To advance quantum information science, physical systems are sought that meet the stringent requirements for creating and preserving quantum entanglement. In atomic physics, robust two-qubit entanglement is typically achieved by strong, long-range interactions in the form of either Coulomb interactions between ions or dipolar interactions between Rydberg atoms\textsuperscript{1–4}. Although such interactions allow fast quantum gates, the interacting atoms must overcome the associated coupling to the environment and crosstalk among qubits\textsuperscript{5–8}. Local interactions, such as those requiring substantial wavefunction overlap, can alleviate these detrimental effects; however, such interactions present a new challenge: to distribute entanglement, qubits must be transported, merged for interaction, and then isolated for storage and subsequent operations. Here we show how, using a mobile optical tweezer, it is possible to prepare and locally entangle two ultracold neutral atoms, and then separate them while preserving their entanglement\textsuperscript{9–11}. Ground-state neutral atom experiments have measured dynamics consistent with spin entanglement\textsuperscript{10,12,13}, and have detected entanglement with macroscopic observables\textsuperscript{14,15}; we are now able to demonstrate position-resolved two-particle coherence via application of a local gradient and parity measurements\textsuperscript{8}. This new entanglement-verification protocol could be applied to arbitrary spin-entangled states of spatially separated atoms\textsuperscript{16,17}. The local entangling operation is achieved via spin-exchange interactions\textsuperscript{9–11}, and quantum tunnelling is used to combine and separate atoms. These techniques provide a framework for dynamically entangling remote qubits via local operations within a large-scale quantum register.

Internal spin states of particles provide robust and long-lived storage for quantum information. Although engineered spin-dependent interactions can realize entangling gates between spins, they also predispose the system to strong environmental coupling and decoherence\textsuperscript{9}. Spin-exchange interactions, which arise from a combination of quantum statistics and spin-independent forces, afford a promising alternative route to entanglement generation, and have been explored with both electrons (in quantum dots) and atoms\textsuperscript{9–11,18–20}. When two particles interact, their interaction energy depends on the spatial symmetry of the two-particle wavefunction. If the particles have spin but are otherwise identical, the symmetry of the two-particle spin state directly determines the spatial symmetry of their wavefunction: for repulsive interactions, two bosons (fermions) of opposite spin in a triplet configuration T experience enhanced (suppressed) interactions, while the converse occurs for the singlet spin state S; see Fig. 1b. By preparing a superposition of the triplet and singlet, dynamical quantum beats result in the exchange of spin between the particles. In our experiment, we prepare atoms of opposing spin in the lowest two motional states (e and q) of an optical tweezer potential that we represent as $|e_r, e_q\rangle$, which results in equal population of the singlet and triplet spatial wavefunctions $\psi (r_1, T_1)$ and $\psi (r_1, T_2)$, respectively. The interference in the contact interaction energy between these states yields spin-exchange dynamics at a rate $\Gamma_{ex}$, which depends on the $s$-wave scattering length and two-particle density (see Methods). Specifically, it gives rise to the dynamics

$$\psi (t) = \left| e_r, e_q \right> \cos (\Gamma_{ex} t / 2\hbar) + i \left| e_r, g_q \right> \sin (\Gamma_{ex} t / 2\hbar)$$ (1)

This evolution is associated with the effective spin-dependent Hamiltonian $H = \hbar \Gamma_{ex} S_r \cdot S_q$, where $S_r$ and $S_q$ are the spin operators for the respective motional states. A spin-entangled state $(|e_r, e_q\rangle + |e_r, g_q\rangle) / \sqrt{2}$ can be created by allowing the state to evolve for an exchange time of $\pi / 2\Gamma_{ex}$.

