Analysis of the blast wave – structure interface phenomenon in case of explosive events

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Abstract. Several structures need their response to blast loads to be investigated to grant an acceptable survivability level. To this purpose, experimental campaigns are complex, but some numerical methods are commonly exploited. The explosion pressure-time history is typically estimated characterizing the detonation of an energetically equivalent explosion involving the detonation of a specific amount of TNT. The pressure-time history at a point, fixed in space, involved by the blast wave is described by the Friedlander equation. A specific approach to modelling the complex blast wave–target structure coupling is reported in the UFC 3-340-02. This method, although consolidated and valid for aerospace, civil and mechanical structure, is not adopted in predictive simulations, which involve naive methods for characterizing that interface phenomenon. This work aims to give an insight on the blast wave–structure interaction event. A methodological approach combining the Friedlander equation and the theory reported in the UFC 3-340-02 is presented, which confidently characterizes the effects of explosive loads on common structures. This fully analytical method may be implemented into numerical codes to perform simple, but effective, preliminary characterizations of the structural response to explosive loads. Finally, a possible application of the proposed method to thin-walled structures is shown.

1. Introduction

The effects of explosions on surrounding structures had not been comprehensively studied until World War II. Prior to that period, only a limited number of papers and research work had been generated, unavailable to the general public because of the decision to keep it classified [1]. Only in 1941, thanks to the work of Sir Geoffrey Taylor, which defined what mechanical effects might be expected in case of nuclear explosions, the topic of explosions started to grow exponentially [2]. A large number of experimental and analytical studies were conducted and later published during the second half of the 20th century, mainly aimed at determining blast effects and the structural response of targets under impulsive loading, the most important and well known of them being the precious research work of Kinney and Graham published in 1962 [3]. The primary aim was to study in great detail the nature of blast waves, their origin and characteristics, while the response of target structures remained a secondary aim. It was thanks to many previously published works that Kinney and Graham in 1962 pioneered the empirical methods for the determination of blast waves characteristics such as overpressure, positive phase duration, arrival time and impulse. Those properties were paramount for the description of the relevant information stored in the Friedlander equation, which had been proposed by Friedlander in 1946 in the work in [4], under the academic direction of Sir Geoffrey Taylor. That equation, which is largely known for accurately describing the pressure-time history of a blast wave, had originally been developed within the larger framework of the analytical solutions of sound pulses diffracted by a semi-infinite plate. Later on, based on those works, many empirical and semiempirical methods were developed for the characterization of far-field explosions, which were accurate and quick in the estimation of the characteristics of a blast. A successful implementation of one of the empirical blast models, which was developed by Kingery and Bulmash [5] [6], is represented by the CONWEP model.
Moreover, that model was implemented in the DYNA2D [8] and DYNA3D [9] general-purpose finite-element programs, which were used for modelling large deformations of structures. A major effort in reporting and comparing several results of those empirical approaches to blast waves characterization was made by Ullah et al. in the work in [1]. The only inputs required by such analytical methods are the nature of the explosive material, its mass, the distance of the target structure from the detonation point and the angle of impact of the blast wave on it. Historically, the explosive used in the experimental campaigns aimed at investigating blast waves was symmetrical 2, 4, 6-trinitrotoluene (TNT). Hence, with all the available analytical methods based on this explosive, the main characteristics of a blast wave are given in terms of equivalent TNT weight.

