Effect of wheel polygonalization on the Degree of Non-linearity of dynamic response of high-speed vehicle system

Chen Shuangxi and Ni Yanting

Abstract
Polygonalization of the wheel describes the growth of out-of-round profiles of the wheels of railway vehicle. This problem was identified in the 1980s but its mechanism is still not well understood. The wheel-rail disturbance formed by wheel polygonalization will accelerate the fatigue fracture of the key parts of rail vehicles and seriously threaten the safety of rail vehicle. This fact has led to significant efforts in detecting and diagnosing wheel polygonalization, in particular in setting the criteria for health monitoring. Currently, the time-domain feature parameters extraction method based on data statistics and frequency-domain feature parameters extraction method based on spectrum estimation are widely applied to detect wheel polygonalization. However, the basis of spectral estimation is the Fourier transform, which is not good at dealing with non-linear vibration systems (such as vehicle-track coupled system). Aiming at the wheel polygonalization problem existing in high-speed train, the non-linear extent of vibration response of vehicle system caused by wheel polygonalization is analyzed based on vehicle-track coupled dynamics and adaptive data analysis method. A typical high-speed train model is established according to the vehicle-track coupled dynamics theory. The wheel polygonalization model is introduced and vehicle system vibration response is calculated by numerical integration. The vibration response signal is decomposed by empirical mode decomposition (EMD) to produce the intrinsic mode functions (IMFs). By calculating the intra-wave frequency modulation of IMFs, that is, the difference between instantaneous and mean frequencies and amplitudes, the non-linearity of the dynamic response is quantified. Influences of wheel polygonalization on the non-linearity of steady-state and unsteady vibration responses of vehicle system are analyzed in detail. An objective criterion for wheel polygonalization health monitoring based on Degree of Non-linearity is proposed, which provides an effective tool for prognostics and health management of trains.

Keywords
Coupled vehicle-track model, wheel polygonalization, vibration response, Degree of Non-linearity, health monitoring

Introduction
As one of the most important parts of railway vehicle, the wheel ensures the running and guiding of the train by connecting the train to the track. It carries the load of the vehicle and applied it to the track. During the service life of the train, the wheel wear is inevitable and predictable, and the wheel is no longer completely round, resulting in unwanted wheel irregularity excitation. Among wheel irregularities, wheel polygonalization is a common kind of wheel out-of-roundness. It is the non-uniform wear in the circumferential direction of the wheel: the radius of the wheel varies periodically along the circumference, and the polygonalization extends to the whole circumferential surface at a certain wavelength.

Kaper first found the phenomenon of wheel polygonalization on the Dutch railway, and later on the ICE high-speed train in Germany and the Stockholm subway in Sweden. Wheel polygonalization of rail vehicle will lead to significant changes of wheel-rail vertical force, lateral force, derailment coefficient, wheel weight reduction rate, and other indexes, which will greatly affect the stability and curve passing performance of the vehicle. Nielsen and Johansson discussed the state-of-the-art in research on experimental detection of wheel/rail impact loads, mathematical models to predict the development and consequences of out-of-round...
wheels, criteria for removal of out-of-round wheels and suggestions on how to reduce the development of out-of-round wheels. Barks and Chiu\textsuperscript{7} gave a review of the effects of out-of-round wheels on track and vehicle components. One of the most dangerous situations that may be caused by wheel out-of-roundness is hunting instability, which can greatly increase wheel-rail forces, accelerate wheel-rail wear, and produce harmful noise. The effect of wheel ovalization on the stability of high-speed trains has been discussed by Zhang et al.\textsuperscript{3} The results show that the hunting motion of the wheelset caused by the wheel ovalization has a great relationship with the combinations of the wheel ovalization extent, the vehicle speed, and the phase angle.

In recent years, the wheel polygonalization problem has been frequently encountered by China’s high-speed trains, causing high-frequency vibrations and wheel-rail impacts. The field test results show that the vibration acceleration of the wheelset reaches 300 g and the vertical and transverse vibration accelerations of the gear-box exceeds 50 g (more than 10 times the normal value). Consequently, the noise in the carriages of the train increases by nearly 10 dB. The wheel-rail interaction caused by wheel polygonization accelerates the fatigue of key components of the railway vehicle, leading to the damage of gearbox cracking and axle box cover bolt breaking, which seriously threatens the safety of the train. For example, in the tracking test of CRH3 high-speed train, Chinese engineers found that the wheel polygonalization phenomenon was prevalent in the operation of this type of train on the Wuhan-Guangzhou line in South China. The detected vibration acceleration of the axle box increased exponentially with the increase of the polygonalization amplitude, and the out-of-limit loosening and the fracture of the bolts of axle box cover were also observed.

With the continuous increase of the running speed and mileage of high-speed trains in China, the wheel polygonalization has become one of the key problems to be solved urgently in today’s railway industry. Because the smoothness of wheel-rail contact directly affects the stability and safety of the vehicle and the service life of the vehicle and track components, the health condition of the wheel is very important for the operation of the vehicle. At present, most of the polygonalization problems are solved by wheel repairing or replacement, which makes cost of maintenance high and do not solve the problem completely. Therefore, it is necessary to study the mechanism of wheel polygonalization in order to predict damages.

The formation mechanism of wheel polygonalization and its effects on vehicle dynamic performance have been studied for decades, but there are still differences on its formation mechanism. There have been some achievements\textsuperscript{6–8} in the study of the relationship between wheel-axle resonance and wheel polygonalization, but the formation of high-order polygonalization has not been explained scientifically and effectively by the theory of wheel-axle resonance. Starting with the track characteristics, Meinke and Meinke\textsuperscript{9} and others\textsuperscript{10–12} explored the response of the rail under external excitation and the connection between local modes and wheel polygonalization, which is of interest regarding the causes of high-order wheel polygonalization. However, there are differences in structures and parameters between different tracks. Therefore, influence of track characteristics on the formation of wheel polygonalization remains to study. Additionally, Wu et al.\textsuperscript{13} and Cai et al.\textsuperscript{14} applied the wheel-rail friction vibration theory to the study of wheel polygonalization with good research value.

