Quantized Circulation of Anomalous Shift in Interface Reflection

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A particle beam may undergo an anomalous spatial shift when it is reflected at an interface. The shift forms a vector field defined in the two-dimensional interface momentum space. We show that, although the shift vector at individual momentum is typically sensitive to the system details, its integral along a closed loop, i.e., its circulation, could yield a robust quantized number under certain symmetry conditions of interest. Particularly, this is the case when the beam is incident from a trivial medium, then the quantized circulation of anomalous shift (CAS) directly manifests the topological character of the other medium. We demonstrate that the topological charge of a Weyl medium as well as the unconventional pair potentials of a superconductor can be captured and distinguished by CAS. Our work unveils a hidden quantized feature in a ubiquitous physical process, which may also offer a new approach for probing topological media.

In the simplest picture, a light beam reflected at a flat sharp interface should follow the law of reflection, which assumes that the reflected and the incident beams meet at the same point on the interface. However, the wave nature of light brings a twist to this simple picture: the reflected beam may acquire an anomalous spatial shift from the incident point, as illustrated in Fig. 1(a). The longitudinal and the transverse components of this shift (defined with respect to the incident plane) represent the well-known Goos-Hänchen effect [1] and Imbert-Fedorov effect [2, 3], respectively. Remarkably, the analogous effects have also been discovered in electronic scattering [4–16], and most recently in Andreev reflection [17–19]. For the latter, the shift occurs at a normal-metal/superconductor interface, between the incident electron beam and the reflected hole beam. These examples demonstrate that the effect is rather ubiquitous for both classical and quantum systems.

In this work, we unveil a hidden feature of this general effect. We construct a character, termed as the circulation of anomalous shift (CAS), which is the integral of the shift vector along a closed path in the interface momentum space. We show that for incident medium satisfying certain symmetries, CAS must take a quantized value, analogous to the circulation in superfluid. This quantization applies for a class of interesting cases. For example, we consider the shift for a beam incident from a trivial medium onto the interface with a Weyl medium, and show that the quantized CAS characterizes the topological charge of the Weyl point. As another example, we show that the CAS for Andreev reflection at the interface between a simple metal and a superconductor can distinguish different superconducting pairing types. The quantized feature makes CAS a robust topological quantity against perturbations. The findings here also provide a new approach for probing nontrivial topological or superconducting states.

**Anomalous shift in interface scattering.** We start by presenting a general formula for this shift. Consider the setup in Fig. 1(a), where a flat interface between two media is formed at the $z = 0$ plane. A beam of particles $\Psi_{k^i}$, incoming from the upper medium (referred to as the incident medium) is scattered at the interface into a reflected beam $\Psi_{k^r}$ and a transmitted beam $\Psi_{k^t}$. The $\Psi$’s are usually modeled by wave packets, and are required to be confined in both real and momentum spaces. $k^\alpha$ ($\alpha = i, r, t$) denote their average momenta, and must share the same conserved component $k_0$ in the interface $(x-y)$ plane. To analyze the scattering, the beam is expanded using the scattering eigenstates. For instance, $\Psi_{k^i} = \int dk' w(k' - k^i) \psi_i^{k'}$, where $w$ is the profile of the beam peaked at $k^i$, and $\psi_i^{k} = e^{ik\cdot r}|u_k^i|$ is the Bloch eigenstate of the incident medium with $|u_k^i|$ being the cell-periodic part. The scattering of each partial wave $\psi_i^r$ into reflected wave $\psi_r^r$ and transmitted wave $\psi_t^t$ is captured by the scattering amplitudes $r$ and $t$, respectively. Hence, the reflected beam can be expressed as

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\(\text{FIG. 1. (a) Schematic figure showing the anomalous spatial shift } \ell \text{ for a beam reflected at an interface. (b) } \ell \text{ forms a vector field in the domain } S \text{ (where reflection occurs) in the interface momentum } k_0\text{-space. The CAS } \kappa_C \text{ is defined for any closed loop } C \in S.\)
\( \Psi_{k_r} = \int dk' w(k' - k) r(k') \psi_{k_r} \), similar for the transmitted one. The anomalous shift is found by comparing the center positions of the beams at the interface.

