Modulus stabilization and IR-brane kinetic terms in gauge-Higgs unification

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Abstract

We discuss the modulus stabilization by the Casimir energy and various effects of IR-brane kinetic terms for the gauge fields in a gauge-Higgs unification model in the warped spacetime, where the Wilson line phase $\theta_H$ is determined as $\theta_H = \pi/2$. We find that the brane kinetic terms with $O(1)$ coefficients are necessary for the modulus stabilization. On the other hand, large brane kinetic terms can deviate 4D gauge couplings from the standard model values and also cause too light Kaluza-Klein (KK) modes. In the parameter region that ensures the modulus stabilization, the KK gluon appears below 1 TeV, which marginally satisfies the experimental bound. The allowed parameter region will be enlarged in a model where a smaller value of $\theta_H$ is realized.

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1 Introduction

The gauge-Higgs unification scenario is an interesting candidate for the physics beyond the standard model, which was originally proposed in Refs. [1] [2] and revived by Refs. [3] [4] as a solution to the hierarchy problem. In this class of models, the Higgs mass is protected against large radiative corrections thanks to a higher-dimensional gauge symmetry [5]. Since the Higgs field is identified with an extra-dimensional component of a higher-dimensional gauge field, this class of models do not require any elementary scalar fields, which often cause a hierarchy problem due to large radiative corrections to their masses.

The gauge-Higgs unification has been first investigated in the flat spacetime [6] [7]. These models have common features that the physical Higgs boson and the Kaluza-Klein (KK) excitation modes become too light to satisfy the experimental bounds unless the Wilson line phase $\theta_H$ along the extra dimension takes a very small value. Besides, the large top quark mass is not realized in simple models although there is an elaborate way to realize it. These difficulties are easily solved in the Randall-Sundrum warped spacetime [9]. The Higgs and KK masses are enhanced by a logarithm of the large warp factor [10], and the top quark mass can easily be realized only by the localization of the mode functions in the extra dimension [11]. Furthermore, the gauge-Higgs unification in the warped spacetime has phenomenologically interesting features [11]-[15]. Hence, we will focus on the Randall-Sundrum spacetime as a background geometry in this paper.

When we discuss extra-dimensional models, the stabilization mechanism for the size of the extra dimension, which is often called the modulus or the radion, must be considered. One of the simplest mechanisms for the modulus stabilization is proposed in Ref. [16]. A five-dimensional (5D) bulk scalar field plays an essential role for the stabilization in this mechanism. Thus it spoils one of the virtues of the gauge-Higgs unification, i.e., no need to introduce an elementary scalar field. There is another way for the modulus stabilization by using the Casimir energy of the bulk fields. The stability by the Casimir energy has been discussed in many papers [17]-[20], and it has been shown that the bulk gauge field can provide a significant contribution to the effective potential [21]. Thus this mechanism is expected to be feasible in the gauge-Higgs unification scenario because the bulk gauge fields are essential ingredients. Besides, this mechanism does not need any elementary

\[1 \text{In Ref.}[8], \text{the top quark mass is realized by using large Clebsch-Gordan coefficients in higher-dimensional representations of the matter multiplets.}\]
scalar fields. Therefore it is an intriguing subject to discuss the modulus stabilization by
the Casimir energy in the gauge-Higgs unification scenario.

However, the authors of Ref. [21] show that the KK tower of a massless gauge boson
provides a negative contribution to the radion mass squared. Since a contribution of the
gluon KK tower is enhanced by the color factor, the radion tends to be tachyonic and
the extra dimension be destabilized. The authors of Ref. [21] also pointed out that a
non-tachyonic radion mass can be realized by introducing gauge kinetic terms localized on
the IR brane. On the other hand, it is also known that such brane kinetic terms affect
relations among various coupling constants and the KK mass spectra in four-dimensional
(4D) effective theory. Hence it is expected that the magnitudes of the brane kinetic terms
receive some constraints from the current experimental results. It is nontrivial whether the
modulus is stabilized or not within the allowed region of the parameter space of a model.

The purpose of this paper is to discuss the modulus stabilization by the Casimir energy
in a specific gauge-Higgs unification model in the warped spacetime, including the IR-brane
kinetic terms for the gauge fields. We also investigate effects of the brane kinetic terms
on the 4D coupling constants and the first KK gluon mass to obtain constraints on the
magnitudes of the brane kinetic terms.

The paper is organized as follows. In the next section, we provide a brief review of
$SO(5) \times U(1)_X$ gauge-Higgs unification model, including the IR-brane kinetic terms for
the gauge fields. In Sec. 3 the one-loop effective potential for the radion and the Higgs
field is shown. In Sec. 4 we calculate the masses of the radion and the Higgs boson in the
presence of the brane kinetic terms. In Sec. 5 we discuss effects of the IR-brane kinetic
terms on the electroweak gauge couplings of fermions, and the mass of the first KK gluon.
Sec. 6 is devoted to the summary and discussions. We collect functions that determine the
mass spectrum in each sector in Appendix A and provide a brief derivation of the one-loop
effective potential in Appendix B.

2 $SO(5) \times U(1)_X$ model

In this section, we briefly review the $SO(5) \times U(1)_X$ gauge-Higgs unification model, which
was first discussed in Ref. [11]. Several similar models with different matter sectors have
been studied so far. Here we consider a model proposed in Ref. [14] as an example.
We consider the 5D gauge theory compactified on an orbifold $S^1/Z_2$. The background metric is given by

$$ds^2 = G_{MN}dx^Mdx^N = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2,$$

where $M, N = 0, 1, 2, 3, 4$ are 5D indices and $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1, 1)$. The fundamental region of $S^1/Z_2$ is $0 \leq y \leq L$. The function $e^\sigma(y)$ is a warp factor, and $\sigma(y) = ky$ in the fundamental region, where $k$ is the inverse AdS curvature radius. The orbifold has two fixed points $y = 0$ and $y = L$, which are called the UV and IR branes, respectively.

