Theory of the fractional microwave-induced resistance oscillations

I.A. Dmitriev1,* , A.D. Mirlin1,2,† and D.G. Polyakov1

1Institut für Nanotechnologie, Forschungszentrum Karlsruhe, 76021 Karlsruhe, Germany
2Institut für Theorie der kondensierten Materie, Universität Karlsruhe, 76128 Karlsruhe, Germany

(Dated: February 1, 2008)

We develop a systematic theory of microwave-induced oscillations in magnetoresistivity of a 2D electron gas in the vicinity of fractional harmonics of the cyclotron resonance, observed in recent experiments. We show that in the limit of well-separated Landau levels the effect is dominated by a change of the distribution function induced by multiphoton processes. At moderate magnetic field, a single-photon mechanism originating from the microwave-induced sidebands in the density of states of disorder-broadened Landau levels becomes important.

PACS numbers: 73.50.Pz, 73.43.Qt, 73.50.Fq, 78.67.-n

Recently, a number of remarkable nonequilibrium phenomena have been discovered in a 2D electron gas (2DEG) under strong ac and dc excitation. Most attention has been attracted to the microwave-induced resistivity oscillations (MIRO) [1], particularly following the spectacular observation of “zero-resistance states” (ZRS) in the minima of the oscillations [2, 3]. Two mechanisms of the MIRO were proposed ("displacement" [4, 5, 6] and “inelastic” [7, 8]), in both of which the MIRO originate from the oscillatory density of states (DOS) ν(ε) of disorder-broadened Landau levels (LLs). Both mechanisms reproduce the observed phase of the ω/ωc oscillations (ω and ωc = eB/mc are the microwave and cyclotron frequencies, respectively). The displacement mechanism [4, 5, 6] accounts for microwave-assisted scattering off disorder in the presence of dc electric field and produces temperature-independent MIRO, in disagreement with the experiments. By contrast, the inelastic mechanism [7, 8] is related to the microwave-induced oscillatory changes in the energy distribution of electrons and yields the MIRO with an amplitude proportional to the inelastic scattering time τin ∝ T−2. At relevant T ∼ 1 K, the inelastic mechanism dominates and the corresponding theory [8] reproduces the experimental findings [1, 2, 3].

Remarkably, in addition to the peak-valley structure near integer ω/ωc ("integer MIRO", or IMIRO), later experiments at elevated microwave power [9, 10] reported similar features near certain fractional values, ω/ωc = 1/2, 3/2, 5/2, 2/3 ("fractional MIRO", or FMIRO), as well as “fractional” ZRS [3] (less pronounced FMIRO features were also observed earlier [11]). It was proposed that the FMIRO can be explained in terms of a multiphoton displacement mechanism [12] or, within the framework of the inelastic mechanism [7, 8], in terms of a series of multiple single-photon transitions [10, 13].

In this Letter, we develop a systematic theory of the fractional MIRO. We demonstrate that in the limit of well-separated LLs the FMIRO are dominated by the multiphoton inelastic mechanism, while at weaker magnetic field a microwave-induced spectral reconstruction (MISR) provides a competing single-photon contribution. Both these mechanisms were disregarded previously. Similarly to the IMIRO, in the fractional case the multiphoton displacement mechanism [12] only gives a parametrically smaller contribution. As far as the mechanism [10, 13] is concerned, it is effective only close to the magnetic field at which LLs start to overlap.

Formalism. We consider a 2DEG in a classically strong B in the presence of a weak dc field, Edc = (Ex, Ey), and a microwave field

\[ \mathbf{E}_ω(t) = \frac{E_ω}{\sqrt{2}} \text{Re} \left[ \left( s_+ + s_- \right) e^{-iωt} \right], \]

where s± with s± = 1 parameterize polarization of the microwaves. The main parameters in the problem are related to each other as follows:

\[ \varepsilon_F ≫ T, \omega, \omega_c, \tau_\text{tr}^{-1} ≫ \tau_\text{in}^{-1}, \]

where τq and τtr are the total and transport disorder-induced scattering times at B = 0, and εF is the Fermi energy. We adopt the approach [6, 7, 8, 14] to the problem, based on the quantum Boltzmann equation (QBE) for the semiclassical distribution function at higher LLs, f(ε, ϕ, t) = \( \sum \mathcal{F}_{nm}(ε) \exp(iωϕ + iωt), \)

\[ (\partial_t + \omega_c \partial_ϕ) f + \tau_\text{in}^{-1} (F_{00} - f_T) = \text{St} \{ f \}, \]

where ϕ is the angle of the kinematic momentum and fT(ε) is the Fermi-Dirac distribution. The QBE allows us to treat the interplay of the disorder, the Landau quantization, and the external fields, which are all included into the impurity collision integral St{f}.

