Measurement and analysis of backlash on harmonic drive

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Abstract. Harmonic drive transmissions are widely used in robotics, space manipulation systems because of their excellent mechanical properties including low backlash, high speed reduction ratio and compact size. Analytical investigation and modeling of backlash and hysteresis for harmonic drive are reported, but no thorough understanding of backlash as well as its attributes is available in literature. In this research, the experimental and analytical studies of backlash are conducted. First, the definition of backlash according to China National Standard (CNS) is given, which also matches the requirements in application. A dedicated apparatus was designed to measure the backlash. The results of this paper offer an insight into backlash of harmonic drive and the results are valuable in the mechanical design of harmonic drive gears as well as the dynamic modeling and precision control of harmonic drive systems.

1. Introduction
Harmonic gear drive transmissions are compact, low-backlash, high-ratio, high-resolution rotary motion transmissions. Military and civil mechanism benefit from these attributes [1]. In space applications, high-stability SADA (Solar Array Drive Assembly) and high-precision antenna GDA (Gimbal Drive Assembly) take advantage of the low-backlash characteristic of harmonic drive to reduce the disturbance caused by vibration of large flexible components. The value of backlash in harmonic drive transmission required is less than 3 arc minutes or even 1 arc minute. The positional errors of harmonic drive transmission can be simply divided into two categories, the kinematic error, which refers to pointing error when the input rotation is unidirectional, and backlash error when reversing rotation is included. Emelyanov [2] carried out a mathematical analysis on the kinematic error and concluded that the error was due to physical imperfections and assembly tolerance of the three principle components in harmonic gear. The results in the report revealed that the kinematic error periodically occurred twice per wave generator rotation. Besides, dynamic models of harmonic drive transmission for various applications were built [3] [4] [5], which all reduced the maximum pointing error through implementing compensation. Though the kinematic error in harmonic drive has been adequately studied, the literature known on backlash error is relatively few, possibly due to the difficult access to sufficient experimental data which will be explored later. Because of the complex source of backlash, we adopt experiment measurement in this paper to investigate the characterization of backlash. When appropriate, the seemingly random test data is explained analytically.

2. Measurement of backlash
Even though few experimental researches were obtained from known literature, the definition of backlash is generally agreed. According to CNS GB/T 12601-90, the backlash of harmonic drive which...
is also called lost motion is the angle lag of the output shaft when the input shaft change the rotational direction in working condition. Hence the harmonic gear should be driven at input end with backlash measured at the output end, as shown in Figure 1. Furthermore, the term “in working condition” in the definition indicated that appropriate inertia or load of torque should be included at the output end. In this section we present a precise characterization on the contribution of backlash and emphasize our research on one particular aspect.

2.1. Mechanism of backlash
Through analysis of transmission principle, the backlash of harmonic drive transmission could be decomposed into two dominant components including lost motion caused by clearance and lost motion caused by elastic deformation. The source of clearance mainly includes tooth clearance between circular spline and flexspline, the clearance from Oldham coupling in the wave generator, while the elastic deformation is mainly due to flexspline under load of torque. The above process could be shown in Figure 2 by analysing the transmission chain. The notation $\Delta \Phi$ is the summation of clearance distributed among the transmission chain. $K_t$ is the torsional stiffness, a variable which increases with increasing load. The research in our experiment in the following sections focus on lost motion caused by clearance.

![Figure 1. Backlash measurement apparatus](image1)

![Figure 2. Harmonic drive transmission model](image2)

![Figure 3. Theoretic clearance calculation diagram](image3)

A method of calculation of tooth clearance for involute profile tooth in design state[6] is shown in Figure 3. A coordinate system $\{0\text{xyz}\}$ is fixed to the center of wave generator with $y$ coaxial with major axis, and $\{0\text{x1y1z1}\}$ is fixed to a meshing flexspline tooth with $y_1$ coaxial with its symmetric line. Since the minimal clearance occurs at tooth top of the flexspline (point $K_1$), it is chosen to calculate the clearance. The coordinates of point $K_1$ and $K_2$ could be defined by the following expression:

$$j \approx \sqrt{(X_{k2} - X_{k1})^2 + (Y_{k2} - Y_{k1})^2}$$

(1)
The center angle or backlash can be expressed as:

\[ \Delta = \frac{l}{\rho} \quad (2) \]

2.2. Measurement apparatus and procedure
The harmonic drive studied in our research is a cup type of 80 size, as shown in Figure 4, and its parameters are listed in table 1. Figure 5 and 6 are photos of the backlash measuring apparatus as well as its profile. In order to minimize the effect of uncertain factors, all testing samples were carefully prepared in assembling except for intended purpose.

