Entanglement evolution across defects in critical anisotropic Heisenberg chains

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Abstract
We study the out-of-equilibrium time evolution after a local quench connecting two anisotropic spin-1/2 XXZ Heisenberg open chains via an impurity bond. The dynamics is obtained by means of the adaptive time-dependent density-matrix renormalization group. We show that the entanglement entropies (von Neumann and Rényi) in the presence of a weakened bond depend on the sign of the bulk interaction. For an attractive interaction ($\Delta_1 < 0$), the defect turns out to be irrelevant and the evolution is asymptotically equivalent to the one without defect obtained by conformal field theory. For a repulsive interaction ($\Delta_1 > 0$), the defect is relevant and the entanglement saturates to a finite value. This out-of-equilibrium behavior generalizes the well-known results for the ground-state entanglement entropy of the model.

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(Some figures may appear in colour only in the online journal)

1. Introduction
Recent times have witnessed a renewed interest in the non-equilibrium dynamics of isolated many-body quantum systems. A particular class of these non-equilibrium problems which is experiencing a dramatic explosion of theoretical activity is that of a sudden quench of a Hamiltonian parameter, principally boosted by the experiments on trapped ultra-cold atomic gases [1–8] in which it has been shown that it is possible to follow the unitary non-equilibrium evolution without any significant coupling to the environment. These experiments fall into two main classes which are usually denoted as global and local quantum quenches. In the former case, a control parameter is suddenly quenched in the whole system (usually in a translationally invariant manner), while in the latter case, the change is only local; see, e.g., [9] for a review. Global quantum quenches are ideal experiments to investigate the intriguing issue of the existence of a stationary state and thermalization [10–18], while local quenches usually reveal the spreading of information and correlations in a cleaner way because they mainly probe universal low-lying excitations. Other interesting effects can be uncovered in
In this paper, we consider the most commonly studied local quench in which two halves of a critical one-dimensional system are initially prepared in their respective ground states and at a given time, let us say \( t = 0 \), they are connected and evolved according to a unitary Hamiltonian dynamics. Previous studies [28–38] showed that a very useful tool to understand the non-equilibrium quench dynamics is the entanglement entropy between two complementary parts. For a general bipartition of a pure state \( |\Psi\rangle \) of a quantum system (i.e. writing the whole Hilbert space of the system as a direct product of two parts \( \mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B \)), the Rényi entropy of the reduced density matrix \( \rho_A = \text{Tr}_B |\Psi\rangle\langle\Psi| \) of the subsystem \( A \) is

\[
S_A^{(\alpha)} = \frac{1}{1-\alpha} \ln \text{Tr} \rho_A^{\alpha}
\]

(1)
is a proper measurement of the entanglement between the two parts. In the limit \( \alpha \to 1 \), \( S_A^{(1)} \) reduces the more studied von Neumann entanglement entropy but the knowledge of the Rényi entropies for any \( \alpha \) gives far more information than the \( \alpha = 1 \) case because it provides the full spectrum of the reduced density matrix [40].

In the ground state of a one-dimensional critical system, whose scaling limit is described by a conformal field theory (CFT), in the case when \( A \) is an interval of length \( \ell \) embedded in an infinite system, the asymptotic large \( \ell \) behavior of the Rényi entropies is given by [41–44]

\[
S_A^{(\alpha)} = \frac{c}{6} \left( 1 + \frac{1}{\alpha} \right) \ln \ell + c'_\alpha,
\]

(2)
where \( c \) is the central charge [45] and \( c'_\alpha \) is a non-universal constant. In the case when the whole system is finite, the Rényi entropies can be obtained by a conformal mapping from the cylinder to the plane and the net effect is just to replace \( \ell \) with the chordal length \( L/\pi \sin(\pi \ell / L) \) [42]. More complicated and model-dependent universal expressions for \( S_A^{(\alpha)} \) have been obtained for the low-lying excited states conformal theories [46, 47] as well.

