Probing the gluon tomography in photoproduction of di-pions

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A sizable \( \cos 4\phi \) azimuthal asymmetry in exclusive di-pion production near \( \rho^0 \) resonance peak in ultraperipheral heavy ion collisions recently has been reported by STAR collaboration. We show that both elliptic gluon Wigner distribution and final state soft photon radiation can give rise to this azimuthal asymmetry. The fact that the QED effect alone severely underestimates the observed asymmetry might signal the existence of the nontrivial correlation in quantum phase distribution of gluons.

Introduction. The study of nucleon/nucleus multiple dimensional partonic structure has been widely recognized as one of the most important frontiers of hadronic physics. Within perturbative QCD factorization framework, 3D partonic structure of nucleon is quantified by transverse momentum dependent distributions (TMDs) and generalized parton distributions (GPDs). The mother distribution of these two, namely quantum phase space Wigner distribution later was introduced \[1\] to accommodate more comprehensive information on nucleon internal structure, such as parton canonical orbital angular momentum \[2–5\], the quantum correlation between the position and momentum of partons. The Fourier transform of the Wigner distribution known as the generalized TMD distributions (GTMDs) \[6, 7\] is also often used in phenomenological studies. Pioneered by a recent work \[8\], a few proposals for extracting the Wigner distribution in hard scattering processes in ep/A collisions for gluon sector \[8–19\] and pp collisions for quark sector \[20\] have been put forward.

One special gluon Wigner distribution commonly referred to as the elliptic gluon Wigner distribution \[8\] has recently attracted much attention \[9–18\]. It describes an azimuthal correlation between the impact parameter and gluon transverse momentum. In the small \( x \) limit, this gluon distribution can be dynamically generated either from a semi-classical model or from small \( x \) evolution. In the kinematic region where nuclear recoil transverse momentum and gluon transverse momentum are comparable, the MV model calculation predicates a rather sizable correlation \[17\], which we view as a very encouraging result. This distribution is proposed to be extracted via a \( \cos 2\phi \) azimuthal asymmetry in diffractive di-jet production in ep/A collisions. To avoid the contributions from the final state soft gluon radiation to the asymmetry \[21–24\], it is necessary to directly measure nuclear recoil transverse momentum instead of the total transverse momentum of di-jet, which imposes a big challenge to experimental measurement.

On the other hand, among many exciting topics \[26–34\], ultraperipheral heavy ion collisions (UPCs) provide us unique chances to investigate partonic structure of nucleus before the advent of the EIC era. In particular, with the discovery of linear polarization of coherent photons \[35–39\], a new experimental avenue is opened to study nucleus structure via the novel polarization dependent observables in photonuclear reactions, which already led to fruitful results \[40–43\]. By coupling with elliptic gluon distribution, linearly polarized coherent photon distribution also plays an important role in inducing the \( \cos 4\phi \) asymmetry in exclusive \( \pi^+ \pi^- \) pair production in ultraperipheral heavy ion collisions recently reported by STAR collaboration \[39\], where \( \phi \) is the azimuthal angle between pion’s transverse momentum and the pion pair’s total transverse momentum.

To be more specific, the \( \cos 4\phi \) asymmetry partially results from an interference contribution. Near the \( \rho^0 \) resonance peak, the di-pion is dominantly from the decay of \( \rho^0 \) which is produced in the coherent photonuclear reaction. However, there is also a non-negligible contribution to the pair invariant mass spectrum from the interference contributions between direct pions production and those from the \( \rho^0 \) decay \[43–48\]. Such an interference effect not only leads to sizable deviation from the Breit-Wigner resonance shape \[44\], but also generates the mentioned \( \cos 4\phi \) azimuthal modulation. The underlying physics of the \( \cos 4\phi \) modulation from the interference effect is that the orbital angular momentum carried by di-pion in the direct production amplitude and in the conjugate \( \rho^0 \) decay amplitude differ by four units angular momentum. In the next section, we explain in details how this mech-
anism is realized through the coupling of elliptic gluon distribution and linearly polarized photon distribution. Though the azimuthal asymmetry can result from the final state soft photon radiation effect [23] as well, our numerical results suggest that the QED effect alone is not sufficient to account for the observed asymmetry.

