CONSTRAINTS ON BLACK HOLE/HOST GALAXY CO-EVOLUTION AND BINARY STALLING USING PULSAR TIMING ARRAYS

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ABSTRACT

Pulsar timing arrays are now setting increasingly tight limits on the gravitational wave background from binary supermassive black holes (SMBHs). But as upper limits grow more constraining, what can be implied about galaxy evolution? We investigate which astrophysical parameters have the largest impact on predictions of the strain spectrum and provide a simple framework to directly translate between measured values for the parameters of galaxy evolution and pulsar timing array (PTA) limits on the gravitational wave background of binary SMBHs. We find that the most influential observable is the relation between a host galaxy’s central bulge and its central black hole, $M_\bullet - M_{\text{bulge}}$, which has the largest effect on the mean value of the characteristic strain amplitude. However, the variance of each prediction is dominated by uncertainties in galaxy stellar mass functions. Using this framework with the best published PTA limit, we can set limits on the shape and scatter of the $M_\bullet - M_{\text{bulge}}$ relation. We find our limits to be in contention with strain predictions using two leading measurements of this relation. We investigate several possible reasons for this disagreement. If we take the $M_\bullet - M_{\text{bulge}}$ relations to be correct within a simple power-law model for the gravitational wave background, then the inconsistency is reconcilable by allowing for an additional “stalling” time between a galaxy merger and evolution of a binary SMBH to sub-parsec scales, with lower limits on this timescale of ~1–2 Gyr.

Key words: galaxies: evolution – gravitational waves – quasars: supermassive black holes

1. INTRODUCTION

Pulsar timing arrays (PTAs) are beginning to place meaningful constraints on the gravitational wave background (GWB) in the nHz–μHz gravitational waveband. In particular, recent upper limits have cut into the range of predicted strengths of the GWB from supermassive black hole (SMBH) binaries (Arzoumanian et al. 2016; Lentati et al. 2015; Shannon et al. 2015).

The characteristic strain spectrum of the GWB, $h_c(f)$, is the quadrature sum of individual sources emitting in a certain GW frequency band. Early predictions of the GWB signal in the PTA band used models of the evolution of active galactic nuclei (e.g., Rajagopal & Romani 1995), and semi-analytical simulations of dark matter halos (e.g., Jaffe & Backer 2003; Wyithe & Loeb 2003). These have been updated with full cosmological simulations (i.e., the Millennium Simulation (Sesana et al. 2009; Ravi et al. 2012)). In recent years it has been recognized that the dominant source of emission in the PTA band may come from the $z \lesssim 2$ binary population (Sesana et al. 2008) and efforts to predict $h_c(f)$ have been based on various combinations of simulated and observed components of cosmological galaxy evolution. In fact, a number of the parameters that make up the astrophysical GWB are well constrained by observation. Thus, the focus has turned toward modeling the GWB based on local observables (Sesana 2013b; McWilliams et al. 2014; Ravi et al. 2015). Current work attempts to understand the effects that both under-efficient inspiral (binary “stalling”) and super-efficient inspiral (i.e., binary coupling with the galaxy core environment through the PTA band) might have on $h_c(f)$ (Sesana 2013a; Ravi et al. 2014; Huerta et al. 2015; Sampson et al. 2015). Inspiral efficiency affects both the predicted amplitude and the spectral shape of the GWB, but it is unclear how the effects described above map into galaxy evolution parameters.

In this work, we aim to provide an accessible “translation” of GW strain directly to the parameters contributing to the largest uncertainties in the GWB spectrum. Our base assumptions are (1) circular SMBH binaries, and (2) dominantly GW-driven inspiral through the nHz–μHz GW band (i.e., a single-power-law GWB). Section 2 lays out the mathematical preliminaries of galaxy evolution parameters that contribute to the strain spectrum. These parameters provide the basis for our construction of the astrophysical GWB. Section 3 describes the state-of-the-art observational constraints on these parameters from the literature used in this work. Section 4 presents how each astrophysical parameter affects the strain amplitude, specifically exploring a direct mapping to the $M_\bullet - M_{\text{bulge}}$ relation. Finally, Section 5 explores an example use-case for the results of this paper.

2. MODEL OF PTA STRAIN SPECTRUM

The polarization- and sky-averaged strain from one circular binary system is

$$h_c = \sqrt{\frac{32}{5}} \left(\frac{G M_c}{c^3}\right)^{5/3} \left(2\pi f_c\right)^{2/3} \frac{c}{D_c}$$

(Thorne 1987, pp. 330–458), where $M_c = (M_1 M_2)^{3/5} / (M_1 + M_2)^{1/5}$ is the chirp mass of the binary and $D_c$ is the proper (co-moving) distance to the binary (e.g., Peters & Mathews 1963). $f_c$ is the frequency of the GWs emitted in the rest frame of the binary, where Earth-observed frequency $f = f_c/(1 + z)$ for a circular binary. In the absence of external influence from the galactic environment, the orbit of such a system changes due to the emission of gravitational radiation at...
a rate (Peters & Mathews 1963)

\[
\frac{dt}{df} = \frac{5}{96} \left( \frac{c^3}{G(1+z)M_c} \right)^{5/3} \frac{1}{\pi^{3/2} f_r^{11/3}} \tag{2}
\]

