Field and lab experimental demonstration of nonlinear impairment compensation using neural networks

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Fiber nonlinearity is one of the major limitations to the achievable capacity in long distance fiber optic transmission systems. Nonlinear impairments are determined by the signal pattern and the transmission system parameters. Deterministic algorithms based on approximating the nonlinear Schrödinger equation through digital back propagation, or a single step approach based on perturbation methods have been demonstrated, however, their implementation demands excessive signal processing resources, and accurate knowledge of the transmission system. A completely different approach uses machine learning algorithms to learn from the received data itself to figure out the nonlinear impairment. In this work, a single-step, system agnostic nonlinearity compensation algorithm based on a neural network is proposed to pre-distort symbols at transmitter side to demonstrate ~0.6 dB Q improvement after 2800 km standard single-mode fiber transmission using 32 Gbaud signal. Without prior knowledge of the transmission system, the neural network tensor weights are constructed from training data thanks to the intra-channel cross-phase modulation and intra-channel four-wave mixing triplets used as input features.
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apacity of optical transmission systems is bound by
fundamental limits in both linear and nonlinear regime.
Recent experiments demonstrated capacities approaching
the Shannon limit in the linear regime. This leaves the Kerr
nonlinearity as one of the major limitations to increasing capacity
per fiber. Nonlinear compensation (NLC) algorithms were
introduced in digital coherent receivers to compensate for
the pattern-dependent and deterministic Kerr nonlinearity.
These digital signal processing (DSP) NLC algorithms were based
on solving the nonlinear Schrödinger equation (NLSE) which
governs the propagation of optical field in the fiber. A variety of
techniques were developed to reduce the complexity of NLC for a
given amount of improvement, some of which reduced the
required complexity by orders of magnitude such as the filtered
DBP. Compared to full-step DBP, at least 0.2 steps per span
(Sps) and 0.6 Sps are found to be sufficient for QPSK and
16QAM, respectively. The first-order linear perturbation of
NLSE has led to the single-step perturbation-based pre/post-
distortion (PPD) algorithm for Gaussian and root-raised cosine
(RRC) pulses. PPD was demonstrated over transoceanic distances to
achieve only slightly less NLC gain than filtered DBP at 0.5 Sps.
Furthermore, after the initial success of filtered DBP and
PPD in reducing the complexity, further reduction of com-
plexity proved difficult to achieve. The common thread among
these algorithms was that they were based on equalizing the
nonlinearity based on a deterministic model of the impairment.

Recently, a different approach was taken using machine
learning algorithms. These algorithms aim to equalize
nonlinear impairments directly by learning from data, rather than
through emulating NLSE. These attempts remained limited in
success in particular for dispersion-uncompensated links, or in
scope until it was demonstrated recently through a field trial on a
live-traffic cable that a simple neural network (NN) can provide
NLC if it was supplemented with nonlinear impairment fea-
tures. A second approach showed through simulations that,
NN can also be used to obtain NLC by treating the DBP steps as
NN layers. However, by design, this approach requires operating
on at least two samples per symbol, which induces additional
complexity.

In this paper it is shown that the NN architecture demon-
strated in ref. can be simplified further so that it can achieve
NLC gain at a complexity lower than filtered DBP algorithms that
are based on solving NLSE, especially at the most critical regime
where the available DSP resources become scarce. Furthermore,
NN not only learns from the received data and generates a black-
box model of the transmission, it also can guide us how to reduce
the complexity through the weights by distinguishing the terms
that contribute significantly from the ones that do not. Another
advantage of the proposed algorithm is that since it relies only on
received data to emulate the transmission model, it works without
prior knowledge of the link parameters. Since the algorithm
becomes free from specifics of the link design, it can be applied
universally to all fiber optical communication links whether they
are short-haul, long-haul, terrestrial, submarine, or whether they
are legacy systems or the state-of-the-art. It is also shown that the
algorithm is versatile and robust enough that while the training
can be performed at the receiver side which is the most practical
case, the equalization can be performed on the transmitter side.
NLC at the transmitter has the benefit of achieving slightly better
Q improvement, but more importantly the possibility of a further
reduction of complexity by calculating nonlinearity features
with look up tables (LUT) rather than real multiplications. Per-
formance of the proposed universal NN-NLC algorithm is
demonstrated in both lab experiment over 2800 km standard
single-mode fiber (SSMF) loop and field trial over 11.017 km
straight-line FASTER cable. Compared with single-step filtered
DBP algorithm, NN-NLC algorithm is capable of achieving ~0.35
dB Q-factor improvement in the 2800 km SSMF transmission and
attaining ~0.08 b/s/Hz higher generalized mutual information
(GMI) after 11,017 km submarine distance. The results show that
NN-NLC has more potential to outperform filtered-DBP espe-
cially in the regime where the degree of computational complexity
is limited.

