Experiences with a two terminal-pair digital impedance bridge

Luca Callegaro, Vincenzo D’Elia, Marian Kampik, Dan Bee Kim, Massimo Ortolano, and Faranak Pourdanesh

Abstract—This paper describes the realization of a two terminal-pair digital impedance bridge and the test measurements performed with it. The bridge, with a very simple architecture, is based on a commercial two-channel digital signal synthesizer and a synchronous detector. The bridge can perform comparisons between impedances having arbitrary phase and magnitude ratio. Bridge balance is achieved automatically in less than a minute. R-C comparisons with calibrated standards, at kHz frequency and 100 kΩ magnitude level, give ratio errors of the order of 10⁻⁶, with potential for further improvements.

I. INTRODUCTION

Coaxial transformer bridges [1], [2] achieve ultimate accuracy in the measurement of impedance ratios in the audio frequency range, and are widely employed in primary metrology laboratories for the realization of electrical resistance and frequency range, and are widely employed in primary metrology laboratories for the realization of electrical resistance and capacitance units and scales. The main drawbacks of such bridges are the number of available measuring points, typically restricted to decadic purely real or imaginary ratios, and the fact that they typically require manual operation. On the other hand, electronic commercial impedance meters (LCR bridges) allow the quick measurement of impedances having arbitrary magnitude and phase angle, but with relative accuracies limited to the 10⁻⁴ range, at best.

Digital bridges [3, Ch. 5] employ digital signal sources as ratio standards to allow the comparison of impedances with arbitrary complex ratios, and can be easily automated. These kind of bridges can therefore provide a way to calibrate impedance standards having arbitrary magnitude and phase angle, which are suitable to be employed in the verification of LCR bridges.

We have implemented a coaxial voltage ratio bridge to perform comparisons of two terminal-pair impedance standards. This bridge, introduced in [4], is here described in full detail, together with test measurements and an expression of the measurement uncertainty.

II. PRINCIPLE OF OPERATION

A. Bridge principle

The schematic diagram of the bridge, well known in the literature (see [3, Ch. 5] and references therein; [5], [6]), is given in Fig. 1. The source output channel \( E_1 \) drives the impedance \( Z_A \) (admittance \( Y_A = 1/Z_A \)); channel \( E_2 \), the impedance \( Z_B \) (admittance \( Y_B = 1/Z_B \)). \( Z_A \) and \( Z_B \) are in series and the null detector D senses the common voltage at the low terminals of \( Z_A \) and \( Z_B \). The bridge balance condition \( V_D = 0, I_D = 0 \) is achieved by adjusting the amplitude and the phase of one of the two channels. At equilibrium, \( E_1 Y_A + E_2 Y_B = 0 \): this implies that the complex impedance ratio \( W = Z_A/Z_B \) is given by

\[
W = \frac{Z_A}{Z_B} = -\frac{E_1}{E_2}.
\]

The pair \( E_1, E_2 \) constitutes the bridge reading.

B. Measurement model

The schematic diagram of Fig. 1 represents an idealized bridge. Fig. 2, instead, shows a circuit model which takes into account the source output impedances and the stray capacitances of the impedance standards in two-terminal pair definition. Assuming that the impedances under comparison are defined at the end of the connecting cables, they can be modeled as two-port \( \Pi \) networks [2, Sec. 5.3.6]. Each \( \Pi \) network comprises the high-to-low transadmittance \( Y_X \) (where \( X = A, B \)), the high-to-shield admittance \( y_{HX} \) and the low-to-shield admittance \( y_{LX} \). Typically, \( y_{HX} \) and \( y_{LX} \) can be regarded as purely capacitive, with an equivalent capacitance of the order of 100 pF.

