Chiral Fermions and AdS/CFT Duality for a Nonabelian Orbifold

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Abstract

We present what we believe is the minimal three-family AdS/CFT model compactified on a nonabelian orbifold $S^5/(Q \times Z_3)$. Nontrivial irreps of the discrete nonabelian group $Q \times Z_3$ are identified with the 4 of $SU(4)$ $R$ symmetry to break all supersymmetries, and the scalar content of the model is sufficient to break the gauge symmetry to the standard model. According to the conformality hypothesis the progenitor $SU(4)^3 \times SU(2)^{12}$ theory becomes conformally invariant at an infra-red fixed point of the renormalization group.
The concept of duality has always proven to be one of the most useful tools in advancing theoretical physics, beginning with the duality between $x$ and $p$ in classical and quantum mechanics. Further examples include wave-particle duality which is closely related to $x$-$p$ duality, the duality between $E$ and $B$ in Maxwell’s equations, the instanton solutions of self-dual Yang-Mills equations, the Kramers-Wannier duality between low and high temperature in a condensed-matter phase transition, and dual resonance models which were the precursor to string theory. More recently there is the duality between weak and strong coupling field theories and then between all the different superstring theories that has led to a revolution in our understanding of strings. Most recently, and equally profound, is the AdS/CFT duality which is the subject of the present article. This AdS/CFT duality is between string theory compactified on Anti-de-Sitter space and Conformal Field Theory.

Until very recently, the possibility of testing string theory seemed at best remote. The advent of $AdS/CFT$ and large-scale string compactification suggest this point of view may be too pessimistic, since both could lead to $\sim 100 TeV$ evidence for strings. With this thought in mind, we are encouraged to build $AdS/CFT$ models with realistic fermionic structure, and reduce to the standard model below $\sim 1 TeV$.

Using AdS/CFT duality, one arrives at a class of gauge field theories of special recent interest. The simplest compactification of a ten-dimensional superstring on a product of an AdS space with a five-dimensional spherical manifold leads to an $N = 4$ $SU(N)$ supersymmetric gauge theory, well known to be conformally invariant [1]. By replacing the manifold $S^5$ by an orbifold $S^5/\Gamma$ one arrives at less supersymmetries corresponding to $N = 2, 1$ or 0 depending [2] on whether $\Gamma \subset SU(2), SU(3), \text{ or } \not\subset SU(3)$ respectively, where $\Gamma$ is in all cases a subgroup of $SU(4) \sim SO(6)$ the isometry of the $S^5$ manifold.

It was conjectured in [3] that such $SU(N)$ gauge theories are conformal in the $N \to \infty$ limit. In [4] it was conjectured that at least a subset of the resultant nonsupersymmetric $N = 0$ theories are conformal even for finite $N$. Some first steps to check this idea were made in [5]. Model-building based on abelian $\Gamma$ was studied further in [6–8], arriving in [8] at an $SU(3)^7$ model based on $\Gamma = \mathbb{Z}_7$ which has three families of chiral fermions, a correct
value for $\sin^2 \theta$ and a conformal scale $\sim 10 \text{ TeV}$.

The case of non-abelian orbifolds bases on non-abelian $\Gamma$ has not previously been studied, partially due to the fact that it is apparently somewhat more mathematically sophisticated. However, we shall show here that it can be handled equally as systematically as the abelian case and leads to richer structures and interesting results.

We consider all non-abelian discrete groups of order $g < 32$. These are described in detail in [9,10]. There are exactly 45 such non-abelian groups. Because the gauge group arrived at by this construction is $\otimes_i SU(Nd_i)$ where $d_i$ are the dimensions of the irreducible representations of $\Gamma$, one can expect to arrive at models such as the Pati-Salam $SU(4) \times SU(2) \times SU(2)$ type by choosing $N = 2$ and combining two singlets and a doublet in the 4 of $SU(4)$. Indeed we shall show that such an accommodation of the standard model is possible by using a non-abelian $\Gamma$.

The procedures for building a model within such a conformality approach are: (1) Choose $\Gamma$; (2) Choose a proper embedding $\Gamma \subset SU(4)$ by assigning the components of the 4 of $SU(4)$ to irreps of $\Gamma$, while at the same time ensuring that the 6 of $SU(4)$ is real; (3) Choose $N$, in the gauge group $\otimes_i SU(Nd_i)$. (4) Analyse the patterns of spontaneous symmetry breaking.

In the present study we shall choose $N = 2$ and aim at the gauge group $SU(4) \times SU(2) \times SU(2)$. To obtain chiral fermions, it is necessary that the 4 of $SU(4)$ be complex $4 \neq 4^\ast$. Actually this condition is not quite sufficient to ensure chirality in the present case because of the pseudoreality of $SU(2)$. We must ensure that the 4 is not just pseudoreal.

