Preservation of atomic coherence in double-well optical lattice in presence of decoherence

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Abstract

We present a quantum interference approach to preserve coherence in the external states of an atom trapped in an optical lattice. We show that this is possible by suitably choosing the initial state of the atom. We demonstrate this in context of decoherence due to spontaneous emission in an one-dimensional optical double-well lattice.

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I. INTRODUCTION

Tunnelling in a double well lattice is a coherent phenomenon. This occurs due to quantum interference between the states of the atoms in the lattice. Grossman et al. [1] has shown that by driving a double-well potential by strong coherent field, one can slow down the tunnelling of the wave packet in one of the wells (left or right) and essentially can have coherent suppression of tunnelling. This has been implied as dynamical localization of wave packet. Once the field is switched off, the tunnelling is resumed, reflecting the preservation of coherence in the wave packet. Such coherent control can also be done in molecular systems by using frequent sequence of short pulses [2].

It is a natural question how any incoherent event would affect a double-well system. Caldeira and Leggett in their seminal paper [3] had modelled the decoherence as a phenomenological "frictional force" and showed that the tunnelling of the wave function is slowed down by the decoherence process. Dynamical localization of two-level atoms in presence of spontaneous emission has been studied in [4], which is an effect of loss of coherence in the atoms.

In this paper, we focus on how to preserve the atomic coherence in the optical lattice in presence of decoherence. We present a technique that relies on a quantum interference approach as originally proposed by Shapiro and Brumer [5]. A suitable choice of initial superposition of the atomic states in the lattice creates different pathways of evolution of the states. The quantum interference of these pathways leads to control in the evolution of the initial state. Note that such a method has been successful in controlling various molecular processes, including photodissociation, scattering cross section etc. [5].

We demonstrate this technique in context of a double-well optical lattice, prepared by a set of four counter-propagating laser fields [6]. Specific choices of field intensity and the polarizations of the participating fields create this lattice. Within the pair, the barrier height and the relative depths of the two potential minima sites are externally controllable. Such lattice has been studied to implement two-qubit phase gate [7] and to demonstrate controlled exchange interaction between atoms [8]. Note that in [9], the spontaneous emission from a two-level atom in double-well lattice has been studied. In the present paper, we propose a coherent control technique to combat the spontaneous emission of atoms, the states of which are allowed to be expanded over all bound energy eigenstates.
The structure of the paper is as follows. In Sec. II, we describe the double-well optical lattice. We discuss how different external initial states of the atoms trapped in the lattice evolve with time. In Sec. III, we investigate the effect of the decoherence. We discuss the control mechanism by choice of initial states. We conclude the paper in Sec. IV.

II. TUNNELLING IN DOUBLE-WELL OPTICAL LATTICE

We start with a double-well optical lattice as described in [6]. A single laser beam initially propagating in the $x$-direction intersects with itself at the position of the cold atom four times through reflection in suitably positioned mirrors. This prepares two pairs of counter-propagating beams in $x$ and $y$ direction. One can have 2-dimensional square lattice if the polarization of the beams are in $x−y$ plane ("in-plane lattice") or in perpendicular plane (parallel to $z$-axis; "out-of-plane" lattice). The double-well lattice in $x−y$ plane can be generated by combining the "in-plane" and "out-of-plane" polarizations. The two-dimensional lattice potential thus constructed can be written as

$$V(x,y) = -\frac{V_{xy}}{4}[(1 - Z_f)V_1(x,y) + Z_fV_2(x,y)],$$

(1)

where $V_{xy}$ is the potential depth, $Z_f$ is the ratio of the intensities of the "in-plane" and "out-of-plane" lattices, and

$$V_1(x,y) = 4 + 2\cos(2kx - 2\theta_{xy} - 2\phi_{xy})$$

$$+2\cos(2ky + 2\phi_{xy}),$$

$$V_2(x,y) = 4 + 4\cos(kx + ky - \theta_z)$$

$$+4\cos(kx - ky - \theta_z - 2\phi_z)$$

$$+2\cos(2kx - 2\theta_z - 2\phi_z) + 2\cos(2ky + 2\phi_z),$$

(2)

where $k = 2\pi/\lambda$, $\theta_{xy}$ ($\theta_z$) and $\phi_{xy}$ ($\phi_z$) are the phases of the in-plane (out-of-plane) lattice.

