Proton spin structure and the axial U(1) problem
Steven D. Bass

ECT*, Strada delle Tabarelle 286, I-38050 Villazzano, Trento, Italy

We emphasise the relation between the spin structure of the proton and the axial U(1) problem. New experiments motivated by the proton spin problem which could shed light on the nature of U(1) symmetry breaking in QCD are discussed.

1. INTRODUCTION
The spin structure of the proton and $\eta'$ physics provide complementary windows on the role of gluonic degrees of freedom in dynamical $U_A(1)$ symmetry breaking in QCD. The small value of the flavour-singlet axial charge $g_A^{(0)}$, which is extracted from the first moment of $g_1$ (the nucleon’s first spin dependent structure function) [1]

\[ g_A^{(0)}|_{p\text{DIS}} = 0.2 - 0.35 \] (1)

and the large mass of the $\eta'$ meson point to large violations of OZI in the flavour-singlet $J^P = 1^+$ channel [2]. The strong (QCD) axial anomaly is central to theoretical explanations of the small value of $g_A^{(0)}|_{p\text{DIS}}$, about 50% of the OZI value 0.6 and the large $\eta'$ mass. In this paper we review the role of the anomaly in the spin structure of the proton and highlight the interplay between the proton’s internal spin structure and axial U(1) dynamics.

In the QCD parton model the proton’s flavour-singlet axial charge $g_A^{(0)}$ receives contributions from the amount of spin carried by quark and gluon partons [4] and from gluon topology [2], viz.

\[ g_A^{(0)} = \left( \sum_q \Delta q - \frac{3}{2\pi} \Delta g \right)_{\text{partons}} + C \] (2)

Here $\frac{1}{2} \Delta q$ and $\Delta g$ are the amount of spin carried by quark and gluon partons in the polarized proton and $C$ is the topological contribution.

The topological contribution is associated with Bjorken $x$ equal to zero and is related to the role of zero-modes in axial U(1) symmetry breaking. It does not contribute to the value of $g_A^{(0)}|_{p\text{DIS}}$ extracted from polarized deep inelastic scattering but does contribute to the value extracted from $np$ elastic scattering. A quality measurement of $np$ elastic scattering could test theoretical ideas about the axial U(1) problem and offers a real possibility to settle the famous Crewther - ’t Hooft debate [5] through experiment.

A positive value of $\Delta g$ acts to reduce the value of $g_A^{(0)}|_{p\text{DIS}}$ extracted from polarized deep inelastic scattering. Measuring the gluon polarization $\Delta g$ is one of the key goals of QCD spin physics [4]. A recent QCD motivated fit to the world $g_1$ data suggests a value $\Delta g = 0.63^{+0.20}_{-0.19}$, in agreement with the prediction [6] based on colour coherence and perturbative QCD.

The interplay between the proton spin problem and the U(1) problem is further manifest in the flavour-singlet Goldberger-Treiman relation [2] which connects $g_A^{(0)}$ with the $\eta'$–nucleon coupling constant $g_{\eta'NN}$. Working in the chiral limit it reads

\[ M g_A^{(0)} = \sqrt{\frac{3}{2}} F_0 (g_{\eta'NN} - g_{QNN}) \] (3)

where $g_{\eta'NN}$ is the $\eta'$–nucleon coupling constant and $g_{QNN}$ is an OZI violating coupling which measures the one particle irreducible coupling of the topological charge density $Q = \frac{\alpha_s}{4\pi} \tilde{G}G$ to the nucleon. ($M$ is the nucleon mass and $F_0$
\( \sim 0.1 \text{GeV} \) renormalises the flavour-singlet decay constant. It is important to look for other observables which are sensitive to \( g_{QNN} \). Gluonic degrees of freedom induce a contact term in the low-energy \( pN \rightarrow pN\gamma' \) reaction with strength proportional to \( g_{QNN}^2 \). The strength of this interaction is presently under experimental study at COSY [14].

In Section 2 we give a brief review of the QCD axial anomaly and its relation to \( g_A^{(0)} \). In Section 3 we discuss QCD anomaly effects in the intrinsic and orbital contributions to the proton’s spin. Section 4 discusses \( \nu p \) elastic scattering as a probe of \( U(1) \) symmetry breaking. Finally, Section 5 highlights the relationship between the proton spin problem and possible OZI violation in \( \eta' \)-production in low-energy proton-proton collisions.

