Multi-dimensional instability of dust-ion-acoustic solitary structure with opposite polarity ions and non-thermal electrons

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Abstract An attempt has been made to study the multi-dimensional instability of dust-ion-acoustic (DIA) solitary waves (SWs) in magnetized multi-ion plasmas containing opposite polarity ions, opposite polarity dusts and non-thermal electrons. First of all, we have derived Zakharov-Kuznetsov (ZK) equation to study the DIA SWs in this case using reductive perturbation method as well as its solution. Small-\( k \) perturbation technique was employed to find out the instability criterion and growth rate of such a wave which can give a guideline in understanding the space and laboratory plasmas, situated in the D-region of the Earth’s ionosphere, mesosphere, and solar photosphere, as well as the microelectronics plasma processing reactors.

Keywords Non-thermal electrons · Multi-ions · Plasma instability · Perturbation

Introduction

Very recently many authors have studied the nonlinear propagation of ion-acoustic (IA) solitary waves (SWs) [1–4] dust-ion-acoustic (DIA) SWs [5–9], dust-acoustic (DA) SWs [10–14] in a multi-ion plasma, as negative ions can be found in space [15, 16] and laboratory [17–19] as well as positive ions. In multi-ion plasmas, there exist fast and slow mode. In fast mode, positive and negative ions oscillate out of phase (180°) where as positive and negative ions oscillate in phase. In the case of low temperature plasmas, both in space (the Earth’s ionosphere, planetary rings, interstellar clouds, planetary atmospheres, interstellar media, protostellar disks, interstellar and circumstellar clouds, asteroid zones, cometary tails and nebula) and laboratory (plasma processing and plasma crystal) some micron or sub-micron sized dust particles might be present. Due to the size effect on secondary emission [20], the dust particles can have the opposite polarity. Larger particles are found to be negatively charged and smaller ones are positively charged [20]. These opposite polarity ions and opposite polarity dusts [10–14] can significantly modify the basic properties of linear and nonlinear DIA SWs and DA SWs. The anomalous dissipation in complex plasmas, which originates from the dust charging process, makes possible the existence of a new kind of shock wave [21–25]. When the dissipation is weak or absent at the characteristic dynamical time scales of the system, the balance between nonlinearity and dissipation is weak at the characteristic dynamical time scales of the system, the balance between nonlinearity and dissipation effects can result in the formation of a symmetrical SWs.

The non-thermal particle distributions turn out to be a characteristic feature in space plasmas, as in auroral zone [26]. The observation of non-thermal electrons made by the Swedish Viking satellite [27] and Freja satellite [28] have shown electrostatic solitary structures in the magnetosphere with density depression. This non-thermal electron populations might be distributed isotropically in velocities or possess a net streaming motion. Cairns et al. [29] considered the influence of non-thermal electrons on the existence conditions of ion-acoustic (IA) solitary structures. The presence of non-thermal electrons allows for the existence of both positive and negative density...
perturbations. The non-thermal electron distribution of Cairns et al. [29] is a more general class of the electron distribution including a population of fast or energetic electrons. The non-thermal electron \( n_e \) can be written as

\[
n_e = \left[ 1 - \frac{4x}{1 + 3x} \psi(1 - \psi) \right] e^\psi,
\]

where \( x \) is a parameter determining the fast particles present in this plasma model and \( \psi \) is the wave potential.

Sayed et al. [9] have studied dust-ion-acoustic (DIA) solitary structures in unmagnetized plasmas containing positive and negative ions with positive and negative dust and Maxwellian distributed electrons. This mode is only valid if a complete depletion of the background electrons and ions is possible, and both positive and negative dust fluids are cold. However, in real dusty plasma, the effect of finite ion temperature cannot be neglected and the electron behavior can be strongly modified by non-linear potential of the localized DIA structures by generating a population of fast energetic electrons [30]. Gill et al. [1] have studied double layered IA SWs containing positive and negative ions of two different temperatures and non-thermal electrons by deriving K-dV and mK-dV equations. Mannan et al. [3] have studied nonplanar (cylindrical and spherical) Gardner solitons and double layers (DLs) in an unmagnetized plasma composed of positive and negative ions, and non-thermal electrons. Mamun and Shukla [4] have studied the effects of the non-thermal distribution of electrons as well as the polarity of the net dust-charge number density on nonplanar DIA SWs. It is found from the different observations that most of the plasmas in reality are magnetized, and it can change its characteristics according to the wave direction.

