Kinematics of elements of a spherical ball bearing suspension

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Abstract. The kinematic constraint equations of elements of a spherical ball bearing suspension, separator and balls are considered. Based on the equation of motion of the separator and the balls, the angular coordinates of the separator and the trajectory of the centers of mass of the balls relative to the bearing surfaces are determined. Analytical expressions for determining the parameters of the separator and the balls motion are obtained for different ratios between the velocities of rotation of the rotor mounted on the outer ring and the inner ring of the suspension.

1. Introduction
Since the early 1960s, a large number of patent descriptions – from various countries, for example, US3517562 Inertial Gyroscope – of gyro devices with a spherical ball bearing suspensions of various designs have appeared, which are designed to measure angular displacements of the longitudinal axis of aircraft, as well as to stabilize optical elements. In work [1], the possibility of using gyroscopes with similar suspensions on board of the dynamic aircraft is indicated.

A spherical ball bearing suspensions (BBS) consist of two bearing rings (supporting members), one of which, or both, have a spherical bearing surface. Between the surfaces of the rings, there is a number of balls of the same radius, placed in a two-row separator. In essence, a BBS is a special double-row ball bearing, in which one of the rings can freely rotate relative to the other around any axis. A schematic diagram of a gyroscope with a BBS is shown in figure 1 [1].

Figure 1. Schematic diagram of a three-degree gyroscope with a BBS: 1 – rotor, 2 – outer ring, 3 – inner ring, 4 – axle, 5 – ball, 6 – cage.
A BBS of the rotor 1 includes outer 2 (consists of two halves) and inner 3 suspension rings (supporting members) and balls 5 located in a separator 6. The inner ring is made in the form of a sphere rigidly fixed to the axle 4, which, in turn, can have a rigid conjunction to the framework, or be mounted on bearings. Rotor 1 rotates with angular velocity $\Omega$ around the axis $z_r$ of the associated coordinate system $x_ry_rz_r$, axle 4 rotates with angular velocity $\omega$.

In general case, the rotor of the gyroscope can be conjuncted to any of the rings of the BBS. The ring to which the rotor is conjuncted can be defined as a bearing supporting member (BSM). In this case, the rotor can be fixed to the BSM rigidly or mounted on bearings. These features completely define the configuration of the mechanical system "rotor-BSM-separator-balls" and can be taken as classification ones [2]. The main design features of a gyroscope with BBS are considered on the basis of the patent descriptions.

In addition to the applications indicated in [1], gyroscopes with BBS can be used as a sensitive element in gyroscopic stabilization systems, as well as in stabilization and control systems of optoelectronic equipment [3].

The accuracy of gyroscopes with BBS significantly depends on the state of the surfaces of the supporting members along which the balls move. The traces of the balls, and in the worst case, the treadmills formed by them, noticeably worsen the surfaces of the supporting members [4]. Therefore, it is important to determine the conditions for the maximum and minimum impact of the balls on their rolling surface.

A feature of the BBS is the absence of a constructive limitation of the angular displacement of the separator, which can be the cause of the destruction of the BBS. To prevent this, the rotor has stops, but when it comes into contact with them, the dynamics of its work is distorted. Therefore, it is advisable to theoretically predict the maximum rotation angles of the separator for the given operating conditions of the BBS.

2. Formulation of the problem
The task is to determine the kinematic parameters of the separator and balls motion relative to the supporting members of the BBS and to analyze the trajectories of the balls on the surfaces of the supporting members for different ratios between their angular velocities of rotation.

3. Theory
The coordinate systems that determine the position of gyroscope elements are shown in figure 2. A coordinate system $xyz$ is associated with the framework, a system $x_ry_rz_r$ is associated with the rotor and the BSM. The position of the coordinate system $x_ry_rz_r$ relative to $xyz$ is determined by the angles $\alpha$, $\beta$, $\varphi$. The position of the coordinate system $x_sz_sz_s$ associated with the separator is determined by the angles $\psi$, $\vartheta$, $\gamma$.