Conformal invariance can also be exploited to predict the behavior of the entanglement entropy for a ‘cut and glue’ local quench when two semi-infinite critical systems are joined together in a translational-invariant/homogeneous way (i.e. in a microscopical model, the added coupling is equal to all the others in the system). It has been found [29] that the entanglement grows logarithmically with time, again with a prefactor given by the central charge, i.e.

\[
S_A^{(\alpha)} = \frac{c}{6} \left( 1 + \frac{1}{\alpha} \right) \ln t + \text{cst.}
\]

(3)

This has been generalized by Stéphan and Dubail [36] to the case when the two parts which are connected at \( t = 0 \) have finite length. In the special case when the two parts have the same length \( L/2 \), the final result is

\[
S_A^{(\alpha)} = \frac{c}{6} \left( 1 + \frac{1}{\alpha} \right) \ln \left| \frac{L}{\pi} \sin \left( \frac{\pi vt}{L} \right) \right| + \text{cst.}
\]

(4)
where we explicitly introduced the speed of the sound \( v \). These results, as well as some for general observables, have been largely tested in the literature both for free and interacting models [32–36, 48–52]. Furthermore, it is worth mentioning that a few proposals to measure the entanglement entropy in real experiments are based on local quantum quenches [53–56].

It is natural to wonder how the previous equilibrium and non-equilibrium results generalize to the case when the two halves are joined with a coupling that is not equal to the others, especially in view of their experimental realizations, where the control of the added bond cannot be perfect. The simpler case of non-interacting fermions turned out to be very peculiar...
and not general enough, as opposite to the homogenous case. Indeed, it has been found that the defect is marginal [57, 58]. For the ground-state entanglement entropy, this implies that the entanglement between the two parts separated by the defect still grows logarithmically with the system size $L$, but with a prefactor $C_\alpha$ that continuously interpolates as a function of the defect strength between $c/6(1 + 1/\alpha)$ in the absence of the defect and 0 when the defect is so strong to divide the system into two parts. This effect was first found numerically [59, 34] and only later was the interpolating function exactly calculated as a function of the defect strength [60–62], which for general $\alpha$ reads

$$C_\alpha(s) = \frac{2}{\pi^2(1 - \alpha)} \int_0^\infty dx \ln \left[ \frac{1 + e^{-2\alpha\omega(x, s)}}{\left(1 + e^{-2\omega(x, s)}\right)^{1/\alpha}} \right], \quad \omega(x, s) = \text{acosh} \left[ \frac{\cosh(x)}{s} \right],$$

where $s \in [0, 1]$ is the transmission amplitude simply related to the defect strength (see below). Finally, very recently, it has been shown that the same interpolating function also enters in the prefactor in equation (4) for the time evolution of the entanglement entropy after a local quench connecting two chains of free spinless fermions [63, 64]. All these results for non-interacting fermions in the presence of a defect still await a full CFT derivation which, also in view of the similar treatment in [66], must clearly be possible.

For bulk interacting spinless fermions, the asymptotic behavior of the entanglement entropy is completely different. Indeed, in equilibrium, an old renormalization group (RG) analysis for one-dimensional interacting fermions (modeled as a Luttinger liquid field theory) shows that the relevance of the defect depends on the sign of the bulk interaction [57, 58]. For an attractive interaction, the defect turned out to be irrelevant, i.e. for large enough distances, the system behaves as if the defect was not present, while for repulsive interaction, the defect is relevant and even a small defect cuts the chain into two almost independent halves. Also, the logarithmic behavior of the entanglement entropy was calculated in [67] to the lowest order in the impurity strength. Finally, a numerical study of the spin-1/2 anisotropic Heisenberg XXZ chain with a central defect was presented in [68] and it was found that the von Neumann entanglement entropy grows logarithmically with $L$ for $\Delta < 0$ (ferromagnetic region corresponding to attractive interaction) with a prefactor equal to 1/3 (i.e. confirm the irrelevance of the defect because $c = 1$) and instead saturates for $\Delta > 0$ (antiferromagnetic region corresponding to repulsive interaction), confirming the relevance of the impurity. The approach to the thermodynamic limit is non-universal and obviously depends on the defect strength. The same qualitative results have also been found for a spin-1 gapless model [69].