Results
Input features. Even though the NN algorithm needs only data to
achieve a working model of the nonlinear impairment, it was
found that providing the NN with nonlinear impairment features
was necessary. These features are provided to the NN by first
calculating the intra-channel cross-phase modulation (IXPM) and
intra-channel four-wave mixing (IFWM) triplets from the received symbols. The triplets originated from the first-order
perturbation of the NLSE that describes the evolution of the
optical field as follows:

\[
\frac{\partial u_{i,j}(t, z)}{\partial z} + i \frac{\beta_2}{2} \frac{\partial^2 u_{i,j}(t, z)}{\partial t^2} = -\frac{8}{9} \left[ |u_{i,j}(t, z)|^2 + |u_{i,j}(t, z)|^2 \right] u_{i,j}(t, z),
\]

where \( u_{i,j}(t, z) \) is the optical field of \( x \) and \( y \) polarization, respectively, \( \beta_2 \) is the group velocity dispersion, and \( y \) is the nonlinear coefficient. In the first-order perturbation theory, the
solution to Eq. (1) consists of both linear \( u_{0,i,j}(t, z) \) and nonlinear perturbation \( \Delta u_{i,j}(t, z) \) terms. Considering nonlinear accumulated dispersion than symbol duration, the nonlinear perturbation
terms for the symbol at \( t = 0 \) can be approximated as:

\[
\Delta u_{i,j}(0, z) = \sum_{m,n} P_0 H_{m+i,m-n}^{*} V_m V_{m+n}^{*} C_{m,n},
\]

where \( P_0, H_m \) and \( V_m \) are, respectively, the launch
power, symbol sequences for the \( x \)- and \( y \)-polarization, and nonlinear perturbation coefficients, \( m \) and \( n \) are symbol indices with respect to the symbol of interest \( H_0 \) and \( V_0 \). The triplet is defined as \( T = H_{m+i,m-n}^{*} + V_m V_{m+n}^{*} \) in this paper. The nonlinear perturbation coefficients \( C_{m,n} \) can be analytically computed given the link parameters and signal pulse duration/shaping factors, whereas the triplets do not depend on the link and can be calculated directly from the received symbols.

The proposed NN-NLC algorithm is divided into two stages:
training and execution stages. In the training stage, the NN learns
from the training data and generates a black-box model of the
transmission link. In the execution stage, the nonlinear impair-
ment is calculated based on the model, and the impairment is
removed from the data.

Training stage. During the training stage it is necessary to have
nonlinear sufficiency therefore the launch power \( P_0 \) should be close to, or larger than the optimum channel power. One draw-
back of data-driven modeling is that the received data is cor-
rupted not only by nonlinear impairments but also by amplified
spontaneous emission (ASE) noise. This can be easily taken care
of by transmitting the same training data repeatedly and aver-
aging out the pattern-independent noise such as ASE, and inter-
channel nonlinearities. The intra-channel nonlinear impairments can be retained. NN-NLC operates on the soft data
output from carrier phase recovery block shown in DSP flowchart of Fig. 1a. At the training stage NN-NLC needs to be imple-
mented at the receiver side whereas the execution stage can be implemented at either the transmitter or receiver side, see Fig. 1. As the training does not have to operate at the data rate unlike the execution stage, and the amount of training data is not excessive,
it can be performed offline to save computation cost. To test the NN-NLC algorithm with experimental data, single-channel 32 Gbaud dual-polarization (DP)-16QAM with RRC 0.01 pulse shaping as shown in Fig. 2 is generated using 64 Gsa/s DAC and transmitted over a recirculating loop testbed consisting of five spans of 80 km SSMF with 0.2 dB/km loss and 17 ps/nm/km dispersion. A digital coherent receiver running at 50-GSa/s with analog bandwidth of 20 GHz downsamples the optical waveforms for offline DSP outlined in Fig. 1a to recover the transmitted symbols. In addition, 50% chromatic dispersion compensation (CDC) is applied at the transmitter to reduce the interaction length between symbols.\(^{15}\). Three uncorrelated datasets each with \(\sim 115\) k symbols are generated for training, cross-validation (CV) and testing. The data pattern used in the training, CV and test datasets is measured to have maximum 0.6% normalized cross-correlation to ensure data independence.

