Novel signatures of the Higgs sector from $S_3$ flavor symmetry

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In an earlier work we analyzed the $CP$-even scalar sector of an $S_3$ flavor model, where we identified some novel decay signatures of an exotic scalar. In this work we extend our analysis by including the complete set of scalars/pseudoscalars, revisiting the potential minimization conditions in a more general setup, setting the spectrum in conformity with the current LHC limits on the scalar mass, and identifying yet another spectacularly novel decay channel which might be revealed from an intense study of rare top decays at the LHC into modes containing multileptons of different flavors.

I. INTRODUCTION

Discrete flavor symmetries constitute an effective way of explaining the masses and mixing of quarks and leptons [1]. These symmetries may be broken at a high scale by vacuum expectation values of scalars, called flavons or familons (see e.g., Ref. [2]). Interesting experimental signatures such as nonstandard decays involving scalars and gauge bosons and/or large flavor changing neutral currents often arise in such scenarios. The flavor group $S_3$ specifically was introduced early in Ref. [3] and has since been used in many different flavor scenarios [4]. Our analysis is based on the flavor symmetry realization introduced in Ref. [5] to describe charged lepton and neutrino masses and mixing. The group structure of $S_3$ favors a maximal atmospheric mixing angle which still makes it a good fit even after the recent measurements of a nonzero $\theta_{13}$ [6]. The group $S_3$ has three irreducible representations: $1$, $1'$, and $2$. The invariants $1$ can be constructed using the multiplication rules $2 \otimes 2 = 1 \oplus 1' \oplus 2$ and $1' \otimes 1' = 1$. We follow the particle assignments [5] as we have in Ref. [7]:

\[
\begin{align*}
(L_\mu, L_\tau) & \in 2, \\
(Q_2, Q_3) & \in 2, \\
(\phi_1, \phi_2) & \in 2, \\
L_e, c^e, \mu^e & \in 1, \\
\tau^e & \in 1', \\
Q_1, w^c, c^c, d^c, s^c & \in 1, \\
b^c, t^c & \in 1', \\
\phi_3 & \in 1.
\end{align*}
\]

The fields $Q_{1/2/3}$ and $L_{e/\mu/\tau}$ refer to the quark and lepton $SU(2)$ doublets of the three generations. This assignment was motivated in Ref. [5] in order to have a reasonably successful reproduction of quark and lepton masses and mixing. An intuitive appreciation of large mixing in the lepton sector vis-à-vis $CP$-even neutral scalars, but $CP$-odd neutral scalars and two sets of charged

1. Two of the three scalars $h_{b,c}$ have standard model (SM)-like couplings except that they can dominantly decay into the third scalar $h_a$, whose couplings are not SM-like.

2. The scalar (pseudoscalar) $h_a (\chi_a)$ has no $(h_a/\chi_a)VV$-type interactions, where $V \equiv W^\pm, Z$.

3. $h_a/\chi_a$ has only flavor off-diagonal Yukawa couplings with one fermion of the third generation.

More specifically, we have extended our previous analysis by including not only the $CP$-even neutral scalars, but all scalar degrees of freedom: three $CP$-even neutral scalars, two $CP$-odd neutral scalars and two sets of charged
The remaining degrees of freedom are neutral (G matrices the masses of the physical scalars/pseudoscalars are obtained. These are denoted by h performed, with the assignments 

By inspection, we found the following conditions on the coefficients c.

The coefficients once the scalars receive vacuum expectation values, the replacement \( h_i \rightarrow (h_1^+, v_1 + h_i + i\chi_i) \) for \( i = 1 \ldots 3 \) is performed, with the assignments \( v_1 = v_2 = v \) and \( v_3 \), which allow for maximal atmospheric mixing. To generate the correct \( W^\pm \) and \( Z \) masses, \( 2v^2 + v_3^2 = v_{SM}^2 \) has to hold, where \( v_{SM} = 246 \text{ GeV} \). After diagonalizing the mass matrices the masses of the physical scalars/pseudoscalars are obtained. These are denoted by \( h_{a,b,c} \) and \( h_{a,b} \).

The remaining degrees of freedom are neutral (\( G^0 \)) and charged (\( G^\pm \)) Goldstone bosons which are eaten up by \( Z \) and \( W^\pm \) respectively.