2. GLUON TOPOLOGY AND \( g_A^{(0)} \)

The flavour-singlet axial charge \( g_A^{(0)} \) is measured by the proton forward matrix element of the gauge invariantly renormalised axial-vector current

\[
J_{\mu 5}^{GL} = \left[ \bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d + \bar{s} \gamma_\mu \gamma_5 s \right] ^{GL}_{\mu 2} \tag{4}
\]

viz.

\[
2m_s g_A^{(0)} = \langle p, s | J_{\mu 5}^{GL} | p, s \rangle \tag{5}
\]

In QCD the axial anomaly induces various gluonic contributions to \( g_A^{(0)} \). The flavour-singlet axial-vector current satisfies the anomalous divergence equation

\[
\partial^\mu J_{\mu 5}^{GL} = 2f \partial^\mu K_\mu + \sum_{i=1}^f 2im_i \bar{q}_i \gamma_5 q_i \tag{6}
\]

where

\[
K_\mu = \frac{g^2}{16\pi^2} \epsilon_{\mu \nu \rho \sigma} \left[ A_\rho \left( \partial^\nu A_\sigma - \frac{1}{3} g f_{abc} A_b^\rho A_c^\sigma \right) \right] \tag{7}
\]

is a renormalised version of the Chern-Simons current and \( f = 3 \) is the number of light-flavours. Eq.(6) allows us to write

\[
J_{\mu 5}^{GL} = J_{\mu 5}^{con} + 2f K_\mu \tag{8}
\]

where

\[
\partial^\mu K_\mu = \frac{g^2}{32\pi^2} G_{\mu \nu} \tilde{G}^{\mu \nu} \tag{9}
\]

and

\[
\partial^\mu J_{\mu 5}^{con} = \sum_{i=1}^f 2im_i \bar{q}_i \gamma_5 q_i \tag{10}
\]

The partially conserved axial-vector current \( J_{\mu 5}^{con} \) and the Chern-Simons current \( K_\mu \) are separately gauge dependent. When we make a gauge transformation \( U \) the gluon field transforms as

\[
A_\mu \rightarrow U A_\mu U^{-1} + \frac{i}{g} (\partial_\mu U) U^{-1} \tag{11}
\]

and the operator \( K_\mu \) transforms as

\[
K_\mu \rightarrow K_\mu \tag{12}
\]

\[
+ i \frac{g}{16\pi^2} \epsilon_{\mu \nu \alpha \beta} \partial^\nu \left( U^\dagger \partial^\alpha U A^\beta \right)
+ \frac{1}{96\pi^2} \epsilon_{\mu \nu \alpha \beta} (U^\dagger \partial^\nu U)(U^\dagger \partial^\alpha U)(U^\dagger \partial^\beta U). \]

Gauge transformations shuffle a scale invariant operator quantity between the two operators \( J_{\mu 5}^{con} \) and \( K_\mu \) whilst keeping \( J_{\mu 5}^{GL} \) invariant.

To make contact with the QCD parton model one would like to isolate the gluonic contribution to \( g_A^{(0)} \) associated with \( K_\mu \) and thus write \( g_A^{(0)} \) as the sum of “quark” and “gluonic” contributions. Here we have to be careful because of the gauge dependence of \( K_\mu \).

Whilst \( K_\mu \) is a gauge dependent operator, its forward matrix elements are invariant under the “small” gauge transformations of perturbative QCD. In deep inelastic processes the internal structure of the nucleon is described by the QCD parton model. The deep inelastic structure functions may be written as the sum over the convolution of “soft” quark and gluon parton distributions with “hard” photon-parton scattering coefficients. Working in light-cone gauge \( A_+ = 0 \) the (target dependent) parton distributions describe a flux of quark and gluon partons carrying some fraction \( x = p_{+\text{parton}}/p_{+\text{proton}} \) of the proton’s momentum into the hard (target independent) photon-parton interaction which is de-
scribed by the hard scattering coefficients. In the QCD parton model one finds

\[ g_A^{(0)} \mid _{\text{partons}} = \left( \sum_q \Delta q - 3\frac{\alpha_s}{2\pi} \Delta g \right) \mid _{\text{partons}} \quad (13) \]

Here, in perturbative QCD, \( \Delta q_{\text{partons}} \) is measured by \( J_{5\mu}^{\text{con}} \) and \( \Delta g_{\text{partons}} \) is measured by \( K_+ \) (in \( A_+ = 0 \) gauge). The polarised gluon contribution to Eq.(13) is characterised by the contribution to the first moment of \( g_1 \) from two-quark-jet events carrying large transverse momentum squared \( \Delta k_T^2 \sim Q^2 \) which are generated by photon-gluon fusion \([4,11]\). The polarised quark contribution \( \Delta q_{\text{partons}} \) is measured with the first moment of the measured \( g_1 \) after these two-quark-jet events are subtracted from the total data set.

