Robust field-dressed spectra of diatomics in an optical lattice

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The absorption spectra of the cold Na₂ molecule dressed by a linearly polarized standing laser wave is investigated. In the studied scenario the rotational motion of the molecules is frozen while the vibrational and translational degrees of freedom are accounted for as dynamical variables. In such a situation a light-induced conical intersection (LICI) can be formed. To measure the spectra a weak field is used whose propagation direction is perpendicular to the direction of the dressing field but has identical polarization direction. Although LICIs are present in our model, the simulations demonstrate a very robust absorption spectrum, which is insensitive to the intensity and the wavelength of the dressing field and which does not reflect clear signatures of light-induced nonadiabatic phenomena related to the strong mixing between the electronic, vibrational and translational motions. However, by widening artificially the very narrow translational energy level gaps, the fingerprint of the LICI appears to some extent in the spectrum.

I. INTRODUCTION

Nonadiabatic processes associated with avoided crossings (ACs) or conical intersections (CIs) always involve nuclear dynamics on at least two coupled potential energy surfaces (PESs) and thus can not be treated within the Born-Oppenheimer (BO) approximation. In this case the electronic states are coupled by the nuclear motion, and the energy exchange between the fast electrons and the slow moving nuclei may become significant. CIs between electronic PESs play a key mechanistic role in chemistry, physics and biology, most notably in photochemistry and photobiology. In several important cases like fragmentation, charge transfer, isomerization processes of polyatomics or radiationless relaxation of the excited electronic states, the CIs can provide very efficient decay channel on femtosecond time scale. CIs show up in different contexts as well, such as in optical and solid state physics. Furthermore, exceptional points – degeneracies between continuum states – can also be found in nature.

CIs can exist only if the molecule possesses at least two independent nuclear vibrational coordinates. In field free diatomics that have only one nuclear degree of freedom CIs are never formed. However, CIs can occur in triatomic systems and they are fairly abundant in truly large polyatomic molecules.

If the laser field is present, then due to the interaction of the diatomics with the field, the rotational nuclear coordinate comes into play and serves as an additional degree of freedom. CIs can be created both by running and by standing laser waves even in diatomic molecules. In the former situation the laser field rotates the molecule due to the interaction of the transition dipole moment of the molecule with the electric field which leads to an effective torque toward the polarization direction of the light and so called “light-induced conical intersections” (LICIs) are formed. The position of these LICIs is determined by the laser intensity while the laser intensity controls the strength of the nonadiabatic coupling. Numerous theoretical and experimental studies have confirmed that the LICIs have strong impact on the dynamical and spectroscopic properties of diatomics. Recently the frequency and intensity dependence of the field-dressed rovibrionic spectrum of the diatomic Na₂ molecule in gas phase was extensively discussed.

Light-induced nonadiabatic phenomena can also occur in optical lattices, which have been widely applied in atomic and diatomic physics so as to cool, trap and control different properties of atoms and molecules. The optical lattice is a periodic potential energy landscape that the molecules experience as a result of the standing wave pattern created by the interference of two counter propagating laser beams. The nonadiabatic phenomenon can emerge when the translational motion of the molecule strongly couples with the electronic, vibrational and rotational motions leading to a periodic array of light-induced CIs. In this case the translational motion provides the new dynamical variable which is necessary to form a branching space for diatomics. Historically, the LICIs have been predicted first in optical lattices for the Na₂ molecule. It was demonstrated that the trapping efficiency of the Na₂ molecule in its lowest electronic state can be significantly reduced due to the presence of the LICI. Later on the intensively studied cold weakly bound rubidium molecule has been considered which plays an important role in the cold-atom physics. As a results, due to the effect of the LICI a significant localization enhancement of the Rb₂ has been revealed. Besides the theoretical studies important experimental papers have also discussed the nonadiabatic
effects [36] and the spectra of ultracold molecules in an optical lattice [37, 38].

In the present work we focus again on the two-atom homonuclear cold sodium molecule and discuss the standing field-dressed spectra of this object. Our aims are twofold. First, by investigating the frequency and intensity dependence of the spectra, we would like to explore to what extent the spectra of a non-rotating Na₂ dressed by an optical lattice and a freely rotating Na₂ dressed by a running laser wave are similar or different. Second, how the impact of the light-induced nonadiabatic phenomena can be visualized on the absorption spectra of the standing field-dressed non-rotating Na₂ molecule.