Note that \( v_1 = v_2 \) is an extremal point if the following conditions are imposed [7]:

To make sure that the extremal point is actually a minimum of the potential, the determinant of the Hessian has to be positive. This statement is equivalent to imposing the condition of positive squared masses of the particles. To keep the potential globally bounded from below the conventional approach is to arrange all the coefficients of the highest-power terms in the potential to be positive definite. This was followed in Ref. [7] where only the \( CP \)-even degrees of freedom were considered. However, this strategy eliminates the allowed possibility of a large part of valid parameter space where the potential is bounded from below although some coefficients still stay negative.

Our present analysis is now more complete in the sense that we deal with the complete spectrum including all neutral and charged degrees of freedom following the potential minimization. Moreover, some parts of the allowed parameter space that were hitherto cut off by the traditional method are now resurrected by our new approach. As a first step, to have an analytical feel we identify some simple-looking relations of the coefficients by inspection that allow the potential to stay positive and also provide the physical scalar masses. To do this the scalar potential in Eq. (2) is factorized into a simplified polynomial in \( \phi_1, \phi_2 \) and \( \phi_3 \), treating them naively as real quantities for calculational ease. There remain three distinct types of terms of order four: \( \phi_1^4 \), \( \phi_1^2 \phi_2^2 \) and \( \phi_i^2 \phi_1 \phi_2 \), where \( i, j, k = 1 \ldots 3 \). Out of the nine terms, only six have independent coefficients, called \( c_{\{1...6\}} \):

The coefficients \( c_{\{1...6\}} \) can be expressed in terms of the potential parameters \( \lambda_{\{1...8\}} \):

By inspection, we found the following conditions on the coefficients \( c_{\{1...6\}} \) from the analytic expressions:

These conditions ensure an acceptable mass spectrum for the neutral scalars/pseudoscalars and charged scalars and keep the potential globally stable. However, this method renders a large part of the parameter space still inaccessible; moreover, the masses obtained by employing Eq. (4) are generally quite light, none exceeding 300 GeV when \( |\lambda_{\{1...8\}}| \leq \pi \).

To obtain a more complete picture we have transformed Eq. (4) into spherical coordinates \((\rho, \theta, \phi)\). The potential then splits into a radial and an angular part. The question of global stability is then reduced to keeping the over-all sign of the angular part of the potential positive definite:

\[
\sin^4 \theta \left\{ (2c_1 - c_3) \cos(4\phi) + 6c_1 + c_3 \right\} + 8c_2 \cos^4 \theta + \sin^2(2\theta) (2c_4 \sin^2 \phi + c_6 \sin(2\phi)) \\
+ 8c_4 \cos^2 \phi \sin^2 \theta \cos^2 \theta + 4c_5 \sin(2\phi) \sin^3 \theta \cos \theta (\sin \phi + \cos \phi) > 0
\]
300 GeV for the scalars/pseudoscalars can be reached even keeping stable parameter space that could not be reached by the conditions of Eq. (6). Consequently, masses well beyond numerically at each point of the parameter space. This allows us to explore the so-far inaccessible territory of imposed on the coefficients $c_{1(10)}$ to solve Eq. (7). We therefore decided to check the positivity of this function numerically at each point of the parameter space. This allows us to explore the so-far inaccessible territory of the stable parameter space that could not be reached by the conditions of Eq. (6). Consequently, masses well beyond 300 GeV for the scalars/pseudoscalars can be reached even keeping $|\lambda_{1(10)}| \leq \pi$. To sum up, our Eq. (6) is an improvement over what we have done in Ref. [7] and subsequently our numerical approach improves the size of the accessible parameter space even further.

Diagonalizing the mass matrix of the pseudoscalars gives the symmetry basis pseudoscalars $\chi_{1,2,3}$ in terms of the physical basis pseudoscalars $\chi_{a,b}$:

$$\chi_{1(2)} = (v/\nu_{SM}) G^0 \mp \left(1/\sqrt{2}\right) \chi_a - v_3/ (\sqrt{2} \nu_{SM}) \chi_b, \quad \chi_3 = (v/\nu_{SM}) G^0 + \sqrt{2} (v_3/\nu_{SM}) \chi_b.$$  \hspace{1cm} (8)

