Using Quantum entanglement to study CP and CPT violations

Yu Shi
Department of Physics, Fudan University
Shanghai, 200433, China
∗E-mail: yushi@fudan.edu.cn

We report some general phenomenological results concerning CP and CPT violations in joint decays of entangled pseudoscalar neutral mesons.1–4

1. Introduction
Quantum entanglement refers to the situation that the quantum state of a composite system is not a direct product of its subsystems. It significance was discovered by Einstein, Podolsky and Rosen.5 Schödinger coined the term and regarded it as “the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought.”6 Pseudoscalar neutral mesons are copiously generated as pairs entangled in the flavor space.7–13 CP and CPT violating parameters can be measured by studying the joint decays of entangled meson pairs.1–4,7,12,14,15 This provides a venue of searching standard model extension.15

2. Single meson states
$|M^{0}\rangle$ and $|\bar{M}^{0}\rangle$ are eigenstates of parity $P$ both with eigenvalue $-1$, and of a characteristic flavor $F$ with eigenvalues $\pm 1$. $C|M^{0}\rangle = -|\bar{M}^{0}\rangle$, $C|\bar{M}^{0}\rangle = -|M^{0}\rangle$. Hence the eigenstates of CP are $|M_{\pm}\rangle = \frac{1}{\sqrt{2}}(|M^{0}\rangle \pm |\bar{M}^{0}\rangle)$, with eigenvalues $\pm 1$. In the Wigner-Weisskopf approximation, the weak decay of a meson can be described by $i\frac{\partial}{\partial t}|M(t)\rangle = H|M(t)\rangle$, where $H = \left(\begin{array}{cc} H_{00} & H_{0\bar{0}} \\ H_{\bar{0}0} & H_{\bar{0}\bar{0}} \end{array}\right)$. It is defined that $\frac{1-\epsilon_{M}}{1+\epsilon_{M}} \equiv \sqrt{\frac{H_{0\bar{0}}}{H_{\bar{0}\bar{0}}}} \equiv \frac{4}{\eta}$, $\delta_{M} \equiv \frac{H_{0\bar{0}}-H_{\bar{0}\bar{0}}}{\sqrt{H_{0\bar{0}}H_{\bar{0}\bar{0}}}}$.

• If CP or T is conserved indirectly, then $\epsilon_{M} = 0$.
• If CPT or CP is conserved indirectly, then $\delta_{M} = 0$. 
The eigenstates $|M_S\rangle$ and $|M_L\rangle$ of $H$, corresponding to the eigenvalues $\lambda_S$ and $\lambda_L$ respectively, are found by diagonalizing $H$. Starting as $|M_S\rangle$, the state of a single meson evolves as $|M_S(t)\rangle = e^{-i\lambda_S t}|M_S\rangle$. Starting as $|M_L\rangle$, the state evolves as $|M_L(t)\rangle = e^{-i\lambda_L t}|M_L\rangle$. Based on this, one obtains $|M^0(t)\rangle$, which starts as $|M^0\rangle$; $|\bar{M}^0(t)\rangle$, which starts as $|\bar{M}^0\rangle$; $|M_+(t)\rangle$, which starts as $|M_+\rangle$; and $|M_-(t)\rangle$, which starts as $|M_-\rangle$.

To characterize direct violations, for decays into flavor eigenstates $|l^\pm\rangle$ with eigenvalue $\pm 1$, we define decay amplitudes $R^+ \equiv \langle l^+|H|M^0\rangle$, $S^+ \equiv \langle l^+|H|\bar{M}^0\rangle$, $S^- \equiv \langle l^-|H|M^0\rangle$, $R^- \equiv \langle l^-|H|\bar{M}^0\rangle$. They can be related to quantities $a, b, c, d$ usually defined.\(^{2,7}\)

- If CP is conserved directly, then $R^+ = R^-$ and $S^+ = S^-$. 
- If CPT is conserved directly, then $(R^+)^* = R^-$ and $(S^+)^* = S^-$. 
- If $\Delta F = \Delta Q$ rule is respected, then we have $S^\pm = 0$.

