Flux of Primordial Monopoles

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Abstract

We discuss how in supersymmetric models with $D$ and $F$-flat directions, a primordial monopole flux of order $10^{-16} - 10^{-18} \text{cm}^{-2}\text{sec}^{-1}\text{sr}^{-1}$ can coexist with the observed baryon asymmetry. A modified Affleck-Dine scenario yields the desired asymmetry if the monopoles are superheavy ($\sim 10^{13} - 10^{18}$ GeV). For lighter monopoles with masses $\sim 10^{9} - 10^{12}$ GeV, the baryon asymmetry can arise via TeV scale leptogenesis.
Unified theories of elementary particle interactions\textsuperscript{1,2} predict the existence of topologically stable magnetic monopoles\textsuperscript{3} with masses and magnetic charges that depend on the details of the underlying theory. In supersymmetric SU(5), for instance, the lightest monopole with mass of order $M_G/\alpha_G \approx 5 \times 10^{17}$ GeV carries one unit $(2\pi/e)$ of Dirac magnetic charge. (Here $M_G \approx 2 \times 10^{16}$ GeV denotes the gauge coupling unification scale, and $\alpha_G \approx 1/25$ is the unified coupling constant at $M_G$). The monopole also has screened color magnetic fields\textsuperscript{4}, and the presence of baryon and lepton number violating superheavy gauge bosons leads to monopole catalysis of nucleon decay\textsuperscript{5}. If the monopole arises from a partial unified model such as $G_{422} \equiv SU(4)_c \times SU(2)_L \times SU(2)_R$\textsuperscript{1} (or $G_{333} \equiv SU(3)_c \times SU(3)_L \times SU(3)_R$), it carries two (or three) quanta of Dirac charge\textsuperscript{1}. The absence in these models of gauge bosons which mediate proton decay means that, in principle, the symmetry breaking scale ($M$) can be considerably below $M_G$\textsuperscript{10}, leading to lighter ($\sim 10^{10}$ GeV) monopoles. In a supersymmetric framework intermediate scales are allowed provided proton decay via dimension five operators is adequately suppressed. Models with intermediate scales have acquired renewed interest because they naturally appear from compactification of superstring theories on Calabi-Yau manifolds and orbifolds\textsuperscript{11}, from D-brane constructions\textsuperscript{10,12}, or from intersecting brane models\textsuperscript{13}.

The fate of primordial monopoles is very closely linked to the history of the very early universe. In hot big bang cosmology without inflation there is a serious cosmological monopole problem pointed out a long time ago\textsuperscript{14}. Here we list some proposed solutions:

- An inflationary epoch\textsuperscript{15,16} reduces the primordial monopole number density to utterly negligible values. The subsequent transition to radiation epoch may give rise to thermally produced monopoles, provided the reheat temperature is not too far below the monopole mass\textsuperscript{17}.

- If the symmetry breaking which gives rise to monopoles experiences only the last 30 or so $e$-foldings of inflation\textsuperscript{18,19}, a measurable flux of monopoles comparable to or somewhat below the Parker bound\textsuperscript{20} and the MACRO experiment limits\textsuperscript{21} may be present in our galaxy. A different scenario with similar objectives is discussed in\textsuperscript{22}.

\textsuperscript{1} Multiply charged monopoles also arise in superstring theories\textsuperscript{8,9}. 

\textsuperscript{2}
Monopoles and antimonopoles get linked either by electromagnetic or Z flux tubes and efficient annihilation occurs as a result of rapid contraction of these tubes. Another resolution may be non-restoration of grand unified symmetry at high temperature. In , it is proposed that if monopoles are produced in association with domain walls, the latter can sweep away monopoles and then disappear. A black hole solution of the cosmological monopole problem is discussed in .

Monopoles arise in supersymmetric models with D and F-flat directions, in which thermal inflation is followed by a huge release of entropy. An initially large monopole number density can be reduced to the Parker bound or somewhat below it .

In this paper we wish to explore this last scenario, previously discussed in , in which monopoles appear after thermal inflation is over and a huge amount of entropy is released. While this certainly helps with monopole dilution, our main challenge is to identify a framework in which an observable flux of primordial monopoles is compatible with the observed baryon asymmetry. This is particularly challenging for superheavy monopoles with a correspondingly large symmetry breaking scale, so that the final temperature after thermal inflation can be quite low, of order a few MeV. Following , we employ a modified Affleck-Dine (AD) scenario to realize the observed baryon asymmetry. Although our discussion is mainly focused on monopoles with mass , it can be adapted, as we will show, to monopoles that are somewhat heavier or significantly lighter .