Additional to the blast wave characterization, a big challenge is also represented by the description of the interaction with the impacted structures. This interaction is particularly important for accurately determining the loads imparted on the impacted structures, since an accurate model of the pressure-time history effectively felt by the structure is needed to characterize its dynamic behaviour. To this purpose, a widespread approach is to consider a combination of the incident pressure and the component which is reflected in the blast wave-structure interaction, i.e., the reflected pressure, as the effective pressure exerted on the structure. This approach is implemented, in a naive way, in the Finite Element numerical codes featuring the CONWEP model [10]. A more refined version of the effective pressure-time history exerted on a structure under blast loading is provided in the UFC 3-340-02 [11]. That model considers the shape and the geometric dimensions of the structure involved in the event, together with some additional information for accurately describing the load exerted on the structure. For instance, the eventual presence of openings, e.g., windows, is considered. However, even though the method presented in the work in [11] is more accurate than the one implemented in the CONWEP equations, to the authors best knowledge no extensive usage of it in predictive simulations has been proposed yet. Alongside the analytical methods introduced above, in the last few decades hydrocode simulations have been proposed as a more accurate and powerful alternative for simulating blast loads and the subsequent effects on structures. Those methods allow getting a description of the evolution in time and space of the properties of an explosive material, governed by the well-known Jones-Wilkins-Lee Equation of State (JWL EOS) [12], exploiting the potentiality of the finite elements Eulerian description. Moreover, that information may then be used to determine the actual load exerted on an eventual structure, which can be of any shape and dimensions, and for characterizing its dynamic response. However, the main drawback of using hydrocode simulations for the calculation of blast wave loads is that the computational time required for the analysis increases enormously with respect to the computational time typical of analytical methods. In fact, while the latter provide instantaneous predictions, the former require up to several days for dealing with complex geometries and simulations of large-scale events involving a multiphysics numerical environment which includes the characterization of the fluid-structure interaction. Hence, hydrocode simulations are impractical at least in the preliminary design phases of a platform, during which many iterations between design improvements and simulation of the benefits may take place. Thus, since in those preliminary phases fast predictive simulations may be preferred to more accurate but time and resources consuming hydrocode analyses, the analytical characterization of the load from blast waves plays a big role in the overall accuracy of those fast predictive methods.

Within this complex framework, this work aims at proposing a fully analytical methodological approach to the blast wave load characterization task, by combining the information stored in the Friedlander equation with the one presented in the UFC 3-340-02, in order to accurately determine the pressure-time history exerted on simple structures. Moreover, the developed method can be easily implemented in numerical codes, such as Finite Element ones, to assess the strength of the blast loaded structure. That allows to couple a simple, yet accurate description of the interaction between blast wave and structure, with the enormous potentiality of the Finite Element method, in order to provide a fast and reliable predictive tool for the preliminary evaluation of explosion effects on structures. This work is organized as follows: Section 2 presents the most relevant theoretical aspects of the proposed method, Section 3 reports a case study in which the effective pressure-time history acting on the front face of a
thin-walled box-like structure determined by the CONWEP algorithm and by the method presented in this work are compared, while Section 4 gives the conclusions and some potential further developments of the presented approach. Finally, the main equations underlying the proposed methodology are reported in the Appendix.

2. Methodology

This Section presents the methodological approach proposed by the authors, which consists of the combination of empirical methods for determining the relevant information stored in the Friedlander equation with the blast wave–structure interface model from the UFC 3-340-02.

A preliminary consideration has to be given. The basic theory behind the method presented in this work has been historically developed for dealing with detonations. Detonation is a mechanical mechanism consisting of the generation of shock waves that, impacting the unexploded material, activate it through shock pressure forces. Materials exploding through detonation are called detonating explosives or high explosive materials, such as TNT. Detonation velocities are always supersonic [13]. However, the process of reconducting an explosion to the equivalent one produced by detonating a specific amount of TNT is also widespread when evaluating the effects of deflagrations (e.g., [14] [15]). Hence, even though based on a theory which was not directly developed for considering both detonations and deflagrations, the methodological approach presented herein may also be used for preliminary evaluations of scenarios involving deflagrations, such as explosions of LPG tanks. However, complex analyses are required for determining the TNT equivalent weight related to deflagrations, typically resulting in the production of ad-hoc scenario-specific solutions. Moreover, models which are able to predict mid-field deflagration effects end up being impractical for the characterization of far-field deflagrations, and vice versa. To conclude, in general, no great accuracy is achieved by estimating near-field deflagration effects via TNT equivalence methods [16] [17].

