Breaking the scale invariance of the primordial spectrum or not: the new WMAP data

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Abstract

It seems that new WMAP data require a fit with a primordial spectrum containing small negative tilt index in addition to the featureless Harrison-Zel'dovich-Peebles spectrum thus implying a broken scale invariance. We show that the data could be otherwise interpreted by a scale invariant primordial spectrum with the scale non-invariant evolution of density contrast using the Press-Schechter formalism. The estimate of the acceleration parameter, as a source of the inhomogeneity of space-time, is made by searching for the minima of the deviation measure defined by the Press-Schechter mass functions for this interpretation compared to the assumptions implicit in the WMAP fit.

Key words: Inhomogeneous cosmologies; Press-Schechter mass-function
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There is a wide agreement in cosmology that the $\Lambda CDM$ model and the inflationary scenario describe in detail a variety of cosmological data. However, general relativity and inflation alone cannot resolve and determine the cosmological constant. The dynamics of the hypothetical inflaton scalar at the scale of $O(10^{-31} \text{cm})$ defines the form but not the magnitude of the primordial fluctuation spectrum at the scale of photon background decoupling $O(10^{25} \text{cm})$.

The fit of three years of WMAP data [1] results in low mass density $\Omega_m = 0.266$ and flat geometry $\Omega_{\Lambda} = 1 - \Omega_m$, but the large scale primordial fluctuation remains disasterously small. This is the confirmation of the COBE and WMAP one year data and there is little chance that future measurements of WMAP or Planck will alter the observed extremely low large scale power of the fluctuations. The negative contribution of the integrated Sachs-Wolfe effect to $TT$

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spectrum, as a possible resolution of the problem, presumes a negative cosmological constant, in apparent contradiction with the overall WMAP fit and other cosmological data.

This paradox could be also resolved by the introduction of a much smaller Hubble constant, as is advocated by Blanchard et al. [2] for the Einstein-de Sitter model. Actually, the estimate of the Hubble constant is sensitive not only on the cosmological model, but also on our detailed knowledge of the mechanisms of standard candles, their galactic environment and their evolution [3,4]. One can expect substantial changes and improvements in this field of research.

The Einstein-Cartan(EC) gravity, as a quantum theory of gravity, predicts a negative cosmological constant [5] assuming the CDM particle to be a fermion and the scale of weak interactions to be a fundamental scale [6]. The EC gravity is the only theory of gravity with definite prediction for the cosmological constant. The phenomena of vorticity and acceleration appears naturally within the EC cosmology [5]. The observed anisotropy in WMAP data has now been confirmed as real but its cosmological origin needs to be clarified [7].

In this note we deal with the scale-noninvariant primordial spectrum inferred from WMAP [1] that has a small negative tilt index as a deviation from the scale-invariant Harrison-Zeldovitch-Peebles spectrum. Instead of assuming constraint for the inflationary scenarios at a scale of $O(10^{-31} cm)$, a scale that is much smaller than the EC cosmology and weak interaction fundamental scale $O(10^{-16} cm)$, we assume a scale-invariant spectrum [8] at a decoupling scale of $O(10^{25} cm)$ and search for a change of the evolving observables in the histogram within an inhomogeneous cosmology up to the present $O(10^{28} cm)$.

To test this idea one could study the CMBR fluctuations beyond Friedmann geometry using an improved CAMB code based on the Ellis-Bruni covariant and gauge-invariant approach [9]. We perform instead the Press-Schechter(PS) analysis in order to constrain the acceleration parameter [10].

From the Press-Schechter mass functions one can derive observables like sub-halo mass functions, cumulative mass functions, galaxy densities, etc [11]. We use the Sheth and Tormen(ST) form [12] confirmed by numerical simulations with cold dark matter. It is necessary to have an evolution equation for the cosmic mass-density contrast for arbitrary inhomogeneous models and then to incorporate the normalized solutions into the ST functions.

