SPONTANEOUS CP VIOLATION IN THE U(3) NJL MODEL

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Abstract

With the help of the functional integration method, the formation of scalar, pseudoscalar condensates, and dynamical symmetry breaking in the U(3) four-fermion model has been investigated. We show the possibility of spontaneous CP symmetry violation in the model under consideration. The bosonization procedures of the model are performed; the propagators of quarks, scalar and pseudoscalar fields are calculated in one loop approximation. The masses of the scalar and pseudoscalar mesons are evaluated.

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1 Introduction

It is possible nowadays to derive the effective quark-meson Lagrangians from the fundamental quantum chromodynamics (QCD) Lagrangian. The reformulation of QCD in terms of hadrons has not been completed yet. Therefore, in the domain of low energy, some phenomenological models are introduced. Local effective chiral Lagrangians (ECL) \(1\), \(2\), \(3\) can describe low energy physics of hadrons with good accuracy. The instanton vacuum theory \(4\) explains the appearance of the chiral condensate which leads to the dynamical symmetry breaking (DSB) and to the effective four-fermion interaction (for two flavours) \(5\) (see also \(6\), \(7\)). So, a contact four-fermion interaction modelling quark interactions, takes into account both quarks and mesons \(8\). In such models, the gluon interactions are neglected and there is no confinement of quarks. Therefore Nambu-Jona-Lasinio (NJL) models are QCD motivated effective models with some shortcomings. In particular, NJL models make it possible to decay the scalar mesons into \(q\bar{q}\).

Our goal is to study the possibility of spontaneous CP symmetry violation in the NJL model. The electric dipole moments of particles violate CP-invariance and, in the framework of QCD, can be explained with the
help of the $\theta$-term. The effect of CP breaking in strong interactions is small, but the investigation of such phenomenon is important. It should be noted that the $\theta$-term is important for the solution of the $U_A(1)$ problem. The axial symmetry, $U_A(1)$, is broken by the QCD anomaly. This may be explained by the interactions of light quarks and instantons which violate the $U_A(1)$ symmetry. There is a region of quark masses\footnote{Mass formulas were obtained in \cite{10} on the basis of the specific relation (constraint) between quadratic and logarithmic diverging integrals. In present paper I use the standard cutoff regularization.}, where CP symmetry is spontaneously broken. The CP violation leads to the exotic phenomena, the possibility of $\eta$ decaying into two pions.

The present work is the generalization of \cite{10} on the case of the U(3) group\footnote{Mass formulas were obtained in \cite{10} on the basis of the specific relation (constraint) between quadratic and logarithmic diverging integrals. In present paper I use the standard cutoff regularization.}.

2 Model and perturbation expansion

We start with a NJL model possessing the internal symmetry group $U(3) \otimes U(3)$ in the chiral limit:

$$\mathcal{L}(x) = -\bar{\psi}(x)(\gamma_\mu \partial_\mu + \hat{m}_0)\psi(x)$$

$$+ \frac{G}{2} \left\{ \left[ \bar{\psi}(x)\lambda^a\psi(x) \right]^2 + \left[ \bar{\psi}(x)\gamma_5\lambda^a\psi(x) \right]^2 \right\},$$

where $\lambda^a$ ($a = 0, 1, ..., 8$) are the Gell-Mann matrices, $\lambda_0 = \sqrt{2/3}I_8$ ($I_8$ is the unit $8 \times 8$-matrix), $\partial_\mu = (\partial/\partial x_i, -i\partial/\partial x_0)$ ($x_0$ is the time), $\gamma_\mu$ are the Dirac matrices, $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$. The $\hat{m}_0$ is the matrix of bare masses of the quark triplet $\psi(x)$:

$$\psi(x) = \text{diag} \left[ u(x), d(x), s(x) \right], \quad \hat{m}_0 = \text{diag} \left( m_u, m_d, m_s \right).$$

The summation over colour quark degrees of freedom $n = 1, 2, ..., N_C$ is implied here. The chiral symmetry is broken by quark masses and dynamically by the appearance of condensates. Therefore, for simplicity, we consider only the formation of the nonet of scalar mesons and the nonet of pseudoscalar mesons $\pi$, $K$, $\eta$, $\eta'$. The octets of vector and pseudovector mesons are ignored here. The $U_A(1)$ symmetry is not broken here as the
Lagrangian (1) at $\hat{m}_0 = 0$ is invariant under $\gamma_5$-chiral transformations. To violate this symmetry "by hands", one can add to the Lagrangian (1) the six-quark interaction due to instantons [6]. On the other hand the $U_A(1)$ anomaly appears because of the non-invariance of the fermion measure in the functional integral [11]. It should be noted that the QCD anomaly, $U_A(1)$, results the existence of a ninth Goldstone boson $\eta'$ with the greater mass compared to $\eta$.

Using the functional integration method [12], the generating functional for Green’s functions

$$Z[\bar{\eta}, \eta] = N_0 \int D\bar{\psi}D\psi \exp \left\{ i \int d^4x \left[ \mathcal{L}(x) + \bar{\psi}(x)\eta(x) + \bar{\eta}(x)\psi(x) \right] \right\},$$

(2)

where $\bar{\eta}, \eta$ are external sources, with the help of the replacement

$$N_0 = N \int D\phi_a D\bar{\phi}_a \exp \left\{ -i \frac{M^2}{2} \int d^4x \left[ \left( \bar{\phi}_a(x) - i \frac{g_0}{M^2} \bar{\psi}(x)\gamma_5 \lambda^a \psi(x) \right)^2 \right. \right.$$

$$+ \left. \left( \phi_a(x) - \frac{g_0}{M^2} \psi(x)\lambda^a \bar{\psi}(x) \right)^2 \right\},$$

(3)

can be cast into

$$Z[\bar{\eta}, \eta] = N \int D\bar{\psi}D\psi D\phi_a D\bar{\phi}_a \exp \left\{ i \int d^4x \left[ -\bar{\psi}(x) \left( \gamma_\mu \partial_\mu + \hat{m}_0 - g_0 \phi_a(x) \right) \right. \right.$$

$$+ \left. \left( \bar{\phi}_a(x) - \frac{g_0}{M^2} \bar{\psi}(x)\lambda^a \psi(x) \right)^2 \right\},$$