The paper is structured as follows. We compute the contributions to the asymmetry from both elliptic gluon distribution and final state soft photon radiation in the next section, followed by the numerical results presented in Sec.III. The paper is summarized in Sec.IV.

\[
\cos 4\phi \text{ azimuthal asymmetry in exclusive pion pair production.}
\]

We start by briefly reviewing the calculation of polarization independent diffractive \( \rho^0 \) production (for the recent experimental studies of the process, see Refs. [49–51]). It is conventionally formulated in the dipole model [22, 63], where the whole process is divided into three steps: quasi-real photon splitting into quark and anti-quark pair, the color dipole scattering off nucleus, and subsequently recombining to form a vector meson after penetrating the nucleus target. Following this picture, one can directly write the \( \rho^0 \) production amplitude (see Refs. [40, 43] for more details),

\[
\mathcal{A}(x_g, \Delta_\perp) = \int d^2b_\perp e^{-i\Delta_\perp \cdot b_\perp} \int \frac{d^2r_\perp}{4\pi} \mathcal{N}(r_\perp, b_\perp) [\Omega^* K](r_\perp) \quad (1)
\]

where \( \mathcal{N}(r_\perp, b_\perp) \) is the dipole-nucleus scattering amplitude, and \( \Delta_\perp \) is nuclear recoil transverse momentum. And \( [\Omega^* K] \) denotes the overlap of photon wave function and the vector meson wave function,

\[
[\Omega^* K](r_\perp) = \frac{N_{eeq}}{\pi} \int_1^\infty dz \{ m_\rho^2 \Omega^* (|r_\perp|, z) K_0(|r_\perp|, e_f) + [z^2 + (1-z)^2] \frac{\partial \Omega^* (|r_\perp|, z)}{\partial |r_\perp|} \frac{\partial K_0(|r_\perp|, e_f)}{\partial |r_\perp|} \} e^{i(z-\frac{1}{2})\Delta_\perp \cdot r_\perp} \]

where \( z \) stands for the fraction of photon's light-cone momentum carried by quark. Quark and antiquark helicities and color sums have been performed in the above formula. \( K_0 \) is a modified Bessel function of the second kind. \( \Omega \) is the scalar part of vector meson wave function. Note that a phase factor \( e^{i(z-\frac{1}{2})\Delta_\perp \cdot r_\perp} \) is included to account for the non-forward correction [14, 54, 55].

One can easily derive the dipion production amplitude by multiplying the \( \rho^0 \) production amplitude with a simplified Breit-Wigner form which describes the transition from \( \rho^0 \) to \( \pi^+ \pi^- \),

\[
\mathcal{M}_\perp = i \mathcal{A}(x_g, \Delta_\perp) \frac{f_{\rho\pi\pi} P_\perp \cdot \hat{k}_\perp}{Q^2 - M_\rho^2 + i M_\rho \Gamma_\rho} \quad (3)
\]

where \( M_\rho \) is \( \rho^0 \) mass. \( Q \) is the invariant mass of the dipion system and \( P_\perp \) is defined as \( P_\perp = (p_{1\perp} - p_{2\perp})/2 \) with \( p_{1\perp} \) and \( p_{2\perp} \) being the produced pions' transverse momenta. The polarization vector of the incident photon is replaced with its transverse momentum \( k_\perp \) by making use of the fact that the coherent photons are naturally linearly polarized. \( f_{\rho\pi\pi} \) is the effective coupling constant and fixed to be \( f_{\rho\pi\pi} = 12.24 \) according to the optical theorem with the parameter \( \Gamma_\rho = 0.156 \text{ GeV} \). With this production amplitude, it is straightforward to obtain the polarization independent cross section \( \frac{d\sigma}{dP.S} \), where \( P.S \) represents the phase space.

