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Fractional dynamics of 2019-nCOV in Spain at different transmission rate with an idea of optimal control problem formulation

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Received 27 March 2021; revised 18 April 2021; accepted 10 July 2021
Available online 29 July 2021

KEYWORDS
2019-nCOV; Fractional mathematical model; Numerical simulations; Optimal control problem; Atangana-Baleanu (AB) fractional derivative
2010 MSC:
92D30; 92C60; 37N25; 26A33

Abstract In this article, we studied the fractional dynamics of the most dangerous deathly disease which outbreaks have been recorded all over the world, called 2019-nCOV or COVID-19. We used the numerical values of the given parameters based on the real data of the 2019-nCOV cases in Spain for the time duration of 25 February to 9 October 2020. We performed our observations with the help of the Atangana-Baleanu (AB) non-integer order derivative. We analysed the optimal control problem in a fractional sense for giving the information on all necessary health care issues. We applied the Predictor-Corrector method to do the important graphical simulations. Also, we provided the analysis related to the existence of a unique solution and the stability of the proposed scheme. The aim and the main contribution of this research is to analyse the structure of novel coronavirus in Spain at different transmission rate and to indicate the danger of this deathly disease for future with the introduction of some optimal controls and health care measures.

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1. Introduction

2019-nCOV is a pandemic induced by SARS CoV-2, also called Severe Acute Respiratory Syndrome Novel Coronavirus (SARS nCoV) which begins from the city Wuhhan of
China in 2019. The transmission of the virus is so quick, and soon it spread in millions of the world population. On 30th January 2020, the World Health Organization (WHO) announced the 2019-nCOV is a public health exigency of international concern. The main symptoms of the coronavirus are coughing, headache, fever, shortness of breath, and tiredness. Recently, scientists around the globe have done outstanding investigations on the transmission dynamics, forecasting, control tactics, and estimation for the infection of COVID-19.

Several mathematicians have given the number of articles to define the structure of COVID-19, in which some are [1–7]. \textit{Anmeen et al.} in [8] studied the structure of Coronavirus-19 with a fresh dynamical model. \textit{Erturk et al.} [9], solved a non-linear 2019-nCOV model via new generalised Caputo type fractional derivative to study the dynamics of the disease in China. \textit{Kumar et al.} [10] gave a new analysis on COVID-19 with the help of a time delay fractional non-linear model in Caputo sense. \textit{Mishra et al.} [11] proposed a non-linear coronavirus model with asymptotic and quarantine classes. In [12], the authors proposed a compartmental mathematical structure for the dispersion of the 2019-nCOV infection by focusing on the transmissibility of super-spreaders individuals, and in [13], fractional mathematical modelling of coronavirus is given. By considering the undetected infections, the authors studied the Mathematical modelling of the spread of 2019-nCOV in [14]. In this work, the authors considered the reported data on China, and they showed the consequence of undetected cases on the coronavirus.

There is a large number of papers that appeared in the literature by considering the mathematical modelling with fractional derivative operators which are widely studied the dynamical behaviour of various pandemics by considering the different scenarios. Geometric and physical interpretations of fractional derivatives are still not well-defined but the number of research papers have been given in this field to solve this problem. In this matter, \textit{Podlubny et al.} in [15] proposed some novel analysis by giving the fence basis shapes for Riemann-Liouville and Caputo derivatives. Some other studies on this topic can be seen from [16,17]. Many authors considered the data of different countries and applied different fractional operators to analyse the study. The interesting readers can be referred to the recently published papers in this direction [18,19]. Mathematical models, employing the system of ODEs with integer-order, have been manifested significantly for explaining the structure of biological phenomena. Atangana-Baleanu derivate is the advance version of all other non-classical derivatives like Caputo and Caputo-Fabrizio. This derivative is defined with the Mittag-Leffler kernel memory which is advanced to singular kernel property. Fractional order operators are more useful to describe a real life phenomena which can be concluded from these following studies. A brief study on the non-integer order dynamical model of HIV infection with the optimal control strategies is given in [20]. Also, \textit{Naik et al.} [21] studied the Chaotic structure of a non-integer order HIV-1 model with AIDS-related cancer cells and in [22] proposed the dynamical modelling of primary infection of HIV-1 under immune control in Caputo non-integer order sense. \textit{Owolabi et al.} [23] analysed the non-integer order operator technique on a multi-mutation and immanent resistance model. In [23], a study on multi-mutation fractional mathematical modelling with drug resistance model is analysed. Some dynamics of non-integer order maps with power-law, exponential decay and Mittag-Leffler memory are given in [24]. In [25], a computational framework of multi-species non-integer order reaction-diffusion model with Atangana-Baleanu (AB) sense is analysed. Some chaotic systems are analysed via AB derivative sense in [26]. In [27], authors simulated a non-linear dynamical model via Atangana-Baleanu and Caputo-Fabrizio non-integer order operative sense. An interesting study on simulations for non-integer order Benney-Lin equation arising in falling film is given in [28]. In [2] some analysis related to the structure of the bats-hosts-reservoir-people COVID-19 model are given. Some studies related to Hilfer fractional order neutral stochastic systems, their existence, and solutions to non-classical order neutral delay systems can be analysed from [29,30] The investigation of epidemiological structural processes, including memory effects, is suitable because such structures believe on the potency of memory which is artificial by order of fractional derivative operator.

