Propagation of local decohering action in distributed quantum systems

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We study propagation of the decohering influence caused by a local measurement performed on a distributed quantum system. As an example, the gas of bosons forming a Bose-Einstein condensate is considered. We demonstrate that the local decohering perturbation exerted on the measured region propagates over the system in the form of a decoherence wave, whose dynamics is governed by elementary excitations of the system. We argue that the post-measurement evolution of the system (determined by elementary excitations) is of importance for transfer of decoherence, while the initial collapse of the wave function has negligible impact on the regions which are not directly affected by the measurement.

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INTRODUCTION

The theory of quantum measurement begun in the 1920s still remains an active topic of interest (see, e.g. Ref. 1 and references therein). According to von Neumann’s theory of measurement 2, unitary evolution of a system prepared initially in a pure quantum state is interrupted by an instant decohering action of the measuring apparatus, so that the density matrix describing an ensemble of such systems changes radically (it ceases to be a projection operator) and entropy rises. This view has been shown to describe rather accurately the consequences of an act of measurement, but the dynamics of the measurement process itself is lacking. The contemporary theory of quantum measurements, which provides much deeper analysis of the measurement process, is based on the concept of decoherence 3. To be measured, the system has to interact with its environment, which consists of a large number of degrees of freedom. The Hilbert space of the system becomes divided into subspaces corresponding to the same eigenvalue of the system-environment interaction Hamiltonian. As a result of this interaction, coherence between different subspaces is quickly lost, and after the measurement the system appears in a mixed state. The concept of decoherence turned out to be successful in many areas of fundamental physics, such as the study of macroscopic quantum effects 4 and consistent histories interpretation of quantum mechanics 5, so that investigation of this process and related effects is of considerable importance.

At present, decoherence and its consequences for point-like quantum systems have been studied in detail (for review, see Ref. 6), but distributed quantum systems have received significantly less attention. Mostly, linear systems have been investigated, where separation into noninteracting modes is possible, and each mode is considered as an independent oscillator 7. However, this approach is difficult to apply to sufficiently nonlinear systems (e.g., spin systems, or the Bose-Einstein condensate as described by the Gross-Pitaevskii equation) possessing localized soliton-like excitations. For systems where localized excitations prevail, dealing explicitly with real-space coordinates could be a more suitable strategy.

A real-space description of decoherence in distributed systems is a very general and complicated issue. In this paper, we consider only one aspect of the problem, namely, how local properties of different regions in a distributed quantum system are affected by a local measurement, that acts only on some part of the system. Indeed, different regions in the system are not isolated from each other, and correlations between them exist (or can build up). Therefore, in spite of the fact that a local measurement initially affects only one region, other regions can “acquire knowledge” that some part of the system has been measured. In this paper we explicitly show that the decohering influence of the local measurement propagates through the system in the form of a decoherence wave. Dynamics of the decoherence wave is governed by elementary excitations, while the effect of entanglement is very small for macroscopically large systems.

The consideration presented here can be applied to other similar situations, so that a decoherence wave propagating with a characteristic velocity of excitations is likely to be quite common. This phenomenon, being a notable part of any real measurement, is of fundamental interest. Moreover, propagation of decoherence can be also of importance for the design of quantum computers. Such a computer is a system of interacting quantum entities, representing quantum bits (qubits). Fault-tolerant quantum computations involve measurements performed on some qubits and it is important to know how such measurement may affect other qubits 8. Moreover, decoherence is introduced by a dissipative environment of qubits, so that analysis of decoherence propagation may lead to strategies to minimize influences detrimental to performance of the computer.

In this paper we consider a Bose-Einstein condensate of an ideal or weakly non-ideal gas of bosons, which constitutes a good example of a distributed system in a pure quantum state. It can be implemented in reality as a gas of trapped atoms cooled down to very low temperatures 9. Suppose we measure the number of particles in some region of space. If two such measurements are done simultaneously at two different parts of the trap we ob-
tain the trivial result corresponding to the ground-state wavefunction of the condensate. But if the second measurement is carried out after some delay then the result is different and provides information about the propagation of the perturbation induced by the first measurement.

