Optical space-time wave packets having arbitrary group velocities in free space

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The publication of Einstein’s seminal work on special relativity initiated an investigation of the speed of light in materials featuring strong chromatic dispersion [1]. Indeed, the group velocity \( v_g \) of an optical pulse in a resonant dispersive medium can deviate significantly from the speed of light in vacuum \( c \), without posing a challenge to relativistic causality when \( v_g > c \) because the information speed never exceeds \( c \) [1, 2]. Modifying the temporal spectrum in this manner is the basic premise for the development of so-called ‘slow light’ and ‘fast light’ [3] in a variety of material systems including ultracold atoms [4], hot atomic vapors [5, 6], stimulated Brillouin scattering in optical fibers [7], and active gain resonances [8, 9]. Additionally, nanofabrication yields photonic systems that deliver similar control over the group velocity through structural dispersion in photonic crystals [10], metamaterials [11], tunneling junctions [12], and nanophotonic structures [13]. In general, resonant systems have limited spectral bandwidths that can be exploited before pulse distortion obscures the targeted effect, with the pulse typically undergoing absorption, amplification, or temporal reshaping, but without necessarily affecting the field spatial profile.

In addition to temporal spectral modulation, it has been recently appreciated that structuring the spatial profile of a pulsed beam can impact its group velocity in free space [14–16]. In a manner similar to pulse propagation in a waveguide, the spatial spectrum of a structured pulsed beam comprises plane-wave contributions tilted with respect to the propagation axis, which undergo larger delays between two planes than purely axially propagating modes. A large-area structured beam (narrow spatial spectrum) can travel for longer distances before beam deformation driven by diffraction and space-time coupling, but its group velocity deviates only slightly from \( c \); whereas a narrow beam deviates further from \( c \), but travels a shorter distance. Consequently, \( v_g \) is dependent on the size of the field spatial profile, and the maximum group delay observable is limited by the numerical aperture. Only velocities slightly lower than \( c \) (\( \approx 0.99999 c \)) have been accessible in the experiments performed to date with maximum observed group delays of \( \sim 30 \) fs, corresponding to a shift of \( \sim 10 \) µm over a distance of 1 m (or 1 part in \( 10^5 \)).

Here we demonstrate precise and versatile control over the magnitude and sign of the free-space group velocity of a propagation-invariant wave packet by sculpting its spatio-temporal profile. Instead of manipulating separately the field spatial or temporal degrees of freedom and attempting to minimize unavoidable space-time coupling, we intentionally introduce into the wave packet tight spatio-temporal spectral correlations that result in the realization of arbitrary group velocities: superluminal, luminal, or subluminal, whether in the forward direction propagating away from the source or in the backward direction traveling toward it. The group velocity here is the speed of the wave packet central spatio-temporal peak. By judiciously associating each wavelength in the pulse spectrum with a particular transverse spatial frequency, we trace out a conic section on the surface of the light-cone while maintaining a linear relationship between the axial component of the wave vector and frequency. The slope of this linear relationship dictates the wave packet group velocity, and its linearity eliminates any additional dispersion terms. The resulting wave packets propagate free of diffraction and dispersion [17–20], which makes them ideal candidates for unambiguously observing group velocities in free space that deviate substantially from \( c \).

There have been previous efforts directed at the synthesis of optical wave packets endowed with spatio-temporal correlations. Several strategies have been implemented to date, which include exploiting the techniques associated with the generation of Bessel beams, such as the use of annular apertures in the focal plane of a spherical lens [21] or utilizing axicons [22–24] synthesis of X-waves [25] during nonlinear processes such as second-harmonic generation [26] or laser filamentation [27, 28] or through direct filtering of the requisite spatio-temporal spectrum [29, 30]. The reported superluminal speeds achieved with these various approaches in free space have been to date 1.00022\( c \) [23], 1.00012\( c \) [24], and 1.00015\( c \) [31], and 1.111\( c \) in a plasma [22]. Reports on measured subluminal speeds have been lacking [19] and limited to delays of hundreds of femtoseconds over
a distance of 10 cm [32, 33], corresponding to a group velocity of \( \approx 0.999c \). There have been no experimental reports to date on negative group velocities in free space.

