DYNAMO IN HELICAL MHD TURBULENCE:  
Quantum Field Theory Approach

M. Hnatic\textsuperscript{1}, M. Jurcisin\textsuperscript{1,2}, M. Stehlik\textsuperscript{1}

\textsuperscript{1} Institute of Experimental Physics of SAS, Watsonova 47, 043 53, Košice, Slovakia,  
\textsuperscript{2} BLTP, Joint Institute for Nuclear Research, 141 980 Dubna, Russian Federation

Abstract

A quantum field model of helical MHD stochastically forced by gaussian hydrodynamic, magnetic and mixed noices is investigated. These helical noises lead to an exponential increase of magnetic fluctuations in the large scale range. Instabilities, which are produced in this process, are eliminated by spontaneous symmetry breaking mechanism accompanied by creation of the homogeneous stationary magnetic field.

1 Introduction

Quantum field theory method including renormalization group (RG) approach has been successfully used for the theoretical explanation of various phenomena in developed turbulence (see [1] and wherein references).

In this paper the quantum field model of helical MHD is investigated. As a starting point we consider the Navier-Stokes equation for the velocity field and the equation for magnetic field which are driven by gaussian random forces with a given $2 \times 2$ matrix $D$ of the hydrodynamic, magnetic and mixed noise correlators, respectively.

In ref. [2] (see also [3]) the multiplicative renormalizability of the quantum field model of non-helical MHD turbulence has been proved and the RG approach has been applied to study the asymptotic behavior of the model considered. The existence of two infrared-stable fixed points has been established. These points govern the two critical regimes: the magnetic regime and the kinetic one (the later being of the Kolmogorov type).

The critical properties of the helical MHD are not known in the case of an arbitrary noise matrix $D$. To provide the multiplicative renormalizability and consequent application of RG it is necessary to extend the theory adding the extra dissipative terms with new helical Prandtl numbers [4]. Therefore, also a critical behavior of the helical MHD is more complicated. A priori, the existence of the former stable
regime of the Kolmogorov type is not clear. In the following the existence of the critical regimes mentioned above for ordinary MHD is demonstrated for the helical one.

There is an additional problem in the helical MHD: the instability of the theory which is induced by the exponential increase of the magnetic fluctuations in the large scales range (see [5], for example). The elimination of this instability leads to formation of a large-scale magnetic field known as the turbulent dynamo. Removal of the instability in quantum field formulation of helical MHD can be achieved by means of a nice and very well known spontaneous symmetry breaking mechanism followed by the creation of homogeneous stationary magnetic field. The special case, when only the hydrodynamic noise does not vanish, was analyzed in [6].

In this paper the value of spontaneous magnetic field \( c \) in one loop approximation for matrix \( D \) of noises in generic form. This value has been found from the conditions of overall exponential instabilities elimination in the steady state.

The dynamo mechanism is accompanied by the appearance of an "exotic" term in the linearized equation for magnetic field. This term causes the linear growth in time \( t \) of the amplitudes of Alfvén waves for small wave vectors \( k \) in the direction orthogonal to the plane of \( k \) and \( c \). Due to the viscosity terms this growth is transformed into long-lived pulses of the type \( t \exp(-i\beta t) \exp(-\alpha t) \) with small \( \alpha > 0 \) and \( \beta \).

2 The formulation of the problem

The interaction of electrically neutral conductive turbulent incompressible fluid with the magnetic field is described by the stochastically forced MHD equations [2]:

\[
\begin{align*}
\partial_t \mathbf{v} &= \nu \Delta \mathbf{v} - (\mathbf{v} \partial) \mathbf{v} + (\mathbf{b} \partial) \mathbf{b} - \partial p + \mathbf{F}^v \\
\partial_t \mathbf{b} &= \nu' \Delta \mathbf{b} - (\mathbf{v} \partial) \mathbf{b} + (\mathbf{b} \partial) \mathbf{v} + \mathbf{F}^b,
\end{align*}
\]

