Vibration Energy harvesting using single and comb-shaped piezoelectric beam structures: Modeling and Simulation

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Abstract

Of late, many have shown great interests in the area of energy harvesting or energy scavenging. Researchers have been venturing into methods that can generate acceptable level of voltage since decades ago. In line with the spirit of green technology, energy harvesting will be a major contributor towards saving our environment in near future. Vibration energy harvesting, specifically, is getting more and more attention nowadays. With the abundant sources, this type of energy harvesting can generate desired voltage to power any low power devices and wireless sensor; and subsequently high power devices in the future. In this research, unimorph piezoelectric energy harvester is chosen to harvest wideband mechanical energy. The derivation of the mathematical modelling is based on the Euler-Bernoulli beam theory. MATLAB and COMSOL Multiphysics software are used to study the influence of the structure in generating output voltage due to base excitations. Finally, the results of the frequency response are displayed in the form of voltage within frequency range of 0 to 3500 Hz, at which the comb-shaped piezoelectric beam structure shows better performance as there exist more natural frequencies in the specified range of frequency.

Keywords: unimorph, piezoelectric, vibration energy harvesting, optimized piezoelectric structure, comb-shaped structure, COMSOL

Nomenclature

- \( h \) thickness of beam (m)
- \( h_p \) thickness of piezoelectric layer (m)
- \( E \) Young Modulus
- \( m \) mass per unit length (kg/m)
- \( L \) length of the beam (m)
- \( z(x,t) \) transverse displacement along the beam

Greek symbols:
- \( \omega \) angular frequency

Subscripts:
- \( k \) mode number

1. Introduction

In the area of energy harvesting, piezoelectric transducers are widely used to power remote sensors and micro-electromechanical (MEMS) devices at which it can potentially supply ten to hundreds of \( \mu \)W [1]. In wireless sensor, the

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needs to constantly change the electrochemical power supply (almost anywhere) might be tedious and costly. Thus, this has become the major motivation for researchers to further enhance the performance of the piezoelectric energy harvester by optimizing the parameters of the structure which include the geometry, length and etc.

The derivation of the mathematical equations is not restricted to Euler-Bernoulli beam theory alone. Hamilton Principle [2], Rayleigh-Ritz method [3] and single degree-of-freedom [4] models are some of the approach that can be employed in deriving the equation of motions as well as the associated boundary conditions.

By varying the geometry of the beam, the stress is distributed along the beam’s length in order to increase the harvested power. Optimum performances are achieved for excitation very close to the fundamental frequency of the system and for a specific value of the load resistance. Sameh, et.al. [2] concluded that a trapezoidal cantilever beam will produce more power per unit area compared to a rectangular beam as the distribution of strain is uniform. In other words, the shape should be as truncated as possible. In addition, most of the works done in this area used lead zirconate titanate (PZT) due to its abundant vibration accessibility. Other materials include zinc oxide (ZnO), polyvinylidene difluiride (PVDF), lead magnesium niobate-lead titanate (PMN-PT), Lead zinc niobate-lead titanate (PZN-PT) and polypropylene polymer (PP). PMN-PT and PZN-PT, though possess large coupling coefficient were not in favour as the price is very high and the accessibility in commercial market is still low [5]. PVDF may be implied in some cases because of its higher tensile strength, lower stiffness and not brittle like ceramics [6]. The slope of the tapered beam used [3] and the thickness of the beam does also contribute in the performance of the piezoelectric energy harvester [7]. Hence, there a lot of other possibilities that can be explored in order to optimize the power generation of this type of piezoelectric energy harvester.

The main focus of this paper is to study the influence of unimorph single and comb-shaped piezoelectric energy harvesting structure to the output voltage generated. The equations of motions are derived from the Euler-Bernoulli beam theory, and simulation studies are carried out using MATLAB and COMSOL multiphysics software.

2. Mathematical modelling of the piezoelectric energy harvester

In deriving the mathematical modelling of the structure, Euler-Bernoulli or thin beam theory is considered [8 - 12]. General equation of motion for the beam structure is given by Eq. (1).

\[
\frac{\partial^2 M(x,t)}{\partial x^2} + m \frac{\partial^2 z(x,t)}{\partial t^2} = f_0(x,t)
\]  

(1)

2.1. Free Vibration

The determination of the natural frequencies of the beam is done without the presence of the piezoelectric patch. By assuming the patch is very thin compared to the thickness of the beam, the stiffness of the patch can be ignored. The governing equation of motion for undamped free vibrations can be expressed as Eq. (2)

\[
EI \frac{\partial^4 Z(x,t)}{\partial x^4} + m \frac{\partial^2 z(x,t)}{\partial t^2} = 0
\]  

(2)

