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Thermodynamic Analysis and Statistical Investigation of Effective Parameters for Gas Turbine Cycle using the Response Surface Methodology

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\begin{abstract}
In this paper, the statistical analyses are presented to study the thermal efficiency and power output of gas turbine (GT) power plants. For analyzing gas turbine operation and performance, a novel approach is developed utilizing the response surface methodology (RSM) which is based on the central composite design (CCD) method. An attempt is made to study the effect of some operational factors (inlet temperatures of turbine and compressor, inlet pressure of compressor, lower heating value of fuel (LHV) and air mass flow rate) and design parameters (pressure drop of combustion chamber and isentropic efficiency of equipment) on the system’s response. Based on the DoE analysis, regression models are presented to quantify the effects of these parameters on thermal efficiency and net work of the Brayton based gas turbine cycle. The proposed correlations obtained by the analysis have a remarkably satisfactory performance for all simulation data. The error analysis shows a maximum error of 5.5\% in numerical computations of response functions for GT power plants. The optimal point of the thermal efficiency and net power output based on the optimized conditions were found to be 45.71\% and 4.182 MW, respectively.
\end{abstract}

\begin{nomenc}
\begin{itemize}
\item \textit{c_p}: Specific Heat (kJ/kg K)
\item \textit{k}: Specific Heat Ratio
\item \textit{LHV}: Lower heating value (kJ/kg)
\item \textit{\dot{m}}: Mass flow rate (kg/s)
\item \textit{P}: Pressure (bar)
\item \textit{\Delta P}: Pressure drop (bar)
\item \textit{T}: Temperature (\degree C)
\item \textit{W}: Work (kW)
\item \textit{\eta}: Efficiency (\%)
\item \textit{\eta_{th}}: Thermal Efficiency (\%)
\item \textit{\eta_{cc}}: Combustion Chamber Efficiency (\%)
\item \textit{\eta_{f}}: Fuel Efficiency (\%)
\item \textit{\eta_{T}}: Turbine Efficiency (\%)
\item \textit{\eta_{a}}: Air Efficiency (\%)
\item \textit{\eta_{cc}}: Combustion Chamber Efficiency (\%)
\item \textit{\eta_{f}}: Fuel Efficiency (\%)
\item \textit{\eta_{T}}: Turbine Efficiency (\%)
\end{itemize}
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1. INTRODUCTION

Electricity production is one of the most important and most successful parts of the flourishing industries in Iran. Steam power plants, diesel and modern gas turbine plants used in single or combined cycle mode are the main plants used for power generation in Iran. This plays a significant role in the development of economics and industrialization [1]. As the power generation industries use a large portion of the primary energy demand in this country, there is a continuing need for improved energy efficiency, coupled with concerns about its environmental effects. One of the most common engines to produce electricity and power from fuels is the gas
turbine that is readily available and can generate electricity in high amounts. The advancement of design techniques for a low-cost energy plant is demanded in this industry. Thermal efficiency plays an important role in the performance of a Brayton cycle as well as its significant effect on the total cost [2]. Since the basic gas turbine cycles have low thermal efficiencies, it is necessary to consider the improvement of their performance. The behavior of gas turbine power plants, like other energy systems, is usually characterized by suitable models depending on their precision and complexity and introducing working conditions more or less accurately [3].

Barzegar Avval et al. [4] modeled the gas turbine power plant with pre-heater and compared the simulation results with one of the gas turbine power plants in Iran. The thermodynamic modeling and multi-objective optimization of a GT power plant with pre-heater was provided by them. Ust et al. [5] have been working on a performance analysis and an optimization for an irreversible regenerative gas-turbine cogeneration system. A new performance criterion has been introduced and the performance of a regenerative gas-turbine cogeneration system was analyzed in that work.

Rahman et al. [6] developed a strategy and presented the parametric study of thermodynamic performance to determine the performance of gas turbine power plant utilizing the effect of operating conditions. The variations of operating conditions on the performance of gas turbine have been investigated. Wu and Kiang [7] examined the efficiency and power function of the Brayton cycle. Results showed that the power output and the cycle efficiency is strong functions of the component efficiencies. Ibrahim and Rahman [8] investigated the effects of varying operating conditions on the performance of a gas turbine cycle. The results of their simulation model showed that the compression ratio, ambient temperature, air-to-fuel ratio, and turbine inlet temperature have significant effects on the performance of gas turbine cycle. Zhang et al. [9] established a finite time thermodynamic model of the air Brayton cycle and analyzed its waste heat recovery performance. Moreover, they have optimized the compressor's pressure ratio and inlet relative pressure drop by taking power output as the objective function. Nami et al. [10] presented a novel cogeneration cycle and thermodynamically analyzed for the production of power as well as saturated steam as a byproduct. A comprehensive parametric study was also done to show the effects of some important parameters on the system performance.

