Modeling of rotational oscillations in a diesel locomotive wheel-motor block

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Abstract. This article highlights the importance of the effective movement of the rolling stock and the coefficient of friction between the wheels and rails in their use. Factors affecting wheel friction with the rails were considered. During the study, the dependence of the coupling coefficient between the wheel and the rails on the speed of movement was considered. Accordingly, it was found that the coefficient of adhesion decreases with increasing speed.

1. Introduction
Effective use of the movement remains a topical issue today. The problem of efficient movement and use (operation) of rolling stock is inextricably linked with the problem of interlocking of the wheel at the point of support of the rail. Great attention has been paid regularly to increasing the gravitational force on adhesion. Over the years, research has been conducted in various countries on the theory of adhesion and the study of its physical basis. The connection of wheels to the rails, the processes that take place in the movement of the locomotive have been studied by many scientists in their works, and their continuation remains a topical issue.

Experience of foreign and domestic countries shows that [1-4], as a result of self-propulsion of traction forces in the traction transmissions of mainline locomotives, the coefficient of adhesion of rails and wheels decreases, leading to a certain reduction in diesel power. Therefore, in developed countries, including the United States, Great Britain, France, Spain, Germany, Japan, South Korea, China, the Russian Federation and other countries, one of the important factors in improving the efficiency of rolling stock is the introduction of traction modeling of locomotives.

2. Methods
To increase the efficiency of diesel locomotives, it is advisable to study the rotational vibrations that occur between traction motors-reducer-wheel pairs. To do this, it is necessary to create a dynamic model of the locomotive in question. Figure 1 shows a dynamic model of a locomotive [5].

1. The following considerations should be made for mathematical modeling of this dynamic model:

- When the traction generator rotates relative to the stator (diesel) – J1;
- Traction motors anchors; J1, J2, J3, J4, J5, и J6 relative to their stators
- The wheel pairs are driven by the armatures of traction electric motors (TEM) J11, J12, J13, J14, J15, J16, and their stators are supplied with power between points G1, G2, G3, G4, G5, and G6 of the circuit.
2. The torque $M_g$ consumed in the operation of all TEMs is the driving force. The torques achieved by the traction of the wheel pairs are taken at points 1', 2', 3', 4', 5' and 6' and are determined by $M_1$, $M_2$, $M_3$, $M_4$, $M_5$, and $M_6$, which are the moments of resistance forces.

3. The following rotational stiffnesses are taken into account for the model:
   - $K_g$ is the elastic connection of the diesel with the generator armature.
   - $K_1$, $K_2$, $K_3$, $K_4$, $K_5$, and $K_6$ are the elastic couplings at 1, 2, 3, 4, 5, and 6 between the armature of the generator (figure 1) and traction electric motors.
   - $K_{11}$, $K_{21}$, $K_{31}$, $K_{41}$, $K_{51}$, and $K_{61}$ are the flexible connections between the wheel pairs of the TEM anchors and the locomotive section wheel-motor blocks.

4. The model in figure 1 takes into account the following functions of elastic rotational oscillations:
   - $\varphi_r(t)$ - Angular oscillations of the traction generator armature belong to the stator (diesel) through a flexible connection with rotational stiffness $K_g$.

   Rotational oscillations of rotational stiffness in- $K_1$, $K_2$, $K_3$, $K_4$, $K_5$, and $K_6$ are the moments of total inertia $\varphi_1(t)$, $\varphi_2(t)$, $\varphi_3(t)$, $\varphi_4(t)$, $\varphi_5(t)$ and $\varphi_6(t)$ relative to the armature $J$ of the traction generator of TEM anchors $J_1$, $J_2$, $J_3$, $J_4$, $J_5$, and $J_6$.

   In order to simplify the following places, the equations were performed according to a computational scheme with one WMB, G 1.1 points, as shown in figure 1.

   The Lagrangian method and equations were used to model the rotational oscillations in a locomotive wheel-motor block (WMB).
For the dynamic connection between the generator and the 1st wheel pair, construct the following equations [6-7]:

- we obtain the equation of kinetic energy:

\[
T = \frac{1}{2} \int \left[ J_r \left( \frac{d\varphi_r}{dt} \right)^2 + J_1 \left( \frac{d\varphi_1}{dt} + \frac{d\varphi_1}{dt} \right)^2 + J_{12} \left( \frac{d\varphi_r}{dt} + \frac{d\varphi_1}{dt} + \frac{d\varphi_{12}}{dt} \right)^2 \right] dt
\]  

(1)