The situation considered here is related to the problem of broken gauge symmetry and existence of a relative phase of two interfering condensates, which has been extensively discussed recently. If we have a condensate with a definite number of particles, its phase is spread uniformly between 0 and 2π, while a definite phase requires a non-conservation of the number of particles in the condensate. It has been shown that a well-defined phase (evidenced experimentally by appearance of the interference fringes) builds up in the course of the measurement (atoms detection), due to increasing uncertainty in the number of particles in each of the interfering condensates: each detected atom may well belong to either of them. For the circumstances considered in this paper, we have a similar situation: the local phase of the condensate is the same in every region. Identity of the phase throughout the condensate is due to uncertainty in the local number of the particles inside each region. However, when the number of particles in some region is determined by a local measurement, the phase coherence in the condensate as a whole is partially destroyed, what leads to observable consequences, propagation of the decoherence wave in the system. Note that decoherence wave is the same both for the condensate with definite number of particles (with uncertain global phase, the case of non-interacting bosons) and for the condensate with definite global phase (but with uncertain number of condensed particles, the case of weakly interacting bosons): the results for the latter case transform exactly to the results for the former as interaction goes to zero.

We describe the dynamics of the condensate in a linear approximation, i.e. we use the approximation of noninteracting quasiparticles to study a weakly non-ideal Bose-gas. In so doing, we lose the ability to investigate some interesting nonlinear effects, but we gain in clarity of presentation: it is reasonable to start from a simplified (and not totally unrealistic) case to emphasize the main idea.

We do not specify the way of measuring the local density of condensate, and the dynamics of the measurement process is not considered here. Analysis of a specific experimental scheme is a distinct problem, requiring separate study, while here we focus on the post-measurement evolution of the condensate. In principle, the local density of the Bose-condensate can be measured by placing some probe into the trap, which interacts with the condensate so that an entangled state is formed

\[ |X\rangle = \sum_n C_n |n\rangle \otimes |\alpha_n\rangle \]  

(1)

where \(|n\rangle\) is the state of condensate with the number of particles \(n\) in the measured region, and \(|\alpha_n\rangle\) is the state of the probe. If the probe interacts with a large number of environmental degrees of freedom, so that \(|\alpha_n\rangle\) are the eigenstates corresponding to different eigenvalues of the probe-environment interaction Hamiltonian, then the coherence between different probe states is being lost, and the condensate’s state also becomes an incoherent mixture of different states \(|n\rangle\). If the probe (and, correspondingly, the condensate) decoheres quickly enough (as is usually the case) we can consider the measurement as instantaneous and safely use von Neumann’s theory to describe the condensate’s state immediately after the measurement.

Although the situation considered above is in many respects too idealized to apply rigorously to a real experiment, it is detailed enough to capture the essential processes of concern in this paper.

**PROPAGATION OF DECOHERENCE IN BOSE-EINSTEIN CONDENSATE**

To study quantitatively the effect of decoherence propagation, let us consider first an ideal Bose-gas confined by external fields and described by the Hamiltonian

\[ H = \sum_\mu E_\mu \alpha_\mu^\dagger \alpha_\mu, \]  

(2)

where \(\alpha_\mu^\dagger\) and \(\alpha_\mu\) are the boson creation and annihilation operators. \(E_\mu\) are the one-particle energies, and we denote the corresponding one-particle wavefunctions as \(\varphi_\mu(r)\), where \(\mu = 0\) stands for the ground state having minimal energy \(E_0 = 0\). Then, the ground-state eigenfunction of the system of \(M\) bosons can be written as

\[ |\Psi\rangle = \frac{1}{\sqrt{M!}} (\alpha_0^\dagger)^M |0\rangle, \]  

(3)

where \(|0\rangle\) is the vacuum state. For simplicity, we can consider the trap as being divided into a large number \(N_c\) of small cells each having the volume \(V_0\) (it can be considered as the volume directly affected by the measuring apparatus), satisfying the relation \(V_0 \ll V\), where \(V\) is the total volume of the trap. Then, the coordinate \(r\) is understood as a discrete quantity (the number of a cell). This is similar to a general practice in solid-state theory, where \(V_0\) is analogous to the volume of an elementary cell of the crystal [1]. Note that in so doing, the number of one-particle states taken into account becomes equal to \(N_c\), which is finite, though very large. This corresponds to the fact that the number of states inside the first Bril-louin zone equals to the number of lattice cells.

