Application of Definite Solution of Partial Differential Equation in Deep Learning

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Abstract. With the continuous development of human knowledge and the rapid expansion of knowledge data, the traditional learning methods have been unable to meet the needs of the development of human knowledge. In this case, some scholars proposed the theory of deep learning on the basis of the layered operation of human brain. At present, deep learning has been widely applied in various fields, such as image processing, speech recognition and emotion classification. Partial differential equation (pde) is a common mathematical analysis method, which can effectively extract data features, which is consistent with the characteristics of deep learning. Therefore, it is feasible to apply the definite solution of partial differential equation to deep learning. The purpose of this paper is to promote the further development of deep learning by studying the application of partial differential equation in deep learning. This paper first summarizes the concepts of deep learning and partial differential equations, then analyzes the wavelet method of the definite solution problem, and then discusses the feasibility of applying the wavelet method to deep learning. Finally, the development of deep learning based on wavelet analysis is discussed. The research in this paper shows that the solution of the definite solution problem of partial differential equations can take three-dimensional space as the starting point to reconstruct deep learning in an all-round way, which greatly expands the application scope of deep learning theory.

Keywords: Partial Differential Equation, Definite Solution Problem, Deep Learning, Wavelet Method

1. Introduction
In recent years, due to the rapid development of information technology, the speed of information generation, dissemination and update has been greatly improved, and the data information of the society has also shown explosive growth. In order to master a certain information, data information must be effectively mined to realize the depth of information understanding. Based on this, some scholars put forward the theory of deep learning, which can effectively extract different data features by establishing a multi-level learning model. With the deepening of the research on deep learning, some scholars have found the convergence point between the solution of definite solutions of partial differential equations and deep learning, and tried to apply partial differential equations to deep learning, so as to promote the perfection and practical application of deep learning theory.

Because of the long history and wide application of partial differential equations, scholars at home and abroad have made a series of studies on differential partial equations. In [1], the author proposes a wavelet solution method for the definite solution of partial differential equations, and concludes that the wavelet solution method can greatly shorten the solution practice of the definite solution of partial differential equations. In [2], the author proposes a difference method for a class of nonlinear partial differential equations, and applies this algorithm to sewage treatment. In [3], the author focuses on the theory of partial differential equations, especially its main definitions, and relates it to image processing and applies it to the three-dimensional segmentation of lung CT images.

The theory of deep learning was first proposed by Hinton et al. At present, the theory has been widely applied to speech recognition, image processing, emotion classification and other fields. Domestic and foreign scholars have also conducted in-depth studies on the theory and application of deep learning. In [4], the author made a comprehensive analysis of the development, concept, model and model composition of deep learning, and specifically analyzed its application in the field of speech recognition. In [5], the author applies the theory of deep learning to the field of facial expression recognition and forms a multi-layer sensory technology. In [6], the author applies deep learning theory in emotion analysis for the first time, and tries to build an emotion analysis model based on deep learning theory according to the complexity and obscurity of emotion.

To explore solve the problems of partial differential equations to solve in the application of deep learning, promote the continuous improvement of the depth of learning theory, this article first to deep learning, the concept of partial differential equations, and has carried on the summary of relevant specific formula, and then the wavelet method to solve the problem are analyzed, and on this basis on the depth of the wavelet method is applied to study the feasibility of the discussed; Finally, the actual development of deep learning based on wavelet solution is discussed [7-8]. The research in this paper not only makes up the theoretical gap existing in the current research in this field, but also promotes the good application of the solution of definite solutions of differential equations in deep learning, and
lays a theoretical foundation for the future research in related aspects [9].

2. Method

2.1. Deep learning

The proposal of deep learning is based on the layered operation of human brain. In essence, it constructs a deeper neural network to conduct in-depth analysis of various types of data, such as images, words and Numbers. This deep neural network is similar to the structure of the human brain. Different from shallow learning, the learning model of deep learning is multi-level, and the types of training data are diverse and large, which can effectively extract data features and greatly enhance the accuracy of data classification results [10-11]. Deep learning places special emphasis on feature learning, which can transform the initial features into a new space by means of feature layer by layer transformation, so as to realize the convenient classification of data. To some extent, deep learning can be regarded as the product of the evolution of neural network. There are certain connections between the two, but there are also many differences. Deep learning shows great advantages. In deep learning, a network can accommodate all feature data, which promotes the in-depth extraction of data features and the enhancement of network's easy ability. In addition, deep learning can get closer to complex functions by understanding the nonlinear network structure, and input data can be expressed by means of distribution. There are four common deep learning models, namely automatic coding machine, deep neural network, deep convolutional network and recursive neural network [12-13].