In this paper, we study the time evolution of the anisotropic XXZ Heisenberg chain after a local quench connecting two initially separated half-chains with a defect bond, as shown in figure 1. We tackle the problem considering the evolution of an open chain with one modified bond in the center and using the adaptive time-dependent density-matrix renormalization group (tDMRG) [70]. This numerical method is well suited for the calculation of the entanglement entropy since at each step of the algorithm, one needs to reconstruct the reduced density matrix and its spectrum. This paper is organized as follows. In section 2, we present the details of
the model and of the numerical method. In section 3, we report the time evolution of the entanglement (von Neumann and Rényi) entropy. The simulations have been performed for different values of the defect strength $\delta$ and the anisotropic parameter $\Delta$, as well as for a few chain lengths $L$. Finally in the last section, we draw our conclusions.

2. The model, the local quench and the method

We consider the anisotropic XXZ Heisenberg chain with $L$ sites and a defect $\delta$ in the center defined by the Hamiltonian

$$H = \sum_{i=1}^{L-1} J_i \left[ S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z \right],$$

(6)

with

$$J_i = \begin{cases} 1, & i \neq L/2, \\ 1 + \delta, & i = L/2, \end{cases}$$

(7)

and we impose open boundary conditions on the sites $i = 1$ and $i = L$. Here, $S_i^\alpha$ are the local spin-1/2 operators, i.e. in terms of Pauli matrices $S_i^\alpha = \sigma_\alpha^\alpha/2$. In the homogenous case with $\delta = 0$, the model is integrable by means of the Bethe ansatz [71] allowing for an exact characterization of the ground state and all excited states. For $|\Delta| \leq 1$, the model is gapless, conformal (for $|\Delta| \neq -1$) and described by a Luttinger liquid field theory with central charge $c = 1$ and Luttinger parameter $K = \pi/(2 \arccos |\Delta|)$, while for $|\Delta| > 1$, it acquires a gap. For $\Delta \neq 0$, although the model is Bethe ansatz solvable, it is still not known how to use integrability to calculate effectively the entanglement entropies in the gapless phase, in spite of several attempts in the literature [72] (in the gapped phase instead, some exact results are known [42, 73]).

Via a Jordan–Wigner transformation, apart from boundary terms, the chain in equation (6) is mapped into a lattice model of spinless fermions $c_j$ (which satisfy canonical anti-commutation relations $[c_i, c_j^\dagger] = \delta_{i,j}$) with the Hamiltonian

$$H = \sum_{j=1}^{L-1} J_j \left[ c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j + 2\Delta \left( c_j^\dagger c_j - \frac{1}{2} \right) \left( c_{j+1}^\dagger c_{j+1} - \frac{1}{2} \right) \right],$$

(8)

which has an attractive nearest-neighbor interaction for $\Delta < 0$ and repulsive for $\Delta > 0$. The case $\Delta = 0$ corresponds to free spinless fermions (or the XX chain).

In this paper, we consider the non-equilibrium situation in which the chain is initially prepared in the relative ground states of two equal parts of length $L/2$, i.e. in the global ground state of the Hamiltonian (6), (7) with $\delta_0 = -1$. We fix $L$ to be an integer multiple of 4, so that the ground state of each of the two initially separated parts is non-degenerate and has zero magnetization (i.e. the corresponding fermion lattice is half-filled). At time $t = 0$, these two parts are connected by a defect bond $J_{L/2} = J_{\text{def}} = 1 + \delta$ characterized by a deficit $\delta$, while the rest of the chain is left unchanged with $J = 1$. The transmission amplitude $s$ in equation (5) is related to the defect strength as $s = \sin(2 \arctan(1 + \delta))$ [60], for $-1 \leq \delta \leq 0$. The above assumptions are not only technical: away from half-filling, a marginal operator (corresponding to forward scattering in the fermion language) can be present and it is expected to modify the behavior of the entanglement entropies as it is known in equilibrium for other observables [74, 75]. However, a proper analysis of these cases is beyond the goal of this paper.
Using the general CFT result reported in the introduction, for a homogeneous quench (i.e. with $\delta = 0$), the von Neumann entanglement entropy (hereafter, we will refer to $S^{(1)}$ just as $S$ to enlighten the notation) is \[ S_{\text{CFT}} = \frac{1}{3} \ln \left| \frac{\pi}{L} \sin \left( \frac{\pi vt}{L} \right) \right|, \] (9)