The QCD parton model formula (13) is not the whole story. Choose a covariant gauge. When we go beyond perturbation theory, the forward matrix element of \( K_\mu \) is not invariant under "large" gauge transformations which change the topological winding number \([23]\). The issue of "large" gauge transformations means that the spin structure of the proton is especially sensitive to the details of axial U(1) symmetry breaking.

Spontaneous U(1) symmetry breaking in QCD is associated with a massless Kogut-Susskind pole which couples equally and with opposite sign to the two gauge dependent currents \( J_{5\mu}^{\text{con}} \) and \( K_\mu \), thus decoupling from \( J_{5\mu}^{\text{G}} \). Large gauge transformations shuffle the residue of this massless pole between the \( J_{5\mu}^{\text{con}} \) and \( K_\mu \) contributions to \( g_A^{(0)} \).

To see this consider the nucleon matrix element of \( J_{5\mu}^{\text{G}} \)

\[ \langle p, s | J_{5\mu}^{\text{G}} | p', s' \rangle = 2m \left[ \bar{s}_\mu G_A(l^2) + l_\mu l. \bar{G} P(l^2) \right] \quad (14) \]

where \( l_\mu = (p' - p)_\mu \) and \( \bar{s}_\mu = \bar{u}(p,s) \gamma_\mu U(p',s')/2m \). Since \( J_{5\mu}^{\text{G}} \) does not couple to a massless Goldstone boson (the \( \eta^\prime \) is heavy) it follows that \( G_A(l^2) \) and \( G_P(l^2) \) contain no massless pole terms. The forward matrix element of \( J_{5\mu}^{\text{G}} \) is well defined and \( g_A^{(0)} = G_A(0) \). In covariant gauge we can write

\[ \langle p, s | K_\mu | p', s' \rangle = 2m \left[ \bar{s}_\mu K_A(l^2) + l_\mu l. \bar{G} P(l^2) \right] \quad (15) \]

where \( K_P \) contains the massless Kogut-Susskind pole. This massless pole cancels with a corresponding massless pole term in \( (G_P - K_P) \). We may define a gauge-invariant form-factor \( \chi^g(l^2) \) for the topological charge density (9) in the divergence of \( K_\mu \):

\[ 2ml. \bar{G} \chi^g(l^2) = \langle p, s \mid \frac{g_2}{8\pi^2} G_{\mu\nu} \bar{G}^{\mu\nu} | p', s' \rangle. \quad (16) \]

Working in a covariant gauge, we find

\[ \chi^g(l^2) = K_A(l^2) + i^2 K_P(l^2) \quad (17) \]

by contracting Eq.(15) with \( l. \bar{u} \). When we make a gauge transformation \( \delta g_1 \) in \( K_\mu(0) \) is compensated by a corresponding change in the residue of the Kogut-Susskind pole in \( K_P \), viz.

\[ \delta g_1[K_A(0)] + \lim_{l^2 \to 0} \delta g_1[l^2 K_P(l^2)] = 0. \quad (18) \]

Topological winding number is a non-local property of QCD. It is determined by the gluonic boundary conditions at "infinity" \([5]\) — a large surface with boundary which is spacelike with respect to the positions \( z_k \) of any operators or fields in the physical problem — and is insensitive to any local deformations of the gluon field \( A_\mu(z) \) or of the gauge transformation \( U(z) \) — that is, perturbative QCD degrees of freedom. When we take the Fourier transform to momentum space the topological structure induces a light-cone zero-mode which has support only at \( x = 0 \). Hence, we are led to consider the possibility that there may be a term in \( g_1 \) which is proportional to \( \delta(x) \)\(^2\).