We schematically represent the experiment in Fig. 1a, in which separated optical tweezers on the left (L) and right (R) each containing a single atom are dynamically reconfigured to produce spin-exchange dynamics and non-local entanglement. Atoms are combined in the right optical tweezer where spin exchange creates the entangled state $(|e_r, e_q\rangle + |e_r, g_q\rangle) / \sqrt{2}$. Importantly, in our experiments we convert this entanglement into spatial spin correlations by separating the atoms into two tweezers (Fig. 1a) to yield a state $(|e_r, e_q\rangle + |e_r, g_q\rangle) / \sqrt{2}$. Although verification of such entanglement is a standard tool in ion and Rydberg experiments\textsuperscript{1,3,4,21}, spatially resolved detection of the entanglement present in interacting systems of ground-state neutral atoms is challenging. Theoretical proposals have studied ways of detecting spatial\textsuperscript{22} and spin\textsuperscript{23,24} entanglement, and very recently experimental progress has been made using a quantum gas microscope\textsuperscript{25}. We devised a protocol that yields rotations of a two-qubit entangled state on the associated Bloch sphere via a combination of a magnetic-field gradient and global spin rotations (Fig. 1c), allowing detection of basis-independent correlations for arbitrary entangled states of the form $|e_r, e_q\rangle + e^{i\theta} |e_r, g_q\rangle / \sqrt{2}$. Our protocol is applicable to qubit pairs in a large quantum register\textsuperscript{15}, to interacting spins in a Bose–Hubbard chain\textsuperscript{15,16}, and to strongly interacting fermions featuring antiferromagnetic correlations\textsuperscript{21–23}.

The experiment begins by loading two thermal $^{87}$Rb atoms into two separate optical tweezer potentials\textsuperscript{26,27}. Each atom is then separately laser-cooled to the 3D ground state via Raman-sideband cooling, leaving a 3D ground–state fraction of 90(7)% (refs 5, 27, 28); the uncertainty in parentheses here and below indicates the standard error. The atoms are initialized in opposite spin states with a fidelity greater than 90%; however, unlike our previous work\textsuperscript{28}, we dynamically shift to an asymmetric configuration such that the ground state of one optical tweezer is near-resonant with the first (radial) excited state of the other optical tweezer. We then perform adiabatic passage by slowly tuning the relative tweezer depths linearly in time, which coherently transfers the left atom into the right optical tweezer over the 12–ms duration of the ramp (see Methods).
After a desired evolution time in the presence of exchange dynamics, the adiabatic passage is applied in reverse, yielding a motional-state mapping of the excited-state atom back into the ground state of the left well. We can then read out the spin populations to verify the exchange oscillations by ejecting atoms in the $|\uparrow\rangle$ state from the optical tweezers, and imaging the atom occupancy in each of the two tweezers. With this procedure, we can ascertain what the spin and motional degrees of freedom of each of the two atoms were when they occupied the same optical tweezer.

Exchange oscillations in our experiment are shown in Fig. 2b, and show the expected anti-correlated behaviour. To display these data, we exclude from our analysis experiments in which imperfections in the entangled state created by the exchange interaction: homogeneously magnetic-field fluctuations induce a global phase on the two-particle state.

dependence, we prepare $|\psi_g\rangle$ and linearly increase the depth of the tweezer in 5 ms, allow evolution of the exchange, and then ramp back in reverse and perform the second adiabatic passage. We can model the 3D non-separable potential of our optical tweezer trap, and find agreement (Fig. 2c) between the calculated and measured spin-exchange frequency $t_{\text{eff}}/(2\pi\hbar)$ (see Methods).

The measurements presented thus far have shown correlations in single-particle spin states. However, to ensure that future operations can retain and propagate quantum information, one must verify that the phase coherence within the entangled state is preserved upon separating the particles. The entanglement verification protocol for separated atoms is summarized in Fig. 3a. For explanatory purposes, we first focus on the case when the particles are separated after an exchange time of $t_{\text{ent}} = n\pi/2\hbar$, where $n$ is an odd integer. The entangled state after the second adiabatic passage is $|\psi_g\rangle = 1/\sqrt{2}(|\uparrow\rangle_R|\downarrow\rangle_L - i|\downarrow\rangle_R|\uparrow\rangle_L)/\sqrt{2}$, omitting from now on the ground-state ($g$) motional subscripts to simplify notation. The $|\psi_g\rangle$ states correspond to the grey and orange Bloch vectors, respectively, in Fig. 3b. We then apply a magnetic-field gradient that imposes a difference, $\delta B$, in the $|\uparrow\rangle \rightarrow |\downarrow\rangle$ single-atomic-transition energy between the left and right optical tweezers. By applying the gradient for a time $t_{g}$, a transformation $|\psi_{g}\rangle \rightarrow 1/\sqrt{2}(|\uparrow\rangle_L |\downarrow\rangle_R \pm i e^{i2\delta B t} |\downarrow\rangle_L |\uparrow\rangle_R)_{R}$ is achieved. As a function of $t_{g}$, the state rotates between the singlet (pink in
Therefore, by measuring the probability $\langle \sigma \rangle$ that a spin exhibits zero parity after application of a $\pi/2$ pulse, we observe singlet–triplet oscillations whose amplitude characterizes the two-particle coherence. We quantify this probability with the parity