### 2.1 Bulk Lagrangian

The model has gauge fields $A^{(G)}_M, A_M$ and $B_M$ for $SU(3)_C$, $SO(5)$ and $U(1)_X$, respectively. In this article, we consider 5D fermions $\Psi_i$ ($i = 1, 2, \cdots$) belonging to the vectorial representation of $SO(5)$ as matter fields. The 5D bulk Lagrangian is given by

$$\mathcal{L} = \sqrt{-G} \left[ -\frac{1}{4} \text{tr} \left\{ F^{(G)}_{MN}F^{(G)MN} \right\} - \frac{1}{4} \text{tr} \left\{ F^{(A)}_{MN}F^{(A)MN} \right\} - \frac{1}{4} F^{(B)}_{MN}F^{(B)MN} ight. \
+ \sum_i \left\{ i\bar{\Psi}_i \Gamma^N D_N \Psi_i - iM_{\Psi i}\varepsilon(y)\bar{\Psi}_i \Psi_i \right\} \right] + \mathcal{L}_{\text{bd}} + \cdots,$$

where $G \equiv \det(G_{MN})$, $\Gamma^N$ are 5D gamma matrices contracted by the fünfbein, $F^{(G)}_{MN}$, $F^{(A)}_{MN}$ and $F^{(B)}_{MN}$ are field strengths for the $SU(3)_C$, $SO(5)$ and $U(1)_X$ gauge fields, respectively. The covariant derivative of $\Psi_i$ is defined as

$$D_N \Psi_i \equiv \left( \partial_N - \frac{1}{4}\omega^{AB}_N \Gamma_{AB} - ig_C A^{(G)}_N - ig_A A_N - ig_B Q_X B_N \right) \Psi_i,$$

where $\omega^{AB}_N$ are the spin connection, $\Gamma^{AB} \equiv \frac{1}{2} \left[ \Gamma^A, \Gamma^B \right]$, and $g_C, g_A$ and $g_B$ are 5D gauge coupling constants for $SU(3)_C$, $SO(5)$ and $U(1)_X$, respectively. The bulk mass parameters of the fermions $M_{\Psi i}$ are associated with a periodic step function $\varepsilon(y)$, which is required in order for the mass terms to be invariant under the orbifold parity. Terms denoted as $\mathcal{L}_{\text{bd}}$ represent brane-localized terms. The ellipsis in (2.2) denotes the gauge-fixing terms and the ghost terms.

The orbifold boundary conditions at $y_0 \equiv 0$ and $y_1 \equiv L$ are given by

$$A^{(G)}_\mu(x, y_j - y) = A^{(G)}_\mu(x, y_j + y),$$
$$A_\mu(x, y_j - y) = P_j A_\mu(x, y_j + y) P_j^{-1},$$
$$B_\mu(x, y_j - y) = B_\mu(x, y_j + y),$$
$$\Psi_i(x, y_j - y) = P_j \Gamma^5 \Psi_i(x, y_j + y),$$
$$P_j = \text{diag}(-1, -1, -1, -1, +1), \quad (j = 0, 1),$$

(2.4)
which reduce the $SU(3)_C \times SO(5) \times U(1)_X$ symmetry to $SU(3)_C \times SO(4) \times U(1)_X$. The orbifold parities for $A_y^{(G)}$, $A_y$, $B_y$ are opposite to those for $A^{(G)}_\mu$, $A_\mu$, $B_\mu$.

### 2.2 Boundary terms

The boundary conditions in (2.4) can be changed by introducing 4D scalar fields localized on the boundaries whose VEVs give brane-localized masses to the 5D fields. Here we introduce a scalar field $\Phi(x)$ on the UV brane which belongs to $(0, \frac{1}{2})$ representation of $SO(4) \sim SU(2)_L \times SU(2)_R$ and has a charge of $U(1)_X$. Then the $SU(2)_R \times U(1)_X$ symmetry breaks down to $U(1)_Y$, similar to the Higgs mechanism in the standard model. As a result, $A^{1r}_\mu$, $A^{2r}_\mu$ and $A^{3r}_\mu$ acquire large masses at the UV brane. Here

$$
\left( \begin{array}{c}
A^{6a}_M \\
A^Y_M 
\end{array} \right) = 
\left( \begin{array}{cc}
c_\phi & -s_\phi \\
s_\phi & c_\phi
\end{array} \right) 
\left( \begin{array}{c}
A^{3a}_M \\
B_M 
\end{array} \right),
$$

$$
c_\phi \equiv \frac{g_A}{\sqrt{g_A^2 + g_B^2}}, \quad s_\phi \equiv \frac{g_B}{\sqrt{g_A^2 + g_B^2}},
$$

(2.5)

Since the typical energy scale at the UV brane is the Planck scale, it is natural to assume that the VEV of $\Phi$ is much larger than the KK mass scale $m_{KK}$. Then the net effect for low-lying modes is that they effectively obey Dirichlet boundary conditions at the UV brane. Other effects of the introduction of $\Phi$ are irrelevant to the physics below $m_{KK}$.

It is useful to express the $SO(5)$ vector $\Psi = (\psi_1, \cdots, \psi_5)^t$ as

$$
\Psi = \left( \begin{array}{c}
\hat{\psi}_{11} \\
\hat{\psi}_{12} \\
\hat{\psi}_{21} \\
\hat{\psi}_{22} \\
\psi_5
\end{array} \right),
$$

(2.6)

where

$$
\hat{\psi} = \left( \begin{array}{c}
\hat{\psi}_{11} \\
\hat{\psi}_{12} \\
\hat{\psi}_{21} \\
\hat{\psi}_{22}
\end{array} \right) \equiv \frac{1}{\sqrt{2}} \left( \psi_4 + i\psi_5 \cdot \vec{\sigma} \right) i\sigma_2
$$

(2.7)

is a bidoublet for $SU(2)_L \times SU(2)_R$, and $\psi_5$ is a singlet under $SU(2)_L \times SU(2)_R$. Then the quarks in the third generation, for instance, are composed of two 5D Dirac fermions

$$
\Psi_1 = \left[ Q_1 = \left( \begin{array}{c}
T \\
B
\end{array} \right), q = \left( \begin{array}{c}
t \\
b \end{array} \right), t'
\right],
$$

$$
\Psi_2 = \left[ Q_2 = \left( \begin{array}{c}
U \\
D
\end{array} \right), Q_3 = \left( \begin{array}{c}
X \\
Y \end{array} \right), b'
\right],
$$

(2.8)

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2 The introduction of elementary scalar fields is not essential. The boundary conditions can also be changed by condensate of fermion bilinear through some strong dynamics.
and 4D right-handed fermions localized on the UV brane, which belong to the \((\frac{1}{2}, 0)\) representation in \(SU(2)_L \times SU(2)_R\),

\[
\hat{\chi}_1^R = \begin{pmatrix} \hat{T}_R \\ \hat{B}_R \end{pmatrix}, \quad \hat{\chi}_2^R = \begin{pmatrix} \hat{U}_R \\ \hat{D}_R \end{pmatrix}, \quad \hat{\chi}_3^R = \begin{pmatrix} \hat{X}_R \\ \hat{Y}_R \end{pmatrix}.
\]

(2.9)

The \(U(1)_Y\) charges of \(\Psi_1, \Psi_2, \hat{\chi}_1^R, \hat{\chi}_2^R\) and \(\hat{\chi}_3^R\) are \(2/3, -1/3, 7/6, 1/6\) and \(-5/6\), respectively.