The position of the center of mass of the $i$-th ball ($i=1, ..., n$) on the separator is determined by coordinates $\vec{OO}_i = (R_i, \alpha_i, \beta_i)$. Each radius vector $\vec{OO}_i$ is associated with a coordinate system $\hat{x}_i\hat{y}_i\hat{z}_i$, the position of which relative to the coordinate system $x_5y_sz_s$ is determined by the angles $\alpha_i$, $\beta_i$. The coordinate systems $\hat{x}_i\hat{y}_i\hat{z}_i$ move with the separator. The coordinate system $x_iy_iz_i$ is associated with the $i$-th ball, its position relative to the coordinate system $\hat{x}_i\hat{y}_i\hat{z}_i$ is determined by the angles $\psi_i$, $\vartheta_i$, $\gamma_i$.

Relative positions of the coordinate systems can be specified using the direction cosine matrices [5]. These matrices define the transition from one coordinate system to another:

$$
x_{xyz} \rightarrow x_ry_rz_r \rightarrow x_sy_sz_s \rightarrow \hat{x}_i\hat{y}_i\hat{z}_i \rightarrow x_iy_iz_i.
$$
It is assumed that the axis of proper rotation of the rotor coincides with the main axis of inertia of the BSM. Then, for the projections of the vector of angular velocity $\omega_r$ of the rotor in the coordinate system, which is formed by the main axes of inertia of the BSM, the following relationships hold:

$$\omega_{sx} = \omega_{smx} = \omega_x, \quad \omega_{sy} = \omega_{sym} = \omega_y, \quad \omega_{sz} = \omega_{smez} = \Phi, \quad \omega_{sz} = \Phi = \Omega,$$

(1)

where the index "sm" refers to the BSM, the index "r" refers to the rotor, $\omega_{smx}$, $\omega_{sy}$ are the angular velocities of the proper rotation of the BSM and of the rotor rotation respectively.

(a) \hspace{1cm} (b) \hspace{1cm} (c)

Figure 2. Coordinate systems: (a) – for the rotor, (b) – for the separator, (c) – for the ball.

In the case of a rigid conjunction between the rotor and the BSM

$$\omega_{sx}^2 = \omega_{sy}^2 = \Phi = \Omega.$$

The projections of the angular velocity of the rotor on the axes of the coordinate system $x_ry_rz_r$ and the projection of the angular velocity of the separator $\omega_{smez}$, relative to the BSM, on the axes of the coordinate systems $x_sy_sz_s$ and $x_ry_rz_r$ are determined by the following expressions:
The equations of nonholonomic constraints in the suspension [6], provided that a slipping of the balls is absent, are:

\[ \begin{align*}
\omega_i^x &= \alpha \cos \beta \sin \varphi + \beta \cos \varphi; \\
\omega_i^y &= \alpha \cos \beta \cos \varphi - \beta \sin \varphi; \\
\omega_i^z &= \dot{\theta} - \alpha \sin \beta,
\end{align*} \tag{2} \]

\[ \begin{align*}
\omega_{i\text{sm}}^x &= \dot{\theta} \cos \psi \cos \gamma + \psi \sin \gamma; \\
\omega_{i\text{sm}}^y &= -\dot{\theta} \cos \psi \sin \gamma + \psi \cos \gamma; \\
\omega_{i\text{sm}}^z &= \dot{\gamma} + \dot{\theta} \sin \psi,
\end{align*} \tag{3} \]

\[ \begin{align*}
\omega_{i\text{sm}}^{x_i} &= \dot{\gamma} + \dot{\theta} \sin \psi; \\
\omega_{i\text{sm}}^{y_i} &= \psi \cos \delta - \dot{\gamma} \sin \delta \cos \psi; \\
\omega_{i\text{sm}}^{z_i} &= \psi \sin \gamma + \dot{\gamma} \cos \delta \cos \psi. \tag{4} \end{align*} \]

The equations of nonholonomic constraints in the suspension [6], provided that a slipping of the balls is absent, are:

\[ \begin{align*}
\tilde{\omega} \times \overrightarrow{O_i A_i} &= \tilde{V}_{oi} + \tilde{\omega}_{bi} \times \overrightarrow{O_i A_i}; \tag{5} \\
\tilde{\omega}_o \times \overrightarrow{O_i B_i} &= \tilde{V}_{oi} - \tilde{\omega}_{bi} \times \overrightarrow{O_i A_i}, \tag{6} \\
\end{align*} \]

where $\tilde{V}_{oi}$ is a vector of the velocity of the center of mass of the $i$-th ball, $\tilde{\omega}_{bi}$ is a vector of the absolute angular velocity of the $i$-th ball, $\tilde{\omega}_o$ is a vector of the absolute angular velocity of the supporting member.

