

A Framework for Network A/B Test

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ABSTRACT

A/B testing, also known as controlled experiment, bucket testing or splitting testing, has been widely used for evaluating a new feature, service or product in the data-driven decision processes of online websites. The goal of A/B testing is to estimate or test the difference between the treatment effects of the old and new variations. It is a well-studied two-sample comparison problem if each user’s response is influenced by her treatment only. However, in many applications of A/B testing, especially those in HIVE of Yahoo and other social networks of Microsoft, Facebook, LinkedIn, Twitter and Google, users in the social networks influence their friends via underlying social interactions, and the conventional A/B testing methods fail to work. This paper considers the network A/B testing problem and provides a general framework consisting of five steps: data sampling, probabilistic model, parameter inference, computing average treatment effect and hypothesis test. The framework performs well for network A/B testing in simulation studies.

Keywords

Network A/B Testing; Ising Model; Logit Model; Maximum Pseudo-likelihood Estimate; Bootstrapping

1. INTRODUCTION

A/B testing, also known as controlled experiment, bucket testing or splitting testing, has been widely used for evaluating a new feature, service or product in the data-driven decision processes of online websites, especially social networks of Yahoo, Microsoft, Facebook, LinkedIn, Twitter and Google [9, 6, 7, 8]. The goal of A/B testing is to estimate or test the average treatment effect (ATE), which is defined as

\[
\text{ATE} = \frac{\mu_A - \mu_B}{\sigma^2}
\]

where \(\mu_A\) and \(\mu_B\) are the mean responses for groups A and B, and \(\sigma^2\) is the variance of the responses.

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Recent A/B testing in social networks has gained sharpened focus (see [2, 20, 4] among others). The issues in the network A/B testing problem have been addressed from different aspects. Backstrom and Kleinberg [2] described a random-walk-based sampling method for producing samples of users (nodes in the network) that are internally well-connected but also approximately uniform over the population. Ugander et. al. [20] show in a simplified setting how
The significance of the point estimate for ATE is usually then. If two groups of users \(Y\) in a social network is a mixture of treatment effect in the setting of network A/B testing, the responses of users for ATE average responses of group A and B is a consistent estimate which further implying that the difference between the averages and more and more users are involved. Moreover, neither Student’s t test nor Fisher’s exact test is applicable for the comparison of two non-i.i.d. and dependent samples.

2. FROM A/B TESTING TO NETWORK A/B TESTING

Denote by \(V\) the set of users, by \(Z_i = 0\) or 1 whether user \(i \in V\) is assigned to group A or B, and by \(Y_i\), its response. The average treatment effect (ATE) is defined by (1).

\[
\text{ATE} = \mathbb{E}\left[\frac{1}{|V|} \sum_{i \in V} Y_i | Z = 1 \right] - \mathbb{E}\left[\frac{1}{|V|} \sum_{i \in V} Y_i | Z = 0 \right]
\]

If two groups of users \(\{Y_i : i \in A\}\) and \(\{Y_i : i \in B\}\) follow the distributions \(Y | Z = 0\) and \(Y | Z = 1\) i.i.d. respectively, then

\[
\mathbb{E}[Y | Z = 0] = \mathbb{E}\left[\frac{1}{|V|} \sum_{i \in V} Y_i | Z = 0 \right],
\]
\[
\mathbb{E}[Y | Z = 1] = \mathbb{E}\left[\frac{1}{|V|} \sum_{i \in V} Y_i | Z = 1 \right],
\]

and thus the ATE is simplified as

\[
\text{ATE} = \mathbb{E}[Y | Z = 1] - \mathbb{E}[Y | Z = 0].
\]

From the strong law of large number it follows that

\[
\frac{1}{|A|} \sum_{i \in A} Y_i \xrightarrow{a.s.} \mathbb{E}[Y | Z = 0],
\]
\[
\frac{1}{|B|} \sum_{i \in B} Y_i \xrightarrow{a.s.} \mathbb{E}[Y | Z = 1]
\]
as more and more users are involved in the experiment, which further implying that the difference between the average responses of group A and B is an consistent estimate for ATE

\[
\frac{1}{|B|} \sum_{i \in B} Y_i - \frac{1}{|A|} \sum_{i \in A} Y_i \xrightarrow{a.s.} \text{ATE}
\]

The significance of the point estimate for ATE is usually evaluated by a statistical hypothesis test and indexed by a p-value. To statistically compare two i.i.d. samples \(\{Y_i : i \in A\}\) and \(\{Y_i : i \in B\}\), it is natural to apply Student’s t test or Fisher’s exact test.

