Cosmological constraints on dark matter particle production rate

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Abstract

Gravitational particle production has been investigated by using Einstein’s gravitational field equations in the presence of a cosmological constant. To study the mechanism of particle creation, the Universe has been considered as a thermodynamics system and non-equilibrium thermodynamics has been employed. In order to estimate the cosmological parameters with observational data, including SNe Ia, BAO, Planck 2015 and HST, we have chosen a phenomenological approach for the rate of particle creation. A non-zero particle production rate was obtained implying that the possibility of the particle production is consistent with recent cosmological observations. In the 1σ confidence interval, the ratio of $\Gamma/3H_0$ was obtained to be $0.0835 \pm 0.0265$.

Keywords: Gravitational particle production, Particle production rate, non-equilibrium thermodynamics, Cosmological constraints.

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## I. INTRODUCTION

The particle creation mechanism in cosmology was first introduced by Schrödinger [1]. He investigated the effects of particle production on the evolution of the Universe by using the microscopic description of the gravitational particle production in an expanding Universe. About three decades later, Parker and others [2–5] argued this idea again based on the quantum field theory in curved space-time with the motivation to find new consequences of the quantum field theory of fundamental particles. Parker combined quantum mechanics with general relativity and concluded that the time variations of the gravitational field lead to the production of the particle. After that, Hawking investigated particle production by black holes and its compatibility with the laws of thermodynamics [6].

In most cosmological models, the perfect fluid is taken into account while the real fluids are dissipative. Therefore, the description of many cosmological phenomena necessitates non-equilibrium or irreversible thermodynamics. Eckart [7] and Landau and Lifschitz [8] pioneered the generalization of irreversible thermodynamics from Newtonian fluid to relativistic fluid and considered the first order deviation from equilibrium. However, the first-order theory suffers from stability and causality problems.

The second-order deviation from equilibrium thermodynamics is considered by [9–12]. Altering the dissipative phenomena into the dynamical variables that have the causal evolution overcome the causality problem, and the evolution equation limits the propagation speed of dissipative perturbations.

On the other hand, Prigogine [13] investigated the open thermodynamic system in the context of cosmology and concluded that although the particle production has not been achieved from Einsteins gravitational field equations, particle production mechanism is consistent with these equations. In 1992, Calvao [11, 14] offered the covariant formulation of the particle production mechanisms.

The rate of produced particles should be determined by the quantum field theory in curved space-time [3]. The exact functional form of the particle production rate is still not available; therefore, cosmologist have adopted the phenomenological approach and fitted it with observational data [15–18].

In this work, our aim is to constrain the dark matter particle production rate with the observational data. In Section II, we apply non-equilibrium thermodynamics on the homo-
geneous and anisotropic background and study the particle production and its corresponding entropy production. The cosmological constraints used to estimate the free parameters of the model are presented in Section III. Eventually, we present the numerical results in Section IV. Discussion and conclusions are presented in Section V.

II. NON-EQUILIBRIUM THERMODYNAMICS AND GRAVITATIONAL PARTICLE PRODUCTION

Let us consider the expanding Universe with the line element including directional scale factors $A(t)$, $B(t)$ and $C(t)$ known as Bianchi type I (BI) [19]

$$ds^2 = dt^2 - A(t)^2dx^2 - B(t)^2dy^2 - C(t)^2dz^2.$$  (1)

The Einstein gravitational field equations in the presence of the cosmological constant are as follows,

$$\frac{\dot{B}}{B} + \frac{\dot{C}}{C} + \frac{\dot{BC}}{BC} = \kappa T^1_1 + \Lambda,$$  (2)

$$\frac{\dot{A}}{A} + \frac{\dot{C}}{C} + \frac{\dot{AC}}{AC} = \kappa T^2_2 + \Lambda,$$  (3)

$$\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{AB}}{AB} = \kappa T^3_3 + \Lambda,$$  (4)

$$\frac{\dot{AB}}{AB} + \frac{\dot{BC}}{BC} + \frac{\dot{AC}}{AC} = \kappa T^0_0 + \Lambda,$$  (5)

where $T^\mu_\nu$ is the energy-momentum tensor and dot means differentiation with respect to the cosmic time $t$. By introducing the time-dependent function $a(t)$ known as the vacuum scale of the BI Universe

$$a = \sqrt{-g} = ABC,$$  (6)

one can write metric functions explicitly. Also by this vacuum scale, the generalized Hubble parameter can be obtained

$$\frac{\dot{a}}{a} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} := 3H.$$  (7)

