Impact of weak annihilation contribution on rare semileptonic $B^+ \to \pi^+ \ell^+ \ell^-$ decay

A Ali\textsuperscript{1}, A Parkhomenko\textsuperscript{2} and I Parnova\textsuperscript{2}

\textsuperscript{1}Deutsches Elektronen-Synchrotron DESY, D-22607 Hamburg, Germany
\textsuperscript{2}P. G. Demidov Yaroslavl State University, Sovietskaya 14, 150003 Yaroslavl, Russia
E-mail: ahmed.ali@desy.de, parkh@uniyar.ac.ru, parnova.irina@yandex.ru

Abstract. In the Standard Model, $b \to s$ and $b \to d$ flavor-changing neutral currents (FCNCs) are not allowed at the tree level and are induced by loop effects. We consider the rare semileptonic $B^+ \to \pi^+ \ell^+ \ell^-$ decay, where $\ell = e, \mu$ is a charged lepton. The dilepton invariant-mass spectrum and decay rate for $B^+ \to \pi^+ \ell^+ \ell^-$ are calculated in the effective Hamiltonian approach in two cases — by taking into account the weak annihilation diagrams and without this contribution. Our predictions for the branching fraction of the $B^+ \to \pi^+ \mu^+ \mu^-$ decay, being dependent on the choice of the $B \to \pi$ form factors, are in agreement with the existing LHCb results within experimental uncertainties. Moreover, accounting for the weak annihilation contributions allows us to get a better agreement with the experimental data on the entire $q^2$-distribution in the kinematically allowed region, in particular, in its lowest $q^2$-part. This provides an alternative description of the data than a previous analysis, in which the low-$q^2$ enhancement seen experimentally was obtained through the long-distance contributions from the light vector mesons.

1. Introduction
Physics of rare semileptonic decays of $B$-mesons and $\Lambda_b$-baryons plays an important role for testing the Standard Model and in searching for possible New Physics beyond the Standard Model. A majority of these processes are due to $b \to s$ and $b \to d$ FCNCs. At present, the proton-proton collider LHC and the $B$-factory SuperKEKB are the only sources of experimental data on these decays. Branching fractions of the semileptonic $B$-meson decays induced by the $b \to s$ transition, such as $B^\pm \to K^{(*)}\pm \mu^+ \mu^-$, $B^0 \to K^{(*)0} \mu^+ \mu^-$, $B^0_s \to \phi \mu^+ \mu^-$, their lepton-pair invariant mass and angular distributions have been experimentally measured quite precisely \cite{1-3}. However, for the exclusive decays induced by the $b \to d$ neutral current transition, the $B^+ \to \pi^+ \mu^+ \mu^-$ decay is so far the only decay mode observed, presented by the LHCb Collaboration in 2012 \cite{4}. Subsequently, the dimuon invariant mass distribution and the direct $CP$-asymmetry in this decay were measured using the integrated luminosity of 3.0 $fb^{-1}$ \cite{5}. Although, experimental data are in good agreement with the theoretical predictions for the dimuon invariant-mass distribution on $B^+ \to \pi^+ \mu^+ \mu^-$ decay \cite{6-10}, the lowest $q^2$-part of the spectrum, where $q^2$ is the dilepton invariant mass, was seen substantially higher compared to the short-distance-based estimates. This threshold effect was explained by incorporating the long-distance contribution of the light vector mesons $\rho$ and $\omega$ \cite{11}, accounting for the observed dimuon invariant-mass distribution in the low-$q^2$ region. The goal of the present work is to calculate
this distribution for the $B^+ \to \pi^+ \mu^+ \mu^-$ decay by taking into account the weak-annihilation contribution and its impact on the lowest $q^2$-bin.