The objective of formulating mathematical models using fractional differential equations is to enhance and generalize various ordinary differential systems. Hence, describing some real-world phenomena using fractional derivative operator has gained more attractions of many researchers from multiple fields. Recently, a number of studies have been given on the models by using non-classical derivatives to describe the dynamics of deadly diseases. Some applications of fractional derivatives in mathematical epidemic studies can also be analysed from [31–36].

In this research paper, we solved a non-linear mathematical model by using Atangana-Baleanu non-classical derivative to do the all necessary simulations. The paper is divided into sections. Some preliminary results are given in Section 2. The fractional order model structure in AB sense is given in Section 3. The analysis of the existence of a unique solution by the most familiar Picard-Lindelof approach is given in Section 4. Also, we solve the model and analyse the stability results of the proposed approximation method in Section 5. In Section 6, we simulate the plots by using parameter values based on the real numerical data of Spain from February 25 to October 9, 2020. We demonstrate the optional control analysis in the form of the AB fractional operator in Section 7. We conclude our results at the end of the paper.

2. Preliminaries

Here we give the definitions of Atangana-Baleanu derivative and integral.

\textbf{Definition 1} [37]. For a map $F \in H^d(c, d)$, where $d > c$ and $0 \leq q \leq 1$, the non-integer order Atangana-Baleanu (AB) derivative is given as follows:

$$^{AB}D^c_t (F(\xi)) = \frac{B[q]}{1 - q} \int_c^\xi F(\eta)E_q \left[\frac{(\xi - \eta)^q}{q - 1}\right] \, \mathrm{d}\eta,$$

(1)

where $B[q]$ follows $B[0] = B[1] = 1$ represents the normalization mapping and $E_q(.)$ is the Mittag-Leffler function in one-parametrised form.

\textbf{Definition 2} [37]. The non-integer order AB integral for normalization function $B[q]$ is given as
integer. Let dynamics of Coronavirus is proposed as: the disease-free equilibrium $E_0$. In our study, we revised this model in the sense to analyse the structure of 2019-nCOV with the real numerical data of Spain. Consider $v_{p,z} = (\zeta - p)^{\epsilon - 1}(p = 1, 2, \ldots, \zeta - 1)$ & $v_{p,z} = 0$ for $p \geq \zeta, H, T > 0, \epsilon \in T & \epsilon$ is an integer. Let $\sum_{p=1}^{\zeta-1} v_{p,z} |e_p| = 0$ for $k > \zeta \geq 1$. If $|e_c| \leq M \sum_{p=1}^{\zeta-1} v_{p,z} |e_p| + |\eta_0|, \zeta = 1, 2, \ldots, \epsilon,$

then $|e_c| \leq C |\eta_0|, \epsilon = 1, 2, \ldots$ where $C$ is an integer independent of $\epsilon$ & $h$.

3. Model dynamics

Adak et al. [39], proposed a non-linear integer order model to analyse the structure of 2019-nCOV with the real numerical data of Spain. In our study, we revised this model in the sense of the Atangana-Baleanu non-integer order derivative. There are five different compartments susceptible $S(t)$, latent class $L(t)$, infectious population $I(t)$, recovered $R(t)$ and deaths $D(t)$. So the fractional order non-linear model to analyse the dynamics of Coronavirus is proposed as:

4. Existence and uniqueness results

Now, we give the framework of existence of a solution by using Picard-Lindelof approach for the proposed epidemic fractional system in the form of Atangana-Baleanu operator by the use of fixed-point results. Regarding that way, the given fractional model (3) can be re-write in the similar kernel form as follows:

$S(t) - S(0) = G_1(t, S(t))(1 - g) + \frac{\varphi}{\Gamma(\varphi)} \int_{0}^{t} (t - \sigma)^{\varphi-1} G_1(\sigma, S(t))d\sigma,$

