Are $Y(4260)$ and $Z^+_c(4250)$ $D_1D$ or $D_0D^*$ Hadronic Molecules?

Gui-Jun Ding
Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, China

In this work, we have investigated whether $Y(4260)$ and $Z^+_c(4250)$ could be $D_1D$ or $D_0D^*$ molecules in the framework of meson exchange model. The off-diagonal interaction induced by $\pi$ exchange plays a dominant role. The $\sigma$ exchange has been taken into account, which leads to diagonal interaction. The contribution of $\sigma$ exchange is not favorable to the formation of molecular state with $I^G(J^{PC}) = 0^-(1^-)$, however, it is beneficial to the binding of molecule with $I^G(J^{PC}) = 1^- (1^-)$. Light vector meson exchange leads to diagonal interaction as well. For $Z^+_c(4250)$, the contribution from $\rho$ and $\omega$ exchange almost cancels each other. For the currently allowed values of the effective coupling constants and a reasonable cutoff $\Lambda$ in the range 1-2 GeV, We find that $Y(4260)$ could be accommodated as a $D_1D$ and $D_0D^*$ molecule, whereas the interpretation of $Z^+_c(4250)$ as a $D_1D$ or $D_0D^*$ molecule is disfavored. The bottom analog of $Y(4260)$ and $Z^+_c(4250)$ may exist, and the most promising channels to discover them are $\pi^+\pi^-\gamma$ and $\pi^+\chi_{b1}$ respectively.

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I. INTRODUCTION

In the past years, a number of charmonium-like $X$, $Y$, $Z$ states have been observed, which stimulate a lot of discussion about the structures and properties of these resonances. In particular, the $Z^+(4430)$ observed in the $\pi^+\psi'$ invariant spectrum\[1\] carries one unit electric charge. Consequently, it can not be simple charmonium. Recently, two new resonance-like structures $Z^+_c(4051)$ and $Z^+_c(4250)$ in the $\pi^+\chi_{c1}$ mass distribution in exclusive $B^- \rightarrow K^-\pi^+\chi_{c1}$ have been reported by the Belle collaboration\[2\]. Their masses and widths are determined to be $M_1 = (4051 \pm 14_{-10}^{+20})$ MeV, $\Gamma_1 = (82_{-17}^{+21+47})$ MeV, $M_2 = (4248_{-29}^{+44+180})$ MeV and $\Gamma_2 = (177_{-39}^{+54+316})$ MeV respectively, with the product branching fractions $B(B^- \rightarrow K^Z_{1+2}) \times B(Z^++\pi^+\chi_{c1}) = (3.0_{-0.8}^{+1.5+4.7}) \times 10^{-5}$ and $(4.0_{-0.9}^{+2.3+19.7}) \times 10^{-5}$ respectively. Both $Z^+_c(4051)$ and $Z^+_c(4250)$ carry one unit electric charge like $Z^+(4430)$, hence they must be states beyond quark model, if these states are confirmed in future. Since $\pi^+$ is an isovector with negative G-parity, and $\chi_{c1}$ is an isospin singlet with positive G-parity, the quantum numbers of $Z^+_c(4051)$ and $Z^+_c(4250)$ are $I^G = 1^-$. It is remarkable that some states are in the vicinity of the S-wave threshold of two charmed mesons, e.g., $X(3872)$ and $Z^+(4430)$ are very close to the thresholds of $D^*D$ and $D_1D^*$ respectively, therefore it is tempting to interpret these states as molecular states\[3, 4\]. Particularly, $Y(4260)$ and $Z^+_c(4250)$ are close to the $D_1D$ and $D_0D^*$ thresholds, which inspires the theoretical interpretations of $Y(4260)$ as a $D_0D^*$ molecule\[5\] and $Z^+_c(4250)$ as a $D_1D$ molecule\[6\].

$Y(4260)$ was reported by the Babar collaboration in the $\pi^+\pi^-J/\psi$ invariant spectrum of the reaction $e^+e^- \rightarrow \gamma_{ISR}\pi^+\pi^-J/\psi$, which has been confirmed by both the CLEO and Belle collaboration\[7, 8\]. A fit to the peak with a single Breit-Wigner resonance shape yields a mass $M = (4259 \pm 10)$ MeV and the full width $\Gamma = (88 \pm 24)$ MeV. Evidently the state is a vector with $c\bar{c}$ flavor, and its quantum numbers are determined to be $I^G(J^{PC}) = 0^- (1^-)$. Although it is above the threshold for decaying into $D\bar{D}$, $D\bar{D}^*$($D^*\bar{D}$) or $D^*\bar{D}^*$ meson pairs, there is no evidence for $Y(4260)$ in these channels\[9, 10\]. Therefore $Y(4260)$ appears not to be a canonical charmonium.

The observation of the $Y(4260)$ has sparked many theoretical speculations. It has variously been identified as a conventional $\psi(4S)$ based on a relativistic quark model\[11\], a tetraquark $cc\bar{s}s$ state\[12\] which decays predominately into $D_1\bar{D}_s$, or a charmonium hybrid\[13\]. The data on $e^+e^- \rightarrow D_1\bar{D}_s$ show a peaking above threshold around 4 GeV but no evidence of affinity for a structure at 4.26 GeV\[14\]. If these data are confirmed, then the interpretation of $Y(4260)$ as a $cc\bar{s}s$ tetraquark would be ruled out. Moreover, dynamical calculation of tetraquark states indicated that $Y(4260)$ can not be interpreted as P-wave $1^-\gamma$ state of charm-strange diquark-antidiquark, because the corresponding mass is found to be 200 MeV heavier\[15\]. Although the charmonium hybrid is a very attractive interpretation, the lattice QCD simulations predict that lightest charmonium hybrid is about 4.4 GeV\[16\], which is very close to the new charmonium-like state $Y(4360)$\[17\]. As has been proposed in Ref.\[18\], a possible resolution to this issue is that $Y(4360)$ is the candidate of charmonium hybrid, while $Y(4260)$ is a $D_1D$ hadronic molecule.

In Ref.\[20\], Swanson emphasized that we should examine the $D_1D$ molecular interpretation before finally concluding that $Y(4260)$ is a charmonium hybrid. Furthermore, he pointed out that $\pi$ exchange does not lead to a diagonal
interaction in the D1D channel, and certain novel mechanism such as off-diagonal interaction may be required. In Refs.\textsuperscript{22, 23}, Close showed that parity conservation requires the π vertex to link D → D\* and D1 → D0, then the π exchange gives an off-diagonal potential linking D1D → D2D\* or D1D → D1D\*. This π exchange attraction possibly results in a 1−− hadronic molecule near the D1D threshold. In this work, we shall investigate whether Y(4260) and Z\(_{2}^{+}\) (4250) could be hadronic molecule due to the off-diagonal interaction in the framework of heavy quark effective theory. The contribution of σ exchange has been considered, which results in diagonal interaction. The light vector mesons ρ and ω exchange is discussed as well.

The paper is organized as follows. In section II, we present the formalism to include both heavy meson and anti-meson fields in the heavy meson chiral perturbation theory (HM\(_{\chi}\)PT), and the complete Lagrangian is written out explicitly. Section III illustrates the systematic procedure for converting a general T-matrix into an equivalent potential operator. Later we follow this to derive the effective potential. In section IV, we present both the diagonal and non-diagonal potential related with Y(4260) and Z\(_{2}^{+}\) (4250). In section V, we investigate the possible bound states of the D1D and D0D\* system by solving the coupled-channel Schrödinger equations, and the structures of Y(4260) and Z\(_{2}^{+}\) (4250) are discussed. Moreover, the bottom analog of Y(4260) and Z\(_{2}^{+}\) (4250) is studied. We present our conclusions and some relevant discussions in Section VI. Finally, the potential from ρ and ω exchange is shown in the Appendix.

II. FORMALISM FOR THE SYSTEM CONTAINING BOTH MESON AND ANTI-MESON FIELDS IN HM\(_{\chi}\)PT

The strong interaction between pseudo-Goldstone bosons and the mesons containing a heavy quark is described by the so-called heavy meson chiral perturbation theory (HM\(_{\chi}\)PT)\textsuperscript{24, 25, 26}. The heavy meson chiral perturbation theory is constructed starting from the spin-flavor symmetry occurring in QCD in the infinite heavy quark mass limit, and from the chiral symmetry valid in the massless limit for the light quarks. In HM\(_{\chi}\)PT, the heavy-light meson field appears in a covariant form, which is represented by a 4 × 4 Dirac-type matrix. The negative and positive parity doublets containing a heavy quark Q and a light anti-quark of flavor \(a\), can be respectively described by the superfields \(H_{a}^{(Q)}\), \(S_{a}\) and \(T_{a}^{\mu}\) as follows

\[
\begin{align*}
H_{a}^{(Q)} &= \frac{1 + \gamma^{j}}{2} [P_{a}^{\dagger}(Q)\gamma_{\mu} - P_{a}(Q)\gamma_{5}] \\
S_{a}^{(Q)} &= \frac{1 + \gamma^{j}}{2} [P_{a}^{\dagger}(Q)\gamma_{\mu} \gamma_{5} - P_{a}(Q)] \\
T_{a}^{(Q)\mu} &= \frac{1 + \gamma^{j}}{2} [P_{2a}^{\dagger}(Q)\gamma_{\mu} - \frac{3}{2} P_{1a}^{\dagger}(Q)(g^{\mu\nu} - \frac{1}{3}\gamma\gamma^{\mu}\gamma^{\nu})]
\end{align*}
\]

(1)

The above various operators annihilate mesons of four-velocity \(v\) which is conserved in strong interaction processes. The heavy field operators contain a factor \(\sqrt{M_{\mathcal{P}}}\) and have dimension 3/2. Under a heavy quark spin \(SU(2)\) transformation \(S\) and a generic light flavor transformation \(U\) (i.e., \(U \in SU(3)\))

\[
\begin{align*}
H_{a}^{(Q)} &\rightarrow S H_{b}^{(Q)} U_{ba}, & S_{a}^{(Q)} &\rightarrow S S_{b}^{(Q)} U_{ba}, & T_{a}^{(Q)\mu} &\rightarrow S T_{b}^{(Q)\mu} U_{ba}^\dagger
\end{align*}
\]

(2)

The conjugate field, which creates heavy-light mesons containing a heavy quark \(Q\) and a light anti-quark of flavor \(a\), is defined as

\[
\begin{align*}
\bar{\Pi}_{a}^{(Q)} &= \gamma_{0} H_{a}^{(Q)\dagger} \gamma_{0}, & \bar{S}_{a}^{(Q)} &= \gamma_{0} S_{a}^{(Q)\dagger} \gamma_{0}, & \bar{T}_{a}^{(Q)\mu} &= \gamma_{0} T_{a}^{(Q)\mu \dagger} \gamma_{0}
\end{align*}
\]

(3)

which transforms under \(S\) and \(U\) as

\[
\begin{align*}
\bar{\Pi}_{a}^{(Q)} &\rightarrow U_{ab} \bar{\Pi}_{b}^{(Q)} S_{ab}^\dagger, & \bar{S}_{a}^{(Q)} &\rightarrow U_{ab} \bar{S}_{b}^{(Q)} S_{ab}^\dagger, & \bar{T}_{a}^{(Q)\mu} &\rightarrow U_{ab} \bar{T}_{b}^{(Q)\mu} S_{ab}^\dagger
\end{align*}
\]

(4)

The octet of light pseudoscalar mesons can be introduced using the non-linear representation \(\Sigma = \xi^{2}\) and \(\xi = \exp(i\mathcal{M}/f_{\pi})\) with \(f_{\pi} = 132\) MeV. The matrix \(\mathcal{M}\) contains \(\pi, K, \eta\) fields, which is a \(3 \times 3\) hermitian and traceless matrix

\[
\mathcal{M} = \begin{pmatrix}
\frac{\pi^{0}}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \frac{\pi^{+}}{\sqrt{2}} & \frac{K^{+}}{2} \\
-\frac{\pi^{-}}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \frac{\pi^{0}}{\sqrt{2}} & \frac{K^{0}}{2} \\
K^{-} & \bar{K}^{0} & -\sqrt{\frac{2}{3}} \eta
\end{pmatrix}
\]

(5)
Under the chiral symmetry, the field $\xi$ transforms as
\[ \xi \rightarrow g_L \xi U^\dagger = U \xi g_R \]  
where $g_L$ and $g_R$ are left-handed and right-handed global $SU(3)$ transformation respectively.