Haider [5] studied the DIA SWs by deriving K-dV equation, associated with a plasma system containing opposite polarity ions, opposite polarity dusts and non-thermal electrons where restoring forces are provided by the plasma thermal pressure of electrons and the inertia is due to the ion mass. But the instability criterion and growth rate is steel unknown. To do this, Zakharov-Kuznetsov (ZK) equation in Sect. 3. The solitary-wave solution of the ZK equation is obtained in Sect. 4. Instability criterion with growth rate is in Sect. 5 and a brief discussion is finally given in Sect. 6.

### Basic equations

The propagation of DIA SWs have been studied, in the present work, in a collisionless magnetized dusty plasma consist of

(i) opposite polarity (negatively and positively charged) mobile ions,

(ii) opposite polarity stationary dust, and

(iii) non-thermal electrons.

It is also considered that there is an external static magnetic field \( \mathbf{B}_0 \) acting along the \( z \)-direction (\( \mathbf{B}_0 = k \mathbf{B}_0 \)), where \( k \) is the unit vector along the \( z \)-direction which is very strong that the electrons and dusts are moving along the magnetic field direction very fast, i.e. the response of electrons and dusts look like as that in the unmagnetized plasma. The nonlinear dynamics of the DIA SWs propagating in such a multi-component dusty plasma is governed by

\[
\frac{\partial n_x}{\partial t} + \nabla \cdot (n_x \mathbf{u}_x) = 0
\]

\[
\frac{\partial \mathbf{u}_x}{\partial t} + (\mathbf{u}_x \cdot \nabla) \mathbf{u}_x = \nabla \phi - \omega_{ce} \mathbf{u}_x \times \hat{k}
\]

\[
\frac{\partial \mathbf{u}_p}{\partial t} + (\mathbf{u}_p \cdot \nabla) \mathbf{u}_p = -\beta \nabla \phi + \beta \omega_{ce} \mathbf{u}_p \times \hat{k}
\]

\[
\nabla^2 \phi = \mu_0 n_e + n_n - n_{p0} n_p - j \mu_d
\]

where \( n_x (n_p n_p) \) is the ion number density (negative/positive) normalized by its equilibrium value \( n_{x0} \), \( u_x (u_p) \) is the negative (positive) ion fluid speed normalized by \( C_n = (k_B T_e/m_p)^{1/2} \), with \( k_B \) is the Boltzmann constant, \( T_e \) is the temperature of electrons and \( m_p \) being the rest mass of negative ions. \( \phi \) is the DIA wave potential normalized by \( k_B T_e/e \), with \( e \) being the magnitude of the charge of an electron. \( \omega_{ce} \) is the negative ion cyclotron frequency \( \left( e \mathbf{B}_0/m_p \right) \) normalized by plasma frequency \( \omega_{pi} = (4 \pi n_{x0} e^2/m_p)^{1/2} \).

At equilibrium we have

\[
n_{p0} + j \beta n_{d0} = n_{d0} + n_{p0}
\]

where, \( j n_{d0} = n_{d+} - n_{d-} \) with \( n_{d+} \) being the positive dust number density and \( n_{d-} \) being the number density of negative dust, \( j = 1 \) for the condition \( n_{d+} > n_{d-} \) and \( j = -1 \) for the condition \( n_{d+} < n_{d-} \), i.e. the value of \( j \) depends on net charge of dust grain and \( \beta \) is the mass ratio of negative ion to positive ion \( (m_p/m_p) \). We can also write

\[
\mu_0 = \mu_p + j \mu_d - 1
\]

where, \( \mu_0 = n_{x0}/n_{p0}, \mu_p = n_{p0}/n_{p0} \) and \( \mu_d = n_{d0}/n_{d0} \).
Derivation of ZK equations

