Investigation of Mixed-Mode Crack Propagation Behaviour under Impact Loading

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Abstract: The fractured rock masses are often subject to impact loading, giving rise to complicated mixed-mode cracking. The propagation of the cracks may result in structural instability and project disaster and often cannot be understood by studying only the mode I fracture. Discovering the mixed-mode crack behaviour is of great interest to engineering design. Here, we report on an investigation of mixed-mode crack propagation under impact loading. A Single Cleavage Simi-Ellipse configuration was proposed with different pre-crack dip angles. Split Hopkinson pressure bar impact tests were conducted and a high-speed camera was employed to capture the cracking behaviour. With the experimental-numerical method, the dynamic initial stress intensity factors $K_I$ and $K_{II}$ were calculated by the finite element method. The AUTODYN was applied to study mixed-mode crack propagation numerically. The investigation demonstrates that the mixed-mode crack would adjust fracture mode continuously in the whole propagation process companied with several extending decelerations. The crack initiation mode and the dynamic initiation stress intensity factor have a relationship with the impact loading orientation. The AUTODYN can be applied in predicting rock crack propagation, offering new insights into dynamic stability analyses in engineering design.

1. Introduction

The mechanical property of engineering materials has shown increasing importance in determining structure strength and dominating project safety. Discontinuities, such as defects, flaws, and micro-cracks, can be generally found in concrete and rock masses. Under dynamic conditions, cracks nucleate, initiate, and propagate successively, contributing to structural instability. Therefore, investigation of the crack propagation behaviour is of significance in geotechnical engineering.

So far, numerous investigations have been conducted to get a better understanding of crack propagation and fracture mechanism[1-7]. The mode I crack initiation, propagation, and arrest are illuminated by Ravi-Chandar[8]. The dynamic stress intensity factors can be calculated accurately by the universal function method[9]. Loading rate effects on crack propagation behaviour can be discussed[10], and the methods to determine the dynamic crack fracture toughness of mode I crack can be studied by split Hopkinson pressure bar (SHPB) tests[11-13]. Up to now, the majority of works concentrate on the mode I crack, the study on mixed-mode crack propagation is still in its infancy.
However, in practice, complex combinations of loadings with different impact orientations are usually applied to fractured rock and concrete. Mixed-mode fracturing in most cases dominates the damage progress of structures. Therefore, great care must be taken when mixed-mode cracking arises. For mixed-mode crack, several classic configurations have been applied in studying the crack propagation under static loading, for instance, the Semi-Circular Bends (SCB)[14], the Single Edge Notched Bend (SENB)[15], and Asymmetric Four-Point-Bend (AFPB)[16]. But these configurations are developed for quasi-static tests. They are not suitable for an impact test under dynamic loading.

In this paper, a Single Cleavage Simi-Ellipse (SCSE) configuration was proposed to investigate the mixed-mode cracking process under impact loading. The impact test was carried out at the loading rate of ~ 470 GPa/s with the SHPB testing system. Crack extension velocity was detected by a high-speed camera. The fracture phenomena were analysed according to the initiation angle, loading orientation, and fracture feature. The stress intensity factor of dynamic fracture initiation, \( K_I \) and \( K_{II} \), were calculated by the finite element method. The AUTODYN[17] was used to simulate dynamic mixed-mode crack propagation. Thus, the dynamic mixed-mode crack propagation was investigated experimentally and numerically, offering new insights into dynamic stability analyses in engineering design.

2. Experiment

2.1. Experimental System

The propagation of mixed-mode crack is more complicated than that of pure mode I crack. In the process of mixed-mode cracking, crack propagation is hard to be tracked. Therefore, a high-speed camera was adopted to capture the whole crack propagation process. Novel SCSE specimens were prepared, as shown in Figure 1(a). The pre-crack of the specimen was made with different angles for triggering different failure modes. Thus, the mixed-mode initiation could be realized and the effect of the loading orientation could be studied. Meanwhile, due to sufficient space provided by the SCSE specimen, the whole propagation process of mixed-mode cracks can be studied quantitatively.

