Pionic Decays of $D_{sj}(2317)$, $D_{sj}(2460)$ and $B_{sj}(5718)$, $B_{sj}(5765)$

Jie Lu, Wei-Zheng Deng, Xiao-Lin Chen, and Shi-Lin Zhu
Department of Physics, Peking University, Beijing 100871, China
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We estimate pionic decay widths of the narrow charm-strange resonances $D_{sj}(2317)$ and $D_{sj}(2460)$ using the $^3P_0$ model. Their one-pion decays occur through $\eta\pi^0$ mixing while the two-pion decays of $D_{sj}(2460)$ occur through the virtual $f_0(980)$ meson. The mixing between $^3P_1$ and $^1P_1$ states enhances the single pion decay width of $D_{sj}(2460)$ and suppresses its double pion decay width significantly. The two-pion decay width of $D_{sj}(2460)$ is much smaller than its one-pion decay width. As a byproduct, we also calculate pionic decay widths of $B_{sj}(5765)$, $B_{sj}(5765)$ mesons in the $(0^+, 1^+)$ heavy doublet.

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I. INTRODUCTION

In April 2003, Babar Collaboration reported a new charm-strange state $D_{sj}(2317)$ in the $D_s\pi^0$ channel. Its spin-parity is $J^P = 0^+$ and its mass is below the $DK$ threshold 2.36 GeV. Later CLEO Collaboration reported another new charm-strange resonance $D_{sj}(2460)$ in the $D_s\pi^0$ channel with $J^P = 1^+$ below the $D^*K$ threshold 2. It's tempting to classify these two states as the $(0^+, 1^+)$ $P$-wave $c\bar{s}$ doublet. But their observed masses are more than one hundred MeV lower than quark model predictions 3. There have been heated debates about its underlying structure and origin of its low mass in the literature 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29. Although several non-conventional schemes such as $DK$ molecules 6, four quark states 8, 9, 10, 11, 12, 13, 14, 15 and $D\pi$ atom 16 were proposed, evidence is gradually accumulating that these two narrow resonances are ordinary $c\bar{s}$ states.

Another interesting property of $D_{sj}(2317)$ and $D_{sj}(2460)$ mesons is their extremely narrow widths. The decay channels $D_{sj}(2317) \rightarrow D K$ and $D_{sj}(2460) \rightarrow D^* K$ are forbidden by kinematics. Therefore, their possible strong decay modes are one-pion and two-pion decays. The two-pion decay occurs via a virtual meson such as $f_0(980)$. The one-pion decay mode breaks the isospin symmetry and happens through $\eta\pi^0$ mixing 30: $D_{sj}(2317) \rightarrow D_s\eta \rightarrow D_s\pi^0$, $D_{sj}(2460) \rightarrow D_s^*\eta \rightarrow D_s^*\pi^0$. The $\eta - \pi^0$ mixing is described by the isospin violating piece in the chiral lagrangian

$$L_m = \frac{m_\pi^2f_\pi^2}{4(m_u + m_d)}\text{Tr}(\xi m_q\xi + \xi^\dagger m_q\xi^\dagger),$$

where $\xi = \exp(i\hat{\pi}/f_\pi)$, $\hat{\pi}$ the light meson octet and $m_q$ is the light quark mass matrix. Such a mixing is suppressed by the factor $m_u = m_d$. Numerically the isospin violating effect is $O(10^{-2})$ in the amplitude. While the isospin conserving strong decay width is $O(10^2)$ MeV, one would naturally expect the one-pion decay width of $D_{sj}(2317)$ and $D_{sj}(2460)$ to be around several tens keV.

