Pulsar Timing Residual induced by Wideband Ultralight Dark Matter with Spin 0,1,2

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The coherent oscillation of ultralight dark matter induces changes in gravitational potential with the frequency in nanohertz range. This effect is known to produce a monochromatic signal in the pulsar timing residual. Here we discuss a multi fields scenario that produces a wide spectrum of frequencies, such that the ultralight particle oscillation can mimic the pulsar timing signal of stochastic gravitational wave background. We discuss how ultralight dark matter with various spins produces such a wideband spectrum on pulsar timing residuals and perform the Bayesian analysis to constrain the parameters.

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I. INTRODUCTION

The pulsar timing array (PTA) observations are sensitive to the gravitational waves of frequencies around the nanohertz range, including EPTA [1], PPTA [2] and NANOGrav [3] experiments. The PTA observations can probe various astrophysical phenomena including gravitational waves from e.g. supermassive black hole binaries (SMBHs), inflation, cosmic strings, modified gravity, phase transitions etc [4]. Interestingly, the recent NANOGrav has reported some evidence for a stochastic common-spectrum process after the analysis of 12.5 year data set [5–7], which has also been supported in the analysis of PPTA and EPTA data set [8, 9].

Significantly, such kind of common-spectrum can be interpreted as stochastic gravitational waves background (SGWB) from various sources [10, 11]. If the signal is from the SGWB, the characteristic strain of gravitational waves $h_c(f)$ has the amplitude of order $10^{-15}$ around the frequency $f_s \approx yr^{-1} \approx 31.7 \text{ nHz}$, with an extended frequency spectrum [7]. The correlator of the timing residuals is $(R_a R_b) = \Gamma_{ab} \int df S_c(f)$, where $\Gamma_{ab}$ is the overlap reduction function. The spectral density is $S_c(f) = \frac{h_c(f)^2}{12 \pi^2 f^2}$. The measured timing residual of the common-spectrum in frequency domain is

$$ R_c(f) \equiv \sqrt{\frac{S_c(f)}{T_s}} = \frac{1}{\sqrt{3}} \frac{h_c(f)}{2\pi f} \left( \frac{f_s}{f} \right)^{1/2}, \qquad (1) $$

where $T_s$ is the span between the maximum and minimum times of arrival in the pulsar timing array, and $f_s \equiv 1/T_s$. PTA is also sensitive to the non-gravitational wave signals, such as the ultralight dark matter through the timing residual [12–14]. Oscillating ultralight bosons across the cosmic space are interesting dark matter candidates [15–17], which can be either axion-like pseudo scalars or scalar fields such as moduli [18–21]. The ultralight dark photon field (vector) oscillation [22–24], as well as the dark massive graviton oscillation as dark matter from bi-metric gravity [25–28], produced during inflation were also proposed.

The ultralight dark matter can be described by a collection of plane waves with energy $mc^2$ and momentum $mv$, with the typical velocity $v \approx 10^{-3}c$ in the halo. The de Broglie wavelength is

$$ \lambda_{dB} = \frac{2\pi \hbar}{mv} \approx 4 \text{kpc} \left( \frac{10^{-23} \text{eV}}{m} \right) \left( \frac{10^{-3}}{v} \right). \qquad (2) $$
Most of the pulsars in the PTA are located around the distances of order kpc from the earth, within a couple of wavelengths. Especially this ultralight dark matter, also known as the fuzzy dark matter was suggested to resolve small-scale problems of the cold collisionless dark matter models [16, 17]. Notice that these explanations are in tension with the Lyman-alpha forest constraints with a mass around $10^{-22}$ eV [20], although similar constraints on the vector(dark photon)/massive graviton ultralight cases are not that clear yet.

