1 Gauge links, TMD-factorization, and TMD-factorization breaking

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Abstract

In this section, we discuss some basic features of transverse momentum dependent, or unintegrated, parton distribution functions. In particular, when these correlation functions are combined in a factorization formulae with hard processes beyond the simplest cases, there are basic problems with universality and factorization. We discuss some of these problems as well as the opportunities that they offer.

1.1 Introduction

In hard processes, parton distribution functions (PDFs) and fragmentation functions (FFs) are expressed as matrix elements of nonlocal combinations of quark or gluon fields. In the collinear situation that all transverse momenta of partons are integrated over in the definitions, the nonlocality is in essence light-like. These correlation functions are convoluted with the squared amplitude for the partonic subprocess (in essence the partonic cross section) of a hard process. When the transverse momenta of partons are involved, the non-locality in the matrix elements includes a transverse separation, and a transverse momentum dependent (TMD) factorization theorem is needed. In all cases the definitions of the non-perturbative functions include gluon contributions resummed into gauge links that bridge the nonlocality.

It is important to realize that the appearance of the gauge links is a consequence of the systematic resummation of extra gluon contributions in the derivations of factorization, so their structure is dictated by the requirements of factorization.

In processes like \( \ell + H \rightarrow \ell' + h + X \) (semi-inclusive deep inelastic scattering), \( \ell + \bar{\ell} \rightarrow h_1 + h_2 + X \) (annihilation process) or \( H_1 + H_2 \rightarrow \ell + \bar{\ell} + X \) (Drell-Yan process) one has, at leading power in the hard scale, a simple underlying hard process, which is a virtual photon (or weak boson) coupling to a parton line. The color flow from hard part to collinear or soft parts is simple. Additional gluons with polarizations collinear to the parton momenta are resummed into gauge links, which exhibit the interesting behavior that for transverse momentum dependent functions they bridge the transverse separation between the non-local field combinations at lightcone past or future infinity. Which gauge link is relevant in a particular non-perturbative function depends on the color flow in the full process. For a quark distribution function one has a link via (future) lightcone \( +\infty \) if the color flows into the final state, and a link via (past) lightcone \( -\infty \) if the color is annihilated by another incoming parton.

QCD factorization theorems are central to understanding high energy hadronic scattering cross sections in terms of the fundamentals of perturbative QCD. In addition to providing a practical prescription for order-by-order calculations, derivations of factorization provide a solid theoretical underpinning for concepts like PDFs and FFs which are crucial in the quest to expand the basic understanding of hadronic structure. The most natural first attempt at a TMD-factorization formula is simply to extend the classic parton model intuition familiar from collinear factorization. For the semi-inclusive deep inelastic
scattering (SIDIS) cross section, for example, the cross section might be written schematically as
\[ d\sigma \sim |H|^2 \otimes \Phi(x, k_T) \otimes \Delta(z, k'_T) \delta^{(2)}(q + k_T - k'_T). \]

Here \( \Phi(x, k_T) \) is the TMD PDF while \( \Delta(z, k'_T) \) is the TMD FF, with the usual probability interpretations, and \( |H|^2 \) represents the hard part. The momentum \( q_T \) is the small momentum sensitive to intrinsic transverse momenta, \( k_T \) and \( k'_T \), carried by the colliding proton and the produced hadron. The \( \otimes \) symbol denotes all relevant convolution integrals, and the \( x \) and \( z \) arguments are the usual longitudinal momentum fractions.

In a perturbative derivation of factorization, a small-coupling perturbative expansion of the cross section is analyzed in terms of “leading regions”, and the sum is shown order-by-order to separate into the factors of equation (1). The precise field theoretic definitions of the correlation functions, \( \Phi(x, k_T) \) and \( \Delta(z, k'_T) \), should emerge naturally from the requirements of factorization. In the hard part \( |H|^2 \), all propagators must be off-shell by order the hard scale \( Q \) so that asymptotic freedom applies, and small-coupling perturbation theory is valid, with non-factorizing higher-twist contributions suppressed by powers of \( Q \). Such factorization theorems are well-established for inclusive processes that utilize the standard integrated correlation functions (see [1] and references therein), but TMD-factorization theorems involve other subtleties, particularly with regard to the definitions of the of the TMD PDFs and FFs and their associated gauge links (or Wilson lines). We will discuss some of these issues in the next few sections.

