Group Effect of Piles at the Foundation Vibrations

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Abstract. Variation of the dynamic stiffness in pile and pile foundation versus the distance between piles at the vertical vibrations is under consideration. Experimental data from references are involved, as well as the solutions of wave models describing the vertical vibrations of a thin plate with round cuts. We use the data obtained experimentally for the determination of natural frequencies in the cap-bound groups of 3×3 friction piles with different distances between them to verify the reliability of the wave models solutions describing the variation of the dynamic stiff nesses at the vertical vibrations of the pile foundations. We also use the data found at the forced vertical vibrations of the cap-bound groups of 2×2 piles under different loads and at different distances between the piles. Processing of available amplitude-frequency curves involves the solution of inverse problem with the theory of nonlinear vibrations for the dynamic stiffness determination. The correlation between measured and predicted data is evaluated by the description of pile and soil system behavior. It has been found that the relations regarding mutual effect of the piles in the group obtained as a part of wave model solutions and applied for the dynamic stiffnesses computation at the vertical vibrations of pile foundations permit getting satisfactory accurate results. The deviation of the computations experimental results is maximum 15%.

1. Introduction

High accuracy of the evaluated characteristics of the vibrations in the pile foundations installed under the machines with dynamic loadings is always a topical challenge [1-17]. The dynamic interaction between pile, pile foundation and soil is among the least tasks to be considered, and the mutual effect of grouped piles makes it even more complex. Group stiffness can be evaluated via simple summing up the stiffnesses of individual piles in the cases, when the distance between piles is large. However, the piles located close to each other demonstrate the essential group effect and finally their efficiency may change considerably. Both between each other and with soil, we need theories describing processes and experiments to verify the applicability of these theories in order to evaluate the dynamic interaction of piles.

We should note that the activities to determine the amplitude-frequency characteristics of the pile foundations are being performed for a long time but still far from its’ finish. The accurate theoretical problem solution of the dynamic interaction between the pile and soil is complicated by the non-linear character of the process. Thus, approximate approaches are applied. Particularly, there is an approach proposed among them [5]. Many researches are devoted to the interaction of one pile with the soil under the dynamic loading, but at the same time, the behavior of pile groups is studied, too. Numerical
simulation methods involving finite or boundary elements are commonly used to evaluate the dynamic condition of pile structures in complex application environment [6-9].

For most engineering tasks, the interaction between pile and soil is usually successfully explained by the elasticity theory and, as it is shown by many theoretical and experimental works, wave models simulate quite accurately the vibration process of the pile foundations in soil [10]. Today, the solutions, found for the tasks of the vibrating in finite plate with a round cut are used successfully to determine the amplitude-frequency characteristics of the pile foundations under dynamic loadings. However, generalization of this result for the cases, when there are more than one cut is of practical interest. In order to find the link between motions and reactions on the side surface of embedded solid bodies, either in-lined or grouped, the authors of [6, 11, 12] proposed solution for the task with vertical vibrations of plate with several round cuts. They also derived formulas to find stiffness and damping characteristics of the system (see schematic in Figure 1). But the issue of reliability and accuracy of these results still remains open.

Therefore, the present research deals with the analysis of the dynamic stiffnesses varying at the vertical vibrations of pile foundations, due to the distance between piles. We use theoretical evaluations from [11, 12] and compare them with the experimental data from references [5, 13-16]. Among used materials, there are the series of tests’ results performed by the authors with the groups of friction piles $3 \times 3$ bound with a cap [15, 16]; the tests were purposed to determine natural frequencies pile foundations ($s/d=m=2;3;5,d$ is the pile diameter, $s$ is the distance between central axes of neighboring piles). Along with, we used the experimental results for the forced vertical vibration of the cap-bound pile groups $2 \times 2$ at $s/d = 2;3;4$ gathered a field under different loadings [5, 13].

2 Materials and Methods

2.1. Theoretical investigations

Agreement between measured and predicted data was evaluated for the description of the non-linear behavior of pile and soil system.

The inverse problem was solved to determine dynamic stiffness and attenuation, theory of non-linear vibrations was used as the amplitude-frequency curves from [13, 14] were processed.

\[ \text{Figure 1. Arrangement of cuts in a wavering thin plate: a) two neighboring ones, b) according to the } \]

$3 \times 3$ scheme.