The effective QCD Lagrangian is constructed by imposing invariance under both heavy quark spin-flavor transformation and chiral transformation, it is \[ \mathcal{L}_V = ig \langle H^Q \rangle a_{\gamma_5} \bar{H}^Q a + ik \langle T^Q \mu \rangle a_{\gamma_5} \bar{T}^Q a_{\mu} + \left[ ih \langle \xi^Q \rangle a_{\gamma_5} \bar{H}^Q a \right] + \left[ ih \langle \xi^Q \rangle a_{\gamma_5} \bar{T}^Q a_{\mu} + h.c. \right] \]  
where $\langle \cdots \rangle$ means trace over the $4 \times 4$ matrices, the covariant derivative $D_\mu = \partial_\mu + V_\mu$, the vector current $V_\mu$, and the axial current $A_\mu$ are defined by
\[ V_\mu = \frac{1}{2} \left( \xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger \right) \]  
\[ A_\mu = \frac{1}{2} \left( \xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger \right) \]  
In order to describe mesons containing heavy anti-quark $\bar{Q}$, we have to introduce six new fields $P^a_{\alpha \mu}$, $P_a^{(2)}$, $P_{1\alpha \mu}^{(1)}$, $P_{0a}^{(1)}$, $P_{2a}^{(1)}$ and $P_{1\alpha}^{(1)}$ which destroy mesons containing a heavy anti-quark $\bar{Q}$. The phase of the field $P_{a \alpha}^{(1)}$ relative to $P^a_{\alpha \mu}$ to $P^{(2)}_a$ etc can be fixed by the following charge conjugation convention
\[ P_a^{(1)} = -CP_{a \mu}^{(1)} C^{-1}, \quad P_a^{(2)} = CP_a^{(1)} C^{-1}, \quad P_{1\alpha}^{(1)} = CP_{1\alpha \mu}^{(1)} C^{-1}, \quad P_{0a}^{(1)} = CP_{0a \mu}^{(1)} C^{-1}, \quad P_{2\alpha}^{(1)} = CP_{2a \mu}^{(1)} C^{-1}, \quad P_{1\alpha}^{(1)} = CP_{1\alpha \mu}^{(1)} C^{-1} \]  
The mesons containing a heavy anti-quark $\bar{Q}$ and a light quark of flavor $a$ can be included into the theory by applying the charge conjugation operation to the above heavy-light meson superfields $H^Q_a$, $S^{(2)}_a$ and $T^{(1)}_{a \mu}$. The matrix $C$ is the charge conjugation matrix for Dirac spinors with $C = i \gamma^5 \gamma^0$, and the transpose is on the spinor matrix indices. Under the heavy quark spin-flavor transformation $S$ and light quark $SU(3)$ flavor symmetry $U$, $H^Q_a \rightarrow U H^Q_a S^\dagger$, $S^{(2)}_a \rightarrow U S^{(2)}_a S^\dagger$, $T^{(1)}_{a \mu} \rightarrow U T^{(1)}_{a \mu} S^\dagger$  
Similarly the hermitian conjugate fields are defined by
\[ \bar{H}^Q_a = \gamma_0 H^Q_a \gamma_0, \quad \bar{S}^{(2)}_a = \gamma_0 S^{(2)}_a \gamma_0, \quad \bar{T}^{(1)}_{a \mu} = \gamma_0 T^{(1)}_{a \mu} \gamma_0 \]  
Under the symmetry transformation $S$ and $U$
\[ \bar{H}^Q_a \rightarrow S \bar{H}^Q_a U^\dagger, \quad \bar{S}^{(2)}_a \rightarrow S \bar{S}^{(2)}_a U^\dagger, \quad \bar{T}^{(1)}_{a \mu} \rightarrow S \bar{T}^{(1)}_{a \mu} U^\dagger \]  
For the system including both heavy meson and heavy anti-meson field in the HM$\chi$PT, the total effective Lagrangian should be invariant under the charge conjugation transformation. The interaction between the pseudo-Goldstone bosons and the meson containing one heavy anti-quark can be obtained from Eq. (14) by applying the charge conjugation operator
\[ \mathcal{L}_V = ig \langle \bar{H}^Q_a \rangle a_{\gamma_5} \bar{H}^Q a + ik \langle \bar{T}^Q \mu \rangle a_{\gamma_5} \bar{T}^Q a_{\mu} + \left[ ih \langle \xi^Q \rangle a_{\gamma_5} \bar{H}^Q a \right] + \left[ ih \langle \xi^Q \rangle a_{\gamma_5} \bar{T}^Q a_{\mu} + h.c. \right] \]
where \( D'_\mu = \partial_\mu - V_\mu \). After expanding the effective Lagrangian in Eq. (7) and Eq. (14) to the leading order of pseudo-Goldstone field, we can obtain the following effective interactions, which is needed in our work

\[
\mathcal{L}_{DD^*P} = g_{DD^*P}D_b(\partial_\mu M)_{ba}D'^{\mu\dagger}_a + g_{DD^*P}(\partial_\mu M)_{ba}D'^{\mu\dagger}_a + g_{DD^*P}D_b(\partial_\mu M)_{ab}D'^{\mu\dagger}_b + g_{DD^*P}D_b(\partial_\mu M)_{ab}D'^{\mu\dagger}_b + h.c.
\]

\[
\mathcal{L}_{DDP} = g_{DDP}D_b(\partial_\mu M)_{ba}D'^{\mu\dagger}_a + g_{DDP}(\partial_\mu M)_{ba}D'^{\mu\dagger}_a + g_{DDP}D_b(\partial_\mu M)_{ab}D'^{\mu\dagger}_b + g_{DDP}D_b(\partial_\mu M)_{ab}D'^{\mu\dagger}_b + h.c.
\]

\[
\mathcal{L}_{DDP} = ig_{DDP}(D_{\mu\nu\partial_\mu}M_{ba} + ig_{DDP}(\partial_\mu D_{ab})M_{ba} + h.c.
\]

\[
\mathcal{L}_{DD^*P} = g_{DD^*P}D_b(\partial_\mu M)_{ba}D'^{\mu\dagger}_a + g_{DD^*P}(\partial_\mu M)_{ba}D'^{\mu\dagger}_a + \frac{1}{\Lambda_{MD}}(\partial_\mu D^\dagger_{ab})(\partial_\tau M)^{\mu\dagger}_{ab} + h.c.
\]

\[
\mathcal{L}_{DD^*P} = g_{DD^*P}D_b(\partial_\mu M)_{ba}D'^{\mu\dagger}_a + g_{DD^*P}(\partial_\mu M)_{ba}D'^{\mu\dagger}_a + \frac{1}{\Lambda_{MD}}(\partial_\mu D^\dagger_{ab})(\partial_\tau M)^{\mu\dagger}_{ab} + h.c.
\]

In the chiral and heavy quark limit, the above coupling constants are

\[
g_{DD^*P} = -g_{DDP} = -\frac{2g}{f_\pi} \sqrt{M_{DD^*P}}.
\]

\[
g_{DDP} = g_{DDP} = -\frac{h}{f_\pi} \Lambda_{MD}.
\]

\[
g_{DD^*P} = g_{DDP} = -\frac{h}{f_\pi} \Lambda_{MD}.
\]

We would like to stress that the DD^*P coupling constant is the negative of the DD^*P coupling constant, because of the phase convention for charge conjugation chosen in Eq. (19). The effective Lagrangian between \( \sigma \) and heavy meson(anti-meson) are

\[
\mathcal{L}_\sigma = g_\sigma (H_a^{(Q)} \sigma \overline{H}_a^{(Q)} + g_\sigma (S_a^{(Q)} \sigma \overline{S}_a^{(Q)}) + g_\sigma (T_a^{(Q)} \sigma \overline{T}_a^{(Q)}) + \frac{h_\sigma}{f_\pi} (S_a^{(Q)} \gamma^\mu (\partial_\mu \sigma) \overline{H}_a^{(Q)})
\]

\[
+ \frac{h'}{f_\pi} (T_a^{(Q)} \gamma^\mu (\partial_\mu \sigma) \overline{T}_a^{(Q)}) + h.c.\right) + g_\sigma (\overline{H}_a^{(Q)} \sigma H_a^{(Q)}) + g_\sigma (\overline{S}_a^{(Q)} \sigma S_a^{(Q)}) + g_\sigma (\overline{T}_a^{(Q)} \sigma T_a^{(Q)})
\]

\[
+ \left[ - \frac{h_\sigma}{f_\pi} (\overline{H}_a^{(Q)} \gamma^\mu (\partial_\mu \sigma) \sigma \overline{T}_a^{(Q)}) + \frac{h'}{f_\pi} (\overline{H}_a^{(Q)} \sigma \overline{T}_b^{(Q)}) + h.c.\right]
\]

The coupling constants are estimated as follows

\[
g_\sigma = -\frac{g_\pi}{2\sqrt{6}}, \quad g_\sigma' = -\frac{g_\pi}{2\sqrt{6}}, \quad h_\sigma = \frac{g_A}{\sqrt{3}}
\]

where \( g_\pi = -3.73 \) and \( g_A = 0.6 \). As in Ref. [31], we take \( |g_\pi'| = |g_\pi| \) and \( |h_\sigma'| = |h_\sigma| \) approximately when performing the numerical analysis. Expanding the Lagrangian \( \mathcal{L}_\sigma \), we get the interactions associated with \( \sigma \)

\[
\mathcal{L}_{DD\sigma} = g_{DD\sigma} D_a D_a^\dagger \sigma + g_{DD\sigma} D_a D_a^\dagger \sigma
\]

\[
\mathcal{L}_{DDP\sigma} = g_{DDP\sigma} D_a D_a^\dagger \sigma + g_{DDP\sigma} D_a D_a^\dagger \sigma
\]

\[
\mathcal{L}_{DD^*P\sigma} = g_{DD^*P\sigma} D_a D_a^\dagger \sigma + g_{DD^*P\sigma} D_a D_a^\dagger \sigma
\]

\[
\mathcal{L}_{DD^*P\sigma} = g_{DD^*P\sigma} D_a D_a^\dagger \sigma + g_{DD^*P\sigma} D_a D_a^\dagger \sigma
\]

\[
\mathcal{L}_{DD^*P\sigma} = g_{DD^*P\sigma} D_a D_a^\dagger \sigma + g_{DD^*P\sigma} D_a D_a^\dagger \sigma
\]

\[
\mathcal{L}_{DD^*P\sigma} = g_{DD^*P\sigma} D_a D_a^\dagger \sigma + g_{DD^*P\sigma} D_a D_a^\dagger \sigma
\]

\[
\mathcal{L}_{DD^*P\sigma} = g_{DD^*P\sigma} D_a D_a^\dagger \sigma + g_{DD^*P\sigma} D_a D_a^\dagger \sigma
\]

\[
\mathcal{L}_{DD^*P\sigma} = g_{DD^*P\sigma} D_a D_a^\dagger \sigma + g_{DD^*P\sigma} D_a D_a^\dagger \sigma
\]
The relevant coupling constants are

\[ g_{DD} = g_{DD} = -2g_{M} \]
\[ g_{D1} = g_{D1} = -2g''_{M} \]
\[ g_{D1} = -\frac{2\sqrt{6} h_{c}}{3f_{\pi}} \sqrt{M_{D} M_{D}} \]
\[ g_{D^*} = g_{D^*} = 2g_{M} \]
\[ g_{D_{0}} = 2g_{M} \]
\[ g_{D^*} = -\frac{2h_{c}}{f_{\pi}} \sqrt{M_{D^*} M_{D_{0}}} \]

(20)

### III. CONVERTING THE T-MATRIX INTO THE EFFECTIVE POTENTIAL

The T-matrix for \( A(p_1)B(p_2) \to C(p'_1)D(p'_2) \) scattering process can be represented by an equivalent Born-order potential operator \( V_{bn}(r_1 - r_2, \nabla_1, \nabla_2) \) between pointlike particles, the definition of this potential operator is \[35, 36\]. In this section, we will follow the general procedure shown above to derive the effective potential associated with \( Y(4260) \) and \( Z_2^+(4250) \)

\[ \delta^3(p_1 + p_2 - p_1 - p_2) T_{fi}(p_1, p_2, p'_1, p'_2) \]
\[ = \frac{1}{(2\pi)^3} \int d^3r_1 d^3r_2 e^{-i(p'_1 - p_1 + p_2 - p_2)} V_{bn}(r_1 - r_2, \nabla_1, \nabla_2) \]

(21)

where \( T_{fi}(p_1, p_2, p'_1, p'_2) \) is the T-matrix for the process \( A(p_1)B(p_2) \to C(p'_1)D(p'_2) \). In general, \( T_{fi} \) depends on all the involved momentum \( p_1, p_2, p'_1 \) and \( p'_2 \). For convenience, we introduce

\[ P_1 \equiv \frac{1}{2}(p_1 + p'_1), \quad P_2 \equiv \frac{1}{2}(p_2 + p'_2), \quad q \equiv p'_1 - p_1 = p_2 - p'_2 \]

(22)

In the center of mass frame \( P_1 = -P_2 \). The amplitude \( T_{fi} \) can be expanded as a power series in \( P_{1i} \) and \( P_{2i} \)

\[ T_{fi}(p_1, p_2, p'_1, p'_2) = T^{(0)}(q) + T^{(1,0)}(q) P_{1i} + T^{(0,1)}(q) P_{2i} + T^{(1,1)}(q) P_{1j} P_{2j} + ... \]

(23)

This procedure produces the full Breit-Fermi Hamiltonian when it is applied to the photon exchanged electron-electron scattering amplitude expanded to \( O(P^2) \). The leading term \( T^{(0)}(q) \) is a function of \( q \) only, its Fourier transformation gives us a local potential \( V(r) \) that is a function of \( r_1 - r_2 = r \) only. The relation between \( T^{(0)}(q) \) and \( V(r) \) is

\[ V(r) = \frac{1}{(2\pi)^3} \int d^3q T^{(0)}(q) e^{iq\cdot r} \]

(24)

For the higher terms of the T-matrix expansion, \( P_{1i} \) and \( P_{2i} \) are replaced by left- and right-gradients in the equivalent potential operator defined implicitly by Eq. (21) \[35\]. Following this systematic procedure, we can convert a general T-matrix into an equivalent potential operator. In this work, we obtain the local potential by Fourier transforming the leading terms \( T^{(0)}(q) \) of the scattering amplitude \( T_{fi} \), which is common in potential model \[35, 36\].