Following the standard approach \[8, 12, 17, 20\], the spatial shift for the reflected/transmitted beam can be obtained as

\[
\ell' = \left\langle u_{k_r}^s \right| \frac{i}{\partial k} \left| u_{k_r}^s \right\rangle - \left\langle u_{k_r}^i \right| \frac{i}{\partial k} \left| u_{k_r}^i \right\rangle - \frac{\partial}{\partial k} \arg(s),
\]

where \( s = r, t \), and in the last term, we have abused the superscript \( s \) to also denote the scattering amplitude.

In this formula, the first two terms each is the in-plane component of the Berry connection \( \mathcal{A} = \langle u| i \nabla_k | u \rangle \) for the corresponding state, which is an intrinsic band geometric property. The last term shows that the shift depends on the phase but not the magnitude of the scattering amplitude. In Fig. 1(a), we considered a single reflected/transmitted beam. The result also applies when there are multiple scattering channels, simply by inserting the state and the scattering amplitude for the corresponding beam \( s \). We note that the formula closely resembles the result for the side jump at an impurity derived by Sinitsyn, Niu, and MacDonald \[21\]. For the case of a single channel with total reflection, it reduces to the result obtained in Ref. \[22\].

**CAS and its quantization.** Now let us proceed to the concept of CAS. For concreteness, we focus on the shift for the reflected beam and neglect the superscript \( r \) in the following discussion.

As from Eq. (1), the shift \( \ell' \) is a function of the interface momentum \( k_{\parallel} \), which is conserved during scattering. Assuming the equi-energy surface in the incident medium takes a simple convex shape, \( \ell' \) forms a vector field defined in a domain \( S \) (where reflection occurs) in the interface momentum \( k_{\parallel} \)-space [Fig. 1(b)]. Then the CAS \( \kappa_C \) along a closed loop \( C \subseteq S \) is defined as

\[
\kappa_C \equiv \oint_C \ell' \cdot dk_{\parallel}.
\]

Using formula (1), it can be expressed as two contributions

\[
\kappa_C = \Delta \gamma - \oint_C \frac{\partial}{\partial k_{\parallel}} \arg(r) \cdot dk_{\parallel}.
\]

Here, the first contribution \( \Delta \gamma \) stands for the integral of the first two terms along \( C \). Importantly, the second contribution represents the phase winding of the reflection amplitude. As long as the loop does not hit any singularity of \( \arg(r) \), this term must give an integer multiple of \( 2\pi \), namely, we should have

\[
\kappa_C = \Delta \gamma + 2\pi N, \quad N \in \mathbb{Z}.
\]

Due to the \( \Delta \gamma \) term, \( \kappa_C \) need not be quantized for the most general case. However, there are many cases of interest, where \( \Delta \gamma \) vanishes and we indeed have a quantized CAS. This is true (at least) when either of the following conditions is satisfied. (i) The incident medium is “trivial”, in the sense that its Berry connection vanishes. This is the case when the medium has a real representation (i.e., described by a real Hamiltonian). If satisfied, \( \Delta \gamma \) must vanish, and \( \kappa_C = 2\pi N \). The condition is met, e.g., for photonic or phononic systems with \( \mathcal{PT} \) symmetry, and for electronic systems with negligible spin-orbit coupling and \( \mathcal{PT} \) symmetry. (ii) The incident medium has a reflection symmetry \( \mathcal{M}_r \) which connects the incident and the reflected states (this mirror must be parallel to the interface). In this case, even though the Berry connections for the incident and the reflected states may be nonzero, their in-plane components must cancel out and hence \( \Delta \gamma = 0 \).

Under the above conditions, the CAS takes a quantized value, analogous to the circulation in superfluid \[23\]. Importantly, the quantized CAS is solely determined by the scattering amplitude \( r \), or more specifically, by the phase winding of \( r \), which encodes the information of the other medium at \( z > 0 \) (referred to as the target medium). A nontrivial CAS value \(( N \neq 0 )\) indicates generically the presence of vortices in the vector field \( \ell' \), and the locations of these vortices correspond to the singularities in the phase of \( r \). As we shall see, the CAS can manifest the distinct features of the target medium. Moreover, the quantization of CAS endows it with a topological robustness, namely, its value is robust against perturbations on the system. Thus, the quantized CAS provides a powerful way to characterize medium properties.

To illustrate these points, we consider two concrete examples below.

