Influence of the jet opening angle on the derived kinematical parameters of blazar jets having uniform and stratified bulk motion

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Accepted 2007 February 15; Received 2007 February 15; in original form 2006 July 22

ABSTRACT
We present analytical modelling of conical relativistic jets, in order to evaluate the role of the jet opening angle on certain key parameters that are inferred from VLBI radio observations of blazar nuclear jets. The key parameters evaluated are the orientation angle (i.e., the viewing angle) of the jet and the apparent speed and Doppler factor of the radio knots on parsec scales. Quantitative comparisons are made of the influence of the jet opening angle on the above parameters of the radio knots, as would be estimated for two widely discussed variants of relativistic nuclear jets, namely, those having uniform bulk speed and those in which the bulk Lorentz factor of the flow decreases with distance from the jet axis (a ‘spine–sheath’ flow). Our analysis shows that for both types of jet velocity distributions the expectation value of the jet orientation angle at first falls dramatically with increases in the (central) jet Lorentz factor, but it levels off at a fraction of the opening angle for extremely relativistic jets. We also find that the effective values of the apparent speeds and Doppler factors of the knots always decline substantially with increasing jet opening angle, but that this effect is strongest for ultra-relativistic jets with uniform bulk speed. We suggest that the paucity of highly superluminal parsec-scale radio components in TeV blazars can be understood if their jets are highly relativistic and, being intrinsically weaker, somewhat less well collimated, in comparison to the jets in other blazars.

Key words: blazars: general – galaxies: active – galaxies: jets – galaxies: nuclei – galaxies: radio continuum

1 INTRODUCTION
Although Very Long Baseline Interferometry (VLBI) monitoring of the radio knots in blazar jets has revealed several sources containing knots with apparent speeds $v_{\text{app}}$ in excess of $25c$ (e.g., Kellermann et al. 2004; Piner et al. 2006), the typical speeds for blazars known to emit the highest energy γ-ray photons (TeV blazars) are found to be much more modest, with $v_{\text{app}} < 5c$, and their radio knots are often subluminal (e.g., Edwards & Piner 2002; Piner & Edwards 2004). The glaring contrast with the very large bulk Lorentz factors in the parsec-scale blazar jets ($\Gamma > 30c$), as inferred from TeV flux variations (e.g., Krawczynski, Coppi & Aharonian 2002), has been highlighted by several authors (e.g., Giroletti et al. 2004; Gopal-Krishna, Dhurde & Wiita 2004; Piner & Edwards 2004, 2005; Levinson 2006). Similar ultra-relativistic bulk Lorentz factors have also been inferred from the intraday radio variability of some blazars (e.g., Rickett, Kedziora-Chudczer & Jauncey 2002; Macquart & de Bruyn 2006) and, more directly, from VLBI measurements of the brightness temperatures of several blazar nuclei (e.g., Horizon et al. 2004; cf., Kovalev et al. 2005). The large mismatch between the estimates of $\Gamma$ derived from these different types of observations has led some authors to postulate a dramatic jet deceleration between sub-parsec and parsec scales (e.g., Georganopoulos & Kazanas 2003; cf., Piner & Edwards 2005). An alternative approach has been to invoke a ‘spine–sheath’ configuration for the jets such that the fast spine close to the jet axis is surrounded by an appropriately slow moving sheath (e.g., Baan 1980; Komissarov 1990;
These considerations recently led us to undertake an analytical study which showed that the modest apparent speeds of the knots of blazars, which are mostly unresolved by VLBI, can be reconciled with the extremely relativistic bulk motion inferred for TeV, and some other, blazars commonly inferred (e.g., Jorstad et al. 2005) by combining the analytical study which showed that the modest apparent speeds of the knots of blazars, which are mostly unresolved by VLBI, can be reconciled with the extremely relativistic bulk motion inferred for TeV, and some other, blazars commonly inferred (e.g., Jorstad et al. 2005) by combining the

d = \frac{1}{\Gamma(1 - \beta \cdot \hat{n})},
\hspace{1cm}(1)

where \( \Gamma = (1 - \beta^2)^{-1/2} \) and \( \hat{n} \) is the unit vector along the line-of-sight;

\[ \vec{v}_{\text{app}} = \frac{\vec{v} \times \hat{n}}{(1 - \beta \cdot \hat{n})} \]
\hspace{1cm}(2)