2. Theory of rare semileptonic $B$-meson decays

In the framework of effective weak Hamiltonians, rare semileptonic decays of $B$-mesons are induced by the $b \to s$ and $b \to d$ flavor-changing neutral current transitions \[12\]:

$$
\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \left\{ V_{tb}V_{\bar{t}p}^* \sum_{i=1}^{10} C_i O_i + V_{ub}V_{\bar{u}p}^* \left[ C_1 \left( O_1 - O_1^{(u)} \right) + C_2 \left( O_2 - O_2^{(u)} \right) \right] \right\},
$$

where $p = s, d$ is the quark flavor, $V_{qi}V_{\bar{q}i}$ are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, and $C_i(\mu)$ are Wilson coefficients determined at the scale $\mu$. For the operators $O_i(\mu)$, the following basis is chosen (here, we consider operators with the largest Wilson coefficients):

$$
O_1 = \left( \bar{p}_L \gamma_\mu T^A c_L \right) \left( \bar{e}_L \gamma^\mu T^A b_L \right), \quad O_2 = \left( \bar{p}_L \gamma_\mu c_L \right) \left( \bar{e}_L \gamma^\mu b_L \right),
$$

$$
O_7 = \frac{m_b}{2m} \left( \bar{p}_L \sigma^{\mu\nu} b_R \right) F_{\mu\nu}, \quad O_8 = \frac{m_b}{g_{\text{st}}} \left( \bar{p}_L \sigma^{\mu\nu} T^A b_R \right) G^A_{\mu\nu},
$$

$$
O_9 = \frac{e^2}{2g_{\text{st}}} \left( \bar{p}_L \gamma^\mu b_L \right) \sum_\ell \left( \bar{e}_\gamma \gamma_\ell \right), \quad O_{10} = \frac{e^2}{g_{\text{st}}} \left( \bar{p}_L \gamma^\mu b_L \right) \sum_\ell \left( \bar{e}_\gamma \gamma_\ell \right),
$$

where $T^A (A = 1, \ldots, 8)$ are the generators of the color $SU(3)_c$-group, $\psi_{LR}(x) = (1 \mp \gamma_5) \psi(x)/2$ are the left and right components of fermionic fields, $F_{\mu\nu}$ and $G^A_{\mu\nu}$ are the electromagnetic and gluon field strength tensors, respectively, $m_b$ is the $b$-quark mass, and $\sigma^{\mu\nu} = i (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu)/2$.

The matrix elements for the $B \to P$ transition, where $P$ is a pseudoscalar meson, are expressed in terms of three transition form factors $f_+(q^2)$, $f_0(q^2)$, and $f_T(q^2)$, where $q^2$ is the four-momentum transferred to the lepton pair, $q^2 = (p - k)^2$, \[13\]:

$$
\langle P(k) | \bar{p}_\gamma \sigma^{\mu\nu} q_\nu b | B(p_B) \rangle = f_+(q^2) \left[ p_B^\mu + k^\mu - \frac{m_B^2 - m_P^2}{q^2} q^\mu \right] + f_0(q^2) \frac{m_B^2 - m_P^2}{q^2} q^\mu,
$$

$$
\langle P(k) | \bar{p} \sigma^{\mu\nu} q_\nu b | B(p_B) \rangle = i \left[ (p_B^\mu + k^\mu) q^2 - q^\mu \left( m_B^2 - m_P^2 \right) \right] \frac{f_T(q^2)}{m_B + m_P},
$$

$$
\langle P(k) | \bar{p}_\gamma \gamma_5 q_\nu b | B(p_B) \rangle = 0,
$$

where $m_B$ and $m_P$ are the $B$- and pseudoscalar meson masses.