It remains an open question whether the net non-perturbative quantity which is shuffled between the \( J_{5\mu}^{\text{con}} \) and \( K_\mu \) contributions to \( g_A^{(0)} \) under "large" gauge transformations is finite or not. If it is finite and, therefore, physical then we find a net topological contribution \( C \) to \( g_A^{(0)} \)

\[ g_A^{(0)} = \left( \sum_q \Delta q - \frac{3\alpha_s}{2\pi} \Delta g \right) \mid _{\text{partons}} + C \quad (19) \]
The topological term $C$ has support only at $x = 0$.\footnote{Possible $\delta(x)$ terms in deep inelastic structure functions are also found in Regge theory where they are induced by Regge fixed poles with non-polynomial residue \cite{31}.} Topological $x = 0$ polarization is inaccessible to polarized deep inelastic scattering experiments which measure $g_1(x, Q^2)$ between some small but finite value $x_{\text{min}}$ and an upper value $x_{\text{max}}$ which is close to one. As we decrease $x_{\text{min}} \to 0$ we measure the first moment

$$
\Gamma \equiv \lim_{x_{\text{min}} \to 0} \int_{x_{\text{min}}}^{1} dx \ g_1(x, Q^2).
$$

This means that the singlet axial charge which is extracted from polarized deep inelastic scattering is the combination $g_A^{(0)}|_{\text{pDIS}} = (g_A^{(0)} - C)$. In contrast, elastic $Z^0$ exchange processes such as $\nu p$ elastic scattering measure the full $g_A^{(0)}$. One can, in principle, measure the topology term $C$ by comparing the flavour-singlet axial charges which are extracted from polarized deep inelastic and $\nu p$ elastic scattering experiments. A decisive measurement of $\nu p$ elastic scattering may be possible with the MiniBooNE set-up at FNAL \cite{26}.

3. SPIN AND ANGULAR MOMENTUM IN THE TRANSITION BETWEEN CURRENT TO CONSTITUENT QUARKS

One of the most challenging problems in particle physics is to understand the transition between the fundamental QCD “current” quarks and gluons and the constituent quarks of low-energy QCD. Relativistic constituent-quark pion coupling models predict $g_A^{(0)} \simeq 0.6$ (the OZI value and twice the value of $g_A^{(0)}|_{\text{pDIS}}$ in Eq.(1)). If some fraction of the spin of the constituent quark is carried by gluon topology in QCD, then the constituent quark model predictions for $g_A^{(0)}$ are not necessarily in contradiction with the small value of $g_A^{(0)}|_{\text{pDIS}}$ extracted from deep inelastic scattering experiments.

The quark total angular momentum $J_q$ measured through the proton matrix of the angular momentum tensor in QCD can, in principle, be extracted from deeply virtual Compton scattering.

\cite{27}. This $J_q$ is anomaly free in both perturbative and non-perturbative QCD \cite{28,29}. This means that the axial anomaly cancels between the intrinsic and orbital contributions to $J_q$. Furthermore, any zero-mode contributions to the “quark orbital angular momentum”, which is measured by the proton matrix element of $[\bar{q} (\vec{z} \times \vec{D}) q](0)$. Future measurements of “quark orbital angular momentum” from DVCS should be quoted with respect to the factorization scheme and process (polarized deep inelastic scattering or $\nu p$ elastic scattering) used to extract information about the “intrinsic spin”.

4. INSTANTONS AND $U_A(1)$ SYMMETRY BREAKING

The presence or absence of topological $x = 0$ polarization is intimately related to the dynamics of $U_A(1)$ symmetry breaking in QCD.

A key issue in dynamical $U_A(1)$ symmetry breaking is the role of instantons. Whether instantons spontaneously \footnote{We denote the tunneling process by the subscript “inst.”. It is not specified at this stage whether “inst.” denotes an instanton or an anti-instanton.} or explicitly \footnote{To go further one has to be precise how one defines chirality in (21): either through $J_{\mu}^{\text{inst}, l}$ or through $J_{\mu}^{\text{inst}, R}$.} break $U_A(1)$ symmetry depends on the role of zeromodes in the quark-instanton interaction and how one should include non-local structure into the local anomalous Ward identity, Eq. (6). Both scenarios start from ’t Hooft’s observation \cite{30} that the flavour determinant

$$
\langle \det \left[ t^{IL} \tau^j q^j (z) \right] \rangle_{\text{inst.}} \neq 0 \quad (21)
$$

in the presence of a vacuum tunneling process between states with different topological winding number \cite{31}. (We denote the tunneling process by the subscript “inst.”. It is not specified at this stage whether “inst.” denotes an instanton or an anti-instanton.) To go further one has to be precise how one defines chirality in (21): either through $J_{\mu}^{\text{inst}, l}$ or through $J_{\mu}^{\text{inst}, R}$.