$$\Pi(t_g) = \sum_j P_j (-1)^j,$$

where $P_j$ is the likelihood to measure $j$ atoms in the spin-down state $| \downarrow \rangle$. The parity is equivalently the projection of the Bloch vector in Fig. 3b onto the $x$ axis before the $\pi/2$ pulse, and hence the gradient is essential because, although entangled, the states $| \psi_{\uparrow \downarrow} \rangle$ (grey, orange) exhibit zero parity after application of a $\pi/2$ pulse.

**Figure 2** | Direct observation of spin-exchange dynamics between two atoms. (a) Preparation of motional-state configuration and detection in the double-well potential (see key for properties of the atoms). Initially, an atom is prepared in the ground state of each of two spatially separated optical tweezers (left panel). The atoms are combined in one of the optical tweezers via tunnelling (green arrows), allowed to interact for a controlled amount of time during which spin exchange occurs, and then separated into two optical tweezers for detection (right panel). Green and purple backgrounds show the colour coding for states shown in b, b. Using post-selection on our spin preparation, we plot the probability $\langle \sigma \rangle$ that a spin exhibits zero parity after application of a $\pi/2$ pulse, and hence the gradient is essential because, although entangled, the states $| \psi_{\downarrow \uparrow} \rangle$ (grey, orange) exhibit zero parity after application of a $\pi/2$ pulse.

**Figure 3** | Detection of non-local entanglement. (a) Procedure for creating and detecting entanglement. The spin-exchange procedure of Fig. 2a is applied to entangle the atoms (left panel), and then, upon separating them, a magnetic-field gradient and microwaves are introduced (middle panel). The underlying entanglement is revealed by the parity of the final spin configurations of the particles (right panel). See Methods for details. b. Four different Bloch vector orientations in the experiment. Right, the grey and orange orientations correspond to the outcome of the spin-exchange dynamics (after the second adiabatic passage), which yield rotations in the $y$–$z$ plane of the Bloch sphere (left). The pink and blue orientations correspond to the points of peak parity, and are accessed by applying a magnetic-field gradient that rotates the states about the $z$ axis. c. After creating $| \psi_{\downarrow \uparrow} \rangle$, we plot the measured parity $\Pi$ as a function of gradient time $t_g$. The grey bar is the bound on parity oscillation contrast delineating separable and entangled states, which accounts for imperfect spin preparation. The coloured dashed lines are at times when the associated states indicated in b are created. d. Measured parity as the exchange time is varied and correspondingly the atoms are entangled and unentangled. In the lower plot, we set $t_g$ such that it rotates $| \psi_{\downarrow \uparrow} \rangle$ to $| T \rangle$ and then measure the parity. The upper plot is the same experiment without the parity detection, that is, the protocol of Fig. 2. The dashed lines indicate times when the corresponding states indicated in b are produced. In c and d, the error bars in the data plots are the standard error, and the pink swaths show the 95% confidence bands.
We demonstrate the outcome of the verification protocol on the state $|\psi\rangle$ in Fig. 3c. We plot $\Pi(t_f)$ after the microwave $\pi/2$ pulse, and observe oscillations in the parity signal as the gradient time $t_f$ is scanned. The contrast of these oscillations is consistent with what is expected given the exchange oscillation contrast in Fig. 2, and non-vanishing parity oscillation would certify entanglement in the ideal case of perfect spin preparation. However, we have imperfect spin preparation, and the erroneous spin populations outside the $\{\uparrow\downarrow\} \rightarrow \{\downarrow\uparrow\}$ manifold could lead to parity oscillations even in the absence of entanglement. We have derived a condition on the parity oscillation contrast that is necessary and sufficient to certify entanglement, and is the simplest way to see there is entanglement in our system (for a full derivation of this condition and its relation to other entanglement metrics, see Methods). We relate the measured parity contrast, $C$, to the measured probabilities $(P^{\uparrow\uparrow}, P^{\downarrow\downarrow})$ that the spins are erroneously prepared in the same spin-state: if $C > C_{\text{had}} = 4(P^{\uparrow\uparrow}P^{\downarrow\downarrow})^{1/2}$, then the state is entangled. By directly measuring the spin populations (see Methods) and their associated uncertainty, we ascertain $C_{\text{had}} = 0.133(25)$ as indicated by the dashed lines in Fig. 3c. The observed parity oscillation contrast $C = 0.49(4)$ exceeds $C_{\text{had}}$ by more than 7σ, certifying the presence of entanglement in the final state of the separated spins. We verify entanglement without correcting the measured parity for experimental imperfections, such as single-atom loss due to background collisions.

