Scalar dark matter and its connection with neutrino physics

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Abstract. The existence of non-baryonic Dark Matter is well established by cosmological and astrophysical probes, however its detailed nature still remains elusive. Among the extensions of the Standard Model (SM) explaining the DM relic abundance, the simplest one is the inert dark matter, where a scalar field is added to the Standard Model which is stabilized by a $Z_2$ symmetry. We intend to give a brief review of this scenario and its possible connection with neutrino physics. In particular the discrete dark matter mechanism will be outlined. This mechanism consists in an extended SM with a non-Abelian flavor symmetry. When the flavor symmetry is spontaneously broken by the electroweak symmetry breaking mechanism, it explains the neutrino mixing patterns and at the same time renders the dark matter stable.

1. Introduction
Non-baryonic Dark Matter (DM) is one of the most compelling problems of modern cosmology. Despite the fact that its existence is well established by cosmological and astrophysical probes, its nature remains elusive. Still, observations can constraint the properties of dark matter and give some hints about its identity. For instance, a fundamental requirement for a viable dark matter candidate is the stability over cosmological times, that is $\approx 10^{18}$ seconds, and depending on the nature of this, if it decays causes fluxes of photons, anti-protons and/or positrons larger than the observed its life-time should be larger than $\approx 10^{26}$ seconds. This suggests the existence of an exact or slightly-broken symmetry protecting or suppressing its decay.

The inert dark matter model [1] is a minimal extension of the SM that can account for the dark matter in the universe. This model contains an extra $SU(2)$ Higgs doublet which is odd under a $Z_2$ symmetry. This symmetry render stable the lightest member of the $SU(2)$ multiplet which in order to be a good candidate should be electrically neutral.

As the SM is even respect to this $Z_2$ symmetry, the only interactions of this new scalar field, let’s call it $\eta$ will be with the gauge bosons, through the kinetic term and with the Higgs field through the quartic coupling. Therefore, the quartic coupling with the Higgs will give us the so-called “Higgs portal”. This is important because this can lead to the direct detection of the DM in the elastic scattering with the nucleons. This extension has been studied extensively in the past years, see for instance [2, 3, 4, 5, 6]. In this model there are basically two allowed regions of masses for the DM candidate, one which goes from few GeVs to 100’s GeV, and another region of heavy masses [7]. This is basically because in the Higgs potential, there are two $\mu$ parameters (the quadratic terms in the potential), one for the SM Higgs and the other...
one for the inert Higgs. The fact that the SM Higgs acquire vev give us the constraint that we can write this $\mu$ parameter as a combination of the quartic couplings and the vev of the Higgs, therefore this $\mu$ term is related to the electroweak scale. On the other hand, as the vev of the inert is zero, the $\mu$ term for this is free and it can push the mass of this inert scalar to higher scales. We will come back to this point in the discussion of the models with flavor symmetries.

2. The inert scalar model and its connection with neutrino masses

In this section we will discuss briefly the “scotogenic” model which is an extension of the inert model introduced by Ernest Ma [8]. In this model the inclusion of some right-handed (RH) neutrinos odd under the $Z_2$ symmetry play a role in the generation of the light neutrino mass. This of course will give us a Dirac interaction with the Left-Handed leptons. As one condition for the inert DM is that the neutral component has zero vev. This will not contribute to the Dirac mass of the neutrino. The neutrino mass is generated through a loop, where inside the loop enters the inert sector, namely the inert scalar and the right-handed neutrino, see figure 1.

![Figure 1. Neutrino mass generation in the Scotogenic model. The fields charged under the $Z_2$ symmetry are in blue.](image)

| $SU(2)$ | $L$ | $l^c$ | $N$ | $\Phi$ | $\eta$ |
|---------|-----|------|-----|--------|-------|
| $Z_2$   | $+  $| $+   $| $-   $| $+      $| $-     $|

Table 1. Summary of relevant model quantum numbers

The charge assignment under the $SU(2)_L \times U(1)_Y \times Z_2$ of the relevant matter fields is as shown in Table 1. With this charge assignment, the lepton Yukawa Lagrangian is as follows [8]:

$$\mathcal{L}_Y = f_{ij} \bar{L} \Phi^i l_j^c + h_{ij} \bar{L} \eta^i N_j + H.c.$$  \hspace{1cm} (1)