Kaviri et al. [11] analyzed a comprehensive thermodynamic modeling of a dual pressure combined power plant cycle. They performed multi-objective by using the genetic algorithm approach. Finally, the effects of key parameters on objective functions have been investigated. It was found that turbine inlet temperature, compressor pressure ratio and pinch point temperatures are significant design parameters.

Invernezzi et al. [12] have investigated the possibility of increasing the performances of micro-gas turbines through the addition of a bottoming organic Rankine cycle which recovers the thermal power of the exhaust gases typically. They carried out an analysis of the characteristics of different classes of working fluids in order to specify a procedure to select the most suitable fluid which capable to satisfy both environmental and technical concerns. Mohammadi et al. [13] studied an exergoeconomic analysis of a proposed combination of a gas turbine (GT), a steam Rankine cycle (SRC), and an organic Rankine cycle (ORC) to obtain the maximum heat recovery of the GT exhaust gas. They carried out a multi-objective optimization which satisfies the exergy efficiency and product cost rate at the same time. Gonca [14] performed exergetic performance analysis of a gas turbine fuelled with different fuel types. In that study, the variations of turbine design parameters on the performance characteristics with considering heat losses were examined.

Each of the previous studies have played some role in improving the understanding of the effect of a number of design and/or operational factors on thermal efficiency, performance, environmental impact, costs or other demands of gas turbine cycles, through theoretical analyses and experimental investigations. However, in processes where two or more factors take part, usually the effect of one factor on the performance is studied by keeping other factors constant. However, the effect of the factor being studied on the performance may not be the same at all levels of the other factors, which indicates an interaction between the factors. Therefore, when interactions may be present, in order to avoid misleading conclusions, it is necessary to plan the experiments so that the effects of a factor is estimated at several levels of the other factors [15].

Although, thermodynamic study is a powerful tool to investigate and optimize an energy system, but statistical methods can improve the solution. Statistical design of experiments (DoE) refers to the process of planning experiments so that appropriate data are collected and analyzed by statistical methods, resulting in valid and objective conclusions [15]. DoE is a collection of mathematical and statistical techniques for reducing the number of experiments in order to find the effect of parameters (factors) affecting a response in a process [16]. The effect of a factor is defined as the change in response produced by a change in the level of the factor.

Various statistical methods such as design experiment method and response surface methodology have been used to determine the effective parameters in several engineering applications, some of which are presented as follow. Mohammadi and Argghavani [17] used the response surface method to investigate one-dimensional
modeling and optimization of two-stage light gas launcher. Their results indicate the effectiveness of the methodology to verify the process. Okati et al. [18] analyzed the sensitivity of a solar desalinator's primary parameters using design of experiment (DoE) method based on response surface method (RSM) in order to obtain optimal conditions. They presented different diagram to assist the designer to choose suitable design parameters based on the operational conditions. Kazemian et al. [19] introduced a case study focused on the application of Design of Experiments method in an industrial-scale RO desalination plant. Their results show insight into the RO process features and can help designers and operators achieve a higher energy efficiency and better performance in the design and operation of RO units. Wang et al. [20] proposed a novel shell and-tube heat exchanger with fold helical baffles which are used to reduce outer fringe triangle gaps between adjacent baffles. They presented an improved algorithm combining second-order response surface method and multi-objective genetic algorithm based on numerical simulation. Fekri et al. [21] proposed a novel axial flux switching permanent magnet generator for small wind turbine applications. They used the central composite design (CCD) and the response surface methodology (RSM) with 3D finite element techniques to achieve optimal machine dimensions. Behzadmehr et al. [22] studied the effect of entrance geometry of a backward inclined centrifugal fan on the efficiency by the DOE method. Numerical simulation has been used to determine the fan efficiency for different configurations of the entrance parameters by CFD calculation. They finally validated the methodology by comparing the predicted result of DOE and the numerical simulation on a specific fan. Ekren et al. [23] demonstrated the use of Response Surface Methodology (RSM) in optimizing the size of an integrated PV/wind hybrid power system with battery storage. In that work, the size of a PV/wind hybrid energy conversion system with battery storage was optimized using the RSM based on an hourly operating cost. Mamourian et al. [24] studied on a 2-D numerical investigation and a sensitivity analysis of combined turbulent mixed convection and radiation heat transfer using two phase mixture model in a solar heat exchanger. In the study by Yantong Li et al. [25], feasible heating system have been designed to increase the availability of open-air swimming pools in winter. They developed surrogate model to improve the computational efficiency using the response surface approach, in which the dataset is generated from the simulation platform established using the MATLAB and TRNSYS softwares.