The article is arranged as follows. In the next section, we present a brief outline of the theory and the computation details. In the third section results are shown and analyzed. Finally, conclusions are given in the last section.

II. THEORY AND COMPUTATION DETAILS

In the present paper a two step numerical pump-probe simulation is derived to study the weak-field absorption spectrum of the field-dressed homonuclear Na₂ molecule in an optical lattice.

First, we specify the field-dressed states of the system by applying a medium intensity “pump” standing laser wave which mixes the field-free eigenstates of the sodium dimer. The applied standing laser wave can be written as the sum of two counter propagating linearly polarized running waves [39]:

\[ \varepsilon_0 [\sin (k_L Z + \omega_L t) + \sin (k_L Z - \omega_L t)] / 2 = \varepsilon_0 \sin (k_L Z) \cos (\omega_L t) \]

where \( \varepsilon_0 \) is the amplitude, \( \omega_L \) is the frequency and \( k_L \) is the wave vector \( (k_L = \omega_L / c = 2\pi / \lambda_L) \) of the laser field.

Second, we determine the dipole transition amplitudes between the field-dressed states in the framework of first-order time-dependent perturbation theory by using a second weak probe pulse.

The two relevant electronic states of the Na₂ molecule which are involved in the numerical calculations are the ground \( X^1\Sigma_g^+ \) and the first excited \( A^1\Sigma_u^+ \) states (further denoted as \( \psi_1^0 \) and \( \psi_2^0 \), respectively). These are characterized by the \( V_1(R) \) and \( V_2(R) \) potential energy curves (see Fig. 1).

We represent the time-dependent Hamiltonian in the Floquet form [10].

Assuming dipole approximation and only one net photon absorption the well known \((2 \times 2)\) static dressed state form can be obtained for the Hamiltonian [40]:

\[
\begin{align*}
\mathbf{H} &= \begin{pmatrix}
\hat{T}_{R\theta \phi} + \hat{T}_{XYZ} & 0 \\
0 & \hat{T}_{R\theta \phi} + \hat{T}_{XYZ}
\end{pmatrix} \\
&\quad + \left( \frac{1}{2} \varepsilon_0 d(R) \cos \theta \sin(k_L Z) \begin{pmatrix}
V_1(R) \\
V_2(R) - h\omega_L
\end{pmatrix} \right).
\end{align*}
\]

Figure 1: Potential energy curves of the Na₂ molecule. Upper panel: potential energy curves \( V_1 \) and \( V_2 \) associated with the \( X^1\Sigma_g^+ \) and \( A^1\Sigma_u^+ \) electronic states of Na₂. \( V_2 \) is dressed by laser light applying \( h\omega_L \) dressed photon energy. Middle panel: adiabatic (\( V_{\text{upper}}, V_{\text{lower}} \)) one dimensional cuts of the potential energy surfaces along the inter-atomic separation. Laser intensity and wavelength are \( 5 \times 10^8 \) W/cm² and \( \lambda_L = 663 \) nm, respectively. The different horizontal lines provide the appropriate diabatic vibrational energy levels. Lower panel: periodic array of light-induced conical intersection (LICI) points between the \( V_{\text{lower}} \) and \( V_{\text{upper}} \) adiabatic potential energy surfaces. The applied field intensity and wavelength are \( I_0 = 5 \times 10^8 \) W/cm² and \( \lambda_L = 663 \) nm, respectively.
Here $\hat{T}_{R\theta\phi}$ and $\hat{T}_{XYZ}$ are the kinetic energy operators of the molecular rovibrational motion and of the translational motion of the center of mass of the molecule, respectively. These are written in the following form:

$$\hat{T}_{R\theta\phi} = -\frac{\hbar^2}{2\mu} \frac{\partial^2}{\partial R^2} + \frac{\hbar^2}{2\mu R^2}, \tag{3}$$

$$\hat{T}_{XYZ} = -\frac{\hbar^2}{2M} \left( \frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} + \frac{\partial^2}{\partial Z^2} \right), \tag{4}$$

