Optimization of structural health monitoring attributes under variable failure rate condition using teaching learning based optimization and multiple regression

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Abstract. The manufacturing industries prefer to use exponential equations for evaluating reliability, failure rate, etc., while assuming failure rate constant, but in real life failure rate is not always constant. This research work proposes a novel combination of Multiple Regression and Teaching Learning Based Optimization (TLBO). This work starts with regression modeling, to form an objective function from available data; secondly TLBO technique used, to obtain optimum values of structural health monitoring (SHM) attributes through cost minimization and finally, Weibull analysis to check the effect of optimum values of SHM attributes on β. The final results indicated that the β parameter reduces, which symbolized reduction in failure rate & SHM cost and, improvement in system reliability, thus validates the adopted methodology of combing multiple regression with TLBO, thus success of this work become the backbone for intelligent structures for SHM system.

Keywords: failure analysis, TLBO, optimization, condition based maintenance (CBM), regression, structural health monitoring (SHM)

1. Introduction

It was observed in reviewed literature that researchers preferred Weibull analysis [1-4] for analyzing failure, as it is a generalized uncertainty failure model and provides a simple graphical solution. It is useful when there are inadequacies in the data, which means that this method can be used with small samples, for identifying mixtures of failures, problems with the origin other than zero time.

The objective of optimization in SHM [5-8] is to improve availability by improving reliability, enhancing equipment life by reducing frequent outages and saving SHM costs by reducing the failure rate thus decreasing the repair and inventory cost.

The values of certain important parameters [9-12] in the optimization models are problematic to estimate. Keeping this in mind, this research work used a regression modeling technique [13-14] to form an objective function from available data to minimize cost. Here teaching learning-based optimization (TLBO) used due to its simplicity to get optimum values of SHM attributes like reliability and monitoring frequency.
The arrangement of this work is as follows: In section 2, the methodology and analysis part showed. In section 3, the result part showed and then, conclusions part shown in the last section. The contribution and the implication of this study are of two folds: Firstly, this is the first work that covered a combination of multiple regression and TLBO, therefore beneficial for researchers those faces difficulties in analyzing real-life data; secondly, this analysis will helpful for SHM personnel for preparing pin-pointed SHM plan to identify and eliminate the potential failure modes.

2. Methodology & Analysis
This section covered the methodology adopted in present work including analysis. To provide content systematically this sub-section is further divided into several sub-sections. To meet the practical conditions the following assumptions have been adopted in the present work for simplification of the analysis procedure
- Each SHM Task is ready for processing immediately upon arrival into the system.
- Condition monitoring will be carried out after the process on a specific machine is over.
- Each inspection reveals the system deterioration state perfectly.
- A failure may visible, whenever degradation is equal to or greater than a failure threshold level.

2.1 Rank Regression (RR)
This sub-section covered the Rank Regression procedure for Weibull analysis of failure data (Table 1). This compressor worked for 16 hours a day for 365 days. For the remaining 8 hours, one of the remaining compressors was used. Here, the seizure problem occurred 12 times, while improper unload of the compressor and insufficient pressure problem occurred for 7 and 10 times respectively. It is quite oblivious from above that seizure/wear is the most prominent cause of failure. This sub-section followed the steps of RR mentioned below and used Microsoft origin for analysis.

**Step 1.** Rank the times-to-failure in ascending order.
**Step 2.** Obtain their median rank plotting positions using the following:

\[
F(t) = \frac{i-0.3}{N+0.4} \times 100
\]

Where \(i\) is the failure order number and \(N\) is the total sample size.

**Step 3.** Evaluate the values of \(x = \ln(t)\) and \(y = \ln(\ln(1/(1-F(t))))\), and then plot it.

**Step 4.** Draw the finest possible straight line through these points, and then obtain the slope \((\beta)\) of this line and use intercept to evaluate vale of scale parameter \((\eta)\).

| Table 1. Failure Data |
|-----------------------|
| Number of failures (i) | Time-to-failure, hours (t) | Median Rank F(t) | x = ln (t) | y = ln (ln (1/(1-F(t)))) |
| 1 | 724 | 0.056451613 | 6.584791392 | -2.845458285 |
| 2 | 1052 | 0.137096774 | 6.958448393 | -1.914247621 |
| 3 | 1287 | 0.217741935 | 7.160069208 | -1.404170849 |
| 4 | 1371 | 0.298387097 | 7.22329568 | -1.037403987 |
| 5 | 2012 | 0.379032258 | 7.606884531 | -0.741337623 |
2.2 Optimization
This sub-section covered firstly, the formation of the objective function using regression modeling and then, optimization of this objective function using TLBO.