![Figure 1. SCSE specimen and experimental system.](image)

The length of the specimen is 105 mm, the width is 50 mm, and the thickness is 15 mm. The major axis of the semi-ellipse is 52 mm, and the minor axis is 26 mm, as shown in Figure 1(a). The pre-crack was designed with a length of 16 mm, and its tip was sharpened by laser to restrict the width of the pre-crack to 0.2 mm. The dip angles of the pre-cracks are 0°, 30°, and 60°, respectively. As polymethyl methacrylate (PMMA) is often used to investigate the dynamic fracture mechanism[18], PMMA was chosen in the tests. Values of PMMA material parameters are shown in Table 1.

| Material | \( C_d \) (m/s) | \( C_s \) (m/s) | \( \rho \) (kg/m³) | \( E_d \) (GPa) | \( \nu_d \) |
|----------|----------------|----------------|-----------------|----------------|---------|
| PMMA     | 2382           | 1305           | 1187            | 5.61           | 0.28    |

Here, \( C_d \) is the longitudinal wave velocity, \( C_s \) is the transverse wave velocity, \( E_d \) is the dynamic elastic modulus, \( \nu_d \) is Poisson’s ratio, and \( \rho \) is the density.

The SHPB system[19] and high-speed camera system were used to achieve dynamic loading and capture the crack extending process, respectively (Figure 1(b)). Values of the incident and transmitted...
bars are listed in Table 2. Strain signals had been obtained by the strain gauges glued on the midpoint of the incident bar and transmitted bar. Then, the dynamic loading can be determined with Equation (1)[20],

\[
P_{in}(t) = A_i E_i \left[ \varepsilon_i(t) + \varepsilon_r(t) \right]
\]

\[
P_{tr}(t) = A_t E_t \varepsilon_t(t)
\]

(1)

where \(P_{in}(t)\) and \(P_{tr}(t)\) are loads of the incident bar and transmitted bar, respectively. \(\varepsilon_i(t)\), \(\varepsilon_r(t)\), and \(\varepsilon_t(t)\) denote incident strain, reflected strain, and transmitted strain, respectively. \(A_i\) and \(A_t\) are the cross-section area of the incident bar and transmitted bar, respectively.

Table 2. Split-Hopkinson pressure bars.

| Name          | Material | Length (mm) | Diameter (mm) | \(\rho\) (kg/m³) | \(E_d\) (GPa) | \(v_d\) |
|---------------|----------|-------------|----------------|------------------|---------------|--------|
| Incident Bar  | 40CrmoV  | 3000        | 50             | 7600             | 210           | 0.25   |
| Transmitted Bar | 40CrmoV | 3000        | 50             | 7600             | 210           | 0.25   |

In the experiment, the camera shooting frequency is constant at 1/50 μs⁻¹, thus the crack propagating velocity could be calculated with the distance of the running crack measured. The launch velocity of strikers was constant at 6.0 m/s. The obtained dynamic stress curves are shown in Figure 2.

2.2. Results and Discussion

Figure 3. Crack fracture patterns of the specimens. (a) The crack path and initiation angle of Specimen-0°. (b) The crack path and initiation angle of Specimen-30°. (c) The crack path and initiation angle of Specimen-60°.

Three typical fracture patterns with different dip angles are shown in Figure 3. When the dip angle increases from 0° to 60°, the initiation angle rises from 0° to 100.42°. For Specimen-30°, the maximum initiation angle is 86.16°, indicating that the crack initiation is a mixed-mode I/II fracture. For the Specimen-60°, the initiation angle is 100.42°, which means the crack initiation is strongly dominated by mode II fracture. The results reveal that the crack initiation mode has a clear relationship with the dip angle, i.e., the crack initiation mode is strongly influenced by the loading orientation.
The crack path on the three specimens also shows different features related to the dip angle. The crack path of the Specimen-0° is almost a straight line, while two inflection paths can be observed on the Specimen-30° and Specimen-60°. There are several distinct deflection points along the crack paths, marked with A and B in Figure 3. The pictures captured by the high-speed camera show the whole cracking process of each specimen, as shown in Figure 4. For Specimen-0°, the crack runs along the symmetrical axis, leaving a straight line before it stopped (a mode I fracture). For the Specimen-30°, the crack no longer runs straightly. An arc path arises after initiation (Figure 4(b)) and extends towards the symmetrical axis of the specimen. Eventually, a straight path emerges behind point A marked in Figure 4(b). This mode transformation process can be divided into two stages: (1). the mixed-mode I/II propagation; (2). the mode I propagation.

