Analysis of the current flow heterogeneity in the electrode-skin structure in electromyography and electrical stimulation systems

V A Bakhtina¹, A A Levitskiy¹, P S Marinushkin¹, M V Abroskina² and A A Ilinskaya²

¹School of Engineering Physics and Radio Electronics, Siberian Federal University, Krasnoyarsk, Russia
²The Department of Nervous Diseases with a Course of Medical Rehabilitation in Postgraduate Education, Prof. V. F. Voino-Yasenetsky Krasnoyarsk State Medical University, Krasnoyarsk, Russia

sl_507@mail.ru

Abstract. The conductivity of the skin and muscle tissue has a significant effect on the magnitude and distribution of the current in the electrode – skin structure. Addressing this issue this paper presents an analysis of the process of electric current flow using an analytical model in the electrode-skin structure for the electromyography system and functional electromyostimulation. As a result, an expression was established for the potential distribution at the electrode-skin interface and the total contact resistance of the structure was determined.

1. Introduction
In electromyography (EMG) and functional electromyostimulation (FES) systems, the electrical properties of the electrodes and skin play an important role in establishing a connection with the subcutaneous tissues of patients. The current distribution under the electrodes affects the patient's feelings, which can vary from tingling to itching or even painful sensations depending on the degree of receptor activation [1]. The control of the current density distribution under the electrodes is also important for EMG and FES technologies which use miniature electrodes combined into arrays allowing to form the required current density profile below them [2]. Experimental study of the nature of the flow of current in the electrode-skin system is very difficult. The simulation provides the ability to analyze the influence of the properties of the electrodes and the skin on the characteristics of the system and the process of current distribution during FES [1-3].

2. The solution of the boundary problem, describing the current flow in a metal-to-skin planar contact
The quantity and nature of the current distribution in the "electrode - conductive gel - skin - muscle tissue" structure is determined by its parameters. The electrical resistance of the human body consists of the resistance of the skin and the resistance of internal tissues. The skin, bones, adipose tissue, tendons and cartilage have relatively high resistance, while muscle tissue, blood, lymph have low resistance. The upper layer of the skin (epidermis) having a thickness of up to 0.1... 0.2 mm and
consisting mainly of dead cells, has a high resistance, which mainly determines the overall resistance of the human body. The resistance of the lower layers of the skin and the underlying internal tissues is insignificant. So, for example, the specific resistance of dry skin can be from $3 \cdot 10^3$ to $2 \cdot 10^4$ Ohm·m, and blood – about 1 - 2 $\Omega$·m, so that the resistance of a human body with dry clean and intact skin can be between 2 k$\Omega$ to 2 M$\Omega$. In this regard, when considering the mechanism of current distribution in the skin below the surface of the electrodes, the top layer of skin with high resistance is taken into account.

We will build a model describing the processes in the electrode-skin structure using the transmission line method (TLM) similarly to [4-8]. To determine the effect of electrode size on characteristics, it is necessary to analyze the mechanism of current flow in the electrode-skin system. To do this, consider the contact area shown in Fig. 1 where epithelium and conductive layers are indicated, wherein the latter can be attributed not only to the metal of the electrode but also to a special conductive gel.

![Figure 1. Model of the electrode-skin structure. Due to symmetry, we will build a mathematical model for the “right” part of the electrode relative to the connection point. For the left side everything will be mirrored.](image)

When considering the contact area, the following assumptions are used:

a) the length of the contact area $L$ is much greater than the thickness of the layers of metal $d_m$ and skin (epithelium) $d_s$. Assuming that the transverse current distribution is uniform, we will use a one-dimensional model;

b) the metal-skin interface is characterized by the specific contact resistance $\rho_s$ [$\Omega$·m$^2$] and its thickness is much less than the values of $d_m$ and $d_s$;

c) the voltage of the power source $U_0$ is applied to the edges of the metal electrode and the skin layer as far as possible from each other - in the sections $z = 0$ and $z = L$, respectively;

d) the boundary conditions in the model under consideration are taken as:

$$\begin{align}
I_m(z=0) &= I_0, & I_m(z=L) &= 0, \\
I_s(z=0) &= 0, & I_s(z=L) &= I_0,
\end{align}$$

where $I_m$ and $I_s$ are the currents in the metal and skin layer respectively.