The Majorana mass term for the Right-handed neutrino is

$$\frac{1}{2} M_i N_i N_i + H.c.$$  

The Higgs potential in this model is

$$V = m_1^2 \Phi^\dagger \Phi + m_2^2 \eta^\dagger \eta + \frac{1}{2} \lambda_1 (\Phi^\dagger \Phi)^2 + \frac{1}{2} \lambda_2 (\eta^\dagger \eta)^2 + \lambda_3 (\Phi^\dagger \Phi)(\eta^\dagger \eta) + \lambda_4 (\Phi^\dagger \eta)(\eta^\dagger \Phi) + \frac{1}{2} \lambda_5 [(\Phi^\dagger \eta)^2 + H.c].$$  \hspace{1cm} (2)
The relevant term in this potential for the neutrino mass is the term proportional to $\lambda_5$. The Higgs field $\Phi$ acquire a vev breaking the electroweak symmetry while the field $\phi$ remains with zero vev. With this the $Z_2$ symmetry remains a symmetry of the theory stabilizing the lightest particle odd under this symmetry, which we are assuming is the scalar $\phi^0$. The diagram of Fig. 1 give the following contribution to the neutrino mass

$$(M_\nu)_{ij} = \sum_k \frac{h_{ik} h_{jk} M_k}{16\pi^2} \left[ \frac{m_R^2}{m_R^2 - M_k^2} \ln \frac{M_R^2}{M_k^2} - \frac{m_l^2}{m_l^2 - M_k^2} \ln \frac{m_l^2}{M_k^2} \right], \quad (3)$$

where $m_R$ and $m_l$ are the masses of $\sqrt{2}Re\eta^0$ and $\sqrt{2}Im\eta^0$ respectively. If $m_R^2 - m_l^2 = 2\lambda_5 v^2$ is assumed to be small compared to $m_0^2 = (m_R^2 + m_l^2)/2$, then

$$(M_\nu)_{ij} = \frac{\lambda_5 v^2}{8\pi^2} \sum_k \frac{h_{ik} h_{jk} M_k}{m_0^2 - M_k^2} \left[ 1 - \frac{M_k^2}{m_0^2} \ln \frac{m_0^2}{M_k^2} \right]. \quad (4)$$

If $M_k^2 >> m_0^2$, then

$$(M_\nu)_{ij} = \frac{\lambda_5 v^2}{8\pi^2} \sum_k \frac{h_{ik} h_{jk} M_k}{m_0^2} \left[ \ln \frac{M_k^2}{m_0^2} - 1 \right]. \quad (5)$$

If $m_0^2 \approx M_k^2$, then

$$(M_\nu)_{ij} \approx \frac{\lambda_5 v^2}{16\pi^2} \sum_k \frac{h_{ik} h_{jk}}{M_k}. \quad (6)$$

This is a possible scenario where the DM is connected with the Majorana neutrino mass generation. In the following section we will discus another possibility where the DM stability is closed related with the neutrino masses and mixing pattern.

3. Discrete dark matter mechanism

In the past 10 years or so, flavor symmetries has been very popular mainly because they can lead to the prediction of some mixing patterns, the tri-bimaximal [9], bi-maximal [10], Golden ratio [11, 12], for a review see for instance [13]. Most popular models where based in $A_4$ [14, 15] basically because this is the smallest group that contains a triplet irreducible representation, which can accommodate the three leptons.

The discrete dark matter (DDM) mechanism contains a stable particle, where its stability is obtained by means a residual $Z_N$ symmetry. This residual symmetry arise from the spontaneously broken non-Abelian discrete flavor symmetry. In the first model, the group of even permutations of four objects was used, $A_4$, and it was spontaneously broken by the electroweak symmetry breaking mechanism into a $Z_2$ sub-group, $A_4 \rightarrow Z_2$ [16].

The model is obtained by adding to the SM three Higgs doublets which transform as a triplet under $A_4$, and four right handed neutrinos transforming as a singlet and a triplet of the flavor group. In the table 3 we present the relevant quantum numbers for the matter fields.