As in Sesana et al. (2008), we construct the characteristic strain spectrum for binary SMBHs as

\[
h_c^2(f) = \int \int \int \frac{d^4N}{dz \ dm \ dq \ d(lnf)} h_s^2 \ dz \ dm \ dq, \tag{3}
\]

where \(d^4N\) is the number of binaries in a given redshift range \(dz\), mass range \(dm\), and mass ratio range \(dq\) that are emitting in a given GW frequency range \(df \ ln(f)\). \(h_s^2\) is the polarization- and sky-averaged strain from each binary in that range, Equation (1). Note that we expand chirp mass here into mass, \(M_c\), and mass ratio, \(q\), with \(q \leq 1\): \(M_c = M(q^3/(1 + q))^{1/3}\).

The contents of the triple integral encapsulate the total number of binaries in a given frequency interval that, due to the constraints of human lifetime and measurement capabilities, are predicted based on the evolutionary properties of the galaxies that host the relevant SMBHs:

\[
\frac{d^4N}{dz \ dm \ dq \ d(lnf)} = \frac{d^3n}{dz \ dm \ dq} \frac{dV_c}{dz} \frac{dt}{d(lnf)}. \tag{4}
\]

In this expression, \(d^3n\) is the number density of binaries in given ranges of redshift, mass, and mass ratio (\(dz, dm, \) and \(dq\), respectively). The resulting conversion takes the number of binaries per co-moving volume shell, \(dV_c\), and converts it to the number of binaries per Earth-observed GW frequency bin, \(d(lnf)\). This is done by first converting from co-moving volume shell to redshift, and then converting from redshift to Earth-observed time.

Given Equations (1)–(4), \(h_c\) can be written as a power law with dimensionless amplitude \(A_{yr}\) (Jenet et al. 2006):

\[
h_c(f) = A_{yr} \left( \frac{f}{1 \ yr^{-1}} \right)^{-2/3}. \tag{5}
\]

PTA constraints are often quoted as an upper limit on \(A_{yr}\) using this basic \(f^{-2/3}\) model of the expected SMBH binary background. However, PTA constraints are most sensitive to the GW frequency close to \(f \rightarrow 1/T_{obs}\), where \(T_{obs}\) is the total observation time, which for current data sets is around 10 yr. The constraint on \(A_{yr}\) is extrapolated using a spectral model of \(f^{-2/3}\). A super-efficient inspiral from environmental coupling could give rise to a different spectral shape of the GWB (Sesana 2013a; Ravi et al. 2014; Huerta et al. 2015), which may include a “turnover” frequency in the PTA band, where the binary evolution changes from being dominated by energy removal from environmental coupling to being dominated by gravitational radiation (Sampson et al. 2015). In this paper, we work within the single-power-law model, but as PTA limits continue to improve, the extrapolation to a meaningful limit on \(A_{yr}\) will become increasingly dependent on the assumed shape of the spectrum (Arzoumanian et al. 2016).

2.1. Parameters of Galaxy Evolution

There are no current direct observational constraints on the demographics used of SMBH binaries. This work follows the prescription used in Sesana (2013b) and Ravi et al. (2015), where the galaxy merger rate is used as a proxy and each galaxy in a merger is populated with a central SMBH using the observed relationship between SMBHs and their host galaxies.

The number density of galaxy mergers,

\[
\frac{d^2n_G}{dz \ dm \ dq} = \Phi(z, M) \mathcal{R}(z, M, q), \tag{6}
\]

is a combination of two functions: the galaxy stellar mass function (GSMF), \(\Phi(z, M)\), and the galaxy merger rate \(\mathcal{R}(z, M, q)\).

The GSMF is an astronomical observable, and is typically parameterized in terms of either a single or a double Schechter function:

\[
\Phi(z, M) = \frac{dn_G}{dM} = e^{-\frac{M}{M^*}} \left[ \Phi_1^* \left( \frac{M}{M^*} \right)^\gamma_1 + \Phi_2^* \left( \frac{M}{M^*} \right)^\gamma_2 \right]. \tag{7}
\]

The galaxy merger rate is the redshift-dependent rate at which a galaxy of mass \(M\) is involved in a major merger \((q > \frac{1}{2})\) with a second galaxy of mass \(qM\):

\[
\mathcal{R}(z, M, q) = \frac{d^2n_G}{dz \ dq} = \frac{d_{pair}(z)}{dq} \frac{1}{\tau(z, M)} \frac{dt}{dz}. \tag{8}
\]

This is calculated by combining the galaxy pair fraction, \(f_{pair}(z)\), and the merger timescale, \(\tau(z, M)\). The galaxy pair fraction is an astronomical observable. The merger timescale, which approximates the dynamical friction timescale for a dynamically bound pair of galaxies, must be determined through simulations.