Each channel \( k = 1, 2 \) can be modeled with a Thévenin equivalent circuit [7] composed of an ideal voltage source \( E_{kX} \) in series with an output impedance \( z_k \). At equilibrium, when the source \( k \) is connected to the impedance \( Y_X \), the channel output voltage \( V_{kX} \) is

\[
V_{kX} = \frac{1}{1 + z_k (Y_X + y_{HX})} E_{kX}.
\]
It is well known [8] [9, Sec. 8.7] [2, Sec. 7.4.3] that exchanging the standards under comparison in the bridge arms can correct some of the systematic errors. We call forward (F) the configuration where \( Y_A \) is connected to source channel 1 and \( Y_B \) to channel 2; reverse (R) the configuration where \( Y_A \) is connected to channel 2 and \( Y_B \) to channel 1. The equilibrium conditions for the two configurations can be written as

\[
W = \frac{V_{1A}}{V_{2B}} = -\frac{E_{1A}}{E_{2B}} \frac{1 + z_2 (Y_B + y_{HB})}{1 + z_1 (Y_A + y_{HA})} \quad \text{(F),}
\]

\[
W = -\frac{V_{2A}}{V_{1B}} = -\frac{E_{2A}}{E_{1B}} \frac{1 + z_1 (Y_B + y_{HB})}{1 + z_2 (Y_A + y_{HA})} \quad \text{(R),}
\]

Because of source imperfection, the actual ratio \( E_1 / E_2 \) deviates from the reading \( E_1^{(F)} / E_2^{(R)} \). We model this deviation with a complex gain tracking error \( g \), dependent on the channel setting:

\[
\frac{E_1^{(F)}}{E_2^{(R)}} = (1 + g) \frac{E_1}{E_2}.
\]

By taking the geometric average of the forward and the reverse bridge readings, the measurement model can be written as

\[
W = \left[ \frac{1 + g_F E_2^{(F)} E_1^{(R)}}{1 + g_R E_2^{(R)} E_1^{(F)}} \right]^{\frac{1}{2}} \left[ \frac{1 + z_2 (Y_B + y_{HB})}{1 + z_1 (Y_A + y_{HA})} \frac{1 + z_1 (Y_B + y_{HB})}{1 + z_2 (Y_A + y_{HA})} \right]^{\frac{1}{2}},
\]

where

\[
W^{(i)} = \left[ E_{1A}^{(i)} E_{2A}^{(i)} \right]^{\frac{1}{2}} E_{1B}^{(i)} E_{2B}^{(i)}
\]

is the ratio reading and

\[
\epsilon_W = -\frac{1}{2} \Delta g_{FR} + \frac{1}{2} (z_1 + z_2) \left[ (Y_B + y_{HB}) - (Y_A + y_{HA}) \right],
\]

with \( \Delta g_{FR} = g_F - g_R \), is a correction term which accounts for the bridge nonidealities.

Eq. (6) shows, as expected, that even a significant but setting-independent gain tracking error \( g \) is compensated by averaging the two readings, whereas the error due to the output impedance is in general not compensated, even in 1:1 comparisons, because of the presence of the \( y_{HX} \) terms.

### III. IMPLEMENTATION

A coaxial schematic diagram of the bridge is given in Fig. 3; Fig. 4 shows a picture of the assembly. The devices employed...
Source (S). Aivon Oy DualDAC (2 channels, 16 bit resolution, up to 5 MS/s maximum sampling rate, $2^{14}$ maximum sample buffer size; the digital part is optically isolated from the analog one).

Detector (D). Stanford Research mod. 830 lock-in amplifier; an optical output from the source provide the reference signal.

Equalizers (E). Coaxial equalizers on nanocrystalline ferromagnetic cores.

Amplitude and phase of each channel are adjusted by recalculating and uploading new waveform samples. Each sample code is chosen to minimize the quantization error. More refined synthesis strategies can be implemented to improve the resolution [10]. The source implements a double buffer, which allows continuous output even during the upload of a new sample set. The quantities $E_{1x}, E_{2x}$ which appear in (7) are calculated from the Fourier expansions of the quantized waveforms.