Fuzzy dark matter induces an oscillating pressure, with the angular frequency $\omega_c \equiv 2m$. The frequency is

$$f_c = \frac{\omega_c}{2\pi} = \frac{m}{\pi} \simeq 4.8 \text{ nHz} \left( \frac{m}{10^{-23} \text{eV}} \right),$$

which leads to the oscillation period $T_c = f_c^{-1} \simeq 6.6 \text{ yr} \left( \frac{10^{-23} \text{eV}}{m} \right)$. As the cosmological time scale is much larger than $T_c$, the average value of the pressure is zero, and the field behaves like pressure-less cold dark matter. However, here for the PTA observation, the time scale is of several decades and the oscillation in the pressure needs to be considered.

It is noteworthy that the oscillation frequencies of fuzzy dark matter models fell into the sensitive region of PTA observations. It was already proposed earlier that the ultralight boson oscillation has a similar effect as the gravitational wave in the single pulsar timing deviations [12, 14, 23], although for a single field dark matter candidate with the fixed mass, the signal is monochromatic. String theory naturally gives rise to more than one complex scalar field from compactification. Those stabilized fields can form an extended-spectrum [30–32]. For example, if we consider extra dimensions or equivalent discretized theories like clockwork [33, 34], a spectrum of scalars/vectors/tensors can arise and form a spread mass spectrum. Such particle spectrum can produce an extended frequency spectrum in timing residuals.

This paper is organized as follows. In section II, we review how an ultralight field can induce the PTA signals with a frequency-dependent monochromatic spectrum. In section III, we calculate how a multi-fields spectrum can produce an extended timing residual spectrum in PTA, and discuss the extended spectra of ultralight dark matter with various spins. In section IV, we list and discuss the result of Bayesian analysis. In section V, we conclude and discuss related issues. In the following, we take the natural units that $\hbar = 1, c = 1$.

II. OSCILLATION INDUCED TIMING RESIDUALS

In this section, we review the induced timing residual from single field ultraliight dark matter. The ultralight dark matter can be described by the action

$$S = \int \sqrt{-g} \sqrt{-\tilde{g}} \mathcal{L}_f,$$

where $s = 0, 1, 2$ represent the scalar, vector and tensor fields, respectively. And the energy momentum tensor is given by $T_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}}$.

The ultralight particle oscillation induces the oscillation of the energy-momentum tensor, so as to trigger the oscillation in the metric. As an approximation, we consider the flat background, with the perturbed metric in the Newtonian gauge

$$ds^2 = - (1 + 2\Phi) dt^2 + [(1 - 2\Psi) \delta_{ij} + h_{ij}] dx^i dx^j. \quad (4)$$

From the Einstein equations $G_{\mu\nu} = \frac{1}{M_P^2} T_{\mu\nu}$, the trace parts of the perturbations lead to

$$-2\nabla^2 \Phi = \frac{1}{M_P^2} T_{tt}, \quad (5)$$

$$2\Psi + \frac{2}{3} \nabla^2 (\Phi - \Psi) = \frac{1}{3M_P^2} T_{kk}, \quad (6)$$

where the spatial derivatives $\nabla^2 \equiv \delta^i j \partial_i \partial_j$. The traceless part of the metric perturbation $h_{ij}$ satisfies

$$\ddot{h}_{ij} - \nabla^2 h_{ij} = \frac{1}{M_P^2} \left( T_{ij} - \frac{1}{3} \delta_{ij} T_{kk} \right). \quad (7)$$

Here we focus on dark matter oscillations across the coherent length at the kpc scale and neglect the cosmic expansion.

A. Spin-0: massive scalar field

We begin with the massive scalar field $\phi$, with the Lagrangian density

$$\mathcal{L}_\phi = -\frac{1}{2} (\partial \phi)^2 - \frac{1}{2} m^2 \phi^2. \quad (8)$$

