Quantum simulation of an exotic quantum critical point in a two-site charge Kondo circuit

The rich behaviours of bulk materials emerge from microscopic quantum interactions among their many electrons and atoms. When competing interactions favour different collective quantum states, one can often tune from one quantum state to another by applying pressure, electromagnetic fields or chemical doping. In principle, this can even happen at absolute zero temperature: a quantum phase transition\(^1,2\). Remarkably, the zero-temperature quantum critical point (QCP) at a specific value of a tuning parameter controls behaviour over a widening range of parameter values as the temperature is increased, making signatures of criticality experimentally accessible. Furthermore,
seemingly very different systems can behave in the same ‘universal’ way near their respective critical points.

A full microscopic description of the range and character of different phases and the transitions between them is in most cases impossible, given the sheer chemical complexity of real bulk materials. Fortunately, simplified models often capture the essential physics of interest, providing valuable insight into the behaviour of bulk materials and even guiding the design of new materials. Typically, these models describe a set of local sites, each hosting one or a few interacting quantum degrees of freedom, coupled to other sites and sometimes to conducting reservoirs. Calculating the low-energy properties of even these simplified models on clusters of more than a few sites exceeds the capabilities of the most powerful classical computers. Digital quantum computers could work for such calculations, but only once they are scaled to a far greater number of quantum bits than the present state of the art. Highly tunable nanoelectronic circuits based on one or a few semiconductor quantum dots can act as analogue quantum simulators, directly implementing Hamiltonians of interest and thus offering the near-term prospect of more powerful computation than other currently available approaches. These circuits display diverse phenomena, including Coulomb blockade, various Kondo effects, emergent symmetries and fractionalization. Quantum phase transitions with universal properties have also been realized in such circuits.

However, the circuits studied so far cannot fully capture collective behaviour over many sites of a lattice. A long-standing goal is to scale up these circuits to more directly mirror the structure of bulk materials. For example, a four-site Fermi–Hubbard system was recently studied using semi-automated control and tuning capabilities, but scaling up to a larger uniform lattice is daunting. Even with advanced modern fabrication, disorder in doping and lithographic patterning can make the level spectra of two nominally identical quantum dots inequivalent. An array of gates may be used to equalize local chemical potentials across different sites, but many-body ground states are still affected by the full level spectrum of each site, which cannot be fully controlled in these systems.

A recently introduced paradigm for quantum simulation of quantum phase transitions is based on a local site formed from a hybrid metal–semiconductor island. Here we demonstrate that this approach is uniquely advantageous for scaling to larger arrays: because the metal component hosts an effective continuum of single particle states, different islands of the same size behave essentially identically. As a step toward scaling up such systems, the nature of the coupling between neighbouring islands must be understood. In this Article we develop a model to describe the coupling between islands in the simplest two-site system, and validate this model experimentally using transport measurements. We show that this device architecture generates an inter-site interaction that favours a many-body coherent state in which the islands are collectively screened by each other, and become strongly entangled by a charge-Kondo effect. This interaction competes with the usual island–lead interactions, which favour separate Kondo screening of each island by its attached lead. The frustration between these interactions in a system of two identical sites results in a QCP. Measured conductance signatures across the quantum phase transition—which is a variant of the long-sought two-impurity Kondo (2IK) transition—match universal theoretical predictions for our model. Scaling up to many such coupled sites will allow the experimental simulation of lattice models that are beyond the reach of traditional computational techniques.

Our device consists of a circuit containing two coupled hybrid metal–semiconductor islands, each also coupled to its own lead, as illustrated schematically in Fig. 1a. Even though the islands are small enough to have a substantial charging energy, the metal component endows each island with an effective continuum of single particle states. This contrasts with the discrete and individualized level spectrum of pure semiconductor quantum dots noted above. Our circuit is based on a GaAs/AlGaAs heterostructure that hosts a buried two-dimensional electron gas (2DEG). Mesas are lithographically patterned (blue regions in Fig. 1a, outside of which the 2DEG is etched away). The metal components are deposited, bridging the various mesas, then are electrically connected to the 2DEG by thermal annealing. The combination of metal and three surrounding patches of 2DEG form the hybrid metal–semiconductor island. The device is operated in a magnetic field of 4.3 T, corresponding to a quantum Hall filling factor of 2 in the 2DEG bulk; this provides robust, spin-polarized conduction electron channels. The left and right islands are designed to behave identically: the spacing of single-particle states on each island is far below k_B T at our base temperature of 20 mK, and to our experimental resolution their charging energies E_C ≈ 25 μeV are equal (r = 10 μeV is the inter-island capacitive interaction). Lithographically patterned metallic top gates form quantum point contacts (QPCs; black, Fig. 1a). The transmissions τ_L and τ_R control the left and right island–lead tunnel couplings, and τ_c controls the coupling between the islands. Each coupling is through the outermost quantum Hall edge state. QPC voltages are set so that the second, inner channel, is completely reflected. Throughout the experiment we fix τ_L = τ_R = τ and keep all other QPCs closed. Finally, plunger gates (green) control the electrostatic potential, and hence electron occupancy, on each island. We measure the conductance G from lead left to right lead through both islands in series, as a function of the left and right plunger gate voltages V_L and V_R. See Methods for further details of the device and measurement set-up.

We now formulate a model to describe our two-site device at low temperatures below the island charging energies, and when the QPCs are only partially open (that is, when k_B T ≪ E_C ≈ 25 μeV and r < 1). Our low-energy effective model retains only the lowest two macroscopic charge states of each island (n = N or N + 1 electrons on the left island and m = M or M + 1 electrons on the right island), giving four thermally accessible charge configurations (n, m) for the two sites, around a single pair of degenerate triple points (see the experimental charge stability diagram in Fig. 1c, discussed further below). Virtual excitations to higher charge states are neglected. The two-level systems of retained charge states on each island are mapped to charge pseudospins in the effective model, which then takes the form of a modified two-impurity Kondo model. The derivation (Methods and Supplementary Information) follows similar lines to that of the charge-Kondo mapping for a single island introduced theoretically by Matveev and colleagues and validated experimentally by Ifikhar and others. Our proposed double-charge Kondo (DCK) model reads

\[
H_{DCK} = \left( J_L \hat{S}_L^+ \hat{S}_L^- + J_R \hat{S}_R^+ \hat{S}_R^- + J_c \hat{S}_L^+ \hat{S}_R^- \hat{c}_c^+ \hat{c}_c + h.c. \right) + \delta z \hat{S}_L^z \hat{S}_R^z + B_L \hat{S}_L^z + B_R \hat{S}_R^z + H_{elec}
\]

