In this paper, we present the study of the hadroproduction rate of $h_c$ at next-to-leading order in $\alpha_s$ under the nonrelativistic QCD (NRQCD) factorization framework, using long-distance matrix elements obtained from the fit of $\chi_c$ production measurements by LHCb. We consider the problem of NRQCD scale dependence for the first time, and find that, for some experimental conditions, the choice of this scale can substantially affect the final results. Up to this order, color-octet channel still contribute the most part of the production rate.

In the last ten years, many experimental measurements for P-wave quarkonia $h_c, h_b(1^{+/-})$ have been achieved. The related branching ratios, the masses of these quarkonia, as well as the cross sections for $h_c(h_b)$ production via $e^+e^-$ annihilation at the CLEO-c and B-factories predicted a significant yield. Photoproduction of $h_c$ was investigated in Ref. by using a color-octet (CO) long-distance matrix element (LDME) extracted from the decay $B \to \chi_c j + X$; the results indicated a significant cross section at the DESY HERA. $h_c$ production through $e^+e^-$ annihilation was investigated in two recent papers, the former of which included the results for $h_b$ as well. Since next-to-leading order (NLO) corrections noticeably change the behavior of the transverse momentum ($p_t$) distribution of the production rate of P-wave quarkonia through color-singlet (CS) channel, in order to investigate $h_c$ production in nonrelativistic QCD (NRQCD) framework, calculation up to this order is required.

However, no adequate experimental data has come out to extract the CO LDME for $h_c$ production directly. Thanks to the spin symmetry at LO of NRQCD expansion, we can simply estimate this LDME as that of $^3S_1$ state for $\chi_c$ production, of which both LO and NLO values have already been given in several papers; the first three experimental data of CDF while the last one added the LHCb data to carry out a global fit.

In this paper, we study $h_c$ production at hadron colliders at NLO of QCD coupling constant $\alpha_s$. New fitted CO LDMEs will be given for different values of NRQCD scale. Employing these LDMEs, we present numerical results for Tevatron and LHC energy.

In NRQCD framework, at LO of $\alpha_s$, hadroproduction can be expressed as
\begin{equation}
\frac{d\sigma(pp \to h_c + X)}{dp^2} = \frac{d\sigma(pp \to (c\bar{c})_n + X)}{dp^2}(O_n^{h_c})
= \sum_{i,j,n} \int dx_1 dx_2 G(x_1, i) G(x_2, j) \hat{f}(i + j \to (c\bar{c})_n + X)(O_n^{h_c}),
\end{equation}
where $p$ denotes either a proton or an antiproton, the indices $i, j$ run over all partonic species, $n$ denotes a definite $c\bar{c}$ state of certain color and angular momentum, $G(x, i)$ is the parton-distribution-function (PDF) with $x$ being the ratio of the momentum of parton $i$ to that of the proton or antiproton, $\hat{f}$ represents the parton-level short-distance coefficient, which can be evaluated perturbatively in $\alpha_s$ and $v$. Since our calculation is up to LO in $v$, only two channels $^{1}P_1^{[1]}$ and $^{1}S_0^{[8]}$ are involved.

The LO partonic processes are listed as
\begin{equation}
g + g \to c\bar{c}(^{1}P_1^{[1]}) + g,
\end{equation}

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and
\[ g + g \to c\bar{c}(1S^0_0) + g, \]
\[ g + q \to c\bar{c}(1S^0_0) + q, \]
\[ q + \bar{q} \to c\bar{c}(1S^0_0) + g. \] \hfill (3)