As the mechanism of wheel polygonalization has not yet been completely understood, it is particularly important to identify, avoid or restrain the phenomenon. Li et al.\textsuperscript{15} gave an overview of existing methodologies, theories, and applications applied in the data measurement and analysis systems to detect wheel defects in railway on-board health monitoring systems. Zampieri et al.\textsuperscript{16} developed an on-board monitoring system installed on railway vehicles, which can detect malfunctions of several critical components by measuring and processing the signals detected on the axle-box, and on the bogie. Bosso et al.\textsuperscript{17} designed and tested an on-board monitoring system installed on railway vehicles, which can detect anomalies of the running behavior of vehicles and faults at the component level. Sun et al.\textsuperscript{18} proposed a detection framework based on the angle domain synchronous averaging technique, which can be used to monitor the conditions of axle-box bearings.

In on-board monitoring system, the time domain, and frequency domain analysis methods have been widely used to detect wheel polygonalization. In frequency domain, the feature parameters are usually gained based on the power spectral density of vibration acceleration. Unfortunately, the basis of power spectral density is Fourier transform, which is not an adaptive data analysis method and is not good at extracting nonlinear feature parameters. Since the vehicle-track coupled system is a dynamic system with a certain extent of non-linearity, does wheel polygonalization affect the nonlinear degree of vehicle system? When the hunting motion of the train caused by polygonalization is unstable, what is the extent of non-linearity of the system? Can the non-linearity index be used to detect wheel polygonalization? With the help of Huang’s\textsuperscript{19–21} works, the nonlinear degree of the vehicle system can be quantified and these questions have been answered in this study. Based on the adaptive signal processing method and the vehicle-track coupled system dynamics, this work focuses on the relationship between wheel polygonalization and the non-linearity of vehicle system vibration response, and puts forward the objective criterion for wheel polygonalization health monitoring.
Definition of the Degree of Non-linearity of dynamic response

Wheel-rail contact is the main difference between vehicle-track coupled system and other mechanical systems. The wheel-rail contact is characterized by obvious non-linearity, including nonlinear contact geometry, nonlinear creep force, and Hertz nonlinear normal force. The geometric relationship of wheel-rail contact is generally described by the contact parameters such as wheel-rail curvature, contact angle, and rolling angle at the wheel-rail contact point. In addition, the mechanical properties of anti-yaw damper and transverse backstop installed on rail vehicles are also nonlinear. During the service of the train, the wheel-rail wear is inevitable and unpredictable, and the contact parameters change nonlinearly. When the hunting motion of the vehicle is unstable, the wheel flange may contact with the rail, the contact angle becomes larger, and the contact geometry relationship will be highly nonlinear.5

The wheel-rail force consists of normal contact force and tangential creep force. The normal contact force is generally described by the nonlinear Hertz contact theory.23 The mathematical description of tangential creep force is more complex. Kalker22 described the relationship between creepage and wheel-rail creep force during wheel-rail rolling contact under low creepage conditions, and gave a specific numerical solution of creep. Shen et al.23 gave a nonlinear approximation of creep saturation.

Vehicle hunting motion is the essential attribute of vehicle-track coupled system and its stability is the key factor affecting running safety. According to the railway vehicle dynamics, there are two kinds of critical velocity in the railway vehicle system, namely, linear critical velocity and nonlinear critical velocity. The linear critical velocity can be obtained by the linearized mathematical model, which only exists in the ideal state with very small track excitation. In contrast, the nonlinear critical velocity is less than the linear critical velocity and is influenced by track conditions. In practical application, due to the influence of track excitation, the critical velocity of railway vehicles is always between nonlinear critical velocity and linear critical velocity. To ensure a high nonlinear critical velocity, anti-yaw dampers are usually installed between the car body and the bogie of high-speed train, and their nonlinear mechanical behavior is described by function of longitudinal relative velocity and damping force.

Because of the nonlinear characteristics, the vibration response of the vehicle system should also be nonlinear. According to the knowledge of sensor technology, linearity is an important index to describe the static characteristics of the sensor, which is based on the premise that the measured input is in a stable state. Under specified conditions, the percentage of the maximum deviation (between the sensor calibration curve and the fitting line) \( \Delta Y_{\text{max}} \) to the full range output \( Y \) is defined as the Degree of Linearity (or “Nonlinear error”). This index describes the extent of linearity of the quantitative relationship between the output and input of the sensor. In engineering application, the output depends on the initial conditions and loads, so this method is not practical. Since natural systems may not be clearly defined, outputs and inputs are difficult to determine, let alone quantify.

So how should non-linearity be defined? Huang et al.20 suggested to define non-linearity based on intra-wave frequency modulation (the deviation between instantaneous frequency and local average frequency). According to this recommendation, the first step in defining non-linearity is to correctly derive the instantaneous frequency (amplitude) and the local average frequency (amplitude). Gabor24 proposed a method to obtain the time-varying frequency from the signal by using the analytical signal, and defined the instantaneous frequency as the derivative of the phase of the analytical signal. The problem with this method is that the resulting instantaneous frequency may be wildly erratic and negative at times. To overcome the shortcomings of traditional time-frequency analysis methods, Huang et al.19 proposed a time-frequency analysis method called Hilbert-Huang transform (HHT), which consists of Empirical Mode Decomposition (EMD) and Hilbert transform. With the help of empirical mode decomposition, any complex data can be decomposed into a finite number of intrinsic mode functions (IMF) that allow Hilbert transform. The instantaneous frequency obtained by Hilbert transform of IMF corresponds to the fluctuation frequency that can be observed in the signal.