The symmetry breaking by \(\Phi(x)\) on the UV brane can also induce the following fermion mass terms localized there.

\[
L_{\text{fermion}}^{\text{bd}} = 2i\sqrt{-g} \left\{ \sum_{\alpha=1}^{3} \bar{\chi}_{\alpha R} D^{\mu} \chi_{\alpha R} - \sum_{\alpha=1}^{3} \mu_{\alpha} \left( \bar{\chi}_{\alpha R} Q_{\alpha L} - \bar{Q}_{\alpha L} \chi_{\alpha R} \right) - \tilde{\mu} \left( \bar{\chi}_{2R} q_L - \bar{q}_L \chi_{2R} \right) \right\} \delta(y),
\]

(2.10)

where \(\sqrt{-g} \equiv \text{det}(g_{\mu\nu}), g_{\mu\nu}\) is the 4D induced metric on the UV brane. The brane mass parameters \(\mu_{\alpha} (\alpha = 1, 2, 3)\) and \(\tilde{\mu}\) have mass dimensions 1/2. In the subsequent discussions, we suppose that they are much larger than \(m_{\text{KK}}\). Then the ratio \(\tilde{\mu}/\mu_2\) becomes the only relevant parameter for physics below \(m_{\text{KK}}\). In this paper, we neglect the flavor-mixings in quark and lepton sectors for simplicity. They can always be incorporated by promoting the brane mass parameters to matrices.

Besides the above brane-localized mass terms, there can be brane-localized kinetic terms. As we will mention in the next section, the gauge kinetic terms localized on the IR brane are necessary to stabilize the radion. Thus we introduce the following terms on the IR brane.

\[
L_{\text{kin}}^{\text{bd}} = 2\sqrt{-g} \left[ -\frac{\kappa_c}{4k} \text{tr} \left\{ F_{\mu\nu}^{(G)} F^{(G)\mu\nu} \right\} - \frac{\kappa_w}{4k} \text{tr} \left\{ F_{\mu\nu}^{(A)} F^{(A)\mu\nu} \right\} - \frac{\kappa_x}{4k} F_{\mu\nu}^{(B)} F^{(B)\mu\nu} \right] \delta(y - L),
\]

(2.11)

where \(\kappa_c, \kappa_w, \kappa_x\) are dimensionless constants. For simplicity, we do not consider kinetic terms on the UV brane or brane kinetic terms for the 5D fermions in this paper.

### 2.3 Mass spectrum

Now we calculate the mass spectrum \(\{m_n\}\) in the 4D effective theory. It is determined as solutions to the equation,

\[
\rho_I(\lambda_n) = 0,
\]

(2.12)

3 The resulting \(U(1)_Y\) and \(U(1)_{\text{EM}}\) charges of each component are listed in Sec. 2 of Ref. [14] or in Table 2 in Ref. [22].

4 Such terms will be generically induced by quantum loop effects of the bulk fields [23].
where \( I = G, W, nt, 5/3, 2/3, -1/3, -4/3 \) specifies the sectors, and \( \lambda_n \equiv m_n/k \). The functions \( \rho_I(\lambda) \) are listed in Appendix A.

For example, the W and Z boson masses are obtained as the smallest solution to \( \rho_W(\lambda_W) = 0 \) and the second smallest solution to \( \rho_{nt}(\lambda_Z) = 0 \), and are approximately expressed as

\[
\begin{align*}
m_W &= k\lambda_W \simeq \frac{k e^{-kL}}{\sqrt{kL + \kappa_w}} \sin \theta_H, \\
m_Z &= k\lambda_Z \simeq \frac{2s^2_\phi(kL + \kappa_w) + c^2_\phi(kL + \kappa_x)}{s^2_\phi(kL + \kappa_x) + c^2_\phi(kL + \kappa_x)} \frac{k e^{-kL}}{\sqrt{kL + \kappa_w}} \sin \theta_H.
\end{align*}
\]

(2.13)

The masses of the top and bottom quarks are obtained as the lowest solutions to \( \rho_{2/3}(\lambda_t) = 0 \) and \( \rho_{-1/3}(\lambda_b) = 0 \), respectively. In the case of \( M_{\Psi 1} = M_{\Psi 2} \) which we assume in the following, their approximate expressions are simplified as

\[
\begin{align*}
m_t &= k\lambda_t \simeq \frac{k \sqrt{1 - 4c^2_t}}{2 e^{kL}} \sin \theta_H \simeq \frac{1}{2} \frac{(1 - 4c^2_t)(kL + \kappa_w)}{2} m_W, \\
m_b &= k\lambda_b \simeq \frac{\mu}{\mu^2} m_t.
\end{align*}
\]

(2.14)

where \( c_t \equiv M_{\Psi 1}/k = M_{\Psi 2}/k \). The above expressions are valid when \( c_t < 1/2 \). As we will see in the next section, the effective potential determines \( \theta_H = \pi/2 \). Then the realistic top quark mass is obtained by choosing \( c_t \simeq 0.43 \) for \( e^{kL} = 10^{15} \) and \( \kappa_w \ll kL \).

## 3 Radion-Higgs potential

The one-loop effective potential for the radion and the Higgs field is calculated from the formula \( (B.9) \) with \( (B.10) \) in Appendix B obtained by the technique in Ref. 18. Noticing that \( \ln \left\{ 1 - e^{i(\beta - \gamma)\pi} \frac{I_\beta(w e^{-kL} K_\gamma(w))}{K_\beta(w e^{-kL} I_\gamma(w))} \right\} \) is exponentially small for \( w \lesssim O(1) \) unless \( \beta \simeq 0 \), only the gauge fields (\( \beta = 0 \)) and the top and bottom quark multiplets (\( \beta = c_t - \frac{1}{2} \approx -0.03 \)) can contribute to the effective potential \( V \). In other words, only the modes whose mode functions spread over the bulk can give sizable contributions to \( V \). In fact, the contribution of the graviton KK tower is exponentially suppressed because the graviton is localized around the UV brane and \( \beta = 1 \) [18]. Here the orders of the Bessel functions \( \beta \) and \( \gamma \) are determined by the boundary conditions at the UV and IR branes, respectively.

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5 The smallest solution to \( \rho_{nt}(\lambda) = 0 = \lambda = 0 \), which corresponds to the massless photon.

