BARYON DIFFUSION CONSTANT
IN HOT AND DENSE HADRONIC MATTER
BASED ON AN EVENT GENERATOR URASiMA

N. SASAKI, O. MIYAMURA, S. MUROYA* and C. NONAKA

Department of Physics, Hiroshima University,
Higashi-Hiroshima 739-8526, Japan
*Tokuyama Women's College, Tokuyama, 745-8511, Japan

Abstract
We generate the statistical ensembles in equilibrium with fixed temperature and chemical potential by imposing periodic boundary condition to the simulation of URASiMA (Ultra-Relativistic AA collision Simulator based on Multiple Scattering Algorithm). By using the generated ensembles, we investigate the temperature dependence and the chemical potential dependence of the nucleon diffusion constant of a dense and hot hadronic matter.

1 Introduction
Because of the highly non-perturbative property of a hot and dense hadronic state, the thermodynamical properties and transport coefficients has been hardly investigated. In this paper, we evaluate the transport coefficients by using statistical ensembles generated by Ultra-Relativistic A-A collision simulator based on Multiple Scattering Algorithm (URASiMA). Originally, URASiMA is an event generator for the nuclear collision experiments based on the Multi-Chain Model (MCM) of the hadrons. Some of us (N. S. and O. M.) has already discussed thermodynamical properties of a hot-dense hadronic state based on a molecular dynamical simulations of URASiMA with periodic condition. We improve URASiMA to recover detailed balance at temperature below two hundred MeV. As a result, Hagedorn-type behavior in the temperature disappears. This is the first calculation of the transport coefficient of a hot and dense hadronic matter based on an event generator.

2 URASiMA for Statistical Ensembles
In order to obtain equilibrium state, we put the system in a box and impose a periodic condition to URASiMA as the space-like boundary condition. Initial
distributions of particles are given by uniform random distribution of baryons in a phase space. Total energy and baryon number in the box are fixed at initial time and conserved through-out simulation. Running URASIMA many times with the same total energy and total baryons in the box and taking the stationary configuration later than \( t = 150 \, \text{fm/c} \), we obtain statistical ensemble with fixed temperature and fixed baryon number (chemical potential). By using the ensembles obtained through above mentioned manner, we can evaluate thermodynamical quantities and equation of states \([5]\).

3 Transport Coefficients

According to the Kubo’s Linear Response Theory, the correlation of the currents stands for the admittance of the system (first fluctuation dissipation theorem) and equivalently, random-force correlation gives the impedance (Second fluctuation dissipation theorem) \([6]\). As the simplest example, we here focus our discussion to the diffusion constant. First fluctuation dissipation theorem tells us that diffusion constant \( D \) is given by current (velocity) correlation,

\[
D = \frac{1}{3} \int_0^\infty < v(t) \cdot v(t + t') > \, dt'.
\]

(1)

Average \(< \cdots >\) is given by,

\[
< \cdots > = \frac{1}{\text{number of ensembles}} \sum_{\text{ensemble}} \frac{1}{\text{number of particle}} \sum_{\text{particle}} \cdots .
\]

(2)

Figure 1 shows correlation function of the velocity of baryons. The figure indicates that exponential damping is very good approximation. In the case that the correlation decrease exponentially, i.e.,

\[
< v(t) \cdot v(t + t') > \propto \exp\left(-\frac{t'}{\tau}\right),
\]

(3)

with \( \tau \) being relaxation time, diffusion constant can be rewritten in the simple form,

\[
D = \frac{1}{3} < v(t) \cdot v(t) > \, \tau.
\]

(4)
Fig. 1. Velocity correlation of the baryons as a function of time. Lines correspond to the fitted results by exponential function. Normalizations of the data are arbitrary.

Usually, diffusion equation is given as,

$$\frac{\partial}{\partial t} f(t, x) = D \nabla^2 f(t, x), \quad (5)$$

and diffusion constant $D$ has dimension of $[L^2/T]$. Because of relativistic nature of our system, we should use $\beta = \frac{v}{c} = \frac{p}{E}$ instead of $v$ in eq.(1) and $D$ is obtained through,

$$D = \frac{1}{3} \int_0^\infty < \beta(t) \cdot \beta(t + t') > dt' c^2. \quad (6)$$

$$= \frac{1}{3} < \beta(t) \cdot \beta(t) > c^2 \tau. \quad (7)$$

$$= \frac{1}{3} < \left( \frac{p(t)}{E(t)} \right) \cdot \left( \frac{p(t)}{E(t)} \right) > c^2 \tau \quad (8)$$
with \( c \) being the velocity of light. Figure 2 displays the our results of baryon diffusion constant in a hot and dense hadronic matter.

![Graph showing diffusion constant vs temperature](image)

**Fig. 2.** Diffusion constant of baryons.

Our results shows clearer dependence on the baryon number density while dependence on energy density is mild. This results means importance of baryon-baryon collision process for the random walk of the baryons and thus non-linear diffusion process of baryons occurs. In this sense, we can state that baryon number density in our system is still high. In the inhomogeneous big-bang nucleosynthesis scenario, baryon-diffusion play an important roll. The leading part of the scenario is played by the difference between proton diffusion and neutron diffusion\[7\]. In our simulation, strong interaction dominates the system and we assume charge independence in the strong interaction, hence, we can not discuss difference between proton and neutron. However obtained diffusion constant of baryon in our simulation can give some kind of restriction to the diffusion constants of both proton and neutron.

Because fundamental system in URASiMA is high energy hadronic collisions, we use relativistic notations usually. However, diffusion equation \((5)\) is not Lorentz’s covariant and is available only on the special system i.e. local rest frame of the thermal medium. For the full-relativistic description of the space-time evolution of a hot and dense matter, we need to establish relativistic Navier-Stokes equation\[8\]. And taking correlation of appropriate currents, we can easily evaluate viscosities and heat conductivity in the same manner \[9\].
4 Concluding Remarks

Making use of statistical ensembles obtained by an event generator URASiMA, we evaluate diffusion constants of baryons in the hot and dense hadronic matter. Our results show strong dependence on baryon number density and weak dependence on temperature. The temperature in our simulation is limited only small range, i.e., from 100 MeV to 200 MeV, and this fact can be one of the reasons why the change of diffusion constant of temperature is not clear. Strong baryon number density dependence indicates that, for the baryon diffusion process, baryon plays more important roll than light mesons. In this sense our simulation corresponds to high density region and non-linear diffusion process occurs. Calculation of the diffusion constants is the simplest examples of first fluctuation dissipation theorem. In principle, taking correlation of appropriate currents, i.e. energy flow, baryon number current, stress-tensor, etc., we can evaluate any kinds of transport coefficients.

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