Assuming the profile of the scalar field $\phi(x, t) = \phi(x) \cos [mt + \theta_0(x)]$, with the random phase $\theta_0(x)$ and plugging this ansatz into the stress energy tensor. After dropping the sub-dominant part $(\nabla \phi)^2 \sim m^2 v^2 \phi^2$, we obtain the energy density $\rho_{\phi} \equiv -T_{tt} \simeq \frac{1}{2} m^2 \phi^2$, and pressure $p_{\phi} \equiv \frac{1}{2} T_{kk} \simeq -\rho_{\phi} \cos (2 (mt + \theta_0(x))]$. This oscillation in the pressure induces the oscillation in the gravitational potentials. In the metric (4), it is enough to consider the leading oscillating contributions from cos type $\Psi \sim \Psi(x) + \Psi_{\phi} \cos [2 (mt + \theta_0(x))]$, and similarly for $\Phi$. After considering the linearized Einstein equations and $\rho_{\phi} = \alpha_0 \rho_{DM}$, we can reach

$$\Psi_{\phi} = \frac{1}{8M_P^2} \rho_{\phi} \simeq 6.5 \times 10^{-16} \alpha_0 \left( \frac{10^{-23} \text{eV}}{m} \right)^2. \quad (9)$$

For an pulse sent at the point $\{x_0, t_0\}$ with frequency $\omega_0$ and detected at the earth $\{x_0, t_0\}$ with with frequency $\omega_{\phi}(t)$, the Doppler effect leads to

$$z_{\phi}(t) \equiv \frac{\omega_0 - \omega_{\phi}(t)}{\omega_0} \simeq \Psi(x_{\phi}, t_0) - \Psi(x_0, t_0). \quad (10)$$

Considering $k/\omega \simeq v \sim 10^{-3}$, we have neglected higher order terms as discussed in [12]. For the scalar
oscillation, the change of frequency can induce the timing residual in the pulse $R_\phi(t) = \int_{0}^{t} \dot{\phi}(t') dt'$, with the amplitude $\dot{\phi} = \frac{\Psi_\phi}{m} \sin (mD + \theta_\phi - \theta_0)$, and $D \equiv |x_\phi - x_0|$. The average square value of pulsar-timing residual is
\[
\Delta t_\phi = \sqrt{\langle \dot{\phi}^2 \rangle} \simeq \frac{1}{\sqrt{2}} \frac{\Psi_\phi}{m} \simeq 30 \text{ns} \left(\frac{10^{-23} \text{eV}}{m}\right)^3 \alpha_0.
\] (11)

It is induced by the oscillation of gravitational potential background $\Psi_\phi$ in (9).

We can compare this with the timing residuals caused by a monochromatic gravitational wave signal with the strain $h_\phi$ and amplitude $\tilde{h}_\phi = \frac{h_\phi}{\sqrt{2}} \sin \left(\frac{\omega D(1-\cos \theta)}{2}\right)(1 + \cos \theta) \sin (2\psi)$ [35, 36]. After integrating over $\theta, \phi, D$, we have
\[
\Delta t_h = \sqrt{\langle \tilde{h}^2 \rangle} \simeq \frac{1}{\sqrt{6}} h_\phi \omega.
\] (12)
The factor of $\frac{1}{\sqrt{2}}$ from Hellings-Downs relation [37] has been considered. We can indentify (11) with (12). Considering $\omega = 2m$, we obtain
\[
h_\phi = 2\sqrt{3} \Psi_\phi = \frac{\sqrt{3}}{4M_p^2} \frac{\rho_\phi}{m^2} \simeq 5.2 \times 10^{-17} \alpha_0 \left(\frac{f_{\gamma \gamma}}{f}\right)^2,
\] (13)
where the frequency $f = m/\pi$ and $\rho_\phi = \alpha_0 \rho_{DM}$ have been considered. Thus, the scalar field dark matter can mimic the monochromatic gravitational wave in PTA. For the spin one and spin two ultralight dark matter, we briefly summarize the results as below.

### B. Spin-1: massive vector field

For the massive vector field $A_\mu$, we consider the Lagrangian density
\[
L_A = -\frac{1}{4} F^2 - \frac{1}{4} m^2 A^2.
\] (14)
The oscillating solution of the vector field is given by
\[
A_\mu = A_\mu \cos [mt + \theta_1(x)],
\] (15)
where $\theta_1(x)$ is a random phase. Such background oscillation can also induce oscillation in the trace part as well as a traceless part in the metric (4).

