INTRODUCTION

This world is dependent on fossil fuel and there are various environmental problems caused by human activities such as global warming and pollution from municipal, industrial and agricultural waste. It is necessary for sustainable development to treat wastes adequately and to develop renewable energy. Utilization of biomass wastes has attracted a lot of interest because it is a way of waste management and also a source of renewable energy. Anaerobic digestion is one of them and has high potential to produce energy (Noike et al., 2009; Ghosh et al., 1975). Processing methods of anaerobic digestion are classified by solid content of feedstock and fermentation temperature (IEA BIOENERGY, 2001). The dry thermophilic anaerobic digestion is expected to be suitable in Japan because it occurs a small amount of digestate. Anaerobic digestion is also sorted out by the used feedstock. Anaerobic co-digestion, that is the process using the feedstock made from several kinds of substrates, has some advantages to anaerobic digestion from single substrate due to be able to adjust component of feedstock. Nakajima showed that anaerobic co-digestion from food waste with rich nitrogen and paper with rich carbon improves biogas generation because the C/N ratio of the feedstock is optimized (Nakajima et al., 2016). Food waste and paper constitute the second and third largest unused biomass waste in Japan (Excerpts from “Biomass Nippon Strategy”, Cabinet Decision, March 31, 2006). The efficient recycling method of these waste has been required to achieve sustainable development and the dry thermophilic anaerobic co-digestion is one of the important options.

Although anaerobic digestion has many advantages such as the simultaneous treatment of waste, the utilization of digestate as a fertilizer and the robustness to the environment, it is not popular in Japan. There is the problem that reaction intermediates of anaerobic digestion including volatile fatty acid (VFA) and ammonia cause inhibition of methanogenesis easily. Overloading of organic matter leads to the accumulation of reaction intermediates and results in failure of the process. In spite of the biological vulnerability, anaerobic digestion has the robustness to the environment because the process proceeds in the closed reactor. Anaerobic digestion could play the role of supply and demand adjustment in the energy mix with environmental dependent renewable energies like wind and solar power. The numerical optimization of organic loading rate (OLR) has been studied for various purposes. Mendez-Acosta regulated the VFA concentration and total alkalinity (TA) to improve the stability of anaerobic digestion process with the dynamical model (Mendez-Acosta et al., 2016). Mauky developed the feeding management to compensate the divergence between supply and demand energy with ADM1 (Mauky et al., 2016; Batstone et al., 2002). The used models in these studies were proper for each control strategies (e.g. Mendez-Acosta set up a state variable about TA and Mauky used substantially simplified ADM1 to predict daily biogas production). This study aims to make the anaerobic digestion process stable and produce...
desirable biogas by the control. The model needs to represent fermentation state and predict biogas yield accurately to perform the task.

Three features were proposed for waste treatment in this study, that were processing capacity, energy production capacity and stability. The processing capacity represents the ratio of the reduced amount of waste based on mass or organic matter by processing. The energy production capacity is obtained by comparing the output energy and the input energy of process. The stability is brought from the model which represent substrate concentration and predict biogas yield and the control so that any inhibition would not occur and the desirable biogas would be produced. The objective of this study is to develop anaerobic co-digestion process which satisfies all of them.

MATERIALS AND METHODS

Feedstock and processing method

In this study, we used the digestate from a biogas plant of cattle manure in Hokkaido University as initial sludge to fill the horizontal cylindrical reactor (effective volume 235 L). The feedstock of this experiment was food waste disposed at the cafeteria and paper used as copy paper in Hokkaido University.

The C/N ratio of food waste is low because its organic matter is composed of a large amount of protein and contains high nitrogen. It was a problem that anaerobic digestion from ammonia rich feedstock is likely to cause ammonia inhibition (Lauterbök et al., 2012; Angelidaki et al., 1993). Nakajima reported that ammonia inhibition was suppressed and the biogas production rate improved by using paper as an auxiliary material to adjust C/N ratio (Nakajima et al., 2016).