The process in Eq. (2) generates \( {}^1P_{1[1]} \) state, while the three processes in Eq. (3) generate \( {}^1S^0_0 \) state. Both of the two states transit into \( h_c \) through long-distance processes which can not be evaluated perturbatively, and the transition rates are described by LDMEs. The LDME of \( {}^1P_{1[1]} \) can be expressed in terms of radial derivative of the wave function of quarkonium at the origin:
\[ \langle O^{h_c}_{1P_{1[1]}} \rangle = \frac{27}{2\pi} |R''_{h_c}(0)|^2, \]
while that of \( {}^1S^0_0 \) is obtained from the fit of experimental measurements of \( \chi_c \) productions. To evaluate the short-distance coefficients, we notice they are independent of the long-distance asymptotic states, and replace \( h_c \) in Eq. (1) by an on-shell heavy quark pair state with definite quantum numbers, the following equations can be obtained,
\[ d\sigma_m = \sum_n df_m(\frac{1}{n}), \]
where, and through out the rest of this paper, we denote the cross sections \( \sigma(pp \to n + X) \) and short-distance coefficients \( f(pq \to n + X) \) in abbreviation as \( \sigma_n \) and \( f_n \), respectively. The cross section for producing \( m \) can be evaluated perturbatively in \( \alpha_s \) by reading the amplitudes from Feynman diagrams. For \( {}^1P_{1[1]} \) state, up to leading order in \( \alpha_s \), only two states, \( {}^1P_{1[1]} \) and \( {}^1S^0_0 \), are involved, thus, we have
\[ d\sigma_{1P_{1[1]}} = df_{1P_{1[1]}}(O_{1P_{1[1]}^{1P_{1[1]}}}^{1P_{1[1]}}) + df_{1S^0_0}(O_{1S^0_0^{1P_{1[1]}}}) \]. \hfill (6)

At LO in \( \alpha_s \), \( \langle O^m \rangle \) vanishes unless \( m = n \), as a result, the short-distance coefficients can be simply expressed in terms of integrated squared amplitudes.

Up to NLO in \( \alpha_s \), the calculation of \( {}^1S^0_0 \) state production has been described in detail in our previous paper \[25\]. Here we focus on \( {}^1P_{1[1]} \) state production at this order. The processes involved are as follows,
\[ V : g + g \to c\bar{c}(1P_{1[1]}), \]
\[ g + g \to c\bar{c}(1P_{1[1]}) + gg, \]
\[ g + q \to c\bar{c}(1P_{1[1]}) + q\bar{q}, \]
\[ q + \bar{q} \to c\bar{c}(1P_{1[1]}) + gg. \] \hfill (7)

where label \( V \) means one-loop virtual corrections for the process on the right side of it. Summing all the processes, one finds the IR divergences do not cancel, which can be figured out in NRQCD framework by subtracting the divergence coming out of \( \langle O_{1S^0_0^{1P_{1[1]}}} \rangle \). Up to the order maintained in our calculation, the transition rate of \( c\bar{c} \) state \( {}^1S^0_0 \) into \( {}^1P_{1[1]} \) can be calculated in dimensional regularization and \( \overline{MS} \) renormalization scheme as
\[ \langle O_{1S^0_0^{1P_{1[1]}}} \rangle_{NLO} = -\frac{\alpha_s}{3\pi m_c}N_c^2 \frac{N_c - 1}{N_c} \langle O_{1P_{1[1]}^{1P_{1[1]}}} \rangle_{LO}, \]
where \( N_c \) is 3 for SU(3) gauge field and \( u_c^{\epsilon} \) is defined as
\[ u_c^{\epsilon} = \frac{1}{\epsilon_4} - \gamma_E - \frac{1}{3} + \ln(4\pi p_R^2), \] \hfill (8)
with \( \gamma_E \) being Euler’s constant. The divergence in this LDME will cancel that in \( \sigma({}^1P_{1[1]}) \), which can be isolated by using the two-cutoff phase space slicing method \[26\] as
\[ d\sigma_{1P_{1[1]}} = d\sigma_{L+V,1P_{1[1]}} + d\sigma_{S,1P_{1[1]}} + d\sigma_{H,1P_{1[1]}} \]
\[ = (d\sigma_{L+V,1P_{1[1]}} + d\sigma_{H,1P_{1[1]}} + d\sigma_{S,1P_{1[1]}})(\langle O_{1P_{1[1]}^{1P_{1[1]}}} \rangle), \] \hfill (9)
where $\sigma^{L+V}$ represents the summation of LO and virtual correction contributions, and $\sigma^S$ and $\sigma^H$ are real corrections from the gluon-soft and -hard region, respectively. The boundary of the two regions is $E_g = \frac{\pi^2}{\mu^2}$, where $E_g$ is the energy of the soft gluon and $\delta_\sigma$ is an arbitrary positive number small enough to provide the soft approximation with adequate accuracy. Moreover, the gluon-soft region consists of where the soft gluon is attached to an external charm quark (antiquark) line (labeled $S_2$) and where it is attached to other external lines (labeled $S_1$). Noticing that we have summed over all the processes in Eq. (7), the only divergent term left in $d\sigma^S_{1P_1^{[1]}}$ comes from $d\sigma^S_{1P_1^{[2]}}$ which will subtract the divergence in $\langle O_{1S_0^{[8]}}^{1P_1^{[1]}} \rangle$. Neglecting the finite terms in this small region, the soft part can be expressed as