Here, we synthesize ‘space-time’ (ST) wave packets [34–36] using a phase-only spatial light modulator (SLM) that efficiently sculpts the field spatio-temporal spectrum and modifies the group velocity dynamically. The ST wave packets are synthesized for simplicity in the forward direction to \(-4c\) in the backward direction. We observe group delays of \( \approx \pm 30\) ps (three orders-of-magnitude larger than those in [14, 15]), which is an order-of-magnitude longer than the pulse width, and is observed over a distance of only \( \sim 10\) mm. Adding to the uniqueness of our approach, the achievable group velocity is independent of the beam size and of the pulse width. All that is needed to change the group velocity is a reorganization of the spectral correlations underlying the wave packet spatio-temporal structure.

The novelty of our approach is its reliance on a linear system that utilizes a phase-only spatio-temporal Fourier synthesis strategy, which is energy-efficient and precisely controllable [36]. Our approach allows for endowing the field with programmable spatio-temporal spectral correlations that can be tuned to produce – smoothly and continuously – any desired wave packet group velocity. The precision of this technique with respect to previous approaches is attested by the unprecedented range of control over the measured group velocity values over the subluminal, superluminal, and negative regimes in a single optical configuration. Crucially, while distinct theoretical proposals have been made previously for each range of the group velocity (e.g., subluminal [37–39], superluminal [40], and negative [41] spans), our strategy is to the best of our knowledge the only experimental arrangement capable of controlling the group velocity continuously across all these regimes (with no moving parts) simply through the electronic implementation of a phase pattern imparted to a spectrally spread wavefront impinging on a SLM.

**RESULTS**

Concept of space-time wave packets.

The properties of ST light sheets can be best understood by examining their representation in terms of monochromatic plane waves \( e^{i(k_x x + k_z z - \omega t)} \), which are subject to the dispersion relationship \( k_x^2 + k_z^2 = \left( \frac{\omega}{c} \right)^2 \) in free space; here, \( k_x \) and \( k_z \) are the transverse and longitudinal components of the wave vector along the \( x \) and \( z \) coordinates, respectively, \( \omega \) is the temporal frequency, and the field is uniform along \( y \). This relationship corresponds geometrically in the spectral space \( (k_x, k_z, \frac{\omega}{c}) \) to the surface of the light-cone (Fig. 1). The spatio-temporal spectrum of any physically realizable optical field compatible with causal excitation must lie on the surface of the light-cone with the added restriction \( k_z > 0 \). For example, the spatial spectra of monochromatic beams lie along the circle at the intersection of the light-cone with a horizontal iso-frequency plane, whereas the spatio-temporal spectrum of a traditional pulsed beam occupies a two-dimensional (2D) patch on the light-cone surface.

The spectra of ST wave packets do not occupy a 2D patch, but instead lie along a curved one-dimensional (1D) trajectory resulting from the intersection of the light-cone with a tilted spectral hyperplane \( P(\theta) \) described by the equation \( \omega = k_0 + (k_z - k_0) \tan \theta \), where \( k_0 = \frac{\omega}{c} \) is a fixed wave number [36], and the ST wave packet thus takes the form

\[
E(x, z; t) = e^{i(k_0 z - \omega_0 t)} \int dk_x \psi(k_x) e^{i(k_0 x + k_z z - k_0 [z - \sqrt{z^2 - k_0^2} \tan \theta])}
\]

Therefore, the group velocity along the \( z \)-axis is \( v_g = \frac{\partial \omega}{\partial k_c} = \omega \cot \theta \), and is determined solely by the tilt of the hyperplane \( P(\theta) \). In the range \( 0 < \theta < 45^\circ \), we have a subluminal wave packet \( v_g < c \), and \( P(\theta) \) intersects with