where $\mu$ is the reduced and $M$ is the total mass of the two atoms, respectively, $\hat{L}^2(\theta, \phi)$ is the squared angular momentum operator associated with diatomic rotations, $R$ is the molecular vibrational coordinate, and $d(R) = -\left< \psi_2^R \left| \sum_j r_j \right| \psi_2^Z \right>$ is the transition dipole moment which is parallel to the molecular axis. $\theta$ denotes the angle between the polarization direction and the direction of the transition dipole and thus one of the angles of rotation of the molecule. The light is propagated along the $Z$ direction. The potentials $V_1(R)$ and $V_3(R)$ were taken from Ref. [41], whereas the transition dipole moment from Ref. [32].

The form of the Hamiltonian demonstrated in Eq. (2) helps to understand the essence of the light-induced conical intersection phenomenon. In this representation the laser light shifts the $V_3(R)$ excited potential curve downwards by $\hbar \omega_L$ (see the upper panel of Fig. 1) and a crossing is formed between the $V_1(R)$ diabatic ground and the $(V_3(R) - \hbar \omega_L)$ shifted diabatic excited potential energy curves (see the middle panel of Fig. 1). After diagonalizing the potential energy matrix the adiabatic potential curves $V_{lower}$ and $V_{upper}$ can be obtained (middle panel of Fig. 1). These two curves can cross each other, giving rise to a light-induced conical intersection whenever the following two conditions are simultaneously fulfilled: (i) $V_1(R) = V_3(R) - \hbar \omega_L$; (ii) $Z = n\lambda_L/2$ ($n$ is an integer) or $\theta = \pi/2$. The lower panel of Fig. 1 visualizes CIs induced by a laser light for the sodium dimer.

Now, we reduce the three-dimensional problem to a two-dimensional one by assuming that the initial ground electronic and rotational state $\psi_0^R(J = 0)$ is coupled with only the first excited electronic and rotational state $\psi_2^R(J = 1)$, $(\psi_0^R(J = 0)| \cos \theta | \psi_2^R(J = 1)) = \frac{1}{\sqrt{2}}$. Then, we can obtain the field-dressed (FD) eigenstates $\Psi_q^{FD}(R, Z)$ and the corresponding quasi-energies $E_q^{FD}(R, Z)$ by solving the matrix eigenvalue problem with the Hamiltonian of Eq. (2). Therefore, let us build up the Hamiltonian matrix by using basis functions which are the products of plain wave functions describing the translational motion of the system and field-free molecular vibrational eigenfunctions of the ground and first excited electronic states:

$$\varphi_j^{(j)}(R) \Phi_k(Z). \tag{5}$$

Here

$$\varphi_j^{(j)}(R) = \sum_y C_{\nu,\eta}^{(j)} \psi_\eta(R) \tag{6}$$

is the $\nu$th vibrational eigenfunction of the $j$th electronic state, where $j = 1$ and $j = 2$ denote the $X^1\Sigma_g^+$ and $A^1\Sigma_u^+$ electronic states, respectively. The $\psi_\eta(R)$ functions in Eq. (6) stand for the basis function used for expanding the vibrational states. Moreover

$$\Phi_k(Z) = \sqrt{\frac{T}{L}} \exp \left( \frac{i k 2 \pi}{L} Z \right), \quad k = -N, ..., N \tag{7}$$

describes the plain wave functions moving along $Z$ direction, where $i$ is the imaginary unit, $L = \lambda_L$, and $N = 100$.

The resulting field-dressed states in the Floquet framework can be obtained as the linear combination of the products of our basis functions, given spherical harmonics, and the Fourier vectors of the Floquet states, namely

$$|\Psi_q^{FD}\rangle = \sum_{\nu,k} c_{\nu,\eta,k}^{(1)} |\varphi_\nu^{(1)}(R)\rangle |\Phi_k(Z)\rangle |\nu_0,\eta\rangle |n\rangle$$

$$+ \sum_{\nu,k} c_{\nu,\eta,k}^{(2)} |\varphi_\nu^{(2)}(R)\rangle |\Phi_k(Z)\rangle |\nu_1,\eta\rangle |n - 1\rangle, \tag{8}$$

where $c_{\nu,\eta,k}^{(j)}$ are the expansion coefficients gained by diagonalizing the Hamiltonian of Eq. (2) after representing it in the basis set of Eq. (5).