Using modal analysis techniques, the solution of \(z(x,t)\) can be expressed as Eq. (3)

\[
z(x,t) = \sum_{k=1}^{\infty} w_k(x)q_k(t)
\]  

(3)

where \(w_k(x)\) is the \(k\)-th normal mode or characteristic function and \(q_k(t)\) is the generalized coordinate in the \(k\)-th mode. As the left end of the beam is fixed and the right end of it is attached with a mass, the boundary equations employed are as in Eq. (4) [12],

\[
z(0,t) = 0, \quad \frac{\partial z(x,t)}{\partial x} = 0 \quad \text{and} \quad EI \frac{\partial^2 z(x,t)}{\partial x^2} = I_t \frac{\partial^3 z(x,t)}{\partial x \partial t^2}, EI \frac{\partial^3 z(x,t)}{\partial x^3} = m_t \frac{\partial^2 z(x,t)}{\partial t^2}
\]  

(4)

where \(I_t\) and \(m_t\) represents the inertia and mass of the tip mass. The natural frequency, \(\omega_k\) is obtained from Eq. (5) where \(\lambda_k\) is determined from the solution of Eq. (3) with associated boundary equations as in Eq. (4).
\[ \omega_k = \lambda_k^2 \sqrt{\frac{EI}{mL^2}} \]  

Subsequently, it is found out that

\[ w_k(x) = \sin \left( \frac{\lambda_k}{L} x \right) - \sinh \left( \frac{\lambda_k}{L} x \right) + \alpha \left[ \cos \left( \frac{\lambda_k}{L} x \right) - \cosh \left( \frac{\lambda_k}{L} x \right) \right] \]  

where

\[ \alpha = \frac{-\cos \lambda_k - \cosh \lambda_k + \frac{\lambda_k m_i}{mL} (\sin \lambda_k - \sinh \lambda_k)}{-\sin \lambda_k + \sinh \lambda_k - \frac{\lambda_k m_i}{mL} (\cos \lambda_k - \cosh \lambda_k)} \]  

\[ 2.2. \text{ Forced vibration due to base excitation} \]

The base motion, \( z_b(x,t) \), in terms of the translation \( (g(t)) \) and the small rotation of the base \( (h(t)) \) can be expressed as Eq. (8).

\[ z_b(x,t) = g(t) + xh(x,t) \]  

However, according to Euler-Bernoulli beam theory, the rotation of an element is insignificant as compared to the vertical translation and this leads to the assumption of zero base rotation in Eq. (8). Under the influence of the base excitation, the governing equation of motion can be written as

\[ \frac{\partial^2 M(x,t)}{\partial x^2} + c_s I \frac{\partial^5 z_{rel}(x,t)}{\partial x^4 \partial t} + c_a \frac{\partial^2 z_{rel}(x,t)}{\partial t} + m \frac{\partial^2 z_{rel}(x,t)}{\partial t^2} = -m \frac{\partial^2 z_b(x,t)}{\partial t^2} - c_a \frac{\partial z_b(x,t)}{\partial t} \]  

where \( z(x,t) = z_b(x,t) + z_{rel}(x,t) \). In Eq. (9), \( z_b(t) \) is the base displacement, \( z_{rel}(x,t) \) is the transverse deflection of the beam relative to its base, \( M(x,t) \) is the internal bending moment, \( c_s I \) is the equivalent area moment of inertia of the composite cross section, \( c_a \) is the viscous air damping coefficient and \( m \) is the mass per-unit length of the beam.

\[ 2.3. \text{ Mechanical equation of motion with electrical coupling} \]

The stress producing the strain is expressed as Eq. (10)

\[ \varepsilon_T = \varepsilon_b - \varepsilon_p \]  

where \( \varepsilon_T \) is the transverse strain, \( \varepsilon_b \) represents the strain in the beam and the free piezoelectric strain is denoted by \( \varepsilon_p \). In addition, \( \varepsilon_p = d_{31} v(t)/h_p \) where \( d_{31} \) is the piezoelectric moduli, \( h_p \) is the thickness of the piezoelectric layer and \( v(t) \) is the applied voltage to the piezoelectric actuator. Using Hooke’s Law, the relationship between the stress distribution in the piezoelectric material and the beam are as follows:

\[ \sigma_T = E_b \varepsilon_T - E_p (\varepsilon_b - \varepsilon_p) \]  

\[ \sigma_b = E_b \varepsilon_b \]  