In Papers I & II we considered that each portion of the surface of the knot, denoted by the differential solid angle, \( d\Omega' \), will have an angle to the observer’s line of sight that can range between \( \theta - \omega/2 \) and \( \theta + \omega/2 \). Then the values of \( \delta \) and \( \omega_{\text{app}} \) arising from each \( d\Omega' \) patch vary (unlike for the commonly considered case where the jet is assumed to be a pencil beam with \( \omega = 0 \)). Here we generalize those results by considering the possibility that the magnitude of the bulk velocity, \( c\beta \), might also vary as a function of the angular distance (\( r \)) of that patch from the jet axis.

The following basic relations are used for computing the effective values, \( \delta_{\text{eff}} \) and \( \delta_{\text{app}} \), for a knot, as would be determined from VLBI observations which are unable to resolve the width of the jet (i.e., knot):

\[ S_{\text{obs}} = \int_{\Omega} \delta^p(\Omega')S_{\text{em}}(\Omega')d\Omega' \equiv A(\theta)S_{\text{em}}; \hspace{1cm}(3) \]

\[ \delta_{\text{eff}} = A^+(\theta); \hspace{1cm}(4) \]

\[ \beta_{\text{app},\text{eff}} = \frac{1}{S_{\text{obs}}} \int_{\Omega} \beta(\Omega')\delta^p(\Omega')S_{\text{em}}(\Omega')d\Omega'. \hspace{1cm}(5) \]

Here, \( S_{\text{em}} \) and \( S_{\text{obs}} \) are the emitted and the (Doppler boosted) observed flux densities, respectively, \( \Omega' = \beta \cdot \hat{n}/\beta \) denotes the location on the knot given by the direction cosine between the local velocity and the line of sight to the nucleus (which is no longer merely \( \cos \theta \)), \( \Omega \) is the entire solid angle subtended by the knot, \( A(\theta) \) is the flux boosting factor averaged over the radio knot’s cross-section, and \( p \) is defined later in this section. Although we allow for a functional dependence for \( S_{\text{em}} \) in Eqs. (3) and (5) our computations assume that \( S_{\text{em}} \) is actually independent of \( \Omega' \) (i.e., a uniform intrinsic surface brightness across the knot).

In principle, several peaks in a VLBI jet could belong to a common shock/knot yet appear separate only because of their different apparent velocities (owing to their different velocity vectors within a cone). In some cases, this approach has already suggested extremely large bulk Lorentz factors (> 100, cf., Jorstad et al. 2004); however, there is usually a bias against accepting and reporting such extreme speeds. Moreover, such associations are unlikely to be the norm, since there is good evidence that individual bright VLBI knots arise from discrete ejection events in the nucleus, since their trajectories can mostly be extrapolated to begin from the core at the times of emission of discrete flares observed at millimetre wavelengths (e.g., Savolainen et al. 2002). Secondly, due to limited resolving power, the apparent motion of individual VLBI knots is commonly determined by monitoring the shift in the “centroid” position of a given knot (e.g., Cohen et al. 2007). To give expression to this practical situation, we have computed the flux weighted speed (Eq. 5). Clearly, this would no longer be required when the spatial resolution and the dynamic range of the VLBI image undergoes a substantial improvement.

The nominal viewing angle, \( \theta \), to the jet axis measures the angular offset of the circular radio disc’s centre (i.e., the jet axis) from the direction of the AGN core (Fig. 1). The probability distribution of the viewing angle, \( P(\theta) \) (see Eq. 7 below), is used to compute the expectation value of \( \theta \) for any particular combination of \( \Gamma, \omega, p \) and \( q \), with the last two parameters defined below.