Using the field-dressed states of Eq. (8) one can determine the spectrum of the field-dressed molecule. We assume a weak probe pulse whose propagation direction is perpendicular to the direction of the pump pulse but has identical polarization direction. Furthermore, we assume that the transitions induced by this probe pulse can be described by one-photon processes. In this case the transition amplitudes in dipole approximation can be computed in the framework of first-order time-dependent perturbation theory (TDPT) which is a standard procedure of computational molecular spectroscopy [43]. Then, the transition amplitude from a $|\Psi_q^{FD}\rangle$ initial state to a $|\Psi_q^{FD}\rangle$ final state reads
\[
\langle \Psi_{q'}^{\text{FD}} | \hat{d}e | \Psi_{q}^{\text{FD}} \rangle = \langle \Psi_{q'}^{\text{FD}} (R, \theta, \phi, Z) | d(R) \cos \theta | \Psi_{q}^{\text{FD}} (R, \theta, \phi, Z) \rangle = \\
= \sum_{\nu', k'} \sum_{\nu, k} \left( c_{q', \nu', k'}^{(1)} \right)^* c_{q, \nu, k}^{(2)} \langle \varphi_{\nu'}^{(1)} (R) | d(R) | \varphi_{\nu}^{(2)} (R) \rangle \langle \Phi_k' (Z) | \Phi_k (Z) \rangle \langle Y_{0, 0} (\theta, \phi) \rangle \langle Y_{0, 0} (\theta, \phi) \rangle | n' - 1 \rangle \\
+ \sum_{\nu', k'} \sum_{\nu, k} \left( c_{q', \nu', k'}^{(2)} \right)^* c_{q, \nu, k}^{(1)} \langle \varphi_{\nu}^{(2)} (R) | d(R) | \varphi_{\nu'}^{(1)} (R) \rangle \langle \Phi_k (Z) | \Phi_k (Z) \rangle \langle Y_{1, 0} (\theta, \phi) \rangle \langle Y_{1, 0} (\theta, \phi) \rangle | n' - 1 | n \rangle \\
= \frac{1}{\sqrt{3}} \delta_{n', n} \sum_{\nu', k'} \sum_{\nu, k} \left( c_{q', \nu', k'}^{(1)} \right)^* c_{q, \nu, k}^{(2)} \delta_{k', k} \sum_{\eta} \sum_{\eta'} \left( C_{\nu', \eta'}^{(1)} \right)^* C_{\nu, \eta}^{(2)} | \psi_{\eta}^{(1)} (R) \rangle | d(R) | \psi_{\eta'}^{(2)} (R) \rangle \\
+ \frac{1}{\sqrt{3}} \delta_{n'-1, n} \sum_{\nu', k'} \sum_{\nu, k} \left( c_{q', \nu', k'}^{(2)} \right)^* c_{q, \nu, k}^{(1)} \delta_{k', k} \sum_{\eta} \sum_{\eta'} \left( C_{\nu', \eta'}^{(2)} \right)^* C_{\nu, \eta}^{(1)} | \psi_{\eta'}^{(2)} (R) \rangle | d(R) | \psi_{\eta}^{(1)} (R) \rangle \\
\tag{9}
\]

Here, the \( \hat{e} \) unit vector defines the polarization direction of the dressing pump and probe pulses. The first term of the expression stands for absorption, while the second term represents the simulated emission spectra. From now on we will focus only on the absorption, therefore the first term will serve as our working formula.

### III. RESULTS AND DISCUSSION

We discuss the field-dressed absorption spectrum of the Na\(_2\) molecule in an optical lattice obtained from an initial state chosen to be a single \( |\Psi_q^{\text{FD}} \rangle \) field-dressed state.

