Are spacetime horizons higher dimensional sources of energy fields? (The black hole case).

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Abstract

We explore the possibility that spacetime horizons in 4D general relativity can be treated as manifestations of higher dimensions that induce fields on our 4D spacetime. In this paper we discuss the black hole event horizon, as an example (we leave the cosmological case for future discussion). Starting off from the field equations of gravity in 5D and some conditions on the metric we construct a spacetime whose imbedding is a 4D generalization of the Schwarzchild metric. The external region of the imbedded spacetime is found to contain two distinct fields. We discuss the properties of the fields and the potential implications. Taken as they are, the results suggest that the collapse of matter to form a horizon may have non-local consequences on the geometry of spacetime. In general, the use of horizon-confined mass as a coordinate suggests three potential features of our universe. The first is that the observed 4D spacetime curvature and ordinary matter fields may be hybrid features of 5D originating from the mixing of coordinates. Secondly, because the fifth coordinate induces physical fields on the 4D hyperface, the global metric of the universe may not be asymptotically flat. And finally, associating matter with an independent dimension points towards a theory of nature that is scale invariant.

1 Introduction

In an effort to build a unified theory of nature to explain the observed universe, physics has had to look up to higher dimensions. A consequence of this effort
has been the development, in recent years, of higher dimensional models, notably the Superstring models\textsuperscript{1} and M-Theory\textsuperscript{2}. In these models dimensions are usually assumed to be curled up into length scales of order of Planck size, hence rendering them difficult to observe at any energies lower than the Planck energy. This modern view of higher dimensional compactification has its historical roots in Klein’s original interpretation\textsuperscript{3} of (the stronger) Kaluza’s *cylinder condition*\textsuperscript{4} (which we discuss later). Lately, alternatives to compactification such as the Randall-Sundrum mechanism\textsuperscript{5} have been proposed.

Dimensional Compactification is certainly consistent with why we don’t sense the higher dimensions. Whether such a feature is also necessary or whether nature may have other options is still academic. It is however reasonable and may be prudent, at this time, to explore further alternative possibilities. In this respect, one is reminded of the citizens of the legendary 2D world in *Flatland*\textsuperscript{6} and their limitations in perception. These citizens’ understanding of concepts like angular momentum, or the force on an electric charge moving in a magnetic field, requires them to develop a higher dimensional theory involving cross-products. Clearly, the ‘other’ dimensions that form part of the manifold in which Flatland is imbedded do not have to be compact. Flatlanders are just two dimensional beings living in their two dimensional world with their observations in the third dimension restricted by what they have learned to call a horizon. As a result, Flatlanders would interpret effects of their interactions with the higher dimensional world as originating from the horizon.

In this discussion we seek for possible manifestations and implications of higher dimensions based on the assumption that such dimensions are not necessarily compact. In our approach we represent matter as the fifth coordinate, provided such matter is confined in a spacetime horizon. The fifth coordinate is then given by the length $x^4 = h$ associated with the horizon size. The criterion employed in this work to treat a quantity as coordinate representing a dimension, is the quantity’s independence from the other spacetime dimensions. In so far as a spacetime horizon (local or global) signifies a boundary to our observable 4D universe, then the length $h$ associated with the horizon size meets the above criterion and its treatment as a manifestation of a dimension can be justified on this basis.

The possibility of representing matter as a coordinate has been suggested before by Lessner\textsuperscript{7} and by Wesson\textsuperscript{8}. It is however difficult to justify the concept of a fifth dimension without the above criterion, namely the independence of the associated coordinate. From the onset it is clear, for example, that in our approach ordinary matter cannot be justifiably treated as an independent coordinate. This is because the worldline of any such matter (including particles) is associated with a 4D spacetime measure and is therefore not independent of the spacetime coordinates. Such an identification distinguishes our treatment, its results and implications, from the previous treatments. As we point out in the next section, ordinary matter (i.e. not trapped inside an event horizon) is seen to be a hybrid, resulting from the mixing of spacetime coordinates with the fifth coordinate.
We shall demonstrate that given a 5D spacetime geometry in which the fifth coordinate \( x^4 \) is treated as a length \( h \) corresponding to the horizon size, then there is an imbedded 4D spacetime on which fields are induced. In particular, taking the horizon size as that of a black hole mass \( M \) so that \( h = \frac{2GM}{c^2} \), we find that, in the case of the induced 4D geometry, the external space outside the black hole is filled with two distinct fluids. We discuss the properties of these fields and their implications.