(1)

together with the incompressibility conditions

\[
\nabla \cdot \mathbf{v} = 0, \quad \nabla \cdot \mathbf{F}^v = 0, \quad \nabla \cdot \mathbf{F}^b = 0.
\]

(2)

The first equation is the well-known Navier-Stokes equation for the divergence free velocity field \( \mathbf{v}(x) = \{v_i(x, t)\} \) with the additional nonlinear contribution of the Lorentz force (\( p \) is a sum of both hydrodynamic and magnetic pressure per unit mass). The second equation for magnetic field \( \mathbf{b}(x) = \{b_i(x, t)\} \) (it is connected with magnetic induction \( \mathbf{B} \) by the relation \( \mathbf{b} = \mathbf{B}/\sqrt{4\pi \varrho} \), where \( \varrho \) is the fluid density and \( \mu \) is its magnetic permeability) follows from the Maxwell equations for continuous medium. The magnetic diffusion coefficient \( \nu' \) is connected with the coefficient of molecular viscosity by relation \( \nu' = u \nu \) with dimensionless magnetic Prandtl number (PN) \( u^{-1} \).
External random forces $F^v, F^b$ are assumed to have a Gaussian distribution with $<F> = 0$ and they are defined by $2 \times 2$ matrix of the noise correlators $D = <FF>$. The matrix elements are: the hydrodynamic $D^{vv}$ noise, the magnetic $D^{bb}$ one and the mixed $D^{vb}$ one.

The problem (1) is equivalent to a quantum theory with the doubled number of the fields $\Phi = \{v, b, v', b'\}$ and the action functional [7, 8]:

$$S(\Phi) = \frac{1}{2} \left( v'D^{vv}v' + b'D^{bb}b' + \frac{1}{2} b'D^{vb}v' + \frac{1}{2} b'D^{bv}v' \right) + v'[-\partial_t v + v_0 \Delta v] - (v\partial)v + (b\partial)b,$$

(3)

where $v', b'$ are some auxiliary vector fields. Hereafter in the similar expressions, the integration over $x, t$ and the traces over the vector indices are implied. As it is usual in QFT, the action (3) is considered to be unrenormalized with the bare constants marked by the subscript "0". The basic objects of the study are the Green functions of the fields $\Phi$ (the correlation functions and response functions in the terminology of the original problem (1)). They can be determined as functional derivatives with respect to an external sources $A = \{A^v, A^b, A'^v, A'^b\}$ of the generating functional $G(A) = \int D\Phi \exp[S(\Phi) + A\Phi]$, i.e., they are the functional averaged values of the corresponding number of the fields $\phi$ with a weight $\exp[S(\phi)]$. Here, $D\Phi$ denotes the functional measure of the integration over the fields $\Phi$ with all normalization coefficients.

We have to choose a concrete form of $D$ in the wave vector-frequency $(k, \omega)$ representation. The noises are transversal for the incompressible fluid. The action for helical MHD can possess scalar terms as well as pseudoscalar ones. Hence, the tensor structure of all noises is a linear combination of both tensor and pseudotensor. Then the correlators have the form:

$$D^{vv}_{js} = g_0^{v} k^{1-2\epsilon} P^1_{js},$$
$$D^{bb}_{js} = g_0^{b} k^{1-2\epsilon} P^2_{js},$$
$$D^{vb}_{js} = D^{bv}_{js} = g_0'' k^{1-(1+a)\epsilon} P^3_{js}.$$  (4)

Here, $P^r_{js} = P_{is} + i \rho_r \varepsilon_{ijs} k_l / k$, where $P_{is} = \delta_{is} - k_ik_s / k$ stands for transverse projector and $\varepsilon_{ijs}$ is Levi-Civita pseudotensor. Dimensionless real parameters $\rho = \{\rho_1, \rho_2, \rho_3\}$ satisfy the conditions $|\rho| \leq 1, \rho_2^2 \leq |\rho_1 \rho_2|$. The scalar parts of the noises explicitly written in (4) are in a standard power form [2]. The parameters $g_0, g_0', g_0''$ play the role of the bare coupling constants, and $a, \epsilon$ are free parameters of the model. The value $\epsilon = 2$ corresponds to the Kolmogorov energy pumping from infrared region of the small $k$.