It is interesting to note that the mixing coefficients are very simple and just depend on the ratio $v_3/\nu_{SM}$. This is in stark contrast to the mixing of $h_{1,2,3}$ and $h_{a,b,c}$ given in Ref. [7], where the coefficients are complicated functions of the $\lambda_{1(10)}$ parameters of Eq. (2):

$$h_{1(2)} = U_{1(2)b} h_b + U_{1(2)c} h_c \mp \left(1/\sqrt{2}\right) h_a, \quad h_3 = U_{3b} h_b + U_{3c} h_c,$$  \hspace{1cm} (9)

where $U_{ib}$ and $U_{ic}$ are complicated functions of the $\lambda_{1(10)}, v$ and $v_3$. The mixing relations for the charged scalars $h_{a,b}^+$ are obtained by substituting $\chi \rightarrow h^+$ and $G^0 \rightarrow G^+$ in Eq. (8). The masses for the $CP$-even scalars are [7]

$$m_{h_a}^2 = 4\lambda_2 v^2 - 2\lambda_3 v^2 - v_3 (2\lambda_7 v_3 + 5\lambda_8 v),$$

$$m_{h_{bc}}^2 = \frac{1}{2v_3} \left[ 4\lambda_1 v^2 v_3 + 2\lambda_3 v_3^2 + 2\lambda_4 v_3^3 - 2\lambda_8 v^3 + 3\lambda_8 v_3^3 \mp \Delta m^3 \right], \hspace{1cm} (10)$$

where

$$\Delta m^3 = \left[ 8v v_3 \left\{ 2v v_3^2 \left[ (\lambda_5 + \lambda_6 + \lambda_7)^2 - \lambda_4 (2\lambda_1 + \lambda_3) \right] + 2\lambda_8 v^4 (2\lambda_1 + \lambda_3) - 3\lambda_4 \lambda_8 v_3^4 \right. \right.$$  

$$+ 12\lambda_8 v^2 v_3^2 (\lambda_5 + \lambda_6 + \lambda_7) + 12\lambda_8 v_3^3 v_3 + \left. \left\{ 2v^2 v_3 (2\lambda_1 + \lambda_3) + 2\lambda_4 v_3^3 - 2\lambda_8 v^3 + 3\lambda_8 v_3^3 \right\} \right]^{1/2}. \hspace{1cm} (11)$$

We now derive the pseudoscalar squared masses as

$$m_{\chi_a}^2 = -9\lambda_8 v v_3, \quad m_{\chi_b}^2 = -v_{SM}^2 (2\lambda_7 + \lambda_8 v/v_3). \hspace{1cm} (12)$$

The corresponding squared masses for the charged scalars are

$$m_{h_a}^2 = -2\lambda_3 v^2 - v_3^2 (\lambda_6 + \lambda_7) + 5\lambda_8 v v_3, \quad m_{h_c}^2 = -v_{SM}^2 (\lambda_6 + \lambda_7 + \lambda_8 v/v_3). \hspace{1cm} (13)$$

FIG. 1. Scatter plots of masses of $h_a, h_b, h_c$ and $\chi_a$, where $v_3/v = 0.6$. The lines give the current interesting window between 114 GeV (LEP2) and 130 GeV (LHC) [3, 12] a) Masses of $CP$-even scalars $h_a$ and $h_b$. b) Mass of $CP$-even scalar $h_b$ plotted against the mass difference of $h_a$ and $h_b$. The highlighted strip is also disfavored by LHC which rules out a second SM-like Higgs within 550 GeV. c) Mass of $h_b$ compared to that of the $CP$-odd scalar $\chi_a$. 

The allowed ranges for the masses can be found by a random scattering in the parameter space where the couplings in the potential are varied within $\lambda_{\{1,8\}} \in [-\pi, \pi]$ and the ratio $v_3/v$ is fixed to 0.6 (this value was chosen in Ref. [7] for compatibility with the quark masses). The allowed range for the couplings $\lambda_{\{1,8\}}$ has been increased with respect to what was assumed in our previous work [7] to admit a broader mass spectrum. The values are still very much within perturbative bounds. This leads to a $CP$-even mass spectrum similar to Ref. [7], but with higher allowed ranges. The scalar $h_u$ can be as massive as roughly 800 GeV or arbitrarily light as it evades the LEP2 bound due to its maximally nonstandard couplings. The mass of the SM-like scalar $h_b$ is limited within 114–500 GeV, while $h_c$ is still heavier. Both $h_b$ and $h_c$ masses should however satisfy the LEP2 lower bound of 114 GeV [See Fig. 4 for details].

In view of the recent LHC results [12, 13] that hint towards a SM-like Higgs boson at around 125 GeV with a large excluded region above and below, the mass spectrum in this model is compatible with the following scenario:

1. $h_b$ plays the role of the SM-like Higgs boson with a mass of roughly 125 GeV. The Yukawa and gauge couplings of $h_b$ and $h_c$ are SM-like with numerically negligible flavor off-diagonal couplings [7].