The blast pressure-time history at a fixed point in space is well described by the modified Friedlander equation, which reads [18]:

\[ P(t) = P_0 + P_s \left( \frac{t - t_{a}}{t_d} \right) e^{-\frac{t - t_{a}}{t_d}} \]  \hspace{1cm} (2.1)

where \( P(t) \) represents the absolute pressure registered at the point of interest at time \( t \) after the detonation, while \( P_0 \) identifies the undisturbed atmospheric pressure, \( P_s \) the peak overpressure, \( t_a \) the time of arrival of the blast wave at the point of interest, \( t_d \) the positive phase duration and \( b \) the decay coefficient. The classical curve described by equation (2.1) is shown in figure 1.

![Figure 1. Typical blast wave pressure-time history.](image)

As it is visible in figure 1, the pressure-time history is characterized by two distinct phases. After the required time span for the blast wave to reach the point of interest, at time instant B the positive phase starts with a peak overpressure, rapidly decaying to a value below the undisturbed atmospheric pressure, commonly named negative phase. During the positive phase, the impacted structure is crushed by the
blast force, while in the following negative phase a reversed blast wind arises, further damaging the target.

Many empirical methods were developed in the past decades for the characterization of the main properties of the blast wave pressure-time history curve. Since tests were normally conducted at small scales, a scaling law was introduced to evaluate the effects of large-scale explosions. The most used scaling law is the independently formulated law by Hopkinson [19] and Cranz [20]. According to the selected law, the scaled distance \( Z \) is defined as:

\[
Z = \frac{R}{W_{TNT}^{\frac{1}{3}}}
\]

(2.2)

where \( R \) is the distance of the point of interest from the detonation location and \( W_{TNT} \) the already introduced TNT equivalent weight.

The approach proposed in this work characterizes the main properties of the blast wave pressure-time history exploiting consolidated empirical models present in the literature. Below, only those values which are relevant to this work are introduced. The peak overpressure \( P_S \) and the positive phase impulse \( i_S \), which is the area under the curve describing the positive phase, are estimated using the model developed in [3]. The estimation of those values leads to the exploitation of the approach presented in [11] in order to compute the fictitious positive phase duration \( t_{of} \). That approach consists of estimating the fictitious value \( t_{of} \) which allows the approximation of the Friedlander equation positive phase as triangular-shaped, while still preserving the predicted overpressure and impulse. The negative phase is neglected since, according to experimental evidence, it typically contributes to damaging the structure far less extensively than the positive phase [1] [11]. No further focus on those empirical methods is reported herein, since an in-depth analysis of those models is not the main aspect of this work. The interested reader is referred to the vast literature on this topic, such as [1], [3] and [6]. All the equations cited above, which are implemented in the proposed methodology, are extensively reported in the Appendix.

As soon as the blast wave strikes a structure, complex interface mechanisms arise, which involve the formation of a new blast wave reflected by the impacted surface. This reflected blast wave, which interacts with the initial incident wave, typically produces a pressure-time history on the impacted structure characterized by a greater exerted pressure value than the value resulting from the incident blast wave. Hence, an accurate characterization of the reflected pressure-time history is required. Numerical codes implementing an analytical characterization of the blast loads, such as LS-DYNA® featuring the CONWEP model, are only able to compute the reflected pressure arising from a normal impact, exploiting the Kingery-Bulmash relationships [6]. Furthermore, in order to compute the effective pressure exerted on the impacted structure, those codes rely on the following equation [10]:

\[
\text{pressure load} = \text{reflected pressure} \cdot \cos^2(\alpha) + \text{incident pressure} \cdot (1 + \cos(\alpha) - 2\cos^2(\alpha))
\]

(2.3)

which considers the incidence angle \( \alpha \) (defined in figure 2) by means of cosine-like functions. However, that method is unable to capture the abnormal behaviour of the reflected pressure value in case of angles of incidence greater than 40°, which is measured in experimental analyses. This abnormal behaviour is visible in figure 2, where the reflected pressure coefficient, i.e., the ratio between the reflected pressure value \( P_r \) and the peak overpressure \( P_S \), is shown at various angles of incidence. For the sake of clarity, according to the angle of incidence definition reported in figure 2, an impact at an angle of incidence of 0° identifies a normal impact.
Figure 2. (a) Reflected pressure coefficient at various angles of incidence [11]. (b) Incidence angle definition.