- equation of potential energy of elastic deformations

\[
\Pi = \frac{1}{2} \left( K_r \varphi_r^2 + K_1 \varphi_1^2 + K_{12} \varphi_{12}^2 \right)
\]  

(2)

- the work of the driving forces and the forces resisting the rotation of the plenary moments of inertia

\[
\delta A = M_r \delta \varphi_r - M_1 \cdot (\delta \varphi_r + \delta \varphi_1 + \delta \varphi_{12})
\]  

(3)

In view of the above, we derive the Lagrangian method and equations for the angular oscillations generated in the generator(\(\varphi_r\)), 1-wheel pair (\(\varphi_1\)) and traction motors and wheel pairs (\(\varphi_{12}\)) [8]

\[
\frac{\partial}{\partial t} \frac{\partial r}{\partial \varphi_2} - \frac{\partial}{\partial \varphi_2} \frac{\partial r}{\partial \varphi_2} = \varphi_r (J_r + J_1 + J_{12}) + (J_1 + J_{12}) \varphi_1 + J_1 \varphi_1 + K_r \varphi_r =
\]

(4)

\[
M_r - M_1 = \varphi_r (J_r + J_1 + J_{12}) + K_r \varphi_r + (J_1 + J_{12}) \varphi_1 + J_1 \varphi_1 + K_1 \varphi_1 + J_{12} \varphi_{12} = M_r - M_1.
\]

(5)

\[
\frac{\partial r}{\partial \varphi_1} - \frac{\partial}{\partial \varphi_1} \frac{\partial r}{\partial \varphi_1} = (J_r + J_1) \varphi_r + (J_1 + J_{12}) \varphi_1 + K_1 \varphi_1 + J_1 \varphi_1 + J_{12} \varphi_{12} = -M
\]

(6)

The solutions of the above system of Lagrange equations consist of two components:

1) Systems of equations of the same type whose right parts are zero.

2) Download solutions are custom solutions that depend on \(M_r(t)\) and \(M_1(t)\).

The solution of systems of equations of the same type whose right parts are equal to zero is realized in the following view of the functions

\[
\varphi_r(t) = \varphi_r \cos \omega t, \varphi_1(t) = \varphi_1 \cos \omega t, \varphi_{12}(t) = \varphi_{12} \cos \omega t
\]

(7)

Here: \(\varphi_r\), \(\varphi_1\), and \(\varphi_{12}\) are the amplitudes of the generator, 1-wheel pair, concentration traction motors and wheel pairs rotational frequency rotation vibrations.

By placing the derivatives of the values (4) + (6) of the function in the Lagrangian equations, we obtain the following as a result of simplifications:

\[
A_{11} \varphi_r + A_{12} \varphi_1 + A_{13} \varphi_{12} = 0
\]

(8)

\[
A_{12} \varphi_r + A_{22} \varphi_1 + A_{23} \varphi_{12} = 0
\]

(9)

\[
A_{13} \varphi_r + A_{23} \varphi_1 + A_{31} \varphi_{12} = 0
\]

(10)

Here:

\[
A_{11} = K_r - \omega^2 (J_r + J_1 + J_{12}), A_{12} = -\omega^2 (J_1 + J_{12}), A_{13} = -\omega^2 J_{12}
\]

\[
A_{21} = -\omega^2 (J_r + J_1), A_{22} = K_1 - \omega^2 (J_1 + J_{12}), A_{23} = -\omega^2 J_{12}
\]

\[
A_{31} = -\omega^2 J_{12}, A_{32} = -\omega^2 J_{12}, A_{33} = K_{12} - \omega^2 J_{12}
\]

(11)

The system of algebraic equations with respect to \(\varphi_r\), \(\varphi_1\), and \(\varphi_{12}\) was solved approximately by the method of determinants, taking into account the limits on the main diagonals.
Based on the condition \( \Delta = 0 \) excitation of specific oscillations, formulas were obtained to calculate the three frequencies of these oscillations.

\[
\omega_1 = \sqrt{K_r + (J_r + J_1 + J_{12})}
\]

\[
\omega_2 = \sqrt{K_1 + (J_1 + J_{12})}
\]

\[
\omega_3 = \sqrt{K_{12} + J_{12}}
\]

The download functions were implemented in the form of custom functions related to \( M_r(t) \) and \( M_1(t) \)

\[
M_r(t) = M_r \cos \omega t, \quad M_1(t) = M_1 \cos \omega t
\]

