Generation of rogue waves in space dusty plasmas

M.H. Rahman*, A. Mannan, N.A. Chowdhury, and A.A. Mamun

Department of Physics, Jahangirnagar University, Savar, Dhaka-1342, Bangladesh
Email: *rahaman1992phy@gmail.com

Abstract
The basic features of dust-acoustic (DA) waves (DAWs) in four component dusty plasma system (containing inertial cold and hot dust grains, inertialess non-extensive ions and electrons) have been theoretically investigated by deriving the nonlinear Schrödinger equation. The analytic analysis under consideration demonstrates two types of modes, namely, fast and slow DA modes. The unstable domain for the fast DA mode, which can be recognized by the critical wave number (k_c), gives rise to the DA rogue waves (DARWs). It is observed that the amplitude and width of the DARWs are significantly modified by various plasma parameters. The present results should be useful in understanding the conditions for modulational instability of DAWs and generation of DARWs in space dusty plasma systems like Saturn F-rings.

Keywords: Dust-acoustic waves, modulational instability, rogue waves.

1. Introduction

The observational data support the existence of massive charged dust grains not only in astrophysical environments, viz. F-rings of Saturn [1], Earth’s mesosphere, and Jupiter’s magnetosphere [2], but also in many laboratory experiments, viz. ac discharge, Q-machine, and rf discharges [3], etc. The presence of highly charged massive dust grains in plasmas can significantly modify the dynamics of the plasma medium. Dust-acoustic (DA) waves (DAWs), in which mass density of dust grains provides moment of inertia and thermal pressure of the ions and electrons provides restoring force to propagate DAWs, have employed by the physicists to understand various nonlinear electrostatic structures, viz. envelope [4] and rogue profile [5], in dusty plasmas (DP).

Sometimes, highly energetic inertialess particles in space and laboratory plasmas move very fast, due to external force field and wave-particle interaction, compared to their thermal velocity. Such kind of highly energetic inertialess particles are governed by the non-extensive q–distribution function [6] and nonlinear dynamics [7]. A number of authors have studied various nonlinear waves in plasma medium by considering q–distributed inertialess plasma species. Tasnim et al. [8] investigated propagation of DA shock waves (DASHWs) in q–distributed ionic plasma medium and the magnitude of the amplitude of DASHWs decreases with non-extensive parameter q. Ferdousi et al. [9] studied DASHWs in presence of q–distributed ions and found that the polarity and amplitude of the DASHWs depend on non-extensivity of ions. Saha and Chatterjee [10] reported that generation and propagation of DA solitary waves (DASWs) in a two component DP with q–distributed ions. Amour and Tribeche [11] examined the DASWs in a DP with q–distributed electrons and found that the non-extensivity of the electrons makes the DASWs structure more spiky. Emamuddin et al. [12] investigated DAWs in a DP with ions and q–distributed electrons and observed that the amplitude of both positive and negative Gardner solitons increases with non-extensivity. Ghosh et al. [12] studied the effect of the non-extensivity of ions during the head-on collision of DASWs and the phase shift in a DP composed of dust and q–distributed ions.