Here, H_{elec} describes the effectively independent conduction electron reservoirs around each of the three QPCs, \( \hat{S}_{L(R)}^z \) are pseudospin-½ operators for the lowest two charge states of the left (right) islands, \( \hat{S}_{L(R)}^+ \) and \( \hat{S}_{L(R)}^- \) are pseudospin-½ operators for the electronic reservoirs around the left (right) QPCs, and \( \hat{c}_c^+ \) and \( \hat{c}_c^- \) are fermionic annihilation (creation) operators for island electrons to the left and right of the central QPC. The first line of equation (1) describes tunnelling events at the three QPCs. The island–lead coupling terms proportional to \( J_L \) and \( J_R \) favour Kondo screening of the charge pseudospin on the left and right islands by their attached lead, whereas the term proportional to \( J_c \) represents the pseudospin coupling between the two islands, which is correlated with electronic tunnelling across the central QPC. The term proportional to \( L \) describes the inter-island capacitive interaction, and the local Zeeman pseudospin fields \( B_L \) and \( B_R \) describe the effect of plunger gates \( V_L \) and \( V_R \). See Methods for further details of these terms.
The non-trivial physics of the model stems from the specific form of the QPC couplings $J_L$, $J_R$, and $J_C$. The physical origin of these terms is the tunnelling of spin-polarized conduction electrons onto and off the interacting islands. We obtain a description in terms of the effective charge pseudospin-$\frac{1}{2}$ operators given in equation (1) by projecting our model onto the reduced subspace of retained charge configurations $(N, M)/(N + 1, M)/(N, M + 1)/(N + 1, M + 1)$. For the island charge states, processes that interconvert the configurations $(N, m) \rightarrow (N + 1, m)$ are described by the left-island charge pseudospin raising or lowering operators $S^{L\uparrow}_{\tau}$, whereas processes that interconvert $(n, M) \rightarrow (n, M + 1)$ are described by the right-island charge pseudospin operators $S^{R\uparrow}_{\tau}$. In terms of the conduction electrons, $S^{L\uparrow}_{\tau}$ describes tunnelling onto/off the left island from the left lead at the left QPC (Methods). Similarly, $S^{R\uparrow}_{\tau}$ corresponds to tunnelling at the right QPC. $S^{L\uparrow}_{\tau}$ and $S^{R\uparrow}_{\tau}$ obey the standard spin-$\frac{1}{2}$ operator algebra, but the conventional ‘up’ and ‘down’ spin states usually associated with those operators are here replaced by the localization of an electron on the lead or the island, respectively.

Because each tunnelling event $(S^{L\uparrow}_{\tau})$ must change an island’s occupancy $(S^{R\uparrow}_{\tau})$, charge pseudospin flips necessarily accompany QPC tunnelling. These island–lead couplings are the same as those for the single-island set-up [23,24,25,26,27]; the correlated tunnelling manifests as an anisotropic Kondo interaction. The new ingredient here is the inter-island coupling $J_C$. Tunnelling from the right island to the left island at the central QPC via the term $C^L_{\tau}C^R_{\tau}$ is accompanied by a simultaneous charge pseudospin flip of both islands $S^{L\uparrow}_{\tau}S^{R\uparrow}_{\tau}$, because the occupation of the left island increases and the occupation of the right island decreases, $(N, M + 1) \rightarrow (N + 1, M)$.

The DCK model is reminiscent of the 2IK model [19], which captures the competition between Kondo screening of local moments and the Ruderman–Kittel–Kasuya–Yosida (RKKY) exchange interaction. However, the DCK model has a major difference: the inter-site coupling is not a simple exchange interaction, but rather tunnelling combined...
with pseudospin flips on both sites (a detailed discussion of this term is provided in the Methods). This turns out to favour the formation of an inter-site Kondo singlet with many-body entanglement, rather than the simple two-body local spin singlet that would arise from RKKY or simple exchange. When scaled to a lattice of sites, this interaction may produce the lattice coherence effect seen in heavy fermion materials but not so far accounted for in microscopic models. A crucial feature of the present charge-Kondo implementation is that the pseudospin couplings $J_L, J_R$ and $J_e$ are related directly to the experimental QPC transmissions of the device, $t_L, t_R$ and $t_e$, and can be large. By tuning these couplings, one can realize various Kondo effects, and indeed a QCP, at relatively high temperatures. This contrasts with the more familiar coupling of spins between two semiconductor quantum dots, where the effective exchange interactions are perturbatively small. Furthermore, the 2IK model is an oversimplified description of real semiconductor double-dot systems, because it does not account for charge transfer between leads, which is known to destroy the QCP.

Our two-island charge-Kondo system therefore presents a unique opportunity to observe a two-impurity QCP at experimentally relevant temperatures.

**Results**

**Phase diagram and Kondo competition**

The island charging energies $E_{c,L,R}$ and inter-island capacitive interaction $I$ are finite in the physical device, so multiple island charge states play a role. This gives rise to a periodic hexagonal structure of the charge stability diagram as a function of the left and right plunger gate voltages $V_{L,R}$. We convert these to energies $U_{L,R} = (I + \lambda_P) V_{L,R}$ using the experimentally measured capacitive lever arm $I = 50 \mu eV mV^{-1}$, relative to an arbitrary reference $U_{0,L,R}$. $U_{0,L,R}$ are related to the pseudo-Zeeman fields $b_{L,R}$ in equation (1) via $b_L = \Lambda U$, where $U = (U_{L,R}), B = (b_L, b_R)$, and the matrix $X$ accounts for cross-capacitive gate effects.

The experimental stability diagram in Fig. 1c allows us to identify regimes with particular charge states on the two islands. In particular, we see distinct charge degeneracy lines $(N, M)/(N, M + 1)$, $(N, M)/(N + 1, M)$ and $(N + 1, M)/(N, M + 1)$, each of which is associated with single-electron tunnelling at one of the three QPCs (Fig. 1b). This structure, including its characteristic gate periodicity, is reproduced very well by numerical renormalization group (NRG) calculations of the DCK model, generalized to take into account multiple charge states on each island (Methods). We fit $f_{L,R}$ for a given set of experimental transmissions $I_{L,R,C}$, as shown in Fig. 1d for the same temperature ($T = 20$ mK). We note that the periodicity of the diagram extends over a larger range of gate voltages than shown (Supplementary Information).

Along the degeneracy line $(N, M)/(N + 1, M)$, the left island charge pseudospin is freely flipped by tunnelling at the QPC between the left island and lead, giving rise to a Kondo effect due to the first term of equation (1). However, the series conductance from left to right leads through the double island structure is suppressed by this, because the conductive pathway involves virtual polarization of the Kondo singlet through the excited state $N + 1$. This is supported by NRG calculations at $T = 2$ mK (Fig. 1e), which show a Kondo blockade in the series conductance as the temperature is lowered. A similar effect is seen along the degeneracy line $(N, M)/(N, M + 1)$, which corresponds to a Kondo effect involving the right island and the right lead. Along the degeneracy line $(N + 1, M)/(N, M + 1)$, tunnelling with the leads is not involved. Instead we may regard $(N + 1, M)$ as two components $\leftarrow$ and $\rightarrow$ of a collective pseudospin state of the double island structure, which is flipped by electronic tunnelling at the central QPC. This gives rise to a kind of inter-island Kondo effect. The resulting Kondo singlet is disrupted by tunnelling at the leads, and hence the conductance is again suppressed on lowering the temperature. The hexagonal structure of the 2D conductance plots as a function of gate voltage for our two-site charge-Kondo system looks superficially similar to that measured for conventional semiconductor double-quantum-dot systems. However, the behaviour in the vicinity of the triple points (TPs) is very different, as shown below.

**Triple point**

The TP, where the $(N, M)/(N, M + 1)/(N + 1, M)$ charge configurations are degenerate, is a special point in the phase diagram. Here the three Kondo effects described above are all competing (Fig. 1b). At $J_L = J_R = J_e$ in equation (1), the resulting frustration gives rise to a QCP, which will be the main focus of this work. By contrast, conventional semiconductor double quantum dots do not support a QCP (see Supplementary Information for a comparison of the two systems).

At the TP, the series conductance is enhanced because an electron can tunnel from left lead to right lead through the islands, without leaving the ground-state charge configurations. A crude approximation treats the QPCs as three resistors in series, and neglects electron interactions. In this case, the maximum conductance of $G = e^2/3h$ arises when the tunnelling rates at each constriction are equal, $J_L = J_R = J_e$, because then there is no bottleneck in the flow of electrons through the structure. Interestingly, we observe the same maximum conductance experimentally in our device at base temperatures when the three QPCs are opened up, but here in conjunction with critical behaviour (to be discussed in subsequent sections) that is absent in the series resistance model. This indicates strong electron interactions, and is expected from our analysis of the full DCK model: NRG calculations confirm that the conductance is indeed capped at a maximum value of $G = e^2/3h$. This arises at the TP at low temperatures $T/T_K \ll 1$ when $J_L = J_R = J_e = J$, which corresponds to the QCP. Here, $T_K \sim e^2/\lambda_P \sim e^2/3h$ is the Kondo temperature right at the QCP, with $\lambda_P$ the electronic density of states at the Fermi energy (Methods). We find from NRG that the conductance $G(T)$ increases with decreasing $T$ at the QCP, approaching the critical value as $e^2/3h - G(T) \sim (T/T_K)^{2/3}$. Opening up the QPCs (large $J$) boosts the Kondo temperature $T_K$ and means that the regime of small $T/T_K$ (and hence $G \sim e^2/3h$) can be accessed experimentally at our base temperature of 20 mK.

