Heavy-quark contributions to the ratio $F_L/F_2$ at low values of the Bjorken variable $x$

A.Yu. Illarionov\(^1\), B.A. Kniehl\(^2\), A.V. Kotikov\(^3\)

\(^1\)SISSA, via Beirut, 2-4, 34014 Trieste and INFN, Sezione di Trieste, Trieste, Italy
\(^2\)II. Institut für Theoretische Physik, Universität Hamburg, 22761 Hamburg, Germany
\(^3\)BLThPh, JINR, 141980 Dubna (Moscow region), Russia

We study the heavy-quark contributions to the proton structure functions $F_i^2(x, Q^2)$ and $F_i^L(x, Q^2)$, with $i = c, b$, for small values of Bjorken’s $x$ variable and provide compact formulas for their ratios $R_i = F_i^L/F_i^2$ that are useful to extract $F_i^2(x, Q^2)$ from measurements of the doubly differential cross section of inclusive deep-inelastic scattering at DESY HERA. Our approach naturally explains why $R_i$ is approximately independent of $x$ and the details of the parton distribution functions in the low-$x$ regime.

1 Introduction

The totally inclusive cross section of deep-inelastic lepton-proton scattering (DIS) depends on the square $s$ of the centre-of-mass energy, Bjorken’s variable $x = Q^2/(2pq)$, and the inelasticity variable $y = Q^2/(xs)$, where $p$ and $q$ are the four-momenta of the proton and the virtual photon, respectively, and $Q^2 = -q^2 > 0$. The doubly differential cross section is parameterized in terms of the structure function $F_2$ and the longitudinal structure function $F_L$, as

$$\frac{d^2\sigma}{dx\,dy} = \frac{2\pi\alpha^2}{xQ^4} \left\{ \left[ 1 + (1 - y)^2 \right] F_2(x, Q^2) - y^2 F_L(x, Q^2) \right\},$$

(1)

where $\alpha$ is Sommerfeld’s fine-structure constant. At small values of $x$, $F_L$ becomes non-negligible and its contribution should be properly taken into account when the $F_2$ is extracted from the measured cross section. The same is true also for the contributions $F_i^2$ and $F_i^L$ of $F_2$ and $F_L$ due to the heavy quarks $i = c, b$.

Recently, the H1 \cite{1,2,3} and ZEUS \cite{4,5,6} Collaborations at HERA presented new data on $F_c^2$ and $F_b^2$. At small $x$ values, of order $10^{-4}$, $F_c^2$ was found to be around 25% of $F_2$, which is considerably larger than what was observed by the European Muon Collaboration (EMC) at CERN \cite{7} at larger $x$ values, where it was only around 1% of $F_2$. Extensive theoretical analyses in recent years have generally served to establish that the $F_c^2$ data can be described through the perturbative generation of charm within QCD (see, for example, the review in Ref. \cite{8} and references cited therein).

In the framework of Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) dynamics \cite{9}, there are two basic methods to study heavy-flavour physics. One of them \cite{10} is based on the massless evolution of parton distribution functions (PDF) and the other one on the photon-gluon fusion (PGF) process \cite{11}. There are also some interpolating schemes (see Ref. \cite{12} and references cited therein). The present HERA data on $F_c^2$ \cite{2,3,5,6} are in good agreement with the modern theoretical predictions.
In earlier HERA analyses [1][4], $F_C^L$ and $F_F^L$ were taken to be zero for simplicity. Four years ago, the situation changed: in the papers [2][3][5][6], the $F_C^L$ contribution at next-to-leading order (NLO) was subtracted from the data.

In this paper, we present compact low-$x$ approximation formulae [13] for the ratio $R_i = F_i^L/F_i^2$ at leading order (LO) and NLO, which greatly simplify the extraction of $F_i^2$ from measurements of $d^2σ_i/(dx dy)$.

### 2 Parton distribution functions at small $x$

The standard program to study the small $x$ behavior of quarks and gluons is carried out by comparison of the data with the numerical solution of the DGLAP equations fitting the parameters of the $x$ profile of partons at some initial $Q_0^2$ and the QCD energy scale $Λ$ (see, for instance, [14][15]). However, in analyzing exclusively the small $x$ region ($x ≤ 0.01$), there is the alternative of doing a simpler analysis by using some of the existing analytical solutions of DGLAP in the small $x$ limit (see [16] for review). It was done in Refs. [17]-[19], where it was pointed out that the HERA small $x$ data can be interpreted in the so called doubled asymptotic scaling (DAS) approximation related to the asymptotic behavior of the DGLAP evolution discovered in [20] many years ago.