(4)

where $G = g_0^2/M^2$; $g_0$ is the dimensionless bare coupling constant and $M$ is dimensional constant. Eq. (3) may be integrated over the $\bar{\psi}$, $\psi$, and as a result

$$Z[\bar{\eta}, \eta] = N \int D\phi_a D\bar{\phi}_a \exp \left\{ i S[\Phi] + i \int d^4x \int d^4y \bar{\eta}(x)S_f(x,y)\eta(y) \right\},$$

(5)

where the effective action for bosonic collective fields $\Phi_a(x) = \phi_a(x) + ig_0\bar{\phi}_a(x)$ is given by

$$S[\Phi] = -\frac{M^2}{2} \int d^4x \left[ \phi_a^2(x) + \bar{\phi}_a^2(x) \right] - i \text{Tr} \ln \left[ -\gamma_\mu \partial_\mu - \hat{m}_0 + g_0 \Phi_a(x)\lambda^a \right].$$
We have used here the relation \( \det \mathcal{Q} = \exp \text{Tr} \ln \mathcal{Q} \) (the \( \mathcal{Q} \) is an operator). The operator \( \text{Tr} \) in Eq. (5) includes tracing in matrix and space-time variables. The Green function of quarks, \( S_f(x,y) \), obeys the equations

\[
[\gamma_{\mu} \partial_{\mu} + m_0 - g_0 \Phi_a(x) \lambda^a] S_f(x,y) = \delta(x-y). \tag{6}
\]

The fields \( \phi_a(x) \) and \( \tilde{\phi}_a(x) \) form nonets of scalar and pseudoscalar (\( \pi^\pm, \pi^0, K^\pm, K^0, \eta, \eta' \)) mesons.

The symmetric vacuum in the NJL models is not stable \[8\]. The physical vacuum is reconstructed which results in the appearance of condensates and dynamical breaking of the \( U(3) \otimes U(3) \) symmetry. We imply here that the condensates are formed as follows:

\[
\langle \bar{\psi} \psi \rangle \neq 0, \quad \langle \bar{\psi} \lambda^3 \psi \rangle \neq 0, \quad \langle \bar{\psi} \lambda^8 \psi \rangle \neq 0, \quad \langle \bar{\psi} \gamma_5 \psi \rangle \neq 0, \quad \langle \bar{\psi} \gamma_5 \lambda^3 \psi \rangle \neq 0, \quad \langle \bar{\psi} \gamma_5 \lambda^8 \psi \rangle \neq 0. \tag{7}
\]

The vacuum expectation values containing the \( \gamma_5 \) matrix are parity and time reversal odd values, and as a result, they violate CP symmetry. To take into consideration and to determine condensates, the fields have to be “shifted” by the constants. Therefore, we make the substitution in Eqs. (5), (6)

\[
\phi_0(x) = \phi'_0(x) + \sigma_0, \quad \phi_3(x) = \phi'_3(x) + \sigma_3, \quad \phi_8(x) = \phi'_8(x) + \sigma_8, \quad \phi_i(x) = \phi'_i(x),
\]

\[
\tilde{\phi}_0(x) = \tilde{\phi}'_0(x) + \tilde{\sigma}_0, \quad \tilde{\phi}_3(x) = \tilde{\phi}'_3(x) + \tilde{\sigma}_3, \quad \tilde{\phi}_8(x) = \tilde{\phi}'_8(x) + \tilde{\sigma}_8, \quad \tilde{\phi}_i(x) = \tilde{\phi}'_i(x),
\tag{8}
\]

where \( i = 1, 2, 4, 5, 6, 7; \sigma_0, \sigma_3, \sigma_8, \tilde{\sigma}_0, \tilde{\sigma}_3, \tilde{\sigma}_8 \) are coordinate-independent and Lorentz-invariant constants. The fields \( \phi'_a(x), \tilde{\phi}'_a(x) \) in Eqs. (8) represent quantum excitations over vacuum and are assumed to be small. The vacuum expectation values (condensates), \( \sigma_0, \tilde{\sigma}_3 \) and \( \tilde{\sigma}_8 \), break CP symmetry. Below, the condensates \( \sigma_j \) and \( \tilde{\sigma}_j \) for \( j = 0, 3, 8 \) will be obtained from the minimum of the effective potential defining the energy density of the vacuum. To formulate the perturbation theory \[12\], we use the saddle-point method. Taking into consideration Eqs. (8), one may rewrite Eq. (5) as follows:

\[
S[\Phi] = -\frac{M^2}{2} \int d^4x \left[ (\phi'_j(x) + \sigma_j)^2 + (\tilde{\phi}'_j(x) + \tilde{\sigma}_j)^2 + \phi'^2_i + \tilde{\phi}'^2_i \right]
\]
Let us consider the equality
\[ \sum_{\sigma_0, \sigma_3} \frac{\sigma_3}{\sqrt{3}} \] 
Then expanding the logarithm in small fluctuations \( \Phi' \)
the action (5) takes the form
\[ S(\Phi') = \sum_{\sigma_0, \sigma_3} \frac{\sigma_3}{\sqrt{3}} \]
where \( \Phi'(x) = \phi'(x) + i \gamma_5 \bar{\phi}'(x), \)
\( \tilde{m} = \text{diag}(m_{01}, m_{02}, m_{03}), \quad \tilde{m} = \text{diag}(\tilde{m}_1, \tilde{m}_2, \tilde{m}_3), \)
\[ m_{01} = m_u - g_0 \left( \sqrt{\frac{2}{3}} \sigma_0 + \sigma_3 + \frac{\sigma_8}{\sqrt{3}} \right), \quad m_{02} = m_d - g_0 \left( \sqrt{\frac{2}{3}} \sigma_0 - \sigma_3 + \frac{\sigma_8}{\sqrt{3}} \right), \]
\[ m_{03} = m_s - g_0 \left( \sqrt{\frac{2}{3}} \sigma_0 - \frac{2\sigma_8}{\sqrt{3}} \right), \quad \tilde{m}_1 = g_0 \left( \sqrt{\frac{2}{3}} \sigma_0 + \sigma_3 + \frac{\sigma_8}{\sqrt{3}} \right), \]
\[ \tilde{m}_2 = g_0 \left( \sqrt{\frac{2}{3}} \sigma_0 - \sigma_3 + \frac{\sigma_8}{\sqrt{3}} \right), \quad \tilde{m}_3 = g_0 \left( \sqrt{\frac{2}{3}} \sigma_0 - \frac{2\sigma_8}{\sqrt{3}} \right). \]
Let us consider the equality
\[ \text{Tr} \ln \left[ -\gamma_\mu \partial_\mu - \tilde{m} + i \tilde{m} \gamma_5 + g_0 \Phi'(x) \lambda^a \right] \]
\[ = \text{Tr} \ln \left[ -\gamma_\mu \partial_\mu - \tilde{m} + i \tilde{m} \gamma_5 \right] + \text{Tr} \ln \left[ 1 - g_0 S_{0f}(x, y) \Phi'(x) \lambda^a \right], \]
where the Green function \( S_{0f}(x, y) \) obeys the equation
\[ \left[ \gamma_\mu \partial_\mu + \tilde{m} - i \tilde{m} \gamma_5 \right] S_{0f}(x, y) = \delta(x - y). \]
Then expanding the logarithm in small fluctuations \( \Phi'(x) \), the effective action (5) takes the form
\[ S[\Phi'] = -\frac{M^2}{2} \int d^4x \left[ (\phi' + \sigma_j)^2 + (\bar{\phi}' + \bar{\sigma}_j)^2 + \phi' + \phi'' \right] \]
\[ -i \text{Tr} \ln \left[ -\gamma_\mu \partial_\mu - \tilde{m} + i \tilde{m} \gamma_5 \right] + \sum_{n=1}^{\infty} \frac{i}{n} \text{Tr} \left( g_0 S_{0f} \Phi' \lambda^a \right)^n, \]
where
\[ \text{Tr} \left( g_0 S_{0f} \Phi' \lambda^a \right)^n = \text{tr} \left[ g_0 \int d^4x_1 ... d^4x_n S_{0f}(x_n - x_1) \Phi'_{a_1} \lambda^{a_1} \right. \]
\[ \times S_{0f}(x_1 - x_2) \Phi'_{a_2} \lambda^{a_2} ... S_{0f}(x_{n-1} - x_n) \Phi'_{a_n} \lambda^{a_n} \],
where the tr[...] means the tracing in matrices. The terms with \( n = 2, 3 \) in Eq. (12) define decaying and the scattering of mesons. The fields \( \phi' \) and \( \bar{\phi}' \) in the effective action (12), after renormalization, describe the physical scalar and pseudoscalar mesons.
3 Propagators of quarks and mesons

To calculate the masses of quarks and mesons it is necessary to find the propagators of quarks and mesons. The condensates \( \sigma_j \) and \( \tilde{\sigma}_j \) for \( j = 0, 3, 8 \) can be obtained from the requirement that terms linear in fields \( \phi'_a(x) \), \( \tilde{\phi}'_a(x) \), which correspond to the “tadpole” diagrams, are absent in the effective action (12). This leads to the gap equations

\[
\frac{\delta S[\Phi']}{\delta \phi'_j(x)}|_{\phi'_j=0} = -M^2 \sigma_j + ig_0 \text{Tr} \left[ S_{0f}(x,x) \lambda^j \right] = 0,
\]

\[
\frac{\delta S[\Phi']}{\delta \tilde{\phi}'_j(x)}|_{\tilde{\phi}'_j=0} = -M^2 \tilde{\sigma}_j(x) - g_0 \text{Tr} \left[ S_{0f}(x,x) \gamma_5 \lambda^j \right] = 0.
\]

To find a solution of Eq. (11), we write down it in the momentum space:

\[
\left[ i\tilde{p} + \tilde{m} - i\tilde{m}\gamma_5 \right] S_{0f}(p) = 1,
\]

where \( \tilde{p} = p^\mu \gamma_\mu \), \( p_\mu = (p, ip_0) \). It is easy to verify that the solution to Eq. (15) for the Green function is given by

\[
S_{0f}(p) = \text{diag} \left( \frac{-i\tilde{p} + m_{01} + i\tilde{m}_1\gamma_5}{p^2 + m_1^2}, \frac{-i\tilde{p} + m_{02} + i\tilde{m}_2\gamma_5}{p^2 + m_2^2}, \frac{-i\tilde{p} + m_{03} + i\tilde{m}_3\gamma_5}{p^2 + m_3^2} \right),
\]

where

\[
m_1^2 = m_{01}^2 + \tilde{m}_1^2, \quad m_2^2 = m_{02}^2 + \tilde{m}_2^2, \quad m_3^2 = m_{03}^2 + \tilde{m}_3^2.
\]

The poles of the Green function (16) define the dynamical (constituent) masses of \( u, d \) and \( s \) quarks: \( m_1, m_2, m_3 \). The scalar \( (\sigma_j) \) and pseudoscalar \( (\tilde{\sigma}_j) \) condensates contribute to the constituent masses of all quarks. The terms containing \( \tilde{m}_j \) in Eq. (16) violate CP symmetry. Substituting Eq. (16) into Eqs. (14), one obtains a system of gap equations:

\[
M^2 \sigma_0 = g_0 \sqrt{\frac{2}{3}} (I_1 m_{01} + I_2 m_{02} + I_3 m_{03}), \quad M^2 \sigma_3 = g_0 (I_1 m_{01} - I_2 m_{02}),
\]

\[
M^2 \sigma_8 = \frac{g_0}{\sqrt{3}} (I_1 m_{01} + I_2 m_{02} - 2I_3 m_{03}),
\]

\[
M^2 \tilde{\sigma}_0 = -g_0 \sqrt{\frac{2}{3}} (\tilde{m}_1 I_1 + \tilde{m}_2 I_2 + \tilde{m}_3 I_3),
\]
where quadratic diverging integrals are given by

\[ I_j = \frac{i N_C}{4 \pi^4} \int \frac{d^4 p}{p^2 + m_j^2} = \frac{N_C}{4 \pi^2} \left[ m_j^2 \ln \left( \frac{\Lambda^2}{m_j^2} + 1 \right) - \Lambda^2 \right], \quad (19) \]

where \( d^4 p = id^3 p dp_0 \), the \( \Lambda \) is a cutoff and there is no summation in index \( j \) \((j = 1, 2, 3)\) in Eq. (19). The self-consistent equations (18) connect such parameters of a model as the dimensional constant \( G \), condensates \( \sigma_j \) (or dynamical masses of quarks) and a cutoff. The system of six gap equations (18), defining the vacuum expectations \( \sigma_j, \tilde{\sigma}_j \), with the help of Eqs. (10) can be rewritten as

\[
\begin{align*}
(m_u - m_{01}) &= 2m_{01}GI_1, \quad (m_d - m_{02}) = 2m_{02}GI_2, \quad (m_s - m_{03}) = 2m_{03}GI_3, \\
- \tilde{m}_1 &= 2\tilde{m}_1GI_1, \quad - \tilde{m}_2 = 2\tilde{m}_2GI_2, \quad - \tilde{m}_3 = 2\tilde{m}_3GI_3. \quad (20)
\end{align*}
\]