**Probing a Weyl medium.** In the first example, we take a trivial incident medium \(( z < 0 )\) described by a simple two-band semiconductor-like model

\[
H_T = \left( \frac{k^2}{2m} + \Delta \right) \sigma_z,
\]

where \( 2\Delta \) is the gap between the two bands, and the Pauli matrix \( \sigma_z \) here stands for a pseudospin (e.g., orbital) degree of freedom. The target medium \(( z > 0 )\) that we try to probe is a Weyl medium \[24, 25\], described by

\[
H_T = vk_x \sigma_x + w(k_z^2 \sigma_+ + k_z^2 \sigma_-) + U + \delta k^2 \sigma_x,
\]

where \( k_{\pm} = k_x \pm ik_y, \sigma_{\pm} = (\sigma_x \pm i\sigma_y)/2, n \) is a positive integer, \( v, w, U, \) and \( \delta \) are model parameters. The first two terms describe a Weyl point at the origin with a topological charge

\[
\nu = \text{sgn}(v)n.
\]

For the case with \( n = 1 \) and \( w = v \), they reduce to the usual Weyl model \( vk \cdot \sigma \); and the case with \( n > 1 \) corresponds to the so-called multi-Weyl point with higher topological charges \[26, 27\]. \( U \) represents a potential energy shift across the interface. The last term is a small quadratic term added to ensure a well-defined boundary condition with \( H_T \), which does not affect the essential physics, and we may take \( \delta \to 0 \) in the final results.
Because $H_1$ is trivial, according to our theory, the CAS must be quantized. More importantly, as we shall see, the CAS can take a nonzero value, which is determined by the topological charge $\nu$ of the Weyl point inside the target medium.

Let us first consider the conventional Weyl point with $n = 1$. Without loss of generality, we assume the incident beam has energy $E > \Delta$, i.e., in the upper band. Our discussion of CAS is within the domain $S$ with $k_\| < k_M$, where $k_M = \sqrt{2m(E - \Delta)}$ is the maximal interface momentum at $E$. For each $k_\| \in S$, the reflection amplitude $r$ and the shift $\ell$ can be directly calculated using the standard approach [28].

Here, to probe the Weyl point, we are most interested in the situation when the Weyl point energy is close to $E$, as illustrated in Fig. 2(a). In such a case, the equi-energy surface of $E$ in the Weyl medium has a radius $k_W = |(E - U)/w| < k_M$ in the $k_x$-$k_y$ plane, so its projection in the $k_\|$-plane lies within $S$. Now, consider the annulus-shaped region $A \subset S$ with $k_W < k_\| < k_M$. Within $A$, we must have total reflection with $|r|^2 = 1$, because there is no transmitted state in the Weyl medium with such $k_\|$. This indicates that arg($r$) has no singularity in $A$, and $\ell$ is analytic in this region. Therefore, for any two topologically equivalent loops $C_1, C_2 \subset A$, we must have $\kappa_{C_1} = \kappa_{C_2}$.

As any contractible loop in $A$ must have $\kappa = 0$, let us consider a non-contractible loop $C \in A$ that encircles the inner hole of the annulus [see Fig. 2(b)]. According to our analysis, the calculation of $\kappa_C$ can be simplified by noting that $\kappa_C = \kappa_{C_0}$, where $C_0$ is a regular circle with radius $k_\|$ in $A$. Through straightforward calculations [28], we obtain the transverse component of the shift $\ell_\phi(\equiv \ell \cdot \hat{\phi})$,

$$\ell_\phi = \frac{1 - |\zeta|^2}{k_\| \times |\zeta + i\eta e^{-i\phi}|^2}, \quad (8)$$

where $\phi = \text{arg}(k_x + ik_y)$ is the polar angle of $k_\|$, $\zeta = (p_+ + mp\nu)/(nmv + ip_+)$, $\eta = \text{sgn}(E - U)(E - U - yp_0)^{1/2}$, with $p_\pm = [2m(E \pm \Delta) \pm k_\|^2]^{1/2}$ and $p_0 = \text{sgn}(E - U)(E - U)^2 - w^2k_\|^2]^{1/2}/v$. With this result, we obtain

$$\kappa_C = \kappa_{C_0} = k_\| \int_0^{2\pi} \ell_\phi d\phi = -2\pi \nu. \quad (9)$$

This remarkable result shows that an arbitrary loop enclosing the hole (i.e., the equi-energy surface of the Weyl medium) has a nontrivial quantized CAS, which is solely determined by the topological charge $\nu$ of the Weyl point. Moreover, this result is independent of whether the Weyl point is above or below the energy $E$.