#### A. Adiabatic approximation

Let us first choose the \( |\Psi_q^{\text{FD}} \rangle \) field-dressed state according to the so-called “adiabatic approximation” \[41\]. Within this framework the field-dressed state chosen is the one populated adiabatically from the field-free ground electronic and vibrational molecular state by slowly switching on the dressing field. This approach has been used previously in a similar context \[27\] and provides a reasonable approximation as long as the dressing field is turned on much slower than the characteristic timescales of the system. Because we can not suggest a specific \( k \) value (see Eq. \[7\]) for the field-free initial state of an optical lattice experiment, we identified the adiabatically populated field-dressed state as that having the largest \( c_{q, 0, k}^{(1)} \) coefficient, see Eq. \[8\], for a single but any \( k \) value. In Figs. 2 and 3 the field-dressed absorption spectra are presented for several different dressing-light intensities and wavelengths from 1 \( \times \) \( 10^8 \) to 5 \( \times \) \( 10^8 \) W/cm\(^2\) and 660 to 665 nm. Inspecting Fig. 2 we see that for the particular dressing-field wavelength of 663 nm, the presence of the dressing field leads to a decrease in the absorption of the molecule with respect to the field-free case. In addition, tiny discrepancies between the different field-dressed absorption envelopes can be realized. Similarly, the absorption envelope obtained by a 663 nm dressing wavelength (see on Fig. 3) also differs from the other wavelength dressing fields.

These findings are not surprising, because such and even more pronounced deviations have already been visualized on the spectra of freely rotating Na\(_2\) molecules dressed by running wave laser lights \[27\]. For the sake of comparison we present in Fig. 4 some field-dressed absorption spectra of the freely rotating Na\(_2\) for several different light intensities of a 663 nm wavelength dressing-field \[17\]. It can clearly be seen that the field-dressed absorption envelopes are undergoing some remarkable changes via modifying the dressing-field intensities. They can move either up or down relative to the position of the field-free envelope.

The lower panels of Figs. 2 and 4 showing the individual spectral lines of the respective field-dressed spectra, demonstrate significant differences between a non-rotating Na\(_2\) dressed by an optical lattice and a freely rotating Na\(_2\) dressed by a running wave laser fields. In both cases Ant\– Townes-type splittings \[45\] of the field-free peaks can be observed, but the splittings are much more pronounced in the optical lattice case. This is probably related to the fact that the density of translational states in the optical lattice is much higher than the density of rotational states for the freely-rotating Na\(_2\), therefore, the mixing of the different field-free translational states can occur more efficiently than the mixing of the different rotational states during the dressing process.

On the other hand, the spectrum envelope for the freely-rotating Na\(_2\) is more sensitive to the dressing field intensity (see upper panels of Figs. 2 and 3). The changes in the overall spectral peak heights resulting from couplings (induced by the dressing field) between eigenstates of a zeroth-order (field-free) Hamiltonian is referred to as intensity-borrowing \[2, 43\]. It is known that in classical vibronic molecular spectroscopy the intensity-borrowing effect is a clear fingerprint of the presence of a strong nonadiabatic mixing between the different degrees of freedom in the close vicinity of an intrinsic conical intersection. It has been shown that similar situation holds for the dressed state spectra of a freely rotating Na\(_2\) molecule when running laser waves are used for the dressing result-
Figure 2: Absorption spectra of the standing field-dressed $\text{Na}_2$ as a function of dressing-field ($\lambda_d = 663$ nm) intensity, computed using the first expression of Eq. (9). The spectra only show transitions from a single field-dressed state, which was chosen by “adiabatic approximation”. The curves shown in the upper panel are obtained by convolving the stick spectra in the lower panel with a Gaussian function having $\sigma = 50$ cm$^{-1}$.

Figure 3: Absorption spectra of the standing field-dressed $\text{Na}_2$ at six different dressing-field wavelengths and a dressing-field intensity of $I_0 = 5 \times 10^8$ W/cm$^2$. The spectra only show transitions from a single field-dressed state, which was chosen by “adiabatic approximation”.

Figure 4: Absorption spectra of the running field-dressed $\text{Na}_2$ as a function of dressing-field ($\lambda_d = 663$ nm) intensity, computed using the first expression of the last line in Eq. (3) of [27]. The spectra only show transitions from a single field-dressed state, which was chosen by “adiabatic approximation”. The curves shown in the upper panel are obtained by convolving the stick spectra in the lower panel with a Gaussian function having $\sigma = 50$ cm$^{-1}$.

