ABSTRACT

In this article we present results from three on–going projects related to the formation of protoplanets in protostellar discs. We present the results of simulations that model the interaction between embedded protoplanets and disc models undergoing MHD turbulence. We review the similarities and differences that arise when the disc is turbulent as opposed to laminar (but viscous), and present the first results of simulations that examine the tidal interaction between low mass protoplanets and turbulent discs. We describe the results of simulations of Jovian mass protoplanets forming in circumbinary discs, and discuss the range of possible outcomes that arise in hydrodynamic simulations. Finally, we report on some preliminary simulations of three protoplanets of Jovian mass that form approximately coevally within a protostellar disc. We describe the conditions under which such a system can form a stable three planet resonance.

Key words: Protostellar discs; Planets; Binaries.

1. INTRODUCTION

The on–going discovery of extrasolar planets has reinvigorated efforts to understand the formation and evolution of planetary systems (Mayor & Queloz 1995; Marcy & Butler 1996; Marcy, Cochran, & Mayor 1999; Vogt et al. 2002; Santos et al. 2003). As observations are carried out over longer time scales, and with increasing sensitivity, the physical properties of the observed planetary systems are set to diversify significantly. At the present time, however, all of the known planets are Jovian–like gas giants. For this reason, much of the theoretical research currently underway is examining the various stages of formation and evolution of giant protoplanets.

The most widely accepted theory of how giant planets form, the so–called core instability model, suggests that a multi stage process operates. The solid component of protostellar discs gradually coagulates to form a solid core of around 15 Earth masses, onto which a gaseous envelope accretes (e.g. Bodenheimer & Pollack 1986; Pollack et al. 1996). An alternative model suggests that giant protoplanets form via gravitational instability of a protostellar disc (e.g. Boss 2001). In either scenario, interaction between the forming protoplanet and the protostellar disc is likely to significantly affect the evolution, leading to orbital migration.

In the standard picture of disc–planet interactions, a protoplanet exerts torques on a protostellar disc through the excitation of spiral density waves at Lindblad resonances (e.g. Goldreich & Tremaine 1979). These waves carry an associated angular momentum flux which is deposited in the disc where the waves are damped. This process results in a negative torque acting on the protoplanet from the outer disc and a positive torque acting on it from the disc interior to its orbit. For most disc models, the outer disc torque is dominant, leading to inward migration (Ward 1997).

A sufficiently massive protoplanet can open up an annular gap in a viscous disc centred on its orbital radius (Papaloizou & Lin 1984). For typical protostellar disc models the protoplanet needs to be approximately a Jovian mass for gap formation to occur. Recent simulations (Bryden et al. 1999; Kley 1999; Lubow, Seibert, & Artymowicz 1999; D’Angelo, Henning, & Kley 2002) examined the formation of gaps by giant protoplanets, and also estimated the maximal gas accretion rate onto them. The orbital evolution of a Jovian mass protoplanet embedded in a standard laminar viscous protostellar disc model was studied by Nelson et al. (2000). They found that gap formation and accretion of the inner disc by the central mass led to the formation of a low density inner cavity in which the planet orbits. Interaction with the outer disc resulted in inward type II migration on a time scale of a few $\times 10^5$ yr. Gas accretion during migration allows the protoplanet to grow to $\sim 3–4$ Jupiter masses. The disc models in these studies all adopted an anomalous disc viscosity modelled through the Navier–Stokes equations without consideration of its origin.
The most likely origin of the viscosity is through MHD turbulence resulting from the magnetorotational instability (MRI) (Balbus & Hawley 1991) and it has recently become possible through improvements in computational resources to simulate discs in which this underlying mechanism responsible for angular momentum transport is explicitly calculated. This is necessary because the turbulent fluctuations do not necessarily result in transport phenomena that can be modelled with the Navier–Stokes equation.

To this end Papaloizou & Nelson (2003) and Nelson & Papaloizou (2003a) developed models of turbulent protostellar accretion discs and considered the interaction with a giant protoplanet of 5 Jupiter masses. The large mass was chosen to increase the scale of the interaction so reducing the computational resources required. This protoplanet was massive enough to maintain a deep gap separating the inner and outer disc and exert torques characteristic of type II migration. More recent work has expanded the range of protoplanet masses examined (Papaloizou, Nelson, & Snellgrove 2003; Nelson & Papaloizou 2003b). We discuss some of the main points of this work in later sections.

