A new window on Strange Quark Matter as the ground state of strongly interacting matter

Vikram Soni
National Physical Laboratory, K.S. Krishnan Marg, New Delhi 110012, India

Dipankar Bhattacharya
Raman Research Institute, Bangalore 560080, India

If strange quark matter is the true ground state of matter, it must have lower energy than nuclear matter. Simultaneously, two-flavour quark matter must have higher energy than nuclear matter, for otherwise the latter would convert to the former. We show, using an effective chiral lagrangian, that the existence of a new lower energy ground state for two-flavour quark matter, the pion condensate, shrinks the window for strange quark matter to be the ground state of matter and sets new limits on the current strange quark mass.

I. INTRODUCTION

The hypothesis that the true ground state of baryonic matter may have a roughly equal fraction of u,d and s quarks, termed strange quark matter (SQM) is of recent origin [1]. This is based on the fact that at some density, when the down quark chemical potential is larger than the strange quark mass, conversion to strange quarks can occur. This reduces the energy density by having 3 (u, d and s) fermi seas instead of just 2 (u, d), and can yield a state of energy lower than nuclear matter. It is also possible to explain why such a state has escaped detection.

This involves at least two puzzles.

i) Why does ordinary 2 flavour nuclear matter, the observed ground state of baryonic matter, not decay into strange quark matter.

The answer is that this decay is not like the radioactive decay of unstable nuclei, as the nucleons cannot decay one by one as it is not energetically favourable for the nucleon to change into a Λ, but only for the entire nuclear matter to transmute into strange quark matter and this requires a high order of the flavour changing weak interaction which renders the cross section to be exponentially and unobservably small.

ii) Why was this matter not created in the evolution of the universe?

This is due to the fact that as the universe cooled past a temperature equivalent to the strange quark mass, strange quark matter was not the chosen state of high entropy. Since the u and d quarks have almost negligible masses at this scale, as the temperature dropped further the strange quarks were Boltzmann suppressed leaving just the u and d quarks, which as we know converted largely into nucleons. For details we refer the reader to [1, 2, 3].

It is really quite remarkable that the ground state cannot be realised easily! Only if we can produce high baryon density by compression can SQM be realised - for example, in the interior of neutron stars.

We now turn to the theoretical underpinning of the case for SQM being the potential ground state of matter.

We already know, empirically as well as theoretically, the ground state energy of saturation nuclear matter - 930 MeV for the Fe$^{56}$ nuclei. However, for calculating quark matter we take recourse to phenomenological models, which are pointers but foundationally inadequate and here lies the uncertainty.

The usual ground state calculation for SQM treats the quarks as a free fermi gas of current quarks. The volume in which these quarks live comes at a cost of a constant energy density that provides ‘confinement.’ It is equivalently the same constant value of negative pressure and hence is often called the bag pressure term. This is a simple extension of the MIT bag philosophy, where the origin of the constant energy density is the fact that quarks are confined. The bag pressure sets the equilibrium or ground state energy density and the baryon density. It can be fixed from the nucleon sector. Further structure can be introduced by adding interaction between the quarks, e.g., one gluon exchange. Such a phenomenological model has been used by Witten [1] and later by Farhi and Jaffe [2] and others for SQM (see [3] for a review).

II. CHIRAL SYMMETRY

It is clear that such a model is phenomenological and does not, for example, address the issue of the spontaneous breaking of chiral symmetry – an essential feature of the strong interactions.

The quark matter in the bag is in a chirally restored state. This means that as in the case of Superconductivity it costs energy to expel the chiral condensate which characterises the true vacuum state. Clearly, this will act
just like the bag energy density/pressure. However, its value will be determined by the energy density of the chiral condensate. Such a term binds but does not confine. Confinement thus requires further input than just a bag pressure.

All results for the SQM state will depend on the model that is used to describe it and the ground state thereof. In a chiral model we find that there is a plurality of ground states. Of these, we find that one particular ground state has the property of chiral restoration at high density and parallels the MIT bag state used in most previous estimates, where the ground state is a fermi sea of current quarks (chirally restored quark matter or CRQM) with the bag pressure provided by absence of the chiral condensate. This regime sets the connection between the parameters of the chiral model and the MIT bag model used in [1,2,3].

Unlike for the MIT bag case where the bag pressure is a parameter, in our formulation, it is the chiral condensate energy and is given in terms of the parameters of low energy phenomenology - the pion decay constant, \( f_\pi \), which is precisely known and the scalar coupling or the \( \sigma \) mass, which is rather poorly ‘known’.

There are, however, other ground states for this model, in which the pattern of symmetry breaking is different at high density, for example, the pion condensed (PC) ground state in which the chiral symmetry is still spontaneously broken at high density. Such a state has lower ground state energy than the former and thus needs to be considered in the description of quark matter. As we show it is found to influence the regime of existence of SQM importantly.