$$d\sigma^S_{1P_1^{[1]}} = \frac{\alpha_s}{3\pi m_c^2} u_c^s N_c^2 - 1 \frac{\alpha_s}{3\pi m_c^2} u_c s \langle O_{1S_0^{[8]}}^{1P_1^{[1]}} \rangle,$$

where

$$u_c^s = \frac{1}{\epsilon} + \frac{E}{p} \ln \frac{E + p}{E - p} + \ln \left( \frac{4\pi \mu_R^2}{s\delta_\sigma^2} \right) - \gamma_E - \frac{1}{3},$$

and $E$ and $p$ are energy and absolute value of momentum of $h_c$.

Matching Eq. (9) and Eq. (10), at the same time, employing Eq. (8) and Eq. (11), and constrain our calculation up to NLO in $\alpha_s$, we obtain the expression of short-distance coefficient for $1P_1^{[1]}$ state as

$$d\sigma^{NLO}_{1P_1^{[1]}} = d\sigma^{L+V+S_1}_{1P_1^{[1]}} + d\sigma^H_{1P_1^{[1]}} - \frac{\alpha_s}{3\pi m_c^2} N_c^2 - 1 \frac{\alpha_s}{3\pi m_c^2} u_c s \langle O_{1S_0^{[8]}}^{1P_1^{[1]}} \rangle,$$

where

$$u_c = u_c^s - u_c^\delta = \frac{E}{p} \ln \frac{E + p}{E - p} + \ln \left( \frac{\mu_R^2}{s\delta_\sigma^2} \right).$$

It is clearly shown that all the short-distance coefficients are IR divergence free and the cross section for the first process is well defined. To calculate $\sigma^{L+S_0^{[8]}}$ and $\sigma^{S_1^{[1]}}$, we apply our Feynman Diagram Calculation package (FDC) [27] to generate all the needed FORTRAN source.

From the heavy quark spin symmetry of the NRQCD Lagrangian, it is obvious that the LDME $\langle O_{1S_0^{[8]}}^{h_c} \rangle$ for the intermediate state $c\bar{c}(1S_0^{[8]})$ evolving into $h_c$ is exactly the same as that for the intermediate state $c\bar{c}(3S_1^{[8]})$ evolving into $\chi_c$ at LO of $v^2$. It gives

$$\langle O_{1S_0^{[8]}}^{h_c} \rangle \approx \langle O_{1S_0^{[8]}}^{\chi_c} \rangle.$$

Substituting Eq. (13) into Eq. (11), we obtain the complete expression of cross section for $h_c$ production,

$$d\sigma^{NLO}_{h_c} = \langle O_{1S_0^{[8]}}^{h_c} \rangle (d\sigma^{L+V+S_1}_{1P_1^{[1]}} + d\sigma^H_{1P_1^{[1]}}) - \frac{\alpha_s}{3\pi m_c^2} N_c^2 - 1 \frac{\alpha_s}{3\pi m_c^2} u_c \langle O_{1S_0^{[8]}}^{h_c} \rangle d\sigma^{LO}_{1S_0^{[8]}} + \langle O_{1S_0^{[8]}}^{h_c} \rangle d\sigma^{NLO}_{1S_0^{[8]}}.$$