Although quantum chromodynamics (QCD) is widely accepted to be the correct theory of strong interaction, our present understanding of the strong decay mechanism is rather limited. First-principle calculation of decay matrix elements on the lattice still has a long way to go. For some limited special cases, complicated tools such as light-cone QCD sum rules may be used to calculate the coupling constant 25. In order to understand the vast amount of strong decay data, we have to turn to phenomenological strong decay models. Among them, the $^3P_0$ model (or quark pair creation model) is the simplest and most successful one. In this work, we employ this model to calculate the decay amplitude of $D_{sj}(2317)$ and $D_{sj}(2460)$ mesons assuming they are $c\bar{s}$ states. A short review of $^3P_0$ model is given in Section III. The analysis of the one-pion decays is presented in Section III. The two-pion decay mode of $D_{sj}(2460)$ is given in Section IV. The last section is a short summary.

*Electronic address: dwz@th.phy.pku.edu.cn
1Electronic address: zhusl@th.phy.pku.edu.cn

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The decay helicity amplitude of the process $A \rightarrow BC$ in the $^3P_0$ model.

II. THE $^3P_0$ MODEL

A. The model

The $^3P_0$ model was first introduced by Micu in 1969 [31] and further developed by the Orsay group in the 1970s [32]. According to this model, a $q\bar{q}$ pair with $J^{PC} = 0^{++}$ is created from the vacuum when a hadron decays. This created $q\bar{q}$ pair carries the quantum number of the vacuum. The new $q\bar{q}$ pair, together with the $q\bar{q}$ within the parent meson regroups into the outgoing mesons via quark rearrangement process, which is shown pictorially in Fig. I. The transition operator in the nonrelativistic limit reads [32, 33]

$$T = -3\gamma \sum_m \langle 1 \ m; 1 - m|0 \ 0 \rangle (2\pi)^2 \int d^3k_3 d^3k_4 \delta^3(k_3 + k_4) \mathcal{Y}_1^m(k_3 - k_4) \chi_{1-m}^{34} \phi_0^{34} \omega_0^{34} b^I_3(k_3) d^I_4(k_4)$$

where $\gamma$ is the dimensionless pair creation parameter in the model, and indicates the strength of coupling. $\mathcal{Y}_1^m(k) = k^l Y_1^m(\theta_k, \phi_k) = -\epsilon_m \cdot k$ is a solid spherical harmonic function that describes the momentum distribution of the created pair. The meson state is defined as [34],

$$|A(n_A^{2S_A+1}L_A J_{A} M_{J_A})(p_A)\rangle = \sqrt{2E_A} \sum_{M_{L_A}, M_{S_A}} \langle L_A M_{L_A} S_A M_{S_A}|J_{A} M_{J_A}\rangle \times \int d^3k_1 d^3k_2 \delta^3(k_1 + k_2 - p_A) \psi_{n_A L_A M_{L_A}}(k_1, k_2) \chi_{S_A M_{S_A}}^{12} |\phi_A^{12} \omega_A^{12} q_1(k_1)q_2(k_2)\rangle$$

$$\langle A(p_A)|A(p_A')\rangle = 2E_A (2\pi)^3 \delta^3(p_A - p_A').$$

The subscripts 1 and 2 refer to the quark and antiquark within the parent meson $A$ respectively. $k_1$ and $k_2$ are the momentum of the quark and antiquark, $k_A$ is their relative momentum and $p_A$ is the momentum of meson $A$. $S_A = s_{q_1} + s_{q_2}$ is the total spin. $J_A = L_A + S_A$ is the total angular momentum.

The S-matrix reads

$$\langle f|S|i\rangle = I + i(2\pi)^4 \delta^4(p_f - p_i) M_{M_{J_A} M_{J_B} M_{J_C}}.$$

The decay helicity amplitude of the process $A \rightarrow B + C$ in the meson $A$ center of mass frame is

$$M_{M_{J_A} M_{J_B} M_{J_C}}(A \rightarrow BC) = \sqrt{8E_A E_B E_C} \gamma \sum_{M_{L_A}, M_{S_A}, M_{L_B}, M_{S_B}, M_{L_C}, M_{S_C}, m} \langle L_A M_{L_A} S_A M_{S_A}|J_{A} M_{J_A}\rangle \langle L_B M_{L_B} S_B M_{S_B}|J_{B} M_{J_B}\rangle \langle L_C M_{L_C} S_C M_{S_C}|J_{C} M_{J_C}\rangle \times \langle 1 \ m; 1 - m|0 \ 0 \rangle \chi_{S_C M_{S_C}}^{14} \chi_{S_B M_{S_B}}^{14} |\phi_A^{12} \omega_A^{12} \varphi_A^{34} | M_{L_B}, M_{L_C} (p),$$