Since the propagators are off-shell, we introduce form factor at each vertex when writing out the scattering amplitude, the usual form factor is expressed as \[35, 37\]

\[ F(q) = \frac{\Lambda^2 - m^2}{\Lambda^2 - q^2} \]

(25)

where \( \Lambda \) is an adjustable constant within a reasonable range of 1-2 GeV, which models the off-shell effects at the vertices due to the internal structure of the meson. \( m \) and \( q \) are the mass and the four momentum of the exchanged meson respectively.

### IV. THE EFFECTIVE POTENTIALS RELATED WITH Y(4260) AND Z_2^+(4250)

Recently, the meson exchange model based on the HM\( \chi \)PT has been used to study possible heavy flavor molecule \[31, 41\]. In this section, we will follow the general procedure shown above to derive the effective potential associated with \( Y(4260) \) and \( Z_2^+(4250) \) in the framework of HM\( \chi \)PT. From the effective interaction in Eq. (15) and Eq. (19), we can
write down the corresponding scattering amplitude for each diagram, including the form factor at each vertex. Then we get the equivalent potential in momentum space following the general formalism presented in section III. Finally we make Fourier transformation to derive the potentials in coordinate space. Because of parity conservation, pseudoscalar $\pi$ and $\eta$ exchange only contributes to the off-diagonal interaction, whereas $\sigma$ exchange and light vector mesons $\rho$, $\omega$ exchange result in diagonal interaction only. The corresponding scattering diagrams are shown in Fig. 1.

![Scattering Diagrams](image)

**FIG. 1**: The scattering diagrams with pseudoscalars $\pi$, $\eta$ exchange, $\sigma$ exchange, and light vector mesons $\rho$, $\omega$ exchange.

Under the ansatz of $Y(4260)$ as a $D_1D$ or $D_0D^*$ hadronic molecule, we can write down its flavor wavefunction

$$
|Y(4260)\rangle = \frac{1}{2} [ |D_0^0D_1^+\rangle + |D_0^+D_1^-\rangle - |D^0\bar{D}_1^+\rangle - |D^+\bar{D}_1^-\rangle ]
$$

$$
|Y'(4260)\rangle = \frac{1}{2} [ |D_0^0D_1^0\rangle + |D_0^+D_1^0\rangle + |D^0\bar{D}_1^0\rangle + |D^+\bar{D}_1^0\rangle ]
$$

(26)

We stress that the phase convention under charge conjugation is consistent with Eq. (9). In the same way, the flavor wavefunction of $Z_2^+(4250)$ is

$$
|Z_2^+(4250)\rangle = \frac{1}{\sqrt{2}} [ |D_1^+\bar{D}_0^0\rangle + |D^+\bar{D}_1^0\rangle ]
$$

$$
|Z'_2^+(4250)\rangle = \frac{1}{\sqrt{2}} [ |D_0^0\bar{D}_1^0\rangle - |D^+\bar{D}_1^-\rangle ]
$$

(27)
In this case, its quantum number are \( I^G(J^P) = 1^- (1^-) \). Following the procedure discussed above, we can calculate the effective potential in momentum space, it is a lengthy and tedious calculation.

For \( Y(4260) \), the exchange potential in momentum space is

\[
V_{12}(q) = V_{21}(q) = \frac{\sqrt{6} \, g \, h}{6 \, \frac{f_\pi^2}{2}} \left( \frac{\Lambda^2 - m^2_\pi}{q^2 + X_1^2} \right)^2 \frac{q^2}{q^2 + \mu^2_1} + \frac{\sqrt{6} \, g \, h}{54 \, \frac{f_\pi^2}{2}} \left( \frac{\Lambda^2 - m^2_\eta}{q^2 + X_1^2} \right)^2 \frac{q^2}{q^2 + \mu^2_2}
\]

\[
V_{11}(q) = \left( \frac{\Lambda^2 - m^2_\sigma}{q^2 + \Lambda^2} \right)^2 \frac{g_\sigma g''}{q^2 + m^2_\sigma} + \frac{2h^2}{9f^2_\pi} \left( \frac{\Lambda^2 - m^2_\sigma}{q^2 + X_2^2} \right)^2 \frac{q^2}{q^2 + \mu^2_3}
\]

\[
V_{22}(q) = \left( \frac{\Lambda^2 - m^2_\sigma}{q^2 + \Lambda^2} \right)^2 \frac{g_\sigma g' \sigma}{q^2 + m^2_\sigma} + \frac{h^2}{3f^2_\pi} \left( \frac{\Lambda^2 - m^2_\sigma}{q^2 + X_3^2} \right)^2 \frac{q^2}{q^2 + \mu^2_4}
\]

(28)

where we have include the monopole form factor in Eq. 25 to regularize the potential. The diagonal potential \( V_{11}(q) \) and \( V_{22}(q) \) is induced by \( \sigma \) exchange, and the non-diagonal potential \( V_{12}(q) \) or \( V_{21}(q) \) arises from the pseudo-Goldstone bosons \( \pi \) and \( \eta \) exchange. The effective potential from \( \rho \), \( \omega \) exchange is shown in the Appendix. The potential for \( Z_2^+ \)(4250) in momentum space is

\[
V_{12}(q) = V_{21}(q) = -\frac{\sqrt{6} \, g \, h}{18 \, \frac{f_\pi^2}{2}} \left( \frac{\Lambda^2 - m^2_\sigma}{q^2 + X_1^2} \right)^2 \frac{q^2}{q^2 + \mu^2_1} + \frac{\sqrt{6} \, g \, h}{54 \, \frac{f_\pi^2}{2}} \left( \frac{\Lambda^2 - m^2_\eta}{q^2 + X_1^2} \right)^2 \frac{q^2}{q^2 + \mu^2_2}
\]

\[
V_{11}(q) = \left( \frac{\Lambda^2 - m^2_\sigma}{q^2 + \Lambda^2} \right)^2 \frac{g_\sigma g''}{q^2 + m^2_\sigma} + \frac{2h^2}{9f^2_\pi} \left( \frac{\Lambda^2 - m^2_\sigma}{q^2 + X_2^2} \right)^2 \frac{q^2}{q^2 + \mu^2_3}
\]

\[
V_{22}(q) = \left( \frac{\Lambda^2 - m^2_\sigma}{q^2 + \Lambda^2} \right)^2 \frac{g_\sigma g' \sigma}{q^2 + m^2_\sigma} + \frac{h^2}{3f^2_\pi} \left( \frac{\Lambda^2 - m^2_\sigma}{q^2 + X_3^2} \right)^2 \frac{q^2}{q^2 + \mu^2_4}
\]

(29)

The various parameters appearing in the above formulas are defined as follows.

\[
X_1^2 = \Lambda^2 - (M_D^* - M_D)(M_{D_1} - M_{D_0})
\]

\[
X_2^2 = \Lambda^2 - (M_{D_1} - M_D)^2
\]

\[
X_3^2 = \Lambda^2 - (M_{D_0} - M_D)^2
\]

\[
\mu^2_1 = m^2_\pi - (M_{D_1} - M_D)
\]

\[
\mu^2_2 = m^2_\eta - (M_{D_1} - M_D)
\]

\[
\mu^2_3 = m^2_\eta - (M_{D_0} - M_D)
\]

\[
\mu^2_4 = m^2_\eta - (M_{D_0} - M_D)^2
\]

(30)

After performing Fourier transformation, we obtain the potential forms in configuration space, For \( Y(4260) \), the potential in coordinate space is

\[
V_{12}(r) = V_{21}(r) = \frac{\sqrt{6} \, g \, h}{6 \, \frac{f_\pi^2}{2}} Z(\Lambda, X_1, \mu_1, m_{\pi, r}) + \frac{\sqrt{6} \, g \, h}{54 \, \frac{f_\pi^2}{2}} Z(\Lambda, X_1, \mu_2, m_{\eta, r})
\]

\[
V_{11}(r) = g_\sigma g'' H(\Lambda, m_{\sigma, r}) + \frac{2h^2}{9f^2_\pi} Z(\Lambda, X_2, \mu_3, m_{\sigma, r})
\]

\[
V_{22}(r) = g_\sigma g' H(\Lambda, m_{\sigma, r}) + \frac{h^2}{3f^2_\pi} Z(\Lambda, X_3, \mu_4, m_{\sigma, r})
\]

(31)

The potential in coordinate space for \( Z_2^+ \)(4250) is

\[
V_{12}(r) = V_{21}(r) = -\frac{\sqrt{6} \, g \, h}{18 \, \frac{f_\pi^2}{2}} Z(\Lambda, X_1, \mu_1, m_{\pi, r}) + \frac{\sqrt{6} \, g \, h}{54 \, \frac{f_\pi^2}{2}} Z(\Lambda, X_1, \mu_2, m_{\eta, r})
\]

\[
V_{11}(r) = g_\sigma g'' H(\Lambda, m_{\sigma, r}) - \frac{2h^2}{9f^2_\pi} Z(\Lambda, X_2, \mu_3, m_{\sigma, r})
\]

\[
V_{22}(r) = g_\sigma g' H(\Lambda, m_{\sigma, r}) - \frac{h^2}{3f^2_\pi} Z(\Lambda, X_3, \mu_4, m_{\sigma, r})
\]

(32)

Here the functions \( H(\Lambda, m, r) \) and \( Z(\Lambda, X, \mu, m, r) \) are defined as

\[
H(\Lambda, m, r) = \frac{1}{4\pi r} (e^{-mr} - e^{-\Lambda r}) - \frac{\Lambda^2 - m^2}{8\pi \Lambda} e^{-\Lambda r}
\]

\[
Z(\Lambda, X, \mu, m, r) = \frac{1}{4\pi r} (X^2 e^{-\Lambda r} - \mu^2 e^{-\mu^2 r}) + \frac{\Lambda^2 - m^2}{8\pi} (X - \frac{2}{r}) e^{-X r}
\]

(33)
configuration is easier to bind than the I\(_G\) third of the former. This is consistent with results from chiral quark model\[39\], consequently the I\(_G\) of the off-diagonal potential related with Y(4260) are larger than that associated with Z\(_2^+\) exchange, this is mainly because \(m_\pi\) is smaller than \(m_\sigma\). Moreover, the magnitude of the off-diagonal potential related with Y(4260) are larger that associated with Z\(_2^+\) exchange, this is mainly because \(m_\pi\) is smaller than \(m_\sigma\). Moreover, the magnitude of the off-diagonal potential related with Y(4260) are larger than that associated with Z\(_2^+\) exchange, this is mainly because \(m_\pi\) is smaller than \(m_\sigma\). Moreover, the magnitude of the off-diagonal potential related with Y(4260) are larger than that associated with Z\(_2^+\) exchange, this is mainly because \(m_\pi\) is smaller than \(m_\sigma\). Moreover, the magnitude of the off-diagonal potential related with Y(4260) are larger than that associated with Z\(_2^+\) exchange, this is mainly because \(m_\pi\) is smaller than \(m_\sigma\). Moreover, the magnitude of the off-diagonal potential related with Y(4260) are larger than that associated with Z\(_2^+\) exchange, this is mainly because \(m_\pi\) is smaller than \(m_\sigma\). Moreover, the magnitude of the off-diagonal potential related with Y(4260) are larger than that associated with Z\(_2^+\)

\[\text{FIG. 2: (color online)The effective potential for the D}_1\text{D and D}_0\text{D}^*\text{ system from pseudoscalar } \pi, \eta \text{ exchange and scalar } \sigma \text{ exchange. Solid line represents the non-diagonal potential } V_{12}(r) \text{ (or } V_{21}(r))\text{, short dashed and dash dotted lines respectively correspond to the diagonal potential } V_{11}(r) \text{ and } V_{22}(r). (a) and (b) are related with Y(4260), and (b) shows the long range behavior of the potential. (c) and (d) are related with Z\(_2^+\)(4250), and (d) is the long range shape of the potential.}

V. THE STRUCTURES OF Y(4260) AND Z\(_2^+\)(4250) AND THE BOTTOM ANALOG

A. The bound states of the D\(_1\)D and D\(_0\)D\(^*\) system with the structure of Y(4260) and Z\(_2^+\)(4250)

With the above effective potential, we shall explore whether there are bound states with \(I^G(J^{PC}) = 0^-(1^-)\) or \(I^G(J^{PC}) = 1^-(1^-)\) in the D\(_1\)D and D\(_0\)D\(^*\) system, by means of solving the two channels coupled Schrödinger equation. There are various methods to integrate the coupled-channel Schrödinger equation numerically. In this work we shall employ two packages MATSCS\[42\] and FESSDE2.2\[43\] to perform the numerical calculation so that the results obtained by one program can be checked by another. The first package is a Matlab software, and the second is written in Fortran77. Both packages can fastly and accurately solve the eigenvalue problem for systems of coupled Schrödinger equations, and the results obtained by two codes are the same within error.

