Analysis of Mathematics Modeling Student Ability in Algebraic Critical Thinking and Form of the Scaffolding

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Abstract - This research aimed to describe the mathematical modeling abilities of students based on the ability to think algebraically, the obstacles experienced and the form of scaffolding provided. The research uses descriptive and explorative qualitative methods. Data collection was carried out by written tests and in-depth interviews. The subjects were students of the Mathematics Education Study Program, Semarang State University in 2018/2019. The results obtained by subjects who have the ability to think critically high algebra are able to do mathematical modeling without experiencing obstacles that require scaffolding. Subjects who have the ability to think critically are able to do mathematical modeling, but it is not appropriate in suppose into variable so that modeling is made less effective because it is too much. Nevertheless, the subject is able to overcome the obstacles experienced without being given scaffolding. As for subjects who have low critical thinking skills are not able to do mathematical modeling well. This is because the subjects are not able to connect the suppose into variable that made to determine the form of information algebra and the core problem. To overcome these obstacles the subject was given an explanation scaffolding.

Keywords: mathematical modeling, critical thinking, algebraic thinking

I. INTRODUCTION

In Learning Mathematics students not only learn mathematical material, but also learn to deal with problems and challenges in life. One provision should be owned by the students in dealing with problems and challenges of living is high-level thinking skills. This also applies to students studying in tertiary institutions. Heong et al. (2011), states that higher-order thinking skills are an important aspect in the teaching and learning process, especially in higher education institutions. Students with higher-order thinking skills can learn, improve their performance and reduce their weaknesses.

Conklin (2012: 14), states the characteristics of higher order thinking skills include critical thinking and creative thinking. Critical and creative thinking are two very basic human abilities because both can encourage a person to always look at every problem faced critically and try to find answers creatively so that a new thing that is better and useful for his life is obtained. Thus, developing critical thinking skills is very supportive of developing higher-order thinking. The importance of critical thinking skills for students has also been investigated by Chukwuyenum (2013: 24), who stated critical thinking skills are an effective way to improve students' understanding of mathematical concepts.

Another ability in mathematics that must be mastered by students is the ability to think algebra. According to Gibson (2014), algebra is the beginning of a journey that provides the ability to solve more complex problems. The importance of algebraic thinking was also raised by Windsor (2010) who argues with the ability to think algebraically to expand the thinking needed to solve problems. This opinion is in line with Greene, as quoted by Ontario (2013) which states that algebraic thinking is the key for students in the progress of their mathematical and scientific knowledge.

The importance of the ability to think critically and think algebra apparently was not followed by the ability of students to think critically and algebra. This can be seen in the results of Trends in International Mathematics and Science Study (TIMSS) where one of the domain dimensions of content is algebra and one of the domains of cognitive dimensions is critical thinking. Based on TIMSS results in 2015, the average gain is 397. Indonesia ranks 50th out of 54 participating countries. Mathematical achievements in Indonesia based on the TIMSS survey show that Indonesia's average score is still below the International average score of 500. The achievement of a low average percentage by Indonesian students is in the domain, one of which is algebra indicating the weakness of Indonesian students in understanding the concept of algebra. According Rochmad et al. (2014) one of the causes of student’s weakness in understanding algebraic concepts is the lack of
students’ critical and creative thinking skills. This is also supported by Agoestanto et al. (2015), (2017) which states that students’ critical thinking skills are still low, where students with low critical thinking skills have low algebraic thinking abilities. Agoestanto et al. (2019) states that one of the causes of students’ errors in thinking algebra is the lack of student understanding related to algebraic modeling.

The lack of mastery in critical thinking and algebra also continues in college. This can be seen from the results of testing the questions on the 2019 UNNES FMIPA comprehensive exam, the question refers to higher-order thinking skills, including critical thinking and Algebra problems, getting a passing average score of 62.09%. Rochmad (2018) indicated that student criticality was caused partly by inaccuracies in changing from written language to mathematical language or mathematical modeling.

Mathematical modeling is an important step in solving mathematical problems. According to Verschaffel et al. (2002), mathematical modeling is a process in which real situations and relationships in these situations are expressed using mathematics. Mathematical modeling is the process of turning a real problem into a mathematical form. Thus, mastery in mathematical modeling is the initial stage that students must master in solving problems including algebra problems. Mastery of mathematical modeling is also an important part in the algebraic critical thinking indicator especially in the indicator of interpreting information. However, the ability to model from real world to mathematical language is still not fully mastered by students. This indicates there are still obstacles of students in mathematical modeling. Barriers experienced by students in mathematical modeling need to be analyzed. By analyzing the barriers in the modeling can be used as a material consideration lecturer in determining the appropriate lesson plan as an attempt to improve the understanding of all students, and help students overcome the barriers. One of the efforts to be made so that students can overcome barriers to learning is to provide scaffolding.