Consider the superpotential

\[ W_{\text{inf}} \supset \lambda \left( \bar{\phi} \phi \right)^n + \beta H_u H_d \left( \bar{\phi} \phi \right)^m M_*^{2n-1}, \]  

where the scalar components of \( \phi \) and \( \bar{\phi} \), called flatons, acquire non-zero vevs \( M_* \) and break the underlying gauge symmetry \( G \). \( M_* \) is the cutoff scale, \( H_u,d \) are the MSSM Higgs doublets, and \( n, m \) are integers suitably chosen to yield the desired symmetry breaking scale \( M \) and the MSSM \( \mu \) term respectively. The second term in Eq. also plays an important role in reheating after thermal inflation.

The zero temperature effective potential of flatons (we use the same notation for the
superfield and its scalar component) is given by

\[ V(\phi) = \mu_0^4 - M_s^2 \left( |\phi|^2 + |\bar{\phi}|^2 \right) + n^2 \left[ \frac{\lambda \phi^2 |\phi|^{n-1}}{M_*(2n-3)} \right]^2 + n^2 \left[ \frac{\lambda \phi^2 |\bar{\phi}|^n}{M_*(2n-3)} \right]^2 + A_\lambda |\phi|^{2n} + c.c. \quad (2) \]

where \( \mu_0^4 \) is the false vacuum energy density, such that \( V(M) = 0 \) at \( |\langle \phi \rangle| = M \), \( M_s \) is the soft supersymmetry breaking mass parameter, and \( |A_\lambda| \lesssim M_s \). Minimizing the effective potential along the \( D \)-flat direction \( |\langle \phi \rangle| = |\langle \bar{\phi} \rangle| \) yields the symmetry breaking scale\(^2\),

\[ M = |\langle \phi \rangle| = \left[ \frac{M_s^{n(2n-3)}}{2(2n-1)n\lambda} \left( |A_\lambda| + \sqrt{|A_\lambda|^2 + 4(2n-1)M_s^2} \right) \right]^{1/2(2n-1)}, \]

\[ \sim \left[ \frac{M_s M_*(2n-3)}{\sqrt{(2n-1)n\lambda}} \right]^{1/2(n-1)}, \quad (3) \]

where \( n \geq 2 \) and for simplicity, we assume \( |A_\lambda| < M_s \). Some typical values of \( M_s, M_s \) and \( M \) that we will consider in this paper are listed in Table I. For non-zero temperature \( T \) the effective potential gets an additional contribution given by \[42\]

\[ V_T(\phi) = \left( \frac{T^4}{2\pi^2} \right) \sum_i (-1)^F \int_0^\infty dx x^2 \ln \left( 1 - (-1)^F \exp \left\{ - \left( x^2 + \frac{M_i^2(\phi)}{T^2} \right)^{1/2} \right\} \right), \quad (4) \]

where the sum is over all helicity states, \( (-1)^F \) is \( \pm 1 \) for bosonic and fermionic states, respectively, and \( M_i(\phi) \) is the field-dependent mass of the \( i \)th state. For \( \phi \ll T \) the temperature-dependent mass term is \( \sigma T^2 |\phi|^2 \), where \( \sigma \approx 0.1 \). For \( T > T_c = M_s/\sqrt{\sigma} \) the potential

\[ V(\phi) = \mu_0^4 + 2(-M_s^2 + \sigma T^2)|\phi|^2 + 2n^2 \lambda^2 |\phi|^{2(n-1)} \frac{M_*(2n-3)}{M_s^{2(2n-3)}} + A_\lambda |\phi|^{2n} \frac{M_*(2n-3)}{M_s^{2n-3}} + c.c. \quad (5) \]

develops a minimum at \( |\phi| = 0 \), the gauge group is unbroken and hence there are no monopoles. The false vacuum energy density \( \mu_0^4 (\sim M_s^2 M^2) \) drives thermal inflation and the universe experiences roughly \( \ln(\mu_0/T_c) \) e-foldings (\( \sim 12 \) for \( M \sim 10^{14} \) GeV).

As the temperature \( T \) falls below the critical value \( T \sim T_c \), the mass-squared term for \( \phi \) turns negative, and \( \phi \) rolls from the origin to \( M \), thereby ending thermal inflation. Monopoles are expected to arise as a consequence of symmetry breaking through the Kibble Mechanism\[43\]. The field \( \phi \) performs damped oscillations about the minimum at \( M \) and subsequently decays. During these oscillations, the universe is matter dominated with energy density

