The Most Precise Extra-Galactic Black-Hole Mass Measurement

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ABSTRACT

I use archival data to measure the mass of the central black hole in NGC 4526, \( M = 4.70 \pm 0.14 \times 10^8 M_\odot \). This 3\% error bar is the most precise for an extra-galactic black hole and is close to the precision obtained for Sgr A* in the Milky Way. The factor 7 improvement over the previous measurement is entirely due to correction of a mathematical error, an error that I suggest may be common among astronomers.

Subject headings: methods: statistical — black hole physics

1. Introduction

Davis et al. (2013) have reported a mass measurement of the central black hole in NGC 4526 of \( M = 4.5^{+4.2}_{-3.0} \times 10^8 M_\odot \) (3\sigma) based on interferometric CO emission measurements from its central molecular ring. They rightly regard their measurement as extremely important because it opens the way to mass production of black hole mass measurements, particularly in the era of ALMA.

However, due to a mathematical error, these authors have vastly overstated their measurement errors and so vastly underestimated the power of their technique. In brief, they evaluated their likelihood contours using \( \chi^2/\text{dof} \) rather than \( \chi^2 \) itself, where dof equals “degrees of freedom”.

Here I correct this error, thereby obtaining the most precise mass measurement of an extra-galactic black hole. I argue that the CO technique can be much more efficient than previously realized. I also suggest that the substitution of \( \chi^2/\text{dof} \) for \( \chi^2 \) may be common among astronomers.
2. Data

I used a Xerox\textsuperscript{TM} machine to enlarge Figure 2 from Davis et al. (2013) and measured the horizontal extent of the "1 \(\sigma\)" error ellipse, finding lower and upper boundaries at 3.65 and 5.75 \(\times 10^8\) \(M_\odot\), respectively. I therefore adopt 4.70 \(\pm 1.05 \times 10^8\) \(M_\odot\) as the "1 \(\sigma\)" mass estimate conveyed by this figure. I note that the nearest grid point evaluated by Davis et al. (2013) to my adopted central value is at 4.5 \(\times 10^8\) \(M_\odot\), which is indeed the grid point with the lowest \(\chi^2\) reported by those authors.

3. Analysis

The contours of Figure 2 of Davis et al. (2013) reflect \(\chi^2/dof\). To recover \(\chi^2\), one should multiply by \(dof\). There are 68 data points and two parameters, hence \(dof = 68 - 2 = 66\). However, Davis et al. (2013) report \(\chi^2_{\text{min}} = 86.36\) and \(\chi^2_{\text{min}}/dof = 1.27\). Hence, they are in fact computing \(\chi^2/dof = \chi^2/68\). We should therefore multiply by 68 rather than 66.

Before proceeding, we must evaluate \(\chi^2\) at the actual minimum of the error ellipse rather than the nearest grid point. While the nearest grid point is extremely close to the error-ellipse axis in the ordinate direction, it is displaced from the center by \((4.70 - 4.50)/1.05 = 0.19\) "\(\sigma\)" in the abscissa direction. Hence, at the ellipse center, the quantity that Davis et al. (2013) are calling "\(\chi^2/dof\)" should be lower by 0.19\(^2 = 0.036\), implying \(\chi^2_{\text{min}} = 83.91\).

Before evaluating the true error bar, we must take note of the fact that \(\chi^2_{\text{min}}/dof = 83.91/66 = 1.27\) is greater than unity. That is, the expected (1 \(\sigma\)) range of this parameter is \(1\pm(2/dof)^{1/2} = 1\pm0.174\), so that the actual value is too high by 1.6\(\sigma\). There are three possible reasons for this high value: 1) normal statistical fluctuations, 2) underestimated errors, 3) inadequacy of the model. To be conservative, I adopt explanation (2) and assume that the measurement errors have been slightly underestimated, i.e., by a factor \(\sqrt{1.27} = 1.127\).