For decays into CP eigenstates $|h^\pm\rangle$ with eigenvalue $\pm 1$, we define decay amplitudes $Q^+ \equiv \langle h^+|H|M_+\rangle$, $X^+ \equiv \langle h^-|H|M_+\rangle$, $X^- \equiv \langle h^+|H|M_-\rangle$, $Q^- \equiv \langle h^-|H|M_-\rangle$. These newly defined quantities are convenient.

- If CP is conserved directly, then $X^\pm = 0$.
- If CPT is conserved directly, then $X^\pm$ is purely imaginary.

3. Entangled states

The $C = -1$ entangled state of a pair of pseudoscalar mesons is $|\Psi_\pm\rangle = \frac{1}{\sqrt{2}}(|M_0\rangle_m|M_0\rangle_b - |M^0\rangle_m|\bar{M}^0\rangle_b)$, which is produced for kaons at $\phi$ resonance,\(^7\) and for B mesons by $\Upsilon(4S)$ resonance\(^8,9\) and by $\Upsilon(5S)$ resonance with a large branch ratio.\(^{10-12}\) The $C = +1$ entangled state is $|\Psi_+\rangle = \frac{1}{\sqrt{2}}(|M_0\rangle_m|M_0\rangle_b + |\bar{M}_0\rangle_m|\bar{M}_0\rangle_b) = \frac{1}{\sqrt{2}}(|M_+\rangle_m|M_+\rangle_b - |M_+\rangle_m|M_-\rangle_b)$, which is produced for B mesons by $\Upsilon(6S)$ resonance with some branch ratio\(^{10-12}\) and above $\Upsilon(4S)$ resonance.\(^{13}\)

Although physically a single meson cannot be in a CP eigenstate $|M_\pm\rangle$ because of CP violation, the entangled states can be exactly written in terms of $|M_\pm\rangle$, this makes the expression $|M_\pm(t)\rangle$ and the decay amplitudes $Q^\pm$ and $X^\pm$ meaningful and useful.

Starting from $|\Psi_\pm\rangle$, the entangled meson pair decay to certain products at $t_a$ and $t_b$, hence the time-dependent state is $|\Psi_\pm(t_a, t_b)\rangle = \frac{1}{\sqrt{2}}(|M_0(t_a)\rangle_m|M^0(t_b)\rangle_b - |M^0(t_a)\rangle_m|M_0(t_b)\rangle_b) = \frac{1}{\sqrt{2}}(|M_-(t_a)\rangle_m|M_+(t_b)\rangle_b - |M_+(t_a)\rangle_m|M_-(t_b)\rangle_b)$. Similarly, starting as $|\Psi_+\rangle$, $|\Psi_+(t_a, t_b)\rangle = \frac{1}{\sqrt{2}}(|M^0(t_a)\rangle_m|M_0(t_b)\rangle_b + |M^0(t_a)\rangle_m|M_0(t_b)\rangle_b) = \frac{1}{\sqrt{2}}(|M_+(t_a)\rangle_m|M_+(t_b)\rangle_b - |\bar{M}_-(t_a)\rangle_m|M_-\rangle_b\rangle).$
4. Joint decay rates

For state $|\Psi(t_a, t_b)\rangle$, the joint rate that particle a decays to $f$ at $t_a$ while particle b decays to $g$ at $t_b$ is $I(f, t_a; g, t_b) = |\langle f | \mathcal{H}_a | \Psi(t_a, t_b) \rangle|^2$, where $\mathcal{H}_a$ and $\mathcal{H}_b$ represent the Hamiltonians governing the weak decays of a and b, respectively. In experiments, it is more convenient to use the integrated rate $I(f, g, \Delta t) = \int_0^\infty I(f, t_a; g, t_a + \Delta t)dt_a$. Then one can find the asymmetry between the joint decays to $f$ and $g$ and the joint decays to $f'$ and $g'$, $A(f, g, \Delta t) \equiv \frac{|\langle f, g | \Delta t \rangle| - |\langle f, g' | \Delta t \rangle|}{|\langle f, g | \Delta t \rangle| + |\langle f, g' | \Delta t \rangle|}$.