By placing the derivatives of the values (4)-(6) of the function (16) in the Lagrangian equations, we obtain the following simplifications:

\[
A_{11}\phi_r + A_{12}\phi_1 + A_{13}\phi_{12} = M_r - M_1
\]

\[
A_{12}\phi_r + A_{22}\phi_1 + A_{23}\phi_{12} = -M_1
\]

\[
A_{13}\phi_r + A_{23}\phi_1 + A_{31}\phi_{12} = -M_1
\]

This system was solved by the method of identifiers

\[
\phi_r = \frac{1}{\Delta} \begin{vmatrix}
M_r - M_1, & A_{12}, & A_{13} \\
-M_1, & A_{22}, & A_{23} \\
-M_1, & A_{32}, & A_{33}
\end{vmatrix}
\]

\[
\phi_1 = \frac{1}{\Delta} \begin{vmatrix}
A_{11}, & M_r - M_1, & A_{13} \\
A_{21}, & -M_1, & A_{23} \\
A_{31}, & -M_1, & A_{33}
\end{vmatrix}
\]

\[
\phi_{12} = \frac{1}{\Delta} \begin{vmatrix}
A_{11}, & A_{12}, & M_r - M_1 \\
A_{21}, & A_{22}, & -M_1, \\
A_{31}, & A_{32}, & -M_1
\end{vmatrix}
\]

At (20)-(22) the approximate values of the amplitude of oscillations along the main diagonal were obtained, taking into account the limits

\[
\phi_r = (M_r - M_1) + [K_r - \omega^2 (J_r + J_1 + J_{12})]
\]

\[
\phi_1 = -M_1 + [K_1 - \omega^2 (J_1 + J_{12})]
\]

\[
\phi_{12} = -M_1 + [K_{12} - \omega^2 (J_1 + J_{12})]
\]

A system of equations based on (4)-(6) was used to obtain a special solution

\[
(J_r + J_1 + J_{12})\phi_r + (J_1 + J_{12})\phi_1 + K_r\phi_r + J_{12}\phi_{12} = M_r - M_1
\]

\[
(J_r + J_1 + J_{12})\phi_r + (J_1 + J_{12})\phi_1 + K_1\phi_1 + J_{12}\phi_{12} = -M_1
\]

\[
J_{12}\phi_r + J_{12}\phi_1 + J_{12}\phi_{12} + K_{12}\phi_{12} = -M_1
\]

In the right part of the system of equations with the rotational frequency of the wheel pair, the cases of pulsed change of functions are considered

\[
M_r(t) - M_1(t) = B_{11}(1 - \cos \omega_n t)
\]
The system of equations was solved using the operational calculation method, taking into account the following initial conditions

$$\varphi_r(0) = \varphi_1(0) = \varphi_{12}(0) = 0 \varphi_r(0) = \varphi_r = \omega_r, \varphi_1(0) = \varphi_1$$

The system of equations was solved using the operational calculation method

$$\varphi_r(t) \leftarrow \varphi_r(p), \varphi_1(t) \leftarrow \varphi_1(p), \varphi_{12}(t) \leftarrow \varphi_{12}(p) \text{ then}$$

$$\varphi_r(t) \leftarrow \varphi_r(p)p^2 - p\omega_k, 1 - \cos \omega_k t \leftarrow \frac{\omega_k}{\omega_n^2 + p^2}, \varphi_1(t) \leftarrow p^2 \varphi_1(p),$$

$$\varphi_{12}(t) \leftarrow p^2 \varphi_{12}(p)$$

A system of equations for the images was obtained, taking into account (26)-(33)

$$\begin{align*}
\{ (J_r + J_1 + J_{12})p^2 + K_r \varphi_r(p) + (J_1 + J_{12})p^2 \varphi_1(p) + J_{12}p^2 \varphi_{12}(p) & = p(J_r + J_1 + J_{12})\omega_k + \\
B_{11}\omega_k^2 & = B_{11}(p) \\
p^2(J_1 + J_{12})\varphi_r(p) + \varphi_1(p)K_1 - p^2(J_1 + J_{12}) & + p^2J_{12}\varphi_{12}(p) = (J_1 + J_{12})\omega_k p + B_{21}\omega_k^2 + \\
(p^2 + \omega_k^2) & = B_{21}(p) \\
p^2J_{12}\varphi_r(p) + p^2J_{12}\varphi_1(p) + \varphi_{12}(K_{12} - p^2J_{12}) & = J_{12}p\omega_k + B_{31}\omega_k^2 + (p^2 + \omega_k^2) = B_{31}(p)
\end{align*}$$