Once the number of galaxy mergers has been calculated in given ranges of redshift, mass, and mass ratio, the mass of each black hole in the merger is calculated using SMBH–galaxy scaling relations that have been observed in the local universe. Again, we use the prescription described in Sesana (2013b) to calculate \(M_{bulge}\) for each galaxy, assuming that some fraction of the total mass is in the central bulge. This fraction, \(f_{bulge}\), depends on galaxy morphology and mass. The SMBH mass is then calculated using a relationship of the form

\[
\log_{10}M = \alpha + \beta \log_{10} \left( \frac{M_{bulge}}{10^{11} M_\odot} \right). \tag{9}
\]

Additionally there is an intrinsic scatter, \(\epsilon\), associated with each measurement of this relation, which is the natural scatter of individual galaxies around the trend line shown above.

Combining Equations (3)–(9), this model gives a prediction for \(A_{yr}\):

\[
A_{yr}^2 = \int \int \int \frac{\Phi \ d_{pair}}{\tau} \frac{dV_c}{dz} \left( \frac{dt}{d(lnf)} \right) h_s^2 \ dz \ dm \ dq, \tag{10}
\]
3. OBSERVATIONAL CONSTRAINTS

Sesana (2013b) presented the first systematic investigation of the GWB using the method described in Section 2 and utilized the mean values of many different observations. This work seeks to update that investigation in a complementary way to what was done in Ravi et al. (2015), which used fewer, but more complete observations and focused closely on the uncertainties in the galaxy merger rate as well as other uncertainties inherent to the specific observations used to determine the GSMF and the $M_\ast - M_{\text{bulge}}$ relation. This work attempts to find a balance between these two approaches, using multiple current and complete observations, while also attempting to determine which details are more relevant to $A_{\text{yr}}$ predictions.

3.1. Galaxy Stellar Mass Function

The total GSMF is fairly well constrained by observation out to $z = 1.5$ (Tomczak et al. 2014); however, there is still debate about how the total GSMF is broken down by galaxy type (star-forming versus quiescent). Quiescent galaxies dominate the GW radiation that PTAs are sensitive to because they host larger black holes than star-forming galaxies. Therefore, the breakdown of galaxy type as a function of mass and redshift has a large impact on the overall signal. We do not assume any kind of clustering effect on mergers, so a quiescent galaxy’s likelihood of merging with another quiescent is based only on the fraction of quiescent galaxies in given ranges of redshift, mass, and mass ratio. Ravi et al. (2015) shows that while there is some possibility for contamination of color-selected GSMFs, the overall effect on the amplitude of the GWB is negligible compared to other contributions.

In this paper, we use the GSMF measured by the CANDELS/ZFOURGE survey (Tomczak et al. 2014), and the full UltraVISTA survey (Ilbert et al. 2013) for $z > 0.2$. The ZFOURGE survey provides the deepest GSMF measurements to date (Tomczak et al. 2014), while the UltraVISTA survey covers a wide field and so they complement each other well. These two surveys show consistent results with the updated GSMF from Muzzin et al. (2013) used by Ravi et al. (2015). This paper uses the SDSS/GALEX observations of the GSMF in the local universe ($z < 0.2$; Moustakas et al. 2013).

3.2. Galaxy Merger Rate and Timescale between Mergers

The observed galaxy merger rate is a combination of the observed galaxy pair fraction $f_{\text{pair}}$ and an analytically calculated merger timescale $\tau$ (Equation (8)). The galaxy pair fraction is fairly well constrained in the local universe ($z \sim 0.1$), but it is less constrained with increasing redshift (Keenan et al. 2014). $f_{\text{pair}}$ is expected to peak and turn over somewhere around $z \sim 2$–3 for the most massive galaxies in which we are interested (Conselice 2014). In this paper, we use the galaxy pair fraction from the GAMA survey (Robotham et al. 2014) and the combined value resulting from analysis of the RCS1, UKIDSS, and 2MASS surveys (Keenan et al. 2014). While these two papers agree on the pair fraction in the local universe ($z \sim 0.1$), they differ by a factor of two in the redshift dependence of the pair fraction, with Keenan et al. (2014) showing a lower value. Here we keep the values of $f_{\text{pair}}$ fixed out to $z = 3$, and find that the values of $f_{\text{pair}}$ reached are approximately at both the high and low ends of predictions, respectively (Conselice 2014). While both Sesana (2013b) and Ravi et al. (2015) use the $f_{\text{pair}}$ measurement from Xu et al. (2012), it is not used in this work because Robotham et al. (2014) note that an observation of the local pair fraction from Xu et al. (2012), which sets both the low fraction at $z = 0$ and the high slope of the redshift dependence of the presented $f_{\text{pair}}$ relation, has been shown to be unrealistic. The revised measurement of Xu et al. (2012) is shown to be consistent with Robotham et al. (2014).