2.2. Solution of definite solutions of partial differential equations

Partial differential equation (pde) is a common mathematical analysis method, which can effectively extract data features. The definite solution problem of partial differential equation means that the extraction method of data features of a certain differential partial equation is fixed in the set function space, and the final solution result of the definite solution problem is also unique and stable, which is a special steady phenomenon in the function. The classification of differential partial equations is usually divided according to the different conditions of definite solution, including three main types of boundary value, initial value and mixed initial boundary value problems. The problem of definite solution of partial differential equations is characterized by the following three aspects: existence, uniqueness and stability of solutions. The common formulas of the differential partial equation are as follows:

\[ F(x_1, x_2, ..., x_n, u, D_u, ..., \frac{\partial^p u}{\partial x_1 \partial x_2 ... \partial x_3}) = 0 \] (1)
In the above formula, $u(x_1,x_2... X_n)$ represents an unknown function that contains a total of $n$ independent variables; $F$ is the constant function, and $u$ is the classical solution. The solution obtained under this equation is called weak solution, which can effectively reflect the discontinuity of data information in deep learning. The formula for the definite solution of partial differential equation is as follows:

$$p_{ij} \frac{\partial^2 u}{\partial x_i \partial x_j} + q_i \frac{\partial u}{\partial x_i} + ku + f = 0 \quad (2)$$

In this equation, $p_{ij}$ is the final solution of the definite solution problem of partial differential equation, $q$ represents the coefficient of the unknown function, and $K$ and $f$ represent the continuous function in the set of real Numbers.

3. Wavelet method of definite solution problem

Through access to relevant data, found that most researchers to solve the problems of partial differential equations of wavelet method is applied to image processing, and deep learning theory is widely used in image processing, so this paper tries to use the wavelet method to micro problem solving partial differential equations, and apply it to the depth of learning. The specific solution steps are as follows: first, the discrete difference processing is carried out, which is mainly aimed at the time direction of the differential partial equation, and the time direction is set as $t$. Secondly, according to the above steps, the suitable wavelet basis is selected and expanded at all times to obtain the algebraic equations related to the time direction. It's important to note that the wavelet basis here is usually orthogonal or biorthogonal; In the end, because algebraic equations have pathologies in general, the iterative and regularization methods are used to solve fixed-point problems. In the practical application of deep learning, wavelet method can effectively solve the problem of time error in deep learning, so as to realize the accurate analysis of deep learning results.

4. Discuss

4.1. Feasibility analysis of the application of wavelet method in deep learning

In order to ensure the operability of the application of wavelet transform in deep learning and the accuracy of the analysis and calculation results, it is necessary to carry out the feasibility analysis of the application of wavelet transform in deep learning. It is concluded that the biorthogonal wavelet basis based wavelet method is more effective in deep learning. The data results of specific analysis are shown in table 1 and figure 1. The data in the chart is the result of the author's experimental arrangement.
Table 1. Practical measurement of wavelet in deep learning

| Applicable wavelet | Daubechious     | Biorthogonal   |
|--------------------|-----------------|----------------|
| Maximum absolute error | 3.675978       | 1.973699       |
| Minimum absolute error     | 0.773812       | 0.002326       |
| Calculation time (s)         | 193.11         | 65.015         |

*Data came from the in-depth analysis of financial data in the experiment

Figure 1. Biorthogonal wavelet base wavelet solution and true solution

It can be seen from the data in table 1 that the computational effect of biorthogonal wavelet in deep learning is significantly better than that of ordinary wavelet in deep learning. The reason for this phenomenon is that the ordinary wavelet N=5 consists of a support range of -4 to 5; While the biorthogonal wavelet N=3.5, the support range of the wavelet function constituted by the biorthogonal wavelet is between -3 and 4, which Narrows the data error range of deep learning to some extent. The specific analysis results of the wavelet and true solutions of biorthogonal wavelet bases in deep learning are shown in figure 1.