\[ \epsilon, \alpha \text{ and } \sigma \text{ of } A_\lambda, \phi \text{ and } \bar{\phi} \text{ are taken to satisfy the relation } \epsilon + n\alpha + n\sigma \pi = \pi. \]
\[ \rho \sim M_s^2 \langle \phi \rangle^2 \left( \frac{t_c^2}{t^2} \right), \] where \( t_c \) represents the cosmic time at the phase transition. A large amount of entropy is released by the decay of \( \phi \) field which helps dilute the initial monopole density [29]. To estimate the initial monopole number density \( n_M \) one usually makes the plausible assumption that a correlation size volume \( \sim \xi^3 (\sim T_c^{-3}) \) contains on the order of one monopole [14, 43]. Then [14, 43]

\[ r_{in} \equiv \left[ \frac{n_M}{T^3} \right]_{initial} \sim \frac{P}{(4\pi/3) \xi^3 T_c^3} \sim 10^{-2}, \] (6)

where \( P \) is a geometric factor of order 1/10. It has been suggested [44] that \( \xi \) may be somewhat larger than \( T_c^{-1} \), in which case fewer monopoles would be produced. A more drastic reduction in the initial monopole number density is achieved by assuming that a single monopole is produced per horizon volume [45]. Since the horizon size during monopole production is larger than \( T_c^{-1} \) by a factor \( M_P/M \), a large suppression (by a factor of order \( (M/M_P)^3 \)) of the monopole number density becomes possible. We will not use this last suppression mechanism for superheavy monopoles, but will keep in mind the fact that the initial monopole number density estimates can have large uncertainties in them.

One may ask whether \( r_{in} \) can be reduced through monopole-antimonopole annihilation [14, 46]. In the context of flaton models this has been investigated in [29], which reached the conclusion that there is no significant annihilation during the epoch of \( \phi \) domination.

We now consider monopole dilution through entropy production from the decay of the flaton field. If the entropy increases by a factor \( \Delta \), the final number density of monopoles is given by

\[ r_{final} \equiv \left[ \frac{n_M}{T^3} \right]_{final} = r_{in} \Delta^{-1} g_*(T_{final}) g_*(T_c), \] (7)

where \( g_*(T) \) represents the degrees of freedom at \( T \). The flaton decay proceeds predominantly via the coupling \( \beta H_u H_d (\overline{\phi \phi})^m / M_s^{2m-1} \). The decay width \( \phi \rightarrow H_u^2, H_d^2 \) is given by

\[ \Gamma_\phi \simeq \frac{1}{8\pi} \beta^4 \left( \frac{M}{M_s} \right)^{4(2m-1)} \frac{M^2}{m_\phi}, \] (8)

where \( m_\phi = 2 \sqrt{2(n-1)} M_s \) is the flaton mass. The final temperature after thermal inflation can be expressed as

\[ T_f \simeq 0.3 \sqrt{\Gamma_\phi M_P} \simeq 0.036 \beta^2 \left( \frac{M}{M_s} \right)^{2(2m-1)} \frac{M}{(n-1)^{1/4}} \sqrt{\frac{M_P}{M_s}}, \] (9)
where $M_P = 2.4 \times 10^{18}$ GeV is the reduced Planck mass. Note that the last term in Eq. (1) generates the effective MSSM $\mu$ term $\beta(M/M_s)^{2n-1}M$. Eq. (9) can be rewritten as

$$T_f \simeq 0.036 \left( \frac{\mu}{M} \right) \frac{1}{(n-1)^{1/4}} \sqrt{\frac{M_P}{M_s}},$$

$$\simeq 0.01 \left( \frac{\mu}{1 \text{ TeV}} \right)^2 \left( \frac{10^{14} \text{ GeV}}{M} \right) \left( \frac{950 \text{ GeV}}{M_s} \right)^{1/2} \text{GeV} \quad \text{(for } n = 4; m = 3).$$ (10)

The entropy released by flaton decay is estimated to be

$$\Delta \simeq \frac{3\mu_0^4}{g_*(T_c)T_c^3T_f},$$

$$\simeq 9 \times 10^{23} \left( \frac{\sigma}{0.1} \right)^{3/2} \left( \frac{M}{10^{14} \text{ GeV}} \right)^3 \left( \frac{1 \text{ TeV}}{\mu} \right)^2 \left( \frac{950 \text{ GeV}}{M_s} \right)^{1/2},$$ (11)

where we have used Eq. (10) and $g_*(T_c) \sim 200$. Using Eqs. (11) and (7) we get

$$r_{\text{final}} \simeq 6 \times 10^{-28} \left( \frac{r_{\text{in}}}{10^{-2}} \right)^{3/2} \left( \frac{10^{14} \text{ GeV}}{M} \right)^3 \left( \frac{\mu}{1 \text{ TeV}} \right)^2 \left( \frac{M_s}{950 \text{ GeV}} \right)^{1/2},$$ (12)

for $g_*(T_f) \sim 10^{[47]}$.