FIG. 1: The meson decay $A \rightarrow BC$ in the $^3P_0$ model.
where the spatial integral $I_n(A, BC)$ is defined as

$$I_{ML_A.M_LC}^{m_n} (p) = \frac{(2\pi)^2}{4!} \int d^3k_1 d^3k_2 d^3k_3 d^3k_4 \delta^3(k_1 + k_2 - p_A) \delta^3(k_1 + k_3 - p_B) \delta^3(k_2 + k_4 - p_C)$$

$$\times \psi_{nLBM_LC}(k_1, k_3) \psi_{nCL.M_LC}(k_2, k_1) \psi_{nALM_A}(k_1, k_2) \mathcal{Y}_{m}^3 (k_3 - k_4)$$

(6)

We employ the harmonic-oscillator wave functions for S-wave states [34]:

$$\psi_{L=0}(k_1, k_2) = \left( \frac{R^2}{\pi} \right)^{3/4} \exp \left[ -\frac{1}{8}(k_1 - k_2)^2 R_B^2 \right]$$

(7)

where the parameter $R$ is the meson radius. For the P-wave state, we use [34]

$$\psi_{L=1}(k_1, k_2) = i \sqrt{\frac{2}{3\pi^1/4}} \mathcal{Y}_{lm}(k_1 - k_2) \left[ -\frac{1}{8} R^2(k_1 - k_2)^2 \right].$$

(8)

The spin matrix element can be written in terms of Wigner’s 9j symbol [32]

$$\langle \chi_{S_C M_S C} \chi_{S_B M_S B} \chi_{S_A M_S A}^{1-m} | \chi_{S_B M_S B} \chi_{S_A M_S A}^{1-m} \rangle$$

$$= \sum_{S_{BC}, M_L} \langle S_C M_S C : S_B M_S B | S M_S | S_A M_S A | 1 - m \rangle \left[ 3(2S_B + 1)(2S_C + 1)(2S_A + 1) \right]^{1/2} \left\{ \begin{array}{ccc} 1 & 1 & 1 \\ S_C & S_B & S_A \end{array} \right\} \right\} \times \langle \phi_{C}^{32} \phi_{B}^{14} \phi_{A}^{12} \phi_{0}^{34} \rangle$$

(9)

The relevant flavor matrix element for $D_{sJ}(2317) \to D_s \eta$ is

$$\langle \phi_{C}^{32} \phi_{B}^{14} | \phi_{A}^{12} \phi_{0}^{34} \rangle = \left\langle \phi_{C}^{32} \phi_{B}^{14} \phi_{A}^{12} \phi_{0}^{34} \right\rangle = \frac{1}{\sqrt{3}} \left\{ u\bar{u} + d\bar{d} + s\bar{s} \right\} \left\{ c_1\bar{s}_2 \right\} \left\{ c_1\bar{s}_3 \right\} \left\{ u\bar{u} + d\bar{d} - 2s\bar{s} \right\} = \frac{2}{\sqrt{18}}$$

(10)

With the Jacob-Wick formula the helicity amplitude can be converted into the partial wave amplitude [36]:

$$\mathcal{M}^{JL}(A \to BC) = \frac{\sqrt{2L + 1}}{2J_A + 1} \sum_{M_J A, M_J C} \langle L0J M_J | J_A M_J A \rangle \langle J_B M_J B J_C M_J C \rangle \mathcal{M}^{J L M_J A M_J B M_J C}(k_B)$$

(10)

where $M_J = M_J B + M_J C$, $J = J_B + J_C$. The decay width in terms of partial wave amplitudes using the relativistic phase space is:

$$\Gamma = \frac{1}{8\pi} \frac{P}{M_A} \sum_{J L} |\mathcal{M}^{J L}|^2.$$

(11)