Equations (2-4) with (7) give

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{X_1}{a},$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{X_2}{a},$$

$$\frac{\dot{AB}}{AB} + \frac{\dot{BC}}{BC} + \frac{\dot{AC}}{AC} = \kappa T^0_0 + \Lambda,$$
where $X_1, X_2, X_3$ are the integration constant. As $a \to \infty$, there is an isotropic expansion in all directions \[20\]. After the equations \[(8)\] are integrated, the explicit expression of the metric functions is obtained as follows, \[21\]

\[
\begin{align*}
A(t) &= Y_1 a^{1/3} \exp \left[ X_1 \frac{\int dt}{a(t)} \right], \\
B(t) &= Y_2 a^{1/3} \exp \left[ X_2 \frac{\int dt}{a(t)} \right], \\
D(t) &= Y_3 a^{1/3} \exp \left[ X_3 \frac{\int dt}{a(t)} \right].
\end{align*}
\] \[(9)\]

Here $Y_i$ and $X_i$ are arbitrary constants that satisfy the following relations:

\[
Y_1 Y_2 Y_3 = 1, \quad X_1 + X_2 + X_3 = 0.
\] \[(10)\]

Finally, a little calculation on the equations \[(2,3)\] gives the evolution equation of $a(t)$ \[21\]

\[
\dot{a}^2 = 3(\kappa \rho + \Lambda) a^2 + C_1.
\] \[(11)\]

Modified energy-momentum tensor for relativistic fluid including particle production is given by the following equation

\[
T_{\mu\nu} = (\rho + p + \Pi)u_\mu u_\nu + (p + \Pi)g_{\mu\nu},
\] \[(12)\]

where $u_\mu$ is the four-vector of velocity such that $u_\mu u^\mu = -1$, $\rho$ and $p$ are energy density and equilibrium pressure, respectively, and $\Pi$ is the pressure associated with the created particle. In a closed thermodynamic system, particle number ($N_\mu = n u^\mu$) is conserved. Laws of energy conservation ($T_{\mu\nu,\nu} = 0$) and particle numbers conservation ($N_{\mu,\mu} = 0$) lead to the following equations

\[
\dot{n} + \theta n = 0,
\] \[(13)\]

\[
\dot{\rho} + \theta (\rho + p + \Pi) = 0,
\] \[(14)\]

where $\theta = u^{\mu,\mu} = (\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}) = 3H$, and $\dot{n} = n_{\mu,\mu} u^\mu$. In \[22\] authors introduce the second-order non-equilibrium thermodynamics with entropy flow vector $S^\mu$

\[
S^\mu = s N^\mu - \frac{\tau \Pi^2}{2 \zeta T} u^\mu,
\] \[(15)\]
where entropy per particle is denoted by $s$, fluid temperature by $T$, relaxation time by $\tau$ and coefficient of bulk viscosity by $\zeta$. Gibbs equation for this system

$$T ds = \left( \frac{\rho}{n} \right) + pd\left( \frac{1}{n} \right),$$

(16)
gives the following relation for variation of the entropy per particle

$$\dot{s} = -\frac{\Pi \theta}{nT}.\quad (17)$$

From the conservation laws (13,14) and (17) it is easy to show that

$$S^{\mu \cdot \mu} = -\frac{\Pi}{T} \left[ \theta + \frac{\tau}{\zeta} \dot{\Pi} + \frac{1}{2} \Pi^2 T \left( \frac{\tau}{\zeta T} \dot{u}^\mu \right) \right].$$

(18)