From Eq. (10) we note that if $M$ increases above $10^{15}$ GeV, the $\mu$ parameter and $M_s$ also have to increase (see Fig. 1) in order that the final temperature $T_f$ stays above 10 MeV for successful nucleosynthesis. Thus, some fine tuning of the soft supersymmetry breaking higgs parameters will be required to implement electroweak breaking. Monopoles with masses greater than or of order $10^{17}$ GeV can gravitationally clump and will be considered later.

We should make sure that the final monopole number density is consistent with the MACRO bound [21] which is more stringent than the well known Parker bound [48, 49]. For superheavy magnetic monopoles ($10^{11}$ GeV $\lesssim m_M \lesssim 10^{17}$ GeV) moving with speed $v_M \sim 3 \times 10^{-3}c (10^{16} \text{ GeV}/m_M)^{1/2}$ [47, 48], the MACRO limit on the flux $F_M$ is

$$F_M \lesssim 10^{-16} \text{cm}^{-2}\text{sec}^{-1}\text{sr}^{-1},$$ (13)

which corresponds to $r_{\text{final}} \lesssim 2 \times 10^{-26} (3 \times 10^{-3}c/v_M)(m_M/10^{16} \text{ GeV})^{1/2}$. For $n = 4, m = 3$ in Eq. (1) and with the remaining parameters given in Table I (corresponding to $M = 10^{14}$ GeV), an entropy release factor $\Delta$ of order $10^{22}$ saturates the bound in Eq. (13), with $r_{\text{in}} \sim 10^{-2}$ (see Eq. (6)). The variation of $M_s$ and $M$ with the initial monopole number density given by Eq. (9), such that the monopole flux bound ($F_M \simeq 10^{-17} \text{cm}^{-2}\text{sec}^{-1}\text{sr}^{-1}$) is saturated, is shown in Fig. 2(a). For $F_M = 10^{-18} \text{cm}^{-2}\text{sec}^{-1}\text{sr}^{-1}$, $M_s$ vs. $M$ is plotted in Fig. 2(b).
| $M$ (GeV) | $M_s$ (GeV) | $M_s$ (GeV) | $T_f$ (GeV) | $\mu$ (GeV) | $\lambda$ | $\beta$ | $r_{in}$ | $F_M$ (cm$^{-2}$sec$^{-1}$sr$^{-1}$) |
|----------|-----------|-----------|-----------|-----------|--------|--------|--------|-----------------|
| $10^{14}$ | 950       | $10^{16}$ | 0.01      | $10^3$   | 0.01   | 0.1    | $10^{-2}$ | $10^{-17}$ |
| $10^{15}$ | 3000      | $6.4 \times 10^{16}$ | 0.3       | $1.8 \times 10^4$ | 0.0003 | 0.02   | $10^{-2}$ | $10^{-18}$ |
| $10^{16}$ | 3000      | $10^{18}$ | 0.03      | $2 \times 10^4$ | 0.0003 | 0.02   | $10^{-2}$ | $10^{-16}$ |
| $2 \times 10^{16}$ | 2500 | $2.4 \times 10^{18}$ | 0.01      | $1.6 \times 10^4$ | 0.0003 | 0.02   | $10^{-2}$ | $10^{-17}$ |

TABLE I: A set of parameter values for which the expected monopole flux is at or below the MACRO bound.

The release of such a large amount of entropy (Eq. (11)) certainly washes away any pre-existing baryon asymmetry. Also, for $M \gtrsim 10^{13} - 10^{14}$ GeV the final temperature is quite low$^3$ ($T_f \simeq 0.01$ GeV), so that the sphalerons are ineffective, and the standard leptogenesis scenario$^5$ does not apply. A different mechanism for generating the desired baryon asymmetry must then be found. This problem has arisen before and discussed by several authors$^{33, 53, 54}$. In$^{33}$, for instance, new particles beyond the MSSM spectrum are considered whose out of equilibrium decay can give rise to the baryon asymmetry. Here we rely on a modification of the AD$^{55}$ scenario proposed in$^{37}$ (and subsequently in$^{56}$) in which a dilution factor $\Delta^{-1} \sim 10^{-17} - 10^{-18}$ and symmetry breaking scale $M \sim 10^{10-11}$ GeV are considered$^4$. In our case, as we saw in Eq. (11), the monopole problem requires an even greater amount of entropy production, especially if $r_{in} \sim 10^{-2}$.

To implement the scenario discussed in$^{37}$ in our case, we couple $\phi$ to the right-handed neutrino superfields $N_i$. In the MSSM notation, consider the couplings$^5$

$$W = W_{inf} + Y_i L H_d e + Y_D L H_u N + \lambda_\phi \phi N^2 + \cdots,$$  \hspace{1cm} (14)

where we have suppressed all generation and group indices, and the ellipsis represent terms in the superpotential which will not participate in the analysis. We assume that both during and after thermal inflation, the squark fields do not acquire non-zero vevs.