Dependencies for currents flowing in the metal layer and in the skin layer from the $z$ coordinate can be expressed as follows (see. Fig. 1):

$$\begin{align}
I_m(z) &= \frac{W d_m}{\rho_m} \frac{dU_m(z)}{dz}, & I_s(z) &= \frac{W d_s}{\rho_s} \frac{dU_s(z)}{dz},
\end{align}$$

where $W$ is the width of the contact area, $\rho_m$ and $\rho_s$ are the specific resistances of the metallic and skin layers respectively, $dU_m$ and $dU_s$ are the voltage drop across the elementary area in the layers of metal and skin respectively.
Due to the fact that the current “branches off” from the metal into the skin in the dz section of the contact area z, the current in the metal layer decreases by dl (where dl is the current flowing through the electrode-skin interface in the dz section), and the current in the epithelial layer is increased by the same amount. Therefore, it is legitimate to write the current balance ratios in the form:

\[
\begin{align*}
I_m(z + dz) - I_m(z) &= dl, \\
I_s(z + dz) - I_s(z) &= dl.
\end{align*}
\]

The current dl flowing through the metal-skin interface in the section z under the action of the potential difference \(U_e(z) = U_s(z) - U_m(z)\) is determined by the relations:

\[
dl(z) = \frac{U_e(z)}{\rho_c} dz \quad \text{or} \quad \frac{dl_e(z)}{dz} = \frac{U_e(z)}{\rho_c}.
\]

To find the currents flowing in the metal layer and in the skin layer in the section (z + dz) we write the equations obtained by decomposing the Taylor series of the expression (2) keeping the first two terms of the series:

\[
\begin{align*}
I_m(z + dz) &\approx \frac{W_d}{\rho_m} \left[ dU_m(z) + \frac{d^2U_m(z)}{dz^2} dz \right], \\
I_s(z + dz) &\approx \frac{W_d}{\rho_s} \left[ dU_s(z) + \frac{d^2U_s(z)}{dz^2} dz \right].
\end{align*}
\]

Replacing in equations (5) the terms \(dU_m/dz\) and \(dU_s/dz\) with the help of (2) and simplifying the result obtained taking into account (3) and (4) we get:

\[
\frac{d^2U_m(z)}{dz^2} = -\frac{\rho_m}{d_m} U_e; \quad \frac{d^2U_s(z)}{dz^2} = \frac{\rho_s U_e}{d_s}.
\]

From the system (6), taking into account \(U_e(z) = U_s(z) - U_m(z)\), it follows:

\[
\frac{d^2U_e(z)}{dz^2} = \xi^2 U_e;
\]

where

\[
\xi^2 = \frac{1}{\rho_c} \left( \frac{\rho_m}{d_m} + \frac{\rho_s}{d_s} \right).
\]

Solution of the (7) has the following form:

\[
U_e(z) = C_1 e^{\xi z} + C_2 e^{-\xi z},
\]

where the coefficients \(C_1\) and \(C_2\) are found from the boundary conditions (1) taking into account the relations for \(I_m(z)\) and \(I_s(z)\). The first derivative of \(dU_e/dz\) is determined from (9):

\[
\frac{dU_e}{dz} = \xi \left( C_1 e^{\xi z} - C_2 e^{-\xi z} \right).\]

For the boundary condition at \(z = 0\) according to (1) and (2):

\[
\frac{dU_m}{dz} = 0, \quad \frac{dU_m}{dz} = \frac{I_m \rho_m}{W_d}.
\]
For the boundary condition at \( z = L \) in accordance with (1) and (2):

\[
\frac{dU_m}{dz} = 0, \quad \frac{dU_s}{dz} = \frac{I_0 \rho_m}{Wd_s}.
\]  

