Measurement of \( CP \) asymmetry in \( B^0_s \to D^{\mp}_s K^\pm \) decays

LHCb collaboration

Abstract

A measurement of the \( CP \)-violating parameters in \( B^0_s \to D^{\mp}_s K^\pm \) decays is reported, based on the analysis of proton-proton collision data corresponding to an integrated luminosity of 6 fb\(^{-1}\) at a centre-of-mass energy of 13 TeV. The measured parameters are \( C_f = 0.791 \pm 0.061 \pm 0.022 \), \( A_f^{\Delta \Gamma} = -0.051 \pm 0.134 \pm 0.058 \), \( A_f^{\Delta F} = -0.303 \pm 0.125 \pm 0.055 \), \( S_f = -0.571 \pm 0.084 \pm 0.023 \) and \( S_f^{\Delta F} = -0.503 \pm 0.084 \pm 0.025 \), where the first uncertainty is statistical and the second systematic. Together with the value of the \( B^0_s \) mixing phase \(-2\beta_s\), these parameters are used to obtain a measurement of the CKM angle \( \gamma \) equal to \((74 \pm 12)\)\(^\circ\) modulo \( 180\)\(^\circ\), where the uncertainty contains both statistical and systematic contributions. This result is combined with the previous LHCb measurement in this channel using 3 fb\(^{-1}\) resulting in a determination of \( \gamma = (81^{+12}_{-11})\)\(^\circ\).
1 Introduction

Measurements of the CP asymmetries in $B_{(s)}^0 \rightarrow D_{(s)}^{(*)}\pi\bar{h}^\pm$ decays, where \( h = \pi \) or \( K \), have been performed by the LHCB [1,2], BaBar [3,4] and Belle [5,6] collaborations. These measurements are of particular interest as they constrain elements of the CKM quark-mixing matrix, in which all Standard Model (SM) CP-violation effects arise from a single complex phase \( \theta_3 \). The unitarity constraint of the CKM matrix, relevant to the \( b \rightarrow u \) and \( b \rightarrow c \) transitions in the above decays, can be written as \( V_{ub}V_{cb}^* + V_{cd}V_{td}^* + V_{td}V_{ub}^* = 0 \), where \( V_{ij} \) are the matrix elements. This constraint can be represented as a triangle in a complex plane in which the internal angle \( \gamma \) is defined by \( \gamma = \phi_3 \equiv \arg(-V_{ub}V_{cb}^*/V_{cd}V_{td}^*) \) and can be probed both indirectly, under the assumption of unitarity, and directly in tree-level processes [9,11]. The consistency between these determinations provides a powerful validation of the SM picture of CP violation. The most accurate determination of the angle \( \gamma \) in tree-level processes is currently obtained by combining LHCB measurements of \( B^+, B^0 \) and \( B_{(s)}^0 \) decays to final states with a \( D \) meson and one or more light mesons. Results from both time-integrated and time-dependent analyses are used, as well as constraints from charm-meson decays [12].

The decay-time-dependent analyses of \( B_{(s)}^0 \rightarrow D_{(s)}^\mp K^\pm \) and \( B^0 \rightarrow D^{(*)\mp}\pi^\pm \) tree-level decays are sensitive to the angle \( \gamma \) in the interference of mixing and decay amplitudes [13,16], which for the \( B_{(s)}^0 \rightarrow D_{(s)}^\mp K^\pm \) decay proceed through the leading-order Feynman diagrams shown in Fig. 1. In these decays, the CP-violating parameters are functions of a combination of the angle \( \gamma \) and the relevant mixing phase \( \beta_{(s)} \), namely \( \gamma + 2\beta \) \( (\beta = \phi_1 \equiv \arg(-V_{cd}V_{cb}^*/V_{td}V_{ub}^*)) \) in the \( B^0 \) system and \( \gamma - 2\beta_{(s)} \) \( (\beta_{(s)} \equiv \arg(-V_{ts}V_{tb}^*/V_{cs}V_{cb}^*)) \) in the \( B_{(s)}^0 \) system. In \( B^0 \rightarrow D^{(*)\mp}\pi^\pm \) decays, the ratio between the interfering decay amplitudes is small, \( r_{D^{(*)}\pi} = |A(B^0 \rightarrow D^{(*)-}\pi^+)/A(B^0 \rightarrow D^{(*)+}\pi^-)| \approx 0.02 \), which limits the sensitivity to the CKM angle \( \gamma \) [17]. By contrast, the ratio is larger for \( B_{(s)}^0 \rightarrow D_{(s)}^\mp K^\pm \) decays, \( r_{D_{(s)}K} = |A(B_{(s)}^0 \rightarrow D_{(s)}^-K^+)/A(B_{(s)}^0 \rightarrow D_{(s)}^+K^-)| \approx 0.4 \), since both \( b \rightarrow cs\bar{u} \) and \( b \rightarrow u\bar{c}s \) amplitudes have similar magnitudes, of \( \mathcal{O}(\lambda^2) \), where \( \lambda \approx 0.23 \) is the sine of the Cabibbo angle [18,19].

This paper presents a measurement of the CP-violating parameters in \( B_{(s)}^0 \rightarrow D_{(s)}^\mp K^\pm \) decays using a data set of proton-proton \( (pp) \) collisions recorded with the LHCB detector at a centre-of-mass energy \( \sqrt{s} = 13 \text{ TeV} \) during the Run 2 data-taking period of the LHC (2015–2018). This data set corresponds to an integrated luminosity of \( 6 \text{ fb}^{-1} \). The decays of the \( D_{(s)}^- \) meson into the final states \( K^-\pi^+\pi^- \), \( \pi^-\pi^+\pi^- \) and \( K^-K^+\pi^- \) are analysed. The analysis strategy is similar to that of Ref. [1], with the selection, fit model and determination of systematic uncertainties reoptimised. These improvements profit from better trigger and reconstruction performances of the LHCB experiment throughout Run 2 [20].

The determination of the CP-violating parameters is achieved using a two-stage fitting procedure. At the first stage, the \( B_{(s)}^0 \rightarrow D_{(s)}^\mp K^\pm \) signal is statistically separated from background components using the sPlot technique, where the signal weights are determined from a two-dimensional fit to the \( m(D_{(s)}^\mp K^\pm) \) and \( m(h^+h^-) \) distributions [21], where \( h \) denotes either a kaon or a pion in the different \( D_{(s)}^- \) decays. Each fit component is factorised using the product of the probability density functions (PDFs) modelling the \( m(D_{(s)}^\mp K^\pm) \) and \( m(h^+h^-) \) invariant-mass distributions since their correlations are

\footnote{Inclusion of charge-conjugate modes is implied throughout, except where explicitly stated.}
determined to be negligible in simulation samples. A systematic uncertainty is assigned to account for the impact of any remaining correlations. The two-dimensional fit is performed simultaneously to all $D_s^-$ final states considered in this analysis and to three data-taking periods (2015–2016, 2017, 2018), where the 2015 sample is fitted together with 2016 data due to its limited size. In the second stage, an unbinned maximum-likelihood fit to the decay-time distribution of the background-subtracted $B_s^0 \rightarrow D_s^+ K^- \pm$ signal is performed to determine the $CP$-violating parameters. In the decay-time fit the data-taking periods are fitted simultaneously, while the $D_s^-$ final states are combined.

Finally, the results of the present analysis are combined with those of Ref. [1], which uses an integrated luminosity of 3 fb$^{-1}$ recorded at $\sqrt{s} = 7$ and 8 TeV during the Run 1 data-taking period (2011–2012) with the external inputs updated to match the values used in the present analysis.