2 The 5D field equations

2.1 The fifth dimension

We begin by introducing the physical basis for the fifth coordinate. One can build an intuitive sense of the fifth dimension by drawing an analogy with the time dimension\(^9\). In the Minkowiski spacetime, the time dimension is associated with a length coordinate written as \( x^0 = ict \), where \( c \) is a universal constant (the upper bound speed for propagation of physical information). Because of the large value of this constant, we do not usually sense time as a dimension except at high velocities (comparable to \( c \)) where such relativistic phenomena as length contraction and time dilation manifest themselves.

Analogously, in our present approach, one can associate the fifth dimension with a coordinate

\[
x^4 = 2\sqrt{\varepsilon \kappa M} = \sqrt{\varepsilon h},
\]

constructed from a mass \( M \) confined in an event horizon of size \( h \). Here, the parameter \( \varepsilon = \pm 1 \) identifies the coordinate as space-like or time-like and \( \kappa = \frac{G}{c^2} \approx \frac{4\pi}{m_p} \) is a universal constant with units of length per unit mass and gives the length scale physically associated with maximum untrapped mass. In MKS units \( \kappa = 7.42 \times 10^{-28} m/kg \). Because of the small value of \( \kappa \), the observable effects of the fifth dimension (analogous to time dilation/length contraction) should be favored by high density fields. One notes, however, that \( \kappa \) is independent of velocity. Thus, even at ordinary densities (e.g. earth density) effects of higher dimensions should be readily observable in our universe, as deviations from Minkowskian spacetime, provided a large enough interval associated with the fields is considered. With regard to the specific nature of the (trapped-mass)-spacetime coordinate mixing effects, one expects a bending of spacetime analogous to the special relativistic length contraction and a ‘dilution’ effect of the mass coordinate analogous to the time dilation. Such effects would give rise to spacetime curvature and introduce matter fields in 4D. The suggestive implication, then, is that ordinary 4D spacetime curvature and ordinary matter/radiation are manifestations of higher dimensions.

That curvature is imposed on the 4D spacetime is a known feature of general relativity. The only addition our treatment suggests is that such imposition results from the mixing of the fifth coordinate with the spacetime ones. On the other hand, the observation that ordinary matter fields in 4D also originate
from such coordinate mixing is a broad concept to be a subject of this initial
and future discussions. We will also comment on the cosmological and other
implications of these effects later in the conclusion of this discussion.

2.2 The equations

To proceed, we first give a summary of the equations which set the frame-
work for the remaining discussion. These equations can be derived from a five
dimensional action of the form

\[ S = \frac{1}{16\pi G} \int \sqrt{-g} (R + \ell) d^5x \]  

(2)

by varying the former with respect to the 5D metric \( g_{ab} \). Here \( R \) and \( \bar{G} \) are the
5D Ricci scalar and Newton’s constant, respectively and \( g \) is the determinant
of the full 5D metric whose line element is

\[ dS^2 = g_{ab} dx^a dx^b, \quad (a, b = 0, ..., 4). \]  

(3)

Further, \( \ell \) is a Lagrangian density which is invariant with respect to arbitrary
transformations of all the five coordinates \( x^a \) (including the mass coordinate
\( h = 2\pi M \)) and represents the fields that may result (as pointed out above)
from the mixing of these coordinates. In our notation, lower case lettering is
used for spacetime indices (Roman for 5D and Greek in 4D) and from now on
we stick to the geometrized units \( 8\pi G = c = 1 \), unless otherwise stated.