![Fig. 1. Spatio-temporal spectral engineering for arbitrary control over the group velocity of an optical wave packet in free space. a-c, Conic-section trajectories at the intersection of the light-cone with a spectral hyperplane \( P(\theta) \) are spatio-temporal spectral loci of ST wave packets with tunable group velocities \( v_g \) in free space. a, subluminal; b, superluminal; c, negative-superluminal \( v_g \). For each case we plot the projection of the spatio-temporal spectrum onto the \((k_x, k_z)\)-plane restricted to \( k_z > 0 \) (dashed white curve). d, Projections of the spatio-temporal spectra from (a-c) onto the \((k_x, \frac{\omega}{c})\)-plane. The slope of each projection determines the group velocity of each ST wave packet along the axial coordinate \( z \), \( v_g = -\cot \theta \).](image)
the light-cone in an ellipse [Fig. 1a]. In the range 45°<θ<90°, we have a superluminal wave packet v_g>c, and P(θ) intersects with the light-cone in a hyperbola [Fig. 1b]. Further increasing θ reverses the sign of v_g such that the wave packet travels backwards towards the source v_g<0 in the range 90°<θ<180° [Fig. 1c]. These various scenarios are summarized in Fig. 1d.

Experimental realization

We synthesize the ST wave packets by sculpting the spatio-temporal spectrum in the (k_x, ω)-plane via a two-dimensional pulse shaper [36, 42]. Starting with a generic pulsed plane wave, the spectrum is spread in space via a diffraction grating before impinging on a SLM, such that each wavelength λ occupies a column of the SLM that imparts a linear phase corresponding to a pair of spatial frequencies ±k_x that are to be assigned to that particular wavelength, as illustrated in Fig. 2a; see Methods. The retro-reflected wave front returns to the diffraction grating that superposes the wavelengths to reconstitute the pulse and produce the propagation-invariant ST wave packet corresponding to the desired hyperplane P(θ). Using this approach we have synthesized and confirmed the spatio-temporal spectra of 11 different ST wave packets in the range 0°<θ<180° extending from the subluminal to superluminal regimes. Figure 3 shows the measured spatio-temporal spectral intensity |E(k_x,λ)|^2 (Fig. 3a) for a ST wave packet having θ=53.2° and thus lying on a hyperbolic curve on the light-cone corresponding to a positive superluminal group velocity of v_g=1.34c. The spatial bandwidth is ∆k_x=0.11 rad/µm, and the temporal bandwidth is ∆λ=0.3 nm. This spectrum is obtained by carrying out an optical Fourier transform along x to reveal the spatial spectrum, and resolving the temporal spectrum with a diffraction grating. Our spatio-temporal synthesis strategy is distinct from previous approaches that make use of Bessel-beam-generation techniques and similar methodologies [21–24, 40], nonlinear processes [26–28], or spatio-temporal filtering [29, 30]. The latter approach utilizes a diffraction grating to spread the spectrum in space, a Fourier spatial filter then carves out the requisite spatio-temporal spectrum, resulting either in low throughput or high spectral uncertainty. In contrast, our strategy exploits a phase-only modulation scheme that is thus energy-efficient and can smoothly and continuously (within the precision of the SLM) tune the spatio-temporal correlations electronically with no moving parts, resulting in a corresponding controllable variation in the group velocity.

To map out the spatio-temporal profile of the ST wave packet I(x,z,t)=|E(x,z,t)|^2, we make use of the interferometric arrangement illustrated in Fig. 2a. The initial pulsed plane wave (pulse width ~100 fs) is used as a reference and travels along a delay line that contains a spatial filter to ensure a flat wave front (see Methods and Supplementary Material). Superposing the shorter reference pulse and the synthesized ST wave packet (Eq. 1) produces spatially resolved interference fringes when they overlap in space and time – whose visibility reveals the spatio-temporal pulse profile. The measured intensity profile I(x,0,τ)=|E(x,0,τ)|^2 of the ST wave packet having θ=53.2° is plotted in Fig. 3b; τ is the delay in the reference arm. Plotted also are the pulse profile at the beam center I(0,0,τ)=|E(x=0,0,τ)|^2 whose width is ≈4.2 ps, and the beam profile at the pulse center I(x,0,0)=|E(x,0,0)|^2 whose width is ≈16.8 µm. Previous approaches for mapping out the spatio-temporal profile of propagation-invariant wave packets have made use of strategies ranging from spatially resolved ultrafast pulse measurement techniques [24, 32, 33] to self-referenced interferometry [29, 36].