and also the anisotropy strength dependence of the sound velocity is exactly known \[ v = \frac{\pi \sin(\arccos \Delta)}{2 \arccos \Delta}, \] (10)

which is valid in the gapless phase $\Delta \in [-1, 1]$. Note that equation (9) is periodic in time with period $t_r = L/v$. We will refer to $t_r$ as the revival time. The existence of the revival time is a manifestation of the well-known light-cone effect \[28, 10, 29, 36]\]. Indeed, $t_r$ is the time needed for an excitation moving with speed $v$ starting from the center to arrive to the boundary, being reflected elastically and return to the center. In a local quench and in the thermodynamic limit, since the excess of energy of the initial state (compared to the ground state) is intensive, only low-lying universal excitations can be populated and these all have the same speed $v$. Clearly, in systems with a finite number of degrees of freedom, quasi-particles with different velocities could be excited even by a small energy excess resulting in imperfect revivals (the precise interference due to finite-sized scaling effects in conformal systems is largely discussed in \[36\] to which we remand for concrete examples). Note that since $v$ in equation (10) tends to zero approaching $\Delta = -1$, then $t_r$ becomes very large and no revival will be numerically accessible in this limit. However, for $\Delta = -1$, the model is not conformal-invariant anymore because the dynamical critical exponent becomes $z = 2$.

2.1. Method

We study the local quench dynamics of the XXZ chain by means of extensive tDMRG simulations \[70\]. The algorithm initially performs a static subroutine \[76\] which selects the initial state as the tensor product of the non-degenerate ground states of the two halves. In the decimation process, we keep a number of states such that the energy precision is at least of the order of $10^{-12}$. Subsequently, we perform the evolution in the presence of the impurity bond using the time-adapting block-decimation procedure implemented in the tDMRG code \[70\]. We use the second-order Suzuki–Trotter decomposition of the evolution operator with the time step $d_t = 5 \times 10^{-2}$. For each time step, the local evolution operator is applied sequentially on each bond starting from the left boundary of the chain. We adapt in time the number of states used to describe the reduced Hilbert space retaining at each local step all those eigenvectors of the reduced density matrix corresponding to eigenvalues larger than $10^{-16}$, up to a maximum value $\chi_{\text{MAX}} = 200$ (clearly, the effective maximal value used by the algorithm strongly depends on the simulation parameters). As well known (see, e.g., \[43, 77\]), the computational complexity of the time evolution of a quantum system on a classical computer using the tDMRG algorithm is essentially set by the growth of the bipartite entanglement. As the entanglement increases with time, we have to enlarge the dimension $\chi$ of the reduced Hilbert space in order to optimally control the truncation error. In spite of this refined adaptive choice of $\chi$, the truncation procedure is the main source of error of the algorithm for the largest system sizes, but if we would let $\chi$ grow without restrictions, the algorithm would easily get stuck in never ending computations. In the following, we will only consider the case of weakened bonds, i.e. $\delta \in [-1, 0]$ in equation (7).

To test the accuracy of the algorithm, we benchmarked it in the XX case ($\Delta = 0$) which is a simple free-fermion hopping model and wherein one can exactly calculate the finite-sized
entanglement entropy from the one-particle correlation matrix $\langle c_i^\dagger c_j \rangle$ [78, 43], even for a chain with defects [59]. In figure 2, we report the numerical results for the time evolution of the entanglement entropy in a XX chain of $L = 64$ spins after a local quench with a defect bond. In the figure, we confront the tDMRG data with the exact numerical calculations for three different values of the defect strength $\delta$. The relative error increases with time as expected, but it remains bounded below 1% (in this case, this error is essentially due to the accumulation of the truncation errors).