To the best of authors’ knowledge, there is no study in which the combined effect of all time-dependent operational and design factors on the performance of gas turbine plant have been considered. Therefore, in the present work, thermal efficiency and network are formulated in terms of all important system design and operational parameters through the DoE method by the response surface methodology in a gas turbine cycle. To this end, the central composite design (CCD) is used to study the effect of operational factors including inlet temperatures of turbine and compressor, inlet pressure of compressor, lower heating value of fuel (LHV) and air mass flow rate, as well as design parameters including the pressure drop of combustion chamber and isentropic efficiency of equipment on the system’s response. In this regards, two new predictive correlations for the performance and power output of the GT power plants are also presented. Based on the DoE analysis, regression models are presented to quantify the effects of variable parameters on thermal efficiency and network. Selected contour line plots are also presented, which illustrate the collective effects of the parameters on responses, and can be helpful to researchers and designers and operators of GT plant for finding the most suitable conditions for a plant design and operation, based on pre-defined priorities.

The presented solution can be built into thermal systems for comprehensive techno-economic evaluation of power cycle based processes, where changes in operation parameters are to be considered, in order to enhance the understanding of the process and its optimal control. These findings are expected to help designers in finding optimum conditions of GT cycle and the presented solution can be built into systems for comprehensive techno-economic evaluation.

2. BRIEF PLANT DESCRIPTION

The gas turbine cycle mainly works based on the Brayton cycle. This power cycle is a thermodynamic cycle used as reference by gas turbine systems and jet engines. In this cycle, air from the ambient atmosphere is compressed to a higher pressure and temperature by the compressor. In the combustion chamber, air is heated further by burning the fuel-air mixture in the air flow. Then, the combustion products expand in the turbine either to near atmospheric pressure or to a pressure required by the jet engines. (Figure 1). The following assumptions are the basis for future calculations of the energy-balance equations for various parts of the GT plant.

- Compressors and turbines have isentropic efficiencies
- All processes are assumed to be steady state- steady flow

The classical thermodynamic equations for the Brayton cycle are cited in many references [13, 26].

Energy balance equation:

- Air compressor
\[ \dot{m}_a = \dot{m}_1 = \dot{m}_2 \]  
\[ T_2 = T_1 \left( 1 + \frac{1}{\eta_{comp}} \left( \frac{k-1}{k} - 1 \right) \right) \]  
\[ r_p = \frac{p_2}{p_1} = \frac{p_3}{p_4} \]  
\[ W_c = \dot{m}_a c_{p,a} (T_2 - T_1) \]  
\[ \dot{m}_g = \dot{m}_3 = \dot{m}_2 + \dot{m}_f \]  
\[ \frac{\eta_{cc}}{\dot{m} c_{p,a} (T_3 - T_4)} \]  
where \( k = 1.33 \) [27] and the \( c_{p,a} \) and \( c_{p,g} \) are considered to be a temperature variable function as [4]:

\[ c_{p,a}(T) = 1.04841 - \left( 3.83717 \times 10^{-3} \right) T + \left( 9.4537 \times 10^{-7} \right) T^2 - \left( 5.49031 \times 10^{-10} \right) T^3 + \left( 7.9298 \times 10^{-13} \right) T^4 \]  
\[ c_{p,g}(T) = 0.991615 - \left( 6.99703 \times 10^{-3} \right) T + \left( 2.71298 \times 10^{-6} \right) T^2 - \left( 1.22442 \times 10^{-9} \right) T^3 \]  
\[ W_{GT} = \dot{m} c_{p,g}(T_3 - T_4) \]  
\[ \eta_{th} = \frac{\dot{P}_{net}}{\dot{m} c_{p,a}(LHV)} \]  

According to Equations (1) to (13), a computational computer code was performed in the EES software. For validation, the numerical results derived from the computational code based on data shown on Table 1 were compared with theoretical findings in literature [13] and shown in Table 2. Good consistencies are seen between the computed pressure and temperature at different points of the power cycle by the computer code with those reported in literature [13].

### 3. METHODOLOGY

In engineering, many phenomena are modeled on their own theories. However, many phenomena do not have the ability to have a mathematical model due to the large number of controlling factors, unknown mechanisms or computational complexity. Response surface methodology (RSM) is one of the exploration approaches in designing experiments and engineering related sciences. It is a set of mathematical and statistical methods beneficial for the modeling and analysis of problems in which a response parameter is influenced by several variables and the objective is desired to be optimized. In this procedure, it is examined to find a way to estimate interactions, quadratic effects and even the localized surface of the response using a suitable test design.