4.2. Influence of the solution of definite solutions of partial differential equations on deep learning

(1) Realize accurate mining of deep learning data

Can see from table 1, and the correlation analysis results is fully capable of wavelet method is applied to the depth of learning, especially the biorthogonal wavelet based wavelet method, can realize all kinds of characteristics of the data in the deep learning effective extraction, the extraction after
completion of the data, can according to the characteristics of each type of data to determine the effect of deep learning. Different from other methods used in this paper, the solution method proposed by partial differential equation with the help of the definite solution problem can achieve a comprehensive understanding of relevant data, not to miss any useful data, so as to ensure the accuracy of analysis results of deep learning to a great extent.

(2) A new construction of deep learning model is realized

Already mentioned above mainly has four kinds of common deep learning model, automatic coding machine, the depth of neural network, the depth of the convolution and the recursive neural network, but the relationship between these four models and is not quite close together, deep learning theory and the application scope of confined in a certain range, it is difficult to achieve further breakthroughs. Is used to solve the problems of partial differential equations to solve the wavelet method can better solve the problem, the wavelet method to build up the depth study of the relationship between the data, establish a stereoscopic 3 d data model, as a support, deep learning existing models are no longer exist independently, but through related data to establish a model of organic whole.

5. Conclusion

In this paper, the partial differential equation of definite solution problem solving in depth study on the application of the related research, the results show that using wavelet algorithm solve the problems of partial differential equations can be directly applied to the practical application of deep learning, the algorithm can achieve both the depth study of a full range of reconstruct, greatly expanded the application scope of deep learning theory. Based on the existing research conclusions of this paper, the next research work is proposed:

(1) Due to the relatively complex data information contained in the deep learning theory, it is far from enough to expand the theory and application field of deep learning only by relying on the wavelet algorithm proposed in this paper. It is hoped to establish a systematic solution model for the definite solution of partial differential equations and build its deep connection with deep learning.

References

[1] Zeng weili, tan xianghua, lu xiaobo. Region-based fourth order partial differential equation denoising method [J]. Journal of southeast university, 2017, 27(2):154-158.

[2] Li liang, guo shuxu, Chen guofa. Image restoration algorithm based on partial differential equations [J]. Journal of jilin university, 2017, 30(1):72-78.

[3] Shi baoli, huang lihong. Multiplicative noise denoising model based on fourth-order partial
differential equations [J]. Journal of hunan university, 2018, 38(7):83-87.

[4] Li yunhui, huang yonghui, Lin yingzhen. Numerical solution of a class of nonlinear partial differential equations [J]. Practice and understanding of mathematics, 2017, 41(13):174-179.

[5] Lu xueqin, cui minggen. A new algorithm for solving a class of second-order nonlinear partial differential equations [J]. Journal of computational mathematics, 2019, 31(2):111-118.

[6] Suo haibin, tang weilong, shen fu. Application of numerical solutions to nonlinear partial differential equations in chemical engineering [J]. Petroleum University journal (natural science), 2017, 20(3):68-71.

[7] Li haifeng, li chunguo. Comparative analysis of deep learning structures and algorithms [J]. Journal of hebei university, 2017, 32(5):538-544.

[8] Bateman, H. The Solution of Partial Differential Equations by Means of Definite Integrals[J]. Proceedings of the London Mathematical Society, 2017, 33(1):451-458.

[9] I. Kalenyuk, Z. M. Nytrebych, G. Kuduk, et al. Integral Problem for a Partial Differential Equation of High Order in an Infinite Strip[J]. 2018, 231(10):1-12.

[10] D.Gunzburger, R.A. Nicolaides. On substructuring algorithms and solution techniques for the numerical approximation of partial differential equations[J]. Applied Numerical Mathematics, 2017, 2(5):243-256.

[11] Balasundaram, S, Bhattacharyya, P. K. On existence of solution of the Dirichlet problem of fourth order partial differential equations with variable coefficients[J]. Quarterly of Applied Mathematics, 2017, 41(3):311-317.

[12] Vladimir Druskin, Leonid Knizhnerman. On application of the Lanczos method to solution of some partial differential equations[J]. Journal of Computational & Applied Mathematics, 2017, 50(3):255-262.

[13] Brigida Molina, Marcos Raydan. Preconditioned Barzilai-Borwein method for the numerical solution of partial differential equations[J]. Numerical Algorithms, 2018, 13(1):45-60.