3. Time evolution of the entanglement entropy

By means of tDMRG, we calculate numerically the time evolution of the entanglement entropies (von Neumann and Rényi) for spin chains of length $L = 32, 64, 128$. For the smaller system sizes $L = 32, 64$, we consider many possible values of the defect strength, namely all values of $\delta = -0.1m$ with $m = 0, 1, \ldots, 9$ and $\delta = -0.05$, and several values of the bulk anisotropy parameter $\Delta \in (-1, 1]$, namely all $\Delta = 0.1m$ with $m = -9, -8, \ldots, 10$ (i.e. only values of $\Delta$ such that the scaling limit of the chain is described by a CFT with $c = 1$).

The data for the von Neumann entanglement entropy in spin chains of length $L = 64$ as a function of time are shown in figure 3, where in the left panel, we fix $\Delta = -0.5$ and report all the values of $\delta$, while in the right panel, we fix $\delta = -0.4$ and show all the considered values of $\Delta \in [-0.6, 1]$ up to $t = 2L/v$. For $L = 128$, the simulations are computationally more demanding. Thus, we focus on few most significative values of $\Delta$ and $\delta$ and only study the non-equilibrium dynamics up to the revival time $t_r = L/v$ since the universal part of the evolution should be periodic with
the period $t_r$. The simulations allow us to draw the following general scenario which we anticipate before the careful analysis of the data. The non-equilibrium behavior after the local quench is reminiscent of the equilibrium one: the entropies evolve in time according to the CFT prediction (9), provided that $\Delta < 0$, i.e. attractive interaction; conversely, for repulsive interactions ($\Delta > 0$), the entanglement growth is suppressed and the suppression increases with the repulsive coupling constant $\Delta$. Thus, as for the equilibrium counterpart, there is a fundamental difference between repulsive and attractive interactions: even out-of-equilibrium, the operator associated with the impurity turns out to be irrelevant for $\Delta < 0$ and relevant for $\Delta > 0$. Obviously, the ‘velocity of the RG flow’ toward its asymptotic limit strongly depends on the defect strength. In the following, we report and discuss the results for von Neumann and Rényi entropies to support this scenario in two separate subsections.

### 3.1. von Neumann entropy

We now report the numerical data for the von Neumann entanglement entropy discussing separately the cases of attractive and repulsive interactions. Figure 4 shows the results for attractive interaction $\Delta < 0$ (note that for $\Delta = -0.8$, the data for $L = 128$ do not reach the revival time $L/v$ because this would have been computationally too demanding since $v$ is very small; cf equation (10)). In order to show accurately that, in the thermodynamic limit, independent of the defect strength $\delta \in (-1, 0)$, the entanglement entropy is described by the CFT prediction (9), we have subtracted the expected conformal formula (9) to the time evolution of the entanglement entropy. For small ($\delta = -0.05$) and moderate ($\delta = -0.4$) defects, the difference $S - S_{\text{CFT}}$ clearly tends to a constant value (independent of $v$ and $L$) as long as the rescaled variable $vt/L$ is sufficiently far from the boundaries $vt/L \sim 0$ or 1, where finite-sized effects are enhanced. However, when the defect gets stronger (e.g. $\delta = -0.8$), deviations from a constant behavior are evident. Increasing the system sizes, these curves tend to flatten for both values of the anisotropic parameter reported in the figure, suggesting that in the thermodynamic limit, the difference would be perfectly constant for any $\Delta$. A proper
Figure 4. Differences between the entanglement entropy evolution and the CFT prediction (9) in the attractive regime ($\Delta < 0$) for two different values of $\Delta$ and four representative values of the defect bond. Dotted lines are for $L = 32$, dashed lines for $L = 64$ and full lines for $L = 128$. Increasing the system size, these differences tend to flatten, showing that for $\Delta < 0$, a defect $\delta \in (-1, 0)$ is always irrelevant.

Finite-sized scaling analysis is made difficult by the non-uniform approach as a function of $vt/L$, but already a qualitative look of the data leaves no doubts on the asymptotic result. Note that for $\Delta = -0.5$, larger system sizes are required for the differences to get constant compared to $\Delta = -0.8$. Figure 3 shows that this is a general trend as a function of $\Delta$, i.e. the smaller $|\Delta|$, the larger the system size required to observe the conformal behavior. The origin of this dependence on $\Delta$ is easily understood. For $\Delta = 0$, the time evolution of the entanglement entropy is not given by the CFT formula (9), but the logarithm has a defect-dependent prefactor [63]. Thus, for small $\Delta$, increasing $L$ there is a crossover between the $\Delta = 0$ behavior and the asymptotic result.