However, the i.i.d. assumption (or SUTVA) does not hold in the setting of network A/B testing, the responses of users in a social network is a mixture of treatment effect, network effect and spill-over effect. Figure 1 illustrates a small social network of 8 users (nodes) and 9 friendship relationships (edges). Five users belong to group A (red), while the other three users belong to group B (blue). Solid lines present the network effect, namely the peer-to-peer influence within either group A or B. Dashed lines present the spill-over effect, namely the peer-to-peer influence across group A and B.

3. A FRAMEWORK FOR NETWORK A/B TESTING

This paper aims to provide a general framework for network A/B testing, which consists of 5 steps:

- data sampling,
- probabilistic model,
- parameter inference,
- ATE computation, and
- hypothesis test.

Such a framework is compatible to many existing studies on the network A/B testing problem and enables a more comprehensive solution.
3.1 Data Sampling

As before, we denote by $G(V, E)$ the underlying social (sub-)network of users involved in the A/B test, by $Z = \{Z_i : i \in V\}$ the vector of assignment, and by $Y = \{Y_i : i \in V\}$ the vector of response. The step of data sampling is to sample a triplet $(G, Z, Y)$ for A/B test in real or simulated a social network.

![Figure 2: Data sampling process.](image)

3.2 Probabilistic Model

Markov Random Field is the standard model for the social network in which random variables $Y = \{Y_i : i \in V\}$ have a jointly distribution in an undirected graph $G(V, E)$ given the treatment/control assignment $Z$. Denote by $\theta$ the parameter of the model, and by $P_\theta(Y|Z; G)$ the joint distribution of $Y$ given $Z$ on social network $G(V, E)$. It is worth noting the Markov property in these models: $Y_i$ relies on other variables through its neighborhood $\{Y_j : j \in N(i)\}$ only. Formally speaking, let $n(i) = \{j \in V : (i,j) \in E\}$ be the neighborhood of user $i$, then

$$P_\theta(Y_i|\{Y_j : j \neq i\}, Z; G) = P_\theta(Y_i|Z_i, \{Z_j, Y_j : j \in n(i)\}; G).$$

One special case of Markov Random Field is Gaussian Graphical Model, in which $Y$ follows a multivariate Gaussian distribution from the global view and each $Y_i$ follows an univariate Gaussian distribution conditional on its neighborhood from the local view. Examples are the additive linear models in (4) like (2).

$$Y_i|\{Z_j, Y_j : j \in n(i)\}; G = \alpha_0 + (\alpha_1 - \alpha_0)Z_i + \beta \sum_{j \in n(i)} Z_j + \gamma \frac{\sum_{j \in n(i)} Y_j}{|n(i)|} + \epsilon_i \tag{2}$$

where $\epsilon_i \sim N(0, \sigma^2)$ and the parameters $\theta = (\alpha_0, \alpha_1, \beta, \gamma, \sigma)$. Here $\alpha_0, \alpha_1$ characterize the intensity of treatment effect of variations A and B, respectively. $\beta$ and $\gamma$ together characterize the intensity of network effect and spill-over effect.