The group of even permutations of four objects, $A_4$, has two generators: $S$ and $T$, which obeys the properties $S^2 = T^3 = (ST)^3 = 1$. $S$ is a $Z_2$ generator while $T$ is a $Z_3$ generator. $A_4$ has four irreducible representations, three singlets $1$, $1'$, and $1''$ and one triplet. The generators of $A_4$ for each irreducible representation are

$${}1 \quad S = 1 \quad T = 1$$
$${}1' \quad S = 1 \quad T = \omega^2$$
$${}1'' \quad S = 1 \quad T = \omega$$

$${}3 \quad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad (7)$$

1 There is the possibility of the $N$ to be the DM candidate when it is the lightest particle charged under the $Z_2$ symmetry [8]
\( \omega^3 = 1 \). The Yukawa Lagrangian of the model is

\[
\mathcal{L} = y_e L_e \ell^c H + y_\mu L_\mu \ell^c H + y_\tau L_\tau \ell^c H + + y_{e}^\prime L_e (N_T \eta)_1 + y_{\mu}^\prime L_\mu (N_T \eta)_1^\prime + + y_{\tau}^\prime L_\tau (N_T \eta)_1^\prime + y_{e}^\prime \prime L_e N_4 H + M_1 N_T N_T + M_2 N_4 N_4 + h.c. \tag{8}
\]

This way \( H \) is responsible for quark and charged lepton masses, the latter automatically diagonal\(^2\). Neutrino masses arise from \( H \) and \( \eta \). One solution for the minimum of the Higgs potential [16] is

\[
\langle H^0 \rangle = v_h \neq 0, \quad \langle \eta_i^0 \rangle = v_\eta \neq 0, \quad \langle \eta_{2,3}^0 \rangle = 0, \tag{9}
\]

which means the vev alignment for the \( A_4 \) triplet of the form \( \langle \eta \rangle \sim (1, 0, 0) \). This alignment is invariant under the \( S \) generator\(^3\), see eq. (7), which means that the minimum of the potential breaks spontaneously \( A_4 \) into a \( Z_2 \) subgroup generated by \( S \). All the fields in the model singlets under \( A_4 \) are even under the residual \( Z_2 \), the triplets transform as:

\[
N_1 \to +N_1, \quad \eta_1 \to +\eta_1 \\
N_2 \to -N_2, \quad \eta_2 \to -\eta_2 \\
N_3 \to -N_3, \quad \eta_3 \to -\eta_3. \tag{10}
\]

The DM candidate is the lightest particle charged under \( Z_2 \) i.e. the lightest combination of the scalars \( \eta_2 \) and \( \eta_3 \), which we will denote generically by \( \eta_{DM} \). We list below all interactions of \( \eta_{DM} \):

(i) Yukawa interactions

\[
\eta_{DM} \nu_i N_{2,3}, \tag{11}
\]

where \( i = e, \mu, \tau \).

(ii) Higgs-Vector boson couplings

\[
\eta_{DM} \eta_{DM} Z \zeta, \quad \eta_{DM} \eta_{DM} W \zeta, \quad \eta_{DM} \eta_{2,3} W \zeta, \quad \eta_{DM} \eta_{2,3} W \zeta, \quad \eta_{DM} \eta_{2,3} \zeta. \tag{12}
\]

(iii) Scalar interactions from the Higgs potential:

\[
\eta_{DM} A_1 A_2 h, \quad \eta_{DM} A_1 A_3 h_1, \quad \eta_{DM} A_1 A_2 h_1, \quad \eta_{DM} A_1 A_3 h, \quad \eta_{DM} A_2 A_3 h_3, \quad \eta_{DM} A_1 A_2 h, \quad \eta_{DM} A_1 A_3 h_1. \tag{13}
\]

After electroweak symmetry breaking, the vevs the Higgs fields acquire vacuum expectation values, \( v_h \) and \( v_\eta \) for the singlet and the first component of the triplet respectively, additional terms are obtained from those in Eq. (13) by replacing \( h \to v_h \) and \( h_1 \to v_\eta \). These vertices

\(^2\) For quark mixing angles generated through higher dimension operators see reference [17].