For self-consistent comparisons, recent observations of the galaxy pair fraction use one of two formulas for $\tau$ (Kitzbichler & White 2008; Lotz et al. 2011), which differ by a factor of two. The timescale of Kitzbichler & White (2008) is a lower limit for the galaxy merger timescale because it is derived from a merger timescale for a dark matter halo from the Millennium Simulation. Lotz et al. (2011) calculates $\tau$ from a set of hydrodynamical simulations, which incorporates gas and dust into the galaxy merger, unlike Kitzbichler & White (2008). Xu et al. (2012) combined the mass and redshift dependence from Kitzbichler & White (2008) with the results from Lotz et al. (2011) to give a description of the major-merger timescale for $z < 1$, which is combined with the pair fractions described above to give two different galaxy merger rates used in this work. Since this paper is focused on the effects of observable galaxy evolution parameters on the GWB we do not investigate different formulations of $\tau$. However, in Ravi et al. (2015) the removal of the mass and redshift dependence of $\tau$ was found to produce a slightly higher value for $A_{\text{yr}}$.

3.3. Relation between SMBH Mass and Galaxy Bulge Mass

In the past few years, there have been observations of a steeper relationship than previously observed for the correlation between bulge mass and the mass of the resident SMBH. The steeper values are largely due to the recent measurements of the high-mass end of $M_{\text{bulge}}$, which in turn houses the most massive SMBHs that will contribute to the GW signal in the PTA band (Kormendy & Ho 2013 (KH13); McConnell & Ma 2013)
Figure 2. Predictions of $A_{\gamma}$ calculated using Equation (10). The solid black line is the total distribution calculated using the errors reported with the measurements we have chosen to use, described in Section 3. Additionally, the total distribution is broken down by the $M_*-M_{\text{bulge}}$ relation, with Kormendy & Ho (2013) shown in magenta, which is understood to predict higher values of $A_{\gamma}$ because the measurement is weighted strongly by large mass observations, as seen in Figure 1, while McConnell & Ma (2013) shown in blue is not and thus creates smaller black hole masses. The breadth of the distributions results from the observed variance in our input parameters, and also from the fact that we use multiple formulations for the GSMF and $f_{\text{pair}}$. The most recent upper limits from each PTA are shown as points directly above the distribution of $A_{\gamma}$, and above that we show the predictions from similar models in Sesana (2013b) and Ravi et al. (2015).

As we aim to predict the most realistic GWB signal as relevant to PTAs, in our simulation we thus consider only the $M_*-M_{\text{bulge}}$ relations that include these massive galaxy measurements. Figure 1 shows a demonstration of how the parameters of Equation (9) vary with the inclusion or exclusion of measurements at the high-mass end, and various other considerations. It is unclear whether a single power law best describes this relationship over the full mass range (Graham 2016, p. 263), but it appears to be sufficient for the most massive systems, and so in this work we do not consider the broken power-law prescriptions in Scott et al. (2013). Additionally, we note that the high-mass relation in Scott et al. (2013) is almost identical to the relation in Kormendy & Ho (2013).

Measurements of the $M_*-M_{\text{bulge}}$ relation also include $\epsilon$, the “intrinsic scatter,” which is the natural scatter of individual galaxies around the trend line described by $\alpha$ and $\beta$ in Equation (9). This parameter plays a critical role in predictions of $A_{\gamma}$, as a way of accounting for the outliers in the distribution. The galaxies containing overmassive black holes will contribute more to the GW signal in the PTA band than most galaxies of similar mass, and thus they are a necessary inclusion in any prediction of $A_{\gamma}$.

4. RESULTS

Combining all of the observational constraints from Section 3 into our model described in Equation (10), we calculate a range of predictions for $A_{\gamma}$. The inputs are allowed to vary within the reported observational errors for each combination of GSMF, $f_{\text{pair}}$, and $M_*-M_{\text{bulge}}$ relation used in this paper. Each combination is run 500 times for a total of 4000 predictions of $A_{\gamma}$. Figure 2 shows a plot of this distribution for major mergers, $q > \frac{1}{3}$, between galaxies with a stellar mass in the range $10^{10}-10^{12} M_\odot$ and lying in a redshift range $z < 3$. Additionally, we include the predictions from Sesana (2013b) and Ravi et al. (2015), and find that all ranges of $A_{\gamma}$ are similar. Figure 2 also indicates where recent 95% confidence upper limits from various PTAs fall with respect to this model’s predictions. The most recently published upper limit comes from 11 years of data taken by the Parkes Pulsar Timing Array (PPTA, Shannon et al. 2015), which quotes a 95% upper limit on $A_{\gamma}$ of $1.0 \times 10^{-15}$. The North American Nanohertz Observatory for Gravitational Waves (NANOGrav) has released nine years of data, which produce a 95% upper limit of $A_{\gamma} < 1.5 \times 10^{-15}$ (Arzoumanian et al. 2016), and the European Pulsar Timing Array (EPTA) quoted a 95% upper limit on $A_{\gamma}$ of $3.0 \times 10^{-15}$ (Lentati et al. 2015). PTA upper limits are typically shown to “rule out” some portion of the predicted range of $A_{\gamma}$. However, it is often unclear what that means in terms of limits on the input parameters for SMBH evolution. We aim to provide clarity by determining how, and by how much, each parameter affects the resulting prediction of $A_{\gamma}$.