### 1.1 Decay rates and $CP$-violating parameters

Following the conventions of Ref. [1], the time-dependent decay rates of an initially produced flavour eigenstate $B_s^0$ or $\bar{B}_s^0$ decaying to final state $f$ can be written as

$$
\frac{d\Gamma_{B_s^0 \rightarrow f}(t)}{dt} = \frac{1}{2} |A_f|^2 (1 + |\lambda_f|^2) e^{-\Gamma_s t} \left[ \cosh \left( \frac{\Delta \Gamma_s t}{2} \right) + A_f^\Gamma \sinh \left( \frac{\Delta \Gamma_s t}{2} \right) \right] + C_f \cos (\Delta m_s t) - S_f \sin (\Delta m_s t),
$$

(1)

$$
\frac{d\Gamma_{\bar{B}_s^0 \rightarrow f}(t)}{dt} = \frac{1}{2} |A_f|^2 \frac{p}{q} (1 + |\lambda_f|^2) e^{-\gamma_s t} \left[ \cosh \left( \frac{\Delta \Gamma_s t}{2} \right) + A_f^\gamma \sinh \left( \frac{\Delta \Gamma_s t}{2} \right) \right] - C_f \cos (\Delta m_s t) + S_f \sin (\Delta m_s t),
$$

(2)

where $\lambda_f \equiv (q/p)(\bar{A}_f/A_f)$, $A_f(\bar{A}_f)$ is the amplitude of a $B_s^0(\bar{B}_s^0)$ decay to the final state $f \equiv D_s^- K^+$ and the complex coefficients $p$ and $q$ describe the mixing of the light, $|B_L\rangle$, and heavy, $|B_H\rangle$, mass and flavour eigenstates according to

$$
|B_L\rangle \equiv p|B_s^0\rangle + q|\bar{B}_s^0\rangle \quad \text{and} \quad |B_H\rangle \equiv p|B_s^0\rangle - q|\bar{B}_s^0\rangle,
$$

(3)

with the normalisation condition $|p|^2 + |q|^2 = 1$. Here, $\Gamma_s$ is the $B_s^0$ decay width or inverse $B_s^0$ lifetime, $\Delta \Gamma_s \equiv \Gamma_{B_L} - \Gamma_{B_H}$ is the decay-width difference between the light and heavy mass eigenstates and $\Delta m_s \equiv m_{B_H} - m_{B_L}$ is the mixing frequency in the $B_s^0$ system.
Similar equations can be obtained for decays into the CP-conjugate final state $\bar{f} \equiv D_\pi^+ K^-$ by replacing $C_f$ by $C_f$, $S_f$ by $S_f$, and $A_f^{\Delta r}$ by $A_f^{\Delta r}$. The CP-asymmetry parameters can be written as

$$C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} = -C_f = -\frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2},$$

$$A_f^{\Delta r} = \frac{-2 \text{Re}(\lambda_f)}{1 + |\lambda_f|^2}, \quad A_f^{\Delta r} = \frac{-2 \text{Re}(\lambda_f)}{1 + |\lambda_f|^2},$$

$$S_f = \frac{2\text{Im}(\lambda_f)}{1 + |\lambda_f|^2}, \quad S_f = \frac{2\text{Im}(\lambda_f)}{1 + |\lambda_f|^2}.$$

The relation $C_f = -C_f$ results from the conditions $|q/p| = 1$ and $|\lambda_f| = |1/\lambda_f|$, which imply no CP violation both in mixing, in agreement with current measurements \[22\], and decay. The second assumption is motivated by the fact that only a single amplitude contributes to each initial-to-final-state transition. Finally, the CP observables are related to the magnitude of the amplitude ratio $r_{D_s K}$, the strong-phase difference $\delta$ between the amplitudes $A(B_s^0 \to D_s^- K^+)$ and $A(B_s^0 \to D_s^- K^+)$ and the weak-phase difference $\gamma - 2\beta_s$ by the following equations

$$C_f = \frac{1 - r_{D_s K}^2}{1 + r_{D_s K}^2},$$

$$A_f^{\Delta r} = \frac{-2 r_{D_s K} \cos(\delta - (\gamma - 2\beta_s))}{1 + r_{D_s K}^2}, \quad A_f^{\Delta r} = \frac{-2 r_{D_s K} \cos(\delta + (\gamma - 2\beta_s))}{1 + r_{D_s K}^2},$$

$$S_f = \frac{2 r_{D_s K} \sin(\delta - (\gamma - 2\beta_s))}{1 + r_{D_s K}^2}, \quad S_f = \frac{2 r_{D_s K} \sin(\delta + (\gamma - 2\beta_s))}{1 + r_{D_s K}^2}.$$

These observables are used to extract $\gamma, \delta$ and $r_{D_s K}$ while fixing $-2\beta_s$, as discussed in Sec. 4. The combined Run 1 and Run 2 result is also expressed in terms of $\gamma - 2\beta_s$. This combined quantity offers complementary sensitivity on a potential new physics phase in $B_s^0 - \bar{B}_s^0$ mixing.

## 2 Detector and software

The LHCb detector \[23\] is a single-arm forward spectrometer covering the pseudorapidity range $2 < \eta < 5$, designed for the study of particles containing $b$ or $c$ quarks. The detector includes a high-precision tracking system consisting of a silicon-strip vertex detector surrounding the $pp$ interaction region, a large-area silicon-strip detector located upstream of a dipole magnet with a bending power of about 4 T m, and three stations of silicon-strip detectors and straw drift tubes placed downstream of the magnet. The tracking system provides a measurement of the momentum, $p$, of charged particles with a relative uncertainty that varies from $0.5\%$ at low momentum to $1.0\%$ at 200 GeV/c. The minimum distance of a track to a primary $pp$ collision vertex (PV), the impact parameter (IP), is measured with a resolution of $(15 + 29/p_T) \mu$m, where $p_T$ is the component of the momentum transverse to the beam, in GeV/c. Different types of charged hadrons are distinguished using information from two ring-imaging Cherenkov detectors. Photons, electrons and hadrons are identified by a calorimeter system consisting of scintillating-pad and preshower detectors, an electromagnetic and a hadronic...
calorimeter. Muons are identified by a system composed of alternating layers of iron and multiwire proportional chambers. The online event selection is performed by a trigger, which consists of a hardware stage, based on information from the calorimeter and muon systems, followed by a software stage, which applies a full event reconstruction.

At the hardware trigger stage, events are required to have a muon with high $p_T$ or a hadron, photon or electron with high transverse energy in the calorimeters. The software trigger requires a two-, three- or four-track secondary vertex with a significant displacement from any primary $pp$ interaction vertex. At least one charged particle must have a transverse momentum $p_T > 1.6\text{GeV}/c$ and be inconsistent with originating from a PV. A multivariate algorithm is used for the identification of secondary vertices consistent with the decay of a $b$ hadron.

Simulation is required to model the effects of the detector acceptance and the imposed selection requirements. In the simulation, $pp$ collisions are generated using PYTHIA, with a specific LHCb configuration. Decays of unstable particles are described by EVTGEN, in which final-state radiation is generated using PHOTOS. The interaction of the generated particles with the detector, and its response, are implemented using the GEANT4 toolkit as described in Ref. 32.

### 3 Candidate selection

The selection criteria are similar to those used in Ref. [1], but updated to reflect improvements in reconstruction performance. Samples of signal $B^0 \rightarrow D_s^0 K^\pm$ and two control channels $B^0_s \rightarrow D_s^- \pi^+$ and $B^0 \rightarrow D^- \pi^+$ decays are selected by combining a $D^- (s)$ candidate with a particle, referred to as “companion” in the following, consistent with the hypothesis of being either a kaon or a pion. The $D^-$ meson is reconstructed using the $D^- \rightarrow K^+ \pi^- \pi^-$ decay. The $D_s^-$ meson is reconstructed using the final states $K^- \pi^+ \pi^-$, $\pi^- \pi^+ \pi^-$ and $K^- K^+ \pi^-$, with the latter further subdivided into $D_s^- \rightarrow \phi \pi^-$, $D_s^- \rightarrow K^*(892)^0 K^-$, and the remaining regions of the phase space denoted as the nonresonant $D_s^- \rightarrow (K^- K^+ \pi^-)_{NR}$ component. The separation between these five decay modes is based on kinematic and particle identification (PID) requirements, allowing for the optimisation of the signal selection while accounting for the different background contributions in each sample. Furthermore, each decay mode undergoes a combination of PID and kinematic vetoes to suppress cross-feed backgrounds from $B^0 \rightarrow D^- h^+$ or $\bar{B}^0 \rightarrow D^- h^+$ decays, as well as background contributions containing $J/\psi$, $D^0$ and $K^{*0}$ decays. The distinction between $B^0 \rightarrow D_s^- \pi^+$ and $B^0_s \rightarrow D_s^+ K^\pm$ decays is achieved with mutually exclusive requirements on the PID information of the companion track.