With the following choice for creation pressure

$$\Pi = -\zeta \left[ \theta + \frac{\tau}{\zeta} \dot{\Pi} + \frac{1}{2} \Pi^2 T \left( \frac{\tau}{\zeta T} \dot{u}^\mu \right) \right],$$

(19)

the second law of thermodynamics is satisfied [23]

$$S^{\mu \cdot \mu} = \frac{\Pi^2}{\zeta T} \geq 0.$$\quad (20)

In open thermodynamic system, the particle number is not preserved ($N^{\mu \cdot \mu} \neq 0$) [24, 25]; thus, equation (13) should be modified as follows,

$$\dot{n} + \theta n = n \Gamma,$$\quad (21)

where $\Gamma$ is the particle production rate. In such a case, the Gibbs equation gives

$$\dot{\rho} + \theta \left( 1 - \frac{\Gamma}{\theta} \right) (\rho + p) = nT \dot{s},$$\quad (22)

which means that to recover the energy conservation equation (14), the entropy per particle must be constant ($\dot{s} = 0$). Gibbs equation with conservation laws (14,21) gives

$$nT \dot{s} = -\Pi \theta - \Gamma (\rho + p),$$\quad (23)

so under the adiabatic condition, the creation pressure is as follows

$$\Pi = -\frac{\Gamma}{\theta} (\rho + p).$$\quad (24)
Therefore, in the adiabatic process, the creation pressure $\Pi$ is linearly related to the particle production rate $\Gamma$. So under the adiabatic condition, dissipative fluid is equivalent to a perfect fluid with variable particle number. On the other hand, from equations (11, 14, 24) it can be deduced that

$$\dot{H} + \frac{3}{2} H^2 (1 + \omega) \left( 1 - \frac{\Gamma}{3H} \right) = 0,$$

which means that regardless of the amount of equation of state, $\Gamma = 3H$ leads to a late time de Sitter ($\dot{H} = 0$).

In an open thermodynamic system, the entropy change ($dS$) in addition to the entropy flow ($d\dot{f}S$) involves the entropy production ($d\dot{p}S$) \[26\]

$$dS = d\dot{f}S + d\dot{p}S,$$

where $d\dot{p}S \geq 0$. In the homogeneous Universe, the entropy flow change is zero and so the entropy change is only due to the entropy production

$$\frac{dS}{dt} = \frac{d\dot{p}S}{dt} = \frac{d(nS)\dot{V}}{dt} = S\dot{\Gamma},$$

After integration, we have

$$S(t) = S_0 \exp \left[ 3 \int_{a_0}^{a} \frac{\Gamma da}{\dot{\theta}/\theta} \right],$$

where $S_0$ and $a_0$ are the present value of entropy and scale factor, respectively.

In the BI Universe dominated by pressureless matter, baryonic matter and the energy of the quantum vacuum, Friedmann equation \[11\] is as follows

$$\frac{H^2(a)}{H_0^2} = \Omega_b a^{-3} + \Omega_c a^{-3} \exp \left( 3 \int_1^{a} \frac{\Gamma da}{\dot{\theta}/\theta} \right) + C_1 a^{-2} + \Omega_\Lambda,$$
only particle production for dark matter is considered here. To go ahead it is necessary to specify the particle production rate $\Gamma$. In fact, $\Gamma$ is determined by studying the irreversible particle production in quantum field theory in curved space-time. The nature of the particles produced affects the particle production rate $\Gamma$ since the nature of the dark matter particles is not yet known, a phenomenological choice for particle production rate seems to be a feasible solution. A general phenomenological choice for particle production rate is $\Gamma = 3\beta H f(a)$ where $f(a)$ is an arbitrary function of the scale factor $a$, and $\beta$ is a non-negative parameter. Following Nunes [27], we work with the phenomenological ansatz as follows,