For the three body process $A \to BCD$, the width is

$$\Gamma = \frac{1}{(2\pi)^5} \frac{1}{16M_A^2} \int |\mathcal{M}|^2 |p_B| |p_C^*| dm_{CD} d\Omega_B d\Omega_C.$$

(11)

### B. Input Parameters

Our convention follows Ref. [37] where the value of $\gamma = 6.9$ is $\sqrt{96\pi}$ times larger than that used by others groups [37, 38]. The masses of $\eta$, $J_0(980)$ and charm-strange mesons are taken from PDG [34]. Since the $(0^+, 1^+)$ beauty-strange mesons have not been discovered yet, we use the theoretical estimate of their masses from Ref. [4] in our calculation. For the $(0^+, 1^+)$ $D_{sJ}$ and $B_{sJ}$ mesons, we use the same parameter $R$ from Ref. [37]. We collect all these values in Table [1].

### III. SINGLE PION DECAYS OF $D_{sJ}(2317)$ AND $D_{sJ}(2460)$

$D_{sJ}(2317)$ and $D_{sJ}(2460)$ lie below the $DK$ and $D^*K$ thresholds. They can only have pionic decay modes. The isospin-violating single pion decay occurs through $\eta - \pi^0$ mixing as shown in Fig. [2].
TABLE I: Value of mass and $R$ in our calculation.

| mass (GeV) | 1.968 | 2.112 | 2.317 | 2.460 | 2.460 | 5.370 | 5.417 | 5.718 | 5.765 | 5.765 | 0.548 | 0.980 |
|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $R$ (GeV$^{-1}$) | 1.37 | 1.59 | 1.89 | 1.85 | 1.89 | 1.59 | 1.89 | 1.85 | 1.89 | 1.47 | 2.7 |

The decay amplitude of $D_{sj}(2317) \to D_s + \pi^0$ is

$$
\mathcal{M}^{JL}(D_{sj}(2317) \to D_s + \pi^0) = \mathcal{V}^{JL}(D_{sj}(2317) \to D_s + \eta) \cdot \frac{i}{m_\pi^2 - m_\eta^2} \cdot \mathcal{V}(\eta - \pi^0).
$$

(12)

The the $\mathcal{V}^{JL}(D_{sj}(2317) \to D_s + \eta)$ term is can be calculated using $^3P_0$ model. The $\eta - \pi^0$ mixing is determined by the up down quark mass difference $^{30}$:

$$
\frac{i}{m_\pi^2 - m_\eta^2} \cdot \mathcal{V}(\eta - \pi^0) = i \frac{\sqrt{3}}{4} \frac{m_d - m_u}{m_s - m_u + m_d} = i \frac{\sqrt{3}}{4} \delta_{\pi \eta},
$$

which vanishes in the limit of exact isospin symmetry. The isospin violating factor $\delta_{\pi \eta} = \frac{1}{2}$. $^{39}$.

In the heavy quark limit, there is one $1^+$ state with $j_i = L + s_q = \frac{1}{2}^+$ in the $(0^+, 1^+)$ doublet. We denote it as $|1^+, j_i = \frac{1}{2}^+\rangle$. The other $1^+$ state $|1^+, j_i = \frac{3}{2}^+\rangle$ belongs to $(1^+, 2^+)$ with $j_i = \frac{3}{2}^+$. $j_i$ is a good quantum number when $m_Q \to \infty$. Therefore $j_i = L + s_q = \frac{3}{2}^+$ and $j_i = L + s_q = \frac{5}{2}^+$ states don’t mix with each other. They are exact mass eigenstates. Moreover, it was shown that the mixing of these two states is small even if one considers the $1/m_c$ correction in Ref. $^{40}$. In other words, the physical state $D_{sj}(2460)$ is quite close to $|1^+, j_i = \frac{3}{2}^+\rangle$.