In [11, 12] considering warping axisymmetric vibrations of the infinitely thin layer with one round cut (radius $r_0$) described by the equation of elastic medium motion at zero volume forces in the cylindrical system of coordinates $(r,t)$ as
with the boundary condition on the contour

\[ w(r_0,t) = w_0 e^{i\omega t}, \]

the authors determine that the reaction of single-thickness soil layer applied to the pile side surface is described as

\[ S_{w_0}(a_0)w_0 e^{i\omega t} = \mu w_0 e^{i\omega t}(S_{w_{1,0}} + iS_{w_{2,0}}), \]

where the real \( S_{w_{1,0}} \) and imaginary \( S_{w_{2,0}} \) dimensionless parts of \( S_{w_0} \) can be presented as

\[ S_{w_{1,0}}(a_0) = 2\pi a_0 \frac{J_0(a_0)J_1(a_0) + Y_0(a_0)Y_1(a_0)}{J_0^2(a_0) + Y_0^2(a_0)} C, \]

\[ S_{w_{2,0}}(a_0) = -\frac{4}{J_0^2(a_0) + Y_0^2(a_0)} \]

(1)

Here, \( J_n, Y_n \) are the Bessel 1st and 2nd kind functions, \( w = w(r,t) \) is the motion along the axis \( z, \rho \) is the density; \( \mu \) is the Lame coefficient equivalent to the shear module \( G, a_0 = \omega \eta_0 / \sqrt{\mu / \rho} \) is the dimensionless frequency.

The layer warping vibrations with several round cuts in line are described in [6]; the cuts radii are \( r_0 \), the centers are located within the distance of \( s > r_0 \) or \( m \) diameters from each other, \( m \) (see schematic in Figure 1a), as well as for the inner cut in line as it is shown in Figure 1b, so the reaction of the single-thickness soil layer attached to the pile side surface, according to [11]

\[ S_{w_1}(a_0, ma_0)w_0 e^{i\omega t} = \mu w_0 e^{i\omega t}(S_{w_{1,1}} + iS_{w_{2,1}}), \]

where \( S_{w_{1,1}}, S_{w_{2,1}} \) are the real and imaginary dimensionless components, so \( S_{w_1} \) can be presented as

\[ S_{w_{1,1}}(a_0, ma_0) = S_{w_{1,0}} - \frac{3}{2}S_{w_{1,cor}}(a_0, ma_0), \]

\[ S_{w_{2,1}}(a_0, ma_0) = S_{w_{2,0}} - \frac{3}{2}S_{w_{2,cor}}(a_0, ma_0), \]

\[ S_{w_{1,cor}}(a_0, ma_0) = \pi a_0 \frac{J_0(a_0)J_1(a_0) + Y_0(a_0)Y_1(a_0)}{J_0^2(a_0) + Y_0^2(a_0)} C, \]

\[ S_{w_{2,cor}}(a_0, ma_0) = -\frac{4}{J_0^2(a_0) + Y_0^2(a_0)} \]

\[ C = \sum_{n=1}^2 \sum_{j=1}^2 \frac{J_{2n-1}(a_0)[Y_{2n-2}(a_0)Y_{2j}(a_0)] - Y_{2n-1}(a_0)[Y_{2n-2}(a_0)Y_{2j}(a_0)]}{J_0^2(a_0) + Y_0^2(a_0)} \]

\[ \sum_{n=1}^2 \sum_{j=1}^2 \frac{J_{2n-1}(a_0)[Y_{2n-2}(ma_0)Y_{2j}(ma_0)] - Y_{2n-1}(a_0)[Y_{2n-2}(ma_0)Y_{2j}(ma_0)]}{J_0^2(a_0) + Y_0^2(a_0)} \]

(2)

here, opposite to (1), there are additive terms regarding the effect of neighboring cuts.

