The possible $\Sigma^0$-$\Lambda$ mixing in QCD Sum Rule

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Abstract

We calculate the on-shell $\Sigma^0$-$\Lambda$ mixing parameter $\theta$ with the method of QCD sum rule. Our result is $\theta(m_{\Sigma^0}^2) = (-)(0.5 \pm 0.1)$ MeV. The electromagnetic interaction is not included.

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Isospin independence and charge symmetry is only approximate in the strong interaction. It is believed that the up and down quark mass difference and the electromagnetic interaction cause all the isospin violations \cite{1,2}. Experimentally a strong signature for the $\rho^0$-$\omega$ mixing has been observed in the cross-section measurement of the reaction $e^+e^- \rightarrow \pi^+\pi^-$ \cite{3}. The effect from the electromagnetic interaction is of the opposite sign and much smaller than the experimentally determined $\langle \rho^0|H_{\text{CSB}}|\omega \rangle$, while the current
quark mass difference plays a dominant role \([4]\). Strong evidence for \(\pi-\eta-\eta'\) has also been obtained from studies of \(\eta'\) \([5]\), \(\psi'\) \([6]\) and \(\psi\) \([7]\) decays. In this work we study the possible \(\Sigma^0-\Lambda\) mixing with the method of QCD sum rules \([8]\). The \(\rho^0-\omega\) mixing has been analysed within the same framework \([8, 9]\). The isospin symmetry breaking sources in such an approach come from the current quark mass difference \(\delta m = m_u - m_d \neq 0\) and and the quark condensate difference \(\gamma = \frac{\langle 0 | \bar{d}d | 0 \rangle}{\langle 0 | \bar{u}u | 0 \rangle} - 1 \neq 0\). We do not take into account of the electromagnetic interaction in the present work.

We may study the \(\Sigma^0-\Lambda\) mixing through the mixed propagator in the QCD vacuum

\[
i \int d^4xe^{ipx} \langle 0|\Sigma^0(x)\bar{\Lambda}(0)\rangle|0\rangle = (-1) \frac{(\hat{p} + m_{\Sigma^0})\theta(\hat{p} + m_{\Lambda})}{(p^2 - m_{\Sigma^0}^2 + i\epsilon)(p^2 - m_{\Lambda}^2 + i\epsilon)} \tag{1}
\]

where the mixing parameter \(\theta(p^2)\) is defined through the following effective Lagragian:

\[
L_{\text{mix}} = \theta(\bar{\Psi}_{\Sigma^0}\Psi_{\Lambda} + \bar{\Psi}_{\Lambda}\Psi_{\Sigma^0}) \tag{2}
\]

and may be measured through the decays like \(\psi \rightarrow \Lambda\Sigma^0 + \bar{\Lambda}\Sigma^0\) in the future experiment.

In order to calculate the mixing parameter we study the two-point correlator at the quark level as:

\[
\Pi(p) = i \int d^4x e^{ipx} \langle T\{\eta_{\Sigma^0}(x), \bar{\eta}_\Lambda(0)\}\rangle|0\rangle e^{ipx}, \tag{3}
\]

The \(\eta_{\Sigma^0}\) and \(\eta_\Lambda\) are the currents with \(\Sigma^0\) and \(\Lambda\) quantum numbers

\[
\eta_{\Sigma^0}(x) = \epsilon^{abc} \frac{1}{\sqrt{2}} \{[u^aT(x)C\gamma_\mu d^b(x)]\gamma_5\gamma^\mu s^c(x) + [d^aT(x)C\gamma_\mu u^b(x)]\gamma_5\gamma^\mu s^c(x)\}, \tag{4}
\]

\[
\eta_\Lambda(x) = \epsilon^{abc} \sqrt{\frac{2}{3}} \{[u^aT(x)C\gamma_\mu s^b(x)]\gamma_5\gamma^\mu d^c(x) - [d^aT(x)C\gamma_\mu s^b(x)]\gamma_5\gamma^\mu u^c(x)\}, \tag{5}
\]

where \(u^a(x)\), \(T\) and \(C\) are the quark field, the transpose and the charge conjugate operators. \(a, b, c\) is the color indices. The interpolating currents couple to the baryon states with the overlap amplitude \(\lambda\).