The $B^0_s$ candidate is associated with the PV with the smallest impact parameter $\chi^2$, calculated as the difference in $\chi^2$ for the vertex fit of the PV with and without the considered particle ($\chi^2_{IP}$). The decay-time resolution of the $B^0_s$ candidate is enhanced through a kinematic fit, which constrains the candidate to originate from the associated PV. Similarly, the measured values of the $D_s^+ K^\pm$ invariant mass are obtained by constraining the $D_s^-$ invariant mass to the world-average value [19]. The $B^0_s$ and $D_s^-$ candidates are required to have invariant masses within $[5300, 5800] \text{MeV}/c^2$ and $[1930, 2015] \text{MeV}/c^2$, respectively.

Contributions from $b$-hadron decays that do not include a charm hadron are suppressed by imposing a $D_s^-$ flight-distance significance requirement. This quantity is defined as the
distance between the $B^0$ and $D_s^−$ decay vertices divided by its uncertainty.

To suppress combinatorial background due to random track combinations, a gradient-boosted decision tree (BDTG) algorithm \cite{34, 35} is employed. The BDTG classifier is trained using the $B^0_s \rightarrow D^−\pi^+$ control sample reconstructed in the $D_s^− \rightarrow K^-K^+\pi^−$ final state, as detailed in Ref. \cite{36}. Since all channels in this analysis exhibit similar kinematics and no PID information is used in the BDTG classifier, the resulting BDTG algorithm performs equally well on other $D_s^−$ decay modes. The classifier uses several track-related variables, including the transverse momentum of the companion particle, the radial flight distance of both the $b$- and $c$-hadron candidates and the companion and $b$-hadron’s minimum $\chi^2_{IP}$. A detailed description can be found elsewhere \cite{1,37}. Fewer than 0.5% of the events passing the selection requirements contain more than one signal candidate, and in such cases, all candidates are used in the analysis.

4 Two-dimensional invariant-mass fit

The selected $B^0_s \rightarrow D_s^\mp K^\pm$ candidates are fitted using a two-dimensional unbinned extended maximum-likelihood fit to the $m(D_s^\mp K^\pm)$ and $m(h^-h^+h^\mp)$ distributions, in order to statistically remove background components using the sPlot technique \cite{21} in the subsequent decay-time fit.

The signal and background PDFs for the invariant-mass fit are derived from the simulated samples after being corrected to better reproduce data. Specifically, the $B^0 \rightarrow D^−\pi^+$ control mode is used to correct for differences between simulation and data in the distributions of the $B^0_s$ and $D_s^−$ vertices’ uncertainty on the $z$-position and to correct for a shift between data and simulation in the $m(D_s^\mp K^\pm)$ invariant mass. The PID distributions in the simulation are corrected to match those in data using $D_s^*^+ \rightarrow D_s^0\pi^+$ and $\Lambda_0^0 \rightarrow p\pi^−$ calibration samples. More information about this procedure is provided in Ref. \cite{38}.

The shape of the $m(D_s^\mp K^\pm)$ distribution for signal candidates is modelled using the sum of a double-sided Hypatia function \cite{39} and a Johnson $S_U$ function \cite{40}, sharing a common peak position. This combination effectively describes the main peak and the radiative tail. The signal PDFs are separately derived from simulated $B^0_s \rightarrow D_s^\mp K^\pm$ candidates for each $D_s^−$ decay mode and data-taking period. The signal shapes are fixed in the data fit with two exceptions. Separate peak parameters are used for the three data-taking periods, which are left free in the fit. Furthermore, the widths of the Hypatia functions are fixed to the values determined from simulation, while the widths of the Johnson $S_U$ functions are left free. This adjustment compensates for mass-resolution differences between simulation and data.

The $m(h^-h^+h^\mp)$ signal distribution is also described using the combination of a double-sided Hypatia function and a Johnson $S_U$ function, sharing a common peak position. The signal PDFs are derived from simulation for each $D_s^−$ decay mode and data-taking period. Similar to the $m(D_s^\mp K^\pm)$ invariant-mass parameterisation, only the common peak position and the width of the Johnson $S_U$ function are free parameters in the fit to data.

The combinatorial background comprises random track combinations that do not originate from $D_s^−$ meson decay, as well as backgrounds containing a true $D_s^−$ decay combined with a random companion track. The functional form of the combinatorial background is motivated by the upper $m(D_s^\mp K^\pm)$ invariant-mass sidebands, [5600, 6800] MeV/c$^2$, with
The signal yield, confirmed to be unbiased through data-like pseudoexperiments, is found to be optimal to describe the combinatorial background component for the $D_s^- \to K^- K^+ \pi^-$ decay modes, while for the $D_s^- \to K^- \pi^+ \pi^-$ and $D_s^- \to \pi^- \pi^+ \pi^-$ final states a single exponential is found to be sufficient. The combinatorial component in the $m(h^- h^+ h^\mp)$ distribution is described by the sum of an exponential and the $D_s^- \to \pi^- \pi^+ \pi^-$ signal shape, where the peak position is shared with the signal itself, modelling the contributions from random track combinations and true $D_s^-$ mesons paired with a random track, respectively. The exponent and the fraction between the exponential and the signal shape are free to vary in the fit to data.

In the two-dimensional fit, besides the signal and the combinatorial background, the following background contributions are considered: fully reconstructed $B^0 \to D_s^- K^+$ decays, companion-track misidentified $B_s^0 \to D_s^- \pi^+$ and $\Lambda^0 \to D_s^- p$ decays, companion-track misidentified partially reconstructed $B_s^0 \to D_{s^-}^* \pi^+$, $B_s^0 \to D_{s^-} \rho^+$ and $\Lambda^0 \to D_{s^-}^* p$ decays, where the neutral $\gamma$ or $\pi^0$ from $D_{s^-}^* \to D_s^- \gamma/\pi^0$ and $\rho^+ \to \pi^+ \pi^0$ is not reconstructed. Furthermore, the $B^0 \to D^- K^+$, $B^0 \to D^- \pi^+$, $\Lambda^0 \to \Lambda_c^- K^+$ and $\Lambda^0 \to \Lambda_c^- \pi^+$ components are included, where a misidentified final state causes the $c$-hadron to be reconstructed as a $D_s^-$ meson.

In the fit to the $m(D_s^\pm K^\mp)$ distribution, double-sided Hypatia functions are used for the fully reconstructed $B^0 \to D_s^- K^+$ and the partially reconstructed $B_s^0 \to D_{s^-}^* \pi^+$ contributions, the sum of a double-sided Hypatia and a Johnson $S_U$ function is used to describe the $B_s^0 \to D_{s^-} \pi^+$ decays, the contribution from partially reconstructed $B_s^0 \to D_{s^-} \rho^+$ decays is modelled using the sum of two exponential functions. For the $m(D_s^\pm K^\mp)$ distribution of the remaining partially or fully reconstructed backgrounds, the shapes of the distributions in the simulation are defined using a nonparametric kernel estimation method [41], corrected to match the PID efficiency and kinematic distributions observed in the data.

In the fit to the $m(h^- h^+ h^\mp)$ distribution, the signal shape is used for the $B^0 \to D_s^- K^+$, $B_s^0 \to D_{s^-} \pi^+$, $B_s^0 \to D_{s^-}^* \pi^+$, $B_s^0 \to D_{s^-} \rho^+$, $\Lambda^0 \to D_s^- p$ and $\Lambda^0 \to D_s^- p$ contributions, whereas the other background components are described using a nonparametric kernel estimation method.

