Application of IIR Digital Filter with A New LMS Adaptive Algorithm

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Abstract. Any system will be inevitably disturbed by noise. The existence of noise affects the processing and application of real signals. How to effectively suppress and eliminate noise has always been a hot spot in the industrial field. In order to design a filter which can properly adjust its parameters according to different environments to obtain the optimal value, this paper designs an adaptive IIR digital filter, and studies as well as improves the adaptive algorithm. A new variable step size LMS algorithm based on multi-scale wavelet transform is proposed in this paper. The wavelet transform domain algorithm belongs to LMS transform domain algorithm and is a new time-frequency analysis method developed, so it has the characteristics of multi-resolution analysis. This adaptive IIR filter designed is applied to suppression and cancellation of noise, and simulation experiments are performed on the MATLAB software platform, so that better illustrating the superiority of this improved algorithm.

1. Introduction

Noise interference is everywhere. In engineering design, the signal will inevitably introduced into noise in the process of propagation. The existence of noise affects the processing and application of real signals. Therefore, the suppression and cancellation of noise has become an extremely important task in the digital signal processing.

In practical applications, it is frequently difficult to obtain complete knowledge about signal characteristics. Therefore, more and more attention has been paid to the application of adaptive filtering technology [1] to noise suppression and cancellation. Adaptive filter is a time-varying nonlinear filter. The complexity of optimization algorithm makes most of adaptive filtering to be digital filters. Most of adaptive filtering can satisfy the processing operations of the unknown parameters required, and use the feedback of error signal to improve the coefficients of filter, so as to better match the changing parameters.

The adaptive IIR filter is a recursive filter with an infinite impulse response, it has better performance than the adaptive FIR filter with the same number of coefficients [2], so an adaptive IIR digital filter is selected and designed in this paper.

The research of adaptive digital filtering algorithm has achieved fruitful results so far. The most widely used typical algorithm is the least mean square adaptive filtering [3~8] algorithm. However, it converges slowly and has low accuracy, which affects its tracking performance. Therefore, a variable step size LMS algorithm and a transform domain LMS algorithm are proposed.
The wavelet transform domain algorithm [9, 10] belongs to LMS transform domain algorithm. It has the characteristics of multi-resolution analysis. The algorithm firstly transforms the vector of the input signal into multi-scale space to reduce the correlation between the signals. And then, the adaptive update coefficient’s equation of LMS algorithm is used to update the coefficients, so as to implement the iteration. At the same time, using the Mallat fast algorithm, the amount of calculation becomes faster.

To suppress and eliminate noise [11], this paper proposes a new algorithm combining multi-scale wavelet transform and variable step size LMS to design an IIR adaptive digital filter.

2. Principle of adaptive digital filter

LMS algorithm [1] as an application of stochastic gradient descent method, it was pioneered by Widrow and Hoff. But this algorithm is generally applied to FIR filters. This paper will improve the classical LMS adaptive algorithm, so that its algorithm can adapt to design an IIR filter.

The update parameters adopt an adaptive IIR filter with output error mode, and its structure is shown in Fig.1, where x(n) is the input signal; d(n) is the reference output signal; y(n) is the output signal; and e(n) is the error signal, that is, the difference between the signal d(n) and the signal y(n). The function of adaptive algorithm minimizes the mean square value of e(n) by updating the weighted filter.

![Figure 1. IIR Filter structure](image)

When A(n, q) = 1, the filter becomes an FIR filter. In order to satisfy the stability of system and ensure the convergence of the algorithm, an approximate gradient algorithm, Pseudo-Linear Regression Algorithm [12], is used here.

