \( \sqrt{-t} F_2 / F_1 \). In Fig. 2 the form factor data, scaled by \( t^2 \), are displayed. The data rather show a dipole behaviour in that range of \( t \) than the asymptotically expected \( \sim t^{-3} \) fall off. Nothing is wrong with that result. The range \( 2 \text{ GeV}^2 \lesssim -t \lesssim 6 \text{ GeV}^2 \) is simply too small for probing the asymptotic behaviour. Soft physics still dominates, that is all.

Soft mechanisms like the handbag can easily produce a power behaviour of form factors. Consider for instance the following ansatz for the GPD \( H \) at zero skewness and for valence quarks

\[
H_0^q(x,t) = q_v(x) \exp \left[ t f_q(x) \right], \tag{1}
\]

which is advocated for in \cite{13}. Here, \( q_v \) is the usual parton distribution for valence quarks. As shown in \cite{13}, at large \( t \) the dominant contribution to the Dirac form factor, \( F_1 \), comes from a rather narrow region of large \( x \). Thus, one may take a large-\( x \) approximations of \( (1) \): \( q_v \sim (1 - x)^{\beta_q} \) and \( f_q \sim A_q (1 - x)^2 \).
where $A_q$ is a soft parameter of the order of $1 \text{ GeV}^{-2}$. The power $\beta_q$ is provided by the phenomenological parton distributions \cite{[14]}: $\beta_u \simeq 3.4$, $\beta_d \simeq 5$. The function (1) has a maximum at

$$1 - x_s = \sqrt{\frac{\beta_q}{2A_q|t|}}.$$  (2)

Hence, the sum rule

$$h_{q,0}(t) = \int_0^1 dx H_q^0(x,t),$$  (3)

can be evaluated in the saddle point approximation and one finds the power law

$$h_{q,0} \sim |t|^{-(1+\beta_q)/2}.$$  (4)

With $\beta_u \simeq 3.4$ we see that the $u$-quark contribution to the Dirac form factor, $h_{1,0}^q$, falls slightly faster than $t^{-2}$ while the $d$-quark contribution drops as $t^{-3}$. It is to be stressed that this is not an asymptotic result but holds provided the saddle point (2) lies in region in which the bulk of the contribution to the Dirac form factor is accumulated. To this region characterized by $1 - x \sim \Lambda / \sqrt{|t|}$ ($\Lambda$ is a typical hadronic scale of order 1 GeV), the Feynman mechanism applies for which the struck quark carries most of the proton momentum and thus large internal virtualities of order $t$ are avoided. As shown in \cite{[13]} the ansatz (1) works quite well for $t$ values up to $30 \text{ GeV}^2$. For very large values of $t$ one may expect the soft contribution to be damped by Sudakov factors and the leading-twist contribution may take the lead.

### 3 Handbag factorization for wide-angle Compton scattering

Consider Mandelstam variables $s$, $-t$ and $-u$ that are large as compared to $\Lambda^2$. The contribution from the handbag diagram shown in Fig. 1 is calculated in a symmetrical frame which is a c.m.s. rotated in such a way that the momenta of the incoming ($p$) and outgoing ($p'$) proton momenta have the same light-cone plus components. Hence, the skewness defined as

$$\xi = \frac{(p - p')^+}{(p + p')^+},$$  (5)

is zero. The bubble in the handbag is viewed as a sum over all possible parton configurations as in deep inelastic lepton-proton scattering. The crucial assumption in the handbag approach is that of restricted parton virtualities, $k_i^2 < \Lambda^2$, and of intrinsic transverse parton momenta, $k_{\perp i}$, defined with respect to their parent hadron’s momentum, which satisfy $k_{\perp i}^2 / x_i < \Lambda^2$, where $x_i$ is the momentum fraction parton $i$ carries.

One can then show \cite{[2]} that the subprocess Mandelstam variables $\hat{s}$ and $\hat{u}$ are the same as the ones for the full process, Compton scattering off protons, up to corrections of order $\Lambda^2/t$:

$$\hat{s} = (k_j + q)^2 \simeq (p + q)^2 = s,$$

$$\hat{u} = (k_j - q')^2 \simeq (p - q')^2 = u.$$  (6)

The active partons, i.e. the ones to which the photons couple, are approximately on-shell, move collinear with their parent hadrons and carry a momentum fraction close to unity. Thus, like in deep virtual Compton scattering, the physical situation is that of a hard parton-level subprocess, $\gamma q \rightarrow \gamma q$, and a soft emission and reabsorption of quarks from the proton.
The light-cone helicity amplitudes \[15\] for wide-angle Compton scattering then read \[2\] [16]:

\[
M_{\mu'^\prime, \mu'}(s,t) = \frac{e^2}{2} \left[ \delta_{\nu'^\prime, \nu} T_{\mu'^\prime, \mu'}(s,t) \left( R_{\nu}(t) + R_A(t) \right) + \delta_{\nu'^\prime, \nu} T_{\mu'^\prime, -\mu'}(s,t) \left( R_{\nu}(t) - R_A(t) \right) + \delta_{-\nu'^\prime, -\nu'} \frac{1}{2m}(T_{\mu'^\prime, -\mu'}(s,t) + T_{\mu', \mu'}(s,t)) \right] R_T(t). \tag{7}
\]