A. Effective chiral lagrangian

We consider this issue in the framework of an intermediate Chiral Lagrangian that has chiral SSB. Such an effective Lagrangian has quarks, gluons and a chiral multiplet of \( [\bar{\pi}, \sigma] \) that flavor couples only to the quarks. For \( SU(2)_L \times SU(2)_R \) chiral symmetry, we have

\[
L = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} - \sum \bar{\psi} \left( D + g_\nu (\sigma + i\gamma_5 \bar{\tau} \bar{\pi}) \right) \psi - \frac{1}{2} (\partial \mu \sigma)^2 - \frac{1}{2} (\partial \mu \bar{\pi})^2 - \frac{\lambda^2}{4} (\sigma^2 + \bar{\pi}^2 - (f_\pi)^2)^2
\]  

(1)

The masses of the scalar (PS) and fermions follow on the minimization of the potentials above. This minimization yields

\[
<\sigma>^2 = f_\pi^2
\]  

(2)

where \( f_\pi \) is the pion decay constant. It follows that

\[
m_\sigma^2 = 2\lambda^2 (f_\pi)^2 \quad m_q = m = g <\sigma> = gf_\pi
\]  

(3)

This theory is an extension of QCD by additionally coupling the quarks to a chiral multiplet, \( [\bar{\pi}, \sigma] \) [4,5,6]. This Lagrangian has produced some interesting physics at the mean field level [4,6]:

1. It provides a quark soliton model for the nucleon in which the nucleon is realized as a soliton with quarks being bound in a skyrmion configuration for the chiral field expectation values [5,6].

2. Such a model gives a natural explanation for the ‘proton spin puzzle’. This is because the quarks in the background fields are in a spin, isospin singlet state in which the quark spin operator averages to zero. On the collective quantization of this soliton to give states of good spin and isospin the quark spin operator acquires a small non zero contribution [8].

3. Such a Lagrangian also seems to naturally produce the Gottfried sum rule [8].

4. Such a nucleon can also yield from first principles (but with some drastic QCD evolution), structure functions for the nucleon which are close to the experimental ones [10].

5. In a finite temperature field theory such an effective Lagrangian also yields screening masses that match with those of a finite temperature QCD simulation with dynamical quarks [11].

6. This Lagrangian also gives a consistent equation of state for strongly interacting matter at all density [4,12].

We shall first briefly establish the parameters of the above effective Lagrangian and the specific connection with the MIT bag model of confinement used in previous treatments of SQM.

As already pointed out above, the nucleon in this model is realised as a soliton in a chiral symmetry broken background with quark bound states [4,6,7]. This sets the value of the yukawa coupling, \( g \), required to fit the nucleon mass in a Mean Field Theory (MFT) treatment to be, \( g = 5.4 \).

For the nucleon the dependence on the scalar coupling, \( \lambda \), is marginal as long as it is not too small. Further, in MFT, the QCD coupling does not play a role; only if 1 gluon exchange is included does the QCD coupling enter.

There are no other parameters except \( f_\pi \), the pion decay constant which is set to 93 MeV.
III. PRELIMINARIES FOR SQM

The connection to MIT bag description of quark matter is set as follows. The last term in the above lagrangian, the potential functional,

\[ \frac{\lambda^2}{4} (\sigma^2 + \vec{\pi}^2 - (f_\pi)^2)^2 \]

is minimized by the VEV's

\( <\sigma> = f_\pi, <\vec{\pi}> = 0 \)

and is equal to zero at the minimum.

In MFT at high density (as we shall see), when chiral symmetry is restored, \( <\sigma> = 0, <\vec{\pi}> = 0 \), this term reduces to a constant energy density term equal to

\[ \frac{\lambda^2}{4} (f_\pi)^4 \]

Besides, due to chiral symmetry restoration the constituent mass of the quarks also vanishes, leaving free massless quarks. This reduced lagrangian for high density is no different from MIT bag quark matter with

\[ B = \frac{\lambda^2}{4} (f_\pi)^4 \]

This completes the identification of the bag pressure term in this model. It shows that bag pressure is automatically generated by chiral restoration and is controlled simply by the scalar coupling or equivalently the sigma mass.

We first briefly describe the logical basis for the investigation of the QM ground states vis a vis the usual nuclear matter ground state at saturation density. Here we follow Farhi and Jaffe \[2\].

1. We fix coordinates by noting that SQM can be the true ground state only if its energy per baryon, \( E_B \), is lower than the lowest value found in nuclei, 930 MeV for iron, as done by Farhi and Jaffe, \[2\].

2. We calculate the 2 flavour quark matter ground states and fix a lower bound for the only free parameter in our lagrangian, the scalar coupling; or equivalently, we get a lower bound on the chiral condensate pressure (or bag pressure) from the condition that the 2 flavour quark matter state must have higher \( E_B \) than nuclear matter - otherwise nuclear matter would be unstable to conversion to the 2 flavour QM. As pointed out in \[2\] this condition is that bulk 2 flavour quark matter must have \( E_B > 934 \) MeV.

