Kinematics and dynamics analysis of the main motion system of reciprocating machine tools

Zijiang Yang¹, Xiangning Pan¹*, Yuan Wang² and Weifeng Tang¹

¹School of Basic Sciences for Aviation, Naval Aviation University, YanTai, China
²Yantai Productivity Promotion Center, Yantai, China

*Corresponding author e-mail: 531519045@qq.com

Abstract. To reveal the kinematic and dynamic characteristics of the main motion system of reciprocating machine tools, the paper adopts vector equation method to set up motion mathematical models for the main motion system, carries on the kinematics simulation analysis by using MATLAB Simulink, and get these kinematic characteristic curves. The force analysis of the components in the main motion system is carried out on the basis of motion analysis. First, force analysis of the components in the main motion system is carried out on the basis of motion analysis; Second, the dynamic calculation matrix is obtained by the assembly of the equilibrium equations of the mechanics. Finally, the analytical solutions of the main motion system dynamics and the dynamic characteristic curves of the main motion system of the machine tool is obtained by using Simulink. All the calculated data will be the data foundation for fatigue life prediction of reciprocating machine tools in the future.

1. Introduction

To achieve better performance, major manufacturers and related research institutes of reciprocating machine tool have done a great deal of research on its crushing mechanism, dynamic features of cutter, dynamic features of vibration, and creative design of structure, which plays a positive role in the optimization and improvement of saw machine performance. However, there is only a little of research on the kinematic and dynamic features of reciprocating machine tool[1-8].

The paper adopts a new vector equation method complying with MATLAB Simulink to analyze the kinematics and dynamics of the main motion system of reciprocating machine tool.

2. Structure characteristics of the main motion system of the reciprocating machine tools

The main motion system of the reciprocating machine tools is like Figure 1. The system is a typical crank-slider mechanism, its machining function is realized by the cutter head on the slide block.

Figure 1. Main motion system
Main technical parameters:
Crank rotation rate $\omega = 0 \sim 3 \pi \text{ rad/s}$
Feedrate of the workpiece $V = 0 \sim 700 \text{mm/h}$
Reciprocating stroke $S = 600 \text{mm}$
Length of crank $R = 300 \text{mm}$
Length of connecting rod $L = 4000 \text{mm}$

3. Analysis of the main motion system based on kinematics

3.1. Theoretical analysis of the main motion system based on kinematics

The main motion system of the reciprocating machine tool is simplified as a "crank-slider" mechanism, and a mechanical model of the mechanism is set up on the basis of this mechanism, which is shown in Figure 2.

![Figure 2. Mechanism motion diagram of main motion system](image)

The motion equations of three components including slider 1, crank 2 and link 3 as shown in Fig. 2 are expressed as:

$$\dot{R}_1 + \dot{R}_3 = \dot{R}_1$$  \(1\)

The angle between the vector and the X axis is $\theta$, and the vector $R$ is decomposed into two components, the X direction and the Y direction. The new formulas are expressed as:

$$\begin{align*}
\begin{cases}
\dot{r}_x = r \cos (\theta) \\
\dot{r}_y = r \sin (\theta)
\end{cases}
\end{align*}$$  \(2\)

Replace formula 2 to Formula 1:

$$\begin{align*}
\begin{cases}
r_2 \cos \theta_2 + r_3 \cos \theta_3 = r_1 \\
r_2 \sin \theta_2 + r_3 \sin \theta_3 = 0
\end{cases}
\end{align*}$$  \(3\)

Solve the first derivative and the two derivative of formula 3:

$$\begin{align*}
\begin{cases}
-r_2 \omega_2 \sin \theta_2 - r_3 \omega_3 \sin \theta_3 = \dot{r} \\
r_2 \omega_2 \cos \theta_2 + r_3 \omega_3 \cos \theta_3 = 0
\end{cases}
\end{align*}$$  \(4\)

$$\begin{align*}
\begin{cases}
-r_2 \omega_2 \sin \theta_2 - r_3 \omega_3^2 \cos \theta_2 - r_3 \omega_3 \sin \theta_3 - r_2 \omega_2^2 \cos \theta_2 + r_3 \omega_3^2 \sin \theta_3 = \ddot{r}_1 \\
r_2 \omega_2 \cos \theta_2 - r_3 \omega_3^2 \sin \theta_2 + r_3 \omega_3 \cos \theta_3 - r_3 \omega_3^2 \sin \theta_3 = 0
\end{cases}
\end{align*}$$  \(5\)
Formula 4 and Formula 5 are expressed in matrix form:

\[
\begin{bmatrix}
  r_1 \sin \theta_3 & 1 \\
  -r_1 \cos \theta_3 & 0
\end{bmatrix}
\begin{bmatrix}
  \alpha_3 \\
  \dot{\alpha}_3
\end{bmatrix}
= \begin{bmatrix}
  -r_2 \omega_2 \sin \theta_2 \\
  r_2 \omega_2 \cos \theta_2
\end{bmatrix}
\]  \hspace{1cm} (6)

\[
\begin{bmatrix}
  r_2 \sin \theta_2 & 1 \\
  -r_2 \cos \theta_2 & 0
\end{bmatrix}
\begin{bmatrix}
  \alpha_3 \\
  \dot{\alpha}_3
\end{bmatrix}
= \begin{bmatrix}
  -r_3 \alpha_3 \sin \theta_3 - r_2 \omega_2^2 \cos \theta_2 - r_2 \omega_2^2 \cos \theta_3 \\
  r_3 \alpha_3 \cos \theta_3 - r_2 \omega_2^2 \sin \theta_2 - r_3 \omega_3^2 \sin \theta_3
\end{bmatrix}
\]  \hspace{1cm} (7)

3.2. Simulation analysis of the main motion system based on kinematics

According to the above theoretical analysis, the matrix equation is obtained by using MATLAB to write the sawing speed and acceleration simulation program respectively, and the program is embedded into the Simulink through the function module [9-12]. The kinematics simulation framework diagram is shown in Figure 3.

The kinematics simulation takes the crank angular velocity \( \omega_3 \) as input, and the displacement \( r_1 \), velocity \( \omega_1 \) and acceleration \( a_1 \) of the slider as output. The initial conditions refer to a common processing parameter, as shown in Table 1.

| Parameter | \( r_1 \) | \( r_2 \) | \( r_3 \) | \( \omega_2 \) | \( \omega_3 \) | \( \theta_2 \) | \( \theta_3 \) |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Numerical value | 4000mm | 300mm | 4000mm | 9.425rad/s | 0.851rad/s | 0 | 0 |

The simulation time is set to 2s, and the results is shown in Figure 4.
As shown in Figure 4, the results of the kinematic simulation can be obtained:

1) The cycle period of slider displacement $r_1$, slider velocity $v_1$ and slider acceleration $a_1$ is 0.67s, slider velocity is positive rotation, and the slider displacement and slider acceleration are cosine.

2) At the initial time, the slider is located at the right end, the velocity is 0, and the trend is pointing to the left. At this moment, the maximum acceleration of the slider is 38.70 $m/s^2$.

3) The speed of the slider increases with the rotation of the crank. At the time 0.17s, the maximum speed can reach 3.76$m/s$, the acceleration is reduced to 0.

4) At 0.33s time, the velocity of the slider is reduced to 0, the acceleration is 38.70 $m/s^2$, and the slider moves to the left end, as shown in Figure 4. The movement trend turned to the right, and the return motion began.

The results of the kinematic simulation of the "crank-slider" mechanism accord with the actual situation, which not only proves the accuracy of the modeling, but also lays a foundation for the dynamic analysis on the further.

4. Analysis of the main motion system based on dynamics

4.1. Theoretical analysis of the main motion system based on dynamics

The main motion system of the reciprocating machine tool is simplified as a "crank-slider" mechanism, and a mathematical model of the mechanism is set up on the basis of this mechanism, which is shown in Figure 2.

From the kinematics analysis of the previous section, the maximum acceleration of the slider can reach 38.70 $m/s^2$, which is close to 4 times of the gravitational acceleration. It is necessary to carry out dynamic analysis of the main motion system.

In engineering applications, the input component is equipped with a flywheel with a larger inertia, and the torque that the driving motor can provide is far greater than the required torque for the mechanism operation. Based on the above two points, the hypothesis of constant rotation of crank is made.

The force in the vertical direction of the slider has been balanced by other parts of the machine tools, so it is ignored. The main motion system is only subjected to two external forces, one is the torque $T$ that drives the crank, and the other is the horizontal sawing resistance $F_{f}$ acting on the slider, and the mechanism force diagram is shown in Figure 5.

