Electric field control of multiferroic domain wall motion
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(Dated: 8 April 2014)

The dynamics of a multiferroic domain wall in which an electric field can couple to the magnetization via inhomogeneous magnetoelectric interaction is investigated by the collective-coordinate framework. We show how the electric field is capable of delaying the onset of the Walker breakdown of the domain wall motion, leading to a significant enhancement of the maximum wall velocity. Moreover, we show that in the stationary regime the chirality of the domain wall can be efficiently reversed when the electric field is applied along the direction of the magnetic field. These characteristics suggest that the multiferroic domain wall may provide a new prospective means to design faster and low-power-consumption domain wall devices.

I. INTRODUCTION

Manipulation of magnetic properties by an external electric field has long been a big challenge in the quest for novel spintronic devices. In conventional ferroelectric or ferromagnetic materials, the controlled motion of ferroic domain walls (DWs) is essential to achieve the desired functionalities. Usually, the motion of DW in magnetic materials are driven by a magnetic field or spin-polarized current. A key concept in the context of DW motion is the so-called Walker breakdown, which distinguishes the two regimes with high- and low-mobility and sets a limit to the DW velocity. To achieve fast and efficient control of DW motion, various attempts have been made to prevent this breakdown process, such as applying a transverse field, or considering a perpendicular magnetic anisotropy and the spin-orbit coupling effect. In this context, a way to manipulate the dynamics of the DWs by an electric field, which is critical for developing low-power-consumption spintronic devices, would be extremely appealing. Recently, it has been shown that modulating the magnetic anisotropy by an applied electric field is possible, and thus will allow for the electric field control of DW dynamics in ultrathin metallic ferromagnets. Nevertheless, the search for alternative schemes allowing fast and energy-efficient DW propagation is of great relevance in advanced spintronics research.

Multiferroic materials, which exhibit simultaneously ferroelectric and magnetic orders, may provide a promising arena to realize electric control of magnetization and even for DW motion. Multiferroics display a particularly rich variety of magnetoelectric (ME) cross-coupling effects. An intriguing scenario of ME coupling is that spiral spin orders can by themselves produce electric polarization, which is called the spin-current mechanism, or equivalently the inhomogeneous magnetoelectric interaction. Therefore, a nonzero electric polarization can be induced not just in bulk but also within local magnetic textures, like magnetic DWs and vortices. The possibility of such a magnetoelectricity in ferromagnetic Néel walls has been anticipated theoretically and recently demonstrated experimentally. Especially, this ME coupling also enables the electric field couple to the magnetization with its spatial gradients, which necessarily presents in metallic as well as insulating ferromagnets. The influence of this coupling on the spin waves of the multiferroics has recently been explored in Ref. However, the relevance of the electric field to the motion of a multiferroic DW, even though it is crucial to future ME multiferroic devices based on DW control, still remains unclear.

In this paper we identify the dynamical nature of a prototypical type of multiferroic DW, that is, a magnetic DW simultaneously displaying an electric polarization. This is a good basis for studying the dynamical properties of the multiferroic DWs, in which the electric field can couple to the magnetization via the inhomogeneous magnetoelectric interaction. We derive the equations of motion for electric field controlled DW dynamics in such a multiferroic DW. We report two main findings. The first one is that the magnetic DW velocity can be considerably enhanced due to the delay of the occurrence of Walker breakdown by an applied electric field. This electric-field-modulated higher DW speed implies faster device operation, which is one of the main aim of the conventional DW device applications. The second finding is that the electric field can be used to control the switching of the DW chirality. This control of the chirality could provide an additional degree of freedom, which can be useful in future magnetoelectric logic devices.

The paper is organized as follows: In Sec. II we present the model for a multiferroic DW. We obtain the equations of motion for the multiferroic DW dynamics using the collective coordinate description. In Sec. III we discuss how the electric field influences the DW velocity and the chirality switching. At the end, in Sec. IV we present a brief summary of the results obtained in this work.

