Coronal fast ignition by laser: relativistic critical density increase and constraints on maximum laser wavelength

P Mulser¹, H Ruhl², R Schneider¹ and D Batani³

¹ TQE: Theoretical Quantum Electronics, Tech. Univ. Darmstadt, Germany
² Institute of Theoretical Physics I, Ruhr-Universität Bochum, Germany
³ Fisica del Plasma, Dipartimento di Fisica G. Occhialini, Università Milano Bicocca

E-mail: peter.mulser@physik.tu-darmstadt.de

Abstract. The problem of the relativistic critical density increase under irradiation of a linearly polarized laser beam is analyzed and its relevance for fast ignition is shown. The corresponding cycle-averaged $\gamma$ factor is determined from numerical simulations and from there an analytical expression is extracted for the mean relativistic critical density. The resulting maximum wavelengths for fast ignition are determined for two different models.

1. Motivation
Coronal fast ignition is characterized by an upper density limit for the deposition of the driver energy. If the driver is a laser with frequency in the infrared - near UV region the limit is given by the critical density $n_c$. Therefore in standard coronal ignition [1, 2] the flux of the energetic electrons has to travel a long distance up to the compressed core thereby undergoing sensitive diffusive attenuation. For example, are 15 kJ energy enough to ignite a pellet (“free ignition energy”), typically overall 70 kJ must be deposited in a diffusive model in the corona to compensate also for lateral losses. Unfortunately, hole boring has proven not to reduce noticeably the coronal dimension[3]. Rather is cone guided fast ignition more advantageous to bring the two regions, i.e. of deposition and ignition, closer together and, in addition, to provide for better coupling of the laser beam [4]. This scheme is a variant of coronal ignition, with energy deposition bound by $n_c$. The advantages of fast ignition with powerful lasers are well known: decoupling of compression from ignition, less requirements on pulse shaping of the compression pulse, lowering of symmetry constraints on peak compression, explicitly shown in [1]. Coupling of the laser energy to the pellet at the highest possible density is of fundamental importance. Here we phase the question of relativistic deposition density increase and the existence of a critical density in the traditional sense of classical optics, not evident a priori, in presence of high nonlinearities and caviton formation [5]. Then, maximum laser wavelength and concomitant $\lambda^2$ are estimated. Their compatibility with energy flux requirements are shown.

2. Relativistic deposition density increase
Rise of the cut off of a relativistic wave is attributed to the increase of the electron mass $m_e$ in the refractive index $\eta = [1 - n_e/n_c]^{1/2}$ at electron density $n_e$. For circular polarization it is given by $\gamma = (1 + \hat{a}^2)^{1/2}$, $\hat{a} = e\hat{A}/m_e\omega$, $\hat{A} = -(1/\omega)E$, $(\hat{A}, \hat{E})(x, t) = (\hat{A}, \hat{E})(x, t) \exp(-i\omega t)$; $\hat{A}$, $\hat{A}$,
vector potential and electric field with associated amplitudes \( E, \mathbf{E} \). Thus, for the relativistic critical density under circular polarization holds \( n_c = \gamma n_e \). As in the linearly polarized wave the mass of the electron in vacuum increases by \( \gamma = (1 + \alpha^2/2)^{1/2} \) it has been tacitly concluded that this is the correct \( \gamma \) factor for the relativistic increase of \( n_c \) under linear polarization \([7, 8, 9]\). However, in numerical studies of \([8]\) anomalous penetration, i.e., weaker penetration than resulting from the mass increase above was found. On the other hand it is well known that in linear optics \( \omega_p \) is a Lorentz invariant because electron mass and electron density transform by the same factor \( \gamma \), for \( n_e \) expressively noticed in \([10]\). Under the apparently natural assumption of the flow \( \mathbf{v} \) induced by a transverse wave to be divergence-free at high laser intensity, from \( \frac{dn_e}{dt} = -n_e \nabla \mathbf{v} = 0 \) follows \( n_e \to \gamma n_{e0} \) when the electron density at rest \( n_{e0} \) is set into motion by the laser field. Hence, as \( m_e \to \gamma m_e, \omega_p \) and the refractive index \( \eta \) become Lorentz-invariant quantities also in this case and nothing would change in comparison with the weak field situation. If \( n_e \) does transform differently there must exist a physical reason for it.