2. The scalars/pseudoscalars $h_a$ and $\chi_a$ have nonstandard interactions that hide them from standard searches, as will be discussed in the following sections. In particular, $h_a$ and $\chi_a$ can be very light.

3. All other scalar/pseudoscalar masses, including the charged scalars, can be above 550 GeV. We however note that the existing limits on charged scalar masses are not so stringent and the parameters of our potential can be arranged to admit a much smaller mass for them, though this is not the main focus of our present work.

III. COUPLINGS OF THE SCALARS/PSEUDOSCALARS

| $h_u^+ W^+$ | $h_b^+ W^+$ | $\chi_a Z$ | $\chi_b Z$ | $W^+ W^+$ | ZZ |
|-------------|-------------|-------------|-------------|-----------|-----|
| $h_a$       | ✓           | –           | –           | ✓         | –   |
| $h_b$       | –           | ✓           | ✓           | ✓         | ✓   |
| $h_c$       | –           | ✓           | ✓           | ✓         | ✓   |

TABLE I. Three-point vertices involving at least one neutral scalar/pseudoscalar and gauge bosons. A checkmark indicates that the vertex exists.

It is worth noting that among the couplings listed in Table I the ones involving $h_a$ do not depend on any parameters of the scalar potential, while the couplings of $h_b$ and $h_c$ to the gauge bosons are complicated functions of the scalar mixing parameters, which we refer to in our tables by putting checkmark signs without displaying their explicit forms. The $h_a \chi_a Z$ coupling has a simple form $h_a \chi_a Z \sim -\frac{2}{3} G q_e$, where $G = \sqrt{g^2 + g'^2}$ and $q_e$ is the momentum transfer. As stated in Ref. [7], $h_a$ stands out because it does not couple to pairs of gauge bosons via the three-point vertex. As a result, neither the LEP2 lower limit of 114 GeV nor the electroweak precision test upper limit of around 200 GeV applies to it. The same is true for the pseudoscalar $\chi_a$. For certain kinematic regions, the coupling $h_a \chi_a Z$ is important for collider searches as we shall see later. Table II contains the other gauge-scalar-scalar and the triple-scalar vertices. Note that $h_a$ couples only off-diagonally to the other scalars/pseudoscalars. The $h_a^+ h_b^-$ couplings depend only on $v_3/v$, while the other triple scalar couplings are complicated functions of the scalar mixing parameters.

For illustration, we first write the Yukawa Lagrangian of the $CP$-even scalars in the basis $\{h_1, h_2, h_3\}$ with the couplings $f_i$ for leptons and $g_i$ for quarks as in Ref. [7]:

$$L_{\text{Yuk}} = f_1 e e^c h + f_2 e^c h + f_3 e^c h + f_4 H^c (\mu h_2 + \tau h_1) + f_5 H^c (\mu h_2 + \tau h_1) + g_1 u u^c h + g_2 d d^c h + g_3 s s^c h + g_4 t t^c (c h_2 + t h_1) + g_5 t t^c (c h_2 + t h_1) + g_6 d d^c h + g_7 s s^c h + g_8 t t^c (c h_2 + t h_1) + g_9 b b^c (c h_2 + t h_1) + H.c. \quad (14)$$
We then rotate the scalars in the Yukawa Lagrangian to their physical basis \( \{ h_a, h_b, h_c \} \) which gives the Yukawa matrices \( Y_{(a,b,c)} \). The individual mixing matrices for up- and down-type quarks contain large angles as a consequence of \( S_3 \) symmetry and the particle assignments \( b \). Specifically, the doublet representation of \( S_3 \) generates maximal mixing when \( v_1 = v_2 \). Now, the Cabibbo-Kobayashi-Maskawa matrix involves a relative alignment of those two matrices which yields small mixing for quarks. Similarly, the Pontecorvo-Maki-Nakagawa-Sakata matrix is given by the relative orientation of the mixing matrices for the charged leptons and neutrinos. Since the neutrino mass matrix generated in the present context by a type-II seesaw mechanism turns out to be diagonal, the large mixing angles in the lepton sector survive. There are two generic textures of Yukawa couplings in this model \( b \). 