2.2.1 Regression Modeling
The values of some key parameters in the optimization models are problematic to guesstimate due to the absence of sufficient sample data. Therefore, some researchers replaced these with observed values or the skilled experience, while the outcomes usually have large mistakes compared with the actual values. As this research work based on real-life data and to mitigate the above-mentioned points, a regression model using Minitab developed to find the relation between available variables. Table 2 contains data for monitoring interval and the corresponding value of SHM costs obtained from this heavy industrial setup. The value of reliability evaluated using time, β and η.

Table 2. Monitoring Interval, Corresponding Reliability, and SHM Cost

| S.No. | Time (t) | SHM Cost (C) | Reliability (R(t)) |
|-------|----------|--------------|-------------------|
| 1     | 480.0    | 13273.97     | 0.95563           |
| 2     | 960.0    | 16723.98     | 0.87719           |
| 3     | 1440.0   | 19241.97     | 0.78377           |
| 4     | 1920.0   | 24273.97     | 0.68500           |
| 5     | 2400.0   | 26923.97     | 0.58727           |
| 6     | 2880.0   | 28923.99     | 0.49485           |
| 7     | 3360.0   | 29119.97     | 0.41041           |
| 8     | 3840.0   | 33967.99     | 0.33540           |
| 9     | 4320.0   | 39223.97     | 0.27033           |
| 10    | 4800.0   | 33423.97     | 0.21505           |
| 11    | 5280.0   | 36173.97     | 0.16895           |
| 12    | 5760.0   | 31223.97     | 0.13117           |

2.2.2 Teaching Learning Based Optimization (TLBO)
This sub-section covered the procedure adopted for TLBO [12]. TLBO method separated into two phases viz., teacher phase and learner phase. The steps of these phases are as follows:

Teacher Phase

Step 1: Determine the objective function.
Step 2: Choose the value of the design variables within the limit given. Randomly generate the population and store it in table 1.

\[ X_i = \text{Lower Limit} + r \times (\text{Upper Limit} - \text{Lower Limit}) \]  

(2)

Step 3: Find the difference mean

\[ \text{Difference}_j = r_i (X_{j,\text{best},i} - \text{TF} \times M_{i,j}) \]  

(3)

Where \( X_{j,\text{best},i} \) is the result of the best learner in subject \( j \), i.e. the value of \( x_j \) corresponding to the optimum
value of the function. $TF$ is the teaching factor, which decides the value of mean to be changed, usually considered as 1 for simplicity, $ri$ is the random number in the range $[0, 1]$.

**Step 4:** Add the difference mean in the element of table I and the value of function corresponding to these Values in table II.

**Step 5:** Now, compare the values of $f(x)$ of Tables I and II, consider the best values of $f(x)$ and place in Table III. This finishes the teacher phase.

**Learner Phase**

**Step 1:**

$$X''_{j,P,I} = X'_{j,P,I} + ri(X'_{j,Q,I} - X'_{j,P,I}), \text{if } X'_{\text{total-P,I}} < X'_{\text{total-Q,I}} \quad (4)$$

$$X''_{j,P,I} = X'_{j,P,I} + ri(X'_{j,Q,I} - X'_{j,P,I}), \text{if } X'_{\text{total-Q,I}} < X'_{\text{total-P,I}} \quad (5)$$

Where, $X''_{j,P,I}$ is accepted if it gives a better function value. The Equation for minimization problems. Here table 4 is obtained. Now compare the values of table III & IV and put best values in table V. In this way, first iteration of TLBO completed.

2.2.3 Optimizing Objective Function Using TLBO

In this research work, the equation obtained with regression modeling considered for optimization (Section 3.2.1). The equation obtained for minimization of SHM cost was as follows:

$$\text{Min. } C (R, t) = 44761 - 1.85t - 34205R + 4.36tR \quad (6)$$

Where,

$C$ = SHM Cost, $t$ = Monitoring Frequency in hrs, $R$ = Reliability

**Step 1:** In this research work equation (Equation 6) obtained from regression modeling considered as an objective function.

**Step 2:** Teacher Phase: Here there are two design variable viz. reliability ($R$) and condition monitoring time ($t$). The population size is randomly selected as 10 and generated using Equation (2) and stored in table 3.