In the quark model, the basis states are $^1P_1$ and $^3P_1$ states. Generally speaking, $D_{sj}(2460)$ should be the linear combinations of $^1P_1$ and $^3P_1$ states $^{41}$

$$
|1^+, j_i = \frac{1}{2}^+\rangle = \cos \theta |^1P_1\rangle + \sin \theta |^3P_1\rangle,
$$

(13)

$$
|1^+, j_i = \frac{3}{2}^+\rangle = -\sin \theta |^1P_1\rangle + \cos \theta |^3P_1\rangle.
$$

(14)

The mixing angle $\theta_{HQ} = -\tan^{-1}\sqrt{2}$ in heavy quark limit $^{37}$. With finite charm quark mass, the mixing angle is not precisely known. However, $\theta$ is not expected to differ dramatically from $-\tan^{-1}\sqrt{2}$ according to Ref. $^{40}$.

The the partial wave decay amplitude and width of $D_{sj}(2317, 2460)$ are listed in Table III where

$$
W = (2\pi)^{\frac{5}{2}} \gamma_\delta e^{-\gamma} \frac{1}{V} \left( R^2_{AB} R^2_{BC} \right)^{\frac{1}{2}} \frac{1}{V} \sqrt{8SM_A E_B E_C \exp \left( -\frac{1}{8} \frac{p^2 R^2_{AB} + R^2_{BC}}{V} \right)},
$$

(15)

$$
V = R^2_{A} + R^2_{B} + R^2_{C}.
$$

(16)

The dependence of the single pion decay width of $D_{sj}(2317)$ on the mixing angle is shown by Fig. 3. The mixing between $^3P_1$ and $^1P_1$ states enhance $D_{sj}(2460)$’s single pion S-wave decay width, and decreases the D-wave width. We ignore the D-wave width because it is much smaller than the S-wave decay width.

FIG. 2: Pionic decay of $D_{sj}(2317)$ via $\eta - \pi^0$ mixing.

The dependence of the single pion decay width of $D_{sj}(2317)$ on the mixing angle is shown by Fig. 3. The mixing between $^3P_1$ and $^1P_1$ states enhance $D_{sj}(2460)$’s single pion S-wave decay width, and decreases the D-wave width. We ignore the D-wave width because it is much smaller than the S-wave decay width.
TABLE II: The single-pion decay width (in keV) of $D_{sJ}(2317, 2460)$ and $B_{sJ}(5718, 5765)$.

| Decay mode          | Amplitude                                                                 | $\Gamma$(keV) |
|---------------------|---------------------------------------------------------------------------|----------------|
| $D_{sJ}(2317) \to [D_s\pi^0]_S$ | $-\frac{1}{2}W\left[\frac{12}{5} + \frac{p^2}{1 - \frac{6}{5}}\right]$ | 32             |
| $B_s(0^+ \to [B_s(0^-)\pi^0]_S)$ | $\frac{1}{4}\sqrt{\pi}W\left[\frac{12}{5} + \frac{p^2}{1 - \frac{6}{5}}\right]\left(-\cos\theta + \sqrt{2}\sin\theta\right)$ | 35             |
| $B_s(1^+ \to [B_s(1^-)\pi^0]_S)$ | $W'\cos\theta + \frac{1}{4\sqrt{2}}W'\left[4\left(1 - 5\frac{R_A^2}{R_B^2}\right) + (1 - \frac{6}{5})R_A^2 k_B^2\right]\sin\theta$ | 38($\theta_{HQ}$) |

FIG. 3: Variation of $B_s(5765)$’s (dashed curve) and $D_{sJ}(2460)$’s (solid curve) single pion S-wave decay width with the mixing angle.

IV. DOUBLE PION DECAYS OF $D_{sJ}(2460)$

Parity and angular momentum conservation forbid the double-pion decay mode of $D_{sJ}(2317)$ mesons. In contrast, $D_{sJ}(2460)$ can decay into $D_s \pi\pi$ and $D^*_s \pi\pi$ final states. There is no reliable way to calculate the multiple pion decay width. One has to rely heavily on specific models to estimate the double-pion decay width. For example, the double pion decay of $D_{sJ}(2460)$ meson was assumed to occur via a virtual $\sigma$ meson in Ref. [4].