Here we illustrate results obtained in [18][19]: the small $x$ asymptotic PDF form in the framework of the DGLAP equation starting at some $Q_0^2$ with the flat function:

$$xf_a(x, Q_0^2) = A_a \quad \text{(hereafter } a = q, g),$$

where $xf_a$ are the leading-twist PDF parts and $A_a$ are unknown parameters that have to be determined from data. We neglect the non-singlet quark component at small $x$.

We would like to note that HERA data [21] show a rise of $F_2$ at low $Q^2$ values ($Q^2 < 1$GeV$^2$) when $x → 0$. This rise can be explained naturally by incorporation of higher-twist terms in the analysis (see [19] and Fig.1).

We shortly compile the LO results (the NLO ones may be found in [18][19]), which are:

$$f_a(x, Q^2) = f_a^+(x, Q^2) + f_a^-(x, Q^2),$$

$$f_q^+(x, Q^2) = \left( A_q + \frac{4}{9} A_g \right) I_0(σ) e^{-d_+(1)s} + O(ρ),$$

$$f_q^-(x, Q^2) = -\frac{4}{9} A_g e^{-d_-(1)s} + O(ρ),$$

where $e = (\sum e_i^2/f)$ is the average charge square and $d_+(1) = 1 + 20f/(27β_0)$ and $d_-(1) = 16f/(27β_0)$ are the regular parts of $d_+$ and $d_-$ anomalous dimensions, respectively, in the limit $n → 1$.

The functions $I_ν (ν = 0, 1)$ are the modified Bessel functions $I_ν$ and the

\[1\] For a quantity $k(n)$ we use the notation $\tilde{k}(n)$ for the singular part when $n → 1$ and $\tilde{k}(n)$ for the corresponding regular part.
variables $\sigma$ and $\rho$ are given by

$$\sigma = 2\sqrt{\hat{d}_+ s \ln(x)}, \quad \rho = \sqrt{\frac{\hat{d}_+ s}{\ln(x)}} = \frac{\sigma}{2 \ln(1/x)}, \quad \hat{d}_+ = -\frac{12}{\beta_0}, \quad (8)$$

where $\beta_0$ is the first coefficient of the QCD beta function and $s = \ln[a_s(Q_0)/a_s(Q)]$, with $Q_0$ being the initial scale of the DGLAP evolution, and $a_s(\mu) = \alpha_s(\mu)/(4\pi)$ is the couplant with the renormalization scale $\mu$.

3 Master formula

We now derive our master formula for $R_i(x, Q^2)$ appropriate for small values of $x$, which has the advantage of being independent of the PDFs $f_a(x, Q^2)$. In the low-$x$ range, where only the gluon and quark-singlet contributions matter, while the non-singlet contributions are negligibly
small, we have\(^2\)

\[ F^l_k(x, Q^2) = \sum_{l=+,-} C^l_{k,g}(x, Q^2) \otimes x f^l_g(x, Q^2), \]  

(9)

where \(l = \pm\) labels the usual + and − linear combinations of the gluon contributions, \(C^l_{k,g}(x, Q^2)\) are the DIS coefficient functions, which can be calculated perturbatively in the parton model of QCD, and the symbol \(\otimes\) denotes convolution according to the usual prescription, \(f(x) \otimes g(x) = \int_x^1 dy y f(y) g(x/y)\). Massive kinematics requires that \(C^l_{k,g} = 0\) for \(x > b_i = 1/(1 + 4a_i)\), where \(a_i = m_i^2/Q^2\). We take \(m_i\) to be the solution of \(\overline{\mu}_i(m_i) = m_i\), where \(\overline{\mu}_i(\mu)\) is defined in the modified minimal-subtraction (\(\overline{\text{MS}}\)) scheme. 