The nontrivial CAS in Eq. (24) indicates the existence of vortex in the phase winding of $r$ inside the central region with $k_\| < k_W$. The vortex center corresponds to the singularity in arg($r$), which occurs here at $r = 0$. In Fig. 2(c), we plot the numerical result for arg($r$), which confirms the presence of a vortex, and the vortex center coincides with the zero-point of $r$. In the limit of $E \gg \Delta, (E - U)$, we can derive an analytic expression for the shift around the vortex center $k_\|$, which exhibits the behavior [28]

$$\ell \propto \nu(q_y - q_x)/q^2, \quad (10)$$
where $q$ is the momentum measured from $k_\parallel^*$. This confirms the nontrivial circulation pattern around the vortex. In addition, one can easily see that the result in (24) applies to any loop $C \in S$ that encloses the point $k_\parallel^*$. The above analysis can be directly extended to multi-Weyl points with $n > 1$. The key finding is that our result (24) remains valid [28], namely, the CAS is nontrivial, and the quantization integer $-\kappa_C/(2\pi)$ just corresponds to the topological charge of the Weyl point. In Fig. 3, we plot the calculation results for double-Weyl ($n = 2$) and triple-Weyl ($n = 3$) points. One observes that there exist $n$ vortices inside the $k_\parallel < k_W$ region, which are responsible for the nontrivial quantized CAS.

**Probing a superconductor.** As a second example, we consider the CAS for Andreev reflection at a normal-metal/superconductor interface. Here, we take the model of a simple metal for the incident medium ($z < 0$), so its Bogoliubov-de Gennes (BdG) Hamiltonian is given by [29]

$$H_T = \left(\frac{k^2}{2m} + \frac{U}{E_F}\right)\tau_z + (\Delta \tau_+ + \Delta^* \tau_-), \tag{12}$$

where $\Delta$ is the superconducting pair potential and $\tau_\pm = (\tau_z \pm i\tau_y)/2$.

Here, we focus on the shift $\ell$ during Andreev reflection for an incident electron beam with excitation energy $\varepsilon$. Again, because the incident medium ($H_I$) is trivial, the shift is solely determined by the Andreev-reflection amplitude $r_A$, and its CAS must be quantized. In the following, we shall investigate how the different pairing types (encoded in $\Delta$) affect the value of CAS.

We first consider the $s$-wave pair potential with $\Delta$ given by a constant $\Delta_0$. Straightforward calculation [28] shows that the shift only has a longitudinal component [Fig. 4(a)], with

$$\ell = \frac{2(p_+^2 - p_-^2)\tan \alpha}{p_sp_n[4p_s^2p_n^2 + (p_s^2 + p_n^2)^2 \tan^2 \alpha]}k_\parallel, \tag{13}$$

where $p_n = (2m(E_F - k_\parallel^2)^{1/2})$, $p_s = [2m(E_F - U) - k_\parallel^2]^{1/2}$, and $\alpha = -i\cosh^{-1}(\varepsilon/\Delta_0)$. Meanwhile, one can check that $\arg(r_A)$ has no singularity within $S$. Thus, the CAS vanishes in this case.

The situation is dramatically different for chiral pair potentials, described by $\Delta = \Delta_0 e^{-i\chi \phi}$ with $\chi$ a nonzero integer. For this case, $\ell$ has not only a longitudinal component [given by the same expression as (13)], but also a transverse component

$$\ell_\phi = -\frac{\chi}{k_\parallel}. \tag{14}$$

One immediately notes that the CAS is nontrivial for any simple loop $C$ encircling the origin [see Fig. 4(b)] and

$$\kappa_C = -2\pi \chi, \tag{15}$$

i.e., the quantized CAS is solely determined by the chirality of the pairing. Figure 4(c) further confirms the vortex in the phase winding of $r_A$. Note that different from the case in Fig. 2, the singularity here is due to the fact that $\phi$ in the pair potential is undefined at $k_\parallel = 0$.