The integration was carried out numerically by dividing the circular radio disc (i.e., the radio knot) of diameter \( \omega \) and centred at \( \theta \) into small pixels of angular size equal to 0.01\( \omega \). For each pixel \((i,j)\) in the disc, its angular off-
Jet opening angle and blazar parameters

3 DISCUSSION

Fig. 2 shows a steep initial decline of $\langle \theta \rangle$ with increasing $\Gamma_0$, regardless of the chosen values of $\omega$ or $q$. This decline continues until the regime of ultra-relativistic jets ($\Gamma_0 \geq 30$) is reached, whereafter $\langle \theta \rangle$ becomes essentially independent of $\Gamma_0$. The near constancy of $\langle \theta \rangle$ for extremely relativistic jets is particularly striking for the conical jets having a uniform $\Gamma$ and at least a moderate opening angle ($\omega \sim 5^\circ$). Further, in this ultra-relativistic regime, the tendency for $\langle \theta \rangle$ to increase with $\omega$ is found to accelerate with the increase in the opening angle $\omega$. Interestingly, these trends found for the uniform $\Gamma$ jets are also shared by both representative forms of spine–sheath jets considered here. Quantitatively, for $\omega = 5^\circ$, ultra-relativistic jets of all the three forms (i.e., $q = 0, 1$ and 2), have $\langle \theta \rangle$ in the narrow range from $1^\circ$ to $2^\circ$. The corresponding range of $\langle \theta \rangle$ for $\omega = 10^\circ$ is $2^\circ$ to $4^\circ$, for all $\Gamma$s above $\sim 30$. From Fig. 2, it is also evident that in a typical radio flux-limited sample of blazars, a larger opening angle of an ultra-relativistic jet would correspond to a considerably lower $\langle \theta \rangle$; hence a milder fore-shortening due to projection is typically expected (see Paper II for an
Figure 2. Expectation values of the viewing angle, $\langle \theta \rangle$, against the jet (central) Lorentz factor, $\Gamma_0$, for uniform (top) and transversely structured (middle and lower panels) jets. The left panels correspond to $p = 2$ and the right to $p = 3$. Solid, dashed and dot-dashed lines are for $\omega = 10^\circ, 5^\circ$, and $1^\circ$, respectively.

expanded discussion of this last point for uniform velocity jets).

We now examine the influence of $\omega$ on the effective apparent speed of a radio knot, $c\beta_{\text{eff}}$, and its effective Doppler factor, $\delta_{\text{eff}}$ (Eqs. 4 & 5). These are displayed in Fig. 3 ($p = 3$) and in Fig. 4 ($p = 2$) for the computed expectation values $\langle \theta \rangle$ (which is a function of $\omega, \Gamma_0, p$ and $\delta$ and hence varies along each curve, as explained above). Good analytical fits to simple exponential curves were found to all the computed values of $\beta_{\text{eff}}$ and they are quoted within those figures. Exponential functions also gave good fits to all but two of the 18 cases considered for $\delta_{\text{eff}}$; in those cases the displayed polynomial fits were found to be substantially better.

Firstly, it is seen that the decline of $\beta_{\text{eff}}$ with $\omega$ is sharper for the knots associated with jets of higher $\Gamma$ (for both uniform and stratified types). On the other hand, for
Jet opening angle and blazar parameters

Figure 3. The effective speeds (left) and effective Doppler factors (right) as functions of the full opening angle of the jet, with \( p = 3 \). The top, middle and bottom panels correspond to \( \Gamma_0 = 20, 50, \) and 100, respectively. The \( \times \) symbols, closed circles and open circles are the computed values for \( q = 0, 1, 2 \), respectively. The solid, dashed and dot-dashed curves give the corresponding labelled exponential function fits to the results except that polynomial fits are given to the \( \delta_{\text{eff}} \) values for the \( \Gamma_0 = 100 \) and \( q = 1, 2 \) cases. In each panel, the open square symbol gives the usual result for \( \omega = 0 \) and constant \( \Gamma \).
well collimated jets (i.e., $\omega \lesssim 0.5^\circ$), $\beta_{\text{eff}}$ for the uniform $\Gamma$ case would typically be 1.5 to 2 times higher than for the $q = 1$ and between 2 and 4 times higher for the $q = 2$ (the two spine–sheath cases). Thus, the fastest spine component of the jet flow, which is near the jet axis, would be substantially concealed in the VLBI measurements.