A clearer transformation of crack mode was shown in Figure 4(c). Compared with other specimens, Specimen-60° shows two legible deflection points (Point A and B) that illustrate the adjustment of a crack path from the outset, and the transfer from mixed-mode to mode I. Here, one conclusion can be drawn that mixed-mode cracking is a progressive change process, and the mode transformation in the cracking can be featured by the deflection points.

Based on the crack propagation tracking, the propagating length of the crack in every 50 μs can be measured. The average crack propagation velocity can thus be precisely calculated. Interestingly, crack
propagation shows distinct velocity deceleration at the point at A and B. For instance, in Figure 5 (c), the extending velocity decreases at the time 100 μs and 250 μs, corresponding to Point A and B shaped (in Figure 4(c)), respectively. In contrast, for Specimen-30° in Figure 5(b), the velocity deceleration at the transformation point (Point A about 150 μs) is not conspicuous. This is due to the duration of mode transformation of the Specimen-30° is a shorter process, compared with the Specimen-60°, that cannot be captured by the high-speed camera with the shutter speed of 50 μs.

Though the velocity deceleration cannot be found in the velocity curve of the Specimen-30°, there is clear evidence that supports the velocity change at the mode transformation point. The Specimen-30°’s fractured surface at A point (in Figure 4(b)) was scanned by an electron microscope. A deceleration trace can be seen in the A point zone in Figure 6(a). Those fragments (in Figure 6(b)) are mainly shaped by tensile fracture, which means the mode of the crack propagation had changed in this zone.

![Deceleration trace](image)

![Fragments](image)

**Figure 6.** Fractography of the crack deceleration zone at A point of Specimen-30°. (a) Deceleration trace on the fracture surface. 300×. (b) Fragments in the deceleration trace zone. 1500×.

### 3. Dynamic Initiation Stress Intensity Factor

#### 3.1. Determination of the crack initiation time

![Determining the crack initiation time](image)

![Numerical model in ABAQUS](image)

**Figure 7.** The determination of DISIF. (a) The determination of the crack initiation time. (b) Numerical model in ABAQUS: $P_i(t)$, $P_r(t)$, and $P_t(t)$ are the incident wave, reflected wave, and transmitted wave, respectively. $r_{OA}$ equals a quarter of $r_{OB}$ in the CPS6 element.

The crack initiation time plays a significant role in calculating the dynamic initiation stress intensity factor (DISIF). A strain gauge technique was used to measure the exact initiation time of cracking (Figure 7(a)). A strain gauge was tightly glued on the pre-crack tip, which is long enough to cover the potential area where the crack passes through. The mixed-mode crack initiation time obtained is 143.4
μs for Specimen-0°, 162.2 μs for Specimen-30°, and 174.4 μs for Specimen-60°, suggesting that the initiation time of a crack decreases with its dip angle. In other words, compared with the mode I crack, mixed-mode cracks performed more competently in the face of dynamic loads.

3.2. Determination of Dynamic Initiation Stress Intensity Factor
The experimental-numerical method is an efficient way to realize high accuracy numerical calculation by introducing experimental data to the numerical modeling process[13, 21]. In this paper, ABAQUS was used to calculate the dynamic stress intensity factors (DSIF). A numerical model was established according to the practical dimension of the experimental specimen. The crack tip meshed with quarter-point triangular elements (CPS6), and quadrilateral elements (CPS8) were applied in the rest of the model, shown in Figure 7(b).