The invariant-mass fit is performed simultaneously across the different $D_s^-$ decay modes and periods of data taking. For each $D_s^-$ decay mode, the PDF is constructed from the sum of signal and background contributions. Most background yields are allowed to vary freely in the fit, except for those with an expected contribution below 2% of the signal yield, specifically: $B^0 \to D^- K^+$, $B^0 \to D^- \pi^+$, $\Lambda^0 \to \Lambda_c^- K^+$ and $\Lambda^0 \to \Lambda_c^- \pi^+$. In such cases, the yields are fixed to the results of dedicated fits to $B^0 \to D^- \pi^+$ and $\Lambda^0 \to \Lambda_c^- \pi^+$ candidates, corrected by the known branching fractions and selection efficiencies relative to the selection of the $B_s^0 \to D_s^\pm K^\mp$ candidates [19].

The $D_s^\pm K^\mp$ and $h^\mp h^+ h^\mp$ invariant-mass distributions, summed over the $D_s^-$ decay modes and the data-taking periods, are shown in Fig. 2 with the results of the fit overlaid. The signal yield, confirmed to be unbiased through data-like pseudoexperiments, is determined to be $20.949 \pm 180$, where the uncertainty is statistical only. The invariant-mass fit to the $B_s^0 \to D_s^- \pi^+$ control mode is reported in Ref. [36].
The bias was determined from the analysis of the same calibration sample of prompt $B^0$ decays which follow a $D_s^\mp K^\pm$ signal and the shaded-stacked histograms show the different background contributions.

The goal of the decay-time fit is to determine the $CP$-violating parameters $C_f$, $S_f$, $S_f'$, $A_f^{S\mp}$ and $A_f^T$ from the decay-time distribution of $B^0_s \rightarrow D_s^\mp K^\pm$ decays which follow a PDF obtained from Eqs. 1 and 2. The decay-time distribution of the signal candidates in the reconstructed $B^0_s \rightarrow D_s^\mp K^\pm$ decays is obtained using the sPlot technique following the two-dimensional mass fit. Several experimental effects are accounted for in the fit to describe the observed decay rates, such as the decay-time resolution and acceptance and the performance of the flavour-tagging algorithms to determine the initial $B^0_s$ flavour.

To account for the decay-time resolution, the signal PDF is convolved with a Gaussian function whose width, $\sigma_t$, is evaluated for each candidate based on the decay-time uncertainty estimated by the vertex fit, $\delta_t$. The decay-time resolution model is calibrated using a control sample of artificial $B^0_s \rightarrow D_s^\mp K^\pm$ decays obtained by combining a $D_s^-$ meson originating promptly from the $pp$ interaction point with an oppositely charged companion meson coming from the same origin. The selection requirements are the same as for the signal, except for those that depend on the displacement from the $pp$ collision point. These $B^0_s$ candidates have a decay vertex compatible with a lifetime of zero, up to resolution and alignment effects, thus their decay-time distribution determines the decay-time resolution. The sample of prompt $D_s^-$ candidates is divided into ten equally populated bins of decay-time uncertainty. In each subsample, the decay-time resolution is determined from a fit to the decay-time distribution. The resulting decay-time resolutions as a function of decay-time uncertainty are then fitted with a linear function, $\sigma_t = p_0 + p_1 \delta_t$. The resulting average decay-time resolution is about 46 fs.

A bias in the measured $B^0_s$ decay time, originating from residual detector misalignment, has been identified in the analysis of $B^0_s \rightarrow D_s^+ \pi^+$ decays for the $\Delta m_s$ measurement [36]. This bias was determined from the analysis of the same calibration sample of prompt $D_s^-$.
mesons used for the study of the aforementioned decay-time resolution model. Corrections for the differences between the prompt $D_s^-$ and signal samples were obtained using intentionally misaligned simulated samples. This measurement takes the bias obtained in the $\Delta m_s$ measurement with a correction to account for differences in selection criteria between the two analyses. The decay-time bias, determined for each period of data taking, is corrected for in the decay-time fit and amounts to about $-3\text{fs}$.

The initial flavour of the $B^0_s$ meson is needed to determine its contribution to the corresponding decay rate. This information is provided by flavour-tagging algorithms which exploit different processes correlated with the $b$-hadron production in $pp$ collisions. Beauty quarks are predominantly produced through $b\bar{b}$ pairs. While one of these $b$ quarks leads to the signal $B^0_s$ meson, the other leads to a $b$ hadron that decays independently. The decay chain of the other $b$ hadron is exploited by the opposite-side (OS) tagging algorithms \[42\] to determine the initial flavour of the signal $B^0_s$ meson. The OS muon and OS electron taggers exploit the semileptonic decay of the $b$ hadron, and the OS kaon and the OS charm taggers identify remnants from $b \rightarrow c \rightarrow s$ and $b \rightarrow c$ transitions, respectively. Furthermore, the OS vertex-charge tagger reconstructs an effective charge of a displaced vertex from the OS $b$-hadron decay \[43\]. Each of these algorithms infers the initial $B^0_s$ meson flavour from the charge of either a reconstructed tagging particle or, in the case of the OS vertex tagger, of a reconstructed vertex. Additionally, the same-side (SS) kaon tagger determines the initial flavour of the $B^0_s$ signal from the charge of kaons originating from the $s\pi^0$ pair produced in the fragmentation process that leads to the signal $B^0_s$ meson \[44\]. In addition to the tag decision, representing the determined flavour, the algorithms provide an estimate of the probability that the decision is wrong, the estimated mistag probability, $\eta$. This estimate does not necessarily match the correct mistag probability, $\omega$, of the data sample. Hence, a calibration is performed using a control sample of flavour-specific $B^0_s \rightarrow D_s^- \pi^+$ decays to provide the measured mistag following the procedure described in Ref. \[36\]. The calibration functions are defined as

\begin{equation}
\omega^{\text{tag},y}(\eta^{\text{tag},y}) = \sum_{i=0}^{1} \left( f_{i}^{\text{tag},y} + \frac{1}{2} \Delta f_{i}^{\text{tag},y} \right) \cdot \left( \eta^{\text{tag},y} - \langle \eta^{\text{tag},y} \rangle_{i} \right) \quad \text{for } B^0_s ,
\end{equation}

and

\begin{equation}
\omega^{\overline{\text{tag}},y}(\eta^{\overline{\text{tag}},y}) = \sum_{i=0}^{1} \left( f_{i}^{\overline{\text{tag}},y} - \frac{1}{2} \Delta f_{i}^{\overline{\text{tag}},y} \right) \cdot \left( \eta^{\overline{\text{tag}},y} - \langle \eta^{\overline{\text{tag}},y} \rangle_{i} \right) \quad \text{for } B^0_s ,
\end{equation}

where $\langle \eta^{\text{tag},y} \rangle$ is the average mistag probability, index $i$ identifies each of the two flavour-tagging calibration parameters $f_{i}^{\text{tag},y}$. The parameters $\Delta f_{i}^{\text{tag},y}$ are introduced to allow for different calibrations for $B^0_s$ and $\overline{B}^0_s$ candidates. Each tagger ($\text{tag} = \text{OS, SS}$) is independently calibrated on each data sample $y$ (2015–2016, 2017, 2018).

The tagging information is included in the PDF used in the decay-time fit, for which the tagging decision assigns candidates to the corresponding decay rate described by Eqs. \[1\] and \[2\]. The mistag probability causes a reduction of the oscillation amplitude by a dilution factor $D = (1 - 2\omega)$. In the fit, the calibration is performed by constraining the function of the predicted mistag described by Eqs. \[6\] and \[7\]. Only two taggers are constrained in the fit: the SS kaon tagger and the OS combination, where the individual tag decisions and mistag estimates of all OS taggers are combined into a single decision and mistag estimate.