\[
\langle 0|\eta_{\Sigma^0}(0)|\Sigma\rangle = \lambda_{\Sigma}\nu_{\Sigma}(p), \tag{6}
\]

\[
\langle 0|\eta_\Lambda(0)|\Lambda\rangle = \lambda_{\Lambda}\nu_{\Lambda}(p). \tag{7}
\]
where the $\nu(p)$ is a Dirac spinor.

The correlation function $\Pi(p)$ may be expressed as

$$
\langle 0 | T \bar{\eta}(x) \eta(0) | 0 \rangle = -\frac{2}{\sqrt{3}} i \epsilon^{abc} \epsilon^{a'b'c'} \{ \gamma_5 \gamma^\mu S^{a'a'}_s(x) \gamma_\nu C[S^{b'b'}_u(x)]^T C \gamma_\mu S^{c'c'}_d(x) \gamma^{b'c'} \}
$$

(8)

where $i S^{a'b'}_q(\hat{x})$ is the quark propagator [10].

$$
i S^{a'b'}_q(\hat{x}) = \langle 0 | T [q^a(x) \bar{q}^b(0)] | 0 \rangle
$$

$$
= \frac{i \delta^{ab}}{2\pi^2 \hat{x}^2} + \frac{\lambda_s^2}{32\pi^2} g_s G^{\mu \nu} \frac{1}{x^2} (\sigma^{\mu \nu} \hat{x} + \hat{x} \sigma^{\mu \nu}) - \frac{\delta^{ab}}{12} \langle \bar{q} q \rangle
$$

$$
+ \delta^{ab} \frac{g_s}{192} \langle \bar{q} \bar{q} \sigma \cdot G q \rangle - \frac{\langle \bar{q} q \rangle \langle G^2 \rangle \hat{x}^4}{29 \times 33} \delta^{ab} - \frac{m_q \delta^{ab}}{4 \pi^2 x^2}
$$

$$
+ \frac{m_q}{32 \pi^2} g_s \lambda_s^2 G^{\mu \nu} \sigma^{\mu \nu} \ln(-x^2) - \frac{\delta^{ab} \langle G^2 \rangle}{29 \times 33} m_q x^2 \ln(-x^2)
$$

$$
+ \frac{i \delta^{ab} m_q \langle \bar{q} q \rangle \hat{x}}{48} - \frac{i m_q \langle g_s \bar{q} \sigma \cdot G q \rangle \delta^{ab} x^2 \hat{x}}{29 \times 33}
$$

$$
+ \ldots
$$

(9)

At the hadronic level

$$
\Pi(p) = (-) \lambda_{\Sigma^0} \lambda_{\Lambda} \frac{(\hat{p} + m_{\Sigma^0}) \theta(p^2)(\hat{p} + m_{\Lambda})}{(p^2 - m_{\Sigma^0}^2 + i\epsilon)(p^2 - m_{\Lambda}^2 + i\epsilon)}.
$$

(10)

The diagrams with nonzero contribution are presented in Fig. 1. In the limit of exact isospin symmetry $\Pi(p)$ vanishes. There are two isospin symmetry breaking parameters $\delta m$ and $\gamma$. Each diagram is either propotional to $\delta m$ or to $\gamma$. We explicitly keep the current quark mass term in the quark propagator up to order $O(m_q)$, which is denoted by a cross. The current quark mass enters a diagram either through expanding the free quark propagator up to $O(m_q)$ or through the equation of motion. Since the strange quark mass is not small, we keep terms like $m_s (m_d - m_u)$. The calculation is standard as in the QCD sum rule analysis of the baryon mass. Equating the correlator $\Pi(p)$ at the quark level and $\Pi(p)$ at the hadronic level we arrive at two sum rules corresponding to two different structures 1 and $\hat{p}$. Here we present the final result after Borel transformation.
For structure 1:

\[
\frac{1}{\sqrt{3}} \left\{ \delta m M_B^6 E_3 L^{-\frac{2}{3}} + \gamma a M_B^4 E_2 - \frac{5}{8} \delta m M_B^4 E_1 L^{-\frac{2}{3}} - \frac{1}{3} \delta m a_s M_B^4 E_1 L^{-\frac{2}{3}} \\
- \frac{1}{3} \gamma m_s a a_s M_B^2 E_0 + \frac{1}{3} \delta m a^2 M_B^2 E_0 - \frac{1}{3} \delta m a M_B^2 E_0 + \frac{7}{48} \delta m a_s m_s M_B^2 E_0 L^{-\frac{2}{3}} - \frac{5}{8} \delta m a m \gamma m_s M_B^2 \ln \frac{M_B^2}{\mu^2} E_0 L^{-\frac{2}{3}} \right\}
= (2\pi)^4 \lambda_S \lambda_A e^{-\frac{m^2}{M_B^2}} 2m^2 \theta(m_{\Sigma^0}) (1 + A_1 M_B^2)
\]

For structure \( \hat{p} \):

\[
-\frac{1}{\sqrt{3}} \left\{ \delta m m_s M_B^6 E_3 L^{-\frac{2}{3}} + 4\gamma m_s a M_B^4 E_1 L^{-\frac{2}{3}} + 4\delta m a_s M_B^4 E_1 L^{-\frac{2}{3}} + \frac{2}{3} \gamma a a_s M_B^2 E_0 L^{\frac{2}{3}} \\
- \frac{2}{3} \gamma m_s a m_2 M_B^2 E_0 L^{-\frac{2}{3}} - \frac{1}{3} \delta m a_s m_2 M_B^2 E_0 L^{-\frac{2}{3}} + \frac{1}{3} \delta m a m_2 M_B^2 E_0 L^{-\frac{2}{3}} \right\}
= (2\pi)^4 \lambda_S \lambda_A 2m e^{-\frac{m^2}{M_B^2}} \theta(m_{\Sigma^0}) (1 + A_2 M_B^2)
\]

where \( \delta m = m_d - m_u \) and \( \gamma = \frac{\langle \tilde{d}d \rangle}{\langle uu \rangle} - 1 \), \( m = \frac{m_{\Sigma^0} + m_A}{2} = 1.15 \text{GeV} \) is the average mass. \( m_s = 150 \text{MeV} \) is the strange quark mass. \( y = \frac{W^2}{M_B^2} \) and the factors, \( E_n(y) = 1 - e^{-y} \sum_{k=0}^{n} \frac{1}{k!} y^k \), are used to subtract the continuum contribution [1]. \( W^2 = 3.4 \text{GeV}^2 \) is the continuum threshold which is determined together with the overlap amplitude \( (2\pi)^4 \lambda^2 = 1.88 \text{GeV}^6 \), \( (2\pi)^4 \lambda_A^2 = 1.64 \text{GeV}^6 \) from the mass sum rules [2, 3]. We adopt the “standard” values for the various condensates \( b = \langle 0 | g_s^2 G^2 | 0 \rangle = 0.474 \text{GeV}^4 \), \( a = -(2\pi)^2 \langle 0 | \bar{u}u | 0 \rangle = 0.55 \text{GeV}^3 \), \( a_s = -(2\pi)^2 \langle 0 | \bar{s} \gamma^5 s | 0 \rangle = 0.55 \times 0.8 \text{GeV}^3 \), \( am_2 = (2\pi)^2 g_s \langle 0 | \bar{u} \gamma^5 u | \gamma \sigma \cdot G u | 0 \rangle \), \( m_0^2 = 0.8 \text{GeV}^2 \).