**Table 3. Randomly Generated Population and Corresponding Objective Function**

| Learner | $t$  | $R$     | Cost (C) |
|---------|------|---------|----------|
| 1       | 418  | 0.59496 | 24721.7161 |
| 2       | 438  | 0.83355 | 17030.553  |
| 3       | 668  | 0.92554 | 14563.5437 |
| 4       | 519  | 0.68541 | 21907.1608 |
| 5       | 466  | 0.52092 | 27139.3216 |
| 6       | 380  | 0.28082 | 34917.9143 |
| 7       | 339  | 0.60595 | 24302.8838 |
| 8       | 514  | 0.24347 | 36027.9107 |
| 9       | 283  | 0.77052 | 18833.1453 |
| 10      | 358  | 0.11542 | 40331.0699 |

**Step 3 & 4:** The difference mean for reliability and monitoring time was evaluated using Equation (3), and then this difference mean was added in the element of table 3, the value of the function corresponding to these values was evaluated and stored in table 4.

**Table 4. Values after Adding Difference Mean**

| Learner | $t$  | $R$     | MC     |
|---------|------|---------|--------|
| 1       | 552  | 0.77522 | 19088.8468 |
| 2       | 571  | 0.99999 | 11990.0485 |
| 3       | 730  | 0.99999 | 12388.6102 |
| 4       | 652  | 0.86567 | 16405.8114 |
Step 5: Now, compare the values of \( f(x) \) of Tables I and II, consider the best values of \( f(x) \) and place in Table III. This finishes the teacher phase.

| Learner | \( t \) | \( R \) | \( MC \) |
|---------|--------|--------|--------|
| 1       | 552    | 0.77522| 19088.8468|
| 2       | 571    | 0.99999| 11990.0 |
| 3       | 730    | 0.99999| 12388.6 |
| 4       | 652    | 0.86567| 16405.8 |
| 5       | 600    | 0.70118| 21500.8 |
| 6       | 513    | 0.46108| 29071.9 |
| 7       | 472    | 0.78621| 18613.7 |
| 8       | 647    | 0.42373| 30265.7 |
| 9       | 417    | 0.95078| 13196.2 |
| 10      | 491    | 0.29568| 34371.8 |

Step 6: Learner Phase: Here any student/learner can interrelate with any other student for information transfer. This interaction can be done randomly. Hereafter applying equations (4 & 5), table 6 is obtained.

| Learner | \( t \) | \( R \) | \( MC \) | Interaction |
|---------|--------|--------|--------|-------------|
| 1       | 568    | 0.98201| 12551.3837| 1 & 2 |
| 2       | 443    | 0.99999| 11667.2135| 2 & 3 |
| 3       | 730    | 0.99999| 12388.6102| 3 & 4 |
| 4       | 695    | 0.99999| 12300.8098| 4 & 5 |
| 5       | 670    | 0.92207| 14674.7888| 5 & 6 |
| 6       | 480    | 0.76020| 19461.2338| 6 & 7 |
| 7       | 330    | 0.99999| 11384.8899| 7 & 8 |
| 8       | 461    | 0.90862| 14654.3488| 8 & 9 |
| 9       | 356    | 0.99999| 11450.8632| 9 & 10|
| 10      | 540    | 0.73686| 20293.1046| 10 & 1 |

Now compare the values of Table 5 & 6 and put the best values in Table 7. In this way, the first iteration of TLBO completed.
| Learner | \( t \) | \( R \) | \( MC \) |
|---------|----------|--------|--------|
| 1       | 568      | 0.98201| 12551.4|
| 2       | 443      | 0.99999| 11667.2|
| 3       | 730      | 0.99999| 12388.6|
| 4       | 695      | 0.99999| 12300.8|
| 5       | 670      | 0.92207| 14674.8|
| 6       | 480      | 0.76020| 19461.2|
| 7       | 330      | 0.99999| 11384.9|
| 8       | 461      | 0.90862| 14654.3|
| 9       | 356      | 0.99999| 11450.9|
| 10      | 540      | 0.73686| 20293.1|

Similarly, multiple iterations performed and after optimization for minimum SHM cost of Rs. 11221.5, optimum values of monitoring time as 265 hours and reliability value 0.99 obtained.

2.3 Validation of Research Approach

This section covered the approach for validating the results. Here, the results were validated using collected data of next year for the same screw compressor element.