TABLE III: The double-pion decay width (in keV) of $D_{sJ}(2317, 2460)$ and $B_{sJ}(5718, 5765)$. The subscript $A$ refers to $D_{sJ}(2460)$ and $B_s(1^+)$ respectively, $B$ refers to $D_s$, $D^*_s$, $B_s(0^-)$ and $B_s(1^-)$ respectively, and $X$ refers to the $f_0(980)$.

| Decay mode          | $\mathcal{V}(A \to [Bf_0(980)]_L)$ | $\Gamma_1$ | $\Gamma_2$ |
|---------------------|-------------------------------------|------------|------------|
| $D_{sJ}(2460) \to [D_s f_0(980)]_L \to [D_s \pi\pi]_L$ | $W'\left(4\left(2 - 5\frac{R_A^2}{R_B^2}\right) + (1 - \frac{6}{5})R_A^2 k_B^2\right)\cos\theta + W'\sin\theta$ | 2.6        | 6.9        |
| $B_s(0^+ \to [B_s f_0(980)]_L \to [B_s(0^-)\pi\pi]_L$ | $W'\cos\theta + \frac{1}{4\sqrt{2}}W'\left[4\left(1 - 5\frac{R_A^2}{R_B^2}\right) + (1 - \frac{6}{5})R_A^2 k_B^2\right]\sin\theta$ | 0.45        | 1.0        |
| $B_s(1^+) \to [B_s f_0(980)]_L \to [B_s(1^-)\pi\pi]_L$ | $W'\cos\theta + \frac{1}{4\sqrt{2}}W'\left[4\left(1 - 5\frac{R_A^2}{R_B^2}\right) + (1 - \frac{6}{5})R_A^2 k_B^2\right]\sin\theta$ | 0.055       | 0.13       |

In our opinion, $f_0(980)$ is well established and less controversial than the sigma meson. Moreover, its width is measured to be 40-100 MeV [3]. $f_0(980)$ mainly decays into pions. This process can be described by the effective Lagrangian

$$\mathcal{L}_{f_0(980)\pi\pi} = g_{f_0(980)\pi\pi} f_0[2\pi^+ \pi^- + \pi^0\pi^0].$$

The coupling constant $g_{f_0\pi\pi}$ is 0.83 $\sim$ 1.3 GeV. In the following we assume the double pion decays occur through the decay chain with the help of a virtual $f_0(980)$ meson [4]:

$$D_{sJ}(2460) \to D_s + f_0(980) \to D_s + 2\pi \text{ and } D_{sJ}(2460) \to$$
sum rule calculation \[25\].

single pion decay widths available in literature in Table IV. Our results agree very well with a recent light-cone QCD

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D

The dependence of their double pion decay widths with the mixing angle is presented in Fig. 5.

For the pure academic purpose, we also estimate the doubly isospin-violating P-wave decay:

$$D^*_s + f_0(980) \rightarrow D_s + 2\pi.$$ We further assume that the decay orbital angular momentum \( L \) exists between \( f_0(980) \) and \( D_s \) (or \( D^*_s \)) only. We want to remind the readers that the above working assumption is only used to make a rough estimate of \( D_s(2460) \)'s double-pion width.

We first consider the decay chain \( D_{sj}(2460) \rightarrow D_s + f_0(980) \rightarrow D_s + 2\pi \). The decay amplitude is

$$\mathcal{M} \left( D_{sj}(2460) \rightarrow D_s + 2\pi \right) = \mathcal{V} \left( D_{sj}(2460) \rightarrow D_s + f_0 \right) \cdot \frac{i}{k_{f_0} - m_{f_0}^2} \cdot \sqrt{\lambda_{\pi \pi}} \cdot g_{f_0 \pi \pi}$$