Vectors $\tilde{V}_{oi}$ and $\tilde{\omega}_{bi}$ are determined by the following expressions:

\[ \begin{align*}
\tilde{V}_{oi} &= (\tilde{\omega} + \tilde{\omega}_{i\text{sm}}) \times \overrightarrow{O_i O_i} ; \\
\tilde{\omega}_{bi} &= \tilde{\omega} + \tilde{\omega}_{i\text{sm}} + \tilde{\omega}_{i\text{bis}}, \tag{7} \end{align*} \]

where $\tilde{\omega}_{i\text{bis}}$ is a vector of the angular velocity of the $i$-th ball relative to the separator.

The projections of the angular velocity $\tilde{\omega}_{i\text{bis}}$ on the axes of the coordinate system $\tilde{x}_i, \tilde{y}_i, \tilde{z}_i$ are as follows:

\[ \begin{align*}
\omega_{i\text{bis}}^{x_i} &= \dot{\vartheta}_i + \dot{\gamma}_i \sin \psi_i; \\
\omega_{i\text{bis}}^{y_i} &= \psi_i \cos \vartheta_i - \dot{\gamma}_i \sin \vartheta_i \cos \psi_i; \\
\omega_{i\text{bis}}^{z_i} &= \psi_i \sin \vartheta_i + \dot{\gamma}_i \cos \vartheta_i \cos \psi_i. \tag{9} \end{align*} \]

The projections of the angular velocity $\tilde{\omega}_o$ on the axis of the coordinate system $xyz$ are as follows:

\[ \begin{align*}
\omega_o^x &= \psi_{\text{sm}} + \dot{\varphi}_{\text{sm}} \cos \vartheta_{\text{sm}}; \\
\omega_o^y &= \dot{\varphi}_{\text{sm}} \sin \vartheta_{\text{sm}} \sin \psi_{\text{sm}} + \dot{\vartheta}_{\text{sm}} \cos \psi_{\text{sm}}; \\
\omega_o^z &= -\varphi_{\text{sm}} \sin \vartheta_{\text{sm}} \cos \psi_{\text{sm}} + \dot{\vartheta}_{\text{sm}} \sin \psi_{\text{sm}}, \tag{10} \end{align*} \]

where $\psi_{\text{sm}}, \vartheta_{\text{sm}}, \gamma_{\text{sm}}$ are the Euler angles defining the position of the supporting member.

The projections of the angular velocity of the supporting member on the axis of the coordinate system $x, y, z$ are determined in accordance with the transformation
where $A$ is the direction cosine matrix that determines the transition from the coordinate system $xyz$ to the coordinate system $x_iy_iz_i$.

Equations (5), (6), taking into account (7) and (8), can be transformed to the following form:

$$
\omega_{\text{sm}} = \frac{(\omega - \omega) \cdot (R - \mathbf{r}_b)}{2R}, \tag{12}
$$

$$(2\mathbf{r}_b \cdot \omega) (R^2 - \mathbf{r}_b^2) \times \mathbf{e}_{z_i} = 0, \tag{13}
$$

where $\mathbf{r}_b$ is the radius of the ball, $\mathbf{e}_{z_i}$ is the unit vector along the axis $z_i$.

The vector equation (12) corresponds to three scalar constraint equations, and the vector equation (13) corresponds to the $2n$ constraint equations ($n$ – number of balls). Thus, equations (12), (13) are non-integrable equations that connect generalized velocities and, hence, the variations of generalized coordinates [6]. Taking into account (4) and (9), these $2n + 3$ equations have the following form:

$$
\begin{align*}
\dot{\vartheta} + \dot{\gamma} \sin \psi & = \frac{r_{\text{in}}}{2R} (\omega^x_i - \omega^x_i) ; \\
\dot{\psi} \cos \vartheta - \dot{\gamma} \sin \vartheta \cos \psi & = \frac{r_{\text{in}}}{2R} (\omega^y_i - \omega^y_i) ; \\
\dot{\psi} \sin \vartheta + \dot{\gamma} \cos \vartheta \cos \psi & = \frac{r_{\text{in}}}{2R} (\omega^z_i - \omega^z_i),
\end{align*}
$$