There are different solutions of gap equations (20), (21). We are interested here in the possibilities of CP violation \( (\tilde{m}_j \neq 0) \). Therefore consider the case when Eqs. (21) have non-trivial solutions. It follows from Eqs. (21) that if three vacuum expectations \( \tilde{m}_j \) \((j = 1, 2, 3)\) do not equal zero, \( \tilde{m}_j \neq 0 \), then \( I_1 = I_2 = I_3 \), and therefore \( m_1 = m_2 = m_3 \). This is not interesting case because the strange quark \( s \) is much heavier than the \( u \) and \( d \) quarks. Another solution is \( \tilde{m}_3 = 0, \tilde{m}_1 \neq 0, \tilde{m}_2 \neq 0 \). Then from Eqs. (21), we arrive at the case \( m_1 = m_2, \tilde{m}_1 = \tilde{m}_2 \), i.e. isotopic symmetry is not broken, and the gap equation becomes \( 2g_0^2 I_1 = -M^2 \) \((I_1 = I_2)\). Comparing this equation with Eqs. (20), one makes a conclusion that \( m_u = m_d = 0 \), i.e. the chiral limit for the light quarks is realized. We expect that pions \((\pi^\pm, \pi^0)\) will be massless Goldstone particles in this case. Requiring \( m_s \neq 0 \), one arrives from Eqs. (20) to two gap equations

\[
(m_s - m_{03}) = 2m_{03}GI_3, \quad -1 = 2GI_1. \quad (22)
\]

At the same time, if there is no CP violation, \( \tilde{m}_1 = \tilde{m}_2 = \tilde{m}_3 = 0 \), we can analyze the case \( m_u \neq 0, m_d \neq 0, m_s \neq 0 \), and gap equations (20) are valid (see [5], [6] for other studies). We pay attention here in the case \( \tilde{m}_1 = \tilde{m}_2 \neq 0, \tilde{m}_3 = 0, m_u = m_d = 0 \), which requires (see Eqs. (10))

\[
\sigma_3 = \tilde{\sigma}_3 = 0, \quad \tilde{\sigma}_0 = \sqrt{2}\tilde{\sigma}_8. \quad (23)
\]
The independent parameters here are the current quark mass \( m_s \), the cutoff \( \Lambda \), and the dimensional constant \( G \).

From Eq. (12), one may obtain the part of effective action which does not depend on coordinates:

\[
S[\sigma, \tilde{\sigma}] = -\frac{M^2}{2} \int d^4x \left[ (\sigma_j)^2 + (\tilde{\sigma}_j)^2 \right] - i \text{Tr} \ln \left( -\gamma_\mu \partial_\mu - \tilde{m} + i\tilde{m}\gamma_5 \right). \tag{24}
\]

We can use the relation \( S[\sigma, \tilde{\sigma}] = -\int d^4x V_{\text{eff}} \) for the constant fields. As a result, one may get from Eq. (24), with the help of Eq. (10), the effective potential

\[
V_{\text{eff}} = \frac{M^2}{4g_0^2} \left[ (m_{01} - m_u)^2 + (m_{02} - m_d)^2 + (m_{03} - m_s)^2 \right] + \frac{iN_C}{8\pi^4} \int d^4p \ln \left( p^2 + m_1^2 \right) \left( p^2 + m_2^2 \right) \left( p^2 + m_3^2 \right). \tag{25}
\]

We will keep all parameters to be nonzero for the possibility to study as the case with CP violation as well as the case without CP breaking. Eqs. (18) or (20), (21) may be obtained from the condition of effective potential (25) to realize the minimum:

\[
\frac{\partial V_{\text{eff}}}{\partial m_{0j}} = \frac{\partial V_{\text{eff}}}{\partial \tilde{m}_j} = 0 \quad (j = 1, 2, 3). \tag{26}
\]

To obtain the mass spectrum of mesons, one needs to evaluate the terms in Eq. (12), quadratic in fields \( \phi'_a, \tilde{\phi}'_a \). From Eq. (12) we find

\[
S^{(2)}[\Phi'] = -\frac{M^2}{2} \int d^4x \left[ \phi'^2_a + \tilde{\phi}'^2_a \right] + \frac{i}{2} \text{Tr} (g_0S_0\Phi'_a\lambda^a)^2 \\
\equiv -\frac{1}{2} \int d^4x d^4y \phi'_A(x)\Delta^{-1}_{AB}(x, y)\phi'_B(y). \tag{27}
\]

In the momentum space the inverse meson symmetric propagator is given by

\[
\Delta^{-1}_{AB}(p) = -ig_0^2 \text{tr} \left[ \int \frac{d^4k}{(2\pi)^4} S_0f(k)T_A S_0f(k - p)T_B \right] + \delta_{AB}M^2_A, \tag{28}
\]

where \( T_A = (\lambda^a, i\gamma_5\lambda^a), \phi'_A = (\phi'_a, \tilde{\phi}'_a) \), and we use the notation \( A = (a, \tilde{a}) \).
Evaluating the traces in Eqs. (28), we obtain the nonzero elements of the inverse propagators of the scalar ($\Phi'_a(x)$) mesons:

$$
\Delta^{-1}_{00}(p) = M^2 + g_6^2 \frac{2}{3} (I_1 + I_2 + I_3) + \frac{1}{3} \left( p^2 + 4m_{01}^2 \right) I_{11}(p)
$$

$$
+ \frac{1}{3} \left( p^2 + 4m_{02}^2 \right) I_{22}(p) + \frac{1}{3} \left( p^2 + 4m_{03}^2 \right) I_{33}(p),
$$