#### TABLE 1. Parameters of the gas turbine cycle

| Parameter                                      | Unit   | Value |
|-----------------------------------------------|--------|-------|
| compressor inlet temperature (\( T_1 \))     | °C     | 20    |
| compressor inlet pressure (\( P_1 \))        | bar    | 1.01  |
| Pressure ratio (\( r_p \))                   | --     | 12    |
| Gas turbine isentropic efficiency (\( \eta_t \)) | %     | 88    |
| Compressor isentropic efficiency (\( \eta_c \)) | %     | 85    |
| Combustion chamber efficiency (\( \eta_{cc} \)) | %     | 95    |
| Lower heating value (LHV)                     | KJ/Kg  | 49482 |
| Turbine inlet temperature (\( T_1 \))        | °C     | 900   |
| Air mass flow rate (\( \dot{m} \))           | Kg/s   | 500   |
| Combustion chamber pressure drop (\( \Delta P \)) | %     | 4     |

#### TABLE 2. Comparison of some parameters of gas turbine cycle compared to reference[13]

| point  | \( T(\text{C}) \) | \( P(\text{bar}) \) | \( \dot{m}(\text{kg/s}) \) |
|--------|------------------|---------------------|-----------------------------|
|        | EES code         | Ref.                | EES code         | Ref.                | EES code         | Ref.                |
| 1      | 20               | 20                  | 1.01             | 1.01                | 500               | 500                  |
| 2      | 376.8            | 374.65              | 12.12            | 12.16               | 500               | 500                  |
| 3      | 900              | 900                 | 11.64            | 11.67               | 506.2             | 508.54               |
| 4      | 452.6            | 452.71              | 1.03             | 1.03                | 506.2             | 508.54               |
After numerical simulation of the gas turbine cycle by the computer code and ensuring the accuracy of computations, it is necessary to determine the appropriate mathematical model to investigate the role of each desired parameters. The selection of a suitable experimental design is the first step in this methodology. RSM designs, such as the Box-Behnken design (BBD) and the central composite design (CCD), are applied to fit quadratic equations. The next step is the selection of independent variables and their design ranges.

Table 3 shows the ranges of independent variables for the analysis of thermal efficiency ($\eta_{th}$) and network ($W_{net}$) as the response functions by the central composite design. Analysis of variance (ANOVA) is a suitable way to evaluate the model's fitness. At this step, F-value (Fisher-Sandcore value), P-value (Probability value) and sum of the square are important parameters. According to the amounts of these parameters by the quadratic and cubic models, which are reported in Tables 4 and 5, the degree of polynomial equations was determined. In addition, these statistical indicators were used to identify the significant model terms.

### Table 3. Independent parameters and their level ranges

| Factor | Name | Units | Min (-1) | Max (+1) | Mean | Std. Dev. |
|--------|------|-------|----------|----------|------|-----------|
| A      | $P_1$ | bar   | 0.9      | 1.013    | 0.95 | 0.051413  |
| B      | $T_1$ | C     | 15       | 40       | 27.5 | 11.37449  |
| C      | $\eta_c$ | % | 70       | 100      | 85   | 13.64939  |
| D      | $\eta_{cc}$ | % | 85       | 100      | 92.5 | 6.824693  |
| E      | $\eta_t$ | % | 85       | 100      | 92.5 | 6.824693  |
| F      | $T_3$ | C     | 1000     | 1300     | 1150 | 136.4939  |
| G      | LHV  | kJ/kg | 45000    | 50000    | 47500| 2274.898  |
| H      | $\Delta p$ | % | 0        | 4        | 2    | 1.819918  |
| J      | $\dot{m}$ | kg/s | 1        | 10       | 5.5  | 4.094816  |

### Table 4. Analysis of variance for network ($W_{net}$)

| Source | Sum of Squares | Mean Square | F-value | p-value |
|--------|----------------|-------------|---------|---------|
| Model  | 200.35         | 18.21       | 132.88  | < 0.0001|
| B ($T_1$) | 0.4968                  | 0.4968         | 3.62    | 0.0589  |
| C ($\eta_c$) | 0.0479                  | 0.0479         | 0.3479  | 0.5552  |
| D ($\eta_{cc}$) | 9.93                        | 9.93          | 72.45   | < 0.0001|
| F ($\eta_t$) | 4.97                        | 4.97          | 36.24   | < 0.0001|
| G($T_3$) | 8.55                          | 8.55          | 62.4    | < 0.0001|
| K ($\dot{m}$) | 161.91                         | 161.91       | 1181.22 | < 0.0001|

