Time-modulation of entangled two-body weak decays with massive neutrinos

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Abstract. In recent experiments at the GSI, Darmstadt, time-modulated orbital Electron Capture (EC) decays of H-like $^{140}$Pr$^{58+}$, $^{142}$Pm$^{60+}$, and $^{122}$I$^{52+}$ ions with one electron in the K-shell, coasting in the ESR storage ring with velocity $\beta = 0.71$, were observed. The EC-branches observed using time-resolved Schottky Mass Spectroscopy show exponential decay curves time-modulated with periods $T = 7.06(8)$ s, $T = 7.10(22)$ s, and $T = 6.16(3)$ s and amplitudes $a = 0.18(3)$, $a = 0.23(4)$, and $a = 0.16(2)$ for $^{140}$Pr, $^{142}$Pm, and $^{122}$I decays, respectively. The $\beta^+$ branch of $^{142}$Pm shows no modulation with $a < 0.03$. As origin of the modulation, we propose quantum beats produced by massive neutrino mass-eigen states emitted in the two body weak decay. From the modulation frequency a value for the difference of the quadratic mass values $m_2^2 - m_1^2 = 2.22(3) \times 10^{-4}$ eV$^2$ is deduced, which is 2.9 times larger than the value derived by the KamLAND antineutrino oscillation experiment. The origin of the small modulation amplitudes is discussed as the result of partial restoration of the interference terms which are expected to cancel for the usual assumed unitarity of the neutrino flavour mixing matrix.

1. Introduction
The heavy ion accelerator and storage ring facility of the GSI, Darmstadt, offers unique opportunities for weak decay studies of highly ionized atoms [1]. In this report we focus on the first results of studying the time evolution of quasi free two body weak decays. In this way we investigate the properties of massive neutrinos by observing the appearance time of the daughter nucleus, detected with 100% efficiency, and entangled with the massive neutrinos. We focus here on the orbital electron capture of hydrogen like ions, (H-like), with one electron in the K-shell, and observe such decays in a storage ring quasi-free without the complication of such decays in a solid. Our aim was to search for quantum beat like phenomena possibly introduced by mixed massive neutrinos. Its properties can then be studied without observing the neutrino directly due to the quantum-entanglement with the daughter nucleus thus avoiding the small detection efficiency of neutrinos.

For our first experiments [2] we selected allowed Gamow-Teller transitions, GT ($1^+ \rightarrow 0^+$), with strong branches to $0^+$ ground-states of stable even-even nuclei, with EC/$\beta^+$ branches close to unity, and half-lives in the range of minutes for the observation of beats with modulation times of several seconds and a time resolution in the 100 ms range. Experiments with H-like $^{140}$Pr and $^{142}$Pm ions fulfill the entire requirements. In addition they have quite different decay energies ($Q_{EC}$-values) and life-times and thus allow a detailed comparison of the time evolution of the decays with different neutrino energies in different time ranges. Both $Q_{EC}$-values of
several MeV are large enough to be well resolved by measuring the time correlated change of the revolution frequency when the mother-ion decays into the daughter-ion without change of their charge. In a recent experiment we investigated also the \((1^+ \rightarrow 0^+)\) GT transition of H-like \(^{122}\text{I}\) in order to learn about the A-dependence of the time evolution of such two body decays [3, 4].

2. Experimental results

![Figure 1](image1.png)

**Figure 1.** Time dependence of the EC decay rate of \(^{140}\text{Pr}\) and \(^{142}\text{Pm}\) showing periodic modulations of about 7 s (dashed curve) of the exponential decays (solid curve). The insets show Fourier frequency spectra with maxima at frequencies of \(f = 0.14 \text{ s}^{-1}\) [2] (upper two panels). Time dependence of the EC decay rate of \(^{122}\text{I}\) with a periodic modulation of about 6 s (lower panel left). \(\chi^2(\omega)\) for the fit of the \(^{122}\text{I}\) data with a modulated EC decay rate as function of the modulation frequency \(\omega\) with all other parameters free (lower panel right) [3, 4].