6 The functions \( I_\gamma(u) \) and \( K_\gamma(u) \) are defined in (A.4).
If we introduce the UV-brane kinetic terms, they effectively shift $\beta$ from zero and make the gauge field contributions negligible. So we do not consider the UV-brane kinetic terms in this paper. Then the effective potential is expressed as the following form.

$$V(kL, \theta_H) = \frac{k^4}{16\pi^2} \left[ \tau_{UV} + \tau_{IR} e^{-4kL} + e^{-4kL} \int_0^\infty dw \ w^3 v_{\text{eff}}(w; kL, \theta_H) \right], \quad (3.1)$$

where dimensionless constants $\tau_{UV}$ and $\tau_{IR}$ are associated with tensions at the UV and IR branes, and cannot be determined in the context of the 5D field theory. The integrand is given from (B.10) by

$$v_{\text{eff}}(w; kL, \theta_H) = 24 \ln \left\{ 1 - \frac{I_0(we^{-kL}) K_0^{\kappa w}(w)}{K_0(we^{-kL}) I_0^{\kappa w}(w)} \right\} + 9 \ln \left\{ 1 - \frac{I_0(we^{-kL}) K_0^{\kappa w}(w)}{K_0(we^{-kL}) I_0^{\kappa w}(w)} \right\}$$

$$+ 3 \ln \left\{ 1 - \frac{e^2 I_0(we^{-kL}) K_0^{\kappa x}(w)}{K_0(we^{-kL}) I_0^{\kappa x}(w)} \right\}$$

$$- 24 \ln \left\{ 1 - \frac{I_{\kappa -\frac{1}{2}}(we^{-kL}) K_{\kappa -\frac{1}{2}}^{\kappa x}(w)}{K_{\kappa -\frac{1}{2}}(we^{-kL}) I_{\kappa -\frac{1}{2}}^{\kappa x}(w)} \right\}$$

$$+ 6 \ln \left\{ 1 + \frac{e^{kL} \sin^2 \theta_H}{2w^2 F_{\kappa 0,0} F_{1,1}} \right\}$$

$$+ 3 \ln \left\{ 1 + \frac{e^{kL} \sin^2 \theta_H}{2w^2 F_{\kappa 0,0} F_{1,1}} \right\}$$

$$- 12 \ln \left\{ 1 + \frac{e^{kL} \sin^2 \theta_H}{2w^2 F_{\kappa 0,0} F_{1,1}} \right\}, \quad (3.2)$$

where the arguments of $F_{\alpha,\beta}$ are all $we^{-kL}$. We have neglected a small dependence on $|\mu|/\mu_2|^2 = (m_b/m_t)^2$, and used an approximation (B.8). The contribution from the region $w \gg \mathcal{O}(1)$ is negligible in the integral. The above $V$ can be understood as the effective potential for the radion and the Higgs field by promoting the parameters $kL$ and $\theta_H$ to 4D dynamical fields $kL(x)$ and $\theta_H(x)$.

From the stationary condition for $kL$, we obtain

$$\tau_{IR} = \int_0^\infty dw \ w^3 \left\{ \frac{\partial_{kL} v_{\text{eff}}}{4} - v_{\text{eff}} \right\}. \quad (3.3)$$

By means of this equation, we can always choose $\tau_{IR}$ so that the potential has a stationary point at a desired value of $kL$. In fact, a large warp factor $e^{kL} = 10^{15}$ is realized by an $\mathcal{O}(1)$ value of $\tau_{IR}$.

From (3.2), we can see that $\theta_H = \pi/2$ always satisfies the stationary condition for $\theta_H$. As shown in Ref. [14], it is a minimum of the potential along the $\theta_H$-direction for relatively large values of the warp factor in the absence of the brane kinetic terms. This is also true for $\kappa_{c,w,x} \neq 0$. 

8
4 Modulus stabilization

Now we consider the stabilization of the size of the extra dimension. In this section, we assume a value of $\tau_{\text{IR}}$ so that $e^{kL} = 10^{15}$ is a stationary point of the potential along the $kL$-direction. Then the AdS curvature scale $k$ is determined by $k \simeq e^{kL} \sqrt{kL} + \kappa_w m_W / \sin \theta_H$, which is obtained from (2.13), and the typical KK mass scale $m_{\text{KK}}$ is estimated as

$$m_{\text{KK}} \equiv \frac{\pi k}{e^{kL} - 1} \simeq \frac{\pi \sqrt{kL + \kappa_w m_W}}{\sin \theta_H}.$$ (4.1)

The second derivatives of the potential are given as

$$\partial^2_{kL} V = \frac{k^4 e^{-4kL}}{16\pi^2} \int_0^\infty dw \; w^3 \left\{ \partial^2_{kL} v_{\text{eff}} - 4 \partial_{kL} v_{\text{eff}} \right\},$$

$$\partial_{kL} \partial_{\theta_H} V = \frac{k^4 e^{-4kL}}{16\pi^2} \int_0^\infty dw \; w^3 \left\{ \partial_{kL} \partial_{\theta_H} v_{\text{eff}} - 4 \partial_{\theta_H} v_{\text{eff}} \right\},$$

$$\partial^2_{\theta_H} V = \frac{k^4 e^{-4kL}}{16\pi^2} \int_0^\infty dw \; w^3 \partial^2_{\theta_H} v_{\text{eff}}.$$ (4.2)

In the first equation, we have used (3.3).

Note that there is no radion-Higgs mixing in our model because $\partial_{kL} \partial_{\theta_H} V \propto \cos \theta_H$ vanishes at $\theta_H = \pi/2$. Thus, the radion mass is calculated by canonically normalizing the radion kinetic term in the Einstein-Hilbert term as

$$m_{\text{rad}}^2 = \frac{e^{2kL} - 1}{3M_5^2} \cdot k^2 \partial^2_{kL} V \simeq \frac{k^5 e^{-2kL}}{48\pi^2 M_5^2} \int_0^\infty dw \; w^3 \left\{ \partial^2_{kL} v_{\text{eff}} - 4 \partial_{kL} v_{\text{eff}} \right\},$$ (4.3)

where $M_5$ is the 5D Planck scale. The right-hand sides are evaluated at the minimum of the potential $(e^{kL}, \theta_H) = (10^{15}, \pi/2)$. Since the 4D Planck scale $M_{\text{Pl}}$ is related to $M_5$ through

$$M_{\text{Pl}}^2 \simeq \frac{M_5^3}{2k},$$ (4.4)

we obtain

$$m_{\text{rad}} \simeq \frac{kL + \kappa_w m_W^2 e^{kL}}{4\sqrt{6\pi}} M_{\text{Pl}} \left\{ \int_0^\infty dw \; w^3 \left( \partial^2_{kL} v_{\text{eff}} - 4 \partial_{kL} v_{\text{eff}} \right) \right\}^{1/2}.$$ (4.5)

The Higgs mass is calculated as

$$m_H^2 = \frac{g_4^2 (e^{2kL} - 1)}{4k} \partial^2_{\theta_H} V \simeq \frac{g_4^2 k^3 e^{-2kL}}{64\pi^2} \int_0^\infty dw \; w^3 \partial^2_{\theta_H} v_{\text{eff}}.$$ (4.6)

Namely,

$$m_H \simeq \frac{g_4 (kL + \kappa_w) m_W}{8\pi} \left\{ \int_0^\infty dw \; w^3 \partial^2_{\theta_H} v_{\text{eff}} \right\}^{1/2}.$$ (4.7)
Figure 1: The masses of the radion and the Higgs boson as functions of the coefficients of the brane kinetic terms. The solid, dotdashed, dotted and dashed lines represent the case of $(\kappa_w, \kappa_x) = (1, 1)\kappa_c, (2/3, 1/3)\kappa_c, (1/3, 2/3)\kappa_c$ and $(0, 0)$, respectively.

where

$$ g_4 \equiv \frac{g_A\sqrt{k}}{\sqrt{kL + \kappa_w}} $$

is the 4D effective weak gauge coupling.