We speculate that the same DCK model (and hence the same $G = e^2/3h$ maximum conductance) may be realized in a system of two coupled large semiconductor quantum dots, but only if the level spacing can be made very small ($\sim k_B T$) such that transport between QPCs through the dots is incoherent and the dot states or the QPC transmissions are fully spin-polarized. However, these conditions are typically not met in conventional semiconductor double-quantum-dot systems, whose maximum series conductance is instead $e^2/h$ (or $2e^2/h$ with spin degeneracy). Our observed $e^2/3h$ maximum conductance is thus already a distinctive feature of this set-up.

Our theoretical analysis of the DCK model also reveals an unusual residual $T = 0$ entropy of $\Delta S = \ln(\sqrt{3})$ (Supplementary Information), establishing the QCP as a non-Fermi liquid with exotic fractional (anyonic) excitations. This entropic signature could, in principle, be observed experimentally via the techniques introduced in refs. 40 and 41.

**Conductance line cuts**

We first focus on the behaviour of conductance near the TPs. Specifically, in Fig. 2a we take cuts along the line between TPs, $U_{L,R} = U_L = U$, for different $t_e$ at fixed $t_L = t_R \equiv t$. $U = 0$ is chosen to be the high-symmetry point between TPs. The experimental data are compared with the corresponding NRG simulations of the device in Fig. 2b. As every pair of triple points is indistinguishable (there are no even/odd occupancy effects in the DCK system, unlike in semiconductor double dots with spin), we extract the experimental line cuts of Fig. 2a from averages over multiple pairs of TPs.

Experiment and theory are seen to match very well in Fig. 2. At large $t_e > 0.7 U_c > 0.4$, the positions of the split conductance peaks are captured by the theory, and at smaller $t_e (U_c)$, the widths of the merged conductance peaks are also reproduced. We note that, for the largest transmissions $t_e > 0.9$, although the qualitative behaviour of the
**Universal scaling**

We now turn to the behaviour near the QCP, resulting from frustrated island–lead and inter-island Kondo effects. We focus on parameter regimes with large $r$ and $\tau_C$, such that the corresponding critical Kondo temperatures are large. This allows experimental access to the universal regime $T/T_c \ll 1$. At the QCP with $\tau_C \approx r_C^*$, our theoretical analysis predicts $G \approx e^2/3h$. Moreover, non-trivial behaviour is observed in the vicinity of this singular point, where perturbations drive the system away from the QCP and towards a regular Fermi-liquid state. The associated conductance signatures are entirely characteristic of the QCP, the latter of which we explore next.

Despite some uncertainty regarding the precise TP location, the very existence of a QCP implies an underlying universality, in terms of which conductance signatures in its vicinity can be quantitatively analysed.

**Experimental line cuts**

Experimental line cuts are still captured by NRG, the peak heights are underestimated. This is because a diverging number of island charge states contribute to transport as $r, \tau_C \to 1$, but only a finite number of these can be retained in practice in the NRG calculations. The experimental quantum simulator can therefore provide results in this regime that are inaccessible to classical computation. Away from the limit of perfect transmission, however, our results validate the DCK model as an accurate description of the physical device.

The TP positions in the space of $(U_i, U_b)$ depend on $\tau_C$, and at large $\tau_C$ are well separated such that they can be easily identified from the conductance peak maxima. However, even at the experimental base electron temperature of 20 mK, thermal broadening complicates the experimental analysis of the TP behaviour at small $\tau_C$, where the TPs are close together and the two conductance peaks are merged. Care must thus be taken to estimate the TP positions from the full stability diagram, and to disentangle the influence of adjacent TPs. Estimates can, however, be validated from NRG by going to much lower temperatures where the peaks sharpen up (compare Fig. 1d,e).

We see clear non-monotonicity as a function of $\tau_C$ of the maximum conductance in each line cut in Fig. 2a. This is a good indicator that the TP conductance is also non-monotonic. This is expected due to the competition between different Kondo interactions controlled by $\tau_C$, but would not be the case for Fermi-liquid resistors in series. Taking the critical point with completely frustrated interactions to be at $r_C^*$ (a monotonic function of $r$ but not necessarily $\tau_C^* = \tau_C$), Supplementary Information, we expect lower conductance for both $\tau_C > r_C^*$ and $\tau_C < r_C^*$ at low temperatures. This is because, in both cases, the ground state is a Fermi-liquid Kondo state, the formation of which blocks series transport across the device. The island–lead Kondo effects, which dominate for $\tau_C < r_C^*$, renormalize the effective QPC transmissions to $r \to 1$ and $\tau_C \to 0$ on lowering the temperature. By contrast, the inter-island Kondo effect, which dominates for $\tau_C > r_C^*$, renormalizes $r \to 0$ and $\tau_C \to 1$. Only when $\tau_C = r_C^*$ is the low-temperature conductance Kondo-enhanced, because in this case island–lead and inter-island Kondo singlets cannot form simultaneously (all of the renormalized QPC transmissions remain finite due to the frustration). We note that the temperature dependence of the conductance enhancement at $\tau_C = r_C^*$, as well as the conductance suppression away from this point, have a non-trivial Kondo form characteristic of the QCP, the latter of which we explore next.

**Fig. 2 | Conductance line cuts between TPs.** a, Experimental measurements (a) and NRG-calculated (b) line cuts for $r = 0.38$ (figures 3 in the model) along the line $U_i = U_b = U$ for different $\tau_C (\hbar)$. Insets: representative 2D $\rho_S$ sweeps from which the line cuts are extracted. The model parameters in b are optimized to fit the experiment, and multiple charge states are retained for each island (Methods).

where $a$ and $b$ are constants. We expect universal behaviour of the system as a function of $T/T_c$ when $T/T_C \to 1$, the system is firmly within a Fermi-liquid regime, and in the opposite limit the system is near the QCP. From NRG we can calculate the universal conductance crossover as a function of $T/T_C$ in the close vicinity of a TP. This is shown as the solid lines in Fig. 3 and has asymptotic behaviour $e^2/3h - G(T) \cdot (T/T_C)^{4/3}$ for $T/T_C \ll 1$ and $G(T) \cdot (T/T_C)^{4/3}$ for $T/T_C > 1$ (Supplementary Information).

However, because equation (2) is calculated with the minimal DCK model, where each island is restricted to two charge states, it cannot capture the full behaviour of our experiment. The DCK model contains a single pair of TPs, whereas the charge-stability diagrams of Fig. 1 exhibit a periodicity in gate voltage space and feature repeated TP pairs. In particular, starting at one TP and then increasing $\Delta U$, we must eventually encounter another TP. This implies a periodicity in the Fermi-liquid scale $T^*$ that is not captured by the universal result, equation (2), which holds only for small values of the perturbation. Phenomenologically, the simplest form to correctly capture the observed
periodicity, while also reducing to the known behaviour of equation (2) in the close vicinity of any given TP, is

$$T^* = a|\Delta U|^{1/2} + b|\cos(2\pi U/\delta) - \Delta TP|^{1/2}.$$  

Here, $\delta$ is the spacing between pairs of TPs and $\Delta TP$ is related to the splitting of the two TPs in a given pair, such that $\Delta TP = \cos(2\pi U/\delta)$. We define $U = 0$ as being halfway between a pair of TPs, as in Fig. 2. In fact, this periodic form of $T^*$ was recently proved as an exact result by theory is both striking and a direct signature of the critical point. The scaling collapse as a function of $T^*/T$ and strong quantitative agreement with the non-trivial universal conductance curve obtained by theory is both striking and a direct signature of the critical point. Significant, the collapse is over the entire range of $T^*/T$ for each line cut, with limitations only at small $T^*/T$ due to the finite $R/T_c$ mentioned previously. We attribute the breakdown in scaling for the data collected at the highest temperature of 75 mK to be due to a breakdown of the assumption $k_B T < E_c$, on which the derivation of the DCK model relies.