At the instant \(t = 0\) we perform measurement of the number of bosons in the cell \(r = 0\). This observable is represented by the operator \(N = a_0^\dagger a_0\), where

\[ a(r) = \sum_\mu \varphi_\mu(r) \alpha_\mu. \]  

(4)

is the boson field operator. Eigenvalues of the operator \(N\) are \(n = 0, 1, 2\ldots\) and, suppose, the measurement has given
us one of them. According to von Neumann’s theory, it corresponds to the action of the operator \( W_n \) on the system, where

\[
W_n = \delta_{n,N} = \int_0^{2\pi} \frac{d\phi}{2\pi} \exp\{i\phi(n - N)\}
\]

(5)
is a projector onto the state with the number of particles \( n \) in the measured region. The operator \( W_n \) has the value equal to unity on this state and it has zero value on all others states. Further development of the system is to be described by the density matrix of the system \( U(t) \), since the measurement interrupts unitary evolution and casts the system into mixed quantum state. According to the standard theory of measurement \([8,9]\), the density matrix at the time \( t \) is

\[
U(t) = \sum_{n=0}^{\infty} \exp(-iHt) W_n U_{in} W_n^\dagger \exp(iHt),
\]

(6)

where \( U_{in} = |\Psi\rangle\langle\Psi| \) is the density matrix before the measurement.

To trace propagation of decoherence in the system, we study evolution of the one-particle density matrix

\[
\rho(r, r', t) = \text{Tr} \left[ U(t) a^\dagger(r') a(r) \right].
\]

(7)

This quantity describes local properties of the Bose-Einstein condensate; in particular, the average number of particles resulting from the second measurement, which is performed at the point \( r \) at the instant \( t \), is given by the value \( \rho(r, r, t) \).

To simplify calculations, we use the fact that the total number of particles is large, \( M \gg 1 \), so that operators \( a_0 \) and \( a_0^\dagger \) acting on the state \( |\Psi\rangle \) can be replaced by the number \( \sqrt{M} \) with relative accuracy \( 1/\sqrt{M} \); this is a standard approximation in the theory of Bose-Einstein condensation \([4]\). Therefore, Eq. (4) can be rewritten as

\[
a(r) = \sqrt{n_B(r)} + \bar{a}(r), \quad \bar{a}(r) = \sum_{\mu \neq 0} \varphi_\mu(r) \alpha_\mu \quad (8)
\]

where \( n_B(r) = M \varphi_0^2(r) \) is the average number of condensate particles contained in the volume \( V_0 \) at the cell \( r \). The expression for the one-particle density matrix can be written as

\[
\rho(r, r', t) = \sum_{n=0}^{\infty} \rho_n(r, r', t),
\]

(9)

\[
\rho_n(r, r', t) = \langle \Psi | W_n^\dagger a^\dagger(r') a(r) W_n | \Psi \rangle,
\]

where \( a(r, t) = \exp(iHt) a(r) \exp(-iHt) \). The operator product in Eq. (8) is to be ordered normally, i.e. it is to be rewritten in such a way that all \( a^\dagger \) stand to the left of all \( a \) in each term of the Taylor series expansion. In so doing, we take into account that

\[
[a(r, t), a^\dagger(0)] = \sum_{\mu \neq 0} \varphi_\mu(r) \varphi_\mu^*(0) e^{-iE_\mu t} = g(r, t).
\]

(10)

Note that for a system containing a large number of particles \( M \gg 1 \), the function \( g(r, t) \) can be replaced by the Green’s function

\[
G(r, t) = \sum_{\mu} \varphi_\mu(r) \varphi_\mu^*(0) e^{-iE_\mu t}
\]

(11)

with accuracy of order of \( 1/M \), since \( G(r, t) = g(r, t) + \varphi_0(r) \varphi_0^*(0) \). Performing the calculations, we obtain

\[
\rho_n(r, r', t) = p_n \left[ \sqrt{n_B(r)} - G(r, t) \sqrt{n_0} \right] \times \left[ \sqrt{n_B(r')} - G^*(r', t) \sqrt{n_0} \right] + p_{n-1} n_0 G(r, t) G^*(r', t),
\]