The amplitude modulation of the nonlinear propagation in a dispersive media, due to carrier wave self interaction or non-linearity of the medium, is an intrigue mechanism. The modulational instability (MI) of nonlinear propagation, which leads to generate freak waves [13], giant waves, rogue waves (RWs), and envelope solitons, is governed by the nonlinear Schrödinger equation (NLSE). The novelty of RWs have attracted the attention of numerous researchers in various fields, viz. optics [15], super-fluid helium [13], hydrodynamics, stock market [16], and plasma physics [7]. For first time in 1977, Watanabe [17] experimentally observed the MI self-modulation of a nonlinear ion wave packet. Subsequently, a number of theoretical investigations have been done to understand the effect(s) of various plasma parameters, viz. plasma species temperature, number density [18], charged state [19], and other factors, on the MI characteristics of the nonlinear propagation in plasma medium. Bouzit and Tribeche [18] reported that the DA RWs (DARWs) structures are very sensitive to any change in the restoring force acting on the dust particles. El-Taibany and Kourakis [19] studied MI of DAWs in an unmagnetized warm DP medium and observed the effects of dust charge variation, dust temperature, and constituent plasma particle concentration on the MI of DAWs. Moslem et al. [20] examined that the amplitude of the DARWs increases with the increase of q in a non-extensive plasma. Selim et al. [21] investigated the propagation of nonlinear DARWs in presence of cold and hot dust grains as well as iso-thermal electrons and non-thermal ions and found that non-
thermal decreases the nonlinearity of the plasma medium and amplitude of the DARWs. Bains et al. [21] analyzed the MI of the DAWs in the presence of \( q \)-distributed electrons and ions and observed that the instability occurs at higher value of \( k \) with an addition of negative dust. To the best of our knowledge, the effects of cold and hot dust as well as \( q \)-distributed electrons and ions on the MI of DARWs have not been investigated. Therefore, in our present investigation, our aim is to examine the MI of the DAWs, generation of the DARWs, and the effects of the \( q \)-distributed electrons and ions on the DARWs in DP system composed of inertial cold and hot ions and inertialless \( q \)-distributed electrons and ions.

The manuscript is organized as the following fashion: The basic model equations are presented in Sec. 2. The MI is given in Sec. 3. Finally, a brief discussion is provided in Sec. 4.

2. Model Equations

We consider an unmagnetized four component DP system which consists of inertial \( q \)-distributed electrons (charge \( -e \); mass \( m_e \); number density \( n_e \) and ions (charge \( +e \); mass \( m_i \); number density \( n_i \), inertially negatively charged cold dust grains (charge \( q_d \); mass \( m_d \); number density \( n_d \)) as well as negatively charged hot dust grains (charge \( q_h = -e Z_h \); mass \( m_h \); number density \( n_h \); adiabatic pressure \( P_h \)). where \( Z_e \) (\( Z_h \)) is the charge state of the negatively charged cold (hot) dust grains. The quasi-neutrality condition at equilibrium is

\[
n_0 = n_{e0} + Z_n n_{d0} + Z_h n_{h0},
\]

where \( n_{e0}, n_{h0} \) is the number densities of the negatively charged cold (hot) dust grains at equilibrium and \( n_{h0} > n_o \). The normalized governing equations of the system can be written as:

\[
\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e u_e) = 0,
\]

\[
\frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} = \frac{\partial \Phi}{\partial x},
\]

\[
\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0,
\]

\[
\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = \sigma_c
\]

\[
\frac{\partial^2 \Phi}{\partial x^2} = -(\mu - 1) n_e - \mu n_i + n_d + n_h.
\]

The normalizing parameters are defined as: \( n_c = N_c / n_{e0}, n_h = N_h / n_{h0}, u_c = U_c / C_{dc}, u_i = U_i / C_{dc}, x = X / \lambda_{Ddc}, t = T \omega_{pdc}, \Phi = e \Phi_{kg T_i}, C_{dc} = (Z_e k^2 \rho_i / m_i \sqrt{\mu}), \lambda_{Ddc} = (k^2 / 4 \pi \epsilon \rho_i Z_{n0}), \omega_{pdc} = (4 \pi e^2 Z_n n_{e0} / m_e), p_h = P_h / \rho_{h0} n_{h0}^2, \mu_h = n_{h0} / 2 \rho_{h0} T_h, \gamma = (N + 2) / N, \sigma = Z_n m_i / m_h, \lambda = Z_h n_{h0} / Z_n n_{d0}, \mu = n_i / \rho_{h0} n_{d0}, \delta = 3 \lambda T_i / \lambda T_h, \Phi = \Phi_{kg T_i}, \lambda_{Ddc}, \omega_{pdc}, T_i, T_h, \) and \( \Phi_{kg T_i} \) is the dimensional number densities of cold dust, hot dust, cold dust fluid speed, hot dust fluid speed, space co-ordinate, time co-ordinate, electrostatic wave potential, Boltzmann constant, sound speed of the negatively charged cold dust, ions temperature, electrons temperature, negatively charged hot dust grains temperature, and the equilibrium adiabatic pressure of the negatively charged cold dust grains, respectively. It may be noted here that we consider \( m_c = m_i \) and \( T_c > T_h \). \( N \) is the degree of freedom and for one-dimensional case \( N = 1 \), hence \( \gamma = 3 \). The number density of the \( q \)-distributed electrons and ions can be given by the following normalized equation, respectively,