If we proceed our calculation to infinite order of $\alpha_s$, the $\mu_A$ dependence can be totally absorbed into the CO LDME. As a result, this scale actually can be any positive value holding the convergence of $\alpha_s$ expansion. If our results dramatically depend on $\mu_A$, the dropped terms in higher orders must contribute significantly, and the calculation up to this order does not reach a sufficient accuracy. Up to NLO, the condition of $\mu_A$ independence requires

$$\frac{\alpha_s}{3\pi m_c^2} N_c^2 - 1 \frac{\alpha_s}{3\pi m_c^2} d\sigma^{LO}_{1S_0^{[8]}} \propto d\sigma^{NLO}_{1S_0^{[8]}},$$

as well as that the proportional ratio be universal for all the processes. Actually, in most of the cases, this condition can not be satisfied.

To show the problem of $\mu_A$ dependence, at the same time, to determine whether $\alpha_s$ expansion up to this order have got an adequate accuracy, one should carry out the calculation at different values of this scale. The fitting should also be carried out at each particular scale.

In the numerical calculation, we have the following common choices as $|R_{K^0}^i(0)|^2 = 0.075$ GeV$^5$ [28] for both LO and NLO calculation, $\langle O_{1S_0^{[8]}}^{h_c} \rangle_{LO} = 0.0098$ GeV$^3$, the same as Ref. [20], and $m_c = 1.5$ GeV. The soft cutoff $\delta_s$
FIG. 1: Fit of the LHCb [24] data with the branching ratio of \( \chi_c \) to \( J/\psi \) multiplied. The values of \( \mu_\Lambda \) are \( m_c \), \( m_c/2 \) and \( \Lambda_{QCD} \) for upper, middle and lower figures, respectively.

independence is checked in the calculation and \( \delta_s = 0.001 \) is used. Since the energy scale of most of the phase space region exceeding b-quark mass, \( \Lambda_{QCD}|_{n_f=5} = 0.226 \) GeV is used. We employ CTEQ6M [29] as PDF and two-loop \( \alpha_s \) running for NLO calculation, and CTEQ6L1 [29] and one-loop \( \alpha_s \) running for LO.

We employ LHCb [24] data for \( \chi_c \) production, excluding \( p_t < 7 \) GeV points, to obtain the value of the CO LDME. The CM energy and rapidity cut are \( \sqrt{s} = 7000 \) GeV and \( 2 < y < 4.5 \), respectively. The CO LDMEs are obtained as

\[
\begin{align*}
\langle O h_{c[8]} \rangle_{m_c} &= (0.73 \pm 0.01) \times 10^{-2} \text{ GeV}^3 \\
\langle O h_{c[8]} \rangle_{m_c/2} &= (0.57 \pm 0.01) \times 10^{-2} \text{ GeV}^3 \\
\langle O h_{c[8]} \rangle_{\Lambda_{QCD}} &= (0.28 \pm 0.01) \times 10^{-2} \text{ GeV}^3
\end{align*}
\]

(18)

where the values of \( \mu_\Lambda \) are labeled at the foot of LDMEs.

Fig. 1 shows \( p_t \) distributions of the production rates for \( \chi_c \) generated \( J/\psi \), from which we can see that, for all the three values of the scale, our calculations can fit the experimental data well. Employing these LDMEs, \( h_c \) production rate for three experimental conditions are presented in the following.

For Tevatron energy, i.e. \( \sqrt{s} = 1.96 \) TeV, and rapidity cut condition \( |y| < 0.6 \), denoted as experimental condition I, \( h_c \) production rates are presented in Fig. 2. As is shown in these figures, \( 1S_0^{[8]} \) channel contribute most of the positive part of the total production, just as the situation for LO. Despite the contributions from the two channels, \( 1S_0^{[8]} \) and \( 1P_1^{[1]} \), vary significantly for different choices of the NRQCD scale, the final results are not affected noticeably.
FIG. 2: $h_c$ production at the Tevatron. The CM energy and rapidity cut are $\sqrt{s} = 1960$ GeV and $|y| < 0.6$, respectively. The values of $\mu_A$ are $m_c$, $m_c/2$ and $\Lambda_{QCD}$ for upper, middle and lower figures, respectively.