Our aim is to calculate the dissipative dc current, \( j_j = (j_x, j_y) \), which is expressed through the first angular harmonic F10 as \( j_j ≡ j_x i_jy - i_{jy} j_x = 2eJ_F \int dεν(ε) F_{10}(ε), \) where υF is the Fermi velocity. Provided τin ≫ τq, the leading contribution to the MIRO comes from microwave-induced changes in the isotropic part F00 of the distribution, governed by the equation F_{00}(ε) - f_T(ε) = τin(\text{St}\{F_{00}(ε)\})t, where the angular brackets denote averaging over both the angle ϕ and the period of the microwave field. The first angular harmonic F_{10}, which
defines the current, is in turn expressed as \(i\omega_0 F_{10}(\varepsilon) = \langle \exp(-i\varphi) \rangle_{\text{St}(F_{10}(\varepsilon))} \rangle_{\varepsilon, \omega} \). As we show below, the above procedure captures both inelastic and displacement contributions to the MIRO \([13]\), yielding

\[
\frac{j_-(\omega)}{2\sigma D} = \int d\Omega \int d\varepsilon K_\Omega(\varepsilon) \langle \hat{\nu}(\varepsilon) \rangle_{\varepsilon, \omega} [F_{00}(\varepsilon - \Omega) - F_{00}(\varepsilon)],
\]

\[F_{00}(\varepsilon) = \frac{F_{00}(\varepsilon) - f_T(\varepsilon)}{\tau_m} = \int \frac{d\Omega}{2\pi} K_\Omega(\varepsilon) \langle \hat{\nu}(\varepsilon - \Omega) \rangle [F_{00}(\varepsilon - \Omega) - F_{00}(\varepsilon)].
\]

(3)

Here \(\sigma^D = e^2 v_F^2 n / \omega_0^2 \tau_r\) is the Drude conductivity, \(\nu_0 = m / 2\pi\), and \(\hat{\nu}(\varepsilon) = \nu(\varepsilon) / \nu_0\) is the dimensionless DOS. The operators \(K_\Omega(\varepsilon)\) and \(K_\Omega(\varepsilon) = K_{10}(\varepsilon) \omega \tau \tau F / \pi e v_F\) can be found to any desired order in the fields \(E_{dc}\) and \(E_\omega\) from the Wigner transform of the kernel \(K(\Omega, t, \varphi)\) of the kernel of the collision integral \(S(t)\) \([14]\). \(K_{nm}(\varepsilon) = \langle e^{-i\varepsilon t} \rangle_{\varepsilon, \omega} \rangle_{\varepsilon, \omega}\). To find the leading contributions to the FMIRO at half-integer \(\omega / \omega_c\), we calculate \(K_{nm}\) to first order in \(E_{dc}\) and fourth order in \(E_\omega\), which gives

\[
\left\{ \begin{array}{l}
K^0_{00}(\varepsilon) \\
K_{10}(\varepsilon) 
\end{array} \right\} = \sum_{n=-2}^{2} \delta(\Omega - n\omega) \left\{ 2\pi A_n / \tau_R \right\}.
\]

(5)

Using the notation \(E_{\pm} = E_x \pm iE_y\) and

\[
E_{\pm} = s_{\pm}(2\tau_r / \tau) \varepsilon e \omega v_F / \omega(\omega_c \pm \omega), \tag{6}
\]

and introducing the dimensionless microwave power, \(P_{\omega} = (E_+^2 + E_-^2) / 2\), express the coefficients \(A_n = A_{n-\omega} = O(P_{\omega})\) and \(B_n = B_{n-\omega} = O(E_{dc} P_{\omega})\) as

\[
A_1 = P_{\omega} / 4 - 2A_2, \\
A_2 = 3P_{\omega}^2 / 32 + 3 E_+^2 E_-^2 / 64, \\
B_1 = E_+ - 2B_1 - 2B_2, \\
B_2 = 3P_{\omega} E_+ / 2 + 3 E_+ E_- / 4 - 4B_3, \\
B_3 = 45(2P_{\omega}^2 E_+ E_- + 2E_+^2 E_- + 2E_+ E_- E_+) / 64. \tag{7}
\]