As expected for each prediction, $A_{\gamma}$ is dominated by binary sources with chirp masses larger than $10^8 M_\odot$, as seen in Figure 3, which shows the cumulative contribution to $A_{\gamma}$ from increasing chirp masses. Figures 4 and 5 show that the majority of the signal is produced at $z \lesssim 1.5$, also as expected, but there is a lot of variance at lower redshift, which is discussed further in Section 4.1. Figure 4 shows the contribution from binaries at different redshifts for both GSMFs used in this paper. While the general trend is the same for each, the differences in them follow directly the way that each GSMF handles the abundance of massive early-type galaxies, specifically in the range $0.5 < z < 1.5$.

At higher redshifts the amount of signal is dominated by the total number of binaries, which is set in part by $f_{\text{pair}}$. The effect of different $f_{\text{pair}}$ on the redshift distribution of the contribution to $A_{\gamma}$ is shown in Figure 5. While both values of $f_{\text{pair}}$ used in this paper are the same at $z = 0$, the redshift dependences vary by a factor of two. However, as described in Section 3.2, the two values trace the upper and lower ends of predictions for $f_{\text{pair}}$ at high redshift. An interesting feature of Figure 5 is that
the steeper \( f_{\text{pair}} \) from Robotham et al. (2014) contributes less at lower redshifts than the shallower \( f_{\text{pair}} \) from Keenan et al. (2014). This trend is directly related to the number of binaries contributing to \( A_{\text{yr}} \) at higher redshifts.

4.1. Relative Effects of GSMF, \( f_{\text{pair}} \), and \( M_*-M_{\text{bulge}} \)

Each parameter in Equation (10) that has an observational constraint also has an associated error. These errors obviously affect the variance in the prediction of the final strain spectrum. In Figure 6, we show the 1\( \sigma \) range of predictions for \( A_{\text{yr}} \) given by each combination of observational parameters. The percentage contribution to the total error is broken down for each parameter in a given combination, and shown underneath the predicted range of \( A_{\text{yr}} \). Below, we discuss how the breakdown of the errors in individual observational parameters impacts the final range of \( A_{\text{yr}} \).

Given that \( A_{\text{yr}}^2 \propto M_*^{2/3} \), the parameters that affect the binary chirp mass should have substantial impact on the prediction of \( A_{\text{yr}}^2 \) (Sesana & Vecchio 2010). Accordingly, we find that the \( M_*-M_{\text{bulge}} \) relation sets the mean value for \( A_{\text{yr}}^2 \) predictions, as can be seen by the breakdown in Figure 2. This is the relation that provides the translation from galaxy population to BH mass and therefore sets the mean value of the \( M_* \) distribution. Section 4.2.1 discusses in detail how the \( M_*-M_{\text{bulge}} \) measurement parameters contribute to \( A_{\text{yr}}^2 \).

The GSMF plays a large role in determining the variance of a certain prediction. All mass functions used in this paper are parameterized using a double Schechter function (Equation (7)) for \( z < 1.5 \), so there are a lot observed parameters included in each calculation. Looking at each Schechter parameter and the error bars associated with it reveals that uncertainty in \( M_* \) has the largest effect on \( A_{\text{yr}}^2 \) predictions, contributing 45% of the GSMF error for both observations used in this paper. \( M_* \) is the mass at which the Schechter function transitions between the high-mass exponential decay and the lower-mass slopes \( \alpha_1 \) and \( \alpha_2 \). The error from the local GSMF, \( z < 0.2 \) (Moustakas et al. 2013), is minimal at 2% of the GSMF error, but the next redshift bin, \( 0.2 < z < 0.5 \), in both GSMF observations used in this paper provides the largest influence by redshift on the GSMF error contribution to \( A_{\text{yr}}^2 \) at 55% for Tomczak et al. (2014) and 72% for Ilbert et al. (2013).
The host galaxy's mass is used to estimate each black hole's mass using \( f_{\text{bulge}} \). \( f_{\text{bulge}} \) for quiescent galaxies is described with two values—one that sets the fraction of mass in the central bulge for massive galaxies, \( M > 10^{11} M_\odot \), and one that sets fraction for galaxies with stellar masses of \( 10^9 M_\odot \). Given the dominance of the signal from binaries with chirp masses above \( 10^8 M_\odot \), the effect of the quiescent galaxy’s \( f_{\text{bulge}} \) on \( A_\nu \) was calculated. Figure 7 demonstrates the dependence of \( A_\nu \) on \( f_{\text{bulge}} \) for quiescent galaxies, making it clear that there is not a strong dependence (less than a factor of \( \sqrt{2} \)).

The pair fraction, \( f_{\text{pair}} \), has the least impact on \( A_\nu \). Although the parameter itself has a large range, it only impacts the number of sources (\( A_\nu^2 \propto N \)), rather than the masses of those sources.

### 4.2. Translating GWB Limits and Astrophysical Parameters

The overarching goal of this paper is to provide a mapping of GWB limit/measurement values to specific parameters of galaxy formation. Below, we look at how areas of parameter space correspond to specific values of \( A_\nu \) with an associated error range. In our discussions below we aim to answer the following two questions:

1. Given a PTA upper limit on the GWB, what specifically does this mean for galaxy/SMBH evolution?
2. Given a new observation of the \( M - M_{\text{bulge}} \) relation, are the new values compatible with the best PTA limit? If not, how can they be reconciled?