with \( \lambda_{\pi + \pi} = 2, \lambda_{\pi \pi^0} = 1 \). The vertex \( \mathcal{V} \left( D_{sj}(2460) \rightarrow D_s + f_0 \right) \) can be obtained in \( ^3P_0 \) model, which is the function of momenta of \( D^*_s \) momentum \( k_B \). The process \( D_{sj}(2460) \rightarrow D^*_s + f_0(980) \) is similar. We list the amplitudes and widths of double pionic decays of \( D_{sj}(2460) \) and \( B_s(5765) \) in Table III where

$$W' = (2\pi)^{\frac{3}{2}} \frac{2\sqrt{3}}{27} \gamma \left( R_A^2 R_B^2 R_X^2 \right)^{\frac{1}{2}} \left( \frac{2}{V} \right)^{\frac{3}{2}} \sqrt{8E_A E_B E_X} \left| k_B \right| \exp \left[ -\frac{1}{8} k_B^2 R_A^2 R_B^2 + R_X^2 V \right].$$

The dependence of their double pion decay widths with the mixing angle is presented in Fig. 4.

For the pure academic purpose, we also estimate the doubly isospin-violating P-wave decay: \( D_{sj}(2460) \rightarrow D_{sj}(2317) + \eta \rightarrow D_s(1869) + \eta \rightarrow D_s(1869) + \pi^0 + \pi^0 \). Numerically the width of the above decay mode is \( 10^{-6} \) eV, which is tiny and completely buried by the isospin conserving double-pion decay mode.

V. DISCUSSION

Assuming both \( D_{sj}(2317) \) and \( D_{sj}(2460) \) are ordinary \( c\bar{s} \) mesons, we have calculated their strong decay width using the \( ^3P_0 \) model. The single pion decays are isospin-violating and occur through \( \eta - \pi^0 \) mixing. We have collected the single pion decay widths available in literature in Table III. Our results agree very well with a recent light-cone QCD sum rule calculation \[25\].
TABLE IV: Single-pion decay widths (in keV) of \( D_{sj}(2317) \) and \( D_{sj}(2460) \) mesons from various theoretical approaches.

| Resonance                      | this work | [25] | [42] | [4] | [43] | [44] | [8] | [45] |
|-------------------------------|-----------|------|------|-----|------|------|-----|------|
| \( D_{sj}^+(2317) \to D_s \pi^0 \) | 32        | 34-44 | 7 ± 1 | 21.5 | ~ 10 | 16   | 10-100 | 150 ± 70 |
| \( D_{sj}^+ (2460) \to D_s^* \pi^0 \) | 35        | 35-51 | 7 ± 1 | 21.5 | ~ 10 | 32   | 150 ± 70 |

The double pion decays of \( D_{sj}(2460) \) are allowed by isospin symmetry. But they are suppressed by three-body phase space. Under a rather crude assumption that such decays occur with the help of a virtual \( f_0(980) \) meson, we have estimated the double pion decay width to be around \( 2.6 \sim 6.9 \text{ keV} \) for \( D_{sj}(2460) \to D_s + 2 \pi \) mode and \( 0.055 \sim 0.13 \text{ keV} \) for \( D_{sj}(2460) \to D_s^* + 2 \pi \) mode, depending on the total pionic width of \( f_0(980) \). The double pion decay width of \( D_{sj}(2460) \) is numerically much smaller than its single pion width because of the cancellation from the mixing of \( ^3P_1 \) and \( ^1P_1 \) states.

Putting the single and double pion decay modes together, the strong decay width of \( D_{sj}(2317,2460) \) is less than 50 keV. Combining the radiative decay width, the total width of \( D_{sj}(2317,2460) \) is less than 100 keV. Both resonances are extremely narrow if they are \( c\bar{s} \) states. A precise measurement of their total widths may help distinguish theoretical models of their quark content.

As an straightforward extension, we also calculate the pionic widths of \( B_{sj} \) mesons in the \((0^+,1^+) \) doublet. With masses estimated in Ref. [4], their single and double pion decay widths are listed in Table III.

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