The majority of the planets so far detected orbit around single solar–type stars, but there have been also been detections in binary systems [e.g. γ Cephei (Hatzes et al. 2003), 16 Cygni B (Cochran et al. 1997)]. Most field stars appear to be members of binary systems (Duquennoy & Mayor 1991). For the longer period systems planets may orbit around one member of the binary. In the shorter period systems they could orbit stably around both stars (i.e. circumbinary planets).

The majority of T Tauri stars, whose discs are thought to be the sites of planet formation, also appear to be in binary or multiple systems, (Ghez, Neugebauer & Matthews 1993; Leinert et al. 1993; Mathieu et al. 2000). Most have sufficiently large separations that it is expected that each component will have its own circumstellar disc. For shorter period systems, however, one expects the existence of a circumbinary disc, a number of which have been observed (e.g. DQ Tau, AK Sco, UZ Tau, GW Ori, GG Tau).

The confirmed existence of planets in binary systems, combined with the fact that binary systems appear to be common, and to be present during the T Tauri phase, means that it is of interest to explore how stellar multiplicity affects planet formation, and post-formation planetary orbital evolution, including formation in circumbinary discs. Previous work examined the stability of planetary orbits in binary systems using N–body simulations (Dvorak 1986; Holman & Wiegert 1999). This work showed that there is a critical ratio of planetary to binary semimajor axis for stability, depending on the binary mass ratio, \( q_{\text{bin}} \), and eccentricity \( e_{\text{bin}} \). A recent paper (Quintana et al. 2002) explored the late stages of terrestrial planet formation in the α Centauri system.

This work concluded that the binary companion can help speed up planetary accumulation by stirring up the planetary embryos, thus increasing the collision rate. Recent work by Kley & Burkert (2000) examined the effect that an external binary companion can have on the migration and mass accretion of a giant protoplanet forming in a circumstellar disc. They found that for sufficiently close companions, both the mass accretion rate and the orbital migration rate could be increased above that expected for protoplanets forming around single stars.

The question of how protoplanet evolution is affected in circumbinary discs has been examined recently by Nelson (2003a). This work explored the evolution of Jovian mass protoplanets forming in circumbinary discs using hydrodynamic simulations of a binary star plus protoplanet system interacting with a viscous protostellar disc. The models apply primarily to binaries with orbital periods of \( \sim 1 \) yr, and semimajor axes of \( \sim 1 \) AU that have protoplanets forming at a radius of a few AU in the circumbinary disc, although the results can be scaled to apply to different parameters. It is well known that a giant protoplanet embedded in a disc around a single star undergoes inward migration driven by the viscous evolution of the disc (e.g. Nelson et al. 2000). The work by Nelson (2003a) examines how this process is affected when the central star is replaced by a close binary system, and delineates the various modes of behaviour that arise depending on the properties of the system (e.g. binary mass ratio \( q_{\text{bin}} \), binary eccentricity \( e_{\text{bin}} \), etc). We present some of the main results of this work in later sections.

A number of the known extrasolar planets exist in multi–planet systems. Three of these systems contain a pair of planets that are in mean motion reso-
nance (GJ876, HD82943, 55 Cancri). The most likely explanation for these resonant systems is approximately coeval formation of the planet pair, followed by disc–induced differential migration leading to resonant capture (e.g. Snellgrove, Papaloizou, & Nelson 2001; Lee & Peale 2002). In later sections we present some preliminary results that examine the plausibility of three–planet resonances being discovered in which the three planets are of approximately Jovian mass. The Jovian satellite system displays such a configuration with Io, Europa, and Ganymede all participating in the Laplace resonance (e.g Peale, Cassen, & Reynolds 1979). Here Io and Europa are in 2:1 resonance, and Europa and Ganymede are simultaneously in 2:1 resonance, leading to a 4:2:1 relationship between the mean motion of Io, Europa, and Ganymede. Our preliminary calculations indicate that three planet resonances may indeed be established, but that the 4:2:1 relation is unstable. However, a situation in which one of the planet pair is in a 3:1 resonance, leading to a 6:2:1 or 6:3:1 relation between the mean motions, appears to be stable, suggesting that such configuration may be found among the population of extrasolar planets.

This article is organised as follows. In section 2 we present the results of simulation of high and low mass protoplanets interacting with turbulent accretion discs. In section 3 we describe the results of simulations that examine the evolution of giant protoplanets forming in circumbinary accretion discs. In section 4 we present some preliminary results of three planet systems leading to the formation of three–planet resonances. Finally we summarise the results presented in this article in section 5.