Controlling the group velocity of a space-time wave packet

We now proceed to make use of this interferometric arrangement to determine v_g of the ST wave packets as we vary the spectral tilt angle θ. The setup enables synchronizing the ST wave packet with the luminal reference
pulse while also uncovering any dispersion or reshaping in the ST wave packet with propagation. We first synchronize the ST wave packet with the reference pulse and take the central peak of the ST wave packet as the reference point in space and time for the subsequent measurement. An additional propagation distance $L_1$ is introduced into the path of the ST wave packet, corresponding to a group delay of $\tau_g/L_1/v_g \gg \Delta \tau$ that is sufficient to eliminate any interference. We then determine the requisite distance $L_2$ to be inserted into the path of the reference pulse to produce a group delay $\tau_g=L_2/c$ and regain the maximum interference visibility, which signifies that $\tau_g=\tau_c$. The ratio of the distances $L_1$ and $L_2$ provides the ratio of the group velocity to the speed of light in vacuum $L_1/L_2=v_g/c$.

In the subluminal case $v_g<c$, we expect $L_1<L_2$; that is, the extra distance introduced into the path of the reference traveling at $c$ is larger than that placed in the path of the slower ST wave packet. In the superluminal case $v_g>c$, we have $L_1>L_2$ for similar reasons. When considering ST wave packets having negative-$v_g$, inserting a delay $L_1$ in its path requires reducing the initial length of the reference path by a distance $-L_2$ preceding the initial reference point, signifying that the ST wave packet is traveling backwards towards the source. As an illustration, the inset in Fig. 3b shows the same ST wave packet shown in the main panel of Fig. 3b observed after propagating a distance of $L_1=10$ mm, which highlights the self-similarity of its free evolution [36]. The time axis is shifted by $\tau_g=24.88$ ps, corresponding to $v_g=(1.36\pm4\times10^{-4})c$, which is excellent agreement with the expected value of $v_g=1.34c$.

The results of measuring $v_g$ while varying $\theta$ for the positive-$v_g$ ST wave packets are plotted in Fig. 4. The values of $v_g$ range from subluminal values of 0.5$c$ to the superluminal values extending up to $32c$ (corresponding to values of $\theta$ in the range $0^\circ<\theta<90^\circ$). The case of a luminal ST wave packet corresponds trivially to a pulsed plane wave generated by idling the SLM. The data is in excellent agreement with the theoretical prediction of $v_g=\cot\theta$. The measurements of negative-$v_g$ ($90^\circ<\theta<180^\circ$) are plotted in Fig. 4, inset, down to $v_g=-4c$, and once again are in excellent agreement with the expectation of $v_g=\cot\theta$.

FIG. 3. Spatio-temporal measurements of ST wave packets. a,b, Spatio-temporal (a) spectrum $|\tilde{E}(k,\lambda)|^2$ and (b) intensity profile $I(x,0,\tau)$ for a ST wave packet with superluminal group velocity $v_g=1.34c$, corresponding to a spectral hyperplane with $\theta=53.2^\circ$. b, The yellow and orange lines depict the pulse profile at $x=0$, $I(0,0,\tau)$, and beam profile at $\tau=0$, $I(x,0,0)$, respectively. The inset shows the spatio-temporal intensity profile after propagating for $z=10$ mm confirming the self-similar evolution of the ST wave packet. On the right, the normalized time-integrated beam profile $\bar{I}(x,0)=\int d\tau I(x,0,\tau)$ is given.

FIG. 4. Measured group velocities for ST wave packets. By changing the tilt angle $\theta$ of the spectral hyperplane $P$, we control $v_g$ of the synthesized ST wave packets in the positive subluminal and superluminal regimes (corresponding to $0^\circ<\theta<90^\circ$). We plot $v_g$ on a logarithmic scale. Measurements of ST wave packets with negative-$v_g$ (corresponding to $\theta>90^\circ$) are given as points in the inset on a linear scale. The data in the main panel and in the inset is represented by points and both curves are the theoretical expectation $v_g=\cot\theta$. The error bars for the measurements are too small to appear, and are provided in Table S1 in the Supplementary Material.
DISCUSSION