The tagging efficiency of the full sample is $\varepsilon = (80.30 \pm 0.07)\%$ with an average mistag fraction of $\omega = (36.21 \pm 0.02 \pm 0.17)\%$, where the first uncertainty is due to the finite
size of the calibration sample and the second is due to the uncertainty of the calibration parameters. This results in a tagging power of $(6.10 \pm 0.02 \pm 0.15)\%$, which indicates the remaining statistical power of the flavour-tagged sample, relative to a perfectly tagged sample. For comparison, the tagging power achieved in Run 1 was $(5.80 \pm 0.25)\%$\footnote{1}. Since the CP-violating parameters depend on the decay-time acceptance, the latter needs to be determined. For flavour-specific $B^0 \to D^-\pi^+$ decays, where $C_f = -C_{\bar{f}} = 1$ and $S_f = S_{\bar{f}} = 0$, the decay-time acceptance can be determined from a fit to the decay-time distribution with $\Gamma_s$ and $\Delta \Gamma_s$ parameters fixed to the combination of LHCb results\footnote{45}. In the $B^0 \to D^\pm K^\mp$ fit, the decay-time acceptance is fixed to the result obtained from the $B^0 \to D_s^-\pi^+$ data fit, corrected by the decay-time acceptance ratio of the two modes estimated from simulation, which is weighted to match the data as described in Sec.\footnote{4}. The decay-time acceptance is modelled using segments of cubic B-splines, which are implemented analytically in the decay-time fit\footnote{46}. The spline boundaries, also known as knots, are chosen in order to accurately model the features of the decay-time acceptance shape. The signal decay-time PDF is then adjusted by multiplying by the decay-time acceptance model.

The decay-time fit requires additional inputs in the form of the following parameters

\begin{align}
\Delta m_s &= (17.7683 \pm 0.0057) \text{ ps}^{-1}, \\
\Gamma_s &= (0.6563 \pm 0.0020) \text{ ps}^{-1}, \\
\Delta \Gamma_s &= (0.085 \pm 0.004) \text{ ps}^{-1}, \\
A_{\text{prod}} &= (-0.33 \pm 0.32)\% , \\
A_{\text{det}} &= (0.96 \pm 0.15)\% ,
\end{align}

which are fixed to their central values in the baseline decay-time fit and varied within their uncertainties to determine the associated systematic uncertainties. The values of $\Delta m_s$, $\Gamma_s$ and $\Delta \Gamma_s$ are based on LHCb measurements\footnote{36, 45}. The production asymmetry, $A_{\text{prod}}$, is fixed to the value obtained in Ref.\footnote{36} and is defined as the relative difference in the $B^0_s$ and $\bar{B}^0_s$ production cross-section. The parameter $A_{\text{det}}$ is defined as the relative difference in detection efficiencies between the $D_s^-\pi^+$ and the $D_s^+\pi^-$ final states. This is evaluated following the method described in Ref.\footnote{47}, where the detection asymmetry is evaluated for $K^+\pi^-$ pairs using $D^+ \to K^-\pi^+\pi^-$ and $D^+ \to K^0\pi^+$ decays. For this measurement, an average asymmetry over the $D_s^-$ final states is calculated using the signal yields obtained from the invariant-mass fit. Its value is compatible with the $K^+\pi^-$-pair detection asymmetry in Ref.\footnote{48}, where the asymmetry was evaluated for $B^+ \to D\pi^+$ decays. The detection and the production asymmetries contribute to the decay-time PDF with multiplicative factors of $(1 \pm A_{\text{prod}})$ and $(1 \pm A_{\text{det}})$ to the decay rates defined by Eqs.\footnote{1} and \footnote{2} depending on the tagged initial state and the reconstructed final state.

The CP-violating parameters are determined in a weighted maximum-likelihood fit to the flavour-tagged decay-time distributions. The fit is performed simultaneously to all five $D_s^-$ final states and three data-taking periods, where calibrations of the decay-time resolution and the mistag probability are constrained individually for each data-taking period. The fitted covariance matrix is corrected following an asymptotically correct approach described in Ref.\footnote{49} to provide good coverage of the uncertainties following the use of the sPlot method. The resulting CP-violating parameters are listed in Table\footnote{1} and the corresponding statistical correlation matrix is given in Table\footnote{2}.

The decay-time distribution and mixing asymmetry of the $D_s^-K^+$ and $D_s^+K^-$ final
Table 1: Values of the CP-violating parameters obtained from the decay-time fit to $B^0_s \rightarrow D^\mp_s K^\pm$ candidates. The first uncertainty is statistical and the second is systematic.

| Parameter | Value |
|-----------|-------|
| $C_f$     | $0.791 \pm 0.061 \pm 0.022$ |
| $A_{fR}^\Delta$ | $-0.051 \pm 0.134 \pm 0.058$ |
| $A_{fI}^\Delta$ | $-0.303 \pm 0.125 \pm 0.055$ |
| $S_f$     | $-0.571 \pm 0.084 \pm 0.023$ |
| $S_{f\bar{f}}$ | $-0.503 \pm 0.084 \pm 0.025$ |

Table 2: Statistical correlation matrix of the CP observables. Other fit parameters have negligible correlations with the CP observables.

| Parameter | $C_f$ | $A_{fR}^\Delta$ | $A_{fI}^\Delta$ | $S_f$ | $S_{f\bar{f}}$ |
|-----------|-------|-----------------|-----------------|-------|-----------------|
| $C_f$     | 1     | 0.134           | 0.130           | 0.039 | 0.022           |
| $A_{fR}^\Delta$ | 1     | 0.501           | -0.108          | -0.036 |
| $A_{fI}^\Delta$ | 1     | -0.056          | -0.067          |       |
| $S_f$     | 1     | 0.006           |                 |       |
| $S_{f\bar{f}}$ | 1     |                 |                 |       |

The mixing asymmetry is defined as the relative difference between events flavour-tagged as $B^0_s$ and $\bar{B}^0_s$, decaying to the $D^\mp_s K^\pm$ or $D^\mp_s K^-\bar{K}^+$ final state as a function of decay time. The CP observables are represented in the $\left( \Re \left[ 2 \lambda_f/(1 + |\lambda_f|^2) \right] , \Im \left[ 2 \lambda_f/(1 + |\lambda_f|^2) \right] \right)$ Cartesian plane in Fig. 4. The agreement of the $(-A_{fR}^\Delta, S_f)$ and $(-A_{fI}^\Delta, S_{f\bar{f}})$ contours with the $q_1-C_f^2$ band indicates that the results are in good agreement with the constraint $C_f^2 + S_f^2 + A_{fR}^\Delta = 1$ that relates to Eq. 5. Using the statistical and systematic uncertainties reported in Table 1 and the corresponding correlations, CP violation in the interference of mixing and decay, i.e. $S_f \neq -S_{f\bar{f}}$, is observed with a significance of 8.6 $\sigma$.

The dependence of the CP observables on the values of the $\Gamma_s$, $\Delta \Gamma_s$ and $\Delta m_s$ parameters is provided in Appendix A.

### 6 Systematic uncertainties

Systematic uncertainties are evaluated due to various factors such as the modelling of the invariant-mass fit, fixed parameters in the decay-time fit, namely those in Eq. 8 and the limited knowledge of the decay-time resolution and acceptance. Additionally, the effect of ignoring correlations among observables is assessed. Table 3 summarises the various contributions to the systematic uncertainty, which are described below.

The systematic uncertainty due to fixed parameters in the invariant-mass fit is determined by repeating fits to the data, in which the fixed parameters for signal and background shapes are varied by ±1 standard deviation. Additionally, fixed background yields and relative fractions among background components are varied. For each parame-
Figure 3: (Top) Decay-time distribution of $B^0_s \rightarrow D^{\mp}_s K^\pm$ signal candidates, where the background is statistically subtracted using the $s$Plot technique. (Bottom) Mixing asymmetry, $A_{\text{mix}}$, for the (blue) $D^-_s K^+$ and the (red) $D^+_s K^-$ final states, folded into one mixing period, $2\pi/\Delta m_s$. In both plots, the curves show the result of the decay-time fit.

ter, the average difference of the $CP$ observables between the baseline and the modified fit is taken as the systematic uncertainty. After evaluating single contributions, all sources are added in quadrature.