Operators $O_7$, $O_9$, and $O_{10}$ give the dominant contributions to the decay amplitude. Knowing these contributions, it is possible to calculate the $B \to P\ell^+\ell^-$ directional branching fraction \[9\]:

$$
\frac{d\text{Br} \left( B \to P\ell^+\ell^- \right)}{dq^2} = S_P \frac{2G_F^2 \alpha_{\text{em}}^2 T_B}{3(4\pi)^5 m_B^3} |V_{tb}V_{\bar{t}p}|^2 \lambda^{3/2}(q^2) F^{BP}(q^2) \sqrt{1 - 4m_\ell^2/q^2},
$$

$$
F^{BP}(q^2) = F^{BP}_{97}(q^2) + F^{BP}_{10}(q^2), \quad \lambda(q^2) = \left( m_B^2 + m^2 - q^2 \right)^2 - 4m_B^2 m_\ell^2,
$$

$$
F^{BP}_{97}(q^2) = \left( 1 + \frac{2m_\ell^2}{q^2} \right) \frac{C_{97}^{\text{eff}}(q^2) f_{97}^{BP}(q^2)}{f_{97}^{BP}(q^2) + \frac{2m_b}{m_B + m_P} C_{97}^{\text{eff}}(q^2) f_{97}^{BP}(q^2) + L_{97}^{BP}(q^2)},
$$

$$
F^{BP}_{10}(q^2) = \left( 1 - \frac{4m_\ell^2}{q^2} \right) \frac{C_{10}^{\text{eff}} f_{10}^{BP}(q^2)}{f_{10}^{BP}(q^2) + \frac{6m_\ell^2}{q^2} \left( m_B^2 - m_P^2 \right)^2 \lambda(q^2) + \frac{C_{10}^{\text{eff}} f_{10}^{BP}(q^2)}{C_{10}^{\text{eff}} f_{10}^{BP}(q^2)}}},
$$

$\text{Br} \left( B \to P\ell^+\ell^- \right)$
where $S_{\pi}$ is an isospin factor of the final meson, in particular, $S_{\pi+} = 1$ and $S_{\pi^0} = 1/2$ for the $\pi$-mesons, $C^\text{eff}_{7,9,10}$ are effective Wilson coefficients including the NLO QCD corrections [14], and $L^B_{A}(q^2)$ is a term responsible for the Weak Annihilation (WA) contribution. The WA contribution is calculated within the so-called Large Energy Effective Theory (LEET), and for the $B \to P\ell^+\ell^-$ decay, it can be expressed as follows [15]:

$$L^B_{A}(q^2) = Q_q \frac{\pi^2}{3} \frac{4f_B f_\pi}{m_b} \lambda_{B,\cdots}^{-1}(q^2) C_{34}, \quad L^B_{A}(q^2) = -Q_q \frac{\pi^2}{3} \frac{4f_B f_\pi}{m_b} \lambda_{B,\cdots}^{-1}(q^2) C_{12},$$

where $Q_q$ is the spectator-quark electric charge, $f_B$ and $f_\pi$ are the $B$- and $\pi$-meson decay constants, respectively, $C_{34} = C_3 + 4(C_4 + 12C_5 + 16C_6)/3$ is a combination of the Wilson coefficients of the QCD penguin operators, and $C_{12} = 3C_2$. Here, $\lambda_{B,\cdots}^{-1}(q^2)$ is the first inverse moment of the $B$-meson Light-Cone-Distribution-Amplitude (LCDA), defined as:

$$\lambda_{B,\cdots}^{-1}(q^2) \equiv \frac{e^{-q^2/(m_B\omega_\pi)}}{\omega_0} \left[ i\pi - \text{Ei} \left( q^2/(m_B\omega_0) \right) \right], \quad \text{Ei} (z) = \int_{-\infty}^{\infty} \frac{dt}{t} e^t,$$

where $\text{Ei} (z)$ is the Exponential integral [16]. The differential branching fraction [5] involve three $B \to P$ form factors. They are discussed for the $B \to \pi$ case in the next section.