\[
n_e = [1 + a(q - 1)\phi] e^{\frac{q-1}{2}},
\]

\[
n_i = [1 - (q - 1)\phi] e^{\frac{q-1}{2}},
\]

where \( a = T_c / T_e \) \( (T_c > T_e) \). It may be noted here that (a) \( q > 1 \) \( (q < 1) \) stands for sub-extensive (super-extensive) electrons and ions; (b) \( q = 1 \) stands for Maxwellian electrons and ions. By substituting (6) and (7) into (1), and expanding up to third order of \( \phi \), we get

\[
\frac{\partial^2 \Phi}{\partial x^2} = -1 - \lambda + n_e + \lambda n_h + \gamma_1 \phi + \gamma_2 \phi^2 + \gamma_3 \phi^3 + \cdots,
\]

where

\[
\gamma_1 = \frac{(q + 1)(\alpha \mu - a - \alpha \lambda + \mu)}{2},
\]

\[
\gamma_2 = \frac{(q + 1)(q - 3)(\alpha^2 + \alpha^2 \lambda + \mu - \alpha^2 \mu)}{8},
\]

\[
\gamma_3 = \frac{(q + 1)(q - 3)(3 q - 5) \alpha^2 \lambda \mu - 3 \lambda^2 \mu + \lambda^2 \alpha \mu)}{48}.
\]

To investigate the MI of the DAWs, we employ the reductive perturbation method to derive the appropriate NLSE. The independent variables are stretched as \( \xi = \epsilon (x - v_{gb} \tau) \) and \( \tau = \epsilon^2 t \), where \( \epsilon \) is a small parameter and \( v_{gb} \) is the group velocity of the wave. The dependent variables can be expressed as:

\[
\Lambda(x, t) = \Lambda_0 + \sum_{m=1}^{\infty} \sum_{l=-\infty}^{\infty} \Lambda^{(m)}(\xi, \tau) \exp(i \Gamma \tau),
\]

where \( \Lambda^{(m)} = [n_{e1}, u_{e1}, n_{d1}, u_{d1}, \phi_{m}] \), \( \Lambda_0 = [1, 0, 1, 0, 0]^T \), \( \Gamma = (k y - \omega \tau) \), and \( \omega \) is the real variables presenting the carrier wave number (frequency), respectively. We are going parallel as done in Chowdhury et al. [22] work to find successively dispersion relation, group velocity, and NLSE. The DAWs dispersion relation

\[
\omega^2 = k^2 D + k^2 \sqrt{D^2 - 4 M},
\]

where \( D = (1 + \sigma \lambda + 2 \gamma_1 + \delta \lambda^2), M = (\gamma_1 + \delta \kappa^2), \) and \( E = \delta \kappa^2 \). In order to obtain real and positive values of \( \omega \), the condition \( D^2 > 4 M \) must be satisfied and positive (negative) sign in (10) is referred to fast DA mode \( \omega_f \) (slow DA mode \( \omega_s \)). The group velocity \( v_g \) of DAWs can be written as

\[
v_g = \frac{2 \omega S^2 - 2 S^2 \omega_3 + \sigma \delta \kappa^2 \omega_3 + \sigma \lambda \omega_5 - \sigma \delta \kappa \omega_3}{2 (k S^2 + \sigma \lambda \kappa \omega_3)}.
\]

where \( S = \delta \kappa^2 - \omega_3^2 \). Finally, the following NLSE:

\[
\frac{\partial \Phi}{\partial \tau} + \frac{P \partial^2 \Phi}{\partial x^2} + Q(\Phi)^2 \Phi = 0,
\]
where $\Phi = \phi_0^{(1)}$ for simplicity and $P$ ($Q$) is the dispersion (nonlinear) coefficient, and is written by

\[
P = \frac{F_1}{2\alpha_0 k^2 (S^2 + \alpha_0 \sigma^2)},
\]

\[
Q = \frac{F_2}{2k^2 (S^2 + \alpha_0 \sigma^2)},
\]

where

\[
F_1 = S^3 (\omega - v_3 k)(\omega - v_3 k - 2\omega v_3 k + 2\nu k^2)
+ \lambda \sigma \omega \delta (\delta k^2) + \omega v_3 k^2 (2\omega v_3 k^2 + kS - k\omega^2 - \delta k^2)
+ (\omega - k v_3)(2\nu k \omega^2 - \delta \omega k^2 - \omega^3 + v_3 kS) - S^3 \omega^4,
\]

\[
F_2 = 3\gamma S^2 \delta \omega - \omega S^2 k^2 (A_1 + A_6) - 2S^2 k^3 (A_2 + A_7)
- \omega S^2 k^3 (A_3 + A_8) - 2\lambda \sigma k^2 \omega^2 (A_4 + A_9)
+ 2\gamma S^2 \delta \omega (A_5 + A_{10}),
\]

\[
A_1 = \frac{3k^4 - 2A_0 k^2 \omega^2}{w_3^2},
\]

\[
A_2 = \frac{A_1 \omega^4 - k^4}{k \omega^2},
\]

\[
A_3 = \frac{2A_5 \sigma \omega^2 k^2 - 3\delta \sigma^2 k^6 - 3\sigma^2 \omega_2 k^4}{2\omega^4},
\]

\[
A_4 = \frac{A_3 S^2 - \omega \sigma^2 k^4}{k \omega^2},
\]

\[
A_5 = \frac{3\gamma S^2 \omega^4 - 3\lambda \omega_2 k^4 \omega^6 - \delta \lambda \omega_2 \sigma^2 \omega^4 k^6}{2\omega^4 \omega^2 S^2 (2S^2 - 4\omega \sigma k^2 - \sigma \omega k^2 - \gamma_1 S \omega^2)},
\]

\[
A_6 = \frac{2\nu_3 k^3 + \omega k^2 - A_{10} \omega^3}{\mu_3},
\]

\[
A_7 = \frac{A_6 \nu_3 \omega^3 - 2k^3}{\mu_3^3},
\]

\[
A_8 = \frac{2\nu_3 \omega k^3 + \sigma \omega k^3 + \delta \sigma k^2 - \sigma A_{10} \omega^2 S^2}{S^2 (v_3^2 - \delta)},
\]

\[
A_9 = \frac{A_8 \nu_3 S^2 - 2 \omega \sigma \omega^2 k^3}{S^2},
\]

\[
A_{10} = \frac{F_3}{S^2 (v_3^2 - \delta) + \lambda \sigma v_3^2 - \gamma_1 S v_3^2 (v_3^2 - \delta)},
\]

\[
F_3 = 2\gamma S^2 v_3^2 \omega (v_3^2 - \delta) + S^2 (v_3^2 - \delta) (2\nu_3 k^3 + \omega k^2)
+ \lambda \nu_3 \omega^2 (2\nu_3 \sigma \omega^2 k^3 + \delta \sigma^2 \omega^4 k^4 + \sigma^2 \omega^4 \delta). \tag{13}
\]