3 Renormalization

In a standard way, one can solve the primary infrared problem for the physical value $\epsilon = 2$ by the transfer to the region of small values $\epsilon$, where the ultravi-
Figure 1: One-loop Feynman diagrams which are UV-divergent. Only diagrams related to the Green functions $<v'v>$, and $<b'b>$ (first and second line) can contain $\Lambda - UV$ divergences.

Olet (UV) divergences appear. They can be eliminated by the addition of the appropriate counterterms to the action $[3]$. The counterterms are formed of the superficial UV divergences, which are present in one-particle irreducible Green functions $[9]$. The following 1-PI Green functions possess the UV divergences: $<v'v>$, $<b'b>$, $<v'b>$, $<b'v>$, $<v'bb>$. Corresponding diagrams are shown in Fig. 1 and Fig. 2 and counterterms have the form: $\nu v' \Delta v$, $\nu b' \Delta b$, $\nu'(b\partial)b$, $\nu \nu' \Delta b$ and $\nu b' \Delta v$. The last two of them are not present in the primary action $[3]$. For this reason, it is necessary to consider the extended theory with the additional cross dissipative terms $\nu v v' \Delta b$, $w v b' \Delta v$ and helical magnetic Prandtl numbers $v^{-1}$, $w^{-1}$. Besides UV divergences mentioned above which manifest themselves like the poles of $\epsilon$ ($\epsilon-UV$ divergences), another divergences proportional to the UV cutoff $\Lambda$ can appear in the Green functions $<v'v>$, $<v'b>$, $<b'v>$, $<b'b>$. They acquire the form of $\Lambda v' \rot v$, $\Lambda v' \rot b$, $\Lambda b' \rot v$ and $\Lambda b' \rot b$. These $\Lambda - UV$ divergences generate the instability of the model, which causes exponential growth in time of the corresponding response functions. Therefore, their direct insertion into the action $[3]$ is not allowed and one has to find an effective way to eliminate them. One can make a natural assumption that finally the energy of the unstable large scale magnetic fluctuations must be transformed into the energy of large scale mean magnetic field.

The model of the helical MHD under consideration describes steady state, therefore it is reasonable to consider the new vacuum state with zero mean values of fields $v, v', b'$, and non-vanishing time-independent mean field $<b> \equiv c \neq 0$.

In a quantum field theory the appearance of non-zero vacuum value of field is associated with spontaneous symmetry breaking $[10]$, and as a standard, the value itself is determined from requirement of minimum of potential energy at the tree level.
In the case considered here the situation is more complicated and rather technically different, because unstable \( \Lambda \)-terms appear at the next (one-loop) level, consequently, the nonvanishing value of mean magnetic field in the steady state must be calculated in this order of a perturbation scheme. By straightforward calculations and/or from the symmetry analysis of the given one-loop Feynman diagrams, one can find that only \(< b'b >\) contains \( \Lambda \)-terms. Here all \( \Lambda \)-divergences can be eliminated by means of the shift of \( b \), namely \( b(x) \rightarrow b(x) + c \), and, on the other hand, \( \epsilon \)-\( \text{UV} \) divergences are compensated by means of five independent renormalization constants \( Z_i, i = 1, \ldots, 5 \) in extended model of helical MHD. As a result one obtains model with renormalized action:

\[
S_R^h(\Phi) = \frac{v'Dvv'}{2} + \frac{b'Dbb'}{2} + \frac{v'Dbb'}{2} + \frac{b'Dvv'}{2} - v'[\partial_t v - Z_1 v \Delta v - Z_4 v \Delta b + (v \partial)v - Z_3 (b \partial)b - Z_3 (c \partial)b] - b'[\partial_t b - Z_2 w \Delta b - Z_5 w \Delta v + (v \partial)b - (b \partial)v - (c \partial)v] - (5)
\]