The masses of the involved mesons are taken from PDG\[44\]: \(M_{D} = 1869.3\text{MeV, } M_{D^*} = 2006.7\text{MeV, } M_{D_0} = 2422\text{MeV, } M_{D_0} = 2308\text{MeV, } m_\pi = 135\text{MeV, } m_\eta = 547.5\text{MeV, } m_\sigma = 600\text{MeV, } m_\rho = 775.5\text{MeV and } m_\omega = 782.65\text{MeV.}

The effective coupling constants in HM\(_\chi^P\)T have been studied from various phenomenological and theoretical approaches, and the estimates for \(g, \tilde{h}\) are listed in Table\[4]\ It is obvious that there are still large uncertainties in their values. In the following, we shall first consider whether one pseudoscalar \(\pi\) and \(\eta\) exchange can result in a bound state in the D\(_1\)D and D\(_0\)D\(^*\) system, then the contribution of \(\sigma\) exchange is included.

The numerical results with only one pseudoscalar exchange are presented in Table\[4]\ For several typical values of \(g\tilde{h}\), we vary the cutoff \(\Lambda\) from a small value until we find a solution which lies below the D\(_1\)D threshold. Here the

\[\text{FIG. 2: (color online)The effective potential for the D}_1\text{D and D}_0\text{D}^*\text{ system from pseudoscalar } \pi, \eta \text{ exchange and scalar } \sigma \text{ exchange. Solid line represents the non-diagonal potential } V_{12}(r) \text{ (or } V_{21}(r))\text{, short dashed and dash dotted lines respectively correspond to the diagonal potential } V_{11}(r) \text{ and } V_{22}(r). (a) and (b) are related with Y(4260), and (b) shows the long range behavior of the potential. (c) and (d) are related with Z\(_2^+\)(4250), and (d) is the long range shape of the potential.}
of the parameter $gh$, one notes that the magnitude of $M$ increases with $\Lambda$, whereas the reverse is true for $r_{\text{rms}}$ and $R$. The bound state mass is sensitive to the parameter $gh$ as well, larger $gh$ is helpful to form a molecular state. From the numerical results in Table III we see that one can get a molecular state consistent with $Y(4260)$, given appropriate value for $gh$ and a reasonable cutoff $\Lambda$ in the range 1-2 GeV. However, the existence of a bound state with $I^G(J^P) = 1^- (1^-)$ require that the value of $\Lambda$ should be at least larger than 4 GeV. The cutoff parameter $\Lambda$ is a typical hadronic scale, which is generally expected to be in the range 1-2 GeV. If $\Lambda$ is required to be much larger than 2 GeV in order to form a bound state, we tend to conclude that such a bound state should not exist. Therefore, it is not appropriate to assign $Z_2^+(4250)$ as a $D_1D$ or $D_0D^*$ molecule, if only the non-diagonal interaction from $\pi$ and $\eta$ exchange is considered.

Then we include the contribution coming from $\sigma$ exchange, which leads to only the diagonal interaction. The corresponding numerical results are shown in Table III and Table IV. The radial wavefunctions $\chi(r) = rR(r)$ for certain certain parameter values are shown in Fig. 3. The wavefunction corresponding to other solutions in Table II, III and IV has similar shape with that in Fig. 3. We find that the $\sigma$ exchange interaction has significant effects, the variations of $M$, $r_{\text{rms}}$ and $R$ with respect to $\Lambda$ have the same pattern as those in the only pseudoscalar exchange case. Varying the parameters $gh$, $g_\sigma g_\sigma^\prime\prime$, $g_\sigma g_\sigma^\prime$ and $h_\sigma$ in the reasonable range results in large change of the predictions, which indicates that the results are sensitive to the effective coupling constants. We can see that large $gh$, negative $g_\sigma g_\sigma^\prime\prime$ and $g_\sigma g_\sigma^\prime$ are favorable to binding the molecular states. Comparing the results in Table III and IV, we find that $\sigma$ exchange is against the formation of bound state with $I^G(J^P) = 0^- (1^-)$, nevertheless, it is beneficial to the formation of $I^G(J^P) = 1^- (1^-)$ molecular state. As for $Y(4260)$, the conclusion reached with only pseudoscalar exchange remains. $Y(4260)$ could be accommodated as a molecule state for appropriate effective coupling constants and cutoff. A $I^G(J^P) = 1^- (1^-)$ bound state around 4250 MeV requires $\Lambda$ should be at least 3 GeV, therefore we conclude that the interpretation of $Z_2^+(4250)$ as a $D_1D$ or $D_0D^*$ molecule is disfavored. This conclusion is consistent with the general observations from chiral quark model. It is found that the isoscalar channel is easier to bind that the isovector channel for the same components [51].

**B. The bottom analog of $Y(4260)$ and $Z_2^+(4250)$**

The bottom analog $Y_{bb}$ and $Z_{bb}^+$, respectively denote the states obtained by replacing both the charm quark and antiquark with bottom quark and antiquark in $Y(4260)$ and $Z_2^+(4250)$. The above calculation can be easily extended to study these states. The shape of both the diagonal and non-diagonal potential is similar to that of the charm system, except that the former is larger than the latter in magnitude. Furthermore, Since the kinetic energy is greatly reduced because of the heavier mass of B meson, a molecular state is more easily formed. We choose the same set of parameters as in the previous section. The numerical results with only pseudoscalar $\pi$, $\eta$ exchange are shown in Table VI and the results with both pseudoscalar and $\sigma$ exchange are listed in Table VII and VIII. As is expected, the magnitude $M$ of the bottom analog is larger than that of the corresponding charmed state for the same parameters. The variation of $M$, $r_{\text{rms}}$ and $R$ with $\Lambda$ is the same as the charm system, large $gh$, negative $g_\sigma g_\sigma^\prime\prime$ and $g_\sigma g_\sigma^\prime$ are beneficial to molecule formation as well. From the results in Table VI, VII and VIII we note that both the bottom analog $Y_{bb}$

| Reference | $g$ | Remark |
|-----------|-----|--------|
| [45] | $0.59 \pm 0.07 \pm 0.01$ | combining the CLEO’s results on $D^*$ decay width |
| [46] | $0.46 \pm 0.04$ | through a constituent quark-meson model |
| [47] | $0.53$ | including one loop corrections without positive parity states |
| [48] | $0.65$ | including one loop corrections with positive parity states |
| [49] | $0.44 \pm 0.16$ | from QCD sum rule |
| [50] | $0.39 \pm 0.16$ | from QCD sum rule |
| [51] | $0.32 \pm 0.02$ | |
| [28] | $0.75$ | from non-relativistic quark model |

| Reference | $h$ | Remark |
|-----------|-----|--------|
| [28] | $|h| = 0.87$ | from non-relativistic quark |
| [50] | $0.91 \pm 0.5$ | in a constituent quark-meson model in soft pion limit |
and $Z_{bb}^+$ may exist.

Since $Y(4260)$ has a large branch ratio into $\pi^+\pi^-J/\psi$, the bottom analog $Y_{bb}$ should be searched for in the $\pi^+\pi^-\Upsilon$ channel. Specifically, the state $Y_{bb}$ can be searched for at B factories and future Super B factory via initial state radiation $e^+e^- \rightarrow \gamma_{ISR}\pi^+\pi^-\Upsilon$ or by $e^+e^- \rightarrow \pi^+\pi^-\Upsilon$ direct scan\cite{52}. Furthermore, $Y_{bb}$ may be searched for at Tevatron via $p\bar{p} \rightarrow Y_{bb} \rightarrow \pi^+\pi^-\Upsilon$, and LHC is more promising. Similarly, for the bottom analog $Z_{bb}^+$, the most hopeful discovery channel would be $Z_{bb}^+ \rightarrow \pi^+\chi_{b1}$, where $\chi_{b1}$ is in turn detected by its decay into $\gamma Y_{bb}$. Because of the large mass of this state, it is difficult to produce such state via decay of certain particle (i.e., $Z_{2}^+(4250)$ is produced in B decay\cite{2} ), consequently large hadron collides such as Tevatron and LHC are good place to search for this state.

VI. CONCLUSION AND DISCUSSIONS

In this work, we have performed a dynamical study of $Y(4260)$ and $Z_{2}^+(4250)$ simultaneously to see whether they could be $D_1D$ or $D_0D^*$ hadronic molecule. We have employed the HM$_1$PT, which combines the heavy quark symmetry and the chiral symmetry. Since both the heavy meson and heavy anti-meson are involved, the interaction related with heavy anti-meson has been included explicitly, and the total effective Lagrangian is invariant under the charge conjugation transformation.

The off-diagonal interaction from pseudoscalar $\pi, \eta$ exchange plays a dominant role, which is a straightforward support to the off-diagonal interaction mechanism proposed by Swanson and Close. $\sigma$ exchange leads to only diagonal interaction, its contribution has been taken into account in this work. We find that $\sigma$ exchange is not favorable to the formation of molecular state with $I^G(J^{PC})=0^-(1^{-}-)$, whereas it is helpful to the binding of molecule with $I^G(J^{PC})=1^-(1^{-})$. For appropriate value of the effective coupling constants and a reasonable cutoff $\Lambda$, $Y(4260)$ could be accommodated as a $D_1D$ and $D_0D^*$ molecule. However, the existence of a molecule around 4250 MeV with $I^G(J^{PC})=1^-(1^{-})$ requires that $\Lambda$ should be at least 3 GeV, given the currently allowed values of the coupling constants. Consequently, the interpretation of $Z_{2}^+(4250)$ as a $D_1D$ or $D_0D^*$ molecule is disfavored. Its structure should be studied further. Through calculating the masses of excited heavy tetraquarks with hidden charm in the diquark-antidiquark picture, the authors in Ref.\cite{52} suggested that $Z_{2}^+(4250)$ could be the charged partner of the $1^-$ 1P state SS or as the $0^- 1P$ state of the $(S\bar{A} \pm \bar{S}A)/\sqrt{2}$ tetraquark. QCD sum rule analysis for $Z_{2}^+(4250)$ is performed in the Ref.\cite{52}.

The effective potential from vector meson $\rho, \omega$ exchange has been presented analytically. Because of the accidental coincidence of $m_{\rho}$ and $m_{\omega}$, the contribution from $\rho$ and $\omega$ exchange almost cancels in the potential related with $Z_{2}^+(4250)$. For $Y(4260)$, the situation is not the same. A number of effective coupling constants are involved. Because some of them have not been determined so far, we can not give a quantitative estimate about the contribution from vector meson exchange. Qualitatively, it should be smaller than the contribution coming from pseudoscalar and $\sigma$ exchange in magnitude. It is necessary and interesting to examine the effect of vector meson exchange on $Y(4260)$ in
future. The bottom analog of Y(4260) and Z_{bb}^+(4250) denoted by Y_{bb} and Z_{bb}^+ respectively may exist. Y_{bb} can be searched for in e^+e^− → γ_{ISR}π^+π^−Υ or by e^+e^− → π^+π^−Υ direct scan. The direct production of Y_{bb} at Tevatron or LHC via p\bar{p} → Y_{bb} → π^+π^−Υ is a hopeful approach as well. For Z_{bb}^+\rightarrow π^+\lambda_{bb}.