2. TURBULENT DISC – PROTOPLANET INTERACTIONS

Most of the previous work examining the interaction between protostellar disc models and embedded protoplanets have used the Navier–Stokes equations to simulate laminar disc models with an anomalous viscosity coefficient (e.g. Bryden et al. 1999; Nelson et al. 2000). The most likely source of disc viscosity, however, is MHD turbulence that arises because of the MRI (Balbus & Hawley 1991). We present simulations of magnetic, turbulent discs interacting with protoplanets of different mass in the following sections.

2.1. Giant Protoplanets

A detailed discussion of the results presented in this section may be found in Nelson & Papaloizou (2003a). The underlying disc model used is described in detail in Papaloizou & Nelson (2003). The disc was a cylindrical disc model with a locally isothermal equation of state such that the effective aspect ratio $H/R = 0.1$ throughout. The initial disc model was initiated with a zero–net flux vertical magnetic field that varied sinusoidally with radius, and was allowed to run until a statistically steady state turbulent disc model was obtained. The initial value of the volume averaged plasma $\beta$ was $< \beta > \approx 1000$, where $\beta$ is the ratio of the thermal gas pressure to the magnetic pressure. The final turbulent state generated a volume averaged value of the Shakura–Sunyaev stress parameter $< \alpha > \approx 5 \times 10^{-3}$. A protoplanet of mass $M_p = 5$ Jupiter masses was placed in the disc at a radius $r_p = 2.2$. Prior to placing the protoplanet in the disc, a partial gap was made in the disc in order to avoid the generation of transient features in the flow associated with the gap opening process.

A snapshot of the final state of the disc with the protoplanet is shown in figure 1 after $\sim 100$ planetary orbits. The protoplanet showed a clear tendency towards gap formation by deepening and widening the initially imposed partial gap during the simulation. Broadly speaking, the visual appearance of the disc in figure 1 differs significantly from that obtained in an equivalent simulation with a laminar, viscous disc (e.g. Nelson et al. 2000; Nelson & Papaloizou 2003a), with the turbulent disc showing a less regular and more time dependent structure, in which the spiral waves have a more diffused appearance.

In the region of the wakes the disc and its turbulence

Figure 2. This figure shows a close-up of the mid-plane density distribution for a 5 Jupiter mass protoplanet in a turbulent disc

Figure 3. This figure shows magnetic field vectors for equivalent region shown in fig.2.
are strongly perturbed. Magnetic field vectors in the disc midplane in the neighbourhood of the planet are illustrated in figure 3 along with a corresponding density plot for the equivalent region in figure 2. An inspection of the magnetic field vectors indicates that these tend to line up along the location of the wakes but in a somewhat broadened region slightly behind the shocks. In this way an ordered structure appears to be imposed on the flow and magnetic field by the protoplanet. The magnetic stress is largely communicated in these ordered regions, leading to a significant change in the magnetic contribution to the stress parameter $\alpha$ there (Nelson & Papaloizou 2003a).

In addition to running the full MHD turbulent disc model, we have also run some 2D laminar $\alpha$ disc models for the purposes of comparison. Two 2D model were run with $\alpha = 0$ and $\alpha = 5 \times 10^{-3}$, respectively. For initial conditions in these 2D models, we took the midplane density distribution of the 3D turbulent model just prior to the addition of the planet, switched off the magnetic field, and introduced the required value of $\alpha$ in the Navier–Stokes viscosity. The implementation of the Navier–Stokes viscosity is described in Nelson et al. (2000).