**Integer MIRO.** Before proceeding to the mechanisms of the FMIRO, it is instructive to show how the results \([8]\) for the displacement and inelastic contributions to the MIRO at leading order \(P_{\omega}\) are reproduced within the present formalism. To this end, we put \(A_2 = B_2 = 0\) and calculate \(F_{00}\) to first order in \(A_1 = P_{\omega} / 4\), which, according to Eqs. \((4)-(7)\), gives

\[
F_{00} - f_T = \frac{\tau_m}{\tau_q} \sum_{\Omega = 0}^{\pm \omega} \hat{\nu}(\varepsilon - \Omega) \langle f_T(\varepsilon - \Omega) - f_T(\varepsilon) \rangle.
\]

(8)

The result for the current \([3]\) at order \(P_{\omega}\) has the form

\[
\frac{j_-(\omega)}{2\sigma D} = B_0 \langle \tilde{\nu}^2(\varepsilon) \rangle_{\varepsilon} + B_1 F_1(\omega) + \frac{\tau_m}{\tau_q} E \omega A_1 F_2(\omega), \tag{9}
\]

where the functions \(F_1(\Omega)\) and \(F_2(\Omega)\), defined as

\[
F_1(\Omega) = 2 \partial_\Omega \langle \hat{\nu}(\varepsilon) \rangle_{\varepsilon},
\]

\[
F_2(\Omega) = \Omega \partial_\Omega \langle \hat{\nu}^2(\varepsilon) \rangle_{\varepsilon} (\hat{\nu}(\varepsilon + \Omega) + \hat{\nu}(\varepsilon - \Omega)). \tag{10}
\]

(11)

oscillate with \(\Omega / \omega_c\). Here we assumed that the Shubnikov–de Haas oscillations are exponentially suppressed by temperature, \(2\pi T \gg \omega_c\), so that the energy integration in Eq. \((6)\) is effectively replaced by the average \(\langle \ldots \rangle_{\varepsilon}\) over the period \(\omega_c\) of the DOS.

In Eq. \((9)\), the first term describes the dark current together with a non-oscillatory (displacement) correction induced by microwaves, while the second and third terms are the displacement \([6]\) and inelastic \([8]\) contributions to the MIRO, which oscilate with \(\omega / \omega_c\). In the limit of separated LLs, \(\omega, \tau_R \gg 1\), the DOS is a sequence of semicircles of width \(2\Gamma = 2(2\omega / \pi \tau)_{1/2}\), i.e.,

\[
\hat{\nu}(\varepsilon) = \tau_\varepsilon \Re \sqrt{\varepsilon^2 - (\delta \varepsilon)^2},
\]

where \(\delta \varepsilon\) is the detuning from the center of the nearest LL. In this limit, calculation of \(F_1(\Omega)\) and \(F_2(\Omega)\) from Eqs. \((10)\) and \((11)\) yields

\[
\frac{F_1(\Omega)}{2 \langle \tilde{\nu}^2(\varepsilon) \rangle_{\varepsilon}} = \frac{4 \Omega \omega_c}{\pi^2} \sum_n \text{sgn}(\tilde{\Omega}_n) \Phi_2(\tilde{\Omega}_n). \tag{12}
\]

Here \(\tilde{\Omega}_n = (\Omega - n\omega_c) / \Gamma\) and \(\langle \tilde{\nu}^2(\varepsilon) \rangle_{\varepsilon} = 16 \omega_c / 3 \pi^2 \Gamma\). The parameterless functions \([12]\) \(H_1(x)\), \(H_2(x)\), and \(\Phi_2(x)\) are nonzero at \(0 < x < 2\) (see Fig. \((1)\)).

\[
H_1(x) = 2 + x \left[ (4 + x^2)E(X) - 4x K(X) \right] / 8, \tag{14}
\]

\[
H_2(x) = 3x \left[ (2 + x)E(X) - 4x K(X) \right] / 8, \tag{15}
\]

\[
4\pi \Phi_2(x) = 3x \arccos(x - 1) - x(1 + x) \sqrt{x(2 - x)}, \tag{16}
\]

where \(X \equiv (2 - x) / (2 + x)^2\) and the functions \(E\) and \(K\) are the complete elliptic integrals of the first and second kind, respectively.