FIG. 3: The comparison of the results for the three different choices of NRQCD scale. The CM energy and rapidity cut are $\sqrt{s} = 1960$ GeV and $|y| < 0.6$, respectively. Distinction of the final results for the three scale choices is presented more explicitly in Fig. 3. We can see that, in low $p_t$ region, different values of this scale bring in little distinction, while in high $p_t$ region, the three curves differ, but not much (the largest is about twice the amplitude of the smallest). Since large logarithm term $\ln(m_c/p_t)$ may ruin $\alpha_s$ expansion for both too large and too small $p_t$, besides, in small $p_t$ region, relativistic corrections contribute a nonlinear remarkable part, only medium $p_t$ region is considered up to this order in this paper. Restricted to this region, our results do not depend on NRQCD scale significantly. We can say the dependence of this scale has been absorbed into the CO LDME, which turns out to be a scale-relevant quantity.
FIG. 4: $h_c$ production at the LHC. The CM energy and rapidity cut are $\sqrt{s} = 7000$ GeV and $|y| < 2.4$, respectively. The values of $\mu_A$ are $m_c$, $m_c/2$ and $\Lambda_{QCD}$ for upper, middle and lower figures, respectively.

FIG. 5: The comparison of the results for the three different choices of NRQCD scale. The CM energy and rapidity cut are $\sqrt{s} = 7000$ GeV and $|y| < 2.4$, respectively.

For LHC energy $\sqrt{s} = 7$ TeV, while rapidity range being $|y| < 2.4$, denoted as experimental condition II, we got similar conclusion; NRQCD scale dependence is not severe to ruin the accuracy of prediction, as is shown in Fig. 4 and Fig. 5.

However, at the same LHC energy, for $2.5 < y < 4.0$, denoted as experimental condition III, when $\mu_A = m_c$ and $m_c/2$, we got unphysical results, i.e. negative cross sections. The dependence of NRQCD scale is so dramatic that we cannot make a definite conclusion for this experimental condition. Only for $\mu_A = \Lambda_{QCD}$, the cross section turns out to be positive throughout the whole range of $p_T$. The comparison of the results for the three different choices
FIG. 6: $h_c$ production at the LHC. The CM energy and rapidity cut are $\sqrt{s} = 7000$ GeV and $2.5 < y < 4$, respectively. The values of $\mu$ are $m_c$, $m_c/2$ and $\Lambda_{QCD}$ for upper, middle and lower figures, respectively.

FIG. 7: The comparison of the results for the three different choices of NRQCD scale. The CM energy and rapidity cut are $\sqrt{s} = 7000$ GeV and $2.5 < y < 4$, respectively.

The comparison of the results for the three different choices of NRQCD scale is also presented, as is shown in Fig. 7. We notice that, for a proper determined NRQCD scale $m_c$, i.e. it is comparable to other scales in this calculation and in perturbative region, even for medium $p_t$, where no resummation is required, we come across unphysical results. This indicates that higher order corrections must be sizeable, and up to this order, the results are not reliable.

In order to investigate the origin of this discrepancy of the three curves for the third experimental condition, we
FIG. 8: The comparison of the value of \( r \) for the three different experimental conditions for \( h_c \), as well as \( \chi_c \), as a function of \( p_t \). The CM energy for LHC and Tevatron experiment are \( \sqrt{s} = 7000 \text{ GeV} \) and \( \sqrt{s} = 1960 \text{ GeV} \), respectively.

define the proportional ratio for Eq.17 as

\[
\frac{d^2N_{LO}}{dp_t} = \frac{r}{\left( \frac{\alpha_s N_c^2 - 1}{3\pi \frac{N_c^2}{N_c}} \right)},
\]

