Observation of $\mathcal{P}\mathcal{T}$-symmetry and exceptional point in ion trap

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Abstract

The theory of parity time ($\mathcal{P}\mathcal{T}$) symmetry has been quite successful in classical system. And classical $\mathcal{P}\mathcal{T}$-symmetric systems exhibited many interesting features. However, $\mathcal{P}\mathcal{T}$-symmetric experiments of quantum systems are progressing more slowly than $\mathcal{P}\mathcal{T}$-symmetric experiments of classical systems. Hence, we constructed a passive quantum $\mathcal{P}\mathcal{T}$-symmetric system by using the internal states of $^{40}\text{Ca}^+$ ion. Comparing with other systems, the experimental setup of single ion trap is easy to implement, meanwhile our single ion trap is a real quantum system and has a long coherence time. In this paper, we show the phenomenon of $\mathcal{P}\mathcal{T}$-symmetry-breaking transitions by controlling the dissipation factor of states, which is achieved through adjusting the power of laser beam. Moreover, we also investigated the quantum phase transition near exceptional point (EP) of $\mathcal{P}\mathcal{T}$-symmetric system. The experimental results enable ion trap to be a successful device for investigating quantum $\mathcal{P}\mathcal{T}$-symmetry.

1. Introduction

In conventional quantum mechanics, hermitian is sufficient condition to ensure that physical observables have real eigenvalues [1]. However, a class of non-hermitian Hamiltonians $H_{\mathcal{P}\mathcal{T}}$, which are invariant under the parity and time-reversal, satisfying $\mathcal{P}\mathcal{T}$-symmetry can still have real eigenvalues. Meanwhile, $\mathcal{P}\mathcal{T}$-symmetric non-hermitian Hamiltonians satisfies commutation relation $[H_{\mathcal{P}\mathcal{T}}, \mathcal{P}\mathcal{T}] = 0$, where $\mathcal{P}$ is parity-reversal operator and $\mathcal{T}$ is time-reversal operator. Until now, the conception of $\mathcal{P}\mathcal{T}$ symmetry has been considered as a complex extension of conventional quantum mechanics [2, 3]. Note that, any eigenfunction of $H_{\mathcal{P}\mathcal{T}}$ is eigenfunction of the $\mathcal{P}\mathcal{T}$ operator, when $H_{\mathcal{P}\mathcal{T}}$ possesses entire real eigenvalues. On the contrary, if $H_{\mathcal{P}\mathcal{T}}$ does not exhibit entire real eigenvalues, the system is in the $\mathcal{P}\mathcal{T}$-symmetry breaking region. Thus the system can exhibit many interesting phenomena [4], especially when the eigenvalues of $H_{\mathcal{P}\mathcal{T}}$ are equal to zero, namely, the $\mathcal{P}\mathcal{T}$-symmetric system is at exceptional point (EP). Such as Local $\mathcal{P}\mathcal{T}$-symmetry violates the no-signaling principle [5], phonon laser [6], quantum brachistochrone problem [7, 8], and so on.

On the one hand, in order to verify the $\mathcal{P}\mathcal{T}$-symmetric theory, researchers found that paraxial wave equation of classical optics and Schrödinger’s equation...
of quantum mechanics have a similar mathematical structure [9], thus one can simulate \( \mathcal{P}\mathcal{T} \)-symmetric theory with classical optic [10]. Naturally, in classical optic filed, the theory of \( \mathcal{P}\mathcal{T} \)-symmetry has been fully explored, and many new optical devices, which are designed by using \( \mathcal{P}\mathcal{T} \)-symmetric theory, have excellent or interesting optical characteristics [11,12,13,14,15,16,17,18]. \( \mathcal{P}\mathcal{T} \)-symmetric optical devices have achieved great success in classical optic filed, which also inspired the investigation of \( \mathcal{P}\mathcal{T} \)-symmetric theory in other fields, such as acoustics [19], classical mechanics [20], atomic system [21,22,23], and so on.