Whereas in Fig. 3c we varied the parity detection parameters via $t_f$ in Fig. 3d we measure the dependence of the parity on the exchange time at fixed $t_f$, thereby observing oscillations as the exchange interactions periodically entangle and unentangle the two atoms. We fix $t_f$ in the parity detection such that the entangled state $|\psi\rangle$ (grey lines in Fig. 3b, d) is rotated to a peak in $\Pi$, corresponding to the creation of the triplet (blue lines in Fig. 3b, c). Because this $t_f$ amounts to a $\pi/2$ rotation about the $z$ axis of the Bloch sphere, it will also rotate $|\psi\rangle$ to the singlet, which corresponds to maximal negative parity. In the lower panel of Fig. 3d, we show how the parity measured under these conditions oscillates at the exchange frequency $t_{\text{exc}}/(2\hbar)$. For comparison, in the upper panel, we show the measured exchange oscillations (purple, green) without the parity detection. At the linear points of the exchange oscillations, one expects maximal entanglement corresponding to states $|\psi\rangle$ (grey) and $|\psi\rangle$ (orange) and thus the extremal values of the parity. At the minima and maxima of the exchange oscillations, the atoms are unentangled and the parity vanishes.

We have demonstrated the entanglement of remote qubits using spin-exchange interactions and a general protocol for detecting spin entanglement in a variety of systems. In a large register, tuning the entanglement phase could be achieved by a far-detuned focused probe that changes the local effective magnetic field experienced by a single qubit, and the qubits can be arranged to allow the passage of the optical tweezers without perturbing the qubits. Although in this work we focus on quantum information applications, the ability to control spin and motion of individual neutral atoms will allow the study of intriguing microscopic models in condensed-matter physics, such as the Kondo lattice model.

**Online Content** Methods, along with any additional Extended Data display items and Source Data, are available in the online version of the paper; references unique to these sections appear only in the online paper.

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To theoretically model the AP procedure, we consider a situation in which two particles with opposite spins are initially localized in the ground motional states of the left and right wells, denoted $L_g$ and $R_g$, respectively. During the AP, the bias between wells $\Delta$ is tuned so that $L_g$ is near resonance with an excited level of the right well, denoted $R_g$. In what follows, we will measure the bias $\Delta$ with respect to this resonance position, that is, the resonance occurs at $\Delta = 0$. Further, we consider that the bias range is such that tunnel couplings to all other motional states (for example, $L_g \rightarrow R_g$ tunnelling) are negligible, and so we can restrict ourselves to the set of motional states spanned by $L_g$ and $R_g$, and the single $L_g \rightarrow R_g$ tunnelling resonance at $\Delta = 0$. Because of the mixing of spin components due to the spin-exchange interaction, it is most convenient to use the singlet-triplet basis \( \{S_g, R_g, R_g, L_g\} \), where $S_g$ and $T$ denote singlet and triplet states and the last two labels are the motional states of the two particles. In this basis, the Hamiltonian is

\[
\hat{H} = \begin{pmatrix}
\Delta & -i \epsilon & 0 & 0 \\
-i \epsilon & 0 & 0 & 0 \\
0 & 0 & \Delta & -i \epsilon \\
0 & 0 & -i \epsilon & 2U_{\text{eg}}
\end{pmatrix}
\]