The processing method used in this study is the dry thermophilic. The total solid (TS) content and volatile solid (VS) of feedstock were about 40% and 35%. The sludge in the reactor was kept at the thermophilic temperature. Furthermore, the dry thermophilic process adopted in this experiment is expected to reduce the emissions of digestate because it is the treatment with no water addition.

Processing flow

Figure 1 shows the anaerobic co-digestion processing flow used in this study. Food waste was mixed with return digestate and ground by a disposer and paper was cut in a shredder. Feedstock was made by blending those in the mass ratio 2.5:1 to adjust its C/N ratio to around 40 and supplied to the horizontal cylindrical reactor. The sludge was heated to maintain around 52°C and stirred regularly to be degassed and mixed with the feedstock.

Some amount of digestate (discharged digestate) was pulled out at feeding time and the residuals except return digestate was taken out as surplus digestate. Biogas was collected by the gas trap bag through the desulfurization equipment. Various experimental terms using this processing flow were conducted for about half a year. In this paper, the period when biogas generation was the most significant and the feedstock condition seems to be optimal is discussed. Figure 2 shows the mass and organic matter of feedstock and the feeding time. Fermentation time was 336
hours and the OLR was 3.96 g-VS/L-digester/day.

**Measurement items**

Biogas generation per hour [L/h] was measured by a wet gas meter manufactured by Shinagawa and recorded using HIOKI LR 5000. The amount of biogas [mol] was calculated using the ideal gas law with the gas temperature in the reactor, the atmospheric pressure, the volume of biogas and the measured gas concentration. The composition of the biogas occurred at 10:00 and 14:00 on weekdays was measured with Geotech BIOGAS 5000, especially methane, carbon dioxide and others. In order to estimate the value of a whole day from the measured gas concentration, it was regarded that the concentration at 0:00 to 11:00 was measured at 10:00, the concentration at 12:00 to 23:00 was measured at 14:00.

**Evaluate points**

The processing capacity was analyzed from material circulation. Mass reduction rate (MRR) is the reduction in mass of waste and organic degradation rate (ODR) is the one in organic matter of waste. They are given by Eq. 1.

\[
\text{MRR, ODR} \ [\%] = \left(1 - \frac{\text{Surplus digestate}}{\text{Food waste + Paper}}\right) \times 100
\]  

(1)

The energy production capacity was assessed by focusing on biogas apart from digestate. Specific biogas yield (SBY) is the biogas generation per mass of the feedstock, specific methane yield (SMY) is the methane gas generation from unit of organic matter and volumetric methane yield (VMP) is the methane gas generation per fermentation time and volume of the reactor (Banks and Heaven, 2013). They were calculated by Eqs. 2-4.

\[
\text{SBY} \ [\text{Nm}^3/\text{kg}] = \frac{\text{Biogas yield} \ [\text{Nm}^3]}{\text{Mass of feedstock} \ [\text{kg}]}
\]  

(2)

\[
\text{SMY} \ [\text{Nm}^3/\text{g-VS}] = \frac{\text{Methane gas yield} \ [\text{NL}]}{\text{Organic matter of feedstock} \ [\text{g-VS}]}
\]  

(3)

\[
\text{VMP} \ [\text{L}]/\text{L-digester/day} = \frac{\text{Methane gas yield} \ [\text{L}]}{\text{Volume of reactor} \ [\text{L-digester}] \cdot \text{Fermentation time [day]}}
\]  

(4)

Energy production rate (EPR) is the proportion of the output energy from the input energy. It was determined by Eq. 5.

\[
\text{EPR} \ [%] = \frac{\text{Output energy} \ [\text{kJ}]}{\text{Input energy} \ [\text{kJ}]} \times 100 = \frac{\text{Energy production by biogas} \ [\text{kJ}]}{\text{Energy consumption} \ [\text{kJ}]} \times 100
\]  

(5)

\[
\text{Energy production by biogas} \ [\text{kJ}] = \text{Lower heating value of methane} \ [\text{kJ/NL}]
\]
yield [\(-\)] \(a(t)\) substrate input [g/L-h]. Output equation refers to gas flow rate. It means that gas is generated with substrate degrading (Hend et al., 2015).