In Fig. 1, we show the radion mass and the Higgs boson mass as functions of the coefficients of the brane kinetic terms $\kappa_{c,w,x}$. In the absence of the brane kinetic terms, the radion mass is tachyonic and the modulus is not stabilized. If we turn on them, the radion mass squared monotonically increases as a function of $\kappa_{c,w,x}$. Due to the color factor, the gluon provides the largest contribution to the radion mass. In the case of $(\kappa_w, \kappa_x) = (1, 1)\kappa_c, (2/3, 1/3)\kappa_c, (1/3, 2/3)\kappa_c$ and $(0, 0)$, the radion becomes non-tachyonic for $\kappa_c \geq 0.21, 0.24, 0.25$ and $0.30$, respectively. When $\kappa_c = 0$, much larger values of $\kappa_{w,x}$ are necessary to stabilize the modulus. For example, $\kappa_w \geq 7.2, 8.6, 12$ when $\kappa_w = \kappa_x/2$, $\kappa_w = \kappa_x$, $\kappa_w = 2\kappa_x$, respectively. We find that $O(1)$ values of $\kappa_{c,w,x}$ lead to the radion mass around 1 GeV.

5 Effects of IR-brane kinetic terms

Although the brane kinetic terms for the gauge fields are necessary for the modulus stabilization, large brane kinetic terms can deviate the weak boson couplings to the fermions from the standard model values, as shown in Ref. [24]. This is because they repel the

\footnote{This constant is an approximate expression of the actual gauge coupling calculated as an overlap integral of the mode functions.}
mode functions of the gauge bosons away from the IR brane, where the custodial symmetry $SO(4)$ exists. The Weinberg angle $\theta_W$ is defined by the ratio of the W and Z bosons as

$$\sin^2 \theta_W \equiv 1 - \frac{m_W^2}{m_Z^2} \simeq \frac{s_\phi^2(kL + \kappa_w)}{2s_\phi^2(kL + \kappa_w) + c_\phi^2(kL + \kappa_x)}.$$  

We have used (2.13). The value of $s_\phi$, or $g_A/g_B$, is determined for given values of $\kappa_w$ and $\kappa_x$ so that the above defined Weinberg angle takes the correct value $\sin^2 \theta_W \simeq 0.22$.

On the other hand, the Weinberg angle is also defined by the ratio of the gauge couplings of the fermions to the photon and the W boson. For example, let us consider the gauge couplings of the quarks in the first generation. Then

$$L^{(4)}_{\text{gauge}} = e A_{\mu}^{(0)} \left\{ \frac{2}{3} (\bar{u}_L \gamma^\mu u_L + \bar{u}_R \gamma^\mu u_R) - \frac{1}{3} (\bar{d}_L \gamma^\mu d_L + \bar{d}_R \gamma^\mu d_R) \right\}$$

$$+ \frac{g_{ud,L}^{(W)}}{\sqrt{2}} (W_\mu \bar{d}_L \gamma^\mu u_L + \text{h.c.}) + \frac{g_{ud,R}^{(W)}}{\sqrt{2}} (W_\mu \bar{d}_R \gamma^\mu u_R + \text{h.c.})$$

$$+ \frac{1}{\cos \theta_W} Z_\mu \left\{ g_{ud,L}^{(Z)} \bar{u}_L \gamma^\mu u_L + g_{uR}^{(Z)} \bar{u}_R \gamma^\mu u_R + g_{dL}^{(Z)} \bar{d}_L \gamma^\mu d_L + g_{dR}^{(Z)} \bar{d}_R \gamma^\mu d_R \right\}$$

$$+ \cdots.$$  

(5.2)

Each coupling constant is given as overlap integral of the relevant mode functions. For example, the electromagnetic coupling constant $e$ is calculated as

$$e = \frac{g_A \sqrt{k s_\phi}}{\sqrt{2s_\phi^2(kL + \kappa_w) + c_\phi^2(kL + \kappa_x)}} \simeq \frac{g_A \sqrt{k}}{\sqrt{kL + \kappa_w}} \sin \theta_W.$$  

(5.3)

The absolute value of the 5D coupling $g_A$ is fixed for given values of $\kappa_w$ and $\kappa_x$ so that $e$ takes the observed value. We have used (5.1) in the second equality. From this, we can read off the approximate expression of the weak gauge coupling $g_4$ shown in (4.8). We do not show the explicit forms of other coupling constants here, but they are obtained from those given in Ref. [22] by modifying the mode functions of the gauge bosons including the brane kinetic terms. In contrast to the standard model, the W boson couplings of the right-handed quarks do not completely vanish although they are negligibly small. Then the Weinberg angle is defined, for example, by

$$\sin \theta_W \equiv \frac{e}{g_{ud,L}^{(W)}}.$$  

(5.4)
By utilizing (5.1) and (5.3), we can estimate the $\rho$ parameter defined by

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2 \vartheta_W} = \frac{\cos^2 \theta_W}{\cos^2 \vartheta_W}. \quad (5.5)$$

In Fig. 2 we show this as functions of $\kappa_w$ and $\kappa_x$. Current electroweak data fitting favors the $\rho$ parameter being close to one, i.e., $1.00989 \leq \rho^{\text{exp}} \leq 1.01026$ [25]. Similar to the result in Ref. [24], the brane kinetic terms for $SO(4) \subset SO(5)$ and $U(1)_X$ deviate the Weinberg angle in the opposite directions. The former reduces the value of $\rho$ (and thus $\vartheta_W$) while the latter raises them. Therefore there is a parameter region where $\rho$ stays within the experimental error even for large $\kappa_w$ and $\kappa_x$.

Since the mode functions of the W and Z bosons are no longer constants after the electroweak symmetry breaking occurs, the universality of the gauge couplings to them is generically violated. As pointed out in Ref. [22], however, such violation remains less than 1% except for the top quark due to the left-right symmetry the model has. This is true even in the presence of the brane kinetic terms. The largest violation appears in the Z boson coupling of the top quark. For instance, in the case of $(\kappa_w, \kappa_x) = (5, 0), (5, 5), (0, 5)$, it deviates from the Z boson coupling of the up quark by 7-8% for the left-handed component and 16-18% for the right-handed component. The universality violation for the first two generations are less than percent level.

