Review of the “Bottom-Up” scenario

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Abstract

The thermalization of a longitudinally expanding color glass condensate with Bjorken boost invariant geometry is investigated within microscopical parton cascade BAMPS. Our main focus lies on the detailed comparison of thermalization, observed in BAMPS with that suggested in the “Bottom-Up” scenario. We demonstrate that the tremendous production of soft gluons via $gg \rightarrow ggg$, which is shown in the “Bottom-Up” picture as the dominant process during the early preequilibration, will not occur in heavy ion collisions at RHIC and LHC energies, because the back reaction $ggg \rightarrow gg$ hinders the absolute particle multiplication. Moreover, contrary to the “Bottom-Up” scenario, soft and hard gluons thermalize at the same time. The time scale of thermal equilibration in BAMPS calculations is of order $\alpha_s^{-2}(\ln \alpha_s)^{-2}Q_s^{-1}$. After this time the gluon system exhibits nearly hydrodynamical behavior. The shear viscosity to entropy density ratio has a weak dependence on $Q_s$ and lies close to the lower bound of the AdS/CFT conjecture.

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I. INTRODUCTION

It is of great interest to understand the mechanisms of quick equilibrium of quark matter at RHIC, indicated by the success of employing simple ideal hydrodynamics [1] in describing the large values of the elliptic flow $v_2$ measured in Au+Au collisions [2, 3]. When the quark gluon matter becomes sufficiently dilute due to the strong longitudinal expansion pQCD, bremsstrahlung processes are essential for momentum isotropization [4, 5].

The importance of pQCD bremsstrahlung was first raised in the “Bottom-Up” scenario [6], which describes the thermal equilibration of a Color Glass Condensate (CGC) [7, 8] characterized by a saturation scale $Q_s$. The main idea of the “Bottom-Up” scenario is that while the hard gluons with transverse momenta of order $Q_s$ degrade as the condensate evolves in space time, soft gluons with transverse momenta much smaller than $Q_s$ are populated due to pQCD $gg \rightarrow ggg$ bremsstrahlung and start to dominate the total gluon number. As soon as the radiated soft gluons achieve thermal equilibrium and build up a thermal bath, the hard ones begin to lose their energy to the thermal bath and subsequently thermalize. A parametric time scale for overall thermalization is given by $\tau_{th} \sim \alpha_s^{-13/5} Q_s^{-1}$ [6].

Thermal equilibration of gluonic matter with an idealistic CGC initial condition in presence of pQCD $gg \leftrightarrow ggg$ processes is investigated in the present work for the first time within transport calculation using the parton cascade BAMPS [12]. In this work we apply a one dimensional expanding geometry, which is adequate to the assumption of Bjorken boost invariance [13]. For the initial gluon distribution of CGC we employ an idealized and boost-invariant form [9, 10] $f(x, p) = \frac{c}{\alpha_s N_c} \tau \delta(y - \eta) \Theta(Q^2_s - p^2_T)$, with $N_c = 3$ for SU(3) and $c \simeq 0.4$ [14, 15]. The initial gluons are produced at eigentime $\tau \sim \frac{1}{Q_s}$. This form of initial condition is a simplified form of CGC inspired by earlier studies in [9, 10]. Particles with transverse momenta $p_\perp > Q_s$ are missing here. Their presence would modify the thermalization process, but as well lead to an even faster thermalization of the hard momentum region. Including a more realistic distribution in the hard region of transverse spectrum is a task for future work.
II. RESULTS: THERMALIZATION OF A CGC

The way that thermalization proceeds within the parton cascade calculations does not resemble the “Bottom-Up” scenario [6]. The strong parametric enhancement of the total gluon number at early times, as predicted by “Bottom-Up” scenario, is not observed in the cascade calculations. Instead, gluon annihilation occurs during the first $0.3 - 0.75 \text{ fm/c}$ for $Q_s = 2 - 4 \text{ GeV}$, as shown in Fig. 1, which is due to domination of the $3 \rightarrow 2$ processes at the early stage of evolution. This indicates that the initial CGC is oversaturated for the chosen values of $\alpha_s$ and $Q_s$. The gluon number begins to increase after $t \sim 0.3 - 0.75 \text{ fm/c}$ (depending on the value of $Q_s$) when the system is close to kinetic equilibrium and a quasi-hydrodynamical cooling sets in.

After the expansion starts, energy flows immediately into both the soft ($p_t < p_{\text{soft}} = 1.5 \text{ GeV}$) and hard momentum region ($p_t > Q_s$) where the populations rapidly increase, as demonstrated in Fig. 3(a) for calculations with $\alpha_s = 0.3$ and $Q_s = 3 \text{ GeV}$. The presence of a thermal bath of soft gluons seems not to be a necessary condition for the equilibration of hard gluons, since they achieve an exponential shape on a short time scale and almost as quick as the soft gluons, as Fig. 3(a) demonstrates. Again, this is different from the picture invoked in the “Bottom-Up” scenario.