where all parameters are renormalized counterparts of bare ones. The action (5) generates Green functions without divergences. In this case Feynman rules have the following form

\[
\Delta_{12}^{vv'} = \frac{MP_{12}}{LM - SV}, \quad \Delta_{12}^{bb'} = -\frac{VP_{12}}{LM - SV}, \quad \Delta_{12}^{vb'} = \frac{SP_{12}}{LM - SV}, \quad \Delta_{12}^{bv'} = \frac{LP_{12}}{LM - SV},
\]

\[
\Delta_{12}^{vv} = \frac{M^+ P_{12}}{L^+ M^+ - S^+ V^+}, \quad \Delta_{12}^{bb} = -\frac{V^+ P_{12}}{L^+ M^+ - S^+ V^+}, \quad \Delta_{12}^{vb} = \frac{SP_{12}}{LM - SV}, \quad \Delta_{12}^{bv} = \frac{LP_{12}}{LM - SV},
\]
\[
\Delta v'_{12} = -\frac{S^+ P_{12}}{L^+ M^+ - S^+ V^+}, \quad \Delta v''_{12} = \frac{L^+ P_{12}}{L^+ M^+ - S^+ V^+},
\]
\[
\Delta v v'_{12} = \frac{D_{12} v M^+ - D_{12} v M^+ - D_{12} b v M^+ + D_{12} b v V^+}{(L^+ M^+ - S^+ V^+)(L M - S V)},
\]
\[
\Delta b b'_{12} = \frac{D_{12} b S^+ - D_{12} b S L^+ - D_{12} b L S^+ + D_{12} b b L^+}{(L^+ M^+ - S^+ V^+)(L M - S V)},
\]
\[
\Delta v b_{12} = \frac{-D_{12} v M^+ + D_{12} b v V^+ + D_{12} b v L^+}{(L^+ M^+ - S^+ V^+)(L M - S V)},
\]
\[
\Delta b v_{12} = \frac{-D_{12} v M^+ + D_{12} b v V^+ + D_{12} b v L^+}{(L^+ M^+ - S^+ V^+)(L M - S V)}.
\]

where

\[
L = -i \omega + \nu k^2, \quad M = -i \omega + \nu u k^2, \quad V = \nu \nu k^2 - i \gamma, \quad S = \nu w k^2 - i \gamma,
\]
\[
D_{mn}^{vv} = g_1 \nu \nu k^4 4-d-2 \epsilon (P_{mn} + i \rho_1 \varepsilon_{mn} k^2 k),
\]
\[
D_{mn}^{vb} = g_3 \nu \nu k^4 4-d-2 \epsilon (P_{mn} + i \rho_3 \varepsilon_{mn} k^2 k),
\]
\[
D_{mn}^{bb} = D_{mn}^{vb},
\]
\[
D_{mn}^{b b} = g_2 \nu \nu k^4 4-d-2 \epsilon (P_{mn} + i \rho_2 \varepsilon_{mn} k^2 k),
\]

with

\[
\gamma = c \cdot k.
\]

and vertices are defined by the expressions:

\[
v' \cdot (v \cdot \partial)v = v' t_{ij} v_j v_i / 2,
\]
\[
v' \cdot (b \cdot \partial)b = v' t_{ij} b_j b_i / 2,
\]
\[
\mathbf{b}' \cdot (\mathbf{b} \cdot \partial)v - \mathbf{b}' \cdot (v \cdot \partial)b = b' t_{ij} b_j v_i,
\]

where

\[
t_{ij}^k = i(k_j \delta_{il} + k_l \delta_{ij}), \quad \bar{t}_{ij}^k = i(k_j \delta_{il} - k_l \delta_{ij}).
\]