However, HHT needs to be improved from two key points. First, because Hilbert transform is limited by Bedrosian’s theorem and Nuttall’s theorem, instantaneous frequency should not be derived directly from Hilbert transform. It is recommended to use the “direct quadrature method” (DQ) and orthogonal Hilbert transform, which consists of Hilbert-Huang transform (HHT), which consists of Empirical Mode Decomposition (EMD) and Hilbert transform. With the help of empirical mode decomposition, any complex data can be decomposed into a finite number of intrinsic mode functions (IMF) that allow Hilbert transform. The instantaneous frequency obtained by Hilbert transform of IMF corresponds to the fluctuation frequency that can be observed in the signal.

The mean frequency of narrowband signal has a long history and the zero-crossing method is the basic method to calculate the mean frequency of signal. However, the result of this method is relatively crude because the defined frequency is constant over the zero-crossings period. Huang et al.20 proposed an improved generalized zero-crossing (GZC) method, which takes all zero-crossing points and local extrema as control points, and improves the time resolution to 1/4 wave period. The GZC frequency of each point along the time axis is defined as:

\[
\text{IF}_{\text{z}} = \frac{1}{12} \left( \frac{1}{T_1} + \sum_{j=1}^{2} \frac{1}{T_2} + \sum_{j=1}^{4} \frac{1}{T_4} \right)
\]  \( \text{(1)} \)
where $T_1$, $T_2$, and $T_4$ are the quarter, half and full period, respectively.

The GZC local mean amplitude at each point along the time axis is defined as:

$$A_{zc} = \frac{1}{12} \left( \frac{1}{A_1} + \sum_{j=1}^{2} \frac{1}{A_2^j} + \sum_{j=1}^{4} \frac{1}{A_4^j} \right)$$

where $A_1$, $A_2$, and $A_4$ are the amplitudes calculated according to $1/4$ wave, half wave and full wave, respectively.

According to the suggestion of Norden Huang, the nonlinear extent of the signal can be described according to the deviation between the instantaneous frequency and the local average frequency, that is, the Degree of Non-linearity (DN) of a single component can be defined as:

$$DN = \text{std} \left( \frac{IF - IF_{zc}}{IF_{zc}} \cdot A_{zc} \right)$$

where $IF$ and $IF_{zc}$ are instantaneous frequency and GZC frequency respectively, $A_{zc}$ is the GZC local mean amplitude and $\bar{A}_{zc}$ the mean value of $A_{zc}$.

This method provides an effective tool for quantifying the nonlinear degree of nonlinear models (such as Duffing model, Stokes model, Rossler model, and Lorenz model) and nonlinear systems (such as nonlinear mechanical systems and structural systems). It has been introduced into the signal analysis of vibration response of vehicle-track coupled system in Chen and Lin.

**Nonlinear model of coupled vehicle-track system**

According to the theory of vehicle-track coupled system dynamics, railway vehicle system and track system should be regarded as an overall large-scale system, and the wheel-rail relationship is the key link between the two subsystems, so that the essence of railway wheel-rail system can be reflected more objectively. Vehicle-track coupled dynamics includes vertical dynamics, transverse dynamics, longitudinal dynamics, and space coupled dynamics. In this study, a high-speed train-track space coupled system model is established, including Chinese CRH2 high-speed train sub-model and ballastless track sub-model, as illustrated in Figures 1 and 2. The nonlinear factors considered in vehicle system modeling include the nonlinear wheel rail contact...
force, the nonlinear stiffness of transverse backstop and the nonlinear damping force of anti-yaw damper. The established model is described as follows.

**Nonlinear vehicle-track model**

The considered train model is supported by two two-axle bogies at each end and each bogie is supported by two wheelsets. The car body, bogie, and wheelset each have five degrees of freedom (vertical, lateral, rolling, sloshing, and nodding motion), so the vehicle system is a lumped mass system with 35 degrees of freedom. The lumped mass system includes the mass and moment of inertia of the two bogies, and the unspring mass and moment of inertia of the four wheelsets. As shown in Figure 1, \( M_c, M_t, M_w \) are car body mass, bogie mass and wheelset mass, respectively; \( K_{px}, K_{py}, K_{pz} \) are stiffness of the primary suspension in the longitudinal, lateral and vertical direction, respectively; \( C_{px}, C_{py}, C_{pz} \) are damping of the primary suspension in the longitudinal, lateral and vertical direction, respectively; \( K_{sx}, K_{sy}, K_{sz} \) are stiffness of the secondary suspension in the longitudinal, lateral and vertical direction, respectively; \( C_{sx}, C_{sy}, C_{sz} \) are damping of the secondary suspension in the longitudinal, lateral and vertical direction, respectively. 

When the high-speed train is running at a relatively high speed on the track, even the small transverse movement of the vehicle may cause violent hunting movement, which greatly reduces the stability and safety of the train. In general, high-speed trains are equipped with anti-yaw dampers to slow down the hunting movement between the bogie and the car body. The main function of anti-yaw damper is to restrain the hunting movement of high-speed main to ensure the stability and safety of the train. The nonlinear longitudinal damping force of anti-yaw damper\(^{27} \) is written as

\[
F_{\text{sx}(L,R)}(t) = \begin{cases} \frac{F_{\text{xmax}} v_{\text{sx}(L,R)}}{v_0}, & v_{\text{sx}(L,R)} < v_0 \\ \text{sign}(v_{\text{sx}(L,R)}), & v_{\text{sx}(L,R)} \leq v_0 \end{cases}
\]

Where \( F_{\text{xmax}} \) is the max damping force of anti-yaw damper, \( v_0 \) is the unload velocity of anti-yaw damper.