The expressions describing the reaction of the boundary (not corner) cut (schematic in Figure 1b) were found in [12]

\[ S_{w_2}(a_0, ma_0)w_0 e^{i\omega t} = \mu w_0 e^{i\omega t}(S_{w_{1,2}} + iS_{w_{2,2}}) \]

\[ S_{w_{1,2}}(a_0, ma_0) = S_{w_{1,0}} - \frac{5}{4}S_{w_{1,cor}}(a_0, ma_0), \]

\[ S_{w_{2,2}}(a_0, ma_0) = S_{w_{2,0}} - \frac{5}{4}S_{w_{2,cor}}(a_0, ma_0), \]

and for the corner cut

\[ S_{w_3}(a_0, ma_0)w_0 e^{i\omega t} = \mu w_0 e^{i\omega t}(S_{w_{1,3}} + iS_{w_{2,3}}) \]

\[ S_{w_{1,3}}(a_0, ma_0) = S_{w_{1,0}} - \frac{7}{8}S_{w_{1,cor}}(a_0, ma_0), \]

\[ S_{w_{2,3}}(a_0, ma_0) = S_{w_{2,0}} - \frac{7}{8}S_{w_{2,cor}}(a_0, ma_0), \]

Thus, the dynamic stiffnesses \( S_{w_j} (j=1,2,3) \) are described by the complex functions which depend on the vibration frequency \( \omega, \) cut size \( r_0, \) as well as on the density \( \rho \) and medium shear modulus \( \mu. \) The
reactions go ahead respective motions by the time intervals $\Delta t$ which are found from the relations $\Delta t = \arctan \left( \frac{S_{w_{2,j}}}{S_{w_{1,j}}} \right)$. The parameters characterizing the motion amplitude can be estimated from the relation $A_j = (S_{w_{1,j}}^2 + S_{w_{2,j}}^2)^{0.5}$.

**Figure 2.** Variation of the relative stiffness at $a_0 = 0.05$ (dashed curves) and $a_0 = 0.35$ (solid curves) from the distance between the piles and their position in a group $\bullet$ ($j=1$), $\Delta$ ($j=2$), $\blacksquare$ ($j=3$).

Figure 2 illustrates the variation of the relative dynamic stiffness $S_{w_{1,j}}/S_{w_{10}}, j=1,2,3$ at $a_0 = 0.05,0.35$ versus the distance between the piles and their position in the group in accordance with the schematic in Figure 1b. The presented results lead to the conclusion that the reducing distance between the piles may cause the stiffness reduction up to 40%.

2.2. Experimental researches

The results of the field measurements’ series are used to verify the presented theoretical evaluations of the dynamic interaction for the system “pile-soil-pile”. The experiments were carried out with the cap-bound groups of friction piles $3 \times 3$ (schematics in Figure 3 and Figure 1b) and were purposed to determine the natural frequencies of the pile foundations published in [15, 16].

All three test pile foundations were made as a monolithic reinforced cap with the sizes $1.0 \times 1.0 \times 0.2$ m supported by 9 rigidly fastened piles with the diameter $d = 76$ mm ($r_0 = d/2$), the working length $M = 1.4$ m; the piles were made from metal tubes, the wall diameter was $3.5$ mm. The distance between pile axes was $2d$, $3d$ and $5d$. Mass of each pile foundation $M$ was 690 kg. The cap did not contact with the soil. The impulse loading was carried out by steel ballast – a 6 kg paralleled piped freely falling on the surface of each tested pile foundation from the height of 0.5 m.

**Figure 3.** Experimental researches of tested foundations: a) $2d$ – 152 mm; b) $3d$ – 228 mm; c) $5d$ – 380 mm.
The soil is the test field contained down to 9.3 m from loessial slightly wet sand clay, the density $\rho = 1.7 \, \text{t/m}^3$ and deformation modulus $E = 14 \, \text{MPa}$, medium-hard loam is the sub-soil. No ground water in the field. The value of the cross waves rate for the test field soil was found experimentally as $V_s = 146 \, \text{m/s}$.

3. Results

Investigation results [15, 16] for the natural frequencies of the pile foundations determination in the test field are presented in Figure 4 and in Table 1.

**Figure 4.** Natural frequencies of the pile foundations $3 \times 3$ at different $s/d$: o – experiment, • – average value.