**I** Lorentz contraction and quasineutrality. Exactly speaking the oscillatory motion of the electron fluid induced by a laser can never be divergence-free because under such an assumption by the charge density \( \rho_e = -e(\gamma n_{e0} - n_0) = -e(\gamma - 1)n_{e0} \) an electric field is induced which for symmetry reasons must point into wave propagation direction. Thus, it depends on the available time interval to what degree charge compensation establishes. In the circularly polarized wave the available time is half the pulse length extending over many oscillations and hence, \( n_e \approx n_{e0} \). Instead, in the linearly polarized wave the available time is a quarter laser cycle long only and restoring of quasineutrality is expected to be incomplete, e.g., compare the tenuous plasma case \((\omega_p \ll \omega)\) where \( n_{c0} = (1 + 3\alpha^2/8)^{1/2}, \, i.e., \, 3/8 < 1/2 \) for this reason \([11, 12, 13]\). The dispersion of propagating plane waves of arbitrary strength and linear polarization in constant plasma density, or density slowly changing in space, under Lorentzian gauge is governed by the set of equations

\[
\begin{align*}
\square A &= -J/e_0c^2, \\
\partial_0 A^\alpha &= 0, \\
A &= (A, \phi/c), \\
J &= (j, c\rho_e), \\
\rho_e &= e(n_{e0} - n_e) = e_0(\nu_0^2/c^2 - 1)\partial^2\phi/\partial x^2, \\
\partial\phi/\partial x &= (e^2/v_\phi^2)\partial A_x/\partial x, \\
j_y &= -(e^2n_{e0}/\gamma m_e)A_y - e_0[ev_\phi(1 - c^2/v_\phi^2)\partial^2 A_x/\partial x^2]A_y/\gamma m_e.
\end{align*}
\]

\( \phi \) is the scalar potential and \( v_\phi \) the constant phase velocity. The current density \( j_y \) follows from the canonical momentum conservation \( \gamma m_e v_y = eA_y \) for a cold electron fluid initially at rest. In a long pulse circular polarization is compatible with \( A_x = \phi/c = 0, \) i.e., perfect quasineutrality, \( n_e = n_{e0} \), in accordance with \([6]\) whereas linear polarization is not \( (j_y \neq 0 \) in \((1)\) 2nd term \( \neq 0 \). The dispersion of waves described by \((1)\) close to cut-off is studied in \([13]\) under the assumption of constant plasma density and it has been found that in most cases the electron motion is 8-like as in the tenuous plasma. In presence of plasma density profile steepening, partial light beam reflection and extremely nonuniform electron fluid expansion and recession during one cycle \([14]\), accompanied by frequency Doppler shift in the frame of co-moving critical layer and, eventually, by caviton formation \([15]\), the 8-shape electron motion may transform into quasiperiodic and chaotic orbits with unknown Lorentz factor \( \gamma \). The question of the existence of a critical density and its relativistic increase under such circumstances and missing quasineutrality can be answered only ‘experimentally’, i.e., by particle-in-cell (PIC) and similar simulation procedures (e.g., Vlasov). We use the 3D PSC PIC \([16]\).

**II** The \( \gamma \) factor. A fully ionized target of 90 times critical density with heavy ions (to suppress hole boring, of no interest in the context) is exposed to linearly polarized laser irradiance of \( I\lambda^2 = 10^8 \text{Wcm}^{-2}\text{µm}^2 \), \( s = 18 - 22 \) under perpendicular and p-45° incidence and, when the interaction has become stationary, all field quantities are averaged over two laser cycles. A typical result for \( I\lambda^2 = 10^{21} \text{Wcm}^{-2}\text{µm}^2 \) is presented in figure 1. In contrast to strongly fluctuating pictures taken at single time instants, after averaging sufficiently quiescent shapes in
Figure 1. Plane target of density $n_0 = 9 \times 10^{22} \text{cm}^{-3}$ irradiated by $I\lambda^2 = 10^{21} \text{Wcm}^{-2}\mu\text{m}^2$ in p-polarization under 45° after 80 fs. Lorentz boost (density upshift). Quantities averaged over two cycles as functions of space coordinate $x$: laser field $|E|^2$ bold, electron density $n_e$ dotted, absorption $j_yE_y$ oscillating around 0.0 solid bold, $j_xE_x \cong 0$ dashed bold; thin dashed lines underneath are the static electric field $E_x$ (positive) and the static magnetic field $B$.