As mentioned in the previous section, the brane kinetic term for the gluon is necessary to stabilize the modulus. On the other hand, it is well known that such a term lowers the first KK gluon mass $m_{g_1}$ compared to the typical KK scale $m_{KK}$ [21, 26]. Fig. 3 shows $m_{g_1}$ as a function of $\kappa_c$. The first KK gluon becomes lighter than 600 GeV for $\kappa_c > 1$.

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\[8\] We have to include the loop contributions when $\rho$ is compared with $\rho^{\text{exp}}$.\]
Such a light colored particle is problematic and contradicts the results of Tevatron searches for resonant $t\bar{t}$ production. Since the experimental lower bound on $m_{g1}$ is 800-900 GeV according to Refs. [26, 27, 28], a possible maximal value of $\kappa_c$ is bounded by around 0.5 from Fig. 3. Recalling that $\kappa_c \gtrsim 0.2 - 0.3$ is required in order to stabilize the modulus, the light KK gluon appears below 1 TeV, which marginally satisfies the experimental bound.

The situation will be improved if we consider models in which smaller values of $\theta_H$ are realized by changing the matter sector. Such a model is proposed in Ref. [28], for example. Then, the KK mass scale raises by a factor of $1/\sin \theta_H$ (see (4.1)), and the deviation of each gauge coupling from the standard model becomes smaller than in our model. On the other hand, the modulus stabilization is expected to occur for $O(1)$ values of $\kappa_c$ because a largest contribution to $m_{\text{rad}}^2$ comes from the gluon KK tower, which is independent of the change of the matter sector, unless we consider a model with a large number of exotic fermion fields.

### 6 Summary and discussions

We have discussed the modulus stabilization of $S^1/Z_2$ by the Casimir energy and effects of IR-brane kinetic terms for the gauge fields in the context of gauge-Higgs unification in the warped spacetime. In the absence of the brane kinetic terms, the modulus is not stabilized due to a large negative contribution of the gluon loop to the radion mass squared $m_{\text{rad}}^2$. This can be cured by introducing the brane kinetic terms for the gauge fields at the IR brane. Especially the brane kinetic term for the gluon provides a sizable positive contribution to $m_{\text{rad}}^2$ due to a large color factor. The modulus is actually stabilized for $\kappa_c \gtrsim 0.2 - 0.3$, and the radion obtains a mass of $O(1 \text{ GeV})$ for $O(1)$ values of $\kappa_c$. In the case of $\kappa_c = 0$, much larger

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9 The precise bound depends on a detail of a model under consideration.
values of $\kappa_{w,x}$ are necessary for the modulus stabilization. For example, $\kappa_w \gtrsim 7.2, 8.6, 12$ is needed when $\kappa_w = \kappa_x/2$, $\kappa_w = \kappa_x$, $\kappa_w = 2\kappa_x$, respectively.

As for the modulus stabilization, we can also cancel the large negative contribution of the gluon to $m_{\text{rad}}^2$ by introducing extra colored fermions that belong to the $SU(3)_C$ adjoint representation and have boundary conditions such that they do not have zero-modes. However, in order for them to give a sizable contribution to $m_{\text{rad}}^2$, their mode functions must obey the Neumann boundary conditions at the UV brane, and thus their lightest modes can be heavy at most 400 GeV. Such light colored particles are already excluded by the experiments.

The IR-brane kinetic terms also affect 4D coupling constants among light modes and the lightest KK masses. The brane kinetic term for $SO(4)$ reduces the $\rho$ parameter and that for $U(1)_X$ raises it. Thus the $\rho$ parameter can remain within the experimental error even for a large value of $\kappa_w$ if $\kappa_x$ takes an appropriate value. The universality violation of the gauge couplings to the W and Z bosons are tiny except for the top quark due to the left-right symmetry $SU(2)_L \times SU(2)_R$, which is checked in Ref. [22] in the case of no brane kinetic terms. We have checked that this is also true in the presence of the IR-brane kinetic terms for the gauge fields. The masses of the lightest KK gauge bosons monotonically decrease as $\kappa_{\text{c,w,x}}$ increase. In fact, in the parameter region that ensures the modulus stabilization, the KK gluon with the mass $m_{g1} \lesssim 1$ TeV appears, which marginally satisfies the experimental bound $m_{g1} \gtrsim 800 - 900$ GeV [26, 27].

The allowed parameter region will be enlarged if we consider models in which smaller values of $\theta_H$ are realized, just like a model in Ref. [28]. In such models, the KK modes become heavier and the deviation of each gauge coupling from the standard model is smaller than in our model, while the modulus is stabilized for $O(1)$ values of $\kappa_c$. We should also note that the radion is mixed with the Higgs boson when $\theta_H \neq \pi/2$. In addition, the IR-brane kinetic terms also affect the violation scale of the tree-level unitarity. One of the authors showed that, in the absence of the brane kinetic terms, the tree-level unitarity will be violated around the KK mass scale in a gauge-Higgs unification model in the warped spacetime, irrespective of the values of $\theta_H$ [29]. This means that the perturbative calculation will no longer be reliable when the KK modes start to propagate. Inclusion of the IR-brane kinetic terms may delay the unitarity violation to higher energy scales, just like in the 5D Higgsless model [24]. We will discuss these issues in a subsequent paper.

Acknowledgments

The authors would like to thank R. Kitano for a useful discussion.


\section*{A \ Mass spectrum}

Here we collect the expressions of the functions $\rho_I(\lambda)$ that determine the mass spectrum by (2.12) in our model. These functions are written in terms of functions defined by

\begin{equation}
F_{\kappa,\alpha,\beta}(\lambda) \equiv J_\alpha(\lambda)Y^\kappa_\beta(\lambda z_L) - Y_\alpha(\lambda)J^\kappa_\beta(\lambda z_L), \tag{A.1}
\end{equation}

where $z_L \equiv e^{\kappa L}$, and

\begin{equation}
J^\kappa_\beta(u) \equiv J_\beta(u) - \kappa u J_{\beta+1}(u), \quad Y^\kappa_\beta(u) \equiv Y_\beta(u) - \kappa u Y_{\beta+1}(u). \tag{A.2}
\end{equation}

For a calculation of the effective potential, we also define

\begin{equation}
\hat{F}_{\kappa,\alpha,\beta}(w) \equiv I_\alpha(w)K^\kappa_\beta(wz_L) - e^{-i(\alpha-\beta)\pi}K_\alpha(w)I^\kappa_\beta(wz_L), \tag{A.3}
\end{equation}

where

\begin{equation}
I^\kappa_\beta(u) \equiv I_\beta(u) + \kappa u I_{\beta+1}(u), \quad K^\kappa_\beta(u) \equiv K_\beta(u) - \kappa u K_{\beta+1}(u). \tag{A.4}
\end{equation}