As we now explain the two choices lead to different phenomenology.

Quark instanton interactions flip quark “chirality” so that when we time average over multiple scattering on an ensemble of instantons and anti-instantons the spin asymmetry measured in
polarized deep inelastic scattering is reduced relative to the asymmetry one would measure if instantons were not important. Topological $x = 0$ polarization is natural in the spontaneous symmetry breaking scenario where any instanton induced suppression of $g_A^{(0)} |_{pDIS}$ is compensated by a shift of flavour-singlet axial-charge from partons carrying finite momentum $x > 0$ to a zero-mode at $x = 0$ so that the total $g_A^{(0)}$ is conserved. It is not generated by the explicit symmetry breaking scenario where the total $g_A^{(0)}$ (rather than the chirality measured by $J_{\text{con}}^\mu$) is sensitive to the quark-instanton interaction. Comparing the values of $g_A^{(0)}$ extracted from $\nu p$ elastic and polarized deep inelastic scattering could provide valuable information on $U_A(1)$ symmetry breaking in QCD.

5. PROTON SPIN STRUCTURE AND THE $\eta'$-NUCLEON INTERACTION

Motivated by the flavour-singlet Goldberger-Treiman relation (3), further insight into the relationship between the proton’s internal spin structure and $U_A(1)$ dynamics may come from studying possible OZI violation in the $\eta'$-nucleon system.

Working with the $U_A(1)$-extended chiral Lagrangian for low-energy QCD one finds a gluon-induced contact interaction in the $pp \to pp\eta'$ reaction close to threshold:

$$L_{\text{contact}} = -\frac{i}{F_0^2} g_{QNN} \tilde{m}_{\eta_0}^2 \epsilon_{\eta_0} \left( \bar{p}_\gamma 5 p \right) \left( \bar{p} p \right)$$

Here $\tilde{m}_{\eta_0}$ is the gluonic contribution to the mass of the singlet $0^-$ boson and $\epsilon$ is a second OZI violating coupling which also features in $\eta'N$ scattering. The physical interpretation of the contact term (22) is a “short distance” ($\sim 0.2\text{fm}$) interaction where glue is excited in the interaction region of the proton-proton collision and then evolves to become an $\eta'$ in the final state. This gluonic contribution to the cross-section for $pp \to pp\eta'$ is extra to the contributions associated with meson exchange models. There is no reason, a priori, to expect it to be small.

Since glue is flavour-blind the contact interaction (22) has the same size in both the $pp \to pp\eta'$ and $pn \to pn\eta'$ reactions. CELSIUS have measured the ratio $R_{\eta'} = \sigma(pn \to pn\eta')/\sigma(pp \to pp\eta')$ for quasifree $\eta$ production from a deuteron target up to 100 MeV above threshold. They observed that $R_{\eta'}$ is approximately energy-independent $\equiv 6.5$ over the whole energy range — see Fig.1. The value of this ratio signifies a strong isovector exchange contribution to the $\eta$ production mechanism. This experiment should be repeated for $\eta'$ production. The cross-section for $pp \to pp\eta'$ close to threshold has been measured at COSY. Following the suggestion in [13] new experiments at COSY have been initiated to carry out the $pn \to pn\eta'$ measurement. The more important that the gluon-induced process (22) is in the $pp \to pp\eta'$ reaction the more one would expect $R_{\eta'} = \sigma(pn \to pn\eta')/\sigma(pp \to pp\eta')$ to approach
unity near threshold after we correct for the final state interaction \cite{33} between the two outgoing nucleons. (After we turn on the quark masses, the small $\eta - \eta'$ mixing angle $\theta \simeq -18$ degrees means that the gluonic effect (22) should be considerably bigger in $\eta'$ production than $\eta$ production.) $\eta'$ phenomenology is characterised by large OZI violations. It is natural to expect large gluonic effects in the $pp \rightarrow pp\eta'$ process.

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