**Nonlinear wheel-rail contact model**

According to the dynamics theory of vehicle-track coupled system, the interaction between the vehicle and track system is realized by wheel-rail forces compatibility at the wheel-rail interfaces. The wheel-rail force consists of normal contact force and tangential creep force. The normal contact force is described by the following nonlinear Hertz theory:

\[
N_{(L,R)}(t) = \begin{cases} \left( \frac{1}{2} \left( \delta Z_{(L,R)}(t) - z_{\text{min}}(t) \right) \right)^{1/2}, & \left( \delta Z_{(L,R)}(t) - U_w(t) \right) > 0 \\ 0, & \left( \delta Z_{(L,R)}(t) - U_w(t) \right) \leq 0 \end{cases}
\]

Where, \( \delta Z_{(L,R)}(t) \) is the elastic deformation of wheel compression in normal direction, \( z_{\text{min}}(t) \) is the wheel/rail irregularity. If \( \left( \delta Z_{(L,R)}(t) - U_w(t) \right) > 0 \), wheel and rail are in close contact with each other, and \( \left( \delta Z_{(L,R)}(t) - U_w(t) \right) \leq 0 \) stands for wheel-rail separation. \( G \) is the Hertzian wheel-rail contact coefficient.
Kalker described the relationship between wheel-rail creep force and creepage in rolling contact at low creepage, and proposed a detailed method for numerical solution of creep force. Shen et al. gave the following nonlinear approximate calculation formula of creep force saturation:

\[ F_s = \frac{m N(L, R) j}{F_0 S} + \frac{m N(L, R) j^2}{C_0/C_1 + C_2/C_3}, F_s \leq 3 \mu N(L, R) j \]

(6)

where \( F_s = (F_1^2 + F_2^2)^{1/2} \), \( \mu \) is the friction coefficient, \( F_1/F_2 \) is longitudinal/lateral creep force.

When a reduction coefficient is defined as

\[ e = \frac{F_s'}{F_s} \]

(7)

The nonlinear creep forces and spin moment are given by

\[ F_{\tau j} = e F_1, F_{\tau j} = e F_2, M_{wj} = e M_3 \]

(8)

where \( M_3 \) is the spin moment.

Therefore, the vertical force \( P_j \), lateral force \( Q_j \), vertical moment \( M_{\tau j} \) and lateral moment \( M_{\mu j} \) are expressed in terms of \( N_j, F_{\tau j}, F_{\mu j}, M_{wj} \), and \( \delta_j \) read

\[ P_j = N_j \cos \delta_j + F_{\tau j} \sin \delta_j, Q_j = -N_j \sin \delta_j + F_{\mu j} \cos \delta_j, M_{\tau j} = M_{wj} \cos \delta_j, M_{\mu j} = -M_{wj} \sin \delta_j \]

(9)

where \( \delta_j \) indicates the contact angle between wheelset \( j \) and the rail, \( n/\tau \) denotes the normal/tangential direction of wheel-rail contact point, as shown in Figure 3.

**Model of wheel polygonalization**

Wheel out-of-roundness includes local out-of-roundness and circumferential out-of-roundness. The local out-of-roundness is mainly characterized by flat scar and peeling, which is caused by braking thermal damage and rolling contact fatigue. Circumferential non-roundness is mainly characterized by wheel polygonalization, such as eccentricity, ovalization, triangulation, quadrilateral, and high-order polygonalization caused by wheel wear (or machining).

As shown in Figure 4, the irregularity function of wheel polygonalization is defined as

\[ z_{wj}(t) = R_d \left(1 - \cos \left(\frac{n_{nor} \theta_d(t) \pi}{180}\right)\right) \]

(10)

where \( R_d = (R_b - R_a)/2 \) is the radial deviation of single wheel, \( R_b \) (\( R_a \)) is the half of the short (long) axis of wheel polygonalization. \( \theta_d = \theta(t) - \theta_i(t) \) is the angle difference between \( \theta_i(t) \) and \( \theta_i(t) \), \( \theta_{nrt}(t) \) is the angle between the left (right) wheel rolling contact radius and the horizontal axis, \( n_{nor} \) is the order of wheel polygonalization, \( n_{nor} = 1, 2 \ldots \) for wheels eccentricity, ovality, etc.

**Numerical method and parameters**

The improved explicit numerical integration method proposed by Zhai and Cai is used to calculate the vibration response of the high-speed train track coupled model. When solving the dynamic equation, both initial and boundary conditions are set to zero. The proposed wheel polygonalization model is similar to the model used by the SIMPACK software, and results are in good agreement with those of Zhang et al. The effectiveness of the model of vehicle-track coupled dynamics and the improved explicit numerical integration method has been demonstrated by several studies. Parameters of train and track used in this work are given in Table 2.

**Results and discussion**

Whether the vibration response of a vehicle system is stable or not is related to the order \( n_{nor} \), radial
difference $R_d$ and angle difference $\theta_d$ of wheel polygonalization. Therefore, our analytical work involves two statements (or states or possibilities): stable dynamic response and unstable dynamic response.

**Analysis of stable dynamic response**

Theoretically, if $\theta_d = 0$, only vertical vibration of the vehicle system can be obviously observed when the train is running on a straight track without track excitation. However, if $\theta_d \neq 0$, transverse force of wheel axle and transverse displacement of wheelset is no longer zero, and the transverse vibration of the vehicle system can be easily observed. When the angle difference $\theta_d(t) = \frac{360}{n_{m}} - \theta_d(t)$, the irregularity function of wheel polygonalization is

$$z_{\theta_d}(t) = R_d \left( 1 - \cos \left( \frac{\theta_d(t)}{180} \pi \right) \right)$$

Obviously, the value of the irregularity function when the angle difference $\frac{360}{n_{m}} - \theta_d(t)$ is equal to that of the function when the angle difference is $\theta_d$. In other words, The function is distributed symmetrically with respect to $\theta_d(t) = \frac{180}{n_{m}}$. As shown in Figure 5, five irregularity function curves of wheel polygonalization with $R_d = 1.5 \text{ mm}$ and $\theta_d$ in the range of $0 - 360$ degrees, all of which are symmetrically distributed with respect to $\theta_d(t) = \frac{180}{n_{m}}$. Therefore, the angle difference $\theta_d$ considered in the range of $0 - 180$ degrees is representative enough.

