The operation of combining sets for images in optical echo holography

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Abstract. The paper considers the implementation of logical operations with sets presented as images in the accumulated photon echo mode. It is shown that the photon echo mode may be used to perform the operation of combining sets.

1. Introduction
Currently, the development of processors that are multifunctional devices allowing to perform integral transformations on signals in real time using echo holography is a subject of great interest [1-4]. Unlike the common hologram recording method, echo holograms (EG) are formed not by interference of the object and reference fields but as a result of atomic states interference. Superposition states arise during the transition process and exist until the end of relaxation of nonequilibrium atomic polarization formed under the action of coherent radiation. Therefore, recording the dynamic echo holograms under the conditions of coherent nonlinear interaction of the object and reference laser pulse with a resonant medium allows to store and play the information on the dynamic processes related to the states change in space and time. Such holograms contain the information on the spatial and temporal structure of the object wave field. The capability of echo holograms to reconstruct or invert the wave front and the temporal form of object laser pulses, which might be used in the systems of operational data processing [5-9], is of particular interest.

The application of stimulated echo holograms allows recording, reconstructing, and transforming images. As the images are sets of points, the logical operations with sets are suitable for their transformation.

The optimal options of filtration and transformation of images are related to the multipulse excitation of the resonant medium. Recording information in the accumulated long-lived echo hologram (ALEH) is the most efficient. In this case, a sequence of pairs of laser pulses affects the resonant medium. Each of these pairs together with a read pulse cause the generation of long-lived photon echo (LPE) response at the same moment of time. Therefore, a signal of ALEH generated by the resonant medium after the impact of N pairs of excitation pulses and one read pulse is a superposition of LPE signals. The paper considers the implementation of combining sets $A \cup B$ operation for images using ALEH. The other logical operations with images may be implemented by the selection of excitation modes and the sequence of images recording [10].
2. The operation of combining sets for images by using ALEH

The order of excitation laser pulses during the formation of ALEH is given in figure 1.

![Diagram](image)

**Figure 1.** The order of excitation laser pulses during the formation of ALEH. $P_n^{(i)}$ – pairs of identical pulses, $P$ – read pulse, $P_e$ – signal of ALEH. $\tau$ – time interval between the pulses in the pairs, $\tau'$ – time interval between the pairs, $\tau''$ – time interval between the last recording pair and the read pulse.

In figure 1 is given the case when the resonant medium is subjected to the action of a series of consecutive identical pairs of pulses. In each pair of pulses in the given sequence the phase difference is the same. Then the generated responses are formed in phase. In the ALEH mode one may perform logical operations with images. As an example, the implementation of combining sets $A \cup B$ operation using ALEH is possible. It is well known that the sum of A and B sets is a set consisting of all the elements belonging to at least one of the sets A, B (figure 2).

![Diagram](image)

**Figure 2.** Euler-Venn diagrams for the operation of combining sets.

For the implementation of the logical operation of combining sets in the ALEH mode, the images, that are embedded in each pair of the excitation pulses, are considered as A and B sets.

Let us write down the electric field strength of the $\eta$-th excitation laser pulse as
\[ E_\eta (\vec{r}, t) = U_\eta (\vec{r}) e^{i\omega t} + k c, \quad (0 \leq t \leq \Delta t_\eta) \]  

(1)

where \( \Delta t_\eta \) is the duration of \( \eta \)-th excitation laser pulse while \( U_\eta (\vec{r}) \) describes the spatial structure of the \( \eta \)-th excitation laser pulse. Let us consider the approximations at which \( U_\eta (\vec{r}) \) may be expanded in spherical or plane waves. The image on the transparency will be considered as a set of points with radius vectors \( \vec{r}_n \). Each point radiates a spherical wave. The aggregation of waves in the location of \( j \)-th optical center in a sample with \( \vec{r}_{0j} \) radius vector provides a value of the optical center’s resonant transition perturbation. Then the electric field strength of an object laser pulse in the point \( \vec{r}_{0j} \) may be written down as

\[ E_j = \sum_n A_n \exp \left\{ i\vec{k}_n^j \cdot (\vec{r}_{0j} - \vec{r}_n) - i\omega t + i\varphi_n \right\} \left| \vec{r}_{0j} - \vec{r}_n \right| \]  

(2)

where \( \vec{k}_n^j = \frac{\omega}{c} \vec{n}_n \), \( \vec{n}_n = \frac{\vec{r}_{0j} - \vec{r}_n}{\left| \vec{r}_{0j} - \vec{r}_n \right|} \), \( \varphi_n \) – initial phases of the spherical waves; \( \exp \{ i\varphi_n \} \) may be included in complex amplitudes \( A_n \). If \( \left| \vec{r}_{0j} - \vec{r}_n \right| \) is significantly greater than the size of the sample, the decomposition over spherical waves (2) transforms into decomposition over the plane waves:

\[ E_j = \sum_n \epsilon_n \exp \left\{ i\vec{k}_n \cdot \vec{r}_{0j} - i\omega t \right\}, \]  

(3)

where \( \epsilon_n \) – the waves’ electric field strength amplitudes from individual points of the object.