#### 4.2.1. GW Limits and the Black Hole–Host Galaxy Relation

The parameter space that characterizes the black hole–host galaxy relation is encompassed by \( \alpha \), \( \beta \), and \( \epsilon \). The last of these is the natural scatter of individual galaxies around the trend described by intercept \( \alpha \) and slope \( \beta \). This \( \epsilon \) plays a critical role in \( A_\nu \) predictions; as \( \epsilon \) increases, higher masses are present in a few binaries, which increases the distribution of chirp masses and effectively weights \( A_\nu \) upward. We demonstrate the implications of this in Figure 8, which shows the dependence calculated numerically; for any combination of \( \alpha \) and \( \beta \), changes in \( \epsilon \) affect \( A_\nu^2 \) in the same way.

In Figure 9, we show contours of constant \( A_\nu \) in \( \alpha - \beta \) space, while keeping \( \epsilon \) equal to zero, as this is the most common way that the \( M - M_{\text{bulge}} \) relation is reported. Yet, for a specific value of \( \alpha \), changing \( \beta \) across the entire range of observed values translates to only a <20% change in \( A_\nu \). It is thus \( \alpha \) and \( \epsilon \) that have the most impact on a prediction of \( A_\nu \), and so we combine the results of Figures 8 and 9 in Figure 10 to show \( A_\nu \) as a function of \( \alpha \) and \( \epsilon \), while holding \( \beta = 1 \).
4.2.2. GW Limits and Stalling Binaries

The assumption that galaxy mergers and binary black holes form at the same cosmological time has until now been implicit in the model used in this paper and others. Yet, as PTA upper limits become inconsistent with measured astronomical parameters, the assumptions of this model must be questioned. It is straightforward to ease this assumption by allowing for a “stall” in the binary SMBH formation. Let us introduce a variable, \( T_{\text{stall}} \), that is a measure of the time between the galaxy merger, which occurs approximately on a dynamical friction timescale, and the binary SMBH entering the PTA band. This stalling timescale creates a redshift offset between the galaxy merger and binary SMBH’s in-band GW emission. This is incorporated into Equation (4) as follows:

\[
\frac{d^4 N}{dz \, dM \, dq \, d(ln f)} = \frac{d^3 n}{dz_1 \, dM \, dq} \, \frac{dV_c}{dz_2} \frac{dz_2}{dt} \frac{dt}{d(ln f)} \quad (11)
\]

where \( z_1 \) is the redshift of the galaxy merger and \( z_2 \) is the redshift at which the binary is emitting in the PTA band, while \( T_{\text{stall}} \) is the proper time between \( z_1 \) and \( z_2 \). Figure 11 shows how \( A_{\nu} \) changes with different values of \( T_{\text{stall}} \). Obviously as the stalling timescale reaches values nearing the Hubble time, \( A_{\nu} \) falls to zero as no systems are expected to ever enter the PTA band. The meaning and use of limits on \( T_{\text{stall}} \) are described further in Section 5.2.

5. LIMITING \( M_*-M_{\text{bulge}} \) AND STALLING TIMESCALE WITH PTA CONSTRAINTS

The following section describes a method by which PTA constraints can be extended beyond \( A_{\nu} \) and into the parameter space of galaxy evolution. Similar goals have been proposed using different methods in Middleton et al. (2016). We note that a key difference between this work and that of Middleton et al. (2016) is that we interpret PTA limits in the context of other observational constraints on galaxy evolution, whereas the method of Middleton et al. (2016) assumes no outside knowledge of the SMBH binary population, except for a general form of the SMBH merger rate density (which is well motivated), to formulate their constraints. As such, they were unable to place meaningful limits on the binary population, while this work is able to place tighter constraints by utilizing a well established range of prior information.

5.1. Constraining \( \alpha, \beta, \) and \( \epsilon \) with PTAs

It is common for PTA constraints to be quoted as a single number, \( A_{\nu} \), which represents the 95% upper limit on a GWB of a given spectral index: for binary SMBHs, \( \alpha_{\nu} = -2/3 \) (Equation (5)). However, quoting a single number is only for simplicity; the actual result produced by PTAs for a limit on a power-law GWB is a probability distribution for the value of the strain amplitude.

Here we describe the use of this probability distribution to obtain direct limits on \( \alpha, \beta, \) and \( \epsilon \). We then demonstrate how an inconsistency with an observed value can be used to place a lower limit on \( T_{\text{stall}} \).