Where \( E_b \) is the beam’s Young Modulus and \( E_p = Cz \), at which \( C \) represents the strain gradient determined by moment equilibrium around the natural axis. The moment equilibrium about the neutral axis, at which the PZT patch covers the entire length of the beam, is expressed as

\[ M(x,t) = EI \frac{\partial^2 z_{rel}(x,t)}{\partial x^2} + \partial v(t)[H(x) - H(x-L)] \]  

where the bending stiffness of the composite cross section, referred as \( EI \) is given by;
\[ EI = B \left[ E_b \frac{h^3}{12} + E_p \frac{h}{3} \left( \frac{h}{2} + h_p \right)^3 - \frac{h^3}{8} \right] \]  

(14)\#

and

\[ g = -E_a B d_{31} \left[ \frac{1}{2} (h + h_p) \right] \]  

(15)\#

2.4. Electrical circuit equation with mechanical coupling

In order to obtain the electrical circuit equation with mechanical coupling, the following piezoelectric constitutive relation is considered.

\[ D_3 = d_{31} \sigma_1 + \varepsilon_{33}^T E_3 \]  

(16)\#

where \( D_3 \) is the electric displacement, \( \sigma_1 \) is the stress and the permittivity at constant stress is denoted by \( \varepsilon_{33}^T \). In this equation, \( E_3 \) is the applied voltage and is given by \( E_3(t) = -v(t)/h_p \). The permittivity at constant strain replaces the permittivity component through \( \varepsilon_{33}^S = \varepsilon_{33}^T - d_{31} E_3 \). Thus, Eq. (16) can be written as

\[ D_3 = d_{31} E_p \varepsilon_1(x,t) - \varepsilon_{33}^S \frac{v(t)}{h_p} \]  

(17)\#

The average bending strain, \( \varepsilon_1(x,t) \) is expressed as

\[ \varepsilon_1(x,t) = \left( \frac{h}{2} + h_p \right) \frac{d^2 z_{rel}(x,t)}{dx^2} \]  

(18)\#

Hence, Eq. (17) becomes

\[ D_3 = d_{31} E_p \left( \frac{h}{2} + h_p \right) \frac{d^2 z_{rel}(x,t)}{dx^2} + \varepsilon_{33}^S B \frac{v(t)}{h_p} \]  

(19)\#

Integrating the electric displacement over the length of the piezoelectric layer yields the amount of the electric charge \( q(t) \) generated and subsequently, the value of current generated by the PZT, \( i(t) \). Hence, the voltage across the resistive load is given as

\[ v(t) = R i(t) = -R_L \int_{x=0}^{x=L} d_{31} E_p B \left( \frac{h}{2} + h_p \right) \frac{d^3 z_{rel}(x,t)}{dx^3} - \varepsilon_{33}^S B \frac{v(t)}{h_p} \]  

(20)\#

2.5. Output Voltage

Using partial differential equation as well as the orthogonality conditions, the modal response of the beam is obtained as

\[ \frac{d^2 q_k(t)}{dt^2} + 2 \zeta_k \omega_k \frac{dq_k(t)}{dt} + \omega_k^2 q_k(t) + \chi_k v(t) = N_k(t) \]  

(21)\#

where the modal coupling term, \( \chi_k \) and the mechanical damping term, \( \zeta_k \) is given as;

\[ \chi_k = S \left. \frac{dv_k(x)}{dx} \right|_{x=L}, \quad \zeta_k = \frac{C_s I \omega_k}{2EI} + \frac{C_a}{2m \omega_k} \]  

(22)\#

Thus, the ratio of the voltage output to the base acceleration, \( z_b(x,t) = Y_0 exp^{j\omega t} \) or better known as the voltage FRF, is given by
\[
\begin{align*}
\frac{v(t)}{-\omega^2 Y_0 e^{j\omega t}} &= \sum_{k=1}^{\infty} \frac{jm\omega^3 \varphi_k \gamma_k^w}{\omega_k^2 - \omega^2 + j2 \zeta_k \omega_k \omega} + \frac{1 + j \omega^2 c}{\tau_c} \\
\end{align*}
\]  \hfill (23)\#

where

\[
\gamma_k^w = \int_{x=0}^{x=L} w_k(x)dx, \varphi_k = -\frac{d_{31} E_p h_p}{\varepsilon_3^S L} \left( \frac{h}{2} + h_p \right) \frac{d w_k(x)}{dx}, \tau_c = \frac{R_e \varepsilon_3^S BL}{h_p}
\]  \hfill (24)

Eq. (24) represents the voltage generated using single beam piezoelectric energy harvesting structure.

2.5.1 Frequency response for comb-shaped piezoelectric energy harvester

For combed shaped structure, the voltage produced is summation of all the voltage FRFs’ of all the beams. Mathematically, it can be expressed as

\[
\begin{align*}
\frac{v(t)}{-\omega^2 Y_0 e^{j\omega t}} \bigg|_{\text{comb-shaped}} &= \sum_{n=1}^{\text{num}} \sum_{k=1}^{\infty} \frac{jm\omega^3 \varphi_k \gamma_k^w}{\omega_k^2 - \omega^2 + j2 \zeta_k \omega_k \omega} + \frac{1 + j \omega^2 c}{\tau_c} \\
\end{align*}
\]  \hfill (25)\#

Where \text{num} represents the total number of beams.