The azimuthally averaged surface density distribution for these models, plus the midplane density distribution for the turbulent model, is plotted in figure 4. Each of the models have been run for a total time of $\approx 2050$ time units, corresponding to $\sim 100$ planetary orbits. It is clear that the $\alpha = 0.0$ 2D model (dashed line) has the deepest and widest gap, as expected. It is also apparent that the turbulent model (dotted line) has a deeper and wider gap than the 2D $\alpha = 5 \times 10^{-3}$ model, even though a volume averaged estimate for the underlying turbulent disc model yields an effective $\alpha \approx 5 \times 10^{-3}$ (see figure 18 in Papaloizou & Nelson 2003). Thus, the turbulent model behaves as if it has a somewhat smaller $\alpha$ than reasonable estimates suggest it has. This may arise for the following reasons. First, a Navier–Stokes viscosity with anomalous viscosity coefficient provides a source of constantly acting friction in the disc, such that it can induce a steady mass flow into the gap region. The turbulence, however, does not operate as a constant source of friction that generates steady inflow velocities. Instead it generates large velocity fluctuations that may be much larger than the underlying inflow velocity arising from the associated angular momentum transport. Results presented in Papaloizou & Nelson (2003) indicate that a process of time averaging the turbulent velocity field is required over long time periods before these fluctuations can be averaged out to reveal the underlying mass flow. The disc material in the vicinity of the planet experiences periodic high amplitude perturbations induced by the planet on a time scale much shorter than the required averaging time scale, so that the disc response is expected to differ from that in the case of a disc with Navier–Stokes viscosity. A second plausible reason for the apparently lower $\alpha$ is that the existence of the magnetic field in the turbulent disc allows for field lines to connect across the gap region, and to enable angular momentum transport across the gap. In this way the magnetic field actually helps the planet to maintain the gap (Nelson & Papaloizou 2003a). The differences in gap structure found here suggest that the accretion rate onto a protoplanet in an MHD turbulent disc is likely to be less than previous estimates based on laminar $\alpha$ disc models indicate [e.g. Bryden et al. (1999); Kley (1999); Lubow, Seibert, & Artymowicz (1999); Nelson et al. (2000); D’Angelo, Henning, & Kley (2002)], but not by a large factor.

The velocity field within the Hill sphere of the planet is shown in the left hand panel of figure 4 for the turbulent disc. An equivalent plot for a laminar disc is shown in the right hand panel of this figure. The material that enters the Hill sphere of the protoplanet and becomes gravitationally bound to it is normally expected to circulate around it by virtue of its angular momentum. In the calculation presented here the protoplanet does not accrete gas, and is modelled as a softened point mass. Consequently it is expected that material that enters the Hill sphere will form a hydrostatic ‘atmosphere’ around the planet that is supported through pressure and rotation. In figure 4 we would expect to see circulation occurring in a clockwise fashion for both the MHD turbulent disc and the laminar disc. However, it is apparent that the circulating pattern has been disrupted in the magnetised disc, indicating that magnetic braking may have been responsible for this (Nelson & Papaloizou 2003a). The expected circulation is observed in the non magnetic run performed at the same numerical resolution as shown in the right hand panel of figure 4. Magnetic braking of material that forms a circumplanetary disc inside the Hill sphere will have significant implications for the mass accretion onto the protoplanet.

### 2.2. Low mass protoplanets

In this section we present the results of simulations that examine the interaction of turbulent discs with
low mass, embedded protoplanets. A detailed discussion of the results presented in this section is given in Papaloizou, Nelson & Snellgrove (2003) and Nelson & Papaloizou (2003b). The underlying disc model used in the simulations with low mass protoplanets differed from that described in section 2.1. The disc was a cylindrical disc model with a locally isothermal equation of state. The effective aspect ratio took a constant value $H/r = 0.07$. The disc model was initiated with a zero–net flux toroidal magnetic field that varied sinusoidally with radius. The initial value of the volume averaged plasma $\beta$ parameter $<\beta> \approx 30$. The disc model was evolved until a statistically steady state turbulent disc was obtained. The final turbulent state had an associated volume averaged stress parameter $<\alpha> \approx 7 \times 10^{-3}$ (Papaloizou, Nelson, & Snellgrove 2003).

Once the final turbulent state had been obtained, low mass protoplanets were inserted into the disc model. These had mass ratios $q = m_p/M_* = 10^{-5}, 3 \times 10^{-5}, \text{and} \ 10^{-4}$, respectively. The gravitational potential of the protoplanets was softened using a softening parameter $b = 0.3H_p$, where $H_p$ is the disc semi–thickness at the position of the protoplanet. The primary aim of these simulations was to examine the tidal torques exerted on non gap forming protoplanets by turbulent accretion discs to estimate the corresponding migration rates.