We focus on the dynamics of the atomic states in one dimension, e.g., in $x$ direction. The potential in this direction resembles a double well potential and is given by

$$V(x, y = 0) = -\frac{V_{xy}}{4}[6 + 2(1 - Z_f)\cos(2kx - 2\theta_{xy})$$

$$+2\cos(2kx - 2\theta_z) + 8\cos(kx - \theta_z)].$$

(3)
FIG. 1: Structure of the double-well $V(x)$ in coordinate space for $Z_f = 0.05$ (solid line) and $Z_f = 0.1$ (dashed line). The other parameters are $V_{xy} = 36E_R$, $\delta\theta = \pi/2$, $\delta\phi = 0$, and $E_R = 3.5$ kHz. Clearly, $Z_f = 0.05$ represents higher barrier.

A typical form of $V(x)$ is shown in Fig. 1. The width of each double well is $\lambda$. The phase-differences $\delta\theta = \theta_z - \theta_{xy}$ and $\delta\phi = \phi_z - \phi_{xy}$ control the tilt of the optical lattice and $Z_f$ controls the barrier height of the double-well.

The eigenfunctions of the lattice can be calculated using Fourier expansion method, which involves expansion of the eigenfunction in terms of its Fourier components as follows:

$$\phi(x) = \sum_n d_ne^{2\pi inkx} .$$

The normalization assures that $\sum_n |d_n|^2 = 1$. We here consider only the states with zero quasi-momentum $q = 0$. To solve for the eigenfunction of the Hamiltonian $H = (p_x^2/2m) +$
$V(x)$, where $p_x$ is the momentum of the atom of mass $m$ in the lattice, we use the above expression of $\phi(x)$ in the eigenvalue equation $H\phi(x) = E\phi(x)$, to obtain a set of linear algebraic equations of $d_n$'s. The eigenvalues of the corresponding matrices are the same as those of $H$. We found that the Hamiltonian $H$ has two near-degenerate lowest energy wave functions $|0\rangle$ and $|1\rangle$.

We next study the propagation of the wave functions using the Schrodinger equation $i\dot{\psi} = H\psi$. This equation can be solved quite efficiently using split-operator-Fourier transform method. In our numerical simulations, we consider 20 double wells ranging from $x = -9.75\lambda$ to $x = 10.25\lambda$ and a grid size of 512.

In the following, we will consider two different initial conditions: (i) Superposition of the eigenstates $|0\rangle$ and $|1\rangle$ as $|L\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$. (ii) Superposition of a few eigenstates including $|0\rangle$ and $|1\rangle$, that can be written as $|\psi_G(x)\rangle = \sum_l c_l|l\rangle$, where $|l\rangle$ represents the energy eigenfunctions of the Hamiltonian $H$. We will investigate the evolution of these two initial states. Various eigenstates lead to a quantum interference of the corresponding time-independent amplitudes. We investigate the effect of this interference in the decay rate of the initial coherence of the state.

We consider the initial states in such a way that the wavefunction resides in the left wells. We calculate the time-evolution of the probability that the wave function remains in the initial left wells which is given by

$$P_L(t) = \sum_i \int_{x_{\text{min},i}}^{x_{\text{max},i}} dx \rho(x, x; t),$$

where $x_{\text{min},i}$ and $x_{\text{max},i}$ are the lower and upper limits of the $i$th left well and $\rho(x, x; t)$ represents the diagonal elements of the density matrix of the atom. We show in Fig. 2 the evolution of $P_L(t)$ for the state $|L\rangle$. Clearly, the wave function tunnels through the barrier back and forth between the left and the right well. The characteristic time-scale of this tunnelling is found to be $\sim 2/\delta$ where $\delta$ is the energy splitting of the of the states $|0\rangle$ and $|1\rangle$. Note that the tunnelling occurs because of the coherence between the wave function at left and right well.

Next, for the state $|\psi_G(x)\rangle$, we choose the amplitude coefficients $c_l$ for the zero-quasi-momentum eigenstate $|l\rangle$ as $c_0 = -0.6785$, $c_1 = -0.677i$, $c_2 = 0.0924$, $c_3 = 0.10107i$, $c_4 = 0.15$, $c_5 = -0.1602i$, $c_6 = 0.0976$, $c_7 = 0.05777i$, $c_8 = 0.0305$, and $c_9 = -0.02056i$. The
FIG. 3: Variation of the probability that the single wave packet remains in the initial left well (solid line) at $x = 0$ and in the right well (dotted line) with time for (a) $Z_f = 0.05$ and (b) $Z_f = 0.1$. The parameters are $\sigma = 0.1\lambda$, $V_{xy} = 36E_R$, $\delta\theta = \pi/2$, $\delta\phi = 0$, and $E_R = 3.5$ kHz.

overlap with the other eigenstates is negligible. This state is equivalent to a Gaussian wave packet $\psi_G(x) \equiv \exp(-x^2/2\sigma^2)$ which is peaked at the left well with center at $x = 0$. The width $\sigma$ of this wave packet is much less than that of the left well. For the above set of values of $\{c_i\}$, $\sigma = 0.1\lambda$. We show in Fig. 3, the temporal behavior of the probability $P_L$. Clearly, the wave packet tunnels, but never returns back to the initial left well completely. It slowly disperses over both the wells in the lattice. The figure for the probability in the initial left well is complemented using the figure for the probability in the corresponding right well.