Exploiting the low-\(x\) asymptotic behaviour of \(f^l_g(x, Q^2)\) \(^2\) \(^\text{22}\),

\[ f^l_g(x, Q^2) \mid_{x \to 0} = \frac{1}{x + \delta_l} \tilde{f}_g(x, Q^2), \]  

(10)

where the rise of \(\tilde{f}_g(x, Q^2)\) as \(x \to 0\) is less than any power of \(x\), Eq. \(^9\) can be rewritten as \(^2\) \(^\text{23} \text{ 24}\)

\[ F^l_k(x, Q^2) \approx \sum_{l=+,-} M^l_{k,g}(1 + \delta_l, Q^2) x f^l_g(x, Q^2), \]  

(11)

where

\[ M^l_{k,a}(n, Q^2) = \int_0^{b_i} dx x n^{-2} C^l_{k,a}(x, Q^2) \]  

(12)

is the Mellin transform, which is to be analytically continued from integer values \(n\) to real values \(1 + \delta_l\) \(^2\) \(^\text{25}\).

In the DAS approach \(^3\), one has \(M^l_{k,a}1, Q^2) = M^l_{k,a}1, Q^2)\), if \(M^l_{k,a}(n, Q^2)\) are devoid of singularities in the limit \(\delta_l \to 0\), as we assume for the time being. Such singularities actually occur at NLO, leading to modifications to be discussed in Section \(^\text{15}\). Defining \(M^l_{k,a}(1, Q^2) = M^l_{k,a}(1, Q^2)\) and using \(^9\), Eq. \(^11\) may be simplified to become

\[ F^l_k(x, Q^2) \approx M_{k,g}(1, Q^2) x f^l_g(x, Q^2). \]  

(13)

In fact, the non-perturbative input \(f_g(x, Q^2)\) does cancels in the ratio

\[ R_i(x, Q^2) \approx \frac{M_{L,g}(1, Q^2)}{M_{2,g}(1, Q^2)}, \]  

(14)

which is very useful for practical applications. Through NLO, \(M_{k,g}(1, Q^2)\) exhibits the structure

\[ M_{k,g}(1, Q^2) = e_i^2 a_s(\mu) \left\{ M_{k,g}^{(0)}(1, a_i) + a_s(\mu) \left[ M_{k,g}^{(1)}(1, a_i) + M_{k,g}^{(2)}(1, a_i) \ln \frac{\mu^2}{m_{ij}^2} \right] \right\} + O(a_s^3). \]  

(15)

where Inserting Eq. \(^15\) into Eq. \(^14\), we arrive at our master formula

\[ R_i(x, Q^2) \approx \frac{M_{L,g}^{(0)}(1, a_i) + a_s(\mu) \left[ M_{L,g}^{(1)}(1, a_i) + M_{L,g}^{(2)}(1, a_i) \ln(\mu^2/m_{ij}^2) \right]}{M_{2,g}^{(0)}(1, a_i) + a_s(\mu) \left[ M_{2,g}^{(1)}(1, a_i) + M_{2,g}^{(2)}(1, a_i) \ln(\mu^2/m_{ij}^2) \right]} + O(a_s^2). \]  

(16)

\(^2\)Here and in the following, we suppress the variables \(\mu\) and \(m_i\) in the argument lists of the structure and coefficient functions for the ease of notation. Moreover, a further simplification is obtained by neglecting the contributions due to incoming light quarks and antiquarks in Eq. \(^15\), which is justified because they vanish at LO and are numerically suppressed at NLO for small values of \(x\). One is thus left with the PGF contribution.

\(^3\)The singular PDF behavior has been considered recently in \(^2\) \(\text{20}\).
We observe that the right-hand side of Eq. (16) is approximately independent of $x$, a remarkable feature that is automatically exposed by our procedure. In the next two sections, we present compact analytic results for the LO ($j = 0$) and NLO ($j = 1, 2$) coefficients $M_{k,g}^{(j)}(1, a)$, respectively.

4 LO results

The LO coefficient functions of PGF can be obtained from the QED case [27] by adjusting coupling constants and colour factors, and they read [28, 29]

$$C_{2,g}^{(0)}(x, a) = -2x\{[1 - 4x(2 - a)(1 - x)]\beta - [1 - 2x(1 - 2a) + 2x^2(1 - 6a - 4a^2)]L(\beta)\},$$

$$C_{L,g}^{(0)}(x, a) = 8x^2[(1 - x)\beta - 2axL(\beta)],$$

where

$$\beta(x) = \sqrt{1 - \frac{4ax}{1 - x}}, \quad L(\beta) = \ln \frac{1 + \beta}{1 - \beta}.$$  \hspace{1cm} (18)