In Ref. [13] one can find the derivation of the evolution for the density contrast as a function of the cosmic mass-density, cosmological constant and the
acceleration parameter $\Sigma$ that describes the inhomogeneity of spacetime:

\[
d s^2 = d t^2 - a^2(t) [(1 - \Sigma) d r^2 + r^2 (d \theta^2 + \sin^2 \theta d \phi^2)] - 2 \sqrt{\Sigma} a(t) d r d t,
\]

(1)

\[
a^\mu \equiv u^\nu \nabla_\nu u^\mu, \quad a^\mu a_\mu = -\Sigma \frac{a^2}{a^2}, \quad a \equiv \frac{d a}{d t}.
\]

(2)

\[
\delta'' + \frac{3}{2} a^{-1} \Omega_m a^{-3} + 2 \Omega_\Lambda \Omega_m a^{-3} + \Omega_\Lambda \delta' - \frac{3}{2} \Omega_m^{4/3} \Sigma^{1/2} a^{-6-1/4} (\Omega_m a^{-3} + \Omega_\Lambda)^{-3/2} \delta = 0,
\]

(3)

\[
\delta' \equiv \frac{d \delta}{d a}, \quad a(z) = 1 / (1 + z).
\]

(4)

In the derivation of this equation the term with $\Sigma$ contains $R$-dependence explicitly and violates the scale-invariance [13]. It is easy to verify that the solution to this equation is reduced to the standard one for vanishing $\Sigma$ and positive cosmological constant:

\[
\delta(a) = \frac{1}{x_0} \sqrt{1 + x^3} \int_0^x \frac{d x x^{3/2}}{(1 + x^3)^{3/2}}, \quad x = x_0 a, \quad x_0 = \left( \frac{\Omega_\Lambda}{\Omega_m} \right)^{1/3}
\]

(5)

The detailed form of ST mass function that will be used in our analysis [14] is:

\[
n_{ST}(M) d M = A \left( 1 + \frac{1}{\nu^2 \sigma(M)} \right) \sqrt{2 \rho_m} \frac{d \nu}{M d M} \exp \left( - \frac{\nu^2}{2} \right) d M,
\]

(6)

with $\nu = \sqrt{a_0 D(a) \delta_c(M)}$, $a_0 = 0.707$, $A \approx 0.322$ and $q = 0.3$; $\sigma(M)$ is the variance on the mass scale $M$, $\delta_c$ is the linear threshold for spherical collapse for a flat universe ($\delta_c = 1.686$), $\rho_m$ is the background density, $D(a) = \delta(a)/\delta(1)$ is the linear growth factor normalized as $D(1) = 1$, $W_{TH}(kR)$ is a top-hat filter in Fourier space with $R$ defined by $M = 4\pi \rho_m R^3/3$ and:

\[
\sigma^2 = \frac{1}{(2\pi)^3} \int d^3 k P(k) W_{TH}^2(kR),
\]

(7)

\[
W_{TH}(x) = \frac{3}{x^2} \left( \frac{\sin x}{x} - \cos x \right).
\]

(8)

The present-time ($a = 1$) power spectrum is $P(k) = C k^{1 + \alpha_t} T^2(k)$ where $C$ is the normalization, $\alpha_t$ the index of tilt and $T(k)$ is the transfer function. We take the transfer function $T(k)$ in the following form
\[ T^2(k) = \frac{\ln^2(1 + 2.34p)}{(2.34p)^2} \times [1 + 3.89p + (16.1p)^2 + (5.46p)^3 + (6.71p)^4]^{-1/2} \]

(9)

where \( p = k/(\Gamma h_0 \text{Mpc}^{-1}) \), \( \Gamma = \Omega_m h_0 \exp[-\Omega_b(1+\sqrt{2h_0}/\Omega_m)] \), \( \Omega_b \) is the baryon density and \( h_0 \) is the Hubble parameter.

For the spectra normalized to \( \sigma_8 \) (the rms density fluctuation smoothed with \( R = 8h_0^{-1} \text{Mpc} \)) there is no need to introduce a correction factor (see ref. [11,14]) because it does not affect the calculation of \( \sigma \).