As already mentioned above, our discussion is based on the assumption that
spacetime horizons can be treated as manifestations of higher dimensions. With
this assumption we investigate the possibility that such horizons are generators
of matter fields on the 4D spacetime. One does expect that the field equations
derived from Eq. (2) should give a 5D geometry \( g_{ab} \) whose foliation yields a
family of 4-hyperfaces with physically meaningful interpretation. In particular,
we suppose in this particular discussion that the metric induced on the 4D
hyperface describes the spacetime outside a static

source which is represented
by the horizon size. This puts constraints on the properties of such a metric
induced on the 4D manifold, and hence on the foliation character of the 5D
space. Thus we expect that the resulting 4D spacetime should:

- (i) have spherical symmetry;
- (ii) reduce to the Schwarzschild solution\(^{10}\) on a constant \( h \) surface;
- (iii) be asymptotically flat (as \( h \to 0 \)).

In passing, one notes with regard to condition (iii), that since in a cosmo-
logical sense \( h \) never realistically goes to zero, and since its expected effect is to
induce fields on 4D spacetime, then the geometry of the universe could never be
globally flat. This suggests, independently, that the 4D universe should, neces-
sarily, be bathed in some vacuum, \( \Lambda \), with an associated non-vanishing energy
density $\rho_A$. This issue will be revisited in a future discussion of the cosmological case.

Condition (i) demands that the geometry of the full 5D manifold should admit a metric of the form

$$dS^2 = e^\nu dt^2 - e^\mu dr^2 - R^2 d\Omega^2 + \varepsilon e^\psi dh^2;$$

(4)

where, condition (ii) implies that the metric coefficients can, at most, be functions of $r$ and $h$ only.

Before proceeding, it is worthwhile to comment on this dependence of the induced 4-metric $g_{\mu\nu}$ on the fifth coordinate, $h$. In his original work, Kaluza\(^4\) assumed that geometric and physical objects in 4D are independent of the fifth coordinate. This assumption is expressed in the cylinder condition which holds that all derivatives of the 4D metric with respect to the fifth coordinate, must vanish, i.e. $\partial_{(\alpha=4)} [g_{\mu\nu}] = 0$. Later, Klein\(^3\) introduced a weaker condition by letting the full 4-metric $g_{\mu\nu} (x^\mu, x^4) = g^0_{\mu\nu} (x^\mu) X (x^4)$, be separable in the variables $x^\mu$ and a compactified $x^4$. This view has been introduced in the modern string theories. It holds that the higher dimensions are curled up into a length scale of order of Planck size, hence rendering them difficult to observe at any energies lower than the Planck energy. Throughout our treatment the cylinder condition is relaxed. As we shall find it is the relaxation of this condition which, in our approach, facilitates the introduction of matter fields into the 4D spacetime from higher dimensions.

With the form of the above line element one finds that the only surviving components of the 5D Ricci tensor, $R_{ab}$, are:

\begin{align*}
R_{00} &= e^{\nu-\mu} \left[ \frac{\nu''}{4} + \frac{\nu'}{4} \frac{\mu'}{4} + \frac{\nu' \psi'}{4} + \frac{\nu R'}{R} \right] + \varepsilon e^{\nu-\psi} \left[ -\frac{\nu}{4} - \frac{\nu^2}{4} + \frac{\mu R'}{R} - \frac{\nu R'}{R} \right]; \\
R_{11} &= -\frac{\nu''}{2} - \frac{\psi''}{2} - \frac{\nu^2}{4} + \frac{\psi^2}{4} + \frac{\nu' \psi'}{4} + \frac{\nu' R'}{R} + \frac{\mu R'}{R} - \varepsilon e^{\nu-\psi} \left[ -\frac{\mu}{4} - \frac{\mu^2}{4} + \frac{\nu R'}{R} + \frac{\nu R'}{R} \right]; \\
R_{22} &= 1 - R^2 e^{-\mu} \left[ \left( \frac{R'}{R} \right)^2 + \frac{R''}{R} + \frac{R'}{2R} (\nu' - \mu') \right] + \varepsilon R^2 e^{-\mu} \left[ \left( \frac{R'}{R} \right)^2 + \frac{R''}{R} + \frac{R'}{2R} (\nu - \mu + \psi) \right]; \\
R_{33} &= \sin^2 \theta (R_{22}); \\
R_{44} &= -\frac{\nu''}{2} - \frac{\psi''}{2} - \frac{\nu}{4} - \frac{\mu}{4} + \frac{\nu^2}{4} + \frac{\psi^2}{4} - \frac{\nu \psi}{4} - \frac{\nu R'}{2R} - \frac{\mu R'}{2R} + \varepsilon e^{\nu-\mu} \left[ \frac{\nu''}{2} + \frac{\psi^2}{4} + \frac{\nu' \psi'}{4} + \frac{\mu' \psi'}{4} + \frac{\mu R'}{R} \right]; \\
R_{14} &= -\frac{\nu''}{2} + \frac{\nu}{4} + \frac{\psi''}{4} + \frac{\mu'}{4} + \frac{\nu' \psi'}{4} + \frac{\mu R'}{R} + \frac{\psi R'}{R} - \frac{2 R'}{R}. 
\end{align*}