Hence, the exploitation of cosine-like functions to account for the effect of the angle of incidence may oversimplify the events. Moreover, the relationship in equation (2.3) does not take the geometric dimensions and shape of the impacted surface into account. Thus, the approach presented in this work involves the fitting of the experimental curves describing the reflected pressure coefficient (figure 2) and of those relating the reflected impulse $i_r$ to the positive phase one, in order to accurately capture the physical phenomenon of the blast wave reflection on the impacted surface. Moreover, this method allows capturing the eventual abnormal behaviour registered at incidence angles greater than 40°. Note that these fittings of the experimental results are valid over the wide range $4.8\text{KPa} \leq P_s \leq 34473.8\text{KPa}$.

Finally, the reflected pressure time duration $t_{rf}$ is determined according to the procedure described in the work in [11]. Thereby a fictitious time duration is determined which leads to the predicted reflected pressure and impulse values by adopting a triangular-shaped time history. Overall, some analyses have been conducted to determine the lowest value of the scaled distance for which the empirical equations employed guarantee an accurate blast wave characterization with respect to experimental evidence and hydrocode simulations. That value has been identified in $Z = 1\text{m} \cdot \text{kg}^{-1/3}$. Note that these analyses are not reported here for the sake of brevity.

The modelling of the interaction of blast waves with complex structures is not straightforward. As already anticipated above, in commercial numerical codes, such as LS-DYNA®, the problem of accurately characterizing that interface is not properly addressed. In fact, those numerical codes implement analytical models for the blast wave characterization which only involve the application of a combination of the reflected pressure and the incident pressure impacting the structure, without considering the geometric properties of the latter (see equation (2.3)). A more refined model of the blast wave–structure interaction is proposed in the work in [11] for a large number of structural configurations, such as box-like and hemispherical buildings. Moreover, that theory also allows the evaluation of the eventual interaction with openings in the surface of the structure. In this work, for the sake of brevity, the only model for the determination of the effective pressure-time history exerted on the front panel of a box-like structure is reported (figure 3).
Figure 3. Box-like structure considered.

The theory underlying the approach presented in this work is based on some assumptions, which are reported below [11]:
- The peak overpressure $P_S$ is 200 psi or less;
- In case of a surface burst, the Mach stem extends above the height of the structure.

For a detailed analysis of surface bursts and the Mach stem, the interested reader is referred to the work in [21]. Under those assumptions, figure 4 presents the pressure-time history acting on the front face of the impacted structure.

Figure 4. Effective pressure exerted on the front panel of a box-like structure.

The effective pressure-time history exerted on the front face of the box-like structure (solid line) is clearly different from the only contribution of the reflected blast wave (dashed line). In particular, at the time of arrival, identified with $t = 0$ in figure 4, the instantaneous pressure value felt by the surface is the reflected pressure value. This value decays linearly up to the clearing time $t_c$, which is the time required to relieve the reflected pressure from the impacted surface. The value of time $t_c$ is uniquely determined by the geometric dimensions of the front panel of the structure and by the sound speed in the air at the thermodynamic conditions in the reflected region. From time $t_c$ up to the fictitious positive phase duration time $t_{of}$, the effective pressure felt by the structure is determined by the combination of the overpressure value of the positive phase of the incident blast wave and that of the force determined by the blast wind behind the blast front, which gives rise to the dynamic pressure $q$. The dynamic pressure value, computed according to the procedure developed by Sir Geoffrey Tailor in the work in [2], has to be multiplied by a drag coefficient $C_D$, which holds the unity value under the hypotheses reported above. For an in-depth insight into the meaning of the values reported in figure 4, the interested reader is referred to chapter 2-15 of the work in [11]. Note that, as already stated before, the negative phase contribute is neglected in this work.