The mentioned \( \cos 4\phi \) azimuthal asymmetry can be induced by soft photon radiation from the final state charged pion particles. The corresponding physics from the final state photon radiation is captured by the soft factor which enters the differential cross section formula via [21–24].

\[
\frac{d\sigma(q_\perp)}{dP.S} = \int d^2q_\perp \frac{d\sigma_0(q_\perp)}{dP.S} S(q_\perp - q'_\perp) \quad (4)
\]

where \( \sigma_0 \) is the leading order Born cross section whose full expression can be found in Ref. [40, 43]. The soft factor is expanded at the leading order as [24].

\[
S(l_\perp) = \delta(l_\perp) + \frac{\alpha_e}{\pi Q^2 l_\perp} \{ c_0 + 2c_2 \cos 2\phi + 2c_4 \cos 4\phi + \ldots \} \quad (5)
\]

where \( \phi \) is the angle between \( P_\perp \) and soft photon transverse momentum \( -l_\perp \). When the final state particle mass is much smaller than \( P_\perp \), there exists the analytical expressions for the coefficients \( c_0 \approx \ln \frac{Q^2}{m_\gamma^2} \), \( c_2 \approx \ln \frac{Q^2}{m_\gamma^2} + \delta y \sinh \delta y - 2 \cos^2 \frac{\phi}{2} \ln [2(1 + \cosh \delta y)] \ldots \) where \( Q \) is the invariant mass of the di-pion system and \( \delta y = y_1 - y_2 \) is the difference between two pions’ rapidities. For the current case, the mass of the final state charged particles is the same order of the invariant mass. We thus proceed numerically and obtain \( c_0 \approx 3.5, c_2 \approx 1.1, \) and \( c_4 \approx 0.42 \). The rapidity dependence of these coefficients is quite mild for RHIC kinematics and is neglected.

Following the standard procedure, the soft factor in Eq. (5) can be extended to all orders by exponentiating the azimuthal independent part to the Sudakov factor in the transverse position space. The resummed cross section takes the form [21–24].

\[
\frac{d\sigma(q_\perp)}{dP.S} = \frac{d^2r_\perp}{(2\pi)^2} \left[ 1 - \frac{20\alpha_c c_2}{\pi} \cos 2\phi + \frac{\alpha_c c_4}{\pi} \cos 4\phi \right] \times e^{ir_\perp \cdot q_\perp} e^{-\text{Sud}(r_\perp)} \int d^2q'_\perp e^{ir_\perp \cdot q'_\perp} \frac{d\sigma(q'_\perp)}{dP.S}. \quad (6)
\]

Here \( \phi_\perp \) is the angle between \( r_\perp \) and \( P_\perp \). In the following numerical estimation, we took into account power corrections from the soft factor following the method outlined in Ref. [24]. The Sudakov factor at one loop is given by [24].

\[
\text{Sud}(r_\perp) = \frac{\alpha_e}{\pi} c_0 \ln \frac{P^2}{\mu_r^2} \quad (7)
\]

with \( \mu_r = 2e^{-\gamma_E}/|r_\perp| \). The Sudakov factor plays a critical role in yielding perturbative high \( q_\perp \) tail of lepton pair produced in UPCs [37, 54].
We now turn to derive the direct di-pion production amplitude. In the collinear factorization, the transition from quark-antiquark pair to dipion pair is described by the nonperturbative and universal dipion distribution amplitude (DA) \[57, 59\]. For the current case, it is necessary to go beyond the collinear factorization and take into account quark and antiquark finite transverse momenta (or transverse position) effects in order to preserve the information of the orbital angular momentum, which is essential for yielding the non-trivial azimuthal modulation. To this end, we assume that the transverse position of \(\pi^+\) is the same as the leading \(u\) or \(\bar{d}\) quark during this non-perturbative transition, though the longitudinal momentum is re-distributed during the transition. The similar requirement applies to \(\pi^-\) as well. We notice that our assumption for the transverse spatial distribution differs from the analysis made in Ref. \[60\]. This difference might be attributed to the fact that quark and anti-quark transverse momenta have been integrated out in the work \[60\], while transverse momenta of quark and the corresponding pion are implicitly assumed to be the same in our treatment. By making this crude approximation, the amplitude reads,