Fig. 1 shows the measured decay spectra for H-like \(^{140}\text{Pr}\) and \(^{142}\text{Pm}\) ions (upper panels) [2]. A fit with a pure exponential decay function,

\[
\frac{dN_{EC}(t)}{dt} = N(0) \cdot \lambda_{EC} \cdot e^{-\lambda t} \quad (1)
\]

as indicated with the solid curve, failed to reproduce the data. The expected exponential decay shows a superimposed periodic time modulation. To account for this modulation the data was fitted with the function:

\[
\frac{dN_{EC}(t)}{dt} = N(0) \cdot \lambda_{EC}(t) \cdot e^{-\lambda t} \quad (2)
\]
with a time dependent decay probability

\[ \lambda_{EC}(t) = \lambda_{EC} [1 + a \cos(\omega t + \phi)] \]  

(3)

representing a periodic modulation of the partial EC-decay rate \( \lambda_{EC}(t) \) with an amplitude \( a \), an angular-frequency \( \omega \), and a phase \( \phi \). For details of the measuring procedure of the decays in the storage ring using time-depandent, single ion Schottky Mass Spectroscopy (SMS) and the results of all fit parameters of both data sets see reference [2] Table 2. Here we quote briefly the main results.

The total decay constants \( \lambda \) obtained from both fitting functions, Eqs. (1) and (2) agree within their error margins for the \(^{140}\)Pr data and for the \(^{142}\)Pm data. All thus deduced decay constants \( \lambda \) agree within two standard errors with the corresponding literature values for neutral atoms corrected for the missing electrons and taking into account the observed “enhancement” of the EC decay of H-like ions [5, 6]. From the angular frequencies 0.890(11) s\(^{-1}\) and 0.885(31) s\(^{-1}\) we extract the periods of modulation as \( T = 7.06(8) \) s and \( T = 7.10(22) \) s in the laboratory frame for the decays of \(^{140}\)Pr and \(^{142}\)Pm, respectively. Note that the modulation periods are within their errors the same for both decays and thus independent of the energies of the emitted neutrinos; in contrast they indicate scaling with the mass of the ions. Also the amplitudes \( a = 0.18(3) \), \(^{140}\)Pr, and \( a = 0.23(4) \), \(^{142}\)Pm, agree within their error margins; the average of both systems is \( \langle a \rangle = 0.20(2) \). The results for the phases of the modulation reflect the uncertainties introduced by the difference and variation of the cooling times of the daughter ions as outlined in [2].

In the following we show a still preliminary result gained with the goal to complement the observations so far reported in [2]. In order to test our measuring method and determine whether the observed modulation are characteristic for the two body EC decay, the simultaneously observed decay spectra of the \( \beta^+ \) decay of \(^{142}\)Pm, which is about three time stronger than the EC branch, was analysed [3, 4]. It shows a pure exponential decay, whereas the decay spectrum of the EC branch gained with the same data set showed the expected modulation with angular frequency of 0.90(2) s\(^{-1}\) and an amplitude of \( a = 0.18(5) \). At a modulation frequency of \( \omega = 0.90 \) s\(^{-1}\) only a vanishing modulation amplitude of \( a = 0.03(3) \) is indicated for the \( \beta^+ \) branch [3, 4], as expected theoretically for a continuous neutrino spectrum [7].

In August 2008 we performed a further experiment [3, 4] to study the decay spectrum of the EC decay branch of the \((1^+ - 0^+)\) GT-transition of H-like \(^{122}\)I\(^{52+}\) to the ground state of stable \(^{122}\)Te\(^{52+}\). The main purpose of this experiment was studying the A-dependence of the observed modulation frequency. Fig. 1 (Lower panel left) shows an EC decay spectrum of H-like \(^{122}\)I\(^{52+}\) of a very recent analysis [3], and a fit of a time modulated exponential decay to the data. It shows a modulation with an angular frequency \( \omega = 1.038(6) \) s\(^{-1}\), an amplitude \( a = 0.16(2) \). In the lower right panel of Fig. 1 we show the mean square deviation, \( \chi^2(\omega) \), of the fit of the data to a modulated decay spectrum eq. (2) as function of the angular frequency \( \omega \) with all other parameters free. A clear minimum of \( \chi^2(\omega) \) is seen at \( \omega = 1.038(6) \) s\(^{-1}\) with a statistical relevance of about 6\( \sigma \). So the reduced value of the observed modulation period of \( T = 6.13(6) \) s for \(^{122}\)I compared with 7.06(8) s for \(^{140}\)Pr indicates that \( T \) scales with the mass number A of the decaying system.