$L(t) - L(0) = G_2(t, L(t))(1 - g) + \frac{\varphi}{\Gamma(\varphi)} \int_{0}^{t} (t - \sigma)^{\varphi-1} G_2(\sigma, L(t))d\sigma,$

$I(t) - I(0) = G_3(t, I(t))(1 - g) + \frac{\varphi}{\Gamma(\varphi)} \int_{0}^{t} (t - \sigma)^{\varphi-1} G_3(\sigma, I(t))d\sigma,$

$R(t) - R(0) = G_4(t, R(t))(1 - g) + \frac{\varphi}{\Gamma(\varphi)} \int_{0}^{t} (t - \sigma)^{\varphi-1} G_4(\sigma, R(t))d\sigma,$

$D(t) - D(0) = G_5(t, D(t))(1 - g) + \frac{\varphi}{\Gamma(\varphi)} \int_{0}^{t} (t - \sigma)^{\varphi-1} G_5(\sigma, D(t))d\sigma.$

Here, we obtain the subsequent iterative equations

$S_{n+1}(t) = (1 - g)G_1(t, S_n) + \frac{\varphi}{\Gamma(\varphi)} \int_{0}^{t} (t - \sigma)^{\varphi-1} G_1(\sigma, S_n)\sigma d\sigma,$

$L_{n+1}(t) = (1 - g)L_0 G_2(t, L_n) + \frac{\varphi}{\Gamma(\varphi)} \int_{0}^{t} (t - \sigma)^{\varphi-1} G_2(\sigma, L_n)d\sigma,$

$I_{n+1}(t) = (1 - g)I_0 G_3(t, I_n) + \frac{\varphi}{\Gamma(\varphi)} \int_{0}^{t} (t - \sigma)^{\varphi-1} G_3(\sigma, I_n)d\sigma,$

$R_{n+1}(t) = (1 - g)\frac{\varphi}{\Gamma(\varphi)} \int_{0}^{t} (t - \sigma)^{\varphi-1} G_4(\sigma, R_n)d\sigma,$

$D_{n+1}(t) = (1 - g)\frac{\varphi}{\Gamma(\varphi)} \int_{0}^{t} (t - \sigma)^{\varphi-1} G_5(\sigma, D_n)d\sigma.$

Now we consider that we can obtain the exact solutions by approaching the limit $n$ goes to $\infty$.

### Table 1: Parameters Description [39].

| Parameter | Description | values |
|-----------|-------------|--------|
| $\lambda$ | birth rate | 1165 |
| $\zeta$ | transmission rate of S to L | 0.542, 0.315, 0.192, 0.151, 0.164, 0.183, 0.271, 0.193 |
| $\eta$ | natural death rate | 3.33 $\times 10^{-5}$ |
| $\kappa$ | transmission rate of exposed to infectious | 0.192 |
| $\phi$ | recovery rate | 0.095 |
| $\theta$ | death rate due to covid infection | 0.11 |
Theorem 1. For AB sense, we discuss the existence and uniqueness analysis of the given model by using the Picard-Lindelof scheme.

Proof. Let $Q_1 = \sup_{C_{\epsilon, \alpha}} \| G_1(t, S)\|$, $Q_2 = \sup_{C_{\epsilon, \alpha}} \| G_2(t, L)\|$, $Q_3 = \sup_{C_{\epsilon, \alpha}} \| G_3(t, I)\|$, $Q_4 = \sup_{C_{\epsilon, \alpha}} \| G_4(t, R)\|$, $Q_5 = \sup_{C_{\epsilon, \alpha}} \| G_5(t, D)\|$, where

$$C_{\epsilon, \alpha} = \{ t - c, t + c \} \times [S - k_1, S + k_1] = B_1 \times D_1,$$

$$C_{\epsilon, \alpha} = \{ t - c, t + c \} \times [L - k_2, L + k_2] = B_1 \times D_2,$$

$$C_{\epsilon, \alpha} = \{ t - c, t + c \} \times [I - k_3, I + k_3] = B_1 \times D_3,$$

$$C_{\epsilon, \alpha} = \{ t - c, t + c \} \times [R - k_4, R + k_4] = B_1 \times D_4,$$

$$C_{\epsilon, \alpha} = \{ t - c, t + c \} \times [D - k_5, D + k_5] = B_1 \times D_5,$$

using the Picard operator as $\xi : C(D_1, D_2, D_3, D_4, D_5, B_1) \rightarrow C(D_1, D_2, D_3, D_4, D_5, B_1)$, defined:

![Fig. 1](plots.png)
\( \xi \phi(t) = \phi_0(t) + \Delta(t, \phi(t))(1 - g) \)
\[ + \frac{q}{\Gamma(q)} \int_0^t (t - s)^{q-1} \Delta(s, \phi(s))ds, \]
where \( \phi(t) = \{ \{ S(t), L(t), I(t), R(t), D(t) \} \}, \phi_0(t) = \{ S_0, L_0, I_0, R_0, D_0 \} \) and \( \Delta(t, \phi(t)) = \{ \mathcal{G}_c(t, S(t)), \mathcal{G}_s(t, L(t)), \mathcal{G}_l(t, I(t)), \mathcal{G}_r(t, R(t)), \mathcal{G}_d(t, D(t)) \}. \) Further we consider that the solution of the given fractional non-linear model are bounded within the given time-range, \( \| \phi(t) \|_\infty \leq \max\{k_1, k_2, k_3, k_4, k_5 \} \),
\[ \| \phi(t) - \phi_0(t) \| = \| \Delta(t, \phi(t))(1 - g) \|
\[ + \frac{q}{\Gamma(q)} \int_0^t (t - s)^{q-1} \Delta(s, \phi(s))ds\]
\[ \leq \| \Delta(t, \phi(t))(1 - g) \| + \frac{q}{\Gamma(q)} \int_0^t (t - s)^{q-1} \Delta(s, \phi(s))ds\]
\[ \leq (1 - g) \beta \max\{Q_1, Q_2, Q_3, Q_4, Q_5 \} \leq bQ \leq k, \]
here we need that \( b \leq 1 \). So by using the results of fixed-point theorem associated to Banach space together defined metric, we get***
\[ \| \xi \phi_1 - \xi \phi_2 \| = \sup_{t \in [a, b]} | \phi_1(t) - \phi_2(t) |. \]
Now we have
\[ \| \xi \phi_1 - \xi \phi_2 \| = \| (1 - g) \{ \Delta(t, \phi_1(t)) - \Delta(t, \phi_2(t)) \} \|
\[ + \frac{q}{\Gamma(q)} \int_0^t (t - s)^{q-1} \{ \Delta(s, \phi_1(s)) - \Delta(s, \phi_2(s)) \}ds\]
\[ \leq \| (1 - g) \beta | \phi_1(t) - \phi_2(t) | + \frac{q}{\Gamma(q)} \int_0^t (t - s)^{q-1} \| \phi_1(s) - \phi_2(s) \|ds\]
\[ \leq \{ (1 - g) \beta + \frac{q}{\Gamma(q)} \} \| z_1(t) - z_2(t) \|
\[ \leq \beta \| \phi_1(t) - \phi_2(t) \|, \]
where \( \beta < 1 \). Because \( \phi \) is conraction map, we got \( \beta |z_1| < 1 \), then the proposed operator \( \xi \) is also a conraction. So, the given non-linear model defining by AB fractional derivative specified in Eq. (3) has a set of unique solution. \( \Box \)

5. Solution of the model in AB sense

After giving all theoretical analysis related to the existence of the solution and before deriving the most important optimal control problem, here we derive the solution of the adopted model by the most common and very easy and familiar technique called Predictor-Corrector method in the sense of AB fractional derivative. For this purpose, we follow the method given in [40]. Here we generalise the fractional model in a single fractional initial value problem given as follows:
\[ \{ \begin{align*}
& 4\beta P^p_t \mathcal{L}(t) = \mathcal{G}(t, \mathcal{L}(t)), \quad 0 \leq t \leq T, \\
& \mathcal{L}(0) = \mathcal{L}_0.
\end{align*} \]
(9)

This corresponding fractional Volterra integral equation:
\[ \mathcal{L}(t) = \mathcal{L}_0 + \frac{q}{\Gamma(q)} \int_0^t (t - s)^{q-1} \mathcal{G}(s, \mathcal{L}(s))ds. \]
(10)