A snapshot of a simulation with $q = 3 \times 10^{-5}$ is shown in figure 6. This corresponds to a protoplanet of $m_p \approx 10$ Earth masses orbiting a solar mass star. The protoplanet is located at $(x,y) = (-3,0)$, and is only just visible in this figure since the density fluctuations generated by the turbulence are in fact of higher amplitude than the spiral wakes generated by the protoplanet. The torque per unit mass exerted on the protoplanet was calculated as a function of time for all simulations, and is shown in figure 7 for the case with $q = 3 \times 10^{-5}$. This figure clearly shows that the force exerted on the protoplanet by the turbulent disc is dominated by high amplitude fluctuations. The total torque exerted on the protoplanet is seen to oscillate between positive and negative values, suggesting that the migration in this case is likely to occur as a ‘random walk’, rather than as a monotonic inward drift normally associated with type I migration (Ward 1997). Figure 8 shows the time evolution of the running time average of the torque per unit mass. The straight line shows the torque obtained from an equivalent simulation with a laminar disc model. The upper line shows the running time average of the torque due to the inner disc, the lower line the torque due to the outer disc, and the middle line the running average of the total torque. The running mean of the total torque is apparently not converging to a well defined value on the time scale of the simulation, and for a large part of the simulation indicates that the protoplanet would migrate outwards on average. Treating the turbulent fluctuations as having a Gaussian distribution, we can estimate the time for the running mean to converge as a function of the amplitude of the fluctuations. Such an estimate yields a time scale of $\approx 100$ planetary orbits for con-
Figure 7. This figure shows the time evolution of the torque per unit mass exerted on \( q = 3 \times 10^{-5} \) protoplanet. The torque is clearly dominated by strong fluctuation due to the turbulence.

vergence, longer than we are currently able to evolve the simulation (Nelson & Papaloizou 2003b).

The simulations with \( q = 10^{-5} \) and \( q = 10^{-4} \) showed qualitatively similar results to those described above. Although we are unable to run simulations for sufficient length to definitively calculate the migration times of protoplanets in turbulent discs, it is clear that the picture of migration that emerges differs significantly from that obtained in laminar disc models. These results offer the possibility that the rapid migration rates of giant protoplanet cores may in fact be slowed down or stopped by interaction with the density fluctuations in a turbulent disc.

3. GIANT PLANETS FORMING IN CIRCUMBINARY DISCS

In this section we present the results of simulations that examine the evolution of giant protoplanets that form in circumbinary discs. A fuller discussion of this work is presented in Nelson (2003a). We consider the interaction between a coplanar binary and protoplanet system and a two-dimensional, gaseous, viscous, circumbinary disc within which it is supposed the giant protoplanet formed. We do not consider the early evolution of the protoplanet in this work or address the formation process itself, but make the assumption that a Jovian mass protoplanet is able to form and examine the dynamical consequences of this.

Each of the stellar components and the protoplanet experience the gravitational force of the other two, as well as that due to the disc. The disc is evolved using the hydrodynamics code NIRVANA (Ziegler & Yorke 1997). The planet and binary orbits are evolved using a fifth-order Runge–Kutta scheme (Press et al.)

Figure 8. This figure shows the time evolution of the running time average of the torque per unit mass. The upper line corresponds to the inner disc torque, the lower line gives the outer disc torque, and the

Figure 9. This figure shows surface density contours for run in which the planet is ejected by the binary.
The force of the planet on the disc, and of the disc on the planet, is softened using a gravitational softening parameter $b = 0.5a_p(H/r)$, where $a_p$ is the semimajor axis of the planet, and $H/r$ is the disc aspect ratio. We assume that the mass of the protoplanet is fixed, and so do not allow accretion of matter from the disc onto the protoplanet.

We adopt a disc model in which the effective aspect ratio $H/r = 0.05$, and the Shakura–Sunyaev viscosity parameter $\alpha = 5 \times 10^{-3}$ (Shakura & Sunyaev 1973). The surface density $\Sigma$ is initialised to have an inner cavity within which the planet and binary orbit (Nelson 2003a). Simulations initiated with no inner cavity show that one is formed by the action of the binary system and planet clearing gaps in their local neighbourhood. As the planet migrates in towards the central binary these gaps join to form a single cavity. The disc mass is normalised through the choice of $\Sigma$ such that a standard disc model with $\Sigma(r) = \Sigma_0 r^{-1/2}$ throughout would contain about 4 Jupiter masses interior to the initial planet radius $r_p$ (assumed in physical units to be 5 AU). Thus the disc mass interior to the initial planet radius would be about twice that of a minimum mass solar nebula model.

The total mass of the binary plus protoplanet system is assumed to be $1 \, M_\odot$. Dimensionless units are used such that the total mass of the binary system plus planet $M_{\text{tot}} = 1$ and the gravitational constant $G = 1$. The initial binary semimajor axis is $a_{\text{bin}} = 0.4$ in all simulations, and the initial planet semimajor axis $a_p = 1.4$. The simulations are initiated with the binary system having an initial eccentricity, $e_{\text{bin}}$, and the protoplanet is initially in circular orbit. The binary mass ratio $q_{\text{bin}} = 0.1$ for all simulations presented in this section, but larger values were considered in Nelson (2003a). The unit of time quoted in the discussion of the simulation results below is the orbital period at $R = 1$.