$$
\Delta^{-1}_{11}(p) = \Delta^{-1}_{22}(p) = M^2 + g_6^2 (I_1 + I_2) + \frac{1}{2} \left( p^2 + 4m_{01}^2 \right) I_{11}(p) + \frac{1}{2} \left( p^2 + 4m_{02}^2 \right) I_{22}(p),
$$

$$
\Delta^{-1}_{33}(p) = M^2 + g_6^2 (I_1 + I_2) + \frac{1}{2} \left( p^2 + 4m_{01}^2 \right) I_{11}(p) + \frac{1}{3} \left( p^2 + 4m_{02}^2 \right) I_{22}(p),
$$

$$
\Delta^{-1}_{44}(p) = \Delta^{-1}_{55}(p) = M^2 + g_6^2 (I_1 + I_3) + \frac{1}{2} \left( p^2 + 4m_{01}^2 \right) I_{11}(p) + \frac{1}{2} \left( p^2 + 4m_{03}^2 \right) I_{33}(p),
$$

$$
\Delta^{-1}_{66}(p) = \Delta^{-1}_{77}(p) = M^2 + g_6^2 (I_2 + I_3) + \frac{1}{2} \left( p^2 + 4m_{01}^2 \right) I_{11}(p) + \frac{2}{3} \left( p^2 + 4m_{03}^2 \right) I_{33}(p),
$$

$$
\Delta^{-1}_{88}(p) = M^2 + g_6^2 \frac{2}{3} (I_1 + I_2 + 4I_3) + \frac{1}{6} \left( p^2 + 4m_{01}^2 \right) I_{11}(p)
$$

$$
+ \frac{1}{6} \left( p^2 + 4m_{02}^2 \right) I_{22}(p) + \frac{2}{3} \left( p^2 + 4m_{03}^2 \right) I_{33}(p),
$$

$$
\Delta^{-1}_{03}(p) = g_6 \sqrt{\frac{2}{3}} (I_1 - I_2) + \frac{1}{\sqrt{6}} \left( p^2 + 4m_{01}^2 \right) I_{11}(p) - \frac{1}{\sqrt{6}} \left( p^2 + 4m_{02}^2 \right) I_{22}(p),
$$

$$
\Delta^{-1}_{08}(p) = g_6^2 \frac{\sqrt{2}}{3} (I_1 + I_2 - 2I_3) + \frac{\sqrt{2}}{6} \left( p^2 + 4m_{01}^2 \right) I_{11}(p)
$$

$$
+ \frac{\sqrt{2}}{6} \left( p^2 + 4m_{02}^2 \right) I_{22}(p) - \frac{\sqrt{2}}{3} \left( p^2 + 4m_{03}^2 \right) I_{33}(p),
$$

$$
\Delta^{-1}_{38}(p) = \Delta^{-1}_{03}(p).
$$

One can get from Eq. (28) the inverse propagators of pseudoscalar ($\bar{\Phi}'_a(x)$) mesons:

$$
\Delta^{-1}_{00}(p) = M^2 + g_6^2 \frac{2}{3} (I_1 + I_2 + I_3) + \frac{1}{3} \left( p^2 + 4\bar{m}_1^2 \right) I_{11}(p)
$$

$$
+ \frac{1}{3} \left( p^2 + 4\bar{m}_2^2 \right) I_{22}(p) + \frac{1}{3} \left( p^2 + 4\bar{m}_3^2 \right) I_{33}(p),
$$

$$
\Delta^{-1}_{11}(p) = \Delta^{-1}_{22}(p) = M^2 + g_6^2 (I_1 + I_2)
$$

$$
+ \left[ p^2 + (m_{02} - m_{01})^2 + (\bar{m}_1 + \bar{m}_2)^2 \right] I_{11}(p),
$$

$$
\Delta^{-1}_{33}(p) = M^2 + g_6^2 (I_1 + I_2)
$$

$$
+ \left[ p^2 + (m_{03} - m_{01})^2 + (\bar{m}_1 + \bar{m}_3)^2 \right] I_{11}(p),
$$

$$
\Delta^{-1}_{44}(p) = \Delta^{-1}_{55}(p) = M^2 + g_6^2 (I_1 + I_3)
$$

$$
+ \left[ p^2 + (m_{03} - m_{02})^2 + (\bar{m}_2 + \bar{m}_3)^2 \right] I_{11}(p),
$$

$$
\Delta^{-1}_{66}(p) = \Delta^{-1}_{77}(p) = M^2 + g_6^2 (I_2 + I_3)
$$

$$
+ \left[ p^2 + (m_{01} - m_{02})^2 + (\bar{m}_1 + \bar{m}_2)^2 \right] I_{11}(p),
$$

$$
\Delta^{-1}_{88}(p) = M^2 + g_6^2 \frac{2}{3} (I_1 + I_2 + 4I_3) + \frac{1}{6} \left( p^2 + 4\bar{m}_1^2 \right) I_{11}(p)
$$

$$
+ \frac{1}{6} \left( p^2 + 4\bar{m}_2^2 \right) I_{22}(p) + \frac{2}{3} \left( p^2 + 4\bar{m}_3^2 \right) I_{33}(p),
$$

$$
\Delta^{-1}_{03}(p) = g_6 \sqrt{\frac{2}{3}} (I_1 - I_2) + \frac{1}{\sqrt{6}} \left( p^2 + 4\bar{m}_1^2 \right) I_{11}(p) - \frac{1}{\sqrt{6}} \left( p^2 + 4\bar{m}_2^2 \right) I_{22}(p),
$$

$$
\Delta^{-1}_{08}(p) = g_6^2 \frac{\sqrt{2}}{3} (I_1 + I_2 - 2I_3) + \frac{\sqrt{2}}{6} \left( p^2 + 4\bar{m}_1^2 \right) I_{11}(p)
$$

$$
+ \frac{\sqrt{2}}{6} \left( p^2 + 4\bar{m}_2^2 \right) I_{22}(p) - \frac{\sqrt{2}}{3} \left( p^2 + 4\bar{m}_3^2 \right) I_{33}(p),
$$

$$
\Delta^{-1}_{38}(p) = \Delta^{-1}_{03}(p).
where the quadratic diverging integrals read