The flavour-tagging parameters are constrained to the values found in the $B^0_s \rightarrow D^-_s \pi^+$ decay-time fit. Systematic uncertainties are assigned by performing a single $B^0_s \rightarrow D^{\mp}_s K^\pm$ decay-time fit, where the uncertainties on the flavour-tagging parameters are enlarged to account for their systematic effects. This fit accounts for both the variation of the fit strategy used in the $B^0_s \rightarrow D^-_s \pi^+$ data fit, as described in Ref. [36], and the portability of the flavour-tagging calibration to $B^0_s \rightarrow D^{\mp}_s K^\pm$ decays, studied in simulation. This alternative decay-time fit is compared to the baseline fit and the difference in the results

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Figure 4: Representation of the CP observables in the \((\text{Re}[2\lambda_f/(1 + |\lambda_f|^2]), \text{Im}[2\lambda_f/(1 + |\lambda_f|^2)])\) Cartesian plane. The contours shown correspond to 39.3% of the distribution.

is assigned as a systematic uncertainty.

The remaining sources of systematic uncertainty are evaluated using pseudoexperiments, where each sample is generated according to the baseline results of the invariant-mass fit and, for the signal component, of the decay-time fit. The decay-time distributions of the backgrounds are generated to resemble the properties observed in data or found in simulation. The pseudoexperiments are processed using the same fit procedure as the data. To assess the systematic uncertainty, the parameter in question, considered for the systematic effect, is varied within its uncertainty. The values obtained from the baseline fit are compared to the values from the fits with the modified model. A distribution of the resulting differences is formed, taking into account the correlations between the baseline and modified pseudoexperiments. The systematic uncertainty is assigned as the mean and width of this distribution added in quadrature.

The systematic uncertainty related to the uncertainty on \(\Delta m_s\), which is fixed in the baseline fit, is evaluated in a fit to pseudoexperiments in which the \(\Delta m_s\) value is shifted by one standard deviation. The difference between the result of the modified fit and baseline value is assigned as the systematic uncertainty. In a similar way, the systematic uncertainty related to the uncertainty on the uncorrelated parameter \(A_{\text{det}}\) is determined.

The systematic uncertainty from the limited knowledge of the decay-time resolution is obtained by repeating the fit to pseudoexperiments using a narrower or wider decay-time resolution model. The largest deviation between the baseline and modified fits is taken as the systematic uncertainty.

The correlation among the parameters \(\Gamma_s\), \(\Delta\Gamma_s\) and the decay-time acceptance obtained from the \(B^0_s \rightarrow D_s^-\pi^+\) data necessitates a combined treatment of the corresponding systematic uncertainties. The pseudoexperiments are fitted with modified values of the spline coefficients, as well as the \(\Gamma_s\) and \(\Delta\Gamma_s\) parameters, which are sampled from the corresponding multidimensional correlated Gaussian distributions centred on their baseline values. The combined correlated systematic uncertainty is determined from the average deviation of these modified fits with respect to the baseline fits.
Table 3: Systematic uncertainties on the $CP$ observables, expressed as a fraction of the corresponding statistical uncertainties. The value “—” indicates that the contribution is negligible.

| Source                                      | $C_f$ | $A_f^{\Delta \Gamma}$ | $A_{\bar{f}}^{\Delta \Gamma}$ | $S_f$ | $S_{\bar{f}}$ |
|---------------------------------------------|-------|------------------------|-------------------------------|-------|---------------|
| Invariant-mass fit                          | 0.045 | 0.095                  | 0.121                         | 0.088 | 0.112         |
| Flavour tagging                             | 0.256 | 0.026                  | 0.028                         | 0.012 | 0.070         |
| Oscillation frequency $\Delta m_s$         | 0.006 | 0.005                  | 0.004                         | 0.108 | 0.101         |
| Detection asymmetry $A_{\text{det}}$        | 0.001 | 0.079                  | 0.082                         | 0.007 | 0.007         |
| Decay-time resolution model                 | 0.195 | 0.008                  | 0.008                         | 0.054 | 0.166         |
| Decay-time acceptance, $\Gamma_s$, $\Delta \Gamma_s$ | 0.006 | 0.397                  | 0.400                         | 0.009 | 0.009         |
| Decay-time acceptance simulation            | 0.004 | 0.064                  | 0.064                         | —     | 0.004         |
| Decay-time bias                             | 0.062 | 0.027                  | 0.046                         | 0.188 | 0.167         |
| Neglecting correlations                     | 0.137 | 0.081                  | 0.054                         | 0.135 | 0.043         |
| **Total**                                   | 0.358 | 0.430                  | 0.439                         | 0.277 | 0.293         |

As the decay-time acceptance obtained from the $B^0_s \to D^-_s \pi^+$ data is corrected by the simulation, an additional source of systematic uncertainty arises from the limited size of the simulation samples that are used to determine the correction. To account for this, pseudoexperiments are fitted with a modified decay-time acceptance correction, that is sampled from a multidimensional Gaussian representing the uncertainties and correlations of the corrections. The average deviation of these modified fits with respect to the baseline fits is taken as a systematic uncertainty.

A decay-time bias is observed in the $\Delta m_s$ measurement from the $B^0_s \to D^-_s \pi^+$ decays [36], and corrected for in the baseline fit. A systematic uncertainty related to the uncertainty on the bias, taken to be $\pm 1 \text{ fs}$, is obtained using pseudoexperiments. This value includes both the uncertainty of the decay-time bias as obtained in the $B^0_s \to D^-_s \pi^+$ decays and the correction to account for the differences in selection criteria between the two analyses [36].

The impact of neglecting the correlations among decay time and decay-time uncertainty with the $B^0_s$ and $D^-_s$ invariant masses in the $sPlot$ method is studied with a dedicated set of pseudoexperiments using a bootstrapping method [50]. The method preserves correlations among observables. The results of the decay-time fits performed on samples with and without these correlations are compared and the mean difference is assigned as the systematic uncertainty.

Additional cross-checks are performed to further validate the results. The data sample is split into subsets according to the two LHCb dipole magnet polarity orientations, the year of data taking and the $B^0_s$ meson momentum. In addition, the data sample is split into decay-time bins and the invariant-mass fit is performed on each subsample. This is followed by the combined decay-time fit. In all cases, no significant deviations among the results are observed. A closure test using a large sample of simulated signal candidates provides an estimate of the intrinsic bias related to the decay-time fit procedure. No significant bias is found.

Several other potential systematic effects are examined but are found to be negligible. No significant systematic effect associated with the production asymmetry $A_{\text{prod}}$ is observed. The decay-time fit is repeated by varying the knot positions in the decay-time acceptance...
Table 4: Correlation matrix of the total systematic uncertainties of the CP observables.

| Parameter | $C_f$ | $A_f^{\Delta \Gamma}$ | $A_f^{\Delta \Gamma}$ | $S_f$ | $S_f$ |
|-----------|-------|------------------------|------------------------|-------|-------|
| $C_f$     | 1     | 0.008                  | 0.012                  | -0.080| -0.246|
| $A_f^{\Delta \Gamma}$ | 1     | 0.878                  | 0.004                  | -0.022|       |
| $A_f^{\Delta \Gamma}$ |       | 1                      | -0.002                 | -0.022|       |
| $S_f$     |       |                        | 1                      | 0.085 |       |
| $S_f$     |       |                        |                        |       | 1     |

description. No significant changes with respect to the baseline result are found. The precision on the world-average value for the oscillation frequency $\Delta m_s$ is dominated by the LHCb measurement [36], which uses the same $B^0_s \rightarrow D_s^- \pi^+$ sample used as a control channel in this analysis. The imperfect knowledge of the particles’ momentum and the longitudinal dimension of the detector are encompassed by the systematic uncertainty on $\Delta m_s$, therefore these sources are not further considered.

The resulting systematic uncertainties are listed in Table 3 and expressed as fraction of the corresponding statistical uncertainties. The total systematic correlation matrix, reported in Table 4, is obtained by adding the covariance matrices corresponding to each source.