3. We calculate the SQM with the parameters established in 2 above and see if for SQM, \( E_B \) is smaller than that given in 1, above. If this is the case, and as \( E_B \) increases monotonically with the scalar coupling (or the chiral condensate pressure), we get an upper bound on the chiral condensate pressure (or bag pressure), when \( E_B \) crosses beyond 930 MeV. SQM can then exist, as the true ground state, in this interval between the two bounds.

IV. TWO FLAVOUR QUARK MATTER

We shall now consider in Mean Field Theory the phases of 2 flavour quark matter in the \( SU(2)_L \times SU(2)_R \) chiral model above. We shall then extend the model to 3 flavours (u, d, s) to describe SQM.

A. The space uniform phase

We now turn to the phase in which the pattern of symmetry breaking is such that the expectation values of the meson fields are uniform. At zero density they are just the VEVs.

\[ <\sigma> = f_\pi \]
\[ <\vec{\pi}> = 0 \]

For arbitrary density we allow the expectation value to change in magnitude, as it becomes a variational parameter that is determined by energy minimization at each density.

\[ <\sigma> = F \]
\[ <\vec{\pi}> = 0 \]

Such a pattern of symmetry breaking simply provides a constituent mass to the quark \( m = g <\sigma>= gF \) and the quarks are in plane wave states as opposed to the bound states in the nucleonic phase \[12\].

The mean field description of this phase is simple. The energy density

\[ \epsilon_\rho = \Sigma_{u,d} \frac{1}{(2\pi)^3} \int \frac{d^3k}{\sqrt{m^2 + k^2 + \frac{\lambda^2}{4} (\sigma^2 - f_\pi^2)^2}} \]

where \( m = g <\sigma>= gF \) and the degeneracy \( \gamma = 6 \). We shall use \( g = 5.4 \) as determined from fixing the nucleon mass in this model at 938 MeV \[2,6\]. The integral above runs up to the \( ^u \) and \( ^d \) fermi momenta.

For neutron matter (without \( \beta \) equilibrium) we have the relations

\[ k_{u}^f = (\pi^2 n_u)^{\frac{1}{3}} = (\pi^2 \rho_B)^{\frac{1}{3}} \]
\[ k_{d}^f = (2\pi^2 \rho_B)^{\frac{1}{3}} \]
\[ E_B = \frac{\epsilon_\rho}{\rho_B} \]

where \( \rho_B \) is the baryon density. At any density the ground state follows from minimising the free energy w.r.t. \( <\sigma> = F \).

As shown in the figures of \[12,13\] this phase begins at zero \( \rho_B \), with \( E_B = 3g f_\pi \), which then falls till chiral restoration occurs at some \( \rho_X \). After this, as density is increased the \( E_B \) continues to drop and goes to a minimum and then starts rising corresponding to a massless quark fermi gas.
In the chirally restored phase the EOS is very simple and parallels the MIT bag description of \( \rho \)

\[
\rho_B > \rho_X \quad (12)
\]

\[
\epsilon_p = \frac{3}{4\pi^2} \rho_B \frac{\rho}{\alpha} + \frac{\lambda^2}{4} f_\pi^4 
\quad (13)
\]

The last term above is just the bag energy density, and

\[
\alpha = (1 + 2^\dagger)
\quad (14)
\]

This phase has two features, a chiral restoration at \( \rho_X \) following, with increasing density, by an absolute minimum in \( E_B \), at \( \rho_C > \rho_X \).

Since \( E_B \) decreases monotonically with density till the chiral restoration density, \( \rho_X \), and then continues to decrease till the minimum is reached at \( \rho_C \), this implies that the density regime till \( \rho_C \) is unstable and has negative pressure. This has been recently conjectured \( [14] \) as the density at which self-bound droplets of quarks form, which may be related to nucleons. Further, since at this density chiral symmetry is restored, these 'nucleons' will be like those in the MIT bag model in which chiral symmetry is unbroken inside the nucleons.

We would like to clarify this issue.

From the comparison of this phase with the nucleon and nucleonic 'phase' arising from the same model (see \( [6,12] \)), it is clear that the nucleonic phase is always of lower energy than the uniform phase above, up to a density of roughly 3 times the nuclear density, which is above the chiral restoration density in the uniform phase. Further, the minimum in the nucleonic phase occurs very much below the minimum in the uniform phase.

The chiral restoration density in the uniform phase is thus not of any physical interest as matter will always be in the lower energy nucleonic phase and so the identification of nucleon as a quark droplet at the density at which the minimum occurs in the uniform phase is not viable. Clearly, the nucleon is a quark soliton of mass \( M = 938 \) MeV and falls at the zero density limit in the nucleonic phase.