A special case is Ising/logistic Model, in which $Y_i \in \{-1, +1\}$ to present whether user $i$ gives negative or positive response. In this paper, we propose a variant of Ising model (5) to fit negative/positive responses in the network A/B testing.

$$P_\theta(Y_i|Z_i; G) \propto \exp \left( \alpha_0 \sum_{i \in A} Y_i + \alpha_1 \sum_{i \in B} Y_i + \beta_0 \sum_{(i,j) \in E, i,j \in A} Y_i Y_j + \beta_1 \sum_{(i,j) \in E, i,j \in B} Y_i Y_j + \gamma \sum_{(i,j) \in E, i \in A, j \in B} Y_i Y_j \right) \tag{3}$$

where the parameters $\theta = (\alpha_0, \alpha_1, \beta_0, \beta_1, \gamma)$. Here $\alpha_0, \alpha_1$ characterize the intensity of treatment effect of variations A and B, respectively; $\beta_0, \beta_1$ for network effect among groups A and B, respectively; and $\gamma$ for spill-over effect across groups A and B.

From the local view, each $Y_i$ follows a logistic model (4) or (5) conditional on its neighborhood and its group assignment $Z_i$. The logistic function $\text{logistic}(x) = 1/(1 + \exp(-x))$.

$$P_\theta(y_i = +1|Z_i = 0, \{Z_j, Y_j : j \in n(i)\}; G) \tag{4}$$

$$=\text{logistic} \left( \frac{2\alpha_0 \sum_{j \in n(i) \cap A} y_j + 2\gamma \sum_{j \in n(i) \cap B} y_j}{\text{treatment effect} + \text{network effect} + \text{spill-over effect}} \right)$$

$$P_\theta(y_i = +1|Z_i = 1, \{Z_j, Y_j : j \in n(i)\}; G) \tag{5}$$

$$=\text{logistic} \left( \frac{2\alpha_1 \sum_{j \in n(i) \cap A} y_j + 2\gamma \sum_{j \in n(i) \cap B} y_j}{\text{treatment effect} + \text{network effect} + \text{spill-over effect}} \right)$$

3.3 Parameter Inference

This subsection presents how to infer parameters $\theta$ by fitting $K$ triplets $(G^{(k)}, Z^{(k)}, Y^{(k)})$ to the model $P_\theta(Y|Z, G)$. It is challenging to yield the Maximum Likelihood Esti-
mate (MLE) by solving the optimization problem (6).

$$\max_{\theta} \prod_{k=1}^{K} p_{\theta}(Y^{(k)}|Z^{(k)}; G^{(k)})$$

(6)

The difficulties are that the normalizing constant of the probability function $P_{\theta}(Y|Z; G)$ is usually unknown in Markov Random Field, and that the probability function $P_{\theta}$ depends on the network structure $G$, which varies for different triplets $(G^{(k)}, Z^{(k)}, Y^{(k)})$.

A remedy is to replace the likelihood function $P_{\theta}(Y|Z; G)$ in the objective function of (6) with a pseudo-likelihood function $\prod_{i \in V} P_{\theta}(Y_i|Z_i, \{Z_j, Y_j: j \in n(i)\}; G)$, which is the product of conditional probabilities of $Y_i$. It results in the Maximum Pseudo-likelihood Estimate (MPLE), which is given by (7).

$$\max_{\theta} \prod_{k=1}^{K} \prod_{i \in V^{(k)}} p_{\theta}(Y_i^{(k)}|Z_i^{(k)}, \{Z_j^{(k)}, Y_j^{(k)}\}_{j \in n(i)}; G^{(k)})$$

(7)

The MPLE is doable since the conditional probabilities of $Y_i$ are available in either (2) for Gaussian Graphical Model or (4), (5) for Ising/logistic Model. Moreover, solving MPLE is equivalent to fit linear or logistic regression just like all $\sum_{k=1}^{K} |V^{(k)}|$ users being independent samples.

3.4 Computing Average Treatment Estimate

Proceed to compute the ATE(\theta) given \theta. It involves the computation of two terms, which are in essence complicated integrals with respect to two joint probability functions $P_{\theta}(Y|Z = 1; G)$ and $P_{\theta}(Y|Z = 0; G)$. Although the normalizing constants in the two joint probability functions are known for neither Gaussian Graphical Model nor Ising/logistic Model, we can still approximately compute the two terms by Gibbs sampling with the conditional probabilities like (2), (4), (5).