An alternative understanding of these results makes use of the fact that tilting the plane \(P(\theta)\) from an initial position of \(P(0)\) corresponds to the action of a Lorentz boost associated with an observer moving at a relativistic speed of \(v_g = \tan \theta\) with respect to a monochromatic source \((\theta = 0)\) [17, 18, 42]. Such an observer perceives in lieu of the diverging monochromatic beam a non-diverging wave packet of group velocity \(v_g\) [43]. Indeed, at \(\theta = 90^\circ\) a condition known as ‘time-diffraction’ is realized where the axial coordinate \(z\) is replaced with time \(t\), and the usual axial dynamics is displayed in time instead [17, 42, 44, 45]. In that regard, our reported results here on controlling \(v_g\) of ST wave packets is an example of relativistic optical transformations implemented in a laboratory through spatio-temporal spectral engineering.

Note that it is not possible in any finite configuration to achieve a delta-function correlation between each spatial frequency \(k_x\) and wavelength \(\lambda_x\) instead, there is always a finite spectral uncertainty \(\delta \lambda\) in this association. In our experiment, \(\delta \lambda \sim 24\) pm (Fig. 3a), which sets a limit on the diffraction-free propagation distance over which the modified group velocity can be observed [34]. The maximum group delay achieved is limited by the propagation-invariant length, which is dictated by the radius of the temporal bandwidth \(\Delta \lambda\) to the spectral uncertainty \(\delta \lambda\). The finite system aperture ultimately sets the value of \(\delta \lambda\). For example, the size of the diffraction grating determines its spectral resolving power, the finite pixel size of the SLM further sets a lower bound on the precision of association between the spatial and temporal frequencies, and the size of the SLM active area determines the maximum temporal bandwidth that can be exploited. Of course, the spatial tilt angle \(\theta\) determines the proportionality between the spatial and temporal bandwidths, which then links these limits to the transverse beam width. The confluence of all these factors then determine the maximum propagation-invariant distance and hence the maximum achievable group delays. Careful design of the experimental parameters helps extend the propagation distance [46], and exploiting a phase plate in lieu of a SLM can extend the propagation distance even further [47]. Note that we have synthesized here optical wave packets where light has been localized along one transverse dimension but remains extended in the other transverse dimension. Localizing the wave packet along both transverse dimensions would require the addition of an additional SLM to extend the spatio-temporal modulation scheme into the second transverse dimension that we have not exploited here.

Finally, a different strategy has been recently proposed theoretically [48] and demonstrated experimentally [49] that makes use of a so-called ‘flying focus’, whereupon a chirped pulse is focused with a lens having chromatic aberrations such that different spectral slices traverse the focal volume of the lens at a controllable speed, which was estimated by means of a streak camera.

We have considered here ST wave packets whose spatio-temporal spectral projection onto the \((k_x, k_z)\)-plane is a line. A plethora of alternative curved projections may be readily implemented to explore different wave packet propagation dynamics and to accommodate the properties of material systems in which the ST wave packet travels. Our results pave the way to novel schemes for phase matching in nonlinear optical processes [50, 51], new types of laser-plasma interactions [52, 53], and photon-dressing of electronic quasiparticles [54].

METHODS

Determining conic sections for the spatio-temporal spectra

The intersection of the light-cone \(k_x^2 + k_z^2 = (\omega/c)^2\) with the spectral hyperplane \(P(\theta)\) described by the equation \(\omega = k_\theta + (k_z - k_\theta)\tan \theta\) is a conic section: an ellipse \((0^\circ < \theta < 45^\circ\) or \(135^\circ < \theta < 180^\circ)\), a tangential line \((\theta = 45^\circ)\), a hyperbola \((45^\circ < \theta < 135^\circ)\), or a parabola \((\theta = 135^\circ)\). In all cases \(v_g = \tan \theta\). The projection onto the \((k_x, k_z)\)-plane, which is the basis for our experimental synthesis procedure, is in all cases a conic section given by

\[
\frac{1}{k_1^2}(\omega_1^2 - k_x^2) + \frac{k_x^2}{k_3^2} = 1, \tag{2}
\]

where \(k_1, k_2\), and \(k_3\) are positive-valued constants: \(k_1 = |\tan \theta| / (1 + \tan \theta)|, k_2 = k_\theta / \sqrt{1 + \tan \theta}|, \text{and } k_3 = \sqrt{1 + \tan \theta}|.\) The signs in the equation are \((-\pm, +)\) in the range \(0^\circ < \theta < 45^\circ\) (an ellipse), \((-\pm, -)\) in the range \(45^\circ < \theta < 90^\circ\), and \((+\pm, +)\) in the range \(90^\circ < \theta < 135^\circ\).