![Figure 5. Mechanism force diagram](image)

The force analysis of each member of the "crank-slider" mechanism is carried out separately, and the force at the node is decomposed into the components of X direction and Y direction. The force diagram is shown in Figure 6.
**Figure 6.** Force diagrams of each component

Mechanical model of the crank:

\[
\begin{align*}
F_{12,x} + F_{32,x} &= m_2 a_{2,x} \\
F_{12,y} + F_{32,y} &= m_2 a_{2,y} \\
-F_{32,x} r_2 \sin \theta_2 + F_{32,y} r_2 \cos \theta_2 + M_{12} &= I_2 a_2
\end{align*}
\]  

Mechanical model of the connecting rod:

\[
\begin{align*}
-F_{32,x} + F_{43,x} &= m_3 a_{3,x} \\
-F_{32,y} + F_{43,y} &= m_3 a_{3,y} \\
F_{43,x} (r_3 - r_3) \sin \theta_3 + F_{43,y} (r_3 - r_3) \cos \theta_3 &- F_{32,x} r_3 \cos \theta_2 + F_{32,y} r_3 \sin \theta_2 = I_3 a_3
\end{align*}
\]  

Mechanical model of the slider:

\[
\begin{align*}
F_{34,x} + F_{4s} &= m \ddot{r}_i \\
F_{34,y} + F_{14,y} &= 0
\end{align*}
\]  

In order to get the mechanical analytical solution of the "crank-slider" mechanism including the forces $F_{12,x}, F_{12,y}, F_{32,x}, F_{32,y}, F_{43,x}, F_{43,y}, F_{34,y}$ and $F_{14,y}$, we should use the analysis method of the kinematics theory of the upper section and use the joint equation set to solve them.

Makes $B_2 = \cos \theta_2$, $D_2 = \sin \theta_2$, $B_3 = \cos \theta_3$, $D_3 = \sin \theta_3$. The above 14 equations are assembled into matrix form, as shown in Figure 7.
4.2. Simulation analysis of the main motion system based on dynamics
The simulation analysis method is similar to the previous section. The dynamics simulation framework diagram is shown in Figure 8.

The simulation results are shown in Figure 9.
Changing curve of $M_{12}$

Figure 9. Force changing curves of each component

The following conclusions can be drawn from the results of mechanism dynamics analysis:

1) The torque $M_{12}$ of the main shaft presents a trend of positive and negative alternation. During the whole cycle, the main motion system satisfies the law of conservation of energy.

2) The change rule of X direction component and Y direction component of hinge joint of crank and connecting rod are cosine and positive rotation respectively. The X component reaches 1.95N at $t=0s$, which is the positive maximum, reaches 1.65N at $t=0.17s$ which is the reverse maximum. The X component reaches the positive maximum at $t=0.08s$ and reaches the reverse maximum at $t=0.50s$, and the maximal value is 3.65N.

3) The cycle of the X component of the connecting rod two hinged joints is the same, but the amplitude is different, $F_{23.x} > F_{32.x}$, which is caused by the joint action of the huge alternating inertia force and the horizontal sawing resistance of the connecting rod during the movement.

4) The X component of the force in the hinge joint of the slider and the connecting rod reaches 2.45N at $t=0s$, which is the reverse maximum; reaches 2.15N at $t=0.33s$, which is the positive maximum. The Y component reaches the positive maximum and the reverse maximum at $t=0.08s$ and $t=0.25s$ respectively, and the maximum value is 900N.

5. Conclusion
The kinematic model of the main motion system of reciprocating machine tool has been established. Through theoretical analysis and simulation analysis, the law of slider movement has been acquired. The maximum acceleration of slider can reach $38.70\, m/s^2$ under normal circumstances.

The dynamic analysis of the main motion system of the reciprocating machine tool have been carried out, and the force change law of each component in the main motion system has been acquired under the action of the horizontal sawing resistance of the slider. The X-component and Y-component of the force vector on the hinged end of crank and connecting rod reach the peak at different times; The X-component and Y-component of the force vector on the hinged end of crank and slider also reach the peak at different times.

The kinematics and dynamics analysis of the main motion system of the reciprocating machine tool have been carried out with MATLAB Simulink, and the curve charts and table data have been
acquired. These lay the data foundation for fatigue life prediction of all components of the main motion system.

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