Toroidal, compression and vortical dipole strengths in $^{124}$Sn

J Kvasil$^1$, A Repko$^1$, V O Nesterenko$^2$, W Kleinig$^{2,5}$, P-G Reinhard$^3$ and N Lo Iudice$^4$

1 Institute of Particle and Nuclear Physics, Charles University, Prague, Czech Republic
2 Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, Moscow Region 141980, Russia
3 Institut für Theoretische Physik II, Universität Erlangen, D-91058 Erlangen, Germany
4 Instituto Nazionale di Fisica Nucleare, Monte S Angelo, Via Cinzia, I-80126 Napoli, Italy
5 Permanent address: Technische Universität Dresden, Institut für Analysis, D-01062, Dresden, Germany.

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Abstract
The toroidal, compression and vortical dipole strength functions in semi-magic $^{124}$Sn (and partly in doubly magic $^{100,132}$Sn) are analyzed within the random-phase-approximation method with the SkT6, SkI3, SLy6, SV-bas and SkM$^*$ Skyrme forces. The isoscalar ($T = 0$), isovector ($T = 1$) and electromagnetic (‘elm’) channels are considered. Both convection $j_c$ and magnetization $j_m$ nuclear currents are taken into account. The calculations basically confirm the previous results obtained for $^{208}$Pb with the force SLy6. In particular, it is shown that the vortical and toroidal strengths are dominated by $j_c$ in the $T = 0$ channel and by $j_m$ in the $T = 1$ and ‘elm’ channels. The compression strength is always determined by $j_c$. It is also shown that the ‘elm’ strength (relevant for the ($e,e'$) reaction) is very similar to the $T = 1$ one. The toroidal mode resides in the region of the pygmy resonance. So, perhaps, this region embraces both irrotational (pygmy) and vortical (toroidal) flows.

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(Some figures may appear in colour only in the online journal)

1. Introduction

The toroidal and compression flows have been the subject of intense investigation for many years, see e.g. the review [1]. The toroidal mode (TM) is known to be determined by the second-order correction to the leading long-wave part of the transition $E_\lambda$ operator [2–4]. The compression mode (CM) [5–7], being different from TM by construction, is related, nevertheless, to TM and also represents the second-order correction [4, 6]. Despite the extended studies on the topic, the vorticity of the modes is still a subject of discussion [4]. Besides, a possible coexistence of the dipole TM and pygmy mode (PM) becomes actual [8].

In hydrodynamics (HD), the vorticity is defined by the curl of the velocity field [9]. Then TM is vortical and CM is irrotational [4]. However, nuclear models deal with the nuclear current rather than the velocity field, which causes alternative definitions of the vorticity. In [10, 11], the nuclear current component $j_\lambda^{(f)}(r)$, arising in the multipole decomposition of the current transition density $\langle f | j_\lambda^{(nuc)}(r) | i \rangle$, was treated as unconstrained by the continuity equation and thus suitable as a measure of the vorticity. Following this definition, the TM and CM are mixed flows with both vortical and irrotational elements. In our recent study [4], the relevant vortical operator was derived and related in a simple manner to the CM and TM operators. The vortical, toroidal and compression $E1$ strengths were numerically explored with the separable random-phase approximation (SRPA) [12, 13] in $^{208}$Pb with the force SLy6 [14].

In this paper, the toroidal, compression and vortical $E1$ strengths are investigated in semi-magic $^{124}$Sn within a wide range of Skyrme forces (SkT6 [15], SV-bas [16], SkM$^*$ [17], SLy6 [14], SkI3 [18]). In addition to $T = 0$ and 1 channels, the electromagnetic (‘elm’) strength relevant for the ($e,e'$) reaction...
Explicit expressions for \( \hat{j}_{\text{vor}}(\vec{r}) \) and \( \hat{j}_{\text{com}}(\vec{r}) \) can be found elsewhere, see e.g. [20] and equations (C1)–(C4) in [4].