(12)

where \( n_0 = n_B(0) \), and \( p_n = e^{-n_0} n_0!/(n!) \) is the Poisson distribution function. Summation over \( n \) can be performed explicitly, yielding

\[
\rho(r, r', t) = \sqrt{n_B(r)n_B(r')} - G(r, t) \sqrt{n_B(r)n_0} - G^*(r', t) \sqrt{n_B(r)n_0} + 2n_0 G(r, t) G^*(r', t) G(r, t).
\]

(13)

This result shows that the measurement made at the point \( r = 0 \) produces a decohering perturbation which propagates over the trap in the form of a decoherence wave, and this propagation is governed by the Green’s function \( G(r, t) \). It can be explicitly demonstrated by considering an example of the gas consisting of free Bose-particles of mass \( m \). The corresponding Green’s function at the distances \( r \gg V_0^{1/3} \) and times \( t \gg mV_0^{2/3}/\hbar \) is \([12]\)

\[
G(r, t) = V_0 \left( \frac{m}{2\pi \hbar^2} \right)^{3/2} \exp \left( \frac{i m r^2}{2\pi \hbar^2} \right).
\]

(14)

Local density of the condensate after the measurement is given by the value

\[
\rho(r, r, t) = n_B + 2n_B V_0^2 \left( \frac{m}{2\pi \hbar^2} \right)^{3/2} \cos \left( \frac{m r^2}{2\pi \hbar^2} \right),
\]

(15)

where \( n_B = M/V \) is density of the condensate before the measurement, which is independent on position \( r \). This is an observable effect, which, in principle, can be detected experimentally.

The entropy of the system, being initially zero, after the measurement is

\[
S = - \text{Tr} [U(t) \ln U(t)] = - \sum_{n=0}^{\infty} p_n \ln p_n > 0,
\]

(16)

which is a clear indication of the decohering effect of measurement. The increase of entropy of condensate as a whole happens only at the instant of measurement and
further evolution, being unitary, keeps it constant (decoherence only propagates in the system from one region to another). Note that local entropy (in contrast to the one-particle density matrix, where the decoherence propagation is clearly seen) can be hardly used to track the decoherence wave. The value of the local entropy is nonzero even in the initial pure state, while the total entropy of the system is zero. It happens because of “negative entropy” stored in the form of correlations between different parts of the condensate (for more detailed discussion see Ref. [13]).

The results obtained can be qualitatively interpreted as follows. The measurement performed at \( r = 0 \) leads to localization of some number of particles within the cell \( r = 0 \). The localized particles acquire rather large momenta, of order \( \hbar/V_0^{1/3} \); the average number of such particles is \( n_0 = n_B(0) \). Immediately after being localized, these particles start to propagate over the trap, and their propagation is governed by the Green’s function \([14]\). Because of indistinguishability of particles in the trap, we can not say that these are “the same” particles which were measured at \( r = 0 \), so that the effect we consider is not a physical motion of some separate particles in the trap, but is the propagation of the decohering influence of the measurement through the system.

An interesting feature of the decoherence propagation can be illustrated by the gas of bosons trapped in a parabolic external potential, so that each particle is represented by an isotropic harmonic oscillator of eigenfrequency \( \Omega \). In this case, provided that \( r \gg V_0^{1/3} \) and \( V_0 \ll (\hbar/\Omega)^{3/2} \sim V \), the Green’s function has the form \([14]\)

\[
G(\mathbf{r}, t) = V_0 \left( \frac{\Omega}{2\pi \hbar \sin \Omega t} \right)^{3/2} \exp \left( \frac{i \Omega t^2}{2\pi \hbar \cot \Omega t} \right) \tag{17}
\]

where the particles are assumed to have unitary mass. This function is periodic in time with the period \( 2\pi/\Omega \). Therefore, the decoherence propagation is also periodic in time with the same period. In the general case of Bose-gas trapped in a finite volume, the decoherence propagation becomes a quasiperiodic process, according to Eq. \([14]\).