\( L = \frac{\ln \left( \frac{m_{\Sigma^0}}{\Lambda_{\text{QCD}}} \right)}{\sqrt{\frac{\Lambda_{\text{QCD}}}{m_{\Sigma^0}}} \ln \left( \frac{M_B^2}{\Lambda_{\text{QCD}}} \right)} \), \( \Lambda_{\text{QCD}} \) is the QCD parameter, \( \Lambda_{\text{QCD}} = 100 \text{MeV} \), \( \mu = 0.5 \text{GeV} \) is the normalization point to which the used values of condensates are referred.

We further improve the numerical analysis by taking into account of the renormalization group evolution of the sum rules (11) and (12) through the anomalous dimensions of the various condensates and currents. \( A_1 \) and \( A_2 \) are constants to be determined from the sum rule. They arise from the mixing with the excited states \( \Sigma^{0*} - \Lambda \) or \( \Sigma^0 - \Lambda^* \) which were first introduced in the QCD sum rules analysis of the nucleon magnetic moments [1]. The working interval for the Borel mass \( M_B^2 \) is \( 1.3 \text{GeV}^2 \leq M_B^2 \leq 2.5 \text{GeV}^2 \) where both the continuum contribution and power corrections are controllable. Moving the factor
on the right hand side to the left and fitting the new sum rule with a straight line approximation we may extract the mixing parameter $\theta$.

Various theoretical approaches \[2, 10, 14, 15, 16, 17, 18\] yield consistent results for the quark mass difference, $\delta m = 3.2 \pm 0.4$ MeV. And the difference of the up and down quark condensate has been analyzed with the chiral perturbation theory \[14\], the QCD sum rules for scalar and pseudoscalar mesons \[17, 18, 19\], effective models of QCD incorporating the dynamical breaking of chiral symmetry \[20, 21\], and the QCD sum rules for baryons \[10\]. The numerical results from the above approaches are $\gamma = -(6 - 10) \times 10^{-3}$ \[14\], $-(10 \pm 3) \times 10^{-3}$ \[17\], $-(7 - 9) \times 10^{-3}$ \[20, 21\] and $-6.57 \times 10^{-3}$ \[10\].

In both of the sum rules (11) and (12) the contribution from $\delta m$ and $\gamma$ has opposite sign. The mixing parameter from (11) and (12) would contradict each other if $\gamma$ is too large or too small. In Fig. 2 the Borel mass dependence of the mixing parameter and the fitting straight line is shown. Through the intersection with the Y-axis, we can obtain the value of $\theta$ directly. With $\gamma = -(7 \pm 1) \times 10^{-3}$ and $\delta m = 3.0 \pm 0.4$ MeV, our final result is $\theta(m_{\Sigma^0}^2) = (-0.5 \pm 0.1)$MeV. It can be seen from Fig. 2 that (11) and (12) yields almost the same value for $\theta$. In summary we have calculated the on-shell $\Sigma^0$-$\Lambda$ mixing parameter, which may be measured in the future experiment.

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Figure Captions

Fig. 1 Relevant diagrams in the QCD sum rules analysis of the $\Sigma^0$-$\Lambda$ mixing up to dimension seven. The current quark mass correction is denoted by a cross. If two crosses appear in the same diagram, one of them comes from the strange quark.

Fig. 2 (a) The Borel mass dependence of the mixing parameter. The solid curve is the QCD sum rule prediction for $\gamma = -7 \times 10^{-3}$ from equation (11) for the structure 1 with $\delta m = 3.0\text{MeV}$ after the numerical factor $(2\pi)^4 \lambda_{\Sigma^0}^2 \lambda_{\Lambda}^2 e^{-\frac{m^2}{M^2_B}}$ is moved to the left hand side. The dotted curve is the fitting straight line. The Borel mass $M^2_B$ is in unit of GeV$^2$. The mixing parameter $\theta$ is unit of MeV.

(b) The Borel mass dependence of the mixing matrix for the structure $\hat{p}$. Notations are the same as in (a).