In this paper, only dipole transitions \( g.s. \rightarrow I^+ = 1^- \) are considered. Taking into account the center-of-mass corrections (c.m.c.) in the \( T = 0 \) channel, the dipole operators to be used in the calculations read [4]

\[
\hat{M}(\text{vor}, 1\mu) = -\frac{i}{5c} \sqrt{2} \int d^3 r \hat{j}_{\text{mac}}(\vec{r}) \cdot r^2 \hat{Y}_{12\mu}(\vec{r}) ,
\]

\[
\hat{M}(\text{tor}, 1\mu) = -\frac{2}{2c\sqrt{3}} \int d^3 r \hat{j}_{\text{mac}}(\vec{r}) \cdot r^2 \hat{Y}_{12\mu}(\vec{r}) + (r^2 - \delta_{T,0}(r^2)_{0}) \hat{Y}_{10\mu}(\vec{r}) ,
\]

\[
\hat{M}(\text{com}', 1\mu) = \frac{1}{10} \int d^3 r \hat{\rho}(\vec{r}) \times r^{\lambda+1} \hat{Y}_{\lambda,\lambda+1;1\mu} (\vec{r}) ,
\]

where \( (r^2)_{0} = \int d^3 r r^2 \hat{\rho}(\vec{r}) \) is the ground state squared radius. Note that the vortical operator has no c.m.c.

The calculations were performed within the SRPA [12, 13]. The model is fully self-consistent since both the mean field and residual interaction are derived from the same Skyrme functional. Moreover, the residual interaction includes all the functional contributions and the Coulomb (direct and exact) terms. The self-consistent factorization of the residual interaction dramatically reduces the computational effort while keeping the accuracy of nonseparable RPA. SRPA has been successfully applied to the description of electric [4, 21–23] and magnetic [24–26] giant resonances as well as \( E1 \) strengths near the particle thresholds [27, 28].

For all the modes, the strength function reads

\[
S_\alpha(E1; E) = 3 \sum_\nu |\langle \Psi_\nu | \hat{M}_\alpha(E10) | \Psi_0 \rangle|^2 \zeta(E - E_\nu) ,
\]

where \( \zeta(E - E_\nu) \) is a Lorentzian weight with the averaging parameter \( \Delta = 1 \text{ MeV} \) (chosen for the convenient comparison of the strength function to the experimental data); \( \hat{M}_\alpha(E1) \) is the transition operator of the type \( \alpha = \{ \text{vor}, \text{tor}, \text{com}, \text{com}' \} \); \( \Psi_0 \) is the ground state wave function; and \( E_\nu \) and \( |\Psi_\nu \rangle \) are the energy and wave function of the RPA \( \nu \)-state.

The \( T = 0, T = 1 \) and ‘elm’ channels are defined by the proper choice of the proton and neutron effective charges \( e_{n,p}^{\text{eff}} \) and gyromagnetic factors \( g_{n,p}^{\text{eff}} \) as

\[
T = 0: e_{n}^{\text{eff}} = e_{p}^{\text{eff}} = 1, \quad g_{n,p}^{\text{eff}} = \frac{\xi}{2} (g_n + g_p) ,
\]

\[
T = 1: e_{n}^{\text{eff}} = -e_{p}^{\text{eff}} = -1, \quad g_{n,p}^{\text{eff}} = \frac{\xi}{2} (g_n - g_p) ,
\]

\[
e_{\text{elm}}: e_{n}^{\text{eff}} = 0, \quad e_{p}^{\text{eff}} = 1, \quad g_{n,p}^{\text{eff}} = \xi g_{n,p} .
\]
Here $g_n = -3.82$ and $g_p = 5.58$ are free neutron and proton gyromagnetic ratios; $\zeta \approx 0.7$ is the quenching factor.

For neutrons in the semi-magic nucleus $^{124}\text{Sn}$, the zero-range pairing forces are used at the BCS level (HF + BCS) [29]. More details of the calculations are found in [4].

3. Results and discussions

Results of the calculations are given in figures 1–6. In figure 1, the SRPA accuracy is inspected for the photoabsorption in $^{124}\text{Sn}$. Unlike the strength functions (12), the photoabsorption $\sigma$ is computed with an energy-dependent averaging parameter $\Delta(E)$ in the Lorentzian weight; for more details see [28].

A representative set of Skyrme forces with various isoscalar effective masses is used: SkT6 ($m^* = 1$), SV-bas ($m^* = 0.9$), SkM* ($m^* = 0.79$), SLy6 ($m^* = 0.69$) and SkI3 ($m^* = 0.58$). Both SRPA and HF + BCS results are depicted.