It is interesting to note that the much sharper fall of $\beta_{\text{eff}}$ with $\omega$ found for the uniform $\Gamma$ case (especially in Fig. 2) ensures that already for modest jet opening angles, the value for $\beta_{\text{eff}}$ of such jets would drop below the corresponding values for both of the spine–sheath models considered here. For $p = 3$ this crossover is seen to occur near $\omega = 6^\circ$, 2.5$^\circ$ and 1.3$^\circ$, for $\Gamma_0 = 20$, 50 and 100, respectively, such that beyond these modest jet opening angles, $\beta_{\text{eff}}$ rapidly approaches mildly relativistic (or, even sub-relativistic) values in the case of extremely relativistic jets with uniform $\Gamma$.

Figure 4. As in Fig. 3 for $p = 2$, except that exponential fits are made to all results.
This steep fall of $\beta_{\text{eff}}$ was first pointed out in Paper I for the case of uniform $\Gamma$ jets and we find here a similar, albeit milder, dependence for the stratified ultra-relativistic jets as well (Figs. 3 and 4). It is also worth noting that while for well collimated ultra-relativistic jets $\beta_{\text{eff}}$ is more strongly suppressed (relative to the uniform $\Gamma$ case) when $\Gamma$ is more peaked towards the axis (i.e., $q = 2$); the opposite is found for the jets having a significant opening angle ($\omega \sim 2^{\circ}$ to $\sim 10^\circ$), with the exact cross-over value of $\omega$ depending on $\Gamma_0$ and $p$. In summary, the provided conical jet is moderately wide ($\omega \sim 10^\circ$), the measured apparent speeds of the VLBI knots would typically remain under 10$c$, even if $\Gamma_0$ were extremely large ($\sim 100$), not only for the uniform $\Gamma$ jets but even for the stratified jets.

The right columns of Figs. 3 and 4 illustrate the $\omega$-dependence of the ‘effective Doppler factor’ ($\delta_{\text{eff}}$) for the uniform $\Gamma$ jets and for the two stratified jet forms; recall that $\delta_{\text{eff}}$ is the cube root (for the $p = 3$ case), or square root (for $p = 2$) of the flux boosting factor, $\Delta_{\text{eff}}$, averaged over the radio knot (Eq. 4). Again, each point of these profiles is meant for the corresponding value of $\omega$, as discussed above. It is seen that for cases of very good collimation ($\omega \lesssim 0.5^\circ$), the uniform $\Gamma$ jets would typically have 2 to 4 times larger $\delta_{\text{eff}}$ compared to the stratified jets, implying roughly an order-of-magnitude stronger Doppler boost. As expected for stratified jets, a sharper spine–sheath contrast (i.e., a larger $q$) goes to a lower $\delta_{\text{eff}}$.

Further, as for $\beta_{\text{eff}}$, a significant reduction in $\delta_{\text{eff}}$ with $\omega$ is the typical expectation for both kinds of jet, the dependence being stronger for extremely relativistic jets, particularly the uniform $\Gamma$ type (Figs. 3 and 4). This leads to a situation where extremely relativistic jets of both uniform and stratified $\Gamma$ types end up with comparable $\delta_{\text{eff}}$ values, once $\omega$ exceeds about $5^\circ$, although the $\delta_{\text{eff}}$ values for the uniform jets do remain larger than those for the stratified jets over the wide parameter space considered here. These still very high values of $\delta_{\text{eff}}$ mean that rapid variability in TeV $\gamma$-ray emission is to be expected in any of our ultrarelativistic models as the variability timescale is proportional to $\delta_{\text{eff}}^{-1}$. Significant changes in TeV fluxes on sub-hour timescales have been reported for Mrk 421 (e.g., Blažejowski et al. 2005).