\[
y(t) = \left( k_{s} + \frac{1}{Y} \mu + k_{g} b \right) W n(t)
\]

where: \(y(t)\): gas flow rate [L/h], \(W\): sludge volume [L], \(k_{s}\), \(k_{g}\): biogas generation coefficient. The specific growth rate that appears in these equations is given by the modified Monod equation (Nagatani et al., 1973).

\[
\mu = \frac{\mu_{max} \alpha(t)}{K_{s} + \alpha(t) + a_{s}(t)}
\]

where: \(\mu_{max}\): maximum specific growth rate [h \(^{-1}\)], \(K_{s}\): dissociation constant [g/L], \(a\) inhibition coefficient [\(-\)]

Consequently, this model has 8 parameters in four equations. The equations were solved by fourth order Runge-Kutta method giving an initial condition and calculation time step. The initial value of bacteria concentration was the organic matter of the sludge and there was no remaining substrate before feeding. This is because we considered the convenience of the model developed in this study. When applied to an actual biogas plant, it is difficult to measure the bacterial concentration and residual substrate in every plant. If the biogas generation is smaller than the value after the feedstock is added, it is considered that there is no significant effect, therefore the bacterial concentration is assumed to be the organic matter concentration that can be easily measured. The calculation time step was one hour which was the same as the measurement interval of biogas generation.

Parameters in the model were estimated in order of magnitude of influence on results (\(\mu_{max}, a, b, Y, K_{s}, a, k_{s}, k_{g}\)) with calibration data from a week in June. The model was validated with the experimental data. At last, the predictive ability of the model was assessed by comparing standard error of calibration (SEC) and standard error of prediction (SEP). SEC and SEP were specified by Eqs. 10 and 11.

\[
SEC = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left( y_{exp} - y_{model} \right)^{2}}
\]

where: \(y_{exp}\): measured biogas yield [L/h], \(y_{model}\): estimated biogas yield [L/h], \(N\): number of data

\[
SEP = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left( \frac{y_{exp} - y_{model}}{y_{model}} \right)^{2}}
\]

where: \(y_{exp}\): average value of measured biogas yield [L/h], \(y_{model}\): average value of estimated biogas yield [L/h], \(N\): number of data

RESULTS AND DISCUSSION

Experimental results

The results are shown in Fig. 4 as mass basis material circulation, focusing on feedstock, biogas, and digestate. The volume of biogas was changed into the mass with the ideal gas law. 11.6 kg of biogas was generated and 19.7 kg of surplus digestate was occurred from 36 kg of the feedstock. Since treated waste is the surplus digestate and untreated waste is the feedstock in this process, MRR was 45.3%. It was high because total solids (TS) of the feedstock was higher than 40% and moisture content of the sludge decreased due to using paper with high solid content as an auxiliary material and adding no water. Figure 5 shows organic basis material circulation. Feedstock and digestate were converted into the organic matter in this process. ODR was 90.8% in the same way as MRR. The fine value was obtained because fermentation progressed well by adjusting the C/N ratio of the feedstock. It suggested that adjusting components of feedstock is a good way of optimization of organic degradation. Considering these values, this process occurs digestate less than usual and has a high processing capacity.