**Two-photon FMIRO.** At order \(E_\omega^2\), microwaves do not produce any oscillatory structure near the fractional values of \(\omega / \omega_c = 1 / 2, 3 / 2, \ldots\) [see Eq. \((9)\)]. Moreover, in the limit \(\Gamma \ll \omega_c\), the oscillatory terms \([19]\) finite only in the narrow intervals \((\omega - N\omega_c) < 4\Gamma\) around integer \(\omega / \omega_c = N\). Solution to Eqs. \((4)-(7)\) at order \(E_\omega^2\) in the regions where \(\nu(\varepsilon) \nu(\varepsilon + \omega) = 0\) gives

\[
\frac{j_-(2\varepsilon)}{2\sigma D} = B_0 \langle \tilde{\nu}^2(\varepsilon) \rangle_{\varepsilon} + B_2 F_1(2\omega) + \frac{\tau_m}{\tau_q} E \omega A_2 F_2(2\omega). \tag{17}
\]
Here $A_2 \propto P_2^2$ and $B_2 \propto E_{dc}P_2$ are given by Eq. (7). The doubling of the arguments of the functions $F_1$ and $F_2$ in Eq. (14) [as compared to the IMIRO case, Eq. (9)] reflects the two-photon nature of the effect and leads to the emergence of the FMIRO at half-integer $\omega/\omega_c$. The form of the inelastic (the last term in Eq. (17)) contribution to the FMIRO is identical to that in the IMIRO case, Eq. (9), and the same holds for the displacement contribution (the second term). In both the integer and fractional cases the inelastic term is a factor $\omega_c^2/\tau_\eta$ larger than the displacement contribution.

With increasing microwave power, the current in the minima becomes negative, $j \cdot E_{dc} < 0$, indicating a transition to the ZRS [17]. Remarkably, like in the IMIRO case [8], the leading-order approximation for the inelastic effect, Eq. (17), is sufficient to describe the photore- sponse even at such high power, since the second-order term $\propto (P_2^2/\tau_\eta)^2$ remains small in the parameter $\Gamma/\omega_c$.

![FIG. 2: Illustration of possible contributions to the FMIRO for $\omega/\omega_c = 1/2 + \Gamma/2$ and $\omega/\Gamma = 7$. Single-photon transitions within and between LLs (thick lines) are forbidden, while two-photon processes are allowed. The microwave-induced sidebands (solid lines) and (dashed lines) make single-photon processes possible [18].](image)

**Microwave-induced spectral reconstruction (MISR).** Above, it was assumed that the spectrum of the 2DEG is not modified by the microwaves. It turns out that, unlike the IMIRO, in the FMIRO case the MISR is relevant, since it provides additional channels for electron transport in the gaps for single-photon absorption. The microwave field enters the equations for the spectrum of the 2DEG at high LLs [8] in the combination $[\sum_{\pm} E_{\pm} \cos(\varphi \pm \omega t)]^2$, suggesting a representation of the retarded Green function and self-energy in the form $\{G_n, \Sigma\} = \sum_{\nu} \sum_{\mu} \sum_{\mu'} \tilde{\Sigma}_{\mu\nu, \mu'}(\varepsilon, t)\tilde{G}_{n\mu\nu}(\varepsilon)$, where the operators $\hat{n}$ and $\hat{\varphi}$ obey $[\hat{n}, \hat{\varphi}] = -i$. Wigner-transforming Eqs. (2.7), (2.15) of Ref. 14 yields

$$[\varepsilon - \mu \omega - (2\nu + n + 1/2)\omega_c]G_{n\mu\nu}(\varepsilon) = \delta_{\mu, \nu} \delta_{n, 0} / 2\pi$$

$$+ \sum_{\mu', \nu'} \sum_{\mu''} \sum_{\nu''} \frac{\omega_c^2}{\tau_\eta} \sum_{n} G_{n\mu\nu}(\varepsilon) = \delta_{\mu, \nu} \delta_{n, 0} / 2\pi$$

$$\times [2\Sigma_{\mu''\nu''}(\varepsilon) - \Sigma_{\mu''\nu''}(\varepsilon + \omega) - \Sigma_{\mu''\nu''}(\varepsilon - \omega)],$$