For ultralight vector particle, the trace part contributes to the timing residuals, which do not depend on the polarization of the ultralight vector field. Its magnitude is $1/3$ of the scalar case, as derived in [23],
\[
h_A = \frac{\sqrt{3}}{12M_p^2} \frac{\rho_A}{m^2}.
\] (16)

Thus, the vector field dark matter can also produce the pulsar timing signal at the monochromatic frequency.

For the traceless part of spacial perturbation $h_{ij}$ in the metric (4), following the calculation in [4, 23], the photon propagating null geodesic with the perturbed metric is $d\tau^2 = -\frac{\mu^\mu}{\alpha^\mu} \xi^\mu$. Then we have the modified frequency at the linear level: $\frac{df_\phi}{d\chi} = -\frac{1}{2} h_{ij} \omega_0 n_i n^j$, where $\omega_0$ is the unmodified frequency and $n^j$ is the unit three-vector. After the integration, we can arrive at:
\[
z_A(t) \equiv \frac{\omega_0 - w_A(t)}{\omega_0} = \frac{1}{2} h_{ij} [h_{ij}(t_A) - h_{ij}(t_0)].
\] (17)

For the traceless part, the residual depends on the polarization direction as $h_A = \frac{\sqrt{3}}{6M_p^2} (1 + 3 \cos 2\theta) \frac{\rho_A}{m^2}$ [23].

The vector oscillation as discussed in [23] has a preferred direction at each spatial point. It is interesting to discuss whether all the polarization directions aligned or not within a coherent patch. Notice that the current pulsar timing array roughly spans the same length as the ultralight dark matter coherent length $\sim$ kpc. If vector polarization has a preferred direction, we can already see it from the single pulsar timing residues, as pointed out in [23]. If the polarization directions for the vector oscillation are stochastic and randomly selected at each point, it is useful to discuss the average isotropic effect. We can then average over the sphere $(\theta, \phi)$ of all possible directions, and the traceless part above averages to zero.

### C. Spin-2: massive tensor field

For a massive spin-2 field $M_{\mu\nu}$, according to the dark matter candidate in Bi-metric gravity [38], we consider the Fierz-Pauli Lagrangian density
\[
L_M = \frac{1}{2} M_{\mu\nu} \mathcal{E}^{\mu\nu\rho\sigma} M_{\rho\sigma} - \frac{1}{4} m^2 (M_{\mu\nu} M^{\mu\nu} - M^2),
\] (18)
where $\mathcal{E}^{\mu\nu\rho\sigma}$ is the Liebernowicz operator, which is defined via $\mathcal{E}^{\mu\nu} = \delta^\mu_{\nu} \delta^\rho_{\sigma} \Box - \delta^\rho_{\mu} \delta^\nu_{\sigma} \Box + g_{\rho\sigma} \nabla^\mu \nabla^\nu - \delta^\rho_{\mu} \nabla^\nu \nabla^\sigma - \delta^\mu_{\nu} \nabla^\rho \nabla^\sigma$, and $M = g^{\mu\nu} M_{\mu\nu}$. The oscillating solution of the spin-2 field is given by
\[
M_{ij} = M \cos [mt + \theta_2(x)] \xi_{ij},
\] (19)
where $\theta_2(x)$ is a random phase, and $\xi_{ij}$ is a symmetric and traceless polarization matrix with unit norm.

Now if we consider its impact on the pulsar arriving residual, one can refer to the interaction as below.
\[
S_{int} = -\frac{\alpha_2}{2M_p} \int d^4 x \sqrt{-g} M_{\mu\nu} T^{\mu\nu},
\] (20)
where $\alpha_2$ characterizes the mixing parameter between the spin-2 field and the matter field. The contribution of $M_{0\mu}$ are in higher order and higher derivative, so we neglect them and work with $\tilde{g}_{ij} = \delta_{ij} + \frac{\alpha_2}{M_p} M_{ij}$. For a free photon with unperturbed four-momentum $\xi^\mu = (\omega_0, \omega_0 n^j)$, we
have the geodesics $\frac{d^2x}{dt^2} = -\frac{\alpha_2^2}{2M^2} \Lambda_{ij} n^i n^j$, where $\lambda$ is the affine parameter. Then keeping only the linear terms in $\alpha_2$ and performing the integral, we have

$$z_M(t) = \frac{\omega_0 - u_M(t)}{\omega_0} = \frac{\alpha_2}{2M_P} n^i n^j [M_{ij}(t_M) - M_{ij}(t_0)],$$

(21)

where $\omega_0$ is the unperturbed frequency at the pulsar.