The first-order polygonalization of wheels, wheels eccentricity, is studied first. The hypothesis is made that the eccentricity exists only on the wheelset $w_1$, while the other wheelsets are completely round. The vehicle runs at $350 \text{ km/h}$ speed on a straight track with track excitation, and the data of the track excitation model are derived from the measured track irregularity spectrum of Beijing-Tianjin high-speed railway. Figure 6 shows the vertical vibration acceleration of the wheelset $w_1$ considering track irregularity and wheel eccentricity ($R_d = 0.5 \text{ mm}$). The track excitation has a significant influence on the calculation results of the vehicle dynamics model. The blue curve is the vibration acceleration of the polygon wheel, and the red curve is that of ideal round wheel. The vertical vibration curve of the polygon wheel shows remarkable periodic characteristics, and its maximum value is 1.4 times that of the ideal round wheel.

### Table 2. Parameters of vehicle-track system.

| Parameters | Value | Parameters | Value | Parameters | Value |
|------------|-------|------------|-------|------------|-------|
| $M_c(kg)$  | 396,000 | $K_{px}(MN \cdot m^{-1})$ | 14.7 | $m_c(kg \cdot m^{-1})$ | 60 |
| $M_s(kg)$  | 3200  | $K_{py}(MN \cdot m^{-1})$ | 6.5 | $E(N \cdot m^{-1})$ | $2.1 \times 10^{11}$ |
| $M_s(kg)$  | 2100  | $K_{pz}(MN \cdot m^{-1})$ | 1.176 | $I_z(m^3)$ | $3.217 \times 10^{-5}$ |
| $l_{cx}(kg \cdot m^2)$ | 128,000 | $C_{xin}(kN \cdot s \cdot m^{-1})$ | 20 | $l_z(m^3)$ | $5.24 \times 10^{-6}$ |
| $l_{cy}(kg \cdot m^2)$ | 194,000 | $K_{px}(MN \cdot m^{-1})$ | 0.2 | $K_{dx}(MN \cdot m^{-1})$ | 60 |
| $l_{cz}(kg \cdot m^2)$ | 167,000 | $K_{py}(MN \cdot m^{-1})$ | 0.2 | $K_{dy}(MN \cdot m^{-1})$ | 30 |
| $l_{sx}(kg \cdot m^2)$ | 2600 | $K_{pz}(MN \cdot m^{-1})$ | 1.145 | $K_{dz}(MN \cdot m^{-1})$ | 70 |
| $l_{sy}(kg \cdot m^2)$ | 1750 | $C_{yn}(kN \cdot s \cdot m^{-1})$ | 60 | $C_{pz}(kN \cdot s \cdot m^{-1})$ | 195 |
| $l_{sz}(kg \cdot m^2)$ | 3200 | $C_{yx}(kN \cdot s \cdot m^{-1})$ | 10 | $C_{zx}(kN \cdot s \cdot m^{-1})$ | 75 |
| $l_{wx}(kg \cdot m^2)$ | 720 | $l_x(m)$ | 8.75 | $F_{x_{max}}(kN \cdot s \cdot m^{-1})$ | 60 |
| $l_{wy}(kg \cdot m^2)$ | 80 | $l_y(m)$ | 1.25 | $C_{wy}(kN \cdot s \cdot m^{-1})$ | 100 |
| $l_{wz}(kg \cdot m^2)$ | 980 | $l_z(m)$ | 1.25 | $C_{wy}(kN \cdot s \cdot m^{-1})$ | 100 |
Figure 7 shows the curves of the vibration response of the wheelset \( w_1 \) and the evaluation index of vehicle dynamic performance changing with \( R_d \) at a speed of 350 km/h. When \( R_d \) increases from 0.2 to 1.0 mm, wheelset vertical acceleration increases from 4.7 g \((g = 9.8 m/s^2)\) to 29 g, wheel-rail vertical force from 143 to 566 kN, derailment coefficient \( Y_Q \) from 0.17 to 0.32 and wheel raise \( D_Z \) from 0.3 to 0.55 mm. When \( R_d \) is > 1.0 mm, the wheel-rail impact force increases rapidly, which leads to rapid increase of the vibration acceleration. When \( R_d \) is > 1.2 mm, the derailment coefficient \( Y_Q \) is > 1.0, and the wheel raise \( D_Z \) is > 3 mm, the trend of wheel derailment is significant. It should be noted out that these dynamic responses vary nonlinearly with \( R_d \), and the nonlinear change is significant when the wheels impact the rails at high frequency.

Figures 8 to 10 show the maximum vertical acceleration of wheelset, the maximum transverse acceleration of bogie and the maximum transverse displacement of wheelset \( w_1 \) corresponding to different angle differences \( \theta_d (R_d = 0.5, 1.0, 1.5 \text{mm}) \). While the vertical acceleration of the wheelset decreases with the increase of \( \theta_d \), the transverse acceleration of the bogie increases with the increase of \( \theta_d \). As shown in Figure 10, when \( R_d = 0.5 \) and 1.0 mm, the transverse displacement of the wheel is relatively small, and the vehicle movement is stable. When \( R_d = 1.5 \) mm, the maximum transverse displacement of the wheel exceeds 15 mm and the motion of vehicle may become unstable. Figure 11 shows the time domain response curve of the corresponding transverse displacement of the wheelset influenced by \( \theta_d \). When \( \theta_d = 30^\circ, 60^\circ, 90^\circ \), the vibration response of the wheelset is convergent after a period of track excitation, but at \( \theta_d = 120^\circ, 150^\circ, 180^\circ \), the vibration response curve no longer converges and the hunting motion of the vehicle becomes unstable. In fact, when the hunting motion is unstable, the extent of non-linear vibration of vehicle system is further intensified and the risk of derailment becomes high.