Finally, we consider the $d_{x^2-y^2}$-wave case. Here we find that $\Delta = \Delta_0 \cos(2\phi)$. Figure 4(d) shows the numerical result for $\ell$, which generally has both longitudinal and transverse components. In the figure, the four gray-colored sectors mark the so-called suppressed zones [19], corresponding to the four nodes of the $d$-wave pairing gap, where $\varepsilon > |\Delta|$ and the shift vanishes [28]. Excluding the suppressed zones from $S$, then within $S$ there is no isolated singular point. Thus, for a loop $C \in S$, we should have $\kappa_C = 0$.

**Discussion.** Quantized quantities are always fascinating (and rare) in physics. Here, we reveal a new member in an ubiquitous process. Moreover, the quantized CAS encodes the topological information of the medium, therefore may offer a new approach for characterization. As we have shown, this could be particularly useful when using a trivial incident medium to probe a nontrivial target medium, which actually conforms with the common experimental setup. Note that this setup is distinct from the previous works on shift in Weyl semimetals [12, 13, 15, 16], where the Weyl semimetal is taken as the incident medium, so the quantization does not occur there.

The experimental detection of the anomalous shift has been well developed in the field of optics [32–36]. Photonic crystals simulating Weyl and other topological band

![FIG. 4. Calculated $\ell$ field in Andreev reflection for (a) $s$-wave, (b) chiral $p$-wave, and (d) $d_{x^2-y^2}$-wave pair potentials. (c) shows the phase of $r_A$ for the chiral $p$-wave case. Here, we take $\varepsilon = 0.02$ eV, $m = 0.04m_e$, $\Delta_0 = 0.04$ eV, $E_F = 0.1$ eV, and $U = 0.3$ eV.](image-url)
structures have also been achieved [37–40]. Hence, the circulation pattern, the vortex, and the quantized CAS should be easily probed in the optical context. Meanwhile, several methods for detecting the shift in electronic systems were also proposed, such as by engineering specific junction geometry [13, 17, 41] and by enhancing the shift via multiple reflections [12, 19]. The shift there can typically reach $10 \sim 100$ nm [19, 41], much larger than the lattice scale. It is particularly interesting to probe the shift for superconductors (in Fig. 4), which can help to characterize unconventional superconductivity.

**SUPPLEMENTAL MATERIAL FOR “QUANTIZED CIRCULATION OF ANOMALOUS SHIFT IN INTERFACE REFLECTION”**

**CALCULATION DETAILS FOR THE FIRST EXAMPLE**

Below, we present more details about the calculation for the first example in the main text, i.e., when the interface is formed by a trivial incident medium and a Weyl target medium.

**Derivation of Eq. (8) and (9) in main text**

With Eq. (5) and Eq. (6) in the main text, the scattering state can be written as

$$
\Psi_{k}(r) = \begin{cases}
\psi_{k_+}^+(1) e^{ik_x x + ik_y y + ik_z^0 z}, & z < 0, \\
t + \psi_{k_+}^+ + t_- \psi_{k_-}^- & z > 0.
\end{cases}
$$

The expressions for the basis states $\psi_{k_+}^+$ and $\psi_{k_-}^-$ in the incident medium ($z < 0$) are given by

$$
\psi_{k_+}^+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{ik_x x + ik_y y + ik_z^0 z},
$$

$$
\psi_{k_-}^- = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{ik_x x + ik_y y + ik_z^0 z},
$$

$$
\psi_{k_+}^+ = \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{ik_x x + ik_y y + ik_z^0 z}.
$$

Here, $k_+ = p_+ = \sqrt{2m(E + \Delta) - k^2}$, $k_- = -p_+ = -ip_+$, with $p_\pm = \sqrt{2m(E \mp \Delta) - k^2}$. Meanwhile, the basis states in the target medium are given by (here we keep a general $n$ for the Weyl medium)

$$
\psi_{k_-}^- = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\beta_-} e^{ik_x x + ik_y y + ik_z^0 z},
$$

$$
\psi_{k_+}^+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\beta_+} e^{ik_x x + ik_y y + ik_z^0 z}.
$$

Here, $k_{z,\pm} = \mathrm{sgn}(E) \pm (1/\sqrt{2})^{1/2}$ $(\sqrt{2}d)^{-1}$ $\sqrt{v^2 - 2\delta^2 k^2_\parallel - 2\delta k \cos(\phi) \pm \Lambda}$, $\beta_{\pm} = \arctan(\frac{w k^2 \sin(n \phi)}{\delta k^2 + \delta^2 k^2_\parallel \pm w k^2 \cos(n \phi)})$, $\eta_- = \xi_-$ and $\eta_+ = \frac{\mathrm{sgn}(k_{z,\pm})^n \mathrm{sgn}(E - U) \eta_\pm}{\Lambda}$, with $\Lambda = \sqrt{(v^2 + 2\delta k \cos(n \phi))^2 + 4\delta^2 (E - U)^2 + 4\delta^2 (v^2 k^2_\parallel - w^2 k^2_\parallel)}$.