Figure 2. Critical density increase by factor $\gamma$ as a function of $I\lambda^2$ at normal (LHS pair) and 45 incidence (RHS pair); ocher: 'experimental', green: analytic. $\gamma = (1 + \frac{I\lambda^2}{10^{18}})^{1/2}$ dashed.

all field quantities are obtained. The critical density is to be expected at the saddle point of the evanescent branch of $|E|^2$ close to half the maximum which in nearly all cases coincides with the maximum of absorption $jE$. Thus the density at this point is taken as the “experimental” critical density $n_{\text{cr}}$ and its relativistic increase $\gamma = n_{\text{cr}}/n_c$, respectively. We compare these values with a tentative ‘theoretical’ $\gamma$ factor from the formula $\gamma = [1 + (1 + r)^2\hat{a}^2/2]^{1/2}$, with $r\hat{a}$ the reflected normalized wave amplitude. From the last max $|E|^2 = M$ and min $|E|^2 = m$ the factor $r = (\sqrt{M} - \sqrt{m})^2/(M - m)$ is obtained. The two $\gamma$ factors are compared in figure 2, green ‘theory’, left: 90° incidence, ocher PSC, left: 90° incidence, right: corresponding quantities under 45° incidence, all taken at $t = 80$ fs. We conclude that up to the intensities considered an average critical density makes sense, in contrast to an instantaneous one, and its approximate relativistic increase is given by the “theoretical” $\gamma$ which, alternatively, is approximated by the best fit $\gamma = (1 + I\lambda^2/10^{18})^{1/2}$ (see figure 2). Vacuum-like (8-shape) electron orbits (i) and instantaneous quasineutrality (ii) would be sufficient for the “theoretical” $\gamma$ to be correct.

Figure 3. Evolution of fast electron spectrum ($p_x > 0$) and its interaction with the return current ($p_x < 0$) after 40, 80 and 120 fs in the boosted frame of target from figure 1 at $I\lambda^2 = 10^{22}$.