### Table 5. Analysis of variance for thermal efficiency ($\eta_{th}$)

| Source | Sum of Squares | Mean Square | F-value | p-value |
|--------|----------------|-------------|---------|---------|
| Model  | 6.99E+05       | 30381.28    | 4728.04 | < 0.0001|
| A ($P_1$) | 0.0174                  | 0.0174        | 0.0027  | 0.9586  |
| B ($T_1$) | 10983.02                  | 10983.02      | 1709.21 | < 0.0001|
| C ($\eta_c$) | 8994.89                  | 8994.89       | 1399.81 | < 0.0001|
| D ($\eta_{cc}$) | 3.04E+05                  | 3.04E+05      | 4730.14 | < 0.0001|
| E ($\eta_t$) | 7.05                         | 7.05         | 1.1     | 0.2968  |
| F ($\eta_{th}$) | 3.34E+05                  | 3.34E+05      | 5190.56 | < 0.0001|
| G($T_3$) | 765.87                     | 765.87       | 119.19  | < 0.0001|
| H (LHV) | 4.11                          | 4.11        | 0.6393  | 0.4254  |
| J ($\Delta p$) | 0.0413                     | 0.0413      | 0.0064  | 0.9362  |
| AH     | 28.79                   | 28.79       | 4.48    | 0.0361  |
| BC     | 575.65                   | 575.65      | 89.58   | < 0.0001|
| BD     | 2364.97                  | 2364.97     | 368.04  | < 0.0001|
| BF     | 64.64                    | 64.64       | 10.06   | 0.0019  |
| BG     | 363.04                   | 363.04      | 56.5    | < 0.0001|
| CD     | 11792.26                 | 11792.26    | 1835.15 | < 0.0001|
| CF     | 5434.2                   | 5434.2      | 845.69  | < 0.0001|
| CG     | 1796.15                  | 1796.15     | 279.52  | < 0.0001|
| DF     | 103.86                   | 103.86      | 16.16   | < 0.0001|
| DG     | 9602.1                   | 9602.1      | 1494.31 | < 0.0001|
| EJ     | 133.37                   | 133.37      | 20.76   | < 0.0001|
| FG     | 775.78                   | 775.78      | 120.73  | < 0.0001|
| D²     | 486.52                   | 486.52      | 75.71   | < 0.0001|
| G²     | 193.48                   | 193.48      | 30.11   | < 0.0001|

### 4. RESULT AND DISCUSSION

Based on the analysis of variance, the mass flow rate for the response function of the thermal efficiency and the compressor inlet pressure, combustion chamber efficiency, lower heating value of fuel and combustion chamber pressure drop for the dependent variable of the output work had the least effect compared to the other parameters, and consequently should be eliminated on final model.
A summary of the statistical results of the models are shown in Table 6. For thermal efficiency, the Predicted R-squared of 0.9982 is in reasonable agreement with the Adjusted R-squared of 0.9986. Similarly, for network of gas turbine cycle, the Predicted R-squared of 0.8916 is in reasonable agreement with the Adjusted R-squared of 0.9023. Adeq. Precision measures the signal to noise ratio, such that a ratio of more than 4 is desirable. The ratios presented in Table 6 represent a suitable signal. These models can be used to navigate the design space.

The quadratic models to predict the response functions $\eta_{th}$ and $W_{net}$ in terms of the input variables (actual factors) are presented by Equations (14) and (15), respectively.

\[(\eta_{th})^{LS} = -1008.55226 - 159.27016 P_1 - 3.56795 T_2 - 75.24325 r_p + 11.59979 \eta_c - 0.167154 \eta_{cc} + 4.0372 \eta_t + 1.12313 T_3 - 0.003282 LHV - 6.30367 \Delta P + 0.003357 P_1 \times LHV - 0.084827 T_1 \times r_p + 0.022925 T_1 \times \eta_e + 0.007587 T_1 \times \eta_t + 0.000898 T_1 \times T_3 + 0.319943 r_p \times \eta_c + 0.434302 r_p \times \eta_t + 0.012497 r_p \times T_3 + 0.008007 \eta_e \times \eta_t - 0.003849 T_3 \times \eta_c + 0.068051 \Delta P \times \eta_{cc} - 0.002180 T_3 \times \eta_t - 0.050116 \eta_e^2 - 0.000316 T_3^2 \]  

\[
\ln(W_{net}) = -14.46262 - 0.043355 T_1 - 0.303348 r_p + 0.072836 \eta_c + 0.093198 \eta_t + 0.007659 T_3 + 0.626492 \eta_t + 0.000333 T_1 \times T_3 + 0.000255 T_3 \times r_p - 0.000047 T_3 \times \eta_e - 0.000587 T_3 \times \eta_t - 0.034408 \eta_t^2
\]  