The boundary condition at $z = 0$ are given by

$$
\frac{1}{2m} \sigma_z \partial_z \Psi \bigg|_{z=0} = \frac{\tilde{v}}{\delta} \sigma_z \Psi(0) + \delta \sigma_z \partial_z \Psi \bigg|_{z=0},
$$

which ensures the current conservation [?]. When taking the limit $\delta \to 0$, one obtains

$$
k_{z,\pm}^* = p_0, \quad k_{z,\pm}^* = \frac{\tilde{v}}{\delta}, \quad \eta_\pm = \left|\frac{\mathrm{sgn}(k_{z,\pm})^n \eta_\pm}{\Lambda}\right|.
$$

with $p_0 = \mathrm{sgn}(E - U) \sqrt{(E - U)^2 - w^2 k^2_\parallel}$ and $\eta = \mathrm{sgn}(E - U) \sqrt{(E - U)^2 - v^2 k^2_\parallel}$.

The reflection amplitude is solved straightforwardly from the boundary condition combined with (17) and (18). The result is

$$
r(k_\parallel) = e^{-2i\gamma} \frac{1 - i\nu \zeta e^{n\phi}}{\xi^* - i\nu \zeta e^{n\phi}},
$$

with $\zeta = \frac{p_+ - mv}{p_- - mv} \times \frac{mv + ip_+}{mv + ip_-}$ and $\vartheta = \arctan \left(\frac{p_+}{mv}\right)$.

In the annulus-shaped region $A \subset S$ with $k_W < k_\parallel < k_M$, there is no transmission into the Weyl medium, so

$$
|\nu| = 1 \Rightarrow r = e^{i\vartheta}.
$$

Thus, by Eq. (1) in the main text, the transverse component of the shift is given by

$$
\ell_\phi = -\frac{1}{k_\parallel} \partial_\phi \varphi = \frac{n(1 - |\zeta|^2)}{k_\parallel \times |\xi| + i\nu e^{-i\nu \phi}}.
$$

This reduces to Eq. (8) of the main text when $n = 1$. With Eq. (23), the CAS for a general $n$ can be obtained as

$$
\kappa_C = \kappa_C^k = k_\parallel \int_0^{2\pi} \ell_\phi d\phi = -2\pi \nu,
$$

where $\nu = \nu(n)$. Thus, the result applies not only for conventional Weyl point with $n = 1$, but also for multi-Weyl points with $n > 1$.

**Shift vector field around the vortex center**

The vortex center here corresponds to the zero point of $r$. By using Eq. (21) with $n = 1$ and $w = v$, and taking
the limit $E \gg \Delta$, $(E - U)$, we can obtain an analytic expression of $r$ around its zero point $k_{\parallel}^*$, given by

$$ r(q) = \frac{\eta(k_{\parallel}^*)}{(k_{\parallel}^*)^2} \xi \left[ (\gamma k_{\parallel}^* - ik_{\parallel}^*)q_x + (\gamma k_{\parallel}^* + ik_{\parallel}^*)q_y \right], $$

(25)

where $q$ is measured from $k_{\parallel}^*$, $\xi = \frac{mv + \sqrt{2m(E - \Delta)}}{mv - \sqrt{2m(E - \Delta)}}$ and $\gamma = \frac{|E - U|}{\sqrt{(E - U)^2 - 2\Delta k_{\parallel}^2}}$. Thus, one can directly obtain the anomalous shift around the vortex center as

$$ \ell(q) = -\partial_q \arg(r) = \text{sgn}(v) f \frac{(q_y - g_x)}{q^2}, $$

(26)

where $f$ is a factor independent of $q$. This gives the result in Eq. (10) in the main text. For multiple-Weyl models with $n = 2, 3$, analytic calculation becomes more tedious. Alternatively, we have numerically checked that there exist $n$ vortices, and the shift around each vortex center has the same behavior as

$$ \ell(q) \propto \text{sgn}(v) \frac{(q_y - g_x)}{q^2}. $$

(27)

