Identification of Students Errors in Solving Indirect Analogical Problems Based on Analogical Reasoning Components

K Kristayulita¹, T Nusantara², A R As’ari², and C Sa’dijah²

¹Student of Doctoral Degree in Mathematics Education, Universitas Negeri Malang, Malang, Jawa Timur, 65145, Indonesia
²Departement of Mathematics, Universitas Negeri Malang, Malang, Jawa Timur, 65145, Indonesia
³Jurusan Tadris Matematika, Universitas Islam Negeri Mataram, Nusa Tenggara Barat, 83127, Indonesia

Email: kristayulita@uinmataram.ac.id

Abstract. The purpose of this study was to identify students’ errors in solving indirect analogical problems based on analogical reasoning components. Using qualitative design approach, the study was conducted at two schools in two different regions of Nusa Tenggara Barat, Indonesia. The instrument used was analogical problems consisting of two problems where the source problem was symbolic quadratic equation problem and the target problem was trigonometric equation problem. The findings of this study indicate that students could make error in each of structuring, mapping, applying, or verifying stage. But, errors can occur also in the combination of those analogical reasoning components, such as structuring-applying.

1. Introduction
The ability of analogical reasoning is important in learning mathematics[1–7]. Analogical reasoning allows one to solve problems that are not clear, new, and complex[8, 9]. When the analogy is identified, the problem is more easily resolved. This suggests that analogical reasoning provides benefits in solving difficult and abstract mathematical problems. Analogical reasoning allows one to make connections to transfer solutions from known problems to new problems of unknown solutions[9, 10]. Specific similarities between source problems and target problems that can be identified by students through analogical reasoning help students resolve the target problems[5].

Researchers have done a lot of research on analogical reasoning. Analogical reasoning studies that used analogical problems have been investigated[11–13]. There are several similarities to analogical problem used by them, that are (1) there are two types of problem: source problems and target problems; (2) source problems and target problems of the same concept; and (3) solution procedures are similar between source problems and target problems. Characteristics of the analogies problems can be said to be a direct analogical problem. That is, conceptual and procedural in solving direct source problems used in solving target problems.

Analogical reasoning ability to solve direct analogical problems is not the same as solving indirect analogical problems. Indirect analogical problems have a very different problem between the source
and the target problem. Indirect analogical problems used in this study has characteristics such as (1) the mathematical concepts between source problems and target problems different; (2) procedures in solving source problems are used to solve target problems; and (3) the context of source problems and target problems are different. Problem solving between source problems and target problems has similarities, where conceptually and procedurally resolving source problems can be used to solve target problems. Indirect analogical problems have not been used in the study of analogical reasoning.

Meanwhile, several studies discussed students making errors in solving analogical problem. The pattern of errors indicated by elementary school children in transferring isomorphic and non-isomorphic problems[14]. The types and causes of analogical reasoning errors made by students in solving mathematical problems[15].The pattern of analogy errors that arise from the featural distraction of children and why they make it[16].Student errors in undefined problem solving on space analytic geometry[17].Errors analogical reasoning based on Newman's procedures[18].

Preliminary research results conducted, there are errors analogical reasoning in solving indirect analogical problems. The process of resolving the analogies problems using analogical reasoning components are structuring, mapping, applying, and verifying[19]. This study focuses more on identifying indirect analogies problems based on analogical reasoning components. The results of this study, further qualitatively to describe the process of the occurrence of student analogical reasoning errors in solving analogies problems. In addition, the research results also provide information to teachers to develop indirect analogies problems with the aim of improving children's critical thinking through analogical reasoning.

2. Methods
This study apply the descriptive qualitative approach. Following are the description of the method used in this study.

2.1. Subject
Participants are grade 12 high school students. They have received lessons on quadratic equations. Overall, participants were 148 students from two schools in two different regions of Nusa Tenggara Barat, Indonesia.

2.2. Materials
The analogical problem consists of source problems and target problems. First, source problems are related to the problem of quadratic equations. Quadratic equations used because students have completed learning about it; That is, they do have formal training and previous experience with problem-solving quadratic equations. Second, target problems are related to the problem of trigonometric equations. Trigonometric equations used because students are studying them.

2.3. Procedure and Analysis
Data retrieval procedure by requesting the subject to do each task. First, the subject is working on source problems with the problem of quadratic equations. Furthermore, the results of source problems answers are taken and collected from the students. Secondly, the subject is working on target problems related to the problem of trigonometric equations. And then the results of target problems answers are taken and collected from the students as well. Third, classify the results of students' answers from source problems and target problems that are true or false. Fourth, classify the location of the student's answer error on target problems based on analogical reasoning component. Furthermore, the results of the study were analyzed qualitatively.

3. Results
Based on the results of research conducted from two high schools. Correct answered source problems are 108 students (72.97%) and wrong answered are 40 students (27.03%). The complete data can be seen in Figure 1. Based on Figure 1, the results of student responses to source problems illustrate that students understanding algebraic material related to root determination of quadratic equations.
Although, there is one class that has a very low level of understanding of topics about quadratic equations. This is evident from the results of student responses related to given source problems. Students who do so are students who have less experience learning in solving the problem of quadratic equations. Students do not understand what concept or formula is used to solve the problem.