As the training pattern is fixed at the transmitter side and the ASE noise is an independent additive Gaussian noise, the recovered symbols can be easily synchronized using a framer in practice to align the symbols in the same order such that the ASE noise can be reduced after averaging, while keeping the nonlinear interaction intact. Multiple waveform acquisition is processed, noise can be reduced after averaging, while keeping the nonlinear practice to align the symbols in the same order such that the ASE recovered symbols can be easily synchronized using a framer in execution stage. Note that the training can be done at much slower pace than data rate to allow deep-learning algorithm to locate the appropriate NN models and compute the optimum tensor weights prior to the execution stage.

The impact of activation function. A single-channel 32Gbaud DP-16QAM is simulated over \(40 \times 80\) km SSMF with 50% chromatic dispersion.

Parameter optimization of ML model. There are various ML models from simple linear regression to sophisticated deep-learning models used for solving a variety of problems. Fully connected neuron network is selected to demonstrate the effectiveness of NN for NLC at a similar or even lower complexity than existing DSP algorithm. The optimized feed-forward NN model shown in Fig. 4 is constructed from an input layer with \(2N_t\) triplets nodes, 2 hidden layers consisting of 2 and 10 nodes, respectively, and two output nodes corresponding to the real and imaginary parts of the estimated nonlinearity. The optimization of the number of hidden layers and the number of nodes in each layer is carried out to simplify the complexity of the NN architecture without degrading the BER performance of the derived models on the CV datasets. Note that the triplets are separated into real and imaginary parts before being fed into the NN model. Although Eq. (2) describes linear relationships among these IFWM/IXPM triplets due to the first-order perturbation, non-linear activation function in the neuron nodes is found in our study to achieve better performance than linear function. The impact of activation function is explained in the next section. A dropout layer with probability of 0.5 is placed after the 2nd hidden layer during training only to avoid overfitting. Applying Adam learning algorithm\(^{23}\) with a learning rate of 0.001 and batch size of \(B=100\), the network is trained by transmitting known but randomly generated patterns, and searching for the best node tensor parameters that minimize the mean square error (MSE) between the transmitted and received symbols after NN-NLC, i.e.,

\[
MSE = \frac{1}{B} \sum_{i=1}^{B} \left| H_i - \left( \hat{H}_{i,\text{NL}} \right) \right|^2,
\]

where \(H_i\) and \(\hat{H}_{i,\text{NL}}\), respectively, are the received symbols and estimated nonlinearity for pol-x. Although the model is trained for x-polarization data, same weights can be used to obtain a similar performance improvement for the y-polarization data too. Note that the training can be done at much slower pace than data rate to allow deep-learning algorithm to locate the appropriate NN models and compute the optimum tensor weights prior to the execution stage.
The NLC performance between PPD and NN-NLC algorithm as a function of the number of triplets. The calculation of perturbation coefficients are based on ref.13. The Q improvement over the CDC is plotted in Fig. 5d as a function of the number of triplets at the optimum channel power of 1 dBm. The NN-NLC with Leaky-ReLU outperforms PPD by ~0.15 dB at ~2000 triplets thanks to the nonlinear activation function in the neuron nodes.

