Determine the parameter $B_K$ of the $K^0 - \bar{K}^0$ system by means of the precise Cabibbo-Kobayashi-Maskawa matrix elements

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Abstract

Taking use of the relation between weak $CP$ phase and the other three mixing angles in Cabibbo-Kobayashi-Maskawa (CKM) matrix postulated by us before, the uncertainty coming from the weak interaction can be reduced, furthermore, by means of the relation between the calculation result about mixing parameter $\epsilon$ from the box diagrams and the related experimental data in $K^0 - \bar{K}^0$ system, the parameter $B_K$ can be extracted. We take $V_{ud}$, $V_{ub}$ and $V_{tb}$ as inputs, when we let them vary in the ranges $0.9745 \leq V_{ud} \leq 0.9760$, $0.0018 \leq V_{ub} \leq 0.0045$ and $0.9991 \leq V_{tb} \leq 0.9993$, we find the permitted window for $B_K$ is $0.444 \leq B_K \leq 1.242$. With the more precise measurement on the CKM matrix elements in the future, we can determine $B_K$ more precisely.

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The heavy flavor physics has aroused a great interests in recent years [1-4]. Through the study of heavy flavor physics, we can extract much useful information. On one hand, the weak decays of heavy flavours can provide a direct way to determine the weak mixing angles and to test the unitarity of the standard Cabibbo-Kobayashi-Maskawa (CKM) matrix [5-6]. On the other hand, study on the semileptonic and nonleptonic decays of heavy mesons can tell us some information about non-perturbative QCD and its long range character. But unfortunately, these two tasks are entangled. When we want to determine the weak mixing angles and the CP violation phase precisely, we must know how to reliably evaluate hadronic matrix elements. On the contrary, if we can reduce some uncertainty coming from the weak interaction, we will be able to determine the relevant parameters of hadronic matrix element more precisely.

As a example, we consider the $K^0 - \bar{K}^0$ system in this work. Within some approximations, the CP violation parameter $\epsilon$ can be calculated [7-12]

$$|\epsilon| \simeq \frac{G_F^2 m_K f_K^2 B_K M_W^2}{\sqrt{2}(12\pi^2)\Delta m_K} \left[ \eta_1 S(x_c) I_{cc} + \eta_2 S(x_t) I_{tt} + 2\eta_3 S(x_c, x_t) I_{ct} \right]$$

where $G_F$ is the Fermi constant, $\eta_1 = 1.38, \eta_2 = 0.57, \eta_3 = 0.47$ are QCD corrections [8],

$I_{ij} \equiv Im(V_{id}^* V_{is} V_{jd}^* V_{js})$, $x_i = m_i^2/M_W^2$ and

$$S(x) \equiv \frac{x}{4} \left[ 1 + \frac{3 - 9x}{(x-1)^2} + \frac{6x^2 ln(x)}{(x-1)^2} \right]$$

$$S(x, y) \equiv xy \left[ \frac{1}{4} + \frac{3}{2(1-y)} - \frac{3}{4(1-y)^2} \right] \frac{ln(y)}{y-x} + (y \leftrightarrow x) - \frac{3}{4(1-x)(1-y)}.$$ (3)

The experimental values of the relevant parameters are [9,13]

$$|\epsilon| = (2.28 \pm 0.02) \times 10^{-3} \quad f_K = 160 MeV \quad \Delta m_K = 3.49 \times 10^{-15} GeV$$

$$m_K = 0.4977 GeV \quad m_t = 175 \pm 6 GeV/c^2 \quad M_W = 80.34 \pm 0.10 GeV/c^2.$$ (4)

The quantity $B_K$ should be in principle renormalization scale independent, its value obtained by various groups with different methods is listed in Table 1 [14-27].

From Table 1, we find that the value of $B_K$ is very undefined. In fact, because we are in ignorance of the low-energy QCD, we can not determine it precisely. However, if we can understand the weak interaction very well, through the study on the heavy flavor physics,
Table 1: $B_K$ value obtained by various groups with different methods

| $B_K$          | Method                              | Ref.   |
|---------------|-------------------------------------|--------|
| 0.37          | lowest-order chiral perturbation theory | [17]   |
| 0.4 ± 0.2     | next-to-leading $1/N_c$ estimate, $o(p^2)$ | [19]   |
| 0.42 ± 0.06   | $o(p^4)$ chiral perturbation theory  | [20]   |
| 0.60-0.80     | NJL model with spin-1 interactions   | [21]   |
| 0.39 ± 0.10   | QCD-hadronic duality                | [22,23]|
| 0.5 ± 0.1 ± 0.2 | QCD sum rules (3-point functions)   | [24]   |
| 0.55 ± 0.25   | QCD sum rules (3-point functions)   | [25]   |
| 0.58 ± 0.22   | Laplace sum rule                     | [26]   |
| 0.90 ± 0.03 ± 0.14 | lattice                             | [27]   |

Conversely, we can determine the relevant parameters of the strong interaction such as $B_K$ more precisely.