However, the theory presented above is valid in case the peak overpressure $P_S$ is 200 psi or less. This is not particularly limiting, since common explosions lead to overpressures compatible with that hypothesis. Nonetheless, two further considerations may be given to relax that assumption. The first
one is related to the value of the drag coefficient $C_D$, which has to be properly investigated in case of high overpressure ranges [11]. In fact, at peak overpressure values higher than 200 psi the assumption $C_D = 1$ is not consistent with the physics of the phenomena involved. The second important consideration in adopting the effective pressure-time history described above is related to the short duration of the pressure pulse, which may lead to inaccurate results. In order to overcome this second issue, a solution is proposed in the UFC 3-340-02. The impulse under the solid curve shown in figure 4 may be compared to the one coming from the reflected wave only (dashed curve in the same figure). Whichever curve gives the smallest value of the impulse should be used in calculating the wall loading in those extreme events analyses [11].

3. Case study
This Section presents a detailed comparison of the effective pressure-time history computed through three different approaches. The structure considered in this case study is a thin-walled box-like structure. The first approach considered is the one described in Section 2, which includes both the refined prediction of the reflected blast wave characteristics and the detailed modelling of the interface phenomena. The second method exploited for building the comparison only involves the characterization of the reflected pressure and impulse as described in the previous Section, directly applying them on the front face of the box, without modelling the blast-wave structure interaction. Lastly, the prediction made employing the CONWEP equations implemented in LS-DYNA® is compared to the results from the other approaches. As already anticipated in Section 2, prior analyses showed that both the empirical equations implemented in the CONWEP method and the ones presented in this work agree with experimental and hydrocode results for scaled distance values $Z > 1 \text{m} \cdot \text{kg}^{-1/3}$. Hence, the scenario investigated in this case study consists of the free-field detonation of 20 kg of TNT at a 6m stand-off distance from the front face of the structure, which has dimensions 500mm$x$500mm. Thereby no limitations of the empirical equations arise, since it holds $Z = 2.2 \text{m} \cdot \text{kg}^{-1/3}$. The box is floating in the air, therefore ground reflection effects are excluded from the simulation. Initially, a normal impact, i.e., an impact characterized by a null angle of incidence, is considered. The selected configuration is shown in figure 5.

![Figure 5. Scenario considered in the case study.](image)

The results from the three selected methods are shown in figure 6.
As expected, the peak effective pressure values are quite similar: the CONWEP algorithm predicts a value of $483 \, kPa$, while the method presented in this work estimates a value of $517 \, kPa$. Considering the CONWEP value as the reference value, the absolute error between the two peak pressures is 7%. Such a minimal discrepancy is expected since, as already mentioned above, the CONWEP equations predict the normal reflected pressure exploiting the Kingery-Bulmash equations, while a different set of empirical relationships are implemented in the proposed software package. Note that in case of a normal impact, the CONWEP equation determining the effective pressure load only considers the reflected blast wave (see equation (2.3)). Comparing the impulses imparted on the front panel of the structure, the method proposed in this work predicts a value of $474 \, Pa \cdot s$, while the only reflected pressure impulse and the impulse computed with the CONWEP approach, which should be extremely similar, hold $586 \, Pa \cdot s$ and $577 \, Pa \cdot s$, respectively. Considering the CONWEP impulse value as the reference value, the error characterizing the reflected impulse computed by the empirical characterization presented in this work is 2%. However, considering the accurate description of the blast wave-structure interface phenomena as shown in the previous Section, the effective impulse acting on the structure is reduced to the 82% of the one from the CONWEP characterization. Those are expected results, since the CONWEP method directly applies the only impulse coming from the reflected blast wave to the impacted structure in case of normal impacts (see equation (2.3) with null incidence angle $\alpha$), while the complete approach proposed in this work further processes that value according to the procedure reported in Section 2. All the three methods predict a peak incident overpressure $P_i \cong 0.16 \, MPa \cong 23 \, psi$. That value, according to figure 2, should produce a reflected pressure peaking at an incidence angle slightly greater than 40°. That peak, however, cannot be captured by the CONWEP method, which is only able to determine the reflected value at a normal incidence. Hence, a comparison of the three methods is reported in case the same blast wave considered above impacts the structure at an incidence angle of 41°. The comparison of the results from the three selected methods is shown in figure 7.