In the case of the data presented in Fig. 3a, we comment that the limit of large $r$, $\tau_c$ is, in fact, not well satisfied. The use of equation (3) here is reasonable, but only conjecture. However, our experimental results do suggest that equation (3) holds more generally. This illustrates the power of quantum simulation to obtain results beyond the reach of other methods.

Although most of this work has focused on line cuts along the axis connecting TPs ($U_1 = U_2$), in fact equation (2) holds for gate detuning $\Delta U$ away from the TP in any direction in the space of ($U_1$, $U_2$). In Fig. 3c we present line-cut data along the orthogonal direction, parallel to $U_1 = -U_2$, for the same couplings as plotted in Fig. 3b. The periodic structure of the charge-stability diagram is different along these cuts and so we cannot use equation (3) (no simple form is known from theory). However, the effect of neighbouring TPs and higher charge states is less pronounced as $\Delta U$ is increased in this direction, and so the simpler form of equation (2) is quite sufficient in practice. We again see excellent data collapse to the universal theory curve at all temperatures considered (and now even the 75 mK data). This further strengthens the sense of universality in this system, because the same behaviour is observed by perturbing the QCP in different directions around the TP—an emergent isotropy in gate space not present in the bare model.

**Discussion**

In this Article we have presented strong evidence for a quantum phase transition in a two-site circuit. By exploiting the charge-Kondo

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paradigm, our device maps to a variant of the celebrated two-impurity Kondo model, here featuring a phase in which the local moments on the two islands are screened collectively by many-body effects driven by conduction electron scattering. This may have relevance for the emergence of lattice coherence in Kondo lattice systems.

We have formulated a model to describe the two-island charge-Kondo device, and demonstrate quantitative agreement between NRG calculations and experimentally measured conductance, including in the universal regime of the exotic QCP.

Our work on the crucial role of the inter-island interaction paves the way for a host of other studies. Opening each of the islands to a second lead (already present but not used in the existing device) would produce two sites, each hosting a two-channel Kondo (2CK) state at low temperatures. Our existing device allows the coupling between two such 2CK states to be studied. Alternatively, by preparing a single 2CK state on one island, the associated Majorana zero mode localized on that island could conceivably be transferred to the other island by gate-voltage tuning. This could eventually allow for the manipulation and even braiding of anyonic excitations arising from Kondo interactions in nanoelectronic circuits.

Unlike for tunnel-coupled semiconductor quantum dots, there is no clear roadblock to scaling this platform to more complex uniform clusters of coupled charge-Kondo islands, and ultimately lattices. This provides a way of examining with unprecedented control some of the most subtle collective dynamics of real correlated materials, and introducing a flexible set of effective interactions. Such scaled-up charge-Kondo clusters would act as analogue quantum simulators with capabilities beyond classical computation: three coupled islands and introducing a flexible set of effective interactions. Such scaled-up charge-Kondo islands, and ultimately lattices. This provides a way of examining with unprecedented control some of the most subtle collective dynamics of real correlated materials, and introducing a flexible set of effective interactions.

Our work on the crucial role of the inter-island interaction paves the way for a host of other studies. Opening each of the islands to a second lead (already present but not used in the existing device) would produce two sites, each hosting a two-channel Kondo (2CK) state at low temperatures. Our existing device allows the coupling between two such 2CK states to be studied. Alternatively, by preparing a single 2CK state on one island, the associated Majorana zero mode localized on that island could conceivably be transferred to the other island by gate-voltage tuning. This could eventually allow for the manipulation and even braiding of anyonic excitations arising from Kondo interactions in nanoelectronic circuits.

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The reservoirs, denoted $f_{αi}$, are incoherent precisely because $δ$ is large. We treat the electrons on each island and on the right lead. We neglect the incoherent transport, and model the electrons as distinct electronic continua in the thermodynamic limit. The physics is insensitive to the precise number of electrons in these reservoirs, because this number is very large. In lead $α = L, R$ are denoted $f_{αi}$. See Extended Data Fig. 1 for a schematic of the set-up. The conduction electrons are described by $H_{\text{elec}}$, which is given by

$$H_{\text{elec}} = \sum_{a, \gamma, k} ε_{a, \gamma, k} c_{a, \gamma, k}^\dagger c_{a, \gamma, k} + \sum_{a} ε_c c_{a}^\dagger c_{a},$$

where $a = L, R$ (for left or right) and $γ = l, i$ (for lead or island). The dispersion $ε_c$ describes the single-particle energy of an electron with momentum $k$ (all reservoirs assumed for simplicity, but this is inconsequential); we take the density of states to be constant, $\nu = 1/2D$, inside a band of half-width $D$.

Electronic tunnelling at the QPCs is described by $H_{\text{QPC}}$ and is given by

$$H_{\text{QPC}} = \sum_{a \neq L, R} J_{a} \left( c_{a}^\dagger c_{L} + c_{L}^\dagger c_{a} + \text{h.c.} \right) + J_{C} \left( c_{C}^\dagger c_{R} + \text{h.c.} \right),$$

where the localized orbitals at the QPC positions are defined as $f_{αi} = \sum \xi_{αj} f_{αj}$ and $c_{a} = \sum \xi_{c, a} c_{a}$, with expansion coefficients $\xi_c$. Tunnelling events at the QPCs described by $H_{\text{QPC}}$, change the charge-state configurations $(n, m)$ of the islands (here $(n, m)$ is the number of electrons on the left (right) island). We denote the corresponding macroscopic charge states of the islands as $[n, m]$, such that the number operators for the left and right islands are given by

$$\hat{N}_L = \sum_{n, m} n | n, m \rangle \langle n, m |,$$
$$\hat{N}_R = \sum_{n, m} m | n, m \rangle \langle n, m |.$$

The island occupancies are then changed by ladder operators

$$\hat{N}_L^\pm = \sum_{n, m} (n \pm 1, m) | n, m \rangle \langle n, m |,$$
$$\hat{N}_R^\pm = \sum_{n, m} (n, m \pm 1) | n, m \rangle \langle n, m |.$$

See Supplementary Information for more details.

Using these operators, we describe the effect of the gate-controlled local potentials as

$$H_{\text{gate}} = B_L \hat{N}_L + B_R \hat{N}_R,$$

and the effect of electronic interactions as

$$H_{\text{int}} = t_{C}^2 \hat{N}_L^2 + t_{L}^2 \hat{N}_R^2 + U_{C} \hat{N}_C \hat{N}_L \hat{N}_R.$$

The full microscopic Hamiltonian then follows as $H = H_{\text{elec}} + H_{\text{QPC}} + H_{\text{gate}} + H_{\text{int}}$.