7 Interpretation

The measured CP-violating parameters listed in Table 4 can be interpreted in terms of the CKM angle $\gamma$, the strong phase difference $\delta$, the magnitude of the amplitude ratio $r_{D_s K}$ and the mixing phase $\beta_s$. This is achieved using a frequentist approach detailed in Refs. [51,52]. First, a likelihood function is defined as

$$L(\vec{\alpha}) = \prod_i f(\vec{A}_{i,\text{obs}}|\vec{\alpha})$$

where the function $f(\vec{A}_{i,\text{obs}}|\vec{\alpha})$ is assumed to follow a multivariate Gaussian distribution

$$f(\vec{A}_{i,\text{obs}}|\vec{\alpha}) \propto \exp \left( -\frac{1}{2} (\vec{A}(\vec{\alpha}) - \vec{A}_{i,\text{obs}})^T V_i^{-1} (\vec{A}(\vec{\alpha}) - \vec{A}_{i,\text{obs}}) \right),$$

where $\vec{A}_{i,\text{obs}}$ is the vector of observables ($C_f, A_f^{\Delta \Gamma}, A_f^{\Delta \Gamma}, S_f, S_f$), $V_i$ is the experimental covariance matrix, and $i$ is an index labelling the different measurements to be combined. The vector function $\vec{A}(\vec{\alpha})$ encodes the dependency of the observables on the parameters of interest $\vec{\alpha} = (\gamma, \beta_s, \delta, r_{D_s K})$, following Eq. 5. A fit is performed to find the set of values $\vec{\alpha}_{\text{min}}$ that minimise the function $\chi^2(\vec{\alpha}) = -2 \ln L(\vec{\alpha})$. An ensemble of pseudoexperiments is generated to determine the best-fit values and confidence intervals of the parameters $\vec{\alpha}$, as detailed in Ref. [53]. This method is referred to as the Plugin method and is used throughout this paper for the results of $\gamma$, $\delta$ and $r_{D_s K}$.

As discussed in Sec. 1, the CP observables determined from $B^0_s \rightarrow D_s^+ K^-$ decays are functions of the weak-phase difference $(\gamma - 2\beta_s)$. Therefore, in order to determine $\gamma$, the value of $\beta_s$ has to be taken from independent measurements. The value of $\beta_s$ is
Table 5: Updated values of the CP observables from the decay-time fit of the Run 1 analysis with updated values of the nuisance parameters $\Delta m_s$, $\Gamma_s$ and $\Delta \Gamma_s$. The first uncertainty is statistical and the second is systematic.

| Parameter | Value       |
|-----------|-------------|
| $C_f$     | 0.75 ± 0.14 ± 0.04 |
| $A_f^{\Delta \Gamma}$ | 0.38 ± 0.28 ± 0.15 |
| $A_{\bar{f}}^{\Delta \Gamma}$ | 0.30 ± 0.28 ± 0.15 |
| $S_f$     | −0.53 ± 0.21 ± 0.06 |
| $S_{\bar{f}}$ | −0.45 ± 0.20 ± 0.06 |

obtained through the relation $\phi_s = -2\beta_s$, which assumes that the contributions from loop diagrams in $B_s^0 \to J/\psi \phi$ decays and physics beyond the SM are negligible. The value $\phi_s = -0.031 \pm 0.018$ rad is used, which is taken from the combination of LHCb measurements presented in Ref. [45].

The $CP$-violating parameters obtained from the Run 2 $B_s^0 \to D_s^\mp K^\pm$ data correspond to the following parameters:

$$\gamma = (74 \pm 12) ^\circ ,$$
$$\delta = (346.9^{+6.8}_{-6.6}) ^\circ ,$$
$$r_{D_s K} = 0.327^{+0.039}_{-0.037} ,$$

where the phases $\gamma$ and $\delta$ are determined up to a global shift of 180°, where the statistical and systematic uncertainties are summed in quadrature. Figure 5 shows the confidence-level (CL) plot for $\gamma$, as well as the two-dimensional contours of $\gamma$ versus $r_{D_s K}$ and $\delta$.

8 Combination with the results of the Run 1 analysis

The results of this measurement are combined with those obtained using Run 1 data, presented in Ref. [1]. To achieve that, the main systematic uncertainties of the Run 1 measurement related to the external parameters $\Gamma_s$, $\Delta \Gamma_s$ and $\Delta m_s$ are recomputed using the more precise values considered in this analysis. These external parameters only affect the time-dependent part of the analysis, which is therefore updated with the new values. The following steps are repeated: the determination of the decay-time acceptance from $B_s^0 \to D_s^- \pi^+$ data, the flavour-tagging calibration, which is required because of its correlation with the $\Delta m_s$ parameter in the fit to $B_s^0 \to D_s^- \pi^+$ data and the decay-time fit to $B_s^0 \to D_s^\mp K^\pm$ candidates to extract the values of the $CP$ observables. Other aspects of the analysis, such as the decay-time resolution, obtained from calibration data, are unchanged with respect to Ref. [1]. The updated values of the $CP$ observables from the Run 1 data set are reported in Table 5. Following the same procedure as described in Sec. 7 the values of the parameters $\gamma$, $\delta$ and $r_{D_s K}$ corresponding to the Run 1 measurement
Figure 5: Contour plots of (top left) $r_{D_s K}$ vs. $\gamma$ and (top right) $\delta$ vs. $\gamma$, where the contours correspond to the confidence levels (CL) of 68% and 95%. The bottom plot shows the $1 - CL$ curve for the angle $\gamma$, with the 68.3% and 95.4% CL intervals indicated with horizontal and vertical lines. The results correspond to $\gamma = (74 \pm 12)^\circ$ and $\gamma = (127^{+18}_{-26})^\circ$ for the Run 2 and Run 1 analysis, respectively, where updated values of $\Gamma_s$, $\Delta\Gamma_s$ and $\Delta m_s$ are also propagated to the latter.

are

$$\gamma = (127^{+18}_{-26})^\circ,$$
$$\delta = (358^{+14}_{-15})^\circ,$$
$$r_{D_s K} = 0.364^{+0.095}_{-0.094}.$$

The Run 2 and the updated Run 1 results are compared in Fig. 5. The compatibility of $\gamma$, $\delta$ and $r_{D_s K}$ parameters between the updated Run 1 and the Run 2 results corresponds to a $p$-value of 12%.

The $CP$-violating parameters of the two data sets have been combined following the methods presented in Ref. [12]. The two measurements are treated as independent and the optimal values of $\gamma$, $\delta$ and $r_{D_s K}$ are obtained via the likelihood fit described in Sec. [7].
Figure 6: Contour plots arising from the combined extraction of $\gamma$, $\delta$ and $r_{DsK}$ from the $CP$ parameters from the Run 1 and Run 2 data sets. (Top left) $r_{DsK}$ vs. $\gamma$ and (top right) $\delta$ vs. $\gamma$, the contours correspond to the confidence levels (CL) of 68% and 95%. The bottom plot shows the $1 - CL$ curve for the angle $\gamma$, with the 68.3% and 95.4% CL intervals indicated with horizontal and vertical lines.

The resulting values are

$$\gamma = (81^{+12}_{-11})^\circ,$$
$$\delta = (347.6 \pm 6.3)^\circ,$$
$$r_{DsK} = 0.318^{+0.034}_{-0.033}.$$  

The corresponding $1 - CL$ curve for $\gamma$ is shown in Fig. 6 as well as the two-dimensional contours of $\gamma$ versus $r_{DsK}$ and $\delta$. Finally, the value of the relative weak-phase difference in $B_s^0 \to D_s^\mp K^\pm$ decays is determined to be $\gamma - 2\beta_s = (79^{+12}_{-11})^\circ$, providing complementary sensitivity to $\phi_s$ on a potential new physics phase in $B_s^0 - \bar{B}_s^0$ mixing.