Then the following relation holds.

\begin{equation}
F_{\kappa,\alpha,\beta}(iw) = -\frac{2}{\pi}e^{i(\alpha-\beta)\pi/2}\hat{F}_{\kappa,\alpha,\beta}(w). \tag{A.5}
\end{equation}

The asymptotic behavior of $\hat{F}_{\kappa,\alpha,\beta}(w)$ for Re $w \gg 1$ is

\begin{equation}
\hat{F}_{\kappa,\alpha,\beta}(w) = -\frac{e^{w(z_L-1)}}{2w\sqrt{z_L}}e^{i(\beta-\alpha)\pi} \left(1 + O(w^{-1})\right). \tag{A.6}
\end{equation}

For the gauge bosons, $\rho_I(\lambda)$ are given by

**Gluon sector**

$$\rho_G(\lambda) = \lambda F^\kappa_{0,0}(\lambda). \tag{A.7}$$

**W boson sector**

$$\rho_W(\lambda) = F^\kappa_{1,0}(\lambda) \left( F^\kappa_{0,0}(\lambda)F^0_{1,1}(\lambda) - \frac{2\sin^2\theta_H}{\pi^2\lambda^2 z_L} \right). \tag{A.8}$$

**Neutral sector**

$$\rho_{\text{nt}}(\lambda) = c^2_\kappa \lambda F^\kappa_{0,0}(\lambda)F^\kappa_{1,0}(\lambda) \left( F^\kappa_{0,0}(\lambda)F^0_{1,1}(\lambda) - \frac{2\sin^2\theta_H}{\pi^2\lambda^2 z_L} \right) + s^2_\kappa \lambda F^\kappa_{1,0}(\lambda)F^\kappa_{0,0}(\lambda) \left( F^\kappa_{0,0}(\lambda)F^0_{1,1}(\lambda) - \frac{4\sin^2\theta_H}{\pi^2\lambda^2 z_L} \right). \tag{A.9}$$

Especially, when $\kappa_w = \kappa_x$, the function $\rho_{\text{nt}}(\lambda)$ can be factorized as $\rho_\gamma(\lambda) \rho_Z(\lambda)$, and the corresponding KK tower is decomposed into the following two sectors.
Photon sector
\[ \rho_\gamma(\lambda) = \lambda F^\kappa w_{0,0}(\lambda). \] (A.10)

Z boson sector
\[ \rho_Z(\lambda) = F^\kappa w_{1,0}(\lambda) \left( F^\kappa w_{0,0}(\lambda) F^0_{1,1}(\lambda) - \frac{2(1 + s^2) \sin^2 \theta_H}{\pi^2 \lambda^2 z_L} \right). \] (A.11)

\( \hat{4} \)-component sector
\[ \rho_4(\lambda) = F^0_{1,0}(\lambda). \] (A.12)

For quarks, there are four sectors according to the \( U(1)_{\text{EM}} \) charge \( Q_{\text{EM}} \).

\( Q_{\text{EM}} = \frac{5}{3} \) sector
\[ \rho_{5/3}(\lambda) = F^0_{c+\frac{1}{2},c-\frac{1}{2}}(\lambda). \] (A.13)

\( Q_{\text{EM}} = \frac{2}{3} \) sector
\[ \rho_{2/3}(\lambda) = \left( F^0_{c+\frac{1}{2},c-\frac{1}{2}}(\lambda) \right)^2 \left( F^0_{c+\frac{1}{2},c+\frac{1}{2}}(\lambda) F^0_{c-\frac{1}{2},c-\frac{1}{2}}(\lambda) - \frac{2 \sin^2 \theta_H}{(1 + r) \pi^2 \lambda^2 z_L} \right), \] (A.14)

where
\[ r \equiv \left| \frac{\mu}{\mu_2} \right|^2 = \left( \frac{m_b}{m_t} \right)^2. \] (A.15)

\( Q_{\text{EM}} = -\frac{1}{3} \) sector
\[ \rho_{-1/3}(\lambda) = \left( F^0_{c+\frac{1}{2},c-\frac{1}{2}}(\lambda) \right)^2 \left( F^0_{c+\frac{1}{2},c+\frac{1}{2}}(\lambda) F^0_{c-\frac{1}{2},c-\frac{1}{2}}(\lambda) - \frac{2 \sin^2 \theta_H}{(1 + r) \pi^2 \lambda^2 z_L} \right). \] (A.16)

\( Q_{\text{EM}} = -\frac{4}{3} \) sector
\[ \rho_{-4/3}(\lambda) = F^0_{c+\frac{1}{2},c-\frac{1}{2}}(\lambda). \] (A.17)

Here we have assumed that the two 5-plet fermions in each generation have a common bulk mass, \( i.e., c \equiv M_{\Psi_1}/k = M_{\Psi_2}/k \), and all the brane mass parameters are assumed to be sufficiently large.

The lepton sector has a similar structure to the quark sector. (See Ref. [22].)
B One-loop effective potential

Here we derive the effective potential for the radion and the Higgs field at one-loop level. By using the dimensional regularization, it is calculated as

\[ V = \sum_I \frac{(-)^{2m_I} N_I}{2} \sum_n \int \frac{d^D p}{(2\pi)^D} \ln \left( p^2 + m_{I n}^2 \right) \]

\[ = \sum_I \frac{(-)^{2m_I} N_I}{(4\pi)^{D/2}} \frac{\pi}{D\Gamma(D/2)\sin(\pi D/2)} \sum_n m_{I n}^D, \quad (B.1) \]

where \( D = 4 + \epsilon, \eta_I = 0 \) (1/2) for bosons (fermions), \( N_I \) is a number of degrees of freedom for a particle in sector \( I \). The KK mass eigenvalues \( m_{I n} \) are solutions to

\[ \rho_I(\lambda_{I n}) = 0, \quad (B.2) \]

where \( \lambda_{I n} \equiv m_{I n}/k \), and the functions \( \rho_I(u) \) are listed in Appendix A. These masses depend on \( \theta_H \) and the warp factor \( z_L = e^{kL} \).