Figure 1 shows good agreement with the experimental data [19] for all the forces. The best agreement is for SLy6. A discrepancy with the experiment around the particle emission threshold (9–10 MeV) may be explained by neglecting the coupling with complex configurations, which is expected to be strong in this particular energy region [30].

Note a small energy upshift of the double-peak HB + BCS strength on decreasing the effective mass $m^*$ from SkT6 to SkI3 (this may be explained by spreading the mean-field spectra with decreasing $m^*$). As seen from figure 1, the residual interaction drastically shifts the strength to a higher energy, neutralizes the $m^*$ effect and transforms the double-peak structure to a single-peak one.

Figures 2 and 3 show the toroidal and vortical strength functions in $^{124}\text{Sn}$ for the $T = 0$ and 1 channels and the same set of Skyrme forces. The cases of the total (8), convection and magnetization nuclear currents are considered. It is seen that, in agreement with the previous SLy6 results for $^{208}\text{Pb}$ [4], the isoscalar (isovector) strengths are dominated by the convection $j_c$ (magnetization $j_m$) current. This is explained by the destructive (constructive) interference of the neutron and proton $j_m$-contributions in the $T = 0$ ($T = 1$) channels.

As mentioned in section 2, the toroidal and vortical strengths represent the second-order $E1$ corrections to the leading first-order $E1$ response (e.g. photoabsorption). These second-order strengths are less collective [4] and,
Figure 4. The compression strength in $^{124}$Sn for the $T = 0$ (left panel) and $T = 1$ (right panel) channels, generated by the operator (11). Since the $j_m$-contribution is zero, only the $j_{\text{nuc}} = j_c$ case is plotted.

Figure 5. Toroidal (left) and vortical (right) strengths in $^{124}$Sn, computed with the total (black bold line), convection (red thin line) and magnetization (blue dash line) nuclear current. The $T = 0$ (upper plot), $T = 1$ (middle plot) and electromagnetic (bottom plot) channels are considered. Only the force SLy6 is used.

The isotopic dependence of isoscalar toroidal, vortical and compression strengths is illustrated in figure 6. In addition to $^{124}$Sn, the neutron-deficit $^{100}$Sn and neutron-excess $^{132}$Sn doubly magic nuclei are considered. The neutron excess (skin) over the $N = Z$ nuclear core is zero in $^{100}$Sn and essential in $^{132}$Sn. Figure 6 shows that the most strong isotopic effect takes place for CM. There is a considerable growth of the CM high-energy branch from $^{100}$Sn to $^{132}$Sn, which may be related to increasing the neutrons participating in the dipole compression. The CM low-energy branch located at the pygmy resonance region 6–10 MeV grows from $^{100}$Sn to $^{132}$Sn even more. This indicates a strong dominance of neutron oscillations in the region. Note that the dipole...
4. Conclusion

The toroidal, vortical and compression dipole strength functions in $^{100,124,132}$Sn isotopes were analyzed in the framework of the self-consistent separable Skyrme-RPA approach [12, 13]. A representative set of five Skyrme forces with essentially different isoscalar effective mass $m^*$ was used. The isoscalar ($T = 0$), isovector ($T = 1$) and electromagnetic (‘elm’) channels were inspected. The calculations generally confirm the previous results for $^{208}$Pb obtained for the Skyrme force SLy6. In particular, it was corroborated that the toroidal and vortical strengths in the $T = 0$ ($T = 1$) channels are mainly provided by the convection (magnetization) nuclear current. A close similarity of $T = 1$ and ‘elm’ channels was established. This means that the $E1(T = 1)$ TM represents an unusual case of the electric collective motion determined by the magnetization current and this case can, in principle, be explored in the $(e,e')$ reaction.

The comparison of the results for $^{100,124,132}$Sn shows that both low- and high-energy branches of the CM considerably depend on the neutron excess. The enhanced dipole strength in the low-energy region, often called pygmy resonance, is strongly correlated with the rotational low-energy CM and vortical TM (probably located in the nuclear $N \approx Z$ core). Altogether, the pygmy resonance region shows an impressive coexistence of various irrotational and vortical flows.

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