In the optical lattice, the different degree of intensity borrowing is most probably related to the fact that while the transition amplitude induced by the probe pulse is a function of the rotational $\theta$ coordinate, it is not a function of the translational $Z$ coordinate, see Eq. (3) in Ref. [27] and Eq. (9). Therefore, the mixing of the different translational states in the light-dressed states can lead to slight peak splittings, but do not lead to significant changes in the summed overall transition amplitudes of Eq. (9). Therefore, the impact of the light-induced conical intersection is not so pronounced.

As already stated before, during the above simulations an adiabatic turn-on of the dressing field was assumed, i.e., the field-free ground state was assumed to change into a single light-dressed state in an adiabatic fashion. This light-dressed state was used as the initial state when computing the field-dressed spectra. However, such an adiabatic approximation is only valid if the turn-on time of the dressing field is much longer than the characteris-
tic timescales of the system under study. For the freely rotating Na$_2$ this can be a reasonable approximation, as the smallest rotational transition energy of approximately 0.3 cm$^{-1}$ stands for a timescale of around 100 ps. On the other hand, the timescale corresponding to the very dense, nearly continuous energy levels of the translational motion in an optical lattice (see Fig. 4) is very long, around a few seconds. Such a slow turn-on time for the dressing field is not realistic, thus the adiabatic approximation seems to be not so adequate for the optical lattice case.

B. Second adiabatic approximation

Another possible approach to select the initial light-dressed state is to choose the one in which the sum of the field-free basis coefficients \( \sum_k |c^{(1)}_{\nu=0,k}|^2 \) is the largest, i.e. the field-dressed eigenvector with the largest contribution from the field-free \( \varphi^{(1)}_{\nu=0} \) is selected (see Eq. 8). Here \( k \) numbers the basis functions related to the translational motion (see Eq. 7). With such a selection process, we aim to represent a physical scenario in which the turn-on time is adiabatic with respect to the timescale of the vibrational and electronic degrees of freedom, but instantaneous with respect to the timescale of the translational degree of freedom. This selection criteria was applied during the investigation of the molecular localization in an optical lattice as well (we shortly refer to it as “second adiabatic approach”). Curves of the field-dressed absorption spectra for several different light intensity and wavelength values obtained by this framework are displayed in Figs. 5 and 7. The most striking result is the robustness of the spectra. We obtained a significantly different picture than for the situation of the previously studied freely-rotating Na$_2$ molecule. The absorption envelopes are almost identical, not only at different intensities but also when compared with the field-free one. In contrast, the height of the peaks of the stick spectra are substantially different and each of which differ prominently from the height of the field-free peaks. In other words, new transitions appear due to the standing laser field formed dressed states. Here again, the densely packed small spikes can be understood as the fingerprint of the Autler–Townes effect. The presence of the huge number of small spikes results in a rich structure in the line spectra, despite the fact that the envelopes do not change their shape. The latter fact, however, allows us to suggest that the intensity borrowing effect – which can be seen as the clear fingerprint of the light-induced conical intersection – is missing now. Investigating in detail the initial field-dressed states obtained with the "adiabatic approximation" or the "second adiabatic approximation" reveals that while in the "adiabatic approximation" the overall contribution of the \( \varphi^{(1)}_{\nu=0} \) state is around 91.63\% (for \( I_0 = 10^8 \) W/cm$^2$ and \( \lambda_L = 663 \) nm), it is almost 99.98\% for the "second adiabatic approximation". This suggests, that for the intensity borrowing effect to be seen in the optical lattice setup represented in our simulations, the initially prepared field-dressed state should be somewhat contaminated by the excited electronic state manifold. This can be rationalized by the previously mentioned fact that the mixing of only translational states does not lead to significant changes in the transition amplitudes induced by the probe pulse.