We have considered the following cases of the final states: (1) the decay products are flavor eigenstates $|l^\pm\rangle$, with the equal-flavor asymmetry $A(l^+l^+, l^-l^-, \Delta t)$ and the unequal-flavor asymmetry $A(l^+l^-, l^-l^+, \Delta t)$; (2) the decay products are CP eigenstates $|h^\pm\rangle$, with the equal-CP asymmetry $A(h^+h^+, h^-h^-, \Delta t)$ and the unequal-CP asymmetry $A(h^+h^-, h^-h^+, \Delta t)$; (3) the decay products $|h_1\rangle$ and $|h_2\rangle$ are CP conjugates.

5. General results on joint decays of $|\Psi_-\rangle^2$

**Theorem 1** If the equal-flavor asymmetry is nonzero, then there exists one or two of the following violations: (1) CP is violated indirectly, (2) both CP and CPT are violated directly.

**Theorem 2** If the equal-flavor asymmetry is nonzero while CP is assumed to be conserved both directly and indirectly, then in addition to indirect CP violation, we can draw the following conclusions: (1) $|q/p| \neq 1$, i.e. $T$ must also be violated indirectly; (2) $|\langle l^+ | \mathcal{H} | M_0 \rangle| \neq |\langle l^- | \mathcal{H} | M^0 \rangle|$, $|\langle l^- | \mathcal{H} | M_0 \rangle| \neq |\langle l^+ | \mathcal{H} | M^0 \rangle|$, despite $\langle l^+ | \mathcal{H} | M^0 \rangle = \langle l^- | \mathcal{H} | M_0 \rangle^*$ and $\langle l^+ | \mathcal{H} | M_0 \rangle = \langle l^- | \mathcal{H} | M^0 \rangle^*$.

**Theorem 3** If the unequal-flavor asymmetry is nonzero, then CP must be violated, directly or indirectly or both.

**Theorem 4** If the unequal-flavor asymmetry is nonzero for $\Delta t \neq 0$ while CPT is assumed to be conserved both directly and indirectly, then we can draw the following conclusions: (1) $|\langle l^+ | \mathcal{H} | M_0 \rangle| \neq |\langle l^- | \mathcal{H} | M^0 \rangle| \neq |\langle l^- | \mathcal{H} | M_0 \rangle| \neq |\langle l^+ | \mathcal{H} | M^0 \rangle|$, (2) $\langle l^- | \mathcal{H} | M^0 \rangle = \langle l^+ | \mathcal{H} | M_0 \rangle^*$, which means $\Delta F = \Delta Q$ rule must be violated.

**Theorem 5** The equal-CP asymmetry is always a constant independent of $\Delta t$.

**Theorem 6** For $\Delta t = 0$, the unequal-CP asymmetry vanishes, no matter whether CP or CPT is violated.

**Theorem 7** If any equal-CP joint decay rate is nonzero, then CP must be violated, directly or indirectly or both.
6. General results on joint decays of $|\Psi_+\rangle^3$

**Theorem 8:** If the unequal-flavor asymmetry is nonzero, then CP must be violated indirectly.

**Theorem 9:** If the $I(l^+, l^-, \Delta t)$ and $I(l^-, l^+, \Delta t)$ depend on the first order of $\epsilon_M$, then CP must also be violated directly.

**Theorem 10:** If the unequal-flavor asymmetry depends on the first or second order of $\epsilon_M$, then $\Delta F = \Delta Q$ rule is violated.

**Theorem 11:** The deviation of the unequal-CP joint decay rate $I(h_1, t_a; h_2, t_b)$ or $I(h_2, t_a; h_1, t_b)$ from zero implies direct CP violation.

**Theorem 12:** Suppose $|h_1\rangle$ and $|h_2\rangle$ are CP conjugates. If $I(h_1, h_2; \Delta t)$ and $I(h_2, h_1; \Delta t)$ depend on the first order of $\epsilon_M$, then CP is violated directly.

In addition, we have derived various quantitative relations of the indirect violating parameters with the decay asymmetries of $|\Psi_\pm\rangle^1,2$ with those of $|\Psi_+\rangle^3$ and with four asymmetries defined for some time-ordered integrated rates of $|\Psi_\pm\rangle$ and $|\Psi_+\rangle$.

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