**Figure 6.** Case study results – incidence angle 0°.
Figure 7. Case study results – incidence angle 41°.

The method proposed in this paper predicts an effective pressure peak value of 550kPa, which is greater than the one registered at null incidence angle, i.e., 517kPa. However, as expected, the results from the CONWEP equations provide a peak effective pressure value lower than the one estimated at a null angle of incidence, i.e., 370kPa instead of 483kPa. That effective pressure value may be introduced in equation (2.3), along with the incident pressure peak value and the incidence angle, i.e., \( P_2 \cong 0.16\text{MPa} \) and \( \alpha = 41^\circ \), respectively, to determine the peak reflected pressure value computed via the CONWEP approach. It turns out that the peak reflected pressure value in this scenario holds 483kPa, which is exactly identical to the value identified in the normal impact case study. This result is coherent with the theory underlying the blast wave characterization method implemented in LS-DYNA®, which only allows the determination of the normal reflected pressure, which is further introduced in equation (2.3) to determine the peak reflected pressure value at the blast arrival (figure 4), the CONWEP effective pressure is compared to the reflected one determined by the methodology proposed in this work, to determine eventual differences in the applied peak effective pressure. The result of the sensitivity analysis is reported in figure 8. As expected, differently from experimental evidence, no peak is provided by the CONWEP approach at incidence angles in the range \( \alpha = [40^\circ; 50^\circ] \), while the proposed methodology successfully detects that abnormal behaviour.
4. Conclusions

Characterizing and assessing explosive events is a challenging task both in the research and in the more applicative fields. Hence, many empirical methods were developed in the past decades and were successfully integrated into commercial numerical codes. However, they lack of accuracy both in the description of the reflection phenomenon and in the characterization of the blast wave–structure interaction. Yet, an accurate description of those events is obtainable from hydrocode simulations, which are, however, time and resources consuming, thus not directly applicable in the initial design phases of a platform. The method proposed in this work aims at combining the capabilities of the consolidated empirical methods with the accurate blast wave–structure interaction description reported in the work in [11]. That method is ready to be easily implemented into finite element codes for fast, reliable preliminary predictive analyses of explosive phenomena and their effects on simple structures. An advantage brought by the proposed method is that no hydrocode simulations are needed to accurately capture the blast wave–structure interface phenomena, which are complex in nature, thus reducing the enormous time and resources consumption required by such analyses. However, if a complex scenario with more than one structure and many blast wave reflections is considered, at the moment the only suitable and accurate approach is still represented by hydrocode simulations. The presented approach also includes in the prediction of the reflected blast wave properties the abnormal behaviour seen experimentally for incident blast waves at angles greater than 40°. The method presented in Section 2 has been applied in Section 3 to a case study involving the free-field detonation of TNT in front of a thin-walled box-like structure. The results from the proposed method, compared to the classic CONWEP algorithm results, provide a slightly different pressure-time history exerted on the front panel of the target structure. That difference may reveal to be effective in evaluating different design solutions for structures subject to blast waves. Nevertheless, not all open issues have been solved. One possible extension of this approach may be the more accurate characterization of the effective pressure-time history exerted on a structure in case of peak overpressure values above 200 psi. Moreover, integrating the proposed approach with empirical theories on the deformation of simple elements, such as the theories in the works in [22] and [23], may lead to the development of a stand-alone software package with real-time preliminary predictive capabilities.
Appendix

This Appendix is aimed at introducing the equations underlying the methodological approach described in this work. Those relationships are taken from the quite ample literature on the blast wave empirical characterization and joined in order to provide an accurate, yet fast method for analytically characterizing a blast wave and the interface phenomena arising in the interaction with a structure.