So following the method mentioned in [40] for \( q \in [0, 1], 0 \leq t \leq T \) and setting \( h = \frac{T}{N} \) for \( n = 0, 1, 2, \ldots, N \in \mathbb{Z}^+ \), the Corrector formula of the projected problem is
\[ \mathcal{L}_{i+1} = \mathcal{L}_0 + \frac{\phi h^q}{\Gamma(q + 2)} \sum_{j=0}^i \mathcal{G}_{i+1, j}(\mathcal{L}_j, \mathcal{L}_{i+1}) \]
(11)
where
\[ a_{i, j} = \begin{cases} (i - j)(i + 1)^q & \text{if } j = 0, \\
(i - j + 2)^q + (i - j)^q - 2(i - j + 1)^q & \text{if } 1 \leq j \leq i, \\
1, & \text{if } i = i + 1.
\end{cases} \]
(12)
and
\[ a_{i+1, i+1} = 1 + \frac{(1 - g) \Gamma(q + 2)}{\phi h^q} \]
(13)
The predictor formula is derived as
\[ \mathcal{L}_{i+1} = \mathcal{L}_0 + \frac{h^q}{\Gamma(q + 2)} \sum_{j=0}^i \mathcal{G}_{i+1, j}(\mathcal{L}_j, \mathcal{L}_{i+1}), \]
(14)
where
\[ \mathcal{B}_{i+1, j} = \begin{cases} -(i - j)^q + (i - j + 1)^q & j = 0, \\
1 & \text{otherwise}.
\end{cases} \]
(15)

Thus the corrector formulae for the proposed model (3) are
\[ \mathcal{S}_{i+1} = \mathcal{S}_0 + \frac{\phi h^q}{\Gamma(q + 2)} \sum_{j=0}^i \mathcal{B}_{i+1, j} \begin{cases} (i + 1) \mathcal{S}_{i+1} - \eta \mathcal{S}_{i+1} + \sum_{j=0}^i (i + 1) \mathcal{S}_{i+1} \end{cases}, \]
(16)
where
\[ \mathcal{S}_{i+1} = S_0 + \frac{\phi h^q}{\Gamma(q + 2)} \sum_{j=0}^i \mathcal{B}_{i+1, j} \begin{cases} (i + 1) \mathcal{S}_{i+1} - \eta \mathcal{S}_{i+1} + \sum_{j=0}^i (i + 1) \mathcal{S}_{i+1} \end{cases}, \]
(17)
where
\[ \mathcal{L}_{i+1} = L_0 + \frac{\phi h^q}{\Gamma(q + 2)} \sum_{j=0}^i \mathcal{B}_{i+1, j} \begin{cases} (i + 1) \mathcal{L}_{i+1} - \eta \mathcal{L}_{i+1} + \sum_{j=0}^i (i + 1) \mathcal{L}_{i+1} \end{cases}, \]
(18)
where
\[ \mathcal{L}_{i+1} = L_0 + \frac{\phi h^q}{\Gamma(q + 2)} \sum_{j=0}^i \mathcal{B}_{i+1, j} \begin{cases} (i + 1) \mathcal{L}_{i+1} - \eta \mathcal{L}_{i+1} + \sum_{j=0}^i (i + 1) \mathcal{L}_{i+1} \end{cases}, \]
(19)
where
5.0.1. Stability analysis

Theorem 2. The numerical method (15) and (16) is conditionally stable.

Proof. Let \( \tilde{L}_0, \tilde{L}_i (i = 0, \ldots, i + 1) \) and \( \tilde{L}_{i+1}^p (i = 0, \ldots, N - 1) \) be perturbations of \( L_0, L_i, \) and \( L_{i+1}^p \), respectively. Then, the following perturbation equations are defined by using Eq. (3)

\[
\tilde{L}_{i+1}^p = L_0 + \frac{h^q}{\Gamma(q)} \sum_{j=0}^{i} b_{i+1,j} (G(t_j, L_j) - G(t_i, L_i)),
\]

(17)

Using the Lipschitz condition, we obtain

\[
|\tilde{L}_{i+1}| \leq \xi_0 + \frac{\xi h^q M}{\Gamma(q + 2)} \left( a_{i+1} |\tilde{L}_{i+1}^p| + \sum_{j=0}^{i} a_{i,j} |L_j| \right),
\]

(19)

Fig. 2 Plots of \( S(t), L(t), I(t), R(t), \) and \( D(t) \) for \( \xi = 0.315 \) at derivative order \( q = 1, 0.95, 0.90, 0.85 \) by solid, dashed, dot-dashed, and dotted line notations, respectively.
where \( G_0 = \max_{0 \leq |x| \leq N} \left\{ |L_0| + \frac{\alpha M b_{i+1}}{\Gamma(\varphi+2)} |L_0| \right\} \). Also, from Eq. (3.18) in [38] we write

\[
|L_{i+1}| \leq \gamma_0 + \frac{\alpha M}{\Gamma(\varphi+2)} \sum_{j=1}^{i} \left( a_{i+1} + \frac{\alpha M b_{i+1} h_{j+1}}{\Gamma(\varphi+2)} \right) |L_j|
\]