One can follow the treatment in Bi-metric gravity, absorb $M_{ij}$ in the metric and calculate the delay in the pulses arriving time. For the stochastic background we also need to average over celestial sphere of the spin-2 polarization. The average square value of pulsar-timing residual is $\Delta t_M \simeq \frac{2m_M}{\sqrt{5}\sigma_M} \sin (mt + \theta_2)$ [38]. This corresponds to the gravitational strain

$$h_M = \frac{\alpha_2}{M_P} \frac{\mathcal{M}}{\sqrt{5}} = \frac{\alpha_2 \sqrt{2\rho_M}}{M_P} \frac{\mathcal{M}}{\sqrt{5}m}$$

(22)

Here $\rho_M$ can be set as before $\rho_{DM} = 0.4 \text{ GeV/cm}^3$.

III. WIDEBAND MASS SPECTRUM

It is reasonable to assume there are more than one scalar/axion field in nature, e.g. the multi-scalar/axions scenario [30], especially from N-flation. We can see the discussion on axions as inflatons from compactification [31, 32], where the mass distribution satisfies random matrix Marcenko-Pastur law [39]. The fuzzy dark matter can also be realized in string compactification, where the dark matter mass is suppressed by the large volume $\mathcal{L}$. For the multi-fields model suggested before in Nflation [31, 32], the mass distribution of a large number of axions is described by a random matrix theory. Here, however, we do not treat axion as inflaton and fix the axion masses to be around $10^{-2} \text{eV}$. This is very far from the axion mass discussed in the N-flation models.

We briefly discuss the mass distribution from the random matrix theory here. The mass matrix can be captured by $M = R^T R$, where $R$ is the $L \times N$ rectangular matrix. Following Marcenko-Pastur law from 1967 [39], the eigenvalue probability distribution is $P(m^2) = \frac{\sqrt{m^2 - m^2_+} \sqrt{(m^2_+ - m^2)}}{2 \pi^2 m^2},$ for $m^2 \leq m^2 \leq m^2_+$, with $m^2_+ = \sqrt{\delta^2 (1 \pm \sqrt{\beta})}$, $\beta = \frac{N}{L}$. The probability density is zero outside this range, and we also have $\langle m^2 \rangle = \delta^2$. In more general cases of compactification, one can identify $\beta$ with the number of (real) axions involved in inflation divided by the total number of moduli chiral multiplets. The value $\beta$ is rather model-dependent. For example, one can refer to [31, 32] for a discussion in N-flation KKLT model. For the multi-component fuzzy dark matter, we assume a similar distribution with some generic values of the parameters.

Now we consider a spectrum of ultralight particles described by the multi-fields model with total number $N$, satisfying a distribution $P(m) = dm/dm$, and $\int dm P(m) = 1$. The axion number in the mass range $[m, m + dm]$ is $NP(m) dm$. Thus, we have $P(m) dm = \mathcal{P}(m^2) dm^2$, which leads to $P(m) = 2m \mathcal{P}(m^2)$. The Marcenko-Pastur distribution is

$$P_B(m) = \frac{m}{\pi^2 \delta^2} \sqrt{\left(1 - \frac{m^2}{m^2_+}\right) \left(\frac{m^2}{m^2} - 1\right)}.$$  

Besides, as a comparison in the Bayesian analysis, we also consider the Rayleigh distribution

$$P_\sigma(m) = \frac{m}{\sigma^2 - \frac{m^2}{2\pi^2}}.$$  

It is a distribution for nonnegative-valued random variables. In the next section, we will use these distributions for the Bayesian analysis with the observed data.