Performing the Mellin transformation in Eq. (12), we find (see details in [13])

$$M_{2,g}^{(0)}(1, a) = \frac{2}{3}[1 + 2(1 - a)J(a)], \quad M_{L,g}^{(0)}(1, a) = \frac{4}{3}b[1 + 6a - 4a(1 + 3a)J(a)],$$

where

$$J(a) = -\sqrt{b} \ln t, \quad t = \frac{1 - \sqrt{b}}{1 + \sqrt{b}}.$$  \hspace{1cm} (20)

At LO, the low-$x$ approximation formula thus reads

$$R_i \approx 2b_i \frac{1 + 6a_i - 4a_i(1 + 3a_i)J(a_i)}{1 + 2(1 - a_i)J(a_i)}.$$  \hspace{1cm} (21)

5 NLO results

The NLO coefficient functions of PGF are rather lengthy and not published in print; they are only available as computer codes [30]. For the purpose of this letter, it is sufficient to work in the high-energy regime, defined by $x \ll 1$, where they assume the compact form [31]

$$C_{k,g}^{(j)}(x, a) = \beta R_{k,g}^{(j)}(1, a),$$

with

$$R_{2,g}^{(1)}(1, a) = \frac{8}{9}C_A[5 + (13 - 10a)J(a) + 6(1 - a)I(a)], \quad R_{2,g}^{(2)}(1, a) = -4C_AM_{k,g}^{(0)}(1, a),$$

$$R_{L,g}^{(1)}(1, a) = -\frac{16}{9}C_Ab\{1 - 12a - [3 + 4a(1 - 6a)]J(a) + 12a(1 + 3a)I(a)\},$$

where $C_A = N$ for the colour gauge group SU($N$), $J(a)$ is defined by Eq. (20), and

$$I(a) = -\sqrt{b} \left[\zeta(2) + \frac{1}{2} \ln^2 t - \ln(ab) \ln t + 2 \text{Li}_2(-t)\right].$$  \hspace{1cm} (24)
Here, $\zeta(2) = \pi^2/6$ and $\text{Li}_2(x) = -\int_0^1 \frac{dy}{1+xy} \ln(1-xy)$ is the dilogarithmic function.

As already mentioned in Section 3 the Mellin transforms of $C_{k,g}(x,a)$ exhibit singularities in the limit $\delta_t \to 0$, which lead to modifications in our formalism, namely in Eqs. (13) and (16). As was shown in Refs. [24, 18, 19], the terms involving $1/\delta_t$ depend on the exact form of the subasymptotic low-$x$ behaviour encoded in $f_g^i(x,Q^2)$, as

$$\frac{1}{\delta_t} = \frac{1}{f_g^i(x,Q^2)} \int_{\hat{\delta}}^1 \frac{dy}{y} f_g^i(y,Q^2),$$  \hspace{1cm} (25)$$

where $\hat{x} = x/b$. In the generalized DAS regime, given by Eqs. (3)-(7), we have

$$\frac{1}{\delta_t} \approx \frac{1}{\hat{\delta}_+} I_1(\sigma(x)) \left( - \frac{1}{\hat{\delta}_-} \ln \left( 1 + \frac{\hat{\delta}_-}{x} \right) \right).$$  \hspace{1cm} (26)$$

Because the ratio $f_g^-(x,Q^2)/f_g^+(x,Q^2)$ is rather small at the $Q^2$ values considered, Eq. (13) is modified to become

$$F_k^1(x,Q^2) \approx \bar{M}_{k,g}(1,Q^2)x f_g(x,Q^2),$$  \hspace{1cm} (27)$$

where $\bar{M}_{k,g}(1,Q^2)$ is obtained from $M_{k,g}(n,Q^2)$ by taking the limit $n \to 1$ and replacing $1/(n-1) \to 1/\hat{\delta}_+$. Consequently, one needs to substitute

$$M_{k,g}^{(j)}(1,a) \to \bar{M}_{k,g}^{(j)}(1,a) \hspace{1cm} (j = 1,2)$$  \hspace{1cm} (28)$$

in the NLO part of Eq. (16). Using the identity

$$\frac{1}{I_0(\sigma(x))} \int_{\hat{\delta}}^1 \frac{dy}{y} \beta(x/y) I_0(\sigma(y)) \approx \frac{1}{\hat{\delta}_+} - \ln(ab) - J(a) \frac{J(a)}{b},$$  \hspace{1cm} (29)$$

we find the Mellin transform $I_2$ of Eq. (22) to be

$$\bar{M}_{k,g}^{(j)}(1,a) \approx \left[ \frac{1}{\hat{\delta}_+} - \ln(ab) - J(a) \frac{J(a)}{b} \right] R_{k,g}^{(j)}(1,a) \hspace{1cm} (j = 1,2).$$  \hspace{1cm} (30)$$

The rise of the NLO terms as $x \to 0$ is in agreement with earlier investigations [32].