3.1. Numerical Results

The results of the simulations can be divided into three categories (Mode 1, Mode 2, and Mode 3), which are described below, and are most strongly correlated with changes in the binary mass ratio, $q_{\text{bin}}$, and binary eccentricity $e_{\text{bin}}$. Changes to the disc mass and/or protoplanet mass appear to be less important. In some runs the planet enters the 4:1 mean motion resonance with the binary. The associated resonant angles in the coplanar case are defined by:

\[
\psi_1 = 4\lambda_s - \lambda_p - 3\omega_s \\
\psi_2 = 4\lambda_s - \lambda_p - 3\omega_p \\
\psi_3 = 4\lambda_s - \lambda_p - 2\omega_s - \omega_p \\
\psi_4 = 4\lambda_s - \lambda_p - 2\omega_p - \omega_s
\] (1)

where $\lambda_s$, $\lambda_p$ are the mean longitudes of the secondary star and protoplanet, respectively, and $\omega_s$, $\omega_p$ are the longitudes of pericentre of the secondary and protoplanet, respectively. When in resonance $\psi_3$ or $\psi_4$ should librate, or all the angles should librate. In principle the protoplanet is able to enter higher order resonances than 4:1, such as 5:1 or 6:1, since its initial location lies beyond these resonance locations. However, none of the simulations presented here resulted in such a capture. Test calculations indicate that capture into higher order resonances requires slower planetary migration rates than those that arise in these simulations. For significantly faster migration rates the planet may pass through the 4:1 resonance (Nelson 2003a).

\[\text{Figure 10. This figure shows semimajor axes and eccentricities for run in which planet is scattered by the binary.}\]

3.1.1. Mode 1 – Planetary Scattering

A number of simulations resulted in a close encounter between the protoplanet and binary system, leading to gravitational scattering of the protoplanet to larger radii, or into an unbound state. We label this mode of evolution as ‘Mode 1’. Typically the initial scattering causes the eccentricity of the planet to grow to values $e_p \approx 0.9$, and the semimajor axis to increase to $a_p \approx 6 - 8$. In runs that were continued for significant times after this initial scattering, ejection of the planet could occur after subsequent close encounters.

We will illustrate this mode of evolution using a simulation with $m_p = 3$ Jupiter masses and $q_{\text{bin}} = 0.1$. A series of snapshots of the simulation are shown in figure 9. Mode 1 evolution proceeds as follows. The protoplanet migrates in towards the central binary due to interaction with the circumbinary disc, and temporarily enters the 4:1 mean motion resonance with the binary. The migration and eccentricity evo-
evolution is shown in figure 11, and the resonance angles are shown in figure 11. The resonant angle $\psi_3$ librates with low amplitude, indicating that the protoplanet is strongly locked into the resonance. The resonance drives the eccentricity of the protoplanet upward, until the protoplanet has a close encounter with the secondary star during or close to periapse, and is scattered out of the resonance into a high eccentricity orbit with significantly larger semimajor axis. We note that being in resonance normally helps maintain the stability of two objects orbiting about a central mass. However, when one of the objects is a star, the large perturbations experienced by the planet can cause the resonance to break when the eccentricities are significant. Once out of resonance, the chances of a close encounter and subsequent scattering are greatly increased. This provides a method of forming ‘free–floating planets’.

3.1.2. Mode 2 – Near–resonant Protoplanet

A mode of evolution was found in some of the simulations leading to the protoplanet orbiting stably just outside of the 4:1 resonance. We label this mode of evolution as ‘Mode 2’. Mode 2 evolution is illustrated by a simulation for which $m_p = 1$, $q_{bin} = 0.1$, and $e_{bin} = 0.1$. The evolution of the orbital elements are shown in figure 12. Here, the protoplanet migrates inwards and becomes weakly locked into the 4:1 resonance, with the resonant angle $\psi_3$ librating with large amplitude. The resonance becomes undefined and breaks when $e_p = 0$ momentarily during the high amplitude oscillations of $e_p$ that accompany the high amplitude librations of $\psi_3$. The protoplanet undergoes a period of outward migration through interaction with the disc by virtue of the eccentricity having reattained values of $e_p \simeq 0.17$ once the resonance is broken. Calculations by Nelson (2003b) show that gap–forming protoplanets orbiting in tidally truncated discs undergo outward migration if they are given eccentricities of this magnitude impulsively, due to the sign of the torque exerted by the disc reversing for large eccentricities. The outward migration moves the planet to a safer distance away from the binary, helping to avoid instability. Once the protoplanet has migrated to just beyond the 4:1 resonance the outward migration halts, since its eccentricity reduces slightly, and the planet remains there for the duration of the simulation. The system achieves a balance between eccentricity damping by the disc and eccentricity excitation by the binary, maintaining a mean value of $e_p \simeq 0.12$ (Nelson 2003a). The torque exerted by the disc on the protoplanet is significantly weakened by virtue of the finite eccentricity (Nelson 2003b), preventing the planet from migrating back towards the binary. Continuation of this run in the absence of the disc indicates that the planet remains stable for over $6 \times 10^6$ orbits. This is in good agreement with the stability criteria obtained by Holman & Wiegert (1999) since the protoplanet lies just outside of the zone of instability found by their study.