The control program adjusts the source S and reads the detector D via a GPIB interface; it is written in C language under the LabWindows/CVI© environment. At start-up, the user set the source sampling rate and the number of samples per sine wave period. The user can then set the initial amplitudes (relative to the full-scale defined by the DAC reference voltage) and phases of the two output channels. The bridge equilibrium can be achieved either by manually adjusting the settings of one channel or, more conveniently, by invoking an automatic balancing routine. This routine, described in detail in Ref. [11, Sec. IV], operates iteratively: after reading D, it recalculates the channel settings by means of a root-finding algorithm based on the secant’s method. Each iteration has a duration which mainly depends on the detector’s time constant, but further delays are introduced when D performs an automatic range adjustment. The duration of a typical iteration is of about 1 s. The balancing routine stops when the voltage magnitude detected by D falls below a predefined threshold. The total adjustment time is typically less than 1 min.

IV. EXPERIMENTAL

A. Some properties of the source employed

In model (6)–(8), the parameters which account for source nonidealities are $z_1$, $z_2$ and $\Delta \eta_{\text{FR}}$. The impedances $z_1$ and $z_2$ were measured with an LCR meter Agilent mod. 4284A; for frequency $f$ up to about 20 kHz, the output impedance $z_k$ can be modelled with a resistance $r_k = 100(50) \, \mu \Omega$ in series with an inductance $l_k = 4(1) \, \mu \text{s}$.

The term $\Delta \eta_{\text{FR}}$ has undergone a preliminary evaluation [12] for $W$ ratios close to $-1$. The span of $|\Delta \eta_{\text{FR}}|$ is less than $2 \times 10^{-6}$ for $W \approx -1$ within a range of $2 \times 10^{-4}$. $\Delta \eta_{\text{FR}}$ becomes more significant for values of $|W|$ far from unity; however, a full characterization of this parameter has not yet been completed.

Other nonidealities not considered in the model of Sec. II-B were evaluated and found negligible. The relative stability of $E_1/E_2$ over time of the source employed was tested with the bridge itself, by substituting the impedance standards with an inductive voltage divider (which has a negligible ratio drift). Results are reported in [13]; the Allan deviation of the amplitude ratio at 1 kHz is $10 \, \text{nV} \, V^{-1}$ over 30 min; phase difference fluctuations are dominated by flicker noise beyond 100 s, with an Allan deviation of 40 nrad. The crosstalk between the channels is lower than $-125 \, \text{dB}$ up to 16 kHz.

B. Impedance measurements

The bridge was tested with the impedance standards listed in Tab. I, calibrated as two terminal-pair standards (at the end of the connecting cables).

Tab. II reports the measurement results. For each comparison, the reported values are:

- The types and the nominal values of the impedances $Z_A$ and $Z_B$;
- The measurement frequency $f$ chosen to have an angular frequency close to a decadic value;
- The real and the imaginary parts of $W$ as computed from the measurement model (the operators Re and Im denote the real and the imaginary parts, respectively); $|W| \approx 1$ for all measurements;
- A reference ratio $W_{\text{ref}} = Z_{A\text{ref}}^2/|Z_{B\text{ref}}|$ obtained by independent two-terminal pair calibrations traceable to the national standards of capacitance and resistance;
- The real and the imaginary parts of the deviation $\delta = W - W_{\text{ref}}$ of the bridge measurement from the reference ratio.

Comments:

- In $C-C$ comparisons, $\text{Re} \, W$ is related to the capacitance ratio, while $\text{Im} \, W$ is related to the difference of the phase angles.
- In $R-C$ comparisons, $\text{Im} \, W$ is related to the principal parameter of the impedances (the resistance and the capacitance), whereas $\text{Re} \, W$ is related to the secondary parameter (i.e., the resistor time constant and the capacitor phase angle); the fact that $\text{Re} \, \delta > \text{Im} \, \delta$ can be possibly due to the mediocre knowledge of these secondary parameters, for which INRIM does not have primary national standards.
- For the comparison in row 7 of Tab. II between a 100 k$\Omega$ resistor and 1 nF capacitor at the frequency of 1592.36 Hz, a complete uncertainty evaluation has been carried out and it is described in Sec. V. The estimated uncertainty of $W$ is in the $10^{-6}$ range, for both the real and the imaginary parts, and $\delta$ is compatible with

| Label | Description |
|-------|-------------|
| 1 nF, *1 nF | General Radio mod. 1404-A, sealed $N_2$ |
| 10 nF, *10 nF | Custom realization, C0G solid dielectric [14] |
| 100 k$\Omega$ | Agilent 42039A |
| 10 k$\Omega$ | Agilent 42038A |

TABLE I

STANDARDS EMPLOYED DURING THE MEASUREMENTS. THE ASTERISK * DENOTES THE SECOND OF TWO STANDARDS OF THE SAME MODEL.
the uncertainties of $W$ and $W_{\text{ref}}$ within an interval of confidence corresponding to a coverage factor of 2.