The labels \(\mu(\nu)\), \(\mu'(\nu')\) denote the helicities of the incoming and outgoing photons (protons in \(M\) and quarks in the subprocess amplitude \(T\)), respectively. The mass of the proton is denoted by \(m\). The Compton form factors \(R_i\) represent \(1/x\)-moments of GPDs at zero skewness. This representation which requires the dominance of the plus components of the proton matrix elements, is a non-trivial feature given that, in contrast to deep inelastic lepton-nucleon and deep virtual Compton scattering, not only the plus components of the proton momenta but also their minus and transverse components are large here. For Compton scattering the hard scattering has been calculated to next-to-leading order (NLO) perturbative QCD \[16\]. The infrared singularities occurring to this order can be absorbed into the Compton form factors. To NLO the dominance of the plus components of the proton matrix elements, is a non-trivial feature given that, in \(A\) phenomenological analysis of the GPDs\[1\] [2], 2\[19\]. The ansatz used in \[13\] is already given in \[1\]. The profile function is assumed to read

\[
f_q(x) = [\alpha' \log(1/x) + B_q](1 - x)^{n+1} + A_q x(1 - x)^n, \tag{10}\]
Fig. 3 Region of $x$ (white region) which accounts for 90% of $F_p^1(t)$ in the best fit to (1) at a scale $\mu = 2$ GeV. The upper and lower shaded $x$-regions each account for 5% of $F_p^1(t)$. The thick line shows the average $\langle x \rangle_t$.

with $n = 1, 2$ and $\alpha' = 0.9$ GeV$^{-2}$ is the usual Regge slope. In an analogous way the GPDs $\tilde{H}$ and $E$ are parameterized with the additional complication that the forward limit of $E$ is not accessible in deep inelastic lepton-nucleon scattering and, hence, is to be determined in that analysis too. The free parameters of this approach are fitted to the available data on the nucleon form factors $F_{1,2}^{p,n}$ and $F_A$ at all $t$. The fit to the form factors through the sum rules

$$F_{1}^{p(n)}(t) = e_u(d) \int_0^1 dx H_u^v(x, t) + e_d(u) \int_0^1 dx H_d^v(x, t),$$

and analogously for the other form factors, only allows an access to the valence quarks at zero skewness. An example of the fit to the data is given in Fig. 2. It turns out in the analysis presented in [13] that the contributions from a rather limited range of $x$ dominate the form factors. The range is shifted towards larger $x$-values with increasing $-t$. This property of the parameterization (1), (10) leads to the power behaviour discussed in Sect. 2.

The quality of the fits is very similar in the two cases $n = 1$ and 2 and the results for GPDs and related quantities agree very well with each other. Substantial differences between the two results only occur for $x$-values outside the range which is sensitive to the form factor data (see Fig. 3). It is the physical interpretation of the results which favours the fit with $n = 2$. Indeed the average distance between the struck quark and the cluster of spectators becomes unphysical large for $x \to \infty$ in the case of $n = 1$.

As an example of the results for the GPDs $H$ is shown in Fig. 4 at two values of $t$. While at small $t$ the GPD still reflects the behaviour of the parton distribution it exhibits a pronounced maximum at larger $t$. This maximum moves towards $x = 1$ with increasing $-t$ and reflects the repeatedly mentioned property that only a limited range of $x$ contributes to the form factors substantially. The results for the GPDs $\tilde{H}$ and $E$ behave similarly. Noteworthy differences are that $\tilde{H}$ and $E$ for $u$ and $d$ quarks have opposite signs and that $E_u^u$ and $E_d^d$ are of about the same magnitude at least for smaller values of $t$.

The lowest moments of the GPD $H$ defined by

$$h_{n,0}^H(t) = \int_0^1 dx \, x^{n-1} \, H^v_1(x, t),$$

are shown in Fig. 5. The different powers with which the $u$ and $d$-quark moments drop correspond to the large-$x$ behaviour of the parton distributions as I discussed in Sect. 2, see (4). Note also the large difference in magnitude between the $u$ and $d$ moments. Strengthened by the charge factor the $u$-quark contribution dominates the proton form factor in the GeV region, the $d$-quark contribution amounts to less than 10%.
Fig. 4  Result for the valence GPDs $H^u_v(x, t)$ at $\mu = 2$ GeV obtained in the analysis presented in [13]. Dashed lines indicate the regions where $x < x_{\text{min}}(t)$ or $x > x_{\text{max}}(t)$.

![Fig. 4](image)

Fig. 5  The first three moments of valence GPDs $H^u_v$ (left) and $H^d_v$ (right), scaled with $t^2$. The error bands denote the parametric uncertainty resulting from the fit to the Dirac form factors $F^p_1$ and $F^n_1$.

![Fig. 5](image)

Its contribution to the neutron form factor is about 30%. High quality neutron form factor data above 3 GeV$^2$ would allow for a direct examination of the different powers.