The system of equations was solved as an algebraic system with respect to the following images

$$\Delta_p = \begin{bmatrix}
p^2(J_r + J_1 + J_{12}) + K_r, & p^2(J_1 + J_{12}), & p^2J_{12} \\
p^2(J_1 + J_{12}), & [p^2(J_1 + J_{12}) + K_1], & p^2J_{12} \\
p^2J_{12}, & p^2J_{12}, & p^2J_{12} + K_{12}
\end{bmatrix}$$

$$\varphi_r(p) = \frac{1}{\Delta_p} \begin{bmatrix}
B_{11}(p), & p^2(J_1 + J_{12}), & p^2J_{12} \\
B_{21}(p), & [p^2(J_1 + J_{12}) + K_1], & p^2J_{12} \\
B_{31}(p), & p^2J_{12}, & p^2J_{12} + K_{12}
\end{bmatrix}$$

$$\varphi_1(p) = \frac{1}{\Delta_p} \begin{bmatrix}
[p^2(J_1 + J_{12}) + K_r] & B_{11}(p), & p^2J_{12} \\
p^2(J_1 + J_{12}), & B_{21}(p), & p^2J_{12} \\
p^2J_{12}, & B_{31}(p), & p^2J_{12} + K_{12}
\end{bmatrix}$$

$$\varphi_{12}(p) = \frac{1}{\Delta_p} \begin{bmatrix}
[p^2(J_r + J_1 + J_{12}) + K_r] & p^2(J_1 + J_{12}), & B_{11}(p) \\
p^2(J_1 + J_{12}), & [p^2(J_1 + J_{12}) + K_1], & B_{21}(p) \\
p^2J_{12}, & p^2J_{12}, & B_{31}(p)
\end{bmatrix}$$

In the approximate solution of (35)-(38), only the limits along the main diagonal of the determinants were taken into account

$$\varphi_r(p) = B_{11}(p) + \int \frac{p^2(J_r + J_1 + J_{12}) + K_r}{p^2(J_1 + J_{12}) + K_1}\,dp + \int \frac{p^2J_{12}}{p^2J_{12} + K_{12}}\,dp$$

Development of algorithms and calculations of auto-vibrations of conversion of locomotives WMBs.
3. Results
Solve the system of equations for the amplitude of rotational oscillations with the coefficients $A_{11}…A_{33}$ according to the formulas (8)-(38) according to the formula (11) and 50 rad/sec the stepwise calculation in the range from $\omega = 50$ rad/sec to $\omega = 300$ rad/sec [9,10].

$$A_{11} = K_1 - \omega^2(j_1 + j_1 + j_1) = 7.55 \cdot 10^4 - 50^2(52.16 + 19.55 + 17) = -14.6 \cdot 10^4$$
$$A_{12} = -\omega^2(j_1 + j_2) = -50^2(19.55 + 17) = -9 \cdot 10^4$$
$$A_{13} = -\omega^2 j_2 = -50^2 \cdot 17 = -4.25 \cdot 10^4$$
$$A_{21} = -\omega^2 (j_1 + j_2) = -50^2(19.55 + 17) = -9 \cdot 10^4$$
$$A_{22} = K_1 - \omega^2 (j_1 + j_2) = 5.87 \cdot 10^6 - 50^2(19.55 + 17) = 578 \cdot 10^4$$
$$A_{23} = -\omega^2 j_2 = -50^2 \cdot 17 = -4.25 \cdot 10^4$$
$$A_{31} = -\omega^2 j_2 = -50^2 \cdot 17 = -4.25 \cdot 10^4$$
$$A_{32} = \omega^2 j_2 = 50^2 \cdot 17 = 4.25 \cdot 10^4$$
$$A_{33} = K_2 - \omega^2 j_2 = 5.87 \cdot 10^6 - 50^2 \cdot 17 = 583 \cdot 10^4$$