\[ \Delta_{33}^{-1}(p) = M^2 + g_0^2 (I_1 + I_2) + \frac{1}{2} \left( p^2 + 4\tilde{m}_1^2 \right) I_{11}(p) + \frac{1}{2} \left( p^2 + 4\tilde{m}_2^2 \right) I_{22}(p), \]
\[ \Delta_{44}^{-1}(p) = \Delta_{55}^{-1}(p) = M^2 + g_0^2 (I_1 + I_3), \]
\[ + \left[ p^2 + (m_{03} - m_{01})^2 + (\tilde{m}_1 + \tilde{m}_3)^2 \right] I_{13}(p), \]
\[ \Delta_{66}^{-1}(p) = \Delta_{77}^{-1}(p) = M^2 + g_0^2 (I_2 + I_3), \]
\[ + \left[ p^2 + (m_{03} - m_{02})^2 + (\tilde{m}_2 + \tilde{m}_3)^2 \right] I_{23}(p), \]
\[ \Delta_{88}^{-1}(p) = M^2 + \frac{g_0^2}{3} (I_1 + I_2 + 4I_3) + \frac{1}{6} \left( p^2 + 4\tilde{m}_1^2 \right) I_{11}(p), \]
\[ + \frac{1}{6} \left( p^2 + 4\tilde{m}_2^2 \right) I_{22}(p) + \frac{2}{3} \left( p^2 + 4\tilde{m}_3^2 \right) I_{33}(p), \]
\[ \Delta_{03}^{-1}(p) = g_0^3 \sqrt{\frac{2}{3}} \left( I_1 - I_2 \right) + \frac{1}{\sqrt{6}} \left( p^2 + 4\tilde{m}_1^2 \right) I_{11}(p) - \frac{1}{\sqrt{6}} \left( p^2 + 4\tilde{m}_2^2 \right) I_{22}(p), \]
\[ \Delta_{08}^{-1}(p) = \frac{g_0^3 \sqrt{2}}{3} (I_1 + I_2 - 2I_3) + \frac{\sqrt{2}}{3} \left( p^2 + 4\tilde{m}_1^2 \right) I_{11}(p), \]
\[ + \frac{\sqrt{2}}{6} \left( p^2 + 4\tilde{m}_2^2 \right) I_{22}(p) - \frac{\sqrt{2}}{3} \left( p^2 + 4\tilde{m}_3^2 \right) I_{33}(p), \quad \Delta_{03}^{-1}(p) = \sqrt{2} \Delta_{38}^{-1}(p). \]

Non-diagonal scalar-pseudoscalar elements of inverse propagators are given by

\[ \Delta_{00}^{-1}(p) = -\frac{4}{3} \left[ m_{01}\tilde{m}_1 I_{11}(p) + m_{02}\tilde{m}_2 I_{22}(p) + m_{03}\tilde{m}_3 I_{33}(p) \right], \]
\[ \Delta_{80}^{-1}(p) = \Delta_{08}^{-1}(p) = \frac{2\sqrt{2}}{3} \left[ 2m_{03}\tilde{m}_3 I_{33}(p) - m_{01}\tilde{m}_1 I_{11}(p) - m_{02}\tilde{m}_2 I_{22}(p) \right], \]
\[ \Delta_{30}^{-1}(p) = \Delta_{03}^{-1}(p) = \Delta_{38}^{-1}(p) = \sqrt{2}\Delta_{83}^{-1}(p) \]
\[ = 2\sqrt{\frac{2}{3}} \left[ m_{02}\tilde{m}_2 I_{22}(p) - m_{01}\tilde{m}_1 I_{11}(p) \right], \]

where the quadratic diverging integrals read

\[ I_{ij}(p) = \frac{ig_0^2 N_C}{4\pi^4} \int \frac{d^4k}{(k^2 + m_i^2) \left[ (k^2 - p)^2 + m_j^2 \right]} \]
\[ = \frac{g_0^2 N_C}{4\pi^2} \left[ \ln \left( \frac{\Lambda^2}{m_i^2} \right) - 1 - \int_0^1 dx \ln \frac{m_j^2 + x(m_i^2 - m_j^2) + p^2x(1-x)}{m_i^2} \right], \]

and there is no summation in indexes \( i, j \). Inverse propagators (29)-(31) define spectrum of mass for the general case including CP violation.
4 Effective action and mass spectrum of mesons

Poles of the propagators (28) give the masses of mesons which can be estimated by numerical calculations. Here we make some evaluations of meson masses. From Eqs. (29)-(31), one can find the effective action of the mesonic "free" fields

\[ S_{\text{free}}[\Phi] = -\frac{1}{2} \int d^4x \left[ (\partial_\mu \Phi_A(x))^2 + m_{AB}^2 \phi_A(x) \phi_B(x) \right] , \tag{33} \]

where \( A = (a, \tilde{a}) \), \( \phi_\tilde{a} \equiv \tilde{\phi}_a \). The eigenvalues of the symmetric mass matrix \( m_{AB}^2 \) define the mass spectrum. To obtain the mass matrix, we renormalize fields \( \tilde{\phi}_a(x) = Z^{-1/2} \phi'_a(x) \), \( \phi_a(x) = Z^{-1/2} \phi'_a(x) \), and the constant \( g^2 = Z g_0^2 \), so that the variables \( g \phi_a \), \( g \tilde{\phi}_a \) are the renormalization-invariant values. It follows from Eq. (32) that the renormalization constant can be defined as follows

\[ Z^{-1} = \frac{g_0^2 N_C}{4 \pi^2} \left[ \ln \left( \frac{\Lambda^2}{m_1^2} \right) - 1 \right] . \tag{34} \]

It is seen from Eq. (34) that the expansion in \( g^2/4\pi^2 \), corresponds to the \( 1/N_C \) expansion. We imply here a cutoff \( \Lambda \) is chosen in such a way that \( g^2/4\pi^2 < 1 \). Using the gap equations (20), (21), in the leading order, we find from Eqs. (29) the elements of the mass matrix for scalar mesons:

\[ m_{00}^2 = g^2 \frac{2}{3} \left( \frac{m_u I_1}{m_u - m_{01}} + \frac{m_d I_2}{m_d - m_{02}} + \frac{m_s I_3}{m_s - m_{03}} \right) + \frac{4}{3} \left( m_{01}^2 + m_{02}^2 + m_{03}^2 \right) , \]

\[ m_{11}^2 = m_{22}^2 = g^2 \left( \frac{m_u I_1}{m_u - m_{01}} + \frac{m_d I_2}{m_d - m_{02}} \right) + (m_{02} + m_{01})^2 + (\tilde{m}_1 - \tilde{m}_2)^2 , \]