(5)

Here the over head diamond '⋄' means differentiation with respect to the fifth coordinate while the prime '′', as usual, denotes differentiation with respect to the radial coordinate.
Now, conditions (i) and (ii) suggest that, as an ansatz, we take a 5D line element of the form

\[ dS^2 = \left( 1 - \frac{h}{r} \right) dt^2 - \left( 1 - \frac{h}{r} \right)^{-1} dr^2 - r^2 d\Omega^2 - \varepsilon \phi dh^2, \]  

where \( \phi = \phi(r,h) \) is a lapse function and \( \varepsilon = \pm 1 \), as mentioned before, identifies the \( h \) coordinate as either spacelike or timelike. In this particular work we assume a coordinate system in which \( \phi \) is scaled to unity.

Using Eqs. (5) and (6) we find the only surviving components of the 5D Ricci tensor \( R_{ab} \) are

\[
\begin{align*}
R_{00} &= -\frac{1}{2\varepsilon r^2 \left( 1 - \frac{h}{r} \right)}; \\
R_{11} &= -\frac{1}{2\varepsilon r^2 \left( 1 - \frac{h}{r} \right)^3}; \\
R_{14} &= -\frac{1}{2r^2 \left( 1 - \frac{h}{r} \right)}; \\
R_{44} &= \frac{1}{2r^2 \left( 1 - \frac{h}{r} \right)^2}.
\end{align*}
\]  

Further, the 5D Ricci scalar is given as

\[ R = R^a_a = \frac{1}{2r^2 \varepsilon \left( 1 - \frac{h}{r} \right)^2} \]  

Using Eqs. (7) and (8) above to build the 5D Einstein tensor \( G_{ab} = R_{ab} - \frac{1}{2} R g_{ab} \) we find its surviving components to be

\[
\begin{align*}
G_{00} &= -\frac{3}{4\varepsilon r^2 \left( 1 - \frac{h}{r} \right)}; \\
G_{11} &= -\frac{1}{4\varepsilon r^2 \left( 1 - \frac{h}{r} \right)^3}; \\
G_{14} &= -\frac{1}{2r^2 \left( 1 - \frac{h}{r} \right)}; \\
G_{22} &= \frac{r^2}{4\varepsilon r^2 \left( 1 - \frac{h}{r} \right)^2}; \\
G_{33} &= \sin^2 \theta G_{22}; \\
G_{44} &= \frac{1}{4r^2 \left( 1 - \frac{h}{r} \right)}.
\end{align*}
\]
3 The 4+1 splitting and the induced fields

In order to isolate physical information from the above full 5D results it is worthwhile comparing such results with those from a 4 + 1 splitting of the Kaluza-Klein theory. This comparison will manifest features in the preceding results which suggest the existence of two distinct fields induced on the 4D hyperface. To this end we start with an overview of a Kaluza-Klein theory with the cylinder condition relaxed, i.e. $\partial_{(a=4)} [g_{\mu\nu}] \neq 0$. In such an approach one can, in general, institute a 4 + 1 split of the 5D metric $dS^2 = g_{ab}dx^a dx^b$. This foliation leaves an induced 4D metric, $g_{\mu\nu} (x^a)$, $(\mu, \nu = 0, .., 3)$ which can be related to the 5D metric by

$$g_{ab} = \begin{pmatrix} g_{\mu\nu} & N_{\mu} \\ N_{\nu} & \varepsilon \phi^2 + N_\alpha N^\alpha \end{pmatrix},$$

where $N^\mu (x^a)$ is a shift vector (historically associated with the electromagnetic fields).