To obtain an effective charge-Kondo model from $H$, we make two simple and well-controlled approximations—both of which follow from the original charge-Kondo proposals of Matveev and others. First, we relax the exact constraints that $N_L = \sum f_{αl}^\dagger c_{αl} + \sum c_{αl}^\dagger f_{αl}$ and promote $N_L$ and $N_R$ to independent quantum degrees of freedom. This is a good approximation when the islands host a very large number of electrons in a quasi-continuum, because the dynamics of $N_L$ and $N_R$ should be unaffected by the precise number of electrons on the islands (that is, the physics is controlled by changes in the occupations rather than the occupations themselves). This limit is well satisfied in practice by the experimental set-up. In this case we replace $f_{αl} \rightarrow f_{αl} N_L$ and $c_{αl} \rightarrow c_{αl} N_R$. The second approximation is to project the full Hamiltonian $H$ onto a reduced subspace of thermally accessible island charge configurations to obtain an effective model $H_{\text{eff}} = \mathcal{P}H$. This is a valid approximation at low temperatures $T \ll T_{\text{C}}$ provided the QPCs are not opened up (a macroscopic number of charge states are involved near perfect transmission), so we additionally require $\nu f_{\text{C}, \text{R}} \ll 1$ for the projection step).

**DCK model.** The DCK model, equation (1), is obtained (up to an irrelevant constant) by following the above steps, when we retain only the charge configurations $(n, m)$ with $n = \text{Nor} N + 1$ and $m = \text{Mor} M + 1$. That is, $H_{\text{DCK}} = \mathcal{P}H$ with the projector

$$P = |N, M \rangle \langle N, M | + |N + 1, M + 1 \rangle \langle N + 1, M + 1 | + |N + 1, M \rangle \langle N + 1, M | + |N + 1, M + 1 \rangle \langle N + 1, M + 1 | .$$

We now introduce pseudospin-$1/2$ raising/lowering operators to describe transitions between the two retained charge states of each island:

$$\hat{S}_L^+ = \sum_{m = 0}^{M - 1} |N + 1, m \rangle \langle N + 1, m | + \sum_{m = 0}^{M} |N + 1, m \rangle \langle N + 1, m | .$$

The effective DCK model then reads

$$H_{\text{DCK}} = \langle J_L \hat{N}_L^\dagger f_{αl}^\dagger c_{αl}+ + J_R \hat{N}_R^\dagger f_{αl}^\dagger c_{αl} + \text{h.c.} \rangle + \langle J_C \hat{N}_C^\dagger c_{R} + \text{h.c.} \rangle$$

$$+ \langle J_{\text{gate}} \hat{N}_L^\dagger \hat{N}_R + \hat{N}_L \hat{N}_R^\dagger + H_{\text{int}} \rangle .$$

We obtain equation (1) by further defining the pseudospin-$1/2$ operators $\hat{S}_R = f_{αl}^\dagger c_{αl}$ and $\hat{S}_L = (\hat{S}_R)^\dagger$.

**Minimal model at the TP.** At the TP, we can further project the model onto the restricted three-dimensional subspace of charge configurations $A = |N, M \rangle, B = |N + 1, M \rangle$, and $C = |N, M + 1 \rangle$ using the projector $P = \sum_{A, B, C} |A \rangle \langle A |$. This leads to a minimal model valid close to the TP:

$$H_{\text{TP}} = \langle J_L \hat{S}_L A | B \rangle \langle B | + \langle J_C \hat{S}_C C | B \rangle \langle B | + \langle J_{\text{gate}} \hat{N}_L A | C \rangle \langle C | + \text{h.c.} \rangle$$

$$+ \langle H_{\text{gate}} \rangle + \text{h.c.} \rangle .$$

where we have additionally defined $\hat{S}_C = c_{R}^\dagger c_{R}$ and $H_{\text{gate}} = \sum_{\text{qubit}} \Delta U(|\gamma\rangle \langle \gamma |)$ describes gate-voltage detuning away from the TP. The QCP is realized
by setting \( f_L = f_R = J \) and \( H_{gen} = 0 \) in equation (9), and is illustrated schematically in Extended Data Fig. 1b. Finite \( H_{gen} \) then generates a Fermi-liquid crossover on the scale of \( T^* \) as discussed in the main text. Equation (9) is solved by NRG and used to obtain the universal conductance curves near the TP shown in Fig. 3, and to extract numerically the dependence of \( T_K \) on \( J \) as mentioned in the main text.

At the QCP of the DCK model, enhanced series conductance between leads proceeds via the following mechanism (blue arrows in Extended Data Fig. 1b). We start from the charge configuration \( |4\rangle = |N, M \rangle \). First, an electron tunnels from the left lead (\( f_L \)) onto the left side of the left island (\( f'_{L} \)), thus flipping the left-island charge pseudospin from ‘down’ to ‘up’ (\( c^\dagger \rightarrow c \), meaning \( N + 1 \)). This converts \( |4\rangle \rightarrow |3\rangle \). In the second step, an electron tunnels from the right side of the left island (\( c_L \)) to the central QPC to the left side of the right island (\( c_R \)). This simultaneously lowers the charge pseudospin of the left island back to ‘down’ and raises the charge pseudospin of the right island to ‘up’ (\( c_R^\dagger \rightarrow c_R \), meaning \( N + M + 1 \)). This step converts \( |3\rangle \rightarrow |2\rangle \). In the final step, an electron tunnels from the right side of the right island (\( f_R \)) onto the right lead (\( f'_{R} \)), which also flips the charge pseudospin on the right island back to ‘down’ (\( c_R^\dagger \rightarrow c_R \), meaning \( N + M + 1 \)). Overall, an electron is transferred from the left lead to the right lead, but the device charge configuration has been ‘reset’ (\( c_L \rightarrow c_R \)), ready for transfer of the next electron. In the DCK model, \( U(1) \) charge is separately conserved in the three ‘channels’ \( f_L, f_R, c_L \) and \( c_R \) (whereas only a global \( U(1) \) symmetry applies in the physical device). The transport mechanism described above leaves an extra electron in the \( c_L \) and \( c_R \) and an extra hole in \( f_L \) and \( f_R \). Overall charge conservation is still maintained, as this is enforced by the charge pseudospin dynamics of \( S_L \) and \( S_R \). The generation of particle–hole pairs in the island reservoirs is irrelevant for the pseudospin dynamics, because the island reservoirs are treated as electronic continua in the thermodynamic limit, and calculations are performed in the grand canonical ensemble.

**Generalized multi-level DCK model.** To capture the periodicity in the experimental charge-stability diagram (Fig. 1a) and to relax the constraints the experimental charge-stability diagram (Fig. 1c) and to relax the dependence of \( T_K \) on \( J \) as discussed in the main text.

Equation (10) reduces to the DCK model equation (1) for \( N = M = 0 \). Only when two charge states are accessible per island. By contrast, the effect of the projection is entirely removed by taking the formal limit \( N, N, M, M \rightarrow \infty \). The generalized model therefore allows us to interpolate between these limiting cases by tuning \( N \) and \( M \). Although a relatively small number of charge states can be retained for numerical calculations in practice (see below for details), one can check post hoc that results are converged with respect to increasing \( N \) and \( M \) for a given set of physical model parameters.

**NRG calculations**

To simulate the experimental two-island charge-Kondo device using NRG \(^{34,47}\), we employ the effective models developed above. Universal results near the critical TP presented in Fig. 3 of the main text were obtained by NRG using equation (9). Figures 1 and 2 feature NRG results using the generalized multi-level model equation (10), which allows to capture the full phase diagram and gate-voltage periodicity.

NRG involves discretizing the conduction electron Hamiltonian \( H_{elec} \) logarithmically, mapping to semi-infinite tight-binding Wilson chains, and diagonalizing the discretized model iteratively. \( N_{\text{elec}} \) of the lowest energy states are retained at each step, resulting in an RG procedure that reveals the physics on progressively lower energy scales.

Standard NRG cannot be used in our case, however, due to the complexity of the models at hand, with six spinless conduction electron channels. Instead, we use the ‘interleaved NRG’ (iNRG) method \(^{49,50}\) which involves mapping \( H_{elec} \) to a single generalized Wilson chain. This dramatically lowers the computational cost of such calculations, and brings the numerical solution of the models within reach. For all iNRG calculations presented in this work, we use a logarithmic discretization parameter \( A = 4 \), retain \( N_{\text{elec}} = 35,000 \) states at each iteration, and exploit all Abelian quantum numbers. When using the generalized model equation (10) we used \( N = M = 7 \), corresponding to 16 retained charge states per island. By contrast, the universal critical physics of the TP can be obtained from equation (9), retaining just three charge states for the entire two-site system.