3.1.3. Mode 3 – Eccentric Discs

A mode of evolution was found in which the planetary migration was halted before the protoplanet could approach the central binary and reach the 4:1 resonance. This only occurred when the central binary had an initial eccentricity of $e_{bin} \geq 0.2$. The migration stalls because the circumbinary disc becomes eccentric. We label this mode of evolution as ‘Mode 3’. We illustrate this mode of evolution by presenting the results of a simulation with $m_p = 1$ Jupiter mass, $q_{bin} = 0.1$, and $e_{bin} = 0.2$. Figure 13 shows snapshots of the surface density at different times during the simulation, with the disc becoming noticeably eccentric. Interaction between the protoplanet and the eccentric disc leads to a dramatic reduction or even reversal of the time–averaged torque driv-
ing the migration. This is because the disc–planet interaction becomes dominated by the $m = 1$ surface density perturbation in the disc rather than by the usual interaction at Lindblad resonances in the disc. Figure 14 shows the evolution of the semimajor axis and eccentricity of the planet, and shows that the migration stalls. Simulations of this type can be run for many thousands of planetary orbits without any significant net inward migration occurring. Such systems are likely to be stable long after the circumbinary disc has dispersed, since the planets remain in the region of stability defined by the work of Holman & Wiegert (1999), and are probably the best candidates for finding stable circumbinary extrasolar planets. Interestingly, spectroscopic binary systems with significant eccentricity are significantly more numerous than those with lower eccentricities (e.g. Duquennoy & Mayor 1991; Mathieu et al. 2000), suggesting that circumbinary planets may be common if planets are able to form in circumbinary discs.

4. THREE PLANET RESONANT SYSTEMS

The are a number of multiplanet systems among the known extrasolar planets. In at least two, and possibly three, of these multiplanet systems, two planets appear to be in mean motion resonance. The planets in the systems GJ876 and HD82943 appear to be in 2:1 resonances, while two of the planets in the 55 Cancri system appear to be in 3:1 resonance. The existence of these mean motion resonances can be explained by a model in which two planets form in the disc approximately coevally, and are driven into resonance by differential inward migration due to interaction with the protostellar disc (e.g. Snellgrove, Papaloizou, & Nelson 2001; Lee & Peale 2002; Nelson & Papaloizou 2002).

In this section we present some preliminary calculations that address the question of whether three-planet resonances may be formed, and remain stable, by a process which is analogous to the above scenario when each of the protoplanets is of Jovian mass. Namely, approximately coeval formation of three planets which are then driven into a three-
planet resonance through interaction with the protoplanetary disc. We examine this question by means of 2-D numerical simulations very similar to those described in section 3 using the same code and basic disc models.

We have considered a number of different scenarios, which we describe below:

(i) Coeval formation of three planets, with the outer two forming a 2:1 resonance, followed by inward migration of the resonant pair until the middle and inner planet themselves become locked in a 2:1 resonance. We refer to this as a 4:2:1 resonance.

(ii) Formation of two planets which differentially migrate and lock into 2:1 resonance, followed by formation of a third planet at larger radius that migrates in and locks into 2:1 resonance with the middle planet. This is another example of a 4:2:1 resonance.

(iii) A scenario very similar to scenario (i), except the middle and inner planet form a 3:1 resonance. We call this a 6:2:1 resonance.

(iv) A scenario similar to scenario (ii), except the outer and middle planets form a 3:1 resonance. We call this a 6:3:1 resonance.