II. THEORETICAL MODEL

The system under consideration is schematically depicted in Fig. I. We will focus on a case of one-dimensional insulating Néel-type DW when magnetic easy (hard) axis is taken to be along the $\hat{z}$ ($\hat{y}$) direc-
with the electric polarization \( \mathbf{P} \) induced within the DW given as

\[
\mathbf{P} = \gamma_0 \int \frac{d^3x}{a^3} \mathbf{S} (\nabla \cdot \mathbf{S}) - (\mathbf{S} \cdot \nabla) \mathbf{S},
\]

where \( \gamma_0 \) is the magnetoelectric coupling coefficient. One can learn immediately from Eq. (7) that inhomogeneous magnetoelectric interaction induces electric polarization \( \mathbf{P} \) within the Néel wall so that it is actually multiferroic.

To study the dynamics of a rigid planar DW, we employ a well-known collective coordinate description.\(^{49}\) In this approach, the position \( X \) and angle \( \phi_0 \) in Eq. (9) of the wall is regarded as time-dependent collective coordinates \( (X(t), \phi_0(t)) \). The chirality of the DW is determined by the tilt angle \( \phi_0(t) \). The domain wall is described by a Lagrangian of local spins given by

\[
\mathcal{L} = -\frac{\hbar NS}{\lambda_{DW}} (X\dot{\phi} + v_\perp \sin^2 \phi_0 - gBX - \gamma E \cos \phi_0).
\]

Here, \( N = 2A\lambda_{DW}/a^3 \) is the number of spins in the wall region with \( A \) being the cross-sectional area of the system, \( v_\perp = \lambda_{DW}K_\perp S/2h \) and \( \gamma = \pi S\gamma_0/2h \). To derive the equations of motion of a DW, one further needs to introduce the dissipation function \( \mathcal{W} \) to incorporate the Gilbert damping, which is written as

\[
\mathcal{W} = \int \frac{d^3x}{a^3} \frac{\hbar}{2S} \dot{\mathbf{S}}^2 = \frac{a\hbar NS}{2} \left[ \left( \frac{X}{\lambda_{DW}} \right)^2 + \dot{\phi}_0^2 \right],
\]

where \( a \) is the Gilbert damping parameter. We then utilize the generalized Euler-Lagrange equation

\[
\frac{d}{dt} \frac{\delta \mathcal{L}}{\delta \dot{q}} - \frac{\delta \mathcal{L}}{\delta q} = -\frac{\delta \mathcal{W}}{\delta \dot{q}},
\]

where \( q \) represents \( X(t) \) and \( \phi_0(t) \) and the last term describes the energy dissipated. The equations of motion for the collective coordinates, derived from the Eqs. (9),

\[
\mathcal{H}_E = -\mathbf{E} \cdot \mathbf{P},
\]
and are given as follows:

\[
\frac{\dot{X}}{\lambda_{DW}} - \alpha \dot{\phi}_0 = \frac{v_1}{\lambda_{DW}} \sin 2\phi_0 + \frac{\gamma E}{\lambda_{DW}} \sin \phi_0, \quad (12a)
\]

\[
\phi_0 + \alpha \frac{\dot{X}}{\lambda_{DW}} = gB. \quad (12b)
\]

These equations provide a basic description of the multiferroic DW dynamics under the magnetic and electric fields. Thus the application of the electric field on the DW introduces an additional spin torque proportional to \(\sin \phi_0\) into the equations of motion. This new term will act as a chirality stabilizer, influencing significantly the DW dynamics. In what follows, we solve Eqs. (12) numerically, and calculate the values for \(X\) and \(\phi_0\) after a sufficiently long time. The average terminal velocity of the DW is defined as \(v_{DW} = \langle \dot{X} \rangle\), where the angular brackets refer to a long-time average. To do the numerical simulation, we take a fixed value for the Gilbert damping parameter \(\alpha = 0.02\). The initial DW tilt angle is set to \(\phi_0(0) = \phi_0(t = 0) = 0\) throughout, so the initial chirality of the DW is clockwise.