On the other hand, the conception of \( \mathcal{P}\mathcal{T} \)-symmetry was first proposed in quantum physics, but the progresses of experiments of quantum \( \mathcal{P}\mathcal{T} \)-symmetry significantly slower than counterparts in classical physics. Generally, the quantum \( \mathcal{P}\mathcal{T} \)-symmetric Hamiltonian can be simulated by balancing gain and dissipation in open quantum system, but this experiment is more difficult than its classical counterpart. Because it not only balances the gain and dissipation of energy, but also balances the gain and dissipation of quantum properties, for example, the quantum coherence [24]. Therefore, it is worth investigating and discussing which quantum systems and experimental schemes can be used to realize \( \mathcal{P}\mathcal{T} \)-symmetry. At present, there are two major ways to achieve \( \mathcal{P}\mathcal{T} \)-symmetric Hamiltonian in quantum system: a) The whole system is constructed by the auxiliary system and the target system, and the Hamiltonian \( H \) of the whole system is hermitian. Hence, one can design parameters of the whole system subtly, and make the effective Hamiltonian of the target system satisfying \( \mathcal{P}\mathcal{T} \)-symmetry [25]. But the current experimental scheme is too complicated to be extended. b) Analogizing the passive \( \mathcal{P}\mathcal{T} \)-symmetry in optics [26], quantum systems can also realize the passive \( \mathcal{P}\mathcal{T} \)-symmetry. However, There is a lack of demonstration of the quantum properties of \( \mathcal{P}\mathcal{T} \)-symmetric system in the current experiments [27].

In this paper, we successfully simulated the \( \mathcal{P}\mathcal{T} \)-symmetric system by using the internal state of single ion, which is a natural quantum system. Hence, comparing with the classical experimental schemes of \( \mathcal{P}\mathcal{T} \)-symmetry [26], our experimental results show not only the phenomena observed in the classical optical schemes, but also some unique properties of quantum systems. Comparing with the quantum experimental scheme of \( \mathcal{P}\mathcal{T} \)-symmetry [25], our experimental scheme is easier to implement and more extensible. For example, in order to realize a controllable stationary environmental dissipation \( \gamma \) in a two-level system, four time-dependent experimental parameters must be controlled simultaneously in experimental scheme of article [25], which makes the accuracy of the experiment very high, and greatly increases the difficulty of the experiment. Our experimental scheme only needs a time-independent experimental parameter to realize a controllable environmental dissipation \( \gamma \). Moreover, comparing with other quantum systems, ion trap system has a longer coherence time \( T_2 \), which is convenient for future quantum \( \mathcal{P}\mathcal{T} \)-symmetry experiments.

In Sec.II, we introduce fundamental theory of the passive \( \mathcal{P}\mathcal{T} \)-symmetry, and the setup of the passive \( \mathcal{T}\mathcal{P} \)-symmetric experiment in ion trap. In Sec.III, we show the results of experiment.
2. The passive $\mathcal{PT}$-symmetry

2.1. Fundamental theory of the passive $\mathcal{PT}$-symmetry

In general, the passive $\mathcal{PT}$-symmetric system, in which there is only dissipation, are experimentally easy to achieve. Hence, one needs some mathematical techniques to obtain effective $\mathcal{PT}$-symmetric Hamiltonian. For example, there is a non-hermitian Hamiltonian of two level system in open quantum system,

$$H_{\text{eff}} = -i\gamma|1\rangle\langle 1| + \frac{\Omega}{2}\sigma_x,$$

where $\sigma_i$ ($i = x, y, z$) is Pauli operator, $\Omega$ is coupling strength, and $\gamma$ is environmental dissipation. Then one can factor out a global dissipation $-i\frac{\gamma}{2}I$. And the eq.(1) becomes the following,

$$H_{\text{eff}} = -i\frac{\gamma}{2}I - i\frac{\gamma}{2}\sigma_z + \Omega\sigma_x = -i\frac{\gamma}{2}I + H_{\mathcal{PT}},$$

where $H_{\mathcal{PT}} = -i\frac{\gamma}{2}\sigma_z + \Omega\sigma_x$ is $\mathcal{PT}$-symmetric Hamiltonian. Note that, eigenvalues of Hamiltonian $H_{\mathcal{PT}}$ is $\lambda_{\pm} = \sqrt{(\frac{\Omega}{2})^2 - (\frac{\gamma}{2})^2}$, which indicates EP of the system. When $\Omega > \gamma$, the symmetry of $\mathcal{PT}$-symmetric system is unbroken, and the dynamical evolution of this system is unitary. On the contrary, when $\Omega < \gamma$, the symmetry is broken. The dynamic equation of this effective non-hermitian Hamiltonian $H_{\text{eff}}$ can be described as,