Often, VLBI observations reveal different apparent speeds for the radio knots in the same jet. This is usually interpreted by postulating that the knots reflect ‘pattern speeds’ which can be substantially different from the underlying speed of the jet (e.g., Vermeulen & Cohen 1994; Cohen et al. 2007). In our picture, such variations can be readily understood in terms of surface brightness distributions across the different knots being dissimilar. Moreover, the observed lack of sources with large apparent speeds but low brightness temperatures (e.g., Kovalev et al. 2005; Homan et al. 2006) also suggests that the intrinsic pattern speed should be broadly correlated to the jet’s bulk speed.

As noted in Paper I, for the case of fully collimated jets the bulk Lorentz factor would have to quickly drop by 1 to 2 orders of magnitude between sub-parsec (TeV) and parsec (radio-VLBI) scales in order to reconcile the very large Doppler factors inferred from the compactness argument with the marginally superluminal (even sub-luminal) motion observed for the VLBI knots (Sect. 1). At present there is no direct observational evidence for such drastic deceleration (and the concomitant dissipation) occurring on sub-parsec scales in blazar nuclei. Rather, for Cygnus A, where sub-parsec radio knot motions can be measured from millimetre VLBI and the jets are close to the plane of the sky (so that small changes in direction would not yield significant apparent speed changes) the evidence favors modest acceleration, not deceleration, on the sub-parsec scale (Bach et al. 2005). One aim of our model is to eliminate the need for such a massively rapid deceleration.

A key question is: why is the Lorentz factor dichotomy so striking only for TeV blazars (e.g., Piner & Edwards 2004)? Essentially all blazars show two correlated peaks in their spectral energy distributions and are now usually classified by the frequency at which the lower frequency (synchrotron) peak is strongest (e.g., Padovani & Giommi 1995; Sambruna, Maraschi & Urry 1996). Now, according to the popular scheme unifying high energy peaked (HBL) and low energy peaked blazars (LBL) (e.g., Sambruna et al. 1996; Fossati et al. 1998), TeV emission, which is an HBL characteristic, would be more common among lower luminosity blazars. This hypothesis has been supported by a recent study, which indicates that the synchrotron peaks for powerful blazars (the Flat-Spectrum Radio Quasars) all remain below the rather high energy ($\gtrsim 1$ keV) which is characteristic of HBLs and TeV blazars (Padovani 2007).

We also recall that extragalactic jets tend to be less well collimated for lower luminosity sources, although this is only well established on the kpc scales where their opening angles can be routinely measured (e.g., Bridle 1984). Opening angles on sub-parsec scales can be measured fairly unambiguously only for very nearby sources such as M87 (Biretta et al. 2002) and Centaurus A (Horishita et al. 2006) where the jets lie rather close to the plane of the sky; both of these are indeed relatively weak sources with wide jets. On the scale of nuclear jets Blandford (1993) gives a theoretical argument for this weak/wide correlation. Assuming it does hold, then the correlation between HBL properties and intrinsically weaker jets would mean that the jet opening angle should be larger for TeV emitting jets. Likely consequences of this are: (a) the probability of detecting TeV blazars would be enhanced, since, even though the TeV emission itself is probably beamed very sharply, its effective beaming angle is in fact much larger, and actually more like the jet opening angle (see, Phinney 1985; Blandford 1993; Begelman, Rees & Sikora 1994); (b) the wider jet would mean a bigger reduction in the apparent velocity of the radio knot (Paper I and Figs. 3 and 4). We suggest that the combination of these factors is probably responsible for the intriguing preference of TeV blazars to display slower motions of their VLBI knots.