$$\Gamma = 3\beta H \left[ 5 - 5 \tanh(10 - 12a) \right].$$

\(30\)
III. COSMOLOGICAL CONSTRAINT

In order to place cosmological constraints on the six free parameters of the model (30), including $\Omega_b$, $\Omega_c$, $\Omega_\Lambda$, $\beta$, $C_1$ and $H_0$, we run the CosmoMC package [28, 29], that uses Markov Chain Monte Carlo (MCMC) algorithm to calculate the likelihood of cosmological parameters by using SNe Ia, BAO, Planck 2015 and HST observations. The likelihood function is defined as $\mathcal{L} \propto e^{-\chi^2/2}$, such that $\chi^2$ represents the difference between observation and theory. The total likelihood is obtained by multiplying the separate likelihoods of SNe Ia, CMB, BAO, and HST data; thus, $\chi^2_{\text{tot}} = \chi^2_{\text{SN}} + \chi^2_{\text{CMB}} + \chi^2_{\text{BAO}} + \chi^2_{\text{HST}}$. For more details about cosmological constraint see [30, 31].

A. Type Ia Supernovae (SNe Ia)

Type Ia supernovae have the same absolute magnitude and therefore these standard candles are a powerful tool for exploring the history of the expansion of the Universe. For our purpose, we employ Joint Light-curve Analysis (JLA) dataset, which in total comprises 740 SNe Ia data points in the redshift range $0.01 \leq z \leq 1.3$ [32]. 118 SNe Ia within the redshift range $0 \leq z \leq 0.1$ from a combination of various subsamples [33–38], 374 SNe Ia from Solon Digital Sky Survey (SDSS) within the redshift range $0.3 \leq z \leq 0.4$ [39], 239 SNe Ia from the Supernova Legacy Survey (SNLS) within the redshift range $0.1 \leq z \leq 1.1$ [40], and 9 SNe Ia from Hubble Space Telescope within the redshift range $0.8 \leq z \leq 1.3$ [41] comprise the JLA collection.

B. Baryon Acoustic Oscillations (BAO)

BAO’s standard ruler has provided another tool to probe the expansion history of the Universe. Cosmological perturbations in baryon-photon primordial plasma generate pressure waves that affect anisotropies of the CMB and the large scale structures of matter. The observed peak in the large scale correlation function measured by the luminous red galaxies of Solon Digital Sky Survey (SDSS) at z=0.35 [42] and z=0.278 [43] reveals the baryon acoustic oscillations at 100$h^{-1}$ Mpc as well as in the two-degree Field Galaxy Redshift Survey (2dFGRS) at z=0.2 [44], six-degree Field Galaxy Redshift Survey (6dFGRS) at z=0.106 [45], z=0.44, z=0.60 and z=0.73 by WiggleZ team [46], the SDSS Data Releases 7
main Galaxy sample at $z=0.15$ \cite{47}, the Data Releases 10 and 11 Galaxy samples at $z=0.57$ \cite{48}.

C. Cosmic Microwave Background (CMB)

Acoustic peaks of the temperature power spectrum of the cosmic microwave background radiation provide useful information about the expansion history of the Universe. The physics of decoupling affects the amplitude of the acoustic peaks and the physics of between the present and the decoupling changes the locations of peaks. Further on to probe the entire expansion history up to the last scattering surface, we will include the CMB data from Planck 2015 \cite{49,50} i.e. a joint observation of $low l + TT$ temperature fluctuations angular power spectrum.