In the paraxial limit where \(k_\max^\theta \ll k_\theta\), the conic section in the vicinity of \(k_z = 0\) can be approximated by a section of a parabola,

\[
\frac{\omega}{\omega_0} = 1 + f(\theta)\frac{k_x^2}{2k_3^2}, \tag{3}
\]

whose curvature is determined by \(\theta\) through the function \(f(\theta)\) given by

\[
f(\theta) = \frac{\tan \theta}{\tan \theta - 1}. \tag{4}
\]

Spatially resolved interferograms for resolving the spatio-temporal intensity profiles

We take the ST wave packet to be \(E(x,t,z) = e^{(ik_z - \omega_0)z} \varphi(x, z - v_g t)\) as provided in Eq. 1, and that of
We make use of the following representations of the wave packet, as would be registered by a CCD, for example, by retro-reflected wave from is then directed through the lens of focal length \( f \approx \omega / v_g \). Note that the spectral function \( \tilde{\psi}(k_x) \) of the ST wave packet determines the coherence length of the observed spatio-temporal interferogram, which we thus expect to be on the order of the temporal width of the ST wave packet itself.

The visibility of the spatially resolved interference fringes is given by

\[
v(x, \tau) = \frac{2|\psi(x, \tau)|}{I_{\text{ST}}(x) + I_x}.
\]

The squared visibility is then given by

\[
v^2(x, \tau) \approx \frac{4|\tilde{\psi}(\omega)|^2 |\psi(x, \tau)|^2}{I_{\text{ST}}(x) + I_x} \propto |\psi(x, \tau)|^2,
\]

where the last approximation requires that we can ignore \( I_{\text{ST}}(x) \) with respect to the constant background term \( I_x \) stemming from the reference pulse.

### Details of the experimental setup

#### Synthesis of ST wave packets.

The input pulsed plane wave is produced by expanding the horizontally polarized pulses from a Ti:sapphire laser (Tsunami, Spectra Physics) having a bandwidth of \( \approx 8.5 \) nm centered on a wavelength of 800 nm, corresponding to pulses having a width of \( \approx 100 \) fs. A diffraction grating having a ruling of 1200 lines/mm and area \( 25 \times 25 \) mm\(^2\) in reflection mode (Newport 10HG1200-800-1) is used to spread the pulse spectrum in space and the second diffraction order is selected to increase the spectral resolving power, resulting in an estimated spectral uncertainty of \( \delta \lambda \approx 24 \) pm. After spreading the full spectral bandwidth of the pulse in space, the width size of the SLM (\( \approx 16 \) mm) acts as a spectral filter, thus reducing the bandwidth of the ST wave packet below the initial available bandwidth and minimizing the impact of any residual chirping in the input pulse. An aperture \( A \) can be used to further reduce the temporal bandwidth when needed. The spectrum is collimated using a cylindrical lens \( L_{1-y} \) of focal length \( f = 50 \) cm in a \( 2f \) configuration before impinging on the SLM. The SLM imparts a 2D phase modulation to the wave front that introduces controllable spatio-temporal spectral correlations. The retro-reflected wave from is then directed through the lens \( L_{1-y} \) back to the grating \( G \), whereupon the ST wave packet is formed once the temporal/spatial frequencies are superposed; see Fig. S1. Details of the synthesis procedure are described elsewhere [36, 42, 46, 47].
Spectral analysis of ST wave packets.