The 5D theory can then be related to its 4D sector through a system of 15 equations which can be divided into three groups. In this treatment we only discuss gravitational fields and as a result we are only interested in the one system of 10 equations for the 4D Ricci tensor,

$$R_{\mu\nu} = \frac{1}{\phi} [D_\mu D_\nu \phi + \varepsilon (L_N K_{\mu\nu} - \partial_h K_{\mu\nu}) + \varepsilon \phi (KK_{\mu\nu} - 2K_{\mu\lambda}K^\lambda_{\nu})],$$

where $K_{\mu\nu} = \frac{1}{2\phi} [D_\mu N_\nu + D_\nu N_\mu - \partial_h g_{\mu\nu}]$ is the extrinsic curvature tensor induced on the 4D spacetime, $L_N K_{\mu\nu}$ is its Lie derivative and $D_\mu$ is a covariant derivative operator. For the same reason we gauge the $N_\alpha$ out and choose a coordinate system with $\phi = 1$. With this the only surviving contribution to the extrinsic curvature term,

$$K_{\mu\nu} = -\frac{1}{2} \partial_h g_{\mu\nu},$$

results from the relaxation of the cylinder condition.

One can now apply this approach to our ansatz (Eq. (6)). Here, the 4 + 1 foliation of the full 5D spacetime gives a family of 4D hyperfaces with an induced metric $g_{\mu\nu}$ whose line element is

$$ds^2 = \left( 1 - \frac{h}{r} \right) dt^2 - \left( 1 - \frac{h}{r} \right)^{-1} dr^2 - r^2 d\Omega^2,$$

and which we also take to be the physical metric. Then from Eqs. (12) and (13) the explicit extrinsic curvature of the induced 4D spacetime is

$$K_{\mu\nu} = -\frac{\varepsilon}{2} \partial_h [g_{\mu\nu} (x^a, h)] = -\frac{\varepsilon}{2r} \text{diag.} \begin{bmatrix} 1, & \frac{1}{(1-h)}, & 0, & 0 \end{bmatrix},$$

\[1\] In general, the induced metric $\tilde{g}_{\mu\nu}$ and the physical metric $g_{\mu\nu}$ can be related by $\tilde{g}_{\mu\nu} (r, h) = \Omega (r, h) g_{\mu\nu} (r, h)$, where $\Omega (r, h)$ is a warp factor.
and, clearly, the associated curvature scala vanishes,

\[ K = g^{\mu\nu}K_{\mu\nu} = 0. \] (15)

Using Eqs. (14) and (15) in (11) one finds the components of the 4D Ricci tensor to be

\[ R_{\mu\nu} = \frac{-\varepsilon}{2r^2 (1 - \frac{h}{r})^2} \text{diag.} \left[ (1 - \frac{h}{r}), \frac{1}{(1 - \frac{h}{r})}, 0, 0 \right]. \] (16)

The resulting Ricci scala is, manifestly traceless,

\[ R = -[g^{\mu\nu}(\partial_h K_{\mu\nu}) + 2g^{\alpha\nu}K^\mu_\alpha K_{\mu\nu}] = 0. \] (17)

Consequently, Eqs. (16) and (17) recover the information originally contained in the 4D sector of (5D Ricci tensor, Eq. (7)). Now, using Eqs. (16), (17) and (13) we can construct a 4D Einstein tensor \( G_{\mu\nu} \), and through the Bianchi identities, the associated 4D field equations \( G_{\mu\nu} = -\tau_{\mu\nu} \) give the matter fields as

\[ \tau_{\mu\nu} = \frac{1}{2\varepsilon r^2 (1 - \frac{h}{r})^2} \text{diag} \left[ (1 - \frac{h}{r}), \frac{1}{(1 - \frac{h}{r})}, 0, 0 \right]. \] (18)