3.5 Hypothesis Test

Along with a point estimate for ATE(\theta), a p-value is desired to indicate the confidence of the estimate. Unfortunately, to the best of our knowledge, there exists no existing method for hypothesis test ATE > 0 so far. In this paper, we propose to construct a p-value by the bootstrapping method. Specifically,

1. Use data

$$(G^{(1)}, Z^{(1)}, Y^{(1)}), \ldots, (G^{(K)}, Z^{(K)}, Y^{(K)})$$

2. Randomly shuffle $Z^{(k)}$ as $\tilde{Z}^{(k)}$, use bootstrapping data sample

$$(G^{(1)}, \tilde{Z}^{(1)}, Y^{(1)}), \ldots, (G^{(K)}, \tilde{Z}^{(K)}, Y^{(K)})$$

3. Repeat step 2 many times, and compare ATE(\theta) to the bootstrapping distribution of ATE(\theta).

If variants A and B do have significant treatment effect, ATE(\theta) is expected to be extremely large compared to the bootstrapping distribution of ATE(\theta), which results from randomly pairing the response $Y_i$ and $Z_i$ on the network $G$. The p-value is the area beyond ATE(\theta) under the curve of the bootstrapping distribution. Figure 3 illustrates the whole scheme of our bootstrapping method.

![Figure 3: The scheme of the bootstrapping method for p-value construction.](image)

It is also worth noting that to test ATE(\theta) > 0 is equivalent to test $\alpha_1 > \alpha_0$ in the Ising/logistic model \[3\], if $\beta_0 \approx \beta_1$. Indeed, assigning all users to group A, i.e. $Z = 0$, in the model \[3\] yields

$$P_{\theta}(Y|Z = 0; G) \propto \exp \left( \alpha_0 \sum_{i \in V} Y_i + \beta_0 \sum_{j \in n(i)} Y_j \right)$$

(8)

\[8\] is an exponential family. $E_{\theta} \left[ \sum_{i \in V} Y_i | Z = 0 ; G \right]$ is the expectation of the sufficient statistic $\sum_{i \in V} Y_i$, is increasing with respect with its coefficient $\alpha_0$. Similarly, assigning all users to group B, i.e. $Z = 1$, in the model \[3\] yields

$$P_{\theta}(Y|Z = 1; G) \propto \exp \left( \alpha_1 \sum_{i \in V} Y_i + \beta_1 \sum_{j \in n(i)} Y_j \right)$$

(9)

$E_{\theta} \left[ \sum_{i \in V} Y_i | Z = 1 ; G \right]$ is increasing with respect with its coefficient $\alpha_1$. If $\beta_0 \approx \beta_1$, the increasing functions

$$\alpha_0 \mapsto E_{\theta} \left[ \frac{1}{|V|} \sum_{i \in V} Y_i | Z = 0 ; G \right]$$

$$\alpha_1 \mapsto E_{\theta} \left[ \frac{1}{|V|} \sum_{i \in V} Y_i | Z = 1 ; G \right]$$

are same. Therefore,

$$E_{\theta} \left[ \frac{1}{|V|} \sum_{i \in V} Y_i | Z = 1 ; G \right] - E_{\theta} \left[ \frac{1}{|V|} \sum_{i \in V} Y_i | Z = 0 ; G \right] > 0$$

(ATE(\theta)

if and only if $\alpha_1 > \alpha_0$.

4. SIMULATION EXPERIMENTS

We generate $K = 100$ social networks $G^{(1)}, \ldots, G^{(K)}$, each of which contains 100 users. Next, we randomly assign
Let $Z_i^{(k)} \sim \text{Bernoulli}(1/2)$ i.i.d., and let $Y_i^{(k)}$ interact with each other in $G^{(k)}$ by Gibbs sampling on Ising/logistic model \cite{1}. Finally, we got a simulation data set of a network A/B test for $\sum_{k=1}^{K} |V^{(k)}| = 10,000$ users. The methods presented in Section 2 are evaluated in three scenarios.