$$\dot{\rho}_{\text{eff}}(t) = -i[\Omega\sigma_x, \rho_{\text{eff}}(t)] - \{\gamma|1\rangle\langle 1|, \rho_{\text{eff}}(t)\},$$

where $[,]$ and $\{,\}$ denote the commutator and anticommutator, respectively. In this paper, the eq.(3) will be simulated by using the internal states of $^{40}\text{Ca}^+$. 

2.2. Experimental setup of the passive $\mathcal{PT}$-symmetry

First, in quantum region, we use $^{40}\text{Ca}^+$ ion trap system to realize the passive $\mathcal{PT}$-symmetry. The energy level diagram of $^{40}\text{Ca}^+$ quantum system is described in FIG.1. Because of short lifetime of $P_{+\frac{3}{2}}$ and according to selection rules, the population of $D_{+\frac{3}{2}}$ can be pumped into $S_{+\frac{1}{2}}$ by 854nm laser. Therefore, the whole system can be considered as three-level system. The state $|0\rangle$, $|1\rangle$ and $|2\rangle$ correspond to energy level $S_{-\frac{1}{2}}$, $D_{+\frac{3}{2}}$ and $S_{+\frac{1}{2}}$ of $^{40}\text{Ca}^+$ ion respectively in our experimental setup. Obviously, We can easily find the master equation for a three-level system. If only the dynamical evolution of the subsystem consisting of $|0\rangle$ and $|1\rangle$ is considered, the following equation can be obtained,

$$\dot{\rho}(t) = -i\frac{\Omega}{2}(|0\rangle\langle 1| + |1\rangle\langle 0|, \rho(t)) - \gamma_{12}|1\rangle\langle 1|\rho(t) - \gamma_{12}\rho(t)|1\rangle\langle 1|,$$

where $\gamma_{12}$ is the intensity of spontaneous radiation from $|1\rangle$ to $|2\rangle$, and $|2\rangle$ is considered as environment. It’s not hard to see that eq.(4) and eq.(3) are the same. Thus the simulation of $\mathcal{PT}$-symmetric system can be realized by using $^{40}\text{Ca}^+$ ion trap.
Figure 1: (Color online) Four energy levels of $^{40}\text{Ca}^+$ ion are used in our experiment. Because of short lifetime of $P_{3/2}$, the whole system can be considered as three-level system, where the state $|0\rangle$, $|1\rangle$ and $|2\rangle$ correspond to energy level $S_{-1/2}$, $D_{1/2}$ and $S_{1/2}$ respectively.
3. Experimental results

3.1. \( \mathcal{PT} \)-symmetry-breaking transitions

According to experimental setup and fundamental theory, experimental results require post-processing of data, and the method is as follows,

\[
\rho(t) = U(t)\rho(0)U^\dagger(t) = \exp(-\gamma t)\rho_{\mathcal{PT}}(t)\\
\rho_{\mathcal{PT}}(t) = \exp(\gamma t)\rho(t),
\]

where \( U(t) = \exp(-iH_{eff}t) \). So dynamic characteristics of \( \mathcal{PT} \)-symmetric system can be described by \( \rho_{\mathcal{PT}} \). The dynamical evolution of \( \rho(t) \) can be obtained by solving the master equation of the two-level system,