A. Fuzzy dark matter

The ultralight scalar dark matter can also be described by the summation of multi fields $\Phi(x, t) = \Sigma f \phi(x) \cos [mt + \theta_1(x)]$. Their oscillations will induce the perturbations in the metric. We consider the distribution of the multi fields with spectral density $P(m)$. In continuous limit, the total profile for the fields is $\tilde{\Phi}(x, t) = \int dm P(m) \tilde{\phi}(m) \cos [mt + \theta(m)]$, where $\theta(m)$ is the random phase. The distributions of energy density and pressure are given by $\tilde{\rho}(m) \simeq \frac{1}{2} m^2 \tilde{\phi}(m)^2 P(m)$, and $\tilde{\rho}(m) \simeq - \tilde{\phi}(m) \cos [2 (mt + \theta_m)]$. The total energy density can be written as

$$\rho_\phi \equiv \int dm \tilde{\rho}(m) = \int dm \frac{1}{2} m^2 \tilde{\phi}(m)^2 P(m).$$

(25)

To extend the spectrum, we can either assume $\tilde{\phi}(m)^2$ or $P(m)$ as different distributions. Notice that different forms of $\tilde{\phi}(m)^2$ also depend on the thermal history of the universe, and axion potentials. It is rather model-dependent, see e.g. [15]. Here we take a convenient choice $\tilde{\phi}(m)^2 = \frac{2 \rho_\phi}{m^2}$, which leads to

$$\tilde{\rho}(m) \simeq \rho_\phi P(m), \quad \int dm P(m) = 1.$$  

(26)

We can make the ansatz for the oscillation part of the potential in the frequency space $\Psi_{osc}(x, t) = \int dm \tilde{\Psi}(m) \cos (2mt + \theta_2).$ After considering the Einstein equations, we can reach the differential gravitational potential induced by the differential axions energy density $d\rho(m)$ in the mass range $[m, m + dm]$. We can use the result (9) in the previous section directly

$$\tilde{\Psi}(m) = \frac{1}{8M_P^2} \tilde{\rho}(m) m^2 = \frac{1}{8M_P^2} \frac{\rho_\phi P(m)}{m^2}.$$  

For each mode we have the result from the monochromatic case in (13), thus we reach the wideband distribution

$$h_\phi^f(f) = \frac{\alpha_0}{M_P} \frac{\sqrt{3} \rho_{DM}}{4\pi f} P(\pi f),$$

(27)

where $m = \pi f$ and $\rho_{DM} \equiv \alpha_0 \rho_{DM}$ have been used.
B. Fuzzy dark photon

Other interesting cases include the spin-1 and spin-2 ultralight dark matter, which can also induce signals in the pulsar timing residual, see e.g. [23] and [38]. One thing to notice is that both spin-1 and 2 ultralight fields are polarized to certain directions and they are known to produce anisotropic signals in pulsar timing residual. However, for the extended frequency spectrum with multi-fields, we can assume their polarized directions are randomly selected and the average effect is isotropic. Here we generalize the extended spectrum case to the spin-1 and spin-2 fields in this and following subsection.

The dark photon background can produce a directional anisotropic residual signal [23] or be isotropic with a distinct signature in the two pulsar correlation. We can simply extend the spectrum of the trace part of dark photon induced residuals from previous section. The extended spectrum is then given by

$$h_c^A(f) = \frac{\alpha_1}{M_P^2} \frac{\sqrt{3} \rho_{DM}}{12\pi f} P(\pi f), \quad (28)$$

where \(\rho_A \equiv \alpha_1 \rho_{DM}\) has been used. Since it is simply \(1/3\) of the scalar case \(h_c^0(f)\), we will not fit them with the data later.

As we discussed already, to produce the isotropic signal, we can look at a model where the polarizations of the vector fields are randomly selected across the space. Then we need to integrate over all polarization angles. To distinguish this isotropic signal from the stochastic gravitational-wave background, we can further look at two pulsar signal correlations, known as the Hellings and Downs curve for the stochastic GW [37]. One can see more discussion of these different modes and correlation curves in [23, 38, 41, 42].