To derive a dynamical equation for the nonlinear propagation of the electrostatic waves in a dusty plasma, under consideration, have to employed in Eqs. (1–5). To do so, the following stretched coordinates [31–37] are introduced

\[
\begin{align*}
X &= \epsilon^{1/2} x \\
Y &= \epsilon^{1/2} y \\
Z &= \epsilon^{1/2} (z - V_0 t) \\
\tau &= \epsilon^{3/2} t
\end{align*}
\]

where \( \epsilon \) is a smallness parameter \((0 < \epsilon < 1)\) measuring the weakness of the dispersion and \( V_0 \) is the Mach number (the phase speed normalized by \( C_p \)). \( n_s, u_s, \) and \( \phi \) can be expanded about their equilibrium values in a power series of \( \epsilon \), viz.,

\[
\begin{align*}
n_s &= 1 + c n_s(1) + \epsilon^2 n_s(2) + \cdots \\
u_x &= \epsilon^{3/2} u_x(1) + \epsilon^2 u_x(2) + \cdots \\
u_y &= \epsilon^{3/2} u_y(1) + \epsilon^2 u_y(2) + \cdots \\
u_z &= \epsilon u_z(1) + \epsilon^2 u_z(2) + \cdots \\
\phi &= \epsilon \phi(1) + \epsilon^2 \phi(2) + \cdots
\end{align*}
\]

where \( s \) represents the species \((n \) for negative ions and \( p \) for positive ions).

Using the stretched coordinates and Eq. (8) into Eqs. (2–5) and (1), one can develop equations in various powers of \( \epsilon \). To the lowest order of \( \epsilon \) is

\[
\begin{align*}
u_{xx} &= -\frac{1}{\alpha_{cn}} \frac{\partial \phi(1)}{\partial X}, & u_{xy} &= \frac{1}{\alpha_{cn}} \frac{\partial \phi(1)}{\partial Y} \\
\nu_{yy} &= \frac{1}{\alpha_{cn}} \phi(1), & u_{yz} &= \frac{\beta}{V_0} \phi(1) \\
n_{ix} &= -\frac{1}{V_0} \phi(1), & n_{ip} &= \frac{1}{V_0} \phi(1)
\end{align*}
\]

Equating the coefficients of \( \epsilon \) from Poissons equation, we get

\[
0 = \mu_0 \left[ \phi(1) - \frac{4\alpha}{1 + 3\alpha} \phi(1) \right] + n_{ix}^{(1)} - \mu_p n_{ip}^{(1)}
\]

Using the value of \( n_{ix}^{(1)} \) and \( n_{ip}^{(1)} \) from Eq. (9) into Eq. (10), we can get the linear dispersion relation

\[
V_0 = \sqrt{\frac{(1 + \beta \mu_p)(1 + 3\alpha)}{\mu_0(1 - \alpha)}}
\]

Again, following the same procedure, one can obtain the next higher order continuity equation and the \( z \)-component of momentum equation as

\[
\begin{align*}
\frac{\partial n_s^{(1)}}{\partial \tau} - V_0 \frac{\partial n_s^{(2)}}{\partial Z} + \frac{\partial u_{sx}^{(1)}}{\partial X} + \frac{\partial u_{sx}^{(2)}}{\partial Y} + \frac{\partial u_{sx}^{(2)}}{\partial Z} \\
+ \frac{\partial}{\partial Z} \left[ n_s^{(1)} u_s^{(1)} \right] &= 0 \\
\frac{\partial u_{sx}^{(1)}}{\partial \tau} - V_0 \frac{\partial u_{sx}^{(2)}}{\partial Z} + \frac{\partial u_{sx}^{(2)}}{\partial X} - \frac{\partial \phi(1)}{\partial Z} &= 0 \\
\frac{\partial u_{sy}^{(1)}}{\partial \tau} - V_0 \frac{\partial u_{sy}^{(2)}}{\partial Z} + \frac{\partial u_{sy}^{(2)}}{\partial Y} + \frac{\partial \phi(1)}{\partial Z} &= 0
\end{align*}
\]