6 Results

As for our input parameters, we choose $Q_0^2 = 0.306 \text{ GeV}^2$, $m_c = 1.25 \text{ GeV}$ and $m_b = 4.7 \text{ GeV}$. While the LO result for $R_t$ in Eq. (21) is independent of the unphysical mass scale $\mu$, the NLO formula (16) does depend on it, due to an incomplete compensation of the $\mu$ dependence of $a_s(\mu)$ by the terms proportional to $\ln(\mu^2/Q^2)$, the residual $\mu$ dependence being formally beyond NLO. In order to estimate the theoretical uncertainty resulting from this, in [13] we put $\mu^2 = (Q^2)\xi$ and vary $\xi$. Besides our default choice $\xi = 1 + 4a_t$, we also considered the extreme choice $\xi = 100$, which is motivated by the observation that NLO corrections are usually large and negative at small $x$ values. A large $\xi$ value is also advocated in Ref. [34], where the choice $\xi = 1/x_0^2$, with $0.5 < \Delta < 1$, is proposed.

We now extract $F_2^i(x,Q^2)$ ($i = c,b$) from the H1 measurements of the cross sections in Eq. (1) at low ($12 < Q^2 < 60 \text{ GeV}^2)$ [34] and high ($Q^2 > 150 \text{ GeV}^2)$ [34] values of $Q^2$ using
such as the values determined by H1. We refrain from showing our results for other popular choices, the theoretical uncertainty related to the freedom in the choice of $T_{del}$ functions and absorbed into the
at low \cite{3} and high \cite{2} values of $Q^2$ (in GeV$^2$) at various values of $x$ (in units of $10^{-3}$) using our approach at NLO for $\mu^2 = \xi Q^2$ with $\xi = 1 + 4a_c$. The LO results agree with the NLO results within the accuracy of this table. For comparison, also the results determined in Refs. \cite{2,3} are quoted.

| $Q^2$ | $x$  | $F_2^\gamma(x,Q^2) \cdot 10^4$ (H1) | $F_2^b(x,Q^2) \cdot 10^4$ | $F_2^\gamma(x,Q^2) \cdot 10^4$ (H1) | $F_2^b(x,Q^2) \cdot 10^4$ |
|-------|------|---------------------------------|----------------|---------------------------------|----------------|
| 12    | 0.197| 435 ± 78                        | 431           | 45 ± 27                         | 45            |
| 12    | 0.800| 186 ± 24                        | 185           | 48 ± 22                         | 48            |
| 25    | 0.500| 331 ± 43                        | 329           | 123 ± 38                        | 123           |
| 25    | 2.000| 212 ± 21                        | 212           | 61 ± 24                         | 61            |
| 60    | 2.000| 369 ± 40                        | 368           | 190 ± 55                        | 190           |
| 60    | 5.000| 201 ± 24                        | 200           | 130 ± 47                        | 130           |
| 200   | 0.500| 202 ± 46                        | 202           | 413 ± 128                       | 400           |
| 200   | 1.300| 131 ± 32                        | 130           | 214 ± 79                        | 212           |
| 650   | 1.300| 213 ± 57                        | 214           | 243 ± 124                       | 238           |
| 650   | 3.200| 92 ± 28                         | 91            | 125 ± 55                        | 125           |

Table 1: Values of $F_2^\gamma(x,Q^2)$ and $F_2^b(x,Q^2)$ extracted from the H1 measurements of $\sigma^{\gamma\gamma}$ and $\sigma^{bb}$ at low \cite{3} and high \cite{2} values of $Q^2$ (in GeV$^2$) at various values of $x$ (in units of $10^{-3}$) using our approach at NLO for $\mu^2 = \xi Q^2$ with $\xi = 1 + 4a_c$. The LO results agree with the NLO results within the accuracy of this table. For comparison, also the results determined in Refs. \cite{2,3} are quoted.

the LO and NLO results for $R_i$ derived in Sections \cite{4} and \cite{5} respectively. Our NLO results for $\mu^2 = \xi Q^2$ with $\xi = 1 + 4a_i$ are presented for $i = c, b$ in Table 1, where they are compared with the values determined by H1. We refrain from showing our results for other popular choices, such as $\mu^2 = 4m_i^2$, $Q^2$ and even $\mu^2 = 100Q^2$ because they are very similar. We observe that the theoretical uncertainty related to the freedom in the choice of $\mu$ is negligibly small and find good agreement with the results obtained by the H1 Collaboration using a more accurate, but rather cumbersome procedure \cite{2,3}.