As mentioned in section 2, the first step to analyze the non-linearity of vibration is to decompose the
A dynamic response by using EMD (or EEMD) to yield IMF components. For brevity, detailed decomposition is omitted here. The instantaneous frequency and amplitude of IMFs can be deduced by DQ method, to construct the time-frequency-amplitude spectrum of IMFs. Typical local instantaneous spectrum of the vertical vibration acceleration of the wheelset $w_1$ is shown in Figure 12. If the train is assumed to be running on a straight unexcited track, the feature frequency of wheels eccentricity is stable at 34 Hz, as shown in Figure 12(a). When the excitation of track irregularity is taken into account in the model, the wheels eccentricity feature frequency fluctuates in a large range, which implies a significant non-linearity of the vibration response, as shown in Figure 12(b). Therefore, the influence of track irregularity should be considered in the non-linearity analysis.

Figure 13 shows the DN of the vertical vibration acceleration of the wheelset corresponding to $R_d$ of wheels eccentricity. Figures 15 and 16 show the DN of the transverse acceleration of the bogie corresponding to different $R_d$, $\theta_d$ and running speed. It can be seen that the influence of $R_d$ and running speed on DN is significant whereas the influence of $\theta_d$ is not obvious. Figure 17 shows the DN of the transverse acceleration of the car body corresponding to the $\theta_d$ and the running speed. When the speeds are 150 and 250 km/h, the DN is $<0.1$ and the nonlinear degree of the vibration response is small. When the speed reaches 350 km/h, the transverse vibration response of the car body caused by wheel-rail high-frequency impact is highly nonlinear and the DN value exceeds 0.2.

The analysis indicates that if DN of the wheel eccentricity feature frequency component of the wheelset’s vertical acceleration exceeds 0.2, DN of the bogie transverse acceleration exceeds 0.1 or DN of the car body transverse acceleration exceeds 0.2, the train should be considered unhealthy.

In this second part, the second order wheel polygonalization, wheels ovality, is studied. It is assumed that the ovality exists only on wheelset $w_1$ whereas other wheelsets are perfectly round. The train is assumed to be running on a straight track with track excitation and the data of the track excitation model are derived from the measured track irregularity spectrum of Beijing-Tianjin high-speed railway. The detailed dynamic response calculation and local instantaneous spectrum analysis steps are omitted here, only the DN is given.

Figure 18 shows the DN of the transverse acceleration of bogie $t_1$ affected by $R_d$ and $\theta_d$ of wheels ovality. If $R_d < 0.5$ mm, the DN value does not exceed 0.1. If $R_d \geq 0.5$, the wheels impact rail, and the maximum value of DN is close to 0.3, indicating that the
Figure 11. Transverse displacement of wheelset $w_1$ influenced by $\theta_d$ of wheels eccentricity with $R_d = 1.5$ mm. The running speed is 350 km/h.

Figure 12. Local instantaneous spectrum of vertical acceleration of $w_1$ influenced by track excitation. $R_d$ of the wheel eccentricity is 1.0 mm. The running speed is 350 km/h: (a) without track excitation and (b) with track excitation.
wheel-rail impact leads to a sharp increase in the Degree of Non-linearity of vibration response. It should be noted that wheel-rail impact does not always lead to hunting instability.

Figure 19 shows DN of transverse acceleration of bogie $t_1$ affected by running speed. When the running speed is $150 \text{ km/h}$, the extent of non-linearity is small (DN $\leq 0.1$). When the speed reaches $250 \text{ km/h}$, DN is in the range of $0.1-0.2$. When the speed reaches $350 \text{ km/h}$, the wheels impacts the rail, the maximum DN exceeds 0.2, and the vibration is highly nonlinear.

Figure 13. DN for vertical acceleration of wheelset $w_1$ influenced by $R_d$ of wheels eccentricity with and without track irregularity. The running speed is $350 \text{ km/h}$.

Figure 14. DN for vertical acceleration of $w_1$ influenced by $\theta_d$ and $R_d$ of wheels eccentricity. The running speed is $350 \text{ km/h}$.

Figure 15. DN for transverse acceleration of bogie $t_1$ influenced by $\theta_d$ and $R_d$ of wheels eccentricity. The running speed is $350 \text{ km/h}$.

Figure 16. DN for transverse acceleration of bogie $t_1$ influenced by speed and $\theta_d$ of wheels eccentricity with $R_d = 1.5 \text{ mm}$.

Figure 17. DN for transverse acceleration of carbody influenced by speed and $\theta_d$ of wheels eccentricity with $R_d = 1.5 \text{ mm}$.

Figure 18. DN for transverse acceleration of bogie $t_1$ influenced by $R_d$ and $\theta_d$ of wheels ovality. The running speed is $350 \text{ km/h}$.
Next, the influence of high-order \( n_{nor} > 2 \) wheel polygonalization on the Degree of Non-linearity of vehicle system vibration response is studied. It is assumed that the wheel polygonalization exists only on the wheelset \( w_1 \) and other wheels are ideally round. Due to the limited space, the detailed dynamic response and local instantaneous spectrum calculation results are omitted here, and only DN are given. Figure 20 shows the DN of transverse acceleration of bogie \( t_1 \) influenced by running speed and the third order wheel polygonalization \( (R_d = 0.1 \text{ mm}) \). When the running speed is \(< 300 \text{ km/h} \), DN is around 0.05, and the vibration is nearly linear. When the running speed reaches 350 km/h, the wheel impact the rail and the maximum DN increases to 0.1, indicating that the Degree of Non-linearity almost doubles.

The calculation also indicates that: if \( n_{nor} > 3 \), \( R_d > 0.1 \text{ mm} \), and the running speed of the train reaches 350 km/h, the wheels may lose contact with the rail and produce high-frequency wheel-rail impact. Similar results can be obtained by analyzing higher order wheel polygonalization. Therefore, wheel polygonalization must be strictly controlled.