To the next higher order of \( \epsilon \), i.e., equating the coefficients of \( \epsilon^2 \), we can express \( x \)– and \( y \)-components of the momentum equations for both negative and positive ions and Poisson’s equation as

\[
\begin{align*}
\nu_{sx}^{(2)} &= -\frac{V_0}{\alpha_{cn}} \frac{\partial^2 \phi(1)}{\partial Z \partial X}, & u_{sx}^{(2)} &= -\frac{V_0}{\alpha_{cn}} \frac{\partial^2 \phi(1)}{\partial Y Z} \\
\nu_{sy}^{(2)} &= \frac{V_0}{\beta \alpha_{cn}} \frac{\partial^2 \phi(1)}{\partial Z \partial Y}, & u_{sy}^{(2)} &= \frac{V_0}{\beta \alpha_{cn}} \frac{\partial^2 \phi(1)}{\partial X Z} \\
\left( \frac{\partial^2 \phi(1)}{\partial X^2} + \frac{\partial^2 \phi(1)}{\partial Y^2} + \frac{\partial^2 \phi(1)}{\partial Z^2} \right) &- \mu_0 \left[ \frac{1}{1 + 3\alpha} \phi(1) \right] \\
- \mu_0 \left[ \frac{1}{2} \left( \phi(1) \right)^2 + \mu_p \phi(2) - n_p^{(2)} \right] &= 0
\end{align*}
\]

Now, using (9–17), we can readily obtain

\[
\begin{align*}
\frac{\partial \phi(1)}{\partial \tau} + A \phi(1) \frac{\partial \phi(1)}{\partial Z} + \frac{1}{2} A \frac{\partial}{\partial Z} \left[ \frac{\partial^2 \phi(1)}{\partial Z^2} \right] \\
+ D \left( \frac{\partial^2 \phi(1)}{\partial X^2} + \frac{\partial^2 \phi(1)}{\partial Y^2} \right) \phi(1) &= 0
\end{align*}
\]

where

\[
\begin{align*}
A &= \frac{V_0}{\mu_0} \left( \frac{1 + 3\alpha}{1 - \alpha} \right) \\
B &= \frac{3}{2} \left( \frac{\beta \mu_p - 1}{V_0} \right) \frac{\mu_0}{2} \\
D &= 1 + \frac{\beta \mu_p}{\beta}
\end{align*}
\]

SW solution of the ZK equation

To study the properties of the solitary waves propagating in a direction making an angle \( \delta \) with the \( Z \)-axis, i.e., with the external magnetic field and lying in the \( (Z-X) \) plane, we first rotate the coordinate axes \( (X, Z) \) through an angle \( \delta \), keeping the \( Y \)-axis fixed. Thus, we transform our independent variables to
This transformation of these independent variables allows us to write the ZK equation in the form

\[
\frac{\partial \phi^{(1)}}{\partial t} + \frac{\partial \phi^{(1)}}{\partial \xi} + \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} + \frac{\partial^3 \phi^{(1)}}{\partial \xi^2 \partial \rho} + \frac{\partial^3 \phi^{(1)}}{\partial \xi \partial \eta^2} = 0
\]

(21)

where

\[
\delta_1 = AB \cos \delta \\
\delta_2 = \frac{1}{2} A (\cos^3 \delta + D \sin^2 \delta \cos \delta) \\
\delta_3 = -AB \sin \delta \\
\delta_4 = -\frac{1}{2} A (\sin^3 \delta + D \sin \delta \cos^2 \delta) \\
\delta_5 = A \left[ D \left( \sin \delta \cos^2 \delta - \frac{1}{2} \sin^3 \delta \right) - \frac{3}{2} \sin \delta \cos^2 \delta \right] \\
\delta_6 = -A \left[ D \left( \sin^2 \delta \cos \delta - \frac{1}{2} \cos^3 \delta \right) - \frac{3}{2} \sin^2 \delta \cos \delta \right] \\
\delta_7 = \frac{1}{2} AD \cos \delta \\
\delta_8 = -\frac{1}{2} AD \sin \delta
\]