![Figure 4. Three main components of harmonic gear](image)

Table 1. Parameter of harmonic drive

| Item                | Value | Unit   |
|---------------------|-------|--------|
| Ratio               | 160:1 |        |
| Design backlash     | 1     | arc minute |
| Flex spline dimension | 80    | mm     |
| Modulus             | 0.25  | mm     |

![Figure 5. View of measurement apparatus](image)

![Figure 6. Profile of backlash measurement apparatus](image)

3. Characteristic of backlash

3.1. General characteristics
The test and analysis of harmonic drive backlash is a hard task due to the phenomenon that units from the same batches may show completely different error signatures and even the same unit shows different backlash in different assembly. What makes it worse is that the test method restricts the number of test data, which makes it difficult to analyze its discipline. However, it is generally agreed that, the backlash is involved with the following sources:

- machining error of gear tooth of flex spline and circular spline;
- machining error of cam profile;
- assemble error of the three components;
- clearance of coupling;
- and flexibility of flexspline and other parts.

Because the harmonic drive is rigidly connected to the input and output shaft and the test is carried out in quasi-static without load of torque, the effects of the last two terms can be removed. In fact, their influences are either relatively clear or sufficiently studied. In the subsequent sections, the machining error and assembly tolerance which have important influences on backlash are experimentally tested using the apparatus described above, and the results are analysed to give a physical explanation.

3.2. Assembly dependence
Another important reason for low backlash characteristic of harmonic drive besides multi-teeth engagement is the existing angle between flexspline and circular spline along rotation axis, as shown in Figure 7. The angle causes higher depth of meshing at outer area of the spline, which results in smaller backlash of harmonic gear with no or little load. When the load of torque increases, the cup flexes to allow more width of flexspline to engage with the circular spline, and the stiffness increases, which explained the variable torsional stiffness discussed in section mechanism of backlash.

![Figure 7. Harmonic drive cross section](image)

Increasing the dimension of the major axis of wave generator could increase the meshing depth of tooth and consequently decrease the backlash or increase the torsional stiffness of harmonic drive. Johnson and Head [7] increased the torsional stiffness of harmonic drive by using wave generator of different major diameter. Increase in insert depth of wave generator into flexspline could similarly achieve the same objective. Harmonic drive with wave generator inserted at different axial position were tested, one of the results were shown in Figure 8. The values on the two curves were tested at the same 26 points uniformly distributed on an output revolution. To enable a visual comparison and minimize the disturbance of irregularity, the results were sorted in ascending order. As shown in figure, with 3 more millimeters into the flexspline along axis, the backlash is obviously reduced, especially at positions of large backlash.

3.3. Life dependence
The backlash requirement is to be satisfied throughout the life, and in systems with backlash compensation, the value should remain stable during the whole life. The wear contains two opposite aspects on backlash: 1) wear of tooth crest and tooth surface could lead to increases of clearance; 2) accumulations of debris in wave generator and tooth slot decrease the backlash. Both the phenomenon has been found in the life test of five units. The accumulations of
debris especially metal debris caused zero backlash or even minus value backlash, and abnormal wear were found in subsequent test after disassembly. For the units which passed the life test of about 4 million revolutions at the input end, variation of backlash is shown in Figure 9.

![Dependence of backlash on penetration depth of wave generator](image1.png)

**Figure 8.** Dependence of backlash on penetration depth of wave generator

![Dependence of backlash on life](image2.png)

**Figure 9.** Dependence of backlash on life

4. Conclusions
In this paper new experimental measurement results and analysis were presented which give new insight into the behavior of backlash in harmonic drives. A dedicated backlash measurement apparatus as well as a test procedure were proposed which conformed to application requirements. Extensive measurements were carried out using the apparatus. Besides, we characterized the dependence of the backlash on assembly and life, revealing the mechanism of backlash under different conditions. The measurement results obtained in this paper are very useful in mechanical design of harmonic drive gear and precision dynamical control systems involving backlash compensation.

5. References
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