Through the above analysis, it can be concluded that:

1. Generally, the wheel is in close contact with the rail, DN of polygonalization feature frequency of transverse acceleration of the bogie is \(< 0.05 \), the Degree of Non-linearity is relatively small and is hardly affected by \( \theta_d \).
2. When the wheel impact the rail, DN of polygonalization feature frequency of transverse acceleration of the bogie exceeds 0.1 and the Degree of Non-linearity of vibration increases significantly.

In summary, if the DN of polygonalization feature frequency of the transverse acceleration of bogie exceeds 0.1, the train should be considered unhealthy.

**Analysis of unstable vehicle hunting motion**

The polygonization of wheel is characterized by the periodic deviation of the diameter of the wheel rolling circle along the circumferential direction, which leads to the asymmetric wheel-rail contact. This defect will seriously reduce the nonlinear critical velocity of the vehicle and aggravate the transverse vibration of the wheelset. The high-order wheel polygonalization can greatly disturb the wheel-rail normal force and the running state of the wheelset, and therefore causes the wheel-rail creep force to fluctuate. It results in a wheel circumferential non-uniform wear which has an unpredictable and destructive impact on the vehicle hunting stability. When polygonalization exists on the two wheels of the same axle, height difference between the left and right sides is easily generated, which tends to make the vehicle shaking head and the wheelset producing transverse hunting displacement and transverse acceleration on the rail surface. This transverse hunting motion can aggravate the wears of the wheel and rail with risk of derailment in serious situations.

The previous discussion shows that wheel-rail impact does not always result in vehicle hunting instability, as the critical velocity of modern high-speed train is high. The calculation method of the critical velocity is made as follows: first, the train runs on a finite length track with irregularity excitation. Second, the train runs on an ideally smooth track. By observing whether the transverse vibration of the vehicle can be attenuated to the equilibrium position or not, the critical velocity of the system can be determined. The results show that the critical velocity decreases with the increase of \( R_d \). For wheel eccentricity, when \( R_d \) increases from 0.5 to 1.5 mm, the critical velocity of the train decreases from more than 500 km/h to \(< 250 \text{ km/h} \), as shown in Figure 21. The calculation also shows that the critical velocity does not guarantee the health of train. For example, for the fifth order wheel polygonalization with \( R_d = 0.1 \text{ mm} \), the critical velocity of the vehicle exceeds 500 km/h, but the high-frequency wheel-rail impact can
be observed at the speed of 250 km/h, as shown in Figure 22. Among all the suspension components, anti-yaw damper is the most important equipment to ensure a high critical velocity of the train. When all anti-yaw dampers fail, the critical velocity of the train will drop below 200 km/h. Therefore, when the train suffers from heavy wheel wear, anti-yaw damper failure, and other defects, the critical velocity will decline undoubtedly.

According to the theory of railway vehicle dynamics, the Hopf bifurcation method is generally used to analyze the nonlinear stability of the hunting motion of the vehicle. If the amplitude bifurcation is observed in the limit cycle of the hunting motion, the vehicle system becomes unstable. In railway engineering applications, the hunting stability of vehicles on the line is generally evaluated by tests in Europe and China, and the time domain response of transverse vibration of bogie frame is commonly used for evaluation. In China, the objective criterion for evaluating the hunting stability of the train is established as follow: the transverse acceleration signal of the bogie frame is measured and filtered by 10 Hz low-pass filtering. If there are six consecutive peaks reaching or exceeding 8 m/s², the hunting motion of the bogie is considered unstable.

This criterion is applied to study hunting stability of trains influenced by wheel polygonalization (Rd = 1.5 mm, θd = 0°) and anti-yaw damper failure. Figure 23 gives the results of the transverse stability analysis of the bogie. After 10 Hz low-pass filtering, the six continuous peak values of the transverse acceleration curve of the bogie frame do not exceed 8 m/s². According to the criterion applied to the transverse vibration, the train is stable. Obviously, this criterion does not work under all circumstances. Similar results can be obtained for trains with high-order wheel polygonalization. Therefore, for trains with polygonal wheels, it is necessary to explore other reasonable hunting stability evaluation and vehicle health monitoring criterion.
This work tries to explore the evaluation criterion from the perspective of Degree of Non-linearity. The DN of the vehicle system response in the unstable state of hunting motion is studied. It is assumed that the train runs at 350 km/h speed on a straight track with track excitation and that the data of the track excitation model are derived from the measured track irregularity spectrum of Beijing-Tianjin high-speed railway. It is also assumed that one anti-yaw damper between the bogie $t_1$ and the car body fails. Considering the first five-order polygonalization of the wheel, the $R_d$ of each order polygon are 1.5, 0.5, 0.1, 0.1, 0.1 mm, respectively. For brevity, the detailed dynamic response calculation process and data decomposition steps are omitted and only the results are given. Figure 24 shows the DN value corresponding to the feature frequency of hunting instability. When the train hunting is unstable, the hunting frequency fluctuates near 2 Hz and the DN of the feature frequency is $>0.1$, and sometimes $>0.2$. It is also indicated that DN of hunting feature frequency tends to decrease with the increase of $\theta_d$.

In summary, when the hunting motion of the vehicle is unstable, (1) DN of hunting feature frequency of the bogie’s transverse acceleration exceeds 0.1; and (2) DN of hunting feature frequency tends to decrease with the increase of $\theta_d$. It can be concluded that if the DN of hunting feature frequency of the bogie’s transverse acceleration exceeds 0.1, the train should be considered unhealthy.

When vehicle hunting motion becomes unstable, DN of the hunting frequency of bogie’s transverse acceleration may exceed 0.1.

Beside the elucidation of the wheel polygonalization mechanism, it is particularly important to establish criteria which help to effectively anticipate, and therefore avoid or restrain, wheel polygonalization that may lead to damages and disasters. Based on the results, an objective criterion for wheel polygonalization health monitoring is proposed: if the Degree of Non-linearity of polygonalization frequency of bogie’s transverse acceleration exceeds 0.1, or if the Degree of Non-linearity of hunting frequency of bogie’s transverse acceleration exceeds 0.1, the train should be considered unhealthy.