\[
Y_a = \begin{pmatrix}
0 & 0 & Y_{13} \\
0 & 0 & Y_{23} \\
Y_{31} & Y_{32} & 0
\end{pmatrix}, \quad Y_{b,c} = \begin{pmatrix}
Y_{11} & Y_{12} & 0 \\
Y_{21} & Y_{22} & 0 \\
0 & 0 & Y_{33}
\end{pmatrix}. \tag{15}
\]

Here \( Y_a \) symbolically describes the Yukawa couplings for \( h_a, \chi_a \) and \( h_a^+ \), while \( Y_{b,c} \) describe the couplings for \( h_b, h_c, \chi_b \) and \( h_b^+ \). The pattern holds both for leptons and quarks \( b \) and reproduces the observed masses and mixing \( b \). The off-diagonal couplings in \( Y_{b,c} \) are numerically small and can be controlled by one free parameter which keeps processes like \( \mu \rightarrow e\gamma \) and meson mixing well under control. The largest off-diagonal coupling in \( Y_a \) is \( (h_a/\chi_a)ct \) which is about 0.8; it leads to viable production channel of \( h_a \) via \( t \) decays as described in the next section. The next largest couplings are \( (h_a/\chi_a)sb \approx 0.02 \) and \( (h_a/\chi_a)\mu\tau \approx 0.008 \). The \( \chi_a\mu\tau \) coupling induces an interesting decay channel potentially observable at the LHC. Note that since the \( h_a^+tb \) coupling does not exist the mass of \( h_a^+ \) is not constrained by the LHC searches in the \( t \rightarrow h^+b \) channel in the mass window of 80–160 GeV \( b \) [44] [15].

IV. OBSERVING \( h_a \) AT THE LHC

If kinematically allowed the dominant production of \( h_a \) occurs through \( t \rightarrow h_a c \) [Fig. 2(a)]. The subsequent decay channels depend crucially on the mass of the pseudoscalar \( \chi_a \); if \( m_{h_a} < m_{\chi_a} \), \( h_a \) decays dominantly into \( b \) and \( s \) quarks, or \( \tau \) and \( \mu \) [see Fig. 2(b)]. The branching ratio (BR) for \( t \rightarrow h_a c \) is about 0.17(0.06) for \( m_{h_a} = 130(150) \) GeV. Then \( h_a \rightarrow \mu\tau \) proceeds with a BR of 10% and \( h_a \rightarrow bs \) with 90%.

A spectacular channel opens up when \( h_a \rightarrow \chi_a Z \) is kinematically accessible [Fig. 2(c)]. The BR of \( h_a \rightarrow Z\chi_a \) is almost 100% due to the numerical dominance of the gauge coupling over the Yukawa couplings involving light fermions, followed by \( \chi_a \rightarrow \tau\mu \) with a BR of \( \sim 10\% \), and a \( Z \rightarrow \mu\mu \) BR of \( \sim 3\% \). If two \( h_a \) are produced from \( tt \)
pairs, this could lead to a characteristic signal with up to six muons with the tau tags. The BRs for $t \rightarrow ch_a$ and subsequently $h_a \rightarrow \chi_a Z \rightarrow \tau\mu\mu\mu$ are plotted in Figs. 3(a)–(c). For these plots $m_{\chi_a} = 20$ GeV has been assumed, which is allowed by current data. The BR peaks for $m_{h_a} = 110$ GeV once the kinematic threshold is crossed and then falls sharply for larger masses due to phase space constraints.

V. CONCLUSIONS

This is a natural extension of our previous work [7] where we studied only the scalar sector assuming the pseudoscalars to be too heavy to be relevant. In this work we have analyzed the complete scalar/pseudoscalar sector of an $S_3$ flavor model. We deal with three $CP$-even, two $CP$-odd and two sets of charged scalar particles. In this work we have improved our potential minimization technique which enabled us to explore a larger region of the allowed parameter space. It is possible to arrange the mass spectrum in full compatibility with the current LHC data, with the scalar $h_a$ mimicking the Higgs-like object lurking around 125 GeV. The specific scalar (pseudoscalar) with prominent non-standard gauge and Yukawa interactions, namely $h_a (\chi_a)$, evade standard searches at LEP/Tevatron/LHC and hence can be rather light. The other scalars/pseudoscalars can be arranged to stay beyond the current LHC reach (e.g., 550 GeV). In particular, we have identified a promising channel for $h_a$ search involving up to six muons in the final state with the tau tags. We urge our experimental colleagues to dig out this information which is probably buried in the existing data.

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