Application of mathematical methods of analysis in selection of competing information technologies

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Abstract. The article discusses the use of qualimetry methods using the apparatus of mathematical analysis in the formation of the integral index that allows one to select the best option among competing information technology. The authors propose the use of affine space in the evaluation and selection of competing information technologies.

1. Introduction
In modern conditions of the knowledge economy, which are characterized by information society, the main factor of formation and development of the economy is human potential [1-2]. For countries with innovative economy, human capital is the most important production factor in the creation of the newest technologies that are the drivers of development of industry, science, culture, technosphere safety, etc.

In modern conditions, one of the rapidly developing areas of the knowledge economy is the industry of information technology [3-4]. The core of information technologies is contemporary achievements in computer technology and the latest high technologies such as electrical engineering, robotics, communications, a distribution system of news, software, technology of teamwork and training, etc. They are aimed at solving the urgent problems of effective organization of the information process in all areas of human life with the aim of reducing labor costs, energy, time and material resources. It significantly improves the quality and competitiveness of the final result of the organizations activities.

Quality as an economic, philosophical, technical, social category is the most complex manifestation of human activities [5]. Indeed, the authors use the notion of quality when selecting items to meet different needs, the evaluation of the same objects, planning production, production of new equipment, consideration of works of art, the assessment of political decisions, the introduction of new technologies [6], etc.

Any innovative technology is characterized by a set of certain properties. It conditionally forms its quality. Any property can be assessed both quantitatively and qualitatively; that is why it requires the development of a model for removing the integral indicator with the use of qualimetry methods and mathematical apparatus. This model is reflecting a feature of the technology itself. The obtained results can be used when comparing competing technologies. In this case, let us consider the application of an affine space as a way to solve the problem.
2. Materials and methods

Each object of information technology, for instance, software, depending on the goals and objectives, stages of its life cycle, customer requirements, wishes of the society, can have the combination of those characteristics or parameters that can be combined into one list as follows:

\[ P = \{ q_1, q_2, \ldots, q_i, \ldots, q_n \} \]  

(1)

Depending on the existing models of formation, parameters can be both deterministic and random variables in this list. The first group includes, for example, parameters such as efficiency, service life, security of technology, and the second group includes reliability, risk, a level of customer satisfaction, etc.

The set of numeric values of the parameters will correspond to specified list \( P \):

\[ Z = \{ z_1, z_2, \ldots, z_i, \ldots, z_n \} \]  

(2)

where, \( z_i \) is the value of parameter \( q_i \), \( i = 1, n \).

From the point of view of affine space \( A \), where the values are the coordinates of point \( Z \), for instance, different points \( Z \) will correspond to the software of the same affine space (Fig. 1).

The feature of the affine space is that any points are equal.

Each pair of points of affine space \( A \) uniquely determines vector \( \vec{V} \) which belongs to associated linear space \( L \). For instance, vector \( \vec{V} = \overrightarrow{Z_0Z} = (\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_i, \ldots, \vec{v}_n) \) corresponds to points: \( Z_0 = \{z_{i0}, z_{20}, \ldots, z_{n0}\} \) and \( Z = \{z_1, z_2, \ldots, z_n\} \).

\( A_0, A \) – two competing objects of information technology. They correspond to points \( Z_0, Z \); that is why, comparing projects \( A_0, A \) is based on the difference in their values of the parameters, i.e. the vector \( \vec{V} \).

Let us turn to dimensionless coordinates in linear space as follows:

\[ \vec{V} = \sum_{i=1}^{n} \vec{v}_i e_i = \sum_{i=1}^{n} \frac{\vec{v}_i}{z_i} \zeta_i e_i = \sum_{i=1}^{n} v_i \zeta_i, \]  

(3)

where, \( \{e_i\} \) is the standard basis of a linear space; \( \{\zeta_i\} \), \( \zeta_i = z_i^* e_i \) is the physical (dimensional) basis of the same space.
3. Results
From the point of view of quality of information technologies, many parameters from list R (1) can be divided into three categories: "A", "B" and "C". The category "A" consists of such parameters, the increase of which leads to improving the quality indicators of information technology. If the increase in the parameter value leads to deterioration of quality, it is belonged to the type "B". In other cases, it is a category "C".

A variety of category parameters "A" and "B" can be merged if one carries out the computation of vector components by the following formulas:

\[ v_j = \frac{z_j - z_{i0}}{z_j}, \]

\[ v_j = \frac{z_j0 - z_j}{z_j}. \]

Then vector \( \vec{V} \) can be represented as \( \vec{V} = \vec{V}_A + \vec{V}_C \), the decomposition of which corresponds to the representation of associated linear space \( L \) into a direct sum of two subspaces \( L = L_A \oplus L_C \), \( \text{dim} L_A = m \), \( \text{dim} L_C = p \), \( m + p = n \). Thus, each of conditions \( \vec{V}_i \geq 0 \), \( i = 1, m \); \( \vec{V}_k = 0 \), \( k = 1, p \); \( n = m + p \) selects target set \( S \) in subspaces \( L_A \) and \( L_C \). If conditions are in this target, set \( S \) means the achievement of quality. (Fig. 2).

Target set \( S \) for category "C" consists of one point \( Z_0 \).

\[ \text{Type "A"} \]

\[ \text{Type "C"} \]

\[ \text{Figure 2. Subspace } L_A, L_C \text{ and target set} \]

Thus, the efficiency of the object of information technologies, assessed through the integrated quality indicator can be linked with the position of point \( Z \). The possible cases are:

- \( Z \in S \), removal \( Z \) from \( Z_0 \) of set \( S \) corresponds to increasing the quality and therefore to the effectiveness of the information technologies;
- \( Z \not\in S \), removal \( Z \) from \( S \) corresponds to a reduction of quality.

