Abel inversion of asymmetric plasma density profile at Aditya Tokamak

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Abstract. In Aditya tokamak, at Institute for Plasma Research, till now, multi-channel microwave interferometer system is used to measure the chord averaged plasma density at predefined radial position. An inversion code is developed to determine the local density profile from the chord average density measurement of radially asymmetric plasma. The radial density profile is interpolated using Spline interpolation analytical technique for symmetric plasma density profile. Code implements the Slice and Stack method to determine localized density from asymmetric averaged plasma density measurement from interferometer. Inverted results are tested with various monotonically varying asymmetric radial density profiles of the plasma shots. It also provides the poloidal picture of plasma density distribution with circular constant density surfaces. Localized density measurements, which is very important for successful operation of tokamak, is in agreement with observation of other diagnostics.

1. Introduction:

Multi cord microwave interferometer is very efficient, non-invasive plasma diagnostics system on fusion devices like Tokamak. Seven channel fixed frequency microwave interferometer is designed and installed on Aditya Tokamak at Institute for Plasma Research. It probes the plasma in O-mode with 100GHz fixed frequency Gunn oscillator and homodyne detection technique Figure 1. [1]. It provides the change in the phase because of change in plasma density over complete viewing chord.

The change in phase can be can be written as

$$\phi(x) = \left[ \frac{2\pi}{n_c} \int_0^{n_c} \int_0^\infty \frac{e(r)}{\lambda} dy \right] \left[ \frac{2\pi}{n_c} \int_0^{n_c} \int_0^\infty \frac{e(r)}{\lambda} dy \right]$$

Where

- $n_e$ is electron density.
- $n_c$ is cutoff density for probing frequency.
- $\lambda$ is probing wavelength.
Interferometer measures the line averaged plasma density. To understand the particle dynamics of various plasma phenomena like micro-instabilities, measurement of local plasma density is essential for accurate understanding. Abel inversion technique gives liberty to determine local plasma density from line averaged interferometer measurement.

2. Abel Inversion Techniques:
To extract the localized plasma density, various techniques are been proposed like analytical and slice & stack method etc. [1, 2, 3].

2.1. Analytical Methods.
The analytical technique uses the spline interpolation to determine the plasma density radial profile. It considers the plasma to be symmetric about the axis.
The phase shift due to the column can be expressed as [2]

\[ \phi(x) = \left[ \frac{2\pi}{n c \lambda} \right] \int_{y_0}^{y_0} n_e(r) dy = \left[ \frac{2\pi}{n c \lambda} \right] \int_0^a \left( \frac{n_e(r)}{x_0 \sqrt{r^2 - x^2}} \right) r dr \]  

(1)

Abel inversion of above equation is given by

\[ n_e(r) = -\frac{\lambda n c a}{\pi^2} \int_r^\infty \frac{\phi'(x)}{\sqrt{x^2 - r^2}} \]  

(2)

The plasma density profile can be interpolated using spline interpolation as

\[ \phi_i(x) = A_i - B_i(x - x_i) + C_i(x - x_i)^2 + D_i(x - x_i)^3 \]  

(3)

And

\[ \phi_i'(x) = \alpha_i - \beta_i x + \gamma_i x^2 + \delta_i x^3 \]  

(4)

Where

\[ \alpha_i = A_i - B_i x_i + C_i x_i^2 + D_i x_i^3 \]
\[ \beta_i = B_i + 2C_i x_i + 3D_i x_i^2 \]
\[ \gamma_i = C_i + 2D_i x_i \]
\[ \delta_i = D_i \]  

(5)

Substituting these values in equation (2)
\[ a \int_{\phi(x)} \frac{\phi'(x)}{r \sqrt{x^2 - r^2}} dx = \frac{a}{r} \int \frac{1}{\sqrt{x^2 - r^2}} dx + 2 \gamma_i \int \frac{x_i}{r \sqrt{x^2 - r^2}} dx + 3 \delta_i \int \frac{x_i^2}{r \sqrt{x^2 - r^2}} dx \]  

Terms in above equation can be solved using standard integration.

\[ a \int \frac{1}{r \sqrt{x^2 - r^2}} dx = \text{cosh}^{-1} \left[ \frac{x}{r} \right] \Bigg|_a^a = \log \left[ \frac{x + \sqrt{x^2 - r^2}}{r} \right] \Bigg|_a^a \]  

\[ a \int \frac{x}{r \sqrt{x^2 - r^2}} dx = \sqrt{x^2 - r^2} \]  

\[ a \int \frac{x^2}{r \sqrt{x^2 - r^2}} dx = \frac{x}{2} \sqrt{x^2 - r^2} + x^2 \ln \left[ \frac{x + \sqrt{x^2 - r^2}}{r} \right] \Bigg|_a^a \]  

Substituting these values in (6), \( n_e(r) \) can be determined.

2.1.1. **Limitation:** The analytical method assumes the plasma density to be symmetrical at minor radius around the cylindrical axis. To consider the asymmetry in plasma density profile additional term has to be added in equation which will make equation very complicated.

2.1.2 **Advantages:** This method is very easy to implement on circularly symmetric plasma and produce very accurate results.