\[ m_{33}^2 = g^2 \left( \frac{m_u I_1}{m_u - m_{01}} + \frac{m_d I_2}{m_d - m_{02}} \right) + 2 \left( m_{01}^2 + m_{02}^2 \right) , \]

\[ m_{44}^2 = m_{55}^2 = g^2 \left( \frac{m_u I_1}{m_u - m_{01}} + \frac{m_s I_3}{m_s - m_{03}} \right) + (m_{03} + m_{01})^2 + (\tilde{m}_1 - \tilde{m}_3)^2 , \]

\[ m_{66}^2 = m_{77}^2 = g^2 \left( \frac{m_d I_2}{m_d - m_{02}} + \frac{m_s I_3}{m_s - m_{03}} \right) + (m_{03} + m_{02})^2 + (\tilde{m}_2 - \tilde{m}_3)^2 , \]

\[ m_{88}^2 = g^2 \left( \frac{m_u I_1}{m_u - m_{01}} + \frac{m_d I_2}{m_d - m_{02}} + \frac{4 m_s I_3}{m_s - m_{03}} \right) + \frac{2}{3} \left( m_{01}^2 + m_{02}^2 + 4 m_{03}^2 \right) , \tag{35} \]
\[ m_{08}^2 = \frac{g^2 \sqrt{2}}{3} (I_1 + I_2 - 2I_3) + \frac{2 \sqrt{2}}{3} (m_{01}^2 + m_{02}^2 - 2m_{03}^2), \]
\[ m_{03}^2 = \sqrt{2} m_{38}^2 = g^2 \sqrt{2} \left( \frac{2}{3} (I_1 - I_2) + 2 \right) \left( m_{01}^2 - m_{02}^2 \right). \]

One can obtain from Eqs. (30) the elements of the mass matrix for pseudoscalar mesons:

\[ m_{0\bar{0}}^2 = \frac{g^2 \sqrt{2}}{3} \left( \frac{m_u I_1}{m_u - m_{01}} + \frac{m_d I_2}{m_d - m_{02}} + \frac{m_s I_3}{m_s - m_{03}} \right) + \frac{4}{3} \left( \tilde{m}_1^2 + \tilde{m}_2^2 + \tilde{m}_3^2 \right), \]
\[ m_{0\bar{8}}^2 = \frac{g^2 \sqrt{2}}{3} (I_1 + I_2 - 2I_3) + \frac{2 \sqrt{2}}{3} \left( \tilde{m}_1^2 + \tilde{m}_2^2 - 2\tilde{m}_3^2 \right), \]
\[ m_{0\bar{3}}^2 = \sqrt{2} m_{38}^2 = g^2 \sqrt{2} \left( \frac{2}{3} (I_1 - I_2) + 2 \right) \left( \tilde{m}_1^2 - \tilde{m}_2^2 \right), \]
\[ m_{1\bar{1}}^2 = m_{2\bar{2}}^2 = g^2 \left( \frac{m_u I_1}{m_u - m_{01}} + \frac{m_d I_2}{m_d - m_{02}} \right) + (m_{02} - m_{01})^2 + (\tilde{m}_1 + \tilde{m}_2)^2, \]
\[ m_{4\bar{4}}^2 = m_{5\bar{5}}^2 = g^2 \left( \frac{m_u I_1}{m_u - m_{01}} + \frac{m_s I_3}{m_s - m_{03}} \right) + (m_{03} - m_{01})^2 + (\tilde{m}_1 + \tilde{m}_3)^2, \]
\[ m_{6\bar{6}}^2 = m_{7\bar{7}}^2 = g^2 \left( \frac{m_d I_2}{m_d - m_{02}} + \frac{m_s I_3}{m_s - m_{03}} \right) + (m_{03} - m_{02})^2 + (\tilde{m}_2 + \tilde{m}_3)^2, \]
\[ m_{3\bar{3}}^2 = g^2 \left( \frac{m_u I_1}{m_u - m_{01}} + \frac{m_d I_2}{m_d - m_{02}} \right) + 2 \left( \tilde{m}_1^2 + \tilde{m}_2^2 \right), \]
\[ m_{8\bar{8}}^2 = \frac{g^2}{3} \left( \frac{m_u I_1}{m_u - m_{01}} + \frac{m_d I_2}{m_d - m_{02}} + \frac{4m_s I_3}{m_s - m_{03}} \right) + \frac{2}{3} \left( \tilde{m}_1^2 + \tilde{m}_2^2 + 4\tilde{m}_3^2 \right). \]

Using Eqs. (31), we find non-diagonal scalar-pseudoscalar elements of the mass matrix

\[ m_{50}^2 = -\frac{4}{3} (m_{01} \tilde{m}_1 + m_{02} \tilde{m}_2 + m_{03} \tilde{m}_3), \]
\[ m_{80}^2 = m_{08}^2 = \frac{2 \sqrt{2}}{3} (2m_{03} \tilde{m}_3 - m_{01} \tilde{m}_1 - m_{02} \tilde{m}_2), \]
\[ m_{30}^2 = m_{38}^2 = \sqrt{2} m_{58}^2 = \sqrt{2} m_{38}^2 = 2 \left( \frac{2}{3} (m_{02} \tilde{m}_2 - m_{01} \tilde{m}_1) \right). \]
It is seen from Eqs. (35)-(37) the Goldstone nature of pseudoscalar mesons: if bare masses of quarks are zero, \( \sigma_0 = \sigma_3 = \sigma_8 = 0, \tilde{m}_j = 0 \), all pseudoscalar mesons are massless. We recall that if \( \tilde{m}_j \neq 0 (j = 1, 2) \), gap equations require the chiral limit: \( m_u = m_d = 0 \). If there is no CP violation (\( \tilde{m}_j = 0 \)), one can consider the case \( m_u \neq 0, m_d \neq 0 \) to have nonzero pion masses. It follows from Eq. (37) that there is mixing of scalar \( \phi_a(x) \) and pseudoscalar \( \tilde{\phi}_a(x) \) fields due to the CP-violating condensates \( \tilde{m}_j \). The fields \( \tilde{\phi}_0 \) and \( \tilde{\phi}_8 \) are also mixed corresponding to \( \eta - \eta' \) mixing. Pions, connected with the fields \( \tilde{\phi}_i(x) (i = 1, 2, 3) \), acquire nonzero masses due to the presence of the CP-violating condensates even for zero current masses \( m_u = m_d = 0 \). At the same time in the case \( \tilde{m}_1 = \tilde{m}_2, \tilde{m}_3 = 0 \) there is less contribution of CP violating condensates to the masses of scalar mesons.