III. EFFECT OF DECOHERENCE

We next consider the effect of decoherence on the evolution of these initial atomic states in the double-well. The decoherence can occur due to spontaneous emission and the fluctuation of the intensity of the laser fields. In this paper, we focus on the case of spontaneous emission. The atom emits a photon of certain wavelength $\lambda'$ while in the excited electronic states. This imparts a recoil momentum $p' = \hbar c/\lambda'$ on the lattice [10], leading to an extra phase of the instantaneous wave function as follows:

$$\psi(x, t) \rightarrow \psi(x, t)e^{-ip'.\vec{r}/\hbar}.$$  \hspace{1cm} (6)

Due to the randomness of the spontaneous emission, the atom emits a photon at any random direction. Thus, $\vec{p}'$ can be random and the time at which the phase kick occurs can also
FIG. 4: Temporal variation of the probability that the wave function $|L\rangle$ remains in the initial left well for (a) kick strength $m = 10$: kick rates 10 Hz (thick solid line), 100 Hz (dashed line), and $10^4$ Hz (thin solid line), (b) kick strength $m = 100$: kick rates 1 Hz (thick solid line), 100 Hz (dashed line), and $10^4$ Hz (thin solid line). We chose $Z_f = 0.1$. The other parameters are the same as in Fig. 2.

FIG. 5: Temporal variation of the survival probability of the wave function $|L\rangle$ for (a) kick strength $m = 10$: kick rates 10 Hz (thick solid line), 100 Hz (dashed line), and $10^4$ Hz (thin solid line), (b) kick strength $m = 100$: kick rates 1 Hz (thick solid line), 100 Hz (dashed line), and $10^4$ Hz (thin solid line). The other parameters are the same as in Fig. 4.

be random. Further, because we consider an one-dimensional optical lattice in $x$ direction, we consider the components of the recoil momentum only at $x$ direction. Thus in spherical coordinate, the random phase can be written as $\exp[-i k' x \sin(\theta) \cos(\phi)]$, where $k' = 2\pi/\lambda'$, and $\theta$ and $\phi$ represent the random direction of the outgoing photon. We write $\lambda' = \lambda/m$, where $m$ defines the strength of the phase kick. Because, the wavelength of the standing wave in an optical lattice has to be much larger than the atomic wavelength and the detuning
FIG. 6: Temporal variation of the probability that the single Gaussian wave packet $\psi(x)$ remains in the initial left well [Figs. (a) and (b)] and the corresponding right well [Figs. (c) and (d)] for kick rates $10 \text{ Hz}$ (thick solid line), $100 \text{ Hz}$ (dashed line), and $10^4 \text{ Hz}$ (thin solid line) – (a), (c) : $m = 10$ ; (b), (d) : $m = 100$. The other parameters are as in Fig. 3(b).

FIG. 7: Variation of the survival probability of the single Gaussian wave packet with time for kick rates $10 \text{ Hz}$ (thick solid line), $100 \text{ Hz}$ (dashed line), and $10^4 \text{ Hz}$ (thin solid line): (a) $m = 10$, (b) $m = 100$. The other parameters are the same as in Fig. 3(b).
of the laser from the trapped atom, it is reasonable to consider a large value of \( m \). Further, note that we are considering the decoherence in the external states of the atom (due to its trapping in a lattice), that occurs due to the decoherence in the internal states (electronic states) of the atom. We here focus on how to reduce the effect of this decoherence in the external state of the atom. We here analyze the time-evolution of the initial wave function from the first principle using the Schrodinger equation, not in the framework of master equation. In case of master equation, one takes average over the bath degrees of freedom and thus loses the essence of the physics that deals with random phase kick. Our numerical procedure comprises of the following steps: We calculate the wavefunction at each \( dt \) time-
interval. (i) We evolve the wave function for a time $\delta t$ ($\leq dt$) if there occurs a phase kick at random time $\delta t$. (ii) Next we incorporate a random phase on the instantaneous wave function at the time $\delta t$, according to Eq. (6). (iii) The wave function again evolves for the time-interval $dt - \delta t$. If multiple phase kicks occur in a given interval $dt$, we evolve the wave function under the action of the Hamiltonian $H$ during the time-intervals between two random kicks. (iv) We find the wavefunction at $dt$ time-interval. We use a Monte Carlo method over $N = 50$ iterations. Thus at any time $t$, the density matrix of the atoms can be written as $\rho(x, x'; t) = (1/N) \sum_{k=1}^{N} \rho_k(x, x'; t)$, where $\rho_k(x, x'; t) = \psi_k(x, t)\psi_k^*(x', t)$ and $\psi_k(x, t)$ is the instantaneous wave function of the atom for the $k$th iteration. The expression for the probability $P_L(t)$ in the initial left well can then be written as Eq. (5). In the following, we also investigate the variation of the survival probability as given by