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APPENDIX A: THE POTENTIAL FROM LIGHT VECTOR MESONS ρ AND ω EXCHANGE

The light vector mesons nonet can be introduced by using the hidden gauge symmetry approach, and the Lagrangian containing these particles is as follows \[27, 32, 33\]

\[\mathcal{L}_V = i\beta (H_b^{(Q)}\overleftrightarrow{\partial} (\nu \mu - \rho \mu))_{ba} \overleftrightarrow{H}_a^{(Q)} + i\lambda H_b^{(Q)} \sigma_{\mu\nu} F_{\mu\nu}(\rho)_{ba} \overleftrightarrow{H}_a^{(Q)} + i\beta_1 S_b^{(Q)} \sigma_{\mu\nu} F_{\mu\nu}(\rho)_{ba} S_a^{(Q)} + i\lambda_1 S_b^{(Q)} \sigma_{\mu\nu} F_{\mu\nu}(\rho)_{ba} S_a^{(Q)} + i\beta_2 T_b^{(Q)} \rho_{ba} S_a^{(Q)} + i\lambda_2 T_b^{(Q)} \sigma_{\mu\nu} F_{\mu\nu}(\rho)_{ba} T_a^{(Q)} \rho_{ba} S_a^{(Q)} + i\zeta_1 H_b^{(Q)} \sigma_{\mu\nu} F_{\mu\nu}(\rho)_{ba} S_a^{(Q)} + i\zeta_2 T_b^{(Q)} \rho_{ba} S_a^{(Q)} + i\mu_1 T_b^{(Q)} \sigma_{\mu\nu} F_{\mu\nu}(\rho)_{ba} T_a^{(Q)} \rho_{ba} S_a^{(Q)} + h.c.\] (A1)

where \( F_{\mu\nu}(\rho) = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu + [\rho_\mu, \rho_\nu] \), and \( \rho_\mu \) is defined as

\[\rho_\mu = \frac{g_\nu}{\sqrt{2}} V_\mu\] (A2)

\( V_\mu \) is a hermitian 3 × 3 matrix analogous to Eq. 3 containing \( \rho, K^*, \omega \) and \( \phi \),

\[ V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \frac{\rho^+}{\sqrt{2}} & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & K^{*0} & \phi \end{pmatrix} \] (A3)

By imposing the KSRF relations, one obtains \( g_{\nu} \simeq 5.8 \). For the same reason, the interaction between the light vector resonances and heavy anti-mesons should be included via applying charge conjugation transformation

\[\mathcal{L}_V' = -i\beta (\overleftrightarrow{H}_a^{(Q)} \overleftrightarrow{\partial} (\nu \mu - \rho \mu))_{ba} H_b^{(Q)} + i\lambda (\overleftrightarrow{H}_a^{(Q)} \sigma_{\mu\nu} F_{\mu\nu}(\rho)_{ab} H_b^{(Q)} - i\beta_1 (S_a^{(Q)} \nu \mu (\nu \mu - \rho \mu))_{ab} S_b^{(Q)}) + i\mu_1 (\overleftrightarrow{H}_a^{(Q)} \gamma_{\mu} F_{\mu\nu}(\rho)_{ab} T_b^{(Q)\mu}) \] (A4)
where we have used the property $C V_{\mu} C^{-1} = -V_{\mu}^T$. Then the effective interactions relevant to the concerned tree level scattering diagrams are as follows

$$
\mathcal{L}_{DDV} = ig_{DDV}(D_b \leftrightarrow D^\dagger_a)V_{\mu a}^\dagger + ig_{DDBV}((D_b \leftrightarrow D^\dagger_a)V_{\mu b}^\dagger
\mathcal{L}_{D_1D_1V} = ig_{D_1D_1V}(D^\dagger_{1b} \leftrightarrow D^\dagger_{1a})V_{\mu a}^\dagger + ig_{D_1D_1V}(D^\dagger_{1b} D^\dagger_{1a} - D^\dagger_{1a} D^\dagger_{1b})(\partial_\mu V_\nu - \partial_\nu V_\mu)_{ab}
\mathcal{L}_{D_1D_1V} = ig_{D_1D_1V}(D^\dagger_{1b} \leftrightarrow D^\dagger_{1a})V_{\mu a}^\dagger + ig_{D_1D_1V}(D^\dagger_{1b} D^\dagger_{1a} - D^\dagger_{1a} D^\dagger_{1b})(\partial_\mu V_\nu - \partial_\nu V_\mu)_{ab}
\mathcal{L}_{DD_1V} = g_{DD_1V}D^\dagger_{1b} V_{\mu ba} D^\dagger_{1a} + g'_{DD_1V}(D^\dagger_{1b} \leftrightarrow D^\dagger_{1a})(\partial_\mu V_\nu - \partial_\nu V_\mu)_{ab}
\mathcal{L}_{D^\dagger D_1V} = g_{D^\dagger D_1V}D^\dagger_{1b} V_{\mu ba} D^\dagger_{1a} + g'_{D^\dagger D_1V}(D^\dagger_{1b} \leftrightarrow D^\dagger_{1a})(\partial_\mu V_\nu - \partial_\nu V_\mu)_{ab}
\mathcal{L}_{DD_1V} = g_{DD_1V}D^\dagger_{1b} V_{\mu ba} D^\dagger_{1a} + g'_{DD_1V}(D^\dagger_{1b} \leftrightarrow D^\dagger_{1a})(\partial_\mu V_\nu - \partial_\nu V_\mu)_{ab} + h.c.
\mathcal{L}_{D^\dagger D_1V} = g_{D^\dagger D_1V}D^\dagger_{1b} V_{\mu ba} D^\dagger_{1a} + g'_{D^\dagger D_1V}(D^\dagger_{1b} \leftrightarrow D^\dagger_{1a})(\partial_\mu V_\nu - \partial_\nu V_\mu)_{ab} + h.c.
$$

(A5)

The coupling constants are as follows.55

\begin{align*}
g_{DDV} &= -g_{DDBV} = \frac{1}{\sqrt{2}} \beta g_V \\
g_{D_1D_1V} &= -g_{D^\dagger D_1V} = \frac{1}{\sqrt{2}} \beta_2 g_V \\
g'_{D_1D_1V} &= -g'_{D^\dagger D_1V} = \frac{5 \lambda g_V}{3 \sqrt{2}} M_{D^*} \\
g_{DD_1V} &= -g_{D^\dagger D_1V} = \frac{2}{\sqrt{3}} \zeta_1 g_V \sqrt{M_D M_{D^*}} \\
g'_{DD_1V} &= -g'_{D^\dagger D_1V} = \frac{1}{\sqrt{3}} \mu_1 g_V \\
g_{D^\dagger D^\dagger V} &= -g_{DD_1V} = \frac{1}{\sqrt{2}} \beta_2 g_V \\
g'_{D^\dagger D^\dagger V} &= -g'_{DD_1V} = \frac{1}{\sqrt{2}} \beta_1 g_V \\
g_{D_1D_1V} &= -g_{DD_1V} = -\sqrt{2} \lambda g_V M_{D^*} \\
g_{D_1D_1V} &= -g_{DD_1V} = -\sqrt{2} \lambda g_V M_{D^*} \\
g_{D^\dagger D_1V} &= -g_{D^\dagger D_1V} = \frac{1}{\sqrt{2}} \beta_1 g_V \\
g'_{D_1D_1V} &= -g'_{D_1D_1V} = \frac{1}{\sqrt{2}} \beta_1 g_V \\
g_{D^\dagger D_1V} &= -g_{D^\dagger D_1V} = -\zeta_1 g_V \sqrt{2 M_D M_{D^*}} \\
g'_{D^\dagger D_1V} &= -g'_{D^\dagger D_1V} = -\zeta_1 g_V \sqrt{2 M_D M_{D^*}} \\
\end{align*}

(A6)
From the above effective interactions, following the general procedure presented in section III, we can calculate the effective potential from $\rho$ and $\omega$ exchange. For Y(4260), the potential in coordinate space is

\[
V_{12}^{\rho,\omega}(r) = V_{21}^{\rho,\omega}(r) = 0
\]

\[
V_{11}^{\rho,\omega}(r) = \frac{1}{4} \beta_2 g_0^2 \left[ 3H(\Lambda, m_\rho, r) + H(\Lambda, m_\omega, r) \right] - \frac{\beta_2 g_1^2 (M^2_D + M^2_{D_1})}{32M^2_D M^2_{D_1}} \left[ 3G(\Lambda, m_\rho, r) + G(\Lambda, m_\omega, r) \right]
\]

\[
- \frac{1}{6} \left[ \zeta_1 + \frac{\mu_1 (M^2_D - M^2_{D_1})}{2 \sqrt{M_D M_{D_1}}} \right]^2 \left[ 3Y(\Lambda, X_2, \mu_5, m_\rho, r) + Y(\Lambda, X_2, \mu_6, m_\omega, r) \right]
\]

\[
+ \frac{g_1^2 \mu^2 (M_D + M_{D_1})^2}{72M_D M_{D_1}} \left[ 3Z(\Lambda, X_2, \mu_5, m_\rho, r) + Z(\Lambda, X_2, \mu_6, m_\omega, r) \right]
\]

\[
- \frac{g_1^2 \zeta_1^2}{18} \left[ \frac{3}{m^2_\rho} Z(\Lambda, X_2, \mu_5, m_\rho, r) + \frac{1}{m^2_\omega} Z(\Lambda, X_2, \mu_6, m_\omega, r) \right]
\]

(A7)

The potential in coordinate space for $Z_2^\pm(4250)$ is

\[
V_{12}^{\rho,\omega}(r) = V_{21}^{\rho,\omega}(r) = 0
\]

\[
V_{11}^{\rho,\omega}(r) = \frac{1}{4} \beta_2 g_0^2 \left[ H(\Lambda, m_\rho, r) - H(\Lambda, m_\omega, r) \right] - \frac{\beta_2 g_1^2 (M^2_D + M^2_{D_1})}{32M^2_D M^2_{D_1}} \left[ G(\Lambda, m_\rho, r) - G(\Lambda, m_\omega, r) \right]
\]

\[
- \frac{1}{6} \left[ \zeta_1 + \frac{\mu_1 (M^2_D - M^2_{D_1})}{2 \sqrt{M_D M_{D_1}}} \right]^2 \left[ Y(\Lambda, X_2, \mu_5, m_\rho, r) - Y(\Lambda, X_2, \mu_6, m_\omega, r) \right]
\]

\[
+ \frac{g_1^2 \mu^2 (M_D + M_{D_1})^2}{72M_D M_{D_1}} \left[ Z(\Lambda, X_2, \mu_5, m_\rho, r) - Z(\Lambda, X_2, \mu_6, m_\omega, r) \right]
\]

\[
- \frac{g_1^2 \zeta_1^2}{18} \left[ \frac{3}{m^2_\rho} Z(\Lambda, X_2, \mu_5, m_\rho, r) - \frac{1}{m^2_\omega} Z(\Lambda, X_2, \mu_6, m_\omega, r) \right]
\]

(A8)

where the parameters $\mu_i (i = 5, 6, 7, 8)$ are given by

\[
\mu_5^2 = m_\rho^2 - (M_{D_1} - M_D)^2
\]

\[
\mu_6^2 = m_\omega^2 - (M_{D_1} - M_D)^2
\]

\[
\mu_7^2 = m_\rho^2 - (M_{D_0} - M_D)^2
\]

\[
\mu_8^2 = m_\omega^2 - (M_{D_0} - M_D)^2
\]

(A9)
The new functions $G(\Lambda, m, r)$ and $Y(\Lambda, X, \mu, m, r)$ are defined as follows

$$G(\Lambda, m, r) = \frac{1}{4\pi} \frac{1}{r} \left( \Lambda^2 e^{-\Lambda r} - m^2 e^{-mr} \right) + \frac{\Lambda^2 - m^2}{8\pi} \left( \Lambda - \frac{2}{r} \right) e^{-\Lambda r}$$

$$Y(\Lambda, X, \mu, m, r) = \frac{1}{4\pi} \frac{1}{r} \left( e^{-mr} - e^{-Xr} \right) - \frac{\Lambda^2 - m^2}{8\pi X} e^{-Xr}$$

(A10)

As is demonstrated in Eq. (A7) and Eq. (A8), light vector mesons $\rho$ and $\omega$ exchange leads to diagonal interaction, and the off-diagonal components of the effective potential are zero because of parity conservation. For $Z^+_2$ (4250), it is obvious that the potential coming from $\rho$ exchange almost cancels that from $\omega$ exchange, because of the accidental coincidence of $m_\rho$ and $m_\omega$, i.e., $m_\rho \approx 775.5$ MeV and $m_\omega \approx 782.7$ MeV.

There are a number of parameters $\beta$, $\beta_1$, $\beta_2$, $\mu$, $\mu_1$, $\zeta$ and $\zeta_1$ involved in the effective potential. The information about the effective coupling constants between the heavy meson and the light vector mesons is very scarce until now, especially those related with the P-wave heavy mesons. By vector meson dominance, $\beta$ is estimated to be about 0.9 [15]. Ref. [27] gives $\mu = -0.1\text{GeV}^{-1}$ and $\zeta = 0.1$. The remaining parameters have not been determined as far as we know, and we even don’t know the ranges which they are in. So at present we can not give a quantitative estimate about the vector meson exchange contribution to the potential associated with $Y(4260)$. Since the light vector meson mass $m_\rho$, $m_\omega$ is larger than $m_\pi$, $m_\rho$ and $m_\pi$, we expect that the potential induced by vector meson exchange should be smaller than that due to pseudoscalar and scalar exchange in magnitude. In principle, we can determine these coupling constants following the methods of QCD sum rule, non-relativistic potential model and so on, by means of which certain coupling constants in HMxPT have been estimated. In future, if we could get a reliable estimate about these coupling constants from both phenomenological and theoretical approaches. The effective potential arising from $\rho$, $\omega$ exchange and its effect on the structure of $Y(4260)$ could be analyzed in the same way as in section V.