C. Model system

By scaling artificially the total mass (denoted by \( M \) in Eq. 3) of the Na$_2$ molecule in an optical lattice without changing the reduce mass (\( \mu \)) related to the vibrational motion, an increased separation of the translational energy levels can be achieved. Fig. 8 shows the translational energy levels of such a model system wherein the actual mass is obtained by reducing the mass of the Na$_2$ molecule to its 20000 portion. The resulting object shows a close similarity with the previously studied rotating Na$_2$ molecule in the sense that the rotational energy levels of the molecule are much further apart from each other than the very dense, almost continuous energy levels belonging to the unscaled translation motion in the lattice. Studying the field-dressed spectra of this model system with the scaled total mass, it can be realized that similarly to the field-dressed spectra of the freely-rotating Na$_2$ molecule (see in Fig. 4) the absorption envelopes, even if not so significantly, but differ from each other. Due to some numerical problems that emerge during the
construction of the model, we could manage a reduction of the mass of the Na$_2$ molecule only to its 20000 portion, but even in this case it is clearly visible that the envelopes belonging to different intensity values noticeably differ from each other. The robustness of the spectrum disappears, the absorption envelopes are moving, although these modifications are much less sensitive to the change of the wavelength and intensity of the dressing field than those of the freely-rotating Na$_2$. Even though the “second adiabatic approach” was applied for the selection of the field-dressed state, which keeps the absorption envelopes of the unscaled Na$_2$ in an optical lattice fixed – the envelopes of the absorption spectra are still changed as a result of the artificial widening of the translational energy level gaps. This means that besides the always existing Autler-Townes-type splittings the intensity borrowing slowly appears as well, representing the signature of the light-induced nonadiabatic phenomena. This effect can be traced back once again to the contamination of the initial field-dressed state with the excited electronic state manifold, contribution of the $\varphi^{(1)}_{\nu=0}$ state is 98.69\% (for $I_0 = 10^8$ Wcm$^{-2}$ and $\lambda_L = 663$ nm).

Obtained results demonstrate that irrespective of the evaluation algorithm used for the selection of the field-dressed molecular states, the very dense energy level structure of the translational motion taking place in an optical lattice play an essential role in the robustness of the field-dressed spectrum.

IV. CONCLUSIONS

Assuming a weak probe pulse, we simulated the standing laser field-dressed absorption spectra of non-rotating Na$_2$ molecules located in an optical lattice. The direction of the applied probe pulse was set to be perpendicular to the direction of the dressing pulse and the polarization direction of the two fields were set the same. In this case the dependence of the translational coordinate $Z$ in the probe pulse does not appear. Applying the proposed algorithm of [35] for the selection of a single dressed state which is the most similar to the field-free ground electronic and vibrational state of the molecule, a robust field-dressed absorption spectrum for the Na$_2$ molecule was obtained. Although, the present model of the optical lattice provides almost identical envelopes of the absorption spectra
in the studied wavelength and intensity ranges, it seems that this robustness can be influenced by changing the density of the states and the method of selecting the initial field-dressed state, which is of course, not a trivial decision.

We think that the present results are likely to change if the probe pulse is applied in a different direction, because changes may appear in the picture due to the emerging translational coordinate (Z) dependence of the probe pulse. Furthermore, by envolving an additional degree of freedom in the probe stage might allow us to visualize the signature of the light-induced nonadiabatic phenomena in the standing field-dressed spectra, which is hardly present in this model. A more accurate method for the standing laser field-dressed spectroscopy can be constructed if rotations are included or if one considers spectra obtained from not a single field-dressed state, but from the combination of several of them. To continue, we plan to go in this direction. However, it is a great challenge, as one has to solve the time-dependent dynamical Schrödinger equation for the pump stage so as to determine the contributions of the different field-dressed states composing the initial wave function measured by the proceeding probe process.

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[46] In the dressed state representation the interaction between the molecule and the electromagnetic field is obtained by shifting the energy of the appropriate electronic states by $n\hbar\omega_L$ ($n = 1, 2, \ldots\infty$). As one net photon absorption is assumed, $n = 1$.
[47] For the calculation of the spectra the first term of the last line in Eq. (3) of [27] is applied.
Figure 8: Top panel: absorption spectra of the standing field-dressed model based system (the mass is obtained by reducing the mass of the Na$_2$ molecule to its 20000 portion) at different dressing-field intensities ($\lambda_L = 663$ nm). The spectra only show transitions from a single field-dressed state, which was chosen by the “second adiabatic approximation”. The curves shown in the top panel are obtained by convolving the stick spectra in the middle panel with a Gaussian function having $\sigma = 50$ cm$^{-1}$. Bottom panel: translational energy levels of a model system wherein the actual mass is obtained by reducing the mass of the Na$_2$ molecule to its 20000 portion and $I_0 = 5 \times 10^8$ W/cm$^2$.