where we used $\hat{\omega}^2/2\mu \omega = c^2\nu \hat{\varphi}$ ($\hat{n} + 2\nu$). The coefficients $p_{\mu\nu} = p_{-\mu, -\nu}$ have nonzero values for $\{\mu, \nu\} = \{-1, 0, 1\}$:

$$p_{00} = P_2/4, \quad p_{10} = p_{01} = 0, \quad p_{11} = 0, \quad p_{-1, -1} = E_c^2.$$
Here \( F_3(\Omega) \) is an even function of \( \tilde{\Omega}_n = (\Omega - n\omega_c)/\Gamma \):

\[
F_3(\Omega) = \sum_{\pm} \left( [\tilde{\nu}^2(\varepsilon) - \nu^2(\varepsilon \pm \Omega)] \partial_\nu [\tilde{\nu}(\varepsilon) \Re s(\varepsilon)] \right)_\varepsilon \\
= \frac{2\omega_c}{3\Gamma} (\tilde{\nu}^2(\varepsilon))_\varepsilon \left[ 1 + \sum_n \Phi_3(\tilde{\Omega}_n) \theta(2 - |\tilde{\Omega}_n|) \right].
\]

(26)

The parameterless function \( \Phi_3(x) \) (Fig. 1) reads

\[
-\pi \Phi_3(x) = \sqrt{x(2-x)} \left[ 1 - 5x + 2x^2 + \frac{x}{3} \right] \arccos(x-1).
\]

The contribution of the oscillatory sidebands \((23)\) is a factor \(\sim (\omega_c\tau_q)^{1/2}\) larger than the contribution \((24)\).

In contrast to the contributions \((17)\) and \((24)\), it is an even and strongly non-monotonic function of the detuning from the resonance (see Fig. 1). Both these circumstances favor the possibility of observing experimentally the changes in the shape of the FMIRO that should be induced by the contribution \((25)\) at not too large \(\omega_c\tau_q\).

Summary and discussion. We have shown that in the limit of separated LLs, \(\omega_c\tau_q \gg 1\), the fractional features in the photoresponse of a 2DEG are dominated by the multiphoton inelastic mechanism \(\delta\) [the last term in Eq. \((17)\)], while the displacement multiphoton contribution \(\delta\) [the second term in Eq. \((17)\)], considered in Ref. 12, is negligible provided \(\omega_c\tau_m/\tau_q^2\Gamma \gg 1\). The main corrections \((24), (25)\) to the multiphoton inelastic effect originate from the microwave-induced sidebands \((22), (23)\) in the spectrum of a 2DEG (see Fig. 2). In the limit \(\omega_c\tau_q \gg 1\), the sideband contributions are small in the parameter \(\Gamma/\omega_c\) and only slightly modify the shape and the amplitude of the FMIRO, which are dominated by the multiphoton inelastic mechanism. However, at \(\omega_c\tau_q \sim 1\) all three inelastic contributions \((17), (24), (25)\) become comparable in magnitude.

At \(\omega_c < 4\Gamma\), single-photon processes both within and between LLs become allowed. Here, the FMIRO are dominated by the resonant series of single-photon transitions \((10, 13)\) in the framework of the single-photon inelastic mechanism \((8)\). Albeit this effect only exists in the crossover region \(\omega_c \sim \Gamma\), it appears at order \((\tau_m P_c/\tau_q)^2\) and is thus detectable at smaller \(P_c\) than the above contributions. Finally, in the regime of overlapping LLs, \(\omega_c\tau_q \ll 1\), the FMIRO become exponentially weaker than the IMIRO: while the \(B\)-damping of the IMIRO is described by the factor \(\delta\) \(\sim\) \(\exp(-2\pi/\omega_c\tau_q)\) \(\ll 1\) \((8)\), the FMIRO appear at order \(\delta\) \(\ll\) \(\frac{\Delta}{\omega_c}\) \((13, 14)\).

To conclude, there are competing mechanisms of the fractional oscillations, each of which is effective in a different region of magnetic field. The experimental observations \((10)\) should be attributed to the single-photon inelastic mechanism \((13, 14)\) since the FMIRO in Ref. 10 were only observed for microwave frequencies below a certain threshold. By contrast, in Ref. 9 no frequency threshold was reported and strongly developed ZRS were observed, which favors the explanation in terms of the mechanisms we consider here. It would be of interest to perform experiments on the FMIRO in the regime of well-separated LLs, where gaps in the single-photon absorption spectrum were observed \((24)\). In particular, to distinguish between the different mechanisms, we suggest to measure and compare the power and temperature dependencies of the photoresponse at \(\Gamma \ll \omega_c\) and \(\Gamma \sim \omega_c\).