### B. The Pion Condensed phase

Here we shall consider another realization of the expectation value of \( < \sigma > \) and \( < \bar{\sigma} > \) corresponding to pion condensation. This phenomenon was first considered in the context of nuclear matter.

Such a phenomenon also occurs with our quark based chiral \( \sigma \) model and was first considered at the Mean Field Level by Kutschera and Broniowski in an important paper \( [13] \). Working in the chiral limit they found that the pion condensed state has lower energy than the uniform symmetry breaking state (phase 2) we have just considered for all density. This is expected, as the ansatz for the PC phase is more general than for phase 2.

The expectation values now carry a particular space dependence

\[
< \sigma > = F \cos (\bar{q} \cdot \tilde{r}) 
\quad (15)
\]

\[
< \sigma > = F \sin (\bar{q} \cdot \tilde{r}) 
\quad (16)
\]

\[
< \sigma > = 0 
\quad (17)
\]

\[
< \sigma > = 0 
\quad (18)
\]

Note, when \( |\bar{q}| \) goes to zero, we recover the uniform phase 2.

The Dirac Equation in this background is solved in \( [13] \) and reduces to

\[
H\chi(k) = (\bar{\alpha} \cdot \bar{q} - \frac{1}{2} m \bar{\gamma}_5 \bar{\gamma}_3 + \beta m)\chi(k) = E(k)\chi(k) 
\quad (19)
\]

where \( m = gF \).

The extra term has been recast in terms of the relativistic spin operator, \( \bar{\alpha} \gamma_5 \). It is evident that if spin is parallel to \( \bar{q} \) and \( \tau_3 = +1 \) (up quark) this term is negative and if \( \tau_3 = -1 \) (down quark) it is positive. For spin antiparallel to \( \bar{q} \) the signs for \( \tau_3 = +1 \) and \( -1 \) are reversed.

The spectrum for the hamiltonian is the quasi particle spectrum and can be found to be

\[
E_{(-)}(k) = \sqrt{m^2 + k^2 + \frac{1}{4} q^2 - \sqrt{m^2 q^2 + (\bar{q} \cdot \bar{k})^2}} 
\quad (20)
\]

\[
E_{(+)}(k) = \sqrt{m^2 + k^2 + \frac{1}{4} q^2 + \sqrt{m^2 q^2 + (\bar{q} \cdot \bar{k})^2}} 
\quad (21)
\]

The lower energy eigenvalue \( E_{(-)} \) has spin along \( \bar{q} \) for \( \tau_3 = 1 \), or has spin opposite to \( \bar{q} \) for \( \tau_3 = -1 \). The higher energy eigenvalue \( E_{(+)} \) has spin along \( \bar{q} \) and \( \tau_3 = -1 \), or has spin opposite to \( \bar{q} \) and \( \tau_3 = +1 \).

In this background the fermi sea in no longer degenerate in spin but gets polarized into the states above. The quasi particles are, however, states of isospin. We describe matter at a given fermi energy of u and d quarks set by their respective densities and by charge neutrality (corresponding to say neutron like matter).

First we fill up all the lower energy, \( E_{(-)}(k) \), states and then we have a gap and start filling up the \( E_{(+)}(k) \) states till we get to \( E_p \), the fermi energy corresponding to a given density for each flavour.

\[
\rho_i = \frac{1}{(2\pi)^3} \gamma (\int d^3k \Theta (E_p - E_{(-)}(k)) + \int d^3k \Theta (E_p - E_{(+)}(k))) 
\quad (22)
\]
we can now write down the equation of state as in Ref. 13. It is found that the PC state is always lower in energy than the uniform phase 2. For the explicit numbers and figures we refer the reader to [13].

We briefly remark on some features of this phase:

1. The 2 flavour PC state is quite different from the uniform phase: unlike the 2 flavour CRQM states considered in [2], it cannot be recovered from 3 flavour CRQM by taking the strange quark mass to infinity. As we shall see in the next sections, this gives a new feature - a maximum strange current quark mass for SQM to be the true ground state.

2. The reason that the PC phase has energy lower than the uniform \( < \sigma > \) condensate is perhaps best understood in the language of quarks and anti-quarks. To make a condensate a quark and anti-quark must make a bound state and condense. For a uniform \( < \sigma > \) condensate the \( q \) and \( \bar{q} \) must have equal and opposite momentum. Therefore, as the quark density goes up the system can only couple a quark with \( k > k_f \) and a \( \bar{q} \) with the opposite momentum. This costs much energy so the condensate can only occur if \( k_f \) is small, at low density. On the other hand, the pion condensed state is not uniform. So at finite density, if we take a quark with \( k = k_f \) the \( \bar{q} \) can have momentum \( k = |k_f - \bar{q}| \), which is a much smaller energy cost.

3. Since the pion condensate is a chirally broken phase, the chiral restoration shifts from very low density in the uniform phase to very high density: \( \sim 10 \rho_{nuc} \). This is a signature of this phase.