\(^3\) \( H \) is in the 1 representation of \( A_4 \) and its vev also respect the generator \( S \).
are relevant for direct detection [18]. The phenomenology of dark matter of this model has been studied in detail in [18]. The model accommodates WMAP and collider constraints and is consistent with Xenon100 and CDMS on one hand and CoGent or DAMA on the other hand [18].

The allowed region with this constraints for the DM mass vs the mass of the Higgs$^4$ is presented in figure 2.

Figure 2. Regions in the plane DM mass ($M_{DM}$) - lightest Higgs boson $M_H$ allowed by collider constraints and leading to a DM relic abundance compatible with WMAP measurements.

The plot for direct DM detection is presented in Figure [?]

4. Neutrino masses phenomenology
The model contains four heavy right-handed neutrinos it is a special case, called (3,4), of the general type-I seesaw mechanism. After electroweak symmetry breaking, it is characterized by Dirac and Majorana mass-matrix:

$$m_D = \begin{pmatrix} x_1 & 0 & 0 & x_4 \\ x_2 & 0 & 0 & 0 \\ x_3 & 0 & 0 & 0 \end{pmatrix}, \quad M_R = \begin{pmatrix} M_1 & 0 & 0 & 0 \\ 0 & M_1 & 0 & 0 \\ 0 & 0 & M_1 & 0 \\ 0 & 0 & 0 & M_2 \end{pmatrix}. \quad (14)$$

where $x_1$, $x_2$, $x_3$ and $x_4$ are respectively proportional to $y^c_1$, $y^c_2$, $y^c_3$ and $y^c_4$ of eq. (16) and are of the order of the electroweak scale, while $M_{1,2}$ are assumed to be close to the unification scale. Light neutrinos get Majorana masses by means of the type-I seesaw relation and the light-neutrinos mass matrix has the form:

$$m_{\nu} = -m_{D_{3\times4}}M_{R_{4\times4}}^{-1}m_{D_{3\times4}}^T \equiv \begin{pmatrix} y^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{pmatrix}. \quad (15)$$

$^4$ When this paper was published the mass of the Higgs was unknown, but as we can see from the figure the range of the DM mass remains more or less the same.
This texture of the light neutrino mass matrix has a null eigenvalue $m_3 = 0$ corresponding to the eigenvector $(0, -b/c, 1)^T$, implying a vanishing reactor mixing angle $\theta_{13} = 0$ and inverse hierarchy. The atmospheric angle, the solar angle and the two square mass differences can be fitted. The model implies a neutrinoless double beta decay effective mass parameter in the range $0.03$ to $0.05$ eV at $3\sigma$, within reach of upcoming experiments. The fact that only two RH neutrinos are $Z_2$ even tell us that basically in the even sector we have a two RH neutrino seesaw and this is the reason why we have a massless state.

In a modification of this model [24] we found that as is expected, by adding an extra RH neutrino we can generate a mass for the lightest state. In Tab.(3) we summarized the model quantum numbers. In contrast to [16], the right-handed neutrino $N_4$ is assigned to $1'$ instead of 1 and we introduced one more right-handed neutrino $N_5$ assigned to $1''$.

| $SU(2)$ | $L_e$ | $L_\mu$ | $L_\tau$ | $\ell_e^c$ | $\ell_\mu^c$ | $\ell_\tau^c$ | $N_T$ | $N_4$ | $N_5$ | $H$ | $\eta$ |
|--------|------|--------|--------|--------|--------|--------|--------|--------|--------|------|------|
| $A_4$ | 1    | 1'     | 1''    | 1      | 1      | 1''    | 3      | 1      | 1      | 1    | 2    |

Table 3. Summary of relevant model quantum numbers

5 Note that if we were to stick to the minimal (3,3)-type-I seesaw scheme, with just 3 $SU(2)$ singlet states, one would find a projective nature of the effective tree-level light neutrino mass matrix with two zero eigenvalues, hence phenomenologically inconsistent. That is why we adopted the (3,4) scheme.
The operators needed to generate neutrinos masses are the following:

\[ \mathcal{L} = y_1^\nu L_{\nu}(N_T)_{1}\varepsilon_y + y_2^\nu L_\mu(N_T)_{1}\varepsilon_y + \]
\[ + y_3^\nu L_r(N_T)_{1}\varepsilon_y + y_4^\nu N_T H + y'_5^\nu \mu_3 N_5 H + M_1 N_T N_T + M_2 N_4 N_5 + \text{h.c.} \]