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$^3$ $T_f$ as low as this can lead to conflict with LSP cosmology$^{50}$. One way out may be to introduce an axino LSP$^{51}$.

$^4$ For nonequilibrium effects in AD mechanism, see$^{57}$.

$^5$ In practice, the superpotential is invariant under $G$, and $N_i$ belong in some appropriate $G$ representation. Later we will provide an example for $G$ and discuss how the terms in Eq. (14) arise.
Consider the zero temperature $F$-term potential

\begin{equation}
V_F = \left| Y_D H_u N + Y_t H_d e \right|^2 + \left| Y_D L H_u + 2\lambda_\phi \psi N \right|^2 + \left| Y_t L e + \beta H_u \frac{\langle \phi \bar{\phi} \rangle^3}{M_s^2} \right|^2 \\
+ \left| Y_D L N + \beta H_d \frac{\langle \phi \bar{\phi} \rangle^3}{M_s^2} \right|^2 + \left| 4\lambda \frac{\langle \phi \bar{\phi} \rangle^3}{M_s^2} \phi + 3\beta H_u H_d \frac{\langle \phi \bar{\phi} \rangle^2}{M_s^2} \phi \right|^2 \\
+ \left| 4\lambda \frac{\langle \phi \bar{\phi} \rangle^3}{M_s^2} M_s^2 \phi + \lambda_\phi N^2 + 3\beta H_u H_d \frac{\langle \phi \bar{\phi} \rangle^2}{M_s^2} \phi \right|^2 ,
\end{equation}

where we have used the same notation for superfields and their scalar components and set $n = 4, m = 3$ in Eq. (1). The $D$-terms ($V_D$) and the supersymmetry breaking parts of the potential ($V_{\text{susy}}$) are

\begin{equation}
V_D + V_{\text{susy}} = g^2 \left( |H_u|^2 - |H_d|^2 - |L|^2 \right) + m_\phi^2 |L|^2 - m_{H_u}^2 |H_u|^2 - 2M_s^2 |\phi|^2 \\
+ \left( A_D Y_D L H_u N + A\lambda_\phi \lambda_\phi N^2 + A\beta \beta H_u H_d \frac{\langle \phi \bar{\phi} \rangle^3}{M_s^2} + A\lambda \lambda \frac{\langle \phi \bar{\phi} \rangle^4}{M_s^2} + \cdots \right) + \text{c.c. , (16)}
\end{equation}

where the soft masses and $A$-terms are all taken to be of order $M_s$, and $D$-flatness along $\phi, \bar{\phi}$ direction has been taken into account. For simplicity we will confine ourselves to a single generation picture.

We assume that initially all fields are held at zero during thermal inflation. To implement the scenario, the AD-field, parameterized along the $D$-flat direction by $L H_u = \psi^2/2$, acquires a vev $\sim m_\psi^2/|Y_D|^2$ provided we set $m_\psi^2 = (m_{H_u}^2 - m_\phi^2)/2 > 0$ ($\phi$ is still at zero). Assuming this happens before the flaton field starts to roll down, the term $A_D Y_D L H_u N + \text{c.c.}$ induces a vev for $N$ which, in turn, triggers the slow roll of $\phi$ towards its minimum. As $\phi$ increases, $N$ follows its instantaneous minimum

\begin{equation}
N \approx -\frac{Y_D L H_u}{2\lambda_\phi \phi} .
\end{equation}

Using Eq. (17) the effective potential reduces to\(^6\)

\begin{equation}
V = V_F + V_D + V_{\text{susy}} ,
\end{equation}

\begin{equation}
= 2 \left| 4\lambda \frac{\langle \phi \bar{\phi} \rangle^3}{M_s^2} \right|^2 |\phi|^{14} - 2M_s^2 |\phi|^2 + \left( \frac{1}{2} \left| \frac{\beta}{M_s^2} \right|^2 |\phi|^{12} - m_\psi^2 \right) |\psi|^2 + \left| \frac{Y_D^2 \psi^3}{4\lambda_\phi \phi} \right|^2 \\
+ \left[ \left\{ \left( A\lambda_\phi - A_D + 2\lambda^* \frac{\langle \phi \bar{\phi} \rangle^3}{M_s^2} \right) \frac{Y_D^2 \psi^4}{8\lambda_\phi \phi} + A\lambda \lambda \frac{\langle \phi \bar{\phi} \rangle^4}{M_s^2} \right\} + \text{c.c. } \right] .
\end{equation}