And, last but not least, decoherence propagation is a wave process, possessing both amplitude and phase. Existence of coherent waves in the system without quantum coherence is not unusual, the same property is shared, e.g., by the sound wave propagating in the classical fluid. Therefore, in principle, an interference of two decoherence waves is possible.

Above, we have considered the system of noninteracting bosons. Now, let us investigate the case of weakly interacting particles, i.e. a weakly non-ideal Bose-gas contained in a trap of large volume \( V \). We assume no external potential acting on the particles, so that the one-particle states are simple plane waves

\[
\varphi_k(\mathbf{r}) = \sqrt{\frac{V_0}{V}} \exp(i\mathbf{k}\cdot\mathbf{r}), \tag{18}
\]

where the normalization reflects the fact that the trap is divided into cells of volume \( V_0 \ll V \). This system is described by the Hamiltonian

\[
H = \sum_k E_k \alpha_k^\dagger \alpha_k + \frac{1}{2V} \sum_{k_1+k_2=k_1'+k_2'} v(k_1-k_1') \alpha_{k_1'}^\dagger \alpha_{k_2}^\dagger \alpha_{k_2'} \alpha_{k_1}, \tag{19}
\]

where \( v(k) \) is the Fourier transform of the interaction potential (which is assumed to be repulsive). Since the interaction is small, new Bose operators can be introduced according to Bogoliubov transformation

\[
\alpha_k = \xi_k \cosh \chi_k + \xi_{-k}^\dagger \sinh \chi_k, \quad \alpha_k^\dagger = \xi_k \sinh \chi_k + \xi_{-k}^\dagger \cosh \chi_k,
\]

with the parameters \( \chi_k \) defined as

\[
\tanh 2\chi_k = -\frac{v(k)n_B}{E_k + v(k)n_B}, \tag{21}
\]

where \( n_B \) is the average number of particles belonging to Bose-Einstein condensate contained in the volume \( V_0 \). Provided that the interaction is small (or the gas density \( M/V \) is small), almost all particles belong to the condensate, so we can take \( n_B = MV_0/V \) with relative accuracy of order of \( \sqrt{v(0)M/V} \) \([14]\). By using the Bogoliubov transformation, we pass to the ideal gas of new excitations with the dispersion law

\[
\omega_k = \sqrt{E_k^2 + 2E_k v(k)n_B}. \tag{22}
\]

Again, we consider dynamical behavior of the one-particle density matrix. The calculation procedure remains essentially the same as for the ideal Bose-gas. In so doing, we obtain the result:

\[
\rho_n(\mathbf{r}, \mathbf{r}', t) = \frac{n_B}{(n!)^2} \frac{\partial^n}{\partial z^n \partial z'^n} \left\{ \begin{array}{c}
[1 + (z - 1)G(\mathbf{r}, t)] \\
\times [1 + (z' - 1)G^*(\mathbf{r}', t)] \\
\times \exp \left[ n_B X(z, z') \right]
\end{array} \right\}_{z=z'=0} \tag{23}
\]

where the following notations were used,

\[
X(z, z') = B(zz' - 1) + (1 - B)(z + z' - 2) + A \left[ (z - 1)^2 + (z' - 1)^2 \right], \tag{24}
\]

\[
A = \frac{V_0}{2V} \sum_k \frac{v(k)n_B}{\omega_k}, \tag{25}
\]

\[
B = \frac{V_0}{2V} \sum_k \left[ 1 + \frac{E_k + v(k)n_B}{\omega_k} \right],
\]

\[
\frac{\partial^n}{\partial z^n \partial z'^n} \left\{ \begin{array}{c}
[1 + (z - 1)G(\mathbf{r}, t)] \\
\times [1 + (z' - 1)G^*(\mathbf{r}', t)] \\
\times \exp \left[ n_B X(z, z') \right]
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\]

\[
B = \frac{V_0}{2V} \sum_k \left[ 1 + \frac{E_k + v(k)n_B}{\omega_k} \right],
\]
and $G(r, t)$ is the Green’s function of the weakly interacting Bose-gas:

$$G(r, t) = \sum_k \exp(ikr) \times \left\{ \cos \omega_k t - i \frac{E_k + v(k)n_B}{\omega_k} \sin \omega_k t \right\}.$$  