3. Simulation and results

The simulation is mainly done in Matlab. To show the effect of the existence of the piezoelectric patch to the frequency, the structure is modelled in COMSOL Multiphysics software. It is assumed that the piezoelectric patch used is of type Lead Zirconate Titanate (PZT-5A). Note that the inertia of the tip mass is neglected in this study.

3.1. Natural frequencies

The parameters used in the free vibration simulation are as listed in Table 1.

| Beam Parameters | Value | PZT Parameters | Value |
|-----------------|-------|----------------|-------|
| Length, L (m)   | 0.5   | Length, L (m)  | 0.5   |
| Width, b (m)    | 0.04  | Width, b (m)   | 0.04  |
| Thickness, h (m)| 0.003 | Thickness, h (m)| 0.254 x 10^{-3} |
| Young Modulus, Y [N/m^2] | 6.9 x 10^{10} | Young Modulus, Y [N/m^2] | 66 x 10^9 |
| Area Moment of Inertia, I [m^4] | 9 x 10^{-11} | Piezoelectric constant, d_{31} [m/V] | -190 x 10^{-12} |
| Mass density, \rho [kg/m^3] | 2700 | Permittivity, \varepsilon_{33}, [F/m] | 15.93 x 10^{-9} |

The list of the first five natural frequencies (in Hz) is listed as in Table 2. It shows that the presence of the tip mass does lower the values of the natural frequencies. In addition, as the thickness of the piezoelectric patch is small compared to the beam’s thickness, only slight differences are observed between the natural frequencies of the beam (without the PZT-5A patch) and with the presence of the PZT-5A patch.

| Mode number, k | COMSOL’s results | MATLAB’s results | COMSOL’s result | MATLAB’s results |
|----------------|------------------|-----------------|----------------|-----------------|
| Without tip mass |                  |                 |                |                 |
| With tip mass |                  |                 |                |                 |
Figure 1 shows simulation of the single beam piezoelectric energy harvesting structure in COMSOL Multiphysics software.

\[ \begin{array}{cccc}
1 & 9.8095 & 9.7955 & 3.9483 & 3.2280 \\
2 & 61.4630 & 61.4125 & 44.8897 & 44.2060 \\
3 & 172.0418 & 171.9567 & 141.4301 & 140.6029 \\
4 & 336.9728 & 336.9664 & 292.7188 & 291.9077 \\
5 & 556.6970 & 557.0296 & 498.7922 & 498.2234 \\
\end{array} \]

3.2. Frequency response to output voltage

Comparisons are made between single beam and comb-shaped beam structures, both with and without the presence of the tip mass. The parameters used for the PZT layer are as in Table 2. In this paper, for comb-shaped beam structure, it consists of four beams with different dimensions.

3.2.1 Single beam structure

The output voltage of single beam structure is plotted as in Figure 2. It can be seen that as the frequency increase, the peak of the plot decreases. In addition to that, as the tip mass lowers the natural frequency, the peak in figure 2(b) are at lower frequency values. The peaks of the plot occur at approximately around the natural frequencies.

3.2.2 Comb-shaped structure

In order to simulate the voltage output for comb-shaped structure, the width and the thickness of the beam are held constants at 10 mm and 1 mm respectively. The length of the beam used is 0.03m, 0.05m, 0.07m and 0.1m. The figure below shows the FRF of the voltage generated by comb-shaped piezoelectric beam structure. The main reason of employing...
this type of structure is to maximize the generation of output voltage. This is since, the number of peaks of the plot is higher as the natural frequencies of the four beams are different. Theoretically, the amount of voltage generated is higher at which the peak occur (around the natural frequency). Figure 3 shows the voltage produced by individual beam and the sum of all beams as function of frequencies. Broadband vibration energy harvesting is possible with the comb-shaped piezoelectric beams.

![Figure 3 Voltage response for individual beam and total beams in comb-shaped structure](image)

4. Conclusion

This paper presents the simulation studies of single and comb-shaped piezoelectric beam structure for energy harvesting. The derivations of the mathematical equations are based on Euler-Bernoulli Beam theory. The harvester is assumed to experience lateral vibration from its base excitation. The presence of tip mass is considered in mathematical derivation shown above. It is seen that comb structure can be used to harvest broadband vibration energy. In future analysis, a wider focus can be made to include optimization of parameters such as the geometry, thickness and the total mass of the system, in order to maximize harvested energy.

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