\(^6\) For simplicity $e$ and $H_d$ are assumed to be at their zero location.
It follows, after some algebra, that for \( \phi \ll M \), \( \psi \) is given by

\[
|\psi|^2 \simeq 4 \sqrt{\frac{1}{3}} \left( \frac{\lambda_\phi}{Y_D^2} \right) M_s |\phi|,
\]

(19)

and its phase is related with that of \( \phi \) from the term

\[
\left( \frac{A_\lambda \phi}{2} - A_D + 2 \lambda^* \left( \frac{\bar{\phi} \bar{\phi}^*}{M_5^5} \phi \right) \right) + \text{c.c.}
\]

(20)

Note that at this stage the third term in Eq. (20) is negligible.

When \( \phi \) becomes comparable to \( M \) (with the temperature still around \( T_c \)), it yields a positive mass squared term for \( \psi \), provided that

\[
\frac{1}{2} \beta^2 \frac{|\phi|^2}{M_5^{10}} - m_\psi^2 > 0.
\]

(21)

This helps the \( LH_u \) direction to reach its true minimum at zero. Simultaneously, the phase of \( \psi \) gets an important contribution from the third term inside the bracket of Eq. (20). This change then initiates an angular momentum in \( LH_u \) which turns out to be the amount of lepton asymmetry \( (n_L = i(\psi^* \dot{\psi} - \psi \dot{\psi}^*) \) generated from the AD mechanism. The difference between \( (A_\lambda \phi/2M_s - A_D/M_s) \) and \( 2 \lambda^* \) would be the source of \( CP \)-violation.

To protect \( n_L \) from being erased by potential lepton-number violating processes, in Ref. \[37\] it is assumed that the AD field should completely decay while the universe is still dominated by the energy of the flaton field. We assume that the decay width of the AD field is such that the electroweak sphalerons \[58\] can convert a fraction of \( n_L \) into \( n_B \), the baryon number density.

The decay of \( \phi \) yields a final temperature of around 0.01 GeV. The release of entropy (estimated earlier) responsible for the dilution of monopoles also dilutes the baryon asymmetry. The final baryon asymmetry is approximately given by

\[
\frac{n_B}{s} \simeq \frac{1}{3} \frac{n_L}{s_{\text{in}}} \Delta^{-1} \simeq \frac{n_L}{s_{\text{final}}},
\]

\[
\sim \frac{n_L}{\rho_\phi} T_f \sim \frac{4}{3} \frac{1}{3} \frac{\lambda_f}{Y_D^2} \frac{T_f}{M} \frac{m_\psi n_L}{M_s n_\psi},
\]

(22)

\[\text{From the tabulated values of } \lambda, \beta \text{ (corresponding to } M = 10^{14} \text{ GeV)} \text{ and Eq. (22) we notice that this happens before } \phi \text{ actually reaches its minimum (as } \lambda \ll \beta).\]

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where we have used Eq. (19). The seesaw relation for the light neutrino mass is

\[ m_\nu \simeq \frac{Y_D^2 v^2}{\lambda_\phi M}, \tag{23} \]

where we employ a basis in which the light neutrino mass matrix is diagonal and, for simplicity, only a single \( LH_u \) flat direction is assumed. Substituting Eq. (23) in Eq. (22),

\[ \frac{n_B}{s} \sim \frac{4}{3} \sqrt{\frac{1}{3}} \frac{v^2}{m_\nu M^2} \frac{m_\nu T_f}{M_s}, \tag{24} \]

where \( n_L/n_\psi \sim O(1) \) is assumed.

To obtain the required \( n_B/s \), the neutrino mass \( m_\nu \) in Eq. (24) turns out to be several orders of magnitude below the scale for atmospheric and solar neutrino oscillations. From Eq. (24),

\[ \frac{n_B}{s} \sim 10^{-10} \left( \frac{v}{174 \text{ GeV}} \right)^2 \left( \frac{2 \times 10^{-7} \text{ eV}}{m_\nu} \right) \left( \frac{10^{14} \text{ GeV}}{M} \right)^2 \left( \frac{T_f}{0.01 \text{ GeV}} \right), \tag{25} \]

where we have set \( m_\psi \sim M_s \), and from the observed baryon to photon ratio \( n_B/n_\gamma \simeq (6.0965 \pm 0.2055) \times 10^{-10} \), \( n_B/s \simeq (n_B/n_\gamma)/7.04 \). In Fig. 3 we display the allowed region for \( m_\nu \), with the range of \( M \) and \( T_f \) restricted by the monopole flux corresponding to Fig. 2 (a).