Using Feynman rules defined above one can immediately calculate the diagrams which are shown in Fig. 1 and Fig. 2. To extract the \( \Lambda \)– terms it is enough to keep only linear in wave vector part of the diagrams. The response functions \( < v'v >, \quad < v'b >, \quad < b'v > \) do not possess any linear divergent terms. \( \Lambda \)-divergent part and term \( \sim c \) of the response function \( < b'b > \) at the one-loop level are of the form:

\[
< b'b > \sim i k_m \varepsilon_{iml} (g \rho_1 C_1 + g' \rho_2 C_2 + g'' \rho_3 C_3) \times
\]
\[
\times \left[ \nu \Lambda \delta_{jt} - |c| \frac{3\pi}{8} \sqrt{\frac{(1 + u)^2 - (v + w)^2}{(1 + u)^2(u - vw)}} (\delta_{jt} + e_j e_t) \right],
\]

6
where \( \mathbf{e} \equiv \mathbf{c}/|\mathbf{c}| \), \( C_1 = (w(v+w) - u(1+u))/\xi \), \( C_2 = (1+u-v(v+w))/\xi \), 
\( C_3 = 2(uw-w)/\xi \) and \( \xi = 6(1+u)^2(u-vw)^2 \). From requirement of vanishing of \( \Lambda \)-term in (11) one determines the value of spontaneous field

\[
|\mathbf{c}| = \frac{8\nu}{3\pi} \sqrt{\frac{(1+u)^2 - (v+w)^2}{(1+u)^2(u-vw)}} \Lambda. \tag{12}
\]

One can see from this equation that magnitude of spontaneous field is independent of coupling constants \( g, g', g'' \), which characterize an intensity of random noises, and helical parameters \( \rho_1, \rho_2, \rho_3 \). In such a way we obtain the renormalized Green functions which are finite as \( \Lambda \rightarrow \infty \) formally, as it is usual in the field theory. But in real problems a natural maximal cutoff exists. In the developed turbulence the Kolmogorov dissipative length \( l_D = \Lambda^{-1} \) plays the role of a minimal scale. This length can be expressed in terms of basic phenomenological parameters - viscosity \( \nu \) and energy dissipation rate \( \varepsilon \); \( l_d = \nu^{3/4} \varepsilon^{-1/4} \). Then from (12) one obtains \( |\mathbf{c}| \sim (\nu \varepsilon)^{1/4} \) and it determines the order of magnitude of the spontaneous field \( \mathbf{c} \).

### 4 Corrections to the Alfvén waves

In order to understand the role of the last term in (11) let us consider the linearized MHD equations which follow from (5) in infrared limit of small wave vector \( \mathbf{k} \):

\[
\partial_t \mathbf{v} = -\nu k^2 \mathbf{v} + (\nu \nu k^2 + i\gamma) \mathbf{b} \\
\partial_t \mathbf{b} = (-w\nu k^2 + i\gamma) \mathbf{v} - w\nu k^2 \mathbf{b} + i\chi [\mathbf{k} \times \mathbf{e}] (\mathbf{b} \cdot \mathbf{e}). \tag{13}
\]

Here \( \chi \equiv 3(g\rho_1 C_1 + g'\rho_2 C_2 + g''\rho_3 C_3)|\mathbf{c}| \sqrt{((1+u)^2 - (v+w)^2)/(u-vw)/(8\pi(1+u))} \) and \( \gamma \equiv (\mathbf{k} \cdot \mathbf{c}) \). To solve Eq. (13) we define a basis of orthonormal vectors \( \mathbf{n} \equiv \mathbf{k}/k \), 
\( \mathbf{l} \equiv (\mathbf{e} - \mathbf{n} \cos \delta)/\sin \delta \), \( \mathbf{m} \equiv [\mathbf{n} \times \mathbf{e}] \sin \delta \), where \( \delta \) is the angle between vectors \( \mathbf{n} \) and \( \mathbf{e} \). The transversal fields \( \mathbf{v}, \mathbf{b} \) can be decomposed with respect to the vectors \( \mathbf{v}, \mathbf{b} : \mathbf{v} = v_l \mathbf{l} + v_m \mathbf{m}, \mathbf{b} = b_l \mathbf{l} + b_m \mathbf{m} \). Time dependent amplitudes \( v_l, v_m, b_l, b_m \) satisfy equation (13) and are given by the following expressions:

\[
v_l = \frac{1}{c} \left[ c_1 e^{\lambda_2 t}(\lambda_2 - d) + c_2 e^{\lambda_1 t}(\lambda_1 - d) \right], \tag{14}
\]

\[
b_l = c_1 e^{\lambda_2 t} + c_2 e^{\lambda_1 t}, \tag{15}
\]

\[
v_m = \frac{1}{2\lambda_1 - a - d} \\
\times \left( e^{\lambda_2 t} \left( -c_1 be \left( \frac{1}{2\lambda_1 - a - d} + t \right) + c_3 (\lambda_1 - a) - c_4 b \right) \right) + \\
+ \left( e^{\lambda_1 t} \left( -c_2 be \left( \frac{1}{2\lambda_1 - a - d} - t \right) + c_3 (\lambda_1 - d) + c_4 b \right) \right), \tag{16}
\]

\[
b_m = \frac{1}{2\lambda_1 - a - d} \left( e^{\lambda_2 t} \left( c_1 e \left( \frac{a-d}{2\lambda_1 - a - d} - 1 + 2t(\lambda_1 - d) \right) \right) \right).
\]
\[- c_3 c + c_4 (\lambda_1 - d) \right) + e^{\lambda_1 t} \left( c_2 e^{t \left( a - d \over 2 \lambda_1 - a - d \right) + 1 + 2t (\lambda_1 - a)} \right) + c_3 c + c_4 (\lambda_1 - a) \right), \]

where \( c_1, c_2, c_3 \) and \( c_4 \) are constants of integration and we have defined

\[
\lambda_{1,2} = \frac{1}{2} (a + d \pm \sqrt{(a + d)^2 - 4(ad - bc)}) ,
\]

\[
a = -\nu k^2 ,
\]

\[
b = i\gamma - \nu \nu k^2 ,
\]

\[
c = i\gamma - w\nu k^2 ,
\]

\[
d = -\nu \nu k^2 ,
\]

\[
e = -i k \sin^2 \delta \frac{3}{4(2\pi)^2} |c| (g_1 \rho_1 C_1 + g_2 \rho_2 C_2 + g_3 \rho_3 C_3) \frac{K}{\xi_1 (1 + u)} ,
\]

where

\[
K = \sqrt{(1 + u)^2 - (v + w)^2} \overline{u - vw} ,
\]

\[
\xi_1 = (1 + u)^2 (u - vw) .
\]

The functions \( \lambda_{1,2} \) are complex with negative real parts for the physical values of Prandtl numbers \( u^{-1}, v^{-1}, w^{-1} \), and they suppress the linear increase of amplitudes. Therefore, the last term in (11) results in the appearance of specific long-lived pulses of Alfvén waves. They are orthogonal polarized with respect to the spontaneous field and try to restore the isotropy broken by them.

5 Short remarks to the RG analysis and critical regimes

After successful elimination of all UV divergences we can use the renormalization group procedure and arrive to the set of RG Gell-Mann-Low equations for five invariant charges:

\[
s \frac{d\bar{g}_i(s)}{ds} = \beta_{\bar{g}_i}(\bar{g}(s), \epsilon) , \quad \bar{g}_i(s) \big|_{s=1} = g_i \equiv g, g', u, v, w ,
\]

where \( s = p/m \) (\( m \) is scale setting parameter). RG \( \beta \)-functions are expressed via renormalization constants \( Z_i \), which have been calculated in one-loop approximation. Note that the \( \beta \)-functions are finite in the limit of \( \epsilon \to 0 \).