### 4.1 Scenario I: Different Treatment Effect, Same Network Effect

In the first scenario (Table 1), the treatment coefficients are set as $\alpha_0 = 0.0$ and $\alpha_1 = 0.1$ such that

\[
P_0(Y_i = +1|Z_i = 0, n(i) = \emptyset) = 0.50
\]

\[
P_0(Y_i = +1|Z_i = 1, n(i) = \emptyset) = 0.55
\]

That means that user $i$, if assigned to group A, has probability of 0.50 to give positive response without the influence of neighbors; the treatment $B$ can increase the chance by 10%. Other parameters $\beta_0 = \beta_1 = \gamma = \alpha_1/10$ means that the treatment effect is roughly equal to network and/or spill-over effects of 10 neighbors.

| True Value | MPLE  |
|------------|-------|
| $\alpha_0 = 0.00$ | $\hat{\alpha}_0 = -0.00160$ |
| $\alpha_1 = 0.10$ | $\hat{\alpha}_1 = 0.09372$ |
| $\beta_0 = 0.01$ | $\hat{\beta}_0 = 0.00927$ |
| $\beta_1 = 0.01$ | $\hat{\beta}_1 = 0.00443$ |
| $\gamma = 0.01$ | $\hat{\gamma} = 0.00304$ |

Table 1: True parameter and MPLE in Scenario I where variations A and B have different treatment effect, but same network effect.

Results show that $\hat{\alpha}_1 - \hat{\alpha}_0 = 0.095$ is quite close to the true value $\alpha_1 - \alpha_0 = 0.1$. Next, the bootstrapping method in Figure 4 gives a p-value of $< 0.01$, indicating the difference is significant (Figure 4).

### 4.2 Scenario II: Same Treatment Effect, Same Network Effect

The second scenario is an A/A test in which the treatment coefficients $\alpha_0 = \alpha_1 = 0.05$ (Table 2). The estimated value $\hat{\alpha}_1 - \hat{\alpha}_0 = 0.004$ is close to 0. And the bootstrapping method in Figure 4 gives a p-value of $= 0.048$, indicating the difference is insignificant (Figure 5).

| True Value | MPLE  |
|------------|-------|
| $\alpha_0 = 0.05$ | $\hat{\alpha}_0 = 0.0448$ |
| $\alpha_1 = 0.05$ | $\hat{\alpha}_1 = 0.0488$ |
| $\beta_0 = 0.01$ | $\hat{\beta}_0 = 0.00761$ |
| $\beta_1 = 0.01$ | $\hat{\beta}_1 = 0.00027$ |
| $\gamma = 0.01$ | $\hat{\gamma} = 0.00607$ |

Table 2: True parameter and MPLE in Scenario 2 where variations A and B have same treatment effect, and same network effect.

### 4.3 Scenario III: Same Treatment Effect, Different Network Effect

In the third scenario, variation A and B have the same treatment effect on individual users ($\alpha_0 = \alpha_1 = 0.05$), but variation B has stronger network effect ($\beta_1 = 0.05$ v.s. $\beta_0 = 0.01$) and thus is more widespread in the social network. As shown in Table 3, the estimated value $\hat{\beta}_1 - \hat{\beta}_0 = 0.039$ is close to the true value 0.04. And the bootstrapping method in Figure 5 gives a p-value of $= 0.03$, indicating the difference is significant (Figure 5).

| True Value | MPLE  |
|------------|-------|
| $\alpha_0 = 0.05$ | $\hat{\alpha}_0 = 0.0445$ |
| $\alpha_1 = 0.05$ | $\hat{\alpha}_1 = 0.0485$ |
| $\beta_0 = 0.01$ | $\hat{\beta}_0 = 0.00767$ |
| $\beta_1 = 0.05$ | $\hat{\beta}_1 = 0.04626$ |
| $\gamma = 0.01$ | $\hat{\gamma} = 0.01093$ |

Table 3: True parameter and MPLE in Scenario III where variations A and B have same treatment effect, but different network effect.

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