\[
W(n) = [a_1(n), a_2(n), ..., a_{N-1}(n), b_0(n), b_1(n), ..., b_{M-1}(n)]^T
\]

\[
\phi(n) = [y(n-1), y(n-N+1), x(n-1), ..., x(n-M+1)]^T
\]

\[
e(n) = d(n) - y(n) = W^T(n)\phi(n)
\]

\[
R^{-1}(n+1) = \frac{1}{\lambda} [R^{-1}(n) - \frac{R^{-1}(n)\phi(n)\phi(n)^T R^{-1}(n)}{\lambda + \phi(n)^T R^{-1}(n) \phi(n)}]
\]

\[
W(n+1) = W(n) + \mu R^{-1}(n+1)\phi(n)e(n)
\]

W(n) is the tap coefficient vector of IIR filter; \(\phi(n)\) is the input signal; \(y(n)\) is the output signal; \(d(n)\) is the expected signal; \(e(n)\) is the error signal; and \(R(n)\) is the identity matrix in the iterative formula. In order to make the algorithm have better convergence effect, this paper replaces the fixed step size \(\mu\) with the variable step size factor \(\mu(n)\) in the LMS, and proposes a variable step size LMS algorithm based on the cosine function[13] by establishing the nonlinear function relationship between the step factor \(\mu\) and the error signal \(e\). Let \(y=-\cos|x|+1\), the function diagram is shown in Fig.2.
Figure 2. The picture of $y = -[\cos(|x|) + 1]$ function

It can be seen from Fig.2 that $y$ is a smooth nonlinear curve passing through the origin, and the independent variable $x$ changes relatively gently at the initial stage and close to zero, while the $y$ changes relatively rapidly with $x$ in the transition.

Replace $x$ with $e(n)$ and $y$ with $\mu(n)$, then $e(n)$ is a function of $\mu(n)$, which gives a new step factor:

$$\mu(n) = -\beta[\cos(\alpha|e(n)|) + 1] \quad (6)$$

$\lambda_{\text{max}}$ represents the maximum eigenvalue of the autocorrelation matrix of the input signal; $\beta$ is the parameter that controls the value range of step curve, and the value is positive, which affects the convergence speed of the algorithm; $\alpha$ is the parameter that controls the shape of the step curve when the error is close to zero.

The iteration formula for the new algorithm at this point is:

$$R^{-1}(n+1) = \frac{1}{\lambda_{\text{max}}} [R^{-1}(n) - \frac{R^{-1}(n) \phi(n) \phi(n)^T R^{-1}(n)}{\lambda_{\text{max}}} + \phi(n) \phi(n)^T] \quad (7)$$

$$W(n+1) = W(n) + \mu(n)R^{-1}(n+1)\phi(n)e(n) \quad (8)$$

3. Transform Domain LMS Algorithm Based on Wavelet Transform Styling

To improve the convergence and tracking speed of the LMS algorithm, the input signal can be decomposed into multi-scale space by wavelet transform to reduce the input signal’s eigenvalue of autocorrelation matrix [14].

For the IIR adaptive filtering structure, its output $y(n)$ is not only related to the input $x(n)$ at time $n$, but also related to the output, since it has a feedback loop. Including:

$$y(n) = \sum_{m=0}^{M-1} x(n-m)c(m) + \sum_{m=1}^{N-1} y(n-m)c(m) \quad (9)$$

In the above formula, $c(m)$ is the weight coefficient of filter, whose length is $(M+N-1)$. For an IIR filter, it can be expressed by wavelet and scaling function, where $J = \log_2 (M+N-1)$.

$$c_i(m) = \sum_{j=1}^{J} \sum_{k=\infty}^{\infty} w_{j,k} \psi_j^\alpha(m) + \sum_{l=0}^{\infty} v_{j,k} \phi_j^\alpha(m), i = 0, 1, ..., M+N-2 \quad (10)$$
Fig. 3 is a schematic diagram of Mallat wavelet decomposition. \( C_0 \) is the original signal vector, \( D_j \) \((j = 1, 2, ..., J)\) and \( C_j \) \((j = 1, 2, ..., J)\) respectively represent the detail signal and the approximating signal after wavelet decomposition.