\[
\mathcal{M}_d = i \int d^2b_{1\perp} e^{i\Delta_{1\perp} b_{1\perp}} \int \frac{d^2r_{\perp}}{4\pi} \int dz \Psi^{\gamma \to q\bar{q}}(r_{\perp}, z, \epsilon_{\perp}) N(r_{\perp}, b_{1\perp}) \Phi(z, \zeta)e^{ir_{\perp} \cdot P_{\perp}} (8)
\]

where \(\Phi\) is a universal nonperturbative function describing the transition from a quark-antiquark pair to a di-pion system. \(\zeta\) is the longitudinal momentum fraction carried by \(\pi^+\). We stress that any non-perturbative model for the quark-pair to pion-pair transition would generate a cos \(4\phi\) azimuthal asymmetry, as long as it conserves angular momentum.

We proceed to explicitly work out the angular structure residing in \(\mathcal{M}_d\) by first writing the polarization dependent photon wave function,

\[
\Psi^{\gamma \to q\bar{q}}(r_{\perp}, z, \epsilon_{\perp}) = \frac{e_{p\perp}}{2\pi} \delta_{ij} \left\{ \delta_{\sigma\sigma'} |m_q(e_{\perp}^r + i\alpha \epsilon_{\perp}^z) - \delta_{\sigma\sigma'} |(1-2z)i\epsilon_{\perp}^z \hat{r}_{\perp} + \epsilon_{\perp}^z \times \hat{r}_{\perp}| \frac{\partial}{\partial |r_{\perp}|} \right\} K_0(|r_{\perp}| e_f) (9)
\]

where \(\sigma\) and \(\sigma'\) are the quark and antiquark helicities, i, j are the color indices. And \(e_{\perp}^r = k_{\perp} \equiv k_{\perp}/|k_{\perp}|\) as explained above. \(m_q\) and \(e_{\perp}^q\) denote quark mass and quark’s electric charge number with flavor \(q\) respectively. And \(e_f = 140\text{MeV}\) is determined by fitting to HERA data.

We proceed to discuss the symmetric properties of \(\Phi(z, \zeta)\) in order to have some guidance for parameterizing this non-perturbative function. The invariance of the scattering amplitude under the charge parity transformation requires

\[
\Phi(z, \zeta) = -\Phi(1 - z, 1 - \zeta) . \quad (10)
\]

On the other hand, the final state produced di-pion is a C-odd state since the orbital angular momentum quantum number carried by di-pion is either 1 or 3 in the current case. This implies that

\[
\Phi(z, \zeta)e^{ir_{\perp} \cdot P_{\perp}} = -\Phi(1 - z, 1 - \zeta)e^{-ir_{\perp} \cdot P_{\perp}} . \quad (11)
\]

Using these relations, one obtains,

\[
\Phi(z, \zeta)e^{ir_{\perp} \cdot P_{\perp}} = \frac{\Phi(z, \zeta) + \Phi(z, 1 - \zeta)}{2} e^{ir_{\perp} \cdot P_{\perp}} - \Phi(z, 1 - \zeta) e^{-ir_{\perp} \cdot P_{\perp}} + e^{ir_{\perp} \cdot P_{\perp}} (12)
\]

where the contribution in the second line decouples after convoluting with the photon wave function and the dipole amplitude in Eq. \[8\]. More discussions on the symmetric properties for the integrated di-pion DA can be found in Ref. \[61\]. We are then motivated to parametrize the \(z\) integration in Eq. \[8\] as the following,

\[
\int dz (1 - 2z) \Phi(z, \zeta) = \int (1 - 2z) \Phi(z, \zeta) + \Phi(z, 1 - \zeta) = \delta_{\sigma, -\sigma} \delta^{ij} \zeta (1 - \zeta) C \quad (13)
\]