To obtain the spatio-temporal spectrum $|\tilde{E}(k_x,\lambda)|^2$ plotted in Fig. 3a in the main text, we place a beam splitter BS$_2$ within the ST synthesis system to sample a portion of the field retro-reflected from the SLM after passing through the lens $L_{1-y}$. The field is directed through a spherical lens $L_{4-s}$ of focal length $f=7.5$ cm to a CCD camera (CCD$_2$); see Fig. S1. The distances are selected such that the field from the SLM undergoes a $4f$ configuration along the direction of the spread spectrum (such that the wavelengths remain separated at the plane of CCD$_2$), while undergoing a $2f$ system along the orthogonal direction, thus mapping each spatial frequency $k_x$ to a point.

Reference pulse preparation.

The reference pulse is obtained from the initial pulsed beam before entering the ST wave packet synthesis stage via a beam splitter BS$_1$. The beam power is adjusted using a neutral density filter, and the spatial profile is enlarged by adding a spatial filtering system consisting of two lenses and a pinhole of diameter 30 µm. The spherical lenses are $L_{5-s}$ of focal length $f=50$ cm and $L_{6-s}$ of focal length $f=10$ cm, and they are arranged such that the pinhole lies at the Fourier plane. The spatially filtered pulsed reference then traverses an optical delay line before being brought together with the ST wave packet.

Beam analysis.

The ST wave packet is imaged from the plane of the grating G to an output plane via a telescope system comprising two cylindrical lenses $L_{2-x}$ and $L_{3-x}$ of focal lengths 40 cm and 10 cm, respectively, arranged in a $4f$ system. This system introduced a demagnification by a factor $4\times$, which modifies the spatial spectrum of the ST wave packet. The phase pattern displayed by the SLM is adjusted to pre-compensate for this modification. The ST wave packet and the reference pulse are then combined into a common path via a beam splitter BS$_4$. A CCD camera (CCD$_1$) records the interference pattern resulting from the overlap of the ST wave packet and the reference pulse, which takes place only when the two pulses overlap also in time; see Fig. S2.

Details of group-velocity measurements

Moving CCD$_1$ a distance $\Delta z$ introduces an extra common distance in the path of both beams. However, since the ST wave packet travels at a group velocity $v_g$ and the reference pulse at $c$, a relative group delay of $\Delta \tau = \Delta z/(c - v_g)$ is introduced and the interference at CCD$_1$ is lost if $\Delta \tau \gg \Delta T$, where $\Delta T$ is the width of the ST wave packet in time. The delay line in the path of the reference pulse is then adjusted to introduce a delay $\tau = \Delta \tau$ to regain the interference. In the subluminal case $v_g < c$, the reference pulse advances beyond the ST wave packet, and the interference is regained by increasing the delay traversed by the reference pulse with respect to the original position of the delay line. In the superluminal case $v_g > c$, the ST wave packet advances beyond the reference pulse, and the interference is regained by reducing the delay traversed by the reference pulse with respect to the original position of the delay line.

For a given value of $\Delta z$, we fit the temporal profile $I(0,0,\tau)$ to a Gaussian function to determine its center from which we estimate $\Delta \tau$. For each tilt angle $\theta$, we repeat the measurement for three different values of $\Delta z$ and set one of the positions as the origin for the measurement set: 0 mm, 2 mm, and 4 mm in positive subluminal case (Fig. S4a); 0 mm, 5 mm, and 10 mm in the positive superluminal case (Fig. S4b); and 0 mm, -5 mm, and -10 mm in the negative-$v_g$ case (Fig. S4c). Finally, we fit the obtained values to a linear function, where the slope corresponds to the group velocity. The uncertainty in estimating the values of $v_g$ ($\Delta v_g$ in Table S1 and error bars for Fig. 4 in the main text) are obtained from the standard error in the slope resulting from the linear regression.