[1] K. Abe et al. [Belle Collaboration], Phys. Rev. Lett. 100, 142001 (2008), arXiv:0708.1790 [hep-ex].
[2] R. Mizuk et al. [Belle Collaboration], arXiv:0806.4098 [hep-ex].
[3] N. A. Tornqvist, Phys. Lett. B 590, 209 (2004), hep-ph/0402237; F. E. Close and P. R. Page, Phys. Lett. B 578, 119 (2004), hep-ph/0309253; C. Y. Wong, Phys. Rev. C 69, 055202 (2004), hep-ph/0311088; E. S. Swanson, Phys. Lett. B 588, 189 (2004), hep-ph/0311229; C. E. Thomas and F. E. Close, Phys. Rev. D 78, 034007 (2008), arXiv:0805.3553 [hep-ph].
[4] G. J. Ding, arXiv:0711.1485 [hep-ph]; G. J. Ding, W. Huang, J. F. Liu and M. L. Yan, arXiv:0808.3622 [hep-ph]; J. L. Rosner, Phys. Rev. D 76, 114002 (2007), arXiv:0706.3406 [hep-ph]; C. Meng and K. T. Chao, arXiv:0708.4222 [hep-ph]; X. Liu, Y. R. Liu, W. Z. Deng and S. L. Zhu, Phys. Rev. D 77, 094015 (2008), arXiv:0803.1295 [hep-ph].
[5] R. M. Albuquerque and M. Nielsen, arXiv:0804.4817 [hep-ph].
[6] S. H. Lee, K. Morita and M. Nielsen, arXiv:0809.0600 [hep-ph]; S. H. Lee, K. Morita and M. Nielsen, arXiv:0808.3168 [hep-ph].
[7] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 95, 142001 (2005), arXiv:hep-ex/0506081.
[8] Q. He et al. [CLEO Collaboration], Phys. Rev. D 74, 091104 (2006), arXiv:hep-ex/0611102.
[9] C. Z. Yuan et al. [Belle Collaboration], Phys. Rev. Lett. 99, 182004 (2007), arXiv:0707.2541 [hep-ex].
[10] K. Abe et al. [Belle Collaboration], Phys. Rev. Lett. 98, 092001 (2007), arXiv:hep-ex/0608018.
[11] G. Pakhlova et al. [Belle Collaboration], Phys. Rev. D 77, 011103 (2008), arXiv:0708.0082 [hep-ex].
[12] B. Aubert et al. [BABAR Collaboration], Phys. Rev. D 77, 011103 (2008), arXiv:0708.0082 [hep-ex].
[13] F. J. Llanes-Estrada, Phys. Rev. D 72, 031503 (2005), arXiv:hep-ph/0507035.
[14] L. Maiani, V. Riquer, F. Piccinini and A. D. Polosa, Phys. Rev. D 72, 031502 (2005), arXiv:hep-ph/0507062.
[15] S. L. Zhu, Phys. Lett. B 625, 212 (2005), arXiv:hep-ph/0507025; E. Kou and O. Pene, Phys. Lett. B 631, 164 (2005), arXiv:hep-ph/0507119; F. E. Close and P. R. Page, Phys. Lett. B 628, 215 (2005), arXiv:hep-ph/0507199.
[16] M. S. Dubrovin [CLEO Collaboration], arXiv:0705.3476 [hep-ex].
[17] D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Lett. B 634, 214 (2006), arXiv:hep-ph/0512230.
[18] C. W. Bernard et al. [MiLC Collaboration], Phys. Rev. D 56, 7039 (1997), arXiv:hep-lat/9707008; Z. H. Mei and X. Q. Luo, Int. J. Mod. Phys. A 18, 5713 (2003), arXiv:hep-lat/0206012; G. S. Bali, Eur. Phys. J. A 19, 1 (2004), arXiv:hep-lat/0308015.
[19] X. L. Wang et al. [Belle Collaboration], Phys. Rev. Lett. 99, 142002 (2007), arXiv:0707.3690 [hep-ex].
[20] G. J. Ding, J. J. Zhu and M. L. Yan, Phys. Rev. D 77, 014033 (2008), arXiv:0708.3712 [hep-ph].
[21] E. Swanson, AIP Conf. Proc. 814, 203 (2006) [Int. J. Mod. Phys. A 21, 733 (2006)], arXiv:hep-ph/0509327.
[22] F. E. Close, arXiv:0801.2646 [hep-ph].
[23] F. E. Close, In the Proceedings of 5th Flavor Physics and CP Violation Conference (FPCP 2007), Bled, Slovenia, 12-16 May 2007, pp 029, arXiv:0706.2709 [hep-ph].
[24] G. Burdman and J. F. Donoghue, Phys. Lett. B 280 (1992) 287.
[25] M. B. Wise, Phys. Rev. D 45 (1992) 2188.
[26] T. M. Yan, H. Y. Cheng, C. Y. Cheung, G. L. Lin, Y. C. Lin and H. L. Yu, Phys. Rev. D 46, 1148 (1992) [Erratum-ibid. D 55, 5851 (1997)].

[27] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio and G. Nardulli, Phys. Rept. 281, 145 (1997), arXiv:hep-ph/9605342.

[28] A. F. Falk and M. E. Luke, Phys. Lett. B 292, 119 (1992), arXiv:hep-ph/9206241.

[29] B. Grinstein, E. E. Jenkins, A. V. Manohar, M. J. Savage and M. B. Wise, Nucl. Phys. B 380, 369 (1992), arXiv:hep-ph/9204207.

[30] W. A. Bardeen, E. J. Eichten and C. T. Hill, Phys. Rev. D 68, 054024 (2003), arXiv:hep-ph/0305049.

[31] X. Liu, Y. R. Liu, W. Z. Deng and S. L. Zhu, Phys. Rev. D 77, 094015 (2008), arXiv:0803.1295 [hep-ph].

[32] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio and G. Nardulli, Phys. Lett. B 292, 371 (1992), arXiv:hep-ph/9209248.

[33] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio and G. Nardulli, Phys. Lett. B 299, 139 (1993), arXiv:hep-ph/9211248.

[34] http://www.feyncalc.org/.

[35] T. Barnes and G. I. Ghandour, Phys. Lett. B 118, 411 (1982).

[36] V. B. Berestetskii, E. M. Lifshitz, L. P. Pitaevskii, Quantum Electrodynamics, Pergamon Press, New York, 1982.

[37] T. Barnes, N. Black, D. J. Dean and E. S. Swanson, Phys. Rev. C 60, 045202 (1999), arXiv:nucl-th/9902068.

[38] T. Barnes, N. Black and E. S. Swanson, Phys. Rev. C 63, 025204 (2001), arXiv:nucl-th/0007025.

[39] N. A. Tornqvist, Z. Phys. C 61, 525 (1994), arXiv:hep-ph/9310247.

[40] M. P. Locher, Y. Lu and B. S. Zou, Phys. Rev. D 55, 1421 (1997).

[41] Y. R. Liu, X. Liu, W. Z. Deng and S. L. Zhu, arXiv:0801.3540 [hep-ph], Euro. Phys. J. C 56, 63 (2008); X. Liu, Y. R. Liu, W. Z. Deng and S. L. Zhu, Phys. Rev. D 77, 034003 (2008); X. Liu, Z. G. Luo, Y. R. Liu and S. L. Zhu, arXiv:0808.0073 [hep-ph].

[42] V. Ledoux, M. Van Daele, G. Vanden Berghe, Comput. Phys. Comm. 176 (2007) 191-199.

[43] A. G. ABRASHKEVICH, D. G. ABRASHKEVICHG, M. S. KASCHIEV and I.V.Puzynin, Comput. Phys. Comm. 85 (1995) 40-64; Comput. Phys. Comm. 85 (1995) 65-81; Comput. Phys. Comm. 115 (1998) 90-92.

[44] Particle Data Group, W.-M. Yao et al., Journal of Physics G 33, 1 (2006).

[45] C. Isola, M. Ladisa, G. Nardulli and P. Santorelli, Phys. Rev. D 51, 6177 (1995), arXiv:hep-ph/9410266.

[46] A. Deandrea, R. Gatto, G. Nardulli and A. D. Polosa, JHEP 9902, 021 (1999), arXiv:hep-ph/9901266.

[47] S. Fajfer and J. F. Kamenik, Phys. Rev. D 74, 074023 (2006), arXiv:hep-ph/0606278.

[48] P. Colangelo, G. Nardulli, A. Deandrea, N. Di Bartolomeo, R. Gatto and F. Feruglio, Phys. Lett. B 339, 151 (1994), arXiv:hep-ph/9406295.

[49] V. M. Belyaev, V. M. Braun, A. Khodjamirian and R. Ruckl, Phys. Rev. D 51, 6177 (1995), arXiv:hep-ph/9410280.

[50] A. D. Polosa, arXiv:hep-ph/9909371.

[51] E. S. Swanson, Phys. Rept. 429, 243 (2006), arXiv:hep-ph/0601110.

[52] G. W. S. Hou, arXiv:hep-ph/0611153.

[53] D. Ebert, R. N. Faustov and V. O. Galkin, arXiv:0808.3012 [hep-ph].

[54] Z. G. Wang, arXiv:0807.3592 [hep-ph], arXiv:0807.2581 [hep-ph].

[55] The following effective Lagrangian are obtained by expanding Eq. (A1) and Eq. (A3) term by term, then they are checked by the program FeynCalc. [34].
\[ Y_\text{cc} \text{ with } I^G(J^P) = 0^- (1^-) \] \[ Z^+_\text{cc} \text{ with } I^G(J^P) = 1^- (1^-) \]

| \( gh \) | \( \Lambda(\text{GeV}) \) | \( M(\text{MeV}) \) | \( r_{\text{rms}}(\text{fm}) \) | \( R \) | \( \Lambda(\text{GeV}) \) | \( M(\text{MeV}) \) | \( r_{\text{rms}}(\text{fm}) \) | \( R \) |
|-----|------|------|---------|----|------|------|---------|----|
| 0.23 | 3.3  | -4.04 | 1.39    | 2.63 | 14.5 | -2.40 | 1.76    | 3.27 |
|      | 3.4  | -12.20| 0.82    | 1.71 | 14.6 | -7.53 | 0.98    | 2.02 |
|      | 3.5  | -23.86| 0.63    | 1.41 | 14.7 | -14.71| 0.71    | 1.60 |
| 0.35 | 2.3  | -3.29 | 1.56    | 2.90 | 9.6  | -3.53 | 1.45    | 2.76 |
|      | 2.4  | -11.32| 0.86    | 1.74 | 9.7  | -9.24 | 0.89    | 1.87 |
|      | 2.5  | -24.95| 0.63    | 1.40 | 9.8  | -16.99| 0.68    | 1.54 |
| 0.54 | 1.6  | -1.79 | 2.17    | 3.88 | 6.3  | -5.53 | 1.17    | 2.29 |
|      | 1.7  | -10.33| 0.95    | 1.84 | 6.4  | -12.13| 0.80    | 1.71 |
|      | 1.8  | -24.66| 0.67    | 1.41 | 6.5  | -20.82| 0.63    | 1.45 |
| 0.85 | 1.2  | -7.43 | 1.15    | 2.11 | 4.0  | -3.45 | 1.50    | 2.81 |
|      | 1.3  | -22.69| 0.76    | 1.46 | 4.1  | -9.28 | 0.93    | 1.88 |
|      | 1.4  | -46.18| 0.57    | 1.24 | 4.2  | -17.42| 0.71    | 1.53 |