The experimental quantity of interest is the series d.c. linear response differential conductance

\[
G = \left. \frac{df}{dV_b} \right|_{V_b \to 0},
\]

where we take \( I = -eV_bJ_{NRG} \) to be the current into the right lead due to a voltage \( V_b \) applied to the left lead. Here \( K_{NRG} = \frac{2e}{3} \) and \( K_{NRG} = \sum f_{LR} \). An a.c. voltage bias on the left lead can be incorporated by a source term in the Hamiltonian, \( H_{bias} = -eV_{0}\cos(\omega t)N_1 \) where \( \omega \) is the a.c. driving frequency. The d.c. limit is obtained as \( \omega \rightarrow 0 \).

The geometry of the device means that the conductance cannot be related to a spectral function. Instead we use the Kubo formula\(^{51} \):

\[
G = \left. \frac{d}{d\omega} \right|_{\omega \to 0} \frac{2\pi \Im K(\omega)}{\omega},
\]

where \( K(\omega) = \langle NL_2 N_2 \rangle \) is the Fourier transform of the retarded current–current correlator \( K(t) = \langle -i\theta(t) | NL_2 N_2 | t \rangle \). Within iNRG, \( \Im K(\omega) \) may be obtained from its Lehmann representation with the full density matrix technique\(^{52} \). The numerical evaluation is substantially improved by utilizing the identity \( \Im K(\omega) = -2\omega \Im \langle NL_2 N_2 \rangle \) (ref. \(^{53} \)). We use iNRG to calculate the conductance through the device from equation (12) at a given temperature \( T \), as a function of \( I_m \) and \( B_k \).

Although NRG employs a discretized representation of the conduction electron part of the model \( H_{elec} \) (the Wilson chains), the renormalization group structure of the model is exploited in NRG to nevertheless obtain highly accurate approximations to the exact continuum result for equilibrium physical quantities\(^{34,47,50}\).
Device
The device was fabricated on a GaAs/AlGaAs heterostructure with 95-nm-deep 2DEG, density of 2.6 × 10^{11} cm^{-2} and mobility of 2.0 × 10^{5} cm^{2} V^{-1} s^{-1}. A scanning electron microscopy (SEM) image of an equivalent device is shown in Extended Data Fig. 2. High transparency of the small ohmic contacts is crucial, so we took special steps to ensure cleanliness of the interface with the GaAs heterostructure. Before any fabrication was done on the heterostructure, we dipped it in HCI 3.7% to remove any oxide layer that had built up. After writing the ohmic layer pattern using electron-beam lithography and developing the poly(methyl methacrylate) (PMMA) resist, we used a light oxygen plasma etch to remove residual PMMA scum. Next, before evaporating the ohmic stack, we used the following chemical treatment procedure: dip in tetramethylammonium hydroxide 2.5% for 20 s, 5 s in water, 5 s in HCI 37% and 5 s in water (separate from the first cup of water). Afterwards, we quickly moved the chip into a KJL Lab 18 electron-beam evaporator (with a load-lock), and pumped down to a vacuum of 10^{-8} torr. Reducing the time in air is important to prevent substantial oxide layer growth. Finally, we ran a low-power in situ argon etch for 20 s. Only after this did we evaporate the ohmic stack (107.2-nm Au, 52.8-nm Ge and 40-nm Ni, in order of deposition).

Experimental set-up
The device was cooled with a +300-mV bias on all the gates to reduce charge instability by limiting the range of voltages we needed to apply. To reduce thermoelectric noise causing unwanted voltage biasing across the device, each lead had a central ohmic contact (between the source and measurement contacts) and all those central ohmics were shorted to each other on chip. The shorted ohmics were connected to a single line and grounded at room temperature. Measurements were made at low frequencies (<100 Hz) using an SR830 lock-in amplifier. The 14-mV output of the SR830 was converted to a current bias using a 100-MΩ resistor, and the current was then converted to a voltage on chip by the quantum Hall resistance (\(h/2e^2\)). A measurement of either the series transmitted voltage or the reflected voltage was amplified by an NF SA-240F5 room-temperature voltage amplifier. The series conductance of interest could then be simply extracted from this voltage as explained in the following. For most reported measurements, we sourced at S2 and measured at M2 (Fig. 1a). The series conductance was then related to the reflected voltage, \(V_R\), by equation (13):

\[
G = \frac{e^2}{3h} \frac{V_R - V_{S2,L=0}}{V_{S2,L=0} - V_{S2,L=\infty}}
\]  

(13)

Measuring in this way eliminates the need for precise knowledge of many settings in a given set-up—excitation amplitude, amplifier gain, line resistances and so on. For arbitrary sourcing and measurement configurations, the relations between measured voltages and the desired conductances can be calculated straightforwardly through Landauer–Büttiker formalism.

Electronic temperature
The electronic temperature is determined from dynamical Coulomb blockade (DCB) measurements as outlined in ref. 1. The zero-bias suppression of the conductance across a QPC when series-coupled to another QPC of low resistance can be fit to a known theoretical form that directly depends on the electron temperature\(^{12}\). In our device we measure through two QPCs across a single island \(t_r = 0\), with one QPC partially transmitting and the other set to fully transmit a single edge mode.

Calibrating QPC transmissions
A standard procedure to measure the transmission of each QPC is to measure the series conductance while varying the applied gate voltage of the QPC, with each other QPC set to fully transmit \(n\) edge states, acting as an \(h/ne^2\) resistor. For our experiments of central interest, we must then adjust each QPC to a desired transmission. We cannot naïvely use the gate voltages that produced that transmission with all other QPCs open, because changing the voltage applied to any gate capacitively affects all other QPCs. However, we can calibrate this capacitive effect by measuring how much each QPC’s transmission curve shifts as we vary each other QPC’s gate voltage by a known amount. This is done for each pair of QPCs we use in the experiment, and with this information we can systematically determine the appropriate gate voltages to set. However, even this procedure fails in our device. When transmission is measured in series through a QPC and an additional resistance of order \(h/e^2\), DCB suppresses the conductance relative to that expected from ohmic addition of the ‘intrinsic’ transmissions\(^{11,14}\). The intrinsic transmissions can be recaptured by applying a source–drain bias that is large compared to a relevant charging energy (Extended Data Fig. 3a).

Alternatively, a measurement pathway that does not go through the metal island (for example, measuring \(t_r\) through the plunger gate \(P_l\)) effectively shorts the circuit to ground, so the intrinsic transmission is recovered even at zero source–drain bias.\(^{19,20}\) The measured QPC transmission at low bias avoiding the island should then be the same as the measurement at high bias through the island. Empirically, this is not the case in our device. Instead, repeating the measurement through \(P_l\) at high bias shows exact agreement with the high bias measurement through the island. Bias-dependent measurements through \(P_l\) show a zero-bias suppression and a high bias plateau of conductance consistent with DCB suppression, contrary to the expectation that DCB should be negligible in this configuration (Extended Data Fig. 3c).

We suspect that this residual DCB effect is due to impedances in our measurement set-up, external to the device. We connect our measurement lines, which ground the device through highly resistive coaxial lines and discrete filters located immediately above the connection, to the sample through large ohmic contacts. This is in contrast to earlier work\(^{19,20}\) on this type of system, in which a cold ground is used, effectively creating a very low impedance path to ground.