**Calculation Details for the Second Example**

The second example is about a normal-metal/superconductor (NS) interface. The calculation follows the standard approach [19, 30]. The scattering states are given by

$$ \Psi_{k}(r) = \begin{cases} \psi_{k^e} + r\psi_{k^e} + r_A \psi_{k^h}, & z < 0, \\ t_+ \psi_{k^e} + t_- \psi_{k^e}, & z > 0. \end{cases} $$

(28)

The basis in the incident medium ($z < 0$) are

$$ \psi_{k^e} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{i k_{x} x + i k_{y} y + i k_{z}^{*} z}, \quad \psi_{k^e} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{i k_{x} x + i k_{y} y - i k_{z}^{*} z}, $$

(29)

where $k_{z}^{*} = \sqrt{2m(E_{F} \pm \varepsilon) - k_{\parallel}^2}$. And the basis in the target medium (S region) ($z > 0$) are given by

$$ \psi_{k^e} = \frac{1}{N} \left( e^{-i \varphi_k} \right) e^{i k_{x} x + i k_{y} y + i k_{z}^{e} z}, $$

(30)

with $\varphi_k = \arg(\Delta)$ denoting the phase of pair potential, $k_{z}^{e, \pm} = \pm \sqrt{2m(E_{F} - U \pm \varepsilon^2 - |\Delta|^2) - k_{\parallel}^2}$, $\alpha = -i \cos^{-1} \left( \frac{\varepsilon}{\Delta} \right)$, and $N$ is a normalization parameter. Notice that $\alpha$ is real for $\varepsilon < |\Delta|$, whereas it becomes imaginary for $\varepsilon > |\Delta|$.

The boundary condition is given by

$$ \Psi(z = 0^-) = \Psi(z = 0^+), \quad \frac{1}{2m} \partial_z \Psi \bigg|_{z=0^-} = \frac{1}{2m} \partial_z \Psi \bigg|_{z=0^+}, $$

(31)

With the boundary condition and the basis, all the four scattering amplitudes can be solved. Here, we are only interested in the Andreev reflection, for which the amplitude is

$$ r_A = 2(k_{z}^{S, +} - k_{z}^{S, -}) e^{i \varphi_k} \left[ e^{i \alpha} (k_{z}^{S, +} + k_{z}^{S, -}) (k_{z}^{S, +} - k_{z}^{h}) e^{-i \varphi_k} \right]^{-1}. $$

(32)

The mean field requirement of superconductivity is that $E_{F} - U > |\Delta|$, $\varepsilon$. Under this condition, the above expression is reduced to

$$ r_A = \frac{2p_s p_n e^{i \varphi_k}}{|2p_n \cos \alpha + i (p_s^2 + p_n^2) \sin \alpha|} e^{i \alpha'}, $$

(33)

with

$$ p_n = \sqrt{2m(E_{F} - k_{\parallel}^2)}, \quad p_s = \sqrt{2m(E_{F} - U - k_{\parallel}^2)}, $$

(34)

$$ \alpha' = -\arctan \left[ \frac{(p_n^2 + p_s^2) \tan \alpha}{2p_n p_s} \right]. $$

(35)

One observes that as $p_n$ and $p_s$ are real parameters, so the phase of $r_A$ has a rather simple expression, given by

$$ \arg(r_A) = \begin{cases} \alpha' + \varphi_k, & \varepsilon < |\Delta|, \\ \varphi_k, & \varepsilon > |\Delta|. \end{cases} $$

(36)

and then the anomalous shift in Andreev reflection is

$$ \ell = \begin{cases} -\frac{\partial}{\partial k_{\parallel}} (\alpha' + \varphi_k), & \varepsilon < |\Delta|, \\ -\frac{\partial}{\partial k_{\parallel}} \varphi_k, & \varepsilon > |\Delta|. \end{cases} $$

(37)

According to above equation, one knows that the anomalous shift for Andreev reflection has dependence on both the magnitude (included in $\alpha'$) and phase ($\varphi_k$) of the pair potential. Particularly, when the excitation energy is larger than superconducting gap, e.g. $\varepsilon > |\Delta|$, the shift for Andreev reflection is solely determined by the phase of the pair potential. Since Andreev reflection is typically sizable when the excitation energy is lower than the pairing gap, i.e., $\varepsilon \leq \Delta_0$, we will focus on this regime in the following discussion of the three types of pair potentials.