In cases where there is a more complex color flow such as is often the case when the underlying hard process involves multiple color flows and/or if the incoming partons are gluons, this can potentially lead to a more complex gauge link structure including traced closed loops or looping gauge links. For situations in which only one TMD correlation function is studied, these structures have been examined in [2] for two-to-two partonic subprocesses.

In situations that involve several TMD functions, factorization using separate TMD functions fails completely, as we will discuss in the last section.

2 Review: Collinear Factorization and Simple Gauge Links

To understand the issues that arise in defining TMDs, it is instructive to start with a review of the definition of the standard integrated quark PDF of collinear factorization. It is
\[ f(x; \mu) = \text{F.T.} \langle p | \bar{\psi}(0, w^-, 0_i) \gamma^+ V_{[0, w]}(u_J) \psi(0) | p \rangle. \]

Our symbol “F.T.” is a short-hand for the Fourier transform from coordinate space to momentum space. The basic structure of the definition is evidently that of a number density; a quark is extracted from the proton state at position 0 and propagates to position \( (0, w^-, 0_i) \). A few subtleties should be noted, however. One is that the definition contains UV divergences which must be renormalized. This gives dependence on an extra scale \( \mu \), and ultimately results in the well-known DGLAP evolution equations for the integrated PDF. The other is that, for a gauge invariant definition, the PDF must contain a path ordered exponential of the gauge field that connects the points 0 and \( (0, w^-, 0_i) \). This is the gauge link and its formal definition is
\[ V_{[0, w]}(u_J) = P \exp \left( -igt^a \int_0^w d\lambda u_J : A^a(\lambda u_J) : \right). \]
Figure 1: (a) Target-collinear gluons in a graph for SIDIS. (b) Factorization of extra gluons into gauge link contributions.

Figure 2: (a) Simple light-like gauge link structure in integrated the PDF. (b) First try at a gauge link structure for the TMD PDF. In both of these diagrams, the thick red arrows represent the main light-like legs of the gauge link. In (b), the then dotted link at connecting the main legs at light-cone minus infinity points in the transverse direction, which is perpendicular to the page.

The path of the gauge link is determined by the light-like vector $u_J = (0, 1, 0_t)$. That is, the gauge link follows a straight path connecting $0$ and $(0, w^-, 0_t)$ along the exactly light-like minus direction. In Feynman graph calculations, the contribution from the gauge link corresponds to the so-called "eikonal factors," which have definite Feynman rules that follow naturally from factorization proofs. After a sum over graphs, and the application of appropriate approximations and Ward identity arguments, extra collinear gluons like those shown in figure 1(a) for SIDIS factor into gauge link contributions. In figure 1(b), the eikonal factors are shown as gluon attachments from the target-collinear bubble to a double line.

To aid in the discussions of how the gauge link should be modified in more complicated circumstances, it is useful to visualize the coordinate space geometry of the gauge link structure. For example, for the integrated PDF in equation (2), the gauge link follows a straight path in the exactly light-like minus direction, as illustrated by the Minkowski-like diagram in figure 2(a).
3 TMD definitions

The most natural first try at extending the PDF definition in equation (2) to the TMD case is to simply leave the integration over transverse momentum in the TMD PDF definition undone. That is, instead of Eq. (2) one may try

$$\Phi(x, k_t) = \text{F.T.} \langle \tilde{p} | \bar{\psi}(0, w^-, w_t) \gamma^+ U_{[0,w]}(u_J) \psi(0) | p \rangle.$$ (4)

The separation is now 0 and $(0, w^-, w_t)$ — it has acquired a transverse component and the Fourier transform is now in both $w^-$ and $w_t$. As a result, the structure of the gauge link $U_{[0,w]}(u_J)$ must also be modified from the simple straight light-like $V_{[0,w]}(u_J)$ gauge link of equation 2. The eikonal attachments on either side of the cut in figure 1 still give minus-direction Wilson lines, but now in order to have a closed link there must also be a small transverse detour at light-cone infinity. This detour arises naturally from boundary terms that are needed as subtractions to make higher twist contributions gauge invariant [3]. The first try at a coordinate space picture of the gauge link should therefore be more like the hook shaped line shown in figure 2(b).