It is apparent that the result in Eq. (18) is different from that expressed by the 4D sector of the 5D Einstein tensor in Eq. (9) and which we write as \( \tilde{G}_{\mu\nu} \). The difference arises because the \( R_{44} \) component in the 5D theory contributes to the non-vanishing of the 5D trace in Eq. (8). This then projects onto the 4D sector a cosmological substrate fluid \( \theta_{\mu\nu} \). The result is that the total matter fields \( T_{\mu\nu} \) in the 4D sector of \( G_{ab} \) (Eq. (9)) can now be represented as a sum of two fluids,

\[ T_{\mu\nu} = \tau_{\mu\nu} + \theta_{\mu\nu}, \] (19)

with \( \tau_{\mu\nu} \) given by Eq. (18) and with \( \theta_{\mu\nu} = g_{\mu\nu}, \) where \( \theta \) is obtainable from 5D Ricci scalar Eq. (8) as

\[ \theta = \frac{1}{2} g^{44} R_{44} = \frac{1}{4r^2 \varepsilon (1 - \frac{h}{r})^2}. \] (20)

### 4 Properties of the induced fields

We now take a brief look at some features of the fields \( \theta_{\mu\nu} \) and \( \tau_{\mu\nu} \) obtained in the preceding section and comment on the potential implications. On the 4D hyperface both these fields would fill the region of the black hole spacetime usually referred to as the external Schwarzschild solution.

As we have noted above the \( \theta_{\mu\nu} \) field originates from the non-vanishing contribution of \( R^2_4 \) to the 5D trace. It is a substrate field with a cosmological
character. The field represents a perfect fluid as can be inferred by writing $\theta_{\mu\nu}$ in the perfect fluid form $\theta_{\mu\nu} = (\rho + p) u^\mu u^\nu - p g_{\mu\nu}$, with $u^0 = (-g_{00})^{-\frac{1}{2}} = (1 - \frac{h}{r})^{-\frac{1}{2}}$ and $u^{i=1,2,3} = 0$. Then one easily verifies that the field behaves like a cosmological fluid with a negative pressure and an equation of state

$$p_\theta = -\rho_\theta.$$  \hfill (21)

On the other hand, the $\tau_{\mu\nu}$ field comes from the induced 4D Ricci tensor. This field is traceless and represents an anisotropic fluid with the only surviving pressure term being in the radial component. Further, there is no energy transport, as indicated in Eq. (18) by the absence of off-diagonal (momentum) terms. All these features can be made apparent by writing the field in a quasi-perfect fluid form

$$\tau_{\mu\nu} = \frac{1}{\varepsilon r^2} \left( \frac{1}{1 - \frac{h}{r}} \right)^2 u^\mu u^\nu - \frac{1}{2\varepsilon r^2} \left( \frac{1}{1 - \frac{h}{r}} \right)^2 \left[ \delta^\mu_0 \delta^\nu_0 + \delta^\mu_1 \delta^\nu_1 \right],$$ \hfill (22)

with $u^0 = (-g_{00})^{-\frac{1}{2}} = (1 - \frac{h}{r})^{-\frac{1}{2}}$ and $u^{i=1,2,3} = 0$. One can infer from Eq. (22) that the radial pressure satisfies the equation of state,

$$p_\tau = \rho_\tau,$$ \hfill (23)

where the fluid density $\rho_\tau$ and the pressure $p_\tau$ are functions of the radial coordinate and are, respectively, given by

$$\rho_\tau = \frac{1}{2\varepsilon (r - h)^2},$$ \hfill (24)

and

$$p_\tau = \frac{1}{2\varepsilon (r - h)^2}.$$ \hfill (25)