This last condition means that many of our 45 candidates for $\Gamma$ do not lead to chiral fermions. For example, $\Gamma = Q_{2n} \subset SU(2)$ has irreps of appropriate dimensionalities for our purpose but it will not sustain chiral fermions under $SU(4) \times SU(2) \times SU(2)$ because these irreps are all, like $SU(2)$, pseudoreal. Applying the rule that 4 must be neither real nor pseudoreal leaves a total of only 19 possible non-abelian discrete groups of order $g \leq 31$.

\footnote{Note that were we using $N \geq 3$ then a pseudoreal 4 would give chiral fermions.}
The smallest group which avoids pseudoreality has order $g = 16$ but gives only two families. The technical details of our systematic search will be postponed to a future publication. Here we shall present only the simplest interesting non-abelian case which has $g = 24$ and gives three chiral families in a Pati-Salam-type model \([1]\).

Before proceeding to the details of the specific $g = 24$ case, it is worth reminding the reader that the Conformal Field Theory (CFT) that it exemplifies should be free of all divergences, even logarithmic ones, if the conformality conjecture is correct, and be completely finite. Further the theory is originating from a superstring theory in a higher-dimension (ten) and contains gravity \([12–14]\) by compactification of the higher-dimensional graviton already contained in that superstring theory. In the CFT as we derive it, gravity is absent because we have not kept these graviton modes - of course, their influence on high-energy physics experiments is generally completely negligible unless the compactification scale is “large” \([15]\); here we shall neglect the effects of gravity.

To motivate our model it is instructive to comment on the choice of $\Gamma$ and on the choice of embedding.

If we embed only four singlets of $\Gamma$ in the $4$ of $SU(4)$ then this has the effect of abelianizing $\Gamma$ and the gauge group obtained in the chiral sector of the theory is $SU(N)^9$. These cases can be interesting but have already been studied \([6,7]\). Thus, we require at least one irrep of $\Gamma$ to have $d_i \geq 2$ in the embedding.

The only $\Gamma$ of order $g \leq 31$ with a $4$ is $Z_5 \times Z_4$ and this embedding leads to a non-chiral theory. This leaves only embeddings with two singlets and a doublet, a triplet and a singlet or two doublets.

The third of these choices leads to richer structures for low order $\Gamma$. Concentrating on them shows that of the chiral models possible, those from groups of low order result in an insufficient number (below three) of chiral families.

The first group that can lead to exactly three families occurs at order $g = 24$ and is $\Gamma = Z_3 \times Q$ where $Q(\equiv Q_4)$ is the group of unit quarternions which is the smallest dicyclic group $Q_{2n}$. 
There are several potential models due to the different choices for the 4 of SU(4) but only the case 4 = (1α, 1′, 2α) leads to three families so let us describe this in some detail:

Since Q × Z₃ is a direct product group, we can write the irreps as $R_i \otimes \alpha^a$ where $R_i$ is a Q irrep and $\alpha^a$ is a Z₃ irrep. We write Q irreps as 1, 1', 1'', 2 while the irreps of Z₃ are all singlets which we call $\alpha, \alpha^2, \alpha^3 = 1$. Thus $Q \times Z₃$ has fifteen irreps in all and the gauge group will be of Pati-Salam type for $N = 2$.

If we wish to break all supersymmetry, the 4 may not contain the trivial singlet of $\Gamma$. Due to permutational symmetry among the singlets it is sufficiently general to choose $4 = (1\alpha^a_1, 1'\alpha^a_2, 2\alpha^a_3)$ with $a_1 \neq 0$.

To fix the $a_i$ we note that the scalar sector of the theory which is generated by the 6 of SU(4) can be used as a constraint since the 6 is required to be real. This leads to $a_1 + a_2 = -2a_3(\text{mod } 3)$. Up to permutations in the chiral fermion sector the most general choice is $a_1 = a_3 = +1$ and $a_2 = 0$. Hence our choice of embedding is

$$4 = (1\alpha, 1', 2\alpha) \quad (1)$$

with

$$6 = (1'\alpha, 2\alpha, 2\alpha^2, 1'\alpha^2) \quad (2)$$

which is real as required.

We are now in a position to summarize the particle content of the theory. The fermions are given by

$$\sum_I 4 \times R_I \quad (3)$$

where the $R_I$ are all the irreps of $\Gamma = Q \times Z_3$. This is:

$$\sum_{i=1}^{3} [(2_1\alpha^i, 2_2\alpha^i) + (2_3\alpha^i, 2_4\alpha^i) + (2_2\alpha^i, 2_1\alpha^i) + (2_4\alpha^i, 2_3\alpha^i) + (4\alpha^i, 4\alpha^i)]$$

$$+ \sum_{i=1}^{3} \sum_{a=1}^{4} [(2_a\alpha^i, 2_a\alpha^{i+1}) + (2_4\alpha^i, 4\alpha^{i+1}) + (4\alpha^i, 2_a\alpha^{i+1})] \quad (4)$$

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It is convenient to represent the chiral portions of these in a given diagram (see Figure 1).