In calculation, the stress waves obtained in the experimental tests were used. The DSIF curves drawn by ABAQUS are illustrated in Figure 8. As the crack initiation time had been obtained, the DISIFs could be determined by picking the value of the DSIF at the exact initiation time. It can be seen that the Specimen-0° is the mode I initiation as the value of $K_{II}$ is constant 0 MPa·m$^{1/2}$, and the $K_I$ is 1.72 MPa·m$^{1/2}$. The Specimen-30° is a tensile-dominated initiation with $K_I$ equals 1.52 MPa·m$^{1/2}$, and $K_{II}$ equals -1.03 MPa·m$^{1/2}$. The Specimen-60° is a shear-dominated initiation with $K_I$ equals -1.39 MPa·m$^{1/2}$, and $K_{II}$ equals -0.53 MPa·m$^{1/2}$. Therefore, the initiation mode of the specimens under different dip angles can be distinguished by the values of $K_I$ and $K_{II}$.

![Figure 8. Dynamic stress intensity factor curves by ABAQUS. (a) DSIF curves of Specimen-0°. (b) DSIF curves of Specimen-30°. (c) DSIF curves of Specimen-60°.](image)

4. Mixed-mode Crack Propagation by AUTODYN
Predicting dynamic mixed-mode crack propagation can provide insights into dynamic stability analyses in engineering design. AUTODYN is a good choice to solve multifarious problems characterized by both geometric and material nonlinearity with high precision[22, 23].

4.1. Model in AUTODYN
In the numerical simulation, a linear equation of state was used to describe the brittle materials under dynamic loading. The linear equation of the state is expressed by Equation (2)[24].

$$P = k \cdot \left( \frac{\rho}{\rho_0} - 1 \right)$$  \hspace{1cm} (2)

where $P$ denotes the pressure of the material, $k$ denotes the bulk modulus, $\rho_0$ denotes reference density, and $\rho$ denotes current density.

The principal stress criterion was chosen as the fundamental failure criterion to describe the failure of the mixed-mode crack fracture. When the major tensile principal stress of the elements exceeds the maximum dynamic tensile strength of the material, the element would damage immediately (also applies to shear cases). This failure criterion can be described by Equation (3) below[25].
\[ \sigma_{11} \geq \sigma_T, \quad \tau_{12} \geq \tau_T, \]  

where \( \sigma_{11} \) denotes the tensile principal stress, \( \sigma_T \) denotes the dynamic tensile strength of the material, \( \tau_{12} \) denotes the shear principal stress, and \( \tau_T \) denotes the dynamic shear strength of the material.

Figure 9. Numerical model in AUTODYN.

Noteworthily, meshing is a crucial work affecting calculation efficiency and computation accuracy. A refined meshing strategy was applied in the modeling, as shown in Figure 9. The area around the crack tip and the potential crack extension path was filled with regular hexahedron grids with a length of 0.2 mm. The material parameters measured in laboratory experiments were applied in this simulation directly to reproduce the real physical condition of the specimen.

4.2. Numerical Simulation Results

Figure 10. Comparison of crack propagation paths. (a) Experimental and numerical results of Specimen-0°. (b) Experimental and numerical results of Specimen-30°. (c) Experimental and numerical results of Specimen-60°.

The simulation results are illustrated in Figure 10. The crack paths and the initiation angles agree well with the ones observed in experimental tests. Those mixed-mode characteristics can be accurately featured in the numerical simulation. A perfectly straight line can be seen on the surface of the Specimen-0°, and the deflection points are exhibited along the crack paths of the Specimen-30° and Specimen-60°. Especially, for the Specimen-60°, the mode transformation can be directly captured in Figure 10(c).

5. Conclusion

The mixed-mode crack propagation behaviour under impact loading was investigated in both experimental and numerical aspects. Testing results were analyzed and the finite difference method was
adopted to predict the propagation of mixed-mode cracks. The main conclusions of this study are as follows:

1. The mixed-mode initiated crack would adjust fracture mode continuously in the whole propagation process accompanied with several extending decelerations. The mixed-mode crack always transforms from mixed-mode to pure mode I during the whole crack propagation process.

2. The crack initiation mode and the dynamic initial stress intensity factors have a relationship with the impact loading orientation.

3. The AUTODYN can be extensively applied in numerical predicting rock fracture behaviour and engineering analysis.

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