Table 6. Models summary statistics for response functions

| Response function | $\eta_{th}$ | $W_{net}$ |
|-------------------|-------------|-----------|
| Units             | %           | MW        |
| Study Type        | Response Surface |
| Design Type       | Central Composite |
| Design Model      | Quadratic |
| Analysis          | Polynomial |
| Minimum           | 8.911       | 0.04919   |
| Maximum           | 45.71       | 14.96     |
| Mean              | 30.67       | 1.63      |
| Std. Dev.         | 2.53        | 0.3702    |
| R-squared         | 0.9988      | 0.9092    |
| Adjusted R-squared| 0.9986      | 0.9023    |
| Predicted R-squared| 0.9982   | 0.8916    |
| Adeq. Precision   | 283.6319    | 38.8825   |
| Transform         | Power       | Natural Log |
| Model             | Reduced Quadratic |

To demonstrate the effect of operational parameters on both response functions, the three and two dimensional contours based on equations (14) and (15) can be plotted. For space saving, only the 3D contours of thermal efficiency and 2D form for power output are drawn in Figures 2 and 3, respectively. These contours can be useful in understanding the effects of selected independent variables more clearly. Also, surface response maps can be considered as a way to predict the thermal efficiency and network of gas turbine cycle for different values of input variables within the desired ranges shown in Table 3. Besides, contour graphs are helpful in determining the type of interaction between the studied variables.

As shown in Figure 2, it is seen that increasing in pressure ratio and the turbine and compressor isentropic efficiency leads to increase in thermal efficiency, while the increase of compressor inlet temperature has an opposite effect. On the other hand, a complicated and dual behavior is seen for the cycle efficiency as a function of the turbine inlet temperature, the pressure drop and efficiency of combustion chamber. Figure 2 reveals that by increasing the turbine inlet temperature, the thermal efficiency will be increased up to a maximum value and then the trend is reversed during a sweep behavior. The position of this return point is under the influences of many affecting parameters such as the compressor inlet temperature, isentropic efficiencies of turbine and compressor and the pressure ratio. Among the above parameters, the change in the inlet temperature of compressor and the pressure ratio have a greater effect on determining the return point position of the turbine inlet temperature. For example, at the pressure ratio of 10, the return point takes place at 1150°C, and this temperature decreases while the pressure ratio gets higher values. It is worth mentioning that the temperature of the return point increases by increasing the compressor inlet temperature.

In a similar way as it is shown in Figure 3, an increasing trend is seen for the network by increasing in the turbine inlet temperature, and the isotropic efficiencies of the compressor and turbine.

5. NUMERICAL OPTIMIZATION

In the present study, the RSM optimizer is also used to optimize the gas turbine cycle parameters. The RSM optimizer is a tool used for the optimization of multi objectives like the thermal efficiency and network of power cycles. The main purpose of applying a statistical method (RSM) is the accurate determination of optimized operating condition for gas turbine cycle. First, the goals should be identified for optimization process, which are included: (1) maximum thermal efficiency, (2) maximum network of cycle.
(a) variations of $\eta_a$ against inlet pressure of compressor and LHV

(b) variations of $\eta_a$ against inlet temperature of compressor and pressure ratio

(c) variations of $\eta_a$ against inlet temperature and isentropic efficiency of compressor

(d) variations of $\eta_a$ against inlet temperature of compressor and isentropic efficiency of turbine

(e) variations of $\eta_a$ against inlet temperature of compressor and turbine

(f) variations of $\eta_a$ against pressure ratio and isentropic efficiency of compressor
(g) variations of $\eta_\text{th}$ against pressure ratio and isentropic efficiency of turbine
(h) variations of $\eta_\text{th}$ against isentropic efficiency of turbine and compressor

(i) variations of $\eta_\text{th}$ against inlet temperature and isentropic efficiency of turbine
(j) variations of $\eta_\text{th}$ against inlet temperature of turbine and pressure ratio

(k) variations of $\eta_\text{th}$ against inlet temperature of turbine and isentropic efficiency of compressor
(l) variations of $\eta_\text{th}$ against pressure drop and isentropic efficiency of combustion chamber

Figure 2. The 3D contours and corresponding 2D contours of thermal efficiency of cycle ($\eta_\text{th}$) against effective variables (other variables kept constant at the mean values mentioned Table 3)
Figure 3. The 3D contours and corresponding 2D contours of net work \( W_{\text{net}} \) against effective variables (other variables kept constant at the mean values mentioned Table 3).