$$
\begin{align*}
\dot{\vartheta} + \dot{\gamma} \sin \psi & = \frac{r_{\text{out}} r_{\text{in}}}{2R r_b} [b_1 (\omega^x_i - \omega^x_i) + b_2 (\omega^y_i - \omega^y_i) + b_3 (\omega^z_i - \omega^z_i)] ; \\
\dot{\psi} \cos \vartheta - \dot{\gamma} \sin \vartheta \cos \psi & = \frac{r_{\text{out}} r_{\text{in}}}{2R r_b} [b_4 (\omega^x_i - \omega^x_i) + b_5 (\omega^y_i - \omega^y_i) + b_6 (\omega^z_i - \omega^z_i)],
\end{align*}
$$

where $r_{\text{out}} = R + \mathbf{r}_b$, $r_{\text{in}} = R - \mathbf{r}_b$, $b_{k,l}$ are the elements of the direction cosine matrix defining a transition from the coordinate system $x_iy_iz_i$ to the coordinate system $x_iy_iz_i$.

Equations (14), (15) define the kinematics of the separator and of the $i$-th ball.

The parameters of the trajectories of the centers of mass of the balls can be estimated under the following assumptions:

- there is no proper rotation of the rotor by the angles $\alpha$ and $\beta$, that is $\alpha = \alpha_0$, $\beta = \beta_0$;

- angular velocity of the proper rotation of the rotor is constant ($\omega^\phi_0 = \phi = \Omega_0$);

- angular velocity of the forced rotation of the supporting member about the axis $x$ is constant, that is $\omega^z_0 = \omega_0$.

The nonholonomic constraints equations, when angles $\psi$ and $\vartheta$ are small, take the following form:

$$
\begin{align*}
\dot{\vartheta} + \dot{\gamma} \psi & = \bar{R} \omega_0 (\beta_0 \sin \varphi - \alpha_0 \cos \varphi) ; \\
\dot{\psi} - \dot{\gamma} \vartheta & = \bar{R} (\beta_0 \cos \varphi + \alpha_0 \sin \varphi) ; \\
\dot{\gamma} & = \bar{R} (\omega_0 - \Omega_0),
\end{align*}
$$

where $\bar{R} = \frac{r_{\text{in}}}{2R}$, $\varphi = \Omega_0 t$.
The solution of system (16) for the initial conditions \( \psi(0) = \psi_0, \ \vartheta(0) = \vartheta_0 \) is:

\[
\begin{align*}
\psi(t) &= \frac{\mathcal{R}_0}{\Omega_0 + \gamma} \bigl( \beta_0(\sin \varphi + \sin \gamma) + \alpha_0(\cos \gamma - \cos \varphi) \bigr) + \vartheta_0 \sin \gamma + \psi_0 \cos \gamma; \\
\vartheta(t) &= \frac{\mathcal{R}_0}{\Omega_0 + \gamma} \bigl( \beta_0(\cos \gamma - \cos \varphi) - \alpha_0(\sin \varphi + \sin \gamma) \bigr) + \vartheta_0 \cos \gamma - \psi_0 \sin \gamma,
\end{align*}
\]

where \( \gamma = \mathcal{R}(\omega_0 - \Omega_0)t \).

The coordinates of the center of mass of each of the balls in the coordinate system \( \mathbf{r}_i \) for small angles \( \psi \) and \( \vartheta \) are determined by the equations:

\[
\begin{align*}
x_{br} &= x_{bs} \cos \gamma - y_{bs} \sin \gamma + z_{bs} \psi; \\
y_{br} &= x_{bs} \sin \gamma + y_{bs} \cos \gamma - z_{bs} \vartheta; \\
z_{br} &= x_{bs}(\vartheta \sin \gamma - \psi \cos \gamma) + y_{bs}(\vartheta \cos \gamma + \psi \sin \gamma) + z_{bs},
\end{align*}
\]

where \( x_{bs}, y_{bs}, z_{bs} \) are the coordinates of the center of mass of the \( i \)-th ball in the coordinate system \( \mathbf{r}_s \).

Equations (17), (18) make it possible to obtain the characteristics of the motion of the centers of mass of the balls, or, taking into account that the projections of these trajectories onto the bearing surface are the traces of movement of the balls, the characteristics of the traces – the number and amplitude of oscillations. Obviously, the numerical values of the characteristics of the traces of the balls, which are in one row of the separator, are the same.