**TABLE II**: The mass, the root of mean square radius(rms) and the ratio(R) between the DD\(_1\) and D\(^*\)D\(_0\) components for the bound state solutions of the DD\(_1\) and D\(^*\)D\(_0\) system with one pseudoscalar exchange, and the mass is measured with respect to the D\(_1\)D threshold \( M_D + M_{D_1} \approx 4291.3 \text{MeV} \).
\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline
\multicolumn{1}{|c|}{gh} & \multicolumn{1}{|c|}{y_{ch}} & \multicolumn{1}{|c|}{J^P_{PC}} & \multicolumn{1}{|c|}{Y_{ch}} & \multicolumn{1}{|c|}{Z_{ch}^+} & \multicolumn{1}{|c|}{M(MeV)} & \multicolumn{1}{|c|}{r_{rms}(fm)} & \multicolumn{1}{|c|}{R} & \multicolumn{1}{|c|}{M(MeV)} & \multicolumn{1}{|c|}{r_{rms}(fm)} & \multicolumn{1}{|c|}{R} \\
\hline
0.58 & 0.58 & 0.35 & -3.43 & 1.48 & 2.81 & 6.1 & -5.43 & 1.09 & 1.47 \\
0.35 & -10.87 & 0.84 & 1.81 & 6.2 & -15.82 & 0.66 & 1.00 \\
0.50 & -21.42 & 0.62 & 1.50 & 6.3 & -29.69 & 0.51 & 0.84 \\
0.58 & -0.58 & 0.35 & -3.65 & 1.50 & 3.49 & 5.9 & -4.22 & 1.33 & 2.52 \\
0.35 & -9.70 & 0.93 & 2.31 & 6.0 & -11.84 & 0.80 & 1.59 \\
0.50 & -18.27 & 0.70 & 1.86 & 6.1 & -22.48 & 0.60 & 1.25 \\
0.23 & 0.58 & 0.35 & -7.02 & 1.02 & 1.80 & 5.9 & -8.06 & 0.87 & 0.95 \\
0.35 & -15.40 & 0.72 & 1.41 & 6.0 & -18.32 & 0.61 & 0.73 \\
0.50 & -26.34 & 0.58 & 1.25 & 6.1 & -31.34 & 0.50 & 0.64 \\
0.58 & 0.58 & 0.35 & -3.81 & 1.37 & 2.32 & 3.9 & -11.21 & 0.74 & 0.65 \\
0.35 & -9.37 & 0.88 & 1.67 & 4.0 & -25.48 & 0.54 & 0.50 \\
0.50 & -16.74 & 0.68 & 1.42 & 4.1 & -43.76 & 0.45 & 0.43 \\
0.58 & -0.58 & 0.35 & -2.15 & 1.91 & 3.38 & 5.2 & -8.56 & 0.90 & 1.48 \\
0.35 & -9.48 & 0.93 & 1.85 & 5.3 & -19.71 & 0.62 & 1.13 \\
0.50 & -20.76 & 0.66 & 1.46 & 5.4 & -34.24 & 0.50 & 0.98 \\
0.35 & 0.58 & 0.35 & -6.47 & 1.11 & 2.32 & 5.7 & -5.35 & 1.15 & 1.69 \\
0.35 & -13.31 & 0.80 & 1.78 & 5.8 & -12.98 & 0.75 & 1.17 \\
0.50 & -22.33 & 0.64 & 1.53 & 5.9 & -23.10 & 0.59 & 0.96 \\
0.58 & -0.58 & 0.35 & -3.13 & 1.60 & 3.37 & 3.8 & -8.99 & 0.88 & 1.12 \\
0.35 & -7.05 & 1.06 & 2.36 & 3.9 & -20.74 & 0.62 & 0.78 \\
0.50 & -12.35 & 0.82 & 1.92 & 4.0 & -36.29 & 0.50 & 0.64 \\
0.35 & 0.58 & 0.35 & -2.86 & 1.91 & 3.38 & 5.2 & -8.56 & 0.90 & 1.48 \\
0.35 & -9.48 & 0.93 & 1.85 & 5.3 & -19.71 & 0.62 & 1.13 \\
0.50 & -20.76 & 0.66 & 1.46 & 5.4 & -34.24 & 0.50 & 0.98 \\
0.35 & 0.58 & 0.35 & -6.47 & 1.11 & 2.32 & 5.7 & -5.35 & 1.15 & 1.69 \\
0.35 & -13.31 & 0.80 & 1.78 & 5.8 & -12.98 & 0.75 & 1.17 \\
0.50 & -22.33 & 0.64 & 1.53 & 5.9 & -23.10 & 0.59 & 0.96 \\
\hline
0.58 & 0.35 & 0.35 & -9.66 & 0.95 & 2.15 & 5.0 & -6.18 & 1.12 & 2.38 \\
0.35 & -19.82 & 0.70 & 1.70 & 5.1 & -14.42 & 0.76 & 1.68 \\
0.35 & -33.45 & 0.56 & 1.48 & 5.2 & -25.58 & 0.58 & 1.38 \\
0.50 & -3.90 & 1.45 & 3.20 & 3.5 & -4.65 & 1.29 & 2.55 \\
0.35 & -10.06 & 0.93 & 2.18 & 3.6 & -13.97 & 0.77 & 1.54 \\
0.35 & -18.66 & 0.70 & 1.77 & 3.7 & -27.47 & 0.57 & 1.18 \\
\hline
0.35 & -8.07 & 0.99 & 1.74 & 5.0 & -7.24 & 0.95 & 1.25 \\
0.35 & -18.21 & 0.70 & 1.36 & 5.1 & -16.94 & 0.66 & 0.94 \\
0.35 & -31.84 & 0.56 & 1.21 & 5.2 & -29.51 & 0.53 & 0.82 \\
0.35 & -7.83 & 0.99 & 1.77 & 3.5 & -5.98 & 1.03 & 1.19 \\
0.50 & -16.23 & 0.72 & 1.42 & 3.6 & -17.51 & 0.65 & 0.80 \\
0.35 & -27.23 & 0.59 & 1.26 & 3.7 & -37.14 & 0.51 & 0.66 \\
0.35 & -17.20 & 0.74 & 1.59 & 5.0 & -21.48 & 0.63 & 1.17 \\
0.35 & -29.48 & 0.59 & 1.38 & 5.1 & -33.64 & 0.52 & 1.03 \\
0.35 & -5.08 & 1.27 & 2.51 & 3.4 & -5.68 & 1.14 & 1.80 \\
0.35 & -11.34 & 0.87 & 1.84 & 3.5 & -15.32 & 0.72 & 1.16 \\
0.35 & -19.83 & 0.69 & 1.55 & 3.6 & -28.71 & 0.56 & 0.93 \\
\hline
\end{tabular}
\caption{The mass, the root of mean square radius (rms) and the ratio (R) between the DD\textsubscript{1} and D*D\textsubscript{0} components for the bound state solutions of the DD\textsubscript{1} and D*D\textsubscript{0} system with both one pseudoscalar exchange and \sigma exchange, and the mass is measured with respect to the D\textsubscript{1}D threshold M_{DD} \simeq 4291.3\text{MeV}.}
\end{table}
| gh | $g_s g'_s$ | $g_v g'_v$ | $h_s$ | A(GeV) | M(MeV) | $r_{rms}(\text{fm})$ | R | A(GeV) | M(MeV) | $r_{rms}(\text{fm})$ | R |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.58 | 0.58 | 0.58 | 1.8 | -2.42 | 1.84 | 3.21 | 4.2 | -9.99 | 0.86 | 1.59 |
| 1.9 | -11.03 | 91 | 1.75 | 4.3 | -21.32 | 0.62 | 1.25 |
| 2.0 | -24.76 | 0.66 | 1.30 | 4.4 | -36.09 | 0.50 | 1.10 |
| 2.1 | -8.24 | 1.02 | 1.92 | 3.2 | -12.62 | 0.78 | 1.38 |
| 0.50 | 2.2 | -19.25 | 0.71 | 1.47 | 3.3 | -26.99 | 0.57 | 1.08 |
| 2.3 | -34.54 | 0.57 | 1.30 | 3.4 | -45.95 | 0.47 | 0.95 |
| 0.58 | -0.58 | 0.58 | 1.8 | -6.29 | 1.19 | 2.42 | 4.1 | -13.88 | 0.78 | 1.83 |
| 0.35 | 1.9 | -16.88 | 0.78 | 1.69 | 4.2 | -24.83 | 0.61 | 1.50 |
| 2.0 | -32.31 | 0.60 | 1.43 | 4.3 | -38.79 | 0.50 | 1.32 |
| 0.50 | 2.0 | -5.65 | 1.25 | 2.51 | 3.1 | -11.78 | 0.85 | 1.87 |
| 0.58 | -0.58 | 0.58 | 1.8 | -5.35 | 1.25 | 2.11 | 4.1 | -13.94 | 0.74 | 1.18 |
| 0.35 | 1.9 | -15.93 | 0.78 | 1.44 | 4.2 | -25.68 | 0.58 | 1.00 |
| 2.0 | -31.41 | 0.60 | 1.23 | 4.3 | -40.44 | 0.48 | 0.91 |
| 0.50 | 2.0 | -4.61 | 1.33 | 2.20 | 3.1 | -12.19 | 0.79 | 1.16 |
| 0.58 | -0.58 | 0.58 | 1.8 | -9.78 | 0.98 | 1.88 | 3.9 | -8.33 | 0.98 | 1.84 |
| 0.35 | 1.9 | -22.04 | 0.70 | 1.46 | 4.0 | -16.85 | 0.71 | 1.42 |
| 2.0 | -39.09 | 0.56 | 1.28 | 4.1 | -28.01 | 0.58 | 1.22 |
| 0.50 | 2.0 | -9.66 | 0.98 | 1.87 | 3.0 | -10.72 | 0.87 | 1.60 |
| 0.58 | -0.58 | 0.58 | 1.3 | -11.86 | 0.93 | 1.72 | 3.2 | -10.84 | 0.86 | 1.67 |
| 0.35 | 1.4 | -28.73 | 0.67 | 1.34 | 3.3 | -22.29 | 0.64 | 1.34 |
| 0.50 | 1.4 | -34.80 | 0.58 | 1.31 | 3.2 | -37.53 | 0.52 | 1.04 |
| 0.58 | 0.58 | 0.58 | 1.3 | -5.27 | 1.31 | 2.29 | 2.6 | -9.21 | 0.94 | 1.76 |
| 0.50 | 1.4 | -16.97 | 0.81 | 1.52 | 2.7 | -21.74 | 0.65 | 1.32 |
| 0.50 | 1.5 | -34.96 | 0.62 | 1.28 | 2.8 | -38.91 | 0.52 | 1.13 |
| 0.58 | 0.58 | 0.58 | 1.3 | -14.66 | 0.87 | 1.71 | 3.1 | -11.85 | 0.86 | 1.99 |
| 0.35 | 1.4 | -32.74 | 0.64 | 1.38 | 3.2 | -22.30 | 0.66 | 1.61 |
| 0.50 | 1.5 | -58.63 | 0.52 | 1.24 | 3.3 | -35.98 | 0.54 | 1.41 |
| 0.58 | 0.58 | 0.58 | 1.3 | -7.58 | 1.14 | 2.13 | 2.5 | -7.56 | 1.07 | 2.38 |
| 0.35 | 1.4 | -20.52 | 0.76 | 1.53 | 2.6 | -17.94 | 0.73 | 1.70 |
| 0.50 | 1.5 | -53.33 | 0.53 | 1.20 | 3.4 | -37.36 | 0.52 | 1.19 |
| 0.58 | 0.58 | 0.58 | 1.3 | -5.27 | 1.31 | 2.29 | 2.6 | -9.21 | 0.94 | 1.76 |
| 0.35 | 1.4 | -16.97 | 0.81 | 1.52 | 2.7 | -21.74 | 0.65 | 1.32 |
| 0.50 | 1.5 | -34.96 | 0.62 | 1.28 | 2.8 | -38.91 | 0.52 | 1.13 |
| 0.85 | 0.58 | 0.58 | 1.3 | -14.13 | 0.87 | 1.56 | 3.1 | -10.85 | 0.85 | 1.44 |
| 0.35 | 1.4 | -32.23 | 0.64 | 1.26 | 3.2 | -21.69 | 0.64 | 1.17 |
| 0.50 | 1.5 | -58.16 | 0.52 | 1.13 | 3.3 | -35.80 | 0.53 | 1.05 |
| 0.58 | 0.58 | 0.58 | 1.3 | -7.04 | 1.15 | 1.90 | 2.5 | -6.58 | 1.08 | 1.74 |
| 0.35 | 1.4 | -19.98 | 0.76 | 1.38 | 2.6 | -17.47 | 0.71 | 1.23 |
| 0.50 | 1.5 | -39.26 | 0.59 | 1.20 | 2.7 | -32.64 | 0.56 | 1.03 |
| 0.58 | 0.58 | 0.58 | 1.3 | -4.75 | 1.40 | 2.51 | 3.0 | -11.33 | 0.87 | 1.75 |
| 0.35 | 1.3 | -16.97 | 0.82 | 1.56 | 3.1 | -21.18 | 0.67 | 1.42 |
| 0.50 | 1.4 | -36.26 | 0.62 | 1.29 | 3.2 | -33.99 | 0.56 | 1.25 |
| 0.58 | 0.58 | 0.58 | 1.3 | -9.47 | 1.03 | 1.87 | 2.5 | -13.90 | 0.81 | 1.62 |
| 0.35 | 1.4 | -23.58 | 0.72 | 1.41 | 2.6 | -26.68 | 0.62 | 1.30 |
| 0.50 | 1.5 | -44.09 | 0.58 | 1.23 | 2.7 | -43.54 | 0.51 | 1.14 |