We thank S.I. Dorozhkin and M.A. Zudov for information about the experiments, and I.V. Gorny for discussions. This work was supported by INTAS Grant No. 05-1000008-8044, by the DFG-CFN, and by the RFBR.

[*] Also at A.F. Ioffe Physico-Technical Institute, 194021 St. Petersburg, Russia.
[†] Also at Petersburg Nuclear Physics Institute, 188300 St. Petersburg, Russia.
[1] I.V. Pechenezhskii, S.I. Dorozhkin, and I.A. Dmitriev , Phys. Rev. B 71, 245320 (2007).
[2] I.A. Dmitriev, M.G. Vavilov, and I.L. Aleiner, Phys. Rev. Lett. 90, 046807 (2003).
[3] V.I. Ryzhii, Sov. Phys. Solid State 11, 2078 (1970).
[4] A.C. Hurst, S. Sachdev, N. Read, and S.M. Girvin, Phys. Rev. Lett. 91, 086803 (2003).
[5] M.G. Vavilov and I.L. Aleiner, Phys. Rev. B 69, 035303 (2004).
[6] I.A. Dmitriev, A.D. Mirlin, and D.G. Polyakov, Phys. Rev. Lett. 91, 226802 (2003).
[7] I.A. Dmitriev, M.G. Vavilov, I.L. Aleiner, A.D. Mirlin, and D.G. Polyakov, Phys. Rev. B 71, 115316 (2005).
[8] M.A. Zudov, R.R. Du, L.N. Pfeiffer, and K.W. West, Phys. Rev. B 73, 041303(R) (2006).
[9] S.I. Dorozhkin, J.H. Smet, K. von Klitzing, V. Narayanamurti, W.B. Johnson, and V. Umansky, Nature 420, 646 (2002).
[10] M.A. Zudov, R.R. Du, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. 90, 046807 (2003).
[11] V.I. Ryzhii, Sov. Phys. Solid State 11, 2078 (1970).
[12] A.C. Hurst, S. Sachdev, N. Read, and S.M. Girvin, Phys. Rev. Lett. 91, 086803 (2003).
[13] M.G. Vavilov and I.L. Aleiner, cond-mat/0608632
[14] M.A. Zudov, Phys. Rev. B 69, 041304(R) (2004).
[15] X.L. Lei and S.Y. Liu, Appl. Phys. Lett. 88, 212109 (2006).
[16] I.V. Pechenezhskii, S.I. Dorozhkin, and I.A. Dmitriev, JETP Lett. 85, 86 (2007).
[17] I.A. Dmitriev, A.D. Mirlin, and D.G. Polyakov, Phys. Rev. B 75, 245320 (2007).
[18] M.A. Zudov, R.R. Du, L.N. Pfeiffer, and K.W. West, Phys. Rev. B 73, 041303(R) (2006).
[19] I.V. Pechenezhskii, S.I. Dorozhkin, and I.A. Dmitriev, JETP Lett. 85, 86 (2007).
[20] I.A. Dmitriev, A.D. Mirlin, and D.G. Polyakov, Phys. Rev. B 75, 245320 (2007).
[21] As far as the MIRO in the Hall resistivity are not concerned and \(\omega_c\tau_q \gg 1\), contributions to the photocurrent involving higher temporal and angular harmonics \(F_{nm}\) (different from \(F_{00}\) and \(F_{10}\)) can be neglected \((14)\).
[22] Equations \((14), (15)\) reproduce Eqs. (4.17), (6.26) in Ref. 4 after correction of a typo in the argument of the elliptic integrals \(E\) and \(K\).
[23] A.V. Andreev, I.L. Aleiner, and A.J. Millis, Phys. Rev. Lett. 91, 056803 (2003).
[24] To avoid confusion, the “oscillatory sidebands” \((23)\) have nothing to do with the DOS given by \(g_{0,\pm 1}\).
[25] The contribution of \(g_{0,\pm 1}\) is neglected since \(\Pi_{0,\pm 1} \sim O(\omega_c/\varepsilon_F)\).
[26] S.I. Dorozhkin, J.H. Smet, V. Umansky, and K. von Klitzing, Phys. Rev. B 71, 201306(R) (2005).