2.3.1 Weibull Analysis  This section covered the Weibull analysis of failure data (Table 8). This work followed the procedure mentioned in Section 3.1 and used Microsoft origin for analysis.

| Number of failures (i) | Time-to-failure, hours (t) | Median Rank \( F(t) \) | \( x = \ln(t) \) | \( y = \ln(\ln(1/(1-F(t)))) \) |
|------------------------|-----------------------------|------------------------|----------------|---------------------|
| 1                      | 424                         | 0.056452               | 6.048802       | -2.84546           |
| 2                      | 827                         | 0.137097               | 6.717593       | -1.91425           |
| 3                      | 1167                        | 0.217742               | 7.062422       | -1.40417           |
| 4                      | 1296                        | 0.298387               | 7.166693       | -1.0374            |
| 5                      | 2301                        | 0.379032               | 7.741052       | -0.74134           |
| 6                      | 3347                        | 0.459677               | 8.115675       | -0.48518           |
| 7                      | 4027                        | 0.540323               | 8.30085        | -0.25202           |
| 8                      | 4454                        | 0.620968               | 8.401521       | -0.03032           |
| 9                      | 4821                        | 0.701613               | 8.480732       | 0.190094           |
| 10                     | 5026                        | 0.782258               | 8.5223         | 0.42163            |
| 11                     | 5066                        | 0.862903               | 8.530231       | 0.68666            |
| 12                     | 5096                        | 0.943548               | 8.536214       | 1.055834           |

2.3.2 Effect on SHM Cost  The SHM cost data for two consecutive years was also collected from this industrial setup and discussed in the result and discussion section.
2.3.3 Effect on Reliability and Failure Rate

This research work utilized the β and η obtained from Weibull analysis before & after optimization to evaluate the values of Reliability and Failure Rate for two consecutive years. The target was to check whether the above research methodology provided higher reliability and a lower failure rate in the long run or not.

3. Result & Discussion

The Weibull analysis performed for failure data by plotting failures and drawn a least-squares regression line with the help of Microsoft Origin (Figure 1). The obtained values for β and η are 1.529778765 and 3624.384377 respectively, which means that the equipment was in the wear-out phase and needs more attention; therefore this research work proceeded to obtain optimum monitoring frequency.

R-square (must be ≥ 70%) is a determination coefficient. It represents the % of the variation in a response variable that is explained from its relationship with one or more soothsayer variables. In the present case, the p-value was less than 0.001 and the R-square value is 94.27%.

Figure 2 showed the multiple regressions using MINITAB for the cost (C) with reliability (R) and time (t). In this research work, the p-value is less than 0.001 and the R-square value is 94.27%, therefore the data is correct. Figure 6 shows the multiple regressions for the cost (C) with reliability (R) and time (t). The output model equation (see figure 3) of this analysis was used for optimization purposes using TLBO.

![Figure 1. Weibull Analysis](image-url)
Figure 2. Multiple Regression Summary Report

First, the objective function obtained after performing regression modeling, and then TLBO optimization used with population size 5, with two design variables. Hereafter TLBO optimization, for minimum SHM cost of Rs. 11221.5, the obtained optimum values of frequency of SHM and targeted reliability were 265 hours and 0.99 respectively. These values recommended to the SHM department of this heavy industrial setup and collected the SHM data again for the next session.

Here the results were validated using data of next consecutive year. The Weibull analysis (Rank Regression) for failure data, performed by plotting failures and drawn a least-squares regression line with the
help of Microsoft Origin, with the help of Microsoft Origin (Figure 4). The values of β found were above 1 but below the previous year β (Table 9), which means the equipment performed better after optimizing monitoring frequency.

Table 9. Weibull Analysis Result

|                      | Before Optimization (BO) | After Optimization (AO) |
|----------------------|---------------------------|-------------------------|
| B (Shape Factor)     | 1.52977                   | 1.28389                 |
| η (Scale Factor)     | 3624.38                   | 3694.31                 |

Figure 4. Weibull Analysis

The SHM cost data for the two consecutive years was also collected from this industrial setup and found to be reduced from, which shows the success of this methodology.

This research work utilized the β and η obtained with Weibull analysis before & after optimization to evaluate the values of Reliability and Failure Rate for two consecutive years (Table 10). The target was to check whether the above research methodology provided higher reliability and a lower failure rate in the long run or not. Figure 5 and 6 based on table 10 indicates that the equipment gives better reliability in the long run and lower failure rate, after using optimum values of monitoring frequency while targeting reliability of 0.99. It is evident from the graphs that after changing monitoring frequency from 480 hours to 265 hours the reliability increases and failure rate decreases.