In contrast to the usual parton distribution which only provide information on the longitudinal distribution of quarks inside the nucleon, GPDs also give access to the transverse structure of the nucleon by Fourier transforming the GPD with respect to $\sqrt{-t}$. This is discussed in some detail in [13]. One can now also evaluate the valence quark contribution to the orbital angular momentum the quarks inside the proton carry by exploiting Ji’s sum rule [20]. A value of -0.17 has been found in [13] for the average valence quark contribution to the orbital angular momentum.
5 Results for Compton factors

With the results for the GPDs at hand one can now evaluate the Compton form factors

\[ R_V(t) \approx \sum_q e_q^2 \int_0^1 \frac{dx}{x} H^q_v(x,t), \]

\[ R_A(t) \approx \sum_q e_q^2 \int_0^1 \frac{dx}{x} H^q_A(x,t), \]

\[ R_T(t) \approx \sum_q e_q^2 \int_0^1 \frac{dx}{x} E^q(x,t), \]  \hspace{1cm} (13)

Contributions from sea quarks are neglected. Numerical results for the Compton form factors are shown in Fig. 6. Approximately the form factors \( R_i \) behave \( \sim t^{-2} \). The particular flat behaviour of the scaled form factor \( t^2 R_T \) is a consequence of a cancellation between the \( u \) and \( d \)-quark contributions. The result for \( R_T \) is however subject to rather large uncertainties, given the considerable freedom one encounters in extracting \( E \) from the Pauli form factor alone. The ratio \( R_T/R_V \) behaves differently from the corresponding ratio of their electromagnetic analogues \( F_2 \) and \( F_1 \).

Inserting these Compton form factors into Eqs. (8) and (9), one is now able to predict the Compton cross section in the wide-angle region as well as the helicity correlation \( A_{LL} = K_{LL} \). The results for \( s = 11 \text{ GeV}^2 \) are shown in Fig. 7. The inner band for the curve of \( d\sigma/dt \) reflects the parametric errors of the form factors, essentially that of the vector form factor which dominates the cross section. The other form factors contribute less than 10\%. The outer band indicates an estimate of the target mass corrections, see \[21\]. The prediction for the cross section from the handbag approach are substantially larger than those from a leading-twist calculation \[8\]. The JLab E99-114 collaboration \[22\] will provide accurate cross section data soon which will allow for a crucial examination of the predictions from the handbag mechanism.

The JLab E99-114 collaboration \[22\] has presented a first measurement of \( K_{LL} \) at a c.m.s. scattering angle of 120° and a photon energy of 3.23 GeV. This still preliminary data point is in fair agreement with the predictions from the handbag given the small energy at which they are available. The kinematical requirement of the handbag mechanism \( s, -t, -u \gg \Lambda^2 \) is not well satisfied and therefore one has to be aware of large dynamical and kinematical corrections. It is to be stressed that the leading-twist approach \[8\] leads to a negative value for \( K_{LL} \) at angles larger than 90° in conflict with the JLab result \[22\].

![Fig. 6 Results for the scaled Compton form factors obtained in \[13\].](image-url)
The handbag approach also applies to wide-angle photo- and electroproduction of pseudoscalar and vector mesons. The amplitudes again factorize into a parton-level subprocess, $\gamma q \rightarrow M q$, and form factors which represent $1/x$-moments of GPDs [3]. Their flavor decomposition differs from those appearing in Compton scattering, see (13). Here, it reflects the valence quark structure of the produced meson. Since the GPDs and, hence, the form factors for a given flavor, $R^i_q$, $i = V, A, T$ are process independent they are known from the analysis of Ref. [13] for $u$ and $d$ quarks (if the contribution from sea quarks can be ignored). Therefore, the soft physics input to calculations of photo-and electroproduction of pions and $\rho$ mesons within the handbag approach is now known.

6 Summary

The treatment of wide-angle exclusive reactions is not simple within QCD and careful analyses are required. While the dominance of the leading-twist contribution for which all valence quarks of the involved hadrons participate in the partonic subprocess is expected for asymptotically large momentum transfer, it seems that the handbag mechanism, characterized by one active parton, dominates for momentum transfer of the order of $10$ GeV$^2$. The approximate power behaviour seen in many exclusive observables cannot safely be considered as evidence for the dominance of the leading-twist contribution. A careful investigation of the perturbatives logs as well as an understanding of the normalization is required. Soft mechanisms like the handbag can also explain the power behaviour. There are many interesting predictions from the handbag mechanism, some are in fair agreement with experiment, others still awaiting their experimental examination.

In Ref. [13] a first analysis of the GPDs has been performed in analogy to those of the usual parton distribution, see for instance [14]. A physically motivated parameterization of the GPDs have been fitted to the available data on the form factors of the nucleon. This analysis therefore provides information only on the GPDs at zero skewness and for valence quarks. This suffices to evaluate the soft physics input to wide-angle exclusive reactions but not for deep virtual exclusive processes where the GPDs at non-zero skewness are required. For these processes we still have to rely on models, see for instance the review [23].
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