Furthermore, students answered target problems related to the problem of trigonometric equations. The given target problems is different from source problems and it is expected that students' can solve target problems related to trigonometric equation problem by using analogical reasoning. Because some of the target problems solve use the concept of resolving source problems. However, students make mistakes in solving target problems. Based on the analogical reasoning component, the error that arises in resolving target problems can be seen in Figure 2.

Based on Figure 2, errors often appear on the applying component. Students also make mistakes on structuring. Then, the student made a mistake on the verifying component. Errors in structuring components will affect the occurrence of errors in the components of mapping, applying, and verifying. The results showed that in solving analogies problem on target problems (trigonometric equations problems), students make many mistakes. In addition, 28 students do not answer target problems (trigonometric equations problems).

Students who do not answer target problems (trigonometric equations problems) are students who really do not know how to solve the problem. The results also showed that students do not understand the problems given, so students do not have a concept picture that will be used to answer problems. Students make mistakes not only on one component of analogical reasoning such as structuring, mapping, applying, or verifying. However, students can make a combination of two or three of analogical reasoning components. Some results of students answered target problems.

Based on Figure 3, the student's errors are (1) the student can not change $\cos 2x$, (2) the student writes the $\sin x$ value equal to $- \cos x$, (4) the student cannot solve the problem of quadratic equation,
(5) students often do not write the symbol "," , (6) the student directly solves by using the quadratic equation concept. In Figure 4, the student's errors: (1) improper operation, (2) did not specify the factorization of quadratic equations correctly, (3) could not determine the value of the roots of quadratic equations.

4. Discussions
The results showed that the error in solving indirect analogical problem can appear on the structuring, mapping, applying, and verifying sections. Structuring errors such as students cannot convert \( \cos 2x \) to \( 1 - 2 \sin^2 x \) or \( \cos^2 x - 1 \). Students are unable to convert symbolic representations to symbolic. This makes students fail to remember the concept used in solving the problem. The sign of a lack of understanding of the basic concepts of a topic resulted in a failure in using formal procedures to solve certain types of problems[20]. In addition, students incorrectly use the concept of problem solving. Student misconceptions can be caused because students do not understand problems. Two essential types of knowledge that children acquire are conceptual understanding and procedure skills[21]. That is, students must understand the concept of mathematics correctly so that mistakes do not happen.

Mapping errors, students fail to see an analogy between source problems and target problems. Furthermore, students are unable to transform the representation of changing the form of the problem to be very similar to a known problems. Many reasons students do that, such as not knowing what concept is used, do not understand the problem before, and wrong to mapping. When students are not aware of what concepts are used in solving target problems, students do not transfer the concept from source problems to target problems. When mapping is done, two things are obviously done: 1) the corresponding concepts in both situations of the problem are mapped and 2) some of the conceptual structures of one problem are transferred into another matter on the basis of the analogies conclusion[22]. In analogical reasoning, mapping component is an important process. Mapping is the process of finding elements in the target that are in accordance with the source[23]. Mapping is the process of building harmony of nature, relationship, or explanation[24]. Mapping systematically is making the one-to-one relationship between source problems and target problems[7, 25]. A person who does mapping does not need to pay attention to the same number of relationships or all relationships to source problems to be mapped to target problems[4, 26].

Furthermore errors mapping, students can not match the concepts between the two problems (source problems and target problems) and there is no conceptual transfer of source problems to target problems. Students do not do mapping, it can be said that students do not make analogical reasoning in solving target problems. Errors are caused by some obstacles when mapping[27]. There are three major obstacles in the process of mapping according to multiconstraint theory [4, 28]: 1) structural similarity barriers that have two components: structural consistency and one-to-one mapping; 2) the barrier of semantic similarity associated with the obstacle in the meaning of the sentence on the problem of the target being faced; and 3) centered on pragmatic, centering on common ground. If the obstacles arise then the mapping process will occur error. Then this obstacle leads to the reasoning process of someone's analogy having an error.

Errors applying, student unable to transform problem solving procedure of source problems in solving target problems. In addition, students do not have the algorithms skill or procedural skills, this type of error does not necessarily result from carelessness or does not know how to solve it, it can also be caused by students who applying "failed strategies"[29]. This error resulted in the student being unable to solve target problems correctly. Students who are able to use procedural skills well able to solve problems very good. The ability of students to use the procedure properly can improve the ability and understanding of mathematical concepts that are mastered by students[30].

The verifying component in solving indirect analogical problems is not done by the students. Students are not able to solve the problem completely. The verifying component is a step to verify whether the answers are obtained in accordance with the question or not. A person who has an obstacle in mapping is bound to encounter an obstacle in producing analogy conclusions[31].
5. Conclusion

Based on analogical reasoning components, errors occurring in structuring, mapping, applying, and verifying sections. Students can make mistakes on one analogical reasoning component such as structuring, mapping, applying, or verifying. In addition, students can make mistakes on some components such as structuring and applying. These results provide benefits to further explore how the thinking of students who make mistakes in analogical reasoning components like (1) errors in structuring, applying, or verifying, and (2) errors in structuring and applying.

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