It is clear from (Table 11) the after optimization (AO) 1% reliability increases to 10784.07641 cycles with $\beta = 1.283892151$ and $\eta = 3694.310699$ as compared to before optimization (BO) 9835.373402 with $\beta = 1.529778765$ and $\eta = 3624.384377$. It means that the remaining useful life is increasing. Generally, β increases above 1 then the failure rate curve slowly-2 turns to a bell curve.
Table 10. Reliability and Failure Rate Comparison before & after Optimization

| Time  | Before Optimization | After Optimization |
|-------|---------------------|--------------------|
|       | Reliability (R(t))  | Failure Rate       |
| 0     | 1.000000            | 0.000000           |
| 480   | 0.955634            | 0.000145           |
| 960   | 0.877191            | 0.000209           |
| 1440  | 0.783767            | 0.000259           |
| 1920  | 0.684997            | 0.000301           |
| 2400  | 0.587269            | 0.000339           |
| 2880  | 0.494850            | 0.000374           |
| 3360  | 0.410413            | 0.000405           |
| 3840  | 0.335401            | 0.000435           |
| 4320  | 0.270330            | 0.000463           |
| 4800  | 0.215048            | 0.000490           |
| 5280  | 0.168953            | 0.000515           |
| 5760  | 0.131165            | 0.000539           |
| 6240  | 0.100671            | 0.000563           |
| 6720  | 0.076420            | 0.000585           |
| 7200  | 0.057398            | 0.000607           |
| 7680  | 0.042669            | 0.000628           |
| 8160  | 0.031404            | 0.000649           |
| 8640  | 0.022890            | 0.000669           |
| 9120  | 0.016527            | 0.000688           |
| 9600  | 0.011824            | 0.000707           |
| 10080 | 0.008383            | 0.000726           |
| 10560 | 0.005892            | 0.000744           |
| 11040 | 0.004105            | 0.000761           |
| 11520 | 0.002836            | 0.000779           |
| 12000 | 0.001944            | 0.000796           |
| 12480 | 0.001321            | 0.000813           |
| 12960 | 0.000891            | 0.000829           |
| 13440 | 0.000596            | 0.000845           |
| 13920 | 0.000396            | 0.000861           |
| 14400 | 0.000261            | 0.000877           |
| 14880 | 0.000171            | 0.000892           |

Table 11. Reliability Comparison

| Reliability | Cycles (BO)   | Cycles (AO)   |
|-------------|--------------|--------------|
| 0.01        | 9835.373402  | 10784.07641  |
| 0.1         | 6251.881928  | 6598.23661   |
| 0.5         | 2852.223526  | 2817.685094  |
| 0.9         | 832.4712588  | 741.361782   |
| 0.99        | 179.1765132  | 140.2014532  |
4. Conclusion

The presented research work has been implemented successfully on the screw compressor of a heavy industrial environment. This section showed the conclusion obtained and are as follows:

- This work has covered the Weibull analysis of failure data with the help of Microsoft Origin and obtained values of $\beta$ and $\eta$ as 1.529778765 and 3624.384377 respectively.
- A $\beta > 1$ means that the failure risk increases over time. The higher the $\beta$, the stronger the time relationship.
- Hereafter optimization for minimum SHM cost of Rs. 11221.5, the optimum values of frequency of SHM and reliability were 265 hours and 0.99 respectively.
- Some recommendations to be considered during SHM inspections were also suggested.
This work covered the Weibull analysis of failure data for next consecutive year with the help of Microsoft Origin. The values of $\beta = 1.283892151$ and $\eta = 3694.310699$ which means the equipment performed better after optimizing monitoring frequency.

It is evident from the graphs that after changing monitoring frequency from 480 hours to 265 hours the reliability increases and failure rate decreases.

Apart from this, the total SHM cost also reduced from which showed the success of this methodology.

In this research work, an effort has been done, to prepare a framework for industries especially heavy ones, to analyze and reduce wastage of SHM resources including human effort, spare part, etc. and prepare a roadmap for other industries too. This work firstly highlighted the prominent problems faced by the researchers who preferred researching a live industrial environment, like difficulty in getting all the required data for the standard objective functions. From the reviewed literature, it was observed that researchers used some pre-existed values in place of unavailable ones, but it usually leads to error.

In this research work, this issue was addressed and resolved by combining multiple regressions to form objective function and TLBO for optimization, through an industrial case study. The final results indicated that the $\beta$ parameter was reduced, which symbolized a reduction in failure rate & SHM cost and, improvement in system reliability. The obtained results validated the adopted methodology of combing multiple regressions with TLBO. Apart from this, this analysis will be helpful for SHM personnel for preparing pinpointed SHM plans, to eliminate the potential failure modes. Furthermore, this is a generalized approach; therefore, a similar approach could be used in other types of equipment or other industries too. Apart from this, other optimization techniques like a neural network, ant colony optimization may also use. The success of this work become the backbone for intelligent structures for SHM system.

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