In order to assess the significance of and the theoretical uncertainty in the NLO corrections to $R_i$, we show in Fig. 2 the $Q^2$ dependences of $R_c$, $R_b$, and $R_t$ evaluated at LO from Eq. \cite{21} and at NLO from Eq. \cite{16} with $\mu^2 = 4m_i^2$, $Q^2 + 4m_i^2$. We observe from Fig. 2 that the NLO predictions are rather stable under scale variations and practically coincide with the LO ones in the lower $Q^2$ regime. On the other hand, for $Q^2 \gg 4m_i^2$, the NLO predictions overshoot the LO ones and exhibit a strong scale dependence. We encounter the notion that the fixed-flavour-number scheme used here for convenience is bound to break down in the large-$Q^2$ regime due to unresummed large logarithms of the form $\ln(Q^2/m_i^2)$. In our case, such logarithms do appear linearly at LO and quadratically at NLO. In the standard massless factorization, such terms are responsible for the $Q^2$ evolution of the PDFs and do not contribute to the coefficient functions. In fact, in the variable-flavour-number scheme, they are $\overline{MS}$-subtracted from the coefficient functions and absorbed into the $Q^2$ evolution of the PDFs. Thereafter, the asymptotic large-$Q^2$ dependences of $R_i$ at NLO should be proportional to $\alpha_s(Q^2)$ and thus decreasing. This is familiar from the Callan-Gross ratio $R = F_L/(F_2 - F_L)$, as may be seen from its $(x, Q^2)$ parameterizations in Ref. \cite{35}. Fortunately, this large-$Q^2$ problem does not affect our results in Table 1 because the bulk of the H1 data is located in the range of moderate $Q^2$ values.

The ratio $R_c$ was previously studied in the framework of the $k_t$-factorization approach \cite{29} and found to weakly depend on the choice of unintegrated gluon PDF and to be approximately $x$ independent in the low-$x$ regime (see Fig. 8 in Ref. \cite{29}). Both features are inherent in our approach, as may be seen at one glance from Eq. \cite{16}. The prediction for $R_c$ from Ref. \cite{29}, which is included in Fig. 2 for comparison, agrees well with our results in the lower $Q^2$ range,
Figure 2: $R_c$, $R_b$, and $R_t$ evaluated as functions of $Q^2$ at LO from Eq. (21) (dot-dashed lines) and at NLO from Eq. (16) with $\mu^2 = 4m_i^2$ (dashed lines) and $\mu^2 = Q^2 + 4m_i^2$ (solid lines). For comparison, the prediction for $R_c$ in the $k_t$-factorization approach (dot-dot-dashed line) [29] is also shown.

which supports the notion that the $k_t$-factorization approach partially accounts for the higher-order contributions in the low-$x$ regime.

7 Conclusions

In this paper, we observed a compact formula [13] for the ratio $R_i = F_i^q/F_i^l$ of the heavy-flavour contributions to the proton structure functions $F_2$ and $F_L$, valid through NLO at small values of Bjorken’s $x$ variable. We demonstrated the usefulness of this formula by extracting $F_2^c$ and $F_2^b$ from the doubly differential cross section of DIS recently measured by the H1 Collaboration [2,3] at HERA. These results agree with those extracted in Refs. [2,3] well within errors. In the $Q^2$ range probed by the H1 data, NLO predictions agree very well with the LO ones and are rather stable under scale variations. Since we worked in the fixed-flavour-number scheme, our results are bound to break down for $Q^2 \gg 4m_i^2$, which manifests itself by appreciable QCD correction factors and scale dependences. As is well known, this problem is conveniently solved by adopting the variable-flavour-number scheme, which we leave for future work. Our approach also simply explains the feeble dependence of $R_i$ on $x$ and the details of the PDFs in the low-$x$
regime.

Acknowledgments. One of the authors (A.V.K.) would like to express his sincerely thanks to the Organizing Committee for the kind invitation. He was supported in part, by Heiserberg-Landau program and by the Russian Foundation for Basic Research (Grant N 08-02-00896-a).

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