9 Conclusion

The $CP$-violating parameters that describe the $B_s^0 \to D_s^\mp K^\pm$ decay rates have been measured using a data set corresponding to an integrated luminosity of 6 fb$^{-1}$ of $pp$
collisions recorded with the LHCb detector. Their values are found to be

\[ C_f = 0.791 \pm 0.061 \pm 0.022 , \]
\[ A_f^\Gamma = -0.051 \pm 0.134 \pm 0.058 , \]
\[ A_{f^*}^\Gamma = -0.303 \pm 0.125 \pm 0.055 , \]
\[ S_f = -0.571 \pm 0.084 \pm 0.023 , \]
\[ S_{f^*} = -0.503 \pm 0.084 \pm 0.025 , \]

where the first uncertainties are statistical and the second are systematic. \( CP \) violation in the interference between \( B^0 - \bar{B}^0 \) mixing and \( B^0_s \to D^+_s K^- \) decays is observed with a significance of 8.6 \( \sigma \). The results are used to determine the CKM angle \( \gamma \), the strong-phase difference \( \delta \) and the magnitude of the ratio \( r_{D_s K} \) between the \( B^0_s \to D^+_s K^- \) and the \( B^0_s \to D^-_s K^+ \) decay amplitudes, leading to

\[ \gamma = (74 \pm 12)^\circ , \]
\[ \delta = (346.9^{+6.8}_{-6.6})^\circ , \]
\[ r_{D_s K} = 0.327^{+0.039}_{-0.037} , \]

where all angles are given modulo 180\(^\circ\), and uncertainties shown are the combination of the statistical and systematic contributions.

The results of the present analysis are combined with those from the previous LHCb analysis \[1\], which is updated to account for improved determinations of \( \Gamma_s \), \( \Delta \Gamma_s \) and \( \Delta m_s \) values. The following values of \( \gamma \), \( \delta \) and \( r_{D_s K} \) are found from the combination:

\[ \gamma = (81^{+12}_{-11})^\circ , \]
\[ \delta = (347.6 \pm 6.3)^\circ , \]
\[ r_{D_s K} = 0.318^{+0.034}_{-0.033} . \]

This value of \( \gamma \) represents the most precise determination of \( \gamma \) in \( B^0 \) meson decays and is in good agreement with the most recent LHCb \( \gamma \) combination \[12\].

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Appendices

A  Dependence of the $CP$-violating parameters on external parameters for Run 2 data

The $CP$-violating parameters depend on external parameters such as the mixing frequency, $\Delta m_s$, the $B_s^0$ decay width, $\Gamma_s$, and the decay-width difference, $\Delta \Gamma_s$, which are fixed in the baseline $B_s^0 \to D_s^\mp K_s^\pm$ decay-time fit. The central values of these parameters might evolve over years, thus for any future combinations of the CKM angle $\gamma$ it is important to evaluate the shift of the $CP$-violating parameters as a function of external parameters. This dependence on external parameters is calculated separately for Run 1 and Run 2 data-taking periods.

The dependence of the $CP$-violating parameters $A_{\Delta \Gamma f}$ and $A_{\Delta \Gamma \bar{f}}$ on $\Delta \Gamma_s$ is shown in Fig. 7 in the interval $\Delta \Gamma_s \in [0.075, 0.095]$ ps$^{-1}$. The horizontally and vertically hatched bands represent the statistical uncertainty of the $A_{\Delta \Gamma f}$ and $A_{\Delta \Gamma \bar{f}}$ parameters, while circles (squares) denote the difference of the $A_{\Delta \Gamma f}$ ($A_{\Delta \Gamma \bar{f}}$) parameter with respect to the baseline result. The interval is extended with respect to the baseline range, $\Delta \Gamma_s \in [0.081, 0.089]$ ps$^{-1}$, to account for the differences in the $\Delta \Gamma_s$ determination obtained with and without constraints from effective lifetime measurements [54]. The decay-time fit is repeated for ten alternative $\Delta \Gamma_s$ values within the extended interval. For these alternative fits the decay width, $\Gamma_s$, is fixed to be $\Gamma_s = 0.6563$ ps$^{-1}$. The difference of the $CP$-violating parameters with respect to the baseline result is evaluated. A small dependence is observed for the $A_{\Delta \Gamma f}$ and $A_{\Delta \Gamma \bar{f}}$ parameters. The first derivatives of the $A_{\Delta \Gamma f}$ and $A_{\Delta \Gamma \bar{f}}$ parameters with respect to the $\Delta \Gamma_s$ variable are determined to be $0.400 \pm 0.012$ ps and $3.13 \pm 0.07$ ps, respectively, for Run 2. For Run 1, the corresponding values are $-4.26 \pm 0.10$ ps and $-3.34 \pm 0.08$ ps. The derivatives are found to be negligible for the $C_f$, $S_f$ and $S_{\bar{f}}$ parameters.

The dependence of the $CP$-violating parameters on the mixing frequency, $\Delta m_s$, is obtained from pseudoexperiments, where the $\Delta m_s$ value is shifted by one standard deviation with respect to the baseline value. In these alternative fits, the $\Delta \Gamma_s$ parameter is fixed to be $\Delta \Gamma_s = 0.085$ ps$^{-1}$. The first derivatives of $S_f$ and $S_{\bar{f}}$ computed with respect to the $\Delta m_s$ variable are evaluated to be $1.712 \pm 0.011$ ps and $-1.653 \pm 0.011$ ps, respectively, for Run 2. For Run 1, the corresponding values are $1.6 \pm 0.9$ ps and $-1.5 \pm 1.0$ ps. The dependencies are found to be negligible for the $C_f$, $A_{\Delta \Gamma f}$ and $A_{\Delta \Gamma \bar{f}}$ parameters.

B  Total statistical and systematic correlation matrices for the updated Run 1 result

For the combination of Run 1 and Run 2 results described in Sec. 8 the decay-time fit of the Run 1 measurement is updated with more recent values of input parameters used in the present analysis of Run 2 data. For reference, the correlation matrix of the five parameters extracted in the updated fit is given in Table 6. Additionally, the update implies the need to reevaluate some of the systematic uncertainties. The systematic uncertainties in the Run 1 measurement introduced by the limited knowledge of the
Figure 7: Dependence of the CP-violating parameters $A_f^\Delta\Gamma$ and $A_f^{\Delta\bar{\Gamma}}$ on the $\Delta\Gamma_s$ parameter. The triangles, circles and squares denote the differences in the $A_f^\Delta\Gamma$ and $A_f^{\Delta\bar{\Gamma}}$ parameters with respect to the baseline result. For comparison, the orange and blue bands visualise the size of the statistical uncertainty of the $A_f^\Delta\Gamma$ and $A_f^{\Delta\bar{\Gamma}}$ parameters as obtained in the corresponding fits to Run 1 and Run 2 data, respectively.

Table 6: Statistical correlation matrix of the CP observables in the updated fit of the Run 1 data set.

| Parameter | $C_f$ | $A_f^\Delta\Gamma$ | $A_f^{\Delta\bar{\Gamma}}$ | $S_f$ | $S_{\bar{f}}$ |
|-----------|-------|--------------------|---------------------------|-------|--------------|
| $C_f$     | 1     | 0.114              | 0.098                     | 0.018 | -0.054       |
| $A_f^\Delta\Gamma$ | 1     | 0.546              | -0.088                    | -0.024|               |
| $A_f^{\Delta\bar{\Gamma}}$ | 1     |                   | -0.051                    | -0.024|               |
| $S_f$     |       |                   |                           | 1     | 0.001        |
| $S_{\bar{f}}$ |       |                   |                           |       | 1            |

parameters $\Delta m_s$, $\Gamma_s$, $\Delta\Gamma_s$ and the decay-time acceptance model are evaluated with the same pseudoexperiment-based approach as described in Sec. 6. The total systematic uncertainty, with updated contributions, is reported together with the updated statistical uncertainty in Table 5. For completeness, the correlation matrix of the combined and partially updated systematic uncertainty of the Run 1 measurement is provided in Table 7.
Table 7: Systematic correlation matrix of the $CP$ observables in the updated fit of the Run 1 data set.

| Parameter | $C_f$ | $A_f^{\Delta f}$ | $A_{\bar{f}}^{\Delta f}$ | $S_f$ | $S_{\bar{f}}$ |
|-----------|-------|-------------------|--------------------------|-------|--------------|
| $C_f$     | 1     | 0.07              | 0.05                     | 0.04  | -0.01        |
| $A_f^{\Delta f}$ | 1     | 0.53              | 0.02                     | 0.02  |              |
| $A_{\bar{f}}^{\Delta f}$ |       | 1                 | 0.03                     | 0.03  |              |
| $S_f$     |       |                   | 1                        | 0.02  |              |
| $S_{\bar{f}}$ |       |                   |                          | 1     |              |

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