In order to make the discussion more explicit and to realize a scenario with superheavy symmetry breaking scale corresponding to \( n = 4, m = 3 \), we consider a realistic model with gauge symmetry \( G_{422} \equiv SU(4)_c \times SU(2)_L \times SU(2)_R \), whose breaking yields monopoles which carry two quanta of Dirac magnetic charge. By the same token, there also can exist color singlet states with charge \( \pm e/2 \). We have checked that \( M \) as low as \( 10^{14} \text{ GeV} \) is compatible with proton lifetime limits. Indeed, for \( M \sim 10^{14} \text{ GeV} \), we estimate the proton lifetime to be of order \( 10^{34} \) yrs, taking into account the operators discussed in [61]. The quarks and leptons are unified in the representations \( F_i = (4, 2, 1)_i \); \( \overline{F}_i = (\overline{4}, 1, 2)_i \) of \( G_{422} \), where the subscript \( i (= 1, 2, 3) \) denotes the family index. The Higgs sector consists of

\[ h = (1, 2, \overline{2}) ; \quad \overline{\phi} = (\overline{4}, 1, 2) ; \quad \phi = (4, 1, 2). \tag{26} \]

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8 From Eq. (19), it is seen that the effective superpotential responsible for generating the neutrino mass is \( W_\nu = -(Y_D LH_u)^2/(2\lambda_\phi) \) after integrating out \( N \).

9 If \( G_{422} \) is replaced by \( G_{333} \equiv SU(3)_c \times SU(3)_L \times SU(3)_R \) we find monopoles carrying three quanta of Dirac charge.
The superpotential $W$ is given by

$$W = \lambda \frac{(\phi \bar{\phi})^4}{M_*^2} + \beta h \frac{(\phi \bar{\phi})^3}{M_*^2} + Y F h \bar{F} + \gamma \phi \frac{\bar{F} F}{M_*} + \kappa \phi \frac{F F}{M_*}$$

$$+ a_1 D \phi \bar{\phi} + a_2 D \bar{\phi} \bar{\phi} + \frac{a_3}{M_*} D D \left( \frac{\phi \bar{\phi}}{M_*} \right)^2 + a_4 F F D \frac{\phi \bar{\phi}}{M_*^2} + \cdots, \quad (27)$$

where a discrete symmetry $Z_4 \times Z_8$ (see Table II) has been introduced to realize the desired $D$ and $F$-flat direction\(^\text{10}\). The color sextet superfield $D = (6, 1, 1) = (D_c, \bar{D})$, where $D_c = (3, 1, 1/3)$ and $\bar{D} = (\bar{3}, 1, -1/3)$, is introduced to provide heavy mass to the components $d_c H, d_c H$ of $\phi, \phi$\([60, 62]\).

The interaction of $\phi$ with $N$ in this particular example is determined by the term $(\gamma \phi^2 + \kappa \bar{\phi}^2)N^2/M_*^2$ in the superpotential instead of $\lambda \phi^2 N^2$ in Eq. (14). Thus as long as $\phi$ is held at zero, $N$ does not acquire any vev through the term $AYFh\bar{F}$ and from $V_F$ even when the AD field has a vev\(^\text{11}\). The $\phi$ field starts to roll down when $T \sim M_*$ as discussed in the context of Eq. (5). As $\phi$ increases, $N$ would receive a vev $\sim (Y(\psi)^2/4(\gamma + \kappa) |\phi|)(M_*/|\phi|)$ similar to Eq. (17), with $\lambda_\phi \sim (\gamma + \kappa)(|\phi|/M_*) = \gamma'(|\phi|/M_*)$. The relation in Eq. (19) now becomes

$$|\psi|^2 \simeq 4 \sqrt{\frac{1}{3}} \left( \frac{\gamma'}{Y_B^2} \right) \frac{M_*^2 M}{M_*} |\phi|. \quad (28)$$

The rest of the discussion for an estimate of the observed baryon asymmetry is very similar to what we already have before Eq. (25).

The baryogenesis scenario we have discussed, following \[37\], can also be employed for monopoles with masses $\sim 5 \times 10^{17}$ GeV, corresponding to GUT symmetry breaking scale $M \sim 2 \times 10^{16}$ GeV. The gravitational force on such monopoles exceeds the magnetic force and it seems plausible that they would clump in the galaxy, and perhaps even contribute

\(^{10}\) To avoid topologically stable domain walls we will assume that the discrete symmetry is explicitly broken by higher dimensional operators.