**Table 1.** Measured frequencies.

| Distance between piles | Measured frequency $f_z$, Hz (the average value) |
|-----------------------|-----------------------------------------------|
| $2d$                  | 82.90                                         |
| $3d$                  | 91.36                                         |
| $5d$                  | 101.05                                        |

The natural frequencies $\lambda_z$ at the vertical vibrations of the pile foundations and stiffness $K_z$, in the presence of damping, are related as

$$\lambda_z = 2\pi f_z \sqrt{\frac{K_z}{M}}, \quad K_z = \lambda_z^2 M,$$

where, $M$ is the mass of the structure.

The stiffness of the pile groups were found for various $s$ involving the measurement results from Table 1 and formula

$$K^g(m) = [2\pi f_z(m)]^2 M.$$  \hspace{1cm} (2)

Theoretical evaluations of the stiffness for the pile groups were found with the relations

$$K^g(m) = \mu h S^g_w(a_0, ma_0).$$ \hspace{1cm} (3)

Hence, it follows that the value of the grouped pile stiffness factor $S^g_{w_1}$ is related with the dimensionless frequency of vibrations $a_0$ and pile position in the cap. Foundations from $3 \times 3$ piles ($h=1.25m$), $S^g_{w_1}$, is determined for the considered pile in accordance with the schematic in Figure 1b by the formula

$$S^g_{w_1}(a_0, ma_0) = S_{w_{1,1}}(a_0, ma_0) + 4S_{w_{1,2}}(a_0, ma_0) + 4S_{w_{1,3}}(a_0, ma_0),$$

where $\omega = 2\pi f_z$ is the angular frequency of vibrations, $\mu = V_s^2 \rho$ is the shear modulus $V_s$ is the cross wave rate in the soil. In the cases under consideration, the dimensionless vibration frequency $a_0 = 0.15$ is the average value within the range from 0.13 to 0.17.
The results obtained by the engineering calculations involving the formulas (2), (3) are shown in Figure 5. The calculated curve and dots, corresponding to the measurement results, illustrate the varying stiffnesses in respect to the value at $s/d=5$. The values, found by the formula (3), show the maximal difference with the test data, which is below 14%. This result vindicates that the calculations in the framework of approximations permit used obtaining the satisfactory agreement with experimental findings.

As an extra check of the relations for the theoretical evaluations, we use the results of the dynamic tests in the field performed to determine the resonance frequencies and pile foundations amplitudes at different levels of the vertical harmonic excitation with the cap for friction pile groups $2\times2$ [5, 13, 14, 17]. All test pile foundations were made as a monolithic reinforced cap with the sizes $0.57\times0.57\times0.25$ m supported by 4 rigidly fastened concrete piles, the diameter $d=100$mm and working length $h=1.5$ m. The distance between pile axes $s$ was 2d, 3d and 4d. The caps do not contact with the soil. The tests were carried out at different eccentric moment’s $m_erm_e=0.0187, 0.0278, 0.0366$ and 0.0450 kg·m, where $m_e$ is the mass of the eccentric rotating part in the vibrator, and $r_e$ are the mass eccentricities. The methodology of vibration tests is described in [13]. The mass of each pile foundation is $M_0= 1200$ kg including the rotating part mass. The shear modulus value $\mu$ is found for the test field soil: from $14\times10^6$ N/m$^2$ to $26\times10^6$ N/m$^2$ and cross wave rates $V_s$ within the range 95 – 150 m/s, which depend on the depth [5]. Table 2 present the resonance frequencies $f_{res}$ and amplitudes $A_{res}$.

### Table 2. Resonance frequencies $f_{res}$ and amplitudes $A_{res}$.

| $m_erm_e$,kg·m | $s/d=2$ | $s/d=3$ | $s/d=4$ |
|----------------|---------|---------|---------|
| A_{res}, (mm) | $f_{res}$, (Hz) | A_{res}, (mm) | $f_{res}$, (Hz) | A_{res}, (mm) | $f_{res}$, (Hz) |
| 0.0187        | 0.0358  | 29.61   | 0.0317  | 35.45   | 0.0262  | 38.21  |
| 0.0278        | 0.0510  | 29.22   | 0.0422  | 34.41   | 0.0381  | 36.71  |
| 0.0366        | 0.0633  | 28.95   | 0.0589  | 33.35   | 0.0501  | 35.18  |
| 0.0450        | 0.0832  | 28.46   | 0.0707  | 32.73   | 0.0619  | 33.50  |

During experiments, the researches often have to solve the inverse problems of non-linear vibrations [17]. First, the effective mass, stiffness and damping of the pile and soil system are determined by the measured non-linear amplitude-frequency curves. Then the amplitude-frequency curves are calculated with the theory of non-linear vibrations involving the found values of parameters; the results are compared to the experimental data.