As each of the excitation laser pulses is an image carrier, the spatial phase synchronism at the formation of an LPE response is

\[ \vec{k}_{en}^j = -\vec{k}_{1n}^j + \vec{k}_{2n}^j + \vec{k}_{3n}^j \]  

(4)

where \( \vec{k}_{en}^j \) – wave vectors of plane waves of the spatial expansion of the wave fronts of object laser pulses for each \( j \)-th pair.

As in [3], a response in the ALEH mode may be obtained in both reversed and unconverted modes (figure 3).
Figure 3. Spatial phase synchronism at the formation of a response in the ALEH mode; a) reversed mode, b) unconverted mode.

Thus, only those components of the decomposition of the response field exist for which the values of the decomposition expansion amplitudes of the pulses field, corresponding to the wave vectors given in the picture, are different from 0.

Having made the calculations similar to those in [3] for the electric field strength of the ALEH response in the presence of n pairs of excitation pulses, one derives the spatial structure of the ALEH response:

$$I \sim EE^*$$  \hfill (5)
\[ E_{ALEH} \approx \sum_{j=1}^{n} E_j(\vec{r}) e^{i\phi_j} \]  

where

\[ E_j \approx \frac{1}{V} \sum_{n, \omega, \Delta} \int_{V} dV \int_{-\infty}^{\infty} g(\Delta) d\Delta \sin \theta_1^{(j)} \sin \theta_2^{(j)} \sin \theta_3 \times \]

\[ \times \left[ \sum_{n} \epsilon_1^{(n)} e^{-i\Delta_n^{(n)}} \right] \left[ \sum_{n} \epsilon_2^{(n)} e^{-i\Delta_n^{(n)}} \right] \left[ \sum_{n} \epsilon_3^{(n)} e^{-i\Delta_n^{(n)}} \right] e^{-\frac{1}{2}\left(\Delta^{(j)}_{1} - \Delta^{(j)}_{2} - \Delta^{(j)}_{3}\right)\vec{r}} \]

\( \theta_1^{(j)}, \theta_2^{(j)} \) – area of the first and second pulses in j-th pair, \( V \) – volume of the sample part excited, \( g(\Delta) \) – distribution of optical centers for frequencies, \( \Delta = \omega - \Omega_0 \), \( \omega \) – frequency of laser radiation, \( \Omega_0 \) – frequency of resonant transition, \( \epsilon_n^{(j)} \) – amplitudes of the electric field strength of the plane waves of the expansion of wave fronts of the object laser pulses in each j-th pair, \( \Delta \phi_j \) – phase of j-th pair.

The figure 4 provides an order of the laser pulses excitation in the ALEH mode in the case when the corresponding image is stored in each pair of \( (P_1^{(j)}, P_2^{(j)}) \) (figure 1) pulses. A numerical calculation of the ALEH response, using the expressions (5-6), will contain an image that includes all the transparent parts of A and C sets. The images could be of any structure and are chosen for convenience as alternating transparent and non-transparent strips given in figure 4.

The obtained ALEH response could be presented as E1 set which is the result of the logical operation of combining sets \( A \cup B = E_1 \).

| Excitation pulses | Echo |
|-------------------|------|
| A \( P_1^{(j)} \) | B \( P_2^{(j)} \) | C \( P_2^{(j)} \) | B | B | E1 | P3 | P2 |
**Figure 4.** The order of the excitation laser pulses in the ALEH mode. $P_n^{(i)}$ – excitation pulses, $P_3$ – read pulse, $P_e$ – echo signal. A, B, C – sets, $E_1$ is the result of combining the sets.

The obtained response in the ALEH mode will contain an image which is the result of combining the sets $A \cup C = E_1$. This operation is true for both unconverted and reversed modes.

The creation of echo holographic processor implies the development of methods for the implementation of logical operations by the processor itself.

### 3. Conclusions

In this paper, the operation of combining the sets presented as images using the ALEH has been considered. The implementation of logical operations with images could allow the pattern recognition, detection of various elements and objects on pictures as well as conducting computational operations in the echo holographic processor.

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