According to Wood, Bruner & Ross as quoted by Anghileri (2006: 33) scaffolding is the support or assistance provided by an advance, for children until the children can learn independently. Whereas Amiripour et al. (2012) argue that scaffolding is an effective method for improving learning outcomes in mathematics learning.

Anghileri (2006: 38) argues that there are three levels in scaffolding namely environmental provisions, the interaction of teachers / lecturers increasingly geared to support students / student learning, and developing conceptual thinking. Meanwhile, according to Roehler & Cantlon as quoted by Bikmaz et al. (2010: 25), and Hogan & Pressley as quoted by Lange (2002), that there are five different techniques in scaffolding, namely modeling of desire behavior, offering explanations, inviting student participation, verifying and clarifying student understanding, inviting students to contribute clues. In this study synthesized the form of scaffolding proposed by Anghileri, Roehler & Cantlon, and Hogan & Pressley, first giving key instructions, second inviting student participation by asking questions and the third giving explanations.

From the opinions and descriptions above, the formulation of the problem in this study are (1) how the mathematical modeling ability of students in algebraic critical thinking, (2) how the obstacles experienced by students in mathematical modeling, (3) how the form of scaffolding is given to overcome obstacles. The purpose of this research is to describe the mathematical modeling abilities of students in algebraic critical thinking, describe the obstacles experienced, and find a form of scaffolding to overcome obstacles.

II. METHOD

This research is a kind of descriptive and explorative qualitative research. This research will start from extracting data in the form of detailed stories that are revealed as is in accordance with the views of research subjects in modeling algebraic forms. Furthermore, students’ obstacles in modeling algebraic forms will be explored to find the appropriate type of scaffolding that students need. The research was conducted at the Mathematics Education Study Program of the Faculty of Mathematics and Natural Sciences, Semarang State University, with the subject of research being students of Mathematics Education who took the Introduction to Basic Mathematics subject in 2018/2019.

The research instrument consisted of tests of algebraic critical thinking skills, interview guidelines, scaffolding instruction sheets and scaffolding interview guidelines. Data analysis includes data reduction, data presentation, making conclusions, and data verification. In data reduction, the data obtained are summarized and focused based on the ability of mathematical modeling according to high, medium, and low levels in the ability to think critically algebra. The classification is based on the value of (N) 0≤ N ≤ x – SB as the low group, the value of x – SB < N ≤ x + SB as the medium group and the value of N > x + SB as the high group. In presenting data, data on the results of student work and interview results are presented in the form of narrative texts. The conclusion presented is a description of the ability of mathematical modeling in algebraic critical thinking according to high, medium and low levels, obstacles experienced and forms of scaffolding.

III. RESULTS AND DISCUSSION

After the test the critical thinking skills of 42 students algebraic get the average of 60.02 and standard deviation 15.5, with 6 students included in the upper group, 28 students included in the mid
group, and 8 students included in the lower group. Then from each group 2 students were chosen as research subjects. The six research subjects can be seen in Table 1.

| No | Code | Group |
|----|------|-------|
| 1  | ST-1 | Upper |
| 2  | ST-2 | Upper |
| 3  | SD-1 | Middle |
| 4  | SD-2 | Middle |
| 5  | SR-1 | Lower |
| 6  | SR-2 | Lower |

Of these 6 subjects then interviewed to check the truth of grouping based on the results of written tests and the results there are no changes in the grouping of research subjects. Then the six subjects were analyzed in their mathematical modeling abilities. The problems given are as follows.

"If the length and width of a rectangle are reduced by reducing each 10 cm, then the width is 20 cm less than the length. If the length and width are enlarged by adding 5 cm each, the length is twice the width. Determine the total length and width of the original rectangle!"

From these problems revealed the results of the subject's written work and the results of the interview.

a. The Ability in Mathematical Modeling at Upper Group of Algebraic Critical Thinking

Research Subject ST-1

The results of the written work ST-1 for the above problem are as follows.

![Figure 1. The result of the written work ST-1](image)

The ST-1 steps in making mathematical modeling based on test and interview results are as follows.

ST-1 understands the problem by reading the problem slowly then writing down the information that is known and what is being asked. Then ST-1 plans to model the problem in algebraic form by writing down the problem information foremost using $p$ as the variable of the initial length and using $l$ as the variable of the initial width of rectangle. After supposing the problem information into variable, ST-1 can model the information is known and what is being asked correctly.