$$F(t) = \int dx \int dx' \psi(x, t = 0)\rho(x, x'; t)\psi^*(x', t = 0),$$

and the variation of the purity of the wave function as given by

$$M(t) = \int dx \int dx' \rho(x, x'; t)\rho(x', x; t).$$

A. Case of the state $|L\rangle$

We first investigate the evolution of the state $|L\rangle$ which is a superposition of the two lowest-eigenenergy zero-quasi-momentum states $|0\rangle$ and $|1\rangle$. For a small rate of random phase kicks ($= 1$ Hz or 10 Hz), the atoms tunnel back forth between the left and the right well. This means that the state remains almost coherent. This is true for both strong ($m = 100$) and weak kicks ($m = 10$). However for a larger kick rate ($\sim 100$ Hz), the larger the kick strength $m$, the more the wavefunction gets diffused throughout the double-well. One can see from Fig. 4 that the probability $P_L(t)$ that the wavefunction remains in the initial left well becomes $\sim 0.5$ at longer times. This is a signature of the decoherence. We have also plotted the survival probability $F(t)$ of the initial wave function to see the nature of decoherence. In Fig. 5, we show how $F(t)$ decays very rapidly with times if one increases the kick strength as well as the kick rate. The similar nature of the variation of probability being in the initial wells and the survival probability reflects the fact the decoherence occurs more due to population redistribution among the wells, rather than any dephasing.
B. Case of the Gaussian wave packet

We next consider a Gaussian wave packet with a width $\sigma = 0.1\lambda$, such that the wave packet exactly fits into the left well located at $x = 0$. At the boundaries of the double-well, the wave packet almost vanishes. This wave packet is a superposition of several eigenfunctions of the lattice, as mentioned before.

We show in Fig. 6 the temporal variation of the probability that the wave packet remains in the initial left well. Clearly, for larger kick strength and larger kick rate, the wave packet gets diffused throughout the lattice. The much larger kick rate leads to faster diffusion without any oscillation. However, for weak kick strength [see Figs. 6(a), 6(c)], the wave packet starts oscillating back and forth between the initial left well and the right well. This result is complemented by the plot of the survival probability in Fig. 7, which shows that the state is preserved for the corresponding parameters.

A comparison of the Figs. 5(a) and 7(a) reveals that for an initial Gaussian wave packet, a moderate kick strength preserves the survival probability for longer times, whereas for an initial state $|L\rangle$, this vanishes rapidly. This observation leads us to the main result of our paper: A suitable superposition of several wave functions provides longer preservation of the state of the system than a superposition of a fewer wave functions. Note that the Gaussian wave packet is made up of several eigenfunctions of the system, where the state $|L\rangle$ is a superposition of only two eigenfunctions.

As the survival probability does not refer coherence, we further choose to study the temporal behavior of the purity $M(t)$. $M(t) = 1$ refers to a pure state. In Figs. 8 and 9, we have shown that the purity in the Gaussian wave packet decays in a slower rate than that in the state $|L\rangle$ for different sets of values of kick rate and kick strength. For a moderate kick rate (100 Hz) and a moderate kick strength ($m = 10$), the Gaussian wave packet exhibits a purity $\sim 0.8$, which is much larger than that ($\sim 0.3$) of the state $|L\rangle$ at a time $t \sim 40/E_R$ [see Fig. 8(a)]. This further verifies the fact that a suitable superposition of the energy eigenfunctions exhibits a better preservation of coherence in the external states of the atoms.
IV. CONCLUSIONS

In conclusions, we show how the atomic coherence in an optical double-well lattice can be preserved by a suitable control technique. We choose an extended lattice in one dimension and model the decoherence as a sequence of random phase kicks during spontaneous emission. We show that upon suitable choice of particular initial superposition of the lattice eigenfunctions, one can preserve the coherence in atomic external state even in the presence of decoherence in its internal states. This control relies upon the quantum interference between the several pathways that are led due to initial choice of the superposition states.

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