1. Editor’s note

This issue contains a relatively large number of abstracts of papers dealing directly with selection principles. This is, in part, due to the stimulus created by the last SPM conference, followed by a special issue of Topology and its Applications that is under preparation.

Pay special attention to Section 2.12 below, that announces a solution of the classic Malyhin’s Problem. This problem is closely related to the question of existence of $\gamma$-sets (one of the central themes of selection principles). In addition to its main result, this paper establishes that the existence of Malyhin groups need not imply that of $\gamma$-sets. Indeed, the result is quite flexible and may have additional consequences in the realm of SPM.

With best regards,

Boaz Tsaban, tsaban@math.biu.ac.il
http://www.cs.biu.ac.il/~tsaban

2. Long announcements

2.1. A characterization of the Menger property by means of ultrafilter convergence. We characterize various Menger-related properties by means of ultrafilter convergence, and discuss their behavior with respect to products.

http://arxiv.org/abs/1210.2118
Paolo Lipparini

2.2. Topological spaces compact with respect to a set of filters.

http://arxiv.org/abs/1210.2120
Paolo Lipparini If $\mathcal{P}$ is a family of filters over some set $I$, a topological space $X$ is sequencewise $\mathcal{P}$-compact if, for every $I$-indexed sequence of elements of $X$, there is $F \in \mathcal{P}$ such that the sequence has an $F$-limit point. Countable compactness, sequential compactness, initial $\kappa$-compactness, $[\lambda, \mu]$-compactness, the Menger and Rothberger properties can all be expressed in terms of sequencewise $\mathcal{P}$-compactness, for appropriate choices of $\mathcal{P}$.

We show that sequencewise $\mathcal{P}$-compactness is preserved under taking products if and only if there is a filter $F \in \mathcal{P}$ such that sequencewise $\mathcal{P}$-compactness is equivalent to $F$-compactness. If this is the case, and there exists a sequencewise $\mathcal{P}$-compact $T_1$ topological space with more than one point, then $F$ is necessarily an ultrafilter.
2.3. **Comparing weak versions of separability.** Our aim is to investigate spaces with sigma-discrete and meager dense sets, as well as selective versions of these properties. We construct numerous examples to point out the differences between these classes while answering questions of Tkachuk [30], Hutchinson [17] and the authors of [8].

http://arxiv.org/abs/1210.4986

Daniel T. Soukup, Lajos Soukup, Santi Spadaro

2.4. **Productively Lindelöf and indestructibly Lindelöf spaces.** There has recently been considerable interest in productively Lindelöf spaces, i.e. spaces such that their product with every Lindelöf space is Lindelöf. Here we make several related remarks about such spaces. Indestructible Lindelöf spaces, i.e. spaces that remain Lindelöf in every countably closed forcing extension, were introduced by Tall in 1995. Their connection with topological games and selection principles was explored by Scheepers and Tall in 2010. We find further connections here.

http://arxiv.org/abs/1210.8010

Haosui Duanmu, Franklin D. Tall, Lyubomyr Zdomskyy

2.5. **Indestructibility of compact spaces.** In this article we investigate which compact spaces remain compact under countably closed forcing. We prove that, assuming the Continuum Hypothesis, the natural generalizations to $\omega_1$-sequences of the selection principle and topological game versions of the Rothberger property are not equivalent, even for compact spaces. We also show that Tall and Usuba’s ”$\aleph_1$-Borel Conjecture” is equiconsistent with the existence of an inaccessible cardinal.

http://arxiv.org/abs/1211.1719

Rodrigo R. Dias and Franklin D. Tall

2.6. **Some observations on compact indestructible spaces.** Inspired by a recent work of Dias and Tall, we show that a compact indestructible space is sequentially compact. We also prove that a Lindelöf Hausdorff indestructible space has the finite derived set property and a compact Hausdorff indestructible space is pseudoradial.

http://arxiv.org/abs/1211.3581

Angelo Bella

2.7. **Reflecting Lindelöf and converging $\omega_1$-sequences.** We deal with a conjectured dichotomy for compact Hausdorff spaces: each such space contains a non-trivial converging omega-sequence or a non-trivial converging $\omega_1$-sequence. We establish that this dichotomy holds in a variety of models; these include the Cohen models, the random real models and any model obtained from a model of CH by an iteration of property K posets. In fact in these models every compact Hausdorff space without non-trivial converging $\omega_1$-sequences is first-countable and, in addition, has many $\aleph_1$-sized Lindelöf subspaces. As a corollary we find that in these models all compact Hausdorff spaces with a small diagonal are metrizable.

http://arxiv.org/abs/1211.2764

Alan Dow and Klaas Pieter Hart
2.8. Selections, games and metrisability of manifolds. In this note we relate some selection principles to metrisability and separability of a manifold. In particular we show that $S_{\text{fin}}(\mathcal{K}, \mathcal{O})$, $S_{\text{fin}}(\Omega, \Omega)$ and $S_{\text{fin}}(\Lambda, \Lambda)$ are each equivalent to metrisability for a manifold, while $S_1(D, D)$ is equivalent to separability for a manifold.