After observing the process and work results of ST-1 accompanied by interviews when solving problems, it appears that ST-1 does not experience obstacles that require scaffolding. This is because ST-1 is able to determine variables as an suppose in accordance with the problem, then able to change the information that is known to be more detailed in the form of algebra, so that in the mathematical modeling process ST-1 is not given scaffolding.

Research Subject ST-2

The ST-2 steps in making mathematical modeling based on test and interview results are as follows.

ST-2 understands the problem by reading the problem then being able to write information that is known and what is asked correctly. Then ST-2 plans the modeling of the problem in algebraic form by writing down the problem information using $x$ as the variable of the initial length and using $y$ as the variable of the initial width of rectangle. After determining the suppose of known information, ST-2 can model the information that is known and what is asked correctly.

After observing the process and work results of ST-2 accompanied by interviews when solving problems, it appears that ST-2 does not experience obstacles that require scaffolding. This is because ST-2 is able to determine the variable as an suppose in accordance with the problem, then able to change the information that is known to be more detailed in the form of algebra, so that in the mathematical modeling process ST-2 is not given scaffolding.

Triangulation

Based on the description above, it can be concluded that subjects at high TKBKA can make mathematical modeling and do not experience any obstacles at all. The subject can determine the variable as an suppose of known information and can change it to be more detailed in the form of algebra, so that the subject is not given scaffolding.

This finding is consistent with the results of research conducted by Facione (2013), which shows that students who have high critical thinking skills are able to identify existing problems and then link information in problems to determine resolution strategies. In this case linking information in the problem to determine the completion strategy is done through a mathematical modeling process that will be used in the next step of completion. Likewise, according to Lehrer & Schauble (2003) by expressing real situations in mathematical language effectively, students must have higher level mathematical abilities, which means students with high algebraic
critical thinking skills will have high mathematical modeling abilities. Similarly, Hasan (2019) revealed that when asked to solve algebraic problems, one must think harder to understand how quantities (numbers) are generalized (formulated) in the form of symbols and letters, relationships between symbols, and manipulation of symbols.

b. The Ability in Mathematical Modeling at Middle Group of Algebraic Critical Thinking

- Research Subject SD-1

The results of the written work SD-1 for the above problem are as follows.

After observing the process and results of SD-1 work accompanied by interviews when solving problems, it appears that SD-1 is still not appropriate when doing algebra modeling. But when asked to mention more appropriate modeling, SD-1 can correct the lack in making algebraic modeling. So, at this stage SD-1 is not given scaffolding.

- Triangulation

Based on the description above, it can be concluded that the subject at TKBKA intermediate is able to do mathematical modeling but it is still not quite right, where the modeling is made too much so it is not effective when used in the next completion step. This finding is in accordance with the results of research conducted by Agoestanto et al. (2019), which shows that group students are likely to make mistakes when writing mathematical models of known problems. This is because students are not careful in implementing problems in mathematical modeling. The obstacle of this middle group subject is also strengthened by Kolovou & Heuvel-Panhuizen (2010) which states that the source of student difficulties in algebra is one of them being different conceptions of variables.

c. The Ability in Mathematical Modeling at Lower Group of Algebraic Critical Thinking

- Research Subject SR-1

The results of the SR-1 written work for the above problem are as follows.

After observing the process and results of SD-2 work accompanied by interviews when solving problems, it appears that SD-2 is still not appropriate when doing algebra modeling. But when asked to mention more appropriate modeling, SD-2 can correct the lack in making algebraic modeling. So, at this stage SD-2 is not given scaffolding.

The steps of SD-2 in making mathematical modeling based on test and interview results are as follows.

SD-2 understands the problem by reading the problem then writing down the information that is known and what was asked correctly. Then SD-2 plans to model the problem in algebraic form by writing down the problem information using the variable, but the variables chosen are too many. So, the modeling was made ineffective.

The steps of SR-1 in making mathematical modeling based on test and interview results are as follows.

SR-1 understand the problems by reading the problems then can mention information that is known and what is being asked correctly. Then SR-1 plans to model the problem in algebraic form by writing down the problem information using the variable first, but the suppose is made incomplete because it does not write the rectangular information as it is known. This
is because SR-1 is often not aware of important information in the problem so that the suppose made are not in accordance with the problem. In addition, the SR-1 is not able to connect the suppose that made to determine the algebraic form of information that is known and what is being asked, were indicated subject often difficult to interpret modeling made.

After observing the process and work outcomes of SR-1 accompanied by interviews when solving problems, it appears that SR-1 encountered obstacles when modeling the problem and has not been able to change the detailed problem information in algebraic form. Based on the obstacles encountered, SR-1 was given scaffolding as needed so that these obstacles could be overcome. The scaffolding provided is a type of giving an explanation by mentioning variables as an suppose of problem information and explaining the form of algebraic information that is known and what is being asked.