We now look for a steady state solution of this ZK equation in the form

\[
\phi^{(1)} = \phi_0(Z)
\]

(23)

where

\[ Z = \xi - u_0 t \]

in which \( u_0 \) is a constant speed normalized by the ion-acoustic speed \( (C_i) \). Using this transformation we can write this ZK equation in steady state form as

\[
-u_0 \frac{d \phi_0}{dZ} + \delta_1 \phi_0 \frac{d \phi_0}{dZ} + \delta_2 \frac{d^3 \phi_0}{dZ^3} = 0
\]

(24)

Now, using the appropriate boundary conditions, viz., \( \phi^{(1)} \rightarrow 0, \quad (d\phi^{(1)}/dZ) \rightarrow 0, \quad (d^2\phi^{(1)}/dZ^2) \rightarrow 0 \) as \( Z \rightarrow \pm \infty \), the SWs solution of this equation is given by

\[
\phi_0(Z) = \phi_m \text{sech}^2(\kappa Z)
\]

(25)

where \( \phi_m = 3u_0/\delta_3 \) is the amplitude and \( \kappa = \sqrt{u_0/4\delta_3} \) is the inverse of the width of the solitary waves. As \( u_0 \rightarrow 0 \), it is clear from (11), (19), and (22) that depending on the sign of \( B \), the solitary waves will be associated with either positive potential \( (\phi_m > 0) \) or negative potential \( (\phi_m < 0) \). Therefore, there exists solitary waves associated with positive (negative) potential when \( B > 0 \) \( (B < 0) \). Figure 1 represents the variation of the phase speed \( V_0 \) of DIA SWs with \( \mu_p \) and \( \mu_d \) for positive as well as negatively charged stationary dust particles. This figure indicates that the phase speed is larger for negatively charged dust (upper surface) than for positively charged dust (lower surface). It is also found from this figure that the phase speed decreases with increasing the number density of both positive ions and dust particles in our present dusty plasma system. Figures 2,3 represent the variations of the potential \( \phi_m \) of DIA SWs with various parameters. From Fig. 2, it has been found that the SWs associated with negative potential only for propagation angle \( (\delta \rightarrow 0^\circ-90^\circ) \) and it associated with positive potential for the propagation angle \( (\delta \rightarrow 90^\circ-180^\circ) \). That means below the propagation angle \( 90^\circ \) the solitary wave structure associates with dip shape and above this value the solitary wave structure associates with hump shape. Figure 3 indicates that the potential of the SW decreases with increasing the number density of both positive ions and dust particles but the potential is larger for negatively charged dust (upper surface) than for positively charged dust particles. Figures 4 and 5 show the variation of the width of SWs with propagation angle \( \delta \), \( \mu_p \) and \( \mu_d \) respectively. From Fig. 4, it has been found that for lower limit of the angle \( (0^\circ-50^\circ) \) the width increases with it and decreases for higher limits of the angle \( (50^\circ-90^\circ) \) for both positively and negatively charged dust particles. It is also clear from this figure that the external magnetic field leads to a decrease in the potential width, i.e., a stronger
magnetic field leads to steeper and thus narrower soliton profiles. Similar observations are found in the work of Haider [5] and the work of Sabetkar and Dorranian [11, 12, 14] while studying the DIA and DA SWs with the presence of magnetic field. The width of the SWs is lower for positive dust grains than for negative dust grains, thus the positive dust grains makes the soliton profile more steeper. Figure 5 shows how the width changes with $\mu_p$ and $\mu_d$ and this figure indicate that the width decreases with increasing the value of both the number density of positive ions and dust particles as found in the work of Haider [5]. From the above analysis we have seen that the magnetic field has an effect only on the width of the SW but it has no effect on the amplitude of the waves.