\[
\rho_{00}(t) = e^{-\gamma t}((e^{-\frac{it\Omega_{\mathcal{PT}}}{2}} - e^{-i\frac{t\Omega_{\mathcal{PT}}}{2}})\gamma + (e^{-\frac{it\Omega_{\mathcal{PT}}}{2}} + e^{i\frac{t\Omega_{\mathcal{PT}}}{2}})\Omega_{\mathcal{PT}})^2,\\
\langle \sigma_y(t) \rangle = e^{-\gamma t}((e^{-\frac{it\Omega_{\mathcal{PT}}}{2}} - e^{i\frac{t\Omega_{\mathcal{PT}}}{2}})\Omega_{\mathcal{PT}}(e^{-\frac{it\Omega_{\mathcal{PT}}}{2}} + e^{i\frac{t\Omega_{\mathcal{PT}}}{2}}) + \gamma(e^{-\frac{it\Omega_{\mathcal{PT}}}{2}} - e^{i\frac{t\Omega_{\mathcal{PT}}}{2}}))^2,\\
Tr(\rho(t)) = e^{-\gamma t}((e^{t\Omega_{\mathcal{PT}}} + e^{-t\Omega_{\mathcal{PT}}}) + \gamma\Omega_{\mathcal{PT}}(e^{t\Omega_{\mathcal{PT}}} - e^{-t\Omega_{\mathcal{PT}}}) - 2\Omega^2),\\
\Omega_{\mathcal{PT}} = \sqrt{\gamma^2 - \Omega^2},
\]

where \( \rho_{00} \) is population of \( |0\rangle \), \( \langle \sigma_y \rangle \) is expectation value of \( \sigma_y \) and \( Tr(\rho) \) is the total number of population.

Firstly, in our experiment, the initial state is \( |0\rangle \), and the coupling strength \( \Omega \) is fixed, which is about \( 32 \times 2\pi(\text{kHz}) \). Then, we need to make sure that dissipation \( \gamma \) is less than \( \Omega \), if we want to obtain the dynamic characteristics in \( \mathcal{PT} \)-symmetry region. The dynamic characteristics of state \( |0\rangle \) (FIG.2(a), (d)), the total number of population (FIG.2(b), (e)) and dynamical evolution of \( \langle \sigma_y \rangle \) (FIG.2(c), (f)) under the certain dissipation \( \gamma \), which is about \( 2\pi(\text{kHz}) \), are presented respectively in FIG.2. Note that the dynamic characteristics of \( \mathcal{PT} \)-symmetric system is described in FIG.2(d), (e) and (f). As expected by the theory, dynamical evolution of \( \mathcal{PT} \)-symmetric system reveals oscillation behavior.

Secondly, in order to observe the phenomenon of \( \mathcal{PT} \)-symmetry breaking, we should make dissipation \( \gamma \) exceed coupling strength \( \Omega \). In this situation, eigenvalues of \( H_{\mathcal{PT}} \) are no longer real numbers, and dynamical evolution of \( \mathcal{PT} \)-symmetric system will show exponential increase. FIG.3 shows that the dynamic characteristics of \( \mathcal{PT} \)-symmetric system are consistent with the \( \mathcal{PT} \)-symmetric theory. In FIG.3, we also show the dynamical evolution of state \( |0\rangle \) (FIG.3(a),(d)), the total number of population (FIG.3(b),(e)) and dynamical evolution of \( \langle \sigma_y \rangle \) (FIG.3(c),(f)) respectively.