D. Hubble Space Telescope (HST)

Another independent constraint that can be applied to the estimation of the model parameters is the Hubble parameter observational data obtained based on different ages of the galaxies \cite{51}. Because the Hubble constant is included in many cosmological and astrophysical calculations, NASA/ESA built the Hubble Space Telescope (HST) to measure precise $H_0$, and one of the three major HST projects was designated measurement $H_0$ with an accuracy of 10%. Freedman and his colleagues have obtained a new high-accuracy calibration of the Hubble constant based on our analysis of the Spitzer data available to date, combined with data from the Hubble Key Project. There was found a value of $H_0 = 74.3$ with a systematic uncertainty of $\pm 2.1$ and a statistical uncertainty of $\pm 1.5 \text{ km s}^{-1} \text{Mpc}^{-1}$ \cite{52}. Riess et al. determined the Hubble constant from optical and infrared observations of over 600 Cepheid variables by using the Wide Field Camera 3 on the Hubble Space Telescope HST. They reported Hubble constant value of $H_0 = 73.8 \pm 2.4 \text{km s}^{-1} \text{Mpc}^{-1}$ including systematic errors \cite{53}. Efstathiou reanalyzed the Riess et al. \cite{53} Cepheid data using the revised geometric maser distance to NGC 4258 of Humphreys et al. \cite{54}. He concluded that $H_0$ based on the NGC 4258 maser distance is $H_0 = 70.6 \pm 3.3 \text{km s}^{-1} \text{Mpc}^{-1}$, compatible within $1\sigma$ with the recent determination from Planck, also assuming that the $H$-band period-luminosity relation is independent of metallicity $H_0 = 72.5 \pm 2.5 \text{km s}^{-1} \text{Mpc}^{-1}$ \cite{55}.
IV. NUMERICAL RESULTS

We have constrained the parameters of the model, using observational data, including SNe Ia, BAO, Planck 2015 and HST. In Table I, the main results of the statistical analysis are summarized. In the $1\sigma$ confidence interval, the best fit of the $\beta$ parameter is obtained $\beta = 0.0085 \pm 0.0027$ for joint analysis SNe Ia + BAO + HST + Planck 2015, which corresponds to $\Gamma/3H_0 = 0.0923^{+0.0304}_{-0.0344}$ and $\Gamma/3H_0 = 0.0835 \pm 0.0265$, respectively.

This result not only confirms the possibility of particle production, but shows that the cosmic scenario involving particle production is consistent with observational data.

Table I: The best fit of the model parameters in $1\sigma$ confidence interval, with Planck TT + low $l$, and SNe Ia + BAO + HST + Planck TT + low $l$.

| Parameter | SNe Ia+BAO+HST +Planck TT + low $l$ | Planck TT+low $l$ |
|-----------|----------------------------------|-----------------|
| $\Omega_b h^2$ | $0.02192 \pm 0.00018$ | $0.02191 \pm 0.00019$ |
| $\Omega_c h^2$ | $0.11789 \pm 0.00096$ | $0.11764 \pm 0.00099$ |
| $C_1$ | $-0.0051 \pm 0.0023$ | $-0.0126^{+0.013}_{-0.0086}$ |
| $\beta$ | $0.0085 \pm 0.0027$ | $0.0094^{+0.0031}_{-0.0035}$ |
| $H_0$ | $67.85 \pm 0.70$ | $65.3 \pm 4.3$ |
| $\Omega_\Lambda$ | $0.6999 \pm 0.0054$ | $0.680^{+0.037}_{-0.028}$ |

For Planck 2015, $\beta = 0.0085 \pm 0.0027$ for joint analysis SNe Ia + BAO + HST + Planck 2015, which corresponds to $\Gamma/3H_0 = 0.0923^{+0.0304}_{-0.0344}$ and $\Gamma/3H_0 = 0.0835 \pm 0.0265$, respectively.

In Figure 4, we have plotted the 1D likelihoods and 2D contours for model parameters.
FIG. 4: 1D likelihoods for each model parameter, and 2D contours for this parameters in 1σ and 2σ confidence intervals, with Planck TT + low l (red), and SNe Ia + BAO + HST + Planck TT + low l (blue).

with Planck TT + low l (red), and SNe Ia + BAO + HST + Planck TT + low l (blue), where contours represent confidence intervals of 68% and 95%.

Figure 5 displays the effective equation of state with the best fit of model parameters along with the effective equation of state of the ΛCDM model. The present value of the effective equation of state for this scenario is obtained $\omega_{\text{eff}}(z = 0) = -0.69$. As the figure shows, at late time, the effective equation of state for this model tends to -1, which corresponds to late time de Sitter. In our study, this cosmic scenario showed a deviation from ΛCDM in the recent past.