where \(\zeta (1 - \zeta)\) is a commonly used parametrization that respects particle-antiparticle symmetry. The constant \(C\) is determined by fitting to the polarization averaged cross section which receives the sizable contribution from the interference term. We close the discussion about di-pion distribution amplitude with a final remark. \(\Phi(z, \zeta)\) does not correspond to the normal leading twist collinear di-pion distribution amplitude. Actually, the polarization independent leading twist dipion DA decouples from the process under consideration once averaging over the polarizations of the incident photon. Instead, the high twist collinear DAs that contribute to the observable are related to the functions \(\frac{\partial}{\partial r_{\perp}} \Phi(z, \zeta)e^{ir_{\perp} \cdot P_{\perp}}|_{r_{\perp} \to 0}\) for the p wave final state and \(\frac{\partial}{\partial r_{\perp}} \frac{\partial}{\partial r_{\perp}} \frac{\partial}{\partial r_{\perp}} \Phi(z, \zeta)e^{ir_{\perp} \cdot P_{\perp}}|_{r_{\perp} \to 0}\) for the f wave final state. Here we refrain ourselves from giving the explicit matrix element definition for these dipion DAs as we do not attempt to extensively discuss the properties of high twist dipion DAs in this work.

We now study the angular correlation in the dipole amplitude \(N(b_{\perp}, r_{\perp})\) which reads in the MV model \[62, 63\],

\[
N(b_{\perp}, r_{\perp}) = 1 - \alpha_s C_F \frac{1}{8\pi} \int d^2x_{\perp} \mu_A(x_{\perp}) \left[ \ln \left( \frac{b_{\perp}^2 + r_{\perp}^2}{b_{\perp}^2 + r_{\perp}^2} \right) \right]^2 (14)
\]

where \(\mu_A(x_{\perp})\) is the color source distribution. If \(\mu_A(x_{\perp})\) is not uniformly distributed in the transverse plane of nucleus, a nontrivial angular correlation between \(b_{\perp}\) and \(r_{\perp}\) can arise. By Taylor expanding the above expression and ignoring high order harmonics, one obtains,

\[
N(b_{\perp}, r_{\perp}) \approx 1 - \exp \left[ -Q^2 b_{\perp}^2 r_{\perp}^2 / 4 \right]
\]

\[
+ E \left( b_{\perp}^2 r_{\perp}^2 / 4 \right) 2 \cos(2\phi_b - 2\phi_r) \frac{Q^2 b_{\perp}^2 r_{\perp}^2}{4} e^{-Q^2(z_{\perp}/z_{1\perp})^2} \quad (15)
\]
The coefficient $E\left(b^2, r^2\right)$ has been explicitly computed in the MV model for different kinematic regions $^{10, 12}$ $^{10, 17}$. In this work, it is treated as a free parameter since the kinematics of interest is in the non-perturbative region.

At this step, one can carry out the integration over the azimuthal angles of $r_\perp$ and $b_\perp$. The direct production amplitude is re-written as,

$$\mathcal{M}_d = i\zeta (1 - \zeta) \cos (2\phi_\Delta + \phi_k - 3\phi_P)\mathcal{E}(x_g, \Delta_\perp) + i\zeta (1 - \zeta) \cos (\phi_k - \phi_P)\mathcal{A}_d(x_g, \Delta_\perp) + ... \quad (16)$$

where we only keep the angular structures which contribute to the azimuthal independent interference cross section and the cos $4\phi$ azimuthal modulation. $\phi_k$, $\phi_P$ and $\phi_\Delta$ represent the azimuthal angles of different transverse momenta respectively. $\mathcal{E}(x_g, \Delta_\perp)$ and $\mathcal{A}_d(x_g, \Delta_\perp)$ are given by,

$$\mathcal{E}(x_g, \Delta_\perp) = -\frac{e^q}{2} N_C e^{i\delta \phi} \int dB_1^2 dB_2^2 \frac{Q^2 g^2}{4} e^{-\frac{Q^2 r^2}{4}} \times J_2(|\Delta_\perp||b_\perp|)J_0(|r_\perp||P_\perp|) \frac{\partial K_0(|r_\perp| e_f)}{\partial |r_\perp|} \quad (17)$$