3. Modulational instability

The stability of the DAWs depends on the sign of the nonlinear ($P$) and dispersive ($Q$) coefficients \[22, 23, 24, 25\]. Modulationally stable domain occurs for the DAWs when $P$ and $Q$ are opposite sign ($P/Q < 0$). On the other hand, modulationally unstable domain occurs for the DAWs when $P$ and $Q$ are same sign ($P/Q > 0$). The point, in which $P/Q$ curve coincides with the $k$-axis in $P/Q vs k$ graph, is known as critical/threshold wave number ($k_c$) and this $k_c$ recognizes the stable/unstable domain for the DAWs. The stability of the DAWs for the fast and slow DA modes can be observed in Figs. 1 and 2, respectively. It is obvious from Fig. 1 that the $k_c$ increases (decrease) with the increase of hot dust concentration $n_{io}$ (cold dust concentration $n_{co}$) when the charge state of the cold ($Z_c$) and hot ($Z_h$) dust remain constant (via $\lambda$). Figure 2 shows that the stable domain of the DAWs increases with the increase in the value of hot dust mass density ($m_h$), but decreases with increase of the cold dust mass density ($m_c$) for constant value of hot dust temperature ($T_h$), ion temperature ($T_i$), and cold dust charge state ($Z_c$), respectively (via $\delta$).

The rogue waves solution of the NLSE \[12\] in the unstable domain, which developed by Darboux Transformation Scheme, can be written as \[26, 27\]:

\[
\Phi(\xi, \tau) = \sqrt{\frac{2P}{Q}} \left[ \frac{4(1 + 4iPr)}{1 + 16P^2 + 4\xi^2} - 1 \right] \exp(2iPr). \tag{13}
\]

The solution \[13\] implies that the concentration of high energy occurs (due to the wave-particle interaction) within a small region. The effects of $q$-distributed ion concentration on the shape of the DAWs can be observed from Fig. 3 and it is obvious that the amplitude of the DAWs increases with ion concentration $n_{io}$ for constant values of $Z_c$ and $n_{co}$ (via $\mu$). Physically, the positively charged ions enhance the nonlinearity of the plasma medium, and increase the amplitude of the electrostatic potentials. So, the number density of $q$-distributed ion plays a vital role to control the shape of the DAWs.

The DAWs are so much sensitive to change in the values...
of ion temperature ($T_i$) and electron temperature ($T_e$). It can be shown from Fig. 3 that the height of the DARWs increases (decreases) with ion (electron) temperature. The physics of this result is that the nonlinearity of the plasma medium enhances with ion temperature, that leads to generate a high energetic DARWs.

The effects of electrons and ions non-extensivity can be observed from Figs. 4 and 5 and it is obvious that (a) the amplitude and width of DARWs increase with an increase $q$ (for both $q > 0$ and $q < 0$) and the physics of this result is that the nonlinearity of the plasma medium increases with $q$; (b) the amplitude of DARWs is independent to the sign of $q$, but dependent on the magnitude of $q$ and this is a good agreement with Chowdhury et al. work; (c) a comparison between DARWs potential for $q > 0$ and $q < 0$ can be observed from these two figures; (d) the magnitude of the electrostatic potential for same interval of positive $q$ is not equal as negative $q$.

4. Discussion

We have investigated the MI of a four component realistic DP medium by using standard NLSE. The nonlinear and dispersive coefficients of the NLSE can be recognized the stability of the DAWs for fast and slow DA modes. The $k_c$ value, which determines the stability conditions of DARWs, totally depends on dust masses, charge state of dusts, and number density of the cold and hot dust. The core results from our present investigation can be summarized as follows:

1. Both $\omega_f$ and $\omega_s$ admit modulationally stable and unstable domain for DAWs.
2. The stable domain of the DAWs increases with the increase in the value of $m_h$, but decreases with $m_c$ for constant value of $T_h$, $T_i$, and $Z_c$ (via $\delta$).
3. The amplitude of the electrostatic rogue profile decreases with ion concentration for constant values of $Z_c$ and $n_{c0}$ (via $\mu$).
4. The amplitude and width of DARWs increase with an increase $q$ (for both $q > 0$ and $q < 0$).
5. The amplitude of DARWs is independent to the sign of $q$, but dependent on the magnitude of $q$.

The present results may help in understanding the conditions of MI of DAWs and generation of DARWs in four component space DP system, viz. F-rings of Saturn.

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