Physics across the phase diagram of QCD

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Abstract. In this talk I present some lattice computations which constrain finite size effects in fireballs resulting from heavy-ion collisions. The results show that early stages of evolution are not constrained by spatial size, whereas freezeout is. The second conclusion is compatible with results of the analysis of fluctuations of conserved quantities, which is a finite-size scaling analysis.

There is incontrovertible evidence now that QCD at zero chemical potential ($\mu = 0$) has no phase transition at finite temperature ($T$) [1]. However, there are phase transitions in nearby unphysical theories which throw shadows on the thermal behaviour of QCD. One such theory is the version of QCD with massless quarks (the chiral limit) which has a second order phase transition. The other is the version with quark masses all larger than the QCD scale $\Lambda$ (the quenched theory) which has a first order phase transition. Squeezed between these two platonic worlds, real world QCD shows a gentle crossover between the hadronic world at low $T$, and the quark gluon plasma at high $T$: the entropy density in units of temperature changes from about unity at $T = 100$ MeV to about 10 at $T = 200$ MeV [2].

As with any gentle change, if one tries to pin down a point at which the change occurs, then different definitions yield different results. On using a measure of deconfinement (technically, the temperature at which the Polyakov loop susceptibility peaks) one finds $T_c(P) = 170 \pm 4 \pm 2$ MeV. If, on the other hand, one uses a measure of chiral symmetry breaking (technically, the temperature at which the chiral susceptibility peaks) then one finds $T_c(\chi) = 152 \pm 3 \pm 3$ MeV. In both these cases the first error is statistical and the second is an estimate of the error in setting the scale [3].

A direct comparison of lattice predictions for fluctuation variables, which we discuss later, with experimental data finds $T_c(P) = 175^{+1}_{-4}$ MeV [4], in agreement with lattice results of [1]. With this established, it would seem to be useful to explore the physics of this gentle crossover in various ways.

In order to interpret data from heavy-ion collisions one has to make an assumption that the fireball produced in the collisions thermalize. This assumption is tested in various ways. However, it would be good to know whether the fireball is large enough, compared to QCD scales, that thermodynamics is applicable. This involves the measurement of microscopic lengths scales called screening lengths, $\xi$, in QCD. The physics is intrinsic to any plasma; for any conserved charge, mobile charge carriers in a plasma screen a free charge within a typical scale $\xi$. As a result, at length scales much larger than $\xi$, details of interactions can be neglected and thermodynamics is applicable.
Figure 1. Screening lengths with valence hypercube smeared staggered quarks (HYP) and thin-link staggered sea quarks. Shown are the inverse of the screening lengths in units of the temperature for colour singlet scalar (S), pseudo-scalar (PS), vector (V), axial vector (AV) obtained with a $\bar{q}q$ source and a spin half (N) $qqq$ source. The horizontal lines indicate the values expected in a theory of free quarks, and the scale $T_c$ is $T_c(P)$.

Recently an extended study of screening lengths has been completed in QCD with dynamical staggered quarks with improved scaling properties [5] (see Figure 1). At temperatures fairly close to $T_c(P)$, for example, at $T = 1.2T_c(P)$, it was observed that all screening lengths are compatible with those in a theory of weakly interacting quarks [6, 7]. As a result at a temperature of about $2T_c(P)$, which could be about the initial temperature of the fireball at large collision energies, all quark correlation lengths are about 0.2 fm. If the fireball is produced in central Pb-Pb collisions, then its size, $L$, is no less than 6 fm. Since $L \gg \xi$, there is no obstruction to using thermodynamic models for the fireball if the dynamics is fast enough to equidistribute energy among various modes. Collisions at higher energies produce a fireball initially at higher $T$ and closer to $\mu = 0$, so the lattice results indicate that thermodynamics could be applicable at all such beam energies. One may have to re-examine this at lower beam energies, where the initial $T$ is lower and $\mu$ is appreciable.

As the fireball expands and cools, one may ask whether finite volume effects become more or less important; the fireball expands, but as it cools the correlation lengths become significantly longer. At the highest RHIC energies freezeout occurs at $T \simeq 165 = 0.94T_c(P)$ MeV, where the smallest $\xi \simeq T$ (see Figure 1). In central collisions, the fireball size at freezeout could be around 10 fm, which means $L/\xi \simeq 8$. As a result, thermodynamics should be good in central collisions, but finite size effects should be important in medium centrality and peripheral collisions.

The study of screening has yielded a little surprise recently [8]. In the quenched theory, where there is a first order deconfining phase transition, no precursors to the transition had been observed. In a study of screening masses in this theory at $T = 0.95T_c(P)$ the screening of various hadrons and their parity partners was examined recently using improved Wilson quarks. In agreement with previous observations [9] it was found that there is no observable thermal effect in the screening of mesons. Extending this measurement to nucleons we again observed no
Figure 2. Screening lengths in quenched QCD with clover improved Wilson valence quarks (the horizontal bar denotes the value, and the vertical bar denotes the error). Shown is the inverse of the product of the zero temperature mass and the screening length at $T = 0.95T_c(P)$ in colour singlet pseudo-scalar (PS), vector (V), axial vector (AV) meson channels and in the nucleon (N) channel. Also shown are the splitting of the V and AV and the N and its parity partner $N^-$, both in units of the zero temperature mass difference between them. The horizontal line shows the values expected if there is no thermal effect.

thermal effects. However, the parity partner of the nucleon seems to be strongly affected by the thermal medium, as a result of which the mass splitting between these two hadrons decreases significantly (see Figure 2). This is something completely new and unexpected, and is worth checking in future.

Information of relevance to experiments is available from lattice computations at finite chemical potential [10]. This comes in the form of a quantitative description of finite size effects. The need for detailed understanding of finite size effects is shown by observations of event-to-event fluctuations of conserved charges.

The argument is simple. It has long been known that thermodynamic fluctuations can be seen only when the volume observed becomes comparable to microscopic dimensions. In the first approximation the spectrum of fluctuations is Gaussian, with computable higher order corrections. The further the deviation is from Gaussian, the more is the importance of microscopic physics. The STAR experiment measured the cumulants of baryon number and found that cumulants up to the 4th order were non-vanishing [11]. In a Gaussian distribution cumulants of order 3 or higher are zero; so finite volume effects studied through these cumulants give information on QCD [12].

Ratios of cumulants which are thermodynamic variables of state were constructed in [12, 13], and computed in lattice QCD in [10]. Agreement of these quantities in QCD and experiment was demonstrated in [11] and this agreement was used to extract $T_c(P)$ in [4].

Ingredients in the lattice computations are the quark number susceptibilities (QNS) [14]
Figure 3. The QNS computed with staggered quarks and lattice spacings $a = 1/(N_t T)$.

and their higher order analogues [15]. In [15, 10] these were computed with a lattice spacing $a = 1/(4T)$ and $a = 1/(6T)$. Now these have been extended to $a = 1/(8T)$ [16] (see Figure 3). Interestingly, the change in the QNS due to this last step is marginal. Computations of the non-linear susceptibilities and further checks are now under way. They will form useful inputs to further analysis of heavy-ion experiments.

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