III. RESULTS AND DISCUSSION

A. Electric field mediated DW velocity

Let us discuss how the electric field affects the field-driven DW motion based on the equations of motion we derived previously. The numerical simulation results of Eqs. (12) are presented in Figs. 2-4. It is important to note that when the electric field is switched off, the equations of motion in Eqs. (12) are reduced to those of a DW purely driven by a magnetic field, whose behaviours are well known.\(^{3,4,40}\) In that case, the DW motion is characterized by the existence of two dynamic regimes, separated by a threshold field called Walker field.\(^{3}\) That is, for an external field smaller than the Walker field \(B_W = \alpha K L S / 2gh\), the DW moves with a constant velocity which increases linearly with the external magnetic field up to \(B_W\). In this regime, the DW chirality which describes the sense of rotation of the spins in the wall is preserved during the motion. For fields \(B > B_W\), the Walker breakdown occurs and the DW undergoes oscillatory motion, which makes the DW velocity decrease rapidly. Such a behaviour was originally predicted by Schryer and Walker\(^{2}\) and was observed experimentally for example by Beach et al.\(^{2}\).

We first show in Fig. 2 the time-averaged precession velocity \(\langle \dot{\phi}_0 \rangle\), as a function of magnetic field \(B\) for various values of electric fields applied along \(+\hat{z}\) direction. We find that \(\langle \dot{\phi}_0 \rangle = 0\) up to a threshold applied field, even when the external electric field is switched on. This zero precession velocity means that the wall angle \(\phi_0\) tilts out of the plane until it reaches a certain angle. From then on, it no longer changes. In the regime where \(\langle \dot{\phi}_0 \rangle = 0\) the wall moves at a constant velocity. As \(\langle \dot{\phi}_0 \rangle\) becomes finite, the wall tilt angle \(\phi_0\) starts precessing, causing an oscillatory motion that slows down the domain wall. In Fig. 2, we clearly see that the zero \(\langle \dot{\phi}_0 \rangle\) regime (stationary regime) is significantly extended by the application of an electric field.

Figure 3(a) shows the time-averaged DW velocity \(v_{DW}\) as a function of \(B\) for several applied electric fields. When
the electric field is switched on, the $v_{DW}(B)$ curves show similar behavior to that of the conventional magnetic field driven model. For each applied $E$, $v_{DW}$ reaches a maximum velocity, namely Walker velocity ($v_W$). The corresponding threshold magnetic field is Walker field ($B_W$), and above $B_W$ the $v_{DW}$ drops abruptly. More specifically, the Walker field $B_W$ increases with $E$ and there is no change of DW mobility. It seems that the presence of an electric field surely acts as a chirality stabilizer and plays a pivotal role to delay the onset of the Walker breakdown and allows for higher attainable DW velocities. Figures 3(b) and (c) summarize the increase of both the Walker field $B_W$ and the Walker velocity $v_W$ with the magnitude of $E$. It is clearly shown that both $B_W$ and $v_W$ exhibit a nearly linear behavior, so we have an scenario where the maximum velocity of the wall is substantially enhanced by the application of an electric field.

To elucidate the role of the electric field in the suppression of Walker breakdown, we further examine analytically the DW dynamics for $B$ smaller than $B_W$, since in such a stationary regime $\partial \phi_0 / \partial t = 0$ as $t \to \infty$. From Eqs. (12), we obtain