(2)

where $\epsilon$ is the tunnelling amplitude for the process $L_g \rightarrow R_g$ and the initial state at large negative $\Delta$, \( \{L_g, |LR\rangle\} \), is an equal weight superposition of \( \{S_g, L_g, R_g\} \) and \( \{T_g, L_g, R_g\} \). The AP process is described by separate tunnelling avoided crossings in the singlet and triplet channels, as shown in Extended Data Fig. 1. Provided that the ramping procedure is adiabatic with respect to these avoided crossings, it will transfer the initial state into an equal weight superposition of \( \{S_g, R_g, R_g\} \) and \( \{T_g, R_g, R_g\} \) at large positive $\Delta$. This pair of eigenstates, which correspond to the two particles occupying the same well, have an asymptotic energy splitting of $2U_{\text{eg}} \gg |\Delta| \gg \epsilon$. Note that the position of the bias resonance for the triplet channel is shifted by $2U_{\text{eg}}$ with respect to the resonance in the singlet channel. The energy offset does not affect the degree of adiabaticity of the ramping procedure and just gives rise to a phase shift between the singlet and triplet components.

**Entanglement verification based on parity oscillations.** Here we derive a criterion for verifying entanglement generated by spin-exchange interactions in a two-atom system. Our strategy will be to assume an unentangled (separable) density matrix, and from this assumption establish a constraint on experimentally measurable quantities: the parity oscillation contrast (Fig. 3c) and the populations of different spin states. Experimental violation of this constraint thus verifies entanglement.

In the experiment, spin exchange occurs between two atoms occupying a single optical tweezer, and then those atoms are separated into two tweezers. Because the APs in the experiment are imperfect, the atoms may sometimes end up in the same tweezer after the attempted separation. For clarity of presentation, we first consider the idealized case of perfect AP fidelity; hence $\rho$ in what follows describes states immediately after the second AP in which one atom occupies each tweezer.

At the end of this section we carefully consider the effects of AP failure, and show that they do not affect our claims of entanglement.

Because the measured spin-coherence time in the experiment is much less than the time between when we prepare the initial spin state and when we complete the APs, $\rho$ cannot have any coherences between states with different total spin $S^z = S^z_L + S^z_R$. Working in a basis that diagonalizes both $S^z_L$ and $S^z_R$,

\[ \{|L\rangle \equiv |L_L, L_R \rangle, |R\rangle \equiv |R_L, R_R \rangle, |L_L, R_R \rangle, |L_R, R_L \rangle \} \]

the most general density matrix satisfying this condition can be written

\[
\rho = \begin{pmatrix}
P(1|1, \rho) & 0 & 0 & 0 \\
0 & P(1|1, \rho) & 0 & 0 \\
0 & 0 & P(1|1, \rho) & 0 \\
0 & 0 & 0 & P(1|1, \rho)
\end{pmatrix}
\]

(3)

The populations $P(1|1, \rho)$ and $P(1|1, \rho)$ are, respectively, the total probabilities of having both atoms in the $|L\rangle$ state or both atoms in the $|R\rangle$ state, and are non-zero owing to imperfect initial spin preparation. Because these probabilities are conserved by the APs and the spin exchange, their measured values before the AP, referred to as $P(1|1)$ and $P(1|1)$ in the main text, can safely be used in equation (3):

\[
P(1|1, \rho) = P(1|1) \quad \text{and} \quad P(1|1, \rho) = P(1|1).
\]