4. Since this phase is always lower in energy than the uniform phase we go directly from the nucleonic phase to the PC phase completely bypassing the uniform phase, and thus all the interesting features and conjectures for the uniform phase are never realized.

5. Another feature of this \( \pi \) condensate is that since we have a spin isospin polarization we can get a net magnetic moment in the ground state.

V. THE THREE FLAVOUR STATE

The extension of the above to 3 flavours or SU(3) chiral symmetry needs some clarification.

The generalized Dirac Equation for the SU(3) case is considerably more complicated and involves a singlet \( \xi_0 \) and an SU(3) octet \( \xi_a \) of scalar fields and a singlet \( \phi_0 \) and an SU(3) octet \( \phi_a \) of pseudoscalar fields, that interact with the quarks as shown in [15].

\[
H\psi(k) = (-i\vec{\sigma} \cdot \vec{\partial} - g\beta(\sqrt{2/3}(\xi_0 + i\phi_0\gamma_5) + \lambda^a(\xi_a + i\phi_a\gamma_5)))\psi = E\psi
\]  

(26)

In the chiral limit, the spontaneous symmetry breaking pattern is not unique. We choose the pattern in which the \( SU(3)_L \times SU(3)_R \) chiral symmetry breaks down to a vector SU(3). For the uniform case, we have

\[
< \xi_0 > = \sqrt{3/2}f_\pi
\]

(27)

\[
< \xi_a > = 0
\]

(28)

\[
< \phi_0 > = 0
\]

(29)

\[
< \phi_a > = 0
\]

(30)

This gives a constituent mass \( m = gf_\pi \) for all (u, d and s) quarks. The explicit symmetry breaking strange quark mass term with mass \( m_s \), is then added to \( H \). The strange quark mass, \( M_s \), then turns out to be the sum of the constituent and explicit mass, \( M_s = gf_\pi + m_s \).

A. The three flavour Pion Condensed phase

For describing strange quark matter we use the 3 flavour Pion Condensed state. This is a more versatile state than the one used in [2] (3 flavour CRQM), the latter being in a subset of the former.

Next, we formulate the symmetry breaking in the presence of the pion condensate. This is given as follows:

\[
< \xi_0 > = \sqrt{3/2}F(1 + 2\cos(\bar{q}\cdot\vec{r})) / 3
\]  

(31)
\[
\begin{align*}
\langle \xi_8 \rangle &= -\sqrt{3}F(1 - \cos(q, \vec{r})) / 3 \\
\langle \phi_0 \rangle &= 0 \\
\langle \phi_3 \rangle &= F(\sin(q, \vec{r}))
\end{align*}
\]

(32) (33) (34)

and all other fields have expectation value zero.

This gives exactly the PC hamiltonian equation for the u,d sector and yields a simple mass relation above for the strange quark: \(M_s = gF + m_s\); when \(q = 0\) and \(m_s = 0\) we recover the chiral limit above.

We may now simply add the 2 flavour PC results for the energy density and density derived above to the strange quark energy density which arises from the single particle relation,

\[E_s = \sqrt{M_s^2 + k^2}\]

The strange quark energy density is given by Baym [10], eq. (8.20)

\[\epsilon_s = \frac{3}{\pi^2} M_s^4 (x_s n_s (2x_s^2 + 1) - \ln(x_s + n_s))\]

(35)

where \(x_s = k_f^3 / M_s\) and \(n_s = \sqrt{1 + x_s^2}\); \(k_f^3\) is the fermi momentum for the strange quarks.

The total energy density of the quarks for the 3 flavour PC is given by

\[\epsilon = \epsilon_u + \epsilon_d + \epsilon_s + \frac{1}{2} F^2 q^2 + \frac{\lambda_1^2}{4} (F^2 - f_\pi^2)^2\]

(36)

From the effective potential given in [15] for the SU(3) case, there is an extra factor of 3/2 that multiplies the last term. This can be absorbed, as we have done, by a redefinition: \(\lambda_1 = A\lambda\), where \(A = \sqrt{3}/2\).

### B. \(\beta\) equilibrium in the PC phase

We have the following general chemical potential relations for quark matter

\[E_u^f = \mu_u\]

(37)

\[E_d^f = \mu_d = \mu_s\]

(38)

\[\mu_e = \mu_d - \mu_u\]

(39)

\[n_e = \frac{\mu_e^3}{3\pi^2}\]

(40)

The charge neutrality condition, below, further reduces the number of independent chemical potentials to one.

\[
2n_u(\mu_u, q, F) - n_d(\mu_d, q, F) - n_s(\mu_s) - n_e = 0
\]

(41)

The baryon density is

\[\rho_B = \frac{n_u(\mu_u, q, F) + n_d(\mu_d, q, F) + n_s(\mu_s)}{3}\]

(42)

\[n_s = (k_f^3) / (\pi^2)\]

(43)

For matter in \(\beta\) equilibrium we need to add the electron energy density to the quark energy density above

\[\epsilon_e = (1/4\pi^2)\mu_e^4\]

The total energy density is

\[\epsilon = \epsilon_\rho + \epsilon_e\]

The energy per baryon, \(E_B = \epsilon / \rho_B\), then follows.