In terms of Bayesian statistics, a PTA produces a posterior on \( A_{\nu} \),

\[
p(A_{\nu}|\text{PTA}) \propto p(A_{\nu}) \, p(\text{PTA}|A_{\nu}), \quad (12)
\]

where \( p(A_{\nu}) \) is the prior distribution and \( p(\text{PTA}|A_{\nu}) \) is the likelihood. We are interested in producing a posterior on parameters from our model. As the \( M_*-M_{\text{bulge}} \) relation provides the most influential parameters on \( A_{\nu} \), here we calculate the posteriors on \( \alpha, \beta, \) and \( \epsilon \):

\[
p(\alpha, \beta, \epsilon|\text{PTA}) \propto p(\alpha) \, p(\beta) \, p(\epsilon) \, p(\text{PTA}|\alpha, \beta, \epsilon). \quad (13)
\]

Our model gives us a way of translating \( \alpha, \beta, \) and \( \epsilon \) into a value of \( A_{\nu} \), which means the two likelihoods are equivalent,

\[
p(\text{PTA}|\alpha, \beta, \epsilon) = p(\text{PTA}|A_{\nu}(\alpha, \beta, \epsilon)). \quad (14)
\]

We do need to include other observational parameters in our model, specifically the GSMF and the galaxy merger rate. We will represent all of these parameters with \( \theta \), which we can...
We can set an upper limit in the upper limit, we can set an upper limit in relevant observational measurements of these parameters shown in Figure 13. This upper limit and relevant observational measurements of these parameters are shown in Figure 13.

Figure 12. Translation of the marginalized posterior distribution of $A_{\text{yr}}$ into the black hole–host galaxy parameter space, which is characterized by an intercept $\alpha$, a slope $\beta$, and an intrinsic scatter $\epsilon$. $\beta$ is not informed by the distribution of $A_{\text{yr}}$, while both $\alpha$ and $\epsilon$ are, with a limit on $\alpha$ being more strongly set. The curves show the $1\sigma$, $2\sigma$, and $3\sigma$ contours. Since $\beta$ is not strongly informed by the upper limit, we can set an upper limit in $\alpha$–$\epsilon$ space by marginalizing over $\beta$. This upper limit and relevant observational measurements of these parameters are shown in Figure 13.

marginalize over, giving

$$p(\alpha, \beta, \epsilon | \text{PTA}) \propto \int d\theta \; p(\theta) \; p(A_{\text{yr}}(\alpha, \beta, \epsilon, \theta) | \text{PTA}).$$

(15)

Figure 12 shows the translation of the recent upper limit from Shannon et al. (2015) into the parameter space that characterizes the $M_{\bullet}$–$M_{\text{bulge}}$ relation. We can set a 95% upper limit in the 2D parameter space, $\alpha$–$\epsilon$, by integrating the distribution, shown in Figure 13. Similar work has been done in Middleton et al. (2016), where an attempt is made to reconstruct a parameterized form of the black hole merger rate density from a posterior on $A_{\text{yr}}$ from PTA data. This kind of mapping from PTA data to astrophysical parameters is the next step forward in analyzing PTA data and is already being used in recent papers on PTA limits (Arzoumanian et al. 2016).

5.2. Reconciling PTA Limits with $M_{\bullet}$–$M_{\text{bulge}}$ Measurements

Figure 13 demonstrates that the measurements of Kormendy & Ho (2013) and McConnell & Ma (2013) are inconsistent with the upper limit on the GWB by Shannon et al. (2015). We can thus attempt to reconcile this discrepancy by considering whether a nonzero $T_{\text{stall}}$ can make these two results consistent. If we assume the measured values are correct, then we can do a direct translation of the PPTA upper limit into a probability distribution on $T_{\text{stall}}$. This is seen in Figure 14, and we set a lower limit of $T_{\text{stall}} > 2.3$ Gyr using Kormendy & Ho (2013) and a lower limit of $T_{\text{stall}} > 1.2$ Gyr using McConnell & Ma (2013).

We reiterate that, in our formulation, $\tau$ essentially represents a dynamical friction timescale (the merger timescale), while $T_{\text{stall}}$ represents the additional time it takes the binary to enter the PTA waveband. Our limits on $T_{\text{stall}}$ could be capturing a longer merger timescale, such as those discussed in Ravi et al. (2015). However, since there is no concrete definition for each of these parameters in the literature, we choose to make the straightforward assumption that after a standard estimate of the dynamical friction timescale, the remaining time for binary evolution is captured by $T_{\text{stall}}$.

There are a limited number of observational and theoretical limits on stalling timescales in the literature. Originally, the apparently “missing” inspiral mechanisms were taken to imply that binary SMBHs might stall for up to a Hubble time (Begelman et al. 1980), although for a few systems with sufficient gas it was shown that binary SMBHs can inspiral efficiently (e.g., Mayer et al. 2007; Cuadra et al. 2009). More recently, numerical simulations have demonstrated that with mild triaxiality or axisymmetries in merging galaxies, binary SMBH coalescence timescales can be pushed to $\lesssim 2.4$ Gyr, and contrary to previous inference, timescales have been found to
be much less (<0.5 Gyr) for the most massive galaxies (Khan et al. 2011, 2013; Preto et al. 2011). Furthermore, Burke-Spolaor (2011) placed an observational upper limit on the stalling timescale of the most massive binary SMBH systems (\(\gtrsim 10^8 M_\odot\)) of <1.25 Gyr at 50% confidence (i.e., a few Gyr at 95% confidence). The latter, observationally derived value is consistent with our lower limits using the relations of both McConnell & Ma (2013) and Kormendy & Ho (2013). However, the theoretical result that inspiral due to host axisymmetries can take place in \(\lesssim 2.4\) Gyr, or even <0.5 Gyr for the most massive galaxies we’re probing here, is in contention with both of these limits. More recent work has shown that for “dry” mergers, where there is little gas present, the total merger time is >1 Gyr (Sesana & Khan 2015), which is consistent with our limits.