The central purpose of this work is to determine $B_K$ by use of the experimental results on $K_0 - \overline{K_0}$ system and the CKM matrix with the uncertainty coming from the weak CP phase $\delta$ being reduced.

In Ref. [28], we find that the weak CP phase is related to the other three mixing angles, the relation can be described by

$$\sin \delta_{13} = \frac{(1 + s_{12} + s_{23} + s_{13}) \sqrt{1 - s_{12}^2 - s_{23}^2 - s_{13}^2 + 2s_{12}s_{23}s_{13}}}{(1 + s_{12})(1 + s_{23})(1 + s_{13})}$$  \hspace{1cm} (4)

where $s_{ij}$ and $\delta_{13}$ are the parameters in the standard parametrization [11-12]

$$V_{KM} = \begin{pmatrix}
  c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}s_{13} \\
  s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}s_{13}
\end{pmatrix}$$ \hspace{1cm} (5)

with $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$ for the “generation” labels $i, j = 1, 2, 3$. Here, the real angles $\theta_{12}, \theta_{23}$ and $\theta_{13}$ can all be made to lie in the first quadrant. The phase $\delta_{13}$ lies in the range $0 < \delta_{13} < 2\pi$. In following, we will make the three angles $\theta_{ij}$ lie in the first quadrant.

According to Eq.(4), the weak CP phase is fully determined by the three mixing angles. Hence, the uncertainty coming from the weak interaction is reduced. In following, we begin to determine $B_K$ with this result is used. The program is

1. From three CKM matrix elements which are measured precisely, we calculate the three mixing angles.
2. Based on Eq.(4), the weak CP phase $\delta_{13}$ can be solved. Hereon, we can determine all the CKM matrix elements, and then, we can calculate $I_{ij}(i, j = c, t)$.

3. According to Eq.(1) and the relevant experimental results on $K_0 - \overline{K}_0$ system, we determine $B_K f_K^2$ or $B_K$.

The other parameters used in this work are

$$G_F = 1.166392 \times 10^{-5} GeV^{-2} \quad m_c = 1.5 GeV$$

We take $V_{ud}, V_{ub}$ and $V_{tb}$ as inputs, When we let them vary in the ranges [13]

$$0.9745 \leq V_{ud} \leq 0.9760 \quad 0.0018 \leq V_{ub} \leq 0.0045 \quad 0.9991 \leq V_{tb} \leq 0.9993 \quad (6)$$

We find a range for $B_K$ as following

$$0.444 \leq B_K \leq 1.242 \quad (7)$$

However, if we let $V_{ud}, V_{ub}$ and $V_{tb}$ vary in more narrow ranges, such as

$$0.9749 \leq V_{ud} \leq 0.9756 \quad 0.0025 \leq V_{ub} \leq 0.0038 \quad 0.9991 \leq V_{tb} \leq 0.9993 \quad (8)$$

then, we find a range for $B_K$ as

$$0.520 \leq B_K \leq 0.902 \quad (9)$$

In conclusion, by use of the postulation on the relation between weak $CP$ phase and the other three mixing angles in CKM matrix, the uncertainty coming from the weak interaction has been reduced, furthermore, based on the relation between the calculation result about mixing parameter $\epsilon$ from the box diagrams and the related experimental data in $K^0 - \overline{K}^0$ system, the parameter $B_K$ is extracted.

We take $V_{ud}, V_{ub}$ and $V_{tb}$ as inputs from the Data Book, when all the three input parameters are on the 90% CL, we find $0.444 \leq B_K \leq 1.242$. So, it is consistent with most of the results listed in Table 1, except those obtained by lowest-order chiral perturbation theory and $O(p^4)$ chiral perturbation theory.

When the more narrow ranges for the inputs being scanned, we will get a more narrow permitted window for $B_K$. Hence, with the more precise measurement on the CKM matrix elements in the future, based on our postulation Eq.(4), we will be able to determine $B_K$ more precisely.
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