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SUPPLEMENTARY MATERIAL

FIG. S1. Detailed experimental setup. The experimental arrangement shown schematically in Fig. 2a in the main text is presented here in detail. The setup comprises four sections for ST wave packet synthesis and characterization. The acronyms on all the optical components are provided in the inset box, and the focal lengths of the lenses and details of the experiment are provided in the text.
FIG. S2. Detailed procedure for restructuring the spatio-temporal wave packet profile from interferometric measurements. 
a, Time-integrated beam profile of a ST wave packet recorded by a slow detector (CCD$_1$) at a fixed propagation distance. In absence of the reference or if the reference and ST wave packet do not overlap in time, off-axis spatial interference along $x$ is absent. 
b, Spatially-resolved interference fringes along the $x$-axis for a delay $\tau \approx -7$ ps, resulting from the overlap of the reference and ST wave packet in time. 
c, The visibility of the spatially resolved interference fringes changes as the delay is scanned around the center of the ST wave packet. 
d, A set of measurements with small delay increments are taken to obtain the visibility $\nu$ near the vicinity of any selected $\tau$. Each set of such measurements yields a single line in the spatio-temporal profile $I(x,0,\tau)$ given in Fig. 2b in the main text.
FIG. S3. **Space-time-diagram for determining the group velocity of ST wave packets.** The experiment starts with the ST wave packet and reference pulse overlapping in space and time, resulting in high-visibility spatially resolved fringes (Fig. S2b). A common distance $\Delta z$ is introduced into the path of the ST wave packet and the reference pulse by moving CCD1 (Fig. S1), which results in a loss of the interference fringes (Fig. S2a). A delay $\Delta \tau$ is then introduced into the path of the reference pulse to regain the visibility of the spatial resolved interference fringes (Fig. S2c).

FIG. S4. **Space-time-diagram measurements of ST wave packets to estimate their group velocities.** a, Positive subluminal ST wave packets lie below the light line $v_g = c$ (blue-dashed). A retardation of the reference pulse is required to obtain maximum visibility. b, Positive superluminal ST wave packets lie above the light line with a positive slope. An advancement of the reference pulse is required to obtain maximum visibility. The luminal ST wave packet, which coincides – as expected – with the light line (data points represented by stars), is produced by idling the SLM. c, Negative superluminal ST wave packets lie above the light line with a negative slope $v_g = -c$. An advancement of the reference pulse is required to obtain maximum visibility. Note that the required advancement here exceeds that of the case of positive superluminal ST wave packets. a-c, In all cases, three measurements are taken at three points in $z$ and the required advancement or retardation of the reference pulse that gives the maximum visibility is recorded. One of the observation points set to be origin ($z=0$). The black lines are linear fits.
**TABLE S1.** Measurement results and theoretical expectation for the group velocity $v_g$ of ST wave packets in free space, arranged in order of increasing value of $\theta$ in the range $0^\circ < \theta < 180^\circ$. The theoretical values correspond to $v_g = \tan \theta$, and $\Delta v_g$ is the uncertainty in the measured value of $v_g$.

| $\theta$ | Wave packet type | $v_g$  | $\Delta v_g$ | Theory | Conic section |
|----------|------------------|--------|--------------|--------|---------------|
| (1) 26.6°| Positive subluminal | 0.49c  | $\pm 3 \times 10^{-4}c$ | 0.50c  | ellipse       |
| (2) 34.6°| Positive subluminal | 0.68c  | $\pm 2 \times 10^{-5}c$ | 0.69c  | ellipse       |
| (3) 45°  | Positive luminal    | $c$    | $\pm 3 \times 10^{-5}c$ | $c$    | line          |
| (4) 53.3°| Positive superluminal | 1.36c  | $\pm 4 \times 10^{-4}c$ | 1.34c  | hyperbola     |
| (5) 61.1°| Positive superluminal | 1.80c  | $\pm 3 \times 10^{-4}c$ | 1.81c  | hyperbola     |
| (6) 73.4°| Positive superluminal | 3.29c  | $\pm 2 \times 10^{-4}c$ | 3.36c  | hyperbola     |
| (7) 80.7°| Positive superluminal | 6.17c  | $\pm 0.02c$    | 6.14c  | hyperbola     |
| (8) 86.4°| Positive superluminal | 14.86c | $\pm 1.07c$    | 15.9c  | hyperbola     |
| (9) 88.5°| Positive superluminal | 32.86c | $\pm 2.18c$    | 39.21c | hyperbola     |
| (10) 104.2°| Negative superluminal | $-3.94c$| $\pm 0.03c$    | $-3.94c$| hyperbola     |
| (11) 120.6°| Negative superluminal | $-1.66c$| $\pm 5 \times 10^{-3}c$ | $-1.69c$| hyperbola     |