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Coordinate-wise transformation of probability distributions to achieve a Stein-type identity.

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Summary: It is shown that for any given multi-dimensional probability distribution with regularity conditions, there exists a unique coordinate-wise transformation such that the transformed distribution satisfies a Stein-type identity. A sufficient condition for the existence is referred to as copositivity of distributions. The proof is based on an energy minimization problem over a totally geodesic subset of the Wasserstein space. The result is considered as an alternative to Sklar’s theorem regarding copulas, and is also interpreted as a generalization of a diagonal scaling theorem. The Stein-type identity is applied to a rating problem of multivariate data. A numerical procedure for piece-wise uniform densities is provided. Some open problems are also discussed.

MSC:
60E05 Probability distributions: general theory
62E10 Characterization and structure theory of statistical distributions

Keywords:
copositive distribution; copula; energy minimization; optimal transportation; Stein-type distribution; Wasserstein space

Full Text: DOI

References:
[1] Alfonsi, A.; Jourdain, B., A remark on the optimal transport between two probability measures sharing the same copula, Stat. Probab. Let., 84, 131-134 (2014) · Zbl 1296.60023 · doi:10.1016/j.spl.2013.09.035
[2] Amari, S., Nagaoka, H.: Methods of Information Geometry, American Mathematical Society (2000) · Zbl 0960.62005
[3] Ambrosio, L., Gigli, N., Savaré, G.: Gradient Flows – in Metric Spaces and in the Space of Probability Measures, Birkhäuser (2005) · Zbl 1100.35002
[4] Borwein, JM; Lewis, AS; Nussbaum, RD, Entropy minimization, DAD problems, and doubly stochastic kernels, J. Funct. Anal., 123, 264-307 (1994) · Zbl 0815.15021 · doi:10.1006/jfan.1994.1089
[5] Butucea, C.; Delmas, J.; Dutfoy, A.; Fischer, R., Maximum entropy copula with given diagonal section, J. Multivar. Anal., 137, 61-81 (2015) · Zbl 1329.62246 · doi:10.1016/j.jmva.2015.01.003
[6] Chen, L.H.Y., Goldstein, L., Shao, Q.: Normal Approximation by Stein’s Method, Springer (2011) · Zbl 1213.62027
[7] Chernozhukov, V.; Galichon, A.; Hallin, M.; Henry, M., Monge-Kantorovich depth, quantiles, ranks and signs, Ann. Stat., 45, 1, 223-256 (2017) · Zbl 1426.62163
[8] De Rossi, A.; Rodino, L., Strengthened Cauchy-Schwarz inequality for biorthogonal wavelets in Sobolev spaces, J. Math. Anal. Appl., 299, 49-60 (2004) · Zbl 1058.42026 · doi:10.1016/j.jmaa.2004.06.005
[9] Fallat, S.; Lauritzen, S.; Sadeghi, K.; Uhler, C.; Wermuth, N.; Zwiernik, P., Total positivity in Markov structures, Ann. Stat., 45, 3, 1152-1184 (2017) · Zbl 1414.60010 · doi:10.1214/16-AOS1478
[10] Fathi, M., Stein kernels and moment maps, Ann. Probab., 47, 4, 2172-2185 (2019) · Zbl 1466.60044 · doi:10.1214/18-AOP1305
[11] Fortuin, CM; Kasteleyn, PW; Ginibre, J., Correlation inequalities on some partially ordered sets, Comm. Math. Phys., 22, 89-103 (1971) · Zbl 0346.06001 · doi:10.1007/BF01651330
[12] Gebelein, H., Das statistische Problem der Korrelation als Variations- und Eigenwertproblem und sein Zusammenhang mit der Ausgleichsrechnung, Z. Angew. Math. Mech., 21, 6, 364-379 (1941) · Zbl 0026.33402 · doi:10.1002/zamm.1941020604
[13] Hallin, M.; On distribution and quantile functions, ranks and signs in \(\{(\text{mathbb\{R\}})^n \times \text{d}\) : a measure transportation approach, preprint (2017)
[14] Hua, L.: Multivariate Extremal Dependence and Risk Measures, Ph. D. Thesis in the University of British Columbia (2012)
[15] Jaynes, ET, Information theory and statistical mechanics, Phys. Rev., 106, 4, 620-630 (1957) · Zbl 0084.43701 · doi:10.1103/PhysRev.106.620
[16] Joe, H., Relative entropy measures of multivariate dependence, J. Am. Stat. Assoc., 84, 157-164 (1989) · Zbl 0677.62054 · doi:10.1080/01621459.1989.10478751
[17] Joe, H., Dependence Modeling with Copulas (2014), Boca Raton: CRC Press, Boca Raton · Zbl 1346.62001 · doi:10.1201/b17116

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