C. Fuzzy dark graviton

One can refer to [27, 38] and the previous section for the bi-metric graviton ultralight dark matter model construction. The local oscillation of such fields with a wide-band can then be \(\int dm M(m) \cos (nt + \theta_m(x)) \varepsilon_{ij} P(m)\), where \(\theta_m(x)\) is a random phase, and \(\varepsilon_{ij}(x)\) is unit-norm, traceless and symmetric polarization tensor to be averaged over. We also have the total dark matter density as \(\rho_M = \int dm \frac{1}{2} m^2 M^2 P(m)\). Again we take a simplified and convenient ansatz that \(M^2 = \frac{2\mu}{m}\). With the replacement of \(m \rightarrow 2\pi f\), we then have

$$h_c^M(f) = \frac{\alpha_2}{M_P^2} \frac{m M^2(m)}{\sqrt{3}} = \frac{\alpha_2}{M_P^2} \frac{2\sqrt{\rho_M}}{\sqrt{3}} P(2\pi f). \quad (29)$$

Notice here is different from the scalar and vector case where \(m \rightarrow \pi f\), and \(\rho_M\) will set to be \(\rho_{DM}\).
We list the best fitting parameters in Table I. For the parameter choice, we consider the mass range for ultra-light particles down to $10^{-25}$ eV and fit $\alpha_0$, $\alpha_2$ as free parameters. CMB data have shown the ultralight axions below $10^{-25}$ eV can only compose a very small fraction of dark matter [43–45]. Since we consider a wide spectrum of particles, we do not require $\alpha_0 = \rho_0 / \rho_{DM}$ to be strictly one in the fitting. Also notice that the best fitted spectrum of spin-1 particle is identical to the spin-0 case, with $3\alpha_0$ as the overall factor.

FIG. 3. The results of Bayesian analysis on the first five frequency bins of NANOGrav 12.5-yr data set for the spin-0 and spin-2 ultralight dark matter with Marcenko-Pastur distribution (23). Here $\alpha_i$ and $\beta_i$ are dimensionless, and $\delta_i$ is plotted with the unit of Hz.

FIG. 4. The results of Bayesian analysis on the first five frequency bins of NANOGrav 12.5-yr data set for the spin-0 and spin-2 ultralight dark matter with Rayleigh distribution (24). Here $\alpha_i$ is dimensionless, and $\sigma_i$ is plotted with the unit of Hz.
V. DISCUSSION AND CONCLUSION

The ultralight dark matter can produce the pulsar timing signals at the monochromatic frequency. Here we generalize it to an extended-spectrum, provided a multi-fields scenario. Then the induced effects are similar to the isotropic stochastic gravitational-wave background in PTA. We also discuss the higher spin cases with vector/dark photon oscillation and spin-2 oscillation. We can see different signals: either with an angular dependence in single pulsar timing residuals as suggested in [23], or isotropic signal with a special curve in two-point timing residuals correlation, provided that the background oscillation polarizations are randomly selected over the galactic scale [41, 42]. The later isotropic cases for both spin-1 and spin-2 can mimic the stochastic gravitational-wave background. The ultralight dark matter can also arise from emergent and holographic scenarios [46–50], which would be interesting to study their signals in PTA. For other modes and correlation curves in the metric and pulsar timing array, see e.g. [51–56].

In summary, we discuss PTA signals from the ultralight dark matter oscillation. Especially we generalize it to an extended-spectrum, provided a common-spectrum process in the search for the ultralight scalars as cosmological dark matter, with this generalization, the ultralight dark matter can also be a viable candidate explaining the claimed common-spectrum at nano-Hz in NanoGrav 12.5 years dataset. This study provides more interpretations for the NanoGrav results along with [37–92] and more means to identify the ultralight dark matter in current and future PTA data. In the future, FAST and SKA can further probe or constrain such wideband ultralight dark matter.

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