The Resistance of a Single Protrusion at a constant current

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Abstract. The article deals with a method for measuring resistance of a single spherical asperity. There is obtained an approximate analytical relation between the electrical resistance of a single bump constriction and the contact spot radius, conductor radius, asperity height and width. Electrical resistance is determined from the solution to the Laplace equation for electrical field potential inside a conductor. The calculations are made using the finite-element method.

1. Introduction
The critical task of improving the reliability of electronic facilities is prevention of functional errors and faults in electric contacts. The importance of electric contacts in current technology grows due to the large number of them being used, and miniaturization of electronics.

Efforts by foreign and national scientists address the electric contact theory. All papers describe an electric contact as a contact of two rough surfaces [1-6]. Asperities of two surfaces form multiple contact spots through which electric current flows.

The main quality criterion of contacts is transient resistance. The analysis of said references shows that transient resistance consists of resistance of single asperities connected in parallel. A single asperity resistance on metal contact areas is due to current lines being constricted to the contact area. An asperity resistance on quasi-metal contact areas is due to the constriction resistance and surface film resistance.

Developing an improved method for measuring transient resistance is a relevant task which solution will decrease heat release and increase the reliability of electric contacts.

2. Problem Description
Transient resistance of electric contacts is largely defined by the constriction resistance, i.e. the constriction of current lines to contact spots. American scientist R. Holm was the first to describe this phenomenon [1]. R. Holm obtained a formula for determining the constriction resistance of a single-point round-spot contact: \[ R_s = \rho / (2a) \]. Here \( \rho \) is specific resistance of contact materials; \( a \) is a
contact spot radius. This formula does not account the impact that sizes of a single asperity have on the constriction resistance.

*This article objective* is to derive an analytical relation between the DC constriction resistance of a single electric contact asperity and the contact spot radius, conductor radius, asperity height and width.

### 3. Mathematical Model

For the purpose of theoretical research into the impact of various parameters on the constriction resistance of a single asperity, we studied a cylindrical conductor of radius $R$ (figure 1) with a spherical asperity peak. It is assumed that the asperity peak and depression have identical shape and sizes. The surface of the irregularity in question consists of the following parts: 1 – a circle having radius $a$ (contact spot of two interfacing asperities); 2 – a spherical surface having radius $r$ (the asperity peak); 3 – a conical surface (a transient part between the peak and the depression of the asperity); 4 – a spherical surface having radius $r$; 5 – a cylindrical surface having radius $R$. Length $L$ of the cylindrical part is determined on the assumption that size does not affect the constriction resistance. In this case, current lines on surface $A$ go along the conductor centerline.

**Figure 1.** Axial section of a single asperity

In the DC case, distribution of potential $\varphi(\psi, r, z)$ in the conductor is described by the Laplace equation [7] that in cylindrical coordinates looks as follows:

$$\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \varphi}{\partial \psi^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0$$

(1)

where $\varphi$ is electric field potential; $r, \psi, z$ are cylindrical coordinates.

A solution to the equation (1) can be found on the assumption that the potential energy of DC field functional is minimal [8]:

$$W(\varphi) = \frac{1}{2} \int_{V^*} \text{grad}(\varphi) \cdot \text{grad}(\varphi) dV$$

(2)

The integration is carried out over the whole three-dimensional region $V^*$ of definition.

The problem is question is axisymmetric. Hence, $\frac{\partial \varphi}{\partial \psi} = 0$. In this case:

$$\text{grad}(\varphi) \cdot \text{grad}(\varphi) = \left( \frac{\partial \varphi}{\partial r} \right)^2 + \left( \frac{\partial \varphi}{\partial z} \right)^2 = g^T \cdot g$$

(3)

where $g = \left[ \frac{\partial \varphi}{\partial r}, \frac{\partial \varphi}{\partial z} \right]^T$.

The distribution of electric potential $\varphi$ is determined with the finite-element method [8] using trigonal circular elements. The potential inside the finite element is of linear distribution:

$$\varphi(r, z) = [r, z] \cdot [K]^{-1} \cdot [\varphi]$$

(4)
where \[
\begin{bmatrix}
1 & r_1 & z_1 \\
1 & r_2 & z_2 \\
1 & r_3 & z_3
\end{bmatrix}
\]
is a matrix of node coordinates; \\
\[
\begin{bmatrix}
\varphi_1 \\
\varphi_2 \\
\varphi_3
\end{bmatrix}^T
\]
is a column matrix of potentials in the finite element nodes. \\
Substituting (4) into (3) and (2), we derive the equation for the finite-element potential energy functional:
\[
W(\varphi) = \frac{1}{2} [\varphi]^T \cdot [S] \cdot [\varphi]
\]  
(5)

The above transformations are used to define the influence coefficient matrix of trigonal circular finite elements:
\[
[S] = V \cdot [R]^T \cdot [R], \quad [R] = [C] \cdot [K]^{-1}, \quad [C] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]  
(6)

where \( V = 2\pi \left( \frac{r_1 + r_2 + r_3}{3} \right) F \) is circular element volume; \( F \) is trigonal element area.

There are more than 50,000 finite elements used for one conductor. Within one element, the potential changes linearly while the potential gradient is constant.