**Instability analysis**

We now study the instability of the obliquely propagating SWs, discussed in the previous section, by the method of small-$k$ perturbation expansion technique [33–38]. We first assume that

$$\psi^{(1)}(Z, \zeta, \eta, \tau) = \phi_0(Z) + \psi(Z, \zeta, \eta, \tau)$$

(26)

where $\phi_0(Z)$ is defined by (25), and for a long-wavelength plane wave perturbation in a direction with direction cosines $(l_{\zeta}, l_{\eta}, l_z)$, $\psi(Z, \zeta, \eta, \tau)$ is given by

$$\psi(Z, \zeta, \eta, \tau) = \phi(Z)e^{-i\omega t + i k(l_{\zeta}\zeta + l_{\eta}\eta + l_z Z)}$$

(27)

in which $l_{\zeta}^2 + l_{\eta}^2 + l_z^2 = 1$, and for small $k$, $\phi(Z)$ and $\omega$ can be expanded as

$$\psi(Z) = \phi_0(Z) + k\phi_1(Z) + k^2\phi_2(Z) + \cdots$$

(28)

$$\omega = k\omega_1 + k^2\omega_2 + \cdots$$

(29)

Now, substituting (26) into (21), and linearizing with respect to $\psi$, we can express the linearized ZK equation in the form.
where $K$ is another integration constant, and $x_1$ and $\beta_1$ are given by

\[
x_1 = (\omega_1 + l_1 u_0) - \frac{1}{2} \phi_m \mu_1 + 2k^2 \mu_2
\]
\[
\beta_1 = \frac{1}{2} \phi_m \mu_1 - 6k^2 \mu_2
\]
\[
\mu_1 = \delta_1 l_1 + \delta_3 l_2, \quad \mu_2 = 3\delta_2 l_1 + \delta_3 l_2
\]

Now, following the same procedure, the general solution of this first-order equation, for $\varphi_1(Z)$ not tending to $\pm \infty$ as $Z \to \pm \infty$, can be written as

\[
\varphi_1(Z) = K f + \frac{iC_1}{8\delta_2 \kappa Z} \left[ a Z + \frac{2}{3} (3x_1 + \beta_1) \phi_0 \right]
\]

where $k = x_1 + \beta_1$. The second-order equation, i.e., the equation with terms involving $k^2$ obtained from (30) after substituting (27–29), can be written as

\[
\left[ -u_0 \frac{d}{dZ} + \delta_1 \frac{d}{dZ} \frac{d}{dZ} + \delta_2 \frac{d^3}{dZ^3} \right] \varphi_2(Z) = i\omega_2 \varphi_0(Z)
\]

\[
+ i[\omega_1 + l_2 u_0] \varphi_1(Z) - i\mu_1 \phi_0 \phi_1(Z)
\]

\[
+ \mu_3 \frac{d\phi_0(Z)}{dZ} - i\mu_2 \frac{d^2 \phi_1(Z)}{dZ^2}
\]

where

\[
\mu_3 = 3\delta_2 l_1^2 + 2\delta_3 l_1 l_2 + \delta_6 l_2^2 + \delta_7 l_1^3
\]

The solution of this second-order equation exists if the right-hand side is orthogonal to a kernel of the operator adjoint to the operator

\[
- u_0 \frac{d}{dZ} + \delta_1 \frac{d}{dZ} \varphi_0 + \delta_2 \frac{d^3}{dZ^3}
\]

This kernel, which must tend to zero as $Z \to \pm \infty$, is $\phi_0 = \phi_m \text{sech}^2(\kappa Z)$. Thus, we can write the following equation determining $\omega_1$:

\[
\int_{-\infty}^{\infty} \varphi_0 \left[ i\omega_2 \varphi_0(Z) + i[\omega_1 + l_2 u_0] \varphi_1(Z) - i\mu_1 \phi_0 \phi_1(Z)
\]

\[
+ \mu_3 \frac{d\phi_0(Z)}{dZ} - i\mu_2 \frac{d^2 \phi_1(Z)}{dZ^2} \right] dZ = 0
\]