$$\mathcal{A}_d(x_g, \Delta_\perp) = -\frac{e^q N_C}{2} \int dB_1^2 dB_2^2 \left(1 - e^{-\frac{Q^2 r^2}{4}}\right) \times J_0(|\Delta_\perp||b_\perp|)J_1(|r_\perp||P_\perp|) \frac{\partial K_0(|r_\perp| e_f)}{\partial |r_\perp|} \quad (18)$$

where $\mathcal{A}_d$ contributes to the polarization independent interference cross section, which will be compared with STAR data to fix the constant $\mathcal{C}$. As there is no experimental evidence suggesting that the azimuthal dependent amplitude must be real, we insert a phase on the right hand side of Eq. $^{17}$ To have a maximal asymmetry, we assume $\delta \phi = \pi/2$ in the following numerical calculation.

Since the two incoming nuclei take turns to act as the coherent photon source and the target, one should sum over these two possibilities on the amplitude level. The observed diffractive pattern at low transverse momentum crucially depends on such double slit interference effect $^{40, 41, 64}$ $^{66}$. We noticed that the similar double slit interference effect has been discovered in spin space $^{67, 68}$. Following the method outlined in Refs $^{40}$, we also include the double slit interference effect in the calculation of the interference cross section from direct production and $\rho$ decay production. The azimuthal dependent part of which is given by,

$$\frac{d\sigma}{dP \cdot S} = \frac{\zeta (1 - \zeta) M_P \Gamma_P}{2(2\pi)^7(Q^2 - M^2)^2 + M^2 (2\pi)^2} \times \int d^2 \Delta_\perp d^2 k_\perp d^2 k_\perp' \delta^2 (k_\perp + \Delta_\perp - q_\perp)$$

$$\times \cos (3\phi_P - \phi_k - 2\phi_\Delta) \cos (\phi_P - \phi_\Delta') \left\{ e^{i\delta \phi}(\Delta_\perp - \Delta_\perp')\mathcal{A}^*(x_2, \Delta_\perp) \mathcal{E}(x_2, \Delta_\perp) F(x_1, k_\perp) F(x_1, k_\perp') \right. \right.$$\right.

$$+ e^{i\delta \phi}(\Delta_\perp + \Delta_\perp')\mathcal{A}^*(x_1, \Delta_\perp) \mathcal{E}(x_1, \Delta_\perp) F(x_2, k_\perp) F(x_2, k_\perp') \right.$$\right.

$$+ e^{i\delta \phi}(\Delta_\perp - \Delta_\perp')\mathcal{A}^*(x_2, \Delta_\perp) \mathcal{E}(x_1, \Delta_\perp) F(x_1, k_\perp) F(x_2, k_\perp') \right.$$\right.

$$+ e^{i\delta \phi}(\Delta_\perp + \Delta_\perp')\mathcal{A}^*(x_1, \Delta_\perp) \mathcal{E}(x_2, \Delta_\perp) F(x_2, k_\perp) F(x_1, k_\perp') \left\} + c.c. \quad (19)$$

where $dP \cdot S = d^2 p_1 \perp d^2 p_2 \perp dy_1 dy_2 d^2 b_\perp$. $y_1$ and $y_2$ are the final state pions’ rapidities. $k_\perp$, $\Delta_\perp$, $k_\perp'$ and $\Delta_\perp'$ are the incoming photon’s transverse momenta and nucleus recoil transverse momenta in the amplitude and the conjugate amplitude respectively. $b_\perp$ is the transverse distance between the center of the two colliding nuclei. $F(x, k_\perp)$ describes the probability amplitude for finding a photon carries certain momentum. The squared $F(x, k_\perp)$ is just the standard photon TMD distribution $f(x, k_\perp)$. Using the equivalent photon approximation, one has,