The scalar and the lepton charged current sectors of our model are the same as in [16] and we refer to that paper for all the details. After electroweak symmetry breaking, two of the Higgs doublets contained in the $A_4$ triplet $\eta$ do not take vev and we have:

\[ \langle H^0 \rangle = v_h \neq 0, \quad \langle \eta^H_1 \rangle = v_\eta \neq 0 \quad \langle \eta^H_{2,3} \rangle = 0. \] (16)

As a consequence, the Dirac and Majorana neutrino mass matrices are:

\[ m_D = \begin{pmatrix}
  y_1^\nu v_\eta & 0 & 0 & 0 \\
  y_2^\nu v_\eta & 0 & 0 & y_5^\nu v_h \\
  y_3^\nu v_\eta & 0 & y_3^\nu v_h & 0
\end{pmatrix} \quad m_M = \begin{pmatrix}
  M_1 & 0 & 0 & 0 \\
  0 & M_1 & 0 & 0 \\
  0 & 0 & M_1 & 0 \\
  0 & 0 & 0 & M_2
\end{pmatrix}. \] (17)

Using the See-Saw type-I formula we get the following light neutrino mass matrix:

\[ m_\nu = \begin{pmatrix}
  a^2 & ab & ac \\
  ab & b^2 & bc + k \\
  ac & bc + k & c^2
\end{pmatrix}, \] (18)

where we defined:

\[ a = \frac{y_1^\nu v_\eta}{\sqrt{M_1}} \quad b = \frac{y_2^\nu v_\eta}{\sqrt{M_1}} \quad c = \frac{y_3^\nu v_\eta}{\sqrt{M_1}} \quad k = \frac{y_3^\nu y_5^\nu v_h^2}{M_2}. \] (19)

This mass matrix is only compatible with a normal spectrum. Among all possible correlations, we found that the $(\theta_{13} - \theta_{23})$ one is quite interesting, as shown in fig. (4). As we can see, there is a strong correlation between the atmospheric and the reactor mixing angles. The other interesting neutrino observable that we want to discuss is the effective mass $|m_{ee}|$ entering the neutrinoless double beta decay rate. Since this is only compatible with the normal hierarchy, it is expected to be quite small for small $m_{\nu_1}$. The result of our simulation can be found in fig. (5), where we can see that a lower bound $(|m_{\nu_1}|, |m_{ee}|) \sim (2 \cdot 10^{-3}, 4 \cdot 10^{-4})\ eV$ can be set.

The extension of this mechanism in such a way we can include the quarks is not trivial, we did some attempts, first with the group $D_4$ where the quarks transform as different singlets representation [30]. In that model some flavor changing neutral currents are induced but under control. Again in that model a massless neutrino state is predicted as well as a zero reactor mixing angle. All this models are ruled out by the results of Daya-Bay on the reactor neutrino mixing angle.

5. A Discrete Dark Matter model for quarks and leptons

We looking for a group $G$ that contains at least two irreducible representations of dimension larger than one, namely $r_a$ and $r_b$. We also require that all the components of the irreducible representation $r_a$ transform trivially under an abelian subgroup of $G \supset Z_N$ (with $N = 2, 3$) while at least one component of the irreducible representation $r_b$ is charged with under
Figure 4. Correlation among the $\theta_{13}$ and $\theta_{23}$ angles as predicted by the model.

Figure 5. Values of the effective mass $|m_{ee}|$ (in eV) allowed in our model. The two dashed horizontal lines represent the experimental sensitivities of some of the forthcoming experiments while the dashed vertical line is the upper limit from tritium $\beta$-decay experiment. For references to experiments see [25, 26, 27, 28, 29].

$Z_N$. The stability of the lightest component of the matter fields transforming as $r_b$ is guaranteed by $Z_N$ giving a potential $^6$ DM candidate.