The important question is which measurement transmission is relevant for the Kondo interactions. Although previous work\(^{18,21}\) found the difference between the intrinsic transmission and the zero-bias transmission through an open plunger gate to be small, and thus both equally valid, we find that the measurement through \(P_l\) at zero bias empirically works best in our device. Without any residual DCB, we believe this should be the same as the intrinsic transmission. However, the residual DCB appears to not only suppress the measured QPC transmissions, but all measured conductances of our system in any configuration, at zero bias. In particular, when measuring through either one or both islands, the conductance when using the intrinsic transmission at charge degeneracy points, where Kondo interactions are most important, is consistently lower than both expectations and previous results seen in ref. 18. With the single island, we see a suppression consistent with DCB: the series conductance we measure at a Coulomb blockade peak is lower than that measured in ref. 18 for the same intrinsic transmissions and temperature, with the smallest differences in conductances being near the maximum \((0.5e^2/h)\) or minimum \((0e^2/h)\) possible conductances. Furthermore, the differences reduce at higher temperatures. These two features are similar to that seen in DCB measurements.

If instead we use not the intrinsic transmission, but the zero-bias transmissions through \(P_l\), we see remarkably consistent agreement with previous measurements of the two-channel Kondo model for different transmissions and temperatures. Our interpretation is that, although both Kondo and DCB renormalize conductance, we can work in a ‘DCB-renormalized space’ by explicitly setting the DCB-renormalized transmissions. These DCB-renormalized transmissions then act as the transmissions that are in turn important for the Kondo interactions. Importantly, we must use the transmissions measured in an environment in which the QPC sees the same electromagnetic impedance.
as in the measurements of interest. This means using the zero-bias transmissions measured through \( P_a \), where DCB is caused by only the external impedances, and not the zero-bias transmissions measured through the island, where there is additional suppression due to the resistance of the second QPC in series.

On the left island, the QPC we use does not have an adjacent pathway through \( P_a \), but, due to an equivalence of the island–lead QPCs, a consistent mapping can be made from the intrinsic transmission to the DCB-renormalized one that would be measured through the plunger gate. However, we are unable to make this mapping for the inter-island QPC as we observe differences in the DCB renormalization when measured through the islands (Extended Data Fig. 3b). Similarly, we are unable to measure the extraneous DCB renormalization of the inter-island QPC due to the device geometry. In our results in the main text, we therefore report intrinsic values for \( \tau_c \) and the DCB-renormalized values (measured through \( P_a \)) for \( \tau_c \). This may be why the maximum conductance does not appear at \( \tau = \tau_c \), and in any case it means that the \( \tau_c \) relevant for Kondo physics grows with increasing temperature.

The conclusions of the main text are not sensitive to the choices described above regarding which values (intrinsic or DCB-renormalized) we report for each QPC transmission. None of our results rely on precise quantitative knowledge of the transmission settings, because \( \tau \) depends only on detuning of one transmission relative to another. This would not be the case in any future work exploring universal scaling as a function of \( T/T_c \), as \( T_c \) depends directly on the quantitative transmissions.

### Fitting universal curves

To calculate \( \cos 2\pi \tau V - \Delta_{\text{TP}}^{1/2} \) and thus \( \tau \) first requires extracting \( \Delta_{\text{TP}} \). As mentioned in the main text, these relate to the TP periodicity and splitting, respectively. The data of Fig. 3 are extracted from charge-stability diagrams over larger gate-voltage ranges as in Fig. 1c. The line cuts of Fig. 3a specifically come from averages over multiple pairs of TPs, whereas the rest of Fig. 3 uses singular line cuts. Using either line cuts from the fully stable diagram or from other stability diagrams taken with exactly the same \( \tau_c \) values, we can extract \( \Delta_{\text{TP}} \). There is uncertainty in the TP splitting, as identifying the TPs is non-trivial, which we will discuss further below. Finally, we must also determine the unknown scaling prefactor \( b \) of equation (2). We do this by least-squares-fitting \( b \cos 2\pi \tau V - \Delta_{\text{TP}}^{1/2} \) of the experimental data to \( T/T_c \) of the universal curve, with \( b \) as a free parameter. Each \( \tau \) line cut is independently fit, and we average the resulting \( b \) values and apply the same rescaling to each curve. We find \( b \approx 1 \) mK works best. A similar procedure is done in Fig. 3b, except we fit only the 20-mK data to the fully universal curve, obtaining \( b \approx 3 \) mK. The elevated temperature data are then rescaled with the same obtained \( b \). For every curve, we also fit a constant shift \( a(\tau_c) \) to take into account that there is a finite \( T_c \), even at \( \Delta U = 0 \), due to a detuning from the critical couplings. These fits exclude the first few data points at the lowest \( T/T_c \), where we do not expect agreement with the universal curve due to a finite \( T/T_c \) (with \( T_c \) the critical Kondo temperature before).

In Extended Data Fig. 4a, we show the \( \tau_c \) values used in the scaling collapse data of Fig. 3a in the main text. To resolve the TP peaks we need a sufficiently large \( \tau_c \), the needed value of which grows with \( \tau \). To reach a conductance close to \( e^2/3h \) and have split TPs, \( \tau_c \) must be made extremely close to 1. Although one might guess that the resulting measured line cuts would be essentially identical, we see that in the full shape (Extended Data Fig. 4b) there are large changes to the line cuts, even for tiny changes in \( \tau_c = 1 \). Contrast this with Fig. 2a of the main text, where \( \tau_c \) is far from 1—large changes in \( \tau_c \) are needed for comparable changes in the line cuts.

In Fig. 3b, to roughly match the conductance at \( \Delta U = 0 \) across different temperatures, we adjust the transmissions at each temperature. If the transmissions were instead held constant, the conductance would decrease with increasing temperature due to a larger \( T/T_c \). By increasing \( \tau, \tau_c \) with temperature, \( T/T_c \) is maintained roughly constant. In our measurement, \( \tau = [0.78, 0.78, 0.81, 0.82] \) at \( T = [20, 26, 46, 75] \) mK while \( \tau_c = 0.9 \) for all \( T \). As described in the Calibrating QPC transmissions section of Methods, we know that this corresponds to a slightly increasing \( \tau_c \) with temperature, although the exact variation is unknown. That \( \tau_c \) varies between temperatures does not affect the conclusion, because the scaling as a function of \( \Delta U \) is unaffected.

Finally, as previously mentioned, in some cases the TPs are not easily identifiable, depending on \( \tau \) and \( \tau_c \). Although in Fig. 3 we try to choose line cuts in which the TPs are relatively well separated, it is not always possible depending on the couplings and temperature. Using NRG calculations at extremely low temperatures to determine the TP locations, and comparing them to the location of the conductance peaks at higher temperatures, we can estimate the difference between conductance peak and TP position. We find from our NRG calculations that choosing a point \(-1 \mu eV \) away from the peak conductance for the two lowest temperatures of Fig. 3b, \(-2 \mu eV \) away for the 46-mK data and \(-1 \mu eV \) away for the 75-mK data, is a good approximation. The large offset of the 75-mK data is simply due to there only being a single conductance peak centred between the two TPs at this temperature. We find that this set of offsets is equivalent to what we would obtain by assuming the TP splitting, and thus \( \Delta_{\text{TP}} \), constant for all the temperatures, which is valid because \( \tau, \tau_c \) do not vary much between them. For all of the data used in Fig. 3a, where the TPs are well separated, we find that using the conductance peak location as the TP location is a good approximation. Due to a combination of an imperfect identification of the TP location and finite spacing of measurements in gate voltage, there will still be some error in choosing the true TP location. We estimate this uncertainty in \( \Delta U \) to be \(-0.25 \mu eV \) for all data taken at the two lowest temperatures, and \(-1 \mu eV \) for the two higher temperatures. Although any uncertainty in \( \Delta U \) creates a nonlinear error when scaling \( \Delta U \) to \( \tau \), we find that changes within the uncertainty of where we set \( \Delta U = 0 \) do not substantially change our results. We see this by offsetting \( \Delta U \) and redoing the same scaling collapse analysis.