$$F(x, k_\perp) = \frac{Z\sqrt{\alpha_s}}{\pi} |k_\perp| \frac{F(k_\perp^2 + x^2 M_p^2)}{(k_\perp^2 + x^2 M_p^2)} \quad (20)$$

where $x$ is constrained according to $x = \sqrt{\frac{p^2 + m^2}{s}} (e^{y_1} + e^{y_2})$. $M_p$ is proton mass. $Z$ is the nuclear charge number. $F$ is the nuclear charge form factor parametrized with the standard Woods-Saxon distribution. Note that the incoming photon carry the different transverse momenta in the amplitude and the conjugate amplitude since we fixed $b_\perp$ $^{50}$ $^{69}$ $^{73}$.

The emergence of this nontrivial azimuthal correlations can be intuitively understood as follows: the linear polarization photon state is the superposition of different helicity states. As illustrated in Fig. $^{[1]}$ the incoming photon carries 1 unit spin angular momentum in the amplitude while it carries -1 unit spin angular momentum in the conjugate amplitude. The orbital angular momentum transferred to quark pair via the two gluon exchange is 2 unit angular momentum as far as the elliptic gluon distribution is concerned. As a result, the orbital angular momentum carried by the produced dipion system is 3 unit in the amplitude and -1 in the conjugate amplitude correspondingly. Such interference amplitude leads to a cos $4\phi$ angular modulation. If one averages over coherent photon polarization, this mechanism gives rise to cos $2\phi$ asymmetry as well, though the dominant contribution to it comes from different source $^{40}$.

**Numerical estimations.** We now introduce models/parametrizations that are necessary for numerical
where we adopt the GBW model for the pion production. In our numerical calculation, and the notation used for the decay of \( \rho^0 \) meson in the amplitude, and are from direct production in the amplitude. The color neutral exchange in the amplitude described by the elliptic gluon distribution effectively carries two unit orbital angular momentum. The incident photon is linearly polarized.

calculations. First of all, the dipole-nucleus scattering amplitude (the azimuthal independent part) is parametrized in terms of dipole-nucleon scattering amplitude \( N(r_{\perp}) \) [74,75].

\[
N(b_{\perp}, r_{\perp}) \approx 1 - [1 - 2\pi B_p T_A(b_{\perp}) N(r_{\perp})]^4
\tag{21}
\]

where we adopt the GBW model for \( N(r_{\perp}) \). We also made the numerical estimates with a more sophisticated treatment for the nuclear thickness function \( T_A(b_{\perp}) \) determined with the Woods-Saxon distribution in our numerical calculation, and \( B_p = 4\text{GeV}^{-1} \). For the scalar part of vector meson function, we use "Gauss-LC" wave function also taken from Ref. [74,75]:

\[
\Omega^2(|r_{\perp}|, z) = \beta z(1 - z) \exp \left[ -\frac{r_{\perp}^2}{2R^2} \right] \quad \text{with} \quad \beta = 4.47, \quad R^2 = 21.9\text{GeV}^{-2}.
\]

The nuclear thickness function is estimated with the Woods-Saxon distribution, \( F(k^2) = \int d^3r e^{i\vec{k} \cdot \vec{r}} \frac{C^0}{1 + \exp[(r-R_{WS})/d]} \) where \( R_{WS} \) (Au: 6.38fm) is the radius and \( d \) (Au:0.535fm) is the skin depth. \( C^0 \) is the normalization factor.

UPCs events measured at RHIC are triggered by detecting accompanied forward neutron emissions. The impact parameter dependence of the probability for emitting any parameter of neutrons from an excited nucleus (referred to as the "X_n" event) is described by the function, \( P(b_{\perp}) = 1 - \exp \left[ -P_{1n}(b_{\perp}) \right] \) with \( P_{1n}(b_{\perp}) = 5.45 \times 10^{-5} \frac{2(\Delta - Z)}{A^{1/3}} \) fm². Therefore, the "tagged" UPC cross section is defined as,

\[
2\pi \int_{2R_A}^{\infty} b_{\perp} db_{\perp} P^2(b_{\perp}) d\sigma(b_{\perp}, ...) \tag{22}
\]

With all these ingredients, we are ready to perform numerical study of the cos4\( \phi \) azimuthal asymmetry for RHIC kinematics.