There are two likely interpretations of this result. First, if we assume that the \(M_{\text{bulge}}\) relations here hold, we must infer either that something else is reducing the expected GW background (for instance, a turnover due to environmental coupling or a longer merger timescale than assumed here) or that triaxiality and axisymmetry are not prevalent enough to be the dominant force in driving pre-GW-dominant inspiral in massive galaxies. Second, it is possible that the parameterizations of \(M_{\text{bulge}}\) relations have been overestimated. As a more general consideration for this analysis, it is worth noting here that it has been suggested that recent black hole–host relations might err on using an upper-limit mass value for many black holes in their fits, for a range of valid values (Merritt 2013, see, e.g., the discussion in Chapter 3). Thus, it is possible that the measurements of McConnell & Ma (2013) and Kormendy & Ho (2013) have arrived at disproportionately high values, particularly for \(\alpha\) and \(\epsilon\). Recent work has proposed that there is a bias in these measurements as well (Shankar et al. 2016), and we are currently working to assess what impact this effect would have on our results. We note that a moderate downward shift of both the relations of McConnell & Ma (2013) and Kormendy & Ho (2013) in \(\alpha\) and/or \(\epsilon\) would render fits consistent with current PTA limits (e.g., Figure 13).

As previously noted, there could exist a deviation from a single-power-law GWB spectrum due to essentially the same additional complexity to PTA analysis (e.g., Ravi et al. 2014; Arzoumanian et al. 2016; Huerta et al. 2015; Sampson et al. 2015). An investigation of that possibility is beyond the scope of this paper.

6. CONCLUSIONS

We inspected the amplitude variance of the nHz–\(\mu\)Hz waveband GWB, as formulated from state-of-the-art observations of massive galaxies, massive galaxy mergers, and SMBHs in the \(z \lesssim 3\) universe. In the course of this analysis we provided simple reference plots that can be used to map PTA limits on the GWB to the parameters of cosmological evolution that most impact the power-law GWB prediction (Figures 8–11).

We found that the vastly different observations of the intercept (\(\alpha\)) and scatter (\(\epsilon\)) of the \(M_{\text{bulge}}\) relation have the most impact on the range of predictions of GWB amplitude. We used the most constraining PTA upper limit on the GWB of \(A_\nu < 1.0 \times 10^{-15}\) from Shannon et al. (2015) to compare with our mapping to these parameters, and found that the \(M_{\text{bulge}}\) relations of McConnell & Ma (2013) and Kormendy & Ho (2013) are inconsistent with the PTA limit. Both of these measurements include the high-mass SMBHs that are expected to contribute the majority of signal to the PTA gravitational waveband (Kormendy & Ho 2013; McConnell & Ma 2013), making them the appropriate measurements for this comparison.

As discussed in Section 5.2, this inconsistency can be reconciled in a number of ways. First, we can include a moderate amount of “stalling” in the inspiral of the binary SMBH, in which the pair slows its evolution for at least 1.2 and 2.3 Gyr for the \(\alpha\), \(\epsilon\) values of McConnell & Ma (2013) and Kormendy & Ho (2013), respectively, which are in contention with theoretical work that has shown how axisymmetries may allow inspiral efficiencies of <0.5 Gyr for the most massive pairs.

Observationally, the uncertainties in \(M_{\text{bulge}}\) and similar relations are largely due to small sample sizes. One important conclusion drawn from this analysis is that better constraints on the \(M_{\text{bulge}}\) relation will greatly tighten GWB predictions, motivating further SMBH measurements to be made in the \(\gtrsim 10^8 M_\odot\) range. Furthermore, it has been pointed out that the masses used for many SMBH–host galaxy relations may be overestimated, potentially solving the discrepancy we have found, as recently noted in Shankar et al. (2016). However, this only points more strongly toward a need for better characterization of these relations to allow an accurate prediction of the GWB, which we intend to pursue in more detail in future work. The host galaxy relation is most critical to be improved if we are to tighten the constraints that PTAs can put on effects such as binary stalling, wandering SMBHs, and environmental interactions from the influence of gas or stellar dynamics during the GW-dominant regime. Finally, while the redshift evolution of this relation was not considered in this work, it is clear that strong evolution could further heighten its impact on GWB predictions (see, e.g., Ravi et al. 2015).

We conclude by reiterating several caveats. We have considered only a power-law GWB, in which the binary inspiral is decoupled from its environment throughout the nHz–\(\mu\)Hz GW band. However, a quantification of super-efficient evolution due to strong environmental coupling poses an equaly large, if not much larger, source of uncertainty in the low-frequency end of the PTA band. This must be better assessed via observational and theoretical work.

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