http://arxiv.org/abs/1212.0589  
David Gauld

2.9. Infinite games and cardinal properties of topological spaces. Inspired by work of Scheepers and Tall, we use properties defined by topological games to provide bounds for the cardinality of topological spaces. We obtain a partial answer to an old question of Bell, Ginsburg and Woods regarding the cardinality of weakly Lindelöf first-countable regular spaces and answer a question recently asked by Babinkostova, Pansera and Scheepers. In the second part of the paper we study a game-theoretic version of cellularity, a special case of which has been introduced by Aurichi. We obtain a game-theoretic proof of Shapirovskii’s bound for the number of regular open sets in an (almost) regular space and give a partial answer to a natural question about the productivity of a game strengthening of the countable chain condition that was introduced by Aurichi. As a final application of our results we prove that the Hajnal-Juhász bound for the cardinality of a first-countable ccc Hausdorff space is true for almost regular (non-Hausdorff) spaces.

http://arxiv.org/abs/1212.5724  
Angelo Bella and Santi Spadaro

2.10. Weak covering properties and selection principles. No convenient internal characterization of spaces that are productively Lindelof is known. Perhaps the best general result known is Alster’s internal characterization, under the Continuum Hypothesis, of productively Lindelof spaces which have a basis of cardinality at most $\kappa_1$. It turns out that topological spaces having Alster’s property are also productively weakly Lindelof. The weakly Lindelof spaces form a much larger class of spaces than the Lindelof spaces. In many instances spaces having Alster’s property satisfy a seemingly stronger version of Alster’s property and consequently are productively X, where X is a covering property stronger than the Lindelof property. This paper examines the question: When is it the case that a space that is productively X is also productively Y, where X and Y are covering properties related to the Lindelof property.

http://arxiv.org/abs/1212.6122  
L. Babinkostova, B. A. Pansera and M. Scheepers

2.11. Asymptotic dimension, decomposition complexity, and Haver’s property C.

http://arxiv.org/abs/1301.3484  
Alexander Dranishnikov and Michael Zarichnyi The notion of the decomposition complexity was introduced in [GTY] using a game theoretical approach. We introduce a notion of straight decomposition complexity and compare it with the original as well with the asymptotic property C. Then we define a game theoretical analog of Haver’s property C in the classical dimension theory and compare it with the original.
2.12. Malykhin’s Problem. We construct a model of ZFC where every separable Fréchet–Urysohn group is metrizable. This solves a 1978 problem of V. I. Malykhin. 

www.matmor.unam.mx/~michael/preprints_files/Frechet-malykhin.pdf

Michael Hrušák and Ulises Ariet Ramos-García

2.13. A new class of spaces with all finite powers Lindelof. 

http://arxiv.org/abs/1302.5287

Natasha May, Santi Spadaro and Paul Szeptycki

We consider a new class of open covers and classes of spaces defined from them, called ”iota spaces”. We explore their relationship with epsilon-spaces (that is, spaces having all finite powers Lindelof) and countable network weight. An example of a hereditarily epsilon-space whose square is not hereditarily Lindelof is provided.

2.14. Topologically invariant σ-ideals on the Hilbert cube. 

http://arxiv.org/abs/1302.5658

Taras Banakh, Michal Morayne, Robert Ralowski, Szymon Zeberski

We study and classify topologically invariant σ-ideals with a Borel base on the Hilbert cube and evaluate their cardinal characteristics. One of the results of this paper solves (positively) a known problem whether the minimal cardinalities of the families of Cantor sets covering the unit interval and the Hilbert cube are the same.

2.15. Topological spaces compact with respect to a set of filters. II. 

http://arxiv.org/abs/1303.0815

Paolo Lipparini

If \( P \) is a family of filters over some set \( I \), a topological space \( X \) is sequencewise \( P \) compact if, for every \( I \)-indexed sequence of elements of \( X \), there is \( F \in P \) such that the sequence has an \( F \)-limit point. As recalled in Part I, countable compactness, sequential compactness, initial \( \kappa \)-compactness, \([\lambda, \mu]\)-compactness, the Menger and Rothberger properties can all be expressed in terms of sequencewise \( P \) compactness, for appropriate choices of \( P \). We show that a product of topological spaces is sequencewise \( P \) compact if and only if so is any subproduct with \( \leq |P| \) factors. In the special case of sequential compactness, we get a better bound: a product is sequentially compact if and only if all subproducts by \( \leq s \) factors are sequentially compact.

2.16. Selective covering properties of product spaces. 

http://arxiv.org/abs/1303.3597

Arnold W. Miller, Boaz Tsaban, Lyubomyr Zdomskyy

We study the preservation of selective covering properties, including classic ones introduced by Menger, Hurewicz, Rothberger, Gerlits and Nagy, and others, under products with some major families of concentrated sets of reals.