We first compute the azimuthal averaged cross section and compare it with STAR data to fix the coefficient \( C \approx -10 \) which determines the relative magnitude between the direct pion pair production and that via \( \rho^0 \) decay. We then are able to compute the cos4\( \phi \) asymmetry from the elliptic gluon distribution. The QED and the elliptic gluon distribution contributions to the asymmetry are separately presented in Fig. 2. If we only take into account the final state soft photon radiation effect, the theory calculation severely underestimates the experimental data. To match the STAR data [35,39], a rather large value of the coefficient \( E = 0.4 \) in the Eq. [13] which is roughly one order of magnitude larger than the perturbative estimate for \( E \) [10,17], has been used in our numerical calculation. Since we are dealing with the deep non-perturbative region, it is hard to tell whether such large value for \( E \) is reasonable or not. Moreover, there is a lot of uncertainties associated with the transition from quark pair to di-pion. Other non-perturbative model for describing this transition might lead to a much larger asymmetry with the same value of \( E \). Nevertheless, as demonstrated in Fig. 2 it is clear that the elliptic gluon distribution is a necessary element to account for the observed asymmetry (around 10% ).

We also compute the cos4\( \phi \) azimuthal asymmetry in the process \( \gamma + A \to A' + \pi^+ + \pi^- \) for EIC kinematics with the same set parameters. It is shown in Fig. 3 that the contribution from the elliptic gluon distribution to the asymmetry flips the sign as the result of the absence of the double slit interference effect in eA collisions. It would be very interesting to test this predication at the future EIC. In view of the recent findings [23,24], this might be the only clean observable to probe the gluon Wigner function at EIC, because it is free from the contamination due to the final state soft gluon radiation effect.

**Conclusion.** We studied cos4\( \phi \) azimuthal asymmetry in exclusive di-pion production near \( \rho^0 \) resonance peak in UPCs. Both the final state soft photon radiation effect and the elliptic gluon distribution can give rise to such an asymmetry. It is shown that the QED effect alone, which can be cleanly computed, is not adequate to describe the STAR data. On the other hand, with some model dependent input, a better agreement with the preliminary STAR data is reached after including the elliptic gluon distribution contribution, though the theory calculation still underestimates the measured asymmetry. This thus leads us to conclude that the observed cos4\( \phi \) asymmetry might signal the very existence of the non-trivial quantum correlation encoded in elliptic gluon distribution.

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be smaller than 0.1GeV.

The contributions from the final state soft photon radiation and elliptic gluon distribution to the asymmetry are shown separately.

FIG. 2: The asymmetry is plotted as the function of $q_\perp$ for RHIC energy $\sqrt{S} = 200$GeV. The rapidities $y_1, y_2$ of produced pions are integrated over the region $[-1, 1]$ and $Q$ is integrated over the region $[0.06\text{GeV}, 1\text{GeV}]$. The contributions from the quasi-real photon emitted from electron beam is required to be smaller than 0.1GeV.

The rapidities $y_1, y_2$ of produced pions are integrated over the region $[2, 3]$ and the invariant mass of di-pion $Q$ is integrated over the region $[0.6\text{GeV}, 1\text{GeV}]$. Transverse momentum carried by the quasi-real photon emitted from electron beam is required to be smaller than 0.1GeV.

FIG. 3: The asymmetry in photon production of di-pion in eA collisions at EIC is plotted as the function of $q_\perp$ for the center of mass energy $\sqrt{S} = 100$GeV. The rapidities $y_1, y_2$ of produced pions are integrated over the region $[2, 3]$ and the invariant mass of di-pion $Q$ is integrated over the region $[0.6\text{GeV}, 1\text{GeV}]$. Transverse momentum carried by the quasi-real photon emitted from electron beam is required to be smaller than 0.1GeV.

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