Our experimental results also show that, with the change of dissipation \( \gamma \), the evolution of \( |0\rangle \) shows different changing trends in the symmetry and symmetry breaking region. In the \( \mathcal{PT} \)-symmetry region, the amplitude of population of \( |0\rangle \) decreases faster with the increase of \( \gamma \) (FIG.4(a)). However, in the \( \mathcal{PT} \)-symmetry breaking region, the attenuation of state \( |0\rangle \) is suppressed with the
Figure 2: (Color online) Dynamical evolution of system in the initial state $|0\rangle$, when $\gamma < \Omega$. Coupling strength $\Omega$ is about $32 \times 2\pi (kHz)$, and dissipation $\gamma$ is about $2\pi (kHz)$. The black dots represent experimental datas, and the orange line represents theoretical fit. (a), (b) and (c) show original datas from experiment. (d), (e) and (f) are the dynamical evolution of $PT$-symmetric system. (a) and (d) show dynamical evolution of state $|0\rangle$. (b) and (e) show dynamical evolution of the total number of population. (c) and (f) show dynamical evolution of $\langle \sigma_y \rangle$. The error bars are the standard deviation of the measurements.
Figure 3: (Color online) Dynamical evolution of system in the initial state $|0\rangle$, when $\gamma > \Omega$. Coupling strength $\Omega$ is about $32 \times 2\pi$ (kHz), and dissipation $\gamma$ is about $47 \times 2\pi$ (kHz). The black dots represent experimental data, and the orange line represents theoretical fit. (a), (b) and (c) show original data from experiment. (d), (e) and (f) are the dynamical evolution of $PT$-symmetric system. (a), (d) show dynamical evolution of state $|0\rangle$. (b) and (e) show dynamical evolution of the total number of population. (c) and (f) show dynamical evolution of $\langle \sigma_y \rangle$. The error bars are the standard deviation of the measurements.
Figure 4: (Color online) Dynamical evolution of experimental system in the initial state $|0\rangle$. Coupling strength $\Omega$ is about $32 \times 2\pi (\text{kHz})$. (a) The amplitude of population of $|0\rangle$ decreases faster with the increase of dissipation. (b) With the increase of dissipation, the attenuation of state $|0\rangle$ is suppressed, when the $\mathcal{PT}$-symmetry has broken. (c) The population of state $|0\rangle$ changes with dissipation at a fixed time $t = \frac{2\pi}{\Omega}$. The error bars are the standard deviation of the measurements.
increase of $\gamma$ (FIG.4(b)). FIG.(4(c)) shows that the population of $|0\rangle$ varies with dissipation at a fixed time $t = \frac{2\pi}{\Omega}$. This experimental phenomenon is similar to the classic experiment [26]. Note that, the lowest point in FIG.4(c) is not the point with zero eigenvalue of $\mathcal{P}\mathcal{T}$-symmetric system, that is, not EP. The experimental results in Reference [26] also show that there is a certain difference between the dissipation corresponding to the minimum energy value and EP. However, Reference [26] attributed the difference to the accuracy of the experiment. The explanation given in this paper is that the actual oscillation period of the $\mathcal{P}\mathcal{T}$-symmetric system in the symmetric region is $t = \frac{2\pi}{|\Omega_{\mathcal{PT}}|}$, and the oscillation period will change with the change of dissipation $\gamma$. However, when processing experimental data, we selected the population number at a fixed time $t = \frac{2\pi}{\Omega}$. As a result, the dissipation corresponding to the lowest point in Fig.4(c) is slightly different from EP. On the whole, there are reasons to believe that the $\mathcal{P}\mathcal{T}$-symmetry change of the system has obvious influence on the evolution of experimental system.

For reasons outlined above, we obtain phenomenon of $\mathcal{P}\mathcal{T}$-symmetry-breaking transitions by controlling dissipation $\gamma$, which is achieved through adjusting the power of laser beam.

3.2. Exceptional point of $\mathcal{P}\mathcal{T}$-symmetric system

In general, $\mathcal{P}\mathcal{T}$-symmetry-breaking transition is a kind of quantum phase transition. And the phase transition point of the $\mathcal{P}\mathcal{T}$-symmetric system is EP. Hence, In order to describe the quantum phase transition process well, the order parameters of the system need to be defined as follows [28],

$$\Sigma_Z = \int_0^\infty \frac{\langle \sigma_{PT}^z(t) \rangle}{Tr<\rho_{PT}(t)>} dt,$$

$$\Sigma_Y = \int_0^\infty \left| \frac{\langle \sigma_{PT}^y(t) \rangle}{Tr<\rho_{PT}(t)>} \right| dt.$$

The order parameters $\Sigma_Z$ reveals the energy of the $\mathcal{P}\mathcal{T}$-symmetric system along with the change of $\gamma$, and the order parameters $\Sigma_Y$ reveals the quatum coherence [24] of the $\mathcal{P}\mathcal{T}$-symmetric system along with the change of $\gamma$. It can be seen from FIG.5 that the variation trend of order parameters in different phases is obviously different, and the experiment is in good agreement with the theory. FIG.5(a) shows the variation of $\Sigma_Z$ with $\gamma$. When the symmetry of the system is broken, the population of the system will be bound to state $|0\rangle$ and unable to transition to state $|1\rangle$ with the increase of $\gamma$. Since it is easy to measure the system energy in classical experiments, the experimental results of only $\Sigma_Z$ cannot reflect the advantages of ion trap quantum system. Therefore, compared with previous quantum experiments which are used for simulating $\mathcal{P}\mathcal{T}$-symmetric system [25, 27], we further investigated the quantum coherence of $\mathcal{P}\mathcal{T}$-symmetric systems (FIG.5(b)). It is not difficult to find from FIG.5(a) that the quantum coherence of $\mathcal{P}\mathcal{T}$-symmetric system near EP reaches the maximum value, and EP can significantly enhance the quantum coherence of the system. For reasons outlined above, our experimental results of $\Sigma_Z$ and $\Sigma_Y$ show that the quantum
\( \mathcal{PT} \)-symmetric system can be successfully simulated by \(^{40}\text{Ca}^+ \) ion trap system.