(12)

Substituting (11) into (10) we obtain:

\[
- \frac{I_0 \rho_m}{Wd_m} = \xi (C_1 - C_2).
\]  

(13)

From (10) and (12) it follows:

\[
\frac{I_0 \rho_m}{Wd_s} = \xi \left( C_1 e^{\alpha z} - C_2 e^{-\alpha z} \right).
\]  

(14)

Using the relations (13) and (14) we find:

\[
C_1 = \frac{I_0}{Wd_s} \frac{\rho_m}{d_m} \frac{d_s}{d_m} e^{\alpha z} - e^{-\alpha z}, \quad C_2 = \frac{I_0}{Wd_s} \frac{\rho_m}{d_m} e^{\alpha z} - e^{-\alpha z}.
\]  

(15)

From (9) and (15) at \( z = L \), we have:

\[
U_s (L) = \frac{I_0}{Wd_s} \left[ \frac{\rho_m}{d_m \text{sh}(\xi L)} + \frac{\rho_s}{d_s \text{th}(\xi L)} \right].
\]  

(16)

Next, we find the current distribution in the skin layer under the electrode metallization layer. From equation (2) it follows:

\[
\frac{dU_s}{dz} = \frac{\rho_s}{Wd_s} I_s(z), \quad \frac{d}{dz} \left( \frac{dU_s}{dz} \right) = \frac{\rho_s}{Wd_s} \frac{dI_s}{dz} = \frac{\rho_s}{d_s \rho_c} U_c,
\]  

(17)

whence taking into account (6):

\[
\frac{dI_s}{dz} = \frac{W}{\rho_c} U_c.
\]  

(18)

Integrating (18) from 0 to \( z \) taking into account the boundary condition (1) \( I_s(0) = 0 \) and (9), we find:

\[
I_s(z) = \frac{W}{\rho_c} \int_0^z U_c dz = \frac{W}{\rho_c \xi} \left[ (C_1 e^{\xi z} - C_2 e^{-\xi z}) - (C_1 - C_2) \right].
\]  

(19)

The potential distribution \( U_s(z) \) in the epithelium layer under the electrode can be determined using equation (2):

\[
\frac{dU_s}{dz} = \frac{\rho_s}{Wd_s} I_s(z).
\]  

(20)

Integrating (20) over \( z \) from 0 to \( z \), using the boundary condition \( U_s(0) = 0 \) and the dependence of \( I_s(z) \) (18):

\[
\int_0^z \frac{dU_s}{dz} dz = \frac{\rho_s}{Wd_s} \int_0^z I_s(z) dz = \frac{\rho_s}{d_s \rho_c \xi} \int_0^z [C_1 e^{\xi z} - C_2 e^{-\xi z}) - (C_1 - C_2)] dz = \frac{\rho_s}{d_s \rho_c \xi} \left[ U_s(z) - (C_1 - C_2) \xi z - (C_1 + C_2) \right].
\]  

(21)
From here we find \( U_s(z) = 0 \) for \( z = L \):

\[
U_s(L) = \frac{\rho_c}{d_s} \frac{1}{\rho_c \xi^2} [U_c(L) - (C_1 - C_2) \xi L - (C_1 + C_2)],
\]

or

\[
U_s(L) = I_0 \frac{\rho_c}{d_s} \frac{L^3}{W \rho_c} \frac{1}{\xi^2 L^2} \left[ \frac{\rho_m}{d_m} + \left( \frac{\rho_m}{d_m} - \rho_m \right) \frac{\text{ch}(\xi L) - 1}{\xi L \text{sh}(\xi L)} \right].
\]

We introduce the following notation: \( r_m = \frac{\rho_m}{d_m} \) \((L/W)\) is the bulk resistance of the metal layer, \( r_s = \frac{\rho_s}{d_s} \) \((L/W)\) is the bulk resistance of the skin layer, \( r_c = \frac{\rho_c}{L W} \) is the surface resistance of the boundary metal-leather section, as well as taking into account (8):

\[
F(\xi L) = \frac{\text{ch}(\xi L) - 1}{\xi L \text{sh}(\xi L)}, \quad \xi L = \left( \frac{r_m + r_s}{r_c} \right)^{1/2}.
\]