At $t = 0.5 \text{ fm/c}$ the transverse spectrum looks almost thermal. After $t \approx 0.5 \text{ fm/c}$ the cooling of the system sets in and the spectra evolve in a way, characteristic for a hydrodynamical expansion, as shown in Fig. 3(b) for $\alpha_s = 0.3, Q_s = 3 \text{ GeV}$.

We extract the time scale when the system is more or less thermalized from Fig. 2 where the scaled effective temperature $Tt^{1/3}$ is shown. We define the time scale of thermalization as the time, at which $Tt^{1/3}$ becomes a constant, which corresponds to a one-dimensional...
ideal hydrodynamical expansion. The times extracted from Fig. 2(constant $\alpha_s = 0.3$) read: 1.2, 0.75, 0.55fm/c (resp. for $Q_s = 2, 3, 4$ GeV). For fixed $Q_s = 3$ GeV and various $\alpha_s$ the thermalization times are extracted in same manner: 1.75, 1.0, 0.75 fm/c (resp. for $\alpha_s = 0.1, 0.2, 0.3$).

From the values presented here we observe that the time scale of thermalization is of order $\alpha_s^{-2}(\ln \alpha_s)^{-2}Q_s^{-1}$. The 1/$Q_s$ scaling is consistent with the “Bottom-Up” result. The dependence on $\alpha_s$ proves to be weaker than the “Bottom-Up” prediction but consistent with the scaling obtained in \[16\]. The reason for the quick thermalization observed in our calculations is the gluon bremsstrahlung, which favors large-angle radiation due to the LPM suppression\[4, 5, 12\]. Thus, the pQCD $gg \leftrightarrow ggg$ processes are dominant for the thermal equilibration.

The ideal hydrodynamic behavior of the system at late times indicates that the ratio of the shear viscosity to the entropy density is small. Figure 4 shows the $\eta/s$ ratio in the calculations with $\alpha_s = 0.3$ and $Q_s = 2, 3, 4$, respectively. The shear viscosity is calculated using the Navier-Stokes approximation: $\eta = \frac{1}{4} (T_{xx} + T_{yy} - 2T_{zz})$. The entropy density is calculated by $s = 4n - n \ln(\lambda)$, where $n$ is the gluon density and $\lambda = n/n_{th}$ denotes the gluon fugacity.

The $\eta/s$ ratio is nearly constant and has a weak dependence on $Q_s$, $\eta/s \approx 0.15$, which is exactly the same as that obtained in full 3+1 dimensional BAMPS calculations with $\alpha_s = 0.3$ and minijets type initial conditions for Au+Au collisions at RHIC energies \[17\]. This verifies that the $\eta/s$ ratio determines the behavior of the late dynamics and, thus, depends only on the coupling $\alpha_s$, but not on initial conditions. The smallness of the $\eta/s$ ratio, which is close
to the AdS/CFT lower bound\cite{18}, corresponds to the efficiency of the pQCD $gg \leftrightarrow ggg$ processes in thermal equilibration, because the $\eta/s$ ratio is inversely proportional to the total transport collision rate which is for $gg \leftrightarrow ggg$ processes 6 − 7 times larger than that of $gg \rightarrow gg$ collisions\cite{16}.

III. CONCLUSION

Using the parton cascade BAMPS, we found that several aspects of the real thermalization might be different compared to the “Bottom-Up” scenario. A strong increase in soft gluon number, as predicted in the “Bottom-Up” scenario, is not observed and thermal equilibration of soft and hard gluons occurs roughly on the same time scale due to $2 \rightarrow 3$ and $3 \rightarrow 2$ processes, respectively. In agreement with the “Bottom-Up” scenario, the thermalization time proves to be proportional to $Q_s^{-1}$, however, its proportionality to $\alpha_s^{-13/5}$ is not seen but is much weaker: $\tau_{th} \sim (\alpha_s \ln \alpha_s)^{-2} Q_s^{-1}$. The differences arise because the back reactions of bremsstrahlung, $3 \rightarrow 2$ processes, play a significant role. They are completely absent in the “Bottom-Up” scenario.

The quick thermalization and the smallness of the $\eta/s$ ratio observed in the present calculations with the CGC initial conditions are consistent with the findings from the previous studies \cite{1, 2, 4, 5, 11, 12} using the Glauber-type minijets initial conditions. This demonstrates that independent of the chosen initial conditions, the pQCD bremsstrahlung processes (and the back reactions) dominate the dynamical equilibration and then keep the system behaving like a nearly perfect fluid.

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