3. Form factor parameterizations

Among the available parameterizations of the $B \to \pi$ transition form factors in the literature, we consider the following three:

The first is the Ball-Zwicky (BZ) parameterization [17] ($i = +, T$):

$$f_i(q^2) = \frac{r_{1,i}^{(i)}}{1 - q^2/m_{B_i}^2} \frac{r_{2,i}^{(i)}}{1 - q^2/m_{\text{fit}}^2}, \quad f_0(q^2) = \frac{f_0(0)}{1 - q^2/m_{\text{fit}}^2},$$

where $m_{B_i} = 5.235$ GeV is the mass of the vector $B^*_i$-meson [18], $f_{+,0,T}(0)$, $r_{2,+}^{(i)}$, and $m_{\text{fit}}$ are model parameters, and $r_{2,+}^{(i)} = f_{+,T}(0) - r_{2,+}^{(i)}$.

The second is the Boyd-Grinstein-Lebed (BGL) parameterization [19] ($i = +, T$):

$$f_i(q^2) = \frac{1}{P_i(q^2)\phi_i(q^2, q_0^2)} \sum_{k=0}^{N} a_k^{(i)} \left[ z(q^2, q_0^2) \right]^k, \quad z(q^2, q_0^2) = \sqrt{\frac{m_{B_i}^2 - q^2}{m_{B_i}^2}} - \sqrt{\frac{m_{B_i}^2 - q_0^2}{m_{B_i}^2}},$$

where $P_{i=+,T}(q^2) = z(q^2, m_{B_i}^2)$ and $P_{0}(q^2) = 1$ are the Blaschke factors, $\phi_i(q^2, q_0^2)$ is an outer function [19], $m_+ = m_B + m_{\pi}$, and $q_0^2 = 0.65(m_B - m_{\pi})^2$.

The third is the Bourrely-Caprini-Lellouch (BCL) parameterization [20] ($i = +, T$):

$$f_i(q^2) = \frac{1}{1 - q^2/m_{B_i}^2} \sum_{k=0}^{N-1} b_k^{(i)} \left[ z(q^2, q_0^2) \right]^k \left[ (-1)^{k-N} \frac{k}{N} \left[ z(q^2, q_0^2) \right]^N \right],$$

$$f_0(q^2) = \sum_{k=0}^{N-1} b_k^{(0)} z(q^2, q_0^2)^k, \quad q_0^2 = m_+ (\sqrt{m_B} - \sqrt{m_{\pi}})^2.$$

Here, the form factors are calculated by truncating the series at $N = 4$. 

\[3\]
Figure 1. Dilepton invariant-mass distribution for $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay without taking into account the WA contribution for the BZ (a), BGL (b) and BCL (c) parameterizations of the form factors. The green areas indicate the uncertainty due to the factorization scale.

Figure 2. The same as in figure 1 but including the WA contribution.

4. Numerical analysis of $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay

Having specified the BZ [17], BGL [6] and BCL [10] parameterizations, we calculate the distributions in the dimuon invariant mass for the decay $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ in the large-recoil limit in two cases: without taking into account the WA contributions and taking them into account, shown in figures 1 and 2 respectively. As can be seen in figure 2, in the region $q^2 \lesssim 0.5\text{ GeV}^2$, the WA contribution gives sizable enhancement over the short-distance perturbative contribution of about 30-45%, independent of the parameterization chosen. In the rest of the $q^2$-region considered here, this contribution is insignificant.

Next, we compare the dimuon invariant mass distribution for each type of parameterization with the latest experimental data on the $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay [5] in the entire $q^2$-region. The corresponding spectra are presented in figures 3 and 4. The contribution of Weak Annihilation diagrams plays an important role in the low $q^2$-region, in agreement with the experiment, as can be seen in figure 4. Note that in the region of $q^2 \lesssim 1\text{ GeV}^2$ we did not take into account the vector $\rho$- and $\omega$-resonances in these figures, which can also increase the theoretical prediction in this region [11].