The parity is measured after first applying a magnetic-field gradient for a variable time $t_g$ and then applying a $\pi/2$ microwave pulse, which transforms $\rho \rightarrow \widehat{P}(\eta) \rho$. After some algebra, the parity of $\rho(t_g)$ can be written $P(t_g) = 2\Re(e^{-i\theta_0 \epsilon})$, which oscillates as a function of $t_g$ with a contrast of $C = 4\epsilon$.
Our goal is to derive a constraint on $C$ in terms of the measured quantities $P^{1\downarrow}$, $P^{1\uparrow}$, under the assumption that $\rho$ is separable. If $\rho$ were a product state $\rho_L \otimes \rho_R$, where

$$
\rho_{L(R)} = \begin{pmatrix}
\rho_L^{\downarrow\downarrow} & \rho_L^{\uparrow\downarrow} \\
\rho_L^{\downarrow\uparrow} & \rho_L^{\uparrow\uparrow}
\end{pmatrix}
$$

then

$$
F = |p_L^{\uparrow\uparrow}| \leq \left| \rho_{L}^{\uparrow\uparrow} \right|^2 = \left| \rho_{L}^{\uparrow\uparrow} \right|^2 = (P^{1\downarrow} P^{1\downarrow})^{1/2} = (p_L^{\uparrow\uparrow})^{1/2}
$$

By definition, a separable state can be written as a classical mixture of product states, $\rho = \sum_j \lambda_j \rho_j^{\downarrow\downarrow} \otimes \rho_j^{\uparrow\uparrow}$, in which case

$$
F = \sum_j \lambda_j = \sum_j \left| \rho_j^{\uparrow\uparrow} \right|^2 \leq \sum_j \lambda_j \left| \rho_j^{\uparrow\uparrow} \right|^2 = \left( \sum_j \lambda_j \rho_j^{\uparrow\uparrow} \right)^{1/2} = (p_L^{\uparrow\uparrow})^{1/2}
$$

Here, the second inequality is the constraint on $|\epsilon|$ derived for a product state in equation (5) applied to each state in the classical mixture, and the first and third are triangle inequalities. Using $C = 4|\epsilon|$, we therefore have guaranteed entanglement whenever

$$
C > 4(P^{1\downarrow} P^{1\uparrow})^{1/2}
$$

We can also verify entanglement by using an entanglement witness associated with the fidelity of $\rho$ in the maximally entangled state $|\Psi_+\rangle = (|1\downarrow\downarrow\rangle + i |1\downarrow\uparrow\rangle)/\sqrt{2}$, $F = \langle \Psi_+ | \rho | \Psi_+ \rangle$. As shown in ref. 1, $\rho$ is entangled if $F > 1/2$. In terms of equation (3), the fidelity can be written

$$
F = \frac{1}{2} \left[ P^{\downarrow\uparrow} + P^{\downarrow\downarrow} \right] + \frac{1}{4} = \frac{1}{2} + \frac{1}{4} \left( p_L^{\uparrow\uparrow} + p_L^{\down\downarrow} \right)
$$

Therefore $F > 1/2$ is equivalent to $C > 2(p_L^{\uparrow\uparrow} + p_L^{\down\downarrow})$, which agrees with equation (7) when $p_L^{\uparrow\uparrow} = p_L^{\down\downarrow}$. Equation (7) can also be derived by applying the Peres–Horodecki criterion to $\rho$, and thus it is actually both sufficient and necessary for entanglement33,34. Thus, in contrast with the fidelity-based entanglement witness $C > 2(p_L^{\uparrow\uparrow} + p_L^{\down\downarrow})$, the right-hand-side of equation (7) is as small as possible for any $p_L^{\uparrow\uparrow}$ and $p_L^{\down\downarrow}$, resulting in the greatest possible confidence in entanglement for a particular measured contrast. We also note that the extent to which equation (7) is satisfied, $\frac{1}{4}(C - 4(p_L^{\uparrow\uparrow} p_L^{\down\downarrow})^{1/2}) > 0$, is a direct measurement of the concurrence in the density matrix $\rho$ (ref. 16).

**Imperfect adiabatic passages.** As mentioned above, the APs in the experiment are not perfect. For our purposes, we define success as any situation in which the atoms end up in different tweezers, which occurs either if both of the individual APs succeed or if they both fail. Conversely, we define failure as any situation in which both atoms end up in the same tweezer, which happens if one of the APs is successful while the other is not. Although the states resulting from failure have not been considered in deriving equation (7), they do not contribute to the parity oscillation contrast, since two atoms in the same tweezer are not sensitive to a magnetic-field gradient. Therefore, we intuitively expect that imperfect AP can only lower the measured contrast, such that equation (7) still implies entanglement.