For the pion condensed state, the ground state energy and the baryon density depend on the variational parameters, the order parameter or the expectation value,

\[\rho = \sqrt{\phi^2 + |\sigma|^2}\]

and the condensate momentum, \(|q|\). To define the free energy at a fixed baryon density then requires some care. However, we are only interested in the absolute minimum of the energy per baryon, \(E_B\), for all density. We can then simply minimize \(E_B\) with respect the variational parameters, \(F\) and \(|q|\) and the one independent chemical potential to get the absolute minimum. Note that this is for a given a value of

\[B = \lambda_1^4 (f_\pi)^4 / 4\]

(44)

### VI. RESULTS FOR THE MEAN FIELD THEORY 3 FLAVOUR PC STATE (PCSQM)

#### A. PCSQM without one gluon exchange

The new window for SQM is established thus:

We maintain the minimum permissible limit on \(E_B\) for 2 flavour quark matter to be 934 MeV. Since the PC is a lower energy state than the chirally restored QM (CRQM) considered in Ref. 2 we find that the lower bound on \(B^{1/4}\) goes up, closing the window on SQM. The results are given in table II. This lower bound for 2 flavour PC is found to be

\[B^{1/4}_2 = 148\text{ MeV}\]

whereas, for the case of 2 Flavour CRQM considered by Farhi and Jaffe, it was

\[B^{1/4}_2 = 145\text{ MeV}\]

We then calculate the minimum \(E_B\) for 3 flavour QM. For this we use our generalised PCSQM as the state. It is important to note that the rather particular 3 flavour CRQM lies within its variational reach. This provides us with the upper bound on the bag pressure \(B_{\lambda_1}\). The maximum upper bound on \(B\) naturally occurs for the \(m_s = 0\) case and is found to be almost the same as in Ref. 2.

\[B^{1/4}_{\lambda_1} = 162.5\text{ MeV}\]

This is because the minimum occurs in the 3 flavour CRQM state, as given in [2], with \(F = 0\).

Some new features compared to Ref. 2:
is the expectation value of the $\sigma >$ of the density of strange quarks to that of $u$-quarks and $< \sigma >$ is the expectation value of the $\sigma$ field. These results are without 1-gluon exchange.

\[
\begin{array}{cccc}
m_s & B^{1/4} & \mu_u & n_s/n_u < \sigma > \\
\text{(MeV)} & \text{(MeV)} & \text{(MeV)} & \text{(MeV)}
\end{array}
\]

\[
\begin{array}{cccc}
0 & 162.7 & 309 & 0.96 & 9.55 \\
50 & 161.4 & 307 & 0.96 & 1.45 \\
100 & 158.9 & 304 & 0.92 & 0.01 \\
150 & 154.9 & 298 & 0.80 & 0.01 \\
200 & 150.6 & 290 & 0.63 & 0.01 \\
250 & 147.7 & 260 & 0.00 & 26.31
\end{array}
\]

Whereas in \cite{2}, the 2 Flavour CRQM threshold is virtually the limit of $m_s \to \infty$ for 3 flavour CRQM, in our case it is not, as our 2 flavour PC ground state is of lower energy and different. In this case the 3 flavour CRQM becomes of higher energy than the 2 flavour PC ground state at a finite $m_s$.

We thus find the limit

\[ m_s < 250 \text{ MeV} \]

for SQM to exist as the ground state, simply from the constraint on 2 flavour QM.

We also find that for this limiting $m_s$, the absolute minimum of $E_B$ is the 2 flavour PC state with zero strange quark density. But, for masses somewhat below the limiting mass (with the condition that SQM be the actual ground state), the absolute minimum occurs in the 3 flavour CRQM state as given in \cite{2}, with $F = 0$. A summary of the results appears in Fig. 1.

B. Results for the three flavour Pion Condensed phase (with 1 gluon exchange interaction and $\alpha_{QCD} = 0.6$)

We next consider the case in which 1 gluon exchange interaction is included, following Farhi and Jaffe. This is with a view to estimating the effects of including such 'perturbative' interactions. No attempt will be made at rigour. Our approximate scheme for this case is best regarded as an estimate.

One reason it is difficult to do an analytic calculation of the interaction energy for the PC is that the quark propagators in the presence of the pion condensate are far more complicated than in the case of free fermi sea quarks.

We first note that in the limit of the condensate, $F \to 0$, all the PC results go smoothly to the free fermi sea results. We find that the condensate expectation values $F$ in the regime of interest to us are such that the value of $F$ at the minimum makes $m = gF$ fall below the relevant quark chemical potentials. So, as a first approximation, we use the free fermi sea results for the given chemical potential and mass.