Let us consider the action of the harmonically changing force with the amplitude proportional to the frequency square $\omega$ on the pile foundation

$$P_x=r_erm_e\omega^2,$$
then the motion equation can be written for the system under consideration as follows

\[ M \dddot{x} + \Phi K_x \dot{x} + K_x x = m_{re} \omega^2 \sin \omega t. \]

Here, \( M \) is the effective mass which includes \( M_0 \) and the mass of the attached soil vibrating together with the pile foundation, \( K_x \) — dynamic stiffness, \( \Phi \) — decay modulus. The solution of this equation is written as

\[ z = A \sin(\omega t + \delta), \]

\[ \tan(\delta) = \frac{\Phi \omega K_x / M}{K_x / M - \omega^2}, \]

where \( A \) is described as the variation of the maximal amplitude by the formula

\[ A = \frac{m_{re} \omega^2}{M \left( (K_x / M - \omega^2)^2 + (\Phi \omega K_x / M)^2 \right)^{1/2}}. \quad (4) \]

It follows from (4) that the amplitude depends in a complex manner on the impact force frequency. In the onset regime, the vibration amplitude is \( A_{\text{res}} = m_{re} f_0 / M \). Before the system comes into onset regime, resonance amplitude collision is possible

\[ A_{\text{res}} = \frac{A_{\text{ref}}}{\Phi \sqrt{K_x / M (1 - \Phi^2 K_x / 2M)}}, \quad \Phi^2 K_x / M < 2. \quad (5) \]

at the frequency

\[ \omega_{\text{res}} = \sqrt{\frac{K_x / M}{1 - \Phi^2 K_x / 2M}}, \quad \Phi^2 K_x / M < 2. \quad (6) \]

The relations (5), (6) permit evaluating \( \Phi \) and \( K_x / M \), if the values \( A_{\text{res}} \) and \( f_{\text{res}} \) (\( \omega_{\text{res}} = 2\pi f_{\text{res}} \)) are found during the measurements on site

\[ K_x / M = \omega_{\text{res}}^2 \sqrt{1 - (A_{\text{ref}} / A_{\text{res}})^2}, \quad \Phi = \omega_{\text{res}}^{-1} \sqrt{2 - 2 \sqrt{1 - (A_{\text{ref}} / A_{\text{res}})^2}}. \]

Figure 6 presents the experimental frequency curves for the pile group at \( s = 4d \) for different \( m_{re} \).

With the known \( m_{re} \), according to [12], the effective mass \( M \) can be evaluated using the experimental values for \( A_{\text{ref}} - M = m_{re} f_0 / A_{\text{ref}} \), which are specified for the following computations. The value \( M \) found in this manner is 3200 kg, which is much more than the structure mass \( M_0 = 1200 \) kg.

![Figure 6](image-url)
curves – results of calculations.

Using the data from Table 2 with the resonance frequencies $f_{\text{res}}$ and amplitudes $A_{\text{res}} \ (s=4d)$, we determine the values $K_g^p/M$ for the pile group at various $s$. Computational results for $K_g^p/M$ and $\Phi$ are shown in Figure 6a. As extra verification, the computations were performed for $s=2d$ at the same $M = 3200 \ \text{kg}$ and respective $K_g^p/M$ and $\Phi$ (Figure 6b). The data presented that the theoretical results satisfactorily agree with the measurement data from the references.

