Fuzzy Evaluation Method for Comprehensive Performance of Urban Public Traffic Networks

Hailian Li 1,2, Mengkai Lin 1,2 and Qicai Wang 1,2
1. Key Laboratory of Road & Bridge and Underground Engineering of Gansu Province, Lanzhou Jiaotong University, Lanzhou 730070, China
2. National and Provincial Joint Engineering Laboratory of Road & Bridge Disaster Prevention and Control, Lanzhou Jiaotong University, Lanzhou 730070, China
Email: 17834040@qq.com

Abstract. Aiming at the comprehensive performance evaluation of urban public transport network, this paper established an evaluation index system of urban public transport network with target layer, criterion layer and index layer based on analyzing its subjective and objective influencing factors. Otherwise, a method for determining the weight of the comprehensive performance evaluation index of the city bus network and its evaluation model were presented by using fuzzy mathematics principle. The validity and feasibility of the model and method are further validated by the evaluation of the comprehensive performance of the existing bus network in Lanzhou New District. The evaluation results provide theoretical support for the further optimization of urban bus network.

1. Introduction
The evaluation of urban public transport network is to judge the relative satisfaction of the network characteristics, technical indicators, economy, society and environment of the public transport network. At present, the evaluation index of the public transport network mostly adopts the total transmission volume, daily average passenger flow, passenger flow intensity, route repetition coefficient, line network coverage, non-linear coefficient, travel time and so on. However, for a long time, people often use qualitative methods to evaluate such public transport network, that is, the subjective feelings of the evaluators to determine the "good" and "bad" of the public transport network. Therefore, it has obvious defects: the evaluation results cannot determine the "good" and "bad" of the public transport network, and the evaluation basis is not scientific, realistic and comparable. So it is very important to find a more scientific evaluation method. Based on the characteristics of urban public transport network and the knowledge of fuzzy mathematics, this paper establishes a comprehensive evaluation method of urban public transport network, and makes a comprehensive evaluation of the existing public transport network in Lanzhou.

2. Evaluation Index System of Urban Public Transport Network
The evaluation index system of urban public transit network should be able to independently reflect the characteristics of a specific aspect of the network planning, and be related to the public transit network scheme and the optimization goal. The smaller the number of evaluation indicators, the better, and each evaluation index should be measurable, or obtained by quantitative method, or can be obtained by qualitative classification and comparison method. The evaluation index should be scientific, realistic, comparable and measurable[1]. Through comprehensive analysis and consideration, the evaluation index system of urban public transport network is obtained, as shown in Table 1.
3. Comprehensive Evaluation Model of Urban Public Transport Network

Comprehensive evaluation is to apply the principle of fuzzy transformation and the principle of maximum membership degree, and take into account the various indicators of the evaluated scheme and make a comprehensive evaluation of it [2-4].

3.1. Mathematical Principle of Comprehensive Evaluation of Urban Public Transport Network

(1) The function \( f : [0,1]^n \rightarrow [0,1] \) defining \( n \) variables is a judgment function [5,6].

(2) Let \( f \) be the evaluation function, and \( f(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} x_i a_i, \sum_{i=1}^{n} a_i = 1, a_i \geq 0 \).

(3) Comprehensive evaluation model \( W = B \cdot V^T \), where \( (A, R, V^T) \) is introduced below.

### Table 1. Evaluation index system of urban public transport network

| Target layer | Comprehensive evaluation of the city’s public transportation network |
|--------------|---------------------------------------------------------------------|
| Criteria layer | Network technology performance technology Service Level Benefit level Sustainable development |
|               | Bus stop coverage ratio On-time rate Car integrity rate Line network development adaptability |
| Indicator layer | Bus operation line network density Average waiting time of Peak full load rate Cost of 100 km per car Land development value along the line |
| The density of the bus line network Running speed Full labor productivity Fitting degree of land usage |

3.2. Steps for Comprehensive Evaluation

(1) Establish the set of factors. The set of factors that affect the value (score) of the evaluation object is called a factor set, and the factor set is a normal set, usually represented by the letter \( u = \{u_1, u_2, u_3, \ldots, u_m\} \). All of these factors should be ambiguous [7,8].

(2) Establish the set of weight. Generally speaking, the importance of each factor affecting the value of the evaluation object is not the same. In order to distinguish, a given factor \( u_i (i = 1, 2, 3, \ldots, m) \) should be given a corresponding weight \( a_i (i = 1, 2, 3, \ldots, m) \), and each weight is composed of a set.

\[
A = \{a_1, a_2, a_3, \ldots, a_m\} \quad \text{or} \quad A = \frac{a_1}{u_1} + \frac{a_2}{u_2} + \frac{a_3}{u_3} + \cdots + \frac{a_m}{u_m} \quad (1)
\]

All of them are satisfied \( \sum_{i=1}^{n} a_i = 1, a_i \geq 0 \). \( A \) and \( \sim A \) are called factor weight sets.