The contribution from the 1 gluon exchange interaction to the free energy at given density or the Thermodynamic Potential (TP), including renormalization group corrections, \cite{18}, is given in \cite{2}. The expression for the 1 gluon exchange or interaction energy, for free fermi seas of quarks, \cite{18}, is given in Baym \cite{16} (eq. 8.20). Though we expect the interaction contribution (in the absence of renormalization group corrections) to both (a) and (b) to be the same, we find that the two expressions are different. Results for these two cases are summarized in Table II.

We briefly give a comparison with the results of the case without the 1 gluon exchange interaction. In our notation the results for (a) are given first (without brackets) and the results for (b) appear alongside in brackets.

The lower bound for 2 flavour PC is found to be

\[ B^{1/4}_{2\sigma} = 141.5 (156.7) \text{ MeV} \]

For (a) this is down, from the case without 1 gluon exchange, where it was 148 MeV. This indicates that gluon exchange is repulsive, even with the constituent quark mass generated by the condensate, whereas for (b) it is up and the 1 gluon exchange is attractive.

We note that for the case of 2 Flavour CRQM (with
TABLE II: Same as table I, but interaction energies due to 1 gluon exchange included.

| $m_s$ (MeV) | (a) Farhi-Jaffe model |  | (b) Baym model |  |
|---|---|---|---|---|
| | $B^{1/4}$ (MeV) | $\mu_u$ (MeV) | $n_u/n_u < \sigma >$ (MeV) | $B^{1/4}$ (MeV) | $\mu_u$ (MeV) | $n_u/n_u < \sigma >$ (MeV) |
| 0 | 150.3 | 274 | 0.83 | 22.08 | 166.1 | 302 | 0.69 | 36.53 |
| 50 | 147.7 | 274 | 0.96 | 0.33 | 161.6 | 292 | 0.54 | 34.45 |
| 100 | 144.6 | 280 | 0.89 | 0.01 | 157.2 | 282 | 0.28 | 35.54 |
| 150 | 140.8 | 244 | 0.00 | 32.91 | 155.8 | 272 | 0.00 | 40.67 |
| 200 | 140.8 | 244 | 0.00 | 32.91 | 155.8 | 272 | 0.00 | 40.68 |
| 250 | 140.8 | 244 | 0.00 | 32.91 | 155.8 | 272 | 0.00 | 40.67 |

gluon exchange) considered by Farhi and Jaffe, we find

$$B^{1/4}_< = 132 \text{ MeV}$$

is even more down, from the case without 1 gluon exchange, where it was $145 \text{ MeV}$.

This clearly shows that the effect of gluon exchange is more repulsive for this case as the quarks are massless as opposed to the PC case when they have a mass, $m = gF$.

We note that our value, $B^{1/4}_< = 132 \text{ MeV}$, is more than that given in the figure 1(c) in [2]. This is due to the difference in the way energy density and density are defined for us and in [2]. They begin with the TP to order $\alpha$, derive a density which includes interaction to order $\alpha$, and use this density to define the energy density. The difference between our case and theirs is $O(\alpha^2)$.

The maximum upper bound on $B$ naturally occurs for the $m_s = 0$ case

$$B^{1/4}_> = 150.3 \ (166.1) \ \text{MeV}$$

Interestingly, in this case the minimum in $E_B$ comes from a new ground state and is genuinely different. It does not occur either in the 2 flavour PC state nor does it occur in the 3 flavour CRQM state as given in Farhi and Jaffe, with $F = 0$, as was the case in the absence of 1 gluon interaction. In this case the minimum is lower than either of the these states and comes from a true merger of the two; it has a non zero value of, $F = 22.1 \ (36.5) \ \text{MeV}$, and also a ratio of the strange quark density to the u quark density of 0.83 (0.69).

For comparison for the 3 flavour CRQM state of [2]

$$B^{1/4}_> = 144.5 \text{ MeV}$$

We find the maximum allowed limit on $m_s$, with gluon exchange included, for both (a) and (b), moves down to

$$m_s < 150 \text{ MeV}$$

for SQM to be the absolute ground state.

We remark that the cases (a) and (b) show the same trends. The difference is that the allowed values of $B$ are shifted up in (b).

Some of these results are summarized in Fig. 2.

FIG. 2: Similar to Figure 1 but with 1-gluon exchange included. The exchange energy is computed using two alternative prescriptions, by Farhi-Jaffe [2] and Baym [16], as indicated. The solid lines represent the constraints imposed by taking into account the PC phase in these two cases. The dashed line is for CRQM, obtained from the data presented by Farhi and Jaffe [2]. $\alpha_{QCD}$ is set to 0.6. The PC phase places a strong constraint on the strange quark mass if SQM should represent the absolute ground state of matter.

VII. CONCLUSIONS

We have found that the existence of a new and lower energy state of 2 flavour quark matter, the pion condensed state, has a significant effect on the window of opportunity for SQM to be the true ground state of matter.