With these designations (23) takes the form:

\[
U_s(L) = I_0 \frac{r_s}{r_c + r_m} \left[ r_m + (r_c - r_m) F(\xi L) \right] = I_0 r_s \frac{F(\xi L) + (r_m / r_s) [1 - F(\xi L)]}{1 + r_m / r_s}.
\]

From (25) it is possible to find the resistance of the skin layer under the electrode metallization layer \( R_s = U_s(L)/I_s(L) \) with regard to \( I_s(L) = I_0 \):

\[
R_s = \frac{U_s(L)}{I_s(L)} = I_0 \frac{r_s}{r_c + r_m} \left[ r_m + (r_c - r_m) F(\xi L) \right] = I_0 r_s \frac{F(\xi L) + (r_m / r_s) [1 - F(\xi L)]}{1 + r_m / r_s}.
\]

Under the condition \( \rho_m / d_m \ll \rho_s / d_s \) (respectively \( r_m \ll r_s \)), the relation (26) is converted to the form:

\[
R_s \approx r_s F(\xi L).
\]

The distribution of the current \( I_m(z) \) in the metal layer can be found in the same way as for \( I_s(z) \). From equations (3) and (4) it follows:

\[
\frac{dI_m}{dz} = -\frac{dI_c}{dz} = -\frac{U_c W}{\rho_c}.
\]

Integrating (28) over \( z \) interval from 0 to \( z \), using the boundary condition \( I_m(0) = I_0 \)

\[
\int_0^z \frac{dI_m}{dz} dz = -\frac{W}{\rho_c} \int_0^z U_c(z) dz,
\]

\[
I_m(z) = I_m(0) - \frac{W}{\rho_c} \int_0^z U_c(z) dz.
\]

And since in any section \( z \) the total current \( I_0 \) along \( z \) is a sum of \( I_m \) and \( I_s \), then:

\[
I_m(z) = I_0 - I_s(z).
\]

We define the impedance of the contact area as
3. Results and discussion
A numerical example of calculation in accordance with the obtained relations is shown in Fig. 2. The obtained current distributions of $I_s/I_0$ show that at high conductivity of the electrodes (gel) compared to skin (at low $\psi$), the highest current density through the electrode/gel-skin interface is observed at the edges of the electrodes. At the same time, on a significant part of the electrode, the current $I_s$ is almost constant, i.e. in this area the current from the electrode penetrates the skin to an extremely small extent. With an increase in the resistance of the electrodes (gel), that is, with an increase in $\psi$, there is a more noticeable increase in the current density through the electrode/gel-skin interface in the middle part of the electrode. The calculations also show that with increasing parameter $\rho_c$, the dependence of $L/I_0$ on $z$ gradually approaches linear, which indicates that the current density through the electrode/gel-skin interface under the electrode is equalized. This confirms the assumption that it is possible to equalize the current through the use of more highly resistive electrodes and gels.

\[ R_c = \frac{U_c(L) - U_m(0)}{I_0}. \]  \hspace{1cm} (32)

Figure 2. Normalized dependence of the $L/I_0$ current in the skin layer from the $z$ coordinate for different ratios of electrode and skin parameters ($\psi = (\rho_m/d_m)/(\rho_s/d_s)$) for $\rho_c = 1 \, [\Omega \cdot m^2]$, $L = 5 \text{ mm}$. The parameters of conductive gels were taken from [9-11].

4. Conclusion
The analysis of the current flow mechanism in a metal-to-skin planar contact allowed to obtain relations describing the laws of potential distribution at the contact boundary and current densities in the skin layer, as well as expressions for the total contact resistance. Calculations based on the above relations may be conducted at different parameters of the electrode-skin structure. On the basis of the presented methodological approach, the resulting model can be further extended to reflect the more detailed structure of the skin layers.

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