(a) variations of net work against inlet temperature of compressor and turbine

(b) variations of net work against pressure ratio and inlet temperature of turbine

(c) variations of net work against isentropic efficiency and inlet temperature of turbine

(d) variations of net work against isentropic efficiency of compressor and inlet temperature of turbine

The numerical optimization was carried out by keeping all the parameters in the range and maximizing the responses. The best-fit model for each response was used for the purpose of multiple response optimizations. Hence, the nonlinear regression models based on CCD were used for the response functions. All the optimum conditions were obtained from a numerical optimization procedure using the software for maximizing both the response functions at a single optimum condition.

A desirability function based optimization technique is used in this work [15, 28, 29]. The desirability function (DF) is an approach utilized to a numerical optimization; allots a score to a series of responses and selects factors settings for maximizing that score.

In this approach, each response \( y_i \) is first converted into an individual desirability function \( d_i \), which varies between 0 and 1 \((0 \leq d_i \leq 1)\). Here, \( d(y_i)=0 \) represents a completely undesirable value of \( y_i \), whereas \( d(y_i)=1 \) represents a completely desirable or ideal response value.

The individual desirability is then combined using the geometric mean, which gives the overall desirability \( D_o \), as calculated by Equation (16)

\[
D_o = \sqrt[\text{m}]{D_1 \times D_2 \times \ldots \times D_m}
\]  

(16)

where \( m \) represents the number of responses. The geometric mean of overall desirability for the present work, consisting of two responses, is mentioned below by Equation (17).
\[ D_0 = (D_{\text{in}} \times D_{\text{net}})^{1/2} \]  

Depending on whether a particular response \( y_i \) is to be minimized, maximized, or assigned a target value, different desirability functions \( d(y_i) \) can be used. In this case, both response functions are to be maximized.

\[ d(y_i) = \begin{cases} 
0 & \text{if } y_i < L \\
\frac{y_i - L}{T - L} & \text{if } L \leq y_i \leq T \\
1 & \text{if } y_i > T 
\end{cases} \]  

where \( L \) and \( T \) are the lower limit and the target values, respectively, and \( r \) is the weight. The value of weight is taken as \( 1 \) \((r = 1)\) to make the desirability function linear.

In order to numerical optimization, all parameters are constrained within the specified range except the compressor inlet pressure, which is set to its maximum value.

Optimal mode selection should be performed with respect to the first two columns data on Table 7, showing the most desirable values. For this purpose, by comparing the parameter values in these two columns, it can be seen that the main difference is in the values of \( \Delta P \).

Since, the more ideal system has less pressure drop, the values of the second column can be selected as the optimal parameters. Therefore, the optimal points of the thermal efficiency and network of cycle based on the optimized conditions were found to be 45.71% and 4.182 MW, respectively.

### TABLE 7. optimal points for thermal efficiency and net work based on operational and design parameters based on the desirability function

| \( P_1 \) | \( T_1 \) | \( r_3 \) | \( \eta_i \) | \( \eta_c \) | \( \eta_r \) | \( T_3 \) | LHV | \( \Delta P \) | m | \( \eta_{\text{th}} \) | \( W_{\text{net}} \) | Desirability |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.013 | 15.267 | 12 | 99.973 | 86.825 | 100 | 1145.917 | 49335.85 | 3.457 | 9.064 | 45.704 | 4.27 | 0.909 |
| 1.013 | 15.154 | 11.862 | 99.999 | 87.831 | 99.986 | 1123.774 | 48435.2 | 0.049 | 8.591 | 45.71 | 4.182 | 0.907 |
| 1.013 | 15.657 | 11.847 | 99.959 | 99.999 | 99.991 | 1128.235 | 46575.84 | 4 | 8.485 | 45.625 | 4.163 | 0.906 |
| 1.013 | 16.838 | 12 | 100 | 93.095 | 100 | 1217.166 | 48394.52 | 4 | 8.715 | 45.223 | 4.437 | 0.906 |
| 1.012 | 16.046 | 12 | 100 | 97.547 | 100 | 1222.219 | 45490.97 | 3.592 | 8.898 | 45.214 | 4.479 | 0.905 |
| 1.013 | 15.177 | 12 | 99.734 | 90.827 | 99.85 | 1171.239 | 46559.13 | 2.692 | 9.785 | 45.428 | 4.242 | 0.905 |
| 1.012 | 15 | 12 | 99.442 | 90.291 | 100 | 1158.873 | 45011.29 | 1.393 | 8.947 | 45.554 | 4.268 | 0.904 |
| 1.013 | 19.993 | 11.938 | 100 | 85.707 | 99.988 | 1134.298 | 45000.07 | 0.001 | 9.289 | 45.528 | 4.128 | 0.904 |
| 1.013 | 15.021 | 12 | 100 | 99.842 | 100 | 1251.614 | 48366.19 | 3.281 | 7.915 | 45.006 | 4.367 | 0.902 |
| 1.013 | 16.378 | 12 | 100 | 91.439 | 99.481 | 1163.754 | 47328.7 | 3.339 | 9.046 | 45.191 | 4.245 | 0.902 |
| 1.01 | 16.101 | 11.874 | 100 | 88.257 | 100 | 1152.922 | 49962.05 | 0.597 | 8.138 | 45.574 | 4.144 | 0.900 |
| 1.013 | 23.653 | 12 | 100 | 92.95 | 100 | 1167.77 | 48568.04 | 2.823 | 9.861 | 45.232 | 4.098 | 0.900 |