Figure 6 shows the gas flow rate during the experimental period. 8.76 Nm\(^{3}\) of biogas was generated and its methane concentration was about 49.0%. Consequently, SBY was 0.24 Nm\(^{3}\)/kg. The fermentation state was good in the experimental period because of the optimum value of the C/N ratio and the organic matter content in the feedstock was high. In the same way, SMY was 0.3 NL/g-VS and VMP was 1.52 L/L-digester/day. Each substrate has a maximum theoretical value of SMY based on its carbon. It is reported that food waste and cellulose become around 0.4 and 0.3 NL/g-VS in first 10 days of 100-day test (Char-
In this study, the same value was obtained and it is considered that fermentation inhibition did not occur. It is known that VMP represents a linear increase and the point of inflection when the organic loading increases (Charles and Sonia, 2013). Inflection point of VMP was observed to be around 1.60 L/L-digester/day in other experimental terms using this processing flow. Therefore, the digester worked with its maximum productivity during this experiment. From the above, the feedstock condition was optimal for this process. The energy consumed for stirring and warming the digester was 48.3 kWh. Assuming that the lower heating value of methane is 35,900 kJ/Nm³ and electricity generation is performed with the fuel cell which has an energy efficiency of 40%, EPR becomes 34.8%. Improvement of heat insulation of the digester would be an issue in the future.

The state space model

Table 1 shows estimated values of the parameters. Figure 7 demonstrates the predicted gas flow rate with them and the state variables are shown in Fig. 8. The feedstock was added once at 0 hour and the mass of organic matter in feedstock was 1.5 kg-VS in the calibration period. Gas flow rate after 72 hours of the experimental data was comparatively steady because substrate had been decomposed mostly at that time. Biogas was generated from bacteria instead of substrate during the late fermentation, the phenomenon is known as autolysis. The state variables of the model revealed unobserved values that the substrate concentration was almost 0 g/L and the bacteria concentration was decrementing by autolysis at that time.

The simulation results of the experimental period with the estimated parameters are shown in Figs. 9 and 10. During the period, the bacteria concentration remained almost maximum value (100 g/L) that is the parameter \( n_{\text{max}} \) in Eq. 6 and Table 1. The substrate concentration and specific growth rate decreased sharply after the feedstock was added and became almost 0 after 72 hours which was the same tendency as calibration period. While the bacteria

| Table 1 Estimated parameters. |
|-----------------------------|
| \( \mu_{\text{max}} \) | \( n_{\text{max}} \) | \( b \) | \( Y \) | \( K_s \) | \( a \) | \( \lambda_1 \) | \( k_0 \) |
| 0.2 | 100 | 0.01 | 15 | 15 | 0.1 | 0.5 | 0.05 |

Fig. 6 Gas flow rate.

Fig. 7 Predicted gas flow rate with calibration data.

Fig. 8 State variables with calibration data. (a) bacteria concentration, (b) substrate concentration, (c) specific growth rate.
concentration remained almost maximum value the sub-
strate concentration changed below 12 g/L even after the
feeding. It indicates that the death of bacteria did not
become dominant and the substrate concentration was not
reached the value in which inhibition would occur. These
findings proved that the feedstock condition was optimal.

SEC and SEP were calculated each day with calibra-
tion data and prediction data in order to perform validation
of the model. Figure 11 shows daily variations of SEC and
SEP. These are relative values and the accuracy of the
model could be discussed by comparing them. The average
values of SEC and SEP in first six days were 4.64 and 5.72
respectively, which were not largely different from each
other, therefore it could be said that the model had fine
accuracy. Both SEC and SEP were gradually decreasing as
soon as feedstock was supplied, but SEP has improved on
the fourth day when the feedstock was added four times.
The accuracy of the first-order kinetic model at the first
stage of fermentation has a problem because there is the
lag time from organic degradation to biogas generation. It
is considered that the prediction accuracy improved
because the reactor became a steady state on the fourth day
of SEP when the feedstock was added multiple times. This
model would be improved by taking into consideration of
the lag time and then become more useful to produce desir-
able biogas.

The developed model could predict the measured gas
flow rate and the unobserved fermentation state precisely
because it has state variables about bacteria concentration
and substrate concentration. Although there are some con-
cerns such as the physical meaning of the state variables
and the accuracy of predicted gas flow rate under the spe-
cific conditions, the anaerobic digestion process can be
advanced by predicting bacterial and substrate concentra-
tions and biogas generation using this model and control-
ling the fermentation state to stabilize. There are future
tasks to measure the bacterial and substrate concentration
at various time and reflect them in the calibration method
and the process stabilization method and to improve the
constitutive equations of the model to represent the reac-
tion lag time. Especially in relation to prediction accuracy
of bacterial and substrate concentration, it is important to
accumulate measurement data when supplied for feedstock
at fermentation has stopped and reflect it in the process sta-

Fig. 9 Predicted gas flow rate with validation data.