To show the impact of the WA and long-distance contributions, we plot the differential branching fraction of $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay in bins of the dimuon invariant mass squared in figure 5 for the BCL (left plot) and BZ (right plot) parameterizations. Our results are marked by the green color, while the predictions by Hambrock, Khodjamirian and Rusov [11], in which the WA contribution is not included, are in the red color. It is important to note that in figure 5 we take into account the uncertainties from the form factor parameters. As can be seen, the BCL parameterization results into a greater uncertainty in the low $q^2$-region and works fine in $q^2 \geq 11.5\text{ GeV}^2$ as the form factor coefficients are fixed from the Lattice data. The BZ parameterization gives a good description of the entire $q^2$-spectrum. We also estimated the total and partial branching fractions in the interval $[4m^2_\mu, 8\text{ GeV}^2]$ for the parameterizations
Figure 3. Dilepton invariant-mass distribution for $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ without taking into account the WA contribution for the BZ (a), BGL (b) and BCL (c) parameterizations. The green areas indicate the uncertainty due to the factorization scale and the crosses show existing experimental data from LHCb [5].

Figure 4. The same as in figure 3 but including the WA contribution.

Figure 5. Differential branching fraction of the $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay in bins of dimuon invariant mass squared by taking into account the WA contribution for the BCL (a) and BZ (b) parameterizations. The red areas indicate theoretical predictions from [11] (called HKR15), the green areas are our predictions (APP), and the experimental data from LHCb [5] are shown by the crosses.

considered, and have collected the results in table 1. For both parameterizations, our estimates are in agreement with the experimental value $\text{Br}_{\text{exp}} = (1.83 \pm 0.29) \times 10^{-8}$ [5] within uncertainties.
Table 1. Our estimates for the total and partial branching ratios for \( B^+ \to \pi^+ \mu^+ \mu^- \) decay in the BCL and BZ parameterization with and without the WA contribution in comparison with the corresponding LHCb measurement [5].

|            | BCL                     | BZ                     |
|------------|-------------------------|------------------------|
|            | Total \([4m_{\mu}^2, 8 \text{ GeV}^2]\) | Total \([4m_{\mu}^2, 8 \text{ GeV}^2]\) |
| \(B_{th} \times 10^8\) | 1.78\text{+0.63\text{-0.48}} | 0.65\text{+0.47\text{-0.33}} | 2.20\text{+0.58\text{-0.44}} | 0.78\text{+0.23\text{-0.18}} |
| \(B_{th}^{WA} \times 10^8\) | 1.86\text{+0.66\text{-0.51}} | 0.72\text{+0.49\text{-0.36}} | 2.28\text{+0.59\text{-0.45}} | 0.86\text{+0.24\text{-0.19}} |
| \(B_{exp} \times 10^8\)       | Total: 1.83 \pm 0.29     |

5. Summary and outlook

Taking into account the Weak-Annihilation contribution, theoretical predictions for the dimuon invariant-mass spectrum in the \( B^+ \to \pi^+ \mu^+ \mu^- \) decay, obtained in the effective weak Hamiltonian framework based on the SM, agree well with the existing experimental data on this decay in the entire \( q^2 \)-region, including low-\( q^2 \). Nevertheless, there is still some room to include the long-distance contributions from the light neutral vector mesons, and we hope to return to an analysis of the data including all three pieces. Present data on \( B^+ \to \pi^+ \mu^+ \mu^- \) decay corresponds to an integrated luminosity of 3.0 fb\(^{-1}\), collected by the LHCb at the center-of-mass energies of 7 and 8 TeV [3]. With the updated measurements from LHCb, including data collected at 13 TeV, as well as forthcoming data anticipated from the Belle-II experiment at SuperKEKB, will enable us to undertake a precise test of the Standard Model in exclusive FCNC \( b \to d \) transitions.

Acknowledgments

A.P. and I.P. acknowledge financial support by the Russian Foundation for Basic Research and National Natural Science Foundation of China according to the joint research project (No. 19-52-53041). The part of work done by I.P. on the analysis of the transition form factors is supported by the Russian Science Foundation (Grant No. 18-72-10070). This research is partially supported by the “YSU Initiative Scientific Research Activity” (Project No. AAAA-A16-116070610023-3).

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