Figure 6 shows the deceleration parameter with the best fit of model parameters along with the effective equation of state of the ΛCDM model. The present value of deceleration parameter for this scenario is obtained $q(z = 0) = -0.65$. The transition from deceleration to acceleration era occurred in $z_t = 0.69$. Our analysis of this cosmological model revealed a deviation from the ΛCDM model in the recent past.

In Figure 7 we plotted the evolutionary behaviors of the density parameters of Cold Dark Matter, $\Omega_{CDM} = \frac{8\pi G \rho_{CDM}}{3H^2}$, and vacuum density, $\Omega_{\Lambda} = \frac{8\pi G \rho_{\Lambda}}{3H^2}$. The figure displays the fact that as the density parameter of CDM is decreased, the vacuum density is increased, during
V. CONCLUSIONS

In this paper, we examined the particle creation from the perspective of non-equilibrium thermodynamics in Bianchi type I Universe and tried to explain the evolution of the Universe by choosing a phenomenological approach for the particle production rate.

To constrain the free parameters of the model, we use the CosmoMC package. Our analysis includes observational data such as supernovae type Ia from JLA, cosmic microwave background from Planck 2015, baryon acoustic oscillation from SDSS, 2dFGRS, 6dFGRS, and observational Hubble data from HST. The results are as follows:
FIG. 7: he best fits of the dimensionless density parameters of CDM, $\Omega_{CDM} = \frac{8\pi G \rho_{CDM}}{3H^2} \Lambda$, $\Omega_{\Lambda} = \frac{8\pi G \rho_{\Lambda}}{3H^2}$ using the full data sets.

* Figure 1 shows that entropy production has started to increase near the present time. Figure 2 shows that in early times $\Gamma/3H \ll 1$ and such as the entropy production near the present starting to increase and at late time $\Gamma/3H < 1$.

* In 1σ confidence interval the best fit of the $\beta$ parameter is obtained $\beta = 0.00 \pm 0.00$ for Planck 2015, $\beta = 0.0085 \pm 0.0027$ for Planck 2015 + SNe Ia + BAO + HST. This value of the $\beta$ implies that $\Gamma/3H = 0.0$ and $\Gamma/3H = 0.0835$, respectively. Thus, the possibility of particle production is approved as consistent with recent cosmological observations.

* Contribution of anisotropy in this model entered with $C_1$ obtained for Planck 2015 data is $C_1 = -0.00 \pm 0.00$ and for joint analysis Planck 2015 + SNe Ia + BAO + HST data is $C_1 = -0.0051 \pm 0.0023$.

* In this cosmic scenario the best fit of the cosmological parameters, $\Omega_b h^2 = 0.02192 \pm 0.00018$, $\Omega_c h^2 = 0.11789 \pm 0.00096$, and $\Omega_{\Lambda} = 0.6999 \pm 0.0054$ is consistent with similar values in the $\Lambda CDM$ model. Figure 4 displayed 1D likelihoods and 2D contours for model parameters with Planck TT + low $l$ (red), and joint analysis SNe Ia + BAO + HST + Planck TT + low $l$ (blue), in 1σ and 2σ confidence intervals.

* Effective equation of state for Planck 2015 data is $\omega_{eff}(z = 0) = -0.0$, and for joint analysis Planck 2015 + SNe Ia + BAO + HST data is $\omega_{eff}(z = 0) = -0.69$. Figure 5 shows EoS changes it is clear that at late time, it behaves like the $\Lambda CDM$ model.
* The deceleration parameter $q$ reveals a transition from an early matter-dominant ($q = 0.5$) epoch to the de Sitter era ($q = -1$) at late time, as expected. The present value of the accelerating epoch starts at transition redshift $z_t = 0.69$. The deceleration parameter $q(z = 0) = -0.65$ obtained.

* Evolutionary behaviors of the density parameters of CDM, $\Omega_{CDM} = \frac{8\pi G \rho_{CDM}}{3H^2}$, and vacuum density, $\Omega_\Lambda = \frac{8\pi G \rho_\Lambda}{3H^2}$ plotted in Figure 7 show that as the $\Omega_\Lambda$ is increased, $\Omega_{CDM}$ is decreased, during the history of the Universe.

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