Our methods include the projection method introduced by the authors in an earlier work, as well as several new methods. Some special consequences of our main results are (definitions provided in the paper):

1. Every product of a concentrated space with a Hurewicz \( S_1(\Gamma, O) \) space satisfies \( S_1(\Gamma, O) \). On the other hand, assuming the Continuum Hypothesis, for each Sierpiński set \( S \) there is a Luzin set \( L \) such that \( L \times S \) can be mapped onto the real line by a Borel function.
(2) Assuming Semifilter Trichotomy, every concentrated space is productively Menger and productively Rothberger.

(3) Every scale set is productively Hurewicz, productively Menger, productively Scheepers, and productively Gerlits–Nagy.

(4) Assuming $\mathfrak{d} = \aleph_1$, every productively Lindelöf space is productively Hurewicz, productively Menger, and productively Scheepers.

A notorious open problem asks whether the additivity of Rothberger’s property may be strictly greater than $\text{add}(\mathcal{N})$, the additivity of the ideal of Lebesgue-null sets of reals. We obtain a positive answer, modulo the consistency of Semifilter Trichotomy with $\text{add}(\mathcal{N}) < \text{cov}(\mathcal{M})$.

Our results improve upon and unify a number of results, established earlier by many authors.

3. Short announcements

3.1. Productivity of $[\mu, \lambda]$-compactness.

http://arxiv.org/abs/1210.2121

Paolo Lipparini

3.2. Almost Souslin Kurepa trees.

http://www.ams.org/journal-getitem?pii=S0002-9939-2012-11461-3

Mohammad Golshani

3.3. On two topological cardinal invariants of an order-theoretic flavour.

http://arxiv.org/abs/1212.5725

Santi Spadaro

3.4. P-spaces and the Volterra property.

http://arxiv.org/abs/1212.5726

Santi Spadaro

3.5. Hereditarily supercompact spaces.

http://arxiv.org/abs/1301.5297

Taras Banakh, Zdzislaw Kosztolowicz, Slawomir Turek

3.6. Hindman’s Coloring Theorem in arbitrary semigroups.

http://arxiv.org/abs/1303.3600

Gili Golan, Boaz Tsaban
4. Unsolved problems from earlier issues

**Issue 1.** Is \((\Omega_0) = (\Omega_1)\)?

**Issue 2.** Is \(U_{\text{fin}}(O, \Omega) = S_{\text{fin}}(\Gamma, \Omega)\)? And if not, does \(U_{\text{fin}}(O, \Gamma)\) imply \(S_{\text{fin}}(\Gamma, \Omega)\)?

**Issue 4.** Does \(S_1(\Omega, T)\) imply \(U_{\text{fin}}(\Gamma, \Gamma)\)?

**Issue 5.** Is \(p = p^*\)? (See the definition of \(p^*\) in that issue.)

**Issue 6.** Does there exist (in ZFC) an uncountable set satisfying \(S_{\text{fin}}(B, B)\)?

**Issue 8.** Does \(X \not\in \text{NON}(M)\) and \(Y \not\in D\) imply that \(X \cup Y \not\in \text{COF}(M)\)?

**Issue 9 (CH).** Is \(\text{Split}(\Lambda, \Lambda)\) preserved under finite unions?

**Issue 10.** Is \(\text{cov}(M) = o\)? (See the definition of \(o\) in that issue.)

**Issue 12.** Could there be a Baire metric space \(M\) of weight \(\aleph_1\) and a partition \(U\) of \(M\) into \(\aleph_1\) meager sets where for each \(U' \subset U, \bigcup U'\) has the Baire property in \(M\)?

**Issue 14.** Does there exist (in ZFC) a set of reals \(X\) of cardinality \(\mathfrak{d}\) such that all finite powers of \(X\) have Menger’s property \(S_{\text{fin}}(O, O)\)?

**Issue 15.** Can a Borel non-\(\sigma\)-compact group be generated by a Hurewicz subspace?

**Issue 16 (MA).** Is there \(X \subseteq \mathbb{R}\) of cardinality continuum, satisfying \(S_1(B_\Omega, B_\Gamma)\)?

**Issue 17 (CH).** Is there a totally imperfect \(X\) satisfying \(U_{\text{fin}}(O, \Gamma)\) that can be mapped continuously onto \(\{0, 1\}^\mathbb{N}\)?)

**Issue 18 (CH).** Is there a Hurewicz \(X\) such that \(X^2\) is Menger but not Hurewicz?

**Issue 19.** Does the Pytkeev property of \(C_p(X)\) imply that \(X\) has Menger’s property?

**Issue 20.** Does every hereditarily Hurewicz space satisfy \(S_1(B_\Gamma, B_\Gamma)\)?

**Issue 21 (CH).** Is there a Rothberger-bounded \(G \leq \mathbb{Z}^\mathbb{N}\) such that \(G^2\) is not Menger-bounded?

**Issue 22.** Let \(\mathcal{W}\) be the van der Waerden ideal. Are \(\mathcal{W}\)-ultrafilters closed under products?

**Issue 23.** Is the \(\delta\)-property equivalent to the \(\gamma\)-property \((\Omega_1)\)?