4. Example of calculation

Since the parameters of the trajectories of the centers of mass of all balls are the same, let us set \( i = 1 \) and choose, for example, \( \alpha_0 = 30^\circ \), which corresponds to the following coordinates of the center of mass of the ball in the system \( \mathbf{r}_s \):

\[
x_{bs} = \frac{1}{2} R; \ y_{bs} = 0; \ z_{bs} = \frac{\sqrt{3}}{2} R.
\]

We further use the solution (17), substitute it into equation (18), and as a result we obtain:

\[
\begin{align*}
x_{br} &= \frac{R}{2} \bigl[ \cos \gamma + \sqrt{3} \bigl( \frac{\mathcal{R}_0}{\Omega_0 + \gamma} \bigl( \beta_0(\sin \varphi + \sin \gamma) + \alpha_0(\cos \gamma - \cos \varphi) \bigr) + \vartheta_0 \sin \gamma + \psi_0 \cos \gamma \bigr) \bigr]; \ (19) \\
y_{br} &= \frac{R}{2} \bigl[ \sin \gamma - \sqrt{3} \bigl( \frac{\mathcal{R}_0}{\Omega_0 + \gamma} \bigl( \beta_0(\cos \gamma - \cos \varphi) - \alpha_0(\sin \varphi + \sin \gamma) \bigr) + \vartheta_0 \cos \gamma - \psi_0 \sin \gamma \bigr) \bigr]; \ (20) \\
z_{br} &= \frac{R}{2} \bigl[ \sqrt{3} - \frac{\mathcal{R}_0}{\Omega_0 + \gamma} \bigl( \beta_0(\sin \varphi + \vartheta) + \alpha_0(1 - \cos(\varphi + \vartheta)) \bigr) - \psi_0 \bigr]. \ (21)
\end{align*}
\]

Let consider three variants of movement:

1) there is no forced rotation of the supporting member ( \( \omega_0 = 0 \));
2) angular velocities of rotation of the rotor and of the supporting member are equal ( \( \Omega_0 = \omega_0 \));
3) proper angular velocities of rotation of the rotor and of the supporting member are different ( \( \Omega_0 \neq \omega_0 \)).

For the first variant of movement, the trajectory of the center of mass of the ball, as well as the trace of the ball, is defined by the equation of a circle (figure 3)

\[
x_{br}^2 + y_{br}^2 = \frac{R^2}{4} \left( 1 + \sqrt{3} \psi_0 \right)^2 + 3 \delta_0^2
\]

centered on the axis \( z_r \) at a distance.
from the origin. The traces of all balls are parallel, and their developments on the plane is a family of parallel straight lines. This mode of movement is unfavorable for the operation of the BBS due to increased wear of the bearing surfaces.

For the second variant of movement, equations (19), (20), (21) take the following form:

\[
\begin{align*}
x_{br} &= \frac{R}{2}[1 + \sqrt{3}(\bar{R}(\beta_0 \sin \varphi + \alpha_0(1 - \cos \varphi)) + \psi_0 \cos \gamma)]; \\
y_{br} &= -\frac{\sqrt{3}R}{2}[\bar{R}(\beta_0(1 - \cos \varphi) - \alpha_0 \sin \varphi) + \vartheta_0]; \\
z_{br} &= \frac{R}{2}[\sqrt{3} - \bar{R}(\beta_0 \sin \varphi + \alpha_0(1 - \cos \varphi)) - \psi_0].
\end{align*}
\]

From system of equations (22), it follows that the trajectories of the centers of mass of the balls are ellipses, the semiaxes of which are proportional to the angles \(\alpha_0, \beta_0\), and the position on the bearing surface is determined by the angles \(\psi_0, \vartheta_0\).

For the actual dimensions of elements of the BBS, the trajectories of the traces of the balls are located in the vicinity of the points of their contact with the bearing surfaces. This mode of movement is unfavorable for the operation of the BBS due to the increased wear of the contacting elements.

For the third variant of the movement, the character of the trajectories of the centers of mass of the balls and their traces depend on the ratio between the angular velocities \(\Omega_0\) and \(\omega_0\) and on the value of \(\bar{R}\).

Depending on the ratio between the velocities \(\Omega_0\) and \(\omega_0\), closed and open trajectories of the centers of mass of the balls and their traces are possible. Obviously, open traces are preferable to reduce the wear of the suspension elements, since in this case the trace of each ball is not repeated during subsequent revolutions of the separator.