The scalars are given by

$$\sum_I 6 \times R_I$$

and are:

$$\sum_{i=1}^{3} \sum_{j=1,j\neq i}^{3} \left[ (2_1 \alpha^i, 2_2 \alpha^j) + (2_2 \alpha^i, 2_1 \alpha^j) + (2_3 \alpha^i, 2_4 \alpha^j) + (2_4 \alpha^i, 2_3 \alpha^j) + (2_2 \alpha^i, 2_1 \alpha^j) + (2_4 \alpha^i, 2_3 \alpha^j) \right]$$

$$+ \sum_{i=1}^{3} \sum_{j=1,j\neq i}^{3} \sum_{a=1}^{4} \left[ (2_a \alpha^i, 4 \alpha^j) + (4 \alpha^i, 2_a \alpha^j) \right] + (4 \alpha^i, 4 \alpha^j) \right]$$

which is easily checked to be real.

The gauge group $SU(4)^3 \times SU(2)^{12}$ with chiral fermions of Eq.(4) and scalars of Eq.(5) is expected to acquire conformal invariance at an infra-red fixed point of the renormalization group, as discussed in [4].

To begin our examination of the symmetry breaking we first observe that if we break the three $SU(4)$s to the totally diagonal $SU(4)$, then chirality in the fermionic sector is lost. To avoid this we break $SU_1(4)$ completely and then break $SU_a(4) \times SU_{a^2}(4)$ to its diagonal subgroup $SU_D(4)$. The first of these steps can be achieved with VEVs of the form $[(4_1, 2_b \alpha^k) + h.c.]$ where we are free to choose $b$, but $k$ must be 1 or 2 since there are no $(4_1, 2_b \alpha^{k=0})$ scalars. The second step requires an

$SU_D(4)$ singlet VEV from $(4_a, 4_{a^2})$ and/or $(4_a, 4_{a^2})$. Once we make a choice for $b$ (we take $b = 4$), the remaining chiral fermions are, in an intuitive notation:

$$\sum_{a=1}^{3} \left[ (2_a \alpha^1, 1_D) + (1, 2_a \alpha^{-1}, 4_D) \right]$$

which has the same content as as a three family Pati-Salam model, though with a separate $SU_L(2) \times SU_R(2)$ per family.

To further reduce the symmetry we must arrange to break to a single $SU_L(2)$ and a single $SU_R(2)$. This is achieved by modifying step one where $SU_1(4)$ was broken. Consider
the block diagonal decomposition of $SU_1(4)$ into $SU_{1L}(2) \times SU_{1R}(2)$. The representations $(2_a \alpha, 4_1)$ and $(2_a \alpha^{-1}, 4_1)$ then decompose as

$$(2_a \alpha, 4_1) \rightarrow (2_a \alpha, 2, 1) + (2_a \alpha, 1, 2) \text{ and } (2_a \alpha^{-1}, 4_1) \rightarrow (2_a \alpha^{-1}, 2, 1) + (2_a \alpha^{-1}, 1, 2).$$

Now if we give VEVs of equal magnitude to the $(2_a \alpha, 2, 1)$, $a = 1, 2, 3$, and equal magnitude VEVs to the $(2_a \alpha^{-1}, 1, 2)$ as well as $(2_a \alpha^{-1}, 2, 1)$ and $(2_a \alpha^{-1}, 1, 2)$ isures that both $SU(2_a \alpha)$ and $SU(2_a \alpha^{-1})$ are broken and that only three families remain chiral. The final set of chiral fermions is then $3[(2,1,4) + (1,2,\bar{4})]$ with gauge symmetry $SU_{1L}(2) \times SU_{1R}(2) \times SU_D(4)$.

To achieve the final reduction to the standard model, an adjoint VEV from $(\bar{1}_a, 4_\alpha)$ and/or $(4_\alpha, \bar{1}_a)$ is used to break $SU_D(4)$ to the $SU(3) \times U(1)$, and a right handed doublet is used to break $SU_R(2)$.

While this completes our analysis of symmetry breaking, it is worthwhile noting the degree of constraint imposed on the symmetry and particle content of a model as the number of irreps $N_R$ of the discrete group $\Gamma$ associated with the choice of orbifold changes. The number of gauge groups grows linearly in $N_R$, the number of scalar irreps grows roughly quadratically with $N_R$, and the chiral fermion content is highly $\Gamma$ dependent. If we require the minimal $\Gamma$ that is large enough for the model generated to contain the fermions of the standard model and have sufficient scalars to break the symmetry to that of the standard model, then $\Gamma = Q \times Z_3$ appears to be that minimal choice \[16\].

Although a decade ago the chances of testing string theory seemed at best remote, recent progress has given us hope that such tests may indeed be possible in AdS/CFTs. The model provided here demonstrates the standard model can be accomodated in these theories and suggests the possibility of a rich spectrum of new physics just around the TeV corner.

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Figure 1

Quiver diagram for the chiral fermions in the $S^5/(Q \times Z_3)$ orbifold. The arrows continue around the diagram, focusing on 4’s and diverging on doublets. Arrows pointing toward 4’s give $(4, 2)$ type terms, while those pointing away give $(\overline{4}, 2)$’s.
This figure "quiverbmp3.JPG" is available in "JPG" format from:

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