When determining the constriction resistance of a single asperity (see figure 1), the boundaries of the shaded portion of the conductor are subject to the following boundary conditions: on surface \( A \): \( \varphi = \varphi_A \); on surface \( B \): \( \varphi = \varphi_B \); on other surfaces: \( \vec{n} \cdot \vec{\nabla} \varphi = 0 \).

The first two conditions are Dirichlet boundary conditions; the third one is Neumann homogeneous natural boundary condition. When solving problems with the finite-element method, Neumann conditions are met by default.

The constriction resistance of one of the contacting bodies is found from the formula:

\[
R_S = R_{AB} - R_{AB}^*,
\]
\[
R_{AB} = \frac{\varphi_A - \varphi_B}{I} = \frac{(\varphi_A - \varphi_B)}{S_A \left[ \frac{\partial \varphi}{\partial z} \right]}, \quad R_{AB}^* = \rho \frac{L}{S_A}
\]  
(7)

where \( R_{AB} \) is resistance of the conductor portion from section \( A \) to section \( B \) (see figure 1); \( R_{AB}^* \) is resistance of a cylindrical conductor having length \( L \) and radius \( R \); \( \rho \) is resistivity of the conductor material; \( I \) is current intensity in the conductor; \( S_A = \pi R^2 \) is area of surface \( A \) of the conductor cross-section;
\( \vec{n} \) is unit vector directed along symmetry axis \( z \).

The finite element potential gradient projected onto symmetry axis \( z \) is found from the formula:

\[
\frac{\partial \varphi}{\partial z} = [0, 0, 1] [K]^{-1} \cdot [\varphi]
\]  
(8)

When determining \( R_S \) the length of the cylindrical portion of the conductor is \( L = 0.8R \). In this case, current on surface \( A \) is distributed almost uniformly while current lines are parallel to the conductor centerline. Radius \( r \) of spherical surfaces has no significant influence on \( R_S \). According to [9], equivalent radius of asperity peak curvatures is \( r = (5...50)h \). In our calculations, radius of spherical surfaces 2 and 4 (see figure 2) is \( r = 30h \).
4. Theoretical Study Results

It follows from (7) that the relative constriction resistance of a single asperity $\tilde{R}_S = R_S \cdot \frac{a}{\rho}$ depends only on 3 relative parameters $\tilde{a} = a / b$, $\tilde{h} = h / b$, $\tilde{b} = b / R$.

The given calculations of the relation between $\tilde{R}_S$ and $\tilde{a}$, $\tilde{h}$, $\tilde{b}$ allowed for obtaining an approximate relation between the constriction resistance of a single asperity and relative contact spot radius $\tilde{a}$, relative height $\tilde{h}$ and width $\tilde{b}$ of an asperity:

$$R_S = \frac{\rho}{4a} \left[ 1 - 1.3\tilde{a} + \tilde{h}(1 - \tilde{b}) + (0.9\tilde{a} + 12.5\tilde{a}\tilde{b}^2 + 0.6\tilde{h}\tilde{b}) \right]$$ (9)

The formula (9) has the following range of definition: $(0; 0.7) \subseteq \tilde{a}$, $(0; 1) \subseteq \tilde{h}$, $(0; 1) \subseteq \tilde{b}$. The discrepancy between $R_S$ determined from the formula (9) and those obtained using the finite-element method does not exceed 5%.

Figure 2 describes the relations between the relative constriction resistance $\tilde{R}_S$ of a single asperity and relative contact spot radius $\tilde{a}$ for various values of $\tilde{h}$ and $\tilde{b}$. The given relations show that the relative constriction resistance can be determined from Holm’s formula $\tilde{R}_S = \frac{\rho}{4a} \cdot \frac{a}{\rho} = 0.25$ if $\tilde{a} < 0.1$. An increase in relative radius $\tilde{a}$ of a contact spot decreases $\tilde{R}_S$ when $\tilde{b} = 0.7$ and increases the same when $\tilde{b} = 0.1$.

Figure 3 describes the relations between relative constriction resistance $\tilde{R}_S$ of a single asperity and relative height $\tilde{h}$ for various values of $\tilde{a}$ and $\tilde{b}$. The given relations show that the constriction resistance can be determined from Holm’s formula for $\tilde{h} > 0.9$ and $\tilde{b} > 0.7$, and for $\tilde{h} < 0.1$ and $\tilde{b} < 0.1$. An increase in relative height $\tilde{h}$ increases $\tilde{R}_S$. 

Figure 2. Relation between the relative constriction resistance and relative radius $\tilde{a}$ of a contact spot for $\tilde{b} = 0.7$ (a) and $\tilde{b} = 0.1$ (b): 1 – $\tilde{h} = 0$; 2 – $\tilde{h} = 0.5$; 3 – $\tilde{h} = 1.0$.
5. Conclusions

1. The constriction resistance of a single asperity depends on the contact spot radius, conductor radius, asperity height and width.
2. There is obtained an approximate analytical relation between the constriction resistance of a single asperity and the given values.
3. The constriction resistance of a single asperity can be determined from Holm’s formula in the following cases: 1) \( \bar{a} < 0.1 \); 2) \( \bar{h} > 0.9 \) and \( \bar{b} > 0.7 \); 3) \( \bar{h} < 0.1 \) and \( \bar{b} < 0.1 \).
4. Based on the asperity height, an increase in the contact spot radius may both increase and decrease the relative constriction resistance.
5. An increase in the asperity height increases the relative constriction resistance.

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