The smallest group with this property we found is $\Delta(54)$, isomorphic to $(Z_3 \times Z_3) \rtimes S_3$. This group contains four triplet irreducible representations, $3_{1,2,3,4}$, in addition, $\Delta(54)$ contains four different doublets $2_{1,2,3,4}$ and two singlet irreducible representations, $1_{\pm}$. The product rules for the doublets are as follows:

$^6$ Other requirements must fulfilled in order to have a viable DM candidate, such as neutrality, correct relic abundance, and consistency with constraints from DM search experiments.
• The product of two equal doublets

\[ 2_k \times 2_k = 1_+ + 1_- + 2_k \]  

(21)

• The product of two different doublets give us the other two doublets, for instance:

\[ 2_1 \times 2_2 = 2_3 + 2_4 \]  

(22)

Of the four doublets \(2_1\) is invariant under the \(P \equiv (Z_3 \times Z_3)\) subgroup of \(\Delta(54)\), while the others transform non-trivially, for example \(2_3 \sim (\chi_1, \chi_2)\), which transforms as \(\chi_1 (\omega^2, \omega)\) and \(\chi_2 (\omega, \omega^2)\) respectively, where \(\omega^3 = 1\). We can see that by taking \(r_a = 2_1\) and \(r_b = 2_3\) that \(\Delta(54)\) is a perfect choice for our purpose [31].

| \(L_e\) | \(L_D\) | \(e_R\) | \(l_D\) | \(H\) | \(\chi\) | \(\eta\) | \(\Delta\) |
|--------|--------|--------|--------|------|------|------|------|
| \(SU(2)\) | 2 | 2 | 1 | 1 | 2 | 2 | 3 |
| \(\Delta(54)\) | 1_+ | \(2_1\) | 1_+ | \(2_1\) | 1_+ | \(2_3\) | \(2_1\) |

Table 4. Lepton and higgs boson assignments of the model.

| \(Q_{1,2}\) | \(Q_3\) | \((u_R, c_R)\) | \(t_R\) | \(d_R\) | \(s_R\) | \(b_R\) |
|--------|--------|--------|--------|--------|--------|--------|
| \(SU(2)\) | 2 | 2 | 1 | 1 | 1 | 1 |
| \(\Delta(54)\) | \(2_1\) | 1_+ | \(2_1\) | 1_+ | 1_- | 1_+ |

Table 5. Quark gauge and flavor representation assignments.

In this way we can choose the scalars in the model to belong to the \(2_3\) for instance while the active scalars to belong to the \(2_1\) [31]. The relevant quantum numbers of the model are in Tables 4 and 5. In table 4 L\(_D\) \(\equiv (L_{\mu}, L_{\tau})\) and l\(_D\) \(\equiv (\mu_R, \tau_R)\). There are five \(SU(2)_L\) doublets of Higgs scalars: the \(H\) is a singlet of \(\Delta(54)\), while \(\eta = (\eta_1, \eta_2) \sim 2_3\) and \(\chi = (\chi_1, \chi_2) \sim 2_1\) are doublets. In order to preserve a remnant \(P\) symmetry, the doublet \(\eta\) is not allowed to take vacuum expectation value (vev).

Let's start discussing the quark sector. In Ref. [16, 18, 24] quarks were considered blind under the flavor symmetry to guarantee the stability of the DM. Consequently the generation of quark mixing angles was difficult, see [17].

A nice feature of our current model is that with \(\Delta(54)\) we can assign quarks to the singlet and doublet representations as shown in table 4, in such a way that we can fit the CKM mixing parameters.

The resulting up- and down-type quark mass matrices in our model are given by

\[
M_d = \begin{pmatrix}
ra_d & rb_d & rd_d \\
-a_d & b_d & d_d \\
0 & c_d & e_d
\end{pmatrix}, \quad M_u = \begin{pmatrix}
ra_u & bu & du \\
bu & au & rd_u \\
cu & rc_u & eu
\end{pmatrix}.
\]