Now, substituting the expressions for $\varphi_0(Z)$ and $\varphi_1(Z)$ given by (35) and (38), respectively, and then performing the integration, we arrive at the following dispersion relation:

\[
\omega_1 = \Omega - l_2 U_0 + (\Omega^2 - \Upsilon)^{1/2}
\]

this equation is valid only for first order approximation, i.e., $\omega = k\omega_1$, where

\[
\Omega = \frac{2}{3} (\phi_m \mu_1 - 2\mu_2 k^2)
\]

\[
\Upsilon = \frac{16}{45} (\phi_m^2 \mu_1^2 - 3\phi_m \mu_1 \mu_2 k^2 - 3\mu_2^2 k^4 + 12\delta_2 \mu_3 k^4)
\]

It is clear from the dispersion relation (43) that there is always instability if $(\Upsilon - \Omega^2) > 0$. Thus, using (19), (22), (37), (40), (41), and (45), we can express the instability criterion as

\[
S_t > 0
\]
The Eq. (49) represents that the growth rate $u$ speed for $b$

Fig. 6 Plot of $S_i = 0$ surface plot (i.e., variation of $\omega_{ci}$ with $l_\|$, $l_\perp$, and $l_q$ for $\beta = 1$, $\mu_p = 1$ and $\delta = 10^\circ$) above which the solitary waves become unstable and below which the SWs become stable

Fig. 7 Showing the variation of growth rate $\Gamma$ with $\delta$ and $\omega_{ci}$ for $u_0 = 0.1$, $\mu_p = 1$, $l_\| = 0.5$, $\beta = 1$ and $l_q = 0.5$

discussed in the work of Allen and Rowlands [39, 40] based on different cases. Using these techniques Williams and Kourakis [41] studied stability analysis with growth rate of solitary structures. The instability analysis using small-$k$ perturbation technique and process of obtaining growth rate in present form is very suitable to understand, because it express instability criterion and growth rate in terms of basic plasma parameters like ratio of number density, cyclotron frequency, direction cosine etc. which are used to describe the concern plasma situation. Many authentic authors have studied instability criterion as well as growth rate using this perturbation technique [36–38, 42] as it is a well established technique. The nonlinear variations of $\Gamma$ with $\delta$, $\omega_{ci}$, $l_\|$, and $l_\perp$ are shown in Figs. 7 and 8. Figure 7 shows how $\Gamma$ changes with $\delta$ and $\omega_{ci}$ for $u_0 = 0.1$, $l_\| = 0.5$, $l_\perp = 0.5$ and $\mu_p = 2$, and Fig. 8 shows the variation of $\Gamma$ with $l_q$ and $l_\perp$ for $u_0 = 0.2$, $\delta = 10^\circ$, $\omega_{ci} = 0.3$ and $\mu_p = 0.1$. These figures represent that the growth rate ($\Gamma$) of the unstable perturbation changes proportionally with direction cosine ($l_q$, $l_\perp$) but inversely with $\delta$ and $\omega_{ci}$.

Results and discussions

Here a magnetized dusty plasma system consisting of non-thermal electrons, cold mobile positive as well as negative ions, and opposite polarity stationary dust has been considered. The ZK equation by using the reductive perturbation method has been derived. Then their multi-dimensional instability by the small-$k$ perturbation expansion technique have analyzed. The results, which have been found in this investigation may be pointed out as follows:
The effects of non-thermal negative ions have been found to modify the nature of the DIA SWs structures in a dusty plasma.

The nonlinear DIA solitary wave structure associated with positive as well as negative potential depending on constant coefficient A and B.

The magnitude of the external magnetic field $B_0$ has no effect on the solitary wave amplitude. However, it has a direct effect on the width of the SWs, and it is found that the width of the waves decreases as the magnitude of magnetic field $B_0$ increases, i.e. the magnetic field makes the solitary structures more spiky.

The phase velocity of the SWs becomes slower with increasing the values of positive ions and with dust grains. On the other hand phase velocity of the SWs becomes faster with increasing the values of both $\alpha$ and $\beta$. But the magnitude of phase velocity is larger for negative dust grains than for positive dust grains.

The nonlinear wave and its perturbation mode. Whether the SWs are stable or unstable. However, the results which we obtained may be useful for understanding the localized electrostatic disturbances in some space (viz. auroral plasma, Saturns E- and F-rings, etc.) [42–45] and laboratory [45–47] dusty plasmas.

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