The improved LMS algorithm based on multi-scale wavelet transform firstly is to transform \((M+N-1)\) discrete input signals \( \phi_{M+N-1}(n) \) into the wavelet domain for discrete orthogonal wavelet multi-scale decomposition. According to the Mallat algorithm, for discrete signals \( C_0 \) with length \( M+N-1 \), there is always decomposition as shown in Fig. 3. Where \( C_j \) and \( D_j \) are wavelet transform matrices composed respectively of conjugated orthogonal mirror low-pass filter \( \{h_k\} \) and high-pass filter \( \{g_k\} \). \( P \) is orthogonal wavelet matrix.

\[
\begin{align*}
D(n) &= P \phi(n) \\
P &= [G_0, G_1 H_0, G_2 H_1 H_0, ..., G_{J-1} H_{J-2} H_{J-1} H_{J-2} ..., H_1 H_0]^T
\end{align*}
\]

4. A New Variable Step Size LMS Adaptive Filtering Algorithm Based on Multiscale Wavelet Transform

In this paper, a new variable step size LMS algorithm based on multi-scale wavelet transform (MSWT-VS-LMS) is proposed by combining with the variable step size LMS algorithm and the transform domain LMS algorithm. The structure of this algorithm is shown in Fig. 4. The wavelet function uses Haar [10] wavelet. In this figure, \( \phi(n) \) is the input of the filter at time \( n \); \( \phi_0(n) \) is the input vector of \( \phi(n) \) delayed; \( D_j \) is the detail signal sequence after the signal sequence \( \phi_0 \) is decomposed by \( J \)-scale wavelet; \( \phi_J \) is the approximating signal sequence after the signal sequence is decomposed by \( J \)-scale wavelet; \( W_j \) is the adaptive filter weight vector of the detailed signal sequence after the \( J \)-time decomposition of the signal sequence \( \phi_0 \); \( U \) is the adaptive filter weight vector of the approximating signal sequence after the \( J \)-time decomposition of the signal sequence \( \phi_0 \); \( F_j \) represents the output of the \( j \)th adaptive filter at time \( n \).

It can be concluded that the new adaptive algorithm proposed in this paper:

Definition of vector:
\[ W(n) = [a_1(n), a_2(n), \ldots, a_{N-1}(n), b_0(n), b_1(n), \ldots, b_{M-1}(n)]^T \]  
(13)

\[ \varphi(n) = [y(n-1), \ldots, y(n-N+1), x(n-1), \ldots, x(n-M+1)]^T \]  
(14)

\[ D(n) = P\varphi(n) \]  
(15)

Initialization coefficient:
\[ W(n) = 0, \quad R(0) = \delta I \]  
(16)

Iterate, generate error signals and update the tap coefficients:
\[ W(n) = 0, \quad R(0) = \delta I \]  
(17)

\[ \mu(n) = -\beta [\cos(\alpha|e(n)|) + 1] \]  
(18)

\[ R^{-1}(n+1) = \frac{1}{\lambda} \left[ R^{-1}(n) - \frac{R^{-1}(n)\varphi(n)^\top R^{-1}(n)}{\mu(n)} + \frac{\varphi(n)^\top R^{-1}(n)\varphi(n)}{\mu(n)} \right] \]  
(19)

\[ W(n+1) = W(n) + \mu(n)R^{-1}(n+1)\varphi(n)e(n) \]  
(20)

5. Application simulation experiment
This paper mainly uses MATLAB to simulate the noise cancellation application in adaptive filtering. The advantages of the proposed new algorithm are verified by simulation analysis and comparison between the classical LMS algorithm and the proposed new algorithm in adaptive noise suppression and cancellation.

In the simulation, the original signal of \( y(n) = \sin(0.05\pi n) \) and the Gaussian white noise with an average value of 0 and a signal-to-noise ratio of 5dB are input to the IIR filter system. The number of simulations is \( g = 10 \), and the number of the input signal’s sampling points is \( n = 512 \); the order of time domain IIR tap filtering is 32; and in the variable step size formula (18), let \( \beta = 0.04 \), \( \alpha = 10 \); in the formula (19), let \( \lambda = 1.0004 \). The simulation results are shown in Fig.5 to Fig.8 below.