For the orthogonal line cuts, we instead fit the scaling prefactor \( b \) in equation (2) of the 20-mK data to the universal curve, finding \( b = 0.0086 \) mK \( \mu eV^{-3/2} \). As in the parallel line cuts, all higher temperatures use the same prefactor.

### Metal–2DEG interface

To apply our model to our experimental system, the edge modes in the 2DEG transmitted through a QPC must then be perfectly transmitted into the metal component. For this to occur, first, the edge mode must not bypass the metal. To enforce this, following ref. 10, trenches are etched in the 2DEG below the island so the metal is the only conducting path bridging otherwise separate regions of 2DEG on different mesas. Second, reflection of the edge mode from the metal must be minimal. We verify near-perfect transmission by relating measured voltages to the transmission into the metal, \( \tau^R \). In principle it is possible to extract the separate transmission probabilities for each metal–2DEG interface, but here we report an average. Unlike in the work of Iftikhar and colleagues13, we find we must take into account the non-negligible resistance to ground in our set-up—through cryostat lines and filters intended to maintain a low electron temperature—which generates a finite transmitted voltage \( V_{\text{trans}} \), even with all QPCs closed:

\[
\frac{V_{\text{trans}}^{T=0}}{V_{\text{trans}}^{R=0}} = 1 - \frac{R(1)}{4}.
\]

Here, \( \tau^R_{a,4} \) are the unused QPCs in our experiment, representing the transmissions of the bottommost QPCs of the left and right islands, respectively. In our device, we measure

\[
\tau^R_{1} = 1.0004 \pm 0.0098 \quad \tau^R_{1} = 0.9947 \pm 0.0417.
\]
There is a much larger uncertainty on the left island, due to a noisier measurement contact MI.

Measurement uncertainty
A few factors contribute to uncertainty in the reported conductance. Due to slow drift in the electric potential, we cannot reliably sit at the same position in the charge-stability diagram. This reduces the ability to average over time at a particular configuration of $P_L, P_R$. Based on the standard deviation in conductance in a configuration where each QPC was fully open, this error is \(-0.001e^2/h\).

There is a second uncertainty in that the conductance for a given pair of TPs varies between periods in $P_L, P_R$. We observe that repeated measurements centre around a particular value, with a few clear outliers of much lower conductance. To reduce this effect, we take the median conductance over a few periods. Reducing the error is limited by needing to re measure the full charge-stability diagram multiple times. For example, the data in Fig. 2a are extracted from 141 charge-stability diagrams, of which about only 40 had no clear charge switching events along the line between TPs. The errors of Fig. 2a for a particular $r_0$ and $U$ (median absolute deviation of \(-0.006e^2/h\)) are such that the exact $r_0$ line cut in which the conductance peaks at $U = 0$ is unknown, but the overall non-monotonic behaviour is unchanged.

Charging events may also infrequently shift the set transmission of one of the QPCs. Thus, for each 2D sweep we recalibrate each QPC to ensure we are at the transmission stated. The QPC calibration procedure relies on the accurate determination of the crosstalk between QPCs. Although, in principle, this can be done very accurately, we cannot explicitly verify the transmissions in the experimental configurations used in our main results. However, we can verify that the procedure works by looking at the symmetry of the charge-stability diagrams, which depends on the two island—lead QPCs having equal transmissions. From this we estimate that the transmissions are off from their intended values by at most 0.03.

Finally, our conversion from voltage to conductance could introduce errors due to uncertainties in our measurement electronics. However, we can calibrate out these uncertainties by normalizing our measured voltages by the reflected voltage with all QPCs closed (Methods). Any error is probably due to any imperfections in the metal–2DEG interface, as our conversion from voltage to conductance assumes $r_0 = 1$. When this assumption is broken, the reported series conductance values would both be off from the true series conductance, and the true series conductance would not be the right comparison to our NRG results, as the DCK model also assumes a perfect metal–2DEG transparency. Using our measurements of $r_0$ above, we estimate that our reported series conductance values have an error of \(-1\%\), coming from $|1 - r_0^\prime| \sim 0.01$.

Data availability
All data used in this work are available in the Stanford Digital Repository at https://doi.org/10.25740/mx151nn9365. Source data are provided with this Paper.

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Author contributions
W.P. and L.P. performed the measurements. L.P. fabricated the device. A.K.M. developed the theory and carried out NRG calculations. W.P., L.P., C.L.H., M.A.K., A.K.M. and D.G.-G. analysed the data. A.C. and U.G. grew the heterostructure that hosts the 2DEG on which these samples are built. D.G.-G. supervised the project.

Competing interests
The authors declare no competing interests.

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Extended Data Fig. 1 | Schematic illustrations of the models discussed. a, The DCK model consists of six effectively independent spinless conduction electron reservoirs (blue for island, red for lead), described by fermionic operators $f_{\alpha}^{}$ and $c_{\gamma}^{}$ for $\alpha = L, R$ and $\gamma = I, i$. Tunneling occurs at each of the three QPCs controlled by $J_{L,C,R}$. The island charging energy $E_{\alpha}^{}$ correlates electrons $f_{\alpha}^{}$ and $c_{\gamma}^{}$ on the same island either side of the metal component (black bar). b, At a TP, Eq. (9) describes the system at low temperatures. The three retained charge states of the two-island structure (denoted $|A\rangle$, $|B\rangle$, $|C\rangle$) are interconverted by QPC tunneling. The frustrated QCP arises when $J_{L}^{} = J_{R}^{} = J_{C}^{}$. The conductive pathway $|A\rangle \rightarrow |B\rangle \rightarrow |C\rangle$ corresponds to transport from left lead to right lead, and is illustrated with the blue arrows (the flow is reversed $|C\rangle \rightarrow |B\rangle \rightarrow |A\rangle$ by changing the sign of the applied bias voltage).
Extended Data Fig. 2 | SEM micrograph of nominally identical device. The acceleration voltage in the SEM was 5 kV.
Extended Data Fig. 3 | Dynamical Coulomb blockade of QPC transmissions.

a. Measured QPC transmissions $\tau_R, \tau_L$ as a function of a source-drain bias $V_{SD}$ for different QPC gate voltages. The measured transmission is extracted by measuring the series conductance when in series with the inter-island QPC and opposite island-lead QPC set to fully transmit a single channel ($\tau_{C,L/R} = 1$). The measured transmissions clearly dip at zero bias, consistent with dynamical Coulomb blockade (DCB) behavior. The high bias behavior ($V_{SD} \approx 50\mu V$) recovers the 'intrinsic' transmission of the QPC, unrenormalized by DCB.

b. DCB measurements comparing the right island-lead QPC to the inter-island QPC. It is clear there is a substantial difference in the DCB-renormalization at zero bias between the two, likely due to the device geometry.

c. Comparison of measuring $\tau_R$ through both islands (blue lines, as in a, b) and through the adjacent plunger gate $P_R$ (red lines). While typically we would expect no significant bias dependence when measuring through $P_R$, we in fact see DCB-like behavior.

d. Comparing the two measurement pathways of c at fixed source-drain bias as a function of the QPC gate voltage. The 'through the island' (blue) measurements have been shifted by 9mV to account for the large capacitive cross-talk effect when switching between the two different measurement pathways. That the high bias traces match well is indicative that there is indeed DCB-renormalization of the transmission when measuring through $P_R$. Empirically, using the zero bias, 'through the plunger gate' measurement of the transmission (solid red line), best captures the relevant transmissions in the Kondo interactions of our system.
Extended Data Fig. 4 | Semi-universal \( \tau_C \) values. \( \mathbf{a} \), Measured inter-island transmission as a function of an applied gate voltage. The markers correspond to the inter-island transmissions used in Fig. 3a of the main text. \( \mathbf{b} \), Original line cuts in which the truncated data used in Fig. 3a are extracted from.