The gauge link structure in equation 4, with its two exactly light-light legs and a transverse link at infinity is commonly cited as the gauge link that is necessary for the definition of the TMD PDFs. However, there are a number of further subtleties, and we will find that the definition needs to be modified. One complication is that rapidity divergences, which in collinear factorization would cancel in the sum of graphs, remain uncanceled in the definition of the TMD correlation functions. Rapidity divergences correspond to gluons moving with infinite rapidity in the direction opposite the containing hadron, and remain even when infrared gluon mass regulators are included. (For a more complete review of these and related issues, see for example [4].) The most common way to regularize the light cone divergences is to make the gauge links slightly non-light-like. In the coordinate space picture, the gauge link therefore becomes more like the tilted hook shape in figure 3. This introduces a new arbitrary rapidity parameter – the “tilt” of the gauge link. A generalization of renormalization group techniques is needed to recover predictability in the factorization formula. A system of evolution equations for the TMD case was developed by Collins, Soper and Sterman (CSS) and has been successfully applied to specific processes [5].

A complete treatment of TMD-factorization involves soft gluons, which give rise to an extra “soft factor” $S(q)$ in the factorization formula of equation (1). The TMD-factorization formula then becomes

$$d\sigma \sim |\mathcal{H}|^2 \otimes \Phi(x, k_T) \otimes \Delta(z, k'_T) \otimes S(h_T) \delta^{(2)}(q_T + k_T - k'_T - h_T).$$ (5)
The soft factor describes the role of gluons with nearly zero center-of-mass rapidity. One difficulty with the usual presentation of the CSS formulation is that the explicit appearance of a soft factor seems somewhat counter to the basic parton model intuition wherein all non-perturbative effects are associated with functions for each external hadron with simple and specific probabilistic interpretations. A natural hope is that, with an appropriate sequence of redefinitions, the role of the soft gluons can be absorbed into the definitions of the PDFs and FFs. The recent work of Collins [6] has shown how this is possible. Indeed, this treatment of the soft factor is necessary for a completely correct factorization derivation with fully consistent definitions for the correlation functions.

While the CSS formalism has been implemented for specific spin independent processes (see, for example, [7]) , much work remains to be done in tabulating and classifying the TMDs. This is especially true for cases that involve spin. Work in this direction has been started in [8].

4 TMD-factorization breaking

The discussion of the last few sections has focused on situations where factorization is known to hold. There are also, however, situations where TMD-factorization is now known to break down [2, 9, 10, 11, 12]. The key issue is the failure of the usual Ward identity arguments that ordinarily allow eikonalized gluons to be factorized and identified with a particular gauge link structure in the definitions of the TMDs. A hint of what leads to TMD-factorization breaking is already suggested by the well-known overall relative sign flip in the Sivers function for SIDIS as compared to the Drell-Yan (DY) process [13]. The difference comes because in the SIDIS TMD-factorization formula, the gauge link in the Sivers function (a spin-dependent TMD PDF) is future pointing, whereas it is past pointing in the the DY case. At the level of Feynman graphs, the difference can be seen in the fact that the “extra” gluons which contribute to the gauge link attach before the hard scattering in one case, and after the hard scattering in the other (see figure 4). This illustrates that the direction of the flow of color through the eikonal lines is a critical factor in the definition of the correlation functions.

In the more complicated hadro-production processes, \( H_1 + H_2 \rightarrow H_3 + H_4 + X \), where \( H_3 \) and \( H_4 \) may be either jets or hadrons, a reasonable first approach would be to traced the flow of color through the eikonal factors and use analogous arguments to what we used for SIDIS and DY in the previous section. Looking at graphs like figure 5, one finds that
the resulting structures are not simply the future or past pointing gauge links familiar from SIDIS or DY, but rather are complicated and highly process dependent objects \[2\]. That this corresponds (at least) to a breakdown of universality is most directly seen in an explicit spectator model calculation. For example, one may consider an Abelian scalar-quark / Dirac spectator model with multiple flavors as in \[9\]. Then, in addition to the standard gauge link attachments, there are extra gluon attachments that do not cancel in a simple Ward identity argument, and which give contributions that are not consistent with having a simple gauge link like what is found SIDIS (figure 3) or DY (opposite pointing).