In the case of repulsive interactions (XXZ antiferromagnet with $\Delta > 0$), let us first discuss what we would expect if, in analogy with equilibrium, the defect would be relevant. For any $\Delta$, the entanglement entropy should initially grow, but at some given time $t^*$, the presence of the relevant defect prevents further passage of information between the two halves and the entanglement saturates to a constant value (up to a possible revival at $L/v$). Clearly, $t^*$ is not universal and its value would depend both on $\delta$ and $L$. When $\delta$ is small enough, we expect this saturation time to be so large that for any accessible size $L$, it would be very difficult to find a perceptible difference with the CFT behavior at $\delta = 0$. Increasing $\delta$, the saturation time $t^*$ decreases, even if for small and moderate values of $L$, the difference with the CFT evolution should remain small. Further increasing $\delta$, the saturation time $t^*$ should become very short and the entanglement quickly saturates to its asymptotic value.

If this scenario is correct, it does not make sense to subtract the conformal formula to tDMRG data, and so we simply report the entanglement entropy evolution for two values of $\Delta$ in figure 5, supplementing the data in the left panel of figure 3 for $L = 64$. From these two figures, it is clear that the previously conjectured scenario for the time evolution is confirmed by the numerics. For all values of $\Delta$, the growth of the entanglement entropy observed in homogeneous systems is more and more suppressed as the absolute value of the defect strength increases. For the smallest impurity we considered ($\delta = -0.05$), the entanglement entropy is very close to the conformal prediction, but the lower quality of the fit (that we do not report in the plot) compared to the data at $\delta = 0$ suggests that some tiny differences can be already present. As the defect becomes stronger (e.g. $\delta = -0.4$), the curves...
flatten substantially and slowly approach a horizontal line in the thermodynamic limit. For a very strong defect ($\delta = -0.8$), the entanglement entropy becomes flat in a very short time. Of course, also in this case the bulk interaction plays a role because the saturation limit is approached faster for larger values of $\Delta$. As in the case of attractive interactions, this is due to the fact that for $\Delta = 0$, there is a logarithmic growth of the entanglement for any $\delta$ and so there is a crossover in $L$ from the non-interacting behavior to the saturation in the interacting chain.

Finally, we also discuss the revival of the entanglement entropy. In the right panel of figure 3, we show the time evolution up to $2L/v$ (i.e. for two full asymptotic periods) for $L = 64$, $\delta = -0.4$ and several values of $\Delta$. For attractive interaction $\Delta < 0$, the quasi-periodic behavior is evident, but during the second period some oscillations in time are superimposed to the CFT prediction (9). These have been already observed for $\Delta = 0$ (see, e.g., [36]), even in the absence of the defect, and they are mainly due the interference between excitations with non-perfectly equal velocities. It has been shown [36] that the amplitude of these oscillations tends to zero increasing the system size for $\Delta = 0$ and $\delta = 0$. From the figure, it is evident that for $L = 64$, the amplitude of the oscillations gets smaller with decreasing $\Delta$; thus, we expect that they should vanish in the thermodynamic limit for all $\Delta < 0$, as also corroborated by the data for $L = 32$ not shown here. For $\Delta > 0$, the effect of the slower quasi-particles is different. Indeed, figure 3 shows that the interference effect gives rise to oscillations which make the curve flatter in time during the second period. Thus, the second period entanglement entropy is closer to the thermodynamic limit compared to the first period value. Note in fact that in figure 3 for $\Delta > 0$ and fixed $\delta = -0.4$, the average over the second period is slightly larger than the average over the first one; similarly as in figure 5, the entanglement for $L = 128$ is larger than the one for $L = 64$ at $\delta = -0.4$.