Declaration of conflicting interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding
The author(s) received no financial support for the research, authorship, and/or publication of this article.

ORCID iD
Chen Shuangxi https://orcid.org/0000-0001-7992-094X

References
1. Kaper HP. Wheel corrugation on Netherlands railways (NS): origin and effects of “polygonization” in particular. *J Sound Vib* 1988; 120(2): 267–274.
2. Nielsen J and Johansson A. Out-of-round railway wheels—a literature survey. *Proc IMechE, Part F: J Rail and Rapid Transit* 2000; 214(2): 79–91.
3. Johansson A. Out-of-round railway wheels—causes and consequences, an investigation including field tests, out-of-roundness measurements and numerical simulations. Chalmers Univ Technol 2005; 2323: 1–43.
4. Barks DW and Chiu WK. A review of the effects of out-of-round wheels on track and vehicle components. *Proc IMechE, Part F: J Rail and Rapid Transit* 2005; 219(3): 151–175.
5. Zhang XS, Xiao XB and Jin XS. Influence of high-speed railway wheels ovalization on vehicle stability. *Chin J Mech Eng* 2008; 44(3): 50–56.
6. Brommundt E. A simple mechanism for the polygonalization of railway wheels by wear. *Mech Res Commun* 2012; 24(4): 435–442.
7. Meywerk M. Polygonalization of railway wheels. *Arch Appl Mech* 1999; 69(2): 105–120.
8. Morys B. Enlargement of out-of-round wheel profiles on high speed trains. *J Sound Vib* 1999; 227(5): 965–978.
9. Meinke P and Meinke S. Polygonalization of wheel treads caused by static and dynamic imbalances. *J Sound Vib* 1999; 227(5): 979–986.
10. Dekker H. Vibrational resonances of nonrigid vehicles: polygonalization and ripple patterns. *Appl Math Model* 2009; 33(3): 1349–1355.

Figure 24. DN for transverse acceleration of bogie $t_1$ influenced by wheel polygonalization and anti-yaw damper failure. The running speed is 350 km/h.
11. Peng B, Iwnicki S, Shackleton P, et al. The influence of wheelset flexibility on polygonal wear of locomotive wheels. Wear 2019; 432–433: 102917.

12. Ma CZ, Gao L, Cui RX, et al. The initiation mechanism and distribution rule of wheel high-order polygonal wear on high-speed railway. Eng Fail Anal 2021; 119: 104937.

13. Wu X, Rakheja S, Cai W, et al. A study of formation of high order wheel polygonalization. Wear 2019; 424–425: 1–14.

14. Cai WB, Chi MR, Wu XW, et al. Experimental and numerical analysis of the polygonal wear of high-speed train. Wear 2019; 440–441: 1–14.

15. Li C, Luo S, Cole C, et al. An overview: modern techniques for railway vehicle on-board health monitoring systems. Veh Syst Dyn 2017; 55(7): 1045–1070.

16. Zampieri N, Bosso N and Gugliotta A. Innovative monitoring systems for onboard vehicle diagnostics. In: Proceedings of the third international conference on railway technology: research, development and maintenance, vol. 110, Cagliari, Sardinia, Italy, 5–8 April 2016.

17. Bosso N, Gugliotta A and Zampieri N. Design and testing of an innovative monitoring system for railway vehicles. Proc IMechE, Part F: J Rail and Rapid Transit 2018; 232: 445–460.

18. Sun Q, Chen C, Kemp AH, et al. An on-board detection framework for polygon wear of railway wheel based on vibration acceleration of axle-box. Mech Syst Signal Process 2021; 153: 107540.

19. Huang NE, Shen Z, Long SR, et al. The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis. Proc R Soc Lond A 1998; 454: 903–995.

20. Huang NE, Wu Z, Long SR, et al. On instantaneous frequency. Adv Adapt Data Anal 2009; 1(2): 177–229.

21. Wu Z and Huang NE. Ensemble empirical mode decomposition: a noise-assisted data analysis method. Adv Adapt Data Anal 2008; 1(1): 1–41.

22. Kalker JJ. Three dimensional bodies in rolling contact. Dordrecht: Kluwer Academic Publishers, 1990.

23. Shen ZY, Hedrick JK and Elkins JA. A Comparison of alternative creep force models for railway vehicle dynamic analysis. In: Proceedings of 8th International Association for Vehicle System Dynamics (IAVSD) Symposium, Cambridge, MA, 15–19 August 1983, pp.591–605.

24. Gabor D. Theory of communication. J Inst Electr Eng Part III 1946; 93: 429–444.

25. Bedrosian EA. A product theorem for Hilbert transforms. Proc IEEE 1963; 51: 868–869.

26. Nuttall AH. On the quadrature approximation to the Hilbert transform of modulated signals. Proc IEEE 1966; 54: 1458–1459.

27. Chen SX and Lin JH. Nonlinearity and non-stationarity analysis of dynamic response of vehicle-track coupling system enhanced by Huang transform. Measurement 2014; 55: 305–317.

28. Zhai WM and Cai CB. Coupling model of vertical and lateral interactions between railway vehicle and track. Veh Syst Dyn 1996; 26(1): 61–79.

29. Zhai WM and Cai CB. Train/track/bridge dynamics interactions: simulation applications. Veh Syst Dyn 2002; 37(Suppl. 1): 653–665.

30. Zhai WM, Cai CB and Wang KY. Numerical simulation and field experiment of high-speed train-track-bridge system dynamics. Veh Syst Dyn 2004; 41: 677–686.

31. Sedighi MH and Shirazi KH. A survey of Hopf bifurcation analysis in nonlinear railway wheelset dynamics. J Vibroeng 2012; 14(1): 344–351.