(3) Establish an evaluation set. The evaluation set is a collection of various evaluation results that the evaluator may make to the evaluation object, expressed by \( V: V = \{v_1, v_2, v_3, \ldots, v_n\} \). In the evaluation of the bus line network, the evaluation set can be taken as \( V = \{\text{very good, good, average, poor, very poor}\} \) [9].

(4) Univariate fuzzy evaluation. The evaluation is made from one factor alone to determine the
degree of membership of the evaluation object to the elements in the evaluation set, which is called single factor fuzzy evaluation. Under normal circumstances assume that the evaluation object is evaluated according to the i-th factor $u_i$ in the factor set, and the membership degree of the j-th factor $v_j$ in the evaluation set is $\gamma_{ij}$, and the result can be expressed as a fuzzy set:

$$R = \frac{v_{1i}}{v_1} + \frac{v_{2i}}{v_2} + \frac{v_{3i}}{v_3} + \cdots + \frac{v_{ni}}{v_n}, (i = 1, 2, 3, \ldots, n)$$

(2)

$R_i$ is called the single factor fuzzy evaluation set, and the fuzzy matrix $R$ composed of the membership degree of each single factor evaluation set is called the single factor fuzzy evaluation matrix.

$$R = \begin{bmatrix}
    R_{11} & R_{12} & R_{13} & \cdots & R_{1n} \\
    R_{21} & R_{22} & R_{23} & \cdots & R_{2n} \\
    R_{31} & R_{32} & R_{33} & \cdots & R_{3n} \\
    \vdots & \vdots & \vdots & \cdots & \vdots \\
    R_{ni} & R_{n2} & R_{n3} & \cdots & R_{nn}
\end{bmatrix}$$

(3)

(5) Fuzzy comprehensive evaluation. Generally speaking, the same thing has multiple attributes, and different aspects of things reflect their different characteristics. Therefore, when evaluating things, you cannot consider only one factor, but must consider all aspects of things, so in order to comprehensively consider all factors. For the influence of the value of the evaluation object, a fuzzy comprehensive evaluation is needed. If the factors are of equal importance, that is, the weights $a_i$ of the weight set are the same. Then only the elements in the $R$ matrix can be added together, and the "scores" of the elements in the evaluation set are respectively obtained. If the weights of the factors are not equal, then the fuzzy matrix operation $B = A \cdot R$ needs to be performed, so the final result of the fuzzy evaluation is $W = B \cdot V^T$.

4. Example of Comprehensive Evaluation of Bus Line Network

In order to make a reasonable evaluation of the urban public transport network in Lanzhou, the author conducted a field passenger inquiry survey on the urban public transport network in Lanzhou New Area from Dec. 10, 2018 to Dec. 25, 2018, and obtained a lot of valid data about the elements of the public transport network. Then, a quantitative comprehensive evaluation of the overall level of the public transport network in Lanzhou is made by using the fuzzy evaluation method proposed in this paper. The specific process of fuzzy evaluation is as follows:

(1) Factor set (taking the most directly related elements of the five bus line network to the citizens of the city as a factor set)[10,11].

$$u = \{ \text{Convenience}(u_1), \text{Punctuality}(u_2), \text{Rapidity}(u_3), \text{Economy}(u_4), \text{Comfort}(u_5) \}$$

Among them, convenience mainly refers to the convenience of travel. Its basic contents include: the rationality of line network layout, the density of line network, the size of transfer coefficient, the frequency of departure, the rationality of station setting, and the travel needs of different capacities. The two factors that have the greatest impact on convenience are network density and departure frequency. Punctuality is generally expressed by punctuality rate, that is, the ratio of the number of running vehicles on time to the total number of vehicles. In addition to its own factors, the factors affecting punctuality include external driving environment, such as road conditions, degree of intersection congestion, whether there is a dedicated lane, etc. Rapidity refers mainly to the operating speed, that is, the moving speed of buses between stations, followed by the speed of transfer routes
and transit vehicles. In addition, the travel time of each bus (from the starting point to the end point) and the travel interval (that is, the departure interval of the two buses) also constitute an important part of the rapidity. Economy is mainly for passengers, reasonable and cheap fares for passengers are the main factors attracting passengers in public transport; comfort mainly refers to the occupancy rate of passengers, congestion degree, temperature in the car and the smoothness of the bus.