![Graph](image)

Figure 5: (Color online) The initial state is state \( |0\rangle \), and the order parameters \( \Sigma_Z \) and \( \Sigma_Y \) change with \( \gamma \). The error bars are the standard deviation of the measurements.

4. Summary

In summary, we successfully simulated the quantum \( \mathcal{PT} \)-symmetric hamiltonian by using the ion trap system, and showed the phenomenon of \( \mathcal{PT} \)-symmetry-breaking transions by controlling the environmental dissipation \( \gamma \). We also investigated the effect of EP of the \( \mathcal{PT} \)-symmetric system. We found that the population of the system can be quickly emptied near EP, and will be bound to state \( |0\rangle \), when \( \gamma >> \omega \). Therefore, qualitatively, the experimental results of \( \mathcal{PT} \)-symmetry may help us to find the rapid cooling mechanism in ion trap systems. We also found that in the vicinity of EP, the quantum coherence of \( \mathcal{PT} \)-symmetric system can be enhanced significantly. Finally, the experimental results enable ion trap to be a successful device for investigating quantum \( \mathcal{PT} \)-symmetry. And the long coherence time of ion trap system will help further research on quantum \( \mathcal{PT} \)-symmetry in the future.
5. Supplement

when $\gamma \to \Omega$, limit of $\rho_{00}$ and $\frac{\partial \rho_{00}}{\partial t}$

\begin{align}
\rho_{00}(\gamma \to \Omega) &= \frac{1}{4}e^{-\Omega}(t\Omega + 2)^2 \\
\frac{\partial \rho_{00}}{\partial \gamma}(\gamma \to \Omega) &= \frac{1}{24}t^3\Omega^2e^{-\Omega}(t\Omega + 2)
\end{align}

(11)

(12)

when $\gamma < \Omega$ and $T \to \infty$, $M_z = \frac{1}{T} \int_0^T \langle \sigma_z(t) \rangle dt$

\begin{align}
M_z &= -\csc(\alpha) \log(-\cos(T\Omega \cos(\alpha)) + \cos(2\alpha - T\Omega \cos(\alpha)) + 2) \\
&\approx 0,
\end{align}

(13)

when $\gamma < \Omega$ and $T \to \infty$, $M_y = \frac{1}{T} \int_0^T \langle \sigma_y(t) \rangle dt$

\begin{align}
M_y &= \frac{\csc \left( \frac{\alpha}{2} \right) \sec \left( \frac{\alpha}{2} \right) \sec(\alpha) \left( -\alpha + T\Omega \cos(\alpha) - 2\cos(\alpha) \tan^{-1} \left( \sin \left( \frac{1}{2} T\Omega \cos(\alpha) \right) \sec \left( \frac{1}{2} T\Omega \cos(\alpha) \right) \right) \right)}{2T\Omega} \\
&\approx \frac{\csc \left( \frac{\alpha}{2} \right) \sec \left( \frac{\alpha}{2} \right) \left( t\Omega \right)}{2T\Omega} - \frac{2\tan^{-1} \left( \sin \left( \frac{1}{2} T\Omega \cos(\alpha) \right) \sec \left( \frac{1}{2} T\Omega \cos(\alpha) \right) \right)}{t\Omega \sin(\alpha)} \\
&= \frac{1}{\sin(\alpha)} - \frac{\cos(\alpha)}{\sin(\alpha)} \\
&= \frac{\Omega}{\gamma} - \frac{\sqrt{\Omega^2 - \gamma^2}}{\gamma},
\end{align}

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Author Contributions
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Additional Information