(26)

Again, we see that the decoherence wave propagating in the system follows the dynamics of the Green’s function [23]. Dynamic behavior of $G(r, t)$ at large times $t$ and large distances $r$ can be analyzed by the method of stationary phase [13]. According to this method, the value of the function $G(r, t)$ at the point $r$ at the instant $t$ is determined by those excitations which have a group velocity $u(k) = \frac{d\omega_k}{dk}$ obeying the requirement $u(k) = \frac{r}{t}$.

The excitations with large wavevectors $k$ are subject to considerable damping [14], so that at large distances only the undamped long-wavelength excitations determine the dynamics of the Green’s function. These excitations represent sound propagating in the Bose-gas with the velocity $c = \sqrt{\hbar v(0)/m}$, so the decoherence wave in a system of weakly interacting bosons propagates with the sound velocity $c$.

This result can be interpreted in the same way as the decoherence wave in an ideal Bose-gas. The measurement affects the particles situated at $r = 0$. Due to the interparticle interaction, the decohering perturbation is transferred to other regions of the system. The decoherence transfer is governed by the undamped excitations present in the system, i.e. by the long-wavelength excitations traveling with the sound velocity $c$.

**DISCUSSION**

Summarizing, we have studied the decohering influence of a local measurement performed on a distributed quantum system. We show that the decohering perturbation exerted on the measured region propagates over the system by forming a decoherence wave, whose dynamics is determined by the Green’s function of the system. This result, although not totally unexpected, is not as trivial as it might seem, since decoherence is a rather peculiar effect, and the decohering impact of a measurement can be quite different from other physical influences (see, e.g. the discussion in Ref. [15]).

The usual scenario for few-particle systems is based on the Einstein-Podolsky-Rosen (EPR) situation [17] of strong entanglement, when, e.g. two particles with spins 1/2 form a singlet state

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle).$$  

(27)

If the first spin has been measured, and as a result of this measurement has been cast in the state $|\uparrow\rangle$ (here again we use von Neumann’s theory of instantaneous measurement), then the transfer of decoherence is instant: the second spin immediately occurs in the state $|\downarrow\rangle$. In distributed systems this effect is also present: the wave function of the system collapses immediately after the measurement. But the impact of the collapse upon the one-particle density matrix (and even $s$-particle density matrix, for $s \ll M$) is practically unobservable for the system of macroscopic size (where $M \gg 1$): the change in the density matrix element $\rho(r, r', t)$ immediately after the measurement is of order of $n_0/M$ (provided, of course, that $r, r' \neq 0$), and the same is true for the $k$-particle density matrix $\rho(r_1, \ldots, r_k; r'_1, \ldots, r'_k)$ if $k \ll M$. This result is rather obvious: localization of the number $n_0$ of particles in some cell can not affect noticeably other cells if the total number of particles is macroscopically large. Therefore, the post-measurement evolution of the system, which is governed by the Green’s function, becomes important since it provides much more noticeable changes in the density matrix elements: in Eq. (15) the term corresponding to the decoherence wave does not go to zero as $M \to \infty$.

Obviously, it happens because in the EPR-like situation the entanglement is very “stiff”, so that each state of one particle determines completely the state of the other. But in the many-particle system there is no one-to-one correspondence, since the total number of degrees of freedom is much larger than the number of degrees of freedom fixed during the measurement. This difference is the reason for the different dynamics of decoherence propagation.

Finally, we remark that another aspect of decoherence in distributed systems has been studied within the context of decoherent quantum histories [18,19]. Although the effects studied there, as well as systems considered and methods used, are different from those investigated here, it is interesting to note that local properties of distributed quantum systems are often “intrinsically” decoherent [13] if a coarse enough description is used. For the effects considered here, sufficient coarse graining leads to averaging of the oscillating Green’s function over the spatial scale of several oscillations, so that the details of the decoherence wave becomes negligible. Therefore, the intrinsic structure of the decoherence wave can be distinguished only at fine scales, where coherence of the Green’s function holds.

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