| $m_e r_o \ (\text{kg} \cdot \text{m})$ | $s/d=2$ | $s/d=3$ | $s/d=4$ |
|-----------------------------------|---------|---------|---------|
|                                   | $K_g^p/M, \ (1/s^2)$ | $\Phi, \ (s)$ | $K_g^p/M, \ (1/s^2)$ | $\Phi, \ (s)$ | $K_g^p/M, \ (1/s^2)$ | $\Phi, \ (s)$ |
| 0.0187                            | 3.41$\times 10^{-4}$ | 0.89$\times 10^{-3}$ | 4.88$\times 10^{-4}$ | 0.84$\times 10^{-3}$ | 5.62$\times 10^{-4}$ | 0.95$\times 10^{-3}$ |
| 0.0278                            | 3.32$\times 10^{-4}$ | 0.94$\times 10^{-3}$ | 4.57$\times 10^{-4}$ | 0.97$\times 10^{-3}$ | 5.18$\times 10^{-4}$ | 1.00$\times 10^{-3}$ |
| 0.0366                            | 3.25$\times 10^{-4}$ | 1.01$\times 10^{-3}$ | 4.31$\times 10^{-4}$ | 0.94$\times 10^{-3}$ | 4.71$\times 10^{-4}$ | 1.05$\times 10^{-3}$ |
| 0.0450                            | 3.15$\times 10^{-4}$ | 0.96$\times 10^{-3}$ | 4.14$\times 10^{-4}$ | 0.98$\times 10^{-3}$ | 5.31$\times 10^{-4}$ | 1.10$\times 10^{-3}$ |

The values of $K_g^p/M$ for the pile group permit evaluating the natural frequencies $\omega_x$ at the vertical vibrations pile foundations and damping $-\lambda_x=\sqrt{K_g/M}$. The results, given in Table 3 lead to the conclusion about the reducing natural frequencies at the rising excitation intensity, which agrees with the conclusions of [17] about the non-linear behavior of the considered “piles-soil” system. The decay modulus varies weakly; its average value is evaluated as $\Phi \approx 0.97-10^{-3}$. The experiment proves that the analytical methods permit describing the major peculiarities of the amplitude-frequency behavior of the pile groups at low vertical vibration actions. In the considered cases, the effective mass and damping preserve their values as the excitation intensity rises. Figure 7 presents the variation of dynamic stiffness, determined by experimental results in respect to the maximal value at $s/d=4$.

![Figure 7](image)

**Figure 7.** Relative stiffness variation of pile group $2 \times 2$ versus the distance between piles in the group: experiment at different values $m_e r_o = 0.0187 \ (\blacktriangledown), \ 0.0278 \ (*)$, $0.0366 \ (\blacktriangle)$, $0.0450 \ (\square) \ (\text{kg} \cdot \text{m})$ [13], curves – computation results.

Theoretical evaluation of the stiffness for pile group is found with the relations

$$K_g^p(m) = 4\mu h S_w l_3(a_0, m a_0),$$

at the dimensionless vibration frequency $a_0 = 0.08$, the average value within the range $0.06 - 0.1$. The closely spaced curves in Figure 7 illustrate the variation of the computed stiffnesses in respect to the maximal value at $s/d=4$ for different $m_e r_o$.

It follows from the comparison of the theoretical evaluation and experimental findings that the considered analytical method permits predicting the peculiarities of the dynamic interaction between piles in the group. The maximal difference between experimental results and theoretical evaluations
which reduce the interaction effect is about 15% at $s/d=2$, whereas $s/d=3$, the difference in results is maximum 5%. We should take into consideration, that at the same time we ignore the interaction under the pile end, which is essential. However, the final conclusion of the satisfactory agreement is valid; it is evident though that theoretical evaluations development of interacting pile groups between each other and the soil needs further improvement.

4. Discussion
As a conclusion, we should note that the theoretical results have been compared with the experimental findings for the quantitative estimation of the interaction between cap-bound friction piles at the vertical vibrations. Previously found analytical expressions for the computations of grouped piles’ dynamic stiffness have been used with the regard of their position and the distance between them.

It is established by the performed investigations results that the relations, obtained in the framework of the wave models involving the mutual effect of the grouped piles and used to compute the dynamic stiffnesses at the vertical vibrations of pile foundations, getting the results of satisfactory accuracy, which is indicated by comparison with the experimental findings.

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