$$\frac{B}{\alpha B_0} = \sin 2\phi_0 + \Delta \sin \phi_0,$$

where $B_0 = K_z S / 2gh$ and $\Delta = E / E_0$ with $E_0 = 2 \gamma$. As long as this equation is satisfied, the DW will propagate without oscillatory motion. The Walker field $B_W$ is determined from the maximum of the r.h.s. of Eq. (13). Equation (13) shows that $B_W$ depends not only on the sign of $\Delta$ (or $E$) but also on the initial tilt angle $\phi_0 (= \phi_0(t = 0))$ (either 0 or $\pi$). The Walker field $B_W$ is found to be associated with the tilt angle $\phi_{W} = \arccos[(-\Delta + \sqrt{\Delta^2 + 32}) / 8]$. Then we can obtain the Walker field as $B_W(\phi_0 = 0, E > 0) = B_{W1}$ with

$$B_{W1} = \frac{16 - \Delta^2 + |\Delta| \tilde{\Delta}^{1/2}}{16\sqrt{2}} (3|\Delta| + \tilde{\Delta}) \alpha B_0, \tag{14}$$

where $\tilde{\Delta} = (\Delta^2 + 32)^{1/2}$. For a large $E$ one has simply $B_W \sim \alpha E$, and further, the Walker velocity $v_W \sim E$.

The analytical results of $B_W$ and $v_W$ versus $E$ are shown as red solid lines in Figs. 3(b) and (c), respectively, which are both consistent with the numerical simulations.

B. Electric field induced DW chirality switching

In this section, we investigate the effect of the applied electric field on the chirality of DW. We show that a reliable control of the chirality switching of a moving DW can be achieved by the application of an electric field along the direction of the magnetic field. Each DW has two possible chiralities: clockwise (CW) and counterclockwise (CCW). This DW chirality can be used as an information unit. Therefore, a controllable switching of the DW chirality is desirable.

We now change the direction of the applied electric field to align the $-\hat{z}$ direction, i.e., parallel to the magnetic field. The numerical results are shown in Fig. 4. Figure 4(a) illustrates the $v_{DW}(B)$ curves for various $E$. We can see that a negative $E$ also results in a suppression of the Walker breakdown and an increase of the DW velocity, similar to the action of a positive $E$ discussed in Sec. III. On the other hand, the chirality of the DW is determined by the tilt angle $\phi_0(t)$. It should be noted that $\phi_0(t)$ increases from the initial tilt angle $\phi_0$ but eventually becomes saturated to a constant value in the limit $t \to \infty$ below the Walker breakdown. Hence, controlling the terminal tilt angle $\phi_0$ can be used to switch the chirality of the DW. For a moving DW driven purely by a magnetic field, its initial chirality is preserved below the Walker field, and the chirality switching is difficult to achieve in a controllable way. However, it can be shown that this picture will not hold for multiferroic DW when $E$ is included. Figure 4(b) shows the terminal DW tilt angle $\phi_{DW} (= \phi_0(t \to \infty))$ as a function of the external magnetic field $B$ in the presence of electric field $E$. We can see that below the Walker field, the DW tilt angle $\phi_{DW}$ initially increases with $B$ from zero, and then suddenly jumps to a value larger than $\pi$ at a critical field $B_c$ for each $E$. Therefore, the switching of the DW chirality from the initially clockwise to terminally counterclockwise has clearly occurred at these critical magnetic

![FIG. 4. (color online) (a) $B$ dependence of the DW velocity $v_{DW}$ for several choices of $E$ applied along the $-\hat{z}$ axis. The color dots mark the critical fields $B_c$ indicating the chirality switching for each case. (b) The terminal DW tilt angle $\phi_{DW}$ as a function of $B$. The sudden jumps of $\phi_{DW}$ denote the chirality switching occurring, from CW to CCW. (c) Component of the spin along the $x$-axis (spin modulation direction) at the DW center. Below $B_c$, $S_x < 0$ (red solid line) means CW chirality, and above $B_c$, $S_x < 0$ (blue dash line) means CCW chirality. The applied $E = -0.5 E_0$. (d) The dependence of $B_c$ with the field strength $|E|$.](image-url)
fields. Fig. 4(c) further shows the x-component of the spin at the DW center for the applied fields \( B \) near the critical field \( B_c \). The applied electric field is chosen as \( E = -0.5E_0 \). The signs of the spin component along the x-axis indicate that below \( B_c \) (i.e., \( B = 0.0132B_0 \)), the DW chirality is CW, when \( B \) reaches up to \( B_c \), the DW chirality switches to CCW. The positions of \( B_c \) extracted from the \( \phi_{DW}(B) \) curves are shown as blue spheres in Fig. 4(d). It is shown that \( B_c \) decreases with \( |E| \) down to zero at \( |E| = 2E_0 \).