\[
\leq \gamma_0 + \frac{\alpha M C_2}{\Gamma(\varphi+2)} \sum_{j=1}^{i} (i+1-j) \varphi^{-1} |L_j|
\]

where \( \gamma_0 = \max_{0 \leq |x| \leq N} \left\{ |L_0| + \frac{\alpha M b_{i+1}}{\Gamma(\varphi+2)} |L_0| \right\} \). Substituting \( |L_{i+1}| \) from Eq. (20) into Eq. (19) results

\[
|L_{i+1}| \leq \gamma_0 + \frac{\alpha M C_2}{\Gamma(\varphi+2)} \sum_{j=1}^{i} (i+1-j) \varphi^{-1} |L_j|
\]

where \( \gamma_0 = \max_{0 \leq |x| \leq N} \left\{ |L_0| + \frac{\alpha M b_{i+1}}{\Gamma(\varphi+2)} |L_0| \right\} \). \( C_2 \) is a positive constant depends only on \( \varphi \) (Lemma 1) and \( h \) is taken to be small.
enough. Justifying Lemma 2 concludes $|\hat{L}_{j+1}| \leq C\gamma_j$, which finishes the proof. □

6. Graphical simulations

Now we perform the plots for all given classes by using the above-mentioned technique. Here we used the eight different values of the transmission rate $\zeta$ which was cited from [39]. We first used $\zeta = 0.542$ which is calculated on the foundation of COVID-19 reported cases in Spain from 25th February to 24th March. Similarly, we used $\zeta = 0.315, 0.192, 0.151, 0.164, 0.183, 0.271, 0.193$ based on cases of COVID-19 from 25th March to 21st April, 22nd April to 19th May, 20th May to 16th June, 17th June to 14th July, 15th July to 11th August, 12th August to 8th September

Fig. 4  Plots of $S(t), L(t), I(t), R(t)$ and $D(t)$ for $\zeta = 0.151$ at derivative order $\varrho = 1, 0.95, 0.90, 0.85$ by solid, dashed, dot-dashed, and dotted line notations, respectively.
and, 9th September to 9th October, respectively. The motivation and the advantage to use different disease transmission rates is to explore the dynamics of the COVID-9 much clearly against the proposed time intervals. Currently, COVID-19 is continuously spreading all over the nations and changing the transmission speed each and every month. So concerning different transmission rates give more rationality to the proposed model. In the collection of Plots 1, we given the figures of $S(t)$, $L(t)$, $I(t)$, $R(t)$ and $D(t)$ at different fractional order values for $\zeta = 0.542$. We can see that the peaks of latent and infected individuals shifted to the right side of the time scale when the value of fractional order decrease. We observed the plots at the fractional order values $\varrho = 1, 0.95, 0.90, 0.85$. Similarly in Fig. 2 and 7, we derived the plots for $\zeta = 0.315$ and $\zeta = 0.271$. In these both cases, the disease show the same nature as for $\zeta = 0.542$. But when we simulate the graphs at

![Plots of S(t), L(t), I(t), R(t) and D(t) for \(\zeta = 0.164\) at derivative order \(\varrho = 1, 0.95, 0.90, 0.85\) by solid, dashed, dot-dashed, and dotted line notations, respectively.](image-url)
\(\zeta = 0.192, 0.151, 0.164, 0.183\) and \(0.193\) then we can see that the nature of latent and infected individuals has changed. In these cases, the infected population is sharply decreased and did not show any peaks which can be seen from the family of Figs. 3–6 and 8. In the assembly of Fig. 9, we show the connection between infected and latent class (in Fig. 9a) and infected and recovered class (in Fig. 9b) for \(\zeta = 0.315\). Similarly, we give the relationship between infected and latent class (in Fig. 9c) and infected and recovered class (in Fig. 9d) for \(\zeta = 0.192\). At the last, we show the \(I(t)\) versus \(L(t)\) and \(I(t)\) versus \(R(t)\) plots for \(\zeta = 0.271\) in Fig. 9e–f. The mathematical relevance of results which are given in the group of Fig. 9 is to analyse the behaviour of infection with respect to the latent and recovered individuals. We can see that when recovery increases then respectively the infection goes down. Some where the latent and infectious individuals growths mutually.

![Graphs](image-url)

**Fig. 6** Plots of \(S(t), L(t), I(t), R(t)\) and \(D(t)\) for \(\zeta = 0.183\) at derivative order \(\varphi = 1, 0.95, 0.90, 0.85\) by solid, dashed, dot-dashed, and dotted line notations, respectively.
The behaviour of the given model in fractional order sense is fully analysed by the above figures. We simulated that in the all cases when the fractional order value reduces then the infection peak starts to shifted later time period. We have done the all above graphical simulations by the help of Mathematica software. From the above plots we can say that the nature of COVID-19 is changing in Spain month by month when the transmission rate change. By the above concerns, the importance of the given model is clearly described.