Fig. 10 State variables with validation data. (a) bacteria concen-
tration, (b) substrate concentration, (c) specific growth
rate.
bilization method. Then, it is necessary to perform control under various conditions to put it into practical use.

**CONCLUSION**

In this study, three features were proposed for waste treatment to achieve sustainable development. Processing capacity, energy production and stability were evaluated from the viewpoint of the material circulation, the biogas yield and the state space model of anaerobic digestion process respectively. The feedstock used in the developed anaerobic co-digestion process was a mixture of food waste and paper. MRR, ODR and SBY were 45.3%, 90.8% and 0.24 Nm³/kg obtained from the experimental results. The process had satisfactory processing capacity and energy production capacity because the components of the feedstock was adjusted. The state space model of anaerobic digestion process based on mass balance theory including two state variables about fermentation state was composed from two differential equations as state equation and one algebraic equation as output equation. The model could predict fermentation state and biogas yield with precious because the average values of SEC and SEP in first six days were 4.64 and 5.72. The model-based control of the process to make anaerobic digestion process stable and produce desirable biogas is a future issue.

**REFERENCES**

Angelidaki, I., Ahring, B. K. 1993. Thermophilic anaerobic digestion of livestock waste: the effect of ammonia. Appl. Microbiol. Biotechnol. 38: 560–564.

Banks, C. J., Heaven, S. 2013. Optimisation of biogas yields from anaerobic digestion by feedstock type. In “The Biogas Handbook” (ed. By Wellinger, A., Murphy, J., Baxter, D.). Woodhead Publishing Limited, Cambridge, p 131–161.

Batstone, D., Keller, J., Angelidaki, I., Kaluzhnyi, S., Pavlostatias, S., Rozzi, A., Sanders W. T., Siegrist, H., Vavilin, V. A. 2002. The IWA anaerobic digestion model no. 1. Water Sci. Technol. 46: 65–73.

Ghosh, S., Conrad, J. R., Klass, D. L. 1975. Anaerobic acidogenesis of wastewater sludge. J. Water Pollut. Control Fed. 47: 30–45.

Hend, M., Hatem, K. 2015. Regulation of biogas production through waste water anaerobic digestion process modeling and parameters optimization. Waste Biomass Valorization 6: 29–33.

IEA BIOENERGY. 2001. Biogas and More!: Systems and Markets Overview of Anaerobic Digestion. AEA Technol. Environ., Oxfordshire, pp 20.

Lauterbök, B., Ortner, M., Haider, R., Fuchs, W. 2012. Counteracting ammonia inhibition in anaerobic digestion by removal with a hollow fiber membrane contactor. Water Res. 46: 4861–4869.

Mauky, E., Weinrich, S., Nagel, H. J., Jacobs, H. F., Liebetran, J., Neles, M. 2016. Model predictive control for demand-driven biogas production in full scale. Chem. Eng. Technol. 39: 652–664.

May, R. M. 1976. Simple mathematical models with very complicated dynamics. Nature 261: 459–467.

Mendez-Acosta, H. O., Palacios-Ruiz, B., Alcaraz-Gonzalez, V., Gonzalez-Alvarez, V., Garcia-Sandoval, J. P. 2016. A robust control scheme to improve the stability of anaerobic digestion processes. J. Process Control 20: 375–383.

Nagatani, M. 1973. Reaction rate of microorganisms. J. Brew. Soc. Jpn. 68: 829–834.

Nakajima, S., Shimizu, N., Ishiwata, H., Ito, T. 2016. The startup of thermophilic anaerobic digestion of municipal solid waste. J. Jpn. Inst. Energy 95: 645–647.

Niike, T. 2009. Anaerobic Digestion. Gihodo, Tokyo, pp 283.