The general curve describing the pressure-time history at a fixed point in space due to a blast wave passing through it is shown in figure 1. The positive phase of the curve, in which an overpressure to the undisturbed ambient pressure arises, is characterized by a peak overpressure value $P_S$ and a total specific impulse $i_S$. The variables $P_S$ [Pa] and $i_S$ [Pa $\cdot$ s] are given by the following empirical equations [3]:

$$P_S = P_0 \cdot 808 \cdot \left[ 1 + \left( \frac{Z}{4.5} \right)^2 \right] \frac{1}{\sqrt{1 + \left( \frac{Z}{0.048} \right)^2}} \frac{1}{\sqrt{1 + \left( \frac{Z}{0.32} \right)^2}} \frac{1}{\sqrt{1 + \left( \frac{Z}{1.35} \right)^2}} \quad (A.1)$$

$$i_S = 6.7 \cdot \frac{\left[ 1 + \left( \frac{Z}{0.23} \right)^2 \right]^4 \cdot \left[ 1 + \left( \frac{Z}{1.55} \right)^2 \right]^3}{\sqrt{WTNT}} \quad (A.2)$$

where $Z$ and $WTNT$ are the scaled distance and the equivalent TNT weight introduced in Section 2, respectively. These two variables are exploited for determining the fictitious positive phase duration $t_{of}$ [s], which defines a triangular-shaped positive phase, equivalent in the peak overpressure and specific impulse values to the one described by the Friedlander equation [11]:

$$t_{of} = 2 \cdot \frac{i_S}{P_S} \quad (A.3)$$

As the blast wave strikes the target structure, the wave is reflected by the impacted surface giving rise to a reflected blast wave. That wave needs to be characterized in terms of peak pressure $P_r$, impulse $i_r$ and fictitious time duration $t_{rf}$. Within the methodological approach proposed in this work, in order to capture the abnormal behaviour of the reflected pressure, which is typically registered at incidence angles greater than 40°, a fit of the curves in figure 2 is computed and used for retrieving the reflected pressure coefficient value $c_{r,\alpha}$. Thus, the reflected peak pressure directly follows [11]:

$$P_r = c_{r,\alpha} \cdot P_S \quad (A.4)$$

A similar procedure is followed for determining the reflected impulse. The experimental curves determining the scaled reflected impulse $i_r$ [Pa $\cdot$ s $\cdot$ kg$^{-1/3}$] related to a specific couple of incident overpressure $P_S$ and incidence angle $\alpha$ values, which are reported in the work in [11], are fit and implemented in the framework. Thus, the reflected impulse is determined as [11]:

$$i_r = \sqrt[3]{\frac{1}{C}} \cdot \sqrt{WTNT} \quad (A.5)$$

Finally, according to the very same procedure already described for determining the fictitious positive phase time duration $t_{of}$, the fictitious reflected pressure time duration $t_{rf}$ [s] is given by [11]:

$$t_{rf} = 2 \cdot \frac{i_r}{P_r} \quad (A.6)$$

Moreover, right behind the blast wave front some wind blows determining the dynamic pressure $q$ [Pa], which is computed as [2]:

$$q = \frac{1}{2} \rho \frac{V^2}{2} \quad (A.7)$$
where \( t \) identifies the time variable and \( P_2(t) \) the positive phase pressure value at time \( t \).

As it regards the interface phenomena arising as the blast wave strikes the front face of a box-like structure, the clearance time value \( t_c \) must be estimated. To recall, that is the time needed for relieving the reflected pressure from the reflecting surface. The equation governing the value of \( t_c [s] \) reads [11]:

\[
t_c = \frac{4S}{(1+R)C_r}
\]  \hspace{1cm} (A.8)

where \( S \) and \( R \) are geometry-dependent parameters and \( C_r \) is the sound velocity in the reflected wave region. The latter is estimated from the fit of empirical results reported in the work in [11]. Moreover, the expressions of the clearing distance \( S \) and of the parameter \( R \) depend on the type of explosion considered in the assessment. For instance, if a hemispherical explosion is considered, i.e., an explosion right above the ground, those relationships read [11]:

\[
S = \min\left(H, \frac{B}{2}\right)
\]  \hspace{1cm} (A.9)

\[
R = \frac{S}{\max\left(H, \frac{B}{2}\right)}
\]  \hspace{1cm} (A.10)

where \( H \) and \( B \) are the height and the breadth of the reflecting surface, respectively (figure 3).

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