7. Optimal control problem formulation

Since the starting date of COVID-19, governments of all countries are very serious about the health care measures. So many countries like China, India, and Brazil etc. had applied strict lockdown for the population to not walking freely in the open environment but still this virus has infected a big amount of population all over the world. In this part of the simulations, our major target is to reduce the population of Coronavirus...
infected individuals at the same time minify the cost $J(\xi)$ allied with these techniques. Regarding the fractional order optimal control problems, a number of research papers have been proposed by mathematicians. Some fractional order optimal control techniques for the pine forests of the wilt epidemic are given in [41]. Ali et al. in [42] proposed a research on the optimal control aspects of the fractional order zika virus model. Some novel analyses on the new techniques of optimal controls are organised in [43]. A new algorithm for analysing the optimal control strategies by using a SIRV disease dynamical model is mentioned in [44]. Optimal controls in the concern of preventive measures of coronary heart disease are utilized in [45]. Authors in [46] derived the optimal control model for tuberculosis infection in Atangana-Baleanu derivative sense. By getting a high motivation from these mentioned researches and for the fulfillment of optimal controls for the proposed model, here we define a control function $\xi = (\xi_1, \xi_2, \xi_3)$, where $\xi_1(t)$ is the control for the change in the nature of the suscep-
tible population by following the all heath care measures and suggestions, $\xi_2(t)$ is for the use of all pre-treatments to the exposed or latent individual who is becoming to be infected and $\xi_3(t)$ is the function of control for increment of the potency of treatment for the infectious population.

To establish the optimal control framework or problem (OCP), we are ignoring the death term $D(t)$, because there is no mean of deaths in the optimal control analysis. Now to analyse the state model mentioned in (22), along-with the set of acceptable control function $\Omega = \{(\xi_1(.), \xi_2(.), \xi_3(.))|\xi_i is Lebesgue measurable on [0, 1], 0 \leq (\xi_1(.), \xi_2(.), \xi_3(.)) \leq 1, i = 1, 2, 3\}$ we have the objective functional specified by

$$J((\xi_1(.), \xi_2(.), \xi_3(.)) = \int_0^T \left[ \phi I(t) + \frac{1}{2} (k_1 \xi_1^2(t) + k_2 \xi_2^2(t) + k_3 \xi_3^2(t)) \right] dt.$$
where the constants \( k_1, k_2 \) and \( k_3 \) are associative cost measure with the controls \( \xi_1, \xi_2 \) and \( \xi_3 \). Then we evolve the optimal controls \( \xi_1, \xi_2 \) and \( \xi_3 \) for minimizing the cost function.

\[
J\left(\xi_1, \xi_2, \xi_3\right) = \int_0^T \mu(S, L, I, R, \xi_1, \xi_2, \xi_3, t) dt,
\]

subject to constraint,

\[
\frac{AB}{\partial S}S(t) = \gamma_1, \quad \frac{AB}{\partial R}R(t) = \gamma_2, \quad \ldots \quad \frac{AB}{\partial T}T(t) = \gamma_4,
\]

where \( \gamma_j = \gamma(S, L, I, R, \xi_1, \xi_2, \xi_3, t), \quad j = 1, 2, 3, 4 \) and the defined initial conditions are satisfied by \( S(0) = S_0, L(0) = L_0, I(0) = I_0 \) and \( R(0) = R_0 \).

Next, let us consider the below revised form of cost function

\[
J = \int_0^T \left[ H(S, L, I, R, \xi_1, \xi_2, \xi_3, t) + \sum_{i=1}^4 \left( \frac{\partial H}{\partial \xi_i} \right) \xi_i \right] dt,
\]

where the units \( i \) equal to 1 to 3 and \( j \) equal to 1 to 4. Then the Hamiltonian is established by:

\[
H(S, L, I, R, \xi_1, \mu_1, t) = \mu(S, L, I, R, \xi_1, t)
\]

where \( j = 1, 2, 3, 4 \) and \( i = 1, 2, 3 \). From Eqs. 25 and 26, the necessary and sufficient illustrations for the fractional optimal control problem (FOCP) are defined as follows:

\[
\frac{AB}{\partial \xi_1}H = \frac{\partial H}{\partial S}, \quad \frac{AB}{\partial \xi_2}H = \frac{\partial H}{\partial L}, \quad \frac{AB}{\partial \xi_3}H = \frac{\partial H}{\partial T},
\]

\[
\frac{AB}{\partial \xi_1}L = \frac{\partial L}{\partial \xi_1}, \quad \frac{AB}{\partial \xi_2}L = \frac{\partial L}{\partial \xi_2}, \quad \frac{AB}{\partial \xi_3}L = \frac{\partial L}{\partial \xi_3}.
\]