(2) Weight set. According to the opinions of citizens, statistical survey analysis, we can give the evaluation weights of the five components of the bus network.

\[ A = \{a_1, a_2, a_3, a_4, a_5\} = \{0.35, 0.25, 0.2, 0.1, 0.1\} \]

In order to more clearly represent the corresponding relationship between weights and elements, above weight set can also be expressed by:

\[ A = \frac{a_1}{u_1} + \frac{a_2}{u_2} + \frac{a_3}{u_3} + \frac{a_4}{u_4} + \frac{a_5}{u_5} = 0.35 + 0.25 + 0.2 + 0.1 + 0.1 \]

Obviously this weight set satisfies the following condition:

\[ \sum_{i=1}^{5} a_i = 1, \quad a_i \geq 0, \quad (i = 1, 2, 3, 4, 5). \]

(3) Evaluation set. In this case, the evaluation set is:

\[ V = \{v_1, v_2, v_3, v_4, v_5\} = \{\text{very good, good, average, poor, very poor}\} \]

expressed as a vector \( V = \{2, 1, 0, -1, -2\} \) (called the evaluation vector).

(4) Single factor fuzzy evaluation. According to the method described above, the membership degree of each factor relative to different comment levels is given by 10 expert scoring methods, and the corresponding single factor fuzzy evaluation set \( R_i \) is obtained as follows:

\[ R_1 = \frac{0.13}{v_1} + \frac{0.18}{v_2} + \frac{0.26}{v_3} + \frac{0.28}{v_4} + \frac{0.15}{v_5}, \quad R_2 = \frac{0.09}{v_1} + \frac{0.12}{v_2} + \frac{0.34}{v_3} + \frac{0.3}{v_4} + \frac{0.3}{v_5} \]

\[ R_3 = \frac{0.13}{v_1} + \frac{0.12}{v_2} + \frac{0.23}{v_3} + \frac{0.35}{v_4} + \frac{0.15}{v_5}, \quad R_4 = \frac{0.18}{v_1} + \frac{0.24}{v_2} + \frac{0.32}{v_3} + \frac{0.14}{v_4} + \frac{0.12}{v_5} \]

\[ R_5 = \frac{0.05}{v_1} + \frac{0.09}{v_2} + \frac{0.12}{v_3} + \frac{0.38}{v_4} + \frac{0.36}{v_5} \]

The single factor evaluation matrix \( R \) is composed of the membership degree of each single factor evaluation set.

\[
\begin{bmatrix}
0.13 & 0.18 & 0.26 & 0.28 & 0.15 \\
0.09 & 0.12 & 0.34 & 0.36 & 0.09 \\
0.13 & 0.14 & 0.23 & 0.35 & 0.15 \\
0.18 & 0.24 & 0.32 & 0.14 & 0.12 \\
0.05 & 0.09 & 0.12 & 0.38 & 0.36
\end{bmatrix}
\]

(5) Fuzzy comprehensive evaluation. Because the weights of factors are not equal, the fuzzy matrix calculation \( B = A \cdot R \) is needed.

\[
\begin{bmatrix}
0.13 & 0.18 & 0.26 & 0.28 & 0.15 \\
0.09 & 0.12 & 0.34 & 0.36 & 0.09 \\
0.13 & 0.14 & 0.23 & 0.35 & 0.15 \\
0.18 & 0.24 & 0.32 & 0.14 & 0.12 \\
0.05 & 0.09 & 0.12 & 0.38 & 0.36
\end{bmatrix}
\times
\begin{bmatrix}
0.35 & 0.25 & 0.2 & 0.1 & 0.1 \\
0.13 & 0.14 & 0.23 & 0.35 & 0.15 \\
0.18 & 0.24 & 0.32 & 0.14 & 0.12 \\
0.05 & 0.09 & 0.12 & 0.38 & 0.36
\end{bmatrix} =
\begin{bmatrix}
0.117 & 0.154 & 0.266 & 0.31 & 0.153 \\
0.09 & 0.12 & 0.34 & 0.36 & 0.09
\end{bmatrix}
\]

The final evaluation result of the fuzzy matrix is:
Therefore, the comprehensive evaluation results of the existing public transport network in Lanzhou New Area are between "general" and "poor", but more inclined to "general".

5. Conclusion
With the rapid development of urban transportation, the qualitative study of urban transportation can no longer meet the needs of real life. As a subject of studying fuzzy phenomena, the scientific, accurate, intuitive and convenient evaluation of fuzzy mathematics has been widely recognized. So it is feasible to use the fuzzy evaluation method to evaluate the urban public transport network, but the difficulty of the evaluation method lies in the determination of the weight of each factor and the evaluation matrix. Therefore, in the process of establishing the evaluation system, all factors should be considered as far as possible, and each factor should be measurable and comparable.

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