4.1. 4:2:1 Resonances

Both scenarios (i) and (ii) above could potentially lead to the formation of a 4:2:1 resonance. In this subsection we present the results of a simulation that examines scenario (i). This simulation consisted of three Jovian mass protoplanets being inserted into a protostellar disc model on fixed circular orbits for a period of \( \approx 200 \) orbits of the middle planet until partial gap clearing had occurred. The protoplanets were then allowed to evolve under the gravitational influence of the disc. The evolution is shown in figure 15 which shows snapshots of the evolution at four separate times. The orbital evolution is shown in figure 16. The outer planet migrates inward the most rapidly and enters the 2:1 resonance with the middle planet. This resonant pair then migrate in towards the innermost planet. As they approach the orbital radius at which the middle planet becomes locked into 2:1 resonance with the innermost planet, there is a strong gravitational interaction that leads to scattering of the planets. Figure 16 shows that the middle and outer planet interchange during this encounter leading to the middle planet being flung out to larger radii in the disc. All simulations that examine scenario (i) result in similar behaviour, with no 4:2:1 resonance ever forming. However, interesting results are obtained which show that the kind of interaction described here may help some planets survive migration in a disc when they would otherwise migrate into the central star by being ejected to larger radii. The evolution after ejection is likely to involve gap formation and slow inward type II migration.

Simulations that test scenario (ii) suggest that 4:2:1 resonances can be established, but that they are never stable for more than a few hundred orbits. This is primarily because the middle planet has its eccentricity pumped up to large values by virtue of undergoing resonant interaction with both the inner most and outer most planets. These will be described in a forthcoming paper (Nelson & Papaloizou 2003c).
Figure 17. This figure shows contour plots for run designed to examine scenario (iii) described in text.

Figure 18. This figure shows semimajor axes and eccentricities for three planets in run designed to examine scenario (iii) described in text.

Figure 19. This figure shows the resonant angles (upper two panels) associated with 2:1 resonance for outer two planets in scenario (iii) run described in text. The lowest panel shows the difference in the

Figure 20. This figure shows the resonant angles (upper three panels) associated with 3:1 resonance for middle and inner planet in scenario (iii) run described in text. The lowest panel shows the difference in the longitude of pericentre for the middle and inner planet in this run.
4.1.1. 6:2:1 and 6:3:1 Resonances

Simulations have been performed to examine the formation of 6:2:1 and 6:3:1 resonances. These indicate that such resonances may be formed quite easily, and remain stable over long time periods. In this section we present one simulation that examined the possibility of scenario (ii).

The initial setup was similar to that described in section 4.1 except that the inner two planets were more widely separated. The outer most planet migrated inward the most rapidly and locked into 2:1 resonance with the middle planet. The resonant pair then migrated in towards the inner most planet until the middle and inner planet became locked into 3:1 resonance. Figure 14 shows snapshots of the disc and planet evolution. The orbital evolution is shown in figure 15, which shows the converging orbits, and the resulting slow-down in migration as the three planets become resonantly locked leading to an effective increase in the inertia of the system that is being driven inward by the disc. The resonant angles for the outer two planets are shown in figure 16 showing the establishment of the 2:1 resonance, and the resonant angles for the inner two planets are shown in figure 17 showing the establishment of the 3:1 resonance. Also shown in the lowest panels of figures 15 and 16 are the differences in the longitudes of pericentre for each of the planet pair. Figure 18 shows that the middle and inner planet becomes apsidally locked when the outer two planets enter the 2:1 resonance, since the eccentricity driving by the resonance acts as an effective ‘tuning’ mechanism for the establishment of a secular resonance between the inner two planets. We note that such a secular resonance is observed in the Upsilon Andromeda system.

5. SUMMARY

In this article we have presented the results from three distinct projects. The first examined the interaction between embedded protoplanets and turbulent, magnetised discs. Broadly speaking we find rather similar behaviour for high mass, gap forming planets when compared with previous work on laminar but viscous discs. However, low mass protoplanets behave rather differently, and appear to undergo ‘stochastic migration’ due to interaction with the background turbulence. The result appears to be that low mass protoplanets will undergo a random walk in turbulent discs, instead of monotonic inward migration.

We presented the results of simulations that examined the formation of giant protoplanets in circumbinary discs. We find that under a wide range of conditions stable circumbinary planets may be maintained. In particular, if the binary system is significantly eccentric, then disc induced inward migration of the protoplanet does not occur.

Finally, we presented some preliminary calculations of three protoplanets forming in a disc, and examined the conditions under which three–planet resonances could be established and maintained. We found that 4:2:1 configurations could be formed, but quickly became unstable. However, 6:2:1 and 6:3:1 configurations could form and remain stable over very long periods.

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