### 6. CONCLUSION

The present work is focused on modeling and optimization of gas turbine cycle. The design of experiments with response surface methodology have been used to study the effect of some operational factors. These include: inlet temperatures of turbine and compressor, inlet pressure of compressor, lower heating value of fuel (LHV) and air mass flow rate and design parameters such as pressure drop of combustion chamber and isentropic efficiency of equipment on thermal efficiency and network of gas turbine cycle. To this end, the method of Central Composite Design of RSM has been adopted. For thermal efficiency, the Predicted R2 of 0.9982 is in reasonable agreement with the Adjusted R2 of 0.9986. Similarly, for network of gas turbine cycle, the Predicted R2 of 0.8916 is in reasonable agreement with the Adjusted R2 of 0.9023.

The main results of the study can be summarised as follows:

- The thermal efficiency and the net work of the cycle always increase by increasing the pressure ratio and the compressor and turbine isentropic efficiencies.
- Decreasing in the compressor inlet temperature leads to increases in the thermal efficiency and the cycle net work.
- The contours of thermal efficiency reveal that by increasing the input temperature to the turbine, the thermal efficiency will be increased up to a maximum value and then the trend is reversed.
- The changes in the inlet temperature of compressor and the pressure ratio have a greater effect on the return point position of the turbine inlet temperature.
- The thermal efficiency and net work of cycle based on the optimized conditions were found to be 45.71% and 4.182 MW, respectively.

Therefore, it is expected that this work will be a significant help in quantifying the effects of
design and operational factors on the performance and costs of GT units and assists in optimizing system design and operation.

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چکیده
در این مقاله، تحلیل آماری نیروگاه توربین گاز (GT) به منظور مطالعه راندمان حرارتی و توان تولید سیکل ارائه شده است. برای تحلیل عملکرد توربین گاز، رویکرد جدیدی بر اساس روش سطح ورما (RSM) و مبایل روش طراحی کامپوزیت مرکزی (CCD) به کار گرفته شده است: تأثیر برخی پارامتر‌های عملکرد از قبیل دمای ورودی توربین و کمپرسور، فشار ورودی به کمپرسور، اثر حرارتی پایین سوخت و دیگر پارامتر‌های حرارتی شامل راندمان آزمایشی در تحقیقات و انتقال حرارت و تحقیقاتی اجرا و بررسی شده است. اثرات برخی پارامترهای عملکردی نظیر دمای ورودی توربین و کمپرسور، فشار ورودی به کمپرسور، اثر حرارتی پایین سوخت و دیگر پارامتر‌های حرارتی شامل راندمان آزمایشی در تحقیقات و انتقال حرارت و تحقیقاتی اجرا و بررسی شده است.

بر اساس تحلیل روش طراحی آزمایشی (DoE) مدل‌های رگرسیون با ارائه گریزی اثرات این پارامترها بر روی راندمان حرارتی و کار خالص چرخه توربین گاز مبنی بر سیکل بیراکون به عنوان تابع پاسخ ارائه شده. روابط پیشنهاد شده از تحلیل نتایج، عملکرد رضایت‌بخش و قابل ملاحظه‌ای برای تمام نمادهای شبیه‌سازی شده با حداکثر خطا 0/5 درصدی برابر گزارش گردیده است.

شماره بهینه شده به ترتیب 71/45 و MW 182/54/4/182 مشابه آنها به دست آمده است.