**s-wave**

For a s-wave superconductor with $\Delta = \Delta_0$, $\alpha = -i \cosh^{-1} \left( \frac{\varepsilon}{\Delta_0} \right)$ and $\varphi_k = 0$. Hence by Eq. (37), the shift for Andreev reflection is obtained as

$$ \ell = \frac{2(p_n^2 - p_s^2)^2 \tan \alpha}{p_s p_n [4p_n^2 p_s^2 + (p_s^2 + p_n^2)^2 \tan^2 \alpha]} (k_{x}, k_{y}), $$

(38)

which gives the Eq. (13) in the main text.
chiral \( p \)-wave

For a chiral \( p \)-wave pair potential, with \( \Delta = \Delta_0 e^{-i\phi} \) \([\phi = \arg(k_x + ik_y)]\), we have \( \varphi_k = -i \cos^{-1} \left( \frac{\Delta_0}{r} \right) \) and \( \varphi_k = \chi \phi \). Compared with the s-wave case, the shift here has one more contribution from the phase \( \varphi_k \). A direct calculation gives

\[
\ell = \left\{ \begin{array}{ll}
\frac{2(p_x^2 - p_y^2)^2 \tan \alpha}{p_x p_y [4p_x^2 p_y^2 + (p_x^2 + p_y^2)^2 \tan^2 \alpha]} (k_x, k_y) + \chi \frac{k_y}{k_\parallel} (k_y, -k_x), \\
\end{array} \right.
\]

which reduces to the result of s-wave when \( \chi = 0 \). The second term above gives the transverse component

\[
\ell_\phi = -\frac{\chi}{k_\parallel} \phi.
\]

One can easily observe that the result also applies for higher-order chiral pair potentials with \( |\chi| > 1 \).

\[ \text{d-wave} \]

For the \( d_{x^2-y^2} \) pair potential described by \( \Delta = \Delta_0 \cos 2\phi \), we have \( \varphi_k = 0 \). The pairing gap has four nodes. Around the nodes, the excitation energy \( \varepsilon \) would be larger than the superconducting gap \( |\Delta| \), where the shift vanishes according to Eq. (37). These are the suppressed zones. The Andreev reflection amplitude can be obtained as

\[
r_A = \frac{2p_x p_y \Delta_0 \cos 2\phi}{2p_x p_y + i(p_x^2 + p_y^2) \sqrt{\Delta_0^2 \cos^2 2\phi - \varepsilon^2}},
\]

One finds that at the node position (where \( \cos 2\phi = 0 \)), \( r_A \) has two zero lines along \( k_x = \pm k_y \) located at the center of the suppressed zones. And there is no isolated zero point of \( r_A \). For \( \varepsilon < |\Delta_0 \cos 2\phi| \), straightforward calculation gives

\[
\ell = \Pi \left[ \frac{(p_x^2 - p_y^2)^2 \sqrt{\Delta_0^2 \cos^2 2\phi - \varepsilon^2}}{p_x^2 p_y^2 (p_x^2 + p_y^2)} (k_x, k_y) \right]

+ \frac{\Delta_0^2 \sin 4\phi}{k_\parallel \sqrt{\Delta_0^2 \cos^2 2\phi - \varepsilon^2}} (k_y, -k_x),
\]

with \( \Pi = \frac{2p_x p_y (p_x^2 + p_y^2) \varepsilon}{4p_x^2 p_y^2 + (p_x^2 + p_y^2)^2 \Delta_0^2 \cos 2\phi - \varepsilon^2} \). Thus, a complete expression of the shift can be written as

\[
\ell = \Theta(|\Delta_0 \cos 2\phi|) - \varepsilon \Pi \left[ \frac{(p_x^2 - p_y^2)^2 \sqrt{\Delta_0^2 \cos^2 2\phi - \varepsilon^2}}{p_x^2 p_y^2 (p_x^2 + p_y^2)} (k_x, k_y) \right]

+ \frac{\Delta_0^2 \sin 4\phi}{k_\parallel \sqrt{\Delta_0^2 \cos^2 2\phi - \varepsilon^2}} (k_y, -k_x).
\]

The shift here also has both longitudinal and transverse components.

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