Both the energy density (Eq. 24) and pressure (Eq. 25) are positive definite with the result that the fluid is a ‘normal’ field, obeying all the standard energy conditions. The pressure $p_\tau$ supports the fluid, holding it in hydrostatic equilibrium by providing an outward force $F(r)$, at any point in the region outside the horizon. This force is given by

$$F(r) = -\frac{1}{\rho_\tau} \frac{\partial p_\tau}{\partial r} = \frac{2}{r - h},$$ \hfill (26)

and is, clearly, due to a non-vanishing pressure gradient $\frac{\partial p_\tau}{\partial r}$ in the radial direction. The force, $F(r)$, whose origins are in the higher dimension, is nevertheless gravitational in nature as can be verified by setting the gravitational coupling constant $G$ to zero.
5 Conclusion

In this paper we have explored the possibility that spacetime horizons may be manifestations of higher dimensions and that such objects may be generators of energy fields in our 4D spacetime. Treating the length associated with the horizon size as an independent fifth coordinate in an uncompactified 5D theory we have constructed a set of solutions, one in 5D gravity and its 4D counterpart in Einstein gravity sector for a black hole with a horizon (length) size \( h = \frac{2GM}{c^2} \).

The 4D solution shows the space outside a black hole to be filled with two distinct fluids. First, we find a substrate field projected on the 4D from the fifth dimension by the non-vanishing 5D trace element, \( R_4^4 \). The field has a cosmological character, behaving as a perfect fluid with an equation of state, \( p = -\rho \), similar to that of a cosmological constant. It differs from the cosmological constant only in its dependence on the radial coordinate. We interpret the influence of the negative pressure to imply that for the hole, the external 4D spacetime appears to expand away. Conversely, for an observer in the external 4D spacetime, the black hole would appear to contract away with the radial coordinate.

The second field, on the other hand, is matter-like in that it originates from the non-vanishing components of the 4D sector of the Ricci tensor. Because the two angular components of this tensor vanish, the resulting fluid is anisotropic, having only a radial pressure component. Both the radial pressure and the density are functions of the radial coordinate and obey an equation of state of the form \( p_r (r) = \rho_r (r) \). The pressure support holds the fluid in hydrostatic equilibrium with a radial force \( F (r) = -\frac{1}{\rho_r} \frac{d\rho_r}{dr} \) at each point. This force does not, however, constitute a “fifth force” as can be seen from its coupling constant. The force is gravitational and vanishes on setting Newton’s gravitational constant \( G \) to zero.

One notes that our 4D solution is a generalization of the Schwarzschild solution, but only for a black hole. It cannot, for example, describe the outside of a regular star, whether radiating or not. This is because the mass of a regular star occupies a physical 4-volume and therefore cannot be treated as an independent coordinate. Taken as they are, these results would suggest possible non-local consequences on the geometry of spacetime resulting from the collapse of matter to form a black hole horizon (and the implied eventual future-directed singularity). The physical implications, then, would be that the global geometry of the spacetime outside a black hole differs from that of an uncollapsed star of the same mass.

In closing we mention some new features and implications that the general treatment of mass as a fifth coordinate makes manifest. One of the improvements that Einstein made to the theory of gravitation (see for example MTW) was the realization that Newton’s concept of gravity as a force could be replaced by spacetime curvature. This approach did, among other things, solve the problem of action at a distance in Newton’s theory. In our present approach, the view of gravity as inducing curvature on the 4D manifold does not change. The
only inference that our approach makes is that spacetime curvature and ordinary matter/radiation fields in the 4D spacetime are manifestations of a higher dimension, resulting from the mixing of the fifth coordinate with the 4D spacetime coordinates. There are two other interesting implications of this approach. The first relates to the expected effects of the fifth dimension on the geometry of the universe. Consistent with the foregoing arguments one expects that the cosmological effect of the fifth dimension will be a cosmological field with a non-vanishing energy density induced on the 4D sector of the 5D manifold. Consequently, the global metric defined on the 4D universe may not be asymptotically flat. This point, which will be taken up again in a future discussion, does seem to naturally justify the cosmological constant and possibly predict inflation. Lastly, taking mass as a dynamical coordinate is a stepping stone (as our solutions in Eqs. (6) and (13) indicate) towards a scale invariant theory of nature.

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