where \( n \) is either \( 1S_0^1 \) or \( 2S_1^1 \). We calculate this parameter for \( h_c \) productions for the three experimental conditions, as well as that for \( \chi_c \) production for the experimental condition we used in the fit. Fig. 8 compares the values of \( r \) for the four conditions. In the case of \( \chi_c \) production, the dependence of \( r \) on \( p_t \) is flat, which explains the fact that, the experiment can be fitted well for the all three choices of the scale. At small and medium \( p_t \), the curve for \( \chi_c \) is close to that for \( h_c \) in condition I and II. This results in slight \( \mu_A \) dependence in the two conditions, just as shown in Fig. 3 and 4. However, for condition III, the value of \( r \) is quite (about two and a half time) below that for \( \chi_c \), which causes the scale dependence problem, as is shown in Fig. 7. In high \( p_t \) region, the curves for condition I and II are quite above that for \( \chi_c \), the corresponding effect is that, in this \( p_t \) region, the dependence of the production rates of \( h_c \) on NRQCD scale is more remarkable than in low \( p_t \) region. By contrast, this problem for condition III becomes milder in this \( p_t \) region.

In summary, we present the QCD NLO theoretical predictions of the \( h_c \) production at the Tevatron and the LHC. At this order, CO channel counts for most of the positive contributions to the production rate of \( h_c \), while the CS channel contributes a small positive or negative part. LO curves are above NLO ones in low \( p_t \) region for all the three conditions. For experimental conditions I and II, LO and NLO curves get close to and intersect with each other as \( p_t \) increases. Using the CO LDMEs for different values of NRQCD scale, we study the NRQCD scale (\( \mu_A \)) dependence of \( h_c \) hadroproduction rate. In medium \( p_t \) region, where perturbative calculation is available, for the first two conditions, the final results depend on \( \mu_A \) slightly, as a result, theoretical predictions up to this order are reliable. The production rates are physical for all the choices of \( \mu_A \) considered in this paper for these two conditions, in contrast to a negative production rate in low (and medium) \( p_t \) range for condition III while setting \( \mu_A = m_c \) (and \( m_c/2 \)). In the third condition, the theoretical prediction remarkably depend on \( \mu_A \) in medium \( p_t \) range, as a result, theoretical prediction for this experimental condition fails at this order. Our calculations for LO results agree with Ref. 14 when employing the same choices of parameters, yet cannot accord with Ref. 12, 13 despite having tried all the possible choices of parameters.

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[13] K. Sridhar, Phys.Lett. B674, 36 (2009), 0812.0474.
[14] C.-F. Qiao, D.-L. Ren, and P. Sun, Phys.Lett. B680, 159 (2009), 0904.0726.
[15] S. Fleming and T. Mehen, Phys.Rev. D58, 037503 (1998), hep-ph/9801328.
[16] J.-X. Wang and H.-F. Zhang, Phys.Rev. D86, 074012 (2012), 1207.2416.
[17] Y. Jia, W.-L. Sang, and J. Xu (2012), 1206.5785.
[18] G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys.Rev. D51, 1125 (1995), hep-ph/9407339.
[19] P. L. Cho and A. K. Leibovich, Phys.Rev. D53, 150 (1996), hep-ph/9505329.
[20] P. L. Cho and A. K. Leibovich, Phys.Rev. D53, 6203 (1996), hep-ph/9511315.
[21] Y.-Q. Ma, K. Wang, and K.-T. Chao, Phys.Rev. D83, 111503 (2011), 1002.3987.
[22] B. Gong, L.-P. Wan, J.-X. Wang, and H.-F. Zhang (2012), 1205.6682.
[23] F. Abe et al. (CDF Collaboration), Phys.Rev.Lett. 79, 578 (1997).
[24] R. Aaij et al. (LHCb Collaboration) (2012), 1204.1462.
[25] B. Gong, X. Q. Li, and J.-X. Wang, Phys.Lett. B673, 197 (2009), 0805.4751.
[26] B. Harris and J. Owens, Phys.Rev. D65, 094032 (2002), hep-ph/0102128.
[27] J.-X. Wang, Nucl.Instrum.Meth. A534, 241 (2004), hep-ph/0407058.
[28] E. J. Eichten and C. Quigg, Phys.Rev. D52, 1726 (1995), hep-ph/9503356.
[29] J. Pumplin, D. Stump, J. Huston, H. Lai, P. M. Nadolsky, et al., JHEP 0207, 012 (2002), hep-ph/0201195.