**Triplets selection.** After cleaning up the ASE noises in the received training dataset, IXPM & IFWM triplets are calculated according to Eq. (2). In systems with large dispersion, symbols can overlap and interact with thousands of neighboring symbols. In order to keep the complexity low, the number of triplets should be kept low by only including the ones that contribute the most. This requires establishing a selection criterion:10,13. In previous work18, nonlinear perturbation coefficients \( C_{m,n} \) were used as a way to estimate which triplets contributed the most. Only the triplets that satisfied the criterion \( C_{m,n} > \kappa \) was retained where \( \kappa \) was a free parameter to adjust the trade off between complexity and performance. Even though \( C_{m,n} \) were not used in the execution stage, their computation during the training stage still required accurate link and transmission system parameters18. Considering that the \( C_{m,n} \) has a hyperbolic dependence on \( m \) and \( n \), we propose to choose all the triplets with index pairs \( m \) and \( n \) that satisfies the following criterion that is independent of link parameters

\[
|n| \leq \min \left\{ \left\lfloor \frac{\rho |L/2|}{|m|} \right\rfloor, \left\lceil \frac{L}{2} \right\rceil \right\},
\]

where \( \min[\cdot] \) takes the minimum of its arguments, \( \lfloor \cdot \rfloor \) stands for the ceiling function, \( L \) determines the largest value of \( m \) and \( n \), and \( \rho \) controls the maximum of the \( m, n \) product. The values required for \( L \) and \( \rho \) are not too restrictive as long as a sufficiently large number of triplets are chosen to initialize the training. Through iterative trimming during the training stage, as discussed below, the excess triplets can be removed before the execution stage.

**Execution stage.** During the training stage, the performance of the model is checked against the CV dataset only to optimize the NN model parameters. Afterwards the learned model is applied to the uncorrelated test dataset for all channel powers in the execution stage. The block diagram of the proposed NN-NLC is shown in Fig. 4. Given the symbol of interest \( \hat{H}_k \) centered at the middle of pattern length \( L \), the IXPM and IFWM terms are

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**Fig. 3** Denoising. a The impact of de-noising by averaging over the training datasets on the Q-factor at SNR = 18.4 dB after 2800 km. b Received constellation before de-noising, and c after de-noising.

**Fig. 4** The block diagram of the proposed NN-NLC. Illustrated for pol-x only. The diagram in the dashed box describes the optimized NN architecture with two hidden layers used in the paper.
calculated and fed into the NN model described in the dashed box of Fig. 4 to estimate the nonlinearity. The estimated $H_{0,\text{NL}}$ is then scaled by the channel power ($P_{\text{ch}}$) of the test dataset with respect to the reference channel power ($P_{\text{ref}}$) of the training data used for deriving the model, i.e.,

$$\alpha = 10^{0.1 \times \left(P_{\text{ref}} - P_{\text{ch}}\right)}. \quad (5)$$

The estimated $H_{0,\text{NL}}$ is subtracted from the original symbol of interest before being sent to next DSP block, for instance, the FEC decoding in Fig. 1a.

**Complexity.** Since the complexity of real multiplications could be four times as much as addition operation, only real multiplication is taken into account when comparing the complexity of the NLC algorithm. The NN model shown in the dashed box of Fig. 4 requires $2N_t \times 2 + 2 \times 10 + 10 \times 2 = 4N_t + 40$ real multiplications because of three cross-layer tensor interaction. Note that the activation function Leaky ReLU() in the hidden nodes and DPM/IFWM triplets computation are assumed to be implemented in LUT. After scaling the estimated nonlinearity term, the number of real multiplication per symbol for the proposed NN-NLC shown in Fig. 4 can be summarized as

$$4N_t + 42. \quad (6)$$

Therefore, reducing the number of triplets $N_t$ is the most effective way to lower the complexity of the NN-NLC algorithm in our model.

As shown in Fig. 6a, with the initial $N_t = 1929$ triplets, some of the input tensor weights $W_{m,n}$ in the trained model show much smaller contribution to the signal nonlinearity than the center ones. As a result, the number of triplets $N_t$ can be further reduced.