3.2. Rényi entropies

In this section, we check that the general scenario drawn above on the basis of the von Neumann entropy carries over to the time evolution of the Rényi entropies for general $\alpha$, limiting to report data for $\alpha = 2, 3, \infty$ (the case $\alpha = \infty$ corresponds to the logarithm of the largest eigenvalues of the reduced density matrix, i.e. the so-called single copy entanglement [79]).
In figure 6, we report the results for the attractive case $\Delta < 0$. In analogy with the von Neumann entropy, we plot the difference $S^{(\alpha)} - S^{(\alpha)}_{\text{CFT}}$. For all values of $\alpha$, $\Delta$ and $\delta$, the irrelevance of the defect is confirmed by these data; indeed for small $|\delta|$, the CFT prediction is already approached even for relatively small values of $L$, while for large $|\delta|$, the difference tends to a constant only in the limit of large $L$. However, there is also an unexpected effect. Indeed, in figure 6, for a small value of $\delta$, there are subleading oscillations on top of the leading CFT behavior. The presence of these oscillations is well known and it has been observed in the absence of the defect in all previous studies (see, e.g., [32, 33, 36]) and explained as the non-equilibrium counterpart of the equilibrium unusual corrections to the scaling [80, 81] of the form $L^{-K/n}$, with $K$ being the Luttinger parameter. The frequency of these oscillations is related to $k_F$ (i.e. the filling) and does not depend on the coupling strength $k$ [80]. The interesting new effect is that in the presence of the defect, such corrections are largely suppressed and almost disappear already for $\delta = -0.4$. This means that the CFT predictions become more accurate in the presence of a defect (that however must not be too large to observe the flattening of the curves in figure 6). We stress that these oscillations are not directly related to the ones discussed in the previous section due to quasi-particles with velocity different from $v$ which in figure 6 starts manifesting only close to the revival time.

The data for the repulsive case are reported in figure 7. Even in this case, the picture drawn from the study of the von Neumann entropy straightforwardly carries over to the Rényi: for the small impurities, the Rényi entropies stay very close to the conformal prediction, but as
the defect becomes stronger, the entropies flatten and approach quickly a saturation value for any $\alpha$. Even in the repulsive case, it is evident that the oscillating corrections to the scaling present for small $|\delta|$ are largely suppressed as the defect strength increases.

4. Conclusions

In this paper, we have studied the time evolution of the entanglement entropies after a local quench connecting two anisotropic spin-1/2 XXZ Heisenberg chains via an impurity bond. By means of tDMRG, we showed that, at half-filling, the asymptotic behavior of the entanglement entropies for large systems depends only on the sign of bulk interactions. For attractive interaction ($\Delta < 0$), it turns out that the defect is irrelevant and the evolution is asymptotically equivalent to the conformal one without the impurity. For repulsive bulk interaction ($\Delta > 0$), the defect becomes relevant and the entanglement entropy saturates to a finite value after a finite time which are both not universal and depend on the defect strength.

We have only determined time evolution of the entanglement entropy between the two initially disconnected parts, but we can safely conclude that the same picture should carry over to all other observables which can be determined just by exploiting the relevance/irrelevance of the defect according to the sign of the bulk interaction. We mention that the correspondence between the equilibrium and out-of-equilibrium situations resulting from a local perturbation for observables different from the entanglement entropies has a long history [74, 75] such as the
connection between equilibrium Anderson orthogonality catastrophe and the x-ray absorption spectra (corresponding to a local perturbation). Furthermore, while we have only considered the XXZ spin chain, in view of the Luttinger liquid universality, the same conclusions apply to all other models in the same universality class, such as one-dimensional gases of spinless bosons (both the continuum Lieb–Liniger and the Bose–Hubbard models in the superfluid phase), quantum wires junction, etc. Oppositely, for multi-component continuum or lattice models of interacting fermions (such as the Hubbard model) or Fermi–Bose mixtures, the phenomenon of spin-charge separation could lead to different universal features, e.g. reminiscent of what observed in [82] for different initial states. For these multi-component models, also shell-filling effects [47] could strongly affect the non-equilibrium dynamics of the entanglement entropy. Therefore, it would be really interesting to investigate local quenches in these models both in the absence and in the presence of defect bonds.

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