Therefore, it is clear that there is at least a violation of universality in the hadro-production of hadrons. The natural next approach to try is to maintain a basic factorization structure, but to loosen the requirement that the TMDs be universal, resulting in a kind of “generalized” TMD-factorization formalism. That is, the cross section might still be expected to factorize order-by-order into a hard part and well-defined, albeit non-universal, matrix elements for each separate external hadron \[11\]. However, a careful order-by-order consideration of multiple gluons in the derivation of TMD-factorization shows that even this is not possible \[12\]. If, for example, one extends the model of \[9\] to allow the gluons to carry color (while still considering a hard part that involves only the exchange of a colorless boson) then it is straightforward to see that the flow of color spoils the possibility of factorizing the graph into TMD PDFs with separate gauge links for each TMD, regardless of what kind of gauge link geometries are allowed. (See figure 6 for an illustration.) Therefore, the problem with factorization in the hadro-production of hadrons is more than just a problem with universality – separate correlation functions cannot even be defined in a way that is consistent with factorization.

Because the graphs in figure 5 illustrate the failure of factorization in terms of a failure of color flow to factorize, it is tempting to conclude that the TMD-factorization breaking is purely an issue of complicated color flow. This is an oversimplification of the issue, however. The root of the problem is a failure of Ward identity arguments, which normally allow “extra” gluons to be factorized after a sum over graphs. The Ward identity arguments are only valid after an appropriate sequence of contour deformations on the momentum integrals. In the case of hadro-production of hadrons the necessary deformations are prohibited. In other cases where the direction of color flow may at first appear to pose a problem for factorization (such as in $e + p \to h_1 + X$ and $e + p \to h_1 + h_2 + X$), the necessary contour deformations are possible and factorization holds. (See the explanation in chapter 12 of \[6\].)
Figure 6: A sample graph that illustrates the entanglement of color that leads to a failure of factorization. The overall color factor of the first graph is non-zero, but the product of two one-gluon contributions to the matrix elements with Wilson loops is exactly zero.

5 Conclusions

We have discussed the basic status of issues related to the definitions of TMDs with a focus on the complications that can arise when determining the gauge link structures that are consistent with factorization. We have also described the breakdown of TMD-factorization in the case of hadro-production of hadrons. To summarize, we list the status of TMD-factorization for various well-known processes with a check mark for processes where factorization appears to be valid and !! where it has been shown to fail:

✓ Semi-inclusive deep inelastic scattering ($e^+ + p \rightarrow h_1 + X$).

✓ Drell-Yan (up to overall minus signs for some spin-dependent TMDs).

✓ Back-to-Back hadron or jet production in $e^+ e^-$ annihilation.

✓ Back-to-back hadron or jet production in DIS ($e^+ + p \rightarrow h_1 + h_2 + X$).

!! Hadro-production of back-to-back jets or hadrons ($H_1 + H_2 \rightarrow H_3 + H_4 + X$).

In cases where TMD-factorization is valid, there is still much work left to be done (and much potential insight to be gained) in terms of implementing the evolution of precisely defined TMDs [8]. Much already exists for the case of unpolarized scattering, but even here the most complete and formal identification of evolution effects with separate TMDs has only recently been clarified in [6]. For polarization dependent functions, it is also important to include evolution, but to date there has been very little work that accounts for evolution in actual fits to data.
Finally, the experimental search for TMD-factorization breaking effects opens the possibility of new and exciting insights into the transverse dynamics of hadronic collisions. The breakdown of TMD-factorization in the hadro-production of hadrons implies that unexpected and exotic correlations between partons in different hadrons can exist. Calculations that allow for experiments to distinguish between factorization and factorization-breaking scenarios are therefore very important, and a quantitative understanding of factorization (via the methods of [14], for example) are part of the next step toward understanding hadronic structure in high energy collisions.

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