Integral quality level \( Q \) of the object of information technology towards another similar quality of object \( A_0 \) is a function of the length of the vector of \( \| \vec{V} \| \): \( \vec{V} = \vec{Z}_0 \vec{Z} \):

\[ Q = f(\| \vec{V} \|). \]

Function \( f \) in equation (6) cannot be arbitrary and must satisfy the following conditions:

- the parallel translation of the pair of points \( Z_0, Z \) must not change the value of \( Q \);
- if three points \( Z_0, Z_1 \) and \( Z_2 \) are on the same line, so that \( \vec{Z}_0 \vec{Z}_2 = \vec{Z}_0 \vec{Z}_1 + \vec{Z}_1 \vec{Z}_2 \);
• \( f(Z_0 Z) = 0 \), if \( Z_0 = Z \) and \( f(Z_0 Z) > 0 \) when \( Z_0 \neq Z \).

Considering the formulation of the problem of evaluation of competing facilities, there is only one dependence \( f \), since any numerical function of the length of the vector which has invariance, additivity and positivity differs from the length of the vector by only constant factor:

\[
Q = k\left(\|\vec{v}\|\right),
\]

where \( k \) is a constant multiplier, \( k \neq 0 \).

Formula (7) allows one to write the expression for indicator of quality \( Q \), which estimates the level of the hit in target set \( S \):

\[
Q = \alpha \sqrt{\sum_{i=0}^{n} v_i^2} - \beta \sqrt{\sum_{j=0}^{n} v_j^2} - \gamma \sqrt{\sum_{k=0}^{n} v_k^2},
\]

where \( I \) is the set of indices \( i \), for which \( v_i \geq 0 \); \( J \) is the set of indices \( j \), for which \( v_j < 0 \); the index "\( k \)" is the parameter of type \( \alpha > 0, \beta > 0, \gamma > 0 \).

The introduction of weighting factors in formula (8) leads to the following form:

\[
Q = \alpha \sqrt{\sum_{i=0}^{n} \delta_i v_i^2} - \beta \sqrt{\sum_{j=0}^{n} \delta_j v_j^2} - \gamma \sqrt{\sum_{k=0}^{n} \delta_k v_k^2}.
\]

The setpoint of metric (distance) for a fixed basis is equivalent to specifying matrix \( g = [g_{ij}] \), where the length of the vector is calculated by the formula:

\[
\|\vec{v}\| = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} g_{ij} V_i V_j}.
\]

In formula (8), metric \( \sqrt{\sum_{i=1}^{n} (V_i)^2} \) was used on condition that \( g_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \). Metric \( \sqrt{\sum_{i=1}^{n} \delta_i (V_i)^2} \) was used in formula (9), on condition that \( g_{ij} = \begin{cases} \delta_i, & i = j \\ 0, & i \neq j \end{cases} \).

In the calculation of indicators according to formula (9), the authors lose the information about the location of points in space and about their evolution over time. That is why let us introduce the reference direction, which is either a vector or a pair of points (Fig. 3).

![Figure 3. The basic directions of S and L](image-url)
and $Z_{inf}$. If one chooses the endpoint $Z_{sup}$, there are two basic directions of $S$ and $L$. Then, the configuration of points $\{Z_i\}$ defines a system of vectors emanating from starting point $Z_0$, which defines the following formula to determine the angle between vectors $\vec{V}_i = (v_{1i}, v_{2i}, \ldots, v_{ni})$ and $\vec{V}_j = (v_{1j}, v_{2j}, \ldots, v_{nj})$:

$$\Theta_j = \arccos \frac{\sum_{p=1}^{n} \sum_{q=1}^{n} S_{pq} v_p v_q}{\|\vec{V}_i\| \|\vec{V}_j\|}.$$

(11)

If one puts the angles from the basic direction defined by the vector, it is possible to obtain a visualization (Fig. 4).

![Figure 4. Visualization](image)

The selected method of visualization accurately shows the deflection only from the basic direction. The angles of matrix $[\Theta_{ij}]$ completely describe the mutual location of the vectors.

The scalar indicator of integral quality $Q$ (9) and angle $\Theta$ (11) allow us to enter a vector parameter of the quality of the object of information technologies (Fig. 5):

$$\vec{Q} = (Q, \Theta),$$

(12)

where scalar $Q$ is considered as the length of the vector.

The vector index of quality $\vec{Q}$ (12) is a random variable, characterized by parameters such as mathematical expectation and variance that are determined respectively by:

$$M[\vec{Q}] = (M[Q], M[\Theta]),$$

(13)

$$D[\vec{Q}] = (D[\vec{Q}], D[\Theta]).$$

(14)

![Figure 5. Vector indicators of quality](image)

The point on the plane with coordinates $(M[\vec{Q}], M[\Theta])$ defines the center of dispersion $R$, and dispersion $(D[\vec{Q}], D[\Theta])$ defines the dimensions of the area of dispersion (Fig. 6).
4. Conclusion

The quality parameters of information technology fill the coordinates of point Z, forming an integral indicator. The application of the method of affine space allows one to choose an alternative version of information technology, the best one for the organization, for example, the same type of software product, but of different manufacturers. Overall, the method is generic and can be used for the assessment of any objects.

References

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