3. Discussion of the results

The periodically modulated exponential EC decay has now been observed for three H-like systems \(^{140}\)Pr, \(^{142}\)Pr, and \(^{122}\)I. The amplitude of the modulation is within the error margin the same in all three systems. Its periods scale with the mass number A of the decaying systems and are independent of their \( Q_{EC} \)-values and their half-lives. The three-body \( \beta^+ \) decay of \(^{142}\)Pm is according to preliminary data not modulated with a limit of its amplitude \( a = 0.03(3) \) [4]. This result, though preliminary, is most important because it indicates that the modulation is directly
connected to the weak decay properties, as it is present for the two-body decay branch only, and absent for the three body decays [7]. It excludes experimental effects and also nuclear properties of the initial state [8, 9]; it thus points to an effect caused by the neutrinos of the EC branch. A further important result is that the time averaged decay probability of the modulated decays is fully consistent with the decay constant of neutral atoms after proper correction [2, 5, 6]. This excludes the influence of possible oscillatory transitions with a period of 7 s between the $F = 1/2$ hyperfine ground state of the H-like system and its $F = 3/2$ excited state which is inert against weak decays. The measured EC/β$^+$ ratios for H- and He-like decays of $^{140}$Pr agrees within 3% with theoretical predictions assuming mass-less neutrinos [5, 6]. This excludes neutrino flavour oscillations as reason for the time modulation of the EC decays because their time averaged decay constants would be reduced in the case of oscillations relative to that of the amplitudes of EC decays of H-like heavy ions.

So what is the origin of these so called “GSI Oscillations?” Strongly controversial arguments have been put forward, one which connects the observations with quantum beats of flavour mixed massive neutrinos [10, 11, 12] and the others claim that the effect has nothing to do with neutrinos [8, 9, 13, 14, 15]. The latter is based essentially on two arguments that interference terms in the decay probability cannot appear in the present GSI experiment:

i) The final mass-eigenstates of the neutrinos are orthogonal and thus asymptotically being identified by different energies and momenta with the consequence that the interference terms are not observable. We have shown [16] that this assertion does not respect the time differential observation of the development of the decay in our experiments which induces energy and momentum uncertainties leading to an overlap of indistinguishable decay paths and thus to interference in time which may be classified as “neutrino quantum beats” [17].

ii) There is a serious objection against our approach [16], based on the argument that the “neutrinos are not observed” in our experiments. Various authors put forward as a “theorem”, that only the squares of the flavour transition amplitudes should be incoherently added in case that the “neutrinos are not observed”, and concluded that the observed time modulation of the EC decay has nothing to do with neutrinos. As we have argued experimentally that the observed modulation must originate from the two-body decay involving the emission of massive neutrinos we have proposed possible ways to get around this “non-neutrino theorem” which is based on unitarity of the flavour mixing matrix [18, 19].

In our theory approach, published recently [16] the amplitude $A(m \rightarrow d + \nu_e)(t)$ of the EC decay of a H-like mother ion $m$ into a daughter ion $d$ and an electron neutrino $\nu_e$ is defined as the sum of the amplitudes of the emission of massive neutrino mass eigenstates $m \rightarrow d + \nu_j$ as follows:

$$A(m \rightarrow d + \nu_e)(t) = \sum_j U_{ej} A(m \rightarrow d + \nu_j)(t)$$  \hspace{1cm} (4)

where the coefficients $U_{ej}$ take into account that only electron neutrinos contribute to the transition. As a result of time dependent perturbation treatment using wave packets for the daughter ion wave functions for description of their time dependent observation a time modulation of the EC decay probability is predicted in the form

$$\lambda_{EC}(t) = \lambda_{EC} [1 + a \cos(\omega_2 t)]$$  \hspace{1cm} (5)

with the modulation amplitude $a$ determined by the neutrino flavour mixing angle $\theta_{12}$

$$a = \sin(2\theta_{12})$$  \hspace{1cm} (6)
and the modulation frequency $\omega_{21}$ determined by energy and momentum conservation of the decay

$$\omega_{21} = (m_2^2 - m_1^2)/2M_m$$  \hspace{1cm} (7)

Note, $\omega_{21}$ is equal to the recoil energy differences of the neutrinos with mass $m_2$ and $m_1$ emitted by the mother nucleus with mass $M_m$ in the cm system. Note also that, as observed, the modulation frequency scales as the inverse of the mother masses $M_m$ and a numerical value

$$m_2^2 - m_1^2 = 2.12 \times 10^{-4} \text{ eV}^2$$  \hspace{1cm} (8)

is derived as averaged value from the three decays observed. It is 2.9 larger than the value reported by the KamLAND collaboration for oscillations of antineutrinos from fission products. A possible solution of this difference in terms of neutrino and antineutrino mass corrections of opposite signs induced by virtual lepton-$W$ boson loops in the Coulomb field of heavy nuclei is proposed in [20].