\(^{11}\) The AD field, $\psi$, in this example is a flat direction chosen along the neutral components of $F$ and $h (H_u^0)$.
to the dark matter in the universe. (This may not be plausible for monopoles that catalyze nucleon decay). However, this latter possibility is disfavored by the MACRO bound \[9\]. To see this, assume that the monopole energy density \( \rho_M \lesssim \rho_B \), the baryon energy density. Taking a local density enhancement factor of order \( 10^5 \), the local flux is estimated to be (with \( v_M = 10^{-3}c \) )

\[
F_M \lesssim 6 \times 10^{-13} \left( \frac{10^{17}\text{GeV}}{m_M} \right) \text{cm}^{-2}\text{sec}^{-1}\text{sr}^{-1}; \quad m_M \sim 10^{17} - 10^{18}\text{GeV},
\]

which is in strong disagreement with the MACRO bound. Thus, \( \rho_M \lesssim 10^{-4} \rho_B \), to be consistent with the MACRO bound. To achieve a monopole flux \( \lesssim 10^{-17}\text{cm}^{-2}\text{sec}^{-1}\text{sr}^{-1} \), we estimate that \( r_{\text{final}} \lesssim 10^{-31} \). The parameters chosen to achieve this are displayed in Table I.

Let us now consider ‘lighter’ monopoles with masses of order \( 10^9 - 10^{10} \text{GeV} \), corresponding to symmetry breaking scales \( \sim 10^8 - 10^9 \text{GeV} \). (In models with \( D \) and \( F \)-flat directions the symmetry breaking scale is expected to be \( \gtrsim 10^8 \text{GeV} \)). In this case the amount of entropy released following thermal inflation is considerably smaller. Namely, from Eq. (11), \( \Delta \sim 10^6 \), so that a monopole flux of order \( 10^{-17}\text{cm}^{-2}\text{sec}^{-1}\text{sr}^{-1} \) (this is consistent with present bounds from MACRO \[21\], SLIM \[63\] and AMANDA \[64\] experiments) requires that the initial relative monopole number density \( r_{in} \) must be sufficiently small, as shown in Fig. 4 (see discussion following Eq. (6)). Following \[40\], the observed baryon asymmetry can now be explained via TeV scale leptogenesis, since the final temperature \( T_f \sim \text{TeV} \). An initially large lepton asymmetry can survive the moderate amount of dilution so as to produce a final baryon asymmetry consistent with observations. Finally, we see from Fig. 4 that a flux bound of intermediate mass monopoles \( (\sim 10^{11} - 10^{13} \text{GeV}) \) requires that \( r_{in} \sim 10^{-17} - 10^{-7} \). The baryon asymmetry here can be achieved either through TeV scale leptogenesis \[41\] or the original AD scenario \[55\] where \( T_f \) is of order 100 GeV. For monopoles of mass \( \sim 10^{13} - 10^{14} \text{GeV} \), the baryon asymmetry can be generated through the modified Affleck-Dine mechanism.

To summarize, magnetic monopoles appear in a variety of unified gauge models with a wide range of masses. In models of thermal inflation, monopoles with masses of order \( 10^9 - 10^{18} \text{GeV} \) appear when the cosmic temperature is in the TeV range. Their subsequent dilution through the release of huge amount of entropy poses a challenge for baryogenesis. We have discussed a class of realistic models in which the observed baryon asymmetry can
co-exist with a primordial monopole flux which can be detected with large scale detectors such as ICE CUBE \[65\].

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FIG. 1: $M_s$ vs $M$ (for superheavy monopoles), with final monopole flux $F_M = 10^{-17}$ cm$^{-2}$sec$^{-1}$sr$^{-1}$. Here $100$ GeV $\leq \mu \leq 20$ TeV.
FIG. 2: $M_s$ versus $M$ with $\lambda = 10^{-2}$ and $10^{-3} \leq \beta \leq 1$. Here $r_{in} = 10^{-2}$. The monopole flux $F_M = 10^{-17}\text{cm}^{-2}\text{sec}^{-1}\text{sr}^{-1}$ for plot (a) and $10^{-18}\text{cm}^{-2}\text{sec}^{-1}\text{sr}^{-1}$ for plot (b).

FIG. 3: $m_\nu$ versus $M$ for $F_M = 10^{-17}\text{cm}^{-2}\text{sec}^{-1}\text{sr}^{-1}$ and $r_{in} = 10^{-2}$ (corresponding to Fig. 2(a)), with $n_B/s \sim (0.837 - 0.895) \times 10^{-10}$.

FIG. 4: $r_{in}$ vs. $M$ (for relativistic monopoles), with $F_M = 10^{-17}\text{cm}^{-2}\text{sec}^{-1}\text{sr}^{-1}$, $100 \text{ GeV} \leq M_s \leq 1 \text{ TeV}$ and $100 \text{ GeV} \leq \mu \leq 2 \text{ TeV}$. $r_H$ represents the relative monopole number density assuming production of one monopole per horizon volume.