- The other comparisons listed in Tab. II show that the magnitude of $\delta$ is minimum for $W \approx 1 + j0$ and frequencies in the kHz range or below, while it increases for purely imaginary values of $W$ and for higher frequencies. Even though a full uncertainty evaluation for these comparisons has not been carried out, the behaviour of the deviation is expected because both the uncertainties of $W$ and $W_{\text{ref}}$ strongly depend on $W$ (see also Sec. V), on the values of the impedances and on the measurement frequency.

V. Uncertainty

Since the measurement model (6)–(8) is a complex-valued function of complex-valued input quantities, an expression of the bridge measurement uncertainty has to be carried out in the context of the Supplement 2 of the Guide to the expression of uncertainty in measurement [15]. The calculations were performed with the Metas.UnclLib [16] software package.

An example of uncertainty budget is reported in Tab III for a comparison between a 100 kΩ resistor and 1 nF capacitor at the frequency of 1592.36 Hz, which corresponds to $W \approx 0 + j$. The last row of Tab III reports the corresponding deviation $\delta$ and its associated uncertainty. The value of $\delta$, reported also in row 7 of Tab II, and its uncertainty are compatible with $0 + j0$ within an interval of confidence corresponding to a coverage factor of about 2.

Some notes about the evaluation of the uncertainties of the model input quantities:

- The measurements of Tab. II and the uncertainty budget of Tab. III correspond to $|W| \approx 1$, for which we have a characterization of $\Delta g_{\text{FR}}$. We assigned $\Delta g_{\text{FR}} = 0$ with an uncertainty compatible with the source specifications given in Sec. IV-A.

- $Y_A$ and $Y_B$ are known from their nominal values, with negligible uncertainty for what concerns the correction term $\varepsilon_W$ given by (8);

- $y_{HA}$ and $y_{HB}$ include also the connections, and are considered as pure capacitances, $y_{HA} = j2\pi f c_{HA}$, with $c_{HA} = 200 \mu F$; an uncertainty is included to take into account variations in cable lengths and differences between models;

- The uncertainty of $W^{(i)}$ is the type A uncertainty related to the measurement repeatability.

The uncertainty expression can be extended to arbitrary $W$ values provided that sufficient information about the input quantities is given. As an example, Fig. 5 shows a color plot of the magnitude $|u(W)|/|W|$ as a function of $W$, calculated for $Z_B = 100 \text{k}\Omega$, $z_k$ and $y_{HX}$ as given in Tab. III, and $|W|$ between 0.1 and 10; for convenience the plot is given as a Smith chart, that is, the cartesian coordinates correspond to the conformal mapping $(W - 1)/(W + 1)$. Since, at the moment, the characterization of $\Delta g_{\text{FR}}$ is not complete, the plot does not take into account this specific contribution. Indeed, different values of $Z_B$, $z_k$ and $y_{HX}$ will lead to a different but analog plot. In particular, the uncertainty is expected to increase toward lower values of $|W|$ because, for fixed $Z_B$, $Z_A$ decreases.

VI. Conclusions

The digital coaxial voltage ratio bridge realized allows to measure two terminal-pair impedances having arbitrary
magnitude ratio and phase difference in the audio frequency range.

The development of this bridge is part of the European Metrology Research Programme (EMRP) Project SIB53 AIM QuTE, Automated impedance metrology extending the quantum toolbox for electricity. Deliverables of the project include the development of more accurate digital sources which will increase the accuracy of the bridge here described, and interlaboratory comparisons with special standards that will allow the validation of the bridge measurements.

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