To obtain the diagonal matrix \( m_{AB} \), one can make the transformation of the rotation group for fields \( \phi_a(x), \tilde{\phi}_a(x) (a = 0, 3, 8) \). For the simple mixing of fields \( \tilde{\phi}_0(x), \tilde{\phi}_8(x) \), one obtains

\[
\tilde{\phi}_0'(x) = \tilde{\phi}_0(x) \cos \theta_P - \tilde{\phi}_8(x) \sin \theta_P, \\
\tilde{\phi}_8'(x) = \tilde{\phi}_0(x) \sin \theta_P + \tilde{\phi}_8(x) \cos \theta_P, 
\]

where \( \tan 2\theta_P = \frac{2 \tilde{m}_0^2}{m_{08}^2 - m_{88}^2} \). The masses of bosonic fields \( \tilde{\phi}_0'(x), \tilde{\phi}_8'(x) \) became:

\[
m_{00}^2 = m_{00}^2 \cos^2 \theta_P + m_{88}^2 \sin^2 \theta_P - m_{08}^2 \sin 2\theta_P, \\
m_{88}^2 = m_{00}^2 \sin^2 \theta_P + m_{88}^2 \cos^2 \theta_P + m_{08}^2 \sin 2\theta_P. 
\]

We consider the case \( m_1 = m_2 \) when the isotopic symmetry is conserved. It follows then from Eqs. (10) that this requires the vacuum expectation value \( \sigma_3 = 0 \). The relation \( m_1 - m_u = -g(\sqrt{2}\sigma_0 + \sigma_8)/\sqrt{3} \) (after the renormalization of the constant \( g_0 \)) is treated as the quark level version of the Goldberger-Treiman identity \[15\] with the pion decay constant \( (\sqrt{2}\sigma_0 + \sigma_8)/\sqrt{3} = f_\pi = 93 \text{ MeV} \). We imply here a very small possible contribution of CP violating condensates to the real masses of mesons. Putting here the value of the constant \[16\] \( g = 3.628 \), one finds that the parameter of expansion is \( g^2/4\pi^2 = 1/N_C = 1/3 \). Using the freedom in the choice of the bare quark mass, we set \( m_u = m_d = 5.3 \text{ MeV} \). From the Goldberger-Treiman relation one obtains the constituent masses of the light quarks.
$m_1 = m_2 = 342.7$ MeV ($\tilde{m}_j = 0$). It follows from Eq. (34): the covariant cutoff $\Lambda$ is given by $\Lambda = e m_1 = 931.5$ MeV. To find the constituent mass of the $s$-quark, we find from the gap equations (20) the self-consistent relation

$m_1(m_s - m_3)I_1 = m_3(m_u - m_1)I_3$. Setting the free parameter of the $s$-quark current mass $m_s = 166$ MeV, for a given cutoff, one obtains the dynamical strange quark mass: $m_3 = 570$ MeV. With the help of these masses and the cutoff $\Lambda$, we calculate from Eqs. (36) the masses of $\pi$, $K$ mesons and quark condensates

$$m_\pi = 139 \text{ MeV}, \quad m_K = 494 \text{ MeV},$$

$$\langle \pi u \rangle = \langle \pi d \rangle = m_1 I_1 = (-252 \text{ MeV})^3, \quad \langle \pi s \rangle = m_3 I_3 = (-268 \text{ MeV})^3.$$ (40)

Masses of $K$ mesons are degenerated here as well as masses of pions. The masses and condensates (40) are agreed with the phenomenology. The pseudoscalar $\eta' - \eta$ mixing angle, evaluated from Eqs. (38), is $\theta_P = -35^\circ$. Masses of $\eta$, $\eta'$ and their mixing angle are not described correctly here because we did not take into consideration anomaly and the axial symmetry $U_A(1)$ is not broken. It is easy to verify that the Gell-Mann–Oakes–Renner [17] relation $f_\pi^2 m_\pi^2 = -2m_u \langle \pi u \rangle$ is approximately valid.

From Eqs. (35) we obtain the elements of the mass matrix corresponding to the nonet of scalar mesons

$$m_{00} = 938 \text{ MeV}, \quad m_{11} = m_{22} = m_{33} = 699 \text{ MeV},$$

$$m_{44} = m_{55} = m_{66} = m_{77} = 1013 \text{ MeV}, \quad m_{88} = 1128 \text{ MeV}. \quad (41)$$

The mixing angle of the $\phi_0$ and $\phi_8$ fields is $\theta_S = -35^\circ$. Scalar and pseudoscalar fields are not mixed in the case (see Eqs. (37)) when the equality $\tilde{m}_j = 0$ is valid. We do not identify here the scalar mesons $\phi_k(x)$ with the nonet of known scalar mesons: $\sigma(560)$, $f_0(980)$, $\kappa(900)$, $a_0(980)$ due to their complicated nature: there are contributions of four-quark states and gluons in these mesons [18, 19, 20].

5 Conclusion

The model under consideration can describe CP violation in strong interactions. There is a contribution of CP violating condensates, $\tilde{m}_j$, to constituent masses of u, d and s quarks and to masses of scalar and pseudoscalar
mesons. If the current masses of quarks equal zero, and the CP-violating condensate $\bar{m}_j = 0$, all pseudoscalar mesons $\pi$, $K$, $\eta$, $\eta'$ become massless Goldstone bosons. Masses of all K-mesons are degenerated in the case $m_{01} = m_{02}$. In this model, the appearance of CP-violating condensates leads to the chiral limit: $m_u = m_d = 0$. In the particular case $\bar{m}_j = 0$, when there is no CP violation, the model gives reasonable dynamical quark masses, masses of $\pi$, $K$ mesons and quark condensates. At the same time $\eta$ and $\eta'$ mesons can not be described correctly in the framework of the model considered because the $U_A(1)$ symmetry is not broken. To take into consideration the $U_A(1)$-anomaly, one may generalize the model by including the determinant six-quark interaction (due to instantons) violating $U_A(1)$ symmetry.[6]

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