The Gell-Mann-Low equations (19) have been solved numerically for the various initial values of the invariant charges \( g_i \). It provides the possibility to analyze the
attracting regions of infrared fixed points. There are two infrared-stable fixed points: the Gaussian $g_i^* = 0$ and nontrivial $g^* \neq 0$, $u^* = 1.393$, $g^* = u^* = v^* = w^* = 0$. The latter provides the existence of asymptotic critical regime of the Kolmogorov type. The RG approach improves expressions of simple perturbation theory and leads to the replacement of the original charges by the invariant ones, and, in critical regime by their values at the corresponding fixed points. Keeping in mind that $c$ is connected with magnetic induction $B$ by the relation $c = B / \sqrt{4\pi \rho \mu}$ (remember that $\rho$ denotes density and $\mu$-permeability of fluid) in above asymptotic region the magnitude of mean spontaneous magnetic induction is equal to $\frac{16}{3} \sqrt{\frac{\rho \mu}{\pi u_\ast}} (\nu \varepsilon)^{1/4}$.

6 Conclusion

In the paper the correlation and response functions of velocity and magnetic fluctuations have been studied. Generally, these functions possess singularities, which can be eliminated by a proper renormalization procedure. As a result, RG equations have been obtained and their solution have been found in the range of small wave numbers. This solution corresponds to the famous Kolmogorov scaling law. In helical MHD, where mirror symmetry of the system under consideration is stochastically broken, the non-vanishing mean magnetic field is spontaneously generated. This phenomena is accompanied by the linear growth in time $t$ of the amplitudes of Alfvén waves for small wave vectors. Due to the viscosity terms this growth is transformed into long-lived pulses of the type $t \exp(-i\beta t) \exp(-\alpha t)$ with small $\alpha > 0$ and $\beta$.

Acknowledgments

This work was supported by Slovak Academy of Sciences within the project 7232. M.H. gratefully acknowledges the hospitality of the N.N.Bogoliubov Laboratory of Theoretical Physics at JINR Dubna.

References

[1] Adzhemyan, L.Ts., Antonov, N.V., Vasil’ev, A.N. (1999) The Field Theoretic Renormalization Group in Fully Developed Turbulence, (Gordon and Breach Sci. Publ., The Netherlands).

[2] Adzhemyan, L.Ts., Vasil’ev, A.N., Hnatich, M. (1985) Quantum-field renormalization group in the theory of turbulence: Magnetohydrodynamics, Teor. Mat. Fiz., 64, pp.196-207

[3] Fournier, J.D., Sulem, P.L., Pouquet, A. (1982) Infrared properties of forced magnetohydrodynamic turbulence, J. Phys. Math. Gen., A 15, pp.1393-1420.
[4] Hnatich, M., Stehlik, M. (1992) Renormalization group in gyrotropic magnetic hydrodynamics, In ”Renormalization group ’91”. Eds. Shirkov D.V., Priezzhev V.B., World Scien. Pub., Singapore, 204.

[5] Vajnshtein, S.I., Zeldovich, Ja.B., Ruzmajkin, A.A. (1980) Turbulent Dynamo in Astrophysics, (Nauka, Moscow).

[6] Adzhemyan, L.Ts., Vasil’ev, A.N., Hnatich, M. (1987) Turbulent dynamo as spontaneous symmetry breaking, Teor. Mat. Fiz., 72, pp.369-383

[7] De Dominicis, C., Martin, P.C. (1979) Energy spectra of certain randomly-stirred fluids, Phys. Rev., A 19, pp.419-422

[8] Adzhemyan, L.Ts., Vasil’ev, A.N., Pis’mak, Yu.M. (1983) Renormalization-group approach in the theory of turbulence: The dimensions of composite operators, Teor. Mat. Fiz, 57, pp.268-281

[9] Collins, J.C. (1984) Renormalization. Cambridge University press, London.

[10] Vasiliev, A.N. (1998) Quantum field renormalization group in theory of critical phenomena and stochastic dynamics, (Sankt Peterburg), (in Russian).