TABLE IV: The continuing of Table III.
| \( gh \) | \( \Lambda (\text{GeV}) \) | \( M (\text{MeV}) \) | \( r_{\text{rms}} \) (fm) | \( R \) | \( \Lambda (\text{GeV}) \) | \( M (\text{MeV}) \) | \( r_{\text{rms}} \) (fm) | \( R \) |
|-----|------------|---------|-------------|-----|------------|---------|-------------|-----|
| 0.23 | 1.8        | -5.60   | 0.81        | 3.08| 6.2        | -7.90   | 0.63        | 2.53|
|      | 1.9        | -14.96  | 0.53        | 2.03| 6.3        | -13.45  | 0.49        | 2.04|
|      | 2.0        | -28.27  | 0.42        | 1.63| 6.4        | -20.22  | 0.40        | 1.76|
| 0.35 | 1.4        | -8.76   | 0.70        | 2.57| 4.2        | -5.17   | 0.88        | 3.08|
|      | 1.5        | -21.93  | 0.49        | 1.79| 4.3        | -10.31  | 0.57        | 2.29|
|      | 1.6        | -40.65  | 0.39        | 1.48| 4.4        | -16.95  | 0.45        | 1.89|
| 0.54 | 1.1        | -11.29  | 0.66        | 2.35| 2.9        | -6.55   | 0.73        | 2.81|
|      | 1.2        | -29.05  | 0.47        | 1.65| 3.0        | -12.79  | 0.54        | 2.13|
|      | 1.3        | -54.93  | 0.38        | 1.38| 3.1        | -20.87  | 0.44        | 1.77|
| 0.85 | 0.8        | -3.49   | 1.13        | 4.14| 2.0        | -7.38   | 0.72        | 2.72|
|      | 0.9        | -18.98  | 0.58        | 1.94| 2.1        | -14.97  | 0.53        | 2.03|
|      | 1.0        | -46.04  | 0.43        | 1.46| 2.2        | -25.03  | 0.44        | 1.69|

**TABLE V:** The mass, the root of mean square radius(rms) and the ratio(R) between the BB and B\( ^* \)B\( _0 \) components for the bound state solutions of the BB and B\( ^* \)B\( _0 \) system with one pseudoscalar exchange, and the mass is measured with respect to the B\( _3 \)B threshold \( M_B + M_{B_3} \approx 11004 \text{MeV} \).
| $gh$ | $g_\sigma g_\sigma'$ | $g_\sigma$ | $g_\sigma'$ | $b_\sigma$ | $A$(GeV) | $M$(MeV) | $r_{rms}$(fm) | $R$ | $A$(GeV) | $M$(MeV) | $r_{rms}$(fm) | $R$ |
|-----|-----------------|---------|---------|--------|--------|--------|---------|-----|--------|--------|---------|-----|
| 0.58 | 0.58 | 2.4 | -8.16 | 0.65 | 2.47 | 3.1 | -4.84 | 0.76 | 2.21 |
| 2.5 | -17.22 | 0.47 | 1.89 | 3.2 | -14.28 | 0.59 | 1.41 |
| 2.6 | -29.14 | 0.39 | 1.63 | 3.3 | -27.06 | 0.36 | 1.11 |
| 3.8 | -4.40 | 0.87 | 3.44 | 2.3 | -11.53 | 0.52 | 1.42 |
| 0.50 | 3.9 | -9.54 | 0.59 | 2.50 | 2.4 | -27.04 | 0.38 | 0.94 |
| 4.0 | -16.34 | 0.46 | 2.07 | 2.5 | -47.66 | 0.31 | 0.74 |
| 0.58 | -0.58 | 2.2 | -5.63 | 0.84 | 3.87 | 2.9 | -5.32 | 0.83 | 1.43 |
| 2.3 | -12.07 | 0.59 | 2.73 | 3.0 | -11.82 | 0.57 | 2.78 |
| 2.4 | -20.93 | 0.47 | 2.19 | 3.1 | -20.94 | 0.44 | 2.05 |
| 3.5 | -7.67 | 0.73 | 3.76 | 2.2 | -11.16 | 0.60 | 2.84 |
| 0.50 | 3.6 | -12.17 | 0.58 | 3.02 | 2.3 | -23.19 | 0.44 | 1.78 |
| 3.7 | -17.83 | 0.49 | 2.57 | 2.4 | -39.83 | 0.35 | 1.29 |
| 0.23 | 2.3 | -8.15 | 0.65 | 2.17 | 3.0 | -10.11 | 0.52 | 1.16 |
| 0.35 | 2.4 | -16.87 | 0.48 | 1.69 | 3.1 | -21.07 | 0.39 | 0.88 |
| 2.5 | -28.20 | 0.40 | 1.46 | 3.2 | -34.78 | 0.33 | 0.75 |
| 0.58 | -0.58 | 2.1 | -5.02 | 0.88 | 3.59 | 2.8 | -7.04 | 0.71 | 2.64 |
| 2.2 | -11.16 | 0.61 | 2.48 | 2.9 | -14.49 | 0.51 | 1.78 |
| 2.3 | -19.60 | 0.48 | 1.98 | 3.0 | -24.96 | 0.41 | 1.38 |
| 3.3 | -8.23 | 0.70 | 3.10 | 2.1 | -8.20 | 0.67 | 2.43 |
| 0.50 | 3.4 | -12.63 | 0.57 | 2.56 | 2.2 | -19.40 | 0.46 | 1.40 |
| 3.5 | -18.04 | 0.49 | 2.21 | 2.3 | -35.11 | 0.37 | 0.99 |
| 0.58 | 0.58 | 1.6 | -8.48 | 0.68 | 2.46 | 2.7 | -8.38 | 0.61 | 2.01 |
| 1.7 | -19.80 | 0.49 | 1.81 | 2.8 | -19.05 | 0.43 | 1.46 |
| 1.8 | -35.44 | 0.39 | 1.53 | 2.9 | -33.04 | 0.35 | 1.21 |
| 1.9 | -8.79 | 0.66 | 2.43 | 2.1 | -11.60 | 0.54 | 1.72 |
| 0.50 | 2.0 | -17.95 | 0.49 | 1.89 | 2.2 | -26.56 | 0.40 | 1.19 |
| 2.1 | -30.10 | 0.41 | 1.63 | 2.3 | -46.60 | 0.33 | 0.96 |
| 0.58 | -0.58 | 1.5 | -5.95 | 0.84 | 3.48 | 2.5 | -6.02 | 0.79 | 3.99 |
| 1.6 | -15.06 | 0.57 | 2.32 | 2.6 | -13.16 | 0.55 | 2.65 |
| 1.7 | -28.17 | 0.45 | 1.85 | 2.7 | -23.07 | 0.44 | 2.02 |
| 1.8 | -9.31 | 0.69 | 2.89 | 2.0 | -10.45 | 0.62 | 2.38 |
| 0.50 | 1.9 | -17.68 | 0.52 | 2.23 | 2.1 | -22.30 | 0.45 | 1.95 |
| 2.0 | -28.77 | 0.43 | 1.89 | 2.2 | -38.74 | 0.37 | 1.47 |
| 0.35 | 1.6 | -12.71 | 0.58 | 1.94 | 2.6 | -10.35 | 0.55 | 1.44 |
| 0.35 | 1.7 | -25.80 | 0.44 | 1.54 | 2.7 | -21.35 | 0.41 | 1.10 |
| 1.8 | -43.15 | 0.37 | 1.35 | 2.8 | -35.32 | 0.35 | 0.94 |
| 0.50 | 1.8 | -6.49 | 0.76 | 2.51 | 2.0 | -8.08 | 0.62 | 1.59 |
| 1.9 | -14.61 | 0.54 | 1.85 | 2.1 | -21.54 | 0.42 | 1.02 |
| 2.0 | -25.57 | 0.43 | 1.56 | 2.2 | -39.85 | 0.35 | 0.89 |
| 0.50 | 1.5 | -8.80 | 0.71 | 2.68 | 2.4 | -6.03 | 0.78 | 3.09 |
| 0.35 | 1.6 | -19.49 | 0.51 | 1.94 | 2.5 | -13.31 | 0.54 | 2.05 |
| 0.50 | 1.7 | -34.19 | 0.42 | 1.61 | 2.6 | -23.26 | 0.43 | 1.58 |
| 0.50 | 1.8 | -13.96 | 0.58 | 2.23 | 2.0 | -16.80 | 0.50 | 1.77 |
| 1.9 | -23.92 | 0.47 | 1.84 | 2.1 | -31.57 | 0.40 | 1.27 |

TABLE VI: The mass, the root of mean square radius($r_{rms}$) and the ratio($R$) between the $BB_1$ and $B^*B_0$ components for the bound state solutions of the $BB_1$ and $B^*B_0$ system with both one pseudoscalar exchange and $\sigma$ exchange, and the mass is measured with respect to the $B_1B$ threshold $M_{B_1} + M_{B_0} \simeq 11004$MeV.
| gh  | $g_x g'_x$ | $g_y g'_y$ | $h_x$ | $A$(GeV) | $M$(MeV) | $r_{rms}$(fm) | $R$ |
|-----|-----------|-----------|--------|----------|----------|--------------|-----|
| 0.58| 0.58      |           |        | 1.2      | -15.49   | 0.58         | 2.0 |   |
|      |           |           |        | 1.3      | -33.34   | 0.44         | 1.56|   |
| 0.50 |           |           |        | 1.2      | -6.93    | 0.79         | 2.73|   |
|      |           |           |        | 1.3      | -18.44   | 0.53         | 1.87|   |
|      |           |           |        | 1.4      | -35.15   | 0.43         | 1.55|   |
| 0.58| -0.58     |           |        | 1.1      | -6.49    | 0.84         | 3.19|   |
|      |           |           |        | 1.2      | -19.23   | 0.55         | 2.03|   |
|      |           |           |        | 1.3      | -38.42   | 0.43         | 1.62|   |
|      |           |           |        | 1.2      | -10.20   | 0.69         | 2.60|   |
|      |           |           |        | 1.3      | -23.12   | 0.51         | 1.90|   |
|      |           |           |        | 1.4      | -41.24   | 0.41         | 1.60|   |
| 0.54|           |           |        | 1.1      | -5.38    | 0.88         | 2.98|   |
|      |           |           |        | 1.2      | -17.94   | 0.55         | 1.81|   |
|      |           |           |        | 1.3      | -37.11   | 0.43         | 1.44|   |
|      |           |           |        | 1.2      | -8.84    | 0.71         | 2.35|   |
|      |           |           |        | 1.3      | -21.63   | 0.51         | 1.69|   |
|      |           |           |        | 1.4      | -39.68   | 0.41         | 1.42|   |
| -0.58| 0.58     |           |        | 1.1      | -7.79    | 0.78         | 2.81|   |
|      |           |           |        | 1.2      | -21.69   | 0.52         | 1.85|   |
|      |           |           |        | 1.3      | -42.15   | 0.42         | 1.50|   |
|      |           |           |        | 1.2      | -12.19   | 0.65         | 2.30|   |
|      |           |           |        | 1.3      | -26.32   | 0.49         | 1.73|   |
|      |           |           |        | 1.4      | -45.74   | 0.40         | 1.49|   |
| -0.58| -0.58    |           |        | 0.9      | -15.10   | 0.63         | 2.09|   |
|      |           |           |        | 1.0      | -37.85   | 0.46         | 1.53|   |
|      |           |           |        | 1.1      | -71.44   | 0.37         | 1.31|   |
|      |           |           |        | 0.9      | -12.39   | 0.67         | 2.24|   |
|      |           |           |        | 1.0      | -31.80   | 0.48         | 1.60|   |
|      |           |           |        | 1.1      | -60.44   | 0.39         | 1.36|   |
| 0.58| 0.58      |           |        | 0.9      | -16.45   | 0.62         | 2.10|   |
|      |           |           |        | 1.0      | -40.19   | 0.45         | 1.56|   |
|      |           |           |        | 1.1      | -74.91   | 0.37         | 1.35|   |
|      |           |           |        | 0.9      | -13.70   | 0.65         | 2.29|   |
|      |           |           |        | 1.0      | -34.09   | 0.48         | 1.64|   |
|      |           |           |        | 1.1      | -63.85   | 0.39         | 1.40|   |
| 0.85|           |           |        | 0.9      | -15.93   | 0.62         | 2.01|   |
|      |           |           |        | 1.0      | -39.56   | 0.45         | 1.47|   |
|      |           |           |        | 1.1      | -74.25   | 0.37         | 1.27|   |
|      |           |           |        | 0.9      | -13.17   | 0.66         | 2.15|   |
|      |           |           |        | 1.0      | -33.43   | 0.48         | 1.54|   |
|      |           |           |        | 1.1      | -63.13   | 0.39         | 1.31|   |
|      |           |           |        | 0.9      | -17.27   | 0.61         | 2.02|   |
|      |           |           |        | 1.0      | -41.88   | 0.45         | 1.51|   |
|      |           |           |        | 1.1      | -77.70   | 0.37         | 1.30|   |
| -0.58| -0.58    |           |        | 0.9      | -14.47   | 0.64         | 2.16|   |
|      |           |           |        | 1.0      | -35.71   | 0.47         | 1.57|   |
|      |           |           |        | 1.1      | -66.52   | 0.38         | 1.35|   |

**TABLE VII:** The continuing of Table VI.