(23)

where \(r = \langle \chi_2 \rangle / \langle \chi_1 \rangle\). Note that the Higgs fields \(H\) and \(\chi\) are common to the lepton and the quark sectors and in particular the parameter \(r\). In order to fit of all quark masses and mixings provided \(r\) lies in the range of about \(0.1 < r < 0.2\) [31]. We turn to the leptonic sector, the charged leptons are basically equal to the up quarks under the flavor symmetry, so the charged
lepton mass matrix is of the same form of the up quark mass matrix in Eq. (23). The neutrino masses are generated through the type-II see-saw mechanism [32]. For that we include in the model an $SU_L(2)$ Higgs triplet scalar field $\Delta \sim 2_1$. Regarding dark matter, note that the lightest $P$-charged particle in $\eta_{1,2}$ can play the role of “inert” DM [2], as it has no direct couplings to matter. The conceptual link between dark matter and neutrino phenomenology arises from the fact that the DM stabilizing symmetry is a remnant of the underlying flavor symmetry which accounts for the observed pattern of oscillations. Choosing the vev alignment $\langle \Delta \rangle \sim (1, 1)$ and $\langle \chi_1 \rangle \neq \langle \chi_2 \rangle$, consistent with the minimization of the scalar potential one finds that

$$M_\nu \propto \begin{pmatrix} 0 & \delta & \delta \\ \delta & \alpha & 0 \\ \delta & 0 & \alpha \end{pmatrix},$$

(24)

where $\delta = y_a \langle \Delta \rangle$, $\alpha = y_b \langle \Delta \rangle$ [31]. Note that this matrix has two free parameters and gives us a neutrino mass sum rule of the form $m_{\nu_1} + m_{\nu_3} = m_{\nu_2}$ in the complex plane, which has implications for neutrinoless double beta decay [33], as seen in Fig. (6).

Figure 6. Effective neutrinoless double beta decay parameter $m_{ee}$ versus the lightest neutrino mass. The thick upper and lower branches correspond the “flavor-generic” inverse (yellow) and normal (gray) hierarchy neutrino spectra, respectively. The model predictions are indicated by the green and red (darker-shaded) regions, respectively. They were obtained by taking the $3\sigma$ band on the mass squared differences. Only these sub-bands are allowed by the $\Delta(54)$ model.

We now turn to the second prediction. For simplicity, we consider in what follows only real parameters and we fix the intrinsic neutrino CP–signs [34] as $\eta = diag(-, +, +)$, where $\eta$ is defined so that the CP conservation condition in the charged current weak interaction reads $U^* = U\eta$, $U$ being the lepton mixing matrix. The correlations we have are presented in Fig. 7.

6. Conclusions

We have present here some extensions of the SM where the generation of the neutrino mass and the DM are connected. First we present the scotogenic model where the neutrino mass is generated at one loop involving the dark sector. We have present the DDM mechanism, in particular we have discussed the stability of the dark matter particle arises from a flavor symmetry. The $A_4$ non-abelian discrete group accounts for both the observed pattern of neutrino mixing and for DM stability. We have analyzed the constraints that follow from electroweak precision tests, collider searches and perturbativity. We have also analyzed the prospects for direct and indirect dark matter detection and found that, although the former already excludes a large region in parameter space, we cannot constrain the mass of the DM candidate.
Figure 7. The left figure (right figure) is the correlation for normal hierarchy (inverse hierarchy). The shaded (yellow) curved band gives the predicted correlation between solar and reactor angles when the solar and atmospheric squared mass splittings are varied within 2σ for the normal hierarchy spectrum. The solid (black) line gives the global best fit values for $\theta_{12}$ and $\theta_{13}$, along with the corresponding two-sigma bands, from Ref. [35]. The dashed lines correspond to the central values of the recent published reactor measurements [36, 37]. Note that $\theta_{23}$ is also within 2σ.

All of the above relies mainly on the properties of the scalar sector responsible for the breaking of the gauge and flavor symmetries. The motivation of our approach is to link the origin of dark matter to the origin of neutrino mass and the understanding of the pattern of neutrino mixing, two of the most outstanding challenges in particle physics today. At this level one may ask what are the possible tests of this idea in the neutrino sector. We have studied other possibilities based on other flavor groups, giving as predictions mainly one massless neutrino and a very tiny neutrino reactor mixing angle. When we move to a bigger group where the quarks and charged leptons also transform as non-trivial representation of the flavor group, it is possible to generate the correct neutrino mixing parameters and at the same time, the quark mixing angles can be accommodated.

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