3. Constraints on wavelengths and $I\lambda^2$

The evolution of the momentum spectrum $p_x(x)$ associated with figure 1 at 40, 80 and 120 fs clearly shows the strong interaction of fast electron jets of periodicity $\omega$ and $2\omega$ with the return current; see also [17]). As a consequence, partial thermalization occurs and a diffusive energy transport model may be appropriate. Hence we may base our further analysis
on the constraints for ignition obtained from a flux limited diffusive Spitzer model [1, 2]. There we have found: minimum absorbed laser intensity $q_0 = 10^{21}$ W/cm$^2$, deposited at DT density of $(4 - 5)g$/cm$^3$, however not inferior to $1g$/cm$^3$, corresponding to particle density $n_d = 2.5 \times 10^{23}$ cm$^{-3}$. Below this limit and $50 - 70$ kJ energy input no ignition was possible. For these constraints the maximum laser wavelength is determined now. Starting from the $50\%$ absorption ($\lambda/\lambda_d$ wavelength of Nd it holds $n_c(\gamma) = n_c(\lambda) = \lambda^2 \lambda_d^2/\lambda^2$. Combining it with the best fit from figure 2 for $\gamma(\lambda) = (1 + 2q/10^{18})^{1/2}\lambda/\lambda_d$ at the minimum flux $q_0$ under the assumption of $50\%$ absorption ($I = 2q_0$) the requirement $n_c = n_d$, safety factor $s = 4 - 5$, leads to the condition $\lambda/\lambda_d = (n_c/\lambda_d)(1 + 2q_0/10^{18})^{1/2}$. Inserting the corresponding numerical values from above yields $\omega = 1.1\omega_d$ for $s = 1$, i.e., the maximum possible wavelength to produce ignition is close to $\lambda_d$ and excludes, for example, the use of the fundamental iodine laser wavelength of $\lambda = 1.315\mu$m. The associated $\gamma$ value amounts to $\gamma = 40/s$, in agreement with figure 2. Finally, the mean electron energy $<E>$ in order to be stopped in the pellet should not exceed $(3 - 5)$MeV. To show the consistency with this condition the overall energy balance $n_c < E > u = q_0$, $n_c = n_d$, for $<E>$ and its average flow velocity $u$ must be fulfilled at any laser wavelength in order to absorb the intensity $I = q_0$. In the worst case of $s = 1$ it yields $u = 0.87c$ and $<E> = 0.95$ MeV, i.e. an energy well below the above limit. At $\lambda_d$ the mean oscillation energy at $I = 2q_0$ amounts to $E_{os} = 14.6$ MeV and hence only a small fraction (typically less than $10\%$) is converted into fast electrons which may not be stopped. The argument holds also for the harmonics of low order $N$ because both $<E>$ and $E_{os}$ decrease by $1/N$. This proves the consistency of the assertion above on the maximum laser wavelength. Alternatively, inspired from [4] one may apply a ballistic model for the fast electrons of minimum flux density $q_0 = 10^{20}$ W/cm$^2$. If $q_0$ deposited at $3\omega_d$ with $R = 0.5$ one finds $\gamma = 6.7$, $n_{cr} = 6.7 \times 10^{22}$ cm$^{-3}$, exceeding solid DT density, mean kinetic energy per particle $<E> = 0.37$ MeV. For comparison, $a = 4.3$ and $E_{os} = 1.1$ MeV, again exceeding $<E>$. If such a ballistic scheme worked it would have the advantage of operating at lower laser intensities and thus producing much less energetic electrons. 

Summarizing it has been shown that a cycle averaged critical density in the traditional sense of linear optics exists. Its relativistic increase over four decades can be described by a simple formula extracted from PIC simulations under normal and 45° incidence. It is the result of a dynamic interplay between relativistic mass increase along complex orbits, Lorentz contraction, and quasineutrality. The critical density increase contributes significantly to reducing the constraints on the laser beam for fast coronal, and in particular, fast cone guided ignition.

References

[1] Hain S and Mulser P 2001 Phys. Rev. Lett. 86 015
[2] Mulser P and Bauer D 2004 Laser Part. Beams 22 5
[3] Mulser P and Schneider R 2004 Laser Part. Beams 22 157
[4] Kodama R et al 2002 Nature 412 798; 418 933
[5] Sircombe N J, Arber T and Dendy R O 2005 Phys. Plasmas 12 012303
[6] Akhiezer A I and Polovin R V 1956 Sov. Phys. JETP 3 696
[7] Krue W L 1988 The Physics of Laser Plasma Interactions (Addison Wesley Pub.) p 42
[8] Sakagami H and Mima K 1996 Phys. Rev. E 54 1870
[9] Haizi B et al 2003 Phys. Plasmas 10 1483
[10] Lontano M, Bulanov S V and Koga J 2002 Phys. Plasmas 8 5113
[11] Kaw P and Dawson J 1970 Phys. Fluids 13 472
[12] McKinstrie C J and Forslund D W 1987 Phys. Fluids 30 904
[13] Pesch T C and Kull H J 2007 Phys. Plasmas August issue
[14] Baeva T, Gordonko S and Pukhov A 2006 Phys. Rev. E 74 046404
[15] Weber S, Lontano M, Passoni M et al 2005 Phys. Plasmas 12 112107
[16] Ruhl H in Bonitz M et al (eds) 2006 Computational Methods in Many Body Physics ISBN 1-58949-009-6
[17] Sakagami H, Johzaki T, Nagatomo H, and Mima K 2006 J. Phys. IV France 133 421