Our aim is to find the best fit to the ST mass functions defined by a tilted primordial spectrum in the Friedmann geometry. Thus, we introduce the following deviation measure \( \lambda \):

\[
\lambda = \left[ \frac{1}{i_{\text{max}} \times j_{\text{max}}} \sum_{i=1}^{i_{\text{max}}} \sum_{j=1}^{j_{\text{max}}} \epsilon_{ij}^2 \right]^{1/2},
\]

(10)

\[
\epsilon_{ij} = \left[ n_{ST}^{(1)}(M_i, z_j) - n_{ST}^{(2)}(M_i, z_j) \right]/n_{ST}^{(1)}(M_i, z_j).
\]

Here \( n_{ST}^{(1)} \) represents ST functions with a tilted spectrum index \( \alpha_t \neq 0 \) and \( \Sigma = 0 \), \( n_{ST}^{(2)} \) has \( \alpha_t = 0 \) and \( \Sigma \neq 0 \), while the parameters \( \Omega_m, \sigma_8, h_0 \) remain unaltered. The summations are over redshifts discretized linearly and over masses logarithmically. Because of the limited validity of the PS formalism we use \( z_{\text{max}} = 4 \), and owing to the normalization of the growth function to redshift zero and low sensitivity at small redshifts, we arbitrarily set \( z_{\text{min}} = 0.5 \).

Using the parameters of WMAP [1] we plot the deviation measure \( \lambda \) as a function of the acceleration parameter \( \Sigma \) (see Fig. 1). The minimum represents (see Fig. 2) the best fit, \( n_{ST}^{(2)} \), and gives us a clue on how large the acceleration parameter \( \Sigma \) could be \( (\Sigma(\lambda_{\text{min}}) = 0.353 \times 10^{-3}) \).

Varying upper and lower bounds on the redshift we obtain (rendering all other parameters as in Fig. 1) \( \Sigma(\lambda_{\text{min}}) = 0.371 \times 10^{-3} \) for \( z \in [1, 4] \) and \( \Sigma(\lambda_{\text{min}}) = 0.167 \times 10^{-3} \) for \( z \in [0.5, 3] \).

The dependence of \( \Sigma(\lambda_{\text{min}}) \) on \( \sigma_8 \) is not significant but it depends sensitively on the magnitude of the index of tilt, as an example \( \Sigma(\lambda_{\text{min}}) = 1.08 \times 10^{-3} \) for \( \alpha_t = -0.08 \) (other parameters as in Fig. 1).

Similarly, one can find the results for the Einstein-de Sitter and Einstein-Cartan cosmologies: \( \Omega_m = 1, \ h_0 = 0.5, \ \Sigma(\lambda_{\text{min}}) = 1.20 \times 10^{-3} \) and \( \Omega_m = 2, \ h_0 = 0.4, \ \Sigma(\lambda_{\text{min}}) = 2.05 \times 10^{-3} \) (other parameters as in Fig. 1).
Fig. 1. Deviation measure ($\lambda$) as a function of the acceleration parameter ($\Sigma$) for $\Omega_m = 0.266$, $\Omega_b = 0.044$, $\sigma_8 = 0.77$, $h_0 = 0.71$, $\alpha_t = -0.05$, $z \in [0.5, 4]$, $M \in [10^{14}, 10^{16}]M_\odot$, $i_{max} = j_{max} = 20$.

To conclude, it seems that it is possible to constrain the acceleration parameter $\Sigma$ from the WMAP data within a PS formalism to be of the order of $O(10^{-3})$. This is in accordance with the estimate of vorticity $\omega_0 = O(10^{-13} \text{yr}^{-1})$ [15] and the relationship in the Einstein-Cartan cosmology $\omega_0 \simeq \Sigma H_0$ [5]. As mentioned above there is a universal fundamental constant of distance for both particle physics and EC cosmology. Therefore, which theory of gravity correctly describes physical reality, may be determined very soon by the pilot run of the LHC in 2007 [6,16].

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Fig. 2. $n_{ST}(M)$ mass functions presented at $z = 2$ for parameters of Fig. 1, where curve denoted by triangles(crosses) is $n_{ST}^{(1)}(n_{ST}^{(2)}(\Sigma(\lambda_{\text{min}})))$.

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