Here let us define a generalized zeta function as

\[ \hat{\zeta}(D) \equiv \sum_n \lambda_n^D. \quad (B.3) \]

This is well-defined for \( \text{Re} \ D < -1 \). Following the technique of Ref. [18], this is analytically continued to the region \( \text{Re} \ D < 1 \) and can be traded for the following integral.

\[ \hat{\zeta}(D) = \frac{D}{\pi} \sin \left( \frac{\pi D}{2} \right) \int_0^\infty dw \ w^{D-1} \ln \frac{\rho_I(iw)}{\rho_I^{\text{asp}}(iw)}, \quad (B.4) \]

where \( \rho_I^{\text{asp}}(u) \) is a \( \theta_H \)-independent analytic function that satisfies

\[ \frac{\rho_I(u)}{\rho_I^{\text{asp}}(u)} = 1 + \mathcal{O}(u^{-1}), \quad (B.5) \]

for \( \text{Im} \ u \gg 1 \). For instance,

\[ \rho_W(iw) = -\frac{8i}{\pi^3} \hat{F}_{1,0}^{\kappa_{\text{IR}}}(w) \left\{ \hat{F}_{0,0}^{\kappa_{\text{UV}}}(w)\hat{F}_{1,1}^0(w) + \frac{\sin^2 \theta_H}{2w^2 z_L} \right\}, \]

\[ \rho_W^{\text{asp}}(iw) = -\frac{i}{\pi^3} \frac{e^{3u(z_L-1)}}{w^3 z_L^{3/2}} (1 + \kappa_w z_L)^2. \quad (B.6) \]

Here we have used (A.6). In general, \( \rho_I^{\text{asp}}(u) \) can be expressed as \( f_I^{\text{UV}}(u) f_I^{\text{IR}}(uz_L) \), where function forms of \( f_I^{\text{UV}} \) and \( f_I^{\text{IR}} \) are independent of \( \theta_H \) and \( z_L \).
Note that, except for the neutral sector, a $\theta_H$-independent part $\rho_{0I}(iw) \equiv \rho_I(iw)|_{\theta_H=0}$ has a form of a product of
\[
\hat{K}_{\alpha,\beta}(w) = e^{-i(\alpha-\beta)\pi} K_\alpha(w) I^\kappa_\beta(w_{zL}) \left\{ 1 - e^{i(\alpha-\beta)\pi} \frac{I_{\alpha}(w) K^\kappa_\beta(w_{zL})}{K_\alpha(w) I^\kappa_\beta(w_{zL})} \right\}.
\] (B.7)
Thus we can define $K_I(w)$ and $\mathcal{I}_I(w)$, which are products of $e^{-i\alpha\pi} K_\alpha(w)$ and $e^{i\beta\pi} I^\kappa_\beta(w)$ respectively, so that $\rho_{0I}(iw)/K_I(w)\mathcal{I}_I(w_{zL})$ becomes a product of $\left\{ 1 - e^{i(\alpha-\beta)\pi} \frac{I_{\alpha}(w) K^\kappa_\beta(w_{zL})}{K_\alpha(w) I^\kappa_\beta(w_{zL})} \right\}$.

For the neutral sector, we define $\mathcal{K}_{nt} \equiv \frac{4\pi}{\kappa_0} \{K_0 K_1\}^2$ and $\mathcal{I}_{nt} \equiv \frac{4}{\pi} I^w_{0s} (I^w_{0s})^2 I_1$. Then,
\[
\ln \frac{\rho_{nt}(iw)}{\mathcal{K}_{nt}(w)\mathcal{I}_{nt}(w_{zL})} = \ln \frac{2^3 \hat{K}_{0,0}(w) \hat{K}_{1,0}(w) + s^2 \hat{K}_{1,0}(w) \hat{K}_{0,0}(w)}{-K_0(w) K_1(w) I^w_{0s}(w_{zL}) I^w_{0s}(w_{zL})} + \ln \left\{ 1 - \frac{I_0(w) K^\kappa_{0s}(w_{zL})}{K_0(w) I^\kappa_{0s}(w_{zL})} \right\}
\]
\[
\simeq \ln \left\{ 1 - \frac{I_0(w) K^\kappa_{0s}(w_{zL})}{K_0(w) I^\kappa_{0s}(w_{zL})} \right\} + \ln \left\{ 1 - \frac{I_1(w) K_{1s}(w_{zL})}{K_1(w) I_{1s}(w_{zL})} \right\}
\]
\[
+ \ln \left\{ 1 - \frac{I_0(w) K^\kappa_{0s}(w_{zL})}{K_0(w) I^\kappa_{0s}(w_{zL})} \right\} + \ln \left\{ 1 - \frac{I_1(w) K_{1s}(w_{zL})}{K_1(w) I_{1s}(w_{zL})} \right\},
\] (B.8)
for $w \lesssim \mathcal{O}(1)$.

Therefore, (B.1) is rewritten as
\[
V = \sum_I (-2)^{n_I} N_I k^D (4\pi)^{D/2} \Gamma(D/2) \int_0^\infty dw \ w^{D-1} \ln \frac{\rho_{I}(iw)}{\rho_{I_{\text{asp}}}(iw)}
\]
\[
= \sum_I (-2)^{n_I} N_I k^D (4\pi)^{D/2} \Gamma(D/2) \int_0^\infty dw \ w^{D-1} \left( \ln \frac{K_I(w) \mathcal{I}_I(w_{zL})}{f_{I_{\text{UV}}}(iw) f_{I_{\text{IR}}}(iw_{zL})} \right)
\]
\[
+ \ln \frac{\rho_{0I}(iw)}{\mathcal{I}_I(w_{zL})} + \ln \frac{\rho_{0I}(iw)}{\rho_{0I}(iw)}
\]
\[
= \frac{k^D}{(4\pi)^{D/2} \Gamma(D/2)} \left\{ \tau_{\text{UV}} + \frac{\tau_{\text{IR}}}{z_{L}} + \frac{1}{z_{L}^2} \int_0^\infty dw \ w^{D-1} v_{\text{eff}}(w; kL, \theta_H) \right\},
\] (B.9)
where
\[
\tau_{\text{UV}} \equiv \sum_I (-2)^{n_I} N_I \int_0^\infty dw \ w^{D-1} \ln \frac{K_I(w)}{f_{I_{\text{UV}}}(iw)}
\]
\[
\tau_{\text{IR}} \equiv \sum_I (-2)^{n_I} N_I \int_0^\infty dw \ w^{D-1} \ln \frac{\mathcal{I}_I(w)}{f_{I_{\text{IR}}}(iw)}
\]
\[
v_{\text{eff}}(w; kL, \theta_H) \equiv \sum_I (-2)^{n_I} N_I \left\{ \ln \frac{\rho_{0I}(iw/z_{L})}{\mathcal{K}_I(w/z_{L}) \mathcal{I}_I(w)} + \ln \frac{\rho_{I}(iw/z_{L})}{\rho_{I}(iw/z_{L})} \right\}.
\] (B.10)
Note that the third term in the brace in (B.9) is finite while the others diverge when we set $D = 4$. The divergent constants $\tau_{\text{UV}}$ and $\tau_{\text{IR}}$ can be absorbed in the renormalization of the tensions of the UV and IR branes, respectively. Only the second term of $v_{\text{eff}}(w; kL, \theta_H)$ in (B.10) has a $\theta_H$-dependence, and corresponds to a contribution calculated in Ref. [30].


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