In order to better understand this remarkable chirality switching process, we take the following analytical analyses. For a negative \( E \), we can obtain from Eq. \[ 15 \] the Walker field \( B_W(\phi_0 = 0, E < 0) = B_{W2} \), with

\[
B_{W2} = \frac{(16 - \Delta^2 - |\Delta|\tilde{\Delta})^{1/2}(-3|\Delta| + \tilde{\Delta})\alpha B_0}{16\sqrt{2}}, \tag{15}
\]

For smaller negative \( E \) we have \( B_W(\phi_0 = 0, E < 0) \sim -\alpha E \). Interestingly, if setting the initial tilt angle \( \phi_0 = \pi \) (i.e., the initial chirality is CCW), we can also obtain the Walker field as \( B_W(\phi_0 = \pi, E < 0) = B_{W1} \). In this case, the terminal tilt angle \( \phi_{DW} \) is larger than \( \pi \). Moreover, it is easy to see that for \( E < 0 \), \( B_W(\phi_0 = 0, E < 0) \) is smaller than \( B_W(\phi_0 = \pi, E < 0) \). Such a difference between the two Walker fields \( B_W(\phi_0 = 0, E < 0) \) and \( B_W(\phi_0 = \pi, E < 0) \) enables the chirality of a moving DW to be switched. That is, when \( B \) increases beyond the first Walker field \( B_W(\phi_0 = 0, E < 0) \), the Walker breakdown process does not occur until \( B \) further reaches the higher Walker field \( B_W(\phi_0 = \pi, E < 0) \). In the meantime, the initial tilt angle \( \phi_0 \) switches from 0 to \( \pi \) at the first threshold field \( B_W(\phi_0 = 0, E < 0) \) and thus the DW chirality can be switched. The \( E \) dependence of \( B_W(\phi_0 = 0, E < 0) \) is shown in Fig. 4(d) as red solid line, which is in excellent agreement with the numerical results \( B_c \) extracted from the \( \phi_{DW}(B) \) curves. However, for \( E > 0 \), \( B_W(\phi_0 = 0, E > 0) > B_W(\phi_0 = \pi, E > 0) \), we have only one threshold field \( B_W(\phi_0 = 0, E > 0) \) and hence there is no DW chirality switching process. The switching of the DW chirality with the application of electric field can be read by a magnetic field sensor since the stray field near a DW depends on its chirality. It should be noted that the controlled chirality switching of a moving DW can also be achieved by applying an oblique magnetic field as proposed by Seo et al.\[41 \].

Nevertheless, we here offer a more efficient alternative to flip the chirality of the DW with the help of an external electric field.

**IV. SUMMARY**

We considered multiferroic DW systems that exhibit both a coexistence and a coupling of electric polarization and a magnetic DW. The effects of electric field on the DW dynamics via the inhomogeneous magnetoelectric interaction have been investigated. We have revealed the dynamical nature of a multiferroic DW and demonstrated the efficiency of an electric field control of magnetic DW motion. In particular, we showed that the electric field can achieve not only a nearly linear enhancement of the maximum wall velocity, but also a controllable switching of DW chirality. This control of the motion of the multiferroic DWs via electric fields can be useful for designing low-power and high-speed DW-based magnetoelectric memory and logic devices.

**ACKNOWLEDGEMENTS**

This work was supported by NSF-China (Grant Nos. 11074216 and 11274272), and the Fundamental Research Funds for the Central Universities in China.

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