![Figure 5. The useful signal](image1)

![Figure 6. The useful signal](image2)
6. Conclusion
Filters play a significant role in many projects. The adaptive IIR digital filter is widely used as well, so it is necessary to improve its adaptive algorithm. This paper introduces the basic principles of adaptive IIR digital filter, the LMS algorithm for IIR structure, the variable step size LMS algorithm and the principle of wavelet transform. A new adaptive LMS algorithm designing an IIR filter is proposed to be used in noise suppression and cancellation systems. Finally, the classical LMS algorithm, the improved variable step size LMS algorithm based on wavelet transform for IIR filter is analyzed and applied to simulate the suppression and elimination of noise. Generally speaking, this adaptive algorithm has greatly improved the convergence speed, tracking speed and filtering effect.

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References
[1] S Haykin. Adaptive Filter Theory. 4th edn. NJ: Prentice-Hall, 2002.
[2] Du Yong, Lu Jian gong, Li Yuan zhou.MATLAB and FPGA digital filter realization [M]. Beijing:
[3] W.P. Ang, B. Farhang-Boroujeny, A new class of gradient adaptive step-size LMS algorithms. IEEE Trans on Signal Processing. 49, 805-810, 2001.

[4] J.K. Hwang, Y.P. Li, Variable step-size LMS algorithm with a gradient-based weighted average. IEEE Signal Process. Lett. 16, 1043-1046, 2009.

[5] OW Kwong, ED Johnston, A variable step-size LMS algorithm. IEEE Trans on Signal Processing. 40 (7), 1633-1642, 1992.

[6] T. Aboulnasr and K. Mayyas, A robust variable step-size LMS-type algorithm: analysis and simulations, IEEE Signal Process. vol.45, no.3, pp.631-639, March 1997.

[7] R.W. Harris, D.M. Chabries, and F.A. Bishop, A variable step-size algorithm, IEEE Trans. Acoust. Speech Signal Process, vol.ASSP-34, no.2, pp.309-316, April 1986.

[8] R.H. Kwong and E.W. Johnston, A variable step-size LMS algorithm, IEEE Trans on Signal Processing. 40 (6), 1633-1642, 1992.

[9] Qi, Yan, Ren, Li. Novel Image Enhancement Algorithm Based on Wavelet Multiscale [C]. // 2010 Third International Conference on Intelligent Networks and Intelligent Systems. 2010: 39-42.

[10] Tereshchenko, T., Yammenko, Y., Veretiuk, A., et al. Wavelet transform at oriented basis for network traffic forecasting [C]. // 2013 IEEE 33rd International Scientific Conference on Electronics and Nanotechnology. 2013: 450-454.

[11] Jin, Jingjing, Wang, Xu, Li, Shilong, et al. Wavelet Transform Adaptive De-noising Algorithm and Application Based on a Novel Variable Step Function [C]. // Knowledge Discovery and Data Mining, WKDD, 2009 Second International Workshop on; Moscow, TBD, Russia. 2009: 80-83.

[12] Mamadou Mboup, Mehdi Ashari, Phillip A. Regalia. Existence of stationary points for pseudo-linear regression identification algorithms [J]. IEEE Transactions on Automatic Control, 1999, 44 (5): 994-998.

[13] Shaojun Huang, Jianru Wan, Runqing Bai, et al. A Novel Algorithm of Instantaneous Symmetrical Components Based on Trigonometric Function Transformation [C]. // 2012 IEEE Innovative Smart Grid Technologies - Asia. [v.2]. 2012: 1564-1569.

[14] Attallah S. The wavelet transform-domain LMS algorithm: a more practical approach [J]. IEEE Transactions on Circuits and Systems, II. Express briefs, 2000, 47 (3): 209-213.