**ACKNOWLEDGMENTS**

We thank the referee, Amir Levinson, for suggestions which significantly improved the presentation of these results. PS would like to thank the Indian Academy of Science, Bangalore, for the award of a summer student fellowship and the National Centre for Radio Astrophysics (NCRA-TIFR), Pune for the facilities provided for his summer project. PJW is grateful for continuing hospitality at the Princeton University Observatory; his efforts were supported in part by NSF grant AST-0507529 to the University of Washington through a subcontract to Georgia State University.
REFERENCES

Baan W., 1980, ApJ, 239, 433
Bach U., Kadler M., Krichbaum T. P., Middelberg E., Alef W., Witzel, A., Zensus J. A., 2005, in Romney J. D., Reid, M. J., eds., Future Directions in High Resolution Astronomy: The 10th Anniversary of the VLBA, ASP Conf. Ser. 340, Astr. Soc. Pacific, San Francisco, p. 30.
Begelman M. C., Rees M. J., Sikora M., 1994, ApJ, 429, L57
Biretta J., Junor W., Livio, M., 2002, New Astr. Rev., 46, 239
Blandford R. D., 1993, in Burgarella D., Livio M., O’Dea C., eds., Astrophysical Jets, Cambridge Univ. Press, Cambridge, p. 15
Blazejowski M., et al., 2005, ApJ, 630, 130
Bridle A. H., 1984, AJ, 89, 979
Cohen M. H., 1989, in Maraschi L., Maccacaro T., Ulrich M.-H., eds., BL Lac Objects, Springer-Verlag, Berlin, p. 13
Cohen M. H., Lister M. L., Homan D. C., Kadler M., Kellermann K. I., Kovalev Y. Y., Vermeulen R. C., 2007, ApJ, in press (astro-ph/0611642)
Edwards P. G, Piner B. G, 2002, ApJ, 579, L67
Fomalont E. B., Windhorst R. A., Kristian J. A., Kellermann K. I., 1991, AJ, 102, 1258
Georganopoulos M., Kazanas D. 2003, ApJ, 594, L27
Giroletti M. et al., 2004, ApJ, 600, 127
Gopal-Krishna, Dhurde S., Wiita P. J., 2004, ApJ, 615, L81 (Paper I)
Gopal-Krishna, Wiita P. J., Dhurde S., 2006, MNRAS, 369, 1287 (Paper II)
Homan D. C., et al., 2006, ApJ, 642, L115
Horiiuchi S., et al., 2004, ApJ, 616, 110
Hughes P. A., Aller H. D., Aller M. F., 1985, ApJ, 298, 296
Jorstad S. G., et al., 2004, AJ, 127, 3115
Kaiser C. R., 2006, MNRAS, 367, 1083
Kellermann K. I., et al., 2004, ApJ, 609, 539
Komissarov S. S. 1990, Soviet Astron. Lett., 16, 284
Kovalev Y. Y., et al., 2005, AJ, 130, 2473
Krawczynski H., Coppi P. S., Aharonian F., 2002, MNRAS, 336, 721
Laing R. A., 1993, in Burgarella D., Livio M., O’Dea C., eds., Astrophysical Jets, Cambridge Univ. Press, Cambridge, p. 95
Levinson A., 2006, Int. J. Mod. Phys., A21, 6015
Lind R. K., Blandford R. D., 1985, ApJ, 295, 358
Macquart J.-P., de Bruyn A. G., 2006, A&A, 446, 185
Marscher A. P., Gear W. K., 1985, ApJ, 298, 114
Meier D. L., 2003, New Astr. Rev., 47, 667
Mészáros P., 2002, ARA&A, 40, 137
Padovani P., 2007, Ap&SS, in press (astro-ph/0610545)
Phinney S., 1985, in Miller J. S., ed., Astrophysics of Active Galaxies and Quasi-stellar Objects, University Science Books, Mill Valley, 453
Piner B. G., Edwards P. G., 2004, ApJ, 600, 115
Piner B. G., Edwards P. G., 2005, ApJ, 622, 168
Rickett B. J., Kedziora-Chudczer L., Jauncey D. L., 2002, ApJ, 581, 103
Sambruna R. M., Maraschi L., Urry C. M., 1996, ApJ, 463, 444
Savolainen T., Wilk K., Valtaoja E., Jorstad S. G., Marscher A. P., 2002, A&A, 394, 851.