2.2 Slice & Stack method:

During actual tokamak operation, even during simplest ohmic heating plasma, plasma continuously moves about its ideal center position and condition of circular symmetry gets frequently violated. Hence application of analytical method on asymmetric plasma will generate erroneous results. Using the slice and stack method, plasma asymmetry can be taken in consideration and results in better accuracy can be obtained.

In Slice & Stack method, the plasma region is divided in small constant density zones as shown in Figure 3.
If the flux surface geometry is known by any means, then that may be considered, else here circular flux surfaces are considered. The assumption is that, the density remains constant over particular zone.

As shown in the Figure 3. If we know the position of chord, length segment of each chord in different constant density zone can be calculate. For k-zones and j-channel, we get the length matrix \( L_{k \times j} \), for k number of channels we get square matrix of \( L_{k \times k} \). The number of constant density regions, \( n_e \), can be varied and the number of chord will be double \( 2n \). In Figure 3, plasma is divide in only 9 zones for clarity of concept. In actual calculation plasma is divided in zones more than 30 zones.

Consider only either side from center chord including center channel, density equation can be given by [3]

\[
[\phi(x)]_k = \left[ \frac{2\pi}{nc \lambda} \right] \times [n_e(r)]_1 \times k \times \left[ L_{k \times k} \right] \text{ Where } l=k
\]

(10)

As \([L]_{k \times k}\) is square matrix, the local density can be determined as

\[
[n_e(r)]_1 \times k = [\phi(x)]_k \times \left[ L_{k \times k} \right]^{-1} \times \left[ \frac{2\pi}{nc \lambda} \right]
\]

(11)

Here \( n_e(r)_k \) is the constant in density zone \( k \).

To improve accuracy of measurement, the chord density profile is interpolated using various interpolation techniques, like spline and polynomial fitting, and used artificial chord. The local density profile using various techniques is compared in Figure 4.
2.2.1 Consideration of asymmetry: The new able inversion code is developed to address the symmetric as well as asymmetric plasma density profile using Slice & Stack method. The edges of the equal density surfaces are determined layer by layer. Circular density zones are considered. Then the length matrices are generated layer by layer. Various interpolation techniques are used to improve the resolution.

Here the plasma over all is considered circular, which is basically limited by circular limiter at 25cm minor radius in Aditya tokamak. Inside the plasma, density peak can be situated at any location and it is not pre-assumed. This consideration gives the freedom and flexibility to handle any asymmetry. SOL consideration is taken in account with density from probe measurement as boundary condition.
As shown in figure 3, the density of any zone can be written as, [3]

\[
    n_k = \left[ n_{tot} - \sum_{l=1}^{k-1} n_l * L_{lk} \right] / L_{kk}
\]

(12)

Where \( n_{tot} \) is the chord density measurement from interferometer.

Density for particular location is determined at location inside from any one edge, say left. Then code search for location on another side, say right, where the localized density comes to be same. This is how the equal density zones are generated step by step from outside towards inside. The number of chord is automatically determined depending on step length to determine the next density surface.

This technique is tested for various kinds of asymmetries in density profiles which are shown in Figure 5.
2.2.2 Advantages: This method does not presume any peak location. Equal density zones are not predefined depending on interferometer peak. Chords are not forced to pass through center of predefined density zones\[4\]. It calculates all chord location depending on actual interferometer system measurements. Hence this logic has very large degree of freedom. It can be applied to almost any asymmetric non-hollow profile. And processed data is much closely dependent on actual experimental measurements.

2.2.3 Assessment: Localized density data was not available from any other system with which this measurement can be absolutely compared. But with this method, we were successfully able to presume the overlapping of cutoff layer surface for reflectometer, which agrees well with correlation reflectometer measurement on Aditya tokamak\[5\].

2.2.3 Limitations: The assumption of circular constant density zones becomes invalid when the density profile gets hollow. In this case different density profile should be implemented with
discontinuous surfaces. Consideration of circular density profile is not always valid, actual density surface data can be implemented if it is known through another technique.

3. Conclusion:
Numerical code is generated for inversion of Symmetric and Asymmetric density profile of plasma is on Aditya Tokamak. Code is tested over various simulated as well as on actual asymmetric plasma density profiles. Code can include various interpolation methods like spline interpolation as well as polynomial fitting. The position of density zones are not presumed, which gives high degree of freedom and the inverted density profile is influenced only by actual measurement and decreased the dependence on assumptions of zone locations. The inverted profile is well in agreement with observations of other diagnostics.

4. Importance & Future work:
Localized density measurement is essential to understand various plasma dynamics phenomenon. This technique provides much accurate local density determination with asymmetric density profiles and hence will be very useful in real-time control of nuclear fusion machines. Generation of micro and macro instabilities can be presumed with real-time localized density measurement and evolution of steep density gradient inside the plasma. These measurements can be use as real time feedback. Hence the instabilities can be effectively suppressed during its growth before it disrupts the plasma, or even before it is generated, which is essential for continuous successful operation of nuclear fusion machines.

To improve the performance further, the shape of constant density flux surfaces has to be studied more rigorously. Quadrature phase detection technique should be implemented for more accurate phase detection.

5. References:
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