We work with an effective chiral lagrangian. Unlike the MIT bag case [2], where the bag pressure is a parameter, in our formulation it is the chiral condensate energy and is given in terms of the parameters of low energy phenomenology - the pion decay constant, $f_\pi$, which is precisely known and the scalar coupling or the $\sigma$ mass, which is rather poorly known. However, it is of interest that SQM is related to parameters of low energy phenomenology. It requires more detailed work to firm up this connection. Furthermore, we have found a new and interesting constraint on the existence of SQM as the true ground state, that comes from the current mass of
the strange quark.
Our finding is that

1. Without including 1 gluon exchange the new PC ground state limits the bounds on the bag pressure $B$, allowing

$$148 \text{ MeV} < B^{1/4} < 162.5 \text{ MeV}$$

instead of the result of [2]

$$145 \text{ MeV} < B^{1/4} < 162.5 \text{ MeV}$$

and cuts down the allowed parameter space of the explicit or current strange quark mass to

$$m_s < 250 \text{ MeV}$$

2. With $\alpha_{QCD} = 0.6$ and including 1 gluon exchange the new PC ground state, with some simplifying approximations strongly limits the bounds on the bag pressure, $B$, allowing

$$141.5(156.7) \text{ MeV} < B^{1/4} < 150.3(166.1) \text{ MeV}$$

instead of the result of [2]:

$$128.5 \text{ MeV} < B^{1/4} < 144.5 \text{ MeV}$$

and it further cuts down the allowed parameter space of the explicit strange quark mass to

$$m_s < 150 \text{ MeV}$$

This is a rather punishing constraint.

These results are obtained in the 2 flavour chiral limit with some approximations and also assuming that all the bag pressure comes from the chiral condensate. Adding a confinement pressure will raise the energy, $E_B$, for SQM and may shrink the window further.

This is, by no means, the last word on possible ground states even in this model–for example, we may have a kaon condensate. However, we shall not investigate this here.

Acknowledgements

We would like to thank W. Broniowski for sharing with us the program on the Pion Condensed state which is the mainstay of this paper. We genuinely thank Judith McGovern for her ever willing help with the intricacies SU(3) chiral model and more. We thank Bob Jaffe for very prompt clarifications on the expressions in their paper, which is the benchmark for this one. We further thank Mike Birse and M. Kutschera. VS thanks the Raman Research Institute for its hospitality.

[1] E. Witten, Phys. Rev. D, 30, 272 (1984).
[2] E. Farhi and R.L. Jaffe, Phys. Rev. D, 30, 2379 (1984).
[3] C. Alcock and A. Olinto, Ann. Rev. Nucl. Part. Sci., 38, 161 (1988).
[4] V. Soni, Mod. Phys. Lett. A, 11, 331 (1996), A. Manohar and H. Georgi, Nucl. Phys. B, 234, 203 (1984).
[5] S. Kahana, G. Ripka and V. Soni, Nucl. Phys. A, 415, 351 (1984), M.C. Birse and M.K. Banerjee, Phys. Lett., 134B, 284, (1984).
[6] V. Soni, ‘The nucleon and strongly interacting matter’, Invited talk at DAE Symposium in Nuclear Physics, Bombay, Dec 1992 and references therein.
[7] M.C. Birse, Soliton Models in Nuclear Physics, Progress in Particle and Nuclear Physics, Vol. 25, 1 (1991), and references therein.
[8] see for example, R. Johnson, N.W. Park, J. Schechter, V. Soni and H. Weigel, Phys. Rev. D42, 2998 (1990), J. Stern and G. Clement, Mod Phys. Lett. A3, 1657 (1988).
[9] see for example, J. Stern and G. Clement, Phys. Lett. B, 264, 426 (1991), E.J. Eichten, I. Hinchcliffe and C. Quigg, Fermilab-Pub 91/272 - T.
[10] D. Diakonov, V. Petrov, P. Pobylitsa, M. Polyakov and C. Weiss, Nucl. Phys. B, 480, 341 (1996), Phys. Rev. D, 56, 4069 (1997).
[11] A. Gocksch, Phys. Rev. Lett. 67, 1701 (1991).
[12] V. Soni, Phys. Lett., 152B, 231 (1985).
[13] M. Kutschera, W. Broniowski and A. Kotlorz, Nucl. Phys. A, 516 566 (1990).
[14] M. Alford, K. Rajgopal and F. Wilczek, Phys. Lett B, 422, 247 (1998), Nucl. Phys. B, 537, 443 (1999).
[15] J.A. McGovern and M.C. Birse, Nucl. Phys. A, 506, 367 (1990).
[16] G. Baym and S. Chin, Phys. Lett., 62B, 241 (1976), G. Baym in NORDITA lectures, ‘Neutron Stars and the Properties of Matter at high density’, (1977).