Moreover, \( \mu_i(T) = 0, \quad j = 1, 2, 3, 4 \) are the lagranges-type multipliers Eqs. 27 and 28 show the necessary conditions in the form of a Hamiltonian for the optimal control mentioned above.

7.1 Fractional order optimality conditions

Further consider the Hamiltonian function in the form:

\[
H(S, L, I, R, \xi_i, \mu_i) = I + \frac{1}{\lambda} \left( k_1 \xi_1^2 + k_2 \xi_2^2 + k_3 \xi_3^2 \right) + \mu_1 \left[ L - (1 - \xi_2) \xi_3 S \right] + \mu_2 \left[ (1 - \xi_2) \xi_3 S \right] + \mu_3 \left[ \phi L - (\phi + \theta + \eta + \xi_3) I \right] + \mu_4 \left[ \eta R + \xi_3 S \right],
\]

where \( \mu_i, \quad j = 1, 2, 3, 4 \) giving the langrangars multipliers called co-states.

Theorem: If \( \xi_1, \xi_2 \) and \( \xi_3 \) are optimal controls for the proposed optimal problem, if \( S^*, L^*, I^*, R^* \) are compatible optimal ways or the solution of the state model, then the variables \( \mu_1, \ldots, \mu_4 \) of co-state exists, such that therewith the proposed control model is contented, then the given below conditions are agreed:

Co-state model system:

\[
\begin{align*}
\mu_1 &= \left[ 1 - \xi_2 \right] + \left( \eta + \xi_3 \right) \xi_1 - \left[ 1 - \xi_2 \right] \xi_3, \\
\mu_2 &= \left( \kappa + \eta \right) \xi_1 - \kappa \xi_3, \\
\mu_3 &= \left[ 1 - \xi_2 \right] \xi_3 + \left( \phi + \theta + \eta + \xi_3 \right) \xi_1 - \phi \xi_3, \\
\mu_4 &= \eta \xi_3 - \eta R + \xi_3 S.
\end{align*}
\]

with conditions of transversality \( \mu_j(T) = 0, \quad j = 1, 2, 3, 4 \) and optimality conditions specified by

\[
\begin{align*}
\xi_1^i(t) &= \min \left\{ 1, \left( \frac{\mu_1^i - \mu_2^i}{\beta_1^i} \right) \right\}, \\
\xi_2^i(t) &= \min \left\{ 1, \left( \frac{\mu_3^i - \mu_4^i}{\beta_2^i} \right) \right\}, \\
\xi_3^i(t) &= \min \left\{ 1, \left( \frac{\mu_3^i - \mu_4^i}{\beta_3^i} \right) \right\}.
\end{align*}
\]

8. Conclusion

In our work, we have successfully analysed the fractional dynamics of one of the most deadliest epidemics of this decade named COVID-19 with the help of Atangana-Baleanu non-classical derivatives. We provided the analysis related to the existence of a unique solution with the help of the Picard-Lindelof approach. We have used a non-linear non-classical order model to describe the dynamics of coronavirus. We have formulated the fractional optimal control problem to give the information related to the effects of all necessary heath care opportunities. The derivation of the solution of the given model is done with the help of the Predictor-Corrector method and the stability of the proposed scheme are also given. We have used the numerical values of the given parameters based on the real data of COVID-19 cases in Spain for the time span of February 25 to October 9, 2020. We have given the all necessary plots to study the dynamics practically in which we first simulated the graphs of every class separately at different transmission rate and then plotted some relations between them. The main contribution of our article is to give a brief idea of the structure of novel coronavirus in Spain by using different infection transmission rate and to indicate the danger of this deadly diseases for future precautions with the introduc-
tion of some health care measures. In our study, we have
reminded the all important optimal controls and instructed
the all heath care issues. In future, we will try to re-simulate
our optimal control problem graphically for a better under-
standing of these results.

Declaration of Competing Interest

The authors declare that they have no known competing
financial interests or personal relationships that could have
appeared to influence the work reported in this paper.

Acknowledgments

Taif University Researchers Supporting Project number
(TURSP-2020/160), Taif University, Taif, Saudi Arabia.

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