The following are excerpts of an interview with SR-1 regarding the scaffolding process.

Based on the results of the written answers, SR-1 has not yet defined the variables used in algebraic equations. For this reason, the researcher gives scaffolding key instructions to SR-1 in order to write the definition of the variable used first as follows.

P: "Let us write the suppose first, and what is it?"

SR-1: "long = p, width = l"

Based on the answers above, researchers provide scaffolding inviting participation to obtain more appropriate answers.

S(2,1): "Try again to read the essence of the problem and its algebraic form. Do you observe what words you write about are lacking?"

SR-1: "original size"

The results of the scaffolding above causes SR-1 to write the suppose according to the problem. Furthermore SR-1 is still not able to connect the suppose made to determine the algebraic form of problem information. To that end, researchers provide scaffolding inviting participation to SR-1 in order to write the correct form of algebra.

S(2,1): "Based on information that is known to measure 20 cm in width, less than the length. Then the longer is the length or width?"

SR-1: "Width measurement"

Researchers asked the same thing to SR-1 up to two times, but SR-1 was still unable to give the right answer. So, the researcher gave an explanation scaffolding to SR-1 like the following interview excerpt.

S(3,1): "l is indeed 20 cm less than p, meaning p is longer than l. When talking about the size of the width of 20 cm lack of length, the equation l = p − 20 is correct. But the position and not there, the position of p and l is when each size is reduced by 10 cm. If you write the algebraic form, it means that the initial width is equal to 20 cm. Should be l and p reduced by 10 cm"

After being given a scaffolding SR-1 correct the second algebraic information equation according to the explanation given by the researcher.

Research Subject SR-2

The steps SR-2 in making mathematical modeling based on test and interview results are as follows.

SR-2 understands the problem by reading the problem then can mention information that is known and asked correctly. Then SR-2 plans the modeling in algebraic form by writing down the problem information using the variable first, but the suppose is made incomplete because it does not write the rectangular information as it is known. In addition, SR-2 is not able to connect the suppose made to determine the algebraic form of information that is known and what is asked. When modeling from information known to SR-2, it gives rise to new variables that have not been defined before, so the modeling is not clear.

After observing the process and the work of SR-2 accompanied by interviews when solving problems, it appears that SR-2 encountered obstacles when modeling the problem and has not been able to change the detailed problem information in algebraic form. Based on the obstacles encountered, SR-1 was given scaffolding as needed so that these obstacles could be overcome. The scaffolding provided is an explanation by explaining that the initial length and width of the rectangle just be assumed by variables p and l, then explaining the algebraic form of the second information.

Triangulation

Based on the description above, it can be concluded that the subjects at the lower group of algebraic thinking cannot make mathematical modeling. The subject can determine the variable as an suppose, but is unable to relate it to change the information that is known and asked into algebraic form. According to Crouch & Haines (2004), one aspect of student difficulties in making modeling is connecting real-world problems with mathematical models. In addition, students also still have difficulty creating a relationship between reality and mathematics, simplifying and constructing reality, and problems related to solutions (Maas, 2006). The obstacles experienced by the lower group subjects in modeling are also in accordance with opinion of Blum & Ferri (2009) which states that the modeling process cannot be separated from other mathematical abilities such as reading and communicating, designing and implementing problem solving strategies, or working mathematically such as reasoning. So, students who
are in the lower group will experience obstacles in modeling. In line with Jupri & Drivers (2016) who revealed that students in Indonesia are still experiencing difficulties in turning problems into symbolic mathematics.

Based on the obstacles encountered, the subjects in the lower TKBKA were given a scaffolding explanation so that the obstacles could be overcome. The form of scaffolding is given to students so that they are accustomed to be careful in identifying problems, so that a comprehensive understanding of the problem exists so that students are able to do modeling well. Schukajlow et al. (2015), states that the use of scaffolding is useful to direct the workings of students in solving modeling problems. Whereas Belland (2017) states that successful scaffolding can help students learn to complete target tasks independently.

IV. CONCLUSIONS
The conclusion in this study is that subjects who have high algebra critical thinking skills are able to do mathematical modeling without experiencing obstacles that require scaffolding. Subjects who have the ability to think algebraically are being able to do mathematical modeling, but it is not appropriate to do the modeling using variables so that the modeling is made less effective because it is too much. Nevertheless, the subject is able to overcome the obstacles experienced without being given scaffolding. The subjects who have low algebra critical thinking skills are not able to do mathematical modeling well. This is because the subject is not able to connect the suppose made to determine the form of information algebra and the core problem. To overcome these obstacles the subject was given an explanation scaffolding.

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