| $\omega$ | 50       | 100      | 150      | 200      | 250      | 300      |
|---------|----------|----------|----------|----------|----------|----------|
| $A_{11}$ | -14.6$\cdot 10^4$ | -81.16$\cdot 10^4$ | -192$\cdot 10^4$ | -347.3$\cdot 10^4$ | -547$\cdot 10^4$ | -791$\cdot 10^4$ |  
| $A_{12}$ | -9$\cdot 10^4$ | -36.5$\cdot 10^4$ | -82.2$\cdot 10^4$ | -146$\cdot 10^4$ | -228.5$\cdot 10^4$ | -329$\cdot 10^4$ |  
| $A_{13}$ | -4.25$\cdot 10^4$ | -17$\cdot 10^4$ | -38.25$\cdot 10^4$ | -68$\cdot 10^4$ | -106.25$\cdot 10^4$ | -153$\cdot 10^4$ |  
| $A_{21}$ | -9$\cdot 10^4$ | -36.55$\cdot 10^4$ | -82.2$\cdot 10^4$ | -146$\cdot 10^4$ | -228.5$\cdot 10^4$ | -329$\cdot 10^4$ |  
| $A_{22}$ | -578$\cdot 10^4$ | 550.4$\cdot 10^4$ | -504.7$\cdot 10^4$ | -440.8$\cdot 10^4$ | -358.5$\cdot 10^4$ | 258$\cdot 10^4$ |  
| $A_{23}$ | -4.25$\cdot 10^4$ | -17$\cdot 10^4$ | -38.25$\cdot 10^4$ | -68$\cdot 10^4$ | -106.25$\cdot 10^4$ | -153$\cdot 10^4$ |  
| $A_{31}$ | -4.25$\cdot 10^4$ | -17$\cdot 10^4$ | -38.25$\cdot 10^4$ | -68$\cdot 10^4$ | -106.25$\cdot 10^4$ | -153$\cdot 10^4$ |  
| $A_{32}$ | 4.25$\cdot 10^4$ | 17$\cdot 10^4$ | 38.25$\cdot 10^4$ | 68$\cdot 10^4$ | 106.25$\cdot 10^4$ | 153$\cdot 10^4$ |  
| $A_{33}$ | 583$\cdot 10^4$ | 570$\cdot 10^4$ | 549$\cdot 10^4$ | 519$\cdot 10^4$ | 481$\cdot 10^4$ | 434$\cdot 10^4$ |  

Table 1. Results.

For the first WMB TEM anchor rotation speed we find the determinant according to formula (12) at the values $\omega = 50$ rad/sec and $A_{11} = -14.6 \cdot 10^4$; $A_{12} = -9 \cdot 10^4$; $A_{13} = -4.25 \cdot 10^4$; $A_{21} = -9 \cdot 10^4$; $A_{22} = 578 \cdot 10^4$; $A_{23} = -4.25 \cdot 10^4$; $A_{31} = -4.25 \cdot 10^4$; $A_{32} = 4.25 \cdot 10^4$; $A_{33} = 583 \cdot 10^4$.

$$\Delta = \begin{vmatrix}
-14.6 \cdot 10^4 & -9 \cdot 10^4 & -4.25 \cdot 10^4 \\
-9 \cdot 10^4 & -587 \cdot 10^4 & 4.25 \cdot 10^4 \\
-4.25 \cdot 10^4 & 4.25 \cdot 10^4 & 583 \cdot 10^4 
\end{vmatrix} = -4956.86 \cdot 10^4$$

We determine the amplitude of the torque of the TEM armature according to the following formula

$$M_1 = \frac{102N}{\omega} = \frac{102 \cdot 305}{50} = 622kNm; M_1 = 0.5 M_{11} = 0.5 M_{12} = 0.5 \cdot 622 = 311kNm.$$

We determine the angle of rotation $\varphi_1$ of the armature of the first WMB TEM's anchor according to the formula (20) at the calculated values of the coefficients $A_{11}…A_{33}$ and $K_1 = 7.55 \cdot 10^5Nm$;

$$K_{11} = 5.87 \cdot 10^7Nm; K_{12} = 5.87 \cdot 10^7Nm;$$ for one section of the locomotive.

$$\varphi_1 = \frac{1}{4956.86 \cdot 10^{15}} \begin{vmatrix}
0 & -9 \cdot 10^4 & -4.25 \cdot 10^4 \\
311 & 578 \cdot 10^4 & 4.25 \cdot 10^4 \\
311 & 4.25 \cdot 10^4 & 583 \cdot 10^4 
\end{vmatrix} = -4.8 \cdot 10^{-5}$$
Table 2. Calculated values of the angle of rotation of the armature of the traction electric motor of the first wheel motor block.

| φ/ω | 50   | 100   | 150   | 200   | 250   | 300   |
|-----|------|-------|-------|-------|-------|-------|
| φ₁  | -3.22417 | -1.64976 | -1.1422 | -0.90111 | -0.76441 | -0.67317 |
| φ₁₁ | 5.371453 | 2.79651 | 1.994972 | 1.645243 | 1.484492 | 1.421658 |
| φ₁₂ | 5.275793 | 2.59639 | 1.671076 | 1.164929 | 0.797201 | 0.456201 |

Figure 2. Computational research on wheel motor block.

4. Conclusion
The results show that the thrust created by the traction forces of traction locomotives of mainline locomotives depends on the speed of rotation of the wheelsets.

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