**Fig. 5** The impact of activation function on the NN-NLC algorithm. a Plots of different activation functions. b The BER optimization trace of DNN over the CV dataset in the training with different activation functions. c The Q-factor of DNN with different activation functions as a function of channel power. d The Q-factor improvement of PPD and NN-NLC with Leaky ReLU w.r.t. the optimum CDC Q

**Fig. 6** Density plots. The density plot of the input layer weights of the NN model at a initial $N_t = 1929$ and b $N_t = 615$ triplets after iterative trimming ($\kappa = -22 \text{ dB}$). Colorbar shows the magnitude of the weights
by only keeping those weights larger than a specified threshold $\kappa$, i.e., $W_{m,n} > \kappa$. After trimming off the weights $W_{m,n}$ that are less than $\kappa = -22$ dB, the remaining 615 triplets are re-trained in the NN model and the new density plot of the input tensor weights $W_{m,n}$ is shown in Fig. 6b. Figure 7 plots the impact of the trimming threshold $\kappa$ on the performance improvement of the NN-NLC as a function of received SNR after 2800 km transmission. At the optimum received SNR, the NN-NLC algorithm at trimming threshold $\kappa < -15$ dB achieves $>0.5$ dB Q improvement over CDC. Larger Q improvement at the highest received SNR further confirms the NN model is able to accurately predict the signal nonlinearity. At the optimum received SNR, Q value improvement is adjusted from 0.2 to 0.4 dB by varying the complexity through adjusting $\kappa$ from $-35$ to $-15$ dB, as shown in Fig. 7. Training the NN is more practical at the receiver side, however, computation of triplets at the receiver side requires the use of soft symbols. To reduce complexity further, it is better to execute the NN-NLC block at the transmitter side and use a LUT to store the triplet values rather than calculate them online. The LUT size could scale as large as $M^3$ where $M$ is the constellation size. Figure 1b shows the DSP block diagram of the NN-NLC at the transmitter side. Figure 8a shows the constellation generated after applying pre-distortion calculated by the NN at the transmitter side with $\kappa$ set to $-22$ dB.

Performance of the transmitter side compensation is compared with the receiver side compensation in Fig. 9a. Even though the NN is trained at the receiver side, the transmitter side compensation performs better in all the ranges but especially at the high complexity end. This improvement is expected considering that at the transmitter side the NN model works on the clean transmitted symbols. Moreover, once the non-linearity is mitigated at the transmitter, the receiver DSP algorithm works on the signals with reduced nonlinearity and cycle slip rate. Tx-side NN-NLC outperforms PPD by more than 0.4 dB, which is attributed to the learning features and nonlinear Leaky ReLU() activation functions. Compared to the recovered constellation without NN-NLC algorithm shown in Fig. 8c, the transmitter-side NN-NLC can significantly improve the constellation quality as plotted in Fig. 8b.

One of the most important criteria for a practical NLC algorithm is its low complexity while providing a significant Q
improvement. Complexity of the NN-NLC was compared with NLC algorithms based on perturbation approach whose complexity also scales with the number of triplets. To establish the performance of the NN-NLC with respect to existing NLC algorithms in terms of the performance-complexity trade off, the comparison is extended to the filtered-DBP technique. Filtered-DBP is chosen for comparison because it is a well-established, low-complexity technique where the complexity versus improvement trade-off can be easily adjusted and it provides gain as long as it has sufficient number of steps. In filtered-DBP, the number of steps can be reduced by filtering the intensity waveforms by a low-pass filter (LPF) at the nonlinear phase rotation stage. The optimal bandwidth for the Gaussian-shaped LPF is found to be 5, 1, 1, and 0.5 GHz for 1, 5, 7, and 35 spans per step (SpS). The optimum scaling factor used to de-rotate the signal’s phase is ~0.7 for all cases. The Q performance improvement over CDC is plotted in Fig. 9b to show that Tx-side NN-NLC is capable of matching the performance of the proposed NN-NLC is further tested on an 11,017 km Undersea cable performance.

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**Data availability**

The datasets generated during this current study are not publicly available due to restrictions from commercial privilege, but portions of the data are available from the corresponding author on reasonable request.

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Author contributions
S.Z. and F.Y. formulated the problem and developed the algorithms, S.Z., F.Y., K.N. and T.I. took the lab and field data, V.K., L.J., V.V., K.N., T.I., E.M. and Y.I. managed the field trial, all the co-authors contributed to the development of the concept, setting the direction, and writing the manuscript.

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