Our approach [16] was criticised recently by Gal [18] and Yazaki [19] who showed that by using the total set of orthogonal neutrino flavour wave function in calculation of the transition probability, and requiring unitarity of the flavour mixing matrix, the interference terms cancel. There are in addition two experimental reasons to use the complete set of neutrino flavour functions with definite lepton charges: i) The indication of a finite phase $\phi_{\text{EC}}$ in the measured modulation pattern which is expected to be $\phi_{\text{EC}} = 0$ in our approach [16]. ii) The small value of the modulation amplitude which is given in our approach as

$$a = \sin(2\theta_{12})$$

is not observed in our experiments.

Thus we have proposed recently [17] an approach to define the amplitude of EC decays as coherent superposition of the amplitude $A(m \to d\nu_\alpha)(t)$ given by:

$$A(m \to d\nu)(t) = \sum_\alpha e^{-i\phi_\alpha} A(m \to d\nu_\alpha)(t)$$  \hspace{1cm} (9)

with the flavour charge index $\alpha$ running over $\alpha = e, \mu$ and $\tau$. In addition we have introduced arbitrary phases $\phi_\alpha$ to take care of the possible different phase shifts of the lepton flavour components. For the amplitude we are using the following general expression:

$$A(m \to d\nu_j)(t) = \sum_\alpha \sum_j U_{\alpha j} U_{ej} e^{-i\phi_\alpha} A(m \to d\nu_j)(t)$$  \hspace{1cm} (10)

with

$$A(m \to d\nu_j)(t) = -\delta_{M_{F,-\frac{1}{2}}} \sqrt{6M_m M_G T} \langle \psi_1^{(Z)} | \sqrt{2E_d(k_j')} E_j(k_j) e^{i(\Delta E_j - i\varepsilon)t} \Phi_d(k_j + q_j)$$  \hspace{1cm} (11)

with notations as defined in [17].

Using this formalism we get the following interference term in the transition rate as function of the time $t$:

$$\sum_{l>j} 2\text{Re} \left[ \sum_{\beta \alpha} U_{\beta l}^* U_{e \alpha} e^{i(\phi_{\beta} - \phi_{\alpha})} U_{\alpha l} U_{e j} e^{i\omega_{21}t} \right] = \frac{1}{\sqrt{2}} \sin(2\theta_{12}(\sin \phi_{\mu e} - \sin \phi_{\tau e}) \sin(\omega_{21}t)$$  \hspace{1cm} (12)

which is modulated as $\sin(\omega_{21}t)$ with a reduced amplitude

$$a_{\text{EC}} = \frac{1}{\sqrt{2}} \sin(2\theta_{12}(\sin \phi_{\mu e} - \sin \phi_{\tau e})$$  \hspace{1cm} (13)
determined by the mixing angle $\theta_{12}$ (For simplicity it was assumed that $\theta_{13} = 0$ and $\theta_{23} = \pi/4$), and the difference of $\sin \phi_{\mu e}$ and $\sin \phi_{\tau e}$. Note that the interference term vanishes for $\phi_{\mu e} = \phi_{\tau e}$ as well as for the special case that both phase differences are zero. The origin of these CP violating phase shifts are so far unknown. They may contain also charged lepton-nucleus Coulomb phase shifts.

Since the GSI experiment did observe modulations with an amplitude $a = 0.20(2)$, Kleinert and Kienle [22] concluded recently that for the partial restoration of the interference term the unitarity of the flavor mixing matrix must be violated. From the observed modulation amplitude a violation of about 10% is derived [22]. A possible origin for such a unitarity violation could be that there are more than three families of leptons in nature and that universality of weak interaction is not valid for all of them, such as for the proposed sterile neutrinos.

More data will be needed to find out which mechanism causes the observed modulation, such as direct comparison of the modulation of bound $\beta$ decay and EC decay of $^{108}$Ag as proposed by us in this context [23]. In such a case the modulation produced by antineutrinos and neutrinos can be compared directly, especially the effect of the Coulomb field which has opposite sign for neutrinos and antineutrinos could be studied.

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