This intuition can be formalized by introducing a projector, $\hat{K}$, onto the states with one atom in each tweezer. Defining $f$ as the success probability, we can then form projections of the true experimental density matrix, denoted by $\rho_{\exp}$, into the subspaces defined by success or failure

$$
\hat{K} \rho_{\exp} (1 - \hat{K}) \rho_{\exp} (1 - \hat{K}) = (1 - f) \rho_{\exp}
$$

Note that, by the choice of pre-factors, both $\rho_{\exp}$ and $\rho_{\exp}$ are properly normalized density matrices. Importantly, $\rho_{\exp}$ has precisely the form given in equation (3); we can therefore directly apply the arguments above, deducing that $\rho_{\exp}$ is entangled whenever $|\epsilon| > \sqrt{p_L^{\uparrow\uparrow} p_L^{\down\downarrow}}$. The only difference from before is in the relationship between $|\epsilon|$ and the measured parity oscillation contrast. The application of a magnetic-field gradient and the subsequent π/2 pulse does not mix the two subspaces partitioned by $\hat{K}$, and therefore the measured parity can be written $P(t_f) = \beta P_{\exp}(t_f) + (1 - \beta) P_{\exp}$. Importantly, $P_{\exp}$ is independent of $t_f$ because both atoms in the same tweezer are not sensitive to a magnetic-field gradient, and therefore the measured parity oscillation contrast is given simply by $C = f|\epsilon|$. Thus the effect of imperfect APs is that, in terms of the measured contrast, the condition $|\epsilon| > \sqrt{p_L^{\uparrow\uparrow} p_L^{\down\downarrow}}$ now reads $C/f > 4\sqrt{p_L^{\uparrow\uparrow} p_L^{\down\downarrow}}$ (compare with equation (7)).

The criterion used in the main text, $C > 4\sqrt{p_L^{\uparrow\uparrow} p_L^{\down\downarrow}}$, is therefore a conservative way to verify entanglement in $\rho_{\exp}$, since $C/f$ is strictly larger than $C$.

The fidelity can now be written $F = \langle \Psi_+ | \rho_{\exp} | \Psi_+ \rangle = \langle \Psi_+ | f \rho_{\exp} | \Psi_+ \rangle$, where $F_{\exp}$ is the fidelity of the (projected and normalized) density matrix $\rho_{\exp}$ and can be equivalently written as $F_{\exp} = \frac{1}{2} + |\epsilon| - \frac{1}{2} (p_L^{\uparrow\uparrow} + p_L^{\down\downarrow})$. Similar to before, entanglement of $\rho_{\exp}$ is now guaranteed by $F_{\exp} > 1/2$, or equivalently $|\epsilon| > 2(p_L^{\uparrow\uparrow} + p_L^{\down\downarrow})$. Using $4|\epsilon| = C/f$, we can extract $F_{\exp}$ from the measured contrast ($C = 0.49(4)$) and measured success probability ($f = 0.69(2)$), obtaining $F_{\exp} = 0.634(17)$. Note that the actual fidelity $F$ is reduced from this value by the success probability $f$, and in fact below 1/2. However, this is not inconsistent with entanglement in $\rho_{\exp}$ as the fidelity-based witness for $\rho_{\exp}$, written in terms of the actual fidelity, is $F > 1/2$.

33. Peres, A. Separability criterion for density matrices. Phys. Rev. Lett. 77, 1413–1415 (1996).
34. Horodecki, M., Horodecki, P. & Horodecki, R. Separability of mixed states: necessary and sufficient conditions. Preprint at http://arXiv.org/abs/ quant-ph/9605038 (1996).
Extended Data Figure 1 | Adiabatic energy eigenstates $E$ as a function of the double-well bias $\Delta$ in units of the ground-excited tunnelling $J_{eg}$. At large positive bias, the triplet and singlet eigenstates corresponding to two particles in the same well are split by $J_{eg}$. The dashed and solid lines denote the energies of the states that asymptotically connect to the states labelled in the figure through the AP process. See Methods for details.