Let determine the number of oscillations of the center of mass and the trace of the ball and the amplitude of oscillations for a complete revolution of the separator. In accordance with (19), (20), (21), the oscillation period is

\[
T = \frac{2\pi}{\frac{d}{dt}(\varphi + \gamma)} = \frac{2\pi}{\Omega_0(1 - \frac{\Omega_0 - \omega_0}{\bar{R}})}.
\]

the period of complete revolution of the separator is determined by the expression

\[
\text{Figure 3. Trajectory of motion of the center of mass of the ball when } \omega_0 = 0: 1 - \text{bearing surface, } 2 - \text{ball, } 3 - \text{trajectory.}\
\]
$$T_c = \frac{2\pi}{\Omega_0 (1 - \frac{\omega_0}{\Omega_0})}.$$

Consequently, the number of oscillations of the separator, that is the center of mass and the trace of the ball, is

$$N = \frac{T_c}{T} = \frac{1 - \bar{R} (1 - \frac{\omega_0}{\Omega_0})}{\bar{R} (1 - \frac{\omega_0}{\Omega_0})} - 1. \quad (23)$$

In accordance with (23), if $\omega_0 / \Omega_0 = 1/2$ and $\bar{R} = 1/3$ the number of oscillations is 5, if $\omega_0 / \Omega_0 = 1/3$ and $\bar{R} = 1/3$ the number of oscillations is 3.5.

To determine the amplitude of oscillations of the centers of mass of the balls (traces on the bearing surfaces), we use figure 4. If angles of oscillation are small, a length of the arc can be replaced by a length of the line. The length of the latter will be

$$A = \left[2 (R^2 - (R^2 - z_{br1}^2)^{1/2})^2 - (z_{br2}^2)^{1/2} - z_{br1} - z_{br2}\right]^{1/2}. \quad (24)$$

Figure 4. Determination of the amplitude of oscillations of the center of mass of the ball: 1, 2 – positions of the ball after half of the oscillation period.

Values of $z_{br1}$ and $z_{br2}$ are determined using (21) for the angles $\gamma$ corresponding to the maximum and minimum values of $z_{br}$. The values of amplitudes of the traces of the balls on the bearing surfaces can be calculated according to (24) by replacing $R$ with $r_{in}$ or $r_{out}$.

Figure 5 shows the developments of trajectories of the center of mass of the ball for the following initial conditions and two variants of values of the parameters and angular velocities of the elements of the BBS:

$$\alpha_0 = 3^\circ, \ \beta_0 = 2^\circ, \ \psi(0) = \delta(0) = \gamma(0) = 0^\circ, \ \bar{R}_1 = 0.55, \ \bar{R}_2 = 0.5, \ \bar{R}_3 = 1/3, \ \bar{R}_4 = 1/3,$$

where index "1" corresponds to the first variant, index "2" – to the second variant.

To reduce the wear of surfaces of elements of the BBS, the number of oscillations of the center of mass should not be an integer, but especially a multiple of the number of balls in a row. Otherwise, the trajectories will repeat.
5. Results
Assuming the absence of slipping of the balls, the equations of nonholonomic constraints between the elements of a spherical BBS, converted into a scalar form, determine the kinematic equations of motion of the separator and the balls along the bearing surfaces. The solutions of these equations determine the angular coordinates of the separator and the parameters of the traces of the balls on the surfaces of the supporting members for various ratios of their angular velocities of rotation.

6. The discussion of the results
The results obtained make it possible to determine the amplitude of angular oscillations of the separator and, at the stage of designing of a gyroscope with a BBS, to establish the maximum angle of rotation of the separator, relative to the supporting member, at which the balls maintain contact with the latter. There are such ratios between the angular velocities of rotation of the inner and outer supporting members, at which the trajectories of the traces of the balls are open, which reduces the wear of the bearing surfaces.

7. Conclusion
The performance of a spherical BBS depends on the kinematics of its elements – separator and balls.
Provided that rotation angles of the separator are small and a drift of the gyroscope (constant increase of the angle of deviation of the rotor from its initial position) is absent, the nonholonomic constraints equations in scalar form take the form of integrable differential equations.

The analytical solution of the geometric differential constraints equations makes it possible to determine the limiting angles of rotation of the separator and the trajectory of motion of the center of mass of the balls, therefore, their traces on the surfaces of the supporting members.

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