If there were no such correction, then the true error would be smaller than the reported one by \(\sqrt{68}\), because Davis et al. (2013) divided their \(\chi^2\) values by 68. However, taking account of the slightly underestimated errors, the true correction factor is \(\sqrt{68/1.27} = 7.32\). Therefore, I finally derive a mass estimate for the central black hole of NGC 4526 of

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M = 4.70 \pm 0.14 \times 10^8\ M_\odot. \tag{1}
\]
4. Discussion

4.1. Most Precise Mass Measurement

The first point to note is that the statistical error bar in Equation (1) is only 3%. This is not quite at the level the statistical errors for the Milky Way’s own central black hole $4.4 \times 10^6 M_\odot$ (Ghez et al. 2013) or $4.28 \pm 0.07 \times 10^6 M_\odot$ (Gillessen et al. 2009) at fixed $R_0 = 8.3$ kpc. However, it is more precise than any extra-galactic black hole measurement. For example, the spectacular nearby mega-maser NGC 4258 yields only a 6% statistical-error black hole mass measurement $3.3 \pm 0.2 \times 10^7 M_\odot$ at fixed distance 7.28 Mpc (Stopis et al. 2009).

4.2. Precision vs. Accuracy

Normally in astronomy, one is more concerned about accuracy than precision, both in the sense that one is finally interested in how close the reported measurements are to the true quantities and in the sense that estimating accuracy usually requires quite a bit more effort than estimating precision.

In the present case, the mass estimate scales with the assumed distance of NGC 4526, which is fixed at 16.4 Mpc in the analysis of Davis et al. (2013), who cite the surface brightness fluctuation distance measurement by Tonry et al. (2001). The latter work quotes a 10% distance error (and a distance of 16.9 Mpc). In principle, one might hope to do better, but even for galaxies in the Hubble flow, there is an error floor due to uncertainty in the Hubble constant. In addition, there are many other sources of systematic error, such as assumptions within the modeling about the geometrical structure of both the CO gas and the stars. In brief, there is no immediate hope of achieving accuracies at the level of this 3% precision.

Nevertheless, I would argue that precision is indeed the key parameter of interest. The point is that Davis et al. (2013) could have achieved 10% precision in only about 1/10 of their observing time. While they do not specify exactly how much observing time they used, they do say that similar precision could have been achieved by 5 hrs of ALMA integration for an object at 75 Mpc.

Now, 5 hrs of ALMA observing time cannot simply be scaled to 30 minutes at a cost of a factor 3 in precision because ALMA relies on the rotation of the Earth to fill in the UV plane. However, with some ingenuity, one could organize observations of, say, six objects, and sequentially rotate through them over 5 hours. That is, a vast program of precision black-hole mass measurement could be undertaken at relatively little cost.
4.3. $\chi^2$ vs. $\chi^2$/dof

This is the second paper in as many months for which I noticed that the authors calculated confidence contours based on changes in $\chi^2$/dof rather than $\chi^2$. Given that it is not often that I read a paper in which there is sufficient information to determine which parameter was used, I think that this error might be quite common. More striking than the errors themselves is the fact that I am the only one who seems to notice them. The two papers had between them 15 co-authors and at least 4 referees. In both cases, the effect of the error was to radically degrade the precision of the reported measurement and so should have jumped out to anyone with a stake in the paper or its results.

For the record, for Gaussian-distributed errors (which, from the central limit theorem, usually applies to the case of many dof), likelihood scales as $L \sim \exp(-\Delta \chi^2/2)$ independent of the number of dof. It is therefore from $\chi^2$ itself that one calculates the relative likelihood of models.

On the other hand, $\chi^2$/dof is useful primarily to check the quality of the data and the completeness of the model space. If the model space covers the system being modeled, and the error bars are correctly estimated, then $\chi^2$/dof should be about unity. Strictly speaking, $\chi^2$ will under these conditions be distributed as a “$\chi^2$ distribution”, which has a mean and standard deviation of $\pm \sqrt{2\text{dof}}$. Hence, if the observed value of $\chi^2$ lies well outside these bounds, it demonstrates that either 1) the errors have been misestimated and/or 2) the model space is inadequate. See Gould (2003) for a more detailed discussion.

I thank Chris Kochanek for helpful discussions. This work was supported by NSF grant AST 1103471.

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